Cosmology
and
Nonlinear Supersymmetric General Relativity ∗

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December 2005

Abstract

We discuss cosmological implications of nonlinear supersymmetric (NLSUSY) general relativity (GR) of the form of Einstein-Hilbert (EH) action for empty spacetime, where NLSUSY GR is obtained by the geometrical arguments on new spacetime just inspired by NLSUSY. The new action of NLSUSY GR is unstable and breaks down spontaneously to EH action with Nambu-Goldstone (NG) fermion matter. We show that NLSUSY GR elucidates the physical meanings of the cosmologically important quantities, e.g., the spontaneous SUSY breaking scale, the cosmological constant, the dark energy and the neutrino mass and describe naturally the paradigm of the accelerated expansion of the present universe.

∗Based on the talk given by K. Shima at International Europhysics Conference on High Energy Physics, July 21st-27th, 2005, Lisboa, Portugal
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1 Introduction

SUSY[1][2][3] and its spontaneous breakdown[4][5] are conformed to the rationale of spacetime, therefore necessary to be accessed not only from the low energy particle physics but also from the cosmology. Despite the success of the standard model(SM), facing so many arbitrary parameters and mysterious textural features, e.g. the CP violation phase, mixing angles, generation structures, tiny neutrino masses, etc., we are tempted to suppose that they may be certain composites and/or that they should be attributed to the particular geometrical structure of spacetime which unifies the (classical) notions: the object and the background space-time manifold irrespective of being classical and quantum.

From the simplicity and the beauty of nature, the NLSUSY[2] invariant coupling of graviton with the spin $\frac{1}{2}$ fermion is anticipated for materializing the standard model(SM) with the spontaneous breakdown of SUSY[6].

We have shown group theoretically in ref.[6] that among the massless irreducible representations of all SO(N) super-Poincaré(SP) groups N=10 SP group is the only one that contains the SM with just three generations of quarks and leptons, where by extending the Gell-Mann’s idea[7] we have decomposed 10 spinor supercharges into $10 = \tilde{5} + \tilde{5}^*$ with respect to SU(5). Surprisingly, the fundamental representation $\tilde{5}$ has the same quantum number as the $\tilde{5}$ of observed quarks-leptons multiplet of SU(5) grand unified theory(GUT)[8], i.e. $\tilde{5}_{SGM} = \tilde{5}_{GUT}$. Regarding 10 supercharges as the hypothetical fundamental spin 1/2 particles(superons)-quintet and anti-quintet, we have proposed the composite superon-graviton model(SGM) for nature[9][10]. In SGM, all (observed) elementary particles except graviton are assumed to be superon(supercharge)-composites and are eigenstates of SO(10) SP symmetry in the sense that as shown partly in Sec. 3 each state of superon(supercharge)-composites can be recasted algebraically into the equivalent local field of the supermultiplet of the corresponding linear(L) SUSY explicitly. We have attempted to reproduce the Feynmann diagramms of SU(5) GUT in SGM picture by replacing the sigle-line of the propagator of a particle into the multiple-lines of the constituents of each particle[11]. Despite SU(5) GUT-like features $\tilde{5}_{SGM} = \tilde{5}_{GUT}$, the SUSY-composite picture of SGM, i.e. the constituent(superon) number conservation at the vertex, gives new remarkable insights into the proton decay, R-parity, neutrino oscillations, · · ·, etc. These group theoretical arguments suggest the NLSUSY invariant coupling of graviton with the spin $\frac{1}{2}$ fermion(superon) and the spin $\frac{1}{2}$ field-current(supercharge) identity, i.e. N=10 NLSUSY Volkov-Akulov(VA) model[2] coupled with gravity. SUSY and its spontaneous breakdown can be encoded in a self-contained way.
2 Nonlinear Supersymmetric General Relativity (NLSUSY GR)

