Assessment of the impact of the main technological characteristics of wells on the power consumption of pumps

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Abstract. The issue of assessing the impact of the main technological characteristics of wells on the power consumption of pumps is one of the important issues. Based on the analysis of the data obtained in the article, the electric energy consumption of the well pump device the rotational speed of the pump (ω); the density of the solution (liquid) (ρ); the pressure generated by the pump (H); the performance of the pump aggregate (q); depth of the well (H); hydrodynamic resistance (dp). Also, on the basis of the STATISTICA program, the calculation work is carried out, the binding function of the pumps is determined to what extent the factor affects the electricity consumption, and is described in the Pareto diagram.

1. Introduction

Pumping systems are one of the largest consumers of electrical energy in both industrial and residential applications. Electrical motors utilize around 22% of the energy consumed by these systems. The issue of pumping system energy efficiency is a hot topic among scholars [1]. The energy consumption of the pump system is investigated using a mathematical model, data from the manufacturer's catalogues, and experimental data [2].

Several studies have been conducted with the goal of operating pumping systems and estimating pressure losses in pipes, yielding valuable experimental results from the testing of pumping equipment already built by reputable international firms; the main limitation of most of these results is that they were obtained using water as the test fluid. Studies on the modeling of centrifugal pumps have been published, which are mostly devoted to the study of the transport of sugary honeys from the sugar industry, and where correction factors for the characteristic curves are obtained. There have also been established mathematical models that allow for the estimate of load and design capacity as a function of the exit blade angle, specific velocity, and viscosity of the fluid other than water. The flow rates of the pumps employed are heavily influenced by the rheological features of the fluid transported; nevertheless, due to a lack of information about these characteristics, they are selected and used under the assumption that the fluids' rheological behavior is Newtonian [3, 4].

A mathematical model is a formalized description of a system (or operation) in some abstract language, for example, in the form of a set of mathematical relationships or an algorithm scheme, i.e., a mathematical description that provides an imitation of the operation of systems or devices at a level close enough to their real one, behavior obtained during field testing of systems or devices [5].

The search for optimal conditions, both in full-scale and in a model experiment, is one of the most common scientific and technical tasks. They occur at the moment when the possibility of carrying out the
process is established and it is necessary to find the best (optimal in a sense) conditions for its implementation [6].

Mathematical modeling is also used in this study to assess the impact of the main technological characteristics of wells on the power consumption of pumps.

2. Methods

The experimental study contains three stages [7]: a) stage of setting the task, b) stage of planning and conducting the experiment, and c) analysis and interpretation of results.

The target function is the electrical energy consumption of the well pump $W$. $W$ must meet the following requirements [7]:

a. the parameter must be measured at any change (combination) of technological process modes.

b. the parameter must be statistically effective, i.e. measured with the highest accuracy.

c. the parameter should be informational, i.e. it should comprehensively characterize the technological process.

Based on the analysis of the data obtained by us, it is revealed that the electrical energy consumption $W$ of the borehole pumping unit is a function of:

$$W = f(\omega, \rho, H, h, \Delta p, \theta)$$

(1)

where the factors are:

- $\omega$ - the speed of rotation of the pump shaft (liquid), round/min;
- $\rho$ - the density of the solution (liquid), $kg/m^3$;
- $H$ - the pressure generated by the pump, $m$;
- $Q$ - the performance of the pumping unit, $m^3/hour$;
- $h$ - the depth of the well, $m$;
- $\Delta p$ - hydrodynamic resistance, $N/m^2$;
- $\theta$ - the level of filling of filters, $N/m^2$.

