Study of flow through and around a square cylinder array

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Abstract. This paper performs the effect of solid volume fractions ($0.09<\text{SVF}<0.64$) on the flow through and around a square cylinder array at $\text{Re}=63832$. A delayed vortex shedding occurs in $8\times8$ array (SVF) while the Strouhal number is slightly higher than the solid one. For the intermediate SVF ($0.25<\text{SVF}<0.49$), the separated shear layers (SSLs) contain large-scale turbulent eddies but their ends cannot interact to generate wake billows. The maximum of root mean square of transverse velocity along wake center-line decreases monotonously as SVF decreases, and several ways of defining the spatial extent of the wake region are discussed.

1. Introduction

The flow past arrays of long solid obstacles is broadly involved in industrial and scientific applications. Frequent occurrences include multiple bridge piles, pebble-bed reactors, transpiration cooling in a gas turbine, and arrays of turbines in a wind farm [1]. The circular and square cylinders are often chosen to model the solid obstacles for their geometric simplicity, and most of the generic flow features can be captured to facilitate fundamental fluid dynamics research.

Nicolle & Eames investigated the flow through and around a two-dimensional (2D) circular cylinder array with 7 to 133 isolated cylinders at $\text{Re}=2100$ [2]. Three flow regimes are conducted: (1) $\text{SVF}<0.05$, the force statistics of the little cylinders are similar to a solid cylinder, and the vorticity dissipates rapidly with wake mixing. (2) $\text{SVF}>0.15$, the wake of the array is similar to the solid ones of the same scale. (3) $0.05<\text{SVF}<0.15$, a shear layer generates on the shoulder of array while the force on the isolated cylinder is steady. Chang et al. [3] applied fully three-dimensional (3D) large eddy simulation to study the effects of solid volume fraction on the flow structure at $\text{Re}=10000$. There is no intermediate flow regime as mentioned above. In these investigations, as the number of isolated cylinder increases, e.g. $N>20$, the region containing the array is relatively equivalent to a porous medium with uniform porosity. The gap spacing has a great influence on the flow structure.

As well as circular cylinder array qualitatively and quantitatively investigated, many numerical and experimental works have been devoted to the square cylinder array. Jue [4] and Chen et al. [5] studied the vortex shedding behind a porous square cylinder. Their investigation reveals that the vortex street formation is delayed with the increment of spacing gap between the single cylinders inside. Dhinakaran et al. [6] focused on the flow past a 2D porous square cylinder based on Open FOAM with Darcy-
Forchheimer-Brinkman model, providing an insight on the onset of vortex shedding behind a porous bluff body at \( Re = 41-150 \) for different permeability (\( Da = 10^{-6}-10^{-2} \)).

Although abundant articles are available on the flow past a circular or square cylinder array, few concentrations have been given to the square cylinder array at high \( Re \), especially in three dimensional research. In this study, we performed the wake characteristics of the 3D square cylinder arrays and detailed force analysis on isolated cylinders. The paper is structured as follows: the numerical methodology and results along with validation by grid sensitivity and literature is outlined in section 2. Section 3 elaborates the LES results of different cylinder arrays at the mid plane.

2. Methodology

2.1. Numerical setup

The Wall-adapting local-eddy viscosity (WALE) subgrid scale model based on the square of velocity gradient tensor in LES is employed in present solution to simulate flow past and around cylinder arrays. The governing equations applied here are the space-filtered Navier-Stokes equations under incompressible assumption:

\[
\frac{\partial \overline{u_i}}{\partial t} + \frac{\partial \overline{u_i u_j}}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 \overline{u_i}}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, (i=1,2,3)
\]

\[
\frac{\partial \overline{u_i}}{\partial x_i} = 0
\]

The influence of the small scales on the resolved scales is exerted through the sub-grid scale stress \( (\tau_{ij}) \), which must be modeled to obtain a closure for (1). It is usually based on eddy-viscosity models:

\[
\tau_{ij} - \frac{1}{3} \tau_{kk} \delta_{ij} = -2\mu \overline{S_{ij}}
\]

\[
\mu_r = (C_w \Delta)^2 \frac{OP_1}{OP_2 + \varepsilon}
\]

