Formation Control Strategy for Nonholonomic Intelligent Vehicles Based on Virtual Structure and Consensus Approach

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Abstract

This paper proposes a novel formation control strategy for nonholonomic intelligent vehicles based on virtual structure and consensus approach. The formation model is obtained based on the coordinate transformation and the virtual structure technique. The controllers are designed by using nonholonomic target tracking technique and leader-following consensus protocol. Depending on virtual structure approach and coordinate transformation, the formation control problem of multiple nonholonomic intelligent vehicles is converted into the target tracking and state consensus stabilization problem. Simulation and real-world experimental results show the correctness and effectiveness of the strategy.

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Keywords: Formation control; Nonholonomic intelligent vehicle; Virtual structure; Consensus

1. Introduction

With the development of automation technology, much more multi-agent collaboration technologies are used in the field of intelligent transportation. And the formation control of intelligent vehicles is a prominent research subject. For complex urban traffic environment, the formation control of intelligent vehicles can greatly increase the efficiency of urban transport, and can reduce the probability of accidents. The conventional control approaches include leader-follower approach by Guo et al. [1], behavior based approach by Balch et al. [2], potential function

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Peer-review under responsibility of the Department of Transportation Engineering, Beijing Institute of Technology
doi:10.1016/j.proeng.2016.01.276
approach by Wachter et al. [3], graph theoretic approach by Larsen et al. [4], virtual structures approach by Yoshioka and Namerikawa, etc [5]. For more information, please refer to the survey the paper by Cui et al. [6].

Leader-follower approach [1, 7, 8] was to keep constant relative distance and angle of leader robot and follower robot. This approach needs distance and location information of leader robot. So followers have to equip sensors for locating the leader robot. Park et al. [7] estimated the location of leader robot using infrared camera and infrared reflection tag. When the leader robot moved according to the planned path, the follower robot recognized the leader robot's tag and estimated its location. And the follower robot moved for maintaining a certain distance and angle. Zhang et al. [9] designed the leader to move along a predefined trajectory while the followers were to maintain a desired distance and orientation to the leader.

Virtual structures control approach [10] was developed to force an ensemble of intelligent vehicles to behave as if they were particles embedded in a rigid structure. Intelligent vehicles motion by tracking virtual points on a rigid structure, when the formation moves. Kawakami and Namerikawa [11] proposed virtual structure based target-enclosing strategies for multiple nonholonomic agents converging to the formation while they were tracking the target-object moving in 2D plane.

Consensus [12] of a group of agents means they asymptotically achieve a common dynamic procedure of some interesting variables through a local distributed cooperative protocol. According to requirements of the common variables, consensus problems could be categorized as state consensus and output consensus problems. Taking into account nonholonomic mechanical systems by Colbaugh et al. [13], conventional consensus algorithms cannot meet the requirement of intelligent vehicles formation control of nonholonomic constraints. Khoo et al. [14] proposed a systematic solution to the leader-follower consensus of a class of nonholonomic chained form systems. And Listmann et al. [15] present novel formation control laws based on artificial potential fields and consensus algorithms for a group of unicycles enabling arbitrary formation patterns for these nonholonomic vehicles. Dong and Farrell [16] considered the consensus of multiple nonholonomic systems converged to a desired trajectory and cooperative control laws were proposed and analyzed with the aid of results from graph theory and Lyapunov analysis. Cao [17] considered formation control of nonholonomic mobile robots and proposed decentralized linear controllers based on cascade design for a group of nonholonomic mobile robots.

In this paper, we propose a novel formation control strategy for nonholonomic intelligent vehicles based on virtual structure and consensus technique. There are some differences and advantages of the control strategy in this paper with others. Firstly, we select a leader from the intelligent vehicles as the origin of coordinate and define a relative coordinate system based on it. Then, we convert the other intelligent vehicles into the relative coordinate system by the coordinate transformation method. In the coordinate system, we initialize the coordinate point of each intelligent vehicle like a rigid structure. We design a control law of target tracking to make sure to track the target point in the rigid structure. So, intelligent vehicles can form a fixed structure formation, which do not need to consume a large amount of communication costs and time-delay and simplifies the complexity of the problem, than that by Cao [17]. Secondly, we design a control strategy to make all the vehicles achieving a common value of velocities and maintaining a constant spacing between the vehicles. The leading vehicle is assumed to travel freely in the two-dimensional coordinate system. We propose the control input protocol of the followers by using a leader-following consensus design approach. Finally, this paper is to develop a novel formation controller that is capable of combining target tracking with consensus design approach to track a formation, which improves the convergence speed and increases the stability of the system.

