Spin-Flip Transistor

Gerrit E. W. Bauer, Yuli V. Nazarov
Delft University of Technology, Department of Applied Physics and DIMES,
Lorentweg 1, 2628 CJ Delft

Arne Brataas
Harvard University, Lyman Laboratory of Physics, Cambridge, MA 02138
(November 13, 2018)

The recently developed semiclassical theory for magneto electronic circuits is applied to a transistor-like device consisting of a normal metal island and three magnetic terminals. The electric current between source and drain can be controlled by the magnetization of a “base” reservoir up to distances of the order of the spin-flip diffusion length.

I. INTRODUCTION

Magnetoelectronics is concerned with the integration of ferromagnetic metals into conventional electronic circuits which can be applied to e.g. improved magnetic field sensors and non-volatile magnetic random access memories (MRAMs). Different magnetoelectronic devices with three or four terminals (“spin transistors”) have been realized [1,2] and proposed [3,4]. In the present manuscript we will analyze in detail the active magnetoelectronic device of Brataas et al. [4], which we call the spin-flip transistor.

Transport in hybrid metallic systems in the presence of long-range correlations in an order parameter can be described by a generalization of Kirchhoff’s theory of electronic circuits when the electronic phase is sufficiently scrambled in parts of the system, the “nodes”. This approach has been pioneered in Ref. [5] for electronic networks with superconducting elements. It has recently been adopted also for magnetoelectronic circuits [6], like the Johnson spin transistor [1] or the 4-terminal mesoscopic spin valve of Jedema et al. [2]. The circuit theory can be derived from a given Stoner Hamiltonian in terms of the Keldysh non-equilibrium Green function formalism in spin space [6]. The basic physics is provided by dividing the system into reservoirs, resistors and nodes which can be real or fictitious. In order to arrive at a useful formalism, an isotropy assumption has to be introduced for the nodes in which the electron distributions may taken to be isotropic. This implies the presence of sufficient disorder (or chaotic scattering). Inelastic or dephasing scattering is not required but, at least in the nodes, not forbidden either. Electron charges and spins are accumulated or depleted in the nodes as a function of the applied voltages. Because the spin-accumulation is not necessarily collinear to the spin-quantization axis, at each node the electron distribution can be denoted as ̂f, where the hat ( ̂ ) denotes a 2×2 matrix in spin-space. The external reservoirs are assumed to be in local equilibrium so that the distribution matrix ̂f = 1fα is diagonal in spin-space and equal to the local chemical potential fα in reservoir α, where 1 is the unit matrix. The direction of the magnetization of the ferromagnetic nodes will be denoted by the unit vector mα. The current matrix ̂Iαβ through a contact connecting two neighboring nodes can be calculated as a function of the distribution matrices on the adjacent nodes and the 2×2 conductance tensor composed of the spin-dependent conductances G↑ and G↓

\[ G^s = \frac{e^2}{\hbar} \left( M - \sum_{nm} |r_{nm}^s|^2 \right) = \frac{e^2}{\hbar} \sum_{nm} |t_{nm}^s|^2, \]

and the mixing conductance

\[ G^{s,-s} = \frac{e^2}{\hbar} \left( M - \sum_{nm} r_{nm}^s (r_{nm}^s)^* \right), \]

where r_{nm}, t_{nm} are the reflection and transmission coefficients in a spin-diagonal reference frame and M the number of modes in the absence of reflections. Spin-flips in the contacts have been disregarded. In the following the node is taken to be a normal metal. When its size in the transport direction is smaller than the spin-flip diffusion length l_{sf} = \sqrt{D \tau_{sf}} (1\mu m in Cu [2]), where D is the diffusion coefficient, the spin-current conservation law

\[ \sum_\alpha ̂I_{\alpha\beta} = \left( \frac{\partial ̂f^N}{\partial t} \right)_{\text{rel}} = \frac{i \text{Tr} ̂f^N - 2 ̂f^N}{2\tau_{sf}} \]
allows computation of the circuit properties as a function of the applied voltages. The right hand side of Eq. (2) can be set to zero when the spin-current in the node is conserved, i.e., when an electron resides on the node sufficiently shorter than the spin-flip relaxation time $\tau_{sf}$.

