Index ARQ Protocol for Reliable Contents Distribution over Broadcast Channels

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Abstract—In the present paper, we propose a broadcast ARQ protocol based on the concept of index coding. In the proposed scenario, a server wishes to transmit a finite sequence of packets to multiple receivers via a broadcast channel with packet erasures until all of the receivers successfully receive all of the packets. In the retransmission phase, the server produces a coded packet as a retransmitted packet based on the side-information sent from the receivers via feedback channels. A notable feature of the proposed protocol is the decoding process at the receiver side has low decoding complexity because only a small number of addition operations are needed in order to recover an intended packet. This feature may be preferable for reducing the power consumption of receivers. The throughput performance of the proposed protocol is close to that of the ideal FEC protocols, which are often used for reliable content distribution over a broadcast channel. In an ARQ protocol, a receiver sends a request for retransmission to the sever if a receiver receives a broken packet or detects a packet loss. The server resends the corresponding packets when it receives a request from a receiver. Simple protocols for single-to-single communication, such as a Go-Back-N protocol and a selective-repeat protocol, can also be used for broadcast channels. However, direct application of such protocols to a broadcast channel often causes significant degradation of the throughput performance because such protocols do not consider packet losses in distinct receivers.

On the other hand, a specialized ARQ protocol using Forward Error Correcting (FEC) code provides excellent throughput performance over broadcast channels. Metzner proposed an ARQ protocol based on Reed-Solomon codes for broadcast channels [1]. His protocol yields much higher throughputs than those of single-to-single ARQ protocols applied to a broadcast channel. Chandran and Lin presented a Selective-Repeat ARQ protocol for broadcast channels [2]. Sakakibara and Kasahara introduced the concept of hybrid ARQ based on GMD decoding to Metzner’s protocol and reported improved throughput performance [3]. Recently, growing demand for real-time multicast, such as video streaming, have stimulated research on FEC-based protocols based on Reed-Solomon codes or sparse graph codes. For example, digital fountain codes [4] [8] provide high throughput performance without introducing complex encoding/decoding operations at the server and receiver sides [4] [5].

The concept of index coding proposed by Bark and Kol [9] has had a significant impact on research into coding for broadcast channels. Index coding is a coding technique to achieve better bandwidth efficiency of a broadcast channel. Index coding can use side-information to improve the throughput of the protocol. The concept of index coding is described as follows. Let us assume a broadcast channel with one server and multiple receivers. The server has a packet sequence, and each receiver knows a part of the packet sequence, which is referred to as side-information. A receiver is assumed to have its own need for packets, i.e., each receiver wants to know a part of the packet sequence. The server perfectly knows the desired part of the packet sequence and side-information for each receiver. Based on such knowledge, the server can produce coded packets by combining the original packets, which are sent to the channel. Appropriately coded packets using index coding can satisfy all of the demands of receivers and reduce the number of packets to be sent. Theoretical

I. INTRODUCTION

Recent strong demand for handing data sets of extremely large size has produced a number of situations in which a server must send a huge file to multiple clients over an unreliable channel. A simple example is the distributed backup of a mission-critical file system. In order to avoid a devastating incident due to a natural disaster, distributed backups at distant locations are of critical importance. It is common to use multicast protocols to reduce the bottleneck traffic at the server for such applications. Since IP multicast is based on the User Data Protocol (UDP), which is not so reliable over an IP network, an IP multicast protocol, such as a reliable multicast protocol, is proposed in order to ensure reliable content distribution. Another possible scenario is content distribution, such as HD video streams in cellular wireless systems. Suppose that a base station (i.e., server) wishes to share a large file or bitstream with multiple mobile terminals. In such a case, careful design of a protocol is necessary in order to achieve sufficient throughput while maintaining a certain degree of reliability.

These situations can be abstracted as a problem of sharing identical content with multiple receivers over an unreliable broadcast channel. A broadcast channel is a channel over which receivers can listen to what a server has sent over common media: wireless signals on a specific band or a multicast network. In order to achieve high reliability (i.e., low error probability) of data and high throughput, several coding techniques and protocols have been proposed for broadcast channels [1] [2] [3] [4] [5] [6].

