The thermal Sunyaev-Zel’dovich (tSZ) effect directly measures the thermal pressure of free electrons integrated along the line of sight and thus contains valuable information on the thermal history of the universe. However, the redshift information is entangled in the projection along the line of sight. This projection effect severely degrades the power of the tSZ effect to reconstruct the thermal history. However, this otherwise lost redshift information can be recovered by the tSZ tomography technique, namely by cross correlating the tSZ effect with galaxies of known redshifts, or alternatively with dark matter distribution reconstructed from lensing tomography. For this purpose, we investigate in detail the 3D distribution of the gas thermal pressure and its relation with the matter distribution. We compare our adiabatic hydrodynamic simulation with the one including radiative cooling and star formation with supernova (SN) feedback. We confirm that these additional processes of gasphysics suppress the pressure power spectrum and thus the tSZ angular power spectrum significantly. On the other hand, they only affect the 3D pressure-matter correlation coefficient $r(k)$ at $\sim 1 - 2\%$ level. Since $r(k)$ is a key input in the tSZ tomography, this finding makes the tSZ tomography robust against uncertainties in the understanding of cooling and SN feedback.

Subject headings: cosmic microwave background - large-scale structure of universe - galaxies: clusters: general - methods: numerical

1. INTRODUCTION

Free electrons in the universe reveal their existence in the CMB sky through their inverse Compton scattering of CMB photons. The induced secondary CMB temperature anisotropies, proportional to the electron thermal energy integrated along the line of sight, are the well known thermal Sunyaev-Zel’dovich effect (Sunyaev & Zeldovich 1972, 1980). Since massive galaxy clusters contain large reservoirs of hot electrons, the generated tSZ effect can thus overwhelm the primary CMB around cluster scales. For this reason, the thermal SZ effect of dozens of galaxy clusters has been measured by various experiments (refer to Carlstrom et al. 2002; Reese et al. 2002; Jones et al. 2005; LaRoque et al. 2006; Bonamente et al. 2006 for reviews). On the other hand, blindly detecting the tSZ effect in random directions of sky is much more difficult, since the expected signal is overwhelmed by the primary CMB fluctuations. Currently there are only a few tentative detections through the observed small scale CMB power excess (Dawson et al. 2002, 2006; Mason et al. 2003; Goldstein et al. 2003; Runyan et al. 2003; Kuo et al. 2004; Readhead et al. 2004; Bond et al. 2005; Reichardt et al. 2008). However, the situation will be significantly improved in the next few years by ongoing and proposed ground surveys such as SZA, ACT, APEX, SPUD and the Planck satellite.

Precision mapping of the SZ sky is of great importance to both cosmology and astrophysics. The SZ effect is a powerful finder of galaxy clusters at high redshifts. The efficiency of free electrons to generate the thermal SZ effect is redshift independent. Photons originated from redshift $z$ suffer a factor of $1 + z$ energy loss caused by the cosmic expansion. On the other hand, CMB photons that electrons scatter at redshift $z$ are a factor of $1 + z$ more energetic than CMB photons today. These two effects cancel out exactly and enables galaxy clusters to be detected at high redshift without extra effort, opposite to the X-ray cluster finding. An exciting advance in this area is the recent discovery of 3 new galaxy clusters through the tSZ effect by the SPT group (Staniszewski et al. 2008).

The tSZ effect is also a powerful probe to the thermal history of the universe, since it directly probes the thermal energy of intergalactic medium (IGM) and intracluster medium (ICM). Numerical simulations show that, the amplitude of the SZ signal is sensitive to the amount of radiative cooling and energy feedback (Springel et al. 2001; da Silva et al. 2001; White et al. 2002; Lin et al. 2004). However, it is not straightforward to extract information on these astrophysical processes from the tSZ measurements alone. First of all, research shows that there exist great degeneracies between different competing processes. Even worse, the tSZ effect only measures the electron thermal energy projected along the line of sight. The redshift information of these astrophysical processes is thus entangled in the projection.

Zhang & Pen (2001) proposed to recover the redshift information by cross correlating the tSZ effect with galaxies with at least photo-$z$ information. The idea is that galaxies in a given redshift bin should strongly correlate with the tSZ signal from the same redshift bin. A key link between the measured cross correlation and the gas pressure auto-correlation that we want to extract is the cross correlation coefficient $r$ between the thermal energy and the galaxy number density. Assuming a constant $r$, the time resolved thermal energy distribution can be reconstructed self consistently. This SZ tomography technique would be applicable in reality, since SZ surveys often have follow-ups of galaxy surveys. For example, the dark

\[ \text{Draft version April 9, 2010} \]
energy survey will cover the SPT sky and measure photometric redshifts of $\sim 10^8$ galaxies up to $z = 1.3$. The lensing tomography also helps to reconstruct the 3D matter distribution, which can also be correlated with the SZ map to make the SZ tomography.

In the current paper, we reformulate this SZ tomography technique and explore the possibility to improve its robustness. We no longer approximate $r$ as a constant. Rather, we rely on numerical simulations to quantify its scale and redshift dependence. We are able to show that, $r$ is insensitive to gas physics such as radiative cooling and supernova (SN) feedback. Namely, $r$ calculated from adiabatic hydrodynamic simulations should be sufficiently accurate, even with the presence of radiative cooling and SN feedback. We are then able to take this $r$ as input to perform the SZ tomography.

The paper is organized as follows. In §2 we introduce the tSZ effect and explain the SZ tomography technique. We then analyze our high precision hydrodynamic simulations to quantify the dependence of various SZ statistics on the extra gas physics in §3. Although most quantities are sensitive to these gas physical processes, we find that the cross correlation coefficient $r$ only weakly depends on them (§3.3). This feature allows us to use $r$ calculated from the adiabatic simulations as the input of the SZ tomography, despite the existence of complicated gas physics in the real universe. We discuss and make conclusion in §4.

