Bits of String and Bits of Branes

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ABSTRACT

String-bit models are both an efficient way of organizing string perturbation theory, and a possible non-perturbative composite description of string theory. This is a summary of ideas and results in string-bit and superstring-bit models, as presented in the Strings ’96 conference.
What are string-bits?

Technically speaking, string-bits are:

- point particles in $d$ space + 1 time dimensions, transforming in the adjoint representation of $U(N_c)$.
- subject to Galilean invariant (nonrelativistic) dynamics.
- able to form closed chain bound states, which are $U(N_c)$ singlets.

These properties guarantee that in the continuum limit:

$$N \to \infty, \quad mN = \text{constant},$$

- $N_c \to \infty$ corresponds to free relativistic light-cone string theory in $D = d + 2$ dimensions.
- The $1/N_c$ expansion corresponds to string perturbation theory.

In this context,

- The $d$-dimensional Galilei group is understood as the light-cone subgroup of the $d + 2$-dimensional Poincaré group.
- The string coupling constant is given by $1/N_c$. 
Philosophically, there are two answers to this question, a conservative one and a radical one:

1. String-bits are a useful way of thinking about perturbative (light-cone) string theory. String-bit models incorporate both free string dynamics and perturbative string interactions at large $N_c$, and therefore offer an extremely efficient discretization of string theory. Key features:
   - Bosonic string-bit model with continuum properties of bosonic string theory.
   - SUSY string-bit model with continuum properties of type IIB superstring theory.
   - String interaction vertices, including necessary contact terms for the supersrtring, directly from string-bit interactions.
   - Logarithmic growth of free strings with energy.

2. String-bits are the fundamental degrees of freedom of string theory. String-bit models describe the dynamics of string constituents for any value of $N_c$, and therefore represent possible non-perturbative formulations of string theory. Key features:
   - String theory is a low energy effective theory.
   - Dimensional reduction: $d + 2 \rightarrow d + 1$.
   - Gauge (diffeomorphism) and Poincaré invariance are abandoned in favor of a global $U(N_c)$ and Galilean invariance.
   - Stability of discrete chains of bits from SUSY.
   - Free energy and dissociation transition at high temperature.
   - The physical size of an interacting string?
String-bit models and perturbative string theory

**Bosonic model** [1,2]

The string-bit dynamics are described by a Galilean invariant $N_c \times N_c$ matrix field theory with hamiltonian:

$$
H = \frac{1}{2m} \int dx \text{Tr} |\nabla \phi|^2 + \frac{T_0^2}{2mN_c} \int dx dy V(x-y) \text{Tr}[\phi^\dagger(x)\phi^\dagger(y)\phi(y)\phi(x)]
$$

The matrix field $\phi(x)^\beta_\alpha$ acts as an annihilation operator for a string-bit, and its conjugate $\phi^\dagger(x)^\beta_\alpha$ acts as a creation operator. The mass of each bit is $m$.

A single closed chain of $N$ bits is described by the Fock state:

$$
|\psi_N\rangle = \int dx_1 \cdots dx_N \text{Tr}[\phi^\dagger(x_1) \cdots \phi^\dagger(x_N)]|0\rangle \psi_N(x_1, \ldots, x_N),
$$

where the wavefunction $\psi_N(x_1, \ldots, x_N)$ is cyclically symmetric. The action of $H$ on this state produces two terms corresponding to a single-chain state and a two-chain state:

$$
H|\psi_N\rangle = |h\psi_N\rangle + \frac{1}{N_c} \sum_{k=1}^N |v_k\psi_{k,N-k}\rangle,
$$

where $h$ is the coordinate space (first-quantized) hamiltonian

$$
h = \frac{1}{2m} \sum_{k=1}^N \left[ p_k^2 + T_0^2 V(x_{k+1} - x_k) \right],
$$

and $v_k = \frac{T_0^2}{2m} V(x_{k+1} - x_1)$. 
Single-chain $N$-bit wavefunction is an eigenstate of $\hbar$,
\[ \hbar \psi_N(x_1, \ldots, x_N) = E \psi_N(x_1, \ldots, x_N), \]
\[ \iff \text{Physical bound chain } \iff V(x) \text{ strong enough to bind.} \]

Harmonic model $V(x) = x^2$ exactly solvable, excitation spectrum:
\[ E_n = \frac{2T_0}{m} \sin \frac{n\pi}{N}. \]

In the continuum limit the hamiltonian becomes:
\[ \hbar \to \frac{1}{2T_0} \int_0^{mN/T_0} d\sigma [\mathcal{P}(\sigma)^2 + T_0^2 x'(\sigma)^2]. \]

Finite energy modes occur for $n \ll N$ and for $N - n \ll N$:
\[ E_n \to \frac{2\pi n T_0}{mN}, \quad E_n \to \frac{2\pi (N - n) T_0}{mN}. \]
\[ \iff \text{Light-cone hamiltonian } (p^-) \text{ and right and left-moving spectra of bosonic string, where the longitudinal momentum is } p^+ = mN. \]

The extra dimension $x^-$ then emerges as the conjugate of $p^+$.

