Research Article

Consensus of Multi-Agent Systems with Switching Disconnected Topologies via Elementary-Unit-Based Dwell Time Approach

Jian Sun,1 Chen Guo,1 Qihe Shan,2 and Yanming Wu3

1Marine Electrical Engineering College, Dalian Maritime University, Dalian, Liaoning 116026, China
2School of Navigation, Dalian Maritime University, Dalian, Liaoning 116026, China
3The School of Automation, Shenyang Aerospace University, Shenyang, Liaoning 110136, China

Correspondence should be addressed to Jian Sun; m_sunjian@163.com and Chen Guo; guoc@dlmu.edu.cn

Received 6 November 2021; Accepted 29 April 2022; Published 28 May 2022

Academic Editor: Driss Mehdi

Copyright © 2022 Jian Sun et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In this study, a leader-following consensus problem is investigated for a class of multi-agent systems with switching disconnected topologies. Different from the existing results on switching disconnected topologies, the multi-agent systems considered in this study are unstable. In this situation, the disconnected agents can disperse over some periods even if there exists control protocol on them. To break through this challenge, we draw lessons from which an appropriate switching law can stabilize the switching unstable systems and propose a novel approach called elementary-unit-based dwell time (EUBDT) approach. Based on this approach, each switching interval is considered to consist of a certain number of elementary time segments. Then, by analyzing the local variation of the error states within each elementary time segment, the stabilization properties of switching behaviors are derived to compensate the divergence within the switching intervals. Based on this, the sufficient conditions for the leader-following consensus can be obtained by using a novel kind of piecewise time-varying Lyapunov functions (PTVLFs). Moreover, a time-scheduled controller is designed for such system. Finally, a numerical simulation is given to illustrate the theoretical approach.

1. Introduction

In the last decade, leader-following consensus control of multi-agent systems has been extensively investigated because of their large applications in biological systems, spacecraft formation, and robot manipulators [1–6]. Because of the broad applications, leader-following consensus of multi-agent systems plays an important role in the automatic control field [7, 8]. Accordingly, a lot of efforts have been put into their analysis for leader-following consensus problem in recent years [9–11].

In the field of multi-agent systems, the communication topology may not be fixed because of changes in the agent relations or failures in communication channels [12, 13]. Various results have studied the leader-following consensus problem under the switching directed topologies [13–20]. In the early works, the efforts mainly investigated the switching topologies where all the topologies are connected [13–17]. In recent years, some works have investigated the consensus problem under switching topologies in which the topology is frequently connected [18–20]. In these works, the error state decreases when the topology is connected and increases when the topology is disconnected. The overall consensus can be guaranteed by a relatively long connected topology. If we consider the severe situation that all the topologies are disconnected, this promising idea will not be applicable. How to find general methods to guarantee the consensus of such systems has aroused the interest of researchers in recent years.

Without loss of generality, switching disconnected topologies are more realistic, since the disconnected topology can exist all the time [21, 22]. Some works have explored the leader-following consensus problem under switching disconnected topologies [21–24]. In [21], the sufficient conditions for the leader-following consensus control problem under switching disconnected topologies are proposed.
Then, the algebraic criteria of consensus control under switching disconnected topologies are developed in [22]. Based on this, the consensus problem of multiple linear systems with switching disconnected topologies via the event-triggering control is investigated in [23]. Until now, most existing works on the switching disconnected topologies are concentrated on the critical stable (or stable) multi-agent systems (Assumption 3 in [21], Assumption 2 in [22], Assumption 3 in [23], and Assumption 1 in [24]). In these results, the connected agents are close to each other and the disconnected agents are not diverging away from each other. The global consensus can always be achieved through cooperative control under different topologies. However, in practice, if we consider the unstable multi-agent systems such as distributed voltage control of microgrid [25], multi-link manipulators driven by DC motor [26, 27], the disconnected agents will disperse though there exists the control protocol on them. Accordingly, reaching the consensus of unstable multi-agent systems under switching disconnected topologies is challenging. This study aims to overcome this challenge and reach the consensus control for such systems.

In the field of switching systems, the switching unstable systems can be stabilized by designing an approximate switching strategy [28, 29]. The main idea is that the stabilization properties at switching instants are utilized to offset the divergence of the unstable systems within the switching periods. Drawing lessons from this idea, we propose a novel approach named elementary-unit-based dwell time (EUBDT) approach to tackle the leader-following consensus problem of unstable multi-agent systems under switching disconnected topologies. Based on the EUBDT approach, we divide the switching intervals into a certain number of elementary time segments and analyze the local variation of error states within each elementary time segment. Then, the stabilization properties of switching behaviors can be obtained to offset the divergence of error states within the switching intervals. Then, by confining the dwell time constraints on each topology, the overall leader-following consensus can be reached. Finally, a simulation example is developed to illustrate the theoretical approach. To sum up, to illustrate the main contribution clearly, the following flow diagram is given.

In Figure 1, the main problem, challenge, approach, theoretical breakthrough, and application values have been condensed. It can be seen that the main problem and the research values of this study are illustrated clearly.

Notation: $\mathcal{S}^m$ refers to the set of $m$-vectors. $\mathcal{N}^*$ and $\mathcal{N}$ refer to the set of positive integers and natural number, respectively. The notation $Q < 0$ represents that $Q$ is negative definite. $I_N$ refers to the $N \times N$ identity matrix. $\otimes$ refers to the Kronecker product.

## 2. Preliminaries

The multi-agent systems are expressed by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ is a set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of directed edges, and $\mathcal{A} = [a_{ij}]_{N \times N}$ is an adjacency matrix. An edge $e_{ij} = (v_i, v_j) \in \mathcal{E}$ represents that an arrow from $i$ to $j$ in the graph, which implies that agent $j$ can acquire information from agent $i$. If $e_{ij} \in \mathcal{E}$, $i \neq j$, we will have $a_{ij} = 1$, otherwise, $a_{ij} = 0$. A directed path in $\mathcal{G}$ denotes a sequence of nodes $[v_1, v_2, \ldots, v_q]$ such that $(v_i, v_{i+1}) \in \mathcal{E}$, $i \in \{1, 2, \ldots, q - 1\}$. If $\mathcal{G}$ has a directed spanning tree, it will imply that at least a node has a directed path to all the other nodes. The Laplacian matrix of the graph is denoted as $\mathcal{L} = \mathcal{D} - \mathcal{A} \in \mathbb{R}^{N \times N}$, in which the matrix $\mathcal{D} = [d_{ii}] \in \mathbb{R}^{N \times N}$ with $d_{ii} = \sum_{j=1}^{N} a_{ij}$ and

$$l_{ij} = \sum_{k=1,k\neq i}^{N} a_{ik}; \quad i = j, -a_{ij}; i \neq j. \quad (1)$$

Consider a group of $N$ agents, whose dynamics is expressed as

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, 2, \ldots, N, \quad (2)$$

where $x_i(t) \in \mathbb{R}^n$ represents the state of agent $i$, $u_i(t) \in \mathbb{R}^p$ represents the control input of agent $i$, and $A$ and $B$ are constant matrices.

