Radiative $B - L$ symmetry breaking in supersymmetric models

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We propose a scheme where the three relevant physics scales related to the supersymmetry, electroweak, and baryon minus lepton ($B - L$) breakings are linked together and occur at the TeV scale. The phenomenological implications in the Higgs and leptonic sectors are discussed.

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Nonvanishing neutrino masses and the existence of non-baryonic dark matter (DM) represent the only two firm observational evidences of new physics (NP) beyond the Standard Model (SM). The energy scale(s) related to such NP are unknown with theoretical proposals ranging from scales close to the electroweak (EW) scale (TeV NP) to much higher scales (GUT or Planck NP). A possible criterion to follow is to link such NP scale(s) to the breaking of symmetries associated to the new particles appearing in the enlarged NP particle spectrum. For instance, in the case where NP is identified with supersymmetry (SUSY), the energy scale at which the breaking of SUSY occurs (in the observable sector) and the typical mass scale for the SUSY particles have to be linked to the (EW) scale if SUSY is called to provide the correct ultraviolet completion of the SM to avoid the gauge hierarchy problem. In turn, the presence of SUSY particles at the TeV scale could provide a solution to the DM problem through the presence of the stable lightest SUSY particle in models with the discrete symmetry called R parity.

In the case of neutrino masses, the new particles which are involved are likely to be the right-handed (RH) neutrinos and the relevant symmetry to be broken should be the difference of the baryon ($B$) and lepton ($L$) quantum numbers ($B - L$). Indeed, the (Majorana) mass of the RH neutrino breaks $L$ or $B - L$ and, once present, one is naturally lead to light neutrino masses through a see-saw mechanism. However, at variance with the SUSY case, here the breaking scale of B-L is left undetermined by the request of obtaining a phenomenologically viable neutrino mass spectrum.

In this Letter we propose a possible link between the $B - L$ and EW scales in SUSY models with a see-saw mechanism for neutrino masses. Once we are in a SUSY context, we can nicely correlate the EW and SUSY scales through the mechanism of radiative breaking of the EW symmetry. Indeed, it was shown [1] that radiative corrections may drive the squared Higgs mass from positive initial values at the GUT scale to negative values at the EW scale. In such a framework, the size of the Higgs vacuum expectation value (VEV) responsible for the EW breaking is determined by the size of the top Yukawa coupling and of the soft SUSY breaking terms. Analogously, we show that in a SUSY see-saw scheme it is possible to radiatively induce the breaking of $B - L$ having the scale of such breaking directly linked to the size of some (large) RH neutrino Yukawa coupling and of the soft SUSY breaking scale. In particular, we prove that for such Yukawa coupling of the order of the top quark Yukawa coupling, the radiative mechanism leads to a $B - L$ breaking scale of the same order as the scale of the SUSY soft breaking terms, i.e. a TeV breaking of $B - L$.

Our result nicely fits with a minimal extension for the SM based on TeV scale gauge $B - L$ that has been recently proposed [2]. It was shown that this type of models can account for current experimental results of light neutrino masses and their large mixing [3]. In addition, the extra-gauge boson and extra-Higgs predicted in this model have a rich phenomenology and can be detected at the LHC [4]. A non-vanishing vacuum expectation value (VEV), $v'$, that breaks the $B - L$ gauge symmetry was obtained in analogy with what happens for the EW breaking. However, in such construction the scale of the scalar potential leading to $v'$ was set by hand to be of $\mathcal{O}(1)$ TeV, much in the same way that the VEV responsible for the breaking of the EW symmetry arises from an ad hoc choice of the $\mu$ and $\lambda$ parameters of the SM scalar potential. In this Letter we construct a supersymmetric version of $G_{B-L} = SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ model, which has been analyzed in Ref. [2, 3, 4]. We work out the renormalization group equation (RGE) for the relevant parameters in the $B - L$ sector, in particular the squared mass of the extra Higgs bosons. We study their evolution from GUT to TeV scale and show that the squared mass of one of these Higgs bosons can be pulled down to negative values leading to the spontaneous breaking of the $B - L$ symmetry. The spontaneous breaking of the gauge $B - L$ symmetry is going to occur at a scale of $\mathcal{O}(1)$ TeV or slightly higher when the following three conditions are met: (i) The soft SUSY breaking terms associated to the $B - L$ sector are of order TeV. (ii) The analog of the Higgs mixing term $\mu$ in the MSSM, namely the mixing parameter of the new Higgs superfields involved in the $B - L$ breaking, $\mu'$, is of the same size as the soft SUSY breaking terms. (iii) The Yukawa coupling of the right-handed neutrino, $Y_N = M_N / v'$ is of order unity. A relevant remark is in order. In building our extension of the MSSM, we introduce
in the superpotential of the theory a new parameter, $\mu'$, which has the dimension of a mass, in addition to the Higgs mixing $\mu$ parameter of the MSSM. As known, this latter parameter is present in the SUSY invariant part of the theory and hence its scale is not directly set by the scale of the soft breaking parameters. Why $\mu$ should then be at the TeV scale and not, for instance, at the superlarge scale of supergravity breaking constitutes the so-called $\mu$ problem. A possible suggestion to obtain a $\mu$ scale of the order of the EW scale is known as the Giudice-Masiero mechanism [5]. Here we are advocating that this same mechanism could be responsible also for the origin of the $\mu'$ parameter, hence implying a similar mass scale for both of them.