2. THE THERMAL SZ EFFECT AND THE CMB TOMOGRAPHY

The tSZ effect induces a new source of CMB temperature fluctuations with the amplitude

$$\frac{\Delta T(\theta)}{T_{\text{CMB}}} = g(x)y(\theta).$$

(1)

Here, $\theta$ is the direction on the sky. $g(x)$ describes the spectral dependence. In the non-relativistic limit,

$$g(x) = \left(\frac{x^4 + 1}{x^4 - 1} - 4\right),$$

(2)

where $x \equiv h\nu/k_B T_{\text{CMB}} = \nu/56.84 \text{ GHz}$ and $\nu$ is the observed frequency of CMB photons. The Comptonization parameter $y(\theta)$ is

$$y = \frac{\sigma_T}{m_e c^2} \int ad\chi n_e k_B T_e,$$

(3)

where $n_e k_B T_e$ is the hot electron pressure. $\chi$, $n_e$, $k_B$, $T_e$, and $\sigma_T$ are the comoving diameter distance, number density of free electrons, the Boltzmann constant, electron temperature and the Thompson scattering cross section respectively.

The spectral dependence of the tSZ effect is unique. The tSZ effect shows as CMB temperature decrements at $\nu < 218 \text{ GHz}$ and as increments at $\nu > 218 \text{ GHz}$. The spectral function $g(x) \rightarrow -2$ at the Rayleigh-Jeans band $x \ll 1$ and $g(x) \rightarrow x - 4$ at $x \gg 1$. This unique spectral dependence allows a clean separation of the tSZ effect from other CMB components in multi-band CMB surveys.

The $y$ parameter contains key information on the thermal history of the universe. However, since it only measures the projected electron thermal energy along the line of sight, the redshift information is smeared by this projection effect. Our SZ tomography technique aims to recover the otherwise lost redshift information in the tSZ effect.

2.1. The SZ tomography

One of the most widely used statistical quantities of the tSZ effect is the angular power spectrum $C_l^{\text{tSZ}}$. Throughout this paper, unless specified, we will focus on the Rayleigh-Jeans limit $\Delta T/T = -2y$. Under the Limber’s approximation [Limber 1954], $C_l^{\text{tSZ}}$ is related to the 3D thermal pressure power spectrum $\Delta_p^2(k,z)$ by the following relation,

$$\frac{P_2}{2\pi} C_l^{\text{tSZ}} = \int_0^{\chi_{\text{CMB}}} \Delta_p^2(k = \frac{1}{\chi}, z) W(z)^2 \chi d\chi. $$

(4)

We have adopted the flat cosmology in the above expression. The weighting function is

$$W(z) = -2\sigma_T a \frac{n_e k_B T_e}{m_e c^2}.$$

(5)

The pressure power spectrum $\Delta_p^2(k, z)$ is that of the fractional thermal pressure fluctuations $\delta_T \equiv n_e k_B T_e / (n_e k_B T_e) - 1$. $W(z)$ tells us the overall thermal energy of the universe and the $\Delta_p^2$ tells us the clustering of the thermal energy.

Our SZ tomography technique aims to reconstruct the time resolved $\Delta_p^2(k, z) W(z)^2$. This quantity tells us the overall amplitude of the thermal energy and the clustering strength at redshift $z$. The tSZ effect is correlated with tracers of the large scale structure. Given the redshift information of these tracers, such as galaxies, we can manage to recover the redshift information of the tSZ effect. The original tomography is presented in the variation formalism [Zhang & Pen 2001], while in the current paper, we reformulate it in a more straightforward manner. The key idea of the SZ tomography is that, galaxies distributed in a certain redshift range correlate with the tSZ signals contributed by the IGM in the same redshift range. Given the photometric redshift information, we are able to split galaxies into different redshift bins. The galaxy number distribution in the $i$-th redshift bin is $n_i(z)$, which is related to the photo-z distribution $n_i(z')$ by

$$n_i(z) = \int_{z-\Delta z_i/2}^{z+\Delta z_i/2} p(z'|z)^n_i(z') dz'$$

(6)

Here, $p(z'|z)$ is the photo-z probability distribution function. Due to non-negligible photo-z error, the real galaxy distribution is wider than $\Delta z_i$. However, it can still be narrow, centered at $z$. We normalize $n_i$ such that $\int_0^\infty n_i(z) dz = 1$. The tSZ-galaxy cross power spectrum is then

$$\frac{P_2}{2\pi} C_l^{\text{tSZ}-g} = \int_0^{\infty} \Delta_p^2(k = \frac{1}{\chi}, z) W(z,n_i(z)) dz$$

(7)

Notice that the integral in Eq. (7) is over the entire redshift range. For a sufficiently narrow redshift bin, the galaxy distribution $n_i(z)$ peaks at $z$, and we can replace the integral with $\sigma^2_{\text{z}} + (\Delta z)^2/2$ where $\sigma^2_{\text{z}}$ is the photo-z error. For typical values $\sigma^2_{\text{z}} = 0.05(1+z)$ and $\Delta z = 0.2$, the effective r.m.s. width is $\sim 0.1-0.2$. Over this redshift width, the functions $\Delta_p^2, W$

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6 http://www.darkenergysurvey.org/

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7 This relation assumes no catastrophic error in photo-z measurement. With the existence of catastrophic error, the peak redshift can be shifted by outstanding outliers.
and $\chi$ vary slowly. We thus have an approximation

$$\frac{I^2}{2\pi} C^{\text{SZ}-g}_{i} \simeq \frac{\Delta^2_{p_{k}}(k = \frac{l}{\chi_i}, z_i)}{\Delta^2_{r}(k, z)} W_{\text{SZ}}(z_i) \chi_i \int^\infty_0 n_i(z)dz$$

$$= \frac{\Delta^2_{p_{k}}(k = \frac{l}{\chi_i}, z_i)}{\chi_i} W_{\text{SZ}}(z_i) \chi_i .$$

(8)

Here, $\chi_i \equiv \chi(z_i)$. It is clear from the above equation that the cross correlation between the thermal SZ maps with galaxy distribution picks up and, to a good approximation, only picks up relevant information within the given redshift bin. This is a key step to recover the redshift information of the thermal SZ effect.

