Subspace-Contrastive Multi-View Clustering

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Most multi-view clustering methods based on shallow models are limited in sound nonlinear information perception capability, or fail to effectively exploit complementary information hidden in different views. To tackle these issues, we propose a novel Subspace-Contrastive Multi-View Clustering (SCMC) approach. Specifically, SCMC utilizes a set of view-specific auto-encoders to map the original multi-view data into compact features capturing its nonlinear structures. Considering the large semantic gap of data from different modalities, we project multiple heterogeneous features into a joint semantic space, namely the embedded compact features are passed through the self-expression layers to learn the subspace representations, respectively. In order to enhance the discriminability and efficiently excavate the complementarity of various subspace representations, we use the contrastive strategy to maximize the similarity between positive pairs while differentiate negative pairs. Thus, the graph regularization is employed to encode the local geometric structure within varying subspaces for optimizing the consistent affinity matrix. Furthermore, to endow the proposed SCMC with the ability of handling the multi-view out-of-samples, we develop a consistent sparse representation (CSR) learning mechanism over the in-samples. To demonstrate the effectiveness of the proposed model, we conduct a large number of comparative experiments on ten challenging datasets, and the experimental results show that SCMC outperforms existing shallow and deep multi-view clustering methods. In addition, the experimental results on out-of-samples illustrate the effectiveness of the proposed CSR.

CCS Concepts: • Computing methodologies → Cluster analysis.

Additional Key Words and Phrases: Multi-view clustering, subspace clustering, multi-view fusion, contrastive learning.

1 INTRODUCTION

With the growing popularity of data generation and feature extraction, multi-view or multimedia data are available in large quantities. To be specific, multi-view data refer to various feature representations from multiple

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aspects of objects. For instance, an image can be characterized by wavelet texture (WT), local binary pattern (LBP), histogram of oriented gradient (HOG), etc. A piece of document can be expressed in numerous languages. Researchers generally believe that multi-view data consist of rich and useful heterogeneous information, so the technologies related to multi-view analysis [8, 35, 54, 62, 68] are receiving increasing attention. Multi-view clustering (MVC) [15] is one of the representative technologies, which aims to explore the complementary and consistent information embedded in multi-view data to boost the clustering performance.

Currently, there are extensive multi-view clustering methods. For example, graph-based MVC [40, 46, 70] learned the connectivity graph matrices to reveal the relationship of samples, then the designed fusion schemes were developed to merge these graph matrices into a global graph. Spectral embedding-based MVC [14, 28, 47] exploited low-dimensional spectral embedding with orthogonal constraint for each view, which portrayed important components of data, then a consensus representation was further merged. The goal of nonnegative matrix based MVC [21, 22, 61] was to factorize a nonnegative discrete cluster indicator matrix from varying representations, thus the $arg \max (\cdot)$ function was adopted to acquire the data labels without post-processing. Among multitudinous MVC methods, multi-view subspace clustering is a research hotspot and widely studied for its superior performance, which absorbs theory from conventional subspace clustering [3] and further develops it. The works [16, 30, 36] were classic multi-view subspace clustering approaches, which aimed to explore a uniform underlying subspace representation from multiple feature spaces. These shallow models have yielded promising clustering results, but most real-world data are high-dimensional and nonlinear, shallow models might not been equipped with the ability to fetch nonlinear structures.

Auto-Encoder (AE) is an effective unsupervised deep representation learning paradigm, which non-linearly maps the original data features into a compact feature space via the encoders, then passes the compact representations through the decoders to reconstruct the data. AE is frequently used to condense data information in clustering tasks. [19, 56] were two well-known deep embedding learning methods, which used Kullback-Leibler (KL) divergence regularization to maximize the similarity of soft assignments and target distributions. During the past few years, AE is also introduced to multi-view subspace clustering. Sun et al. [45] used self-supervised strategy to improve the unified subspace representation learning. Zhu et al. [49] simultaneously learned a set of view-specific self-expression representations, then which were combined into a common self-expression representation. Wang et al. [50] learned a unified subspace representation from multi-view discriminative feature spaces. Cui et al. [10] proposed the spectral supervisor to guide the learning of consensus subspace representation. The clustering performance of the above deep multi-view subspace clustering approaches are excellent, but their abilities of exploiting the association between multiple subspace representations still need to be improved. For instance, [45, 50] directly learn the consistent self-expression representation from multi-view latent features refined by AE, which could not capture the disparate characteristics of varying views, thus failing to utilizing the complementary information. [49] applies a Hilbert Schmidt Independence Criterion (HSIC) regularizer to reinforce the diversity of different views, this indistinguishable alienation of different views may render it difficult to obtain the agreement of them. As for [10], a weighted fusion layer is used to integrate all self-expression representations, which does not harness the view correlations in insightful ways. Contrastive learning [18] is an emerging self-supervised strategy that aims to maximize the similarity between positive pairs whereas minimize the similarity between negative pairs. In multi-view clustering scenarios, there is a natural contrastive relationship between varying views, thus giving rise to some multi-view contrastive clustering methods [20, 26, 48, 58, 63]. These methods enhance the discrimination of latent representations by maximizing the similarity of positive samples and minimizing the similarity of negative sample pairs from different views, belonging to the feature-level calibration. Nonetheless, an important objective reality in multi-view data is that there may be large modality gap of data under different views, which can drive the distance between instance pairs to be extremely huge, rendering the contrast process difficult. Hence, how to mitigate modal isolation and improve the contrast quality is an vital motivation of this paper. Additionally, most of multi-view clustering methods cannot handle the
multi-view out-of-samples, which are not involved in the construction of the similarity graph or the training of the clustering network. To cluster the out-of-samples, most clustering algorithms or networks have to be reexecuted or retrained, which is time-consuming and laborious.

We are inspired by the idea of contrastive learning, and propose a Subspace-Contrastive Multi-View Clustering (SCMC) method. Specifically, in order to perceive the nonlinear structures in multi-view data, we employ view-specific AEs to encode the initial features into multiple compact spaces, wherein the respective subspace representations are further learned through the self-expression layers, such that the semantic information of data belonging to disparate modalities can be unified into a common semantic space. Thus, we consider the same sample under different views as the positive pairs, and the rest of pairs are considered as negative, Fig. 2 illustrates the manner of constructing positive and negative pairs. By pairwise contrasting multiple subspace representations, we bring the sample affinities of positive pairs closer together and the sample affinities of negative pairs further apart, belonging to the structure-level calibration. This operation enhances the discriminability of each subspace representation and explores the complementary information within them, which is also different from the discrimination-induced regularization achieved by the indistinguishable mutual exclusion between various representations in literatures [50, 69]. To obtain a consistent affinity matrix, we use a weighted fusion scheme to merge multiple subspace representations. Moreover, the graph regularization is applied to encode the local structures inside the learned subspaces, further fine-tuning the suitable affinities between samples. In
addition, we further propose an extension mechanism for the multi-view out-of-samples, namely, the consistent sparse representation (CSR) learning method over the multi-view in-samples, thus directly achieving the clustering for out-of-samples instead of retraining the clustering network.

Finally, abundant experiments on ten challenging datasets are implemented to verify the effectiveness of the proposed SCMC. The major contributions of this paper are summarized as follows:

- We nonlinearly map the multi-view data into compact feature subspaces via the auto-encoders, then regard different subspace representations as contrast entities and perform the structure-level contrastive learning, thus exploring the complementarity between heterogeneous views.
- We obtain an initial unified affinity matrix through the weighted aggregation, to capture the local geometric structures of multiple subspaces, the graph regularization is utilized to further fine-tune the affinities between instances. Thus, the inter-view consistency is well guaranteed.
- We propose an extension mechanism to handle the multi-view out-of-samples, that is, the consistent sparse representations of out-of-samples over the in-samples are learned, thus directly obtaining the clustering results for the out-of-samples.
- To demonstrate the validity of the proposed SCMC, we carry out comprehensive experiments on ten multi-view datasets, and the experimental results show that SCMC possesses advanced data clustering capability compared with other multi-view clustering methods. Moreover, the proposed CSR is also verified to be effective via the designed experiments.

The rest of this paper are structured as follows. Section 2 briefly reviews the related works. In Section 3, we explicate the proposed SCMC and CSR. In Section 4, we discuss the differences between the existing works and this paper. Experimental details are narrated in Section 5. Finally, the conclusion is summarized in Section 6.

Fig. 2. The diagram of constructing the positive and negative pairs. Let us take three views as an example, the first data point of view 2 and the same data points of the other two views are positive pairs (connected by solid lines). The first data point of view 2 and the remaining data points of the other two views and its own view are negative pairs (connected by dashed lines).

2 RELATED WORKS

2.1 Multi-view Subspace Clustering

Multi-view subspace clustering [5] leverages heterogeneous features of data to group samples into a union of diverse subspaces. Self-expression based subspace learning technology has gained widespread attention due to its concise but sound feature characterization capabilities. Some works aimed at exploring a shared subspace representation. For instance, Cai et al. [4] explicitly modeled the consistency and the specificity of multi-view data, and learned a well-structured common affinity matrix. Li et al. [25] proposed a kernel completion schema to learn
the compact and low-redundant subspace representations. Wang et al. [53] relied on the information bottleneck theory to discard the superfluous information in raw data, and explored a common subspace representation via the view-common encoder network. Chao et al. [6] leveraged the multiple imputation and ensemble technology to cope with the incomplete multi-view data. Wang et al. [52] adopted the Frobenius norm and $l_2,1$-norm to enhance the robustness of consistent graph matrix. For capturing the high-order correlations among views, tensor-oriented methods have been researched. Ji et al. [23] proposed an enhanced tensor nuclear norm to differentiate the contributions of diverse singular values. Qin et al. [42] projected the initial multi-view features into nonlinear subspaces, then captured the high-order correlations via minimizing the rank of representation tensor. Wang et al. [51] considered the existence of data noise in multi-view data and removed it with the help of entropy-regularized tensor learning. To improve the abilities for matching the complex data distributions, some researchers used neural networks to model multi-view data. [50, 69] used deep network frameworks to learn the subspace representations, then they all adopted an exclusive regularization term to boost the complementary information among varying views. Du et al. [11] learned the discriminative multi-view features via adversarial training, based on which the consistent subspace representation was explored. Gao et al. [17] aimed at learning the semantic-invariant representations among multiple heterogeneous features, and sought the optimal cluster divisions by the reinforcement learning.

2.2 Contrastive Learning
Contrastive learning is one of the research hot spots of the self-supervised learning paradigm over recent years, its central concept is to enhance the similarity between positive instance pairs and weaken the similarity between negative instance pairs. In practice, [7, 43, 67] were proposed in computer vision and natural language processing filed, which successively enhanced the discrimination of data representations by means of contrastive learning. Owing to the presented favorable performance, contrastive learning has gained attention and been applied in clustering field. Li et al. [27] simultaneously contrasted the instance-level and cluster-level representations to strength the separability of samples belonging to different clusters. Liu et al. [34] adopted the data preprocessing and multilayer perceptions to achieve the simple contrastive graph clustering, alleviating the burden of high computational complexity. Furthermore, researchers have extended single-view contrastive clustering to multi-view cases, Xu et al. [58] conducted data reconstruction in a low-level space and punished the consistent objectives via a contrastive scheme in a high-level space. Yan et al. [60] proposed a structure-guided contrastive schema to align common and view-specific semantic information. Furthermore, some works also made the progress for combining the contrastive learning and multi-view subspace clustering. Du et al. [12] performed the pairwise contrast between multiple heterogeneous features via binary cross-entropy loss. Cheng et al. [9] contrasted the multi-view features in the latent space and utilized the HSIC regularization to strengthen the diversity of different subspace representations. Zhang et al. [66] proposed a multi-view clustering-driven contrast regularizer, which regarded the neighboring nodes of a node as the positive samples as well.

3 THE PROPOSED METHOD
In this section, we first explain the motivations for proposing the SCMC method. Second, we present the objective functions of the proposed SCMC. Thus, the specific network architectures are summarized, which are also graphically illustrated in Fig. 1 for better comprehension. Further, we conduct a series of analyses on the proposed method, including the optimization method, the training details, the extension for the out-of-samples, and the analysis of time complexity.
3.1 Motivation

1) Existing deep multi-view subspace clustering methods [50, 69] enhance the discrimination of different representations through an exclusive regularization term, but this undifferentiated disparity of all representations may render the models difficult to acquire the agreement across views. Inspired by contrastive learning, we differentially treat samples from various views, construct cross-view positive and negative pairs, and strengthen the discrimination of subspace representations by bringing positive pairs closer and separating negative pairs.

