Multiscale structure-phenomenological study of magnetorheological suspensions in blood

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Abstract. The rheological equations for dilute suspensions of rigid axisymmetric elongated particles possessing a permanent magnetic moment with blood as a carrier fluid are derived within the frame of a structure-phenomenological approach. The Stokes fluid with couple stresses and the uniaxial dumbbell are used, respectively, as the rheological and hydrodynamic models of blood and suspended particles. The influence of an external magnetic field on a rheological behaviour of the suspensions under study is examined.

1. Introduction

Suspensions in blood of magnetically sensitive particles can appear upon an addition to blood of particles of drugs based on magnetic carriers [1, 2]. In particular, such suspensions arise on addition of ferro- or ferrimagnetic micro- and nanoparticles coated with either polysaccharides or proteins designed for diagnostic or hyperthermic treatment of cancer.

While solving medical problems through the use of suspensions in blood, the possible consequences of biomechanical intervention into the human body should be remembered. In order to carry out investigations in this area, a magnetobiorheological model of dilute suspension in blood of axially symmetric elongated microparticles possessing permanent magnetic moment is constructed in our paper owing to the multiscale combination of microfluidic and structure-continual studies of such suspensions within the frames of the structure-phenomenological approach [3].

2. The rheological model of dilute suspension of elongated magnetic particles in blood

For using the structure-phenomenological method [3] in the construction of constitutive equations defining the stressed state in gradient flows of a dilute suspension in blood, we assume that such a suspension satisfy the assumptions of Einsteinian microfluidic study [4]. In particular, the double inequality \( l \ll d \ll \tilde{l} \) is fulfilled; here \( d \) is the characteristic size of axially symmetric suspended particles; \( l \) is the characteristic size of red blood cells; \( \tilde{l} \) is the characteristic size of macroflow region of the suspension.

The inequality \( l \ll d \) allows us to consider the interaction of blood with suspended particles as a hydrodynamic interaction. At the first scale level of modeling, we model blood by classical continuum and use, following [5], the Stokes fluid with couple stresses [6]

\[
\tau_{ij} = -p \delta_{ij} + 2\mu \dot{d}_{ij},
\]  

(2.1)
\[ \mu_{ij} = 4\eta K_{ij} + 4\eta' K_{ji} \quad (2.2) \]

as a rheological model of blood. The theory of Stokes [6] is the simplest generalization of the Newtonian theory of viscous fluids that takes into account the presence of couple stresses in gradient flows of liquid media. The viscous and couple stresses \( \tau_{ij} \) and \( M_{ij} \) in rheological model of Stokes [6] are completely defined by the field of velocity \( \nu_i \). In rheological equations (2.1) and (2.2), \( \tau_{ij} \) is the symmetric part of the stress tensor \( \tau_i \); \( \mu_{ij} \) is the deviatoric part of the couple stress tensor \( M_{ij} \); \( p \) is the pressure; \( \mu, \eta, \eta' \) are rheological constants; \( d_{ij} \) is the strain rate tensor, \( d_{ij} = (1/2)(\nu_{i,j} + \nu_{j,i}) \); \( K_{ij} \) is the gradient tensor of angular microrotational velocities of the fluid particles that is determined as the gradient of the vorticity vector \( \omega_i = (1/2)\epsilon_{ijsr}v_{s,r} \); \( K_{ij} = \omega_{j,i} \); \( \delta_{ij}, \epsilon_{ijk} \) are the Kronecker and Levi-Civita symbols.

As a hydrodynamic model of suspended particles, a uniaxial dumbbell with axis \( L \) is utilized. According to [7], the friction coefficient \( \xi \) of dumbbell beads in the Stokes fluid (2.1), (2.2) does not depend on the flow around the beads:

\[ \xi = \xi_N (1 + B) \quad B = \frac{2 + \nu}{r_0^2 + (2 + \nu) \eta_0} \quad (2.3) \]

In equation (2.3), \( \xi_N \) is the friction coefficient of a dumbbell bead of radius \( r \) in slow translational motion in the Newtonian fluid with dynamic viscosity \( \mu \), \( \xi_N = 6\pi \mu r \); \( \nu = \eta' / \eta \); \( r_0 = r / l_0 \), here \( l_0 = \eta / \mu \).

In order to model blood by the Stokes fluid (2.1), (2.2), the material characteristic length \( l_0 \) is evaluated as function of the haematocrit value of blood \( C_b \) on the base of results obtained in [5] and [8] (Table 1).

| \( l_0 \) \( \cdot 10^6 \), \( m \) | 6.666 | 8.000 | 14.815 |
|---|---|---|---|
| \( C_b \), % | 6 | 13 | 40 |

Table 1. The dependence of parameter \( l_0 \) on the haematocrit value of blood \( C_b \).

