Production of axion-like particles via vector boson fusion at future electron-positron colliders

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Abstract One kind of particularly interesting pseudoscalar particles, called axion-like particles (ALPs), have rich physical phenomenology at high- and low-energy collider experiments. After discussing most of single production channels of ALP at electron-positron colliders, we investigate the possibility of detecting this kind of new particles through the $W^+W^-$ fusion process $e^+e^- \rightarrow \nu_{e}\nu_{e}(\rightarrow \gamma\gamma)$ at the CLIC. The $3\sigma$ and $5\sigma$ bounds on the ALP parameter space at the three energy stages of the CLIC are obtained. We find that the bounds given by the CLIC are complementary to the existing experiments exclusion regions.

Keywords axion-like particles · vector boson fusion · electron-positron collider

PACS

1 Introduction

After discovery of the Higgs boson in the CMS and ATLAS experiments [1, 2], experiments at the LHC have made great achievements and demonstrated that the standard model (SM) is a correct model to explain most of observed phenomena at the electroweak scale. However, deviations of experimental measurements from the SM predictions at or below the percent level are beyond the achievable precision at the LHC and is expected to be performed at the planned $e^+e^-$ colliders with high luminosity and energy. It is well known that, compared to the LHC, the future $e^+e^-$ colliders, such as the ILC [3–7], CLIC [8, 9], FCC-ee [10, 11], and CEPC [12–14] have higher luminosity and more clean experimental environment, which can not only study the SM observables at unprecedented accuracy, but would be very useful to discover the evidence of new physics beyond the SM.

The discovery of the Higgs boson has stimulated interest in searching for additional (pseudo) scalar particles in high- and low-energy experiments. Axion-like particles (ALPs) are pseudoscalar bosons that naturally appear in many extensions of the SM as pseudo Nambu-Goldstone bosons arising in explicit global symmetry breaking [15–21]. The properties of ALPs, such as their masses and coupling strengths to the SM particles, are model-dependent and have been extensively investigated. High-energy collider experiments provide possibilities to explore ALPs with masses about $M_a \geq 1$ GeV [22–32]. The constraints on a wide range of the ALP parameter space are obtained using the available data, such as the LEP, Tevatron and LHC data, the future prospects of discovering ALPs at running or future collider experiments are studied. For instance, searches for ALPs via photon fusion are investigated at the LHC and $e^+e^-$ colliders [33–41]. The capabilities of these colliders to probe the ALP parame-
ter space are discussed and the expected bounds on the ALP-photon coupling for a wide range of masses are obtained.

ALPs can also be produced via massive vector boson fusion (VBF) processes (i.e. $W^+W^-$-fusion and $ZZ$-fusion processes) at high-energy $e^+e^-$ colliders [42]. Since the production cross sections of these processes depend logarithmically on the center-of-mass (c.m.) energy, and are generally smaller than those of other processes at low-energy $e^+e^-$ colliders, there are very few studies about ALP productions via VBF processes with $V = W^\pm, Z$. In this work, we will consider the possibilities of detecting ALPs via VBF processes at high-energy $e^+e^-$ colliders like the CLIC. We will focus on ALPs that only couple to the electroweak gauge bosons and perform a careful investigation of the ALP signals at high-energy $e^+e^-$ colliders from VBF processes. The relevant Feynman diagrams are shown in Fig. 1. Our numerical results show that VBF processes can extend the $e^+e^-$ sensitivity to a region of the ALP parameter space that is not covered by other experiments.

The rest of this paper is organized as follows. After summarizing the effective description of ALP interactions with the electroweak gauge bosons, we calculate and compare the production cross sections of ALPs at high-energy $e^+e^-$ colliders and perform a careful investigation of the ALP signals and the relevant SM backgrounds from $W^+W^-$ fusion process, sensitivity projections of the CLIC to the ALP parameter space are presented in Sec. 3. We report our conclusions in Sec. 4.

