Calculation of the protective current density distribution of a cathodic protection system with galvanic anodes in terms of double-layer electrolyte

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Abstract

This paper considers the effect of the discontinuity of electrolyte electrical conductivity on the distribution of potential and protective current density in cathodic protection systems with galvanic anodes. Its aim is to present a simultaneous application of both analytical and numerical models for the calculation of the distribution of protective potential and current density in the cases of homogeneous and double-layer electrolyte. For non-linear boundary conditions at the electrode surface (secondary distribution of protective current density), the indirect boundary element method is used, because of the complexity of calculation for which collocation at the point method was used. The calculation is further complicated due to the nonlinearity of boundary conditions at the electrode surfaces. In order to show the importance of this analysis, calculations for the observed system as well as of errors caused by neglecting the boundary discontinuity of the soil conductivity are provided. Based on these calculations and error analyses, the impact of the double-layer electrolyte on the correct calculation of the cathodic protection system with galvanic anodes is evaluated. This paper also provides the analysis when the double-layered nature of the electrolyte can be practically ignored, which is of great importance to designers of these systems.

Keywords: cathodic protection, protection current density, single-layer electrolyte, double-layer electrolyte.
1 Introduction

Cathodic protection (CP) is a technique that prevents corrosion of underground metallic structures. The factors that may affect the parameters of the modern CP systems are indispensable data for project documentation. They describe a multi-disciplinary phenomenon that complicates the definition of CP system parameters. The potential distribution of anode strings is determined for each specific case and it depends on the real polarization curves. The polarization curves, in the mathematical sense, define the boundary conditions for determining the value of the electrode potential and the current density. Calculation of the electrode potential and the current density can be done by numerical techniques. The boundary element method (BEM) is mostly used for the calculation of CP system parameters. The application of BEM for the calculation of the protective potential and the current density of long pipelines, with either larger or smaller insulation damage, is well covered by Riemer and Orazem [1]. Riemer developed a calculation method that takes into account the potential drop in the pipeline, i.e. the attenuation that is not balanced over the entire pipeline [2]. Adey et al. incorporated a full 3D model for calculating the potential of the CP system, also including neighboring objects that have connections to the anodes [3]. Peratta et al. applied the multi-layer (ML) BEM method for the calculation of the parameters of a CP system in multi-layer soil [4]. Liu and Wang proposed a line model-based fast BEM for the calculation of parameters of the cathodically protected pipelines placed in multi-layered soil [5].

The calculations of the distribution of both the potential and protective current density form the basis of a CP system project. The factors that complicate the calculation are the non-linear boundary conditions on electrode surfaces. In addition, the calculation needs to take into account the occurrence of any boundary discontinuity of electrical soil conductivity. The electrolyte, in which the metallic pipeline is buried, is mostly inhomogeneous (layered) and has two or more layers of different values of electrolyte conductivity [6]. Based on what is mentioned above, it becomes clear that the exact analytical solution of this problem is only partial and superficial.

2 Theory and implementation

This paper indicates that there is a substantial difference between methods used for determining the parameters of CP system, as well as in the treatment of boundary conditions, i.e. between:

- The exact analytical method that treats the primary distribution of protective current density, taking into account the characteristics of electrolyte and geometry of the system [7, 8], and
- The numerical method that treats the secondary distribution of protective current density, i.e. taking into account the kinetic effects that are specific to the electrode surface and result in nonlinear boundary conditions [9, 10].
2.1 Primary distribution of the protective current density

2.1.1 Homogeneous electrolytes

As previously mentioned, when modeling the primary distribution of the protective current density, only the electrical and geometrical characteristics of corrosive elements are relevant. In this section of the paper, the authors analyze an example of galvanic anode placed in a homogeneous electrolyte. If the DC source is installed as galvanic anode at some location in the electrolyte with coordinates \((x', y', z')\), then the potential \(\phi\) at any point \(P\) in the electrolyte with coordinates \((x, y, z)\), at distance \(r\), can be defined as:

\[
\phi(x, y, z) = \frac{1}{4\pi\gamma_u} \int_{S} \frac{J(x', y', z')}{r} \, dS, \tag{1}
\]

where: \(\gamma_u\) is the electric conductivity of the electrolyte, \(J(x, y, z)\) the surface density of the protective current, \(r\) the distance between the source and observation points and \(dS\) the integration path.

