Electromagnetic interaction vertex of $\Delta$ baryons in hard-wall AdS/QCD model

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Abstract

We consider an interaction of a spin-3/2 field with the electromagnetic field in the framework of the hard-wall model of AdS/QCD. We write Lagrangian for this interaction including all kinds interaction terms in the bulk of AdS$_5$ space and present the scattering matrix element in integrals over the fifth coordinate. Comparing the current matrix element obtained in the boundary of this space with the one known from field theory, we find the vertex function coefficients for the $\gamma^\ast \Delta \Delta$ interaction vertex. As an example, we apply the obtained coefficients to the computation of the charge form factor $G_{E0}$ for the $\Delta^+$ baryon and compare the result with the one obtained in the field theory.

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I. INTRODUCTION

The idea of the AdS/CFT (Anti de-Sitter/Conformal Field Theory) correspondence (or gravity-gauge duality) is successfully used for solving problems in different areas of theoretical physics. Originally formulated for a super Yang-Mills theory this duality connects fields in the bulk of the AdS space with current correlators on the boundary of this space which then are corresponded to the ones in the field theory in flat space-time with less dimension. In application to QCD the duality idea is valuable for solving low-energy problems, when usual perturbation theory does not work. Though QCD is not conformal theory, there are some QCD models based on this duality, such as holographic QCD and AdS/QCD models. In order to build a model of AdS/QCD one should break the conformal symmetry. This can be done either by imposing boundary condition on a solution to equation of motion over extra dimension or by warping the metric of the AdS space. In the first case, the boundary condition cuts a slice of the AdS space and gives mass spectrum, which then is corresponded to the mass spectrum of observed particles. Such a model is called a hard-wall model and is successfully applied to calculation of physical quantities of mesons and baryons. In the second case, which is called a soft-wall model, there are no sharp boundaries and propagation of a particle at long distances of extra dimension is suppressed by introduction of a dilaton field.

The hard-wall model was extended in by inclusion of spin-3/2 fields and the model was applied to calculations of the meson-baryon transition couplings and transition form factors for interactions of Δ baryons with nucleons, π and ρ mesons. Here we include interactions of spin-3/2 fields with a photon field into the hard-wall model and consider γ∗ΔΔ interaction vertex within this model. In Section II we briefly describe the hard-wall model with the spin-3/2 field introduced in . In Section III we present some relevant formulas of the isotopic structure for Δ baryons and expressions of multipole form factors of these particles. We construct in the bulk of the AdS5 space an interaction Lagrangian for the spin-3/2 field with a gauge field, which includes all possible interactions of the gauge field with the spin-3/2 field and produces on the boundary the matrix element of the electromagnetic current of the spin-3/2 field. This matrix element has the same tensorial structure as the one for the Δ-baryon’s electromagnetic current obtained in within the QCD framework. From the matching of tensorial structures in matrix element expressions obtained here and in
we obtain the vertex function coefficients in integrals over the additional dimension. These coefficients allow us to study electromagnetic form factors of $\Delta$ baryons in the framework of the hard-wall model that we perform in Section IV.

II. HARD-WALL MODEL FOR SPIN-3/2 FIELD

Let us express the main features of the hard-wall model of the AdS/QCD described in [11]. Gravity theory in this model is given by a 5-dimensional (5D) anti de-Sitter space (AdS$_5$) and with the metric chosen in Poincare coordinates as

$$ds^2 = \frac{1}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right).$$

(2.1)

Here, $z$ is the fifth coordinate and it extends from 0 to $\infty$, which are called, correspondingly, the ultraviolet (UV) and the infrared (IR) boundaries of the AdS space. $\eta_{\mu\nu}$ is the metric of 4D flat Minkowski space ($\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, $\mu = 0, 1, 2, 3$). In the hard-wall model $z$ is cut off at the bottom by $\varepsilon$ ($\varepsilon \rightarrow 0$) and at the top by $z_m = 1/\Lambda_{QCD}$, ($\varepsilon \leq z \leq z_m$), where $\Lambda_{QCD}$ corresponds to the confinement scale of QCD. The latter cutoff breaks conformal symmetry at a slice of the AdS$_5$ space. Hereafter, under the boundary we shall mean the cut off IR boundary.

In this background we introduce a gauge filed $V^M(x, z)$ and a spin-3/2 field $\Psi_A$ that interact with each other. The gauge field $V^a_M(x, z)$ in the bulk theory corresponds to the current of spin-3/2 field $J^a_\mu(x) = \overline{\psi}(x) \gamma_\mu T^a \psi(x)$ on the boundary. The flavor symmetry group for the model is $SU(2)_L \times SU(2)_R$ and the gauge symmetry group is $SU(2)$. The gauge field $V^M$ has left $V^M_L$ and right $V^M_R$ components, transforming under flavor symmetry as representations of $SU(2)_L$ and $SU(2)_R$ groups respectively.

Let us write the bulk-to-boundary propagator for the gauge field, which is called profile function and the solution to equation of motion for the Rarita-Schwinger field in the background (2.1).

A. Bulk-to-boundary propagator for gauge field

The action for the bulk gauge field is written in the following form [12]:

$$S = -\frac{1}{2g_5^2} \int d^5x \sqrt{G} Tr \left( \mathcal{F}_L^2 + \mathcal{F}_R^2 \right),$$

(2.2)
where $F_{MN} = \partial_M \mathcal{V}_N - \partial_N \mathcal{V}_M - i [\mathcal{V}_M, \mathcal{V}_N]$, $G$ is the determinant of the metric tensor and $g_5 = 2\pi$. In the axial-like gauge the fifth component of $\mathcal{V}_M$ is gauged away, $\mathcal{V}_z = 0$. In the momentum space, the $z$-dependence of $\mathcal{V}_\mu(x,z)$ is separated by the anzats:

$$
\tilde{\mathcal{V}}_\mu(q,z) = \mathcal{V}_\mu(q) \frac{V(q,z)}{V(q,z)}.
$$

(2.3)

The equation of motion over $z$ leads to the next equation for $V(q,z)$,

$$
z \partial_z \left( \frac{1}{z} \partial_z V(q,z) \right) + q^2 V(q,z) = 0
$$

(2.4)

and solution to this equation is expressed via Bessel functions $J_m$ and $Y_m$:

$$
V(q,z) = \frac{\pi}{2} qz \left[ \frac{Y_0(qz_m)}{J_0(qz_m)} J_1(qz) + Y_1(qz) \right].
$$

(2.5)

Remark, the bulk to boundary propagator obtained here is the propagator for the free gauge field. In the hard-wall model a gauge field interacts with the spin 3/2 field and this should lead to changes in the equation of motion of gauge field. Here we shall neglect this backreaction of spin 3/2 field and use this bulk to boundary propagator in our calculations.

B. Rarita-Schwinger field in the bulk

As is known from the AdS/CFT correspondence of spin-3/2 fields that in order to obtain one spin-3/2 field with left- and right-handed components in the boundary theory one needs to introduce two Rarita-Schwinger (R-S) fields $\Psi^M_1$ and $\Psi^M_2$ in the bulk theory. One of the bulk R-S fields gives left-handed component of the boundary field and the second one gives the right-handed component of this field. On the boundary extra components of bulk fields are eliminated by boundary conditions, that give a mass spectrum of excited states of this field. For a clearness let us present in brief some formulas from this AdS/CFT correspondence.

