Electron-electron interaction in a MCS model with a purely spacelike Lorentz-violating background

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One considers a planar Maxwell-Chern-Simons electrodynamics in the presence of a purely spacelike Lorentz-violating background. Once the Dirac sector is properly introduced and coupled to the scalar and the gauge fields, the electron-electron interaction is evaluated as the Fourier transform of the Möller scattering amplitude (derived in the non-relativistic limit). The associated Fourier integrations can not be exactly carried out, but an algebraic solution for the interaction potential is obtained in leading order in $v^2/s^2$. It is then observed that the scalar potential presents a logarithmic attractive (repulsive) behavior near (far from) the origin. Concerning the gauge potential, it is composed of the pure MCS interaction corrected by background contributions, also responsible for its anisotropic character. It is also verified that such corrections may turn the gauge potential attractive for some parameter values. Such attractiveness remains even in the presence of the centrifugal barrier and gauge invariant $A \cdot A$ term, which constitutes a condition compatible with the formation of Cooper pairs.

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I. INTRODUCTION

In the latest years, Lorentz-violating theories have been in focus of great interest and investigation [1]-[6]. Despite the intensive activity proposing and discussing the consequences of a Lorentz-violating electrodynamics, some experimental data and theoretical considerations indicate stringent limits on the parameters responsible for such a breaking [2], [3]. These evidences put the Lorentz-violation as a negligible effect in a factual (1+3)-dimensional electrodynamics, which raises the question about the feasibility of observation of this effect in a lower dimension system or in another environment distinct from the usual high-energy domain in which this matter has been generally regarded so far.

Condensed Matter Systems (CMS) are low-energy systems sometimes endowed with spatial anisotropy which might constitute a nice environment to study Lorentz-violation and to observe correlated effects. Indeed, although Lorentz covariance is not defined in a CMS, Galileo covariance holds as a genuine symmetry in such a system (a least for the case of isotropic low-energy systems). Having in mind that a CMS may be addressed as the low-energy limit of a relativistic model, there follows a straightforward correspondence between the breakdown of Lorentz and Galileo symmetries, in the sense that a CMS with violation of Galileo symmetry may have as counterpart a relativistic system endowed with breaking of Lorentz covariance. Considering the validity of this correspondence, it turns out that an anisotropic CMS may be addressed as the low-energy limit of a relativistic model in the presence of a spacelike Lorentz-violating background.

The attainment of an attractive electron-electron ($e^-e^-$) potential in the context of a planar model incorporating Lorentz-violation is a point that sets up a clear connection between such theoretical models and condensed matter physics. Theoretical planar models able to provide attractive $e^-e^-$ interaction potentials may constitute a suitable framework to deal with the condensation of Cooper pairs, a fundamental characteristic of superconducting systems. The Maxwell-Chern-Simons theories [7] were addressed in the beginning 1990s as a theoretical alternative to accomplish this objective, without success. In a recent calculation [8], new results concerning an electron-electron interaction were also obtained in the context of a noncommutative extension of the MCS electrodynamics, revealing a non-relativistic potential nearly identical to the MCS outcome. Actually, it is known that the MCS-Proca models [9] may better provide an attractive interaction due to the action of the intermediation played by the Higgs sector. Another well defined feature of a planar superconductor concerns the symmetry of the order parameter (standing for the Cooper pair), which is described in terms of a spatially anisotropic d-wave [10]. A field theory model able to account for an anisotropic $e^-e^-$ interaction is
the first step to the achievement of anisotropy for the order parameter. This is exactly the expected result to be obtained in the case of a Lorentz-violating model in the presence of a purely spacelike background, where the $e^-e^-$ scattering potential may be identified with the one evaluated in the context of a CMS endowed with a privileged direction in space.

The investigation of the $e^-e^-$ interaction can be suitably considered in the context of a Lorentz-violating planar framework. In fact, in a very recent paper [11], the low-energy Möller interaction potential was carried out for the case of a planar electrodynamics [3], arising from the dimensional reduction of the Carroll-Field-Jackiw model [5], for a purely timelike background. With this purpose, the Dirac sector was included in this planar model, so that to make feasible the consideration of the low-energy Möller scattering (adopted as the appropriate tool to analyze the non-relativistic electron-electron interaction). The interaction potential obtained revealed to be composed of a scalar and a gauge contributions. The scalar one (coming from the scalar intermediation) has presented a logarithmic attractive (repulsive) behavior near (far from) the origin. On the other hand, it has been shown that gauge potential, associated with the gauge intermediation, is composed of the Maxwell-Chern-Simons (MCS) usual interaction [7] corrected by background depending terms. One has also noted that these corrections lead to a gauge potential endowed with attractiveness (for some parameters values) even in the presence of the centrifugal barrier and the $A^2$—gauge invariant term stemming from the Pauli equation. Thereby, one has shown that these results bypass the controversy involving the pure Maxwell-Chern-Simons potential (see Hagen and Dobroliubov [7]) concerning the possibility of attractiveness, and yields a strong indication that it may occur the formation of Cooper pairs in this theoretical framework.

