Kondo effect in a one-electron double quantum dot: Oscillations of the Kondo current in a weak magnetic field

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We present transport measurements of the Kondo effect in a double quantum dot charged with only one or two electrons, respectively. For the one electron case we observe a surprising quasi-periodic oscillation of the Kondo conductance as a function of a small perpendicular magnetic field \( B_L \) \( \lesssim \) 50 mT. We discuss possible explanations of this effect and interpret it by means of a fine tuning of the energy mismatch of the single dot levels of the two quantum dots. The observed degree of control implies important consequences for applications in quantum information processing.

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The Kondo effect describes a bound state formed by interactions between a localized magnetic impurity and itinerant conduction band electrons shielding the localized spin. This results in an increased density of localized states at the Fermi energy, causing anomalous low temperature properties. In case of a degenerate ground state of a quantum dot (QD), the Kondo effect manifests itself as an enhanced conductance within the Coulomb blockade region \( U \). This was first observed on large QDs with half integer spin \( \frac{1}{2}, \frac{3}{2} \), and later, for a total spin of \( S = 1 \), where the triplet states of a QD are degenerate \( \frac{3}{2}, \frac{1}{2} \). On a double quantum dot (DQD) a two-impurity Kondo effect was studied \( 2, \frac{3}{2} \).

In this article we present the results of Kondo effect differential conductance (KDC) measurements on a DQD charged with one or two electrons in a perpendicular magnetic field \( B_L \). For only one electron \( (N = 1) \) in the DQD we observe a quasi-periodic structure of the KDC with a characteristic scale of \( B_0 \sim 10 \text{ mT} \). In contrast for \( N = 2 \) the KDC is found to be a monotonic function of \( B_L \). We discuss possible explanations for this effect that imply consequences in quantum information processing.

Our sample is fabricated from an AlGaAs/GaAs heterostructure. It embeds a two-dimensional electron system (2DES) with carrier density \( n_e \sim 1.8 \times 10^{15} \text{ m}^{-2} \) and electron mobility \( \mu \sim 75 \text{ m}^2/\text{Vs} \) (at \( T = 4.2 \text{ K} \)) below its surface. Figure 1(b) shows Ti/Au-gates created by electron beam lithography. They are used to locally deplete the 2DES to define a one electron QD. The gate design is optimized for transport measurements through a QD charged by only few electrons \( 10 \). By decreasing the voltages applied to gates \( g_G \) and \( g_X \) (with respect to the 2DES) while increasing the voltages on the side gates \( g_l \) and \( g_r \) we deform the QD into a DQD (sketched in Fig. 1(b)) \( 11, 12 \). The DQD is tuned to the regime of strong coupling to the leads and an order of magnitude stronger interdot tunnel coupling of \( 2t_0 \sim 240 \mu \text{eV} \) between the adjacent QDs \( 11 \). Measurements are performed in a dilution refrigerator at an electron temperature \( T_{\text{2DES}} \sim 0.1 \text{K} \).

A nearby quantum point contact (QPC) is used to detect the charge distribution of the DQD shown in the stability diagram in Fig. 1(a) \( 13 \). It displays a lock-in measurement of the differential transconductance \( \Delta G = \partial I / \partial U \) as a function of the dc voltages applied to gates \( g_L \) and \( g_R \). In the lower left corner region in Fig. 1(a) the DQD is uncharged (compare figure caption) \( 11 \).

The differential conductance of the DQD is plotted in Fig. 1(c) as a function of the applied bias voltage \( U_{\text{SD}} \) and the center gate voltage \( U_{\text{GC}} \). The DQD is tuned such that the variation in \( U_{\text{GC}} \) (x-axis) causes a shift in the stability diagram approximately along the arrow in Fig. 1(a). Hence, a charge between \( N = 0 \) and 3 electrons, marked in Fig. 1(c) by numerals, is distributed symmetrically between the adjacent QDs. Within the diamond-shaped regions transport is impeded by Coulomb blockade (CB). Nevertheless, strong coupling of our DQD to its leads allows for inelastic co-tunneling causing an enhanced conductance within the CB regions at \( U_{\text{SD}} = 0 \), \( e.g. \) for \( N = 1 \) at \( \left| eU_{\text{SD}} \right| \lesssim 2t_0 \sim 240 \mu \text{eV} \) \( 14 \).

In addition, at \( U_{\text{SD}} = 0 \) an increased differential conductance is visible in the CB regions for \( N = 1, 2 \) or 3. We assign this zero bias anomaly to the Kondo effect on a DQD, here charged with only a few electrons. The observed KDC is small compared to the unitary limit \( (G \ll 2e^2/h) \). This is due to the tunnel barrier hindering electron transport between the two adjacent QDs and to an asymmetric coupling to the leads \( 11 \). For \( N = 1 \) or 3 the KDC of the DQD can be described by the spin
1/2 Kondo effect, but for \( N = 2 \) the threefold degenerate triplet states lead to the KDC. This suggests that the exchange coupling separating the triplet states from the singlet ground state is smaller than either the Kondo or the electron temperature.

