Chiral Symmetry Breaking, Duality in the $\bar{Q}q$ Channel and $b \to \bar{c}cs$ Decays

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Abstract

We address the issue of the quark-hadron duality in the spectral densities induced by the heavy-light quark currents $\bar{Q}q$. In the limit $m_Q \to \infty, \ m_q \to 0$ we observe an enhancement of the physical spectral density compared to the quark one in the scalar and axial channels, due to the Goldstone meson contributions. This may imply that the scale where duality sets in is higher in these channels than in the vector (pseudoscalar) case. Implications for the nonleptonic decays of $B$ mesons (the $b \to \bar{c}cs$ transition) are considered. We discuss in detail the decay pattern and obtain an independent estimate of the “wrong” sign $D$ yield.

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1 Introduction

A large and rapidly growing number of applications of the heavy quark theory requires predictions in the Minkowski domain. The inclusive width of the heavy flavour hadrons is one the best known examples. All predictions of this type are based on the so-called QCD duality (for a recent discussion see, e.g. Ref. [1]).

The issue of duality is as old as QCD itself. Because of its complexity it was virtually neglected for a long time, basically after the classical paper of Poggio et al. [2]. Recently the interest to this question was revived by the necessity of having QCD predictions valid up to per cent accuracy in several problems of practical importance. The first attempts at quantifying possible deviations from duality were presented in Refs. [3, 4].

It is not easy to estimate the scale of the duality violations from fundamental QCD. Therefore, it is of paramount importance to gain experience in various particular problems. The present work investigates duality violations in an applied aspect. We will be mostly concerned with the spectral density in the two-point function induced by the heavy-light currents $\bar{Q}\Gamma q$, where $Q$ and $q$ are the light and heavy quarks, respectively. It turns out that in the limit $m_Q \to \infty$, $m_q \to 0$ the spectral density in the scalar (axial) channel is very peculiar: it starts below the position of the ground-state resonance, and is enhanced by the Goldstone meson contribution. Qualitatively the emerging picture is quite different from that taking place in the “conventional” vector (pseudoscalar) channel, where the ground state resonance is physically the lowest-lying state. A key role in this phenomenon belongs to the spontaneous breaking of the chiral symmetry for the light quarks.

The formulation of the problem to be discussed below is, in a sense, complementary to Ref. [3]. If in the latter work one approaches from the high-energy side (i.e. one considers the oscillation and asymptotic zones, in the nomenclature of Ref. [3], see Sect. 5.2), where duality is defined point-by-point; here we mostly deal with the resonance zone. Correspondingly, by duality we mean here that certain finite-range integrals over the hadronic spectral density are equal to the same integrals over the quark spectral density. The equality (or, rather, deviations from it) is checked by saturating the hadronic contribution by exclusive modes (cf. Ref. [4]). The size of an integration interval large enough for duality to take place characterizes the scale of structures and disturbances due to low-lying resonances.

In the second part of the paper, as a practical application we consider a particular physical process, namely $b \to c\bar{c}s$. Due to a relatively small energy release in this transition it is a natural suspect [3, 6, 7] for duality violations, which might be relevant in the so-called “semileptonic branching ratio versus $n_c$ problem” [8]. The model of Ref. [3] estimates possible violations of duality in the $b \to c\bar{c}s$ only at the per cent level. If so, they can be neglected, and the duality-based predictions [3, 10] should be valid. The above model is admittedly crude, however, and independent estimates are badly needed. Since we observe a strong enhancement of the spectral density in the axial $\bar{c}s$ channel, associated with the soft kaon contribution, a corre-
sponding enhancement in the $\bar{B} \to D^* \bar{D} K$ could be naturally expected. *A priori*, it could then be duality violating.

We suggest to use factorization, and the so-called generalized small velocity (GSV) limit in the $b \to c \bar{c} s$ transition. Both approximations are described at length below; here we just note that using them opens all decays of the type $\bar{B} \to D \bar{D} + S$-wave kaons (kaons and pions) for applications of the soft pion technique [1]. The most economic way to implement the idea is by using the effective Lagrangian approach, which combines the chiral and heavy-quark symmetries [12, 13, 14].

Surprisingly, our estimates of the decay modes $\bar{B} \to D^{(*)} \bar{D}^{(*)} \bar{K}$, where the kaon is treated as a soft Goldstone boson, give no indications on significant duality violations. We hasten to add, though, that the accuracy of the above estimates is not high.

The mechanism we discuss does not differentiate noticeably between the $B$ meson and $\Lambda_b$ baryon decays. Thus, it adds nothing new to the problem of the $\Lambda_b$ lifetime. However, the $1/N_c$-motivated factorization used in the $B$ decays may no longer be applicable when a heavy baryon is produced in the final state of $\Lambda_b$ decays. This may provide a mechanism for the observed lifetime difference, but we will not go into this issue here.

An interesting lesson follows for the lattice analysis of the heavy-light systems in the scalar and axial channels, where the quenched approximation is expected to give results different from those with light dynamical quarks.

The paper consists of two distinct parts. In the first part we discuss general features of the spectral densities corresponding to the two-point functions induced by $\bar{Q} q$ in the limit $m_Q \to \infty$, $m_q \to 0$. (Sect. 2). Implications for the transition $b \to \bar{c} c s$ are considered in the second part (Sects. 3 and 4). Our conclusions are summarized in Sect. 5.

## 2 Spectral densities and duality in the scalar and pseudoscalar $\bar{Q} q$ channels

As a preparatory step let us consider the spectral densities for the two-point functions induced by the current $\bar{Q} \gamma_\mu (1 - \gamma_5) q$. Since our consideration at this stage focuses on qualitative aspects, it is convenient to work in the limit where the mass of the heavy quark $m_Q \to \infty$ while that of the light quark $m_q \to 0$. In the chiral limit all mesons ($\pi, K, \eta$) belonging to the Goldstone octet are massless. The effects due to the finite quark masses will be incorporated later, when we pass to $b \to \bar{c} c s$. The vector and pseudoscalar $\bar{Q} q$ mesons are degenerate. The same is valid for the scalar and axial ones. The splitting between the first and the second multiplets is of order $\Lambda_{QCD}$ (practically, about 500 MeV).

The first analysis of the two-point functions

$$\Pi^{(V)}_{\mu\nu} = i \int d^4 x e^{i q x} \langle 0 | V_\mu(x) V_\nu^\dagger(0) | 0 \rangle = \Pi g_{\mu\nu} + \ldots,$$
\[ \Pi^{(A)}_{\mu\nu} = i \int d^4x e^{iqx} \langle 0 | A_\mu(x), A_\nu^\dagger(0) | 0 \rangle = \tilde{\Pi} g_{\mu\nu} + \ldots, \]

\[ \Pi^{(S)} = i \int d^4x e^{iqx} \langle 0 | S(x), S^\dagger(0) | 0 \rangle, \]

\[ \Pi^{(P)} = i \int d^4x e^{iqx} \langle 0 | P(x), P^\dagger(0) | 0 \rangle, \]

\[ \text{(1)} \]

dates back to the early days of QCD \[15\]. The currents \( V_\mu, A_\mu, S \) and \( P \) are defined as

\[ V_\mu = \bar{Q} \gamma_\mu q, \quad A_\mu = \bar{Q} \gamma_\mu \gamma^5 q, \quad S = \bar{Q} q, \quad P = \bar{Q} i \gamma^5 q. \]

\[ \text{(2)} \]

Although the weak currents are \( V - A \), the scalar and pseudoscalar two-point functions appear as their longitudinal parts,

\[ S = \frac{-i}{m_Q - m_q} \partial_\mu V_\mu, \quad P = \frac{1}{m_Q + m_q} \partial_\mu A_\mu. \]

Already in Ref. \[15\] it was noted that the quark condensate correction is enhanced in the heavy-light quark case compared to, say, the classical \( \rho \)-meson sum rule \[16\], producing stronger disturbances of the spectral density at small energies and, thus, leading to stronger deviations from duality \[1\]. First of all, its dimension in the former case is 3, while in the latter case it is 6. Second, the numerical coefficient is larger. Further studies \[17, 18, 19\] confirmed this observation in a more quantitative way. Actually, Ref. \[17\] was the first to introduce the heavy quark limit. It was noted there that in this limit the spectral densities in the (transverse) vector and pseudoscalar channels are degenerate. The same is valid for the scalar and (transverse) axial channels. For this reason we will limit ourselves in this section only to \( S \) and \( P \) channels.

Even more remarkable is the fact \[17\] that using the standard “first resonance plus parton-like continuum” model of the spectral density produces an abnormally large residue of the ground-state resonance in the scalar (axial) channels, an order of magnitude larger than that in the pseudoscalar (vector) channels. This observation, which went unnoticed, must be viewed as a hint that something unusual takes place in the scalar channel. Now we are ready to explain this anomaly.

As a matter of fact, the standard model of the spectral density mentioned above does not work in the scalar (axial) channels because of an enhancement at low energies due to the contribution of the \( D\pi \) intermediate state. (The mesons of the type \( D \) and \( D^* \) built from \( Q\bar{q} \) and degenerate in the limit \( m_Q \to \infty, m_q \to 0 \) will be generically referred to as \( D \)'s. The massless pseudoscalar mesons will be generically referred to as \( \pi \)'s).