Having as main motivation the encouraging outcomes achieved in Ref. [12], in this work one searches for the electron-electron potential in the context of a Lorentz-violating planar electrodynamics endowed with a purely spacelike background, $v^\mu = (0, v)$. As this kind of background fixes a 2-direction in space, it will certainly lead to an anisotropic behavior, one consequence of the directional dependence of the solutions in relation to the fixed background ($v$). By determining such $e^-e^-$ potential, one can investigate two expected properties concerning the $e^-e^-$ interaction: attractiveness and anisotropy, which are relevant due its possible connection with high-$T_c$ superconducting systems. The procedure here adopted is the same one developed in Ref. [11], that is, one carries out the $e^-e^-$ interaction potential stemming from the Möller scattering amplitude associated with the scalar and the gauge intermediations, exhibiting and pointing out the corrections induced by the fixed background, on the pure Maxwell-Chern-Simons result. With this purpose, one starts from the planar Lagrangian defined in Ref. [11], in which the Dirac sector has been already included. One then carries out the $e^-e^-$ Möller scattering amplitude (from which the interaction potential is derived according to the Born approximation) following the general guidelines set up in Refs. [3, 4, 11]. The potential here attained is composed of a scalar and a gauge contribution as well, since the $e^-e^-$ interaction is equally mediated by the massless scalar and the massive gauge fields. The scalar potential maintains the logarithmic behavior (asymptotically repulsive and attractive near the origin) of the purely timelike case, being different only by the presence of anisotropy. For as the gauge potential, it is given by a lengthy expression composed of the pure MCS interaction and many background depending terms which imply the presence anisotropy, among other features. It is possible to show that these corrections are able to turn this potential attractive for some values of the relevant parameters, behavior which remains even in the presence of the centrifugal barrier ($l/mr^2$) and the $A \cdot A$ gauge invariant term. Furthermore, the total interaction (scalar and gauge potentials) may always be attractive with a suitable adjust of the coupling parameters. This outcome constitute the essential condition to promote the condensation of Cooper pairs, which shows that this theoretical framework may be useful to address some properties of superconducting systems.

This paper is organized as follows. In Sec. II, one briefly presents the structure of reduced planar model (derived in Ref. [3]) which is here adopted as stating point. This model is supplemented by the Dirac field. In Sec. III, one presents the spinors which fulfill the two-dimensional Dirac equation and that are used to evaluate the Möller scattering amplitude associated with the Yukawa and the gauge intermediations. The corresponding interaction potentials are carried out, and the results are discussed. In Sec. IV, one concludes with the Final Remarks.
II. THE PLANAR LORENTZ-VIOLATING MODEL

The starting point is the planar Lagrangian obtained from the dimensional reduction of the CFJ-Maxwell electrodynamics [4], which consists in a Maxwell-Chern-Simons electrodynamics coupled to a massless scalar field $\phi$ and to a fixed background 3-vector $(\bar{\psi}^\mu)$ through a Lorentz-violating term, derived from the dimensional reduction of the Carroll-Field-Jackiw model [5]. One then regards the additional presence of a fermion field $\psi$:

$$L_{1+2} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{s}{2} \varepsilon_{\mu\nu\kappa} \partial^\nu A^\kappa - \frac{1}{2} \partial^\mu \phi \partial^\nu \phi + \phi \varepsilon_{\mu\nu\kappa} v^\mu \partial^\nu A^\kappa - \frac{1}{2\alpha} (\partial^\mu A^\mu)^2 + \bar{\psi} (i\not{D} - m_c) \psi - y \phi \bar{\psi} \psi. \tag{1}$$

Here, the covariant derivative, $D\psi \equiv (\not{D} + ie_3 A)\psi$, states the minimal coupling, whereas the term $\phi \bar{\psi} \psi$ reflects the Yukawa coupling between the scalar and fermion fields, with $y$ being the constant that measures the strength of the electron-phonon coupling. The mass dimensions of the fields and parameters are the following: $[\varphi] = [A^\mu] = 1/2, [\psi] = 1, [s] = [v^\mu] = 1, [e_3] = [y] = 1/2$. One then notes that the coupling constants, $e_3, y$, both exhibit $[mass]^{1/2}$ dimension, a usual result in $(1+2)$ dimensions. Furthermore, in Ref. [4], the propagators of the scalar $(\varphi)$ and gauge $(A_\mu)$ fields were properly evaluated and used as starting point to analyze the causality and unitarity. Such analysis has revealed a model totally stable, causal and unitary (at a classical level) for both spacelike and timelike backgrounds. This result has demonstrated that this planar model bypasses the problems concerning the stability and causality presented by the original Carroll-Field-Jackiw (CFJ) model [5] in $(1+3)$ dimensions, which indicates that this model may undergo consistent quantization procedures, a necessary condition to address condensed matter systems. The knowledge of the propagators$^1$ evaluated in Ref. [4] is essential to the calculations of this work.

III. THE MÖLLER SCATTERING AMPLITUDE AND THE INTERACTION POTENTIAL

The two-particle interaction potential is given by the Fourier transform of the two-particle scattering amplitude in the low-energy limit (Born approximation). In the case of the nonrelativistic Möller scattering, one should consider only the t-channel (direct scattering) [10] even for indistinguishable electrons, since in this limit one recovers the classical notion of trajectory. From Eq. (1), there follow the Feynman rules for the interaction vertices: $V_{\psi\varphi\psi} = iy; V_{A\psi\psi} = ie_3 \gamma^\mu$, so that the $e^- e^-$ scattering amplitude are written as:

$$-i M_{\text{scalar}} = \pi(p_1)(iy)u(p_1)\langle(\varphi\bar{\psi})\rangle \pi(p_2)(iy)u(p_2), \tag{2}$$

$$-i M_A = \pi(p_1)(ie_3 \gamma^\mu)u(p_1)\langle[A_\mu A_\nu]\rangle \pi(p_2)(ie_3 \gamma^\nu)u(p_2),$$

with $\langle(\varphi\bar{\psi})\rangle$ and $\langle[A_\mu A_\nu]\rangle$ being the scalar and photon propagators. Expressions (2) and (3) represent the scattering amplitudes for electrons of equal polarization mediated by the scalar and gauge particles, respectively. The spinors $u(p)$ stand for the positive-energy solution of the Dirac equation $(\not{D} - m)u(p) = 0$. The $\gamma^\mu$ matrices satisfy the $\mathfrak{so}(1,2)$ algebra, $[\gamma^\mu, \gamma^\nu] = 2i\varepsilon^{\mu\nu\alpha}\gamma_\alpha$, and correspond to the $(1+2)$-dimensional representation of the Dirac matrices, that is, the Pauli ones: $\gamma^\mu = (\sigma_z, -i\sigma_x, i\sigma_y)$. Regarding these definitions, one obtains the

