Pair Production and Correlated Decay
of Heavy Majorana Neutrinos in $e^+e^-$ Collisions

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Abstract

We consider the process $e^+e^- \rightarrow N_1N_2$, where $N_1$ and $N_2$ are heavy Majorana particles, with relative $CP$ given by $\eta_{CP} = +1$ or $-1$, decaying subsequently via $N_1, N_2 \rightarrow W^\pm e^\mp$. We derive the energy and angle correlation of the dilepton final state, both for like-sign ($e^+e^-$) and unlike-sign ($e^-e^+$) configurations. Interesting differences are found between the cases $\eta_{CP} = +1$ and $-1$. The characteristics of unlike-sign $e^+e^-$ dileptons originating from a Majorana pair $N_1N_2$ are contrasted with those arising from the reaction $e^+e^- \rightarrow N\bar{N} \rightarrow W^+e^-W^-e^+$, where $N\bar{N}$ is a Dirac particle-antiparticle pair.

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I. INTRODUCTION

In an interesting paper \[1\], Kogo and Tsai have analysed the reaction $e^+e^- \rightarrow N_1N_2$, where $N_{1,2}$ are heavy Majorana neutrinos, and compared the cases where the relative $CP$ of $N_1$ and $N_2$ is $\eta_{CP} = +1$ and $-1$. It was found that the two cases differ in threshold behaviour, in angular distribution, and in the dependence on the spin-directions of $N_1$ and $N_2$. A comparison was also made between the Majorana process and the Dirac process $e^+e^- \rightarrow N\bar{N}$, where $N\bar{N}$ is a Dirac particle-antiparticle pair. A related analysis was carried out in Ref. \[2\]. (The contrast between Majorana neutrinos and Dirac neutrinos has been the subject of several other papers (e.g. \[3–6\]) and monographs (\[7,8\]).)

In the present paper, we examine how the differences between the cases $\eta_{CP} = +1$ and $-1$ propagate to the decay products of $N_1$ and $N_2$, assuming the decays to take place via $N_{1,2} \rightarrow W^\pm e^\mp$. We focus on the like-sign lepton pair created in the reaction chain $e^+e^- \rightarrow N_1N_2 \rightarrow W^+e^-W^+e^-$, which is a characteristic signature of Majorana pair production. We derive, in particular, the correlation in the energies of the $e^-e^-$ pair, and in their angles relative to the $e^+e^-$ axis. Interesting differences are found between the cases $\eta_{CP} = +1$ and $-1$. We also examine the behaviour of the unlike-sign dileptons $e^+e^-$, comparing the Majorana cases with dileptons created in the production and decay of a Dirac $N\bar{N}$ pair, i.e. $e^+e^- \rightarrow N\bar{N} \rightarrow W^+e^-W^-e^+$. 

II. CHARACTERISTICS OF THE REACTION $e^+e^- \rightarrow N_1N_2$

The analysis of Ref. \[1\] was carried out in the context of the simple production mechanism for $e^+e^- \rightarrow N_1N_2$ shown in Fig. 1, and we begin by recapitulating the essential results. The interaction Lagrangian is taken to be

$$
\mathcal{L}_1(x) = -\frac{g}{2\cos \theta_W} \left[ \bar{\nu} (x) \gamma_\mu (c_V - c_A \gamma_5) \nu (x) \\
+ \alpha_N \bar{N}_1(x) \gamma_\mu \frac{1}{2}(1 - \gamma_5) N_2(x) \\
+ \alpha_N \bar{N}_2(x) \gamma_\mu \frac{1}{2}(1 - \gamma_5) N_1(x) \right] Z^\mu (x) , \quad (2.1)
$$
where \( c_V, c_A \) and \( \alpha_N \) may be regarded as real phenomenological parameters. (For the standard Z-boson, \( c_V = -1/2 + 2 \sin^2 \theta_W, \ c_A = -1/2 \)). The matrix element for Majorana neutrinos (with momenta and spins as indicated in Fig. 1) is

\[
\mathcal{M}_m = -i \alpha_N \left( \frac{g}{2 \cos \theta_W} \right)^2 j_\mu^e \Delta_\mu^\nu_Z \left[ \bar{u}_{t_1}(q_1) \gamma_\nu \frac{1}{2} (1 - \gamma_5) v_{t_2}(q_2) \lambda_2 \\
- \bar{u}_{t_2}(q_2) \gamma_\nu \frac{1}{2} (1 - \gamma_5) v_{t_1}(q_1) \lambda_1 \right], \tag{2.2}
\]

where

\[
j_\mu^e = \bar{v}_{s_2}(p_2) \gamma_\mu (c_V - c_A \gamma_5) u_{s_1}(p_1) \tag{2.3}
\]

and

\[
\Delta_\mu^\nu_Z = \frac{g^{\mu\nu} - q^\mu q^\nu/m_Z^2}{q^2 - m_Z^2 + i m_Z \Gamma_Z}. \tag{2.4}
\]

Assuming \( CP \)-invariance the factors \( \lambda_1, \lambda_2 \) in Eq. (2.2) are such that \( \lambda_1 \lambda_2^* = +1(-1) \) when \( N_1 \) and \( N_2 \) have the same (opposite) \( CP \)-parity [9]. Rewriting the second term in Eq. (2.2) as

\[
\bar{u}_{t_2}(q_2) \gamma_\nu \frac{1}{2} (1 - \gamma_5) v_{t_1}(q_1) = \bar{u}_{t_1}(q_1) \gamma_\nu \frac{1}{2} (1 + \gamma_5) v_{t_2}(q_2), \tag{2.5}
\]

we observe that the current of the Majorana neutrinos is pure axial vector when \( N_1 \) and \( N_2 \) have the same \( CP \)-parity (\( \eta_{CP} = \lambda_1 \lambda_2^* = +1 \)), and pure vector when they have opposite \( CP \) (\( \eta_{CP} = \lambda_1 \lambda_2^* = -1 \)). In comparison, the matrix element for the Dirac process \( e^+e^- \rightarrow N\bar{N} \) is

\[
\mathcal{M}_d = -i \alpha_N \left( \frac{g}{2 \cos \theta_W} \right)^2 j_\mu^e \Delta_\mu^\nu_Z \bar{u}_{t_1}(q_1) \gamma_\nu \frac{1}{2} (1 - \gamma_5) v_{t_2}(q_2). \tag{2.6}
\]