In order to simplify the mathematical model, the observed anode string, which is shown in Fig. 1, is equivalented with one anode, having total length equal to the sum of the lengths of individual anodes. When solving the primary distribution of the protective current density of CP systems with galvanic anodes, it is necessary to add a set of equations that take into account the boundary conditions at the interface between anode strings and the electrolyte. The simplest case is the one that considers the potential at the complete outer surface of the anode string as a constant value (this condition can only be used in the primary distribution because...
of neglecting the polarization on the surfaces of the galvanic anodes). Similarly, the potential change in the electrolyte is constant as well. The boundary conditions can be written as:

\[ \varphi = \text{const} \]  \hspace{1cm} (2.a)

\[ \frac{\partial \varphi}{\partial \mathbf{n}} = \text{const} \]  \hspace{1cm} (2.b)

\[ \mathbf{n} \times \left( \frac{\mathbf{j}_u}{\gamma_u} - \frac{\mathbf{j}_a}{\gamma_a} \right) = 0 \]  \hspace{1cm} (2.c)

\[ \mathbf{n} \times \left( \mathbf{j}_u - \mathbf{j}_a \right) = 0 \]  \hspace{1cm} (2.d)

where \( \mathbf{n} \) is the normal unit vector on the boundary surfaces, \( \gamma_a \) the electrical conductivity of air, \( \mathbf{j}_a \) the current density in air at discontinuity boundary of electrical conductivity, \( \mathbf{j}_u \) the current density in electrolyte at the boundary of electrical conductivity discontinuity.

Because of its low electrical conductivity (\( 10^{-18} \) S/m), the air is treated as an insulator. Therefore, the mathematical analysis of the problem considers the mirror source at a distance \( h \) away from the electrolyte/air boundary, with the same protective current density as the original, placed in the electrolyte (Fig. 2(a)).

![Figure 2: Equivalent anode in (a) single-layer electrolyte and (b) double-layer electrolyte.](image)
expression for calculating the potential in the electrolyte, in the vicinity of the anodic string, is:

$$
\phi = \frac{I}{4\pi \gamma_u} \left( \ln \frac{z + l + h + \sqrt{(z + l + h)^2 + r^2}}{z - l - h + \sqrt{(z - l - h)^2 + r^2}} + \ln \frac{z - h + \sqrt{(z - h)^2 + r^2}}{z + h + \sqrt{(z + h)^2 + r^2}} \right)
$$

where: $l$ is the total length of the equivalent anode string, $h$ the depth of burying anode string, $I$ the total current intensity and $r$ can be expressed as follows:

$$
r = x^2 + y^2.
$$

### 2.1.2 Double-layer electrolytes

Soil as the electrolyte is mainly composed of more layers, which differ in chemical and geological composition. When expecting a large difference in conductivity of the electrolyte layers, it is advisable to use a double-layer model of the electrolyte. In a case of simple geometry of electrodes, such as those considered in the paper, the easiest way to find a solution is by using a method of images.

In order to estimate the exact scalar value of the electric potential of the equivalent anodic strings, it is necessary to determine the position of equivalent galvanic strings, as well as their current densities. Furthermore, it is necessary to set appropriate boundary conditions. The boundary conditions applicable in this situation are almost identical to the boundary conditions for a homogeneous electrolyte. The equivalent anode string, as well as its images, has constant potential, the potential change within each medium is constant and on all the boundaries between the media boundary conditions given by eqns (2.c) and (2.d) apply. The double-layer model considers high-order mirror images that have a direct impact on the potential of the equivalent anode (Fig. 2(b)). The impact of the high-order image attenuates as the distance between the galvanic string and image increases, i.e. as the order of the image increases. Therefore, when calculating the potential of the anode surface, it is good enough to consider second order images. The effects of mirror images can be simply calculated as a sum of effects of individual images. The easiest way of calculating the potential of the equivalent galvanic string, buried in the double-layer electrolyte, is by using the following relations:

$$
\phi(x, y, z) = \frac{I}{4\pi \gamma_u} \left[ f_1 + \alpha f_2 + \beta f_3 + \alpha\beta(f_4 + f_5) \right],
$$

where:

$$
\alpha \approx 1
$$

$$
\beta = \frac{\gamma_u - \gamma_i}{\gamma_u + \gamma_i}
$$

$$
f_1 = \ln \frac{z + l + h + \sqrt{(z + l + h)^2 + r^2}}{z - l - h + \sqrt{(z - l - h)^2 + r^2}}
$$
where \( \gamma_l \) is the soil conductivity of the lower layer of double layer soil and \( d \) the distance between anode and lower layer of electrolyte.

