Heavy-baryon chiral perturbation theory and reparametrization invariance

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Abstract

We examine the constraints imposed by reparametrization invariance on heavy-baryon chiral perturbation theory (HBChPT). We study the case of 3 flavors and consider both the strong and weak ($|\Delta S|=1$) interaction sector. Some of the parameters in the HBChPT Lagrangian are fixed as a consequence of reparametrization invariance. We discuss the consequences for the calculation of hyperon weak radiative decays in HBChPT. (hep-ph/9611260).

1 Introduction

Chiral perturbation theory (ChPT) is an effective field theory for the description of processes involving hadrons at energies well below the chiral symmetry breaking scale, $\Lambda_{\chi}$. The Lagrangian of ChPT is the most general Lagrangian, in terms of the relevant hadronic degrees of freedom at that energy scale, consistent with the symmetry properties of underlying QCD. In ChPT, amplitudes are expanded in external momenta and quark masses. Every term in the Lagrangian contains an arbitrary coefficient. However, for a process at low energies only a limited number of these parameters are relevant. By comparing to experimental data one can then try to fit these parameters and, if possible, try to use these fitted values to predict other experimental observables.

Chiral perturbation theory with baryons is complicated by the baryon mass in the chiral limit, which is of the same order of magnitude as $\Lambda_{\chi}$. By using heavy-baryon chiral perturbation theory (HBChPT) this complication can be dealt with. As was pointed out by Ecker and Mojžis a general Lagrangian constructed starting directly in the heavy-baryon formulation is due to the presence of the external four-velocity $v^\mu$ not necessarily Lorentz invariant, or equivalently, not necessarily reparametrization invariant. Reparametrization invariance constrains the number of independent terms in an effective Lagrangian. In this paper we will study the constraints of reparametrization invariance on the general HBChPT Lagrangian considered in Ref. Reparametrization invariance of the HBChPT Lagrangian can be ensured by matching it with the fully relativistic Lagrangian. This approach we will follow in this paper.

This paper is organized as follows. In the next section we will start with the relativistic ChPT Lagrangian in both the strong and weak interaction sectors, and consider the case of 3 flavors. In Sect. 3 we will then use a path integral method to arrive at the corresponding HBChPT Lagrangian. We will compare the result with the HBChPT Lagrangian, derived earlier. In Sect. 4 we will discuss the consequences of the constraints on the analysis of hyperon weak radiative decay. Finally, Sect. 5 contains a summary and our conclusions.
where ˙\(D\) semi-leptonic decays the values in the expansion of the general ChPT Lagrangian. One would expect that terms of the form ˙\(D\) and \(\Gamma\) appearing in \(L\) will again go through all the terms in the Lagrangian. The Lagrangian \(L_s^{(1,0)}\) before by Krause [11]. In his analysis Krause takes \(m_u = m_d = 0\). This Lagrangian has been analyzed before by Krause [11]. In his analysis Krause takes \(m_u\) and \(q^2\) to be of the same order in the chiral expansion, as is customary in the meson sector. However, arguments below will indicate that in the baryon sector the relative counting between \(q\) and \(m_s\) might be different. Since we use a different expansion systematic we will again go through all the terms in the Lagrangian. The Lagrangian \(L_s^{(1,0)}\) in Eq. (1) can be readily constructed. Using the same notation for the fields as Krause, it is given by

\[
L_s^{(1,0)} = \langle \bar{B}i\gamma^\mu [D_\mu, B] \rangle - \hat{m}_s \langle \bar{B}B \rangle + D \langle \bar{B}i\gamma^5\gamma^\mu \{\Delta_\mu, B\} \rangle + F \langle \bar{B}i\gamma^5\gamma^\mu [\Delta_\mu, B] \rangle,
\]

where \(\hat{m}_s\) is the baryon mass in the chiral-SU(3) limit and \(\langle \ldots \rangle\) denotes a trace in flavor space. The parameters \(D\) and \(F\) can be fitted to experimental data. For example, Jenkins and Manohar [9] found by fitting to baryon semi-leptonic decays the values \(D = 0.61\) and \(F = 0.40\).

