Experimental hints of Gravity in Large Extra Dimensions? *

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Abstract

Recent conjectures suggest the universe may have large extra dimensions, through which gravity propagates. This implies gross departures from Newton’s law of gravity at small length scales. Here I consider some implications for particle dynamics on scales comparable to the compactification radius, $R_c \lesssim 1$ mm. During planet formation, coalescence of micron sized dust grains to planetesimals is a rate critical step. Blum et al (2000) found dust grain aggregates form low fractal dimension structures in microgravity, consistent with high angular momentum coalescence. I consider the effects of non-Newtonian gravity on dust aggregation on scales less than $R_c$ and show they naturally coalesce into low dimensional structures with high specific angular momentum. We infer $R_c \approx 80$ microns.

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Recent conjectures have postulated that the hierarchy problem in physics may be resolved if two (or more) of the extra dimensions postulated by extensions of the standard model of particle theory, are compactified on mesoscopic scales - with effective radii much larger than the Planck scale. A particularly interesting possibility is in the case of $n = 2$ mesoscopic compactification, in which case the implied scale, $R_c$, for the extra dimensions is of the order of 0.1 mm. In the simplest theory, the standard model gauge fields are restricted to (or near) the 3-dimensional brane on which we normally live, and only gravity propagates into the bulk of the large extra dimensions (LEDs). The resulting theory has a number of interesting astrophysical implications.

An immediate implication of LEDs is that Newton’s law fails on small scales, and is replaced by an effective potential gradient

$$\nabla \Phi(r) = -(n + 1) \frac{m_1 m_2}{M_{pl}^{n+2} r^{n+2}} \quad r \ll R_c$$

where $n$ is the number of large extra dimensions, and $M_{pl-n}$ is the higher dimensional Planck mass, implying an effective 4-D Planck mass $M_{pl} \sim M_{pl-n}^{1/n} R_c^{n/2}$. The laboratory experimental constraints on deviations from Newton’s law on scales less than 1 cm are weak, so the conjecture is not directly excluded by direct experiments, although recent experiments have constrained $R_c < 218 \mu m$, and future experiments should detect $R_c$. If correct, LEDs have many implications for physics on different scales, some of which will be tested in the near future.

It is generally accepted that planets form through an aggregation of smaller grains. The protoplanetary disk is expected to contain sub–micron sized dust (and ice) grains which segregate through sedimentation to the mid–plane of the disk. The disk may be turbulent on some scales, and in general there is drag on the grains due to the gas orbiting at slightly sub–Keplerian velocities. The basic picture for planet formation has planetesimals form by dust aggregation, with runaway accretion onto the largest planetesimals once they reach mesoscopic sizes and the gravitational field of the largest planetesimals dominates the local potential. The accretion of planetesimals builds masses up to several times the mass of the Earth, at which point direct accretion of gas is thought to runaway to produce the Jovian planets. A problem is presented, because observational evidence and indirect theoretical arguments require that the time scale for dust grains to assemble into large planetesimals must be very short, of order $10^6$ years or shorter.

A critical step in this process is the growth from sub–micron sized dust grains to $\sim 1$ cm (see eg review by Ruden), at which point radial drag allows grains to effectively sweep up a large volume rapidly, and to grow to the point where the self–gravity of the largest planetesimals produces rapid coalescence. In order for small grains to grow rapidly enough, it is necessary to postulate that they form loose aggregates with fractal dimension $D_f \leq 2$ in order for the grains to effectively sweep up smaller grains. Yet if the grain–grain velocity is large enough for significant growth to occur, we expect compactification or fragmentation during grain collisions, which slows down growth, as the geometric cross–section of the grains is reduced.

Recently Blum et al (2000) experimentally measured aggregation of micron sized dust grains in microgravity, in the CODAG experiment on STS-95. Rather surprisingly, they...
found dust grain growth up to $\sim 50\mu m$ producing branched linear structures with $D_f \sim 1.3$, much lower than predicted. The implied grain-grain coalescence time scale is then almost independent of grain mass ($\tau \sim m^{0.06}$), the dust mass function is dominated by large grains, and aggregation processes are dominated by large grains.

To explain this process, Blum et al impose an ad hoc cut-off on aggregation, where sticking is restricted to high impact parameter collisions ($b/a > 0.65$, for impact parameter $b$ on grain radius $a$). They conjecture that thermal rotation biases grain impact to large impact parameters. An alternative conjecture is that non-Newtonian gravitational attraction biases collisions to large effective impact parameters for $a < R_c$.

