Numerical Modelling of Plate Load Tests on Unsaturated Silt Loams in Laboratory Stands

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Abstract. The paper summarizes the results of the numerical modelling of plate load tests on unsaturated silt loams in laboratory conditions using two different constitutive models and different software packages. The first part of the paper presents laboratory measurements on large specimens of compacted unsaturated silt loams with a constant degree of saturation during the experiments. It comments on the issues of the classical interpretations of static plate load tests including the influence zone theory and further issues with calibration of numerical models using the parameters gained from the classical approach. The second part of the paper dedicated to the problems of numerical modelling of these soils presents results of the Cam-Clay constitutive model modified for capturing the influence of the moisture content implemented in the SIFEL software package and results of the Hypoplastic constitutive model for clays implemented in the GEO 5 software package. The paper presents a comprehensive analysis of the advantages and drawbacks of both constitutive models in details and comments on the possible issues when using them for more topologically complex tasks.

1. Introduction
Interpretation of results of numerical modelling of unsaturated soils for complex tasks presents an enormous challenge on engineers to avoid neglecting known inaccuracies of the approximations due to the used constitutive models and to minimise the potential impacts of uncertainties. While the results of numerical modelling of saturated soils (fine, cohesive as well as coarse, cohesionless) can be interpreted with a relatively high degree of certainty, the unsaturated soil mechanics and the numerical modelling of unsaturated soils still present a room for further improvement, however significant improvement in the field of coupled heat and moisture transport modelling has been achieved using both, phenomenological models [1], [2] as well as complex micro mechanical based models [3], [4].

The presented work focused on challenges linked with numerical modelling of unsaturated soils subjected to moisture content changes, i.e. modelling of three phased medium. The paper is founded on the results of the full scale laboratory experiments on reconstituted silt loams Czech Republic origin [5], which were used for calibration and confirmation and are presented in the second chapter. The third chapter of the paper introduces the problems resulting from the interpretation of static plate load tests and the acquisition of material parameters from such widely used and accepted experiments, which are usually carried out in-situ without sufficient information about the reach of the applied loading and the changes related to the moisture content variations. The second part of the paper is devoted to a brief introduction of the used constitutive models and results obtained.
2. Full scale laboratory experiments

Governing idea of the experiments was to provide experimental results easily comparable with in-situ measured values. With similar loading/unloading behaviour measured on unsaturated soils in-situ and in laboratory appropriate analogy could be used to predict the effect of wetting, suction cancellation or groundwater table variations. Obtained results were calibration and confirmation of numerical models as shown in chapter 4 in this paper.

The stand for the tested soil sample was a massive reinforced concrete box without the top covering part, see figure 1. The bottom part contains a system of pipes 12.5 mm in diameter and is connected to the large water storage tank. The side walls were 200 mm thick and the box is constricted by steel beams in two levels. A steel frame is attached to the box to take the reaction force and additional small frame presents an inertial body to which the deformations are measured. The internal dimensions of the box are 1.0 x 1.0 x 1.0 m. The stand was designed and constructed strictly for this purpose while taking into account the effect of vibrations during the soil sample compaction as well as the impact of the load and water. Although the stand served well with respect to the needs of this thesis, adjustments recommended regarding the water tightness and pore pressure measurements are presented at the end of this chapter.

![Figure 1. Laboratory stand for plate load test including lateral pressure measurement and shear zone settlement](image)

The load was applied through a hydraulic jack to the steel plate 20 mm thick and 300 mm in diameter (70.685.10^3 mm^2 surface area). As the maximum admissible load for the reaction frame was set on 80 kN, the plate was regarded as rigid within the load interval for analytical and numerical purpose. The applied load was measured in the hydraulic system (calibrated manometer, with confirmation using pressure cell). Settlement of the plate was measured by two dial / digital gauges installed on the plate with a guaranteed accuracy of 0.01 mm and 0.001 mm respectively. Settlement of the shear zone was measured by two dial gauges in 50 mm and 100 mm distance from the edge of the rigid plate, see figure 2.
2.1. Silt loams – description of the tested soils

