Weak Ergodicity Breaking in the Schwinger Model

Jean-Yves Desaules,1 Debasish Banerjee,2,3 Ana Hudomal,1,4 Zlatko Papić,1 Arnab Sen,5 and Jad C. Halimeh6,7,*

1 School of Physics and Astronomy, University of Leeds, Leeds LS2 9JT, UK
2 Theory Division, Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Kolkata 700064, India
3 Homi Bhabha National Institute, Training School Complex, Anushaktinagar, Mumbai 400094, India
4 Institute of Physics Belgrade, University of Belgrade, 11080 Belgrade, Serbia
5 School of Physical Sciences, Indian Association for the Cultivation of Science, Kolkata 700032, India
6 Department of Physics and Arnold Sommerfeld Center for Theoretical Physics (ASC), Ludwig-Maximilians-Universität München, Theresienstraße 37, D-80333 München, Germany
7 Munich Center for Quantum Science and Technology (MCQST), Schellingstraße 4, D-80799 München, Germany

(Dated: April 21, 2022)

As a paradigm of weak ergodicity breaking in disorder-free nonintegrable models, quantum many-body scars (QMBS) can offer deep insights into the thermalization dynamics of gauge theories. Having been first discovered in a spin-1/2 quantum link formulation of the Schwinger model, it is a fundamental question as to whether QMBS persist for \( S > 1/2 \) since such theories converge to the lattice Schwinger model in the large-\( S \) limit, which is the appropriate version of lattice QED in one spatial dimension. In this work, we address this question by exploring QMBS in spin-\( S \) \( U(1) \) quantum link models (QLMs) with staggered fermions. We find that QMBS persist at \( S > 1/2 \), with the resonant scarring regime, which occurs for a zero-mass quench, arising from simple high-energy gauge-invariant initial states. We furthermore find evidence of detuned scarring regimes, which occur for finite-mass quenches starting in the physical vacua and the charge-proliferated state. Our results conclusively show that QMBS exist in a wide class of lattice gauge theories in one spatial dimension represented by spin-\( S \) QLMs coupled to dynamical fermions.

Introduction.—Quantum many-body scars (QMBS) form an intriguing paradigm of ergodicity breaking in interacting systems that are typically expected to thermalize due to their nonintegrability and spatial homogeneity [1–8]. QMBS comprise eigenstates of low entanglement entropy [9, 10], many of which reside in the middle of the spectrum, and are often separated roughly equally in energy [11, 12]. These eigenstates are nonthermal, forming a “cold” subspace that is weakly connected to the rest of the Hilbert space. Consequently, quenches starting in initial states with high overlap with these nonthermal states (see, e.g., Fig. 1) will not show typical thermalization, but instead exhibit long-lived coherent dynamics, persisting significantly beyond local relaxation times [13, 14]. This behavior is of particular interest to fundamental investigations of the eigenstate thermalization hypothesis (ETH) [15–20], as it has been linked to novel mechanisms for avoiding thermalization in closed quantum systems based on spectrum generating algebras [21–24] and embedding of nonthermally excited states [25] (see recent reviews [26, 27]). Moreover, given that many of these QMBS constitute high or even infinite-temperature states, when the system is initially prepared in them it will not dephase its information, which is pertinent to applications in quantum memory and information processing [28–30].

QMBS are also relevant to gauge theories [31–35], which describe the interactions of elementary particles mediated by gauge bosons through an extensive set of local constraints [36–38]. A paradigmatic example of the latter is Gauss’s law in quantum electrodynamics (QED), where the distribution of charged matter strictly specifies the allowed configurations of the surrounding electromagnetic field [39]. Recently, a concerted experimental

FIG. 1. (Color online). Quantum many-body scars in the spin-\( S \) \( U(1) \) QLM: Overlap of the extreme vacuum \( |0_\mu\rangle \) with the eigenstates of the quench spin-\( S \) \( U(1) \) QLM Hamiltonian (1) at \( \mu = g = 0 \). At all considered values of \( S \), distinctive towers of eigenstates arise that are equally spaced in energy (see insets). These are a hallmark of quantum many-body scars. These results are obtained from exact diagonalization calculations, where \( L \) denotes the number of matter sites, and periodic boundary conditions are employed.
effort has emerged for the implementation of gauge theories in synthetic quantum matter (SQM) devices [40–50]. This has been facilitated in large part due to the great progress achieved in the precision and control of SQM setups [51, 52], making the quantum simulation of gauge theories a realistic endeavor [53–59]. Due to the complexity involved in these experiments, the implementations often focus on quantum link formulations of gauge theories, where spin-$S$ operators model the gauge fields, which in QED span an infinite-dimensional Hilbert space [60]. This has allowed the first large-scale realization of the spin-1/2 U(1) quantum link model (QLM) in (1 + 1)−D using ultracold atoms [49, 50].

