OPTIMIZING MULTI-OBJECTIVE DECISION MAKING HAVING QUALITATIVE EVALUATION

HAMED FAZLOLLAHTABAR
PhD. Student of Industrial Engineering
Iran University of Science and Technology, Tehran, Iran

MOHAMMAD SAIDI-MEHRAJAD
Faculty of Industrial Engineering, Iran University of Science and Technology
Tehran, Iran

(Communicated by Hussein A. Abbass)

Abstract. We develop a ranking process for multi-objective decision making. For optimizing the multi-objective problem having both quantitative and qualitative objectives, weight assessment is important to convert the problem into the corresponding single objective problem. Therefore, a ranking process is proposed to simultaneously obtain the objective weights and the evaluation of alternatives with multiple objectives. Several new concepts are developed to handle the dynamism in distance computation and ranking of decisions in a multi-objective model having qualitative evaluations. The proposed process is illustrated in a numerical example.

1. Introduction. Mathematical modelling is a powerful technique to solve multi-objective decision making problems. The essentials of the approaches in the literature (Hussein and Abo-Sinna, 1993 and 1995; Kacprzyk, 1997, Kacprzyk and Esogbue, 1996; Kacprzyk and Sugianto, 1998) are usually converting the multi-objective problem into a single-objective problem, by an aggregate of hybrid objective values that expresses the performance of the particular stage decision. This aggregation may take on various forms from a pessimistic minimum to an optimistic maximum, through all intermediate cases exemplified by an average, for example, appropriate aggregation operators in (Kacprzyk and Esogbue, 1996; Kacprzyk and Sugianto, 1998) were developed to aggregate objective and subjective aspects for solving a socioeconomic regional planning problem.

Human factors play a very important part in virtually all real-life problems, so the weight of objective is one of the primary components and a significant parameter to express decision makers knowledge, experience and judgment preference in MODM. From a structural viewpoint, the weight-assessing methods can be categorized into two types: subjective methods, such as, AHP and Delphi, and objective methods, such as the extreme weight approach, random weight approach and entropy method. For the first, the value of weight is highly dependent on decision-maker experience and judgments, so the methods are with strong subjectivity; for the second, the
weight is computed from the outcomes without asking the perceptions of the decision makers. For multi-objective decision making problems, the weight assessment is more complicated due to the changing situations at each stage. It is difficult, even impossible, for decision makers to immediately provide precise values for objective weights, especially for some real time decision problems, for example, real time flood control or automated guided vehicle operation problems. To overcome the above weaknesses, the weights for decision problems can be initially provided by an objective method and then adjusted according to decision situation changes at different circumstances. A convenient weight-assessing approach should be developed to obtain the initial weights in the dynamic process of decision makings.

The motivations to conduct this study are listed below:

- Several works are done in the past to handle homogenous multi-objective problems where objectives were either quantitative or qualitative, but in this study we work out a model to handle both quantitative and qualitative objectives;
- In the current methods different objectives are integrated using initial solutions, but in this work the concept of intensity for each objective is introduced;
- In past studies, to model uncertainty in MODM fuzzy concepts were developed but here using the cross entropy approach uncertainty and dynamism of real time changes of qualitative objectives are considered;
- In other researches, preference or utility of objectives were effective in the priority of the objectives, but this work considers weighing, preference and ranking of objectives at the same time in a computational mechanism.

This paper aims to evaluate multiple hybrid objectives of the alternatives and provide the initial objective weights at the same time in the dynamic decision making process. The main work is devoted to present an integrated computational mechanism to handle different qualitative and quantitative objectives based on severity, preference and ranks of objectives. Also, dynamism and uncertainty of the environment is considered by cross entropy approach.

The remainder of our work is organized as follows. Next, we review the related literature. In Section 3, a ranking process for MODM is developed after performing an optimization procedure, in which the initial objective weights are elicited from the information implicit in the relative severity degrees of alternatives. In Section 4, we test the proposed MODM methodology in a numerical example for applicability and effectiveness purposes. We conclude in Section 5.

2. Literature review. The multi-objective decision-making discipline (e.g., Chan-kong and Haimes, 2008), focuses on the aspects of the goals and objectives that are measurable. Ehrgott and Gandibleaux (2002) reviewed methods of multi-objective combinatorial optimization, with a focus on optimizing and balancing among portfolios with quantified objectives. For reasons of tractability, typical applications of MODM analyze two objectives at a time. For example, Badri et al. (1998) and Hodgson et al. (1997) emphasized selection of the two most important conflicting quantified objectives. Sanchez et al. (2005) developed a model for representing and aggregating quantitative and qualitative (ordinal) objective functions in project selection. The conditions that the aggregation function must satisfy were also discussed. Frohwein et al. (1999) presented an MODM analysis enabling the graphical comparison of projects and provided information on the tradeoff that comes with the combinatorial selection of the projects. Lambert et al. (2003) developed an MODM decision aid specialized for resource allocation in highway guardrails.
Other MODM applications in transportation and infrastructure investments were described by Lambert et al. (2001) and Tsang et al. (2002).

Niemeier et al. (1995) compared five optimization models constructed for selecting an optimal subset of projects for a statewide transportation programming process. Each of the models built on a basic linear programming formulation in which a maximization of benefits and minimization of costs was pursued. Lee (2000) described that cost-benefit analysis is frequently used in decisions for resource allocation to portfolios of dissimilar projects. Golabi (1987) used a utility function for evaluating and selecting a group of dissimilar research proposals with an analytical solution using integer programming. The work assessed the merits and technical qualities using measures of effectiveness. Utility and value functions including multi-attribute utility functions (DeGroot, 1970) are in principle able to account for difficult-to-quantify aspects of portfolio performance.

Shang et al. (2004) used the analytical network process (ANP) with scoring and weighting functions of transportation projects. Meade and Presley (2002) used the analytic network process to support the selection of projects in a research and development environment. The analytic network process and similar weighting approaches find a middle ground between multi-objective analysis and utility theory, but are widely criticized for the absence of fundamental principles in the construction of their value functions. Furthermore, the analytic network process may not be amenable to handling combinatorial decision spaces such as those that occur in resource allocation. Lambert et al. (2005, 2006, 2007) explored solutions to MODM problems in part developing graphical decision aids that elicit tradeoffs among costs, benefits, and risks in the choices among project portfolios. In particular, a relationship between the allocated resources and the broad and conceptual system goals was illuminated, complementing the traditional approaches of multi-objective optimization involving quantified objectives. Lambert et al. (2005, 2006, 2007) stopped short of formalizing the developed approach.