In the free-quark approximation the four spectral densities (for \( \Pi^{(V,P)} \) and \( \Pi^{(S,A)} \)) are all identical and equal to

\[ \text{Im} \Pi = \frac{N_c}{2\pi} \varepsilon^2; \]

\[ \text{(3)} \]
only chirally odd condensate corrections distinguish between the first and the second pairs of above currents. Here \( \varepsilon \) is the excitation energy measured from the quark threshold (i.e. from the heavy quark mass \( m_Q \)). Logarithmic factors due to anomalous dimensions and \( \alpha_s \) corrections are neglected in Eq. (3). Although they play a role numerically, see e.g. Ref. [18], qualitatively they are not important. Equation (3) will set a natural scale for the spectral density in the heavy-light channels. Its important feature is a strong suppression at low energies, as \( \varepsilon^2 \).

Now, let us turn to the physical (hadron-saturated) spectral densities. In the pseudoscalar channel we encounter a more or less familiar picture. The physical spectral density starts from the ground-state pseudoscalar \( D \), then there is a gap, and then multiparticle intermediate states (continuum) add up into a curve that is expected to be relatively close to the quark one. The continuum actually is a sum over broad resonances, the radial excitations of \( D \).

The first resonance in the pseudoscalar (vector) channel is situated at \( \varepsilon = \bar{\Lambda} \) where \( \bar{\Lambda} = M_D - m_c \); the continuum threshold is at \( \varepsilon = \varepsilon_c \sim 2\bar{\Lambda} \), see Fig. 1. The onset of “continuum” approximately coincides with the position of the first radial excitation in the given channel. The parameter \( \bar{\Lambda} \) above is a basic parameter of the heavy quark theory [20]. Numerically \( \bar{\Lambda} \sim 500 \) MeV [21, 22, 23]. (We recall that although we keep using the name “\( D \) mesons” so far we work in the limit \( m_Q \to \infty \).)

The scalar (axial) resonances we are interested in are \( P \)-wave in the language of the naive quark model and \( 1/2^+ \) in the language of HQET. Due to the chiral symmetry breaking they lie higher\(^\text{2}\) (For a concise review of the higher \( \bar{Q}q \) states see Ref. [24].) The heavy quark symmetry implies that the scalar \( D_0^\ast \) and the axial \( D_1 \) are degenerate; one expects both \( D_0^\ast \) and \( D_1 \) at \( \varepsilon \sim 2\bar{\Lambda} \). What is remarkable is that the scalar (transverse axial) spectral density is non-vanishing and large below \( D_0^\ast \) (\( D_1 \)), in the interval from \( \bar{\Lambda} \) to \( \sim 2\bar{\Lambda} \), since it receives an enhanced contribution from the states \( D\pi \) (in the scalar channel) and \( D^\ast \pi \) (in the axial channel). The pions are strongly coupled to the \( \bar{Q}q \) current, in the \( S \) wave.

Consider for definiteness \( \Pi^{(S)} \). In the low-energy limit the pion is soft and can be reduced by using the soft-pion technique. Then,

\[
\langle 0|\bar{Q}q|D\pi\rangle_{\vec{k}\to 0} = \frac{1}{f_\pi}\langle 0|\bar{Q}i\gamma_5 q|D\rangle = -\frac{f_Q}{f_\pi} m_Q. \tag{4}
\]

Here \( f_Q \) is defined as

\[
\langle 0|\bar{Q}\gamma_\mu\gamma_5 q|D\rangle = if_Q p_\mu ,
\]

and we do not differentiate between \( M_D \) and \( m_Q \) in the limit \( m_Q \to \infty \); \( \vec{k} \) is the pion momentum. The spectral density takes the form

\[
\text{Im} \Pi^{(S)} = \frac{1}{2} |\langle 0|\bar{Q}q|D\pi\rangle|^2 \times \text{phase space} = \frac{f_Q^2 m_Q}{f_\pi^2} \frac{1}{8\pi} \left( N_f - \frac{1}{N_f} \right) (\varepsilon - \bar{\Lambda}) , \tag{5}
\]

\(^2\)The scalar resonance will be referred to as \( D_0^\ast \). The axial \( 1/2^+ \) resonances are sometimes denoted as \( D_1' \), to distinguish them from the axial \( 3/2^+ \) resonances denoted as \( D_1 \). Since in the present paper we will discuss only the axial \( 1/2^+ \) resonances, the prime will be omitted. The primes are reserved for radial excitations.
Figure 1: A sketch of the spectral density versus $\varepsilon/\bar{\Lambda}$ in the pseudoscalar channel. To set the scale we show also the free quark spectral density represented by the parabola.

Figure 2: A sketch of the spectral density versus $\varepsilon/\bar{\Lambda}$ in the scalar channel.
where $N_f = 3$ is the number of massless flavours ($N_f^2 - 1$ is the number of Goldstone mesons) and $\varepsilon - \Lambda$ is the energy measured from the position of the ground state $D$.

Comparing Eqs. (3) and (4) reveals the remarkable enhancement of the spectral density at low energies that was mentioned above. First, the dependence on $\varepsilon$ is parametrically different. Due to the $S$-wave nature of the matrix element $\langle 0|S|D\pi \rangle$ the $D\pi$ contribution to the spectral density vanishes as $O(\varepsilon)$ while the free quark expression (3) is $O(\varepsilon^2)$. Second, as is well known, the static coupling $f_Q$ is rather large numerically \cite{18, 25}. The combination $f_Q^2 m_Q$ stays constant in the limit $m_Q \to \infty$ \cite{17}, modulo the hybrid logarithms \cite{26}, and this constant is approximately 0.2 to 0.25 GeV$^3$ \cite{18, 23}. Treating Eq. (5) literally we get that the $D\pi$ contribution to the spectral density at $\varepsilon = 2 \bar{\Lambda}$ exceeds the free quark expression by a factor of $\sim 1.5$ (Fig. 2). The integral over the $D\pi$ spectral density up to $\varepsilon = 2 \bar{\Lambda}$ is approximately 1.2 times the integral over the free quark spectral density from 0 to $\varepsilon = 2 \bar{\Lambda}$.

Of course, at a certain energy, the soft-pion result (4) becomes invalid – the momentum-dependent part of the interaction will cut off the amplitude. The cut off presumably occurs near the position of the resonance $D_0^*$. The excess of the spectral density below $D_0^*$ and at $D_0^*$ has to be compensated by an extended gap immediately after $D_0^*$. This gap stretches, presumably, up to $\varepsilon \sim 3 \bar{\Lambda}$. In general, the situation is very similar to what occurs with the two-pion contribution in the spectral density induced by the gluonic current $\alpha_s G^2$ \cite{27}, where a strong low-energy enhancement is supplemented by an extended gap, and the scale of the duality violation is quite large.

Note that the $D\pi$ contribution we have calculated has proper dependence on parameters $N_c$ and $N_f$. In the limit $N_c \to \infty$ it decouples compared to Eq. (3), as it should. There is no decoupling, however, if $N_f/N_c$ stays fixed. The pattern we observe presents another example, similar to Ref. \cite{27}, that in the low-energy domain the $1/N_c$ counting rules should be taken with caution. There exist certain mechanisms that can totally upset the $1/N_c$ estimates at $N_c = 3$ (although at academically large $N_c$ the $1/N_c$ counting will work, of course).

Thus, in the pseudoscalar channel we expect the duality interval to be $\sim 2 \bar{\Lambda}$, while in the scalar channel its size is expected to be larger, $\sim 3 \bar{\Lambda}$. By the duality interval we mean the following: the minimal interval of energy $(0, \varepsilon_0)$ needed to make the smeared resonance contribution (approximately) equal to the quark-gluon one. In other words, in the smearing integrals over the spectral densities from 0 to $\varepsilon_0$ the upper limit $\varepsilon_0$ should be chosen at $\sim 2 \bar{\Lambda}$ in the pseudoscalar channel and $\sim 3 \bar{\Lambda}$ in the scalar one.

We pause here to make a few comments. First, if we consider the sum of the scalar plus pseudoscalar heavy-light channels (or vector plus axial) the quark condensate correction $\langle \bar{q}q \rangle$ cancels, leaving us with no hint that the duality violation scale is larger in this case than in the classical $\rho$ meson sum rule, at least at the level of analyzing the first terms in the operator product expansion. This fact is rather alarming, since in the weak inclusive decays we deal with the $V - A$ currents, and the above cancellation is quite typical. The model of Ref. \cite{3} gives no clues in this case.
either, since it admittedly omits all effects specifically related to the spontaneous breaking of the chiral symmetry. At an observational level we detect here a possible signal that the scale of the duality violations for $V - A$ currents may be larger than can be inferred from the analysis of the lowest-dimension condensates. Does this mean that some other condensates, of higher dimension, must be important? And how can they be identified? These questions still remain open. It is clear that the sum rules for the heavy-light currents have to be reanalyzed with emphasis on this aspect. In particular, an updated analysis of the scalar (axial) channel is more than in order. One has to replace the standard “lowest resonance plus continuum” pattern by the more complicated picture described above. The impact seems to be obvious. The residue of the $D_0^* (D_1)$ state will go down with respect to the prediction of Ref. [17]. A part of the spectral density will be pumped out from the resonance into the non-resonant $D\pi$ background.

