On Time-dependent Backgrounds in Supergravity and String Theory

Alex Buchel\textsuperscript{1,*}, Peter Langfelder\textsuperscript{1,2†}, and Johannes Walcher\textsuperscript{1‡}

\textsuperscript{1}Kavli Institute for Theoretical Physics
University of California
Santa Barbara, CA 93106, USA

\textsuperscript{2}C. N. Yang Institute for Theoretical Physics
State University of New York
Stony Brook, NY 11794-3840, USA

Abstract

Time-dependent solutions of supergravity and string theory are studied. The examples are obtained from de Sitter deformation of gauge/gravity dualities, analytical continuation of static solutions, and "exactly solvable" worldsheet models. Among other things, it is shown that turning on a Hubble parameter in the background of a confining gauge theory in four dimensions can restore chiral symmetry. Some of the solutions obtained from analytical continuation have the interpretation of a universe with a bounce separating a big bang from a big crunch singularity. In the worldsheet context, it is argued why string propagation close to a Milne-type cosmological singularity might be physically non-singular.

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\textsuperscript{*}buchel@kitp.ucsb.edu
\textsuperscript{†}peter.langfelder@sunysb.edu
\textsuperscript{‡}walcher@kitp.ucsb.edu
1 Introduction and Outline

The understanding of space-like singularities is a timely question. More broadly, one of the important open problems in string theory is the correct treatment of time-dependent backgrounds. It is generally hoped that there is a way in which string theory will solve the conceptual and technical problems associated with time dependence in a full fledged quantum mechanical and gravitational theory. In this context, the practical point of view, that we shall adopt, is to neglect the conceptual difficulties and to try to naively extend the ordinary rules of the game to the time-dependent setting.

Consider for instance spacetime singularities. In recent years, motivated and guided by the AdS/CFT correspondence, many new lessons have been learned about time-like singularities and their resolution in supergravity and string theory. Firstly, the goal of exhibiting new tests of the duality has led to the construction of many new exact supergravity backgrounds. This includes metrics on non-compact Calabi-Yau spaces with and without fluxes, $G_2$-holonomy metrics, etc. Secondly, the correspondence with gauge theory has hinted at the correct interpretation and resolution of singularities. The mechanisms include confinement, chiral symmetry breaking, generation of a mass gap, etc. These recent advances have complemented the existing knowledge about singularities from closed strings and D-branes. It is not unreasonable to hope that these lessons will also teach us something about space-like singularities.

In this paper, we collect a number of ideas on the treatment of time-dependent problems in string theory. One way of introducing time dependence in the context of the gauge/gravity duality was suggested in [16]. The basic idea of [16] (we will review this in detail in section 2) was to consider a deformation of the ordinary correspondence by turning on a Hubble parameter (cosmological constant), so that the gauge theory effectively lives on a de Sitter space. In the conformal case, this translates simply into a different slicing of the dual AdS space. But in non-conformal examples such as $\mathcal{N} = 1$ SYM in four dimensions, there is an interesting interplay between the scale of the theory $\Lambda$ and the Hubble scale $H$. For instance, for $H \ll \Lambda$, the theory is a minor deformation from the flat case. On the other hand, for $H \gg \Lambda$, one observes the more drastic phenomenon of restoration of chiral symmetry. In this respect, the de Sitter deformation of the correspondence is very similar to turning on a finite temperature. In [16], these effects were studied for the de Sitter deformation of the Klebanov-Strassler background, which realizes $\mathcal{N} = 1$ SYM theory by D5 and D3 branes on the conifold.
In this paper, in section 3, we will consider the (1, 1) little string theory realizing NS5-branes wrapped on \( S^2 \). We give a careful discussion of the physics involved and also study chiral symmetry restoration in the de Sitter background.

In sections 4 through 6, we give a somewhat more general discussion of time-dependent supergravity backgrounds, focusing on those that can be obtained from analytical continuation of known static solutions. This is motivated in part by the works in refs. [24] and [25]. In [24], it was proposed to construct time-dependent backgrounds of string theory as spacelike branes. These S-branes are defined as the gravitational backreaction of the decay of unstable D-brane systems. However, the explicit solutions found in [24], and later in [26, 27] (see [28, 29] for earlier work on such solutions, recent related work includes [30, 34]), are singular, and the singularities not well understood. As we will see in section 3, some of these solutions are actually best thought of as analytical continuations of ordinary D-branes, or \( iD \)-branes, for short. In this respect, this is an alternative to the strategy used in [25], where the analytical continuation of ordinary black holes was considered. A different type of analytical continuation is discussed in section 5. Explicitly, we will present a method using T-duality at an intermediate step that allows the analytical continuation of backgrounds with non-zero (NS-NS) flux.

Finally, in section 7, we enter the third current circle of ideas for studying time dependence in string theory, using the exact worldsheet approach. The most prominent example is probably the so-called Milne universe, which is a \( \mathbb{Z} \) orbifold of two-dimensional Minkowski space. The Milne universe owes its physical interest to its rôle in ekpyrotic cosmology [33], and has been studied in a number of recent works [36, 37, 59], see also [40, 44] for recent work on related null orbifolds. It turns out that the singularity of Milne type also appears in an effective description of certain gauged WZW models (see, for instance, [53, 54]), which have been intensively studied in the past [46–52]. Here, using this connection, we present an observation that can help explain the non-singularity of string propagation near Milne-type singularities. The basic physical intuition is that near a spacetime singularity, the most relevant physical states are the lightest ones. At a Milne-type singularity probed by strings, these states are the winding modes of the string. One then observes that the effective background metric seen by winding modes in the gauged WZW is less singular than that seen by ordinary particles.

We will end the paper in section 8 with a brief summary.
De Sitter deformation of the gauge/gravity correspondence

The construction of time-dependent backgrounds for string theory is a rather complicated problem. As in the static case, one possible starting point is a solution of the corresponding supergravity equations of motion. Then, given a supergravity solution, one should check whether the truncation of the full string theory in this background to the supergravity modes was consistent at all. This necessitates small curvatures and small string coupling. One then has to face the problems of singularities and the classical and quantum stability of the background.

In [16] it was proposed to use the deformation of the gauge/gravity correspondence of Maldacena [1] (see [2] for a review) as a tool for the construction of a large class of consistent time-dependent string theory backgrounds. The basic idea is to replace the flat spacetime background on which the gauge theory is defined by a genuinely time-dependent background, while still keeping gravity on the brane non-dynamical. This is a deformation of the gauge theory in the generalized sense as explained, for instance, in [15]. On the gravity side, this deformation requires a change in the asymptotics of the supergravity background. In fact, all of the wrapped brane solutions also belong to the class of deformations in which one replaces the flat gauge theory background with a general curved space. The major simplification, compared to the direct approach outlined above, comes from the fact that the duality allows us to think about a certain class of supergravity backgrounds in terms of a dual strongly coupled gauge theory. In particular, we can recast questions of singularities, stability, consistent truncation to the supergravity modes, etc., in the dual gauge theory language where, as we argue below, they appear to be more tractable.

More specifically, consider the simplest gauge/gravity duality realized [1] by a system of $N$ D3-branes in a flat type IIB string theory background. At small 't Hooft coupling $g_sN \ll 1$, the system is best described by open strings and realizes $SU(N)$ $\mathcal{N} = 4$ supersymmetric gauge theory. In the limit of strong 't Hooft coupling this gauge theory has a perturbative description as type IIB supergravity on $AdS_5 \times S^5$, with $N$ units of RR 5-form flux through the $S^5$. Being an exact correspondence, the duality guarantees that any consistent (physical) deformation on the gauge theory side visible in the large $N$ limit should translate into a consistent background on the su-

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$^1$An interesting alternative approach to generate time-dependent backgrounds by exploiting the supergravity duals to confining gauge theories was recently discussed in [45].
pergravity side. Now the simplest way to introduce time dependence is to deform the correspondence by turning on a non-zero cosmological constant $H \neq 0$ on the gauge theory side. In other words, we wish to replace the flat four-dimensional Minkowski space $\mathcal{M}_4$ by four-dimensional de Sitter space $\mathcal{M}_4^H$, with the metric

$$
(ds_{\mathcal{M}_4})^2 = -dt^2 + dx^2 \rightarrow (ds_{\mathcal{M}_4^H})^2 \equiv -dt^2 + \frac{1}{H^2} \cosh^2 Ht \, d\Omega_3^2.
$$

We expect that much like in the usual correspondence, the supergravity dual to the gauge theory on $\mathcal{M}_4^H$ will feature $\mathcal{M}_4^H$ in the asymptotics of the type IIB supergravity background. The background must therefore be genuinely time dependent. Provided that the gauge theory on $\mathcal{M}_4^H$ is a consistent quantum field theory, the resulting time-dependent supergravity background will be consistent as well.

Unfortunately, our understanding of interacting quantum field theories (QFT) in general time-dependent backgrounds is still rather rudimentary. (For recent developments see [18,19].) Thus, it appears that we are replacing a difficult question of classical consistency of certain time-dependent supergravity backgrounds with an equally complicated (and unsolved) problem in the interacting QFT. This is true for example for the de Sitter deformation of the gauge/gravity correspondence for the $\mathcal{N} = 4$ gauge theory described above. It is not obvious that the $\mathcal{N} = 4$ gauge theory exists as a consistent QFT in the de Sitter background. Moreover, the deformed supergravity dual appears to be simply a different slice of the original $AdS_5 \times S^5$ background [16]. The difficulty for finding a simple plausibility argument for the consistency of this gauge theory on four-dimensional de Sitter space can be traced back to the fact that this is an interacting QFT in which scale invariance has been broken by the introduction of a single scale, $H$. Therefore, it is not easy to evaluate the importance of the effects on the IR and UV dynamics. As a result, it is not clear that $H \neq 0$ is a “smooth deformation” of the $H = 0$ duality.

Fortunately, one can argue that the situation is not so grim with the deformed duality for a non-conformal field theory which already has an intrinsic scale $\Lambda$. Let us assume that this gauge theory on $d$-dimensional Minkowski spacetime $\mathcal{M}_d$ is a well-defined QFT at energy scales much larger than $\Lambda$. Since the background metric deformation due to $H \neq 0$ in (2.1) does not affect short-distance properties of the metric, we expect that local field theories on $\mathcal{M}_d^H$ should have the same ultraviolet dynamics as those on $\mathcal{M}_d$.

Rather, it seems that the Hubble scale $H$ should be thought of as an infrared cutoff
on the gauge theory dynamics. This is supported by the non-conformal examples of the de Sitter deformed gauge/gravity duality discussed in [16]. Indeed, de Sitter deformation resolves the infrared singularity in the supergravity dual to (1,1) little string theory (LST; for a review of LST, see, e.g., [4]) and to Klebanov-Tseytlin $\mathcal{N} = 1$ gauge theory [5]. These resolutions are similar to the finite-temperature deformations discussed in [13] and [14], respectively. Ultimately, this is not a surprise since a vacuum state in an accelerating universe has a nonzero Gibbons-Hawking temperature $T_{\text{GH}} = H/2\pi$, analogous to the Hawking temperature of a black hole. The identification of $H$ with an infrared cutoff on the gauge theory is also supported by the conclusions reached in related studies of de Sitter gravity in warped compactifications of type IIB string theory [17].

