Non-relativistic Model for the Semileptonic $\Lambda_b \to \Lambda_c$ Decay

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Abstract

We calculate the decay width for $\Lambda_b \to \Lambda_c e\bar{\nu}$ in the frame work of a nonrelativistic quark (NRQ) model of heavy baryons where the light quarks play the role of spectators. Our calculation does not make an explicit use of the heavy quark symmetry. The branching ratio for the above process as calculated here agrees reasonably well with the experimental value.

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I. INTRODUCTION

Non-relativistic quark (NRQ) models have played an important role in understanding various properties of hadrons. In the recent past they have been used in the literature to study semileptonic decay of heavy mesons [1]. Such calculations can partly be justified from the fact that the heavy quark in a heavy hadron acts as a source of static colour field.

In this Letter we extend such calculations to the case of baryons containing a single heavy quark. Specifically we look at the semileptonic decay of $\Lambda_b \rightarrow \Lambda_c e \bar{\nu}$. The study of this decay provides an opportunity to measure the CKM matrix element $V_{cb}$ and also to test the limits of the NRQ model calculations. Even though we are dealing with systems containing heavy quarks, no explicit use of the heavy quark symmetry is used in our calculation. This enables us to express quantities of physical interest explicitly in terms of the masses of heavy quarks (and other parameters of the model.) Our calculation thus by construction incorporates some of the finite mass corrections.

This paper is organised as follows: the next section sets forth the review of the underlying kinematics. In section III, we discuss the NRQ model as a tool for calculating the form factors. Section IV concludes the paper with the results and discussions.

II. KINEMATICS

The matrix element for the process $\Lambda_b \rightarrow \Lambda_c e \bar{\nu}$ is given by,

$$A_{s's} = \frac{G_F}{\sqrt{2}} V_{cb} L^\mu H^{(s's)}_\mu,$$

with the leptonic and hadronic parts being,

$$L^\mu = \bar{u}_e(p) \gamma^\mu (1 - \gamma_5) v_\nu(p'),$$

$$H^{(s's)}_\mu = \bar{s} \gamma_\mu s.$$
\[ H^{(s')\mu} = \langle K, s' | J_{\text{had}}^\mu | P, s \rangle. \] (3)

\( P, K, p \) and \( p' \) are the four momenta corresponding to \( \Lambda_b, \Lambda_c, e \) and \( \bar{\nu} \) respectively. The corresponding decay rate \(^{2,3}\) is given by

\[ \frac{d\Gamma_{s's}}{dyd\Omega_e d\Omega_{\Lambda_c}} = \frac{1}{2} \frac{|K|}{(4\pi)^5} |A_{s's}|^2, \] (4)

where \( y \equiv \frac{q^2}{m_{\Lambda_b}^2} \) is a dimensionless kinematic variable. The four momentum of the virtual \( W \) is given by \( q = p + p' = P - K \). With both the parent and daughter baryons possessing spin \( \frac{1}{2} \), the spin components \( s, s' \) of \( \Lambda_b \) and \( \Lambda_c \) can be either \( +\frac{1}{2} \) or \( -\frac{1}{2} \). The solid angle of the electron in the \( e\bar{\nu} \) centre of mass frame (\( e\bar{\nu} \) frame) is denoted by \( d\Omega_e \) while \( d\bar{\Omega}_{\Lambda_c} \) denotes the solid angle of the daughter baryon \( \Lambda_c \) in the rest frame of parent baryon \( \Lambda_b \). In the parent rest frame the quantities are denoted by a tilde and in the \( e\bar{\nu} \) frame without the tilde.