The differential cross section for \( e^+e^- \rightarrow N_1N_2 \), for general masses \( m_1 \) and \( m_2 \), and for arbitrary polarizations \( \vec{n} \) and \( \vec{n}' \) of the two neutrinos is given in the Appendix. In Sec. 5 we compare our formulas with those of Ref. [1], and with special cases treated in other papers. Here we specialise to the case \( m_1 = m_2 = m_N \), for which the cross-section \( (d\sigma/d\Omega) \) in the cases \( \eta_{CP} = +1 \) and \( -1 \) is
\[
\left(\frac{d\sigma}{d\Omega}\right)_+ = \frac{1}{2} \sigma_0 \beta^3 \left\{ f_1 \left[ (n_y n'_y - n_x n'_x) S^2 + (1 + n_z n'_z)(1 + C^2) \right] - f_2 \left[ 2(n_x + n'_x) C \right]\right\}, \quad (2.7)
\]

\[
\left(\frac{d\sigma}{d\Omega}\right)_- = \sigma_0 \beta \left\{ f_1 \left[ 2 - \beta^2 + C^2 \beta^2 + n_z n'_z (\beta^2 + C^2 (1/\gamma^2 + 1)) \right] + n_x n'_x S^2 (1/\gamma^2 + 1) \right\}
\]

\[-n_y n'_y S^2 \beta^2 + (n_x n'_y + n'_x n_z) 2 S C/\gamma^2\right] - f_2 \left[ 2(n_x + n'_x) S/\gamma^2 + 2(n_z + n'_z) C \right]\right\}. \quad (2.8)
\]

For comparison, the differential cross section of the Dirac process $e^+ e^- \rightarrow N \bar{N}$ is

\[
\left(\frac{d\sigma}{d\Omega}\right)_d = \frac{1}{2} \sigma_0 \beta \left\{ f_1 \left[ (1 + C^2 \beta^2) - (n_x + n'_x) \beta (1 + C^2) \right]
\right. \\
\left. - (n_x + n'_x) S C \beta/\gamma + n_z n'_z (C^2 + \beta^2) + (n_x n'_y + n'_x n_z) S C/\gamma + n_z n'_x S^2/\gamma^2 \right] + f_2 \left[ 2 C \beta - (n_x + n'_x) C (1 + \beta^2) - (n_x + n'_x) S/\gamma + 2 n_z n'_x C \beta + (n_x n'_y + n'_x n_z) S / \beta / \gamma \right]\right\}. \quad (2.9)
\]

The symbols in Eqs. (2.7)-(2.9) are defined as follows:

\[
\sigma_0 = \frac{G_F^2 \alpha_N^2}{512 \pi^2} \left| \frac{m_Z^2}{s - m_Z^2 + i m_z \Gamma_z} \right|^2 s, \quad \beta = (1 - 4 m_N^2 / s)^{1/2}, \quad \gamma = (1 - \beta^2)^{-1/2}
\]

\[
C = \cos \theta, \quad S = \sin \theta, \quad f_1 = 2 \left( c_V^2 + c_A^2 \right), \quad f_2 = 4 c_V c_A,
\]

\(\theta\) being the scattering angle of \(N_1\) (or \(N\)) with respect to the initial \(e^-\) direction. The co-ordinate axes are defined so that the momentum- and spin-vectors of \(N_1\) and \(N_2\) in the \(e^+ e^-\) c.m. frame have the components

\[
q_1^\mu = (\gamma m, 0, 0, \gamma \beta m), \quad t_1^\mu = (\gamma \beta n_z, n_x, n_y, \gamma n_z), \quad (2.11)
\]

\[
q_2^\mu = (\gamma m, 0, 0, -\gamma \beta m), \quad t_2^\mu = (-\gamma \beta n'_z, n'_x, n'_y, \gamma n'_z).
\]

Inspection of Eqs. (2.7)-(2.9) reveals several interesting features:

(a) The Majorana cases '+-' and '−' have different dependence on the spin-vectors \(\vec{n}\) and \(\vec{n}',\) and different angular distributions, even after the spins \(\vec{n}\) and \(\vec{n}'\) are summed over. These differences stem from the fact that the matrix element \(\mathcal{M}_m\) in Eq. (2.2) effectively involves an axial vector current \(\bar{N}_1 \gamma_\mu \gamma_5 N_2\) when \(\lambda_1 \lambda_2^* = +1\) and a vector current \(\bar{N}_1 \gamma_\mu N_2\) when \(\lambda_1 \lambda_2^* = -1\).
(b) The Majorana cases ' + ' and ' − ' differ from the Dirac case ' d ', in which the current of the $N\bar{N}$-pair has a V–A structure $\bar{N}\gamma_{\mu}\frac{1}{2}(1 - \gamma_5)N$. This difference persists even if the spins of the heavy neutrinos are summed over, in which case

$$
\sum_{\vec{n},\vec{n}'} (\frac{d\sigma}{d\Omega})^+ = 2\sigma_0 \beta^3 \left[ f_1 (1 + C^2) \right],
$$

$$
\sum_{\vec{n},\vec{n}'} (\frac{d\sigma}{d\Omega})^- = 4\sigma_0 \beta \left[ f_1 (2 - \beta^2 + C^2 \beta^2) \right],
$$

$$
\sum_{\vec{n},\vec{n}'} (\frac{d\sigma}{d\Omega})^d = 2\sigma_0 \beta \left[ f_1 (1 + C^2 \beta^2) + f_2 (2\beta C) \right].
$$

(2.12)

Whereas the spin-averaged Majorana cross sections are forward-backward symmetric, the Dirac process has a term linear in $\cos \theta$, with a coefficient proportional to $f_2 = 4 c_V c_A$. Eq. (2.12) also shows that the threshold behaviour is $\beta^3$, $\beta$ and $\beta$ for the cases ' + ', ' − ' and ' d ' respectively. In the asymptotic limit $\beta \rightarrow 1$ the Majorana cases ' + ' and ' − ' have the same angular distribution $(1 + C^2)$, distinct from that of the Dirac process.

(c) In the high energy limit $\beta \rightarrow 1$, the Dirac process $e^+e^- \rightarrow N\bar{N}$ has a spin-dependence given by

$$
(\frac{d\sigma}{d\Omega})^d = \frac{1}{2} \sigma_0 \beta \left[ 1 - (n_z + n'_z) + n_z n'_z \right] \left[ f_1 (1 + C^2) + 2 f_2 C \right].
$$

(2.13)

The fact that only the longitudinal components ($n_z$ and $n'_z$) of the $N, \bar{N}$ spins appear in this expression is consistent with the expectation that relativistic Dirac neutrinos are eigenstates of helicity. The fact that the cross section (2.13) vanishes when $n_z = -1$, $n'_z = +1$ confirms the expectation that for a V–A current the $N$ and $\bar{N}$ are produced in left-handed and right-handed states, respectively. By comparison, the Majorana processes $e^+e^- \rightarrow N_1 N_2$, for $\eta_{CP} = +1$ and $-1$, have the high energy behaviour ($\beta \rightarrow 1$)

$$
(\frac{d\sigma}{d\Omega})^\pm = \frac{1}{2} \sigma_0 \left\{ f_1 [(1 + C^2)(1 + n_z n'_z) + S^2(n_y n'_y - n_x n'_x)] - 2 f_2 C (n_z + n'_z) \right\},
$$