The calculation results of the potential distribution in the vicinity of the anode string, according to eqns (3) and (5), are presented below. The total length of the equivalent anode string is 6 m and it is buried in a single-layer of soil (Fig. 3(a)) with soil resistivity \( \rho_u = 50 \ \Omega \text{m} \). In the second example (Fig. 3(b)), the anode string, with the same characteristics, is buried in the upper layer of double-layer soil. The soil resistivity of the upper layer is \( \rho_u = 50 \ \Omega \text{m} \) and that of the lower layer \( \rho_l = 1000 \ \Omega \text{m} \). The total thickness of the upper layer is 10 m and the depth of anode string burial is 2.8 m. When modeling this example, second order images are taken into account.

\[
f_2 = \ln \frac{z - h + \sqrt{(z - h)^2 + r^2}}{z + h + \sqrt{(z + h)^2 + r^2}} \quad (5.d)
\]

\[
f_3 = \ln \frac{z + 2l + h + 2d + \sqrt{(z + 2l + h + 2d)^2 + r^2}}{z - 2l - h - 2d + \sqrt{(z - 2l - h - 2d)^2 + r^2}} \quad (5.e)
\]

\[
f_4 = \ln \frac{z - 2d - h - l + \sqrt{(z - 2d - h - l)^2 + r^2}}{z + 2d + h + l + \sqrt{(z + 2d + h + l)^2 + r^2}} \quad (5.f)
\]

\[
f_5 = \ln \frac{z + 3h + 3l + 2d + \sqrt{(z + 3h + 3l + 2d)^2 + r^2}}{z - 3h - 3l - 2d + \sqrt{(z - 3h - 3l - 2d)^2 + r^2}}, \quad (5.g)
\]
2.2 Secondary distribution of protective current density

When calculating the secondary distribution of the protective current density, the kinetics of the electrode processes must be taken into account. Processes of hydrogen separation, dissolving of metal and oxygen reduction, simultaneously occur on the cathode surface. As a result of electrochemical reactions, the expression for the polarization current density on the cathode surface can be written as:

\[ j_C = j_{0,Fe} \times 10^{-\frac{\phi - \phi_{Fe}}{\beta_{Fe}}} - j_{lim,O_2} \left( 1 + 10^{-\frac{\phi - \phi_{O_2}}{\beta_{O_2}}} \right)^{-1} - j_{0,H_2} \times 10^{-\frac{\phi - \phi_{H_2}}{\beta_{H_2}}}, \]  

(7)

where \( j_C \) is the sum of all partial current densities for cathode surface (bare steel), \( j_{0,Fe} \) the current density corresponding to the metal dissolution reaction, \( j_{lim,O_2} \) the threshold current density of oxygen reduction, \( j_{0,H_2} \) the current density corresponding to the reaction of hydrogen separation, \( \phi_{Fe}, \phi_{O_2}, \phi_{H_2} \) are the corrosion potentials for the corresponding reactions, \( \beta_{Fe}, \beta_{O_2}, \beta_{H_2} \) are Tafel’s coefficients (slopes) and \( \phi \) is the potential difference of interface metal/electrolyte.

Galvanic anodes are made from materials that are more electrically active than the cathode materials. Therefore, the dominant reaction on the anode surface is oxygen reduction. Mathematically, this can be defined as follows:

\[ j_A = j_{lim,O_2} \left( \frac{\phi - \phi_{Zn}}{10^{-\frac{\phi - \phi_{Zn}}{\beta_{Zn}}} - 1} \right), \]  

(8)

where \( j_A \) is the current density for zinc anode, \( \phi_{Zn} \) the corrosion potential of zinc and \( \beta_{Zn} \) the Tafel’s coefficient (slope) for zinc.

The diagram in Fig. 4 presents the boundary conditions on the electrode surfaces of the CP system with galvanic anodes. It is noticeable that these are highly nonlinear boundary conditions, especially on a cathode surface. The values of the protective potential of the cathode surface depend on the operating time of the CP system. At the beginning, i.e. at the time of CP commissioning, the cathode and anode surfaces get on the potential defined by points A and B, respectively. In this situation, the voltage drop in the electrolyte is defined by \( j_{0,Fe} \times 10^{-\frac{\phi - \phi_{Fe}}{\beta_{Fe}}} \). After some time, i.e. after polarization of steel and increase of the resistance of the anodic string, the potentials of the cathode and anode are defined by points A₁ and B₁, respectively.

