Dark Matter with Time-Dependent Mass

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Abstract

We propose a simple model in which the cosmological dark matter consists of particles whose mass increases with the scale factor of the universe. The particle mass is generated by the expectation value of a scalar field which does not have a stable vacuum state, but which is effectively stabilized by the rest energy of the ambient particles. As the universe expands, the density of particles decreases, leading to an increase in the vacuum expectation value of the scalar (and hence the mass of the particle). The energy density of the coupled system of variable-mass particles ("vamps") redshifts more slowly than that of ordinary matter. Consequently, the age of the universe is larger than in conventional scenarios.

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1 Introduction

The Big Bang model has proven extraordinarily successful as a framework for interpreting the structure and evolution of the universe on large scales. Within that framework, the cold dark matter scenario (featuring massive particles which bring the density of the universe to its critical value, and a scale-free spectrum of Gaussian density perturbations) has provided an elegant theory of structure formation, which unfortunately seems to fall short of perfect agreement with observation. Although the precise extent to which CDM disagrees with observation is arguable, there are two important areas in which the discrepancies are particularly troubling: predicting an age for the universe which is larger than the ages of the oldest globular clusters, and matching the COBE-normalized power spectrum of density fluctuations as measured by microwave background anisotropy experiments and direct studies of large-scale structure.

One way in which the simple CDM scenario may be modified, affecting the age of the universe as well as the evolution of density fluctuations, is to imagine that the closure density is provided by something different than (or in addition to) nonrelativistic particles. In a flat Robertson-Walker universe with metric
\[ ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2) \] (1)
and energy-momentum tensor
\[ T^\mu_\nu = \text{diag}(-\rho, p, p, p) , \] (2)
the Friedmann equations imply that the time derivative of the scale factor \( a \) satisfies
\[ \dot{a}^2 = \frac{8\pi G}{3} a^2 \rho . \] (3)
The evolution of \( \dot{a} \) is therefore dependent on how the energy density \( \rho \) scales with \( a \); if \( \rho \propto a^{-n} \), we have
\[ \frac{\ddot{a}}{a} = \frac{4\pi G}{3} (2 - n) \rho . \] (4)
Hence, the more slowly the energy density decreases as the universe expands, the more slowly the expansion will decelerate, implying a correspondingly older universe for any given value of the expansion rate today — for a flat universe dominated by such an energy density, the age is \( t_0 = \frac{2}{n} H_0^{-1} \), where \( H = \dot{a}/a \) is the Hubble parameter and the subscript 0 refers to the present time. (Eq. 4) can also be derived by positing an equation of state \( p = w \rho \) and
using energy conservation; the two parameterizations are related by \( n = 3(1 + w) \). The energy density in a species of ordinary “matter” (a nonrelativistic particle species \( X \)) can be written \( \rho_X = m_X n_X \), where \( m_X \) is the mass of the particle and \( n_X \) is its number density. The energy density of a matter-dominated universe is therefore proportional to \( a^{-3} \), as the mass stays constant while the number density is proportional to the volume; the age of such a universe is \( t_0 = \frac{2}{3}H_0^{-1} \).

Although there is some controversy over the value of the Hubble constant, most recent determinations are consistent with a value \( H_0 = 70 \pm 10 \) km/sec/Mpc, or \( H_0^{-1} = (14 \pm 2) \times 10^9 \) yr. The upper limit on the age of a matter-dominated flat universe is therefore \( t_0(\text{MD}) \leq 11 \times 10^9 \) yr. Meanwhile, calculations of the ages of globular clusters imply an age \( t_{GC} \sim 15 \times 10^9 \) yr, with a lower bound of \( t_{GC} \geq 12 \times 10^9 \) yr. The apparent discrepancy between these values may be resolved by a revision in distance determinations to globular clusters, as suggested by recent measurements by the Hipparcos satellite; while this would be the simplest solution, further work is necessary to accept it with confidence.

Alternative resolutions are provided by models in which the density parameter \( \Omega = 1 \), and some or all of the unseen energy density resides in a component which redshifts more slowly than nonrelativistic matter. The most popular such alternative is the introduction of a cosmological constant \( \Lambda \), for which \( \rho_\Lambda = \text{constant} \). Such models have some attractive features, but are also plagued with both theoretical and observational disadvantages. A popular variation on this theme is to invoke a slowly-rolling scalar field, or equivalently a cosmological constant whose value varies with time, or simply an unspecified smooth component. More speculative possibilities include a network of cosmic strings or stable textures. We will not enumerate the good and bad qualities of each of these scenarios, noting only that none are sufficiently compelling to discourage the exploration of still further models.

