Rotating and vibrating tetrahedrons in heavy nuclei

Andreas Heusler
Gustav-Kirchhoff-Strasse 7/1, 6120 Heidelberg, Germany
E-mail: A.Heusler@mpi-hd.mpg.de

Abstract.

In the heavy nucleus $^{208}$Pb the $3^-$ yrast and $4^+$ yrast states ($E_x = 2615$ and $4324$ keV) are described by a rotating tetrahedron and eight more states at $4 < E_x < 8$ MeV by rotating and vibrating tetrahedrons. The stiffness in three vibration modes is hundred times higher than for corresponding states in $^{16}$O by comparing the excitation energies from the prediction by the algebraic cluster model [Nucl. Phys. A 957 (2017) 154] and the splitting of a $2^+$ doublet predicted to be degenerate.

1. Introduction

The lowest excited state in the heavy nucleus $^{208}$Pb has spin $3^-$ [1, 2]. It is described by the shell model with difficulty and was assumed to be a collective octupole vibration [3]. The $2^+$ yrast and $4^+$ yrast states were similarly assumed to be collective excitations. All three states are strongly excited by electromagnetic probes [1].

For nearly fifty years a search for double octupole vibrations was pursued. Indeed a $0^+$ state in $^{208}$Pb was discovered [1] at exactly twice the excitation energy of the $3^-$ yrast state. Yet among the lowest 150 states identified with high reliability [4] the other members of the double octupole multiplet with spins $4^+$ and $6^+$ were not found at excitation energies $E_x < 6.2$ MeV.

A perfect tetrahedron built from fermions may rotate and in addition vibrate in the dilation, flattening, and torsion mode as deduced by Wheeler [5]. The Pauli exclusion principle together with the symmetry relations derived from group theory [6] predicts the leading members of the

| Nucleus | $I^+$ | $(v_1 v_2 v_3)$ | $E_x$ [Eq. (1)] | $[\text{W.u.}]$ | $B(E4)/B(E3)$ |
|---------|-------|-----------------|-----------------|-----------------|----------------|
| $^{16}$O | 3$^-$ | 000             | 6130 $\kappa_r = 511$ $\kappa_r/\omega_3 = 0.21$ | $B(E3) = 13.5(7)$ |                |
| $^{16}$O | 0$^+$ | 100             | 6049 $\omega_1 = 4033$ $\omega_1/\omega_3 = 1.65$ |                |                |
| $^{16}$O | 2$^+$ | 010             | 6917 $\omega_2 = 1926$ $\omega_2/\omega_3 = 0.79$ | $B(E2) = 3.1(1)$ | 0.23           |
| $^{16}$O | 1$^-$ | 001             | 7117 $\omega_3 = 2438$ $\omega_3/\omega_3 = 1$ | $B(E1) = 0.00035(2)$ | $3 \cdot 10^{-5}$ |
| $^{208}$Pb | 3$^-$ | 000             | 2615 $\kappa_r = 218$ $\kappa_r/\omega_3 = 0.12$ | $B(E3) = 33.8(6)$ |                |
| $^{208}$Pb | 0$^+$ | 100             | 5241 $\omega_1 = 3494$ $\omega_1/\omega_3 = 1.98$ |                |                |
| $^{208}$Pb | 2$^+$ | 010             | 4086 $\omega_2 = 1389$ $\omega_2/\omega_3 = 0.79$ | $B(E2) = 8.4(5)$ | 0.25           |
| $^{208}$Pb | 1$^-$ | 001             | 4842 $\omega_3 = 1762$ $\omega_3/\omega_3 = 1$ | $B(E1) = 0.021(3)$ | $6 \cdot 10^{-4}$ |
rotation band to be the $0^+$ ground state, a $3^-$ state, a $4^+$ state, and a $6^\pm$ doublet. The spin sequence in the dilation mode is the same as for the pure rotation. The leading members of the flattening band are a $2^\pm$ doublet and a $4^\pm$ doublet. The leading members of the torsion band are a $1^-$ state and a $2^+$ state followed by a $3^\pm$ doublet.

Figure 1. Level scheme of states with rotating and vibrating tetrahedrons (left) for $^{16}$O [7], (right) for $^{208}$Pb. The values $v_1, v_2, v_3$ from Eq. (1) are given at bottom for each of the four rotating and vibrating bands. The neutron threshold $S(n)$ is used to adapt the two scales for the excitation energy. The splitting of the $2^\pm$ (010) doublet is marked by a dotted circle.

