Analysis of elastic supports and rotor flexibility for dynamics of a cantilever impeller

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Abstract. Turbochargers of modern internal combustion engines experience severe service conditions. In particular the compressor impeller is driven at extremely high angular velocity (30,000 rpm and more). Furthermore the increased requirements for the air intake dictate ever larger dimensions of the rotor disk up to 250 mm. These two factors lead to a pressing problem of stiffness, strength and dynamic stability of the newly designed units. This problem is analyzed with the help of a newly created parametric model of cantilever impeller rotor dynamics. This model accounts for the flexibility of the drive shaft and the impeller disk, quasistatic inertial and aerodynamic loads. The rotordynamics analysis is performed with a specific focus on the support behavior. The effect of initial gap and resilient characteristics of bearings and bushings is taken into account. As a result various components of displacements can be evaluated for different values of the disk diameter and angular velocity. It is critical for the design that neither radial nor axial displacement should not exceed given bounds. Furthermore, critical speeds of the studied rotor system have been determined for the various configurations of the disk and shaft dimensions. The proposed solution is to lower the first one or two eigenfrequencies of the system below the operating domain, so that the rotor becomes stable to any external excitations. Altogether, the developed mechanical analysis tool has proved itself as a useful tool for the engineering practice that allows to eliminate fatal flaws at early design stages.

1. Introduction

The issues of the dynamics and durability of rotor systems do not lose relevance. This is because their operating modes are increasing. Accordingly, the elastic deformations of these rotor systems elements increase, which leads to the change of the gaps with the stator part of the turbomachines. Centrifugal stresses also increase, which can lead to problems with the strength of individual elements of the rotor systems. In addition, critical modes can be implemented when these elements are rotated.

In many cases, all these problematic issues are resolved consistently. It builds models of stress-strain state and dynamic processes that are not directly related. Therefore, there are problems with purposeful variation of parameters, which are based on the criteria of ensuring the strength, rigidity and detachment from the critical rotational speeds of the rotor systems. This article is aimed at solving this problem.
2. Literature Review
Methods for analyzing the dynamics of rotor systems have developed significantly. This is due to the fact that the issue of debugging from the critical speeds of rotation is primary in the design of these machines. Different models are used in researches: hard disk, hard shaft, elastic supports. Thus, the related bending and torsional vibrations of rotor system with nonlinear friction are investigating in [1-3]. The papers [4-7] describe the parametric instability of flexible rotor system at an angle of the periodic motion basis. In [8-10], the authors proposed a method for reducing the amplitude of turbochargers oscillations with passive and active magnetic bearings in resonances. In [11] an engineering method is proposed to determine the radial yield on example of cylindrical roller bearings. However, the task of debugging from critical rotation modes does not solve all problems of rotor systems. For example, in [12-15] processes of work environment flow and wall wear are modeled. However, in many cases an analysis of other processes and states is required. There are many constructions in which the impeller (disk with blades) has a large axial size and is cantilevered relative to the supports. In this case, changing its design parameters affects not only the strength and stiffness of the centered forces, but also the critical rotational speed of the rotor system as a whole. Thus, the task becomes connected. In such a situation, it is desirable to build a unified model for the study of stress-strain state and critical rotational speeds of the rotor system. In addition, this model should also have an algorithm for determining the sensitivity to varying individual parameters. For this purpose, it is possible to involve the finite-difference method, as is done in [16-20].

3. Research methodology
The objective of this work is to build a parametric model and to study the stress-strain state and critical rotational speeds of rotor systems.

The following tasks have been set and resolved:
1. Formation of approach to construction of parametric models of stress-strain state and critical rotational speeds of rotor systems.
2. Investigation of peculiarities of stress-strain state, natural frequencies and oscillations of rotor systems.
3. Analysis of the effect of varying parameters on the strength, stiffness and critical speed characteristics of rotor systems.

