The Effective Lagrangian for the Seesaw Model of Neutrino Mass and Leptogenesis

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Abstract

The effective Lagrangian for the seesaw model is derived including effects due to \textit{CP} violation. Besides the usual dimension-5 operator responsible for light neutrino masses, a dimension-6 operator is obtained. For three or less heavy neutrino generations, the inclusion of both operators is necessary and sufficient for all independent physical parameters of the high-energy seesaw Lagrangian to appear in the low-energy effective theory, including the \textit{CP}-odd phases relevant for leptogenesis. The dimension-6 operator implies exotic low-energy couplings for light neutrinos, providing a link between the high-energy physics and low-energy observables.

1 Introduction

There is mounting experimental evidence for neutrino masses and mixings from oscillation experiments \cite{1}. It is possible that the light neutrino masses take natural (i.e. non-fine-tuned) values, in contrast to all other fermion masses except the top quark mass, if the smallness of neutrino masses is explained by the seesaw mechanism \cite{2}. In the minimal seesaw model \cite{3}, gauge singlet fermions with Majorana masses of order $M$ much larger than the electroweak scale couple to the massless weak doublet neutrinos of the Standard Model (SM) through Yukawa couplings to the Higgs scalar doublet. Upon spontaneous symmetry breakdown of the electroweak gauge symmetry, these Yukawa interactions generate a Dirac neutrino mass between the heavy singlet and weak doublet neutrinos. The otherwise

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massless weakly-interacting neutrinos develop small masses \( \sim -m^2_{\text{Dirac}}/M \). The conditions for a seesaw mechanism of light neutrino mass to occur, namely the existence of heavy Majorana gauge singlet fermions with Yukawa couplings to the massless weakly-interacting neutrinos, naturally arise in the context of grand unified theories and partially unified theories. It is striking that present neutrino oscillation data indicates a seesaw scale \( M \) of new physics that is comparable to the scale at which the Standard Model gauge couplings are converging.

Present solar and atmospheric neutrino oscillation data imply two distinct mass differences for light neutrinos. In a seesaw model, this requires at least two heavy Majorana singlet neutrinos. As soon as two or more Majorana neutrinos are present in the seesaw model, an attractive scenario opens up for solving the puzzle of the matter-antimatter asymmetry of the universe: leptogenesis at the scale \( M \) and baryogenesis from the lepton asymmetry [3].

Recent observations of acoustic peaks in the Cosmic Microwave Background (CMB) [4] have confirmed and refined the estimation of the baryon number \( B \) in the universe resulting from big bang nucleosynthesis. The ratio of the baryon to photon density \( \eta = (n_B - n_{\bar{B}})/n_\gamma = n_B/n_\gamma \), extracted from the CMB, is

\[
\eta = (6.0^{+1.1}_{-0.8}) \times 10^{-10}.
\]

The Sakharov conditions [5] for baryogenesis are departure from thermal equilibrium, and violation of \( B, CP \) and \( C \). Electroweak baryogenesis is ruled out in the SM since \( CP \)-violation in the quark sector is insufficient by many orders of magnitude [6], and in addition the Higgs mass seems to be too large [7]. In practice, any efficient mechanism for baryogenesis beyond the SM acting at high energies should break both \( B \) and \( B-L \), where \( L \) denotes lepton number; otherwise any baryon asymmetry produced at high energy can be washed out by non-perturbative effects, the sphalerons [8], which preserve \( B-L \), well before the electroweak transition [9]. It is interesting that a baryon asymmetry points to \( (B-L) \)-breaking interactions, and hence to \( L \)-violating neutrino masses.

An excess of lepton density in the early universe can be generated dynamically in the seesaw model by decay of the heavy Majorana neutrinos into light leptons and the Higgs boson, since lepton number \( L \), \( CP \) and \( C \) are violated by the decay. The expansion of the universe naturally provides the necessary out-of-thermal-equilibrium condition required for leptogenesis. The SM interactions recycle about half of this lepton asymmetry into a baryon asymmetry by active sphaleron processes. The seesaw model of leptogenesis is extremely elegant and highly economical, since it requires only a minimal extension of the SM to include sterile neutrinos.