2) The semantic information gap between different modalities in multi-view data [57] could be very large. For example, the HOG feature of an image describes completely different semantic information from the LBP feature, then the corresponding feature representations are extremely disparate. Most current multi-view contrastive clustering methods [9, 58] are based on feature-level contrast and may suffer from the above drawback. In light of this, we explore the subspace representations of all views to unify the semantic information from heterogeneous views into a joint semantic space, achieving the structure-level contrast.

3.2 Objective Function

1) Reconstruction and Subspace Losses: A multi-view dataset is denoted by \( \{X^{(v)}\}_{v=1}^V \). Specifically, \( X^{(v)} \in \mathbb{R}^{N \times d^{(v)}} \) is feature matrix of the \( v \)-th view, where \( N \) and \( d^{(v)} \) represent the number of instances and the feature dimension, respectively. For the original feature \( X^{(v)} \), it may be high-dimensional and nonlinear, which poses difficulties for the downstream tasks. Hence, we use multiple view-specific encoders to nonlinearly map \( X^{(v)} \) into a latent low-dimensional space. For the \( v \)-th view, its encoder is formulated as

\[
C^{(v)} = \epsilon_v(X^{(v)} | W_e^{(v)}, b_e^{(v)}),
\]

where \( C^{(v)} \) is the embedding feature after \( X^{(v)} \) passing the encoder \( \epsilon_v(\cdot) \), \( W_e^{(v)} \) and \( b_e^{(v)} \) indicate the weight matrix and bias vector in the encoder, respectively. Mathematically, nonlinearity [33, 44] expresses the fact that the dependent variables do not have a linear or direct relationship with the independent variables, and the variation of output is not proportional to the variation of input. In addition, [33, 44] indicate that the output of each layer for a neural network-based model is actually a high-level semantic feature embedding. Under the action of activation functions such as Relu, Sigmoid, etc, the data is nonlinearly mapped to a compact representation at each layer. Finally, with the guidance of loss function, the output of last layer is a discriminative representation that is strongly correlated with the downstream tasks.

Current multi-view contrastive clustering tends to first project multi-view data into compact feature spaces and then perform contrast between them. However, the semantic information of data from heterogeneous views can be very disparate. Even if some works such as [58] consider projecting data from different views into a unified feature space through a shared network, the distance between pairs could still be very large. It is difficult to pull together the positive pairs of different modalities. Subspace representation is not only an informative low-dimensional representation form of data, but it also contains an important property, i.e., it portrays the affinity relationship between sample points. For example, given a subspace representation \( Z \in \mathbb{R}^{N \times N} \), \( Z_{ij} \) measures the affinity between the \( i \)-th instance and the \( j \)-th instance. Thus, the \( i \)-th row \( Z_i \) can be considered as a low-dimensional subspace representation of the \( i \)-th instance, but also as the affinities of the \( i \)-th instance with other instances. Therefore, there are still differences in subspace representations \( \{Z^{(v)}\}_{v=1}^V \) from diverse views, but their semantic information remains consistent, alleviating the dilemma in the contrast process. We learn the subspace representation \( Z^{(v)} \) of the \( v \)-th view by

\[
\min_{C^{(v)}, Z^{(v)}} ||C^{(v)^T} - C^{(v)^T}Z^{(v)}||_F^2.
\]
In the networks, $Z^{(o)}$ is coefficient matrix of the learnable self-expression layer, which is achieved via one-layer fully connected layer without the bias part. The original data is projected to the latent space via the encoder, how to ensure that the features in the latent space maintain highly informative is a key issue. In the literature [49], the authors proved that the essence of the reconstruction loss is to the maximize the lower bound of the mutual information between original features and latent features, which guaranteed that the latent features contain as much important information as possible in the original feature space. Inspired by the previous works, we introduce the reconstruction loss to guide the training of encoders, thus obtaining the compact and informative latent features. After the embedding feature $C^{(o)T}$ passes through the self-expression layer $Z^{(o)}$ to obtain $C^{(o)T}Z^{(o)}$, we reconstruct the data via feeding it into the decoder $d^{(o)}(\cdot)$, the decoding process is formulated as

$$\hat{X}^{(o)} = d^{(o)}(C^{(o)T}Z^{(o)}|W^{(o)}_d, b^{(o)}_d),$$

where $\hat{X}^{(o)}$ denotes the reconstructed data, $W^{(o)}_d$ and $b^{(o)}_d$ represent the coefficient matrix and the bias vector of the decoder network, respectively. For $V$ views, the reconstruction loss and subspace representation learning loss are computed by Eq. (4) and Eq. (5).

$$L_{Re} = \min_{\hat{X}^{(o)}} \sum_{o=1}^{V} ||X^{(o)} - \hat{X}^{(o)}||^2_F$$

$$L_{Sub} = \min_{C^{(o)},Z^{(o)}} \sum_{o=1}^{V} ||C^{(o)T} - C^{(o)T}Z^{(o)}||^2_F.$$  \hspace{1cm} (4)

(5)

2) Contrastive Loss: There is a natural contrast between multiple subspace representations $\{Z^{(o)}\}_{o=1}^{V}$. To exploit this property, we first construct positive and negative pairs across views. For $Z^{(o)}_j$, it mutually forms a positive pair with the same instances under different views, while its negative samples contain all the instances except the positive samples. Fig. 2 provides a graphical illustration of how positive and negative pairs are constructed. Summarily, there are $V - 1$ positive instances and $V(N - 1)$ negative instances for $Z^{(o)}_j$. Thus, we apply cosine distance to measure the similarity between pairs, the mathematical form is expressed as follows

$$\Theta(Z^{(o)}_i, Z^{(o)}_j) = \frac{\left(Z^{(o)}_i \cdot Z^{(o)}_j\right)^T}{\|Z^{(o)}_i\| \cdot \|Z^{(o)}_j\|}.$$  \hspace{1cm} (6)

Taking one view as an example, to achieve the goal of narrowing the positive pairs and widening the negative pairs, we formulate the problem as

$$\ell_{c} = - \min_{Z^{(o)},Z^{(k)}} \sum_{k=1,k\neq o}^{V} \sum_{l=1}^{N} \exp\left(\Theta(Z^{(o)}_i, Z^{(k)}_j)/\tau\right)/\sum_{j=1}^{N} \left(\exp\left(\Theta(Z^{(o)}_i, Z^{(o)}_j)/\tau\right) + \exp\left(\Theta(Z^{(o)}_i, Z^{(k)}_j)/\tau\right)\right),$$

where $\tau$ denotes the temperature parameter. We can observe that the numerator is about the calculation of positive pairs, while the denominator is about the calculation of negative pairs. The contrastable loss of $V$ views is computed by

$$L_{Con} = \min_{Z^{(o)},Z^{(k)}} \frac{1}{NV} \sum_{o=1}^{V} \ell_{c}.$$  \hspace{1cm} (7)

\hspace{1cm} (8)
3) Fusion Loss: For aggregating the complementary information in different views, we integrate multiple subspace representations into a consistent affinity matrix in a weighted fusion manner. Specifically, a set of weight coefficients are assigned to varying views, and they can be adaptively optimized in the back propagation process. The problem is written as

\[
A = f(\{Z^{(v)}\}_v \mid \Omega) = \sum_{v=1}^{V} \omega^{(v)} Z^{(v)}
\]

\[
\text{s.t. } \sum_{v=1}^{V} \omega^{(v)} = 1, \omega^{(v)} \geq 0,
\]

where \(f(\cdot)\) denotes the fusion function, \(\Omega\) represents the learnable coefficients in the fusion function. In addition, an important assumption of graph embedding theory \([39, 59]\) is that two samples closer to each other in the original space retain this property in the new low-dimensional space. We follow this assumption and consider that two instances similar in subspaces under any view, they should have higher affinity in the unified space. Thus, the following minimum problem can be obtained

\[
\mathcal{L}_{Fu} = \min_{Z^{(v)}, A} \sum_{v} \sum_{i} \sum_{j} \|Z^{(v)}_i - Z^{(v)}_j\|_2^2 A_{ij} + \sum_{v} \sum_{i} A_{ii}
\]

\[
\text{s.t. } A_{i1} = 1, A_{ij} \geq 0, A_{ii} = 0,
\]

where \(A_{ij}\) is the \((i,j)\)-th element in the uniform affinity matrix \(A\), \(A_{i}\) denotes the \(i\)-the row of \(A\), \(1\) is a vector with all elements of 1. The constraints on \(A\) aim to avoid the trivial solutions. One may think that a unified affinity \(A\) can be learned directly through Eq. (10), and the weighted fusion mechanism seems to be unnecessary. Our intension is to initially obtain a consistent affinity matrix \(A\) through Eq. (9) to avoid that all elements of \(A\) are zeros, which makes Eq. (10) unable to optimize; then we use Eq. (10) to further fine-tune the affinities between samples. At present, we give the final objective function of the proposed SCMC, which is written as

\[
\mathcal{L} = \mathcal{L}_{Re} + \gamma_1 \mathcal{L}_{Sub} + \gamma_2 \mathcal{L}_{Con} + \gamma_3 \mathcal{L}_{Fu}
\]

\[
\text{s.t. } \sum_{v=1}^{V} \sum_{k=1,k\neq v}^{V} \sum_{i=1}^{N} \sum_{j=1}^{N} \exp(\Theta(Z^{(v)}_i, Z^{(k)}_j) / \tau)
\]

\[
\text{log } \sum_{j=1}^{N} \left( \exp(\Theta(Z^{(v)}_i, Z^{(j)}_i) / \tau) + \exp(\Theta(Z^{(v)}_i, Z^{(k)}_j) / \tau) \right)
\]

\[
+ \gamma_2 \sum_{v=1}^{V} \text{Tr}(Z^{(v)} A Z^{(v)T}) + \gamma_3 ||A||_F^2,
\]

\[
\text{s.t. } \sum_{j=1}^{N} A_{ij} = 1, A_{ij} \geq 0, A_{ii} = 0,
\]
where $\gamma_1$, $\gamma_2$, and $\gamma_3$ are three nonnegative trade-off parameters. After the optimization based on back propagation, the consistent affinity $A$ is obtained, we perform the spectral clustering algorithm on the matrix $(A + A^T)/2$ to get the data labels.

We summarize how the complementarity and consistency between views are captured by the proposed SCMC. We first nonlinearly project the original features of each view into the subspaces to unify their semantic meanings. Then, the strategy of contrastive learning is adopted to capture the complementary information between different views. We specify that a sample forms the positive instance pairs with the same samples from other views while forming the negative instance pairs with other samples. Guided by the contrastive learning loss, the positive pairs are approaching while the negative pairs are alienating. Essentially, the view-invariant features are learned via the contrast manner while enhancing the discrimination of the samples in the feature space, thus capturing the complementarity hidden in different views. Further, to explore the consistency across different views, we use the graph regularization to learn the view-shared affinity matrix. Specifically, the graph embedding theory reveals to us that two data points close together in the original space retain the property in the projected subspace. Therefore, we constrain two samples similar in either subspaces should have similarity in the view-shared affinity matrix.

**Algorithm 1** Subspace-Contrastive Multi-View Clustering

- **Input**: Multi-view data $\{X^{(v)}\}_{v=1}^V$, parameters $\gamma_1$, $\gamma_2$, $\gamma_3$, and number of clusters $c$.
- **Output**: Consistent affinity matrix $A$.

1. Initialize multiple view-specific AEs with the parameters after pre-training, initialize the learning rate to 0.0001, the training epochs to 500.
2. **for** epoch = 1 to training epochs **do**
   3. Compute the reconstruction loss $L_{Re}$ by Eq. (4);
   4. Compute the subspace learning loss $L_{Sub}$ by Eq. (5);
   5. Compute the contrast loss $L_{Con}$ by Eq. (8);
   6. Obtain the initial consistent affinity matrix $A$ by Eq. (9);
   7. Compute the local structures loss $L_{Da}$ by Eq. (10);
   8. Compute the overall objective loss $L$ by Eq. (11) and update the network parameters via back propagation;
3. **end for**
4. Performing the spectral clustering algorithm on $\frac{(A + A^T)}{2}$ to acquire the data labels.

