Quenching of $N = 28$ shell gap and a novel type of low-lying quadrupole mode in the vicinity of neutron-rich $N = 28$ isotones

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We discuss the low-lying $2^+$ states of $N = 28$ isotones ($^{48}\text{Ca}$, $^{46}\text{Ar}$, $^{44}\text{S}$ and $^{42}\text{Si}$) based on the canonical-basis time-dependent Hartree-Fock-Bogoliubov theory in which the pairing is taken into account. The quadrupole mode with very small excitation energies which emerges in $^{46}\text{Ar}$, $^{44}\text{S}$ and $^{42}\text{Si}$ due to the strong quadrupole correlations triggered by the quenching of the $N = 28$ shell gap. The importance of the quadrupole correlations between the protons and neutrons, and the role of the pairing correlation to reinforce the lowest $2^+$ mode are discussed. It is also shown that the observed $2^+$ energies and B(E2) are plausibly explained by our calculations.

**KEYWORDS:** Shell quenching, $2^+$ state, Pairing, Time-dependent mean field theory

**Introduction**

The low-energy excited modes are quite sensitive to the underlying shell structure and the pairing correlations, and hence, the quenched magic shell gaps of unstable nuclei should generate unique collective modes. One of the interesting example is the $N=28$ shell gap that is known to be quenched in the vicinity of $^{44}\text{S}$, and has been paid considerable experimental [1] and theoretical attention [2].

Since there is a $\Delta l = 2$ difference of orbital angular momentum between $f_{7/2}$ and $p_{3/2}$ states, the quench of $N=28$ shell gap will induce the strong quadrupole correlation in the low-lying state. Furthermore, the protons in Si, S and Ar isotopes occupy the middle of the $sd$-shell, and hence, the strong quadrupole correlation should also exist in the proton side. Therefore, when the $N=28$ shell gap is quenched, the strong quadrupole correlations among protons and neutrons will be ignited and can be expected to lead to a novel variety of the excitation modes. Indeed, various exotic phenomena such as the shape transition in Si and S isotopes and the coexistence of various deformed states are theoretically suggested [3].

We investigate the low-lying quadrupole excitation modes in $^{46}\text{Ar}$, $^{44}\text{S}$ and $^{42}\text{Si}$ generated by the strong quadrupole correlation between protons and neutrons. To access the quadrupole modes, we apply the Canonical-basis time-dependent Hartree-Fock-Bogoliubov (Cb-TDHF) theory [4] which has been successfully applied to the study of the dipole and quadrupole resonances of isotopes [4–6].

**Formulation**

**Cb-TDHF equations**

The Cb-TDHF can describe self-consistently the dynamical effects of pairing correlation which has a significant role to generate the low-lying quadrupole strength as reported in Ref. [7]. By assuming the diagonal form of pairing functional, the Cb-TDHF equations are derived from the full TDHF equation represented in the canonical basis $\{\phi_i(t), \phi_j(t)\}$ which diagonalize a density matrix. The Cb-TDHF equations describe the time-evolution of the canonical pair $\{\phi_i(t), \phi_j(t)\}$, its occupa-
tion probability $\rho(t)$ and pair probability $\kappa(t)$,

$$i\hbar \frac{\partial}{\partial t} \phi(t) = (\hat{h}(t) - \eta(t)) \phi(t), \quad i\hbar \frac{\partial}{\partial t} \Psi(t) = (\hat{h}(t) - \eta(t)) \phi(t),$$

$$i\hbar \frac{\partial \rho(t)}{\partial t} = \kappa(t) \Delta_0^+(t) - \kappa^*_0(t) \Delta_0(t),$$

$$i\hbar \frac{\partial \kappa(t)}{\partial t} = (\eta(t) + \eta(t)) \kappa(t) + \Delta_0(t) (2\rho(t) - 1),$$

(1)

where $\eta(t) \equiv \langle \phi(t) | h(t) | \phi(t) \rangle$, and the $h(t)$ and $\Delta_0(t)$ are the single-particle Hamiltonian and the gap energy, respectively. We apply the Skyrme interaction with SkM* parameter set to $ph$-channel and the simple pairing form

$$\Delta_0(t) = \sum_{k>0} G_{kl} \kappa_k(t) \equiv \sum_{k>0} G_0 f(\epsilon_k^0) f(\epsilon_l^0) \kappa_k(t),$$

(2)

where $f(\epsilon)$ is a cutoff function, and $\epsilon_1^0, \epsilon_1^0$ mean the single-particle energies of the canonical states $\phi_1(t = 0), \phi_1(t = 0)$, respectively. We choose $G_0, f(\epsilon_0)$ as a constant value in the real-time evolution. The detail form of $f(\epsilon)$ and evaluation of $G_0$ is used as same as Ref. [4,8]. In accordance with Ref. [9], we also apply the absorbing boundary condition to eliminate unphysical modes. The canonical basis $\phi_1(\vec{r}, \sigma; t) = \langle \vec{r}, \sigma | \phi(t) \rangle$ with $\sigma = \pm 1/2$ is represented in the three-dimensional coordinate discretized in a square mesh of 1 fm in a sphere with radius of 18 fm including 6 fm for the absorbing potential whose the depth is $-3.75$ MeV.