Furthermore, gauge theories with a finite mass gap $m_{\text{gap}} \neq 0$ for $H = 0$, should remain stable when defined on $\mathcal{M}_d^H$, provided $H/m_{\text{gap}}$ is sufficiently small. The argument is identical to the one given in [12]. The original dual supergravity background had a mass gap, and thus a small deformation (which can be smoothly turned off) should not produce a tachyonic instability.

Altogether, these arguments suggest that non-conformal gauge theories with a mass gap, defined on $\mathcal{M}_d^H$, would have the same IR and UV properties as their $H = 0$ cousins, provided $H \ll \min\{m_{\text{gap}}, \Lambda\}$. This singles out such theories as toy models for studies of consistent time-dependent backgrounds in string theory from the perspective of the deformed gauge/gravity duality.

Basically, there are three approaches for constructing supergravity duals to confining four-dimensional gauge theories: a mass deformation of $\mathcal{N} = 4$ SYM [6], the gauge theory on the world-volume of regular and fractional D3-branes [7], and the effective low-energy $d = 4, \mathcal{N} = 1$ SYM theory arising from the compactification of (1,1) LST on $S^2$ [8]. These theories have a consistent UV completion and a mass gap. Thus, according to the above arguments, we expect that the de Sitter deformation of any of these backgrounds would give rise to a consistent time-dependent string theory background. By far the simplest of these backgrounds is the one discussed by Maldacena and Nuñez (MN) [8]. In the next section we discuss de Sitter deformations of the MN background. We demonstrate that the Hubble parameter $H$ can be smoothly turned off. The deformed geometry has small curvatures, small string coupling, and asymptotically (the ultraviolet from the perspective of the dual gauge theory) approaches the BPS solution of MN. For reasons explained above, we expect it to be stable. We also
comment on the chiral symmetry restoration phase transition for MN gauge theory in de Sitter space, relegating a few technical details to the appendix.

3 De Sitter deformation of the Maldacena-Nuñez background

We begin with a brief review of the MN construction for the supergravity dual to four dimensional \( \mathcal{N} = 1 \) \( SU(n) \) super Yang-Mills theory [8].

Recall that (1,1) little string theory [3,4] is realized on the world-volume of NS5-branes in type IIB string theory. The theory has sixteen supercharges, and for \( n \) NS5-branes reduces in the IR to six-dimensional \( \mathcal{N} = (1,1) SU(n) \) SYM. Consider wrapping NS5-branes on \( S^2 \). Such a compactification will completely break the supersymmetry unless the supersymmetry generators are appropriately twisted. The correct twisting is obtained by embedding the \( SO(2) \) spin connection of \( S^2 \) into the \( SO(4) \sim SU(2)_R \times SU(2)_L \) R-symmetry group of the flat LST. It is easy to show that by identifying \( U(1) \) connection of the \( S^2 \) with a \( U(1)_R \subset SU(2)_R \subset SO(4) \) we preserve four supercharges, or \( \mathcal{N} = 1 \) SUSY in the remaining four flat directions. Furthermore, the four massless scalars representing transverse zero modes of flat NS5-branes upon twisting become spinors on the \( S^2 \). Thus the only massless mode in the IR are the \( d = 4 \) gauge fields and the gauginos.

The supergravity dual [8] to this theory has the string frame metric

\[
    ds_{st}^2 = dx_4^2 + n \left( d\rho^2 + G^2 d\Omega_2^2 + \frac{1}{4} \sum_a (\omega_a - A_a)^2 \right), \quad (3.1)
\]

where \( \Omega_2^2 \) is a round \( S^2 \) (parameterized by \( (\tilde{\theta}, \tilde{\phi}) \)) which the branes wrap, and \( \omega_a \) are the \( SU(2) \) left-invariant one forms on the \( S^3 \) (parameterized by \( (\theta, \phi, \psi) \)) transverse to the NS5-branes,

\[
    \omega_1 = \cos \phi \, d\theta + \sin \phi \sin \theta d\psi \\
    \omega_2 = -\sin \phi \, d\theta + \cos \phi \sin \theta d\psi \\
    \omega_3 = d\phi + \cos \theta d\psi.
\]

Also in (3.1), \( A_a \) are the \( SU(2)_R \) gauge fields on the \( S^2 \) realizing the twist,

\[
    A_1 = a \, d\tilde{\theta} \\
    A_2 = -a \sin \tilde{\theta} \, d\tilde{\phi} \\
    A_3 = \cos \tilde{\theta} \, d\tilde{\phi}. \quad (3.3)
\]
Finally, there is a dilaton $\Phi = \ln g_s$, and an NS-NS 3-form flux

$$H_3 = n \left[ -\frac{1}{4}(\omega_1 - A_1) \wedge (\omega_2 - A_2) \wedge (\omega_3 - A_3) + \frac{1}{4} \sum_a F_a \wedge (\omega_a - A_a) \right],$$

(3.4)

where $F_a = dA_a + \epsilon_{abc}A_b \wedge A_c$. Altogether, the background is parameterized by three functions $G, a, g_s$ of the radial coordinate $\rho \in [0, +\infty)$. The type IIB supergravity equations of motion can be solved analytically in this case with the result

$$G^2 = \frac{1}{4} \left( 4\rho \coth(2\rho) - \frac{4\rho^2}{\sinh^2 2\rho} - 1 \right),$$

$$g_s = g_0 \left[ \frac{2G}{\sinh 2\rho} \right]^{1/2},$$

$$a = \frac{2\rho}{\sinh 2\rho},$$

(3.5)

where $g_0$ is a free parameter. The background (3.1)-(3.5) is nonsingular, has small curvatures in $\alpha'$ units, and a small string coupling provided $n \gg 1$. It correctly reproduces the IR properties of the dual gauge theory: confinement, chiral symmetry breaking, and a mass gap in the spectrum. In the UV (large $\rho$), the leading asymptotics of the solution (3.5) are those of the flat LST.

**De Sitter deformation**

To study the de Sitter deformation of the MN model, we replace, following

$$dx_4^2 \quad \rightarrow \quad F^2 \left[ -dt^2 + \frac{1}{H^2} \cosh^2 Ht \ d\Omega_3^2 \right],$$

(3.6)

in the string frame metric (3.1). The physical reason for introducing an extra warp factor $F \equiv F(\rho)$ is analogous to the appearance of a warp factor $G$ in the holographic dual to LST on $S^2$. In the MN model, the size of the $S^2$, which is fixed from the LST perspective, becomes $\rho$-dependent in the supergravity dual. This is simply due to the holographic interpretation of the radial coordinate $\rho$ as the RG scale at which we probe the gauge theory, with large values of $\rho$ corresponding to the UV (or short distances) in the gauge theory. The growth of the corresponding $S^2$ in the supergravity dual simply reflects the fact that we “measure” little string theory on an $S^2$ of fixed size with shorter and shorter reference scales. Also, the decompactification of the $S^2$ in the supergravity at $\rho \rightarrow \infty$ is necessary to match the flat LST UV limit of the MN model. Clearly, the same must be true for the $H$-deformation. A fixed scalar curvature
(= 12H^2) of the gauge theory background dS_4 must “flow” in the holographic dual. It must approach zero as \( \rho \to \infty \), so we expect \( F(\rho \to \infty) \to \infty \).

Finally, we assume the same ansatz for the 3-form flux (3.4), and \( g_s \equiv g_s(\rho) \). With this ansatz, the type IIB supergravity equations of motion for the deformed MN model are reduced to

\[
0 = \left[ \frac{a'F^4}{g_s^2} \right]' - \frac{aF^4(a^2 - 1)}{g_s^2 G^2}
\]

\[
0 = \frac{(G^2)'F^4}{g_s^2} + \frac{F^4}{2g_s^2 G^2} \left\{ (a^2 - 1)^2 + G^2[(a')^2 - 4] \right\}
\]

\[
0 = \left[ G^2 F^4 \left( \frac{1}{g_s^2} \right)' \right]' - \frac{F^4}{4g_s^2 G^2} \left\{ (a^2 - 1)^2 + 2G^2[8G^2 + (a')^2] \right\}
\]

\[
0 = \left[ \frac{(F^4)'G^2}{g_s^2} \right]' - 12nH^2 F^2 G^2.
\]

(3.7)

There is also a first order constraint coming from fixing the reparametrization invariance (the choice of \( \rho \)),

\[
0 = F^2 \left\{ 2G^2 \left[ 8G^2 (g_s')^2 + 4g_s^2(G')^2 - 4(G^2)'(g_s^2)' - g_s^2(a')^2 \right] 
\]

\[
+ g_s^2 \left[ (a^2 - 1)^2 - 8G^2(1 + 2G^2) \right] \right\} + 16g_sG \left\{ 3g_sG \left( (F')^2 - nH^2 \right) 
\]

\[
+ 2(F')^2 g_s^2 \left( \frac{G'}{g_s} \right)' \right\}.
\]

(3.8)

Notice that (3.3) solves (3.7), (3.8) with \( F \equiv 1 \) and \( H = 0 \). We have not been able to find an analytical solution to (3.7), (3.8) for \( H \neq 0 \), so we proceed with the asymptotic analysis and numerical interpolation.

There are two classes of regular solutions to (3.7), (3.8), representing physically distinct deformations of the MN background. With a radial coordinate \( \rho \in [0, \infty) \) they are realized as different boundary conditions for the warp factors \( F \) and \( G \). Namely, the two possibilities are

\[
(a) : \quad F(\rho = 0) = 0 \quad G(\rho = 0) \neq 0
\]

\[
(b) : \quad F(\rho = 0) \neq 0 \quad G(\rho = 0) = 0.
\]

(3.9)

As we show below, to obtain a nonsingular solution with the boundary condition (b) we must turn on nonabelian gauge fields (3.3) (i.e. \( a(\rho) \) is nontrivial), as opposed

\[ \text{The prime denotes derivative with respect to } \rho. \]
to turning on $A_3$ only. This is easily interpreted if we recall what happens in the
supersymmetric solution, $H = 0$, $F = 1$. There, non-singularity requires turning on
a non-trivial $a$, which breaks the $\mathbb{Z}_{2N} \subset U(1)_\phi$ corresponding to shifts in $\phi$ in (3.2)
down to $\mathbb{Z}_2 \subset U(1)_\phi$. (The first breaking to $\mathbb{Z}_{2N}$ is due to worldsheet instantons.)
The result is in complete agreement with the dual low energy effective $SU(N)$ SYM,
where the $U(1)_\phi$ is identified with the chiral $U(1)_R$ symmetry, broken by the gaugino
condensate to a $\mathbb{Z}_2 \subset \mathbb{Z}_{2N} \subset U(1)_R$ in the IR, [8]. Thus we identify boundary condition
(b) in (3.9) as corresponding to $\mathcal{N} = 1$ $SU(N)$ SYM theory in $dS_4$ in a phase of
spontaneously broken chiral symmetry. It is this phase of the gauge theory that we
expect to correspond to a consistent time-dependent string theory background in the
supergravity dual, at least for small $H$.