The experiments were carried out on silt loams samples reconstituted in the laboratory from excavated soils. The soil was inserted into the model stand by layers 200 mm thick. The first layer was placed on nonwoven geotextile (200 g/m²) protecting the outlets from pipelines. Each layer was compacted by a vibrating plate. The time over which was the vibrating plate acting on each layer was estimated by a compacting experiment carried out in advance. The soil contained a natural amount of moisture as it was kept in plastic covers after being removed from the site. The time of storage was kept as short as possible. The moisture content was monitored in selected intervals during the time of storage. Some text.

The soil specimens were classified as saSi according to EN ISO 14688 or F5 – F, i.e. silt loam according to old Czech standard CSN 731001. Grain distribution of the three soil samples is shown in figure 3 from which is also clear that the portion of fine particles is very close to F3 - FS class. The optimal moisture content and maximal dry soil density were evaluated using standard Proctor test.

Figure 2. Experiment setup – settlement gauges and lateral pressure probe, constraining frame and reaction frame

Figure 3. Grain distribution of the tested silt loam (three large specimens)
This particular soil was obtained from the excavation in the Prague city district near Prague Castle (Kings Park) and built in the model with natural moisture content being close to 7%. Compacting experiment prescribed 19 minutes duration of compaction with a vibrating plate for each 200 mm thick layer while achieving soil density approximately 1550 kg.m\(^{-3}\). This soil is considered highly collapsible and moisture sensitive with high volumetric deformations due to swelling and shrinkage.

3. Interpretation of results from the static plate load test

It was experimentally confirmed that different results for the same soils will be obtained when using plates with different diameters and loading forces and employing generally used Bousinesq formula (1) to obtain secant modulus of the subsoil [6].

\[
E_0 = \frac{\pi}{2} \left(1 - \nu^2\right) \left(\frac{f_z r}{s_{tot}}\right)
\]

where \(r\) is the radius of the plate, \(s_{tot}\) represents final settlement, \(f_z\) is the load magnitude and \(\nu\) is the Poisson’s ratio.

As the formula was derived assuming the infinite half-space. It is very useful as it allows for explicit estimation of secant modulus, but it is also limiting as it neglects the phenomenon of hysteresis of soil’s load / unload memory. Therefore, it should only be used, when the depth of the influence zone exceeds approximately 2 times the diameter. For shallow influence zones, the resulting secant moduli will be overestimated, i.e. the subsoil would seem to be stiffer than it is.

3.1. Influence zone depth for circular loads

The governing idea for estimating the depth of the influence zone is the pre-consolidation of the soil, which is generally caused by the excavation and the soil’s ability to memorize the highest load it was subjected to. In this particular case, the pre-consolidation was achieved by the compaction process.

The problem in estimating the influence zone depth \(H\), according to above mentioned assumptions, can be substituted by the problem of calculating the vertical stress for complex geotechnical problems using, for instance, elastic layer solution in Westergard manner [7]. The following formula describes how to calculate the depth of the influence zone when \(r\) represents the radius of the plate.

\[
\frac{\gamma h}{f_z} = F_r(\beta) \rightarrow \beta = \frac{r}{H} \sqrt{\frac{2-2\nu}{1-2\nu}} \rightarrow H
\]

The \(F_r(\beta)\) function can be introduced as follows and plotted in figure 4:

\[
F_r(\beta) = 1 - \beta \int_1^0 \frac{t}{\sqrt{t^2-1} \cosh(t^2 \beta)} \, dt
\]

When applying the elastic layer theory presented [7] for the circular foundation we yield the depth of influence zone \(H = 0.36\) m, which does not satisfy the twice as width assumption leading to overestimation of \(E\). The differences in the secant moduli are significant. While the equation 1 yields for 400 kPa loading and 1.25 mm settlement \(E_0 = 73\) MPa (for \(\nu = 0.4\)), for 0.36 m deep influence zone and 300 kPa pre-consolidation pressure we obtain \(E_0 = 37\) MPa, which was used in numerical modelling as a starting point for the calibration process and as it is shown in the next chapter it was also approximately the mean value for both approaches.
Figure 4. \( F_{E}(\beta) \) function for the circular load area

