Measurement Error Analysis of Combined Doppler Sonar Using Adaptive Algorithm

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Abstract:
This paper presents a measurement method combining conventional and coherent Doppler sonar using an adaptive algorithm to reduce measurement error at a wide range of SNRs. In our previous paper, we proposed a combined method to provide accurate and precise velocity using a fixed range of ambiguity velocity. The combined method worked well at high SNRs, but at low SNRs it was not as accurate. In order to provide accurate velocity at a wide range of SNRs, an adaptive algorithm for the range of ambiguity velocity is proposed at navigators’ request. The results of theoretical and numerical error analyses showed that the combined method using the adaptive algorithm provides accurate and precise velocities at a wide range of SNRs.

Classification: Signal processing; Miscellaneous (Observations, Measurements, etc.)
Keyword: measurement of velocity, combined Doppler sonar, velocity ambiguity, adaptive algorithm, error analysis

1. Introduction
Recently, ocean exploration has attracted a great deal of attention. Both the position and velocity of ships and underwater vehicles are important data in drawing the topography of the seabed and determining the locations of objects. Conventional Doppler sonar (CNDS), up until now, has been used on moving vehicles to provide velocity information without ambiguity, but CNDS cannot provide precise velocity at low signal-to-noise ratios (SNR). In order to improve the precision of velocity, a smoothing operation which requires a few seconds time lag has been introduced to CNDS. From the results of a questionnaire survey, navigators who operate vessels over 50,000 gross tonnages generally use CNDS or their sense of sight to obtain velocity information. However, they are not always satisfied with the current accuracy of CNDS, and identified a need for more accurate velocity information, within a few cm/s, in the situation of docking or anchoring. From the experimental results in the situation of docking, there was a time lag of a few seconds to obtain the velocity information from CNDS. The time lag is mainly caused by the smoothing operation, and depends on the number of velocity measurements.
used to obtain the average. For safety navigation in the situation of docking or anchoring, it is desired to provide more accurate information of velocity without a time lag, even under the circumstance of a low SNR.

Pulse-to-pulse coherent Doppler sonar (CHDS) was developed to provide accurate velocity measurement with a short time lag \(^3\)\(^5\), but the occurrence of range and velocity ambiguities has limited more general application of CHDS. In our previous paper\(^6\), a combined method (CMDS) of CNDS and CHDS to provide accurate velocity using a fixed range of ambiguity velocity was proposed. Accordingly, the fixed range of ambiguity velocity allowed the velocity measurements by CMDS to be as precise as those by CHDS at high SNR, because the velocity error of CNDS is much smaller than the range of ambiguity velocity. However, at low SNR, the proposed method has the same performance level as CNDS, because the velocity error of CNDS becomes larger than the range of ambiguity velocity, and the CMDS error is dominated by impulsive noises. In the previous paper\(^6\), CMDS using the fixed range of ambiguity velocity could not provide accurate and precise velocity at wide SNR, especially at low SNR.

In our previous analytical results\(^6\), the measurement error of CMDS (CMDS error) was calculated by means of the CHDS error, the CNDS error, and the range of ambiguity velocity at each SNR. The CHDS error increased as the range of ambiguity velocity increased, but the error associated with impulsive noise was decreased as the range of ambiguity velocity increased. Accordingly, we can find a minimum CMDS error at each SNR; from this we can decide the optimum range of ambiguity velocity based on the minimum CMDS error. In order to provide precise and accurate velocity at a wide range of SNRs, we propose an adaptive algorithm to decide the optimum range of ambiguity velocity for CMDS.

In this paper, first the structure of CMDS using the adaptive algorithm of the range of ambiguity velocity and the decision algorithm of the integer factor is introduced. In regard to CMDS, if a velocity exists near a boundary of a decision of integer factor, the measured velocity by CNDS would easily exceed the boundary, and generate a lot of impulsive noise. We call the above effect a near-boundary-error effect. Accordingly, we introduce a variable shift to reduce the error effect near the boundary. Subsequently, the theoretical error analysis of the proposed CMDS was carried out. In our theoretical error analysis, we assumed that the two measurement errors by CNDS and CHDS are statistically independent.

