Selection Of Best Social Media Using Group Decision With Picture Fuzzy Graphs

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Abstract-In this dissertation, we study the picture fuzzy graph (PFG) in the decision-making problem based on picture fuzzy relation and some types of picture fuzzy graphs such as complete picture fuzzy graphs, strong picture fuzzy graphs, complement picture fuzzy graphs are described and also study the Pythagorean fuzzy graph in decision making environment. Further, we present an application of Pythagorean Fuzzy Graphs and Picture Fuzzy Graphs in decision making environment.

Keywords: Pythagorean Fuzzy Graph, Picture Fuzzy Graph, Picture Fuzzy Preference Relation, Picture Fuzzy Weighted Averaging Operator, Picture Fuzzy Weighted Geometric Operator.

1. Introduction

In this paper, we find the best social media for developing the career and studies vice in the current COVID-19 situation using picture fuzzy graphs. In this critical situation, peoples have to consider our safety and develop their business, students also concentrated their studies. In this way, we choose that kind of social media for those peoples. In the previous chapter, we discuss Face book, what’s App, and YouTube. Here, we have seen about Google Meet, which is one of the popular Social media also, and is more suitable for studying students business application like webinars and video conferences in the current scenario. With Google Meet, you can host video calls with up to 150 people.

2. Definition

DEFINITION 1.1:

Let M be a non-empty finite set and \( \delta : M \rightarrow [0, 1] \), let \( \Omega : M \times M \rightarrow [0, 1] \) such that \( \Omega(a, b) \leq \delta(a) \land \delta(b) \forall a, b \in M \times M \). The pair \( G = (\delta, \Omega) \) is called a fuzzy graph over the set.
DEFINITION 1.2:
Let \( G^* = (V, G) \) be a graph. A pair \( G = (X, Y) \) is called a Pythagorean fuzzy graph on \( G^* \) where \( X = (\theta_x, \chi_x, \phi_x) \) on \( V \) and \( Y = (\theta_y, \chi_y, \phi_y) \) on \( E \subseteq V \times V \) such that for each \( ab \in E \):
\[
\theta_y(a, b) \leq \min (\theta_x(a), \theta_x(b)), \phi_x(a, b) \geq \max (\phi_x(a), \phi_x(b))
\]

DEFINITION 1.3:
Let \( G^* = (V, E) \) be a graph. A pair \( G = (X, Y) \) is called a picture fuzzy graph on \( G^* \) where \( X = (\theta_x, \chi_x, \phi_x) \) is a picture fuzzy set on \( V \) and \( Y = (\theta_y, \chi_y, \phi_y) \) is a picture fuzzy set on \( E \subseteq V \times V \) such that for each arc \( ab \in E \) as shown in Table 1.

DEFINITION 1.4:
A picture fuzzy preference relation (PFPR) \( H \) on a set of choices \( V = \{v_1, v_2, \ldots, v_n\} \) is described by a matrix \( H = (h_{ij})_{n \times n} \subset V \times V \) where
\[
h_{ij} \left( \theta_{ij}, \chi_{ij}, \phi_{ij} \right)
\]
for all \( i, j = 1, 2, \ldots, n \).

DEFINITION 1.5:
Let \( \alpha_j (j = 1, 2, 3, \ldots, n) \) be a collection of Pythagorean Fuzzy Numbers. The picture fuzzy weighted averaging (PFWA) operator is a mapping \( P^n \rightarrow P \) such that \( PFWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \oplus_{j=1}^{n} (\omega_j \alpha_j) \)
where \( \omega = (\omega_1, \omega_2, \ldots, \omega_n) \) be the weight vector of \( \alpha_j = (\alpha_1, \alpha_2, \ldots, \alpha_n) \) and \( \sum_{j=1}^{n} \omega_j = 1 \).

