Unconventional Fermi surface in two-dimensional systems of Dirac fermions

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At the low energy regime, the decay rate of two-dimensional massless Dirac fermions due to interactions can be written as \( \text{Im} \Sigma(\omega) \propto |\omega|^2 \) at zero temperature. We find that the fermion system has: I) no sharp Fermi surface and no well-defined quasiparticle peak for \( 0 < x < \frac{1}{2} \); II) a sharp Fermi surface but no well-defined quasiparticle peak for \( \frac{1}{2} \leq x \leq 1 \); III) both sharp Fermi surface and well-defined quasiparticle peak for \( x > 1 \). In the presence of long-range gauge/Coulomb interaction or certain massless boson mode, the system exhibits unusual behavior belonging to class II.

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Landau’s Fermi liquid (FL) theory is the standard model of quantum many-body physics [1]. During the past five decades, this theory has been applied successfully to a wide range of condensed matter systems. In any normal FL, there are always sharp Fermi surface and well-defined quasiparticle peak at the Fermi energy. Generally, the fermion decay rate can be written as \( \text{Im} \Sigma(\omega) \propto |\omega|^2 \) for small energy \( \omega \) at zero temperature. The quasiparticles are well-defined only when the decay rate vanishes faster than \( \omega \) does upon approaching the Fermi surface. Hence, an interacting fermion system is a normal FL for \( x > 1 \) and a non-FL for \( x \leq 1 \). One powerful quantity to characterize the FL is the renormalization factor \( Z \), also called the quasiparticle residue, which is defined by the real part of the retarded fermion self-energy \( \text{Re} \Sigma(\omega) \) as

\[
Z = \left( 1 - \frac{\partial \text{Re} \Sigma(\omega)}{\partial \omega} \right)_{\omega=0}^{-1}.
\] (1)

It is easy to check that \( Z = 0 \) for \( 0 < x \leq 1 \) but \( 0 < Z < 1 \) for \( x > 1 \). Therefore, the residue \( Z \) vanishes in a normal FL and has a finite value in a non-FL. Physically, \( Z \) measures the discontinuity of fermion momentum occupation number \( n(k) \) at Fermi energy, which signals the presence of a sharp Fermi surface.

According to the conventional wisdom, the finiteness of residue \( Z \) is sufficient to guarantee the presence of a sharp Fermi surface and well-defined quasiparticle peak. However, the opposite reasoning may not be correct. In particular, the presence of sharp Fermi surface does not necessarily require that \( Z \) is finite. Indeed, there is an interesting possibility that a system may have a sharp Fermi surface even though \( Z = 0 \). Recently, Senthil [2] studied the unusual properties of the quantum critical point of a Mott insulator to metal phase transition. In his scenario, although \( Z = 0 \) at the critical point, there can be a sharp Fermi surface, dubbed as critical Fermi surface, that is characterized by a kink singularity, i.e., the discontinuity in the derivative of the fermion occupation number \( n(k) \) at the Fermi energy [2].

On the experimental side, it has long been known that there is Fermi surface (or Fermi arc) in the normal state of underdoped high temperature superconductors despite the absence of well-defined Landau quasiparticles [3-5].

In this paper, we propose that this possibility can be realized in some planar interacting systems composed of massless Dirac fermions. In recent years, the Dirac fermions with linear dispersion have been investigated extensively because they are the low-energy excitations of many strongly correlated systems, including \( d \)-wave high temperature superconductor [6], quantum Hall system [7], graphene [8], and iron-based superconductor [9]. In different physical systems, these fermions may experience gauge, Coulomb, or other kinds of interactions. We show that the Dirac fermion system has a sharply defined Fermi surface but no quasiparticle peak when the exponent \( x \) of decay rate satisfies \( \frac{1}{2} \leq x \leq 1 \). In contrast to the scenario of Senthil, the Fermi surface in our case is defined by the divergence of the derivative of \( n(k) \) at the Fermi energy, rather than a kink singularity. When the Dirac fermions couple to massless boson modes, such as gauge field, Coulomb potential, or certain order parameter fluctuation, they exhibit such unconventional behavior. For \( 0 < x < \frac{1}{2} \), the fermion system has no sharp Fermi surface and no quasiparticle peak, which loses any similarity to a free fermion gas. For \( x > 1 \), there are both Fermi surface and quasiparticle peak, so the system is just a normal FL.