The remainder of the paper is organized as follows. Section 2 briefly states the nonholonomic intelligent vehicle model and describes the method of coordinate transformation. In Section 3, we provide a formation method based on a rigid structure and analyze the stability of the formation control algorithm. A numerical discussion of the communication problem and simulation is given in Section 4 to demonstrate theory result, and Section 5 concludes the paper.

2. Description of formation

As shown in Fig.1, a typical nonholonomic vehicle model lies in a Cartesian coordinate system. The Cartesian coordinate system \( \{O,X,Y\} \) is the global coordinate system where \((x, y)\) is the coordinate of the vehicle. The vehicle model contains two front drive wheels which decide the linear velocity \( v \), and a back guide wheel which is
responsible for the angular velocity $\omega$. The symbol $d$ denotes the distance between the centroid $(x, y)$, and drive shaft, i.e., the shaft of the two front drive wheels. And $\theta$ is the angle between the line velocity and the positive $X$-axis.

![Diagram of a vehicle model](image)

**Fig. 1.** Nonholonomic vehicle model.

In the paper, the state $(x, y, \theta, v, \omega)$ is constructed to represent the Cartesian position, orientation, linear and angular velocity of the $i$th vehicle, $i = 0, 1, 2, \ldots, N$. The 0th vehicle is the leader of the vehicle formation and the others are followers.

According to the mathematical relationship, the kinematics model of the vehicle is described as follows

$$
\dot{q}_i = S(\theta)u_i
$$

where $q_i = (x, y, \theta)^T$ is the pose of the $i$th vehicle in the global coordinate system, $u_i = [v, \omega]^T$ is the control input of motion, $S(\theta)$ is the parameter matrix and described as follows:

$$
S(\theta) = \begin{bmatrix}
\cos \theta & -d \sin \theta \\
\sin \theta & d \cos \theta \\
0 & 1 
\end{bmatrix}
$$

In the process of formation, the centroid of the leader, i.e., of the 0th vehicle, is set as the origin of a new local relative coordinate system. Let $q_i = (x_i, y_i, \theta_i)^T$, $i = 0, 1, 2, \ldots, N$, denote the pose of the $i$-th vehicle in the new local relative coordinate system. A coordinate transformation method is described to convert the global coordinate $q_i$ into the relative coordinate $q_i - q_0$, which is shown as follows:

$$
q_i = R(\theta_0)(q_i - q_0)
$$

where $R(\theta_0)$ is a rotation matrix

$$
R(\theta_0) = \begin{bmatrix}
\cos \theta_0 & \sin \theta_0 & 0 \\
-\sin \theta_0 & \cos \theta_0 & 0 \\
0 & 0 & 1
\end{bmatrix}
$$
Based on the virtual structure approach by Tan K H [10], we set $G_i = (\bar{x}_i, \bar{y}_i, \bar{\theta}_i)^T$ as the target point of the $i$th follower vehicle. The point $G_i$ is defined in the leader coordinate system and it is a known constant by the formation description. The target tracking control algorithm is to design a velocity control input such that we can guarantee tracking the points in the rigid structure.

$$\lim_{i \to \infty} \| q_i - G_i \| = 0$$  \hspace{1cm} (5)

The flow chart of vehicle formation program for virtual structures and consensus technique is shown in Fig.2.