The excitation energies are calculated [7] with four parameters $\kappa_r, \omega_1, \omega_2, \omega_3$,

$$
E_x^{\text{calc}}(J, v_1, v_2, v_3) = E_x^{\text{rot}}(J) + E_x^{\text{vib}}(v_1, v_2, v_3),
$$

$$
E_x^{\text{rot}}(J) = \kappa_r J(J+1),
$$

$$
E_x^{\text{vib}}(v_1, v_2, v_3) = \omega_1 \left(v_1 + \frac{1-\delta_{0v_1}}{2}\right) + \omega_2 \left(v_2 + \frac{2-2\delta_{0v_2}}{2}\right) + \omega_3 \left(v_3 + \frac{3-3\delta_{0v_3}}{2}\right). \quad (1)
$$

Here $\delta$ is the Kronecker symbol and $J$ the angular momentum. $v_1, v_2, v_3$ count the number of phonons for the fundamental vibrations; they are lateron shortly written as $(v_1v_2v_3)$. 
Figure 2. Spectra of (top) $^{208}$Pb$(d, d')$, (middle) $^{208}$Pb$(p, p')$ – on $g_{7/2}$ resonance in $^{209}$Bi ($E^{res} = 17.43$ MeV), (bottom) $^{208}$Pb$(p, p')$ – on $g_{9/2}$ resonance ($E^{res} = 14.92$ MeV). The 4142 $2^-$ state was first observed by Wagner et al [9]. The 4230 $2^-$ yrare state is described by the one-particle one-hole configuration $g_{9/2}f_{5/2}$ [8].

2. Identification

The ratio of the excitation energies for the $3^-$ and $4^+$ yrast states in $^{208}$Pb with $E_x(4^-)/E_x(3^+)$ = 1.6538 deviates from the predicted ratio $5/3$ [Eq. (1)] by less than 1%. Therefore the $3^-$ and $4^+$ yrast states are interpreted as the first excited states in the rotation band. The large electromagnetic moments confirm the interpretation. The ratio $B(E3, ^{16}O)/B(E3, ^{208}Pb)$ = 0.40 ± 0.02 agrees within the uncertainties with the ratio of the moments of inertia as given by the ratio of the parameters $\kappa_r$ of the excitation energies of the $3^-$ yrast state, $\kappa_r(^{16}O)/\kappa_r(^{208}Pb)$ = 0.426 (Fig. 1). The ratios $B(E4)/B(E3)$ for $^{16}O$ and $^{208}Pb$ are similar, too (Table 1).

The 4086 $2^+$ and 4842 $1^-$ states are interpreted as the heads of the flattening and torsion bands. The failure of all shell model calculations to describe these states is thus solved.
The search for double octupole vibrations was based on the misleading interpretation of the 2615 3− state as a vibration; it is now described as a rotating tetrahedron. Therefore the 5241 0+ state previously described as a “double octupole” state is identified with the head of the dilation mode.

The comparison of the 1−, 3−, 2+, 4+ yrast states and the 5241 0+ state in 208Pb to the corresponding states in 16O corroborates the interpretation by rotating and vibrating tetrahedrons (Fig. 1, Table 1). The relative values of the parameters for the three vibration modes are similar for 16O and 208Pb, the ratio ω3/ω2 is even exactly the same. The difference of the ratio κω3 by a factor two should be compared to the ratio 13 of the masses. It shows that the nucleus 208Pb is more compact than 16O.

The knowledge of the four parameters allows to predict five more tetrahedral states in 208Pb by Eq. (1), see Fig. 1 and Table 2. The 7137 4− and 7020 3− states are known from 207Pb(d, p) and 207Pb(d, pγ) [1]. (The spin 1− for the 7020 state is excluded by the re-interpretation of the few data points for 207Pb(d, p) [10].) The 5715 2+ state was previously counted as a member of the double octupole multiplet which is shown to be a miscomprehension. The 7838 3− state is known as a resonance in the neutron capture on 207Pb [11]. It has one of the biggest cross sections among all resonances up to 0.5 MeV above the neutron threshold S(n) = 7368 keV.

The 4142 2− state was observed by 208Pb(p, p′) with E_p = 35 MeV [9]. By the criteria set up for a firm identification of a nuclear state [4] this level was assumed to be spurious because no other data supported the observation. Recent 208Pb(d, d′) spectra confirm the existence of the 4842 2− state and the ongoing reanalysis of the 208Pb(p, p′) data taken in 1968-1969 [8] adds confidence (Fig. 2).