3. Theorem
The aggregated value by using PFWA operator is also a Pythagorean Fuzzy Numbers, where
\[
PFWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left\{ 1 - \prod_{j=1}^{n} \left( 1 - \theta_{ij} \right)^{\gamma}, \prod_{j=1}^{n} \left( \chi_{ij} \right)^{\nu}, \prod_{j=1}^{n} \left( \phi_{ij} \right)^{\nu} \right\}
\]
Here, we use picture fuzzy weighted averaging PFWA operator [3,4] to combine the individual PFPR. Thus, we have
\[
PFWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left\{ 1 - \prod_{j=1}^{n} \left( 1 - \theta_{ij} \right)^{\gamma}, \prod_{j=1}^{n} \left( \chi_{ij} \right)^{\nu}, \prod_{j=1}^{n} \left( \phi_{ij} \right)^{\nu} \right\}
\]
Here, we consider five alternatives, \( A_1 \) - FaceBook(FB), \( A_2 \) - Youtube(YT), \( A_3 \) - Whatsapp(WA), \( A_4 \) - Instagram (IG), \( A_5 \) - Twitter(TR). To choose a suitable social platform, three experts \( E_k (k = 1, 2, 3) \) are evaluate the alternative to choice the best alternative using picture fuzzy graphs. The experts analyze each pair of choices and give singular decisions utilizing the accompanying PFPRs. The picture fuzzy digraphs \( D_k \) corresponding to PFPRs given below,
In this section, we use picture fuzzy graphs to find the best social media for peoples with interest, neutral, disinterest towards a social medias are given in $D_1$, $D_2$, and $D_3$ are described in Table 2. Now, we use PFWA operator [3,4] to combine the individual PFPR. Thus, we have

$$PFWA = \left( 1 - \prod_{j=1}^{n} (1 - \theta) \right)^{\frac{1}{n}}, \prod_{j=1}^{n} (\chi)_{\phi}^{\frac{1}{n}}, \prod_{j=1}^{n} (\phi)_{\chi}^{\frac{1}{n}}$$

Where

$$1 - \prod_{j=1}^{n} (1 - \theta)_{\phi}^{\frac{1}{n}} = \left[ 1 - \left( (1 - 0.3)_{\phi}^{\frac{1}{3}} \times (1 - 0.3)_{\chi}^{\frac{1}{3}} \times (1 - 0.3)_{\phi}^{\frac{1}{3}} \right) \right]$$

$$= [1 - (0.8879 \times 0.8879 \times 0.8879)]$$

$$= 0.3001$$

$$\prod_{j=1}^{n} (\chi)_{\phi}^{\frac{1}{n}} = \left[ (0.3)_{\phi}^{\frac{1}{3}} \times (0.3)_{\phi}^{\frac{1}{3}} \times (0.3)_{\phi}^{\frac{1}{3}} \right] = [0.6694 \times 0.6694 \times 0.6694]$$

$$= 0.3001$$

$$\prod_{j=1}^{n} (\phi)_{\chi}^{\frac{1}{n}} = \left[ (0.3)_{\chi}^{\frac{1}{3}} \times (0.3)_{\chi}^{\frac{1}{3}} \times (0.3)_{\chi}^{\frac{1}{3}} \right] = [0.6694 \times 0.6694 \times 0.6694]$$

$$= 0.3001$$

$$1 - \prod_{j=1}^{n} (1 - \theta)_{\chi}^{\frac{1}{n}} = \left[ 1 - \left( (1 - 0.2)_{\chi}^{\frac{1}{3}} \times (1 - 0.5)_{\phi}^{\frac{1}{3}} \times (1 - 0.4)_{\phi}^{\frac{1}{3}} \right) \right]$$

$$= [1 - (0.9283 \times 0.7937 \times 0.8434)]$$

$$= 0.3786$$

$$\prod_{j=1}^{n} (\chi)_{\phi}^{\frac{1}{n}} = \left[ (0.3)_{\phi}^{\frac{1}{3}} \times (0.2)_{\phi}^{\frac{1}{3}} \times (0.3)_{\phi}^{\frac{1}{3}} \right] = [0.6694 \times 0.5848 \times 0.6694]$$

