Cold-preventing simulation of low-temperature protective clothing based on an unsteady one-dimensional heat conduction model

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Abstract: With the development of science and technology, people are constantly exploring the extreme cold regions. In order to enable people to work normally in extremely cold weather, scientists have been studying low-temperature protective composite materials to protect workers in ultra-low ambient temperatures. This paper studies the cold-proof effect of low-temperature protective clothing composed of three materials. First of all, this paper uses the principle of heat transfer to establish an unsteady one-dimensional heat conduction partial differential equation model for the experimenter, cryogenic protective clothing and the system composed of the external air environment. Secondly, this paper determines the boundary value conditions and boundary value conditions of the definite solution, as well as the continuous conditions of the contact surface between the materials in the protective clothing. On this basis, this paper uses the finite difference method to convert the differential equation into a display format to solve the problem. The final result is that the experimenter’s persistence time with 15°C as the holding limit outdoors is 662.5s, and 10°C as the holding limit. It is 675.3s. Finally, the sensitivity analysis of the model is carried out by setting the temperature fluctuation.

1. Introduction

The temperature distribution boundary conditions of the model are continuous, and the heat conduction is only conducted on the thickness x, that is, the study is perpendicular to the skin surface. This process considers the convective heat generation from the surface of the protective clothing to the external environment, the ultra-low temperature raises the condition to ignore the heat radiation, the internal air flow is small, and the internal air and people are also convective heat transfer. Therefore, this paper simplifies the three-dimensional model of the protective clothing and the human body into a one-dimensional model, and conducts research from the thickness direction, and the model is established as shown in the following figure:
2. Parameter setting
The basic parameter settings of protective clothing set up in this paper are shown in Table 1.

| Physical name               | Parameter value |
|-----------------------------|-----------------|
| Ambient temperature         | -40°C           |
| Thickness of the middle layer | <45mm           |
| Material temperature range  | 14.7°C—25°C     |
| Body temperature            | >15°C           |

3. One-dimensional unsteady heat conduction model

3.1 Heat transfer control equation
According to the law of conservation of energy, the partial differential equation for unsteady heat transfer can be obtained:

$$\rho c_i \frac{\partial T}{\partial t} = k_i \frac{\partial^2 T}{\partial x^2} + F_i, i = (1, 2, 3)$$

Among them, the left term represents the change of thermodynamic energy during the heat transfer process, the first term on the right represents the internal heat conduction of the object, and the second term F represents the heat generation of the object, and i=1,2,3 respectively represent 3 regions. Among them, only region 2 will generate heat between 14.05°C and 26.018°C, and the remaining regions F are zero.

3.2 Boundary conditions and initial value conditions
For the low-temperature protective clothing heat transfer model, both ends meet the third type of boundary conditions, that is, the transferred heat is taken away by thermal convection, and Newton's cooling law is satisfied.


\[
\begin{align*}
-k_1 \frac{\partial T_i}{\partial x} \bigg|_{x=x_0} &= h_{i,2} \left( T_p - T_i \bigg|_{x=x_0} \right), \\
-k_2 \frac{\partial T_i}{\partial x} \bigg|_{x=x_1} &= h_{i,1} \left( T_i \bigg|_{x=x_1} - T_e \right), \\
T(x,0) &= T_p 
\end{align*}
\]

Among them, \( h_{i,1} \) represents the convective heat transfer coefficient between the outer insulation layer and the external environment, \( h_{i,2} \) represents the convective heat transfer coefficient between the inner fabric layer and the human skin, and \( T_p \) represents the surface of the human body temperature (unit: K), \( T_e \) represents the ambient temperature (unit: K).

### 3.3 Continuous condition

The heat transfer model has two contact surfaces. It is assumed that the contact of each material is good, there is no contact thermal resistance, and the boundary condition of the contact surface is continuous, namely:

\[
\begin{align*}
T(x^-,t) &= T(x^+,t), \\
\frac{\partial T}{\partial x}(x^-,t) &= \frac{\partial T}{\partial x}(x^+,t) 
\end{align*}
\]

### 3.4 Model summary In summary

The models are summarized as follows:

- **Governing equation**: \( \rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + F_i \)
- **Boundary conditions**: \( -k \frac{\partial T_i}{\partial t} \bigg|_{x=x_0} = h_{i,2} \left( T_p - T_i \bigg|_{x=x_0} \right) \)
    \( -k \frac{\partial T_i}{\partial t} \bigg|_{x=x_1} = h_{i,1} \left( T_i \bigg|_{x=x_1} - T_e \right) \)
- **Continuous condition**: \( T(x^-,t) = T(x^+,t) \)
- **Initial value conditions**: \( T(x,0) = T_p \)

### 4. Finite difference method

#### 4.1 Introduction

For the combination of partial differential equations, the conditions for determining the solution are more complicated, and it is often impossible to directly obtain an analytical solution. Therefore, we use numerical discretization to discretize the above partial differential equations. Usually, the finite difference method is used to solve partial differential equations.

