The importance of the mixed phase in hybrid stars built with the Nambu-Jona-Lasinio model

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We investigate the structure of hybrid stars based on two different constructions: one is based on the Gibbs condition for phase coexistence and considers the existence of a mixed phase (MP), and the other is based on the Maxwell construction and no mixed phase is obtained. The hadron phase is described by the non-linear Walecka model (NLW) and the quark phase by the Nambu-Jona-Lasinio model (NJL). We conclude that the masses and radii obtained are model dependent but not significantly different for both constructions.

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I. INTRODUCTION

Understanding the processes involved in the supernova explosions, in the creation of stellar compact objects and in their temporal evolution requires a huge multidisciplinary effort with investigations in areas as distinct as nuclear and particle physics, thermodynamics, quantum field theory and astrophysics.

In the present work we concentrate on the description of neutron stars. From very low densities up to the high densities present in their core, the constitution of these compact objects is a great source of speculation. At low densities there can be neutrons, protons, electrons and possibly neutrinos (at finite temperatures). At high densities, stellar matter can be much more complex, including hyperons, kaons and even deconfined quarks.

Many works considering the construction of equations of state (EoS) used to describe compact objects have already been done \cite{1,2}. Once a certain EoS is obtained, it serves as input to the Tolman-Oppenheimer-Volkoff equations (TOV) \cite{3} and the output gives the structure of the compact stars, characterized by their mass and radius. An appropriate EoS or an inadequate one can only be chosen or ruled out once astronomical observations are used as constraints. Although some observational results are known, many uncertainties exist. It is still unknown whether the neutron stars are composed only of hadrons and leptons, necessary to ensure chemical equilibrium and charge neutrality \cite{4}, if they are quark stars \cite{5} or even hybrid stars, containing both hadron and quark matter in their interior \cite{6,7}. Each one of these possibilities is represented by a great variety of relativistic and even non-relativistic models used to built the EoS.

We next investigate hybrid stars only, whose existence is a source of intense discussions in the literature \cite{2,6–9}. The discussion presented in \cite{7} is particularly interesting because the existence of quark stars is shown to be questionable within the calculations performed (which depend strongly on a specific parametrization). Moreover, it is also pointed out that the possibility of a mixed population (or hybrid stars) is compatible with the calculations of model dependent quark matter nucleation, what reinforces the interest in the calculations of hybrid stars as compact objects. Recent calculations show the importance of the nucleation mechanism in the process of phase transition from hadronic to quark matter \cite{10,11}.

The main reason for the present work is the fact that many astrophysicists claim that the mixed phase is only a hypothetical choice and cannot be checked. Moreover, some authors calculated macroscopic quantities as radii and masses for hybrid stars with and without the mixed phase and concluded that the differences were not significant \cite{12,13} or that the region corresponding to the hadron-quark mixed phase is too narrow \cite{14}. Although hybrid stars have been obtained with different combinations of models for the hadron and the quark phases, most of the discussions on the use of Gibbs and Maxwell constructions have been based on the MIT bag model \cite{15} for the description of the quark phase. The MIT bag model \cite{15} is a very simple model that does not reproduce some of the necessary features of QCD at high densities, as chiral symmetry, for instance. As it is easily checked on the literature, all results for compact stars are model dependent. Hence, before completely ruling out the need for the Gibbs construction and the consequent existence of the mixed phase in hybrid stars, it is desirable that another calculation with a different model for the quark phase is considered. That is the basis of the calculations and discussion that follows.

In the present paper, the hadron phase is described by the non-linear Walecka model (NLW) \cite{16} and the quark phase by the Nambu-Jona-Lasinio model (NJL) \cite{17}. Two different constructions are made: one with a mixed phase (MP) and another without the mixed phase, where the hadron and quark phases are in direct contact. In the first case, neutron and electron chemical potentials are continuous throughout the stellar matter, based on the standard thermodynamical rules for phase coexistence known as Gibbs conditions. In the second case, the electron chemical potential suffers a discontinuity because only the neutron chemical potential is imposed to be continuous. The condition underlying the fact that only a single chemical potential is common to both phases is known as Maxwell construction. In our approach we ignore surface and Coulomb effects for the structure in