### 3.3 Network Architecture

Based on the introduction of the objective function above, we sketch the network architecture of the proposed SCMC herein. $V$ three-layer encoders embed multi-view data $\{X^{(v)}\}_{v=1}^V$ into compact features $\{C^{(v)}\}_{v=1}^V$. Next, $\{C^{(v)}\}_{v=1}^V$ pass the $V$ self-expression layers to obtain the matrix $\{C^{(v)}Z^{(v)}\}_{v=1}^V$, respectively. Concretely, the self-expression layer is achieved by a one-layer linear layer discarding the bias part. Then, $\{C^{(v)}Z^{(v)}\}_{v=1}^V$ is fed into $V$ decoders symmetrical to the encoders’ structures to decode the reconstructed data $\{\hat{X}^{(v)}\}_{v=1}^V$, respectively. In the encoding and decoding processes, the activation function $Relu(\cdot)$ is adopted. Furthermore, we contrast the learned subspace representations $\{Z^{(v)}\}_{v=1}^V$ with each other, and fuse them into a consistent affinity matrix $A$ with a group of learnable weights, then the nonnegative $A$ is obtained via $Relu(\cdot)$ function. For fine-tuning the affinities between instances, the graph regularization is leveraged to protect the local structures within multiple subspace representations.
3.4 Optimization of Multiple Variables

Herein, we derive the gradient of each variable in each loss. For simplicity, the bias of linear layer is not considered. Furthermore, the activation function Relu(·) is used, that is, $\text{Relu}(x) = x$ if $x > 0$, otherwise $\text{Relu}(x) = 0$, so the activation function is not presented in the derivation. We recall the four proposed losses, which are written as

$$L_{\text{Re}} = ||X^{(a)} - \hat{X}^{(a)}||^2_F$$

$$L_{\text{Sub}} = ||C^{(a)^T} - C^{(a)^T}Z^{(a)}||^2_F$$

$$L_{\text{Con}} = -\sum_{k=1}^{V} \sum_{k\neq v}^{N} \log \frac{\exp\left(\Theta(Z_{i}^{(a)^T}, Z_{j}^{(k)^T})/\tau\right)}{\sum_{j=1}^{N}\exp(\Theta(Z_{i}^{(a)^T}, Z_{j}^{(a)^T})/\tau)+\exp(\Theta(Z_{i}^{(a)^T}, Z_{j}^{(k)^T})/\tau)}$$

$$L_{\text{Fu}} = \sum_{i} \sum_{j} ||Z_{i}^{(a)^T} - Z_{j}^{(a)^T}||_2^2 A_{ij} + ||A||_2^2 + (A_i - 1),$$

where $C^{(a)}$ and $\hat{X}^{(a)}$ are computed by

$$C^{(a)} = X^{(a)}W^{(a)}_e$$

$$\hat{X}^{(a)} = Z^{(a)^T}C^{(a)}W^{(a)}_d = Z^{(a)^T}X^{(a)}W^{(a)}_e W^{(a)}_d.$$

For the reconstruction loss $L_{\text{Re}}$, taking the derivative of $C^{(a)}$, we have

$$\frac{\partial L_{\text{Re}}}{\partial C^{(a)}} = \sum_{k,l} \frac{\partial L_{\text{Re}}}{\partial \hat{X}^{(a)}} \frac{\partial \hat{X}^{(a)}}{\partial C^{(a)}}.$$  \hspace{1cm} (14)

Expanding $\frac{\partial \hat{X}^{(a)}}{\partial C^{(a)}}$ leads to

$$\frac{\partial \hat{X}^{(a)}}{\partial C^{(a)}} = \sum_{a,b} \frac{\partial (Z^{(a)^T}C^{(a)}W^{(a)}_d)}{\partial C^{(a)}} W^{(a)}_d = \sum_{a,b} \frac{\partial Z^{(a)^T}}{\partial C^{(a)}} W^{(a)}_d = Z^{(a)^T}W^{(a)}_d.$$

Thus, it has

$$\frac{\partial L_{\text{Re}}}{\partial C^{(a)}} = \sum_{a,b} \frac{\partial L_{\text{Re}}}{\partial \hat{X}^{(a)}} Z^{(a)^T}W^{(a)}_d \delta_{ij} = \frac{\partial L_{\text{Re}}}{\partial C^{(a)}} = Z^{(a)^T} \frac{\partial L_{\text{Re}}}{\partial \hat{X}^{(a)}} W^{(a)}_d.$$  \hspace{1cm} (16)

where $\frac{\partial L_{\text{Re}}}{\partial \hat{X}^{(a)}} = -2(X^{(a)^T} - \hat{X}^{(a)^T}).$

Taking the derivative with respect to $Z^{(a)}$, we have

$$\hat{X}^{(a)^T} = W^{(a)^T}C^{(a)^T}Z^{(a)}.$$  \hspace{1cm} (17)

$$\frac{\partial L_{\text{Re}}}{\partial C^{(a)}} = \sum_{k,l} \frac{\partial L_{\text{Re}}}{\partial \hat{X}^{(a)}} \frac{\partial \hat{X}^{(a)^T}}{\partial C^{(a)}}.$$  \hspace{1cm} (18)

Focusing on the $\frac{\partial \hat{X}^{(a)^T}}{\partial Z^{(a)}}$, it can be derived

$$\frac{\partial \hat{X}^{(a)^T}}{\partial Z^{(a)}} = \sum_{a,b} (W^{(a)^T}C^{(a)^T})_{ka}Z_{al} = (W^{(a)^T}C^{(a)^T})_{ka}Z_{al} = (W^{(a)^T}C^{(a)^T})_{ka}Z_{al}.$$  \hspace{1cm} (19)

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if \( l = j, \delta_{lj} = 1 \), otherwise \( \delta_{lj} = 0 \). Thus, we have

\[
\frac{\partial \mathcal{L}_{Re}}{\partial Z_{ij}^{(v)}} = \sum_{k,l} \frac{\partial \mathcal{L}_{Re}}{\partial X_{kl}^{(v)}} \left( W_d^{(v)} C_d^{(v)} \right)_{li} \delta_{lj} \Rightarrow \frac{\partial \mathcal{L}_{Re}}{\partial Z_{ij}^{(v)}} = \frac{\partial \mathcal{L}_{Re}}{\partial X_{kl}^{(v)}} C_d^{(v)} W_d^{(v)}.
\] (20)

Similar to the process of taking the derivatives for \( C^{(v)} \) and \( Z^{(v)} \), we can obtain

\[
\frac{\partial \mathcal{L}_{Re}}{\partial W_e^{(v)}} = X^{(v)} Z^{(v)} \frac{\partial \mathcal{L}_{Re}}{\partial X^{(v)}} \quad \frac{\partial \mathcal{L}_{Re}}{\partial W_d^{(v)}} = W_e^{(v)T} X^{(v)} Z^{(v)} \frac{\partial \mathcal{L}_{Re}}{\partial X^{(v)}}.
\] (21) \( (22) \)

For the subspace loss \( \mathcal{L}_{Sub} \), taking the derivative with respect to \( C^{(v)} \), \( Z^{(v)} \), and \( W_e^{(v)} \), respectively, we have

\[
\frac{\partial \mathcal{L}_{Sub}}{\partial C^{(v)}} = -2Z^{(v)} (C^{(v)} - Z^{(v)T} C^{(v)}).
\] (23)
\[
\frac{\partial \mathcal{L}_{Sub}}{\partial Z^{(v)}} = -2C^{(v)} (C^{(v)T} - C^{(v)} Z^{(v)}).
\] (24)
\[
\frac{\partial \mathcal{L}_{Sub}}{\partial W_e^{(v)}} = X^{(v)T} \frac{\partial \mathcal{L}_{Sub}}{\partial C^{(v)}}.
\] (25)

For the contrastive loss \( \mathcal{L}_{Con} \), taking the derivative of \( Z_i^{(v)} \), it can be obtained

\[
\frac{\partial \mathcal{L}_{Con}}{\partial Z_i^{(v)}} = \sum_{k=1,k \neq v}^V \left( \frac{1}{t_0 t_1} Z_i^{(k)^T} \right) - \left( \frac{1}{t_2 t_3} \right) Z_i^{(v)^T} Z_i^{(v)^T} - \sum_j (A_j Z_j^{(v)} + B_j Z_j^{(k)} - C_j Z_j^{(v)}),
\] (26)

where \( t_0 = ||Z_i^{(v)}||^2, t_1 = ||Z_i^{(k)}||^2, t_2 = ||Z_i^{(v)}||^2, t_3 = t_0^3 \) and \( A_j, B_j, C_j \) are corresponding coefficients.

For the fusion loss \( \mathcal{L}_{Fu} \), when taking derivative of \( Z^{(v)} \), we can obtain

\[
\frac{\partial \mathcal{L}_{Fu}}{\partial Z^{(v)}} = 2 \sum_{\alpha=1}^A (Z^{(v)} L_A + Z^{(v)} L_A^T).
\] (27)

When taking the derivative of \( A_j \), we have

\[
\frac{\partial \mathcal{L}_{Fu}}{\partial A_j} = \sum_{\alpha} \sum_j ||Z_j^{(\alpha)} - Z_j^{(v)}||^2 + 2A_i + I^T.
\] (28)

From the above derivation, we can observe that each variable of each loss has its derivative. Through the iterative optimization, the overall loss becomes decreasing and converges.

### 3.5 Training Details

The training process of the network is divided into two steps: Pre-training and overall training. First, we pre-train \( V \) three-layer AEs to initial their parameters. The purpose of pre-training is to mitigate the difficulties of training the overall network caused by all zeros in the AEs’ parameters and the possibility of generating trivial solutions. Second, we train the overall network, the parameters of \( V \) AEs, \( V \) self-expression layers, a group of learnable weights, and the uniform affinity matrix are iteratively optimized. Adam is adopted as the optimizer, and the learning rate is set to 0.0001. The experiments are run on a server equipped with Intel(R) Core(TM) i9-10980XE, RTX 3090 GPU, and 128G RAM. Algorithm 1 provides a summary of the main steps of the proposed SCMC.

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3.6 Extension for the Out-of-Samples

In the practical applications, some samples fail to participate in similarity construction or deep network training, these unseen samples are called out-of-samples [2]. Common clustering methods cannot directly cluster the out-of-samples unless the clustering methods are reexecuted or the network is re-trained, this challenge is equally faced in the multi-view clustering. Hence, how to empower the proposed SCMC with the capability of handling the out-of-samples without retraining the network is a matter we consider.

Herein, we propose a consistent sparse representation construction method over the in-samples for the out-of-samples for bridging the gap. Specifically, the sparsity theory [13, 41] states that any data point can be fitted by a linear combination of a set of basis. Furthermore, assuming that all samples (including in-samples and out-of-samples) are independently and identically distributed, thus the subspaces spanned by the in-samples can approximate the subspaces spanned by all samples. Given the multi-view in-samples \( \{X^{(v)}\}_{v=1}^{V} \) and the multi-view out-of-samples \( \{\hat{X}^{(v)}\}_{v=1}^{V} \), we first use the proposed SCMC to generate the cluster labels over the multi-view in-samples \( \{X^{(v)}\}_{v=1}^{V} \). Then, we expect to explore the consistent sparse representation (CSR) of multi-view out-of-samples in the subspaces spanned by the multi-view in-samples. For the \( i \)-th out-of-sample \( \hat{x}^{(i)} \), the objective function of learning the consistent sparse representation is formulated as

\[
\min_{h_i} \sum_{v=1}^{V} ||\hat{x}^{(i)} - h_i X^{(v)}||^2_2 + ||h_i||^2_p
\]

where \( h_i \) denotes the consistent sparse representation for the \( i \)-th out-of-sample, \( \lambda \) denotes the trade-off parameter. To obtain the optimal solution of \( h_i \), we take the derivation of \( h_i \) and have

\[
h_i^* = \sum_{v=1}^{V} \hat{x}^{(i)} X^{(v)t} \left( \sum_{v=1}^{V} X^{(v)t} X^{(v)} + \lambda I \right)^{-1}.
\]

Then, we compute the distance between the \( i \)-th out-of-sample and the \( j \)-th subspace, which is expressed as

\[
d_j(x_i) = \sum_{v=1}^{V} ||\hat{x}^{(i)} - \sigma_j(h)(X^{(v)})||^2_2,
\]

where the elements of \( \sigma_j(h) \) are the entries in \( h \) related to the \( j \)-th subspace. Naturally, the label belonging to the subspace closest to the \( i \)-th out-of-sample is its label, i.e.,

\[
\rho(\hat{x}_i) = \arg \min_j (d_j(\hat{x}_i))_{j=1}^{c},
\]

where \( \rho(\hat{x}_i) \) denotes the assigned label for the \( i \)-th out-of-sample, \( c \) is the number of clusters. More importantly, the proposed mechanism of handling the out-of-samples can be applied as a plugin to any of multi-view clustering methods.