The second remark refers to the lattice calculations of the heavy-light systems (spectra and coupling constants). Pseudoscalar and scalar channels should be drastically different with respect to inclusion of the light dynamical quarks. If the pseudoscalar channel seems to be relatively stable, unquenching quarks and making light quarks really light must produce a dramatic effect in the scalar one. Of course, this is not the first time the dynamic light quarks lower the threshold. For instance, in the $\rho$ meson case there is the two-pion cut as well. This state, however, is only relatively weakly coupled to the current $\bar{q}\gamma_\mu q$, since the pions are in the $P$ wave; therefore, speaking in practical terms, this contribution is rather unimportant (although at asymptotically large separations it will dominate anyway). This is not the case for the current $\bar{Q}q$, where the $D\pi$ intermediate state is not only lower in mass than $D_0^* (D_1)$, but also strongly coupled to the current. Therefore, its impact in the two-point function should be essential and the correlator must drastically change once the light quarks are unquenched and made light. Studying this problem on the lattice seems to be a nice testing ground for various approximations routinely made within this approach.

3 Nonleptonic $B$ decays ($b \to \bar{c}cs$): outlining the problem

Having established the enhancement of the $D\pi$ intermediate state in Im$\Pi^{(S)}$ (or $D^*\pi$ in Im$\Pi^{(A)}$) near the threshold, it is natural to turn to nonleptonic decays of $B$ mesons, the $b \to \bar{c}cs$ transition. Indeed, in this case the $\bar{c}\Gamma_\mu s$ current produces charmed/strange hadronic states with the quantum numbers $1^-, 0^+$ (the vector current) and $1^+, 0^-$ (the axial current). The $1^+$ and $0^+$ channels were shown above to be responsible for an enhanced production of Goldstone mesons in the $S$ wave. Of course, in the actual $B$ decays the situation is not quite the same as in the limit $m_Q \to \infty$, $m_q \to 0$ considered in Sect. 2. Suffice it to mention that the actual values of the $c$ and $s$ quark masses are such that the axial ground-state meson $D_{s1}$,
shown in Fig. 2 at the end of the shoulder (at \( \varepsilon \sim 2\bar{\Lambda} \)), turns out to be almost exactly at its beginning, presumably barely above the threshold of \( D^*K \). The \( 1/2^+ \) charmed strange mesons have not been detected experimentally so far. Moreover, the actual value of \( f_2^B \) is also noticeably lower than its static value. Also, the energy release in the light particles in the case at hand is \( \leq 1.5 \) GeV. It is quite clear that the kaon mass is not negligible in the phase space. For this reason in the nonleptonic \( B \) decays a dedicated analysis is needed.

First, however, a few remarks regarding the general situation with the \( b \to \bar{c}ccs \) transition are in order. This transition came under renewed scrutiny recently on purely phenomenological grounds, in an attempt to find a solution of the "branching ratio versus \( n_c \) problem". In Ref. \[7\] it was assumed that the theoretical understanding of \( b \to c\bar{u}d \) is solid. Then, from the measured semileptonic branching ratio of approximately \( 10.5\% \), the branching ratio of \( b \to \bar{c}ccs \) was predicted to be close to \( 30\% \), with the corresponding charm yield \( n_c \approx 1.3 \). Moreover, using the most naive duality estimates in conjunction with the parton model for \( b \to \bar{c}cs \), it was suggested \[7\] that approximately \( 1/2 \) of \( b \to \bar{c}cs \) hadronizes in the channels of the type \( \bar{B} \to D^{(*)}\bar{D}^{(*)}KX \), and only the remaining \( 1/2 \) goes to \( D^{(*)}\bar{D}^{(*)}X \), the channels on which the attention was focused previously. If so, about \( 15\% \) of the \( \bar{B} \) decays have to end up with the "wrong" sign \( D \)'s and \( K \)'s in the final state.

This prediction was confirmed by recent results from the CLEO, ALEPH and DELPHI experiments \[28\], reporting the yield of the "wrong" sign \( D \)'s at the 10\% level. The corresponding value of \( n_c \) is close to 1.24 \[28\]. Additionally, ALEPH has recently reported a value for \( n_c \) in \( Z \to bb \) \[29\] of 1.23 \pm 0.07.

Although the situation can be looked at quite optimistically, serious questions are still to be answered. The first question is, of course, purely experimental. As was emphasized in Ref. \[28\], the charm yield \( n_c \), as computed in the usual way from the measurements at \( \Upsilon(4S) \), remains unchanged: \( n_c = 1.10 \pm 0.06 \) \[30\]. The contradiction is obvious, suggesting that the experimental situation is still not settled.

We clearly cannot comment on this aspect, and quickly pass to what is known theoretically. Since the leading nonperturbative corrections are expected to play no significant role in the issue \[8\], the focus of theoretical analysis is shifted towards perturbative calculations in \( b \to \bar{c}ccs \). The first dedicated calculation of the gluon corrections was carried out in Ref. \[31\]. The most advanced analysis existing today is presented in Refs. \[9, 10\] (see also references therein). To quote representative values of the predicted parameters, let us assume that \( \alpha_s(M_Z) = 0.11 \) and \( \mu = m_b/2 \), in \( \overline{MS} \); then \[9\] \( n_c = 1.28 \) (the corresponding \( \text{BR}_{\text{sl}}(B) \) is slightly lower than \( 11\% \)). Within a somewhat different procedure of treating the ratio of the quark masses \( m_c/m_b \) (but the same values of the parameters as above), the theoretical numbers become \[32\] \( n_c = 1.23 \) and \( \text{BR}_{\text{sl}}(B) = 11.5\% \). Thus, it is fair to say that at the current level of understanding the theoretical prediction for \( n_c \) is close to 1.25. Without the \( \mathcal{O}(\alpha_s) \) correction the charm yield is 1.15. Thus, the inclusion of the \( \mathcal{O}(\alpha_s) \) gluon correction enhances \( \text{BR}(b \to \bar{c}cs) \) by a factor \( \sim 1.6 \).

In the approximation of Ref. \[9\] factorization is explicit. This statement will be
explained in more detail below (Sect. 3), where we will introduce the factorization hypothesis, one of the key elements of our consideration. The impatient reader may turn to Ref. [33] for very clear explanations. Here we just note that, in any perturbative calculation respecting factorization, the transitions corresponding to the vector $\bar{c}\gamma_\mu s$ and the axial $\bar{c}\gamma_\mu\gamma_5 s$ currents necessarily have equal probabilities, provided very small effects $\propto m_s^2$ are neglected. If so, the result [4] implies that the branching ratios of $b \to \bar{c}cs$, with $\bar{c}s$ from the vector and axial currents, respectively, are 12 to 13% each.

It is clear that the theoretical predictions [2] discussed above are quite compatible with the newest experimental trend. The basic assumption underlying the theoretical analysis is the validity of the quark-hadron duality. Strong violations of duality in $b \to \bar{c}cs$ would destroy the predictive power of the existing theory. Although at the moment no sources for such large violations were identified [3], in the absence of a complete theory it is obviously desirable to have as many independent confirmations as possible. We will analyze below the transition $b \to \bar{c}cs$ from this point of view.

The transition $b \to \bar{c}cs$ is singled out in this aspect for the following reason. The relative smallness of the energy release, which alerts one regarding duality violations, can be turned into an advantage. Indeed, in this case the number of the hadronic channels saturating the physical decays is not large, and one can try to estimate these channels one by one to see whether they add up to the quark-gluon result. As a by-product one could hope to get a more direct estimate for the $\bar{B} \to D^{(*)}\bar{D}^{(*)}\bar{K}X$ rate. The goal here is to check whether the 10% yield reported experimentally is well understood theoretically, invoking as few unsubstantiated assumptions as possible.

### 4 Nonleptonic $B$ decays ($b \to \bar{c}cs$): analysis of exclusive modes

The relevant part of the weak Lagrangian contains two structures, with the “direct” and “twisted” colour flow

$$L = \frac{G_F}{\sqrt{2}} V_{cb} V_{sc} \left[ a_1(\bar{b}\Gamma_\mu c)(\bar{c}\Gamma_\mu s) + a_2(\bar{b}\Gamma_\mu s)(\bar{c}\Gamma_\mu c) \right]. \quad (6)$$

We will disregard the second term, with the twisted color structure, for the following reason. First, the value of $|a_2/a_1|$ is rather small numerically (see e.g. the review paper [34]), approximately 0.1. Therefore, the square of the second term contributes at the level of 0.01. The interference with the first term is suppressed by $N_c$ and is thus expected to show up at the level of corrections $\sim 0.2/3$ in the probability

\[3\] In Ref. [33] some of the $O(\alpha_s^2)$ corrections were estimated, and at this level nonfactorizable terms appear. For the purpose of our analysis one can safely use factorization, at least as a starting point.
of the decay modes we are interested in. The estimates to be presented below have intrinsic theoretical uncertainties of this order of magnitude or larger.

