Phase diagram of a one-dimensional exciton-polariton condensate

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Exciton-polaritons

Strong coupling hamiltonian

\[
\hat{H} = \sum_k \left\{ E_{X,k} \hat{a}^{\dagger}_{X,k} \hat{a}_{X,k} + E_{C,k} \hat{a}^{\dagger}_{C,k} \hat{a}_{C,k} + \hbar \Omega_R \left( \hat{a}^{\dagger}_{C,k} \hat{a}_{X,k} + \hat{a}^{\dagger}_{X,k} \hat{a}_{C,k} \right) \right\}
\]

- cavity photons \( \hat{a}_C \)
- excitons in quantum well \( \hat{a}_X \)
- coupling rate \( \Omega_R \)

\( \Rightarrow \) Exciton-polaritons

\[
E_{UP/LP} = \frac{E_X + E_C}{2} \pm \sqrt{\left( \hbar \Omega_R \right)^2 + \left( \frac{E_X - E_C}{2} \right)^2}
\]
Exciton-polaritons condensation

Driven dissipative steady-state

- losses of photons
  \[\Rightarrow\] finite lifetime of polaritons
- external laser driving (incoherent pumping)
  \[\Rightarrow\] driven-dissipative steady state

Non equilibrium phase transition

- external laser driving \(>\) threshold
  \[\Rightarrow\] Bose-Einstein Condensation
- Dynamics and statistical properties
  \(\neq\) equilibrium BEC

Kasprzak et al, 2006. Nature, 443(7110)
Model for non equilibrium condensate

Mean-field dynamics of the condensate described by generalized Gross-Pitaevskii equation:

\[
\begin{align*}
    \langle \xi(t, x) \rangle &= 0 \\
    \langle \xi^*(t, x) \xi(t', x') \rangle &= 2\sigma \delta(x - x') \delta(t - t')
\end{align*}
\]
Non-equilibrium condensate: KPZ dynamics of the phase

Density-phase decomposition $\psi = \sqrt{\rho} \ e^{i\theta}$

The dynamics of the phase $\theta = \operatorname{Arg}(\psi)$ is mapped to the Kardar-Parisi-Zhang equation:

$$\partial_t \theta = \nu \partial_x^2 \theta + \frac{\lambda}{2} (\partial_x \theta)^2 + \sqrt{D} \eta$$

![Graphs and plots showing the evolution of density, phase angle, and width over time]
Non-equilibrium condensate: KPZ dynamics of the phase

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$$\partial_t \theta = \nu \partial_x^2 \theta + \frac{\lambda}{2} (\partial_x \theta)^2 + \sqrt{D} \eta$$

The Kardar-Parisi-Zhang universality class (non-equilibrium rough interfaces, randomly stirred fluids, ...)

$$w(t)^2 = \sum_x (h(x, t) - \bar{h}(t))^2 \sim t^{2/3}$$
The decay of coherence displays the universal scaling behaviour of KPZ

\[ g^{(1)}(x,t) = \frac{\langle \psi^*(x_0,t_0) \psi(x_0+x,t_0+t) \rangle}{\langle |\psi(x_0,t_0)|^2 \rangle^{1/2}} \frac{\langle |\psi(x_0+x,t_0+t)|^2 \rangle^{1/2}}{\langle |\psi(x_0+t_0)|^2 \rangle^{1/2}} \]

\[ |g^{(1)}(x,t)| \approx e^{-\frac{1}{2} C_\theta(x,t)} \]

\[ C_\theta(x,t) = C_0 t^{2\beta} g^{(KPZ)}(\alpha_0 \frac{x}{t^{1/z}}) \]

\[ \beta = 1/3, \ z = 3/2 \ \text{KPZ exponents (1d)} \]

Experiment → see talk of J.Bloch
More non-equilibrium behaviours of exciton-polaritons

- Compact KPZ for discrete systems \(\Rightarrow\) de-synchronisation instability: \textbf{space-time vortices}