The action for the Rarita-Schwinger field $\Psi_A$ in AdS$_5$ is the extension of the 4D-action into 5D and is given as

$$
S = \int d^5x \sqrt{G} \left( i \bar{\Psi}_A \Gamma^{ABC} \sigma^B \Psi_C - m_1 \bar{\Psi}_A \Psi^A - m_2 \bar{\Psi}_A \Gamma^{AB} \Psi_B \right).
$$

(2.6)

Here, $\Gamma^{ABC}$ and $\Gamma^{AB}$ are antisymmetric products of $\Gamma$ matrices: $\Gamma^{ABC} = \frac{1}{3} \sum_{\text{perm}} (-1)^p \Gamma^A \Gamma^B \Gamma^C = \frac{1}{2} (\Gamma^B \Gamma^C \Gamma^A - \Gamma^A \Gamma^C \Gamma^B)$, $\Gamma^{AB} = \frac{1}{2} (\Gamma^A \Gamma^B - \Gamma^B \Gamma^A)$. The covariant
derivative $D_B$ is defined by the following shift:

$$D_B = \partial_B - \frac{i}{4} \omega_B^{MN} \Sigma_{MN} - i (\mathcal{V}_B) \mathcal{T}^a,$$

where $\Sigma_{MN} = \frac{1}{4} \Gamma_{MN}$. For the metric (2.1) the vielbein $e^A_M$ has been chosen as $e^A_M = \frac{1}{z} \eta^A_M$ and the spin connection $\omega_B^{MN}$ has following non-zero components $\omega_5^A = -\omega_5^A = \frac{1}{z} \delta^A_\mu$ ($\mu = 0, 1, 2, 3$). When transforming from the non-coordinate frame into the coordinate one (orthogonal frame) the vector-spinor $\Psi_A$ transforms via inverse vielbein $e^M_A = \frac{1}{z} \eta^M_A$ as $\Psi_A = e^M_A \Psi_M$.

The equation of motion obtained from the action (2.6) will give us 5D Rarita-Schwinger equations in the AdS$_5$ space (2.1):

$$i \Gamma^A (D_A \Psi_B - D_B \Psi_A) - m_- \Psi_B + \frac{m_+}{3} \Gamma^B \Gamma^A \Psi_A = 0,$$

where $m_\pm = m_1 \pm m_2$ are masses of spinor harmonics or the R-S field on $S^5$ of AdS$_5 \times S^5$ [14]. For $\Gamma$ matrices it is convenient to use chirality basis [11],

$$\Gamma^5 = -i \gamma^5 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \quad \Gamma^0 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \Gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad (i = 1, 2, 3).$$

(2.8)

In the four-dimensional theory, the R-S field $\psi_\mu$ is a 4D vector-spinor and contains extra components, which correspond to the states of spin 1/2 particle. These extra components are projected out by application of the additional Lorentz-covariant constraint

$$\gamma^\mu \psi_\mu = 0.$$

In a five-dimensional theory, it is necessary to impose similar condition in order to project out extra components of the 5D R-S filed $\Psi_M$ corresponding to the states of the spin-1/2 field

$$e^M_A \Gamma^A \Psi_M = 0.$$  

(2.9)

This constraint, together with the free equation of motion, gives another constraint $\partial^M \Psi_M = 0$. The 5D Rarita-Schwinger filed $\Psi_M$ has one more extra spin-1/2 component $\Psi_z$. It can be gauged away in the unitary gauge or it can be mapped to the longitudinal component of the massive spin-1 vector mesons in the boundary theory. In this model, there is no extra spinor states in the boundary theory into which the $\Psi_z$ could be mapped. In ref. [11], the condition $\Psi_z = 0$ was chosen in order to eliminate this extra spin-1/2 degree of freedom and
we shall follow it. Taking into account these conditions in Rarita-Schwinger equation (2.7) gives the next set of Dirac equations for the remaining components

\[(iz\Gamma^A \partial_A + 2i\Gamma^5 - m_-) \Psi_\mu = 0. \tag{2.10}\]

It is useful to make further calculations in left and right components of vector-spinor which have properties \(\gamma^5 \Psi_\mu^L = \Psi_\mu^L\) and \(\gamma^5 \Psi_\mu^R = -\Psi_\mu^R\). Fourier transformation for these bulk vector-spinors is written as following

\[\Psi_{L,R}^\mu (x, z) = \int d^4p \ e^{-ip \cdot x} F_{L,R}(p, z) \psi_{L,R}^\mu (p) \tag{2.11}\]

and the 4D vector-spinor \(\psi^\mu (p)\) obeys 4D Dirac equation

\[p \psi^\mu (p) = |p| \psi^\mu (p). \tag{2.12}\]

Here, \(|p| = \sqrt{p^2}\) for a time-like four-momentum \(p\). Then the 5D Dirac equation (2.11) will lead to equations for \(F_{L,R}\) over the fifth coordinate \(z\):

\[\left(\partial_z^2 - \frac{4}{z} \partial_z + \frac{6 \pm m_- - m_+^2}{z^2}\right) F_{L,R} = -p^2 F_{L,R}. \tag{2.13}\]

The normalizable solution to this equation for non-zero modes (\(|p| \neq 0\)) is expressed in terms of Bessel functions of first kind

\[F_{L,R} = C_{L,R} z^{5/2} J_{m_- + \frac{1}{2}} (|p| z), \tag{2.14}\]

where \(C_{L,R}\) are normalization constants. Value of \(m_-\) can be found from the relation \(|m_-| = \Delta_{3/2} - 2\), which is given by the AdS/CFT correspondence of the R-S field. Scaling dimension \(\Delta_{3/2}\) for the spin-3/2 composite operator is \(\Delta_{3/2} = 9/2\) and \(|m_-| = 5/2\). For the left and the right R-S fields the mass \(|m_-|\) has values \(m_- = \pm 5/2\), correspondingly.

In order to get only left-handed component of the R-S field on the boundary of the AdS space, we should impose a boundary condition on \(\Psi^M\) at \(z = z_m\), which eliminates the right-handed component of this vector-spinor on that boundary:

\[\Psi^M_R (x, z_m) = 0. \tag{2.15}\]

This condition gives a mass spectrum \(m_n\) for the boundary spin-3/2 field, which is expressed in terms of zeros of the Bessel function \(J_{m_- + \frac{1}{2}} (|p| z)\). Then boundary condition (2.15) will lead to

\[J_3 (M_n z_m) = 0\]
and the spectrum of excited states will be

\[ M_n = \frac{\alpha_n^{(3)}}{z_m}. \]  

(2.16)

Here \( \alpha_n^{(3)} \) are zeros of the Bessel function \( J_3(x) \).

In order to get the right-handed component on the boundary we introduce another spin-3/2 field in the bulk of the AdS space-time. Obviously, all formulas for the left-handed component are applied to the second R-S field. This time the left-handed component of the bulk field \( \Psi^M \) is eliminated by the boundary condition:

\[ \Psi^M_L(x, z_m) = 0. \]  

(2.17)

Remark, for this component it should be made replacement \( m_- \rightarrow -m_- \) in the formulas above. The condition (2.17) leads to

\[ J_{-3}(M_n z_m) = 0 \]

and this gives the same spectrum of excited states for the left-handed components of the field \( \Psi^M \) as for the right-handed components. The normalization constants \( C_{L,R} \) were found in [8] and are equal to

\[ |C_{L,R}| = \frac{\sqrt{2}}{z_m J_2(M_n z_m)}. \]