For similar argument, the galaxy angular power spectrum is

$$\frac{I^2}{2\pi} C^{g}_{i} = \int^\infty_0 \Delta^2_{r}(k = \frac{l}{\chi_i}, z) n_i(z) \frac{dz}{d\chi} dz$$

$$\simeq \Delta^2_{r}(k = \frac{l}{\chi_i}, z_i) \chi_i \int^\infty_0 n_i(z) \frac{dz}{d\chi} dz .$$

(9)

The next step is to recover the thermal SZ contribution in the given redshift bin with the correct weighting, namely, $\Delta^2_{p_{k}}(k, z) W_{\text{SZ}}^2(z)$. This step requires a key input, namely the cross correlation coefficient $r$ between fluctuations in the gas pressure and the galaxy distribution, defined by

$$r(k, z) = \frac{\Delta^2_{p_{k}}(k, z) W_{\text{SZ}}^2(z)}{[\Delta^2_{r}(k) W_{\text{SZ}}(z)] \Delta^2_{g}(k, z)} .$$

(10)

Combining Eq. 8 and Eq. 9 we obtain

$$\Delta^2_{p_{k}}(k, z) W_{\text{SZ}}^2(z) = r^2(k, z) \frac{[\Delta^2_{p_{k}}(k, z) W_{\text{SZ}}^2]}{\Delta^2_{g}(k, z)} .$$

(11)

In this relation, the angular power spectra $C^{\text{SZ}-g}_{i}$ and $C^{g}_{i}$ are evaluated at $l = k \chi_i$. Both the angular power spectra can be measured directly by combining the SZ surveys and the galaxy surveys. The quantity $n_i(z)$ is given by the galaxy surveys, and $\chi$ can be calculated given the cosmology model. As long as $r(k, z)$ can be figured out, one can apply this equation to do the thermal SZ tomography.

We summarize the thermal SZ tomography as follows.

- Measure the cross power spectrum $C^{\text{SZ}-g}_{i}$ between the thermal SZ effect and the galaxy distribution in a given redshift bin.
- Measure the galaxy power spectrum $C^g_{i}$ in this redshift bin.
- Reconstruct the thermal SZ contribution from this redshift bin through Eq. 11.

A key input required in this SZ tomography is the quantity $r$. Zhang & Pen (2001) adopted a simplification that $r$ is a constant. They further showed that, the value of $r$ can be measured combining Eq. 11 and the measured SZ power spectrum. This approach is self consistent and able to provide a quick realization of the SZ tomography. However, improvements must be made to do precision SZ tomography. As shown in the assumption that $r$ is a constant is only accurate at the level of $\sim 20\%$. Furthermore, to obtain the value of $r$ unbiasedly from the SZ power spectrum, the galaxy surveys must well cover the SZ redshift range ($z \leq 2$). This is challenging for galaxy surveys.

Since $r(k, z)$ is such a key quantity in the SZ tomography, a natural question arises: *Can one robustly predict $r$?* This seems challenging, given the fact that various complicated gastrophysical processes affect the tSZ effect. Surprisingly, robust prediction on $r$ is likely feasible. The key point is that, these gastrophysics affects $\Delta^2_{p_{k}}$ and $\Delta^2_{g}$ in basically the same way, so that these effects roughly cancel out in $r$ by definition. To quantify the dependence of $r$ on these gastrophysics, we compare two sets of hydrodynamic simulations with and without radiative cooling, star formation and SN feedback. These simulations confirm the above naive speculation and find that, these additional gastrophysics can only affect $r$ at 1% level, despite the fact that they alter the SZ power spectrum by $\sim 40\%$.

3. SIMULATIONS

The tSZ statistics has been studied by both semi-analytical models (Cooray 2000; Zhang & Pen 2001; Komatsu & Seljak 2002; Zhang & Wu 2003; Zhang et al. 2004; Zhang & Sheth 2007) and numerical simulations (Persi et al. 1995; Refregier et al. 2000; Refregier & Teyssier 2002; da Silva et al. 2000, 2001; Seljak et al. 2001; Springel et al. 2001; White et al. 2002, 2004; Lin et al. 2004; Zhang et al. 2004; Roncarelli et al. 2004; Hallman et al. 2007, 2009). Beyond the gravitational heating mechanism, some works (da Silva et al. 2001; Springel et al. 2001; White et al. 2002; Lin et al. 2004; Roncarelli et al. 2007; Scannapieco et al. 2008) have incorporated additional gastrophysics such as radiative cooling, preheating, SN/AGN feedback. These studies found that these processes suppress the small scale SZ power spectrum. But none of them addressed the dependence of the cross correlation coefficient $r$ on these processes, whose investigation is the key goal of this paper.