Next, in order to evaluate the proposed CMDS using an adaptive algorithm, numerical error analysis was carried out. From the results of the chi square test, we show that the measurement errors by CNDS and CHDS are statistically independent. Since the error characteristics of the theoretical and numerical results are fairly consistent with each other, our theoretical results can be used to evaluate the proposed CMDS. Consequently, we show that the proposed CMDS can provide accurate and precise velocity at a wide range of SNRs. Finally, we summarise our research.

2. Adaptive Type of CMDS

2.1 CMDS using an Adaptive Algorithm

In our previous paper\(^6\), we proposed a method of CMDS using fixed range of ambiguity velocity that provides accurate and precise velocity at high SNRs, but this method was significantly and negatively affected by the impulsive noises at low SNRs. In order to provide accurate and precise velocity at a wide range of SNRs, CMDS using
an adaptive algorithm for the range of ambiguity velocity is proposed.

A functional block diagram of CMDS using the adaptive algorithm of the range of ambiguity velocity is shown in Fig. 1. Signals are received by hydrophone and sent to CNDS and CHDS. In the CNDS, coarse velocity including noise \( (v_n + \epsilon_n) \) is measured and forwarded to the adaptive algorithm for the range of ambiguity velocity and the decision algorithm of integer factor. The adaptive algorithm of the range of ambiguity velocity should determine the optimum range of ambiguity velocity \( (\Delta v_a) \) based on the minimum CMDS error. By means of the optimum range of ambiguity velocity and the signal received by hydrophone, the precise velocity including noise \( (v_h + \epsilon_h) \) is then measured by CHDS. With the use of \( \Delta v_a \), \( (v_n + \epsilon_n) \) and \( (v_h + \epsilon_h) \), a decision algorithm for integer factor \( n \) including a variable shift technique is proposed. The variable shift technique changes the range of the variable to a suitable range to eliminate any error effect near the boundary. After carrying out the variable shift, the most probable integer factor \( \hat{n} \) is decided by means of a derived inequality, and the optimum range of ambiguity velocity \( (\Delta v_a) \) is also used. Finally, by combining the most probable range of ambiguity velocity and the precise velocity including noise \( (v_h + \epsilon_h) \), a precise and accurate measurement of velocity by CMDS \( (v_m) \) is obtained.

2.2 Adaptive Algorithm of range of Ambiguity velocity

At low SNR, the CMDS error using a fixed range of ambiguity velocity included many impulsive noises due to wrong decisions for the integer factor. When the range of ambiguity velocity is wider, the probability of a wrong decision for the integer factor becomes lower. Therefore, if the range of ambiguity velocity were made larger, the CMDS error using a variable range of ambiguity velocity would become smaller. On the other hand, the CMDS error also depends on CHDS error. As the range of ambiguity velocity is wider, the CHDS error becomes larger. Therefore, a wide range of ambiguity velocity does not necessarily reduce the CMDS error. As CMDS with an adaptive algorithm can take advantage of the above two contrary characteristics of the CMDS error at a variable range of ambiguity velocity, it can provide accurate and precise velocity at wide range of SNRs.

In the process of CMDS using the adaptive algorithm, the most important step is to determine the optimum range of ambiguity velocity based on the minimum CMDS error. The flow chart for the adaptive algorithm for the range of ambiguity velocity is shown in Fig. 2. From the measurement results of coarse velocity including noise, standard deviations \( (\sigma_n) \) are calculated, and SNRs are estimated by means of the frequency characteristics of the measurement results. On the other hand, the range of ambiguity velocity of CHDS is deter-
mined using dual time intervals, where the minimum value of the range of ambiguity velocity is zero, and the maximum value of the range of ambiguity velocity is $\Delta v_u$. Next, the range of ambiguity velocity $\Delta v$ limited to the range $(0, \Delta v_u]$, is used to calculate the standard deviation of the CHDS error ($\sigma_h$) at the estimated SNR. From the results of $\sigma_n$, $\sigma_h$ and $\Delta v_u$, the values of the standard deviation of the CMDS error ($\sigma_m$) can be calculated. From 0 to $\Delta v_u$ at intervals of $\Delta v$, the CMDS error ($\sigma_m$) is obtained for each range of ambiguity velocity. Consequently, the optimum range of ambiguity velocity $\Delta v_a$ is selected based on the minimum CMDS error $\sigma_m$.