The curvature of particular spacetime, the ultimate physical entity, materializes the general relativity of nature. We extend the geometrical arguments of Einstein general relativity (EGR) on Riemann spacetime to new (called SGM hereafter) spacetime where the tangent spacetime possesses local NLSUSY degrees of freedom (d.o.f), i.e. besides the ordinary SO(3,1) Minkowski coordinate $x^a$ the SL(2C) Grassman coordinates $\psi$ for the coset space $\text{superGL}(4,R)/\text{GL}(4,R)$ turning subsequently to the NG fermion (called superon hereafter) dynamical d.o.f. are attached at every curved spacetime point. (Note that locally homomorphic SO(3,1) and SL(2C) are non-compact groups for spacetime d.o.f which is analogous to SO(3) and SU(2) compact groups for gauge d.o.f. of 't Hooft-Polyakov monopole[12][13].) We have obtained the following NL-SUSY GR (N=1 SGM) action[9] of the vacuum EH-type in SGM spacetime.

$$L(w) = -\frac{c^4}{16\pi G} |w|(\Omega + \Lambda),$$

(1)

where $w^a_\mu(x)$ invertible and $\Omega$ are the unified vierbein and the unified scalar curvature of SGM spacetime, respectively. Accordingly $s_{\mu\nu} \equiv w^a_\mu \eta_{ab} w^b_\nu$ and $s^{\mu\nu}(x) \equiv w^a_\mu(x)w^a_\nu(x)$ are unified metric tensors of SGM spacetime[9][10]. The explicit expression of $\Omega$ is obtained by just replacing $e^a_\mu(x)$ by $w^a_\mu(x)$ in Ricci scalar $R$ of EGR [10]. $e^a_\mu$ describing the local SO(3,1) and $t^a_\mu(\psi)$ describing the local SL(2C) are the ordinary vierbein of EGR and the mimic vierbein of the stress-energy-momentum tensor of superons $\psi(x)$, respectively. $G$ is the Newton gravitational constant, $\Lambda$ is a (small) cosmological term, and $\kappa$ of NLSUSY VA model [2] with the dimension (length)$^4$ is now related to $\kappa^2 = (\frac{c^4}{8\pi G})^{-1}$. It is remarkable that the constant $\kappa$, i.e. the strength of the superon-vacuum coupling constant in the (low energy theorem) particle physics viewpoint, is determined by the quantities of spacetime, $G$ and $\Lambda$, in the NLSUSY GR(SGM) scenario. SGM contains hierarchic two mass scales, $\frac{1}{G}$ (Planck scale) in the first term and $\kappa^2 \sim \frac{\Lambda}{G}$ in the second term describing the curvature energy and the vacuum energy of SGM, respectively.

NLSUSY GR action (1) is invariant under the new NLSUSY transformation[14];

$$\delta_\zeta \psi = \frac{1}{\kappa} \zeta - i\kappa (\bar{\zeta} \gamma^\rho \psi) \partial_\rho \psi,$$

$$\delta_\zeta e^a_\mu = i\kappa (\bar{\zeta} \gamma^\rho \psi) \partial_\rho e^a_\mu,$$

(3)

consequently GL(4R) transformations on $w^a_\mu$

$$\delta_\zeta w^a_\mu = \xi^\nu \partial_\nu w^a_\mu + \partial_\mu \xi^\nu w^a_\nu,$$

$$\delta_\zeta s_{\mu\nu} = \xi^\kappa \partial_\kappa s_{\mu\nu} + \partial_\mu \xi^\kappa s_{\kappa\nu} + \partial_\nu \xi^\kappa s_{\mu\kappa},$$

(4)
where $\zeta$ is a constant spinor, $\partial_{[\rho}e^{a}_{\mu]} = \partial_{\rho}e^{a}_{\mu} - \partial_{\mu}e^{a}_{\rho}$, and $\xi^\rho = i\kappa^2(\bar{\zeta}\gamma^\rho\psi)$. The commutators of two new NLSUSY transformations (3) on $\psi$ and $e^{a}_{\mu}$ are GL(4R), i.e. new NLSUSY (3) is the square-root of GL(4R):

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]\psi = \Xi^\mu\partial_\mu\psi, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}]e^{a}_{\mu} = \Xi^a\partial_\mu e^{a}_{\mu} + e^{a}_{\rho}\partial_\mu \Xi^\rho;$$

(5)

where $\Xi^\mu = 2i\kappa(\bar{\zeta}_2\gamma^\mu\zeta_1) - \xi^\rho_1\xi^\sigma_2 e^{a}_{\mu}(\partial_\rho e^{a}_{\sigma})$. They show the closure of the algebra.