4. Modified Cam Clay (SIFEL code)

The implementation of the Modified Cam Clay (MCC) model prepared in the SIFEL was in detail described in [8] and the following description tends to introduce it in a more general manner. The model follows Lewis and Schrefler’s approach [3] of coupled heat and moisture transfer while employing Darcy’s and Fick’s laws for moisture transfer, Fourier’s law for heat transfer, standard mass and energy balance equations and modified concept of effective stress according to [9], i.e. one stress variable as shown in following equation

\[
(1-n)\sigma_{ij}^s = (1-n)S_w\sigma_{ij}^w + (1-n)S_g\sigma_{ij}^g + \sigma_{ij}^{ef}
\]  

(4)

where \( \sigma_{ij}^s \) is the stress in grains, \( \sigma_{ij}^w \) is the stress in the liquid phase (water), \( \sigma_{ij}^g \) is the stress in gas, \( \sigma_{ij}^{ef} \) is the effective stress, \( nS_w \) is a volume fraction of water and \( nS_g \) is a volume fraction of gas.

In order to describe the deformation of a porous skeleton or actually the rearrangement of grains, the standard constitutive equation is written in the rating form.

\[
\dot{\sigma}_{ij}^{ef} = D_{sk} \left( \dot{\varepsilon} - \dot{\varepsilon}_0 \right)
\]

(5)

where \( D_{sk} \) is a tangential matrix of porous skeleton while, \( \dot{\varepsilon} \) represents the strain rate while \( \dot{\varepsilon}_0 \) represents strains indirectly associated with stress changes, such as shrinkage and swelling, creep, etc. and also involves the strain of the bulk material due to pore pressure changes. Combining equations 4 and 5 while assuming negligible shear stress in fluid we obtain

\[
\dot{\sigma}_{ij} = D_{sk} \dot{\varepsilon} - \alpha \delta_{ij} \left( \dot{S} \right)
\]

(6)

where \( \alpha \) represents the Biot’s constant and suction \( s \) is defined in agreement with volume fractions as follows

\[
s = S_w p^w + S_g p^g
\]

(7)

The constitutive model reflects a non-linear behaviour observed in isotropic compression tests. The results are usually presented in a semi-logarithmic scale as shown in the following figure 5.
Figure 5. Normal consolidation and swelling line describing the behaviour of soil during the isotropic compression test

The used yield function and yield condition are described in equation 8 while the visualization of yield surface in mean-deviatoric stress plane is shown in the following figure 6.

\[ f(\sigma_{ij}^{\text{ef}}) = q^2 + M^2 \sigma_{m}^{\text{ef}} (\sigma_{m}^{\text{ef}} - p_c) = 0 \]  \hspace{2cm} (8)

where \( p_c \) represents the pre-consolidation pressure or hardening parameter respectively which in fact determines the diameter of the yield surface ellipsoid along the \( \sigma_{m}^{\text{ef}} \) (mean effective stress) axis. \( M \) is the material parameter related to the friction angle which determines the slope of the critical state line and consequently the radius in the deviatoric plane. It is assumed to be \( M = 6. \sin \varphi / (3 - \sin \varphi) \) for triaxial compression test and \( M = 6. \sin \varphi / (3 + \sin \varphi) \) for triaxial extension test.

