Combined inverse Fourier transformation of magnetic resonance and intensity-curvature functional images

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Abstract
This research reports an image processing technique used to merge Magnetic Resonance Imaging (MRI) or Magnetic Resonance Angiography (MRA) with their intensity-curvature functional (ICF). Given a two-dimensional MR image, six 2D model polynomial functions were fitted to the image, and six ICF images were calculated. The MR image and its ICF were direct Fourier transformed. The phase of MR image is estimated pixel-by-pixel as arctangent of ratio between imaginary and real components of k-space and is called phase ratio. The phase of ICF is the phase of inverse Fourier transformation and is called base phase. The two values of phase were summed up and used to reconstruct ICF images through inverse Fourier transformation. The reconstructed ICF image is the combination of MR and ICF. Data obtained with T2-MRI and MRA indicate that the image processing technique elucidates two findings: improvement of vessel detection in T2-MRI; and change of the contrast of T2-MRI and MRA. ICF and reconstructed ICF images are sharper than T2-MRI and MRA are. The degree of sharpness of such images causes improvement of vessel detection and also changes the contrast. The practical implication and potential clinical benefit that the technique offers is to allow expansion of T2-MRI functionality because reconstructed ICF images allow detection and highlight of either hypo-intense tissue or hyper-intense tissue.

KEYWORDS
base phase, intensity-curvature functional, inverse Fourier transformation, magnetic resonance imaging, phase ratio

1 | INTRODUCTION

The literature review opens with the topic of vessel imaging and detection, then introduces to the reader Magnetic Resonance Imaging (MRI) protocols in use with details relevant to pathology studied and magnet strength. Then, the review connects with the state of the art in vessels imaging which is susceptibility weighted imaging (SWI).
1.1 Vessels imaging, detection, MRI protocols, and contrast enhancement through MRI

Human brain vessel imaging is customarily performed using magnetic resonance angiography (MRA), MRI, computed tomography (CT), ultrasound (US) with color Doppler, high-resolution intracranial vessel wall MR imaging (IVW-MRI imaging), and computed tomographic angiography (CTA). Within the aforementioned array of choices, MRA remains the modality which serves as the most important and primary diagnosis tool. MRA scans are used to find the first remarkable evidence of pathology and also to elucidate and/or confirm the clinical condition of patients, before investigating further the patient with additional and diverse MRI protocols.

Indeed, high resolution-MRI is used in combination with T1-weighted, T2-weighted protocols, proton density imaging, and contrast enhanced T1-weighted MRI. T2-weighted MRI is also used in combination with SWI. FLAIR pre- and post-contrast agent injection is used in conjunction with T1 and T2-MRI. MR Imaging of the human brain at 1.5 T, 3 T, and 7 T is rapidly developing as diagnostic tool and various and diverse MRI protocols are used. T1-weighted MRI is used to image cortical micro-infarcts. T1-weighted MRI is also utilized to visualize intracranial arterial vessel wall and vessel wall lesions or in combination with T2-MRI and T2*-weighted MRI, it can help distinguish and classify intracranial atherosclerotic plaques. FLAIR can be utilized at 1.5-3 T to detect cerebral micro-bleeds and can image white matter and cortical lesions using 7 T. MRA is used to visualize intracranial perforating arteries at 7 T. Contrast enhanced T1-weighted MRI performed at various magnet strengths is utilized to image human brain metastases caused by lung cancer. Perfusion Weighted Imaging is used to visualize glioblastoma in the human brain. Diffusion Weighted Imaging (DWI) can be used to detect acute infarctions at 3 T since the modality does not offer sufficient image quality at 7 T. Diffusion Tensor Imaging (DTI) using 1.5 T offers viable option for the identification of medial longitudinal fasciculus in the human brain. State of the art techniques for the generation of synthetic MRI data and their alignment with existing MRI is gaining popularity in view of the development of deep learning algorithm named cycle-consistent generative adversarial network (CycleGAN). State of the art MRI protocols for the study of the vasculature of the human brain nowadays include SWI. In the past 15 years SWI-based MR venography has gained momentum and is now considered as one of the leading MR techniques used to image the vasculature of the human brain. On the basis of SWI, current trends have progressed to calculation of susceptibility maps, which are also used as instrument for the creation of human brain venograms. Human brain vessel detection is relevant to relative orientation between vessel and magnetic field and also relevant to vessel intrinsic magnetic susceptibility (which determines the MRI signal). Brain magnetic susceptibility has been demonstrated to be anisotropic while introducing the computational technique called susceptibility tensor imaging. Feasible ways to enhance vessels detection and contrast enhancement in MRI include: 1. Use of contrast agent injection. 2. Use of ultra or very high magnetic fields which cause enhanced susceptibility effects in the vasculature territory, or like SWI: 3. Use of the properties of paramagnetic deoxyhemoglobin which causes contrast between the vessels and the surrounding tissues.

1.2 The proposed technique

The intensity-curvature functional (ICF) is a property of the model polynomial function. An immediate application of ICF is resampling which yields to the ICF image. The ICF image has been proposed as image space and k-space filter in MRI. To calculate the ICF of the model function requires the model polynomial function to be second-order differentiable and to have non-null sum of second-order partial derivatives calculated at the origin of the pixel coordinate system. The visual appearance of ICF is high pass filtered signal and ICF of T2-weighted MRI of the human brain yields information about the vasculature. The motivation of this research is to improve vessels detection and to change the contrast of Magnetic Resonance of the human brain. Two techniques were chosen among the MR spectrum. T2-weighted MRI, which is not indicated for vessels detail detection, and MRA which is among the best techniques for vessels imaging. The purpose of this paper is to report a technique that merges MR image and ICF image. MR image and its ICF are direct Fourier transformed. The phase of the MR image is calculated pixel-by-pixel as the arctangent of ratio between imaginary and real components of k-space and is called phase ratio. The ICF is inverse Fourier transformed using as phase the sum of: 1. The phase of MR image (phase ratio), and 2. The phase of ICF. The phase of ICF is the phase of the direct Fourier transformation and is called base phase. Merging MR and ICF is determined through reconstruction with inverse Fourier transformation of ICF using as phase the sum of base phase and phase ratio. The combination MR and ICF is
called reconstructed ICF image. When comparing ICF and reconstructed ICF images to T2-MRI and MRA, this research shows that: 1. Is possible to improve vessels detection and change the contrast of T2-MRI; and 2. Is possible to change the contrast of MRA images.