A prominent example is Automatic Repeat reQuest (ARQ) protocols, which are often used for reliable content distribution over a broadcast channel. In an ARQ protocol, a receiver sends a request for retransmission to the sever if a receiver receives a broken packet or detects a packet loss. The server resends the corresponding packets when it receives a request from a receiver. Simple protocols for single-to-single communication, such as a Go-Back-N protocol and a selective-repeat protocol, can also be used for broadcast channels. However, direct application of such protocols to a broadcast channel often causes significant degradation of the throughput performance because such protocols do not consider packet losses in distinct receivers.
aspects of linear index coding are discussed in Bar-Yossef et al. [10], who showed that the minrank of the side-information graph gives the shortest code length of linear index coding. A number of theoretical studies on index coding have been conducted. For example, the relationship between index coding and network coding is discussed in [11]. From a practical point of view, finding appropriate combinations of packets is the most difficult part of the index coding process. Bark and Kol [9] proposed a greedy type algorithm for searching large cliques in a given side-information graph. Several efficient algorithms based on a graph algorithm or SAT solver are discussed in [12].

In the present paper, for reliable content distribution over broadcast channels with symbol (i.e., packet) erasure, we propose an ARQ protocol, referred to as index ARQ protocol, based on the concept of index coding. The goal of the proposed scenario is to share a packet sequence with all receivers participating in the protocol. Due to packet erasure, some of the received packets are missing at certain receivers. The proposed protocol does not rely on conventional FEC to compensate such packet losses. However, unlike conventional ARQ protocols, the proposed protocol performs an encoding procedure similar to index coding to produce a retransmitted packet. The states of the receivers are fed back to the server and such state information is used to make an appropriate coded packet. A coded packet is constructed by superimposing several packets over a finite field. This coding process resembles index coding based on the greedy clique algorithm proposed by Bark and Kol [9]. Successfully received packets at each receiver act as side-information, and these packets can be used to improve the bandwidth efficiency of the system. In the proposed protocol, one packet may compensate several packet losses at several receivers. Another notable feature of the proposed protocol is that the decoding process at the receiver side has low decoding complexity because a small number of addition operations are needed in order to recover an intended packet. This feature may be preferable for reducing power consumption in receivers.

II. PRELIMINARIES

In this section, we introduce the notation and definitions used throughout the present paper.

A. Broadcast channel

Figure 1 represents the broadcast channel assumed in the present paper. A server $S$ wishes to share a packet sequence $p = (p_1, p_2, \ldots, p_n) \in \mathbb{F}_q^n$ with $m$ receivers $R_1, \ldots, R_m$. The symbol $\mathbb{F}_q$ denotes the finite field with $q$ elements, where $q$ is a prime power. An element in $p$, $p_i$, is said to be a packet. Each receiver wishes to obtain whole packets in $p$. In other words, the goal of communication over this channel is to distribute $p$ to all of the receivers.

For simplicity, we assume that the server $S$ can send a coded packet (or an uncoded packet) $x^t \in \mathbb{F}_q^n$ to the channel at the discrete time instant $t \in \mathbb{N}$, where $\mathbb{N}$ is the set of positive integers. The time interval between two consecutive packet transmissions is assumed to be sufficient to accommodate a packet. In other words, two consecutive transmitted packets never collide with each other. The receiver $R_i (i \in [1, m])$ receives the received packet $y_i^t = x^t$ with probability $1 - \epsilon(0 < \epsilon < 1)$; otherwise $y_i^t = E$ with probability $\epsilon$, where the symbol $E$ represents the erasure symbol. An erasure can be considered as the occurrence of a packet loss on the channel. The occurrences of erasures (i.e., packet losses) are assumed to be independent (i.e., the channel is memoryless). The notation $[a, b]$ represents the set of consecutive integers from $a$ to $b$.

In the initial state of the protocol, none of the receivers have knowledge of the contents of the packet sequence $p$. The server continues to send a sequence of coded packets $x^1, x^2, x^3, \ldots$ until all of the receivers successfully obtain all of the packets in $p$. At time $t$, the packet indices corresponding to the packets that were successfully received by $R_i$ are denoted by $K_i^t \subseteq [1, n]$ (known indices). The indices of unknown packets are represented as $W_i^t \subseteq [1, n]$ (wanted indices). Based on these definitions, $K_i^t \cup W_i^t = [1, n]$ holds for any $i \in [1, m]$ and for any $t \in \mathbb{N}$.