It is assumed in the paper that the suspended particles modeled by uniaxial dumbbells are magnetically sensitive, namely, they possess a permanent magnetic moment \( p_i = Pn_i \), where \( P \) is the value of permanent magnetic moment; \( n_i \) is the unit vector characterizing the orientation of an axially symmetric suspended particle as well as the orientation of its dumbbell model. It is also assumed that the suspension is diluted to such an extent that the interaction between the magnetic fields of the suspended particles, as well as the hydrodynamic interaction between them is not taken into account.

Such a modeling allows us to perform the microfluidic study of the considered suspension in blood at a scale of suspended microparticles using the procedures of the Einstein’s method [4]. So, we obtain the constitutive equation for vectors \( n_i \) and \( N_i \)

\[ N_i = d_{ik}n_k + d_{km}n_k n_m n_i + \frac{p}{W} (N_i - n_k H_k n_i), \quad (2.4) \]

where \( N_i \) is the vector characterising the angular velocity of suspended particles with respect to the carrier fluid, \( N_i = \dot{n}_i - \omega_{ik}n_j \); here, the dot over \( n_i \) denotes the local time derivation; \( \omega_{ik} \) is the velocity vortex tensor, \( \omega_{ik} = (1/2)(v_{i,k} - v_{k,i}) \); \( W \) is the rotational friction coefficient of a uniaxial
dumbbell in blood as a carrier fluid of the suspension, \( W = (1/2)\xi L^2 \); \( H_i \) is the external magnetic field strength.

In the frame of this microfluidic study we also obtain the rate of mechanical energy dissipation per unit volume of the suspension

\[
\Phi = \Phi_0 + n_0 \frac{\xi L^2}{2} (N_i N_j) - 2d_{ij}(N_i n_j) + d_{ij}d_{ik}(n_i n_k),
\]

(2.5)

where \( \Phi_0 \) is the rate of mechanical energy dissipation per unit volume of the carrier fluid of the suspension in the absence of suspended particles; \( n_0 \) is the number of suspended particles per unit volume of the suspension; the angular brackets \( \langle \rangle \) denote averaging with the use of the distribution function \( F \) of angular positions of suspended particles, which satisfies the equation

\[
\frac{\partial F}{\partial t} + \frac{\partial}{\partial n_j}(F\dot{n}_j) = 0.
\]

(2.6)

According to the structure-phenomenological approach [3] used in the paper, the inequality \( d << \bar{I} \) allows us to model the considered suspension in blood at the scale of the region of suspension’s macroflow by microcontinuum with two internal microparameters \( n_i \) and \( N_i \) defining the orientation and relative angular velocity of suspended microparticles. In this case, the rheological equation defining the stress \( T_{ij} \) in the modeled suspension is postulated phenomenologically. According to [3], the most general form of such an equation is

\[
T_{ij} = t_{ij} + n_0[(a_0 + a_i d_{km}(n_k n_m))\delta_{ij} + a_2(n_i n_j) + a_3 d_{km}(n_k n_i n_j n_j) + a_4 d_{ij} + a_5 d_{jk}(n_k n_j) + a_6 d_{ik}(n_i n_j) + a_7(n_i N_j) + a_8(n_j N_i)],
\]

(2.7)

where \( t_{ij} \) is the stress tensor in the carrier fluid of the suspension in the absence of the suspended particles; \( n_0 \) is the quantity of suspended particles per unit volume of the suspension; \( a_i \) \((i = 0, ..., 8)\) are constant phenomenological coefficients.

The phenomenological coefficients \( a_i \) \((i = 0, ..., 8)\) in the equation (2.7) are found from comparing the rate of mechanical energy dissipation per unit volume of the suspension defined by the equation (2.5) that was determined in the microfluidic part of the theory, with the rate determined in the same way as in [3] within the framework of the structure-continual approach. As a consequence of this, we obtain the rheological equation of a dilute suspension with blood as the carrier fluid

\[
T_{ij} = t_{ij} + \frac{1}{2} n_0 \xi L^2 (d_{ik}(n_k n_j) - \langle n_i n_j \rangle).
\]

(2.8)

It is demonstrated in [9] that such an equation as (2.4) has the stationary solution \( \langle \dot{n}_i = 0, N_i = -\omega_k n_k \rangle \) for steady-state shear flows of a suspension in the presence of the steady-state magnetic field \( H_j \). This means that the suspended dumbbell particles of the considered suspension in blood acquire the same stationary orientation defined by the constitutive equation

\[
W_{ij} = -\frac{1}{2} d_{km} n_k n_m n_i + P(H_i - n_k H_k n_i) = 0
\]

(2.9)

under above-mentioned conditions. The rheological equation for stress (2.8) in such an anisotropic suspension takes the form

\[
T_{ij} = t_{ij} + n_0 W_{ij} n_k n_i n_j.
\]

(2.10)

\( W \) is a single rheological parameter in equations (2.9) and (2.10), which characterizes the interaction of suspended particles with blood modeled by the Stokes fluid (2.1), (2.2).

