

The hidden-charm strong decays of the $Z_c$ states

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Inspired by BESIII’s measurement of the decay $Z_c(3900)^{±} \rightarrow \rho^± \eta_c$, we calculate the branching fraction ratio between the $\rho \eta_c$ and $\pi J/\psi$ decay modes for the charged states $Z_c(3900)$, $Z_c(4020)$ and $Z_c(4430)$ using a quark interchange model. Our results show that (i) the ratio $R_{Z_c(3900)}^{\rho \eta_c} / R_{Z_c(3900)}^{\pi J/\psi}$ is 1.3 and 1.6 in the molecular and tetraquark scenarios respectively, which is roughly consistent with the experimental data $R_{Z_c(3900)}^{\rho \eta_c} / R_{Z_c(3900)}^{\pi J/\psi} = 2.2 ± 0.9$.

TABLE I: The theoretical predictions of $R_{Z_c(3900)}$ in various models.

| Model         | Molecular | Tetraquark |
|---------------|-----------|------------|
| Experiment    | 0.046±0.025 [46] | 230±330 [46] |
|               | 1.78±0.41 [47]  | 0.27±0.40 [46] |
|               | 0.12 [48]     | 1.86±0.35 [44] |
|               | 0.070 [45]    | 1.28±0.37 [44] |
|               | 0.059 [51]    | 2.2 [51]    |
|               | 1.08±0.88 [49] | 0.95±0.40 [48] |
|               | 0.66 [35]     | 0.17 [50]   |

I. INTRODUCTION

Since 2003 the Belle Collaboration observed the first charmonium-like state $[1]$, $X(3872)$, an explosion in the observation of new hadronic states began. Dozens of charmonium-like states (or XYZ states) $[2]$ have been reported by several major experimental collaborations such as BESIII, LHCb, Belle, BaBar, CDF and so on; see Ref. $[3–8]$ for a review. The properties of the charged $Z_c$ states cannot be explained by the naive quark model and make them manifestly exotic. It seems that we have a new zoo of exotic hadrons. How to understand their internal structures remains a great challenge.

The charged charmonium-like state $Z_c(4430)$ $[9–14]$ was first observed in 2007, which has trigged extensive theoretical speculations, such as the molecular state $[15]$, the first radial tetraquark excitation $[16–18]$, threshold cusp effects $[19]$, and triangle singularities $[20]$. In 2003, $Z_c(3900)$ $[21–25]$ and $Z_c(4020)$ $[24]$ were observed. There are also many model-dependent interpretations of their inner structures, such as the $D^*(c)\bar{D}^*(c)$ molecular states $[25–50]$, the $S$-wave tetraquark states $[16,41–50]$, and the kinematical effects $[57–52]$. Very recently, the BESIII Collaboration reported the first evidence of the decay $Z_c(3900)^{±} \rightarrow \rho^± \eta_c$ with a statistical significance of 3.9σ in the $\pi^± \pi^± \eta_c$ final state $[43]$. The BESIII Collaboration also gave the ratio between the partial widths of the $\rho^± \eta_c$ and $\pi^± J/\psi$ decay modes at $\sqrt{s} = 4.226$ GeV $[43]$

$$R_{Z_c(3900)}^{\exp} = \frac{B(Z_c(3900)^{±} \rightarrow \rho^± \eta_c)}{B(Z_c(3900)^{±} \rightarrow \pi^± J/\psi)} = 2.2 ± 0.9.$$

Before the BESIII’s measurement $[43]$, the relative decay rate was predicted either in the molecular or tetraquark scenarios within the framework of a covariant quark model $[44]$, the phenomenological Lagrangian field theory $[45]$, the nonrelativistic effective field theory $[46]$, the light front model(LFM) $[47]$, QCD sum rules $[48–50]$, etc. Later, the author of Ref $[51]$ studied the decay properties of the $Z_c(3900)$ as a compact tetraquark state and a hadronic molecular state through the Fierz rearrangement of the Dirac and color indices. We collect the theoretical predictions in Table I which differ greatly.

In the present work we calculate the ratio between the $\rho \eta_c$ and $\pi J/\psi$ decay modes for the charged states $Z_c(3900)$, $Z_c(4020)$ and $Z_c(4430)$ in two different scenarios. In scenario I, we take the $Z_c(3900)$, $Z_c(4020)$ and $Z_c(4430)$ as the $D\bar{D}$, $D^*\bar{D}$ and $D(2S)\bar{D}$ molecular states with spin-parity $J^P = 1^+$, respectively. In scenario II, we treat the $Z_c(3900)$, $Z_c(4020)$ and $Z_c(4430)$ as the tetraquark states. Our results show that in the molecular and tetraquark scenarios, the ratios for $Z_c(3900)$ are $R_{Z_c(3900)}^{\rho \eta_c} ≃ 1.3$ and $R_{Z_c(3900)}^{\pi J/\psi} ≃ 1.6$, respectively, which are both in the range of experimental result, $R_{Z_c(3900)}^{\rho \eta_c} / R_{Z_c(3900)}^{\pi J/\psi} = 2.2 ± 0.9$. For $Z_c(4020)$, the ratio is about $R_{Z_c(4020)}^{\rho \eta_c} / R_{Z_c(4020)}^{\pi J/\psi} \sim 2$ in both two scenarios. As to the $Z_c(4430)$, we find that the ratio in the molecular scenario $R_{Z_c(4430)}^{\rho \eta_c} / R_{Z_c(4430)}^{\pi J/\psi} \sim 1.4$ is slightly smaller than that in the tetraquark scenario $R_{Z_c(4430)}^{\rho \eta_c} / R_{Z_c(4430)}^{\pi J/\psi} \sim (1.7 – 1.4))$.

