Possibility of higher dimensional anisotropic compact star

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Abstract We have provided here a new class of interior solutions for anisotropic stars admitting conformal motion in higher dimensional noncommutative spacetime. The Einstein field equations are solved by choosing a particular density distribution function of Lorentzian type \cite{1} under noncommutative geometry. Several cases with dimensions 4\(D\) and higher, e.g. 5\(D\), 6\(D\) and 11\(D\) have been discussed separately. An overall observation is that the model parameters, such as density, radial pressure, transverse pressure, anisotropy all are well behaved and represent a compact star with radius \(4.17\) km. However, emphasis has been given on the acceptability of the model from physical point of view. As a consequence it is observed that higher dimensions, i.e. beyond 4\(D\) spacetime, exhibit several interesting yet bizarre features which are not at all untenable for a compact stellar model of strange quark type and thus dictates a possibility of its extra dimensional existence.

Keywords General Relativity; noncommutative geometry; higher dimension; compact star

1 Introduction

To model a compact object it is generally assumed that the underlying matter distribution is homogeneous and isotropy i.e. perfect fluid obeying Tolman-Oppenheimer-Volkoff (TOV) equation. The nuclear matter of density \(\rho \sim 10^{15}\) gm/cc, which is expected at the core of the compact terrestrial object, becomes anisotropic in nature as was first argued by Ruderman \cite{2}. Later on Bowers and Liang \cite{3} showed that anisotropy might have non-negligible effects on such parameters like equilibrium mass and surface redshift. Very recently other theoretical advances also indicate that the pressure inside a compact object is not essentially isotropic in nature \cite{4,5,6,7,8,9,10}.

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Thus pressure inside a compact object can be decomposed into two components, namely, the radial pressure $p_r$ and the transverse pressure $p_t$, where $p_t$ is in the orthogonal direction to $p_r$. Now, the measure $\Delta = (p_t - p_r)$ is defined as the pressure anisotropy. If $\Delta > 0$ i.e. $p_t > p_r$ then the force is repulsive in nature whereas for $\Delta < 0$ i.e. $p_t < p_r$ the force is of attractive character. So it is reasonable to consider pressure anisotropy to develop our model under investigation.

In the recent years the extension of General Relativity in higher dimension has become a topic of great interest. As a special mention in this line of thinking we note that ‘Whether the usual solar system tests are compatible with the existence of higher spatial dimensions’ has been investigated by Rahaman et al. [11]. Some other studies in higher dimension are done by Liu and Overduin [12] for the motion of test particle whereas Rahaman et al. [13] have investigated higher dimensional gravastar.

One of the most interesting outcomes of the string theory is that the target spacetime coordinates become noncommuting operators on D-brane [14,15]. Now the noncommutativity of a spacetime can be encoded in the commutator $[x^\mu, x^\nu] = i\theta^{\mu\nu}$, where $\theta^{\mu\nu}$ is an anti-symmetric matrix and is of dimension $(\text{length})^2$ which determines the fundamental cell discretization of spacetime. It is similar to the way as the Planck constant $\hbar$ discretizes phase space [16]. So the energy density of the static spherically symmetric smeared and particle like gravitational source has the form [1]

$$\rho = \frac{M}{\pi^2 (r^2 + \phi)^{n+2}},$$

where $M$ is the total mass of the source.

In the literature lots of studies are available on noncommutative geometry, e.g. Nazari and Mehdipour [1] used Lorentzian distribution to analyze ‘Parikh-Wilczek Tunneling’ from noncommutative higher dimensional black holes. Besides this investigation some other noteworthy works are on galactic rotation curves inspired by a noncommutative-geometry background [17], stability of a particular class of thin-shell wormholes in noncommutative geometry [18], higher-dimensional wormholes with noncommutative geometry [19], noncommutative BTZ black hole [20], noncommutative wormholes [21] and noncommutative wormholes in $f(R)$ gravity with Lorentzian distribution [22].

