The Higgs Boson Mass in Gauge-Mediated Supersymmetry-Breaking Models with Generalized Messenger Sectors

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Abstract

The lighter neutral scalar Higgs mass is examined in gauge-mediated supersymmetry-breaking models in which the messenger sector responsible for SUSY breaking is allowed to involve more general sets of $SU(3) \otimes SU(2) \otimes U(1)$ multiplets than those contained in the $SU(5) 5 + \bar{5}$ or $10 + \bar{10}$ multiplets. The largest mass for the lighter scalar Higgs is found to be 132 GeV when the breaking parameter $\Lambda$ is taken to be 100 TeV. Thus the predictions for the lightest Higgs mass can be tested at the upgraded Tevatron runs for these general classes of GMSB models.
1 Introduction

One of the main goals at present and the next generation of colliders is to search for evidence of the Higgs boson of the Standard Model (SM). Theories of supersymmetry (SUSY) possess extended Higgs sectors with multiple Higgs bosons. Common to all models of low energy supersymmetry where the couplings remain perturbative up to very high energies, however, is the prediction of a relatively low mass Higgs boson \[1\]. Thus the search for the lightest Higgs boson is a crucial test for SUSY theories. The question arises as to the kinds of limits that can be placed on the mass of the lightest Higgs boson in various SUSY models.

In this paper, we explore the values that the mass of the lightest Higgs boson can have in theories of gauge-mediated supersymmetry-breaking (GMSB). GMSB models provide an alternative to models where supersymmetry breaking is communicated to the visible sector by gravitational interactions \[2\]. The defining characteristic of GMSB models is that the SUSY breaking is communicated to the visible sector by gauge interactions. The messenger sector is composed of some set of superfields with SM couplings, but which are not part of the spectrum of the Minimal Supersymmetric Standard Model (MSSM). In minimal models of GMSB, these superfields are assumed to form complete SU(5) multiplets so that the apparent unification of the gauge couplings can be preserved. However, the superfields that act as messengers need not form complete GUT multiplets. Other superfields can be present at the messenger scale that do not act as messengers and have little or no effect on the masses of the MSSM particles, but can still participate in ensuring gauge coupling unification \[3\]. The messenger sector in these generalized GMSB models thereby contains superfields that transform as \(SU(3) \otimes SU(2) \otimes U(1)\) multiplets, but not as complete GUT multiplets. We will explore the values that the mass of the lightest Higgs boson can acquire in minimal GMSB models and the generalized GMSB models discussed in \[3\].

We seek the answers to two questions. First, is there a bound on the lightest Higgs mass in these general classes of \(SU(3) \otimes SU(2) \otimes U(1)\) multiplets as we vary the parameters of the models? Second, are the predictions for the lightest Higgs mass still within the detectable range of the upcoming Tevatron runs?

2 Generalized Messenger Sector

Here we will outline the generalization of the messenger sector as suggested by \[3\].

We assume that the messenger fields couple to a single chiral superfield \(S\), so that the superpotential contains the term

\[ W \ni \lambda_i S \Phi_i \Phi_i. \]  

(1)

The induced gaugino mass parameters are (for \(a = 1, 2, 3\))

\[ M_a = \frac{\alpha_a}{4\pi} \frac{F}{S} \sum_i n_{ai} g(x_i) \]  

(2)

where

\[ x_i = \left| \frac{F}{\lambda_i S^2} \right| \]  

(3)

where \(S\) and \(F\) here represent the vacuum expectation values (VEVs) of the chiral scalar superfields \(S\) and \(F\), respectively. The function \(g\) is defined by

\[ g(x) \equiv \frac{1}{x^2} [(1 + x) \log(1 + x) + (1 - x) \log(1 - x)] \]  

(4)

and \(n_{ai} = 1\) for a pair of \(\overline{N} + N\) of SU(\(N\)), and \(n_{ai} = 3\) for an antisymmetric two-index tensor of SU(\(N\)). We use the GUT normalization for \(\alpha_1\) so that \(n_1 = \frac{8}{3} Y^2\) for a pair with hypercharge \(Y = Q - T_3\).