We notice that the ratios $\frac{R_{Z_c(3900)}^{\rho \eta_c}}{R_{Z_c(4020)}^{\rho \eta_c}}$ and $\frac{R_{Z_c(3900)}^{\pi J/\psi}}{R_{Z_c(4020)}^{\pi J/\psi}}$ are about 12.5 and 24.2, respectively in the molecular sce-
nario. However both ratios are about 1.2 in the tetraquark scenario. In other words, these ratios are very sensitive to the underlying dynamics of Z_{c}(3900) and Z_{c}(4020), which may be helpful to pin down their inner structures.

This paper is organized as follows. In Sec. II, we give an introduction of the quark-exchange model and calculate the transition amplitudes in the molecular and tetraquark scenarios. Then we discuss and compare our results in the two scenarios in Sec. III. We give a short summary in Sec. IV.

II. MODEL INTRODUCTION

A. Decay width

For a four-quark state (F, for short) decaying into two particles labelled as C and D, the decay width in the rest frame of the initial particle has the form

$$d\Gamma = \frac{|\vec{p}_f|}{32\pi^2M^2}|\mathcal{M}(F \rightarrow C + D)|^2 d\Omega. \quad (2)$$

Here, M represents the mass of the initial four-quark state F; \(\vec{p}_f\) denotes the three-momentum of the meson C in the final state; \(\mathcal{M}(F \rightarrow C + D)\) is the transition amplitude of the two-body decay \(F \rightarrow C + D\), which is related to the T-matrix via

$$\mathcal{M}(F \rightarrow C + D) = -(2\pi)^{3/2} \sqrt{2E_C} \sqrt{2E_D} T, \quad (3)$$

where \(E_C\) and \(E_D\) denote the energy of the final mesons C and D, respectively. The T-matrix reads

$$T = \langle \psi_{CD}(\vec{p}_C)|V_{eff}(\vec{k}, \vec{p})|\psi_{AB}(\vec{k}) \rangle$$
$$= \langle \psi_{CD}(\vec{p}_C)|V_{eff}(\vec{k}, \vec{p})|\psi_{AB}(\vec{k}) \rangle. \quad (4)$$

Here, \(\psi_{CD}(\vec{p}_C)\) represents the relative spatial wave function between the final mesons C and D; \(\psi_{AB}(\vec{k})\) is the normalized relative spatial wave function between the constituent clusters A and B. In molecular scenario, the constituents represent mesons, while in tetraquark scenario the constituents represent the diquark \([cq]\) and antidiquark \([\bar{c}\bar{q}]\). \(V_{eff}(\vec{k}, \vec{p})\) denotes the effective potential, which is in the general case a function of the initial and final relative momentum \(\vec{k}\) and \(\vec{p}_C\).

The four-quark state may be a superposition of terms with different orbital angular momenta \(^4\). Thus, the relative spatial wave function in the momentum space has the form

$$\psi_{AB}(\vec{k}) = \sum_l R_l(k)Y_{lm}(\hat{k}). \quad (5)$$

Then, the Eq. (4) can be written as

$$T = \frac{1}{(2\pi)^3} \int d\vec{k} \int d\vec{p}\delta(\vec{p} - \vec{p}_C)V_{eff}(\vec{k}, \vec{p}) \sum_l R_l(k)Y_{lm}(\hat{k})$$

where

$$M_{ll} = \int P_l(\mu) d\mu \int d\vec{k} V_{eff}(\vec{k}, \vec{p}_C, \mu) R_l(k)k^2. \quad (7)$$

In this equation, \(P_l(\mu)\) is Legendre function and \(\mu\) represents the cosine of the angle between the momenta \(\vec{k}\) and \(\vec{p}_C\).

Finally, with the relativistic phase space, the decay width of two-body decay progress reads

$$\Gamma_{\pi J/\psi} = \frac{E_C E_D |\vec{p}_C|}{(2\pi)^3 M} |M_{ll}|^2. \quad (8)$$

For the \(\rho p_c\) decay mode, we further consider the decay width of the \(\rho\) meson, and get

$$\Gamma_{\rho p_c} = \frac{1}{N} \int ds \frac{E_C E_D |\vec{p}_C|}{(2\pi)^3 M} \frac{1}{(s - m_c^2)^2 + (m_\rho \Gamma_\rho)^2} \quad (9)$$

with

$$N = \int ds \frac{1}{(s - m_c^2)^2 + (m_\rho \Gamma_\rho)^2}. \quad (10)$$

Here, \(m_\rho\) and \(\Gamma_\rho\) stand for the mass and total decay width of the \(\rho\) meson, respectively. \(s\) denotes the square of the \(\rho\) meson invariant mass spectrum.