In the following we concentrate on the practical application of the circuit theory to the spin-flip transistor. We consider the limit of long $\tau_{sf}$, which is experimentally relevant even at room temperature [3]. In order to simplify results we will confine attention to half-metallic (fully polarized) ferromagnetic metals (HMF) where appropriate. Our model for the interface between a normal metal and half-metallic ferromagnet reads:

$$\hat{G} = \begin{pmatrix} G & G \\ G & 0 \end{pmatrix},$$  
(3)

so $G^\uparrow = G^{\uparrow\downarrow} = G$, $G^\downarrow = 0$.

II. CURRENT MATRIX FOR F|N CONTACT

The particle current on the normal side of an F|N contact and directed into the normal metal is [4]

$$\hat{f}^{(\tilde{m})} = G^\uparrow \hat{a}_m^\uparrow (\hat{f}^{\uparrow} - \hat{f}^{\downarrow}) \hat{u}_m^\uparrow + G^\downarrow \hat{a}_m^\downarrow (\hat{f}^{\downarrow} - \hat{f}^{\uparrow}) \hat{u}_m^\downarrow + G^{\uparrow\downarrow} \hat{a}_m^\uparrow \hat{f}^{\downarrow} \hat{a}_m^\downarrow - (G^{\uparrow\downarrow})^* \hat{a}_m^\downarrow \hat{f}^{\uparrow} \hat{a}_m^\uparrow,$$

where

$$\hat{a}_m^\uparrow = \frac{1}{2} \left( \hat{1} + \hat{\sigma} \cdot \hat{m} \right); \quad \hat{a}_m^\downarrow = \frac{1}{2} \left( \hat{1} - \hat{\sigma} \cdot \hat{m} \right)$$

are the spin-$\frac{1}{2}$ rotation matrices. In the following we take the ferromagnet as a reservoir at equilibrium $\hat{f}^{\uparrow} = \hat{f}^{\downarrow}$. Some insight can be gained by re-writing the current and the distribution function in the form of a scalar particle and a vectorial spin contribution. The distribution function on the normal node can be written as

$$\hat{f}^{\downarrow} = \begin{pmatrix} f^{\uparrow\uparrow} \\ (f^{\uparrow\downarrow})^* \\ f^{\downarrow\uparrow} \\ f^{\downarrow\downarrow} \end{pmatrix} = C\hat{1} + \hat{S} \cdot \hat{\sigma},$$

where $C = (f^{\uparrow\uparrow} + f^{\downarrow\downarrow})/2$ is the local chemical potential and

$$\hat{S} = \frac{1}{2} \begin{pmatrix} f^{\uparrow\uparrow} + (f^{\uparrow\downarrow})^* \\ f^{\uparrow\downarrow} - (f^{\uparrow\downarrow})^* \\ f^{\downarrow\uparrow} - f^{\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} \text{Re} f^{\uparrow\downarrow} \\ \text{i \text{Im}} f^{\uparrow\downarrow} \\ \frac{1}{2} (f^{\uparrow\uparrow} - f^{\downarrow\downarrow}) \end{pmatrix} \equiv \begin{pmatrix} M_R \\ M_i \\ \Sigma \end{pmatrix},$$

(7)

the spin accumulation vector. The junction parameters can be rewritten as $P = (G^{\uparrow} - G^{\downarrow})/2$, $G = G^{\uparrow} + G^{\downarrow}$, $R = \text{Re} G^{\uparrow\downarrow}$, and $\Im = \text{i \text{Im}} G^{\uparrow\downarrow}$. The matrix-current through an F|N interface $\hat{I} = (IC\hat{I} + IS \cdot \hat{\sigma})/2$ can then be expanded into vector components in terms the scalar charge current:

$$I_C = G(f^{\uparrow} - C) - 2P\hat{m} \cdot \hat{S}$$

and the vector spin current:

$$\frac{1}{2}I_s = [P(f^{\uparrow} - C) - \left( \frac{G}{2} - R \right) S \cdot \hat{m} - R \hat{S} - \Im (S \times \hat{m})$$

