Government regulation of emergency supplies under the epidemic crisis

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ABSTRACT
This paper constructs a multi-oligopoly model of emergency supplies and analyses the market equilibrium results under normal conditions and epidemic conditions. The impacts of the degree of change in market demand, externalities, the material cost of emergency supplies and government regulation on the equilibrium results, especially on the prices of emergency supplies, are discussed. The results show that an increase in material cost will lead to low output and social welfare and a high price, under either normal conditions or epidemic conditions. Moreover, under epidemic conditions, the degree of change in market demand, externalities, material cost and the presence and mode of government regulation all have multiple and complex influences on the equilibrium results. Under epidemic conditions, both government output and price regulation can increase the supply of emergency supplies. In addition, when market demand changes drastically, consumer surplus and social welfare can be enhanced by the implementation of regulations. Particularly, price regulation is more effective when there is a high material cost.

ARTICLE HISTORY
Received 18 January 2021
Accepted 13 September 2021

KEYWORDS
Epidemic crisis; emergency supplies; government regulation; oligopoly

JEL CLASSIFICATION:
L11; L13; L53

1. Introduction
The public health crisis has always been a hot spot of global concern (Metcalf & Lessler, 2017). At present, COVID-19 is raging around the world. The demand for some important emergency supplies, such as masks and protective clothing, has surged, and prices have soared. The issue of how to reasonably regulate the production or prices of emergency supplies and ensure the supply of emergency supplies has become extremely prevalent regarding the prevention and control of the epidemic. Taking China as an example, the prices of emergency supplies, such as masks and their raw materials, such as melt-blown, nonwoven fabrics, saw a surge at the beginning of the epidemic. However, as the government has implemented a series of
regulatory policies and the market supply continues to increase, the prices of emergency supplies have come down, which indicates that this a successful example of regulation. Regarding emergency supplies during the epidemic, what impact will government regulations have on the supply of emergency supplies, consumer surplus, and social welfare as a whole? Should the government regulate and how should it do so? To answer these questions, this article constructs an oligopoly model composed of multiple emergency material manufacturers, discusses the equilibrium results under the implementation of output regulation and price regulation, derives the rationality and boundary conditions of government regulation, and analyses the impacts of the degree of change in market demand and material costs, thereby providing a reference for fighting the pandemic and strengthening the management of emergency supplies.

Compared with the existing research, the contributions of this paper are mainly as follows. First, this paper combines the characteristics of emergency supplies in the COVID-19 pandemic, especially factors such as positive externalities and the material cost of emergency supplies, it constructs a multi-oligopoly model to investigate the role of government regulations, and it expands the range of application of the oligopoly model. Second, this paper compares the equilibrium results under no government regulation, output regulation, and price regulation and makes a marginal contribution as to whether the government should regulate and how it should do so. Third, unlike the traditional social welfare design, which is the sum of consumer surplus and producer surplus, externalities are incorporated into the social welfare function, and the dynamic changes in the characteristics of emergency supplies during an epidemic are examined.

The remainder of this paper is organized as follows. Section 2 reviews the literature. Section 3 sets up a multi-oligopoly model of emergency supplies. Section 4 examines the equilibrium results of the four situations (Model NN, Model EN, Model EG, and Model EP) and analyses the impact of the material cost, the degree of change in market demand, and the number of enterprises, then compares the four situations. The main conclusions, according to the previous analysis and discussion, are obtained in Section 5.

### 2. Literature review

Regarding the prices of emergency supplies in a public crisis, there is controversy within the academic community. Some scholars believe that the government should not regulate the prices of emergency supplies in case of disaster. Chapman et al. (2014) analyse the distribution and supply of natural disasters and find that it is necessary to maximize the demand to build a response system. Therefore, due to changes in costs and markets, price increases are reasonable, and government control is ineffective, which can cause an even ‘greater disaster’. Etienne & David (2020) believe that government price regulation is unnecessary because market stability can be maintained by allocating consumer spending between high-price and low-price stores. Tomas et al. (2010) believe that real options between companies in the supply chain can be constructed to withstand the uncertain risks caused by disasters, thereby providing a basis for corporate price decisions without government intervention.
Cavallaro et al. (2014) show in their study that the recovery of production and supply after an earthquake is relatively slow, but the prices are relatively stable, especially in emergency supplies. They also believe that the government does not need to intervene.

However, some scholars hold different views. Zhang and Li (2013) point out that the government should provide subsidies to enterprises to ensure the supply of emergency supplies by analysing the relationship between the optimal reserve of emergency supplies and government subsidy policies. Xiao et al. (2017) find that government regulation can reduce product prices and increase consumer surplus considering the situation of remanufacturing. Zhao et al. (2018) compare the impacts of government regulation and market incentives on coping with food safety and show that they have a synergistic effect (Etienne et al., 2017). Nie et al. (2020) consider that major infectious diseases harm society as a whole, and timely control is very important. The higher the cost of treatment is, the more important government intervention is. Thus, should the government regulate and how it should do so are the main content of the discussion.

In addition, many scholars believe that emergency supply management-related activities should be led by the government to ensure effectiveness and timeliness (Chen et al., 2019c; Guan et al., 2019; Liu et al., 2017; Nonell & Borrell, 2001; Zhang & Chen 2016) find that strengthening government emergency power is important to reducing the loss caused by disasters. Gersovitz and Hammer (2004) propose an objective function for evaluating infection costs and intervention measures and explore the scope of government subsidies/tax interventions to maximize private welfare. Culpepper and Walter (2008) find that a government can avoid price gouging by passing legislation, allocating emergency supplies more efficiently. Wang et al. (2019) examine the effects of pricing decisions and government regulation and suggest that the government should regulate. Zhang et al. (2017) conduct an analysis of emergency rescue material distribution strategy under the joint management of government and suppliers and believe that the government can restrain the behaviour of enterprises so that the distribution of emergency supplies can be steadily improved and both parties can achieve long-term mutual benefit and win-win results (Wang et al., 2014).

In the existing research on emergency supplies, the study of changes in market demand and cost is an important research direction. Serel (2017) makes price decisions, formulates production policies based on supply capacity affected by uncertainty and assumes that demand cannot be met at the same time. Liu et al. (2021) find that the optimal price is related to the uncertainty of demand. Bozorgi-Amiri et al. (2013) propose a relief material logistics system considering the uncertainty of demand and cost. Furthermore, Yang et al. (2021) study the impact of uncertainty, as well as the incompleteness of information. Therefore, the change in market demand and cost are incorporated into our model to better reflect reality. In addition, there is much literature on other aspects of emergency supplies, such as site selection, logistics, distribution, and dispatch (Caunhye et al., 2015; Liu et al., 2013, 2020; Luscombe & Kozan, 2016; Pacheco & Batta, 2016; Wang et al., 2020). These studies provide support and further enrich the direction of this article. By using the oligopoly model, we can
investigate the mutual game between multiple market players in a specific market and obtain the optimal decision under the subgame perfect Nash equilibrium, which is an effective method by which to examine the decision-making behaviour of market players (Chen et al., 2020b). This article uses an oligopoly model to explain the impacts of government regulation on emergency supplies in an epidemic based on existing research.

3. The model

A multi-oligopoly model of the supply of emergency supplies is constructed to analyse government regulation of emergency supplies under epidemic conditions. Specific assumptions are as follows:

Assumption 1. Multiple enterprises are engaged in the production of certain emergency supplies (such as masks). The number of enterprises is \( n \) and \( n > 3 \). The products are homogenous, and enterprises compete with each other for output. Under normal conditions, the demand function of emergency supplies is \( p = a - \sum_{i=1}^{n} q_i \) (Chen et al., 2019b). When an epidemic occurs, the demand for emergency supplies will increase significantly, and the price will rise accordingly. At this point, the demand function becomes \( p = \beta(a - \sum_{i=1}^{n} q_i), \beta > 1 \), which represents the degree of change in market demand caused by the epidemic. The more drastically the demand changes, the larger the value of \( \beta \) is. In addition, the production of emergency supplies is limited by the availability of materials because with an increase in production, there will be a shortage of raw materials and an increase in their price, such as raw materials for masks. Therefore, we assume that the cost function is \( c_i = \frac{d}{2}(q_i)^2 \), \( d > 0 \), which reflects the cost of materials. The higher the value of \( d \) is, the more expensive the materials are. The profit functions are \( \pi_i = pq_i - c_i \). Under normal conditions, the social welfare function is \( sw = \sum_{i=1}^{n} \pi_i + \delta cs, \delta > 1 \) (Chen et al., 2020a), including the positive externalities of emergency supplies. When the epidemic occurs, the externalities of emergency supplies will increase. For example, the role of masks under epidemic conditions is much greater than that under normal conditions. Therefore, we assume that the social welfare function becomes \( sw = \sum_{i=1}^{n} \pi_i + \delta cs, \delta < r \) and that \( cs \) represents consumer surplus, which is expressed as \( cs = \frac{(\sum_{i=1}^{n} q_i)^2}{2} \) (Chen et al., 2019a). Both \( \delta \) and \( r \) represent the ratio of concern felt by consumers in the different conditions.

