Local Cascade Ensemble for Multivariate Data Classification

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Abstract

We present LCE, a Local Cascade Ensemble for traditional (tabular) multivariate data classification, and its extension LCEM for Multivariate Time Series (MTS) classification. LCE is a new hybrid ensemble method that combines an explicit boosting-bagging approach to handle the usual bias-variance tradeoff faced by machine learning models and an implicit divide-and-conquer approach to individualize classifier errors on different parts of the training data. Our evaluation firstly shows that the hybrid ensemble method LCE outperforms the state-of-the-art classifiers on the UCI datasets and that LCEM outperforms the state-of-the-art MTS classifiers on the UEA datasets. Furthermore, LCEM provides explainability by design and manifests robust performance when faced with challenges arising from continuous data collection (different MTS length, missing data and noise).

1 Introduction

The prevalent deployment and usage of sensors in a wide range of sectors generate an abundance of multivariate data which have proven to be instrumental for researches, businesses and policies. In particular, multivariate data that integrates temporal evolution has received significant interests over the past decade, driven by automatic and high-resolution monitoring applications (e.g. healthcare \cite{23}, mobility \cite{20}, natural disasters \cite{13}). The rising importance of multivariate data analysis reflects the growing complexity of the world, as more and more factors influence the decision-making processes.

In our study, we address the issue of multivariate data classification. The objective is to learn the relationship between a multivariate sample and its label. More specifically, we study the traditional (tabular) multivariate data classification and the Multivariate Time Series (MTS) classification settings. A time series is a sequence of real values ordered according to time; and when a set of co-evolving time series are recorded simultaneously by a set of sensors, it is called a MTS.

To undertake the task of the general multivariate classification, no single classifier can claim to be superior to any of the others \cite{41} (known as the “No Free Lunch theorem”). Thus, the combination of different classifiers - an ensemble method - is often considered a good method to obtain a better generalizing classifier. There are three main reasons that justify the use of ensembles over single classifiers \cite{10}: statistical (reduce the risk of choosing the wrong classifier by averaging when the amount of training data available is too small compared to the size of the hypothesis space), computational (local search from many different starting points may provide a better approximation to the true unknown function than any of the individual classifier), and representational (expansion of the space of representable functions).

Ensemble methods are the current state-of-the-art classifiers for traditional multivariate data classification (Random Forest \cite{6}, Extreme Gradient Boosting \cite{8}) as well as for univariate time series
classification (HIVE-COTE [24]). However, there is no ensemble method among the state-of-the-art MTS classifiers. Hence, we propose a new ensemble method for multivariate data classification that would perform well on both traditional multivariate data and MTS. In addition to meeting the performance requirement, our approach is robust with varying MTS input data quality (different MTS length, missing data and noise), which often arises in continuous data collection systems. Moreover, modern machine learning methods are required to provide some explanations of their decisions [17], which is particularly difficult for ensemble methods and for multivariate time series. Our approach also tackles this challenge and proposes an explainable method for MTS classification.

The construction of an ensemble method involves combining accurate and diverse individual classifiers. There are two complementary ways to generate diverse classifiers. First, they can be generated through learning different parts of the training data [27]. However, this approach does not ensure a bias-variance tradeoff. Second, classifiers diversity can also be emphasized through the creation of different training sets by probabilistically changing the distribution of the original training data [37]. Within this approach, there are two well-known methods that modify the distribution of the original training data with complementary effects on the bias-variance tradeoff: bagging [5] (variance reduction) and boosting [24] (bias reduction). Nevertheless, a combination of bagging and boosting does not benefit from the diversification of learning different parts of the training data. An ensemble method which adopts these two ways to generate diverse classifiers is called an hybrid ensemble method. As far as we have seen, we have developed in [14] the first hybrid ensemble method (Local Cascade Ensemble - LCE). However, [14] does not show how LCE behaves on public (e.g. UCI) multivariate datasets since it was only applied on a proprietary dataset. In addition, [14] did not address the challenges MTS classification faces (explainability and varying input data quality - different TS length, missing data, noise).

In this paper, we present in detail and thoroughly examine the behavior of LCE, and its extension to MTS classification, LCEM. The new hybrid ensemble method combines a boosting-bagging approach to handle the bias-variance tradeoff and a divide-and-conquer approach to learn different parts of the training data. Our study shows that:

- LCE outperforms the state-of-the-art classifiers on the UCI datasets [11];
- LCEM outperforms the state-of-the-art MTS classifiers on the UEA datasets [1];
- LCEM provides explainability by design through identifying the time window used to classify the whole MTS;
- LCEM manifests robust performance when faced with challenges arising from continuous data collection (different MTS length, missing data and noise).

2 Background and Related Work

In this section we first introduce the background of our study. Then, we present the state-of-the-art classification methods on which we position our algorithm and we end with a similar presentation for MTS.

2.1 Background

We address the issue of supervised learning for classification. Classification consists in learning a function that maps an input data to its label: given an input space $\mathcal{X}$, an output space $\mathcal{Y}$, an unknown distribution $P$ over $\mathcal{X} \times \mathcal{Y}$, a training set sampled from $P$, compute a function $h^*$ such as:

$$h^* = \arg \min_h \mathbb{E}_{(x,y) \sim P} [h(x) \neq y]$$
Our classifier is based on a new way to handle the bias-variance tradeoff in ensemble methods. The bias-variance tradeoff defines the capacity of the learning algorithm to generalize beyond the training set. The bias is the component of the classification error that results from systematic errors of the learning algorithm. A high bias means that the learning algorithm is not able to capture the underlying structure of the training set (underfitting). The variance measures the sensitivity of the learning algorithm to changes in the training set. A high variance means that the algorithm is learning too closely the training set (overfitting). The objective is to minimize both the bias and variance.

We perform classification on two types of datasets: traditional (tabular) multivariate data and MTS. In the traditional multivariate data setting, in contrast to the MTS one, there is no explicit relationship among records or variables and every record has the same set of variables (also called attributes or dimensions). A multivariate time series (MTS) \( M = \{ x_1, ..., x_d \} \in \mathbb{R}^{d \times l} \) is an ordered sequence of \( d \in \mathbb{N} \) streams with \( x_i = (x_{i,1}, ..., x_{i,l}) \), where \( l \) is the length of the time series and \( d \) is the number of multivariate dimensions. We address MTS generated from automatic sensors with a fixed and synchronized sampling along all dimensions. An example of a MTS dataset is given at the top of Fig 2. This dataset contains \( n \) MTS with 2 dimensions and a length of 5.

### 2.2 Classification

In machine learning, the most popular (and often best performing) classifiers belong to the following classes: k-nearest neighbors, regularized logistic regressions, support vector machines, neural networks and ensemble methods. As previously discussed, ensemble methods are usually well generalizing classifiers and thus, we position our contributions into this class. The other classes constitute our competitors and the algorithms evaluated are presented in section 4.2.

Ensemble methods are structured around two approaches (explicit, implicit) which have their own strengths and limitations. Therefore a hybrid ensemble method is encouraged [27]. The implicit approach involves creating diverse classifiers on the original training data. There are two methods adopting an implicit approach: Mixture of Experts (ME) [19] and Negative Correlation Learning (NCL) [25]. ME uses a divide-and-conquer algorithm to split the problem space, and each individual classifier learns a part of the training data. The advantage of this method is that each individual classifier is concerned with its own individual error. However, individual classifiers are trained independently so there is no control over the bias-variance tradeoff. NCL is an ensemble method which is trained on the entire training data simultaneously and interactively to adjust the bias-variance tradeoff. Individual classifiers interact through the correlation penalty terms of their error functions. The correlation penalty term is a regularization term that is integrated into the error function of each individual classifier. This term quantifies the amount of error correlation and is minimized during the training, which leads to negatively correlated individual classifiers and balances the bias-variance tradeoff. The disadvantage of this method is that each classifier is concerned with the whole ensemble error due to the training of each classifier on the same data. Some studies combine NCL and ME features to address their limitations [16, 28]. [28] proposed an augmented version of ME by integrating a regularization term in the error function (NCL) to better balance the bias-variance tradeoff.