Chen and Fu (2005) developed a fuzzy dynamic programming approach for multi-objective decision making problem. Fuzzy dynamic programming usually converts the uncertainties in the problems into corresponding deterministic one by an aggregate of hybrid objective values that expresses the performance of the particular stage decision.

Data Envelopment Analysis (DEA) is a managerial powerful tool to evaluate the relative efficiency of each decision making unit (DMU). Nowadays, multi-objective DEA models in static environment are an attractive technique for evaluation quantity and quality aspects of performance analysis because there is some weakness in single objective DEA such as one-dimensional performance analysis and also it is important to consider the decision maker(s) preference over the potential adjustments of various inputs and outputs when DEA is employed. Jafarian-Moghaddam and Ghoseiri (2011) presented a fuzzy dynamic multi-objective DEA model in which data were changing sequentially.

In the process of multi-criteria decision support system, people usually rank the alternatives or select a best alternative according to evaluation criteria of alternatives. However, an engineering or management decision information is often vague, imprecise, and uncertain, by nature. Designers usually present many alternatives in designing phase. The subjective characteristics of the alternatives are generally uncertain and need to be evaluated based on the decision-makers insufficient knowledge and judgments. The nature of this vagueness and uncertainty is fuzzy, rather
than random, especially when subjective assessments are involved in the decision-making process. Fuzzy set theory offers a possibility for handling these sorts of information involving the subjective characteristics of human nature in the multicriteria decision-making process. The Limitations and deficiencies of existing score functions of intuitionistic fuzzy set are analyzed by Wang and Li (2011). Based on the two theories of intuitionistic fuzzy set and cross entropy, with the adoption of cross entropy of the degree of membership from the degree of non-membership handling the effect of hesitancy degree, two improved methods handling multi-criteria fuzzy decision-making problem were provided.

Guikema and Milke (2003) proposed a multi-attribute optimization model for infrastructure systems projects selection that was based on a combination of multi-attribute utility theory, mixed-integer optimization, and sensitivity analysis. Other notable efforts in multiple criteria decision-making and multi-objective optimization methods and their applications in engineering systems domain were Dicdican and Haimes (2005), Briggs and Little (2008), Daniels et al. (2001), Parnell et al. (2001), and Weisbin et al. (2004).

3. **A weighted ranking process for MODM.** In a multi-objective decision system, there is a finite decision (alternative and not a solution) set \( D = (d_1, d_2, \ldots, d_n) \) consisting of \( n \) candidate decisions, and each alternative is described by the objective set \( O = (o_1, o_2, \ldots, o_m) \) involving \( m \) qualitative and quantitative objectives. Selection of the most satisfying decision is limited to the decision set, the relative severity degree matrix of hybrid objectives should be computed for the global evaluation of decisions. Note that, severity degree is the occurrence intensity of a decision for an objective and it is in \((0,1)\) interval. Let's supposing

\[
S = (S_{ij})_{m \times n} \quad (1)
\]

where \( S_{ij} \) is the relative severity degree of decision \( d_j \) with respect to objective \( o_i \), \( i = 1,2,\ldots,m \) and \( j = 1,2,\ldots,n \). Though optimal and non-optimal are two opposite concepts, there is no absolute distinction but an intermediary transition between them, especially for the problems of MODM with limited decisions. Therefore, the decision space \( (D.S) \) is hypothetically categorized into \( c \) rankings in an ordinal way starting from those including the most preferred decisions to those including the least preferred decisions. The partition vector is supposed to be

\[
P = (P_1, P_2, \ldots, P_c) \quad (2)
\]

where \( p_h \) \((h = 1,2,\ldots,c)\) actually represents the relative severity degree of the center of ranking \( h \). For simplicity, the relative severity degrees of rankings from 1 to \( c \) are assumed to decrease linearly from 1 to 0, so the normalized severity membership vector is expressed as,

\[
P = (1, \frac{c-2}{c-1}, \ldots, \frac{c-h}{c-1}, \ldots, 0), h = 1,2,\ldots,c. \quad (3)
\]

In order to select the most satisfying decision among \( n \) decisions, we need to estimate the relative severity degree vector of each decision belonging to each ranking, supposing

\[
\alpha_j = (\alpha_{1j}, \alpha_{2j}, \ldots, \alpha_{cj}), \quad (4)
\]

s.t.

\[
\sum_{h=1}^{c} \alpha_{hj} = 1, a_{hj} \in [0,1] \quad (5)
\]
where $\alpha_{hj}$ describes the relative severity degree of decision $d_j$ belonging to ranking $h$, $h = 1, 2, ..., c$ and $j = 1, 2, ..., n$.

Generally speaking, different objectives are of different weight importance, the weight vector is denoted by

$$w = (w_1, w_2, ..., w_m), \sum_{i=1}^{m} w_i = 1, w_i \geq 0$$ (6)

According to pattern recognition, the optimal relative severity degree of each decision can be obtained by minimizing the sum of its squared distances to ranking centers. Due to stochastic nature of the decisions and the real-time concept of data involved in multi-objective decision making leading to dynamic changes in ranking centers, here cross entropy weighted distance is used to represent the distance between decision $d_j$ to ranking $h$.

$$dis_{hj} = \left\{ \sum_{i=1}^{m} \left[ w_i (S_{ij} - P_h) \right]^2 \right\}^{\frac{1}{2}}$$ (7)

As stated the severity degree of alternatives with respect to objectives is considered stochastic and the rankings are also stochastic. Therefore the term $(s_{ij} - p_{ij})$ should be computed using a stochastic methodology being considered here as cross entropy (CE).