Assumption 2. Under normal conditions, the output decision is denoted by Model NN, and the equilibrium results are denoted by superscript NN. Furthermore, the output decision of emergency supplies under epidemic conditions can be divided into three cases. The first is that the government does not control the output of emergency supplies, and the output of emergency supplies is decided by enterprises themselves to maximise profits, which is denoted by Model EN, with the equilibrium results denoted by superscript EN. The second is that the government regulates the production of emergency supplies, and the output decision pursues the maximization of social welfare, which is expressed by Model EG, and the equilibrium results are denoted by superscript EG. The third is that the government regulates the price of
emergency supplies to ensure that the price is consistent with normal conditions, which is represented by Model EP, and the equilibrium results are denoted by superscript EP. These hypotheses are based on the fact that in a crisis, the government will introduce effective policies to ensure supply and severe crackdowns on hoarding and arbitrary price increases.

Based on the above assumptions, we first discuss the production and pricing of emergency supplies under normal conditions, and then we study three cases under epidemic conditions and make comparisons.

4. Model analysis

4.1. Model NN

Under normal condition, according to profit maximization, \( \frac{\partial \pi_i^{NN}}{\partial q_i^{NN}} = 0 \) should be satisfied as:

\[
\frac{\partial \pi_i^{NN}}{\partial q_i^{NN}} = a - dq_i^{NN} - 2q_i^{NN} - \sum_{j \neq i}^{n} q_j^{NN} = 0 \tag{1}
\]

Based on Equation 1, outputs can be derived as:

\[
q_i^{NN} = \frac{a}{d + n + 1} \tag{2}
\]

Lemma 1. Under normal conditions, the equilibrium results are

\[
p^{NN} = \frac{a(d + 1)}{d + n + 1}, \quad \pi_i^{NN} = \frac{a^2(d + 2)}{2(d + n + 1)^2}, \quad cs^{NN} = \frac{n^2a^2}{2(n + d + 1)^2}, \quad sw^{NN} = \frac{na^2(n\delta + d + 2)}{2(n + d + 1)^2}.
\]

Proposition 1 can be obtained by analysing the effects of \( d \) and \( n \).

Proposition 1:

1. The effects of \( d \) are:
   \[
   \frac{\partial \pi_i^{NN}}{\partial d}|_{d = 0} < 0; \quad \frac{\partial \pi_i^{NN}}{\partial d}|_{d = n - 3} > 0; \quad \text{if} \quad 0 < d \leq n - 3, \quad \text{then} \quad \frac{\partial \pi_i^{NN}}{\partial d} \geq 0; \quad \text{if} \quad d > n - 3,
   \]
   then \( \frac{\partial \pi_i^{NN}}{\partial d} < 0; \quad \frac{\partial \pi_i^{NN}}{\partial d} < 0; \quad \frac{\partial \pi_i^{NN}}{\partial d} < 0. \)

2. The effects of \( n \) are:
   \[
   \frac{\partial q_i^{NN}}{\partial n} < 0; \quad \frac{\partial p^{NN}}{\partial n} < 0; \quad \frac{\partial \pi_i^{NN}}{\partial n} < 0; \quad \frac{\partial cs^{NN}}{\partial n} > 0; \quad \frac{\partial sw^{NN}}{\partial n} > 0.
   \]

Proof. See Appendix A.
Proposition 1 shows that under normal conditions, a high material price reduces production because enterprises need to pay more costs and the price of emergency supplies increases. As output decreases and price rises, consumer surplus and social welfare decrease. The impact of decreased output and increased price of emergency supplies on enterprises’ profits is uncertain due to the impact of production cost. If \( d > n - 3 \), profits will decrease when the cost of material increases. Otherwise, profits will increase. Under normal conditions, excessively high material costs are harmful to enterprises, consumers, and society as a whole. The government should encourage enterprises to reduce material costs through management and innovation.

In addition, the number of enterprises in the market will also have an impact on the equilibrium results. The more enterprises there are in the market, the less each enterprise can produce, and the resulting lower prices for emergency supplies lead to lower profits and increased consumer surplus and social welfare. This is in line with market reality. When the number of enterprises in the market is large, the competition between them will be more intense, and their competitive strategy is a low-price strategy. This shows that maintaining competitive conditions in the emergency supply market will be conducive to consumer surplus and social welfare.

### 4.2. Model EN

When an epidemic occurs, if there is no government regulation, equilibrium outputs can be derived by satisfying \( \frac{\partial \pi_{EN}}{\partial q_i} = 0 \).

\[
q_{i En} = \frac{\beta a}{d + \beta n + \beta}
\]

Lemma 2. Under epidemic conditions, without government regulation, in other words when enterprises make decisions by themselves, the equilibrium results are

\[
p_{En} = \frac{\beta a (d + \beta)}{d + \beta n + \beta}, \quad \pi_{i En} = \frac{\beta^2 a^2 (d + 2\beta)}{2(d + \beta n + \beta)^2}, \quad c_s_{En} = \frac{n^2 a^2 \beta^2}{2(\beta n + d + \beta)^2}, \quad s_w_{En} = \frac{n \beta^2 a^2 (nr + d + 2\beta)}{2(\beta n + d + \beta)^2}.
\]

Proposition 2 can be obtained by analysing the effects of \( d \) and \( \beta \).

**Proposition 2:**

1. The effects of \( d \) are:

\[
\frac{\partial q_{i En}}{\partial d} < 0; \quad \frac{\partial p_{En}}{\partial d} > 0; \quad \text{if } 0 < d \leq \beta(n - 3), \text{ then } \frac{\partial \pi_{i En}}{\partial d} \geq 0; \quad \text{if } d > \beta(n - 3), \text{ then } \frac{\partial \pi_{i En}}{\partial d} < 0; \quad \frac{\partial c_s_{En}}{\partial d} < 0;
\]
If $1 < \beta \leq \frac{2rn}{n-3}$, then $\frac{\partial \text{sw}^{EN}}{\partial d} < 0$; if $\beta > \frac{2rn}{n-3}$ and $0 < d \leq \beta(n - 3) - 2rn$, then $\frac{\partial \text{sw}^{EN}}{\partial d} \geq 0$.

If $\beta > \frac{2rn}{n-3}$ and $d > \beta(n - 3) - 2rn$, then $\frac{\partial \text{sw}^{EN}}{\partial d} < 0$.

2. The effects of $\beta$ are:

$$\frac{\partial q_i^{EN}}{\partial \beta} > 0; \frac{\partial p^{EN}}{\partial \beta} > 0; \frac{\partial \pi_i^{EN}}{\partial \beta} > 0; \frac{\partial cs^{EN}}{\partial \beta} > 0; \frac{\partial \text{sw}^{EN}}{\partial \beta} > 0.$$

3. The effects of $n$ are:

$$\frac{\partial q_i^{EN}}{\partial n} < 0; \frac{\partial p^{EN}}{\partial n} < 0; \frac{\partial \pi_i^{EN}}{\partial n} < 0; \frac{\partial cs^{EN}}{\partial n} > 0; \frac{\partial \text{sw}^{EN}}{\partial n} > 0.$$

**Proof.** See Appendix B.

Proposition 2 shows that under epidemic conditions, an increase in material cost will lead to a decrease in output and an increase in price. The reason is that an increase in material cost decreases output and a shortage of supplies, thus leading to an increase in price and a reduction in consumer surplus. The impact of material cost on profit is related to the degree of change of market demand. If $d > \beta(n - 3)$, then profits will decrease when the material cost increases. Otherwise, profits will increase. To judge whether government regulation should be carried out, the issue of whether it can promote social welfare should be considered. The impact of material costs on social welfare is uncertain and is related to the degree of change in market demand, the externalities of emergency supplies, and the number of enterprises. When $1 < \beta \leq \frac{2rn}{n-3}$, higher costs will lead to lower social welfare. The reason is that market demand is highly sensitive to costs, and rising costs hurt profits and consumer surplus. When $\beta > \frac{2rn}{n-3}$, if $0 < d \leq \beta(n - 3) - 2rn$, then the externalities of emergency supplies are positively correlated with social welfare, and vice versa. In the case of excessive costs, although the profits of enterprises increase with the increase in raw material costs, this increase will be smaller than the decrease in consumer surplus, which will reduce social welfare. We find that the material cost threshold ($d = \beta(n - 3)$) in Model EN is higher than the material cost threshold ($d = \beta(n - 3) - 2rn$) in Model EG, which means that social welfare will be damaged when maximizing profit. Thus, when an epidemic occurs, the government needs to intervene from the perspective of the entire society.

In addition, the degree of change in market demand will also have an impact on the equilibrium results. An increase in market demand will lead to an increase in the output and prices of emergency supplies, thus increasing the profits of enterprises. The increase in profits will encourage enterprises to expand production and alleviate the shortage of emergency supplies, thus increasing consumer surplus and social welfare. Although the increase in market demand for emergency supplies can promote social welfare, the impact of material cost on social welfare will also be constrained by the degree of change in market demand. Therefore, it is necessary to
comprehensively consider both the material cost and the degree of change in market demand to maximize social welfare. The number of enterprises in the market has the same effect on the equilibrium results as those found under normal conditions.

### 4.3. Model EG

In this scenario, the government will regulate enterprises; then, $\frac{\partial q_{EG}^*}{\partial p_i} = 0$, and equilibrium outputs are

$$q_i^{EG} = \frac{\beta a}{(2\beta - r)n + d} \quad (4)$$

**Lemma 3.** With government output regulation, the equilibrium results are:

$$p_i^{EG} = a[(\beta - r)n + d]\beta \quad \text{and} \quad \pi_i^{EG} = \frac{a^2 \beta^2 [2(\beta - r)n + d]}{n(2\beta - r) + d}, \quad c_s^{EG} = \frac{n^2 \beta^2 a^2}{2[(2\beta - r)n + d]^2}, \quad s_w^{EG} = \frac{n \beta^2 a^2}{2[(2\beta - r)n + d]}.$$  

To ensure the production of enterprises, $q_i^{EG} > 0$, $p_i^{EG} > 0$ and $\pi_i^{EG} > 0$ should be satisfied, and then $d > \max \{0, (r - \beta)n, 2(r - \beta)n\}$ can be derived.