$^1$ The gauge propagator is given by:

$$\langle A^\mu(k) A^\nu(k) \rangle = \left\{ -\frac{1}{k^2-s^2} \Theta^{\mu\nu} - \frac{\alpha(k^2-s^2)\Xi(k) + s^2(v.k)^2}{k^2(k^2-s^2)\Xi(k)} \varphi^{\mu\nu} - \frac{s}{k^2(k^2-s^2)\Xi(k)} S^{\mu\nu} + \frac{s^2}{(k^2-s^2)\Xi(k)} \Lambda^{\mu\nu} - \frac{1}{k^2-s^2} T^{\mu\nu} + \frac{s}{(k^2-s^2)\Xi(k)} \left[ Q^{\mu\nu} - Q^{\nu\mu} \right] + \frac{i s(v.k)}{k^2(k^2-s^2)\Xi(k)} \left[ \Sigma^{\mu\nu} + \Sigma^{\nu\mu} \right] - \frac{i s(v.k)}{k^2(k^2-s^2)\Xi(k)} \left[ \Phi^{\mu\nu} - \Phi^{\nu\mu} \right] \right\},$$

whereas the scalar propagator is:

$$\langle \varphi(0) \varphi(0) \rangle = \frac{1}{k^2-s^2} \Xi(k), \text{ where: } \Xi(k) = k^4 - (s^2 - v \cdot v) k^2 - (v \cdot k)^2. \tag{4}$$

The involved 2-rank tensors are defined as follows: $\Theta^{\mu\nu} = \eta^{\mu\nu} - \omega^{\mu\nu}, \omega^{\mu\nu} = \partial_\mu \partial_\nu / \Box, S^{\mu\nu} = \varepsilon_{\mu\nu\kappa} \partial^\kappa, Q^{\mu\nu} = v_\nu T_{\mu} - T_{\mu} v_\nu = S_{\mu\nu} v^\nu, A_{\mu\nu} = v_\nu v_\mu, \Sigma^{\mu\nu} = v_\mu \partial_\nu, \Phi^{\mu\nu} = T_{\mu} \partial_\nu.$
Møller scattering should be easily analyzed in the center of mass frame, where the momenta of the incoming particle, \( E, p \) and \( -E, -p \), the coordinate vector, \( \mathbf{r} \), and the transfer 3-momentum arising from this convention are explicitly written in Ref. [11].

\[
\mathcal{M}_{\text{scalar}} = -y^2 \frac{[k^2 + s^2]}{k^2 [k^2 + s^2 + v^2 \sin^2 \alpha]},
\]

where \( \alpha \) is the angle defined by the vectors \( \mathbf{v} \) and \( \mathbf{k} \). Taking into account the Born approximation, the potential associated with the Yukawa interaction reads as,

\[
V_{\text{scalar}}(r) = -y^2 \left( \frac{2\pi}{\alpha} \right)^2 \int e^{i \mathbf{k} \cdot \mathbf{r}} \frac{[k^2 + s^2]}{k^2 [k^2 + s^2 + v^2 \sin^2 \alpha]} \frac{d^2 \mathbf{k}}{k^2}.
\]

Such Fourier integration above can not be exactly carried out. However, this integration may be solved in the regime in which \( s^2 \gg v^2 \). As far as this condition holds, the following approximation,

\[
\frac{[k^2 + s^2]}{k^2 [k^2 + s^2 + v^2 \sin^2 \alpha]} \approx \frac{1}{k^2} - \frac{v^2 \sin^2 \alpha}{k^2 [k^2 + s^2]},
\]

is valid in first order in \( v^2/s^2 \). In order to solve Eq. (8), two other angles are of interest: \( \varphi \) and \( \beta \) - defined respectively by the relations: \( \cos \varphi = \mathbf{r} \cdot \mathbf{k}/r \), \( \cos \beta = \mathbf{r} \cdot \mathbf{v}/r \). While the background vector, \( \mathbf{v} \), sets up a fixed direction in space, the coordinate vector, \( \mathbf{r} \), defines the position where the potentials are to be measured; so, \( \beta \) is the (fixed) angle that indicates the directional dependence of the fields in relation to the background direction. Being confined into the plane, these angles satisfy a simple relation: \( \alpha = \varphi - \beta \), whose consideration leads to: \( \sin^2 \alpha = c_2 + c_1 \cos^2 \varphi + c_3 \sin 2\varphi \), with: \( c_1 = (1 - 2 \cos^2 \beta) \), \( c_2 = \cos^2 \beta \), \( c_3 = -(\sin 2\beta)/2 \). This expression allows the evaluation of the angular integration on the \( \varphi \)-variable enclosed in Eq. (6), given below:

\[
\int_0^{2\pi} e^{ikr \cos \varphi} \sin^2 \alpha d\varphi = 2\pi \left[ (c_1 + c_2) J_0(kr) - \frac{c_1}{kr} J_1(kr) \right].
\]