3.1. Cross entropy for distance computation. Let $X = (X_1, ..., X_n)$ be a random vector taking values in some space $\chi$. Let $(f(\cdot; \nu))$ be a family of probability density functions (pdfs) on $\chi$, with respect to some base measure $\mu$, where $\nu$ is a real-valued parameter (vector). Thus,

$$EH(X) = \int_{\chi} H(X) f(x; \nu) \mu(dx),$$ (8)

for any function $H$. Note that $E$ stands for expected value. For simplicity $\mu(dx)$ is considered to be $d_x$. Let $S$ be some real function on $\chi$. Suppose we are interested in the probability that $S(X)$ is greater than or equal to some real number $\xi$ under $f(\cdot; \mu)$. This probability can be expressed as

$$l = P_u(S(X) \geq \xi) = E_u I_{S(X) \geq \xi},$$ (9)

where $I$ is an indicator random variable. A straightforward way to estimate $l$ is to use Monte Carlo simulation. Draw a random sample $X_1, ..., X_n$ from $f(\cdot; u)$; then

$$\frac{1}{N} \sum_{i=1}^{N} I_{S(X_i) \geq \xi},$$ (10)

is an unbiased estimator of $l$. This way is a time consuming simulation effort. An alternative way is based on importance sampling (Smith et al., 1997; Srinivasan, 2002). Take a random sample $X_1, ..., X_n$ from an importance sampling density $g$ (as relative severity $s_{ij}$) on $\chi$, and estimate $l$ using the likelihood ratio estimator.

$$l = \frac{1}{N} \sum_{i=1}^{N} I_{S(X_i) \geq \xi} \frac{f(X_i; u)}{g(X_i)}.$$ (11)

The best way to estimate $l$ is to use change of measure with density

$$g^*(x) = \frac{I_{S(X) \geq \xi} f(X; u)}{l}$$ (12)
Therefore, we obtain from (11)
\[ I_{S(X_i;\geq\xi)} \frac{f(X_i;u)}{g^*(x_i)} = l, \text{ for all } i. \] (13)

\( l \) is a constant value with zero variance. Here, the problem is that \( g^* \) (optimal severity) depends on the unknown parameter \( l \). It is convenient to choose a \( g \) in the family of densities \( f(.;\nu) \). Now, we need to choose the reference parameter \( \nu \) such that the distance between the density \( g^* \) in (12) and \( f(.;\nu) \) is minimal. A usual measure of distance between two densities, say \( g \) (as severity \( s_{ij} \)) and \( h \) (degree of ranking \( p \)), is the Kullback-Leibler distance (Kullback and Leibler, 1951; Kullback, 1959) or the cross entropy between \( p \) and \( h \). The Kullback-Leibler distance is defined as,
\[
D(g,h) = E_g \ln \left( \frac{g(x)}{h(x)} \right) = \int g(x) \ln g(x) dx - \int g(x) \ln h(x) dx.
\] (14)

Minimizing the Kullback-Leibler distance between \( g^* \) in (12) and \( f(.;\nu) \) is equivalent to choosing \( \chi \) such that \(-\int g^*(x) \ln f(x,v) dx\) is minimized. which is solving the following maximization problem,
\[
\max_v \int g^*(x) \ln f(x,v) dx,
\] (15)

Substituting \( g^* \) from (12) into (15) we obtain the maximization program
\[
\max_v \int \frac{I_{S(X;\geq\xi)}}{l} \ln f(x,v) dx,
\] (16)

which is,
\[
\max_v D(x) = \max_v E(u) I_{S(X;\geq\xi)} \ln f(x,v).
\] (17)

Again, using importance sampling with a change of measure \( f(.;w) \) we can rewrite (17) as
\[
\max_v D(v) = \max_v E_w I_{S(X;\geq\xi)} W(X;u,w) \ln f(x,v),
\] (18)

for any reference parameter \( w \), where,
\[
W(x;u,w) = \frac{f(x,u)}{f(x,w)},
\] (19)

Is the likelihood ratio at \( x \), between \( f(.;u) \) and \( f(.;w) \). The optimal solution of (18) can be written as:
\[
\nu^* = \arg\max_v E_w I_{S(X;\geq\xi)} W(X;u,w) \ln f(X;\nu),
\] (20)

We can solve \( v^* \) by solving the following stochastic program
\[
\max_v D(v) = \max_v \frac{1}{N} \sum_{i=1}^{N} I_{S(X_i;\geq\xi)} W(X_i;u,w) \ln f(X_i;\nu). \]
(21)

where \( X_1, ..., X_n \) is a random sample from \( f(.;w) \).

This way we obtain an estimated probability for the distance between decision and the ranking centers. A substantial advantage of the proposed CE algorithm is to be capable for data collections without any known probability distribution function. The following algorithm is composed to facilitate the CE computations:

Algorithm 1: Cross entropy method for computing distance between relative severity and degree of ranking.
Step 1. Define $\nu_0 = u$. Set $t = 1$ (iteration)

Step 2. Generate a sample $X_1, ..., X_n$ from density $f(\cdot; \nu_{t-1})$ and compute the sample $(1-q)$-quantile $\xi_t$ of the performances according to $\xi_t = S_{(1-q)N}$, provided $\xi_t$ is less than $\xi$. Otherwise set $\xi_t = \xi$. Note that,

\[ P_{\nu_{t-1}}(S(X) \geq \xi_t) \geq q, \]

\[ P_{\nu_{t-1}}(S(X) \leq \xi_t) \geq 1 - q. \]

Step 3. Use the same sample $X_1, ..., X_n$ to solve the stochastic program

\[ \max_{\nu} D(\nu) = \max_{\nu} \frac{1}{N} \sum_{i=1}^{N} I_{S(X_i) \geq \xi W(X_i; u, \nu_T^0) \ln f(X_i; \nu)}. \]

Denote the solution by $\nu_t$.

Step 4. If $\xi_t < \xi$, set $t = t + 1$ and reiterate from Step 2. Else proceed with Step 5.

Step 5. Estimate probability $l$ using likelihood ratio estimate

\[ l^* = \frac{1}{N} \sum_{i=1}^{N} I_{S(X_i) \geq \xi W(X_i; u, \nu_T^0)}, \]

where $T$ denotes the final number of iterations.

After computing the $(s_{ij} - ph$ stochastic distance, from now on we call it dis.CE, and employ it in the remainder of computations.

On the other hand, $\alpha_{hj}$ can be considered as weight for the above distance $dis_{hj}$, and multiplying $dis_{hj}$ by $\alpha_{hj}$ can result in a more complete description of the difference between alternative $d_j$ to ranking $h$, therefore, Eq. (7) can be rewritten as

\[ DIS_{hj} = \alpha_{hj} \left( \sum_{i=1}^{m} [w_i (dis.CE)]^2 \right)^{\frac{1}{2}}. \]

(22)

and the distance of alternative $d_j$ to all $c$ rankings is further computed by

\[ f_j(\alpha_j, w) = \sum_{h=1}^{c} DIS_{hj}^2 = \sum_{h=1}^{c} \alpha_{hj}^2 \sum_{i=1}^{m} [w_i (dis.CE)]^2, \]

(23)

where $f(\Lambda, W) = (f_1(\alpha_1, w), f_2(\alpha_2, w), ..., f_n(\alpha_n, w))$.