The dynamics of the leader is shown as

$$\dot{x}_0(t) = Ax_0(t), \quad (3)$$

where $x_0(t) \in \mathbb{R}^n$ represents the leader’s state. The main objective is to guarantee the consensus of followers (2) and the leader (3), which can be illustrated as

$$\lim_{t \to \infty} \|x_i(t) - x_0(t)\| = 0, \quad \forall i = 1, 2, \ldots, N. \quad (4)$$

We use a diagonal matrix $\mathcal{D} = \text{diag}[b_1, b_2, \ldots, b_N]$ to denote the access between the agents and the leader, where $\mathcal{D} \in \mathbb{R}^{N \times N}$. If the agent $i$ can receive the leader’s information, it will be $b_i = 1$, $i = 1, 2, \ldots, N$; otherwise, $b_i = 0$. For convenience, a matrix $\mathcal{K} = \mathcal{D} + \mathcal{B}$ is utilized to denote the information-exchange matrix.

Define a piecewise constant function $\sigma(t) : [1, 2, \ldots, m] \rightarrow \mathbb{R}$ as the switching signal of switching topologies, which satisfies the switching sequence $\{T_{p}, p = 0, 1, \ldots\}$ with $T_0 = 0$ and $\lim_{t \to \infty} T_p = \infty$. Moreover, denote $T_i = T_{p+1} - T_p$ as the dwell time of the $p$th topology. The dwell time $T_p$ is constrained by minimum dwell time $T_{\min}$ and maximum dwell time $T_{\max}$, which is denoted as $T_{\min} \leq T_p \leq T_{\max}$. It can be observed that the topology can only be switched within the interval $t \in [T_p + T_{\min}, T_{p+1})$. For convenience, we use $\mathcal{D}[\max(T_{\min}, T_{p})]$ to denote the set of all the switching policies in the framework of dwell time $T_p \in [T_{\min}, T_{\max}]$. Therefore, $\{\mathcal{G}^1, \mathcal{G}^2, \ldots, \mathcal{G}^m\}$, $m \geq 1$ denotes the set of all the possible directed graphs under the switching graphs. The information-exchange matrices can be denoted as $(\mathcal{K}_1, \mathcal{K}_2, \ldots, \mathcal{K}_m)$, $m \geq 1$. The communication topology under the switching function $\sigma(t)$ satisfies the following assumption.

Assumption 1. The switching graph $\mathcal{G}^{\sigma(t)}$ is not connected. The union of all the graphs $\mathcal{G} = \bigcup \mathcal{G}^1 \cup \mathcal{G}^2 \cup \ldots \cup \mathcal{G}^m$ has a directed spanning tree with the leader as the root, where $\sigma(t) \in \mathcal{N}^*$. Based on this, we consider the controller adopting the following form:
Few works have considered the unstable multi-agent systems under switching disconnected topologies.

Main Problem

The disconnected unstable agents will disperse even if there exists the control protocol on them.

Main Challeng

The theory of switching unstable systems.

Main Approach

Propose a novel EUBDT approach.

Application Values

Distributed voltage control of microgrid
Multi-link manipulators driven by DC motors.

Theoretical Breakthrough

Break through the limitation that the system matrix is required stable or critical stable under switching disconnected topologies.

Figure 1: The flow diagram of the main contribution.

\[ u_{\sigma(t)}(t) = K \left[ \sum_{j=1}^{N} a_{ij}^{\sigma(t)}(x_j(t) - x_i(t)) \right] + b_{ij}^{\sigma(t)}(x_0(t) - x_i(t)) \]

where \( K \) is the gain matrix, \( a_{ij}^{\sigma(t)} \) represents the adjacency element of \( \mathcal{G}^{\sigma(t)} \), \( b_{ij}^{\sigma(t)} = 1 \) when agent \( i \) can receive the leader’s information under the graph \( \mathcal{G}^{\sigma(t)} \) and \( b_{ij}^{\sigma(t)} = 0 \), otherwise.

Define \( e_i(t) = x_i(t) - x_0(t) \) as the tracking error for agent \( i \). Then, the error system of agent \( i \) under the switching signal \( \sigma(t) \) can be obtained as

\[ \dot{e}_i(t) = A e_i(t) + BK \left[ \sum_{j=1}^{N} a_{ij}^{\sigma(t)}(e_j(t) - e_i(t)) - b_{ij}^{\sigma(t)} e_{ij}(t) \right]. \]

Define \( e(t) = [e_1(t)^T, e_2(t)^T, \ldots, e_N(t)^T]^T \), then the error systems for \( \sigma(t) = r \) can be given by

\[ \dot{e}(t) = (I_N \otimes A)e(t) - (H \otimes BK)e(t), \quad t \in [T_p, T_{p+1}], \quad p \in \mathcal{N}. \]

Therefore, the control objective is to achieve the stability of error system (7) under switching laws \( \sigma(t) \).

Remark 1. Most existing works on the consensus problem under switching disconnected topologies require that the multi-agent systems are critical stable (or stable), which is reflected in that the system matrix \( A \) in these works contains no positive real part eigenvalues (Assumption 3 in [21], Assumption 2 in [22], Assumption 3 in [23], and Assumption 1 in [24]). In this case, the connected agents will close to each other and the disconnected agents will not disperse. Then, the overall consensus can be achieved by the jointly connected topology. However, this idea cannot be applied to the unstable multi-agent systems such as distributed voltage control of microgrid and multi-link manipulators driven by DC motor, since the disconnected agents will always diverge even if there exists the control protocol on them. Thus, it is challenging or even impossible to achieve consensus of all the agents. How to overcome this difficulty and achieve the seemingly impossible consensus is the main work of this study.

3. Main Results

In this section, the EUBDT approach and the conditions of consensus control are presented and proved.

