We present the joint helicity amplitudes for $J/\psi \to \Lambda \bar{\Lambda}$, $\Lambda \bar{\Lambda}$ decays to different final states in the helicity frame. Two observables to search for $CP$ violation in $J/\psi \to \Lambda \bar{\Lambda}$ can be expressed with the information of helicity angles of baryon and antibaryon. Four decay parameters of $\Lambda$ and $\bar{\Lambda}$, namely, $\alpha^-$, $\alpha^+$, $\alpha^0$ and $\bar{\alpha}^0$, can be obtained with the joint helicity amplitude equations by the likelihood fit method. With the data sample of $10^{10}$ $J/\psi$ decays accumulated by BESIII, the precision of the measurements is estimated to be about $10^{-3}$.

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1. Introduction

$\Lambda \bar{\Lambda}$ decay is one of the octet baryon-antibaryon pairs decays of $J/\psi$ and other charmonium states. The branching ratio of $J/\psi \to \Lambda \bar{\Lambda}$ has been measured by BES$^{[1]}$ and CLEO$^{[2]}$ collaboration. The world average value is $(1.61 \pm 0.15) \times 10^{-3}$$^{[3]}$. It is noted that this decay channel is very special to study the $CP$ invariance not only in $J/\psi \to \Lambda \bar{\Lambda}$ but also in the nonleptonic decay of $\Lambda (\bar{\Lambda})$.

$CP$ violation in $J/\psi \to \Lambda \bar{\Lambda}$ decay is studied in Ref.$^{[4]}$. Two observables are suggested to test $CP$ invariance,

$$A_{J/\psi} = \theta(\hat{p} \cdot (\hat{q}_1 \times \hat{q}_2)) - \theta(-\hat{p} \cdot (\hat{q}_1 \times \hat{q}_2)),$$  \hspace{1cm} (1)

$$B = \hat{p} \cdot (\hat{q}_1 \times \hat{q}_2),$$  \hspace{1cm} (2)
where $\theta(x)$ is 1 if $x > 0$ and is zero if $x < 0$. $\mathbf{\hat{p}}$, $\mathbf{\hat{q}_1}$ and $\mathbf{\hat{q}_2}$ are the momentum unit vectors of $\Lambda$, proton and anti-proton. Any nonzero values for them signal CP violation. Beside $J/\psi \rightarrow \Lambda \bar{\Lambda}$, the measurement can be carried in other charmonium states decay to $\Lambda \bar{\Lambda}$ experimentally.

CP violation can also be studied in nonleptonic decays of $\Lambda$. Nonleptonic hyperon decays have long been known as an ideal laboratory to study the parity violation[5]. Considering a nonleptonic decay of the hyperon $Y$, the angular distribution of the baryon in the center-of-mass (CM) system of $Y$ takes the form $\frac{dN}{d\Omega} \propto 1 + \alpha_Y \mathbf{P} \cdot \mathbf{\hat{p}}$, where $\mathbf{P}$ is the polarization vector of the hyperon, $\mathbf{\hat{p}}$ is the momentum unit vector of the baryon, and $\alpha_Y$ is the hyperon decay parameter, which characterizes the parity violation in the decays. Taking $\Lambda \rightarrow p\pi^-$ as an example, a CP-odd observable, $A_\Lambda$, can be defined as

$$A_\Lambda = \frac{\alpha_- + \alpha_+}{\alpha_- - \alpha_+},$$

where $\alpha_-$ is the decay parameter of $\Lambda \rightarrow p\pi^-$, $\alpha_+$ is the decay parameter of $\bar{\Lambda} \rightarrow \bar{p}\pi^+$. If $\Lambda$ decays to $n\pi^0$ and $\bar{\Lambda}$ decays to $\bar{n}\pi^0$, $\alpha_-$ and $\alpha_+$ should be replaced by $\alpha_0$ and $\bar{\alpha}_0$. If CP is conserved, this observable vanishes for $\alpha_- = -\alpha_+$ or $\alpha_0 = -\bar{\alpha}_0$. Any nonzero value implies evidence for CP asymmetry in $\Lambda$ decays. This asymmetry has been previously performed at $p\bar{p}$ colliders by the R608[6] and PS185[7] collaborations, and at an $e^+e^-$ collider by DM2[8] collaboration. The latest result is measured by BES[9] collaboration, although the precision has been improved much, it is insufficient to observe CP violation at the level predicted by the standard model.

