Abstract

The IEEE 802.11 protocol is a popular standard for wireless local area networks. Its medium access control layer (MAC) is a carrier sense multiple access with collision avoidance (CSMA/CA) design and includes an exponential backoff mechanism that makes it a possible target for probabilistic model checking. In this work, we identify ways to increase the scope of application of probabilistic model checking to the 802.11 MAC. Current techniques do not scale to networks of even moderate size. To work around this problem, we identify properties of the protocol that can be used to simplify the models and make verification feasible. Using these observations, we directly optimize the probabilistic timed automata models while preserving probabilistic reachability measures. We substantiate our claims of significant reduction by our results from using the probabilistic model checker PRISM.

1. Introduction

The IEEE 802.11 protocol [9] is a popular standard for wireless networks. Its medium access control layer (MAC) is a carrier sense multiple access with collision avoidance (CSMA/CA) design and includes an exponential backoff mechanism that makes it an ideal target for probabilistic model checking. This protocol has been modeled using a range of techniques such as finite state machines [20] and probabilistic timed automata [15].

The 802.11 protocol suffers from a potential livelock problem, demonstrated formally in [20], which is mitigated only by the presence of a finite retry limit for each data packet. The livelock arises because it is possible, although improbable, for two stations to behave symmetrically and continuously collide until they drop their respective packets on exceeding the retry limit. In such a scenario, it is useful to bound the probability of such pathologically symmetric behavior. This motivates the application of probabilistic model checking to the problem of computing probabilities of desired and undesired behavior in the protocol. Two primary properties of interest are: the probability of the number of retries reaching a certain count and the probability of meeting a soft deadline.

A recent solution to the problem of obtaining these probabilities has been proposed in [15]. It models a limited (but critical) aspect of the protocol using Probabilistic Timed Automata (PTA) [14] and exploits available tools, namely, the Probabilistic Symbolic Model Checker (PRISM) [13] for computing the probability values and the real time model checker Uppaal [18] as a proof assistant. Results on the probability of the backoff counter on a station reaching a particular value and the probability of a packet being transmitted within a certain deadline are presented. This work, however, suffers from scalability problems. The model assumes only two stations (sender destination pairs). When we extended the models to 3 stations (and 3 corresponding destinations), which is a practical sized network topology, we found it computationally infeasible to model check properties of interest. These problems are compounded by an inaccurate assumption that the packet length can vary on every retransmission.

The aim of this work is twofold. First, we present a more accurate and scalable model for the protocol. Second, we set up a logical framework to exploit protocol specific redundancies. Under this framework, we perform a number of provably correct optimizations that reduce the generalized multi station model. The optimizations involve abstracting away the deterministic waits, and considering only a subset of the allowed packet sizes that nevertheless captures all the relevant behavior. In addition, we duplicate the model reduction technique of [15] for the multi station problem.

Our reduced models are immediately verifiable in PRISM and require no further tools. However, the option of using tools like RAPTURE [6] on the reduced PTA models remains. Our results show a reduction in state space over the existing solution for two stations. We are also able to successfully model check a
topology of three station that was infeasible with the current models.

The organization of the paper is as follows. We begin with the modeling formalism used in this paper. We present the scalable models for the multi station 802.11 problem and discuss the behavior of the protocol. Next, we present a notion of equivalence in probabilistic systems that abstracts away deterministic deterministic paths in the system but preserves probabilistic reachability. We give sufficient requirements for equivalence both at the level of untimed probabilistic systems and probabilistic timed automata. Based on this framework, we present our set of reductions to the generalized model for the multi station problem. We also show that we can verify soft deadlines inspite of these optimizations. We conclude with results that detail state space reduction as well as case studies for a three station topology.

2. Modeling formalism for the 802.11 protocol

In order to efficiently model and verify the 802.11 protocol, we need a modeling formalism that can represent the protocol at sufficient depth and, at the same time, must be amenable to transformations for more efficient verification. We have been guided by the existing work in [15] in our choice of Probabilistic Timed Automata to model the 802.11 protocol.

We introduce Probabilistic Timed Automata (PTA) [14], Probabilistic Systems (PS) [15][19] and fully probabilistic systems (FPS). All these have been surveyed in [12] with special reference to their relationship in the context of probabilistic model checking.