The ratio of the excitation energies for the 3− and 4+ yrast states in 208Pb,

\[ R_{4/3} = \frac{E_x(4^+)}{E_x(3^-)} = \frac{5}{3} \times 0.9923, \]  

(2)

deviates from the prediction for a rotating tetrahedron by less than 1%.

### Table 2. Comparison of states in 16O and 208Pb.

| (v1v2v3) | Jπ | Nucleus | E_x keV | E_x(calc) keV | δE_x keV | δE_x/E_x 10^-4 | Nucleus | E_x keV | E_x(calc)[?] keV | δE_x/E_x 10^-4 |
|----------|-----|---------|---------|--------------|---------|-----------------|---------|---------|-----------------|-----------------|
| (000) 0^+ | 208Pb | 2615 | a | 16O | 0 | 0 |
| (000) 3^- | 208Pb | 4324 | 4358 | -34 | -80 | 16O | 10356 | 10220 | -130 |
| (100) 0^+ | 208Pb | 5241 | a | 16O | 6049 | 6050 |
| (100) 3^- | 208Pb | 7838 | 7856 | -18 | -23 | 16O | 11600 | 10970 | -570 |
| (010) 2^+ | 208Pb | 7137 | 7136 | +1 | +2 | 16O | 11907 | 11690 | -430 |
| (001) 1^- | 208Pb | 5715 | 5713 | +2 | +3 | 16O | 7117 | 6850 | +400 |
| (001) 2^+ | 208Pb | 7021 | 7020 | -1 | -1 | 16O | 9844 | 8460 | +1650 |
| (001) 3^- | 208Pb | 7137 | 7136 | +1 | +2 | 16O | 1080 | 10870 | -170 |

(a) The excitation energy defines the parameters κr, ω1, ω2, ω3 by Eq. (1).
3. Conclusion

Rotating and vibrating tetrahedrons are identified in the doubly magic nucleus \(^{208}\text{Pb}\). By comparison to \(^{16}\text{O}\) the tetrahedron in \(^{208}\text{Pb}\) is stiffer by a factor hundred if the relative deviation of the excitation energies from the predictions by the model calculation [Eq. (1)] is considered (Table 2).

The dilation mode head is predicted to be a degenerate \(2^\pm\) doublet. Indeed close to the 4086 \(2^+\) yrast state in \(^{208}\text{Pb}\), the 4142 state first observed by Wagner et al [9] is verified (Fig. 2). The low splitting of 0.2\% of the \(2^\pm\) doublet in \(^{208}\text{Pb}\) is explained by the stiffness of the tetrahedron. In contrast, the separation of the \(2^-\) state from the \(2^+\) state in \(^{16}\text{O}\) by 30\% indicates a rather floppy tetrahedron.

Data for other heavy nuclei but \(^{16}\text{O}\) and \(^{208}\text{Pb}\) do not allow to clearly identify rotating and vibrating tetrahedrons. The knowledge of excitation energies, spin, and parity is often rather limited [12]. However a low-lying \(3^-\) state with a large B(E3) value is often found in heavy nuclei [13].

Acknowledgments

Thanks are given to P. von Brentano for encouragement during more than 50 years.

References

[1] Martin M J 2007 Nucl. Data Sheets \textbf{108} 1583
[2] Elliott L G, Graham R L, Walker J, Wolfson J L 1954 Phys. Rev. \textbf{93} 356
[3] Hamamoto I 1970 Nucl. Phys. A \textbf{155} 362
[4] Heusler A, Jolos R V, Faestermann T, Hertenberger R, Wirth H-F, von Brentano P 2016 Phys. Rev. C \textbf{93} 054321
[5] Wheeler J A 1937 Phys. Rev. \textbf{52} 1083
[6] Tisza L 1933 Z. Phys. \textbf{82} 48
[7] Bijker R and Iachello F 2017 Nucl. Phys. A \textbf{957} 154
[8] Heusler A, Glöckner H-J, Grosse E, Moore C F, Solf J, Brentano P von 2014 Eur. Phys. J. A \textbf{50} 92
[9] Wagner W T, Crawley G M, Hammerstein G R, McManus H 1975 Phys. Rev. C \textbf{12} 757
[10] Valnion B D 1998 PhD Thesis Universität München (Herbert Utz Verlag, München)
[11] Horen D J, Auchampaugh G F, Harvey J A 1978 Phys. Lett. B \textbf{79} 39
[12] National Nuclear Data Center, Brookhaven 2017 Evaluated Nuclear Structure Data File http://ie.lbl.gov/ensdf/.
[13] Spear R H 1989 At. Data Nucl. Data Tables \textbf{42} 55