However, a combination of implicit approaches does not benefit from the diversification of generating classifiers by perturbing the distribution of the original training data (explicit approach). There are two methods adopting an explicit approach with complementary effects on the bias-variance tradeoff (bagging [5] - variance reduction, boosting [34] - bias reduction). Bagging is a method for generating multiple versions of a predictor (bootstrap replicates) and using these to get an aggregated predictor. Boosting is a method for iteratively learning weak classifiers and adding them to create a final strong classifier. After a weak learner is added, the data weights are readjusted, allowing future weak learners to focus more on the examples that previous weak learners misclassified. Bagging and boosting methods have been combined [22] but without integrating the diversification benefit of an implicit approach.

There is a study which combines the explicit boosting method with the implicit ME divide-and-conquer principle [12]. Nonetheless, the only bias reduction distribution change of boosting does not ensure a bias-variance tradeoff. Therefore, we propose a hybrid ensemble method called Local Cascade
Ensemble (LCE). LCE combines an explicit boosting-bagging approach to handle the bias-variance tradeoff and an implicit divide-and-conquer approach (decision tree) to learn different parts of the training data. We detail LCE and its extension for MTS classification (LCEM) in section 3. We present in the next section the state-of-the-art MTS classifiers.

2.3 MTS Classification

We can categorize the state-of-the-art MTS classifiers into three families: similarity-based, feature-based and deep learning methods.

Similarity-based methods make use of similarity measures (e.g., Euclidean distance) to compare two MTS. Dynamic Time Warping (DTW) has been shown to be the best similarity measure to use along the k-Nearest Neighbors (k-NN) [35]. This approach is called kNN-DTW. There are two versions of kNN-DTW for MTS: dependent (DTW_D) and independent (DTW_I). Neither dominates over the other [35]. DTW_I measures the cumulative distances of all dimensions independently measured under DTW. DTW_D uses a similar calculation with single-dimensional time series; it considers the squared Euclidean cumulated distance over the multiple dimensions.

Feature-based methods include shapelets and bag-of-words (BoW) models. Shapelets models use subsequences (shapelets) to transform the original time series into a lower-dimensional space that is easier to classify. gRSF [21] and UFS [40] are the current state-of-the-art shapelets models in MTS classification. They relax the major limiting factor of the time to find discriminative subsequences in multiple dimensions (shapelet discovery) by randomly selecting shapelets. gRSF creates decision trees over randomly extracted shapelets and shows better performance than UFS on average (14 MTS datasets) [21]. On the other hand, BoW models (LPS [3], mv-ARF [39], SMTS [2] and WEASEL+MUSE [33]) convert time series into a bag of discrete words, and use a histogram of words representation to perform the classification. WEASEL+MUSE shows better results compared to gRSF, LPS, mv-ARF and SMTS on average (20 MTS datasets) [33]. WEASEL+MUSE generates a BoW representation by applying various sliding windows with different sizes on each discretized dimension (Symbolic Fourier Approximation) to capture features (unigrams, bigrams, dimension identification).

Then, deep learning methods use Long-Short Term Memory (LSTM) and/or Convolutional Neural Network (CNN) to extract latent features. [15] proposed the current state-of-the-art model (MLSTM-FCN) consisting of a LSTM layer and a stacked CNN layer along with Squeeze-and-Excitation blocks to generate latent features. MLSTM-FCN is shown to be better than WEASEL+MUSE on large datasets (relative to the one tested) on average (20 MTS datasets) [33].

Therefore, in this work we choose to evaluate the performance of LCEM in comparison to similarity-based methods results published in the UEA archive (ED, DTW_D, DTW_I) [1] and to the best-in-class for each feature-based and deep learning category (WEASEL+MUSE and MLSTM-FCN classifiers). As previously introduced, in addition to meeting the performance requirement, MTS classifiers are facing two particular challenges: the lack of explanations (explainability) supporting their predictions and the varying input data quality (different TS length, missing data, noise). First, there is no mathematical definition of explainability. A definition proposed by [29] states that the higher the explainability of a machine learning algorithm, the easier it is for someone to comprehend why certain decisions or predictions have been made. Some model-agnostic approaches exist to provide post-hoc explainability [31, 26, 18] and approximate the relative impact on algorithm predictions of the different parts of the dataset for each sample (local explainability). These approaches approximate the decision surface of a classifier using an explainable linear model. In the case of time series data, it has been shown that time windows of the time series can be used as explanatory variables of the linear model [18]. However, the explanations from the surrogate models cannot be perfectly faithful with respect to the original model [52], which is a prerequisite for numerous applications. On the other hand, some simpler models, such as decision trees, provide faithful explanations with their explainability “by design”, as the model itself is relatively easy to understand by a human. This explainability often comes at the expense of classification performance. None of the state-of-the-art MTS classifiers reconciles performance and explainability by design. Similarity-based methods are explainable by design and provide the distance...
between two MTS for each timestamp. However, the explainability of these methods remains weak as it does not indicate which part of the MTS is discriminative. Moreover, similarity-based methods are often less accurate than other MTS classification methods. WEASEL+MUSE and MLSTM-FCN classifiers show better performance than similarity-based methods but are not explainable. LCEM provides explainability by design through identifying the time window used to classify the whole MTS.

Table 1: The state-of-the-art MTS classifiers - overview.

| Similarity-Based | Deep Learning | Feature-Based | Ensemble |
|------------------|---------------|---------------|----------|
| ED               | DTW           | MLSTM-FCN     | WEASEL+MUSE | LCEM     |

Output

- Performance: ∼
- Explainability: √

Input

- Varying TS Length: √
- Missing Data: √
- Noise: √

Finally, none of the state-of-the-art MTS classifiers handles the three varying data quality aspects. Table 1 presents an overview of the challenges addressed by the state-of-the-art MTS classifiers and how we position the extension of our ensemble method for MTS (LCEM). We evaluate the classification performance of LCEM and its ability to handle the challenges MTS classification faces in section 5.2.

3 Algorithm

We first explain how the hybrid ensemble method LCE has been designed and then we present its extension for multivariate time series LCEM. Finally, we detail LCE and LCEM properties and implementations.

3.1 LCE

First of all, LCE is an improved hybrid (explicit and implicit) version of an implicit cascade generalization approach [35]: Local Cascade (LC) [16]. Among the implicit approaches, LC is one of the easiest to augment with explicit techniques. LC uses a decision tree as a divide-and-conquer method, which is compatible with the explicit bagging/boosting approaches. This criteria has motivated the choice of LC algorithm as the starting point for our hybrid ensemble method. We present in this section LC and our proposed LCE. Figure 1 illustrates the different algorithms.

LC is a combined implicit approach (negative correlation learning and mixture of experts) based on a cascade generalization [35]. Cascade generalization uses a set of classifiers sequentially and at each stage adds new attributes to the original dataset. The new attributes are derived from the class probabilities given by a classifier, called a base classifier (e.g. class probabilities $H_0(D)$, $H_1(D_{01})$ in Figure 1). The bias-variance tradeoff is obtained by negative correlation learning: at each stage of the sequence, classifiers with different behaviors are selected. It is recommended in cascade generalization to begin with a low variance algorithm to draw stable decision surfaces ($H_0$ in Figure 1) and then use a low bias algorithm to fit more complex ones ($H_1$ in Figure 1). LC [16] applies cascade generalization locally following a divide-and-conquer strategy based on mixture of experts. The objective of this approach is to capture new relationships that cannot be discovered globally. The LC divide-and-conquer method is a decision tree. When growing the tree, new attributes (class probabilities from a base classifier) are computed at each decision node and propagated down the tree. In order to be applied as a predictor, local cascade stores, in each node, the model generated by the base classifier.
As discussed in the previous sections, MTS classification has received significant interest over the past decade driven by automatic and high-resolution monitoring applications. MTS classification faces two major challenges: explainability and varying input data quality (different TS length, missing data, noise). A subset of the MTS can be characteristic of the event we aim to predict and can be adequate for the prediction. LCE can be adapted for MTS classification based solely on the discriminative part of a MTS.
and to provide explainability through the identification of this part only. In addition, our new hybrid ensemble method LCE can handle the varying input data quality challenge based on its tree-based learning. Therefore, we propose LCEM, an extended version of LCE for MTS classification. LCEM adds one parameter to LCE, the time window size, which gives the estimated size of a discriminative part of the MTS. In the following sections, we first present how dividing the time series into time windows is used to help LCEM classify MTS based on their discriminative part and then how it provides explainability.