On conformal motion there are many earlier works performed by some of us and the present one can be treated as a sequel of those studies. A class of solutions for anisotropic stars admitting conformal motion has been studied by Rahaman et al. [23]. In a very recent work Rahaman et al. [24] have also described conformal motion in higher dimensional spacetime. Charged gravastar admitting conformal motion has been studied by Usmani et al. [25] whereas relativistic stars admitting conformal motion has been analyzed by Rahaman et al. [26]. Inspired by these earlier works on conformal motion we are looking forward for a new class of solutions of anisotropic stars under the framework of General Relativity with noncommutative spacetime.

It is familiar to search for the natural relationship between geometry and matter through the Einstein field equations where it is very convenient to use inheritance symmetry. The well known inheritance symmetry is the symmetry under conformal Killing vectors (CKV) i.e.

$$L_\xi g_{ik} = \psi g_{ik}$$

(2)
where \( L \) is the Lie derivative of the metric tensor, which describes the interior gravitational field of a stellar configuration with respect to the vector field \( \xi \), and \( \psi \) is the conformal factor. It is supposed that the vector \( \xi \) generates the conformal symmetry and the metric \( g \) is conformally mapped onto itself along \( \xi \). It is to note that neither \( \xi \) nor \( \psi \) need to be static even though one consider a static metric \[27, 28\]. We also note that (i) if \( \psi = 0 \) then Eq. (2) gives the Killing vector, (ii) if \( \psi = \) constant it gives homothetic vector and (iii) if \( \psi = \psi(x, t) \) then it yields conformal vectors. Moreover it is to be mentioned that for \( \psi = 0 \) the underlying spacetime becomes asymptotically flat which further implies that the Weyl tensor will also vanish. So CKV provides a deeper insight of the underlying spacetime geometry.

The plan of our present investigation is as follows: in Section 2 we have formulated the Einstein field equations for the interior spacetime of the anisotropic star. In Section 3 we have solved the Einstein field Equations by using the density function of Lorentzian distribution type in higher dimensional spacetime as given by Nozari and Mehdipour \[1\]. We have considered the cases \( n = 2, 3, 4 \) and \( 9 \) i.e. \( 4D \), \( 5D \), \( 6D \) and \( 11D \) spacetime in Section 4 to find out expressions for physical parameters. In Section 5 we have explored physical properties with interesting features of the model and presented them with graphical plots for comparative studies among the results of different dimensional spacetime. Finally the paper has been finished with some concluding remarks in Section 6.

2 The interior spacetime and the Einstein field equations

To describe the static spherically symmetric spacetime (in geometrical unit \( G = 1 = c \) here and onwards) in higher dimension the line element can be given in the standard form

\[
ds^2 = -e^\nu(r)dt^2 + e^\lambda(r)dr^2 + r^2d\Omega_n^2,
\]

where

\[
d\Omega_n^2 = d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2 + \sin^2 \theta_1 \sin^2 \theta_2 d\theta_3^2 + ... + \prod_{i=1}^{n-1} \sin^2 \theta_i d\theta_n^2.
\]

The energy momentum tensor for the matter distribution can be taken in its usual form \[29\]

\[
T^\mu_\nu = (\rho + p_r)u^\mu u_\nu - p_r g^\mu_\nu + (p_t - p_r)\eta^\mu_\nu,
\]

with \( u^\mu u_\mu = -\eta^\mu_\mu = 1 \).

Now for \( n \geq 2 \) dimensional spacetime the Einstein equations can be supplied as \[30,31\]

\[
e^{-\lambda} \left[ \frac{n \lambda'}{2r} - \frac{n(n - 1)}{2r^2} \right] + \frac{n(n - 1)}{2r^2} = 8\pi \rho,
\]

\[
e^{-\lambda} \left[ \frac{n(n - 1)}{2r^2} + \frac{nu'}{2r} \right] - \frac{n(n - 1)}{2r^2} = 8\pi p_r,
\]
\[
\frac{1}{2} e^{-\lambda} \left[ \frac{1}{2} (\nu')^2 + \nu'' - \frac{1}{2} \lambda' \nu' + \frac{(n-1)}{r} (\nu' - \lambda') + \frac{(n-1)(n-2)}{r^2} \right] - \frac{(n-1)(n-2)}{2r^2} = 8\pi p_t, \tag{8}
\]

where \( \rho \) is the energy density, \( p_r \) and \( p_t \) are respectively the radial and transverse pressure of the fluid. Here \( \tau \) denotes the differentiation with respect to the radial parameter \( r \).