The first experimental observation of QMBS was achieved in a Rydberg-atom setup [1] that maps to the spin-1/2 U(1) QLM [32]. It remains an open question whether QMBS persist at larger link spin lengths ($S > 1/2$) in the QLM formulation of QED, and, if they do, what their form will be. In this Letter, we show that QMBS survive at any value of $S$. For a zero-mass quench, QMBS arise when the system is prepared in the *extreme vacua* of the spin-$S$ U(1) QLM, which are the most highly excited vacuum states of lattice QED. Furthermore, we find that preparing the system in the physical (least excited) vacua or in the charge-proliferated state can still lead to *detuned* scarring behavior for certain massive quenches, similar to the case of $S = 1/2$ that has recently been demonstrated experimentally in a tilted Bose–Hubbard optical lattice [7]. Our results indicate that QMBS may persist toward the limit of lattice QED, $S \to \infty$.

**U(1) quantum link model.**—The Schwinger model, or QED in (1 + 1) dimensions, is possibly the simplest gauge theory with dynamical matter and that shows non-trivial phenomena like confinement [61, 62]. A possible discrete version of the Schwinger Hamiltonian on a lattice is provided by the Kogut–Susskind formulation, which is also reached in the large-$S$ limit of the QLMs being studied here. The $(1 + 1)$−D spin-$S$ U(1) QLM is given by the Hamiltonian [63, 64]

$$\hat{H} = \sum_{j=1}^{L} \left[ \frac{J}{2a\sqrt{S(S+1)}} (\hat{\sigma}_j^+ \hat{s}_{j,j+1}^z \hat{\sigma}_{j+1}^- + \text{H.c.}) + \frac{\mu}{2} (-1)^j \hat{s}_{j}^z + \frac{g^2 a}{2} (\hat{s}_{j,j+1}^z)^2 \right].$$

(1)

Here, $J = 1$ sets the energy scale, $\mu$ is the fermionic mass, and $g^2$ is the electric-field coupling strength. Throughout this work, we will set the lattice spacing to $a = 1$ and employ periodic boundary conditions, with $L$ denoting the number of lattice sites. The matter field on site $j$ is represented by the Pauli operator $\hat{\sigma}_j^z$, and the electric (gauge) field at the link between sites $j$ and $j + 1$ is represented by the spin-$S$ operator $\hat{s}_{j,j+1}^z$. The generator of the U(1) gauge symmetry of Hamiltonian (1) is

$$\hat{G}_j = \frac{\hat{\sigma}_j^z + (-1)^j}{2} + \hat{s}_{j,j-1}^z - \hat{s}_{j,j+1}^z,$$

(2)

which can be interpreted as a discretized version of Gauss’s law relating the matter occupation on site $j$ to the electric-field configuration on its neighboring links. We will work in the *physical* sector of Gauss’s law: $\hat{G}_j |\phi\rangle = 0, \forall j$.

Due to the gauge symmetry imposed by the generator (2), one can integrate out the matter fields in the Hamiltonian (1), resulting in a constrained spin system. For $S = 1/2$, this corresponds to the PXP model [32]. For larger $S$, the resulting model differs from generalizations of the PXP model already explored in the literature [13, 65]. In the joint submission [66], we derive the relevant constrained spin-$S$ model corresponding to the spin-$S$ U(1) QLM for any value of $S$ [67]. Exact diagonalization (ED) techniques resolving the translation and spatial-inversion symmetries have been employed to study the eigenstates of these models. Time-evolution results are obtained either directly from the ED results or by time-evolving the initial state using sparse matrix exponential techniques.

**Resonant scarring.**—The physical vacuum of $\hat{H}$ is its ground state at $\mu \to \infty$ and $g^2 > 0$. In the case of half-integer $S$, there are two doubly degenerate physical vacua. These can be defined on a two-site two-link unit cell using as quantum numbers the eigenvalues $\sigma_j^z$ and $s_j^z$ of the matter and electric field operators $\hat{s}_j^z$ and $\hat{s}_{j,j+1}^z$, respectively, explicitly reading $|\sigma_1^z, s_1^z, 2, \sigma_2^z, s_2^z, 3\rangle = |+1, \pm 1/2, -1, \pm 1/2\rangle$. For integer $S$, the physical vacuum is nondegenerate, and reads $|\sigma_1^z, s_1^z, 2, \sigma_2^z, s_2^z, 3\rangle = |+1, 0, -1, 0\rangle$. Henceforth, we will denote $|0_+\rangle = |+1, +1/2, -1, +1/2\rangle$ for half-integer $S$ and $|0_+\rangle = |+1, 0, -1, 0\rangle$ for integer $S$, with the subscript denoting the sign of $g^2$.

On the other hand, the *extreme vacua* of $\hat{H}$ are high-energy states that can be realized as doubly degenerate ground states of Eq. (1) at $\mu \to -\infty$ and $g^2 < 0$: $|\sigma_1^z, s_1^z, 2, \sigma_2^z, s_2^z, 3\rangle = |+1, \pm S, -1, \pm S\rangle$. Henceforth, we will denote $|0_-\rangle = |+1, +S, -1, +S\rangle$, with the subscript again indicating the sign of $g^2$.