\underline{Large $N_c$}:
Two-chain term gives a 1-chain $\iff$ 2-chain transition at $O(1/N_c)$:
\[ \langle \psi_N | H | \psi_L, \psi_{N-L} \rangle \overset{N \to \infty}{\sim} \frac{1}{N_c} (mT_0)^{-3/2} N^{3/2 - d/8}, \]
\[ \iff \text{finite in continuum } \iff d = 24 \iff D = 26. \]

We stress that this vertex was derived from the same term in $H$ which gave the \textbf{free} string tension $T_0$. 
Limitations:
This bosonic model is of little use however, because:

1. Chains are unstable, and decay into smaller chains through the above vertex.

The ground state energy of a long chain is:

$$E_{G.S.} = d \sum_{n=1}^{N-1} E_n = (d/m) \left[ aN + \frac{b}{N} + O\left(\frac{1}{N^2}\right)\right].$$

In this model $b < 0$, so two chains are lighter than one, hence the instability. Continuum interpretation: $b = M_{tachyon}^2$.

2. Long range interaction between separate chains,

$$V(x_i - y_j) \sim (x_i - y_j)^2/N_c^2,$$ precludes a well defined S-matrix.

3. The bit interaction term in $H$ is not unique, e.g. add to it:

$$H'_{I} = \frac{\lambda}{N_c} \int dx dy U(x - y) : \text{Tr}[\phi^\dagger(x)\phi(x)\phi^\dagger(y)\phi(y)] :.$$  

Different matrix ordering in the trace implies

$$H'_{I}|\psi_N\rangle = \frac{1}{N_c} \sum_k |u_k \psi_{k,N-k}\rangle,$$

no $O(1)$ single chain term. Same free string limit, slightly different string interactions.
**Supersymmetric models** [3,4]

In continuum string theory SUSY resolves the first issue by removing the Tachyon. SUSY string bit models will resolve the first and second issues, and will reduce somewhat the third.

First, we extend the Galilei group to an $\mathcal{N} = 1$ Super-Galilei algebra by adding two supercharges $Q, R$, transforming as spinors under $SO(d)$ and satisfying among other things

$$\{Q^A, Q^B\} = mN\delta^{AB}, \quad \{Q^A, R^B\} = \frac{1}{2}\alpha_i^{AB} P^i, \quad \{R^A, R^B\} = \delta^{AB} H/2.$$ 

For simplicity, let $d = 1 \implies$ drop spinor indices.

The model is defined by the supercharges:

$$Q \sim \int dx \ Tr\phi^\dagger(x)\psi(x) + h.c.$$ 

$$R \sim \int dx \ Tr\phi^\dagger(x)\psi'(x) + \frac{1}{N_c} \int dxdy \ W(y-x)Tr\phi^\dagger(x)\rho(y)\psi(x) + h.c.,$$

where $\psi$ is the fermionic matrix field, and $\rho = [\phi^\dagger\phi + \psi^\dagger\psi]$ is the bit density matrix. The hamiltonian is computed from the superalgebra:

$$H = \frac{1}{2m} \int dx Tr[|\nabla\phi|^2 + |\nabla\psi|^2]$$

$$+ \frac{1}{2mN_c} \int dxdy \{[W^2(y-x) + W'(y-x)]Tr\phi^\dagger(x)\rho(y)\phi(x)$$

$$+ \text{other two-body terms}\}$$

$$+ \frac{1}{2mN_c^2} \int dxdydz \{W(y-x)W(z-x) : Tr\phi^\dagger(x)\rho(z)\rho(y)\phi(x) :$$

$$+ \text{other three-body terms}\}.$$ 

Note the presence of three-body terms, which were absent in the bosonic model, but are required by supersymmetry. These will give rise to superstring “contact terms” in the continuum limit.
The action of $H$ on a single SUSY chain state,

$$|\Psi_N\rangle = \int \prod dx_i d\theta_i \ Tr \prod [\phi^\dagger(x_i) + \psi^\dagger(x_i)\theta_i]|0\rangle \Psi_N(x_1, \theta_1 \ldots, x_N, \theta_N)$$

gives rise to several single-chain states, two-chain states and three-chain states.

$N_c \to \infty$:

Only the original single chain survives, acted on by the super-coordinate space Hamiltonian:

$$h = \frac{1}{2m} \sum_{k=1}^{N} \left\{ p_k^2 + W^2(x_{k+1} - x_k) \right\} + W'(x_{k+1} - x_k) [\theta_k \pi_k - \pi_k \theta_k + \pi_{k+1} \theta_k - \theta_{k+1} \pi_k - i(\theta_k \theta_{k+1} + \pi_k \pi_{k+1})].$$

$W(x) = T_0 x \implies$ SUSY harmonic model, exactly solvable.