III. ELECTROMAGNETIC CURRENT MATRIX ELEMENT FOR \( \Delta \) BARYONS

A. \( \Delta \) baryons and electromagnetic form factors

The model which was described above is applicable to any spin-3/2 fields that differ from one another by some other quantum number. The simplest known realistic model for spin-3/2 particles is the model based on the gauge group \( SU(2) \) and the flavor group \( SU(2)_L \times SU(2)_R \). These are states of the spin-3/2 field with isospin-3/2, i.e., the multiplet of \( \Delta \) baryons. There are four kinds of \( \Delta \) baryons (\( \Delta^{++}, \Delta^+, \Delta^0, \Delta^- \)) which differ from each other by the isospin projection. In order to get a difference in the current matrix element for different kinds of those baryons, we should take into account the known isotopic structure of the electromagnetic current of \( \Delta \) baryons. To this end we can utilize formulas for the isospin operator and wave functions for \( \Delta \) baryons used in [16, 17]. The electromagnetic current of \( \Delta \) baryons can be divided into isoscalar and isovector parts
\[ J_\mu = J_\mu^{(s)} \frac{I}{2} + J_\mu^{(v)} \frac{T^{(3)}}{2} \]  

(3.1)

with

\[ T^{(3)} = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix} \]  

(3.2)

Here, \( I \) is the four dimensional unit matrix and \( \frac{1}{2} T^{(3)} \) is the isospin operator. The \( T^{(3)} \) and other basic matrices \( T^{(1,2)} \) can transit to Pauli matrices by means of spin-3/2 to 1/2 transition matrices, explicit form of which can be found in [17]. In this representation, which is called isoquartet representation, the isotopic part \( \zeta^{(i)} \) of the wave function of \( \Delta \) baryons are eigenvectors of \( T^{(3)} \) operator:

\[ \zeta^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \zeta^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \zeta^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \zeta^{(4)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \]  

(3.3)

Eigenvalues of corresponding eigenstates are \( T_i^{(3)} = 3, 1, -1, -3 \). Electric charges \( Q_i \) of these baryons are defined by eigenvalues of the charge operator

\[ Q = \frac{I}{2} + \frac{1}{2} T^{(3)}. \]  

(3.4)

\( \Delta^{++}, \Delta^+, \Delta^0 \) and \( \Delta^- \) baryons have a charge \( Q_1 = 2, Q_2 = 1, Q_3 = 0 \), and \( Q_4 = -1 \) respectively. Magnetic moments of these baryons also have isoscalar and isovector parts:

\[ \mu^{(i)} = \mu_s \frac{1}{2} + \mu_v \frac{T_i^{(3)}}{2} \]

There are two following relations between \( \mu^{(i)} \):

\[ \mu^{(1)} + \mu^{(3)} = 2\mu^{(2)} \]  

(3.5)

\[ \mu^{(4)} + \mu^{(2)} = 2\mu^{(3)} \]

In the field theory framework the electromagnetic form factors of spin-3/2 particles had been studied in [15]. The general structure of the matrix element of the electromagnetic current of these particles was found to be of the following form:

\[ \langle pt, st | j^\mu (0) | p, s \rangle = \overline{u}_a (pt, st) O^{\alpha \beta \gamma} u_\beta (p, s), \]  

(3.6)
where \( p', p, s', s \) are momenta and spin states of a final and an initial baryon, \( u_\alpha \) is a vector-spinor describing it. The Lorentz-covariant tensor \( O^{\alpha \mu \beta} \) has explicit form:

\[
O^{\alpha \mu \beta} = g^{\alpha \beta} \left[ a_1 \gamma^\mu + \frac{a_2}{2m} P^\mu \right] + \frac{q^\alpha q^\beta}{(2m)^2} \left[ c_1 \gamma^\mu + \frac{c_2}{2m} P^\mu \right],
\]

where \( P = p' + p \) and \( q = p' - p \). Coefficients \( a_i \) and \( c_i \) are independent covariant vertex function coefficients. Multipole form-factors of the \( \Delta \) baryons were expressed by means of these coefficients in the following way:

\[
G_{E0} (q^2) = \left( 1 + \frac{2}{3} \tau \right) [a_1 + (1 + \tau) a_2] - \frac{1}{3} \tau (1 + \tau) [c_1 + (1 + \tau) c_2],
\]

\[
G_{E2} (q^2) = [a_1 + (1 + \tau) a_2] - \frac{1}{2} \tau (1 + \tau) [c_1 + (1 + \tau) c_2],
\]

\[
G_{M1} (q^2) = \left( 1 + \frac{4}{5} \tau \right) a_1 - \frac{2}{5} \tau (1 + \tau) c_1,
\]

\[
G_{M3} (q^2) = a_1 - \frac{1}{2} (1 + \tau) c_1,
\]

where \( \tau = -q^2 / (2m)^2 \). \( G_{E0}, G_{E2}, G_{M1}, \) and \( G_{M3} \) are called charge \((E0)\), electroquadrupole \((E2)\), magnetic-dipole \((M1)\) and magnetic-octupole \((M3)\) form factors respectively. Thus, it is enough to find \( a_i \) and \( c_i \) coefficients in order to study form factors \((3.8)-(3.11)\) and related physical quantities. Our aim is to find within hard-wall model of AdS/QCD the expressions corresponding to these coefficients.

**B. Vertex function coefficients in hard-wall model**

In order to obtain an expression for the electromagnetic interaction vertex of the \( \Delta \) baryons within AdS/QCD model, we should start from the interaction Lagrangian of the Rarita-Schwinger field with the gauge field in the bulk theory. Note that this Lagrangian besides simple interaction term will contain different interaction terms which should lead to contributions of interactions with different multipole moments in the boundary theory. So, it is awaited that such Lagrangian will be significantly complicated. Let us determine the interaction Lagrangian in the AdS\(_5\) space-time with metric \((2.1)\). First, it should match the gauge symmetry group in the bulk theory with the flavor symmetry group in the boundary theory. According to the AdS/CFT correspondence principle, the gauge symmetry of the vector field in the bulk theory is the same as the flavor symmetry of the corresponding
current in the boundary theory. So, the isotopic structure of the vector field $\mathcal{V}_N$ in the bulk of the AdS$_5$ should be same as the isotopic structure of the barionic current on the boundary of the AdS$_5$, i.e., be as following

$$\mathcal{V}_N = V_N^{(s)} \frac{I}{2} + V_N^{(v)} \frac{\mathcal{T}^{(3)}}{2}. \quad (3.12)$$

Here $V_N^{(s)}$ and $V_N^{(v)}$ are isoscalar and isovector parts of the vector field. In the simplest case (for the photon) these parts are equal $V_N^{(s)} = V_N^{(v)} = V_N$ and the isotopic part of $\mathcal{V}_N$ is factored out as the charge operator $Q$:

$$\mathcal{V}_N = V_N Q.$$ 

In this representation the field strength tensor $\mathcal{F}_{MN}$ becomes the one for abelian field $\mathcal{F}_{MN} = \partial_M \mathcal{V}_N - \partial_N \mathcal{V}_M$ and has the same isotopic structure as $\mathcal{V}_N$:

$$\mathcal{F}_{MN} = F_{MN} \frac{I}{2} + F_{MN} \frac{\mathcal{T}^{(3)}}{2} = F_{MN} Q \quad (3.13)$$

