Study of the optical properties of V$_2$O$_5$-Li$_2$O-P$_2$O$_5$ glass system

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Abstract. In the last two decades, several applications became more dependent on the nonlinearity phenomena in glasses, for example: laser glass has low nonlinear refractive index ($n_2$), while higher values serve in optical switching. In this study, Vanadium-Lithium-Phosphate glass samples [V$_2$O$_5$-Li$_2$O-P$_2$O$_5$] were prepared by the melt-quenching technique. The amorphous structure of these glasses is examined by XRD analysis. The recorded reflectance and transmittance were investigated over the range from 200 to 2500 nm for the five samples. The two parts of the calculated refractive indices are estimated via an analytical technique. The dispersion parameters, such as Sellmeier gap energy, dispersion energy as well as Abbe’s number, were deduced. From the obtained results, the parameters for absorption dispersion, namely: the optical energy gaps for indirect transition as well as the Urbach energy were extracted. Moreover, the other parameters such as: the molar refractivity, the reflection loss and density are calculated. The measured optical parameters were found to vary nonlinearly with increasing vanadium oxide (V$_2$O$_5$), in the presence of Li$_2$O in the phosphate matrix. In the aim of enhancing the performance in this glass system, the effects of ascending doping with a transition metal oxide (V$_2$O$_5$) on both the predicted nonlinear and the linear optical parameters, are investigated and analyzed.

1. Introduction
Historically, in 1932, Zachariasen proposed the following four rules that allowed the selection of the most suitable oxides that have tendency to form glasses;
An oxygen (O) atom is ordinarily linked to two glass forming atoms.
The oxygen polyhedra have tendency to share corners with each other (not edges or faces).
The coordination number of the glass-forming atoms is considerably small.
The polyhedra are linked in a 3D network.
In comparison with silicate and borate glasses, the peculiar and exquisite physical properties of phosphate glasses aim to promote applications in various domains.
These properties include: high thermal expansion coefficient, low melting temperature, near normal preparation temperature (glass sealing candidates), low transition temperature, significant refractive indices (n) as well as conventional high transmission in UV range (e.g. laser sensing tech., optical data transmission, etc.). The accompanying outstanding optical properties offer advantages, specifically in the molding of the optical elements and nuclear waste storage glass [1], among other technical and biological applications [2]. Though transition metal oxides have a mixed dual role, as modifiers and formers, their semiconducting properties, photo-conducting, optical absorption and
electrical memory switching seem to motivate the vanadate glasses to contribute in unceasing glass systems for advanced applications. On the other hand, the hygroscopic nature of theses glasses leads to poor chemical stability; which calls for adding the alkali, transition metal or rare earth ions [3,4]. Industrial applications, based on such improvements, include: pigment manufacturing, impounding agents for hard water treatment and separation for clay processing [3,5].

From the previous work [6,7], the structural units of phosphate glasses fall into $Q^n$ groups; where $n$ is the number of bridging oxygen (BO) per unit cell, figure 1. These groups start with ultraphosphate ($Q^5$), metaphosphate ($Q^4$), pyrophosphate ($Q^3$) then orthophosphate ($Q^2$), creating non bridging oxygens (NBOs), upon adding modifying oxide to $P_2O_5$ [3,8,9,10]. According to spectroscopic analysis [11,12] transition metal ions (TMIs) in alkali phosphate glasses seem to prefer the low valence state (octahedral) than the high valence state (tetrahedral). This indicates that P-tetrahedra are the basic building blocks of vitreous phosphates which are linked through covalent BOs to the amorphous phosphate. Hence, a network of ($Q^i$) tetrahedral units, with three BOs ($P-O-P$) and one NBO ($P=O$) for the short range structure of $P_2O_5$ glass. Principally, in a single component glass system, an oxygen of a very short ($P=O$) bond can be distinguished on each network configured (O) cation; with a purport $\pi$ -- bond character, which accounts for the additional valence electrons [13]. The polymeric structure can be altered for 3D random network to a linear phosphate chain upon adding a modifier oxide (alkali or alkali earth) [3,14]. Experimental and theoretical studies [15,16] deduced that upon increasing the modifier concentration, the long chains of phosphate are reduced causing a break in the network coherency results in NBOs. The $P_2O_5$ glasses therefore tend to disintegrate forming structural elements, consisting of rings and chains, which reduces the oxide modified glasses to become relatively stable. The addition of vanadium pentoxide to the mixed divalent oxide phosphate glasses tends to improve the durability of phosphate with the benefit that it can exhibit the luster and appearance of lead glass; which is costly and fragile.