3.1. The Elementary-Unit-Based Dwell Time Approach.

Most results on the consensus control problem under switching disconnected topologies are concentrated on the stable or critical stable multi-agent systems [21–24]. In this situation, the connected agents will close to each other and the disconnected agents will not disperse. If we consider unstable multi-agent systems such as distributed voltage control of microgrid and multi-link manipulators driven by DC motor, the disconnected agents will disperse under the disconnected topology even if there exist the control protocol on them. Thus, how to stabilize the error states under switching disconnected topologies is challenging. In order to break through this challenge, one has to utilize the stabilization properties of switching behaviors to stabilize the error states. It is widely known that the Lyapunov functions are effective tools to describe the dynamics of multi-agent systems [19–23]. If the multiple Lyapunov functions are constructed to describe the error dynamics of unstable multi-agent systems under switching disconnected topologies, they will probably divergent within the switching intervals. Thus, we consider utilizing the “decline” characteristics at the transition instants to offset the divergence within the switching intervals. Suppose that \( V_{\sigma(t)}(t) \) are multiple nonnegative functions under switching law \( \sigma(t): \{1, 2, \ldots, l\} \). Then, we propose the following useful lemma first.
Lemma 1. Consider the multiple nonnegative functions $V_{\sigma(t)}(t)$ for $\sigma(t) \in \{1, 2, \ldots, l\}$. If there exist constants $0 < \mu < 1$ and $\alpha > 0$ such that

$$V_q(t) < e^{\alpha(t-\tau_p)}V_q(T_p), \quad \forall t \in [T_p, T_{p+1}),$$

(8)

$$V_r(T_p^-) \leq \mu V_r(T_p^-), \quad r \neq q,$$

(9)

$$\ln \mu + \alpha \tau_{\text{max}} < 0,$$

(10)

where $\tau_p = T_{p+1} - T_p$, $p = 0, 1, 2, \ldots$, then global convergence of functions $V_{\sigma(t)}(t)$ can be reached.

Proof: Define a Lyapunov function as $V(t) = \sum_{k=1}^n e_k(t)V_k(t)$, where

$$e_k(t) = \begin{cases} 1, & \text{if } \sigma(t) = k, \\ 0, & \text{otherwise.} \end{cases}$$

(11)

Assuming that $\sigma(t) = q$ when $t \in [T_p, T_{p+1})$, then according to (8), one has $V(t) < e^{\alpha(t-T_p)}V(T_p)$, $\forall t \in [T_p, T_{p+1})$. If the functions switch from mode $q$ to mode $r$ when $t = T_{p+1}$, we can derive $V(T_{p+1}) \leq \mu V(T_{p+1})$ from (9). Then, we can further obtain $V(T_{p+1}) \leq \mu e^{\alpha(T_{p+1} - T_p)}V(T_p)$. Considering $\tau_p = T_{p+1} - T_p$, condition (10) guarantees $\mu e^{\alpha(T_{p+1} - T_p)} < 1$. Then, one has

$$V(t) < e^{\alpha(t-T_p)}V(T_p^-)$$

$$< \mu e^{\alpha(t-T_p)}V(T_p^-)$$

$$< \mu^2 e^{\alpha(t-T_p)}V(T_p^-)$$

$$< \cdots < \mu^p e^{\alpha(t-T_p)}V(T_0).$$

(12)

Because $p \geq (t - T_0)/\tau_{\text{max}}$, one further has

$$V(t) < e^{\alpha(t-T_p)}V(T_0), \quad t \in [T_p, T_{p+1}).$$

(13)

where $e = \rho \ln(\mu + \alpha \tau_{\text{max}})$. From (10), we have $e < 0$. Therefore, the global convergence of $V_{\sigma(t)}(t)$ is reached. The proof is completed.

From Lemma 1, it can be concluded that despite the divergence of all the Lyapunov functions, we can utilize the stabilization characteristics at switching instants to guarantee the global convergence. In order to derive the stabilization properties at switching instants, the elementary-unit-based dwell time (EUBDT) approach is proposed in this study. The EUBDT approach can be divided into three steps. The first step is to divide the switching interval into a certain number of elementary time segments. The second step is to analyze the local variation of error state within each elementary time segment. The last step is to derive the stabilization properties at switching instants to offset the divergence within the switching intervals. Considering that the dwell time of the switching interval $[T_p, T_{p+1})$ is uncertain, we divide $[T_p, T_{p+1})$ into the certain interval $[T_p, T_p + \tau_{\text{min}})$ and uncertain interval $[T_p + \tau_{\text{min}}, T_{p+1})$. It is obvious that the communication topology can only be switched within the uncertain interval. Assume that the certain interval $[T_p, T_p + \tau_{\text{min}})$ consists of $M$ elementary time segments and each elementary time segment is denoted as $[t_{p,l}, t_{p,l+1})$, where $l \in \{1, 2, \ldots, M - 1\}$. The length of each elementary time segment is denoted as $h$, which satisfies $h = \tau_{\text{min}}/M$. Under the EUBDT method, the corresponding piecewise time-varying Lyapunov functions (PTVLFs) is constructed as

$$V_r(t) = e^{\alpha t}(I_N \otimes \mathcal{P}_r(t))e(t), \quad t \in [T_p, T_{p+1}),$$

(14)

where $\mathcal{P}_r(t)$ is the piecewise time-varying matrix, which is described as follows.

For the certain interval $[T_p, T_{p+1})$ with $l \in \{1, 2, \ldots, M - 1\}$ and $M \in \mathcal{N}^+$, $\mathcal{P}_r(t)$ is given by

$$\mathcal{P}_r(t) = (1 - \rho_1(t))\mathcal{P}_{r,l} + \rho_1(t)\mathcal{P}_{r,l+1}, \quad t \in [t_{p,l}, t_{p,l+1}),$$

(15)

where $\mathcal{P}_{r,l} > 0$ and $\rho_1(t) = (t - t_{p,l+1})/h$. For the uncertain interval $[T_p + \tau_{\text{min}}, T_{p+1})$, $\mathcal{P}_r(t)$ is expressed as

$$\mathcal{P}_r(t) = \mathcal{P}_{r,M}.$$  

(16)

Remark 2. The main advantage of the piecewise time-varying Lyapunov function in (14) is that it can be utilized to derive the “decline” properties at switching instants by adjusting the time-varying Lyapunov matrix $\mathcal{P}_r(t)$. Such “decline” properties can be utilized to offset the divergence made by disconnected topology and unstable systems. If the classic time-invariant Lyapunov functions in [20–24] are utilized to describe the states, the Lyapunov matrix will be fixed and the “decline” properties will be hard to derive.