Arrays of coupled oscillators (in 1d and 2d)

R. Lauter, A. Mitra, and F. Marquardt, Physical Review E 96, (2017)

Space-time vortices in neq condensates

L. He, L. M. Sieberer, and S. Diehl, Physical Review Letters 118, (2017)

- \textbf{Instability} and pattern-formation \(\iff\) Complex Ginzburg-Landau Equation

Spatio-temporal chaos in CGLE

V. García-Morales and K. Krischer, Contemporary Physics 53, 79 (2012)

Space-time patterns in gGPE

N. Bobrovska et al., Physical Review B 99, (2019).
Study of the phase diagram of one-dimensional EP condensate

\[ p = \frac{P}{P_{th}} \]

\[ \mu = 2g_R n_R \]

\[ \frac{\sigma}{\sigma_0} \]

F.V. et al, in preparation (2023)
Effect of the noise: space-time vortex activation

- $\sigma = 0.1\sigma_0$
- $\sigma = 5\sigma_0$
- $\sigma = 25\sigma_0$

Coherent KPZ phase

Space-time vortices
Defects vs KPZ

- With defects, the interface $\text{Arg}(\psi) = \theta(x, t) + 2\pi N$ has singularities but is KPZ piecewise.
- The temporal coherence is not affected by random phase slips:
  \[
g^{(1)}(t) = \frac{\langle \psi^*(t_0) \psi(t + t_0) \rangle}{\langle |\psi|^2 \rangle} \sim \langle e^{i[\theta(t_0 + t) - \theta(t_0)]} \rangle = g_0 e^{-\alpha t^{2/3}}
\]
- While the time-unwrapped phase $\theta^{(UW)}(x_0, t)$ is:
  \[
  C_{\theta}^{(UW)}(t) = \langle \left[ \theta^{(UW)}(t_0 + t) - \theta^{(UW)}(t_0) \right]^2 \rangle \sim t^1
  \]
Solitons: pattern formation from instability

Spatio-temporal dynamics of $\psi = \sqrt{\rho} e^{i\theta}$

Soliton-like defects:

Characterisation of the transition:

- Number of solitons:
  \[ N_s \simeq \frac{1}{\pi} \int_x |\partial_x \theta| \, dx \]

- Average density:
  \[ \bar{\rho} = \frac{1}{L} \int_x |\psi|^2 \, dx \]

$\Rightarrow (\mu, p)$ section of phase diagram
Signature of solitons: momentum distribution

Momentum distribution $n_k = \langle |\psi(k)|^2 \rangle$

- KPZ: bogoliubov quasi-particles
- solitons: non-trivial behaviour $n_k \sim k^{-\alpha}$ ($\alpha = 6 \div 7$) at intermediate $k$
- vortices: spectral broadening for low $k$, but unclear signature
Summary

Non-equilibrium condensate of exciton-polaritons:
- Universal critical behaviour (KPZ)
- Rich phase diagram
- Long-lived soliton defects
- Short coherence
- Density-phase coupled dynamics
- No density dynamics
- Large coherence
- KPZ universality of the phase
- Random localized defects
- Piece-wise coherence
- Resilient KPZ scaling of the coherence
• Understand modifications to the phase diagram due to **finite size** for experiments

• Extend analysis to understand the phase diagram of EP in two dimensions

• Accessing with EP the regimes of one-dimensional KPZ equation: new **inviscid fixed point** with dynamic scaling exponent $z = 1$ (→ see talk of L.Canet)
Thank you

Thanks for the attention!

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Jacqueline Bloch
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Coherence in the soliton regime

- Space-time behaviour

\begin{align*}
|g_1(x, t = 0)| = 1.16, \\
p = 1.26, \\
p = 1.37, \\
p = 1.47, \\
p = 1.58, \\
p = 1.68, \\
p = 1.79, \\
p = 1.89
\end{align*}
Trapping of single solitons with pump spatial profile

Extra: Pump confinement