Linear response calculation with a time-dependent scheme

We use the linear response calculation to investigate excited modes. We add a numerical small amplitude to the ground state, and extract the information of excited states from small nuclear density fluctuations. In order to induce quadrupole responses, we add a weak instantaneous external field $V_{ext}(\vec{r}, t) = \eta \hat{F}_K(\vec{r}) \delta(t)$ to initial states of the time evolution. Here the quadrupole external field acting on proton, neutron, isoscalar (IS) and isovector (IV) channels are given as $\hat{F}_K(\vec{r}) = (1_{\tau_1}, 1 \text{ or } \tau_1) \otimes (r^2 Y_{2K} + r^2 Y_{2K})/\sqrt{2(1 + \delta_{K0})}$. The amplitude of the external field is so chosen to be a small number $\eta = 1 \sim 3 \times 10^{-3}$ fm$^{-2}$ to guarantee the linearity. The strength function $S(E; \hat{F}_K)$ in each channel is obtained through the Fourier transformation of the time dependent expectation value $\hat{F}_K(t) \equiv \langle \Psi(t) | \hat{F}_K | \Psi(t) \rangle$:

$$S(E; \hat{F}_K) \equiv \sum_n |\langle \Psi_n | \hat{F}_K | \Psi_0 \rangle|^2 \delta(E_n - E) = \frac{1}{\pi \eta} \text{Im} \int_0^\infty (\hat{F}_K(t) - \hat{F}_K(0))^\dagger e^{i(E + i\Gamma/2) t} dt,$$

where $|\Psi_0 \rangle$, $|\Psi_n \rangle$ and $|\Psi(t) \rangle$ are the ground and excited states and a time-dependent many body wave function represented in the canonical form, respectively. $\Gamma$ is a smoothing parameter set to 1 MeV.

We also performed unperturbed calculations in which $h(t)$ in Eq.(1) is replaced with the static single particle Hamiltonian $h(t = 0)$ computed using the ground state density. By comparing the results obtained by the fully self-consistent and unperturbed calculations, we investigate the effects of the residual interaction and the collectivity of the excited states.

Results

Figure 1 shows strength functions of quadrupole vibrational modes (IS, IV, proton, neutron) for $^{48}$Ca and $^{46}$Ar. The strength functions in the IS and IV channels shown in Fig. 1 (a) have two peaks at 3.40 and 9.12 MeV in addition to the IS giant quadrupole resonance (GQR) at 17 MeV and the IVGQR having broad distribution around 30 MeV. The properties of these two peaks below 10 MeV become clear by comparing the results in the proton and neutron channels (Fig. 1 (b) and (c)). Their
strengths in the neutron channel are much larger than those in the proton channel showing the dominance of neutron excitation and they are understood as the neutron single-particle excitations across the $N=28$ shell gap. Indeed, they correspond to the 3.86 and 9.06 MeV peaks in the unperturbed results (dashed line) that are the neutron single-particle excitations of $f_{7/2}$ ($\epsilon_{f_{7/2}} = -10.41$ MeV) $\rightarrow p_{3/2}$ ($\epsilon_{p_{3/2}} = -6.55$ MeV) and $f_{5/2}$ ($\epsilon_{f_{5/2}} = -1.36$ MeV), respectively. On the other hand, in the proton channel, there is no peak below 10 MeV in the unperturbed result, since the proton excitation with $\Delta l = 2$ costs much larger energy due to the $Z=20$ shell closure. In proton channel, the peaks of the fully self-consistent results seem to be induced due to the neutron $2^+$ excitation.