Figure 6. Yield surface of the Cam-Clay model in the mean-deviatoric stress plane

4.1. Material parameters and results
Two different approaches were employed to simulate the plate load test. The first approach focuses on the good approximation of the loading path while the unloading path is not considered for the calibration. The second approach neglects the initial loading cycle and focuses on the calibration of the rest of the experiment. Following table 1 summarizes the calibrated model parameters and figures 7 and 8 present selected results.
Table 1. Material parameters of the MCC model

| Parameter       | Approach no. 1 | Approach no. 2 |
|-----------------|----------------|----------------|
| λ               | 0.148          | 0.148          |
| κ               | 0.037          | 0.037          |
| ε₀              | 2              | 2              |
| \(p_{e₀} (OCR)\) | 300 kPa        | 280 kPa        |
| M               | 0.69           | 0.69           |
| E               | 25 MPa         | 55 MPa         |

Regular mesh consisting of quadrilateral finite elements using linear interpolation functions was employed. Loading plate is assumed to be rigid and the problem is simplified due to axial symmetry. The unchanging initial tangential stiffness matrix limits the possibilities of the code but also stabilizes and accelerates the computations.

Figure 7. Displacement and deformed mesh (First approach left – scaled 10x; Second approach right – scaled 50x)

Figure 8. Comparison of results for both approaches (settlement of the rigid plate)
5. Hypoplasticity for clays (GEO 5 FEM code)
The constitutive model used for simulation of unsaturated silt loams with steady no-flow boundary condition was developed by Masin [10] and is based on the combination of classical critical state models and generalized hypoplasticity principles.

The non-linear behaviour of the soil in this model is governed by generalized hypoplasticity while as the limit stress criteria Matsuoka–Nakai failure surface [11] was selected. The normal compression line for the isotropic compression is similar to the normal consolidation line (NCL) from the Modified Cam Clay model presented in chapter 4. The intergranular strain concept was not involved within carried out calculations.

\[ \lambda^* \] and \[ \kappa^* \] [10]

In the figure 9 above quantity, \( p_{cr} \) is defined as the mean stress at the critical state line at the current void ratio and \( p_{cr}^* \) is the equivalent pressure at the isotropic normal compression line. The calibration procedure is in detail described in [12].

5.1. Material parameters and results
As in the case of MCC model, the behaviour of the specimen before the first loading cycle was found to be difficult to simulate as the overall pre-consolidation should be distributed in layer (due to compaction). This time only the second approach was applied, i.e. the initial loading cycle was neglected and calibration procedure focused only on the rest of the experiment (Table 2).

| Parameter | Approach no. 2 | Interpretation of the parameters |
|-----------|----------------|----------------------------------|
| \( \lambda^* \) | 0.18 | position of the normal consolidation line for isotropic compression in the \( \ln (1+e) \) vs. \( \ln p \) plane |
| \( \kappa^* \) | 0.003 | slope of the normal consolidation line for isotropic compression in the \( \ln (1+e) \) vs. \( \ln p \) plane |
| \( N \) | 1.4 | slope of the unloading line for isotropic compression in the \( \ln (1+e) \) vs. \( \ln p \) plane |
| \( \varphi_c \) | 22° | critical state friction angle |
| \( r \) | 0.01 | parameter controlling shear stiffness |
Figure 10. Horizontal (left) and vertical (right) displacement on the deformed specimen for the load 500 kPa

In contrast to the MCC model calculated the horizontal stress and measured lateral pressure are very close (Figure 10, 11).

Figure 11. Comparison of the measured and calculated settlement of the rigid plate using the hypoplastic constitutive model

6. Conclusions
The presented results demonstrate the capabilities of the used constitutive models to approximate the static plate load test which was performed on silty loams in laboratory conditions. Despite the demanding calibration process, both of the models failed to successfully approximate the entire experiment which includes two load / unload cycles not mentioning the follow up experiments which involved significant moisture changes and suction cancellation processes.
Single loading and unloading cycle, however, were both constitutive models and both software packages able to capture very well including the settlement of the shear zone around the rigid plate. The calibration process was less difficult due to other laboratory tests on small samples which allowed for fitting most of the parameters. In spite of that, the final set of parameters clearly demonstrates that neglecting the influence zone depth and using semi-infinite based formulas could significantly overestimate the secant modulus of soil and lead to dangerous underestimation of the deformation, in this case, settlement of the foundation structure.

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