2 | METHODS AND MATERIALS

2.1 | Ethics statement

The study and corresponding informed consent were approved by the Department of Radiology, General Hospital 8-mi Septemvri, Boulevard 8th September, Skopje 1000, Republic of North Macedonia. This study was conducted according to the principles expressed in the Declaration of Helsinki. The 10 subjects who partook in the present study provided signed consent. The informed consent of this study included a statement declaring the possibility of publishing anonymized data and/or images of the subjects enrolled in it. The author of this article, Carlo Ciulla, provided the MRA and consented to its use.

2.2 | Model polynomial functions

MRI images were fitted with six model polynomial functions: 1. Bivariate cubic, 2. Bivariate Lagrange (two different functions), 3. Bivariate quadratic B-Spline, 4. Bivariate cubic B-Spline, 5. Bivariate linear. The mathematics of the model polynomial functions were reported elsewhere while describing the ICF calculated from MRI.27-29 Table 1 shows the abbreviations used in this manuscript when referring to model polynomial functions.27-29

T2-MRI and MRA images were used to calculate their ICFs (one for each model polynomial function) and they were direct Fourier transformed. The ICFs were also direct Fourier transformed. The inverse Fourier transform used two values of the phase. The base phase which is the phase used commonly for Fourier transformations. And, the phase ratio, which was calculated using the arctangent of the ratio between imaginary and real components of MR k-space. The inverse Fourier transformation of ICF used the sum of these two values of the phase to accomplish to calculate the reconstructed ICF image. The reconstructed ICF image is the combination of MR and ICF. Figure 1 illustrates the aforementioned process.

As shown in Table 2, the reconstructed ICF image was compared to T2-MRI: 1. to check for improved vessel detection and change of contrast; and to MRA: 2. to check for change of contrast. Note that the comparison of the ICF and MRI was reported elsewhere.27,30 Table 3 reports the acquisition parameters of the MR acquisitions. MRA slices were determined by maximum intensity projections.

| TABLE 1 | The abbreviations of the model polynomial functions |
| --- | --- |
| Model polynomial function | Bivariate cubic | Bivariate cubic Lagrange | Bivariate quadratic B-Spline | Bivariate cubic B-Spline | Bivariate cubic Lagrange | bivariate linear |
| | B32D | G42D | H32D | H42D | LGR2D | SRE2D |

FIGURE 1 | The signal processing technique used to merge MR with ICF of MR
Reconstructed ICF image: merge of MR & ICF

| Parameters                  | T2-MRI acquisition | MRA acquisition |
|-----------------------------|--------------------|-----------------|
| TE (time to echo)           | 96 ms              | 6.9 ms          |
| TR (repetition time)        | 4720 ms            | 48 ms           |
| FOV (field of view)        | 204 × 250          | 230 × 230       |
| Pixel matrix size           | 262 × 320          | 512 × 512       |
| # Of slices/images          | 28                 | 15              |
| Slice thickness             | 5 mm               | 1.4 mm          |

To assess difference existing between ICF images the results section presents quantitative evaluation conducted using Z-Test on histogram plots of ICF images. The Z-Test provides standardization (scaling) of the independent variable (bin value). The bin value in the histograms is the frequency of occurrence of the pixel intensity value. The Z-Test was chosen because it provides scaling, normalizes the variability existing among the ICF images, and at the same time allows quantitative evaluation aimed to assess and quantify the aforementioned difference even if ICF images are not normally distributed.

3 | THEORY

Let \( f(i, j) \) be the pixel intensity of MR image and ICF image. The direct discrete Fourier transformation yields the real and imaginary components \( \text{Re}(i, j) \) and \( \text{Im}(i, j) \) of the k-space as:

\[
\text{Re}(i, j) = \sum_{i=0}^{N_x} \sum_{j=0}^{N_y} f(i, j) \cdot (\cos \varphi + \sin \varphi) \quad (1)
\]

\[
\text{Im}(i, j) = \sum_{i=0}^{N_x} \sum_{j=0}^{N_y} f(i, j) \cdot (-\sin \varphi + \cos \varphi) \quad (2)
\]

The base phase is:

\[
\varphi = \frac{2.0 \pi x y}{N_x N_y} \quad (3)
\]

where \( x \) and \( y \) are the spatial coordinates of the image. \( N_x, N_y \) are the number of pixels of the image along \( x \) and \( y \) directions respectively. Let the imaginary and real components of the k-space of MR be defined by \( \text{Re}_{MR}(i, j) \) and \( \text{Im}_{MR}(i, j) \) and they are calculated with Equations (1) and (2) respectively. The phase ratio is \( \varphi_1 \):

\[
\varphi_1 = \frac{2.0 \pi \cdot \text{atan2}(\text{Im}_{MR}(i, j), \text{Re}_{MR}(i, j))}{N_x N_y} \quad (4)
\]

The phase ratio is summed to the base phase \( \varphi \). The sum is the phase \( \omega \) as per Equation (5).

\[
\omega = \varphi + \varphi_1 \quad (5)
\]
\( \omega \) is used to inverse Fourier transform the combination of MR and ICF which is indicated in Equations (6) and (7). \( \omega \) varies on a pixel-by-pixel basis. Let \( \text{Re}_{\text{ICF}}(i, j) \) and \( \text{Im}_{\text{ICF}}(i, j) \) be the real and imaginary components of the k-space of the ICF calculated with the direct discrete Fourier transformation as per Equations (1) and (2). The real and imaginary components of the reconstructed ICF image are:

\[
\text{Real}(i, j) = \sum_{i=0}^{N_x} \sum_{j=0}^{N_y} \text{Re}_{\text{ICF}}(i, j) \cdot (\cos \omega + \sin \omega) \quad (6)
\]

\[
\text{Imaginary}(i, j) = \sum_{i=0}^{N_x} \sum_{j=0}^{N_y} \text{Im}_{\text{ICF}}(i, j) \cdot (-\sin \omega + \cos \omega) \quad (7)
\]

\[ g(i, j) = \sqrt{\text{Real}(i, j)^2 + \text{Imaginary}(i, j)^2} \quad (8) \]

The coefficients \((\cos \omega + \sin \omega)\) and \((-\sin \omega + \cos \omega)\) embed the characteristics of the k-space of MR. The result of inverse Fourier transform is reconstructed image pixel \( g(i, j) \) as per Equation (8).