### 2 Single production of ALP at $e^+e^-$ colliders

The ALP interactions with the SM fermions and gauge bosons arise through five-dimensional operators, and their masses can be treated independently of their couplings [42], which can be described via an effective Lagrangian. The effective interactions of ALP with electroweak gauge bosons, which are related to our calculation, are given by the following dimension-5 effective Lagrangian [27]:

$$
L^{D=5}_{\text{eff}} = \frac{1}{2} \left( \partial_\mu \alpha \right) \left( \partial^\mu \alpha \right) - \frac{M_\alpha^2}{2} \alpha^2 + C_{WW} \frac{a}{f_\alpha} W^\mu W^{\nu} A_\nu + C_{BB} \frac{a}{f_\alpha} B^\mu B^\nu,
$$

where $X_{\mu\nu}$ denotes the field strength tensor for $SU(2)_L$ or $U(1)_Y$, $X_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} X_{\alpha\beta} \epsilon$ with $\epsilon^{0123} = 1$ and $X \in \{B, W\}$. The ALP field and mass are denoted by $\alpha$ and $M_\alpha$, respectively. The interaction of the ALP with two photons is contributed by the last two terms of Eq. 1. The dimensionful couplings $g_{\gamma\gamma}$, $g_{WW}$, $g_{\gamma Z}$ and $g_{ZZ}$ control the coupling strength of the interactions between ALP and electroweak gauge bosons ($W^\pm, Z, \gamma$), which are given by

$$
g_{\gamma\gamma} = \frac{4}{f_\alpha} (c_{\theta_W}^2 C_{BB} + s_{\theta_W}^2 C_{WW}),
$$

$$
g_{WW} = \frac{4}{f_\alpha} C_{WW},
$$

$$
g_{\gamma Z} = \frac{4}{f_\alpha} (s_{\theta_W} C_{BB} + c_{\theta_W} C_{WW}),
$$

$$
g_{ZZ} = \frac{4}{f_\alpha} s_{2\theta_W} (C_{WW} - C_{BB}),
$$

(2)

where $s_{\theta_W} = \sin \theta_W$, $c_{\theta_W} = \cos \theta_W$ with $\theta_W$ being the weak mixing angle. To simplify our analysis, we will assume $C_{WW} = C_{BB}$ in this work and there are $g_{\gamma Z} = 0$, $g_{\gamma\gamma} = g_{WW}$ and $g_{ZZ}$.

In this case, ALPs can be singly produced via $e^+e^-$ annihilation and VBF processes at $e^+e^-$ colliders. We use FeynRules [44] to generate the UFO model file corresponding to the effective Lagrangian Eq. 1. The cross sections of $e^+e^-$ annihilation and VBF processes at $e^+e^-$ colliders are calculated, which are shown in Fig. 2. One can see from Fig. 2 that the ALP-strahlung processes ($\gamma a$ and $Z a$) are the main production channels. However, their cross sections become independent of $s$ in the high-energy limit $M_\gamma^2 \ll s$ and thus saturate quite soon and flatten out. The cross sections of VBF processes depend only logarithmically on $s$. Therefore, the cross section of $W^+W^-$ fusion process grows with $s$ increasing and can reach sizable values at high energies. In principle, the cross section of the $ZZ$ fusion process follows the same trend as that of the $W^+W^-$ fusion process, while its value is about an order of magnitude smaller than that of the $W^+W^-$ fusion process, due to the fact that the neutral current couplings are smaller than the charged current couplings. The $\gamma\gamma$ fusion process has been extensively discussed in Ref. [39]. The cross section of the $ZZ$ fusion process $e^+e^- \rightarrow e^+e^- a$ is the smallest. In Ref. [45], the similar conclusion is given by exploring the mass and cross section of Higgs with ultimate precision at the FCC-ee.