Furthermore, in considering the first term it is reasonable to accept, at least at this stage of the analysis, the factorization approximation. By this we mean that in treating the hadronic matrix elements, the $b\Gamma_{\mu}c$ bracket of the effective Lagrangian, will be factored out from the $\bar{c}\Gamma_{\mu}s$ bracket. The corresponding hadronic subsystems are assumed not to communicate with each other through the soft gluon exchanges, although inside the subsystems all these exchanges are taken into account in full. Note, that we also automatically include all hard gluons (with off-shellness larger than $m_b$), through the factor $a_1$. The standard theoretical justification for the factorization hypothesis is the $1/N_c$ counting. All non-factorizable contributions are suppressed by $1/N_c$. Note that the modes with the hidden charm (e.g. $J/\psi K$) will not concern us here.

Certainly, we are well aware that the non-factorizable contributions must be present (see e.g. [34]); their effect is noticeable in the fine structure of the nonleptonic decays and in some special modes, but otherwise it is quite modest. For instance, in $B \to D\bar{D}$ the non-factorizable contributions were estimated to be less then 10% of the factorizable part [35]. We will ignore the non-factorizable terms in the present work.

Another theoretical tool which will help us is the generalized small velocity limit, i.e. the assumption that two charmed mesons we deal with in the $b \to \bar{c}cs$ transition, in the final state, are slow. Kinematically this means that

$$M_B - 2M_D \ll M_D.$$  \hspace{1cm} (7)

This is not the first time the GSV limit is exploited in the context of $B$ decays, see e.g. [33, 36]. Using the GSV limit, in combination with factorization, will allow us to disregard excitations in the $b\Gamma_{\mu}c$ bracket, and exploit the well developed formalism of the soft Goldstone mesons for the modes of the type $D\bar{D}K$.

Although in purely theoretical aspect the GSV limit is an excellent tool, in the actual $B$ decays Eq. (7) is valid only marginally. We are neither too close to the GSV point nor too far from it (cf. e.g. [33]). Under the circumstances, in kinematical factors we will keep the terms containing the $D$ meson velocities, while omitting $\vec{v}^2$ terms when they are additionally suppressed. In the future, with more phenomenological input, this approximation can be improved, including all terms quadratic in the (spatial) velocities.

To warm up let us consider two-particle decays. This exercise is not new (see Ref. [37]), and we repeat it merely to introduce our notation and explain the choice of numerical values of the relevant parameters.

Thus, factorization and the GSV limit are starting elements of our analysis. If so, then the bracket $b\Gamma_{\mu}c$ is responsible for the $B \to D$ ($B \to D^*$) transition. Production of excitations by this bracket – either resonances (say, radial excitations of $D$ and $D^*$), or nonresonant states of the type $D\pi$ – is suppressed by the velocity squared of the charmed meson. The $D\pi$ production by $b\Gamma_{\mu}c$ can be studied within the chiral
perturbation theory \[38\]. One has to consider pole graphs which, in addition to the velocity suppression, are proportional to the $D^* D \pi$ coupling constant $g$. The latter was calculated within the QCD sum rules \[39\], and turns out to be rather small, $g \sim 0.3$. Therefore, here and below we will consistently disregard all contributions to the decay rate that are proportional to $\vec{v}^2$ and $g^2$.

Limiting ourselves to the $D$ and $D^*$ states in the $\bar{b} \Gamma_{\mu} \bar{c}$ bracket gives us, by itself, a predictive power since the form factors of the $B \to D^{(*)}$ transitions at zero recoil are normalized to unity \[40, 41\], and near zero recoil are well approximated by the first derivative of the Isgur-Wise function $\xi$ \[42\].

The second bracket, $\bar{c} \Gamma_{\mu} s$, creates sometimes $\bar{D}_s$ or $\bar{D}_s^*$ states, sometimes radial excitations of $\bar{D}_s$ and $\bar{D}_s^*$, and sometimes nonresonant $D^{(s)} \bar{K}$ pairs. The axial current, in its transverse part, can produce $\bar{D}_{s1}$ and excitations, while the longitudinal part of the vector current can produce $D_{s0}^*$ and excitations. Let us first concentrate on $\bar{D}_s$ and $\bar{D}_s^*$.

(i) $B \to D^{(*)} \bar{D}_s^{(*)}$. “Wrong” spin correlations

The amplitudes for two-body transitions are given by

\[
A(\bar{B} \to D \bar{D}_s) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 \left( 2 \sqrt{M_B M_D f_{D_s} M_{D_s}} \right) \left[ \frac{M_B - M_D}{M_{D_s}} \right] (1 + vv')\xi(vv') ,
\]

(8)

\[
A(\bar{B} \to D^* \bar{D}_s^*) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 \left( 2 \sqrt{M_B M_{D^*} f_{D_s} M_{D^*_s}} \right) \times \\
\left\{ \frac{(\epsilon' \epsilon'')}{2} + \frac{1}{2} (\epsilon' v)(\epsilon'' v') + \frac{i}{2} \epsilon^\rho_\mu \chi_\mu \epsilon^\nu_\rho \epsilon''_\nu v' v" \right\} \xi(vv') ,
\]

(9)

\[
A(\bar{B} \to D \bar{D}_s^*) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 \times \\
\left( 2 \sqrt{M_B M_D f_{D^*_s} M_{D_s}} \right) \left[ \frac{M_B + M_D}{2 M_{D_s}} \right] (\epsilon'' v) \xi(vv') .
\]

(10)

\[
A(\bar{B} \to D^* \bar{D}_s) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 \times \\
\left( 2 \sqrt{M_B M_{D^*} f_{D^*_s} M_{D^*_s}} \right) \left[ \frac{M_B + M_{D^*}}{2 M_{D^*_s}} \right] (\epsilon' v) \xi(vv') .
\]

(11)

Here we omitted inessential overall phase factors in front of some amplitudes. The
constants $f_{D_s}$ are defined as,

\[ \langle D_s | \bar{c} \gamma_\mu \gamma_5 s | 0 \rangle = -i f_{D_s} M_{D_s} v'_\mu, \quad \langle D^*_s | \bar{c} \gamma_\mu s | 0 \rangle = f_{D_s} M_{D^*_s} \epsilon''_\mu. \]

The pseudoscalar and vector constants are identical in the limit $m_c \to \infty$, but may differ a little, due to preasymptotic terms, for the actual $c$ quarks. For simplicity the difference between the pseudoscalar and vector constants are neglected within any hyperfine multiplet. Anyway, we do not know them to that accuracy. Moreover, $v$ is the four-velocity of the decaying $B$ meson, $v'$ is the four-velocity of the $D$ meson and $v''$ is the four-velocity of $\bar{D}^*_s$,

\[ vv' = \frac{M^2_B + M^2_D - M^2_{D_s}}{2 M_B M_D}, \quad vv'' = \frac{M^2_B + M^2_{D_s} - M^2_D}{2 M_B M_{D_s}}, \quad v'v'' = \frac{M^2_B - M^2_D - M^2_{D_s}}{2 M_D M_{D_s}}, \]

and likewise for other decays, $\epsilon'$ and $\epsilon''$ are the polarization vectors of $D^*$ and $\bar{D}^*_s$, respectively.

In the limit of slow $D$'s, when $vv'$, $vv''$, $v'v'' \rightarrow 1$, these expressions essentially simplify. Thus, in Eq. (8) the square bracket becomes unity, and in Eqs. (9), (10) and (11) the square brackets tend to zero – as $\sqrt{vv''} - 1$ in Eq. (10) and as $\sqrt{vv'} - 1$ in Eq. (11). Thus, in this limit we have a rigid spin correlation. If $D$ is produced by the $\bar{b}c$ bracket, the $\bar{c}s$ bracket will produce a pseudoscalar $\bar{D}_s$; if $D^*$ is produced by the $\bar{b}c$ bracket we get, in association, a vector $\bar{D}^*_s$. This is because in this limit the $B \to D^*$ transition is caused by the zeroth (time) component of the current, whereas $B \to D^*$ is due to the spatial component. The “wrong”-spin transitions, Eqs. (10) and (11), switch off in this limit. Table 1 gives the values of the parameters $y = vv'$, $v'v''$ and $vv''$ for various transitions in the ground state and excited resonances. In the GSV limit all these kinematic parameters reduce to unity.

This table also gives the values of “block factors”. They are defined as

\[
\text{block factor} = \left( \frac{A^2}{\sqrt{\frac{G_F}{2}} V_{cb} V_{cs}^* a_1 \left( 2 \sqrt{M_B M_D f_{D_s} f_{D^*_s}} \right)^2} \right)^{1/2}
\]

where the summation over the polarizations of $D^*_s$ is implied, where applicable; for the excited $D_s$ states considered below $M_{D_s}$ and $f_{D_s}$ in the denominator will be understood as the mass and the coupling of the respective pseudoscalar. We included here the IW function $\xi(vv')$, see below. In the first decay the block factor is just the square bracket in Eq. (8).