We will further consider the charge-proliferated state, which is the ground state of Eq. (1) at $\mu \to -\infty$ and $g^2 > 0$. For half-integer $S$, it is nondegenerate and reads $|\text{CP}\rangle = |-1, -1/2, +1, +1/2\rangle$. For integer $S$, we obtain two doubly degenerate ground states: $|\text{CP}\rangle = |+1, -1, +1, 0\rangle$ and $|0_-\rangle = |+1, 0, +1, +1\rangle$.

With respect to the eigenstates of Hamiltonian (1) at $\mu = g = 0$, we find through ED that only $|0_-\rangle$ exhibits the overlap behavior indicative of scarring for general $S$; see Fig. 1. Just as in the known case of $S = 1/2$, we also see at other values of $S$ signatures of 2SL + 1 towers equally spaced in energy (see insets), exhibiting large
overlap with $|0_−\rangle$, particularly in the middle of the spectrum. The overlap of the top band of states can be further enhanced by considering a truncated version of the QED gauge field [66]. Note how for all values of $S$ that we consider, there is a prominent zero-energy mode with the largest overlap. The presence of these eigenstates is evidence of weak ergodicity breaking in the model. Due to the scaling term $1/\sqrt{S(S+1)}$, the ground-state energy $E_0$ is approximately independent of $S$ at $\mu = g^2 = 0$, and we find numerically that $E_0 \approx -0.32L$. As the spectrum is symmetric around zero, we can use this along with the number of towers to get the approximate energy spacing between towers as $\Delta E \approx -2E_0/(2SL) \approx 0.32/S$. We note that the various approximation schemes for scarred eigenstates in the PXP model also show good results for larger $S$ [66].

The presence of scarred eigenstates can be detected via the dynamics of the fidelity, $F(t) = |\langle \psi(0) | \psi(t) \rangle|^2$ with $|\psi(t)\rangle = e^{-iHt} |\psi(0)\rangle$ and $|\psi(0)\rangle$ the initial state. In the case of $S = 1/2$, $|0_+\rangle = |0_−\rangle$, as the last term of Eq. (1) is an inessential energy constant since $(\hat{s}_3^z)_{j+1}^2 = 1$. Quenching this vacuum state with $\hat{H}$ at $\mu = 0$ is known to lead to scarring behavior for $S = 1/2$ [1, 32], and this is exhibited in the revivals of the fidelity, shown in the top left panel of Fig. 2. For comparison, we have included the fidelity dynamics for $|\psi(0)\rangle = |\text{CP}\rangle$, which shows no revivals, in agreement with what is established in the literature for this quench when $S = 1/2$ [26].

However, once the link spin length is $S > 1/2$, we find that the fidelity dynamics exhibits revivals only when the system is initialized in the extreme vacuum, $|\psi(0)\rangle = |0_−\rangle$, whereas neither the physical vacuum $|0_+\rangle$ nor the charge-proliferated state $|\text{CP}\rangle$ give rise to scarring behavior; see Fig. 2 for $S = 1, 3/2, 2$. We have checked that the other vacua $|\sigma_1^z, \sigma_{1,2}^z, \sigma_{2,3}^z\rangle = |+1, \pm M, -1, \pm M\rangle$ with $1/2 < M < S$ and higher-energy charge-proliferated states are also not scarred states [66]. From the previous estimate of the energies of scarred towers, we expect the revival period to be $T \approx 6.25\pi S$. However, in practice the energy spacing between towers varies along the spectrum. So the relevant energy spacing to consider is the one near $E = 0$, where the scarred states have the higher overlap with $|0_−\rangle$. This provides an estimate of $T \approx 5.13\pi S$, which much more accurately agrees with the numerical data.

We explore the effect of scarring on the dynamics of the mid-chain entanglement entropy $S_{L/2}(t)$, shown in Fig. 3 for $S = 1/2$ to 2. In all cases, starting in the extreme vacuum leads to an anomalously low $S_{L/2}(t)$ exhibiting significantly slower growth, whereas preparing the system in the charge-proliferated state leads to a rapid increase in the entanglement entropy. Except for the case of $S = 1/2$ where the extreme and physical vacua are the same, starting in the physical vacuum leads to behavior qualitatively similar to that of the charge-proliferated state, with a rapid growth in $S_{L/2}(t)$. These findings are consistent with nonthermal scarred dynamics only when the initial state is prepared in the extreme vacuum.
Of great experimental interest in the observation of scarring behavior are local observables, which are expected to show persistent oscillations up to times sufficiently longer than the fastest timescales of the system [1]. In the following, we explore this by computing the dynamics of the chiral condensate

$$\mathcal{C}(t) = \frac{1}{2} + \frac{1}{2L} \sum_{j=1}^{L} (-1)^j \langle \psi(t) | \hat{\sigma}_j^z | \psi(t) \rangle,$$