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the mixed phase so the leptons are taken into account as free Fermi gases. However, it is worthy pointing out that the energy density in mixed phases should depend on the electromagnetic and surface contributions and this is commonly known as finite size effects. In [13, 14, 18] it was shown that for a weak surface tension the EoS resembles the one obtained with a Gibbs construction while for a strong surface tension, the Maxwell construction was reproduced. Unfortunately, the surface energy coefficient is not well described yet [19]. The differences between stellar structures obtained with both constructions are discussed through out the present paper. A similar calculation was done in [18], where the effects of different lepton fractions on protoneutron stars with trapped neutrinos were investigated. Although the result for zero temperature was also presented, its validity when trapped neutrinos are enforced is only academic because the neutrino mean free path at $T=0$ is larger than the neutron star radius. While in [18] no hyperons were included in the hadronic phase, they are also taken into account in the present paper for two parametrizations of the NLW model. Notice, however, that $s$ quarks were also considered in the quark phase described in [18].

The fact that the NJL model incorporates chiral symmetry and that the strange quark appears only in densities much higher than the $u$ and $d$ quarks are the main reasons for the differences. Hence, the calculations for the hybrid stars are here done with the NJL model so the previous conclusions on the mixed phase are confirmed or refuted. The consequences of the inclusion of the $s$-quark in the NJL model at quite high densities is also seen once a comparison between the two versions of the NJL model, i.e., SU(2) and SU(3) is performed. Whenever the SU(2) version of the NJL is used to describe quark matter, the corresponding hadron phase is strangeness free, i.e., no hyperons are considered. Two parameter sets are used for each case considered so that the model dependence can be established.

The paper is organized as follows: In Sec. 2 we show the lagrangian densities of the models considered and describe the formalism used; in Sec. 3 we present and discuss the results; in Sec. 4 we draw our final conclusions.

II. THE FORMALISM

We next give some of the main equations related to the two models used in our investigation. Detailed calculations are extensively available in the literature and hence are omitted in the present paper. Two possible systems are studied: one comprehends 8 baryons in the hadron phase and 3 quarks in the quark phase and the other includes only protons and neutrons in the hadron phase and the corresponding $u$ and $d$ quarks in the quark phase. In most cases, our studies refer to a hadron matter with all 8 baryons and a quark matter with the 3 possible quarks.

A. The NJL model

The NJL model is defined by the lagrangian density

$$\mathcal{L}_{NJL} = \bar{q}(i\gamma^{\mu}\partial_{\mu} - m)q + g_{S} \sum_{a=0}^{8} [(\bar{q}\lambda^{a}q)^{2} + (\bar{q}i\gamma_{5}\lambda^{a}q)^{2}]$$

$$- g_{D}\{det[\bar{q}_{i}(1+\gamma_{5})q_{j}] + det[\bar{q}_{j}(1-\gamma_{5})q_{i}]\},$$

where $q = (u, d, s)$ are the quark fields and $\lambda_{a}$ (0 ≤ $a$ ≤ 8), are the U(3) flavour matrices. The model parameters are the current quark mass matrix $m = diag(m_{u}, m_{d}, m_{s})$, the coupling constants $g_{S}$ and $g_{D}$, and the cutoff in 3-momentum space $\Lambda$.

The thermodynamical potential density is given by $\Omega = \varepsilon = \sum_{i} \mu_{i}\rho_{i} - \Omega_{0}$, where the energy density is

$$\varepsilon = -2N_{c} \int \frac{d^{3}p}{(2\pi)^{3}} \frac{p^{2} + m_{i}M_{i}}{E_{i}} \theta(\Lambda^{2} - p^{2})$$

$$- 2g_{S} \sum_{i=\text{u,d,s}} \langle \bar{q}_{i}q_{i} \rangle^{2} + 2g_{D} \langle \bar{u}u \rangle \langle \bar{d}d \rangle \langle \bar{s}s \rangle - \varepsilon_{0}.$$  

