Analytical theory of neutral current sheets with a sheared magnetic field in collisionless relativistic plasma

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Abstract. We derive and describe analytically a new wide class of self-consistent magnetostatic structures with sheared field lines and arbitrary energy distributions of particles. To do so we analyze superpositions of two planar current sheets with orthogonal magnetic fields and cylindrically symmetric momentum distribution functions, such that the magnetic field of one of them is directed along the symmetry axis of the distribution function of the other. These superpositions satisfy the pressure balance equation and allow one to construct configurations with an almost arbitrarily sheared magnetic field. We show that most of previously known current sheet families with sheared magnetic field lines are included in this novel class.

1. Introduction

Quasistationary neutral current sheets are important elements of various structures in collisionless plasma [1, 2, 3, 4], including space plasma or the relativistic plasma of neutron stars’ magnetosphere and wind. Most analytical studies of these structures are limited by planar configurations of magnetic field lines and ignore the effect of shear (see, e.g., [5, 6, 7]), although the latter appears naturally in the system ‘rotating magnetosphere – outgoing wind’ [3, 4].

In this report we present a new wide class of neutral self-consistent magnetostatic structures with sheared field lines using superpositions of planar current sheets with orthogonal magnetic fields for widely tunable momentum and arbitrary energy distributions of particles. We consider particle distributions as functions of generalized particle momenta and restrict ourselves to the case when the total particle distribution function expands into the sum of two cylindrically symmetric distribution functions with orthogonal axes. In this simple, yet poorly explored situation the equations of magnetostatics break down into two independent nonlinear Grad–Shafranov equations, each corresponding to a current sheet with a planar magnetic field.

Following our initial suggestion in [8], we analyze various superpositions of two neutral current sheets with orthogonal magnetic fields, in which the field of one of them is parallel to the current in the other. They satisfy the ‘plasma + field’ pressure balance equation and allow us to construct configurations with an almost arbitrarily sheared magnetic field. It turns out that one-dimensional periodic helical structures and localized current sheets exist with either monotonous magnetic field vector rotation, or with several switches in it.
We show that the major part of previously known current sheets with sheared magnetic field lines, e.g. [9, 10, 11, 12], are particular cases of this novel wide class of current sheets.

2. Analytical description of self-consistent current sheets with the magnetic shear

The analysis of stationary current structures in a collisionless plasma is based on the recently developed method of particle motion invariants, details on which could be found in [8]. Let all quantities depend on a single spatial coordinate \( x \) and the magnetic field \( \vec{B} \) be orthogonal to the \( x \)-axis, so that it can be described with two components of the vector potential \( \vec{A} = (A_y, A_z) \):

\[
B_{x,y} = \pm \frac{dA_{y,z}}{dx}.
\]

The basic fact is that in the case of a neutral plasma, with zero electrostatic potential, any distribution function of the form

\[
f_\alpha = f_\alpha^y (\varepsilon, p_y + \frac{q_\alpha}{c} A_y) + f_\alpha^z (\varepsilon, p_z + \frac{q_\alpha}{c} A_z),
\]

with orthogonal axes \( y \) and \( z \), the equations of magnetostatics for the vector potential \( \vec{A} \) break down into two independent equations of the Grad–Shafranov type:

\[
\frac{d^2 A_{y,z}}{dx^2} = -\frac{dU^{(y,z)}}{dA_{y,z}}.
\]

Here the functions \( U^{(y,z)}(A_{y,z}) = 4\pi P_{xx}^{(y,z)} + \text{const} \) are called the Grad–Shafranov potentials and

\[
P_{xx}^{(y,z)} = \sum_\alpha \int \int \int p_x v_x f_\alpha^{(y,z)} d^4p
\]

is the \( xx \)-component of the pressure tensor, \( v_x = p_x c^2/\varepsilon \).

Thus the use of simple superpositions (1) of two one-dimensional sheets, in which the magnetic field of one sheet is directed along the symmetry axis of the distribution function of the other, allows us to construct a wide class of self-consistent current sheets with a sheared magnetic field. Obviously, a planar external field may stand for the magnetic field of each sheet. The resultant solutions to (2), \( \vec{B}\{0, B_y, B_z\} \), may have rather diverse shear spatial profiles, determined by the shape of the respective Grad–Shafranov potentials [8] (Fig. 1). In the following we describe only a few representative combinations (see Fig. 2-5).