The experimental discovery of \( CP \)-violation in the lepton sector would be a major breakthrough. Low-energy Majorana \( CP \)-odd phases, requiring at least two active light neutrino families, may contribute [10] to the total neutrinoless double beta decay amplitude \(^1\). The “CKM-like” \( CP \)-odd phases, requiring at least three active light neutrino families, may be detected in neutrino oscillations, provided the large mixing angle MSW solution [12] to the solar neutrino deficit is confirmed, and the angle \( \theta_{13} \) is not orders of magnitude below its present limit [13]. It is of prime importance to determine what is the connection, if any, between the \( CP \)-odd phases of the high-energy seesaw Lagrangian and those to be measured in on-going or future experiments.

\(^1\)For a recent pessimistic appraisal of the practical possibility of extracting them in the future, see ref. [11].
Establishing whether light neutrino masses are the result of the seesaw model requires finding an experimental signature of the seesaw model beyond the existence of light neutrino masses. In this work we construct an effective Lagrangian of the seesaw theory which is valid for energies below the seesaw scale. We search for the minimal set of higher dimensional operators, compatible with the symmetries of the problem, which are necessary to take into account the leading effects involving the parameters of leptogenesis. The RG running of the effective Lagrangian to low energies will be the subject of future investigation [14].

2 Integrating out the heavy neutrino

We consider the minimal extension of the Standard Model with $n$ light generations in which $n'$ right-handed neutrinos $N_R$ are added to the field content. The most general gauge invariant renormalizable Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{N_R},$$

where $\mathcal{L}_{SM}$ is the SM Lagrangian, while

$$\mathcal{L}_{N_R} = i\overline{N_R} \slashed{D} N_R - \bigg( \overline{\ell_L} \phi Y_\nu N_R + \overline{N_R} Y_{\nu}^c \phi^\dagger \ell_L \bigg) - \frac{1}{2} \left( \overline{N_R}^c M N_R + \overline{N_R} M^* N_R^c \right).$$

$\mathcal{L}_{N_R}$ contains kinetic energy and Majorana mass terms for the right-handed neutrinos as well as Yukawa interactions between right-handed neutrino singlets, left-handed lepton doublets $\ell_L$, and the Higgs boson scalar doublet. Since the right-handed neutrinos $N_R$ are color and SU(2) singlets with hypercharge $Y = 0$, the covariant derivative reduces to $D_\mu = \partial_\mu$ in the kinetic energy terms. In addition, lepton-number violating Majorana mass terms are allowed by the gauge symmetries. The Majorana mass matrix $M$ is an $n' \times n'$ complex symmetric matrix with eigenvalues of $O(M)$. Note that the charge conjugate of the chiral fermion field appearing in the Majorana mass term is defined by $\overline{\psi}_{R} \equiv C \overline{\psi}_{R}^T$. The Yukawa interactions are written in terms of the Higgs boson doublet $\overline{\phi}$ which is related to the standard scalar doublet $\phi$ by $\overline{\phi} = i \tau_2 \phi^*$. $Y_\nu$ is the $n \times n'$ matrix of neutrino Yukawa couplings.

The Majorana mass matrix $M$ has, in general, $n'$ complex eigenvalues $M_i = e^{i \theta_i} |M_i| \equiv \eta_i |M_i|$ which depend on the Majorana phases $\theta_i$ of the heavy Majorana neutrinos. We can work in the basis in which $M$ is real and diagonal. In this case, the Majorana neutrino mass eigenstates $N_i = N^c_i$ are given by

$$N_i \equiv e^{i \theta_i/2} N_{Ri} + e^{-i \theta_i/2} N_{Ri}^c = \sqrt{\eta_i} N_{Ri} + \sqrt{\eta_i^*} N_{Ri}^c,$$