We begin with the following free action

\[
\mathcal{L}_0 = \sum_{i=1}^{N} \Psi_i^{\dagger} \left( \partial_x - iv_F \sigma \cdot \partial \right) \Psi_i, \tag{2}
\]

where spinor field \( \Psi \) describes massless Dirac fermions and \( v_F \) is the fermion velocity. The coupling of Dirac fermions with various singular boson modes can result in unusual behaviors.

We first consider the interaction of Dirac fermions with an abelian gauge field, which corresponds to three-dimensional quantum electrodynamics (QED\(_3\)). After proper modifications, this model can be used to study the low-energy properties of \( d \)-wave high temperature superconductor [4, 10] and spin liquid state [11]. Recently, we studied the decay rate of massless Dirac fermions due to gauge interaction. At zero chemical potential \( \mu = 0 \), the fermion decay rate is always divergent within perturbation expansion method. We calculated the decay rate by the self-consistent Eliashberg equation approach and
found \[12\] that it is of the form \(\text{Im} \Sigma(\omega) \propto |\omega|^{1/2}\) at zero temperature. At finite \(\mu\), the longitudinal component of gauge field becomes massive due to static screening, but the transverse component remains massless. In this case, the decay rate is free of divergence and depends on energy as \(\text{Im} \Sigma(\omega) \propto \mu^{-1/3}|\omega|^{2/3}\) at zero temperature \[12\]. Obviously, the chemical potential only changes the coefficient of fermion decay rate. When applied to cuprate superconductor, the gauge field may couple to an additional scalar field \[14\]. In the superconducting ground state, the gauge field acquires a finite mass via the Anderson-Higgs mechanism. It is easy to obtain that, \(\text{Im} \Sigma(\omega) \propto |\omega|^3\), which has the same energy dependence as the decay rate yielded by contact four-fermion interaction \[15\].

In the context of graphene, the massless Dirac fermions experience the Coulomb interaction, which is unscreened due to the vanishing density of states at Fermi energy \[7\]. From previously analysis \[12, 16\], we know that the decay rate caused by Coulomb interaction is \(\text{Im} \Sigma(\omega) \propto |\omega|\), which is clearly marginal FL behavior \[17\].

In the vicinity of continuous quantum phase transition, the massless Dirac fermions interact with the strong fluctuation of order parameter \[18\]. Such interaction can be described by a Yukawa coupling between a spinor field and a scalar field. In particular, at the critical point between two superconducting phases in \(d\)-wave cuprate superconductor \[19\], the fermion decay rate due to massless order parameter fluctuation is \(\text{Im} \Sigma(\omega) \propto |\omega|\).

In general, after including the self-energy correction due to various interactions, the full retarded Green function of Dirac fermion is

\[
G(\omega, \mathbf{k}) = \frac{1}{\omega - \xi_k - \text{Re} \Sigma(\omega) - i \text{Im} \Sigma(\omega)},
\]

where \(\xi_k = \varepsilon_k - \mu\) with \(\mu\) being the chemical potential and \(\varepsilon_k = v_F |\mathbf{k}|\) energy of the Dirac fermion. To simplify the problem, here we assume that the self-energy depends only on energy \(\omega\). The spectral function, defined as \(A(\omega, \mathbf{k}) = -\frac{1}{\pi} \text{Im} G(\omega, \mathbf{k})\), has the form

\[
A(\omega, \mathbf{k}) = -\frac{1}{\pi} \frac{\text{Im} \Sigma(\omega)}{|\omega - \xi_k - \text{Re} \Sigma(\omega)|^2 + [\text{Im} \Sigma(\omega)]^2}.
\]

The fermion momentum occupation number is given by

\[
n(\mathbf{k}) = \int_{-\infty}^{0} d\omega A(\omega, \mathbf{k}).
\]

In order to make a general analysis, we write the zero-temperature decay rate of Dirac fermions as

\[
\text{Im} \Sigma(\omega) = C|\omega|^x,
\]

where \(x\) is a tuning parameter and \(C\) a negative constant. The real part of self-energy can be obtained from the Kramers-Kronig relation

\[
\text{Re} \Sigma(\omega) = \frac{1}{\pi} \int_{-\infty}^{+\infty} d\omega' \frac{|\omega'|^{x-1}}{\omega' - \omega}.
\]