We define the convenient numbers

\[ N_a \equiv \sum_i n_{ai}. \]  

(5)
The requirement that the MSSM gauge couplings should stay perturbative ($\alpha_i \leq 0.2$) up to the unification scale amounts to

\begin{align}
N_1 &\leq 4, \\
N_2 &\leq 4, \\
N_3 &\leq 4.
\end{align}

We shall therefore consider messenger sectors that contain up to four sets of SU(5) multiplets $5 + \overline{5}$, or one $10 + \overline{10}$, or one $5 + \overline{5}$, or one $10 + \overline{10}$. We will also consider generalized messenger sectors consisting of various multiplets of $SU(3) \times SU(2) \times U(1)$ as in [3]. Consider fields that transform under $SU(3) \times SU(2) \times U(1)$ as

\begin{align}
Q + \overline{Q} &= (\mathbf{3}, \mathbf{2}, \frac{1}{2}) + (\mathbf{3}, \mathbf{2}, -\frac{1}{2}), \\
U + \overline{U} &= (\mathbf{3}, \mathbf{1}, -\frac{2}{3}) + (\mathbf{3}, \mathbf{1}, \frac{2}{3}), \\
D + \overline{D} &= (\mathbf{3}, \mathbf{1}, \frac{1}{3}) + (\mathbf{3}, \mathbf{1}, -\frac{1}{3}), \\
L + \overline{L} &= (\mathbf{1}, \mathbf{2}, -\frac{4}{3}) + (\mathbf{1}, \mathbf{2}, \frac{2}{3}), \\
E + \overline{E} &= (\mathbf{1}, \mathbf{1}, 1) + (\mathbf{1}, \mathbf{1}, -1).
\end{align}

The number of sets of $Q + \overline{Q}$ fields will be denoted by $n_Q$, the number of sets of $U + \overline{U}$ by $n_U$, etc. The number of sets of SU(5) $5 + \overline{5}$ will be denoted by $n_5$, and the number of $10 + \overline{10}$ by $n_{10}$. Their contributions to the $N_a$ are given by

\begin{align}
N_1 &= \frac{1}{2} (n_Q + 8n_U + 2n_{10} + 3n_L + 6n_E) + n_5 + 3n_{10}, \\
N_2 &= 3n_Q + n_L + n_5 + 3n_{10}, \\
N_3 &= 2n_Q + n_U + n_{10} + n_5 + 3n_{10}.
\end{align}

Requiring gauge coupling unification as well as gauge coupling perturbativity, we have the requirement that $(n_Q, n_U, n_{10}, n_L, n_E)$ be less than $(1,0,2,1,2)$, or $(1,1,1,1,1)$, or $(1,2,0,1,0)$, or $(0,0,4,4,0)$.

Using these fields, the possible combinations satisfying this condition with $N_1 \neq 0$, $N_2 \neq 0$, $N_3 \neq 0$ are

\begin{align*}
&\{1\}, \{2\}, (\mathbf{1}, 3, 2), (\mathbf{2}, 3, 3), (\mathbf{3}, 4, 2), (1, 1, 1), (1, 3, 4), (\mathbf{2}, 1, 1), (\mathbf{4}, 3, 2), (\mathbf{2}, 2, 1), (\mathbf{4}, 1, 2), (\mathbf{4}, 4, 4), (\mathbf{2}, 2, 2), (\mathbf{2}, 4, 2), (\mathbf{4}, 1, 1), (\mathbf{4}, 1, 4), (\mathbf{4}, 3, 1), (\mathbf{4}, 3, 4), (\mathbf{4}, 2, 3), (\mathbf{4}, 4, 3), (\mathbf{4}, 1, 2), (\mathbf{4}, 3, 2), (\mathbf{4}, 2, 4), (\mathbf{4}, 4, 4), (\mathbf{3}, 3, 3), (\mathbf{3}, 3, 4), (\mathbf{3}, 4, 3), (\mathbf{3}, 4, 4), (\mathbf{3}, 3, 1), (\mathbf{3}, 3, 2), (\mathbf{3}, 4, 2), (\mathbf{3}, 4, 3), (\mathbf{3}, 4, 4), (\mathbf{3}, 3, 4), (\mathbf{3}, 4, 3), (\mathbf{3}, 4, 4), (\mathbf{3}, 3, 1), (\mathbf{3}, 3, 2), (\mathbf{3}, 4, 2), (\mathbf{3}, 4, 3), (\mathbf{3}, 4, 4), (\mathbf{3}, 3, 4), (\mathbf{3}, 4, 3), (\mathbf{3}, 4, 4), (\mathbf{3}, 3, 1), (\mathbf{3}, 3, 2), (\mathbf{3}, 4, 2), (\mathbf{3}, 4, 3), (\mathbf{3}, 4, 4), (\mathbf{3}, 3, 4), (\mathbf{3}, 4, 3), (\mathbf{3}, 4, 4), (\mathbf{3}, 3, 1), (\mathbf{3}, 3, 2), (\mathbf{3}, 4, 2), (\mathbf{3}, 4, 3), (\mathbf{3}, 4, 4), (\mathbf{3}, 3, 4), (\mathbf{3}, 4, 3), (\mathbf{3}, 4, 4).
\end{align*}

