Prediction on Lubrication and Friction of Mold Flux Based on Inverse Problem in a Continuous Slab Casting Process

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The uniformity of strand solidification and lubrication in continuous casting mold has a great effect on the slab quality and production stability. In this work, an inverse problem model, based on online measured mold temperature, was developed to simulate mold heat transfer and strand solidification. Through quantifying the local heat flux and its distribution between the mold and strand shell, the temperature field and strand solidified behavior inside mold were obtained. Following Newton law of viscosity, Coulomb law of friction and mixed lubrication theory, the mathematical model of mold flux lubricated and frictional behavior was proposed in view of various contact states between mold flux and strand surface. The non-uniform distributions of slag film thickness, lubrication status and frictional stress were further investigated. The inverse problem model could reflect the non-uniformity of solidification and lubrication, and therefore would provide a worthwhile calculation method for exploring the complex lubricated and frictional behavior inside mold.

KEY WORDS: mold flux; lubrication; friction; inverse problem; slab continuous casting.

1. Introduction

In slab continuous casting process, mold is the core component of continuous caster. Heat transfer, solidification and mechanics behavior in mold are key factors determining the slab quality and casting process. Understanding the status of these key factors within the mold in real time becomes very important for casting process optimization and slab quality control.1) Local status such as friction details could not be obtained readily for the sake of hostile condition on current available detection means.2,3) Therefore, numerical simulation is a necessarily important way to investigate the details of such local lubricated and frictional behavior.4–6)

In conventional research, a variety of numerical models were developed to simulate heat transfer behavior between strand and mold, most of which with heat flux empirical formula as boundary condition.7) As expected, these models can’t simulate the real non-uniform status inside mold, which has less guiding significance for continuous casting production. Thomas et al. calculated heat flux applying classical heat resistance method, and the liquid mold flux and solid mold flux thickness were further obtained.8) Focusing on the lubricated behavior of mold flux, models were developed to simulate the distribution of mold flux layer. On the basis of N-S equation and Bikerman equation, Yamauchi et al. built a model to investigate the thickness of liquid slag layer precisely.8) Ramirez-Lopez et al. simulated the flow and variation of liquid slag layer in a cycle.9) However, the heat transfer and lubricated behavior is non-uniform, which is closely related to casting conditions and changes instantaneously. Therefore, expanded understanding of the non-uniform behavior inside mold contributes to control the steel solidification and surface defects.

In the present work, a trial of combing inverse heat transfer model10,11) and lubrication model has been performed to predict the non-uniformity and local information of lubricated behavior of mold flux. It may provide a worthwhile calculation method for exploring the complex process inside continuous casting mold.

2. Experiment Details and Mathematical Model

2.1. Experiment Details

The experiment was conducted on a curved slab caster which produced slab with the thickness of 143 mm. The length of mold is 1 200 mm. Target meniscus height is 82 mm and mold flux is manually added. To detect mold sticking and heat transfer behavior inside of mold, 24 pairs of thermocouples are embedded into mold wall as shown in Fig. 1, which are placed on two different heights away from mold top: 12 on level 190 mm away and 12 on level 390 mm away. Among each group of the 12 thermocouples, 10 were placed on wide plates and 2 were placed on narrow plates. The detecting frequency of mold temperature is 1 Hz. The value of temperature in this study is detected over a time of steady casting state, and the casting parameters are also measured.

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2.2. Heat Transfer Inverse Problem Model

Concerning the heat transfer characteristics of the continuous casting slab, the assumptions are made as follows:

a) Heat transfer along the casting direction is neglected, heat transfer is transformed to two-dimensional unsteady state problem;

b) The top and bottom of copper plates are considered adiabatic;

c) The physical properties of copper mold change are approximated to constant, considering they have little change while temperature blows 400°C.