According to equations (2.3), taking into account the couple stresses \( \mu_{ij} \) arising in blood modeled by the Stokes fluid (2.1), (2.2) leads to an increase in the rotational friction coefficient of the
suspended dumbbell particles: \( W = W_N (1 + B) \), as compared with the value \( W_N = (1/2) \xi N L^2 \) in a suspension with the Newtonian model of blood as the suspension carrier fluid.

3. Magnetobiorheological behavior of dilute suspensions in blood of magnetic particles.

The obtained constitutive equations (2.9), (2.10) are used to study the rheological behavior of the considered suspension in the steady simple shear flow \( v_x = 0, \ v_y = K x, \ v_z = 0 \ (K = const) \) in the presence of the cross magnetic field \( H_x = H, \ H_y = H_z = 0 \ (H = const) \). It allows to investigate the influence of the shear rate \( K \) of the flow, the haematocrit value \( C_b \) of blood and the strength \( H \) of the external magnetic field on effective viscosity of the considered suspension \( \mu_a \equiv (T_{xy} + T_{yx})/(2K) \) and on the non-zero difference of normal stresses \( \sigma_1 \equiv T_{yy} - T_{xx} \) arising in it.

The calculations of the numerical values of the characteristic viscosity of the suspension \( [\mu_a] \equiv (\mu_a - \mu)/(PH) \) and \( \sigma_1^*/K \) as functions of \( \alpha = KW_N/(PH) \) (Figures 1 and 2) show that the increase of the shear rate \( K \) of the simple shear flow at the fixed strength \( H \) of the cross magnetic field leads to the pseudoplastic decrease of the effective viscosity \( \mu_a \) of the suspension and to the increase of the difference \( \sigma_1 \) (the Weissenberg effect).

**Figure 1:** The dependences of \([\mu_a]\) and \(\sigma_1^*/K\) on \(\alpha\) at \(\bar{p} = 10\), curves 1 correspond to the suspension with the Newtonian carrier fluid of the viscosity \(\mu\); curves 2–4 correspond to the suspension in blood with the haematocrit values \(C_b = 6\%, 13\%, 40\%\), respectively.

**Figure 2:** The dependences of \([\mu_a]\) and \(\sigma_1^*/K\) on \(\alpha\) at \(C_b = 40\%\), curves 1–3 correspond to \(\bar{p} = 3, 5, 10\), respectively.
The following designations are used: \( \sigma_1^* = \sigma_1 / (\mu V) \); \( V \) is the volume concentration of suspended particles, \( V = (4/3)n_0 \pi a b^2 \), where \( a \) and \( b \) are semiaxes of ellipsoid of revolution that is equivalent to the dumbbell as a hydrodynamic model of suspended particles in the Newtonian carrier fluid with the viscosity \( \mu \); \( \bar{\rho} = a/b \). The calculation was made at the length \( L = 4 \cdot 10^{-5} \text{m} \) of suspended particles with axial ratios \( \bar{\rho} = 3, 5, 10 \) of them.

The calculations made in the paper show as well that the considered suspension in blood exhibits the magnetorheological properties. We obtain that an increase of the strength \( H \) of the magnetic field at the fixed shear rate \( K \) of the simple shear flow leads to the increase of the effective viscosity \( \mu_a \) of the considered suspension in blood and leads in addition to the decrease of the difference \( \sigma_1 \) of normal stresses. The detected variations of rheological characteristics of the suspension are the consequences of variations of orientation of suspended particles under the action of hydrodynamic forces and an external magnetic field.

The constructed rheological model allows us to investigate biorheological properties of the considered suspension. Curves 2–4 on Fig.1 show that the effective viscosity \( \mu_a \) of the studied suspension in blood and the difference \( \sigma_1 \) of normal stresses in it are augmented with increasing the haematocrit value \( C_b \) of blood holding \( K \) and \( H \) fixed.

4. Conclusions.
The structure-phenomenological method of modeling used in the paper allowed to express explicitly the macrorheological (continual) characteristics of dilute suspension in blood of magnetically sensitive microparticles through the parameters describing microrheological, microstructural and physical properties of blood and microparticles suspended in it. Thanks to such an advantage of the structure-phenomenological method, the biorheological and magnetorheological studies of the considered suspension have been made possible. The results obtained in the paper may be used in medicine. In particular, the change of the strength \( H \) of an external magnetic field may be used as a control factor of the rheological properties of a suspension formed on addition of magnetically sensitive elongated particles to blood.

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