3.2. Optimal estimation of severity degree. To get the optimal estimation of the relative severity degree of each decision to each ranking, we establish the following objective function to minimize the distances between each alternative and each ranking, i.e.,

\[ \min[f(\Lambda, W) = (f_1(\alpha_1, w), f_2(\alpha_2, w), ..., f_n(\alpha_n, w))], \]

(24)

s.t.

\[ \sum_{i=1}^{m} w_i = 1, w_i \geq 0, i = 1, 2, ..., m, \]

\[ \sum_{h=1}^{c} \alpha_{hj} = 1, 0 \geq \alpha_{hj} \geq 1, j = 1, 2, ..., n. \]

(25)

Since each decision in the alternative set is in a state of fair competition, there exists no preference relation between the decisions, i.e., each decision is of equal importance in the alternative set. Therefore, the objective function (24) can be turned into

\[ \min[f(\Lambda, W) = \sum_{j=1}^{n} f_j(\alpha_j, w)]. \]

(26)
We can establish the following Lagrange function:

$$L(\Lambda, w, \lambda_1, \lambda_2) = \sum_{j=1}^{n} f_j(\alpha_j, w) - \lambda_1 \left[ \sum_{h=1}^{c} \alpha_{hj} - 1 \right] - \lambda_2 \left[ \sum_{i=1}^{m} w_i - 1 \right]$$

which is similar to the distribution of a unit mass on axis $j$. Setting the above equations to zero, the iteration model is then obtained as

$$\sum_{j=1}^{c} \sum_{h=1}^{c} \left[ \alpha_{hj}^2 \right] + \sum_{i=1}^{m} \left[ w_i \left( \text{dis.CE} \right) \right]^2 - \lambda_1 \left[ \sum_{h=1}^{c} \alpha_{hj} - 1 \right] - \lambda_2 \left[ \sum_{i=1}^{m} w_i - 1 \right],$$

where $\lambda_1$ and $\lambda_2$ are Lagrange multipliers. By differentiating, we can get the derivatives

$$\frac{\delta L(\Lambda, w, \lambda_1, \lambda_2)}{\delta \alpha_{hj}} = 2\lambda_{hj} \sum_{i=1}^{m} w_i \left( \text{dis.CE} \right)^2 - \lambda_1,$$

$$\frac{\delta L(\Lambda, w, \lambda_1, \lambda_2)}{\delta \lambda_1} = \sum_{h=1}^{c} \alpha_{hj} - 1,$$

$$\frac{\delta L(\Lambda, w, \lambda_1, \lambda_2)}{\delta \lambda_2} = \sum_{i=1}^{m} w_i - 1$$

Setting the above equations to zero, the iteration model is then obtained as

$$\alpha_{hj} = \left[ \sum_{k=1}^{m} \sum_{i=1}^{n} \left[ w_i \left( s_{ij} - p_h \right) \right]^2 \right]^{-1}$$

$$w_i = \left[ \sum_{k=1}^{m} \sum_{h=1}^{c} \left[ \alpha_{hj} \left( s_{kj} - p_h \right) \right]^2 \right]^{-1}$$

where $h = 1, 2, ..., c$ and $j = 1, 2, ..., n$. The model can simultaneously provide not only relative severity degrees of decisions but also initial objective weights directly from the information of decisions, involving $m$ qualitative and quantitative objectives. In the situations where the weight vector is known, $\alpha_{hj}$ can be computed by directly substituting the weight vector into Eq. (29) and Eq. (30) is not used, thus the proposed model is eliciting initial objective weights from the information implicit in alternatives, and can provide more precise recognition results of alternatives compared with the previous models. Through the above model, we have the optimal estimation of the relative severity degree of each decision belonging to each ranking, forming the matrix

$$\Lambda^* = \begin{bmatrix} \alpha_{11}^* & \alpha_{12}^* & \cdots & \alpha_{1n}^* \\ \alpha_{21}^* & \alpha_{22}^* & \cdots & \alpha_{2n}^* \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{c1}^* & \alpha_{c2}^* & \cdots & \alpha_{cn}^* \end{bmatrix} = (\alpha^*_{hj})_{c \times n}$$

In terms of the maximization rule of severity degree, the ranking of a decision is the one related to the maximum severity degree. However, under some conditions, the rule cannot identify the optimal ranking due to consideration of the maximum severity degree and ignorance of other severity degrees.

3.3. Cross entropy for distance computation.

Here we use a new index, rank feature value, to indicate the ranking of a decision. The column vector in matrix (31)

$$\alpha_j^* = (\alpha_{1j}^*, \alpha_{2j}^*, \ldots, \alpha_{cj}^*), \sum_{h=1}^{c} \alpha_{hj}^* = 1,$$

represents the relative severity degrees of decision $d_j$ to all the $c$ rankings. This can be regarded as the distribution of relative severity degree of decision $d_j$ on the axis of ranking variable $h$, which is similar to the distribution of a unit mass on axis $j$. 
in mechanics. The idea is to use the expectation value of variable \( h \) to represent the ranking of a decision in that the expectation value combines all the information that the decision belongs to all the rankings. Given that \( \alpha_{hj}^* \) means the relative severity degree of decision \( d_j \) to ranking \( h \), the expectation ranking to which alternative \( d_j \) belongs can be calculated by

\[
H_j = \sum_{h=1}^{c} \alpha_{hj}^* h, \tag{33}
\]

which also represents the center of the shape encircled by ranking variable \( h \) and \( \alpha_{hj} \) on the \( h \sim \alpha_{hj} \) coordinate plane. Therefore \( H_j \) is called rank feature value and can be considered as an index of the performance of decision \( d_j \). The ranking of the rank feature values from the smallest to the biggest means the ranking of decisions from the most preferred to the least preferred, in other words, the smaller the rank feature value is, the more preferred the decision is. Therefore, it is convenient to rank the decisions and identify the most satisfying one.

The performing process of the proposed weighted ranking model is formulated as follows:

**Algorithm 2: The weighted ranking process for multi-objective decision making.**

**Step 1.** Computing the relative severity degree matrix \( S \) and setting the initial value to the weight vector \( w^0 = (w_1^0, w_2^0, ..., w_m^0) \) such that \( \sum_{i=1}^{m} w_i^0 = 1 \) and \( w_i^0 \geq 0 \). Setting values to the total number of ranking \( c \), the partition vector \( p \), computation precise \( \delta \) and letting iteration number \( k = 0 \).

