Fire and Heat Spreading Model Based on Cellular Automata Theory

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Abstract. The distinctive feature of the proposed fire and heat spreading model in premises is the reduction of the computational complexity due to the use of the theory of cellular automata with probability rules of behavior. The possibilities and prospects of using this model in practice are noted. The proposed model has a simple mechanism of integration with agent-based evacuation models. The joint use of these models could improve floor plans and reduce the time of evacuation from premises during fires.

1. Introduction
The problem of ensuring the safety of people in fires is becoming increasingly important. In recent years, the number of fires in Russia has increased to 240-300 thousand a year. Fires often lead to injuries and casualties among people [1]. In this regard, it is important to predict dangerous fire factors and to adopt preemptive solutions to eliminate fire hazards [2, 3].

However, it is not possible to exclude fire hazards completely. Therefore, modeling the dynamics of the fire spread in the premises is important. For simulation of fire in rooms, thermodynamic models are widely used [4]. However, such models, as a rule, are either computationally complex or give just averaged characteristics of fire and heat spread in the output. Therefore, using them to study the dynamics of the development of fire in the premises is not always appropriate.

Thus, the problem of new models creation which would be, on the one hand, quite simple in terms of implementation and computational complexity, and, on the other hand, reproduce a rather realistic fire and heat dynamics, arises. These models should also be simple in terms of perception by the researcher. In case of cumbersome systems of differential equations usage, the researcher is forced to abstract from the physical sense because of their complexity, which is associated with the risk of making a mistake in the development of the model, both at the stage of its construction and at the stage of its implementation.

These circumstances determine the relevance, scientific novelty and practical significance of this study.

2. Fire spreading model
Let the area, given in the Cartesian coordinate system $xOy$, be divided into cells with side $dl = 0.05$ m. Then $C = \{c_{ij}\mid 0 \leq i < n, 0 \leq j < m\}$ is a network that divides the room into cells; $n, m$ is number of cells horizontally and vertically, respectively; $c_{ij}$ is a cell with coordinates $(i, j)$.

Let us assume that at time $t$ each cell is in one of the following states.

1. **Non-combustible.** A cell of this type corresponds to a section of a wall or floor of a non-combustible material. Let us denote such cells as $c_{ij} \in NC$.

2. **Ignition is possible.** A cell of this type corresponds to a section of a wall or floor made of combustible material, which is not burning at time $t$ and did not do it before. Let us denote such cells as $c_{ij} \in I_t$.

3. **Burning.** A cell of this type corresponds to a section of a wall or floor made of combustible material, which is currently burning. Let us denote such cells as $c_{ij} \in B_t$.

4. **Burning finished.** A cell of this type corresponds to a section of a wall or floor made of combustible material, which in the past burned, but is not and will not burn, since all the combustible substances are consumed. Let us denote such cells as $c_{ij} \in F_t$.

Let there be some set $B_0$ of cells inside which combustion occurred at time $t = 0$. All other cells belong to either the $NC$ set or the $I_t$ set, depending on whether a section of the wall or floor related to it is made of a combustible or non-combustible material.

The probability of cell $c_i$ transition from $I_t$ state to state $B_{t+1}$ is determined by the ratio [5]:

$$P_{ij} = v_{ij} f_{ij} dt / 4 dl,$$

where $v_{ij}$ is the linear velocity of fire spread for cell $c_{ij}$ determined by the material of the wall or floor section to which the cell corresponds; $dt$ is the model time step; $f_{ij}$ is a parameter characterizing the burning of neighboring cells at time $t$.

$$f_{ij} = 2 n'_{ijx} + n'_{ijx},$$

where $n'_{ijx}$ is the number of cells in which burning occurs at time $t$ located orthogonally and $n'_{ijx}$ diagonally, with respect to $c_{ij}$.

The cell transits from the $B_t$ state to the state $F_{t+1}$ if the combustible mass ends inside the cell. Suppose that for each cell $c_{ij}$ the combustible load $m_{ij}$ in kg is given, $\psi_{ij}$ is the burn-up rate of the combustible load in kg/s.

From the moment of the cell ignition, the mass of the combustible substance $m_{ij}$ within it changes by the law:

$$m_{ij}^{t+1} = m_{ij}^t - \psi_{ij} dt,$$

where $m_{ij}^t$ is the mass of the combustible load of cell $c_{ij}$ in time $t$.