B. Effective potential

1. The molecular scenario

The \(J^P\) quantum number of the \(Z_c(3900), Z_c(4020)\) and \(Z_c(4430)\) is \(1^+\). In the molecular scenario, we treat them as the loosely bound \(S\)-wave \(DD^*, D^*D^*\) and \(D(2S)D^*\) molecular states according to their mass spectra, respectively. At Born order, the effective potential \(V_{eff}(\vec{k}, \vec{p}_C, \mu)\) is related to the reacting amplitude of the meson-meson scattering process,

$$A(12) + B(34) \rightarrow C(13) + D(24), \quad (11)$$

where 1(3) and 2(4) denote the c(\bar{c}) quark and \(\bar{q}_{\bar{q}}\) quark, respectively. In the quark interchange model \[^{[52,56]}\], the scattering Hamiltonian of the processes \(DD^*/D(2S) + D^*\) is estimated by the sum of the interactions between the inner quarks as illustrated in Fig. \[^{[4]}\]. Moreover, the short-range interactions are dominant in the scattering processes of two open-charmed mesons into a ground charmonium state plus a light-flavor meson. Thus, the scattering potential can be approximated by the one-gluon-exchange

\[^{1}\] We assume the orbital excitation is between the diquark and antidiquark.
(OGE) potential $V_{ij}$ at quark level

$$V_{ij} = \frac{\Lambda_i \Lambda_j}{2} \left\{ \frac{4\pi\alpha_s}{q^2} + \frac{6\pi b}{q^2} - \frac{8\pi\alpha_s}{3m_i m_j} s_i \cdot s_j e^{-\frac{q^2}{4\alpha_s}} \right\}, \quad (12)$$

where $\Lambda_i(\Lambda_j^T)$ represents the quark (antiquark) generator; $q$ is the transferred momentum; $b$ denotes the string tension; $\sigma$ is the range parameter in the hyperfine spin-spin interaction; $m_i (m_j)$ and $s_i (s_j)$ correspond to the interacting constituent quark mass and spin operator; $\alpha_s$ is the running coupling constant,

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \ln(A + Q^2/B^2)}. \quad (13)$$

In this equation, $Q^2$ is the square of the invariant masses of the interacting quarks. The parameters in Eqs. (12)-(13) are fitted by the mass spectra of the observed mesons $^{[57]}$, and their values are listed in Table II.

![Diagram](image)

**FIG. 1:** Diagrams for the scattering process $AB \rightarrow CD$ in the molecular scenario.

**TABLE II:** The parameters $^{[57]}$ used in the quark model.

| Parameter | Value |
|-----------|-------|
| $b$       | 0.18 GeV$^2$ |
| $\sigma$  | 0.897 GeV |
| $A$       | 10    |
| $B$       | 0.31 GeV |
| Constituent quark mass $m_u$ | 0.334 GeV |
| $m_c$     | 1.776 GeV |

In the quark model, the color-spin-flavor-space wave function for a meson is

$$\Psi = \omega_c \phi_f \chi_s \psi(\vec{p}) \quad (14)$$

where $\omega_c$, $\phi_f$, $\chi_s$, and $\psi(\vec{p})$ represent the wave functions in the color, flavor, spin, and momentum space, respectively. Here, the wave functions of the mesons are determined by fitting the mass spectra in the Godfrey-Isgur model $^{[58]}$.

According to the decomposition of the meson wave functions, the effective potential can be given as the product of the factors,

$$V_{\text{eff}}(\vec{k}, \vec{p}_c, \mu) = I_{\text{color}} I_{\text{flavor}} I_{\text{spin-space}}. \quad (15)$$

Here, $I$ with the subscripts color, flavor and spin-space represent the overlaps of the initial and final wave functions in the corresponding space. The color factor $I_{\text{color}}$ reads

$$I_{\text{color}} = \langle \omega_c^{(12)}(13) \omega_c^{(24)}(21) \rangle = \frac{\Lambda_i}{2} \frac{\Lambda_j}{2} \langle \omega_c^{(12)}(13) \omega_c^{(24)}(21) \rangle. \quad (16)$$

Its value in different diagrams in Fig. 1 is listed in Table III. For the flavor factor $I_{\text{flavor}}$, its value is simply unity for all diagrams considered in this paper.

**TABLE III:** The color factor $I_{\text{color}}$ within the molecular scenario.

| Diagram | $I_{\text{color}}$ |
|---------|-------------------|
| 12(C1)  | $-\frac{4}{3}$    |
| 14(T1)  | $\frac{4}{3}$     |
| 32(T2)  | $\frac{4}{3}$     |
| 34(C2)  | $-\frac{4}{3}$    |

For the $S$-wave decay process, the spin and space factors can be decoupled. The spin factor $I_{\text{spin}}$ reads

$$I_{\text{spin}} = \langle \chi_s^{(13)}(13) \chi_s^{(24)}(21) \rangle \langle \tilde{O}_s \chi_s^{(12)}(13) \chi_s^{(24)}(21) \rangle, \quad (17)$$

where $S(S')$ stands for the total spin of the initial(final) system. The spin operator $\tilde{O}_s$ equals to unitary for the Coulomb and linear interactions, and equals to $s_c \cdot s_f$ for the spin-spin interaction. We collect the values of color-spin factors $I_{\text{color}}, I_{\text{spin}}$ in Table IV.

