Synchronization and associative memory of FitzHugh-Nagumo neuronal networks with randomly distributed time delays

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Abstract Synchronization and associative memory in a neural network composed of the widely discussed FitzHugh-Nagumo neurons is investigated in this paper. Based on the reality of the microscopic biological structure in the neural system, the couplings among those neurons are accompanied with randomly distributed time delays which models the times needed for pulses propagating on the axons from the presynaptic neurons to the postsynaptic neurons. The memory is represented in the spatiotemporal firing pattern of the neurons, and the memory retrieval is accomplished with the fluctuations of the noise in the system.

1. Introduction
One of the most fundamental and central questions in neuroscience is how objects and thoughts are represented. The idea of distributed representation a group of neurons is reasonable one [1-6]. The strong mutual excitations between neurons belonging to the same assembly acquired by learning stimulate them to synchronous firing. Temporal correlation among the neurons is the working mechanism for the feature binding. Evidence for this oscillatory activity was found in different levels till potential recordings. Synchronous firing is also used for the purpose of associative memory by lots of researchers. Patterns are stored in the connection coefficients of the network and the stored patterns are retrieved using the neural dynamics[7-10]. The couplings among those neurons are accompanied with time delays that model the time lag from the presynaptic neuron to the postsynaptic neuron, and the memory is represented in the spatiotemporal firing pattern of the neurons. Meanwhile, the physiological environment where neurons operate is thought to have several sources of randomness, such as, thermal noise, stochastic properties of synapses and the sum of enormous presynaptic inputs, thus the effect of the fluctuation may not be neglected. Generally, stochastic resonance is a well-known phenomenon where a weak input signal is enhanced by its background fluctuation and observed in many systems [11-13].

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In this paper we compose a network using the widely discussed neuronal model of FitzHugh-Nagumo (FN), we discussed Hodgkin-Huxley model in another paper, then we showed that the widely used spiking neuronal models are suitable for achieving the function of associative memory. Considering the time delays from the presynaptic neuron to the postsynaptic neuron as randomly distributed variants and discuss their effects on synchronization and associative memory. By a thorough adjusting of the parameters as coupling strength and the intensity of the noise shows that the synchronization and associative memory attains its optimal performance at an intermediate noise intensity, which is reminiscence of the stochastic resonance observed in the coupled oscillator networks.

2. FitzHugh-Nagumo Equations and Network

We study the Fitzhugh-Nagumo [14] neuronal model and the network composed of them. The FN model assumed that the sodium channel adapt rapidly so that the system is described by a set of only two differential equations, it is relatively simpler than the Hodgkin-Huxley model, but we find that in some aspects it is relative easy to achieve our purpose

\[
\tau \frac{du_i}{dt} = -v_i + u_i - \frac{1}{3} I_i(t) + \xi_i(t) + \sum_{j=1}^{N} W_{ij} [u_j(t - dp_j) - u_{\mu_j}] \quad i = 1, 2, 3, \ldots, N
\]

\[
\frac{dv_i}{dt} = u_i - \beta v_i + \gamma
\]

where \( \xi_i(t) = \xi_i(t') \) is the Gaussian white noise with strength \( D \), which represents the fluctuation in the system. Note that a single FN neuron shows the characteristic of the spiking neuron, namely, it has a stable rest state, and with an appropriate amount of disturbance it will generate a pulse with a characteristic magnitude of height and width. \( u_{\mu_j} \) is the equilibrium value of \( u_i \). We store \( p \) patterns \( \xi_i(t) (i = 1, 2, \ldots, N, \mu = 1, 2, \ldots, p) \) in the above \( N \) neurons, the connection coefficients \( W_{ij} \) are learned by the Hebbian rule, so they can be defined as

\[
W_{ij} = \frac{\mu}{N a (1 - a)} \sum_{\mu=1}^{p} \xi_i^\mu (\xi_j^\mu - a)
\]

Where \( w \) is the strength of the connections \( W_{ij} \), \( N \) is the number of neurons composing the network, \( a \) is the mean of potentials of all the neurons, \( p \) is the number of patterns or memories stored in the connection, \( \xi_i^\mu \) are vector, their element have the value of 0 or 1 with the probability distribution function described by

\[
P(\xi_i^\mu) = (1 - a) \delta(\xi_i^\mu) + a \delta(\xi_i^\mu - 1)
\]
where \( I \) is the strength of the external input, \( x_i \) is the binary factor which determines whether the input is injected to the \( i \)th neuron or not, and \( \Theta(t) \) is Heaviside’s step function which takes \( I \) for \( t > 0 \) and otherwise takes \( 0 \). In the following, \( I \) is fixed with a small value that each neuron cannot fire without \( \xi_i \). Using the binary factors \( x_i \), the input overlap \( m_{in}^{\mu} \), which measures the correlation between the pattern \( \zeta^\mu = (\zeta^\mu_1, \zeta^\mu_2, \ldots, \zeta^\mu_N) \) and the external input \( I(t) = (I_1(t), I_2(t), \ldots, I_N(t)) \), is defined as
\[
m_{in}^{\mu} = \frac{1}{Na(1-a)} \sum_{i=1}^{N} (\zeta_i^{\mu} - a)(x_i - a)
\]
With this function we can judge if the network be able to repair the input signals or not, and the degree of the repairing.