(2.14)
\[
\left( \frac{d\sigma}{d\Omega} \right)_- = \frac{1}{2} \sigma_0 \left\{ f_1 \left[ (1 + C^2)(1 + n_zn'_z) + S^2(n_xn'_x - n_yn'_y) \right] - 2f_2C \left( n_z + n'_z \right) \right\}. \quad (2.15)
\]

Contrary to the Dirac case, the Majorana reactions have an explicit dependence on \( n_x, n_y, n'_x, n'_y \), reflecting the fact that a relativistic Majorana particle with \( m_N \neq 0 \) is not necessarily an eigenstate of helicity, and can have a spin pointing in an arbitrary direction. The Majorana cases '+' and '-' differ in the sign of the term proportional to \( S^2 \), which contains the transverse (x- and y-) components of the neutrino spins. It is with the purpose of exposing the subtle differences in the spin state of the \( N_1N_2 \) and \( N\bar{N} \) systems that we investigate in the following sections the dilepton final state created by the decays of the heavy neutrinos via \( N_{1,2} \rightarrow W^\pm e^\mp \) and \( N(\bar{N}) \rightarrow W^+e^-(W^-e^+) \).

III. LIKE-SIGN DILEPTONS: THE REACTION \( e^+e^- \rightarrow N_1N_2 \rightarrow W^+W^+e^-e^- \)

As seen in the preceding, the spin state and the angular distribution of the Majorana pair produced in \( e^+e^- \rightarrow N_1N_2 \) depends on the relative \( CP \)-parity \( \eta_{CP} \) of the two particles. We wish to see how these differences manifest themselves in the decay products of \( N_1 \) and \( N_2 \). To this end, we assume that \( m_N > m_W \), and that the simplest decay mechanism is \( N_{1,2} \rightarrow W^\pm e^\pm \). In particular, the reaction sequence \( e^+e^- \rightarrow N_1N_2 \rightarrow W^+W^+e^-e^- \) leads to the appearance of two like-sign leptons in the final state, an unmistakable signature of Majorana pair production. (For the purpose of this paper we assume that the \( W \)-bosons decay into quark jets, thus avoiding the complications of final states with 3 or 4 charged leptons.)

We have calculated the amplitude of the process \( e^+e^- \rightarrow N_1N_2 \rightarrow W^+W^+e^-e^- \), depicted in Fig. 2, assuming a decay interaction (\( \alpha'_N \) and \( \alpha''_N \) being real parameters)

\[
\mathcal{L}_2(x) = - \frac{g}{\sqrt{2}} \left[ \alpha'_N \bar{\epsilon}(x) \gamma_\mu \frac{1}{2}(1 - \gamma_5) N_1(x) W^{\mu-}(x) + \alpha'_N \bar{N}_1(x) \gamma_\mu \frac{1}{2}(1 - \gamma_5) e(x) W^{\mu+}(x) + \alpha''_N \bar{\epsilon}(x) \gamma_\mu \frac{1}{2}(1 - \gamma_5) N_2(x) W^{\mu-}(x) \right]
\]
\[ + \alpha''_N \tilde{N}_2(x) \gamma_\mu \frac{1}{2} (1 - \gamma_5) e(x) W^{\mu+}(x) \]\]. \hspace{1cm} (3.1)

This amplitude has the form (see Appendix for details)

\[ M = iA \ j_\mu^* \ \Delta_\mu^\nu \ \frac{1}{q_1^2 - m_f^2 + \im \Gamma_1} \cdot \frac{1}{q_2^2 - m^2 + \im \Gamma_2}. \]

\[ \left[ m_2 \lambda_2 \bar{u}_t(k_1) \gamma_\rho \not\!\!q_1 \gamma_\nu \gamma_\sigma \frac{1}{2} (1 + \gamma_5) v_t(k_2) \right. \]

\[ - m_1 \lambda_1 \bar{u}_t(k_2) \gamma_\sigma \not\!\!q_2 \gamma_\nu \gamma_\rho \frac{1}{2} (1 + \gamma_5) v_t(k_1) \left. \right] \epsilon^{\mu \rho \nu}(k_3) \epsilon^{\sigma \nu \rho}(k_4). \hspace{1cm} (3.2) \]

where

\[ A = \alpha' \alpha''_N \ \frac{g^4}{8 \cos^2 \theta_W}. \hspace{1cm} (3.3) \]

Using the narrow-width approximation for the \( N_1, N_2 \) propagators, and specializing to the case \( m_1 = m_2 = m_N \), we obtain the following expression for the squared matrix element (summed over final and averaged over initial spins), the subscript in \( M_{\pm} \) denoting \( \eta_{CP} = \pm 1 \) \( (q = p_1 + p_2, \ l = p_1 - p_2) \):

\[ |M_{\pm}|^2 = \frac{|A|^2}{2} \left( \frac{1}{(s - m^2_Z)^2} \frac{\pi}{m_N \Gamma_n} \delta(q^2_1 - m_N^2) \frac{\pi}{m_N \Gamma_n} \delta(q^2_2 - m_N^2) \frac{m_N^2}{m_W^2} \right) \]

\[ \left\{ f_1 \cdot \left( \frac{\pm(m_N^2 - m_W^2)^2(m_N^2 + 2m_W^2)^2 \cdot s \mp 4(m_N^2 - 2m_W^2)^2.}{s (k_1k_2)(q_1q_2) - s (k_1q_2)(k_2q_1) - (k_1k_2)(q_1q_2) + (k_1k_2)(q_1l)(q_2l)} \right. \right. \]

\[ + (k_1q_2)(k_2q_2)(q_1q_2) - (k_1q_2)(k_2l)(q_1l) + (k_1q_2)(k_2q_1)(q_2q) - (k_1l)(k_2q_1)(q_2l) \]

\[ - (k_1q_2)(k_2q_1)(q_1q_2) + (k_1l)(k_2l)(q_1q_2) \pm m_N^2 ((k_1q_2)(k_2q_2) - (k_1l)(k_2l)) \]

\[ + 2 (m_N^2 - m_W^2)(m_N^2 - 2m_W^2)^2 \left[ (k_1q_2)(q_2q) - (k_1l)(q_2l) + (k_2q)(q_1l) - (k_2l)(q_1l) \right] \]

\[ + 8 m_W^2(m_N^2 - m_W^2)^2 \left[ (q_1q)(q_2q) - (q_1l)(q_2l) \right] \]

\[ - 2f_2 \cdot (m_N^2 - m_W^2)(m_N^4 - 4m_W^4) \left( \pm (k_1q)(q_1l) - (k_1q)(q_2l) \right. \]

\[ \pm (k_1l)(q_1q) + (k_1l)(q_2q) \pm (k_2q)(q_2l) - (k_2q)(q_1l) \]

\[ \pm (k_2l)(q_1q) + (k_2l)(q_2q) \right) \right\}. \hspace{1cm} (3.4) \]