**TABLE IV:** The values of color-spin factors for the diagrams $^{[1, T1, T2, C2]}$ within the molecular scenario. Here, $D^{*(0)} / D(2S)D^*$ is the shorthand for $D^{*(0)} / D(2S)D^* + c.c.$

| Initial state | Final state | Coul & linear | Hyperfine |
|--------------|-------------|---------------|-----------|
| $D\bar{D}$   | $\eta_c \rho$ | $\frac{2}{3}[-1, 1, 1, -1]$ | $\frac{1}{4}[3, -1, 3, -1]$ |
| $D^* \bar{D}$| $J/\psi\pi$  | $-\frac{2}{3}[-1, 1, 1, -1]$ | $\frac{-1}{16}[3, -1, 3, -1]$ |
| $D(2S) \bar{D}$ | $\eta_c \rho$ | $\frac{2}{3}[-1, 1, 1, -1]$ | $\frac{1}{16}[3, -1, 3, -1]$ |
|              | $J/\psi\pi$  | $-\frac{2}{3}[-1, 1, 1, -1]$ | $\frac{1}{16}[3, -1, 3, -1]$ |

As to the spatial factor $I_{\text{space}}$, its expression reads

$$I_{\text{space}}^{C1} = \int \int d\vec{q} d\vec{p}_c \psi_A(-\vec{q} - \vec{p}_3 - \vec{p}_c - f\tilde{k}) \psi_B(\vec{p}_3 + f\tilde{k})$$

2 The interactions in Eq. (12) are the Fourier transformation of the potential in Ref. $^{[57]}$. In the following, we perform our calculations in the momentum space for the purpose of simplification. The constant potential in the spatial space does not contribute due to the cancelation of the form factors and we just omitted the term in Eq. (13).
TABLE V: The corresponding values of the harmonic oscillator strength α between the constituent mesons A and B in the molecular scenario.

| \(\sqrt{\langle r_{\text{mean}} \rangle^2} \) (fm) | 1.0 | 1.2 | 1.5 | 1.7 | 2.0 | 2.4 | 3.0 |
|---|---|---|---|---|---|---|---|
| \(\alpha \) (GeV) | 0.21 | 0.18 | 0.16 | 0.14 | 0.12 | 0.10 | 0.08 |

2. The tetraquark scenario

For comparison, we further study the decays of the \(Z_c(3900)\), \(Z_c(4020)\) and \(Z_c(4430)\) as tetraquark states \(c\bar{c}q\bar{q}\) \(\text{[18]}\).

\[
Z_c(3900) : \quad \frac{1}{\sqrt{2}} \left\{ \left[ cu \right]_{\chi_3^+}^{1s_1} [c\bar{d}]_{\chi_3^0}^{1s_0} \right|_L^1 + \left[ cu \right]_{\chi_3^+}^{1s_1} [c\bar{d}]_{\chi_3^0}^{1s_0} \right|_L^1 \right\},
\]

\[
Z_c(4020) : \quad \frac{1}{\sqrt{2}} \left\{ \left[ cu \right]_{\chi_3^+}^{1s_1} [c\bar{d}]_{\chi_3^0}^{1s_0} \right|_L^1 + \left[ cu \right]_{\chi_3^+}^{1s_1} [c\bar{d}]_{\chi_3^0}^{1s_0} \right|_L^1 \right\}.
\]

where \(Z_c(4430)\) is interpreted as the first radial excitation of the \(Z_c(3900)\).

Similar to the molecular case, the \(V_{\text{eff}}(\bar{k}, \bar{p}_c, \mu)\) can be approximated by the interaction between the inner quarks, as shown in Fig. 2.

\[\text{FIG. 2: Diagrams for the scattering process } AB \rightarrow CD \text{ in the tetraquark scenario.}\]

The calculation of the \(V_{\text{eff}}(\bar{k}, \bar{p}_c, \mu)\) in the tetraquark scenario is similar to that in the molecular scenario. We can obtain the effective potential \(V_{\text{eff}}(\bar{k}, \bar{p}_c, \mu)\) with Eq. \(15\) as well. The flavor factor \(I_{\text{flavor}}\) and spin factor \(I_{\text{spin}}\) are the same as those in the molecular scenario. For the color factor \(I_{\text{color}}\), there is a difference between the two scenarios. In the molecular scenario, the initial four-quark state is composed of two mesons, of which the color configurations are \(1_c-1_c\). However, in the tetraquark scenario, the initial four-quark state is composed of diquark \([c\bar{q}]\) and antidiquark \([\bar{c}\bar{q}]\), of which the color configurations are \(3_c-3_c\). The difference in color configurations may result in quite different decay properties. The values of color-spin factors are collected in Table VI.

To calculate the space factor \(I_{\text{space}}\), we need the wave function of the initial tetraquark state,

\[
\psi(\vec{k}_c, \vec{k}_R, \vec{k}_X) = \psi_A(\vec{k}_c, \alpha_c) \psi_B(\vec{k}_R, \alpha_R) \psi_{AB}(\vec{k}_X, \alpha_X) \times \left[ \chi_{\chi_3^+} (cu) \chi_{\chi_3^0} (c\bar{d}) \right]_L^1 \left[ \phi_{\chi_3} (cu) \phi_{\chi_3} (c\bar{d}) \right]_L^1 \text{, (26)}
\]

where \(\vec{k}_{c/R}\) denotes the momentum between the \(c\bar{c}\) and \(u\bar{d}\) quarks in the diquark (antidiquark), and \(\vec{k}_X\) is the one between the diquark \([c\bar{q}]\) and antidiquark \([\bar{c}\bar{q}]\). The \(\alpha\) with the subscripts represents the oscillating parameter along the corresponding Jacobi coordinates.

