Quantum electrodynamics for vector mesons

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Quantum electrodynamics for \( \rho \) mesons is considered. It is shown that, at tree level, the value of the gyromagnetic ratio of the \( \rho^+ \) is fixed to 2 in a self-consistent effective quantum field theory. Further, the mixing parameter of the photon and the neutral vector meson is equal to the ratio of electromagnetic and strong couplings, leading to the mass difference \( M_{\rho^0} - M_{\rho^\pm} \sim 1 \) MeV at tree order.

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The question of the intrinsic magnetic moment of (elementary) particles of arbitrary spin \( s \) has been discussed controversially in the literature and is still of great interest. On the one hand, low-energy theorems and the optical theorem require that the gyromagnetic ratio \( g \approx 2 \) for a particle with arbitrary spin \( s \) different from zero (at least for particles which do not participate in strong interactions) 1. On the other hand, general arguments have been given that the minimal coupling leads to \( 1/s \) for this quantity 2. Finally, the investigations of Ref. 3 regarding the theory of charged vector mesons interacting with the electromagnetic field suggested that the gyromagnetic ratio depends on a free parameter, thus allowing it to take any value. Below, we will address this question from the point of view of effective field theory (EFT).

In Ref. 4 we have shown how the universal coupling of the \( \rho \) meson and the Kawarabayashi-Suzuki-Riadzuddin-Fayyazuddin (KSRF) relation 5, 6 follow from the requirement that chiral perturbation theory of pions, nucleons, and \( \rho \) mesons is a consistent EFT. Although EFT’s are non-renormalizable in the traditional sense, the general principles of EFT 4 require that all ultraviolet divergences can be absorbed into the redefinition of fields and parameters of the most general Lagrangian 5. Imposing the renormalizability in this sense one finds that not all parameters of the most general Lagrangian are free but satisfy consistency conditions 7. In this Letter we will use similar arguments for the effective Lagrangian including, in addition, the interaction with photons to show that the gyromagnetic ratio is fixed to \( g = 2 \) at tree level. Furthermore, the mixing parameter of the photon and the neutral vector meson is also fixed and leads to \( M_{\rho^0} - M_{\rho^\pm} \sim 1 \) MeV at tree order.

We start with the chirally invariant effective Lagrangian including vector mesons in the form given by Weinberg 10, containing all interaction terms which respect Lorentz invariance, the discrete symmetries, and chiral symmetry. The electromagnetic interaction is introduced by adding all terms with photon fields which are allowed by U(1) gauge invariance,

\[
\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{M_0^2}{2} V_\mu V^{\mu} + \frac{c_0}{2} B^{\mu\nu} V^3_{\mu\nu} + \frac{\kappa_0}{2} \epsilon^{3ab} B^{\mu\nu} V^b_{\mu} V^a_{\nu} \\
+i\bar{\Psi} \gamma^\mu \left( \partial_\mu + ie_0 \frac{1 + \gamma^3}{2} B_\mu \right) \Psi \\
-m_0 \bar{\Psi} \gamma^\mu \left( g_0 \bar{\Psi} \gamma^\mu \frac{\tau^a}{2} \Psi V^a_{\mu} + \mathcal{L}_1 \right). \tag{1}
\]

Here, \( B_\mu \) is a U(1) gauge vector field, \( V^a_\mu \) (\( a = 1, 2, 3 \)) denote the Cartesian components of an isospin triplet of vector fields, and \( \Psi \) is an isospin doublet of nucleon fields with mass \( m_0 \) 11. Furthermore,

\[
B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \\
V^3_{\mu\nu} = \partial_\nu V_\mu - \partial_\mu V_\nu, \\
G_{\mu\nu} = V^a_{\mu\nu} + \epsilon^{abc} V^b_\mu V^c_\nu + e_0 \epsilon^{3ab} (B_\mu V^b_\nu - B_\nu V^b_\mu).
\]

All fields and coupling constants in Eq. (1) are bare quantities. From the point of view of EFT it is not consistent to consider a minimal coupling only (see, e.g., Ref. 12). Using symmetry arguments only, \( c_0 \) and \( \kappa_0 \) are free parameters of the most general effective Lagrangian 12. They contribute to the mixing of the photon and the neutral vector meson, and to the magnetic (and quadrupole) moment of the charged vector mesons 5, respectively. Finally, \( \mathcal{L}_1 \) contains an infinite number of interaction terms which are allowed by symmetries 5, 6.