![Formation control program flow chart.](image)

**3. Controller Design**

Assume $p_{\alpha} = \begin{bmatrix} e_{x_i} & e_{y_i} & e_{\theta} \end{bmatrix}^T$ is the target tracking error of the $i$th follower vehicle. It is described as follows:

$$p_{\alpha} = R(\theta_{\alpha})(q_{\alpha} - G_i)$$  \hspace{1cm} (6)

where $R(\theta_{\alpha})$ is a matrix to calculate the target tracking error, $q_{\alpha}$, $G_i$, and $\theta_{\alpha}$ are defined in section 2. Obviously $R(\theta_{\alpha})$ is reversible because of $|R(\theta_{\alpha})| = 1$.

$$\begin{aligned}
\begin{cases}
\dot{x}_{\alpha} = v_i \cdot \cos \theta_{\alpha} \\
\dot{y}_{\alpha} = v_i \cdot \sin \theta_{\alpha} \\
\dot{\theta}_{\alpha} = \omega_i
\end{cases}
\end{aligned}$$  \hspace{1cm} (7)
where $v_i$ and $\omega_i$ is the linear and angular velocity of the $i$th vehicle, $\overline{v}_i$ and $\overline{\omega}_i$ is the linear and angular velocity of the target point. Based on the virtual structure approach, the formation formed by these points $G_i$ is a rigid structure. And these points have been initialized in formation description. So, we can get $\overline{v}_i = 0, \overline{\omega}_i = 0$ and the equation (8) can be rewritten to equation (11) as follows:

$$
\begin{bmatrix}
\dot{e}_{\alpha_i} \\
\dot{e}_{\nu_i} \\
\dot{e}_{\theta_i}
\end{bmatrix} =
\begin{bmatrix}
\omega_i e_{\nu_i} + v_i - \overline{v}_i \cos e_{\theta_i} \\
-\omega_i e_{\nu_i} + \overline{v}_i \sin e_{\theta_i} \\
\omega_i - \overline{\omega}_i
\end{bmatrix}
$$

(8)

3.1. Step 1: target tracing

The angular velocity of the $i$th follower can be designed as follows

$$
\omega_i = -\alpha_1 e_{\theta_i} - \alpha_2 \sum_{j=1, j \neq i}^{N} a_{ij} (\theta_j - \theta_i)
$$

(10)

where $\alpha_1 > 0$ is the weight of the $i$th follower angle error, $\alpha_2 > 0$ is the weight of consensus input between the $i$th follower and the other $N-1$ followers, $a_{ij}$ is the gain required to be designed. If the $i$th follower can receive the information from the $j$th follower, then $a_{ij} > 0$, otherwise $a_{ij} = 0$.

The linear velocity $v_{\nu_i}$ of the $i$th follower can be designed as follows

$$
v_{\nu_i} = -\alpha_3 e_{\alpha_i} + \alpha_4 \omega_i e_{\nu_i}
$$

(11)

where $\alpha_3 > 0$, $\alpha_4 > -1$.

Through (10) and (12), we can get the closed-loop system as follows

$$
\begin{bmatrix}
\dot{e}_{\alpha_i} \\
\dot{e}_{\nu_i} \\
\dot{e}_{\theta_i}
\end{bmatrix} =
\begin{bmatrix}
-\alpha_5 (1 + \alpha_4) \cdot \overline{\omega}_i \\
-\alpha_4 \\
\overline{\omega}_i
\end{bmatrix} e_{\theta_i}
$$

(12)

In order to prove the asymptotical stability of the tracking control system, we can consider the following Lyapunov function:

$$
V = \alpha_1 e_{\alpha_i}^2 + \alpha_5 (\alpha_4 + 1) e_{\nu_i}^2
$$

(13)

It is obvious that $V \geq 0$. And $V = 0$ if and only if $e_{\alpha_i} = 0, e_{\nu_i} = 0$. Its derivation is derived as follows
Thus, we can confirm that the system error $p_{ie} = [e_{ix}, e_{iy}, e_{i\theta}]^T$ is bounded from the equation (14).

3.2. Step 2: formation motion

The leading vehicle is assumed to travel freely with $(v_d, \omega_d)$, where $v_d$ and $\omega_d$ are linear time-varied. This paper design formation controller $\Phi_i$ for the $i$th follower, including the angular velocity controller $\Phi_{i\theta}$ based on PID algorithm, and the linear velocity controller $\Phi_i^v$ based on consensus algorithm.

$$\Phi_{i\theta} = \omega - \alpha_s(\omega - \omega_d) - \alpha_\theta(\dot{\omega} - \dot{\omega}_d)$$

(15)

where $\alpha_s$ is the proportional gain, $\alpha_\theta$ is the differential time.

