SU(3) Mass Splittings for $\bar{q}q$ Mesons and $qqq$ Baryons

M. D. Scadron*, G. Rupp†, E. van Beveren** and F. Kleefeld†

*Physics Department, University of Arizona, Tucson, AZ 85721, USA
†Centro de Física das Interacções Fundamentais, Instituto Superior Técnico, Lisbon, Portugal
**Departamento de Física, Universidade de Coimbra, Coimbra, Portugal

Abstract. By comparing SU(3)-breaking scales of linear mass formulae, it is shown that the lowest vector, axial-vector, and scalar mesons all have a $\bar{q}q$ configuration, while the ground-state octet and decuplet baryons are $qqq$. Also, the quark-level linear $\sigma$ model is employed to predict similar $\bar{q}q$ and $qqq$ states. Finally, the approximate mass degeneracy of the scalar $a_0(980)$ and $f_0(980)$ mesons is demonstrated to be accidental.

1. INTRODUCTION

In the quark model, one usually assumes that pseudoscalar ($\mathcal{P}$), vector ($\mathcal{V}$), and axial-vector ($\mathcal{A}$) mesons are $\bar{q}q$, whereas octet ($\mathcal{O}$) and decuplet ($\mathcal{D}$) baryons are $qqq$ states. However, it is often argued [1] that the light scalar ($\mathcal{S}$) mesons are non-$\bar{q}q$ candidates, in view of their low masses. In this short paper, we shall show that the ground-state meson nonets $\mathcal{P}$, $\mathcal{O}$, $\mathcal{V}$, and $\mathcal{A}$ are all $\bar{q}q$, hence including the light scalars, while the lowest $\mathcal{O}$ and $\mathcal{D}$ baryons are $qqq$ states.

In Sec. 2, SU(3) mass splittings for loosely bound $\mathcal{V}$, $\mathcal{A}$, and $\mathcal{S}$ states are shown to have symmetry-breaking scales of 16%, 8%, and 25%, respectively, using linear mass formulae. We apply the latter formulae to $qqq \mathcal{O}$ and $\mathcal{D}$ states in Sec. 3, leading to SU(3)-breaking scales of 13% and 12%, respectively. Then in Sec. 4, we employ the quark-level linear $\sigma$ model (L$\sigma$M) to predict similar $\bar{q}q$ and $qqq$ states as in Secs. 2 and 3. Next in Sec. 5, we study the $\mathcal{S} \bar{q}q$ states and argue why the $\mathcal{V}$ states have slightly higher masses, on the basis of the nonrelativistic quark model. Moreover, the approximate mass degeneracy of the $\mathcal{S} a_0(980)$ and $f_0(980)$ mesons is shown to be just accidental. We summarize our results in Sec. 6.

2. MASS SPLITTINGS FOR SU(3) $\mathcal{V}$, $\mathcal{S}$, $\mathcal{A}$ GROUND STATES

Although meson masses are expected to appear quadratically in model Lagrangians, while they must appear so for $\mathcal{P}$ states [2], for $\mathcal{V}$, $\mathcal{O}$, $\mathcal{A}$ states a Taylor-series linear form for SU(3) mass splittings is also possible. Thus consider a Hamiltonian density $H = H(\lambda_0) + H_{ss}(\lambda_8)$ using Gell-Mann matrices. Then the vector-meson-nonet masses
\[ m_{\gamma} = m^0_{\gamma} - \delta m_{\gamma} d_{\delta s} \] are

\begin{align*}
  m_{\rho,\omega} &= m^0_{\gamma} - \frac{\delta m_{\gamma}}{\sqrt{3}} \approx 776 \text{ MeV} , \\
  m_{K^*} &= m^0_{\gamma} + \frac{\delta m_{\gamma}}{2\sqrt{3}} \approx 892 \text{ MeV} , \\
  m_{\phi} &= m^0_{\gamma} + \frac{2\delta m_{\gamma}}{\sqrt{3}} \approx 1020 \text{ MeV} ,
\end{align*}

with \( \phi \approx \delta s \). Measured vector masses \[1\] suggest average mass splittings

\[ m^0_{\gamma} \approx 850 \text{ MeV} , \quad \delta m_{\gamma} \approx 140 \text{ MeV} , \quad \] (2)

giving an \( SU(3) \)-breaking scale of \( \delta m_{\gamma} / m^0_{\gamma} \approx 16\% \).

