Probing the strong gravity regime with eLISA: Progress on EMRIs

Carlos F. Sopuerta
1 Institut de Ciències de l’Espai (CSIC-IEEC), Campus UAB, Facultat de Ciències, Edifici C5, 2a planta, parells, 08193 Bellaterra, Spain

Abstract. The capture of a stellar-mass compact object by a supermassive black hole and the subsequent inspiral (driven by gravitational radiation emission) constitute one of the most important sources of gravitational waves for space-based observatories like eLISA/NGO. In this article we describe their potential as high-precision tools that can be used to perform tests of the geometry of black holes and also of the strong field regime of gravity.

1. Introduction

General Relativity (GR) provides a good description of physical phenomena that involves gravity for which we have experimental probes, from the submillimeter scales to cosmological scales (which require to include a dark matter and a dark energy component). However, we have tested gravity only in specific regimes (with respect to the strength of the gravitational field) and for specific length/time scales [a review on experimental tests of the validity of GR can be found in Will (2006)]. If we look at the strength of the gravitational field, then we can say that observations to date have tested gravity only in the weak and mild gravity regimes, but not in the strong gravity regime. To make this statement more precise let us consider the dimensionless Newtonian potential associated with a self-gravitating system: \( \phi \equiv \phi_N/c^2 \) with \( c \) being the speed of light. Tests in the weak gravitational regime are those done, for instance, with observations in the Solar system (they can measure different precessional effects) and the typical values of \( \phi \) they can reach are \( \phi \sim \frac{GM_\odot}{c^2 1\text{AU}} \sim 10^{-8} \). Stronger gravitational fields can be found in pulsars and in particular in binary systems involving at least one pulsar. In particular, the best evidence to date of the existence of gravitational radiation comes from observations of the well-known Hulse and Taylor binary pulsar (PRS B1913+16) [Hulse & Taylor (1975); Weisberg & Taylor (2005); Weisberg et al. (2010)]. For binary pulsar we have two types of gravitational fields, the self-gravity of the pulsar, which is very strong and the gravitational attraction of the binary. The first one is the self-gravity of a neutron star (NS), and for this we have that \( \phi_1 \sim \frac{GM_{\text{NS}}}{c^2 r_{\text{NS}}} \sim 0.2 \), while the second one is (using the Hulse-Taylor pulsar) \( \phi_2 \sim \frac{GM_\odot}{c^2 r_{\text{periastron}_{\text{Hulse-Taylor}}} \sim 10^{-6} \). By observing the dynamics of the binary system we can test \( \phi_2 \) and also how the inner structure of the pulsar affects it, i.e. how \( \phi_1 \) may change this dynamics. It turns out that in General Relativity the inner structure of the

---

1Here, \( G \) denotes the gravitational Newton constant and \( M_\odot \) the mass of the Sun.
individual components of a binary system only affects the orbital dynamics to the fifth post-Newtonian (PN) order \( \text{[Damour (1987)]} \) and hence the effect of \( \phi_1 \) has not been seen but has been used to put constraints on alternative theories of gravity. In particular, millisecond pulsar observations have been used to constrain scalar-tensor theories of gravity, where the internal structure of the compact objects appears at lower PN order [see, e.g. \text{Damour (2007a,b); Stairs (2003)}], usually due to violations of the strong equivalence principle. So, we are testing gravitational fields with \( \phi \sim 10^{-8} \) (Solar system) and \( \phi \sim 10^{-6} \) (millisecond pulsars). In contrast, gravitational wave observations of the merger of compact binaries (NS+NS, NS+BH, BH+BH) will probe the strong field regime \( \phi \sim 1 \), which is what we will refer here as the strong gravitational regime.

There are different ways to observationally access this regime. We can try to use electromagnetic observations, mainly in the high-energy part of the spectrum, but there is the drawback that light itself is strongly affected by the gravitational field of these systems. Moreover, the electromagnetic emission comes from matter distributions like accretion disks whose physics is quite complex and makes it difficult to produce a precise modeling of the system to be compared with observations. Nevertheless, tests of the strong field regime have been proposed using the electromagnetic spectrum (see, e.g. \text{Johannsen (2012)}).