Taking into account these preliminary results, one shall now carry out the integrations on the \( \mathbf{k} \)-variable, obtaining the following scalar interaction potential:

\[
V_{\text{scalar}}(r) = \frac{y^2}{(2\pi)} \left\{ \left[ 1 - \frac{v^2}{s^2} \right] \ln r - \frac{v^2}{s^2} \sin^2 \beta K_0(sr) - \frac{v^2 \cos 2\beta}{s^4} \frac{1}{r^2} \left[ 1 - sr K_1(sr) \right] \right\}.
\]

Near the origin, \( r \to 0 \), the modified Bessel functions behave as \( K_0(r) \to -\ln r \), \( K_1(sr) \to 1/sr + sr \ln r/2 \), apart from constant terms. In such a way, the potential \( V_{\text{scalar}} \) goes like:

\[
\lim_{r \to 0} V_{\text{scalar}}(r) = \frac{y^2}{(2\pi)} \left[ 1 - \frac{v^2}{2s^2} (1 + \sin^2 \beta) \right] \ln r.
\]
Far from the origin, \( r \to \infty \), the Bessel functions decay exponentially whereas the logarithmic function increases. In this limit, one has:

\[
\lim_{r \to \infty} V_{\text{scalar}}(r) = \frac{y^2}{(2\pi)} \left[ 1 - \frac{v^2}{2s^2} \right] \ln r. \tag{11}
\]

Remaking the condition \( s^2 >> v^2 \) under which this solution was derived, the scalar potential turns out always attractive near the origin and repulsive asymptotically. Thereby, both near and far form the origin the scalar potential exhibits a logarithmic behavior corrected by the background term, with explicit directional dependence in terms of the angle \( \beta \). Such a result implies an attractive (repulsive) interaction near (far from) the origin. This logarithmic asymptotic behavior also indicates the absence of screening concerning the scalar sector of this model. The existence of unscreened solutions is ascribed to the presence of a massless-like term, \( 1/|k^2| \), in the body of the scattering amplitude.

In comparing the solution here attained with the scalar potential valid for a purely timelike background, given in Ref. \([11]\), it is instructive to point out that both possess a similar logarithmic behavior for \( r \to 0 \) and \( r \to \infty \). The difference lies mainly in the directional dependence on the \( \beta \)-angle, responsible for the anisotropy, absent in the purely timelike case.

**B. The gauge potential**

Although the propagator of the gauge sector is composed by eleven terms, only six of them will contribute to the scattering amplitude, namely \( \theta^{\mu
u}, S^{\mu
u}, \Lambda^{\mu
u}, T^\nu T^\nu, Q^{\mu
u}, Q^{\nu \mu} \). This is a consequence of the current-conservation law (\( k_{\mu}J^{\mu} = 0 \)). The first two terms yield, in the non-relativistic limit, the Maxwell-Chern-Simons (MCS) scattering amplitude, already carried out in Refs. \([7]\). The other four terms lead to background depending scattering amplitudes. In order to obtain the total scattering amplitude mediated by the gauge field, one must previously evaluate the following current-current amplitude terms,

\[
j^{\mu}(p_1)(S_{\mu\nu})j^{\nu}(p_2) = j^{(0)}(p_1)S_{\mu\nu}j^{(0)}(p_2) + j^{(i)}(p_1)S_{\mu\nu}j^{(0)}(p_2), \tag{12}
\]

\[
j^{\mu}(p_1)(T_\mu T_\nu)j^{\nu}(p_2) = j^{(0)}(p_1) \left[ (\vec{\nabla} \cdot \vec{\nabla}) (\vec{k} \cdot \vec{k}) - (\vec{\nabla} \cdot \vec{k})^2 \right] j^{(0)}(p_2), \tag{13}
\]

\[
j^{\mu}(p_1) (\Lambda_{\mu\nu})j^{\nu}(p_2) = j^{(i)}(p_1)\left[ \nu \nu j^{(i)}(p_2), \tag{14}
\]

\[
j^{\mu}(p_1) (Q_{\mu\nu} - Q_{\nu\mu})j^{\nu}(p_2) = j^{(i)}(p_1)\nu j^{(0)}(p_2) - j^{(i)}(p_2)\nu j^{(0)}(p_1) (\vec{\nabla} \times \vec{k}), \tag{15}
\]

which carried out in the non-relativistic limit, with \( \nu^\mu = (0, 0, v) \) and \( k^\mu = (0, k) \), lead to:

\[
j^{\mu}(p_1)(S_{\mu\nu})j^{\nu}(p_2) = \frac{k^2}{m} - \frac{2i}{m}k \times p, \quad j^{\mu}(p_1)(T_\mu T_\nu)j^{\nu}(p_2) = [v^2k^2 \sin^2 \alpha], \tag{12}
\]

\[
j^{\mu}(p_1) (\Lambda_{\mu\nu})j^{\nu}(p_2) = -\frac{v^2 k^2}{4m^2} e^{i\theta}, \quad j^{\mu}(p_1) (Q_{\mu\nu} - Q_{\nu\mu})j^{\nu}(p_2) = \frac{v^2 k^2}{4m^2} [1 - e^{i\theta}],
\]

where the vector \( p = \frac{1}{2}(p_1 - p_2) \) is defined in terms of the 2-momenta \( p_1, p_2 \) of the incoming electrons, and \( \theta \) is the scattering angle in the CM frame. The total scattering amplitude associated with the gauge sector is obviously given by:

\[
\mathcal{M}_{\text{gauge}} = \mathcal{M}_{\text{MCS}} + \mathcal{M}_\Lambda + \mathcal{M}_{TT} + \mathcal{M}_{QQ}.
\]