3.7 Analysis of Time Complexity

In this subsection, we analyse the time complexity of the proposed SCMC. Specifically, the time complexity mainly arises from four aspects, including the encoding-decoding process, the calculation of self-expression layer, the implementation of contrastive loss, and the computation of graph regularization. Suppose the sizes of hidden layers in the \( o \)-th encoder are \( d^{(o)}_1, h_1, ..., h_i, p \), respectively. Then, the sizes of hidden layers in the \( v \)-th decoder are in reverse order. First, multi-view features \( \{X^{(v)}\}_{v=1}^{V} \) are passed through the encoders, it takes the \( O((\sum_{v=1}^{V} d^{(o)}_1 + V(h_1 h_2 + ... + h_i p))N) \) complexity, and so is the decoding. Let \( h_{max} = \max(\sum_{v=1}^{V} d^{(o)}_1, V h_1 h_2, ..., V h_i p) \), the encoding complexity is simplified as \( O(h_{max} N) \). Second, the intermediate latent features \( \{C^{(v)}\}_{v=1}^{V} \) are fed into the self-expression layer, which produces the \( O(p N^2) \) complexity. Third, when the contrast of two views is performed,
the $\mathcal{O}(\#^3 + N(\# - 1))$ complexity is consumed. Considering $V$ views, it costs the $\mathcal{O}(V(V - 1)(\#^3 + N(\# - 1)))$ complexity. Fourth, computing the graph regularization incurs $\mathcal{O}(VN^3)$ complexity. In summary, the overall time complexity of the proposed SCMC is $\mathcal{O}(h_{max}N + pN^2 + V(V - 1)(\#^3 + N(\# - 1)) + VN^3)$.

4 DISCUSSION

At present, there is a lot of works on multi-view clustering, some of which are relevant to the proposed SCMC, it is necessary to discuss the differences between them.

- Compared to the traditional shallow multi-view subspace clustering methods [16, 29, 36], we use neural networks to perceive the nonlinear structures in multi-view data, and expect the multi-view data with complex distributional properties to separate well in subspaces. Then, the contrastive learning strategy and graph regularization are leveraged to explore the complementary and consistent information.

- Compared to the deep multi-view subspace clustering methods [10, 45, 50, 69], which either use the feature fusion or exclusion-induced regularizers to mine the complementary information in multi-view data, we adopt the contrastive learning strategy to explore the cross-view complementarity. The main benefit of the introduction of contrastive learning is that it differently treats different sample pairs, bringing the positive sample pairs closer together while pulling the negative sample pairs farther apart, which facilitates both the learning of view-invariant features and the enhancement of the feature discrimination in the latent space.

- Existing multi-view subspace clustering methods [9, 12, 58] with contrastive mechanism perform the contrast in the latent feature space, aiming to align the latent representations of positive sample pairs in different views while alienating the latent representations of negative sample pairs, which belongs to the feature-level calibration. In contrast, we perform the contrast at the self-expression representation level, it is worth noting that the self-expression representation depict the affinity between samples. We bring the sample affinities of positive sample pairs closer under different views while pulling the sample affinities of negative sample pairs away, which belongs to the structure-level calibration.

- The focus of this work [11] is to explore the multi-view and multi-level self-expression representations. To enhance the discrimination of latent features output by the encoder networks, the adversarial training strategy is used to assist the training of encoder networks. Our work aims to bring the same samples closer together and different samples farther apart by contrasting the self-expression representations from different views, thus strengthening the discrimination of self-expression representations.

5 EXPERIMENTS

5.1 Multi-view Datasets

Table 1 summarizes the statistics of eight test multi-view datasets. Specifically, ALOI\(^1\) contains 1,079 images from 10 objects, 4 features are extracted from these images: Haralick texture feature, HSV color histograms, Color similarities, and RGB color histograms. GRAZ02\(^2\) is object categorization dataset, which is composed of 1,476 images with 4 kinds of objects. 6 visual features are extracted, including Wavelet texture, Local binary pattern, SIFT feature, pyramid HOG, SURF feature, GIST feature. NUS-WIDE\(^3\) is a famous image database, we select two subsets from it. NUS-WIDE-v1 contains 1,600 images of 8 categories, 6 views are Color histograms, Edge direction histograms, Block-wise color moments, Bag of words, Color correlograms, Wavelet textures, respectively. NUS-WIDE-v2 consists of 2,000 images from 31 objects, each image is represented from 5 features:

\(^1\)https://elki-project.github.io/datasets/multi-view
\(^2\)http://www.emt.tugraz.at/~pinz/data/GRAZ_02
\(^3\)https://lms.comp.nus.edu.sg/wp-content/uploads/2019/research/nuswide
Table 1. Statistics of eight datasets.

| Dataset       | Views | Samples | Clusters | Features          |
|---------------|-------|---------|----------|-------------------|
| ALOI          | 4     | 1,079   | 10       | 64 / 64 / 77 / 13 |
| GRAZ02        | 6     | 1,476   | 4        | 512 / 32 / 256 / 500 / 500 / 680 |
| NUS-WIDE-v1   | 6     | 1,600   | 8        | 64 / 144 / 73 / 128 / 225 / 500 |
| NUS-WIDE-v2   | 5     | 2,000   | 31       | 65 / 226 / 145 / 74 / 129 |
| Reuters       | 5     | 1,500   | 6        | 21,531 / 24,892 / 34,251 / 15,506 / 11,547 |
| UCI           | 3     | 2,000   | 10       | 240 / 76 / 6     |
| WikipediaArticles | 2   | 693     | 10       | 128 / 10          |
| Youtube       | 6     | 2,000   | 10       | 2,000 / 1,024 / 64 / 512 / 64 / 647 |
| Animals       | 2     | 5,000   | 50       | 4,096 / 4,096    |
| Cifar10       | 2     | 10,000  | 10       | 768 / 324        |

Color Histogram, Edge distribution, Color correlation, Color moments, and Wavelet texture. Reuters contains 1,500 documents with 5 languages, each sample is represented as a bag of words that extracted by the TFIDF-based weighting means. UCI is comprised of 2,000 handwritten numeric images ranging in [0, 9], three views are FOU feature, PIX feature, and MOR feature, respectively. WikipediaArticles is a document dataset organized by editors, which contains 693 short articles with 10 classes and 2 views. Youtube is a multi-view video games dataset with 2,000 instances divided into 10 classes. Each entry has 6 features including HOG feature, CH feature, MFCC feature, VS feature, SS feature, and HME feature. Animals contains 5,000 samples with 50 classes, two kinds of features are extracted from DECAF and VGG19 networks, respectively. Cifar10 is famous image dataset consisted of 10 categories of objects, 10,000 samples is randomly selected, and each item has two features: CH feature and HOG feature. Following the most of deep clustering works, we use the all samples in a dataset as the training set and also use all samples as the test set, i.e., the training set is the test set. When the training process using the training samples is completed, we adopt the spectral clustering algorithm over the learned consistent affinity matrix to generate the predicted labels of training samples. Finally, the values of evaluation metrics are computed through comparing the ground truth and predicted labels.

In addition, we explain what complementarity and consistency are in the test datasets. In [57, 68], the authors clarified the meaning of complementarity and consistency in multi-view data. The feature representations from different views characterize different aspects of a same object, one view possesses some unique characteristics that other views do not possess. The data information between multiple views is complementary with respect to the complete data information. Although the characteristics of different views are diverse, they all agree on a consistent latent feature space. In other words, the feature representations of different views can be regarded as the specific projections from the consistent latent representations. Fig. 3 illustrates the ideal stated above. For each multi-view dataset, its complementarity is reflected in the feature representations from multiple domains, taking the ALOI dataset as an example, four kinds of feature representations from four domains consist of the dataset, including Haralick texture feature, HSV color histograms, Color similarities, and RGB color histograms. Its consistency is reflected by the consistent affinity matrix learned via the proposed SCMC.

4https://archive.ics.uci.edu/ml/datasets.html
5http://lig-membres.imag.fr/grimal/data.html
6http://archive.ics.uci.edu/ml/datasets
7https://www.cs.toronto.edu/~kriz/cifar.html

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Fig. 3. Illustration of multi-view data. The feature representations $X^{(1)}$ and $X^{(2)}$ of View 1 and 2 depict different aspects for the same object, and they can be regarded as the feature representations obtained from a consistent latent feature through different projections.

5.2 Baseline and Compared Multi-view Methods

To illustrate the validity of the proposed SCMC, we select the K-means algorithm as the baseline method. In practice, multiple features are concatenated together to form a unified feature representation, then it is fed into K-means to obtain the clustering labels. In addition, we collect thirteen state-of-the-art multi-view clustering approaches as the compared methods, which are briefly introduced as follows.

- **AMGL** [37] proposed a auto-weighted multi-view fusion mechanism, then solved the consensus spectral embedding.
- **SwMC** [38] proposed a self-weighted fusion method of multiple graphs, obtaining the consistent graph with exact connection components.
- **TBGL** [55] learned a low-rank bipartite graph tensor representation, then acquired the unified graph via auto-weighted integration.
- **CSMSC** [36] learned the consensus subspace representation while the specific representations of different views were also explored.
- **MCGC** [65] designed a disagreement cost function to reinforce the consensus among different graphs.
- **SM$^2$SC** [64] pursued the view consistency via a variable splitting and a multiplicative decomposition module.
- **MvDSCN** [69] proposed the diversity and uniformity networks to capture the view-specific and consistent information, respectively.
- **LMVSC** [24] used anchor graph embedding to fit the global affinity matrix, thus resulting in linear computational complexity.
- **CGL** [28] optimized the spectral embedding matrices in a low-rank tensor space.
- **DMSC-UDL** [50] learned a unified subspace representation while enhanced the discrimination between diverse views.
- **EOMSC-CA** [32] fused the anchor graph scheme and graph construction into a uniform model.
- **CoMSC** [31] employed the eigendecomposition to obtain the robust representations of multi-view data, from which the consistent self-expressive representation was explored.
• **MFLVC** [58] proposed a novel contrastive multi-view clustering framework that simultaneously learned low-level and high-level features from multi-view data.

**CSMSC, SMSC, LMVSC, EOMSC-CA, and CoMSC** are shallow models based on subspace learning. **MvDSCN** and **DMSC-UDL** are deep subspace models. **MCGC** is a graph based model. CGL is a low-rank tensor learning based model. **MFLVC** is a deep model using contrast strategy. We run the codes published by the authors and set the parameters according to the intervals suggested in these papers.

In the proposed SCMC, we use two kinds of three-layer AEs to encode the multi-view data. The dimensions of each layer of one AE are \(d^{(o)}, 500\), \([500, 200]\), and \([200, c]\), respectively. The dimensions of each layer of another AE are \([d^{(o)}, 200]\), \([200, 100]\), and \([100, c]\), respectively. \(d^{(o)}\) and \(c\) denote the feature dimension of the \(v\)-th view’s data matrix and the number of clusters. The temperature parameter \(\tau\) is fixed to 0.1. \(y_1\), \(y_2\) are tuned in \(\{500, 1000\}\), \(\{0.01, 0.025, 0.03, 0.06, 0.3, 0.4\}\), respectively. \(y_3\) is set to 0.01. Herein, we illustrate how the selected values of \(y_1\), \(y_2\), and \(y_3\) are obtained. To seek the best parameters, the two-stage grid search strategy is adopted. Specifically, in the first stage, namely, the coarse-grained stage, we empirically set the values of \(y_1\), \(y_2\), and \(y_3\) in \(\{0.001, 0.01, 0.1, 1, 10, 100, 1000\}\), and perform the proposed SCMC on the small-size dataset WikipediaArticles to find the general parameter intervals that can yield the acceptable clustering results. After the coarse-grained stage, we locate the appropriate values of \(y_1\), \(y_2\), and \(y_3\) in \(\{100, 1000\}\), \(\{0.01, 1\}\), respectively. To cope with different datasets having different characteristics, we carry out the fine-grained grid search in the second stage on each dataset. For instance, we tune the value of \(y_1\) in \([100, 1000]\) with a step of 100, tune the value of \(y_2\) in \([0.01, 1]\) with a step of 0.01.

### 5.3 Evaluation Metrics

Seven dominant clustering evaluation metrics are adopted to quantify the clustering performance, which are Accuracy (ACC), Normalized Mutual Information (NMI), Purity, Adjusted Rand Index (ARI), F-score, Precision, and Recall respectively. In view of the algorithm stability, each experiment is run ten times, then the mean values are reported. The ranges of ACC, NMI, Purity, F-score, Precision, and Recall are all \([0, 1]\), while the range of ARI is \([-1, 1]\). For all metrics, higher values correspond to better clustering effects.

