Observation of a continuous time crystal
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Time crystals are classified as discrete or continuous depending on whether they spontaneously break discrete or continuous time translation symmetry. Although discrete time crystals have been extensively studied in periodically driven systems, the experimental realization of a continuous time crystal is still pending. We report the observation of a limit cycle phase in a continuously pumped dissipative atom-cavity system that is characterized by emergent oscillations in the intracavity photon number. The phase of the oscillation was found to be random for different realizations, and hence, this dynamical many-body state breaks continuous time translation symmetry spontaneously. Furthermore, the observed limit cycles are robust against temporal perturbations and therefore demonstrate the realization of a continuous time crystal.

Fig. 1. CTC in an atom-cavity system. (A) Schematic drawing of the atom-cavity system pumped transversely with an optical pump lattice, blue detuned with respect to an atomic transition. (Inset) The photon field (blue) and the atomic density (red) of the limit cycle dynamics, based on simulations. The blue color shading of the time axis indicates the intracavity photon number. (B) Single experimental realization of the limit cycle phase for δat/2π = −3.8 kHz and ε1 = 1.25 Erec. The vertical dashed black line indicates the start of the 10 ms holding time, during which the pump strength is held constant. The black line indicates the time trace of the pump strength ε1, and the blue line indicates the time evolution of the intracavity photon number Np(t). (C) Normalized and rescaled single-sided amplitude spectrum of Np calculated from the data shown in (B). (D) Distribution of the time phase in the limit cycle phase for δat/2π = −5.0 kHz and ε1 = 1.25 Erec. The error bars indicate the phase uncertainty within our discrete Fourier transform resolution of 100 Hz. However, the uncertainty with regard to the radial dimension—the amplitude uncertainty—is negligible small. For clarity, we removed the error bars, around 30%, which are overlapping. (Bottom) The evolution of the intracavity photon number for two specific experimental realizations, marked with “1” and “2” at top, which have a time phase difference of almost π.
wave field with a wavelength $\lambda_p = 792.55 \text{ nm}$ (Fig. 1A). This wavelength is blue detuned with respect to relevant atomic D$_1$ transition of $^{87}$Rb at a wavelength of $794.98 \text{ nm}$. The cavity operates in the recoil resolved regime (27)—its field decay rate $\kappa = 2\pi \times 3.4 \text{ kHz}$ is smaller than the recoil frequency $\omega_{\text{rec}} = 2\pi \times 3.7 \text{ kHz}$. The cavity resonance frequency $\omega_c$ is shifted because of the refractive index of the BEC by an amount of $\delta_c = N_c U_0/2$, where $U_0 = 2\pi \times 1.3 \text{ Hz}$ is the maximal light shift per intracavity photon. We define the effective detuning as $\delta_{\text{eff}} = \delta_c - \delta_p$, where $\delta_c = \omega_p - \omega_0$ is the detuning between the pump field frequency $\omega_p$ and the resonance frequency of the empty cavity $\omega_0$.

