the Non-Gaussianity of Racetrack Inflation Models

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Abstract

In this paper, we use the result in [7] to calculate the non-Gaussianity of the racetrack models in [3, 5]. The two models give different non-Gaussianities. Both of them are reasonable.

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1 Introduction

In cosmology, the inflation paradigm plays an important role. The central idea of inflation is very simple. But it has proved difficult to discriminate among the large number of different models that have been developed to date [1]. Both the simplest classes of inflation models and the most complicated classes may predict Gaussian-distributed perturbations and nearly scale-invariant spectra of the primordial density perturbations [2]. However it is believed that the deviation away from the Gaussian statistics may be a
potential powerful discriminant between the competing inflationary models. On the other hand, physicists are always trying to understand cosmological inflation within the deeper theory, e.g. string theory. In the past two years, continued progress has been made in identifying how the inflation arises from within string theory. Recently, the racetrack inflation models, which are based on the Calabi-Yau compactification of type IIB String, attract the attention. Ref. [3] suggests a simple racetrack inflation model, where only a single Kähler modulus is used. And in Ref. [5] a complicated racetrack inflation model is given with two Kähler moduli (See [5] for comparison between the two models.). In this paper, we try to calculate the non-Gaussianities, the non-linear parameter $f_{NL}$, of the two models.

The racetrack models give effectively multi-field inflation models, and, for a multiple-field inflation model, the expression of $f_{NL}$ is very complicated. Fortunately, in [13], Lyth and Rodriguez have shown that the non-Gaussianity of the curvature perturbation in multiple field models can be simply expressed in the so-called “$\delta N$-formalism” [4]. Further, in [6], the authors have given the expression of $f_{NL}$ involving the metric of the field space explicitly. Further more, in [7], $f_{NL}$ has been expressed in terms of the slow-rolling parameters. In this paper, we would adopt the result in [7] to calculate the non-linear parameter. Here we should note that the result in [7] is obtained by neglecting the non-adiabatic perturbations and the intrinsic non-Gaussianity of the fields (See [7] for details.). Of course, for the two racetrack models, due to the interacting terms in the effective actions the non-adiabatic perturbations would be generated unavoidably. And the intrinsic non-Gaussianity of the field perturbations exists too. However, the assumption of the Gaussian-distributed and adiabatic perturbations is in good agreement with the observation [8, 9]. So we suppose that in the racetrack models, this assumption is still good enough. Then we expect the dominant non-Gaussianity would be obtained by using the result in [7].

In this paper, we first give a brief summary of the result in [7]. Then we calculate the non-linear parameter of the two racetrack inflation models.

2 the non-linear parameter

This section we summarize the result in [7]. For the background, the effective action of the simple coupling system of Einstein gravity and scalar fields with
an arbitrary inflation potential $V(\varphi)$ is

$$S = \int \sqrt{-g} d^4x \left[ \frac{M_p^2}{2} R - \frac{1}{2} G_{IJ} \partial_\mu \varphi^I \partial^\mu \varphi^J - V(\varphi) \right], \quad (1)$$

where $G_{IJ} \equiv G_{IJ}(\varphi)$ represents the metric on the manifold parameterized by the scalar field values. And $8\pi G = M_p^{-2}$ represents the reduced Planck mass. Units are chosen such that $c = \hbar = 1$. And the Friedmann-Robertson-Walker metric is used,

$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j. \quad (2)$$

The non Gaussianity of the curvature is expressed in the form:

$$\zeta = \zeta_g - \frac{3}{5} f_{NL} (\zeta_g^2 - \langle \zeta_g^2 \rangle), \quad (3)$$

where $\zeta_g$ is Gaussian, with $\langle \zeta_g \rangle = 0$. And $f_{NL}$ is the non-linear parameter.

On the other hand, the curvature perturbation can be expressed as the difference between an initial space-flat fixed-$t$ slice and a final uniform energy density fixed-$t$ slice (see [11, 13, 12] for details),

$$\zeta(t, x) = \delta N, \quad (4)$$

where $N = \int H dt = \int \frac{\dot{a}}{a} dt$ is the integrated number of e-folds. Here and after, one dot denotes the derivative with respect to the time, $\dot{a} = da/dt$. Expanding the curvature perturbation to the second order [12], we get

$$\zeta \simeq N_I(t) \delta \varphi^I(x) + \frac{1}{2} N_{IJ}(t) \delta \varphi^I(x) \delta \varphi^J(x), \quad (5)$$

where $N_I = \frac{\partial N}{\partial \varphi^I}, \ N_{IJ} = \frac{\partial^2 N}{\partial \varphi^I \partial \varphi^J}$.

