Generalized inference for the common mean of several lognormal populations

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Abstract

A hypothesis testing and an interval estimation are studied for the common mean of several lognormal populations. Two methods are given based on the concept of generalized p-value and generalized confidence interval. These new methods are exact and can be used without restriction on sample sizes, number of populations, or difference hypotheses. A simulation study for coverage probability, size and power shown that the new methods are better than the existing methods. A numerical example is given with some real medical data.

Keywords: Lognormal population, Common mean, Generalized variable, Generalized p-value, Generalized confidence interval.

1 Introduction

The statistical analysis that combines the results of several independent is known as meta-analysis and it is used in clinical trails and behavioral sciences.

Consider we have $k$ independent normal populations with means $a\mu + b\sigma_i^2$ and variances $\sigma_i^2$. Also we have a random samples of sizes $n_i$, $i = 1, ..., k$ from each one. We denote these samples by $Y_{ij} \sim N(a\mu + b\sigma_i^2, \sigma_i^2)$, $i = 1, ..., k$, $j = 1, ..., n_i$, where $a \neq 0$, and $b$ are constant. The problem of interest is to combine the summary statistics of samples for statistical inference about the parameter $\mu$. The statistical analysis that combines the results of several independent used in clinical trails and behavioral sciences.
If \( a = 1 \) and \( b = 0 \) then, \( Y_{ij} \sim N(\mu, \sigma^2_i) \) and this problem is known as the common mean for several normal populations. There are some inference for this problem in statistical literature. For example see; Krishnamoorthy and Lu (2003), Lin and Lee (2005). If \( a = 1 \) and \( b = -0.5 \), then \( Y_{ij} \sim N(\mu - 0.5\sigma^2_i, \sigma^2_i) \) and this is equivalent to problem of common mean of several lognormal populations. Our interest in this paper is inference about this problem.

For the common lognormal mean, a few authors proposed approximate methods: Ahmed et al (2001) proposed an estimator and approximate confidence interval for the common lognormal mean; Baklizi and Ebrahem (2005) studied several types of large samples and bootstrap intervals; Gupta and Li (2005) developed procedures for estimating the common mean and investigated the performance of the resulting confidence interval for two lognormal populations.

In this paper, we first propose estimation of \( \mu \) when the variances, \( \sigma^2_i \) are known. Then two methods are given that are applicable for both hypothesis testing and interval estimation for \( \mu \), based on the concepts of generalized \( p \)-value and generalized confidence interval. These methods are based on extending the method of Krishnamoorthy and Lu (2003) and the method of Lin and Lee (2005), which are used for the problem of common mean of several normal populations. Our methods also are applicable for the common mean of several lognormal for the interval mean of \( k \) lognormal populations. This chapter also is devoted to a short review regarding the existing method for inference of the common lognormal mean and application of our two methods for this problem. Finally, we give a numerical example for the common lognormal mean and by Monte Carlo simulation, we compare the coverage probabilities, size and power of these methods for the common mean of two lognormal populations.

Theorem 1.1. Let \( Y_{ij} \sim N(a\mu + b\sigma^2_i, \sigma^2_i) \), \( i = 1, \ldots, k, \, j = 1, \ldots, n_i \), where \( a \neq 0 \), \( b \) are constants and \( \sigma^2_i \)'s are known. The estimator

\[
\hat{\mu} = \frac{\sum_{i=1}^{k} \frac{n_i Y_i}{\sigma^2_i} - nb}{a \sum_{i=1}^{k} \frac{n_i}{\sigma^2_i}} \tag{1.1}
\]

is UMVUE and MLE for \( \mu \) and \( \hat{\mu} \sim N(\mu, 1/(a^2 \sum_{i=1}^{k} \frac{n_i}{\sigma^2_i})) \).

Proof. The probability density function for \( Y_{ij} \) is

\[
f_{Y_{ij}}(y_{ij}) = (2\pi\sigma^2_i)^{-\frac{1}{2}} e^{-\frac{1}{2} \frac{n^2 \mu^2}{\sigma^2_i} \times e^{-\frac{1}{2} \frac{1}{\sigma^2_i} (y_{ij} - b\sigma^2_i)^2} \times e^{\frac{a\mu}{\sigma^2_i} (y_{ij} - b\sigma^2_i)}.}
\]
Since the distribution of $Y_{ij}$ is from exponential family, in the form $A(\mu)B(y)e^{C(\mu)D(y)}$, then

$$T = \sum_{i=1}^{k} \sum_{j=1}^{n_i} \frac{1}{\sigma_i^2} (Y_{ij} - b\sigma_i^2) = \sum_{i=1}^{k} n_i \bar{Y}_i - nb$$

is UMVUE for $E(T) = a\mu \sum_{i=1}^{k} n_i / \sigma_i^2$ and $\hat{\mu} = T / \sum_{i=1}^{k} n_i / \sigma_i^2$ is UMVUE for $\mu$ (see Casella and Berger, 1990, page 263). It is easy to prove the rest of the theorem.

