Coulomb excitation of $^{11}$Be reexamined

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Abstract

The study of Coulomb excitation of $^{11}$Be to the first excited state in intermediate energy collisions with heavy targets is presented. The existing experimental data are reanalysed by including the probability of projectile survival into the calculation of Coulomb excitation cross sections. The survival probabilities are calculated using a recently developed global optical model potential tailored in line with the double folding model. The extracted $B(E1)$ values for the transition $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$ in $^{11}$Be are found to be slightly larger than those obtained so far using the $b_0$-recipe in Coulomb excitation theory.

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The Coulomb excitation process has been proven to be a useful tool to study nuclear structure (see, e.g., Ref.[1]). It is an attractive test of the different nuclear models as the Coulomb interaction is very well known. Recently a number of experiments has been done to study the structure of light neutron rich nuclei, such as $^{11}$Be [2, 3, 4], $^{15}$C, $^{17}$C, $^{19}$C and several oxygen isotopes, using the Coulomb excitation process [5].

The isotope $^{11}$Be is one of the most studied halo nuclei nowadays. The reason why $^{11}$Be is an attractive system to study by Coulomb excitation lies in the peculiar structure of the spectrum of this nucleus. Both the $1^+$ ground state and the only bound excited state $1^-$ are weakly bound and have energy difference of 0.32 MeV, thus making the E1 transition in $^{11}$Be the fastest known between bound states.

There have been several experiments that measured the excitation of the first excited state in $^{11}$Be and extracted the corresponding $B(E1)$ value. The adopted value $ca$ $0.116 e^2fxm^2$ was obtained by Millener et al. in Ref.[6] by averaging the results of three experiments on the lifetime of the excited state using a Doppler-shift technique.

The experiment done in GANIL [2] studied the inelastic scattering of $^{11}$Be on a lead target at a bombarding energy of 45 MeV/u. The obtained cross section was only 40% of the one predicted by calculations for pure Coulomb excitation. To explain this large difference it was proposed that the higher order effects may contribute, but calculations done in Ref.[7] showed that the cross section falls by only 4% if coupling to continuum was taken into account. The inclusion of monopole and quadrupole modes of nuclear excitation also performed in the work [7] resulted in the slight increase of the cross section less than 2%.

Similar experiment was done by Nakamura et al. in Ref.[3] at $E = 64$ MeV/u. The $B(E1)$ value extracted from the cross section assuming pure Coulomb excitation was comparable with the result of the Millener analysis. Later the inelastic scattering of $^{11}$Be was measured at MSU [4] on lead and gold targets for energies 60 and 58 MeV/u, respectively. The extracted $B(E1)$ value confirmed that of [3] and agreed, at least marginally, with the lifetime experiments. For summary of this see Table II.

In the analysis performed in the mentioned papers, the authors used the formalism of pure Coulomb excitation and excluded nuclear processes approximately by using a low impact parameter cutoff $b_0$. In this work we show that the inclusion of the full fledged survival probability $|S(b)|^2$ in conjunction with $b_0$, though slightly improves the agreement with the
data, it does allow one to perform a more realistic smooth cutoff based analysis.

In the calculation of the Coulomb excitation cross section, the important ingredient of the model is the minimum value of the impact parameter, $b_0$, from which the integration of the excitation amplitude is performed. This $b_0$ is roughly determined by the sum of the projectile and target radii. $^{11}$Be, being a halo nucleus, has a very diffuse structure, which makes the definition of the minimum impact parameter obscure.

To improve upon the calculation we multiply the probability of the Coulomb excitation at a given impact parameter by the survival probability $|S(b)|^2$ calculated for the system under consideration. In this work we take the S-matrix from the optical model. The calculation of the elastic S-matrices requires the knowledge of the densities of the projectile and the target. The halo density of $^{11}$Be was calculated in the framework of the particle-rotor model, which includes the excitation of the rotational $2^+$ state of the core $^{10}$Be.

As was mentioned above, the contribution of nuclear excitation is small. This is so since nuclear effects are limited to a very small impact parameter region around the grazing value. In fact, as shown in Refs. 9, 10, 11, the Coulomb excitation was found to be by far the dominant piece of the cross section. Further, in Ref. 4, an experiment was done with light targets to study the importance of nuclear excitation. The results were 4.0 mb for carbon target and 1.7 mb for beryllium target. Since the nuclear cross section corresponds to the area of a ring around grazing $b_0$, it scales with $A$ as $A^{1/3}$. Accordingly, we find for the lead target approximately 10 mb nuclear excitation contribution implying a mere 3% effect. Therefore in the following we ignore the nuclear excitation effect.