Note that the isotopic part of the bulk Rarita-Schwinger field is defined in the same representation as in the wave function of $\Delta$ baryons, i.e., we define this part in the isoquartet representation:

$$\Psi^M = \psi^M \zeta^{(i)}. \quad (3.14)$$

It is obvious that in any terms of interaction including these fields, like $\mathcal{V}_N \Psi^M$, $\mathcal{F}_{MN} \Psi^M$ and so on, the value of charge $Q_i$ will be factored out:

$$\mathcal{V}_N \Psi^M = Q_i V_N \psi^M \zeta^{(i)}.$$

Interaction Lagrangian in the bulk is constructed by composing hermitian Lorentz scalars using the 5D R-S fields $\Psi_{1,2}^M$, $\overline{\Psi}_{1,2}^M$, the gauge field $\mathcal{V}_N$, the field stress tensor $\mathcal{F}_{MN}$ and its dual tensor $\widetilde{\mathcal{F}}_{MBN}$, their derivatives $\partial_M \Psi^A_{1,2}$, $\partial_M \overline{\Psi}^A_{1,2}$, $\partial^A \mathcal{F}^{MB}$, $\partial^A \widetilde{\mathcal{F}}_{MBN}$ and matrices $\Gamma^A$, $\Gamma^{AB}$ and $\Gamma^{ABC}$. Main requirement for constructing a Lagrangian term is to obtain from it on the boundary a matrix element, which has tensorial structure coinciding with one of structures in (3.7). Let us collect all possible Lorentz scalars, which obey this requirement. It will consist of the following terms of interaction of the vector-spinor with the gauge field:

a) terms describing simple interaction vertices

$$\mathcal{L}_0 = \overline{\Psi}_1^M \Gamma^N \mathcal{V}_N \Psi_{1M} + \overline{\Psi}_2^M \Gamma^N \mathcal{V}_N \Psi_{2M}, \quad (3.15)$$
b) terms describing an interaction with magnetic dipole moment

\[ \mathcal{L}_1 = \left\{ \eta^{(s)} \left[ \overline{\Psi}_1^M \Gamma^{NP} F_{NP} \frac{I}{2} \Psi_{1M} - \overline{\Psi}_2^M \Gamma^{NP} F_{NP} \frac{I}{2} \Psi_{2M} \right] + \eta^{(v)} \left[ \overline{\Psi}_1^M \Gamma^{NP} F_{NP} \frac{T^{(3)}}{2} \Psi_{1M} - \overline{\Psi}_2^M \Gamma^{NP} F_{NP} \frac{T^{(3)}}{2} \Psi_{2M} \right] \right\}, \tag{3.16} \]

Coefficients \( \eta^{(s)} \) and \( \eta^{(v)} \) introduced here take into account difference in contributions of isoscalar and isovector parts of the magnetic moment of the \( \Delta \) baryon \[ [9]. \]

Other Lorentz invariants are constructed by means of fields and derivatives, which produce one (or more) of required tensorial structures on the boundary. We classify them as following:

c) terms constructed by \( \mathcal{V}^M \)

\[ \mathcal{L}_2 = i k_2 \left[ (\partial_M \overline{\Psi}_{1A}) \mathcal{V}^M \Psi_1^A - \overline{\Psi}_{1A} \mathcal{V}^M (\partial_M \Psi_1^A) - (1 \rightarrow 2) \right], \tag{3.17} \]

\[ \mathcal{L}_3 = i k_3 \left[ (\partial_M \overline{\Psi}_{1A}) \Gamma^{AB} \mathcal{V}^M \Psi_1^B - \overline{\Psi}_{1A} \Gamma^{AB} \mathcal{V}^M (\partial_M \Psi_1^B) - (1 \rightarrow 2) \right], \tag{3.18} \]

\[ \mathcal{L}_4 = k_4 \left[ \overline{\Psi}_{1A} \Gamma^{AMN} \mathcal{V}_M \Psi_{1N} - (1 \rightarrow 2) \right], \tag{3.19} \]

d) terms constructed by \( \mathcal{F}^{MN} \):

\[ \mathcal{L}_5 = \frac{i k_5}{2} \left[ \overline{\Psi}_{1M} \mathcal{F}^{MN} \Psi_{1N} - \overline{\Psi}_{2M} \mathcal{F}^{MN} \Psi_{2N} \right] + h.c., \tag{3.20} \]

\[ \mathcal{L}_6 = \frac{i k_6}{2} \left[ (\partial_M \overline{\Psi}_{1A}) \mathcal{F}^{MN} (\partial_N \Psi_1^A) - (\partial_N \overline{\Psi}_{1A}) \mathcal{F}^{MN} (\partial_M \Psi_1^A) - (1 \rightarrow 2) \right] + h.c., \tag{3.21} \]

\[ \mathcal{L}_7 = \frac{i k_7}{2} \left[ (\partial_M \overline{\Psi}_{1A}) \mathcal{F}^{MN} (\partial^A \Psi_{1N}) - (\partial^A \overline{\Psi}_{1A}) \mathcal{F}^{MN} (\partial_M \Psi_{1N}) - (1 \rightarrow 2) \right] + h.c., \tag{3.22} \]

\[ \mathcal{L}_8 = \frac{i k_8}{2} \left[ (\partial_A \overline{\Psi}_{1M}) \Gamma^A \mathcal{F}^{MN} \Psi_{1N} - \overline{\Psi}_{1M} \Gamma^A \mathcal{F}^{MN} (\partial_A \Psi_{1N}) - (1 \rightarrow 2) \right] + h.c., \tag{3.23} \]

\[ \mathcal{L}_9 = \frac{k_9}{2} \left[ (\partial_A \overline{\Psi}_{1B}) \Gamma^{AMN} \mathcal{F}_{MN} \Psi_1^B - \overline{\Psi}_{1B} \Gamma^{AMN} \mathcal{F}_{MN} (\partial_A \Psi_1^B) + (1 \rightarrow 2) \right] + h.c., \tag{3.24} \]

e) terms constructed by \( \tilde{\mathcal{F}}^{AMN} \)

\[ \mathcal{L}_{10} = \frac{i k_{10}}{2} \left[ \overline{\Psi}_{1A} \Gamma_M \tilde{\mathcal{F}}^{AMN} \Psi_{1N} - (1 \rightarrow 2) \right] + h.c., \tag{3.25} \]

\[ \mathcal{L}_{11} = \frac{k_{11}}{2} \left[ (\partial^C \overline{\Psi}_{1A}) \Gamma_{CM} \tilde{\mathcal{F}}^{AMN} \Psi_{1N} - \overline{\Psi}_{1A} \Gamma_{CM} \tilde{\mathcal{F}}^{AMN} (\partial^C \Psi_{1N}) - (1 \rightarrow 2) \right] + h.c., \tag{3.26} \]

\[ \mathcal{L}_{12} = \frac{i k_{12}}{2} \left[ \overline{\Psi}_{1A} \Gamma_{BAM} \tilde{\mathcal{F}}^{AMN} \Psi_{1N} - (1 \rightarrow 2) \right] + h.c., \tag{3.27} \]

\[ \mathcal{L}_{13} = \frac{i k_{13}}{2} \left[ \overline{\Psi}_{1B} \Gamma_{AMN} \tilde{\mathcal{F}}^{AMN} \Psi_{1N} - (1 \rightarrow 2) \right] + h.c., \tag{3.28} \]
f) terms constructed by $\partial^A \mathcal{F}^{MB}$

$$L_{14} = \frac{ik_{14}}{2} \left[ (\partial_M \Psi_{1A}) (\partial^A \mathcal{F}^{MB}) \Psi_{1B} - \Psi_{1A} (\partial^A \mathcal{F}^{MB}) (\partial_M \Psi_{1B}) - (1 \rightarrow 2) \right] + h.c.,$$  

$$L_{15} = \frac{k_{15}}{2} \left[ \bar{\Psi}_{1A} \Gamma_M (\partial^A \mathcal{F}^{MB}) \Psi_{1B} + (1 \rightarrow 2) \right] + h.c.,$$

g) terms constructed by $\partial^A \tilde{\mathcal{F}}^{MBN}$

$$L_{16} = \frac{ik_{16}}{2} \left[ (\partial_B \bar{\Psi}_{1A}) \Gamma_M (\partial^A \tilde{\mathcal{F}}^{MBN}) \Psi_{1N} + \bar{\Psi}_{1A} \Gamma_M (\partial^A \tilde{\mathcal{F}}^{MBN}) (\partial_B \Psi_{1N}) - (1 \rightarrow 2) \right] + h.c.,$$

$$L_{17} = \frac{k_{17}}{2} \left[ \bar{\Psi}_{1A} \Gamma_{MB} (\partial^A \tilde{\mathcal{F}}^{MBN}) \Psi_{1N} + (1 \rightarrow 2) \right] + h.c.,$$

$$L_{18} = \frac{ik_{18}}{2} \left[ (\partial_M \bar{\Psi}_1^C) \Gamma_{CAB} (\partial^A \tilde{\mathcal{F}}^{MBN}) \Psi_{1N} + \bar{\Psi}_1^C \Gamma_{CAB} (\partial^A \tilde{\mathcal{F}}^{MBN}) (\partial_M \Psi_{1N}) - (1 \rightarrow 2) \right] + h.c.$$  