3.2.1 MTS Dataset Transformation

In order to classify a MTS, the whole series is not always needed: only some parts may be relevant for the classification task, while others can be considered as noise and may even degrade classification performance. Therefore, we introduce a parameter to LCEM defining the time window size, i.e. the size of the subsequence of the MTS expected to be sufficient to assign a label to the MTS. We later discuss suitable methods to set this hyperparameter. LCEM is trained on subsequences of MTS, which require a transformation of the dataset. This transformation is presented in Figure 2. Using a sliding window, all subsequences corresponding to the time window size (MTS length-window size+1 subsequences) are generated. The time aspect is managed by setting the different timestamps as column dimensions. Each subsequence is considered as a new sample, labeled as the original MTS. For example in Figure 2, 4 subsequences (samples) are generated from the first MTS, composed of 2 timestamps (time window size) with 2 dimensions each (4 attributes columns). The 4 subsequence are calculated as: \(5 \text{ (MTS length)} - 2 \text{ (time window size)} + 1\). We present in the next section how we compute the classification performance with the transformed dataset and how this configuration allows a better model explainability.

![Input Dataset](image)

![Dataset Transformed](image)

Figure 2: The dataset transformation (from MTS to a traditional multivariate tabular dataset). ID is the MTS identifier; Timestamp is one element of the time series; AttributeX is the value produced by the sensor X at each timestamp; d is the number of dimensions; n is the number of MTS; T is the time series length and win_size is the time window size. In this example: \(T=5, d=2, \text{win_size}=2\).
3.2.2 Classification

As seen in the previous section, LCEM is trained on subsequences of MTS which sizes are controlled by the time window size parameter. Then, LCEM assigns class probabilities to all subsequences of the MTS. For example, on the upper part of the Figure 3, LCEM assigns class probabilities for each of the 4 subsequences of a MTS. Finally, LCEM determines the class of a MTS based on the subsequence on which it is the most confident. For each MTS, the maximum class probability over the different subsequences is selected to determine the whole MTS classification output. For example, on the lower part of Figure 3, we can observe that LCEM assigns the class 1 to the first MTS (ID=1) based on the highest class probability (0.95 versus 0.6 and 0.7) obtained with the classification of the third subsequence of the MTS. In the case where LCEM is the most confident for a subsequence of a MTS which is not discriminative, it means that the time window size value is not suited for the classification problem and it would lead to poor classification accuracy of LCEM on the training set. A time window size better suited for the classification problem would lead to better accuracy on the training set and would therefore be selected. In our evaluation, without having prior knowledge on the time window size which would suit the classification tasks, we set the time window size by hyperparameter optimization (see section 4.3). The transformation presented and the performance evaluation procedure allow any classifier to perform MTS classification. Therefore, we compare in section 5.2.1 the performance of LCEM to the best two state-of-the-art classifiers applying the same transformation as LCE and to the state-of-the-art MTS classifiers.

![Figure 3: LCEM prediction computation on the example from Figure 2 and illustration of the explainability on the first MTS (ID=1).](image)

3.2.3 Explainability

LCEM provides local explainability by design through the identification of the time window used to classify a MTS. Following the dataset transformation performed (see section 3.2.1), we obtain the class probabilities for every subsequences from LCEM. As mentioned, a subset of the MTS can be characteristic of the event we aim to predict and can be adequate for the prediction. Therefore, our prediction for a MTS is based on the subsequence that has the highest class probability - the subsequence on which LCEM is the most confident. We illustrate the explainability of LCEM with the previous section example in Figure 3. We observe that for the first MTS (ID=1), after performing a
grouping by MTS ID and taking the maximum, class 1 has the highest probability (0.95). We can trace back to the subsequence from which LCEM is predicting this class probability (third subsequence), and show it to the user. This subsequence can help the user to understand why the MTS classifier attributed a particular label to the whole MTS (explainability). We further illustrate the explainability property of LCEM in section 5.2.2 on a synthetic and two UEA datasets.

3.3 Properties

In addition to its explainability by design, LCEM has other interesting properties: phase invariance, interplay of dimensions, different MTS length compatibility, missing data management, noise robustness and scalability.

- **Phase Invariance**: LCEM is not sensitive to the position of the discriminative subsequence in the MTS due to the selection of the subsequence which has the highest class probability to classify the whole MTS. This property improves the generalization ability of the algorithm: in the possible cases when the sequences of events in a MTS change, the classification result is not modified. For example, the classification result would be the same if the discriminative subsequence appears at the beginning or at the end of the MTS;

- **Interplay of Dimensions**: LCEM exploits the relationships among the dimensions through the use of boosting-based classifier as base classifier. It allows LCEM to exploit complex interactions among dimensions at different timestamps to perform classification;

- **Different MTS Length Compatibility**: LCEM handles it in two different ways. If a MTS length is inferior to the maximum length of the MTS in a dataset multiplied by the window size selected, LCEM uses padding of 0 values. Otherwise, no padding is necessary, less samples are generated per MTS but the performance evaluation procedure presented in 3.2.2 remains valid;

- **Missing Data Management**: LCEM naturally handles missing data through its tree-based learning [7]. Similar to extreme gradient boosting [8], LCEM excludes missing values for the split and uses block propagation. During a node split, block propagation sends all samples with missing data to the side minimizing the error. We evaluate this property in our experiments in section 5.2.3;

- **Noise Robustness**: the bagging component of LCEM provides noise robustness through variance reduction by creating multiple predictors from random sampling with replacement of the original dataset. We discuss this property in our experiments in section 5.2.4;

- **Scalability**: as a tree-based ensemble method, LCEM is scalable. Its time complexity is detailed in section 3.4.

Most of the properties of LCEM are coming from LCE. The properties shared between LCE and LCEM are interplay of dimensions, missing data management, noise robustness and scalability.

3.4 Time Complexity

LCE time complexity is determined by the time complexity of multiple decision trees learning and extreme gradient boosting. The time complexity of building a single tree is $O(n(wd)D_t)$, where $n$ is the number of samples after the dataset transformation, $w$ is the time window size, $d$ is the number of dimensions and $D_t$ is the maximum depth of the tree. So the time complexity of creating multiple decision trees with bagging is $O(N_t n(wd)D_t)$, where $N_t$ is the number of trees. Extreme gradient boosting has a time complexity of $O(N_b D_b \|x\|_0 \log(n))$ where $N_b$ is the number of trees, $D_b$ is the maximum depth of the trees and $\|x\|_0$ is the number of non-missing entries in the data. Therefore, LCE has a time complexity of $O(N_t n(wd)D_t 2^{D_t} N_b \|x\|_0 \log(n))$, where $2^{D_t}$ represents the maximum number of nodes in a binary tree. LCEM time complexity is the same as LCE plus the dataset transformation which is linear in the number of samples.
3.5 Implementation

We present LCEM pseudocode in Algorithm 1. LCEM implementation is the same as LCE plus the dataset transformation. A function (LCEM_Tree) builds a tree and the second one (LCEM) builds the forest of trees through bagging, after having transformed the dataset. There are 2 stopping criteria during a tree building phase: when a node has an unique class or when the tree reaches the maximum depth. We set the range of tree depth from 0 to 2 in LCEM as in LCE. This hyperparameter is used to control overfitting. Low bias boosting-based classifier as base classifier justifies the maximum depth of 2. The set of low bias base classifiers is limited to the state-of-the-art boosting algorithm (extreme gradient boosting - XGB [8]).