which is a measure of how strongly the chiral symmetry corresponding to fermions in the theory is (spontaneously) broken by the dynamics of the theory. The results shown in Fig. 4 for $S = 1/2$ to $2$ show strikingly nonthermal behavior for quenches starting in $|0_\mu\rangle$. Indeed, over the timescales we simulate there are persistent oscillations in the signal and no equilibration, in agreement with experimental results for the spin-1/2 $U(1)$ QLM [1, 7]. The oscillations in $\mathcal{C}(t)$ have twice the frequency of the fidelity revivals. This happens when the state comes back to itself, but also when there is state transfer to the other extreme vacuum. As $S$ is increased, we also see more oscillations in-between the large ones. These consist of an alternation of vacua and charge-proliferated states with different value of field [66]. On the other hand, systems prepared in $|0_\mu\rangle$ (for $S > 1/2$) or $|\text{CP}\rangle$ show relatively fast thermalization, where oscillations are quickly suppressed in the dynamics, consistent with the ETH.

As such, we have demonstrated that for a quench at $\mu = g = 0$, the spin-$S$ $U(1)$ QLM (1) exhibits scarring behavior when the system is initially prepared in an extreme vacuum. The underlying scarring mechanism is precocious of a “large” spin of magnitude $SL$ [68]. The relation between scarring and the Hilbert space constraint can be firm up by studying the structure of the adjacency graph of the Hamiltonian [66]. These extreme vacua are product states that can be naturally explored in SQM experiments [49, 50] even though they are otherwise inaccessible in lattice QED.

**Detuned scarring.**—In a recent study [7], it has been shown theoretically and demonstrated experimentally that there are scarring regimes beyond the resonant one discussed above. This was demonstrated in the spin-1/2 $U(1)$ QLM by starting in the charge-proliferated state and performing a quench at finite mass (detuning).

Motivated by the question as to whether QMBS persist in lattice QED for physically relevant initial states, we explore these *detuned* scarring regimes in the spin-3/2 $U(1)$ QLM starting in either the physical vacuum $|0_\mu\rangle$ or the charge-proliferated state $|\text{CP}\rangle$. As shown in Fig. 5, the overlap of these initial states with the eigenstates of the quench Hamiltonian (1) at $\mu = 0.486J$ and $g^2 = 0.6J$ shows distinctive towers equally spaced in energy, similar to the known case of $S = 1/2$ [7]. Also displayed in Fig. 5 is the fidelity dynamics for each of $|0_\mu\rangle$ and $|\text{CP}\rangle$ upon quenching them with this Hamiltonian, where we see persistent revivals up to all considered evolution times. We also arrive at a similar picture for other values of $S$, and in fact we find a wide range of values of $(\mu, g^2)$ over which scarring behavior emerges [66].

Given that $|0_\mu\rangle$ and $|\text{CP}\rangle$ are both physically relevant...
in lattice QED in one spatial dimension, and since the latter has been shown to be achieved at relatively small values of $S$ both in [67, 69, 70] and out of equilibrium [71], our results suggest that QMBS may play a role for understanding dynamics in physically interesting regimes as well.

**Summary.**—In conclusion, we have investigated QMBS in the paradigmatic spin-$S$ U(1) QLM, a staple of modern SQM experiments on lattice gauge theories. We have shown that the regime of resonant scarring for quenches at zero mass, prevalent in the literature in the case of $S = 1/2$, is also present in the case of $S > 1/2$ when the system is initially prepared in an extreme vacuum, where the local electric field takes on its largest possible eigenvalue. This has been done by calculating in ED the overlap of the extreme vacuum with the eigenstates of the quench Hamiltonian, showing clear towers equally spaced in energy, which are a hallmark of QMBS. Furthermore, we have shown consistent revivals in the dynamics of the fidelity, an anomalously low and slowly growing mid-chain entanglement entropy, and persistent oscillations in the dynamics of the chiral condensate over all times. These extreme vacua are not physical as ground states in lattice QED yet are easily implementable product states exhibiting clear scarring behavior that can be explored in SQM experiments.

We have also uncovered detuned scarring in the spin-$S$ U(1) QLM arising from quenches at small nonzero mass and electric-field coupling, when the system is initially prepared in either the physical vacuum, where the local electric field takes on its lowest possible eigenvalue, or the charge-proliferated state. The overlap of these states with the eigenstates of the quench Hamiltonian exhibits distinctive towers at equal spacing in energy, and the corresponding fidelity dynamics shows consistent revivals over all accessible times. These initial states are physically relevant as low-energy states in lattice QED. Given that recent works have shown convergence to the latter limit in and out of equilibrium already at $S \gtrsim 3/2$, our results suggest that this detuned scarring regime may already exist in lattice QED.