\section{Calculation of the Higgs Mass}

If we assume that all $\lambda_i$ are equal, and replace the VEVs with $\Lambda = F/S$ and the messenger scale $M$, then the soft SUSY breaking gaugino and scalar masses at the messenger scale are given by [3]

\begin{equation}
\tilde{M}_a(M) = N_a g \left( \frac{\Lambda}{M} \right) \frac{\alpha_a(M)}{4\pi} \Lambda
\end{equation}

and

\begin{equation}
\tilde{m}^2(M) = 2 f \left( \frac{\Lambda}{M} \right) \sum_{a=1}^{3} k_a N_a C_a \left( \frac{\alpha_a(M)}{4\pi} \right)^2 \Lambda^2
\end{equation}

where the $\alpha_a$ are the three SM gauge couplings and $k_a = 1, 1$ and $3/5$ for SU(3), SU(2), and U(1), respectively. The $C_a$ are zero for gauge singlets and are $4/3, 3/4$ and $Y^2$ for the fundamental representations of SU(3), SU(2) and U(1), respectively (with $Y$ given by $Q = I_3 + Y$). The functions $g(x)$ and $f(x)$ are messenger scale threshold functions. We calculate the sparticle masses at the messenger scale $M$ using Eqs. (9) and (10) and run these to the electroweak scale using the appropriate renormalization group equations [3].

The calculation of the Higgs mass from the sparticle spectrum is carried out as in [6], with one-loop corrections given in [3]. The tree-level potential for the Higgs is

\begin{equation}
V_{\text{tree}} = m^2 H_1^\dagger H_1 + m^2 H_2^\dagger H_2 + n_{12}^2 (H_1 H_2 + H_2 H_1) + ...
\end{equation}
where
\[
\begin{align*}
m_1^2 &= m_{H_u}^2 + \mu^2, \\
m_2^2 &= m_{H_d}^2 + \mu^2, \\
m_{12}^2 &= B\mu.
\end{align*}
\]

The mass of the pseudoscalar Higgs is then
\[
M_A^2 = m_1^2 + m_2^2. 
\]

We can parameterize the corrections to the mass matrix elements for the scalar Higgs sector by
\[
\begin{align*}
M_{11} &= M_A^2 \sin^2 \beta + M_Z^2 \cos^2 \beta + (\Delta M_{11}^2)_\text{1LL} + (\Delta M_{11}^2)_\text{mix} \\
M_{22} &= M_A^2 \cos^2 \beta + M_Z^2 \sin^2 \beta + (\Delta M_{22}^2)_\text{1LL} + (\Delta M_{22}^2)_\text{mix} \\
M_{12} = M_{21} &= -(M_A^2 + M_Z^2) \sin \beta \cos \beta + (\Delta M_{12}^2)_\text{1LL} + (\Delta M_{12}^2)_\text{mix}
\end{align*}
\]