In the above expressions, $N_{c} = 3$, $E_{i} = \sqrt{p^{2} + M_{i}^{2}}$, $\mu_{i}(\rho_{i})$ is the chemical potential (number density) of particles of type $i$, for which $\varepsilon_{0}$ and $\Omega_{0}$ are included in order to ensure $\varepsilon = \Omega = 0$ in the vacuum. The quark condensates and the quark densities are defined, for each of the flavors $i = u, d, s$, respectively, as:

$$\langle \bar{q}_{i}q_{i} \rangle = -2N_{c} \int \frac{d^{3}p}{(2\pi)^{3}} \frac{M_{i}}{E_{i}} \theta(\Lambda^{2} - p^{2}),$$

$$\rho_{i} = \langle \bar{q}_{i}q_{i} \rangle = \frac{P_{i}^{3}}{3\pi^{2}}.$$  

Minimizing the thermodynamical potential $\Omega$ with respect to the constituent quark masses $M_{i}$ leads to three gap equations for the masses $M_{i}$

$$M_{i} = m_{i} - 4g_{S} \langle \bar{q}_{i}q_{i} \rangle + 2g_{D} \langle \bar{q}_{j}q_{j} \rangle \langle \bar{q}_{k}q_{k} \rangle,$$

with cyclic permutations of $i, j, k$.

The pressure can be found from

$$P = -\Omega = -\varepsilon + \sum_{i} \mu_{i}\rho_{i} + \Omega_{0}.$$  

The relations between the chemical potentials of the different particles required by $\beta$-equilibrium are given by

$$\mu_{u} = \mu_{d} = \mu_{u} + \mu_{c}, \quad \mu_{c} = \mu_{\mu}.$$
and for charge neutrality we must impose

$$\rho_e + \rho_\mu = \frac{1}{3} (2 \rho_n - \rho_p - \rho_s).$$ (8)

In order to obtain the NJL SU(2) model we just need to neglect the terms related to the strange quark in equations 2, 4 and 5. It means that $\langle \bar{s}s \rangle = \mu_s = \rho_s = 0$.

The parameter sets of the NJL model used in the present work are given in Table 1.

### B. The non-linear Walecka model

The lagrangian density for the NLW model reads

$$\mathcal{L}_{NLW} = \sum_B \bar{\psi}_B [\gamma_\mu (i\partial^\mu - g_{\nu B}V^\mu - g_{\rho B}B^\mu \cdot b^\nu) - (M_B - g_{\nu B}\phi)] \psi_B + \frac{1}{2} (\partial_\mu \phi \partial^\mu \phi - m_s^2 \phi^2) - \frac{1}{3!} \phi^3 - \frac{1}{4!} \lambda \phi^4$$

$$- \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_v^2 V_\mu V^\mu - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{2} m_p^2 b_\mu \cdot b^\mu, \quad (9)$$

with $\Omega_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ and $B_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu - g_\rho (b_\mu \times b_\nu)$. The hyperon coupling constants are defined as $x_i = \frac{2a_i}{g_\rho}$, $i = s, v, \rho$.

In a mean field approximation the energy density reads

$$\varepsilon = \frac{\gamma}{2\pi^2} \sum_B \int_0^{\infty} \frac{P_F^B}{p^2 dp} \sqrt{p^2 + M_F^2}$$

$$+ \frac{m_v^2 V^2}{2} + \frac{m_p^2 b_0^2}{2} + \frac{m_v^2 \phi^2}{2} + \frac{\kappa}{6} \phi^3 + \frac{\lambda}{24} \phi^4, \quad (10)$$

and the pressure becomes

$$P = \frac{\gamma}{6\pi^2} \sum_B \int_0^{\infty} \frac{P_F^B}{\sqrt{p^2 + M_F^2}^2} dp$$

$$+ \frac{m_v^2 V^2}{2} + \frac{m_p^2 b_0^2}{2} - \frac{m_v^2 \phi^2}{2} - \frac{\kappa}{6} \phi^3 - \frac{\lambda}{24} \phi^4, \quad (11)$$

where $\gamma = 2$ is the spin degeneracy factor.