where \( \mathcal{M}_{\text{MCS}} \) is the Maxwell-Chern-Simons scattering amplitude (for which contribute the terms \( \theta^{\mu\nu}, S^{\mu\nu} \) of the gauge propagator) and the other three are explicitly depending on the background, as exhibited below:

\[
\mathcal{M}_{\text{MCS}} = e_s^2 \left\{ \left( 1 - \frac{s}{m} \right) \frac{1}{k^2 + s^2} - \frac{2s}{m} \frac{k \times p}{k^2 (k^2 + s^2)} \right\}, \quad \mathcal{M}_\Lambda = e_s^2 \frac{s v^2}{4m^2} \frac{k^2}{[k^2 + s^2] \ln (k)} e^{i\theta},
\]

\[
\mathcal{M}_{TT} = e_s^2 v^2 \frac{k^2}{[k^2 + s^2] \ln (k)} \sin^2 \alpha, \quad \mathcal{M}_{QQ} = -e_s^2 \frac{s v^2}{2m} \frac{k^2}{[k^2 + s^2] \ln (k)} [1 - e^{i\theta}],
\]
with the term, \( \Xi(k) = [k^2(k^2 + s^2 + v^2 \sin^2 \alpha)] \), being given in Ref. [4]. The amplitude \( \mathcal{M}_{MCS} \) leads to the well-know Maxwell-Chern-Simons potential (see Refs. [2]),

\[
V_{MCS}(r) = \frac{e^2}{(2\pi)^3} \left[ \left( 1 - \frac{s}{m} \right) K_0(sr) - \frac{2}{ms} [1 - srK_1(sr)] \frac{l^2}{r^2} \right].
\]

(16)

which presents a purely logarithmic behavior near the origin, namely:

\[
V_{MCS}(r) \rightarrow - \left( \frac{e^2}{2\pi} \right) \left[ 1 - \frac{s}{m} - \frac{s}{m} \ln r \right],
\]

(17)

and a typical \(-1/r^2\) behavior in the asymptotic limit. This preliminary MCS result will be corrected by the other background depending contributions, still to be evaluated. Hence, the remaining task consists in carrying out the Fourier transforms of the three amplitudes above. Starting from the \( \mathcal{M}_A \)-amplitude, the corresponding potential is written as follows:

\[
V_A(r) = \frac{1}{(2\pi)^2} e^2 s^2 v^2 \int_0^\infty \int_0^{2\pi} \frac{e^{ikr \cos \phi} e^{i\theta k \cos \phi}}{[k^2 + s^2][k^2 + s^2 + v^2 \sin^2 \alpha]} e^{i\theta k \cos \phi} d\theta dk d\phi.
\]

Again, this integral can not be exactly solved, so that the expansion in first order in \( v^2/s^2 \),

\[
\frac{1}{[k^2 + s^2][k^2 + s^2 + v^2 \sin^2 \alpha]} \approx \frac{1}{[k^2 + s^2]^2} - \frac{v^2 \sin^2 \alpha}{[k^2 + s^2]^3},
\]

(18)

must be adopted. Besides this approximation, an important point concerns the relation existing between the scattering angle \( \theta \) and the integration angle \( \varphi \): \( \theta = (2\varphi - \pi) \), which is decisive for the solution of the relevant angular integration, now read as

\[
\int_0^{2\pi} e^{ikr \cos \phi} e^{i\theta k \cos \phi} d\phi = - (2\pi) [J_2(kr)].
\]

(19)

Considering it and stressing that only the first term (on the right hand side) of Eq. (13) will provide a first order contribution (in \( v^2 \)), the following potential expression comes out:

\[
V_A(r) = \frac{e^2}{(2\pi)^2} v^2 \left\{ - \frac{2}{s^2 r^2} + K_0(sr) + \left( \frac{2}{sr} + \frac{sr}{2} \right) K_1(sr) \right\},
\]

(20)

whence one notes that in first order the directional dependence on the angle \( \beta \) does not appear in this potential, which exhibits a behavior as \(-\ln r\) (and as \(-1/r^2\)) near (and far from) the origin.

As for the interaction potential related to the \( \mathcal{M}_{TT} \)-amplitude,

\[
V_{TT}(r) = \frac{e^2 v^2}{(2\pi)^2} \int_0^\infty \frac{e^{ikr \cos \phi} \sin^2 \alpha}{[k^2 + s^2][k^2 + s^2 + v^2 \sin^2 \alpha]} r^2 k \cdot d^2 k,
\]

the integral can not be exactly solved as well, in such a way the expansion (at first order in \( v^2/s^2 \)),

\[
\frac{\sin^2 \alpha}{[k^2 + s^2][k^2 + s^2 + v^2 \sin^2 \alpha]} \approx \frac{v^2 \sin^2 \alpha}{[k^2 + s^2]^2},
\]

(21)

must be properly considered. The associated angular integration is given by Eq. (9), so that the resulting potential takes then the form (at first order in \( v^2 \)):

\[
V_{TT}(r) \approx \frac{e^2 v^2}{(2\pi)^2} \left\{ \frac{c_1}{2s^2} K_0(sr) - \frac{c_1}{s^4 r^2} + \frac{\sin^2 \beta}{2s^2} r K_1(sr) + \frac{c_1}{s^4 r^2} K_1(sr) \right\},
\]

(22)
where: $c_1 = -(\cos 2\beta) $.