Equating the bispectrums of $\zeta$ obtained by using the two equations (5) and (3), we may get

$$f_{NL} = -\frac{5}{6} \times \frac{G^{IM} G^{KN} N_{I,J} N_{K,M} N_{M,N}}{(N_{IJ} G^{JJ})^2}, \quad (6)$$

where $G^{IJ}$ are the elements of the inverse metric of the field space. In this result, the intrinsic non-Gaussian part, $\sim \langle \delta \varphi^I(k_1) \delta \varphi^J(k_2) \delta \varphi^K(k_3) \rangle$, has been neglected.
Define one parameter, $\varepsilon_I$, as

$$\varepsilon_I \equiv -\frac{V_I M_p}{\sqrt{2V}},$$

(7)

where $V_I \equiv \frac{\partial V}{\partial \varphi^I}$. Then, using the slow-rolling approximation, the slow-rolling parameter, $\varepsilon$, can be expressed as

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} \simeq G^{IJ} \varepsilon_I \varepsilon_J.$$  

(8)

Further, neglecting the non-adiabatic perturbations of the inflation fields, we can get

$$N,I \simeq -\frac{1}{2\varepsilon V} V,I = \frac{\varepsilon_I}{\sqrt{2\varepsilon M_p}}.$$  

(9)

Now, the non-linear parameter may be expressed as

$$f_{NL} \simeq \frac{5M_p}{3\varepsilon} \times \beta,$$  

(10)

with

$$\beta = \frac{\varepsilon^2}{M_p} + \frac{G^{MN,L} G^{IJ} \varepsilon_I \varepsilon_J \varepsilon_M \varepsilon_N}{\sqrt{2}} - \frac{1}{2M_p} G^{IJ} G^{MN} \varepsilon_I \varepsilon_J \varepsilon_M \eta_{NL},$$

(11)

where $G^{MN,L} \equiv \frac{\partial G^{MN}}{\partial \varphi^L}$ and $\eta_{NL} \equiv \frac{V_{IJ} M_p}{V}$, $V,I,J \equiv \frac{\partial^2 V}{\partial \varphi^I \partial \varphi^J}$.

3 non-Gaussianity of the racetrack model with single modulus

In [3], basing on a simple extension of KKLT scenario [14], the authors suggest a racetrack inflationary model in the frame of string theory. This model is equivalent to a double-field inflationary model, with the effective action

$$S = \int \sqrt{-g} d^4x \left[ \frac{1}{2} R - \frac{3M_p^2}{4X^2} (\partial_{\mu} X \partial^{\mu} X + \partial_{\mu} Y \partial^{\mu} Y) - V(X,Y) \right],$$

(12)
with
\[
V(X, Y) = \frac{E}{X^\alpha} + \frac{e^{-aX}}{6X^2} [aA^2(aX + 3)e^{-aX} + 3W_0aA \cos(aY)] \\
+ \frac{e^{-bX}}{6X^2} [bB^2(bX + 3)e^{-bX} + 3W_0bB \cos(bY)] \\
+ \frac{e^{-(a+b)X}}{6X^2} [AB(2abX + 3a + 3b) \cos((a - b)Y)].
\] (13)

In this model, \(X\) and \(Y\) correspond to the scalar fields in Eq. (10), \(\varphi^I, I = 1, 2\). \(E, \alpha, a, A, b, B\) and \(W_0\) are constant parameters of this model. The non-zero components of the field-space metric are \(G_{11} = G_{22} = \frac{3M_p^2}{2X^2}\). In [3], the appropriate values of the parameters have been suggested,
\[
E = 4.14668 \times 10^{-12}, \alpha = 2, \\
a = \frac{2\pi}{100}, A = \frac{1}{50}, b = \frac{2\pi}{90}, B = -\frac{35}{1000}, W_0 = \frac{1}{5000}.
\] (14)

The inflationary saddle point is at
\[
X_{\text{saddle}} = 123.22, \quad Y_{\text{saddle}} = 0.
\] (15)

The units of the values above have been taken to be \(M_p = 1\).

Setting the initial point of the inflation at
\[
X = 123.22, \quad Y = 0.2,
\]
and then calculating the slow-rolling parameters at the COBE scale or equivalently at \(N \simeq 60\) which is the number of e-foldings before the end of the inflation, we get
\[
f_{NL} \simeq -0.16.
\] (16)

The observation [9] shows that the limits on primordial non-Gaussianity are \(-54 < f_{NL} < 114\) at the 95% confidence level. So this result fits the limits. Supposing the contribution of the non-adiabatic part and the intrinsic non-Gaussianity at the same order of this result, we may expect that the total non-Gaussianity of this model should be \(|f_{NL}| \sim 1\). This is still a reasonable result.
4 non-Gaussianity of the racetrack model with two Kähler moduli

In [5], by compactifying to the orientifold of degree 18 hypersurface $\mathbb{P}^4_{[1,1,1,6,9]}$, an elliptic fibered Calabi-Yau over $\mathbb{P}^2$, the authors suggest a racetrack inflation model with two Kähler moduli. The effective action of this model is

$$S = \int \sqrt{-g} d^4x [\frac{1}{2} R - \mathcal{L}_{\text{kin}} - V],$$

with

$$\mathcal{L}_{\text{kin}} = \frac{3}{8(X_2^{3/2} - X_1^{3/2})^2} \times \left\{ \frac{2X_1^{3/2} + X_2^{3/2}}{\sqrt{X_1}} (\partial X_1^2 + \partial Y_1^2) \right.\right.$$

$$\left. - 6 \sqrt{X_1X_2} (\partial X_1 \partial X_2 + \partial Y_1 \partial Y_2) \right.\right.$$

