Pile-up corrections in laser-driven pulsed x-ray sources

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A formalism for treating the pile-up produced in laser-driven pulsed x-ray sources has been developed. It allows the direct use of x-ray spectroscopy without artificially decreasing the number of counts in the detector. The influence of the pile-up on the overestimation of temperature parameters is shown up.

Keywords: pile-up, x-ray spectroscopy, laser ultrashort pulses

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I. INTRODUCTION

Since the first experiments of the interaction of ultrashort and ultraintense laser pulses with matter it was realized that laser-produced plasmas may constitute a pulsed, bright, x-ray source. The alternating electrical field of the laser is high enough to rapidly ionize atoms by barrier suppression ionization, forming a plasma. A substantial part of the plasma electrons are accelerated by several mechanisms, reaching energies up to the MeV range with a maxwellian-like energy distribution. In their interaction with the target material x-ray pulses are produced, which spectrally consist of a continuous bremsstrahlung component and discrete characteristic line emissions.

The x-rays energies range from few tens of keV up to several MeV, depending on the target material. The pulse duration is of the order of hundreds of femtoseconds and the source size can be a few times larger than the laser spot size. These properties make these sources well suited for laser micromachining, medical imaging, spectroscopy, homeland security, and nuclear physics.

Since laser-produced x-rays are emitted in very short bursts, which are orders of magnitude shorter than the detector response time, experimental x-ray spectra are plagued of pile-up artifacts. This phenomenon occurs when two or more photons are detected as a single event. Thus, it represents a loss of information which may lead to a wrong analysis of the physics of the process.

This problem is more severe when the bremsstrahlung spectrum is used to estimate the temperature of the original electron energy distribution, which can be overestimated by more than 30% due to the pile-up phenomena.

A solution to the pile-up problem is to ensure the photon detection rate is sufficiently low for the probability of simultaneous photon detection in a single observation to be below an acceptable maximum value. In laser-generated x-ray emission that can only be achieved by collimating the x-ray beam or increasing the distance from the detector to the source. However, this greatly increases the observation time needed for an adequate signal-to-noise ratio.

In this work we provide a formalism to correct the pile-up in these systems by using the ratio between the count rate and the repetition rate. The model requires the duration of each pulse to be much shorter than the detector response time, so all the pile-up is due to
photons from the same pulse. When this condition is fulfilled the number of counts can be assumed to be given by a Poisson distribution, hence, we will call it a Poisson pile-up. The application of the method presented allows to remove the pile-up distortion in the number of counts in the channels of a detector.

II. POISSON PILE-UP MODEL

Suppose a pulsed source is emitting particles with a frequency $\nu$. Assuming that the interaction probability of a particle with the detector is very small and that the number of emitted particles is much greater than one, the detection is a Poisson process and the number of particles producing a single count at the detector $N$ follows a Poisson distribution. Its rate parameter $\lambda$ can be estimated using the frequency $\nu$ and the count rate of the detector $r$, since

$$E \left[ \frac{r}{\nu} \right] = P(N > 0) = 1 - e^{-\lambda},$$

where $E[\cdot]$ denotes the expected value of a random variable.

If the detector is measuring the energy to characterize the source, which follows an energy distribution $f(E)$, multiple events will be recorded as a single one with an energy given by the sum of the individual energies of the detected particles. The probability density function of the sum of two independent random variables is given by the convolution product

$$f_{A+B}(E) = (f_A * f_B)(E) = \int_R f_A(x)f_B(E-x)\,dx.$$  \(2\)

Repeated application of Eq. 2 describes the total energy distribution for any fixed number of independent events in a single detection. This is sometimes called the convolution power, defined by the recursion

$$f^{*n} \stackrel{\text{def}}{=} \left\{ \begin{array}{ll} f & \text{if } n = 1 \\ f * f^{*n-1} & \text{if } n > 1 \end{array} \right.,$$

for integer $n$.