According to Eq. (1), the range of ambiguity velocity is determined by the carrier frequency $f_c$ and the time interval $\tau$. In order to change the range of the ambiguity velocity, the carrier frequency or the time interval should be changed. Two separate frequencies are used in the multi-frequency method\textsuperscript{7,8). On the other hand, two different time intervals are used in the dual time interval method\textsuperscript{9–11). In this research, the dual time interval method is selected, because the multi-frequency method requires a wide frequency band. The value of the range of ambiguity velocity using the dual time interval method is\textsuperscript{9)}

$$\Delta v = \frac{c}{4f_c(\tau_2 - \tau_1)} ,$$

where $\tau_1$ is the first time interval and $\tau_2 (\tau_2 > \tau_1)$ is the second time interval. Consequently, the range of ambiguity velocity can be varied due to the difference $(\tau_2 - \tau_1)$.

### 2.3 Decision Algorithm of Integer Factor

#### 2.3.1 Variable Shift

As shown in our previous paper\textsuperscript{6), the integer factor $n$ can be estimated by the following inequalities:

$$n - \frac{1}{4} < \frac{v_n + \varepsilon_n}{2\Delta v} \leq n + \frac{3}{4} \quad (v_n > 0) ,$$

$$n - \frac{3}{4} < \frac{v_n + \varepsilon_n}{2\Delta v} \leq n + \frac{1}{4} \quad (v_n \leq 0) .$$

If the value $v_n/(2\Delta v)$ approximates each boundary in inequalities (3) and (3'), the decision variable, $(v_n + \varepsilon_n)/2\Delta v$, will exceed the boundary due to

![Flow chart for adaptive algorithm for the range of ambiguity velocity.](image)
the error component $\varepsilon_n/(2\Delta v)$. An incorrect integer factor may result from this excess error. In order to reduce this excess error, a variable shift is proposed and explained as follows.

In a noise-free process, $v_n$ is the true velocity and $v_h$ is the same true velocity including the most probable range of ambiguity velocity. Accordingly, the relationship between $v_n$ and $v_h$ can be expressed as:

$$v_n - v_h = 2n\Delta v.$$  \hfill (4)

In the actual process of measurement, each measurement noise should be considered, and the difference between the two measurement values by CMDS and CHDS can be expressed as:

$$\frac{(v_n + \varepsilon_n) - (v_h + \varepsilon_h)}{2\Delta v} = n + \frac{\varepsilon_n - \varepsilon_h}{2\Delta v}. \quad (5')$$

We propose using the left side of Eq. (5') as the new decision variable to clear away the error effect near the boundary. The new decision variable is shifted from the previous decision variable with $-(v_h + \varepsilon_h)/2\Delta v$. Accordingly, the previous error effect near the boundary is cleared, and the probability that the integer factor $n$ can be correctly determined becomes independent of the true velocity, $v_n$.

### 2.3.2 Decision Algorithm of Integer Factor

In the decision algorithm of the integer factor, the inequality used to determine the most probable integer factor $\hat{n}$, can be expressed:

$$n - \frac{1}{2} < \frac{v_n + \varepsilon_n - (v_h + \varepsilon_h)}{2\Delta v} \leq n + \frac{1}{2}.$$

(6)

In Fig. 3, a flow chart for the decision algorithm of integer factor $n$ is shown, and the procedure of the decision algorithm is described below.

1) Input the coarse velocity measured by CNDS $(v_n + \varepsilon_n)$, the precise velocity measured by CHDS $(v_h + \varepsilon_h)$, and the range of ambiguity velocity $(\Delta v)$.

2) Carry out the variable shift, and calculate $[(v_n + \varepsilon_n) - (v_h + \varepsilon_h)]/(2\Delta v)$.