Here, $k_s$ are arbitrary real coefficients. Let us remark that because of isotopic symmetry each Lagrangian term consist of four terms corresponding to different values of $T^{(3)}_i$ ($L_s = \sum_{i=1}^{4} L^{(i)}_s$). The interaction vertex of another multiplet of spin-3/2 fields with the photon also can be described by these Lagrangian terms after changes of the isotopic symmetry group. Some terms of chiral symmetry breaking, like following one (see [11])

$$L_{\chi_{SB}} = \frac{i}{2} k_1 \left[ \bar{\Psi}_1^M X^3 \Gamma^{NP} \mathcal{F}_{NP} \Psi_{2M} + \bar{\Psi}_2^M (X^4)^3 \Gamma^{NP} \mathcal{F}_{NP} \Psi_{1M} \right],$$

are possible as well. We did not include here those terms into Lagrangian, since their contributions to the current matrix element are too small in comparison with other terms due to the $X^3$ factor. Thus, the total interaction Lagrangian of the R-S field with the bulk gauge field will consist of the sum of all Lagrangian terms $L_0 - L_{18}$:

$$L_{int} = \sum_{s=0}^{18} L_s$$  

$S$-matrix element $S_{fi}$ of this interaction, in the first approximation, is obtained from this Lagrangian by performing the 5D integration:

$$S_{fi} = \int_{\varepsilon}^{z_m} dz \sqrt{G} \int d^4 x L_{int}.$$  

Integration of each Lagrangian term $L_s$ over $x$ and using the equation of motion (2.12) and the constraint (2.9) leads to momentum integrals of the product of vector-spinors $\bar{\Psi}_a(p')$,  

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$u_\beta (p)$ of final and initial states with a tensor $\tilde{O}^{\alpha \mu \beta}$.

So, $S_{fi}$ in momentum space is reduced to the following form:

$$S_{fi} = \int d^4 p' d^4 p \overline{\psi}_\alpha (p') \varepsilon_\mu \tilde{O}^{\alpha \mu \beta} u_\beta (p), \quad (3.36)$$

where $\tilde{O}^{\alpha \mu \beta}$ is the sum of all contributions $\tilde{O}^{\alpha \mu \beta}$:

$$\tilde{O}^{\alpha \mu \beta} = \sum_{s=0}^{18} \tilde{O}^{\alpha \mu \beta}_s. \quad (3.37)$$

Here $p'$ and $p$ are momenta of final and initial states of R-S filed, which are states after and before interaction with the gauge field.

For a transparency it is useful to present $\tilde{O}^{\alpha \mu \beta}_s$ separately. The obtained expressions of $\tilde{O}^{\alpha \mu \beta}_s$ are the following ones:

$$\tilde{O}^{\alpha \mu \beta}_0 = g^{\alpha \beta} \gamma_\mu \frac{1}{\sqrt{2}} Q_i \int_0^{z_m} d \frac{V(qz)}{z^2} \left[ F^{*}_{1L} (p'tz) F_{1L} (pz) + F^{*}_{1R} (p'tz) F_{1R} (pz) 
+ F^{*}_{2L} (p'tz) F_{2L} (pz) + F^{*}_{2R} (p'tz) F_{2R} (pz) \right]$$

$$\tilde{O}^{\alpha \mu \beta}_1 = g^{\alpha \beta} \left[ m \gamma_\mu - \frac{1}{2} P^\mu \right] \frac{1}{\sqrt{2}} \left( \eta^{(s)} + \eta^{(v)} ) T_i^{(3)} \right) \int_0^{z_m} d \frac{V(qz)}{z} \times [ F^{*}_{1L} (p'tz) F_{1R} (pz) + F^{*}_{1R} (p'tz) F_{1L} (pz) - F^{*}_{2L} (p'tz) F_{2R} (pz) - F^{*}_{2R} (p'tz) F_{2L} (pz) ]$$

$$\tilde{O}^{\alpha \mu \beta}_2 = - g^{\alpha \beta} P^\mu \frac{1}{2 \sqrt{2}} k_2 Q_i \int_0^{z_m} d \frac{V(qz)}{z^3} \times [ F^{*}_{1L} (p'tz) F_{1R} (pz) + F^{*}_{1R} (p'tz) F_{1L} (pz) - F^{*}_{2L} (p'tz) F_{2R} (pz) - F^{*}_{2R} (p'tz) F_{2L} (pz) ]$$

$$\tilde{O}^{\alpha \mu \beta}_3 = g^{\alpha \beta} P^\mu \frac{1}{2 \sqrt{2}} Q_i k_3 \int_0^{z_m} d \frac{V(qz)}{z} \times [ F^{*}_{1L} (p'tz) F_{1R} (pz) + F^{*}_{1R} (p'tz) F_{1L} (pz) - F^{*}_{2L} (p'tz) F_{2R} (pz) - F^{*}_{2R} (p'tz) F_{2L} (pz) ]$$

$$\tilde{O}^{\alpha \mu \beta}_4 = g^{\alpha \beta} \gamma_\mu \frac{1}{\sqrt{2}} Q_i k_4 \int_0^{z_m} d \frac{V(qz)}{z} \times [ F^{*}_{1L} (p'tz) F_{1R} (pz) + F^{*}_{1R} (p'tz) F_{1L} (pz) - F^{*}_{2L} (p'tz) F_{2R} (pz) - F^{*}_{2R} (p'tz) F_{2L} (pz) ]$$
\[\tilde{O}_5^{\alpha\beta} = \left\{ g^{\beta\gamma} \left[ m \left( 1 - \frac{q^2}{4m^2} \right) \gamma^\mu - \frac{1}{2} P^\mu \right] + \frac{1}{2m} q^\alpha q^\beta \gamma^\mu \right\} \frac{1}{\sqrt{2}} k_5 q_i \int_0^{z_m} dz \frac{V(q_z)}{z^3} \times \]

\[\left[ F_{1L}^* (ptz) F_{1R} (pz) + F_{2R}^* (ptz) F_{2L} (pz) - F_{2L}^* (ptz) F_{2R} (pz) - F_{2R}^* (ptz) F_{2L} (pz) \right] \]

\[\tilde{O}_6^{\alpha\beta} = g^{\alpha\beta} P^\mu \frac{1}{2\sqrt{2}} k_6 q_i \int_0^{z_m} dz \frac{1}{z^3} \left\{ q^2 V(q_z) \right\} \left[ F_{1L}^* (ptz) F_{1R} (pz) + F_{2L}^* (ptz) F_{2L} (pz) - F_{2R}^* (ptz) F_{2L} (pz) \right] \]

\[-F_{2L}^* (ptz) F_{2R} (pz) - F_{2R}^* (ptz) F_{2L} (pz) - \partial_z V(q_z) \left[ \partial_z F_{1L}^* (ptz) F_{1R} (pz) + \right. \]

\[+ \partial_z F_{1R}^* (ptz) F_{1L} (pz) + F_{2L}^* (ptz) \partial_z F_{1R} (pz) + F_{1R}^* (ptz) \partial_z F_{1L} (pz) - \]