3.2. The Consensus Conditions and the Controller Synthesis. The previous subsection has developed the EUBDT approach. Considering that the error states are always divergent within the switching intervals due to the coexistence of disconnected topology and unstable multi-agent systems, we need to utilize the cooperative control to guarantee that the exponential divergence rate within a threshold. Then, the following lemma discussing the exponential divergence rate of error system (7) is obtained.

Lemma 2. For given constants $\alpha > 0$ and $h > 0$, if there exist matrices $\mathcal{P}_{r,l} > 0, (l = 0, 1, \ldots, M - 1)$ such that for any $r = 1, 2, \ldots, m$, the following conditions hold:

$$\Sigma_1 = A^T\mathcal{P}_{r,l+1} + \mathcal{P}_{r,l}A + \frac{\mathcal{P}_{r,l+1} - \mathcal{P}_{r,l}}{h}$$

$$- \lambda_1^T K B^T \mathcal{P}_{r,l+1} - \lambda_1^T \mathcal{P}_{r,l+1} BK$$

$$- \alpha \mathcal{P}_{r,l+1} < 0,$$

$$\Sigma_2 = \frac{\mathcal{P}_{r,l+1} - \mathcal{P}_{r,l}}{h} + A^T\mathcal{P}_{r,l} + \mathcal{P}_{r,l+1} A$$

$$- \lambda_2^T K B^T \mathcal{P}_{r,l+1} - \lambda_2^T \mathcal{P}_{r,l+1} BK - \alpha \mathcal{P}_{r,l} < 0.$$  

(17)

(18)
\[
\Sigma_3 = A^T P_{r,M} + P_{r,M}A - X_i^TK^T B^T P_{r,M} - \lambda_i^r P_{r,M}BK - \alpha P_{r,M} < 0.
\]

where \( X_i \), \( i = 1, 2, \ldots, N \) is the \( i \)th positive eigenvalue of \( H_r \). Then, the exponential divergence rate of error system (7) can be limited within \( \alpha \) for the switching interval \( t \in [T_p, T_p + 1) \).

Proof: First of all, we will prove the exponential divergence rate of system (7) is limited within \( \alpha \) for the certain interval \( t \in [T_p, T_p + \tau_{min}) \). Assume \( \sigma (t) = r \) for \( t \in [T_p, T_p + \tau_{min}) \). Then, calculating \( \dot{V}_r (t) \) for \( t \in [t_p,l, t_{p,l+1}) \) , \( l = 1, 2, \ldots, M - 1 \), we have

\[
\dot{V}_r (t) - \alpha V_r (t) = e^T (I_N \otimes \frac{P_{r,l+1} - P_{r,l}}{h}) e(t) + [e^T (I_N \otimes A)e(t) - (H_r \otimes BK)e(t)]^T
\]

\[
- \alpha e^T (I_N \otimes P_r (t)) e(t)
\]

\[
= e^T (I_N \otimes \frac{P_{r,l+1} - P_{r,l}}{h}) e(t)
\]

\[
+ [(I_N \otimes A)e(t) - (H_r \otimes BK)e(t)]^T
\]

\[
\times [I_N \otimes P_r (t) + \frac{t - t_p,l}{h}(P_{r,l+1} - P_{r,l})] e(t)
\]

\[
+ e^T [I_N \otimes \frac{t - t_p,l}{h}(P_{r,l+1} - P_{r,l})] e(t) - \alpha e^T (I_N \otimes P_r (t)) e(t)
\]

Substituting (21) into (20), one obtains

\[
\dot{V}_r (t) - \alpha V_r (t) = e^T (I_N \otimes \hat{P}_r (t)) e(t) + e^T (I_N \otimes P_r (t)) e(t)
\]

\[
+ e^T (I_N \otimes \hat{P}_r (t)) \dot{e}(t) - \alpha e^T (I_N \otimes P_r (t)) e(t).
\]

According to (15), one has

\[
\hat{P}_r (t) = \frac{(P_{r,l+1} - P_{r,l})}{h}.
\]

Discrete Dynamics in Nature and Society 5
Considering that \( e(t) = [e_1^T(t), e_2^T(t), \ldots, e_N^T(t)]^T \), we can further obtain
\[
\dot{V}_r(t) - aV_r(t) \\
\leq \sum_{i=1}^{N} e_i^T(t) \left( \frac{\mathcal{P}_{r,l} - \mathcal{P}_{r,l+1}}{h} \right) e_i(t) \\
+ (\mathcal{A}e_i(t) - \lambda_i^r BKe_i(t))^T \\
\times \left[ \mathcal{P}_{r,l} + \frac{t - t_{pl}}{h} (\mathcal{P}_{r,l+1} - \mathcal{P}_{r,l}) \right] e_i(t) \\
+ e_i^T(t) - \frac{t - t_{pl}}{h} (\mathcal{P}_{r,l+1} - \mathcal{P}_{r,l}) (A - \lambda_i^r BKe_i(t)) e_i(t) \\
- a e_i^T(t) \left[ \mathcal{P}_{r,l} + \frac{t - t_{pl}}{h} (\mathcal{P}_{r,l+1} - \mathcal{P}_{r,l}) \right] e_i(t).
\]  

Due to \( p_i(t) = t - t_{pl}/h \), one obtains
\[
\dot{V}_r(t) - aV_r(t) \\
\leq \sum_{i=1}^{N} e_i^T(t) [p_i(t) \Sigma_1 + (1 - p_i(t)) \Sigma_2] e_i(t).
\]  

Then, conditions (17) and (18) guarantee \( \dot{V}_r(t) - aV_r(t) < 0 \), \( \forall t \in [t_{pl}, t_{pl+1}) \), \( l = 1, 2, \ldots, M - 1 \), which implies that \( V_r(t) < \varepsilon(t) \), \( \forall t \in [t_{pl}, t_{pl+1}) \), \( l = 1, 2, \ldots, M - 1 \). Thus, we can further get
\[
\dot{V}_r(t) < e^\alpha(t-t_{pl}) V_r(t_{pl}) \\
\leq e^\alpha(t-t_{pl}) V_r(t_{pl-1}) \\
\leq \ldots \leq e^\alpha(t-T_p) V_r(T_p).
\]  

