Investigation of the deformation of reinforced structures interacting with soils

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Abstract. Within the framework of the proposed methodology for calculating the processes of interaction of elastic-plastic structures with soils of complex physical nature, based on the use of defining ratios connecting the increments of true stresses and deformations, a calculation is made of the stress-strain state of the lining of the underground tunnel and located in the immediate vicinity of the ventilator unit erecting over them a roadbed.

1. Introduction

The work is devoted to the problem of predicting the processes of nonlinear deformation of interacting elastic-plastic reinforced structures and media [1-4], including in the framework of contact interaction [5-8]. In modern mechanics of a deformable solid, there are different versions of three-dimensional non-linear equations, which are composed taking into account the geometric nonlinearity [9-11]. In most of them, the differences in the components of the metric tensor before and after deformations of the body are taken as a measure of deformations, in accordance with which the energy-compatible stress tensors associated with them are also introduced. However, as studies conducted in recent years have shown, such physical relationships of this type in solving some specific problems lead to the appearance of “false” bifurcation solutions.

2. Statement of the problem

The state of static body balance is described by a variation equation of the principle of possible displacements:

$$
\delta \Pi = \int_{0}^{V} \left( \tau_{1} \delta \varepsilon_{1} + \tau_{2} \delta \varepsilon_{2} + \tau_{3} \delta \varepsilon_{3} + \tau_{21} \delta \sin \gamma_{21} + \tau_{31} \delta \sin \gamma_{31} + \tau_{32} \delta \sin \gamma_{32} \right) \left( 1 + \varepsilon_{1} \right) \left( 1 + \varepsilon_{2} \right) \left( 1 + \varepsilon_{3} \right) dV_{0},
$$

where

$$
\varepsilon_{i} = \frac{\sigma_{yi} \cos \gamma_{yi}}{1 + \varepsilon_{i}}, \quad \tau_{ij} = \sigma_{yi} \cos \gamma_{yi} \sin \gamma_{ij} + \sigma_{yj} \cos \gamma_{ij} \sin \gamma_{yj}, \quad \tau_{21} = \sigma_{21} \cos \gamma_{21} = \sigma_{12} \cos \gamma_{12} = \sigma_{32} \cos \gamma_{32} = \tau_{21}^{12};
$$

$$
\sigma_{y} \text{ - true stresses, } \varepsilon_{i} \text{ and } \sin \gamma_{ij} \text{ - true deformations:}
$$

$$
\varepsilon''_{i} = \varepsilon_{i} = \sqrt{1 + 2 \varepsilon_{i}'}, \quad \varepsilon''_{ij} = \sin \gamma_{ij} = 2 \varepsilon_{ij}' \left( 1 + \varepsilon_{j} \right)^{-1} \left( 1 + \varepsilon_{j} \right)^{-1}.
$$

As a condition of plasticity in the work, the Huber-Mises criterion is used.
\[ \Phi = \sigma_i - H(\sigma_T, \chi) = 0, \]

where \( \sigma_i \) - is the stress intensity, \( \sigma_T \) - is the yield strength, \( \chi \) - is the hardening parameter in the form of Odqvist. Equations of Prandtl-Reuss type connect the components of the increments of true stresses \( \sigma_{ij} \) and true deformations \( \varepsilon^{''}_{ij} \)

\[ \Delta \sigma_{ij} = \frac{E}{1 - 2\mu} \delta_{ij} \Delta \varepsilon^{''}_0 + 2G \Delta \varepsilon^{''}_{ij} - \alpha \frac{3G}{(\sigma_T)^2} \left( H'_f / 3G + 1 \right) \sigma'_j. \]

A technique is implemented that is ideally suited for solving three-dimensional elastoplastic problems according to flow theory and is called the “modified Lagrange incremental approach”, in which the deformation process is represented as a sequence of equilibrium states at corresponding loading levels.

For a number of soils, the limiting state in three-dimensional arrays is well described by the strength condition of the Mises-Botkin

\[ \tau_i + \sigma_O \phi^*_{oct} - c^*_{oct} = 0, \]

which is written through \( \phi^*_{oct} \) - the angle of internal friction on octahedral sites and \( c^*_{oct} \) - the limiting resistance to pure shear. Then for soils, Prandtl-Reuss type relations can be replaced by relations composed of the components of true stresses and true deformations.

\[ \Delta \sigma_{ij} = 2G \left( \Delta \varepsilon^{''}_{ij} + \delta_{ij} \Delta \varepsilon^{''}_0 \right) - \alpha \left( \frac{G}{\tau_i} \sigma'_j + K t g \phi^*_{oct} \delta_{ij} \right) \sum_{kl} \left( \frac{G}{\tau_i} \sigma'_i + K t g \phi^*_{oct} \delta_{il} \right) \Delta \varepsilon^{''}_{kl}. \]