J.C.H. and J.-Y.D. are very grateful to Giuliano Giudici for insightful discussions and valuable comments. J.C.H. acknowledges funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programm (Grant Agreement no 948141) — ERC Starting Grant SimUcQuam, and by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany’s Excellence Strategy – EXC-2111 – 390814868. We acknowledge support by EPSRC grants EP/R020612/1 (Z.P.) and EP/R513258/1 (J.-Y.D.). A.H. and Z.P. acknowledge support by the Leverhulme Trust Research Leadership Award RL-2019-015. A.H. acknowledges funding provided by the Institute of Physics Belgrade, through the grant by the Ministry of Education, Science, and Technological Development of the Republic of Serbia.

Statement of compliance with EPSRC policy framework on research data: This publication is theoretical work that does not require supporting research data.

*jad.halimeh@physik.lmu.de*

[1] Hannes Bernien, Sylvain Schwartz, Alexander Keesling, Harry Levine, Ahmed Omran, Hannes Pichler, Soon-won Choi, Alexander S. Zibrov, Manuel Endres, Markus Greiner, Vladan Vuletić, and Mikhail D. Lukin, “Probing many-body dynamics on a 51-atom quantum simulator.” *Nature* **551**, 579–584 (2017).

[2] Sanjay Moudgalya, Stephan Rachel, B. Andrei Bernevig, and Nicolas Regnault, “Exact excited states of nonintegrable models.” *Phys. Rev. B* **98**, 235155 (2018).

[3] Hongzheng Zhao, Joseph Vovrosh, Florian Mintert, and Johannes Knolle, “Quantum many-body scars in optical lattices,” *Phys. Rev. Lett.* **124**, 160604 (2020).

[4] Hongzheng Zhao, Adam Smith, Florian Mintert, and Johannes Knolle, “Orthogonal quantum many-body scars,” *Phys. Rev. Lett.* **127**, 150601 (2021).

[5] D. Bluvstein, A. Omran, H. Levine, A. Keesling, G. Semeghini, S. Ebadi, T. T. Wang, A. A. Michailidis, N. Maskara, W. W. Ho, S. Choi, M. Serbyn, M. Greiner, V. Vuletić, and M. D. Lukin, “Controlling quantum many-body dynamics in driven Rydberg atom arrays,” *Science* **371**, 1355–1359 (2021).

[6] Paul Niklas Jepsen, Yoo Kyung Lee, Hanzhen Lin, Ivana Dimitrova, Yair Margalit, Wen Wei Ho, and Wolfgang Ketterle, “Catching Bethe phantoms and quantum many-body scars: Long-lived spin-helix states in Heisenberg magnets,” arXiv preprint (2021), arXiv:2110.12043 [cond-mat.quant-gas].

[7] Guo-Xian Su, Hui Sun, Ana Hudomal, Jean-Yves Desaules, Zhao-Yu Zhou, Bing Yang, Jad C. Halimeh, Zhen-Sheng Yuan, Zlatko Papić, and Jian-Wei Pan, “Observation of unconventional many-body scarring in a quantum simulator,” arXiv preprint (2022), arXiv:2201.00521.

[8] Pengfei Zhang, Hang Dong, Yu Gao, Liangtian Zhao, Jie Hao, Qijiang Guo, Jiachen Chen, Jinfeng Deng, Bobo Liu, Wenhui Ren, Yunyan Yao, Xu Zhang, Shibao Xu, Ke Wang, Feitong Jin, Xuhao Zhu, Hekang Li, Chao Song, Zhen Wang, Fangli Liu, Zlatko Papić, Lei Ying, H. Wang, and Ying-Cheng Lai, “Many-body hilbert space scarring on a superconducting processor,” arXiv preprint (2022), arXiv:2201.03438.

[9] Sanjay Moudgalya, Nicolas Regnault, and B. Andrei Bernevig, “Entanglement of exact excited states of Affleck-Kennedy-Lieb-Tasaki models: Exact results, many-body scars, and violation of the strong eigenstate thermalization hypothesis,” *Phys. Rev. B* **98**, 235156 (2018).

[10] Cheng-Ju Lin and Oleksii I. Motrunich, “Exact quantum many-body scar states in the Rydberg-blockaded atom chain,” *Phys. Rev. Lett.* **122**, 173401 (2019).

[11] C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, and Z. Papić, “Weak ergodicity breaking from quantum many-body scars,” *Nature Physics* **14**, 745–749 (2018).