One can now solve the last Fourier transformation for the scattering amplitude $M_{QQ}$, written as follows:

$$ V_{QQ}(r) = \frac{1}{(2\pi)^2} \left( \frac{3}{2m} \right)^{\frac{3}{2}} \int_0^{\infty} \left( \frac{1 - e^{\frac{1}{2}\beta}}{\sqrt{|k^2 + s^2|^2 + \frac{2}{s} v^\prime}} \right) d^3 k, \tag{23} $$

which must be rewritten according to the approximation $\mathbf{1}$ and solved making use of the angular integration $\mathbf{15}$, so that one achieves at first order:

$$ V_{QQ}(r) \approx \frac{e^3}{(2\pi)^2} \frac{v^2}{2m} \left\{ -\frac{2}{s^3} + \frac{3}{2} v^1 K_1(sr) + \frac{1}{s} [K_0(sr) + \frac{2}{sv} K_1(sr)] \right\}. \tag{24} $$

It is interesting to point out that the three potentials, $V_\Lambda, V_{TT}, V_{QQ}$, behave at the same way both near and away from the origin. Indeed, it is easy to show that these potentials behave as a constant for $r \rightarrow 0$, and as $-1/r^2$ for $r \rightarrow \infty$. Regarding that rest mass of the electron represents a large energy threshold before low-energy excitations usually observed in condensed matter physics, one should adopt the following condition $m^2 >> s^2$. Thereby, the potential $V_{TT}$ turns out proportionally more significant that $V_{QQ}$ and $V_\Lambda$, which is the least relevant one, in accordance the order of magnitude of the multiplicative factors $(v^2/4m^2, v^2/2m, v^2)$ which appear in Eqs. $\mathbf{24}$, $\mathbf{25}$, $\mathbf{26}$.

The total gauge potential, $V_{gauge}(r) = V_{MCS} + V_\Lambda + V_{TT} + V_{QQ}$, is then written as a complex combination of Bessel functions and $1/r^2$ terms, explicitly as:

$$ V_{gauge}(r) = \frac{e^3}{(2\pi)^2} \left\{ 1 - \frac{s}{m} + v^2 \left( \frac{1}{2ms} + \frac{1}{4m^2} - \frac{\cos 2\beta}{s^2} \right) K_0(sr) - \left[ \frac{2l}{ms} + v^2 \left( \frac{1}{ms^2} + \frac{1}{2ms^2} - \frac{s^3}{2s^3} \right) \right] \right\} \frac{1}{r^2} $$

$$ + \left[ \left( \frac{2l}{m} + v^2 \left( \frac{1}{s^2m} + \frac{1}{2s^2} - \frac{\cos 2\beta}{s^3} \right) \right] \frac{1}{r} + v^2 \left( \frac{s}{8s^2m} + \frac{\sin^2 2\beta}{s^3} + \frac{3}{4m} \right) \right] K_1(sr). \tag{25} $$

Near the origin, this gauge potential is reduced to a simple expression,

$$ V_{gauge}(r) \rightarrow -\frac{e^3}{(2\pi)^2} \left[ 1 - \frac{s}{m} - \frac{sl}{m} \right] \ln r, \tag{26} $$

which corresponds exactly to the limit of the MCS gauge potential, already established in Eq. $\mathbf{17}$. This is an expected result, once all the potentials $V_\Lambda, V_{TT}, V_{QQ}$ behave as a constant in the limit $r \rightarrow 0$. It is still interesting to observe that the gauge potential derived in the case of a purely timelike background (see Ref. $\mathbf{4}$) also presents this exact dependence, which shows that all background induced corrections are negligible in the proximity of the origin for both time and spacelike backgrounds. Far from the origin the Bessel functions decay exponentially, so that the gauge potential is ruled by the $1/r^2$ terms, which remain as dominant. So, one has:

$$ V_{gauge}(r) \rightarrow -\frac{e^3}{(2\pi)^2} \left[ \frac{2l}{ms} + v^2 \left( \frac{1}{2ms^2} - \frac{1}{ms^3} - \frac{\cos 2\beta}{s^4} \right) \right] \frac{1}{r^2}. $$

This is also similar to the asymptotic behavior of the pure MCS potential, $-(2l/ms)r^{-2}$, supplemented by background corrections, which however do not modify the $1/r^2$ physical behavior. Such analysis indicates that the gauge potential is always attractive in the limit $r \rightarrow \infty$, since one relies on the approximation $s^2 >> v^2$. The attractiveness of this potential near the origin depends on the sign of the coefficient $(1 - s/m - sl/m)$ in much the same way as it occurs with the pure MCS potential: it will be attractive for $s > m/(1 + l)$ or repulsive for $s < m/(1 + l)$. As it is reasonable to suppose that $m >> s$, once the rest mass of the electron represents a large energy threshold before the low-energy excitations usually observed in condensed matter systems, it should hold the condition $s < m/(1 + l)$, compatible with a repulsive gauge potential. Since the potential is always attractive far from the origin, there must exist a region in which the potential is negative (a well region) even in the case $s < m/(1 + l)$. This general behavior is attested in Fig. 1.

The graphics below exhibits a simultaneous plot for the gauge potential expression and for the pure MCS potential, given respectively by Eqs. $\mathbf{16}$ and $\mathbf{20}$. 
FIG. 1: Plot of the pure MCS potential (box dotted line) × plot of the gauge potential (continuous line) for the following parameter values: $s = 20$, $v = 5$, $\beta = \pi/2$, $m_e = 5.10^5$.