As a factor, we will take a value characterizing a particular property of an object or a mode of technological equipment. This value, the numerical value of which is measured within the limits (boundaries) of the change, should affect the power consumption of the pumping unit [8]. The description of the object under study cannot be obtained in the form of an exact function formula that is valid over the entire range of arguments. It can only be approximate and only over a small area in the vicinity of the selected base point. The approximation of the desired mathematical dependence is a polynomial segment of the Taylor series, into which the unknown dependence is decomposed

$$Y = f(X_1, X_2, ..., X_n) = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{j=1}^{n} b_{ij} x_j + \sum_{i=1}^{n} b_{iij} x_{ij} + ....$$

(2)

where

$$b_i = \frac{\partial^2 f}{\partial X_i^2} \Big|_{X=0}; b_{ij} = \frac{\partial^2 f}{\partial X_i \partial X_j} \Big|_{X=0}; b_{iij} = \frac{\partial^2 f}{\partial X_i \partial X_j \partial X_i} \Big|_{X=0}.$$

It is well known, that due to the presence of unmanaged and even uncontrolled input variables $X_i$ - ($\nu, \rho, \delta, \mu, \alpha$), the change in the value of $Y$ ($W$) is random, and therefore equation (2) does not give us an exact relationship between the input and output of the object and is only a conditional mathematical expectation of the random variable $Y$, i.e. the regression equation. The coefficients of the regression equation are determined by the following system of equations [9].

$$\sum_{g=1}^{N} \left( Y_g - \bar{Y} \right)^2 = \sum_{g=1}^{N} \left( Y_g - \sum_{i=0}^{d} b_i x_{ig} \right)^2 = \min .$$

(3)

where $Y_g$ are the experimental values of the output parameter obtained at the $g$-th point of the factor space; $d$ is the number of terms in the regression equation.

As is known, when finding polynomial dependencies, the model adequacy is checked by the Fisher criterion and the significance of coefficients is estimated by the Student's criterion. Before implementing the plan, the following are performed:

a) Collecting information, in the absence of which it is necessary to carry out one-factor cross-sections, i.e. implementation of one-factor dependencies $Y=f(X)$, where $y$ is the response function (performance criterion).
b) The choice of factors forcing performance (independent variables).
c) Appointment of intervals of variation of factors, i.e. area of study of the desired function
d) \( Y = f(X) \).

Encoding of factors. Encoding (converting to relative units). Encoding is performed using the following formula:
\[
X_i = \frac{X_i - x_{iav}}{x_{imax} - x_{imin}} = \frac{X_i - x_{iav}}{\Delta x},
\]
(4)

Where: \( X_i \) and \( x_i \) are encoded and natural variables; \( x_{imin} \) and \( x_{imax} \) is the minimum (lower level) value and the maximum (upper level) value; \( x_{iavr} \) - zero level; \( \Delta x = x_{imax} - x_{iavr} \) - variation interval. Encoded variables take the following values: (+1) - at the upper level, (-1) - at the lower level, (0) - at the zero level.

Expression (3) is the main criterion for checking the correctness of the found regression equation.

3. Results and Discussion

The plan matrix obtained in the STATISTICA program is shown in Table 1.

| Sample | speed, w | density, r | productivity | depth, h | filtering capacity, q | electricity W |
|--------|----------|------------|--------------|---------|-----------------------|--------------|
| 6      | 1.00000  | -1.00000  | 1.00000      | -1.00000| 1.00000               | 16.83        |
| 3      | -1.00000 | 1.00000   | -1.00000     | -1.00000| 1.00000               | 15.54        |
| 7      | -1.00000 | 1.00000   | 1.00000      | -1.00000| -1.00000              | 15.54        |
| 8      | 1.00000  | 1.00000   | -1.00000     | 1.00000 | 1.00000               | 18.5         |
| 2      | 1.00000  | -1.00000  | -1.00000     | 1.00000 | -1.00000              | 15.54        |
| 4      | 1.00000  | 1.00000   | 1.00000      | 1.00000 | -1.00000              | 17.02        |
| 5      | -1.00000 | -1.00000  | 1.00000      | 1.00000 | -1.00000              | 15.91        |
| 1      | -1.00000 | -1.00000  | -1.00000     | 1.00000 | 1.00000               | 15.54        |

The estimation of statistical data of formula (5) – (9) was performed according to the well-known method according to [10-12]. The calculation was performed using the Statistica application software package version 10.0. In accordance with this, estimates of variances are determined by the formula:
\[
SS = \frac{\sum_{i=1}^{m} (Y_{ig} - \bar{Y}_g)^2}{m-1},
\]
(5)

since all variances are obtained from samples of the same volume \( m \), the number of degrees of freedom for all variances is the same and is equal to
\[
df = m-1
\]
(6)

the average square of the effect is determined according to
\[
MS = \frac{SS}{df},
\]
(7)

to test the hypothesis of equality of mathematical expectations in two samples under the assumption of normality of distributions with equal variances, the Student's criterion is used
\[
t = \left( Y_{ig} - \bar{Y}_g \right) \sqrt{\frac{1}{SS \cdot N}},
\]
(8)

Where: \( N \) is the sample size.
The model adequacy hypothesis is tested using $F$ – the Fisher criterion. Fischer's criterion allows us to test the null hypothesis about the equality of two general variances $D_x$ and $D_y$.

