Description of non-linear unloading curve and closure of cyclic stress-strain loop based on Y-U model

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Abstract. This paper proposes a cyclic elasto-plasticity model to describe non-linear unloading-reloading curves and closure of a cyclic stress-strain hysteresis loop based on the Yoshida-Uemori (Y-U) model. The nonlinear unloading stress-strain response is expressed by an equation for the stress-dependent Young’s modulus in the yield surface. The closure of a cyclic stress-strain curve is described by automatically changing Y-U material parameters. The great advantage of the above approaches is that no additional parameters are needed.

1. Introduction
For accurate numerical simulation of sheet metal forming an appropriate material model is needed. Especially for springback simulation, cyclic elasto-plasticity models play a critical role. The present authors proposed a model of large-strain cyclic plasticity, the Yoshida-Uemori (Y-U) model [1-3], and nowadays, it is widely used for sheet metal forming simulation. This model takes into account the degradation of unloading stress-strain slope with increasing plastic strain (so-called ‘plastic-strain dependent Young’s modulus’). This model works well to predict springback, since the amount of springback is directly related to Young’s modulus of the sheet, as well as the Bauschinger effect. Recently, some researchers attempted to describe the nonlinear unloading behavior [4-7], however, most of their models are rather complicated and they need several additional material parameters (or even tensor variables). In the present paper, a simple non-linear elasticity model is presented, which is consistent with the model of plastic-strain dependent Young’s modulus.

The second topic in this paper is the description of closure of cyclic stress-strain hysteresis loop for a case of the subsequent small-strain-range cycling after a large plastic pre-strain. To describe the closure of a cyclic stress-strain hysteresis loop, one of the approaches is the use of multi-LKH (with a stress threshold) components [8], which gives piecewise linear stress-strain responses. The multi-surface modeling [9] is another type of cyclic plasticity model that describes the closure of a cyclic stress-strain hysteresis loop. All these multi-component type models well describe the closure of a cyclic stress-strain loop, however, they need many additional tensor variables, which may increase the computational cost enormously.

In the present paper, an improved version of the Y-U model is presented to describe the closure of the cyclic stress-strain loop. Recently, Sumikawa et al. [10] also proposed a cyclic plasticity model to describe the closure of a stress-strain hysteresis loop based on the Y-U model,
however their model requires several additional material parameters. In contrast, the present model is able to determine necessary parameter changes automatically.

2. Cyclic stress-strain responses calculated by the Y-U model

2.1. Y-U model

With the assumption of a small elastic and large plastic deformation, the rate of deformation $D$ is decomposed as

$$ D = D^e + D^p, $$

where $D^e$ and $D^p$ are the elastic and plastic parts of the rate, respectively. The present constitutive model of plasticity has been constructed within the framework of two-surface modeling, wherein the yield surface moves kinematically within a bounding surface. When the yield function at the initial (non-deformed) state, $f_0$, has a general form:

$$ f_0 = \phi(\sigma) - Y = 0, $$

where $\phi$ denotes a function of the Cauchy stress $\sigma$, and $Y$ is the initial yield strength, the subsequent yield function $f$ is given by the equation:

$$ f = \phi(\sigma - \alpha) - Y = 0, $$

where $\alpha$ denote the backstress. The associated flow rule is written as

$$ D^p = \frac{\partial f}{\partial \sigma} \dot{\sigma}, $$

where $\dot{\sigma}$ is the effective plastic strain rate. The bounding surface $F$ is expressed by the equation:

$$ F = \phi(\sigma - \beta) - (B + R) = 0, $$

where $\beta$ denotes the center of the bounding surface, and $B$ and $R$ are its initial size and isotropic hardening (IH) component. The kinematic hardening of the yield surface describes the transient Bauschinger deformation characterized by early re-yielding and the subsequent rapid change of workhardening rate. The relative kinematic motion of the yield surface with respect to the bounding surface is expressed by

$$ \alpha_* = \alpha - \beta. $$

For the evolution of $\alpha_*$, we assume

$$ \alpha_* = C \left[ \frac{\alpha}{Y} (\sigma - \alpha) - \frac{\alpha}{\sqrt{\alpha_*}} \right] \dot{\sigma}, \quad \alpha_* = \phi(\alpha_*), \quad \alpha = B + R - Y $$

where $C$ is a material parameter that controls the rate of the kinematic hardening. Here, $(\circ)$ stands for the objective rate. For the evolution equations of $\beta$ and $R$, refer to articles [2, 3].