**Step 2.** Substituting \( w^k \) into Eq. (29), we can get \( \alpha_{hj}^k \).

**Step 3.** Substituting \( \alpha_{hj}^k \) into Eq. (30), we can get \( w^{k+1} \).

**Step 4.** If \( \max_{1 \leq i \leq m} |w_i^k - w_i^{k+1}| \leq \delta \), then go to Step 5, otherwise repeat Steps 2C4 and let \( k = k+1 \).

**Step 5.** Substituting \( w^{k+1} \) into Eq. (29), we can get \( \alpha_{hj}^{k+1} \), which constitutes the matrix \( \Lambda^* \). According to Eq. (33), we can get the rank feature value vector \( H = (H_1, H_2, ..., H_n) \). So the ranking of decisions is obtained, and then the most preferred decision can be identified.

The weights obtained through the iteration process are elicited mainly from the information implicit in the relative severity degrees of decisions, therefore sometimes they cannot suitably reflect the physical characteristics of objectives or the preference of decision makers where the objective weights need to be adjusted in terms of knowledge, experience and judgment preference from decision makers, and sometimes the weights are given to model specific situations or reflect specific requirements of the decision maker. Under these situations, the matrix \( \Lambda^* \) can be directly obtained by substituting the adjusted or known weights into Eq. (29). The above iteration model can provide a performance index of each decision representing the aggregation result of multiple objectives as well as the initial objective weights.

**3.4. Ranking.** We assess the objective weights or quantify the relative severity degrees of the decisions regarding qualitative objectives. This method applies complementarity to measure the scales of pairwise comparisons, and constructs the
preference matrix in two steps, including (1) making pairwise comparisons between elements using only three scales to obtain the qualitative ranking of the elements, and (2) using linguistic terms to construct the preference matrix in terms of the qualitative ranking.

For elements $a_k$ and $a_l$ in set $A = (a_1, a_2, ..., a_n)$ which is being compared, we make pairwise comparisons on importance (excellence or other properties) using three scales $0, 0.5$ and $1$, and the following rules:

1. If $a_k$ is more important than $a_l$, then give a scale $e_{kl} = 1$, $e_{lk} = 0$,
2. if $a_k$ and $a_l$ make no distinction, then give a scale $e_{kl} = 0, e_{lk} = 0,5$,
3. if $a_l$ is more important than $a_k$, then give a scale $e_{kl} = 0, e_{lk} = 1$,

where $k = 1, 2, ..., n; l = 1, 2, ..., n$. After making comparisons between $n$ elements, we get a qualitative sorting matrix

$$E = \begin{bmatrix} e_{11} & e_{12} & ... & e_{1n} \\ e_{21} & e_{22} & ... & e_{2n} \\ ... & ... & ... & ... \\ e_{c1} & e_{c2} & ... & e_{cn} \end{bmatrix} = (e^*_{kl})_{n \times n} \quad (34)$$

The matrix satisfies the consistency of comparisons, if $E$ is subject to the following conditions:

1. If $e_{gl} > e_{gk}$, we have $e_{kl} = 0$,
2. If $e_{gk} < e_{gl}$, we have $e_{kl} = 1$,
3. If $e_{gk} = e_{gl} = 0.5$, we have $e_{kl} = 0.5$,

where $g=1, 2, ..., n$. It is easy to construct a consistent matrix when making pairwise comparisons using only three scales. For a consistent matrix $E$, the sum of element values in each row is denoted by

$$Y_k = \sum_{l=1}^{m} e_{kl}, (l = 1, 2, ..., n), \quad (35)$$

where $Y_k$ represents the degree of importance of element $a_k$. The decreasing ranking of $Y_k, (k = 1, 2, ..., n)$ means the decreasing ranking of elements in $A$ in terms of their importance. Before constructing the preference matrix, we reorder the elements $(a_1, a_2, ..., a_n)$ such that $Y_1 \geq Y_2 \geq ... \geq Y_n$.

3.5. Preference matrix. A linguistic set including several linguistic terms is used to describe the relations between elements. We set the quantitative scale to $0.5$ if an element is equally important compared with the first element in the reordered element set, and $1.0$ if an element is incomparably important. Through the two extremes, some linguistic terms are inserted to describe the scales in a linear way. According to complementarity, if $\beta_{ij}$ is the quantitative scale of element $a_i$ to $a_j$ when $a_i$ is compared with $a_j$ on importance, then its counterpart $\beta_{ji} = 1 - \beta_{ij}$. Subscripts $i$ and $j$ represent the serial number which is reordered according to qualitative sorting matrix $E$. Then making pairwise comparisons between the first element and the others in the reordered element set, we have the preference matrix

$$\beta = \begin{bmatrix} \beta_{11} & \beta_{12} & ... & \beta_{1n} \\ \beta_{21} & \beta_{22} & ... & \beta_{2n} \\ ... & ... & ... & ... \\ \beta_{n1} & \beta_{n2} & ... & \beta_{nn} \end{bmatrix} = (\beta_{ij})_{n \times n} \quad (36)$$
Subject to,
\[
\begin{align*}
0 & \leq \beta_{ij} \leq 1, \\
\beta_{ij} + \beta_{ji} & = 1, \\
\beta_{ij} & = 0.5, \; i = j,
\end{align*}
\]  
\tag{37}

The preference matrix $\beta$ is a matrix of $n \times n$ order. The element values in each row of the upper triangular matrix monotonically increase from the left hand to the right hand, and the element values in each column of the lower triangular matrix monotonically decrease from the top down. This leads to the matrix satisfying the requirement of consistency of comparisons. The relative severity degree of element $a_j$ can be computed as below:

\[
s_j = \frac{1 - \beta_{1j}}{\beta_{1j}}, \quad 0.5 \leq \beta_{1j} \leq 1
\]  
\tag{38}

where $j = 1, 2, \ldots, n$. According to Eq. (38), we can transform the quantitative scales to the corresponding relative severity degrees. Now we can directly use the linguistic terms (qualitative scales) and relative severity degrees to the assessment process of the decisions and the objective weights. Thus in practice, it is not necessary to repeat the computation process of the preference matrix, which makes it feasible for multistage multi-objective decision making problems.

4. **Numerical illustrations.** To imply the effectiveness and applicability of the proposed multi-objective optimization methodology, two numerical examples are worked out.