Such illustration confirms the equality inherent to the behavior near and away the origin, at the same time it demonstrates that the presence of the background may turn this potential attractive at some region. However, this result is not definitive once it is known that one should address with care the low-energy potential in order to avoid a misleading interpretation. As discussed in literature (see Hagen and Dobroliubov [7]), in concerning a nonperturbative calculation one must consider not only the centrifugal barrier term $(l^2/m_e r^2)$, but also the gauge invariant $A^2$-term coming from the Pauli equation, 

$$\left[\left(\overrightarrow{p} - e\overrightarrow{A}\right)^2/m_e + e\phi(r) - \frac{\overrightarrow{A} \cdot \overrightarrow{e}}{m_e}\right]\Psi(r, \phi) = E\Psi(r, \phi).$$

The centrifugal barrier term is generated by the action of the Laplacian operator, 

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2}\right]$$

on the total wavefunction $\Psi(r, \phi) = R_{nl}(r)e^{i\phi l}$; on the other hand, the $A^2$-term is essential to ensure the gauge invariance in the nonrelativistic domain. As this term does not appear in the context of a nonperturbative low-energy evaluation, for the same is associated with two-photon exchange processes (see Hagen and Dobroliubov [7]), it must be suitably added up the low-energy potential in order to assure the gauge invariance. In the presence of these two terms, the pure MCS potential reveals to be really repulsive instead of attractive. Hence, to correctly analyze the low-energy behavior of the gauge potential, it is necessary to add up the centrifugal barrier and the $\mathbf{A} \cdot \mathbf{A}$ term to the gauge potential previously obtained, leading to the following effective potential:

$$V_{\text{eff}}(r) = V_{\text{gauge}}(r) + \frac{l^2}{m_e r^2} + \left(\frac{e^2}{m_e}\right)\overrightarrow{A} \cdot \overrightarrow{A}$$

(27)

In order to proceed with this analysis, it is necessary to know the expression for the vector potential, which was not determined in Ref. [6]. This potential may be obtained solving a system of two coupled differential equations read off from Ref. [4], namely: 

$$\nabla^2(\nabla^2 - s^2)\overrightarrow{A} = s\overrightarrow{\nabla} \cdot \rho - s[\nabla(\overrightarrow{\nabla} \times \nabla \phi)]^*, \nabla^2 \phi + (1/s) (\overrightarrow{\nabla} \times \overrightarrow{\nabla})(\overrightarrow{\nabla} \times \overrightarrow{A}) = 0.$$

We proceed decoupling them, yielding the following equation for the vector potential: 

$$[\nabla^2(\nabla^2 - s^2) - (\nabla^* \cdot \overrightarrow{\nabla})(\nabla^* \cdot \overrightarrow{\nabla})]\overrightarrow{A} = s\overrightarrow{\nabla} \cdot \rho.$$ 

The solution for this equation (by the usual methods) leads to an approximated expression in first order in $v^2/s^2$:

$$\overrightarrow{A}(r) = \frac{e}{(2\pi)}\left\{-\frac{1}{sr}(1 - v^2/s^2 \sin^2 \beta - v^2 \cos 2\beta/2s^2) + (1 - v^2/s^2 \sin^2 \beta + v^2 \cos 2\beta/2s^2)K_1(sr)\right\} + \frac{2v^2 \cos 2\beta}{s^3 \rho} K_0(sr) - \frac{4v^2 \cos 2\beta}{s^5 \rho^3} (1 - rK_1(sr)) - \frac{v^2 \sin^2 \beta}{2s} rK_0(sr)\right\}.$$
One should now compare the gauge potential (25) with the effective potential, given by Eq. (27). In this way, one performs a graphical analysis of these two functions for small and large electron mass, as it is shown below:

![Plot of the gauge potential (dotted line) × effective potential (continuous line) for two set of parameters with distinct mass values.](image)

**FIG. 2:** Plot of the gauge potential (dotted line) × effective potential (continuous line) for two sets of parameters with distinct mass values: \((s = 20, v = 5, m = 50, \beta = \pi/2, L = 1)\) and \((s = 20, v = 5, m = 5 \times 10^5, \beta = \pi/2, L = 1)\).

For a large mass value \((m_e = 5 \times 10^5)\), one observes that the effective potential does not differ from the gauge potential (circle dotted curve), so that both graphics result perfectly overlapped. This fact reveals that the terms \(l^2/m_e r^2\), \(A^2/m_e\) are not decisive to alter the behavior of the gauge potential in the regime of large mass \((m_e/s \approx 10^5)\). On the other hand, for a small mass parameter \((m_e/s \approx 1)\), one notes that the gauge potential (box dotted curve) may differ drastically from the effective potential (continuous solitary curve). Therefore, in the regime of small mass the low-energy potential has to be replaced by the effective one in order to yield the gauge invariant correct behavior, requirement not necessary in the regime of large mass.

Another point that deserves to be analyzed concerns the influence of the background direction on the solutions. The graphics in Fig. 3 presents three simultaneous plots of the gauge potential for different values of the angle \(\beta\):

Such an illustration reflects the anisotropy of the system: depending on the value of \(\beta\), the potential may become totally repulsive, or exhibit a region in which is attractive. The interest in such an effect is related to its possible connection with the anisotropic parameter of order of high-\(T_c\) superconductors. An interaction potential whose intensity varies with a fixed direction indeed leads to an anisotropic wavefunction, which certainly requires additional investigation.