What is the physics corresponding to the boundary condition (a) in (3.9)? From
the gauge theory perspective, increasing $H$ should raise the IR cutoff on the dynamics.
Thus, for $H$ large enough we would expect restoration of the chiral symmetry, similarly
to finite temperature restoration of the chiral symmetry in the MN model demonstrated
in [10, 11]. Indeed, this is precisely what is realized by the de Sitter deformed MN
model with boundary conditions (a). We delegate the analysis to Appendix A, and
only mention the results here. Unlike the case of boundary condition (b), here there
are regular solutions to (3.7) both for nontrivial $a$ and for $a \equiv 0$. We don’t have much
to say about solutions with nontrivial $a$. They break chiral symmetry, and since they
do not exist in the $H \to 0$ limit, they can not be identified with excitations of the MN model.

The solutions with boundary conditions (a) and $a \equiv 0$ are much more interesting.
They exist as a globally regular solutions only for $H > H_{\text{critical}}$, where $H_{\text{critical}}$ depends
on the radius of the $S^2$ on which the NS5-branes are wrapped. These solutions thus
realize the restoration of the chiral symmetry of the gauge theory on the de Sitter
background in the supergravity context. Finally, we mention that we have found a
special solution for $H = H_{\text{critical}}$. The details can be found in Appendix A.

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3In the original MN model there is a large class of regular non-BPS excitations with both finite
and infinite four-dimensional energy density, and broken chiral symmetry [11]. Here, we are finding
that there is also an additional large set of globally regular solutions with broken chiral symmetry,
once the Hubble parameter is turned on, $H \neq 0$. 

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**Broken Phase**

We continue with the analysis of boundary conditions (b) in (3.9). For $F(\rho = 0) \neq 0$, by rescaling the Hubble parameter $H$ we can always set $F(\rho = 0) = 1$. There is then a three-parameter family $\{H, A, g_0\}$ of regular solutions at $\rho \to 0$, with asymptotics

$$F = 1 + \frac{nH^2}{2} \rho^2 + O(\rho^4)$$

$$G^2 = \rho^2 \left(1 + \left(-\frac{2}{9} - \frac{2nH^2}{3} - \frac{A^2}{2}\right) \rho^2 + O(\rho^4)\right)$$

$$g_s = g_0 \left(1 - \frac{4 + 3A^2}{12} \rho^2 + O(\rho^4)\right)$$

$$a = 1 + A \rho^2 + O(\rho^4).$$

(3.10)

These solutions are continuously connected to the supersymmetric, nonsingular MN solution, for which

$$\{H, A, g_0\}_{MN} = \left\{0, -\frac{2}{3}, g_0\right\}.$$

(3.11)

Here, $g_0$ is the string coupling at $\rho = 0$. In addition to this supersymmetry preserving solution (3.11), there is an infinite discrete sequence of non-BPS excitation in the MN model with finite four-dimensional energy density [11], $\{H, A, g_0\} = \{0, a_k, g_0\}$, $k = 0, 1, \ldots$ where $a_k \in (-2, -2/3]$ and $a_0 = -2/3$. The generic solution $\{H, A, g_0\} = \{0, A, g_0\}$ has infinite four-dimensional energy density [11].

In general, the boundary conditions (3.10) describe de Sitter deformations of globally regular (non-BPS) excitation of the MN model. To see this, let us look at the $\rho \to \infty$ asymptotics of a generic solution to (3.7) with $G(\rho \to +\infty) \to +\infty$,

$$F = (3nH^2 \rho)^{1/2} + \cdots$$

$$G^2 = \rho + \cdots$$

$$g_s = g_0 \left(\rho^{3/4} e^{-\rho} + \cdots\right)$$

$$a = \Upsilon \rho^{-1/2} (1 + \cdots) + C \rho^{1/2} e^{-2\rho} (1 + \cdots),$$

(3.12)

where $\cdots$ denote corrections which are subdominant as $\rho \to \infty$. We have confirmed by numerical integration that $G(\rho \to \infty) \to +\infty$, provided we start integrating with the boundary condition (b) in (3.9). In this way, the integration constants $\Upsilon$ and $C$ in (3.12) become functions of $H, A$ in (3.10).

Now apart from $F$ (which we argued on physical grounds must diverge as $\rho \to \infty$), all the remaining functions have the same leading asymptotics as a generic $\{0, A, g_0\}$.
solution \([\text{[1]}]\). In fact, apart from \(a\), the leading asymptotics coincide with the BPS solution of MN. Moreover, the gauge potential \(a\) has the same leading asymptotics as in the MN solution provided

\[ \Upsilon = 0. \tag{3.13} \]

In \([\text{[1]}]\), the condition (3.13) was found to be necessary for the non-BPS excitation to have finite four-dimensional energy density. In other word, the discrete set of parameters \(a_k\) are just the zeros of \(\Upsilon(H = 0, A)\),

\[ \Upsilon(H = 0, A = a_k) = 0, \quad k = 0, 1, \ldots. \tag{3.14} \]

However, this “energy” of the gravitational background was computed in \([\text{[1]}]\) using the general definition of \([\text{[23]}]\), valid for static backgrounds. In our time-dependent case, it is not clear why we should impose (3.13) based on considerations of energy, which is not a conserved quantity in de Sitter space. From the perspective of getting a consistent time-dependent string theory background what we need is that the de Sitter deformed geometry smoothly reduces to the supersymmetric MN background as \(H \to 0\). This will be the case for \(\text{any} \ A = -\frac{2}{3} + O(nH^2)\). Generically, \(\Upsilon(H, A = -\frac{2}{3} + O(nH^2)) = O(nH^2) \neq 0\). Actually, it must in principle also be possible to modify \(A = -2/3\) perturbatively in \(nH^2\) to enforce (3.13) for \(H \neq 0\). We verified this numerically for \(nH^2 \sim 1\). The important point is that even without this condition, all our solutions reduce for \(H = 0\) to regular excitations of the MN model.

To summarize, the analyticity in \(H\) of the deformed equations (3.7) and of the boundary conditions (3.10) guarantees that for \(nH^2 \ll 1\), our solutions will be regular deformations of the non-singular MN solution, and thus should describe consistent (and in particular stable) time-dependent backgrounds of string theory. The same analyticity argument implies that for \(nH^2 \ll 1\), the supergravity approximation for the deformed MN geometry is valid if the supersymmetric MN background is a valid supergravity background. The same applies to string loop corrections. They are small in the deformed geometry, provided they were small to begin with.

An interesting open question is what will happen with de Sitter deformation of the non-singular MN solution for \(nH^2 \gg 1\)? Recall that if we deform starting from the non-singular solution, chiral symmetry remains broken. On the other hand, from the dual gauge theory perspective, we expect chiral symmetry restoration in the regime \(nH^2 \gg 1\). Such chirally symmetric solutions indeed exist—they are just solutions with
boundary condition in (3.1). In Appendix A we argue that the chirally symmetric solution exist only above a certain critical value of the Hubble parameter $H_{\text{critical}}$. We identify this as a signature of chiral symmetry restoration in the de Sitter deformed MN model. But: Can this phase transition be seen from the broken phase as well? And if so, what is the critical Hubble parameter in this case? We hope to address these issues in a future publication.

**Discussion**

We have shown here that the gauge/gravity correspondence of Maldacena for UV-consistent gauge theories with a mass gap in the IR is a convenient starting point for constructing consistent time-dependent string theory backgrounds. Unfortunately, this approach is of little practical use in attempts to generate phenomenologically viable cosmologies. The reason for this is a technical problem rather than a conceptual one. One of the simplifications that allowed us to study the de Sitter deformation of the MN model in some detail was that the resulting type IIB equations of motion were ordinary differential equations. We will not go into details on this here, but it is straightforward to verify that the only time-dependent deformation of the Minkowski background which does not introduce time-dependence in various warp-factors in the geometry or for the fluxes and the dilaton are the gauge theory metric deformations

$$ds^2 \rightarrow -dt^2 + e^{2Ht} \, ds^2_{R^d,1},$$

$$(3.15)$$

The first of these is just a de Sitter deformation in the accelerating patch, the second is the de Sitter deformation we have studied here. For generic time-dependent deformations we find, for instance, that the off-diagonal $\rho-t$ component of the Ricci tensor (vanishing in the original background) becomes nonzero. Once time dependence is required in warp factors (in addition to the radial dependence), the equations of motion become partial differential equations. Analyzing the solutions becomes then much more challenging. Thus for all practical purposes “time-dependent” backgrounds from deformations of the gauge/gravity correspondence are actually “de Sitter” backgrounds. But the Universe we live in was not de Sitter at all times! For one reason, the inflation should have stopped (or substantially slowed down) over a period of time to allow the large scale structure formation we observe today. So ultimately, for constructing time-dependent backgrounds with potential phenomenological interest, we will have to
consider alternative methods.

4 Time dependence from analytical continuation

In the previous section we discussed time-dependent backgrounds for string theory from the perspective of the deformation of the Maldacena gauge/gravity correspondence. The presence of a dual gauge theory description of the backgrounds, which here is a gauge theory formulated in de Sitter space, allowed for a simple physical argument for the consistency of the constructed supergravity backgrounds. We explained technical difficulties arising from the attempts of extending this construction to more generic time-dependent string theory backgrounds, as would be desirable for phenomenological reasons. More promising in this regard is the approach recently followed in [25], where it was shown that a double analytical continuation of a higher-dimensional Kerr black hole yields a cosmological solution with a de Sitter phase ending in a Milne phase at late times.

By itself, analytical continuation of a given string theory background does not provide insights into its physical properties, such as stability and consistency of string propagation. Nonetheless, analytical continuation is a very useful technical tool for (at least) constructing time-dependent solutions of the supergravity equations of motion. In the following two sections, we will take this approach and ask what other types of “analytical continuation” we can perform, and what are the resulting time-dependent backgrounds. We concentrate mainly on construction techniques, rather than attempt to present an exhaustive study of the proposed time-dependent backgrounds.

Incidentally, the de Sitter deformations we have studied in the previous section can also be thought of as analytical continuation of the gauge theory background after compactification, in the sequence\(^4\)

\[
R^{d,1} \rightarrow S^{d+1} \rightarrow dS_{d+1}. \tag{4.1}
\]

But while the authors of [25] generated new solutions as analytical continuations of previously known ones, the chain (4.1) requires finding a new supergravity solution at the first step, because of the compactification.

One reason that the analytical continuations in [25] were rather straightforward is that the backgrounds discussed there were purely gravitational. A generic supergravity

\[^4\]This has also been observed by Arkady Tseytlin.
solution is supported by fluxes, and typically, the naively analytically continued metric ends up being supported by imaginary fluxes, to which it is hard to assign a physical meaning. In the next section we demonstrate how beginning with a certain class of backgrounds with fluxes one can perform an analytical continuation resulting in backgrounds with only real fluxes. The additional ingredient that allows achieving this is T-duality or, more generally, mirror symmetry.

The idea is to exploit the fact that supersymmetric string compactification are characterized by (complexified) Kähler moduli and complex structure moduli, which are exchanged under T-duality or mirror symmetry. The only part that can involve fluxes is the real part of the Kähler moduli, which is the NS-NS two-form potential. Therefore, if the original complex structure moduli were purely imaginary, the dual background could be purely gravitational. Of course, to really produce a non-trivial flux this way, the original geometry has to involve a certain fibration, with Kähler moduli varying over some base. Also note that in thinking about the dual space in terms of a purely metrical background, we are neglecting all stringy (e.g., worldsheet instanton) corrections that are crucial for mirror symmetry. The basic example for all this is the NS5-brane in type IIB string theory, which is T-dual to a certain type IIA background involving $\mathcal{N} = 2$ minimal models without fluxes [34].