3 The solution under conformal Killing vector

The conformal Killing vector, as given in Eq. (2), can be written in a more convenient form:

\[
L_\xi g_{ik} = \xi_i g_{ik} + \xi_k g_{ij} = \psi g_{ik}. \tag{9}
\]

Now the conformal Killing equation for the line element (3) gives the following equations:

\[
\xi^1 \nu' = \psi, \tag{10}
\]

\[
\xi^{n+2} = C_1, \tag{11}
\]

\[
\xi^1 = \frac{\psi r}{2}, \tag{12}
\]

\[
\xi^1 \lambda' + 2 \xi^1,1 = \psi, \tag{13}
\]

where \( C_1 \) is a constant.

The above equations consequently gives

\[
e^\nu = C_2^2 r^2, \tag{14}
\]

\[
e^\lambda = \left( \frac{C_3}{\psi} \right)^2, \tag{15}
\]

\[
\xi^i = C_1 \delta^i_{n+2} + \left( \frac{\psi r}{2} \right) \delta^i_1, \tag{16}
\]

where \( C_2 \) and \( C_3 \) are constants of integrations.

Hence Eqs. (6)-(8) take the following form

\[
\frac{n(n-1)}{2\tau^2} \left[ 1 - \psi^2 \frac{2}{C_3} \right] - \frac{m\psi\psi'}{rC_3^2} = 8\pi\rho, \tag{17}
\]

\[
\frac{n}{2\tau^2} \left[ \frac{(n+1)\psi^2}{C_3} - (n-1) \right] = 8\pi p_r, \tag{18}
\]

\[
\frac{m\psi\psi'}{rC_3^2} + n(n-1)\psi^2 \frac{2r^2C_3}{2\tau^2} - \frac{(n-1)(n-2)}{2r^2} = 8\pi p_t. \tag{19}
\]
We thus have three independent equations with four unknowns. So we are free to choose any physically reasonable ansatz for any of these four unknown. Hence we choose density profile \( \rho \) in the form given in Eq. (1) in connection to higher dimensional static and spherically symmetric Lorentzian distribution of smeared matter as provided by Nozari and Mehdipour [1]. This density profile will be employed as a key tool in our present study.

Therefore, solving Eq. (17) and substituting Eq. (1) therein, we get

\[
\psi^2 = C_3^2 - \frac{16MC_3^2\sqrt{\phi}}{n\pi} \frac{1}{r^{n-1}} \int \frac{r^n}{(r^2 + \phi)^{\frac{n+2}{2}}} dr + \frac{A}{r^{n-1}},
\]  

where \( A \) is a constant of integration that can be later on found out under suitable boundary condition.

### 4 Exact solutions of the models in different dimensions

The above set of equations are associated with dimensional parameter \( n \) and hence to get a clear picture of the physical system under different spacetime we are interested to study several cases starting from standard 4D to higher 5D, 6D and 11D as shown below.

#### 4.1 Four dimensional spacetime \((n = 2)\)

The conformal parameter \( \psi(r) \) and the metric potential \( e^\lambda \) are given by

\[
\psi = \sqrt{C_3^2 - \frac{4MC_3^2\sqrt{\phi}}{\pi r} \left[ \frac{1}{\sqrt{\phi}} \arctan \left( \frac{r}{\sqrt{\phi}} \right) - \frac{r}{r^2 + \phi} + \frac{A}{r} \right]},
\]  

\[
e^{-\lambda} = 1 + \frac{A}{C_3^2 r} - \frac{4M\sqrt{\phi}}{\pi r} \left[ \frac{1}{\sqrt{\phi}} \arctan \left( \frac{r}{\sqrt{\phi}} \right) - \frac{r}{r^2 + \phi} \right].
\]

