Quantum gravitational optics: the induced phase

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Abstract
The geometrical approximation of the extended Maxwell equation in curved spacetime incorporating interactions induced by the vacuum polarization effects is considered. Taking into account these QED interactions and employing the analogy between the eikonal equation in geometrical optics and the Hamilton–Jacobi equation for the particle motion, we study the modified structure of the new wave vector. There is a complicated, local induced phase which is believed to be responsible for the modification of the classical picture of a light ray. However, it is not clear whether it is permissible to obtain the main features of quantum gravitational optics through the study of this induced phase or not. We discuss this ambiguity after reviewing the initial principles in conventional and modified geometrical optics and compare the results.

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1. Introduction

Geometrical optics (GO) has proved to be a useful approximation for the semiclassical study of the photonic equation of motion, within the small wavelength limit. In fact the similarity between geometrical optics and particle dynamics has been the guiding principle for developing the semiclassical limit of particle motion, determined by the Hamilton–Jacobi equation. The phase (or eikonal) in GO plays the same role as the action of the particle in mechanics [1]. Application of GO to the electromagnetic wave propagation in a curved spacetime dates back to the studies initiated by Sachs [2]. Considering the photon as a quantum field and incorporating the coupling of the background curvature to the corresponding QED vacuum polarization effects modify the corresponding action. Examining the reflection of this change on the GO formalism leads to the introduction of a correction term (in eikonal equation and) to the photon wave vector. Vacuum polarization is an essential ingredient of QED whose contribution leads to an astonishingly precise agreement between the predicted and observed values of the electron magnetic moment and Lamb shift. Looking for its implications in the
context of the so-called quantum gravitational optics (QGO) has proved to be quite interesting and fruitful. It is found that a wide range of phenomena such as the polarization-dependent propagation of photons as well as superluminal photon velocities become possible. These phenomena have been investigated in different gravitational backgrounds [3–7].

When the background is considered to be the exact pp-wave solution of the Einstein equation, the light propagation is reduced to an exactly integrable problem and there is no need to use an approximate condition [8]. In general, however, the wave solution is only given through the known approximate methods. Looking differently at the above-mentioned phenomena through the formal GO solution and discussing the resulting ambiguities are the main concern in the present paper. The above-mentioned vacuum polarization effects can be interpreted through the new wave vector introduced by the new null cone condition. In the conventional GO we have no choice but to interpret it in terms of a gradient of a new phase, composed of the old phase plus a new term. Generally, this new term is a complicated function of the local parameters appearing in the theory and therefore one could only identify its general properties. However, it is not clear whether it is valid to mix the new term with the old expression given by the geometric optics, though this is the way that one actually makes a new wave vector. To clarify this ambiguity we apply a modification to the initial principles of GO, i.e. the way the complex amplitude and the phase are separated. This procedure has been presented in [11] and applied to formulate the interaction between an electromagnetic vector potential with a gauge field in a smooth inhomogeneous isotropic medium. In a modified geometric optics framework, we can absorb the new added term in the wave vector expression into the amplitude rather than mixing it with the gradient of the old phase to produce a new phase. This is allowed if it is the gradient of a local phase as in the present case. So the wave vector does not change and as long as our observables depend on the phase, no observable physical effect can be detected. We see that choosing a criterion for defining a phase could have physical consequences which make the mentioned effects either revealed or neglected.

In the following section, we review the basic ideas of the GO solution and apply these to the equation of motion appearing in QGO. The interaction-induced phase is introduced and the resulting local effects will be listed. In section 3, we concentrate on the GO ambiguity in separating the phase and the amplitude which is the origin of the postulate brought up in a solution designated by modified GO. We also consider the nature of different interaction-induced phases. Some evidence supporting this postulate are discussed in the last section.

