The Difficulty of the Heating of Cluster Cooling Flows by Sound Waves and Weak Shocks

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We investigate heating of the cool core of a galaxy cluster through the dissipation of sound waves and weak shocks excited by the activities of the central active galactic nucleus (AGN). Using a weak shock theory, we show that this heating mechanism alone cannot reproduce observed temperature and density profiles of a cluster, because the dissipation length of the waves is much smaller than the size of the core and thus the wave energy is not distributed to the whole core.

1 Introduction

The failure of standard cooling flow models indicates that the gas is prevented from cooling by some heating sources. At present, the most popular candidate for the heating source is the active galactic nucleus (AGN) at the cluster center. However, it is not understood how the energy ejected by the AGN is transferred into the surrounding ICM. One idea is that bubbles inflated by AGN jets move outward in a cluster by buoyancy and mix the surrounding ICM [1, 2, 3]. As a result of the mixing, hot ICM in the outer region of the cluster is brought into, and subsequently heats, the cluster center. The other idea is that the dissipation of sound waves created through the AGN activities. In fact, sound waves or weak shocks that may have evolved from sound waves are observed in the Perseus and the Virgo clusters [4, 5]. It was argued that the viscous dissipation of the sound waves is responsible for the heating of a cool core [4, 6]. They estimated the dissipation rate assuming that the waves are linear. However, when the amplitude of sound waves is large, the waves rapidly evolve into non-linear weak shocks [7], and their dissipation can be faster than the viscous dissipation of linear waves. Although it was argued the presence of weak shocks in Ref. [4], their evolution from sound waves was not considered. Numerical simulations of dissipation of sound waves created by AGN activities were also performed [8]. Their results actually showed that the sound waves became weak shocks. However, their simulations were...
finished before radiative cooling became effective. Thus, the long-term balance between heating and cooling is still unknown. In this paper, we consider the evolution of sound waves to weak shocks, and analytically estimate the ‘time-averaged’ energy flux of the propagating waves as a function of distance, explicitly taking account of the dissipation at weak shock fronts and its global balance with radiative cooling. We assume the Hubble constant of $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The detail of the models and results are shown in Ref. [9].

2 Models

We assume that sound waves are created by central AGN activities. The waves propagate in the ICM outwards. These waves, having a relatively large but finite amplitude, eventually form shocks to shape sawtooth waves. If the velocity amplitude is larger than $\sim 0.1$ sound velocity (the Mach number is $\gtrsim 1.1$), those waves steepen and become weak shocks after propagating less than a few wavelengths [10]. These shock waves directly heat the surrounding ICM by dissipating their wave energy. We adopt a heating model for the solar corona based on a weak shock theory ([10, 7], see also [11]). We assume that a cluster is spherically symmetric and steady.

The equation of continuity is

$$\dot{M} = -4\pi r^2 \rho v, \quad (1)$$

where $\dot{M}$ is the mass accretion rate, $r$ is the distance from the cluster center, $\rho$ is the ICM density, and $v$ is the ICM velocity. The equation of momentum conservation is

$$\frac{d}{dr} \left( \frac{v^2}{r^2} \right) = -\frac{GM(r)}{r^2} - \frac{1}{\rho} \frac{dp}{dr} - \frac{1}{\rho c_s^2 (1 + [(\gamma + 1)/2]\alpha_w)} \frac{1}{r^2} \frac{d}{dr} \left( r^2 F_w \right) \quad (2)$$

where $G$ is the gravitational constant, $M(r)$ is the mass within radius $r$, $p$ is the ICM pressure, $c_s$ is the sound velocity, $\gamma (= 5/3)$ is the adiabatic constant, and $\alpha_w$ is the wave velocity amplitude normalized by the ambient sound velocity ($\alpha_w = \delta v_w/c_s$). The wave energy flux, $F_w$, is given by

$$F_w = \frac{1}{3} \rho c_s^3 \alpha_w^2 \left( 1 + \frac{\gamma + 1}{2}\alpha_w \right). \quad (3)$$

