Optimizing the Process of Multi-wire Sawing Silicon Rods with Consolidated Abrasive Diamonds Based on RSM

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Abstract. In order to improve the processing efficiency and processing quality of silicon rods, and to optimize the process parameters during the cutting process of silicon rods, the sawing force of wire saw is studied from the aspects of cutting deformation and friction. Meanwhile, the mathematic model is established between the sawing force and some cutting process parameters. Besides, the quadratic regression model is also established by Box-Behnken center combination experiment and the response surface methodology(RSM), which can reflect the wire saw speed, workpiece feed rate and wire saw radius of the interaction of three process parameters on the wire saw sawing force. The results show that the model can accurately predict the sawing force and optimize the process parameters during the cutting process as well.

1. Introduction

In the many slicing methods, the consolidated abrasive diamond wire saw is widely used in the cutting process of silicon materials in solar cells due to its high cutting speed, low cost of consumables and environment protection [1]. However, the quality of multi-wire saw products is affected by parameters such as wire saw speed. Therefore, it is necessary to study and analyze the sawing force during the wire saw cutting process. The influence of process parameters on the cutting force of wire saw is analyzed, which lays a foundation for optimizing process parameters.

Chaohua Wu et al. [2] established a single abrasive grain sawing force model during the wire saw cutting process, and studied the influence of parameters such as wire saw speed on the deformation of a single abrasive grain by finite element analysis; Hao Wu et al. [3] studied the influence of diamond abrasive shape on the cutting depth and cutting force of SiC by theoretical calculation and finite element analysis. Li Shujuan et al. [4] established the cutting force model of wire saw during the wire saw cutting process. Zhang Liaooyuan [5], Cui Dan et al. [6] studied the effects of different process parameters on the sawing force. Meng Jianfeng et al. [7] analyzed the sawing force from the angle of vibration, and established the transfer function of the wire saw cutting operation equation. Gao Yufei et al. [8] studied the influence of wire speed and feed speed on the surface roughness of silicon wafer.

Firstly, the sawing force of wire saw is studied from the aspects of cutting deformation and friction, and the mathematical model between wire saw sawing force and cutting process parameters is established. Then the response surface methodology is used to analyze the influence of each parameter on the sawing force of the wire saw, and the optimal parameters are obtained. The accuracy of the model is verified by experiments.
2. Mathematical model of sawing force

2.1. Analysis of sawing force of single abrasive particle

Due to its many excellent properties, diamond is especially suitable for processing hard and brittle non-metallic materials such as silicon [9]. Therefore, diamond is usually electroplated on a fine wire to form a wire saw for processing photovoltaic silicon rods. Figure 1 shows a SEM photograph of a wire saw with a diameter of 100um. Therefore, the idealized model is established, that is, the abrasive diamond grains have the same shape and are conical. At the same time, the abrasive diamond particles are evenly distributed on the surface of fine steel wire. The side view of the idealized saw is shown in Figure 2. As shown in the figure, Silicon rod move up with speed $V_s$, and diamond wire move outside (the direction perpendicular to the X axis and Y axis) with speed $V_w$.

![Figure 1](image1.png)  
**Figure 1** Scanning electron micrograph of wire saw with diameter of 100um

![Figure 2](image2.png)  
**Figure 2** Side view of idealized saw

In the process of abrasive diamond sawing silicon rods, the wire saw process can be considered as the extrusion deformation and friction between many diamond abrasive particles and the workpiece to remove the material [10]. The force analysis diagram of a single diamond abrasive at point A is shown in Figure 3. Among them, $F_{nc}$ is the normal sawing force, $F_{ns}$ is the normal friction, $F_{tc}$ is the tangential cutting force, $F_{ts}$ is the tangential friction. The sawing force acting on a single abrasive particle is as follows:

\[
F_{na} = F_{nc} + F_{ns}
\]

\[
F_{ta} = F_{tc} + F_{ts}
\]

In the formula, $F_{na}$ is the normal sawing force, $F_{ta}$ is the tangential sawing force.

![Figure 3](image3.png)  
**Figure 3** The force analysis diagram of the single diamond abrasive grain

The process of consolidating diamond abrasive wire sawing photovoltaic silicon rods is similar to traditional grinding, so the conclusions of grinding can be used to lead to similar results, as shown in Formula (2) and Formula (3).
\[ F_{nc} = KQ_1 \]
\[ Q_1 = \frac{1}{2}bh \]
\[ F_{ns} = \bar{p}Q_2 \]
\[ Q_2 = \frac{1}{8}b^3 \]  

In the above formula, \( K \) is the cutting force per unit area, \( Q_1 \) is the average cross-sectional area of a single diamond abrasive, \( Q_2 \) is the normal projection area of the contact area between a single diamond abrasive particle and the workpiece, \( \bar{p} \) is the average contact pressure between a single diamond abrasive and silicon rod, \( b \) is the average cutting width of a single diamond abrasive, \( h \) is the average cutting height of a single diamond abrasive.