2. Geometrical optics approximation and quantum gravitational optics

As light propagates in a fixed curved background spacetime, its characteristics may be given by some unknown complicated function as the exact result to the Maxwell equation in the presence of gravity. Yet as for many other linear differential equations, the WKB theory provides a simple and general approximation method, consisting of exponentials of the form $A(x) e^{S(x)/\epsilon}$ [10], where $\epsilon$ is a small parameter in theory. Mathematically, if one exploits the WKB solution with the assumption of a locally plane and monochromatic wavelength of order $\epsilon$, the phase is imaginary and the amplitude is a slowly varying function and both depend explicitly on $\epsilon$. The dependence is usually described by expanding $A(x)$ and $S(x)$ as series in powers of $\epsilon$. The formal WKB expansion is represented after combining the two series into a single exponential form. Substituting this approximate solution into the original differential equation, one can determine the different coefficients of powers of $\epsilon$. Some of the terms in the expansion of $S(x)$ can be separated and identified as the complex amplitude in the form of a pre-exponent factor. This point about WKB is our main concern in the following section. If we retain only the most rapidly varying term in the phase and truncate the series, then we
make the GO approximation. The GO solution for the electromagnetic vector potential is in the form of
\[ A^\mu = \text{Re} \{ A^\mu \} e^{i\theta/\epsilon}, \]
and results in the following laws [9]:

- light rays are null geodesics,
- the polarization vector is perpendicular to the ray and is parallelly propagated along the ray.

In each small region of space, we can speak of a direction of propagation normal to a surface at all of whose points the phase of the wave is constant. We identify the wave vector as \( k_\mu = \partial_\mu \theta \), and the polarization vector is introduced through \( A^\mu = A^a_\mu \). The GO solution of the Maxwell equations implies that the wave vector \( k_\mu \) and the polarization vector \( a_\mu \) specified at one point are fixed along the entire ray by their propagation equation. Since both propagation equations are nothing but parallel transport laws, the conditions \( k^2 = 0, a^2 = -1 \) and \( k \cdot a = 0 \), once imposed on the vectors at one point, will be satisfied along the entire ray.

The effect of one-loop vacuum polarization on the Maxwell equations in a fixed curved background spacetime is represented by the following effective equation of motion derived by Drummond and Hathrell [3]:
\[ D_\mu F^{\mu\nu} + \frac{1}{m^2} \left( 2b R^\mu_\lambda D_\mu F^{\lambda\nu} + 4c R^{\mu\nu}_{\lambda\rho} D_\mu F^{\lambda\rho} \right) = 0. \]

Here, \( b = \frac{13}{360} \frac{\alpha}{\pi} \), \( c = -\frac{1}{360} \frac{\alpha}{\pi} \), in which \( \alpha \) is the fine structure constant, and \( m \) is the electron mass. There are some approximations under which this equation of motion was obtained. The first one is the low-frequency approximation in the sense that the derivation is only applicable to wavelengths \( \lambda > \lambda_c \). By this approximation, we ignore terms in the effective action involving higher order field derivatives. The second is a weak-field approximation for gravity which implicitly means that the typical curvature scale, \( L \), is much larger than the electron Compton wavelength, i.e., \( \lambda_c \ll L \).