The radial and transverse pressures are obtained as

\[
p_r = \frac{1}{8\pi r^2} \left[ 2 + \frac{3A}{C_3^2 r} - \frac{12M\sqrt{\phi}}{\pi r} \left[ \frac{1}{\sqrt{\phi}} \arctan \left( \frac{r}{\sqrt{\phi}} \right) - \frac{r}{r^2 + \phi} \right] \right],
\]

\[
p_t = \frac{1}{8\pi} \left[ \frac{1}{r^2} - \frac{8M\sqrt{\phi}}{\pi(r^2 + \phi)^2} \right].
\]

To find the above constant of integration we impose the boundary condition \( p_r(r = R) = 0 \), which gives

\[
A = \frac{4MC_3^2\sqrt{\phi}}{\pi} \left[ \frac{1}{\sqrt{\phi}} \arctan \left( \frac{R}{\sqrt{\phi}} \right) - \frac{R}{R^2 + \phi} \right] - \frac{2}{3}C_3^2 R.
\]
4.2 Five dimensional spacetime \((n = 3)\)

In this case the solution set can be obtained as follows:

\[
\psi = \sqrt{\frac{C_3^2}{3} + \frac{16MC_3^2\sqrt{\phi}}{9\pi r^2} \frac{3r^2 + 2\phi}{(r^2 + \phi)^2}} + \frac{A}{r^2}, 
\]

\[(26)\]

\[
e^{-\lambda} = 1 + \frac{16M\sqrt{\phi}}{9\pi r^2} \frac{3r^2 + 2\phi}{(r^2 + \phi)^2} + \frac{A}{C_3^2 r^2}, \]

\[(27)\]

\[
p_r = \frac{3}{8\pi r^2} \left[ 1 + \frac{32M\sqrt{\phi}}{9\pi r^2} \frac{3r^2 + 2\phi}{(r^2 + \phi)^2} + \frac{2A}{C_3^2 r^2} \right], \]

\[(28)\]

\[
p_t = \frac{1}{8\pi} \left[ \frac{2}{r^2} - \frac{8M\sqrt{\phi}}{\pi (r^2 + \phi)^2} \right], \]

\[(29)\]

with

\[
A = -\frac{C_3^2 R^2}{2} \left[ 1 + \frac{32M\sqrt{\phi}}{9\pi R^2} \frac{3R^2 + 2\phi}{(R^2 + \phi)^2} \right].
\]

\[(30)\]

4.3 Six dimensional spacetime \((n = 4)\)

Here the solutions are as follows:

\[
\psi = \sqrt{\frac{C_3^2}{3} - \frac{MC_3^2\sqrt{\phi}}{2\pi r^2} \left[ \frac{3}{\sqrt{\phi}} \tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) - \frac{5r^3 + 3r\phi}{(r^2 + \phi)^2} \right] + \frac{A}{r^2},
\]

\[(31)\]

\[
e^{-\lambda} = 1 - \frac{M\sqrt{\phi}}{2\pi r^2} \left[ \frac{3}{\sqrt{\phi}} \tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) - \frac{5r^3 + 3r\phi}{(r^2 + \phi)^2} \right] + \frac{A}{C_3^2 r^2}, \]

\[(32)\]

\[
p_r = \frac{1}{4\pi r^2} \left[ 2 - \frac{5M\sqrt{\phi}}{2\pi r^2} \left[ \frac{3}{\sqrt{\phi}} \tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) - \frac{5r^3 + 3r\phi}{(r^2 + \phi)^2} \right] + \frac{5A}{r^2 C_3^2} \right],\]

\[(33)\]

\[
p_t = \frac{1}{8\pi} \left[ \frac{3}{r^2} - \frac{8M\sqrt{\phi}}{\pi (r^2 + \phi)^2} \right], \]

\[(34)\]

and

\[
A = \frac{MC_3^2 \sqrt{\phi}}{2\pi} \left[ \frac{3}{\sqrt{\phi}} \tan^{-1} \left( \frac{R}{\sqrt{\phi}} \right) - \frac{5R^3 + 3r\phi}{(R^2 + \phi)^2} \right] - \frac{2C_3^2 R^3}{5}.
\]

\[(35)\]
4.4 Eleven dimensional spacetime \((n = 9)\)

