Compact QED under scrutiny: it’s first order

G. Arnold\textsuperscript{1}, B. Bunk\textsuperscript{2}, Th. Lippert\textsuperscript{1}, K. Schilling\textsuperscript{1}

\textsuperscript{1}Department of Physics, University of Wuppertal, D-42097 Wuppertal, Germany
\textsuperscript{2}Department of Physics, Humboldt University Berlin, D-10099 Berlin, Germany

We report new results from our finite size scaling analysis of 4d compact pure U(1) gauge theory with Wilson action. Investigating several cumulants of the plaquette energy within the Borgs-Kotecky finite size scaling scheme we find strong evidence for a first-order phase transition and present a high precision value for the critical coupling, $\beta_T$ in the thermodynamic limit.

1. INTRODUCTION

The nature of the phase transition in compact QED has been under debate for long time. We have addressed this problem in high statistics runs to reach a final conclusion in that matter. An important ingredient of our approach is the finite size scaling (FSS) theory à la Borgs-Kotecky (BK) first established a long time ago in the context of strong first order phase transitions \cite{1–3}. According to BK the finite volume partition function at temperature $\beta$ in finite volumes with periodic boundary conditions (neglecting interfacial contributions) has the remarkably simple form

$$Z = e^{-V f_1(\beta)} = e^{-V f_1(\beta)} + e^{-V f_2(\beta)+\ln(X)}.$$ 

The functions $f_1(\beta)$ and $f_2(\beta)$ denote bulk free energy densities in the two coexisting phases 1 and 2. $X$ stands for the asymmetry parameter which is nothing but the relative phase weight in the probability distribution $P(E)$.

A heuristic extension of the BK ansatz to weak first order transition was demonstrated for the 3d 3-state Potts model \cite{4}. The conclusion of our work is based on a validation of BK by perturbative analysis as well as independent ab initio determinations of the gap characteristics.

2. SIMULATION DETAILS

We consider 4d pure U(1) gauge theory with Wilson action $S = -\beta \sum_{n,\nu>\mu} \cos(\theta_{\mu\nu}(n))$, where $\beta$ represents the Wilson coupling and $\theta_{\mu\nu}(n)$ the plaquette angle. We use a lattice of volume $V = L^4$ with periodic boundary conditions.

We have implemented three different algorithms for generating the U(1) gauge field configurations: (a) a local Metropolis (Metro), updating each link separately, (b) a global hybrid Monte Carlo algorithm (HMC) and (c) a combination of the multicanonical and the hybrid Monte Carlo algorithm (MHMC). For details we refer to Refs. \cite{5,6}. The cumulative number of generated configurations at each lattice size $L$ is $> 5 \times 10^6$. We measure the number of tunneling events (flips) as control parameter for the mobility of the algorithms and the integrated plaquette autocorrelation time $\tau_{int}$ which controls the statistical quality of each single Markov chain. Runs differing by coupling, algorithm, HMC parameters or by weight function are considered as independent. Our simulation parameters are listed in Table 1.

3. CUMULANTS

Based on the plaquette operator

$$E = \frac{1}{6V} \sum_{n,\nu>\mu} \cos(\theta_{\mu\nu}(n)), \quad (1)$$

we consider the following cumulants:

$$C_v(\beta, L) = \frac{6V}{\langle E^2 \rangle - \langle E \rangle^2},$$
$$U_2(\beta, L) = 1 - \frac{\langle E^2 \rangle}{\langle E \rangle^2},$$
$$U_4(\beta, L) = \frac{1}{3} \left( 1 - \frac{\langle E^4 \rangle}{\langle E^2 \rangle^2} \right).$$

In addition to their derivatives with respect to $\beta$ we measure higher derivatives of the free energy
Table 1
Simulation details. The HMC subscript denotes length of trajectory.

| L    | β    | Algorithm | #conf × 10^6 | #flips | t_0  |
|------|------|-----------|--------------|--------|------|
| 8    | 1.25 | Metro     | 5.13         | 41748  | 5466 |
|      |      | HMC8      | 1.25         | 5247   | 5963 |
| 10   | 1.25 | Metro     | 7.44         | 3104   | 775  |
|      |      | HMC10     | 1.25         | 490    | 508  |
| 12   | 1.25 | Metro     | 6.42         | 939    | 5963 |
|      |      | HMC12     | 1.25         | 365    | 474  |
| 14   | 1.25 | Metro     | 6.52         | 395    | 7480 |
|      |      | HMC14     | 1.25         | 300    | 1070 |
| 16   | 1.25 | Metro     | 5.42         | 389    | 687  |
|      |      | HMC16     | 1.25         | 344    | 1770 |
| 18   | 1.25 | Metro     | 6.63         | 20     | 5000 |
|      |      | HMC18     | 1.25         | 298    | 3760 |