3) Round $[(v_n + \varepsilon_n) - (v_h + \varepsilon_h)]/(2\Delta v)$, and decide the most probable integer factor $\hat{n}$.

### 3. Theoretical Analysis

#### 3.1 Measurement Error of Decision Algorithm

From our previous paper, the probability density functions of $\varepsilon_n$ and $\varepsilon_h$ can be expressed as

$$p_n(\varepsilon_n) = \frac{1}{\sqrt{2\pi} \sigma_n} e^{-\frac{\varepsilon_n^2}{2\sigma_n^2}} \quad \text{and}$$

(7)

$$p_h(\varepsilon_h) = \frac{1}{\alpha} \int_{-\pi}^{\pi} p(\phi) p\left(\frac{\varepsilon_h}{\alpha} - \phi\right) d\phi,$$

(8)

and the standard deviations of measurement error for CNDS $(\sigma_n)$ and CHDS $(\sigma_h)$ can be expressed as:

$$\sigma_n = \frac{1}{\tau_0 \sqrt{2E_s/N_0}} \times \frac{c}{f_c} \quad \text{and}$$

(9)

$$\sigma_h = \frac{c}{4\pi f_c^2 \tau} \times \sqrt{\int_{-\pi}^{\pi} \phi^2 p_s(\phi) d\phi}.$$  \hfill (10)

From Eq. (1), Eq. (10) can be changed as below:
\[
\sigma_h = \frac{\Delta v}{\pi} \times \int_{-\pi}^{\pi} \phi^2 p_\phi(\phi) d\phi , \quad (10')
\]

where

\[
\begin{align*}
\alpha &= \frac{c}{4\pi f_r} ; \\
p(\phi) &= e^{-\gamma} \left\{ 1 + \sqrt{\pi} \gamma \cos \phi \right\} \\
\times \left[ 1 + \text{erf} \left( \sqrt{\pi} \gamma \cos \phi \right) \right] e^{\cos^2 \phi} \quad (12)
\end{align*}
\]

\( \gamma \) is the SNR of the signal through the low pass filter in CHDS;

\[
p_0(\phi) = \int_{-\pi}^{\pi} p(\phi) p(\phi - \phi)d\phi ;
\]

\( E_s \) is the energy of the signal;

\( \frac{N_0}{2} \) is the noise power per Hertz;

\[
e_0^2 = \frac{(2\pi)^2}{\pi} \int_{-\infty}^{\infty} [a_0(t)]^2 dt ;
\]

\( a_0(t) \) is the envelope of the signal;

\( \phi \) is the phase limited from \(-\pi\) to \(\pi\); and

\( \text{erf}(x) \) is the error function.

Next, in order to obtain the measurement error numerically derived from the variable shift process, the left-side term of Eq. (4) is substituted into inequality (6). Accordingly, inequality (6) becomes

\[
n_d - \frac{1}{2} \leq \frac{2n_0\Delta v + e_n - e_h}{2\Delta v} \leq n_d + \frac{1}{2} , \quad (11)
\]

\[
\Delta n_d - \frac{1}{2} \leq \frac{e_n - e_h}{2\Delta v} \leq \Delta n_d + \frac{1}{2} , \quad (12)
\]

where \( n_0 \) is the true integer factor, \( n_d \) is the decided integer factor using inequality (6), and the decided integer error is defined as \( \Delta n_d \) (\( \Delta n_d = n_d - n_0 \)). In the inequality (12), \( e_n = (e_n - e_h)/(2\Delta v) \) means the error factor used to identify a wrong integer factor.

The measurement error by CNDS, \( e_n \), and by CHDS, \( e_h \), are different velocity errors processed by CNDS and CHDS, respectively. The velocity by CNDS is measured from the Doppler shift frequency, but the velocity by CHDS is measured from the coherent phase shift. Accordingly, we assume that the measurement error by CNDS, \( e_n \), and that by CHDS, \( e_h \), are statistically independent; we verify this assumption later in Sec. 4 by a statistical test. Based on the above assumption, the probability density function of the error factor, \( e_t \), can be obtained using the convolution integral of two random variables\(^{(13)}\) and is expressed as

\[
p_t(e_t) = \int_{-\infty}^{\infty} p_n(e_t + \frac{e_h}{2\Delta v}) p_h \left( \frac{e_h}{2\Delta v} \right) d\frac{e_h}{2\Delta v} . \quad (13)
\]

Eq. (13) shows the probability density function of the new variable obtained using the variable shift.