**Remark 1.1.** If $b = 0$ then $\hat{\mu}$ is the best linear unbiased estimator for $\mu$.

**Remark 1.2.** If $Y_{ij} = \ln(X_{ij}) \sim N(\mu - 0.5\sigma_i^2, \sigma_i^2)$, i.e. $X_{ij}$ is a lognormal variable, then $T = \exp(\hat{\mu} - 1/ \sum_{i=1}^{k} 2n_i / \sigma_i^2)$ is UMVUE for $E(X_{ij}) = e^\mu$, but the MLE of $e^\mu$ is $e^{\hat{\mu}}$.

**Remark 1.3.** If $\sigma_i^2$ are unknown, then we cannot find a closed form for MLE’s of $\mu$; we have to use a numerical approximation.

## 2 Generalized inferences for $\mu$

Suppose $Y_{ij} \sim N(a\mu + b\sigma_i^2, \sigma_i^2)$, $i = 1, \ldots, k$, $j = 1, \ldots, n_i$, where $a \neq 0$, $b$ are constants. For the $i$th population, let

$$\bar{Y}_i = \frac{1}{n_i} \sum_{i=1}^{n_i} Y_{ij}, \quad S_i^2 = \frac{1}{n_i - 1} \sum_{i=1}^{n_i} (Y_{ij} - \bar{Y}_i)^2,$$

be the sample mean and sample variance.

In this section, by using the idea of generalized $p$-value and by extending (i) the method of Krishnamoorthy and Lu (2003) and (ii) the method of Lin and Lee (2005), for the problem of common mean of normal populations, we give two generalized pivot variables for interval estimation and hypothesis testing for $\mu$ and we obtain two generalized $p$-values for testing hypothesis

$$H_0 : \mu \leq \mu_0 \quad \text{vs} \quad H_1 : \mu < \mu_0. \quad (2.1)$$

### 2.1 A weighted linear combination

It is clear that $\bar{Y}_i \sim N(a\mu + b\sigma_i^2, \sigma_i^2/n_i)$, $i = 1, \ldots, k$. Therefore, the generalized pivot variable for estimating $\mu$ based on the $i$th sample is

$$T_i^w = \frac{1}{a} (\bar{y}_i - b(n_i - 1)s_i^2/U_i) - Z_i \sqrt{\frac{(n_i - 1)s_i^2}{n_iU_i}} \quad (2.2)$$

$$= \frac{1}{a} (\bar{y}_i - b\frac{s_i^2}{S_i^2}\sigma_i^2 - Z_i \sqrt{\frac{s_i^2}{n_iS_i^2}\sigma_i^2}),$$
where
\[ Z_i = \frac{\bar{Y}_i - (a \mu + b \sigma_i^2)}{\sqrt{\sigma_i^2/n_i}} \sim N(0, 1), \quad U_i = \frac{(n_i - 1)S_i^2}{\sigma_i^2} \sim \chi^2_{(n_i-1)}, \]
and \((\bar{y}_i, s_i^2)\) is the observed value of \((\bar{Y}_i, S_i^2)\).

The generalized pivot variable for estimating \(\sigma_i^2\) based on the \(i\)th sample is given by
\[ R_i = \frac{(n_i - 1)S_i^2}{V_i} = \frac{s_i^2}{\sigma_i^2}, \quad i = 1, \ldots, k, \]  
(2.3)
where \(V_i = (n_i - 1)S_i^2/\sigma_i^2\) are independent \(\chi^2_{(n_i-1)}\) random variables (Weerahandi, 1995).

The generalized variable that we want to propose is a weighted average of the generalized pivot variables \(T^*_i\) in (2.2). The weights are inversely proportional to the generalized pivot variables \(R_i\) in (2.3) for the variances, and they are directly proportional to the sample sizes. (see Krishnamoorthy and Lu, 2003).