As a final comment, one should remark that the real potential corresponding to the total \(e^-e^-\) interaction comprises the gauge and the scalar contributions:

\[
V_{total}(r) = V_{scalar} + V_{gauge}.
\]

The character attractive or repulsive of this total potential arises from the combination of these two expressions for each radial region. Near the origin, for instance, the total interaction goes as:

\[
V_{total}(r) \rightarrow \frac{1}{(2\pi)} \left\{ -e_3^2 \left[ 1 - \frac{s}{m} - \frac{sl}{m} \right] + y^2 \left[ 1 - \frac{e^2}{2s^2} (1 + \sin^2 \beta) \right] \right\} \ln r.
\]

In the regime of large mass, the total interaction will be attractive near the origin whenever the phonic constant \(y^2\) overcomes the 2-dimensional U(1) coupling, \(e_3^2\) (or repulsive for \(y^2 < e_3^2\)). Far from the origin, the total potential exhibits the same logarithmic behavior stated in Eq. (11). It should be noted that this asymptotic behavior will change solely in the case in which a new mass parameter is introduced in, as it occurs when a
spontaneous symmetry breaking takes place. This is mentioned with more details in the Final Remarks. By adjusting the value of the phonic constant, $y$, one can certainly conclude that the total potential may always be negative at some region regardless the character of the gauge interaction, which is a relevant result concerning the possibility of obtaining $e^-e^-$ bound states in the framework of this particular model.

IV. FINAL REMARKS

In this work, one has considered the Möller scattering in a planar Maxwell-Chern-Simons electrodynamics incorporating a Lorentz-violating purely spacelike background. The interaction potential was calculated as the Fourier transform of the scattering amplitude (Born approximation) carried out in the non-relativistic limit. The interaction potential exhibits two distinct contributions: the scalar (stemming form the Yukawa exchange) and the gauge one (mediated by the MCS-Proca gauge field). The scalar Yukawa interaction turns out to be logarithmically attractive and repulsive near and far from the origin, respectively, in much the same way as verified in the purely timelike case. As for the gauge interaction, it is composed by a pure MCS potential corrected by background-depending contributions, which are able to induce physical interesting modifications despite the smallness of the background before the topological mass ($v^2/s^2 << 1$). Near and far from the origin, this gauge potential goes like the pure MCS counterpart, so that the observed alterations appear in the intermediary radial region. Namely, it is verified gauge potential becomes attractive for some values of the parameters. Such attractiveness remains even in the presence of the centrifugal barrier and gauge invariant $A \cdot A$ term. Besides the possibility of having a gauge interaction attractive, it should be mentioned that the total interaction (scalar plus gauge potential) may always result attractive provided a fine adjust of the coupling constants values ($y,e_3$) is realized. This fact indeed constitutes a promising result in connection with the possibility of obtaining the formation of Cooper pairs. It was also reported that in the regime of a large mass ($m_e/s \simeq 10^5$), the effective low-energy gauge invariant potential becomes equal to the gauge potential with a high precision, whereas in the regime of small mass ($m_e/s \simeq 1$) these two expressions become sensibly different.

The real possibility of obtaining Cooper pairs may be checked by means of a quantum-mechanical numerical analysis of the non-relativistic interaction potential here derived. Such potential should be introduced in the Schrödinger equation, whose numerical solution will provide the corresponding $e^-e^-$ binding energies for each set of parameter values stipulated. One should remark that the values must be chosen in accordance with the
usual scale of low-energy excitations in a condensed matter system. This analysis may be performed for the potentials obtained both in the case of a purely timelike and spacelike background, having in mind also the issue of the anisotropy of the resulting parameter of order.

One must now comment on the validity of the approximation which has been here adopted. At first sight, the higher order terms (in \(v^2\)) are always negligible before the first order ones. Indeed, this is true for terms that decay quickly at large distances. Near the origin, although, it might occur that a high order term (in \(v^2\)) come to increase with \(r\) more rapidly, overcoming a first order term, fact which really is related to its radial dependence in the limit \(r \to 0\). Such a behavior would be observed if a second order term (in \(v^2\)) had a more pronounced power in \((1/r)\) than the first order one. According to all evaluations carried out in second order, this fact was not observed, which confirms the validity of the approximation adopted as well as the outcomes obtained in this work.

The absence of screening, first observed in Refs. [6], [11], is here manifest only in the scalar potential expression by means of the asymptotic logarithmic term, once the gauge sector revealed an asymptotic behavior much different \((\sim 1/r^2)\). Some usual planar models, in \((1+2)\) dimensions, are known for exhibiting a confining (logarithmic) potential as representation of the gauge interaction, such behavior however does not reflect a convenient physical interaction, since it increases with distance. To represent a physical interaction, it may be changed to a condensating potential, which may be attained when the model is properly supplemented by new parameters of mass. The consideration of the Higgs mechanism is a suitable tool able to provide a Proca mass to the gauge field and to induce an efficient screening of the corresponding field strengths and solutions, bypassing this difficulty. In a recent work [12], it was accomplished the dimensional reduction of an Abelian-Higgs Lorentz-violating model endowed with the CFJ term, resulting in a planar Maxwell-Chern-Simons-Proca electrodynamics coupled to a massive Klein-Gordon field \((\phi)\), with a scalar Higgs sector. This resulting Higgs-Lorentz-violating planar model has been analyzed in some theoretical directions, revealing new possibilities and outcomes. The classical solutions for field strengths \((E, B)\) and four-potential \((A^0, A)\) were carried out for a static point-like charge, yielding interesting deviations in relation to the pure MCS-Proca electrodynamics [12]. A particular feature of this model is the presence of totally screened modes (all its physical excitations are non-screened). Furthermore, one can show that it provides stable charged vortex configurations able to bring about the associated Aharonov-Casher effect [14]. The consideration of the Möller scattering in this framework [15] will certainly lead to an entirely shielded interaction potential, with the logarithmic term being replaced by \(K_0\), \(K_1\) functions.

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