- “statistics” modes spectrum = phonon spectrum.

- $E_{G.S.} = 0 \implies$ chains are stable. We stress that this is more than required for stability of continuum strings, and holds for all $N$.

Changing to more familiar fermionic variables,

$$S_k = \frac{1}{\sqrt{2}}(\theta_k + \pi_k) \ , \ \tilde{S}_k = \frac{i}{\sqrt{2}}(\theta_k - \pi_k) \ ,$$

the continuum limit becomes:

$$h \to \frac{1}{2T_0} \int_{P^+/T_0} d\sigma [P(\sigma)^2 + T_0^2 x'(\sigma)^2 - iT_0 S(\sigma) S'(\sigma) + iT_0 \tilde{S}(\sigma) \tilde{S}'(\sigma)],$$

$\implies$ Light-cone Hamiltonian of type IIB superstring.

Note that $\mathcal{N} = 1$ SUSY $\to \mathcal{N} = 2$ SUSY in the continuum limit, since discretization breaks half the supersymmetry.
Large $N_c$:
As in the bosonic model, the non-nearest-neighbor bit interactions will give rise to light-cone superstring vertices. At $O(1/N_c)$ we get a 3-string vertex:

$\sim$

At $O(1/N_c^2)$ the three body interactions will give rise to a 4-string contact term corresponding to the boundary of the moduli space of the 4-string amplitude:

$\sim \mathcal{O}$

and a 2-string contact term corresponding to the boundary of the moduli space of the one loop string propagator:

$\sim \mathcal{O}$

Such contact terms (at least the 4-string vertex) were shown to be necessary in interacting superstring theory by supersymmetry [5]. Here they arise naturally, just like the usual 3-string vertex, from the superstring-bit model.
Short range interactions:
SUSY harmonic model still suffers from long range interactions between separate chains. Can we make the interaction short range, while still maintaining a stringy continuum limit?

Yes! In the $d = 1$ superstring-bit model we can prove a restricted form of UNIVERSALITY. Let:

$$W(x) = T_0 x + \delta W(x).$$

As long as $|\delta W(x)| \ll T_0 |x|$ for $|x| < \sqrt{\alpha'}$, the only effect is to renormalize the string tension:

$$T_0 \rightarrow T_0 + \langle \delta W'(x_2 - x_1) \rangle.$$

So we deform the superpotential as in the figure, resulting in a short range interaction. This is not quite enough for a well defined S-matrix, since the chain separation energy is still large (diverges in the continuum limit). We fix this by replacing the density matrix:

$$\rho(y) \rightarrow \rho(y) - : [\phi\phi^\dagger - \psi\psi^\dagger] :$$

in the equation for the supercharge $R$. This has the effect of freeing the two chains when they are far apart.

The requirement of asymptotic freedom also has the effect of reducing the non-uniqueness of the bit interactions, since only very special interactions will have this property.
String-bits as the fundamental degrees of freedom

- Continuum limit equivalent to low energy limit, so string theory can be thought of as a low energy effective theory of a certain string-bit model.
- Stability at the discrete level allows such an interpretation.
- String bit model can be analyzed for small $N_c$, with possible implications on non-perturbative string behavior.

High temperature: [6]
The binding energy of two bits in a chain is $E_B \sim T_0/m$, so a dissociation phase transition is expected to occur at $T_c \sim T_0/m$. The low temperature phase is a bound chain, whereas the high temperature phase is a gas of nonrelativistic weakly interacting particles in $d$ space dimensions, with a free energy:

$$\frac{F}{VT} \sim T^{d/2}.$$  

For $d = 2$, corresponding to a low temperature phase of 4-dimensional string theory, this gives the result found by Atick and Witten [7] using string perturbation theory. The linear dependence on temperature implies that string theory ultimately contains far fewer degrees of freedom than any relativistic field theory, even though it is not manifest in the perturbative approach. String-bits offer a possible physical realization of this idea.
**String Growth** : [8]

• Physical (transverse) size of perturbative string is infinite:

\[ R_\perp^2 \sim \sum_{n=1}^{\infty} \frac{1}{n} \, . \]

• Finite resolution time \( \epsilon \Rightarrow \)

\[ R_\perp^2 \sim \ln \frac{p^+}{\epsilon} \, . \]

String grows denser with increasing \( p^+ \), invalidating perturbation theory at the Planck density \( T_0/g^2 \).

• Black hole entropy \( \propto A/\hbar \Rightarrow \) seems to require \( R_\perp^2 \sim p^+/\epsilon \) at high \( p^+ \) [9]. This would be a non-perturbative effect.