Gravitational Waves (GWs) offer an alternative way to access the strong gravitational regime as they have a weak interaction with matter and hence they carry almost uncorrupted information from the systems that produced them. At present we have several interferometric ground detectors, like [LIGO] and [VIRGO], whose advanced design models will come online around 2014. They are expected to provide the first detections during the present decade, opening the gravitational-wave window in the high-frequency part of the spectrum, which contains sources like coalescence of compact binaries, supernovae core collapse, pulsar oscillations, and stochastic backgrounds. At the same time, there have been plans for the development of a space-based interferometric GW observatory, first as an ESA-NASA collaboration (LISA) and recently as an ESA-only mission, eLISA/NGO (see \text{Amaro-Seoane et al. (2012b)}). This kind of detectors will operate in the low frequency band (roughly \( 10^{-4} \) – 1 Hz), which is not accessible from the ground (mainly due to the gravity gradient noise). The main sources in this part of the spectrum are: coalescence of (super)massive black holes (MBHs), capture and inspiral of stellar-mass compact objects (SCOs) by a MBH at a galactic center (also known as Extreme-Mass-Ratio Inspirals (EMRIs)), galactic and extragalactic binaries, and stochastic backgrounds. These sources have the potential to provide a wealth of new discoveries that will impact Astrophysics, Cosmology, and even Fundamental Physics, including constraints on galaxy formation models, stellar dynamics around galactic nuclei, formation and evolution of MBHs, tests of the geometry of BHs, and even tests of gravity in the strong field regime. In this write-up we focus precisely on how to test the strong gravity regime with GW observations from space of EMRIs. For a more extended discussion on this see \text{Sopuerta (2010)}. For recent reviews of the potential of LISA for Fundamental Physics and Cosmology see: \text{Hogan (2007); Schutz (2009); Schutz et al. (2009); Babak et al. (2011)}.

2. Extreme-Mass-Ratio Inspirals

EMRIs are extreme-mass-ratio binaries in the stage where the dynamics is driven by GW emission. They are composed of a SCO that inspirals into a MBH located in a
galactic center. The masses of interest for the SCO are in the range \( m_\star = 1 - 10^2 \, M_\odot \), and for the MBH in the range \( M_\bullet = 10^5 - 10^7 \, M_\odot \). Then, the mass-ratio for these systems is in the interval: \( \mu = m_\star / M_\bullet \sim 10^{-7} - 10^{-3} \). As the inspiral proceeds the system loses energy and angular momentum through the emission of GWs, producing a secular decay of the orbit and hence a decrease of the orbital periods. There are several astrophysical mechanisms that can produce EMRIs (see Amaro-Seoane et al. (2007) for a review on several aspects of EMRIs). The most studied mechanism is the gravitational capture of SCOs from a stellar cusp or core that surrounds the MBH. A number of these SCOs will evolve to a point in which interactions with other stellar objects are negligible and hence will become EMRIs. The EMRIs produced by this mechanism are initially very eccentric, with eccentricities in the range \( 1 - e \sim 10^{-6} - 10^{-3} \), but due to GW emission, by the time they enter the band of a space-based detector it would have been substantially reduced, in the range \( e \sim 0.5 - 0.9 \).

To understand why EMRIs are useful systems for fundamental physics we have to look at the number of cycles they spend in band. This number scales with the inverse of the mass ratio, \( \mu^{-1} \). Then, for a detector like LISA, an EMRI can spend more than \( 10^5 \) GW cycles in band during the last year before plunge (Finn & Thorne (2000)). Generic EMRI orbits are very complex (see Poisson’s contribution on EMRI modeling), with high eccentricity and inclined (see Figure 1), so that they contain a high number of harmonics contributing to the GW emission (Barack & Cutler (2004)). Moreover, many of these cycles can take place very near the MBH horizon, especially for highly spinning MBHs (see Amaro-Seoane et al. (2012a)). All this means that EMRI GWs carry a wealth of information about the MBH strong field region.

Figure 1. Example of an EMRI trajectory (left) and of an EMRI waveform (right).