![Figure 1. Phosphate based glass tetrahedral in $Q^i$ terminology.](image-url)

2. Experimental Procedure.

2.1. Preparation Method

Glassy samples of $[V_2O_5 \cdot Li_2O \cdot P_2O_5]$, were prepared by the traditional melt quenching technique. These samples were prepared by mixing together appropriate amounts of NH$_4$H$_2$PO$_4$ (Aldrich. 99.997%), Li$_2$CO$_3$ (Aldrich. 99.9%) and V$_2$O$_5$ (Aldrich. 99.99%). In several earlier studies, NH$_4$H$_2$PO$_4$ and Li$_2$CO$_3$ were used to obtain P$_2$O$_5$ and Li$_2$O [17,18,19,20]. The above mixtures were heated at first in an electric furnace at 250 °C for 1.5h, in alumina crucibles, to remove the volatile products. This was followed by further increase in temperature, up to 950 °C for 1 h; with frequent stirring of the melt to ensure homogeneity. In an attempt to avoid severe shattering of the quenched samples as a direct of thermal stress, the mixtures were poured in turn on preheated 300 °C stainless steel plates. Rectangular samples of thickness 3-mm were annealed for 2 h at 300 °C to relieve residual internal stress and slowly cooled to room temperature.
Table 1. Percentage composition of the prepared samples

| Glass Sample | Composition % |  |
|--------------|---------------|---|
|              | P$_2$O$_5$ | Li$_2$O | V$_2$O$_5$ |
| S$_0$        | 70           | 30     | 0          |
| S$_1$        | 70           | 29.8   | 0.2        |
| S$_2$        | 70           | 29.6   | 0.4        |
| S$_3$        | 70           | 29     | 1          |
| S$_4$        | 70           | 28     | 2          |

2.2. Spectrophotometric Measurements.

Computer aided two-beam spectrophotometer (JASCO Corp., V = 570, Rev. 1.00 UV/VIS/NIR, Japan) was used to measure the Reflectance (R) and Transmission (T) from the plane-parallel glass samples. The resolution limit of the spectrometer is $\delta \lambda = 2$ nm. The accuracy of measuring reflectance and transmittance is $\pm 0.002$ with the incident beam making an angle of $5.0^\circ \pm 0.1^\circ$ to the normal of the external sample faces. The angle of the propagating beam through the sample is within below $5^\circ$. The measurements were carried out at room temperature for the entire spectral range 0.19-2.5 $\mu$m (data $\sim 1160$ points).

3. Factors that affect the linear refractive index.

There are several factors considered as more important in influencing the linear refractive index of oxide glasses:

1- The most important is the dependency of linear refractive index on the density ($\rho$) [21, 22] according to the relation $n = (C_1 \rho + 1)^{1/2}$, where ($C_1$) is a constant.

2- In previous studies [21] other aspects proved to be effective, among them:
   a. polarizability of the first neighbor ions coordinated with it (anion),
   b. field intensity $z = a^2$ (i.e., polarization power), where ($z$) is the ionic charge and ($a$) is the ionic radius,
   c. coordination number of the ion,
   f. fundamental NBO’s,
   g. the oxide ion’s electronic polarizability,
   h. optical basicity of the material.

Moreover, the dependency of linear refractive index on the photon energy, ($E = h \nu$), is given by Wemple and DiDomenico equation (see section 4.4.2).