Figure 4 is a diagram of a consolidated diamond abrasive mitochondrial saw photovoltaic silicon rod, \( R \) is the radius of wire saw, If \( d\theta \) is a very small angle, then its corresponding arc length \( ds \) is:
\[ ds = Rd\theta \]  

In unit time, the volume \( Q_s \) of the silicon rod for section \( ds \) arc saw is:
\[ Q_s = V_s V_w \cos \theta ds \]  

**Figure 4** The schematic diagram that consolidated diamond abrasive saws photovoltaic silicon rods

At the same time, the volume \( Q_s \) of the \( ds \) arc wire sawing silicon rod can be calculated by formula (6) per unit time.
\[ Q_s = V_s V_w C V ds \]  

In the above formula, \( V_s Q_1 \) represents the volume of a single diamond abrasive saw per unit time, \( C \) is the number of diamond abrasives per unit area, and \( V_s ds \) is the area through which the \( ds \) arc length passes per unit time. As can be seen from Figure 3, the relationship between \( b \) and \( h \) can be obtained:
\[ h = \frac{b}{2\tan \alpha} \]  

United equations (2), (5), (6), and (7), the average cutting width \( b \) and the average cutting height \( h \) of a single diamond abrasive can be obtained as follows:
\[ b = \frac{4V_v \cos \theta \tan \alpha}{CV_v} \]  \hspace{1cm} (8)

\[ h = \frac{V_v \cos \theta}{CV_v \tan \alpha} \]  \hspace{1cm} (9)

From formulas (8) and (9), \( F_{ns} \) and \( F_{nc} \) can be obtained and substituted into Formula (1) to obtain \( F_{na} \):

\[ F_{na} = (K + \frac{\bar{p} \tan \alpha}{2}) \frac{V_v \cos \theta}{CV_v} \]  \hspace{1cm} (10)

The relationship between tangential cutting force and normal cutting force is as follows:

\[ F_{tc} = -\frac{\pi}{4\tan \alpha} F_{nc} \]  \hspace{1cm} (11)

The tangential friction produced under normal friction is as follows:

\[ F_{ts} = \mu F_{ns} \]  \hspace{1cm} (12)

After the above derived formula, the tangential sawing force of a single abrasive grain is obtained as follows:

\[ F_{ta} = \left( \frac{\pi K}{2\tan \alpha} + \mu \bar{p} \tan \alpha \right) \frac{V_v \cos \theta}{2CV_v} \]  \hspace{1cm} (13)

2.2. Analysis of cutting force of wire saw

In the process of consolidating diamond abrasive miter saw photovoltaic silicon rod, if the length of photovoltaic silicon rod is \( L \), the contact area \( dA \) of the wire saw corresponding to angle \( d\theta \) is:

\[ dA = LdS = LRd\theta \]  \hspace{1cm} (14)

The cutting force of a wire saw in the differential region is as follows:

\[ dF_n = (K + \frac{\bar{p} \tan \alpha}{2}) \frac{V_v \cos \theta}{CV_v} CLRd\theta \]

\[ dF_t = \left( \frac{\pi K}{2\tan \alpha} + \mu \bar{p} \tan \alpha \right) \frac{V_v \cos \theta}{2CV_v} CLRd\theta \]  \hspace{1cm} (15)

When formula (15) is integrated in the range of \((-\frac{\pi}{2}, \frac{\pi}{2})\), the sawing force of wire saw photovoltaic silicon rod is as follows:

\[ F_n = (2K + \bar{p} \tan \alpha) \frac{V_v}{V_v} LR = K_n \frac{V_v}{V_v} LR \]

\[ F_t = \left( \frac{\pi K}{2\tan \alpha} + \mu \bar{p} \tan \alpha \right) \frac{V_v}{V_v} LR = K_n \frac{V_v}{V_v} LR \]  \hspace{1cm} (16)

Therefore, the sawing force of the wire saw is a function related to \( V_v, V_v, R \) and \( L \), The coefficients \( K_n \) and \( K_n \) mainly depend on the material of the workpiece and the wire saw, and the lubrication between the wire saw and the workpiece.

3. Analysis results of RSM

3.1. Establishment of mathematical model and its significance analysis

The dimension of workpiece \( L \) is fixed when cutting silicon rod with ultra-thin diamond wire cutting machine. Box-Behnken is used to carry out 3 factors and 3 levels experiment. The input factors of this
experiment are wire saw speed $V_s$, workpiece feed rate $V_w$ and wire saw radius $R$, and the output factor is normal sawing force $F_n$. The experiment factors and levels are shown in Table 1.

