Rotational bands in $^{11}$B and identification of diluted states

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Abstract. Differential cross-sections of the $^{11}$B $+$ α inelastic scattering at $E(\alpha) = 65$ leading to the most of the known $^{11}$B states at the excitation energies up to 14 MeV were measured. The data analysis was done by DWBA and in some cases by the modified diffraction model allowing determining the radii of the excited states. The radii of the states with excitation energies less than ~ 7 MeV with the accuracy not less than 0.1-0.15 fm coincide with the radius of the ground state. This result is consistent with the traditional view of the shell structure of the low-lying states in $^{11}$B. Most of the observed high-energy excited states are distributed among four rotational bands. The moments of inertia of band states are close to the moment of inertia of the Hoyle state of $^{12}$C. The calculated radii, related to these bands, are 0.7 - 1.0 fm larger than the radius of the ground state, and are close to the radius of the Hoyle state. These results are in agreement with existing predictions about various cluster structure of $^{11}$B at high excitation energies. The state with the excitation energy 12.56 MeV, $I^{\pi} = 1/2^+$, $T = 1/2$ and the root mean square radius $R \sim 6$ fm predicted in the frame of the alpha condensate hypothesis was not found.

1. Introduction

During long time $^{11}$B nucleus was considered as good example of shell effects in light nuclei. Up to excitation energies ~ 7 MeV states of entire spectrum were described by different variants of shell models. Recently, however, a number of theoretical and experimental works appeared what resulted in increased interest to the $^{11}$B structure. This interest was primarily connected with the predictions [1-3] about cluster configurations of various types co-existed in $^{11}$B and the appearance of some experimental data indicating a greater probability of such a possibility [4].

Particular attention was drawn to the idea that there may be states in $^{11}$B, analogues of the famous $0^+_2$ state with excitation energy 7.65 MeV in $^{12}$C nucleus (the so-called Hoyle state). The Hoyle state properties, consisting of three weakly interacting alpha - clusters were crucial for verification and justification of the theory of alpha - particle condensation in nuclei [5], the main of which was the suggestion about its abnormally large radius. Accordingly, the Hoyle state analogues in $^{11}$B (in which one proton is removed from alpha cluster, and having, therefore, the structure of $\alpha + \alpha + t$), must also have increased size.

Initially it was assumed [1] that the Hoyle state analogue in $^{11}$B is the state $3/2^-$ with excitation energy 8.56 MeV, which is not described by any variant of shell model. However, other possibilities were started to be discussed afterwards. For example, in [3] it was suggested that the true analogue of
the Hoyle state is the state with excitation energy 12.56 MeV, despite contradictory data about its quantum numbers. Moreover, in [3] it was made even more ambitious assumption that this state has a "giant" radius $R_{\text{rms}} \approx 6$ fm, comparable to the radius of the uranium nucleus (!). The radius of 8.56 MeV state was still considered to be abnormally large, and it was predicted that this state is base for rotational band.

In [2, 4] other unusual states in $^{11}$B were predicted, such as quasimolecular states, with the structure of "two alpha – particle core plus three nucleons moving on their orbits". It is expected that the sequence of such states forms the positive parity rotational band, based on the $1/2^+$, $E^* = 6.79$ MeV state. It has been suggested [3] that this state also has an increased radius, 0.6 - 0.8 fm larger than the radius of the ground state.

Thus, recently coexistence of several types of structures was predicted in $^{11}$B, and this nucleus suddenly become in the focus of the entire problem of nucleon clustering in nuclei.

There are a lot of experimental studies of $^{11}$B (see, e.g., [6] and references therein), but they did not affect the excitation energy region of interest for the problem. Due to the fact that many questions about $^{11}$B states remained open, we have undertaken a new study of inelastic $^{11}$B + $\alpha$ scattering at $E(\alpha) = 65$ MeV. We measured the differential cross sections in a wide range of $^{11}$B excitation energies and analyzed them together with the data available at other energies. Some preliminary results have been reported in [7].

