Conceptual understanding and mathematical disposition of college student through Concrete-Representational-Abstract approach (CRA)

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Abstract. The purpose of this study is to examine whether the improvement of mathematical conceptual understanding of students whose learning using Concrete-Representational-Abstract (CRA) approach is better than direct learning, and to know the relationship of mathematical conceptual understanding with mathematical disposition of students. The method used in this research is quantitative. The population in this study is the students of the Department of Mathematics at the second level in Cimahi, the sample is two classes of students who contract Mathematics Statistics courses. The instrument used in this research is a mathematical conceptual understanding test, as well as a questionnaire of mathematical disposition. Data analysis using Mann-Whitney and correlation with the help of SPSS software. The conclusion gained is the improvement of conceptual understanding ability of students who learn to use CRA approach better than students with direct learning, and also there is relationship between conceptual understanding with mathematical disposition of student.

1. Introduction

Conceptual understanding is an important ability possessed by students. Conceptual understanding of mathematics will help and is the initial capital of students in solving mathematical problems. Zulkardi said that Math lesson focuses on the concept [1]. Can be interpreted in learning mathematics should first be able to understand the concept of mathematics so that it can solve the problems and able to apply the learning. The concepts in mathematics from the simplest to the most complex, organized systematically, logically, and hierarchically. Understanding mathematical concepts are fundamental in learning mathematics meaningfully.

Through the ability of mathematical understanding of students able to explain and interpret mathematics, provide images, examples and explanations are more complex and adequate and able to provide more creative descriptions and explanations. While the concept is reflected in the mind of the student is an idea or an understanding. So that student who has the ability to comprehend conceptual mathematics are students who can formulate a strategy of completion, apply simple calculations, use symbols to present the concepts, and transform a form into another form such as fractions in mathematics learning [2].
The structured mathematical tasks that focus on conceptual understanding, procedural fluency, problem-solving in real-world applications, and reasoning has an important role in mathematics learning [3]. The knowledge and ability in conceptual understanding, problem-solving, and teaching methods are as important for effective learning, and focusing on teacher knowledge alone will result in low mathematical skills in mathematics teaching [4].

Conceptual understanding is one of the necessary cognitive knowledge, in addition to cognitive need also the affective ability of students to compensate, one of which is a mathematical disposition. Syahputra [5], suggests one of the requirements of proficient students in mathematics is that students should have a positive disposition attitude, namely the attitude that math is useful in life. Listed in the purpose of learning mathematics in school, which has an attitude appreciate the usefulness of mathematics in life, curiosity, attention, and interest in learning mathematics, as well as resilience and confidence in the solution [6]. The development of interest and interest in mathematics will form a strong tendency called mathematical disposition. Mathematics learning is not only about conceptual, procedural, and application learning, but also related to the development of interest and interest in mathematics as a powerful way to solve problems [7].

Unproductive mathematical disposition capabilities result in inhibition of mathematical achievement [8]. It is important to note that mathematical disposition is not a single sliding scale, but rather a multidimensional manifold of overlapping intellectual and emotional factors related to completing tasks that require mathematical thinking or processes [9]. Mathematical disposition is a combination of attitudes, behaviors, motivations, interests, and real feelings of mathematical achievements that can activate or inhibit mathematical learning. Mathematical disposition is an individual's "tendency or inclination to have or experience particular attitudes, beliefs, feelings, emotions, moods or temperaments with respect to mathematics" [10].

Based on the above description it can be seen that conceptual understanding and mathematical disposition is needed in studying mathematics. Therefore, there needs to be a learning approach that supports it. One is the Concrete-Representational-Abstract (CRA) approach. The CRA approach systematically and explicitly teaches students through three stages of learning, namely concrete, representation, and abstract. Teaching with CRA is a three-stage learning process that stimulates students to understand mathematical concepts through the physical manipulation of concrete objects, then learning through pictorial representation of concrete object manipulations, and ends by solving mathematical problems through abstract notation. Concrete-Representational Abstract The sequence (CRA) has an effective instructional sequence on mathematical concepts for students [11]. In line with Mancl's findings benefit from the CRA sequence for sharing mathematical skills, the current study adds to the CRA literature related to subtraction with regrouping [12]. Explicitly designed CRA Lessons with integrated strategies, that are aligned with the regrouping process can be implemented in a parsimonious manner.

This study aims to know and examine students' conceptual understanding of mathematical learning using Concrete-Representational-Abstract (CRA) approach compared to direct learning and to know the relationship of mathematical conceptual understanding with the mathematical disposition of students.

2. Experimental method
This research is an experimental research conducted on groups of students who were treated (experiment) and the group of students as a comparison or control. The design used is quasi-research (quasi-experiment), where both classes get pretest and posttest. The control class uses direct learning and the experimental class uses CRA approach.