and the Lagrangian in Eq. (3) can be rewritten as

$$\mathcal{L}_N = \frac{1}{2} \overline{N_i} \left( i \partial_\mu - M_i \right) N_i - \frac{1}{2} \left[ \overline{\ell_L} \overline{\phi} Y_\nu \sqrt{\eta} + \overline{\ell_L}^c \overline{\phi}^* Y_{\nu}^c \sqrt{\eta} \right] N_i - \frac{1}{2} \overline{N_i} \left[ \sqrt{\eta} Y_{\nu}^T \overline{\phi}^T \ell_L^c + \sqrt{\eta^*} Y_{\nu}^c \overline{\phi} \ell_L \right]_i,$$

where $\eta$ is the $n' \times n'$ diagonal matrix with elements $\eta_i$. We adopt the convention that Latin indices denote mass eigenstates and Greek indices denote flavor eigenstates throughout this paper.
An effective Lagrangian which is valid at energies less than \( \mathcal{M} \) can be constructed by integrating out the heavy Majorana neutrino fields \( N_i \). The effective Lagrangian has a power series expansion in \( 1/\mathcal{M} \) of the form

\[
L_{\text{eff}} = L_{\text{SM}} + \frac{1}{\mathcal{M}} L^{d=5} + \frac{1}{\mathcal{M}^2} L^{d=6} + \cdots \equiv L_{\text{SM}} + \delta L^{d=5} + \delta L^{d=6} + \cdots ,
\]

where \( L_{\text{SM}} \) contains all \( SU(3) \times SU(2) \times U(1) \) invariant operators of dimension \( d \leq 4 \) and the gauge invariant operators of dimension \( d > 4 \), constructed from the SM fields, account for the physics effects of the heavy Majorana neutrinos at energies \( \leq \mathcal{M} \). The effective Lagrangian is defined through the effective action \([15]\),

\[
e^{iS_{\text{eff}}} = \exp \left\{ i \int d^4x \, L_{\text{eff}}(x) \right\} \equiv \int \mathcal{D}N\mathcal{D}\bar{N} e^{iS} = e^{iS_{\text{SM}}} \int \mathcal{D}N\mathcal{D}\bar{N} e^{i\delta S} ,
\]

obtained by functional integration over the heavy Majorana neutrino fields. The classical equations of motion for the \( N \)-field with solution \( N_0 \) are obtained from

\[
\frac{\delta S}{\delta N_i(x)} \bigg|_{N_0} = 0 , \quad \frac{\delta S}{\delta \bar{N}_i(x)} \bigg|_{\bar{N}_0} = 0 ,
\]

which yield

\[
\bar{N}_0 (i \overset{\rightarrow}{\partial} - M_i) - \left( \ell_L \bar{\phi} Y_\nu \sqrt{\eta^*} + \ell_L^c \bar{\phi}^* Y_\nu^* \sqrt{\eta} \right)_i = 0 ,
\]

\[
(i \overset{\rightarrow}{\partial} - M_i) N_0 - \left( \sqrt{\eta} Y_\nu^T \bar{\phi}^T \ell_L + \sqrt{\eta^*} Y_\nu^T \bar{\phi}^T \ell_L^c \right)_i = 0 .
\]

The effective action is given by

\[
S_{\text{eff}} = S_{\text{SM}} + S_N[N_0] ,
\]

where

\[
S_N[N_0] \approx -\frac{1}{2} \int d^4x \left( \ell_L \bar{\phi} Y_\nu \sqrt{\eta} + \ell_L^c \bar{\phi}^* Y_\nu^* \sqrt{\eta} \right)_i \left( \frac{\delta_{ij}}{i \overset{\rightarrow}{\partial} - M_i} \right) \left( \sqrt{\eta} Y_\nu^T \bar{\phi}^T \ell_L + \sqrt{\eta^*} Y_\nu^T \bar{\phi}^T \ell_L^c \right)_j .
\]

The high-energy Lagrangian \( \mathcal{L} \) does not contain loop corrections with only heavy neutrinos running around the loop, so all contributions to the effective Lagrangian can be obtained by expanding the heavy neutrino propagator in a power series in \( 1/\mathcal{M} \),

\[
\frac{1}{i \overset{\rightarrow}{\partial} - M} = -\frac{1}{\mathcal{M}} - i \frac{\overset{\rightarrow}{\partial}}{\mathcal{M}^2} + \cdots .
\]

