A study of a discrete Bayes and a Kalman filter computational Complexity and performance in the case of 1D robot localization

I Siradjuddin1, *, I M Fitriani1, R A Asmara2, M Junus1, T S Patma1, G A Azhar1 and H Setiawan1

1 Electronics Engineering Department, State Polytechnic of Malang. Jl. Soekarno Hatta No. 9 Malang 65141
2 Information Technology Department, State Polytechnic of Malang. Jl. Soekarno Hatta No. 9 Malang 65141

*indrazno@polinema.ac.id

Abstract. In the robotic field of study, localization is one of the important methods for autonomous mobile robot navigation. Probabilistic approaches have received significant attention in the robotics community. The discrete Bayes and Kalman filters are the fundamental algorithms in probabilistic approach which have to be clearly understood in order to develop more advanced filtering algorithms. This paper discusses discrete Bayes and Kalman filtering algorithms. The mathematical representation of each filter algorithm, in the 1-dimensional case, presented in detail. The algorithms were implemented using python to simulate the probability of the robot position. The algorithm's complexity was analysed with respect to the computational cost and size of memory used. From this study, it has been observed that the Kalman filter is computationally more efficient, and less memory is required.

1. Introduction

The development of the robotic field is increasing rapidly. Developments in the field of robotics have the purpose of making it easier for humans to do work. In the robotic field of study, localization is one of the essential methods for autonomous mobile robot navigation, which the robot has to be able to locate itself in the environment [1]. Robots with these advantages received considerable interest because it will be pretty helpful for people to work in high-risk jobs such as detecting gas leaks.

One of the major problems of the localization process is the rate of accuracy from the robot's position data [2]. The accuracy of the information is necessary for this type of robot because it can decide the navigation system and motion planning in the field. The rate of accuracy becomes more inaccurate because of the uncertainty data from several factors such as sensor, computational time, and the environmental condition [2]. The condition in the environment is unpredictable because the level of uncertainty environment is very high. Some factors that influence the level of uncertainty are structure and density condition in the field. The second factor is the sensors. Most of the sensor such as odometry, laser, and camera has some noise for its data. The rate of uncertainty from the sensor comes from some noise carried by the observation data, and the third factor is robot computation time. Robots are real-time systems. It means the robot has a limited amount of capacity for executing the calculation process. There are a lot of advanced algorithms such as approximate logic is used to be able to achieve timely
response at the expense of accuracy. Therefore, to be able to make a perfect mobile robot, the ability to overcome uncertainty is pretty necessary [3].

The solution is needed to solve the problem such as a Kalman filter to maximize the position estimation result from observation in the localization process [4-6] and Kalman filter is suitable in a real-time system [7]. In the previous research about Mobile Robot Localization: A Review of Probabilistic Map-Based Techniques explained that Kalman filter suitable for linear systems with Gaussian noise and to get better estimation using particle filter [8]. This paper discusses which method that can give the best estimation respect to computational time and size of memory used between discrete Bayes and Kalman filter in the case of 1D Robot Localization.

2. Method

2.1. Discrete bayes
Bayesian is a theory to estimate the probability of an event in the future. The probability distribution of the Bayesian expressed in Equation 1.

\[
p(x|z) = \frac{p(z|x) \cdot p(x)}{p(z)}
\]  

From equation (1), the posterior probability of \( x \) given by \( z \) is equal to the normalized measurement probability of \( z \) given by \( x \) and multiplied by the prior probability of \( x \). The step to determine the position probability of a mobile robot in this study is shown in figure 1.

![Figure 1. Discrete bayes step.](image)

2.2. Kalman filter
Similar to the Bayesian filter, The Kalman filter is also an estimator. In this research, Kalman filter distributes a state characterized by a Gaussian distribution [9] while in discrete Bayes filter distributes by a histogram.
\( \mu \) is the average of Gaussian distribution and \( \sigma^2 \) is the deviation standard of Gaussian distribution. The step to determine the probability of the position of a mobile robot shown in figure 3.

**Figure 2.** Gaussian distribution of Kalman filter.

**Figure 3.** Kalman filter step in the case of 1D robot localization.

3. Results and discussion

In the discrete Bayes simulation \( bel(\chi_{t-1}) \) is determined to be \([1, 0, 1, 0, 0, 1, 0, 1, 0, 0]\) as shown in figure 4. After being given \( u_t \), the result of \( z_t \) determined to be \([1, 0, 1, 0, 0, 1, 0, 1, 0, 0]\).

**Figure 4.** The initial position of discrete Bayes.

**Figure 5.** The posterior probability of discrete Bayes.
With the determination as above, the robot has a problem because the probability of the current position is between $x = 0$ and $x = 5$ with the same probability value of 0.28. This happens because the same results of sensor readings when the robot is on $x = 0$ and $x = 5$, that is [1, 0, 1, 0, 1, 0, 1, 0, 0, 0].

Whereas in the Kalman filter simulation, $\mu_{t-1}$ and $\sigma^2_{t-1}$ determined to be 10 and 1. Then provide 2 provisions for each control (optional), the first provision are $u_t = 40$ and $\sigma^2_r = 10^2$ and the second provision are $u_t = 70$ and $\sigma^2_r = 10^2$. The first results of $\mu_t$ and $\sigma^2_t$ determined to be $\mu_t = 60$ and $\sigma^2_t = 10^2$ and the second results $\mu_t = 140$ and $\sigma^2_t = 20^2$. With this determination, the position probability of the robot is obtained as shown in figure 6.

![Figure 6. The posterior probability of Kalman filter.](image)

| Table 1. The $\mu$ value of posterior result. |
|-----------------------------------------------|
| **Estimation** | **$\mu$ prediction** | **$\mu$ measurement** | **$\mu$ correction** |
| 1 | 50.0 | 60.0 | 55.025 |
| 2 | 125.025 | 140.0 | 129.114 |

Based on figure 6, the first estimation value of correction probability is $\pm 0.057$ and the second is $\pm 0.037$. This indicates that the robot is more confident in estimation 1, that $x$ is 55.025.

In a previous study conducted by Salvador Manuel Malagon-Soldara and friends [8], explain that the solution to get better estimation is using particle filter and Kalman filters are suitable for linear systems and having gaussian noise. Whereas P. Wang, H. Li and B. Himed [10], explain that Bayesian detectors can reduce the computational complexity.

Based on the results of this research, the Kalman filters also give a better estimation in the case of 1D robot localization. In the Kalman filter, to get a better estimate, the variance value should be as small as possible. Discrete Bayes also provide better estimates by breaking discrete into smaller parts so that estimation becomes more accurate.

However, the memory used and computational time depends on the size of the robot working space. In contrast, the memory used and the computational time of the Kalman filter only depend on the number of states [9].

4. Conclusion

Based on the results, it can be concluded that the memory used and the computational time of the Kalman filters is less than the discrete Bayes.

Acknowledgement

We would also like to show our gratitude to the State Politechnic of Malang for the support of this research.
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