4. Results and discussion.

4.1. Density and molar volume.

The calculated density ($\rho$) and the molar volume ($V_m$) of the studied glasses were obtained from the expressions [23,24]:

$$\rho = \frac{x M_{V_2O_5} \rho_{V_2O_5} + (0.3 - x)M_{Li_2O} \rho_{Li_2O} + 0.7M_{P_2O_5} \rho_{P_2O_5}}{x M_{V_2O_5} + (0.3 - x)M_{Li_2O} + 0.7M_{P_2O_5}}$$

$$V_m = \frac{M_{wt}}{\rho}$$

where ($M_{Li_2O}$), ($M_{P_2O_5}$) and ($M_{V_2O_5}$) are the molecular weights of ($Li_2O$), ($P_2O_5$) and ($V_2O_5$). Moreover, ($\rho_{Li_2O}$), ($\rho_{P_2O_5}$) and ($\rho_{V_2O_5}$) are the densities of ($Li_2O$), ($P_2O_5$) and ($V_2O_5$) (2.01, 2.39, and 3.36 g/cm$^3$ respectively). On the other hand, ($M_{wt}$) is the total molecular weight of each sample. The calculated data for the molar volume and the density are tabulated in table (2).
Table 2. shows the molecular weight (M), the density (\(\rho\)), the molar volume (\(V_m\)) and the number of molecules per unit volume (N) of the studied glasses

| Glass Samples | Density (g/cm\(^3\)) | Molecular Weight (g/mole) | Molar Volume \(V_m\) (cm\(^3\)/mole) | \(N(\times 10^{22})\) (cm\(^3\)) |
|---------------|-----------------------|---------------------------|--------------------------------------|-------------------------------|
| S\(_0\)       | 2.373                 | 108.298                   | 45.637                               | 1.319                         |
| S\(_1\)       | 2.375                 | 108.602                   | 45.727                               | 1.316                         |
| S\(_2\)       | 2.377                 | 108.906                   | 45.816                               | 1.314                         |
| S\(_3\)       | 2.382                 | 109.818                   | 46.103                               | 1.306                         |
| S\(_4\)       | 2.391                 | 111.339                   | 46.565                               | 1.293                         |

The density, molecular weight and molar volume of the samples are observed to increase with increasing the dopant content of V\(_2\)O\(_5\) as well as decreasing the Li\(_2\)O content. Alternatively, the number of molecules per unit volume (N) decreases with increasing V\(_2\)O\(_5\). Consequently, the density and the molar volume of the studied glass samples are strongly dependent on the vanadium pentoxide content. The glass network thus tends to break off the bridging oxygen bonds leading to subsequent formation of non-bridging bonds [25,26]. Hence, it can be inferred that the intercalation of V\(_2\)O\(_5\) in [Li\(_2\)O – P\(_2\)O\(_5\)] glass causes the structural change in the glass network.

4.2 Refractive Indices Determination

It is an established fact that when light passes through glass it is divided into processes: transmission, absorption and reflection; the following formula must be verified:

\[
T + R + A = 1
\]

where (T) is the transmittance, (R) is the reflectance and (A) is the absorbance of the material.

Based upon measured data, the obtained results for reflectance and transmittance are displayed in figure 2. The experimental spectral reflectance of the prepared glasses with different concentrations of V\(_2\)O\(_5\) is illustrated in figure 2.a. On the other hand, figure 2.b. shows the experimental spectral transmittance of the prepared samples with various concentrations of V\(_2\)O\(_5\).

![Figure 2. Measured (a) reflectance (R), (b) transmittance (T); of the different prepared glasses vs. wavelength](image)

As an approximation, earlier studies [27] proposed: the relation between the measured (R), (T) and interface (Fresnel) reflectance (\(R_{\text{as}}\)) is
This leads to deduces the imaginary part of the refractive index \( k \) from

\[
k = \frac{\lambda}{4\pi t} \ln \left[ \frac{TR_{as}}{(R-R_{as})} \right]
\]

(5)

and the absorption coefficient \( \alpha \) can be written as

\[\alpha = \frac{4n\kappa}{\lambda}\]

(6)

where \( \lambda \) is the wavelength of light used and \( t \) is the glass sample thickness. Alternatively, the real part of the refractive index is given by

\[
n = \frac{(1+R_{as})}{(1-R_{as})} + \frac{4R_{as}}{(1-R_{as})^2 - k^2}^{1/2}
\]