The semiclassical model is usually used to describe Coulomb excitation at intermediate and high energies. This model assumes the straight line trajectory for the projectile and treats quantum mechanically the absorption of radiation by the nucleus. The formalism of this method was presented in Ref. 12 and later was extended to the relativistic case in Ref. 13. For the high energies the first order perturbation theory is a good approximation to calculate the amplitudes for Coulomb excitation. In the first order perturbation theory the process of Coulomb excitation can be described as emission and absorption of virtual photons [14]. We included only $E1$ multipolarity in our analysis since for the transition $^{1+}_2 \rightarrow ^{1-}_2$ in $^{11}$Be the dipole multipolarity is the dominant one [15].

Using the formalism of virtual photons the Coulomb excitation cross section has the
following form:

\[
\sigma_C = \int_0^{\theta_0} d\sigma_C d\Omega = 2\pi \int_{b_0}^\infty P(b)b db
\]

(1)

with

\[
P(b) = \frac{16\pi^3}{9hc} B(E1)N_{E1}(\omega, b),
\]

(2)

and \(b_0 = a\cot \frac{\theta_0}{2}\), where \(a\) is half the distance of closest approach for head-on Coulomb collision. \(N_{E1}(\omega, b)\) is the number of virtual photons given by

\[
N_{E1}(\omega, b) = \frac{Z_1^2\alpha}{\pi^2} \left( \frac{\xi}{b} \right)^2 \left( \frac{\xi}{\nu} \right)^2 \left[ K_1^2(\xi) + \frac{1}{\gamma^2} K_0^2(\xi) \right],
\]

(3)

where \(Z_1e\) is the charge of the target, \(\xi = \frac{\omega b}{\gamma v}, \omega = E_{ex}/\hbar, E_{ex}\) is the excitation energy of the state in the projectile, \(v\) is the relative energy, \(\gamma\) is the relativistic factor and \(\alpha\) is the fine structure constant. The behavior of the integrand in the Eq. (1) is determined by the impact parameter dependence of the modified Bessel functions of the zero and first order \(K_0\) and \(K_1\).

Coulomb recoil was taken into account using the method of Winther and Alder [13] replacing \(b\) in the expression for \(P(b)\) by \(b' = b + \pi a/2\gamma\). Further, we multiply \(P(b)\) of Eq. (1) by the survival probability \(|S(b)|^2\), calculated from the optical potential NLM3Y of Ref. [16] and then write

\[
\tilde{P}(b) = P(b) \cdot |S(b)|^2.
\]

(4)

The optical potential was calculated from the double folding model, using projectile and target densities and the effective nucleon-nucleon interaction. The effect of the exchange-related non-locality is taken fully into account. The local-equivalent, energy-dependent potential can be written as [16]

\[
V(r, E) = V_N(r, E) - iW(r, E),
\]

where

\[
V_N(r, E) = V_F(r)e^{-4\nu^2/c^2},
\]

where \(c\) is the speed of light and \(\nu\) is the local relative velocity between the two nuclei

\[
v^2(r, E) = \frac{2}{\mu}(E - V_C(r) - V_N(r, E)).
\]

(5)
The local folding potential $V_F(r)$ is obtained in the usual way. For the Coulomb part, $V_C(r)$ we employ a similar folding prescription using the proton densities. The imaginary part of the optical potential was found to be

$$W(r, E) \approx 0.78 V_N(r, E).$$

The determination of the number $0.78$ is discussed in Ref. [17]. This number was obtained by fitting calculated elastic scattering cross sections with experimental data for the following systems: $^{12}\text{C} + ^{12}\text{C}, ^{16}\text{O}, ^{40}\text{Ca}, ^{90}\text{Zr}, ^{208}\text{Pb}; ^{16}\text{O} + ^{208}\text{Pb}, ^{40}\text{Ar} + ^{208}\text{Pb}$ and for the wide range of energies. The calculations showed that the number $0.78$ is approximately system-independent.

We employ the same potential here by using the appropriate halo+core density of $^{11}\text{Be}$.