In the \( e\bar{\nu} \) frame, the energy and three momenta \((E_{\Lambda_b}, \vec{P})\) and \((E_{\Lambda_c}, \vec{K})\) of \( \Lambda_b \) and \( \Lambda_c \) respectively, are easily determined to be

\[ E_{\Lambda_b} = \frac{m_{\Lambda_b}}{2\sqrt{y}}(1 - \frac{m_{\Lambda_c}^2}{m_{\Lambda_b}^2} + y), \] (5)

\[ E_{\Lambda_c} = \frac{m_{\Lambda_b}}{2\sqrt{y}}(1 - \frac{m_{\Lambda_c}^2}{m_{\Lambda_b}^2} - y), \] (6)

\[ |P| = |K| = \frac{k}{\sqrt{y}}, \]

while in parent rest frame \((E_{\Lambda_b} = m_{\Lambda_b}, \vec{P} = 0)\), the energy and three momenta of daughter baryon \( \Lambda_c \) are

\[ E_{\Lambda_c} = \frac{m_{\Lambda_b}}{2}(1 + \frac{m_{\Lambda_c}^2}{m_{\Lambda_b}^2} - y), \] (7)

\[ |\vec{K}| \equiv k = \frac{m_{\Lambda_b}}{2}[(1 - \frac{m_{\Lambda_c}^2}{m_{\Lambda_b}^2} - y)^2 - 4\frac{m_{\Lambda_c}^2}{m_{\Lambda_b}^2} y]^{\frac{3}{2}}. \] (8)
In terms of form factors, the hadronic matrix elements of the current
\[ J_{\text{had}}^\mu = V^\mu - A^\mu \] are written as,
\[
\langle K, s' | V^\mu | P, s \rangle = \bar{u}_{s'}^{\Lambda} \left[ g(q^2) \gamma^\mu + g_+(q^2)(P + K)^\mu + g_-(q^2)(P - K)^\mu \right] u_{s}^{\Lambda_b}, \tag{9}
\]
\[
\langle K, s' | A^\mu | P, s \rangle = \bar{u}_{s'}^{\Lambda} \left[ a(q^2) \gamma^\mu \gamma_5 + a_+(q^2)(P + K)^\mu \gamma_5 + a_-(q^2)(P - K)^\mu \gamma_5 \right] u_{s}^{\Lambda_b}. \tag{10}
\]
Here \( u_{\Lambda_b} \) is the spinor associated with \( \Lambda_b \).

The decay width for the process under consideration is given by the expression
\[
\Gamma = \int_{y_{\text{min}}}^{y_{\text{max}}} \frac{G_F^2 |V_{cb}|^2 k m_{\Lambda_b}^2 y}{96\pi^3} \left( |H_+|^2 + |H_-|^2 + |H_0|^2 \right), \tag{11}
\]
where
\[
H_\pm = \pm (a F_0 \mp g F_-), \tag{12}
\]
\[
H_0 = \left\{ [2a(1 - \frac{1}{2} F_0) - 2 \frac{k}{\sqrt{y}} a_+ F_-]^2 + (2 \frac{k}{\sqrt{y}} g_+ F_0 + g F_+)^2 \right\}^{\frac{1}{2}}, \tag{13}
\]
with
\[
F_\pm = \left[ \frac{(E_{\Lambda_b} + m_{\Lambda_b})(E_{\Lambda_c} + m_{\Lambda_c})}{4 m_{\Lambda_b} m_{\Lambda_c}} \right]^{\frac{1}{2}} \left[ \frac{k}{\sqrt{y}(E_{\Lambda_c} + m_{\Lambda_c})} \pm \frac{k}{\sqrt{y}(E_{\Lambda_b} + m_{\Lambda_b})} \right], \tag{14}
\]
and
\[
F_0 = \left[ \frac{(E_{\Lambda_b} + m_{\Lambda_b})(E_{\Lambda_c} + m_{\Lambda_c})}{4 m_{\Lambda_b} m_{\Lambda_c}} \right]^{\frac{1}{2}} \left[ 1 - \frac{k^2}{y(E_{\Lambda_b} + m_{\Lambda_b})(E_{\Lambda_c} + m_{\Lambda_c})} \right]. \tag{15}
\]
Within the kinematically allowed region, the lower \( y_{\text{min}} \) and upper \( y_{\text{max}} \) limits of \( y \) are 0 and \( (1 - \frac{m_{\Lambda_c}}{m_{\Lambda_b}})^2 \) respectively (neglecting the electron mass).

