Beyond Space-Time

A.M. Polyakov
Joseph Henry Laboratories
Princeton University
Princeton, New Jersey 08544

Abstract

These notes, based on the remarks made at the 23 Solvay Conference, collect several speculative ideas concerning gauge/strings duality, de Sitter spaces, dimensionality and the cosmological constant

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Let me begin with some general views. In fundamental physics we invent dynamical mechanisms, based on the first principles, like the Higgs mechanism, and construct models, based on these mechanisms. In the case of the standard model this path led to a tremendous success. Finding the right mechanisms may be easier than constructing a detailed model. For example, the Yang-Mills theory was at first misapplied to describe the rho mesons, and only ten years later found its right place in QCD. This is a kind of danger we should be aware of.

Another danger is to get distracted by non-dynamical anthropic arguments, which recently acquired some popularity. I find the anthropic principle irrelevant. It is unlikely to uncover fundamental ideas and equations governing the universe. But, in spite of these misanthropic remarks, I believe that in special cases anthropic arguments may be appropriate.

In what follows I shall briefly describe various mechanisms operating in and around string theory. This theory provides a novel view of space-time. I would compare it with the view of heat provided by statistical mechanics. At the first stage the word ”heat” describes our feelings. At the second we try to quantify it by using equations of thermodynamics. And finally comes an astonishing hypothesis that heat is a reflection of molecular disorder. This is encoded in one of the most fascinating relations ever, the Boltzmann relation between entropy and probability.

Similar stages can be discerned in string theory. The first is of course the perception of space-time. The second is its description using the Einstein equations. The third is perhaps a possibility to describe quantum space-time by the boundary gauge theory. Let us discuss in more details our limited but important knowledge of the gauge/string correspondence.

1 Gauge /String correspondence

It consists of several steps. First we try to describe the dynamics of a non-abelian flux line by some string theory. That means, among other things that the Wilson loop $W(C)$ must be represented as a sum over 2d random surfaces immersed in the flat 4d space-time and bounded by the contour $C$. Surprisingly, strings in 4d behave as if they are living in the 5d space, the fifth (Liouville) dimension being a result of quantum fluctuations. More detailed analyses shows that while the 4d space is flat, the 5d must be warped
with the metric
\[ ds^2 = d\phi^2 + a^2(\phi)d\vec{x}^2 \]  
where the scale factor \( a(\phi) \) must be determined from the condition of conformal symmetry on the world sheet \([2]\). This is the right habitat for the gauge theory strings. If the gauge theory is conformally invariant (having a zero beta function) the isometries of the metric must form a conformal group. This happens for the space of constant negative curvature, \( a(\phi) \sim \exp c\phi \) \([3]\). The precise meaning of the gauge/strings correspondence \([4],[5]\) is that there is an isomorphism between the single trace operators of a gauge theory, e.g. \( Tr(\nabla^k F_{\mu\nu} \nabla^l F_{\lambda\rho}...) \) and the on-shell vertex operators of the string, propagating in the above background. In other words, the S- matrix of a string in the 5d warped space is equal to a correlator of a gauge theory in the flat 4d space. The Yang-Mills equations of motion imply that the single trace operators containing \( \nabla_\mu F_{\mu\nu} \) are equal to zero. On the string theory side it corresponds to the null vectors of the Virasoro algebra, leading to the linear relations between the vertex operators. If we pass to the generating functional of the various Yang-Mills operators, we can encode the above relation in the formula
\[ \Psi_{WOE}[h_{\mu\nu}(x),...] = \langle \exp \int dx h_{\mu\nu}(x) T_{\mu\nu}(x) + ... \rangle_{YM} \]  
Here at the left hand side we have the "wave function of everything" (WOE). It is obtained as a functional integral over 5d geometries with the metric \( g_{mn}(x,y) \), where \( y = \exp -c\phi \), satisfying asymptotic condition at infinity \( (y \to 0) \ g_{\mu\nu} \to \frac{1}{y^2}(\delta_{\mu\nu} + h_{\mu\nu}(x)) \). It differs from the "wave function of the universe " by Hartle and Hawking only by the \( y^{-2} \) factor. On the right side we have an expression defined in terms of the Yang-Mills only, \( T_{\mu\nu} \) being its energy-momentum tensor. The dots stand for the various string fields which are not shown explicitly. An interesting unsolved problem is to find the wave equation satisfied by \( \Psi \). It is not the Wheeler-de Witt equation. The experience with the loop equations of QCD tells us that the general structure of the wave equation must be as following
\[ \mathcal{H}\Psi = \Psi \star \Psi \]  
where \( \mathcal{H} \) is some analogue of the loop Laplacian and the star product is yet to be defined. This conjectured non-linearity may lead to the existence of soliton-like WOE-s.
The formula (2), like the Boltzmann formula, is relating objects of very different nature. This formula has been confirmed in various limiting cases in which either LHS or RHS or both can be calculated. I suspect that, like with the Boltzmann formula, its true meaning will still be discussed a hundred years from now.