4 | RESULTS

4.1 | The need of the base phase

Figures 2 and 3 illustrate the need of the base phase for the implementation of the signal processing technique reported in this paper. In the two figures, the base phase is presented in (C) and (C1) and its k-space is shown in (H) and (H1) respectively. The phase of images in (A) and (A1) is calculated as ratio of imaginary and real components of the k-space and is presented in (D) and (D1).

As shown in (I) and (I1), in Figures 2 and 3, the inverse Fourier transformation fails if base phase is not used for signal reconstruction. When the base phase is summed to phase ratio as displayed in (E) and (E1), the inverse Fourier transformation is successful and the reconstructed signal is shown in (J) and (J1). Clearly the ICF seen in (B) and (B1) is not the same as the reconstructed ICF signal shown in (J) and (J1), which is combination of signal (see (A) and (A1)) and ICF of signal (see (B) and (B1)). Note that numerical value of base phase is considerably smaller than numerical value of phase ratio and so the images in (D) and (E) appear very similar. Likewise for (D1) and (E1).

4.2 | ICFs of model polynomial functions and reconstructed ICFs

This section presents two evaluations. One is in relation to the determination of the optimal ICF among the six presented in this paper. The other evaluation is conducted on ICF and reconstructed ICF for each of the six generating model polynomial functions (see Table 1) in order to demonstrate that ICF and reconstructed ICF are not the same.
Figure 3  MRA in (A). (B) ICF calculated with SRE2D model polynomial function. (C) Base phase. (D) Phase ratio of imaginary and real components of k-space of MRA. (E) Sum of (C) and (D). (F) k-space magnitude of (A). (G) k-space magnitude of (B). (H) k-space magnitude of (C). (I) reconstructed ICF image making no use of base phase. (J) reconstructed ICF image making use of the sum of the phase in (C) plus the phase in (D).

Figure 4  T2-MRI in (A). (B) ICF calculated with B32D. (C) ICF calculated with G42D. (D) ICF calculated with H32D. (E) ICF calculated with H42D. (F) ICF calculated with LGR2D. (G) ICF calculated with SRE2D.

Figure 4 shows comparison across ICF images obtained when fitting six model polynomial functions to T2-MRI presented in (A). Table 4 reports the values of minimum and maximum pixel intensity used to visualize the images shown in Figure 4. The comparison is progressed in Figure 5 to the analysis of the histograms of ICF images and their Z-Tests. Figure 5 presents histograms of ICFs (see (A) and (B)) and histograms of Z-Tests (see (C) and (D)). Each histogram of ICF plots the recurrence of image pixel intensity value at each bin (1000 bins were used). Each histogram of Z-Test plots the one-tailed P-value (probability) of recurrence of image pixel intensity value at each bin. The hypothesis that ICF is Z-distributed is supported when the Z-Test is true (valued 1 or almost 1). Figure 4A, B shows that six ICFs are different because histograms in (A) and (B) are not overlapping. Figure 4C shows that histograms of Z-Tests of five ICFs are not the same and moreover that the ICF mostly resembling the Z-distribution is ICF of SRE2D (see histogram in (D)).
**Table 4**  Pixel intensity minimum and maximum values used by ImageJ to display the images in Figure 4

|        | (B) ICF_B32D | (C) ICF_G42D | (D) ICF_H32D | (E) ICF_H42D | (F) ICF_LGR2D | (G) ICF_SRE2D |
|--------|--------------|--------------|--------------|--------------|--------------|--------------|
| **min** | 110.07       | 115.00       | 124          | 212          | 213          | 141.5        |
| **max** | 111.06       | 126          | 46           | 47           | 142          | 156.43       |

**Figure 5** (A) Histograms of ICFs presented in Figure 4B, C, D, E, F. In (A) from left to right: ICF calculated with H32D (gray), ICF calculated with B32D (light blue), ICF calculated with G42D (orange), ICF calculated with LGR2D (blue), ICF calculated with H42D (yellow). In (B) histogram of ICF calculated with SRE2D. In (C) from left to right, histogram of Z-Test on ICF calculated with: H32D (gray), B32D (light blue), G42D (orange), LGR2D (blue), H42D (yellow). Histogram of Z-Test on ICF calculated with SRE2D in (D).

Figures 4 and 5, Table 4 provide description of six ICFs used in this research. The six ICFs are different because calculated from different model polynomial functions. The ICF histograms and the values of max and min pixel intensity reported in Table 4 show that is not possible to set same contrast for all ICFs. The ICF calculated from the bivariate linear model function (SRE2D) is most consistent with the hypothesis of Z distribution (see Figure 5D). The Z-Test in Figure 5D shows that the one-tailed P-value is true for all of the bins of the histogram in (B). The ICF calculated from the bivariate linear model function (SRE2D) is the optimal ICF of the six reported, is a k-space filter and an image space high pass filter.

Figure 6 presents the evidence that ICF images and reconstructed ICF images are not the same. Each sum of squared difference (SSD) image shows the difference between ICF and reconstructed ICF image. Table 5 shows maximum and minimum pixel intensity value of the range used to visualize the images presented in Figure 6. The table provides an evidence that each ICF and each reconstructed ICF has a different range of pixel intensity values.

