Phase Shifts and Mixing Parameters for Elastic Proton-Deuteron Scattering

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Abstract

The eigenphase shifts and mixing parameters for elastic p-d scattering below the deuteron-breakup threshold are calculated for the AV14 two-body potential using two different numerical methods. The excellent agreement confirms that it is possible to perform accurate numerical studies of p-d scattering as well as n-d scattering for low energies. The numerical results can be considered as benchmarks for future calculations.
I. Introduction

The theoretical investigation of the three-nucleon system is an important tool to test our knowledge of the nuclear interaction. This system is simple enough that one can obtain accurate solutions for the $^3$H and $^3$He bound-state systems. Modern realistic nucleon-nucleon two-body potential models that fit the data with a $\chi^2$ per datum very close to unity underbind the trinucleon bound states by 0.5 - 1.0 MeV. A possible solution to this discrepancy is to include the three-nucleon interaction in the model Hamiltonian. Various models of the three-nucleon force have been adjusted so that the correct three-body binding energy is obtained. Since the other bound-state observables scale with the binding energy, the bound-state system cannot be used to test our understanding of the three-body force.

Initially it was believed that the nucleon-deuteron scattering problem could be used to test our understanding of the three-nucleon force; however, it has been shown [1] that most of the scattering data can be reproduced at a good level with only two-body interactions. For low-energy scattering the effects of the three-nucleon force are usually small; there are nevertheless some discrepancies (such as the nucleon analyzing power $A_y$ and the deuteron analyzing power $iT_{11}$) that could be sensitive to these interactions. Since much of the experimental data is for p-d scattering at low energies, where the Coulomb interaction cannot be neglected, it is necessary to solve the scattering equations for this case as well as for the n-d case.

Various groups have published benchmark calculations showing that it possible to obtain accurate solutions of the n-d Faddeev scattering equations for energies above the breakup threshold [2, 3] and using s-wave potentials. Recently, a benchmark calculation for n-d scattering has been published at $E_{\text{lab}} = 3$ MeV (just below the deuteron breakup threshold) using a realistic NN potential [4]. Conversely, at present there has been no thorough study of the accuracy of the solutions for p-d scattering. In this paper we present a detailed comparison of the p-d phase shifts and mixing parameters obtained by two different configuration-space calculations. These results can serve as benchmarks for the low-energy p-d scattering problem. The methods used for these calculations can be extended above the deuteron-breakup threshold, and some preliminary calculations have been published for this case [5].

II. Results

The two methods used for the results in this paper have been described in previous papers. The Pisa group [6, 7, 8] uses the Pair-correlated Hyperspherical Harmonic (PHH) basis to expand the wave function, and the corresponding S-matrix is obtained using the complex form of the Kohn variational principle. The Los Alamos-Iowa (LA-Iowa) group [9, 10] solves the Faddeev-Noyes equations using a spline expansion in configuration space with boundary conditions appropriate to two-body Coulomb scattering in the asymptotic region.

For the two-body potential we choose the Argonne AV14 model [11], since this was used in a previous benchmark paper for n-d scattering [4]. We consider the same parameters and energies in order to show the effects of the Coulomb interaction on the results. There is one small difference from the previous calculations, we use
$h^2/M$ (MeV·fm$^2$) = 41.47 instead of the value of 41.473 used previously. The conventions used for the phase shifts and mixing parameters are the same as in Ref. [4]. For completeness, we show in Table 1 the binding energies and scattering lengths for this potential. The Pisa values shown in the table were previously published in Ref. [8], while the LA-Iowa numbers are the results of recent more accurate calculations. Also, we give in Table 2 the n-d results for our value of $h^2/M$. Our primary results, the p-d phase shifts and mixing parameters, are given in Table 3, where one can see that the two calculations differ by less than 1%, and in many cases (such as the large phase shifts) the differences are much smaller than that.