The next important point is to see how this information about the strong field gravity around the MBH is encoded in the gravitational waveforms. The crucial point here is the separation of scales imposed by the extreme mass ratios involved. In the case of spatial scales it is very clear from the ratio of sizes between the MBH and the SCO. There is also a separation of time scales due to the dynamics of an EMRI. The gravitational backreaction, that is, the effect of the SCO’s gravitational field on its own trajectory is the responsible mechanism for the inspiral of the SCO into the MBH. Then, we have the orbital time scales (for instance the time to go from apocenter to
pericenter and back) and the inspiral time scale (due to backreaction), whose ratio is:

\[ T_{\text{orbital}} / T_{\text{inspiral}} \sim \mu. \]

This means that locally in time the inspiral looks like a geodesic trajectory around the MBH. The constants of motion of this geodesic trajectory (which can be taken to be the orbital parameters, \((e, p, \iota)\), where \(p\) is the semilatus rectum and \(\iota\) is the inclination angle) change slowly during the inspiral. Then, during the long inspiral the trajectory tracks most of the space around the MBH and close to its horizon. As a consequence, these long waveforms encode a map of the geometry of the MBH (which can be fully parametrized in terms of multiple moments) but also the inspiral tells us about how the gravitational backreaction mechanism works and hence we have information about the details of the theory of gravity.

To get an idea of the potential of a GW space-based observatory like LISA we can look at estimates of the parameter estimation studies made for EMRIs. As we have said, EMRI signals are quite long (1–2 yrs or even more) and have significantly high Signal-to-Noise Ratios (SNRs) (\(\text{SNR} \gtrsim 30\)). Assuming GR and that the no-hair conjecture is true, for a typical EMRI system consisting of an SCO with mass \(m_\star = 10 M_\odot\) (a stellar-mass BH) inspiralling into a MBH with mass \(M_\bullet = 10^6 M_\odot\) at SNR = 30, LISA should be able to estimate the main EMRI parameters with the following precision (see Barack & Cutler (2004); this assumes we see the last year of inspiral):

\[ \Delta (\ln M_\bullet), \quad \Delta \left(\ln \frac{m_\star}{M_\bullet}\right), \quad \Delta \left(\frac{S_\bullet}{M_\bullet^2}\right) \sim 10^{-4}, \]

where \(S_\bullet\) denotes the MBH spin (for spinning Kerr BHs: \(0 \leq |S_\bullet/M_\bullet^2| \leq 1\)), and

\[ \Delta e_\circ \sim 10^{-4}, \quad \Delta \Omega_5 \sim 10^{-3}, \quad \Delta \Omega_K \sim 5 \cdot 10^{-2}, \]

where \(e_\circ\) is the initial eccentricity, \(\Delta \Omega_5\) is the error in the angular position of the source in the sky (solid angle), and \(\Omega_K\) is the ellipse error in the determination of the MBH spin direction.

Although in this article we are focussing on EMRIs, some of the ideas can, to some extent, also be applied to Intermediate-Mass-Ratio Inspirals (IMRIs), which instead of a MBH have an intermediate-mass BH (IMBH) with masses in the range \(M_\bullet \sim 10^{2-4} M_\odot\). We have not yet conclusive evidence of the existence of IMBHs but in case they exist, the inspiral of an SCO into an IMBH would be detectable by future advanced ground observatories like the Einstein Telescope [Sathyaprakash et al. (2012)]. Moreover, the inspiral of an IMBH into an MBH should be detectable by space-based observatories. The modeling of these systems is difficult in the sense that the approximations made for EMRI modeling are not going to be accurate enough in order to obtain precise IMRI waveform templates for data analysis purposes. In particular, the non-linear gravitational dynamics is going to play an important role and feedback from Numerical Relativity and post-Newtonian theory is going to be required.

3. Towards tests of the Strong Gravity Regime with EMRIs

From the previous discussion it is clear that EMRIs are high precision tools for gravitational wave astronomy. The question now is what can be done with them apart from measuring with precision their physical parameters assuming the standard paradigm, i.e.: GR is the theory of gravity and the cosmic no-hair conjecture is true and the dark
compact objects at the galactic nuclei can be described by the Kerr family of solutions (Hawking & Ellis (1973)). The important point is that despite all the observational data available, neither we have experimental tests of the strong field gravity nor we have conclusive evidence that the dark compact objects populating galactic centers are BHs as those described by the Kerr solutions (even despite the data coming from our own galactic center (Gillessen et al. (2009))).