For any time $t$, the information on $W_i^t$ (or equivalently $K_i^t$) is fed back to the server via a noiseless feedback channel before $x^{t+1}$ is sent to the channel. In other words, the server $S$ always has perfect knowledge of the known packets for all receivers. Here, we assume that the size of a packet is much larger than the index information communicated via the noiseless feedback channel. This means that the capacity of the feedback channel can be much smaller than that of the forward broadcast channel. For example, in a multicast scenario, the noiseless feedback channel can be implemented using reliable TCP connections.

B. State matrix

As described in the previous section, the server $S$ perfectly knows the states of the receivers. The state matrix is a matrix representation of the knowledge of the server. The definition is given as follows.

**Definition 1 (State matrix):** An $m \times n$ binary matrix $C = \{C_{i,j}\}$ is said to be a state matrix if $(i, j)$ element $C_{i,j} (i \in [1, m], j \in [1, n])$ is given by

$$C_{i,j} = \begin{cases} 1, & R_i \text{ knows the content of packet } p_j, \\ 0, & \text{otherwise}. \end{cases}$$
We here focus on these steps. The function $f^t$ at the server side is summarized as Algorithm 1.

### Algorithm 1 Encoding process at the server side

1: $t := 0$
2: $C^t := 0^{m \times n}$ (0$^{m \times n}$ represents $m \times n$ zero matrix)
3: while $C^t$ contains a zero element do
4: \[
I^t := f^t(C^t),
\]
5: A coded packet is constructed as
\[
x^t := \sum_{j \in I^t} p_j.\]
6: Send $x^t$ to the broadcast channel.
7: $t := t + 1$.
8: $C^t$ is obtained based on the feedback information.
9: end while
due to the definition of the clique matrix and clique indices.

According to [3], a coded packet $x^t \in \mathbb{F}_q$ is encoded by adding packets having indices in $I$. Several packets are superimposed over $\mathbb{F}_q$ to produce a coded packet. This coded packet can thus satisfy the demands from several receivers simultaneously, i.e., a coded packet can compensate multiple unknown packets in several receivers. This provides the advantage of the proposed protocol in terms of throughput and bandwidth efficiency.

### B. Decoding process of index ARQ protocol

The receiver $R_i$ receives symbol $y_i^t$ from the channel. We assume that each receiver knows $I^t$ via the header information attached to a transmitted coded packet. The decoding process at time index $t$ in the receiver $R_i$ is summarized in Algorithm 2.

### Algorithm 2 Decoding process at the receiver $R_i$

1: if $y_i^t = E$ then
2: Quit the decoding process.
3: end if
4: if $I^t \cap \mathcal{W}_i^t = \emptyset$ then
5: Quit the decoding process.
6: else
7: Let $k$ be a unique index in $I^t \cap \mathcal{W}_i^t$.
8: end if
\[
p_k := y_i^t - \sum_{j \in I^t \setminus \{k\}} p_j.
\]
9: $\mathcal{W}_i^{t+1} := \mathcal{W}_i^t \setminus \{k\}$.
10: Send $\mathcal{W}_i^{t+1}$ to the server via the feedback channel.

$k \in \mathcal{W}_i^t$. Note that $R_i$ knows the packet $p_j$ for any $j \in I^t \setminus \{k\}$ due to the definition of the clique matrix and clique indices. The reconstruction rule (7) is an immediate consequence of the encoding rule (6). As an example, encoding and decoding processes are depicted in Fig. 2.
the state matrix with regard to the delay of feedback information.

In the present paper, we use the following strategy to design an index generator. The first phase of the protocol is defined as a sequence of time indices \( (1, 2, \ldots, n) \). The index generator outputs \( \{t\} \) if \( t \) is in the first phase, i.e., \( t \in [1, n] \). In other words, the original packets \( p_1, p_2, \ldots, p_n \) are sent directly to the channel in the first phase. The second phase of the protocol \( (t > n) \) can be considered to be a retransmission phase. In the second phase, unreceived packets due to packet losses are gradually compensated as the decoding process proceeds.

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of the protocol is that every receiver requires to solve an erasure correcting problem based on a Reed-Solomon code that requires a certain computational power at the receiver side.

In the present paper, we adopt throughput as the main performance measure. The throughput of a protocol is directly related to the bandwidth efficiency of the protocol. Let \( N \) be the total number of transmitted packets from the server until the protocol terminates (i.e., all of the receivers obtain the entire packet sequence). Note that this number \( N \) is a random variable depending on the randomness of the erasure channels. The throughput \( \tau \) is defined as \( \tau = E[n/N] \) which represents average amount of information per transmitted packet. In the following subsections, the throughput \( \tau \) is estimated through computer simulations of these protocols.

