Conductivity in Two-Dimensional Disordered Model with Anisotropic Long-Range Hopping.

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Abstract

We consider two-dimensional system of particles localized on randomly distributed sites of squared lattice with anisotropic transfer matrix elements between localized sites. By summing of "diffusion ladder" and "cooperon ladder" type vertices we calculated the conductivity for various sites and particles densities. The model is relevant to the problem of strong nonmagnetic impurities in superconductors with $d_{x^2-y^2}$ symmetry of the order parameter [4].

PACS numbers: 74.20–z, 75.10.Hk.

1 Introduction

The following tight-binding Hamiltonian is considered:

$$H = \sum_{i,j} \left[ t(r_{ij} - r_i)\psi^+(r_i)\psi^+(r_j)\rho(r_i)\rho(r_j) \right]$$

where $\psi^+(r_i), \psi^+(r_j)$ are creating and annihilation operators, $\rho(r_i)$ is the filling number, equal to 1 at the localized sites, and to 0 otherwise. The transfer matrix element has a cross-shape configuration

$$t(r) = (\delta_{x,0} + \delta_{y,0})f(r),$$

with

$$f(r) = J \left( \frac{a}{r} \right)^\gamma \exp(-kr),$$

and $a$ is a lattice constant. We examine the case of a random distribution of impurities on a sites of two-dimensional squared lattice. An impurity potential generates a localized state with strongly anisotropic wave function. As a result the conductivity is carried out due to hoppings of particles between local states on the same vertical or horizontal lines. A similar picture can be realized in 2D $d_{x^2-y^2}$-wave superconductors, where local bound quasiparticle states may arise in the presence of unitary impurities [4].

The plan of this article is as follows. In Chapter 2 we consider the case of low impurity density. In Chapter 3 we calculate the conductivity in the case of high impurity density. In the Conclusion we discuss our results.

2 Low density

We consider now the limit of low impurities concentration ($c \ll 1$). In the case of an external electromagnetic field we should substitute in

$$t(r_i - r_j) \to t(r_i - r_j)\exp(i\int_{r_i}^{r_j} A(r,t)dr).$$

The electric current is defined as usually from the Hamiltonian [1] by varying over a gauge invariant vector-potential $A$

$$j_\alpha(t) = -ie \sum_{i,j}(r_i - r_j)\alpha t_{ij}\Psi_i^+(t)\Psi_j(t)\rho_i\rho_j \exp(i\epsilon A(t)(r_i - r_j)).$$
Where \( t_{ij} = t(r_i - r_j) \). Since we will calculate \( j(\omega) \) we consider that the potential \( A \) depends on time \( t \) only. Using the equation for the Green function \( G(t_1, r_1, t_2, r_2) \)

\[
\frac{\partial}{\partial t_1} G(t_1, r_1, t_2, r_2) = -i T \frac{\partial \Psi(1)}{\partial t_1} \Psi^*(2) > -i \delta(t_1 - t_2) \]

(4)

we obtain after Fourier transformation in the linear over \( A \) approximation

\[
j_a(\omega) = \frac{e^2}{c} \sum_{i,j} t_{i,j}(r_i - r_j)a(r_i - r_j)A_{\beta}(\omega) \int \frac{d\Omega}{2\pi} G(\Omega, r_i, r_i)e^{i\Omega a} - \\
\frac{e^2}{c} \sum_{i,j,k,l} t_{i,j}t_{k,l}(r_i - r_j)a(r_k - r_l)A_{\beta}(\omega) \int \frac{d\Omega}{2\pi} G(\omega + \Omega, r_j, r_k)G(\Omega, r_i, r_i)e^{i\Omega a}.
\]

(5)

The summation in eqn. (5) is taken over impurity sites, and \( \alpha \to +0 \). In order to evaluate (5) in the lowest order with respect to the concentration we examine the case of two random arranged sites. The Green function is found easy

\[
G(\omega, r_i, r_i) = \frac{\omega + \mu}{(\omega + \mu)^2 - t_{1,2}^2},
\]

\[
G(\omega, r_i, r_j) = \frac{t_{1,2}}{(\omega + \mu)^2 - t_{1,2}^2},
\]