ALPs have been investigated through the ALP-strahlung processes $e^+e^- \rightarrow V a$ ($a \rightarrow \gamma\gamma$), where $V = \gamma, Z$ [23, 46, 47]. In Ref. [23], the ALP-strahlung processes have also been studied at the CLIC, but the author assumes $C_{WW} = 0$, considering $\gamma Z a$ coupling, which is opposite to our work. There are previous works about the ALP single production induced by two-photon fusion in future electron-positron colliders [37–39]. Thus, in this paper, we only consider the production of ALP via $W^+W^-$ fusion process $e^+e^- \rightarrow \gamma, \gamma, a$ with $a$ subsequently decaying to photon pairs (see Fig. 3).

### 3 Search for ALP at the CLIC

As we know, the high-energy $e^+e^-$ collider can provide clean experimental environment to explore new physics be-

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1In this case, Ref. [43] has investigated searching for ALPs via ALP-strahlung process at the LHC.
yond the Standard Model (BSM). The future $e^+e^-$ colliders are contemplated to study the properties of the SM observables with an unprecedented precision, and the CLIC is one of them. The CLIC is planned to be built and operated in three energy stages. The first stage is planned to accurately measure the properties of the Higgs boson at $\sqrt{s} = 380$ GeV with $\mathcal{L} = 0.5$ ab$^{-1}$. The second and ultimate stages running at $\sqrt{s} = 1500$ GeV with $\mathcal{L} = 1.5$ ab$^{-1}$ and $\sqrt{s} = 3000$ GeV with $\mathcal{L} = 3$ ab$^{-1}$ are not only able to measure the top Yukawa coupling, the Higgs trilinear self-coupling and the Higgs rare decays, but also can give indirect sensitivities to many new physics scenarios through precision measurements [48]. They will be able to discover new particles through direct detection. We will do calculations and analysis of the signal and background for the process $e^+e^- \rightarrow \nu\nu\gamma\gamma$ at the CLIC with three energy stages.

In this section, we perform the Monte Carlo (MC) simulation to analyze the possibility of exploring ALPs at the CLIC through the $W^+W^-$ fusion process $e^+e^- \rightarrow \nu\nu\gamma\gamma$, where the gauge bosons $W^\pm$ are emitted from the incoming electron/positron beams. Its signal is characterized by two photons, missing transverse energy from the escaping undetected two neutrinos $(\nu\nu\gamma\gamma)$. In our numerical calculations, we take into account the process $e^+e^- \rightarrow \nu\nu\gamma\gamma$ as the dominant SM background, which is mainly induced through electroweak interaction. For the signal, the final two photons mainly come from the decay of pseudo-scalar ALPs (spin-0), while there is no particle with spin-0 in the background that decays to two photons at tree level. So the interference is very small. We have estimated the cross section of pure ALP signal to be $17.25$ fb and the cross section of the interference to be $-0.1549$ fb at $\sqrt{s} = 1500$ GeV, $M_a = 10$ GeV and $g_{\gamma\gamma} = 10^{-3}$ GeV$^{-1}$ without any cut applied. Apparently the cross section of pure ALP signal is two orders of magnitude larger than that of the interference. We have estimated the cross sections of pure ALP signal and interference for multiple ALP benchmark points, and the conclusions are similar. For simplicity, we have neglected the influence of the interference between ALP production and SM background in $e^+e^- \rightarrow \nu\nu\gamma\gamma$.

We look for photons reconstructed in the barrel region $|\eta| < 2.5$. A light ALP tends to decay a pair of collimated photons, which appear essentially as one photon of the c.m. energy $\sqrt{s} = 380$ GeV with $M_a = 10$ GeV and $g_{\gamma\gamma} = 10^{-3}$ GeV$^{-1}$ with any cut applied. Apparently the cross section of pure ALP signal is two orders of magnitude larger than that of the interference. We have estimated the cross sections of pure ALP signal and interference for multiple ALP benchmark points, and the conclusions are similar. For simplicity, we have neglected the influence of the interference between ALP production and SM background in $e^+e^- \rightarrow \nu\nu\gamma\gamma$.