It is due to recall that this section reports data on the study performed at the aim to discern which one among the six ICFs is optimal. Within this context, is necessary to mention that ICFs have diverse and different range of pixel intensity values. Thus, for the appreciation of ICFs high pass filtering effect, is necessary to choose different range for display (min and maximum value of pixel intensity). Table 5 reports the range relevant to the images presented in Figure 6.

4.3 The reconstructed ICF image vs ICF and MR

This section presents the evidence to the claims of this research. The claims are: 1. Vessels detection improvement in T2-MRI is possible through the use of signal processing technique illustrated in Figure 1. 2. The signal processing technique is able to change the contrast of T2-MRI and MRA. The two claims are demonstrated through the sharpness measure called kurtosis. It is seen in this section that kurtosis images of ICF and reconstructed ICF are sharper than T2-MRI. It follows that vessels can be visualized more clearly on ICF and reconstructed ICF images, which present different contrast vs T2-MRI. Likewise, when comparing ICF and reconstructed ICF images with MRA, vessels appear sharper and with different contrast.
**TABLE 5** Minimum and maximum values of pixel intensity used by ImageJ to display the images in Figure 6

|   | ICF B32D | recon_ICF_B32D | SSD between (A) and (B) | ICF G42D | recon_ICF_G42D | SSD between (D) and (E) | ICF H32D | recon_ICF_H32D | SSD between (G) and (H) | ICF H42D | recon_ICF_H42D | SSD between (J) and (K) | ICF LGR2D | recon_ICF_LGR2D | SSD between (M) and (N) | ICF SRE2D | recon_ICF_SRE2D | SSD between (P) and (Q) |
|---|-----------|----------------|------------------------|-----------|----------------|-------------------------|-----------|----------------|------------------------|-----------|----------------|------------------------|-----------|----------------|------------------------|-----------|----------------|------------------------|
| A | min = 182 | max = 184 | min = 364.46 max = 368.48 | min = 131 855.48 max = 136 987.03 | min = 258.07 max = 264.09 | min = 65 487.98 max = 70 857.65 | min = 43 max = 44 | min = 86.21 max = 88.22 | min = 7401.2 max = 7940.8 | min = 111 max = 113 | min = 222.49 max = 224.5 | min = 49 301.84 max = 50 540 | min = 199 max = 201 | min = 398.79 max = 400.8 | min = 158 840.98 max = 161 126.47 | min = 113.11 max = 166.89 | min = 243.06 max = 327.07 | min = 67 775.22 max = 105 899.7 |

**FIGURE 6** In each column, from left to right: ICF (A), (D), (G), (J), (M), (P); reconstructed ICF (B), (E), (H), (K), (N), (Q); SSD between ICF and reconstructed ICF (C), (F), (I), (L), (O), (R). From the top row to the bottom row, the generating model polynomial function is: B32D, G42D, H32D, H42D, LGR2D and SRE2D.
In Figure 7, ICF (E) and reconstructed ICF image (I) provide vessels highlight and high pass filtering effect as opposed to T2-MRI. However, images in (E) and (I) were brightness-contrast enhanced (see values of min and max in Table 6) before computing second moment (see (F) and (J)), fourth moment (see (G) and (K)), and kurtosis31 (see (H) and (L)). This brightness-contrast enhancement might be a source of possible bias. The image in (M) is the SSD32 and it shows that ICF (E) and reconstructed ICF image (I) are different. The vessels indicated by the arrow and the ellipse in Figure 7A are highlighted in the kurtosis images. Kurtosis images measure edge sharpness. Images in (H) and (L) confirm what visually discernible in images (E) and (I). Second moment, fourth moment, kurtosis, and SSD images were computed using $2 \times 2$ pixels convolving window. Each pixel of the aforementioned images results from processing $2 \times 2$ pixels convolving window. The difference between ICF and reconstructed ICF is displayed in Figure 6 in the previous section and is revealed through SSD images. Kurtosis images, which measure edge sharpness, do not reveal such difference (see Figures 7-11).
FIGURE 8  (A) T2-MRI showing vessel structures (see arrows). (B) Kurtosis of (A). (C) ICF of T2-MRI calculated with SRE2D model polynomial function. (D) Kurtosis of (C). (E) Reconstructed ICF obtained merging T2-MRI in (A) and ICF in (C). T2-MRI and ICF are merged in k-space through the procedure illustrated in Figure 1. (F) Kurtosis of (E)

FIGURE 9  (A) T2-MRI showing vessels structures (see arrows). (B) Kurtosis of (A). (C) ICF SRE2D of (A). (D) Kurtosis of (C). (E) Reconstructed ICF obtained merging, through the procedure illustrated in Figure 1, T2-MRI in (A) and ICF in (C). (F) Kurtosis of (E)
FIGURE 10  (A) T2-MRI showing vessels structures (see arrows). (B) Kurtosis of T2-MRI. (C) ICF of T2-MRI calculated with SRE2D model polynomial function. (D) Kurtosis of (C). (E) Reconstructed ICF obtained using T2-MRI in (A) and ICF in (C). (F) Kurtosis of (E)

FIGURE 11  (A) T2-MRI showing vessels structures (see inside the ellipse). (B) Kurtosis of (A). (C) ICF of (A) calculated with SRE2D model polynomial function. (D) Kurtosis of (C). (E) Reconstructed ICF obtained through the procedure illustrated in Figure 1 using T2-MRI in (A) and ICF in (C). (F) Kurtosis of (E)
Kurtosis images calculated from ICF and reconstructed ICF appear richer in details than the kurtosis image calculated from T2-MRI and MRA, thus suggesting that ICF and reconstructed ICF embed more details related to the vessel structures. This effect is visible in Figures 7-11 and Figure 13.

