Production of $\chi_c$ meson pairs with additional emission

I. Babiarz$^1, \ast$, W. Schäfer$^1$, and A. Szczurek$^{1,2}$

$^1$Institute of Nuclear Physics PAS, Krakow, Poland
$^2$Faculty of Mathematics and Natural Sciences, University of Rzeszow, Poland

Abstract. We discuss mechanism of double $\chi_c$ production with large rapidity separation. The first order perturbative correction to $gg \to \chi_c\chi_c$ process includes additional emission of gluon among $\chi_c$ pair. We have considered real gluon distribution as well as virtual correction to the production process of $\chi_c$ pairs. Results for two scalar $\chi_{c0}\chi_{c0}$ and two axial mesons $\chi_{c1}\chi_{c1}$ are shown.

1 Introduction

The production process of $\chi_c$ pairs with extra emission of a gluon is the first order perturbative correction to the Born result for $\chi_c$ pair production, see Fig. 1.

The cross section for the elementary process $gg \to \chi_c\chi_c$, at gluon gluon center mass energy squared $\hat{s}$, shown in the first diagram in Fig. 1, can be written as follows:

$$d\sigma = \frac{1}{16\pi^2\hat{s}^2} |A|^2 \delta^2(q_1 + q_2)d^2q_1d^2q_2,$$

where the squared amplitude is:

$$|A|^2 = \frac{\hat{s}^2}{N_c^2 - 1} I_1(q_1)I_2(q_2),$$

$$I_1(q_1) = \frac{1}{2} \sum_{\lambda_1} |\epsilon_{\mu}(\lambda_1) T_{\mu\nu} n_\nu^-|^2, \quad I_2(q_2) = \frac{1}{2} \sum_{\lambda_2} |\epsilon_\alpha(\lambda_2) T_{\beta\alpha} n_\beta|^2. \tag{2}$$

Here $q_1$ and $q_2$ are transverse momenta of outgoing $\chi_c$. For scalar particle or axial particle $T^{\mu\nu}$ takes the form given in Ref. [1], and $n^+n^- = 1$.

$\ast$e-mail: izabela.babiarz@ifj.edu.pl

*Fig. 1. Diagrams for production $\chi_c$ pairs. In the left panel there is a leading order process and in the right panel there is pair production associated with extra gluon emission.*
The cross section for the inclusive production of $\chi_c$ pair:

$$d\sigma(gg \rightarrow \chi_c \chi_c X) = d\sigma^{(0)}(2 \rightarrow 2) + d\sigma^{(1)}(2 \rightarrow 2) + d\sigma(2 \rightarrow 3),$$

where terms $d\sigma^{(1)}(2 \rightarrow 2)$ and $d\sigma(2 \rightarrow 3)$ correspond to high order mechanisms. The cross section $d\sigma(2 \rightarrow 3)$ is calculated as for a process with large rapidity distance between two $\chi_c(1)$ and an extra gluon emission among them. The cross-section $d\sigma^{(1)}(2 \rightarrow 2)$ includes the virtual corrections to the first order in $\alpha_s Y$ in the BFKL formalism.

2 Real-gluon contribution to the inclusive $gg \rightarrow \chi_c \chi_c X$ process

The cross section for $gg \rightarrow \chi_c g \chi_c$ process is: $d\sigma(2 \rightarrow 3) = \frac{1}{28\pi^5 \hat{s}^2} |A|^2$ $d^2 q_{1\perp} d^2 q_{2\perp} dy$, where $y$ denotes rapidity of the gluon. In the impact factor representation the amplitude for the $gg \rightarrow \chi_c g \chi_c$ reads:

$$|A|^2 = \frac{N_c}{N_c^2 - 1} (16\pi\alpha_s I_1(q_1) \frac{\hat{s}^2}{(q_1 + q_2)^2} I_2(q_2) = \frac{16\pi^3 \hat{s}^2}{N_c^2 - 1} I_1(q_1) \mathcal{K}_\epsilon(q_1, -q_2) I_2(q_2),$$

where $\mathcal{K}_\epsilon$ is derived from real radiative correction to gluon gluon scattering amplitude in the BFKL formalism, obtained using effective Lipatov vertex [2].