**Example 1.** The following assumptions are considered in the proposed numerical example. Assume there are some robots to move through a manufacturing network and thus some measures are effective on finding the optimal paths for the robots. Objectives affecting decisions (alternatives) include one quantitative objective, benefit of routing ($o_1$), and two qualitative objectives: risk ($o_2$) and reliability ($o_3$). Three robots are considered to be evaluated by the objectives to make the optimal decisions namely, A, B and C. the linguistic terms with the corresponding quantitative scales and severity degree are given in Table 1, note that the computations are based on Eqs. (36-38). There are 11 decisions considered for the robots. Benefits of decision making are listed in Table 2.

| Table 1. Linguistic terms and the corresponding qualitative scale and severity degree |
|---------------------------------|-----------|-----------|-----------|----------|-----------|
| Linguistic term                | Equally   | Rather    | Obviously | Extra     | Incomparably |
| Quantitative scale             | 0.5       | 0.65      | 0.75      | 0.9       | 1          |
| Severity degree                | 1         | 0.538     | 0.429     | 0.111     | 0          |

Now we compute the optimal decisions for three robots. For each robot, the different decisions can lead to different benefits being shown in Table 2. The objective $o_1$ is a profit-type objective expected to be maximized, so the relative severity degree can be computed by Eq. (29). Take objective $o_2$ as an example to show the process of computing the relative severity degree of qualitative objectives. The economic influence of allocating the same decision is quantified using the preference concept. A qualitative sorting matrix of the three robots regarding qualitative
Table 2. Linguistic terms and the corresponding qualitative scale and severity degree

| Robot | Decisions          |
|-------|--------------------|
|       | 0  0.1  0.2  0.3  0.4  0.5  0.6  0.7  0.8  0.9  1.0 |
| A     | 0  38  47  57  66  74  84  89  94  97  98          |
| B     | 0  37  49  59  68  74  80  86  91  95  99          |
| C     | 0  27  40  52  63  72  81  87  92  96  100         |

Objective $o_2$ is given as

$$E_2 = \begin{bmatrix} 0.5 & 1 & 0 \\ 0 & 0.5 & 0 \\ 1 & 1 & 0.5 \end{bmatrix}$$

The sums of elements in each row are listed at the right hand of the matrix. Their ranking from the biggest to the smallest represents the qualitative ranking of the three robots from the most preferred to the least preferred, i.e., $C \succ A \succ B$.

Robot C is regarded as the comparison standard, according to the linguistic terms and their severity degrees in Table 1, the following consideration is given: robot C is rather preferred to robot A, and obviously preferred to robot B. According to Table 1, the relative severity degrees of the three robots, A, B, C, are 0.538, 0.429 and 1, respectively. Suppose the economic influence is linear to its corresponding decision, the relative severity degrees of the 11 decisions regarding robot A can be computed by $s_2 = 0.538(0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0) = (0, 0.05, 0.11, 0.16, 0.22, 0.27, 0.32, 0.38, 0.43, 0.49, 0.54)$.

Similarly, the relative severity degrees regarding robot B and C can be obtained. As for objective $o_3$, the relative severity degrees of the three robots are determined as 0.429, 1.0 and 0.111, respectively. The relative severity degree of the 11 decisions can be obtained based on the same supposition. Table 3 shows the relative severity degrees obtained above.

Table 3. Linguistic terms and the corresponding qualitative scale and severity degree

| Robot | Objectives | Decisions | 0  | 0.1  | 0.2  | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  | 0.8  | 0.9  | 1.0  |
|-------|------------|-----------|----|------|------|------|------|------|------|------|------|------|------|
| A     | o1         | 0         | 0.38| 0.47 | 0.57 | 0.66 | 0.74 | 0.84 | 0.89 | 0.94 | 0.97 | 0.98 |
|       | o2         | 0         | 0.05| 0.11 | 0.16 | 0.22 | 0.27 | 0.32 | 0.38 | 0.43 | 0.49 | 0.54 |
|       | o3         | 0         | 0.04| 0.09 | 0.13 | 0.17 | 0.22 | 0.26 | 0.30 | 0.34 | 0.38 | 0.43 |
| B     | o1         | 0         | 0.37| 0.49 | 0.59 | 0.68 | 0.74 | 0.8  | 0.86 | 0.91 | 0.95 | 0.99 |
|       | o2         | 0         | 0.04| 0.09 | 0.13 | 0.17 | 0.22 | 0.26 | 0.30 | 0.34 | 0.38 | 0.43 |
|       | o3         | 0         | 0.10| 0.2  | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  | 0.8  | 0.9  | 1    |
| C     | o1         | 0         | 0.27| 0.4  | 0.52 | 0.63 | 0.72 | 0.81 | 0.87 | 0.92 | 0.96 | 1.0  |
|       | o2         | 0         | 0.1 | 0.2  | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  | 0.8  | 0.9  | 1.0  |
|       | o3         | 0         | 0.03| 0.05 | 0.08 | 0.1  | 0.13 | 0.15 | 0.18 | 0.20 | 0.23 | 0.25 |

Applying Algorithm 2, we perform the allocation process from robot C through robot B to robot A. For simplicity, the weights for the three objectives are not adjusted in the process of computation. The optimal decision and the optimal rank feature values corresponding to each decision are listed in 4. The decision corresponding to the smallest rank feature value is the optimal decision, i.e., $d_1^* =$
0.2. The optimal decision for robot B and robot C can be traced, i.e., 0.5 and 0.3, respectively. The optimal decision sequence is obtained.

Table 4. The rank features obtained for decisions for different robots

| Robot | Decisions | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|-------|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| A     | Rank features | 5.00 | 4.3 | 4.06 | 3.74 | 3.58 | 3.41 | 3.24 | 3.10 | 2.98 | 2.87 |     |
|       | Cumulative rank features | 11.50 | 11.13 | 11.12 | 11.26 | 11.50 | 11.55 | 11.68 | 11.90 | 12.10 | 12.41 | 12.87 |
| B     | Cumulative rank features | 10.00 | 9.42 | 8.98 | 8.65 | 8.27 | 7.97 | 7.76 | 7.37 | 7.07 | 6.82 | 6.50 |
| C     | Cumulative rank features | 5.00 | 4.68 | 4.29 | 3.97 | 3.78 | 3.56 | 3.33 | 3.12 | 2.95 | 2.79 | 2.67 |

Example 2. The following assumptions are considered in the proposed numerical example. Objectives affecting decisions include two quantitative objectives of benefit type namely $o_1$ and $o_2$, and two qualitative objectives $o_3$ and $o_4$. Three robots are considered to be evaluated by the objectives to make the optimal decisions namely, A, B and C. The linguistic terms with the corresponding quantitative scales and severity degree are given in 1, note that the computations are based on Eqs. (36-38). There are 16 decisions considered for the robots. Benefits of decision making are listed in 5.