In Fig. 4, we plot the normalized distributions of $M(\gamma\gamma)$, $\theta(\gamma\gamma)$, $E_T$ and $E_T$ for the signal and background with the signal benchmark points $M_a = 8, 10, 20, 100, 200$ GeV and the fixed parameter $g_{\gamma\gamma} = 10^{-3}$ GeV$^{-1}$ for the c.m. energy $\sqrt{s} = 380$ GeV. In Fig. 5 we show the same normalized kinematic distributions for $M_a = 8, 10, 20, 100, 500, 1000$ GeV and $g_{\gamma\gamma} = 10^{-3}$ GeV$^{-1}$ at $\sqrt{s} = 1500$ GeV, and the same is shown in Fig. 6 for $M_a = 8, 10, 20, 100, 1500, 2000$ GeV and $g_{\gamma\gamma} = 10^{-3}$ GeV$^{-1}$ at $\sqrt{s} = 3000$ GeV. In Figs. 4, 5 and 6, all the signal and background samples are obtained by after applying the above basic cuts. From Figs. 4, 5 and 6, we can see that, in order to optimize the signal significance, some additional cuts of kinematic distributions are needed. From these figures, we can see that it is not suitable to impose restrictions on the normalized $M(\gamma\gamma)$ distribution, hence we do not put any constraint. According to the behavioral characteristics of the other three distributions, we can further impose the following improved cuts on the signal and background events as shown in Table 1.
Table 1: The improved cuts on the signal and background events.

| Cuts                      | √s = 380 GeV | √s = 1500 GeV | √s = 3000 GeV |
|---------------------------|--------------|---------------|---------------|
| Cut 1: Angle between the ALP and the beam axis | 0.6 < θ(γγ) < 2.6 | 0.7 < θ(γγ) < 2.5 | 0.5 < θ(γγ) < 2.6 |
| Cut 2: The missing transverse energy | E_T > 70 GeV | E_T > 150 GeV | E_T > 200 GeV |
| Cut 3: The transverse energy | E_T > 85 GeV | E_T > 160 GeV | E_T > 220 GeV |

Table 2: After different cuts applied, the cross sections for the signal and SM background at the √s = 380 GeV CLIC for M_a = 8, 10, 20, 100, 200 GeV and g_{aγγ} = 10^{-3} GeV^{-1}.

| Cuts | Signal (fb) | Background (fb) |
|------|-------------|-----------------|
|      | M_a = 8 GeV, M_γ = 10 GeV | M_a = 20 GeV, M_γ = 100 GeV | M_a = 200 GeV, M_γ = 1000 GeV |
| Basic cuts | 0.3572 | 0.3950 | 0.2779 | 0.0731 | 12.66 |
| Cut 1 | 0.3436 | 0.3540 | 0.2672 | 0.0612 | 7.4612 |
| Cut 2 | 0.3202 | 0.3325 | 0.2464 | 0.0459 | 5.0182 |
| Cut 3 | 0.3198 | 0.3118 | 0.2443 | 0.0457 | 5.3916 |

Table 3: Same as Table 2 but for the √s = 1500 GeV CLIC.

| Cuts | Signal (fb) | Background (fb) |
|------|-------------|-----------------|
|      | M_a = 8 GeV, M_γ = 10 GeV | M_a = 20 GeV, M_γ = 100 GeV | M_a = 200 GeV, M_γ = 1000 GeV |
| Basic cuts | 6.6575 | 8.2525 | 12.76 | 11.9803 | 1.7862 | 0.1573 | 21.322 |
| Cut 1 | 3.7949 | 5.4389 | 8.6115 | 8.3208 | 1.0745 | 0.0913 | 8.1313 |
| Cut 2 | 3.3201 | 4.9969 | 8.12 | 7.8021 | 1.0991 | 0.0756 | 3.2893 |
| Cut 3 | 3.1913 | 4.8616 | 8.0028 | 7.7769 | 1.0089 | 0.0756 | 3.2211 |

Table 4: Same as Table 2 but for the √s = 3000 GeV CLIC.