Next, in order to obtain the probability of a decided integer error, \( \Delta n_d \), we use the following:

\[
P_t(\Delta n_d) = \int_{\Delta n_d - \frac{1}{2}}^{\Delta n_d + \frac{1}{2}} p_t(e_t) de_t . \quad (14)
\]

Consequently the standard deviation of CMDS error using the variable shift can be expressed\(^{(6)}\) as

\[
\sigma_m = \sqrt{\sigma_n^2 + \sum_{\Delta n_d} P_t(\Delta n_d) \left( 2\Delta n_d \Delta v \right)^2} . \quad (15)
\]

3.2 Measurement Error of Adaptive Algorithm

The core calculation in the adaptive algorithm is to find the optimum range of ambiguity velocity, \( \Delta v_a \), from the minimum CMDS error at any SNR shown before the end in Fig. 2. First, the CMDS error \( (\sigma_m) \) at one SNR is obtained from 0 to \( \Delta v_a \) at intervals of \( \Delta v \). Then the minimum CMDS error \( (\sigma_m) \) can be selected for the SNR, and the optimum range of ambiguity velocity, \( \Delta v_a \), can be obtained. Next, when the range of SNR is changed to a new
value, a new minimum CMDS error is calculated. Consequently, the minimum CMDS error and the optimum range of ambiguity velocity are simultaneously obtained. This calculation process can be expressed as

\[ \Delta v_0 = \arg \min_{\Delta v \in [0, \Delta v_u]} \sigma_m(SNR, \Delta v). \]  

(16)

Next, we determine the standard deviation of CHDS error using the optimum range of ambiguity velocity. Eq. (10′) shows the standard deviation of CHDS error using the range of ambiguity velocity. Therefore, \( \Delta v \) in Eq. (10′) is changed to \( \Delta v_a \). As a result of the change, the standard deviation of the CHDS error using the optimum range of ambiguity velocity is obtained as

\[ \sigma_h = \Delta v_a \times \sqrt{\int_{-\pi}^{\pi} \phi^2 p_\phi(\phi) d\phi}. \]  

(17)

In Eq. (15), \( \sigma_m \) means the standard deviation of CMDS error using a fixed range of ambiguity velocity and the variable shift. Consequently, \( \Delta v \) and \( \sigma_h \) in Eq. (15) are replaced by \( \Delta v_a \) and \( \sigma_{ha} \); then the CMDS error using the optimum range of ambiguity velocity can be expressed as

\[ \sigma_{ma} = \sqrt{\sigma_{ha}^2 + \sum_{\Delta \eta \in Z} P_{\eta}(\Delta \eta, \Delta v_a)^2}. \]  

(18)

4. Error Analysis

In this section, in order to evaluate our proposed CMDS using the adaptive algorithm, we carried out a numerical analysis. A comparison of the theoretical and numerical error analysis results is carried out to verify the equations in theoretical analysis. In order to compare the performance between numerical and experimental results, the conditions were adjusted to the characteristics of the experimental facilities in our laboratory.

4.1 Conditions

In this numerical analysis, the transducer was considered a fixed point, and the hydrophone was assumed to move away from the transducer at a constant velocity as shown in Fig. 4. The sign of the hydrophone’s velocity was determined by the moving direction, which was negative when the hydrophone moved away from the transducer and positive when it moved towards it.