Likewise, the ground-state axial-vector mesons (while slightly ambiguous) still suggest

\begin{align*}
  m_{a_1(1260), f_1(1285)} &= m^0_{\omega} - \frac{\delta m_{\omega}}{\sqrt{3}} \approx 1256 \text{ MeV} , \\
  m_{K_1(1270)} &= m^0_{\omega} + \frac{\delta m_{\omega}}{2\sqrt{3}} \approx 1273 \text{ MeV} , \\
  m_{f_1(1420)} &= m^0_{\omega} + \frac{2\delta m_{\omega}}{\sqrt{3}} \approx 1426 \text{ MeV} .
\end{align*}

Here, we assume the \( f_1(1420) \) is mostly \( \delta s \), because the PDG \[1\] reports \( f_1(1420) \to KK\pi, K^*K \) as dominant, while \( f_1(1285) \to KK\pi, K^*K \) are almost absent. Thus, \( f_1(1285) \) is mostly \( \delta n \), like the nonstrange \( a_1(1260) \) (with \( a_1 \to \sigma\pi \) seen, but \( a_1 \to f_0(980)\pi \) not seen, because \( f_0(980) \) is mostly \( \delta s \)). Then the pattern of Eqs. (3) suggests approximate average mass splittings

\[ m^0_{\omega} \approx 1305 \text{ MeV} , \quad \delta m_{\omega} \approx 98 \text{ MeV} , \quad \frac{\delta m_{\omega}}{m^0_{\omega}} \approx 8\% \quad . \] (4)

Also the scalar masses (not incompatible with Ref. [1]) predicted from the L\( \sigma \)M discussed in Sec. 4 obey the mass-splitting pattern

\begin{align*}
  m_{\sigma_n} &= m^0_{\gamma} - \frac{\delta m_{\gamma}}{\sqrt{3}} \approx 650 \text{ MeV} , \\
  m_{\kappa} &= m^0_{\gamma} + \frac{\delta m_{\gamma}}{2\sqrt{3}} \approx 800 \text{ MeV} , \\
  m_{\sigma_s} &= m^0_{\gamma} + \frac{2\delta m_{\gamma}}{\sqrt{3}} \approx 970 \text{ MeV} .
\end{align*}

Here, \( m_{\sigma_n(650)} \) is near the PDG average \[1\] \( m_{f_0(600)} \), \( m_{\kappa(800)} \) is near the E791 value \[3\] \( 797 \pm 19 \text{ MeV} \), and \( m_{\sigma_s(970)} \) from Sec. 5 and the Appendix is near the PDG value
$m_{f_0(980)}$, which is thus mostly $\bar{s}s$. The masses from Eqs. (5) then give the average mass splittings

$$m_0^{0\phi} \approx 753 \text{ MeV} \quad \delta m_\phi \approx 185 \text{ MeV} \quad \frac{\delta m_0}{m_0^{0\phi}} \approx 25\% \quad .$$

The fact that the $\bar{q}q$ scalars have an $SU(3)$-breaking scale of 25%, about double the scale of $\mathcal{V}$ and $\mathcal{A}$ ground states, further suggests that, whereas the $\mathcal{V}$, $\mathcal{A}$ are $\bar{q}q$ loosely bound states, the $\bar{q}q$ $\phi$ states (with quarks touching in the NJL scheme [4]) are “barely” elementary-particle partners of the tightly bound $\mathcal{P}$ states (discussed in Sec. 4).