| Cuts | Signal (fb) | Background (fb) |
|------|-------------|-----------------|
|      | M_a = 8 GeV, M_γ = 10 GeV | M_a = 20 GeV, M_γ = 100 GeV | M_a = 200 GeV, M_γ = 1000 GeV |
| Basic cuts | 9.6983 | 14.4563 | 27.993 | 32.5997 | 3.5915 | 1.3519 | 0.3323 | 29.1063 |
| Cut 1 | 5.557 | 8.7537 | 20.1513 | 24.8866 | 2.2756 | 0.8178 | 0.2003 | 14.56 |
| Cut 2 | 3.8733 | 7.0935 | 18.4306 | 23.1416 | 2.1683 | 0.7647 | 0.1769 | 5.0493 |
| Cut 3 | 3.3481 | 6.552 | 17.903 | 22.946 | 2.1682 | 0.7647 | 0.1769 | 4.8927 |

After imposing the cuts for a few representative ALP mass benchmark points, we summarize the cross sections of the signal and background in Tables 2, 3 and 4 for √s = 380 GeV, 1500 GeV and 3000 GeV, respectively. From Tables 2, 3 and 4, after applying the basic cuts we can find that Tables 3 and 4 show very large differences in the cross sections between M_a = 8, 10 and 20 GeV, while the variations appear to be minimal in Table 2. For the ALP-photon coupling g_{aγγ} = 10^{-3} GeV^{-1}, the variations in the cross section of signal are minimal in the ALP mass interval 5 GeV ~ 20 GeV without cut applied at √s = 380, 1500 and 3000 GeV. The peak of the distribution shifts towards smaller ∆R(γγ) for the larger c.m. energy at same ALP mass, while the range of |η| for the high c.m. energy is larger than that for the low c.m. energy at same ALP mass. That is to say, for √s = 3 TeV, the signal for M_a = 8, 10 and 20 GeV lie very close to the signal region boundaries, so the lighter the ALP the more the signal is cut out. Therefore, the effects of the basic cuts on the cross sections of M_a = 8, 10 and 20 GeV at √s = 1500 and 3000 GeV are greater than those at √s = 380 GeV. From these tables, one can see that the background is suppressed very efficiently, while the signal still has good efficiency after imposing all cuts.

We use the following Poisson formula [54] to estimate the statistical significance (SS) at the CLIC with the luminosities of 0.5 ab^{-1}, 1.5 ab^{-1} and 3 ab^{-1}, respectively,

$$SS = \sqrt{2L[(S + B) \ln(1 + \frac{S}{B}) - S]},$$

where L represents the integrated luminosity, S and B respectively represent the effective cross sections of the signal and background after imposing all cuts.

In Fig. 7, we present the 3σ and 5σ bounds on the ALP parameter space from the W^+W^- fusion process e^+e^- \rightarrow ℓ^+ℓ^-γγ at the CLIC. The blue, red and green lines depict the bounds coming from the analysis for the three energy stages of the CLIC. This figure shows that the sensitivity bounds on the ALP-photon coupling g_{aγγ} can be im-
proved to 0.8 TeV$^{-1}$ (1.1 TeV$^{-1}$) with the first energy stage, 0.13 TeV$^{-1}$ (0.15 TeV$^{-1}$) with the second energy stage and 0.073 TeV$^{-1}$ (0.091 TeV$^{-1}$) with the third energy stage for the ALP mass interval 5 GeV $\sim$ 340 GeV, 5 GeV $\sim$ 1300 GeV and 5 GeV $\sim$ 2600 GeV at 3$\sigma$ (5$\sigma$) levels, respectively. Comparing our results of the three energy stages at the CLIC with each other, it is obvious that the sensitivity bound increases with the c.m. energy increases.