[12] Michael Schecter and Thomas Iadecola, “Weak ergodicity breaking and quantum many-body scars in spin-1 XY...
magnets,” Phys. Rev. Lett. 123, 147201 (2019).
[13] Wen Wei Ho, Soonwon Choi, Hansne Pichler, and Mikhail D. Lukin, “Periodic orbits, entanglement, and quantum many-body scars in constrained models: Matrix product state approach,” Phys. Rev. Lett. 122, 040603 (2019).
[14] C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, and Z. Papić, “Quantum scarred eigenstates in a Rydberg atom chain: Entanglement, breakdown of thermalization, and stability to perturbations,” Phys. Rev. B 98, 155134 (2018).
[15] Mark Srednicki, “Chaos and quantum thermalization,” Phys. Rev. E 50, 888–901 (1994).
[16] J. M. Deutsch, “Quantum statistical mechanics in a closed system,”Phys. Rev. A 43, 2046–2049 (1991).
[17] Marcos Rigol, Vanja Dunjko, and Maxim Olshanii, “Thermalization and its mechanism for generic isolated quantum systems,” Nature 452, 854–858 (2008).
[18] J. Eisert, M. Friesdorf, and C. Gogolin, “Quantum many-body systems out of equilibrium,” Nature Physics 11, 124–130 (2015).
[19] Luca D’Alessio, Yariv Kafri, Anatoli Polkovnikov, and Marcos Rigol, “From quantum chaos and eigenstate thermalization to statistical mechanics and thermodynamics,” Advances in Physics 65, 239–362 (2016).
[20] Joshua M Deutsch, “Eigenstate thermalization hypothesis,” Reports on Progress in Physics 81, 082001 (2018).
[21] Daniel K. Mark, Cheng-Ju Lin, and Oleksi I. Motrunich, “Unified structure for exact towers of scar states in the Affleck-Kennedy-Lieb-Tasaki and other models,” Phys. Rev. B 101, 195131 (2020).
[22] Sanjay Moudgalya, Nicolas Regnault, and B. Andrei Bernevig, “q-pairing in Hubbard models: From spectrum generating algebras to quantum many-body scars,” Phys. Rev. B 102, 058140 (2020).
[23] Nicholas O’Dea, Fiona Burnell, Anushya Chandran, and Vedika Khemani, “From tunnels to towers: Quantum scars from Lie algebras and q-deformed Lie algebras,” Phys. Rev. Research 2, 043305 (2020).
[24] P. Pakrouski, P. N. Pallegar, F. K. Popov, and I. R. Klebanov, “Many-body scars as a group invariant sector of Hilbert space,” Phys. Rev. Lett. 125, 230602 (2020).
[25] Naoto Shiraishi and Takashi Mori, “Systematic construction of counterexamples to the eigenstate thermalization hypothesis,” Phys. Rev. Lett. 119, 030601 (2017).
[26] Maksym Serbyn, Dmitry A. Abanin, and Zlatko Papić, “Quantum many-body scars and weak breaking of ergodicity,” Nature Physics 17, 675–685 (2021).
[27] Sanjay Moudgalya, B. Andrei Bernevig, and Nicolas Regnault, “Quantum many-body scars and Hilbert space fragmentation: A review of exact results,” arXiv preprint (2021), arXiv:2109.09724.
[28] A. Omran, H. Levine, A. Keesling, G. Semeghini, T. T. Wang, S. Ebadi, H. Bernien, A. S. Zibrov, H. Pichler, S. Choi, J. Cui, M. Rossignolo, P. Rembold, S. Montangero, T. Calarco, M. Endres, M. Greiner, V. Vuletić, and M. D. Lukin, “Generation and manipulation of Schrödinger cat states in Rydberg atom arrays,” Science 365, 570–574 (2019).
[29] Shane Dooley, “Robust quantum sensing in strongly interacting systems with many-body scars,” PRX Quantum 2, 020330 (2021).
[30] Jean-Yves Desaules, Francesca Pietracaprina, Zlatko Papić, John Good, and Silvia Pappalardi, “Quantum many-body scars have extensive multipartite entanglement,”arXiv preprint (2021), arXiv:2109.09724.
[31] Marlon Brenes, Marcello Dalmon, Markus Heyl, and Antonello Scardicchio, “Many-body localization dynamics from gauge invariance,” Phys. Rev. Lett. 120, 030601 (2018).
[32] Federica M. Surace, Paolo P. Mazza, Giuliano Giudici, Alessio Lerose, Andrea Gambassi, and Marcello Dalmon, “Lattice gauge theories and string dynamics in Rydberg atom quantum simulators,” Phys. Rev. X 10, 021041 (2020).
[33] Debashis Banerjee and Arnab Sen, “Quantum scars from zero modes in an abelian lattice gauge theory on ladders,” Phys. Rev. Lett. 126, 226001 (2021).
[34] Adith Sai Arathamottil, Utso Bhattacharya, Daniel González-Cuadra, Maciej Lewenstein, Luca Barbiero, and Jakub Zakrzewski, “Scar states in deconfined $Z_2$ lattice gauge theories,” arXiv preprint (2022), arXiv:2201.10260.
[35] Saptarshi Biswas, Debashis Banerjee, and Arnab Sen, “Scars from protected zero modes and beyond in $U(1)$ quantum link and quantum dimer models,” arXiv preprint (2022), arXiv:2202.03451.
[36] S. Weinberg, The Quantum Theory of Fields, Vol. 2: Modern Applications (Cambridge University Press, 1995).
[37] C. Gatttringer and C. Lang, Quantum Chromodynamics on the Lattice: An Introductory Presentation, Lecture Notes in Physics (Springer Berlin Heidelberg, 2009).
[38] A. Zee, Quantum Field Theory in a Nutshell (Princeton University Press, 2003).
[39] R.P. Feynman and P. Cziffra, Quantum Electrodynamics, A lecture note and reprint series (Basic Books, 1962).
[40] Esteban A. Martinez, Christine A. Muschik, Philipp Schindler, Daniel Nigg, Alexander Erhard, Markus Heyl, Philipp Hauke, Marcello Dalmon, Thomas Monz, Peter Zoller, and Rainer Blatt, “Real-time dynamics of lattice gauge theories with a few-qubit quantum computer,” Nature 534, 516–519 (2016).
[41] Christine Muschik, Markus Heyl, Esteban Martinez, Thomas Monz, Philipp Schindler, Berit Vogell, Marcello Dalmon, Philipp Hauke, Rainer Blatt, and Peter Zoller, “(U(1) Wilson lattice gauge theories in digital quantum simulators,” New Journal of Physics 19, 103020 (2017).
[42] N. Klco, E. F. Dumitrescu, A. J. McCaskey, T. D. Morris, R. C. Pooser, M. Sanz, E. Solano, P. Lougovski, and M. J. Savage, “Quantum-classical computation of Schwinger model dynamics using quantum computers,” Phys. Rev. A 98, 032331 (2018).
[43] Alexander Keesling, Ahmed Omran, Harry Levine, Hannes Bernien, Hansne Pichler, Soonwon Choi, Rmine Samajdar, Sylvain Schwartz, Pietro Silvi, Subir Sachdev, Peter Zoller, Manuel Endres, Markus Greiner, Vladan Vuletić, and Mikhail D. Lukin, “Quantum Kibble–Zurek mechanism and critical dynamics on a programmable Rydberg simulator,” Nature 568, 207–211 (2019).
[44] C. Kokail, C. Maier, R. van Bijnen, T. Brydges, M. K. Joshi, P. Jurcevic, C. A. Muschik, P. Silvi, R. Blatt, C. F. Roos, and P. Zoller, “Self-verifying variational quantum simulation of lattice models,” Nature 569, 355–360 (2019).
[45] Frederik Görg, Kilian Sandholzer, Joaquín Minguzzi, Rémi Desbuquois, Michael Messer, and Tilman
Esslenger, “Realization of density-dependent Peierls phases to engineer quantized gauge fields coupled to ultracold matter,” Nature Physics 15, 1161–1167 (2019).