### 5.4 Experimental Results

To verifying the ability of capturing the nonlinear structures of SCMC, we first perform the traditional method AMGL and the proposed SCMC on the Moon dataset, which is a typical nonlinear multi-view dataset with 200 samples and 2 views, and visualize the clustering results in Fig. 4. It can be seen that AMGL achieves the inferior performance, misclassifying almost of upper part of the dataset. While SCMC obtains the nearly perfect results, demonstrating the its superior ability of capturing the nonlinear structures. Tables 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 provide the numerical results of all experiments, the best results are bolded and the second best results are underlined. These results reveal several interesting phenomenons and they are explained below. From a holistic perspective, the proposed SCMC outperforms other single-view and multi-view clustering methods, which shows that the strategies adopted by the model to improve the representation learning ability is efficient. Especially, on the UCI dataset, SCMC raises the clustering performance by 9%, 2.28%, 8.45%, 9.7%, 8.68%, 14.85%, 1.47% compared to the suboptimal results. MFLVC also leverages the contrast mechanism to improve the information capability of different representations, whose clustering performance achieves favorable status on several datasets such as GRAZ02, Youtube. While SCMC still achieves better performance, this situation may be attributed to the fact that SCMC applies the subspace learning technology to standardizes data from various modalities into a common semantic space, thus alleviating the difficulties in the contrast process. MvDSCN and DMSC-UDL are two deep multi-view subspace clustering models, from all experimental statistics, their clustering outcomes are not stable. For example, their clustering effects on UCI dataset are relatively superior, and both exceed the baseline method...
K-means. Nonetheless, on WikipediaArticles dataset, they both produce inferior clustering results than K-means.

An interesting observation is that although CGL is a conventional shallow model, it yields promising clustering results, probably because it captures the high-order correlations between views by virtue of the low-rank tensor learning. Furthermore, we present the running time of all compared methods on the test time in the Fig. 7. It can be seen that due to the network training consuming time, the proposed SCMC needs relatively longer time on the small datasets, while the running time is more satisfactory on the large datasets than that of some compared methods such as CSMSC and TBGL.

![Fig. 4. The visualizations of AMGL and SCMC on Moon dataset.](image)

### Table 2. Comparison of clustering results (%) on ALOI dataset.

| Datasets | Methods  | ACC | NMI | Purity | ARI | F-score | Precision |
|----------|----------|-----|-----|--------|-----|---------|-----------|
| ALOI     | K-means  | 47.49 | 47.34 | 48.58 | 32.98 | 41.04 | 33.97 |
|          | AMGL     | 52.28 | 54.85 | 55.70 | 33.18 | 41.03 | 34.71 |
|          | SwMC     | 67.10 | 64.99 | 69.42 | 37.29 | 45.71 | 33.77 |
|          | TBGL     | 58.29 | 56.15 | 61.35 | 26.24 | 36.74 | 25.59 |
|          | CSMSC    | 75.66 | 73.32 | 76.68 | 63.61 | 67.42 | 63.80 |
|          | MFCGC    | 83.32 | 74.77 | 83.32 | 65.51 | 69.06 | 66.61 |
|          | SMFSC    | 23.73 | 26.34 | 28.64 | 13.86 | 25.29 | 18.90 |
|          | MvDSCN   | 82.39 | 82.08 | 74.88 | 74.56 | 79.56 | 77.87 |
|          | LMVSC    | 73.31 | 68.62 | 73.59 | 58.50 | 62.89 | 58.77 |
|          | CGL      | 94.89 | 91.54 | 94.89 | 89.18 | 90.25 | 90.03 |
|          | DMS-UDL  | 79.61 | 80.16 | 72.95 | 67.33 | 77.71 | 76.22 |
|          | EOMSC-CA | 65.80 | 76.36 | 66.27 | 47.24 | 54.49 | 39.33 |
|          | CoMSC    | 80.63 | 81.31 | 84.62 | 71.96 | 74.95 | 69.63 |
|          | MFLVC    | 82.63 | 78.57 | 72.85 | 69.59 | 73.89 | 73.84 |
|          | SCMC     | **95.74** | **92.72** | **95.74** | **90.91** | **92.16** | **92.18** |

5.5 Experimental Visualization

We undertake some visualization experiments to directly compare the divergence in representation learning abilities of varying approaches. The t-SNE technology is used to downscale the concatenated UCI dataset to a two-dimensional planes. Fig. 5 visualizes the clustering results of ten multi-view clustering methods on the UCI dataset, and data points belonging to different clusters are painted in different colors. It can be seen that the ten
| Dataset   | Methods | ACC   | NMI   | Purity | ARI   | F-score | Precision |
|-----------|---------|-------|-------|--------|-------|---------|-----------|
| GRAZ02    | K-means | 35.91 | 3.20  | 35.91  | 3.56  | 33.83   | 27.19     |
|           | AMGL    | 46.61 | 5.64  | 46.61  | 6.82  | 35.10   | 29.18     |
|           | SwMC    | 45.54 | 12.56 | 46.68  | 11.16 | 40.23   | 29.18     |
|           | CSMSC   | -     | -     | -      | -     | -       | -         |
|           | MCGC    | 43.02 | 6.92  | 43.02  | 6.44  | 30.10   | 30.03     |
|           | SM²SC   | 47.15 | 12.49 | 47.76  | 11.91 | 34.22   | 34.09     |
|           | MvDSCN  | 40.44 | 8.21  | 44.24  | 8.09  | 31.40   | 31.23     |
|           | LMSC    | 40.79 | 8.29  | 43.50  | 7.82  | 31.17   | 31.04     |
|           | CGL     | 46.46 | 12.54 | 46.46  | 11.43 | 33.87   | 33.72     |
|           | DMSC-UDL| 41.32 | 8.33  | 52.24  | 8.28  | 32.65   | 32.05     |
|           | EOMSC-CA| 42.48 | 12.19 | 46.95  | 10.66 | 33.33   | 33.14     |
|           | CoMSC   | 47.97 | 13.76 | 57.18  | 13.79 | 35.64   | 35.35     |
|           | MFLVC   | -     | -     | -      | -     | -       | -         |
|           | SCMC    | 51.90 | 16.16 | 59.55  | 14.11 | 37.34   | 37.72     |

| Dataset       | Methods | ACC   | NMI   | Purity | ARI   | F-score | Precision |
|---------------|---------|-------|-------|--------|-------|---------|-----------|
| NUS-WIDE-v1   | K-means | 25.44 | 14.88 | 28.69  | 6.79  | 23.71   | 16.26     |
|               | AMGL    | 26.13 | 16.83 | 29.63  | 9.09  | 26.07   | 17.37     |
|               | SwMC    | 33.88 | 17.28 | 33.88  | 11.23 | 25.63   | 19.67     |
|               | TBGL    | 27.19 | 14.43 | 28.63  | 6.40  | 24.33   | 15.81     |
|               | CSMSC   | 34.31 | 19.58 | 38.38  | 13.29 | 24.46   | 23.63     |
|               | MCGC    | 31.75 | 18.87 | 35.63  | 11.35 | 23.40   | 21.42     |
|               | SM²SC   | 32.31 | 19.87 | 36.81  | 12.66 | 23.83   | 23.18     |
|               | MvDSCN  | 31.13 | 16.76 | 35.19  | 10.01 | 23.66   | 23.72     |
|               | LMSC    | 31.88 | 20.63 | 36.19  | 12.38 | 24.34   | 22.19     |
|               | CGL     | 31.35 | 20.36 | 36.91  | 12.57 | 24.52   | 22.31     |
|               | DMSC-UDL| 27.19 | 12.74 | 32.88  | 6.91  | 23.88   | 19.40     |
|               | EOMSC-CA| 32.94 | 20.21 | 34.00  | 11.84 | 27.11   | 19.48     |
|               | CoMSC   | 28.63 | 11.82 | 29.88  | 9.27  | 20.77   | 20.40     |
|               | MFLVC   | 34.81 | 18.68 | 36.13  | 14.33 | 26.55   | 24.62     |
|               | SCMC    | 36.56 | 21.83 | 40.06  | 14.33 | 27.92   | 28.15     |

clusters divided by SCMC rarely involve the samples of other clusters, the segmentation is nearly perfect. On the contrary, the clusters produced by EOMSC-CA are heavily mixed with samples of the other clusters, and even cannot divide enough ten clusters.

Fig. 6 presents the visualizations of consistent affinity representations learned by seven compared multi-view clustering methods and the proposed SCMC. We can observe that SM²SC and DMSC-UDL almost fail to engrave the diagonal-block structures, CGL can clearly highlight important structures on the diagonal, but the boundaries between diagonal-blocks are not apparent. Fortunately, SCMC achieves the clear depiction of diagonal-blocks. In addition, the view-specific and consistent affinity representations produced by SCMC are also visualized in Fig. 8. As Fig. 8 shows, the abilities to characterize the similarities between instances of the affinity matrices of ACM Trans. Knowl. Discov. Data.
Table 5. Comparison of clustering results (%) on four NUS-WIDE-v2 dataset.

| Dataset | Methods | ACC  | NMI  | Purity | ARI  | F-score | Precision |
|---------|---------|------|------|--------|------|---------|-----------|
| NUS-WIDE-v2 | K-means | 15.15 | 19.62 | 25.55 | 4.91 | 9.88   | 11.16     |
|         | AMGL    | 15.90 | 17.63 | 23.50 | 3.34 | 8.83   | 9.18      |
|         | SwMC    | 15.85 | 9.18  | 18.35 | 0.91 | 11.56  | 6.36      |
|         | TBGL    | 14.65 | 4.13  | 15.40 | 0.01 | 11.07  | 5.92      |
|         | CSMSC   | 13.60 | 15.89 | 21.90 | 3.08 | 7.55   | 9.68      |
|         | MCGC    | 15.05 | 15.92 | 21.60 | 4.46 | 8.49   | 11.89     |
|         | SM^2Sc  | 15.15 | 17.16 | 24.75 | 0.71 | 10.05  | 12.04     |
|         | TBGL    | 14.55 | 17.90 | 25.80 | 3.71 | 10.50  | 12.87     |
|         | LMVSC   | 15.40 | 18.13 | 24.35 | 4.66 | 8.76   | 12.04     |
|         | CGL     | 15.40 | 18.13 | 24.35 | 4.66 | 8.76   | 12.04     |
|         | MvDSb   | 14.65 | 17.90 | 25.80 | 3.71 | 10.50  | 12.87     |
|         | LMVSC   | 15.40 | 18.13 | 24.35 | 4.66 | 8.76   | 12.04     |
|         | CGL     | 15.40 | 18.13 | 24.35 | 4.66 | 8.76   | 12.04     |
|         | MFLVC   | 16.35 | 14.16 | 23.15 | 5.35 | 13.31  | 10.45     |
|         | SCMC    | 17.85 | 21.23 | 30.30 | 5.92 | 11.97  | 15.10     |

Table 6. Comparison of clustering results (%) on Reuters dataset.

| Datasets | Methods | ACC  | NMI  | Purity | ARI  | F-score | Precision |
|----------|---------|------|------|--------|------|---------|-----------|
| Reuters  | K-means | 42.47 | 22.09 | 50.67 | 16.19 | 36.63  | 32.40     |
|         | AMGL    | 42.20 | 13.19 | 42.87 | 5.94  | 36.72  | 24.21     |
|         | SwMC    | 44.93 | 28.49 | 52.53 | 10.15 | 36.87  | 26.74     |
|         | TBGL    | 32.07 | 6.27  | 34.27 | 0.84  | 35.02  | 21.92     |
|         | CSMSC   | 50.27 | 30.12 | 58.67 | 24.83 | 42.06  | 39.58     |
|         | MCGC    | 45.40 | 19.06 | 51.27 | 17.66 | 39.37  | 32.21     |
|         | SM^2Sc  | 47.93 | 26.06 | 52.40 | 24.76 | 41.25  | 40.59     |
|         | MvDSb   | 49.20 | 28.59 | 50.67 | 19.71 | 42.46  | 41.46     |
|         | LMVSC   | 47.40 | 26.80 | 51.73 | 19.64 | 41.27  | 33.07     |
|         | CGL     | 44.59 | 21.21 | 48.97 | 20.83 | 37.35  | 38.61     |
|         | DMSC-UDL | 34.93 | 16.50 | 43.00 | 1.85  | 44.42  | 30.66     |
|         | EOMSC-CA | 30.60 | 12.53 | 46.40 | 12.32 | 30.20  | 31.95     |
|         | CoMSC   | -     | -     | -     | -     | -      | -         |
|         | MFLVC   | 43.40 | 29.76 | 60.53 | 24.70 | 38.13  | 44.23     |
|         | SCMC    | 51.80 | 34.47 | 58.13 | 21.83 | 50.97  | 44.93     |

four single views are not good, Fig. 8(e) is generated via averaging all subspace representations, namely $Z_{\text{sum}} = (Z^{(1)} + Z^{(2)} + Z^{(3)} + Z^{(4)})/4$. Though $Z_{\text{sum}}$ can also clearly portray the diagonal-blocks structures, some non-diagonal elements are also easily observed. Its suppression of non-diagonal affinities is insufficient, which means that it does not protect the local geometric structures of data well.