Our obtained bounds and other current existing constraints on the ALP-photon coupling are presented in Fig. 8. The green region comes from different beam dump experiments [55–57]. The dark blue and light blue regions depict the results given by [46, 47] via the diphoton and triphoton final states ($e^+e^- \rightarrow 2\gamma, 3\gamma$) at the LEP. The dark gray and black regions denote the constraints from radiative $\gamma \rightarrow \gamma a$ at the Babar [58] and the diphoton resonance searches at the LHCb [59], respectively. The dark green region labelled as "L3" represents looking for isolated and energetic photons from hadronic decay of Z boson at the L3 collaboration [60]. The exclusion regions also include the results from the LHC resonant $\gamma\gamma$ searches in red [33, 61, 62]. Our projected CLIC sensitivity for $\sqrt{s} = 380, 1500$ and 3000 GeV are enclosed by the cyan, blue and wine lines, respectively. Comparing our exclusion limits with the results of the LEP and LHC, we can conjecture that the CLIC has better potential to search for ALPs in the lower ALP mass interval 5 GeV $\sim$ 50 GeV and high ALP mass interval 2 TeV $\sim$ 2.7 TeV.

4 Conclusions

As a class of attractive particles, ALPs may influence the structure Electroweak phase transition, play an important role in solving the hierarchy problem and serve as cold dark matter (DM). There are excellent motivations to study ALPs. In this paper, we study the CLIC potential to search for ALP via the $W^+W^-$ fusion process $e^+e^- \rightarrow \bar{\nu}_e\nu_e a$ ($\rightarrow \gamma\gamma$). We have performed the numerical calculations and the phenomenological analysis for the signal and relevant SM background and obtained the $3\sigma$ and $5\sigma$ bounds on the ALP parameter space at the three energy stages of CLIC. Comparing our numerical results with other existing bounds, we find that, for the low mass region 5 GeV $\sim$ 50 GeV, the constraint on the ALP-photon coupling $g_{\gamma a}$ can be improved to $5\times10^{-2}$ TeV$^{-1}$. Meanwhile, our result complements the research of ALP-photon coupling by the ATLAS and CMS in the ALP mass range of 2 TeV $\sim$ 2.7 TeV.

In conclusion, we have found that searching for ALPs at the CLIC via the $W^+W^-$ fusion process $e^+e^- \rightarrow \bar{\nu}_e\nu_e a$ ($\rightarrow \gamma\gamma$) can not only cover low mass ALPs, but also have the greater potential to study the high mass ALPs. Thus, studying ALPs through the $W^+W^-$ fusion process $e^+e^- \rightarrow \bar{\nu}_e\nu_e a$ ($\rightarrow \gamma\gamma$) at the CLIC provides a complementary window to the opportunities offered by other high energy collider experiments.

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Appendix

Fig. 1: The Feynman diagrams of the single production of ALP with $V = \gamma, Z$ at $e^+e^-$ colliders.

Fig. 2: The ratio $\sigma/(g_{\gamma\gamma})^2$ as a function of the c.m. energy $\sqrt{s}$ for $M_a = 100, 200, 300$ and $500$ GeV.
Fig. 3: Feynman diagram of the $W^+W^-$ fusion process $e^+e^- \rightarrow \bar{\nu}_e\nu_e (\rightarrow \gamma\gamma)$.

Fig. 4: Normalized distributions of $M(\gamma\gamma)$, $\theta(\gamma\gamma)$, $E_T$, $E_T$ for the signal of selected ALP-mass benchmark points and SM background at the $\sqrt{s} = 380$ GeV CLIC with designed luminosity.
Fig. 5: Same as Fig. 4 but for $\sqrt{s} = 1500$ GeV CLIC with designed luminosity.

Fig. 6: Same as Fig. 4 but for $\sqrt{s} = 3000$ GeV CLIC with designed luminosity.
Fig. 7: The 3σ and 5σ curves in the $M_a - g_{a\gamma\gamma}$ plane from the $W^+W^-$ fusion process $e^+e^- \rightarrow \nu e, a (\rightarrow \gamma\gamma)$ at the 380, 1500 and 3000 GeV CLIC with the designed luminosities.

Fig. 8: The 2σ exclusion limits on the ALP couplings $g_{a\gamma\gamma}$ as a function of $M_a$ from the $W^+W^-$ fusion process $e^+e^- \rightarrow \nu e, a (\rightarrow \gamma\gamma)$ and other current exclusion regions.