The precise measurement of the $\Lambda$ decay parameter also plays an important role in the determination of $\Omega^-$ and $\Xi^-$ decay parameters. To note that the non-polarized $\Omega^-$ or $\Xi^-$ decays can produce polarized $\Lambda$ particle. Namely, $\Lambda$ is the daughter particle in the decays of $\Omega^-$ and $\Xi^-$. In the $\Lambda$ rest frame, the angular distribution of the proton takes the form of $\frac{dN}{d\cos\theta} \propto 1 + \alpha_{\Omega(\Xi)} \alpha_- \cos\theta$. Experimentally, the extraction of $\alpha_{\Omega(\Xi)}$ from the product $\alpha_{\Omega(\Xi)}\alpha_-$ is dependent on the value of $\alpha_-$. The accuracy of $\alpha_{\Omega(\Xi)}$ measurement is dependent on the accuracy of $\alpha_-$. The situation is the same for $\Omega^+$ and $\Xi^+$ which can produce polarized $\bar{\Lambda}$. So, the measurement of $\alpha_-\alpha_+$ plays a unique role in other hyperon decay parameters measurement.

In this paper, based on the study in Refs.[4][10], we detail the information on the CP observables in $J/\psi \rightarrow \Lambda\bar{\Lambda}$, and the decay parameters of $\Lambda$ and $\bar{\Lambda}$ with helicity amplitude analysis. The similar study is applied for $\psi(2S) \rightarrow \gamma\chi_{cJ} \rightarrow \gamma\Lambda\bar{\Lambda}[11]$. Nowadays, $10^{10}$ $J/\psi$ decays have been accumulated by BESIII detector, the advantage of this work is obvious and high accuracy can be achieved for the statistics.
2. Helicity amplitudes analysis of $J/\psi \rightarrow \Lambda \bar{\Lambda}, \Lambda \rightarrow B\pi, \bar{\Lambda} \rightarrow \bar{B}\pi$

The helicity amplitudes for $J/\psi \rightarrow \Lambda \bar{\Lambda}, \Lambda \rightarrow B\pi, \bar{\Lambda} \rightarrow \bar{B}\pi$ decays are constructed in the helicity frame defined as:

1. In $J/\psi \rightarrow \Lambda \bar{\Lambda}$, the z-axis of the $J/\psi$ rest frame is along $\Lambda$ outgoing direction, which changes from event to event. The $e^+ \pi^-$ beam is along the direction of the solid angle $(\theta, \phi)$.

2. For $\Lambda$ decay $\Lambda \rightarrow B\pi$, the solid angle of the daughter particle $\Omega_1(\theta_1, \phi_1)$ is refereed to the $\Lambda$ rest frame, where the z-axis is taken along the outgoing direction of $\Lambda$ in its mother particle rest frame. The helicity frame of $\bar{\Lambda}$ has a similar definition which is described by the solid angle $\bar{\Omega}_1(\bar{\theta}_1, \bar{\phi}_1)$.

Fig. 1 shows the definition of the helicity frame for $J/\psi \rightarrow \Lambda \bar{\Lambda}, \Lambda \rightarrow B\pi, \bar{\Lambda} \rightarrow \bar{B}\pi$.

The joint helicity amplitudes of $J/\psi \rightarrow \Lambda \bar{\Lambda}, \Lambda \rightarrow B\pi, \bar{\Lambda} \rightarrow \bar{B}\pi$ can be expressed by $A_{\lambda,\bar{\lambda}}$, $B_{\lambda B}$ and $\bar{B}_{\lambda B}$ as:

$$|\mathcal{M}|^2 \propto \sum_{\lambda,\bar{\lambda},\lambda',\bar{\lambda}',\lambda B,\bar{\lambda} B} \rho^{\lambda-\bar{\lambda},\lambda'-\bar{\lambda}'}(\theta, \phi) \times$$

$$A_{\lambda,\bar{\lambda}} A^{*}_{\lambda',\bar{\lambda}'} B_{\lambda B} B^{*}_{\lambda B} \bar{B}_{\lambda B} \bar{B}^{*}_{\lambda B} \times$$

$$D^{1/2}_{\lambda,\bar{\lambda} B}(\Omega_1) D^{1/2}_{\lambda',\bar{\lambda}' B}(\Omega_1) \times$$

$$D^{1/2}_{\lambda,\bar{\lambda} B}(\bar{\Omega}_1) D^{1/2}_{\lambda',\bar{\lambda}' B}(\bar{\Omega}_1),$$

where $\lambda, \lambda B, \bar{\lambda} B$ are the helicity values for $\Lambda, \bar{\Lambda}$, baryon and anti-baryon.