To evaluate the convergence of the proposed SCMC, we plot the curves of the objective function values and metric values as the number of training epochs increases on eight datasets in Fig. 9. It can be observed that the objective function values decrease quickly and the metric values increase quickly, though there are fluctuations in the middle of training process, they eventually tend to stabilize.
Table 7. Comparison of clustering results (%) on UCI dataset.

| Datasets | Methods | ACC   | NMI   | Purity | ARI   | F-score | Precision |
|----------|---------|-------|-------|--------|-------|---------|-----------|
| UCI      | K-means | 38.76 | 46.64 | 44.23  | 31.35 | 38.86  | 35.39     |
|          | AMGL    | 76.28 | 78.30 | 78.02  | 68.69 | 72.04  | 67.55     |
|          | SwMC    | 72.95 | 79.09 | 75.55  | 67.17 | 70.80  | 63.85     |
|          | TBGL    | 81.40 | 84.06 | 83.40  | 76.13 | 78.71  | 72.50     |
|          | CSMC    | 87.20 | 77.35 | 87.02  | 74.32 | 78.93  | 77.18     |
|          | MCGC    | 80.20 | 79.74 | 83.55  | 73.90 | 76.63  | 72.87     |
|          | SMSC    | 81.40 | 84.06 | 83.40  | 76.13 | 78.71  | 72.50     |
|          | MvDSCN  | 81.85 | 71.72 | 73.05  | 65.82 | 69.47  | 69.31     |
|          | LMVSC   | 87.75 | 80.55 | 87.75  | 76.58 | 78.93  | 78.27     |
|          | CGL     | 84.20 | 90.52 | 88.55  | 83.22 | 78.93  | 78.27     |
|          | DMSC-UDL| 78.25 | 76.66 | 81.80  | 66.54 | 74.75  | 72.62     |
|          | EOMSC-CA| 54.80 | 67.09 | 55.10  | 46.28 | 53.57  | 59.29     |
|          | CoMSC   | 77.80 | 78.41 | 81.55  | 69.28 | 72.60  | 66.85     |
|          | MFLVC   | 79.95 | 78.36 | 79.95  | 69.73 | 73.31  | 72.51     |
|          | SCMC    | **96.75** | **92.84** | **97.00** | **92.92** | **93.70** | **93.71** |

Table 8. Comparison of clustering results (%) on WikipediaArticles dataset.

| Datasets          | Methods | ACC   | NMI   | Purity | ARI   | F-score | Precision |
|-------------------|---------|-------|-------|--------|-------|---------|-----------|
| WikipediaArticles | K-means | 54.69 | 51.48 | 58.59  | 39.02 | 45.80  | 44.97     |
|                   | AMGL    | 55.99 | 52.69 | 60.89  | 39.31 | 46.10  | 45.03     |
|                   | SwMC    | 51.08 | 44.81 | 58.68  | 19.94 | 31.49  | 23.84     |
|                   | TBGL    | 29.44 | 21.40 | 32.61  | 6.04  | 31.49  | 14.46     |
|                   | CSMC    | 52.03 | 46.87 | 55.70  | 38.36 | 44.91  | 46.16     |
|                   | MCGC    | 54.40 | 40.79 | 56.85  | 31.68 | 38.91  | 40.13     |
|                   | SMSC    | 55.12 | 50.83 | 59.31  | 40.67 | 46.95  | 48.46     |
|                   | MvDSCN  | 39.97 | 32.09 | 47.04  | 21.41 | 31.87  | 32.62     |
|                   | LMVSC   | 55.56 | 47.46 | 57.00  | 33.12 | 41.00  | 37.99     |
|                   | CGL     | 54.16 | 49.83 | 59.42  | 37.09 | 44.11  | 43.21     |
|                   | DMSC-UDL| 44.30 | 35.01 | 47.33  | 22.30 | 35.53  | 36.15     |
|                   | EOMSC-CA| 56.13 | 52.91 | 61.04  | 42.30 | 48.47  | 49.56     |
|                   | CoMSC   | 21.07 | 7.18  | 23.23  | 2.90  | 13.13  | 13.61     |
|                   | MFLVC   | 43.15 | 31.54 | 47.61  | 24.97 | 34.09  | 32.61     |
|                   | SCMC    | **57.86** | **53.74** | **63.20** | **42.54** | **50.61** | **51.01** |

5.6 Ablation Study

The proposed SCMC consists of four loss components, i.e., $\mathcal{L}_{Re}$, $\mathcal{L}_{Sub}$, $\mathcal{L}_{Con}$, and $\mathcal{L}_{Fu}$. We implement experiments to verify the role played by each loss component. Tables 12 and 13 report the ablation results. When SCMC has only reconstruction loss $\mathcal{L}_{Re}$, we obtain the consensus features by averaging multiple embedding features, then they are fed into the K-means algorithm to yield the clustering results. However, the results are inferior, which means that the latent representations embedded only through AEs are not yet well discriminated. When the subspace loss is introduced, i.e., $\mathcal{L}_{Re} + \mathcal{L}_{Sub}$, the performance is somewhat improved. It is worth noting that once the contrast loss is included, the improvements of clustering effects are significant, illustrating that the contrast strategy dose help to augment the discrimination of subspace representations. In general, on the basis of
Table 9. Comparison of clustering results (%) on Youtube dataset.

| Datasets | Methods | ACC  | NMI  | Purity | ARI  | F-score | Precision |
|----------|---------|------|------|--------|------|---------|-----------|
| Youtube  | K-means | 24.56| 15.16| 27.85  | 8.19 | 19.48   | 15.78     |
|          | AMGL    | 27.65| 16.68| 27.80  | 11.47| 24.23   | 16.74     |
|          | SwMC    | 23.00| 11.18| 24.25  | 6.27 | 17.54   | 14.52     |
|          | TBGL    | 20.80| 10.55| 21.10  | 3.72 | 20.49   | 11.71     |
|          | CSMSC   | -     | -    | -      | -    | -       | -         |
|          | MCGC    | 28.35| 13.66| 29.70  | 8.34 | 17.62   | 17.33     |
|          | SM²SC   | 30.58| 18.12| 34.08  | 11.14| 20.11   | 19.83     |
|          | MvDSCN  | 30.55| 17.67| 34.90  | 10.76| 22.77   | 20.41     |
|          | LMVSC   | 29.15| 18.01| 33.30  | 10.06| 19.50   | 18.52     |
|          | CGL     | 32.95| 19.65| 35.39  | 12.52| 21.48   | 20.88     |
|          | DMSC-UDL|29.95| 17.13| 35.30  | 9.39 | 21.54   | 19.11     |
|          | EOMSC-CA|34.86| 37.73| 36.64  | 4.62 | 8.85    | 4.69      |
|          | CoMSC   | 51.42| 61.74| 60.80  | 41.92| 43.23   | 45.06     |
|          | MFLVC   | 53.44| 62.39| 58.96  | 19.59| 22.67   | 13.90     |
|          | SCMC    | 55.84| 66.38| 63.78  | 44.56| 45.79   | 48.63     |
|          | SCMC    | 57.90| 26.12| 44.15  | 18.53| 28.50   | 25.61     |

Table 10. Comparison of clustering results (%) on Animals dataset.

| Datasets | Methods | ACC  | NMI  | Purity | ARI  | F-score | Precision |
|----------|---------|------|------|--------|------|---------|-----------|
| Animals  | K-means | 30.50| 39.28| 34.52  | 17.52| 19.33   | 20.64     |
|          | AMGL    | 43.64| 54.43| 48.74  | 18.11| 21.02   | 13.88     |
|          | SwMC    | 54.04| 62.47| 60.80  | 14.24| 17.63   | 10.45     |
|          | TBGL    | 54.86| 37.73| 36.64  | 4.62 | 8.85    | 4.69      |
|          | CSMSC   | 51.42| 61.74| 60.82  | 41.92| 43.23   | 45.06     |
|          | MCGC    | 53.44| 62.39| 58.96  | 19.59| 22.67   | 13.90     |
|          | SM²SC   | 55.84| 66.38| 63.78  | 44.56| 45.79   | 48.63     |
|          | MvDSCN  | 55.15| 65.99| 60.62  | 44.38| 45.23   | 48.36     |
|          | LMVSC   | 37.90| 57.15| 47.54  | 28.59| 30.22   | 31.18     |
|          | CGL     | 42.57| 53.82| 47.75  | 28.96| 30.73   | 29.01     |
|          | DMSC-UDL|49.16| 62.45| 55.06  | 22.41| 47.20   | 43.35     |
|          | EOMSC-CA|54.88| 67.21| 59.72  | 47.54| 42.94   | 53.24     |
|          | CoMSC   | 52.50| 40.80| 37.96  | 17.30| 19.16   | 20.02     |
|          | MFLVC   | 28.10| 41.45| 29.56  | 18.99| 21.42   | 18.02     |
|          | SCMC    | 57.06| 68.82| 64.16  | 45.49| 48.30   | 51.57     |

$L_{Re} + L_{Subh}$, SCMC performs better with the fusion loss $L_{Fu}$ than with contrast loss $L_{Con}$, which indicates that the contribution of $L_{Fu}$ is greater. Nevertheless, $L_{Con}$ is essential if the optimal performance is to be achieved. To verify the necessity of weighted fusion manner, we use the method of averaging multiple subspace representations for the ablation experiments, that is, the initial consistent affinity matrix is obtained by $A = \sum_{i=1}^{V} Z^{(i)}/V$. The experimental results are showed in the Fig. 10, we can see that the clustering results with weight fusion manner outperforms that of average fusion approach, showing that integrating multiple subspace representations with weight assignment is more beneficial for utilizing the complementary information.

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Table 11. Comparison of clustering results (%) on Cifar10 dataset.

| Datasets | Methods | ACC  | NMI  | Purity | ARI  | F-score | Precision |
|----------|---------|------|------|--------|------|---------|-----------|
| Cifar10  | K-means | 15.84| 3.81 | 16.89  | 1.71 | 13.80   | 11.22     |
|          | AMGL    | 10.74| 0.37 | 10.82  | 0.00 | 18.17   | 10.00     |
|          | SwMC    | 17.04| 6.05 | 17.34  | 1.97 | 18.97   | 11.88     |
|          | TBGL    | 10.59| 0.15 | 10.60  | 0.00 | 18.18   | 10.00     |
|          | CSMSC   | 17.30| 4.26 | 17.79  | 2.16 | 12.23   | 11.88     |
|          | MCGC    | 18.12| 6.17 | 18.57  | 3.01 | 18.96   | 11.53     |
|          | SM²SC   | 16.90| 4.10 | 17.50  | 2.05 | 12.26   | 11.77     |
|          | MvDSCN  | 17.68| 6.65 | 25.40  | 2.62 | 16.07   | 13.04     |
|          | LMVSC   | 18.55| 8.21 | 19.51  | 4.05 | 13.91   | 13.54     |
|          | CGL     | 19.03| 6.74 | 19.84  | 3.67 | 14.46   | 12.91     |
|          | DMSC-UDL| 18.76| 7.46 | 23.05  | 3.39 | 15.96   | 13.39     |
|          | EOMSC-CA| 18.36| 7.90 | 18.43  | 3.38 | 10.95   | 11.18     |
|          | CoMSC   | 17.15| 4.09 | 18.29  | 2.11 | 12.60   | 11.76     |
|          | MFLVC   | 19.62| 6.68 | 24.53  | 3.26 | 13.15   | 13.19     |
|          | SCMC    | 21.22| 10.26| 25.56  | 5.19 | 16.75   | 14.45     |

Fig. 5. Visualization of clustering results of ten multi-view clustering methods on UCI dataset via t-SNE technology.

