The Analysis of Students Mathematical Reasoning in Completing the Word Problem in SMPN 7 Sumedang

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Abstract—Students’ mathematical reasoning ability is important to understand mathematics. This study aims to analyze students' reasoning ability in solving word problems. This research is a descriptive qualitative research with 30 students as the subjects. The results showed that mathematical reasoning ability is close to the low category with an average score of 2.26 with a percentage of 56.5% and this result is in the medium assessment category. The indicator of mathematical reasoning ability investigates the alleged average score of 2.74 with a percentage of 68%, the indicator makes an analogy and generalization of the average score of 2.04 with a percentage of 51.25%, the indicator evaluates the argument and the mathematical evidence of an average score of 2.26 with the percentage of 56.5%, and the indicator validates the logical conclusions about a number of ideas and the relation of the average score of 2.01 to the percentage of 50.25%.

Keywords: mathematical reasoning, word problems

I. INTRODUCTION

Mathematics is one of the compulsory subjects taught in schools. A strong mastery of mathematics from an early age is needed to be able to master and create technology in the future (BSNP, 2006). The process of learning mathematics that optimizes all of students' abilities in the learning process becomes the attention of the world of mathematics education nowadays (Yusepa, Kusumah, Kartasasmita; 2018). According to the National Council of Teachers of Mathematics (NCTM, 2000), there are five standards of the mathematics education process, namely: (1) problem solving, (2) reasoning and evidence, (3) communication, (4) reasoning, and (5) representation. Mathematics serves as a means to practice ways of thinking and reasoning in drawing conclusions and developing the ability to solve problems, both problems in mathematics itself, other fields, and problems in daily life. In accordance with the objectives of mathematics learning and the process of mathematics education, it appears that reasoning is an ability that students need to possess. The Ministry of National Education (2006) further said that the general objectives of learning mathematics are so that: (1) students learn to respect mathematics; (2) students build confidence in their mathematical abilities; (3) students become problem solvers; (4) students learn to communicate mathematically; (5) students learn mathematical reasoning. Mathematical material and mathematical reasoning are two things that cannot be separated; mathematics is understood through reasoning and reasoning is understood and trained through learning mathematics.

Reasoning is defined as a process of thinking specifically thinking logically or thinking to solve problems. Reasoning is the development of something using reason or the process of providing facts with principles based on reason. Krulik, Rudnick, and Milou (2003) revealed that reasoning is part of the thought process, but often thinking and reasoning are used synonymously. The link between thinking and reasoning is presented in Figure 1.1 below.

![Figure 1.1 Hierarchy of Thinking](image-url)

Reasoning is a thinking process that includes basic thinking, critical thinking, and
creative thinking, but does not include recalling. Therefore, reasoning is a thinking process that has certain characteristics, they are: logical thinking patterns or analytical thinking processes. According to Johar (2006), logical thinking patterns mean using certain logic. While analytical is a consequence of certain thinking patterns. This is reinforced by the statement of Subanji (2007); reasoning is a mental/cognitive activity in solving problems by thinking logically and analytically.

Mathematical reasoning is part of mathematical thinking that involves forming generalizations and drawing valid conclusions about ideas and how those ideas are linked. According to Copi (in Sadiq, 2007) Reasoning is an activity or a process of thinking to draw a conclusion or make a new statement based on several statements that are known to be true or that are considered true which are called premises.

According to Brodie K (2010) learning mathematical reasoning is a process that requires teacher guidance and participation of the entire class community. Teacher guidance involves practices that teachers use in class to help students understand mathematics. Mathematical reasoning is a major component of thinking that involves forming generalizations and drawing valid conclusions about ideas and how ideas are related (Artzt & Yaloz, 1999; Peressini & Webb, 1999). Furthermore NCTM (2009) stated that “reasoning is very important for understanding mathematics”. Mathematical reasoning ability is very important to support the success of learning, because of the connection between mathematics and reasoning. This is consistent with the statement of Brodie K (2010) that mathematical reasoning is a key element of mathematics and is very important for learning mathematics in school. According to Yackel and Hanna (2003) Mathematical reasoning is a general activity, which involves methods of induction, deduction, association, and inference as well as how students interact with one another to solve problems.