Where \( \varepsilon = 10^{-6} \), \( \overline{S_{ij}} = (\overline{u_i \partial u_j / \partial x_j} + \overline{u_j \partial u_i / \partial x_i}) \), and \( \Delta \) is the characteristics length scale based on cell volume \( V \)

\[
\Delta = \begin{cases} 
\min(\kappa d, C_w V^{1/3}) & \text{if length scale limit is applied} \\
C_w V^{1/3} & \text{if length scale limit is not applied}
\end{cases}
\]

\( d \) stands for the wall distance and \( \kappa \) is the von Karman constant \( (\kappa = 0.41) \). \( C_w \) is not universal and commonly range from 0.325 to 0.5, and the default value in STAR-CCM+ \( C_w = 0.544 \) works well. Other quantities are:

\[
\bar{OP}_1 = (\overline{S_{ij}^d S_{ij}^d})^{3/2}, \bar{OP}_2 = (\overline{S_{ij}^d S_{ij}^d})^{5/2} + (\overline{S_{ij}^d S_{ij}^d})^{5/2}
\]

\[
S_{ij}^d = \frac{1}{2}(\overline{g_{ij}} ^2 + \overline{g_{ij}} ^2) - \frac{1}{3} \delta_{ij} \overline{g_{kk}}, \overline{g_{ij}} = \frac{\partial \overline{u_i}}{\partial x_j}
\]
WALE model recovers the proper $y^+$ near wall scaling for eddy viscosity without requiring dynamic procedure, and it is also shown that the model can handle the transition. The second-order implicit formulation is applied to advance the equations in time, while the central difference scheme based on finite volume method is adopted for spatial discretization. The SIMPLE algorithm is used to combine with Pressure-velocity coupling in STAR-CCM+.

2.2. Computational and boundary conditions

The computational domain in x-y plane comprises $(6.5D+20.5D) \times 7.5D$, shown in figure 1. The span-wise length $4D$ for high Reynolds number can provide accurate results especially for global quantities if one employs periodic boundary conditions and a suitable grid [7]. Here $5.6D$ is chosen equal to the wind tunnel size. A uniform and constant velocity $U_{in} = 20 m/s$ is imposed at the inlet, and the pressure at the outlet is set zero. No-slip and no penetration boundary conditions are applied to the cylinders. $D$ is $0.05m$ and $d$ is $0.005m$. The distance between the front face of square cylinder and the inlet is $6D$. Several Trim meshes are carried out to obtain the grid sensitivity and validation with previous simulation and experiments in Table 1. Five grids are considered here and ‘Fine-grid-2’ trim mesh is chosen, because its good results in present simulation.

![Figure 1. (a) Computational domain and grid distribution (b) schematic of ordered cylinder array](image-url)

The base size in x-y plane is $6mm$, and grid-refined region in stream-wise and transverse is $3mm$. The grid in the region behind cylinder is $1.5mm$ and the smallest one around the cylinder is $0.75mm$. The maximum dimensionless wall distance $y^+$ is kept bellow 1.

| case            | method       | Re   | St  | $C_p$ |
|-----------------|--------------|------|-----|-------|
| Lyn et al.[8]   | experiments  | 21400| 0.133| 2.1   |
| Luo et al.[9]   | experiments  | 34000| 0.13 | 2.21  |
| Vickery[10]     | experiments  | 100000| 0.133| 2.1   |
| Minguez et al.[11]| LES     | 21400| 0.141| 2.2   |
| Trias et al.[12]| DNS         | 22000| 0.132| 2.18  |
| Bai and Allam[13]| Data Summary | >1000| $\rightarrow 0.134$ | $\rightarrow 2.21$ |
| Wind tunnel     | experiments  | 63832| 0.132| --    |

| present         | Grid         | Time step | St  | $C_p$ |
|-----------------|--------------|-----------|-----|-------|
| Coarse-grid-1, isotropic | 10 million   | 0.0002s   | 0.145| 1.923 |
| Coarse-grid-1, anisotropic | 11.2 million | 0.0002s   | 0.134| 2.352 |
| Medium-grid-1, isotropic  | 15 million   | 0.0002s   | 0.138| 2.108 |
| Fine-grid-1, isotropic    | 20 million   | 0.0002s   | 0.145| 2.246 |
| Fine-grid-2, isotropic    | 25 million   | 0.0002s   | 0.138| 2.123 |

The base size in x-y plane is $6mm$, and grid-refined region in stream-wise and transverse is $3mm$. The grid in the region behind cylinder is $1.5mm$ and the smallest one around the cylinder is $0.75mm$. The maximum dimensionless wall distance $y^+$ is kept bellow 1.
3. Numerical results and discussions

3.1. Mean flow statistics

Figure 2. Mean flow in the near region of different SVF: (a) 0.09, (b) 0.25, (c) 0.36, (d) 0.49, (e) 0.64.