The variance ratio is used as a reference value,

$$F = \frac{D_x}{D_y}, \quad (9)$$

Where: $D_x = \frac{1}{N-d} \sum_{i=1}^{N} (Y_{ix} - \bar{Y}_x)^2$; $D_y = \frac{1}{N} \sum_{i=1}^{N} SS$; $N$ is the sample size; $d$ is the number of terms of the approximating polynomial.

As is known, when determining the value in accordance with the expression (9), the criterion indicator $F$ is less than the tabular $F_{tab}$ one obtained for $q\%$ - the significance level, i.e. $F_{tab} > F$, the null hypothesis is accepted and the mathematical model of borehole pump electric energy consumption is considered adequate at a certain level of confidence. Otherwise, it is rejected and the description (model) is considered inadequate for the object.

In accordance with expressions (5) – (9), Table 2 shows the results of the analysis of variance.

| Factor       | Analysis of variance ANOVA; Variable: Electricity W R- sqr=.99472; Adj=.98153 |
|--------------|----------------------------------------------------------------------------------|
| 2**(5-2)     | draft, MS Residual = .0000625                                                   |
| (1) speed, w | 0.010512 1 0.010512 168.200 0.005893                                              |
| (2) density, r| 0.002813 1 0.002813 45.000 0.021508                                               |
| (3) performance, Q | 0.003613 1 0.003613 57.8000 0.016865                              |
| (4) depth, h | 0.004512 1 0.004512 72.2000 0.013569                                               |
| (5) filterability q | 0.002112 1 0.002112 33.8000 0.028334                          |
| Error        | 0.000125 2 0.000063                                                              |
| Totals       | 0.023687 7                                                                      |

| Factor       | Projected scoring; Var: electricity W R- sqr=.99466; Adj=.98132 |
|--------------|------------------------------------------------------------------|
| 2**(5-2)     | design, MS Residual = .021625                                     |
| DV: Electricity, W |                                                              |
| Regressn Coeff | Value | Coefficient Value |
| Constant     | 16.3025 | 0.000000          |
| (1) speed, w | 0.670000 0.00 | 0.000000 |
| (2) density, r | 0.34750 0.00 | 0.000000 |
| (3) performance, Q | 0.39250 0.00 | 0.000000 |
| (4) depth, h | 0.440000 0.00 | 0.000000 |
| (5) filterability q | 0.30000 0.00 | 0.000000 |
| Predicted    | 16.3025 | 16.07880          |
| -95, % Conf,  | 15.63139 | 16.52620         |
| +95, % Conf,  | 16.97361 | 16.73611         |

Table 3-7 shows the results of evaluating the effect of variance analysis, including: correlation of effects (Table 3), draft samples by category (Table 4), category selection project (Table 5), correlation matrix of factors and variables (Table 6), evaluation of the effects of a mathematical model of power consumption of a borehole pumping unit (Table 7). Effect is the effect of the value of the contribution of each factor to
the consumed electrical energy by the pumping unit; Std. Error - Standard error of the effect estimate; t (df) and p-value - the value of the t-criterion and level p; t-test is used to test the hypothesis that the intercept is zero; F - values of the F-criterion; df is the number of degrees of freedom of the F-criterion; p is the level of significance; Coeff. - coefficients of the equation; Std. Err. Coeff. is the standard error of the coefficients (equations).