On the basis of the above assumptions, the heat transfer equations between mold and slab are established respectively, as follows:

\[
\rho \tau \lambda_{mold} \frac{\partial T_{mold}}{\partial \tau} = \frac{\partial}{\partial x} \left( \lambda_{mold} \frac{\partial T_{mold}}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda_{mold} \frac{\partial T_{mold}}{\partial y} \right) + \cdots (1)
\]

\[
\rho \tau \lambda_{steel} \frac{\partial T_{steel}}{\partial \tau} = \frac{\partial}{\partial x} \left( \lambda_{steel} \frac{\partial T_{steel}}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda_{steel} \frac{\partial T_{steel}}{\partial y} \right) + Q_{source} \cdots (2)
\]

where \( \rho \) is density, kg/m³; \( c \) represents heat capacity, J/(kg·°C); \( \tau \) is time, s; \( x, y \) are coordinates represent the width and the thickness direction, respectively, m. \( \lambda \) represents heat conductivity of metal, W/(m·°C); the subscript of mold and steel represent copper plate and liquid steel, respectively. \( Q_{source} \) is the latent heat source of steel, J/kg, which is dealt with equivalent specific heat method in this paper.

The initial and boundary conditions are given as follows:

Initial condition for steel:

\[
\tau = 0, \quad T = T_p \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (3)
\]

where \( T_p \) is casting temperature, °C.

Boundary condition at interface between mold and cooling water:

\[
q_c = h_u \left( t_n - t_w \right) \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (4)
\]

Boundary condition at interface between mold and strand:

\[
-\lambda \frac{\partial T}{\partial x} = q_s, \quad -\lambda \frac{\partial T}{\partial y} = q_n \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (5)
\]

where \( q_c \) is the heat flux of the interface of mold and cooling water, W/m²; \( t_n \) and \( t_w \) are temperatures of copper plate cold face and cooling water, respectively; \( h_u \) represents the convection heat transfer coefficient of cooling water, W/(m²·°C); \( q_s \) and \( q_n \) are the heat fluxes between mold and strand surface on wide and narrow faces, respectively, W/m². Because \( q_s \) and \( q_n \) are unknown, the inverse algorithm is needed for solving the equation.

Based on the equations, an inverse heat transfer problem model was developed with finite difference method (FDM). In the model, mold temperature field was calculated based on the heat transfer equation and initial condition. After the 2-D slice model move down to mold exit, the RMS (root mean square) of temperature difference between thermocouples calculated and measured values, was evaluated to judge if the calculated temperature was in accord with the real one. As is known, a different heat flux corresponds to a different temperature field. The local heat flux, \( q_{ij}^p \) (where \( i \) and \( j \) represent the row and column numbers of thermocouples installed, respectively, and \( p \) refers to the iterations times) will be adjusted by add a positive or negative value when RMS doesn’t meet the condition, and the temperature of mold will be re-calculate, as shown in formula (6). The RMS would be evaluated and the \( q_{ij}^p \) would be adjusted every time after the calculation until the RMS was met the condition, and when the corresponding \( q_{ij}^p \) can be considered as the actual value of heat flux, which can be used in direct heat transfer model. Figure 2 schematically shows the
flow chart of heat transfer simulation procedure, where IHTP means inverse heat transfer problem, DHTP means direct heat transfer problem, and LFEBP means lubricated and frictional behavior problem model.

\[ q_{l,i}^{m+1} = q_{l,i}^m + \Delta q_{l,i} \] .............................. (6)

Parameters needed in heat transfer calculation are shown in Table 1.

### Table 1. Casting parameters and thermophysical property.

| Item                     | Symbol | Value   | Unit |
|--------------------------|--------|---------|------|
| Carbon content           | C%     | 0.17    | %    |
| Steel liquidus temperature | \( T_{eq} \) | 1517    | °C   |
| Slab geometry            | \( W \times N \) | 1530 x 143 | mm x mm |
| Casting speed             | \( V_c \) | 2.3     | m/min|
| Oscillation frequency    | \( f \) | 188.5   | cpm  |
| Oscillation stroke       | \( S \) | 8.34    | mm   |
| Pouring temperature      | \( T_p \) | 1540    | °C   |
| Steel solidus temperature | \( T_{sol} \) | 1458   | °C   |
| Steel density            | \( \rho_{steel} \) | 7500–1.2(T–\( T_{sol} \)) | kg/m³ |
| Steel heat capacity      | \( c_{steel} \) | (666 + 0.177) | J/(kg·°C) |
| Steel heat conductivity  | \( \lambda_{steel} \) | K(13.86 + 0.0113T) | W/(m·°C) |
| Convective amplification factor | \( K \) | 5–7   | –    |
| Mold density             | \( \rho_{mold} \) | 9800   | kg/m³ |
| Mold heat capacity       | \( c_{mold} \) | 390   | J/(kg·°C) |