2.2.1 Electrode potential and current density calculation

When analyzing CP systems, finding the values of the electrode potential and protection current density on the boundary surface is of particular interest. Therefore, BEM is an ideal method for solving this problem. Electrode potential and current density of the electrode surface of the CP system placed in single or double-layer soil can be calculated by using following integral equation:

\[ c(q) \phi(q) + \int \phi(p) \frac{\partial G(p,q)}{\partial n} \mathrm{d}S = -\rho_s \int \phi(p) G(p,q) \mathrm{d}S + \phi_s, \]  

(9)
Figure 4: Polarization diagram of galvanic element steel/zinc (vs. Cu/CuSO₄).

where \( p \) is the field source point, \( q \) the observation point, \( c(q) \) a constant, \( j(p) \) the surface current density, \( G(p,q) \) the Green’s function, \( \phi(q) \) the electric potential at observation point, \( \phi(p) \) the electric potential at source point, \( \phi_\infty \) is a constant potential on the boundary at infinity, \( \rho_u \) the soil resistivity of upper layer and \( S \) the boundary surface.

The corresponding Green’s functions are:

- For a homogeneous soil:

\[
G(p,q) = \frac{1}{4\pi} \left( \frac{1}{r_{p1,q}} + \frac{1}{r_{p2,q}} \right),
\]

(10)

- For double-layer soil:

\[
G(p,q) = \frac{1}{4\pi} \left[ \frac{1}{r_{p1,q}} + \frac{1}{r_{p2,q}} + \sum_{n=1}^{k} \beta^n \left( \frac{1}{r_{p3,q}} + \frac{1}{r_{p4,q}} + \frac{1}{r_{p5,q}} + \frac{1}{r_{p6,q}} \right) \right],
\]

(11)

where \( r_{pi,q} \) is the distance between the \( i \)-th field source and observation points. The first field source point represents the original one while the others represent the images of the field source point.

Eqn (11) is the Green’s function for double-layer soil when both source and observation points are in the upper layer. For the other situations, such as when the source and observation points are in different layers or when they are both in the lower layer of double-layer soil, Green’s function has a different form that can be found in Berberovic et al. [11].

Integral eqn (9) is solved using the direct boundary element method. The electrode surfaces are discretized using 2D biquadratic boundary elements. The collocation method is applied at a point, so that the weight function is a Dirac
delta function. Applying the collocation method at the point, an integral expression for a boundary element can be written as:

\[
c(q_i)\phi(q_i) + \sum_{e=1}^{n_e} \sum_{i=1}^{9} \left[ \phi(p)N_i(\xi, \eta) \right] \det[J(\xi, \eta)] \frac{\partial G(\xi, \eta; q)}{\partial n} d\xi d\eta
\]

(12)

\[
= -\rho_u \sum_{e=1}^{n_e} \sum_{i=1}^{9} \left[ j_i(p)N_i(\xi, \eta) \right] \det[J(\xi, \eta)] G(\xi, \eta; q) d\xi d\eta,
\]

where \(N(\xi, \eta)\) is the shape function, \(\xi, \eta\) are Gauss points, \(\det[J(\xi, \eta)]\) is the determinant of the Jacobean matrix and \(n_e\) the number of boundary elements.

In order to solve this system of equations, it is necessary to add another integral equation [12], as follows:

\[
\int_S j(p) dS = 0.
\]

(13)

After the application of the collocation method at a point to eqn (13), the latter can be written:

\[
\sum_{e=1}^{n_e} \sum_{i=1}^{9} \left[ j_i(p)N_i(\xi, \eta) \right] \det[J(\xi, \eta)] d\xi d\eta = 0.
\]

(14)

The previously obtained systems of eqns (12) and (14) can be expressed in matrix form [2]:

\[
\begin{bmatrix}
H_{aa} & H_{ac} & -1 \\
H_{ca} & H_{cc} & -1 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\phi_a \\
\phi_c \\
\phi_e
\end{bmatrix}
= \begin{bmatrix}
G_{aa} & G_{ac} \\
G_{ca} & G_{cc} \\
A_c & A_c
\end{bmatrix}
\begin{bmatrix}
j_a \\
j_c
\end{bmatrix},
\]

(15)

where \([H]\) and \([G]\) are matrices of coefficients, \(\{\phi\}\) is the column vector of potential and \(\{j\}\) the column vector of current density. In eqn (15), indices \(c\) and \(a\) represent cathode and anode surface, respectively.

To solve eqn (15) it is necessary to apply an iterative solution technique due to the nonlinear boundary conditions. By solving this system of eqn (15), the values of the protection current density and potential on the electrode surfaces can be obtained.