Christian Schweizer, Fabian Grusdt, Moritz Berngruber, Luca Barbiero, Eugene Demler, Nathan Goldman, Immanuel Bloch, and Monika Aidsドルger, “Floquet approach to Z2 lattice gauge theories with ultracold atoms in optical lattices,” Nature Physics 15, 1168–1173 (2019).

Alexander Mil, Torsten V. Zache, Apoorva Hegde, Andy Xia, Rohit P. Bhatt, Markus K. Oberthaler, Philipp Hauke, Jürgen Berges, and Fred Jendrzejewski, “A scalable realization of local U(1) gauge invariance in cold atomic mixtures,” Science 367, 1128–1130 (2020).

Natalie Klco, Martin J. Savage, and Jesse R. Stryker, “SU(2) non-Abelian gauge field theory in one dimension on digital quantum computers,” Phys. Rev. D 101, 074512 (2020).

Bing Yang, Hui Sun, Robert Ott, Han-Yi Wang, Torsten V. Zache, Jad C. Halimeh, Zhen-Sheng Yuan, Philipp Hauke, and Jian-Wei Pan, “Observation of gauge invariance in a 71-site Bose–Hubbard quantum simulator,” Nature 587, 392–396 (2020).

Zhao-Yu Zhou, Guo-Xian Su, Jad C. Halimeh, Robert Ott, Hui Sun, Philipp Hauke, Bing Yang, Zhen-Sheng Yuan, Jürgen Berges, and Jian-Wei Pan, “Thermalization dynamics of a gauge theory on a quantum simulator,” arXiv preprint (2021), arXiv:2107.13563 [cond-mat.quant-gas].

Immanuel Bloch, Jean Dalibard, and Wilhelm Zwerger, “Many-body physics with ultracold gases,” Rev. Mod. Phys. 80, 885–964 (2008).

Waseem S. Bakr, Jonathan I. Gillen, Amy Peng, Simon Fölling, and Markus Greiner, “A quantum gas microscope for detecting single atoms in a Hubbard-regime optical lattice,” Nature 462, 74–77 (2009).

U.-J. Wiese, “Ultracold quantum gases and lattice systems: quantum simulation of lattice gauge theories,” Annalen der Physik 525, 777–796 (2013).

Mari Carmen Bañuls, Rainer Blatt, Jacopo Catani, Alessio Celi, Juan Ignacio Cirac, Marcello Dalibart, Leonardo Fallani, Karl Jansen, Maciej Lewenstein, Simon Montangero, Christian A. Muschik, Benni Reznik, Enrique Rico, Luca Tagliacozzo, Karel Van Acoleyen, Frank Verstraete, Uwe-Jens Wiese, Matthew Wingate, Jakub Zakrzewski, and Peter Zoller, “Simulating lattice gauge theories within quantum technologies,” The European Physical Journal D 74, 165 (2020).