We consider the minimal supersymmetric version of the $B-L$ extension of the SM based on the gauge group $G_{B-L} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. The particle content of the SUSY $B-L$ includes the following fields in addition to those of the MSSM: three chiral right-handed superfields $(N_i)$, the vector superfield necessary to gauge the $U(1)_{B-L}$ ($Z_{B-L}$), and two chiral SM-singlet Higgs superfields $(\chi_1, \chi_2)$ with $B-L$ charges $Y_{B-L} = -2$ and $Y_{B-L} = +2$, respectively. As in the MSSM, the introduction of a second Higgs singlet $(\chi_2)$ is necessary in order to cancel the $U(1)_{B-L}$ anomalies produced by the fermionic member of the first Higgs $(\chi_1)$ superfield. The $Y_{B-L}$ for quark and lepton superfields are assigned in the usual way.

The interactions between Higgs and matter superfields are described by the superpotential

$$W = (Y_U)_{ij} Q_i H_2 U_j^c + (Y_D)_{ij} Q_i H_1 D_j^c + (Y_L)_{ij} L_i H_1 E_j^c + (Y_e)_{ij} L_i H_2 N_j^c + (Y_N)_{ij} N_i^c N_j^c \chi_1 + \mu H_1 H_2 + \mu' \chi_1 \chi_2. \tag{1}$$

Interestingly enough, the presence of the $B-L$ gauge symmetry, forbids the appearance in the superpotential of the $B$ or $L$ violating terms. Hence, in this model there is no need to impose an additional discrete symmetry (for instance, $R$ parity) to achieve such result.

Assuming flavor and gaugino universality at the grand unification scale, $M_X = 3 \times 10^{16}$ GeV, the SUSY soft breaking Lagrangian at that scale reads

$$\mathcal{L}_{soft} = m_0^2 \left[|\tilde{Q}_i|^2 + |\tilde{U}_i|^2 + |\tilde{D}_i|^2 + |\tilde{L}_i|^2 + |\tilde{E}_i|^2 + |\tilde{N}_i|^2 + |H_1|^2 + |H_2|^2 + |\chi_1|^2 + |\chi_2|^2\right]$$

$$+ \frac{1}{2} Y^U \tilde{Q} \tilde{U} c H_2 + Y^D \tilde{Q} \tilde{D} c H_1 + Y^L \tilde{L} \tilde{E} c H_1$$

$$+ Y^\nu \tilde{L} \tilde{N}^c c H_2 + Y^N \tilde{N} \tilde{N}^c c \chi_1 + B(\mu H_1 H_2) + \mu' \chi_1 \chi_2 + h.c. + \frac{1}{2} M_{1/2} \left[\tilde{g}^a \tilde{g}^a + W^a \tilde{W}^a\right] + \tilde{B} \tilde{B} + \tilde{Z}_{B-L} \tilde{Z}_{B-L} + h.c. \right],$$

where $(Y^A)_{ij} \equiv (Y A)_{ij}$. The tilde denotes the scalar components of the chiral matter superfields and fermionic components of vector superfields. We denote by $H_{1,2}$ and $\chi_{1,2}$ also the scalar components of the Higgs superfields $H_{1,2}$ and $\chi_{1,2}$.

