Probabilistic Approach to the Separation of Grain Material

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Abstract. Possible ways to improve operating elements of grain cleaning machines due to mobilization factors directly influence on qualitative and quantitative aspects of separation are presented. In relation to sieve grain cleaning machine the technological process of grain mixture separation on the sieve is presented as a block diagram. Some variants of mathematical description (models) consideration of grain material separation process are also given. The dependencies between parameters of grain material and uniformity of seed distribution on sieve area are given. Calculation formulas and schemes of definition probability of a particle entering the hole are presented. It is established, that in the range $\beta$ from 0 to 30° probability of particle passage increases from 0.315 to 0.548, that is 1.74 times.

1. Introduction

The Russian Federation is one of the largest producer of grain crops. According to the Government program of agricultural development and regulation of agricultural production, raw material and food in 2013 – 2020 years, gross yield of cereals and legumes on the farms of all categories must reach 135 million tons and provide stable grain export 40–50 million tons in a year.

Qualitative and quantitative grain safety depends upon providing the enterprises of agro-industrial complex with modern equipment for the post-harvest treatment, its technical level and effective usage. Foreign grain cleaning machines of different models have proven themselves well for rural manufacturers but not every company can afford to purchase equipment as the price level of foreign machines and other costs associated with their operation in the current economic environment is not favor of buying machines of foreign brands.

Post-harvest treatment of grain is carried out by production grain cleaning lines through which the grain pass after combine harvesting. Characteristics of freshly cleaned grain heap substantially affect the quality of grain cleaning machines operation. That is why agricultural enterprises of our country need innovative grain cleaning machines for post-harvest treatment of grain. The problem is particularly acute in the conditions of the Western Siberia, where the harvesting and post-harvesting treatment of grain is carried out under high humidity.

Undertaken review and analysis of literature show that at present time in creating new ways of grain cleaning and development of progressive attachments for grain cleaning machines there is no cardinal change for the better [1,2,3,4,5].

The efficiency of grain cleaning machines operation depends largely upon the ratio regimes of sieve mill movement close to optimal (passport). Study the influence of parameters of the sieve mill fluctuations and the detection of dependencies between the parameters of grain material and uniformity of seed distribution on sieve area on separation process of grain material allows to identify ways for increase the productiveness of grain cleaning machines, to reduce terms and to decrease the cost of grain treatment.

From the main ways of increasing the productiveness machines you can select the following: definition of dependencies between the parameters of grain material and uniformity of seed distribution on sieve area, theoretical substantiation of increasing rate movement of grain on the surface, complication of the fluctuation law of the sieve mills, detection of particles interaction on the sieve surface.
2. Definition of dependencies between the parameters of grain material and uniformity of seed distribution on sieve area.

The main task of theoretical study reduces to dependencies detection between the parameters of grain material and uniformity of seed distribution on sieve area with the aim to define rational performances of sieve mill operation.

Theoretical studies of the proceeding phenomena, namely technological process of separation is presented in the form of investigation object, which is characterized by input and output parameters [6].

In relation to sieve grain cleaning machine the technological process of grain mixture separation on the sieve is presented as a block diagram in figure 1.

![Block diagram of technological process](image)

Parameters group \( \mathbf{U} \) is output data characterizing the technological process of separation. In relation to sieve grain cleaning machine it will be the productiveness, completeness of division, power input, seed injury, reliability etc.

Parameters groups \( \mathbf{X}, \mathbf{Y} \) and \( \mathbf{Z} \) are input parameters. Parameters group \( \mathbf{X} \) characterizes the properties of grain material received on the sieve: content of grain mixture, sizes, configuration, moisture, mass of particles, delivery etc.

Parameters group \( \mathbf{Y} \) characterizes construction parameters of the separator. They include: sizes, sieve material, form and sizes of hole, form and sizes of jumpers, installation angle of the sieve etc.

Parameters group \( \mathbf{Z} \) matches to the kinematic regime of separator drive (law of vibrations, oscillation frequency and amplitude etc.).

The task of theoretical investigation is to establish patterns of relationships between input and output parameters of object.

If input influences on the object are not changed over time and space and output parameters are also unchanged, the process is called stationary.

If only one input influence is changed over time and space, the process of corresponding change of output object parameters is called dynamic, at the end time of transition process it became stationary.

Mathematical model of technological process is made for theoretical study of statistical and dynamic characteristics of object.

For mathematical description of investigated model processes at probabilistic approach the most efficient is the mathematical apparatus of substance kinematic theory. Herewith separate grains of bulk material interprets as a specific molecules associated with some interaction forces.

