Black holes – Estimation of their lower and upper mass limits stemming from the model of Expansive Nondecelerative Universe

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Abstract

In this contribution, the thermodynamics of black holes is treated by the model of Expansive Nondecelerative Universe (ENU). Based on entropy considerations and localization of gravitational energy, estimation of both the lower and upper mass limits of black holes is given. Both mass limits are time dependent.

Keywords: Black holes; Expansive Nondecelerative Universe; mass limits; thermodynamics; quantum evaporation; gravity localization

1 Thermodynamics of black holes and a lower limit of their mass

As part of his famous ideas concerning the existence and properties of black holes, combining quantum mechanics and thermodynamics, Hawking [1] formulated the following equation describing the quantum evaporation output $P_{\text{evap}}$ of a black hole with the gravitational radius $r_{BH}$

$$P_{\text{evap}} = \frac{\hbar c^2}{r_{BH}^2}$$

Hypothesis on the quantum evaporation of black holes has been developed by several groups of theoreticians and field researchers, no burst proving a fatal cessation of either primordial or later formed black hole has been, however,
observed. The questions of minimum and maximum mass of black holes are still under discussion and deserve attention. We are convinced that the model of Expansive Nondecelerative Universe (ENU) [2-4] is capable to contribute in offering answers to such questions. In this contribution its potential in this field is documented.

Due to the Vaidya metric application [5], ENU enables to localize and quantify the energy of the gravitational field. For weak field conditions it adopts the form

\[ \varepsilon_g = -\frac{R.c^4}{8\pi.G} = -\frac{3m.c^2}{4\pi.a.r^2} \]  

(2)

where \( \varepsilon_g \) is the density of gravitational field energy generated by a body with a mass \( m \) at a distance \( r \), \( R \) is the scalar curvature and \( a \) is the gauge factor (reaching at present \( a \approx 1.3 \times 10^{26}m \) ) [2].

It is worth mentioning that the magnitude of \( \varepsilon_g \) given by (2) is closed to the \( \theta_0^0 \) component of the Einstein energy-momentum pseudotensor describing the density of gravitational energy also for strong field conditions [6]. As a direct consequence of Vaidya metric application, \( R \neq 0 \) also outside a body. In such a case the gravitational output \( P_g \) of a body at a given cosmological time \( t_c \) is as follows

\[ P_g = -\frac{d}{dt} \int \frac{R.c^4}{8\pi.G} dV = -\frac{m.c^2}{t_c} = -\frac{m.c^3}{a} \]  

(3)

In order not to violate the second law of thermodynamics, it must hold

\[ |P_g| \geq P_{\text{evap}} \]  

(4)

In a limiting case of the outputs being equal and taking into account that

\[ r_{BH} = \frac{2G.m_{BH}}{c^2} \]  

(5)

a lower mass limit of a black hole can be expressed as

\[ m_{BH(\text{min})} = \left( \frac{\hbar.c^3.a}{4G^2} \right)^{1/3} \]  

(6)

that leads to its current value

\[ m_{BH(\text{min})} \approx 10^{12}kg \]  

(7)

The value given by eq. (7) can be obtained by an independent mode, based on the entropy consideration, too.
It has been rationalized by Bekenstein [7] that the entropy of a black hole can be expressed as

$$S_{BH} \approx k \cdot \frac{r_{BH}^2}{l_{Pc}^2} \approx k \cdot 10^{76} \cdot \left(\frac{m_{BH}}{m_{Sun}}\right)^2$$

(8)

where $k$ is the Boltzmann constant, $l_{Pc}$ is the Planck length, and $m_{Sun}$ is the current mass of our Sun. The second law of thermodynamics stipulates that for the irreversible processes (creation of a black hole may set as an example of such processes), the initial state (e.g., star) entropy must be lower than that of the final state (black hole). Suppose, a black hole creation is a consequence of gravitational collapse of a star with the mass $m_{Star}$. The entropy value of the magnitude of Boltzmann constant may be allocated to each degree of freedom of a classical system of particles (for the sake of simplicity, nucleons are considered). For the entropy of the initial system it must, therefore, hold

$$S \approx k \cdot \frac{m_{Star}}{m_n} \approx k \cdot 10^{57} \frac{m_{Star}}{m_{Sun}}$$

(9)

where $m_n$ is the mass of nucleon (usually the proton mass).

Comparing (8) and (9) it can be seen that the process of a collapse accompanied by a black hole creation is associated with an increase in entropy by 19 orders of magnitude and it is, therefore, of a strongly irreversible nature.

It follows further from comparison of (8) and (9) that reducing the initial mass $m_{Star}$, the entropy of a corresponding black hole will drop by a higher extent than that of initial star. Both the value become equal at

$$m_{Star} \approx 10^{12} \text{kg}$$

(10)

It means that at the time being no black holes with a lower mass can be created (and probably no such a black hole can exist). Otherwise the entropy of a black hole would be lower than that of the collapsing star, i.e. the second law of thermodynamics would be violated.

It is worth mentioning that relations (8) and (10) have been obtained in independent ways.