For this arbitrarily chosen higher dimension the solutions can be obtained as

\[
\psi = \sqrt{C_3^2 + \frac{16M\sqrt{\phi}}{2835\pi r^8} (315r^8 + 840r^6\phi + 1008r^4\phi^2 + 576r^2\phi^3 + 128\phi^4)} + \frac{A}{r^8},
\]

\( (36) \)

\[
e^{-\lambda} = 1 + \frac{16M\sqrt{\phi}}{2835\pi r^8} (315r^8 + 840r^6\phi + 1008r^4\phi^2 + 576r^2\phi^3 + 128\phi^4)} + \frac{A}{C_3^2r^8},
\]

\( (37) \)

\[
p_r = \frac{9}{8\pi r^2} \left[ 1 + \frac{16M\sqrt{\phi}}{567\pi r^8} (315r^8 + 840r^6\phi + 1008r^4\phi^2 + 576r^2\phi^3 + 128\phi^4)} + \frac{5A}{C_3^2 r^8} \right],
\]

\( (38) \)

and

\[
A = \frac{C_3^2}{3} \left[ R^8 + \frac{16M\sqrt{\phi}}{567\pi} (315R^8 + 840R^6\phi + 1008R^4\phi^2 + 576R^2\phi^3 + 128\phi^4)} (R^2 + \phi)^\frac{2}{3} \right].
\]

\( (40) \)

---

**Fig. 1** The graphical plot for radial pressure vs radius which has a definite cut-off at 4.17 km

Let us first turn up to find out the structure of the stellar model under consideration, i.e. whether it is a normal star or something else. To do so, primarily as a dry test, we try to figure out the radius of the stellar configuration. It is to note that in Eqs. \((30), (35)\) and \((40)\) the radius of the star \(R\) has been mentioned under
the boundary condition, \( p_r(R) = 0 \), i.e. we get analytical results in the respective cases. So it seems that we can proceed on without further plot descriptions to get \( p_r(R) = 0 \) for all dimensions. However, for 4D case it reveals that the radius of the star is very small with a numerical value of 4.17 km (Fig. 1). This obviously then indicates that the star is nothing but a compact object (see Table 1 of all the Refs. [10,32,33] for comparison with the radius of some of the real compact stars).

5 A comparative study of the physical features of the model

Let us now carry out a comparative study of the physical features based on the solutions set obtained in the previous Section 4. This can be done in different ways. However, in the present investigation the best method we may adopt for comparative study, firstly, in connection to stability of the models for different dimensions which may be considered as most crucial one and secondly, for other physical parameters viz., density, pressures, pressure anisotropy, pressure gradient, conformal parameter and metric potential.

5.1 Stability of the stellar configuration

The Generalized Tolman-Oppenheimer-Volkoff (TOV) equation can be written in the form

\[
- \frac{M_G(r)(\rho + p_r)}{r} \frac{d\rho}{dr} - \frac{dp_r}{dr} + \frac{2}{r}(p_t - p_r) = 0,
\]

(41)

where \( M_G(r) \) is the gravitational mass within the radius \( r \) and is given by

\[
M_G(r) = \frac{1}{2} \sqrt{\nu \nu'},
\]

(42)

Substituting the value of \( M_G(r) \) in above equation we get

\[
- \frac{\nu'}{2}(\rho + p_r) - \frac{dp_r}{dr} + \frac{2}{r}(p_t - p_r) = 0.
\]

(43)

The above TOV equation describe the equilibrium of the stellar configuration under gravitational force \( F_g \), hydrostatic force \( F_h \) and anisotropic stress \( F_a \) so that we can write it in the following form:

\[
F_g + F_h + F_a = 0,
\]