4. Results

4.1. Approach to formation of parametric models of stress-strain state and critical rotational speeds of rotor systems

It is proposed to build a complex model of stress-strain state and critical rotational speeds of rotor systems based on the method of generalized parametric modeling [16]. This approach involves working with models of different types and from different subject areas. It is only important to have an algorithm that, by a set of parameters, allows to get a model of the object under study in a certain format [16].

Furthermore, a special module is required in order to perform the variation of individual object parameters or their sets. The combination of these two capabilities provides the basis for determining the sensitivity of stress-strain characteristics and critical rotational speeds to varying individual parameters. And this, in turn, enables purposeful variation to ensure the criteria of rigidity, strength and stability of the rotor systems.

The proposed approach can be considered as an example of a rotor system, which is shown in Figure 1.
This rotor system is defined by many parameters. They are: the shape and dimensions of the shaft and the blade disc, in addition, it is the material properties, angular rotation speed, distance between the bearing supports etc.

Some of these parameters are specified and some are variable. In different cases, these may be different parameters. We denote the vector of variable parameters by \( \mathbf{p} = \{p_1, p_2, ..., p_N\}^T \), where \( N \) is the number of these parameters. Then, to determine the stress-strain state of the elements of the rotor system, it is necessary to solve the system of equations of the elasticity theory. Using the finite element method [21], this problem is reduced to system of equations

\[
K(\mathbf{p}) \cdot \mathbf{v} = f(\mathbf{p}),
\]

where \( K(\mathbf{p}) \) is the stiffness matrix of the investigated structure;

\( \mathbf{v} \) is an array of node unknowns, such as displacements;

\( f(\mathbf{p}) \) is an array of nodal forces.

The stiffness matrix \( K \) and nodal forces \( f \) depend on the variable parameters \( \mathbf{p} \). Respectively, the solution \( \mathbf{v} = \mathbf{v}(\mathbf{p}) \) depends too. This dependence on the variable parameters is determined during the solution of the system of equations (1).

The known finite element method and the elasticity theory relationships determine the distribution of the displacement vector \( \mathbf{u}_i \), strain tensor \( \varepsilon_{ij} \) and stress tensor \( \sigma_{ij} \) \( (i, j = 1, 2, 3) \). By these distributions it is possible to form the characteristics of stress-strain state \( H_r(\mathbf{p}) = H_r(\mathbf{u}(\mathbf{p}), \varepsilon(\mathbf{p}), \sigma(\mathbf{p})) \), where \( r = 1, 2, ..., M \) \( (M \) is a number of characteristics).

By known ratios [16] sensitivity components are determined

\[
\nabla_{rs} = \frac{\partial H_r}{\partial p_s} \approx \frac{H_r(p_i + \Delta^r) - H_r(p_i)}{\delta_s}.
\]

Here \( \Delta^r \) is increment of parameter vector with one non-zero component with number \( s \), which is equal to \( \delta_s \).

Having an algorithm for determining the sensitivity (2), one can build an optimization algorithm [22]

\[
\mathbf{p}^* = \arg \min I(\mathbf{p}),
\]

where \( I \) is the relevant criterion.

Turning to the problem of critical rotational speed, it reduces to finding the natural frequencies \( p_{c0} \) and oscillations \( \lambda \) of rotor system under study.
\[ C(p, \omega_0) - \omega^2 M(p, \omega_0) \lambda = 0, \]  

where \( C, M \) are stiffness and mass matrices of the system under study; 
\( \omega_0 \) is operating speed of this system.

If a finite element model of rotor system is already constructed, then modification of \( C \) and \( M \) with respect to the dependences on \( \omega_0 \) is a well-known task.

Solution of the system of equations (4) makes it possible to establish dependencies

\[ \omega_i = \omega_i \left( p, \omega_0 \right), \lambda_i = \lambda_i \left( p, \omega_0 \right), i = 1, 2, \ldots \]  

Here \( \omega_i, \lambda_i \) are corresponding natural frequencies and oscillations of the finite element model of the system under study.