In the direction of movement, every intelligent vehicle can get the distance $s$ between the following vehicle and leading vehicle. The intelligent vehicle also can get the velocity $v$ of each other following vehicle and leading vehicle.

Based on the consensus approach by Tan K H [10], we can construct the following consensus protocol.

$$\Phi_{i} = v_d + \sum_{j=1, j \neq i}^{N} a_{ij}(s_j - s_i) + \sum_{j=1, j \neq i}^{N} b_{ij}(v_j - v_i)$$

(16)

where $a_{ij}$ and $b_{ij}$ are the gains required to determine. If the $i$th vehicle can get the information of $j$th vehicle, then $a_{ij} > 0, b_{ij} > 0$, otherwise $a_{ij} = 0, b_{ij} = 0$. Under the relative coordinate system, we can get that the error $e_{ix}$ can be substituted for the distance error $s_i$. So we can rewrite equation (16) as follows

$$\Phi_{i} = v_d + \sum_{j=1, j \neq i}^{N} a_{ij}(e_{ix} - e_{ix}) + \sum_{j=1, j \neq i}^{N} b_{ij}(v_j - v_i)$$

(17)

We can get the last controller based on (10), (11), (15) and (17).

$$u_i = \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}$$

(18)

where

$$v_i = (-\alpha_s e_{ix} + \alpha_4 \omega_d e_{iy}) + v_d + \sum_{j=1, j \neq i}^{N} a_{ij}(e_{ix} - e_{ix}) + \sum_{j=1, j \neq i}^{N} b_{ij}(v_j - v_i)$$

(19)
\[ \omega_i = (\alpha_i e_{\omega_i} - \alpha_z \sum_{j=1,j \neq i}^{N} a_y (\theta_{x_j} - \theta_{y_j})) + (\omega_{\gamma_i}(\omega_i - \omega_{\gamma_j}) - \alpha_z (\omega_i - \omega_{\gamma_j})) \]  

(20)

4. Simulation Results

In this section, we verify the correctness and effectiveness of the proposed control strategy by use of three intelligent vehicles. The trajectory of the leading vehicle is a straight line where \( x_0(t) = t, \ y_0(t) = 2t, \ \theta_0(t) = \pi / 6 \). The input of the leading vehicle are \( \omega_y = 0, \ v_y = \) 2 m/s. The two following vehicles keep a certain distance and angle tracking the leading vehicle where \( d = 5m, \ \theta = \pi / 4 \). Simulation parameters are given as follows: \( \alpha_1 = 1, \ \alpha_2 = 0.5, \ \alpha_3 = 0.3, \ \alpha_4 = 0.5, \ \alpha_5 = 1.5, \ \alpha_6 = 0.28, \ v_y = 1, \ b_y = 0.5 \). The simulation results and effect in the real-world are shown in Fig. 3-7.

Fig. 3. Formation trajectories.

Fig. 4. Velocity errors of the follower.
The formation trajectories of three intelligent vehicles are shown in Fig. 3. The two following vehicles track the leading vehicle in 6s. They can form a fixed structure formation. The two following vehicles keep stable distance and angle with the leading vehicle.

The velocity errors of the follower are shown in Fig. 4. The velocity of the two following vehicles can track the velocity of leading vehicle in 6s. The posture errors of the two followers are shown in Fig. 5-6. The errors converge to zero quickly under the formation control strategy. From those simulation trajectories, we can get that the three intelligent vehicles can form a fixed structure formation. And the system is stable.
The effect of formation on the real vehicles is shown in Fig. 7. We use AmigoBots as intelligent vehicle which was produced by company named U.S. ActiveMedia. It has the same model that we discussed in the paper. The algorithm can be achieved based on C++ programming. The communication networks are built based on WLAN with TCP/IP protocol. In the experiment we choose an intelligent vehicle as the leader and two other intelligent vehicles as the followers. And the two followers are placed in random initial positions. The target spacing and angle are set as $d = 2\text{m}$ and $\theta = \pi / 4$. The result of the experiment shows the three intelligent vehicles can keep formation in this control strategy.