Baryons described by \(L_s^{(1,0)}\) satisfy the equations of motion

\[
i\gamma^\mu [D_\mu, B] - \hat{m}_s B + D i\gamma^5\gamma^\mu \{\Delta_\mu, B\} + F i\gamma^5\gamma^\mu [\Delta_\mu, B] - \frac{2}{3} D i\gamma^5\gamma^\mu \langle \Delta_\mu B \rangle = 0,
\]

which can be rewritten as

\[
i\partial B(x) = \hat{m}_s B(x) + \ldots,
\]

appearing in \(L_s^{(1,0)}\), is of order \(O(q)\). The fact that \([D^\mu, B]\) is of order \(O(1)\) raises an apparent complication in the expansion of the general ChPT Lagrangian. One would expect that terms of the form

\[
\langle \bar{B}i\gamma^\mu [D_\mu, B] \rangle - \hat{m}_s \langle \bar{B}B \rangle
\]

appearing in \(L_s^{(1,0)}\), is of order \(O(q)\). Therefore, a derivative of the baryon field counts as order \(O(\hat{m}_s / \Lambda) = O(1)\). In particular, the covariant derivative acting on \(B\), \([D^\mu, B] = \partial^\mu B + [\Gamma, B],\) is also of order \(O(1)\). Nevertheless, from Eq. (4) the combination

\[
\langle \bar{B}i\gamma^\mu [D_\mu, B] \rangle - \hat{m}_s \langle \bar{B}B \rangle
\]

appearing in \(L_s^{(1,0)}\), is of order \(O(q)\). Therefore, a derivative of the baryon field counts as order \(O(\hat{m}_s / \Lambda) = O(1)\). Therefore, it seems impossible to expand the general Lagrangian with a finite number of terms in each order. However, it can be shown [11] that most terms with covariant derivatives acting on \(B\) can be written as a linear combination of terms with less covariant derivatives acting on \(B\) and terms that are proportional to the equation of motion (3). Terms in an effective Lagrangian that are proportional to the classical equations of motion can be eliminated by a suitable field redefinition [12] (See also Ref. [13] for a discussion on the use of field redefinitions in mesonic ChPT). The Lagrangians before and after such a field redefinition will give the same predictions for any physical amplitude. In the weak interaction sector we will consider an example of how a field transformation can eliminate a term from the Lagrangian.

By using such field redefinitions, it is then possible to cast the general Lagrangian in a form in which each order in its expansion contains a finite number of terms. Of course, for a practical calculation of an amplitude in ChPT it is convenient to write the Lagrangian in a form with the least number of independent terms in each order. In the following we will attempt to give such a minimal set of terms. To do so we will also make use of the Cayley identities, which can relate terms in the Lagrangian with a double trace in flavor space to terms with a single trace in flavor space.
In this way, we have found that the Lagrangians $\mathcal{L}_s^{(0,1)}$, $\mathcal{L}_s^{(2,0)}$, $\mathcal{L}_s^{(1,1)}$, and $\mathcal{L}_s^{(0,2)}$ consist of the terms

\[
\langle \bar{B}(\sigma, B) \rangle; \quad \langle \bar{B}B \rangle \langle \sigma \rangle; \quad \langle \bar{B}i\sigma^\mu(D_\mu, [D_\nu, B]) \rangle; \quad \langle \bar{B}(\Delta^\mu, (\Delta_\nu, [D_\nu, B])) \rangle;
\]

\[
\langle \bar{B}\Delta^\mu \rangle \langle i\sigma_\mu(\Delta_\nu B) \rangle; \quad \langle \bar{B}\Delta^\mu \rangle \langle i\gamma_\mu(\Delta_\nu D_\nu B) \rangle,
\]

\[
\langle \bar{B}\gamma^5(\rho, B) \rangle; \quad \langle \bar{B}\gamma^5 B \rangle \langle \rho \rangle; \quad \langle \bar{B}i\gamma^5\gamma^\mu(\Delta_\mu, B) \rangle \langle \sigma \rangle; \quad \langle \bar{B}\Delta^\mu \rangle \langle i\gamma_\mu(\Delta_\nu D_\nu B) \rangle;
\]