Given such a geometry, $\mathcal{W}$, that is dual to a background with fluxes, $\mathcal{M}$, we can perform naive analytical continuation resulting in a time-dependent, purely gravitational background (neglecting stringy corrections), $i\mathcal{W}$. Finally, dualizing back will result in a new time-dependent background with real fluxes, $\tilde{\mathcal{M}}$. Schematically, we have,

$$\mathcal{M} \rightarrow \mathcal{W} \rightarrow i\mathcal{W} \rightarrow \tilde{\mathcal{M}}.$$  \hspace{1cm} (4.2)

We make this explicit in section 5, where we perform the sequence of transformations (4.2) starting from the exactly soluble string theory background describing the throat geometry of the near-extremal flat NS5-branes in type IIB string theory.

Another procedure for constructing time-dependent solutions can be described as follows. In general, the difficult step in finding supergravity solutions is to find an ansatz and gauge that can be solved by solving reasonable (i.e., ordinary) differential equations. Usually, the solutions of these equations become complex upon analytical continuation. But if instead of continuing the solutions, one continues the original ansatz, one can look for real (physical) solutions of the resulting analytically continued
equations. In terms of the original equations, these are of course imaginary solutions, in general. We will follow this procedure in section 6, where we discuss the analytical continuation of the usual D-brane solutions. The supergravity equations of motion describing flat D-branes are ordinary differential equations in the radial coordinate transverse to the brane. In the continuation, this radial coordinate becomes the time direction. We will refer to the solutions of these equations as \( iD \)-branes. Some of these solutions are special cases of the time-dependent solutions recently discussed in \([26]\). We should also point out that these solutions can not be S-branes in the sense introduced in \([24]\). Certain aspects of obtaining genuine spacelike branes as the decay process of unstable D-branes will be discussed in a separate publication \([56]\).

5 Analytical continuation via T-duality

In this section, we illustrate the idea of analytical continuation of supergravity backgrounds with fluxes following steps (4.2) with the example of the throat (near-horizon) geometry of the near extremal NS5-branes in type IIB string theory.

The relevant NS5-brane geometry was described in \([13]\). The string frame metric is

\[
\begin{align*}
\text{(in units where } \alpha' = 1), \\
d s^2_{st} &= d x_5^2 + k \left( - \tanh^2 \rho \, d t^2 + d \rho^2 \right) + k \, d \Omega_3^2, 
\end{align*}
\]

where we take the \( S^3 \) metric

\[
\begin{align*}
d \Omega_3^2 &= d \theta^2 + \cos^2 \theta \, d \psi^2 + \sin^2 \theta \, d \lambda^2.
\end{align*}
\]

There is also NS-NS two-form potential \( B_2 \) and dilaton \( \Phi \)

\[
\begin{align*}
B_2 &= k \sin^2 \theta \, d \lambda \wedge d \psi, \\
e^{2\Phi} &= \frac{k}{\mu \cosh^2 \rho}.
\end{align*}
\]

In above equations, \( k \) is the number of NS5-branes, and \( \mu \) is related to the energy density above the extremality. The radial coordinate \( \rho \in [0, \infty) \). String propagation in this geometry corresponds to an “exact conformal field theory”

\[
C_1 \times C_2 \times C_3 \equiv [SL(2, \mathbb{R})/U(1)]_k \times SU(2)_k \times \mathbb{R}^5,
\]

where the coset \( C_1 \)

\[
[SL(2, \mathbb{R})/U(1)]_k
\]
parameterized by \((\rho, t)\) in (5.1), is the 2-d black hole of [46]; the 3-sphere \((\theta, \psi, \lambda)\) along with the \(B_2\)-field is described by the \(C_2 \equiv SU(2)_k\) WZW model; and \(\mathbb{R}^5\) is a free CFT describing noncompact directions along the NS5-branes. The central charges of the three conformal theories in (5.4) are

\[
\left[\frac{3(k+2)}{k} - 1 + 2 \cdot \frac{1}{2}\right] \times \left[\frac{3(k-2)}{k} + 3 \cdot \frac{1}{2}\right] \times \left[5 \cdot \frac{3}{2}\right] = 15 ,
\]

respectively. The above background is our starting point, \(M\), in (4.2).

Notice that we cannot perform any analytical continuation of \(M\) directly. For instance, consider the naive analytical continuation \(\rho \rightarrow i\rho\). This is actually a continuation of the coset CFT \(C_1\) to a negative level \(k \rightarrow -k\). As a result, the total central charge of the background would no longer be 15, unless we continue the WZW CFT to negative level as well. However, the latter continuation must be accompanied by an analytical continuation of the 3-sphere \(S^3 \rightarrow H_3\),

\[
(\theta, \psi, \lambda) \rightarrow (i\theta, i\psi, \lambda) ,
\]

in order to have a single time direction. But now, from (5.3), the two-form potential becomes purely imaginary.

To find a physical analytical continuation of this background, following (1.2), we have to find a mirror, purely geometric background, \(W\). In our case the appropriate mirror symmetry is that of the WZW CFT

\[
C_2 \rightarrow W_2 \equiv \left[ U(1) \times \frac{SU(2)_k}{U(1)} \right] / \mathbb{Z}_k .
\]

At the level of the target space metric, the cover of \(W_2\) corresponds to

\[
\begin{align*}
&d s^2_{W_2} = k \, d\psi^2 + k(d\theta^2 + \cot^2 \theta \, d\beta^2) \\
&B^2_{W_2} = 0 \\
&\Phi_{W_2} = -\ln \sin \theta + \text{const} ,
\end{align*}
\]

where \(\psi\) is the \(U(1)\) coordinate, and \((\theta, \beta)\) parameterize the \(\mathcal{N} = 2\) minimal model in (5.8). The \(\mathbb{Z}_k\) orbifold generator acts in the usual way. At this level, the mirror symmetry (5.8) is nothing but T-duality along the Killing vector field

\[
v_K = \frac{1}{k} \left( \frac{\partial}{\partial \beta} + \frac{\partial}{\partial \psi} \right) ,
\]

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where the dual variable is what is called $\lambda$ in (5.2).

Now we can implement the third step in (4.2) by taking

$$W_2 \rightarrow iW_2 \quad : \quad (\psi, \theta, \beta) \rightarrow (\psi, i\theta, i\beta)$$  \hspace{1cm} (5.11)

along with

$$C_1 \rightarrow iC_1 \quad : \quad (\rho, t) \rightarrow (\rho, it).$$  \hspace{1cm} (5.12)

Notice that the transformations (5.11) and (5.12) do not change the central charge, thus $[iC_1] \times [iW_2] \times C_3$ is still a critical superstring background. Furthermore, the background for $iW_2$ becomes

$$ds^2_{\tilde{M}} = dx^2_5 + k \left( \tanh^2 \rho \, dt^2 + d\rho^2 \right) - k \, d\theta^2 + \frac{k}{1 + \coth^2 \theta} \left\{ \coth^2 \theta \, d\psi^2 + d\lambda^2 \right\}$$

$$B_{\tilde{M}}^2 = B_{2(iW_2)}^2 = \frac{k}{1 + \coth^2 \theta} \, d\lambda \wedge d\psi$$

$$\Phi_{\tilde{M}} = \Phi_{iW_2} = -\ln(\cosh \rho \, \cosh \theta) - \frac{1}{2} \ln(1 + \tanh^2 \theta) + \text{const}.$$  \hspace{1cm} (5.13)

The background in (5.14) has everywhere small curvature in string units for $k \gg 1$ and has small string coupling for large negative constant in the dilaton expression. Moreover, it corresponds to an exactly soluble CFT. We do not have a simple argument settling the question of stability of (5.14). Note that the starting point $\mathcal{M}$, eq. (5.1), is unstable, as shown in [57]. However, it is not clear whether this instability has anything to do with instabilities of $\tilde{\mathcal{M}}$. For example, one could literally repeat the
steps of (4.2) starting from \( \mathcal{N} = 2 \) supersymmetric background corresponding to NS5-branes wrapped on \( S^2 \). Finally, we note that the metric written in (5.14) has an apparent singularity at \( \theta = 0 \), as the \( S^1 \) parameterized by \( \lambda \) shrinks to zero size. This is a Milne-type coordinate singularity, some thoughts about which are presented in section 7 below.

6 iD-branes

We now turn to the alternate method for constructing analytically continued solutions from known ones, outlined at the end of section 4. Specifically, we will apply this method starting from the usual D-brane solutions \([33]\). Recall that flat D-branes (or NS5-branes) can be obtained as solutions of dilaton-Einstein-Maxwell gravity with the Einstein-frame action

\[
S = \int d^{10}x \sqrt{-g} \left( R - \frac{1}{2} (\partial \Phi)^2 - \frac{1}{2q!} e^{a\Phi} F_{[q]}^2 \right).
\]

(6.1)

Here, \( F_{[q]} \) is the \( q \)-form field strength sourced by the brane, and \( a \) is the dilaton coupling. For the NS5-brane, \( q = 3 \) and \( a = -1 \), and for a \( D_{8-q} \)-brane, \( a = (5 - q)/2 \). The equations of motions derived from (6.1) are

\[
R_{\mu\nu} - \frac{1}{2} \partial_\mu \Phi \partial_\nu \Phi - \frac{e^{a\Phi}}{2(q - 1)!} \left[ F_{\mu\alpha_1\ldots\alpha_q} F_{\nu}^{\alpha_2\ldots\alpha_q} - \frac{q - 1}{8q} F_{[q]}^2 g_{\mu\nu} \right] = 0
\]

\[
\partial_\mu \left( \sqrt{-g} e^{a\Phi} F_{\mu\alpha_1\ldots\alpha_q} \right) = 0
\]

(6.2)

\[
\frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} e^{a\Phi} \right) - \frac{a}{2q!} e^{a\Phi} F_{[q]}^2 = 0.
\]

The metric ansatz for a flat \( D_{8-q} \)-brane is

\[
d s_E^2 = e^{2A} d\rho^2 + e^{2B} (-dt^2 + dx_{8-q}^2) + e^{2C} d\Omega_q^2,
\]

(6.3)

where \( d\Omega_q^2 \) is a metric on a round \( S^q \) of unit size, and the warp factors \( A, B, \) and \( C \) depend only on the radial coordinate \( \rho \). In addition, the field strength is taken to be

\[
F_{[q]} = b \text{ vol}(S^q),
\]

