Design, modelling and simulation of vibratory micromachined gyroscopes

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Abstract. Among various MEMS sensors, a rate gyroscope is one of the most complex sensors from the design point of view. The gyro normally consists of a proof mass suspended by an elaborate assembly of beams that allow the system to vibrate in two transverse modes. The structure is normally analysed and designed using commercial FEM packages such as ANSYS or MEMS specific commercial tools such as Coventor or Intellisuite. In either case, the complexity in analysis rises manyfolds when one considers the etch hole topography and the associated fluid flow calculation for damping. In most cases, the FEM analysis becomes prohibitive and one resorts to equivalent electrical circuit simulations using tools like SABER in Coventor. Here, we present a simplified lumped parameter model of the tuning fork gyro and show how easily it can be implemented using a generic tool like SIMULINK. The results obtained are compared with those obtained from more elaborate and intense simulations in Coventor. The comparison shows that lumped parameter SIMULINK model gives equally good results with fractional effort in modelling and computation. Next, the performance of a symmetric and decoupled vibratory gyroscope structure is also evaluated using this approach and a few modifications are made in this design to enhance the sensitivity of the device.

1. Introduction

The miniaturization of angular rate sensors has opened up applications in new areas such as vehicle stability, navigation and consumer electronics. This immense potential has led to intense research activities by several groups over the last decade in improving the performance of MEMS rate gyroscopes. Specifically, research in this area is focused on issues such as improving the resolution, increasing the bandwidth and designing appropriate packaging.

The operating principle of MEMS vibratory gyroscopes involves the rotation induced energy transfer between two principal modes of the mechanical structure. Resonant motion of the proof mass in the drive mode induces motion in the sense mode upon rotation due to Coriolis force. The measure of this Coriolis force generated displacement in the sense mode is used to find the rotation rate. MEMS vibratory gyroscope design is a complex task that involves optimization of several geometrical and electrical variables. The main design parameters that need to be evaluated are stiffness of the structure to enable mode-matching, evaluation of damping coefficients and determination of system response to applied rotational rate. Once a process sequence has been decided upon and the geometric constraints are set, the major task of the designer is to decide upon a symmetric structure for further evaluation. Optimization of the structure to
achieve desired performance specifications involves an iterative modelling and simulation procedure. We look at two specific design configurations in this paper, and propose an equivalent circuit model to simulate the gyro performance that enables quick optimization of the design. The equivalent circuit model has been validated with results from Coventor. We also propose a few modifications in design of the sense electrodes of existing gyros [1, 2] that enhance the base capacitance thus allowing for higher resolution.

2. Gyroscope structures

2.1. A structure with in-plane drive mode and out-of-plane sense mode

First, we analyse a surface micromachined polysilicon MEMS rate gyroscope that is designed for a minimum sense rate of 10°/sec. It consists of two identical proof masses, flexures, comb drives and sense electrodes as shown in figure 1. Large electrodes provided below the proof mass sense the out-of-plane motion of the device. The proof mass is driven into resonance in the y-direction (drive) by the electrostatic force generated by the interdigitated comb fingers. When the gyroscope is subjected to an angular rotation in the x-direction, the Coriolis force is induced in the z-direction (sense). The resulting oscillation amplitude in the sense direction is proportional to the rotation rate to be measured, and is detected by the air gap capacitors.

In this design, the in-plane motion (drive mode) of the structure is damped by the slide film effect due to Couette flow, while the out-of-plane motion (sense mode) is strongly damped due to the squeeze film effects. The Q-factor due to squeeze-film \((Q_{sq})\) is lower by more than an order of magnitude than the Q-factor due to the slide film \((Q_{sl})\). When the sense and the drive resonant modes have equal frequencies, the output signal is amplified by the quality factor of the sense mode. Squeeze film effects, however, limit the amplification of the sense mode displacement. One method to increase the quality factor in the sense mode is to have both, the drive and the sense modes in-plane (i.e., z-axis gyro). In-plane designs provide high Q-factors for both the modes, thereby increasing the sense displacements by an order of magnitude. The design of a z-axis symmetric decoupled gyroscope is discussed in the next section.