Let us now discuss how the $B-L$ and electroweak symmetries may be broken in the SUSY $G_{B-L}$. We have to study the scalar potential for the Higgs fields $\chi_{1,2}$ and $H_{1,2}$ and check if there are minima for which $\langle \chi_1 \rangle \neq 0$ and $\langle H_1 \rangle, \langle H_2 \rangle \neq 0$. The scalar potential for $H_{1,2}$ and $\chi_{1,2}$ is

$$V(H_1, H_2, \chi_1, \chi_2) = \frac{1}{2} g^2 \left(\frac{v^2}{2} \right)^2 H_1^2 + \frac{1}{2} g^2 \left(\frac{v^2}{2} \right)^2 H_2^2 + \frac{1}{2} g^{'2} \left(\frac{v^2}{2} \right)^2 \chi_1^2 + m_1^2 \chi_1^2 + m_2^2 \chi_2^2 - m_3^2 (H_1 H_2 + h.c.)$$

$$+ \mu_1^2 |\chi_1|^2 + \mu_2^2 |\chi_2|^2 - \mu_3^2 (\chi_1 \chi_2 + h.c.),(3)$$

where

$$m_i^2 = m_0^2 + \mu^2, \quad i = 1, 2 \quad m_0^2 = -B \mu, \quad (4)$$

$$\mu_1^2 = m_0^2 + \mu^2, \quad i = 1, 2 \quad \mu_0^2 = -B \mu', \quad (5)$$

As can be seen from Eq. (3), the potential $V(H_1, H_2, \chi_1, \chi_2)$ results from the sum of the usual MSSM scalar potential $V(H_1, H_2)$ and of the new potential $V(\chi_1, \chi_2)$ which exhibits an apparent similarity in its structure to $V(H_1, H_2)$. For simplicity, in defining $\mu_1^2$ and $m_1^2$ only one $B$ parameter has been introduced.

As is known, the radiative breaking of the EW symmetry is induced by the running from $M_X$ to the weak scale of $m_3^2$. Given the large value of the top Yukawa coupling, such running succeeds to turn the positive value of $m_3^2$ at $M_X$ to a negative value, hence inducing the desired EW breaking.

Following the same steps as for the minimization of $V(H_1, H_2)$ in the MSSM, one readily obtains for the minimization of $V(\chi_1, \chi_2)$:

$$v^2 = (v_1^2 + v_2^2) = \frac{(\mu_1^2 - \mu_2^2) - (\mu_1^2 + \mu_2^2) \cos \theta}{2 g^{'2} \cos \theta}, \tag{6}$$

where $\langle \chi_1 \rangle = v_1'$ and $\langle \chi_2 \rangle = v_2'$. The angle $\theta$ is defined as $\tan \theta = \tau_1' / \tau_2'$. Consequently, the $Z_{B-L}$ gauge boson acquires a mass [6]:

$$M^2_{Z_{B-L}} = 4 g^{'2} v^2.$$

The boundness from below of the potential $V(\chi_1, \chi_2)$ requires

$$\mu_1^2 + \mu_2^2 > 2 |\mu_3^2|, \tag{7}$$

This represents the stability condition for the potential. Furthermore, to avoid that $\langle \chi_1 \rangle = \langle \chi_2 \rangle = 0$ be a local minimum one has to impose that

$$\mu_1^2 \mu_2^2 < \mu_3^4. \tag{8}$$

It is not possible to simultaneously fulfill both the above conditions for the positive values of $\mu_1^2$ and $\mu_2^2$ as given in Eq. (4). Indeed, if $\mu_1^2 = \mu_2^2 < \mu_3^4$, then the condition Eq. (7) is not satisfied and the scalar potential is unbounded from below in the direction $\langle \chi_1 \rangle = \langle \chi_2 \rangle \rightarrow \infty$.