(6)

where \( \mu \) is the chemical potential. Substituting (6) to (5) we obtain in the case when sites are arranged on the same horizontal chain

\[
j_x(\omega) = \frac{e^2}{2c} (x_1 - x_2)^2 t_{1,2} \frac{\omega^2}{(\omega + i\alpha)^2 - 4t_{1,2}^2} A_x(\omega) = Q(\omega)A_x(\omega).
\]

(7)

The similar equation can be derived for the case of nonzero temperatures. The result differs only by Fermi filling factor. After averaging over imurities sites we obtain for the conductivity \( \sigma(\omega) = iQ(\omega)/\omega \)

\[
\sigma(\omega) = \frac{\pi e^2}{4} x_0^2 L \int x^2 t(x) |n_F(\omega - \mu) - n_F(-\omega - \mu)| \delta(\omega - 2t(x))dx
\]

\[
= \frac{\pi e^2}{8} x_0^2 L \frac{x_0^2 t(x_0) |n_F(\omega - \mu) - n_F(-\omega - \mu)|}{|t'(x_0)|},
\]

(8)

where \( n_F \) is the Fermi distribution function, \( 2t(x_0) = \omega \).

Substituting \( t(x) \) from (5) we get

\[
\frac{\omega}{2J} = \frac{\exp(-\kappa x_0)}{x_0^2},
\]

(9)

\[
\sigma(\omega) = \frac{\pi e^2}{8} x_0^2 L \frac{x_0^2 |n_F(\omega - \mu) - n_F(-\omega - \mu)|}{\gamma + \kappa x_0}.
\]

(10)

In the limit of low frequency we have following asymptotic behaviors

1. \( \kappa = 0, x_0 = (2J/\omega)^{1/\gamma} \):

\[
\omega \gg T, \sigma(\omega) \propto \omega^{-3/\gamma},
\]

(11)

\[
\omega \ll T, \sigma(\omega) \propto \omega^{-3/\gamma + 1},
\]

(12)

2. \( \kappa \neq 0, \kappa x_0 \sim \log(2J/\omega) \):

\[
\omega \gg T, \sigma(\omega) \propto \omega \log^2 \frac{2J}{\omega},
\]

(13)

\[
\omega \ll T, \sigma(\omega) \propto \omega \log^2 \frac{2J}{\omega}.
\]

(14)
3 High density.

3.1 Green function.

In the case of high density of impurities we will assume that distribution function of impurities may be approximate by Gausses distribution with dispersion $g$. That is

$$\rho(r_i) = c + \delta\rho(r_i)$$

$$<\delta\rho(r_i)\delta\rho(r_j) >_\rho = g^2\delta_{ij}. \quad (15)$$

We will assume that concentration $c \leq 1$.

The one-particle Green function for the arbitrary impurity distribution is defined in terms of functional integral as usually

$$G(E, r, r') = (E - t_{ij}\rho(r_i)\rho(r_j))^{-1} =$$

$$i\int D\tilde{\psi}D\psi(r')\psi(r)\exp(iS)$$

$$\int D\psi D\tilde{\psi} \exp(iS), \quad (16)$$

where

$$S = S_0 + S_1, \quad \quad \quad \quad (17)$$

$$iS_0 = i\sum_r \bar{\psi}(r)E\psi(r), \quad \quad (18)$$

$$iS_1 = -i\sum_{r_1, r_2} \bar{\psi}(r_1)t(r_1 - r_2)\rho(r_1)\rho(r_2)\psi(r_2). \quad \quad (19)$$