We first consider the region \(0 < x < 1\). In such region, the real self-energy function is found to be

\[
\text{Re} \Sigma(\omega) = C \text{sgn}(\omega)|\omega|^x I(x),
\]

where \(I(x)\) is

\[
I(x) = \frac{1}{\pi} \lim_{\delta \to 0} \left[ \int_{0}^{\infty} dy \frac{1}{y^{x+1}} + \int_{0}^{1-\delta} dy \frac{1}{y^{x-1}} \right],
\]

When \(0 < x < 1\), the function \(I(x)\) has a finite magnitude and hence the real part of retarded self-energy \(\text{Re} \Sigma(\omega)\) has the same energy dependence as \(\text{Im} \Sigma(\omega)\). In particular, we have \(I(1/2) = 1\) and \(I(2/3) = \sqrt{3}\).

When \(x \geq 1\), there will be divergence when Eq.\(7\) is used to calculate \(\text{Re} \Sigma(\omega)\). To get a finite result, we should introduce a cutoff energy \(\omega_c\) and write

\[
\text{Re} \Sigma(\omega) = \frac{1}{\pi} \int_{-\omega_c}^{\omega_c} d\omega' \frac{|\omega'|^{x-1}}{\omega'^2 - \omega}.
\]

For \(x = 1\), we have

\[
\text{Re} \Sigma(\omega) = C \frac{2}{\pi} \frac{|\omega|^x}{\omega_c^{x-1}},
\]

which in the low-energy regime reduces to

\[
\text{Re} \Sigma(\omega) \approx C \frac{2}{\pi} \frac{|\omega|}{x - 1} \omega.
\]

This corresponds to the marginal FL behavior. For \(x > 1\), the real part of self-energy at low energy is

\[
\text{Re} \Sigma(\omega) \approx C \frac{2}{\pi} \frac{|\omega|^{x-1}}{x} \omega.
\]

![FIG. 1: Spectral function.](image)
that the well-defined quasiparticle peak exists only for \( x > 1 \) where \( Z \neq 0 \). The residue \( Z \) also represents the discontinuity of momentum occupation number \( n(k) \) at \( |k| = k_F \). When \( 0 < x \leq 1 \), there is no discontinuity in \( n(k) \); but when \( x > 1 \), there is a discontinuity in \( n(k) \). In conventional many-body theory, \( Z \) can uniquely differentiate a non-FL from the normal FL. In strongly correlated systems, however, \( Z \) itself can no longer fully characterize the non-FL. In order to see how this happens, we will calculate \( n(k) \) and its derivative for all values of \( x \).

For \( 0 < x < 1 \), the momentum occupation number is

\[
n(k) \propto \int_{-\infty}^{0} d\omega \frac{|\omega|^2}{|\omega - \xi_k + CI(x)|^2 + |C|\omega|^2}.
\]

At Fermi energy, its derivative with respect to \(|k|\) is

\[
n'(k)|_{k_F} \propto \int_{-\infty}^{0} d\omega \frac{|\omega|^2}{|\omega + CI(x)|^2 + |C|\omega|^2}.
\]

As \( \omega \to 0 \), the integrand approaches \(-\frac{1}{|\omega|^2}\). For \( 0 < x < \frac{1}{2} \), it is easy to obtain

\[
n'(k)|_{k_F-0} = \text{finite}, \quad n'(k)|_{k_F+0} = \text{finite}. \quad (13)
\]

For \( \frac{1}{2} \leq x < 1 \), \( n'(k) \) diverges as:

\[
n'(k)|_{k_F-0} \to -\infty, \quad n'(k)|_{k_F+0} \to -\infty, \quad (14)
\]

which indicates that the momentum occupation number drops dramatically at \( k_F \).

Similarly, for \( x = 1 \), we can obtain the following derivative of occupation number

\[
n'(k)|_{k_F} \propto \int_{-\infty}^{0} d\omega \frac{|\omega|^2}{|\omega - C\pi\omega \ln \left( \frac{\omega}{\xi_k} \right)|^2 + |C|\omega|^2}.
\]

As \( \omega \to 0 \), the integrand approaches \(-\frac{1}{|\omega|^2|\ln(\frac{\omega}{\xi_k})|^2}\), thus

\[
n'(k)|_{k_F-0} \to -\infty, \quad n'(k)|_{k_F+0} \to -\infty, \quad (15)
\]

which is similar to the case of \( \frac{1}{2} \leq x < 1 \).