(44)

where

\[
F_g = - \frac{\nu'}{2}(\rho + p_r),
\]

\[
F_h = - \frac{dp_r}{dr},
\]

\[
F_a = \frac{2}{r}(p_t - p_r).
\]

(45)

We have shown the plots of TOV equations for 4D, 5D, 6D and 11D spacetime in Fig. 2. From the plots it is overall clear that the system is in static equilibrium.
under three different forces like gravitational, hydrostatic and anisotropic, e.g. in the case of 4D to attain equilibrium hydrostatic force is counter balanced jointly by gravitational and anisotropic forces. In 5D also the situation is exactly same, only the difference being in the radial distances. In 4D it is near about on 5 whereas in 5D it is near about on the scale 8. This distance factor can also be observed the same in the higher dimensional spacetime though the balancing features between three forces are clearly different there in the respective cases.

5.2 Anisotropy of the model

We have shown the possible variation of radial and transverse pressures in Fig. 3 (top left and right of the panel). Hence the measure of anisotropy $\Delta = (p_t - p_r)$
in 4, 5, 6 and 11 dimensional cases are respectively given as

$$\Delta_4 = \frac{1}{8\pi} \left[ \frac{12M\sqrt{\phi}}{\pi r^3} \left\{ \frac{1}{\sqrt{\phi}} \arctan \left( \frac{r}{\sqrt{\phi}} \right) - \frac{r}{r^2 + \phi} \right\} - \frac{1}{r^2} - \frac{8M\sqrt{\phi}}{\pi (r^2 + \phi)^2} - \frac{3A}{C_4^2 r^3} \right],$$  \hspace{1cm} (46)$$

$$\Delta_5 = \frac{1}{8\pi} \left[ \frac{32M\sqrt{\phi}}{3\pi r^4} \left\{ \frac{3r^2 + 2\phi}{(r^2 + \phi)^{3/2}} \right\} - \frac{1}{r^2} - \frac{8M\sqrt{\phi}}{\pi (r^2 + \phi)^{3/2}} + \frac{6A}{C_4^2 r^4} \right],$$  \hspace{1cm} (47)$$

$$\Delta_6 = \frac{1}{8\pi} \left[ \frac{5M\sqrt{\phi}}{\pi r^5} \left\{ \frac{3}{\sqrt{\phi}} \tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) - \frac{5r^3 + 3r\phi}{(r^2 + \phi)^2} \right\} - \frac{1}{r^2} - \frac{8M\sqrt{\phi}}{\pi (r^2 + \phi)^3} - \frac{10A}{r^3 C_4^{10}} \right],$$  \hspace{1cm} (48)$$

$$\Delta_{11} = \frac{1}{8\pi} \left[ \frac{M\sqrt{\phi}}{4\pi r^{11}} \left\{ \frac{315r^8 + 840r^6\phi + 1008r^4\phi^2 + 576r^2\phi^3 + 128\phi^4}{(r^2 + \phi)^{3/2}} \right\} - \frac{1}{r^2} - \frac{8M\sqrt{\phi}}{\pi (r^2 + \phi)^{3/2}} + \frac{45A}{C_4^{10} r^{11}} \right].$$  \hspace{1cm} (49)$$

All these are plotted in Fig. 3 (bottom panel). From all the plots we see that $$\Delta < 0$$ i.e., $$p_t < p_r$$ and hence anisotropic force is attractive in nature. A detailed study shows that firstly, in every case of different dimension the measure of anisotropy is a decreasing function of $$r$$. Secondly, from 4D onward measure of anisotropy is increasing and via 11D attaining maximum at 5D. However, surprisingly, it is very high and comparable to 4D for 11D spacetime. This observation therefore dictates that 4D configuration represents almost a spherical object as departure from isotropy is very less than the higher dimensional spacetime.

Moreover, in all the above cases of different dimension one can note that the pressure gradient $$\frac{dp}{dr}$$ is a decreasing function of $$r$$ (see Fig. 4).