NLSUSY GR action (1) is invariant at least under the following spacetime symmetries\[10\]

[\text{new NLSUSY}] \otimes [\text{local GL}(4, R)] \otimes [\text{local Lorentz}] \otimes [\text{local spinor translation(LST)}]

and the following internal symmetries for N-superons $\psi_j; (j = 1, 2, ..N)$ extended NLSUSY GR:

[\text{global SO}(N)] \otimes [\text{local U}(1)^N].

(6)

(7)

SGM action $L(w)$ (1) on SGM spacetime is unstable due to the global NLSUSY structure(SP d.o.f.) of tangent spacetime and breaks down spontaneously to EH action with the superon(NG fermion) matter in Riemann spacetime as follows

$$L(e,\psi) = -\frac{c^4}{16\pi G}|e|\{R(e) + \Lambda + \tilde{T}(e, \psi)\};$$

(8)

where $\tilde{T}(e, \psi)$ is the kinetic term and the gravitational interaction of superon. The second and the third terms produces N-extended NLSUSY VA action with $\kappa^2 = \left(\frac{c^4\Lambda}{8\pi G}\right)^{-1}$ in Riemann-flat $\left(e^a_{\mu}(x) \rightarrow \delta^a_\mu\right)$ spacetime. Finally we just mention that by the straightforward extension of SGM argument we can obtain NLSUSY GR with spin $\frac{3}{2}$ NG fermion\[15\] of Baaklini action\[16\].

3 Linearization of NL SUSY

SGM action (8) describing the geometry of the gravitational interactions of superons in Riemann spacetime is highly nonlinear. It is necessary to linearize SGM action (8) to obtain an equivalent and renormalizable field theory, where NLSUSY is recasted to the broken LSUSY defined on the LSUSY supermultiplet fields irreducible representations of SO(N) SP. N=1 VA model is linearized by many authors\[17\][18][19][20][21] As a preliminary for SGM linearization we have shown\[22\] in flat spacetime that N=2 NLSUSY VA model (expanded in $\kappa$):

$$L_{VA} = -\frac{1}{2\kappa^2} \left[ 1 + \kappa^2 f_{\alpha}^{\gamma} a + \frac{1}{2} \kappa^4 (f^{a}_{\mu} b - f^{b}_{\mu} a) - \frac{1}{3!} \kappa^6 \epsilon_{abcd} e^{fgh} f^{b} e f^{c} + \cdots \right];$$

(9)
which is invariant under $N=2$ NLSUSY transformation 
\[
\delta_Q \psi_L^i = \frac{1}{\kappa} \xi_i^L - i \kappa \left( \tilde{\zeta}_L \gamma^a \psi_L^i - \tilde{\zeta}_R \gamma^a \psi_R^i \right) \partial_a \psi_L^i,
\]
is equivalent to the following free action of the spontaneously broken $N=2$ LSUSY:
\[
L_{\text{lin}} = \partial_a \phi \partial^a \phi^* - \frac{1}{4} F_{ab}^2 + i \lambda_R \theta \lambda_R + \frac{1}{2} (D^J)^2 - \frac{1}{\kappa} \xi^J D^J,
\]
where $\psi_{Li}^i (i = 1, 2)$ is superon field, $w^a_b = \delta^a_b + \kappa^2 t^a_b$, $t^a_b = -i \tilde{\psi}_L \gamma^a \partial_b \psi_R + i \tilde{\psi}_R \gamma^a \partial_b \psi_L$, $\psi_R^i = C \psi_{Li}^T$, $\xi^J, (I = 1, 2, 3)$ are arbitrary real parameters of the induced global $SU(2)(SO(3))$ rotation $(\xi^I)^2 = 1$. The last term is the Fayet-Iliopoulos $D$ term indicating spontaneous SUSY breaking with the vacuum expectation value $D^I = \xi^I / \kappa$.

