Formulation Problems of Forming with the Theory of Incomplete Reversibility Creep Strain

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Abstract. The paper presents the formulation of direct and inverse problems of forming a quasi-static deformation based on the theory of incomplete reversibility of creep strain. Solving problems deformation and unloading are offered by the method of finite elements in MSC. Marc system. User subroutines, that determine the development of creep strain on the model of partial reversibility, are develop.

Introduction

There is a class of materials, which is observed experimentally only partial reversibility of deformation during unloading creep [1]. This property must take into account when solving the direct and inverse problems of forming. In earlier papers of the author [2] are formulated problems and constructed methods for solving direct and inverse problems of forming elements of structures, taking into account only the elastic unloading, and did not describe the reversible part of creep deformation after unloading. Considered the formulation is used for the tasks of finding, by forming in creep mode a predetermined aerodynamic shape for high-strength materials.

Formulation of Shaping Problems

Let \( V \subset R^n \) be a bounded domain with a sufficiently regular boundary \( S \). Denote by \( u = (u_1,u_2,u_3), \) \( u = (u_1,u_2,u_3) \) the vectors of current and residual displacements; \( u, \dot{u} \in W^1_0(Q), \) \( Q = V \times \{ 0 \leq t \leq T \} \).

Consider the quasi-static shaping problem with allowance for small deformations and large displacements and rotations (general Lagrangian formulation), including inelastic strain and unloading. The inelastic strain and unloading problems can be represented in the form of a quasi-static variational principle with the functional:

\[
J(u(t), u(t + \tau)) = \frac{1}{\varepsilon_1} \| u(t) - u^*(t) \|_S^2 + a(u(t), u(t)) + a(u(t + \tau), u(t + \tau)) + \frac{1}{\varepsilon_2} \| u(t + \tau) - u^*(t + \tau) \|_S^2,
\]

where \( \varepsilon_1 > 0, \varepsilon_1 \to 0, \varepsilon_2 > 0, \varepsilon_2 \to 0 \),

\[
a(u,v) = \int_V \frac{\partial E(u_{i,j})}{\partial u_{i,j}} v_{i,j} dV, \quad a(u,v) = \frac{\partial E(u_{i,j})}{\partial u_{i,j}} v_{i,j} dV,
\]

where \( u^*(t + \tau) \) is a given residual displacement rate on \( S \) in moment in time \( t + \tau \); \( t \) — time deformation under load; \( \tau \) — time unloading; potential form are given by ([3])
\[
E(u_{i,j}) = \frac{1}{2} \epsilon_{ijkl} \epsilon_{ijkl} - \frac{1}{2} \sigma_{ijkl,k} + \frac{1}{2} \sigma_{ijkl,j} ,
E(u_{i,j}) = \frac{1}{2} \epsilon_{ijkl} \epsilon_{ijkl} - \frac{1}{2} \sigma_{ijkl,k} + \frac{1}{2} \sigma_{ijkl,j},
\]
where the current and residual strain rates are the Green–Lagrange strain tensor rates
\[
\epsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) , 
\epsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}).
\]

Here, \( \epsilon_{ijkl} \) are the components of the symmetric elastic constant tensor; \( \eta_{ijkl} \) are the creep strain rates; \( i, j, k, l = 1, 2, 3 \); dotted letters denote derivatives with respect to time; and double indices with a comma denote differentiation with respect to the coordinate: \( u_{i,j} = \frac{\partial u_i}{\partial x_j} \).

In what follows, the symbol \( \langle . , . \rangle_S \) denotes the inner product in \( L_2(S) \): \( \langle u, v \rangle_S = \int \sum_{i=1}^{3} u_i v_i dS \). The corresponding norm is given by \( \| u \|_S = \sqrt{\langle u, u \rangle_S} = \left( \int \sum_{i=1}^{3} u_i^2 dS \right)^{1/2} \).

Creep velocity components are presented in the form of
\[
\eta_{ij} = \eta_{ij} + \eta_{ij} + \eta_{ij} + \eta_{ij},
\]
where \( \sigma_{ijkl} \) - the intensity of the stress tensor; \( \lambda_{ijkl} \), \( a_{ijkl} \), \( c_{ijkl} \), \( n_{ijkl} \), \( m_{ijkl} \), \( \sigma^* \) - model constants of the first and second stages of creep of the material and reversible after unloading part; \( \mu_{ijkl} \) and \( \mu_{ijkl} \) - Poisson’s ratios for reversible and irreversible components of creep strain; \( \beta_{ijkl} \) - active viscoplastic strain.

Calculation of plastic deformation \( v_{ij} \) is carried out in the principal axes. Components creep strains in the potential form problem of unloading are taking into account only the residual stresses.

Alternative constitutive relations written for the current and residual rates of the first Piola–Kirchhoff stress tensor have the form
\[
\Sigma_{ij} = \frac{\partial E(u_{i,j})}{\partial u_{i,j}} = (\delta_{ik} + u_{i,k}) \sigma_{kj} + u_{i,k} \sigma_{kj},
\]
where \( \Sigma_{ij} \) denotes the difference between solution-corresponding quantities in any two different forms of strain.
Iterative Method for Solving Inverse Shaping Problems

Then iterative process for solving the inverse shaping problem is represented as

\[ u_{i}^{k+1}(T) = u_{i}^{k}(T) + \alpha^{k} (u_{i}^{k}(T + T^*) - u_{i}^{k}(T + T^*)), \quad 0 < \alpha^{k} < 2, \]

where \( T \) - final time under loading in creep mode, \( T^* \) - the final unloading time to recover the final form.

To solve inverse shaping problems for structural elements, such as thin plates, it is sufficient to find a displacement function.

Applying the finite element method [3, 5] to the functional of the variational principle for inverse problems with allowance for the unloading displacement \( w^e = w - w \), we obtain two vector equations

\[ Kw = F, \quad Kw^e = F(\sigma_0), \quad (2) \]

where \( w, w \) are the velocities of the node parameters describing the inelastic strain displacement \( w(x_1, x_2) \) and the residual displacement \( w(x_1, x_2) \); \( K, K \) are the tangent stiffness matrices; and \( F \) is the vector of external forces.

The second equation in (Eq. 2) is an unloading problem with initial stresses and strains obtained by solving the problem loading.

Solving such problems requires the introduction of defining relations of the model incomplete reversibility of creep strain, in which is necessary to divide the strain. This is possible by building user subroutines in MSC.Marc system.

The numerical solution of finite element method in MSC. Marc system for uniaxial tension problem with the material EI698 [1] describes a full refund of the reversible component of the creep strain \( u_{i} \) after unloading (Figure 1, Figure 2).

![Figure 1. The reversible component creep strain during loading.](image-url)
Thus, the iterative method for solving inverse problems of shaping presented in [2], can be generalized for the case of an incomplete reversibility of creep strain.

Given the definition of parameters of the model (Eq.1) to describe the creep strain, solving problem modeling processes forming the wing panels [6] will allow provide for better accuracy.

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