In this paper, we analyze a controlled set of hydrodynamic simulations for the relevant SZ statistics. The cosmology adopted is a ΛCDM cosmology: $\Omega_M = 0.732, \Omega_b = 0.268, \Omega_c = 0.04448, h = 0.71, \sigma_8 = 0.85$. The simulations are run by GADGET2 code (Springel 2005), with 100 $h^{-1}$ Mpc box size, and 512$^3$ particles for both dark matter and SPH particles. The masses of the dark matter and SPH particles are 4.62 x $10^7$ $h^{-1}$ M$_{\odot}$ and 9.20 x $10^7$ $h^{-1}$ M$_{\odot}$ respectively. One simulation only includes gravitational heating and the other also includes radiative cooling and star formation with SN feedback. We refer these two as adiabatic and non-adiabatic run, respectively. For the non-adiabatic run, 7.63% of gas particles turn into stars by $z = 0$, resulting in stellar mass density $\Omega_* = 0.0034$. Both simulations start from redshift $z = 120$. We output 60 snapshots, which are equally spaced in $\ln a$. Refer to Lin et al. (2006), Jing et al. (2006) for more details of the simulations.

We derive the temperature $T_i$ of the $i$-th gas particle as follows

$$k_B T_i = (\gamma - 1) \mu m_p u_i ,$$

(12)

with $\mu = 0.588$ the mean molecule weight, $\gamma = 5/3$ the spe-
cific heat ratio for monatomic gas, and $u_i$ the internal energy per unit mass of the $i$-th particle. When the temperature of a SPH particle is higher than 10000 K, we consider it as ionized, otherwise as neutral. We sum up all the SPH particles to calculate the density-weighted temperature $T_\rho$ at all the available snapshots. From these measurements, we are then able to calculate the $y$ parameter.

We use the fast Fourier transform (FFT) to calculate the power spectra of dark matter, gas pressure and the cross power spectra, which is defined as $\int u(T)\ln T = \mathcal{U}$, where $\mathcal{U}$ is the total internal energy. The additional processes brings the total internal energy down by 12.0%, 10.9%, 11.5% and 16.6% at $z=0, 0.52, 1.02$ and 2.08 respectively. The same arbitrary unit is adopted for both the simulations. For both the adiabatic (solid lines) and non-adiabatic (dashed lines) simulation, the most contributions come from the gas at temperature around 1–2 keV. And the lower the redshift, the more contributions from higher temperature components.

Here $T_\rho = \langle \bar{n}_e T/\bar{n}_e \rangle$ is the density weighted temperature, which is computed by the following equivalent expression

$$T_\rho = \frac{\sum_i T_m_i}{\sum_i m_i} .$$

The sum is over all gas particles, assuming gas with $T_i \geq 10000$ K as ionized, otherwise neutral.

Figure 1 shows the redshift evolution of the density weighted temperature for the adiabatic (solid lines) and the non-adiabatic (dashed lines) simulation. As shown, the additional processes suppress the mass weighted temperature $T_\rho$, which confirms the earlier results by White et al. (2002), though more evident than theirs at $z=0$. We further look into the effect of cooling and feedback in redistributing the energy budget into regions with different temperatures. The logarithmic contribution to the total thermal energy from regions with different temperatures is shown in Fig. 2. We can see clearly that, for the 4 presented redshifts, the most contributions of the total energy come from the regions with $T \sim 1–2$ keV at lower redshifts except $z=2.08$, for both adiabatic and non-adiabatic simulations. The figure also shows that gas with temperature higher than $\sim 2$ keV is slightly affected by these additional gasphysics. Most of these gas lies in clusters, which have deep gravitational potential wells to offset the feedback effect. Also, the star formation rate is relatively low in cluster regions, resulting in less feedback intensity and also less depletion of hot gas by cooling. On the other hand, gas cooler than $\sim 1$ keV has opposite fate. Most of these gas lies in galaxy groups or in IGM, susceptible to cooling and feedback. This suppresses their contribution to the total energy. The combined effect is that, the total thermal energy is suppressed by a factor of 12.0%, 10.9%, 11.5% and
We thank Volker Springel for providing the result of an independent GADGET2 simulation, which allows us to do a preliminary check. The cosmological parameters of this simulation are identical to ours, except the adiabatic one (solid lines). However, the powers at large scales are slightly boosted. **Bottom panel:** The differences between the power spectra defined by \( \Delta P_T(k)/\Delta P_A(k) = \Delta P_{TA}(k) - \Delta P_{AD}(k)/\Delta P_{TA}(k) \)

16.6% at \( z = 0, 0.52, 1.02 \) and 2.08 respectively.

Given \( T_p \) measured at the 60 simulation snapshots, we are able to integrate it over the whole relevant redshift range to calculate \( y \), according to Eq. 13. We adopt a simple linear, while stable, interpolation to model \( T_p \) at redshift falling between two adjacent output redshifts. For both cases, about a half of the \( y \) signal comes from \( z < 1 \). We obtain \( y = 2.07 \times 10^{-6} \) for the adiabatic run and \( y = 1.77 \times 10^{-6} \) for the non-adiabatic run. These values of \( y \) are lower than the results in e.g. (da Silva et al. 2001; White et al. 2002; Zhang et al. 2004), so as the result of \( T_p \). On the other hand, it is higher than some recent simulation results, such as that of (Roncarelli et al. 2007) using GADGET2. Differences in cosmological parameters, such as difference in \( \sigma_8 \) and \( \Omega_b \), can only account for part of the discrepancy, while cosmic variance, differences in the input astrophysics and numerical codes may be responsible for the remains. We postpone further investigation of this issue elsewhere. 9