To determine the regime of the CTC, we measured the time dependence of the intracavity photon number $N_{\text{eff}}(t)$ that emerges in the protocol given below. We show $N_{\text{eff}}(t)$ in Fig. 2A and two derived quantities, the crystalline fraction $\Xi$ and the limit-cycle frequency $\omega_{\text{LC}}$ in Fig. 2, B and C, respectively. In our protocol, the intracavity photon number $N_{\text{eff}}(t)$ was recorded as we linearly ramped the pump strength from 0 to $3.5 \times E_{\text{rec}}$ within 10 ms, while keeping $\delta_{\text{eff}}$ fixed. Initially, for weak pump intensities, the BEC phase was stable, and $N_{\text{eff}}$ was zero. Above a critical value of $\epsilon$, the BEC became unstable toward the formation of a self-organized superradiant phase heralded by a nonzero $N_{\text{eff}}$. This represents a many-body state as the cavity photons mediate a retarded infinite-range interaction between the atoms. Although this superradiant phase transition has been intensively studied for a red-detuned pump (28–31), it has only been realized recently for a blue-detuned pump after its theoretical prediction (32, 33). For blue detuning, the atoms are low-field seeking and localize at the intensity minima of the light field. Nevertheless, the atoms can still self-organize into the superradiant phase, as evident from the blue large areas shown in Fig. 2A. However, the self-organized superradiant phase may become unstable for higher pump strengths because it costs energy for the atoms to localize away from the nodes of the pump lattice. This behavior leads to the disappearance of the self-organized phase for higher pump strengths (32). A phase diagram in fig. S1 in (34) shows a larger range of $\epsilon$, demonstrating the disappearance of the self-organization for strong pumping. In the recoil-resolved regime, because of the retarded character of the cavity-mediated interaction, we additionally observed the emergence of a new dynamical phase or a limit cycle phase characterized by self-sustained oscillations of $N_{\text{eff}}$ as the atoms cycled through different density wave patterns (33, 35). The resolution of the experimental imaging system is insufficient to observe the real-space density of the cloud; instead, simulations of the evolution of the single-particle density by use of a mean-field model are shown in fig. S3 (36). Physically, the limit cycles can be understood as a competition between opposing energy contributions: one coming from the pump lattice potential, and another coming from the cavity-induced all-to-all interaction between the atoms (33). In the superradiant phase, the cavity-induced interaction energy dominates, and the atoms localize at the antinodes. In the limit cycle phase for sufficiently strong pump intensities, localization of low-field–seeking atoms at the antinodes becomes energetically costly, resulting in a decrease in the density modulations and $N_{\text{eff}}$ as the system attempts to go back to the normal homogeneous phase. However, this is unstable toward self-organization because the chosen pump strength already exceeds the critical value, and thus, the cycle starts anew. The regime of recoil-resolution of the cavity, in which the dynamics of the atomic density and the light field evolve with similar time scales, has turned out to be the key ingredient to realize the limit cycle phase. This can be understood by noting that the delayed dynamics of the cavity field, with respect to the atomic density, leads to cavity cooling, which in contrast to broadband cavity

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**Fig. 2. Determining the time-crystalline regime.** (A) (Top) Pump strength protocol. (Bottom) The corresponding intracavity photon number $N_{\text{eff}}$ as a function of $\delta_{\text{eff}}$ and $\epsilon$. The area enclosed by the yellow dashed lines indicates the parameter space spanned in (B) and (C). (B) Relative crystalline fraction $\Xi$ and (C) limit cycle frequency $\omega_{\text{LC}}$ plotted versus $\delta_{\text{eff}}$ and $\epsilon$. To obtain (B) and (C), for fixed $\delta_{\text{eff}}$, the pump strength is ramped to its final value $\epsilon_f$ and subsequently held constant for 10 ms. The relative crystalline fraction $\Xi$ and the corresponding value of $\omega_{\text{LC}}$ identify the time-crystalline state. The parameter space is divided into 20 by 24 plaquettes and averages across 5 to 10 experimental implementations are produced. The white cross indicates the parameter values $\delta_{\text{eff}}/2\pi = -5.0 \text{ kHz}$ and $\epsilon_f = 1.25 E_{\text{rec}}$. The white area in (C) corresponds to data with $\Xi$ below $1/e$. 

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setups restricts the atoms to occupy only a small number of momentum modes. This prevents the system from heating up and entering chaotic dynamics. We observed the limit cycle phase in the region shown in Fig. 2A enclosed by the yellow dashed lines. To further highlight the dynamical nature of this phase, we show a typical single-shot realization in Fig. 1, B and C.