$$\rho^{(i,j)}(\theta, \phi) = \sum_{k=\pm1} D_{i,k}(\Omega) D_{j,k}(\Omega)$$

is the density matrix for the $J/\psi$ produced in $e^+e^-$ annihilation. The element $\rho^{(i,j)}$ is equal to $\rho^{(j,i)*}$ and equal to $(-1)^{(i+j)} \rho^{(-i,-j)}$. The density matrix elements are shown in Tab. II.
Table 1. J/ψ density matrix elements

\begin{align*}
\rho^{(1,1)}(\theta, \phi) &= \frac{1 + \cos^2 \theta}{2} \\
\rho^{(0,0)}(\theta, \phi) &= \sin^2 \theta \\
\rho^{(1,0)}(\theta, \phi) &= \frac{\sin \theta \cos \theta}{\sqrt{2}} e^{-i \phi} \\
\rho^{(1,-1)}(\theta, \phi) &= \sin \theta \cos \theta e^{-2i \phi}
\end{align*}

Table 2. Density matrix elements of $D^{J}_{\lambda,\lambda'}(\Omega)$ with $J = \frac{1}{2}$

\begin{align*}
D^{\frac{1}{2}}_{\frac{1}{2},\frac{1}{2}}(\Omega) &= e^{-i \frac{\phi}{2}} \cos \frac{\theta}{2} \\
D^{\frac{1}{2}}_{\frac{1}{2},-\frac{1}{2}}(\Omega) &= -e^{-i \frac{\phi}{2}} \sin \frac{\theta}{2} \\
D^{\frac{1}{2}}_{-\frac{1}{2},\frac{1}{2}}(\Omega) &= e^{i \frac{\phi}{2}} \sin \frac{\theta}{2} \\
D^{\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2}}(\Omega) &= e^{i \frac{\phi}{2}} \cos \frac{\theta}{2}
\end{align*}

$D^{J}_{\lambda,\lambda'}(\Omega) \equiv D^{J}_{\lambda,\lambda'}(\phi, \theta, 0)$ is the Wigner D-function. The standard Wigner D-function is defined as:

\begin{equation}
D^{j}_{m,m'}(\alpha \beta \gamma) = \sum_{k=0}^{j+m'} (-1)^{j-m'_{_{}}+k} \sqrt{\frac{(j-m')!(j+m')!(j-m')!(j+m'_{_{}})!}{k!(j+m-k)!(j-m-k)!(k+m-m')!}} \times e^{-im'_{_{}} \alpha} e^{-im'_{_{}} \gamma} (\cos \frac{\beta}{2})^{2j+m-m'_{_{}}-2k} (\sin \frac{\beta}{2})^{m-m'_{_{}}+2k},
\end{equation}

The final baryons in Λ and ¯Λ decays are $p, \bar{p}, n, \bar{n}$. The possible helicity values for $p, \bar{p}, n, \bar{n}$ are $\frac{1}{2}$ or $-\frac{1}{2}$, so the density matrix elements of $D^{J}_{\lambda,\lambda'}(\Omega)$ with $J = 1/2$ are shown in Tab. 2.

From parity invariance, $A_{-\lambda, -\lambda'} = A_{\lambda, \lambda'}$. So the decay parameters in Λ and ¯Λ decays are defined as:

\begin{align*}
\alpha_- &= \alpha(\Lambda \to p\pi^-) = \frac{|B_{1/2}|^2 - |B_{-1/2}|^2}{|B_{1/2}|^2 + |B_{-1/2}|^2}, \\
\alpha_+ &= \alpha(\bar{\Lambda} \to \bar{p}\pi^+) = \frac{|\bar{B}_{1/2}|^2 - |\bar{B}_{-1/2}|^2}{|\bar{B}_{1/2}|^2 + |\bar{B}_{-1/2}|^2}, \\
\alpha_0 &= \alpha(\Lambda \to n\pi^0) = \frac{|B_{1/2}|^2 - |B_{-1/2}|^2}{|B_{1/2}|^2 + |B_{-1/2}|^2}.
\end{align*}
\[ \bar{\alpha}_0 = \alpha(\bar{\Lambda} \rightarrow \bar{n} \pi^0) = \frac{|\bar{B}_{1/2}|^2 - |\bar{B}_{-1/2}|^2}{|\bar{B}_{1/2}|^2 + |\bar{B}_{-1/2}|^2}, \tag{9} \]