If the final state is \( e^+e^+ \) instead of \( e^-e^- \), we replace \( f_2 \to -f_2 \) in the above equation.
The expression for $|\mathcal{M}_\pm|^2$ can be integrated over the phase space of $W^+$ and $W^+$ (i.e. over the momenta $k_3(= q_1 - k_1)$ and $k_4(= q_2 - k_2)$, in order to obtain the spectra in the lepton variables $k_1$ and $k_2$. Defining the four-vectors $k_1$ and $k_2$ in the $e^+e^-$ c.m. frame by

$$k_1^\mu = E_1 \left(1, \sin \theta_1 \cos \phi_1, \sin \theta_1 \sin \phi_1, \cos \theta_1 \right),$$

$$k_2^\mu = E_2 \left(1, \sin \theta_2 \cos \phi_2, \sin \theta_2 \sin \phi_2, \cos \theta_2 \right),$$

we have been able to derive the correlated distribution of the energies $E_1$ and $E_2$, as well as the correlation of the variables $\cos \theta_1$ and $\cos \theta_2$ measured relative to the $e^-$ beam direction.

### A. Energy Correlation

The normalized spectrum in the energies of the dilepton pair $e^-e^-$ is ($\mathcal{E}_{1,2} = E_{1,2}/m_N$)

$$\frac{1}{\sigma} \cdot \left(\frac{d\sigma}{d\mathcal{E}_1 d\mathcal{E}_2}\right) = \mathcal{N} \left[a + b(\mathcal{E}_1 + \mathcal{E}_2) + c(\mathcal{E}_1 + \mathcal{E}_2)^2 - c(\mathcal{E}_1 - \mathcal{E}_2)^2\right],$$

where $\mathcal{N}$ is a normalization factor,

$$\mathcal{N} = \left[\mathcal{W}^2 \beta^2(a + b \cdot \mathcal{W} + c \cdot \mathcal{W}^2)\right]^{-1},$$

with $\mathcal{W} = \sqrt{s} \cdot (m_N^2 - m_W^2)/2m_N^2$, $\beta = (1 - 4m_N^2/s)^{1/2}$. The coefficients $a$, $b$, $c$ depend on the relative $CP$ of the $N_1N_2$ system, and take the values

$$a^+ = m_N^2m_W^2(m_N^2 - m_W^2)^2(2s - \frac{(m_N^2 + 2m_W^2)^2}{2m_N^2}) +$$

$$b^+ = \sqrt{s}m_N^3(m_N^2 - 2m_W^2)^2(m_N^2 - m_W^2),$$

$$c^+ = -m_N^6(m_N^2 - 2m_W^2)^2, \quad (\eta_{CP} = +1)$$

$$a^- = 2m_N^2(m_N^2 - m_W^2)^2(s^2 - 2m_N^2) + \frac{m_N^2}{m_W^2}(m_N^2 + 2m_W^2)^2,$$

$$b^- = \sqrt{s}m_N^3(m_N^2 - 2m_W^2)^2(m_N^2 - m_W^2)(s - 2m_N^2),$$

$$c^- = -m_N^6(m_N^2 - 2m_W^2)^2(s - 2m_N^2). \quad (\eta_{CP} = -1)$$

Notice that the ratios $b^+/a^+$ and $b^-/a^-$ are unequal (likewise the ratios $c^+/a^+$ and $c^-/a^-$), although $b^+/c^+ = b^-/c^-$. Thus the energy correlation of the two electrons in the final state is
different for the cases $\eta_{CP} = \pm 1$. This is illustrated in Fig. 3 for the hypothetical parameters $m_N = 500$ GeV, $\sqrt{s} = 1200$ GeV. It may be noted that the factor $f_2 = 4c_Vc_A$ does not appear in the spectrum ($d\sigma/dE_1dE_2$), so that the energy correlation of $e^+e^+$ dileptons is the same as that of $e^-e^-$. In the limit $\beta \to 1$ the term $a^\pm$ dominates and the '+' and '-' cases are no more distinguishable.

B. Angular Correlation

Eq. (3.4) also allows a calculation of the correlated angular distribution of the final state $e^-e^-$ system. Defining the angles $\theta_{1,2}$ as in Eq. (3.5), and integrating over all other variables, we find

$$
\left( \frac{d\sigma}{d\cos \theta_1 d\cos \theta_2} \right)^\pm \sim \beta \cdot \int d\cos \theta_n \left\{ f_1 \cdot \left[ \mp (m_N^2 + 2m_W^2)^2 s \cdot K_1^{\theta_1}K_2^{\theta_2} + (m_N^2 - 2m_W^2)^2 s \cdot \left( \pm K_2^{\theta_1}K_2^{\theta_2} (\cos \theta_n \beta - \cos \theta_1) (\cos \theta_n \beta + \cos \theta_2) \right) \right.ight.$$ 

$$+ K_2^{\theta_1}K_2^{\theta_2} (1 + \beta \cos \theta_n \cos \theta_1) + K_1^{\theta_1}K_2^{\theta_2} (1 - \beta \cos \theta_n \cos \theta_2) \bigg) \right.$$ 

$$+ 2m_W^2 s^2 \cdot K_1^{\theta_1}K_2^{\theta_2} (1 + \cos^2 \theta_n \beta^2) \right.$$ 

$$\left. \left[ \frac{m_N^2 - 2m_W^2}{m_N^2 - 2m_W^2} \right] \cos \theta_n \beta \left( K_1^{\theta_1}K_2^{\theta_2} - K_1^{\theta_1}K_2^{\theta_2} \right) \right.$$ 

$$\left. \frac{K_2^{\theta_1}K_2^{\theta_2} \cos \theta_1 + K_1^{\theta_1}K_2^{\theta_2} \cos \theta_2}{K_2^{\theta_1}K_2^{\theta_2} \cos \theta_1 + K_1^{\theta_1}K_2^{\theta_2} \cos \theta_2} \right\} ,
$$

(3.10)

with

$$
K_1^{\theta_1(2)} = \frac{2A_1^{\theta_1(2)}}{(A_1^{\theta_1(2)} - B_1^{\theta_1(2)})^{3/2}} , \quad K_2^{\theta_1(2)} = \frac{2A_1^{\theta_2(2)} + B_1^{\theta_2(2)}}{(A_1^{\theta_2(2)} - B_1^{\theta_2(2)})^{3/2}} , \quad (3.11)
$$

$$A_1^{\theta_1(2)} = 1 - (+)\beta \cos \theta_n \cos \theta_{1(2)} , \quad B_1^{\theta_1(2)} = \beta \sin \theta_n \sin \theta_{1(2)} .$$

