Simulation of Off-center Crack Dynamic Curving Fracture with Manifold Element

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ABSTRACT

On basis of introducing the method of solving the dynamic problems, this paper adopts numerical manifold method program to simulate the crack propagation of the off-center crack in the simply supported beam. The result shows that the crack propagation path obtained by the numerical manifold method is very correspondent with the experiment result, that is to say producing the evident curving fracture phenomenon. The failure of the material is the result of mixed propagation of crack mode I and II, and mode I plays a dominant role. At the same time, with the increase of the off-center distance, the stress intensity factor of the crack tip gradually decreases.¹

INTRODUCTION

Numerical manifold method is successfully used in numerical simulation study of crack propagation and material failure from its birth [1-2]. But now it is mainly used in the steady crack propagation under static load, rarely in unsteady or fast crack propagation under dynamic load. While under dynamic load, fast crack propagation is a familiar and very dangerous failure style. Because now many engineering structures are under dynamic load, and the propagation velocity of the dynamic crack is much more than that of the static one, the failure of material is very easy to occur if the dynamic crack propagates. Therefore, study on the crack propagation under dynamic load is of very important theoretical and practical significance.

Fracture dynamics should be adopted in study of the crack dynamic propagation, which mainly studies these fracture mechanics problems in which the

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inertia effect can not be ignored [3]. Under dynamic loading, the particles will obtain certain acceleration to produce inertia force besides elasto-plastic deformation in the specimen, which is so called inertia effect. Because of the existence of inertia effect, the solution method of fracture dynamics is evidently different from that of fracture statics. Therefore, in order to simulate the crack dynamic propagation better, fracture dynamics theory must be adopted. On basis of illustration of basic principle of numerical manifold method in solving dynamic problems, its program is adopted to simulate the crack propagation path of an off-center crack in a simply supported beam under impact load. The change law of stress intensity factor of the crack tip with time is analysis during the crack propagation.

NUMERICAL MANIFOLD METHOD OF DYNAMICS

Numerical Manifold Method with Second-Order Total Displacement

The more accurate calculation results of stress and displacement fields in simulation of the crack propagation, therefore, numerical manifold method with second-order total displacement must be adopted. There are two methods in generating second-order total displacement function, one of which is linear weight function and linear cover function, the other is second-order weight function and constant cover function. Because the crack propagation direction is arbitrary, the small element will be produced. When the small element occurs, calculation of the first method is very difficult to converge and the integration error of the physical element will increase when the deformation is large. Therefore, the second-order numerical method which is composed by second-order weight function and constant displacement function is adopted here. Its element is similar to six-node triangle element in finite element method, whose displacement function is shown as Eq. (1).

\[
\begin{align*}
\{u(x, y)\} = & \sum_{i=1}^{6} w_i(x, y) \{u_i\} \\
\{v(x, y)\} = & \sum_{i=1}^{6} w_i(x, y) \{v_i\}
\end{align*}
\]  

(1)

Its matrix composition, calculation methods and proof of its convergence are shown as reference [5].

SOLUTION EQUATION OF DYNAMICS PROBLEMS

The equation solving dynamics problems in numerical manifold method is structure dynamics equations [4]:

\[
M \ddot{\delta} + C \dot{\delta} + K \Delta \delta = \Delta F
\]  

(2)

where, \(M\) is mass matrix, \(C\) is damp matrix, \(\Delta \delta\) is displacement increment, \(\dot{\delta}, \ddot{\delta}\) are velocity and acceleration respectively. \(K = K_e + K_p + K_{cs} + K_f\), where,
$K_e$ is stiffness matrix, $K_{cn}, K_{cs}$ are contact matrices between blocks and discontinuous faces, $K_f$ is constraint matrix. $\Delta F$ is total load increment, $\Delta F = F_p + F_b + F_f - F_0 + F_{cn} + F_{cs} + F_{f_r}$, $F_p$ is external vector, $F_b$ is body force vector, $F_f$ is equivalent load vector caused by known constraint displacement, $F_0$ is initial stress vector, $F_{cn}, F_{cs}$ are equivalent load vectors caused by normal and triangle contact, $F_{f_r}$ is equivalent load vector caused by friction between contact faces.

**PROPAGATION CRITERION OF A CRACK**

For the existing crack propagation, the stress intensity factor criterion is adopted in numerical manifold method, in which the maximum circular stress theory is used to decide the crack propagation direction and fracture toughness $K_{IC}$ is used to decide whether the crack propagates. Therefore, stress intensity factor is very important in crack propagation simulation. In numerical manifold method, the displacement field is firstly solved according to second order manifold element, then the stress intensity factor is solved with singular boundary element, and finally the maximum circular stress criterion is adopted to decide whether the crack propagates and its propagation direction [6]. Suppose the angle between the crack propagation direction and the existed crack direction is $\theta$, then $\theta$ can be solved by Eq. (3).

\[
K_I \sin \theta + K_{II}(3\cos \theta - 1) = 0
\]

When the crack propagation angle $\theta$ is solved, its propagation criterion of the composed crack $I - II$ can be expressed by Eq. (4).

\[
\cos \frac{\theta}{2} (K_I \cos^2 \frac{\theta}{2} - \frac{3}{2} K_{II} \sin \theta) = K_{IC}
\]

**ANALYSIS OF A CALCULATION EXAMPLE**

**Displacement Field and Stress Field of Dynamic Crack Tip**

When dynamic load is acted on the object with a crack, the stress concentration phenomenon also occurs at the crack tip, which can also be described with stress intensity factor. To crack $I$ in a plane, its dynamic stress intensity factor can be defined with the following Eq. (5).

\[
K_{I_{dyn}} (t) = \lim_{r \to a} \sqrt{2\pi r} \sigma_{yy} (r, 0, t)
\]
To the composed dynamic crack I and II in a plane, the format of stress and displacement fields at the crack tip is same as that of static. The only difference is that the static stress intensity factor in static formula is changed into the dynamic one [7]. But because the dynamic stress intensity factor is function of time, the stress and displacement fields of a dynamic problem are also functions of time, which has essential difference with dynamic problems.