The form factors given above can be evaluated within the framework of some model. We employ the NRQ model for this purpose which is described next.
III. NONRELATIVISTIC QUARK MODEL AND FORMFACTORS

In the NRQ model, the parent baryon $\Lambda_b$ contains the heavy quark $b$ and two light quarks $u$ and $d$ (having nearly equal masses). The light quarks are taken to behave as spectators. Similarly the daughter baryon consists of the charmed quark and the same two spectator light quarks. The spatial coordinates of the three quarks in $\Lambda_b$ are denoted by $\vec{r}_b$, $\vec{r}_1$ and $\vec{r}_2$, while those in $\Lambda_c$ are denoted by $\vec{r}_c$, $\vec{r}_1'$ and $\vec{r}_2'$. It is convenient to introduce the Jacobi coordinates $\vec{R}$, $\vec{\rho}$ and $\vec{\lambda}$ defined as
\begin{align}
\vec{R} &= \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_b \vec{r}_b}{m_1 + m_2 + m_b}, \\
\vec{\rho} &= \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2), \\
\vec{\lambda} &= \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_b).
\end{align}

We take $m_1 = m_2 = m_u$ (mass of each light quark contained in the spectator pair.) The corresponding canonically conjugate momenta $\vec{P}$, $\vec{p}_\rho$ and $\vec{p}_\lambda$ are respectively,
\begin{align}
\vec{P} &= \vec{p}_1 + \vec{p}_2 + \vec{p}_b, \\
\vec{p}_\rho &= \frac{1}{\sqrt{2}}(\vec{p}_1 - \vec{p}_2), \\
\vec{p}_\lambda &= \frac{\sqrt{3}}{2} \frac{m_b}{\bar{m}_{\Lambda_b}} (\vec{p}_1 + \vec{p}_2) - \frac{2m_u}{\bar{m}_{\Lambda_b}} \vec{p}_b,
\end{align}
where $\vec{p}_1$ and $\vec{p}_2$ are momenta of light quarks and $\bar{m}_{\Lambda_b} = 2m_u + m_b$. We have taken $m_{\Lambda_b}$ (the physical mass of $\Lambda_b$) = $\bar{m}_{\Lambda_b}$.

In this model, the normalized $\Lambda_b$ baryon state vector is written as,
\begin{align}
|\Lambda_b(P, s)\rangle &= \sqrt{2\bar{m}_{\Lambda_b}} \sum C_{s_1, s_2, s_b}^s d^3 \vec{p}_\lambda d^3 \vec{p}_\rho \phi_{\Lambda_b}(\vec{p}_\lambda, \vec{p}_\rho) \\
&\quad \times |u(p_1, s_1); d(p_2, s_2); b(p_b, s_b)\rangle,
\end{align}
where $\phi_{\Lambda_b}(\vec{p}_\lambda, \vec{p}_\rho)$ is the momentum space wave function, satisfying the normalisation condition,

$$\int d^3\vec{p}_\lambda d^3\vec{p}_\rho |\phi_{\Lambda_b}(\vec{p}_\lambda, \vec{p}_\rho)|^2 = 1.$$  \hspace{1cm} (23)

$C^s_{s_1s_2s_b}$ is the Clebsch-Gordon coefficient for combining three spin-$\frac{1}{2}$ constituent quarks into a spin $\frac{1}{2}$ baryon with the spin component $s$ along the $z$ axis.