$$= 0.2620$$

$$\prod_{j=1}^{n} (\phi)_{\chi}^{\frac{1}{n}} = \left[ (0.4)_{\chi}^{\frac{1}{3}} \times (0.3)_{\chi}^{\frac{1}{3}} \times (0.1)_{\chi}^{\frac{1}{3}} \right] = [0.6694 \times 0.6694 \times 0.6694]$$

$$= 0.2289$$

Now, we found the individual PFPRs is given below and directed model of combined picture fuzzy preference relations is shown in Figure 1.
Fig 1. Directed model of the combined PFPR

D value is shown in Table 3.

Here, we select picture fuzzy numbers whose truthness degrees $\alpha_{ij}, (i,j = 1, 2, 3, 4)$ and the results of partial model is presented in Figure 2

$$\alpha_{ij} \geq 0.3$$

Fig 2. Partial model of the combined PFPR

**Out-degree values**

We compute the out-degrees $(j = 1, 2, 3, 4)$ in a partial directed model as follows:

$$Out - d(A_1) = [0.3786, 0.2620, 0.2289]$$

$$Out - d(A_2) = [2.0318, 0.2586, 0.4433]$$

$$Out - d(A_3) = [0.6086, 0.1587, 0.1816]$$

$$Out - d(A_4) = [0.3001, 0.1586, 0.3914]$$

As indicated by the truthness degrees of $(j = 1, 2, 3, 4)$, we have the positioning of the variables are

$$Out - d(A_j)$$

$$A_2 > A_3 > A_1 > A_4$$

The best alternative is $A_2$.

Now, using a picture fuzzy weighted geometric (PFWG) operator [3, 4]

$$PFWA = \left\{ \prod_{j=1}^{n}(\theta)^{\frac{1}{p}}, 1 - \prod_{j=1}^{n}(1 - \chi)^{\frac{1}{p}}, 1 - \prod_{j=1}^{n}(1 - \phi)^{\frac{1}{p}} \right\}$$

$$\prod_{j=1}^{3}(\theta)^{\frac{1}{3}} = \left[ (0.3)^{\frac{1}{3}} \times (0.3)^{\frac{1}{3}} \times (0.3)^{\frac{1}{3}} \right] = [0.6694 \times 0.6694 \times 0.6694]$$
\[ = 0.3001 \]
\[
1 - \prod_{j=1}^{n} (1 - \chi_{ij})^{\frac{1}{n}} = \left[ 1 - \left\{ (1 - 0.3)^{\frac{1}{n}} \times (1 - 0.3)^{\frac{1}{n}} \right\} \right]
\]
\[ = \left[ 1 - (0.8879 \times 0.8879 \times 0.8879) \right] \]
\[ = 0.3001 \]
\[
1 - \prod_{j=1}^{n} (1 - \phi_{ij})^{\frac{1}{n}} = \left[ 1 - \left\{ (1 - 0.3)^{\frac{1}{n}} \times (1 - 0.3)^{\frac{1}{n}} \right\} \right]
\]
\[ = \left[ 1 - (0.8879 \times 0.8879 \times 0.8879) \right] \]
\[ = 0.3001 \]
\[
\prod_{j=1}^{n} (\theta_{ij})^{\frac{1}{n}} = \left[ (0.2)^{\frac{1}{n}} \times (0.5)^{\frac{1}{n}} \times (0.4)^{\frac{1}{n}} \right] = [0.5848 \times 0.7937 \times 0.7368] = 0.3419
\]
\[
1 - \prod_{j=1}^{n} (1 - \chi_{ij})^{\frac{1}{n}} = \left[ 1 - \left\{ (1 - 0.3)^{\frac{1}{n}} \times (1 - 0.2)^{\frac{1}{n}} \times (1 - 0.3)^{\frac{1}{n}} \right\} \right] = 0.2682
\]
\[
1 - \prod_{j=1}^{n} (1 - \phi_{ij})^{\frac{1}{n}} = \left[ 1 - \left\{ (1 - 0.4)^{\frac{1}{n}} \times (1 - 0.3)^{\frac{1}{n}} \times (1 - 0.1)^{\frac{1}{n}} \right\} \right] = 0.2771
\]

Now, we found the individual PFPRs is given below and the directed model of combined picture fuzzy preference relations shown in Figure 3.