For \( x > 1 \), we have

\[
n'(k)|_{k_F} \propto \int_{-\infty}^{0} d\omega \frac{|\omega|^2}{|\omega - C\pi\omega \ln \left( \frac{\omega}{\xi_k} \right)|^2 + |C|\omega|^2}.
\]

As \( \omega \to 0 \), the integrand approaches \(-\frac{1}{|\omega|^2-\pi^2}\). For \( 1 < x \leq 2 \), the residue \( Z \neq 0 \) and the derivative of \( n(k) \) is

\[
n'(k)|_{k_F-0} \to -\infty, \quad n'(k)|_{k_F+0} \to -\infty. \quad (16)
\]

For \( x > 2 \), although \( Z \neq 0 \), we know that

\[
n'(k)|_{k_F-0} = \text{finite}, \quad n'(k)|_{k_F+0} = \text{finite}. \quad (17)
\]

The dependence of momentum occupation number \( n(k) \) on \( x \) is shown in Fig.2. Apparently, as the parameter \( x \) grows, the interacting system of massless Dirac fermions first develops a sharp Fermi surface at \( x = \frac{1}{2} \) and then develops a finite quasiparticle residue \( Z \) once \( x \) exceeds unity. There is a fundamental difference between the \( n(k) \) functions for \( 0 < x < \frac{1}{2} \) and for \( \frac{1}{2} \leq x \leq 1 \). In the former case, the residue \( Z = 0 \), so there are no well-defined quasiparticles. Moreover, the derivative of \( n(k) \) is continuous at the Fermi energy \( k_F \), so there is also no Fermi surface. In the latter case, the residue \( Z = 0 \) and thus there is no well-defined quasiparticle peak. However, the derivative of \( n(k) \) diverges at \( k_F \), therefore \( n(k) \) drops suddenly as \( |k| \) increases across \( k_F \), which can be identified as the presence of a sharp Fermi surface. To see more details of the evolution of \( n(k) \) with \( x \), we also show the derivative of \( n(k) \) in Fig.3.

In summary, we can divide all the interacting Dirac fermion systems into three classes: I) for \( 0 < x < \frac{1}{2} \), there are no sharp Fermi surface and no well-defined quasiparticle peak; II) for \( \frac{1}{2} \leq x \leq 1 \), there is sharp Fermi surface but no well-defined quasiparticle peak; III) for \( x > 1 \), there are both sharp Fermi surface and well-defined quasiparticle peak.

In QED3, the fermion decay rate has \( x = \frac{1}{2} \) at zero \( \mu \) and \( x = \frac{2}{3} \) at finite \( \mu \). These states fall into the class...
II. To see the Fermi surface evolution with growing $\mu$, we show $n(k)$ in Fig. 4. We can see that $n(k)$ does not evolve continuously from $\mu = 0$ to the case of finite $\mu$. On the contrary, as $\mu$ jumps from zero to a small value, the curve of $n(k)$ changes dramatically. However, the Fermi surface remains sharply defined for arbitrary small value of $\mu$, because $\mu$ does not alter the energy dependence of decay rate $\Gamma$. The marginal FL ($x = 1$) caused by the long-range Coulomb interaction or certain massless order parameter fluctuation also belongs to class II. In the presence of short-ranged gauge/Coulomb interaction, the decay rate has exponent $x > 1$ and thus the behavior falls into class III. Unfortunately, it is currently unclear what kind of interactions can produce behaviors belonging to class I, which deserves further research.

We emphasize that the unconventional Fermi surface studied in this paper is different from that of Senthil. As shown in Fig. 5, the critical Fermi surface proposed by Senthil is defined by the discontinuity in the derivative of $n(k)$. In our problem, however, there is no such kink singularity and the sharp Fermi surface is characterized by the divergence of the derivative of $n(k)$.

Finally, although all the above discussions were made in the case of massless Dirac fermions, the unconventional Fermi surface of class II should also be realized in a number of non-relativistic strongly correlated systems, such as non-relativistic fermion-gauge system with large Fermi surface and quantum critical metals near various symmetry-breaking instabilities, provided that the exponent in fermion decay rate satisfies $\frac{1}{2} \leq x \leq 1$.

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