Let \(\bar{Y} = (\bar{Y}_1, \ldots, \bar{Y}_k)\) and \(V = (V_1, \ldots, V_k)\), with the observed values \(\bar{y}\) and \(v\), respectively. Then, the generalized variable can be expressed as
\[ T(\bar{Y}, V; \bar{y}, v) = \frac{k \sum_{i=1}^{k} \frac{n_i V_i}{(n_i - 1)s_i^2} \left[ \bar{y}_i - b \frac{(n_i - 1)s_i^2}{U_i} - Z_i \sqrt{\frac{(n_i - 1)s_i^2}{n_i U_i}} \right]}{\alpha \sum_{j=1}^{k} \frac{n_j V_j}{(n_j - 1)s_j^2}} - \mu \]  
(2.4)
where the weights are
\[ W_i = \frac{n_i V_i}{\sum_{j=1}^{k} \frac{n_j V_j}{(n_j - 1)s_j^2}}, \quad i = 1, \ldots, k. \]

The distribution of \(T(\bar{Y}, V; \bar{y}, v)\) is an increasing function with respect to \(\mu\). Therefore, the generalized p-value for (2.2) is given by
\[ p = P(T(\bar{Y}, V; \bar{y}, v) \leq T(\bar{y}, v; \bar{y}, v) | \mu = \mu_0) \]  
(2.5)
\[ = P(\sum_{i=1}^{k} W_i T^*_i \leq \mu_0). \]

This generalized p-value can be well approximated by a Monte Carlo simulation using the following algorithm:

**Algorithm 2.1.** For a given \((n_1, \ldots, n_k)\), \(\bar{Y} = (\bar{y}_1, \ldots, \bar{y}_k)\) and \((s_1^2, \ldots, s_k^2)\):
For $j = 1, m$

Generate $U_l \sim \chi^2_{(n_l-1)}$, $l = 1, \ldots, k$

Generate $V_l \sim \chi^2_{(n_l-1)}$, $l = 1, \ldots, k$

Generate $Z_l \sim N(0, 1)$, $l = 1, \ldots, k$

Compute $W_1, \ldots, W_k$

Compute $T_j = \sum_{l=1}^{k} W_l T^*_l$

(end $j$ loop)

Let $\gamma_j = 1$ if $T_j \leq \mu_0$, else $k_j = 0$. Then $\frac{1}{m} \sum_{j=1}^{m} \gamma_j$ is a Monte Carlo estimate of the generalized $p$-value for (2.5).

**Remark 2.1.** $T^* = \sum_{i=1}^{k} W_i T^*_i$ is a generalized pivot variable for $\mu$ and we can use that to obtain a generalized confidence interval for $\mu$.

**Remark 2.2.** If $a = 1$ and $b = 0$, then

$$T(\bar{Y}, V; \bar{y}, v) = \frac{\sum_{i=1}^{k} \frac{n_i V_i}{(n_i-1)s_i^2} \left[ \bar{y}_i - Z_i \sqrt{\frac{(n_i-1)s_i^2}{n_i U_i}} \right]}{\sum_{j=1}^{k} \frac{n_j V_j}{(n_j-1)s_j^2}} - \mu$$

(2.6)

and this generalized variable is introduced by Krishnamoorthy and Lu (2003) for inference on the common mean of several normal populations.

### 2.2 A generalized variable based on UMVUE

From theorem 1, we have

$$Z = |a| \sqrt{\sum_{i=1}^{k} \frac{n_i}{\sigma^2_i} (\bar{\mu} - \mu)} \sim N(0, 1).$$

We know that $R_i = \frac{(n_i - 1)s_i^2}{U_i}$ is a generalized pivot variable for $\sigma^2_i$, $i = 1, \ldots, k$, where $U_i \sim \chi^2_{(n_i-1)}$.

Let $\bar{Y} = (\bar{Y}_1, \ldots, \bar{Y}_k)$ and $U = (U_1, \ldots, U_k)$, with the observed values $\bar{y}$ and $u$, respectively.
We define a generalized variable for $\mu$ based on the UMVUE for $\mu$ in (1.1) by

$$T(\bar{Y}, U; \bar{y}, u) = \frac{k}{a} \sum_{i=1}^{k} \frac{n_i \bar{y}_i}{(n_i - 1)s_i^2} U_i - nb - \frac{Z}{a} \sqrt{\sum_{j=1}^{k} \frac{n_j}{(n_j - 1)s_j^2} U_j} - \mu \quad (2.7)$$

The distribution of $T(\bar{Y}, U; \bar{y}, u)$ is an increasing function with respect to $\mu$, and therefore the generalized $p$-value for testing (2.1) is

$$p = P(T(\bar{Y}, U; \bar{y}, u) \leq T(\bar{y}, u; \bar{y}, u) \mid \mu = \mu_o) = P(T^* \leq \mu_o) \quad (2.8)$$

where

$$T^* = \frac{k}{a} \sum_{i=1}^{k} \frac{n_i \bar{y}_i S_i^2}{\sigma_i^2 s_i^2} - nb - \frac{Z}{a} \sqrt{\sum_{j=1}^{k} \frac{n_j S_j^2}{\sigma_j^2 s_j^2} U_j} \quad (2.9)$$

and $\Phi$ is distribution function of the standard normal variable and expectation is taken with respect to chi-square random variables with $n_i - 1, i = 1, \ldots, k$, degrees of freedom.