Table 4. The results of evaluating the effect of variance analysis, including: draft samples by category

| No | Category selection project | speed, w | Density, r | Productivity, Q | Depth, h | Filterable, q | Electricity, W Means | Electricity, W Std.Dev | Electricity, W N |
|----|---------------------------|----------|------------|----------------|---------|--------------|----------------------|----------------------|-------------------|
| 1  | ALL                       | 1.00     | 0.00       | 0.00           | 0.00    | 0.00         | 1.00                 | 0.00                 | 1.00              |
| 2  | 1 by 3                    | 0.00     | 0.00       | 0.00           | 0.00    | 0.00         | 0.00                 | 0.00                 | 1.00              |
| 3  | 1 by 5                    | 0.00     | 0.00       | 0.00           | 0.00    | 0.00         | 0.00                 | 0.00                 | 1.00              |
| 4  | 2 by 3                    | 1.00     | 0.00       | 0.00           | 1.00    | 0.00         | 0.00                 | 0.00                 | 1.00              |
| 5  | 2 by 5                    | 0.00     | 0.00       | 0.00           | 0.00    | 0.00         | 0.00                 | 0.00                 | 1.00              |
| 6  | 1 by 2                    | 0.00     | 0.00       | 0.00           | 0.00    | 0.00         | 0.00                 | 0.00                 | 1.00              |

Table 5. Category selection project

| No | Category selection project | speed, w | Density, r | Productivity, Q | Depth, h | Filterable, q | Electricity, W Means | Electricity, W Std.Dev | Electricity, W N |
|----|---------------------------|----------|------------|----------------|---------|--------------|----------------------|----------------------|-------------------|
| 1  | ALL                       | 1.00     | 0.00       | 0.00           | 0.00    | 0.00         | 1.00                 | 0.00                 | 1.00              |
| 2  | 1 by 3                    | 0.00     | 0.00       | 0.00           | 0.00    | 0.00         | 0.00                 | 0.00                 | 1.00              |
| 3  | 1 by 5                    | 0.00     | 0.00       | 0.00           | 0.00    | 0.00         | 0.00                 | 0.00                 | 1.00              |
| 4  | 2 by 3                    | 1.00     | 0.00       | 0.00           | 1.00    | 0.00         | 0.00                 | 0.00                 | 1.00              |
| 5  | 2 by 5                    | 0.00     | 0.00       | 0.00           | 0.00    | 0.00         | 0.00                 | 0.00                 | 1.00              |
| 6  | 1 by 2                    | 0.00     | 0.00       | 0.00           | 0.00    | 0.00         | 0.00                 | 0.00                 | 1.00              |
| 7  | 1 by 5                    | 0.00     | 0.00       | 0.00           | 0.00    | 0.00         | 0.00                 | 0.00                 | 1.00              |
| 8  | 1 by 2                    | 0.00     | 0.00       | 0.00           | 0.00    | 0.00         | 0.00                 | 0.00                 | 1.00              |
| ALL|                           | 0.881250 | 0.58172    | 1             | 1       | 1            | 1                    | 1                    | 8                 |

Table 6. The results of evaluating the effect of variance analysis, including: correlation matrix of factors and variables

| Effects | Correlation factors and variables |
|---------|----------------------------------|
| speed, w| 1.00                             |
| density, r| 0.00                            |
| productivity, Q| 0.00  |
| depth, h| 0.00                            |
| filterable, q| 0.00  |
| Electricity, W Means| 0.00  |
| Electricity, W Std.Dev| 0.00  |
| Electricity, W N| 1.00  |

| (1) speed, w| 1.00 |
| (2) density, r| 0.00 |
| (3) performance, Q| 0.00 |
| (4) depth, h| 0.00 |
| (5) filterability q| 0.00 |
| 1 by 2| 1.00 |
| 1 by 3| 1.00 |
| 1 by 4| 1.00 |
| 1 by 5| 1.00 |