For the \(Z_c(3900)\) and \(Z_c(4020)\), the spatial wave function \(\psi\) is estimated by the \(S\)-wave harmonic oscillating wave func-
TABLE VI: The values of the color-spin factors for the diagrams [C1, T1, T2, C2] within the tetraquark scenario.

| Initial state | Final state | Coul & linear | Hyperfine |
|---------------|-------------|---------------|-----------|
| $Z_c(3900)$ $\left[ [cu] \bar{S} = [\bar{c} \bar{d}] S = 1 \right]_{J = 1}$ | $\eta \rho$ | $\frac{1}{3} \sqrt{5} [1, 1, -1, 1]$ | $\left[ \frac{1}{4 \sqrt{5}}, -\frac{1}{4 \sqrt{5}}, \frac{1}{4 \sqrt{5}}, -\frac{1}{4 \sqrt{5}} \right]$ |
| $Z_c(4020)$ $\left[ [cu] \bar{S} = [\bar{c} \bar{d}] S = 1 \right]_{J = 1}$ | $J/\psi \pi$ | $\frac{2}{3} \sqrt{6} [1, 1, 1, 1]$ | $\left[ \frac{1}{6 \sqrt{6}}, -\frac{1}{6 \sqrt{6}}, \frac{1}{6 \sqrt{6}}, -\frac{1}{6 \sqrt{6}} \right]$ |
| $Z_c(4430)$ $\left[ [cu] \bar{S} = [\bar{c} \bar{d}] S = 1 \right]_{J = 1}$ | $J/\psi \pi$ | $-\frac{1}{3} \sqrt{5} [1, 1, -1, 1]$ | $\left[ \frac{1}{4 \sqrt{5}}, -\frac{1}{4 \sqrt{5}}, \frac{1}{4 \sqrt{5}}, -\frac{1}{4 \sqrt{5}} \right]$ |

TABLE VII: The rms of the tetraquark states $\eta \rho$ via imitating the wave function of the tetraquark state $Z_c$ is estimated by an $2S$-wave harmonic oscillating space-wave function $\psi(k, \alpha) = \frac{1}{\pi^{3/4} \alpha^{3/2}} \exp \left( -\frac{k^2}{2\alpha} \right)$. (27)

The $\alpha$ values are taken from Ref. [36], in which the authors presented a systematic study of the tetraquark states $[cu][\bar{c} \bar{d}]$ with the color flux-tube model, and predicted that the charged charmonium-like states $Z_c(3900)$ could be identified as the tetraquark state $[cu][\bar{c} \bar{d}]$ with the quantum numbers $1^3S_1$ and $J^P = 1^+$ as listed in Table VII. For $Z_c(4020)$, we give its wave function via imitating the wave function of $Z_c(3900)$, listed in Table VII as well. It should be remarked that the spin of the diquark $[cu]$ and antidiquark $[\bar{c} \bar{d}]$ both equal to unitary for $Z_c(4020)$.

TABLE VIII: The rms of the tetraquark states $[cu][\bar{c} \bar{d}]$, $\sqrt{\langle R^2 \rangle} (\sqrt{\langle \chi \rangle^2})$ denotes the distance between $c(\bar{c})$ and $u(\bar{u})$ quarks; $\sqrt{\langle X \rangle^2}$ is the distance between the diquark $[cu]$ and antidiquark $[\bar{c} \bar{d}]$; unit of rms is fm.

| States         | tetraquark $\sqrt{\langle R^2 \rangle}$ | $\sqrt{\langle \chi \rangle^2}$ | $\sqrt{\langle X \rangle^2}$ |
|----------------|----------------------------------------|---------------------------------|-----------------|
| $Z_c(3900)$ [36] | $\left[ [cu] \bar{S} = [\bar{c} \bar{d}] S = 1 \right]_{J = 1}$ | 0.90 | 0.90 | 0.48 |
| $Z_c(4020)$    | $\left[ [cu] \bar{S} = [\bar{c} \bar{d}] S = 1 \right]_{J = 1}$ | 0.90 | 0.90 | 0.48 |

As to $Z_c(4430)$, the spatial wave function of the diquark $[cu]$ is replaced by that of $D$, and the spatial wave function of anti-diquark $[\bar{c} \bar{d}]$ is replaced by that of $D^\ast$. The relative spatial wave function between the diquark $[cu]$ and antidiquark $[\bar{c} \bar{d}]$ is estimated by an $2S$-wave harmonic oscillating space-wave function $R_{10}(k) = \frac{\sqrt{\exp \frac{2k^2}{3\alpha_X}}}{\pi^{1/4} \alpha_X^{3/2}} \left( 1 - \frac{2k^2}{3\alpha_X} \right)$. (28)

The value of the harmonic oscillator strength $\alpha_X$ is related to the root mean square radius $r_X$ of the tetraquark state by $\sqrt{\alpha_X} = r_X$. We vary the $r_X$ in the range of $(0.5-2.0)$ fm and the corresponding value of $\alpha_X$ is listed in Table VIII.