\[- \partial_z F_{2L}^* (ptz) F_{2R} (pz) - \partial_z F_{2R}^* (ptz) F_{2L} (pz) - F_{2L}^* (ptz) \partial_z F_{2R} (pz) - F_{2R}^* (ptz) \partial_z F_{2L} (pz) \} \}

\[\tilde{O}_7^{\alpha\beta} = - \left\{ 2g^{\alpha\beta} \left[ m \left( 1 - \frac{q^2}{4m^2} \right) \gamma^\mu - \frac{1}{2} P^\mu \right] + \frac{1}{m} q^\alpha q^\beta \gamma^\mu \right\} \frac{1}{2\sqrt{2}} k_7 q_i \int_0^{z_m} dz \frac{V(q_z)}{z^2} \times \]

\[\left[ F_{1L}^* (ptz) F_{1R} (pz) + F_{1R}^* (ptz) F_{1L} (pz) - F_{2L}^* (ptz) F_{2R} (pz) - F_{2R}^* (ptz) F_{2L} (pz) \right] \]

\[\tilde{O}_8^{\alpha\beta} = 2mg^{\alpha\beta} \left[ m \left( 1 - \frac{q^2}{4m^2} \right) \gamma^\mu - \frac{1}{2} P^\mu \right] + q^\alpha q^\beta \gamma^\mu \frac{1}{\sqrt{2}} k_8 q_i \int_0^{z_m} dz V(q_z) \times \]

\[\left[ F_{1L}^* (ptz) F_{1L} (pz) + F_{1R}^* (ptz) F_{1R} (pz) + F_{2L}^* (ptz) F_{2L} (pz) + F_{2R}^* (ptz) F_{2R} (pz) \right] \]

\[\tilde{O}_{10}^{\alpha\beta} = - g^{\alpha\beta} \gamma^\mu \frac{1}{2\sqrt{2}} k_{10} q_i \int_0^{z_m} dz \frac{\partial_z V(q_z)}{z^2} \times \]

\[\left[ F_{1L}^* (ptz) F_{1L} (pz) - F_{1R}^* (ptz) F_{1R} (pz) - F_{2L}^* (ptz) F_{2L} (pz) + F_{2R}^* (ptz) F_{2R} (pz) \right] \]

\[\tilde{O}_{11}^{\alpha\beta} = - \left\{ g^{\beta\gamma} \frac{q^2}{2} \gamma^\mu - q^{\alpha\gamma} q^\gamma \right\} \sqrt{2} k_{11} q_i \int_0^{z_m} dz \frac{V(q_z)}{z} \times \]

\[\left[ F_{1L}^* (ptz) F_{1R} (pz) + F_{1R}^* (ptz) F_{1L} (pz) - F_{2L}^* (ptz) F_{2R} (pz) - F_{2R}^* (ptz) F_{2L} (pz) \right] \]

\[\tilde{O}_{12}^{\alpha\beta} = \left\{ g^{\alpha\beta} \left[ \frac{q^2}{2m} \gamma^\mu - P^\mu \right] + \frac{1}{m} q^\alpha q^\beta \gamma^\mu \right\} 2\sqrt{2} k_{12} q_i \int_0^{z_m} dz \{ V(q_z) \} \times \]

\[\left[ F_{1L}^* (ptz) F_{1R} (pz) + F_{1R}^* (ptz) F_{1L} (pz) - F_{2L}^* (ptz) F_{2R} (pz) - F_{2R}^* (ptz) F_{2L} (pz) \right] \]

\[- \partial_z V(q_z) \left[ F_{1L}^* (ptz) F_{1L} (pz) - F_{1R}^* (ptz) F_{1R} (pz) - F_{2L}^* (ptz) F_{2L} (pz) + F_{2R}^* (ptz) F_{2R} (pz) \right] \}

\[\tilde{O}_{13}^{\alpha\beta} = g^{\alpha\beta} \gamma^\mu 3\sqrt{2} k_{13} q_i \int_0^{z_m} dz V(q_z) \times \]
Here we have considered initial and final R-S fields on mass shell (baryons with which we want to match R-S fields are on the mass shell as well. In calculations
\[
\tilde{O}_{14}^{\alpha \beta} = -q^\alpha q^\beta P^\mu \frac{1}{2\sqrt{2}} Q_i k_{14} \int_0^{z_m} dz V(qz) z^{-3} \left[ F_{1L}^* (ptz) F_{1R}^* (pz) - F_{2L}^* (ptz) F_{2R}^* (pz) \right]
\]
\[
- F_{2L}^* (ptz) F_{2R}^* (pz) - F_{2R}^* (ptz) F_{2L}^* (pz)
\]
\[
\tilde{O}_{15}^{\alpha \beta} = q^\alpha q^\beta \gamma^\mu \frac{1}{2\sqrt{2}} Q_i k_{15} \int_0^{z_m} dz V(qz) z^{-2} \left[ F_{1L}^* (ptz) F_{1L}^* (pz) + F_{1R}^* (ptz) F_{1R} (pz) \right]
\]
\[
+ F_{2L}^* (ptz) F_{2L}^* (pz) + F_{2R}^* (ptz) F_{2R}^* (pz)
\]
\[
\tilde{O}_{16}^{\alpha \beta} = \frac{1}{2} Q_i k_{16} \int_0^{z_m} dz V(qz) z^{-2} \left[ 2V(qz) \left\{ \partial_z F_{1L}^* (ptz) F_{1L}^* (pz) - \partial_z F_{1R}^* (ptz) F_{1R}^* (pz) \right\}
\]
\[
+ \partial_z F_{2L}^* (ptz) F_{2L}^* (pz) - \partial_z F_{2R}^* (ptz) F_{2R}^* (pz) \right\} -
\]
\[
- \partial_z V(qz) \left[ F_{1L}^* (ptz) F_{1R}^* (pz) - F_{1R}^* (ptz) F_{1R}^* (pz) - F_{2L}^* (ptz) F_{2L}^* (pz) + F_{2R}^* (ptz) F_{2R}^* (pz) \right]
\]
\[
\tilde{O}_{17}^{\alpha \beta} = -q^\alpha q^\beta \gamma^\mu \frac{5}{2\sqrt{2}} Q_i k_{17} \int_0^{z_m} dz V(qz) z^{-1} \times
\]
\[
\left[ F_{1L}^* (ptz) F_{1L}^* (pz) + F_{1R}^* (ptz) F_{1R}^* (pz) + F_{2L}^* (ptz) F_{2L}^* (pz) + F_{2R}^* (ptz) F_{2R}^* (pz) \right]
\]
\[
- \left[ F_{1L}^* (ptz) F_{1L}^* (pz) - F_{1R}^* (ptz) F_{1R} (pz) - F_{2L}^* (ptz) F_{2L} (pz) + F_{2R}^* (ptz) F_{2R} (pz) \right]
\]
\[
+ 2m \left[ F_{1L}^* (ptz) F_{1L}^* (pz) - F_{1R}^* (ptz) F_{1R} (pz) - F_{2L}^* (ptz) F_{2L} (pz) + F_{2R}^* (ptz) F_{2R} (pz) \right]
\]
\[
- \left[ \partial_z F_{1L}^* (ptz) F_{1R}^* (pz) + \partial_z F_{1R}^* (ptz) F_{1L}^* (pz) + F_{1L}^* (ptz) \partial_z F_{1R} (pz) + F_{1R}^* (ptz) \partial_z F_{1L} (pz) \right]
\]
\[
- \partial_z F_{2L}^* (ptz) F_{2R}^* (pz) - \partial_z F_{2R}^* (ptz) F_{2L}^* (pz) - F_{2L}^* (ptz) \partial_z F_{2R} (pz) - F_{2R}^* (ptz) \partial_z F_{2L} (pz) \right]
\]
\[
+ \frac{1}{2} q^\alpha q^\beta \gamma^\mu 2\sqrt{2} k_{18} Q_i \int_0^{z_m} dz \partial_z V(qz) [F_{1L}^* (ptz) F_{1L}^* (pz) - F_{1R}^* (ptz) F_{1R} (pz) -
\]
\[
- F_{2L}^* (ptz) F_{2L}^* (pz) + F_{2R}^* (ptz) F_{2R} (pz)] - q^\alpha q^\beta \gamma^\mu 2\sqrt{2} k_{18} Q_i \int_0^{z_m} dz V(qz) \times
\]
\[
\times \left[ \partial_z F_{1L}^* (ptz) F_{1L}^* (pz) - \partial_z F_{1R}^* (ptz) F_{1R}^* (pz) + F_{1L}^* (ptz) \partial_z F_{1L} (pz) - F_{1R}^* (ptz) \partial_z F_{1R} (pz) \right]
\]
\[
- \partial_z F_{2L}^* (ptz) F_{2L} (pz) + \partial_z F_{2R}^* (ptz) F_{2R} (pz) - F_{2L}^* (ptz) \partial_z F_{2L} (pz) + F_{2R}^* (ptz) \partial_z F_{2R} (pz) \right]
\]
Here we have considered initial and final R-S fields on mass shell (|p| = m = |p'|), since Δ baryons with which we want to match R-S fields are on the mass shell as well. In calculations of some terms we used the formulas (4a) and (4b) from [13].
As was noted above, in our case of the AdS/CFT correspondence, the bulk field $\mathcal{V}_M$ corresponds to the boundary current $j^\mu$ of the spin-3/2 field. In the field theory the S-matrix element in momentum space is written in the form