Considering the one-loop vacuum polarization affecting the photon propagation in a fixed local background, the above view shifts slightly. The null cone and phase velocity modifications result as a direct consequence of assigning the GO solution to the modified Maxwell equations in a curved spacetime. The two-length-scale expansion is typical of quantum field theory calculation in a background field. The general form of the perturbative expansion in the coupling constant is \( f_0(\epsilon) + \alpha f_1(\epsilon) + \alpha^2 f_2(\epsilon) + \cdots \), where \( f_i(\epsilon) \) are functions of a dimensionless parameter \( \epsilon \) defined as the ratio of the physical length scale \( \lambda_c \) to the background scale \( L \), i.e., \( \frac{\lambda_c}{L} \). For the present calculation, an expansion of \( f_i(\epsilon) \) for small \( \epsilon \), like what is given in (1), is inserted in equation (2); in other words, only terms up to \( O(\alpha) \) are taken into account. In this notation, \( D_\mu F^{\mu\nu} \) can be written as
\[ D_\mu F^{\mu\nu} = \text{Re} \left\{ -\frac{1}{\epsilon^2} \left[ k^\lambda (A^\mu + \epsilon B^\mu + \cdots) - k^\mu k_\lambda (A^\nu + \epsilon B^\nu + \cdots) \right] \right. \]
\[ + \left. \frac{i}{\epsilon} k_\lambda [(A^\mu + \epsilon B^\mu + \cdots)^\nu = (A^\nu + \epsilon B^\nu + \cdots)^\mu] + \frac{i}{\epsilon} [k^\lambda (A^\mu + \epsilon B^\mu + \cdots) \lambda - k^\mu (A^\nu + \epsilon B^\nu + \cdots) \lambda] \right. \]
\[ - k^\mu \lambda \lambda (A^\nu + \epsilon B^\nu + \cdots) + \frac{i}{\epsilon} [k^\lambda (A^\mu + \epsilon B^\mu + \cdots) \lambda] - k^\mu (A^\nu + \epsilon B^\nu + \cdots) \lambda] \right. \]
\[ + (A^\mu + \epsilon B^\mu + \cdots)^\nu \lambda (A^\nu + \epsilon B^\nu + \cdots)^\mu \lambda \right\} e^{i\theta/\epsilon}. \]

After applying the expansion equation (3) to the equation of motion, the next step is to collect terms of order \( \frac{1}{\epsilon} \) and \( \frac{1}{\epsilon^2} \) (terms of order higher than \( \frac{1}{\epsilon} \) govern post-geometric corrections).
The leading term, \(O(\epsilon^2)\), results in the modified light cone equation:

\[
\begin{aligned}
k^2 A^\nu - k^\nu k_\mu A^\mu &+ \frac{1}{m^2} \left[ 2b R^\mu_{\lambda \nu} (-k^\nu k_\mu A^\lambda + k_\mu k^\lambda A^\nu) + 4c R^{\mu \nu}_{\lambda \rho} (-k_\mu k^\rho A^\lambda + k^\lambda k_\mu A^\rho) \right] = 0,
\end{aligned}
\]

which for transverse photons with the polarization vector normalized to unity reduces to the following equation [3]:

\[
\begin{aligned}
k^2 + 2b m^2 R^\lambda_{\mu \nu} k^\mu k^\nu &- 8c m^2 R^\lambda_{\mu \rho \nu \sigma} k^\mu a^\nu a^\rho = 0.
\end{aligned}
\]

The subleading term, \(O(\epsilon)\), in the equation of motion gives

\[
\begin{aligned}
-\frac{i}{\epsilon} \left\{ \left( -k^\nu k_\mu B^\mu + k^\mu k_\nu B^\nu \right) &+ \frac{1}{m^2} \left[ 2b R^\mu_{\lambda \nu} (-k^\nu k_\mu B^\lambda + k^\lambda k_\mu B^\nu) \
+ 4c R^{\mu \nu}_{\lambda \rho} (-k^\rho k_\mu B^\lambda + k^\lambda k_\mu B^\rho) \right] \right\} \\
+ \frac{1}{m^2} \left[ 2b R^\mu_{\lambda \nu} \left[ k_\mu (A^\lambda B^\nu - A^\nu B^\lambda) + k^\nu (A^\lambda - k^\lambda A^\nu) + k^\lambda (A^\nu - k^\nu A^\lambda) \right] \right] \\
+ 4c R^{\mu \nu}_{\lambda \rho} \left[ k_\mu (A^\lambda B^\nu - A^\nu B^\lambda) + (k^\nu A^\lambda - k^\lambda A^\nu) + (k^\lambda A^\nu - k^\nu A^\lambda) \right] = 0.
\end{aligned}
\]