3.2. The 3D gas pressure power spectrum and the tSZ power spectrum

Most of the simulated SZ power spectra in the literature are directly calculated from the simulated 2D SZ maps. In this paper we take an alternative approach, as adopted by Refregier et al. (2000); Refregier & Teyssier (2002). For each simulation output, we can calculate the 3D power spectrum of the gas thermal pressure \( \Delta T^2 \). The 2D SZ power spectrum is obtained through Eq. 4 and we interpolate linearly between adjacent snapshots to model \( \Delta T^2(k) \) at any other redshifts.

\[ \Delta T^2(k) \] at \( z = 0, 0.52, 1.02 \) and 2.08 are shown in Figure 3. At small scales, the pressure power spectra of the non-adiabatic simulation (dashed lines) fall below the corresponding adiabatic ones (solid lines). Most of tSZ signals at these scales are contributed by gas in less massive halos. These halos have shallower potential wells and thus SN feedback is easier to deplete gas. This suppression effect is even larger toward higher redshifts where the gravitational potential wells are shallower, leading to as high as 50% reduction at \( z = 2.08 \). The same feedback redistributes gas and moves the clustering power of the thermal energy from small scales to large scales. This is likely the reason that we see enhancement at large scales (Fig. 5). However, at these scales, massive halos contribute most of the signal. Since they have deeper potential wells to fight against feedback and confine gas, the large scale pressure power spectrum is less affected.

The gas pressure power spectrum is often expressed by the gas pressure bias with respect to the matter distribution, defined as \( b_P(k) = \sqrt{\Delta T^2(k)/\Delta \rho^2} \). The measured pressure bias is shown in Figure 4. We find in Figure 4 that the results are in good agreement with the simulation results at \( z = 0 \) by Refregier & Teyssier (2002).

The SZ power spectrum evaluated from the Limber integral is shown in Figure 5. In the existence of the cooling and self-regulated star formation, the tSZ angular power spectrum at large scales is suppressed by \( \sim 20\% \), which is mainly due to the reduction of density weighted temperature \( T_p \). At the same time, the powers are reduced by as large as around \( \sim 40\% \) at very small scales around \( \ell = 10000 \), which is jointly caused by the suppression of both the density weighted temperature and pressure power spectra compared to the adiabatic sim-
ulation. These suppression effects confirm the previous results by da Silva et al. (2001); White et al. (2002). And what’s more, the non-adiabatic power spectrum peaks on a slightly larger scale, about $l \sim 8000$, than the adiabatic one, which may arise from SN feedback that would drive away the gas out to diffuse regions. Springel et al. (2001) also found that energy injection would suppress the power on small scales and push the powers toward larger scales. Roncarelli et al. (2007) considered the same non-gravitational processes as our non-adiabatic one in their simulation, and their power spectrum behaves consistently with ours, despite of their low $y$.

3.3. The correlation coefficients $r(k, z)$

As addressed in [2], a key input of the SZ tomography is the gas pressure-galaxy number density cross correlation coefficient $r$. State of art numerical simulations with purely gravitational heating (adiabatic simulations) is able to model $r(k,z)$ robustly. If we can further quantify the dependence of $r(k,z)$ on additional gastrophysics such as radiative cooling and feedback, we will be able to model $r(k,z)$ robustly for general cases and perform the SZ tomography robustly.

For this purpose, we analyze the behavior of $r(k,z)$ in the two simulations. Since galaxies are tightly correlated with the matter distribution, and the stochasticity is small for the measurement of $r(k,z)$ between the gas pressure and galaxy number density, we are able to approximate galaxy distribution as matter distribution. Namely, what we actually measure is $r_{Pm}$, the cross correlation coefficient between the gas pressure and the matter density. To the first order approximation, these two should be identical. For example, for the often adopted constant galaxy bias model, the two are equal to each other. Even for scale dependent galaxy bias, as long as the stochasticity is negligible, the two should be equal to each other. In future works, we will populate dark matter halos with galaxies and measure the real gas pressure-galaxy cross correlation coefficient.

The result on $r(k,z)$ is shown in Figure 6. We find gas pressure tends to trace the matter distribution faithfully on the very large scales. At the largest scale that our simulation can approach, $k \sim 0.062 \ h \ Mpc^{-1}$, $r(k) \geq 0.9$ for all of the 4 redshifts. At $0.4 \leq k \leq 3 \ h \ Mpc^{-1}$, there’s still great correlation about $0.8 \leq k \leq 0.6$ at $z = 2.08$. Bottom panel: The difference of $r$ between the two simulations: $\Delta r(k)/r_A(k) = (r_A(k) - r_A(k))/r_A(k)$. If cooling and SN feedback are included, there’s only $\sim 1 - 2\%$ change in $r(k)$ at most scales, and only at the smallest scales does $r$ change $\sim 5\%$.

![Fig. 5. — The SZ angular power spectra $C^S_{\ell}$ for the two simulations. Radiative cooling and SN feedback suppress the SZ power spectrum by $\sim 20\%$ at large scales and around $\sim 40\%$ at small scales. The same legends are adopted as in Figure 1.](image1)