\[
\langle \bar{B}i\gamma^5(\Delta_\mu, B) \rangle; \quad \langle \bar{B}(\sigma, B) \rangle; \quad \langle \bar{B}(\rho, B) \rangle; \quad \langle \bar{B} \rangle \times \langle \sigma^2 \rangle; \quad \langle \bar{B}(\sigma, B) \rangle \times \langle \sigma \rangle; \quad \langle \bar{B}(\rho, B) \rangle \times \langle \rho \rangle;
\]

\[
\langle \bar{B} \sigma \rangle \times \langle \sigma \rangle; \quad \langle \bar{B} \rho \rangle \times \langle \rho B \rangle,
\]

respectively. In the above compact notation $(A, B)$ can be the anti-commutator $\{A, B\}$ or the commutator $[A, B]$. Furthermore, $(A, (B, C))$ can be any of the combinations $\{A, \{B, C\}\}$, $\{A, [B, C]\}$, $[A, \{B, C\}]$, or $[A, [B, C]]$. (It is easy to show that $(A, (A, C))$ represents only three independent combinations.) Furthermore, a term $X$ in one of the above lists is an abbreviations for the combination $(X + X^c)/2$, where $X^c$ is the charge conjugated of $X$. From the above expressions it an be easily seen that the Lagrangian $\mathcal{L}_s^{(0,1)}$ consists of 13 independent terms, the Lagrangian $\mathcal{L}_s^{(2,0)}$ consists of 13 independent terms, the Lagrangian $\mathcal{L}_s^{(1,1)}$ consists of 13 independent terms, and the Lagrangian $\mathcal{L}_s^{(0,2)}$ consists of 14 independent terms.

The Lagrangian $\mathcal{L}_s^{(0,1)}$ breaks SU(3) symmetry. By including SU(3) breaking through $\mathcal{L}_s^{(0,1)}$ the equation of motion for free octet baryons reads

\[
(i\partial - \hat{m})B(x) = \Delta m B(x),
\]

where $\Delta m$ is a quantity of order $O(m_\pi)$, determined by the parameters in Eq. (5). As is discussed after Eq. (3), the left-hand side of Eq. (11) counts as order $O(q)$ in the chiral expansion. This indicates that $m_\pi$ is of order $q$, rather than $q^2$ as in the meson sector. We again emphasize that in principle ChPT $m_\pi$ and $q$ should be considered as separate expansion parameters, and only phenomenological analysis will dictate the relative order.

Necessary ingredients in the general weak $(|\Delta S| = 1)$ interaction Lagrangian are the fields $\lambda$ and $\chi$ defined e.g. in Ref. [1]. In the construction of the weak interaction Lagrangian one encounters the same problem as in the strong interaction sector, namely that $[D^\mu, B]$ counts as order $O(1)$ in the chiral expansion. However, also in the weak interaction sector this problem can be dealt with by redefining the baryon field. For example, the term

\[
\alpha \left( \langle \bar{B}i\gamma^5\gamma^\mu(\lambda, [D_\mu, B]) \rangle + \langle \bar{B}i\gamma^5\gamma^\mu(D_\mu, [\lambda, B]) \rangle \right),
\]

can be effectively removed from the Lagrangian by the field redefinition

\[
B \to B + \alpha \gamma^5[\lambda, B].
\]

Under the redefinition (13) the kinetic part of the strong Lagrangian (3) will generate a term that cancels with Eq. (12). The weak interaction Lagrangian can be readily obtained. Its expansion reads

\[
\mathcal{L}_w = \mathcal{L}_w^{(0,0)} + \mathcal{L}_w^{(1,0)} + \mathcal{L}_w^{(0,1)} + \ldots
\]

where $\mathcal{L}_w^{(0,0)}$ is given by

\[
\mathcal{L}_w^{(0,0)} = h_D \langle \bar{B}\{\lambda, B\} \rangle + h_F \langle \bar{B}[\lambda, B] \rangle,
\]