2.3. Prediction Model for Lubricated and Frictional Behavior

Based on the temperature distribution of mold plate and strand, a model was developed to calculate the gap between mold and strand. For the sake of convenient and rapid calculation, ferrostatic pressure, mechanics behavior of solidified shell and friction stress of solid slag are neglected, the shrink of solidification is viewed as the only reason for gap formation.15,16)

\[ \text{Gap} = \sum_{i=1}^{f} \left[ T_{\text{sol}} - T_i \right] E \left( \frac{W}{2} \right) - \left[ z \left( \frac{\theta_{\text{mold}}}{100} \right) \left( \frac{W}{2} \right) \right] \] ............................. (7)

where \( i \) represents solidified shell node, and \( f \) is the latest solidified node near liquid core, \( T_i \) is the node temperature of strand shell, °C; \( E \) is the linear contractive coefficient of steel, 1/°C; \( z \) represents the distance of 2-D slice below meniscus, m; \( \theta_{\text{mold}} \) is the mold taper, %/m; \( T_{\text{sol}} \) is the solidus temperature of steel; \( W \) is the width of mold, m.

\[ E = \sum f_i x_i + 0.12217T + 1.357 \times 10^{-6} T^2 \] ............................. (8)

It should be noted that the gap around the meniscus could not be calculated by formula (7) because of the surface tension between strand and slag which lead to a larger gap around the meniscus, as shown in Fig. 3. Aiming to solve this question, formula (9) and (10) proposed by Bikerman17) are applied to calculate and revise the gap thickness around meniscus:

\[ d_{\text{meniscus}} = -\sqrt{2a^2 - z^2} + \sqrt{a^2 + z^2} \ln \left( \frac{\sqrt{a^2 + z^2}}{z} \right) + \left( 1 - \frac{1}{\sqrt{2}} \ln \left( \sqrt{2} + a \right) \right) a \] ............................. (9)

\[ a^2 = \frac{2\sigma_{s,f}}{\left( \rho_{\text{steel}} - \rho_{\text{slag}} \right) g} \] ............................. (10)

where \( \sigma_{s,f} \) represents the surface tension between steel liquid and slag, N/m; \( \rho_{\text{slag}} \) is slag density, kg/m³; \( g \) is the gravity acceleration, m/s². Formula (9) can be simplified as below:39)

\[ d_{\text{meniscus}} = d_e T_{\text{sl}}^{z \frac{d_1}{d_2}} \] ............................. (11)

In the formula, \( L_s = 7 \) mm, \( d_1 = 15 \) mm, \( d_2 = 0.2 \) mm.

The result of formula (11) is approaching to 0 when \( z \) is lower than 7 mm, the gap calculation formula within the entire height in mold can be described by integrated formula (7) and formula (11) as bellow:

\[ d_{\text{gap}} = \text{Gap} + d_{\text{meniscus}} \] ............................. (12)

The gap between mold and strand is filled with liquid and solid slag absolutely when temperature of strand surface (\( T_{\text{shell}} \)) is higher than slag melting point (\( T_{\text{sol}} \)),38) and temperature distribution in the gap is regarded as linear because the gap is very narrow. So the thickness of liquid and solid slag film can be determined by following equations:

\[ q_{\text{mold}} = q_{\text{slag}} \] ............................. (13)

\[ q_{\text{slag}} R_{\text{int}} = T_2 - T_{\text{mold}} \] ............................. (14)

\[ (T_{\text{shell}} - T_{\text{sol}}) / d_1 = (T_{\text{shell}} - T_2) / d_{\text{gap}} \] ............................. (15)

\[ (T_2 - T_{\text{mold}}) / R_{\text{int}} = (T_{\text{shell}} - T_2) k_{\text{eff}} / d_{\text{gap}} \] ............................. (16)