Figure 1 displays the KDC at \( U_{SD} \approx 0 \) of the DQD as a function of a magnetic field \( B_{\perp} \) perpendicular to the 2DES for \( N = 1 \) and \( N = 2 \), respectively. Each point corresponds to the maximum KDC near zero bias measured at constant gate voltage approximately along the white vertical lines in Fig. 1(c). Three of these traces \( G(U_{SD}) \) are plotted in the inset of Fig. 2. For increasing \( B_{\perp} \) the KDC is expected to monotonically decrease as the spin degeneracy is lifted. A theory by Pustilnik and Glazman provides analytical expressions for the limits \( B \ll B_K \) and \( B \gg B_K \) [15], where the characteristic field \( B_K \) is determined by the Kondo temperature \( k_B T_K = g \mu_B B_K \). For \( B_{\perp} \ll B_K \), the KDC is described by \( G \approx G_0(1 - (B_{\perp}/B_K)^2) \) and for \( B_{\perp} \gg B_K \) by \( G \approx G_0/\ln^2(B_{\perp}/B_K) \). \( G_0 \) is the KDC at \( B_{\perp} = 0 \). The lines in Fig. 2 model these expressions with \( T_K = 0.1 \) K for \( N = 1 \) and \( T_K = 0.12 \) K for \( N = 2 \). \( T_K \) is taken identical for both limits (solid and dashed lines), respectively. Being close to the electron temperature of the 2DES, \( T_K \) cannot be extracted from temperature dependences as usual. Nevertheless, the model curves used here are expected to hold even for \( T_K \sim T_{2DES} \). The agreement with our data is satisfactory.

For \( B_{\perp} \gtrsim 0.5 \) T the decrease of the KDC gets steeper due to a \( B_{\perp} \) dependent decrease of the interdot tunnel coupling, specifically investigated for our DQD [11]. Taking such effects into account does not change the Kondo temperature too much compared to the simple model presented here. However, our simplified model causes the fit-parameter \( G_0 \) to be strongly suppressed compared to \( G_0 \).

Figure 3 shows detailed measurements of the KDC for \( N = 1 \) (main figure) and \( N = 2 \) (inset), as plotted in Fig. 2 but at small \( |B_{\perp}| < 0.1 \) T. All curves are symmetric in respect to the magnetic field direction, despite a small offset of \( B_{\text{offset}} \approx -2 \) mT caused by a residual background magnetic field. As expected, the KDC for \( N = 2 \) (inset) as well as the co-tunneling differential conductance for \( N = 1 \) (not shown) decrease monotonically when the magnetic field increases. Surprisingly, for \( N = 1 \) the KDC shows a non-monotonic behavior. A pronounced local minimum at \( B_{\perp} \approx B_{\text{offset}} \approx 0 \)
is followed by a quasi-periodic oscillation with minima at \( |B_{\perp} - B_{\text{offset}}| \approx 0, 20, 30, 42 \text{ mT} \) (vertical lines in Fig. 3). These oscillations quickly decay with increasing magnetic field and are convolved with the expected decrease of the KDC for \( B \ll B_K \) (dashed line).

One can consider the following possible explanations for the KDC observed in a DQD for \( N = 1 \) to be a nonmonotonic function of \( B_{\perp} \): Namely, (I, II) the leads, (III) nuclear spins, (IV) Aharonov-Bohm (AB) like interferences, or (V) the alignment of energy levels of the two adjacent QDs. All but the last of these possibilities can be ruled out.

(I) Shubnikov-de Haas oscillations in the leads cannot depend on the number of electrons. Thus, in contrast to our findings they should manifest identically in both cases for \( N = 1 \) and 2.

(II) Spin orbit (SO) interaction in the leads may result in a suppression of the KDC in zero magnetic field due to spin entanglement between the electrons in the lead and the dot \([16]\). The SO interaction, however, cannot explain oscillations of the KDC and a quasi-periodic peak structure. Furthermore, the SO interaction should equally affect the dot whether charged with one or two electrons. Experimentally, there is no evidence of a conductance minimum for \( N = 2 \) (see inset of Fig. 3). We conclude, that the influence of SO interaction is negligible for the effect we observed.

(III) In GaAs, \( \sim 10^5 \) nuclear spins form an internal Overhauser field \( B_{\text{nuc}} \sim 10 \text{ mT} \) applied to the electrons on each QD. This field fluctuates on the time scale of \( t_N \sim 10 \text{ ms} \) \([17]\). In our lock-in measurements the data are averaged over a much longer time of \( \sim 300 \text{ ms} \). Hence, the fluctuations of \( B_{\text{nuc}} \) are unlikely to be responsible for the observed oscillations.