![Directed model of the combined PFPR](image)

The D value is shown in Table 4.

Here, we select picture fuzzy numbers whose truthness degrees \( \alpha_{ij} \geq 0.3 \).
Fig 4. Partial model of the combined PFPR

**Out-degree values**

We compute the out-degrees \( j = 1, 2, 3, 4 \) in a partial directed model as follows:

\[
Out - d(A_j) \]

\( j = 1, 2, 3, 4 \)

- \( Out - d(A_1) = [0.3419, 0.2682, 0.2771] \)
- \( Out - d(A_2) = [1.9845, 1.2684, 0.4711] \)
- \( Out - d(A_3) = [0.3001, 0.214, 0.4572] \)
- \( Out - d(A_4) = [0.3001, 0.214, 0.4572] \)

As indicated by the truthness degrees of \( j = 1, 2, 3, 4 \), we have the positioning of the variables are

\[ A_2 > A_3 > A_1 > A_4 \]

The best alternative is \( A_2 \).

Hence, Google Meet (\( A_2 \)) is the best social media for students to their studies in COVID-19.

### 4. CONCLUSION

Picture fuzzy set is a direct extension of fuzzy set and intuitionistic fuzzy Set. Picture fuzzy models deal with uncertainty problems more efficiently, with a constraint of \( 0 \leq \mu + \eta + \nu \leq 1 \), which provides a wider space for assigning one’s own level of liking compared to the intuitionistic fuzzy models. Because fuzzy graph theory can easily handle ambiguous and vague concepts, it has many applications in modeling real-life systems, where the levels of information inherent in the system vary with varying degrees of accuracy. In graph-theoretical concepts, to deal with situations when human ideas are of the types: yes, abstain, no, refuse, the proposed model can reveal a wide range of assessment information. In this dissertation, we have discussed some operations of PFGs and have made several conclusions regarding their degrees. Furthermore, we have described some innovative ideas of PFGs, such as strong and connected with illustrative examples. The PFGs allows one to overcome many of the challenges associated with the analysis of graphs. We have developed the MCGTM problem of choosing the best social media using PFGs when decision-making problems in ambiguous models present themselves to different elements of the state. The applications of PFGs serve us with innovative and optimal results that seem to be highly significant as they give directions
to MCDM. In the future, we plan to extend this study to (i) interval-valued picture fuzzy graphs (ii) hesitant picture fuzzy graphs; and (iii) interval-valued hesitant picture fuzzy.

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Table 1. Formula

\[ \theta_1(a,b) \leq \min(\theta_X(a),\theta_X(b)), \chi_1(a,b) \leq \min(\chi_X(a),\chi_X(b)), \phi_1(a,b) \geq \max(\phi_X(a),\phi_X(b)) \]

Table 2. D1, D2 and D3

\[
D_1 = \begin{bmatrix}
  (0.3,0.3,0.3) & (0.2,0.3,0.4) & (0.1,0.2,0.6) & (0.2,0.5,0.2) \\
  (0.7,0.1,0.2) & (0.3,0.3,0.3) & (0.8,0.1,0.1) & (0.6,0.1,0.2) \\
  (0.1,0.4,0.3) & (0.7,0.2,0.1) & (0.3,0.3,0.3) & (0.2,0.5,0.3) \\
  (0.2,0.2,0.5) & (0.3,0.1,0.6) & (0.1,0.1,0.7) & (0.3,0.3,0.3) 
\end{bmatrix}
\]