For the densities of the heavy targets and the two-parameter Fermi distributions were used

$$\rho(r) = \frac{\rho_0}{1 + \exp \frac{r - R_0}{a}},$$

where $a = 0.56$ fm, $R_0 = 1.31 A^{1/3} - 0.84$ fm and $\rho_0$ is determined by the normalization condition. These densities were obtained in Ref. [16] with the aim of providing a global description of the nuclear interaction, based on an extensive study involving charged distributions extracted from electron scattering experiments and theoretical densities calculated through the Dirac-Hartree-Bogoliubov model. For the density of the core of $^{11}\text{Be}, ^{10}\text{Be}$, we used the Eq.(7) with $a = 0.5$ fm and $R_0 = 1.75$ fm, which gives $R_{rms}(^{10}\text{Be}) = 2.3$ fm.

We use the existing particle + core excitation model to describe the halo structure of a light halo nucleus such as $^{11}\text{Be}$ [8]. The inclusion of the core excitation allows for the coupling between the collective degrees of freedom of the core and the orbital motion of the single neutron. In this calculation we assumed a quadrupole-deformed core (with deformation parameter $\beta_2$) and included only the ground state $0^+$ and the first excited rotational state $2^+$. The interaction between the valence particle and the core was described by a deformed Woods-Saxon potential. The spin-orbit interaction, $V_{so}$, is the standard undeformed one. To reproduce the adopted $B(E1)$ of $^{11}\text{Be}$ we adjusted the model parameters obtained in Ref.[8] and these parameters are presented in Table II. The particle-core model with these parameters gives a radius for $^{11}\text{Be}$ of 3.0 fm, the $B(E1) = 0.116 \text{e}^2\text{fm}^2$.

In Fig[I] we show the calculated squared moduli of the elastic S-matrices for $^{11}\text{Be} + ^{208}\text{Pb}$ at 59.7 MeV/u. It is seen that using the density with the halo reduces the survival probability.
TABLE I: Parameters of the particle-core model used to calculate density of $^{11}\text{Be}$. $V_{so}$ and $V_{w_{s}}^{EVEN/ODD}$ are the strengths of spin-orbit and Woods-Saxon potentials, $a$ and $R_{0}$ are the diffuse-ness and the radius of the potentials. $\beta_{2}$ is the quadrupole deformation parameter and $E^{*}(2^{+})$ is the excitation energy of the core.

| $R_{0}$ (fm) | $a$ (fm) | $V_{so}$ (MeV) | $V_{w_{s}}^{EVEN}$ (MeV) | $V_{w_{s}}^{ODD}$ (MeV) | $\beta_{2}$ | $E^{*}(2^{+})$ (MeV) |
|--------------|---------|---------------|-----------------|-----------------|-----------|------------------|
| 2.483        | 0.65    | 5.0           | 54.65           | 47.82           | 0.67      | 3.368            |

In Ref. [18] the projectile survival probability was determined from the angle dependence of the Coulomb dissociation cross section. The very small number of experimental points (3 in the grazing region) with their rather large error bars is certainly insufficient to make meaningful comparison. In Ref. [18] the survival probability was approximated by the function with the shape $1/(1 + \exp(-(b - b_{0})/a))$ with parameters $b_{0} = 12.3(1.2)$ fm and $a = 0.9(0.6)$ fm. In fact, one should include the size of the error bars in the presentation of the experimental survival probability. For the purpose of completeness we show in Fig.2 the experimentally determined survival probability of Nakamura et al. presented in the shaded area together with ours shown as the full line. It is clear that our calculation of the survival probability agrees reasonably well with Nakamura’s one.

The experimental data on Coulomb excitation of $^{11}\text{Be}$ is summarized in Table II. The fifth column shows the values of $B(E1)$ extracted from the corresponding Coulomb excitation cross section by assuming pure Coulomb excitation and choosing appropriate $b_{0}$, that is without including projectile survival probability. In the analysis of Nakamura et al. [3] Eq. (1) was used with $b_{0} = 12.3$ fm which was determined from the impact parameter dependence observed in the Coulomb breakup of $^{11}\text{Be}$ [18]. Fauerbach et al. in Ref. [4] used the similar formalism to extract $B(E1)$ values from the experimental cross sections with $b_{0}$ obtained from the aperture of the experimental setup.

In our calculation we use $\tilde{P}(b)$ in Eq. (1) and start the integration using the experimental value of $b_{0}$, which is determined by the aperture of experimental setup. Thus for the case of, e.g., $^{11}\text{Be} + ^{208}\text{Pb}$ at 59.7 MeV/u we integrated the probability shown by dotted line starting with $b_{0} = 11.5$ fm.

The theoretical cross sections calculated in this work are presented in columns 6, 7 and
FIG. 1: The projectile survival probability $|S(b)|^2$ calculated for $^{11}\text{Be} + ^{208}\text{Pb}$ at 59.7 MeV/u. The solid line shows the projectile survival probability calculated using the densities of the halo, dashed line stands for the calculation using the global densities of the Eq.(7).