The detected energy distribution $f_\lambda(E)$ is related with the pile-up-undistorted one $f(E)$ by applying the law of total probability to the partition of the sample space which corresponds to the outcomes of the Poisson process conditioned to $N > 0$, since then no detection occurs if no particle arrives,

$$f_\lambda = \frac{1}{1-e^{-\lambda}} \sum_{n=1}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!} f^{*n} = \frac{1}{e^\lambda - 1} \sum_{n=1}^{\infty} \frac{\lambda^n f^{*n}}{n!},$$

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Note that Eq. (4) expresses a convex combination of distributions and hence is well defined.

To recover the undistorted distribution \( f \) it is natural to apply a Fourier Transform (FT) to Eq. (4). Denoting the FT operator as \( F \), applying the convolution theorem, and identifying the exponential series we find

\[
F\{f_\lambda\} = \frac{1}{e^{\lambda} - 1} \left( \sum_{n=0}^{\infty} \frac{\lambda^n F\{f\}^n}{n!} - 1 \right) = \frac{e^{\lambda F\{f\}} - 1}{e^{\lambda} - 1},
\]
and then

\[
f_\lambda = F^{-1}\left\{ \frac{e^{\lambda F\{f\}} - 1}{e^{\lambda} - 1} \right\}.
\]

It is then trivial to recover the original distribution from the piled-up one in the Fourier domain by inverting Eq. (5), i.e.,

\[
f = F^{-1}\left\{ \frac{\ln\left(1 + \left(\frac{e^{\lambda} - 1}{e^{\lambda} - 1}\right) F\{f_\lambda\}\right)}{\lambda} \right\}.
\]

We shall call this process depiling.

It is worth noting that, depending on the convention used to define the FT, an additional factor may appear in the application of the convolution theorem in Eq. (5), which eventually appears in the denominator in Eq. (7). This fact can be simply ignored when performing calculations by imposing normalization on \( f \), regardless the convention used.

Also note that if an energy-dependent efficiency is taken into account to correct the response, this correction has to be applied after depiling the spectrum, i.e., the uncorrected response should be used as the probability density distribution \( f_\lambda \) and the correction should be applied latter to \( f \).

In order to computationally apply the method to a sampled approximation of a density function one has to naturally replace the FT of the functions by the Discrete Fourier Transform (DFT) of the sequences. Cf. e.g. §6 in Ref. 8 for a discussion. A caveat will be discussed later in §III C.

III. NUMERICAL ANALYSIS

A. A Monte Carlo verification of the model

Monte Carlo numerical experiments can be used to verify the proposed model. As a first test we generate probability density functions distorted by pile-up from both uniform
and maxwellian \((f(E) \propto \sqrt{E} \exp(-E))\) distributions in energy, for different values of the rate parameter \(\lambda\) by using Eq. (6). Figure 1 and Figure 2 show the comparison of these distorted distributions with histograms obtained from simulations each consisting of \(10^6\) Poisson samplings with an associated uniform or maxwellian energy distribution sampling respectively. The distributions obtained from Eq. (6) reproduce perfectly the Monte Carlo simulated data, showing that the model accurately generates the piled-up distributions.

![Figure 1: Pile-up of the uniform distribution in [0, 1]. The histogram shows Monte Carlo-simulated data, and the dashed line represents the prediction of Eq. (6).](image1)

![Figure 2: Pile-up of the unit scale parameter maxwellian energy distribution. The histogram shows Monte Carlo-simulated data, and the dashed line represents the prediction of Eq. (6).](image2)

Figure 1 also clearly illustrates the pile-up series in Eq. (4): low values of \(\lambda\) are accurately described by the superposition of the rectangular density in [0, 1] and the triangular density
in $[0, 2]$, which is $f^*$. Greater values of $\lambda$ show contributions for higher powers, which are splines in $[0, n]^9$

More interesting is the depiling process performed by means of Eq. (7) over a piled-up distribution with some noise added to study the influence of the noise on the reproducibility of the original undistorted distribution. The noise was modeled by multiplying each bin in the piled-up spectra by a value sampled from a normal distribution with mean 1 and standard deviation 0.1, the bin size being 0.01 in the uniform distribution and 0.05 in the maxwellian. A 2-values moving average was applied to the piled distribution to slightly reduce the noise and then Eq. (7) is applied. The results are shown in Figure 3, where the shadow region is 1$\sigma$ in the noise-adding process. The results can be further smoothed by increasing the steps of the moving average.