This generalized $p$-value can be well approximated by a Monte Carlo simulation like the algorithm [2,1].

**Remark 2.3.** $T^*$ in (2.10) is a generalized pivot variable for $\mu$ and we can use that to obtain a generalized confidence interval for $\mu$.

**Remark 2.4.** If $a = 1$ and $b = 0$, then

$$T(\bar{Y}, U; \bar{y}, u) = \frac{k}{a} \sum_{i=1}^{k} \frac{n_i \bar{y}_i}{(n_i - 1)s_i^2} U_i - \frac{Z}{a} \sqrt{\sum_{j=1}^{k} \frac{n_j}{(n_j - 1)s_j^2} U_j} - \mu,$$

which is a generalized variable, introduced by Lin and Lee (2005), for the common mean of several normal populations.
Remark 2.5. For testing the hypothesis of the form

\[ H_0 : \mu = \mu_o \quad \text{vs} \quad H_1 : \mu \neq \mu_o, \]

the p-value is

\[ p = 2 \min \{P\{T^* < \mu_o\}, P\{T^* > \mu_o\}\}, \] (2.10)

and \( H_o \) can be rejected when \( p < \alpha \).

3 Methods for Common lognormal mean

Consider independent \( X_{ij} \) with lognormal distribution, for \( i = 1, \ldots, k \), \( j = 1, \ldots, n_i \), and assume that \( \theta_1 = \ldots = \theta_k = \varphi > 0 \), where \( \theta_i = E(X_{ij}) = \exp(\mu_i + \sigma_i^2) \), i.e., the \( k \) lognormal populations have common mean \( \varphi \). Therefore, we have \( Y_{ij} = \ln(X_{ij}) \sim N(\mu - 0.5\sigma_i^2, \sigma_i^2) \), where \( \mu = \ln \varphi \), and to find a confidence interval for \( \varphi \), it is enough to have a confidence interval for \( \mu \), and a hypothesis test for \( \varphi \) is equivalent to a hypothesis test for \( \mu \). For example the hypothesis test

\[ H_0 : \varphi \leq \varphi_o \quad \text{vs} \quad H_1 : \varphi > \varphi_o, \]

is equivalent to

\[ H_0 : \mu \leq \ln \varphi_o \quad \text{vs} \quad H_1 : \mu > \ln \varphi_o. \]

It is useful to review the existing methods for the problem of common lognormal mean.

3.1 Ahmed method

Let \( X_{ij} \sim LN(\theta, \tau_i^2) \), \( i = 1, \ldots, m \), \( j = 1, \ldots, n_i \). Then a combined sample estimate of \( E(X_{ij}) = \theta \) is given by

\[ \tilde{\theta} = \frac{\sum_m n_i \tilde{\theta}_i}{\sum_m n_i / v_i}, \]

where \( \tilde{\sigma}_i^2(1 + 0.5\tilde{\sigma}_i^2) \exp(2\tilde{\mu}_i + \tilde{\sigma}_i^2) \), \( \tilde{\theta}_i = \exp(\tilde{\mu}_i + 0.5\tilde{\sigma}_i^2) \), \( \tilde{\mu}_i = \bar{Y}_i \) and \( \tilde{\sigma}_i^2 = \frac{n_i - 1}{n_i} S_i^2 \).

The estimator \( \tilde{\theta} \) is asymptotically normal with mean \( \theta \) and asymptotic variance \( (\sum_{i=1}^m n_i / v_i)^{-1} \), which can be estimated by \( (\sum_{i=1}^m n_i / v_i)^{-1} \). Therefore, a 100(1 - \( \alpha \))% confidence interval for \( \theta \) is

\[ \tilde{\theta} \pm Z_{\alpha/2} (\sum_{i=1}^m n_i / v_i)^{-1/2}. \] (3.1)
3.2 Baklizi and Ebrahim method

The acceptance set for all \( \theta \) is

\[
\sum_{i=1}^{m} \frac{n_i(\hat{\theta}_i - \theta)^2}{v_i} \leq \chi^2_{\alpha,m}. \tag{3.2}
\]

This is a quadratic function in \( \theta \) whose two roots can be found directly. Since the coefficient of \( \theta^2 \) in this expression is positive, it follows that the set of all values of \( \theta \) between the two roots is the desired confidence interval.