Thus, from the moment of ignition to the cessation of burning in the $c_{ij}$ cell, time passes by the expression $m_{ij} / \psi_{ij}$. Formally, probability $H_{ij}^t$ of a cell transition from the $B_t$ state to the $F_{t+1}$ state is determined by expression $H_{ij}^t = 1$ if $m_{ij} \leq 0$ and $H_{ij}^t = 0$ otherwise. That is, the state of each cell $c_{ij}$ is determined by autonomous probabilistic finite-state automaton $A_{ij}$, whose state diagram is shown in Figure 1.

![Figure 1. The automaton state diagram](image)

The state of the $A_{ij}$ automaton at time $t$ is determined by the set of the family $\{NC, I_t, B_t, F_t\}$, in which cell $c_{ij}$ is located. Thus, let us assume that each cell $c_{ij}$ corresponds to autonomous automaton $A_{ij}$, and the whole network $C$ can be considered as fire spread controlling cellular automaton $A$ [6 – 8].
3. Heat spreading model

Let $\theta_{ij}$ be the value by which the temperature of the air inside the cell can increase, provided that it is completely thermal insulation by burning 1 kg of combustible matter inside the cell

$$\theta_{ij} = \lambda_{ij}/C_{ij},$$

where $\lambda_{ij}$ is the specific heat of combustion of the combustible load inside the cell $c_{ij}$; $C_{ij}$ is the heat capacity of the $c_{ij}$ cell [9].

Let $k_{ij}$ be the coefficient specifying the heat-conducting characteristics of the cell $c_{ij}$, which depends on the material and on the height of the ceilings. The $c_{ij}$ cell can exchange heat with a set of other related cells. Let also $S_{ij}$ be the set of cells connected with the cell $c_{ij}$, $T_{ij}^t$ is the temperature inside the cell $c_{ij}$ at time $t$, $T_s^t$ the temperature inside one of the cells in the set $S_{ij}$ at the model time step $t$, and $k_s$ the coefficient specifying the heat-conducting characteristics inside the cell $s$ from of the set $S_{ij}$. $dt$ is the step of the model time.

Then the temperature in cell $c_{ij}$ at each next step of the model time is determined by the relation:

$$T_{ij}^{t+1} = T_{ij}^t + \frac{k_{ij}}{C_{ij}} \sum_{s \in S_{ij}} k_s (T_s^t - T_{ij}^t) + \psi_{ij} \theta_{ij} dt. \quad (2)$$

To define the set of the connected cells is possible in the different ways adapting the model to various conditions. In the model used the configuration specified by heat spread controlling graph $G = (V, E)$.

Let $W \subset C$ be the set of cells, corresponding to the wall, and for each $c_{ij}$ there is a vertex of graph $G = (V, E) v_{ij} \in V$, and $e_{ijkl} \in E$ is the edge of the graph between vertices $v_{ij}$ and $v_{kl}$.

Then the set of edges $E$ is defined as follows:

$$E = \{ e_{ijkl} \mid ((i = k) \& (j - l \in \{5; 25\})) \lor ((j = l) \& (|i - k| \in \{5; 25\})) \} \cup \forall a, b (a \in [\min(i, k); \max(i, k)] \& b \in [\min(j, l); \max(j, l)] \rightarrow c_{ab} \notin W) \lor [(a = k) \& (j - l = 1)) \lor (j = l) \& (i - k = 1])}. \$$

If there is edge $e_{ijkl}$ in graph $G$, then cells $c_{ij}$ and $c_{kl}$ are connected. Each cell has from 4 to 12 connected cells. Figure 2 illustrates the relative location of the cells connected with the $c_{ij}$ cell, which is far from the walls.

![Figure 2. The set of cells connected with $c_{ij}$](image)

The model user can adjust the speed of fire and heat spread by modifying the algorithm for selecting multiple edges for graph $G$, changing the values of parameters $\psi_{ij}$, $\theta_{ij}$, $\nu$, $C_{ij}$ and $k_{ij}$ or initial conditions $T_{0ij}$. It is also possible to add a third dimension to the model, in which case graph $G$ will have edges containing cells from different layers.