Then, we can make ourselves several questions. An obvious one consists in assuming that GR is the true theory of gravity and then ask the question of whether the dark objects in galactic nuclei have a geometry compatible with the Kerr solution. We will call this the *Kerr hypothesis* and the main problem is to figure out how to test it in practice. The solution must contain a method to discriminate between the Kerr geometry and other possible geometries for collapsed objects, in particular for the case of EMRI signals. As an starting point we are going to assume that the geometry of the dark compact objects is stationary, asymptotically flat, and axisymmetric. Then, within GR, it can be shown that the geometry can be fully characterized by two sets of numbers, the multipole moments, \( M_\ell \) and \( J_\ell \) \((\ell = 0, 1, \ldots)\). The first set, \( \{M_\ell\} \), are the mass moments, which also exist in Newtonian gravity. The second set, \( \{J_\ell\} \), are the current moments, which do not exist in Newtonian gravity. In GR the current moments are a consequence of the fact that not only the mass density gravitates but also the momentum density. The Kerr multipole moments satisfy the following elegant relations:

\[
M_\ell + i J_\ell = M_\bullet \left( i a_\bullet \right)^\ell,
\]

and as expected, they are fully determined by the BH mass \( M_\bullet \) and the BH intrinsic angular momentum (spin) \( S_\bullet \left( a_\bullet = S_\bullet / M_\bullet \right) \). That is, only \( M_0 \) and \( J_1 = M_\bullet a_\bullet \) are independent moments, the rest is a combination of them. For instance, the next mass and current multipoles can be related to these two by the following simple relations: \(-M_\bullet a_\bullet^2 = M_2 = -J_1^2 / M_0\) and \(-M_\bullet a_\bullet^3 = J_3 = M_2 J_1 / M_0\). This discussion about the multipole moments can be used to rewrite the Kerr hypothesis in a way more suitable for tests using GW EMRI observations: *Kerr Hypothesis*: The exterior gravitational field of the dark, compact and very massive objects sitting at the galactic centers can be well described by the vacuum, stationary, and axisymmetric solutions of the General Theory of Relativity whose multipole moments \( \{M_\ell, J_\ell\}_{\ell=0,\ldots,\infty} \) satisfy the Kerr relations of Eq. (3).

What remains is to see how to do this in practice with GW EMRI observations. A first method to experimentally test the strong-field predictions of general relativity was proposed by Poisson (1996) using a simplified model where the inspiral is a sequence of circular orbits in the equatorial plane of the MBH. Later, Ryan, in a series of papers (Ryan (1995, 1996, 1997)), studied how to test experimentally the Kerr hypothesis using the multipole moments. In a first study Ryan (1995) studied to what extend the GW emission of EMRIs depends on the values of the different multipole moments of the central massive object. To that end, Ryan assumed nearly circular and nearly equatorial orbits and that the inspiral takes place in a slowly and adiabatic manner, avoiding analyzing the GW emission in detail. In particular, Ryan showed that the number of cycles that the dominant GW components spend in a logarithmic frequency interval, \( \delta N(f) \equiv f^2 \langle df / dt \rangle \), which contains equivalent information to the GW phase, contains full information of the whole set of multipole moments. Ryan also showed how to extract at least the first three moments, \( (M_0, J_1, M_2) \), which is enough to perform a partial test of the Kerr hypothesis.
In a second study (Ryan (1997)) the goal was to estimate the accuracy of LIGO and LISA in determining the multipole moments from the GWs emitted by an IMRI (LIGO) or an EMRI (LISA). The basic idea is to use waveform template models that include a certain number of multipole moments in the set of source parameters. It is well known that standard EMRI GWs can be described in terms of 14 parameters, hence the waveforms used by Ryan are time series \( h(t; \theta^I) \), with \( \theta^I = \theta^I_{\text{GR}} (I = 1, \ldots, 14) \) begin the standard EMRI parameters and \( \theta^I = \theta^I_{\text{MM}} (I = 15, \ldots, 14+N_{\text{mm}}) \), are the extra parameters containing \( N_{\text{mm}} \) multipole moments (different from the first two, \( M_0 \) and \( J_1 \)). The conclusions of Ryan’s estimations indicate that a space-based detector like LISA can be able to make several tests of the Kerr hypothesis. In the particular case of an SCO with \( m_\bullet = 10M_\odot \) inspiralling into a central massive object with \( M = 10^5M_\odot \), during 2 years before plunge and with an SNR of 10, Ryan found that LISA could measure \( M \) with a fractional error of \( \Delta M/M \sim 10^{-3} \), \( m_\bullet \) with fractional error of \( \Delta m_\bullet/m_\bullet \sim 10^{-3} \), and for the spin Ryan finds \( \Delta(S/M_2) \sim 10^{-3} \). Finally, for the mass quadrupole moment the accuracy found is \( \Delta(M_2/M_3) \sim 0.5 \). Adding many multipole moments in the analysis degrades the precision significantly.