III. RESULTS

Inspired by the recent measurement of the decay $Z_c(3900)^+ \rightarrow \rho^+ \eta_c$ by the BESIII Collaboration, we calculate the ratios between the $\rho \eta_c$ and $\pi J/\psi$ decay modes for the charged states $Z_c(3900)$, $Z_c(4020)$ and $Z_c(4430)$ in the molecular and tetraquark scenarios. Our results and theoretical predictions are presented as follows.

A. The molecular scenario

The mass of $Z_c(3900)$ ($M = 3886.6 \pm 2.4$ MeV) is slightly higher than the mass threshold of $DD^\ast (\sim 3872$ MeV). In the molecular scenario, we take the $Z_c(3900)$ as a $DD^\ast$ resonance molecular state, and calculate its branching fraction ratio between the $\rho \eta_c$ and $\pi J/\psi$ decay modes. Considering the uncertainty of the effective size for the molecular state, we plot the ratio as a function of the root mean square radius $r_{\text{mean}}$ in Fig. 5. The ratio is

$$R_{Z_c(3900)}^{\rho \eta_c} \sim 1.3,$$

which roughly accords with the experiment result $R_{Z_c(3900)}^{\exp} = 2.2 \pm 0.9$ [35] within errors. Meanwhile the ratio is insensitive to $r_{\text{mean}}$ in the range of $(1.0-3.0)$ fm we considered in this work.

With the estimated relative spatial wave function as illustrated in Eq. (24), we further obtain the partial widths of the $\eta \rho$ and $J/\psi \pi$ decay modes and show them in Fig. 4. It is obvious that the partial widths are sensitive to $r_{\text{mean}}$ and vary from one MeV to $O(10^{-2})$ MeV. With $r_{\text{mean}}$ increasing, the partial decay widths become smaller, or even close to zero. This can be easily understood since the larger $r_{\text{mean}}$ means the freer mesons $A$ and $B$. It is more difficult to interact with each
TABLE IX: The partial decay widths (MeV) for the \( Z^{(3900)} \), \( Z^{(4020)} \) and \( Z^{(4430)} \) as the \( D\bar{D}^* \), \( D^*\bar{D} \) and \( (2S)\bar{D}^* \) molecular states, respectively. Their masses are fixed respectively on physical masses, namely 3886.6 MeV, 4024.1 MeV and 4478 MeV.

For \( Z^{(4020)} \), we take it as the \( S \)-wave \( D^*\bar{D}^* \) resonance molecular state since its mass \((M=4024.1 \text{ MeV})\) is slightly about 10 MeV higher than the mass threshold of the \( D^*\bar{D}^* \). With the molecular size varying in the range of \( r_{\text{mean}}=(1.0\sim3.0) \text{ fm} \), we calculate its partial decay width ratio between the \( \eta_\rho \) and \( J/\psi \pi \) modes, and obtain

\[
R^{\text{th}}_{Z^{(4020)}} \sim (2.7 \sim 2.3) \text{,}
\]

with the mass being \( M=4024.1 \text{ MeV} \) (see Fig.3). This value is almost independent of \( r_{\text{mean}} \) we considered in the present work.

We also plot the partial decay widths of the \( \eta_\rho \) and \( J/\psi \pi \) modes versus the molecular size \( r_{\text{mean}} \) in Fig.4. In the figure, we find that the partial widths are about \( O(10^{-1} \sim 10^{-2}) \) MeV, and strongly dependent on \( r_{\text{mean}} \). Fixing \( r_{\text{mean}} \approx 1.37 \) fm estimated by Eq. (30), we obtain

\[
\begin{align*}
\Gamma[Z^{(4020)} \to \eta_\rho] & \sim 0.22 \text{ MeV} , \\
\Gamma[Z^{(4020)} \to J/\psi \pi] & \sim 0.09 \text{ MeV}. 
\end{align*}
\]

The predicted branching ratios are

\[
\begin{align*}
B[Z^{(4020)} \to \eta_\rho] & \sim 1.7\% , \\
B[Z^{(4020)} \to J/\psi \pi] & \sim 0.7\%. 
\end{align*}
\]

which are quite small.

The partial widths of the \( \eta_\rho \) and \( J/\psi \pi \) decay modes for \( Z^{(4020)} \) are smaller than those for \( Z^{(3900)} \).

\[
\begin{align*}
\Gamma[Z^{(3900)} \to \eta_\rho] & \sim 12.5 , \\
\Gamma[Z^{(4020)} \to \eta_\rho] & \sim 24.2. 
\end{align*}
\]

This indicates that the couplings of the \( D\bar{D}^* \) to the \( \eta_\rho \) and \( J/\psi \pi \) channels are stronger than those of the \( D^*\bar{D}^* \). The main difference between the \( D\bar{D}^* \) and \( D^*\bar{D}^* \) is the spin wave function. Our results show that in the molecular scenario, different spin-spin coupling may have a great impact on the strong decay properties. We take the \( J/\psi \pi \) decay mode as an example. In Table IV, the spin factor for the coupling with the \( D\bar{D}^* \) is three times larger than that of the \( D^*\bar{D}^* \) in Fig.1 C1 and Fig.1 T1. The hyperfine interaction is expected to be more important for the \( J/\psi \pi \) decay mode of \( Z^{(3900)} \). Moreover, our calculation shows that the hyperfine interaction for the \( Z^{(3900)} \) plays a quite important role in Fig.1 C1 and even change the sign of its amplitude.