In this paper we propose a simple model in which the dark matter consists of particles \( \psi \) whose rest mass increases with time. This is achieved by having the rest mass derive from the expectation value of a scalar field \( \phi \); the potential for \( \phi \) depends on the number density of \( \psi \) particles, and therefore increases naturally on cosmological timescales as the universe expands. As a result, the particle energy density \( \rho_\psi = m_\psi n_\psi \) decreases more slowly than \( a^{-3} \), resulting in a larger age for the universe. (There is also a contribution from the potential energy of \( \phi \), which redshifts at the same rate.) We discuss some of the cosmological consequences of this proposal, including potential observational tests. The question of structure formation in the presence of such particles, as well as the construction
of realistic particle physics models containing the necessary fields, is left for future work.

After this paper was first submitted, we became aware of earlier an proposal for dark matter with time-dependent mass by Casas, García-Bellido, and Quirós [9]. These authors considered models of scalar-tensor gravity, in which the scalar coupled differently to different species of particles.

2 Scale factor and age of the universe

The model consists of a scalar $\phi$ and a particle species $\psi$, which can be either bosonic or fermionic for the purposes of this work. The mass of $\psi$ is imagined to come from the vacuum expectation value of $\phi$, with the constant of proportionality some dimensionless parameter $\lambda$:

$$m_\psi = \lambda \langle \phi \rangle .$$  \hspace{1cm} (5)

More elaborate dependences of $m_\psi$ on $\langle \phi \rangle$ are certainly conceivable, but for the purposes of this paper we make this simple choice. The dynamics of $\phi$ are determined by a conventional kinetic term and a potential energy $U(\phi)$. The notable feature of the model is that we choose the potential $U(\phi)$ to blow up at $\phi = 0$ and roll monotonically to zero as $\phi \to \infty$. For simplicity we will write

$$U(\phi) = u_0 \phi^{-p} ,$$  \hspace{1cm} (6)

although more complicated forms are again possible. While such a potential seems unusual, this form can arise for example due to nonperturbative effects lifting flat directions in supersymmetric gauge theories [10], as well as for moduli fields in string theory. (In fact this form of potential is not strictly necessary, as the phenomenon we will describe can occur with almost any potential; however, the effects are most dramatic with this choice.)

This model possesses no stable vacuum state; in empty space $\phi$ tends to roll to infinity. We consider instead the behavior of $\phi$ in a homogeneous background of $\psi$’s with number density $n_\psi$. In that case, the dependence of the free energy on the value of $\phi$ comes both from the potential $U(\phi)$ and the rest energy of the $\psi$ particles, which have a mass proportional to $\phi$. The equilibrium value of a homogeneous configuration is therefore one which minimizes an effective potential of the form

$$V(\phi) = u_0 \phi^{-p} + \lambda n_\psi \phi .$$  \hspace{1cm} (7)

(See Fig. 1.) The additional contribution can be thought of as arising because increasing $\phi$
The effective potential at finite density, given by the solid curves, is obtained by adding a contribution linear in \( \phi \) and proportional to the number density \( n_\psi \). This is plotted for two different values of \( n_\psi \), corresponding to two different stages in the evolution of the universe. As the universe expands, \( n_\psi \) decreases, and the equilibrium value of \( \phi \) increases.

In an expanding universe, the number density \( n_\psi \) will change with time; in turn, the mass of both \( \phi \) and \( \psi \) will change, as will the vacuum energy. After the interactions of \( \psi \) have frozen out, the number density can be written as \( n_\psi = n_\psi^0 a^{-3} \), where \( n_\psi^0 \) is the density when \( a = 1 \), which we take to be the present epoch. Then \( \phi \) evolves as

\[
\phi = \phi_0 a^{3/(1+p)} ,
\]

where \( \phi_0 \) is the value of (8) at the present time. In terms of these variables the mass of the \( \phi \) boson is given by

\[
m_\phi^2 = \frac{\partial^2 V}{\partial \phi^2} = \left[ p(p+1)u_0 \phi_0^{-(p+2)} \right] a^{-3(2+p)/(1+p)} ,
\]
and the mass of $\psi$ is

$$m_\psi = \lambda \phi_0 a^{3/(1+p)}.$$  

(11)

Both $\psi$ and $\phi$ are therefore variable-mass particles, or “vamps”; a cosmological model in which vamps are the dominant component of the energy density at late times will be referred to as VDM.