**Proposition 3:**

1. The effects of $d$ are:

$$\frac{\partial q_i^{EG}}{\partial d} < 0; \quad \frac{\partial p_i^{EG}}{\partial d} > 0; \quad \text{if } 1 < \beta < \frac{3r}{2} \text{ and } \max \{0, (r - \beta)n, 2(r - \beta)n\} < d$$

$$\leq (3r - 2\beta)n, \quad \text{then } \frac{\partial \pi_i^{EG}}{\partial d} > 0; \quad \text{if } 1 < \beta < \frac{3r}{2} \text{ and } d > (3r - 2\beta)n, \quad \text{then } \frac{\partial \pi_i^{EG}}{\partial d} < 0; \quad \text{if } \beta \geq \frac{3r}{2}, \quad \frac{\partial \pi_i^{EG}}{\partial d} < 0; \quad \frac{\partial c_s^{EG}}{\partial d} < 0; \quad \frac{\partial s_w^{EG}}{\partial d} < 0.$$

2. The effects of $\beta$ are:

- If $\max \{0, (r - \beta)n, 2(r - \beta)n\} < d \leq nr$, then $\frac{\partial q_i^{EG}}{\partial \beta} \leq 0$; if $d > nr$, then $\frac{\partial q_i^{EG}}{\partial \beta} > 0$; $\frac{\partial p_i^{EG}}{\partial \beta} > 0$; $\frac{\partial c_s^{EG}}{\partial \beta} > 0$; if $\max \{0, (r - \beta)n, 2(r - \beta)n\} < d < nr$, then $\frac{\partial c_s^{EG}}{\partial \beta} \leq 0$; $d > nr$, then $\frac{\partial c_s^{EG}}{\partial \beta} > 0$; $\frac{\partial s_w^{EG}}{\partial \beta} > 0$.

3. The effects of $n$ are:

- If $1 < \beta \leq \frac{r}{2}$, then $\frac{\partial q_i^{EG}}{\partial n} \leq 0$; if $\beta > \frac{r}{2}$, then $\frac{\partial q_i^{EG}}{\partial n} > 0$; $\frac{\partial p_i^{EG}}{\partial n} < 0$; if $1 < n \leq -\frac{bd}{2\beta^2 - 3br + r^2}$, then $\frac{\partial \pi_i^{EG}}{\partial n} \leq 0$; if $\beta > -\frac{bd}{2\beta^2 - 3br + r^2}$, then $\frac{\partial \pi_i^{EG}}{\partial n} > 0$; $\frac{\partial c_s^{EG}}{\partial n} > 0$; $\frac{\partial s_w^{EG}}{\partial n} > 0$.

**Proof.** See Appendix C.
Proposition 3 indicates that rising material costs will cause a shortage of emergency supplies, a price increase, and a decrease in consumer surplus. The relationship between profit and material cost is more complex. When the degree of change in market demand is below a certain level (\(1 < \beta < \frac{3}{2}\)), profit at first decreases and then increases along with the rising material cost. When the degree of change in market demand is above a certain level (\(\beta \geq \frac{3}{2}\)), the rise of the material cost will lead to lower profit. The reason is that although an increase in material cost will cause an increase in marginal cost, the profits of enterprises are also closely correlated to the output. The excessive increase in material cost will lead to a substantial decrease in output. Higher prices for raw materials will reduce consumer surplus and social welfare due to the shortage of emergency supplies. The reason is that the reduction in the number of emergency supplies affects profits and consumer surplus. Therefore, as far as the government is concerned, in the early stage of the epidemic, there will be a situation where the price of emergency supplies will soar and supply will be insufficient, and various measures need to be taken to ensure the supply of emergency supplies as much as possible. For example, a temporary dispatch system for key enterprises can be established to ensure the supply of emergency supplies such as masks.

The impact of the degree of change in market demand on output is more complex. Its impacts on output and consumer surplus are related to material cost, and there is a minimum value. Similar to Model NN and Model EN, the demand for emergency supplies in the market increases enterprises’ profits. The impact of increased demand for emergency supplies on consumer surplus, while it might be uncertain, will lead to an increase in profits and, ultimately, improve social welfare. When an epidemic occurs, government production regulations based on the maximization of social welfare should consider enterprises, consumers, and externalities as a whole. Although consumer surplus may not increase, the positive externalities of emergency supplies increase, along with the additional effect of increased profits ensures social welfare.

The relationship between the number of firms in the market and their outputs, profits, and social welfare also changes. Enterprise outputs are mainly affected by changes in market demand for emergency supplies. When the degree of change in market demand for emergency supplies is lower than a certain level (\(1 < \beta \leq \frac{3}{2}\)), e.g. when the epidemic crisis is serious but people’s market response is not strong enough, the increase in the number of enterprises will reduce the outputs of emergency supplies. Conversely, the overall output of emergency supplies will continue to increase as the number of enterprises increases. The reason is that government output regulation makes the supply of enterprises adjust to the demand of consumers to maximize the supply of emergency supplies during the epidemic and ensure the orderly progress of epidemic prevention and control. Profits first decrease and then increase with an increase in the number of enterprises. The increase in the number of enterprises will enhance social welfare. A sudden decrease in the number of enterprises during the epidemic has an important impact on the shortage of emergency supplies. Therefore, the government must maintain appropriate regulations on the entry of enterprises, allowing entry is more beneficial to both enterprises and society.
Moreover, with government output regulation, Model EG is more complicated than both Model NN and Model EN. Although the increase in market demand for emergency supplies will increase output, profit, and social welfare to a certain extent, it will also affect the relationship between material cost and equilibrium results. Therefore, material costs and other factors should be fully considered to set a reasonable price for emergency supplies when enterprises decide their production.

4.4. Model EP

Price regulation is also a common government regulation. However, is price regulation effective? Compared with output regulation, how does price regulation work? Based on these issues, this section explores government price regulation. In this scenario, the government regulates prices, \( p_{EP} = p_{NN} = \frac{a(d+1)}{d+n+1} \), and equilibrium outputs are:

\[
q_{EP}^i = a(\beta d + \beta n + \beta - d - 1) \quad (d + n + 1)n\beta
\]

Lemma 4. With government price regulation, the equilibrium results are:

\[
p_{EP} = a(d + 1) \quad (d + n + 1), \quad \pi_{EP} = -\frac{a^2(\beta d + \beta n + \beta - d - 1)(\beta d^2 - \beta dn + \beta d - 2\beta n - d^2 - d)}{2(d + n + 1)^2n^2\beta^2},
\]

\[
c_{EP} = \frac{a^2(\beta d + \beta n + \beta - d - 1)^2}{2(d + n + 1)^2\beta^2}, \quad s_{EP} = -\frac{\beta d^2 - \beta dn - d^2}{2n(d + n + 1)^2}\frac{\beta^2}{\beta^2}.
\]

To ensure the production of enterprises, \( q_{EP}^i > 0, \quad p_{EP} > 0 \) and \( \pi_{EP} > 0 \) should be satisfied, and then \( 0 < d < \frac{n\beta - \beta}{\beta + 1 + \sqrt{\beta^3n^2 + 6\beta^2n + \beta^2 - 6\beta n - 2\beta + 1}} \) can be derived.

Proposition 4:
1. The effects of \( d \) are:

\[
\frac{\partial q_{EP}^i}{\partial d} < 0; \quad \frac{\partial p_{EP}}{\partial d} > 0; \quad \frac{\partial \pi_{EP}}{\partial d} < 0; \quad \frac{\partial c_{EP}^i}{\partial d} < 0; \quad \frac{\partial s_{EP}}{\partial d} < 0.
\]

2. The effects of \( \beta \) are:

\[
\frac{\partial q_{EP}^i}{\partial \beta} > 0; \quad \text{if} \quad 1 < \beta < \frac{(d+1)d}{d^2 + d - n}, \quad \text{then} \quad \frac{\partial \pi_{EP}}{\partial \beta} \geq 0; \quad \text{if} \quad \beta > \frac{(d+1)d}{d^2 + d - n}, \quad \text{then} \quad \frac{\partial c_{EP}^i}{\partial \beta} < 0; \quad \frac{\partial s_{EP}}{\partial \beta} > 0.
\]

3. The effects of \( n \) are:

\[
\frac{\partial q_{EP}^i}{\partial n} < 0; \quad \frac{\partial p_{EP}}{\partial n} < 0; \quad \frac{\partial \pi_{EP}}{\partial n} > 0; \quad \frac{\partial c_{EP}^i}{\partial n} > 0; \quad \frac{\partial s_{EP}}{\partial n} > 0.
\]
Proof. See Appendix D.

From Proposition 4, it can be seen that the rising cost of raw materials leads to a decline in the outputs of emergency supplies, a trend of rising prices, and a decline in profits, consumer surplus, and social welfare. Because the cost of raw materials is rising, the price of emergency supplies cannot rise correspondingly because of the price regulation implemented by the government. As a result, enterprises’ incomes are reduced under the joint action of declining output and unchanged price, but enterprises’ costs are increasing, which leads to a decline in their profits. Due to the decrease in enterprises’ outputs after price regulation, consumer surplus decreases. According to the decrease in enterprises’ profits and consumer surplus, social welfare declines, which is even greater due to the increased importance of the externalities of emergency materials.

