Thermal expansion of coexistence of ferromagnetism and superconductivity

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Abstract. The temperature dependence of thermal expansion of coexistence of ferromagnetism and superconductivity below the superconducting transition temperature $T_c$ of a majority spin conduction band is investigated. Majority spin and minority spin superconducting gaps exist in the coexistent state. We assume that the Curie temperature is much larger than the superconducting transition temperatures. The free energy that Linder et al. [Phys. Rev. B76, 054511 (2007)] derived is used. The thermal expansion of coexistence of ferromagnetism and superconductivity is derived by the application of the method of Takahashi and Nakano [J. Phys.: Condens. Matter 18, 521 (2006)]. We find that we have the anomalies of the thermal expansion in the vicinity of the superconducting transition temperatures.

1. Introduction
The coexistence of ferromagnetism and superconductivity has intrigued many researchers since the coexistence of ferromagnetism and superconductivity in UGe$_2$ [1], UCoGe [2] and URhGe [3] was discovered. Recently, Linder and Sudbo find that coexistence of ferromagnetism and superconductivity has energetically been favoured by the free energy of the coexistent state being compared with the free energy of a purely ferromagnetic state and that of a unitary superconducting state [4]. They use the single band Hamiltonian. The ferromagnetic state splits the band into the up-spin band and the down-spin band. The up-spin superconducting gap and the down-spin superconducting gap exist in this model.

On the other hand, theory of the temperature dependence of the thermal expansion coefficient in magnetism is developed. The thermodynamic Grüneisen’s relation between the temperature dependence of the thermal expansion coefficient and the temperature dependence of the magnetic specific heat is not satisfied in the conventional theories of thermal expansion [5, 6]. Takahashi and Nakano [7], and Konno et al. [8] improve the theories of thermal expansion due to ferromagnetic spin fluctuations by taking into account the volume dependence of the free energy explicitly. They show that the temperature dependence of the thermal expansion coefficient is consistent with the temperature dependence of the magnetic specific heat. In other words, thermodynamic Grüneisen’s relation is satisfied.

Thermal expansion of coexistence of ferromagnetism and superconductivity based on the free energy derived by Linder and Sudbo has not been investigated. We apply Takahashi’s method to the coexistent state of ferromagnetism and superconductivity in this paper. The temperature dependence of thermal expansion of the coexistence of ferromagnetism and superconductivity is studied by taking into account the volume $V$ dependence of the free energy explicitly.
This paper is organised as follows. In the next section thermal expansion of coexistence of ferromagnetism and superconductivity will be derived. In section 3, the numerical results will be provided. Section 4 will be devoted to conclusions.

2. The derivation of thermal expansion of coexistence of ferromagnetism and superconductivity

We begin with the following free energy [4]:

\[ F/N = F_0/N + F_T/N \]

with

\[ F_0/N = \frac{IM^2}{2} + \sum_\sigma \frac{\Delta^2_{\sigma,0}}{2g} - \sum_\sigma \int_0^{E_F} d\epsilon N(\epsilon) \sqrt{(\epsilon - \sigma IM - E_F)^2 + \Delta^2_{\sigma,0}} / 2 \]

\[ F_T/N = T \sum_\sigma \int_0^\infty d\epsilon N(\epsilon) \ln(1 + e^{-\sqrt{(\epsilon - \sigma IM - E_F)^2 + \Delta^2_{\sigma,0}}/T}) \]

\[ \sigma = 1(\uparrow) \text{or} -1(\downarrow) \]

\[ \Delta_{k\sigma} = \frac{\Delta_{\sigma,0}}{\sqrt{3/8\pi}} Y_{l=1}^{(\sigma,0)}(\theta, \phi) \]

where \( N(\epsilon) \) is the density of states. \( g, M, IM \) are the effective attractive pairing coupling, the magnetization and the magnetic exchange energy, respectively. \( Y_{l=1}^{(\sigma,0)}(\theta, \phi) \) is the spherical harmonics. \( \Delta_{k\sigma} \) is the superconducting gap. We shall consider \( \sin \phi = 1 \) similar to the A2 phase in liquid \(^3\)He. The superconducting order parameters at \( T = 0 [\text{K}] \) are obtained