(8)

The vector spin current component perpendicular to the magnetization direction $-I_s$ equals the spin-torque exerted by the polarized current on the ferromagnet [5,6]

$$-I_\perp = -I_s + (I_s \cdot \hat{m}) \hat{m} = -2 \text{Re} G^{\uparrow\downarrow}(S \cdot \hat{m}) \hat{m} + 2\text{Re} G^{\uparrow\downarrow}S - 2\text{Im} G^{\uparrow\downarrow}(S \times \hat{m}).$$

(9)

When the magnetization is parallel to the quantization axis, $\hat{m}_{\pm z} = (0, 0, \pm 1)$:

$$\hat{a}_{\pm z}^\uparrow = (\hat{1} + \hat{\sigma}_z)/2 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \quad \hat{a}_{\pm z}^\downarrow = (\hat{1} - \hat{\sigma}_z)/2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

(10)
and $\hat{u}_z^\dagger = \hat{u}_z^\dagger$. It is easy to see that
\begin{equation}
\hat{I}^{(z)} = \left( \begin{array}{cc}
G^\dagger (f^F - f_{↑↑}) & -G^\dagger f_{↑↓} \\
-(G^\dagger f_{↑↓})^* & G^\dagger (f^F - f_{↑↑})
\end{array} \right) ; \hat{I}^{(-z)} = \left( \begin{array}{cc}
G^\dagger (f^F - f_{↑↓}) & -G^\dagger (f_{↑↑})^* \\
-(G^\dagger)^* f_{↑↓} & G^\dagger (f^F - f_{↑↑})
\end{array} \right)
\end{equation}

Non-diagonal elements become important when the magnetization is not collinear to the quantization axis. For $\vec{m}_z = (1, 0, 0)$:
\begin{equation}
\hat{u}_z^\dagger = \left( \hat{1} + \hat{\sigma}_z \right)/2 = \frac{1}{2} \left( \begin{array}{cc}
1 & 1 \\
1 & 1
\end{array} \right) ; \hat{u}_z = \left( \hat{1} - \hat{\sigma}_z \right)/2 = \frac{1}{2} \left( \begin{array}{cc}
1 & -1 \\
-1 & 1
\end{array} \right)
\end{equation}

\begin{equation}
\hat{I}^{(z)} = \left( \begin{array}{cc}
\frac{G}{2} (f^F - C) - PM_R - \Re \Sigma + \Im M_I & P (f^F - C) - \frac{G}{2} M_R + \Re \Sigma - \Im M_I \\
\frac{P}{2} (f^F - C) - \frac{G}{2} M_R - \Re \Sigma + \Im M_I & \frac{G}{2} (f^F - C) - PM_R + \Re \Sigma - \Im M_I
\end{array} \right)
\end{equation}

### III. TWO-TERMINAL (SPIN VALVE) DEVICE

In this section we determine the current through a F|N|F structure with parallel, antiparallel and perpendicularly oriented magnetizations, making use of the current conservation condition Eq. (12). $f_{↑}^F = \Delta \mu$ and $f_{↓}^F = 0$.

#### A. Collinear magnetization

Let us first determine $\hat{I}^N$ for parallel magnetizations in the parallel F$_1^{(z)}$|N|F$_2^{(z)}$ configuration. Spin current conservation requires that $\hat{I}_1^{(z)} + \hat{I}_2^{(z)} = 0$.

\begin{equation}
\left( \begin{array}{cc}
G^\dagger (\Delta \mu - f_{↑↑}) & -G^\dagger f_{↑↓} \\
-(G^\dagger f_{↑↓})^* & G^\dagger (\Delta \mu - f_{↑↑})
\end{array} \right) = \left( \begin{array}{cc}
G^\dagger f_{↑↑} & G^\dagger f_{↑↓} \\
(G^\dagger f_{↑↓})^* f_{↑↓} & G^\dagger f_{↑↓}
\end{array} \right)
\end{equation}

from which we conclude that $f_{↑↑} = 0$ and $f_{↑↓} = \Delta \mu/2$.