For completeness, we include the expressions for the corrections. The one-loop leading-log corrections are
\[
\begin{align*}
(\Delta M_{11}^2)_\text{1LL} &= \frac{g^2 m_A^2 \cos^2 \beta}{m_Z^2 \cos \theta_W} \left[ P_t \ln \left( \frac{M_Q M_U}{m_t^2} \right) \\
&\quad + \left( 36 \frac{m_i^2}{m_Z^2} - 18 \frac{m_i^2}{m_Z^2} \cos^2 \beta \right) \left( P_b + P_f + P_g + P_{2H} \right) \ln \left( \frac{M_Q M_D}{m_i^2} \right) \\
&\quad + \Theta(m_A^0 - m_Z)(P_{1H} - P_{2H}) \ln \left( \frac{m_i^0}{m_i^2} \right) \\
&\quad - 9 \left( 1 + 4e_b \sin^2 \theta_W \right) \frac{m_i^2}{m_Z^2} \left( \frac{m_i^2}{m_Z^2} - \frac{1}{6} \right) \ln \left( \frac{M_D^2}{M_U^2} \right) \\
&\quad + \frac{3}{2} \left( 1 + 4e_t \sin^2 \theta_W \right) \ln \left( \frac{M_D^2}{M_U^2} \right) \right],
\end{align*}
\]
\[
\begin{align*}
(\Delta M_{22}^2)_\text{1LL} &= \frac{g^2 m_A^2 \sin^2 \beta}{96\pi^2 \cos \theta_W} \left[ (P_b + P_f + P_g + P_{2H}) \ln \left( \frac{M_Q M_D}{m_i^2} \right) \\
&\quad + \left( 36 \frac{m_i^2}{m_Z^2} \sin^2 \beta - 18 \frac{m_i^2}{m_Z^2} \sin^2 \beta \right) \left( P_i + P'_{1H} - P'_{2H} \right) \ln \left( \frac{M_U^2}{m_i^2} \right) \\
&\quad + \Theta(m_A^0 - m_Z)(P_{1H} - P_{2H}) \ln \left( \frac{m_i^0}{m_i^2} \right) \\
&\quad - \frac{9}{2} \left( 1 + 4e_t \sin^2 \theta_W \right) \left( \frac{m_i^2}{m_Z^2} \sin^2 \beta - \frac{1}{6} \right) \ln \left( \frac{M_D^2}{M_U^2} \right) \\
&\quad - \frac{3}{2} \left( 1 + 4e_b \sin^2 \theta_W \right) \ln \left( \frac{M_D^2}{M_U^2} \right) \right],
\end{align*}
\]
\[
(\Delta M_{12}^2)_\text{1LL} = (\Delta M_{21}^2)_\text{1LL} = \frac{g^2 m_A^2 \sin \beta \cos \beta}{96\pi^2 \cos \theta_W} \left[ (P_b - 9 \frac{m_i^2}{m_Z^2} \sin^2 \beta) \left( P_i - 9 \frac{m_i^2}{m_Z^2} \sin^2 \beta \right) \ln \left( \frac{M_Q M_U}{m_i^2} \right) \\
&\quad + \left( P_b - 9 \frac{m_i^2}{m_Z^2} \sin^2 \beta \right) \left( P_f + P_g + P_{2H} \right) \ln \left( \frac{M_Q M_D}{m_i^2} \right) \\
&\quad + \Theta(m_A^0 - m_Z)(P_{1H} - P_{2H}) \ln \left( \frac{m_i^0}{m_i^2} \right) \\
&\quad - \frac{9}{2} \left( 1 + 4e_t \sin^2 \theta_W \right) \left( \frac{m_i^2}{m_Z^2} \sin^2 \beta - \frac{1}{6} \right) \ln \left( \frac{M_D^2}{M_U^2} \right) \\
&\quad - \frac{3}{2} \left( 1 + 4e_b \sin^2 \theta_W \right) \ln \left( \frac{M_D^2}{M_U^2} \right) \right],
\]

where $M_U$, $M_D$, and $M_Q$ refer to the soft mass parameters and
\[
\begin{align*}
P_t &= 3(1 - 4e_t \sin^2 \theta_W + 8e_t^2 \sin^4 \theta_W), \\
P_b &= 3(1 + 4e_b \sin^2 \theta_W + 8e_b^2 \sin^4 \theta_W), \\
P_f &= 6 \left[ 3 - 6 \sin^2 \theta_W + 4(1 + 2e_t^2 + 2e_b^2) \sin^4 \theta_W \right], \\
P_g &= -44 + 106 \sin^2 \theta_W - 62 \sin^4 \theta_W, \\
P_g' &= 10 + 34 \sin^2 \theta_W - 26 \sin^4 \theta_W, \\
P_{2H} &= -10 + 2 \sin^2 \theta_W - 2 \sin^4 \theta_W, \\
P_{2H}' &= 8 - 22 \sin^2 \theta_W + 10 \sin^4 \theta_W, \\
P_{1H} &= -9 \cos^2 2\beta + (1 - 2 \sin^2 \theta_W + 2 \sin^4 \theta_W) \cos^2 2\beta.
\end{align*}
\]
and the quark charges are \( e_t = 2/3 \) and \( e_b = -1/3 \). The contributions due to squark mixing are