More recently, Barack & Cutler (2007) did a similar analysis with some improvements with respect to Ryan’s work. First of all, the waveform model is essentially the same used to produce the estimations of Eqs. (1) and (2). In this model, the EMRI system follows, at any instant in time, a Newtonian orbit emitting lowest-order quadrupolar GWs. Nevertheless, the model contains post-Newtonian modifications to secularly evolve the parameters of the orbit in such a way that the model contains all the features that we expect from generic EMRIs. The LISA model used by Barack & Cutler is also more complete than Ryan’s one, including the main features of the LISA constellation motion and a better noise model that includes the confusion noise from galactic and extragalactic binaries. However, Barack & Cutler only include the effects of the mass quadrupole \( M_2 \). The parameter estimation analysis of Barack & Cutler (2007) predicts that the mass quadrupole could be measure with errors in the range [for an EMRI system as in Eqs. (1) and (2) but with SNR = 100]:

\[
\Delta(M_2/M_0^3) \sim 10^{-4} - 10^{-2},
\]

which is a much better error estimate than Ryan’s one, mainly due to the full complexity of the EMRI dynamics encoded in the waveform model of Barack & Cutler (2004).

The program initiated by Ryan is far from being complete. One question that arises is to what extend the multipole moment parametrization of the central object should be carried forward to the parametrization of the EMRI GW signals. First of all, the multipole moments are quantities defined at spatial infinity, far away from the strong field region, which complicates the way in which the generated GWs depend on them. Secondly, the expansion of the gravitational field in terms of multipole moments is such that higher multipoles will contribute less and less to the expansion as long as we evaluate the expansion far away enough from the central compact object. However, we have mentioned that EMRIs can expend a large fraction of cycles very near the last stable orbit, which in turn can be quite close to the horizon for spinning MBHs and prograde orbits. This means that for these situations a high number of multipoles are going to be relevant for the description since most powers of \( M/r \) are going to be of the same order near pericenter, where the field and the GW emission are stronger. Then, the multipole moments as waveform parameters are going to exhibit correlations and the error estimations for them are not going to be good enough for making precise
tests. In summary, it is possible that a space-based GW detector will measure a few multipole moments with high precision, allowing for partial tests of the Kerr hypothesis, but including too many multipoles in the waveform model may be counterproductive for these purposes.

Other ideas to test the strong field region near the dark compact objects in galactic nuclei have been proposed. For instance, Collins & Hughes (2004) initiated a program to construct exact solutions within GR that are almost BHs (they are stationary, axisymmetric, asymptotically flat and vacuum, and have been named *bumpy* BHs) but such that some multipole moments have the wrong value. An interesting feature of these geometries is that they are valid in the strong field region. These bumpy BHs may be used for performing null experiments, i.e. by comparing their properties with measurements of astrophysical sources. The drawback of the initial solutions was that the changes introduced are not smooth and present some pathological strong-field structure. These problems were fixed by Vigeland & Hughes (2010) who also introduced angular momentum (bumpy Kerr BHs). Construction of other bumpy BH geometries has continued and has been extended to other theories of gravity (Vigeland et al. (2011)).

There are other studies that consider geometries for the central compact object alternative to Kerr. Kesden et al. (2005) investigated the possibility of distinguishing between a central MBH and a boson star from signatures in the waveforms produced by the inspiral of an SCO. The main idea is that for a MBH the waveform will end at the plunge whereas for the boson star it will continue until the SCO reaches its center. Barausse et al. (2007) constructed models that add a self-gravitating and homogeneous compact torus to the MBH with comparable mass and spin. It was found that for most cases the emission from these systems is indistinguishable from pure-Kerr waveforms, indicating a possible confusion problem. Moreover, Barausse & Rezzolla (2008) find that in general the dissipative effect of the hydrodynamic drag exerted by the torus on the SCO is much smaller than the one due to radiation reaction. Other exact solutions, like the Manko-Novikov solutions (Manko & Novikov (1992)) have been used (Gair et al. (2008); Apostolatos et al. (2009); Lukes-Gerakopoulos et al. (2010)), where the orbital motion can exhibit chaotic behavior and this can produce observable signatures in the EMRI waveforms.