For transverse photon and after employing equation (4), it transforms into

\[
\begin{aligned}
\nabla^\lambda A^\nu = \frac{1}{2} \left( k^\nu A^\mu_{\mu \nu} - k^\mu A^\nu_{\mu \nu} \right) &+ \frac{1}{m^2} \left[ 2b R^\mu_{\lambda \nu} \left[ k^\nu (A^\lambda - k^\lambda) + k^\mu (A^\nu - k^\nu) \right] \\
+ 4c R^{\mu \nu}_{\lambda \rho} \left[ (k^\mu A^\lambda) - k^\mu (A^\lambda) \right] \right].
\end{aligned}
\]

which determines the first-order variation of \(A^\mu = A^\mu_{0} + A^\mu\) as the vector amplitude along the ray. This is a new result which in the zeroth order guarantees the parallel transportation of the polarization vector along the ray [9].

Equation (5) is an effective light cone equation, representing the wave vector changes induced by QED interactions. It shows that at this level of approximation, the wave vector acquires an additional polarization-dependent component as follows:

\[
\begin{aligned}
k_\mu &\equiv k^{(0)}_{\mu} - \frac{1}{m^2} \left[ 2b R^\mu_{\lambda \nu} k^\lambda - (4c) R^\mu_{\lambda \rho \sigma} a^\lambda a^\rho k^\sigma \right] \\
&\equiv k^{(0)}_{\mu} + k^{(1)}_{\mu}.
\end{aligned}
\]

This means that the effect of vacuum polarization is to change the phase of the electromagnetic field according to

\[
\theta \rightarrow \theta + \Phi
\]

with the extra phase the photon acquires over the trajectory \(c\).

\[
\Phi = \int_c k^{(1)}_{\mu} \, dx^\mu.
\]

\(\Phi\) can be understood as a single-valued \textit{local} phase of order \(\alpha\), which like any other first-order correction term must be calculated along the zeroth-order approximation (i.e., with \(k = k^{(0)}\) and \(\alpha = a^{(0)}\)). This integral form of the phase is the typical form of the phases induced by the interactions, a good example of which is the phase acquired by the electron in the presence
of a topologically non-trivial electromagnetic field [16]. Following the definition of $k_{\mu}$ as a gradient satisfying $k_{\mu,v} = k_{v,\mu}$, one can obtain the photon trajectories corresponding to the new equation of motion (2) as
\[
\nabla_k v^v = k^\mu D^v k_\mu = \frac{1}{2} D^v k^2 = -\frac{1}{m^2} D^v [b R_{\mu\lambda k^\lambda k^\lambda} - (4c) R_{\mu\lambda\sigma\kappa} a^\lambda a^\kappa k^\mu k^\sigma].
\]

(11)

Here the bracket times $-\frac{1}{m^2}$ in the second line can be identified as $\nabla_k \Phi$, which is not generally zero. Note that in the framework of the conventional geometrical optics, the phase characteristics of the potential vector are now determined by the phase $\theta + \Phi$ and equation (5), whereas the vector amplitude $A^\mu = A \alpha^\mu$ variation and consequently the corrections to the polarization vector (of the zeroth order in $\epsilon$) specified by equation (7) are usually ignored. Therefore as long as we work with equation (11), the wave and the polarization vectors on the right-hand side are the zeroth-order (classical) quantities, and we will consider manifestations of the induced phase for a photon in a state of adiabatically invariant polarization vector.

The phase introduced in (8) also affects the propagation of a bundle of rays. For infinitesimally nearby geodesics in the family, the geodesic deviation equation relates the relative acceleration to the Riemann tensor. Let $\xi^\mu$ be the deviation vector normal to the wave vector which connects abreast rays, so that $\xi \cdot k = 0$ and $\nabla_k \xi = \nabla_k k$. The relative velocity of two infinitesimally nearby geodesics is given by $v^\mu = \nabla_k \xi^\mu$ and the relative acceleration is
\[
a^\mu \equiv \nabla_k \nabla_k \xi^\mu = R^\mu_{\nu\lambda\kappa} k^\lambda k^\kappa \xi^\nu.
\]

(12)

The role of the induced phase, $\Phi$, shows up in the study of the perturbative deformation of the bundle’s cross section where effective counterparts for optical scalars, namely effective expansion, shear and vorticity, are introduced in the study of Raychaudhuri equations in QGO [15]. In summary, the following general characteristics could be obtained from the above first-order QGO corrections in the context of GO approximation.