The problem we encounter is reminiscent of what occurs for the electroweak symmetry breaking, i.e. for the $V(H_1, H_2)$
part of the potential \(V(H_1, H_2, \chi_1, \chi_2)\). In that case, the problem of obtaining the desired breaking vacuum while guaranteeing the stability of the potential is solved by noting that the boundary conditions Eq. (4) are valid only at the GUT scale. However, in the running from that large scale down to \(M_W\), one finds that \(m_{\chi_1}^2\) and \(m_{\chi_2}^2\) get renormalized differently if \(H_1\) and \(H_2\) couple with different strengths to fermions. Indeed, \(H_2\) couples to the top quark with a large Yukawa coupling. The running from \(M_X\) down to the weak scale reduces the squared Higgs masses, until eventually the minimization condition is satisfied and the electroweak gauge symmetry is broken.

We consider the \(B-L\) renormalization group equations and analyze the running of the scalar masses \(m_{\chi_1}^2\) and \(m_{\chi_2}^2\). The key point for implementing the radiative \(B-L\) symmetry breaking is that the scalar potential \(V(\chi_1, \chi_2)\) receives substantial radiative corrections. In particular, a negative (mass)\(^2\) would trigger the \(B-L\) symmetry breaking of \(B-L\). We argue that the masses of Higgs singlets \(\chi_1\) and \(\chi_2\) run differently in the way that \(m_{\chi_1}^2\) can be negative whereas \(m_{\chi_2}^2\) remains positive. The renormalization group equation (RGE) for the \(B-L\) couplings and mass parameters can be derived from the general results for SUSY RGEs of Ref. [6]. Here, for simplicity, we neglect the couplings of the first two generations. As is known, neglecting the Yukawa couplings of the first two generations for the SM quark and lepton is quite different from the general results for SUSY RGEs of Ref. [6]. Here, for the \(B-L\) coupling \(h_N\), this is a further assumption. Also it is more convenient to write the RGE in terms of gauge couplings: \(\tilde{\alpha}_i = g_i^2 / 16 \pi^2\) and Yukawa couplings: \(\tilde{Y}_i = Y_i^2 / 16 \pi^2\).

The RGEs for the masses of the \(B-L\) Higgs field \(\chi_1\) and right-handed sneutrino read

\[
\frac{d m_{\chi_1}^2}{dt} = 6 \tilde{\alpha}_{B-L} M_{B-L}^2 - 2 \tilde{Y}_{N_3} (m_{\chi_1}^2 + 2 m_{N_3}^2 + A_{N_3}^2),
\]

\[
\frac{d m_{\chi_2}^2}{dt} = \frac{3}{2} \tilde{\alpha}_{B-L} M_{B-L}^2 - \tilde{Y}_{N_3} (m_{\chi_1}^2 + 2 m_{N_3}^2 + A_{N_3}^2).\]

Since the second Higgs \(\chi_2\) has no interaction with any particle, its evolution is given by

\[
\frac{d m_{\chi_2}^2}{dt} = 6 \tilde{\alpha}_{B-L} M_{B-L}^2.\]

The evolution of these mass parameters depends on the boundary conditions at GUT scale. As mentioned, we assume universal soft SUSY breaking at this scale, i.e.,

\[
m_{\chi_1}^2(0) = m_{\chi_2}^2(0) = m_{N_3}^2(0) = m_0^2,
\]

\[M_{\alpha}(0) = M_{B-L} = M_{1/2},\]

\[a = 1, 2, 3 \text{ for } SU(3)_C, SU(2)_L, U(1)_Y\]

\[A_i(0) = A_{N_3} = A_0, \quad i = t, b, \tau.\]

Fig. 1 reports the result of the running. In this figure, we set \(m_0 = M_{1/2} = A_0 = 200\) GeV and order one \(Y_{N_3} \simeq M_{N_3}/v'\) is assumed. As can be seen from this figure, \(m_{\chi_1}^2\) drops rapidly to negative region, while \(m_{\chi_2}^2\) remains positive. Analogously to the radiative electroweak symmetry breaking, this mechanism works for large Yukawa coupling. It is worth noting the faster drop of \(m_{\chi_2}^2\), in comparison with that of \(m_{N_3}^2\). Indeed, \(m_{\chi_2}^2\) receives a positive contribution in its running only from the \(B-L\) gaugino, while the \(SU(2)_L\) and \(U(1)_Y\) gaugino masses are responsible for the positive contributions in the running of \(m_{N_3}^2\).