(6.4)

where \( b \) is a constant related to the total charge of the brane. Finally, the dilaton \( \Phi \) depends on \( \rho \) only. For the above ansatz the supergravity equations of motion
reduce to a coupled system of second order ordinary differential equations in the variable \( \rho \) \( (p \equiv 9 - q) \)

\[
p \left( B'' + (B')^2 - B'A' \right) + q \left( C'' + (C')^2 - C'A' \right) + \frac{1}{2} \left( \Phi' \right)^2 - \frac{(q - 1)b^2}{16} e^{a\Phi + 2A - 2qC} = 0
\]

\[
B'' - B'A' + p (B')^2 + qB'C' - \frac{(q - 1)b^2}{16} e^{a\Phi + 2A - 2qC} = 0
\]

\[
C'' - C'A' + q (C')^2 + pB'C' - e^{2A - 2C}(q - 1) + \frac{pb^2}{16} e^{a\Phi + 2A - 2qC} = 0
\]

\[
(e^{pB + qC - A\Phi'})' - \frac{1}{2}ab^2 e^{a\Phi + A - qC + pB} = 0
\]

(6.5)

The equations (6.5) have the familiar D-brane solution [33]

\[
\Phi = \frac{q - 5}{4} \ln H
\]

\[
A = \frac{p}{16} \ln H
\]

\[
B = \frac{1 - q}{16} \ln H
\]

\[
C = \ln \rho + \frac{p}{16} \ln H,
\]

with

\[
H = 1 + \frac{b}{(q - 1)\rho^{q-1}}.
\]

(6.6)

(6.7)

Can we analytically continue these solutions to time-dependent ones? Let us consider taking

\[
(t, \rho) \rightarrow (i\rho, it).
\]

(6.8)

Under (6.8) the solution (6.6) becomes unphysical. Indeed, \( e^{2C} \rightarrow -e^{2C} \), and so in order to have a single time direction, we have to analytically continue the \( q \)-sphere transverse to the D-branes to the \( q \)-dimensional hyperbolic space, \( S^q \rightarrow H_q \). If \( q \) is odd as in type IIB string theory, \( \text{vol}(S^q) \rightarrow i\text{vol}(H_q) \). In addition, reality of the harmonic function \( H \) requires real \( b \). Thus the \( q \)-form flux (3.4) becomes purely imaginary. On the other hand, if \( q \) is even as in type IIA string theory, \( \text{vol}(S^q) \rightarrow \text{vol}(H_q) \). But now the reality of \( H \) requires \( b \rightarrow ib \)—so again we end up with an imaginary flux (3.4).

A very simple way to circumvent the above obstacle is to analytically continue the supergravity equations (6.5), rather than their solutions (6.6)! Clearly, the analytically
continued equations are real and their solutions describe time-dependent backgrounds of the form

$$ds^2_L = -e^{2A} dt^2 + e^{2B} (dp^2 + dx_{-(q)}^2) + e^{2C} d\Omega_q^2,$$

(6.9)

with fluxes given by (6.4). Now all the warp factors and the dilaton depend on time. The analytically continued equations may have real (physical!) solutions which were missed by the analytical continuation of the original solutions. The D-brane example discussed in this section is precisely of this type. In the gauge

$$-A + pB + qC = 0,$$

(6.10)

the analytically continued equations (6.3) become just the equations studied in [26] in the context of constructing supergravity solutions corresponding to S-branes. More precisely, the solutions found in [26] fall in different classes depending on the signature of the curvature of the transverse space. The solutions (6.9) are those that have $\sigma = +1$ in [26] (spherical transverse space).

We mention that these iD-branes are rather different from the spacelike branes proposed in [24]. Indeed, the solutions (6.9) have an $SO(q+1)$ “R-symmetry”, instead of $SO(q,1)$ spontaneously broken down to $SO(q)$, as suggested by the original arguments in [24]. Moreover, the fluxes sourced by these branes do not match the S-brane requirements. As explained in [24], even and odd codimension S-branes should have physically very different fluxes. The former should source a flux spreading only on their lightcone, while the flux produced by the latter should spread inside the entire lightcone of the branes. Here, the flux is always distributed over the entire sphere. Finally, it is easy to see that the ansatz (6.3) does not admit any asymptotically flat solution, and so is unlikely to correspond to the dynamical decay of an unstable D-brane system in string theory, which should produce just the closed string vacuum.

The physical meaning of the iD-branes (apart from the fact that they are supergravity backgrounds) is somewhat unclear. One of their properties is that they have a Big Crunch singularity in the future and a Big Bang singularity in the past [24]. To close this section, we briefly discuss a less trivial example of our solution-generating technique. Namely, we present solutions corresponding to wrapped versions of the above iD-branes. The motivation to construct these solutions is to check whether or not the wrapping can resolve the cosmological singularity of the flat branes. Explicitly, we will consider iNS5-branes wrapped on $S^2$ obtained from analytical continuation of
the MN background \[^8\]. The result is a time-dependent supergravity background with five non-compact directions\[^5\].

**iNS5 wrapped on \(S^2\)**

The supergravity equations of motion describing the MN background are given in (3.7) with \(F \equiv 1\) and \(H = 0\). The analytically continued equations \(\rho \to it\) and \(t \to i\rho\) are

\[
0 = \left[ \frac{a'}{g_s^2} \right] + \frac{a(a^2 - 1)}{g_s^2 G^2} \left[ \frac{(G^2)'}{g_s^2} \right] - \frac{(a^2 - 1)^2 - G^2[(a')^2 + 4]}{2g_s^2 G^2} \]

\[
0 = \left[ G^2 \left( \frac{1}{g_s^2} \right) \right]' + \frac{(a^2 - 1)^2 + 2G^2[8G^2 - (a')^2]}{4g_s^2 G^2} \]

\[
0 = 2G^2 \left[ 8G^2 (g_s')^2 + 4g_s^2 (G')^2 - 4 (G^2)' (g_s^2)' - g_s^2 (a')^2 \right] - g_s^2 \left[ (a^2 - 1)^2 - 8G^2 (1 + 2G^2) \right].
\]

These equations describe the time-dependent background

\[
ds_{st}^2 = dx_4^2 + n \left( -dt^2 + G^2 d\Omega_2^2 + \frac{1}{4} \sum_a (\omega_a - A_a)^2 \right),
\]

with a 3-form flux given by (3.4), and a time-dependent string coupling \(g_s\).

For simplicity, we only consider solutions invariant under \(t \leftrightarrow -t\). Nonsingular solutions to (6.11) are characterized by two integration constants \(g_0 \equiv g_s|_{t=0}\) and \(G_0 \equiv G|_{t=0}\). The asymptotics are

\[
G = G_0 + \frac{4G_0^2 + 1}{2G_0} t^2 + O(t^4)
\]

\[
g_s = g_0 \left( 1 + \frac{4G_0^2 + 1}{2G_0} t^2 + O(t^4) \right)
\]

\[
a = a_0 - \frac{a_0(a_0^2 - 1)}{G_0^2} t^2 + O(t^4),
\]

where

\[
a_0 = \sqrt{1 \pm 2 \sqrt{4G_0^4 + 2G_0^2}}.
\]

---

\[^5\]Applying this analytical continuation to NS5-branes wrapped on \(S^3\) constructed in \[^6\] gives rise to a four-dimensional time-dependent superstring background.
Notice that $G$ has positive second derivative at $t = 0$, thus the boundary condition (6.13) describes a cosmological solution of (6.11) with a bounce at $t = 0$. What happens to our solutions at late/early times? We verified numerically for both branches of (6.14) that the bounce always ends up in a Big Crunch and starts at a Big Bang singularity. Physically, it is not surprising that the bounce solutions is singular. We would expect that a nonsingular solution asymptotically would correspond to decompactification of $S^2$, i.e., would describe the flat iNS5 brane of (6.1). It is straightforward to verify that there is no solution of (6.1) with $B = C + \text{const}$, which follows from the metric ansatz (6.12). On the other hand, it can be checked that (6.12) is the most general metric ansatz invariant under time reversal.

7 Strings in cosmological WZW models and Milne universe

In this section, we analyze a worldsheet description of strings at a cosmological singularity. The model we study is related to the well-known Witten’s black hole background [46] by analytic continuation. Hence, it is equivalent to a certain gauged WZW model, based on $SL(2, \mathbb{R})/U(1)$, analytically continued to negative level. The associated sigma-model is a two-dimensional cosmology, with a singularity of Milne type, i.e., it is locally of the form

$$ds^2 = -dt^2 + \lambda^2 t^2 dr^2,$$  \hspace{1cm} (7.1)

where $r$ is periodic with period $2\pi$.

The Milne singularity and similar background have been discussed recently in a number of works [36, 41, 53, 54]. Since the singularities are orbifolds, one is tempted to expect that a natural extension of classical methods should suffice. However, it has not been possible so far to fully understand the singularity using all available methods from ordinary Euclidean orbifolds. For example, the deformation of the theory by a tachyon v.e.v. does not smooth the metric [39]. The existence of closed timelike curves poses genuine problems for physics [53, 54]. Moreover, the null orbifolds of [11] (which do not suffer from closed timelike curves) are unstable to timelike orbifolds [11, 42, 44].

The purpose of this section of our paper is to point out that there is a surprising way in which string propagation near the Milne singularity, obtained as a limit of an “exactly solvable” WZW model, might be less singular than at first expected. More precisely, we observe that the winding modes of the string see a non-singular background metric.
Since winding modes are the lightest near the singularity, it is physically reasonable to use those as a probe of the geometry, and of the presence of a singularity. We hasten to emphasize, however, that the following observations can not be viewed as compelling evidence, let alone a proof, that strings fully resolve the singularity. For example, we do not address the problems associated with closed timelike curves. Moreover, the string coupling in the winding mode picture appears to diverge, invalidating string perturbation theory. Nonetheless, we pursue the question what is the cosmological interpretation of the gauged WZW model, and, assuming that the latter is well-defined, why is the cosmology.

Let us begin by recalling the two-dimensional black hole metric and dilaton \[ \Phi = -\frac{1}{4} \ln (1 - uv)^2, \] (7.2)

where \( k > 0 \). The Penrose diagram for this metric is depicted in fig. 1. A change of coordinates brings the metric into the form

\[ ds^2 = k (d\rho^2 - \tanh^2 \rho \, dt^2), \quad \Phi = -\frac{1}{2} \ln \cosh^2 \rho, \] (7.3)

which covers the asymptotically flat regions of spacetime. An analytical continuation \( t \to i\theta \) yields the popular semi-infinite cigar metric, but this is not the analytical continuation that we are interested in here. Instead, we note that, as was exploited for instance in [52], the metric (7.3) with negative \( k \) has a cosmological interpretation: The scale factor of space goes to zero in a finite proper time. Of course, there is no real singularity there unless one compactifies space, as we will do in a moment. The maximally extended spacetime then is just fig. 1 rotated by 90°.
The important point of ref. [46] was that the metric (7.3) and its analytical continuation to the semi-infinite cigar can be obtained as the leading order effective spacetime metric associated with a certain gauged WZW model [47]. More precisely, the group \( SL(2, \mathbb{R}) \) has two non-degenerate subgroups, a compact \( U(1)_- \) of negative signature, and a non-compact \( U(1)_+ \) of positive signature (we endow the \( SL(2, \mathbb{R}) \) group manifold with the metric of signature \((+,+,−)\)). Starting from the WZW model based on \( SL(2, \mathbb{R}) \) at level \( k > 0 \), one can construct new models by gauging the various subgroups. Gauging \( U(1)_- \) can obviously only give the Euclidean black hole, while gauging \( U(1)_+ \) gives (7.3).