- The polarization degeneracy has been removed.
- Depending on the direction and polarization of photons, superluminal propagation becomes possible.
- The presence of superluminal photons and interactions violating the strong equivalence principle does not necessarily imply causality violation, although there is no reason to prove that causality violation does not occur.
- The photons with classical polarizations acquire equal phases but with different signs. This in turn gives rise to two opposite trajectories, both departed from the zero-order one.
- Due to the polarization sum rule, the sum of averaged velocity shifts for two physical polarizations is zero in Ricci flat spacetimes [12].
- Besides the classical parameters, some quantum corrections of $O(\alpha)$, depending on the change in the curvature along a geodesic, may affect the local parameters such as the focusing, twist and shearing of a bundle of rays and equally the associated Raychaudhuri equation [15].
- Both the wave vector and the polarization vector are transported along the ray according to equations (11) and (7), respectively. This means that any polarization is possible in quantum theory and the final observed state may not retain the transverse nature of the wave. However in all the above points variation of the polarization vector and other post-geometric corrections (of order $\epsilon^0, \epsilon^1, \ldots$) are neglected.

Further details of the above points can be found in [3–5, 12–14].

3 Mathematically, this happens provided that $\Phi$ is a well-defined local function or $k_{\mu}^{(1)}$ can be written as a local exact differential form.
3. QGO and the modified GO

We have outlined the quantum corrections to the propagation characteristics of a photon in a fixed gravitational background through the conventional GO approximation. The relative order of magnitude for these modifications is \( O\left(\frac{\alpha^2}{c^2}L^2\right) \), which is extremely small. We attributed corrections to the photon wave vector (or a phase) induced by the interactions. Multiplying equation (11) by \( k_\nu \), we get

\[
k_\nu \nabla k^\mu = \nabla_k \left[ -\frac{1}{m^2} (b R_{\mu\lambda} k^\mu k^\lambda - (4c) R_{\mu\lambda\sigma\tau} a^\lambda a^\sigma k^\mu k^\tau) \right] = O(\alpha^2).
\]

This means that

\[
k_\nu \nabla k^\mu = \nabla_k \nabla \Phi_1 \approx 0.
\]

This calculation shows that the first-order correction to the wave vector is orthogonal to its zeroth-order direction. The gradient of a polarization-dependent phase is a function of the local values of parameters, so it cannot exceed \( O(\alpha) \). In this interpretation the effects (velocity shift and trajectory splitting), listed in the previous section, are the results of this polarization-dependent phase deviation and follow immediately from the initial principles of GO. We recall that in the conventional GO, the phase characteristics of the wave vector are determined by the phase \( \theta \), and its evolution is governed by the null cone equation, whereas \( A^\mu \) specifies the amplitude and the polarization. Without any criterion for separating the phase and the amplitude, interactions could alter the phase up to the relevant order. But from the GO basic principles discussed in the last section, it is believed that in so far as the amplitude is a complex value, the separation of the phase and the amplitude is a matter of calculation. In light of that, in a modified version of GO suggested by Bliokh and Bliokh [11], the complex amplitude and the phase of the electromagnetic field are separated in the following manner:

\[
A^\mu = \hat{A}^\mu e^{\theta/\epsilon}, \quad \hat{A}^\mu = A a^\mu e^{\Phi/\epsilon}.
\]

(15)