The squared amplitude is independent of $y$, thus one can integrate it out and receive

$$\mathcal{Y} = \int dy \approx \log(\hat{s}/M^2),$$

hence: $d\sigma(2 \rightarrow 3) = \frac{\mathcal{Y}}{16\pi^2(N_c^2 - 1)} I_1(q_1) \mathcal{K}_\epsilon(q_1, -q_2) I_2(q_2) d^2 q_{1\perp} d^2 q_{2\perp}.$

The cross section has a singularity when the transverse momentum of the produced gluon $q_{3\perp}$ vanishes. Because of that fact let us introduce a parameter $\mu$ and a regulator function:

$$F_{\text{suppression}} = \frac{q_{3\perp}^2}{(\mu^2 + q_{3\perp}^2)^2}.$$  

The factor $F_{\text{suppression}}$, which is included in the amplitude, provides that the cross section is integrable in the region of small $q_{3\perp}$. Note, that the singularity appears in the back-to-back kinematical situation.

Figure 2. Differential cross section as a function of rapidity. There are shown results for gluon distribution in the middle of the figure and also distribution of $\chi_c(0)$ with higher order corrections.
3 Virtual corrections

The virtual corrections in the first order in $\alpha_s Y$ means that there are two diagrams in which the two gluons are in octet state and give a ln(\delta) contribution[2]. It leads to a replacement of the gluon propagator by: $\frac{1}{q^2} \rightarrow \frac{1}{q^2} e^{\omega(q)Y}$, whereas: $\omega(q_1) = -\frac{\alpha_s N_c}{4\pi^2} \int d^2 q_\perp \frac{q^2_\perp}{q_\perp^2 (q_\perp - q_1\perp)^2}$.

Subsequently expressing $Q$ by $q_\perp - q_1\perp$ and doing some calculation one can obtain an expression free of singularity, also using the same parameter $\mu$:

$$\omega(q_1) = -\frac{\alpha_s N_c}{2\pi^2 q_1^2} \int d^2 Q^2 \frac{Q^2}{(Q^2 + \mu^2)^2} \frac{1}{\sqrt{4Q^4 + p^4}}.$$  \hspace{1cm} (6)

Then expanding the exponential function and decomposing into two terms the formula for the differential cross section for $d\sigma(2 \rightarrow 2)$, the radiative correction can be written:

$$d\sigma^{(1)}(2 \rightarrow 2) = \frac{Y}{16\pi^2(N_c^2 - 1)} I_1(q_1\perp) K_c(q_1\perp, -q_2\perp) I_2(q_2\perp) d^2 q_1\perp d^2 q_2\perp,$$  \hspace{1cm} (7)

$$K_c(q_1\perp, -q_2\perp) \equiv \delta^{(2)}(q_1\perp + q_2\perp) 2\omega(q_1).$$  \hspace{1cm} (8)

In order to omit the singularity, there we also use the parameter $\mu$ and as before $y_{\text{etot}}$ in purpose to guarantee numerical compatibility with real correction. Here $y_{\text{etot}}$ is applied in the gluon propagator by replacing $Y$ by $Y - 2y_{\text{etot}}$. After summation of all terms in the first order correction in $\alpha_s Y$ the singularity disappears and the single meson transverse momentum is indeed infrared finite.

![Figure 3](image-url). Differential cross section as a function of $\chi_{c1}$ transverse momentum when the second produced particle is $\chi_{c1}$. The dotted-dashed (red) line is for leading order result.

In the Fig. 3 one can notice some effects caused when we include the virtual correction as well as the real gluon distribution at the parton level. Adding the virtual corrections cause reduction of the cross section, while including extra gluon emission cause enhancement. It will be interesting to see these effects for hadronic cross sections.

References

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