The correlation (3.10) has been evaluated for $m_N = 500$ GeV and $\sqrt{s} = 1200$ GeV (using $f_1 = 1 + 4\sin^2 \theta_W + 8\sin^4 \theta_4$, $f_2 = 1 - 4\sin^2 \theta_W$) and is plotted in Fig. 4. There is a clear difference between the cases $\eta_{CP} = \pm 1$. The angular correlation in (3.10) becomes particularly transparent near the threshold $\beta \to 0$, where we obtain the analytic results.
\[ \frac{1}{\sigma^+} \cdot \left( \frac{d\sigma}{d \cos \theta_1 d \cos \theta_2} \right)_{\beta \rightarrow 0}^+ \approx \frac{1}{4} \cdot \left[ 1 + \frac{1}{2} \cdot \frac{f_2}{f_1} \cdot \frac{m_N^2 - 2m_W^2}{m_N^2 + 2m_W^2} \cdot (\cos \theta_1 + \cos \theta_2) \right], \quad (3.12) \]

\[ \frac{1}{\sigma^-} \cdot \left( \frac{d\sigma}{d \cos \theta_1 d \cos \theta_2} \right)_{\beta \rightarrow 0}^- \approx \frac{1}{4} \cdot \left[ 1 + \frac{(m_N^2 - 2m_W^2)^2}{(m_N^2 + 2m_W^2)^2} \cdot \cos \theta_1 \cos \theta_2 \right. \]

\[ + \frac{f_2}{f_1} \cdot \frac{(m_N^2 - 2m_W^2)}{(m_N^2 + 2m_W^2)} \cdot (\cos \theta_1 + \cos \theta_2) \] . \quad (3.13)

Notice that the distribution in the variables \( \cos \theta_1 \) and \( \cos \theta_2 \) becomes flat in the case \( \eta_{CP} = +1 \) when \( f_2/f_1 \) is neglected. By contrast, there remains a nontrivial correlation for \( \eta_{CP} = -1 \), even in the absence of \( f_2 \). As before, the above results for \( e^-e^- \) hold for \( e^+e^+ \) if one replaces \( f_2 \rightarrow -f_2 \).

**IV. UNLIKE-SIGN DILEPTONS: THE REACTION \( e^+e^- \rightarrow N_1N_2 \rightarrow W^+W^-e^+e^- \)**

Proceeding as in Sec. 3, the matrix element for the reaction \( e^+e^- \rightarrow N_1N_2 \rightarrow W^+W^-e^+e^- \) (Fig. 2) is

\[ \mathcal{M}_m = iA \cdot j^e_\mu \Delta_{\mu\nu}^{\rho\sigma} \cdot \frac{1}{q_1^2 - m_1^2 + im_1\Gamma_1} \cdot \frac{1}{q_2^2 - m_2^2 + im_2\Gamma_2} \cdot \]

\[ \left[ \lambda_2 \bar{u}_{t_1}(k_1) \gamma_\rho \not\!q_1 \gamma_\nu \not\!q_2 \gamma_\sigma \frac{1}{2}(1 - \gamma_5) v_{t_2}(k_2) \right] \epsilon_{\lambda_3}^* \epsilon_{\lambda_4}^* \](4.1)

\[ + \lambda_1 m_1 m_2 \bar{u}_{t_1}(k_1) \gamma_\rho \gamma_\nu \gamma_\sigma \frac{1}{2}(1 - \gamma_5) v_{t_2}(k_2) \] \( \epsilon_{\lambda_3}^* \epsilon_{\lambda_4}^* \) .

The same final state, produced via a Dirac particle-antiparticle pair \( (e^+e^- \rightarrow N\bar{N} \rightarrow W^+W^-e^+e^-) \), has the amplitude

\[ \mathcal{M}_d = iA \cdot j^e_\mu \Delta_{\mu\nu}^{\rho\sigma} \cdot \frac{1}{q_1^2 - m_1^2 + im_1\Gamma_1} \cdot \frac{1}{q_2^2 - m_2^2 + im_2\Gamma_2} \cdot \]

\[ \bar{u}_{t_1}(k_1) \gamma_\rho \not\!q_1 \gamma_\nu \not\!q_2 \gamma_\sigma \frac{1}{2}(1 - \gamma_5) v_{t_2}(q_2) \] \( \epsilon_{\lambda_3}^* \epsilon_{\lambda_4}^* \) . \quad (4.2)

Summing (averaging) over final (initial) polarizations, and using the narrow-width approximation for the \( N_1, N_2 \) propagators, we obtain the squared matrix elements given below:

\[ |\mathcal{M}_\pm|^2 = \frac{|A|^2}{2} \cdot \frac{1}{(s - m_Z^2)^2} \cdot \frac{\pi}{m_N \Gamma_n} \delta(q_1^2 - m_N^2) \cdot \frac{\pi}{m_N \Gamma_n} \delta(q_2^2 - m_N^2) \cdot \frac{m_N^2}{m_W^2}. \]
In complete analogy with the discussion of like-sign leptons (Sec. 3), we derive from the above equations the correlation in the energies and angles of the final $e^+e^-$ state.

**A. Energy Correlation**

The distribution in the scaled energies $\mathcal{E}_1$, $\mathcal{E}_2$ has the quadratic form given in Eq. (3.6), where the coefficients in the Majorana cases '+' and '−' and the Dirac case 'd' now have
The values

\[ a^+ = \frac{1}{2} m_N^2 (m_N^2 - m_W^2)^2 \left( \frac{s}{m_N^2} (m_N^4 + 4m_N^4) - 2(m_N^2 + 2m_W^2)^2 \right) , \]
\[ b^+ = -\sqrt{s} m_N^3 (m_N^2 - 2m_W^2)^2 (m_N^2 - m_W^2) , \]
\[ c^+ = m_N^6 (m_N^2 - 2m_W^2)^2 , \]
\[ a^- = \frac{1}{2} m_N^2 (m_N^2 - m_W^2)^2 \left( s(s - 2m_N^2) (1 + \frac{4m_W^2}{m_N^2}) - 4(m_N^2 + 2m_W^2)^2 \right) , \]
\[ b^- = -\sqrt{s} m_N (m_N^2 - 2m_W^2)^2 (m_N^2 - m_W^2) (s - 2m_N^2) , \]
\[ c^- = m_N^4 (m_N^2 - 2m_W^2)^2 (s - 2m_N^2) , \]
\[ a^d = (m_N^2 - m_W^2)^2 \left( 4(s - m_N^2)(s - 4m_N^2) \frac{m_N^4}{m_N^2} \right. \]
\[ - (m_N^2 - 2m_W^2) \left( 2(s - 4m_N^2) m_N^2 - m_N^2 (m_N^2 - 2m_W^2) \right) \right) , \]
\[ b^d = \sqrt{s} m_N (m_N^2 - 2m_W^2) (m_N^2 - m_W^2) \left( 4sm_N^2 - (m_N^2 + 14m_W^2)m_N^2 \right) , \]
\[ c^d = m_N^4 (m_N^2 - 2m_W^2)^2 (s - 3m_N^2) . \] (4.5)