$$S_{fi} = \int d^4p' d^4p j^\mu \varepsilon_\mu.$$

In our case, an expression corresponding to the current $j^\mu$ can be extracted from (3.36):

$$j^\mu = \bar{u}_\alpha (p f) \tilde{O}^{\alpha\mu\beta} u_\beta (p). \quad (3.38)$$

Now we identify the boundary R-S field with the $\Delta$ baryons. Then matrix element (3.36) will describe $\gamma^* \Delta \Delta$ interaction vertex and the matrix element (3.38) will be identified with the matrix element for $\Delta$ baryon current (3.6). This suggestion leads to identification of tensor $\tilde{O}^{\alpha\mu\beta}$ in (3.38) with the $O^{\alpha\mu\beta}$ in (3.7). The terms of $\tilde{O}^{\alpha\mu\beta}$, which have the same tensorial structure as those of $O^{\alpha\mu\beta}$ will be corresponded to each other and integrals over the fifth coordinate $z$ in the summands $\tilde{O}_s^{\alpha\mu\beta}$ will be identified with coefficients $a_i$ or $c_i$ of corresponding term in $O^{\alpha\mu\beta}$. Thus, the coefficients $a_i$ and $c_i$ are expressed as following integrals of profile function $V(qz)$ and $F_{iL,R}(pz)$ over $z$ variable:

$$a_1^{(i)} = \sqrt{2} \int_0^{z_m} dz \ V(qz) \left\{ Q_i \left[ z^{-2} + (4m^2 - q^2) k_9 \right] [F_{i(L)}^{*}(ptz) F_{i(L)}(pz) + F_{2L}^{*}(ptz) F_{2L}(pz)] \right.$$

$$+ \left[ -z^{-1} m \left( \eta^{(s)} + \eta^{(v)} T_i^{(3)} \right) - \left( k_5 z^{-3} + 2m k_8 z^{-2} \right) m Q_i \left( 1 - \frac{q^2}{4m^2} \right) + k_{11} Q_i q^2 z^{-1} + \right.$$

$$+ Q_i \left( -k_4 + z^{-2} m \left( 1 - \frac{q^2}{4m^2} \right) k_7 - \frac{2q^2}{m} k_{12} \right) \left[ F_{iL}^{*}(ptz) F_{2L}(pz) + \right.$$

$$+ F_{2L}^{*}(ptz) F_{iL}(pz) \right] + 6k_{13} Q_i \left[ F_{1L}^{*}(ptz) F_{iL}(pz) - F_{2L}^{*}(ptz) F_{2L}(pz) \right] \right\}$$

$$+ \sqrt{2} \int_0^{z_m} dz \ \partial_z V(qz) \left\{ \left[ z^{-1} m \left( \eta^{(s)} + \eta^{(v)} T_i^{(3)} \right) - \frac{1}{2} j_{10} Q_i z^{-2} - 2q^2 Q_i \left( \frac{1}{m} k_{12} + 2k_{18} \right) \right] \right.$$

$$\times \left[ F_{iL}^{*}(ptz) F_{iL}(pz) - F_{2L}^{*}(ptz) F_{2L}(pz) \right] + \frac{2q^2}{m} k_{18} Q_i \left[ F_{iL}^{*}(ptz) F_{2L}(pz) + F_{2L}^{*}(ptz) F_{iL}(pz) \right]$$

$$- \partial_z F_{iL}^{*}(ptz) F_{2L}(pz) - \partial_z F_{2L}^{*}(ptz) F_{iL}(pz) - F_{iL}^{*}(ptz) \partial_z F_{2L}(pz) - F_{2L}^{*}(ptz) \partial_z F_{iL}(pz) \right\}$$

$$a_2^{(i)} = \sqrt{2m} \int_0^{z_m} dz \ \frac{1}{z^3} V(qz) \left\{ Q_i \left( k_2 + k_5 + k_6 q^2 + \left( 2m k_8 + \frac{1}{2} k_7 \right) z + k_3 z^2 \right) - \right.$$
\[-2 \left( \eta^{(s)} + \eta^{(v)} \mathcal{T}_i^{(3)} \right) z^2 + 8mQ_i z^3 k_{12} \left[ F_{1L}^* (p^t z) F_{2L} (p z) + F_{2L}^* (p^t z) F_{1L} (p z) \right] \]
\[-4m^2 Q_i k_9 z^3 \left[ F_{1L}^* (p^t z) F_{1L} (p z) + F_{2L}^* (p^t z) F_{2L} (p z) \right] \]
\[+ \sqrt{2} m k_6 Q_i \int_0^{z_m} dz \frac{1}{z^3} \partial_z V (q z) \left[ \partial_z F_{1L}^* (p^t z) F_{2L} (p z) + \partial_z F_{2L}^* (p^t z) F_{1L} (p z) \right] \]
\[+ F_{1L}^* (p^t z) \partial_z F_{2L} (p z) + F_{2L}^* (p^t z) \partial_z F_{1L} (p z) \]
\[c_1 = -16 m \sqrt{2} Q_i \int_0^{z_m} dz \partial_z V (q z) \left\{ k_{12} - \frac{1}{8} k_7 z^{-2} + \frac{m}{8} \left( \frac{1}{m} k_5 z^{-3} + 2k_8 z^{-2} + 4z^{-1} k_{11} \right) \right\} \]
\[\times \left[ F_{1L}^* (p^t z) F_{2L} (p z) + F_{2L}^* (p^t z) F_{1L} (p z) \right] + m \left( k_{18} + \frac{1}{4} z^{-2} k_{16} \right) \]
\[\times \left[ \partial_z F_{1L}^* (p^t z) F_{1L} (p z) + F_{1L}^* (p^t z) \partial_z F_{1L} (p z) - F_{2L}^* (p^t z) \partial_z F_{2L} (p z) - \partial_z F_{2L}^* (p^t z) F_{2L} (p z) \right] \]
\[\left\{ -\frac{1}{8} m \left( z^{-2} k_{15} - 5z^{-1} k_{17} \right) \right\} \]
\[- \partial_z F_{1L}^* (p^t z) F_{2L} (p z) - \partial_z F_{2L}^* (p^t z) F_{1L} (p z) - F_{1L}^* (p^t z) \partial_z F_{2L} (p z) - F_{2L}^* (p^t z) \partial_z F_{1L} (p z) \]
\[+ \left( k_{12} - \frac{5}{2} k_{18} + \frac{1}{4} m k_{16} z^{-2} \right) \left[ F_{1L}^* (p^t z) F_{1L} (p z) - F_{2L}^* (p^t z) F_{2L} (p z) \right] \]
\[c_2^{(i)} = 8 \sqrt{2} m^3 Q_i k_{14} \int_0^{z_m} dz \ z^{-3} V (q z) \left[ F_{1L}^* (p^t z) F_{1L} (p z) + F_{2L}^* (p^t z) F_{2L} (p z) \right] \]
\[\text{(3.40)} \]
\[\text{(3.41)} \]
\[\text{(3.42)} \]