Table 12. Ablation results of the proposed SCMC on four datasets.

| Loss  | ALOI | GRAZ02 | NUS-WIDE-v1 | NUS-WIDE-v2 |
|-------|------|--------|--------------|-------------|
|       | ACC  | NMI    | Purity       | ACC         | NMI    | Purity       | ACC         | NMI    | Purity       |
| \mathcal{L}_{A} | 71.36 | 81.26  | 76.09        | 38.28       | 7.30   | 54.47        | 27.00       | 12.05  | 35.06        |
| \mathcal{L}_{F} + \mathcal{L}_{S} | 78.68 | 81.60  | 72.47        | 39.77       | 8.74   | 55.08        | 30.69       | 19.82  | 36.18        |
| \mathcal{L}_{F} + \mathcal{L}_{C} | 88.32 | 84.17  | 79.98        | 45.33       | 12.46  | 56.23        | 32.50       | 19.88  | 36.88        |
| \mathcal{L}_{F} + \mathcal{L}_{S} + \mathcal{L}_{F} | 93.05 | 89.87  | 84.52        | 45.12       | 11.18  | 57.18        | 32.88       | 18.82  | 38.88        |
| \mathcal{L}_{S} + \mathcal{L}_{C} | 89.62 | 86.58  | 85.82        | 45.39       | 11.66  | 58.27        | 33.00       | 17.28  | 36.12        |
| \mathcal{L} | 95.74 | 92.72  | 95.74        | 51.90       | 16.16  | 59.55        | 36.56       | 21.83  | 40.06        |

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Fig. 6. Visualizations of consistent affinity representations learned via eight multi-view clustering approaches on ALOI dataset.

Table 13. Ablation results of the proposed SCMC on four datasets.

| Loss                  | Reuters                        | UCI               | WikipediaArticles | Youtube           |
|-----------------------|--------------------------------|-------------------|-------------------|-------------------|
|                       | ACC   | NMI   | Purity | ACC   | NMI   | Purity | ACC   | NMI   | Purity | ACC   | NMI   | Purity |
| $L_{Re}$              | 41.80 | 17.53 | 45.93  | 70.90 | 69.16 | 73.90  | 23.09 | 9.59  | 29.73  | 23.95 | 13.00 | 29.15  |
| $L_{Re} + L_{Sub}$    | 38.33 | 28.52 | 51.07  | 73.65 | 73.88 | 76.65  | 35.64 | 24.54 | 40.69  | 27.65 | 17.65 | 32.40  |
| $L_{Re} + L_{Sub} + L_{Con}$ | 47.60 | 31.05 | 49.93  | 91.40 | 83.70 | 81.90  | 37.93 | 27.72 | 41.99  | 29.20 | 18.74 | 35.40  |
| $L_{Re} + L_{Sub} + L_{Fu}$ | 48.00 | 29.34 | 51.00  | 90.55 | 90.99 | 86.25  | 56.57 | 54.67 | 62.36  | 31.90 | 18.90 | 37.60  |
| $L_{Sub} + L_{Con} + L_{Fu}$ | 48.60 | 30.10 | 57.53  | 93.00 | 86.78 | 93.15  | 52.23 | 51.89 | 62.04  | 34.40 | 21.17 | 38.75  |
| $\mathcal{L}$        | 51.80 | 34.47 | 58.13  | 96.75 | 92.84 | 97.00  | 57.86 | 53.74 | 63.20  | 37.90 | 26.12 | 41.15  |

Table 14. Ablation results of the proposed SCMC on two datasets.

| Loss                  | Animals                      | Cifar10                      |
|-----------------------|------------------------------|------------------------------|
|                       | ACC   | NMI   | Purity | ACC   | NMI   | Purity |
| $L_{Re}$              | 48.10 | 63.59 | 53.94  | 15.73 | 5.28  | 22.55  |
| $L_{Re} + L_{Sub}$    | 50.34 | 65.87 | 58.46  | 17.10 | 5.60  | 23.63  |
| $L_{Re} + L_{Sub} + L_{Con}$ | 54.84 | 66.60 | 63.20  | 18.26 | 7.00  | 23.57  |
| $L_{Re} + L_{Sub} + L_{Fu}$ | 54.50 | 66.96 | 62.47  | 18.36 | 8.08  | 24.78  |
| $L_{Sub} + L_{Con} + L_{Fu}$ | 52.72 | 68.02 | 62.90  | 18.40 | 7.34  | 24.90  |
| $\mathcal{L}$        | 57.06 | 68.82 | 64.16  | 21.22 | 10.26 | 25.56  |
Fig. 7. Comparison of execution time of all compared multi-view clustering methods on the test datasets. Considering the large gaps between some values of running time, we perform the log(·) with the base of 10.

5.7 Investigation of Subspace-Contrastive Effects

We argue that learning subspace representations of multi-view data unifies the semantic information between different modalities, thus facilitating the contrastive training of positive and negative instance pairs. Some comparative experiments are designed to validate the subspace-contrastive effects.

Firstly, we directly perform the contrast between latent representations learned by multiple view-specific AEs, namely removing the subspace loss $\mathcal{L}_{Sub}$. The reconstruction loss $\mathcal{L}_{Re}$ is transformed as

$$\mathcal{L}_{Re} = \sum_{\nu=1}^{V} \| X^{(\nu)} - d_V(e_\nu(X^{(\nu)}|W_{e}^{(\nu)}, b_{e}^{(\nu)})|W_{d}^{(\nu)}, b_{d}^{(\nu)}) \|_F^2.$$  \hfill (33)

Meanwhile, the fusion loss $\mathcal{L}_{Fu}$, i.e., Eq. (10) is modified as the following form:

$$\mathcal{L}_{Fu} = \min_{C^{(\nu)}, A} \sum_{\nu=1}^{V} \sum_{i} \sum_{j} ||C_i^{(\nu)} - C_j^{(\nu)}||_2^2 A_{ij} + \sum_{i} \sum_{j} A_{ij}^2$$

$$= \sum_{\nu=1}^{V} Tr(C^{(\nu)} L_A C^{(\nu)^T}) + ||A||_F^2$$ \hfill (34)

s.t. $A_i1 = 1, A_{ij} \geq 0, A_{ii} = 0,$
where $C^{(e)}$ is the latent representation learned via $C^{(e)} = \varepsilon_e(X^{(e)}|W^{(e)}, b^{(e)})$. Besides, the weighted fusion scheme is discarded. The designed ablation method is tentatively named Latent-Contrastive Multi-view Clustering (LCMC). After the training ends, the clustering results are obtained via performing the spectral clustering algorithm on the affinity matrix $(A + A^T)/2$. Certainly, to maintain the fairness of experiments, we also train the proposed ACM Trans. Knowl. Discov. Data.
SCMC without the weighted fusion mechanism. Fig. 11 shows the result comparisons under the two contrast methods. Obviously, when executing the contrast at the subspace level, the experimental performance is much better than when executing the contrast between latent representations.

Secondly, to further avoid the influence of fusion loss $\mathcal{L}_{fu}$, we remove the fusion loss $\mathcal{L}_{fu}$ of LCMC and SCMC, and obtain the unified latent and subspace representation via $C = \sum_{v=1}^{V} C^{(v)}/V$, $Z = \sum_{v=1}^{V} Z^{(v)}/V$, respectively. Thus, the clustering results are acquired by adopting the spectral clustering algorithm. We leverage the t-SNE technology.
Table 15. Clustering performance of the compared methods on ALOI dataset with in-samples and out-of-samples.

| Method | In-Samples | Out-of-Samples |
|--------|------------|----------------|
|        | ACC | NMI | Purity | ARI | F-score | Precision | ACC | NMI | Purity | ARI | F-score | Precision |
| K-means | 47.63 | 48.39 | 48.63 | 34.28 | 42.22 | 34.66 | 49.10 | 49.02 | 52.69 | 30.45 | 38.50 | 33.06 |
| SwMC | 67.88 | 65.28 | 71.00 | 38.32 | 46.45 | 34.95 | 70.61 | 65.52 | 70.61 | 40.80 | 48.86 | 35.65 |
| MCGC | 80.38 | 72.97 | 80.37 | 63.38 | 67.16 | 64.62 | 81.00 | 77.95 | 81.00 | 68.86 | 72.24 | 66.19 |
| LMVSC | 71.00 | 73.21 | 71.75 | 63.31 | 67.28 | 61.69 | 73.12 | 73.56 | 74.91 | 58.57 | 63.34 | 54.67 |
| SCMC | **96.13** | **93.72** | **96.12** | **91.55** | **93.25** | **93.55** | **93.55** | **90.53** | **93.55** | **87.47** | **88.72** | **88.37** |

Table 16. Clustering performance of the compared methods on GRAZ02 dataset with in-samples and out-of-samples.

| Method | In-Samples | Out-of-Samples |
|--------|------------|----------------|
|        | ACC | NMI | Purity | ARI | F-score | Precision | ACC | NMI | Purity | ARI | F-score | Precision |
| K-means | 36.56 | 3.40 | 36.56 | 3.93 | 33.55 | 27.42 | 36.67 | 4.69 | 37.33 | 3.57 | 33.36 | 27.18 |
| SwMC | 42.86 | 11.76 | 46.34 | 8.93 | 36.29 | 30.44 | 41.00 | 7.63 | 42.67 | 5.36 | 33.32 | 28.39 |
| MCGC | 45.15 | 9.16 | 45.15 | 8.75 | 31.74 | 31.75 | 41.67 | 8.35 | 44.00 | 7.79 | 30.94 | 31.01 |
| LMVSC | 42.35 | 8.01 | 42.35 | 7.54 | 31.24 | 30.71 | 42.33 | 6.09 | 42.33 | 5.90 | 30.65 | 29.33 |
| SCMC | **49.82** | **15.03** | **57.77** | **13.73** | **36.43** | **36.90** | **49.66** | **15.53** | **59.66** | **13.95** | **36.75** | **37.19** |

method to reduce the dimensionality of C and Z learned on ALOI and UCI datasets, respectively, and visualize the scatter plots. As Fig. 12 shows, many different clusters visualized by latent features adhesive with each other, the overall separation is not good. On the contrary, the cluster shapes visualized by the subspace features are distinct, and diverse clusters are pulled apart. This phenomenon illustrates the subspace features are more discriminative than the latent features, which means that the subspace learning does alleviate the modality separation and help the contrastive training.

5.8 Experiments on Out-of-Samples

To verify the effectiveness of the proposed CSR, we plug it into several multi-view clustering methods SwMC, CSMSC, LMVSC and our SCMC to conduct the experiments. Specifically, 80% of all samples in a multi-view dataset are randomly selected to consist of the in-samples, and the remaining 20% consist of the out-of-samples. K-means is used as the most basic compared method. Actually, K-means does not have the ability to handle the out-of-samples, so we have to perform it on the in-samples and out-of-samples and obtain the clustering results, respectively. As for the other compared methods, we first perform them on the in-samples to obtain their labels, then adopt the proposed CSR mechanism on the out-of-samples to generate their labels. Tables 15, 18, 16, 17 report the experimental results of several compared methods on four multi-view datasets. It can be seen that the proposed SCMC achieves the optimal performance on both in-samples and out-of-samples. Moreover, the proposed CSR has excellent adaptability and can be combined with any multi-view clustering algorithms to yield the superior performance on the out-of-samples.