There are a number of contributions to the energy density of the universe in this model. These include the energy in the scalar $\phi$ particles, in the $\psi$ particles, in the time derivative of the expectation value of $\phi$, in the potential $U(\phi)$, and in ordinary components of matter and radiation. For reasonable values of the parameters, the energies in $\phi^2$ and in $\phi$ quanta are small; the former because $\phi$ is only changing on cosmological timescales, and the latter because the mass of $\phi$ is decreasing with time. (At early times, the $\phi$ bosons are very massive and rapidly decay.) The important new contribution is therefore simply $V(\phi)$, the sum of the fundamental potential and the rest energy in the $\psi$'s. (We assume for now that $\psi$ is nonrelativistic. As we discuss later, it is most likely that the particles were relativistic when they decoupled, but at late times their momenta have redshifted sufficiently that they are slowly moving today.) Both of these components turn out to depend on the scale factor in the same way; the ratio of the energy density in $\psi$ particles to that in the potential for $\phi$ is simply

$$\frac{\rho_\psi}{\rho_{U(\phi)}} = \frac{1}{p}.$$  

(12)

It is therefore convenient to deal with the sum of these two contributions,

$$\rho_V = (1 + p)u_0 \phi^{-p} = \left(\frac{1 + p}{p}\right) \lambda \phi n_\psi,$$  

(13)

which evolves as

$$\rho_V = \rho_{V0} a^{-3p/(1+p)}.$$  

(14)

The parameter $w$ characterizing the effective equation of state of the coupled $\phi/\psi$ system is therefore $w = -1/(1 + p)$.

The energy density $\rho_M$ in ordinary massive particles (baryons plus a possible cold dark matter component) redshifts as $a^{-3}$, more slowly than $\rho_V$, and will therefore be the dominant source of energy density in the universe for intermediate redshifts. The redshift at which $\rho_V = \rho_M$ is given by

$$1 + z_{VM} = \left(\frac{\rho_{V0}}{\rho_{M0}}\right)^{(1+p)/3}.$$  

(15)
The age of the universe, meanwhile, will be larger than in conventional flat models. The age corresponding to a redshift $z$ is given by

$$t = \int_0^a \frac{da'}{a'} \left[ 1 - \Omega_0 + \Omega_{M0} x^{-1} + \Omega_{V0} x^{(2-p)/(1+p)} \right]^{-1/2} dx .$$  \hspace{1cm} (16)

For the limiting case $\Omega_{M0} = 0$, $\Omega_0 = \Omega_{V0} = 1$, we find that the age of the universe now is simply

$$t_0 = \frac{2}{3} H_0^{-1} \left( 1 + p^{-1} \right) .$$  \hspace{1cm} (17)

Fig. 2 plots the age of universes with $\Omega_0 = 1$ as a function of $\Omega_{M0} = 1 - \Omega_{V0}$, for $p = 1$.

An interesting feature of this model, in comparison with alternative theories of rolling scalar fields and unusual equations of state, is that (because $\psi$ is massive and nonrelativistic at late times) it is at least conceivable that the energy density of the universe is dominated solely by baryons and vamps (without any significant cold dark matter component). For illustrative purposes, let us define the “minimal VDM model” as that with $p = 1$, $\Omega_{V0} = 1 - \Omega_{\text{baryon}} = 0.96$, and $H_0 = 70 \text{ km/sec/Mpc}$. This is a flat universe consisting solely of baryons and vamps, with the baryon density consistent with the prediction of Big Bang nucleosynthesis. In this minimal model, vamp-matter equality occurs at a redshift $z_{VM} \sim 7$. The age of the universe turns out to be approximately $17 \times 10^9$ years, in good accord with the (pre-Hipparcos) ages of the oldest globular clusters.

### 3 Particle parameters and abundances

The properties we have deduced to this point depend on the present energy density $\rho_{V0}$, but not on any assumptions about the parameters of the particle physics model in which we imagine the necessary fields and interactions could arise. To understand the formation of large-scale structure in the model, however, it is necessary to know the mass and average velocity of the $\psi$ particles today, and to compute these requires some detailed knowledge of the interactions of our two fields. In the absence of a specific model, we will estimate these quantities under the minimal assumptions that $\psi$ was in thermal equilibrium at some high temperature and has evolved freely ever since.