The substitution of Eq. (12) into Eq. (11) yields the terms of dimension \( \leq 6 \), which suffice for the purposes of this work.
2.1 $d=5$ operator

Eq. (11) yields the $d = 5$ operator of the effective Lagrangian for the seesaw model,

$$\delta \mathcal{L}^{d=5} = \frac{1}{2} c_{\alpha \beta}^{d=5} \left( \overline{\ell}_{L \alpha} \bar{\phi} \right) \left( \phi^\dagger \ell_{L \beta} \right) + \frac{1}{2} \left( c_{\alpha \beta}^{d=5} \right)^* \left( \overline{\ell}_{L \alpha} \bar{\phi} \right) \left( \phi^T \ell_{L \beta} \right),$$  \hspace{1cm} (13)

where

$$c_{\alpha \beta}^{d=5} = \left( Y_\nu^* \frac{\eta}{M} Y_\nu^\dagger \right)_{\alpha \beta}. \hspace{1cm} (14)$$

This expression can be rewritten in terms of $SU(2)$ singlet and triplet components of the leptons using a Fierz identity. For the case under study the singlet term does not contribute because it is proportional to $\bar{\phi}^\dagger i \tau_2 \phi^* = 0$. Using the property $\ell^L \phi^* = \ell^R i \tau_2 \phi \equiv -\bar{\ell} \phi$, Eq. (13) can be reexpressed as

$$\delta \mathcal{L}^{d=5} = -\frac{1}{4} c_{\alpha \beta}^{d=5} \left( \overline{\ell}_{L \alpha} \bar{\tau} \ell_{L \beta} \right) \left( \phi^\dagger \bar{\tau} \phi \right) + \text{h.c.},$$  \hspace{1cm} (15)

which is the well-known ($\Delta L = 2$) $d = 5$ operator $[17]$ that generates Majorana masses for the light weak doublet neutrinos $\nu_L$ when the Higgs doublet develops a non-zero vacuum expectation value $v/\sqrt{2} \simeq 174$ GeV. The Majorana mass matrix of the light neutrinos is given by

$$m_{\alpha \beta} = -\frac{v^2}{2} \left( Y_\nu^* \frac{\eta}{M} Y_\nu^\dagger \right)_{\alpha \beta} = -\frac{v^2}{2} \left( c_{\alpha \beta}^{d=5} \right).$$  \hspace{1cm} (16)

Notice that the Majorana phases of the heavy singlet Majorana neutrinos are inherited by the Majorana mass matrix of the light Majorana neutrinos.

2.2 $d=6$ operator

Eq. (11) yields two $d = 6$ operators which can be shown to be identical, resulting in

$$\delta \mathcal{L}^{d=6} = i \begin{pmatrix} \overline{\ell}_{L \alpha} \bar{\phi} Y_\nu \frac{\bar{\theta}}{M^2} \left( Y_\nu^* \phi^\dagger \ell_{L \beta} \right) \end{pmatrix} = c_{\alpha \beta}^{d=6} \left( \overline{\ell}_{L \alpha} \bar{\phi} \right) i \bar{\theta} \left( \phi^\dagger \ell_{L \beta} \right),$$  \hspace{1cm} (17)

where

$$c_{\alpha \beta}^{d=6} = \left( Y_\nu \frac{1}{M^2} Y_\nu^\dagger \right)_{\alpha \beta}. \hspace{1cm} (18)$$