Furthermore, for each of these subgroups, there is the choice of axial or vector gauging. As discussed in detail in [48, 50, 51], the two ways of gauging are expected to result in isomorphic conformal field theories up to charge conjugation of the left-moving sector of the string. But the effective metrics associated with axial and vector gauging are different, rather as in ordinary T-duality or mirror symmetry. For instance, the metric dual to (7.3) is given by

\[
ds^2 = k(d\rho^2 - \coth^2 \rho \, d\tau^2), \quad \Phi = -\frac{1}{2} \ln \sinh^2 \rho,
\]

which can be viewed as local T-duality on the \( \tau \) isometry.

As explained above, our interest is to view (7.3) or (7.4) as a cosmological background, which formally amounts to taking \( k \) negative and writing

\[
ds^2 = |k|(-dt^2 + \tanh^2 t \, dr^2), \quad \Phi = -\frac{1}{2} \ln \cosh^2 t,
\]

\[
ds^2 = |k|(-dt^2 + \coth^2 t \, dr^2), \quad \Phi = -\frac{1}{2} \ln \sinh^2 t,
\]

respectively. Compactifying the \( r \) direction and taking the limit \( t \to 0 \) obviously results in the Milne universe (7.1) and its conical singularity [52], or its T-dual version with a curvature singularity, i.e.,

\[
ds^2 = -dt^2 + \lambda^{-2} t^{-2} dr^2, \quad \Phi = -\ln |t|.
\]

It is a reasonable question to ask whether any stringy mechanism could resolve or explain the apparent spacelike singularities in either of (7.1), (7.3), (7.4), or (7.7). We will here present an observation concerning the possibility that \( \alpha' \) corrections respectively the presence of winding modes can regulate the singularity.

As reviewed above, the metrics (7.3), (7.4) appear in the \( \sigma \)-model that to first order governs the effective dynamics of strings described by the gauged WZW model. This
was derived in [46] by simply integrating out the gauge field, which is exact in the \( k \to \infty \) limit. In [48], a careful comparison of the zero-mode energy of the string with the target-space Laplacian revealed that the exact metrics should in fact be

\[
\begin{align*}
    ds^2 &= k(d\rho^2 - \frac{\tanh^2 \rho}{1 - \frac{2}{k} \tanh^2 \rho} d\tau^2), \\
    \Phi &= -\frac{1}{2} \ln \cosh^2 \rho \left( 1 - \frac{2}{k} \tanh^2 \rho \right)^{1/2},
\end{align*}
\]

(7.8)

\[
\begin{align*}
    ds^2 &= k(d\rho^2 - \frac{\coth^2 \rho}{1 + \frac{2}{k} \coth^2 \rho} d\tau^2), \\
    \Phi &= -\frac{1}{2} \ln \sinh^2 \rho \left( 1 + \frac{2}{k} \coth^2 \rho \right)^{1/2}.
\end{align*}
\]

(7.9)

It was shown in [49] that the first few terms in a \( 1/k \) expansion of these expressions solve the known low-loop beta-functions of the \( \sigma \)-model. Since \( k \) has dimensions of \( (\text{length})^2 \), these corrections should therefore be viewed as \( \alpha' \) corrections of the \( \sigma \)-model metric. Writing the metrics (7.8), (7.9) might then seem like a resummation of all \( \alpha' \) corrections, which is particularly touchy for (7.4), where the curvature diverges. However, in the absence of a general rule for extracting a metric from an “exactly solvable”, algebraically defined, conformal field theory model such as the gauged WZW model, the method used in [48] seems a reasonable way to proceed.

We now make the same naive analytical continuation as before, and compactify \( r \). This yields the \( \alpha' \) corrected versions of (7.3) and (7.6),

\[
\begin{align*}
    ds^2 &= k(-dt^2 + \frac{\tanh^2 t}{1 + \frac{2}{k} \tanh^2 t} dr^2), \\
    \Phi &= -\frac{1}{2} \ln \cosh^2 t \left( 1 + \frac{2}{k} \tanh^2 t \right)^{1/2},
\end{align*}
\]

(7.10)

\[
\begin{align*}
    ds^2 &= k(-dt^2 + \frac{\coth^2 t}{1 + \frac{2}{k} \coth^2 t} dr^2), \\
    \Phi &= -\frac{1}{2} \ln \sinh^2 t \left( 1 + \frac{2}{k} \coth^2 t \right)^{1/2}.
\end{align*}
\]

(7.11)

The interesting aspect of these metrics is that while the circle in (7.10) still shrinks to zero size at \( t = 0 \), the circle in (7.11) that before was growing indefinitely, causing the curvature to diverge, now reaches a maximum size at \( t = 0 \), of order \( k/\sqrt{\alpha'} \). Note that T-duality of these two metrics is not realized simply as inversion of the size of the circle. This is as expected because \( \alpha' \) corrections should not commute with small/large duality in string theory.

We interpret this as follows. The metric (7.10) governs ordinary particles or point-like strings moving on our two-dimensional cosmology; such objects follow geodesics. The metric has a conical singularity of Milne type near \( t = 0 \), and is not geodesically complete. From the orbifold point of view, and also from the WZW perspective [54],
this suggests the presence of extra regions of spacetime with closed timelike curves (called “whiskers” in ref. [53]). On the other hand, the T-dual metric (7.11) governs pointlike strings in the T-dual space, which from the original point of view are winding strings. The non-singularity of this metric signals that strings wound around the $r$-direction in our two-dimensional cosmology (7.10) might propagate through without too much difficulty.

We thus see an inherently stringy mechanism that makes the origin of this cosmological model look less singular than appears for ordinary particles, and thereby constitutes a partial answer to the question raised above. But clearly, the picture is far from being complete. We end this section with pointing out a few more open issues.

- One might object that the metric (7.11) in the original picture before T-duality still has a shrinking circle, and therefore we have not “resolved” the singularity. However, this is as in ordinary Euclidean orbifolds, even those that can actually be resolved to a smooth manifold, such as the famous $A_1$ singularity which can be blown up to the Eguchi-Hanson space. The only reasonable metric one can write down for the orbifold is the singular one. String theory is only non-singular because twisted/winding modes do not care so much about the original metric.

- One might ask the question what happens if we throw an ordinary particle or a string without winding into this cosmology. In particular, it was shown in [44] that such a particle causes an insurmountable amount of backreaction on the geometry. We do not know how this could be avoided in our picture.

- It might be possible and interesting to see directly a change of the metric (7.7) for the T-dual of Milne for which the circle near $t = 0$ has a finite size. It would be also interesting see what other “exactly solvable” cosmological backgrounds, such as the Nappi-Witten model [55], look like when treated in this way.

- Our discussion has been purely in the context of the bosonic string. In particular, the beta functions that are solved to low order by the above metrics are those for the bosonic $\sigma$-model. For the superstring, worldsheet supersymmetry changes the structure of the beta functions, and it is claimed that actually the tree-level metrics are exact, at least for Euclidean spacetime signature. An obvious question then is whether this is still the case for the Minkowskian spacetime signature, and, if the answer is positive, why adding the fermions has such a drastic effect on the way the winding modes probe the geometry.

- An important issue that we have been suppressing so far concerns the strength of
the string coupling $g_s$ near the singularity, i.e., whether a perturbative description can be expected at all. Clearly, the dilaton in (7.10) is finite and constant near $t = 0$, but blows up in the T-dual version (7.11), which includes $\alpha'$ corrections. We could now perform S-duality $g_s \rightarrow 1/g_s$ in order to end up with a perturbative string description. However, the string frame metric of the target space becomes $ds_{\text{string}}^2 \rightarrow 1/g_s ds_{\text{string}}^2$, so that now the full target space shrinks to zero size with a curvature singularity at $t = 0$. Thus it appears that the our resolution of the locally Milne singularity results in a Big Crunch singularity instead.

8 Summary

In this paper, we have presented a number of attempts at time dependence in string theory and supergravity, with an emphasis on some practical computations. Let us summarize our main results here.

Deformations of the gauge/gravity correspondence are a convenient tool for studying various phenomena associated with singularities in supergravity backgrounds. As we have shown here, following up on [16], these deformations also give a very controllable way of introducing time dependence in supergravity and the dual field theories. For example, chiral symmetry restoration in de Sitter space can be easily studied in the holographic dual. From a phenomenological point of view, however, it is somewhat unfortunate that deformations other than de Sitter space become technically rather involved.

A more straightforward way of introducing time dependence is through analytical continuation. Although one loses some control over stability and well-definedness, it is by far the simplest tool to generate time-dependent solutions of supergravity. One of the practical obstacles in applying the procedure in supergravity is the reality of fluxes. In this paper, we have pointed out that there are in fact some ways in which analytical continuation connects backgrounds with real fluxes only. In principle, this allows the construction of a wide variety of time-dependent backgrounds, of which we have presented a few simpler ones here.

Analytical continuation is also a useful tool at the level of the string worldsheet. In particular, exactly solvable string worldsheets with Euclidean target space allow at least a formal continuation to Minkowskian signature. Under the assumption that

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We would like to thank Gary Horowitz and Joe Polchinski for discussions on this.
one can carry over also certain non-perturbative worldsheet results, we have argued here that they can teach us something about how strings perceive simple cosmological singularities such as the Milne singularity.

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A Chiral symmetry restoration of MN model in de Sitter

In sections 2 and 3 we argued that the chiral symmetry that is broken in the nonsingular supersymmetric MN model [8] should be restored in a de Sitter background for sufficiently large Hubble parameter. In this appendix we present details supporting the claim.

The appropriate equations describing de Sitter deformation of the MN model are given by (3.7) and (3.8), with (a) type boundary condition of (3.9). Unbroken chiral symmetry in the gauge theory implies $a \equiv 0$ in the dual supergravity. Solutions regular as $\rho \to 0$ are characterized by three parameters $H$, $G_0$, and $g_0$, with asymptotics

\begin{align}
F^2 &= nH^2\rho^2 \left(1 - \frac{16G_0^2 + 8G_0 - 1}{240G_0^2}\rho^2 + O(\rho^4)\right) \\
G^2 &= G_0 + \frac{4G_0 - 1}{20G_0}\rho^2 + O(\rho^4) \\
g_s^{-2} &= g_0^{-2} \left(1 + \frac{16G_0^2 + 1}{40G_0^2}\rho^2 + O(\rho^4)\right)
\end{align}

(A.1)

Here, $G_0^{1/2}$ is the size of the $S^2$ at $\rho = 0$, and $g_0$ is the string coupling at $\rho = 0$. It seems that the important parameter in (A.1) is $G_0$, and not $H$, which we have claimed controls chiral symmetry restoration. However, as we discuss below, the Hubble parameter $H$ which fixes the scale of the gauge theory background, is related to the first two parameters by the scale dependence of the gauge theory coupling. For now, we analyze the $G_0$ dependence of the solutions.
Starting from (A.1), we expect that as $\rho \to \infty$ (UV from the gauge theory perspective), we recover the asymptotics of the flat LST for the physical solution representing the chirally symmetric phase of the MN model in the de Sitter. It turns out that the asymptotics at $\rho \to \infty$ depend on whether $G_0 < \frac{1}{4}$ or $G_0 > \frac{1}{4}$. In the first case, the solution develops a singularity at some finite $\rho = \rho^*(G_0)$, while in the latter case we indeed find the flat LST asymptotics.