We see that in the $\bar{B} \to D \bar{D}_s$ transition the block factor which is equal to unity in the GSV limit, is actually close to 1.5, while in the “wrong” spin correlation transition $\bar{B} \to D D^*_s$ the block factor, which vanishes in the GSV limit is actually close to 1.25. This is a manifestation of the fact that in the two-body decays into the lowest-lying $D$'s the GSV approximation is not too good, as was expected, of course.

Table 1 shows that the GSV approximation becomes much better for excited $\bar{D}$'s where the wrong spin transitions are indeed suppressed. The GSV approximation,
Table 1:

| Decay                | $y$  | $v'v''$ | $vv''$ | block factor |
|----------------------|------|---------|--------|--------------|
| $B \rightarrow \bar{D}D_s$ | 1.39 | 2.79    | 1.36   | 1.50         |
| $B \rightarrow \bar{D}D_s^*$ | 1.36 | 2.53    | 1.29   | 1.25         |
| $B \rightarrow \bar{D}^*D_s$ | 1.32 | 2.52    | 1.33   | .93          |
| $B \rightarrow \bar{D}^*D_s^*$ | 1.29 | 2.28    | 1.27   | $\sqrt{3} \cdot 0.94$ |
| $B \rightarrow \bar{D}D_s'$ | 1.25 | 1.82    | 1.13   | 1.22         |
| $B \rightarrow \bar{D}^*D_s'$ | 1.21 | 1.64    | 1.10   | 0.79         |
| $B \rightarrow \bar{D}^*D_s^*$ | 1.19 | 1.64    | 1.11   | 0.58         |
| $B \rightarrow \bar{D}^*D_s''$ | 1.15 | 1.47    | 1.08   | $\sqrt{3} \cdot 1.04$ |
| $B \rightarrow \bar{D}D_s''$ | 1.10 | 1.28    | 1.04   | 1.07         |
| $B \rightarrow \bar{D}D_s'''$ | 1.05 | 1.14    | 1.02   | 0.37         |
| $B \rightarrow \bar{D}D_s''''$ | 1.05 | 1.14    | 1.02   | 0.28         |
| $B \rightarrow \bar{D}D_s'''''$ | 1.006 | 1.02 | 1.002 | $\sqrt{3} \cdot 1.09$ |

quite naturally, significantly improves in the three-particle decays as well, where a part of the overall energy release goes to create, additionally, the kaon mass, and the remainder is shared by three particles, not two.

If instead of $\bar{D}_s$ and $\bar{D}_s^*$ we have their radial excitations, Eqs. (8)–(11) are modified in a minimal way. Apart from the masses and the kinematical factors, one must replace $f_{D_s}$ by the corresponding coupling.

For numerical estimates of the decay rates we need to fix various parameters. First, we take $f_{D_s} \approx 200$ MeV [25, 43]. Now, the Isgur-Wise function $\xi$ must be evaluated at the proper recoil values, which are given in Table 1. We use the linear approximation for the Isgur-Wise function

$$\xi(v'v') = 1 - \rho^2 (vv' - 1),$$  \hspace{1cm} (12)

where $\rho^2$ is the slope parameter. Its numerical value is more or less known, and we put

$$\rho^2 = 0.7.$$  \hspace{1cm}

This value is compatible with the experimental data [44] and with the QCD sum rule calculations [45]. Finally, $a_1 \approx 1.1$.

The relevant decay rates are

$$\Gamma(B \rightarrow D\bar{D}_s) = \Gamma_0 \left(\frac{M_B - M_D}{M_{D_s}} \frac{1 + vv'}{2}\right)^2 \xi^2,$$  \hspace{1cm} (13)

$$\Gamma(B \rightarrow DD_s^*) = \Gamma_0^* \left(\frac{M_B + M_D}{4M_D^2}\right) ((vv'')^2 - 1) \xi^2,$$  \hspace{1cm} (14)
Table 2:

| Decay               | Branching, % |
|---------------------|--------------|
| $B \to D\bar{D}_s$  | 1.1          |
| $B \to D\bar{D}_s^*$| 0.87         |
| $B \to D^*\bar{D}_s$| 0.45         |
| $B \to D^*\bar{D}_s^*$| 1.5         |
| Total Br, %         | 4.0          |

\[
\Gamma(B \to D^*\bar{D}_s) = \Gamma_0 \frac{(M_B + M_{D*})^2}{4M_{Ds}^2}((vv')^2 - 1)\xi^2, \quad (15)
\]

\[
\Gamma(B \to D^*\bar{D}_s^*) = 3\Gamma_0^* \left[ 1 + \frac{vv' - 1}{3} + \frac{(vv')^2 - 1}{4} - \frac{(v'v'')^2 - 1}{12} + \frac{(vv')(vv'') - 1}{3} \right] \xi^2. \quad (16)
\]

Here

\[
\Gamma_0 = \frac{G_F^2}{4\pi}|V_{cb}V_{cs}|^2a_1^2 f_{D_s}^2 M_D M_{D_s}^2 \frac{|\vec{p}|}{M_B},
\]

and $\Gamma_0^*$, $\Gamma_0'$, $\Gamma_0''$ are the same with the $D$ ($D_s$) replaced by $D^*$ ($D_s^*$) for asterisk, and $\vec{p}$ is the c.o.m. momentum of the produced mesons. Numerically the relevant branching ratios are given in Table 2.

So far we basically repeated the calculations of Mannel et al. \[37\]; our estimate of the Isgur-Wise function is different, and our value of $f_{D_s}$ is essentially lower than that accepted in Ref. \[37\]. We note that the total sum of the above four modes amounts to $\sim 4\%$ in the branching ratio. None of these modes produces “wrong” sign $D$’s. The above numerical results are in agreement with the experimental data \[46\],

\[
\Gamma(B^0 \to D\bar{D}_s) = 0.7 \pm 0.4\%, \quad \Gamma(B^0 \to D^*\bar{D}_s^*) = 1.9 \pm 1.2\%, \quad (17)
\]

\[
\text{Br}(B^0 \to D^*D_s) = 1.2 \pm 0.6\%, \quad \text{Br}(B^0 \to D\bar{D}_s^*) = 2 \pm 1.5\%, \quad (18)
\]

\[
\text{Br}(B^+ \to D\bar{D}_s) = 1.7 \pm 0.6\%, \quad \text{Br}(B^+ \to D^*\bar{D}_s^*) = 2.3 \pm 1.4\%, \quad (19)
\]

\[
\text{Br}(B^0 \to D^*\bar{D}_s) = 1 \pm 0.7\%, \quad \text{Br}(B^0 \to D\bar{D}_s^*) = 1.2 \pm 1%\). \quad (20)
\]

Encouraged by this success we can proceed to other exclusive modes which have not been discussed in the literature so far.

(ii) Modes with the radial excitations of $\bar{D}_s^{(*)}$

We got used to the fact that the radial excitations are coupled to corresponding
currents significantly weaker than the ground-state mesons with the given quantum numbers. This is, for instance, the case with the $J/\psi$ and $\Upsilon$ mesons whose coupling to the current $\bar{Q}\gamma_{\mu}Q$ decreases with the excitation number. In the decays $B \to D^{(*)}\bar{D}_s^{(*)}$ (where the prime denotes the first radial excitation and the double prime will denote the second excitation) we expect the opposite trend. This is a specific feature of the $\bar{Q}q$ current. The corresponding parton spectral density grows quadratically with the energy $\varepsilon$, see Sect. 2. If duality takes place, the residues should be roughly proportional to the corresponding areas under the parabola in Fig. 1. To get an idea of the excited state contribution we will assume that $M_{D_s^*} \approx 2.6$ GeV, $M_{D_s^{*'}} \approx 3.1$ GeV, $M_{D_s^{*''}} \approx 2.75$ GeV and $M_{D_s^{*'}} \approx 3.25$ GeV, i.e. are equidistant. Imposing the duality condition in the form

$$\int_{M_{k-1}}^{M_k} dM \text{ Im } \Pi_{\text{pert}}(M) = \int dM \text{ Im } \Pi_k(M), \quad \text{Im } \Pi_k(M) \propto M_k f^2_k \delta(M - M_k)$$

we then obtain

$$\frac{f^2_{D_s^*} M_{D_s^*}}{f^2_{D_s} M_{D_s}} \approx \frac{27 - 8}{8} \approx 2.5, \quad \frac{f^2_{D_s^{*'}} M_{D_s^{*'}}}{f^2_{D_s} M_{D_s}} \approx \frac{64 - 27}{8} \approx 4.5. \tag{21}$$

Note that we use here an “extended” definition of the excited resonances, which need not coincide with the standard Breit-Wigner peaks, with the backgrounds subtracted. Rather, we include backgrounds, collecting, say, in $D_s^*$ everything with the invariant mass lower than $\approx 2.3$ GeV (except the ground state $D_s$, of course). This is a natural operational definition from the point of view of the QCD practitioner.