2 de Sitter Space and Dyson’s instability

Above we discussed the gauge/strings duality for the geometries which asymptotically have negative curvature. What happens in the de Sitter case? It is not very clear. There have been a number of attempts to understand it [6]. We will try here a different approach. It doesn’t solve the problem, but perhaps gives a sense of the right direction.

Let us begin with the 2d model, the Liouville theory. Its partition function is given by

$$Z(\mu) = \int D\varphi \exp\{-\frac{c}{48\pi} \int d^2x \left( \frac{1}{2} (\partial \varphi)^2 + \mu e^{\varphi} \right) \}$$

For large $c$ (the Liouville central charge) one can use the classical approximation. The classical solution with positive $\mu$ describes the AdS space with the scalar curvature $-\mu$. By the use of various methods [7] one can find an exact answer for the partition function, $Z \sim \mu^\alpha$ where $\alpha = \frac{1}{12}[c - 1 + \sqrt{(c - 1)(c - 25)}]$. In order to go to the de Sitter space we have to change $\mu \Rightarrow -\mu$. Then the partition function acquires an imaginary part, $\text{Im}Z \sim \sin \pi \alpha |\mu|^\alpha$. It seems natural to assume that the imaginary part of the Euclidean partition function means that the de Sitter space is intrinsically unstable. This instability perhaps means that due to the Gibbons-Hawking temperature of this space it ”evaporates” like a simple black hole. In the latter its mass decreases with time, in the de-Sitter space it is the cosmological constant. If we define the Gibbons-Hawking entropy $S$ in the usual way, $S = (1 - \beta \frac{\partial}{\partial \beta}) \log Z$, we find another tantalizing relation, $\text{Im}Z \sim e^S$, which holds in the classical limit, $c \to \infty$. Its natural interpretation is that the decay rate of the dS space is proportional to the number of states, but it is still a speculation, since the precise meaning of the entropy is not clear.

For further progress the euclidean field theory, used above, is inadequate and must be replaced with the Schwinger-Keldysh methods.
In higher dimensions we can try once again the method of analytic continuation from the AdS space. The AdS geometry is dual to a conformally invariant gauge field theory. In the strong coupling limit (which we consider for simplicity only) the scalar curvature of the AdS, \( R \propto \sqrt{\lambda} \) (\( \lambda = g_Y^2 N \)). So, the analytic continuation we should be looking for is \( \sqrt{\lambda} \Rightarrow -\sqrt{\lambda} \). In order to understand what it means in the gauge theory, let us notice that in the same limit the Coulomb interaction of two charges is proportional to \( \sqrt{\lambda} \) [8]. Hence under the analytic continuation we get a theory in which the same charges attract each other. Fifty years ago Dyson has shown that the vacuum in such a system is unstable due to creation of the clouds of particles with the same charge. It is natural to conjecture that Dyson’s instability of the gauge theory translates into the intrinsic instability of the de Sitter space. Once again the cosmological constant evaporates.

### 3 Descent to four dimensions

Critical dimension in string theory is ten. How it becomes four? If we consider type two superstrings, the 10d vacuum is stable, at least perturbatively, and stays 10d. Let us take a look at the type zero strings, which correspond to a non-chiral GSO projections. These strings contain a tachyon, described by a relevant operator of the string sigma model. Relevant operators drive a system from one fixed point to another. According to Zamolodchikov’s theorem, the central charge must decrease in the process. That means that the string becomes non-critical and the Liouville field must appear. The Liouville dimension provides us with the emergent ”time” in which the system evolves and changes its effective dimensionality (the central charge). As the ”time” goes by, the effective dimensionality of the system goes down. If nothing stops it, we should end up with the \( c = 0 \) system which has only the Liouville field. It is possible, however, that non-perturbative effects would stop this slide to nothingness [10]. In four dimensions we have the \( B \)-field instantons, described by the formula (at large distances) \( (dB)_{\mu \nu \lambda} = q_\mu q_\nu \frac{x_\lambda}{x^4} \). In the modern language they correspond to the NS5 branes. These instantons form a Coulomb plasma with the action \( S \sim \sum \frac{q_i q_j}{(x_i - x_j)^2} \). As was explained in [10], the Debye screening in this plasma causes ”string confinement”, turning the string into a membrane. Formally this is described by the relation

\[
\langle \exp i \int B_{\mu \nu} d\sigma_{\mu \nu} \rangle \sim e^{-aV} \tag{5}
\]
where we integrate over the string world sheet and $V$ is the volume enclosed by it. There is an obvious analogy with Wilson’s confinement criterion. While the gravitons remain unaffected, the sigma model description stops being applicable and hopefully the sliding stops at 4d.

4 Screening of the cosmological constant

Classical limits in quantum field theories are often not straightforward. For example, classical solutions of the Yang - Mills theory describing interaction of two charges have little to do with the actual interaction. The reason is that because of the strong infrared effects the effective action of the theory has no resemblance to the classical action. In the Einstein gravity without a cosmological constant the IR effects are absent and the classical equations make sense. This is because the interaction of gravitons contain derivatives and is irrelevant in the infrared.