The corresponding three distributions are plotted in Fig. 5. As in the case of like-sign dileptons, the \( e^+ e^- \) pairs have distinct correlations for \( \eta_{CP} = \pm 1 \). A comparison of the Majorana cases with the Dirac case reveals an interesting difference. In the Majorana cases the total \( e^+ e^- \) energy \( Y = \mathcal{E}_1 + \mathcal{E}_2 \) is distributed symmetrically around the mid-point of this variable \( Y_0 = 1/2(Y_{\text{min}} + Y_{\text{max}}) \). By contrast, \( e^+ e^- \) pairs resulting from Dirac \( NN \) primary state have a total energy distribution that is unsymmetric around the mid-point.

### B. Angle Correlation

In analogy to the distribution \( d\sigma/d\cos\theta_1 d\cos\theta_2 \) obtained for \( e^- e^- \) pairs (Eq. 3.10), the result for unlike-sign dileptons \( e^+ e^- \) is

\[
\left( \frac{d\sigma}{d\cos\theta_1 d\cos\theta_2} \right)^\pm \sim \beta \cdot \int d\cos\theta_n \left\{ f_1 \cdot \right.
\]
\[
\left[ \pm 2m_N^2 (m_N^2 + 2m_W^2)^2 s \cdot K_1^\theta K_2^\theta \right.
\]
\[
- 2m_N^2 (m_N^2 - 2m_W^2)^2 s \cdot \left( \pm K_2^\theta (\cos\theta_n, \beta - \cos\theta_1)(\cos\theta_n, \beta + \cos\theta_2) \right) \]
\[ + \mathcal{K}^\beta_1 \mathcal{K}^\beta_2 (1 + \beta \cos \theta_n \cos \theta_1) + \mathcal{K}^\beta_1 \mathcal{K}^\beta_2 (1 - \beta \cos \theta_n \cos \theta_2) \]

\[ + (m_N^4 + 4m_W^4) s^2 \cdot \mathcal{K}^\beta_1 \mathcal{K}^\beta_2 (1 + \cos^2 \theta_n \beta^2) + 8m_N^4 (m_N^2 - 2m_W^2)^2 \cdot \]

\[ \mathcal{K}^\beta_1 \mathcal{K}^\beta_2 (1 - \cos \theta_1 \cos \theta_2) \]

\[ + f_2 \cdot 2(m_N^4 - 4m_W^4) s \cdot \left[ -\mathcal{K}^\beta_1 \mathcal{K}^\beta_2 \cdot s \cos \theta_n \beta \right] \]

\[ + 2m_N^2 \cdot \left[ \cos \theta_n \beta (\mathcal{K}^\beta_2 \mathcal{K}^\beta_1 + \mathcal{K}^\beta_1 \mathcal{K}^\beta_2) : + \right] \left[ \mathcal{K}^\beta_2 \mathcal{K}^\beta_1 \cos \theta_1 - \mathcal{K}^\beta_1 \mathcal{K}^\beta_2 \cos \theta_2 : - \right] \] \tag{4.6}

\[ \left( \frac{d\sigma}{d \cos \theta_1 d \cos \theta_2} \right)^d \sim \beta \cdot \int d \cos \theta_1 \left\{ f_1 \cdot \right. \]

\[ \left[ s^2 m_W^4 \cdot \mathcal{K}^\beta_1 \mathcal{K}^\beta_2 \cdot (1 + \cos \theta_n \beta^2) + m_N^2 m_W^2 (m_N^2 - 2m_W^2) s \cdot \left( \mathcal{K}^\beta_1 \mathcal{K}^\beta_2 (1 + \beta \cos \theta_n \cos \theta_1) + \mathcal{K}^\beta_1 \mathcal{K}^\beta_2 (1 - \beta \cos \theta_n \cos \theta_2) \right) \right. \]

\[ + m_N^4 (m_N^2 - 2m_W^2)^2 \cdot \mathcal{K}^\beta_1 \mathcal{K}^\beta_2 \cdot (1 - \cos \theta_1 \cos \theta_2) \left. \right] + f_2 \cdot \left[ 2s^2 m_W^4 \cdot \mathcal{K}^\beta_1 \mathcal{K}^\beta_2 \cdot \cos \theta_n \beta \right. \]

\[ + m_N^2 m_W^2 (m_N^2 - 2m_W^2) s \cdot \left( \mathcal{K}^\beta_1 \mathcal{K}^\beta_2 \cos \theta_n \beta + \cos \theta_1 \right) + \mathcal{K}^\beta_1 \mathcal{K}^\beta_2 \cos \theta_n \beta - \cos \theta_2 \left. \right] \]

\[ + m_N^4 (m_N^2 - 2m_W^2)^2 \cdot \mathcal{K}^\beta_1 \mathcal{K}^\beta_2 \cdot (\cos \theta_1 - \cos \theta_2) \right\} \] \tag{4.7}

As usual, the indices ' + ', ' − ' and ' d ' differentiate between the Majorana cases \( \eta_{CP} = +1, \)

\(-1 \) and the Dirac case. The angle-correlations expressed by Eqs. (4.6), (4.7) are plotted

in Fig. 6., where the differences between the three cases are obvious. Close to threshold

(\( \beta \rightarrow 0 \)), the correlation between \( \cos \theta_1 \) and \( \cos \theta_2 \) can be presented in analytic form

\[ \frac{1}{\sigma^+} \cdot \left( \frac{d\sigma}{d \cos \theta_1 d \cos \theta_2} \right)^d_{\beta \rightarrow 0} \approx \frac{1}{4} \cdot \left[ 1 + \frac{1}{2} \cdot \frac{f_2}{f_1} \cdot \frac{m_N^2 - 2m_W^2}{m_N^2 + 2m_W^2} \cdot (\cos \theta_1 - \cos \theta_2) \right] \] \tag{4.8}

\[ \frac{1}{\sigma^-} \cdot \left( \frac{d\sigma}{d \cos \theta_1 d \cos \theta_2} \right)^d_{\beta \rightarrow 0} = \frac{1}{\sigma^-} \cdot \left( \frac{d\sigma}{d \cos \theta_1 d \cos \theta_2} \right)^d_{\beta \rightarrow 0} \] \tag{4.9}

\[ \approx \frac{1}{4} \cdot \left[ 1 - \frac{(m_N^2 - 2m_W^2)^2}{(m_N^2 + 2m_W^2)^2} \cdot \cos \theta_1 \cos \theta_2 + \frac{f_2}{f_1} \cdot \frac{(m_N^2 - 2m_W^2)}{(m_N^2 + 2m_W^2)} \cdot (\cos \theta_1 - \cos \theta_2) \right] . \]

In this limit, the cases ' + ' and ' − ' remain distinct, but the case \( \eta_{CP} = -1 \) converges to the

Dirac case.
V. COMMENTS

We comment briefly on some other papers which have a partial overlap with the considerations presented above.