III. Conclusions

The results from two very different numerical methods are in excellent agreement for p-d scattering at energies below the deuteron-breakup threshold. This confirms that in this case it is possible in practice to perform theoretical studies with the same precision as for the n-d problem. These results also provide a benchmark against which researchers in the future can test their techniques.

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References

[1] W. Glöckle, H. Witała, D. Hüber, H. Kamada, and J. Golak, Phys. Rep. 274, 107 (1996).

[2] J. L. Friar, B. F. Gibson, G. Berthold, W. Glöckle, Th. Cornelius, H. Witała, J. Heidenbauer, Y. Koike, G. L. Payne, J. A. Tjon, and W. Kloet, Phys. Rev. C 42, 1838 (1990).

[3] J. L. Friar, G. L. Payne, W. Glöckle, D. Hüber, and H. Witała, Phys. Rev. C 51, 2356 (1995).

[4] A. Kievsky, M. Viviani, S. Rosati, D. Hüber, W. Glöckle, H. Kamada, H. Witała and J. Golak, Phys. Rev. C 58, 3085 (1998).

[5] A. Kievsky, S. Rosati, and M. Viviani Phys. Rev. Lett. 82, 3759 (1999).

[6] A. Kievsky, M. Viviani and S. Rosati, Nucl. Phys. A551, 241 (1993).

[7] A. Kievsky, M. Viviani and S. Rosati, Nucl. Phys. A577, 511 (1994); A. Kievsky, M. Viviani and S. Rosati, Phys. Rev. C 52, R15 (1995).

[8] A. Kievsky, Nucl. Phys. A624, 125 (1997).

[9] C. R. Chen, G. L. Payne, J. L. Friar, and B. F. Gibson, Phys. Rev. C 44, 50 (1991).
[10] J. L. Friar and G. L. Payne, “Proton-Deuteron Scattering and Reactions,” in *Coulomb Interactions in Nuclear and Atomic Few-Body Collisions*, ed. by F. S. Levin and D. A. Micha, (Plenum Press, New York, 1996), p. 97.

[11] R.B. Wiringa, R.A. Smith, T.L. Ainsworth, Phys. Rev. C 29, 1207 (1984).
|                  | Pisa   | LA-Iowa |
|------------------|--------|---------|
| $E_B(^3\text{H})$ | 7.684  | 7.684   |
| $E_B(^3\text{He})$ | 7.033  | 7.033   |
| $^2a_{nd}$       | 1.189  | 1.191   |
| $^4a_{nd}$       | 6.379  | 6.376   |
| $^2a_{pd}$       | 0.941  | 0.945   |
| $^4a_{pd}$       | 13.773 | 13.752  |