Density \((-1)^{n+1} \kappa_n(\beta, L) = \beta^n \mu(\beta, L) \partial \nu_{\beta,L}^n\). Introducing the central moments \(\mu_n = V^{n-1} \langle E - \langle E \rangle \rangle^n\) we can write them as

\[ \kappa_3 = \mu_3, \]
\[ \kappa_4 = \mu_4 - 3V \mu_2^2, \]
\[ \kappa_5 = \mu_5 - 10V \mu_2 \mu_3, \]
\[ \kappa_6 = \mu_6 - 15V \mu_2 \mu_4 - 10V \mu_3^2 + 30V^2 \mu_2^3. \]

For each of the ten cumulants the location \((\beta_0, \beta)\) of its rightmost extremum is determined by reweighting the measured probability distribution \(P(E)\) to different couplings \(\beta\). To calculate the estimates of our cumulants at each lattice size \(L\) we proceed in two steps: i) we determine the error of each individual run performing a jackknife error analysis by subdivision of the run into ten blocks; ii) we calculate the final result by \(\chi^2\) fitting these individual results to a constant.

4. FINITE SIZE SCALING

Let us consider the expansion of the pseudocritical coupling for \(C_v\) in the BK representation it is given by

\[ \beta_{C_v}(V) = \beta_{C_v}(\infty) + \sum_{k=1}^{k_{\text{max}}} B_k V^{-k}. \]  

In order to expose systematic effects in the fit parameter \(\beta_{C_v}(\infty)\), we vary both, the fit range within \(L_{\text{min}} \leq L \leq 18\) and the truncation parameter \(k_{\text{max}}\). Table 2 displays a remarkable stability pattern both for \(\beta_{C_v}\) and \(B_1\) supporting the validity of the \(V^{-1}\)-expansion. Averaging the best fit couplings to a constant we obtain \(\beta_{C_v}(\infty) = 1.0111310(62)\). Performing the same analysis for all ten cumulants yields an average infinite volume transition coupling

\[ \beta_T = 1.0111331(21). \]

Table 2
Transition couplings \(\beta_{C_v}(\infty)\) fitted to Eq (2). Best fits are in bold face letters.

| \(L_{\text{min}}\) | \(k_{\text{max}}\) | \(\chi^2_{\text{red}}\) | \(\beta_{C_v}(\infty)\) | \(B_1\) |
|-----------------|-----------------|-----------------|-----------------|--------|
| 14              | 1               | 1.03            | 1.0111241(13)   | -18.95(14) |
| 12              | 1               | 1.09            | 1.0111144(55)   | -18.24(21) |
| 2               | 0.19            | 1.01111315(57)  | -19.96(53)      |        |
| 10              | 1               | 12.7            | 1.0110945(147)  | -17.18(37) |
| 2               | 0.13            | 1.0111283(25)   | -19.63(15)      |        |
| 3               | 0.21            | 1.01112139(62)  | -20.06(65)      |        |
| 8               | 1               | 108             | 1.0110474(349)  | -15.33(50) |
| 2               | 2.14            | 1.0111159(69)   | -18.70(25)      |        |
| 3               | 0.11            | 1.0111309(25)   | -19.94(17)      |        |
| 4               | 0.10            | 1.0111316(11)   | -20.02(6)       |        |
| 6               | 1               | 970             | 1.0110389(913)  | -12.38(56) |
| 2               | 37.1            | 1.0110792(218)  | -16.84(41)      |        |
| 3               | 2.15            | 1.0111199(55)   | -19.02(22)      |        |
| 4               | 0.10            | 1.0111316(11)   | -20.02(6)       |        |
infinite volume gap $G$

$$\frac{C_{v,max}(V)}{6V} = \frac{1}{4} G^2 + \sum_{k=1}^{\infty} C_k V^{-k}$$

$$G = 0.026721(59).$$

(4)

Scaling of the Binder cumulant yields

$$U_{4,\text{min}} = U + \sum_{k=1}^{\infty} A_k V^{-k}$$

$$U = -5.816(27) \times 10^{-4}.$$

(5)

From $B_1 = -\ln(X)/6V$ we can derive an asymmetry $\ln(X) = 3.21(10)$.