Next, we will prove that the exponential divergence of system (7) is limited within \( \alpha \) for the uncertain interval \( t \in [T_p + \tau_{\min}, T_p + \tau_{\max}] \) when \( \sigma(t) = r \). Calculating \( V_r(t) \) for \( t \in [T_p + \tau_{\min}, T_p + \tau_{\max}] \), it yields
\[
\dot{V}_r(t) - aV_r(t) \\
= e^T(t) (I_N \otimes \mathcal{P}_{r,M}) e(t) + e^T(t) (I_N \otimes \mathcal{P}_{r,M}) e(t) \\
- \alpha e^T(t) (I_N \otimes \mathcal{P}_{r,M}) e(t) \\
= [(I_N \otimes A)e(t) - (\mathcal{H}r \otimes BK)e(t)]T (I_N \otimes \mathcal{P}_{r,M}) e(t) \\
+ e^T(t) (I_N \otimes \mathcal{P}_{r,M}) [(I_N \otimes A)e(t) - (\mathcal{H}r \otimes BK)e(t)] \\
- \alpha e^T(t) (I_N \otimes \mathcal{P}_{r,M}) e(t).
\]  

From the similar guideline of (23), it yields
\[
\dot{V}_r(t) - aV_r(t) \\
\leq \sum_{i=1}^{N} (\mathcal{A}e_i(t) - \lambda_i^r BKe_i(t))^T \mathcal{P}_{r,M} e_i(t) \\
+ e_i^T(t) \mathcal{P}_{r,M} (A - \lambda_i^r BKe_i(t)) e_i(t) \\
- \alpha e_i^T(t) \mathcal{P}_{r,M} e_i(t)
\]

Theorem 1. Suppose that Assumption 1 is satisfied. For given constants \( \alpha > 0, 1 > \mu > 0 \), and \( h > 0 \), if there exist matrices \( P_{r,l} > 0, (l = 0, 1, \ldots, M-1) \) such that for any \( r = 1, 2, \ldots, M-1 \), the following conditions hold:
\[
\Sigma_1 = A^T \mathcal{P}_{r,l+1} + \mathcal{P}_{r,l+1} A + \frac{\mathcal{P}_{r,l+1} - \mathcal{P}_{r,l}}{h} \\
- \lambda_i^r K^T B \mathcal{P}_{r,l+1} - \lambda_i^r \mathcal{P}_{r,l+1} BK \\
- \alpha \mathcal{P}_{r,l+1} < 0,
\]  

\[
\Sigma_2 = A^T \mathcal{P}_{r,l} + \mathcal{P}_{r,l} A + \frac{\mathcal{P}_{r,l+1} - \mathcal{P}_{r,l}}{h} \\
- \lambda_i^r K^T B \mathcal{P}_{r,l} - \lambda_i^r \mathcal{P}_{r,l} BK - \alpha \mathcal{P}_{r,l} < 0,
\]  

\[
\Sigma_3 = A^T \mathcal{P}_{r,M} + \mathcal{P}_{r,M} A - \lambda_i^r K^T B \mathcal{P}_{r,M} \\
- \lambda_i^r \mathcal{P}_{r,M} BK - \mu \mathcal{P}_{r,M} < 0,
\]  

where \( \tau_{\max} \) denotes the maximal dwell time. Then, the stability of error system (7) can be guaranteed under the switching law \( \sigma(t) \in \mathcal{P}_{\tau_{\min}, \tau_{\max}} \).

Proof: From Lemma 2, we can get that conditions (28)–(30) guarantee \( V_\sigma(T_p(t)) < e^{\alpha(t-T_p)} V_\sigma(T_p(t)) \), \( \forall t \in [T_p, T_p+\tau_{\max}] \). According to (31), we have \( V_\sigma(T_p(t)) \leq \mu V_\sigma(T_p(t)) \), which implies that \( V_\sigma(T_p(t)) \leq \mu e^{\alpha(t-T_p)} V_\sigma(T_p(t)) \). Then, from Lemma 1, one has
where \( \bar{p} = p(\ln \mu + \alpha r_{\text{max}}) \). According to (32), one has \( \bar{p} < 0 \). Therefore, the stability of error system (7) is guaranteed. The proof is completed.

**Remark 3.** In Theorem 1, by selecting appropriate values of \( M \) and \( h \), the corresponding maximal dwell time \( \tau_{\text{max}} \) can be computed by

\[
\tau^*_{\text{max}} = \max \{ \tau_{\text{max}} : (28) - (32) \text{ hold} \}. \tag{34}
\]

**Remark 4.** In this study, due to the coexistence of unstable systems and disconnected topologies, some agents inevitably diverge from each other though there exist the control protocols on them. Thus, it is hard to guarantee the exponential convergence for error system within switching intervals and we inevitably leave the error system divergent within the switching intervals. Therefore, we choose \( \alpha > 0 \) to denote the exponential divergence rate of error states within the switching intervals.

**Remark 5.** Reference [28] has studied the stabilization problem of switching unstable systems. It derives the stabilization properties of the switching behaviors to offset the divergence property across the switching intervals and further achieve the overall stability. Drawing lessons from this, the EUBDT approach is proposed in this study to tackle the consensus problem of unstable multi-agent systems under switching disconnected topologies. The main idea is to utilize the stabilization properties at switching instants to offset the divergence of error states within the switching intervals. By analyzing the local variation of error states within each elementary time segment \([t_{p,l-1}, t_{p,l})\), the stabilization properties at switching instants can be derived in condition (31). Then, condition (32) guarantees that the “decline” properties at switching instants are larger than the divergence of error states within the switching intervals. Based on this, the global stability of the error system can be guaranteed.

Next, we focus on the controller synthesis of error system (7) under switching disconnected topologies. A time-scheduled controller is adopted here, which is easy to be calculated. Instead of the controller (5), we will rather consider the following time-scheduled controller:

\[
u_{\sigma(t)}(t) = K_{\sigma(t)}(t) \left[ \sum_{j=1}^{N} a^j(t) x_j(t) - x_{\sigma(t)}(t) \right]
\tag{35}
\]

where \( K_{\sigma(t)}(t) \) is the time-scheduled gain to be determined. Substituting this controller into error system (7), it yields

\[
\dot{e}(t) = (I_N \otimes A) e(t) - (H_\sigma \otimes B K_{\sigma(t)}(t)) e(t),
\tag{36}
\]

Then, the consensus of multi-agent systems (2) and (3) is equivalent to the stability of error system (36). Then, the following theorem can be obtained.