Algorithm 1 LCEM

Require: A dataset \( D \), a set of classifiers \( H \), time window size \( win\_size \), maximum depth of a tree \( max\_depth \), number of trees \( n\_trees \)

1: function LCEM\((D, H, win\_size, n\_trees, max\_depth)\)
2: \( D' \leftarrow \) Dataset\_Transformation\((D, win\_size)\)
3: \( F \leftarrow \emptyset \)
4: for each \( i \) in \([1, n\_trees]\) do
5: \( S \leftarrow \) A bootstrap sample from \( D' \)
6: \( t \leftarrow \) LCEM\_Tree\((S, H, max\_depth, 0)\)
7: \( F \leftarrow F \cup t \)
8: return \( F \)

9: function LCEM\_Tree\((D, H, max\_depth, depth)\)
10: if \( max\_depth \) or uniform class then
11: return leaf
12: else
13: \( D' \leftarrow \) Concatenate\((D, H_{depth}(D))\)
14: Split \( D' \) on attribute maximizing Gini criterion
15: \( depth \leftarrow depth + 1 \)
16: for \( D'^{(j)} \in \mathcal{P}(D') \) do
17: \( Tree_j = \) LCEM\_Tree\((D'^{(j)}, H, max\_depth, depth)\)
18: return tree containing one decision node, storing classifier \( H_{depth}(D) \) and descendant subtrees \( Tree_j \)

4 Evaluation

In this section, we present our evaluation method. We first compare the performance of LCE to the state-of-the-art classifiers. Then, as explained in section 3.2.2, the dataset transformation performed and the performance calculation to extend LCE for MTS classification can be done for any classifier. Therefore, we compare the performance of LCEM to the best two classifiers from the first evaluation applying the same transformation as LCE and to the state-of-the-art MTS classifiers.

4.1 Datasets

4.1.1 Multivariate Data

In the experiments, we benchmark LCE on the UCI datasets [11]. We have randomly selected one dataset per category available on the repository and obtained 26 UCI datasets. The categories are defined according to the dataset topic (life sciences, physical sciences, computer science/engineering, social sciences, business and game), the number of instances (less than 100, 100 to 1,000 and greater than
Table 2: The UCI datasets. Abbreviations: CS - Computer Science.

| Datasets                     | Type               | Instances | Dimensions | Classes | LCE Parameters Trees | Depth |
|------------------------------|--------------------|-----------|------------|---------|----------------------|-------|
| Absenteeism at Work         | Business           | 740       | 19         | 19      | 100                  | 2     |
| Banknote Authentication      | CS/Engineering     | 1,372     | 4          | 2       | 5                    | 1     |
| Breast Cancer Coimbra        | Life Sciences      | 116       | 9          | 2       | 60                   | 0     |
| CNAE-9                       | Business           | 1,080     | 856        | 9       | 20                   | 2     |
| Congressional Voting         | Social Sciences    | 435       | 16         | 2       | 1                    | 1     |
| Drug Consumption (quantified)| Social Sciences    | 1,185     | 12         | 7       | 5                    | 2     |
| Electrical Grid Stability    | Physical Sciences  | 10,000    | 13         | 2       | 40                   | 1     |
| Gas Sensor                   | CS/Engineering     | 58        | 432        | 4       | 100                  | 0     |
| HTRU2                        | Physical Sciences  | 17,898    | 8          | 2       | 60                   | 2     |
| Iris                         | Life Sciences      | 150       | 4          | 3       | 20                   | 2     |
| Leaf                         | CS/Engineering     | 340       | 13         | 30      | 5                    | 0     |
| LSVT Voice Rehabilitation    | Life Sciences      | 126       | 310        | 2       | 5                    | 0     |
| Lung Cancer                  | Life Sciences      | 32        | 56         | 3       | 60                   | 1     |
| Mice Protein Expression      | Life Sciences      | 1,080     | 77         | 8       | 60                   | 1     |
| Musk V1                      | Physical Sciences  | 476       | 166        | 2       | 5                    | 2     |
| Musk V2                      | Physical Sciences  | 6,598     | 166        | 2       | 5                    | 2     |
| p53 Mutants                  | Life Sciences      | 31,159    | 5,408      | 2       | 10                   | 1     |
| Page Blocks Classification   | CS/Engineering     | 5,473     | 10         | 5       | 80                   | 2     |
| Parkinson Disease            | CS/Engineering     | 756       | 753        | 2       | 5                    | 2     |
| Seyton Handwritten Digit     | CS/Engineering     | 1,593     | 256        | 10      | 20                   | 2     |
| Ultrasonic Flowmeter         | CS/Engineering     | 181       | 43         | 4       | 60                   | 1     |
| User Knowledge Modeling      | CS/Engineering     | 403       | 5          | 5       | 40                   | 2     |
| Wholesale Customers          | Business           | 440       | 6          | 2       | 40                   | 0     |
| Wine                         | Physical Sciences  | 178       | 13         | 3       | 100                  | 0     |
| Wine Quality                 | Business           | 1,509     | 11         | 6       | 100                  | 2     |
| Yeast                        | Life Sciences      | 1,484     | 8          | 10      | 80                   | 2     |

Table 3: The UEA MTS datasets. Abbreviations: AS - Audio Spectra, Dims - Dimensions, ECG - Electrocardiogram, EEG - Electroencephalogram, HAR - Human Activity Recognition, MEG - Magnetoencephalography.