In reality, junior high school mathematics teachers rarely pay attention to students' mathematical reasoning ability; the lack of junior high school mathematics reasoning ability is a major problem in mathematics education. Francisco & Maher (2010) stated teachers often underestimate students' mathematical reasoning ability, and then the ability is underutilized as a path of mathematical understanding by teachers and students. The ability of mathematical reasoning is still considered difficult and the results are not satisfying. This statement is reinforced by Suryadi (2007) who concluded that mathematics is considered difficult for students to learn and difficult for teacher to teach are justification or proof, problem solving that requires mathematical reasoning, finding generalizations or conjecture and find the relationship between data and facts provided. Suryadi (2005) further emphasized the weaknesses of junior high school students in discovering general patterns or forms and in making generalizations. In solving mathematical reasoning problems, students have difficulty in interpreting problem situations, applying what they know to solve situations they do not face, explaining their thinking, and communicating mathematically that is expected in accordance with the steps of mathematical reasoning.

In general, reasoning is divided into two; they are inductive reasoning and deductive reasoning. Inductive reasoning is a process of reasoning that derives principles or general rules from observing things or case. Whereas deductive reasoning is the process of reasoning from knowledge of principles or general experience that leads us to get conclusions for special cases. Some activities that are classified as Sumarmo inductive reasoning (2013) are: (1) transductive: the process of drawing conclusions from limited observations is applied to certain cases, (2) Analogy: drawing conclusions based on similarity of data processes, (3) generalization: drawing conclusions based on limited data, (4) conjecturing answers, solutions or trends: Interpolation and extrapolation (5) explanation of existing models, facts, properties, relationships, or patterns, (6) using patterns to analyze situations and construct conjectures.

Some of the activities included in Sumarmo deductive reasoning (2013) are: (1) carrying out calculations based on certain rules or formulas, (2) drawing logical conclusions based on inference rules, checking the validity of arguments, proving, and constructing valid arguments, (3) arranging direct proof, indirect proof and proof by induction.

Word problem is a form of question that presents problems related to everyday life in the form of stories (Hartini, 2008). According to Jonassen (Yudharina, 2015) in solving mathematical word problems, it is not just getting results in the form of answers to questions that are asked, but more importantly students must know and understand the thinking process or steps to get those answers. In solving a word problem, it takes a thinking process or systematic steps that begin by translating the word problem: reading, writing what is known, what is being asked, writing down the mathematical model, and then solving the problem and the final step making conclusions.

Based on observations that have been made at SMPN 7 (a public junior high school) in Sumedang shows that students have not been able to solve reasoning questions in the form of word
problem, students are still unable to understand the purpose of the given word problem, having difficulty in translating word problems into mathematical model form so that the reasoning process is running well.

Reasoning indicators according to NCTM (2000) are: (1) making and investigating mathematical conjectures; (2) developing and evaluating mathematical arguments and proofs; (3) exploring phenomena, justifying results, and using mathematical conjectures in all content areas. Furthermore, reasoning indicators or reasoning that must be achieved by students based on Basic Education Regulation No.506 / C / PP / 2004 (Rizqi, NR and Surya, 2017) are: (1) the ability to present mathematical statements verbally, in writing, in pictures, in diagrams, (2) the ability to present validity, (3) the ability to do mathematical manipulation, (4) the ability to organize evidence, provide reasons/evidence for solutions, (5) the ability to make conclusions on statements, (6) checking for argument errors, (7) find patterns or characters from mathematical symptoms to make generalizations.

From the description above, the mathematical reasoning referred to this study is a process of thinking activity with logical thinking patterns or analytical thinking processes to draw conclusions. While the indicators of mathematical reasoning ability used in this study are as follows: (1) making and investigating mathematical conjecture; (2) Making analogies and generalizations; (3) Evaluating arguments and mathematical proofs; and (4) Describing (validating) logical conclusions about a number of ideas and their interrelations.

II. RESEARCH METHODS

This research is a qualitative-descriptive study with the aim of describing students' mathematical reasoning ability in solving word problems. The research subjects are 30 students of eighth grade. The instrument used in this research is mathematical reasoning ability tests. Data analysis includes calculating scores and percentages of mathematical reasoning achievement, grouping data, and drawing conclusions.