The estimates in Eq. (21) are crude and intended only for the purpose of orientation. However, it is a model for the spectral density that exactly respects duality to the given approximation, and, thus, allows simultaneous estimates of its violations in the actual decay width, where only a limited energy release is available. The contribution of the last open multiplet is actually a reasonable estimate of the scale of duality violations that might occur. On the practical side, to allow a measure of theoretical uncertainty in the excited state residues, we will allow the ratios to float,

$$(f^2_{D_s^*} M_{D_s^*})/(f^2_{D_s} M_{D_s}) = 2.5 x, \quad (f^2_{D_s^{*'}} M_{D_s^{*'}})/(f^2_{D_s} M_{D_s}) = 4.5 x \tag{22}$$

where $x$ will be varied between 0.6 and 1. The values of $x < 1$ reflect a relative enhancement of the lower end of the spectral density by the perturbative and, to some extent, condensate effects. Also the higher end of the spectral density is somewhat suppressed since at these energies relativistic effects become important, and temper the $\varepsilon^2$ growth of the spectral density characteristic to the non-relativistic limit. As we will see later on, the data are more consistent with $x \sim 0.6$.

In Table 3 the branching ratios for the two-particle decays into excited states are quoted for three different values of $x$. Altogether we get that

$$\text{Br}(B \to D^{(*)} + \text{excited } D_s^{(*)}) \sim 7 \text{ to } 12\%. \quad 15$$
Table 3:

| Decay                | Br, % ($x = 0.6$) | Br, % ($x = 0.8$) | Br, % ($x = 1$) |
|----------------------|-------------------|-------------------|-----------------|
| $B \rightarrow DD'_s$| 1.1               | 1.5               | 1.9             |
| $B \rightarrow DD''_s$| 0.5               | 0.65              | 0.8             |
| $B \rightarrow D^*D'_s$| 0.25              | 0.35              | 0.4             |
| $B \rightarrow D^*D''_s$| 2.4               | 3.2               | 4.0             |
| Total $D^{(*)}_s$ Br, % | 4.3               | 5.7               | 7.1             |
| $B \rightarrow DD''''_s$| 1.2               | 1.6               | 2.0             |
| $B \rightarrow DD'''_s$| 0.1               | 0.15              | 0.2             |
| $B \rightarrow D^*D'''_s$| 0.06              | 0.08              | 0.1             |
| $B \rightarrow D^*D''''_s$| 1.1               | 1.5               | 1.9             |
| Total $D^{(*)''}_s$ Br, % | 2.5               | 3.3               | 4.1             |
| Total $D^{(*)}_{excit}$ Br, % | 6.8               | 9.0               | 11.3            |

Let us discuss the decay pattern of these excitations. This issue is interesting not only in connection with the “wrong” sign $D’$’s in the $B$ decays, but by itself as well.

$D'_s$ can decay neither into $\bar{D}K$ nor into $\bar{D}^*K$. The second mode is presumably below the threshold. Even if it is slightly above the threshold, the small energy release, in conjunction with the $P$ wave nature of the decay, strongly suppresses it. Thus, the dominant mode is expected to be $\bar{D}_s\pi\pi$. Therefore, the $\bar{D}'_s$ production does not generate the “wrong” sign $D’$’s.

On the other hand, $\bar{D}''_s$ will presumably decay predominantly into $\bar{D}^*K$. One third of the above goes into $\bar{D}'_s\eta$. Some $\bar{D}^*K$’s are also possible. The latter mode also leads to a “wrong” sign $D$. Its presence may somewhat distort, however, our estimate of $\bar{D}_s\eta$. A competition from the $\bar{D}_s\pi\pi$ mode is generally possible, but unlikely to be essential: the $\bar{D}_s'' \rightarrow \bar{D}_s\pi\pi$ is a three-body decay, with the additional suppression due to the pion momenta. (It can proceed via an $S$-wave $\pi\pi$ resonance; however, the $\sigma$ meson is too broad and leads only to a moderate enhancement, whereas higher resonances do not have much of a phase space.)

As for $\bar{D}''''_s$ and $\bar{D}'''_s$, their dominant decay mode is likely to be $\bar{D}^{(*)}K$ and $\bar{D}^{(*)}_s\eta$. Thus, in the three latter cases we end up with the “wrong” sign $D$’s. Due to $SU(3)_f$ symmetry the yield of $\eta$ is three times smaller than that of $K$’s. The total branching of the corresponding modes is 5 to 9%. The $D\bar{D}\eta$ yield from this mechanism is 1.2 to 2.3%.

(iii) Nonresonant $D^{(*)}\bar{D}^{(*)}K$ modes

So far we discussed the $1^-$ and $0^-$ channels of the $\bar{c}s$ current. Now let us proceed to the positive-parity channels.

As is clear from Sect. 2 the nonresonant $S$ wave kaons are produced by $\bar{c}\gamma_{\mu}\gamma_5s$ in association with $\bar{D}^{(*)}$. The longitudinal part of the current produces $\bar{D}K$, while
the transverse component produces $\bar{D}^*\bar{K}$. We will assume that the kaons can be treated as soft Goldstone mesons. Needless to say, factorization is implied too.

Then, we have to write out the currents $\bar{b}\Gamma^\mu c$ and $\bar{c}\Gamma^\mu s$ in the chiral/heavy quark theory. The second current is given in Ref. [12]. The amplitude of the kaon “bremsstrahlung” in the transition $B \rightarrow D^{(*)}\bar{D}^{(*)}\bar{K}$ consists of two parts – contact and pole (see e.g. graphs of Figs. 1 and 2, respectively, in Ref. [12]; see also [38]). It is not difficult to check that all pole graphs vanish in the GSV limit, when $D$’s spatial velocities are set equal to zero. Additionally they are numerically suppressed by $g$, the $D^*D\pi$ coupling constant, which, according to Ref. [39], is about 0.3. Therefore, at the current level of accuracy, we prefer to omit the pole graphs altogether. In the future, as confidence in the numerical value of $g$ grows and estimates of the excited meson couplings acquire better accuracy, it will be necessary to return to the issue and include the pole graphs in the theoretical predictions.

With the pole graphs discarded, calculating the decay rates of the above transitions becomes trivial. Indeed, the ratio of the amplitude $D^{(*)}\bar{D}^{(*)}\bar{K}$ in the limit $p_K \rightarrow 0$ to the corresponding two-body amplitude $D^{(*)}\bar{D}^{(*)}$ is given by $i/f_\pi$. (Note that we will consistently neglect all SU(3)$_f$ breaking effects everywhere, except the phase spaces. Accounting for SU(3)$_f$ breaking effects in the amplitudes in the first order in $m_s$ is possible; we will defer this exercise as well.) Since the kaons are produced in the $S$ wave, and the contact amplitudes depend neither on the kaon energy nor on the angles, the ratio of probabilities is given merely by the ratio of the three-body to two-body phase spaces $V_{2,3}$,

$$\frac{\Gamma(\bar{B} \rightarrow D^{(*)}\bar{D}^{(*)}\bar{K})}{\Gamma(D^{(*)}\bar{D}^{(*)})} = \frac{1}{f_\pi^2 V_2} V_3. \quad (23)$$

The three-particle phase space is conveniently written out e.g. in Ref. [47].

Note that the above estimate is valid only for the “right” spin correlation modes, $D\bar{D}\bar{K}$ and $D^*\bar{D}^*\bar{K}$. The “wrong” spin correlation modes vanish in the GSV limit, for the same reason as was explained above, and cannot be computed without inclusion of the pole graphs. Their role, however, is significantly reduced in the three-particle decays compared to the two-particle case, since in the three-particle decays we are closer to the GSV limit.

It is convenient to present the ratio $V_3/V_2$ as follows:

$$\frac{V_3}{V_2} = \frac{M_B^2}{32\pi^2} \frac{I_3}{I_2}, \quad (24)$$

where the first factor corresponds to the ratio of the phase spaces with all massless final particles. The dimensionless factors $I_3$ and $I_2$ reflect the finite masses:

$$V_3 = M_B^2 I_3/(256\pi^3), \quad V_2 = \frac{1}{4\pi} \bar{p}/M_B = \frac{I_2}{8\pi}.$$  

Numerically $I_3$ are summarized in Table 4, while the branching ratios for three-particle yield for different nonresonant decay modes are given in Table 5.
Table 4:

| Decay                  | $I_3$       |
|-----------------------|------------|
| $B \to D\bar{D}K$ ($\eta$) | 0.073 (0.068) |
| $B \to D^*\bar{D}'K$ ($\eta$) | 0.039 (0.035) |
| $B \to D\bar{D}'K$ ($\eta$) | 0.014 (0.002) |
| $B \to D^*\bar{D}''K$ ($\eta$) | 0.002 (0.001) |

Table 5:

| Decay                  | $\text{Br, % (}x = 1\text{)}$ |
|-----------------------|-------------------------------|
| $B \to D\bar{D}K$     | 0.5                           |
| $B \to D^*\bar{D}K$   | 1                             |
| $B \to D\bar{D}'K$    | 0.3                           |
| $B \to D^*\bar{D}'K$  | 0.15                          |

Using the numerical values for $I_3$ one can easily estimate the corresponding branching ratios:

$$\frac{\Gamma(\bar{B} \to D\bar{D}K)}{\Gamma(D\bar{D}_s)} \approx 0.5,$$
$$\frac{\Gamma(\bar{B} \to D^*\bar{D}'K)}{\Gamma(D^*\bar{D}_s')} \approx 0.7,$$

and

$$\frac{\Gamma(\bar{B} \to D\bar{D}'K)}{\Gamma(D\bar{D}_s')} \approx 0.2,$$
$$\frac{\Gamma(\bar{B} \to D^*\bar{D}''K)}{\Gamma(D^*\bar{D}_s'')} \approx 0.04.$$ (25)

Here we accounted for the fact that one can have two $\bar{K}$'s, of different charge, in each process at hand, and incorporated the block factors for the two-body modes. The relevant branching ratios, computed directly, are collected in Table 5.