Table 5. Benefit values of different decisions for different robots

| Robot | Decisions | 0.0 | 0.1 | 0.2 | 0.25 | 0.3 | 0.35 | 0.4 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.8 | 0.85 | 0.9 | 1.0 |
|-------|-----------|-----|-----|-----|------|-----|-----|-----|-----|------|-----|------|-----|-----|------|-----|-----|
| A     | 0 | 38 | 47 | 52 | 57 | 61 | 66 | 74 | 79 | 84 | 87 | 89 | 93 | 96 | 98 | 99 |
| B     | 0 | 37 | 49 | 54 | 59 | 63 | 68 | 74 | 77 | 80 | 82 | 86 | 91 | 94 | 96 | 98 |
| C     | 0 | 27 | 40 | 47 | 52 | 58 | 62 | 71 | 75 | 82 | 85 | 91 | 93 | 96 | 97 | 100 |

Now we compute the optimal decisions for three robots. For each robot, the different decisions can lead to different benefits being shown in 5. The objectives $o_1$ and $o_2$ are profit-type objectives expected to be maximized, so the relative severity degree can be computed by Eq. (29). The relative severity degree of the 16 decisions can be obtained. Table 6 shows the relative severity degrees obtained.

Applying Algorithm 2, we perform the allocation process from robot C through robot B to robot A. For simplicity, the weights for the four objectives are adjusted to be 0.21, 0.35, 0.14 and 0.3 respectively, in the process of computation. The optimal decision and the optimal rank feature values corresponding to each decision are listed in 7. The decision corresponding to the smallest rank feature value is the optimal decision, i.e., $d_1^* = 0.25$. The optimal decision for robot B and robot C can be traced, i.e., 0.65 and 0.35, respectively. The optimal decision sequence is obtained.

Through the above process, the multiple quantitative and qualitative objectives of each decision are converted into a single objective represented by its rank feature value; correspondingly, multi-objective programming is turned into single objective programming.

5. Conclusions. We proposed a multi-objective decision model to evaluate the qualitative terms and rank the decisions to find the optimal ones. In the proposed process cross entropy was used to compute the distance between decisions and ranking centers. Also preference analysis of qualitative terms was employed for converting and weighing the decisions and rank feature concept was developed to
Table 6. The relative severity degrees of objectives for different robots

| Robot | Decisions |
|-------|-----------|
| A     | 0         | 0.1 | 0.2  | 0.25 | 0.3  | 0.35 | 0.4  | 0.5  | 0.55 | 0.6  | 0.65 | 0.7  | 0.8  | 0.85 | 0.9  | 1.0  |
|       | 0         | 0.38 | 0.47 | 0.52 | 0.57 | 0.61 | 0.66 | 0.74 | 0.81 | 0.84 | 0.87 | 0.89 | 0.94 | 0.96 | 0.97 |
| O1    | 0         | 0.34 | 0.39 | 0.46 | 0.49 | 0.53 | 0.56 | 0.61 | 0.67 | 0.72 | 0.75 | 0.81 | 0.87 | 0.92 | 0.95 |
| O2    | 0.05      | 0.11 | 0.13 | 0.16 | 0.19 | 0.22 | 0.27 | 0.3  | 0.32 | 0.35 | 0.38 | 0.43 | 0.46 | 0.49 | 0.54 |
| O3    | 0         | 0.04 | 0.09 | 0.11 | 0.13 | 0.15 | 0.17 | 0.22 | 0.24 | 0.26 | 0.28 | 0.30 | 0.34 | 0.36 | 0.38 |
| O4    | 0.37      | 0.49 | 0.54 | 0.59 | 0.63 | 0.68 | 0.74 | 0.77 | 0.80 | 0.82 | 0.86 | 0.91 | 0.94 | 0.95 | 0.99 |
| B     | 0         | 0.37 | 0.49 | 0.54 | 0.59 | 0.63 | 0.68 | 0.74 | 0.77 | 0.80 | 0.82 | 0.86 | 0.91 | 0.94 | 0.95 |
| O1    | 0         | 0.27 | 0.33 | 0.37 | 0.45 | 0.47 | 0.52 | 0.62 | 0.65 | 0.73 | 0.76 | 0.80 | 0.85 | 0.90 | 0.93 |
| O2    | 0.04      | 0.09 | 0.11 | 0.13 | 0.15 | 0.17 | 0.22 | 0.24 | 0.26 | 0.28 | 0.30 | 0.34 | 0.36 | 0.38 | 0.43 |
| O3    | 0         | 0.10 | 0.2  | 0.25 | 0.3  | 0.35 | 0.4  | 0.5  | 0.55 | 0.6  | 0.65 | 0.7  | 0.8  | 0.85 | 0.9  |
| O4    | 0.27      | 0.40 | 0.45 | 0.52 | 0.57 | 0.63 | 0.72 | 0.77 | 0.81 | 0.85 | 0.87 | 0.92 | 0.94 | 0.96 | 1.0  |
| C     | 0         | 0.28 | 0.39 | 0.46 | 0.49 | 0.53 | 0.56 | 0.61 | 0.67 | 0.72 | 0.75 | 0.81 | 0.87 | 0.92 | 0.95 |
| O1    | 0         | 0.10 | 0.2  | 0.25 | 0.3  | 0.35 | 0.4  | 0.5  | 0.55 | 0.6  | 0.65 | 0.7  | 0.8  | 0.85 | 0.9  |
| O2    | 0.03      | 0.05 | 0.06 | 0.08 | 0.09 | 0.10 | 0.13 | 0.14 | 0.15 | 0.17 | 0.18 | 0.20 | 0.21 | 0.23 | 0.25 |