The flavor and spin part of $\Lambda_b$ is given by

$$|\Lambda_b, s\rangle = |ud[I = 0, S = 0]; b\rangle S_s(1),$$  \hspace{1cm} (24)

where

$$S_s(1) = \frac{1}{\sqrt{2}}(|+-+\rangle - |--+\rangle) \text{ with } s = +\frac{1}{2},$$

$$= \frac{1}{\sqrt{2}}(|++-\rangle - |-++\rangle) \text{ with } s = -\frac{1}{2}$$

following the ordering $|s_1s_2s_b\rangle$;

Similarly,

$$|\Lambda_c(K, s')\rangle = \sqrt{2m_{\Lambda_c}} \sum C^s_{s_1s_2s_c} \int d^3\vec{p}_\lambda' d^3\vec{p}_\rho' |\phi_{\Lambda_c}(\vec{p}_\lambda', \vec{p}_\rho')|$$

$$|u(p_1, s_1); d(p_2, s_2); c(p_c, s_c)\rangle,$$  \hspace{1cm} (25)

The Hamiltonian that we consider is given by,

$$H = \sum \frac{p_i^2}{2m_i} + \sum_{i<j} \left( \frac{1}{2} a_1 r_{ij} + a_2 - \frac{2a_x}{3r_{ij}} \right).$$  \hspace{1cm} (26)

Here the “$\frac{1}{2}$ rule” i.e. $V_{qq} = \frac{1}{2}V_{q\bar{q}}$ \hspace{1cm} (4) is applied. The parameter $a_1 = 0.18GeV^2$ \hspace{1cm} (4) is fixed from the meson mass spectrum. It is obvious that the wavefunctions should be independent of the parameter $a_2$. 

5
The Hamiltonian (Eqn. 25) is best re-expressed in terms of Jacobian coordinates introduced in Eqns 16-18. Thus

\[
H = \frac{p_\rho^2}{2m_\rho} + \frac{p_\lambda^2}{2m_\lambda} + \frac{P^2}{2m} + \frac{1}{2}a_1|\sqrt{2}\rho| \\
+ \frac{1}{2}a_1\frac{1}{\sqrt{2}}(\bar{\rho} + \sqrt{3}\bar{\lambda})| + \frac{1}{2}a_1\frac{1}{\sqrt{2}}(\bar{\rho} - \sqrt{3}\bar{\lambda})| + a_2 \\
- \frac{2}{3}\alpha_s\left\{ \frac{1}{|\sqrt{2}\rho|} + \frac{\sqrt{2}}{|(\bar{\rho} + \sqrt{3}\lambda)|} + + \frac{\sqrt{2}}{|(\bar{\rho} - \sqrt{3}\lambda)|} \right\},
\]

(27)

where \( m = m_{\Lambda_b}, m_\rho = m_1 = m_2 = m_u \) and \( m_\lambda = \frac{3m_u m_b}{2m_u + m_b} \). The values of the mass parameters we shall use are \( m_u = 0.35 GeV, m_c = 1.80 GeV, m_b = 5.00 GeV \) and the coupling strength relevant to the b & c hadrons should be taken to be \( \alpha_s = 0.5 \).

The wave function \( \phi_{\Lambda_b}(\vec{p}_\rho, \vec{p}_\lambda) \) is an eigenfunction of the above \( H \) given by Eqn.26. Employing a variational approach with trial wavefunctions taken as the product of normalised Gaussians in the variables \( \rho \) and \( \lambda \),

\[
\phi(\rho, \lambda) = \left( \frac{\alpha_1}{\pi} \right)^{\frac{3}{4}} \left( \frac{\alpha_2}{\pi} \right)^{\frac{3}{4}} exp(-\alpha_1 \rho^2 / 2) exp(-\alpha_2 \lambda^2 / 2).
\]