$\mathcal{L}_w^{(1,0)}$ consists of the terms

\[
\langle \bar{B}i\gamma^5(\lambda', B) \rangle; \quad \langle \bar{B}i\gamma^5(\lambda, (\Delta_\mu, B)) \rangle; \quad \langle \bar{B}i\gamma^5\gamma^\mu(\lambda, (\Delta_\mu, B)) \rangle; \quad \langle \bar{B}i\gamma^5(\{\lambda', \Delta_\mu \}, B) \rangle; \quad \langle \bar{B}i\gamma^5\gamma^\mu([\lambda', \Delta_\mu], B) \rangle;
\]

\[
\langle \bar{B}\lambda \rangle i\gamma^\mu(\Delta_\mu B) \rangle; \quad \langle \bar{B}(\chi) i\gamma^\mu(\Delta_\mu B) \rangle; \quad \langle \bar{B}(\lambda) i\gamma^5\gamma^\mu(\Delta_\mu B) \rangle; \quad \langle \bar{B}(\chi') i\gamma^5\gamma^\mu(\Delta_\mu B) \rangle; \quad \langle \bar{B}(\chi') \rangle \langle \Delta^\mu [D_\mu, B] \rangle,
\]

(16)
and $\mathcal{L}_w^{(0,1)}$ consists of the terms

$$
\langle \bar{B}(\lambda, (\sigma, B)) \rangle; \langle \bar{B}(\lambda', (\rho, B)) \rangle; \langle \bar{B}(\lambda', [\sigma, B]) \rangle; \langle \bar{B}([\lambda, \rho], B) \rangle; \langle \bar{B}(\lambda, B) \rangle \times \langle \sigma \rangle; \langle \bar{B}(\lambda', B) \rangle \times \langle \rho \rangle; \\
\langle \bar{B}(\lambda) \rangle \times \langle \sigma B \rangle; \langle \bar{B}(\lambda') \rangle \times i \times \langle \rho B \rangle; \langle \bar{B}(\lambda) \rangle \times \langle \rho B \rangle.
$$

(17)

A term $X$ in the above lists for $\mathcal{L}_w^{(0,1)}$ and $\mathcal{L}_w^{(1,0)}$ is an abbreviation for $X + X^{\text{cp}}$, where $X^{\text{cp}}$ is $X$ after a charge conjugation and a parity operation. Note that in some cases the chiral order of a given term is not so obvious. For example, the list for $\mathcal{L}_w^{(1,0)}$ does not contain the term

$$
X = \langle \bar{B}\lambda \rangle \langle \Delta^\mu [D_\mu, B] \rangle,
$$

(18)

since $X + X^{\text{cp}}$, given by

$$
X + X^{\text{cp}} = \langle \bar{B}\lambda \rangle \langle \Delta^\mu [D_\mu, B] \rangle + \langle \{\bar{B}, D_\mu\}\lambda \rangle \langle \Delta^\mu B \rangle,
$$

(19)

can be shown to be of order $O(q^2)$ by using Cayley’s identity. In conclusion, we have found that the weak Lagrangians $\mathcal{L}_w^{(0,0)}$, $\mathcal{L}_w^{(1,0)}$, and $\mathcal{L}_w^{(0,1)}$ consist of 2, 19, and 20 independent terms, respectively.

### 3 The non-relativistic limit

To obtain the non-relativistic limit of the above Lagrangian we use the path integral approach of Ecker and Mojžiš [8], who studied the case of 2 flavors. The method is similar to the approach used in heavy-quark theory by Mannel et al. [10]. The first step consist of writing the Lagrangian in terms of the velocity dependent fields

$$
H(v, x) = e^{i v \cdot x} P^+_v B(x), \quad h(v, x) = e^{i v \cdot x} P^-_v B(x),
$$

(20)

where $v$ is a four-velocity satisfying $v^2 = 1$, and $P^+_v$ and $P^-_v$ are the velocity dependent projection operators

$$
P^\pm_v = \frac{1 \pm \hat{v}}{2}.
$$

(21)