Table 1: Comparison of the binding energies in MeV and scattering lengths in fermis determined by the Kohn variational method (Pisa) and the configuration-space Faddeev equations (LA-Iowa) for the AV14 NN potential.
| $J^\pi$ | $\delta_{\Sigma \Lambda}$ | $E_{\text{lab}} = 1 \text{ MeV}$ | $E_{\text{lab}} = 2 \text{ MeV}$ | $E_{\text{lab}} = 3 \text{ MeV}$ |
|--------|----------------|-----------------|-----------------|-----------------|
| $\frac{1}{2}^+$ | $\delta_{\frac{3}{2}^+}$ | -0.999 | -0.999 | -2.573 | -2.573 | -3.903 | -3.901 |
| | $\delta_{\frac{1}{2}^+}$ | -17.696 | -17.702 | -27.866 | -27.864 | -34.822 | -34.812 |
| | $\eta$ | 1.043 | 1.041 | 1.205 | 1.203 | 1.253 | 1.251 |
| $\frac{1}{2}^-$ | $\delta_{\frac{3}{2}^-}$ | -4.195 | -4.195 | -6.654 | -6.651 | -7.532 | -7.526 |
| | $\delta_{\frac{1}{2}^-}$ | 12.384 | 12.383 | 20.558 | 20.551 | 25.052 | 25.026 |
| | $\epsilon$ | 3.742 | 3.742 | 5.394 | 5.393 | 7.256 | 7.255 |
| $\frac{3}{2}^+$ | $\delta_{\frac{5}{2}^+}$ | -47.148 | -47.143 | -61.329 | -61.334 | -70.491 | -70.532 |
| | $\delta_{\frac{3}{2}^+}$ | 0.580 | 0.580 | 1.547 | 1.547 | 2.419 | 2.418 |
| | $\delta_{\frac{1}{2}^+}$ | -1.073 | -1.073 | -2.771 | -2.770 | -4.214 | -4.212 |
| | $\epsilon$ | 0.651 | 0.650 | 0.716 | 0.716 | 0.780 | 0.780 |
| | $\xi$ | 0.545 | 0.546 | 1.010 | 1.010 | 1.438 | 1.437 |
| | $\eta$ | -0.114 | -0.114 | -0.247 | -0.247 | -0.387 | -0.388 |
| $\frac{3}{2}^-$ | $\delta_{\frac{5}{2}^-}$ | 14.285 | 14.287 | 22.722 | 22.721 | 26.398 | 26.382 |
| | $\delta_{\frac{3}{2}^-}$ | -1.307 | -1.309 | -1.970 | -1.972 | -2.761 | -2.766 |
| | $\epsilon$ | -0.184 | -0.185 | -0.269 | -0.269 | -0.258 | 0.258 |
| | $\xi$ | -1.108 | -1.110 | -2.316 | -2.318 | -3.804 | -3.807 |
| | $\eta$ | -1.108 | -1.110 | -2.316 | -2.318 | -3.804 | -3.807 |
| $\frac{5}{2}^+$ | $\delta_{\frac{7}{2}^+}$ | -0.015 | -0.015 | -0.093 | -0.093 | -0.211 | -0.211 |
| | $\delta_{\frac{3}{2}^+}$ | 0.575 | 0.575 | 1.528 | 1.529 | 2.384 | 2.384 |
| | $\delta_{\frac{1}{2}^+}$ | -1.141 | -1.141 | -2.974 | -2.973 | -4.567 | -4.564 |
| | $\epsilon$ | -0.288 | -0.288 | -0.309 | -0.308 | -0.328 | -0.327 |
| | $\xi$ | -0.287 | -0.287 | -0.522 | -0.521 | -0.737 | -0.735 |
| | $\eta$ | -0.870 | -0.870 | -1.573 | -1.572 | -2.156 | -2.154 |
| $\frac{5}{2}^-$ | $\delta_{\frac{7}{2}^-}$ | 13.442 | 13.452 | 22.026 | 22.029 | 26.347 | 26.350 |
| | $\delta_{\frac{3}{2}^-}$ | -0.065 | -0.065 | -0.257 | -0.257 | -0.476 | -0.476 |
| | $\delta_{\frac{1}{2}^-}$ | 0.131 | 0.131 | 0.523 | 0.524 | 0.971 | 0.971 |
| | $\epsilon$ | -0.468 | -0.468 | 0.493 | 0.493 | 0.517 | 0.512 |
| | $\xi$ | 0.414 | 0.415 | 0.728 | 0.729 | 0.985 | 0.985 |
| | $\eta$ | -0.131 | -0.131 | -0.255 | -0.255 | -0.361 | -0.361 |

