We show the conclusions claimed in the manuscript arXiv:1202.5309v1 by Cuoco, Komatsu and Siegal-Gaskins (CKS) are not generally valid. The results in CKS are based on a number of simplifying assumptions regarding the source population below the detection threshold and the threshold flux itself, and do not apply to many physical models of the blazar population. Physical blazar population models that match the measured source counts above the observational threshold can account for $\sim 60\%$ of the diffuse gamma-ray background intensity between 1-10 GeV, while the assumptions in CKS limit the intensity to $\lesssim 30\%$. The shortcomings of the model considered in CKS arise from an over-simplified blazar source model. A number of the simplifying assumptions are unjustified, including: first, the adoption of an assumed power-law source-count distribution, $dN/dS$, to arbitrary low source fluxes, which is not exhibited in physical models of the blazar population; and, second, the lack of blazar spectral information in calculating the anisotropy of unresolved gamma-ray blazar emission. We also show that the calculation of the unresolved blazars’ anisotropy is very sensitive to the spectral distribution of the unresolved blazars through the adopted source resolution threshold value, and must be taken into account in an accurate anisotropy calculation.

INTRODUCTION

The contribution of unresolved blazars to the diffuse gamma-ray background (DGRB) has been of interest for some time (see Ref. [1], hereafter ABH, for a discussion). The recent manuscript by Cuoco, Komatsu, and Siegal-Gaskins [2] (hereafter CKS) has derived limits on the contribution of blazars to the DGRB from a combination of measurements of the DGRB anisotropy, source-count distribution, and intensity. Using a simplistic $dN/dS$ for the blazars, and neglecting any blazar spectral information, the CKS analysis concludes that blazars can contribute no more than $30\%$ of the DGRB intensity, independent of the measured angular correlation power in the DGRB. In this note, we show that the CKS limit on the blazar contribution to the DGRB intensity is not generally valid, and strictly the result of their chosen over-simplified model. Such a model neglects many crucial features of physically-motivated blazar models, e.g. Refs. [1, 3-7]. Importantly, using a physically-constrained source-count distribution above the threshold that is consistent with that measured by the Fermi-LAT collaboration [3] (hereafter FB10) and assumed by CKS, ABH find an intensity contribution to the DGRB between 1-10 GeV of approximately $60\%$, in direct contradiction to the general claim in CKS of a required $\lesssim 30\%$ contribution. In this note, we summarize the reasons for this discrepancy, which reside in a number of invalid assumptions in CKS.

BLAZAR FLUX SOURCE-COUNT DISTRIBUTION FUNCTION

The blazar source count distribution functions $dN/dS$ give the number of blazars expected at a particular flux $S$, defined in this note as

$$S = \int_{1 \text{ GeV}}^{10 \text{ GeV}} dE \frac{dN}{dE},$$

which is consistent with the flux $S$ defined in CKS, though $S$ can be defined in different energy bands. The Large Area Telescope (LAT) on the Fermi Gamma-Ray Space Telescope has measured the $dN/dS$ for blazars to be consistent with a broken power-law over the Fermi-
LAT sensitivity range in FB10. CKS assumes that the full blazar $dN/dS$ follows this faint-end single power-law, down to zero flux. They calculate the diffuse blazar intensity in the $1 - 10$ GeV band as

$$I = \int_{0}^{S_i} \frac{dN}{dS} S \, dS ,$$

and they determine the value of $S_i$, the “flux sensitivity threshold” below which all sources are undetected by the Fermi-LAT, by normalizing this intensity integral to the measured blazar intensity from FB10. This definition of $S_i$ neglects the strong spectral dependence of the Fermi-LAT point source sensitivity. CKS then calculates the Poisson term of the angular power spectrum for undetected blazars as

$$C_P = \int_{0}^{S_i} \frac{dN}{dS} S^2 \, dS ,$$

and compares this value to the measured $C_P$ in the DATA:CLEANED sample from Table II of Ref. [9]. Using the limits on $I$ and $C_P$, they conclude that “unresolved blazars account for only 30% of the IGRB intensity but 100% of the angular power.” However, the CKS analysis makes several simplifications which change these results greatly when examining physical blazar models. Importantly, CKS incorrectly use the same value of $S_i$ in calculating $C_P$, equation (3), as used in $I$, equation (2), even though the point source exclusion limit for the former is the 1FGL catalog, $TS = 25$, and that for the latter DGRB intensity is more conservative, $TS = 50$.

Unlike the broken power-law $dN/dS$ used by CKS, at low fluxes $dN/dS$ is expected to flatten rather than continuing to increase down to zero flux. The blazar model of ABH, for example, exhibits a flattening of source-count distribution at low fluxes, as shown in figure 1. The ABH model was determined by using a luminosity-dependent density evolution (LDDE) model of the gamma-ray luminosity function using the spectral energy distribution (SED) sequence for blazars, which was fit to the Fermi-LAT source-count distribution of FB10 and the Fermi-LAT-measured DGRB of Ref. [10] using the spectrally-dependent flux limit of sources. Rather than continuing to rise as a power-law for low fluxes, this source-count distribution flattens and provides less blazars at low flux than a simple power-law extrapolation would predict, as is shown in figure 1. Additionally, a power-law extrapolation down to zero flux of the type assumed in CKS is mathematically inconsistent, giving a divergent number of blazars within our cosmological horizon, $N = \int_{0}^{S_i} (dN/dS) dS$, while the physical source-count distribution of ABH does not.