(7)

In this work, several analytical expressions are utilized to retrieve the real and imaginary refractive indices; while taking into consideration incoherent multiple reflections inside the sample. Equations (4, 5 and 7) are used to obtain the real \( n \) and imaginary \( k \) refractive index of the studied glasses, from the overall measured reflectance and transmittance of the glass samples. The refractive index dispersion obeys, to a great extent, Sellmeier’s empirical dispersion equation:

\[
n^2(\lambda) = 1 + \sum \frac{B_i \lambda^2}{\lambda^2 - C_i}
\]

(8)

where \( B \) and \( C \) are experimentally determined Sellmeier coefficients.

Figure 3.a shows the behavior of the real part of refractive index \( n \) of studied glass samples within the range 200-2500 nm. On the other hand, Figure 3.b shows the behavior of the imaginary part of refractive index \( k \) of studied glass samples within the range 200-2500 nm.

It can be inferred that, the accommodation of \( V_2O_5 \) readily changes the refractive index \( n \) of the samples nonlinearly.

### 4.3 Determination of the dielectric constant

The complex dielectric function \( \varepsilon \), representing the underlying molecular mechanism, is an indication for the interaction of EM waves with matter [28]. The components of the complex dielectric constant \( \varepsilon_i \) of a material, are expressed in terms of the optical constants \( n \) and \( k \); such that the real part
\[
\varepsilon_1 = n^2 - k^2
\]  \hspace{1cm} (9)

and the imaginary part is \[
\varepsilon_2 = 2nk
\]  \hspace{1cm} (10)

Figure 4.a. shows the real part of the dielectric constant, \((\varepsilon_1)\), versus the photon energy \((\hbar\nu)\) for the studied samples. For all samples the dielectric constants shows an exponential increase with photon energy and randomly change with the increase of the ratio of \(V_2O_5\). This means that the free carrier concentration of the different glass compositions change in the different manner with the change in \(V_2O_5\) content. Sample \((S_2)\) has the highest, \((\varepsilon_1)\), while sample \((S_1)\) has the lowest, \((\varepsilon_1)\). On the other hand, Figure 4.b. shows the imaginary part of the dielectric constant, \((\varepsilon_2)\), versus the photon energy for the same glass compositions. Such determination of the imaginary part of dielectric constant could be employed in determination of the optical relaxation time of glasses.

4.4 Equations for Linear Refractive index Dispersion.

4.4.1 Abbe’s Number.

In glass manufacturing, for chromatic aberration correction, the material dispersion of any glass is described by the Abbe’s number, \((\nu_d)\) which is given by [21,29]:

\[
\nu_d = \frac{n_d-1}{n_F- n_C}
\]  \hspace{1cm} (11)

where \((n_F, n_C\) and \(n_d\) \) are the linear refractive indices at the standard wavelengths \((\lambda_F, \lambda_C\) and \(\lambda_d)\), respectively. F-line is the blue line \((\lambda=479.98nm)\), the C-line is the red line \((\lambda=643.8nm)\) from the cadmium spectrum and d-line is yellow line \((\lambda=587.56nm)\) from the He spectrum. The refractive indices difference \((n_F - n_C)\) is called the mean dispersion. Moreover, materials which have low refractive indices usually have a high Abbe’s number and therefore exhibit low dispersion. Traditionally, the reciprocal of Abbe’s number \((\nu_d)\) represents the dispersive power of a glass material.

4.4.2 The Wemple-DiDomenico dispersion relation

Wemple & DiDomenico (W-D) [30,31] laid a design for the \((n)\) data in a single oscillator model, as a function of the incident photon energy \((E)\) in the transparent region as

\[
E^2 - 1 = \frac{E_D E_d}{E_0^2 - E^2}
\]  \hspace{1cm} (12)

where \((E_0)\) is the so called Sellmeier energy gap and \((E_d)\) is the dispersion energy[21,32]. Conventionally, The single oscillator energy \((E_d)\) is related to the average band gap and corresponds to the distance between the centers of gravity of the CB and VB. \((E_d)\) is the average strength of interband optical
transitions which represents the average excitation energy for electronic transitions as well as the
electronic oscillator strength related to dispersion.