| Datasets                      | Type               | Train     | Test      | Length  | Dims | Classes | LCEM Parameters Window (%) | Trees | Depth |
|-------------------------------|--------------------|-----------|-----------|---------|------|---------|----------------------------|-------|-------|
| Articulatory Word Recognition | Motion             | 275       | 300       | 144     | 9    | 25      | 40                         | 5     | 1     |
| Atrial Fibrillation           | ECG                | 15        | 15        | 640     | 2    | 3       | 20                         | 1     | 0     |
| Basic Motions                 | HAR                | 40        | 40        | 100     | 6    | 4       | 20                         | 1     | 0     |
| Character Trajectories        | Motion             | 1,422     | 1,436     | 182     | 3    | 20      | 80                         | 10    | 2     |
| Cricket                       | HAR                | 108       | 72        | 1,197   | 6    | 12      | 40                         | 20    | 0     |
| Duck Duck Geese               | AS                 | 60        | 40        | 270     | 1,345| 5       | 100                        | 20    | 0     |
| Eigen Worms                   | Motion             | 128       | 131       | 17,984  | 6    | 5       | 100                        | 20    | 1     |
| Epilepsy                      | HAR                | 137       | 138       | 206     | 3    | 4       | 20                         | 1     | 1     |
| Ering                         | HAR                | 30        | 30        | 65      | 4    | 6       | 20                         | 1     | 2     |
| Ethanol Concentration         | Other              | 261       | 263       | 1751    | 3    | 4       | 20                         | 1     | 2     |
| Face Detection                | ECG/MEG            | 5,890     | 3,524     | 62      | 144  | 2       | 100                        | 5     | 2     |
| Finger Movements              | ECG/MEG            | 316       | 100       | 50      | 28   | 2       | 60                         | 5     | 2     |
| Hand Movement Direction       | ECG/MEG            | 320       | 147       | 400     | 10   | 4       | 80                         | 20    | 2     |
| Handwriting                   | HAR                | 150       | 850       | 152     | 3    | 26      | 20                         | 10    | 2     |
| Heartbeat                     | AS                 | 204       | 205       | 405     | 61   | 2       | 80                         | 10    | 0     |
| Insect Wingbeat               | AS                 | 30,000    | 20,000    | 200     | 30   | 10      | 100                        | 10    | 1     |
| Japanese Vowels               | AS                 | 279       | 370       | 29      | 12   | 9       | 40                         | 5     | 1     |
| Libras                        | HAR                | 180       | 180       | 45      | 2    | 15      | 40                         | 60    | 1     |
| LSST                          | Other              | 2,459     | 2,466     | 36      | 6    | 14      | 60                         | 10    | 2     |
| Motor Imagery                 | ECG/MEG            | 278       | 100       | 3,000   | 64   | 2       | 100                        | 20    | 1     |
| NAPTOPS                       | HAR                | 180       | 180       | 51      | 24   | 6       | 40                         | 10    | 0     |
| PenDigits                     | Motion             | 7,494     | 3,498     | 8       | 2    | 10      | 80                         | 80    | 2     |
| PEMSF                         | Other              | 267       | 173       | 144     | 963  | 7       | 100                        | 20    | 1     |
| Phoneone                      | AS                 | 3315      | 3353      | 217     | 11   | 39      | 80                         | 1     | 2     |
| Racket Sports                 | HAR                | 151       | 152       | 30      | 6    | 4       | 60                         | 20    | 0     |
| Self Regulation SCP1          | ECG/MEG            | 268       | 293       | 896     | 6    | 2       | 100                        | 5     | 2     |
| Self Regulation SCP2          | ECG/MEG            | 200       | 180       | 1,152   | 7    | 2       | 100                        | 20    | 2     |
| Spoken Arabic Digits          | AS                 | 6,599     | 2,199     | 93      | 13   | 10      | 80                         | 10    | 1     |
| Stand Walk Jump               | ECG                | 12        | 15        | 2,500   | 4    | 3       | 20                         | 1     | 1     |
| U Wave Gesture Library        | HAR                | 120       | 320       | 315     | 3    | 8       | 60                         | 1     | 0     |
1,000) and the number of dimensions (less than 10, 10 to 100 and greater than 100). The characteristics of each dataset are presented in Table 2. There is no train/test split provided on the repository so we have decided to perform a 3-fold cross-validation.

4.1.2 Multivariate Time Series

We benchmark LCEM on the 30 currently available the UEA MTS datasets [1]. For each dataset, we keep the train/test split provided in the archive. The characteristics of each dataset are presented in Table 3.

4.2 Algorithms

4.2.1 Classifiers

We compare our LCE algorithm, implemented in Python 2.7, to the following classifiers:

- Elastic Net - EN: the logistic regression combining L1 and L2 regularization methods. We use the SGDClassifier public implementation;
- Local Cascade - LC: algorithm has been implemented in Python 2.7 based on the description of the paper [16];
- Multilayer Perceptron - MLP: we consider small MLPs due to the limited size of the datasets and the absence of pretrained networks. We use the implementation available in the package Keras for Python and limit the neural network architecture to 3 layers;
- Random Forest - RF: we use the RandomForestClassifier public implementation;
- Support Vector Machine - SVM: we use the SVC public implementation;
- Extreme Gradient Boosting - XGB: we use the implementation in the xgboost package for Python.

4.2.2 MTS Classifiers

We compare our algorithm LCEM, implemented in Python 2.7, to the following MTS classifiers:

- DTW-1NN-D with and without normalization: the one nearest neighbor classifier with DTW distance based on multi-dimensional points instead of treating each dimension separately. We report the results published in the UEA archive [1];
- DTW-1NN-I with and without normalization: the one nearest neighbor classifier based on the sum of DTW distance for each dimension. We report the results published in the UEA archive [1];
- ED-1NN with and without normalization: the one nearest neighbor classifier with Euclidean distance. We report the results published in the UEA archive [1];
- MLSTM-FCN [15]: we use the implementation available and run it with the setting recommended by the authors in the paper (128-256-128 filters, 250 training epochs, a dropout of 0.8 and a batch size of 128);

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1 sklearn.linear_model.SGDClassifier
2 https://keras.io/
3 sklearn.ensemble.RandomForestClassifier
4 sklearn.svm.SVC
5 https://xgboost.readthedocs.io/en/latest/python/
6 https://github.com/houshd/MLSTM-FCN

12
RFM: Random Forest for Multivariate time series classification. We use the RandomForestClassifier public implementation with the transformation presented in section 3.2.1.

WEASEL+MUSE [33]: we use the implementation available[7] and run it with the setting recommended by the authors in the paper (SFA word lengths l in [2,4,6], windows length in [4:max(MTS length)], chi=2, bias=1, p=0.1, c=5 and a solver equals to L2R LR DUAL);

XGBM: Extreme Gradient Boosting for Multivariate time series classification. We use the implementation in the xgboost package for Python[5] with the transformation presented in section 3.2.1.

4.3 Hyperparameters Optimization

The ranges of LCE hyperparameters are the following: number of trees [1, 5, 10, 20, 40, 60, 80, 100] and maximum depth [0, 1, 2]. As explained in section 3.2, we add the time window size hyperparameter to LCEM. This parameter is expressed as a percentage of the total size of the MTS and the range of time window size percentages is [20%, 40%, 60%, 80%, 100%].

The hyperparameters of the different classifiers presented in section 1.2 are set by hyperopt, a sequential model-based optimization using a tree of Parzen estimators search algorithm [1]. Hyperopt chooses the next hyperparameters decision from both the previous choices and a tree-based optimization algorithm. Tree of Parzen estimators meet or exceed grid search and random search performance for hyperparameters setting. We use the implementation available in the Python package hyperopt[8] and hyperas[9] wrapper for keras.

4.4 Metrics

For each dataset, we compute the classification accuracy. Then, we present the average rank and the number of wins/ties to compare the different classifiers on the same datasets. Finally, we present the critical difference diagram [9], the statistical comparison of multiple classifiers on multiple datasets, to show the overall performance of LCE and LCEM. We use the implementation available in R package scmamp[10].

5 Results

In this section, we begin by evaluating the performance of LCE compared to the state-of-the-art classifiers. Next, we compare the performance of LCEM to the other MTS classifiers. Then, we show that the explainability of LCEM can give insights to the user about LCEM predictions. Finally, we assess the robustness of LCEM to missing data and noise.

5.1 LCE

Table 5 shows the classification results of the 7 classifiers on the 26 UCI datasets. The best accuracy for each dataset is denoted in boldface. We observe that the top 3 classifiers are the ensemble methods: LCE obtains the best average rank (2.2), followed by RF in second position (rank: 2.4) and XGB in third position (rank: 2.7).

First of all, LCE obtains the best average rank with the first position on 38% of the datasets (10 wins/ties). Based on the categorization of the UCI datasets presented in section 4.1.1, we do not observe any influence of the number of instances, dimensions or classes on the performance of LCE relative to other classifiers. Nonetheless, LCE exhibits varying performances across the different dataset

[7]https://github.com/patrickzib/SFA
[8]https://github.com/hyperopt/hyperopt
[9]https://github.com/maxpumperla/hyperas
[10]https://www.rdocumentation.org/packages/scmamp/versions/0.2.55/topics/plotCD
LCE shows its best performance on physical sciences (rank: 1.8, 19% of all datasets) and life sciences (rank: 1.9, 27% of all datasets) datasets while having its worst performance on computer science/engineering (rank: 2.4, 31% of all datasets), business (rank: 2.5, 15% of all datasets) and social sciences (rank: 3.5, 8% of all datasets) datasets. However, none of the classifiers shows a better average rank than LCE on the business and social sciences datasets.