The conditions of chemical equilibrium are also imposed through the two independent chemical potentials $\mu_n$ and $\mu_e$ and it implies that

$$\mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_\Lambda = \mu_n,$$

$$\mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e,$$

$$\mu_{\Sigma^+} = \mu_\pi = \mu_n - \mu_e.$$ (12)

For the charge neutrality, we must have

$$\sum_B q_B \rho_B + \sum_l q_l \rho_l = 0,$$ (13)

where $q_B$ and $q_l$ stand, respectively, for the electric charges of baryons and leptons.

When the system is constituted only of protons and neutrons, the sum on the baryons appearing in the equations above are restricted to the nucleons and the only condition for chemical equilibrium is the last one in (12).

Two sets of parameters were chosen and they are given in Table 2. The following parameters are equal for both sets: $x_s = 0.7$, $x_v = x_\rho = 0.783$, $m_s = 400$ MeV, $m_v = 783$ MeV and $m_\rho = 770$ MeV.

#### 1. Comments on the inclusion of the leptons

As we are dealing with neutral stellar matter in $\beta$-equilibrium in both quark and hadron phases according to eqs. (7)-(8) and (12)-(13) respectively, the electrons and muons have to be introduced. They are normally included as free Fermi gases obeying the following lagrangian density:

$$\mathcal{L}_l = \sum_i \bar{\psi}_i (i \gamma^\mu \partial_\mu - m_l) \psi_i, \quad l = e^-, \mu^-.$$ (14)

Expressions for energy density and pressure in a MFT become:

$$\varepsilon = \frac{1}{\pi^2} \sum_l \int_0^{K_{F_l}} \frac{p^2 dp}{\sqrt{p^2 + m_l^2}},$$ (15)

and

$$P = \frac{1}{3\pi^2} \sum_l \int_0^{K_{F_l}} \frac{p^4 dp}{\sqrt{p^2 + m_l^2}}.$$ (16)

### C. Hybrid stars with mixed phase

We next build a mixed phase (MP) constituted of hadrons and quarks, which interpolates between the hadron (HP) and the quark phase (QP). In the mixed phase charge neutrality is not imposed locally but only globally. This means that quark and hadron phases are not neutral separately, but rather, the system prefers to rearrange itself so that

$$\chi \rho_c^{QP} + (1 - \chi) \rho_c^{HP} + \rho_c^{\\downarrow} = 0,$$

where $\rho_c^{iP}$ is the charge density of the phase $i$, $\chi$ is the volume fraction occupied by the quark phase, and $\rho_c^{\\downarrow}$ is the electric charge density of leptons. According to the Gibbs conditions for phase coexistence, the neutron chemical potentials, the electron chemical potentials and pressures have to be identical in both phases, i.e. [2],

$$\mu_n^{HP} = \mu_n^{QP}, \quad \mu_e^{HP} = \mu_e^{QP} \quad \text{and} \quad P^{HP} = P^{QP}.$$
As a consequence, the energy density and total baryon density (no leptons included) in the mixed phase read
\[ \langle \varepsilon \rangle = \chi \varepsilon^{QP} + (1 - \chi)\varepsilon^{HP} + \varepsilon^t \] \hspace{1cm} (17)
and
\[ \langle \rho \rangle = \chi \rho^{QP} + (1 - \chi)\rho^{HP}. \] \hspace{1cm} (18)

**D. Hybrid stars without mixed phase**

Much simpler than the case above, we just need to find the point where
\[ \mu_n^{HP} = \mu_n^{QP} \quad \text{and} \quad P^{HP} = P^{QP}, \]
and then construct the EoS. In this case the electron chemical potential suffers a discontinuity when passing from the hadron to the quark phase as expected from the simple use of the Maxwell conditions.

**III. RESULTS**

In the graphs shown next SU(3) stands for the quark phase taking into account the strange quark and SU(2) represents the NJL model without the strange quark. GM1 and GM3 represent the hadron phase with their respective set of parameters. Systems without strangeness are described by protons and neutrons in the hadron phase and quarks u and d in the quark phase. Systems with strangeness also accommodate the hyperons in the hadron phase and quark s in the quark phase.

In Fig. 1 the EoS for the pure hadron and pure quark matter are shown. They are the base to build the EoS of the hybrid stars. The kick, or variation, in curvature in the quark system is due to the appearance of the s-quark.