Also in Fig. 1 we plot the scale evolution for the scalar mass \(m_{N_3}^2\). Although \(m_{N_3}^2\) decreases in the running from \(M_X\), it remains positive at the TeV scale. Therefore, the \(B-L\) breaking via a non-vanishing vacuum expectation value for right-handed sneutrino does not occur in the present framework.

The phenomenology of TeV scale neutral gauge boson \(Z_{B-L}\) is very rich and its potential discovery at LHC has been recently analyzed in Ref. [4]. Also, the three SM singlet fermions, \(\nu_{R_i}\) in the superfields \(N_i\), get the following masses:

\[M_{N_i} = v' Y_{N_i} \sim O(\text{TeV}),\]

These three particles play the role of right handed neutrinos. In addition, the electroweak symmetry breaking induces the Dirac mass term:

\[m_D = \frac{v}{\sqrt{2}} Y_{\nu}.\]
Therefore, in addition to the usual five MSSM Higgs bosons, namely one neutral pseudoscalar Higgs bosons $A$, two neutral scalars $h$ and $H$ and a charged Higgs boson $H^\pm$, three new physical degrees of freedom remain. They form a neutral pseudoscalar Higgs boson $A'$ and two neutral scalars $h'$ and $H'$. Their masses at tree level are given by

$$m_{A'}^2 = \mu_1^2 + \mu_2^2,$$

$$m_{h',H'}^2 = \frac{1}{2} \left( m_A^2 + M_{Z_{B-L}}^2 \right) \pm \sqrt{\left( m_A^2 + M_{Z_{B-L}}^2 \right)^2 - 4 m_A^2 M_{Z_{B-L}}^2 \cos 2\theta}$$

Here $\theta = \tan^{-1} \frac{\nu'}{\nu}$ and $\mu_i$ with $i = 1, 2$ are defined in Eq. (5). From the expression of the lightest $B-L$ Higgs boson, one finds the following upper bound

$$m_h \lesssim M_{Z_{B-L}} \cos 2\theta.$$  

However, in analogy with the large radiative corrections to the lightest MSSM Higgs mass due to the top-stop loop, the $N - \tilde{N}$ loop can induce large correction leading to $m_{h'} > m_{h}$. The enlarged sneutrino sector of this model deserves some attention. Indeed, in the present SUSY extension of the $G_{B-L}$ model, a significant mixing between the left-handed and right-handed sneutrinos can be obtained. This would lead to what is known as sneutrino-antisneutrino oscillation [7]. The $12 \times 12$ sneutrino mass matrix, in the basis $(\phi_L, \phi_N)$ with $\phi_L = (\tilde{\nu}_L, \tilde{\nu}_L^*)$ and $\phi_N = (\tilde{\nu}_R, \tilde{\nu}_R^*)$, is given by

$$M^2 = \frac{1}{2} \begin{pmatrix} M_{LL}^2 & M_{LN}^2 \\ M_{NL}^2 & M_{NN}^2 \end{pmatrix}.$$  

The detailed expressions for the $6 \times 6$ matrices $M^2_{AB}$ for $A, B = L, N$ can be found in Ref. [7]. In general, the order of magnitude of the entries of this matrix can be estimated as follows:

$$M^2 = \frac{1}{2} \begin{pmatrix} \mathcal{O}(\nu'^2) & \mathcal{O}(\nu' \nu') \\ \mathcal{O}(\nu' \nu') & \mathcal{O}(\nu^2) \end{pmatrix}.$$  

Since $\nu' \sim \text{TeV}$, the sneutrino matrix elements are of the same order and there is no seesaw type behavior as usually found in MSSM extended with heavy right-handed neutrinos. Therefore a significant mixing among the left- and right- handed sneutrinos is obtained. The phenomenological consequences for such mixing have been studied in [8].

In conclusion, we have shown that in a SUSY extension of the SM where $B - L$ is gauged, it is possible to link together the electroweak, $B - L$ and soft SUSY breakings at a scale of $\mathcal{O}(\text{TeV})$. The ensuing richer TeV phenomenology for the coming LHC and neutrino physics opens new prospects and deserves further attention.

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