\[ \Delta_{\sigma,0}(0) = 2E_0 e^{-1/c\sqrt{1+\sigma M}} \]

where \( \tilde{M} = IM/E_F \) and \( E_F \) is the Fermi energy. \( E_0 \) is the cutoff energy. \( E_0/E_F \) is set to 0.01. The weak-coupling constant \( c = gN(0)/2 \) is set to 0.2. When \( \epsilon > E_0, \Delta_{\sigma,0} \) is zero. The temperature dependence of the superconducting order parameters is as follows:

\[ \Delta_{\sigma,0}(T) = \Delta_{\sigma,0}(0) \tanh(1.74\sqrt{T_{c,\sigma}/T - 1}) \]

where \( T_{c,\sigma} \) is the superconducting transition temperature of the spin \( \sigma \) band. Thermal expansion \( \omega \) is given by \( \omega = -K \frac{\partial\Phi}{\partial T} \). \( K \) is the compressibility. From Eq.(1), thermal expansion of coexistence of ferromagnetism and superconductivity is derived as follows:

\[ \omega/(NE_F) = \omega_0/(NE_F) + \omega_T/(NE_F) \]

with

\[ \omega_0/(NE_F) = -K(\frac{1}{2} E_F (\frac{\partial \ln N(0)}{\partial T}) \tilde{M}^2 + \frac{1}{2} EF \sum_\sigma \frac{\partial}{\partial T}(\frac{1}{2}) \Delta^2_{\sigma,0} - N(0)EF \frac{\partial \ln N(0)}{\partial T} \sum_\sigma \int_0^{T_F} dx \frac{\sqrt{(x - \sigma M - 1)^2 + \Delta^2_{\sigma,0}}}{2} - \frac{1}{2} \sum_\sigma A_\sigma (\sqrt{(-\sigma M)^2} - \sqrt{(-\sigma M - 1)^2 + \Delta^2_{\sigma,0}})] \]

\[ \omega_T/(NE_F) = -K \frac{T}{T_F} N(0)EF \sum_\sigma \left[-\frac{\partial \ln N(0)}{\partial T} \int_0^{\infty} dx \ln(1 + e^{-\frac{T_F}{\tilde{M}} \sqrt{(-\sigma M - 1)^2 + \Delta^2_{\sigma,0}}}) + A_\sigma \ln(1 + e^{-\frac{T_F}{\tilde{M}} \sqrt{(-\sigma M - 1)^2 + \Delta^2_{\sigma,0}}}) \right] \]

\[ + A_\sigma \ln(1 + e^{-\frac{T_F}{\tilde{M}} \sqrt{(-\sigma M - 1)^2 + \Delta^2_{\sigma,0}}}) \]
where $x = \epsilon/E_F$ and $\tilde{\Delta}_{\sigma,0} = \Delta_{\sigma,0}/E_F$. $N(0)$ is the density of states at the Fermi energy. $N$ is the number of the magnetic atoms. The derivatives of $\Delta_{\sigma,0}$ and $\tilde{M}$ do not appear in Eqs.(8), (9), and (10) because of the minimal conditions $\frac{\partial F}{\partial \Delta_{\sigma,0}} = 0$ and $\frac{\partial F}{\partial \tilde{M}} = 0$. We assume that $\frac{\partial x}{\partial V}$ is constant because it is very complicated and because we discuss the temperature dependence of thermal expansion of coexistence of ferromagnetism and superconductivity. $T_F$ is the Fermi temperature. In the next section, the temperature dependence of thermal expansion of coexistence of ferromagnetism and superconductivity is provided numerically.

3. Results
The temperature dependence of thermal expansion of coexistence of ferromagnetism and superconductivity is investigated with Eqs.(8), (9), (10), (11). Fig.1 shows the temperature dependence of thermal expansion of coexistence of ferromagnetism and superconductivity when $I/E_F = 0.1$, $\frac{\partial\ln I}{\partial V} = 0.1$, $E_F \frac{\partial}{\partial V} (\frac{1}{\beta}) = 0.1$, $\frac{\partial x}{\partial x} = 0.1$, $N(0)E_F = 0.1$, and $N(0)E_F \frac{\partial\ln N(0)}{\partial V} = 0.1$.