![Fig. 6. — Top panel: The cross correlation coefficient between the 3-D gas pressure and dark matter in adiabatic simulations (solid lines) and non-adiabatic simulations (dashed lines). For both the simulations, $r(k)$ is greater than 0.9 at $k \sim 0.062 \ h \ Mpc^{-1}$ for all the 4 redshifts, as gas pressure traces dark matter on the very large scales. At intermediate scales where $0.4 \leq k \leq 3 \ h \ Mpc^{-1}$, there’s still great correlation about $\sim 0.8$ at $z = 0$, while $r(k)$ drops by 25% to $\sim 0.6$ at $z = 2.08$. Bottom panel: The difference of $r$ between the two simulations: $\Delta r(k)/r_A(k) = (r_A(k) - r_A(k))/r_A(k)$. If cooling and SN feedback are included, there’s only $\sim 1 - 2\%$ change in $r(k)$ at most scales, and only at the smallest scales does $r$ change $\sim 5\%$.](image2)
the otherwise circular procedure to do the tSZ tomography. This feature assures us that we can rely on adiabatic simulations alone to provide $r$. However, it does not mean that the simulations presented in this paper have already robustly predicted $r$. We expect the function $r(k)$ to be sufficiently smooth with respect to $k$. However, since our box size is only $100h^{-1}$ Mpc, statistical fluctuations induced by cosmic variance are non-negligible and the simulated $r$-$k$ relation shows clear irregularities (Fig. 6). To beat down the cosmic variance, we need more simulations or larger box size. Nonetheless, we only need adiabatic simulations for this improvement.

3.4. The galaxy-matter relation

The $r$ measured above is actually between the gas pressure and the matter distribution. We have shown its robustness against various gastrophysical processes. However, in the proposed SZ tomography, what we really need is the one between the gas pressure and the galaxy distribution. At sufficiently large scale, the galaxy distribution with respect to the matter distribution is believed to be deterministic (e.g. [Bonoli & Pen 2009; Baldau et al. 2009]). This is a generic behavior in the standard model of structure formation and is unlikely to be changed by realistic gastrophysics. Thus at these scales, our result proves that $r$ between gas pressure and galaxies is also robust various gastrophysical processes. However, at small scales, stochasticities between galaxy and matter distribution develop and the cross correlation coefficient between galaxy and matter distribution $r_{mg} \neq 1$. In this case, we should also quantify the impact of various gastrophysical processes such as feedback and cooling on $r_{mg}$.

Unfortunately, since even the state of art simulations are not able to simulate galaxies from first principles, we are not able to robustly quantify the impact of these gastrophysical processes on $r_{mg}$. Instead, we will adopt the halo model and test $r_{mg}$ by varying parameters of the halo occupation distribution (HOD) to study possible impact of these gastrophysical processes. The technical details and results are presented in the appendix. We find that in some cases $r_{mg}$ barely changes and in other cases it can change by $\sim 10\%$. Due to uncertainties in the dependence of HOD parameters on these gastrophysical processes, we are not able to draw any conclusive observation. However, these results do warn us the possibility that these astrophysical processes could significantly change $r_{mg}$ and hence the cross correlation coefficient between the gas pressure and galaxy distribution, despite the fact that they have only minor impact on the cross correlation coefficient between the gas pressure and matter distribution.

The good news is that, even for this worst scenario, we are still able to carry out the thermal SZ tomography robustly, with the help of the lensing tomography. SZ surveys often overlap with lensing surveys. For example, the South Pole Telescope survey basically overlaps with the Dark Energy Survey (DES). Through the lensing tomography of surveys with redshift information, in principle we are able to reconstruct the 3D matter distribution from the nearby universe to $z \sim 2$ where the furthest source galaxies reside. Massey et al. (2007) have clearly demonstrated the feasibility of this technique. Given the capacity of stage III and IV lensing surveys, the 3D matter distribution can be reconstructed with much higher accuracy. We are then able to cross correlate the thermal SZ measurement with the reconstructed matter distribution in a redshift bin to perform the thermal SZ tomography. The thermal SZ tomography can then be carried out as follows.

- Reconstruct the 3D matter distribution from the lensing tomography.
- Measure the cross power spectrum $C^{tSZ-m}_{ll}$ between the thermal SZ effect and the matter distribution in a given redshift bin.
- Measure the matter angular power spectrum $C^{m}_{l}$ in this redshift bin.
- Reconstruct the thermal SZ contribution from this redshift bin through the following relation

$$\Delta^{2}_{l}(k,z)W_{SZ}^{2}(z) = r^{2}(k,z)\frac{\Delta^{2}_{l,m}(k,z)}{\Delta^{2}_{l}(k,z)}$$

$$\simeq r^{2}(k,z) \frac{\int_{0}^{\infty} n_{tSZ}(z)\frac{d\xi}{dz} dz}{2\pi C^{m}_{l}}.$$  

In this case the relevant cross correlation coefficient $r$ is the one between the gas pressure and the matter density, exactly the one that we study in this paper. We have demonstrated its robustness against various gastrophysical processes and hence the robustness of the thermal SZ tomography.

4. CONCLUSION AND DISCUSSION

In this paper, we investigate the feasibility of the tSZ tomography method, which aims to extract the redshift information of the tSZ effect. As explained in §2 future SZ and galaxy surveys will measure the galaxy angular power spectrum $C^{g}_{ll}$ in each redshift bin and the corresponding tSZ-galaxies cross power spectrum $C^{tSZ-g}_{ll}$. Given the cross correlation coefficient $r(k,z)$ between the gas thermal energy and the galaxy number density, we are able to reconstruct the time resolved thermal energy distribution. So the key question in the tSZ tomography is whether we can robustly predict $r(k,z)$, given uncertainties in gastrophysical processes. To quantify the effect of two dominant gastrophysical processes, namely radiative cooling and SN feedback, we compare the result of our adiabatic simulation against the one with cooling and feedback. We find that $r(k,z)$ is insensitive to these additional gastrophysics. The resulting difference in $r(k,z)$ is $\sim 1$--2% in most relevant $k$ and $z$ range, much smaller than the $\sim 30\%$ change in the SZ power spectrum and the gas density weighted temperature. This allows us to neglect the dependence of $r(k,z)$ on these gastrophysical processes and adopt the $r(k,z)$ evaluated from adiabatic simulations as the input of the tSZ tomography. We thus show that the tSZ tomography is feasible in reality.