Figures 8-11, bring additional evidence supporting the advantage of reconstructed ICF image (merge of MR and ICF) vs T2-MRI. ICF image and reconstructed ICF image were calculated when fitting data with SRE2D model polynomial function. Reconstructed ICF images display more richness of details as far as the visualization of the vessels is concerned and also display change of contrast vs T2-MRI. This is a recurring occurrence and this evidence is presented in support of the significance of the proposed image processing technique. Within this context, kurtosis images in Figures 8-11 measure the edge sharpness of the three images: T2-MRI, ICF, and reconstructed ICF. Edges are sharper in kurtosis images calculated from ICF and reconstructed ICF. For each picture, compare (D) and (F), which are kurtosis images of ICF and reconstructed ICF respectively, with (B), which is kurtosis of T2-MRI. The vessel structures are indicated by the arrows (see Figures 8-10) and located inside the ellipse (see Figure 11). Tables 7, 8, 9 and 10 show minimum and maximum pixel intensity used to visualize the images.

Figure 12 presents results obtained using MRA (see (A)) as departing image. Sharpening, increases the contrast and so it makes vessels more perceptible vs the background signal (compare for instance (B), (C), (D) with (A)). As a result, reconstructed ICF images provide change of contrast vs MRA. This aspect extends what already seen for T2-MRI in Figures 7-11. Finally, k-space images in Figure 12 provide qualitative account of the reconstructed images. Similarity of results obtainable through ICF and reconstructed ICF image is illustrated in Figure 13 where an MRA case is analyzed. Figure 13 looks at similarities of ICF and reconstructed ICF image from one perspective: edge sharpness properties. The edge sharpness properties are displayed for: MRA in (D); ICF in (H); and reconstructed ICF image in (L). The structures inside the red ellipses (compare (D) with (H) and (L)) are an evidence that the kurtosis (sharpness measure) is able to unveil the richness of details of ICF and reconstructed ICF images. Table 11 shows the range of pixel intensity used to visualize the images.

Through the two windows on ICF image (see Figure 13E) and the two windows on the reconstructed ICF image (see Figure 13I), is possible to measure the signal-to-noise ratio (SNR). The SNR is approximately 1 in (E) in the region.
Figure 12  Magnetic resonance angiography (MRA) in (A). Reconstructed ICF calculated using G42D in (B). Reconstructed ICF calculated using H32D in (C). Reconstructed ICF calculated using LGR2D in (D). The k-space magnitude of the images in (A) through (D) is labeled (A1) through (D1). Reconstructed ICF images were obtained merging ICF with MRA as illustrated in Figure 1.

Figure 13  MRA in (A). (B) Second moment of (A). (C) Fourth moment of (A). (D) Kurtosis of (A). (E) ICF of (A) calculated with SRE2D. (F) Second moment of (E). (G) Fourth moment of (E). (H) Kurtosis of (E). (I) Reconstructed ICF image obtained merging (A) and (E). (J) Second moment of (I). (K) Fourth moment of (I). (L) Kurtosis of (I). (M) Sum of square differences (SSD) between (E) and (I). Second moment, fourth moment, kurtosis, and SSD images were computed using 2 × 2 pixels convolving windows. Each pixel of the aforementioned images results from processing 2 × 2 pixels. Images in (E), (I) and their kurtosis in (H) and (L) suggest similarity of performance of ICF and reconstructed ICF.

of the signal (see red arrow: Mean = 187.03, SD = 11.45) and in the region of the background signal (see white arrow: Mean = 187.01, SD = 1.12). The SNR is approximately 1 also in (I) in the region of the signal (see red arrow: Mean = 374.05, SD = 22.90) and in the region of the background signal (see white arrow: Mean = 374.02, SD = 2.24).

Figures 7-11 and Tables 6-10 show the visual and numerical evidence of one the claims of this research. The claim is the change of contrast visible in ICF and reconstructed ICF images vs T2-MRI. The visual evidence is provided through the images. The numerical evidence is the max and min value of pixel intensity used to display the images. Similarly, Figure 13 and Table 11 provide the evidence to the claim of change of contrast visible in ICF and reconstructed ICF images vs MRA.
TABLE 1  Minimum and maximum values of image intensity used to visualize the images displayed in Figure 13

|        | (B) Second moment | (C) Fourth moment | (D) Kurtosis |
|--------|-------------------|-------------------|--------------|
| MRA    | min = 0 max = 255 | min = 0 max = 2223 $10^3$ | min = 0.25 max = 0.25 |
| (E) ICF SRE2D | min = 182 max = 197 | min = 0 max = 2341 $10^3$ | min = 0.25 max = 0.34 |
| (I) recon_ICF_SRE2D | min = 361 max = 401 | min = 0 max = 2340 $10^3$ | min = 0.25 max = 0.34 |

Note: The values were read in by ImageJ.

FIGURE 14  T2-MRI in (A). Phase ratio of T2-MRI in (B). (C) ICF of T2-MRI calculated with SRE2D model polynomial function. (D) Phase ratio of ICF. (E) Real component of k-space of (B). (F) Imaginary component of k-space of (B). (G) Real component of k-space of (D). (H) Imaginary component of k-space of (D). Reconstructed T2-MRI in (I). The image in (I) results from the inverse Fourier transformation of the sum of k-space of T2-MRI (not shown) and k-space of phase ratio (shown in (E) and (F)). The image in (J) is difference between T2-MRI (A) and reconstructed T2-MRI (I). The image in (K) results from the inverse Fourier transformation of the sum of k-space of ICF (C) and k-space of phase ratio (D). The image in (L) is difference between ICF of T2-MRI (see (C)) and reconstructed ICF image of T2-MRI (see (K)).

To ascertain the meaning of phase ratio and to validate its use in the image processing technique proposed in this paper, Figure 14 shows in (B) phase ratio of T2-MRI (see (A)), k-space real and imaginary components of phase ratio in (E) and (F) respectively. Figure 14 also shows in (I) reconstructed T2-MRI obtained inverse Fourier transforming the sum of k-space of T2-MRI and k-space of phase ratio. The image in (J) is difference between images in (A) and (I) and such difference gives meaning to phase ratio. Figure 14 also shows ICF of T2-MRI in (C), phase ratio of ICF in (D), k-space real and imaginary components of phase ratio in (G) and (H). Figure 14 finally shows in (K) reconstructed ICF obtained inverse Fourier transforming the sum of k-space of ICF and k-space of the phase ratio. The image in (L) is difference between images in (C) and (K).