Table 7. The rank features obtained for decisions for different robots

| Robot | Decisions |
|-------|-----------|
| A     | 0         | 0.1 | 0.2  | 0.25 | 0.3  | 0.35 | 0.4  | 0.5  | 0.55 | 0.6  | 0.65 | 0.7  | 0.8  | 0.85 | 0.9  | 1.0  |
|       | 0         | 4.3  | 4.6  | 4.9  | 5.1  | 5.3  | 5.6  | 5.9  | 6.1  | 6.4  | 6.7  | 7.0  | 7.3  | 7.6  | 7.9  | 8.2  |
| B     | 0         | 4.3  | 4.6  | 4.9  | 5.1  | 5.3  | 5.6  | 5.9  | 6.1  | 6.4  | 6.7  | 7.0  | 7.3  | 7.6  | 7.9  | 8.2  |
| C     | 0         | 4.3  | 4.6  | 4.9  | 5.1  | 5.3  | 5.6  | 5.9  | 6.1  | 6.4  | 6.7  | 7.0  | 7.3  | 7.6  | 7.9  | 8.2  |

rank the decisions and determine the optimal ones. The effectiveness and applicability of the proposed multi-objective optimization methodology was illustrated in two numerical examples. The results show that the proposed mathematical optimization model is capable to consider both quantitative and qualitative objectives in the process of optimization mapping the linguistic terms into severity degrees. The numerical results related to the examples were determined to be optimal decisions having the minimum rank feature. In example 1 the optimal decisions for different robots were 0.22, 0.5 and 0.3, respectively. Also, in example 2 the optimal decisions were 0.25, 0.65 and 0.35, respectively. This optimization model is able to handle dynamic changes in rankings, weights and severity degrees computing the distances even when the changes do not follow specific statistical distributions.

REFERENCES

[1] M. A. Badri, A. K. Mortagy and C. A. Alysed, A multiobjective model for locating fire stations, European Journal of Operational Research, 110 (1998), 243–260.
[2] C. Briggs and P. Little, Impacts of organizational culture and personality traits on decision-making in technical organizations, Systems Engineering, 11 (2008), 15–26.
[3] V. Chankong and Y. Y. Haimes, Multiobjective Decision Making: Theory and Methodology, Dover, New York, USA, 2008.
[4] J. Daniels, P. W. Werner and T. Bahill, Quantitative methods for tradeoff analysis, Systems Engineering, 4 (2001), 190–212.
[5] M. H. DeGroot, Optimal Statistical Decisions, McGraw-Hill, New York, USA, 1970.
[6] R. Y. Dicdican and Y. Y. Haimes, Relating multiobjective decision trees to the multiobjective risk impact analysis method, Systems Engineering, 8 (2005), 95–108.
[7] M. Ehrgott and X. Gandibleaux, (Editors), Multiple criteria optimization: State of the Art Annotated Bibliographic Surveys, Springer, New York, 2002.
[8] A. O. Esogbue and R. E. Bellman, Fuzzy dynamic programming and its extensions, TIMS/Studies Management Sci., 20 (1984), 147–167.
[9] H. I. Frohwein, J. H. Lambert, Y. Y. Haimes and L. A. Schiff, Multicriteria framework to aid comparison of roadway improvement projects, J Transportation Engineering, 125 (1999), 224–230.
[10] K. Golabi, Selecting a group of dissimilar projects for funding, IEEE Transaction in Engineering Management, 34 (1987), 138–145.
[11] S. D. Guikema and M. W. Milke, Sensitivity analysis for multi-attribute project selection problems, *Civil Engineering Environmental Systems*, 20 (2003), 143–162.

[12] M. J. Hodgson, K. E. Rosing and A. L. G. Storrier, Testing a bicriterion location-allocation model with real world network traffic: The case of Edmonton, *Canada, Multicriteria Analysis, Climaco, J. (Editor), Springer, Berlin*, (1997), 484–495.

[13] M. L. Hussein and M. A. Abo-Sinna, Decomposition of multi-objective programming problems by hybrid fuzzy dynamic programming, *Fuzzy Sets Systems*, 60 (1993), 25–32.

[14] M. L. Hussein and M. A. Abo-Sinna, A fuzzy dynamic approach to the multicriteria resource allocation problem, *Fuzzy Sets Systems*, 69 (1995), 115–124.

[15] J. Kacprzyk, *Multistage Fuzzy Control*, Wiley, Chichester, USA, 1997.

[16] J. Kacprzyk and A. O. Esogbue, Fuzzy dynamic programming: Main developments and applications, *Fuzzy Sets Systems*, 81 (1996), 31–45.

[17] J. Kacprzyk and L. Sugianto, Multistage fuzzy control involving objective and subjective aspects, in L.C. Jain, R.K. Jain (Eds.), *Proceedings of the Second International Conference on Knowledge Based Intelligent Electronic Systems*, Adelaide, Australia, (1998), 564–573.

[18] S. Kullback, *Information Theory and Statistics*, John Wiley and Sons, NY, 1959.

[19] D. B. Lee, Methods for evaluation of transportation projects in the USA, *Transportation Policy*, 7 (2000), 41–50.

[20] L. M. Meade and A. Presley, R and D project selection using the analytic process, *IEEE Transaction in Engineering Management*, 49 (2002), 59–66.

[21] D. A. Niemeier, Z. B. Zabinsky, Z. Zeng and G. S. Rutherford, Optimization models for transportation project programming process, *Journal of Transportation Engineering*, 121 (1995), 14–26.

[22] G. S. Parnell, R. E. Metzger, J. Merrick and R. Eiler, Multiobjective decision analysis of theater missile defense architectures, *Systems Engineering*, 4 (2001), 24–34.

[23] M. Sanchez, N. Agell and G. Ormazabal, Multiple-criteria evaluation for value management in civil engineering, *Journal of Management Engineering*, 21 (2005), 131–137.

[24] J. S. Shang, Y. Tjader and Y. Ding, A unified framework for multicriteria evaluation of transportation, *IEEE Transaction in Engineering Management*, 51 (2004), 300–313.

[25] P. J. Smith, M. Shafi and H. Gao, Quick simulation: A review of importance sampling techniques in communication systems, *IEEE J.Select.Areas Commun.*, 15 (1997), 597–613.

[26] R. Srinivasan, *Importance Sampling - Applications in Communications and Detection*, Springer-Verlag, Berlin, 2002.

[27] S. Y. Chen and G. T. Fu, Combining fuzzy iteration model with dynamic programming to solve multiojective multistage decision making problems, *Fuzzy Sets and Systems*, 152 (2005), 499–512.
[36] J. Q. Wang and J. J. Li, Multi-criteria fuzzy decision-making method based on cross entropy and score functions, *Expert Systems with Applications*, 38 (2011), 1032–1038.

[37] A. R. Jafarian-Moghaddam and K. Ghoseiri, Fuzzy dynamic multi-objective Data Envelopment Analysis model, *Expert Systems with Applications*, 38 (2011), 850–855.

Received November 2012; 1st revision December 2013; 2nd revision July 2014.

*E-mail address: hfazl@iust.ac.ir*

*E-mail address: mehrabad@iust.ac.ir*