There are a number of improvements which can be made in future work. (1) As addressed in the end of §3 We need to run more simulations to reduce the cosmic variance. We may also need to correct for the alias effect to improve the accuracy of $r$ at scales $k \gtrsim 4 \ h \text{Mpc}^{-1}$. (2) In this paper, to evaluate $r(k,z)$, we have approximated galaxy distribution as dark matter distribution for simplification. In the limit that the stochasticity vanishes, our approximation becomes exact. This should be the case at large scales. At small scales, non-vanishing

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10 If $r(k)$ strongly depends on the detailed gastrophysics, the tSZ tomography procedure will become circular. In this case, we need to know the detailed gastrophysics to reliably predict $r(k,z)$ from simulations and then apply back to observations, while we count on observations to tell us the detailed gastrophysics.
stochasticity could result in some systematical errors in the tSZ tomography. In the future work, we will populate halos in our simulations with galaxies and quantify the possible systematics induced by this simplification. (3) Another issue is the AGN feedback, which potentially has larger influence on the tSZ effect, although we do not expect it to significantly modify $r(k, z)$. We shall investigate it in the future and quantify its possible impact on the tSZ tomography.

There’s also a possible limitation that may degrade tSZ tomography method using galaxies surveys. In our non-adiabatic simulation, extra gas dynamics may influence the galaxy-matter relation, resulting in deviations and stochasticities in the correlation coefficient $r(k, z)$. Though we can’t figure out the exact influence of gas dynamics and galaxy selection function, given little knowledge of recipes of galaxy formation at present, we test the possible effects with a generic halo occupation distribution (HOD) model in the appendix. The results of this generic HOD model favor tSZ tomography methods using galaxy survey in some aspects, while disfavor it in other aspects. Thus we can’t pin down the influence of gas dynamics and selection function in galaxy surveys, and further investigations are needed. However, an optimistic news is that we DO demonstrate the robustness of the tSZ tomography, alternatively using tSZ survey with dark matter instead of galaxies, while we can measure the clustering of dark matter to high accuracy on scales of tSZ interest, using lensing tomography. We thus can expect a useful application of tSZ tomography in the future surveys of tSZ effect.

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APPENDIX

THE INFLUENCE OF EXTRA GASTROPHYSICS TO GALAXY-MATTER RELATION

We would like to address a crucial point here in the appendix. The tSZ tomography method employed in this paper is based on an assumption that $r_{mg}(k) = 1$ on relevant scales, i.e. galaxies are fully correlated with dark matter, and the correlation coefficient $r_{mg}$ is not sensitive to gasdynamics. This assumption assures that there’s a deterministic map between the galaxy and matter
distribution, and thus the correlation coefficient of pressure and galaxy is equal to that of pressure and dark matter. Galaxy survey is actually an agency which introduces dark matter distribution in the tSZ tomography method. Therefore, the uncertainties in galaxy clustering of different galaxy samples may introduce stochasticities in estimating the underlying dark matter clustering features. In our simulations, when we emerge extra gasdynamics, dark matter would be influenced little, while galaxies, on the other hand, suffer from these gasdynamical processes. Exact scheme of gasdynamics is needed to model the galaxy formation for both simulations and theories, while it’s beyond our knowledge so far. We thus have no idea how to incorporate detailed gas dynamics in both observations and simulations, given little knowledge of the underlying gasdynamics. Nevertheless, earlier studies of galaxy-matter correlation (Seljak 2000, Gazik & Seljak 2001) showed that $r$ is approximate to unity up to $k \sim 1 - 2 h$ Mpc$^{-1}$, by cross correlating dark matter with galaxy samples selected by different selection criteria. In an alternative point of view, if we can measure matter power spectrum, we can circumvent the galaxy-matter cross correlation while directly correlate dark matter and pressure power spectrum when doing tSZ tomography. Seliak & Warren (2004) proposed to use faint galaxies as dark matter tracer, as they found the linear bias of these galaxies is nearly constant. Pen (2004) suggested to obtain the 3D dark matter clustering using the cross correlation between galaxies with distance information and projected weak lensing dark matter maps. More recently, Baldauf et al. (2009) proposed that by combining galaxy-clustering and galaxy-galaxy lensing measurements, they can reconstruct the dark matter power spectrum at a few percent accuracy up to $k \sim 1$ Mpc$^{-1}$.

However, we would like to investigate this correlation coefficient $r_{mg}$ in a more general way, say, by a HOD model instead of looking into our two specific hydrodynamic simulations. Following the routine of halo model in Cooray & Sheth (2002), we can calculate the power spectrum of dark matter as well as of galaxies. Given the halo model and the halo occupation distribution model (HOD) for galaxies, we can predict the matter power spectrum $P_m(k)$ and galaxy power spectrum $P_g(k)$, as well as the cross power spectrum $P_{mg}(k)$. The cross correlation coefficient is thus simply derived by $r_{mg}(k) = P_{mg}(k)/\sqrt{P_m(k)P_g(k)}$. We won’t describe the details of how to calculate the power spectra, for which readers can refer to Cooray & Sheth (2002) for a review. However, we present in the following paragraphs how we populate galaxies in halos with a generic HOD model.