Below we perform a one-loop order analysis of the \( \rho \)-meson self-energy and the \( \rho \bar{\psi}\psi \) vertex functions. To that end, we first introduce renormalized fields \( A_\mu, \rho^\mu_0, \rho^\pm_\mu \), and \( \psi \) as

\[
B_\mu = \sqrt{Z_A} A_\mu + \delta \lambda \rho^0_\mu, \\
V^3_\mu = \sqrt{Z_0} \rho^0_\mu, \\
V^1_\mu + i V^2_\mu = \sqrt{Z} \rho^\pm_\mu, \\
\Psi = \sqrt{Z} \bar{\psi} \psi, \tag{2}
\]

20 May 2005
with wave-function renormalization constants
\[
Z_A = 1 + \delta Z_A, \\
Z_0 = 1 + \delta Z_0, \\
Z_{\pm} = 1 + \delta Z_{\pm}, \\
Z_\Psi = 1 + \delta Z_\Psi.
\] (3)
The function $\delta \lambda$ allows for a linear superposition of the neutral vector fields. Note that the wave-function renormalization constants for the neutral $\rho$ meson, $Z_0$, and for the charged $\rho$ mesons, $Z_{\pm}$, differ from each other. Finally, $\sqrt{Z_\Psi}$ is a real diagonal $2 \times 2$ matrix. For the renormalization of the masses and coupling constants we write
\[
\begin{align*}
  c_0 &= c + \delta c, \\
  g_0 &= g_s + \delta g, \\
  e_0 &= e + \delta e, \\
  \kappa_0 &= \kappa + \delta \kappa, \\
  M_0^2 &= M^2 + \delta M \rho_0^\nu \\
  m_0 &= m + \delta m,
\end{align*}
\] (4)
where the functions $\delta c$ etc. depend on all renormalized coupling constants (and the renormalization prescription). Using Eqs. (2) - (4) we rewrite the Lagrangian of Eq. (1) as the sum of the basic Lagrangian, the counterterm Lagrangian, and a remainder [14]:
\[
\mathcal{L} = \mathcal{L}_{\text{basic}} + \mathcal{L}_{\text{ct}} + \mathcal{L}_1.
\]
The basic Lagrangian is given by
\[
\mathcal{L}_{\text{basic}} = \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{4} \rho_{\mu \nu} \rho_{\nu \mu} + M^2 \rho_0^\nu \rho_0^\mu + \frac{c}{2} F^{\mu \nu} \rho_{\mu \nu} - \frac{1}{2} \rho_{\mu \nu} \rho_{-\nu \mu} + M^2 \rho_0^\nu \rho_0^\mu - i \kappa F^{\mu \nu} \rho_{\mu}^\nu \rho_{\nu}^\mu + g_s \bar{\psi} \gamma_{\mu} \left( \partial_{\mu} + i \frac{1 + \gamma^0}{2} A_{\mu} \right) \psi - m \bar{\psi} \psi + g_s \bar{\psi} \gamma_{\mu} \tau_{\alpha} \frac{\gamma_\alpha}{2} \psi \rho_{\mu}^0 + \cdots,
\] (5)
with the field-strength tensors
\[
\begin{align*}
  F_{\mu \nu} &= \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}, \\
  \rho_{\mu \nu} &= \rho_{0}^\nu \rho_{0}^\mu + M^2 \rho_0^\nu \rho_0^\mu + \frac{c}{2} F^{\mu \nu} \rho_{0}^\nu \rho_{0}^\mu + \frac{1}{2} \rho_{\mu \nu} \rho_{-\nu \mu} + M^2 \rho_0^\nu \rho_0^\mu - i \kappa F^{\mu \nu} \rho_{\mu}^\nu \rho_{\nu}^\mu + g_s \bar{\psi} \gamma_{\mu} \left( \partial_{\mu} + i \frac{1 + \gamma^0}{2} A_{\mu} \right) \psi - m \bar{\psi} \psi + g_s \bar{\psi} \gamma_{\mu} \tau_{\alpha} \frac{\gamma_\alpha}{2} \psi \rho_{\mu}^0 + \cdots,
\end{align*}
\]
where we only display those counterterms explicitly which are relevant for the subsequent discussion. All remaining terms are included in $\mathcal{L}_1$.

In order to establish relations among the renormalized coupling constants pertaining to the Lagrangian of Eq. (5), we analyze the renormalization of the self-energy of the charged vector mesons and of the coupling constant $g_0$ using dimensional regularization (with $n$ as a spacetime dimension parameter) in combination with the minimal subtraction (MS) scheme (for a definition see, e.g., Ref. [14]). However, our findings below do not depend on the choice of a specific renormalization scheme.

First we analyze the divergent parts of the self-energy of the charged vector mesons to one-loop order (see Fig. 1). In particular, we are interested in terms quadratic in the momenta. These terms consist of counterterm contributions,
\[
\Pi_{\text{ct}}^{\mu \nu} = \delta \Pi_{\pm} (p^2 g^{\mu \nu} - p^\mu p^\nu),
\] (7)
and of contributions of one-loop diagrams,
\[
\Pi_{\text{1 loop}}^{\mu \nu} = A p^2 g^{\mu \nu} - B p^\mu p^\nu,
\] (8)
where $A$ and $B$ depend on the renormalized parameters of the Lagrangian [15]. Perturbative renormalizability requires that the expressions in Eqs. (7) and (8) cancel each other. This condition implies that
\[
A = B.
\] (9)
Equation (9) is not automatically satisfied and imposes constraints on the renormalized parameters of the Lagrangian.