Aside from a change in the number of sources at low flux, the definition of the flux itself leads to a $dN/dS$ at low fluxes which is different from the broken power-law used by CKS. Consider the intensity coming from near the threshold flux $S_i$: $S_1 < S_i < S_2$

$$I_{\text{band}} = \int_{S_1}^{S_2} \frac{dN}{dS} S \, dS .$$

Let us consider the sources’ fluxes to be from a population that changes its spectrum and/or number density from a distribution $N_1$ to $N_2$ near the threshold flux,

$$S = \left\{ \begin{array}{ll}
\int_{E_1}^{E_2} (dN_1/dE) \, dE & \text{for } S \leq S_i \\
\int_{E_1}^{E_2} (dN_2/dE) \, dE & \text{for } S > S_i 
\end{array} \right. .$$

The flux from the sources is spectrally dependent, and could be, e.g., two different average power law spectral distributions

$$\frac{dN_i}{dE} = N_0 \left( \frac{E}{E_0} \right)^{-\Gamma_i} .$$

The intensity calculation therefore changes its form from just below to just above the threshold, and the former lacks any dependence on the latter,

$$I_{\text{band}} = \int_{S_1}^{S_i} \frac{dN_1}{dS} S \, dS + \int_{S_i}^{S_2} \frac{dN_2}{dS} S \, dS .$$

Therefore the contribution from below the threshold has, in general, complete independence from that above the threshold, and it should be clear that the contribution from just below the threshold can be very different from just above the threshold. Therefore, all predictive power of equation (2) (and therefore in CKS) is lost. The same problem can be illustrated with equation (3) which was used by CKS to calculate $C_P$.

Equation (2) is only valid when blindly extrapolating a power-law for the source population to fluxes below the observable flux threshold, and furthermore requires the assumption of an invariant source population spectrum below that threshold. Though neither of these assumptions is valid in general blazar source population models, CKS has adopted both. What is properly needed in understanding the nature of the source population below the threshold flux is a physical picture of the blazar population. One such physical model is provided by the LDDE plus SED sequence model in ABH.

**DIFFUSE INTENSITY, ANGULAR CORRELATION, AND THE THRESHOLD FLUX**

In addition to the simplistic assumptions used in extrapolating $dN/dS$ below the Fermi-LAT sensitivity
threshold, CKS mishandles the calculation of the threshold itself. In CKS, the threshold flux \( S_t \) is calculated using equation 2 and normalizing \( I \) to the measured intensity reported in FB10. The calculation of \( C_P \) is then made using the \( S_t \) calculated from \( I \). However, \( C_P \) is highly sensitive to barely-unresolved sources near the threshold, so the calculation of \( C_P \) is strongly dependent on the chosen value of \( S_t \). A factor of two change in \( S_t \) only changes \( I \) by \( \sim 20\% \) but can change \( C_P \) by a factor of three.

CKS additionally considers the blazar model of Ref. 5 and calculates \( C_P \) for this model. However, they use the \( S_t \) previously calculated for the FB10 blazar model, which was normalized to a significantly different value of \( I \) than Ref. 5 calculates. This flux threshold value is not the correct one for the Ref. 5 blazar model, and therefore, the CKS-calculated value of \( C_P \) for this model is not valid. For comparison, for the ABH blazar model, we find that the blazar intensity from 1-10 GeV is \( 2.2 \times 10^{-7} \text{ ph cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \), approximately 60\% of the DGRB. For this intensity, we find a flux threshold of \( S_t = 2.9 \times 10^{-9} \text{ ph cm}^{-2} \text{ s}^{-1} \), which is much different than the \( S_t = 3.7 \times 10^{-10} \text{ ph cm}^{-2} \text{ s}^{-1} \) from CKS.

Additional problems with CKS are related to equations 2 and 3 for the intensity and anisotropy of the source population. There is an inherent integration and averaging over the source spectrum in these expressions. Figure 1 of FB10 shows the threshold flux for the Fermi-LAT to be not a single flux value but rather a strong function of the blazar spectral index. Depending on the blazar index, the threshold flux can vary by two orders of magnitude. This is important because the blazar intensity \( I \) is more sensitive to hard sources than the blazar anisotropy \( C_P \), so the threshold flux \( S_t \) is, in general, not the same for the calculation of \( I \) and the calculation of \( C_P \).