III. RESULTS AND DISCUSSION

From the obtained results of mathematical reasoning ability test, for each indicator of mathematical reasoning there are students who did not answer at all but also there are students who answer correctly and provide mathematical arguments such as the assessment criteria below:

In general, the criteria for evaluating mathematical reasoning ability is divided into 5, they are:

Table 1 assessment of mathematical reasoning ability

| Score | Criteria |
|-------|----------|
| 0     | Not answering at all or incorrect answers based on process or argument |
| 1     | Giving answers but only writes what is given in the question, what is asked, makes a mathematical model, and makes a solution with the wrong answer and also without giving arguments |
| 2     | Give the right answer but the solution and argument are still not correct |
| 3     | Give correct answers and arguments but do not give mathematical reasons |
| 4     | Provide answers with the correct arguments but have not provided mathematical reasons |

Table 2 categories of students' reasoning ability achievement

| Category | Achievement of Mathematical Reasoning Ability |
|----------|---------------------------------------------|
| High     | $x_i > 70\%$                                |
| Medium   | $55\% < x_i < 70\%$                         |
| Low      | $x_i \leq 55\%$                             |

Of the four indicators of reasoning, the indicator 'drawing a logical conclusion about a number of ideas and their interrelations' is the most difficult indicator for students. Whereas the 'making and investigating conjectures' is the easiest indicator for students. Overall, the achievement score of mathematical reasoning ability is close to the low category, which is an average score of 2.26 with a percentage of 56.5% and is in the medium assessment category.

The average score of the first indicator ‘making and investigating the conjecture’ is 2.74 with an average percentage of 68% on medium criteria, this means that most students are able to make and investigate conjecture. In this indicator, students generally can write and solve word problems and capture the meaning of each sentence so that it is easy to find the relation between facts, concepts, and principles to complete a conclusion.

The second indicator ‘making the analogy and generalization’: some students can draw conclusions based on the similarity of data and also can determine the general form. The average score of this indicator is 2.04 with a percentage of 51.25% which is a low rating category. It means that on the second indicator students have difficulties so that they cannot solve this problem properly. This is due to students’ difficulty in understanding questions and making conclusions based on the similarity of data and drawing general conclusions based on the data or facts provided.
The third indicator ‘evaluates arguments and mathematical proof’ with an average score of 2.26 with a percentage of 56.5% means that the achievement of mathematical ability is moderate, although the achievement is on the medium criteria but this is almost close to the low criteria. On this indicator, overall students have difficulty in answering questions with the correct answers in accordance with mathematical steps as well as the students have difficulty in expressing an argument because of the lack of understanding of the concept as well as the prerequisite material, it is also difficult to do calculations. This is in line with the opinion of Turmudi (2008) that stated the ability of mathematical reasoning is the ability to make arguments that are important in understanding mathematics.

The fourth indicator ‘validating logical conclusions about a number of ideas and their relevance’ with an average score of 2.01 with a percentage of 50.25% with the achievement of mathematical ability of low criteria. This means that overall students have difficulty in terms of the thinking process that connects the facts or know facts to reach logical conclusions. This occurs because students cannot write mathematically and communicate their ideas, the misunderstanding of the concept and also the prerequisite material consequently all the steps in all arguments must be justified by the previous step so that the reasoning process is not running well. This is in line with the results of research (D’Angelo et al., 2014; Baxter et al., 2005; Burns, 2005) stated writing in mathematics can be used to encourage communication related to mathematical ideas and give students the opportunity to show reasoning.

Overall, in answering questions about mathematical reasoning ability in the form of word problem, the students’ scores are quite varied, starting with a minimum score of 0, students cannot answer at all, do not respond, and there are some students who answer, but the results are not correct in interpreting the questions, to write down what is given, students do not understand what is asked in questions. Score 1; students have been able to translate word problems into mathematical sentences; but students still write the solution wrong and also cannot provide the right reasons. Score 2; students give the correct answer but the completion and arguments were not right. Score 3, students can solve the problem and the reason is correct but the reason is not written mathematically, and score 4; students answer perfectly: the answer and the reason are mathematically correct.

IV. CONCLUSION

Based on the results and discussion that has been described, it can be concluded that the students’ mathematical reasoning ability in solving word problems is close to the low category with a percentage of 56.5% and is in the medium category.

Students’ difficulties in working on word problems are: understanding problems, understanding mathematical concepts, making mathematical models of a word problem, using the right formula, making conclusions based on similarity of data and drawing general conclusions, and drawing logical conclusions.

From the results of this research it is expected that teachers can grow students’ reasoning ability by applying appropriate learning models as well as getting used to giving non-routine questions.

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