Up to now we have discussed several ways of testing deviations from the standard EMRI paradigm within GR. Mainly by studying changes in the geometry of the central compact object (one could also study changes in the SCO structure, like adding spin (Huerta & Gair (2011)), although these are in general less important). In what follows, we discuss how to extend these studies by allowing for theories that include a description of the gravitational interaction different from the one of GR. In this sense, it is important to mention that the landscape of theories of gravity is very rich. However, it is not always possible to extract what are the predictions of a given theory for a system like an EMRI, either because of the complexity of the theory or because we lack consistent formulations of the problem. Nevertheless, there are several theories of gravity different from GR where EMRIs have been studied. For instance in the well-known class of scalar-tensor theories of gravity, where it has been suggested that EMRIs may exhibit floating orbits (Yunes et al. (2012)), a very different behavior with respect to GR. To illustrate the potential of EMRI GW observations, in what follows we describe a couple of examples where EMRIs have been studied in two different theories of gravity and the predictions that have been made for LISA observations.
The first example is about EMRIs is Dynamical Chern-Simons Modified Gravity (DCSMG; see [Alexander & Yunes (2009) for a review]). This is a modification of GR introduced first by Jackiw & Pi (2003) that consists in adding to the GR action a gravitational parity-violating term, the Pontryagin invariant, which is a generalization of the well-known three-dimensional Chern-Simons (CS) term. Since in four dimensions the Pontryagin term is a topological invariant and would not contribute to the field equations, in DCSMG it appears coupled to a scalar field. To avoid the restrictive dynamics that comes out of the original model (see [Yunes & Sopuerta (2008)]), in DCSMG we also add the action for this scalar field. This theory can be seen as a low-energy limit of string theory or even of loop quantum gravity, but it can be seen just as a gravitational correction in the spirit of effective field theories.

There are several interesting points about this theory with regard to the description of EMRIs. First, spinning MBHs in this theory are no longer described by the Kerr metric (although non-spinning MBHs are still described by the Schwarzschild metric). There are deviations from Kerr that have been found using a slow-rotation and a small-coupling (these deviations are controlled by a single parameter, $\xi$, that turns out to be a combination of universal coupling constants) approximation ([Yunes & Pretorius (2009); Yagi et al. (2012)]). These corrections affect the multipolar structure of the MBH but, as it happens with the Kerr metric, they still remain completely determined by only two numbers, the mass $M_\bullet$ and spin parameter $a_\bullet$. Therefore, the main spirit of the no-hair conjecture, as far as we know, remains valid. For the lowest order correction (order $\xi \cdot a_\bullet$), the Kerr relations for the multipole moments satisfy different relations for $\ell \geq 4$, which involve the CS parameter $\xi$. However, when we add the next correction (order $\xi \cdot a_\bullet^2$, these relations are modified for lower $\ell$ ([Yagi et al. (2012)]). In what follows, we will describe only results found using the lowest-order correction, whose details can be found in [Canizares et al. (2012b)] and in [Sopuerta & Yunes (2009)]. One important point is that in this theory the effective energy-momentum tensor of the GWs is formally as in GR, with contributions to the radiation reaction mechanism from the CS scalar field. Moreover, in DCSMG distant observers will notice the same GW polarizations as in GR ([Sopuerta & Yunes (2009)]).