Let \( \chi \) be a set of non-negative real valued variables clocked. Call \( Z \) the set of zones over \( \chi \), which is the set of all possible atomic constraints of the form \( x - y \sim c \) and \( (x - y) \sim c \) and their closure under conjunction. Here \( x, y \in \chi, \sim \in \{ <, \leq, >, \geq \} \) and \( c \in \mathbb{N} \), where \( \mathbb{N} \) is the set of natural numbers. A clock valuation \( v \) is the assignment of values in \( \mathbb{R}_{\geq 0} \) (where \( \mathbb{R}_{\geq 0} \) is the set of non-negative reals) to all clocks in \( \chi \). The concept of a clock valuation \( v \) satisfying a zone \( Y \), indicated as \( v \prec Y \), is naturally derived by assigning values to each clock in the zone and checking whether all constraints are satisfied.

Definition 1 A probabilistic timed automaton is a tuple \((L, T, \chi, \Sigma, I, P)\) where \( L \) is a finite set of states, \( I \) is the initial state, \( \chi \) is the set of clocks and \( \Sigma \) is a finite set of labels used to label transitions. The function \( I \) is a map \( I : L \rightarrow Z \) called the invariant condition. The probabilistic edge relation \( P \) is defined as \( P \subseteq L \times Z \times \Sigma \times \text{Dist}(2^X \times L) \), where \( \text{Dist}(2^X \times L) \) is the set of all probability distributions, each elementary outcome of which corresponds to resetting some clocks to zero and moving to a state in \( L \). We call a distinguished (not necessarily non-null) subset \( \Sigma^u \) of the set of events as urgent events.

A critical feature of PTAs that makes them powerful modeling tools is that each transition presents probabilistic choice in the PTA while different outgoing probabilistic transitions from a state present nondeterministic choice in the PTA. Hence, a PTA can model non-determinism, which is inherent in the composition of asynchronous parallel systems.

Composition of PTAs is a cross product of states with the condition that the composed PTAs must synchronize on shared actions. For a detailed description see [15].

A feature of PTAs that is useful for higher-level modeling is urgent channels. Urgent channels are a special set of edge labels (symbols) on which a PTA must synchronize whenever possible.

Definition 2 A probabilistic system (PS), is a tuple \((S, \pi, \Sigma, \text{Steps})\) where \( S \) is a finite set of states, \( \pi \) is the start state, \( \Sigma \) is a finite set of labels and \( \text{Steps} \) is a function \( \text{Steps} : S \rightarrow 2^{\Sigma \times \text{Dist}(S)} \) where \( \text{Dist}(S) \) is the set of all distributions over \( S \).

This is the same as the simple probabilistic automaton of [19].

Definition 3 Given a PTA \( T = (L, T, \chi, \Sigma, I, P) \), the semantics of \( T \) is the Probabilistic System \([T] = (S, \pi, \Sigma, \text{Act}, \text{Steps})\), with the following definitions:

\[ S \subseteq L \times \mathbb{R}_{\geq 0}^{\chi |} \text{ is the set of states with the restrictions } (s, v) \in S \text{ iff (} s \in L \text{ and } v \prec I(l) \text{ and } \pi = (T, 0) \text{.} \]

\[ \text{Act} = \mathbb{R}_{\geq 0} \cup \Sigma. \text{This reflects either actions corresponding to time steps } (\mathbb{R}_{\geq 0}) \text{ or actions from the PTA } (\Sigma). \]

\[ \text{Steps} \text{ is the least set of probabilistic transitions containing, for each } (l, v) \in S, \text{ a set of action distribution pairs } (\sigma, \mu) \text{ where } \sigma \in \Sigma \text{ and } \mu \text{ is a probability distribution over } S. \text{Steps for a state } s = (l, v) \text{ is defined as follows.} \]

\[ I. \text{ for each } t \in \mathbb{R}_{\geq 0} (t, \mu) \in \text{Steps}(s) \text{ iff } \]

\[ 1. \mu(l, v + t) = 1 \text{ and } v + t' \prec I(l) \text{ for all } 0 \leq t' \leq t. \]

\[ 2. \text{ For every probabilistic edge of the form } (l, g, \sigma, -) \in P, \text{ if } v + t' \prec g \text{ for any } 0 \leq t' \leq t, \text{ then } \sigma \text{ is non-urgent.} \]

\[ II. \text{ for each } (l, g, \sigma, p) \in P, \text{ let } (\sigma, \mu) \in \text{Steps}(s) \text{ iff } v \prec g \text{ and for each } (l', v') \in S: \mu(l', v') = \Sigma \chi \sigma v = v \chi : 0 = p(X, l', v') \text{, the sum being over all clock resets that result in the valuation } v'. \]

