Vibration characteristics and stability of a moving membrane with variable speed subjected to follower force

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Abstract. In this paper, the vibration characteristics and stability of a moving membrane with variable speed subjected to follower force are studied. The vibration differential equation of the moving membrane is deduced by the D’Alembert principle. The intermediate variables of the differential equation are discretized by the Differential Quadrature Method, and the state equation with periodic coefficients is obtained. The state equation is solved by the implicit Runge-Kutta method. According to the Floquet theory, the dynamic stability region and instability region of the membrane are obtained, and the influence of the tension ratio, aspect ratio, average velocity and the follower force on the unstable region of the moving membrane are analyzed. This result provides theoretical guidance and basis for the design, manufacture and stability of the printing press.

1. Introduction
During the printing process, the air significantly influences the vibration characteristics related to the overprinting accuracy of the printing membrane. The membrane is moving at a variable speed in engineering projects. In recent years, many scholars have studied the stability of the moving system subjected to the follower force. Hasanshahi and Azadi [1] studied the flutter vibration of beam subjected to the follower force. Robinson [2] studied the dynamic stability of viscoelastic rectangular plates subjected to the uniform tangential force and triangular tangential force. Chen [3] analyzed the steady-state periodic transverse responses and stabilities of axially accelerating viscoelastic strings.
Lewandowski [4] investigated the nonlinear vibration of beams, harmonically excited by harmonic forces. Alidoost [5] proposed an analytical solution for instability of a composite beam with a single delamination subjected to concentrated follower force. Ma [6] proved that the follower force effect decreased the natural frequencies in lower modes, and increased them in higher modes. Zhou al. [7] analyzed the vibration stability of a uniform orthogonal tangential rectangular plate, a functionally graded material plate and a rectangular thin plate with intermediate support and thermo elastic coupling rectangle boards. Azadi [8] analyzed and controlled the flutter vibration of a thermo elastic Functionally Graded Material (FGM) beam subjected to follower force.

2. Dynamic model and establishment of vibration equation

Figure 1 shows the mechanical model of the moving membrane subjected to the tangential uniform distribution. The follower force is \( q_0 \), \( v \) is the moving speed of the membrane in \( x \) direction, and the transverse vibration displacement of membrane is \( z \) direction. \( \bar{w}(x,y,t) \) is transverse vibration displacement, \( T_x \) and \( T_y \) represent the tension on the boundary, \( a \) and \( b \) are the length and width of the membrane, respectively. The membrane density is \( \rho \).

![Figure 1. The mechanical model for the axially moving membrane subjected to follower force](image)

Assuming that the membrane is subjected to an external force \( F(x,y,t) \) along the \( z \) direction, the dynamic equation of the moving membrane is obtained according to the D’Alembert principle.

\[
\rho \left\{ \frac{\partial^2 \bar{w}}{\partial t^2} + 2v \frac{\partial \bar{w}}{\partial x} + \frac{\partial^2 \bar{w}}{\partial y^2} + 2 \frac{\partial^2 \bar{w}}{\partial x^2} + \frac{\partial \bar{w}}{\partial x} \right\} - \frac{T_x}{\partial x^2} - \frac{T_y}{\partial y^2} + q \left( a - x \right) \frac{\partial \bar{w}}{\partial x} = 0
\]

Assuming that the axial velocity of the moving membrane [9] has a small harmonic fluctuation with respect to the average velocity

\[
v_a = v_0 + \psi \sin \Omega t \quad (\psi, \Omega > 0)
\]

Where \( v_0 \) is the axial average velocity, \( \psi \) and \( \Omega \) are the amplitude and frequency of the axial velocity fluctuation respectively. Introduce the dimensionless quantities

\[
\zeta = \frac{x}{a}, \eta = \frac{y}{b}, \omega = \frac{\bar{w}}{a}, \tau = t \left( \frac{T_x}{\rho a^2} \right)^{\frac{1}{2}}, c = v_0 \left( \frac{\rho}{T_x} \right)^{\frac{1}{2}}, c_i = v_1 \left( \frac{\rho}{T_x} \right)^{\frac{1}{2}}
\]

\[
\kappa = \frac{T_y}{T_x}, \gamma = \frac{a}{b}, \Omega = \bar{\Omega} a \left( \frac{\rho}{T_x} \right)^{\frac{1}{2}}, Q = \frac{q_0 a}{T_x}
\]

The dimensionless form of the differential equation is
\[ \frac{\partial^2 w}{\partial \tau^2} + 2(c + c_1 \sin \Omega \tau) \frac{\partial^2 w}{\partial \zeta \partial \tau} + \left(c^2 + c_1^2 \sin^2 \Omega \tau + 2cc_1 \sin \Omega \tau\right) \frac{\partial^2 w}{\partial \zeta^2} + c_1 \Omega \cos \Omega \tau \frac{\partial w}{\partial \zeta} - \frac{\partial^2 w}{\partial \zeta^2} - \kappa r^2 \frac{\partial^2 w}{\partial \eta^2} + Q(1-\zeta) \frac{\partial^2 w}{\partial \zeta^2} = 0 \] (3)

The dimensionless form of the boundary condition is
\[ \begin{cases} w(\zeta,0,\tau) = 0 & w(0,\eta,\tau) = 0 \\ w(\zeta,1,\tau) = 0 & w(1,\eta,\tau) = 0 \end{cases} \] (4)