\[ d_{\text{gap}} = d_l + d_e \] ............................. (17)

where \( R_{\text{int}} \) represents the interface thermal resistance between mold and slag, m²·°C/W; \( k_{\text{eff}} \) represents the effective thermal conductivity, W/(m·°C); \( T_{\text{mold}} \) is the hot face temperature of mold plate, °C. \( q_{\text{mold}} \) and \( q_{\text{slag}} \) are the heat flux of mold wall and slag film, W/m². \( T_2 \) is the temperature of solid slag near mold wall, °C. \( d_{\text{gap}} \) is the gap size calculated by Eq. (12). \( d_l \) and \( d_e \) are the thickness of liquid and
solid slag film, respectively, mm, as shown in Fig. 4.

Based on the mold flux model, friction and lubrication status can be divided into four situations as following according to the contact status among mold, slag and slab:

I. When \( T_{\text{shell}} \) (slab surface temperature) is higher than the \( T_{\text{cry}} \) (slag crystallization temperature), the shell is lubricated by the liquid friction only;

II. When \( T_{\text{shell}} \) (slag solidification temperature) < \( T_{\text{shell}} \) < \( T_{\text{cry}} \), dendritic crystal in slag film is going to precipitate and increase as the temperature decreases, which means the lubricant effect of liquid film begins to disappear and a mixed lubrication status between solid and liquid lubrication on strand shell is established;

III. With the temperature dropping further, liquid slag film disappears without producing air gaps; in other words, the relative movement occurs between the mold wall and the solid slag film and the frictional force is solid friction;

IV. At the lower part of the mold, no liquid film flowing into the gap, the air gap between solid slag film and mold would form with the reason of the larger shrinkage of strand shell, thus, no frictional behavior exists at this stage.

The distribution of lubrication and friction described above is shown in Fig. 3.

Liquid slag film will disappear at a slice \((\varphi, Z_k)\) in longitudinal section as the temperature decrease, where \( \varphi \) and \( Z_k \) represent the positions of gap along the width direction and the height direction, respectively. The condition whether the gap formed or not is described as below: When \( d_{\text{gap}} \) (the width of gap at k slice in longitudinal direction) \( \leq d_{\text{gap}} \) \((\varphi, Z_{k-1})\), air gap does not form, and only solid friction exist, which can be calculated based on Newton’s friction law as follows:

\[
\tau_{\text{sls}} = \eta (\rho_{\text{slag}} g h_0 + \rho_{\text{steel}} g Z_k) \quad \text{........................... (19)}
\]

where \( \eta \) is friction coefficient between slab surface and solid slag, \( h_0 \) is depth of slag bath, m.

When \( d_{\text{gap}} \) (\( \varphi, Z_{k-1} \)) > \( d_{\text{gap}} \) (\( \varphi, Z_k \)), the air gap is going to form, mold plates separate from strand, and no friction exist in this situation.

In mixed lubrication stage, slab shell is lubricated by liquid and solid slag, which can be calculated by mix friction theory, using Eqs. (18) and (19):

\[
\tau_{\text{mix}} = \alpha \tau_{\text{sls}} + (1 - \alpha) \tau_{\text{sls}} \quad \text{........................... (20)}
\]

where \( \alpha \) is the percentage of solid friction, which varies with the slab surface temperature descending and ranges from 0 to 100%.

Based on the four situations and calculation method mentioned above, the total friction force between slab and mold can be computed through integration as follows:

\[
F = \sum (\tau \cdot \Delta A) \quad \text{........................... (21)}
\]

where \( \Delta A \) is the surface area of different contact situations, m².

It should be noted that if slab surface temperature is quite close to slag solidification temperature, liquid slag film hardly exists and \( d_i \) is very small, resulting in that the calculated value of mixed frictional force is larger than that of solid frictional force, which is apparently unreasonable. In this work, strand surface is regarded as solid lubrication stage when the mix frictional force equal to the solid frictional force. Corresponding parameters in friction calculation are shown in Table 2.