Introducing an additional integration over new fields $\chi, \bar{\chi}$ in order to eliminate the second order terms $\rho\rho$, we get

$$e^{iS_1} = \int D\bar{\chi}D\chi \exp\{-i\sum_r \bar{\chi}(r_1)t^{-1}(r_1 - r_2)\chi(r_2)$$

$$+c\sum_r (\bar{\chi}(r)\psi(r) + \bar{\psi}(r)\chi(r))$$

$$+\sum_r \delta\rho(r)(\bar{\chi}(r)\psi(r) + \bar{\psi}(r)\chi(r))/Z \quad (20)$$

where

$$Z = \int D\bar{\chi}D\chi \exp\{-i\sum_{r_1, r_2} \bar{\chi}(r_1)t^{-1}(r_1 - r_2)\chi(r_2)) \quad (21)$$

$$t^{-1}(r) = \frac{1}{V} \sum_k \varepsilon^{-1}(k)\epsilon^{ikr} \quad (22)$$

$$\varepsilon(k) = \sum_r t(r)\epsilon^{ikr} = -J\ln\left(\kappa^2 + 4\sin^2\left(\frac{k_xa}{2}\right)\left(\kappa^2 + 4\sin^2\left(\frac{k_ya}{2}\right)\right)\right) \quad (23)$$

The Green function in terms of new two component field $\varphi$

$$\varphi_1(r) = \psi(r)$$

$$\varphi_2(r) = \chi(r)$$

$$\bar{\varphi}_1(r) = \bar{\psi}(r)$$

$$\bar{\varphi}_2(r) = \bar{\chi}(r) \quad (24)$$

reads

$$\tilde{G}(r_1, r_2) = -i <\bar{\varphi}(r_1) \otimes \bar{\varphi}(r_2) > \quad (25)$$
where \( \hat{\varphi}(\mathbf{r}) = (\varphi_1(\mathbf{r}), \varphi_2(\mathbf{r})) \) and angle brackets are defined as

\[
< \ldots > = \frac{\int D\hat{\varphi} D\hat{\psi} D\hat{D} D\hat{\chi} \ldots e^{iS_{\text{eff}}}}{\int D\hat{\varphi} D\hat{\psi} D\hat{D} D\hat{\chi} e^{iS_{\text{eff}}}}
\]

(26)

with

\[
iS_{\text{eff}} = \sum_{\mathbf{r}} \hat{\varphi}(\mathbf{r}) \left( \frac{iE}{c + \delta \rho(\mathbf{r})} \right) \hat{\varphi}(\mathbf{r}).
\]

(27)

The equation for the Green function after averaging over impurities in the Born approximation reads

\[
\hat{G}(\mathbf{k}) = \hat{G}^0(\mathbf{k}) + \hat{G}^0(\mathbf{k}) \hat{\Sigma}(\mathbf{k}) \hat{G}(\mathbf{k})
\]

(28)

with the bare Green function

\[
[\hat{G}^0(\mathbf{k})]^{-1} = \begin{bmatrix}
E & -ic \\
-ic & -\varepsilon^{-1}(\mathbf{k})
\end{bmatrix}
\]

(29)

and the self-energy \( \hat{\Sigma} \) obtained by summing of diagrams without intersections

\[
\hat{\Sigma}(\mathbf{k}) = g^2 a^2 \int \frac{d\mathbf{k}_1}{(2\pi)^2} \sigma^x \hat{G}(\mathbf{k}_1) \sigma^x.
\]

(30)

The solution of Eqns. (28), (30) is

\[
\Sigma(\mathbf{k}) = \begin{pmatrix}
Q & -iP \\
-iP & R
\end{pmatrix}
\]

\[
\hat{G}(\mathbf{k}) = \frac{1}{(1 + \varepsilon(\mathbf{k}) R)(E - Q) - (c + P)^2 \varepsilon(\mathbf{k})} \begin{pmatrix}
1 + \varepsilon(\mathbf{k}) R & -i(c + P) \varepsilon(\mathbf{k}) \\
-i(c + P) \varepsilon(\mathbf{k}) & -\varepsilon(\mathbf{k})(E - Q)
\end{pmatrix}
\]

(31)