The mechanical analogy for the Harris sheet [6], \( B_{y,z} \propto \tanh((x - d_{1,2})/\lambda_{1,2}) \), is the reflection of a point \( A' \) from the potential wall (Fig. 1b). Profiles of the magnetic field and its shear angle, \( \psi(x) \), defined as the angle between the vector \( \vec{B} \) and the positive \( z \)-axis, of the combinations of a pair of those sheets are shown in Fig. 2 for different relations of spatial scales, \( \lambda_{1,2} \), and shift

**Figure 1.** Various types of the Grad–Shafranov potential (see [8] for details) corresponding to a) periodic sheets; b) isolated sheets (as Harris sheet); c) double sheets (as Nicholson sheet); d) symmetrical shielded sheets. Arrows show the motion of a point \( A' \) in the potential wells.
values, $d_{1,2}$. For the given parameters, with $x$ ranging from $-\infty$ to $\infty$, vector $\vec{B}(x)$ switches its rotation direction once, gathering $\psi = \pi$ radians in total. Similar self-consistent structures with sheared field lines were found in [9, 10] in a different way and only for non-relativistic Maxwell distribution function. Note that in our solutions particle energy distributions may be arbitrary, and similar sheared structures may be obtained for spatial profiles different from the Harris one and defined by other Grad–Shafranov potential profiles $U^{(y,z)}(A_{y,z})$.

The ‘motion’ type of the vector potential in Fig. 1c constitutes a self-consistent structure with a localized field as in Nicholson sheet [7]: $B_z \propto \cosh^{-1}((x-d_2)/\lambda_2)$. Its combination with a Harris-type one is shown in Fig. 3 (cf. [11]), with another localized sheet — in Fig. 5a.

Combining two symmetrical shielded current sheets with $B_{y,z} \propto d/dx \cosh^{-1}((x-d_{1,2})/\lambda_{1,2})$, corresponding to Fig. 1d, one can obtain a localized structure with several switches in the rotation direction of $\vec{B}$ (Fig. 4). Finally, a periodic sheet is presented in Fig. 5b. It is force-free, i.e., $\vec{B}^2(x) \equiv \text{const}$, and the pressure balance equation, following from (2), $P_{xx} + \vec{B}^2/8\pi \equiv \text{const}$, leads to $P_{xx} \equiv \text{const}$. Other examples of force-free sheets, resembling Figs. 4a, 5b, can be found in [12]. A general classification of the considered sheared magnetostatic structures may be easily given on the basis of known classification [8] of the shearless current sheets.

3. Conclusions
We describe and analyze a new wide class of neutral current sheets with a widely tunable momentum and arbitrary energy distributions of particles as well as rather arbitrarily sheared

Figure 2. Current sheet of a type 'Harris + Harris', $B_{y,z} \propto \tanh((x-d_{1,2})/\lambda_{1,2})$:
a) the shear angle $\psi/\pi$: black curve — the ratio between the scales of two sheets, $\lambda_2/\lambda_1$, is 1.2, $d_1 = d_2$; blue — $\lambda_2/\lambda_1 = 20$, the sheets are shifted along each other by $d_1 - d_2 = 0.5\lambda_1$; b) black — total magnetic field of a current sheet with parameters used for the blue curve in a), light blue and orange — fields of orthogonal sheets.

Figure 3. 'Harris + Nicholson' sheet, $B_y \propto \tanh((x-d_1)/\lambda_1)$, $B_z \propto \cosh^{-1}((x-d_2)/\lambda_2)$, $\lambda_2/\lambda_1 = 1.2$: a) black curve — total magnetic field, light blue and orange — fields of orthogonal sheets with $d_1 = d_2$; b) black — the shear angle for the parameters in a), red — for the shifted sheets, $d_1 - d_2 = 0.5\lambda_1$. 
magnetic field. These sheets may have almost arbitrary spatial profile of the field vector’s angle in the plane orthogonal to the axis of the sheet’s inhomogeneity, including several switches in its derivative. We give analytical examples of sheared structures of this kind and demonstrate qualitatively that the suggested class of current sheets may be used to model quasistationary equatorial current structures in pulsar magnetospheres.

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