The energy equation is

$$\rho v \frac{d}{dr} \left( \frac{1}{2} v^2 + \frac{\gamma}{\gamma - 1} \frac{k_B T}{\mu m_H} \right) + \rho \frac{d}{dr} \frac{GM(r)}{r^2} + \frac{1}{r^2} \frac{d}{dr} \left[ r^2 (F_w + F_c) \right] + n_e^2 A(T) = 0, \quad (4)$$

where $k_B$ is the Boltzmann constant, $T$ is the ICM temperature, $\mu (= 0.6)$ is the mean molecular weight, $m_H$ is the hydrogen mass, $F_c$ is the conductive
The Difficulty of Heating

flux, \( n_e \) is the electron number density, and \( \Lambda \) is the cooling function. The term \( \nabla \cdot F_w \) indicates the heating by the dissipation of the waves. The equation for the time-averaged amplitude of the shock waves is given by

\[
\frac{d\alpha_w}{dr} = \frac{\alpha_w}{2} \left[ -\frac{1}{p} \frac{dp}{dr} - \frac{2(\gamma + 1)\alpha_w}{c_s^2} \right] - \frac{2}{r} \frac{1}{c_s^2} \frac{dc_s}{dr},
\]

where \( \tau \) is the period of waves, which we assume to be constant \([7, 10]\). The second term of the right side of equation (5) denotes dissipation at each shock front.

3 Results

For parameters of our model cluster, we adopt the observational data of the Perseus cluster \([12]\). We assume that \( r_s = 280 \) kpc, \( M(r_{1000}) = 3.39 \times 10^{14} M_\odot \), and \( r_{1000} = 826 \) kpc, where the mean density within \( r_\Delta \) is \( \Delta \) times the critical density of the Universe. Waves are injected at the inner boundary \( r = r_0 \), which should be close to the size of bubbles observed at cluster centers. We assume that \( \lambda_0 = r_0 \), where \( \lambda_0 \) is the initial wavelength. If the waves are injected in a form of sound waves with amplitude \( 0.1 \lesssim \alpha_w < 1 \), waves travel about \( \lambda_0 \) before they become shock waves \([10]\). Therefore, for \( r_0 \leq r \leq r_0 + \lambda_0 = 2\lambda_0 \), we assume that \( \nabla \cdot F_w = 0 \) (eqs. \([2]\) and \([4]\)), and that the second term of the right-hand side of equation (5) is zero. The temperature, electron density, and wave amplitude at \( r = r_0 \) are \( T_0 n_e_0 \), and \( \alpha_w_0 \), respectively. Unless otherwise mentioned, the first two are fixed at \( T_0 = 3 \) keV and \( n_e_0 = 0.08 \) cm\(^{-3} \), respectively, based on the observational results of the Perseus cluster \([13]\).

![Fig. 1.](image-url) (a) Temperature, (b) density, and (c) dissipation length profiles for \( \alpha_{w_0} = 0.3 \) (dotted lines), 0.5 (thin solid lines), and 0.7 (dashed lines). Other parameters are \( \tau = 1 \times 10^7 \) yr, \( M = 50 M_\odot \) yr\(^{-1} \), and \( f_c = 0 \). Filled circles are Chandra observations of the Perseus cluster \([13]\). The bold solid lines correspond to a genuine cooling flow model of \( \dot{M} = 500 M_\odot \) yr\(^{-1} \).
In Figure 1, we show the results when $\tau$ is fixed at $1 \times 10^7$ yr, and $\alpha_{w0}$ is changed. For the Perseus cluster, it was estimated that $\alpha_{w0} \sim 0.5$ [4]. The dissipation length is defined as $l_w = \left| \frac{\mathbf{F}_w}{\nabla \cdot \mathbf{F}_w} \right|$. For these parameters, the initial wavelength is $\lambda_0 = 9$ kpc, which is roughly consistent with the Chandra observations [4]. Other parameters are $\dot{M} = 50 M_\odot \text{yr}^{-1}$, and $f_c = 0$, where $f_c$ is the ratio of actual thermal conductivity to the classical Spitzer conductivity. In general, larger $\dot{M}$ reproduces observed temperature and density profiles better. However, large $\dot{M}$ is inconsistent with recent X-ray observations as was mentioned in § 1. For comparison, we show the results of a genuine cooling flow model ($\dot{M} = 500 M_\odot \text{yr}^{-1}$, $\alpha_{w0} = 0$, and $f_c = 0$) and the Chandra observations of the Perseus cluster [13]. Figures 1a and 1b show that only a small region is heated. The jumps of $T$ and $n_e$ at $r = 2\lambda_0 = 18$ kpc are produced by weak shock waves that start to dissipate there. The energy of the sound waves rapidly dissipates at the shocks, which is clearly illustrated in short dissipation lengths, $l_w \sim 2$–15 kpc (Fig. 1c). These dissipation lengths are smaller than those of viscous dissipation for linear waves, which can be represented by $l_v = 420 \lambda_0^3 n_0 T_3^{-2}$ kpc, where the wavelength $\lambda = 9 \lambda_0$ kpc, the density $n = 0.08 n_0$ cm$^{-3}$, and the temperature $T = 3 T_3$ keV [4]. In Figure 1, the ICM density becomes large and the temperature becomes small at $r > 2\lambda_0$ so that the rapid shock dissipation is balanced with radiative cooling. Because of this, waves cannot reproduce the observed temperature and density profiles that gradually change on a scale of $\sim 100$ kpc.