2. Experiment and data analysis

The measurements were carried out at the University of Jyvaskyla K130 cyclotron (Finland) using the LSC (Large scattering chamber). The standard $\Delta E-E$ method was used. 4 telescopes were used simultaneously, each telescope made measurements for 2 degrees. The target was a self-supporting film of $^{11}$B with 0.275 mg/cm$^2$ thickness. $^{12}$C and $^{10}$B were the only impurities in the target. The beam intensity was about 30 nA (for the particles). In the experiment, we used a system of beam monochromatisation, which allowed us to obtain a complete energy resolution about 150 keV.

The measured differential cross sections were analyzed by the Distorted Wave Born Approximation method (DWBA). To determine the radius of the $^{11}$B nucleus it was supposed to use Modified Diffraction Model (MDM), described in details in [8], which allowed us to obtain self-consistent data on the mean-square radii of a large number of excited states. MDM allows determining radii of the nucleus excited states through the difference between diffraction radii of excited and ground states using the expression (1):

$$R^* = R_0 + [R_{\text{dif}}^* - R_{\text{dif}}(0)]$$

where $R_0$ - is the mean square radius of the ground state of the nucleus, $R_{\text{dif}}^*$ and $R_{\text{dif}}(0)$ - diffraction radii calculated from positions of minima and maxima of experimental inelastic and elastic scattering angular distributions, respectively.

3. Results and discussion

3.1. Low-lying states

The differential cross sections for low-lying 2.12, 4.44, 5.02 и 6.74 + 6.79 MeV states in $^{11}$B were measured. It was shown that in the above unresolved doublet of excited states 6.74 MeV state dominates. The DWBA calculations were fulfilled.

Mean square radii of low-lying excited states of $^{11}$B were received using MDM based on experimental data of current work and data [6] at 40 and 50 MeV. As it was expected, $^{11}$B has equal non-enhanced radii in all low-lying states. The radii of low-lying ($E^* < 7$ MeV) states practically coincide with the radius of the ground state.

The received results are consistent with traditional notions about shell structure of the low-lying states in the $^{11}$B nucleus.
3.2. 3/2⁺, 8.56 MeV state and rotational band based on it

As it was mentioned in introduction, one of the main aims of this work was to get information about “abnormal” properties of 8.56 MeV state, and especially to determine its radius. Differential cross section with excitation of this state is shown of figure 1.

![Figure 1. Differential cross sections of $^{11}$B + α scattering at $E_\alpha=65$ MeV. Points correspond to experimental data and solid curves - to calculations: optical model for elastic scattering; DWBA for inelastic scattering with excitation of the 3/2⁺, 8.56 MeV state (angular momentum transfer $L=0, 2$); 5/2⁻, 10.33 MeV ($L=4$); 11.6 MeV; 9/2⁺, 13.14 MeV ($L=4$).](image)

Diffractive structure of angular distributions is shown clearly, however, the difference between DWBA calculations and experimental data takes place in the region of the second minimum. Compared with the elastic scattering, observed minima and maxima are shifted toward smaller angles, which is an indication of the increased radius of the 8.56 MeV state.

AMD - calculations [2] predict the existence of a rotational band, based on the 8.56 MeV state (see figure 2). There and in [4], it was suggested that the band is formed by sequence of states: 10.33 (5/2⁻) - 11.60 - 13.14 (9/2⁺) MeV. However, before these predictions, it was considered [9] that the spin-parity of the 11.60 MeV state is 5/2⁺ and therefore it cannot be a member of this band.

As it was shown in [10], 10.33 and 13.14 MeV states are excited by angular momentum transfer $L = 4$. MDM calculations give $R_{\text{diff}} = 4.8$ fm for these states. This diffraction radius is close to diffraction radius of recently observed state in $^{12}$C with $E^* = 13.75$ MeV (this state is excited by angular momentum transfer $L = 4$ in $^{12}$C ($\alpha, \alpha'$) scattering) and identified [11], as member of rotational band, based on the Hoyle state. It was shown [11] that the radius of the 13.75 MeV state is 0.8 fm larger than the radius of the 14.08 MeV state. Thus an estimation can be done, that the radii of above mentioned excited states of $^{11}$B have radii 0.8 fm larger than radius of the ground state.