We begin with the special case $G_0 = \frac{1}{4}$. With (A.1) we find from (3.7) $G \equiv \frac{1}{2}$.

Introducing a new radial coordinate

$$r \equiv \frac{F}{n^{1/2}H},$$

(A.2)

we find from (3.7) the following first order equation for $f \equiv r[\ln g_s]'$

$$f' = -\frac{(f - 1)(f - 3)(3f + 4r^2)}{r(2r^2 + 3)}.$$  

(A.3)

The boundary condition (A.1) then translates into

$$f = -\frac{4}{5}r^2 - \frac{16}{175}r^4 + O(r^6).$$

(A.4)

Though we can not find an exact analytical solution to (A.3), the $r \to \infty$ asymptotic can be easily extracted

$$f(r \to \infty) = -\frac{4}{3}r^2 + 1 + O(1/r^2).$$

(A.5)

The complete string frame metric of the special $G_0 = \frac{1}{4}$ solution reads

$$ds^2_{st} = nH^2r^2 \left[-dt^2 + \frac{1}{H^2} \cosh^2 Ht \, d\Omega_3^2\right] + n \left(\Delta^2 dr^2 + \frac{1}{4} ds^2_{\tilde{T}^{1,1}}\right)$$

$$\equiv n \left(r^2 dS_4^2 + \Delta^2 dr^2\right) + \frac{n}{4} ds^2_{\tilde{T}^{1,1}},$$

(A.6)

where

$$\Delta^2 = \frac{(f - 1)(f - 3)}{3 + 2r^2},$$

(A.7)

and

$$ds^2_{\tilde{T}^{1,1}} \equiv d\tilde{\theta}^2 + \sin^2 \tilde{\theta} d\tilde{\phi}^2 + d\theta^2 + \sin^2 \theta d\psi^2 + (d\phi + \cos \theta d\psi - \cos \tilde{\theta} d\tilde{\phi})^2.$$  

(A.8)

The $\tilde{T}^{1,1}$ metric above along with the corresponding NS-NS background (3.4) was identified in [20] as target space of the simplest representative in the class of $SU(2) \times SU(2)$.
coset sigma models introduced in [21]. In [20] conformal invariance of this coset was checked in the 3-loop approximation, and the expectation is that this background is an exact NS-NS string solution to all orders in \( \alpha' \). The same coset appears in the “special Abelian solution” of the non-BPS excitation of the MN model [11]. This “special Abelian solution” has an infrared singularity due to the linear dilaton. Thus we are led to the conclusion that our special \( G_0 = \frac{1}{4} \) solution is a de Sitter regularization of the singular “special Abelian solution” of [11]. The exact central charge for the \( T^{1,1} \) coset is [11]

\[
c_{T^{1,1}} = 5 \cdot \frac{3}{2} - \frac{12}{n}. \tag{A.9}
\]

Thus, the sigma-model corresponding to the time-dependent target space metric

\[
ds_5 = n \left( r^2 dS_4^2 + \Delta^2 dr^2 \right) \tag{A.10}
\]

and dilaton

\[
\Phi = \ln g_0 + \int_0^r d\xi \frac{f(\xi)}{\xi} \tag{A.11}
\]

with \( f \) being a solution of (A.3) with boundary condition (A.4), must have central charge

\[
c_5 = 5 \cdot \frac{3}{2} + \frac{12}{n}. \tag{A.12}
\]

The latter can be easily verified as asymptotically, (A.10), (A.11) is a linear dilaton background.

It would be very interesting to compute \( \alpha' \) corrections to (A.10) and to identify the appropriate CFT. The “special Abelian solution” of [11] was shown to be unstable. It would be interesting to check whether its de Sitter regularization, realized by our special \( G_0 = \frac{1}{4} \) solution, is stable.

We now proceed with the general \( G_0 \neq \frac{1}{4} \) case. We could not find an exact analytical solution for general \( G_0 \). For numerical analysis we find it convenient to use a new radial coordinate \( r \equiv \rho^2, \ r \in [0, +\infty) \), and introduce \( f, g, \) and \( h \) by

\[
F^2 \equiv nH^2 f \\
G^2 \equiv g \\
g_s^{-2} \equiv g_0^{-2} h. \tag{A.13}
\]

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With the boundary conditions (A.1) the only parameter in the numerical integration becomes $G_0$. We find qualitatively different $r \to \infty$ asymptotics depending on the value for $G_0$. Namely, for $0 < G_0 < \frac{1}{4}$, we have,

$$
\begin{align*}
    h(r \to r^*) &\to +\infty \\
    g(r \to r^*) &\to 0 \\
    f(r \to r^*) &\to \text{const},
\end{align*}
$$

(A.14)

where $r^* \equiv r^*(G_0)$ is finite, and for $G_0 > \frac{1}{4}$, we find

$$
\begin{align*}
    h(r \to \infty) &\to +\infty \\
    g(r \to \infty) &\to +\infty \\
    f(r \to \infty) &\to +\infty.
\end{align*}
$$

(A.15)

We can now check analytically whether there are asymptotic solutions of (3.7) which have the same qualitative behavior as predicted by the numerical analysis (A.14) and (A.15). Indeed, solutions with such leading asymptotics exist. Explicitly, we find first

$$
\begin{align*}
    h &= \frac{(4\delta_2^2 r^*)^{1/4}}{\sqrt{r^* - r}} [1 + O(r^* - r)] \\
    g &= \frac{1}{2} \sqrt{1 - \frac{r}{r^*}} [1 + O(r^* - r)] \\
    f &= \delta_2 [1 + O(r^* - r)].
\end{align*}
$$

(A.16)

If we identify this solution as corresponding to the $0 < G_0 < \frac{1}{4}$ boundary condition, then $\delta_1, \delta_2, r^*$ will be dependent on $G_0$. Also, there is a solution

$$
\begin{align*}
    h &= \delta_1 e^{2r^{1/2}} [1 + o(1)] \\
    g &\to r^{1/2} [1 + o(1)] \\
    f &\to 3r^{1/2} [1 + o(1)],
\end{align*}
$$

(A.17)

which we identify as corresponding to the $G_0 > \frac{1}{4}$ boundary condition, with $\delta_1$ depending on $G_0$.

To summarize, we have shown using numerics and asymptotic analysis that a chirally symmetric phase (nonsingular solution to (3.7) with (A.1) boundary condition) exists only for

$$
G_0 > G_{\text{critical}} \equiv \frac{1}{4}.
$$

(A.18)
Although this is not an analytical proof, it strongly support the picture of a chiral symmetry restoration phase transition in the de Sitter deformed MN model. So far, this phase transition appears to occur as the (dimensionless) parameter $G_0$ is varied. We now argue that in fact, the relevant physical parameter is the Hubble scale $H$.

In addition to $G_0$ satisfying (A.18), nonsingular deformations are characterized, naively, by two more parameters $g_0$ and $H$. This is one parameter too many: from the perspective of LST wrapped on $S^2$, we expect only two continuous parameters: $g_0$ (the dilaton parameter in the supersymmetric MN model) and the deformation parameter, $H$. In the remainder of the appendix we argue that for fixed \{\(g_0, H\), $G_0$ is not actually an independent parameter, but rather

\[
\ln H \sim G_0^2, \tag{A.19}
\]

and thus, the non-singularity condition (A.18) translates into a lower bound on $H$.

One may observe that rescaling $t \rightarrow \tau \equiv Ht$ entirely removes the $H$ dependence from the background. But this is not very illuminating, as in doing so we are changing the reference energy scale from the LST perspective. Instead, we extract the relation (A.19) from the gauge theory beta-function, keeping the effective four-dimensional gauge theory strong coupling scale (and the string scale) fixed. We don’t know the beta-function of gauge theories in de Sitter space, but for asymptotically free gauge theories the perturbative beta-function should be roughly the same as the standard Minkowski one. The reason is that it is related to the short distance dynamics of the theory, and we expect de Sitter deformation to be irrelevant at small scales. We will use the Minkowski beta-function with the understanding that at best only qualitative physics would come out correct. Now recall that the open string interpretation of the MN model is LST compactified on $S^2$. At low energies the effective description is in terms of $\mathcal{N} = 1$ SYM theory with gauge coupling $g_4$ [8],

\[
\frac{1}{g_4^2} = \frac{Vol_{S^2}}{g_6^2} = \frac{nG_0^2}{2\pi^2}, \tag{A.20}
\]

where $g_6$ is the D5-brane gauge theory coupling. In the supergravity dual, $G$ depends on $\rho$ which is interpreted as the RG running of the effective four-dimensional coupling. Verifying (A.20) requires understanding of UV/IR relation in the MN model, or, in other words, the translation of the radial direction $\rho$ into the RG gauge theory scale $\mu$. This relation cannot be defined unambiguously. Maldacena and Nuñez [8] used $\mu \sim e^{\rho/2}$, and showed that (A.20) agrees with the gauge theory result $1/(g_4^2 n) \sim \ln \mu/\Lambda_{QCD}$.

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up to a numerical coefficient. In the MN model with unbroken chiral symmetry $G^2$ was singular at $\rho = 0$. This singularity reflects the Landau pole in the perturbative gauge theory coupling. On physical grounds we expect that the effective gauge theory Landau pole would be regulated in the de Sitter for large enough Hubble parameter simply because the RG running of the gauge coupling stops at $\mu = \mu_{IR \text{ cutoff}} \sim H$. This fits nicely with the supergravity result for the de Sitter deformed MN model where $G^2 \geq G_0^2 > 0$. Provided we can use (A.20) when $H \neq 0$, we are led to (A.19).

Needless to say, our heuristic arguments can not replace a precise computation. However, such a computation requires (at the very least) an understanding of the precise UV/IR relation in the MN model and a precise understanding of perturbative (non-abelian) gauge theory dynamics in de Sitter space. Both problems are very difficult. For recent progress on the first of them, see, e.g., [62, 63].

References

[1] J. M. Maldacena, “The large $N$ limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2, 231 (1998) [Int. J. Theor. Phys. 38, 1113 (1999)] [arXiv:hep-th/9711200].

[2] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, “Large $N$ field theories, string theory and gravity,” Phys. Rept. 323, 183 (2000) [arXiv:hep-th/9905111].

[3] M. Berkooz, M. Rozali and N. Seiberg, “On transverse fivebranes in M(atrix) theory on $T^5$,” Phys. Lett. B 408, 105 (1997) [arXiv:hep-th/9704089].

[4] O. Aharony, “A brief review of 'little string theories','” Class. Quant. Grav. 17, 929 (2000) [arXiv:hep-th/9911147].

[5] I. R. Klebanov and A. A. Tseytlin, “Gravity duals of supersymmetric SU($N$) x SU($N+M$) gauge theories,” Nucl. Phys. B 578, 123 (2000) [arXiv:hep-th/0002158].

[6] J. Polchinski and M. J. Strassler, “The string dual of a confining four-dimensional gauge theory,” [arXiv:hep-th/0003136].