The phase is separated into the local phase \( \Phi \) and the non-local phase \( \theta \). Then it is postulated that the wave vector is the gradient of the non-local part, \( k_\mu = \partial_\mu \theta \), determined by the entire path covered by the wave, so it is conceptually a non-local or integral quantity, in contrast to the amplitude which is a local quantity, and the polarization is specified by the amplitude eigenvector. In this postulate, the ambiguity in the separation is resolved with a gauge transformation

\[
\theta \rightarrow \theta - \Phi', \quad \Phi \rightarrow \Phi + \Phi'.
\]

(16)

Here \( \Phi' \) is a local phase. We will see that in spite of mathematical appearance, such transformations change the theoretical predictions. So one must be careful about applying different postulations. We believe that a correct postulate in QGO must lead to the true GO limit for the exact solution of equation (2) and the theory to be accepted or rejected, both depend on the experimental results\(^4\), so the physically observable quantities are the judges on different postulations being correct.

Before continuing, we would like to clarify the nature of different interaction-induced phases. These can be associated with the gradient of either of local or non-local phases. If the induced wave vector deviation has no projection along its zeroth-order component (transverse deviation), it is the gradient of a local phase. So it could be a part of the amplitude, under a gauge transformation, with a slight distortion of the phase front. It generally has a small value which does not exceed the particle wavelength. In view of the uncertainty relation, the

\(^4\) To be picky, any positive test result in each case must be interpreted as an illustration and not even a verification.
The wave vector is not a physically measurable quantity in this range. Interactions may induce longitudinal wave vector deviations if the induced phase is a non-local one. These deviations cannot change the direction of the normal to the phase front, and their integral after the transport along the ray does not vanish.

An example of the first group of deviations is that we encounter in the study of QED interactions in a curved background formulated as QGO through the Drummond–Hathrell [1] action or those in an arbitrary anisotropic (but homogeneous) electromagnetic field described by the Euler–Heisenberg action [12]. The second familiar group of phase deviations is that of the electron wave vector in the presence of a topologically non-trivial electromagnetic field (in the form of minimal substitution) leading to the spectacular quantum interference phenomena, known as the Bohm–Aharanov effect. At the heart of these groups of interactions lies a non-integrable phase \( \oint_c A_\mu \, dx^\mu \) which arises after the wavefunction transport around a closed path \( c \). In quantum mechanics there exists an analogous non-local topological phase, the so-called Berry phase, arising from the transport around a closed path in momentum space in the presence of a gauge field or in a smooth inhomogeneous medium.

So according to this modified version of GO,

- the induced phase can be attributed to the change of amplitude and
- the ray shifts would be related to the uncertainty in determining the ray trajectory within the scale of the theory.

So there is no observable physical effect in QGO as long as our observables depend on the ray trajectory and the phase (the integral of the wave vector on the ray). In other words, since the deviations are within the range of uncertainty principle the observability seems to be an ambiguous concept. Also if the classical theory respects the causality, there is no reason to doubt that the QED in curved spacetime remains a causal theory. In the following section we will discuss the evidence indicating that the modified GO must be taken seriously.

4. Discussion

The study of curvature-induced interactions of the QED photon goes back to Drummond and Hathrell [3]. This is believed to be the search basis for a series of strong equivalence principle violation research. In the present paper, we have also considered the characteristics of the QED theory in which the interactions of vacuum polarization effects with the curvature are taken into account. In this way, the photon is treated as a test particle, so its effect on the metric is assumed to be negligible. Any reliable theoretical predictions could be obtained after solving the problem equation (2) exactly. Without such a solution, we rely on the leading order approximation in geometric optics to obtain information about the photon wave vector. It turns out that interpretation of the results, at least to the first order in geometric optics approximation, depends to a large extent on the way the amplitude and the phase of the electromagnetic wave are separated. The ambiguity around the separation criterion in approximate solutions is the origin of the postulation brought up first in [11]. The nature of the phase to be local (non-integrable) or non-local (integrable) has already proved to be an important criterion according to which the separation could happen correctly in modified geometric optics, though the QED effects (the shift in phase velocity and ray trajectory) could be interpreted differently. Any postulation with the aim of resolving QGO ambiguity must result in the GO limit for exact solution of equation (2). Having particular theoretical predictions to be accepted or refuted, both depend on experimental results. A comparison