The basic idea of the finite difference method is to replace the continuous definite solution area with a grid composed of a finite number of discrete points, and use the function of the continuous variable in the continuous definite solution area to be approximated by the discrete variable function defined on the grid. The derivative and integral in the solution condition are respectively the difference quotient and the integral and approximation, so the original differential equation and the definite solution condition
are approximately replaced by algebraic equations, which is the finite difference equations; the original problem can be obtained by solving the equations. For the approximate solution at the discrete point, interpolation is used to obtain the approximate solution of the definite solution problem in the entire area from the discrete solution.

The difference format includes two types, explicit and implicit. For the explicit format, the calculation amount is smaller, but the accuracy and stability are not as good as the implicit format; the implicit difference must solve the simultaneous equations, and the stability and accuracy are higher but the calculation amount is larger. Due to the large amount of data in this question, an explicit difference format is adopted.

Specific solution domain meshing method: Write the solution domain as $\Omega$, then $\Omega = \{(t,x) | 0 \leq x \leq L, 0 \leq t \leq T_0\}$

(1) On the spatial scale, divide the interval $l_i$ into $M_i$, $M_i = \frac{l_i}{\Delta l_i}$, where $\Delta l_i$ is the space step. $i=1,2,3$, representing 3 regions respectively.

(2) On the time scale, the time $T$ is divided into $n$ parts, $n = \frac{T}{\Delta t}$, $\Delta t$ is the time step.

(3) Use parallel lines to divide $\Omega$ into rectangular grids, as shown in the figure below. Discrete the above heat transfer model through the explicit difference format, and the discrete equations can be obtained as follows:

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![Figure 2. Rectangular grid](image)

Discrete the above heat transfer model through the explicit difference format, and the discrete equations can be obtained as follows:

$$
\begin{align*}
\Delta x_i \rho_c c_i \frac{T_i^n - T_i^{n+1}}{2} &= k_j \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{2} + F,
\frac{1}{2} \Delta x_i \rho_c c_1 \frac{T_i^{n+1} - T_i^n}{\Delta t} &= -h_{i2} (T_i - T_e) - k_1 \frac{T_i^n - T_2^n}{\Delta x_i},
\frac{1}{2} \Delta x_i \rho_c c_3 \frac{T_{end}^{n+1} - T_{end}^n}{\Delta t} &= -h_{i3} (T_{end}^n - T_p) - k_3 \frac{T_{end}^{n-1} - T_{end}^n}{\Delta x_3},
\frac{1}{2} \left( \Delta x_j \rho_j c_j + \Delta x_{j+1} \rho_{j+1} c_{j+1} \right) \frac{T_1^{n+1} - T_1^n}{\Delta t} &= k_j \frac{T_{j+1}^n - T_j^n}{\Delta x_j} + k_{j+1} \frac{T_{j+1}^n - T_j^n}{\Delta x_{j+1}}
\end{align*}
$$
4.2 Solution steps
After meshing the solution area, the temperature distribution at each time step and space step can be solved. In this paper, the time step is 0.3s, and the space step is 0.00015m. Assuming that the convective heat dissipation coefficient $h_c$ between the surface of the protective clothing and the external environment is a variable, and the convective heat dissipation coefficient $h_p$ between the human surface and the internal air is a known quantity, explore the time when the heat generation capacity of the protective clothing is fixed and the human body temperature reaches 15°C. The specific steps are as follows:

Step1: Perform spline interpolation according to the data in Annex 1 to explore the heating capacity of the middle layer.

Step2: Determine the values of $h_c$ and $h_p$.

Step3: The difference scheme is solved iteratively.

4.3 Solution result
Compiled with the help of matlab software, the finite difference equations are compiled using matlab language, and the relationship of temperature with time is obtained as shown in the figure below:

![Temperature-time diagram](image)

Figure 3. Temperature-time diagram

It can be seen from the above figure that if the body surface temperature is lowered to 15°C as the limit of persistence, the experimenter is still, and the time that the experimenter can persist in the absence of wind is 662.5s; if the limit is 10°C, the persistence is The time is 675.3s. In order to explore the internal situation of the low-temperature protective clothing, matlab language is used to compile and obtain the spatial temperature map of each part of the low-temperature protective clothing: draw the image of the temperature of the human body-the low-temperature protective clothing system with time and depth, as shown in the figure.

![Insolution Space temperature](image)  ![Inner space temperature diagram](image)

Figure 4. Insolution Space temperature  Figure 5. Inner space temperature diagram
As can be seen from the above figure, the temperature of the low-temperature protective clothing slowly decreases with the increase of time, first rapidly decreasing, and then slowly decreasing. It is consistent with the actual situation: the large temperature difference with the outside world in the early stage and the fast heat transfer rate; the relatively small temperature difference with the outside world in the later stage, and the slow heat transfer rate coincides with the model, which proves the validity of the model.

5. Conclusion
(1) This paper establishes a one-dimensional heat conduction equation for the protective clothing, and according to the law of conservation of energy, determines the relationship between the thickness of each layer of the protective clothing and the protective clothing’s cold-proof effect.

(2) For the established one-dimensional heat conduction model, this paper uses the finite difference method to solve it. Finally, an image of the relationship between the experimenter’s persistence time and ambient temperature while wearing protective clothing was obtained.

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