(i) Our discussion of the production reaction $e^+e^- \rightarrow N_1N_2$ follows very closely that given in Ref. [1]. Our results for $d\sigma/d\Omega$ given in the Appendix (Eqs. (A.1)–(A.6)) essentially coincide with those in this paper, with two minor differences: The angular distributions for the case of two distinct Majorana particles with the same $CP$-parity, as well as for the case of two distinct Dirac particles (Eq. (4E) and (4D) in Ref. [1]), are slightly different from our distributions, presented in the Appendix (Eq. (A.2) and (A.3)).

(ii) The cross section for the Majorana process $e^+e^- \rightarrow N_1N_2$, with $m_1 = m_2$ and $\eta_{CP} = +1$ calculated in Ref. [2] agrees with that obtained in this paper. However the Dirac case $e^+e^- \rightarrow N\bar{N}$ (Eq. (2) of Ref. [2]) differs from our result (Eq. (A.5)), as also noted in Ref. [1].

(iii) The spin-summed differential cross section for the Majorana process $e^+e^- \rightarrow N_1N_2$ (with $m_1 = m_2$, $\eta_{CP} = +1$) calculated in the present paper, as well as in Refs. [1,2], differs from that given in Ref. [1], but agrees with the results given in Refs. [3,6,10].

(iv) Our analysis of heavy Majorana production and decay has been essentially model-independent. Discussions in the context of specific gauge models, based on $SU(2)_L \times SU(2)_R \times U(1)$ or $E(6)$ symmetries, may be found in Refs. [1,11,12].
APPENDIX A: DIFFERENTIAL CROSS SECTION FOR $e^+e^- \rightarrow N_1N_2$

Following Ref. [1], we consider the following five cases, in which $N_1$ and $N_2$ are

A distinct Dirac particles.

B distinct Majorana particles with the same $CP$-parity.

C distinct Majorana particles with opposite $CP$-parity.

D Dirac particle-antiparticle pair.

E identical Majorana particles.

Choosing the $N_1$ direction in the $e^+e^-$ c.m. system to be the z-axis, and the $e^-$-beam direction to be at an angle $\theta$ (scattering angle), the momenta $(q_1, q_2)$ and spins $(t_1, t_2)$ of $N_1$ and $N_2$ have components

\[ N_1 : \quad q_1^\mu = (\gamma m_1, 0, 0, \gamma/\beta m_1) , \]
\[ t_1^\mu = (\gamma/\beta n_z, n_x, n_y, \gamma/\beta m_1) , \]
\[ N_2 : \quad q_2^\mu = (\gamma'/m_2, 0, 0, -\gamma'/\beta m_2) , \]
\[ t_2^\mu = (-\gamma'/\beta n'_z, n'_x, n'_y, \gamma'/n'_z) . \]  \tag{A.1}

The differential cross sections are (with $\beta = (1 - 4m_1^2/s)^{1/2}$, $\beta' = (1 - 4m_2^2/s)^{1/2}$, $\gamma = (1 - \beta^2)^{-1/2}$, $\gamma' = (1 - \beta'^2)^{-1/2}$, $\lambda(x, y, z) = [(x^2 + y^2 + z^2) - 2(xy + yz + zx)]^{1/2}$)

\[
\frac{d\sigma}{d\Omega}_A = \frac{G_F^2\alpha^2_N}{512\pi^2} |R(s)|^2 \left[ 1 - \frac{(m_1^2 - m_2^2)^2}{s^2} \right] \lambda(s, m_1^2, m_2^2) \left\{ f_1 \left[ (1 + C^2\beta \beta') - (\beta n'_z + \beta' n_z)(1 + C^2) - (\beta' n_x/\gamma + \beta n_x'/\gamma')SC 
+ n_z n'_x(C^2 + \beta \beta') + (n_x n'_x/\gamma + n'_x n_z/\gamma')SC + n_x n'_x S^2/\gamma \gamma' \right] 
\right.
+ f_2 \left[ C(\beta + \beta') - (n_z + n'_z) C(1 + \beta \beta') - (n_x/\gamma + n_x'/\gamma')S 
+ n_z n'_z C(\beta + \beta') + S(\beta' n_x n'_x/\gamma + \beta n'_x n_z/\gamma') \right] \right\} , \tag{A.2}
\]
\[
\frac{d\sigma}{d\Omega}_B = \frac{G^2_F\alpha^2_N}{256\pi^2} |R(s)|^2 \left[ 1 - (m^2_1 - m^2_2)^2/s^2 \right] \lambda(s, m^2_1, m^2_2)
\]
\[
\begin{align*}
& f_1 \left[ n_x n'_x S^2 \left( 1/\gamma' - 1 \right) + n_y n'_y S^2 \beta' \beta' + n_z n'_z \left( \beta' - C^2 \left( 1/\gamma' - 1 \right) \right) \right] \\
& + (n_x n'_x - n_x n_z) S C \left( 1/\gamma - 1/\gamma' \right) + C^2 \beta' - 1/\gamma' + 1 \\
& + f_2 \left[ (n_x - n'_x) S \left( 1/\gamma' - 1/\gamma \right) + (n_z + n'_z) C \left( 1/\gamma' - \beta' - 1 \right) \right] \}
\end{align*}
\]

\[
\frac{d\sigma}{d\Omega}_C = \frac{G^2_F\alpha^2_N}{256\pi^2} |R(s)|^2 \left[ 1 - (m^2_1 - m^2_2)^2/s^2 \right] \lambda(s, m^2_1, m^2_2)
\]
\[
\begin{align*}
& f_1 \left[ n_x n'_x S^2 \left( 1/\gamma' - 1 \right) + n_y n'_y S^2 \beta' \beta' + n_z n'_z \left( \beta' + C^2 \left( 1/\gamma' + 1 \right) \right) \right] \\
& + (n_x n'_x + n'_z n_x) S C \left( 1/\gamma + 1/\gamma' \right) + C^2 \beta' + 1/\gamma' + 1 \\
& - f_2 \left[ (n_x + n'_x) S \left( 1/\gamma + 1/\gamma' \right) + (n_z + n'_z) C \left( 1/\gamma' + \beta' + 1 \right) \right] \}
\end{align*}
\]