For brevity of these expressions we have used relations \[F_{1R} (p z) = - F_{2L} (p z), \ F_{2R} (p z) = F_{1L} (p z)\] and have wrote all coefficients in terms of \[F_{1,2L} (p z)\]. Recall that the superscript \((i)\) shows a kind of \(\Delta\) baryons. The obtained here integral expressions for \(a_{1,2}^{(i)}\) and \(c_{1,2}^{(i)}\) can be applied for numerical studies of form factors \((3.8) - (3.11)\) and related physical quantities in the framework of the hard-wall model AdS/QCD.

**IV. NUMERICAL ANALYSIS**

In order to make a comparison with the electromagnetic form factors of \(\Delta\) baryons obtained in the field theory the initial and final R-S fields should be taken on mass shell and the momenta \(p\) and \(p'\) in \(F_{iL,R} \) in \((3.39) - (3.42)\) should be set \(p = p' = m\). The time-like region
of \( Q^2 = - q^2 \) should be considered. In this region the bulk to boundary propagator (2.15) becomes

\[
V(Q, z) = \frac{\pi}{2} Q z \left[ \frac{K_0(Qz_m)}{I_0(Qz_m)} I_1(Qz) + K_1(Qz) \right] \quad (4.1)
\]

and its derivative equals to the following expression:

\[
\partial_z V(Q, z) = \frac{\pi}{2} Q^2 z \left[ \frac{K_0(Qz_m)}{I_0(Qz_m)} I_0(Qz) + K_0(Qz) \right]. \quad (4.2)
\]

As an example of application of (3.39) - (3.42) let us consider the charge form factor \( G_{E0} \) for one of the charged \( \Delta \) baryons, i.e. for \( \Delta^+ \). For this baryon in (3.39) - (3.42) we should take \( Q^2 = 1 \) and \( T^{(3)}_2 = 1 \) and then \( G_{E0} \) will be written in terms of \( Q^2 \) as below:

\[
G^{(2)}_{E0}(Q^2) = \left( 1 + \frac{2}{3}(2m)^2 \right) \left[ a_{1(2)} + \left( 1 + \frac{Q^2}{(2m)^2} \right) a_{2(2)} \right] - \frac{1}{3}(2m)^2 \left[ 1 + \frac{Q^2}{(2m)^2} \right] \left[ c_{1(2)} + \left( 1 + \frac{Q^2}{(2m)^2} \right) c_{2(2)} \right]. \quad (4.3)
\]

The coefficient functions \( a_{i(2)} \) and \( c_{i(2)} \) are reduced to integrals of products of Bessel functions \( J_i(z) \):

\[
A^\pm_n = \sqrt{2} C^2 \int_0^{z_m} dz V(Qz) z^n \left[ J_2^2(mz) \pm J_3^2(mz) \right]
\]

\[
B_n = \sqrt{2} C^2 \int_0^{z_m} dz V(Qz) z^n J_2(mz) J_3(mz)
\]

\[
C_n = \sqrt{2} C^2 \int_0^{z_m} dz V(Qz) z^n J_1(mz) J_2(mz)
\]

\[
D_n = \sqrt{2} C^2 \int_0^{z_m} dz \partial_z V(Qz) z^n \left[ J_2^2(mz) - J_3^2(mz) \right]
\]

\[
E_n = \sqrt{2} C^2 \int_0^{z_m} dz \partial_z V(Qz) z^n \left[ J_1(mz) J_3(z) + J_2^2(z) \right]
\]

\[
F_n = \sqrt{2} C^2 \int_0^{z_m} dz \partial_z V(Qz) z^n J_2(mz) J_3(z)
\]

and then in terms of these integrals the coefficient functions become equal to:

\[
a_{1(2)} = A_3^+ + (4m^2 + Q^2) k_9 A_5^+ + 2m \left( 1 + \frac{Q^2}{4m^2} \right) k_5 B_2 - 2m \left( 1 + \frac{Q^2}{4m^2} \right) (k_7 - 2mk_8) B_3
\]
\[ + \left[ 2m \left( \eta^{(s)} + \eta^{(v)} \right) + 2k_{11}Q^2 \right] B_4 + 2 \left( k_4 + \frac{2Q^2}{m} k_{18} - \frac{2Q^2}{m} k_{12} \right) B_5 + 6k_{13}A_5^- + \]

\[- \frac{1}{2}k_{10}D_3 + m \left( \eta^{(s)} + \eta^{(v)} \right) D_4 + 2Q^2 \left( \frac{1}{m} k_{12} + 2k_{18} \right) D_5 + 4Q^2k_{18}E_5 \quad (4.4) \]

\[ a_2^{(2)} = -2m \left( k_2 + k_5 - k_6Q^2 \right) B_2 - 2m \left( 2mk_8 + \frac{1}{2}k_7 \right) B_3 - 2m \left[ k_3 - \frac{1}{2} \left( \eta^{(s)} + \eta^{(v)} \right) \right] B_4 \]

\[- 16mk_{12}B_5 - 4m^3k_9A_5^+ - 2m^2k_6E_2 \quad (4.5) \]

\[ c_1^{(2)} = 4mk_5B_2 - 4m \left( k_7 - 2mk_8 - 2m^2k_{16} \right) B_3 + 16m^2k_{11}B_4 + 32m \left( k_{12} + m^2k_{18} \right) B_5 \]

\[- 8m^3k_{16}C_3 - 32m^3k_{18}C_5 + 2m^2 \left( k_{15} - 2k_{16} \right) A_3^+ - 2m^2 \left( 5k_{17} + 8k_{18} \right) A_4^+ \]

\[ + 32mk_{18}F_5 - 32m^2k_{18}E_5 - 8m^2k_{16}D_3 - 16m \left( 2k_{12} - 5k_{18} \right) D_5 \quad (4.6) \]

\[ c_2^{(2)} = 8m^3k_{14}A_2^+ \quad (4.7) \]

Main difficulty for numerical estimations is the absence of values of coefficients \( k_i \). We choose all these constants equal to 0.00000001. The dependence of \( G_{E0} \) on \( Q^2 \) is drown in Fig. 1. This dependence is widely studied in the lattice model for QCD in [16, 17, 27–29] and references therein. The behavior of \( G_{E0} \) as a function of \( Q^2 \) for \( \Delta^+ \) baryon obtained here agrees with the one obtained in the lattice QCD framework. Such qualitatively correct dependence of the form factor \( G_{E0} \) on \( Q^2 \) indicates on rightness of inclusion into hard-wall model the interaction of electromagnetic field with the spin-3/2 fields. Similar numerical study can be done for other baryons of this multiplet and for other form factors as well.
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