Figure 2 displays the mean streamline within and close to the cylinder array and normalized vorticity \( \omega / U_wD \) at the mid plane for different values of SVF. The last case with a high SVF (SVF=0.64) has four re-circulation bubbles in the near region and two counter-rotating secondary bubbles on the sides of the large one. Small-scale structures occur along the top and bottom surfaces of the array while none appears inside the array. The flow is diverted laterally as it approaches the upstream faces of the cylinder array, and the separated shear layers (SSLs) interact with each other. It triggers a vortex shedding with a delayed vortex formation region. The 6x6 (SVF=0.36) and 7x7 (SVF=0.49) array have the same characteristics where two pair of bubbles are distributed near the rear edges. The SSLs contain large-scale turbulent eddies but cannot interact to generate vortex shedding. As SVF decreases to 0.25 and 0.09, the blocking effect is weak and no large SSLs of array occur.

Figure 3. Mean turbulent kinetic energy in the near region of different SVF: (a) 0.09, (b) 0.25, (c) 0.36, (d) 0.49, (e) 0.64; in the far region: (f) 0.09, (g) 0.25, (h) 0.36, (i) 0.49, (j) 0.64.

Figure 3(a)-3(j) describe the distribution of the turbulent kinetic energy \( k=0.5 \sum u_i u_i \) for different cases. It should be noted that the level is high inside the array with low SVF, while it is low in the far wake region. For SVF=0.64, there is a peak value region along the wake center-line, corresponding to the vortex shedding. As SVF decreases to 0.49, the SSLS is blend to the center-line but can not interact with each other. This corresponds to the flow through the array, which plays a role similar to a splitter plate, hence the movement of rolling shear layers are precluded. As SVF decreases to 0.36 and 0.25, this phenomenon is more identified and the SSLs contain large-scale structures near the rear edge. For a very low SVF=0.09, the wakes of isolated cylinders have a fully development and results in an intense wake-to-cylinder interaction.
Figure 4. Variation of drag coefficients (red) and Lift coefficients (blue): (a) 3x3 array, (b) first 3 rows of 5x5 array, (c) first 3 rows of 6x6 array, (d) first 4 rows of 8x8 array. Mean dimensionless force $\bar{F}^* / (0.5 \rho U^2 d) = C_{D,i}^* \pm C_{L,i}^*$ acting on the isolated cylinders in the array: (e) 3x3 array, (f) 5x5 array, (g) 6x6 array, (h) 8x8 array. Normalized force on isolated cylinders for a clear sight on force direction: (i) 8x8 array.

The effect of the SVF on the forces acting on the little square cylinders are discussed. Figure 4 displays the temporal variation of $C_{D,i}$ and $C_{L,i}$. When SVF=0.09, the little cylinders on the first and third rows of 3x3 array has a modulation on the cylinders between them. There is a characteristics $C_{D,i}^*$ Strouhal number St=0.108 for all little cylinders and a secondary peak St=0.091 for the cylinders in the center-line. As SVF increases, the oscillation of $C_{D,i}^*$ is suppressed and the values of $C_{D,i}$ have a sharp drop from the first column to the second column. The universal St=0.1, with a secondary peak St=0.082 except for the cylinders at the corners. The vortex shedding is slightly delayed that the major St vary from 0.108 to 0.1. As for the 6x6 array, there is a major St=0.311 for the cylinders of first column except for the corner with St=0.277. No energetic spectra values are captured for the cylinders in 8x8 array. The formation of eddies in the array is impeded by the presence of neighbouring cylinders which constrain the amplitude of the movements of SSLs. Figure 4(e)-4(h) performs the time-averaged dimensionless force vector on the individual cylinders. The length of the vector represents the force magnitude and the direction is a good indicator of a local direction of the bleeding flow in the vicinity of that cylinder. And it also indicates the flow penetrated in to the array transverse laterally. It is obviously seen that the magnitude of force acting on the first column is the largest associated with a large adverse pressure gradient. However, for high SVF (0.36 and 0.64), there are reversed forces acting on some small cylinders due to the complex flow interaction.
Figure 5. Longitudinal profiles of stream-wise velocity $\bar{U}/U_0$ (a) and root mean square of transverse velocity $V_{rms}/U_0$ along the wake center-line.