As to \( Z^{(4430)} \), in molecular scenario, we take it as an \( S \)-wave \( D(2S)\bar{D}^* \) molecular state. Similarly we change the size of the molecular state in the range of \( r_{\text{mean}}=(1.0\sim3.0) \text{ fm} \), and obtain

\[
R^{\text{th}}_{Z^{(4430)}} \sim (1.4 \sim 1.3) ,
\]

for \( Z^{(4430)} \) with a mass of \( M=4478 \text{ MeV} \) (see Fig.3). Meanwhile, the partial widths of the \( \eta_\rho \) and \( J/\psi \pi \) modes as the function of \( r_{\text{mean}} \) for \( Z^{(4430)} \) are shown in Fig.4 as well. According to the figure, the decay properties of the \( D(2S)\bar{D}^* \) molecular state are similar to the \( D^*\bar{D}^* \) molecular state.

Fixing \( r_{\text{mean}} \approx 1.00 \text{ fm} \), we further obtain

\[
\begin{align*}
\Gamma[Z^{(4430)} \to \eta_\rho] & \sim 1.28 \text{ MeV} , \\
\Gamma[Z^{(4430)} \to J/\psi \pi] & \sim 0.94 \text{ MeV}. 
\end{align*}
\]

At present, the charged state \( Z^{(4430)} \) was observed both in the \( \psi' \pi^\pm \) and \( J/\psi \pi^\pm \) channels [9,14], and has not been reported in the \( \eta_\rho \) channel. According to our theoretical predictions, if \( Z^{(4430)} \) is a \( D(2S)\bar{D}^* \) molecular state, the partial width of \( \eta_\rho \) is comparable to that of \( J/\psi \pi \), which indicates this state may be observed in the \( \eta_\rho \) channel as well.

So far, we have obtained the decay ratios in the molecular scenario. We find that the ratios are not sensitive to the relative...
molecular wave function in the loosely bound system while the partial decay widths are very sensitive to the size of the molecules because of the sensitivity of the effective potentials.

### B. The tetraquark scenario

In the tetraquark scenario, we obtain the decay ratio for the $Z_c(3900)$ state

$$R_{Z_c(3900)}^{th} \sim 1.6, \quad (39)$$

which agrees with the experimental result (see table X).

**TABLE X:** The partial decay widths (MeV) for the $Z_c(3900)$ and $Z_c(4020)$ as the tetraquark states. $R^{th}$ and $R^{exp}$ are the theoretical and experimental ratios, respectively.

| state       | $\Gamma[\eta_c \rho]$ | $\Gamma[J/\psi \pi]$ | $R^{th}$ | $R^{exp}$ |
|-------------|------------------------|------------------------|----------|----------|
| $Z_c(3900)$ | 0.23                   | 0.14                   | 1.6      | 2.2 ± 0.9 |
| $Z_c(4020)$ | 0.19                   | 0.12                   | 1.6      | ...      |

The predicted partial decay widths of the $\eta_c \rho$ and $J/\psi \pi$ modes are

$$\Gamma[Z_c(3900) \rightarrow \eta_c \rho] \sim 0.23 \text{ MeV},$$
$$\Gamma[Z_c(3900) \rightarrow J/\psi \pi] \sim 0.14 \text{ MeV}. \quad (40)$$

Via imitating the wave function of $Z_c(3900)$, we estimate the wave function of $Z_c(4020)$ as listed in Table VII. Similarly we fix the mass of $Z_c(4020)$ at $M = 4024.1 \text{ MeV}$ and obtain

$$\Gamma[Z_c(4020) \rightarrow \eta_c \rho] \sim 0.19 \text{ MeV},$$
$$\Gamma[Z_c(4020) \rightarrow J/\psi \pi] \sim 0.12 \text{ MeV}. \quad (41)$$

Then the predicted partial decay widths ratio is

$$R_{Z_c(4020)}^{th} \sim 1.6. \quad (42)$$

The decay properties of $Z_c(4020)$ are similar in the molecular and tetraquark scenarios. Thus, besides the decay ratios, more precise experimental information is required to pin down the inner structure of this state.

As shown in table [X] the partial widths of the $\eta_c \rho$ and $J/\psi \pi$ decay modes for $Z_c(4020)$ are comparable to those for $Z_c(3900),$

$$\frac{\Gamma[Z_c(3900) \rightarrow \eta_c \rho]}{\Gamma[Z_c(4020) \rightarrow \eta_c \rho]} = 1.2, \quad (43)$$
$$\frac{\Gamma[Z_c(3900) \rightarrow J/\psi \pi]}{\Gamma[Z_c(4020) \rightarrow J/\psi \pi]} = 1.2. \quad (44)$$

The ratios are very different from those in Eqs. (35)-(36). As mentioned earlier, in the molecular scenario the hyperfine interaction plays a quite important role for $Z_c(3900)$ in Fig.2-C1 and even changes the sign of its amplitude. Thus the total amplitudes for $Z_c(3900)$ are much larger than those for $Z_c(4020)$. However, in the tetraquark scenario the Coulomb and linear interactions are dominant for both states $Z_c(3900)$ and $Z_c(4020)$. There exists good evidence for $Z_c(3900)$ in the $\eta_c \rho$ and $J/\psi \pi$ channels experimentally and no evidence for $Z_c(4020)$. Our results support that the two states are more likely to be the molecular states.