In Figs. 2 and 3 the EoS for the two types of hybrid stars are shown with parametrizations GM1 and GM3 respectively for the hadron phase. Two parametrizations (SU(3) HK and SU(3) RKH) are used for the quark phase in both figures. In both figures the hyperons and strange quarks are included. The EoS of the hybrid stars with mixed phase are built by the superposition of the EoS for the quark and hadron matter, plus the EoS for the mixed phase. The plateau in the EoS of the hybrid stars without mixed phase shows a vivid phase transition from hadron to quark matter. One can see that GM1 produces a much larger quark phase in both constructions while GM3 gives rise to a very large mixed phase when it is present and a quark phase much smaller than the hadron phase if a Maxwell construction is used. This consideration is true independently of the parametrization used in the quark phase, what means that the size of each phase is basically dependent on the hadron phase parametrization, at least for the choices we have considered. This fact has obvious consequences in the constituents of the stellar matter.

In Figs. 4 and 5 we compare hybrid stars without the mixed phase built with the SU(2) and SU(3) NJL models, to understand the role played by the strange quark. Whenever SU(2) set1 is used, the hadron phase is minimal and the EoS is practically given only by the quark phase. In all the other cases the size of each phase is mainly determined by the hadron phase parametrization, with small variations in the size of each phase. Notice that the u and d quark vacuum masses are not identical, being the smallest for the SU(2) set1 parametrization, as seen in Table 1.

We use all the EoS studied and commented before as input to the TOV equations to obtain the neutron star profiles that are shown in Fig. 6, 7 and Table 3. The tails of the hadron and hybrid stars were obtained with the insertion of the BPS EoS \([24]\). As expected, the maximum masses for the hadron stars are larger than for the hybrid stars. When the Maxwell construction is used, the resulting mass-radius curve shows a kink, produced by the sharp transition in the EoS. A similar result is shown in \([25]\) and it is easier to see in Fig. 7, where we once more compare the hybrid stars without mixed phase built with the SU(2) and SU(3) models. As a consequence of the EoS, GM1 and GM3 produce identical stellar profiles if only protons, neutrons and consequently quarks u and d are considered with SU(2) set1. SU(2) set2 produces different results. If only stars with strangeness are considered, GM1 always result in stars with larger maximum masses and radii than GM3.

In Table 3 we also show the results for the central energy density \(\varepsilon_0\). \(\varepsilon_{min}\) corresponds to the point where the hadron phase disappears, either because of the onset of a mixed phase (whenever \(\varepsilon_{max}\) is also shown) or giving rise to the quark phase (otherwise). If strangeness are considered, in hybrid stars without mixed phase, the central energy density may lie within the hadron phase as in GM3 \(\times\) SU(3)HK and GM3 \(\times\) SU(3)RKH. This means that if a phase transition to the quark phase occurs, the star becomes unstable. On the other hand, if the parametrization GM1 \(\times\) SU(3)RKH or GM1 \(\times\) SU(3)HK is used, the central energy density lies in the quark phase. If a mixed phase is considered, the central energy density always lies in its interior. As a consequence, the pure quark phase is never present. This is the only effect (not possible to infer from astronomical observations) that depends strongly on the choice between Gibbs and Maxwell constructions. Nevertheless, it is well known that the strange quark condensate is very large within the SU(3) NJL model and our results are a consequence of this behaviour. For this specific reason, we have also checked the results for hybrid stars built without strangeness.

If strangeness is not included, SU(2) set1 gives rise to hybrid stars with central energy densities in the quark phase and SU(2) set2 shows unstable solutions after the onset of the quark phase with the GM1 parametrization.

Analyzing the results shown in Table 3 and based on the accuracy of our calculations and the experimental difficulties in the measurements of neutron stars radii, it
is fair to say that the method used to built the EoS, i.e., the more rigorous Gibbs conditions or the simple use of the Maxwell construction give almost indistinguishable results for gravitational masses and radii.