### Table 1. Test factors and levels.

| Factors      | Level | 0  | 1  |
|--------------|-------|----|----|
| $V_s$ / m·min$^{-1}$ |     | 800 | 1000 | 1200 |
| $V_w$ / mm·min$^{-1}$ |     | 0.2 | 0.4 | 0.6 |
| $R$ / um     |     | 50  | 70  | 90  |

Through regression fitting with Design-Expert software, its quadratic regression model is as follows:

$$F_n = 3.69 - 0.75V_s + 1.84V_w + 1.05R - 0.38V_sV_w - 0.22V_wR + 0.52V_sR + 0.11V_s^2 - 0.04V_w^2 - 0.04R^2$$

### Table 2. The test design and results of response surface analysis

| Serial number | $V_s$ / m·min$^{-1}$ | $V_w$ / mm·min$^{-1}$ | $R$ / um | $F_n$ / N |
|---------------|----------------------|-----------------------|----------|-----------|
| 1             | -1                   | 1                     | 0        | 6.765     |
| 2             | 0                    | 0                     | 0        | 3.708     |
| 3             | 0                    | 0                     | 0        | 3.712     |
| 4             | 0                    | 1                     | 1        | 6.958     |
| 5             | 1                    | 1                     | 0        | 4.510     |
| 6             | 0                    | 0                     | 0        | 3.722     |
| 7             | 0                    | 0                     | 0        | 3.698     |
| 8             | 1                    | 0                     | 1        | 3.866     |
| 9             | -1                   | -1                    | 0        | 2.255     |
| 10            | 1                    | 0                     | -1       | 2.148     |
| 11            | 0                    | 0                     | 0        | 3.608     |
| 12            | 0                    | -1                    | -1       | 1.289     |
| 13            | 0                    | -1                    | 1        | 2.319     |
| 14            | -1                   | 0                     | -1       | 3.221     |
| 15            | 1                    | -1                    | 0        | 1.503     |
| 16            | -1                   | 0                     | 1        | 5.799     |
| 17            | 0                    | 1                     | -1       | 3.866     |

Design-Expert software was used to analyze variance of response value $F_n$ in Table 2, and the results were shown in Table 3. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case $V_s, V_w, R, V_sV_w$ are significant model terms. The "Lack of Fit F-value" of 2.32 implies the Lack of Fit is not significant relative to the pure error. Non-significant lack of fit is good. So the model has high stability.

### Table 3. The variance analysis results of normal force model

| Source | Sum of Squares | df | Mean Square | F Value | p-value Prob > F |
|--------|----------------|----|-------------|---------|------------------|
| Model  | 42.39          | 9  | 4.71        | 1396.44 | < 0.0001         |
| $V_s$  | 4.52           | 1  | 4.52        | 1339.94 | < 0.0001         |
| $V_w$  | 27.14          | 1  | 27.14       | 8045.35 | < 0.0001         |
| $R$    | 8.86           | 1  | 8.86        | 2626.79 | < 0.0001         |
| $V_sV_w$ | 0.56       | 1  | 0.56        | 167.44  | < 0.0001         |
3.2. Analysis of response surface figure

Furthermore, the multiple correlation coefficient and adjusted coefficient of determination of the model are close to 1, which further verifies the accuracy of the model. Figure 5-7 shows the effect of the interaction of wire saw speed $V_s$, workpiece feed rate $V_w$ and wire saw radius $R$ on the normal sawing force $F_n$.

![Figure 5](image1)

**Figure 5** $F_n = f(V_s, V_w)$ response surface

![Figure 6](image2)

**Figure 6** $F_n = f(V_s, R)$ response surface ($V_s = 1000\text{ m/min}$)

![Figure 7](image3)

**Figure 7** $F_n = f(V_w, R)$ response surface ($V_w = 0.4\text{ mm/min}$)

As can be observed from the above three figures, when the other two process parameters are fixed, a higher workpiece feed rate or a larger wire saw radius will result in a larger normal sawing force. Conversely, the lower the wire saw speed, the larger the normal sawing force. The optimal process parameters are obtained by Design-Expert software, as shown in Table 4, which verified the correctness of Equation (15). However, in the actual processing, the values of these three process
parameters must be within a reasonable range, otherwise there will be wire breakage, fragmentation and other phenomena.

Table 4. The optimal process parameters

| \(v_s/\text{m} \cdot \text{min}^{-1}\) | \(v_c/\text{mm} \cdot \text{min}^{-1}\) | \(R/\text{um}\) | \(F_n/N\) |
|---|---|---|---|
| 800 | 0.6 | 90 | 8.469 |

4. Conclusions
The model shows that the sawing force of wire saw is a function related to the wire saw speed, the workpiece feed rate, the radius of wire saw and the length of photovoltaic silicon rod. Therefore, within a reasonable range, the mathematical model can accurately predict the sawing force in the cutting process.

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