Altogether we see that the total branching for four possible nonresonant channels is $\sim 2\%$. In the same approximation it is easy to estimate the yield of nonresonant $D^{(*)}_s\eta$ instead of $D^{(*)}\bar{K}$. In the SU(3)$_f$ limit it is $1/3$ of the kaon yield. Taking account of the differences in the phase spaces we get $\text{Br}(B \to D\bar{D}_s\eta) \sim 0.3$ to $0.5\%$.

Altogether the nonresonant channels give $\sim 2 - 3\%$ of the total branching ratio.

(iv) Other modes with nonresonant Goldstone mesons

Invoking the chiral/heavy quark technique and factorization in the same vein as above, it is possible to calculate amplitudes with two soft Goldstone mesons (nonresonant), e.g. $D\bar{D}K\pi$ or $D\bar{D}_s\pi\pi$. For instance, the $D^*\bar{K}\pi$ state will be produced by the vector $\bar{c}\gamma_\mu s$ current, and all relevant matrix elements are already known in the literature [14]. A number of pole graphs are to be taken into account. We did
not attempt this calculation, because the four-particle phase space in the processes at hand is prohibitively small. The decays with two nonresonant Goldstone mesons are rare.

(v) *Resonances with the quantum numbers* $0^+$ and $1^+$ *in the $\bar{c}s$ channel*

Since the non-resonant contributions in the above channels turn out to be rather small, well below the 12% level following from the quark-gluon calculation (plus duality), it is natural to conclude that $D_{s1}$ and $D'_{s1}$ (and $D_{s0}^*$ and $D'_{s0}^*$) play a significant role. The mass of $D_{s1}$ is expected to be $\sim 2.5$ GeV, and it is natural to expect that the mass of $D'_{s1}$ is close to 3 GeV. The masses of $D_{s0}^*$ and $D'_{s0}^*$ are close to those above.

The estimates of the corresponding yield can be approached in a way similar to what has been done for the vector (pseudoscalar) channel. (The effect of the $s$-quark mass may be harder to take into account properly, since, as we have seen, the significant chiral kaon contribution gets suppressed, which may affect the residues of the resonances.) We use the following simplified estimate. The rate of $D_{s1}$ is expected to be approximately the same as that of $D_{s1}^*$. The first excitation of $D_{s1}$ is produced roughly with the same rate as the second excitation of $D_{s1}^*$. The same is valid with respect to $D_{s0}^*$ and $D_{s0}^*$ relative to the excitations of $D_s$.

One may expect that all these resonances decay in the $\bar{D}^{(*)}\bar{K}$ mode; in some cases $K^*$ may appear, instead of $K$. This will still lead to the “wrong” sign $D$. Then one gets $\sim 7$ to $11\%$ depending on the value of $x$.

(vi) *Excited D states produced by $\bar{b}\Gamma_\mu c$ bracket*

Above we considered only decays of $B$ into $D^{(*)}$ and some radially excited $\bar{D}_s$ states. There may also be decays into the excited states of $D$. The total yield for such decays with the radial excitations is quite small. There exist two possibilities: one can produce from the $\bar{b}\Gamma_\mu c$ current either radial excitations of $D^{(*)}$, or $P$-wave mesons.

In the first case the corresponding partial widths are proportional to the square of the Isgur-Wise function of the transition between $B$ and the radial excitations of $D$. However, the corresponding IW function vanishes as $v v' - 1$. Thus the corresponding partial widths are suppressed by a factor $(v v' - 1)^2 \sim \vec{v}^4$ relative to the transitions into $D^{(*)}$. A direct calculation shows that if one takes the masses of the excited $D$ states to lie uniformly below the masses of the excited $\bar{D}_s$ states by 150 MeV, the corresponding recoil values of $v v' - 1$ spread from 0 to 0.2. Thus the corresponding partial widths are suppressed by a factor of order 0.04 relative to the decays into the non-excited states. Our estimate would show that the corresponding contribution is less than 1%; although a larger effect is expected from the power corrections [38, 48], up to 10 to 20% of the elastic transition, they still seem to be small.

Higher rates can be due to the $P$-wave mesons from the $\bar{b}\Gamma_\mu c$ current; in this case the rates are only linear in $v v' - 1$. An upper bound (and a reasonable estimate) of them can be obtained by merely calculating the sum over all decay modes with $D$
and $D^*$, with a “faked” $\rho = 0.25$ (or with the excitation-free IW function $\xi_0(vv') = (1 + vv')/(2vv')^{1/2}$), through the sum rules for the transition amplitudes \cite{18}. We then compare the result obtained to that with the “actual” $\xi$ of Eq.\((12)\):

$$\text{Br}(B \to D^{\text{excit}} + Xcs) \approx \text{Br}(B \to D^{(*)} + Xcs)|_{\xi=\xi_0} - \text{Br}(B \to D^{(*)} + Xcs)|_{\xi=1-\rho^2(vv'-1)}.$$ 

Since for higher excitations of $D^{(*)}$ the phase space is limited, this relation only overestimates somewhat the associated production of excited $D^{(*)}$. In this way we arrive at the conclusion that the contribution of the excited $D^{(*)}$ states from the $b\bar{c}\Gamma_{\mu}c$ bracket is $\lesssim 3\%$.

5 Conclusions

The existence of the Goldstone mesons results in a very peculiar pattern of the heavy-light spectral densities, which turn out to be drastically different, say, in the vector and axial channels. By treating kaons as soft, we are able to calculate the nonresonant $D^{(*)}\bar{D}^{(*)}\bar{K}$ ($D^{(*)}\bar{D}_s^{(*)}\eta$) yield in $B$ decays.

Altogether, summing all two-body and three-body decays modes, we get 23 to 31\% for the total $b \to \bar{c}cs$ yield. The first number corresponds to $x = 0.6$ and the second to $x = 1$. Moreover, if our assumptions about the decay modes of $D''_s$, $D''_s$, $D''_{s1}$, $D'_{s1}$, $D''_{s0}$ and $D''_{s0}$ are justified, they lead predominantly to the “wrong” sign $D$’s. The yield of the “wrong” sign $D$’s then comes out to be 13 to 19\%\footnote{We assume here, rather arbitrarily, that the excited $D^{(*)}$ mesons from the $\bar{b}c$ bracket are produced at the level of 2\%, and this yield is split equally between the “wrong” sign $D$’s and $D_s$.} The yield of the “right sign” $D$’s is estimated to be 10 to 12\%; about 4\% comes from the modes $D^{(*)}\bar{D}_s^{(*)}$, and about the same from the modes with $\eta$ in the final state. Thus, not only we confirm, exploiting a different line of reasoning, the conclusion of Ref. \cite{7} regarding the abundance of the “wrong” sign $D$’s, but we encounter a menace of “overshooting”: the yield of the “wrong” sign $D$’s naturally tends to be too high.

Under the circumstances it is tempting to calibrate theoretical predictions by using the CLEO and ALEPH data on the “wrong” sign $D$’s, $\text{Br}(B \to D^{(*)}\bar{D}^{(*)}\bar{K}) = 10 \pm 4\%$. Combining this result with our estimates we see that the value of $x \approx 0.6$ is somewhat preferred. If so, the theoretically expected branching ratio of the “wrong” sign $D$’s is about 13\%, that of the “right sign” $D$’s is about 10\%, and the total branching ratio of the $\bar{c}cs$ channel is about 23\%. Thus, we see that our model of saturation of inclusive data with two- and three-particle exclusive modes is perfectly consistent with the duality-based prediction of Ref. \cite{1}, $\sim 23\%$ or slightly higher. It is quite remarkable that the exclusive estimates presented above do not contradict the quark-hadron duality. This is not too surprising, of course, since in modelling the resonance yields we respected the constraints imposed by QCD plus factorization. The accuracy of this conclusion is not high, though, at the level of 30\%.

10
As an experimental confirmation of our approach the $D\bar{D}_s\eta + ...$ yield will have to be seen at the 4% level.