The linear fitting of \((n^2 - 1)^{-1}\) versus \(E^2\) is suitable to find \((E_0)\) and \((E_d)\) from

\[
\frac{1}{n^2-1} = \frac{E_0}{E_d} - \frac{E^2}{E_0E_d}
\]

(13)

The refractive index at infinite wavelength, considering only electronic transitions, can be written as [21]:

\[
n_\infty = \sqrt{1 + \frac{E_d}{E_0}}
\]

(14)

4.5 The Reflection loss (R %)
The energy loss of the incident light, from the glass surface, by reflection is defined as "reflection loss". Eventually, it leads to important measurements concerning light transmission characteristics for various colors of glasses. Basically, the fraction reflected by a single surface can be calculated from Fresnel’s formula (specifically in case of normal incidence of light on a glass surface). The reflection losses (R %) of each glass sample can be determined from [21,29]

\[
R\% = \left(\frac{(n_e - 1)}{(n_e + 1)}\right)^2 \times 100
\]

(15)

where \((n_e)\) is the glass refractive index at \(\lambda = 546.1\text{nm}\).

4.6 Molar Refractivity and Polarizability
The molar refractivity \((R_M)\), basically deduced from the refractive index \((n_e)\), molecular weight \((M)\), and the density \((\rho)\) of the studied glass [21, 29], is given as

\[
R_M = \frac{(n_e^2 - 1)}{(n_e^2 + 2)} \left(\frac{M}{\rho}\right)
\]

(16)

From equation (16) and using the Clausius-Mossotti equation [21,33,34], the polarizability \((\alpha_m)\) is calculated from:

\[
R_M = \frac{4\pi NA}{3}\alpha_m
\]

(17)

where \((N_A)\) is Avogadro's number.

Table 3. The measured indices \((n_F, n_D \text{ and } n_C)\), Abbe’s number \((\nu_d)\), mean dispersion, dispersive power of a glass material, reflection loss (R%), molar refractivity \((R_M)\) and polarizability \((\alpha_m)\) of the glass samples.

| Glass Sample | \(n_F\) | \(n_D\) | \(n_C\) | \(n_e\) | \(\nu_d\) | \(\nu_I\) | \(\nu_I\) | Mean dispersion | dispersive power | R% | \(R_M\) | \(\alpha_m\) |
|--------------|--------|--------|--------|--------|---------|---------|---------|----------------|----------------|-----|--------|--------|
| S_0          | 1.433  | 1.378  | 1.364  | 1.394  | 5.787   | 5.546   | 0.068   | 0.1728         | 0.0270         | 10.904 | 4.32   |
| S_1          | 1.132  | 1.127  | 1.118  | 1.133  | 9.479   | 8.881   | 0.014   | 0.1055         | 0.0039         | 3.942  | 1.52   |
| S_2          | 1.518  | 1.471  | 1.443  | 1.489  | 6.53    | 6.288   | 0.0749  | 0.1531         | 0.0386         | 13.226 | 5.24   |
| S_3          | 1.474  | 1.440  | 1.405  | 1.458  | 6.660   | 6.3953  | 0.0688  | 0.1501         | 0.0347         | 12.585 | 4.99   |
| S_4          | 1.373  | 1.374  | 1.351  | 1.385  | 17.434  | 16.923  | 0.0221  | 0.0573         | 0.0260         | 10.920 | 4.33   |

From table (3), it is evident that the measured optical parameters change randomly as the V_2O_5 content increases. On the other hand, the parameters \((E_0)\), \((E_d)\) and \((n_\infty)\) changed randomly with the increase of the concentration of V_2O_5.
Table 4. The dispersive results deduced from the (n) data of the studied samples.