Table 1 shows some basic conditions to carry out the theoretical and numerical analysis. These conditions were appropriate for the experimental facilities in our laboratory. In this analysis, the transducer transmitted a series of square pulses with two time intervals. As the difference between the two time intervals uniquely established the range of ambiguity velocity, we could select any range of ambiguity velocity by adjusting the two time intervals. The received signal was set as the transmitted square pulse with a time delay and the additive white Gaussian noise (AWGN). The

| Pulse envelope, \( a_0(t) \) | Square |
| Pulse length, \( \eta_0 \) (ms) | 0.6 |
| Carrier frequency, \( f_c \) (kHz) | 200 |
| Sampling frequency (MHz) | 10 |
| Sound speed, \( c \) (m/s) | 1500 |
| Maximum of range of ambiguity velocity, \( \Delta v_u \) (m/s) | 1.875 |
| Step of range of ambiguity velocity, \( \Delta v_s \) (m/s) | 0.038 |
| Fixed range of ambiguity velocity, \( \Delta v \) (m/s) | 0.188 |
| Moving velocity of hydrophone, \( v \) (m/s) | −2.500 |

Fig. 4 Apparatus for numerical calculation.
numerical calculations were carried out using 98 velocity samples at every 1 dB SNR between −10 to 19 dB. The definition of SNR is as follows:

\[
SNR = \frac{\text{average signal power}}{\text{average noise power}} = \frac{1}{t_0} \int_0^{t_0} s(t)^2 \, dt \quad \text{and} \quad \frac{1}{t_0} \int_0^{t_0} g(t)^2 \, dt,
\]

(19)

where

\( s(t) \) is the square pulse used in the error analysis;

\( g(t) \) is the generated AWGN; and

\( t_0 \) is the pulse length.

Before performing the numerical analysis, the independence of the two measurement errors by CNDS and CHDS at the SNR of −10 dB was carried out by use of the chi square statistical test. As a result of the test, it was verified that the above two measurement errors were statistically independent based on a 1% level of statistical significance.

4.2 Results of Theoretical Error Analysis

The theoretical error analysis was carried out using the parameters shown in Table 1. In Fig. 5 the standard deviation error by CMDS using a variable shift with a 0.188 m/s range of ambiguity velocity at 3 dB SNR is shown. In Fig. 5 the dashed line shows the calculation of CMDS without a variable shift from our previous paper, and the bold line shows the calculation of CMDS using the variable shift shown in Eq. (15). We found that the CMDS error calculated with a variable shift was reduced and became flat. Our proposed variable shift process was proved to be effective at clearing away the error effect near the boundary.

In Section 3, the measurement error of CMDS using the adaptive algorithm is shown in Eq. (18), and the measurement errors of CNDS, CHDS and CMDS using a fixed range of ambiguity velocity are obtained from Eqs. (9), (10) and (15). Figure 6 shows some comparative results of standard deviation of measurement errors among CNDS, CMDS using fixed ambiguity, CHDS, and CMDS using the adaptive algorithm. In Fig. 6, CMDS using the adaptive algorithm at a velocity of −2.500 m/s could provide much better measurement results than CMDS using the fixed ambiguity proposed in our previous paper at a wide range of SNRs. Our currently proposed CMDS using the adaptive algorithm cannot provide smaller measurement error than CHDS shown in the previous paper.
Although the CHDS error is smaller than the CMDS error using adaptive algorithm, the velocity measurement of CHDS has the property of ambiguity velocity. The measurement range of CHDS has a restricted range of ambiguity velocity (0.188 m/s). The CMDS using the adaptive algorithm does not have the property of ambiguity velocity, and is more reliable and effective than CHDS.

4.3 Results of Numerical Error Analysis

In Fig. 2, as the optimum range of ambiguity velocity based on the minimum CMDS error was selected, our proposed CMDS using the adaptive algorithm was able to provide accurate and precise velocities at a wide range of SNRs. By the numerical conditions, the optimum ambiguity velocities and the minimum CMDS errors using the adaptive algorithm were calculated at a wide range of SNRs.

The CMDS error was calculated using the adaptive algorithm and the conditions in Table 1. In Fig. 7, the relationship between CMDS error and the range of ambiguity velocity is shown, and there is only one optimum range of ambiguity velocity based on the minimum CMDS error, shown as the dot. For a smaller range of ambiguity velocity, less than 1.163 m/s, the impulsive noises are overwhelming. With a larger range of ambiguity velocity, greater than 1.163 m/s, the CHDS error grows progressively. Accordingly, the optimum value of the range of ambiguity velocity was identified as 1.163 m/s, and the minimum CMDS error was obtained as 0.045 m/s. The CMDS error using the fixed range of ambiguity velocity (0.188 m/s) at −10 dB is shown as a circle in Fig. 7.