With dependencies (5), it can be set and solve debugging tasks from dangerous rotation modes. For example, a task will set

\[ \omega_0 \notin \Omega_0, i = 1, 2, \ldots \]  

where \( \Omega_0 \) is a predetermined interval \( \omega_0 \) around the operating speed of the system.

Thus, the proposed approaches make it possible, on the basis of a single parametric model, to set and solve the problems of analysis of stress-strain state and critical rotation speeds of rotor systems. In addition, the basis for solving the problems of substantiating the rational parameters of these rotor systems by the criteria of strength, stiffness and stability of motion is created.

4.2. Analysis of stress-strain state, natural frequencies and oscillations of rotor systems

Let us set some nominal parameters of rotor system under study (see Figure 1): shaft length \( L_0 = 0.24 \) m, shaft diameter \( d_0 = 0.019 \) m, disk diameter \( D_0 = 0.18 \) m, length \( l = 0.12 \) m, distance between bearing supports \( l_0 = 0.1 \) m, shaft material is steel (elasticity modulus \( E = 2 \cdot 10^{11} \) N/m\(^2\), Poisson's ratio \( \nu = 0.3 \)); disk with blades made of aluminum alloy (elasticity modulus \( E = 7.1 \cdot 10^{10} \) N/m\(^2\), Poisson's ratio \( \nu = 0.33 \)); nominal rigidity of bearing supports \( c = 1.96 \cdot 10^7 \) N/m, nominal operating speed \( \omega_0 = 20 \) thousands rpm.

First, a quasi-static analysis of this rotor loaded by centrifugal forces has been performed with a commercial finite element package ANSYS Workbench. Figure 2 shows the obtained distribution of the total displacements of rotor system elements by nominal parameter values. Similarly, von Mises stresses distribution in this system are presented in Figure 3.

It can be seen that the stress-strain state of the rotor system is characterized by a substantially heterogeneous distribution of displacements \( u \) and stresses \( \sigma \). In particular, the largest displacement values are at the periphery of the blade disc. However, the dominant component cannot be distinguished: the axial and radial displacements are proportional to each other. Therefore, it is necessary to control the change in both the radial and axial clearance between the rotor and stator parts of the construction.

For von Mises stresses distribution, a characteristic feature is that they reach maximums at the edge of the blade disc. Given that the mechanical properties of aluminum alloys are relatively low, this creates some problems with the impeller strength.
The next stage was the harmonic analysis of this rotor system. The previously obtained pre-stress data were used to account for the specific effect of stiffening. Figure 4 shows the selected eigenfrequencies and eigenmodes of the rotor system.

It can be seen that the first two self-oscillations mainly correspond to the behavior of the hard disk and the hard shaft on the elastic supports. However, there is some contribution from the deformation of the disk and the shaft, which must be taken into account. It is especially indicated when varying the parameters of the rotor system. If we consider the higher oscillations of the rotor system, they correspond to the excitation of the blades. The natural frequencies of these oscillations are much higher than $\omega_1$, $\omega_2$. Accordingly, the natural bending frequency (bending shaft) is much higher than the lower natural frequencies $\omega_1$, $\omega_2$. 

Figure 2. Distribution of complete displacements of rotor system elements.

Figure 3. Mises stresses distribution in rotor system elements.
Figure 4. Some natural frequencies and oscillations of the rotor system: first, second and third bending and higher vane shapes (4-6).

Thus, it is necessary to determine mainly dependencies $\omega_1(p), \omega_2(p)$. These dependencies should be taken into account when debugging the rotor system from critical operating modes.
4.3. Investigation of stress-strain state, natural frequencies and oscillations of rotor systems

To study the effect on the stress-strain state of the rotor system, two parameters were varied: the relative elasticity modulus of aluminum alloy $p_2 = \rho = E_0 / E_p$ and speed $p_1 = q = \omega / \omega_0$. Relative displacements $H_1 = \bar{w} = w_{\max} (p_1, p_2) / w_{\max} (1, 1)$ and relative stresses $H_2 = \bar{\sigma} = \sigma_{\max} (p_1, p_2) / \sigma_{\max} (1, 1)$ were investigated as controlled characteristics. Here $w$ are complete displacement, and $\sigma$ are von Mises stresses. Figure 5, 6 show the dependencies $H_1 (p_1, p_2), H_2 (p_1, p_2)$ from parameters $p_1, p_2 \in [0,2;5,0]$.