By expanding the baryon field $B$ as

$$
B = \frac{1}{\sqrt{2}} B^i \lambda^i
$$

(22)

with $\lambda^i$ the Gell-Mann matrices (or any basis will do for that matter), it is straightforward to extend the formalism of Ecker and Mojžiš and Bernard et al. to 3 flavors. (Recently, we have received a preprint [14] in which in a similar way the case of 2 flavors has been extended to 3 flavors.) After the above field redefinitions the Lagrangian [8] is given by

$$
\mathcal{L}_w = \tilde{H}^i M_1^{ji} H^i + \tilde{h}^j M_2^{ji} H^i + \tilde{H}^j \gamma_0 (M_1^i)^{ji} \gamma_0 h^i - \tilde{h}^i M_3^{ji} h^i,
$$

(23)

where $M_1$, $M_2$ and $M_3$ are $8 \times 8$ matrices given by

$$
2M_1^{ji} = i v^\mu \langle \lambda^i [D_\mu, \lambda^j] \rangle - 2 i D_S^\mu \langle \lambda^i \{\Delta_\mu, \lambda^j\} \rangle - 2 i F_S^\mu \langle \lambda^i [\Delta_\mu, \lambda^j] \rangle + \ldots,
$$

(24)

$$
2M_2^{ji} = P^-_v \left[ i \gamma^\mu \langle \lambda^i [D_\mu, \lambda^j] \rangle + i D_5^\gamma \langle \lambda^i \{v \cdot \Delta, \lambda^j\} \rangle + i F_5^\gamma \langle \lambda^i [v \cdot \Delta, \lambda^j] \rangle + c_1 \gamma^5 \delta^{ji} (\rho) + c_2 \gamma^5 \langle \lambda^j (\rho, \lambda^i) \rangle + c_3 \gamma^5 \langle \lambda^j [\rho, \lambda^i] \rangle + \ldots \right] P^+_v,
$$

(25)

and

$$
M_3^{ji} = 2 i m \delta^{ji} + \ldots.
$$

(26)
with $S^\mu_v$ is the spin operator defined by

$$S^\mu_v \equiv -\frac{1}{2} P^+_v \gamma^\mu P^+_v,$$  \hspace{1cm} (27)

and $c_1$, $c_2$, and $c_3$, are constants from the first two terms in Lagrangian (8). The dots in the expressions for $M_2$ and $M_3$ represent terms that are not relevant for the present analysis.

After the field redefinition

$$h^i \rightarrow h^i - (M_3^{-1} M_2)^{ij} H^j,$$  \hspace{1cm} (28)

the minus component field $h$ can be easily integrated out from the Lagrangian, using the path integral formalism. The resulting strong Lagrangian $L_{s,v}$, which should give equivalent physical prediction as $L_s$, reads

$$L_{s,v} = \bar{H}^j M_1^{ji} H^i + \bar{H}^j (\gamma_0 M_2^{ji} \gamma_0 M_3^{-1} M_2)^{ij} H^i.$$  \hspace{1cm} (29)

Using

$$(M_3^{-1})^{ij} = \frac{1}{2m} \delta^{ij} + \ldots$$

it is straightforward to expand $L_{s,v}$ in powers of $q$ and $m_s$ (note that each covariant derivatives acting on $H$ counts as order $q$). The expansion of $L_{s,v}$ reads

$$L_{s,v} = L_{s,v}^{(1,0)} + L_{s,v}^{(0,1)} + L_{s,v}^{(2,0)} + L_{s,v}^{(1,1)} + L_{s,v}^{(0,2)} + \ldots.$$  \hspace{1cm} (31)

where $L_{s,v}^{(1,0)}$ is given by

$$L_{s,v}^{(1,0)} = \langle \bar{H} [v \cdot D, H] \rangle - 2i D \langle \bar{H} S^\mu_v [\Delta, H] \rangle - 2i F \langle \bar{H} S^\mu_v [\Delta, H] \rangle,$$  \hspace{1cm} (32)

and $L_{s,v}^{(0,1)}$ consists of the terms

$$\langle \bar{H} (\sigma, H) \rangle; \langle \bar{H} H \rangle \langle \sigma \rangle.$$  \hspace{1cm} (33)