(28)

and with the \( \Lambda_c \) baryon wavefunction expressed with analogous variables written with primes, the value of the parameters \( \alpha_1 \) and \( \alpha_2 \) minimizing the ground state energies for \( \Lambda_b \) and \( \Lambda_c \) are found to be

\[
\alpha_1^b = 0.264 \quad \text{and} \quad \alpha_2^b = 0.653 \quad \text{for} \quad \Lambda_b,
\]

\[
\alpha_1^c = 0.261 \quad \text{and} \quad \alpha_2^c = 0.568 \quad \text{for} \quad \Lambda_c.
\]

In the rest frame of the \( \Lambda_b \) baryon, \( (\vec{P} = 0) \)

\[
\vec{p}_c = \vec{p}_b + \vec{K} = \vec{K} - \sqrt{\frac{2}{3}}\vec{p}_\lambda
\]

(29)

with spectator approximation \( (\vec{p}_1' = \vec{p}_1 \text{ and } \vec{p}_2' = \vec{p}_2) \),

\[
\vec{p}_\rho' = \vec{p}_\rho,
\]

(30)
\[ \vec p_{\lambda}' = \vec p_{\lambda} - \sqrt{\frac{3}{2} \frac{m_u}{m_{\Lambda_c}} \vec K}. \] (31)

Using these baryon state vectors, the hadronic matrix elements are found to be

\[ \langle \vec K', s' | J^\mu | \vec 0, s \rangle = \sum \mathcal{C}_{s_1 s_2 s_c} \mathcal{C}_{s_1 s_2 s_b} \int d^3 \vec p_{\rho'} d^3 \vec p_{\lambda'} d^3 \vec p_{\rho} d^3 \vec p_{\lambda} \]

\[ \delta^{(3)} (\vec p_{\lambda'} - \vec p_{\lambda} + \sqrt{\frac{3}{2} \frac{m_u}{m_{\Lambda_c}} \vec K}) \delta^{(3)} (\vec p_{\rho'} - \vec p_{\rho}) \]

\[ \phi_{\Lambda_c}^* (\vec p_{\rho'}, \vec p_{\lambda'}) \phi_{\Lambda_b} (\vec p_{\rho}, \vec p_{\lambda}) \]

\[ c(\vec p_{c}, s_c) \gamma^\mu (1 - \gamma_5) b(\vec p_{b}, s_b). \] (32)

Writing the relevant matrix elements for the temporal and spatial components of the currents separately we get

\[ \tilde V_{s', s}^0 = (4m_{\Lambda_b} m_{\Lambda_c})^{\frac{1}{2}} \int d^3 \vec p_{\rho} d^3 \vec p_{\lambda} \phi_{\Lambda_c}^* (\vec p_{\rho}, \vec p_{\lambda} - \sqrt{\frac{3}{2} \frac{m_u}{m_{\Lambda_c}} \vec K}) \phi_{\Lambda_b} (\vec p_{\rho}, \vec p_{\lambda}) \chi_{s'} \dagger \varphi_s, \] (33)

\[ \tilde V_{s', s} = (4m_{\Lambda_b} m_{\Lambda_c})^{\frac{1}{2}} \int d^3 \vec p_{\rho} d^3 \vec p_{\lambda} \]

\[ \phi_{\Lambda_c}^* (\vec p_{\rho}, \vec p_{\lambda} - \sqrt{\frac{3}{2} \frac{m_u}{m_{\Lambda_c}} \vec K}) \phi_{\Lambda_b} (\vec p_{\rho}, \vec p_{\lambda}) \]

\[ \chi_{s'} \dagger \{ i \hat{\sigma} \times (\vec K - \sqrt{\frac{3}{2} \vec p_{\lambda}}) \over 2 m_c \} + \left( \sqrt{\frac{3}{2}} \over 2 m_c \right) \]

\[ + i \hat{\sigma} \times \sqrt{\frac{3}{2}} \vec p_{\lambda} \over 2 m_b \} \varphi_s, \] (34)

\[ \tilde A_{s', s}^0 = (4m_{\Lambda_b} m_{\Lambda_c})^{\frac{1}{2}} \int d^3 \vec p_{\rho} d^3 \vec p_{\lambda} \]