As a simple example, the blazar model of FB10, which extrapolates the Fermi-LAT broken power-law \( dN/dS \) below the Fermi-LAT threshold, considers a spread in the blazar spectral indices \( \Gamma \):

\[
\frac{dN}{dS d\Gamma} = f(S) g(\Gamma) \tag{8}
\]

\[
f(S) = \begin{cases} 
A S^{-\beta_1} & S \geq S_b \\
A S_b^{-\beta_1+\beta_2} S^{-\beta_2} & S < S_b 
\end{cases} \tag{9}
\]

\[
g(\Gamma) = \exp \left[ -\frac{(\Gamma - \mu)^2}{2\sigma^2} \right] . \tag{10}
\]

Including the blazar index distribution, equations 2 and 3 become

\[
I = \int_{-\infty}^{\infty} d\Gamma \int_0^{S_t(\Gamma)} dS S \frac{dN}{dS d\Gamma} \tag{11}
\]

\[
C_P = \int_{-\infty}^{\infty} d\Gamma \int_0^{S_t(\Gamma)} dS S^2 \frac{dN}{dS d\Gamma} \tag{12}
\]

Using the 0.1 – 100 GeV band model from table 4 of FB10 and the threshold fluxes in FB10 figure 1, we calculate \( I \) and \( C_P \) for this blazar model. Note that this analysis was done using \( F_{100} = \int_{100}^{\infty} dN/dE \frac{dE}{dE} \) rather than \( S \), to be consistent with FB10 figure 1. The FB10 model gives \( I = 2.4 \times 10^{-6} \text{ ph cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \) and \( C_P = 3.7 \times 10^{-14} (\text{ph cm}^{-2} \text{ s}^{-1})^2 \text{ sr}^{-1} \). Using equations 2 and 3 the equivalent index-independent threshold fluxes for each calculation are \( S_t(I) = 1.7 \times 10^{-8} \text{ ph cm}^{-2} \text{ s}^{-1} \) and \( S_t(C_P) = 3.8 \times 10^{-8} \text{ ph cm}^{-2} \text{ s}^{-1} \). As shown above, the large difference in \( S_t \) significantly affects the calculation of \( C_P \). CKS fails to take this effect into account.

For a full LDDE plus SED sequence blazar model, like ABH, the flux-dependence of the blazar spectrum must also be taken into account. An LDDE-based blazar model which integrates over blazar luminosity and redshift has been considered, but only with a simple power-law energy spectrum, rather than the full blazar SED 9. To do an accurate calculation of the LDDE plus SED model from ABH, an extension of the calculations of Ref. 6 to include the blazar SED must be done 11.

CONCLUSIONS

As shown in ABH and above, using a source-count distribution that is consistent with that measured by the Fermi-LAT collaboration (FB10) and assumed by CKS above the threshold, ABH find an intensity contribution to the DGRB between 1-10 GeV of 60\%, in direct contradiction to the general claim in CKS of a required \( \lesssim 30\% \) contribution. The CKS calculation of the Poisson term of the angular power spectrum for undetected blazars is inadequate. The broken power-law \( dN/dS \) they choose cannot be accurately extrapolated below the Fermi-LAT flux threshold, and doing so leads to unphysical results. CKS also use the incorrect value for the threshold flux when calculating \( C_P \) and comparing model intensity results. Furthermore, the model they consider fails to account for blazars’ spectral properties, which can affect the anisotropy calculation significantly. They assume a spectrally-independent threshold flux for the Fermi-LAT, which does not match the actual Fermi-LAT measurements. For the other model CKS considers, from Ref. 5, they use a value of the Fermi-LAT flux threshold which does not accurately reflect the threshold flux for that model, and therefore this model’s exclusion by the \( C_P \) is questionable.

Forthcoming work should accurately consider the consistency between angular correlations in the DGRB and its intensity, as contributed by blazars in physically-motivated blazar models, and should not rely on unjustified extrapolations and the other unqualified assumptions present in CKS, as described above.

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[1] K. N. Abazajian, S. Blanchet, and J. Harding, Phys.Rev. D84, 103007 (2011), arXiv:1012.1247 [astro-ph.CO]
[2] A. Cuoco, E. Komatsu, and J. Siegal-Gaskins (2012), arXiv:1202.5309 [astro-ph.CO]
[3] M. Cavadini, R. Salvaterra, and F. Haardt (2011), arXiv:1105.4613 [astro-ph.CO]
[4] J. Singal, V. Petrosian, and M. Ajello, Astrophysical Journal (2011), arXiv:1106.3111 [astro-ph.CO]
[5] F. W. Stecker and T. M. Venters, Astrophys.J. 736, 40 (2011), arXiv:1012.3678 [astro-ph.HE]
[6] S. Ando, E. Komatsu, T. Narumoto, and T. Totani, Phys. Rev. D75, 063519 (2007), arXiv:astro-ph/0612467
[7] Y. Inoue and T. Totani, Astrophys. J. 702, 523 (2009), arXiv:0810.3580 [astro-ph]
[8] A. A. Abdo et al. (Fermi-LAT Collaboration), Astrophys. J. 720, 435 (2010), arXiv:1003.0895 [astro-ph.CO]
[9] M. Ackermann, M. Ajello, A. Albert, L. Baldini, J. Ballet, et al. (2012), arXiv:1202.2856 [astro-ph.HE]
[10] A. A. Abdo et al. (Fermi-LAT Collaboration), Phys. Rev. Lett. 104, 101101 (2010), arXiv:1002.3603 [astro-ph.HE]
[11] K. N. Abazajian and J. Harding, in preparation (2012)