The mean of the slightly varying $\mathcal{V}$, $\mathcal{A}$, $\phi$ mass scales in Eqs. (2, 4, 6) is $m_0^{0\phi} = 969$ MeV, $\delta m = 141$ MeV, and the latter are close to the baryon mass-splitting scales which we derive next.

### 3. LOOSELY BOUND $QQQ$ BARYONS

In this same Taylor-series spirit, the $\phi$ baryon $SU(3)$ mass splitting $m_\phi = m_0^{0\phi} - \delta m_\phi (d_{ss}\bar{d}^{\bar{s}i} + f_{ss} f^{\bar{s}i})$ for $d_{ss} + f_{ss} = 1$, predicts (the index $ss$ means semistrong)

$$m_N = m_0^{0\phi} - \frac{\delta m_\phi}{2\sqrt{3}} (-d_{ss} + 3f_{ss}) \approx 939 \text{ MeV} ,$$

$$m_\Lambda = m_0^{0\phi} + \frac{\delta m_\phi}{\sqrt{3}} d_{ss} \approx 1116 \text{ MeV} ,$$

$$m_\Sigma = m_0^{0\phi} - \frac{\delta m_\phi}{\sqrt{3}} d_{ss} \approx 1193 \text{ MeV} ,$$

$$m_\Xi = m_0^{0\phi} + \frac{\delta m_\phi}{2\sqrt{3}} (d_{ss} + 3f_{ss}) \approx 1318 \text{ MeV} .$$

The $(d/f)_{ss}$ ratio can be found from Eqs. (7) as

$$\left( \frac{d}{f} \right)_{ss} = -\frac{3}{2} \frac{m_\Sigma - m_\Lambda}{m_\Xi - m_N} \approx -0.305 \quad , \quad d_{ss} \approx -0.44 \quad , \quad f_{ss} \approx 1.44 .$$

Thus, Eqs. (7) predict the average mass splittings

$$m_0^{0\phi} \approx 1151 \text{ MeV} \quad , \quad \delta m_\phi \approx 150 \text{ MeV} \quad , \quad \frac{\delta m_0}{m_0^{0\phi}} \approx 13\% \quad .$$

The $SU(3)$ $\phi$ baryon masses $m_\phi = m_0^{0\phi} + \delta m_\phi$ have $m_0^{0\phi}$ weighted by wave functions

$$\Psi^{(abc)} \Psi_{(abc)} = \overline{\Delta} \Delta + \overline{\Sigma} \Sigma^* + \overline{\Xi} \Xi^* + \overline{\Omega} \Omega ,$$

and $\delta m_\phi$ is weighted by

$$3 \overline{\Psi}^{(ab3)} \Psi_{(ab3)} = \overline{\Sigma} \Sigma^* + 2 \overline{\Xi} \Xi^* + 3 \overline{\Omega} \Omega .$$
Then the $SU(3)$ $D$ masses are predicted (in MeV) to be
\begin{align*}
m_{\Delta} &= m_0^D \approx 1232, \\
m_{\Sigma^*} &= m_0^D + \delta m_0^D \approx 1385, \quad \text{with } \delta m_0^D \approx 153, \\
m_{\Xi^*} &= m_0^D + 2\delta m_0^D \approx 1533, \quad \text{with } \delta m_0^D \approx 151, \\
m_{\Omega} &= m_0^D + 3\delta m_0^D \approx 1672, \quad \text{with } \delta m_0^D \approx 147. 
\end{align*}

This corresponds to average mass splittings
\begin{equation}
m_0^D \approx 1232 \text{ MeV}, \quad \delta m_0^D \approx 150 \text{ MeV}, \quad \frac{\delta m_0^D}{m_0^D} \approx 12\%. \tag{13}
\end{equation}

It is interesting that both loosely bound $qqq$ $O$ and $D$ symmetry-breaking scales of about 150 MeV are near the $\bar{q}q$ $V, A, S$ mean mass-splitting scale of $\delta m = 141$ MeV. However, the $SU(3)$-breaking scale of 25% for scalars is almost double the 12–16% scales of $V, A, O, D$ states. This suggests that $V, A, O, D, \bar{q}q$ or $qqq$ states are all loosely bound, in contrast with the $\bar{q}q$ $P$ and, of course, the $P$ states (see above). In fact, the latter Nambu–Goldstone $P$ states are massless in the chiral limit (CL) $p^2 = m_\pi^2 = 0$, $p^2 = m_K^2 = 0$, as the tightly-bound measured $[13]$ $\pi^+$ and $K^+$ charge radii indicate $[5]$.