In summary, phase ratio embeds vessel information which is used inverse Fourier transforming the sum of k-space of ICF and k-space of phase ratio, so to obtain images like (K). The difference image in (L) reveals that ICF and reconstructed ICF image are not equal, and validates the use of phase ratio in the image processing technique.
In relation to the images presented in Figure 14, histogram of T2-MRI in (A); histogram of phase ratio of T2-MRI in (B); histogram of k-space of phase ratio in (C) and (D); histogram of reconstructed T2-MRI in (E). The histogram in (F) shows that T2-MRI and reconstructed T2-MRI are not the same. The reconstructed T2-MRI is obtained using the procedure outlined in Figure 1

In relation to the images presented in Figure 14, histogram of ICF of T2-MRI in (A); histogram of phase ratio in (B); histogram of k-space of phase ratio in (C), (D); histogram of reconstructed ICF image in (E). The histogram in (F) shows that ICF and reconstructed ICF image are not the same. The reconstructed ICF image is obtained using the procedure outlined in Figure 1.
Figures 15 and 16 show histograms of the images presented in Figure 14. The two figures determine that: T2-MRI and ICF are different (see (A)); phase ratio is numerically small (see (B)); real part and imaginary part of k-space of phase ratio are different (see (C) and (D)); reconstructed T2-MRI is not the same as reconstructed ICF image (compare (E) in Figure 15 to (E) in Figure 16). And most importantly that: T2-MRI and reconstructed T2-MRI are not the same (see (F) in Figure 15); and ICF and reconstructed ICF image are not the same (see (F) in Figure 16).

5 | DISCUSSION OF THE CONTRIBUTION

5.1 | The image processing technique

The present research relates to vessel imaging, it looks at the computational aspect and is motivated by recent developments, which indicate ICF of MR images as filtering technique able to achieve vessel detection and to change the contrast. Hence, this paper proposes a novel fully computational technique which explores the combination of MR images with ICF images. The technique is based on inverse Fourier transformation of k-space of ICF using as phase the sum of base phase and phase ratio. The base phase is the phase used normally when direct or inverse Fourier transforming the image. The phase ratio is the arctangent of the ratio between imaginary and real component of the k-space of the MR image. The phase ratio is thus the numerical parameter used to merge k-space of ICF with k-space of MR. Reconstructed ICF images are similar to ICF and, when compared to the departing MR, offer level of detail of the vessel structures that are not overtly observable in T2-MRI. Moreover, the change of contrast offered by reconstructed ICF images is additive to the invaluable information already provided by MRA. As recently reported, the ICF was used as k-space filter subtracting the k-space of the ICF from the k-space of MR, and inverse Fourier transforming back the subtraction signal. In this paper a new approach is undertaken. The approach takes into account MR through a measure (phase ratio) and embeds this measure into formulation (see Equations (6) and (7)) merging k-space of ICF and k-space of MR.

5.2 | Novel contribution

The contribution of this paper to the state of the art is two-folded. One is the computational technique used to reconstruct ICF after fusing it with the k-space of MR data. The other one is to be able to visualize and to change the contrast of vessels imaged with T2-weighted MRI. T2-weighted MRI is not indicated for vessel imaging, and in this context, the paper proposes a technique able to improve vessels detection in T2-MRI. This aspect is not reported elsewhere and is thus the main novelty of this research vs recent developments. Moreover, this research makes a neat progress vs the inverse Fourier transformation procedure reported earlier. The present technique consists of merging two images in k-space using phase (see Figure 1), while previous research inverse Fourier transforms the difference between k-space of MRI and k-space of intensity-curvature-term (compare fig. 1 with table 3 in Reference 28). In other words, the present technique encodes MRI phase (called phase ratio) into ICF phase (called base phase) and inverse Fourier transforms into reconstructed ICF image (which embeds MRI information in it), thus fusing ICF and MR image. The reproducibility of results of the present technique is assured by rigorous mathematical methodology implemented for Fourier-based signal reconstruction and by calculus for ICF calculation.

The main contribution of the article is the image processing technique illustrated in Figure 1. The implication of the findings and also the potential medical benefit is to widen T2-MRI functionality. This is because, although T2-MRI is not most apposite for vessel detection, through the contribution provided by the present study, is possible to improve vessel detection post-processing T2-MRI images. Due to recall that consequential to the use of ICF images and reconstructed ICF images both T2-MRI and MRA benefit of change of contrast. And this is the second finding of the present study.

5.3 | Context and literature

The context of this research is human brain vessels imaging, which is nowadays performed with a combination of large arrays of MR-based imaging techniques. In fact, recent trends in research and the parallel introduction of high and ultra-high field MR scanners indicate intracranial vessel wall MR imaging to be among the most promising technique both in
research and diagnostic settings. Hence, relevant literature is: 1. High-resolution intracranial vessel wall MR imaging (IVW-MR imaging) and 2. Susceptibility mapping.

Within the context of IVW-MR imaging the correlation with the present research is the use of T2-weighted MRI. The image reconstruction technique proposed here allows improved vessels detection (vs T2-MRI) and changes the contrast. In short, it is here proposed that the task of vessel visualization through T2-weighted MRI can be enhanced through image processing. The aforementioned task is not easy to achieve through the sole use of T2-MRI. Indeed, the reconstructed ICF image and the ICF are pivotal to the aim of improving vessel detection in T2-MRI. Moreover, it is relevant to report that IVW-MR imaging requires 3 T or ultra-high-field MRI scanners, while the proposed image processing technique can be used on MR acquired at low magnetic fields (1.5 T).