The local dynamics of EMRIs in DCSMG is very similar to the one in GR in the sense that the modified MBH metric has the same symmetries (at order $\xi \cdot a_\bullet$) as the Kerr metric. Therefore, locally in time, the waveforms are dominated by multiples of three fundamental frequencies and one could think that these can lead to a confusion problem between GR and DCSMG EMRI waveforms. What breaks the degeneracy are the radiation reaction effects. Recently, a parameter estimation study of EMRIs has been carried out (see [Canizares et al. (2012b)] and also Canizares’ contribution to this volume [Canizares et al. (2012a)]). In the case where DCSMG is assumed to be the true theory of gravity and for an EMRI with $M_\bullet = 10^6M_\odot$, $a_\bullet/M_\bullet = 0.25$, $e_0 = 0.25$, and $\xi a_\bullet/M_\bullet^2 = 5 \cdot 10^{-2}$, it was found that the expected errors are in the range: $\Delta \log M_\bullet \sim 5 \cdot 10^{-3}$, $\Delta a_\bullet \sim 5 \cdot 10^{-6}$, $\Delta e_0 \sim 3 \cdot 10^{-7}$, and $\Delta \log(\xi \cdot a_\bullet) \sim 4 \cdot 10^{-2}$. In contrast, if we assume that GR is the true theory of gravity and that measurements are compatible with a vanishing $\xi$, we should be able to establish, using LISA EMRI measurements, the following bound on this CS parameter ([Canizares et al. (2012b)]): $\xi^{1/4} < 1.4 \cdot 10^4$km, which is almost four orders of magnitude better than the bound imposed by Solar System experiments.

A completely different study of EMRIs has been done in the context of the higher-dimensional braneworld scenarios of [Randall & Sundrum (1999)]. In these scenarios,
standard model fields live on a 3+1 brane moving in a five-dimensional spacetime where the direction out of the brane is not non-compact. They were proposed as an attempt to solve the hierarchy problem of the physical fundamental interactions. They are also a playground for holography (and the anti-de Sitter/Conformal Field Theory (AdS-CFT) correspondence) since it has been argued that the BH solutions localized on the brane, found by solving the classical bulk equations correspond to quantum-corrected black holes, rather than classical ones. As a consequence, it was conjectured that static black holes can not exist for radius much greater than the Anti-de Sitter (AdS) length of the extra dimension, $L$. The presence of a large number of CFT degrees of freedom accelerates the decay of a black hole via Hawking radiation determining its evolution. It has been found (Emparan et al. (2003)) that $\frac{dm_*}{dt} \approx -2.8 \times 10^{-7} \left(\frac{M_\odot}{m_*}\right)^2 \left(\frac{L}{1 \text{mm}}\right)^2 M_\odot \text{yr}^{-1}$. Therefore, EMRIs where the SCO is a stellar-mass BH can be used to constraint this theory. McWilliams (2010) found that LISA could constrain $L$ to be below 5 microns. However, static solutions for black holes have been found recently (Figueras & Wiseman (2011)) invalidating the hypothesis on which these constraints are based. These hypothesis are based on applying free field theory intuition to the CFT on the brane, which is strongly coupled, and this may explain the apparent contradiction. In any case, the important lesson from this example is that EMRI GW observations have a tremendous potential to test fundamental physics scenarios linked with high-energy physics (for more on this see Cardoso et al. (2012)).

4. Conclusion

The main goal of this article has been to convey the idea that observations of EMRIs by future space-based observatories like eLISA will provide a unique opportunity to make great discoveries in the area of Fundamental Physics, specially for learning about the geometry of the presumed MBHs at the galactic centers and about the strong field regime of gravity that so far has been experimentally unexplored.

Acknowledgments

CFS acknowledges support from the Ramón y Cajal Programme of the Spanish Ministry of Education and Science, contract 2009-SGR-935 of AGAUR, and contracts FIS2008-06078-C03-03, AYA-2010-15709, and FIS2011-30145-C03-03 of MICCIN. We acknowledge the computational resources provided by the BSC-CNS (AECT-2011-3-0007) and CESGA (contracts CESGA-ICTS-200 and CESGA-ICTS-221).

References

Alexander, S., & Yunes, N. 2009, Phys. Rep., 480, 1. 0907.2562
Amaro-Seoane, P., Sopuerta, C. F., & Freitag, M. D. 2012a. 1205.4713
Amaro-Seoane, P., et al. 2007, Class. Quant. Grav., 24, R113. astro-ph/0703495
— 2012b, Class. Quant. Grav., 29, 124016. 1202.0839
Apostolatos, T. A., Lukes-Gerakopoulos, G., & Contopoulos, G. 2009, Phys. Rev. Lett., 103, 111101. 0906.0093
Babak, S., Gair, J. R., Petiteau, A., & Sesana, A. 2011, Class. Quant. Grav., 28, 114001. 111101. 0906.0093
Barack, L., & Cutler, C. 2004, Phys. Rev., D69, 082005. gr-qc/0310125