The scientists, working on kinematic theory of separation are V.M. Tsetsinovsky, E.A. Nepomnyashy, A.A. Russadin and others [7].

V.M. Tsetsinovsky obtained generalized equation of separation kinematic (1):
\[ \varepsilon = 1 - \exp\left(-\int_0^1 P_S \, dt\right), \]

where \( \varepsilon \) – completeness of division;

\[ P_S = f(t) \] – function of the separation length \( t \), defining the intensity of division process in dependence upon the working regime of separator and physical components properties of the mixture.

It is established that the value \( P_s \) for some separators (sieve, trier, pneumatic separator) at relatively low loads and constancy of initial mixture content does not depend on time \( t \) and for this separator is constant value, that is \( P_s = k \). In this case the equation kinematics (1) can be represented as follows:

\[ \varepsilon = 1 - e^{-kt} \] (2)

From the equation (2) it follows that separation process is asymptotic, complete extraction separated component in simple separator is only possible with a long separation \( t = \infty \). In real separators the separation length is measured in seconds and that is why complete separation practically does not occur.

The equation kinematics (2) applies to the cleaning conditions of grain mixture on the sieve, at low loads on the sieve and the movement of grain in a thin layer. That is why it can be used in separation of small parties of grain, or by laboratory classifier, as the coefficient \( k \) is defined by resistance to the passage of small particles from sieve side.

E.A. Nepomnyashy considered the separation as a random markov process and used it for description Kolmogorov – Fokker – Plank’s equation for multidimensional density of probability which is converted by the author to differential equation of the process (3):

\[ \frac{\partial \omega}{\partial t} = \frac{b}{2} \frac{\partial^2 \omega}{\partial z^2}, \]

where \( z \) – generalized particle coordinate;

\( \omega(t_0, z_0; t, z) \) – the density of probability distribution defining the probability that random value \( z(t) \) at time \( t \) will take the meaning \( z < z(t) < z + dz \), if at time \( t_0 \) it was equal to \( z(t_0) \);

\( b \) – the constant coefficient which is the measure of movement disorder of particles under random influence and defines the intensity of mixing.

The task of this study is to define the qualitative dependencies of coefficients, characterized screening of small particles from parameters of process and to check pattern by comparing with reliably established physics of the phenomena.

Herewith it is established that coefficient dependency of screening from layer height is very weak. It is quite natural since according to E.A. Nepomnyashy’s opinion, this coefficient defines the passage rate of grain through the sieve and must not depend on layer thickness of grain on the sieve. So, the author admits that this coefficient is proportional to difference of hole sieve sizes and grains, initial grain concentration and depends on living area of sieve. Nature of obtained dependency of mixing coefficient from layer height does not correspond its physical content. This phenomena is explained by the extraction value depending on clogging of sieve holes during the experiments.

M.V. Kuzmin suggested another model of separation process in structural-logical approach. It means that state of the process is taken logically possible outcomes of which can be any particle of the mixture (figure 2.).

The author comes to decision that the intensity of selection material is characterized by indicator:

\[ P'_4 = \frac{\lambda}{2} t^2 e^{-\lambda t}, \]

where \( t \) – selection time;

\( \lambda \) – value defining outcome of event in corresponding direction.

Maximum meaning \( P'_4 \) is taken when \( t = 2 / \lambda \), that is for intensification of separation process it is necessary to increase the meaning \( \lambda \).
Figure 2 – Scheme of structural-logical approach in relation to separation process of grain (at the threshing unit, straw rack, sieve, trier), suggested by M.V. Kuzmin: \( A_0 \) – the grain with ear; \( A_1 \) – the grain without ear in a heap layer; \( A_2 \) – the grain is on the sieve surface; \( A_3 \) – the grain is near sieve hole; \( A_4 \) – the grain passed through separating attachment.

Despite the author’s logic reasoning, this theory is difficult to use since it is unknown what methods you can find the numerical value \( \lambda \).

B.D. Papin’s research is of particular interest as it is based on queuing theory. The subject of queuing theory is finding probabilistic characteristics of queuing theory system [7].

Transitions from one state to another occur under the influence of random flow of applications \([\lambda(t)]\) and release flow \([\mu(t)]\). Flow of applications is the simplest and has exponential law of timing:

\[
f(\lambda(t)) = \lambda \exp(-\lambda t),
\]

where \( \lambda \) – intensity of flow applications;
\( t \) – current time

Release flow is also the simplest:

\[
f_r(\mu(t)) = \mu \exp(-\mu t),
\]

where \( \mu \) – the density of release flow individual for each type of separators.