### 5.3 Compactness and redshift of the star

At the end of previous Sec. we did a dry test to get a preliminary idea about the structure of the star under consideration and seen that the star actually represents a compact object with a radius 4.17 km. However, for further test of confirmation one can perform a wet test by calculating the compactness factor $[10,32,33]$. To do so first we define gravitational mass of the system of matter distribution as follows:

$$M(r) = \int_0^r \left[ \frac{2\pi^{d+1}}{\Gamma \left( \frac{d+1}{2} \right) \rho} \right] r^d \rho dr. \hspace{1cm} (50)$$

Therefore, the compactness factor and surface redshift of the star can be respectively given by

$$u(r) = \frac{M(r)}{r}, \hspace{1cm} (51)$$

$$z_s = 1 - 2u r^{-1/2} - 1. \hspace{1cm} (52)$$

Hence for different dimension we can calculate the expressions for the above parameters as follows:
Fig. 3 Physical parameters are shown against $r$ for different dimensional spacetime in the above panel: The radial pressure $p_r$ (top left), the transverse pressure $p_t$ (top right) and the anisotropy $\Delta = (p_t - p_r)$ (bottom)

For $n=2$:

$$m(r) = \frac{2M}{\pi} \left[ \tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) - \frac{r\sqrt{\phi}}{r^2 + \phi} \right], \quad (53)$$

$$u(r) = \frac{2M}{\pi r} \left[ \tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) - \frac{r\sqrt{\phi}}{r^2 + \phi} \right], \quad (54)$$

$$z_s = \left[ 1 - \frac{4M}{\pi r} \left( \tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) - \frac{r\sqrt{\phi}}{r^2 + \phi} \right) \right]^{-\frac{1}{2}} - 1. \quad (55)$$

For $n=3$:

$$m(r) = \frac{2M}{3} \left[ 2 - \frac{(3r^2 + 2\phi)\sqrt{\phi}}{(r^2 + \phi)^{\frac{3}{2}}} \right], \quad (56)$$

$$u(r) = \frac{2M}{3 r} \left[ 2 - \frac{(3r^2 + 2\phi)\sqrt{\phi}}{(r^2 + \phi)^{\frac{3}{2}}} \right], \quad (57)$$
Fig. 4 Pressure gradient vs radius relation is shown in the plot for the specified range

\[ z_s = \left[ \frac{M - 6 \phi}{3} \left( 2 - \frac{(3r^2 + 2 \phi)\sqrt{\phi}}{(r^2 + \phi)^{3/2}} \right) \right]^{-\frac{1}{2}} - 1. \] (58)

For \( n = 4 \):

\[ m(r) = \frac{M}{3} \left[ \frac{1}{r^2} \left( r \frac{r\sqrt{\phi}(5r^2 + 3\phi)}{3(r^2 + \phi)^2} \right) \right], \] (59)

\[ u(r) = \frac{M}{3} \left[ \frac{1}{r} \left( r \frac{\sqrt{\phi}(5r^2 + 3\phi)}{3(r^2 + \phi)^2} \right) \right], \] (60)

\[ z_s = \left[ 1 - 2M \left( \frac{1}{r} \frac{1}{\sqrt{\phi}} \left( r \sqrt{\phi}(5r^2 + 3\phi) \right) \right) \right]^{-\frac{1}{2}} - 1. \] (61)

For \( n = 9 \):

\[ m(r) = \frac{M}{3780} \left[ \frac{128 \phi^4 + 576 \phi^3 r^2 + 1008 \phi^2 r^4 + 840 \phi^6 + 315 r^8}{(r^2 + \phi)^{12/2}} \right], \] (62)

\[ u(r) = \frac{M}{3780r} \left[ \frac{128 \phi^4 + 576 \phi^3 r^2 + 1008 \phi^2 r^4 + 840 \phi^6 + 315 r^8}{(r^2 + \phi)^{12/2}} \right], \] (63)

\[ z_s = \left[ 1 - \frac{M}{1890} \left( \frac{128 \phi^4 + 576 \phi^3 r^2 + 1008 \phi^2 r^4 + 840 \phi^6 + 315 r^8}{(r^2 + \phi)^{12/2}} \right) \right]^{-\frac{1}{2}} - 1. \] (64)

The nature of variation of the above expressions for compactness factor and surface redshift of the star can be seen in the Fig. 5 in the left and right panel respectively for all the values of \( n \).