Compared with Model EN, profits and social welfare are negatively correlated with raw material costs when the government introduces price regulation. If the government implements price regulation, then in the initial stage of the outbreak of the epidemic, it will be unfavourable for both enterprises and society. In the later stage of epidemic prevention and control, when the cost of raw materials starts to return to normal gradually, it is beneficial to both enterprises and society. Therefore, price regulation is not suitable to be implemented immediately after the outbreak. There are two reasons. First, at the early stage of the outbreak, the demand for emergency supplies and raw materials surges, and the rise in their prices were in line with the law of the market. However, once the rise is too sharp, market failure could occur, and government intervention is needed. Second, the increase in demand stimulates an increase in supply, but due to the restrictions of production technology and production personnel, the price of raw materials rises due to the increase in value, which does not require government intervention.

Similar to Model EG, consumer surplus is also negatively correlated with raw material cost in Model EP. The reason for this situation is that the increase in market demand after the outbreak of the epidemic is not enough to make up for the subsequent suppression of enterprise outputs by price regulation, which makes consumer surplus decline along with the decline in total output.

In Model EP, the impact of changes in market demand for emergency supplies on profits is different from Model NN, Model EN, and Model EG. When market demand changes less than a certain threshold in the case of the epidemic, \(1 < \beta \leq \frac{(d+1)d}{d+1-d-n}\), an increase in market demand for emergency supplies is conducive to increasing profits. Otherwise, increased market demand for emergency supplies will reduce profits. The reason is that the degree of change in market demand is positively correlated with the output of emergency supplies. Thus, a small increase in market demand will bring an increase in enterprise profits. However, when market demand changes greatly, an increase in enterprise outputs will not be enough to compensate for the price decrease, which will lead to a decline in enterprise profits. It can be seen that price regulation is more effective within an appropriate range. When market demand surges, price regulation will harm the interests of enterprises.
However, it is beneficial to both consumers and society, so it needs to be implemented. When price regulation is implemented, only the influence of the number of enterprises in the market on profits changes compared with Model NN and Model EN. At this time, a decrease in the number of enterprises will also reduce the profits of enterprises. This is because although a decrease in the number of enterprises will increase the outputs of enterprises and the price of emergency supplies, the costs will also increase substantially. After epidemic outbreaks, the number of companies will drop sharply, which will increase production costs. The implementation of price regulation will reduce the profits of enterprises, consumer surplus, and social welfare. It is not wise to implement price regulation immediately after epidemic outbreaks, but appropriate implementation needs to be determined by the specific production conditions of enterprises in the market. When enterprises gradually resume production, the government should gradually implement price regulation to realize price stability.

**Proposition 5:** The effects of $\delta$ and $r$ are:

\[
\frac{\partial o_{sw}^{NN}}{\partial \delta} > 0; \quad \frac{\partial o_{sw}^{EN}}{\partial r} > 0; \quad \frac{\partial o_{sw}^{EP}}{\partial r} > 0;
\]
\[
\frac{\partial o_{qi}^{EG}}{\partial \delta} > 0; \quad \frac{\partial o_{pi}^{EG}}{\partial r} < 0; \quad \frac{\partial o_{cs}^{EG}}{\partial r} > 0; \quad \frac{\partial o_{sw}^{EG}}{\partial r} > 0.
\]

**Proof.** See Appendix E.

The positive externalities of emergency supplies can only promote social welfare under normal conditions, when enterprises make their own decisions and when the government conducts price regulation under epidemic conditions. If the government regulates production, the positive externalities of emergency supplies will have an impact on all equilibrium results. When the positive externalities of emergency supplies increase, then outputs, consumer surplus, and social welfare increase; however, the prices of emergency supplies and profits decline. The positive externalities of emergency supplies also have an impact on the supply and demand for emergency supplies while the government conducts output regulation. The greater the positive externalities of emergency supplies, the easier it is to realize the supply guarantee, and the price does not rise significantly. Therefore, while the government regulates, it is necessary to continue to maintain the externalities of emergency supplies. For example, the call for continuing to wear masks during epidemic prevention and control can not only guarantee people’s health but also support the government regulation of emergency supplies.

### 4.5. Comparisons

We compare the equilibrium results of four cases and classify the scenarios by the degree of change in market demand. When we consider the scenario in which the
demand for emergency supplies changes slightly \((1 < \beta \leq \frac{nr}{n-1})\), Corollary 1 can be obtained.

**Corollary 1:** If \(1 < \beta \leq \frac{nr}{n-1}\), then \(q_i^{EG} > q_i^{EN} > q_i^{NN}\), and the relationship between \(q_i^{EG}\) and \(q_i^{EP}\) is uncertain. The threshold is \(\beta = \sqrt{\left\{ \left[ -\beta^2 + (r+5)(\beta-r)n + \beta - 1 \right] \left[ (r-\beta)n + 1 \right] - \beta^2 n + (r+1)n - \beta - 1 \right\}}\).

\(p^{EP} = p^{NN}\), \(p^{EG} < p^{EN}\) and \(p^{EN} > p^{NN}\), but the relationship between \(p^{EG}\) and \(p^{NN}\) is uncertain. The threshold is \(d = (1-\beta)(1 + \beta n - nr) + \left[ (\beta-1)(1+\beta n - nr)(\beta^2 n - \beta nr - 5Bn + nr - \beta + 1) \right]^2 \) then \(p^{EG} > p^{NN}\); otherwise, \(p^{EG} < p^{NN}\). For profits, \(\pi_i^{EG} < \pi_i^{EN}\), \(\pi_i^{EN} > \pi_i^{NN}\) and the relationship between \(\pi_i^{EG}\) and \(\pi_i^{NN}\) and between \(\pi_i^{EN}\) and \(\pi_i^{EP}\) is uncertain.

The thresholds are \(d = \frac{\sqrt{8\beta^2 n + \beta^2 - 8\beta n - 2\beta + 1 - \beta + 1}}{2(\beta-1)}\) and \(d = \frac{\sqrt{\left\{ \left[ -\beta^2 + (r+5)(\beta-r)n + \beta - 1 \right] \left[ (r-\beta)n + 1 \right] - \beta^2 n + (r+1)n - \beta - 1 \right\} \left[ (r-\beta)n + 1 \right] - \beta^2 n}}{2(\beta-1)}\) respectively. For consumer surplus, \(cs^{EG} > cs^{EN} > cs^{NN}\), but the relationship between \(cs^{EG}\) and \(cs^{EP}\) is uncertain.

As for social welfare, \(sw^{EG} > sw^{EP} > sw^{EN} > sw^{NN}\).

**Proof.** See Appendix D.

From Corollary 1, it can be seen that when \(1 < \beta \leq \frac{nr}{n-1}\), either output regulation or price regulation will increase the output of emergency supplies. In other words, government regulations can be used in the event of an epidemic to guarantee the supply of emergency supplies effectively, which can provide support for government regulations. However, the outputs under the two kinds of government regulatory actions are also different. When the market demand for emergency supplies changes slightly, and the government has implemented regulations, the price is related to the cost of emergency supplies. When the cost of emergency supplies is smaller \((d \leq \frac{(1-\beta)(1+\beta n - nr) + [(\beta-1)(1+\beta n - nr)(\beta^2 n - \beta nr - 5Bn + nr - \beta + 1)]^2}{2(\beta-1)}\) government output regulation has a better effect on the protection of emergency supplies; otherwise, the effect of price regulation is better. The conditions for profits are more complicated, which is reflected in the uncertainty of the relationships between the two forms of regulation and profits under normal conditions and during price regulation, which is mainly related to the cost of emergency supplies. For enterprises, the regulation of output will lead to a decline in their profits. However, when the government implements price regulation, their profits may increase, which mainly depends on their cost management capabilities. This thereby effectively stimulates enterprises to carry out technological innovation and process innovation to reduce production costs and increase their profits. This shows that price regulation can not only directly control prices but also indirectly promote companies to reduce costs. Regardless of whether the government regulates or not, consumer surplus and social welfare will increase. However, if the government regulates, consumer surplus and social welfare are better
than in situations where the government does not regulate. Compared with Model EN, although the profits of enterprises are reduced under government regulation, the prices of emergency supplies are also reduced, thereby alleviating the phenomenon of ‘expensive’ emergency supplies. Therefore, when emergency supplies have large externalities during the epidemic, government regulations are reasonable. However, the reduced profits must be compensated through subsidies and other methods to encourage enterprises to improve production. However, for consumer surplus, when the cost of emergency supplies increases sharply, for the benefit of consumers and society as a whole, price regulations should be implemented, and the increase in costs can be gradually suppressed so that the supply of emergency supplies will return to normal as soon as possible.

Then, considering the scenario of $\beta > \frac{nr}{n-1}$, Corollary 2 can be obtained.

**Corollary 2**: If $\beta > \frac{nr}{n-1}$, then $q_{EP} > q_{EG} > q_{EN} > q_{NN} > q_{EN}$; $p_{EN} > p_{EG} > p_{NN} = p_{EP}$; $\pi_{EN} > \pi_{NN} > \pi_{EG} > \pi_{EP}$; $cs_{EP} > cs_{EG} > cs_{EN} > cs_{EG}$; and $sw_{EG} > sw_{EP} > sw_{EN} > sw_{NN}$.