![Figure 1. Solid Model of the MEMS tuning fork gyroscope](image-url)
2.2. A structure with in-plane drive mode and sense mode

A symmetric and decoupled gyroscope structure, having both the drive and the sense mode in-plane, reported in [1, 2] is a variant of the design proposed by [3]. This structure solves the problem of mechanical coupling and mismatched resonant frequencies at the same time by using a symmetric suspension beam design combined with decoupled vibration modes for stable operation. In this structure, the sensing is done using a pair of comb fingers which are identical to those used for driving the structure. The equivalent circuit model is used to analyze this structure for two specific fabrication processes: the PolyMUMPS process for a structural layer thickness of 2 μm and the SOI process with a structural layer thickness of 10 μm. Variations in the capacitance measurement configurations are explored for the designs reported in [1, 2] and the results for the same are documented in the results and discussion section. The gyroscope designed for the PolyMUMPS process is shown in figure 2(a) and the modified design is shown in figure 2(b). In the modified design the comb fingers on the side-plates on the sensing side are replaced by electrodes laid down beneath the suspended structure. Here, the mode mismatch arising due to the removal of the comb fingers on the sense side is compensated by increasing the width of the side-plate. This gives more than an order of magnitude higher base capacitance than the original design, as each side-plate has about $500 \times 60 \, \mu m^2$ area. This also leads to a significant increase in the change in the sense capacitance for a given rotation rate. The gyroscope designed for the SOI process is shown in figure 3(a) and the modified design is shown in figure 3(b). In this case, vertical electrodes are used to sense the capacitance change caused by the sense motion. The advantage of this configuration is that a significantly larger capacitance change occurs in comparison to the original design, for a given displacement in the sense direction. This design modification also leads to an advantageous $\Delta C_b/C_b$ ratio, which makes the task of measurement easier.

3. Modelling and simulation

The suspended proof mass behaves as a single degree of freedom system for both the drive and the sense motions. The lumped parameters of the structure, namely, the mass and the stiffness, are obtained using energy methods. The boundary conditions of the suspension beam are: the end anchored to the substrate is fixed and the other end connected to the proof mass is guided. The damping due to squeeze-film and slide film is obtained using the closed form formulae [4].

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**Figure 2.** Symmetrical and decoupled microgyroscope: (a) Alper et al. [1] and (b) Modified design.
Table 1. Expressions for lumped parameters of tuning fork gyro shown in figure 1.

| Parameter                              | Expression                                                                 |
|----------------------------------------|---------------------------------------------------------------------------|
| Mass of the structure \( M_d = M_s \)  | \( m_p + m_b + m_c - m_e \)                                               |
| In plane stiffness \( K_d \)           | \( \frac{4Ew^3}{lb} \)                                                  |
| Out of plane stiffness \( K_s \)       | \( \frac{4Ew^2}{lb} \)                                                  |
| Slide film damping \( B_d \)           | \( \frac{\mu A}{(1+2K_c)l_p} \) for rarefied flow                      |
| Squeeze film damping \( B_s \)         | \( \frac{3\mu(L_p)^1n_h[4\ln(\eta) - 3 + \frac{4}{\eta^2} - \frac{1}{\eta^4}] + \frac{8\pi\mu n_h(L_p^2 - L_h^2)}{L_p^4}}{L_p} \) |

by the equation,

\[
M_d\ddot{Y} + B_d\dot{Y} + K_dY = F_e,
\]  

(1)

Here, the electrostatic driving force \( F_e = \frac{2.28nV_eV_{ac}\sin(\omega_d t)}{g^2} \) [3]. When a rotation rate \( \Omega_r \) is applied about the rotational axis, the Coriolis force causes motion along the sense direction (perpendicular to the excitation direction). This motion is described by the equation

\[
M_s\ddot{Z} + B_s\dot{Z} + K_sZ = F_c,
\]  

(2)

where the Coriolis force \( F_c = 2M_s\Omega_r \dot{Y} \sin(\omega_d t) \).

The corresponding change in the sense capacitance in the linearized form is written as,

\[
\Delta C_b = \frac{\epsilon A e}{g^2} Z
\]  

(3)

Using the governing equations (1)-(3) with lumped parameters, reduced order models are developed to design and predict the performance of the gyroscope in the time domain and
frequency domain \[5\]. The SIMULINK model shown in figure 4(a) predicts the transient response of the gyroscope to a rotation rate applied in the x-direction. The equivalent electrical circuit representation of the gyro is shown in figure 4(b). The overall transfer function is obtained using equations (1)-(3) and taking the Laplace transform of the ratio of the output \(\Delta C_b\) to the input \(\Omega_r\) as,

\[
\frac{\Delta C_b}{\Omega_r} = \frac{(G_1\bar{G}_2G_3)s}{[M_dM_s s^4 + (M_d B_s + M_s B_d)s^3 + (M_d K_s + M_s K_d + B_d B_s)s^2 + (B_d K_s + B_s K_d)s + K_d K_s]}
\]