In addition to the lowest mass limit, using (3) the gravitational radius of such a black hole, $r_{BH(\text{min})}$ can be calculated

$$r_{BH(\text{min})} = \left(\frac{l_{Pc}^2}{a}\right)^{1/3}$$

(11)

Introducing the present value of $a$ it is obtained

$$r_{BH(\text{min})} \approx 10^{-15} \text{m}$$

(12)

which represents a nucleus dimension.
2 The lowest mass of particles exerting gravitational influence

Gravitational force is a far-reaching force with ostensibly unlimited range. Due to the existence of hierarchic rotational gravitational systems, the range is, however, actually finite. This is a reason for introducing so called “effective gravitational range” $r_{ef}$, i.e. the distance at which the density of gravitational field of a given body is equal to the critical density of background gravitational field. Critical energy density is in ENU equal to the actual mean energy density, it decreases in time and is given \[2\] by

$$
\varepsilon_{crit} = \frac{3c^4}{8\pi G a^2}
$$

(13)

It follows from (2) and (13) that (in absolute values)

$$
\frac{3c^4}{8\pi G a^2} = \frac{3m c^2}{4\pi a r^2}
$$

(14)

and, in turn

$$
r_{ef} = (r_g a)^{1/2}
$$

(15)

where $r_{ef}$ is the effective gravitational range of a body with the gravitational radius $r_g$.

Based on the above rationalization, it is possible to determine the lightest particle able to exert gravitational influence on its surroundings. The particle has the mass $m_x$ and its gravitational range is identical to its Compton’s wavelength. Stemming from the following relation

$$
\left(\frac{2G m_x a}{c^2}\right)^{1/2} = \frac{\hbar}{m_x c}
$$

(16)

the current mass of the particle is

$$
m_x \cong \left(\frac{\hbar^2}{2G a}\right)^{1/3} \cong 10^{-28}\text{kg}
$$

(17)

and its Compton’s wavelength

$$
\lambda_x = \frac{\hbar}{m_x c} = \left(\frac{\hbar^2}{P c a}\right)^{1/3} \cong 10^{-15}\text{m}
$$

(18)

As follows from (13), it is at the same time also a lower mass limit of a black hole. Turning back to entropy considerations, it is obvious that relation (9) should be modified as follows

$$
S \cong k \frac{m_{Star}}{m_x}
$$

(19)
In such a way, relations (6), (7), (9), and (10) become consistent since the entropy in (9) will be time-dependent. Stemming from (15), an increase of the gravitational range of nucleons in time is an expected phenomenon. Similarly, the degree of freedom and entropy of the system will increase too. It should be pointed out that at the beginning of the matter era it had to hold

$$m_x \simeq m_n$$

(20)

It means that nucleons started to exert their gravitational influence to the environment just at the beginning of the matter era which, inter alia, allowed black holes to be formed. Stemming from the equality of (8) and (19), it is possible to obtain relations (6) and (10).

3 Upper mass limit of black holes

Contrary to all the time present gravitational quanta, there were no elementary particles present the at the beginning of the Universe expansion. This is why it is useful to express the total entropy of the Universe, $S_U$, by a number of gravitational quanta [8] as follows

$$S_U = \frac{m_U c^2}{|E_g|} = \frac{c^5 t_c}{2G|E_g|}$$

(21)

where $m_U$ is the Universe mass and $E_g$ is the mean energy of the gravitational field quantum. The wavefunction of the mean gravitational quantum of the Universe is given by [8]

$$\Psi_g = \exp \left[ i.t \left( t_{Pc}.t_c \right)^{-1/2} \right]$$

(22)

where $t_{Pc}$ is the Planck time. Relation (22) complies with a Schrödinger-like equation for the energy of gravitational quanta $E_g$

$$E_g.\Psi_g = i.\hbar \frac{d\Psi_g}{dt}$$

(23)

and it comes from (22) and (23) that

$$|E_g| = \frac{\hbar}{(t_{Pc}.t_c)^{1/2}}$$

(24)

Based on (21) and (24) for the total entropy of the Universe it follows

$$S_U = \left( \frac{a}{l_{Pc}} \right)^{3/2} \simeq 10^{92}$$

(25)
No body, black holes including, can possess an entropy content higher than that of the whole Universe. This is the reason due to which using (8) and (25) it must hold
\[
\frac{r_{BH}^{2}(\text{max})}{l_{Pc}} = \left(\frac{a}{l_{Pc}}\right)^{3/2}
\]
In (26), \(r_{BH}(\text{max})\) is the maximum gravitational radius of a black hole in the Universe with the gauge factor \(a\). It can be directly obtained from (26) that
\[
r_{BH}(\text{max}) = \left(a^{3}.l_{Pc}\right)^{1/4}
\]
which currently approaches to
\[
r_{BH}(\text{max}) \cong 10^{11}\text{m}
\]
The maximum mass of black holes, corresponding to (28) is then
\[
m_{BH}(\text{max}) \cong 10^{38}\text{kg} \cong 10^{8}m_{\text{Sun}}
\]
which is in excellent agreement with experimental observation [9].

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