The anapole moment in scalar quantum electrodynamics

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The anapole moment of a charged scalar particle is studied in a model independent fashion, using the effective Lagrangian technique, as well as radiatively within the context of scalar quantum electrodynamics (SQED). It is shown that this gauge structure is characterized by a non renormalizable interaction, which is radiatively generated at the one–loop. It is found that the resulting anapole moment for off–shell particles, though free of ultraviolet divergences, is gauge dependent and thus it is not a physical observable. We also study some of its kinematical limits. In particular, it is shown that its value comes out to be zero when all particles are on–shell.

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In a previous communication by some of us, a one–loop SQED calculation for the 3–point off–shell $\phi^– \phi^+ \gamma$ Green function in arbitrary gauge and dimension was reported [1]. It was shown that a new independent gauge structure arises at this level. In this brief report, we show that this non renormalizable gauge structure corresponds to the anapole moment of a charged scalar particle. To the best of our knowledge, this study has not been carried out in the literature so far. The anapole moment, first introduced by Zeldovich [2], is an electromagnetic property that vanishes for on–shell photons and always arises as a quantum fluctuation at one–loop or higher orders within the context of renormalizable theories. It is the only electromagnetic property of Majorana particles. The anapole moment is best known from the studies of Majorana neutrinos [3] and the standard model (SM) Z boson [4], although it is a characteristic of any type of particle\(^1\). Its systematic study for Majorana particles including fields of higher spin has been carried out in [5]. The anapole moment of spin 1/2 non Majorana particles has also been the subject of interest in the literature [6]. In the present work, we focus on the anapole moment associated with a scalar particle within the context of SQED. We study this electromagnetic property of charged scalars in two different ways. Firstly, using the effective Lagrangian formalism [7], we demonstrate the existence of this gauge structure in a model–independent way. Secondly, using general results reported in [8] for a charged scalar, we show that this off–shell electromagnetic structure arises at the one–loop level. Apart from its intrinsic theoretical interest, there are phenomenological motivations to study the electromagnetic properties of scalar particles, as it is expected that new physics show up during the experiments that will be carried out at the Large Hadron Collider (LHC). In particular, if there is a Higgs boson, it is almost certain to be found at this collider and its mass measured by the ATLAS [8] and CMS [9] experiments. If discovered, answers to many related questions would have to be sought. For instance, does it differ from the one predicted by the Standard Model (SM)? Are there more than one Higgs particles? Therefore, it is important to study all types of physical properties of scalar particles, including those that arise as a quantum fluctuation, as is the case of the anapole moment.

As it is well known, the renormalizable structure of SQED induces three– and four–point $\phi^– \phi^+ \gamma$ and $\phi^– \phi^+ \gamma\gamma$ couplings at the level of the classical action. However, it is not difficult to convince ourselves that these couplings develop new components of non renormalizable type at the level of the quantum action. The corresponding effective theory incorporates $U_c(1)$–invariant terms of up to dimension six. To that order, one can write an effective Lagrangian of the form

$$\mathcal{L}_{eff} = \mathcal{L}_{SQED} + \frac{\alpha D}{\Lambda^2} \mathcal{O}_D + \frac{\alpha D D}{\Lambda^2} \mathcal{O}_{DD},$$

(1)

where $\Lambda$ is some relevant energy scale and $\mathcal{L}_{SQED}$ represents the renormalizable Lagrangian given by:

$$\mathcal{L}_{SQED} = (D_\mu \phi^+)\dagger (D^\mu \phi^+) - V(\phi^-, \phi^+ - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{GF}.$$  

(2)

In these expressions, $D_\mu = \partial_\mu - ieA_\mu$ is the electromagnetic covariant derivative, $V(\phi^-, \phi^+)$ is the scalar potential,

\(^1\) The only exception is the neutral scalar particle that coincides with its own antiparticle, which has no electromagnetic properties.
\( \mathcal{L}_{GF} \) is the gauge-fixing term, and the \( \mathcal{O}_i \) are \( U(1) \)-invariant structures of dimension six given by \(^2\)

\[
\mathcal{O}_D = i \left[ \phi^- D_\nu \phi^+ - \phi^+ (D_\nu \phi^-) \right] \partial_\mu F^{\mu \nu},
\]

\[
\mathcal{O}_{DD} = i \left[ \phi^- (D_\mu D_\nu \phi^+) - \phi^+ (D_\mu D_\nu \phi^-) \right] F^{\mu \nu}.
\]