where \(G_1 = 2 \omega_s\); \(\bar{G}_2 = \frac{2.28 \nu_s V_{dc} V_{ac}}{\varphi c}\) and \(G_3 = \frac{\epsilon A_p}{g_e}\). When the drive mode and the sense mode are fully matched (i.e., \(\omega_s = \omega_d\)), the transfer function reduces to

\[
\frac{\Delta C_b}{\Omega_r} = \frac{2Y Q_s}{\omega_s} G_3
\]

where, \(Y\) is the amplitude of the drive mode and \(Q_s = \sqrt{K_s M_s / B_s}\), is the quality factor of the sense mode. The frequency response of the gyroscope is analyzed with the help of these transfer functions.

4. Results and discussion
The behaviour of the tuning fork gyroscope has been simulated in Coventor’s ARCHITECT Saber module. The squeeze film and slide film damping parameters have been extracted using the INTEGRATOR module. The transient analysis and frequency response results obtained from Coventor and the lumped parameter model are shown in figure 5 and 6 respectively. Table 2 makes a comparison of these results. Next, the in-plane gyro is modelled in the same manner and the simulated results of the original designs are compared with the modified designs. Note that the increase in the width of the side plates compensates for the loss of mass due to removal of the sense combs. This ensures that the modes of interest are still perfectly matched. The sense displacements listed are for a rotation rate of 1°/sec. The PolyMUMPS gyro is simulated...
for a pressure of 100 Pa and the SOI gyro at the atmospheric pressure. The results of the modified designs are listed in tables 3 and 4. The modified designs shown in figures 2(b) and 3(b) effectively improve the sensitivity of the gyroscope.

**Figure 5.** Comparison of transient response obtained by the lumped parameter model with Coventor for tuning fork gyro of figure 1.

**Figure 6.** Comparison of frequency response obtained by the lumped parameter model with Coventor for tuning fork gyro of figure 1.

**Table 2.** Comparison of the simulated results obtained using lumped parameter model and Coventor for tuning fork gyro of figure 1.

| Parameter       | Lumped model | Coventor |
|-----------------|--------------|----------|
| y-frequency     | 6380 Hz      | 6472 Hz  |
| z-frequency     | 6380 Hz      | 6472 Hz  |
| y-amplitude     | 14.2 μm      | 12.5 μm  |
| z-amplitude     | 1.02 nm      | 0.796 nm |
| Cap. change (ΔC_b) | 0.444 fF    | 0.352 fF |
Table 3. Comparison of the results obtained using lumped parameter simulations for vibratory gyroscopes of figure 2(a) and 2(b).

| Parameter             | Basic design [1] | Modified design |
|-----------------------|------------------|-----------------|
| x-frequency           | 30764 Hz         | 30764 Hz        |
| y-frequency           | 30764 Hz         | 30764 Hz        |
| x-amplitude           | 4.25 μm          | 4.25 μm         |
| y-amplitude           | 1.85 nm          | 1.85 nm         |
| Base cap. \((C_b)\)  | 7 fF             | 265 fF          |
| Cap. change \((\Delta C_b)\) | 1.74 aF | 8.16 aF |

Table 4. Comparison of the results obtained using lumped parameter simulations for vibratory gyroscopes of figure 3(a) and 3(b).

| Parameter             | Basic design [2] | Modified design |
|-----------------------|------------------|-----------------|
| x-frequency           | 41541 Hz         | 41541 Hz        |
| y-frequency           | 41541 Hz         | 41541 Hz        |
| x-amplitude           | 2.7 μm           | 2.7 μm          |
| y-amplitude           | 0.73 nm          | 0.73 nm         |
| Base cap. \((C_b)\)  | 135 fF           | 59.5 fF         |
| Cap. change \((\Delta C_b)\) | 8.57 aF | 29 aF |

5. Conclusions
The in-plane and out-of-plane natural frequencies obtained using lumped parameters of the suspended structures show a very good agreement with those obtained by the simulations in Coventor. The difference in the response amplitudes in drive and sense modes as well as the capacitance change, is due to the difference in the damping values obtained using the closed-form formulae and Coventor’s INTEGRATOR module. The comparison shows that lumped parameter SIMULINK model gives equally good results with fractional effort in modelling and computation. Further, the modification in the sense capacitance of the symmetric decoupled in-plane gyroscope has led to a significant increase in the sensitivity of the device thus leading to an improvement in the resolution.

References
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