5.9 Parameter Sensitivity Analysis

Three nonnegative trade-off parameters are used to balance the overall objective loss, their impacts on the clustering outcomes are investigated via sensitivity experiments. Specifically, we tune $\gamma_1$, $\gamma_2$, and $\gamma_3$ in $[10, 50, 100, 200, 500, 1000]$, $[0.0001, 0.001, 0.01, 0.1, 0.5, 1]$, and $[0.0001, 0.001, 0.01, 0.1, 0.5, 1]$, respectively. Fig. 13 shows
Table 17. Clustering performance of the compared methods on Reuters dataset with in-samples and out-of-samples.

| Method | ACC | NMI | Purity | ARI | F-score | Precision | ACC | NMI | Purity | ARI | F-score | Precision |
|--------|-----|-----|--------|-----|---------|-----------|-----|-----|--------|-----|---------|-----------|
| K-means | 36.75 | 4.26 | 36.92 | 5.47 | 35.22 | 24.08 | 41.67 | 9.11 | 42.67 | 10.20 | 37.91 | 26.94 |
| SwMC   | 47.75 | 28.40 | 53.75 | 13.48 | 38.97 | 28.46 | 45.00 | 26.62 | 51.67 | 5.96 | 34.64 | 24.90 |
| MCGC   | 46.33 | 18.81 | 51.50 | 18.66 | 40.31 | 32.58 | 44.67 | 22.92 | 53.67 | 15.56 | 37.65 | 31.56 |
| LMVSC  | 46.00 | 24.40 | 50.50 | 18.93 | 41.19 | 32.28 | 47.33 | 27.23 | 52.33 | 16.02 | 38.47 | 31.55 |
| SCMC   | 50.25 | 29.24 | 56.75 | 21.38 | 49.2 | 41.37 | 51.00 | 30.88 | 58.33 | 18.21 | 41.88 | 31.73 |

Table 18. Clustering performance of the compared methods on UCI dataset with in-samples and out-of-samples.

| Method | ACC | NMI | Purity | ARI | F-score | Precision | ACC | NMI | Purity | ARI | F-score | Precision |
|--------|-----|-----|--------|-----|---------|-----------|-----|-----|--------|-----|---------|-----------|
| K-means | 39.12 | 47.01 | 44.56 | 31.93 | 39.41 | 35.75 | 37.75 | 45.03 | 41.25 | 26.25 | 34.52 | 30.88 |
| SwMC   | 73.25 | 79.49 | 75.94 | 68.05 | 71.58 | 64.58 | 75.25 | 78.99 | 77.50 | 66.10 | 70.06 | 60.66 |
| MCGC   | 83.06 | 78.77 | 83.06 | 72.63 | 75.37 | 74.86 | 82.50 | 78.62 | 72.75 | 75.65 | 71.43 |
| LMVSC  | 89.56 | 82.51 | 89.56 | 79.35 | 81.42 | 80.96 | 87.50 | 82.44 | 87.50 | 76.12 | 78.54 | 77.65 |
| SCMC   | 95.56 | 90.75 | 96.19 | 90.41 | 91.49 | 91.51 | 95.00 | 90.56 | 95.00 | 88.82 | 89.93 | 90.15 |

the numerical results of SCMC under different settings of $W_1$, $W_2$, and $W_3$. When evaluating $W_1$ and $W_2$, we fix $W_3$, the same operation is done for evaluating $W_3$. It can be seen that a small $W_1$ is not conducive to learning an informative subspace representation, this is caused by the insufficient penalty for $||C^{(x)} - C^{(x)}Z^{(x)}||_F^2$ in the optimization. Similarly, when $W_2$ is set to be small, the clustering performance is not ideal, because the contrast mechanism has little effects to enhance the discrimination of subspace representations. Interestingly, selecting a relatively larger $W_3$ can degrade the effectiveness of SCMC, this situation may result from over-smoothing.

6 CONCLUSION

In this paper, we propose a novel Subspace-Contrastive Multi-View Clustering (SCMC) approach. In SCMC, the subspace representation of each view is nonlinearly explored through a extraction network. Inspired by the idea of contrastive learning, we regard the subspace representation of each view as a contrastable entity. By pairwise comparison of multiple subspace representations, we exploit the complementary information in them and augment the discriminability of each subspace representation. Guided by the important principle of multi-view consensus, we obtain a consistent affinity matrix by the graph regularization. Furthermore, to handle the out-of-samples, the consistent sparse representation learning method over the in-samples is proposed. In future work, we focus on three important challenges in deep multi-view subspace clustering. First, how to achieve more effective contrast between subspace representations. The proposed SCMC uses a cross-view comparison approach while ignoring the intra-view comparison relationship, which needs to be paid attention to. Second, how to improve the efficiency of subspace representation learning and reduce its memory overhead. Self-expression based subspace representation learning suffers from huge time and space overhead, which severely limits its practical applications, and the development of efficient multi-view subspace learning methods is necessary. Third, how to deal with possible missing values in multi-view data. Due to various negative problems such as sensor failures, multi-view data is often incomplete, which affects the learning of subspace representations, it is significant to study a multi-view subspace learning algorithm that can adaptively deal with missing data.
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APPENDIX

A THE DEFINITION OF EVALUATION METRICS

Following the compared methods [28, 31, 36, 55, 65], we select six frequently-used clustering evaluation metrics to evaluate the clustering results, including ACC, NMI, Purity, ARI, F-score, Precision. To elaborate on which aspect of the clustering results they characterize respectively, their definitions need to be introduced.

Suppose $y_i$ is the $i$-th sample’s true label, $\hat{y}_i$ is the $i$-th sample’s predicted label, and $\Phi(\cdot)$ is the best mapping function from the predicted label to the true label, then ACC is defined as

$$\text{ACC} = \frac{\sum_{i=1}^{n} \psi(y_i, \Phi(\hat{y}_i))}{N},$$  \hspace{1cm} (35)

where $N$ is the number of instances, $\psi(\cdot, \cdot)$ is the binary criterion function and defined as

$$\psi(x, y) = \begin{cases} 
1, & \text{if } x = y; \\
0, & \text{otherwise}
\end{cases}$$

Given that $Y = \{Y_i\}_{i=1}^{c}$ is the true clusters and $\hat{Y} = \{\hat{Y}_j\}_{j=1}^{c}$ is the predicted clusters, then NMI is defined as

$$\text{NMI} = \frac{I(\hat{Y}, Y)}{(F(\hat{Y}) + F(Y))/2},$$  \hspace{1cm} (36)

where $I(\cdot, \cdot)$ denotes the mutual information between two variables, $F(\cdot)$ denotes the entropy of certain variable. The mutual information $I(\cdot, \cdot)$ is computed by

$$I(\hat{Y}, Y) = \sum_{j=1}^{c} \sum_{i=1}^{c} P(\hat{Y}_j \cap Y_i) \log \frac{P(\hat{Y}_j \cap Y_i)}{P(\hat{Y}_j)P(Y_i)},$$  \hspace{1cm} (37)

where $P(\cdot)$ denotes the probability. The entropy $F(\hat{Y}), F(Y)$ are respectively computed by

$$F(\hat{Y}) = -\sum_{j=1}^{c} P(\hat{Y}_j) \log(P(\hat{Y}_j)), F(Y) = -\sum_{i=1}^{c} P(Y_i) \log(P(Y_i)).$$  \hspace{1cm} (38)

Purity indicates the ratio of correctly grouped samples and is calculated as

$$\text{Purity} = \frac{1}{N} \sum_{j=1}^{c} \max_{i} |\hat{Y}_j \cap Y_i|$$  \hspace{1cm} (39)
Assume that TP, FP, TN, FN represent the number of entries correctly predicted as positive instances, the number of entries erroneously predicted as positive instances, the number of entries correctly predicted as negative instances, the number of erroneously predicted as negative instances, respectively. Thus, the definition of Rand Index (RI) is written as

\[ RI = \frac{TP + TN}{TP + FP + TN + FN}. \]  

(40)

RI cannot guarantee that the evaluation value of clustering results of randomized division tends to be zero, adjust RI (ARI) is introduced as follows,

\[ ARI = \frac{RI - E[RI]}{\max(RI) - E[RI]} = \frac{\sum_{ij} (a_i \cdot b_j) - (\sum_i a_i \cdot \sum_j b_j)/n}{\frac{1}{2} (\sum_i (a_i^2) + \sum_j (b_j^2)) - (\sum_i (a_i^2) \cdot \sum_j (b_j^2))/n}, \]

(41)

where \( n_{ij} \) denotes the number of samples overlap between the true \( i \)-th cluster and the predicted \( j \)-th cluster, \( a_i \) denotes the number of samples belonging to the \( i \)-th true cluster, \( b_j \) denotes the number of samples belonging to the \( j \)-th predicted cluster.

Precision denotes the proportion of correctly predicted positive samples to all predicted positive samples, which is defined as

\[ \text{Precision} = \frac{TP}{TP + FP}. \]  

(42)

Recall denotes the proportion of correctly predicted positive samples to all true positive samples, which is defined as

\[ \text{Recall} = \frac{TP}{TP + FN}. \]  

(43)

To balance the Precision and Recall, the F-score is proposed and defined as

\[ \text{F-score} = \frac{2 \cdot \text{Precision} \cdot \text{Recall}}{\text{Precision} + \text{Recall}}. \]  

(44)

It is worthy emphasizing that when calculating the overall values of Precision, Recall and F-score, the values for each class are first calculated separately and then averaged. Furthermore, Recall focuses on the retrieval rate of correctly categorized samples. Metaphorically speaking, it is preferred to misclassify more samples than to miss one. In the scenario of multi-class clustering, the Recall metric is easily biased by certain categories and cannot objectively reflect the sample division, so we discard the metric.

Ultimately, we summarize the characteristics of the six metrics in the following Table 19.

| Metric | Characteristic |
|--------|----------------|
| ACC    | Computes the clustering accuracy from an overall view, but is susceptible to the class imbalance problem. |
| NMI    | Measures the similarity degree between the predicted labels and true labels. |
| Purity | Computes the ratio of correctly clustered samples, but cannot balance the quality of clustering against the number of clusters. |
| ARI    | Measures the similarity degree between the predicted labels and true labels. |
| Precision | Computes the proportion of correctly predicted positive samples to all predicted positive samples for each category, then averages the Precision values of all categories. |
| F-score| Computes proportion of correctly predicted positive samples, is a balanced solution of Precision and Recall, and adapts to the situation of class imbalance. |
Table 20. Clustering results on each view of four datasets via spectral clustering.

| View Index (Method) | ALOI                   | GRAZ02                 | NUS-WIDE-v1            | NUS-WIDE-v2          |
|---------------------|------------------------|------------------------|------------------------|----------------------|
|                     | ACC        NMI   Purity | ACC        NMI   Purity | ACC        NMI   Purity | ACC        NMI   Purity |
| View 1 (SC)         | 57.58  71.98  67.82   | 35.37  3.80  37.26     | 13.19  0.85  13.38     | 12.30  1.83  13.85   |
| View 2 (SC)         | 62.09  62.05  64.23   | 36.04  9.11  40.11     | 19.38  4.03  20.19     | 12.35  1.53  13.25   |
| View 3 (SC)         | 58.33  59.59  60.50   | 28.52  0.23  28.66     | 17.69  3.65  19.38     | 12.65  1.85  13.70   |
| View 4 (SC)         | 51.96  51.04  54.46   | 28.52  0.23  28.66     | 15.69  2.62  16.81     | 12.50  1.82  13.40   |
| View 5 (SC)         | – – –       | 35.77  9.08  20.94     | 20.94  8.86  24.75     | 13.75  4.17  14.80   |
| View 6 (SC)         | – – –       | 36.11  5.88  38.21     | 14.75  7.75  19.75     | – – –     |
| – (SCMC)            | 95.74  92.12  95.74   | 51.90  16.16  59.55    | 36.56  21.83  40.06    | 17.85  21.23  30.30  |

Table 21. Clustering results on each view of four datasets via spectral clustering.

| View Index (Method) | Reuters          | UCI              | WikipediaArticles | Youtube         |
|---------------------|------------------|------------------|-------------------|-----------------|
|                     | ACC        NMI   Purity | ACC        NMI   Purity | ACC        NMI   Purity | ACC        NMI   Purity |
| View 1 (SC)         | 28.00  0.39  28.13 | 30.55  33.42  31.90 | 19.77  6.55  21.93 | 22.20  8.74  22.65 |
| View 2 (SC)         | 27.93  0.34  28.07 | 70.95  64.10  70.95 | 55.12  51.91  60.03 | 22.40  10.51  24.60 |
| View 3 (SC)         | 27.93  0.34  28.07 | 22.25  11.46  23.90 | – – – | 19.20  8.38  20.70 |
| View 4 (SC)         | 26.20  0.38  28.60 | – – – | – – – | 30.75  20.22  33.15 |
| View 5 (SC)         | 27.80  0.28  28.00 | – – – | – – – | 29.90  16.91  31.95 |
| View 6 (SC)         | – – – | – – – | – – – | 27.55  15.16  29.80 |
| – (SCMC)            | 51.80  34.47  58.13 | 96.75  92.84  97.00 | 57.86  53.74  63.20 | 37.90  26.12  41.15 |

Table 22. Clustering results on each view of two datasets via spectral clustering.

| View Index (Method) | Animals          | Cifar10          |
|---------------------|------------------|------------------|
|                     | ACC        NMI   Purity | ACC        NMI   Purity |
| View 1 (SC)         | 28.88  36.63  33.18 | 10.46  0.10  10.54 |
| View 2 (SC)         | 33.76  51.51  40.98 | 11.92  4.61  15.34 |
| – (SCMC)            | 57.06  68.82  64.16 | 21.22  10.26  25.56 |

B THE VERIFICATION OF CAPTURING THE COMPLEMENTARY INFORMATION

To demonstrate that the proposed SCMC can capture the complementary information in different views, we execute the spectral clustering algorithm on each view of the test multi-view datasets and report the clustering results in Tables 20, 21, 22. It can be observed that the clustering results under any of the views are worse than those obtained by the proposed SCMC, which is a good indication that SCMC captures the complementarity information among multiple views.