| Glass Sample | E_d (eV) | E_o (eV) | n_o  |
|--------------|---------|---------|------|
| S_0          | 6.013   | 2.688   | 1.7991|
| S_1          | 3.175   | 5.905   | 1.24  |
| S_2          | 3.98    | 4.71    | 1.3583|
| S_3          | 4.321   | 5.068   | 1.3611|
| S_4          | 4.082   | 5.505   | 1.3196|

4.7 Glass Fermi energy
As the imaginary refractive index, (k), has a steep edge towards the IR side of the spectrum, and can be represented by Fermi–Dirac distribution function [35,36]:

\[ k = \frac{1}{1 + \exp\left(\frac{E_F - E}{k_B T}\right)} \]  

(18)

where \((E_F)\) is the Fermi energy, \((E)\) is the variable photon energy, \((k_B)\) is Boltzmann’s constant and \((T)\) is the absolute temperature at the moment of measuring the sample spectra; hence \((k_B T)\) represents the thermal energy. Eq. (18) can be rewritten as:

\[ K_B T \ln \left(\frac{1}{k} - 1\right) = E_F - E \]  

(19)

The linear fitting of this equation was implemented to determine the glass Fermi energy.

4.8 The Optical Absorption Results

4.8.1 Optical Band Energy \((E_{Og})\)
The absorption coefficient, \((\alpha)\), of each glass sample (as a function of the photon energy \((E)\)) can be computed from the relation [37]:

\[ \alpha E = B \left( E - E_{Og}\right)^m \]  

(20)

The numerical value, \((B)\), is a constant that essentially depends on the assessment of electron transition probability; commonly known as Tauc’s law [38]. The value \((B)\) is in the order of \(10^5\) cm\(^{-1}\) eV\(^{-1}\). The power, \(m\), is a characterizing number for the transition mechanism; \(m = 1/2\) for allowed direct transition, \(m = 3/2\) for direct forbidden and \(m = 2\) for allowed indirect transition, \(m = 3\) for indirect forbidden transitions. the optical band gap energy is \((E_{Og})\). Plotting the relation between \((\alpha E)^{1/2}\) and \((E)\) gives the direct band gap energy, \((E_{Og-d})\), while a relation between \((\alpha E)^{1/2}\) and \((E)\) gives the indirect band gap energy, \((E_{Og-i})\) [31].

4.8.2 Urbach Energy
The values of the absorption coefficient lying between \(10^2 - 10^4\) cm\(^{-1}\), are defined as Urbach’s exponential tail region [39] and are given by:

\[ \alpha = \alpha_0 \exp\left(\frac{E}{E_U}\right) \]  

(21)

where \((\alpha_0)\) is a constant, \((E)\) is basically the photon energy and \((E_U)\) being the Urbach energy. \((E_U)\) is commonly interpreted as the tail width of localized states, within the band gap.

Equation (21) can be rewritten as:
\[
\ln \alpha = \ln \alpha_0 + \frac{E}{E_U}
\]  

(22)

Hence, the Urbach energy is usually obtained from plotting the relation between the energy of photon (E) and the absorption coefficient’s natural logarithm (\(\ln \alpha\)).

4.8.3 The steepness parameter

The broadening of the optical absorption edge due to exciton-phonon or electron-phonon interaction at room temperature, T=300 K, is characterized by the steepness parameter [36,40]:

\[
S = \frac{K_B T}{E_U}
\]

(23)

**Table 5.** The indirect allowed optical energy gap (\(E_{in\_direct}\)), the empirical gap (\(E_{g\_em}\)), the Urbach energy (\(E_U\)), the Fermi energy (\(E_F\)) and the steepness parameter (S) of the samples

| Glass Sample | \(E_{in\_direct}\) (eV) | \(E_F\) (eV) | \(E_U\) (eV) | \(E_{g\_em}\) (eV) | S       |
|--------------|----------------|-------------|-------------|----------------|---------|
| S₀           | 6.275          | 2.154       | 2.275       | 3.671          | 11.373 × 10⁻³ |
| S₁           | 5.218          | 2.720       | 3.261       | 4.426          | 7.934 × 10⁻³  |
| S₂           | 2.706          | 2.383       | 3.238       | 3.440          | 7.991 × 10⁻³  |
| S₃           | 3.254          | 2.358       | 3.092       | 3.512          | 8.368 × 10⁻³  |
| S₄           | 4.351          | 2.352       | 4.050       | 3.692          | 6.388 × 10⁻³  |

The indirect optical band gap values (as seen in Table 5) are observed to change nonlinearly upon increasing the V₂O₅. The nonlinear change in these values may be attributed to the effect of V₂O₅ concentration; which can alter and enhance the formation of NBOs in the glass network.