To solve problems of contact interaction of structures, a special contact element with specific properties is used [12-16]. Depending on the nature of external influences, various variants of their interaction of the contacting surfaces are possible. The resolving equation is written based on the principle of virtual displacements in a variational form:

\[ \sum_{m} \int_{\Omega_m} \{ \varepsilon \}^T \{ \delta \varepsilon \} d\Omega + \sum_{k} \int_{\Omega_k} \{ \varepsilon_H \}^T \{ \delta \varepsilon_H \} d\Omega = \sum_{m} \int_{\Omega_m} \rho \{ g \}^T \{ \delta V \} d\Omega + \int_{S} \{ p \}^T \{ \delta V \} dS. \]

In some problems arising from the calculation of construction or transport facilities, one has to deal with locally supported structures, in particular, with reinforced concrete, in which reinforcement can be placed unevenly and in a special way. It can be irrational and difficult to use the method of averaging properties, since within each element there can be one or two reinforcing links, moreover, located in the most arbitrary way. In this case, it is advisable to use special reinforced finite elements of the continuum, in which reinforcements are discretely located in the form of links, working in tension and compression and rigidly connected with the deformable medium of the main array. Three-dimensional isoparametric finite elements are chosen as the basic finite elements, the reinforcements are assumed to be straight and passing through the finite element in an arbitrary manner. The stiffness of each reinforcement element is distributed according to the degrees of freedom of the element of the environment. From an energy point of view, this means that the potential deformation energy of a reinforced finite element is the sum of the potential deformation energy of the base volume and the potential deformation energy of all the reinforcements. The total stiffness matrix of the element is represented as the sum of the stiffness matrix of the medium element and the stiffness matrices of the reinforcements.

\[ [K^*_y] = [K_y] + \sum_{m=1}^{N} [K^*]_{m}, \]

where the stiffness matrices of each reinforcement are of the form
\[
[K^{(m)}_{ij}] = \frac{EF^{(m)}}{L^2_m} \left( H_i^{(m)} H_j^{(m)} + \frac{4}{3} M_i^{(m)} M_j^{(m)} \right) \{P^m\}^T \{P^m\}^T,
\]
and where \( L^2_m = \{P^m\}^T \{P^m\} \) - link length squared,

\[
\{P^m\} = \sum \left\{ \begin{array}{c} x_i \\ y_i \\ z_i \end{array} \right\}, \quad H_i^{(m)} , H_j^{(m)} = N_i(\xi_2^{(m)}, \eta_2^{(m)}, \zeta_2^{(m)}) - N_i(\xi_1^{(m)}, \eta_1^{(m)}, \zeta_1^{(m)}),
\]

\[
M_i^{(m)} = N_i(\xi_1^{(m)}, \eta_1^{(m)}, \zeta_1^{(m)}) + N_i(\xi_2^{(m)}, \eta_2^{(m)}, \zeta_2^{(m)}) - 2N_i(\xi_3^{(m)}, \eta_3^{(m)}, \zeta_3^{(m)}).
\]

As a practical application of the implemented algorithms, the practical problem of deforming elements of practical structures in a soil environment has been solved.

3. Numerical results

In the process of modeling the phased construction of complex structural elements, industrial and transport facilities in the preparation of power and design schemes to identify emerging stress fields, deformations and displacements requires the introduction of the concept of transforming structures (mechanical systems), which at certain stages of the construction process go from one class to another.

Mathematical modeling of the formation of stress fields, deformations and displacements in the elements of this mechanical system also requires the formulation of the problem of mechanics of a transforming structure. In the mechanical system described above, the transformation of the design scheme occurs discretely during the transition from one stage of construction to another.

At each step of transformation, the necessary calculations have to be carried out taking into account stress fields, displacements and deformations that accumulate in the system at the previous steps. Often, such calculations require the formulation of the corresponding problems of mechanics with regard to geometric nonlinearity, when the deformation process is represented as a sequence of
equilibrium states, and the transition from the current state to the next one is determined by the load increment, changing boundary conditions or the computational domain, etc.

As an example, the calculation of the stress-strain state of the lining of the metro tunnel and the ventilating unit construction located in close proximity to them during the construction of a roadbed above them is given. The problem is also solved in a two-dimensional formulation, and the soil model is assumed to be piecewise constant in height. The lateral and lower boundaries of the region are specified by straight lines, and on them the conditions for the absence of points displacement in the direction perpendicular to the rectilinear boundaries are set. The size of the area is determined during the computational experiment. Figure 1 schematically shows all structural states that are realized during the phased construction of the ventilation unit of the metro station, which determine the discreteness of the transformation of the design schemes.

Figures 2a and 2b show the first main stresses in all concrete structures and the draft of the entire computational domain for one of the calculation options for illustration.

4. Conclusion
The proposed method of solving problems of mechanics with specific applications relates to the modern technology of scientific support for the design and construction of complex objects. Its use allows us to trace the change in the stress-strain state and the field of displacements of the structurally changing computational domain from the beginning to the end of construction. This allows you to more accurately and technically competently make design decisions for various stages of construction work.

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