5. CONSISTENCY CHECKS

An independent leading order perturbative lattice calculation confirms the value of $\ln(X)$ without relying on the validity of the BK ansatz [7]. In the Coulomb phase the partition function can approximately be written as

$$Z \simeq \left(\frac{2\pi}{c_R^2}\right)^{-\frac{9}{2}(V-1)} V^{\frac{3}{2}} \prod_p \left[2(1 - \cos p_\mu)\right]^{-1},$$

leading to the free energy $F_2 = \ln Z = -\frac{3}{2}(V-1)\ln(\frac{2\pi}{c_R^2}) + 2\ln L - \sum_\mu \ln \sum_\mu 2(1 - \cos p_\mu)$. The summation over all momenta $p_\mu$ can be done for asymptotically large $L$ as $\sum_\mu \ln \sum_\mu 2(1 - \cos p_\mu) = a V + 2\ln L - b + O(L^{-2})$ with parameters $a = 1.999708$ and $b = 1.701216$ that can be numerically computed to arbitrary precision. With $F_2(\beta, L) = V f_2(\beta) + \Delta F_2(\beta, L)$ we obtain for the Coulomb phase finite size correction $\Delta F_2 = b + \frac{9}{2} \ln(2\pi/e_R^2) = 3.15(8)$. $e_R$ is taken from a very accurate measurement of the renormalized fine structure constant at the phase transition $\alpha_T = e_R^2 / 4\pi = 0.19(1)$ [8]. Note that the logarithmic correction cancels out for the 4d symmetric system with periodic boundary conditions.

One can argue that the leading finite size effects in the confined phase due to $0^+^+$ gaugeballs and string states are at least 3 respectively 6 orders of magnitude smaller than $\Delta F_2$ on a lattice as small as $L = 16$ [7]. Thus we neglect these contributions and obtain, in perfect agreement, an asymmetry parameter $\ln(X) = \Delta F_2 = 3.15(8)$.

Furthermore our very accurate value of $\beta_T$ (Eq (3)) admits a direct measurement of latent heat and Binder cumulant, performing metastable simulations at $\beta_T$ on lattice sizes up to $32^4$ [9]. Denoting the energy peaks of the probability distribution in the confined and Coulomb phases by $E_1(L)$ and $E_2(L)$ we fit their continuum limit values $E_i = E_i(\infty)$ and find

$$\hat{G} = E_2 - E_1 = 0.026685(54),$$

$$\hat{U} = \frac{(E_2^2 - E_1^2)^2}{12E_2^2 E_1^2} = -5.777(16) \times 10^{-4}.$$ 

Both values are in perfect agreement with the BK results from Eqs (4,5).

6. SUMMARY AND CONCLUSION

All cumulants investigated in our high statistics analysis at $L=6,8,10,12,14,16,18$ can be described by BK first-order FSS. Ab initio measurements confirm the FSS results for the infinite volume gap $G$, the Binder cumulant $U$ and the asymmetry $\ln(X)$. The non vanishing values for $G$ and $U$ lead to the conclusion that the phase transition in compact 4d $U(1)$ theory with Wilson action is first-order.

REFERENCES

1. C. Borgs and R. Kotecky: J. Stat. Phys. 61 (1990) 79.
2. C. Borgs and R. Kotecky: Phys. Rev. Lett. 68 (1992) 1734.
3. C. Borgs, R. Kotecky and S. Miracle–Sole: J. Stat. Phys. 62 (1991) 529.
4. W. Janke and R. Villanova: Nucl.Phys. B489 (1997) 679-696.
5. G. Arnold, Th. Lipper and K. Schilling: Phys.Rev. D59 (1999) 054509.
6. G. Arnold, Th. Lipper and K. Schilling: Nucl.Phys.Proc.Suppl. 83 (2000) 768-770.
7. G. Arnold, Bunk Th. Lipper and K. Schilling: to appear.
8. J. Jersak: Nucl.Phys. B497 (1997) 371-408.
9. G. Arnold, Th. Lippert, K. Schilling and Th. Neuhaus: Nucl.Phys.B (Proc.Suppl.) 94 (2000) 651-656.