The population in this study is the students of the Department of Mathematics at the second level in Kota Cimahi, the sample is two classes of students who contract Mathematics Statistics courses. Samples are grouped into 2 classes: one class becomes experiment (learning using CRA), while one class is again used as control class (learning using direct learning).

The data in this study were collected through tests (pretest and posttest). Furthermore, the two-difference difference test data (t-test) is used to test the difference between the two averages, in this case...
between the average of the experimental class data and the average of the control class data. The test criterion is rejected H0 if \( \text{sig} < \alpha \) with \( \alpha \) significance level has been determined. Furthermore, the correlation test, to see the relationship between the ability of mathematical conceptual understanding and mathematical disposition.

3. Result and discussion
Data analysis was done with the help of SPSS 23 software, description of conceptual ability of mathematical conceptual representation of improvement of student conceptual understanding ability on learning with CRA and direct learning. The statistical descriptions include mean, standard deviation, maximum and minimum values and number of students based on the given learning approach presented in Table 1.

| Variable                      | CRA | PL |
|-------------------------------|-----|----|
|                               | Pretest | Posttest | Pretest | Posttest |
| **Mathematical conceptual understanding** | 7.11  | 34.71  | 7.05   | 19.00   |
| \( \bar{x} \)                  | 4.733  | 7.995  | 5.558  | 11.429  |
| \( s \)                         | 15 | 60    | 20 | 55    |
| \( \text{Max} \)                | 0 | 13    | 0 | 6    |
| **Mathematical disposition**    | 41.37 | 35.46  | 9.897  | 9.822  |
| \( \bar{x} \)                  | 58 | 56    | 19 | 21    |
| \( s \)                         | 58 | 56    | 19 | 21    |
| \( \text{Max} \)                | 58 | 56    | 19 | 21    |
| \( \text{Min} \)                | 58 | 56    | 19 | 21    |

The mathematical conceptual understandings measured in this study include: re-states a concept; classify objects according to certain according to their concepts; provide an example and not an example of a concept; presents concepts in various forms of representation; develop sufficient conditions or sufficient conditions of a concept; use and utilize and select certain procedures or operations; apply concepts or algorithms in problem solving. In general, assessed from the students' conceptual understanding of mathematics is the ability of Representation, providing solutions to problems, giving judgments, and explanations. This is in line with Niemi [13] who assessed mathematical conceptual understanding through representations, problem solutions, justifications, and explanations.

From Table 1, can be described the description of students' conceptual understanding of mathematical, in terms of pretest results have a similar average of 7.11 and 7.05 or 11% of the ideal score. This indicates that the initial ability of Mathematical conceptual understanding of students in control class and experimental class is relatively the same. Post-test show the mean difference with the max value in the experiment class 60 and the control class 55. The average increase when viewed from the pretest value to the post-test value, in the experimental class increased by 27.5 and the control class by 11.95. In other words, a class that uses the CRA approach, its conceptual understanding is better than students who use direct learning. While in the mathematical disposition obtained average experiment class 41.37 and control class 35.46 with standard deviation is not much different.

Furthermore, prior to hypothesis testing on the ability of mathematical evaluation, the prerequisite test is first performed which includes the normality test (Shapiro-Wilk fit test). The statistical test results on the normality of the sample data for the experimental and experimental class data of each sig. is 0.012 and 0.047. Then for the experimental class posttests data and control each sig. is 0.001 and 0.001. This means that the value of each sig. for pretest, and posttest for both experimental and control groups are all less than \( \alpha = 0.05 \). So it can be concluded that each data on pretest and post-test is not normally distributed.

The pretest and posttest scores of mathematically conceptual understanding in the two groups were abnormally distributed, then the test of difference of mean score of mathematical conceptual understanding was done by using non-parametric Mann-Whitney U. Mann-Whitney test was done to
know inferential the quality of mathematical conceptual understanding between the two groups, as well as testing the research hypothesis which states that the achievement of the conceptual understanding of mathematical students who get learning with CRA approach better than students who received learning with direct learning.

**Table 2. Mann-whitney pre-test and post-test test result mathematical conceptual understanding.**

|                                | Pre-test | Post-test |
|--------------------------------|----------|-----------|
| Mann-Whitney U                 | 622,000  | 169,000   |
| Wilcoxon W                     | 1325,000 | 872,000   |
| Z                               | -2.89    | -5.398    |
| Asymp. Sig. (2-tailed)          | .773     | .000      |
| Monte Carlo Sig. (2-tailed)     | .777b    | .000b     |
| 95% Confidence Interval Lower Bound | .768     | .000      |
| Upper Bound                     | .785     | .000      |
| Monte Carlo Sig. (1-tailed)     | .387b    | .000b     |
| 95% Confidence Interval Lower Bound | .377     | .000      |
| Upper Bound                     | .396     | .000      |

a. Grouping Variable: Class  
b. Based on 10000 sampled tables with starting seed 92208573.