$$\left. + \frac{X_1^{3/2} + 2X_2^{3/2}}{\sqrt{X_2}} (\partial X_2^2 + \partial Y_2^2) \right\},$$

and

$$V = \frac{D}{(X_2^{3/2} - X_1^{3/2})^2} + \frac{216}{(X_2^{3/2} - X_1^{3/2})^2} \times \left\{ \frac{B^2 b(bX_2^2 + 2bX_1^{3/2} X_2^{1/2} + 3X_2)e^{-2bX_2}}{X_2} \right.\right.$$

$$\left. + \frac{A^2 a(3X_1 + 2aX_2^{3/2} X_1^{1/2} + aX_1^2)e^{-2aX_1}}{X_1} \right.\right.$$

$$\left. + 3BbW_0X_2 e^{-bX_2} \cos(bY_2) + 3AaW_0X_1 e^{-aX_1} \cos(aY_1) \right.\right.$$

$$\left. + 3ABe^{-aX_1-bX_2} (aX_1 + bX_2 + 2abX_1X_2) \cos(-aY_1 + bY_2) \right\}. \tag{19}$$

In this model, $X_1$, $Y_1$ and $X_2$, $Y_2$ correspond to the scalar fields in Eq.(1), $\varphi^I$, with $I = 1, 2, 3, 4$. And $D$, $W_0$, $a$, $A$, $b$, $B$ are the constant parameters of this model. In [5], the appropriate values of these parameters are suggested,

$$D = 6.21 \times 10^{-9}, \quad W_0 = 5.227 \times 10^{-6}, \tag{20}$$

$$A = 0.56, \quad B = 7.47 \times 10^{-5}, \quad a = \frac{2\pi}{40}, \quad b = \frac{2\pi}{258}. \tag{21}$$

And the inflationary saddle point is at

$$X_1 = 108.96, \quad X_2 = 217.69, \quad Y_1 = 20, \quad Y_2 = 129. \tag{22}$$
As before, the units of the values above are taken to be $M_p = 1$.

In [5], an example of the inflation with 980 e-foldings is suggested. Again, Calculating the slow-rolling parameters at the COBE scale, we get

$$f_{NL} \simeq 0.02.$$  

(23)

Here, we note that we have taken the COBE scale at $N \simeq 60$ as in the section 3 in order to compare between the both models. This is also a reasonable result.

5 Summary and Discussion

Above, we have obtained the non-Gaussianities of the models in [3, 5] by using the result in [7]. Both of the models give the reasonable results, although the two results are different. By considering that in our result, the contribution of the intrinsic non-Gaussianity of the fields and the non-adiabatic perturbations are neglected, we do not think the difference is important.

However, our results illustrate that there exits remarkable difference between multi-field inflation models with the nontrivial metric of the field space and multi-field inflation models with the trivial metric of the field space (or single field inflation models). In [6], it has been revealed that, for a multi-field model with $G_{IJ} = \delta_{IJ}$, the contribution of Eq.(11) is about at the order of $\varepsilon$. But this analysis can not be applied to the models in [3, 5]. In fact, for the model in [3], at the COBE scale, we get

$$\varepsilon \simeq 3 \times 10^{-10}.$$  

And for the model in [5], at the COBE scale, we get

$$\varepsilon \simeq 9 \times 10^{-10}.$$  

So the non-Gaussianities of the two models are very large compared with the values of $\varepsilon$.

This is due to the third term on the right-hand side of Eq.(11). For the first two terms on the right-hand side of Eq.(11), they are expected to be at the order of $\varepsilon^2$. But the third term,

$$G^{JJ}G^{MN} \varepsilon_J \varepsilon_M \eta_{NL},$$
is different. We may expect that this term be at the order of $\varepsilon G^{MN} \eta_{NL}$, basing on the relation, $\varepsilon = G^{IJ} \varepsilon I \varepsilon J$. However, for a multi-field inflation with a non-trivial metric of field space, some components of the metric, $G^{MN}$, may be large. Then, even if $\eta_{IJ}$ is at the order of $\varepsilon$, the contribution of this term would be much larger than the first two terms.

This is just the case for the models in [3, 5]. In the model of [3], $G^{MN}$ is about $10^4$ and $\eta_{IJ}$ is about $10^{-4}$. Considering the prefactors, we expect the result (16) as obtained. For the model of [5], $G^{MN}$ is about $10^4$ and $\eta_{IJ}$ is about $10^{-2}$. Then we may expect the non-Gaussianity be at the order $10^2$. However, this is not the case because the opposite contribution of some components reduces the value. So we just obtain the result (23). But this reduction does not work during the initial stage of the inflation with 980 e-foldings in [3]. Calculating the non-Gaussianity at the initial point of the inflation, we get $f_{NL} \simeq -125$. This value is beyond the limits. Fortunately, for a inflation with 980 e-foldings, the scales of the perturbations generated during the initial stage of inflation are so large that the large non-Gaussianity calculated at the initial point does not matter the observation. However, our analysis above reveals that the large non-Gaussianity may be generated in a multi-field inflation model.

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