The temperature in the extreme and corner cells can be considered equal to external temperature $T_{ext}$ at each step of the model time. Due to such cells, heat exchange with the environment takes place. The use of edges connecting far-away cells allows one to increase the model time step and take into
account the different ways of heat spreading. It is known that air has a low thermal conductivity, so the main mechanisms of heat transfer during a fire are convection and radiation. Let us assume that cells exchanging heat with use of long edges of graph \( G \) do this by convection and radiation.

4. Results and Discussion

Using the developed model, the dynamics of a fire and heat spread in a premises measuring 10 \( \times \) 10 meters was investigated (see Figure 3). The walls are given by the coordinates of the lower-left corner \((x, y)\), and also by the width of the axes \( Ox \) and \( Oy \) \((xWidth, yWidth)\) in the format \((x, y, xWidth, yWidth)\) as follows: \((0.0, 0.0, 10, 0.2); (0.0, 0.2, 0.2, 9.6); (0.0, 9.8, 10, 0.2); (9.8, 0.2, 0.2, 3.8); (9.8, 6.0, 0.2, 3.8); (3.3, 0.2, 0.2, 3.8); (3.3, 6.0, 0.2, 3.8); (6.6, 0.2, 0.2, 3.8); (6.6, 6.0, 0.2, 3.8))

The walls were considered incombustible when modeling, and the floor between the walls was combustible. Thus, all the cells corresponding to the space inside the walls belong to set \( NC \). Set \( B_0 \) is given by a set of 5 cells: a cell with coordinates \((1.25, 5)\) in the middle of the very left room, and also its four orthogonal neighbors. All other cells at time \( t_0 = 0 \) belong to set \( I_0 \). Model time step is \( dt = 0.04c \).

The model uses the following values of the parameters:
\((\forall i \in [0, n))\)(\(\forall j \in [0, m)\))(\(\theta_{ij} = 36000 \ ^\circ C, m_{ij} = 0.1 \ kg, \psi_{ij} = 0.0015 \ kg/s, T_{ij}^0 = 293 \ ^\circ C, v_{ij} = 0.005 \ m/s, k_{ij} = 0.2, D_m = 23 \ m^2/\ kg, c_{ij} \notin NC \rightarrow C_{ij} = 1.0, c_{ij} \in NC \rightarrow C_{ij} = 4.0)\).

Figure 3 shows the distribution of the temperature in the room at time \( t = 270 \) s in the FireSim program window.

The temperature is displayed according to the scale shown in the figure on the right, in the range from \( T_{min} = 20 \ ^\circ C \) (white color, which is also used for lower temperatures) to \( T_{max} = 500 \ ^\circ C \) (black color, which is also used for higher temperatures). Black color also shows the walls. It can be seen from the figure that the velocity of heat propagation through the walls is much less than in the open space. At the exit, one can notice some distortion of the temperature field in the form of light bands, which are located at a distance of 5 and 25 cells from the output and are explained by the method of selecting graph \( G \).

![Figure 3. Distribution of temperature in the room at time t = 270 s](image)

Figure 4 allows us to compare the dynamics of average temperature changes calculated by the proposed model with the data obtained using the model presented in [10]. As can be seen from the chart, the results obtained from these models are correlated with acceptable accuracy.
Figure 4. Dynamics of change in average temperature in rooms:

1, 2, 3 - average temperatures in the first, second and third rooms (left-to-right), respectively, calculated according to the model presented in [10]; 4, 5, 6 - average temperatures in the first, second and third rooms (left-to-right), respectively, calculated with usage of FireSim.

5. Conclusion
A mathematical model of the fire and heat dynamics in premises is proposed, which makes it possible to improve speed of modeling due to usage of a specially defined heat spread controlling graph and fire spread controlling cellular automaton. The model has considerable flexibility, as it can be modified not only by changing parameters and initial conditions, but also by modifications of the graph and the cellular automaton. A software package is developed that allows the user to visualize the rooms and fire and temperature distribution inside them.

The proposed model provides opportunities for effective joint modeling of the fires dynamics and the evacuation of people from different types of premises. Based on the results of computational experiments, it can allow issuing recommendations on changing the geometric parameters of the premises for quick people evacuation in case of fire.

The model can be used as a part of simulator complexes for training the Ministry of Emergency Situations personnel.

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