**Theorem 2.** Suppose that Assumption 1 is satisfied. For given constants \( M \in \mathbb{R}^+ \), \( \alpha > 0 \), \( 1 > \mu > h > 0 \), if there exist matrices \( \mathcal{P}_{r,l} > 0 \), \( X_r(l = 0, 1, \ldots, M - 1) \) such that for any \( r = 1, 2, \ldots, m \), the following conditions hold:

\[
\begin{align*}
\Sigma_1 &= A^T \mathcal{P}_{r+1} + \mathcal{P}_{r+1} A + \frac{\mathcal{P}_{r+1} - \mathcal{P}_r}{h} \\
&\quad - \lambda^r_1 B^T X_{r+1} - \lambda^r_1 T X_{r+1} B - \alpha \mathcal{P}_{r+1} < 0,
\end{align*}
\tag{37}
\]

\[
\begin{align*}
\Sigma_2 &= A^T \mathcal{P}_{r} + \mathcal{P}_{r} A + \frac{\mathcal{P}_{r} - \mathcal{P}_{r+1}}{h} \\
&\quad - \lambda^r_1 B^T X_{r+1} - \lambda^r_1 T X_{r+1} B - \alpha \mathcal{P}_{r+1} < 0,
\end{align*}
\tag{38}
\]

\[
\begin{align*}
\Sigma_3 &= A^T \mathcal{P}_{r+1} + \mathcal{P}_{r+1} A - \lambda^r_1 B^T X_{r+1} - \alpha \mathcal{P}_{r+1} < 0,
\end{align*}
\tag{39}
\]

where \( \tau_{\text{max}} \) denotes the maximal dwell time. Then, consensus of multi-agent systems (2) and (3) can be achieved under the switching law \( \sigma(t) \in \mathcal{D}_{\{\tau_{\text{max}}, \tau_{\text{min}}\}} \) and controller gain \( K_{\sigma(t)} = X_{\sigma(t)}(t) \mathcal{P}^{-1}_{\sigma(t)}(t) \), where \( X_{\sigma(t)}(t) \) is given by

\[
X_{\sigma(t)}(t) = \begin{cases} X_{r,l}(t), & t \in [t_{p,l-1}, t_{p,l}), \\ X_{r,l+1}, & t \in [T_p + \tau_{\text{min}}, T_{p+1}). \end{cases}
\tag{42}
\]

where \( X_{r,l}(t) = \rho_l(t) X_{r,l-1}(t) + (1 - \rho_l(t)) X_{r,l+1}, \rho_l(t) = t - t_{p,l}/h \) and \( l = 1, 2, \ldots, M - 1 \).

**Proof:** First of all, we consider error system (36) for \( t \in [T_p, T_{p+1} + \tau_{\text{min}}) \). Calculating \( V_{\sigma(t)}(t) \) for \( t \in [t_{p,l-1}, t_{p,l}) \), \( l = 1, 2, \ldots, M - 1 \), one obtains

\[
\begin{align*}
V_{\sigma(t)}(t) &= a V_{\sigma(t)}(t) \\
&= e^T(t) \left( I_N \otimes \mathcal{P}_{\sigma(t)}(t) \right) e(t) + 2 e^T(t) (I_N \otimes A - H_{\sigma} \otimes B K_{\sigma(t)}(t)) e(t) \\
&\quad - a e^T(t) (I_N \otimes \mathcal{P}_{\sigma(t)}(t)) e(t).
\end{align*}
\tag{43}
\]

Taking the time-varying parameter (42) into (43), it yields
\[ V_{r,k}(t) - aV_{r,k}(t) \]
\[ = e^T(t) \left( I_N \otimes \frac{\mathcal{P}_{r,l+1} - \mathcal{P}_{r,l}}{h} \right) e(t) + 2ae^T(t) \quad (44) \]
\[ \cdot \left[ I_N \otimes A - \mathcal{H}_r \otimes B \left( \chi_{r,l} + \rho_1(t) \chi_{r,l+1} - \chi_{r,l} \right) \right] e(t) \]
\[ - ae^T(t) \left[ I_N \otimes \left( \mathcal{P}_{r,l} + \rho_1(t) \left( \mathcal{P}_{r,l+1} - \mathcal{P}_{r,l} \right) \right) \right] e(t). \]

From similar guideline of (33), the stability of system (36) is
\[ \begin{align*}
\sum_{i=1}^{N} e_i^T(t) & \left[(1 - \rho_1(t)) \left( \frac{\mathcal{P}_{r,l+1}}{h} - \frac{\mathcal{P}_{r,l}}{h} + A^T \mathcal{P}_{r,l} \right)
\mathcal{P}_{r,l} A - \chi_{r,l}^T B \chi_{r,l} - \chi_{r,l+1}^T B A - a \mathcal{P}_{r,l+1} \right] e_i(t) \\
& + e^T(t) \left[ \rho_1(t) \left( \frac{\mathcal{P}_{r,l+1}}{h} - \frac{\mathcal{P}_{r,l}}{h} + A^T \mathcal{P}_{r,l+1} \right)
\mathcal{P}_{r,l+1} A - \chi_{r,l+1}^T B \chi_{r,l+1} - \chi_{r,l+2}^T B - a \mathcal{P}_{r,l+1} \right] e_i(t) \\
& - a \mathcal{P}_{r,l+1} e_i(t) \end{align*} \]

\[ = \sum_{i=1}^{N} e_i^T(t) \left[(1 - \rho_1(t)) \Sigma_1 + \rho_1(t) \Sigma_2 \right] e_i(t). \]

Then, conditions (37) and (38) can guarantee \( V_r(t) - aV_r(t) < 0 \) for \( t \in [t_{r,l}, t_{r,l+1}] \), \( l = 1, 2, \ldots, M - 1 \). It means that \( V_r(t) < e^{\delta_{r,l}t} V_r(T_p) \) for \( t \in [T_p, T_p + \tau_{\text{min}}] \)

Next, we will prove the exponential divergence of system (36) for \( t \in [T_p + \tau_{\text{min}}, T_{p+1}) \). Calculating \( V_r(t) \) for \( t \in [T_p + \tau_{\text{min}}, T_{p+1}) \), one has
\[ \begin{align*}
\sum_{i=1}^{N} e_i^T(t) & \left[A^T \mathcal{P}_{r,l+1} + \mathcal{P}_{r,l+1} A - \chi_{r,l+1}^T B \right. \\
& \left. - \chi_{r,l+2}^T B \chi_{r,l+1} - a \mathcal{P}_{r,l+1} \right] e_i(t) \\
& = \sum_{i=1}^{N} e_i^T(t) \Sigma_3 e_i(t). \end{align*} \]

Then, condition (39) guarantees \( V_r(t) - aV_r(t) < 0 \) for \( t \in [T_p + \tau_{\text{min}}, T_{p+1}) \).