(32)

where

\[
Q = g^2 a^2 \int \frac{d\mathbf{k}}{(2\pi)^2} \frac{(E - Q) \varepsilon(\mathbf{k})}{(1 + \varepsilon(\mathbf{k}) R)(E - Q) - (c + P)^2 \varepsilon(\mathbf{k})}
\]

\[
R = -g^2 a^2 \int \frac{d\mathbf{k}}{(2\pi)^2} \frac{1 - \varepsilon(\mathbf{k}) R}{(1 + \varepsilon(\mathbf{k}) R)(E - Q) - (c + P)^2 \varepsilon(\mathbf{k})}
\]

\[
P = g^2 a^2 \int \frac{d\mathbf{k}}{(2\pi)^2} \frac{(c + P) \varepsilon(\mathbf{k})}{(1 + \varepsilon(\mathbf{k}) R)(E - Q) - (c + P)^2 \varepsilon(\mathbf{k})}
\]

In the limit of low dispersion \( g^2 \ll 1 \) we obtain

\[
Q^{R,A} = \pm i\gamma/2
\]

\[
R^{R,A} = \mp i\frac{c^2}{E^2} \frac{\gamma}{2}
\]

(33)

\[
P^{R,A} = \pm i\frac{c}{E} \frac{\gamma}{2}
\]

\[
\gamma = 2\pi g^2 a^2 \frac{E^2}{c^2} \nu \left( \frac{E}{c^2} \right)
\]

(34)

Where \( \nu_0(\varepsilon) \) is density of states of the pure model \( (c=1, g=0) \):

\[
\nu(\varepsilon) = \int \frac{d\mathbf{k}}{(2\pi)^2} \delta(\varepsilon - \varepsilon(\mathbf{k})).
\]

(35)

Taking into account that \( Q \ll E, RE \ll 1, P \ll c \), we find for the Green function in the limit \( g^2 \ll 1 \)

\[
\hat{G}^{R,A}(\mathbf{k}) = \frac{1}{E - c^2 \varepsilon(\mathbf{k}) \pm i\gamma/2} \begin{pmatrix}
1 & -i c \varepsilon(\mathbf{k}) \\
-ic \varepsilon(\mathbf{k}) & -\varepsilon(\mathbf{k}) E
\end{pmatrix}
\]

(36)
3.2 Drude formula

The conductivity in our case is defined as in Ch.I in terms of four-particle correlation function

\[
\sigma_E(\omega) = \frac{e^2}{2\pi} \int \frac{d\mathbf{k}_1}{(2\pi)^2} \frac{d\mathbf{k}_2}{(2\pi)^2} v_\alpha(\mathbf{k}_1) v_\alpha(\mathbf{k}_2) K_{E,\omega}(\mathbf{k}_1, \mathbf{k}_2; k_1, k_2)
\]

where \( E \) is taken at the Fermi level,

\[
K_{E,\omega}(\mathbf{k}_1, \mathbf{k}_2; k_1, k_2) = \frac{1}{V} \sum_{x,y,z,t} e^{ik_1(x-y)} e^{ik_2(z-t)} \langle \rho_x \rho_y \rho_z \rho_t G_{11}^R(\mathbf{y}, E + \omega/2) G_{11}^A(\mathbf{t}, x, E - \omega/2) \rangle_\rho
\]

and

\[
v_\alpha(\mathbf{k}) = \frac{\partial \varepsilon(\mathbf{k})}{\partial k_\alpha}
\]

Substituting the solution (36) to (40) we find in the lowest approximation

\[
\sigma_E(\omega) = \frac{e^2}{2\pi} \int \frac{d\mathbf{k}}{(2\pi)^2} v^2(\mathbf{k}) G_{11}^R(\mathbf{k}, E + \omega/2) G_{11}^A(\mathbf{k}, E - \omega/2) = \frac{e^2 e^6}{2\pi} \left( \frac{J}{E} \right)^2 \frac{A(E)}{B(E)}
\]

where

\[
A(E) = \int_{\varepsilon(\mathbf{k}) = E/c^2} \frac{d\mathbf{k}}{v(\mathbf{k})} v^2(\mathbf{k}), \quad B(E) = \frac{d\mathbf{k}}{\varepsilon(\mathbf{k}) = E/c^2 v(\mathbf{k})},
\]

\( v(\mathbf{k}) = \sqrt{v_x^2(\mathbf{k}) + v_y^2(\mathbf{k})} \), and \( d\mathbf{k} \) - element of the length of the Fermi surface.