In the string-bit picture a measure for the size is:

\[ R^2(N) = \frac{1}{N} \sum_{k=1}^{N} \langle \text{G.S.}|(x_k - x_1)^2|\text{G.S.}\rangle \, . \]

• \( N_c \rightarrow \infty \Rightarrow \) In the harmonic model \( R^2(N) \sim \ln N \), but this was shown (numerically) to hold for other nearest-neighbor interactions as well, in particular short-range ones.

• Finite \( N_c \) : String-bit models imply non-nearest-neighbor repulsions, “bits with elbows”. We studied a toy model for elbows, that uses harmonic nearest-neighbor attraction with \( \delta \)-function repulsions:

\[ h = \frac{1}{2m} \sum_{k=1}^{N} \left[ p_k^2 + T_0^2 (x_{k+1} - x_k)^2 + g^2 \sum_{l \neq k}^{N} \delta(x_k - x_l) \right] \, . \]
We used a variational approach using wavefunctions that are exact solutions to an harmonic problem. The variational parameters can be taken to be the mode frequencies $\omega_n$. The variational energy and size are then:

$$E = \frac{1}{2m} \left[ \sum_n \omega_n + \sum_n \frac{\sin^2(\pi n/N)}{\omega_n} + \frac{1}{2} g^2 N^2 \sum_k \left( \sum_n \frac{\sin^2(\pi nk/N)}{\omega_n} \right)^{-1} \right],$$

$$R^2 = \frac{1}{N} \sum_{n=1}^{N-1} 1/\omega_n.$$

Numerical solution of variational problem resulted in the following growth patterns:

\[ \Rightarrow \text{Chains experience a small } N \text{ growth of } R^2 \sim \ln N, \text{ and a large } N \text{ growth of } R^2 \sim N^2, \text{ not the conjectured } R^2 \sim N \propto p^+. \]

Somewhat discouraging, since relativistic string $\iff R^2 \lesssim N$. 
Bits of Branes [6]

- Membrane-bit field: $\phi(x)^{\beta b}_{\alpha a}$, four legged object.
- $U(N_c) \times U(N_c)$ global symmetry.
- Building blocks:

![Building block diagrams]

allow for a rich variety of singlet structures.

- Membranes made by tiling type (b) structures, e.g.

![Membrane diagrams]

- Stringy objects made by connecting type (a) structures, e.g.

![Stringy object diagrams]

(a) Closed string.
(b) Open string attached to closed string, D-1-brane?
(c) Open string attached to two closed strings, two D-1-branes?
Dynamics:

- Free hamiltonian is unique:
  \[ H_0 = \frac{1}{2m} \int dx \nabla \phi^\dagger(x) \frac{\beta^b}{\alpha^a} \cdot \nabla \phi(x) \frac{\alpha^a}{\beta^b}. \]

- Unlike string-bit models, there are many possible membrane-bit interactions that yield a nearest-neighbor interaction pattern for \( N_c \to \infty \), e.g.
  \[
  H_1 = \frac{1}{N_c^2} \int dx dy V_1(x - y) \phi^\dagger(x) \frac{\beta^b}{\alpha^a} \phi^\dagger(y) \frac{\gamma^c}{\beta^b} \phi(y) \frac{\delta^d}{\gamma^c} \phi(x) \frac{\alpha^a}{\delta^d} \\
  H_2 = \frac{1}{N_c} \int dx dy V_2(x - y) \phi^\dagger(x) \frac{\beta^b}{\alpha^a} \phi^\dagger(y) \frac{\delta^d}{\beta^c} \phi(y) \frac{\epsilon^c}{\delta^d} \phi(x) \frac{\alpha^a}{\epsilon^b} \\
  H_3 = \frac{1}{N_c} \int dx dy V_3(x - y) \phi^\dagger(x) \frac{\beta^b}{\alpha^a} \phi^\dagger(y) \frac{\delta^d}{\gamma^c} \phi(y) \frac{\gamma^c}{\delta^d} \phi(x) \frac{\alpha^a}{\beta^c}.
  \]

Different interactions will generally give an \( \mathcal{O}(1) \) result for different kinds of singlet structures.

- Consider a membrane bit model defined by:
  \[ H = H_0 + \lambda_1 H_1 + \lambda_2 H_2 + \lambda_3 H_3. \]

It can be shown that if \( \lambda_2 = \lambda_3 = 0 \), this model does not support pure membrane formation. At generic values of the parameters, both membranes and strings, as well as mixed structures are supported.

\[ \implies \text{parameter space} \sim \text{“moduli” space}. \]

- \( p \)-brane-bits: \( 2p \)-legged objects, can form \( p \)-branes, \( p-1 \)-branes, etc.

\[ \implies \text{Unified and \textbf{democratic} description of all } p \text{-branes as composites of bits.} \]
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