Fig.1 (a) The firing times of all the neurons and (b) the output overlap \( m_{out}^{\mu} \), here \( N = 200 \), \( p = 2 \), \( a = 0.5 \), \( D = 0.001 \), \( dp = 60 \)

3. Memory retrieval with noise and time delay
The associative memory in a pulsed neural network with noise is considered, and the memory retrieval is induced by the suitable amount of noise. Numerical simulations are performed for \( N=200 \), the patterns are defined as
\[
\zeta^\mu_i = \begin{cases} 1, & 1 \leq i \leq 100 \\ 0, & \text{otherwise} \end{cases}
\]
the pattern \( \zeta^2 \) is determined randomly with the element value of 0 or 1, To measure the correlation between the pattern \( \zeta^\mu_i \) and the time series \( u_i(t) \) \( (i=1,2,\ldots,N) \), \( v_i(t) \) is transformed into the binary series \( y_i(t) \). First, let us define the firing time of the \( i \)th neuron as the time when \( v_i(t) \) exceeds a threshold \( v_{th} \), then the time series \( u_i(t) \) is transformed into the \( y_i(t) \)
\[
y_i(t) = \begin{cases} 1, & t < t_i' + d \\ 0, & \text{otherwise} \end{cases}
\]
where \( t_i' \) is the latest firing time of the \( i \)th neuron at time \( t \), the parameter \( d \) is set close to the characteristic width of the output pulse, Then the output overlap \( m_{out}^{\mu} \) between the pattern \( \zeta^\mu_i \) and the binary series \( y(t) = (y_1(t), y_2(t), \ldots, y_N(t)) \) is defined as
\[
m_{out}^{\mu} = \frac{1}{Na(1-a)} \sum_{i=1}^{N} (\zeta_i^{\mu} - a)(y_i - a)
\]
At first, we let the time delays fixed to constants, that is they will not change with the time delay of
the network. The memory is represented by the synchronized periodic firings of the neurons which store
1’s. Numerical simulations are performed for $N = 200, p = 2, a = 0.5$, and $dp = 60$, the simulation
result is shown in Fig.1. The first 100 neurons fired synchronously and the next 100 neurons fired
randomly with less intensity, so the retrieve of the pattern $\zeta^i$ is successful.

Fig.2 (a) The firing times of all the neurons and (b) the output overlap $m_{out}^{i}$ with higher strength of
noise, $D = 0.005$

Fig.3 (a) The firing times of all the neurons and (b) the output overlap $m_{out}^{i}, dp$, runs from 20 to 60.

From the results we can see that the system can enter into the synchronous firing state in a short
time. And also if we let the input pattern corrupted, the system can repair the pattern, that is the
system has the function of associative memory. If there are no noise, the network will not fire, with
small strength of noise the neurons fire barely and sporadically, with middle strength of noise
$D = 0.001$, the first 100 neurons fire synchronously corresponding with the stored pattern, with the
increase of the strength, at $D = 0.005$, we can still see the synchronous firing, but at this level, the
firing rate increase, and irregular fires increase, so the output overplay $m_{out}^{i}$ decrease to 0.8 as seen in
Fig.2. In the middle strength of noise, the output overplay function reaches its maximum value near
almost 1. This is reminiscence of the phenomena of stochastic resonance.

Considering the times needed for the propagation of pulses from the presynaptic neurons to the
postsynaptic neurons are different. We take them as randomly distributed variants and discuss their
effects on synchronization and associative memory. In the time domain of the dynamical equations,
there are randomly distributed variants, intuitively it is impossible to work in the same pace, but as
seen in Fig.3, we can still see the first 100 neurons fire synchronously. In the simulation we let the
time delay last from 20ms till 60ms, the \( m_{\text{out}} \) reach about 0.6. With the increase of the varying range of the time delay, the synchronization become more and more difficult, as seen in Fig.4, \( dp_i \) randomly distribute from 20ms to 200ms.

Fig.4 (a) The firing times of all the neurons and (b) the output overlap \( m_{\text{out}} \), \( dp_i \) runs from 20 to 200.

4. Conclusions and discussion
We discussed the effects of time delay and noise on the synchronization and associative memory. We find that under middle range of noise strength, the neurons in the network can fire synchronously as arranged or learned patterns. This is in accordance with the phenomena of stochastic resonance, and the network has the function of associative memory. To simulate the more realistic conditions, we take the time delays to be randomly distributed variants, we find the neurons still can work in pace, that is they still fire synchronously and the network has the function of associative memory. Till now the time lagging in the neurons are set to be constant, but obviously it is not the case in the real situation of neural system, so this is an improvement to the exist researches.

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