3. Application of Differential Quadrature Method to establish complex characteristic equation

The differential quadrature form [10] of the vibration equation of the moving membrane with variable speed is
\[ \ddot{\mathbf{w}} + 2(c + c_1 \sin \Omega \tau) \sum_{k=1}^{N} A_k^{[1]} \mathbf{w} + \left(c^2 + 2cc_1 \sin \Omega \tau + \frac{1}{2} c_1^2 - \frac{1}{2} c_1^2 \cos 2\Omega \tau - 1\right) \sum_{k=1}^{N} A_k^{[2]} \mathbf{w} \\ + c_1 \Omega \cos \Omega \tau \sum_{k=1}^{N} A_k^{[1]} \mathbf{w} - \kappa r^2 \sum_{m=1}^{N} B_m^{[2]} \mathbf{w} + Q(1-\zeta) \sum_{k=1}^{N} A_k^{[1]} \mathbf{w} = 0 \] (5)

The weight coefficient \( A_k^{[1]}, A_k^{[2]}, B_m^{[2]} \) are obtained using the formula.

The differential quadrature form of the boundary condition is
\[ \begin{cases} w_{ij} = w_{Ny} = 0 & (j = 1, 2, \ldots, N) \\ w_{li} = w_{Ny} = 0 & (i = 1, 2, \ldots, N) \end{cases} \] (6)

Combining equation (5) with boundary condition (6) into a matrix form is
\[ \mathbf{R} \dot{\mathbf{W}} + \mathbf{C} \mathbf{W} + \mathbf{K} \mathbf{W} = 0 \] (7)

In order to address the problem, the equation (7) is transformed into a first-order differential form, and the periodic coefficients of the motion equation are derived. Equation (7) can be written as
\[ \dot{\mathbf{W}} = -\mathbf{R}^{-1} \mathbf{C} \mathbf{W} - \mathbf{R}^{-1} \mathbf{K} \mathbf{W} \] (8)

The formula of \( \mathbf{y} \) can be written as \( \mathbf{y} = [\mathbf{W} \mathbf{W}']^T \), then the equation (8) can be expressed as
\[ \dot{\mathbf{y}} = \mathbf{G}(\tau) \mathbf{y} \] (9)

Where \( \mathbf{G}(\tau) = \begin{bmatrix} 0 & \mathbf{I} \\ \mathbf{-R}^{-1} \mathbf{K} & -\mathbf{R}^{-1} \mathbf{C} \end{bmatrix} \), \( \mathbf{I}_{N \times N} \) is N-order identity matrix, \( \mathbf{G}_{2N \times 2N}(\tau + T) = \mathbf{G}_{2N \times 2N}(\tau) \), \( T \) is the period of the sinusoidal velocity and the periodic function \( \mathbf{G}(\tau) \).

4. Solution of differential equations and Determination of system stable region

Second-order four-level implicit R-K method is applied to solve the rigid equation (9).

Based on the previous derivation, matrix \( \mathbf{A} \) and \( \mathbf{B} \) can be obtained
The number of nodes is taken as $N_x = N_y = 12$, and the appropriate step size is $h$. $\gamma$ can be obtained by the program in MATLAB. According to the Floquet theory [11], the dynamic stability region and instability region of the membrane are determined. When $|\lambda| < 1$, the system is stable. When $|\lambda| > 1$, the system is unstable. When $|\lambda| = 1$, the system is in a critical state. $\lambda$ is the eigenvalue of the dynamic stability equation.

5. Numerical calculation and analysis

As can be seen from the figure 2, when the tension ratio $\kappa = 0.5$, the average velocity $c_0 = 0.5$, and the follower force $Q = 1$, the aspect ratio is $r = 0.5$ and $r = 1$, the boundary of the stable region of the system moves to the upper right of the plane $\Omega - c_1$ as the aspect ratio increases. Therefore, the stable region increases.

![Figure 2](image.png)

**Figure 2.** The effect of the aspect ratio on the stable region ($\kappa = 0.5$, $c_0 = 0.5$, $Q = 1$)

As is shown in figure 3, when the aspect ratio $r = 1$, the tension ratio $\kappa = 0.5$, and the follower force $Q = 1$, the average velocity is $c_0 = 0.5$ and $c_0 = 0.9$, the stable region gradually decreases as the average velocity increases.

![Figure 3](image.png)

**Figure 3.** The effect of the average velocity on the stable region ($r = 1$, $\kappa = 0.5$, $Q = 1$)
As is shown in figure 4, when the aspect ratio $r = 1$, the average velocity $c_0 = 0.5$, and the follower force $Q = 1$, the tension ratio is $\kappa = 0.8$ and $\kappa = 1$, the stable region decreases as the tension ratio increases.

![Figure 4. The effect of the tension ratio on the stable region ($r = 1, c_0 = 0.5, Q = 1$)](image1)

As is shown in figure 5, when the aspect ratio $r = 1$, the tension ratio $\kappa = 0.5$, and the average velocity $c_0 = 0.5$, the follower force is $Q = 0.2$ and $Q = 0.5$, the stable region gradually decreases as the follower force increases.

![Figure 5. The effect of the follower force on the stable region ($r = 1, \kappa = 0.5, c_0 = 0.5$)](image2)

6. Conclusions

The vibration characteristics and stability of a moving membrane with variable speed subjected to follower force are studied. The conclusions are as follows:

1) For moving membrane with variable speed, the stable region becomes larger when aspect ratio increases.

2) For moving membrane with variable speed, the stable region becomes larger when the tension ratio, the average speed and the follower force decreases.
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