If CP invariance is conserved, one has \( B_\lambda = -\bar{B}_{-\lambda}, \alpha_- = -\alpha_+ \) and \( \alpha_0 = -\bar{\alpha}_0 \) can be gotten. The angular distribution of baryons(\( p,n \)) and antibaryons(\( \bar{p},\bar{n} \)) in \( \Lambda(\bar{\Lambda}) \) rest frame can be written as:

\[
\frac{dN}{d\Omega} \propto 1 + \alpha_- |P| \cos \theta_1 \quad \Lambda \rightarrow p\pi^-, \tag{10}
\]

\[
\frac{dN}{d\Omega} \propto 1 + \alpha_+ |P| \cos \theta_1 \quad \bar{\Lambda} \rightarrow \bar{p}\pi^+, \tag{11}
\]

\[
\frac{dN}{d\Omega} \propto 1 + \alpha_0 |P| \cos \theta_1 \quad \Lambda \rightarrow n\pi^0, \tag{12}
\]

\[
\frac{dN}{d\Omega} \propto 1 + \bar{\alpha}_0 |P| \cos \bar{\theta}_1 \quad \bar{\Lambda} \rightarrow \bar{n}\pi^0, \tag{13}
\]

The product of decay parameter and polarization of \( \Lambda(\bar{\Lambda}) \) can be gotten by fitting the angular distribution of the baryons(anti-baryons). If the polarization of \( \Lambda(\bar{\Lambda}) \) could be measured, the decay parameter could be extracted, but in \( e^+e^- \) collision, the average value of polarization for the produced \( \Lambda(\bar{\Lambda}) \) is zero, so, experimentally, the decay parameter can not be measured in this way.

From Eq.(4), the angular distribution parameter of \( \Lambda(\bar{\Lambda}) \) can be defined as:

\[
\alpha = |A_{1/2,-1/2}|^2 - 2 |A_{1/2,1/2}|^2 \tag{14}
\]

if the normalization condition is selected as \( |A_{1/2,-1/2}|^2 + 2 |A_{1/2,1/2}|^2 = 1 \), one has:

\[
|A_{1/2,-1/2}|^2 = \frac{1 + \alpha}{2} \quad \text{and} \quad |A_{1/2,1/2}|^2 = \frac{1 - \alpha}{4}. \tag{15}
\]

Combining Eqs.(4) (6) (7) (14) (15) and integrating over \( \phi \), Eq.(4) is simplified as:

\[
\frac{d|M|^2}{d(\cos \theta)d\Omega_1 d\Omega_2} \propto (1 - \alpha) \sin^2 \theta \times
\]
\[
[1 + \alpha_- \alpha_+(\cos \theta_1 \cos \bar{\theta}_1 + \sin \theta_1 \sin \bar{\theta}_1 \cos(\phi_1 + \bar{\phi}_1))] - (1 + \alpha)(1 + \cos^2 \theta)(\alpha_- \alpha_+ \cos \theta_1 \cos \bar{\theta}_1 - 1), \tag{16}
\]
Eq. (16) is the helicity amplitude equation for $J/\psi \rightarrow \Lambda \bar{\Lambda} \rightarrow p\pi^-\bar{p}\pi^+$. With different final states of $\Lambda$ and $\bar{\Lambda}$ decays, according to Eqs. (6) (7) (8) (9), one can also get helicity amplitude equations for $J/\psi \rightarrow \Lambda \bar{\Lambda}$ with different final states.

- $J/\psi \rightarrow \Lambda \bar{\Lambda} \rightarrow p\pi^-\bar{p}\pi^+$

$$\frac{d|\mathcal{M}|^2}{d(\cos \theta)d\Omega_1 d\bar{\Omega}_1} \propto (1 - \alpha) \sin^2 \theta \times \left[ 1 + \alpha_0 (\cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos (\phi_1 + \bar{\phi}_1)) \right] - (1 + \alpha)(1 + \cos^2 \theta)(\alpha_0 \cos \theta_1 \cos \bar{\theta}_1 - 1), \quad (17)$$

- $J/\psi \rightarrow \Lambda \bar{\Lambda} \rightarrow n\pi^0\bar{n}\pi^0$

$$\frac{d|\mathcal{M}|^2}{d(\cos \theta)d\Omega_1 d\bar{\Omega}_1} \propto (1 - \alpha) \sin^2 \theta \times \left[ 1 + \alpha_0 (\cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos (\bar{\phi}_1 - \phi_1)) \right] - (1 + \alpha)(1 + \cos^2 \theta)(\alpha_0 \cos \theta_1 \cos \bar{\theta}_1 - 1), \quad (18)$$

3. $CP$ violation of $J/\psi \rightarrow \Lambda \bar{\Lambda}$, $\Lambda \rightarrow B\pi$, $\bar{\Lambda} \rightarrow \bar{B}\pi$