3.3 Gupta and Li method

Let \( \theta = (\mu, \sigma_1, \sigma_2) \) be a vector of parameters, where \( \mu = \ln \eta = \mu_i + 0.5\sigma_1^2, \ i = 1, 2 \) and \( \eta \) is the common mean. The joint log-likelihood function based on the log-transformed data of two independent log-normal populations is given by

\[
\ln l(\theta) = \frac{-(n_1 + n_2)}{2} \ln 2\pi - n_1 \ln \sigma_1 - n_2 \ln \sigma_2 - 0.5(t_1 + t_2) + \frac{\mu}{\sigma_1^2}t_1 - \frac{1}{2\sigma_1^2}t_3 - \frac{(\mu - \sigma_1^2/2)n_1}{2\sigma_1^2} + \frac{\mu}{\sigma_2^2}t_2 - \frac{1}{2\sigma_2^2}t_4 - \frac{(\mu - \sigma_2^2/2)n_2}{2\sigma_2^2},
\]

where

\[
(t_1,t_2,t_3,t_4) = (\sum_j \ln x_{1j},\sum_j \ln x_{2j},\sum_j (\ln x_{1j})^2,\sum_j (\ln x_{2j})^2).
\]

Let \( \hat{\mu} \) be MLE for \( \mu \). The asymptotic variance of \( \hat{\mu} \) is

\[
\text{Var}(\hat{\mu}) = \frac{2n_1}{\sigma_1^4} + \frac{n_1(2n_2)}{\sigma_2^4} + \frac{n_2(2n_1)}{\sigma_1^4} + \frac{n_2(\sigma_1^4 + \sigma_2^4)}{\sigma_1^4 + \sigma_2^4},
\]

where \( \hat{\sigma}_1 \) and \( \hat{\sigma}_2 \) are MLEs for \( \sigma_1 \) and \( \sigma_2 \). A 100(1 - \( \alpha \))% confidence interval for \( \eta = e^\mu \) is

\[
\exp(\hat{\mu} \pm Z_{\alpha/2} \times SD(\hat{\mu})). \tag{3.3}
\]

3.4 Generalized inferences

In fact, the problem of common lognormal mean is a special case of our model when \( a = 1 \) and \( b = -\frac{1}{2} \). Thus, the generalized variable in (2.4) becomes

\[
T(\bar{Y}, V; \bar{y}, v) = \frac{\sum_{i=1}^{k} \frac{n_i V_i}{(n_i - 1)s_i^2} \left[ \bar{y}_i + \frac{(n_i - 1)s_i^2}{2U_i} - Z_i \sqrt{\frac{(n_i - 1)s_i^2}{n_iU_i}} \right]}{\sum_{j=1}^{k} \frac{n_j V_j}{(n_j - 1)s_j^2}} - \mu,
\]
and the generalized variable in (2.7) becomes

\[ T(\bar{Y}, U; \bar{y}, u) = \frac{k}{n_i} \sum_{i=1}^{k} \frac{n_i \bar{y}_i}{(n_i - 1)s^2_i} U_i + \frac{n}{2} \frac{Z}{\sqrt{\sum_{j=1}^{k} \frac{n_j}{(n_j - 1)s^2_j} U_j}} - \mu \]

4 Numerical Studies

In this section, we give a numerical example and compare our methods with other methods for the problem of common lognormal mean.

4.1 An example

The data come from the Regenstrief Medical Record System (RMRS) (MCDonald et al, 1988; Zhou et al, 1997) on effects of race on medical charges of patients with type I diabetes who had received inpatient or outpatient care at least two occasions during the period from 1 January 1993, through 30 June 1994. The data set consists of 119 African American patients and 106 white patients. The mean medical charges and their corresponding variance for the African American and white groups are given in Table 1.

| Data          | Patients group | Sample mean $ | Sample variance $^2$ |
|---------------|----------------|---------------|----------------------|
| Original      | African American | $18,850       | 26.897               |
|               | White           | $18,584       | 30.694               |
| Log-transform | African American | 9.06695       | 1.824                |
|               | White           | 8.69306       | 2.629                |

The studies show that (i) lognormal model adequately describes the both data sets. (ii) the variances of the two sets are not equal. (iii) the means of the two sets are equal (see Gupta and Li, 2005). Therefore, the average medical costs for African American patients and white patients are the same. We want to test that this average medical costs is 20,000$, i.e. the hypothesis test

\[ H_0 : \varphi = 20000 \quad vs \quad H_1 : \varphi \neq 20000, \quad (4.1) \]

The p-values for this test, with different methods are given in Table 2 and the confidence intervals are given in Table 3 Therefore, we cannot reject $H_o$. 