\[
\begin{align*}
(\Delta M_{11}^2)_{\text{mix}} &= \frac{3g^2}{16\pi^2\Lambda^2} \cos^2 \beta \left\{ m_\tilde{t}^2 A_{\tilde{t}} A_{\tilde{b}} + \frac{1}{3} A_{\tilde{t}} + A_{\tilde{b}} \right\} \left[ 2h(M_{Q^2}^2, M_{\tilde{t}^2}^2) + A_{\tilde{b}} X_{\tilde{b}} g(M_{Q^2}^2, M_{\tilde{t}^2}^2) \right] \\
(\Delta M_{22}^2)_{\text{mix}} &= \frac{3g^2}{16\pi^2\Lambda^2} \cos^2 \beta \left\{ m_\tilde{t}^2 A_{\tilde{t}} A_{\tilde{b}} + \frac{1}{3} A_{\tilde{t}} + A_{\tilde{b}} \right\} \left[ 2h(M_{Q^2}^2, M_{\tilde{t}^2}^2) + A_{\tilde{b}} X_{\tilde{b}} g(M_{Q^2}^2, M_{\tilde{t}^2}^2) \right] \\
(\Delta M_{12}^2)_{\text{mix}} &= (\Delta M_{21}^2)_{\text{mix}} = -\frac{3g^2}{32\pi^2\Lambda^2} \left\{ m_\tilde{t}^2 A_{\tilde{t}} A_{\tilde{b}} + \frac{1}{3} A_{\tilde{t}} + A_{\tilde{b}} \right\} \left[ 2h(M_{Q^2}^2, M_{\tilde{t}^2}^2) + A_{\tilde{b}} X_{\tilde{b}} g(M_{Q^2}^2, M_{\tilde{t}^2}^2) \right]
\end{align*}
\]

where

\[
\begin{align*}
X_t &= A_t - \mu \cot \beta, \\
X_b &= A_b - \mu \tan \beta, \\
Y_t &= A_t + \mu \tan \beta, \\
Y_b &= A_b + \mu \cot \beta,
\end{align*}
\]

and the functions \( h, B, p_b, \) and \( p_t \) are

\[
\begin{align*}
h(a, b) &= \frac{1}{a - b} \ln \frac{a}{b}, \\
B(a, b) &= \frac{1}{(a - b)^2} \left[ \frac{1}{2} (a + b) - \frac{a b}{a - b} \ln \frac{a}{b} \right], \\
p_b(a, b) &= f(a, b) - 2 e_b \sin^2 \theta W(a - b) g(a, b), \\
p_t(a, b) &= f(a, b) + 2 e_t \sin^2 \theta W(a - b) g(a, b), \\
g(a, b) &= \frac{1}{a - b} \left[ 2 - \frac{a b}{a - b} \ln \frac{a}{b} \right], \\
f(a, b) &= \frac{1}{a - b} \left[ 1 - \frac{b - a}{a - b} \ln \frac{a}{b} \right].
\end{align*}
\]

4 Results

For the various choices of \( N_1, N_2, \) and \( N_3, \) the parameter space of \( \Lambda \) and \( \tan \beta \) was explored. The messenger scale was fixed at \( M = 10^5 \) TeV. Figure 1 shows two examples, for \( N_1 = 1, N_2 = 1, N_3 = 1 \) and for \( N_1 = 1, N_2 = 3, N_3 = 4 \). The contours of \( M_h \) rise sharply for small \( \tan \beta \). They appear to level out for large \( \tan \beta \), but actually begin to turn upward again. A large part of the region considered is excluded for having a negative mass squared for the scalar tau lepton. Such a mass would give a VEV that violates both electric charge and lepton number, and is hence phenomenologically unacceptable.

Using \( M = 10^5 \) TeV, and considering \( \Lambda \) up to 100 TeV, we find that the largest value for the lighter scalar Higgs mass is 132 GeV. The largest values for the mass in the various models with \( \Lambda = 100 \) TeV are given in Table 1. The largest mass typically occurs at \( \tan \beta = 29 \), with little variation on the values of \( N_a \). The maximum value occurs at the maximum value of \( \Lambda \), and would be larger if \( \Lambda \) were allowed to vary beyond 100 TeV. For example, for the model with \( N_1 = N_2 = N_3 = 1 \) and \( \Lambda = 200 \) TeV, \( M_h = 127 \) GeV. For the model with \( N_1 = N_2 = N_3 = 4 \) and \( \Lambda = 200 \) TeV, \( M_h = 139 \) GeV. Notice also that the greatest dependence is on \( N_3, \) due to the large corrections from squark mixing (Eq. 17).