To sum up, conditions (37)–(39) guarantee \( V_r(t) < e^{\delta_{r,l}t} V_r(T_p) \) for \( t \in [T_p, T_{p+1}) \). Then, by the similar guideline of (33), the stability of system (36) is reached from conditions (40) and (41). This also implies that the consensus of systems (2) and (3) is guaranteed. The proof is completed. \( \square \)

### 4. Simulation Examples

In this section, the theoretical approach is illustrated by a simulation example.

Consider the multi-agent system described in (2), where the system matrices are given as
\[ A = \begin{bmatrix} 0 & -1 \\ 1 & 0.1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}. \] 
(47)

The eigenvalues of A are 0.05 + 0.9987i and 0.05 − 0.9987i. It can be observed that the eigenvalues of A contain positive real parts, which implies that the multi-agent systems are unstable. The switching graphs \( \mathcal{G}^1 \) and \( \mathcal{G}^2 \) are shown in Figures 2 and 3 respectively.

It is obvious that \( \mathcal{G}^1 \) and \( \mathcal{G}^2 \) are disconnected and the union of \( \mathcal{G}^1 \) and \( \mathcal{G}^2 \) has a directed spanning tree with the leader as the root, which satisfies Assumption 1. The objective is to design controller (35) and constrain the dwell time \( \{r_{\text{min}}, r_{\text{max}}\} \) for the switching topologies such that the consensus of systems (2) and (3) can be achieved. The initial states are given as \( x_0(0) = [-5 \ 2]^T, x_1(0) = [-2 \ -2]^T, x_2(0) = [-2 \ 5]^T, x_3(0) = [-1 \ -5]^T, x_4(0) = [0 \ 5]^T \).

Then, if we fixed \( \mu = 0.6, M = 3, h = 0.5, \) and \( \alpha = 0.4, \) by calculating conditions (37)–(41), the controller gain \( \mathcal{K}_r(t) = \chi_r(t) \mathcal{P}^{-1}(t) \) can be obtained as follows:

\[ \mathcal{P}_{1,0} = \begin{bmatrix} 2.3604 & 0.0204 \\ 0.0204 & 2.3583 \end{bmatrix}, \]
\[ \mathcal{P}_{1,1} = \begin{bmatrix} 13.0317 & 0.0538 \\ 0.0538 & 13.0521 \end{bmatrix}, \]
\[ \mathcal{P}_{1,2} = \begin{bmatrix} 23.9958 & 0.0872 \\ 0.0872 & 24.0237 \end{bmatrix}, \]
\[ \mathcal{P}_{1,3} = \begin{bmatrix} 35.4834 & 0.1226 \\ 0.1226 & 35.5032 \end{bmatrix}, \]
\[ \mathcal{P}_{2,0} = \begin{bmatrix} 15.8189 & 0.0614 \\ 0.0614 & 15.8264 \end{bmatrix}, \]
\[ \mathcal{P}_{2,1} = \begin{bmatrix} 14.2445 & 0.0638 \\ 0.0638 & 14.2436 \end{bmatrix}, \]
\[ \mathcal{P}_{2,2} = \begin{bmatrix} 12.3907 & 0.0606 \\ 0.0606 & 12.3900 \end{bmatrix}, \]
\[ \mathcal{P}_{2,3} = \begin{bmatrix} 10.1704 & 0.0544 \\ 0.0544 & 10.1714 \end{bmatrix}, \]
\[ \chi_{1,0} = \begin{bmatrix} 8.8749 & -4.4233 \\ -4.4233 & 8.8866 \end{bmatrix}, \]
\[ \chi_{1,1} = \begin{bmatrix} 9.3661 & -5.0156 \\ -5.0156 & 9.9033 \end{bmatrix}. \]
The switching signal 

\[
\begin{align*}
426 & 1 6 14 2001 0 1 881 2 \\
\end{align*}
\]

Figure 4: The switching signal for switching topologies.

\[
\begin{align*}
x_{i1}(t), \ i = 0, 1, 2, 3, 4. \\
x_{01} & \\
x_{11} & \\
x_{21} & \\
x_{31} & \\
x_{41} & \\
\end{align*}
\]

Figure 5: The state $x_{i1}$ of the multi-agent systems.
Moreover, the maximum dwell time can be calculated as \( \tau_{\text{max}} = 2.24s \). The minimum dwell time can be derived as \( \tau_{\text{min}} = Mh = 1.5s \). Based on this, the switching signal is shown in Figure 4, where \( \sigma(t) = 1 \) denotes graph \( \mathcal{G}^1 \) and \( \sigma(t) = 2 \) denotes graph \( \mathcal{G}^2 \).

By imposing the obtained controller and the switching signal on the system, the state trajectories are obtained in Figures 5 and 6.

In Figures 5 and 6, \( x_{i1}(t) \) and \( x_{i2}(t) \) denote the first and second state trajectories of the agent \( i \), respectively. It is obvious that the leader-following consensus is achieved. Furthermore, we define the tracking error for agent \( i \) as \( e_{i1}(t) = x_{i1}(t) - x_{01}(t) \) and \( e_{i2}(t) = x_{i2}(t) - x_{02}(t) \) for \( i = 1, 2, 3, 4 \). Then, \( e_{i1}(t) \) and \( e_{i2}(t) \) are depicted in Figures 7 and 8 respectively.
From Figures 7 and 8, it is obvious that the tracking errors decay to 0, which means that the consensus of the unstable multi-agent systems can be achieved under the switching disconnected topologies.

5. Conclusion

In this study, the consensus control problem of unstable multi-agent systems under switching disconnected topologies was investigated. Drawing lessons from the theory of switching unstable systems, a novel elementary-unit-based dwell time (EUBDT) approach was proposed to divide the switching intervals into a certain number of elementary time segments. Then, by analyzing the local variation of error state within each elementary time segment, the divergence properties made by the coexistence of disconnected topology and unstable systems were overcome. Moreover, the consensus was proven by the utilization of a novel function named PTVLF. Lastly, the proposed results were illustrated by a simulation example. Future works conclude reducing the computation burden of the main theorem.

Data Availability

All of the parameters of the numerical simulation are included in the study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this study.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant nos. 51879027, 51579024, and 62003337), the Postdoctoral Research Foundation of China (Grant no. 3620081006), and the Natural Science Foundation of Liaoning Province (Grant no. 2020-BS-026).

References

[1] H. Ma, X. Jia, N. Cai, and J. Xi, “Adaptive guaranteed-performance consensus control for multi-agent systems with an adjustable convergence speed,” *Discrete Dynamics in Nature and Society*, vol. 2019, Article ID 5190301, 9 pages, 2019.