When antiparallel F$_1^{(z)}$|N|F$_2^{(−z)}$, $\hat{I}_1^{(z)} + \hat{I}_2^{(−z)} = 0$:

\begin{equation}
\left( \begin{array}{cc}
G^\dagger (\Delta \mu - f_{↑↑}) & -G^\dagger f_{↑↓} \\
-(G^\dagger f_{↑↓})^* & G^\dagger (\Delta \mu - f_{↑↑})
\end{array} \right) = \left( \begin{array}{cc}
G^\dagger f_{↑↑} & G^\dagger f_{↑↓} \\
(G^\dagger f_{↑↓})^* f_{↑↓} & G^\dagger f_{↑↓}
\end{array} \right)
\end{equation}

\begin{equation}
f_{↑↑} = \frac{G^\dagger \Delta \mu}{G^\dagger + G^\dagger}; f_{↑↓} = \frac{G^\dagger \Delta \mu}{G^\dagger + G^\dagger}; \Sigma = \frac{P \Delta \mu}{G}
\end{equation}

Since $-f_{↑↓} = (f_{↑↓})^*$ \Re $f_{↑↓} = 0$ and \Im $f_{↑↓}$ is undetermined but irrelevant. The charge current

\begin{equation}
I_C = G \Delta \mu - G^\dagger f_{↑↑} - G^\dagger f_{↑↓} = \frac{2G^\dagger G^\dagger}{G^\dagger + G^\dagger} \Delta \mu
\end{equation}

vanishes for the HMF.

#### B. Non-collinear

$\hat{I}^N$ for normal magnetizations in F$_1^{(z)}$|N|F$_2^{(z)}$ structures follows again from the current conservation condition $\hat{I}_1^{(z)} + \hat{I}_2^{(z)} = 0$ and using Eq. (13):

\begin{equation}
\left( \begin{array}{cc}
G^\dagger (\Delta \mu - f_{↑↑}) & -G^\dagger f_{↑↓} \\
-(G^\dagger f_{↑↓})^* & G^\dagger (\Delta \mu - f_{↑↑})
\end{array} \right) = \left( \begin{array}{cc}
\frac{G}{2} C - PM_R - \Re \Sigma + \Im M_I & \frac{P}{2} C - \frac{G}{2} M_R + \Re \Sigma - \Im \Sigma M_I \\
\frac{P}{2} C - \frac{G}{2} M_R - \Re \Sigma + \Im \Sigma M_I & \frac{G}{2} C - PM_R - \Re \Sigma - \Im \Sigma M_I
\end{array} \right)
\end{equation}

We find a spin accumulation vector
\[ \mathbf{S} = \frac{P \Delta \mu}{2} \left( \begin{array}{cc} \Re \frac{1}{\mathcal{R}^2 - \mathcal{S}^2 + \frac{G^2}{R}} & \Re \frac{-1}{\mathcal{R}^2 - \mathcal{S}^2 + \frac{G^2}{R}} \\ \Re \frac{1}{\mathcal{R}^2 - \mathcal{S}^2 + \frac{G^2}{R}} & \Re \frac{0}{\mathcal{R}^2 - \mathcal{S}^2 + \frac{G^2}{R}} \end{array} \right) \]  

(19)

and a charge current (from left to right):

\[ I_C = G(\Delta \mu - C - \Sigma) = \frac{G}{2} - \frac{P^2}{\mathcal{R}^2 - \mathcal{S}^2 + \frac{G^2}{R}} \rightarrow G \frac{\Delta \mu}{3}. \]  

(20)

IV. THREE TERMINAL DEVICE

We now compute the characteristics of the device with three ferromagnetic terminals attached to a normal metal node. The magnetization of the third (base) terminal can be either collinear or normal to the magnetizations of source and drain. The latter are invariably taken to be antiparallel.