7The UV/IR connection is very subtle even for the gauge/gravity correspondence for D5/NS5 branes [22].
[7] I. R. Klebanov and M. J. Strassler, “Supergravity and a confining gauge theory: Duality cascades and chiSB-resolution of naked singularities,” JHEP 0008, 052 (2000) [arXiv:hep-th/0007191].

[8] J. M. Maldacena and C. Nunez, “Towards the large N limit of pure N = 1 super Yang Mills,” Phys. Rev. Lett. 86, 588 (2001) [arXiv:hep-th/0008001].

[9] A. H. Chamseddine and M. S. Volkov, “Non-Abelian BPS monopoles in N = 4 gauged supergravity,” Phys. Rev. Lett. 79, 3343 (1997) [arXiv:hep-th/9707176].

[10] A. Buchel and A. R. Frey, “Comments on supergravity dual of pure N = 1 super Yang Mills theory with unbroken chiral symmetry,” Phys. Rev. D 64, 064007 (2001) [arXiv:hep-th/0103022].

[11] S. S. Gubser, A. A. Tseytlin and M. S. Volkov, “Non-Abelian 4-d black holes, wrapped 5-branes, and their dual descriptions,” JHEP 0109, 017 (2001) [arXiv:hep-th/0108203].

[12] O. Aharony, E. Schreiber and J. Sonnenschein, “Stable non-supersymmetric supergravity solutions from deformations of the Maldacena-Nunez background,” JHEP 0204, 011 (2002) [arXiv:hep-th/0201224].

[13] J. M. Maldacena and A. Strominger, “Semiclassical decay of near-extremal five-branes,” JHEP 9712, 008 (1997) [arXiv:hep-th/9710014].

[14] A. Buchel, “Finite temperature resolution of the Klebanov-Tseytlin singularity,” Nucl. Phys. B 600, 219 (2001) [arXiv:hep-th/0011146]. A. Buchel, C. P. Herzog, I. R. Klebanov, L. A. Pando Zayas and A. A. Tseytlin, “Non-extremal gravity duals for fractional D3-branes on the conifold,” JHEP 0104, 033 (2001) [arXiv:hep-th/0102104]. S. S. Gubser, C. P. Herzog, I. R. Klebanov and A. A. Tseytlin, “Restoration of chiral symmetry: A supergravity perspective,” JHEP 0105 (2001) 028 [arXiv:hep-th/0102172].

[15] A. Buchel and A. A. Tseytlin, “Curved space resolution of singularity of fractional D3-branes on conifold,” Phys. Rev. D 65, 085019 (2002) [arXiv:hep-th/0111017].

[16] A. Buchel, “Gauge / gravity correspondence in accelerating universe,” Phys. Rev. D 65, 125015 (2002) [arXiv:hep-th/0203041].
[17] P. Berglund, T. Hubsch and D. Minic, “de Sitter spacetimes from warped compactifications of IIB string theory,” Phys. Lett. B 534, 147 (2002) [arXiv:hep-th/0112073].

[18] R. Bousso, A. Maloney and A. Strominger, “Conformal vacua and entropy in de Sitter space,” Phys. Rev. D 65, 104039 (2002) [arXiv:hep-th/0112218].

[19] T. Prokopec, O. Tornkvist and R. P. Woodard, “One loop vacuum polarization in a locally de Sitter background,” arXiv:gr-qc/0205130.

[20] L. A. Pando Zayas and A. A. Tseytlin, “Conformal sigma models for a class of T(p,q) spaces,” Class. Quant. Grav. 17 (2000) 5125 [arXiv:hep-th/0007080].

[21] E. Guadagnini, M. Martellini and M. Mintchev, “Scale Invariance Sigma Models On Homogeneous Spaces,” Phys. Lett. B 194, 69 (1987). E. Guadagnini, “Current Algebra In Sigma Models On Homogeneous Spaces,” Nucl. Phys. B 290, 417 (1987).

[22] A. W. Peet and J. Polchinski, “UV/IR relations in AdS dynamics,” Phys. Rev. D 59, 065011 (1999) [arXiv:hep-th/9809022].

[23] S. W. Hawking and G. T. Horowitz, “The Gravitational Hamiltonian, action, entropy and surface terms,” Class. Quant. Grav. 13, 1487 (1996) [arXiv:gr-qc/9501014].

[24] M. Gutperle and A. Strominger, “Spacelike branes,” JHEP 0204, 018 (2002) [arXiv:hep-th/0202210].

[25] O. Aharony, M. Fabinger, G. T. Horowitz and E. Silverstein, “Clean time-dependent string backgrounds from bubble baths,” arXiv:hep-th/0204158.

[26] C. M. Chen, D. V. Gal’tsov and M. Gutperle, “S-brane solutions in supergravity theories,” arXiv:hep-th/0204071.

[27] M. Kruczenski, R. C. Myers and A. W. Peet, “Supergravity S-branes,” JHEP 0205, 039 (2002) [arXiv:hep-th/0204144].

[28] K. Behrndt and S. Forste, Nucl. Phys. B 430, 441 (1994) [arXiv:hep-th/9403179].
[29] C. Grojean, F. Quevedo, G. Tasinato and I. Zavala C., “Branes on charged dilatonic backgrounds: Self-tuning, Lorentz violations and cosmology,” JHEP 0108, 005 (2001) arXiv:hep-th/0106120.

[30] L. Cornalba, M. S. Costa and C. Kounnas, “A resolution of the cosmological singularity with orientifolds,” Nucl. Phys. B 637, 378 (2002) arXiv:hep-th/0204261.

[31] N. S. Deger and A. Kaya, “Intersecting S-brane solutions of D = 11 supergravity,” JHEP 0207, 038 (2002) arXiv:hep-th/0206057.

[32] C. P. Burgess, F. Quevedo, S. J. Rey, G. Tasinato and C. Zavala, “Cosmological spacetimes from negative tension brane backgrounds,” arXiv:hep-th/0207104.

[33] G. T. Horowitz and A. Strominger, “Black Strings And P-Branes,” Nucl. Phys. B 360, 197 (1991).

[34] H. Ooguri and C. Vafa, “Two-Dimensional Black Hole and Singularities of CY Manifolds,” Nucl. Phys. B 463, 55 (1996) arXiv:hep-th/9511164.

[35] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, “The ekpyrotic universe: Colliding branes and the origin of the hot big bang,” Phys. Rev. D 64, 123522 (2001) arXiv:hep-th/0103239.

[36] G. T. Horowitz and A. R. Steif, “Singular String Solutions With Nonsingular Initial Data,” Phys. Lett. B 258, 91 (1991).

[37] V. Balasubramanian, S. F. Hassan, E. Keski-Vakkuri and A. Naqvi, “A space-time orbifold: A toy model for a cosmological singularity,” arXiv:hep-th/0202187.

[38] L. Cornalba and M. S. Costa, “A New Cosmological Scenario in String Theory,” arXiv:hep-th/0203031.

[39] N. A. Nekrasov, “Milne universe, tachyons, and quantum group,” arXiv:hep-th/0203112.

[40] J. Simon, “The geometry of null rotation identifications,” JHEP 0206, 001 (2002) arXiv:hep-th/0203201.

[41] H. Liu, G. Moore and N. Seiberg, “Strings in a time-dependent orbifold,” JHEP 0206, 045 (2002) arXiv:hep-th/0204168. H. Liu, G. Moore and N. Seiberg, “Strings in time-dependent orbifolds,” arXiv:hep-th/0206182.
[42] A. Lawrence, “On the instability of 3D null singularities,” arXiv:hep-th/0205288.

[43] M. Fabinger and J. McGreevy, “On smooth time-dependent orbifolds and null singularities,” arXiv:hep-th/0206190.

[44] G. T. Horowitz and J. Polchinski, “Instability of spacelike and null orbifold singularities,” arXiv:hep-th/0206228.

[45] S. Kachru and L. McAllister, “Bouncing brane cosmologies from warped string compactifications,” arXiv:hep-th/0205209.

[46] E. Witten, “On string theory and black holes,” Phys. Rev. D 44, 314 (1991).

[47] S. Elitzur, A. Forge and E. Rabinovici, “Some Global Aspects Of String Compactifications,” Nucl. Phys. B 359, 581 (1991).

[48] R. Dijkgraaf, H. Verlinde and E. Verlinde, “String propagation in a black hole geometry,” Nucl. Phys. B 371, 269 (1992).

[49] A. A. Tseytlin, “On the form of the black hole solution in D = 2 theory,” Phys. Lett. B 268, 175 (1991).

[50] M. Rocek and E. Verlinde, “Duality, quotients, and currents,” Nucl. Phys. B 373, 630 (1992) [arXiv:hep-th/9110053].

[51] M. J. Perry and E. Teo, “Nonsingularity of the exact two-dimensional string black hole,” Phys. Rev. Lett. 70, 2669 (1993) [arXiv:hep-th/9302037].

[52] A. A. Tseytlin and C. Vafa, “Elements of string cosmology,” Nucl. Phys. B 372, 443 (1992) [arXiv:hep-th/9109048].

[53] S. Elitzur, A. Giveon, D. Kutasov and E. Rabinovici, “From big bang to big crunch and beyond,” JHEP 0206, 017 (2002) [arXiv:hep-th/0204189].

[54] B. Craps, D. Kutasov and G. Rajesh, “String propagation in the presence of cosmological singularities,” JHEP 0206, 053 (2002) [arXiv:hep-th/0205101].

[55] C. R. Nappi and E. Witten, “A Closed, expanding universe in string theory,” Phys. Lett. B 293, 309 (1992) [arXiv:hep-th/9206078].
[56] A. Buchel, P. Langfelder and J. Walcher, “Does the tachyon matter?,” \texttt{arXiv:hep-th/0207233}.

[57] D. Kutasov and D. A. Sahakyan, “Comments on the thermodynamics of little string theory,” JHEP \textbf{0102}, 021 (2001) \texttt{arXiv:hep-th/0012258}.

[58] J. P. Gauntlett, N. Kim, D. Martelli and D. Waldram, “Wrapped fivebranes and $N = 2$ super Yang-Mills theory,” Phys. Rev. D \textbf{64}, 106008 (2001) \texttt{arXiv:hep-th/0106117}.

[59] F. Bigazzi, A. L. Cotrone and A. Zaffaroni, “$N = 2$ gauge theories from wrapped five-branes,” Phys. Lett. B \textbf{519}, 269 (2001) \texttt{arXiv:hep-th/0106160}.

[60] K. Hori and A. Kapustin, “Worldsheet descriptions of wrapped NS five-branes,” \texttt{arXiv:hep-th/0203147}.

[61] J. M. Maldacena and H. Nastase, “The supergravity dual of a theory with dynamical supersymmetry breaking,” JHEP \textbf{0109}, 024 (2001) \texttt{arXiv:hep-th/0105049}.

[62] R. Apreda, F. Bigazzi, A. L. Cotrone, M. Petrini and A. Zaffaroni, “Some Comments on $N=1$ Gauge Theories from Wrapped Branes,” Phys. Lett. B \textbf{536}, 161 (2002) \texttt{arXiv:hep-th/0112230}.

[63] P. Di Vecchia, A. Lerda and P. Merlatti, “$N = 1$ and $N = 2$ super Yang-Mills theories from wrapped branes,” \texttt{arXiv:hep-th/0205204}. 