So 10.33 (5/2⁻) and 13.14 (9/2⁺) MeV states have properties, similar to 8.56 MeV state and can be members of its rotational band.
Separate discussion is required for question about spin-parity of the 3rd predicted state of rotational band - the 11.60 MeV state, according to \[2, 4\], \(J^\pi = 7/2^-\). But, in review [9] \(J^\pi = 5/2^+\).

In [10] we received using DWBA that this state is excited by angular momentum transfer \(L = 1\) and \(L = 3\), so \(J^\pi = 5/2^+\). If so, the question about the fact of the existence of this band arises. However, this result is not definitive. A significant contribution to the cross section the 11.44 MeV state can give, which has not been completely separated from the 11.60 MeV state, and for which there is no data on its spin and parity. In case \(J^\pi = 7/2^-\), this state should be excited by \(L = 2\) and the use of MDM provides for the latter value \(R_{dif} = 6.11 \pm 0.35\) fm.

3.3. Rotational bands for higher excited states

Following rotational bands were predicted [2, 4] in high excitation energy region in \(^{11}\)B. Main part of these states was observed in our experiment:

- \(K = 3/2^-:\) 8.56 (3/2^-) – 10.33 (5/2^-) – 11.60 – 13.14 (9/2^-) MeV,
- \(K = 1/2^+:\) 6.79 (1/2^+) – 9.88 (3/2^+) – 11.60 (5/2^+) – 13.16 (7/2^+) MeV,
- \(K = 3/2^+:\) 7.98 (3/2^+) – 9.27 (5/2^+) – 10.60 (7/2^+) – 12.63 (9/2^+) MeV,
- \(K = 5/2^+:\) 7.29 (5/2^+) – 9.19 (7/2^+) – 11.27 (9/2^+) MeV.

These rotational bands are shown on figure 2 together with band in \(^{12}\)C, based on the Hoyle state. Received from our experiment data about angular momentums transfer with excitation states, belonging to specified bands, are in agreement with known spin-parities of \(^{11}\)B states. However they could not be determined unambiguously for the states 6.79, 9.88, 10.33, 13.14 and 13.16 MeV because of an insufficient energy resolution. Several special features in the \(J(J + 1)\) dependence of the excitation energies in figure 2 can be seen. Firstly, moments of inertia of the band states are very high and comparable. The largest of them \((2I/\hbar^2 \approx 4.0\)\), by the energy difference between the excitation energies 11.60 and 10.33 MeV) are observed for the higher members of the rotational band \(K = 3/2^-\), for which cluster structure \(2\alpha + t\) is predicted. It is interesting that it is much larger than the moment of inertia of its analogue - the Hoyle state, for which \(2I/\hbar^2 = 2.7\).

![Figure 2](image_url)
Secondly, there is a clear correlation between the moments of inertia and values of radii obtained using MDM from scattering data. Low-lying states of $^{11}$B have "normal" radii and "reduced" moments of inertia about $2I/\hbar^2 \approx 1.1$. These values are close to values for the first excited state of $^{12}$C, 4.44 MeV. "Big" moments of inertia correspond to increased radii. The radii of the states, located at excitation energies above 7 MeV in $^{11}$B, were obtained using MDM. The increased radii were found, at least, for one of the members of each band, and in most cases they are about 0.7 - 1.0 fm larger than the radius of the ground state of $^{11}$B. This leads to the conclusion that all states belonging to the bands under consideration have abnormal size. Theoretical works [1 - 4] suggest a significant deformation of the rotational states of $^{11}$B with $E^* > 7$ MeV and it allows the increase of their radii. Radii and moments of inertia of these states are close to the corresponding values of the Hoyle state in $^{12}$C nucleus.