Table 2: \( p \)-values for hypothesis test of the common lognormal mean \( \varphi \)

| Methods                        | \( p \)-values |
|-------------------------------|----------------|
| Likelihood Ratio Test         | 0.5245         |
| Ahmed                         | 0.5582         |
| Gupta and Li                  | 0.5343         |
| First Generalized \( p \)-value| 0.4348         |
| Second Generalized \( p \)-value| 0.4732         |

Table 3: Interval estimation for the common lognormal mean \( \varphi \)

| Methods                          | Intervals                  | Width   |
|----------------------------------|----------------------------|---------|
| Ahmed                            | (15831.21, 27720.26)       | 11889.14|
| Gupta and Li                     | (16596.91, 28658.17)       | 12061.19|
| Baklizi and Ebrahem              | (14372.59, 29178.79)       | 14806.20|
| First Generalized confidence     | (17286.30, 30701.92)       | 13415.62|
| Second Generalized confidence    | (17090.54, 29998.23)       | 12907.69|

4.2 Simulation study

A simulation study is performed for inference about the common lognormal mean, \( \varphi \). The purpose of the simulation is to compare the size, power and coverage probability of each of the introduced methods with the others existing for two lognormal populations. For this purpose, several data sets from two normal distributions, with means \( \mu - 0.5\sigma_i^2 \) and variances \( \sigma_i^2 \), \( i = 1, 2 \), where \( \mu = \ln \varphi \), were created. For each condition 10000 simulations are used.

The sizes are given in table 4 and the powers in tables 5 and 6, and the coverage probability in tables 7, 8 and 9. These methods are

1. Likelihood ratio test
2. Ahmed method
3. Gupta and Li method
4. Baklizi and Ebrahem method
5. First Generalized variable in (2.3)
6. Second Generalized variable in (2.7)

The tables show that

- The simulated sizes of the two new methods are satisfactory since they are close to the significance level, 0.05.
- The power of the first generalized method is better than other methods when the sample
Table 4: Simulated sizes of the tests for $H_0: \varphi = 1$ vs $H_1: \varphi \neq 1$ at 5% significance level when $\mu = 0$ and $\sigma_1^2 = 1$.

| $\sigma_2^2$ | $n_1$ | $n_2$ | (1)  | (2)  | (3)  | (5)  | (6)  |
|--------------|------|------|------|------|------|------|------|
| 0.1          | 5    | 10   | 0.071| 0.233| 0.099| 0.035| 0.055|
|              | 25   | 25   | 0.075| 0.116| 0.086| 0.059| 0.071|
|              | 30   | 35   | 0.051| 0.081| 0.059| 0.046| 0.055|
|              | 50   | 50   | 0.046| 0.067| 0.052| 0.043| 0.045|
| 0.5          | 5    | 10   | 0.065| 0.274| 0.106| 0.042| 0.051|
|              | 25   | 25   | 0.083| 0.147| 0.096| 0.054| 0.069|
|              | 30   | 35   | 0.056| 0.122| 0.069| 0.054| 0.051|
|              | 50   | 50   | 0.048| 0.095| 0.059| 0.041| 0.045|
| 1            | 5    | 10   | 0.082| 0.331| 0.141| 0.036| 0.054|
|              | 25   | 25   | 0.075| 0.178| 0.092| 0.051| 0.066|
|              | 30   | 35   | 0.054| 0.148| 0.062| 0.046| 0.046|
|              | 50   | 50   | 0.055| 0.113| 0.061| 0.044| 0.045|
| 2.5          | 5    | 10   | 0.092| 0.397| 0.179| 0.034| 0.063|
|              | 25   | 25   | 0.061| 0.208| 0.085| 0.047| 0.059|
|              | 30   | 35   | 0.068| 0.177| 0.078| 0.051| 0.064|
|              | 50   | 50   | 0.048| 0.124| 0.057| 0.047| 0.049|

- The coverage probabilities of our generalized methods are close to the significance level and they are better than the coverage probabilities of existing methods.

sizes are large.
Table 5: Simulated powers of the tests for $H_0 : \varphi = 1$ vs $H_1 : \varphi \neq 1$ at 5% significance level when $\mu = 0.2$ and $\sigma_1^2 = 1$.