The Lagrangian $L_{s,v}^{(2,0)}$ is given by

$$L_{s,v}^{(2,0)} = \frac{1}{2m} \langle \bar{H} [v \cdot D, [v \cdot D, H]] \rangle - \frac{1}{2m} \langle \bar{H} [D^\mu, [D_\mu, H]] \rangle$$

$$- \frac{D}{m} \langle \bar{H} S^\mu_v [D_\mu, \{v \cdot \Delta, H]\} + \bar{H} S^\mu_v \{v \cdot \Delta, [D_\mu, H]\}]$$

$$- \frac{F}{m} \langle \bar{H} S^\mu_v [D_\mu, \{v \cdot \Delta, H\}] + \bar{H} S^\mu_v [v \cdot \Delta, [D_\mu, H]\}] + \Delta L_{s,v}^{(2,0)},$$  \hspace{1cm} (34)

where $D$ and $F$ are the same constants as in Lagrangian (8), and $\Delta L_{s,v}^{(2,0)}$ consists of the terms

$$\langle \bar{H} S^\mu_v (\sigma, [\Delta, H]) \rangle; \langle \bar{H} [S^\mu_v, S^\nu_v] ([\Delta, \Delta, H] \rangle; \langle \bar{H} (\Delta^\mu, (\Delta, H)) \rangle; \langle \bar{H} (v \cdot \Delta, (v \cdot \Delta, H)) \rangle;$$

$$\langle \bar{H} \Delta^\mu \rangle \times (\Delta, H); \langle \bar{H} \Delta^\mu \rangle \times [S^\mu_v, S^\nu_v] \times (\Delta, H); \langle \bar{H} v \cdot \Delta \rangle \times \langle \Delta \Delta \rangle.$$  \hspace{1cm} (35)

The Lagrangian $L_{s,v}^{(1,1)}$ consists of the terms

$$\langle \bar{H} S^\mu_v (\sigma, (\Delta, H)) \rangle; \langle \bar{H} ([v \cdot \Delta, \rho], H) \rangle; \langle \bar{H} S^\mu_v (\Delta_\mu, H) \rangle \times \langle \sigma \rangle; \langle \bar{H} \Delta^\mu \rho \rangle \times S^\mu_v \times \langle \sigma \rangle; \langle \bar{H} v \cdot \Delta \rangle \times \langle \rho, H \rangle; \langle \bar{H} S^\mu_v ([D_\mu, \rho], H) \rangle; \langle \bar{B} S^\mu_v B \rangle \langle [D_\mu, \rho] \rangle;$$  \hspace{1cm} (36)

and the Lagrangian $L_{s,v}^{(0,2)}$ consists of the terms

$$\langle \bar{H} (\sigma, (\sigma, H)) \rangle; \langle \bar{H} ([\rho, \rho], H) \rangle; \langle \bar{H} (\sigma, H) \rangle \times \langle \sigma \rangle; \langle \bar{H} (\rho, H) \rangle \times \langle \rho \rangle; \langle \bar{H} \sigma \rangle \times \langle \sigma, H \rangle; \langle \bar{H} \rho \rangle \times \langle \rho, H \rangle.$$  \hspace{1cm} (37)

Note that the terms

$$\langle \bar{H} S^\mu_v ([D_\mu, \rho], H) \rangle; \langle \bar{B} S^\mu_v B \rangle \langle [D_\mu, \rho] \rangle;$$  \hspace{1cm} (38)
enter $L^{(1,1)}_{w,v}$ through the $1/m$ expansion, i.e., through the second term in the right-hand side of Eq. (23).