We begin by considering the general problem of the motion of an otherwise free particle whose mass $m_\psi = \lambda \phi$ may vary throughout spacetime. The motion of such a particle
Figure 2: Age of the universe in billions of years. The values in this plot are computed for flat universes consisting of only vamps and nonrelativistic matter, with $p = 1$.

extremizes the action

$$S = \int \sqrt{-g_{\mu\nu}} p^\mu d\tau$$

$$= \lambda \int \phi(x^\mu) \left(-g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}\right)^{1/2} d\tau,$$

(18)

where $\tau$ is the proper time along the particle’s trajectory and $p^\mu = m(dx^\mu/d\tau)$ is the particle’s four-momentum. Variation of this action with respect to the path leads to an equation of motion

$$\frac{Dp^\mu}{d\tau} \equiv \frac{dp^\mu}{d\tau} + \Gamma^\mu_{\rho\sigma} \frac{dx^\rho}{d\tau} p^\sigma = -\lambda \nabla^\mu \phi,$$

(19)

which can be written explicitly in terms of the path $x^\mu(\tau)$ as

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma} \frac{dx^\rho}{d\tau} \frac{dx^\sigma}{d\tau} = \left(g^{\mu\nu} + \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}\right) \partial_\nu (\ln \phi).$$

(20)

Since we are assuming that $\phi = \phi(t)$ is constant along spacelike hypersurfaces $t = \text{constant}$ of the metric (11), we can solve explicitly for the motion of a particle obeying (19). In terms of the magnitude of the spacelike 3-momentum,

$$|\vec{p}|^2 = g_{ij}p^i p^j = a^2 \delta_{ij}p^i p^j,$$

(21)

we find

$$|\vec{p}| \propto a^{-1},$$

(22)
just as for conventional (constant-mass) particles. The distinction arises for the velocity; if the four-velocity satisfies $p^\mu = mu^\mu$, the magnitude of the three-velocity $|\vec{u}| = (g_{ij}u^iu^j)^{1/2}$ is proportional to $(a\phi)^{-1}$. Thus, as the particles get more massive with time, they naturally slow down even more rapidly than ordinary test particles.

Although we have not specified any explicit interactions between the vamps and visible matter, we may imagine that such reactions exist as long as they are sufficiently weak that they do not lead to consequences which would have already been observed. As a result of such interactions, we presume that the $\psi$'s were in thermal equilibrium at some high temperature. Their equilibrium phase-space distribution function is either a Fermi-Dirac or Bose-Einstein distribution,

$$f(p) = \frac{g_\psi}{h_P} \frac{1}{e^{E/kT_\psi} \pm 1},$$

where $g_\psi$ is the number of spin degrees of freedom, $h_P$ is Planck’s constant, $k$ is Boltzmann’s constant, $T_\psi$ is the temperature of the $\psi$'s, and $E$ is the energy, given by

$$E = p^0 = (m_\psi^2 + |\vec{p}|^2)^{1/2}.$$

As the temperature and density increase, $|\vec{p}|$ goes up while $m_\psi$ goes down. At sufficiently early times, therefore, the $\psi$ particles were relativistic, and $E \sim |\vec{p}| \propto a^{-1}$. Under these circumstances the $\psi$’s behave like ordinary relativistic particles; their temperature redshifts as $T_\psi \propto a^{-1}$, and their energy density as $\rho_\psi \propto a^{-4}$. When they become nonrelativistic, on the other hand, their kinetic temperature will scale as $T_\psi \sim |\vec{p}|^2/m_\psi \propto a^{-2}\phi^{-1}$; they cool off more rapidly than ordinary matter. (Strictly speaking it is incorrect to speak of a temperature after the particles become nonrelativistic, as the varying rates at which the particles slow down will distort the initially thermal distribution.)

It is reasonable to assume that $\psi$ was relativistic when it decoupled, and we may proceed under this assumption to show that it leads to a consistent picture. In that case the number density of $\psi$ today is given by the standard formula [12]

$$n_{\psi0} = 825 r_\psi \text{ cm}^{-3},$$

where $r_\psi$ is the ratio of $g_{\text{eff}}$, the effective number of degrees of freedom in $\psi$, to $g_{\ast f}$, the total effective number of relativistic degrees of freedom at freeze-out. (In terms of the number of physical degrees of freedom $g$, $g_{\text{eff}} = g$ for bosons and $g_{\text{eff}} = 3g/4$ for fermions.) For simplicity let us consider the case $p = 1$. Then we can directly determine the mass of $\psi$ in terms of the current density parameter $\Omega_{\psi0}$ and Hubble constant $H_0 = 100h$ km/sec/Mpc:

$$m_\psi = 12.7 \Omega_{\psi0} h^2 r_\psi^{-1} a^{3/2} \text{ eV}.$$
In terms of the Yukawa coupling $\lambda$, the other relevant parameters of the model are then

$$u_0 = 1.02 \times 10^{-9} \frac{\Omega_{\psi_0}^2 h^4}{\lambda r_\psi} \ (eV)^5$$