Finally, in Figs. 6 and 7 we have added three lines corresponding to observational constraints. Some properties of the neutron stars are determined by measuring the gravitational redshift of spectral lines produced in neutron star photosphere which provides a direct constraint on the mass-to-radius ratio (M/R). A redshift of z = 0.35 from three different transitions of the spectra of the X-ray binary EXO0748-676 was obtained in \( [27] \). This redshift corresponds to \( M/R = 0.15 \). The top line corresponds to this constraint, whose validity remains controversial \( [28] \). On the other hand, the 1E 1207.4-5209 neutron star, which is in the center of the supernova remnant PKS 1209-51/52 was also observed and two absorption features in the source spectrum were detected \( [29] \). These features were associated with atomic transitions of once-ionized helium in the neutron star atmosphere with a strong magnetic field. This interpretation leads to a redshift of the order of \( z = 0.12 - 0.23 \). This redshift imposes another constraint to the mass to radius ratio given by \( M/R = 0.069 \). This constraint is represented by the two lowest lines. One can see in Figs. 7 and 8 that all the curves obtained are consistent with the measurements of \( [27] \) and \( [29] \) by crossing the 3 lines.

IV. CONCLUSIONS

Assuming that hybrid stars are possible remnants of supernova explosions, their constitution becomes important only if their macroscopic quantities can be constrained to astronomical observations. While some calculations practically exclude the existence of hybrid stars \( [20] \) favoring quark stars, others tend to rule out quark stars and favor hybrid stars \( [7] \). All those conclusions are obviously model dependent and were reached based on the use of the MIT bag model to describe quark matter.

Hybrid stars have a hadron phase, in this paper described by the non-linear Walecka model (NLW) \( [16] \) and a quark phase. Instead of using the usual MIT bag model \( [15] \) to build the quark phase, we have opted to use the NJL model \( [17] \) to check some of the previous results on the existence of the mixed phase inside hybrid stars.

If the hadron phase is constituted of protons and neutrons, the corresponding quark phase has quarks u and d only and the SU(2) version of the NJL model is used. If the baryonic octet is possible in the hadron phase, quarks u, d and s are present in the quark phase described by the SU(3) NJL model.

We have concluded that the results are very model dependent, as expected. The onset of a stable quark phase is practically ruled out. Our calculations suggest that stable neutron stars are either of hadronic nature only or bear a mixed phase in their core. Concerning the existence of the mixed phase (MP), one can see that the stellar measurable results calculated in the present paper (mass, radii, central energy density) depend very little on the choice of the Maxwell or the Gibbs construction. Hence, it is reasonable to claim that the Maxwell construction gives satisfactory results.

The effects of colour superconductivity are out of the scope of the present work, but it is important to mention that they may play an important role in the description of neutron star matter \( [20] \). The colour-flavour-locked phase (CFL) could turn into a superconducting phase (2SC) before matter is hadronized when we read the QCD phase diagram from high to low densities (see a figures \( [20] \), for example). This non-continuous transition from the CFL to the 2SC phase in the presence of realistic strange quark masses would certainly affect the description of hybrid stars.

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FIG. 1: EoS for the pure hadron and pure quark matter.
FIG. 2: EoS for the hybrid star with and without the mixed phase build with the GM1 parametrization, \( n \) stands for nucleons only and \( nh \) for nucleons and hyperons.

FIG. 3: EoS for the hybrid star with and without the mixed phase build with the GM3 parametrization, \( n \) stands for nucleons only and \( nh \) for nucleons and hyperons.

FIG. 4: EoS for the hybrid star without the mixed phase build with the NJL SU(3) and SU(2) model with the GM1 parametrization.

FIG. 5: EoS for the hybrid star without the mixed phase build with the NJL SU(3) and SU(2) model with the GM3 parametrization.

FIG. 6: Mass-radius curves for the hybrid stars with and without the mixed phase.