By applications the author means particles amount of passage fraction taking part in separation process. Release flow - particles amount of passage fraction, for one reason or another were not able to pass through the holes. The author mathematically described the character of objective relation of probabilistic characteristics of different separation process in high and low concentrations of grain flow on operating elements.

Thus, any model is an abstraction of real process that must possess possibility to use for prediction the process where the experimental data are absent; possibility to highlight the main characteristics of the process.

The conclusions obtained during the analysis of process model must not contradict known confirmed by practice theories but must expand the knowledge in the field of study. Choice of investigation method and mathematical model depend upon the laboriousness of theoretical study and reliability of obtained results.

3. Probability penetration of particles into the sieve oblong hole

Standard sieve has oblong holes located on the long side in direction of longitudinal sieve axis which corresponds with grain flow movement.

Particles of grain heap entering the sieve located on it at random. Moreover two variants are possible. The first – when the grain falls on the hole area, then at a certain position of the particle regarding the hole, it can pass through hole.

The second situation, when the grain hits the jumper. In this case, during vibrations of the sieve regarding grain movement will be parallel to the longitudinal edges of the holes and if grain center of gravity is located on the jumper area, then it can’t get in hole zone. Thus, the particles located on the jumpers do not participate in separation. They can get hole zone in the result of random process of interaction with other particles or in the case of special sieve construction, for example with jumpers having cross – section as a triangle or circle.

Therefore, in order to pass through the sieve hole, the particle must be in hole zone and located regarding holes edges definitely. Considering these events as independent, probability passage of particle through the sieve hole will be equal to:

\[
P = P_\gamma \cdot P_\beta ,
\]

where \( P \) – probability passage of particle through the sieve hole;
\( P_\gamma \) – probability of «pass» position of particle on hole edges;
\( P_\beta \) – probability of interaction particle with hole edges due to the trajectory of particle movement on the jumper.

For definition of particle position in which its passage through the sieve hole is not difficult, we take into consideration the design scheme (figure 3). We consider the grain as a form of ellipsoidal oval where large axis equals to the grain length \( a \), and small axis equals to the grain thickness \( b \), central profile is the oval from conjugate radius arcs \( R_1 \) and \( R_2 \). Radiuses value \( R_1 \) and \( R_2 \) at given sizes of large and small oval axis can be calculated and defined graphically [8]. Grain center of gravity coincides with its geometric center \( O \).

![Figure 3 - Design scheme of mutual position of grain and sieve hole](image)

For the accepted conditions limit position of grain regarding sieve hole according to the passage condition through it when grain center of gravity will be located on one longitudinal edge of the hole and the second longitudinal edge of the hole will be tangent to the curve forming ellipsoidal oval. The angle between the edge hole and the long axis of particle describe as \( \gamma \).

Expressing sides of triangular \( \text{OBO}_2 \) through the hole sizes and the grain we will obtain:

\[
\sin \gamma_{\text{mp}} = \frac{2 \cdot (h_0 - R_2)}{a - 2 \cdot R_2},
\]

where \( \gamma_{\text{mp}} \) – limit angle when the grain passage into sieve hole is possible;

\( a \) – the size of long axis of particle;

\( h_0 \) – the width of hole;

\( R_2 \) – small radius of curvature at the top of particle.

Obtained ratio defines the maximum angle \( \gamma_{\text{mp}} \) between longitudinal axis of grain and edge of sieve hole according to the passage. So, grain passage into the hole is possible provided that:

\[
0 \leq \gamma \leq \pm \arcsin \frac{2 \cdot (h_0 - R_2)}{a - 2 \cdot R_2}.
\]

At angles \( \gamma_{\text{mp}} \) larger, than defined ratio (10), theoretically separating capacity of the sieve will approach zero at any direction of moving particle.

For the grain of sizes \( a = 5.2 \) mm, \( b = 2.1 \) mm [8] and radiuses of curvature surface \( R_1 = 2.5 \) mm and \( R_2 = 0.7 \) mm, as well as the sieve of the hole sizes \( 2.2 \times 20 \) mm and the jumper width \( 2.5 \) mm according to the formula (9) critical angle is calculated \( \gamma_{\text{mp}} \), in which grain passage in hole sieve is possible:

\[
0 \leq \gamma_{\text{mp}} \leq \arcsin \frac{2 \cdot (2.2 - 0.7)}{5.2 - 2 \cdot 0.7} = 52^\circ
\]
If we know the statistic characteristics of real particles position on sieve surface regarding its longitudinal axis, then probability of «pass» position of particle regarding hole edge will be equal:

\[ P_{\gamma} = \sum P_{\gamma \gamma} = \frac{N_{\gamma}}{N}, \]  

where \( i \) – the number of independent events in a range \( \gamma; \)

\( P_{\gamma \gamma} \) – probability of each events in a range \( \gamma; \)

\( N_{\gamma} \) – the number of particles on control section of the sieve located in a range \( \gamma; \)

\( N \) – the total number of particles on control section of the sieve.