Next we analyze one-loop contributions to the $\rho^+ \bar{\psi} \psi$ and $\rho^0 \bar{\psi} \psi$ vertices. These vertex functions (amputated Green’s functions) receive contributions from counterterms as well as one-loop diagrams. We again require that the divergent parts have to cancel each other. From this condition we determine the counterterm $\delta g$ as a function of the parameters of the Lagrangian. As this counterterm contributes in both vertex functions we obtain
\[
\begin{align*}
  \mathcal{L}_{\text{ct}} &= - \frac{\delta Z_0}{4} \rho_{\mu \nu} \rho_{\nu \mu} + \delta M^2 \rho_0^\nu \rho_0^\mu + \frac{\delta M^2 + \delta Z_0}{2} \rho_0^\nu \rho_0^\mu + \cdots.
\end{align*}
\]
two expressions,
\[ \delta g = \frac{1}{n - 4} \phi_1(c, \kappa, m, M, c, g_s) \]  
and
\[ \delta g = \frac{1}{n - 4} \phi_2(c, \kappa, m, M, c, g_s). \]  

The particular forms of the functions \(\phi_1\) and \(\phi_2\) have been determined by calculating the loop diagrams of Fig. 2. Equations (10) and (11) result in a condition for the renormalized parameters of the Lagrangian
\[ \phi_1(c, \kappa, m, M, c, g_s) - \phi_2(c, \kappa, m, M, c, g_s) = 0, \]  
which is not automatically satisfied. Solving Eqs. (10) and (11) simultaneously, we obtain
\[ c = e/g_s, \quad \kappa = e. \]  

After taking the different normalization of Ref. [15] into account, the second relation of Eq. (13) agrees with Ref. [17]. Modeling the electromagnetic coupling of the \(\rho\) using vector meson dominance the same result has been obtained in Ref. [17] by arguing that the electromagnetic self-mass should be finite.

Since the magnetic moment of, say, the positively charged \(\rho\) meson reads
\[ \mu = \frac{e}{2M_{\rho+}} \left(1 + \frac{\kappa}{c}\right) \bar{s}, \]
the second equality in Eq. (13) leads to the gyromagnetic ratio \(g = 2\) [13, 14]. Note that in the present case minimal coupling, as considered in Ref. [2], would correspond to \(\kappa = 0\) yielding \(g = 1/s = 1\) rather than \(g = 2\) in agreement with Ref. [1]. In Ref. [1], \(g = 2\) was obtained by considering a dispersion relation for forward Compton scattering, assuming that the spin-dependent amplitude \(f_-(\omega^{-2})\) vanishes at infinity. Renormalizability is a matter of asymptotic behavior at infinite momentum, so it may not be surprising that the renormalizability (in the sense of EFT) is achieved for \(g = 2\). However, we are not able to establish a connection between perturbative renormalizability of the low-energy EFT and the (high-energy) asymptotic behavior of the full amplitude.

Our derivation of Eq. (13) and therefore the value \(g = 2\) hold independently whether or not pion degrees of freedom are included explicitly. As the pionless effective theory can be obtained by integrating out the pion fields, the matching condition together with Eq. (13) requires that the pion loop corrections to the gyromagnetic ratio vanish. We emphasize that this argument only holds if there exist self-consistent effective field theories both with and without explicit pion degrees of freedom.

Calculating the propagator poles at tree level and using Eq. (13), we find
\[ M_{\rho+}^2 = M_{\rho0}^2 \left(1 - \frac{e^2}{g_s^2}\right). \]

Substituting numerical values for \(g_s\) [estimated from the KSRF relation \( \delta \), \( g_s^2 = M_{\rho0}^2/(2F_\pi^2) \), with \(F_\pi = 92.4\) MeV and \(M_{\rho0} = 769\) MeV] and \(e\) we obtain as a prediction for the mass difference
\[ M_{\rho0} - M_{\rho+} \approx 1\) MeV \(\) which has to be compared with the present PDG average of \((0.7 \pm 0.7)\) MeV [22].

To summarize, we have considered quantum electrodynamics of \(\rho\) mesons as an effective field theory described by the most general Lagrangian consistent with all symmetries of the theory. The self-consistency condition of this theory (imposed by the renormalization procedure) constrains the parameters of the Lagrangian. In particular, instead of depending on a free parameter, the gyromagnetic ratio of the charged vector meson is \(g = 2\) (up to loop corrections). The mixing parameter of the photon and the neutral vector meson is equal to the ratio of electromagnetic and strong coupling constants, leading to \(M_{\rho0} - M_{\rho+} \approx 1\) MeV at tree order.

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