\[
\frac{d\sigma}{d\Omega}_D = \left( \frac{d\sigma}{d\Omega}_{A,m_1=m_2} \right) = \frac{G^2_F\alpha^2_N}{512\pi^2} |R(s)|^2 \lambda(s, m^2, m^2)
\]
\[
\begin{align*}
& f_1 \left[ (1 + C^2 \beta') - (n_z + n'_z) \beta (1 + C^2) - (n_x + n'_x) S C / \gamma + n_z n'_z (C^2 + \beta^2) \right] \\
& + (n_x n'_x + n_z n'_x) S C / \gamma + n_z n'_z S / \gamma \right] + f_2 \left[ 2 C \beta - (n_z + n'_z) C (1 + \beta^2) \right] \\
& - (n_x + n'_x) S / \gamma + 2 n_z n'_z C \beta + (n_x n'_z + n'_x n_z) S \beta / \gamma \}
\end{align*}
\]

\[
\frac{d\sigma}{d\Omega}_E = \frac{1}{2} \cdot \frac{d\sigma}{d\Omega}_{B,m_1=m_2} = \frac{G^2_F\alpha^2_N}{512\pi^2} |R(s)|^2 \lambda(s, m^2, m^2) \beta^2
\]
\[
\begin{align*}
& f_1 \left[ (n_y n'_y - n_x n'_x) S^2 + (1 + n_z n'_z) (1 + C^2) \right] - f_2 \left[ 2 (n_z + n'_z) C \right] \}
\end{align*}
\]

APPENDIX B: MATRIX ELEMENTS FOR \( e^+e^- \rightarrow N_1 N_2 \rightarrow e^\pm e^- W^\mp W^+ \)

1. Like Sign Dileptons

The matrix element for the reaction (Fig. 2)

\[
e^+(p_2, t_2) + e^-(p_1, t_1) \rightarrow e^-(k_1)e^-(k_2)W^+(k_3, \lambda_3)W^+(k_4, \lambda_4) \] (B.1)

is
\[ \mathcal{M} = i A j_\mu^e \Delta Z_{\mu\nu} \left\{ \lambda_2 \bar{u}_{t_1}(k_1) \gamma_\rho \frac{1}{2}(1 - \gamma_5) \frac{\hat{g}_1 + m_1}{q_1^2 - m_1^2 + i m_1 \Gamma_1} \gamma_\nu \frac{1}{2}(1 - \gamma_5) \right. \\
\left. - \frac{-\hat{g}_2 + m_2}{q_2^2 - m_2^2 + i m_2 \Gamma_2} \gamma_\sigma \frac{1}{2}(1 + \gamma_5) v_{t_2}(k_2) \right\} \epsilon_{\lambda_3}^{*\sigma}(k_3) \epsilon_{\lambda_4}^{*\sigma}(k_4) . \] (B.2)

Upon rearrangement, this gives the matrix element in Eq. (3.2).

### 2. Unlike Sign Dileptons

The matrix element for the Majorana-mediated process (Fig. 2)

\[ e^+(p_2, t_2) + e^-(p_1, t_1) \rightarrow e^-(k_1)e^-(k_2)W^+(k_3, \lambda_3)W^+(k_4, \lambda_4) \] (B.3)

is

\[ \mathcal{M}_m = -i A j_\mu^e \Delta Z_{\mu\nu} \left\{ \lambda_2 \bar{u}_{t_1}(k_1) \gamma_\rho \frac{1}{2}(1 - \gamma_5) \frac{\hat{g}_1 + m_1}{q_1^2 - m_1^2 + i m_1 \Gamma_1} \gamma_\nu \frac{1}{2}(1 - \gamma_5) \right. \\
\left. - \frac{-\hat{g}_2 + m_2}{q_2^2 - m_2^2 + i m_2 \Gamma_2} \gamma_\sigma \frac{1}{2}(1 - \gamma_5) v_{t_2}(k_2) \right\} \epsilon_{\lambda_3}^{*\sigma}(k_3) \epsilon_{\lambda_4}^{*\sigma}(k_4) . \] (B.4)

Upon rearrangement, this gives the matrix element in Eq. (4.1).
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FIGURE CAPTIONS

1. Feynman diagram for the reaction $e^+e^- \rightarrow N_1N_2$

2. Diagram showing the sequential process $e^+e^- \rightarrow N_1N_2 \rightarrow e^\pm e^-W^\mp W^+$

3. Energy correlation of the $e^-e^-$ lepton pair in the reaction $e^+e^- \rightarrow N_1N_2 \rightarrow e^-W^+e^-W^+$, for the cases (a) $\eta_{CP} = +1$, (b) $\eta_{CP} = -1$. (Parameters for this and succeeding figures: $\sqrt{s} = 1.2$ TeV, $m_N = 500$ GeV.)

4. Angle correlation of $e^-e^-$ dileptons in $e^+e^- \rightarrow N_1N_2 \rightarrow e^-W^+e^-W^+$, for (a) $\eta_{CP} = +1$, (b) $\eta_{CP} = -1$

5. Energy correlation of $e^-e^+$ dileptons in $e^+e^- \rightarrow N_1N_2 \rightarrow e^-W^+e^+W^-$: (a) Majorana pair, $\eta_{CP} = +1$, (b) Majorana pair, $\eta_{CP} = -1$, (c) Dirac $N\bar{N}$-pair

6. Angle correlation of $e^-e^+$ dileptons in $e^+e^- \rightarrow N_1N_2 \rightarrow e^-W^+e^+W^-$: (a) Majorana pair, $\eta_{CP} = +1$, (b) Majorana pair, $\eta_{CP} = -1$, (c) Dirac $N\bar{N}$-pair
FIG. 1.

\[ e_{s_2}^+(p_2) \rightarrow Z^0(q) \rightarrow N_1 t_1(q_1) \]

\[ e_{s_1}^-(p_1) \rightarrow N_2 t_2(q_2) \]

FIG. 2.

\[ e_{s_1}^+(p_1) \rightarrow Z^0(q) \rightarrow N_1(q_1) \]

\[ W_{\lambda_3}^+(k_3) \rightarrow e_{t_2}^-(+)(k_2) \]

\[ e_{s_2}^-(p_2) \rightarrow N_2(q_2) \]

\[ W_{\lambda_4}^{++(-)}(k_4) \]
$e^- e^- \text{ Final State: Energy Correlation}$

FIG. 3.
$e^- e^-$ Final State: Angular Correlation

(a) Majorana (+)

(b) Majorana (−)

FIG. 4.
$e^+e^-$ Final State: Energy Correlation

Majorana ($+$)  

![Graph of Majorana ($+$)](image)

Majorana ($-$)  

![Graph of Majorana ($-$)](image)

Dirac  

![Graph of Dirac](image)

FIG. 5.
$e^+e^-$ Final State: Angular Correlation

Majorana ($+$)  

Majorana ($-$)

Dirac

FIG. 6.