The conductivity can be expressed in terms of a particle density defined as

\[
n(\mathbf{k}) = a^2 \int_{-\pi/a}^{\pi/a} dk_x \int_{-\pi/a}^{\pi/a} dk_y \theta(\frac{c^2}{\varepsilon(\mathbf{k})} - 1).
\]

We obtain in limits of low and high densities following results:

\[
\sigma = \left\{ \begin{array}{ll}
\frac{e^2 e^6}{g^2} \frac{1}{3\pi \log^2(2)} n_0, & \text{for } n_0 \ll 1, \\
\frac{e^2 e^6}{g^2} \frac{1}{4\pi \log^2(\nu_0)} (1 - n_0), & \text{for } (1 - n_0) \ll 1.
\end{array} \right.
\]

the asymptotic behavior in intermediate region \( 0 < n_0 < 1 \) is

\[
\sigma = 2\pi^4 \frac{e^2 e^6}{g^2} \frac{1}{(1 - n_0)^2 \log \left( \frac{1}{1 - n_0} \right)}
\]

with maximum value

\[
\sigma_{\text{max}} \sim \frac{1}{\kappa^{8/3} \log(\kappa)}
\]

reached at \( 1 - n_0 \sim \kappa^{4/3} \).

3.3 The absence of weak localization.

To go beyond the quasiclassical approximation we include contributions to the conductivity from "diffusion-ladder" and "crossed-ladder" or "cooperon" vertices [1, 3]. The addition term is

\[
\delta\sigma_E(\omega) = \frac{e^2}{2\pi} \int \frac{d\mathbf{k}_1}{(2\pi)^2} \frac{d\mathbf{k}_2}{(2\pi)^2} v_\alpha(\mathbf{k}_1) v_\alpha(\mathbf{k}_2) G_{1a}^R(\mathbf{k}_1) G_{c1}^R(\mathbf{k}_2) G_{c2}^A(\mathbf{k}_2) G_{d1}^A(\mathbf{k}_1) K_{ac;bd}(\mathbf{k}_1, \mathbf{k}_2)
\]

where for "diffusion" - vertex contribution we get the equation (see Fig.1a):

\[
K_{ac;bd}^{(D)} = g^2 \sigma_{a}^{\alpha} \sigma_{b}^{\alpha} + g^2 \frac{d}{(2\pi)^2} \sigma_{aa}^{\alpha} G_{aa}^R(\mathbf{l}) K_{ac;bd}^{(D)} G_{b1}^A(\mathbf{l}) \sigma_{d}^{\alpha}.
\]
The solution of this equation does not depend on $k_1$ and $k_2$. Therefore the contribution of a "diffusion" - vertex to the conductivity is equal to zero because

$$
\int \frac{dk_1}{(2\pi)^2} v_\alpha (k_1) G^{R}_{1a}(k_1) G^{A}_{d_1}(k_1) = 0.
$$

(48)

\[K = \ldots \]

Fig. 1. Diffusion a) and crossed-ladder b) vertices.

Now we consider the "cooperon"-vertex contribution. The vertex $K^{(D)}$ obeys the equation (see Fig. 1b)

$$
K^{(C)}_{acbd}(q) = g^2 \sigma^x_{ac} \sigma^x_{bd} + g^2 \int \frac{dl}{(2\pi)^2} \sigma^x_{ac} \sigma^x_{bd} G^{R}_{ca_1}(l) G^{A}_{b_1}(q - l) K^{(C)}_{ac_1;bd_1}(q),
$$

(49)

where $q = k_1 + k_2$.