Then, we observe that the second ranked classifier RF obtains the same number of wins/ties as LCE. RF exhibits better performance than LCE on the computer science/engineering datasets (rank 1.8, 31% of all datasets) which represents half of RF wins/ties. 80% of the wins/ties of RF on the computer science/engineering category are obtained on small datasets (train size < 1000). We can infer that the bagging only (variance reduction) of RF can provide better generalization than LCE bagging-boosting combination on small datasets (wins/ties on small datasets - 54% of the datasets: LCE 6, RF 6). The third ranked classifier XGB gets 5 wins/ties. We do not see any influence of the different dataset categories on XGB wins/ties relative to LCE. Therefore, we conclude that LCE bagging and boosting combination to handle the bias-variance tradeoff exhibits better generalization on average than the bagging only (RF) and boosting only (XGB) algorithms on these 26 UCI datasets.

Next, LC algorithm gets the fifth rank with one win/tie. We do not see any particular influence of the different dataset categories on LC performance. So, the outperformance of LCE compared to LC on the 26 UCI datasets confirms the better generalization ability of an hybrid (explicit and implicit) versus an implicit only approach. The comparison in Table 4 aims to underline the superior performance of LCE compared to LC on the UCI datasets. In order to be comparable, the low bias base classifier in LC is XGB. The depth of a tree is set to 1 for LCE and LC. The results correspond to the average accuracy on test sets with the corresponding standard error. Results show a comparable accuracy variability of LCE compared to LC when the number of trees is set to 1 (standard error of 4.6% versus 4.8%). However, LCE on 1 tree exhibits a higher accuracy than LC (71.8% versus 65.9%). Additionally, through bagging, we observe LCE variability reduction as well as an increase of accuracy (71.8±4.6 with 1 tree versus 74.9±4.1 with 60 trees versus 65.9±4.8 with LC). Therefore, this comparison affirms the superiority of our explicit bias-variance tradeoff approach compared to the implicit approach of LC on the UCI datasets.

Table 4: Average accuracy score of LCE versus LC on test sets of the UCI datasets with the corresponding standard error.

| Trees | 1  | 5  | 10 | 20 | 40 | 60 | 80 |
|-------|----|----|----|----|----|----|----|
| LCE   | 71.8 | 74.1 | 73.6 | 72.8 | 73.2 | 74.9 | 73.9 |
|       | ±4.6 | ±4.3 | ±4.4 | ±4.4 | ±4.5 | ±4.1 | ±4.2 |
| LC    |     |     |     |     |     |     | 65.9 ± 4.8 |

Concerning the other classifiers, EN obtains only one win/tie but gets a better rank on average than SVM (3 wins/ties) and MLP (3 wins/ties).

Finally, we analyze a statistical test to evaluate the performance of LCE compared to the other classifiers. We present in Figure 4 the critical difference plot with alpha equals to 0.05 from results shown in Table 4. The values correspond to the average rank and the classifiers linked by a bar do not have a statistically significant difference. The plot confirms the top 3 ranking as presented before (LCE: 1, RF: 2, XGB: 3), without showing a statistically significant difference between each other. We also observe that the ensemble methods accuracies are statistically different from other classifiers. Therefore, considering that LCE transformation to multivariate time series classification is also applicable to other classifiers, we evaluate the performance of RF and XGB with the same transformation as LCE in comparison to the state-of-the-art MTS classifiers in the next section.
| Datasets                        | LCE | LC  | XGB | RF  | MLP | SVM | EN |
|--------------------------------|-----|-----|-----|-----|-----|-----|----|
| Absenteeism at Work            | 42.7| 27.6| 44.2| 42.0| 28.3| 28.7| 31.7|
| Banknote Authentication         | 99.3| 98.9| 99.6| 99.1| 89.5| 100.0| 98.8|
| Breast Cancer Coinbra           | 71.4| 65.5| 64.6| 64.5| 48.4| 55.2| 57.5|
| CNAE-9                         | 86.2| 51.0| 84.1| 91.6| 95.6| 30.4| 92.2|
| Congressional Voting            | 97.0| 94.0| 96.8| 96.6| 79.5| 87.8| 91.7|
| Drug Consumption (quantified)   | 34.6| 27.9| 37.8| 38.5| 40.3| 40.3| 39.3|
| Electrical Grid Stability       | 100.0| 99.9| 100.0| 100.0| 88.5| 79.1| 96.6|
| Gas Sensor                     | 74.4| 63.3| 74.6| 89.6| 78.7| 61.5| 70.4|
| HTRU2                          | 97.9| 97.8| 97.9| 97.8| 96.8| 91.1| 97.6|
| Iris                            | 96.7| 90.2| 96.7| 96.7| 44.4| 95.4| 83.0|
| Leaf                           | 52.5| 48.7| 61.6| 71.7| 8.5 | 35.2| 56.0|
| LSVT Voice Rehabilitation      | 81.0| 57.1| 77.0| 81.0| 66.7| 66.7| 66.7|
| Lung Cancer                    | 41.1| 47.2| 34.4| 37.2| 37.2| 36.7| 52.8|
| Mice Protein Expression        | 56.7| 40.1| 43.1| 53.1| 13.9| 14.4| 42.9|
| Musk V1                        | 73.3| 63.5| 76.1| 72.5| 57.4| 56.5| 72.3|
| Musk V2                        | 78.8| 74.5| 78.4| 77.5| 84.6| 84.7| 76.3|
| p53 Mutants                    | 96.6| 82.7| 94.8| 95.6| 99.5| 86.5| 81.7|
| Page Blocks Classification     | 97.3| 90.8| 96.5| 96.0| 90.4| 91.1| 94.2|
| Parkinson Disease              | 82.7| 74.2| 82.5| 83.2| 58.2| 74.6| 41.4|
| Semeion Handwritten Digit      | 90.3| 43.2| 90.0| 92.2| 92.1| 36.4| 75.8|
| Ultrasonic Flowmeter           | 59.0| 40.2| 45.2| 49.6| 24.4| 29.8| 45.1|
| User Knowledge Modeling        | 85.6| 80.4| 85.6| 85.6| 29.8| 80.4| 74.6|
| Wholesale Customers            | 91.8| 88.6| 92.5| 91.6| 77.0| 67.7| 83.0|
| Wine                           | 92.8| 96.1| 91.1| 92.8| 35.4| 42.7| 73.4|
| Wine Quality                   | 55.5| 49.2| 54.5| 56.9| 42.1| 41.9| 45.9|
| Yeast                          | 57.1| 35.3| 59.2| 59.6| 28.9| 58.9| 53.2|
| Average Rank                   | 2.2 | 5.0 | 2.7 | 2.4 | 5.3 | 5.2 | 4.7 |
| Wins/Ties                      | 10  | 1   | 1   | 10  | 3   | 3   | 1   |

Table 5: Accuracy results on the UCAI datasets. Abbreviations: LCE - LSTool-CNN, LC - LSTM-CNN, XGB - XGBoost, RF - Random Forest, MLP - Multi-Layer Perceptron, SVM - Support Vector Machine, EN - Ensemble.
5.2 LCEM

5.2.1 Classification Performance

The classification results of the 11 MTS classifiers are presented in Table 6. A blank in the table indicates that the approach ran out of memory or the accuracy is not reported \[1\]. The best accuracy for each dataset is denoted in boldface. We observe that LCEM obtains the best average rank (3.0), followed by RFM in second position (rank: 3.7) and MLSTM-FCN in third position (rank: 3.9).

LCEM gets the first position in one third of the datasets. Using the categorization of the datasets published in the archive website\[11\], we do not see any influence from the different train set sizes, MTS lengths, dimensions and number of classes on LCEM performance relative to the other classifiers on the UEA datasets. Nonetheless, LCEM exhibits weaker performance on average on human activity recognition (rank: 3.6, 30% of all datasets) and motion classification (rank: 5.0, 13% of all datasets) datasets.