In these arguments all fields of LSUSY supermultiplet are the composites of superons $\psi^i_L$, e.g., $\phi(\psi) = \frac{1}{\sqrt{2}} i \kappa \xi_i^L \bar{\psi}_R \sigma^i \psi_L - \sqrt{2} \kappa^3 \xi^i \psi_L \gamma^a \psi_L \bar{\psi}_R \sigma^I \partial_a \psi_L + \cdots$, $\lambda_{Li}(\psi) = i \xi^I (\psi_L \sigma^I) + \kappa^2 \xi^I \gamma^a \psi_R \bar{\psi}_R \sigma^I \partial_a \psi_L + \cdots$, $A_a(\psi) = -\frac{1}{2} \kappa \xi^I (\bar{\psi}_L \sigma^I \gamma_a \psi_L - \bar{\psi}_R \sigma^I \gamma_a \psi_R) + \cdots$, $D^I(\psi) = \frac{1}{\kappa} \xi^I - i \kappa \xi^J (\bar{\psi}_L \sigma^J \sigma^I \psi_L - \bar{\psi}_R \sigma^J \sigma^I \psi_R) + \cdots$, etc. and the familiar LSUSY transformations on the component fields of the supermultiplet are reproduced in terms of the NLSUSY transformations on $\psi^i_L$ contained. When $\xi^I = \xi^3 = 0$, the supermultiplet is the ordinary vector supermultiplet containing the vector $U(1)$ gauge field as expected. It is remarkable that the compact group $SU(2)$, though global in flat space so far, emerges in Riemann-flat spacetime of $N=2$ SGM (9).

This shows that the vacuum of SGM has rich structures manageable, which is favourable to SGM scenario.

4 Cosmology of NLSUSY GR

Now we survey the cosmological implications of NLSUSY GR (or SGM from the composite viewpoints). SGM spacetime is unstable and spontaneously breaks down to Riemann spacetime with superon(massless NG fermion) matter, which is the birth of the universe by the quantum effect in advance of so called the inflation and/or the big bang. The variation of (8) with respect to $e^a_\mu$ gives the equation of motion for $e^a_\mu$ recasted as follows
\[
R_{\mu\nu}(e) - \frac{1}{2} g_{\mu\nu} R(e) = \frac{8\pi G}{c^4} \left\{ \bar{T}_{\mu\nu}(e, \psi) - g_{\mu\nu} \frac{c^4 \Lambda}{8\pi G} \right\},
\]
where $\bar{T}_{\mu\nu}(e, \psi)$ is the stress-energy-momentum tensor of superon(NG fermion) matter including the gravitational interactions. Note that $\frac{c^4 \Lambda}{8\pi G}$ can be interpreted as the (negative) energy density of empty spacetime, i.e. the dark energy density $\rho_D$. (The sign is fixed by the sign of the kinetic term of VA action.) While, on tangent spacetime, the low energy theorem of the particle physics gives the following
superon (massless NG fermion matter)-vacuum coupling

\[< \psi_L^\alpha(q)|J^{M\mu}|0> = i \sqrt{\frac{c^4}{8\pi G}}(\gamma^\mu)_{\alpha\beta}\delta^{LM}\epsilon^{i q x} + \cdots, \quad (12)\]

where \( J^{M\mu} = i \sqrt{\frac{c^4}{8\pi G}}\gamma^\mu \psi^M + \cdots \) is the conserved supercurrent obtained by applying the Noether theorem to NLSUSY VA action\[23\] and \( \sqrt{\frac{c^4}{8\pi G}} \) is the coupling constant \( g_{sv} \) of superon with the vacuum. Further we have seen in the preceding section that the right hand side of (11) for \( N=2 \), which is essentially \( N=2 \) NLSUSY VA action, is equivalent to the broken LSUSY action (10) with the vacuum expectation value of the auxiliary field giving the SUSY breaking mass \( M_{SUSY} \) to a component field of the (massless) LSUSY supermultiplet \( < D > \sim \sqrt{\frac{c^4}{8\pi G}} \sim M_{SUSY}^{-2} \).

We find NL SUSY(SGM) scenario gives interesting relations among the mysterious quantities of the cosmology and the low energy particle physics, i.e., \( \rho_D \sim \sqrt{\frac{c^4}{8\pi G}} \sim (\text{fundamental length of SGM spacetime})^{-4} \sim < D >^2 \) and \( g_{sv} \sim \sqrt{\frac{c^4}{8\pi G}} \sim < D > \). It is natural to suppose that among the LSUSY supermultiplet the neutrino \( \lambda(x) \) (not superon \( \psi(x) \)), which is the stable and the lightest particle, retains mass of the order of the spontaneous SUSY breaking, i.e. \( m_{\nu}^2 \sim \sqrt{\frac{c^4}{8\pi G}} \), then SGM predicts the observed value \( \rho_{D,obs} \sim (10^{-12}\text{GeV})^4 \sim m_{\nu}^4 \). The tiny neutrino mass is the direct evidence of SUSY (breaking), i.e., the spontaneous phase transition of SGM spacetime.