$^{46}$Ar has different nature of the ground and low-lying $2^+$ states. The ground state of $^{46}$Ar is also spherical but superfluid phases appear in both of proton ($\Delta p = 1.16$ MeV) and neutron ($\Delta n = 1.77$ MeV). And if the proton pairing is switched off the ground state is oblately deformed ($\beta = -0.15$). These results imply the weaker neutron-magicity in $^{46}$Ar than in $^{48}$Ca. Actually, the calculated $N=28$ shell gap is slightly reduced in $^{46}$Ar (3.5 MeV) compared to that in $^{48}$Ca (3.86 MeV). The unperturbed strengths in the proton and neutron channels in the Fig. 1 (e) and (f) are quite similar to those of $^{48}$Ca except for minor differences; (1) the reduction of the peak energies below 10 MeV in the neutron channel due to the quenching of the $N=28$ shell gap, and (2) the very weak strength distributed below 10 MeV in the proton channel that are generated by the proton hole-states in $sd$-shell and fluctuated by the pairing correlation. These differences generate a novel type of low-lying collective mode when the residual interaction is switched on. In the fully self-consistent results, very strong peaks emerge around 1 MeV in all channels as shown in Fig. 1 (d)-(f).

The same mechanism also applies to other $N=28$ isotones $^{44}$S and $^{42}$Si. In the present calculation, $^{44}$S has obscure prolate shape ($\beta = 0.08$), while $^{42}$Si has oblate shape ($\beta = -0.19$). Deformation of these nuclei splits the strength functions in the $K=0, 2$ modes, and makes their strength distributions more complicated than those in $^{46}$Ar. However, we could still identify very low-lying peaks which correspond to a couple of $2^+$ states with enhanced collectivity.

To analyze the peak structure, we fit the strength function $S(E; F_K)$ below 25 MeV by the sum of Lorentzian $f_k(E; \hat{F}_K)$:

$$S(E; \hat{F}_K) \simeq \sum_k f_k(E; \hat{F}_K) \equiv \sum_k \frac{a_k^\tau (\Gamma/2)^2}{(E - E(2^+_k)^2 + (\Gamma/2)^2},$$

where $a_k^\tau$ is the amplitude of fitted Lorentzian and $\Gamma = 1$ MeV corresponding to smoothing parameter.
in Eq. (1). The reduced transition probabilities in the proton and neutron channels, that are denoted as \( B_k(E2 \uparrow) \) in the following, are evaluated by integrating \( f_k(E; \hat{F}_K) \) for each \( 2^+_1 \) state,

\[
B_k(E2 \uparrow) \equiv \sum_K |\langle 2^+_1 | \hat{Q}_K | 0^+_1 \rangle|^2 \approx 5 \int_0^{\infty} f_k(E; \hat{F}_{K=0}) \, dE.
\]

This evaluation is reasonable for spherical nuclei such as \(^{48}\text{Ca}\) and \(^{46}\text{Ar}\), but gets worse for deformed nuclei \(^{44}\text{S}\) and \(^{42}\text{Si}\). The detail of strength functions of deformed nuclei are not discussed in this paper.

The results of fitting, evaluated \( B(E2) \) below 5 MeV are shown in Fig. 2. The square and filled circle symbols indicate the calculated \( B(E2 \uparrow) \) of \(^{46}\text{Ar}\) and \(^{48}\text{Ca}\), respectively. The open square symbols show the results without the neutron pairing correlation. By comparing the results with and without the pairing in the neutron channel, it turns out that the pairing correlation enhances \( B(E2 \uparrow) \) \(^7\) by about 20% in this work. The symbols with error bar are experimental data, but the triangle and under triangle show new and old one. Experimentally, the \( 2^+_1 \) state of \(^{48}\text{Ca}\) locates at 3.83 MeV and \( B(E2 \uparrow)=95\pm32 \text{ e}^2\text{fm}^4 \) \(^{10}\) that are in good agreement with the calculation. For \(^{46}\text{Ar}\), our evaluated values nicely explain the observed old values of \( E(2^+_1) = 1.58 \text{ MeV} \) and \( B(E2) = 196 \pm 39 \text{ e}^2\text{fm}^4 \) \(^1\), but for the new experimental value \( B(E2) = 570^{+335}_{-160} \text{ e}^2\text{fm}^4 \) \(^{11}\), they do not. The discrepancy between the old and new \( B(E2) \) data has been an open question now.

**Summary**

We have investigated the low-lying quadrupole vibrational modes of \( \text{N}=28 \) isotones by using \( \text{Cb-TDHF} \) theory. We pointed out the importance of quadrupole correlation between protons and neutrons that couples proton hole states with neutron excitations across \( \text{N}=28 \) shell gap. It is found that the quenching of \( \text{N}=28 \) shell gap and the proton holes in the \( sd \)-shell trigger strong quadrupole correlation and generate the low-lying \( 2^+ \) states in \(^{46}\text{Ar}\).

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