If the solution of this equation has some singularity behavior as function of $q$ (for example "diffusion pole") we may hope to obtain the contribution to conductivity. Now we prove that "cooperon"-vertex has not singularity.

We search the solution of (49) in the form

$$
K^{(C)}_{acbd} = g^2 \sum_{\mu\nu} \sigma^\mu_{ac} \sigma^\nu_{bd} A_{\mu\nu},
$$

(50)

where $\mu, \nu = 0, x, y, z$, and $A$ satisfies the equation

$$
A_{\mu\nu} = \delta_{\mu x} \delta_{\nu x} + \sum_{\alpha\beta} \Lambda_{\mu\nu}^{\alpha\beta} A_{\alpha\beta}
$$

(51)

with

$$
\Lambda_{\mu\nu}^{\alpha\beta} = \frac{g^2}{4} \int \frac{dl}{(2\pi)^2} \left[ Sp(\sigma^x G^{R} \sigma^\mu \sigma^\alpha) \right] \left[ Sp(\sigma^x G^{R} \sigma^\nu \sigma^\beta) \right]
$$

(52)

The matrix $\Lambda$ can be rewritten as

$$
\Lambda_{\mu\nu}^{\alpha\beta} = -\Delta_{\mu}^{\alpha} \Delta_{\nu}^{\beta}
$$

(53)

with

$$
\Delta_{\mu}^{\alpha} = -\frac{1}{2} \frac{c}{E} \left[ Sp(\sigma^x g \sigma^\mu \sigma^\alpha) \right]
$$

(54)

and

$$
g = \begin{bmatrix}
\frac{i}{E/c} & \frac{E/c}{-iE^2/c^2}
\end{bmatrix}
$$

(55)
So the components of matrix $\Lambda$ are

$$\hat{\Delta} = \begin{pmatrix} 1 & i \sinh(\theta) & \cosh(\theta) & 0 \\ i \sinh(\theta) & 1 & 0 & -i \cosh(\theta) \\ \cosh(\theta) & 0 & 1 & -\sinh(\theta) \\ 0 & i \cosh(\theta) & \sinh(\theta) & 1 \end{pmatrix}$$  \hspace{1cm} (56)$$

where

$$c/E = e^\theta$$  \hspace{1cm} (57)$$

The solution of (62) is found in terms of matrices $U$, $B$:

$$A_{\mu\nu} = U_{\mu\mu_1} U_{\nu\nu_1} B_{\mu_1\nu_1},$$  \hspace{1cm} (58)$$

where

$$(U^{-1})_{\mu\mu_1} \Delta_{\mu_1\nu_1} U_{\nu\nu_1} = z_\mu \delta_{\mu \nu}$$  \hspace{1cm} (59)$$

$$B_{\mu\nu} = \frac{1}{1 + z_\mu z_\nu} (U^{-1})_{\mu x} (U^{-1})_{\nu x}$$  \hspace{1cm} (60)$$

$$\det(\hat{\Delta} - z \hat{1}) = z^2(z - 2)^2 = 0$$  \hspace{1cm} (61)$$

The eigenvalues $z_\nu$ are following

$$z_0 = 0$$
$$z_1 = 0$$
$$z_2 = 2$$
$$z_3 = 2$$  \hspace{1cm} (62)$$

As a result we have that for $q = 0$ the function $B$ and consequently $K$ has no singularity. Therefore we do not obtain the phenomenon of "weak localization" in our 2D model due to specific type of a disorder.

4 Conclusions

We have investigated the two-dimensional model with a new type of disorder due to a random distribution of local states with strongly anisotropic overlaps of wave functions. The conductivity of this system was calculated in limits of low and high densities of local states. We have shown that considered type of disorder does not lead to the weak localization phenomenon, as usually in two-dimensional case \cite{5}, due to the absence of logarithmic divergence from the integration over the diffusion pole.

5 Acknowledgments

S.M. thanks A. Balatsky for a discussion. This work was supported by the Russia Fund of Fundamental Research under grant No 960217791.

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