Note that we have \( k_\nu \partial_\nu \Phi = 0 \) as is mentioned in the paragraph below equation (14).
with an exact solution found in the given background may support one’s postulation. The desirable exact solution, however, must be expressed in terms of elementary functions with a well-defined geometric optics limit. The phase characteristics may resolve the problem in favor of either the conventional or the modified GO. Our difficulty in extracting the true QGO behavior from the only available exact solution [8] is that the corresponding equation of electrodynamics has been solved by an ansatz, in which the separation between the amplitude and an arbitrary function of the phase is performed from the beginning. So the evolution of each part is considered separately and this makes the judgment difficult.

Yet, there are some guiding points which convinces one to rely on the modified GO expectancies: firstly, in [6] it is shown that the cosmological constant, \( \Lambda \), and the topological structure parameter, \( k \), failed to enter the velocity shift; secondly, in [15] we have discussed the invariant quantities in the QGO which reduce to their classical counterparts in the limit of zero perturbation and therefore there remains no anomaly in the theory. The effective Raychaudhuri equation, also discussed in [15], has the general form of its classical version. Finally, as shown in (12) the geodesic deviation vector in the modified theory satisfies the same (Jacobi) equation as the classical deviation vector does. It seems that whenever we touch upon the evolution equations for the effective local quantities, there is no difference between them and their classical counterparts. This is not surprising, since \( \nabla_k \nabla_k \Phi = 0 \) and there are no basic changes, at least to this order of approximation. These facts support the non-integrable nature of the QGO phase. In the literature [14], the matter of observability is linked to the limiting condition \( aL^2 > \alpha \lambda^2 \lambda \), i.e., the length difference between paths corresponding to different polarizations, \( aL^2 \), should be larger than the photon wavelength, \( \lambda \). This length discrepancy is given by \( \delta s \approx \delta v \approx aL^2 \), where the essential assumption is that the first-order correction to the velocity must be along its zeroth-order direction, in contrast to the results mentioned in this paper. However we believe that due to the uncertainty relation and the non-integrability of the QGO phase, these interactions cannot be detected.

Acknowledgments

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References

[1] Landau L D and Lifshitz E M 1975 The Classical Theory of Fields (Oxford: Pergamon)
[2] Sachs R K 1961 Proc. R. Soc. A 264 309
[3] Drummond I T and Hathrell S J 1980 Phys. Rev. D 22 343
[4] Daniels R D and Shore G M 1994 Nucl. Phys. B 425 634
[5] Daniels R D and Shore G M 1996 Phys. Lett. B 367 75–83
[6] Cai R-G 1998 Nucl. Phys. B 524 639
[7] Ahmadi N, Khojeini-Moghaddam S and Nouri-Zonoz M 2008 J. Cosmol. Astropart. Phys. JCAP05(2008)015
[8] Balakin A B 1997 Class. Quantum Grav. 14 2881
[9] Misner C W, Thorne K S and Wheeler J A 1973 Gravitation (San Francisco, CA: Freeman)
[10] Bender C M and Orszag S A 1987 Advanced Mathematical Methods for Scientists and Engineers (New York: McGraw-Hill)
[11] Bliokh K Yu and Bliokh Yu P 2004 Phys. Rev. E 70 026605
[12] Shore G M 1996 Nucl. Phys. B 460 379
[13] Shore G M 2001 Nucl. Phys. B 605 455
[14] Shore G M 2003 Contemp. Phys. 44 503
[15] Ahmadi N and Nouri-Zonoz M 2006 Phys. Rev. D 74 44034
[16] Ryder L H 1996 Quantum Field Theory (Cambridge: Cambridge University Press)