Next, we quantitatively identified the area in the parameter space, spanned by the pump strength $\varepsilon$ and the effective detuning $\delta_{\text{eff}}$ where limit cycles can be observed. For fixed $\delta_{\text{eff}}$, we linearly ramped $\varepsilon$ to the desired final value $\varepsilon_f$, using the same slope as for the measurement shown in Fig. 2A, and held $\varepsilon$ constant for 10 ms. The protocol is depicted by the black curve in Fig. 1B. We show in Fig. 1C an example of the normalized and rescaled single-sided amplitude spectrum $N_p(\omega) = \tilde{N}_p(\omega)/N_{p,\text{max}}(\omega_{\text{LC}})$ obtained from $N_p(\omega)$ within the holding time window [0,10] ms in Fig. 1B. $\tilde{N}_p(\omega)$ is the normalized single-sided amplitude spectrum, and $N_{p,\text{max}}(\omega_{\text{LC}})$ is the maximum value of the measured limit cycle amplitude. In the case of pronounced limit cycle dynamics as in Fig. 1C, the single-sided amplitude spectrum shows a distinct peak, with a width associated with the limit cycle lifetime of several milliseconds. The narrowest peaks observed exhibit a $\varepsilon^{-2}$ width $\Delta \omega = 2 \pi \times 1.4$ kHz: The limit cycle frequency $\omega_{\text{LC}}$, plotted in Fig. 2C, is defined as the frequency of the dominant peak in the single-sided amplitude spectrum within the frequency interval $\Delta \omega_{\text{LC}} = [3.5,15.5] \times 2 \pi$ Hz, chosen much larger than $\delta_{\text{LC}}$ $= [\omega_{\text{LC}} - \Delta \omega_{\text{LC}}/2, \omega_{\text{LC}} + \Delta \omega_{\text{LC}}/2]$. The oscillation frequency of a CTC is not necessarily fixed, and robustness refers to the persistence of the CTC in the thermodynamic limit and for a wide range of system parameters [finite-size effects are discussed in the supplementary materials (36)] (22). We calculated a common measure for time crystallinity, the crystalline fraction $\Xi$ (10, 11), as the ratio between the area under the single-sided amplitude spectrum within $\Delta \omega_{\text{LC}}$ and the total area within $\Delta \omega_{\text{LC}}$. That is, $\Xi = \sum_{\omega_{\text{LC}}} N_p(\omega)/\sum_{\omega} N_p(\omega)$. The relative crystalline fraction $\Xi$ shown in Fig. 2B is normalized to the maximum crystalline fraction measured in the parameter space explored in this work. Because of the finite lifetime of the BEC, it is difficult to access the long-time behavior of the system, which makes it experimentally challenging to distinguish between the areas of stable limit cycle, chaos, and possible transient phases. Hence, we define a cut-off or threshold value for the relative crystalline fraction, $\Xi_{\text{cut}} = 1/e$, to identify regions with observable limit cycle dynamics. In Fig. 2C, the frequency response of the limit cycle phase is only shown if its relative crystalline fraction is higher than the cut-off value: $\Xi > \Xi_{\text{cut}}$. The experimental lifetime of our time crystal is limited by atom loss. Furthermore, the short-range contact interaction, due to collisions between the atoms, leads to dephasing of the system and hence melting of the time crystal. Simulations that include contact interactions and phenomenological atom loss can be found in the supplementary materials.