According to the analyses of statistic characteristics of grain on control section of the sieve probability of favorable position of grain regarding hole sieve under the term of grain passage through the hole is calculated.

At limit value of the angle \( \gamma = \pm 54^\circ \), probability of grain passage in hole sieve located on the hole edge amounts

\[ P_{\gamma} = \sum_{\gamma = 54^\circ}^{0} P_{\gamma \gamma} = 0.089 + 0.092 + 0.159 + 0.144 + 0.099 + 0.092 = 0.675 \]

For definition of probability of particles movement in the direction of longitudinal axis of the sieve it will meet longitudinal edge of the hole located at an angle to the direction of grain movement, we use design scheme (fig. 4) where fragment of the sieve consisting of one hole and one jumper is depicted.

The position of grain is considered when the long axis of grain is parallel to the hole edge \((\gamma = 0)\).

The justice of such assumption is confirmed by the analyses results of statistic characteristics of grain distribution on the sieve, as the mathematical expectation of random value \( M_{\gamma} \) approximately equals to the arithmetic mean of observed value \( \gamma_{cp} \) and amounts: \( M_{\gamma} = 1.269^\circ \approx \gamma_{cp} \approx 0 \).

For definition of probability \( P_{\beta} \) we use ratio of elementary sieve area \( ABB'A' \) (fig. 4), formed by area of one hole and by area of one jumper and part of elementary area where the particle is located in the hole zone in pass position or where it may be in pass position at moving on jumper in direction of hole edge [9].

If \( 0 \leq \beta \leq arctg \frac{h_{II}}{l} \), where \( h_{II} \) – width of jumper, \( l \) – hole length; from figure 2 we will write geometrical ratio:

\[ P_{\beta} = \frac{S_{ABC} + S_{CDE}}{S_{ABB'CA'}} \]

after transformation we obtained:

\[ P_{\beta} = \frac{h_{II}}{h_{II} + h_{II}} + \frac{l \cdot tg \beta}{2(h_{II} + h_{II})} \]  

\( (11) \)
Figure 4 – Design scheme of definition $P_\beta$. The arrow indicates the direction of particle moving

If $\arctg \frac{h_o}{l} < \beta \leq \frac{\pi}{2}$, from figure 4 we will write geometrical ratio:

$$ P_\beta = 1 - \frac{S_{\text{EFF}}}{S_{\text{ABB}}} $$

after transformation we obtained:

$$ P_\beta = 1 - \frac{h_o^2 \cdot \ctg \beta}{2 \cdot l \cdot (h_o + h_i)} $$

(12)

thus:

$$ P_\beta = \begin{cases} \frac{h_o}{h_o + h_i} + \frac{l \cdot \tan \beta}{2 (h_o + h_i)} \\ with \quad 0 \leq \beta \leq \arctg \frac{h_o}{l} \end{cases} $$

$$ P_\beta = \begin{cases} 1 - \frac{h_o^2 \cdot \ctg \beta}{2 \cdot l \cdot (h_o + h_i)} \\ with \quad \arctg \frac{h_o}{l} < \beta \leq \frac{\pi}{2} \end{cases} $$

(13)

The probability $P_\beta$ is calculated at initial values: for sieve – $h_o = 2.2$ mm, $h_i = 2.5$ mm, $l = 20$ mm; for grain – $a = 5.2$ mm, $b = 2.1$ mm.

According to the equation results (7) we obtained the dependence $P = f(\beta)$, which is shown in picture 5.

Figure 5 – Dependence of probability particle passage through hole sieve from the angle of inclination of the hole
The graph follows that angle of inclination of the sieve hole to the longitudinal axis helps to increase probability of particle passage in hole sieve. Probability increases at small angle of inclination of the holes. In a range $\beta$ from 0 to $30^\circ$ probability of particle passage increases from 0.315 to 0.548 that is 1.74 times [7].

4. Conclusions
1. Block diagram of technological process of separation in the form of mathematical model of investigation object characterizing input and output parameters is presented.
2. It is established that dependence of increasing the probability of particle passage in oblong sieve hole at small angle of inclination of the holes. It is defined that in a range $\beta$ from 0 to $30^\circ$ probability of particle passage increases from 0.315 to 0.548.

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