Table 7. The results of evaluating the effect of variance analysis, including: correlation matrix of factors and variables
Table 7. The effects of a mathematical model of power consumption of a borehole pumping unit

| Factor | Evaluation of effects; Variable: Electricity W; R-sq=0.99466; Adj: 0.98132 |
|--------|--------------------------------------------------------------------------|
|        | MS residual=0.021625, DV: Electricity, W                                  |
| Grade  | Stand error t(2) p -95, % Pred, meaning +95, % Pred, meaning Coefficient |
| Mean/Interc | 16.30250 0.051992 313.5604 0.006010 16.07880 16.07880 16.30250 0.051992 16.07880 |
| (1) speed, w | 1.34000 0.103983 12.8867 0.005968 0.89260 0.89260 0.67000 0.051992 0.44630 |
| (2) density, r | 0.69500 0.103983 6.6838 0.021660 0.24270 0.24760 0.34750 0.051992 0.12380 |
| (3) performance, Q | 0.78500 0.103983 7.5493 0.017098 0.33760 0.33760 0.39250 0.051992 0.16880 |
| (4) depth, h | 0.88000 0.103983 8.4629 0.013677 0.43260 0.43260 0.44000 0.051992 0.21630 |
| (5) filterability, q | 0.60000 0.103983 5.7702 0.028746 0.15260 0.15260 0.30000 0.051992 0.07630 |

The Pareto diagram is shown in Figure 1. In this diagram, estimates of the effects of variance analysis are arranged by absolute value: from the largest to the smallest. The magnitude of each effect is represented by a column, and the columns are intersected by a line indicating what the effect should be in magnitude (i.e., what the length of the column should be) in order to be statistically significant [13]. In accordance with this, all the factors discussed above are significant for the model of electric energy consumption of a well pumping unit.

Figure 2 shows a contour diagram of the performance versus speed relationship, taking into account the dependent variable of electrical energy. Figure 3 shows a surface diagram for finding the best combination of two data sets (electrical energy and performance).
Figure 2. Contour diagram. Variable: Electrical energy

Figure 3. Surface smoothing diagram. Variable: Electrical energy

Figure 4 shows the Box-Cox transformation. This transformation allows you to convert non-normal dependencies of variables to a normal form. The Box-Cox transform is based on the exponent, lambda ($\lambda$), which ranges from -5 to 5. All values of $\lambda$ are considered and the optimal value for your data is chosen; the "optimal value" is the one that leads to the best approximation of the normal distribution curve. The $y$ transform has the form:

$$y(\lambda) = \begin{cases} 
\frac{y^{\lambda-1}}{\lambda} & \text{if } \lambda \neq 0; \\
\log y & \text{if } \lambda = 0.
\end{cases}$$

(10)
This system of equations only works for positive data. However, Box and Cox did suggest the following formula, which can be used for negative y values:

\[
y(\lambda) = \begin{cases} 
\frac{y^{\lambda} - 1}{\lambda} & \text{if } \lambda \neq 0; \\
\log(y + \lambda_2) & \text{if } \lambda_1 = 0.
\end{cases}
\]

Figure 4. BOX-Coke transformation. Variable: electrical energy.

It is advisable to divide the solution of the regression analysis problem into several stages: preprocessing; choosing the type of regression equations; calculating the coefficients of the regression equation; checking the adequacy of the constructed function to the results of observations.

Tables 2 and 6 allow us to get acquainted with the results of analysis of variance of the regression equation. In the lines of the variance analysis table, the regression equations show sources of variation: Regress. - caused by regression, In the columns of the table: df - the number of degrees of freedom, Mean Squares - the average square, F - the value of the F-criterion, p-level - the probability of a null hypothesis for the F-criterion.

The F-criterion of the resulting regression equation is significant at the 5% level. The probability of the null hypothesis (p-level) is significantly less than 0.05, which indicates the overall significance of the regression equation.

4. Conclusions

Thus, as a result of the regression analysis, the following equation for the electrical energy consumption of the borehole pumping unit W is obtained as a function of the pump unit rotation speed \( \omega \), solution (liquid) density \( \rho \), pump unit performance \( Q \), well depth \( h \), and the level of filterability (taking into account hydrodynamic resistance) \( \Theta \):

\[
W = 16.30251 + 1.34\omega + 0.695\rho + 0.785Q + 0.88h + 0.6\Theta
\]

in other words, a model of electric energy consumption is determined, which is approximated by an equation with the original data set obtained during the experiment. Analysis of the model’s compliance with the initial data allows us to speak about a sufficient degree of accuracy of the results obtained. The
coefficients of the resulting linear regression equation are evaluated as significant. The investigated model is assessed as adequate.

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