Figure 5 performs the center-line profiles of mean stream-wise velocity and root mean square of transverse velocity. An initial sharp decrease of $\bar{U}/U_0$ appears behind the obstacles except for 3x3 case with low SVF=0.09. The location of the minimum value is a good indicator of the end of steady wake region with the black solid arrows. The distance between this and the rear face of array is denoted as $L'$. And then $\bar{U}/U_0$ increases along the center-line where the growth rate increases first and the decreases strongly. The total length of wake region $L''$ is defined from the back of obstacles to a location, where the gradient of mean velocity becomes smaller than a threshold with the blue solid arrows. Figure 5 (a) performs another total wake length $L''$ with red dash arrows, where the gradient of mean velocity reaches maximum. It is obviously seen that $L''$ is smaller than $L'$. The length $L_1$ is defined as the distance of the rear face to the location of max $V_{rms}/U_0$, while $L''$ is to the location where $V_{rms}/U_0$ starts increasing towards the peak. There is a good approximation that $L'' \approx L_1$, and we usually use $L_1$ to define the total wake region rather than $L'$. It is more easy to obtain form the measurements with less ambiguity. In present study, $L_1$ increases monotonously as SVF decreases and the peak value of $V_{rms}/U_0$ is opposite.

3.2. Flow dynamics

The most characteristic feature of flow past bluff body is the Von Karman vortex shedding in the wake region. However, this phenomenon would disappear for the cylinder array if SVF is low. This critical value is different from the size and shape of the cylinder section. It is between 0.49 and 0.64 in present study. The trailing edge bleeding from the array affects the vortex formation in away consistent with the entrainment-detrainment mechanism. The combination of vorticity accumulation and decay gives rise to the small-scale dissipative flow structures. This is investigated qualitatively by the power spectrum density of monitoring points located at $X/D=1, 1.5, 2, 2.5, 3, 4, 6, 8$, at $Y/D=\pm 0.75$ and $Y/D=0$. There is no well-defined peak for the points along the center-line when the array is sparsely arranged, and the whole energy of spectrum decreases as the point away from rear face of the array. $P7(X/D=6)$ and $P8(X/D=8)$ in 8x8 array have a peak $St=0.30$ due to the vortex shedding. The PSD distributions present double major frequencies for 6x6 array ($St=0.244$ and 0.325) and 7x7 array ($St=0.183$ and 0.213), corresponding to the coherent structures in SSLs. The PSD for P1, P3, P5, P7, P8 at $Y/D=0.75$ has the same peak frequency $St=0.15$, which is half of that of P7 and P8 at $Y/D=0$. Figure 7 performs the instantaneous vortical structures for different cases with Omega criterion.
Figure 6. PSD distributions of stream-wise velocity at P1, P3, P5, P7, P8, for X/D=1, 2, 3, 6, 8, Y/D=0: (a) 5x5 array, (b) 6x6 array, (c) 7x7 array, (d) 8x8 array, for Y/D=0.75: (e) 5x5 array, (f) 6x6 array, (g) 7x7 array, (h) 8x8 array. Sample frequency is 5000Hz and sample time is 6000 time steps.

Figure 7. Visualization of flow structures in an instantaneous flow field using omega criterion: (a) 3x3 array, (b) 5x5 array, (c) 6x6 array, (d) 7x7 array, (e) 8x8 array, (f) 8x8 array far wake region

4. Conclusion
This paper performs the effect of SVF on the flow through and around a square cylinder array. The vortex shedding is suppressed when \(SVF \leq 0.49\), while the separated shear layers contain large-scale turbulent eddies with high Strouhal numbers. The SSLs of 8x8 array (SVF=0.64) generates wake billows at X/D=5, with St=0.15. The vortex formation length is elongated (for solid one it is about 1.04) but the shedding frequency is slight enhanced. The maximum of root mean square of transverse velocity along wake center-line decreases monotonously as SVF decreases, and several ways of defining the spatial extent of the wake region are discussed.

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