A. Collinear configuration

Let us consider a bias \( \Delta \mu_3 \) at which there is zero charge current through the base terminal, \( i.e. [I_{\uparrow\uparrow} + I_{\downarrow\downarrow}]_3 = 0. \) For \( \vec{m}_3 = (0, 0, 1) \) this translates into

\[ 0 = G^\uparrow (\Delta \mu_3^{(z)} - f_{\uparrow\uparrow}) + G^\downarrow (\Delta \mu_3^{(z)} - f_{\downarrow\downarrow}) \]  

(21)

\[ \Delta \mu_3^{(z)} = \frac{G^\uparrow f_{\uparrow\uparrow} + G^\downarrow f_{\downarrow\downarrow}}{G^\uparrow + G^\downarrow} = \frac{G^\uparrow (C + \Sigma) + G^\downarrow (C - \Sigma)}{G^\uparrow + G^\downarrow} = C + \frac{2P \Sigma}{G}. \]  

(22)

So in this case matrix-current conservation in the node reads:

\[ \hat{j}_1^{(z)} + \hat{j}_2^{(-z)} + \hat{j}_3^{(z)} = 0 \]  

(23)

We find:

\[ \Delta \mu_3^{(z)} = \frac{(G^\downarrow)^2 + G^\downarrow G^\uparrow + (G^\uparrow)^2}{(G^\uparrow)^2 + 4G^\downarrow G^\uparrow + (G^\downarrow)^2} \Delta \mu \]  

(24)

The source-drain current

\[ \frac{j_3^{(z)}}{G \Delta \mu} = \frac{3G^\downarrow G^\uparrow}{(G^\downarrow)^2 + 4G^\downarrow G^\uparrow + (G^\uparrow)^2} \]  

vanishes again for HMF terminals:

\[ f_{\downarrow\downarrow} = 0; \ f_{\uparrow\uparrow} = \Delta \mu_3^{(z)} = \Delta \mu; \ \hat{i}_C^{(z)} = 0; \ \Sigma = \frac{\Delta \mu}{2}. \]  

(25)

B. Non-collinear configuration

When the third electrode is rotated to the \( x \)-direction, the zero particle current condition for the base contact dictates:

\[ \Delta \mu_3^{(z)} = C + 2PM_R/G \]  

(26)

Matrix-current conservation:

\[ \hat{j}_2^{(1)} + \hat{j}_2^{(2)} + \hat{j}_2^{(3)} = 0 \]  

(27)
leads to:

\[ \Delta \mu^{(x)}_3 = C = \Delta \mu / 2 \]  

\[ I^{(x)}_C = \left( \frac{G}{2} - \frac{2P^2}{G + |G^{\uparrow \downarrow}|^2 / \text{Re} G^{\uparrow \downarrow}} \right) \Delta \mu \]  

For the HMF, \( \Sigma = \frac{\Delta \mu}{2} \) and \( I^{(x)}_C = \frac{G}{2} \Delta \mu \). When (in this limit) the potential at the third electrode is varied \( \Sigma \) does not change, but the length of \( S \) does:

\[ |S|^2 = \Sigma^2 + M^2_R + M^2_I = \left( \frac{\Delta \mu}{4} \right)^2 + \left( \frac{2 \Delta \mu^{(3)} - \Delta \mu}{14} \right)^2 \]  

The spin-accumulation is minimal for the zero charge current condition for the third terminal, which corresponds to a maximum of the spin-current through the third terminal. The minimum spin-accumulation consequently allows a maximum source-drain current without dissipation in the base.