Concerning the K=1/2$^+$ band: this band produces the largest number of questions. In the measured spectra there were not observed any sign of contribution from the first member of the band, the 6.79 MeV state, to the unresolved group with the 6.74 MeV state. The second member, the 3/2$^+$, 9.88 MeV state was not observed at all. Spin-parity of the third member of the band, the 11.60 MeV state has not been established unambiguously. The estimated fourth band member, the 13.16 MeV state was not separated from the 13.14 MeV state. In the angular distribution the 13.16 state probably manifests itself, but probably with angular momentum transfer $L = 1$, which would mean that it cannot have spin 7/2$^+$, and it therefore doesn’t belong to this band.

3.4. Radii of the states in $^{11}$B
The levels of $^{11}$B could be divided in 2 groups from the point of view of their radii values (see figure 3). The resulting radii of states with excitation energies up to ~ 7 MeV within the errors do not differ from the radius of the ground state, which is consistent with the traditional view of the $^{11}$B shell structure.
At higher excitation energies formation of states belonging to rotational bands based on the 3/2−
(8.56 MeV), 3/2+ (7.98 MeV), 5/2+ (7.29 MeV) and maybe 1/2+ (6.79 MeV) states was observed. All
these bands are characterized by large moments of inertia (2I/ћ = 3-4). The radii of rotational states,
deﬁned using MDM, were determined 0.7-1.0 fm larger than the radius of the 11B ground state. The
moments of inertia and the obtained values of the radii are close to the corresponding parameters of
the Hoyle state in the 12C and the rotational band, built on it, indicating cluster structure of these 11B
states. Particularly, the state with excitation energy 8.56 MeV can be considered as an analogue of the
Hoyle state. The predictions of the theory of the coexistence of different structures in the 11B,
including cluster, are conﬁrmed. Regarding the proposed band, based on the 6.79 MeV state, a number
of uncertainties stay, including the fact of such band existence.

3.5. 12.56/12.63 MeV state
Special attention should be provided to the problem of the 12.56 / 12.63 MeV state. Before the work
[4], the only state at excitation energies 12.0 - 12.9 MeV was the state at 12.56 MeV with spin - parity
J = 1/2−, and isospin T = 3/2 [9], the isobar - analogue of the ground state of the 11Be. In [3] it has
been suggested that this state really has a spin - parity J = 1/2+, and isospin T = 1/2 and should be a "true"
 analogue of the Hoyle state in 12C nucleus. The value of the isospin T = 1/2 is supported by the
observation of the state decay with the emission of alpha - particles [12 - 14].

In reaction 7Li (α,α) [4] a strong resonance was indeed found, but with an excitation energy 12.63 ±
0.04 MeV. The state with excitation energy 12.56 MeV was not observed. In addition to the obvious
value of the isospin T = 1/2, the 12.63 MeV state was received having the spin - parity J = 3/2+ or
9/2+. Possible value of the spin 9/2+ made it possible [4] to interpret the 12.63 MeV state, as the last
member of the rotational band with K = 3/2+.

In our experiment state with an excitation energy 12.6 MeV also was observed, so its isospin is T =
1/2. It is possible the state we observed is the state from [4] with excitation energy 12.63 MeV. Most
probable value of the spin-parity according to our data is J = 3/2+. Analysis within MDM gave a
"normal" value of the mean square radius (see ﬁgure 3), so prediction [3] has not been conﬁrmed.

4. Conclusions
The 11B nuclear has a number of excited states with the radii exceeding that of the ground state (size
isomers). They are grouped in rotational bands whose moments of inertia correlate with enhanced radii
values.

Radii and moments of inertia of the low-lying (E* < 7 MeV) states of 11B are close to those of the
ground state.

Radii and moments of inertia of rotational states with E*>7MeV are similar to those of the Hoyle
state rotational band indicating their predicted cluster structure.

The 11B state with E* = 12.6 MeV was observed, and thus its isospin is T = 1/2. However, the
radius of this state, in contradiction with some predictions, is not signiﬁcantly larger than the radius of
the ground state, and thus the hypothesis about the presence of "giant" states in some light nuclei has
not been conﬁrmed.

Established numerous correlations between the values of moments of inertia of rotational band
states and their radii are further conﬁrmation of MDM ability to determine radii of excited states using
experimental differential scattering cross sections.

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