\[
D_2 = \begin{bmatrix}
  (0.3,0.3,0.3) & (0.5,0.2,0.3) & (0.2,0.5,0.2) & (0.1,0.6,0.2) \\
  (0.8,0.0,0.1) & (0.3,0.3,0.3) & (0.6,0.1,0.2) & (0.7,0.2,0.1) \\
  (0.2,0.5,0.2) & (0.6,0.1,0.2) & (0.3,0.3,0.3) & (0.2,0.6,0.1) \\
  (0.3,0.4,0.2) & (0.3,0.1,0.5) & (0.1,0.1,0.6) & (0.3,0.3,0.3) 
\end{bmatrix}
\]

\[
D_3 = \begin{bmatrix}
  (0.3,0.3,0.3) & (0.4,0.3,0.1) & (0.2,0.1,0.5) & (0.1,0.4,0.2) \\
  (0.6,0.1,0.1) & (0.3,0.3,0.3) & (0.7,0.1,0.2) & (0.5,0.2,0.2) \\
  (0.2,0.2,0.5) & (0.5,0.2,0.3) & (0.3,0.3,0.3) & (0.1,0.3,0.4) \\
  (0.2,0.0,0.5) & (0.3,0.4,0.2) & (0.2,0.3,0.4) & (0.3,0.3,0.3) 
\end{bmatrix}
\]

Table 3. D value

\[
D = \begin{bmatrix}
  (0.3001,0.3001,0.3001) & (0.3786,0.2620,0.2289) & (0.1681,0.2154,0.3914) & (0.1349,0.4932,0.1999) \\
  (0.7116,0.0,0.1259) & (0.3001,0.3001,0.3001) & (0.7116,0.0999,0.1587) & (0.6086,0.1587,0.1587) \\
  (0.1681,0.3419,0.3107) & (0.6086,0.1587,0.1816) & (0.3001,0.3001,0.3001) & (0.1681,0.4481,0.2289) \\
  (0.2349,0.0,0.3684) & (0.3001,0.1586,0.3914) & (0.1349,0.1441,0.5517) & (0.3001,0.3001,0.3001) 
\end{bmatrix}
\]
Out-degree values

We compute the out-degrees \((j = 1, 2, 3, 4)\) in a partial directed model as follows:

\[
\begin{align*}
Out - d(A_1) &= [0.3419, 0.2602] \\
Out - d(A_2) &= [1.9845, 1.2684, 0.4711] \\
Out - d(A_3) &= [0.5943, 0.1681, 0.2043] \\
Out - d(A_4) &= [0.3001, 0.214, 0.4572]
\end{align*}
\]

As indicated by the truthness degrees of \((j = 1, 2, 3, 4)\), we have the positioning of the variables are

\[
\text{Out } \rightarrow A_2 > A_3 > A_1 > A_4
\]

The best alternative is \(A_2\).

Hence, Google Meet \((A_2)\) is the best social media for students to their studies in COVID-19.

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Picture fuzzy set is a direct extension of fuzzy set and intuitionistic fuzzy set. Picture fuzzy models deal with uncertainty problems more efficiently, with a constraint of \(0 \leq \mu + \eta + \nu \leq 1\), which provides a wider space for assigning one’s own level of liking compared to the intuitionistic fuzzy models. Because fuzzy graph theory can easily handle ambiguous and vague concepts, it has many applications in modeling real-life systems, where the levels of information inherent in the system vary with varying degrees of accuracy. In graph-theoretical concepts, to deal with situations when human ideas are of the types: yes, abstain, no, refuse, the proposed model can reveal a wide range of assessment information. In this dissertation, we have discussed some operations of PFGs and have made several conclusions regarding their degrees. Furthermore, we have described some innovative ideas of PFGs, such as strong and connected with illustrative examples. The PFGs allows one to overcome many of the challenges associated with the analysis of graphs. We have developed the MCGTM problem of choosing the best social media using PFGs when decision-making problems in ambiguous models present themselves to different elements of the state. The applications of PFGs serve us with innovative and optimal results that seem to be highly significant as they give directions.