**Dynamic Curving Fracture of a Simply Supported Beam**

Dynamic load is a usual dynamic load in practical engineering, which is one of the external loads causing most engineering materials to fail. When the load acts on the structure with a crack, its dynamic fracture behavior is very different from that of the static one. When the impact load acts on an object, the stress wave produced by the load will propagate in it. The stress intensity factor at the crack tip will show different dynamic property and the crack will have different initiation and propagation behavior. Meanwhile, the dynamic behavior of the crack will have different dispersion to the propagation of the stress wave, and there exists interaction between every wave and the cracks. Therefore, study on the dynamic behavior of the structure with the crack under impact load is of very important theoretical significance and engineering value to not only dynamic mechanical behavior but also safety evaluation of the engineering structure. Now the theoretical and experimental study on the crack propagation behavior under impact load is illustrated in many relevant references [8-10]. On basis of the study before, the simulation study on the crack dynamic propagation behavior under the impact load is made with numerical manifold method here.

**A Calculation Model**

In order to compare with concerned experiment results, the experiment model in reference [10] is adopted to simulate. The calculation model is a simply supported beam with an off-center crack, shown as Fig.1. In order to observe the crack curving fracture phenomenon better, the width of the beam is rather large, whose dimension is shown as Fig.1. The external load is a 107N step impact load. The elastic constants E and \( \mu \) of the beam are 100GPa and 0.2, respectively.

![Figure 1. Simply supported beam with an off-center crack (cm).](image)
Dynamic Curving Paths

The simulation results of the beam with different off-center distance \( a \) are shown as Fig.2 (only the right half calculation model is shown in Fig.2).

![Figure 2. Crack propagation path with different off-center distance.](image)

It can be seen from Fig.2, the crack propagation path shows different curving fracture under different off-center distance, whose reason is as follow.

From simulation results, it can be seen that the crack initiation propagation direction is along the direction of the initial crack, but it will produce curving fracture towards the direction of impact load action point after it propagates a certain distance. But it can also be seen that the larger the off-center distance is, the earlier the time of curving fracture is, the more serious curving fracture phenomenon is. Meanwhile, the second curving fracture will happen after the crack propagates a certain distance. The main reason of this curving fracture phenomenon is that the force at the crack tip is mixture load of type I and II when the curving wave produced by the impact load acts on the crack, and the crack propagation belongs to compound crack propagation of type I and II. If the shorter the off-center distance is and the earlier the crack initiation propagation time is, the larger the proportional of type I stress. Therefore, at the beginning of crack initiation, the crack will propagate along the original direction, and with increase of the off-center distance, the curving fracture degree will be more serious. When the crack propagates to upper of the beam, the crack continues to make curving motion because of inertia. But the crack propagation velocity will become lower because of the action of compressive stress. Meanwhile the stress in the beam is newly distributed because of crack propagation, and a new center point occurs below the crack tip. The compressive and tensile stresses are produced below and up this
point respectively, which composes the new bending moment causing the crack to the second curving fracture, and leads to the structure failure finally.

**Change Law of Stress Intensity Factor During Curing Facture**

Fig.3 is the change relationship between two types of stress intensity factor at crack tip and time at different off-center distance. Firstly, from it can be seen that KI is always positive, while KII is positive or negative. In fact, three types of stress intensity factor are positive or negative. When KI>0, it indicates that the crack surface is open, while when KI<0, it indicates that two surfaces of a crack are inset each other, which is impossible of course. In fact, the upper and down lower surfaces will compress each other when the crack closes, in which there will be contact stress and friction force, which is very complicated. Therefore, KI is always larger than zero, which indicates the crack is open under external load. Second, it can be seen that the stress intensity factor is little at the beginning, while it quickly becomes large along with increase of crack propagation length, and eventually the crack runs through. While during this process, KII is rather less, which shows that crack I plays much more role in crack propagation that crack II during curving fracture. Third, from crack propagation time, the time of crack running through is shorter when the off-center distance is shorter, while along with the increase of off-center distance, the crack propagation path also becomes larger, and the time of crack running through also becomes larger. Fourth, to the stress intensity factor during crack propagation, the stress intensity factor of the crack tip gradually becomes little with the increase of off-center distance, which agrees with the theoretical results. Because along with the increase of off-center distance, the force of the crack become little under the same external load, therefore, the stress intensity factor of crack tip also becomes little correspondingly.
CONCLUSIONS

Through the study above, the following conclusions can be obtained.

(1) Numerical manifold method is feasible valid.

(2) The crack propagation in this paper belongs to composite crack propagation of type I and II. When the time of crack initiation is earlier and the off-center distance is shorter, stress intensity factor of type I has much more proportion. Meanwhile, the curving fracture phenomenon is more evident with increase of crack off-center distance.

(3) During the crack curving fracture, the stress intensity factor of type I is always larger than zero, which indicates that the crack is open under external load. Moreover, $K_I$ is always larger than $K_{II}$ during crack propagation, which shows crack of type I has a dominant place. From the viewpoint of stress intensity factor during crack propagation, at the same time, the stress intensity factor of crack tip gradually decreases with the distance of crack off-center distance increasing.

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