Yuri Alexeev, Dave Bacon, Kenneth R. Brown, Robert Calderbank, Lincoln D. Carr, Frederic T. Chong, Brian DeMarco, Dirk Englund, Edward Farhi, Bill Fefferman, Alexey V. Gorshkov, Andrew Houck, Jungsang Kim, Shelby Kimmel, Michael Lange, Seth Lloyd, Mikhail D. Lukin, Dmitri Maslov, Peter Maunz, Christopher Monroe, John Preskill, Martin Roetteler, Martin J. Savage, and Jeff Thompson, “Quantum computer systems for scientific discovery,” PRX Quantum 2, 017001 (2021).

Monika Aidsドルger, Luca Barbiero, Alejandro Bermudez, Titas Chanda, Alexandre Dauphin, Daniel González-Cuadra, Przemysław R. Grzybowski, Simon Hands, Fred Jendrzejewski, Johannes Jüniemann, Gediminas Juzeliūnas, Valentin Kasper, Angelo Piga, Shi-Ju Ran, Matteo Rizzi, Germán Sierra, Luca Tagliacozzo, Emanuele Tirrito, Torsten V. Zache, Jakub Zakrzewski, Erez Zohar, and Maciej Lewenstein, “Cold atoms meet lattice gauge theory,” Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 380, 20210064 (2022).

Erez Zohar, “Quantum simulation of lattice gauge theories in more than one space dimension: requirements, challenges and methods,” Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 380, 20210069 (2022).

Natalie Klco, Alessandro Roggero, and Martin J. Savage, “Standard model physics and the digital quantum revolution: Thoughts about the interface,” arXiv preprint (2021), arXiv:2107.04769 [quant-ph].

Lucas Homeier, Christian Schweizer, Monika Aidsドルger, Arkady Fedorov, and Fabian Grusdt, “z2 lattice gauge theories and Kitaev’s toric code: A scheme for quantum analog simulation,” Phys. Rev. B 104, 085138 (2021).

S Chandrasekharan and U.-J Wiese, “Quantum link models: A discrete approach to gauge theories,” Nuclear Physics B 492, 455 – 471 (1997).

Julian Schwinger, “Gauge invariance and mass. II,” Phys. Rev. 128, 2425–2429 (1962).

Sidney Coleman, R Jackiw, and Leonard Susskind, “Charge shielding and quark confinement in the massive Schwinger model.” Annals of Physics 93, 267–275 (1975).

D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, U.-J. Wiese, and P. Zoller, “Atomic quantum simulation of dynamical gauge fields coupled to fermionic matter: From string breaking to evolution after a quench,” Physical Review Letters 109 (2012), 10.1103/physrevlett.109.175302.

V Kasper, F Hebenstreit, F Jendrzejewski, M K Oberthaler, and J Berges, “Implementing quantum electrodynamics with ultracold atomic systems,” New Journal of Physics 19, 023030 (2017).

Bhaskar Mukherjee, Zi Cai, and W. Vincent Liu, “Constraint-induced breaking and restoration of ergodicity in spin-1 PXP models,” Phys. Rev. Research 3, 033201 (2021).

Jean-Yves Desaules, Ana Hudomal, Debashis Banerjee, Arnab Sen, Zlatko Papić, and Jad C. Halimeh, “Prominent quantum many-body scars in a truncated schwinger model,” (2022), 10.48550/ARXIV.2204.01745.

Torsten V. Zache, Maarten Van Damme, Jad C. Halimeh, Philipp Hauke, and Debashis Banerjee, “Achieving the continuum limit of quantum link lattice gauge theories on quantum devices,” arXiv preprint (2021), arXiv:2104.00025 [hep-lat].

Soonwon Choi, Christopher J. Turner, Hannes Pichler, Wen Wei Ho, Alexios A. Michailidis, Zlatko Papić, Maksym Serbyn, Mikhail D. Lukin, and Dmitry A. Abanin, “Emergent SU(2) dynamics and perfect quantum many-body scars,” Phys. Rev. Lett. 122, 220603 (2019).

Boye Buyens, Simone Montangero, Jutho Haegeman, Frank Verstraete, and Karel Van Acoleyen, “Finite-representation approximation of lattice gauge theories at the continuum limit with tensor networks,” Phys. Rev. D 95, 094509 (2017).

Mari Carmen Bañuls and Krzysztof Cichy, “Review on novel methods for lattice gauge theories,” Reports on Progress in Physics 83, 024401 (2020).

Jad C. Halimeh, Maarten Van Damme, Torsten V. Zache, Debashis Banerjee, and Philipp Hauke,
“Achieving the quantum field theory limit in far-from-equilibrium quantum link models,” arXiv preprint (2021), arXiv:2112.04501 [cond-mat.quant-gas].