\[ \phi_{\Lambda_c}^* (\vec p_{\rho}, \vec p_{\lambda} - \sqrt{\frac{3}{2} \frac{m_u}{m_{\Lambda_c}} \vec K}) \phi_{\Lambda_b} (\vec p_{\rho}, \vec p_{\lambda}) \]

\[ \chi_{s'} \dagger \{ i \hat{\sigma} \cdot (\vec K - \sqrt{\frac{3}{2} \vec p_{\lambda}}) \over 2 m_c \} - i \hat{\sigma} \cdot \sqrt{\frac{3}{2}} \vec p_{\lambda} \over 2 m_b \} \varphi_s, \] (35)

\[ \tilde A_{s', s} = (4m_{\Lambda_b} m_{\Lambda_c})^{\frac{1}{2}} \int d^3 \vec p_{\rho} d^3 \vec p_{\lambda} \]

7
\[
\phi_{\Lambda_c}(\vec{p}_\rho, \vec{p}_\lambda) = \sqrt{\frac{3}{2}} \frac{m_u}{m_{\Lambda_c}} \phi_{\Lambda_b}(\vec{p}_\rho, \vec{p}_\lambda) (1 + \frac{\vec{K}^2}{8m_{\Lambda_c}^2}) \chi_{s'}^\dagger \hat{\sigma} \varphi_s.
\]  

(36)

In order to extract the form factors appearing in Eqns. 9 - 10, we write

their temporal and spatial components separately in the parent rest frame

with \(|\vec{K}| << m_{\Lambda_c}\) (non-relativistic limit) for comparison with results from

NRQ model:

\[
\tilde{V}_{s',s}^0 = \chi_{s'}^\dagger \left[ g(q^2) + (m_{\Lambda_b} + m_{\Lambda_c})g_+(q^2) + (m_{\Lambda_b} - m_{\Lambda_c})g_-(q^2) \right] \varphi_s
\]

(37)

\[
\tilde{V}_{s',s} = \chi_{s'}^\dagger \left[ \frac{g(q^2)}{2m_{\Lambda_c}} + g_+(q^2) - g_-(q^2) \right] \vec{K} + i \frac{\hat{\sigma} \times \vec{K}}{2m_{\Lambda_c}} g(q^2) \varphi_s
\]

(38)

\[
\tilde{A}_{s',s}^0 = \chi_{s'}^\dagger \left[ a(q^2) - (m_{\Lambda_b} + m_{\Lambda_c})a_+ + (m_{\Lambda_b} - m_{\Lambda_c})a_- \right] \left( \frac{\hat{\sigma} \cdot \vec{K}}{2m_{\Lambda_c}} \right) \varphi_s
\]

(39)

\[
\tilde{A}_{s',s} = \chi_{s'}^\dagger \left[ \{a(q^2)\hat{\sigma}[1 + \frac{\vec{K}^2}{8m_{\Lambda_c}^2}]\} - (a_+ + a_-) \left( \frac{\hat{\sigma} \cdot \vec{K}}{2m_{\Lambda_c}} \right) \vec{K} \right] \varphi_s
\]

(40)

Here \(\varphi_s\) and \(\chi_{s'}\) are two component Pauli spinors along z (parent spin direction) and \(\vec{e}'\) (daughter polarization direction), the latter being taken to be arbitrary. It is to be noted that in the \(e\bar{\nu}\) rest frame the parent and the daughter have equal and opposite three momenta.