![Fig. 2. Same as Fig. 1 but for $\tau = 1 \times 10^7$ yr (solid lines), and $2 \times 10^7$ yr (dashed lines). Other parameters are $\alpha_{w0} = 0.5$, $\dot{M} = 50 M_\odot \text{yr}^{-1}$, and $f_c = 0$.](image-url)

In Figure 2, we present the results when $\tau = 2 \times 10^7$ yr. Compared with the case of $\tau = 1 \times 10^7$ yr, the wave energy dissipates in outer regions. However, the dissipation lengths are still smaller than the cluster core size ($\sim 100$ kpc). Note that larger $\tau$ (or $\lambda_0$) means formation of larger bubbles. As indicated in Ref. [14], it is unlikely that the size of the bubbles becomes much larger than 20 kpc; the bubbles start rising through buoyancy before they become larger. On the other hand, when $\tau < 10^7$ yr ($\lambda_0 < 9$ kpc), the waves heat only
The Difficulty of Heating 5

The inclusion of thermal conduction changes the situation dramatically. Figure 3 shows the results when \( f_c = 0.2 \). The models including both wave heating and thermal conduction can well reproduce the observed temperature and density profiles. Figure 3c shows the contribution of the wave heating \((-\nabla \cdot F_w\)) to compensating radiative cooling \((n_e^2 \Lambda)\). Since \(-\nabla \cdot F_w/n_e^2 \Lambda > 1/2\) for \( r \sim 20–30\) kpc, the wave heating is more effective than the thermal conduction in that region.

![Figure 3](image_url)

Fig. 3. (a) Temperature, (b) density, and (c) dissipation strength profiles for \( \alpha_{w0} = 0 \) and \( T_0 = 3\) keV (dotted lines), \( \alpha_{w0} = 0.5 \) and \( T_0 = 3\) keV (solid lines), and \( \alpha_{w0} = 0.5 \) and \( T_0 = 2.5\) keV (dashed lines). Other parameters are \( \tau = 1 \times 10^7 \) yr, \( \dot{M} = 50 M_\odot\) yr\(^{-1}\), and \( f_c = 0.2 \). Filled circles are Chandra observations of the Perseus cluster [13].

4 Discussion

We showed that sound waves created by the central AGN alone cannot reproduce the observed temperature and density profiles of a cluster, because the dissipation length of the waves is much smaller than the size of a cluster core and the waves cannot heat the whole core. The results have been confirmed by numerical simulations [15]. On the other hand, we found that if we include thermal conduction from the hot outer layer of a cluster with the conductivity of 20% of the Spitzer value, the observed temperature and density profiles can be reproduced. The idea of the “double heating” (AGN plus thermal conduction) was proposed in Ref. [16].

However, the fine structures observed in cluster cores may show that the actual conductivity is much smaller than that we assumed [17]; the structures would soon be erased, if the conductivity is that large. If the conductivity is small, we need to consider other possibilities. While we considered successive minor AGN activities, some authors consider that rare major AGN activities...
should be responsible for heating of cool cores [18]. In this scenario, powerful bursts of the central AGN excite strong shocks and heat the surrounding gas in the inner region of a cluster on a timescale of $\gtrsim 10^9$ yr. Moreover, in this scenario, heating and cooling are not necessarily balanced at a given time, although they must be balanced on a very long-term average. This is consistent with the fact that there is no correlation between the masses of black holes in the central AGNs and the X-ray luminosities of the central regions of the clusters [19]. Alternative idea is that cluster mergers are responsible for heating of cool cores [20, 21]. In this “tsunami” model, bulk gas motions excited by cluster mergers produce turbulence in and around a core, because the cooling heavy core cannot be moved by the bulk gas motions, and the resultant relative gas motion between the core and the surrounding gas induces hydrodynamic instabilities. The core is heated by turbulent diffusion from the hot outer region of the cluster. Since the turbulence is produced and the heating is effective only when the core is cooling and dense, fine-tuning of balance between cooling and heating is alleviated for this model.

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