(27)

and

$$m_\phi = 1.00 \times 10^{-6} \frac{\lambda r_\psi}{\Omega_{\psi_0}^{1/2} h} a^{-9/4} \ eV.$$

(28)

The temperature of the $\psi$ particles (while they are still relativistic) is diluted somewhat with respect to that of the photons, due to entropy production subsequent to the freeze-out of $\psi$:

$$T_\psi = \left( \frac{g_{*0}}{g_{*f}} \right)^{1/3} T_{\gamma 0} a^{-1}$$

$$= 3.55 \times 10^{-4} a^{-1} g_{*f}^{-1/3} \ eV.$$  

(29)

Comparing (26) to (29), we find that the $\psi$'s first become non-relativistic at a redshift of

$$z_{NR} = 66.3 \left( \frac{\Omega_{\psi 0} h^2 g_{*f}^{1/3}}{T_\psi} \right)^{2/5}.$$

(30)

4 Further consequences

Although the VDM model helps to alleviate the age problem, there are a number of other cosmological tests that could conceivably rule it out. For example, nucleosynthesis places stringent limits on the number of degrees of freedom contributing to the energy density at $T \sim 1 \ keV$ [13]. Particles which decouple at sufficiently high energies are not constrained by this test, as their number density is diluted by entropy production after decoupling. We do not know the temperature at which $\psi$ decouples, although there is no reason to believe that it isn’t sufficiently high to evade the nucleosynthesis bound. Meanwhile, the energy density in $\phi$ is much less than that in $\psi$ at high temperatures, and is therefore even less constrained.

Any model which increases the age of the universe by changing the behavior of the scale factor with time will be subject to various cosmological tests which are sensitive to that relationship; these are conventionally used to place limits on the cosmological constant [5]. Currently the most promising such tests are direct measurements of deviation from the linear Hubble law using high-redshift Type Ia supernovae [14], and volume/redshift tests provided by the frequency of gravitational lensing of distant quasars by intervening galaxies [15]. These have recently been applied to a number of models with novel dependences of the scale factor on time, very similar to the scenario discussed in this paper. The results to date [16]
seem to indicate that these tests do not rule out the kind of models considered here, but may be able to do so in the near future when more data is available.

Our investigation has been exclusively in the context of an unperturbed Robertson-Walker cosmology. The next step is to introduce perturbations and discuss CMB anisotropies and the formation of structure; work in this direction is in progress. However, it is worth noting some important features of the problem. There are two powerful effects which distinguish the growth of perturbations in a VDM cosmology from conventional cold dark matter, and they tend to affect the power spectrum in opposite ways. The first effect is the effectively negative pressure of the coupled system. At zero temperature, perturbations in vamps grow more rapidly than those in CDM; indeed, perturbations tend to grow even in the absence of gravity. The other effect, meanwhile, is the free streaming of the $\psi$ particles. The $\psi$’s decouple while relativistic, and in some respects act as hot dark matter. They will tend to flow out of overdense regions, damping the growth of perturbations until sufficiently late times. An accurate appraisal of the magnitude of this process requires numerical integration of the evolution equations, as the Boltzmann equation does not simplify as it would for massless or completely nonrelativistic particles [17]. These two competing effects are not the entire story; for example, if the $\psi$’s are fermions they will be prevented from clustering on very small scales by the exclusion principle [18]. The final perturbation spectrum is therefore the result of a number of processes, and cannot be reliably estimated analytically. In addition, of course, the simple model we have investigated here may be modified, either by altering the form of the potential (6) or by introducing other forms of energy in addition to baryons and vamps (e.g., ordinary hot or cold dark matter).

Another direction currently under investigation is the construction of particle physics models in which vamps may arise. A possible origin for the scalar $\phi$ is as one of the moduli of string theory; our understanding of the nonperturbative effects which give potentials to such fields is not sufficiently developed to attempt realistic model building at this time. In supersymmetric gauge theories, however, there are (perturbatively) flat directions whose dynamics are somewhat better understood, and in that context the search for a model may be more hopeful. In such a scenario there are a number of potentially dangerous effects which must be avoided; for example, if the expectation value of $\phi$ breaks supersymmetry, it may lead to gradual variations in the parameters of the standard model as the universe expands. Such variations are tightly constrained by a variety of data [13].
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