Inserting the Fourier transform of the above wavefunctions \(\phi(\vec{p}_\rho, \vec{p}_\lambda)\) in the expressions for the matrix elements of the vector and axial vectors currents [Eqns. 32 - 35] and then comparing with the matrix elements expressed in terms of form factors [Eqns. 36 - 39], the following relations emerge:

\[
g + (m_{\Lambda_b} + m_{\Lambda_c})g_+ + (m_{\Lambda_b} - m_{\Lambda_c})g_- = I(4m_{\Lambda_c}m_{\Lambda_b}) \frac{1}{2} \exp\left( -\frac{3m_u^2 \vec{K}^2}{(\alpha_2^b + \alpha_2^b)m_{\Lambda_c}^2} \right),
\]

(41)

\[
\frac{g}{2m_{\Lambda_c}} + g_+ - g_- = \frac{I}{m_c}(m_{\Lambda_c}m_{\Lambda_b}) \frac{1}{2} \exp\left( -\frac{3m_u^2 \vec{K}^2}{(\alpha_2^b + \alpha_2^b)m_{\Lambda_c}^2} \right) \left[ 1 - \frac{2m_u \alpha_2^b}{m_{\Lambda_c}m_b(\alpha_2^b + \alpha_2^b)(m_b + m_c)} \right],
\]

(42)
\[
\frac{g}{2m_{\lambda_c}} = I \frac{(m_{\lambda_c}m_{\lambda_b})^\frac{1}{2}}{m_c} \exp(-\frac{3m_c^2K^2}{(\alpha_c^2 + \alpha_b^2)m_{\lambda_c}^2})
\]
\[
[1 - \frac{2m_u\alpha_b^2}{m_{\lambda_c}m_b(\alpha_c^2 + \alpha_b^2)}(m_b - m_c)],
\]
\[
a - (m_{\lambda_b} + m_{\lambda_c})a_+ - (m_{\lambda_b} - m_{\lambda_c})a_- = m_{\lambda_c} \frac{I}{m_c} (4m_{\lambda_c}m_{\lambda_b})^\frac{1}{2}
\]
\[
\exp(-\frac{3m_c^2K^2}{(\alpha_c^2 + \alpha_b^2)m_{\lambda_c}^2})[1 - \frac{2m_u\alpha_b^2}{m_{\lambda_c}m_b(\alpha_c^2 + \alpha_b^2)}(m_b + m_c)],
\]
\[
a = I (4m_{\lambda_c}m_{\lambda_b})^\frac{1}{2} \exp(-\frac{3m_c^2K^2}{(\alpha_c^2 + \alpha_b^2)m_{\lambda_c}^2}),
\]
\[
a_+ - a_- = 0,
\]

where the overlap integral \( I \) is given by

\[
I = \left(\frac{1}{\alpha_c^b}\right)^\frac{1}{2}\left(\frac{1}{\alpha_b^b}\right)^\frac{1}{2}\left(\frac{1}{\alpha_c^c}\right)^\frac{1}{2}\left(\frac{1}{\alpha_b^c}\right)^\frac{1}{2}\int d^3\vec{p}_d d^3\vec{p}_\Lambda
\]
\[
\exp[-(\frac{\alpha_c^b + \alpha_b^c}{2\alpha_c^b\alpha_b^c})\vec{p}_d^2]\exp[-(\frac{\alpha_b^c + \alpha_c^b}{2\alpha_b^c\alpha_c^b})\vec{p}_\Lambda^2]
\]
\[
= \frac{8\alpha_c^b\alpha_b^c}{(\alpha_c^b + \alpha_b^c)^2}\left(\frac{\alpha_b^b + \alpha_c^c}{\alpha_b^b + \alpha_c^c}\right)^\frac{1}{2},
\]

with

\[
\vec{p}_\Lambda = \vec{p}_\Lambda - \sqrt{\frac{3}{2m_{\lambda_c}(\alpha_c^2 + \alpha_b^2)}} K.
\]