**Proof.** See Appendix E.

Unlike Corollary 1, the case of Corollary 2 is simple. Compared with normal conditions, regulation in any case during the epidemic will enhance outputs of emergency supplies, consumers, and social welfare, but the impact on the prices of emergency supplies and profits is not the same. When the government does not impose regulations, prices and profits will be increased; if regulations are implemented, profits will decrease, and the prices of emergency supplies and profits will be lower when price regulation is implemented. This shows that when market demand changes greatly under epidemic conditions, government regulations will ensure the supply of emergency supplies, which will have a positive impact on consumers and society. For enterprises, it will cause a loss of their profits. Therefore, it is necessary to implement subsidies and support while carrying out regulations.

### 5. Conclusion

This paper sets up an oligopoly model of emergency supplies production to investigate the equilibrium results under normal conditions and epidemic conditions with and without government regulation, discusses the impacts of the degree of change in market demand, externalities, the material cost of emergency supplies, and the number of enterprises in the market on the equilibrium results, finally compares four models. Conclusions are drawn as follows.

First, the material cost of emergency supplies has an impact on the equilibrium results. Under normal conditions and epidemic conditions, an increase in material cost will lead to a decrease in output and a price increase, which means a shortage of supply and in turn will result in a corresponding change in consumer surplus and
social welfare. In Model EP, this trend will still exist. When the cost of emergency materials is lower, under normal conditions, profit will increase with an increase in cost. In Model EN, unless there is low variation in market demand and lower material cost ($1 < \beta < \frac{1}{2}$ and $0 < d < (r - 2\beta)n$), a rising cost of emergency supplies will reduce consumer surplus. In addition to the smaller externalities of emergency supplies and lower cost of the materials ($\beta > \frac{2\mu}{n-3}$ and $0 < d \leq \beta(n - 3) - 2\mu$), an increase in the material will decrease social welfare. The implication for policy is that strengthening market price supervision is necessary, such as severely cracking down on price raising and price collusion. Through effective control of the cost of raw materials, the goals of standardizing enterprises’ behaviour, stabilizing the market, and ensuring the basic needs of the people in the epidemic can be achieved.

Second, equilibrium results are affected by the degree of change in market demand. By comparing the equilibrium results without government regulation, that is, when enterprises make decisions independently (Model EN), with the results when government output and price regulation (Model EG and Model EP) are implemented when the epidemic occurs, we find, contrary to people’s intuitive understanding, that increasing market demand will reduce social welfare. In Model EN, when the market demand for emergency supplies increases prices rise, which increases production and alleviates the shortage of emergency supplies. When enterprises make decisions independently and when the government carries out output regulation, price is positively correlated with the degree of change in market demand; under normal conditions and when the price is regulated, the price will not be correlated with the degree of change in market demand. When independent decisions are made by enterprises and when price regulation is implemented by the government, then output and consumer surplus will increase with the degree of change in market demand. While output regulation is carried out and the cost is lower ($d < nr$), then output and consumer surplus will be negatively correlated with the degree of change in market demand. When enterprises make independent decisions and when the government regulates output, then profits are positively correlated with the degree of change in market demand. When price regulation is implemented and market demand changes drastically, then profits decline as the degree of change in demand increases. In real life, some companies are also faced with a series of problems, such as increasing transportation costs, increasing store rents, and the risk of loss. Therefore, the policy implication is that the government should establish a perfect price information system to restrict prices within a reasonable range, using appropriately relaxed price control, such as the regulation of purchase-sale price differences. Moreover, the government should play a role in standard guidance to protect normal market changes without excessive intervention, such as adopting temporary intervention measures when the price rises significantly.

Third, the number of enterprises in the market and the positive externalities of emergency supplies will also affect the equilibrium results. In Model NN and Model EN, the greater the number of enterprises in the market is, the smaller the output of each enterprise and the lower the prices of emergency supplies. Profits decline, but consumer surplus and social welfare increase. When output regulation is implemented, the profits of enterprises will first decrease and then increase with the increase in the number of enterprises. An increase in the number of enterprises will improve social welfare. In addition, when the government implements price
regulation, profits will increase as the number of enterprises increases. Therefore, when the epidemic is brought under control and market demand stabilizes, the government needs to loosen regulations to promote and guide normal market competition. Under normal conditions, when enterprises make independent decisions and when the government conducts price regulation, the positive externalities of emergency supplies will only enhance social welfare. In Model EG, higher positive externalities of emergency supplies lead to the increase of outputs, consumer surplus, and social welfare. However, the price of emergency supplies and profits decline. The positive externalities of emergency supplies have an impact on the supply and demand for emergency supplies while the government regulates them. The greater the positive externalities of emergency supplies are, the easier it is to realize the supply guarantee, and the price does not appear to rise sharply. Therefore, the policy implication is that it is necessary to continue to maintain the externalities of emergency supplies. The government should guarantee the supply of emergency materials by allowing the resumption of production and work to increase the production of materials. A critical commodity reserve system and the establishment of a disaster relief material stockpile should be implemented to address the uncertain impact of COVID-2019.

Fourth, whether the government should implement regulation and which modes of regulation it should implement should be considered more comprehensively. Independent decisions of enterprises and government regulations of emergency materials production are reasonable in different situations, considering the cost of materials, the degree of change in market demand, and the externalities. When market demand does not change severely, then government regulations are needed to increase production. The mode of regulation should be decided by the degree of change in the demand for emergency materials. Although the profits of enterprises are reduced during government regulation compared to those under the absence of regulation, the price of emergency supplies is also reduced, thereby alleviating the phenomenon of ‘expensive’ emergency supplies. From the perspective of consumer surplus, when enterprises are independent in decision-making, an increase in market demand will increase consumer surplus. When the government does regulate, especially when facing a high cost of emergency supplies, then price regulation is a more effective policy for the government.

Disclosure statement
No potential conflict of interest was reported by the author.

Funding
This work was supported by the National Social Science Fund in the Later Stage of China under Grant [20FJYB066].

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Appendix A

Proof of Proposition 1

1. The effects of $d$:

\[
\begin{align*}
\frac{\vartheta q_i^{NN}}{\vartheta d} &= -\frac{a}{(n + d + 1)^2} < 0; \\
\frac{\vartheta p^{NN}}{\vartheta d} &= \frac{an}{(n + d + 1)^2} > 0; \\
\frac{\vartheta \pi_i^{NN}}{\vartheta d} &= \frac{a^2(n - d - 3)}{2(n + d + 1)^3}; \\
\end{align*}
\]

If $0 < d \leq n - 3$, then $\frac{\vartheta \pi_i^{NN}}{\vartheta d} \geq 0$; if $d > n - 3$, then $\frac{\vartheta \pi_i^{NN}}{\vartheta d} < 0$; $\frac{\vartheta c_s^{NN}}{\vartheta d} = -\frac{n^2a^2}{(n + d + 1)^2} < 0$; $\frac{\vartheta c_s^{NN}}{\vartheta d} = \frac{a^2n((2\delta - 1)n + d + 3)}{2(n + d + 1)^3} < 0$.

2. The effects of $n$:

\[
\begin{align*}
\frac{\vartheta q_i^{NN}}{\vartheta n} &= -\frac{a}{(d + n + 1)^2} < 0; \\
\frac{\vartheta p^{NN}}{\vartheta n} &= -\frac{a(d + 1)}{(d + n + 1)^2} < 0; \\
\frac{\vartheta \pi_i^{NN}}{\vartheta n} &= -\frac{a^2(d + 2)}{(d + n + 1)^3}; \\
\frac{\vartheta c_s^{NN}}{\vartheta n} &= \frac{na^2(d + 1)}{(d + n + 1)^3} > 0; \\
\frac{\vartheta c_s^{NN}}{\vartheta n} &= \frac{a^2[(d^2 + 3d + (2\delta - 1)nd + 2 + 2(\delta - 1)n]}{2(d + n + 1)^3} > 0. \\
\end{align*}
\]

Appendix B

Proof of Proposition 2

1. The effects of $d$:

\[
\begin{align*}
\frac{\vartheta q_i^{EN}}{\vartheta d} &= -\frac{\beta a}{(\beta n + \beta + d)^3} < 0; \\
\frac{\vartheta p^{EN}}{\vartheta d} &= \frac{\beta^2 an}{(\beta n + \beta + d)^3} > 0; \\
\frac{\vartheta \pi_i^{EN}}{\vartheta d} &= \frac{\beta^2 a^2[\beta(n - 3) - d]}{2(\beta n + \beta + d)^3} > 0; \\
\end{align*}
\]

If $0 < d \leq \beta(n - 3)$, then $\frac{\vartheta \pi_i^{EN}}{\vartheta d} > 0$; if $d > \beta(n - 3)$, then $\frac{\vartheta \pi_i^{EN}}{\vartheta d} < 0$;

\[
\begin{align*}
\frac{\vartheta c_s^{EN}}{\vartheta d} &= -\frac{n^2\beta^2 a^2}{(\beta n + \beta + d)^3} < 0; \\
\frac{\vartheta c_s^{EN}}{\vartheta d} &= \frac{n^2\beta^2 a^2((\beta(n - 3) - 2rn) - d)}{2(\beta n + \beta + d)^3}; \\
\end{align*}
\]

If $\beta > \frac{2rn}{n-3}$ and $0 < d \leq \beta(n - 3) - 2rn$, then $\frac{\vartheta c_s^{EN}}{\vartheta d} > 0$; if $\beta > \frac{2rn}{n-3}$ and $d > \beta(n - 3) - 2rn$, then $\frac{\vartheta c_s^{EN}}{\vartheta d} < 0$.