The $d = 6$ operator renormalizes the neutrino kinetic energy, which can be diagonalized through the redefinition

$$\nu'_\alpha = \left( \delta_{\alpha \beta} + \frac{v^2}{4} c_{\alpha \beta}^{d=6} \right) \nu_\beta,$$  \hspace{1cm} (19)

where $\nu'$ corresponds to the diagonal basis. Thus, one physical impact of the $d = 6$ operator is to modify the couplings of neutrinos to gauge bosons. It does not further modify the effects of the $d = 5$ operator, since the effective Lagrangian is restricted to $\mathcal{O}(1/M^2)$ in this work.
It is instructive to rewrite the $d=6$ operator in a form which allows comparison with the tower of $d=6$ operators invariant under the SM found in the literature [18]. It is possible to promote the partial derivative to a covariant one,

$$\partial^\mu (\bar{\phi}^\dagger\ell_L) = (\partial^\mu \bar{\phi}^\dagger)\ell_L + \bar{\phi}\partial^\mu \ell_L = D^\mu (\bar{\phi}^\dagger\ell_L) = (D^\mu \bar{\phi}^\dagger)\ell_L + \bar{\phi}\partial^\mu \ell_L,$$

(20)

since $\bar{\phi}^\dagger\ell_L$ is a gauge singlet. In addition, the equations of motion can be used to further simplify the operator. The equation of motion for the left-handed lepton doublet is given by

$$i \not\!\!\! D \ell_L - Y_e \phi e_R + \cdots = 0 \quad \text{(21)}$$

where $Y_e$ is the $n \times n$ matrix of the Yukawa couplings involving the right-handed charged leptons $e_R$. The ellipsis stands for terms suppressed in $1/M$ which lead to higher dimension operators that are discarded here. The term proportional to $Y_e$ does not contribute, since $\bar{\phi}^\dagger\phi = -i\phi^T \tau_2 \phi = 0$. After a Fierz transformation, Eq. (17) can be rewritten as

$$\delta L^{d=6} = \frac{i}{2} \epsilon_{\alpha\beta}^{d=6} \left\{ (\ell_{\alpha \mu} \gamma_\mu \ell_{\beta}) (\phi^\dagger D_\mu \phi) - \frac{1}{2} (\ell_{\alpha \mu} \bar{\tau} \gamma_\mu \ell_{\beta}) (\phi^\dagger D_\mu \bar{\tau} \phi + \phi^\dagger \bar{\tau} D_\mu \phi) \right\}, \quad \text{(22)}$$

which contains the $SU(2)$ singlet and triplet operators largely studied in the literature. The singlet operator is well known to modify, when considered alone, the couplings of the $Z$ to neutrinos and charged leptons. The triplet operator by itself modifies the couplings of the $W$ and the $Z$. However, in the particular combination Eq. (22), corrections to the $Z$ couplings of charged leptons cancel and only the $W$ and $Z$ couplings which involve neutrinos are modified. It can be easily verified that the physical consequences of Eq. (22) match those stemming from Eq. (19), as they must.

Inclusion of the $d=6$ operator in the charged current implies that the leptonic mixing matrix of the effective theory is given by

$$U_{\alpha i}^{\text{eff}} = \left( \delta_{\alpha\beta} - \frac{v^2}{4} \epsilon_{\alpha\beta}^{d=6} \right) U_{\beta i},$$

(23)

where $U$ is the usual MNS lepton mixing matrix. Thus, neutrino oscillations are affected by the presence of the $d=6$ operator. We note that the sensitivity of neutrino oscillations to more phases than just the “CKM”-like phase, although with effects suppressed by powers of $1/M^2$, has been pointed out already in Ref. [19] in a general context. Phenomenological bounds for the $d=6$ operator also can be found in the literature, as this operator has been dealt with previously in the context of theories with extra dimensions [20]. For the particular case of a very short baseline $L \simeq 0$, the oscillation probability depends on the coefficient $\epsilon_{\alpha\beta}^{d=6}$ [20]:

$$P(\nu_\alpha \rightarrow \nu_\beta) = \left| \delta_{\alpha\beta} - \frac{v^2}{2} \epsilon_{\alpha\beta}^{d=6} \right|^2.$$  

(24)

\text{From the results of the short baseline experiments [13, 21], we obtain a bound on the seesaw scale,}

$$Y_\nu/M \lesssim 10^{-4} \text{ GeV}^{-1},$$

(25)

which is many orders of magnitude weaker than that obtained from the $d=5$ operator, although independent from it.