The spontaneous symmetry breaking of a many-body system indicates a phase transition. We demonstrated strong evidence that the limit cycle phase emerges through spontaneous breaking of continuous time translation symmetry, and thus, it is a CTC. We repeated the experimental pump protocol shown as the Fig. 1B black line more than 1000 times with fixed $\delta_{\text{eff}}/2\pi = -5.0$ kHz and $\varepsilon_f = 1.25 E_{\text{rec}}$. These parameter values are indicated in Fig. 2C with a white cross. Because of technical instabilities, the number of the atoms in the BEC fluctuates by 5%. This leads to a fluctuating value of $\delta_{\text{eff}}$ and hence of $\omega_{\text{LC}}$. Pictorially, this can be understood by observing that fluctuations in $N_{\text{c}}$ effectively shift the CTC regime in Fig. 2C either up or down. For the parameter values indicated by a white cross in Fig. 2C, the median of $\omega_{\text{LC}}$ is $\omega_{\text{LC}} = 2 \pi \times 9.69$ kHz. Our discrete Fourier transform resolution, set by the 10-ms time window, is 100 Hz. Thus, we only considered experimental runs, which yielded response frequencies of $\omega_{\text{LC}} = \omega_{\text{LC}} \pm 2 \pi \times (50$ kHz. For each single-shot measurement, we obtained the time phase defined as the principal argument $\arg[N_p(\omega_{\text{LC}})]$ of the Fourier transformed intracavity photon number $N_p(\omega_{\text{LC}})$ evaluated at the limit cycle frequency $\omega_{\text{LC}}$. In Fig. 1D, we show the distribution of the observed time phases, which randomly covers the interval $[0,2\pi]$. This corroborates the spontaneous breaking of continuous time translation symmetry in the limit cycle phase. In the bottom of Fig. 1D, we show two specific experimental realizations, which have a time phase difference of almost $\pi$. Simulations representing the BEC as a coherent state show a range of the response frequency distribution of 300 Hz. Because we post-selected our data far below this limit, the origin of the spread over $2\pi$ in the time phase distribution is not due to technical noises but rather to quantum fluctuations. In the supplementary materials, we show a more detailed theoretical analysis to support this argument. The error bars along the angular direction in Fig. 1D indicate the phase uncertainty within 100 Hz of our Fourier limit. The average phase uncertainty is around 0.25$\pi$. The uncertainty in the radial direction corresponding to the oscillation amplitude is, however, negligible. Moreover, we removed 30% of the error bars for clarity in Fig. 1D.

Last, we demonstrated the robustness of the limit cycle phase against temporal perturbations, which is a defining feature of time crystals. We introduced white noise onto the pump signal with a bandwidth of 50 kHz. The noise strength is quantified by $r = \sum_{\omega=0}^{2\pi \times 50}$ kHz $|A_{\text{noisy}}(\omega)|/\sum_{\omega=0}^{2\pi \times 50}$ kHz $|A_{\text{clean}}(\omega)| > 1$, where $A_{\text{noisy}}$ and $A_{\text{clean}}$ are the single-sided amplitude spectrum of the pump in the presence and absence of white noise, respectively. We chose the parameters $\delta_{\text{eff}}/2\pi = -5.0$ kHz and $\varepsilon_f = 1.25 E_{\text{rec}}$ in the center of the stable limit cycle region, indicated by the white cross in Fig. 2C, and added white noise with varying strengths. In Fig. 3, A and B, top, single-shot realizations of the noisy pump signal are shown for weak and strong noise, respectively. The corresponding dynamics of $N_p$ is shown in Fig. 3, A and B, bottom. In Fig. 3E, we show how increasing the noise strength can “melt” the CTC as inferred by the decreasing relative crystalline fractions calculated from single-sided amplitude spectra, similar to those shown in Fig. 3, C.
and D. The system takes time to react to the noise, so that a few oscillations can always be observed before decay sets in. This leads to an offset of 0.4 in the crystalline fraction, even for very strong noise. Nevertheless, we found that the limit cycle phase indeed exhibits robust oscillatory behavior over a wide range of the noise strength. This, together with the observation of spontaneous breaking of a continuous time translation symmetry, suggests that the observed limit cycle phase is a CTC.

We have experimentally demonstrated a CTC and provided a theoretical understanding. This class of dynamical many-body states expands the concepts of long-range order and spontaneous symmetry breaking into the time domain and is therefore of fundamental interest. This result, and the precision and control achieved with our atom-cavity platform, paves the way toward a broad and comprehensive study of dynamical many-body states of bosonic or fermionic quantum matter in the strongly correlated regime. For example, an increased atom-photon coupling could generate a new class of time crystals associated with symmetry-broken periodic entanglement. Furthermore, technological applications, such as toward time metrology, can be envisioned.

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SUPPLEMENTARY MATERIALS

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