The existence of large extra dimensions, in which gravity propagates, changes the dynamics of grain-grain interactions at scales smaller than $R_c$. In the Newtonian regime, for grains size $a$ and density $\rho \sim 1$, the surface escape velocity $v_N \approx 2a\sqrt{G\rho}$ and the self-gravity of a grain is irrelevant for plausible grain dispersions for sizes less than $\sim 100$ m. Grain coagulation requires high grain velocity dispersion so small grains can have large collision rates, implying large velocity gradients or small scale turbulence. Countering this is the problem that at relative velocities $> \sim 100$ cm s$^{-1}$, experiments show that grains do not stick and growth is inhibited.

A lower bound on local dispersion is set by the Brownian motion induced dispersion, $v_B \sim 10^{-5}$ cm s$^{-1}\sqrt{(T/100)/m_{-3}}$, where $m_{-3}$ is the grain mass in milligrams. A 100$\mu m$ grain has a mass of about $0.1 m_{-3}$ if it is compact. The smallest grains have high dispersion due to Brownian motion, but intermediate sized grains, of order 100 microns, have low dispersion and low number densities.

The gas-grain response time is a few msec, so a dust grain approaching within $R_c \sim 100\mu m$ of an aggregate, will explore a range of relative velocities while it traverses the region of non-Newtonian attraction. This favours grain growth if there is no small scale turbulence and gas velocity gradients are small.

With LEDs, the escape velocity of grains smaller than $R_c$ is independent of the grain size and is just $v_6 \sim v_N(R_c) \sim 10^{-4}$ cm s$^{-1}$. Further, orbits in $1/r^3$ potentials are unstable, so any grains approaching within $R_c$ of each other, with instantaneous relative velocities less than $v_6$ become bound and must coalesce. A non-circular orbit in a $r^{-3}$ potential spirals into contact on a dynamical time, so any particle random walking in velocity so that it becomes bound to the aggregate cluster, immediately coalesces with the cluster. For sub-millimeter sized grains, this increases the collision cross-section by $\sim 2$ orders of magnitude, at low velocity dispersion, and makes the issue of sticking and compactification in collisions moot, if the grain-grain velocities are low. Thus in LED modified gravity, grains in lower density dust can coagulate rapidly, if the grain-grain velocity dispersion is small on scales $\lesssim R_c$. In disks, gas drag imposes a size dependent velocity gradient on dust grains, but the grain-grain dispersion is in general much smaller than the differential velocity between grains and gas, in the CODAG experiment the local grain-grain velocity dispersion is small and dominated by Brownian motion.

More importantly, the coalescing grains have much larger relative angular momentum than in the Newtonian case. A typical grain will have an orbital eccentricity of order 0.7 when it becomes bound, and enters the non-Newtonian regime at apocenter $r_a \sim R_c$. So the specific angular momentum of the grain is $l \sim \frac{1}{2}v_{c-Newtonian}(R_c) \times R_c \sim G\rho a^3\sqrt{R_c}$. If the distribution of impact parameters is uniform in area, then the excess angular momentum
observed by Blum et al, is consistent with $R_c \approx 80\mu m$.

Establishing the conditions necessary for rapid planet formation, as required by observations, has required some considerable fine tuning of the disk initial conditions in the models. With the modifications to Newtonian gravity that follow from theories of large extra dimensions, the problem of rapid coalescence at the smallest scales becomes simpler. The stronger gravitational force and resulting orbit instability on scales smaller than the effective compactification radius, allow non–Newtonian gravity to dominate the particle dynamics of this critical early stage of planet formation, allowing rapid grain growth precisely in those cases in which time scales for Newtonian particles to coalesce become prohibitively long. In principle the efficient coalescence of micron sized grains through non–Newtonian gravitational forces can be tested directly in microgravity experiments. The recent experiment of Blum et al is consistent with $R_c \sim 80$ microns, but it is of course quite possible other, Newtonian, effects are the cause of the anomalous dust grain aggregation observed.

IF the orbit instability inside $R_c$ is the cause of the anomalous grain growth, then first order estimates suggest $R_c \approx 80$ microns, the corresponding unification scale for $n = 2$ extra dimensions is about $10^{17}eV$. This should be testable, both through direct measurements of deviation from Newtonian gravity on small length scales; through saturation of grain aggregation at $a \gtrsim R_c$, and a transition to grain fractal dimension $D_f(a > R_c) \sim 2$; and, of course, through direct observation of anomalous particle collisions cross-sections at $\sim 10$ TeV.

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