As is evident, both \( \mathcal{O}_D \) and \( \mathcal{O}_{DD} \) contribute to the 4-point vertex, but only \( \mathcal{O}_D \) contributes to the 3-point coupling. Thus, the most general electromagnetic gauge structure for the \( \phi^+ \phi^\gamma \) vertex is given by

\[
\mathcal{L}_{\phi^+ \phi^\gamma} = ic \mu A^\mu + \frac{i \alpha_D}{\Lambda^2} J_\mu \partial_\nu F^{\nu \mu},
\]

where \( \mu \) is the electromagnetic current given by:

\[
\mu = \phi^- \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^-.
\]

From the above Lagrangian, one can construct the most general 3-point vertex function, which can be written as:

\[
\Gamma_\mu = f_Q(q^2, p^2, k^2)(k + p)_\mu + \frac{f_A(q^2, p^2, k^2)}{\Lambda^2} \left[ q^2 (k + p)_\mu - (k^2 - p^2)q_\mu \right],
\]

where \( f_Q(q^2, p^2, k^2) \) and \( f_A(q^2, p^2, k^2) \) are the form factors associated with the monopole and anapole moments of the \( \phi^\pm \) scalar boson, respectively. In the context of the renormalizable theory, \( f_Q(q^2, p^2, k^2) = 1 \) and \( f_A(q^2, p^2, k^2) = 0 \) at the level of the classical action. As we will show below, the anapole moment is generated at the one-loop level by the renormalizable theory. The new gauge structure satisfies \( q^\mu [q^2 (k + p)_\mu - (k^2 - p^2)q_\mu] = 0 \) ensuring the Ward identity \( q^\mu \Gamma_\mu = S^{-1}(k^2) - S^{-1}(p^2) \) is conserved, \( S^{-1} \) being the scalar propagator. To be able to make quantitative statements about the anapole form factor, we need to know \( f_A(q^2, p^2, k^2) \) explicitly.

![Figure 1: Diagrams contributing to the \( \phi^+ \phi^- \) vertex up to one-loop order.](image)

While the complete non-perturbative expression for the anapole moment is not known \(^3\), it is induced at one-loop level. The one-loop contribution to the \( \phi^- \phi^+ \phi^- \) Green function is given through the diagrams shown in Fig. 1. As it was shown on general grounds in \(^4\), the complete Green function \( \Gamma_\mu \) is made of two independent components, one proportional to the monopole moment \( (k + p)_\mu \) and other associated with a part that satisfies the \( q^\mu \Gamma_\mu^T = 0 \) transverse condition. In the light of the above discussion, this loop induced gauge structure can be identified with the anapole moment:

\[
\Gamma_\mu^T = \frac{f_A(q^2, p^2, k^2)}{\Lambda^2} \left[ q^2 (k + p)_\mu - (k^2 - p^2)q_\mu \right] \equiv \tau(k^2, p^2, q^2) T_\mu(k, p),
\]

where

\[
T_\mu(k, p) = q^2 (k + p)_\mu - (k^2 - p^2)q_\mu.
\]

\(^2\) The \( U(1) \)-invariant \( (D_\mu \phi^+)(D_\nu \phi^+) F^{\mu \nu} \) term is not independent, as it is related to \( \mathcal{O}_D \) and \( \mathcal{O}_{DD} \) through a surface term.

\(^3\) Transverse Ward identities in SQED fail to yield any information on \( f_A(q^2, p^2, k^2) \).
The explicit general expression for \( \tau(k^2, p^2, q^2) \) for the off-shell particles in arbitrary dimensions is given in [1]. In 4-dimensions, it is

\[
\tau(k^2, p^2, q^2) = \frac{\alpha}{8 \pi \Delta^2} \left[ (k^2 + p^2 - 4k \cdot p) \left( k \cdot p J_0 + \ln \left( \frac{q^4}{2p^2} \right) \right) + \frac{(k^2 + p^2)q^2 - 8k^2p^2 \ln \left( \frac{k^2}{p^2} \right)}{p^2 - k^2} \right] - \frac{\alpha \xi'}{8 \pi \Delta^2} \left[ k^2 p^2 J_0 + \frac{2k^2 p^2}{k^2 - p^2} \ln \left( \frac{p^2}{k^2} \right) + \frac{2k \cdot p}{k^2 - p^2} \left( \ln \left( \frac{p^2}{q^2} \right) + k^2 \ln \left( \frac{q^2}{k^2} \right) \right) \right].
\]