[2] Y. Ji, “Distributed consensus of semi-Markovian jumping multiagent systems with mode-dependent topologies,” *Discrete Dynamics in Nature and Society*, vol. 2018, Article ID 6360782, 7 pages, 2018.

[3] J. Sun and Z. Wang, “Consensus of multi-agent systems with intermittent communications via sampling time unit approach,” *Neurocomputing*, vol. 397, pp. 149–159, 2020.

[4] L. Wang, C. Wen, F. Guo, H. Cai, and H. Su, “Robust cooperative output regulation of uncertain linear multi-agent systems not detectable by regulated output,” *Automatica*, vol. 101, pp. 309–317, 2019.

[5] X. Yang, L. Liao, Q. Yang, B. Sun, and J. Xi, “Limited-energy output formation for multiagent systems with intermittent interactions,” *Journal of the Franklin Institute*, vol. 358, no. 13, pp. 6462–6489, 2021.

[6] J. Sun and Z. Wang, “Event-triggered consensus control of high-order multi-agent systems with arbitrary switching topologies via model partitioning approach,” *Neurocomputing*, vol. 413, pp. 14–22, 2020.

[7] L. Jian, J. Hu, J. Wang et al., “Distributed functional observer-based event-triggered containment control of multi-agent systems,” *International Journal of Control, Automation and Systems*, vol. 18, no. 5, pp. 1094–1102, 2020.

[8] Y. Zheng, J. Ma, and L. Wang, “Consensus of hybrid multi-agent systems,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 4, pp. 1359–1365, 2018.

[9] S. Liu, H. Jiang, L. Zhang, and X. Mei, “Distributed adaptive optimization for generalized linear multiagent systems,” *Discrete Dynamics in Nature and Society*, vol. 2019, Article ID 9181093, 10 pages, 2019.

[10] J. Sun, Z. Wang, and X. Fan, “Periodic event-triggered consensus control for multi-agent systems with switching jointly connected topologies,” *IET Control Theory & Applications*, vol. 14, no. 19, pp. 3282–3290, 2020.

[11] L. Wang, J. Xi, M. He, and G. Liu, “Robust time-varying formation design for multiagent systems with disturbances: extended-state-observer method,” *International Journal of Robust and Nonlinear Control*, vol. 30, no. 7, pp. 2796–2808, 2020.

[12] G. Wen, W. Yu, Y. Xia, X. Yu, and J. Hu, “Distributed tracking of nonlinear multiagent systems under directed switching topology: an observer-based protocol,” *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 5, pp. 869–881, 2017.

[13] J. Sun, Z. Wang, and N. Rong, “Sampled-data consensus of multiagent systems with switching jointly connected topologies via time-varying Lyapunov function approach,” *International Journal of Robust and Nonlinear Control*, vol. 30, no. 14, pp. 5369–5385, 2020.

[14] J. Xi, L. Wang, J. Zheng, and X. Yang, “Energy-constraint formation for multiagent systems with switching interaction topologies,” *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 67, no. 7, pp. 2442–2454, 2020.

[15] J. Sun, C. Guo, and L. Liu, “Consensus of piecewise time-varying multi-agent systems with switching topologies,” *Transactions of the Institute of Measurement and Control*, 2022.

[16] J. Liu, J. a. Fang, Z. Li, and G. He, “Time-varying formation tracking for second-order multi-agent systems subjected to switching topology and input saturation,” *International Journal of Control, Automation and Systems*, vol. 18, no. 4, pp. 991–1001, 2020.

[17] F. Sun, M. Tuo, J. Kurths, and W. Zhu, “Finite-time consensus of leader-following multi-agent systems with multiple time delays over time-varying topology,” *International Journal of Control, Automation and Systems*, vol. 18, no. 8, p. 1985, 2020.

[18] H. Yan, Q. Yang, and H. Zhang, “Distributed $H_{\infty}$ state estimation for a class of filtering networks with time-varying switching topologies and packet losses,” *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 12, pp. 2047–2057, 2018.

[19] G. Wen, Z. Duan, G. Chen, and W. Yu, “Consensus tracking of multi-agent systems with Lipschitz-type node dynamics and switching topologies,” *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 61, no. 2, pp. 499–511, 2014.

[20] G. Wen, W. Yu, G. Hu, J. Cao, and X. Yu, “Pinning synchronization of directed networks with switching topologies:
a multiple Lyapunov functions approach,” IEEE Transactions on Neural Networks and Learning Systems, vol. 26, no. 12, pp. 3239–3250, 2015.

[21] W. Ni and D. Cheng, “Leader-following consensus of multi-agent systems under fixed and switching topologies,” Systems & Control Letters, vol. 59, no. 3-4, pp. 209–217, 2010.

[22] B. Zhang and Y. Jia, “Algebraic criteria for consensus problems of general linear multi-agent systems with switching topology,” Journal of the Franklin Institute, vol. 352, no. 4, pp. 1521–1540, 2015.

[23] Z. G. Wu, Y. Xu, R. Lu, and T. Huang, “Event-triggered control for consensus of multiagent systems with fixed/switching topologies,” IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 48, no. 10, pp. 1736–1746, 2018.

[24] B. Cheng, X. Wang, and Z. Li, “Event-triggered consensus of homogeneous and heterogeneous multiagent systems with jointly connected switching topologies,” IEEE Transactions on Cybernetics, vol. 49, no. 12, pp. 4421–4430, 2019.

[25] D. Li, S. S. Ge, G. Ma, and W. He, “Layered formation-containment control of multi-agent systems in constrained space,” International Journal of Control, Automation and Systems, vol. 18, no. 3, pp. 768–779, 2020.

[26] N. Zhao and J. Zhu, “Sliding mode control for robust consensus of general linear uncertain multi-agent systems,” International Journal of Control, Automation and Systems, vol. 18, no. 8, pp. 2170–2175, 2020.

[27] S. Liang, Z. Liu, and Z. Chen, “Leader-following exponential consensus of discrete-time multi-agent systems with time-varying delay and intermittent communication,” International Journal of Control, Automation and Systems, vol. 18, no. 4, pp. 944–954, 2020.

[28] Y. E. Wang, H. R. Karimi, and D. Wu, “Conditions for the stability of switched systems containing unstable subsystems,” IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 66, no. 4, pp. 617–621, 2019.

[29] Z. Wang, J. Sun, and H. Zhang, “Stability analysis of T-S fuzzy control system with sampled-dropouts based on time-varying Lyapunov function method,” IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 50, no. 7, pp. 2566–2577, 2020.