**FIG. 5:** Left (a): the branching fraction ratio between $\eta_c \rho$ and $J/\psi \pi$ for $Z_c(4430)$ in tetraquark scenario. Right (b): the partial decay widths for the $Z_c(4430)$ decaying into the $\eta_c \rho$ and $J/\psi \pi$ channels in tetraquark scenario.

For the $Z_c(4430)$, with the estimated wave function we plot the partial decay width ratio between the $\eta_c \rho$ and $J/\psi \pi$ decay modes as a function of the effective size $r_X$ of the tetraquark state (see Fig. [5]). We find that the ratio slightly depends on $r_X$. Varying the $r_X$ in the range $r_X = (0.5 \sim 2.0) \text{ fm}$, the ratio is

$$R_{Z_c(4430)}^{th} \sim (1.7 \sim 1.4), \quad (45)$$
which is slightly larger than that as a molecular state. According to our results, the branching fraction ratio between the $\eta_\rho$ and $J/\psi\pi$ modes of the $Z_c(4430)$ as a molecule or a tetraquark state is larger than one, which indicates that the $Z_c(4430)$ is more easier to decay into the $\eta_\rho$ channel.

So far, we have calculated the decay ratios of the $Z_c(3900)$, $Z_c(4020)$ and $Z_c(4430)$ decaying into the $\eta_\rho$ and $J/\psi\pi$ channels in the molecular and tetraquark scenarios. Our results show that the decay ratios in both scenarios are similar to each other. We cannot determine the inner structures only with the decay ratios. However, if we look at the partial decay widths, we find that in molecular scenario, the $\Gamma(Z_c(3900)) \rightarrow J/\psi\pi$ are much larger than the $\Gamma(Z_c(4020)) \rightarrow J/\psi\pi$, while they are similar in the tetraquark scenario. In experiments, the $Z_c(3900)$ state is observed in the $J/\psi\pi$ invariant mass spectrum while no significant $Z_c(4020)$ signal is observed. This may support the states $Z_c(3900)$ and $Z_c(4020)$ as the molecules instead of tightly bound tetraquark states.

IV. SUMMARY

In the present work, we calculate the branching fraction ratios between the $\eta_\rho$ and $J/\psi\pi$ decay modes for the charged states $Z_c(3900)$, $Z_c(4020)$ and $Z_c(4430)$ with a quark interchange model. In order to compare the decay properties in different physical scenarios and pin down the inner structure of these three mysterious charmonium-like states, we study the ratios in the molecular and tetraquark scenarios, respectively. Meanwhile, we estimate the absolute partial decay widths for the $\eta_\rho$ and $J/\psi\pi$ decay channels. Our main results are summarized as follows.

For $Z_c(4430)$, the branching fraction ratio as an S-wave $D(2S)\bar{D}^*$ molecule ($R^{th}_{Z_c(4430)} \simeq 1.4$) is slightly smaller than that in the tetraquark scenario ($R^{th}_{Z_c(4430)} \simeq 1.7 \sim 1.4$). We notice that the ratios in both two physical scenarios are larger than 1, which indicates that the $Z_c(4430)$ prefers to decay into the $\rho_\eta$ channel rather than the $\pi J/\psi$ channel. Besides the $\pi J/\psi$ channel, the $\rho_\eta$ may be another interesting channel for the observation of $Z_c(4430)$ in future experiments.

For $Z_c(3900)$, we obtain that the ratios are $R^{th}_{Z_c(3900)} \sim 1.3$ and 1.6 in the molecular and tetraquark scenarios, respectively. Both are comparable with the experimental result. For $Z_c(4020)$, the ratios are $R^{th}_{Z_c(4020)} \sim 2.4$ and 1.6, respectively. The above results show that the ratios in both scenarios are similar to each other. Thus, to investigate the inner structures, considering only the decay ratio $R$ of the $Z_c$ itself is not enough.

In the molecular scenario, the partial decay widths of the $\eta_\rho$ and $J/\psi\pi$ modes for $Z_c(4020)(D^*\bar{D}')$ are smaller than those for $Z_c(3900)(D\bar{D}')$ by one order. In the molecular scenario, different spin-spin coupling may have a great impact on the strong decay properties. On the other side, the partial decay widths of the $\eta_\rho$ and $J/\psi\pi$ modes for $Z_c(4430)(D^*\bar{D}')$ are comparable to those for $Z_c(3900)$ in the tetraquark scenario. At present, there exists good evidence for $Z_c(3900)$ in the $\eta_\rho$ and $J/\psi\pi$ channels experimentally and no evidence for $Z_c(4020)$. Our results indicate that these two states are more likely to be the molecule-like states which arise from the $D(\pi)\bar{D}(\pi)$ hadronic interactions.

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