(IV) Interference effects in the orbital motion of an electron in a double well potential, which determines the DQD, could lead to AB-like oscillations in the amplitude of the tunneling between the two wells. In terms of the magnetic flux the period of the oscillation is the flux quantum \( \Phi_0 = h/e \) \([18]\). The overlap of the wavefunctions centered in the adjacent wells of a DQD is proportional to a relative phase shift \( \langle \psi_1 | \psi_2 \rangle \propto \exp(2\pi i B S_B/\Phi_0) \) of the classically forbidden region \( S_B \sim Ld \) (we assume a rectangular barrier of width \( d \), lateral extension \( L \) and height \( V \) separating the two wells). Therefore, one would expect the period of the oscillations to be of the order \( \Phi_0/S_B \) which is \( \sim 0.5 \text{ T} \) for our DQD. This is far in excess of the typical quasi-period (\( \sim 10 \text{ mT} \)) of the observed oscillations (Fig. 3).

As to AB interferences between different tunneling paths, the area enclosed by a possible AB contour can be estimated to be \( S_{\text{AB}} \sim Ld \) (or even smaller due to the serial configuration of the DQD).

We also consider the possibility of Fano-like oscillations, which would be associated with a leakage current under our gates \([19]\). However, experimentally we exclude leakage currents due to several measurements (not shown here).

(V) We believe that the observed quasi-periodic oscillations can be attributed to the magnetic field effect on the alignment of the energy levels in the two adjacent QDs. There is no reason to expect that the DQD structure is perfectly symmetric and the single electron eigen-energies \( \epsilon_{1,2} \) in the two QDs are exactly identical. The transmission through the barrier corresponding to a one-dimensional motion of an electron is determined by the overlap of the wave functions \( \langle \psi_1 | \psi_2 \rangle \sim \sinh(\kappa_{\perp})e^{-\kappa_{\perp}}/\kappa_{\perp} \), where \( \kappa_{\pm} = (\kappa_1 \pm \kappa_2)/2 \) and \( \kappa_{1,2} = \sqrt{2md^2(V - \epsilon_{1,2})}/h \). The tunneling rate reaches its maximum at resonance for \( \kappa_1 = \kappa_2 \).

Since the eigen-energies and, consequently, the parameters \( \kappa_{1,2} \) depend on the magnetic field, the latter can be used to fine tune the tunneling rate. What is the magnetic field that can compensate a mismatch \( \Delta \epsilon = |\epsilon_1 - \epsilon_2| \) between the ground state eigen-energies of the two wells? Using the 1d Schrödinger equation for a rectangular (or parabolic) double well potential one obtains \( h\omega_c \sim \sqrt{\epsilon_1 \epsilon_2} \sqrt{\Delta \epsilon/W} \), where \( W \) is the energy difference between the two local minima of the double well potential and \( \omega_c = |eB/m| \) is the cyclotron frequency.

At \( B_{\perp} = 0 \) the energy mismatch \( \Delta \epsilon \) is finite. It takes about \( B_0 \approx \pm 12 \text{ mT} \) to align the ground states, which corresponds to the first KDC maximum. The suppression of the mismatch by the magnetic field explains the
pronounced minimum at $B_\perp = 0$. This behavior is ob-

Note that the characteristic magnetic field $B_\perp$ can be very small due to the factor $\sqrt{\Delta \nu/W}$, roughly esti-
mated to be $\sim 10^{-3}$ for our setup. The interdot tunneling 
rate is unaffected by thermal line broadening as long as it 

corresponds to low frequency noise allowing adiabatic 
alignment of the energy levels in both QDs.

The remaining KDC maxima at slightly larger mag-
netic fields probably correspond to alignments of excited 
energy states in the two QDs. This implies the impor-
tance of co-tunneling processes, which are indeed strong 
(compare Fig. 1(c)). Moreover, in order to explain the 
observed quasi-periodic magnetic field dependence, the 
level structures in the two QDs should differ due to some 
anisotropy. From the number of observed KDC maxima we 
conclude that at least three excited states are involved in 
the co-tunneling.

Our model is consistent with the missing KDC oscil-
lations for the doubly occupied DQD ($N = 2$). Indeed, for 
$N = 2$ and a symmetric charge distribution with one 
electron in each dot, transport is determined by the sin-
glet and triplet states. The symmetric charge distribu-
tion allows enhanced elastic co-tunneling decreasing the 
dependence on the misalignment of the single dot ground 
states.

In conclusion, we here presented measurements of the 
Kondo effect on a DQD charged by only one or two elec-
trons. We demonstrate control of the resonant tunnel-
ing in the one electron case by means of a magnetic field 
which appears to be surprisingly small. A non-monotonic 
magnetic field dependence of the KDC is attributed to 
the anisotropy of the DQD. The magnetic field fine tunes 
the alignment of energy levels in the adjacent QDs, mod-
ifying the interdot tunnel splitting. Hence, the magnetic 
field provides an extremely sensitive tool to detect and 
control the anisotropy of a single electron DQD.

Our double dot with strong coupling to the leads is not 
a perfect system for a qubit. However, the influence of 
a very small misalignment of the single dot energy levels 
onto the tunnel probability of the electron will remain 
even for a double dot weakly coupled to the leads, which 
can be used e.g. as a charge qubit [20, 21]. The ob-

erved variations of the interdot tunnel splitting at very 
small magnetic fields have to be taken into account for 
the design of semiconductor nano-devices in the field of 
quantum information processing [22].

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