2.2.2 Calculation of the potential on the soil surface

The mathematical model presented in the previous section is suitable for the calculation of electric potential and current density distribution on the electrode surfaces of the CP system. For the calculation of the electric potential distribution on the soil surface, Fredholm’s integral equation of the first kind can be applied:

\[
\varphi(q) = \rho_u \int_S j(p)G(p, q) dS
\]

(16)

Fredholm’s integral equations of the first kind are solved using the indirect boundary element method. Integral equation (16) is solved in almost identical way as the integral equations (9) and (13). After the collocation method is applied at a point, eqn (16) can be rewritten in the following form:

\[
\varphi(q_i) = \rho_u \sum_{e=1}^{n_e} \sum_{i=1}^{9} [j_i(p)N_i(\xi, \eta)] \det[J(\xi, \eta)] G(\xi, \eta; q) d\xi d\eta
\]

(17)
The eqn (17) can be expressed in matrix form:

\[
\begin{bmatrix}
G_{as1} & G_{cs1} \\
G_{as2} & G_{cs2} \\
G_{as3} & G_{cs3}
\end{bmatrix}
\begin{bmatrix}
j_a \\
j_c
\end{bmatrix}
= 
\begin{bmatrix}
\phi_{s1} \\
\phi_{s2} \\
\phi_{s3}
\end{bmatrix},
\] (18)

where index \( si \) represents the \( i \)-th colocation point on the soil surface.

As can be noted in eqn (18), for the calculation of the electric potential distribution on the soil surface, the current density distribution on the electrode surface must be known.

Fig. 5 shows the distribution of the protective potential of the CP system with galvanic anodes of the non-isolated steel pipeline, for homogeneous soil with soil resistivity \( \rho_u = 50 \ \Omega m \). Fig. 6 shows the same CP system in double-layer soil with a soil resistivity of the upper layer \( \rho_u = 50 \ \Omega m \), thickness of the upper layer \( h = 10 \ m \) and soil resistivity of the lower layer \( \rho_l = 1000 \ \Omega m \). The pipeline has a diameter of 4 m, and is buried at a depth of 3.75 m. Anode strings are placed on both sides of the pipeline at a distance of 5.5 m away from the pipeline axis. The burial depth of the anode strings is 2.8 m, and its length is 6 m.

Figure 5: Potential distribution of CP system in single-layer soil.

Figure 6: Potential distribution of CP system in double-layer soil.
Fig. 7 provides a comparison of protective potential values for single-layer and double-layer soil. It can be noticed that the absolute value of the protective potential increases as the electrical resistivity of the lower layer increases. In such situations, neglecting the multiple layers of soil leads to overprotection of a protected object and high financial cost.

![Figure 7: Comparison of the equipotential lines of the CP system for cases of single-layer and double-layer soil for x = 12.5 m; 0 < y < 25 m.](image)

Fig. 8(a) gives the value of the percentage error of the potential, in the case of neglecting multiple layers of soil. It is noticeable that with the increase of soil resistivity, the percentage systematic error also increases, by over 40%. In these situations, from a technical point of view, neglecting multiple layers has a positive effect, because an object becomes overprotected. On the other hand, from an economic point of view, the cost of equipment increases. Therefore, for the large differences in the values of soil resistivity, between neighboring layers, it is necessary to take into account the multiple layers of electrolyte. Fig. 8(b) shows how the percentage systematic error changes with the change of thickness of the upper layer (the layer in which protected object and anode strings are placed). The considered double-layer soil had an upper layer with variable thickness and soil resistivity $\rho_u = 50 \, \Omega\text{m}$, while the soil resistivity of the lower layer is $\rho_l = 1000 \, \Omega\text{m}$ in the first case, and $\rho_l = 500 \, \Omega\text{m}$ in the second case. In both situations it is noticeable that the error decreases with increasing thickness of the upper layer of soil.

3 Conclusion

The main task in determining the optimal CP system with galvanic anodes is to define technically the correct distribution of the protective current density and potential. Usually, the static current field of the CP system is calculated by solving
the integral field equation. When applying the CP system to underground steel structures, geometries as well as nonlinear mathematical interpretation of electrochemical reactions are very complex. Therefore, the integral field equation can only be solved using numerical techniques. Also, considering the fact that the electrolyte is inhomogeneous, the paper analyzes the effect of discontinuity of electrolyte conductivity on the distribution of the protective potential of the CP system with galvanic anodes. This problem has been solved using the indirect boundary element method. The paper also presents an analysis of the systematic errors that appear when neglecting the fact that the electrolyte is composed of more
layers. It turned out that errors in the protective potential calculation can occur and can exceed 40%. Also, in a case of double-layer soil with an upper layer of large thickness, the soil can be modeled as single-layered. In this case, the soil resistivity of a single-layer is equal to the soil resistivity of the upper layer in a double-layer model. This conclusion follows from the fact that the percentage systematic error decreases with increasing thickness of the upper layer.

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