Then, we observe that the better generalization of LCE bagging-boosting combination compared to bagging only (RF) and boosting only (XGB) is also valid on the MTS datasets (average rank: LCEM 3.0, RFM 3.7, XGBM 4.8). The adaptation of ensemble methods to the MTS datasets (see section 3.2.1) is well performing: the three ensemble methods obtain the highest number of wins/ties (ensemble methods for MTS: 17 - 57% of all datasets, MLSTM-FCN: 11 - 37% of all datasets, WEASEL+MUSE: 4 - 13% of all datasets). The 6 wins/ties of RFM are obtained on small datasets (train size < 500).

As seen in section 5.1, we can infer that the bagging only (variance reduction) of RFM can provide better generalization than LCEM bagging-boosting combination on small datasets (wins/ties on small datasets - 77% of the datasets: LCEM 8, RFM 6). On the time window sizes used, we observe that the choice of LCEM time window is a tradeoff between its bagging and boosting components. LCEM and XGBM use the same time window size on 70% of the datasets. When the time window size is different, LCEM obtains a better accuracy than XGBM on 90% of the cases. Moreover, LCEM employs the same time window size as RFM on half of the UEA datasets. On the other half of the datasets, RFM adopts a slightly bigger time window size than LCEM. RFM uses a bigger time window in 75% of the cases with an average time window difference of 29% between LCEM and RFM. The different choice of LCEM time window size leads to a better accuracy on 75% of the cases compared to RFM. These observations prove that LCEM bias-variance tradeoff can refine the time window size of boosting only and bagging only to obtain a better generalization ability on average.

Concerning the state-of-the-art MTS classifiers, we observe a performance difference between the third (MLSTM-FCN) and fourth (WEASEL+MUSE) classifiers on datasets sizes. MLSTM-FCN outperforms WEASEL+MUSE (rank: 2.6 versus 4.6 for WEASEL+MUSE) on the largest datasets (train size \( \geq 500 \), 23% of all datasets) whereas WEASEL+MUSE slightly outperforms MLSTM-FCN (rank 3.6 versus 3.8 for MLSTM-FCN) on the smallest datasets (train size < 500, 77% of all datasets). LCEM shows the same performance as MLSTM-FCN on the largest datasets (rank 2.6) while outperforming WEASEL+MUSE on the smallest datasets (rank: 3.1 versus 3.6 for WEASEL+MUSE).

\[11\]http://www.timeseriesclassification.com/dataset.php
Therefore, LCEM is better than the state-of-the-art MTS classifiers on both small and large UEA datasets. Last, similarity-based methods obtain the lowest wins/ties counts. Euclidean distance is never in the first position on the UEA datasets. The wins/ties of DTW (DTW-1NN-D normalized: 2, DTW-1NN-D: 3) stem from their outperformance on human activity recognition datasets.

Next, we performed a statistical test to evaluate the performance of LCEM compared to the other MTS classifiers. We present in Figure 5 the critical difference plot with alpha equals to 0.05 from results shown in Table 6. The values correspond to the average rank and the classifiers linked by a bar do not have a statistically significant difference. The plot confirms the top 3 ranking as presented before (LCEM: 1, RFM: 2, MLSTM-FCN: 3), without showing a statistically significant difference between each other. As seen in the evaluation on the UCI datasets, the plot also confirms that there is no statistically significant difference between the ensemble methods LCEM/RFM/XGM on the MTS datasets. We notice that LCEM is the only classifier with a significant performance difference compared to DTW-1NN-D normalized.

Figure 5: Critical difference plot of the MTS classifiers on the UEA datasets with alpha equals to 0.05.

5.2.2 LCEM Explainability

LCEM provides explainability by design through the identification of the time window used to classify the whole MTS. There is no metric to quantify a model explainability. Therefore, we adopt a qualitative approach to analyze LCEM explainability. First, we illustrate the explainability of LCEM on a synthetic dataset, then we show which windows have been used on the UEA datasets of section 5.2.1 and we illustrate it on two UEA datasets (Atrial Fibrillation and Racket Sports).

**Synthetic Dataset**

First of all, we show that LCEM uses and identifies the expected time window to perform the classification on a MTS synthetic dataset. We design a dataset composed of 10 MTS with a length of 100, 2 dimensions and 2 balanced classes. The difference between the 5 MTS belonging to the negative class and the one belonging to the positive class stems from a 20% time window of the MTS. As illustrated in Figure 6, negative class MTS are sine waves and positive class MTS are sine waves with a square signal on 20% of the dimension 1 (see timestamps between 60 and 80).

The classification results show that LCEM with a time window size parameter set to 20% is enough to correctly classify the 20 MTS (accuracy: 100% - number of trees: 10, depth: 1). Moreover, the classification results for the positive class MTS are based on the 20% time window with a square signal on dimension 1. We observe that the maximum class probability for the MTS of positive class is 100% and this probability is reached for samples on the range [62,100] (maximum class probability on the range [0,61]: 92.6%). This range is the expected range. As explained in section 3.2.1, all the samples of the dataset obtained with a 20% sliding window have a piece of the square signal for the timestamps in the range [62,100], which is the information sufficient to correctly classify the MTS in the positive class. Therefore, by taking all the samples of the dataset with the maximum class probability, LCEM allows
the identification of the full parts of the MTS which are characteristic of a class (e.g. the square signal on 20% of the dimension 1 in Figure 6).

**Time Window Size Percentages on the UEA** We then present the LCEM explainability results on the UEA datasets. We begin with illustrating in Figure 7 the distribution of the time window size percentage used by LCEM on the UEA archive per dataset type. We observe that LCEM has a tendency to use particular time window size percentages per dataset type. Most of audio spectra, EEG/MEG and motion datasets have been classified on a time window size > 60% of the MTS lengths. Meanwhile, most ECG and human activity recognition datasets have been classified on a time window size ≤ 60% of the MTS lengths. Therefore, we can induce that the information provided by the whole MTS is useful to discriminate between the different classes on the audio spectra, EEG/MEG and motion datasets. Concerning the ECG and human activity recognition datasets, we can infer that the discriminative information is located in a particular part of the MTS.

**Atrial Fibrillation Dataset** For example, LCEM obtains its best performance on the two ECG datasets using a time window size of 20%. Therefore, we assume that the information necessary for LCEM to classify the MTS in ECG datasets are really condensed compared to the entire MTS available. We illustrate it in Figure 8 by highlighting the 20% time window of the first MTS sample per class in the Atrial Fibrillation test set to gain insights on LCEM classification result. Atrial Fibrillation
dataset is composed of two channels ECG on a 5 second period (128 samples per second). MTS are labeled in 3 classes: *non-terminating atrial fibrilation*, *atrial fibrilation terminates one minute after* and *atrial fibrilation terminates immediately*. LCEM correctly predicts the 3 MTS based on the one second time window (20%) highlighted in Figure 8. There is a unique window for each MTS with the highest class probability (class *non-terminating atrial fibrilation*: 94.6%, *atrial fibrilation terminates one minute after*: 97.7%, *atrial fibrilation terminates immediately*: 97.4%). We can observe in the *non-terminating atrial fibrilation* MTS that the time window highlighted reveals an abnormal constant increase on channel 2 (red line) during one second whereas the other channel keeps the same motif as other windows. On the *atrial fibrilation terminates one minute after* MTS, we observe a smaller decrease in channel 2 than in other windows and a low peak in channel 1. These particular 20% time windows inform the user about LCEM classification outcome, thus providing important information to domain experts.