The analysis of the impact on observables of the imaginary part of the $\epsilon^{d>4}$ coefficients is in progress [14].
2.3 General Lagrangian and RG running

While the $d = 5$ operator of the effective Lagrangian is the unique dimension-five operator compatible with the gauge symmetries of the SM, there are many $d = 6$ operators other than the one derived here. This means that the effective Lagrangian

$$L_{\text{eff}}^{d \leq 6} = L_{SM} - \frac{1}{4} \left[ c_{d=5} \bar{\ell}_L \tau \ell_L \bar{\phi} \phi + \text{h.c.} \right] + i \epsilon_{d=6} \left[ \bar{\ell}_{L \alpha} \bar{\phi} \rightarrow \frac{\partial}{\partial \bar{\phi}} \left( \ell_{L \beta} \bar{\phi} \right) \right], \quad (26)$$

is not the only possible form of the effective Lagrangian with $d \leq 6$ stemming from the general idea of the seesaw mechanism. More complicated scenarios for the heavy neutrino and scalar sectors, or further interactions, can add new operators with dimension $d = 6$ to the Lagrangian \[14\]. Also, the RG evolution of the operator couplings in Eq. (26) from the putative high-energy scale where they are produced down to the electroweak scale induces mixing with other $d = 6$ operators \[14\] and changes the relationship of the $d = 5$ coefficient to the high-energy parameters. Nevertheless, unless very unnatural cancellations are present, our tree-level result should be a tell-tale signature of the seesaw mechanism.

3 Parameter counting

It is necessary and sufficient to consider our tree-level effective Lagrangian Eq. (26) in order to take into account the leading low-energy effects related to leptogenesis. In this section, we show that the number of independent physical angles and $CP$-phases contained in our effective Lagrangian with $d \leq 6$ equals that of the high-energy seesaw Lagrangian, when the number of heavy and light neutrino generations is the same, $n' = n$. The situation for $n' \neq n$ also is discussed. We count how many physical parameters are contained in the effective Lagrangian by analyzing the symmetry structure of the theory, using the method developed in Ref. \[16\].

3.1 Seesaw Lagrangian

First consider the symmetry structure of the high-energy seesaw Lagrangian Eq. (2) with $n$ light lepton families and $n'$ right-handed Majorana neutrinos. The kinetic energy terms are invariant under the chiral transformations

$$\begin{align*}
\ell_L & \rightarrow V_\ell \ell_L, \\
e_R & \rightarrow V_e e_R, \\
N_R & \rightarrow V_N N_R,
\end{align*} \quad (27)$$

where $V_\ell$ and $V_e$ are $n \times n$ unitary matrices and $V_N$ is an $n' \times n'$ unitary matrix. The Yukawa sector and the Majorana mass term explicitly break the chiral invariance unless the coupling matrices transform as

$$\begin{align*}
Y_\ell & \rightarrow Y'_\ell \equiv V_\ell Y_\ell V_\ell^T, \\
Y_e & \rightarrow Y'_e \equiv V_\ell Y_e V_N^T, \\
M & \rightarrow M' \equiv V_N^T M V_N^T.
\end{align*} \quad (29)$$
Eq. (29) defines an equivalence relation between theories with different matrices

\[(Y_e, Y_\nu, M) \leftrightarrow (Y'_e, Y'_\nu, M').\]  (30)

Counting how many physical parameters \(N_{\text{phys}}\) are needed to describe the Yukawa and Majorana mass terms in the seesaw Lagrangian is tantamount to counting how many equivalence classes there are with respect to the transformation Eq. (30). The result is given by

\[N_{\text{phys}} = N_{\text{order}} - (N_G - N_H),\]  (31)