The situation with the cosmological term is quite different, since it doesn’t contain derivatives. Here we can expect strong infrared effects \[11\], see also \[9\] for the recent discussion.

Let us begin with the 2d model \((3)\). The value of $\mu$ in this lagrangian is subject to renormalization. Perturbation theory generates logarithmic corrections to this quantity. It is easy to sum up all these logs and get the result $\mu_{ph} = \mu \left( \frac{\Lambda}{\mu_{ph}} \right)^{\beta}$, with $\beta = \frac{1}{12} [c - 13 - \sqrt{(c - 1)(c - 25)}]$. Here $\Lambda$ is an UV cut-off while the physical (negative) cosmological constant $\mu_{ph}$ provides a self-consistent IR cut-off. We see that in this case the negative cosmological constant is anti-screened.

In four dimensions the problem is unsolved. For a crude model one can look at the IR effect of the conformally flat metrics. If the metric $g_{\mu\nu} = \phi^2 \delta_{\mu\nu}$ is substituted in the Einstein action $S$ with the cosmological constant $\Lambda$, the result is $S = \int d^4x \left[ -\frac{1}{2} (\partial \phi)^2 + \Lambda \phi^4 \right]$. There is the well known non-positivity of this action. This is an interesting topic by itself, but here we will not discuss it and simply follow the prescription of Gibbons and Hawking and change $\phi \Rightarrow i\phi$. After that we obtain a well defined $\phi^4$ theory with the coupling constant equal to $\Lambda$. This theory has an infrared fixed point at zero coupling, meaning that the cosmological constant screens to zero.

There exists a well known argument against the importance of the infrared effects. It states that in the limit of very large wave length the perturbations can be viewed as a change of the coordinate system and thus are simply...
gauge artefacts. This argument is perfectly reasonable when we discuss small fluctuations at the fixed background (see [12] for a different point of view). However in the case above the effect is non-perturbative— it is caused by the fluctuation of the metric near zero, not near some background. In this circumstances the argument fails. Indeed, if we look at the scalar curvature, it has the form $R \sim \varphi^{-3} \partial^2 \varphi$. We see that while for the perturbative fluctuations it is always small because of the second derivatives, when $\varphi$ is allowed to be near zero this smallness can be compensated. In the above primitive model the physical cosmological constant is determined from the equation $\Lambda_{ph} = \frac{\text{const}}{\log \Lambda_{ph}}$ which always has a zero solution. One would expect that in the time-dependent formalism we would get a slow evaporation instead of this zero. The main challenge for these ideas is to go beyond the conformally flat fluctuations. Perhaps gauge/strings correspondence will help.

As it is clear from the list of the references below, these ideas (except for the gauge/strings correspondence) did not attract any attention. Perhaps they don’t deserve it. My best hope, however, is that some of them may serve as small building blocks of the future theory, the vague contours of which we can discern at the horizon.

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References

[1] A. M. Polyakov, “Quantum Geometry Of Bosonic Strings,” Phys. Lett. B 103 (1981) 207.

[2] A. M. Polyakov, “String theory and quark confinement,” Nucl. Phys. Proc. Suppl. 68 (1998) 1 [arXiv:hep-th/9711002].

[3] J. M. Maldacena, “The large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2 (1998) 231 [Int. J. Theor. Phys. 38 (1999) 1113] [arXiv:hep-th/9711200].
[4] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge theory correlators from non-critical string theory,” Phys. Lett. B 428 (1998) 105 [arXiv:hep-th/9802109].

[5] E. Witten, “Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. 2 (1998) 253 [arXiv:hep-th/9802150].

[6] A. Strominger, “The dS/CFT correspondence,” JHEP 0110 (2001) 034 [arXiv:hep-th/0106113].

[7] V. G. Knizhnik, A. M. Polyakov and A. B. Zamolodchikov, “Fractal Structure Of 2d-Quantum Gravity,” Mod. Phys. Lett. A 3 (1988) 819.

[8] J. M. Maldacena, “Wilson loops in large N field theories,” Phys. Rev. Lett. 80 (1998) 4859 [arXiv:hep-th/9803002].

[9] R. Jackiw, C. Nunez and S. Y. Pi, “Quantum relaxation of the cosmological constant,” Phys. Lett. A 347 (2005) 47 [arXiv:hep-th/0502215].

[10] A. M. Polyakov, “Directions In String Theory,” Phys. Scripta T15 (1987) 191.

[11] A. m. Polyakov, ‘Phase Transitions And The Universe,” Sov. Phys. Usp. 25 (1982) 187 [Usp. Fiz. Nauk 136 (1982) 538].

[12] N. C. Tsamis and R. P. Woodard, “Quantum Gravity Slows Inflation,” Nucl. Phys. B 474 (1996) 235 [arXiv:hep-ph/9602315].