To obtain the non-relativistic weak Lagrangian the same steps can be followed. To obtain the $1/m$ part for the weak Lagrangian we need

$$M^2_{\text{weak}} = \frac{1}{2} P_v \left[ d_1 \gamma^5 \langle \lambda, \lambda \rangle + d_2 \gamma^5 \langle \lambda', \lambda \rangle + \ldots \right] P_v^+,$$

(39)

where $d_1$ and $d_2$ are constants corresponding to the first term in Lagrangian (16), and the dots denote terms at least containing one of the fields $D^\mu$, $\Delta^\mu$, $\rho$ or $\sigma$. The resulting non-relativistic weak Lagrangian, obtained in the same way as Eq. (24), then reads

$$L_{w,v} = L^{(0,0)}_{w,v} + L^{(1,0)}_{w,v} + L^{(0,1)}_{w,v} + \ldots,$$

(40)

where the Lagrangian $L^{(0,0)}_{w,v}$ is given by

$$L^{(0,0)}_{w,v} = h_D \langle \bar{H} \{ \lambda, H \} \rangle + h_F \langle \bar{H} \{ \lambda, H \} \rangle,$$

(41)

the Lagrangian $L^{(1,0)}_{w,v}$ consists of the terms

$$\langle \bar{H} i(\lambda, (v \cdot \Delta, H)) \rangle; \langle \bar{H} i S^\mu_\nu(\lambda, (\Delta, H)) \rangle; \langle \bar{H} i(\lambda', (v \cdot \Delta, H)) \rangle; \langle \bar{H} i S^\mu_\nu((\lambda', H)) \rangle; \langle \bar{H} i v \cdot \lambda \rangle; \langle \bar{H} i S^\mu_\nu(\lambda, H) \rangle; \langle \bar{H} i S^\mu_\nu(\lambda, H) \rangle,$$

(42)

and the Lagrangian $L^{(0,1)}_{w,v}$ consists of the terms

$$\langle \bar{H} (\lambda, (\sigma, H)) \rangle; \langle \bar{H} (\lambda', (\rho, H)) \rangle; \langle \bar{H} ((\lambda', \sigma), H) \rangle; \langle \bar{H} (\lambda, H) \rangle \times (\sigma); \langle \bar{H} (\lambda, H) \rangle \times (\rho);$$

$$\langle \bar{H} \lambda \rangle \times \langle \sigma H \rangle; \langle \bar{H} \lambda \rangle \times \langle \rho H \rangle; \langle \bar{H} \lambda \rangle \times \langle \sigma H \rangle; \langle \bar{H} \lambda \rangle \times \langle \rho H \rangle.$$

(43)

The above results can be compared with the results for the general Lagrangian given in Ref. [8], which was derived by starting directly in heavy-baryon formulation of ChPT. This makes it possible to derive the constraints imposed by reparametrization invariance on the HBChPT Lagrangian. Using Eq. (24) one can easily see that as a result of reparametrization invariance the coefficients of the strong interaction terms

$$\langle \bar{H} D^\mu, [D_{\mu}, H] \rangle; \langle \bar{H} v \cdot D, [v \cdot D, H] \rangle; \langle \bar{H} S^\mu_\nu [D_{\mu}, \{ v \cdot \Delta, H \}] + \bar{H} S^\mu_\nu [v \cdot \Delta, [D_{\mu}, H]] \rangle; \langle \bar{H} S^\mu_\nu [D_{\mu}, \{ v \cdot \Delta, H \}] + \bar{H} S^\mu_\nu [v \cdot \Delta, [D_{\mu}, H]] \rangle,$$

(44)

in Ref. [8] are given by $-1/(2m), 1/(2m), -D/m, -F/m$, respectively. In the weak interaction sector, the coefficients of the terms

$$\langle \bar{H} S^\mu [D_{\mu}, \{ \lambda, H \}] \rangle + \langle \bar{H} S^\mu \{ \lambda, [D_{\mu}, H] \} \rangle; \langle \bar{H} S^\mu [\lambda, [D_{\mu}, H]] \rangle + \langle \bar{H} S^\mu [D_{\mu}, \{ \lambda, H \}] \rangle,$$

(45)

in Ref. [8] are both zero. All other terms in the HBChPT Lagrangian derived in Ref. [8] are not constrained by reparametrization invariance.

