Reply to Marinatto’s comment on “Bell’s theorem without inequalities and without probabilities for two observers”

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It is shown that Marinatto’s claim [Phys. Rev. Lett. 90, 258901 (2003)] that the proof of “Bell’s theorem without inequalities and without probabilities for two observers” [A. Cabello, Phys. Rev. Lett. 86, 1911 (2001)] requires four spacelike separated observers rather than two is unjustified.

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In his Comment [1], Marinatto claims that the proof of Bell’s theorem without inequalities in [2] requires four spacelike separated observers rather than two, as asserted in [2]. Marinatto’s claim is based on the fact that to test some of the properties used in the proof, for instance the property

\[ P_\psi(B_2 = B_4 | A_1 A_3 = +1) = 1, \]

one of the observers (Bob) must measure the spin along one direction of his particle 2, B_2, and also the spin along one direction of his particle 4, B_4. Marinatto argues that, since both measurements are not spacelike separated, then measuring B_2 could disturb v(B_4) (the element of reality corresponding to B_4), and measuring B_4 could disturb v(B_2) (the element of reality corresponding to B_2). Therefore, he maintains that, to avoid such possible disturbances, both measurements should be spacelike separated.

However, such a prevention is not needed, because it can be demonstrated that measuring B_2 does not disturb v(B_4), and measuring B_4 does not disturb v(B_2). Let us recall Einstein, Podolsky, and Rosen’s (EPR) criterion for elements of reality: “If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity” [3]. In the scenario described in [2], Alice, by means of a spacelike separated measurement of the spin along one direction of her particle 1, A_1, can use the property

\[ P_\psi(A_1 = B_2) = 0 \]

(2)

to predict with certainty the element of reality v(B_2). Analogously, she, by means of a spacelike separated measurement of A_3 on her particle 3, can use the property

\[ P_\psi(A_3 = B_4) = 0 \]

(3)

to predict with certainty the element of reality v(B_4). Note that, according to EPR’s criterion, what allows Alice to conclude that there is an element of reality, for instance v(B_2), is the fact that the result of Bob’s measurement of B_2 can be predicted with certainty. The fact that close to particle 2 there could exist (or not) a second particle (in this case particle 4) on which Bob could perform one measurement or other does not invalidate Alice’s prediction, since, according to the predictions of quantum mechanics (presumably corroborated by any conceivable experiment), the presence (or absence) of particle 4 close to particle 2, or the fact that Bob performs one experiment or other on particle 4, do not change the result for B_2 predicted by Alice (otherwise this effect could be used to transmit information between spacelike separated regions). Therefore, I must conclude that Marinatto’s argument does not justify the need for more spacelike separated observers. Indeed, the proof of Bell’s theorem without inequalities using four qubits [2] can be transformed into a proof using only two particles [4, 5].

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