FIG. 2: The projectile survival probability $|S(b)|^2$ calculated for $^{11}\text{Be} + ^{208}\text{Pb}$ at 72 MeV/u including halo density. The shaded area represents the projectile survival probability determined from experiment in [18].

8 of Table II. First we assumed the $B(E1)$ values from the previous analysis [3, 4] and calculated Coulomb excitation cross sections without taking into account Coulomb recoil and using density of Eq.(7). We obtained the cross sections slightly larger than the experimental for $^{11}\text{Be} + ^{208}\text{Pb}$ at 64 and 45 MeV/u as we integrated the probability from $b_0$ obtained from
FIG. 3: The impact parameter dependence of $P(b)b$ and $\tilde{P}(b)b$. The case of $^{11}\text{Be} + ^{208}\text{Pb}$ at 59.7 MeV/u is considered. $\tilde{P}(b)b$ was calculated with the halo density (solid line) and with global densities of the work $^{[16]}$ (dashed line) in $|S(b)|^2$, see text for details.

Experimental aperture and not from grazing impact parameter as was done in $^{[3]}$. Taking into account Coulomb recoil reduced the calculated cross section by approximately 2%. The further reduction (approximately 2%) in the calculated cross section was obtained by using the halo densities of $^{11}\text{Be}$.

In Fig. 3 we show the impact parameter dependence of the Coulomb excitation probabilities $P(b)$ and $\tilde{P}(b)$ for the case of $^{11}\text{Be} + ^{208}\text{Pb}$ at 59.4 MeV/u. The inclusion of projectile survival probability reduces the Coulomb excitation probability for impact parameters from 11.5 fm and up to 17 fm. This results in the reduction of the cross section by 4% compared to the result of the pure Coulomb excitation with $b_0 = 11.5$ fm. For the case of $^{11}\text{Be} + ^{197}\text{Au}$ the reduction of the calculated cross section was 5%.

Finally, we extracted the revised values of $B(E1)$ using $\tilde{P}(b)$ and taking Coulomb recoil into account. We found no difference between the values of $B(E1)$ extracted in the work $^{[3]}$ and results of our analysis. The revised $B(E1)$ values obtained from the experimental cross sections of the work $^{[4]}$ are approximately 4% larger than those obtained in $^{[4]}$. The revised $B(E1)$ values are presented in the last column of the Table $^{[10]}$.

In conclusion, a better treatment of nuclear absorption exemplified by the use of an appropriate survival probability in the calculation of the Coulomb excitation cross section
Ref. Target Beam energy $\sigma_C$ (exp.) $B(E1)$ (old) $\sigma_C$ (th. (1)) $\sigma_C$ (th. (2)) $\sigma_C$ (th. (3)) $B(E1)$ (this work)

|  | MeV/u | mb | $e^2$fm$^2$ | mb | mb | mb | $e^2$fm$^2$ |
|---|---|---|---|---|---|---|---|
| [2] | $^{208}$Pb | 45 | 191(26) | 0.047(06) | 197 | 193 | 188 | 0.048(06) |
| [3] | $^{208}$Pb | 64 | 302(31) | 0.099(10) | 314 | 309 | 303 | 0.099(10) |
| [4] | $^{208}$Pb | 59.7 | 304(34) | 0.094(11) | 304 | 295 | 291 | 0.098(11) |
| [4] | $^{197}$Au | 59.7 | 244(25) | 0.079(08) | 244 | 236 | 233 | 0.083(08) |

TABLE II: The experimental and theoretical Coulomb excitation cross sections for different energies and extracted $B(E1)$. The fifth column shows the $B(E1)$ extracted from experimental $\sigma$ using pure Coulomb excitation in Refs. [2, 3, 4]. Columns 6, 7 and 8 show calculated cross section (using $B(E1)$ from column 5) assuming no Coulomb recoil and no halo-type density (th. (1)); with Coulomb recoil, but no halo-type density (th. (2)); with Coulomb recoil and halo-type density (th. (3)). The last column shows the revised values of $B(E1)$ which reproduce experimental cross section using $\tilde{P}(b)$ and taking Coulomb recoil into account.

of $^{11}$Be leads to slightly smaller cross-section and then a slightly (about 4%) larger $B(E1)$ value. We expect similar effect in the Coulomb dissociation cross-section of $^{11}$Be and other halo nuclei.

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