### Table 2. Relative parameters of friction force calculation.

| Item                  | Symbol      | Value | Unit |
|-----------------------|-------------|-------|------|
| Mold taper            | \( \theta_{\text{solid}} \) | 1.2   | %/m  |
| Mold velocity         | \( V_m \)   | 2\( \pi f c \cos2\pi ft \) | m/s  |
| Mold oscillation amplitude | \( s \) | 0.003 | m    |
| Slag density          | \( \rho_{\text{slag}} \) | 3000  | kg/m³|
| Liquid slag pool depth | \( h_0 \)  | 0.015 | m    |
| Solid slag/shell Interface friction coefficient | \( \eta \) | 0.48 | –    |
| Crystallizing temperature of slag | \( T_{\text{cry}} \) | 1201 | °C   |
| Solidifying Temperature of slag | \( T_{\text{sol}} \) | 1090 | °C   |
| Melting Temperature of slag | \( T_{\text{mel}} \) | 1166 | °C   |
| Interface thermal resistance | \( R_{\text{int}} \) | 0.00045 | m²·°C/W |
| Effective heat conduction coefficient | \( k_{\text{eff}} \) | 1.12 | W/(m·°C) |
| Temperature dependent viscosity exponent | \( n \) | 1.24 | –    |
| Viscosity of liquid slag | \( \mu \) | 540(\( F/T_{\text{mel}} \))^n | Pa·s  |
3. Result and Discussion

3.1. Validation of Heat Transfer

Some of the result can be confirmed with plant data in order to show the validity of the present model. Figure 5 shows the comparison of calculated and measured temperatures at the monitoring points. Along the mold width direction, the distribution of temperature is non-uniform and the site having highest temperature value is located about 200 mm away from the corner, the temperature of first row is more than 50°C higher than that of second row. The temperature of first row on two wide faces distributes as the mirror symmetry generally, possibly due to the non-uniformity of slag infiltration and the bias flow of SEN. The temperature of second row is relatively lower because of the gradually increased air gap and shell thickness. Figure 5(c) shows the temperature trend along the casting direction for narrow face. The temperature of narrow face is lower than that of wide face in both two rows; therefore, the calculated value is relative low. The highest temperature occurs at 100 mm away from the meniscus and decreases gradually until to mold exit. Overall, the calculated temperature coincides well as the measuring data and the trend is same as the actual condition, which could reflect the non-uniform distribution of mold temperature filed in real condition.

Figure 6 shows the distribution of transient slab surface temperature of outside radius, which is relatively non-uniformity along crosswise direction and getting worse along the casting direction. At the exit of mold, more than 50°C difference of temperature along crosswise. In slab width direction, temperature in center left of slab is high, where the heat flux is relatively low, and temperature near narrow face is relatively low.

3.2. Distribution of Liquid Slag Film

The liquid slag film distribution is similar to the distribution of temperature, as shown in Fig. 7, which is also the transient state of outside radius. The liquid slag film is relatively thick near the meniscus, where liquid slag can easily flow into the gap between mold and slab. As temperature is dropping along casting direction, the thickness of liquid slag film decreases until disappears when temperature is below the slag solidification temperature. Because of the high temperature in the longitudinal area 400–600 mm away from the left corner where temperature is higher than slag crystallization temperature even at the exit of mold, there is a continuous liquid slag film maintained from the top of mold to the exit (about 0.1 mm at mold exit). Thus, the whole lubricated behavior in this area is liquid lubrication rather than the four alternative situations mentioned above. In other longitudinal area, liquid slag film disappeared at about 300 mm away from mold exit and whole liquid lubrication did not occur merited.

Based on Fig. 7, liquid frictional force in different length and width can be calculated successively. Figure 8 shows that liquid frictional force is inversely proportional to liquid
slag thickness at the moment when mold oscillation velocity is up maximum, which is consistent with Eq. (12). However, the frictional force value fluctuated in a small range rather than keeping in a certain value even the thickness of slag is invariant. The reason is that different slag temperature and their gradient result in the variety of slag viscosity, which leads to the difference of frictional force. In addition, when liquid slag film thickness is lower than 0.05 mm, the calculated frictional force is higher than 35 kPa, which is not consistent with reality, which shows the necessity of model modification.