\section*{4. CONSTITUENT QUARKS AND THE QUARK-LEVEL L$\sigma$M}

Formulating the $P$ and $\sigma$ $\bar{q}q$ states as elementary chiral partners $[6]$, the Lagrangian density of the $SU(2)$ quark-level linear $\sigma$ model (L$\sigma$M) has, after the spontaneous-symmetry-breaking shift, the interacting part $[7]$
\begin{equation}
L^{int}_{L\sigma M} = g \overline{\psi} (\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) \psi + g' \sigma (\sigma^2 + \pi^2) - \frac{\lambda}{4} (\sigma^2 + \pi^2)^2 - f_\pi g \overline{\psi} \psi, \tag{14}
\end{equation}
with tree-order CL couplings related as (for $f_\pi \approx 93$ MeV)
\begin{equation}
g = \frac{m_q}{f_\pi}, \quad g' = \frac{m_\sigma^2}{2f_\pi} = \frac{\lambda}{f_\pi}. \tag{15}
\end{equation}
The $SU(2)$ and $SU(3)$ chiral Goldberger–Treiman relations (GTRs) are
\begin{equation}
f_\pi g = \hat{m} = \frac{1}{2}(m_u + m_d), \quad f_K g = \frac{1}{2}(m_s + \hat{m}). \tag{16}
\end{equation}
Since $f_K/f_\pi \approx 1.22$ $[10]$, the constituent-quark-mass ratio from Eq. (16) becomes
\begin{equation}
1.22 \approx \frac{f_K}{f_\pi} = \frac{1}{2} \left(1 + \frac{m_s}{\hat{m}}\right) \Rightarrow \frac{m_s}{\hat{m}} \approx 1.44, \tag{17}
\end{equation}
which is independent of the value of $g$. In loop order, Eqs. (15) are recovered, along with $[8, 5]$
\begin{equation}
m_\sigma = 2m_q, \quad g = \frac{2\pi}{\sqrt{N_c}}, \quad \text{for } N_c = 3. \tag{18}
\end{equation}
Here, the first equation is the NJL relation [4], now true for the LσM as well. The second
equation in Eq. (18) was first found via the Z = 0 compositeness relation [9], separating
the elementary π and σ particles from the bound states ρ, ω, and a_1.

We first estimate the (non-chiral-limiting) nonstrange and strange constituent quark
masses from the GTRs (16), together with the LσM loop-order result (18):

\[ \hat{m} \approx g f_{\pi} \approx \frac{2\pi}{\sqrt{3}} (93 \text{ MeV}) \approx 337 \text{ MeV}, \]
\[ m_s = \left( \frac{m_s}{\hat{m}} \right) \hat{m} \approx 1.44 \hat{m} \approx 485 \text{ MeV}. \]  

These quark-mass scales in turn confirm the mass-splitting scales found in Secs. 2, 3:
\[ \delta m_{\varphi} = \delta m_{\rho} = \delta m_{\phi} \approx (485 - 337) \text{ MeV} = 148 \text{ MeV}, \]

near 140, 98, 150, and 150 MeV, respectively. Also the SU(3) non-vanishing masses are
predicted as
\[ m_{\varphi}^0 = m_{\rho}^0 \approx m_s + \hat{m} \approx 822 \text{ MeV}, \]
\[ m_{\phi}^0 = m_{\phi}^0 \approx m_s + 2\hat{m} \approx 1160 \text{ MeV}, \]

near the 850, 1151, and 1232 MeV m^0 masses in Secs. 2, 3.