The capacity of a single erasure channel with erasure probability \( \epsilon \) is given by \( 1 - \epsilon \). The coding theorem proved by Shannon [15] guarantees the existence of sufficiently long FEC-codes with coding rates below \( 1-\epsilon \) that achieve arbitrarily small error probabilities. Suppose that a server uses such an FEC-code with a coding rate close to \( 1-\epsilon \). Although such a system suffers from large latency due to long code length, a throughput close to \( 1-\epsilon \) can be achieved. In the following discussion, we use such a system as a benchmark for the throughput performance. The upper bound of the throughput \( \tau = 1-\epsilon \) is referred to hereinafter as the ideal FEC bound.

B. Results of computer experiments

Figure 3 shows the relationship between throughput and erasure probability \( (\epsilon \leq 0 \leq 0.1, \text{number of receivers } m = 100, \text{number of packets } n = 1000) \). The Metzner protocol is confirmed to achieve the best throughput performance among these three protocols at all erasure probabilities. The throughput of the Metzner protocol are quite close to the ideal FEC bound \( \tau = 1 - \epsilon \). This fact indicates that the Metzner protocol provides excellent bandwidth efficiency that is close to optimal. On the other hand, the SR protocol offers poor throughput performance compared with the Metzner protocol. For example, the Metzner protocol provides \( \tau = 0.877 \), whereas the SR protocol yields \( \tau = 0.365 \) at \( \epsilon = 0.1 \). This result implies that the SR protocol cannot achieve a bandwidth efficiency close to the optimal performance, although it is the simplest to implement. The proposed protocol, index-ARQ, provides slightly smaller throughput compared with the Metzner protocol but the difference is fairly small, especially when the erasure probability is small, such as \( \epsilon < 0.05 \). Even for a relatively large erasure probability \( \epsilon = 0.1 \), the proposed protocol achieves 92% of the throughput performance of the Metzner protocol.

Figure 4 shows the case in which \( m = 50 \) and \( n = 2000 \). In this case, we can also observe the same tendency seen in Fig. 3.

In order to observe the relationship between the number of packets and the throughput, we conducted several experiments. Figure 5 shows such a relationship under the condition in which the erasure probability is \( \epsilon = 0.05 \) and the number of receivers is \( m = 100 \). The horizontal axis indicates the number of packets, and the vertical axis indicates the throughput. In the case of the SR protocol, the throughputs are approximately constant, regardless of the number of packets. On the other hand, in the case of the index ARQ protocol and the Metzner protocol, the throughputs increase slightly as the number of packets increases. Moreover, the difference in throughput of these two protocols and the ideal FEC bound becomes negligible as the number of packets increases. The experimental results suggest a system design principle for the index ARQ protocol such that the number of packets should be larger than the number of receivers in order to achieve higher throughput.
V. CONCLUDING SUMMARY

In the present paper, we proposed an ARQ protocol, referred to as index ARQ, for broadcast channels. In the proposed protocol, the server incorporates a coded packet by adding several packets over $\mathbb{F}_q$ based on the knowledge of the demands of all of the receivers. In order to find an appropriate set of indices for the packets to be added, a randomized greedy algorithm is devised. The bandwidth efficiency of the proposed protocol derives from the fact that a coded packet can compensate multiple packet losses in several receivers. A practical advantage of the proposed protocol is the simplicity of its decoding process at the receiver side. A decoding process only constitutes several additions over $\mathbb{F}_q$ and results in a small computational load at the receiver side. If $q = 2^m$, then only exclusive OR operations are required to recover packet losses.

Based on the results of computer experiments, we confirmed that the proposed protocol achieves much higher throughputs than the SR protocol. The proposed protocol requires a certain computational load to encode at the server side. This computational load at the server side can be considered as a cost to be paid in order to achieve better bandwidth efficiency than the SR protocol. The throughput performance of the proposed protocol is close to that of the Metzner protocol and the ideal FEC bound when the erasure probability is in the range of $0 < \epsilon < 0.1$, which implies that the proposed protocol provides approximately optimal throughput performance in such a regime. If a receiver is a mobile terminal with lower computational power or prefers a low power consumption, the proposed protocol would be a preferable choice in order to achieve both high bandwidth efficiency and lower computation load at the receiver side.

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