In the present model, the size of yield surface is $Y$ constant. However, if we carefully observe the stress-strain response during unloading after plastic deformation, we can find out that the stress-strain curve is no longer linear but slightly curved due to very early re-yielding and the Bauschinger effect. In order to describe this, in the model, the following equation of plastic-strain dependent Young’s modulus was presented [1]:

$$ E_{av} = E_o - (E_o - E_a) \left[ 1 - \exp(-\xi \dot{\varepsilon}) \right], $$

where $E_o$ and $E_a$ stand for Young’s modulus for virgin and infinitely large prestrained materials, respectively, and $\xi$ is a material constant.

2.2. Y-U calculations to be improved
Although the Y-U model is able to express cyclic plasticity behavior mostly well, the model would be improved so as to describe the following two stress-strain responses:
- non-linear unloading/reloading stress-strain responses,
- complete closure of cyclic stress-strain hysteresis loop.

Figure 1(a) shows the comparison of experimentally observed cyclic stress-strain response (data on 590 MPa high strength steel [HSS] sheet were taken from Sumikawa et al. [10]) with the corresponding Y-U model calculation for the subsequent small-strain-range cycling after a large plastic re-strain. In the experiment, the stress-strain point after a full cycle reaches just the same point as that of unloading start. However, the Y-U model predicts slightly lower stress after a full cycling compared to the unloading-start stress. This is because the backstress does not return to the unloading-start point after a cycle, as shown in Fig. 1(b). This is a common problem appearing in most of non-linear kinematic hardening models (e.g., Armstrong-Frederick model [11]).

![Cyclic stress-strain responses](image1)

(a) Cyclic stress-strain responses

![Backstress evolution in cyclic loading](image2)

(b) Backstress evolution in cyclic loading

Fig. 1 Stress-strain responses during subsequent small-strain-range cycling after a large pre-strain

3. Non-linear unloading/reloading stress-strain curve

Figure 2 shows a model of stress-dependent Young’s modulus when a stress point exists (at point B) in a yield surface. We assume that Young’s modulus $E$ is determined as a function of stress-point position measured by the length of an extrapolated stress path (B-C in Fig. 1), $\rho$, as

$$E = 2E_{av} - E_0 + \left(\frac{E_0 - E_{av}}{Y}\right)\rho$$  (8)

Under the uniaxial stress-state, Young’s modulus at unloading-start point (A in Fig. 2) is always equal to $E_0$, and $E = 2E_{av} - E_0$ at the final elastic unloading point (C in Fig. 2). In Fig. 2, point B is a current stress state, at which Young’s modulus is $E$, and an arrow schematically shows the direction of unloading. The stress-strain response during elastic unloading and the subsequent reloading, calculated by this model, is shown in Fig. 3. Here, non-linear stress-strain curves are successfully described. Note that this model is consistent with the Y-U model of plastic-strain dependent Young’s modulus, where Eq. (8) uses only initial Young’s modulus $E_0$, yield strength $Y$, and $E_{av}$. This model can be applied to multi-axial stress state, as schematically shown in Fig. 3 (see path $A’ \rightarrow B’ \rightarrow C’$, $\rho = \rho’$).
Fig. 2 Schematic illustration of the variation of Young’s modulus at a stress point in the yield surface

Fig. 3 Non-linear unloading-reloading stress-strain responses in elastic region calculated by the present model