It is observed from the Fig. 5 (left panel) that compactness factors for different dimension are gradually increasing with decreasing \( n \) and maximum for 4D spacetime. Thus, very interestingly, at the center the star is most dense for 4-dimension
Fig. 5 Compactness factor (left) and surface redshift (right) are shown in the plot for the specified range

with a very small yet definite core whereas there seems does not exist any core at all for $11D$ spacetime.

We note that in connection to isotropic case and in the absence of the cosmological constant it has been shown for the surface redshift analysis that $z_s \leq 2^{34,35,36}$. On the other hand, Böhmer and Harko [36] argued that for an anisotropic star in the presence of a cosmological constant the surface redshift must obey the general restriction $z_s \leq 5$, which is consistent with the bound $z_s \leq 5.211$ as obtained by Ivanov [37]. Therefore, for an anisotropic star without cosmological constant the above value $z_s \leq 1$ is quite reasonable as can be seen in the 4D case (Fig. 5, right panel) [32]. In the other cases of higher dimension the surface redshift values are increasing and seem to be within the upper bound [37].

5.4 Some other physical parameters

In this subsection we have shown the panel of the plots for the conformal parameter $\psi(r)$ (top left), the metric potential $e^\lambda$ (top right) and the density $\rho$ (bottom) for 4 and extra dimensional spacetime (Fig. 6). It is observed that for all the physical parameters the features are as usual for 4D, however for extra dimension they take different shapes. A special mention can be done for density where central densities are abruptly decreasing as one goes to increase dimension. Thus, from the plot it reveals that central density is maximum for 4D whereas it is minimum for 11D spacetime showing most compactness of the star for standard 4-dimension. Note that this same result also we observe in Fig 5 (left panel).

6 Conclusion

In the present paper we have studied thoroughly a set of new interior solutions for anisotropic star admitting conformal motion in higher dimensional noncommutative spacetime. Under this spacetime geometry the Einstein field equations are
Fig. 6 Physical parameters are shown against $r$ for different dimensional spacetime in the above panel: The conformal parameter $\psi(r)$ (top left), the metric potential $e^\lambda$ (top right) and the density $\rho$ (bottom)

solved by choosing a particular Lorentzian type density distribution function as proposed by Nozari and Mehdipour [1]. The studies are conducted not only with standard 4D dimensional spacetime but also for three special cases with higher dimension, such as 5D, 6D and 11D. In general it is noted that the model parameters e.g. matter-energy density, radial as well as transverse pressures, anisotropy and others show physical behaviours which are mostly regular throughout the stellar configuration.

Also it is specially observed that the solutions represent a star with radius 4.17 km which fall within the range of a compact star [10,32,33]. However, it has been shown that for a strange star of radius 6.88 km surface redshift turns out to be $Z_s = 0.530334$ [52] whereas the maximum surface redshift for a strange star $HerX - 1$ of radius 7.7 km is 0.022 [33] and that for a compact star 4U 1820 – 30 of radius 10 km turns out to be again 0.022 [10]. Therefore it seems that our compact star may be a strange quark star.

However, through several mathematical case studies we have given emphasis on the acceptability of the model from physical point of view for various structural
aspects. As a consequence it is observed that for higher dimensions, i.e. beyond 4D spacetime, the solutions exhibit several interesting yet bizarre features. These features seem physically not very unrealistic.

Thus, as a primary stage, the investigation indicates that compact stars may exist even in higher dimension. But before placing a demand in favour of this highly intriguing issue of compact stars with extra dimension we need to perform some more specific study and to look at the diversified technical aspects related to higher dimensional spacetime of a compact star. However, in the literature there are some evidences available in favour of ‘Extra Dimensions in Compact Stars’.

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