5. \( \mathcal{S} \) SCALARS AND ACCIDENTAL DEGENERACIES

An almost degenerate case in the nonrelativistic quark model (NRQM) is [10], in the
context of QCD.

\[ m_{\varphi} \approx m_{\varphi} - \frac{2\alpha_{\text{eff}}}{m^2_{\text{dyn}}} \left( \begin{array}{c} L \cdot S \\ r^3 \end{array} \right) = 780 \text{ MeV} - 140 \text{ MeV} = 640 \text{ MeV}, \]

where the ground-state vector mesons have L = 0 and so no spin-orbit contribution to
the mass. This corresponds to m_{σ(650)} \approx m_{ω(782)} - 140 \text{ MeV} = 642 \text{ MeV}. Equivalently,
invoking the I = 1/2 CGC of 1/2, one predicts via the NRQM m_{K^*(892)} \approx m_{K^*(892)} - 70
MeV = 822 \text{ MeV}. Or invoking instead the \( \bar{s}s \) CGC of 1/3, one gets m_{σ(970)} \approx m_{φ(1020)} -
47 \text{ MeV} = 973 \text{ MeV}.

However, for the elementary-particle \( \mathcal{P} \) and \( \mathcal{S} \) states, one should invoke the infinite-momentum-frame (IMF, see Appendix) scalar-pseudoscalar SU(3) equal-splitting laws
(ESLs), reading [11]

\[ m_{\sigma}^2 - m_{\pi}^2 \approx m_{K}^2 - m_{\eta}^2 \approx m_{a_0}^2 - m_{\eta_{\text{avg}}}^2 \approx 0.40 \text{ GeV}^2, \]

where \( m_{\eta_{\text{avg}}} \) is the average \( \eta, \eta' \) mass 753 MeV. These ESLs hold for m_{σ(650)} = 2\hat{m}
and m_{K(800)} = 2\sqrt{m_s \hat{m}} = 809 \text{ MeV}, the NJL-LσM values. Using the ESLs (23) to predict the
a_0 mass, one finds
\[ m_{a_0} = \sqrt{0.40 \text{ GeV}^2 + m_{\eta_{\text{avg}}}^2} \approx 983.4 \text{ MeV}, \]
very close to the PDG value $984.7 \pm 1.2$ MeV. Thus, the nearness of the $a_0(980)$ and $f_0(980)$ masses, the latter scalar being mostly $\bar{s}s$ and so near the vector $\bar{s}s \phi(1020)$ (see above), is indeed an accidental degeneracy. Note that a similar (approximate) degeneracy is found in the dynamical unitarized quark-meson model of Ref. [12], where the same $\bar{q}q$ assignments are employed as here.

6. SUMMARY AND CONCLUSIONS

The usual field-theory picture is that meson masses should appear quadratically and baryon masses linearly in Lagrangian models based on the Klein–Gordon and Dirac equations. However, in Secs. 2 and 3 we have studied both mesons and baryons in a linear-mass $SU(3)$-symmetry Taylor-series sense. Instead, in Sec. 5 we have studied symmetry breaking in the IMF, with $E = \left[p^2 + m^2\right]^{1/2} \approx p \left[1 + m^2/2p^2 + \ldots\right]$. Here, between brackets, the 1 indicates the symmetry limit, and the quadratic mass term means that both meson and baryon masses are $squared$ in the mass-breaking IMF for $\Delta S=1$ ESLs. While the former mass-splitting approach (with linear masses) fits all $\mathcal{V}, \mathcal{A}, \mathcal{F}, \mathcal{G}$, and $\mathcal{P}$ ground-state $SU(3)$-flavor multiplets, so does the latter (with quadratic masses) for the IMF-ESLs. Nevertheless, Nambu–Goldstone pseudoscalars $\mathcal{P}$ always involve quadratic masses. Both approaches suggest that all ground-state mesons ($\mathcal{P}, \mathcal{F}, \mathcal{V}, \mathcal{A}$) are $\bar{q}q$ states, while baryons ($\mathcal{G}, \mathcal{D}$) are $qqq$ states. This picture is manifest in the quark-level $L\sigma M$ of Sec. 4. Finally, the accidental scalar degeneracy between the $\bar{s}s$ $f_0(980)$ and the $\bar{n}n$ $a_0(980)$ was explained in Sec. 5, via the IMF quadratic-mass ESLs — also compatible with mesons being $\bar{q}q$ and baryons $qqq$ states.