3.3. Slab Surface Lubrication and Friction State

Based on the slab surface temperature, liquid slag distribution combined with the numerical model, the distribution of solid, liquid slag and air gap are calculated. Figure 9 shows the distribution of lubricated and frictional behaviors on slab surface at the moment when mold oscillation velocity is up maximum and the tensile friction is up maximum. The lubrication state of slab surface, shown in Fig. 9, is determined by the distribution of slab surface temperature shown in Fig. 6. Along the width direction, upper-middle part of slab is primarily liquid lubrication, the middle or low part of slab is mixed lubrication or solid lubrication. Next to the solid lubricated behavior, air gap appear at the low temperature region and extend to the mold exit. Along the casting direction, mixed lubrication region is relative long in high temperature region, about 400 mm, and solid lubrication distributes as narrow band which is next to the mixed lubrication. Due to the higher casting speed, strand shell has smaller shrink and thinner shell thickness at mold exit, the air gap appears 600 mm away from meniscus. The distribution of lubrication states are consistent to slab temperature and slag film distribution generally, which reflects to heat transfer, solidification process and slag thermal performance.

3.4. Frictional Stress Variation Along Longitudinal and Transverse Direction

Frictional stress variation in different longitudinal sections is shown in Fig. 10. In the longitudinal section 600 mm away from left corner, only liquid and mixed lubrication states act along casting direction for the reason that high temperature leads to the thicker liquid film which still exist at mold exit, and liquid lubrication length is more than 700 mm. In other three longitudinal sections with different distance away from the left corner, because of the lower temperature, the lubrication states act from liquid lubrication to solid lubrication.
air gap, and taking on all four lubrication states: lubrication changed from liquid lubrication to mixed lubrication at the location 350 mm away from meniscus, where frictional force increased rapidly until to the solid lubrication region and disappear after the appearance of air gap. The average frictional force of the three sections is about 7.76 kPa, which is obvious higher than liquid lubrication region. In summary, in liquid lubrication region, the frictional force changed slightly and the value is relative low; in mixed lubrication region, frictional force increased faster because of liquid slag film become thinner and the percent of solid frictional force increased; in solid lubrication region, frictional force is mainly determined by steel static pressure, which varies with the molten steel height.

Figure 11 shows the variation of frictional stress acting on strand surface along transverse direction at 400 and 800 mm below meniscus. For 400 mm section, the non-uniform temperature distribution, which varies around slag crystallization temperature, leads to the alternation between liquid and solid slag. The liquid lubrication leads to a low frictional stress, about 4.51 kPa, although most regions are mixed lubrication. At 800 mm location, lubrication states vary from mixed lubrication to air gap; because of the decreased temperature, and the variation of frictional stress is comparatively obvious. The relatively high solid frictional force lead to higher total frictional value of about 8.72 kPa compared to middle-upper slab, although air gap region is nearly a half of slab width.

4. Conclusions

A novel method which combined inverse heat transfer problem with mold friction calculation model was developed to predict the non-uniform mold friction. Following Newton law of viscosity, Coulomb law of friction and mixed lubrication theory, the mathematical model of mold flux lubricated and frictional behavior was proposed on the basis of considering different contact states between mold flux and strand surface, which could be used to discriminate and quantize lubricated behavior of mold slag in slab continuous casting.

Liquid slag film, which distributes as temperature of slab surface, is relative thick at meniscus, and decrease as temperature dropping until it disappears. Frictional force increases sharply when liquid slag film is close to disappear. In the middle-upper part of mold 400 mm away from meniscus, liquid lubrication is mainly lubrication state and frictional force is relative low. However, in the middle-lower part, because of the low and non-uniform temperature distribution, lubrication states vary from liquid lubrication to air gap and contain four lubrication states; thus the frictional force is relative high. Although the solid lubrication region distributes as narrow band at the bottom of mixed lubrication region, the solid lubrication leads to a serious friction and damage to the slab, which should be paid more attention.

The proposed model could present more precise and accurate information about the non-uniform distribution on mold heat transfer, slag film thickness, lubricated and frictional behavior, and therefore would provide a worthwhile calculation method for exploring the complex process inside mold.

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