| $\sigma_2^2$ | $n_1$ | $n_2$ | (1)  | (2)  | (3)  | (5)  | (6)  |
|-------------|-------|-------|------|------|------|------|------|
| 0.1         | 5     | 10    | 0.528| 0.396| 0.539| 0.447| 0.435|
|             | 25    | 25    | 0.909| 0.831| 0.907| 0.891| 0.882|
|             | 30    | 35    | 0.964| 0.933| 0.961| 0.956| 0.952|
|             | 50    | 50    | 0.995| 0.989| 0.955| 0.995| 0.995|
| 0.5         | 5     | 10    | 0.171| 0.158| 0.156| 0.156| 0.148|
|             | 25    | 25    | 0.381| 0.157| 0.327| 0.385| 0.365|
|             | 30    | 35    | 0.464| 0.215| 0.403| 0.458| 0.439|
|             | 50    | 50    | 0.631| 0.395| 0.585| 0.633| 0.608|
| 1           | 5     | 10    | 0.124| 0.190| 0.128| 0.107| 0.109|
|             | 25    | 25    | 0.225| 0.063| 0.199| 0.225| 0.239|
|             | 30    | 35    | 0.280| 0.087| 0.229| 0.280| 0.267|
|             | 50    | 50    | 0.423| 0.188| 0.376| 0.428| 0.417|
| 2.5         | 5     | 10    | 0.108| 0.219| 0.124| 0.068| 0.076|
|             | 25    | 25    | 0.199| 0.073| 0.155| 0.193| 0.189|
|             | 30    | 35    | 0.215| 0.048| 0.148| 0.219| 0.201|
|             | 50    | 50    | 0.306| 0.101| 0.247| 0.302| 0.283|
Table 6: Simulated powers of the tests for $H_0 : \varphi = 1$ vs $H_1 : \varphi \neq 1$ at 5% significance level when $\mu = 1$ and $\sigma_1^2 = 1$.

| $\sigma_2^2$ | $n_1$ | $n_2$ | (1)   | (2)   | (3)   | (5)   | (6)   |
|------------|------|------|------|------|------|------|------|
| 0.1        | 5    | 10   | 1.000| 1.000| 1.000| 1.000| 1.000|
| 0.1        | 25   | 25   | 1.000| 1.000| 1.000| 1.000| 1.000|
| 0.1        | 30   | 35   | 1.000| 1.000| 1.000| 1.000| 1.000|
| 0.1        | 50   | 50   | 1.000| 1.000| 1.000| 1.000| 1.000|
| 0.5        | 5    | 10   | 0.999| 0.861| 0.999| 0.981| 0.983|
| 0.5        | 25   | 25   | 1.000| 1.000| 1.000| 1.000| 1.000|
| 0.5        | 30   | 35   | 1.000| 1.000| 1.000| 1.000| 1.000|
| 0.5        | 50   | 50   | 1.000| 1.000| 1.000| 1.000| 1.000|
| 1          | 5    | 10   | 0.958| 0.559| 0.946| 0.922| 0.927|
| 1          | 25   | 25   | 1.000| 1.000| 1.000| 1.000| 1.000|
| 1          | 30   | 35   | 1.000| 1.000| 1.000| 1.000| 1.000|
| 1          | 50   | 50   | 1.000| 1.000| 1.000| 1.000| 1.000|
| 2.5        | 5    | 10   | 0.749| 0.186| 0.702| 0.691| 0.686|
| 2.5        | 25   | 25   | 1.000| 0.924| 1.000| 0.998| 0.998|
| 2.5        | 30   | 35   | 1.000| 0.971| 1.000| 1.000| 1.000|
| 2.5        | 50   | 50   | 1.000| 0.997| 1.000| 1.000| 1.000|
Table 7: Simulated coverage probabilities at 5% significance level when $\mu = 0$ and $\sigma^2_1 = 1$.

| $\sigma^2_2$ | $n_1$ | $n_2$ | (2) | (3) | (4) | (5) | (6) |
|--------------|-------|-------|-----|-----|-----|-----|-----|
| 0.1          | 5     | 10    | 0.774 | 0.901 | 0.743 | 0.963 | 0.944 |
|              | 25    | 25    | 0.884 | 0.914 | 0.874 | 0.939 | 0.929 |
|              | 30    | 35    | 0.919 | 0.941 | 0.897 | 0.952 | 0.945 |
|              | 50    | 50    | 0.933 | 0.952 | 0.914 | 0.956 | 0.955 |
| 0.5          | 5     | 10    | 0.726 | 0.894 | 0.735 | 0.957 | 0.947 |
|              | 25    | 25    | 0.853 | 0.904 | 0.865 | 0.945 | 0.954 |
|              | 30    | 35    | 0.878 | 0.931 | 0.884 | 0.946 | 0.939 |
|              | 50    | 50    | 0.905 | 0.942 | 0.907 | 0.959 | 0.955 |
| 1            | 5     | 10    | 0.669 | 0.859 | 0.703 | 0.964 | 0.943 |
|              | 25    | 25    | 0.822 | 0.908 | 0.856 | 0.949 | 0.935 |
|              | 30    | 35    | 0.852 | 0.938 | 0.874 | 0.953 | 0.954 |
|              | 50    | 50    | 0.887 | 0.942 | 0.903 | 0.955 | 0.954 |
| 2.5          | 5     | 10    | 0.603 | 0.821 | 0.642 | 0.962 | 0.937 |
|              | 25    | 25    | 0.792 | 0.915 | 0.813 | 0.953 | 0.943 |
|              | 30    | 35    | 0.823 | 0.922 | 0.842 | 0.947 | 0.938 |
|              | 50    | 50    | 0.876 | 0.943 | 0.882 | 0.952 | 0.949 |