Table 2: Comparison of the n-d phase shifts and mixing parameters (in degrees) determined by the Kohn variational method (Pisa) and the configuration-space Faddeev equations (LA-Iowa) for the AV14 NN potential.
| $J^\pi$ | $E_{\text{lab}} = 1\,\text{MeV}$ | $E_{\text{lab}} = 2\,\text{MeV}$ | $E_{\text{lab}} = 3\,\text{MeV}$ |
|---|---|---|---|
| | Pisa | LA-Iowa | Pisa | LA-Iowa | Pisa | LA-Iowa |
| $1^+_{\frac{1}{2}}$ | $0.787$ | $0.787$ | $2.281$ | $2.281$ | $3.615$ | $3.615$ |
| | $-12.067$ | $-12.616$ | $-23.525$ | $-23.535$ | $-31.400$ | $-31.414$ |
| | $1.189$ | $1.187$ | $1.257$ | $1.256$ | $1.259$ | $1.260$ |
| $1^-$ | $-3.394$ | $-3.393$ | $-6.122$ | $-6.120$ | $-7.405$ | $-7.401$ |
| | $9.431$ | $9.432$ | $17.896$ | $17.894$ | $22.815$ | $22.799$ |
| | $3.128$ | $3.133$ | $4.589$ | $4.593$ | $6.227$ | $6.229$ |
| $3^+$ | $-37.335$ | $-37.323$ | $-53.436$ | $-53.436$ | $-63.673$ | $-63.705$ |
| | $0.454$ | $0.454$ | $1.357$ | $1.356$ | $2.206$ | $2.204$ |
| | $-0.849$ | $-0.849$ | $-2.459$ | $-2.459$ | $-3.905$ | $-3.903$ |
| | $0.819$ | $0.817$ | $0.792$ | $0.796$ | $0.833$ | $0.836$ |
| | $0.533$ | $0.532$ | $0.976$ | $0.974$ | $1.385$ | $1.384$ |
| | $-0.095$ | $-0.093$ | $-0.215$ | $-0.213$ | $-0.343$ | $-0.340$ |
| $3^-$ | $10.957$ | $10.958$ | $20.152$ | $20.150$ | $24.628$ | $24.615$ |
| | $-1.077$ | $-1.080$ | $-1.650$ | $-1.653$ | $-2.327$ | $-2.333$ |
| | $-0.191$ | $-0.190$ | $-0.296$ | $-0.295$ | $-0.322$ | $0.321$ |
| | $-0.998$ | $-0.994$ | $-2.091$ | $-2.092$ | $-3.362$ | $-3.365$ |
| $4^+_{\frac{3}{2}}$ | $-0.011$ | $-0.011$ | $-0.081$ | $-0.081$ | $-0.194$ | $-0.193$ |
| | $0.451$ | $0.451$ | $1.342$ | $1.342$ | $2.176$ | $2.175$ |
| | $-0.905$ | $-0.905$ | $-2.640$ | $-2.639$ | $-4.229$ | $-4.227$ |
| | $-0.378$ | $-0.378$ | $-0.353$ | $-0.355$ | $-0.362$ | $-0.364$ |
| | $-0.318$ | $-0.320$ | $-0.530$ | $-0.529$ | $-0.736$ | $-0.734$ |
| | $-0.997$ | $-1.006$ | $-1.617$ | $-1.621$ | $-2.189$ | $-2.186$ |
| $4^-$ | $10.237$ | $10.246$ | $19.279$ | $19.289$ | $24.216$ | $24.222$ |
| | $-0.051$ | $-0.051$ | $-0.229$ | $-0.228$ | $-0.443$ | $-0.442$ |
| | $0.103$ | $0.103$ | $0.464$ | $0.464$ | $0.899$ | $0.899$ |
| | $-0.234$ | $-0.223$ | $0.259$ | $0.261$ | $0.360$ | $0.361$ |
| | $0.409$ | $0.408$ | $0.729$ | $0.729$ | $0.992$ | $0.992$ |
| | $-0.136$ | $-0.135$ | $-0.258$ | $-0.257$ | $-0.366$ | $-0.365$ |

Table 3: Comparison of the p-d phase shifts and mixing parameters (in degrees) determined by the Kohn variational method (Pisa) and the configuration-space Faddeev equations (LA-Iowa) for the AV14 NN potential.