**Racket Sports Dataset** The second category of datasets where LCEM obtains its best results on a time window size $\leq 60\%$ of the MTS lengths is human activity recognition. As previously done with Atrial Fibrilation, we illustrate it in Figure 9 by highlighting the 60% time window of the first MTS sample per class in the Racket Sports test set to gain insights on LCEM classification result. Racket Sports dataset is composed of 6 dimensions, x/y/z coordinates for both the gyroscope and accelerometer of an android phone, on a 3 second period (10 samples per second). MTS are labeled in 4 classes: *badminton smash*, *badminton clear*, *squash forehand boast* and *squash backhand boast*. We illustrate the explainability of LCEM on the two classes relative to the squash: *squash forehand boast* and *squash backhand boast*. LCEM correctly predicts the 2 MTS based on the 1.8 seconds time window (60%) highlighted in Figure 8. There is a unique window for each MTS with the highest class probability (*squash forehand boast*: 90.3%, *squash backhand boast*: 86.7%). We can observe that for these 2 MTS the window highlighted well correspond to the period of the full movement. Then, we can see a simultaneous steep peak on red and orange dimensions with a steep decrease on green dimension for *squash forehand boast*. Whereas, we can see a simultaneous steep decrease on red and
orange dimensions without a particular variation on the green dimension for squash backhand boast. These particular 60% time windows inform the user about LCEM classification outcome, thus providing important information to domain experts.

These two examples show how LCEM outperforms other MTS classifiers (rank 1 on Atrial Fibrillation and Racket Sports) while offering explainability by design on its predictions.

5.2.3 Effect of Missing Data

None of the state-of-the-art MTS classifiers handles missing data. Missing data are interpolated, which adds a parameter to the problem. LCEM naturally handles missing data through its tree-based learning [7]. Similar to extreme gradient boosting [8], LCEM excludes missing values for the split and uses block propagation. Block propagation sends all samples with missing data to the side minimizing the error.

We present in this section an experiment to illustrate the performance of LCEM in the case of missing data. We have selected three datasets from the most representing type of the UEA datasets (human activity recognition, 30% of the datasets); it is also a type on which LCEM does not obtain the best performance comparing to the other classifiers (rank: 3.6). We choose the three datasets according to the performance of LCEM to show the evolution of accuracies according to different starting points: Basic Motions (LCEM accuracy: 100%, no error), Racket Sports (94.1%, [0,10] percent of error) and U Wave Gesture Library (89.7%, [10,100] percent of error). Then, we randomly removed an increasing proportion of the values for each time series ([5%, 10%, ..., 50%]) of the datasets before transformation (see section 3.2.1). The error rates on test sets over 10 replications are presented in Figure 10.

First, we observe that missing data does not have an effect on LCEM performance (100% accuracy) on the dataset Basic Motions. On the other two datasets, the error rates of LCEM increase progressively with the proportion of missing data. The error rate induced by missing data never exceeds 5% on these 2 datasets when half the data is missing (accuracy difference from 0% to 50% missing data: Racket
Figure 10: Evolution of LCEM error rates with standard errors according to the proportion of missing values on three Human Activity Recognition datasets.

Sports +3.7% and U Wave Gesture Library +1.9%). Finally, LCEM performance is stable: the error rates remain roughly the same across the 10 replications on all proportions of missing values (mean of standard error across Racket Sports/U Wave Gesture Library: 0.34%).

5.2.4 Effect of Gaussian Noise on Classification Accuracy

In this section, we evaluate the robustness of LCEM to Gaussian noise compared to the second and third ranked MTS classifiers according to the number of wins/ties. Therefore, we compare the performance of LCEM to RFM and MLSTM-FCN, with RFM proven to be robust to noise based on bagging [5].

Following the same logic as the section on missing values, we performed an experiment on the same three datasets. These three datasets are from the most representing type of the UEA datasets (human activity recognition, 30% of the datasets) and from different LCEM accuracy categories: Basic Motions (LCEM accuracy: 100%, no error), Racket Sports (94.1%, [0,10] percent of error) and U Wave Gesture Library (89.7%, [10,100] percent of error). Then, after z-normalization of these datasets on each dimension (standard deviation of 1), we added an increasing Gaussian noise with a standard deviation of 0 to 1 to each dimension, which is equivalent to noise levels of 0% to 100%. The average error rates with standard errors on these three datasets are presented in Figure 11.
We observe that LCEM fully exploits its bagging component and is as robust to noise as RFM. LCEM shows lower error rates than RFM on 60% of the noise levels, without having a greater variability across the datasets (average standard error: LCEM 3.7% versus RFM 3.5%). Moreover, LCEM is more robust to noise than MLSTM-FCN. LCEM exhibits lower error rates than MLSTM-FCN on 80% of the noise levels with a lower variability across the datasets (average standard error: LCEM 3.7% versus MLSTM-FCN 5.3%).

5.3 Discussion

We have presented our new hybrid ensemble method LCE for multivariate data classification with its extension LCEM for MTS classification. We have shown that LCE outperforms the state-of-the-art classifiers on the UCI datasets and that LCEM outperforms the state-of-the-art MTS classifiers on the UEA datasets. In addition, LCEM provides explainability by design and manifests robust performance when faced with challenges arising from continuous data collection (different MTS length, missing data and noise).

However, our ensemble method has some limitations. First, LCEM predicts the class of a MTS based on a single window, the one on which it is the most confident, without considering the predictions on the other windows. Some datasets can contain MTS with different windows close to the characteristics of different classes. Therefore, LCEM can have high class probabilities on different windows and when the window on which LCEM is the most confident is characteristic of another class than the expected one, LCEM incorrectly classifies the MTS. To illustrate it, we present in Figure 12 two MTS of the UEA Libras test set. LCEM performed poorly on this dataset and obtained the rank 10/11 (see section 5.2.1). The Libras dataset contains 15 classes of 24 instances each, where each class references a hand movement type in the Brazilian sign language Libras. The hand movement is represented as a bi-dimensional curve performed by the hand in a period of time. We can observe in Figure 12 that the two MTS belonging to the same class have comparable evolution across time but LCEM classifies them into two different classes. The first MTS is correctly classified based on the time window [23,40] with a class probability equals to 93.5%. We can assume that the evolution on this window is characteristic of the class 6. The second MTS also contains a comparable window on the range [23,40] but is incorrectly classified based on another window (range [0,17]) with a class probability of 94.5%. Therefore, LCEM is the most confident on a window characteristic of another class (class 4). LCEM did not consider the predictions on the other windows to take its decision. More particularly, LCEM did not consider the expected window [23,40] to take its decision, where it also gets a high class probability of 86.3%. So, it would be interesting to improve our hybrid ensemble method for MTS classification by considering in the final decision the predictions on the different windows of a MTS.

Moreover, LCEM provides explainability by design through identifying the time window used to classify the whole MTS. However, our black-box hybrid ensemble method LCE is not an easy-to-
understand classifier. So, LCEM explainability relies on human visual analysis of the selected window to identify the pattern characteristic of a MTS class. It would be valuable to integrate into LCEM explainability a post-hoc model-specific approach to mine the pattern characteristic of the time window selected for each MTS.

Finally, we assume in LCEM that a unique window size is suitable to discriminate the different classes. Nonetheless, we can imagine that different classes can be characterized by signals of different lengths. Therefore, it would also be interesting to improve LCEM by integrating the possibility of multiple windows sizes.

6 Conclusion

We have presented LCE, a new hybrid Local Cascade Ensemble, and LCEM its extension for MTS classification. LCE exhibits better accuracy than the state-of-the-art classifiers on the UCI datasets and LCEM shows better accuracy than the state-of-the-art MTS classifiers on the UEA datasets. As tree-based ensemble methods, LCE and LCEM can scale well on larger datasets than the ones tested. In addition, LCEM addresses the challenges MTS classification usually faces. First, it provides explainability by design through the identification of the time window used to classify the whole MTS. Then, LCEM is robust when faced with challenges arising from continuous data collection (different MTS length, missing data and noise).

In our future work, we would like to adapt this approach to the regression task and evaluate it against the state-of-the-art regression methods. To further improve the explainability of LCEM, it would be interesting to analyze the time windows characteristic of each class in the training set in order to determine if they contain some common patterns (ex: high values at the beginning of the window followed by a sharp drop). Such patterns may be even easier to understand for the user, as they would synthesize the important information in the discriminative time windows.

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