Table 8: Simulated coverage probabilities at 5% significance level when $\mu = 0.2$ and $\sigma^2_1 = 1$.

| $\sigma^2_2$ | $n_1$ | $n_2$ | (2) | (3) | (4) | (5) | (6) |
|--------------|-------|-------|-----|-----|-----|-----|-----|
| 0.1          | 5     | 10    | 0.778 | 0.901 | 0.748 | 0.963 | 0.944 |
|              | 25    | 25    | 0.884 | 0.914 | 0.877 | 0.939 | 0.929 |
|              | 30    | 35    | 0.924 | 0.941 | 0.904 | 0.947 | 0.945 |
|              | 50    | 50    | 0.933 | 0.951 | 0.914 | 0.959 | 0.955 |
| 0.5          | 5     | 10    | 0.686 | 0.872 | 0.724 | 0.969 | 0.936 |
|              | 25    | 25    | 0.853 | 0.904 | 0.865 | 0.945 | 0.929 |
|              | 30    | 35    | 0.878 | 0.931 | 0.884 | 0.946 | 0.949 |
|              | 50    | 50    | 0.905 | 0.941 | 0.907 | 0.959 | 0.954 |
| 1            | 5     | 10    | 0.661 | 0.852 | 0.692 | 0.965 | 0.935 |
|              | 25    | 25    | 0.840 | 0.931 | 0.855 | 0.952 | 0.946 |
|              | 30    | 35    | 0.857 | 0.928 | 0.882 | 0.959 | 0.948 |
|              | 50    | 50    | 0.888 | 0.938 | 0.915 | 0.944 | 0.941 |
| 2.5          | 5     | 10    | 0.644 | 0.851 | 0.892 | 0.963 | 0.937 |
|              | 25    | 25    | 0.814 | 0.924 | 0.841 | 0.942 | 0.938 |
|              | 30    | 35    | 0.831 | 0.935 | 0.855 | 0.946 | 0.946 |
|              | 50    | 50    | 0.873 | 0.936 | 0.887 | 0.943 | 0.941 |
Table 9: Simulated coverage probabilities at 5% significance level when $\mu = 1$ and $\sigma^2 = 1$.

| $\sigma^2$ | $n_1$ | $n_2$ | (2) | (3) | (4) | (5) | (6) |
|------------|-------|-------|-----|-----|-----|-----|-----|
| 0.1        | 5     | 10    | 0.771 | 0.901 | 0.743 | 0.963 | 0.944 |
|            | 25    | 25    | 0.884 | 0.914 | 0.877 | 0.939 | 0.929 |
|            | 30    | 35    | 0.924 | 0.941 | 0.904 | 0.947 | 0.949 |
|            | 50    | 50    | 0.929 | 0.942 | 0.912 | 0.945 | 0.947 |
| 0.5        | 5     | 10    | 0.699 | 0.868 | 0.728 | 0.958 | 0.936 |
|            | 25    | 25    | 0.853 | 0.904 | 0.865 | 0.945 | 0.929 |
|            | 30    | 35    | 0.884 | 0.931 | 0.897 | 0.951 | 0.943 |
|            | 50    | 50    | 0.896 | 0.938 | 0.908 | 0.946 | 0.944 |
| 1          | 5     | 10    | 0.667 | 0.839 | 0.724 | 0.958 | 0.937 |
|            | 25    | 25    | 0.827 | 0.917 | 0.855 | 0.948 | 0.939 |
|            | 30    | 35    | 0.870 | 0.932 | 0.889 | 0.962 | 0.951 |
|            | 50    | 50    | 0.884 | 0.937 | 0.894 | 0.949 | 0.951 |
| 2.5        | 5     | 10    | 0.614 | 0.841 | 0.658 | 0.961 | 0.932 |
|            | 25    | 25    | 0.722 | 0.915 | 0.813 | 0.953 | 0.941 |
|            | 30    | 35    | 0.821 | 0.930 | 0.854 | 0.956 | 0.951 |
|            | 50    | 50    | 0.876 | 0.943 | 0.882 | 0.952 | 0.949 |

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