4. Closure of cyclic stress-strain hysteresis loop
Under the uniaxial cyclic stress state, as schematically shown in Fig. 4, the evolution equations of the backstress are expressed as follows, e.g.,
- for \( a \rightarrow b \), \( d\alpha = -Ca\left(1+\sqrt{\alpha_s}/a\right)dp \),
then \( p_{sb} = \frac{2}{C} \left[ \sqrt{\alpha_{r\max}}/a - \ln\left(1+\sqrt{\alpha_{r\max}}/a\right) \right] = \frac{2}{C} \left[ t_{\max} - \ln\left(1+t_{\max}\right) \right] \), \( \quad \ldots (9) \)
where \( t_{\max} = \sqrt{\alpha_{r\max}}/a \),
for $b \rightarrow c$, $d\alpha_c = -Ca\left(1-\sqrt{|\alpha_c/a|}\right)dp$

then

$$p_{bc} = \frac{-2}{C}\left[\sqrt{|\alpha_{\text{max}}/a|} + \ln\left(1-\sqrt{|\alpha_{\text{min}}/a|}\right)\right] = \frac{-2}{C}\left[t_{\text{min}} + \ln(1-t_{\text{min}})\right], \quad(10)$$

where $t_{\text{min}} = \sqrt{|\alpha_{\text{min}}/a|}$.

Thus the plastic strain in the process $a \rightarrow b \rightarrow c$ is

$$p_{ac} = p_{ab} + p_{bc} = \frac{2}{C}\left[t_{\text{max}} - t_{\text{min}} + \ln(1-t_{\text{min}})(1+t_{\text{max}})\right]. \quad(11)$$

In the similar way, the process $c \rightarrow d \rightarrow a$ are analysed by using the slightly different value of $C'$, as

$$p_{ca} = p_{cd} + p_{da} = \frac{2}{C}\left[t_{\text{min}} - t_{\text{max}} + \ln(1-t_{\text{max}})(1+t_{\text{min}})\right]. \quad(12)$$

From the condition of closed hysteresis loop of $a \rightarrow b \rightarrow c \rightarrow d \rightarrow a$, i.e., $p_{ac} = p_{ca}$, the value of $C'$ is determined as follows:

$$C' = C \frac{t_{\text{min}} - t_{\text{max}} - \ln(1-t_{\text{max}})(1+t_{\text{min}})}{t_{\text{max}} - t_{\text{min}} - \ln(1-t_{\text{max}})(1+t_{\text{min}})}. \quad(13)$$

Figure 5 shows the cyclic stress-strain curves calculated by the present model, which successfully describes the complete closure of cyclic hysteresis loop, where the experimental data plots on 590 MPa HSS were taken from Sumikawa et al. [10]. For more precise description of the re-loading stress-strain response, we may also change $a$-value to $a'$ for re-loading process ($c \rightarrow d \rightarrow a$ in Fig. 4), as

$$a' = \xi a, \quad(14)$$

where $1 \leq \xi \leq |\alpha_{\text{max}}/\alpha_{\text{max}}|$ is a correcting parameter. For a given $a'$ value, $C'$ is calculated as follows:

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Figure 4 shows the evolution of backstress under cyclic loading.
\[ C' = C \frac{t'_\min - t'_\max - \ln(1 - t'_\max)(1 + t'_\min)}{t'_\max - t'_\min - \ln(1 - t'_\min)(1 + t'_\max)}, \]  \hspace{1cm} (15)

where \( t'_\min = \sqrt{\alpha'_\min / a} \) and \( t'_\max = \sqrt{\alpha'_\max / a} \).

Fig. 5 Cyclic stress-strain responses calculated by the improved Y-U model (experimental data on 590 MPa HSS were taken from Sumikawa et al. [10])

5. Concluding remarks

The new version of the Y-U model can describe the nonlinear unloading-reloading hysteresis loop by introducing the nonlinear elasticity model, as well as the closure of cyclic stress-strain loop by using automatically changing \( C \)-parameter in Y-U model. The great advantage of the above approaches is that no additional parameters are needed for modeling.

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