5. Conclusion

In this paper, we propose a novel formation control strategy for nonholonomic intelligent vehicles based on virtual structures and consensus techniques. This paper simplifies target tracking approach by model transformation based on coordinate conversion and virtual structure techniques. By using the leader-following consensus protocol, the convergence rate of system is accelerated. The simulation trajectories based on Matlab7.1 and real-world experimental results based on AmigoBot show the correctness and effectiveness of the strategy.

References

[1] Guo, C., Qin, J., Ge, Y., and Chen, Y., 2014. Leader-follower and cascade system based formation control of nonholonomic intelligent vehicles. In 26th Chinese Control and Decision Conference, 3931-3935.
[2] Balch, T., and Arkin, R. C., 1998. Behavior-based formation control for multirobot teams. IEEE Transactions on Robotics and Automation, 14(6), 926-939.
[3] Wachter, L., Murphy, J., and Ray, L., 2008. Potential function control for multiple high-speed nonholonomic robots. In IEEE International Conference on Robotics and Automation (ICRA), 1781-1782.
[4] Larsen, M. B., Smith, R. S., and Blanke, M., 2011. Modeling of tethered satellite formations using graph theory. Acta Astronautica, 69(7), 470-479.
[5] Yoshioka, C, and Namerikawa, T., 2008. Formation control of nonholonomic multi-vehicle systems based on virtual structure. In 17th IFAC World Congress, 5149-5154.
[6] Cui, S, Wu, L, Zhao, L, and Bing, Z., 2010. Research on method of multi-agent formation control. In 2010 IEEE Asia-Pacific Conference on Power Electronics and Design (APED), 59-62.
[7] Park, H., Choi, I., Park, S. K., and Choi, J., 2013. Leader-follower formation control using infrared camera with reflective tag. In IEEE 2013 10th International Conference on Ubiquitous Robots and Ambient Intelligence (URAI), 321-324.

[8] Consolini, L., Morbidi, F., Prattichizzo, D., and Tosques, M., 2007. A geometric characterization of leader-follower formation control. In 2007 IEEE International Conference on Robotics and Automation, 2397-2402.

[9] Zhang, Y., Zeng, L., Li, Y., and Liu, Q., 2009. Multi-robot formation control using leader-follower for MANET. In IEEE 2009 International Conference on Robotics and Biomimetics (ROBIO), 337-342.

[10] Tan, K. H., and Lewis, M. A., 1996. Virtual structures for high-precision cooperative mobile robotic control. In proceedings of the 1996 IEEE/RSJ International Conference on Intelligent Robots and Systems, 132-139.

[11] Kawakami, H., and Namerikawa, T., 2008. Virtual structure based target-enclosing strategies for nonholonomic agents. In IEEE International Conference on Control Applications (CCA), 1043-1048.

[12] Chen, Y., Qu, X., Guo, Y., and Aleksandrov, A. Y., 2014. Output consensus of linear multi-agent systems: linear-transformation-based partial stability approach. In proceedings of the 33rd Chinese Control Conference, 1545-1550.

[13] Colbaugh, R., Trabatti, M., and Glass, K., 1998. Analysis and control of redundant nonholonomic mechanical systems. In IEEE Proceedings of the American Control Conference, 13-15.

[14] Khoo, S., Xie, L., and Man, Z., 2010. Leader-follower consensus control of a class of nonholonomic systems. In IEEE 2010 11th International Conference on Control Automation Robotics & Vision (ICARCV), 1381-1386.

[15] Listmann, K. D., Masalawala, M. V., and Adamy J., 2009. Consensus for formation control of nonholonomic mobile robots. In IEEE International Conference on Robotics and Automation (ICRA), 3886-3891.

[16] Dong, W., and Farrell, J. A., 2008. Consensus of multiple nonholonomic systems. In 47th IEEE Conference on Decision and Control (CDC), 2270-2275.

[17] Cao, K. C., 2009. Formation control of multiple nonholonomic mobile robots based on cascade design. In proceedings of the 48th IEEE Conference on Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference (CDC/CCC), 8340-8344.