B. The impact on temperature estimation

Changes in the spectra due to pile up may induce an error in the evaluation of magnitudes derived from them. The most typical example in laser-acceleration processes is the ‘characteristic temperature’ of the bremsstrahlung energy distribution, derived as the slope in a log-linear scale$^{10}$.

The impact of the pile-up can be studied in a realistic situation using an experimental spectrum taken from Ref. 11. It was obtained from a 1 GW Ti:sapphire laser with a repetition rate of 990 Hz, focalized with an intensity of $5 \times 10^{16}\, \text{W cm}^{-2}$ in an aluminum target. The source was collimated so the pile-up effect in the experimental data was considered negligible ($\lambda < 0.01$). The effective temperature was obtained in the original experiment by fitting the log-scale slope in the range 15 to 30 keV, obtaining a result of 8.2 keV.

From the experimental spectrum one can generate artificially a piled-up spectrum for different values of $\lambda$ using Eq. (6). An example is shown in Figure 4 for $\lambda = 0.7$. The effective temperature is depicted as function of $\lambda$ in the solid line in Figure 5, which makes clear that the effective temperature is increased from its original value as the pile-up effect increases.

Figure 5 also shows the results for the maxwellian (dashed line) and dual exponential (dot-dashed line) distributions, where all the calculations were done in a similar energy range. As in the former case, an overestimation of the temperature occurs in both models.
FIG. 3: Distributions obtained from depiling the piled-up uniform distribution in [0, 1] (a) and the maxwellian distribution (b), both cases with added noise. The shadowed region indicates the order of magnitude of the noise introduced.

The overestimation effect can be further increased depending on the shape of the detector response function. Cf. Ref. [5] where an overestimation of around 15% due to the pile-up was estimated to increase up to around the 35 to 70% range, for a HPGe detector in the 120 to 1000 keV energy range, due to the loss of collection efficiency in the Compton range.

C. The loss of resolution

As a final note, when a coarsely binned spectrum is being considered, some information may not be recovered by the depiling process. This is inherent to the uncertainty in the
FIG. 4: An experimental bremsstrahlung distribution and the simulated pile-up calculated with Eq. (6).

FIG. 5: Relative effective temperature as a function of the rate parameter for experimental (solid line, Ref. 11), maxwellian (dashed line) and dual exponential (dot-dashed line, Ref. 11) distributions.

energy that exists in a histogram: the real (piled) distribution inside each bin is unknown indeed. This can also be thought as an aliasing effect in the Fourier domain, which is the only error appearing in this case (finite energy ‘duration’ waveform) when the FT is replaced by the DFT (cf. p. 105 in Ref. 8).

The exact loss of resolution depends on the (unknown) distribution in each of the bins. However, its magnitude can be estimated by using the piled-up distribution of a uniform distribution in a bin —which, up to scale and location, corresponds to Figure 1—. Let \( \Delta E \) be the size of a bin in an unpiled spectrum. The piled-up uniform distribution extends up to infinite energy, so a confidence-interval-like magnitude is needed. For that purpose
we define $\Delta E_{\lambda,\rho}$ as the minimum energy range such that a fraction of counts $\rho$ from the original interval are found in this new energy range. Thus, $\Delta E_{\lambda,\rho}/\Delta E$ is the relative loss of resolution (with a ‘confidence’ $\rho$). This is shown as a function of $\lambda$ for different values of $\rho$ in Figure 6. The initial linear portion of the curves corresponds to an interval $[0, \bar{\lambda}_\rho]$ where the piled-up change of the distribution is within the energy resolution of the detector in the sense given by $\rho$. This can serve as a quantitative definition of the limit where the loss of resolution becomes non-negligible.

FIG. 6: Estimation of the loss of resolution produced by the binning.

IV. CONCLUSIONS

In summary, a procedure for studying and resolving the pile-up in pulsed-laser driven sources has been described and verified against Monte Carlo simulation and noisy numerical data. The results allow direct spectroscopy to be applied without artificially decreasing the number of counts in the detector to avoid the pile-up.

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