![Figure 1](image1.png)

**Figure 1.** The reduced temperature $T/T_F$ dependence of thermal expansion of coexistence of ferromagnetism and superconductivity when $I/E_F = 0.1$, $\frac{\partial\ln I}{\partial V} = 0.1$, $E_F \frac{\partial}{\partial V} (\frac{1}{\beta}) = 0.1$, $\frac{\partial x}{\partial x} = 0.1$, $N(0)E_F = 0.1$, and $N(0)E_F \frac{\partial\ln N(0)}{\partial V} = 0.1$.

![Figure 2](image2.png)

**Figure 2.** The reduced temperature dependence of the thermal expansion at very low temperatures when $I/E_F = 0.1$, $\frac{\partial\ln I}{\partial V} = 0.1$, $E_F \frac{\partial}{\partial V} (\frac{1}{\beta}) = 0.1$, $\frac{\partial x}{\partial x} = 0.1$, $N(0)E_F = 0.1$, and $N(0)E_F \frac{\partial\ln N(0)}{\partial V} = 0.1$.
dependence of the thermal expansion. Fig.2 shows the temperature dependence of the thermal expansion in the vicinity of $T_{\text{c}, \sigma}$. We have the anomalies of the thermal expansion around $T_{\text{c}, \sigma}$. The temperature dependence originates from the superconducting order parameters $\Delta_{\sigma,0}(T)$ because the magnetization at low temperatures is constant and because $T_{\text{c}, \sigma}$ is much smaller than the Curie temperature. The thermal expansion due to the explicitly thermal part of the free energy $F_T$ is very small because the superconducting transition temperatures are much lower than the Fermi temperature. From Eq.(7) the temperature dependence of the superconducting order parameters $\Delta_{\sigma,0}(T)$ at very low temperatures is obtained

$$\Delta_{\sigma,0}(T) \simeq \Delta_{\sigma,0}(0)(1 - e^{-3.48\sqrt{T_{\text{c}, \sigma}/T}}) \tag{12}$$

This equation is substituted for Eqs.(8) and (9). The temperature dependence of the thermal expansion at low temperatures is obtained

$$\omega \propto -\sum_{\sigma} \Delta_{\sigma,0}^2(0)(1 - 2e^{-3.48\sqrt{T_{\text{c}, \sigma}/T}}) \tag{13}$$

The thermal expansion increases with temperature rise exponentially. Moreover, the thermodynamic Grüneisen’s relation between the temperature dependence of the thermal expansion coefficient and the temperature dependence of the magnetic specific heat is satisfied because we use the same free energy as the free energy when the expression of the specific heat is derived.

4. Conclusions

We investigate the temperature dependence of thermal expansion of coexistence of ferromagnetism and superconductivity. The superconducting gaps similar to the A2 phase in liquid $^3$He are assumed. We find that anomalies of the thermal expansion exist in the vicinity of the superconducting transition temperatures. The thermal expansion at low temperatures increases with temperature rise exponentially. The thermodynamic Grüneisen’s relation between the temperature dependence of the thermal expansion coefficient and the temperature dependence of the magnetic specific heat is satisfied.

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References

[1] Saxena S S, Agarwal P, Ahilan K, Grosch F M, Haselwimmer R K W, Steiner M J, Pugh E, Walker I R, Julian S R, Monthoux P, Lonzarich G G, Huxley A, Sheikin I, Braithwaite D, and Flouque J 2000 Nature 406 587
[2] Huy N T, Gasparini A, Nijs D E de, Huang Y K, Klaasse J C P, Gortenmulder T, Visser A de, Hamann A, Görlac T, and Löhneysen v H 2007 Phys. Rev. Lett 99 067006, and references therein.
[3] Levy F, Sheikin I, Grenier B, Marcenat C, and Huxley A 2009 J. Phys.:Condens. Matter 21 164211, and references therein.
[4] Linder J and Sudbo 2007 Phys. Rev. B 76 054511, and references therein.
[5] Moriya T and Usami K 1980 Solid State Commun. 34, 95
[6] Wohlfarth E P 1977 Physica B91, 305
[7] Takahashi Y and Nakano H 2006 J. Phys. Condens. Matter 18, 521
[8] Konno R, Hatayama N, Takahashi Y, and Nakano H 2009 J. Phys. Conf. Ser. 150, 042100