Though it’s hard for one to exactly predict the galaxy distribution either by theories or by simulations, detailed semi-analytic galaxy formation models provides one with an experienced way to describe the galaxy distribution in the existence of generic gasdynamics. Nevertheless, earlier studies of galaxy-matter correlation (Seljak 2000, Gazik & Seljak 2001) showed that $r$ is approximate to unity up to $k \sim 1 - 2 h$ Mpc$^{-1}$, by cross correlating dark matter with galaxy samples selected by different selection criteria. In an alternative point of view, if we can measure matter power spectrum, we can circumvent the galaxy-matter cross correlation while directly correlate dark matter and pressure power spectrum when doing tSZ tomography. Seliak & Warren (2004) proposed to use faint galaxies as dark matter tracer, as they found the linear bias of these galaxies is nearly constant. Pen (2004) suggested to obtain the 3D dark matter clustering using the cross correlation between galaxies with distance information and projected weak lensing dark matter maps. More recently, Baldauf et al. (2009) proposed that by combining galaxy-clustering and galaxy-galaxy lensing measurements, they can reconstruct the dark matter power spectrum at a few percent accuracy up to $k \sim 1$ Mpc$^{-1}$.

There will be great differences between the HOD parameters of galaxy samples selected by various selection criteria, e.g. luminosity, color, history and morphology. However, we take into account the selection criteria and extra gasdynamics by considering two generic cases, which are framed by changing the number density of galaxies and changing the morphology of galaxy distribution. More specifically in our HOD model, we introduce these two effects by varying the six parameters of HOD adopted here, i.e. $M_{min}, M_B, M_0, N_0, \alpha$ and the concentration of galaxies $c_g$. In practice, we calculate $r_{mg}$ by changing one of the parameters while fixing the others. In the existence of star formation and SN feedback, since it’s hard for gas to cool and condense into stars, we expect less galaxies in the non-adiabatic simulation than the adiabatic one. We set the parameters such that the average galaxy number is less than that in the fiducial HOD model. We thus raise each time one of the mass thresholds, $M_{min}, M_B$ and $M_0$ as twice as that in the fiducial model, while reduce $N_0$ by a half. Yang et al. (2005) found $c_g = 1/3c_m$ by investigating galaxy surveys, and Nagai & Kravtsov (2005) found similar results in hydrodynamical simulations. Inspired by these works, we set the galaxy concentration $c_g$ as 1/3 or 2/3 as that of dark matter $c_m$. For $\alpha$, we reduce it to 0.7 so as to reduce the average galaxy number to test the effect.

We show the result of the cross correlation coefficient $r_{mg}$ in Fig. 7. We find that reducing $N_0$ doesn’t change the $r$ at all, and changing $M_0$ only influence $r$ by around 2 percent, except a 5% drop on scales around $k \sim 5$ h Mpc$^{-1}$. A more extended galaxy distribution in halos, i.e. $c_g = 1/3c_m$ would induce a 10% suppression of $r_{mg}$ at $k \sim 7$ h Mpc$^{-1}$. However, the suppression is
Fig. 7.— The sensitivity of the dark matter-galaxy cross correlation coefficient $r_{mg}$ to different HOD parameters. We change one of the parameters once at a time while keeping others fixed during the calculation. $2M_{\rm min}$, $2M_B$ and reducing $\alpha$ to 0.7 greatly influence $r_{mg}$. However, it’s possible that they cancel each other partly and reduce the stochasticity. Encouragingly, changing $N_0$, $M_0$, $c_g$ would introduce several percent influence on $r_{mg}$ on scales of interest of tSZ effect.

at most 5% on scales of interest of tSZ, say, $k \lesssim 3 \ Mpc^{-1}$. $c_g = 2/3c_m$, as expected, only reduce $r_{mg}$ by at most 3% on small scales. Actually an amplitude of galaxy number $N_0$ won’t introduce any difference of galaxy clustering. While galaxy number is insensitive to $M_0$ or irrelevant to $c_g$, the one halo term of galaxy power spectrum, i.e. on small scales is indeed influenced.

The changes in $M_{\rm min}$, $M_B$ and $\alpha$, on the other hand, lead to great influence in $r_{mg}$. $2M_{\rm min}$, i.e. increasing the mass threshold of host halo of central galaxy, reduces the average galaxy number by nearly a half, from $1.93 \times 10^{-2}h^3\text{Mpc}^{-3}$ in the fiducial model to $9.93 \times 10^{-3}h^3\text{Mpc}^{-3}$, while $2M_B$ only changes that by 3%. Interestingly, they induce opposite effects in $r_{mg}$, both with amplitudes up to 15%-20% on scales of interest. Though $\alpha = 0.7$ reduces the galaxy number by less than 2%, it suppresses $r_{mg}$ by 10% around $k \sim 1 - 2 \ Mpc^{-1}$. Thus from this simple model, it’s possible that the stochasticity between dark matter and galaxies is large, and therefore the effectiveness of tSZ tomography would degrade. However, we hope that the change in $r_{mg}$ would not be so severe. Firstly, the parameters we adopt here are extreme, while in the non-adiabatic simulation, cooling and feedback influence mostly in the central regions of halos, and it would tend to influence small scales power spectrum below cluster scales. Secondly, they lead to opposite effects to $r_{mg}$, it’s thus possible that the effects partly cancel out each other. In earlier works, Guzik & Seljak (2001) showed that galaxy-dark matter correlation coefficients of different galaxy samples, which are selected by luminosity, colour and star formation rate, converge to unity on scales above $1 \ Mpc^{-1}$. Also, they found in the faintest and most abundant samples, $r$ is unity up to $k \sim 3 \ Mpc^{-1}$, which is also indicated by Seljak & Warren (2004) where they found small halos trace dark matter with constant bias. This is encouraging, as in practice, we can select faint galaxy sample to do tSZ tomography. After all, the stochasticity of galaxy-matter relation is an interesting issue, and need to be further investigated.