The predictions for the absolute rates made above do not pretend to have too high an accuracy; suffice it to mention that the rates are proportional to $f_{\bar{D}(s)}^2$, and this quantity carries a noticeable uncertainty. However, it seems certain that, contrary to naive expectations, the modes with $\bar{D}_s$ (the “right” sign $D$’s) constitute a smaller fraction of the $b \to c\bar{c}s$ channel. This seems to be a rather general feature not depending on details of the approximations we rely on. The spectral density emphasizes larger invariant masses of the $\bar{c}s$ system, which then predominantly generates decay chains with strangeness and charm separated.

In our calculations we employed factorization, which must be violated at some level. However, if the cross-talking between two currents $\bar{c}s$ and $b \to c$ appears to be strong, then one must expect dissolving $D_s$ into separate charm and strange particles from the very beginning.

It is worth noting that the relative yield of $D_s^{(*)}$ is more sensitive to the strange quark mass, than the overall $\bar{c}c$ yield.

We point out that, although our calculations do not seemingly suggest a prominent mechanism for differentiating the hadronic widths of $B$ and $\Lambda_b$ in the $b \to c\bar{c}s$ channel, where the calculations naively go in closest analogy to the one presented above, there still is a potential of locating the origin of differences. The standard arguments for factorization based on $1/N_c$ counting rules are not necessarily applicable for the decays involving heavy baryons, in particular when the velocities of the final particles are small. This theoretical possibility must be carefully explored. On the other hand, a detailed experimental information about $\Lambda_b$ hadronic width and, first of all, the value of $n_c$, can provide us with an invaluable direct input.

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References

[1] M. Shifman, Theory of Preasymptotic Effects in Weak Inclusive Decays, in Proc. Workshop on Continuous Advances in QCD, ed. A. Smilga (World Scientific, Singapore, 1994), p. 249 [hep-ph/9405243]; Recent Progress in the Heavy Quark Theory, in Particles, Strings and Cosmology, Proc. V PASCOS Symp., March 1995, Baltimore, ed. J. Bagger (World Scientific, Singapore, 1996) [hep-ph/9505288].

[2] E. Poggio, H. Quinn and S. Weinberg, Phys. Rev. D13 (1976) 1958.
[3] B. Chibisov, R. Dikeman, M. Shifman and N. Uraltsev, Preprint CERN-TH/96-113 [hep-ph/9605465], to appear in IJMPA.

[4] B. Blok and T. Mannel, *Mod. Phys. Lett.* A11 (1996) 1263.

[5] I. Bigi, N. Uraltsev and A. Vainshtein, *Phys. Lett.* B293 (1992) 430.

[6] A. Falk, M. Wise and I. Dunietz, *Phys. Rev.* D51 (1995) 1183.

[7] G. Buchalla, I. Dunietz and H. Yamamoto, *Phys. Lett.* B364 (1995) 188; I. Dunietz, Preprint FERMILAB-PUB-96/104-T [hep-ph 9606247].

[8] I. Bigi, B. Blok, M. Shifman, and A. Vainshtein, *Phys. Lett.* B323 (1994) 408.

[9] E. Bagan, P. Ball, V. Braun and P. Gosdzinsky, *Phys. Lett.* B342 (1995) 362; (E) B374 (1996) 363;
E. Bagan, P. Ball, B. Fiol, and P. Gosdzinsky, *Phys. Lett.* B351 (1995) 546.

[10] M. B. Voloshin, *Phys. Rev.* D51 (1995) 3948.

[11] For a concise review see e.g. A. Vainshtein and V. Zakharov, *Sov. Phys. - Uspekhi* 13 (1970) 73.

[12] M.B. Wise, *Phys. Rev.* D45 (1992) 2188.

[13] G. Burdman and J. Donoghue, *Phys. Lett.* B280 (1992) 287;
T.M. Yan *et al.*, *Phys. Rev.* D46 (1992) 1148.

[14] C.L.Y. Lee, M. Lu, and M.B. Wise, *Phys. Rev.* D46 (1992) 5040.

[15] V. Novikov *et al.*, In Proc. Int. Conf. Neutrino-78, ed. E.C. Fowler, (West Lafayette, 1978) p. C278.

[16] M. Shifman, A. Vainshtein, and V. Zakharov, *Nucl. Phys.* B 147 (1978) 385, 448.

[17] E. Shuryak, *Nucl. Phys.* B198 (1982) 83.

[18] E. Bagan, P. Ball, V. Braun, and H.G. Dosch, *Phys. Lett.* B278 (1992) 457.

[19] Y.-B. Dai, C.-S. Huang, M.-Q. Huang, and C. Liu, [hep-ph/9609436];
P. Colangelo, G. Nardulli, and A. Ovchinnikov, *Phys. Lett.* B269 (1991) 201;
P. Colangelo, G. Nardulli, and N. Paver, *Phys. Lett.* B293 (1992) 207.

[20] M. Luke, *Phys. Lett.* B252 (1990) 447.

[21] M. Neubert, *Phys. Rev.* D45 (1992) 2451;
S. Narison, *Phys. Lett.* B308 (1993) 365.
[22] P. Ball and V. Braun, *Phys. Rev.* **D49** (1994) 2472; E. Bagan, P. Ball, V. Braun, and P. Gosdzinsky, *Phys. Lett.* **B342** (1995) 362.

[23] M. Voloshin, *Int. J. Mod. Phys.* **A10** (1995) 2865.

[24] A. Falk, *The Many Uses of Excited Heavy Hadrons*, Preprint JHU-TIPAC-96014 [hep-ph/9609380].

[25] H. Wittig, *Lattice Results for Heavy Quark Physics*, [hep-ph/9509292]; C. Bernard et al., *fB Quenched and Unquenched*, [hep-lat/9509045].

[26] M. Voloshin and M. Shifman, *Yad. Fiz.* **45** (1987) 463 [Sov. J. Nucl. Phys. **45** (1987) 292]; H. D. Politzer and M. Wise, *Phys. Lett.* **B206** (1988) 681.

[27] M. Shifman, *Z. Phys.* **C9** (1981) 347.

[28] T. Browder (representing CLEO collaboration), Talk at the XXVIII International Conference on High Energy Physics, Warsaw, July 25-31, 1996, and private communication.

[29] D. Buskulic et al. (ALEPH Collaboration), Preprint CERN-PPE/96-117.

[30] T. Browder, K. Honscheid, and D. Pedrini, *Nonleptonic Decays and Lifetimes of b-quark and c-quark Hadrons*, Preprint UH-515-848-96 [hep-ph/9606354], to appear in *An. Rev. Nucl. Part. Sci.* This paper also contains an excellent review of experimental data.

[31] G. Altarelli and S. Petrarca, *Phys. Lett.* **B261** (1991) 303.

[32] P. Ball, private communication.

[33] M. B. Voloshin, Preprint TPI-MINN-96/1-T [hep-ph/9602256].

[34] B. Blok and M. Shifman, *Nucl. Phys.* **B389** (1993) 534.

[35] B. Blok and M. Shifman, *The Fermilab Meeting*, Proc. 1992 DPF Meeting of APS, Eds. C.H. Albright et al. (World Scientific, Singapore) p. 537.

[36] M. Shifman, *Nucl. Phys.* **B388** (1992) 346.

[37] T. Mannel, W. Roberts, and Z. Ryzak, *Phys. Rev.* **D44** (1991) 18; *Phys. Lett.* **B259** (1991) 359.

[38] M. Shifman, N. Uraltsev and A. Vainshtein, *Phys. Rev.* **D51** (1995) 2217.

[39] V. Belyaev, V. Braun, A. Khodjamirian, and R. Rückl, *Phys. Rev.* **D51** (1995) 6177.
[40] S. Nussinov and W. Wetzel, *Phys. Rev.* **D36** (1987) 130.

[41] M. Voloshin and M. Shifman, *Yad. Fiz.* **47** (1988) 801 [Sov. J. Nucl. Phys. **47** (1988) 511].

[42] N. Isgur and M. Wise, *Phys. Lett.* **B232** (1989) 113; *Phys. Lett.* **B237** (1990) 527.

[43] B. Blok and V. Eletsky, *Sov. J. Nucl. Phys.* **42** (1985) 787.

[44] J.D. Richman and P.R. Burchat, *Rev. Mod. Phys.* **67** (1995) 893.

[45] B. Blok and M. Shifman, *Phys. Rev.* **D47** (1992) 2949; E. Bagan, P. Ball, and P. Gosdzinsky, *Phys. Lett.* **B301** (1993) 249; the first calculation of $\rho^2$ was done by M. Neubert, *Phys. Rev.* **D45** (1992) 2451; this reference, however, gives a value of $\rho^2$ larger than unity, which is disfavoured by the current experimental data.

[46] R.M. Barnett *et al.* (Particle Data Group), *Phys. Rev.* **D52** (1996) 1.

[47] B. de Wit and J. Smith, *Field Theory in Particle Physics* (Elsevier, Amsterdam, 1986).

[48] I. Bigi, M. Shifman, N.G. Uraltsev and A. Vainshtein, *Phys. Rev.* **D52** (1995) 196.