V. DISCUSSION

In the HMF limit the physics of the transistor action is easily understood. In the collinear configuration the device is electrically dead: No current can flow into the drain because spin-up states can not penetrate a HMF with spin-down magnetization and source and base reservoir are at the same potential. When the magnetization of the base is rotated by 90 degrees and the potential is lowered to \( \Delta \mu / 2 \) the incoming spin-up current is exactly equal to the outflowing spin-down current. Although there is no direct current between source and drain, the outflowing spin-down current from the base can enter the drain. Effectively we thus have switched on a source-drain charge current by rotating the magnetization. Since the base contact operates as a perfect spin-flip, we suggest the name spin-flip transistor for our device.

Let us consider the ideal case in which magnet number 3 has a negligible anisotropy, thus can be rotated without appreciable energy cost. We still need a voltage source for the base in order to stick to the optimal working point. Let us consider the source-drain conductances in the three terminal device for the \( x \) (on) and \( z \) (off) configurations:

\[ \frac{I^{(z)}_C}{\Delta \mu} = \frac{3G^4 G^{\uparrow}}{(G^{\downarrow})^2 + 4G^4 G^{\uparrow} + (G^{\uparrow})^2} \frac{G}{6} \frac{G^2 - 4P^2}{3G^2 - 4P^2} G \xrightarrow{HMF \to 0} 0 \]  

with

\[ \Delta \mu^{(z)}_3 = \frac{(G^{\downarrow})^2 + G^{\downarrow} G^{\uparrow} + (G^{\uparrow})^2}{(G^{\downarrow})^2 + 4G^4 G^{\uparrow} + (G^{\uparrow})^2} \Delta \mu \]  

and

\[ \frac{I^{(x)}_C}{\Delta \mu} = \frac{G}{2} - \frac{2P^2}{G + |G^{\uparrow \downarrow}|^2 / \text{Re} G^{\uparrow \downarrow}} \xrightarrow{HMF \to G} 4 \]  

with

\[ \Delta \mu^{(x)}_3 = \frac{\Delta \mu}{2} \]  

For a source-drain current-based HMF device very large voltage gains could be realized, since high voltages are necessary in order to maintain a source-drain current in the \( off \) state. The transconductance is only one-half of the source-drain conductance, however:

\[ g = \left| \frac{I^{(x)}_C - I^{(z)}_C}{\Delta \mu^{(x)}_3 - \Delta \mu^{(z)}_3} \right| \xrightarrow{HMF \to G} \frac{G}{2} \]  

It is doubtful whether such a device will be of practical use, since we have not addressed the question of how to realize the controlled rotation of the base terminal which should occur simultaneously with the adjustment of the base voltage. The spin-flip transistor action therefore demonstrates nicely the non-local action of the coherent spin-accumulation rather than being a useful functional property.
VI. ACKNOWLEDGMENT

We acknowledge discussions with Daniel Huertas-Hernando, Wolfgang Belzig, Ke Xia as well as support by FOM and the NEDO joint research program (NTDP-98). A.B. is supported by the Norwegian Research Council.

[1] M. Johnson, Phys. Rev. Lett. 70, 2142 (1993).
[2] F.J. Jedema, A.T. Filip, and B. van Wees, Nature (2001) in press.
[3] S. Datta and B. Das, Appl. Phys. Lett. 6, 665 (1990)
[4] A. Brataas, Yu. V. Nazarov, and G.E.W. Bauer, Phys. Rev. Lett. 84, 2481 (2000).
[5] Yu.V. Nazarov, Phys. Rev. Lett. 73, 1420 (1994); Yu.V. Nazarov, Superlatt. and Microstruc. 25, 1221 (1999).
[6] A. Brataas, Yu. V. Nazarov, and G.E.W. Bauer, cond-mat/0006174
[7] J. A. Katine, F. J. Albert, R. A. Buhrman, E. B. Myers and D. C. Ralph, Phys. Rev. Lett. 84, 3149 (2000) and references therein.
[8] X. Waintal, E.B. Myers, P.W. Brouwer, and D.C. Ralph, Phys. Rev. B 62, 12317 (2000).