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A. KINEMATIC INFINITE-MOMENTUM FRAME

The infinite-momentum frame (IMF) has two virtues: (i) $E = \left[p^2 + m^2\right]^{1/2} \approx p + m^2/2p + \ldots$, for $p \to \infty$, requires $squared$ masses when the lead term $p$ is eliminated, using $SU(3)$ formulae with coefficients $1+3 = 2+2$, as e.g. the Gell-Mann–Okubo linear mass formula $\Sigma + 3\Lambda = 2N + 2\Xi$, valid to 3%; (ii) when $p \to \infty$, dynamical tadpole graphs are suppressed [13]. In fact, $\Sigma^2 + 3\Lambda^2 = 2N^2 + 2\Xi^2$ is also valid empirically to 3%. This squared $qqq$ baryon mass formula can be interpreted as a $\Delta S=1$ ESL, which holds for
both $\mathcal{O}$ and $\mathcal{D}$ baryons [11]:

\[
\begin{align*}
\Sigma\Lambda - N^2 &\approx \Xi^2 - \Sigma\Lambda \approx \frac{1}{2} (\Xi^2 - N^2) \approx 0.43 \text{ GeV}^2, \\
\Sigma^2 - \Delta^2 &\approx \Xi^2 - \Sigma^2 \approx \Omega^2 - \Xi^2 \approx \frac{1}{2} (\Omega^2 - \Sigma^2) \approx 0.43 \text{ GeV}^2.
\end{align*}
\]

(25)

However, the $\bar{q}q$ pseudoscalar and vector $\Delta S = 1$ ESLs have about one half this scale (also empirically valid to 3%), viz.

\[
m^2_K - m^2_\pi \approx m^2_K - m^2_\rho \approx m^2_\phi - m^2_\rho \approx \frac{1}{2} (m^2_\phi - m^2_\rho) \approx 0.22 \text{ GeV}^2,
\]

(26)
as roughly do the $\bar{q}q$ scalars found in Sec. 2, i.e.,

\[
m^2_{K(800)} - m^2_{\sigma(650)} \approx m^2_{\sigma(970)} - m^2_{K(800)} \approx 0.22 - 0.30 \text{ GeV}^2.
\]

(27)

This approximate factor of 2 between Eqs. (25) and Eqs. (26,27) is because there are two $\Delta S = 1$ $qqq$ transitions, whereas there is only one $\Delta S = 1$ transition for $\bar{q}q$ configurations.

So if we take Eq. (27) as physically meaningful, we may write

\[
2m^2_\kappa \approx 2m^2_{\sigma(600)} + m^2_{f_0(980)} \approx m^2_{\sigma(650)} + m^2_{\sigma(970)} \approx 1.32 - 1.36 \text{ GeV}^2,
\]

(28)
yielding $m_\kappa \approx 819$ MeV close to experiment, which again suggests these scalars are $\bar{q}q$ states.

These IMF quadratic mass schemes, along with the NJL-$\sigma$M $\kappa$ mass $m_{K(800)} = 2\sqrt{m_\kappa m} = 809$ MeV, again suggest (as do the empirical scales of Eqs. (26) and (27) vs. Eqs. (25)) that all ground-state meson nonets are $\bar{q}q$, whereas the baryon octet and decuplet are $qqq$ states.

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