where \(N_{\text{order}}\) is the sum of the number of parameters contained in the Yukawa and Majorana mass matrices, \(N_G\) is the number of parameters contained in the matrices of the chiral symmetry group \(G = U(n)_\ell \times U(n)_e \times U(n')_N\) and \(N_H\) is the number of parameters contained in the matrices of the subgroup \(H\) of the chiral symmetry group which remains unbroken by the Yukawa and Majorana mass matrices. In the seesaw model, the chiral symmetry group is completely broken by the Yukawa and Majorana mass terms, so there is no unbroken subgroup \(H\). The number of moduli and phases (real and imaginary parameters) in the Yukawa and Majorana mass matrices and in the chiral symmetry matrices \(V_e, V_\ell\) and \(V_N\) are tabulated in Table 1. The number of physical parameters contained in the seesaw model is computed using Eq. (31), and appears at the bottom of the table. There are \((n + n' + nn')\) physical moduli and \(n(n' - 1)\) physical phases in the Yukawa and Majorana mass matrices of the seesaw model. Of the real parameters, \(n\) are the charged lepton masses, \(n\) are the light Majorana neutrino masses and \(n'\) are the heavy Majorana neutrino masses, whereas the remaining \((nn' - n)\) real parameters are mixing angles.

It is useful to check this counting result for a few special cases. For \(n = n' = 2\), there are 8 moduli (6 masses and 2 mixing angles) and 2 \(CP\)-odd phases, whereas for \(n = n' = 3\), there are 15 moduli (9 masses and 6 mixing angles) and 6 \(CP\)-odd phases. The case \(n = 3\) and \(n' = 2\) results in 11 moduli (8 masses and 3 mixing angles) and 3 \(CP\)-odd phases.

| Table 1: Seesaw Model |
|------------------------|
| Matrix | Moduli | Phases |
| \(Y_e\) | \(n \times n\) | \(n \times n\) |
| \(Y_\nu\) | \(n \times n'\) | \(n \times n'\) |
| \(M\) | \(\frac{n(n+1)}{2}\) | \(\frac{n(n+1)}{2}\) |
| \(V_e\) | \(\frac{n(n-1)}{2}\) | \(\frac{n(n+1)}{2}\) |
| \(V_\ell\) | \(\frac{n(n-1)}{2}\) | \(\frac{n(n+1)}{2}\) |
| \(V_N\) | \(\frac{n(n'-1)}{2}\) | \(\frac{n(n'+1)}{2}\) |
| \(N_{\text{phys}}\) | \(n + n' + nn'\) | \(n(n' - 1)\) |

### 3.2 Low-energy Effective Lagrangian

Consider now the Lagrangian of the effective theory truncated at the \(d = 6\) operators for \(n\) light active families. Consistency with the symmetries of the high-energy Lagrangian implies
that the coefficients of the $d = 5$ and $d = 6$ operators in the effective theory transform under the chiral symmetry as

\[ c^{d=5} \to V^{*}_{} c^{d=5} V^\dagger_\ell, \]
\[ c^{d=6} \to V_\ell \ c^{d=6} V^\dagger_\ell. \]

The chiral symmetry group of the effective theory is $G = U(n)_\ell \times U(n)_e$ since the heavy neutrinos have been integrated out of the theory. The $d = 5$ term breaks the chiral symmetry group $G$ completely, so there is no unbroken subgroup $H$. The real and imaginary parameters contained in the $d = 5$ and $d = 6$ operator coefficient matrices are tabulated in Table 2, using the fact that $c^{d=5}$ is an $n \times n$ complex symmetric matrix and $c^{d=6}$ is an $n \times n$ Hermitian matrix. The number of parameters contained in the $n \times n$ unitary matrices $V_e$ and $V_\ell$ of the chiral symmetry group also are tabulated. The number of physical parameters in the effective Lagrangian up to operators of dimension six is given on the last line of Table 2. The number of physical parameters in the low-energy effective Lagrangian ($d \leq 6$) equals the number of physical parameters in the high-energy seesaw Lagrangian of Table 1 if $n' = n$, demonstrating our assertion that all of the physical parameters of the high energy theory appear in the low-energy effective theory containing the $d = 5$ and $d = 6$ terms only.