It is possible to find an analytic expression approximating the largest mass for the lighter scalar Higgs when \( \Lambda \) is fixed. The strongest dependence is on \( N_3 \) and is logarithmic. The variation with respect to \( N_1 \) and \( N_2 \) is weaker and can be approximated by linear functions. If we parameterize
the dependence of $M_h$ on the $N_a$ by

$$M_h = (a + bN_1 + cN_2 + d\ln N_3) \text{ (GeV)},$$

(20)

then for $\Lambda = 100$ TeV, we have approximately

$$a = 118.3, b = 0.1, c = 0.2, \text{ and } d = 9.1.$$ (21)

For comparison, the values of these coefficients for $\Lambda = 50$ TeV and 200 TeV are given in Table 2. The dependence on $N_3$ is largest, as reflected in the values of $d$. This large, logarithmic dependence is due to the large corrections due to squark mixing (Eq. 17). This dependence decreases as $\Lambda$ increases, while the constant term $a$ also increases.

5 Conclusion

In this work, we have made a detailed investigation of the mass of the lightest MSSM Higgs boson in generalized GMSB models. We have considered a variety of messenger sectors belonging to different vectorlike representations of the SM gauge group, $SU(3) \otimes SU(2) \otimes U(1)$, as well as SU(5). We imposed the condition of perturbativity on the gauge couplings as well as the possibility of unification. For each choice of messenger sector, we studied the variation in the Higgs mass with respect to the GMSB parameters $\tan \beta$ and $\Lambda$. The Higgs mass depends sensitively on $\tan \beta$ and $\Lambda$, but not much on the messenger sector scale, $M$. For $\Lambda$ less than 100 TeV, the bound on the lightest Higgs mass is 132 GeV. For $\Lambda$ equal to 500 TeV, the bound increases to 139 GeV. These bounds are within reach of the forthcoming upgraded Tevatron search. Thus, we conclude that the Tevatron Run 2 and Run 3 searches will be able to explore the lightest Higgs boson for a wide variety of messenger sector content in GMSB models.

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Table 1: Numerical results for the mass of the lighter neutral scalar Higgs boson. Values are for $M = 10^5$ TeV and maximum $\Lambda = 100$ TeV. The largest mass occurs typically near $\tan \beta = 29$ and $\Lambda = 100$ GeV, except for the value marked with $\ast$, for which the region of negative stau mass squared covers that point of parameter space.

| Messenger Sector | $N_1$ | $N_2$ | $N_3$ | $M_h$ (GeV) |
|------------------|-------|-------|-------|-------------|
| $5 \oplus \overline{5}$ | 1     | 1     | 1     | 118.5       |
| $2(5 \oplus \overline{5})$ | 2     | 2     | 2     | 125.1       |
| $3(5 \oplus \overline{5})$ | 3     | 3     | 3     | 129.0       |
| $4(5 \oplus \overline{5})$ | 4     | 4     | 4     | 131.7       |
| $10 \oplus \overline{10}$ | 3     | 3     | 3     | 129.0       |
| $5 \oplus \overline{5} \oplus 10 \oplus \overline{10}$ | 4     | 4     | 4     | 131.7       |

Table 2: Values for the coefficients in Eq. 20 for different values of $\Lambda$.

| $\Lambda$ (TeV) | $a$   | $b$   | $c$   | $d$   |
|-----------------|-------|-------|-------|-------|
| 50              | 108.9 | 0.1   | 0.3   | 9.7   |
| 100             | 118.3 | 0.1   | 0.2   | 9.1   |
| 200             | 127.0 | 0.02  | 0.2   | 8.4   |
Figure 1: Contours of lighter neutral scalar Higgs mass (in GeV) in the $\Lambda$-$\tan \beta$ parameter space with $M = 10^5$ TeV. The shaded region is the area where the stau mass squared is negative. (a) For the case $N_1 = 1$, $N_2 = 1$, $N_3 = 1$. (b) For the case $N_1 = 1$, $N_2 = 3$, $N_3 = 4$. 