The relation between the optimum range of ambiguity velocity and the range of SNR using the adaptive algorithm is shown in Fig. 8. In Fig. 8, as the range of SNR became lower, the optimum range of ambiguity velocity became larger. The reason for this is that CNDS provides a larger measurement error at the lower range of SNR; thus, a larger range of ambiguity velocity is required to decrease the probability of impulsive noise.

Both the numerical and theoretical standard deviations of CNDS, CHDS, CMDS using fixed range of ambiguity velocity, and CMDS using the adaptive algorithm at different SNRs are shown in Fig. 9. Figure 9 (a) shows the comparative results of measurement errors between the four types of Doppler sonar. In particular, in order to present the effect of the adaptive algorithm, the CHDS error and the CMDS using the adaptive algorithm are
magnified and shown in Fig. 9(b). In Fig. 9(a) and (b), it is clear that CMDS error using the adaptive algorithm becomes considerably smaller than the CMDS error using a fixed range of ambiguity velocity. Especially, at low SNR, the proposed adaptive algorithm had a profound effect in reducing the measurement error.

Table 2 shows the quantitative results of the theoretical and numerical errors by these four methods. In Table 2, the average standard deviation errors by these four methods were calculated at every 1 dB SNR from −10 to 19 dB. At the comparatively lower range of SNR from −10 to 4 dB, the effectiveness of the adaptive algorithm was clearly found, and the adaptive algorithm reduced the measurement error to one tenth that of the CMDS error using a fixed range of ambiguity velocity. In the higher range of SNR from 5 to 19 dB, the measurement error, except for CNDS, was almost the same and fairly small. In the wide range of SNR from −10 to 19, the measurement error by CMDS using the adaptive algorithm was about one tenth the measurement errors by CMDS using a fixed range of ambiguity velocity and by CNDS.

The necessary numbers of smoothing to achieve the standard deviation of 0.01 m/s at different SNRs are shown in Fig. 10 for each of the four types of Doppler sonar. The necessary number of data for smoothing ($N_s$) is calculated by the equation

$$\sigma^2 = \frac{\sigma_0^2}{N_s},$$

Table 2. Theoretical and numerical averaged measurement errors by four types of Doppler sonar.

| Method                        | SNR (dB) | −10−4 | −5−1 | 0−4 | 5−9 | 10−14 | 15−19 | 16−19 |
|-------------------------------|----------|-------|------|-----|-----|-------|-------|-------|
| CNDS                          | $\sigma_T$ | 0.214 | 0.120| 0.068| 0.038| 0.021| 0.012| 0.079 |
|                              | $\sigma_N$ | 0.229 | 0.132| 0.068| 0.038| 0.022| 0.014| 0.084 |
| CHDS                          | $\sigma_T$ | 0.003 | 0.002| 0.001| 0.000| 0.000| 0.000| 0.001 |
|                              | $\sigma_N$ | 0.003 | 0.002| 0.001| 0.001| 0.000| 0.000| 0.001 |
| CMDS using fixed range of ambiguity velocity | $\sigma_T$ | 0.251 | 0.139| 0.039| 0.002| 0.000| 0.000| 0.072 |
|                              | $\sigma_N$ | 0.257 | 0.161| 0.036| 0.001| 0.000| 0.000| 0.076 |
| CMDS using adaptive algorithm | $\sigma_T$ | 0.031 | 0.010| 0.003| 0.001| 0.000| 0.000| 0.008 |
|                              | $\sigma_N$ | 0.027 | 0.009| 0.003| 0.001| 0.000| 0.000| 0.007 |

$\sigma_T$: Theoretical standard deviation error (m/s), $\sigma_N$: Numerical standard deviation error (m/s).
where

\[ \sigma \] is the standard deviation of measurement error for each type of Doppler sonar; and

\[ \sigma \] is the reduced standard deviation of measurement error.