2. The effects of $\beta$:

\[
\begin{align*}
\frac{\vartheta q_i^{EN}}{\vartheta \beta} &= \frac{a\beta}{(\beta n + \beta + d)^2} > 0; \\
\frac{\vartheta p^{EN}}{\vartheta \beta} &= \frac{(\beta^2 n + \beta + 2\beta d + d^2)}{(\beta n + \beta + d)^2} > 0;
\end{align*}
\]
3. The effects of $n$:

$$\frac{\partial q_i^{EN}}{\partial n} = -\frac{a\beta^2}{(\beta n + \beta + d)^2} < 0; \quad \frac{\partial p^{EN}}{\partial n} = -\frac{a\beta^2(d + \beta)}{(\beta n + \beta + d)^2} < 0; \quad \frac{\partial \pi^{EN}}{\partial n} = -\frac{a^2\beta^2(2\beta - 3\beta r)n + d}{2[(\beta - r)n + d]^3};$$

if $1 < \beta < \frac{3r}{2}$ and $\max\{0, (r - \beta)n, 2(r - \beta)n\} < d \leq (3r - 2\beta)n$, then $\frac{\partial \pi^{EN}}{\partial d} > 0$; if $1 < \beta < \frac{3r}{2}$ and $d > (3r - 2\beta)n$, then $\frac{\partial \pi^{EN}}{\partial d} < 0$; if $\beta \geq \frac{3r}{2}$, then $\frac{\partial \pi^{EN}}{\partial d} < 0$; $\frac{\partial q_i^{EN}}{\partial d} = -\frac{n\beta a^2(d - nr)}{[2(\beta - r)n + d]^2} < 0$; $\frac{\partial \pi^{EN}}{\partial d} = -\frac{2n\beta a^2}{4[(\beta - r)n + d]^2} < 0$.

2. The effects of $\beta$:

$$\frac{\partial q_i^{EG}}{\partial \beta} = \frac{a(d - nr)}{[(\beta - r)n + d]^2}; \quad \text{if } \max\{0, (r - \beta)n, 2(r - \beta)n\} \leq d \leq nr, \text{ then } \frac{\partial q_i^{EG}}{\partial \beta} \leq 0;$$

if $d > nr$, then $\frac{\partial q_i^{EG}}{\partial \beta} > 0; \frac{\partial p^{EG}}{\partial \beta} = \frac{a[(2\beta^2 - 2\beta r + r^2)n^2 + 2d(\beta - r)n + d^2]}{[(2\beta - r)n + d]^3} > 0$;

$$\frac{\partial \pi^{EG}}{\partial \beta} = \frac{a^2\beta[(2\beta^2 - 3\beta r + 2r^2)n^2 + 3d(\beta - r)n + d^2]}{2[(2\beta - r)n + d]^3} > 0;$$

$$\frac{\partial \pi^{EG}}{\partial \beta} = \frac{n\beta a^2(d - nr)}{[(2\beta - r)n + d]^2}; \quad \text{if } \max\{0, (r - \beta)n, 2(r - \beta)n\} < d < nr, \text{ then } \frac{\partial q_i^{EG}}{\partial \beta} \leq 0;$$

if $d > nr$, then $\frac{\partial q_i^{EG}}{\partial \beta} > 0; \frac{\partial p^{EG}}{\partial \beta} = \frac{a[(2\beta^2 - 2\beta r + r^2)n^2 + 2d(\beta - r)n + d^2]}{[(2\beta - r)n + d]^3} > 0;$$

Applying Proposition 3
3. The effects of $n$ :

$$\frac{\partial q^{EG}_i}{\partial n} = -\frac{a\beta(2\beta-r)}{(d + (2\beta - r))}; \text{ if } 1 < \beta \leq \frac{r}{2}, \text{ then } \frac{\partial q^{EG}_i}{\partial n} \geq 0; \text{ if } \beta > \frac{r}{2}, \text{ then } \frac{\partial q^{EG}_i}{\partial n} > 0;$$


$$\frac{\partial p^{EG}_i}{\partial n} = -\frac{a\beta^2 d}{(2\beta n - nr + d)} < 0; \frac{\partial p^{EG}_i}{\partial n} = -\frac{a^2 \beta^2 (2\beta^2 n - 3\beta nr + nr^2 + \beta d)}{(d + (2\beta - r))}; \text{ if } 1 < n$$

$$\leq -\frac{\beta d}{2\beta^2 - 3\beta r + r^2},$$

then $\frac{\partial p^{EG}_i}{\partial n} \leq 0; \text{ if } n > -\frac{\beta d}{2\beta^2 - 3\beta r + r^2}$ then $\frac{\partial p^{EG}_i}{\partial n} > 0; \frac{\partial p^{EG}_i}{\partial n} = \frac{\beta^2 n d^2 d}{(2\beta n - nr + d)^2} > 0;$$

$$\frac{\partial s^{\text{SWEG}}}{\partial n} = -\frac{\beta^2 (2\beta n - nr - d)}{2(2\beta n - nr + d)^2} > 0.$$

**Appendix D**

**Proof of Proposition 4**

1. The effects of $d$ :

$$\frac{\partial q^{EP}_i}{\partial d} = -\frac{a}{(d + n + 1)^2 \beta} < 0; \frac{\partial p^{EP}_i}{\partial d} = \frac{an}{(d + n + 1)^2} > 0;$$

$$\frac{\partial \pi^{EP}_i}{\partial d} = \frac{a^2 \left\{ [d^2 + 2(n + 1)d - n^2 + 2n + 1](d + n + 1)\beta^2 + [-2d^3 - 6(n + 1)d^2 - 2(5n + 3)n^2 + 4n - 2] \beta + [d^2 + (3n + 2)d + n + 1](d + 1) \right\}}{2(d + n + 1)^3 n^2 \beta^2} < 0;$$

$$\frac{\partial c^{\text{SWEP}}}{\partial d} = -\frac{a^2 (\beta d + \beta n + \beta - d - 1)n}{(d + n + 1)^2 \beta^2} < 0;$$

$$\frac{\partial s^{\text{SWEP}}}{\partial d} = -\frac{a^2 \left\{ [(d^2 + 2(d + 1)n + (d + 1)^2)](d + n + 1)\beta^2 + [2rn^2 + 2(d + r)(3d + 2)n] - 2(3d^2 + 5d + 2)n - 2(d + 1)^2(n + (d + 1))(-2n^2 r + (3d + 1)n + (d + 1)^2) \right\}}{2n(d + n + 1)^2 \beta^3} < 0.$$

2. The effects of $\beta$ :

$$\frac{\partial q^{EP}_i}{\partial \beta} = \frac{a(d + 1)}{(d + n + 1)2 \beta^2} > 0; \frac{\partial \pi^{EP}_i}{\partial \beta} = \frac{-a^2 (\beta d^3 + 2\beta d^2 - \beta d d^2 + \beta d - \beta n d ^2 - d)}{(d + n + 1)^2 n^2 \beta^3};$$

If $1 < \beta \leq \frac{(d+1)d}{d^2 + d - n}$, then $\frac{\partial q^{EP}_i}{\partial \beta} \geq 0$; if $\beta > \frac{(d+1)d}{d^2 + d - n}$, then $\frac{\partial q^{EP}_i}{\partial \beta} < 0;$$

$$\frac{\partial c^{\text{SWEP}}}{\partial \beta} = \frac{a^2 (\beta d + \beta n + \beta - d - 1)(d + 1)}{(d + n + 1)^2 \beta^3} > 0.$$
\[
\frac{\partial \sigma_{SW}^{EP}}{\partial \beta} = - \frac{(\beta - 1)d^2 - (\beta - 1)(nr - 1)d - n[\beta nr + (r + 1)\beta - r]}{n(d + n + 1)^2 \beta^3} (d + 1)a^2 > 0.
\]

3. The effects of \( n \):

\[
\frac{\partial q_i^{EP}}{\partial n} = \frac{-a[(d + n + 1)^2 - (d + 1)(d + 2n + 1)]}{n^3(d + n + 1)^2 \beta} < 0; \frac{\partial p_i^{EP}}{\partial n} = \frac{-a(d + 1)}{(d + n + 1)^2} < 0;
\]

\[
\frac{\partial n_i^{EP}}{\partial n} = \frac{a^2\{((\beta - 1)^2d^4 + (2n + 3)(\beta - 1)^2d^3 + [3(n + 1)\beta - 4n - 3](\beta - 1)d^2 + [(-n^3 - 3n^2 + 1)\beta]^2 + (3n^2 - 2n - 2)\beta + 2n + 1]d - (2n^2 + 3n + 1)\beta - 3n - 1]\beta n]}{n^3(d + n + 1)^3 \beta^2} > 0;
\]

\[
\frac{\partial c_s^{EP}}{\partial n} = \frac{a^2(\beta d + \beta n + \beta - d - 1)(d + 1)}{(d + n + 1)^3 \beta^2} > 0;
\]

\[
\frac{\partial \sigma_{SW}^{EP}}{\partial n} = \frac{a^2\{((\beta - 1)^2d^4 + 3(\beta - 1)^2(n + 1)d^3 + [(n^2 + 6n + 3)\beta]^2 + (2n^2r - 12n - 6)\beta - 2n^2r + 6n + 3]d^2 + [(-n^2 - n^2 + 3n + 1)\beta]^2 + (-2 + 2n^3 + 4n^2 + 4n^2 - 6n)\beta - 4n^2r + 3n + 1]d + 2n^2[(-n^2 - 1)\beta^2 + (nr + r + 2)\beta - r]}{2\beta^2 n^2(d + n + 1)^3} > 0.
\]