| Matrix | Moduli | Phases |
|--------|--------|--------|
| $Y_e$  | $n \times n$ | $n \times n$ |
| $c^{d=5}$ | $\frac{n(n+1)}{2}$ | $\frac{n(n+1)}{2}$ |
| $c^{d=6}$ | $\frac{n(n+1)}{2}$ | $\frac{n(n-1)}{2}$ |
| $V_e$  | $\frac{n(n-1)}{2}$ | $\frac{n(n+1)}{2}$ |
| $V_\ell$ | $\frac{n(n-1)}{2}$ | $\frac{n(n+1)}{2}$ |
| $N_{phys}$ | $n(n+2)$ | $n(n-1)$ |

Note that with an effective Lagrangian containing only the $d = 5$ operator, information is lost: in this case, the number of physical moduli is $n(n + 3)/2$ and the number of physical phases is $n(n - 1)/2$, which does not equal the number of physical parameters of the high-energy seesaw model for any value of $n'$. For example, for $n = 2$, the $d = 5$ effective Lagrangian would contain only 5 moduli (2 charged lepton masses, 2 neutrino masses and one mixing angle) and one phase, to be compared with the 8 moduli and 2 phases of the high-energy Lagrangian for $n = n' = 2$. The addition of the $d = 6$ operator allows to recover the missing parameters.

When some extra symmetry or constraint is imposed in the high-energy Lagrangian (i.e., degenerate heavy neutrinos, $n' < n$, etc.), the low-energy Lagrangian still has the same form, which appears paradoxical since now it contains a larger number of independent parameters than the high-energy theory. The resolution of the paradox is that hypothetical low-energy measurements then are correlated.

Consider for instance the special case of $m'$ degenerate neutrinos among the heavy $n'$ Majorana fields, $m' \leq n'$. The number of physical parameters at high energy decreases, as
shown in Table 3².  

Table 3: Degenerate Seesaw Model  

| $N_{phys}$ | Moduli | Phases |
|------------|--------|--------|
| $\frac{(n+1)(n'+1) - m'(m'+1)}{2}$ | $n(n'-1)$ |

For the simplest case $m' = n' = n$, using the explicit form of the coefficients in the low-energy Lagrangian, Eqs. (15) and (22), it is easy to show that if all the elements of the coefficient matrices, $c^{d=5}$ and $c^{d=6}$, are determined experimentally, most of the information extracted from the values of the $c^{d=6}$ elements is redundant with respect to the information resulting from the measurement of the $c^{d=5}$ elements. For instance, for $n = n' = m' = 2$, the seesaw Lagrangian would contain 6 real parameters and 2 phases. After measuring all the coefficients of the $d = 5$ operator, 5 moduli and 1 phase are determined. Then, the supplementary knowledge of one modulus and one phase of the $c^{d=6}$ coefficient suffices to determine completely the value of the fundamental parameters $M$ and $Y_\nu$ of the high-energy theory. Other models of particular interest will be presented elsewhere [14].  

Finally, for models in which $n' > n$, Tables 1 and 2 illustrate that the number of independent parameters of the seesaw model is larger than that of our effective $d \leq 6$ Lagrangian, Eq. (26). The leading low-energy effects are still given by the latter, as no new $d \leq 6$ operators are generated. In order to fully take into account the remaining parameters of the high-energy theory, operators of $d > 6$ have to be added to the effective Lagrangian [14].  

4 Conclusions  

We have established a generic relationship between the seesaw model, including its leptogenesis-related parameters, and exotic low-energy neutrino couplings. The physical consequences of the low-energy dimension 6 operator are suppressed by two inverse powers of the large seesaw scale, and consequently, there is little practical hope to observe them, unless the seesaw scale turns out to be surprisingly small. The present work allows, though, to quantify the difficulty of the task.  

5 Acknowledgments  

We thank A. Donini and P. Hernández for illuminating discussions and reading the manuscript. M.B.G. and E.J. thank the Aspen Center for Physics for hospitality during the initial stage of this work. We also thank A. Romanino for pointing out an inconsistency in a formula in the original version of this paper. A.B acknowledges MECD for financial support by FPU grant AP2001-0521. A.B and M.B.G were partially supported by CICYT FPA2000-0980 project. E.J. was supported in part by the Department of Energy under grant DOE-FG03-97ER40546.  

²We thank A. Romanino for pointing out an error in our original formulae in this Table.
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