On pretest, the Mann-Whitney test was performed to test the similarity of two median rates from each group. Acceptance of the null hypothesis in this test identifies that any class selected to be a control or experiment class will not be affected by the results of the study. From Table 2, pretest results obtained sig (2-tailed) = 0.773> α = 0.05 means H0 is accepted. So it can be concluded that there is no difference in the ability of early conceptual understanding of mathematical control class students and experimental class.

Furthermore, the posttest is used to test the achievement of students' conceptual understanding of mathematical students who gain a better CRA approach than students who have received direct learning. The hypotheses tested, the mathematical conceptual understanding of students using CRA pendants are better than students who receive direct learning. Based on Table 2, the result of posttest with sig (1-tailed) = 0.000, H0 is rejected. So it can be concluded the achievement of conceptual understanding of mathematical students who get a better CRA approach than students who received direct learning.

This good conceptual understanding is also supported by Hughes [14] study that examines the CRA approach and found that CRA-ordered instruction is an effective lesson for teaching fractions to students studying mathematics, the findings of his research indicate that students who participated in learning using the CRA approach are more effective than traditional instruction, on the fractional retention of student knowledge. Also confirmed by the findings of Mudaly [15], his findings suggest that the use of the CRA instruction sequence is essential for effective mathematics instruction. From these exposures, the results of this study suggest that CRA can help improve students' conceptual understanding of mathematical understanding.

Furthermore, to know the relationship of mathematical understanding and mathematical disposition of students conducted correlation test. This analysis is done to see the positive or not the conceptual understanding of the mathematical disposition of the students. Before the correlation test first tested the normality of data mathematical disposition of students, and obtained sig. CRA class 0.200 and direct learning class get 0.136. Since both classes have more than 0.05 significance then both classes have a normal distribution on the mathematical disposition. Followed by correlation analysis, the hypothesis used is that there is a positive correlation between conceptual understanding with the mathematical disposition of students.
Table 3. Results of correlation test conceptually understanding and mathematical disposition.

| Control Variables | Disposition | PK |
|-------------------|-------------|----|
| Class Disposition | Correlation | 1.000,658 |
| Significance (2-tailed) | . | .000 |
| df | 0 | 69 |
| PK Correlation | .658 | 1.000 |
| Significance (2-tailed) | .000 | . |
| df | 69 | 0 |

*PK: Conceptual understanding

Table 3 on the results of Pearson correlation test results obtained a positive correlation value of 0.658 which means that the strength of the relationship between mathematical conceptual understanding and mathematical disposition of students [16]. From the sig value correlation can be seen 0.000 value, sig value <0.05 which means that H0 is rejected, it means that there is a positive relationship between conceptual understanding and student mathematical disposition.

An important mathematical disposition belongs to the student, National Research Council [17] suggests that students who have a habit of seeing mathematics as sensible, useful, and useful, are confident in their own efficacy that the student has a "productive disposition" attitude. And this disposition is also related to the result of students' mathematical understanding ability. Mathematical disposition cannot change significantly [18]. In addition, mathematical disposition is a broad and complex part of a person that can be influenced by many things, including the family and teachers of mathematics, both implicitly and explicitly [19]. Thus, the numerical scale cannot actually undermine the hidden nuances of mathematical dispositions and try to filter out complex psychological thinking. The mathematical disposition of this researcher was revealed through a questionnaire compared to the students' conceptual comprehension skills. There was found a positive relationship between the conceptual understanding of students with mathematical disposition, meaning that students who have a good understanding have a positive mathematical disposition as well. In learning, student disposition to mathematics is seen from the activity done, such as approach used in solving the problem (task), desire to find alternative way, confidence in solving problem, spirit, and reflex in learning.

4. Conclusion

Based on the results of data analysis and research findings, it can be taken some conclusions that the achievement of students' conceptual understanding of mathematical approach with CRA is better than students who gain direct learning and there is a positive correlation between conceptual understanding with the mathematical disposition of students, so it can mean students who have the good conceptual understanding ability then he has a good disposition skill as well. We student’s conceptual understanding is a capital to solve mathematical problems and requires a positive disposition attitude to students in facing mathematical problems. Students who have a conceptual understanding of mathematics mean that the student has the ability to represent, give solution to the problem, give judgment, and explanation to the given problem, and in the process the student should have attitude, behavior, motivation, interest, and taste to get achievement in mathematical learning or so-called positive mathematical dispositions. Through the characteristics of the CRA approach can help students develop the ability of their conceptual understanding and mathematical disposition. Further research is needed on the effect of CRA instruction sequences on mathematical learning and its compatibility with learning materials.

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