Appendix E

Proof of Proposition 5 The effects of \( \delta \) and \( r \):

\[
\frac{\partial \sigma_{SW}^{NN}}{\partial \delta} = \frac{n^2a^2}{2(d + n + 1)^2} > 0; \frac{\partial \sigma_{SW}^{EN}}{\partial r} = \frac{\beta^2n^2a^2}{2(\beta n + \beta + d)^2} > 0;
\]

\[
\frac{\partial q_i^{EG}}{\partial r} = \frac{a\beta n}{[d + (2\beta - r)n]^2} > 0; \frac{\partial p_i^{EG}}{\partial r} = \frac{-a\beta^2n^2}{(2\beta n - nr + d)} < 0;
\]

\[
\frac{\partial n_i^{EG}}{\partial r} = \frac{-\beta^2n^2a^2r}{[d + (2\beta - r)n]^3} < 0; \frac{\partial c_s^{EG}}{\partial r} = \frac{\beta^2n^2a^2}{(2\beta n - nr + d)^3} > 0;
\]

\[
\frac{\partial \sigma_{SW}^{EG}}{\partial r} = \frac{\beta^2a^2n^2}{2(2\beta n - nr + d)^3} > 0; \frac{\partial \sigma_{SW}^{EP}}{\partial r} = \frac{a^2(\beta d + \beta n + \beta - d - 1)^2}{2(d + n + 1)^2 \beta^2} > 0.
\]
Appendix F

Proof of Corollary 1
If \( 0 < \beta \leq \frac{m}{n-1} \), we can get:

\[
q_i^{EN} - q_i^{NN} = \frac{a\beta(r-\beta)n + \beta}{(d + \beta n + \beta)(d + n + 1)} > 0; q_i^{EG} - q_i^{EN} = \frac{\beta a(r-\beta)n + \beta}{(2\beta - r)n + d}(\beta n + \beta + d) > 0;
\]

\[
q_i^{EP} - q_i^{EN} = a\beta(r-\beta)n^2 + a\{\beta^2(d + 1) - [(r + 1)d + r + 2]\beta + r(d + 1)\}n + a(\beta - 1)(d + 1)
\]

\[
q_i^{EP} - q_i^{EG} = \frac{-a\beta(r-\beta)n^2 + a\{\beta^2(d + 1) - [(r + 1)d + r + 2]\beta + r(d + 1)\}n + a(\beta - 1)(d + 1)}{n(\beta n + \beta + d)} > 0;
\]

\[
p_i^{EN} - p_i^{NN} = \frac{a(\beta - 1)(\beta n + \beta + d)^2 + d}{(d + n + 1)[(\beta n + \beta + d)]^2} > 0; p_i^{EG} - p_i^{EN} = \frac{\beta a(n)\beta - \beta}{(2\beta - r)n + d}(\beta n + \beta + d) > 0;
\]

\[
p_i^{EP} - p_i^{NN} = \frac{a\{-\beta(r-\beta)n^2 + [(d + 1)(\beta^2 - (r + 1)d + r + 2)\beta + r(d + 1)]n + d(d + 1)(\beta - 1)\}}{(d + n + 1)[d + (2\beta - r)n]} > 0;
\]

\[
p_i^{EP} - p_i^{NN} = \frac{a\{-\beta(r-\beta)n^2 + [(d + 1)(\beta^2 - (r + 1)d + r + 2)\beta + r(d + 1)]n + d(d + 1)(\beta - 1)\}}{2\beta - r}] > 0;
\]

\[
\pi_i^{EN} - \pi_i^{NN} = \frac{a^2(\beta - 1)[2(n + 1 + d^2)\beta^2 + d(d + 2)(d + 2n + 2)\beta + d^2(d + 2)]}{2(n + 1 + d)[(n + 1)\beta + d]} > 0;
\]

\[
\pi_i^{EG} - \pi_i^{EN} = -\frac{\beta^2 a^2[(r-\beta)n + \beta][(2n^2 - 2n)\beta^2 + [(2r + d)n - d] + \beta + dnr]}{2[(n + 1)\beta + d]^2[(2\beta - n)r + d]} < 0;
\]

\[
\pi_i^{EG} - \pi_i^{NN} = \frac{-2a^2\beta^2(r-\beta)n^3 + a^2\{4(d + 1)\beta^3 - [(4 + 3)d + 4r + 8]\beta^2 + 4r(d + 2)\beta - r^2(d + 2)\}n + a^2\{2(d + 1)^2\beta^3 - 2(r - 1)d + r(d + 1)\beta^2 - 4d(d + 2)\beta + 2dr(d + 2)\}n + a^2\{d(d + 1)^2\beta^3 - d^2 - 2d\}^2}{2(d + n + 1)^2[d + (2\beta - r)n]} < 0;
\]
\[
\pi_i^{EN} - \pi_i^{NN} = - \frac{a^2 \{2n^2(d + n + 1)\beta^3 + [(2n + 1)d^2 + (n + 1)d - 2n^2 - 2n]\beta^2 + d(d^2 - 2dn - 3n - 1)\beta - \beta^4 - d^2\} \{d(n + 1)\beta + d^2 + d\}(\beta - 1)}{(d + n + 1)^2 \{d(n + 1)\beta + d\}^2 \beta^2 n^2} < 0; \\
\pi_i^{EP} - \pi_i^{NN} = - \frac{a^2 \{[\beta - 1]d^2 + (\beta - 1)d - 2\beta n\} \{d + 1\} (\beta - 1)}{2(d + n + 1)^2 \beta^2 n^2}; \\
\pi_i^{EP} - \pi_i^{EN} = - \frac{a^2 \{[\beta - 1]d^2 + (\beta - 1)d - 2\beta n\} \{d + 1\} (\beta - 1)}{2(d + n + 1)^2 \beta^2 n^2}; \\
\pi_i^{EN} - \pi_i^{NN}; \text{ if } 0 < d \leq \frac{\sqrt{8\beta^n + 8\beta n - 2\beta n + 2\beta n + 2\beta n}}{2(\beta - 1)}, \text{ then } \pi_i^{EN} \geq \pi_i^{NN}; \text{ if } d > \frac{\sqrt{8\beta^n + 8\beta n - 2\beta n + 2\beta n}}{2(\beta - 1)}, \text{ then } \pi_i^{EP} < \pi_i^{EN}; \\
\pi_i^{EP} - \pi_i^{EN}; \\
c^s_{EN} - c^s_{NN} = \frac{dn a^2 (\beta - 1) \{(d + 2n + 2)\beta + \beta\} \{d(n + 1)\beta + d\}^2}{2(n + 1 + d)^2 \{d(n + 1)\beta + d\}^2} > 0; \\
c^s_{EG} - c^s_{EN} = \frac{a^2 \{2\beta^3 n^2 + \beta \{2(d + 1)\beta^2 - (d + 4)\beta + r(d + 2)\} n^2 + d \{3(d + 1)\beta^2 - 1\} d^2 + (r + 3) d\beta + (r + 4)\beta - r(d + 1)\} n + d^2 \{d + 1\} (\beta - 1) + (\beta - 1) d - 2\beta n \} \{d + 1\} (\beta - 1)}{2(d + 2 \beta - r) n^2 (d + n + 1)^2 \beta^2 n^2}; \\
c^s_{EP} - c^s_{EN} = \frac{a^2 \{2\beta^3 n^2 + \beta \{2(d + 1)\beta^2 - (d + 4)\beta + r(d + 2)\} n^2 + d \{3(d + 1)\beta^2 - 1\} d^2 + (r + 3) d\beta + (r + 4)\beta - r(d + 1)\} n + d^2 \{d + 1\} (\beta - 1) + (\beta - 1) d - 2\beta n \} \{d + 1\} (\beta - 1)}{2(d + 2 \beta - r) n^2 (d + n + 1)^2 \beta^2 n^2}; \\
\sqrt{\left\{ \left( - \beta^2 + (r + 5)\beta - r \right) n + \beta - 1 \right\} (\beta - 1) \{(r - \beta) n - 1\} (\beta - 1) - \beta^2 n} \text{ if } 0 < d \leq \frac{\sqrt{\left\{- \beta^2 + (r + 5)\beta - r\right\} n + \beta - 1 \right\} (\beta - 1) \{(r - \beta) n - 1\} (\beta - 1) - \beta^2 n}}{2(\beta - 1)}, \text{ then } c^s_{EP} \leq c^s_{EN}; \text{ if } d > \frac{\sqrt{\left\{- \beta^2 + (r + 5)\beta - r\right\} n + \beta - 1 \right\} (\beta - 1) \{(r - \beta) n - 1\} (\beta - 1) - \beta^2 n}}{2(\beta - 1)}, \text{ then } c^s_{EP} > c^s_{EN}; \\
sw_{EN} - sw_{NN} = \frac{a^2 \{2n^2 + 2\beta n + 2\beta n + 2\beta n + 2\beta n\} \{d + 1\} (\beta - 1)}{2(\beta n + d) \{d + d + 1\} \beta^2 n^2}; \\
sw_{EG} - sw_{EN} = \frac{a^2 \beta^2 n \{(\beta - r) n - \beta\}^2}{2(2\beta - r) n + d(\beta n + \beta + d)} > 0.
\[ s_{EP}^{EN} - s_{EN}^{EP} = - \frac{a^2(\beta - 1)\{2n^2(d + n + 1)\beta^3 + [ - 2n^2 r - (2dr + 3r + 2)n^2 + 2d^2 n - (r - 1)dn - (r + 2)n + d^2 + d\beta]^2[r^2 - (r + 2)d^2 n - (3d - r)n + d^3 - d]\beta - d(d + 1)(d - nr)] \}[(d + n + 1)^2\beta + d^2 + d]}{2(d + n + 1)^2\beta^2 n[(n + 1)^2 + d]} > 0; \]