![Figure 5](image5.png) ![Figure 6](image6.png)

**Figure 5.** Dependence of relative maximum displacements $H_1$ from the relative angular speed at different relative elasticity modulus ($\rho = p_2$).

**Figure 6.** Dependence of relative maximum von Mises stresses $H_2$ from the relative angular speed at different relative elasticity modulus ($\rho = p_2$).

It can be seen that the displacement level increases with the reduction of the elastic modulus of impeller material almost in a linear dependence. However, the displacement rate depends on the rotational speed depending on the parabolic dependence. Also, the level of stresses in the rotor system changes depending on the parabolic rotational speed. But the level of stress on the variation of the elasticity modulus of impeller material depends less than the level of displacement.

These features of the stress-strain state are expected, since the level of displacements and stresses is determined in the case under study by a blade disc, not a shaft.

The determined dependences make it possible to analyze the main tendencies of varying parameters influence on the stress-strain state of the rotor system.

The relative linear dimensions of the whole system $p_3 = \alpha = D / D_0$ and the relative stiffness of the elastic supports $p_4 = \phi = c / c_0$ varied during the analysis of the critical rotational speeds of the rotor system. The relative level of the first natural oscillation frequency relative to the rotation speed $H_3 = \bar{\omega}_1 = \omega_1 (p_3, p_4) / \omega_1 (1, 1)$ and the relative level of the second natural oscillation frequency $H_4 = \bar{\omega}_2 = \omega_2 (p_3, p_4) / \omega_2 (1, 1)$ were controlled.

Figure 7, 8 shows dependences $H_3 (p_3, p_4), H_4 (p_3, p_4)$ from parameters $p_3, p_4 \in [0,2;5,0]$.

As can be seen from the above results, the stiffness of the supports influences the first and second eigenfrequencies quite significantly. These frequencies are also significantly affected by the linear dimensions of rotor system elements.
Thus, when debugging from undesirable operating modes, it should be borne in mind that the pliability of the supports has a strong influence. In order to significantly affect the first natural frequencies, this parameter should be varied. The dimensions of the rotor system elements also affect these frequencies.

5. Conclusion
1. To study the characteristics of strength, stiffness and detachment from critical rotational speeds, the single integrated model was developed. It makes it possible to determine, on a general finite element model, both the stress-strain state and the natural oscillation frequencies of the rotor system.
2. The paper describes the approach to the formation of parametric models of stress-strain state and critical rotational speeds of rotor systems. The advantage of this approach is that the research object can be varied in an automated manner. This creates advantages in determining the sensitivity of the controlled strength, stiffness, and critical rotation speeds to varying individual parameters.
3. It is determined that the studied rotor system is characterized by a significantly uneven distribution of elastic displacements and stresses in the blade disc. The maximum displacements are reached at the periphery of the disk; the maximum voltages are reached in the areas of connection of the blades with the disk.
4. The two lower bending proper oscillations of rotor system under study resemble the oscillations of the hard disk and the hard shaft on elastic supports. At the same time, the elastic deformation of the disc and the shaft in some way distorts the oscillations of this stiffer system.
5. Different parameters are influenced by the characteristics of the stress-strain state and the natural oscillation frequencies of the rotor system: the sizes, physical and mechanical properties of the material of the disk with blades, the pliability of the bearing supports, etc. It is determined that the degree of influence of different parameters is different. Thus, the rotational speed of rotor system is strongly influenced by the stress-strain state. The lower natural frequencies of oscillations are significantly influenced by the pliability of the elastic supports and the dimensions of the system elements.

The developed approach, models and methods are the basis for further studies of the influence of various parameters on the dynamics and strength characteristics of rotor systems.
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