From [eqs. 40 - 45] the form factors are obtained as functions of \( y \);

\[
g = m_{\lambda_c} \frac{I}{m_c} (4m_{\lambda_c}m_{\lambda_b})^\frac{1}{2} \exp[-\alpha\{((\beta - y)^2 - \gamma y\}]
\]
\[
[1 - \frac{2m_u\alpha_b^2}{m_{\lambda_c}m_b(\alpha_c^2 + \alpha_b^2)}(m_b - m_c)],
\]

\[
g_{\pm} = I \left(\frac{m_{\lambda_c}}{m_{\lambda_b}}\right)^\frac{1}{2} \exp[-\alpha\{((\beta - y)^2 - \gamma y\}]
\]
\[
[m_c - m_{\lambda_c} + \frac{2m_u\alpha_b^2}{m_{\lambda_c}m_b(\alpha_c^2 + \alpha_b^2)}(m_b m_{\lambda_c} \mp m_c m_{\lambda_b})],
\]

\[
a = I (4m_{\lambda_c}m_{\lambda_b})^\frac{1}{2} \exp[-\alpha\{((\beta - y)^2 - \gamma y\}],
\]
\[
a_{\pm} = \frac{I}{m_c \Lambda_c} \left( \frac{m_{\Lambda_c}}{m_{\Lambda_b}} \right)^{\frac{1}{2}} \exp\left[ -\alpha ((\beta - y)^2 - \gamma y) \right] \\
\left[ m_c - m_{\Lambda_c} + \frac{2m_u a^b}{m_b (\alpha^c + \alpha^b)} (m_b + m_c) \right],
\]

where

\[
\alpha = \frac{3m_u^2 m_{\Lambda_b}^2}{4m_{\Lambda_c}^2 (\alpha^b + \alpha^c)},
\]
\[
\beta = 1 - \frac{m_{\Lambda_c}^2}{m_{\Lambda_b}^2},
\]
\[
\gamma = \frac{4m_{\Lambda_b}^2}{m_{\Lambda_c}^2},
\]

having numerical values \( \alpha = 0.43, \beta = 0.83 \) and \( \gamma = 0.69 \) for our choice of values of the parameters.

**IV. RESULTS AND CONCLUSION**

In our calculation, we use nonrelativistic wave functions which gives the \( y \) dependence \( \exp[ -\alpha ((\beta - y)^2 - \gamma y) ] \) for all the form factors. The form factors at zero recoil are obtained as \( g = 8.02 \ GeV, g_+ = -0.15, g_- = 0.075, \)
\( a = 7.07 \ GeV \) and \( a_\pm = 0.008 \) from which inserting the common form factor the quantities of physical interest may be calculated. We get the numerical value of decay width and branching ratio (where experimental value of the life time of \( \Lambda_b \) has been used) as \( 5.7 \times 10^{10} \ sec^{-1} \) and 7.6% respectively.

Our results compare favourably well with [2, 3, 7]. As an example, we list the results of Ref. [2] where in order to calculate the \( y \) dependence of the form factors, use has been made of the pole dominated form factors with a pole at \( m^* = 6.0 \ GeV \). They assume the same resonance mass for all the form factors, which yields the same \( y \) dependence for all the form factors. They have found the numerical values of decay width and branching ratio to be \( 5.7 \times 10^{10} \ sec^{-1} \) and 7.7% respectively.
At present, semileptonic decays of bottom baryons are studied mainly from the LEP experiments to extract lifetimes from $Z$ decays. The best experimental value of $B(\Lambda_b \to \Lambda c l^{-}_\nu X) = \Gamma(B(\Lambda_b \to \Lambda c l^{-}_\nu X))/\Gamma_{total}$ (most of these decays are presumed to be three body decays) is $(10.0 \pm 3.0)$%. However, this is not a pure measurement since what experimentalists actually measure is the product branching fraction, viz. $B(\Lambda_b \to \Lambda c l^{-}_\nu X) \times f(b \to \Lambda_b)$ where $f(b \to \Lambda_b)$ is fraction of $b$-flavored baryons produced in $Z \to b\bar{b}$ decays. So our calculated estimate compares reasonably well with experimental value taking into account the uncertainties in the present experimental measurements.

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