The proton decay imposes stringent constraints on (SUSY)GUTs in general, so the spacetime origin of tiny neutrino mass may be worth being considered. NLSUSY GR gives in general \( \Lambda \sim M_{SUSY}^2 \left( \frac{M_{Planck}}{M_{SUSY}} \right)^2 \). The large mass scales and the compact (broken) gauge d.o.f. necessary for the realistic and interacting broken LSUSY model will appear through the linearization of \( T_{\mu\nu}(e, \psi) \) which contains the mass scale \( \Lambda^{-1} \). NLSUSY GR (or SGM) (1) can be easily generalized to spacetime with extra dimensions, which allows to consider the unification in terms of the elementary fields.

5 Discussions

The geometry of new spacetime is described by SGM action (1) of vacuum EH-type and gives the unified description of nature. As proved for EH action of GR\[24\], the energy of NLSUSY GR action of EH-type is positive (for \( \Lambda > 0 \)). NLSUSY GR action (1), \( L(w) \sim \Omega(w) + w\Lambda \), on SGM spacetime is unstable and induces the spontaneous (symmetry) breakdown into EH action with NG fermion (massless superon) matter, \( L(e, \psi) \sim eR(e) + e\Lambda + (\cdots \psi, e \cdots)[10] \), on ordinary Riemann spacetime, for the curvature-energy of SGM spacetime is converted into the energy of Riemann
spacetime and the energy-momentum of superon(matter), i.e. \( w\Omega > eR \), which may be called Big Decay. As mentioned before SGM action possesses two different flat spaces: SGM-flat \( (w^a\mu(x) \rightarrow \delta^a\mu) \) of NLSUSY GR action \( L(w) \) and Riemann-flat \( (e^a\mu(x) \rightarrow \delta^a\mu) \) of SGM action \( \hat{L}(e,\psi) \) allowing the (generalized) NLSUSY VA action. This can be regarded as the phase transition of spacetime from SGM to Riemann (with massless NG fermion matter). Also this may be the birth of the present expanding universe, i.e. the big bang and the rapid expansion (inflation) of spacetime and matter, which is followed by the present acceleration due to \( \Lambda \). And we think that the birth of the present universe by the spontaneous breakdown of SGM spacetime described by vacuum action of EH-type (1) may explain qualitatively the observed critical value(\( \sim 1 \)) of the energy density of the universe. It is interesting if SGM could give new insights into the unsolved problems of the cosmology, e.g. the origins(real shapes) of the big bang, inflation, dark energy, matter-antimatter asymmetry, \( \cdots \), etc.

The linearization of NLSUSY GRT(N=1 SGM) action (1), i.e. the construction of the renormalizable and local broken LSUSY gauge field theory which is equivalent to (1), is inevitable to derive the SM as the low energy effective theory and to test the abovementioned cosmological aspects as well. The linearized theory may have the similarity to SUGRA\(^{25}\)\(^{26}\) with the spontaneous SUSY breaking\(^{27}\)\(^{28}\) but not the same up to the supermultiplet structure, for the new global NLSUSY transformation (3) should be respected throughout. Particularly N=10 must be linearized to test the (composite) SGM scenario for (1). The generalization of the systematics found in ref.[17] to the superspace formalism\(^{29}\) of SUGRA may be useful. By this detour of the linearization we can circumvent the no-go theorem based on the S-matrix arguments\(^{30}\)\(^{31}\) for the gravitational interaction of the high spin massless elementry gauge field. We can expect the dangerous high spin states are massive in the linearized equivalent theory on the true vacuum, for such states are contained in \( \tilde{T}_{\mu\nu}(e,\psi) \). The study of the vacuum structure of SGM action in the broken phase (i.e. NLSUSY GRT action in Riemann spacetime with matter) is crucial for linearizing SGM.

NLSUSY GR with the extra dimensions, which can be constructed straightforwardly and gives another unification framework by regarding the observed particles as elementary, is open. In this case there are two mechanisms for relating the structure of spacetime to the dynamical d.o.f., i.e. by the compactification of Kaluza-Klein type and by the new mechanism, Big Decay, presented in SGM.
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