SWI uses mIP to help visualizing the connectivity of veins and to enhance contrast. Hence, due to the benefits of mIP in SWI, to create 3D maps of the veins of the brain onto 2D projections, may be preferred to 2D image analysis. As opposite to SWI, the signal processing technique here proposed does not make use of minimum intensity projections (mIPs), neither maximum intensity projections (MIPs), and it works with a single 2D MR image at time.

Moreover, as recent research suggests, susceptibility mapping algorithms suffer from the diversity of solutions implemented to solve the field-to-source inverse problem, which is related to accurate estimation of the susceptibility from MRI phase data. On the other hand, the present research uses MR data to enforce high pass filtering, which is indeed: 1. The effect of calculation of ICF on MR image. And, 2. The effect of reconstructed ICF image after fusing ICF with MR. High pass filtering might be suffering though of artefactual peaks (false “ripples”) in the signal-image or by insufficient background removal. This happens for instance when high pass filtering phase images are used in SWI.

6 | DISCUSSION OF THE FINDINGS

This section discusses the findings of this research in relation to the state of the art, while broadening the view on the current state of the art reported in literature in terms of assessment of human brain vascular health. The section also provides instructions on how to replicate results or to make new ones.

6.1 | Findings and literature

This research provides two major findings: 1. Improvement of vessel detection in T2-MRI; and 2. Change of contrast of T2-MRI and MRA. As far as regards the first finding, ICF calculated from T2-MRI images, and reconstructed ICF obtained through signal processing technique illustrated in Figure 1, embed enhanced vessels visibility. Vessels visibility is related to abrupt change of signal intensity which is manifest through spikes of signal and such change can be detected through high pass filtering. ICF and reconstructed ICF are gradient images and they present high pass filtering effect able to highlight spikes of signal in the images. The effect of ICF and reconstructed ICF on T2-MRI was investigated through kurtosis images which provide sharpness (brightness) measurement. Kurtosis images of ICF and reconstructed ICF show enhanced brightness vs T2-MRI and highlight the vessels (see Figures 7-11). These figures show enhanced kurtosis (brightness) of ICF and reconstructed ICF, which signifies that ICF and reconstructed ICF are amplifiers of the T2-MRI signal spikes at the vessels location.

In relation to the second finding, change of contrast of MRA is once again caused by high pass filtering effect of the two gradient images: ICF and reconstructed ICF. MRA images are brought to a new domain: intensity-curvature; which is the combination of signal and second-order partial derivatives of the signal. The intensity-curvature images (ICF and reconstructed ICF) present the characteristic of sharpening the signal at the vessel location, leaving the background noise as it is, thus providing an overall highlight of the image structures (see Figures 12 and 13). Due to recall that blood vessel connectivity is not emphasized by sharpening effect enforced by ICF and reconstructed ICF images. In a nutshell, what is the advantage of ICF and reconstructed ICF over T2-MRI and MRA? 1. To improve T2-MRI vessel visibility; and 2. To render sharper images vs MRA. The key is to change the contrast of the images in proximity of the vessels for both imaging modalities. This is possible because the high pass filtering effect of the intensity-curvature concept.
Black blood MRI is a technique used to visualize inflammation in blood vessel walls and it can be used within the context of intracranial vessel imaging using modified T1 MRI protocol at 3 T. The literature reports black blood MRI able to identify: categories of brain vascular disease; atherosclerotic plaques; and aneurysms. Moreover, 2D T1 MRI was compared to 3D T1 dark blood vessel wall imaging using 3 T as magnet strength, suggesting the 3D technique as more effective for visualization of intracranial artery segments.39

The importance of the assessment of human brain vascular health is crucial to prevent stroke. As far as regards to this, brain and vessel imaging can be determined through: Non-contrast CT (NCCT), magnetic resonance DWI, CTA, carotid Doppler and magnetic resonance (MR) angiogram; and perfusion imaging: either with MR perfusion or CT perfusion; among others.41 Moreover, inflammatory vascular pathology, also known as central nervous system vasculitis (CNSV) can be investigated through combination of high-resolution 3 T vessel wall MR imaging protocol and biopsy.42 Recently, contrast enhanced SWI was used to image sub-voxel vasculature of the human brain.19

Also, within the context of human brain vascular health, T1-MRI is used: 1. To image arterial segments in the human brain at 3 T so to study intracranial atherosclerotic disease; 2. In combination with T2-MRI, to image and detect enlargement of perivascular spaces, which is a feature of small vessel disease.44

Segmentation of human brain vessels is a major step to assess neurological disease and is currently performed using T1-MRI, T2-MRI, and processing Time of Flight (TOF) 3 T MRI with deep learning artificial neural networks. Only recently, research conducted through T2-MRI was able to image human brain vessels related structures. Indeed, the study reported by Ballerini et al., uses T2-MRI to image and segment perivascular spaces (PVS) in the human brain. PVS appear hyper intense (bright) on T2-MRI.12,47

In summary, present state of the art in clinical and research MRI, offers a wide array of choices to assess human brain vessels health.

Within the aforementioned context, and recalling that T2-MRI is not specifically indicated for vessel imaging, this research proposes a signal processing technique which is able to improve vessels detection in T2-MRI and to change the contrast of T2-MRI and MRA. Improvement of vessel detection and change of contrast happen because ICF and reconstructed ICF images are sharper than T2-MRI and MRA. To be able to improve detection of human brain vessels using T2-MRI is advancement which launches T2-MRI at 1.5 T as viable imaging tool for intracranial vessels detection.

### 6.2 Replication, data, and software availability

In order to make possible replication of findings the author commits data and software to the repository: https://github.com/cxc2728/invFT. Programs need to be compiled in Visual Studio C++, and executables can be called from the command prompt. Each program provides the specifics of the command line upon calling the program name from the console. Image data are provided in analyze format: 64 bits Real. To visualize images calculated through the programs, the user can utilize ImageJ (https://imagej.nih.gov/ij/download.html). It is not necessary to change image format when using ImageJ to read an image output from the aforementioned programs.