4.9 Predicted Nonlinear Optical Parameters

In order to evaluate the non-linear part of the refractive index, \((n_2)\), the following empirical formula [29,31] is applied:

\[
n_2(\text{e}^{-13}\text{esu}) = \frac{68 (n_e - 1)(n_e^2 + 2)^2}{\nu_d 1.517 + \frac{(n_e - 1)(n_e^2 + 2)}{6 n_e} \nu_d} \left[ \frac{1}{\nu_d} \right]^{1/2}
\]

(24)

where \((n_r)\) is the refractive index at \((\lambda_e) = 546.1\text{nm}\) and \((\nu_d)\) is the Abbe number for the studied samples. From Boiling’s semi-empirical equation [31,41], the 3rd order susceptibility \((\chi^3)\) of the nonlinear part index of refraction, is written as

\[
\chi^3(\text{e}^{-13}\text{esu}) = \frac{n_{\infty} n_2}{12 \pi}
\]

(25)

Moreover, the non-linear refractive index coefficient \((\gamma)\) is given by

\[
\gamma(\text{cm}^2/\text{W}) = \frac{4 \pi × 10^7}{c n_e n_2(\text{esu})}
\]

(26)

where \((c)\) is the velocity of light.
Table 6. The predicted nonlinear optical parameters.

| Glass Samples | $n_2 (e^{-13} \text{esu})$ | $\chi^3 (e^{-13} \text{esu})$ | $\gamma (e^{-13} \text{esu})$ |
|---------------|--------------------------|-------------------------------|-----------------------------|
| $S_0$         | 46.984                   | 2.241                         | 14.128                      |
| $S_1$         | 7.581                    | 0.249                         | 2.804                       |
| $S_2$         | 54.601                   | 1.966                         | 15.365                      |
| $S_3$         | 48.718                   | 1.758                         | 14.000                      |
| $S_4$         | 11.097                   | 0.388                         | 3.356                       |

The studied samples show random change in the values of the non-linear parameters. Then as the concentration of $V_2O_5$ increases, the values of the non-linear parameters change randomly. The calculated accuracy of the non-linear optical parameters is ±0.001.

5. Conclusions
Vanadium-Lithium-Phosphate [V$_2$O$_5$-Li$_2$O-P$_2$O$_5$] glass samples were prepared by the technique of melt-quenching. The following deductions are evoked from this study:
1. The amorphous nature of these glasses was verified through XRD examination at room temperature.
2. The density of samples increased from $2.373 - 2.391 \text{ g/cm}^3$ as the $V_2O_5$ content was increased.
3. Upon doping with various $V_2O_5$ concentrations, linear and non-linear optical parameters indicated an anomalous behavior of the samples. Thus dispersion parameters can be controlled by the doping concentration of $V_2O_5$.
4. The glass sample ($S_1$), doped with (0.2% $V_2O_5$), exhibits reduced values of (E$_d$) and ($n_\infty$).
5. In addition, glass sample ($S_1$) shows lower values of: reflection loss, mean dispersion, dispersive power of a glass material, molar refractivity and polarizability.
6. For glass sample ($S_2$), similarly doped with (0.4% $V_2O_5$), lower values of (E$_d$), indicate that the covalent bonding nature decreases with increasing the concentration of $V_2O_5$.
7. Moreover, glass sample ($S_1$), doped with (0.2% $V_2O_5$), displays lower values for: non-linear optical parameters , the 3rd order susceptibility and the non-linear refractive index coefficient.
The above deduced inferences find contemporary applications based on the phenomena of non-linearity in modified glass matrices, namely; laser glass with low nonlinear refractive index as well as optical switching with high refractive index.

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