(10)

It is gauge–dependent but free of ultraviolet divergences. All such divergences reside in the longitudinal part of the vertex. Here, we shall analyze different kinematic limits of the transverse vertex in an arbitrary covariant gauge. We summarize our results as follows:

- In the limit \( q^2 \to 0 \) for \( k^2 \neq p^2 \),

\[
\Gamma_T^\mu(q^2 \to 0) = -\frac{\alpha}{4 \pi} \frac{q_\mu}{(k^2 - p^2)^2} \ln \left( \frac{q^2}{p^2} \right) \times \left[ 2(k^4 - p^4) + (k^2 + p^2)^2 \ln \left( \frac{p^2}{k^2} \right) + \xi' \left\{ (k^4 - p^4) + 2k^2 p^2 \ln \left( \frac{p^2}{k^2} \right) \right\} \right],
\]

(11)

where \( \alpha = e^2/(4\pi) \) and \( \xi' = 1 - \xi \). \( \xi = 0 \) corresponds to the Landau gauge and \( \xi = 1 \) to the Feynman gauge. Note that this limit is logarithmically divergent and gauge dependent.

- In the limit \( k^2 \to p^2 \) for \( q^2 \neq 0 \)

\[
\Gamma_T^\mu_k(k^2 \to p^2) = -\frac{\alpha}{4 \pi} \frac{(k + p)_\mu}{(q^2 - 4p^2)} \times \left[ (q^2 - p^2) \left( 2p^2 - q^2 \right) J_0 + 4 \ln \left( \frac{q^2}{p^2} \right) \right] + 2(4p^4 - q^2) + \xi' \left\{ -p^4 J_0 + (4p^2 - q^2) - (2p^2 - q^2) \ln \left( \frac{q^2}{p^2} \right) \right\} \right],
\]

(12)

where

\[
J_0(k^2 \to p^2) = \frac{2}{\Delta} \left[ S_P \left( \frac{q^2 + 2 \Delta}{2p^2} \right) - S_P \left( \frac{q^2 - 2 \Delta}{2p^2} \right) + \frac{1}{2} \ln \left( \frac{2p^2 - q^2 - 2 \Delta}{2p^2 - q^2 + 2 \Delta} \right) \ln \left( \frac{q^2}{p^2} \right) \right],
\]

(13)

with \( \Delta^2 = q^2(q^2 - 4p^2)/4 \) and

\[
S_P(x) = -\int_0^x \frac{dy}{dy} \frac{\ln(1 - y)}{y}.
\]

Therefore, this limit, though gauge dependent, is perfectly finite, corresponding to the fact that there are no kinematic singularities for the 3-point functions in scalar QED. The same is true for spinor QED and QCD for \( k^2 \to p^2 \).

- Moreover, one can show that

\[
\Gamma_T^\mu_k(k^2 \to p^2, q^2 \to 0) = \Gamma_T^\mu_k(q^2 \to 0, k^2 \to p^2) = 0.
\]

(14)

Thus the anapole moment is zero for all the external particles on the mass–shell. We have presented our results for the massless case alone as it is sufficient to analyze the singularity structure of SQED, [11].

The electromagnetic properties of elementary particles constitute observables of important theoretical and phenomenological interest, as they are model independent and respond to \( P, T \), and \( C \) transformations. Therefore, they constitute windows through which new physics effects could show up. In particular in neutrino physics, they can act as a probe of whether neutrinos are Dirac of Majorana particles. In this brief report, within the context of scalar electrodynamics, we have shown that a charged scalar particle develops an anapole moment at the one–loop level, which is finite but gauge–dependent. Some kinematic limits have been derived 4. In particular, it is shown that the

4 Incidentally, this form factor plays an important role in constructing a non perturbative triple-gluon vertex which would ensure gluon mass generation without seagull divergences [12]. Although this form factor is logarithmically divergent as \( q^2 \to 0 \), it still diverges sufficiently slowly for the argument constructed in [12] to hold valid, [13].
anapole moment is zero for all the external particles on the mass–shell. The existence of this electromagnetic property is shown by using the effective Lagrangian technique. We would like to comment that the anapole moment would be the sole electromagnetic property of a scalar particle that is neutral but differs from its own antiparticle, i.e., a particle which is associated with a non self–conjugate scalar operator field.

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