4 Application to hyperon weak radiative decays

As discussed in the previous section some of the coefficients in the general HBChPT Lagrangian are constrained by reparametrization invariance. In particular, the coefficients of the weak-interaction terms in
Eq. (45) are constrained to be zero. In Ref. [15] it was shown that these constants, denoted there by $a_5$ and $a_6$, are crucial for the description of the two charged weak radiative decay channels,

$$\Xi^- \rightarrow \Sigma^- + \gamma,$$

(46)

and

$$\Sigma^+ \rightarrow p + \gamma.$$  

(47)

In leading order, the parity-violating amplitudes ($B$) for the charged channels were both shown to be proportional to a linear combination of $a_5$ and $a_6$, while the parity-conserving amplitudes ($A$) for the charged channels were found to be zero. By fitting the constants $a_5$ and $a_6$ it was possible to give a satisfactory description of the decay rates of the charged channels. However, from the requirement that $a_5$ and $a_6$ are fixed to be zero it follows that in leading order not only $A$, but also $B$ for the two charged channels is vanishing. This result is in agreement with Jenkins et al. [16] who argue that the amplitude $B$ for the charged channels cannot receive any short distance contributions. Therefore, to obtain a non-zero value for both $A$ and $B$ one needs to go at least to the next-to-leading order. In such analysis one should also consider loop-diagrams contributions to the decay as studied by Refs. [16, 17].

In Ref. [15] it was shown that Hara’s theorem, which states that $B = 0$ in the SU(3) symmetric limit, was violated if $a_5$ and $a_6$ are non-zero. Since $a_5$ and $a_6$ must be zero by reparametrization invariance, we conclude that Hara’s theorem, is still satisfied in HBChPT. In the past, the measured value of asymmetry parameter for the $\Sigma^+$ decay ($\alpha = -0.75 \pm 0.08$), has been considered as being inconsistent with Hara’s theorem. Indeed, if $B = 0$ and $A$ is non-zero, the asymmetry parameter defined by

$$\alpha = \frac{\text{Re}(AB^*)}{|A|^2 + |B|^2},$$

(48)

is vanishing. However, in HBChPT both $A$ and $B$ are zero in leading order, making $\alpha$ indeterminate. Therefore, Hara’s theorem is satisfied, but at the same time $\alpha$ is not necessarily close to zero.

5 Summary and discussions

Heavy-baryon chiral perturbation theory (HBChPT) [2] is considered to be an appropriate tool to study processes involving baryons at low kinetic energies. In this papers we have studied the constraints imposed by reparametrization invariance [4] on heavy-baryon chiral perturbation theory (HBChPT). We have considered the case of 3 flavors, and both the strong and the weak interaction sectors. One feasible way to make sure the Lagrangian is reparametrization invariant is to start from the fully relativistic Lagrangian and take non-relativistic limit [3]. By matching the thus obtained Lagrangian with the most general HBChPT Lagrangian the constraints imposed by reparametrization invariance on HBChPT can be easily derived. In this way we established that a total of 4 terms in the strong interaction HBChPT Lagrangian of Ref. [15] are constrained, and a total number of 2 terms in the weak HBChPT Lagrangian are constrained. This is of importance since the total number of free parameters available to fit the experimental data is smaller by the constraints. For example, for the case of hyperon weak radiative decays it is shown that as a consequence of these constraints the leading-order parity-violating amplitudes for both charged channels $\Sigma^+ \rightarrow p + \gamma$ and $\Xi^- \rightarrow \Sigma^- + \gamma$ are vanishing, consistent with Hara’s theorem. Since at the same time also the parity-conserving amplitude is vanishing for these channels, the asymmetry parameter can not be determined in leading-order ChPT.

Acknowledgments

This work was supported by the National Science Council of the Republic Of China under contracts Nos. NSC85-2112-M-007-032, NSC85-2112-M-007-029, and NSC85-2112-M-001-026.
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