\[ s_{EP}^{EN} - s_{EN}^{EG} = - \frac{a^2 \{- r^2 \beta^2 (r - 2\beta)n^3 - [4(2r + 1)d\beta^3 + (8r + 7)\beta^3 - 8(r + 1)d\beta^3 - 8(r + 2)r\beta^2 + 2r + 7)d^3 \beta + 2(r + 5)^2\beta - 2r^2(d + 1)\beta n^4 + [ - 2(2dr + 2r + 3)(d + 1)^2 + 4(r^2 - 1)d^2 \beta^3 + 8r(d + 1)d\beta^3 + (r^2 + 4r + 2)\beta^3 - (r^2 + 4r - 5)d^3 \beta^3 - 2(r^2 + 7r + 2)d\beta^3 - (r^2 + 10r + 12)r\beta^2 + 2r^2(d + 1)(d + r + 3)\beta - r^2(d + 1)^2 n^3 + [(4d^3 + 9d^2 + 6d + 1)\beta^3 - 8d(r + 1)\beta d + 1)\beta^3 - 8(r + 2)(d + 1)\beta^2 + (3r^2 + 14r + 3)d^3 \beta^2 + (6r^2 + 34r + 14)d^2 \beta^3 + (3r^2 + 20r + 12)d\beta^3 6d^2 (r + 1)(d + 1)r\beta - 6d(r + 2)(d + 1)\beta^2 + 3d^2 (d + 1)^2 n^2 + d^2 (d + 1)(4(d + 1)\beta - (3r - 4)\beta - (r + 2)\beta - d(r + 1)n + d^3 (d + 1)^2 \beta - (r^2 + 1)\beta + d^2 + d]}{2(d + n + 1)^2\beta^2 n[(n + 1)^2 + d(2\beta - r)n]} < 0. \]

Appendix G

Proof of Corollary 2

If \( \beta > \frac{rn}{n-1} \), we can obtain:

\[ q_{EN}^{ii} - q_{NN}^{ii} = \frac{ad(\beta - 1)}{(d + \beta n + \beta)(d + n + 1)} > 0; q_{EN}^{ii} - q_{EN}^{ii} = \frac{\beta a(r - \beta)n + \beta}{[(2\beta - r)n + d](\beta n + \beta + d)} > 0; \]

\[ q_{EP}^{ii} - q_{EN}^{ii} = \frac{a(\beta - 1)|\beta(d + n + 1) + d^2 + d|}{n\beta(d + n + 1)((n + 1)^2 + d)} > 0; \]

\[ q_{EP}^{ii} - q_{EG}^{ii} = \frac{-a\beta(r - \beta)n^2 + a\{\beta^2(d + 1) - [(r + 1)d + r + 2\beta + r(d + 1)]n + ad(\beta - 1)(d + 1)\}}{n\beta(d + n + 1)((2\beta - r)n + d)} > 0; \]

\[ p_{EN}^{ii} - p_{NN}^{ii} = \frac{a(\beta - 1)|\beta(d + n + 1) + d^2 + d|}{(n + 1 + d)(n + 1)(\beta + d)} > 0; p_{EG}^{ii} - p_{EN}^{ii} = \frac{\beta^2 an[(r - \beta)n - \beta]}{(2\beta - r)n + d](\beta n + \beta + d)} \]

\[ < 0; \]

\[ p_{EG}^{ii} - p_{NN}^{ii} = \frac{a\{-\beta(r - \beta)n^2 + [(d + 1)\beta^2 - (r + 1)d\beta - r + 2\beta + r(d + 1)]n + d(d + 1)(\beta - 1)\}}{(d + n + 1)(d + (2\beta - r)n)} > 0; \]

\[ \pi_{EN}^{ii} - \pi_{NN}^{ii} = \frac{a^2(\beta - 1)|2n + 1 + d^2|\beta^3 + d(d + 2)(d + 2n + 2)\beta + d^2(d + 2)}{2(n + 1 + d)(n + 1)(\beta + d)^2} > 0; \]

\[ \pi_{EG}^{ii} - \pi_{EN}^{ii} = \frac{\beta^2 a^2(r - \beta)n + \beta][(2n^2 - 2n)\beta^2 + [(2r + d)n - d\beta + dr)]}{2(n + 1)\beta + d^2 (2\beta - n)r + d^2} < 0; \]
\[
\pi_{iG}^E - \pi_{iN}^E = \frac{-2a^2\beta^2(r-\beta)n^3 + a^2\{4(d+1)\beta^3 - [(4+3)d + 4r + 8]\beta^2 + 4r(d+2)\beta - r^2(d+2)\}n^2 + a^2\{2(d+1)^2\beta^2 - 2[(r-1)d + r](d+1)\beta^2 - 2d\}r}{2d + (\beta - r)n^2} < 0;
\]

\[
\pi_{iP}^E - \pi_{iN}^E = -\frac{a^2\{2n^2(d + n + 1)\beta^3 + [(2n + 1)\beta^2 + (n + 1)d - 2n^2 - 2n]\beta^2 + d(d^2 - 2dn)}{(d + n + 1)^2[(n + 1)\beta + d]^2\beta^2 n^2} < 0;
\]

\[
\pi_{iP}^E - \pi_{iN}^E = -\frac{a^2\{[(\beta-1)d^2 + (\beta-1)d - 2bn](d + 1)\beta(-1)\}}{(d + n + 1)^2\beta^2 n^2} > 0;
\]

\[
\pi_{iP}^E - \pi_{iE}^G = -\frac{1}{2d + (2\beta - r)n^2}(d + n + 1)^2\beta^2 n^2 < 0;
\]

\[
\cs_{EN}^E - \cs_{NN}^E = \frac{dn^2a^2(\beta - 1)\{(d + 2n + 2)\beta + d\}}{2(n + 1 + d)^2((n + 1)\beta + d)^2} > 0;
\]

\[
\cs_{EG}^E - \cs_{EN}^E = 2\beta^2 n^2 \{(r - \beta)n + \beta\}((3 \beta - r)n + 2d + \beta) > 0;
\]

\[
\cs_{EP}^E - \cs_{EG}^E = \frac{a^2\{\beta(3 \beta - r)n^2 + [3(d + 1)\beta^2 - (r + 1)d\beta + (r + 2)\beta - r(d + 1)]n + d(d + 1)(\beta - 1)\}}{2d + (2\beta - r)n^2}(d + n + 1)^2\beta^2 n^2 > 0;
\]

\[
\sw_{EN}^E - \sw_{NN}^E = \frac{n\beta^2 a^2(nr + d + 2\beta)(n + d + 1)^2 - na^2(n\delta + d + 2)(\beta n + d + \beta)^2}{2(\beta n + d + \beta)^2(n + d + 1)^2} > 0;
\]

\[
\sw_{EG}^E - \sw_{EN}^E = \frac{a^2\beta^2 n[(\beta - r)n + \beta]^2}{2([\beta - r)n + d)(\beta n + d + \beta)} > 0.
\]

\[
\sw_{EP}^E - \sw_{EN}^E = -\frac{a^2\beta^2 n(\beta - r)n\beta^2}{2d + (n + 1)^2\beta^2 n[(n + 1)\beta + d]^2} > 0;
\]
\[
\begin{align*}
sw_{EP}^{\text{SW}} - sw_{EG}^{\text{SW}} &= -\frac{a^2 \{-r^2 (r-2) \beta n^2 - \{(4(2r + 1) d\beta^3 + (8r + 7) \beta^3 - 8(r + 1) d\beta^2 - 8(r + 2) \beta^2 + 2r + 7) d\beta^2 + 2(r + 5) r^2 \beta - 2r^2 (d + 1) \beta n^4 + \{-2(2dr + 2r + 3)(d + 1) \beta^4 + 4(r^2 - 1) d\beta^3 + 8r(r + 2) d\beta^3 + 4(r^2 + 4r + 2) \beta^3 - (r^2 + 4r - 5) d\beta^2 - 2(r^2 + 7r + 2) d\beta^2 - (r^2 + 10r + 12) r\beta^2 + 2r^2 (d + 1)(dr + r + 3) \beta - r^3 (d + 1)^2 \beta n^3 + \{(4d^3 + 9d^2 + 6d + 1) \beta^4 - 8d(r + 1)d(d + 1) \beta^3 - 8(r + 2)(d + 1) \beta^3 + (3r^2 + 14r + 3)d^2 \beta^2 + (6r^2 + 34r + 14)d^2 \beta^2 + (3r^2 + 20r + 12)d\beta^2 6d^2 (r + 1)(d + 1) r\beta - 6d(r + 2)(d + 1)r\beta + 3dr^2 (d + 1)^2 \beta n^2 + d^2 (d + 1) \beta^2 (3r - 4) d\beta - (r + 2) \beta - r(d + 1) n + d^2 (d + 1)^2 (\beta - 1)^2\} \}}{2(d + n + 1)^2 \beta^2 n [d + (2\beta - r)n]^2} < 0.
\end{align*}
\]