### 6.3 Discussion of observations

To observe difference existing between ICF and reconstructed ICF the reader is referred to k-space images shown in Figure 14. The picture shows in (D) k-space ratio of ICF of T2-MRI. Real and imaginary components of the image in (D) are presented in (G), (H). Details of these k-space components make the difference between ICF (see (C)) and reconstructed ICF (see (K)). Details are indicated by the arrows in (G), (H). They are related to vessels structures visible in T2-MRI image in (A). The inverse Fourier transformation procedure illustrated in Figure 1 merges k-space of ICF and k-space ratio components ((G) and (H)). The fusion brings the vessel information seen in (G), (H) into reconstructed ICF (K), and is demonstrated by the difference image shown in (L). Overall, what is the advantage of the signal processing technique presented in Figure 1? To extract from k-space components vessel topography of T2-MRI and to bring vessel topography into the ICF, which, after inverse Fourier transformation, becomes reconstructed ICF (see (K)). The aforementioned observation gives the natural justification of purpose to the signal processing technique illustrated in this paper.
6.4 Limitations

This section states the limitations of the analysis reported in this paper. First of all, we should recall that high pass filtering characteristics of ICF remains competitive when compared to high pass filtering obtained through reconstructed ICF image, and the two of them are quite similar. A viable use of ICF and reconstructed ICF image is the two of them together vs one preferred to the other one.

The second limitation to be addressed is in relation to possible source of bias that may have been determined because of the image contrast enhancement done before calculation of kurtosis (see Figures 7-11). In evaluating this possible source of bias is necessary however to consider that edge effect created by ICF and reconstructed ICF image improves visibility of vessels vs T2-MRI, and this is an evidence visible in Figures 7-11. Edge effect is a direct consequence of high pass filtering characteristics of ICF and reconstructed ICF images (the intensity-curvature images). Edge effect causes increased brightness seen in kurtosis images calculated from intensity-curvature images vs kurtosis images calculated from T2-MRI. Hence, edge effect is the advantage that makes possible to improve vessel detection in T2-MRI. Indeed, the usefulness of ICF and reconstructed ICF is to be able to detect either hypo-intense tissue or hyper-intense tissue in T2-MRI.

The third limitation due to address is in relation to calculation of ICF and reconstructed ICF images using MRA. The intensity-curvature images calculated from MRA present the disadvantage to disrupt blood vessel connectivity. The disruption happens because of edge effect determined by the aforementioned images. Hence, the characteristic that might be preferred, when looking at intensity-curvature images calculated from MRA, is the change of contrast, and not vessel topography, as it can be seen when evaluating ICF and reconstructed ICF images calculated from T2-MRI.

In a nutshell: 1. Intensity-curvature images improve vessel detection in T2-MRI. 2. Intensity-curvature images change the contrast of T2-MRI and MRA images. An additional limitation is that no expert reader evaluation of the ICF reconstructed images experiment was done in this study.

7 CONCLUSION

It is possible through use of ICF and reconstructed ICF image to improve vessels detection in T2-MRI and to change the contrast of T2-MRI and MRA. Improvement of vessel detection in T2-MRI and change of contrast of T2-MRI and MRA are caused by sharpening effect of ICF and reconstructed ICF images. Present state of knowledge in intensity-curvature image processing is based on evidence provided in support to the use of ICF as image space and k-space filter.\textsuperscript{27-30} The main contribution of this research is the signal processing technique illustrated in Figure 1. Because of applications of this technique presented in this paper, the state of the art on intensity-curvature image processing advances further providing T2-MRI with wider functionality. The observed edge effect, which reconstructed ICF brings in, through inverse Fourier transformation of the k-space of the T2-MRI, is alternative to edge detection. Vessel structure edges are visible in T2-MRI (see (A) in Figure 14), and through the image processing technique, they are automatically superimposed onto ICF images. The ICF image becomes reconstructed ICF image and vessel edges are highlighted. As result, wider T2-MRI functionality is achieved because reconstructed ICF images allow detection and highlight of either hypo-intense tissue or hyper-intense tissue. In either case: hypo-intense tissue or hyper-intense tissue; the signal processing technique departs from an edge visible in T2-MRI images; and determines edge detection, highlight, and enhanced vessel visibility. This capability is consequential to high pass filtering effect of ICF and reconstructed ICF and may provide clinical benefits as shown in Figure 7 where an example of ICF and reconstructed ICF enhanced vessel visibility is provided. Hence, to conclude, the justification of using the signal processing technique is to extract, from vessel topography of T2-MRI, k-space components and to bring vessel topography into the reconstructed ICF image. Extensions of the use of the technique may be in MRI protocols like SWI which is more suited for vessels imaging.

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**CONFLICT OF INTEREST**

The authors declare no potential conflict of interest.

**AUTHOR CONTRIBUTIONS**

Carlo Ciulla: Conceptualization; formal analysis; investigation; methodology; project administration; software; supervision; validation; writing-review and editing.

**ETHICS STATEMENT**

MRI acquisition was consented, and approval was obtained from the subjects to use the images for research purposes after the administration of written consent. The research protocol of MRI data acquisition was approved by the Department of Radiology, General Hospital 8-mi Septemvri, Boulevard 8th September, Skopje 1000, Republic of North Macedonia. This study was conducted according to principles expressed in the 1964 Declaration of Helsinki. The author of this article, Carlo Ciulla, provided the Magnetic Resonance Angiography (MRA).

**DATA AVAILABILITY STATEMENT**

The data and software that support the findings of this study are openly available at https://github.com/cxc2728/invFT. The programs need to be compiled in Visual Studio C++, and executables can be called from the command prompt. Each program provides the specifics of the command line upon calling the program name from the console. Image data is provided in analyze format: 64 bits Real. To visualize images calculated through the programs, the user can utilize ImageJ (https://imagej.nih.gov/ij/download.html). It is not necessary to change image format when using ImageJ to read an image output from the aforementioned programs.

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