From Fig. 9 (a), the standard deviation of CNDS measurement error at −5 dB SNR is 0.175 m/s (= \( \sigma_o \)). The reduced standard deviation of the measurement error is determined to be 0.01 m/s (= \( \sigma \)). According to Eq. (20), the necessary number of data for smoothing is 307. Consequently, in Fig. 10, the necessary number of data for smoothing CNDS at −5 dB SNR is shown as 307.

5. Summary

In this paper, we proposed a combined method using CNDS and CHDS with an adaptive algorithm to provide accurate and precise velocity at a wide range of SNRs. The adaptive algorithm was divided into two processes, the adaptive algorithm of the range of ambiguity velocity, and the decision algorithm of the integer factor. In the decision algorithm of the integer factor, a variable shift for CMDS effectively reduced the measurement error, and the integer factor was correctly decided.

Theoretical and numerical analyses were carried out to evaluate the performance of the proposed method. The theoretical and numerical results proved that the adaptive algorithm could substantially reduce the CMDS error. Consequently, CMDS using the adaptive algorithm could provide accurate and precise velocity at a wide range of SNRs.

References

1) Y. Yoo, Y. Nakama, N. Kouguchi and C. Song, "Experimental result of ship’s maneuvering test using GPS," J. Nav. Port Res. Int. Ed., 33(2), 99-104 (2009).

2) K. Tatsumi, H. Fuji, T. Kuboda, S. Okuda, Y. Arai, N. Kouguchi and K. Yamada, "Performance requirement of ship’s speed in docking/anchoring maneuvering," Proc. the 12th International Association of Institute of Navigation, Jeju, 67-73 (2006).

3) R. Lhermitte and R. Serafin, "Pulse-to-pulse coherent Doppler sonar signal processing techniques," J. Atmos. Ocean Tech., 1(4), 293-308 (1984).

4) L. Zedel, "Modeling pulse-to-pulse coherent Doppler sonar," J. Atmos. Ocean Tech., 25, 1834-1844 (2008).

5) F. Veron and W. K. Melville, "Pulse-to-pulse coherent Doppler measurements of waves and turbulence," J. Atmos. Ocean Tech., 16, 1580-1597 (1999).

6) P. Liu and N. Kouguchi, "Combined method of conventional and coherent Doppler sonar to avoid velocity ambiguity," J. Marine Acoust. Soc. Jpn., 41(3), 103-112 (2014).

7) A. E. Hay, L. Zedel, R. Craig and W. Paul, "Multi-frequency, pulse-to-pulse coherent Doppler sonar profiler," Proc. the 9th IEEE/OES/CMTC Working Conf. on Current Meas.
8) L. Zedel and A. E. Hay, "Resolving velocity ambiguity in multifrequency, pulse-to-pulse coherent Doppler sonar," IEEE J. Oceanic Eng., 35, 847-851 (2010).

9) P. Liu and N. Kouguchi, "Velocity measurement by dual time interval pulse-to-pulse coherent Doppler sonar," Proc. OCEANS’12 MTS/IEEE Yeosu, 6 pages (2012).

10) I. Holleman and H. Beekhuis, "Analysis and correction of dual PRF velocity data," J. Atmos. Ocean Tech., 20, 443-453 (2003).

11) P. Joe and P. T. May, "Correction of dual PRF velocity errors for operational Doppler weather radars," J. Atmos. Ocean Tech., 20, 429-442 (2003).

12) W. B. Devenport, Jr. and W. L. Root, An Introduction to the Theory of Random Signals and Noise (McGraw Hill Book Co. Inc., New York, 1958), p. 165.

13) A. Papoulis, Probability, Random Variables, and Stochastic Process Third Edition (McGraw Hill Book Co. Inc., New York, 1991), p. 136.

14) R. S. Berkowitz, Modern Radar Analysis, Evaluation, and System Design (John Wiley & Sons, Inc., New York, 1965), p. 170.

15) W. B. Devenport, Jr. and W. L. Root, An Introduction to the Theory of Random Signals and Noise (McGraw Hill Book Co. Inc., New York, 1958), p. 77.