FIG. 7: Mass-radius curves for the hybrid stars without the mixed phase build with the NJL SU(3) and SU(2) model.
Parameter set | $\Lambda$ (MeV) | $g_S \Lambda^2$ | $g_D \Lambda^5$ (MeV) | $m_{u,d}$ (MeV) | $M_{u,d}$ (MeV) | $M_s$ (MeV)
--- | --- | --- | --- | --- | --- | ---
SU(2) set1 | 664.3 | 2.06 | - | 5.0 | - | 300 | -
SU(2) set2 | 587.9 | 2.44 | - | 5.6 | - | 400 | -
SU(3) HK | 631.4 | 1.835 | 9.29 | 5.5 | 135.7 | 335 | 527
SU(3) RHK | 602.3 | 1.835 | 12.36 | 5.5 | 140.7 | 367.7 | 549.5

| Parameter set | $\frac{(g_S/m_s)^2}{(fm^2)}$ | $\frac{(g_\omega/m_\sigma)^2}{(fm^2)}$ | $\frac{(g_\rho/m_\rho)^2}{(fm^2)}$ | $\kappa/M$ | $\lambda$ |
|--- | --- | --- | --- | --- | ---|
| GM1 | 11.79 | 7.149 | 4.411 | 0.005894 | -0.006426 |
| GM3 | 9.927 | 4.820 | 4.791 | 0.017318 | -0.014526 |

| Star type | Model | $M_{\text{max}}$ (M$_\odot$) | $M_{b\text{max}}$ (M$_\odot$) | $R$ (Km) | $\varepsilon_0$ ($fm^{-4}$) | $\varepsilon_{\text{min}}$ ($fm^{-4}$) | $\varepsilon_{\text{max}}$ ($fm^{-4}$) |
|--- | --- | --- | --- | --- | --- | --- | ---|
| Hadron | GM1n | 2.390 | 2.892 | 11.992 | 5.595 | - | -
| Hadron | GM1nh | 2.006 | 2.325 | 11.851 | 5.908 | - | -
| Hybrid without m.p. | GM1n $\times$ SU(2) set1 | 1.835 | 2.108 | 11.259 | 6.464 | - | 1.241 |
| Hybrid without m.p. | GM1n $\times$ SU(2) set2 | 2.227 | 2.638 | 13.085 | 4.810 | - | 5.689 |
| Hybrid without m.p. | GM1nh $\times$ SU(3) RHK | 1.970 | 2.276 | 12.542 | 6.615 | - | 6.607 |
| Hybrid without m.p. | GM1nh $\times$ SU(3) HK | 1.906 | 2.189 | 12.821 | 4.538 | - | 4.000 |
| Hybrid with m.p. | GM1nh $\times$ SU(3) RHK | 1.945 | 2.242 | 12.568 | 4.979 | 3.454 | 7.379 |
| Hybrid with m.p. | GM1nh $\times$ SU(3) HK | 1.909 | 2.192 | 12.666 | 4.876 | 2.357 | 5.023 |
| Hadron | GM3n | 2.042 | 2.421 | 10.933 | 7.048 | - | -
| Hadron | GM3nh | 1.710 | 1.946 | 10.980 | 7.151 | - | -
| Hybrid without m.p. | GM3n $\times$ SU(2) set1 | 1.836 | 2.110 | 11.287 | 6.464 | - | 1.303 |
| Hybrid without m.p. | GM3n $\times$ SU(2) set2 | 2.018 | 2.381 | 11.484 | 8.300 | - | 8.295 |
| Hybrid without m.p. | GM3nh $\times$ SU(3) RHK | 1.710 | 1.946 | 10.977 | 7.161 | - | 11.895 |
| Hybrid without m.p. | GM3nh $\times$ SU(3) HK | 1.710 | 1.946 | 10.972 | 7.179 | - | 11.119 |
| Hybrid with m.p. | GM3nh $\times$ SU(3) RHK | 1.704 | 1.938 | 11.176 | 6.820 | 5.424 | 13.437 |
| Hybrid with m.p. | GM3nh $\times$ SU(3) HK | 1.700 | 1.934 | 11.198 | 6.772 | 4.772 | 12.797 |

TABLE I: Parameter sets for the NJL SU(2) and SU(3) models.

TABLE II: Set of parameters for the NLW model.

TABLE III: Maximum gravitational mass $M_{\text{max}}$, baryonic mass $M_{b\text{max}}$, and radius $R$. $\varepsilon_0$ is the central energy density, $\varepsilon_{\text{min}}$ is the energy density where the hadron phase ends, and $\varepsilon_{\text{max}}$ is the energy density where the quark phase begins.