In the helicity frame of $J/\psi \rightarrow \Lambda \bar{\Lambda} \rightarrow p\pi^-\bar{p}\pi^+$, by using the momenta of $p, \pi^-, \bar{p}$ and $\pi^+$, Eqs. (1) (2) can be written as:

$$A_{J/\psi} = \theta (\sin \theta_1 \sin \bar{\theta}_1 \sin (\phi_1 - \bar{\phi}_1)) - \theta (\sin \theta_1 \sin \bar{\theta}_1 \sin (\bar{\phi}_1 - \phi_1)), \quad (20)$$

$$B = \sin \theta_1 \sin \bar{\theta}_1 \sin (\phi_1 - \bar{\phi}_1), \quad (21)$$

With Eq. (20), one can try to search for the $CP$ violation in $J/\psi \rightarrow \Lambda \bar{\Lambda}$. Ref. [12] has shown the result which is consistent with the expectation of $CP$ conservation.

According to Refs. [9][10], the $\Lambda$ angular distribution parameter $\alpha$ and the product of $\Lambda$ and $\bar{\Lambda}$ decay parameters with different final states can
Table 3. Λ angular distribution parameter $\alpha$ and the product of Λ and ¯Λ decay parameters with different final states

| Final states | Λ angular distribution | Λ, ¯Λ decay parameters |
|--------------|------------------------|------------------------|
| $p\pi^-\bar{p}\pi^+$ | $\alpha$ | $\alpha_-$, $\alpha_+$ |
| $p\pi^-\bar{p}\pi^0$ | $\alpha$ | $\alpha_-$, $\bar{\alpha}_0$ |
| $n\pi^0\bar{p}\pi^+$ | $\alpha$ | $\alpha_0$, $\alpha_+$ |
| $n\pi^0\bar{p}\pi^0$ | $\alpha$ | $\alpha_0$, $\bar{\alpha}_0$ |

be determined by using the unbinned maximum likelihood method with Eqs. (16) (17) (18) (19). Tab. 3 shows the undetermined parameters with different final states. In each channel, one can get the product of one Λ decay parameter and one ¯Λ decay parameter.

The advantage of the helicity amplitude analysis for the four channels in Tab. 3 is that four products of Λ and ¯Λ decay parameters can be determined. With these four products, Λ decay parameters $\alpha_-$, $\alpha_+$ and ¯Λ decay parameters $\alpha_0$, $\bar{\alpha}_0$ can be obtained respectively. BESIII has already accumulated $10^{10}$ $J/\psi$ decays, the detection efficiency is simulated at least 5% for pure neutral channels and 30% for pure charged channels with Monte Carlo, the measurements can be done with high precision. The precision of the measurements is shown in Tab. 4.

From Tab. 4 the precision of four products of Λ and ¯Λ decay parameters could be improved by two orders of magnitude compared with the current values[3]. Combining these four products, one can get $\alpha_-$, $\alpha_+$, $\alpha_0$ and $\bar{\alpha}_0$ respectively. The $CP$ invariance could be obtained by Eq. (3) with $\alpha_-$, $\alpha_+$ or $\alpha_0$, $\bar{\alpha}_0$, the precision would be also improved.

4. Summary

The helicity amplitudes for $J/\psi \rightarrow \Lambda\bar{\Lambda}$, $\Lambda(\bar{\Lambda})$ decays to different final states are presented. Two observables to search for $CP$ violation in $J/\psi \rightarrow \Lambda\bar{\Lambda}$ can be expressed in the helicity frame with the information of helicity angles of baryon and antibaryon. Four decay parameters of Λ and ¯Λ can
Table 4. The precision of the measurements

| Final states | Detection | Λ, ¯Λ decay | Precision |
|--------------|-----------|-------------|-----------|
|              | efficiency| parameters  |           |
| pπ−pπ+       | 30%       | α−α+       | 10^{-3}   |
| pπ−nπ0       | 15%       | α−α0       | 10^{-3}   |
| nπ0pπ+       | 15%       | α0α+       | 10^{-3}   |
| nπ0nπ0       | 5%        | α0α0       | 10^{-2}   |

be obtained with the joint helicity amplitude equations by the likelihood fit method which has been already used in Ref. [9]. With the data sample of 10^{10} \( J/\psi \) decays accumulated by BESIII, the precision for \( \alpha_-, \alpha_+, \alpha_0 \) and \( \bar{\alpha}_0 \), compared with the current value, could be improved by two orders of magnitude. It would be helpful for the further study of the \( CP \) invariance both in \( J/\psi \to \Lambda \bar{\Lambda} \) and the nonleptonic decays of \( \Lambda (\bar{\Lambda}) \).

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