A critical result [17], analogous to the region construction result for timed automata, states that it is sufficient to assume only integer increments when all zones are closed (there are no strict inequalities). Hence, the definition given
above is modified to \( S \subseteq L \times \mathbb{N}^{\mid x \mid} \) and \( \text{Act} = \mathbb{N} \cup \Sigma \). Under integer semantics, the size of the state space is proportional to the largest constant used. For the rest of this paper, we will assume integer semantics.

Note that, in the presence of non-determinism, the probability measure of a path in a PS is undefined. Hence, define an adversary or scheduler that resolves non-determinism as follows:

**Definition 4** An adversary of the Probabilistic System \( P = (S, \pi, \text{Act}, \text{Steps}) \) is a function \( f : S \rightarrow \cup_{s \in S} \text{Steps}(s) \) where \( f(s) \in \text{Steps}(s) \).

We only consider simple adversaries that do not change their decision about an outgoing distribution every time a state is revisited, their sufficiency has been shown in [5]. A simple adversary induces a Fully Probabilistic System (FPS) as defined below.

**Definition 5** A simple adversary \( A \) of a Probabilistic System \( P = (S, \pi, \text{Act}, \text{Steps}) \) induces a Fully Probabilistic System (FPS) or Discrete Time Markov Chain \( P^A = (S, \pi, \text{Act}, P) \). Here, \( P(s) = A(s) \), the unique outgoing probability distribution for each \( s \in S \), where we drop the edge label on the transition.

Thus, given a PS \( M \) and a set of “target states” \( F \), consider an adversary \( A \) and the corresponding FPS \( M^A \). A probability space (Prob\(^A\)) may be defined on \( M^A \) via a cylinder construction [11]. A path \( \omega \) in \( M^A \) is simply a (possibly infinite) sequence of states \( \pi s_1 s_2 \ldots \) such that there is a transition of non-zero probability between any two consecutive states in the path. For model checking, we are interested in

\[
\text{ProbReach}^A(F) \overset{\text{def}}{=} \text{Prob}^A\{\omega \in \text{Path}^A_\infty \mid \exists i \in \mathbb{N} \text{ where } \omega(i) \in F\},
\]

\( F \) is the desired set of target states, \( \omega(i) \) is the \( i \)th state in the path \( \omega \) and \( \text{Path}^A_\infty \) represents all infinite paths in \( M^A \). Define \( \text{MaxProbReach}^M(F) \) and \( \text{MinProbReach}^M(F) \) as the supremum and infimum respectively of \( \{\text{ProbReach}^A(F)\} \) where the quantification is over all adversaries.

3. Logic Formulas Under Consideration

Properties of interest at the PTA level are specified using Probabilistic Computational Tree Logic (PCTL) formulas [7]. We limit ourselves to restricted syntax (but non-trivial) PCTL formulas, expressible as \( P_{\leq \lambda} \langle \varphi \rangle p \), where \( \sim \in \{<, >, \leq, \geq\} \) and \( \lambda \) is the constant probability bound that is being model checked for. These PCTL formulas translate directly into a probabilistic reachability problem on the semantic Probabilistic System corresponding to the PTA. The reason for this restriction is that, in the case of the 802.11 protocol, the properties of interest, including the real time ones, are all expressible in this form. For example, in the case of a probabilistic timed automaton \( A \), the PCTL formula \( P_{\leq 0.5} \langle \varphi \rangle p \) directly translates to maximum probabilistic reachability on the induced Markov decision process \([A]\) from a well-defined start state. We mark the target states as those where the proposition \( p \) is true. The model checker returns true when this maximum probability is smaller than 0.5. Under this restricted form of PCTL, we indicate numerical equivalence using the following notation.

**Definition 6** Two probabilistic systems \( P_1 \) and \( P_2 \) are equivalent under probabilistic reachability of their respective target states \( F_1 \) and \( F_2 \), denoted by \( P_1 \equiv_{\mathbb{F}_1, \mathbb{F}_2} P_2 \) when \( \text{MaxProbReach}^P_1(F_1) = \text{MaxProbReach}^P_2(F_2) \) and \( \text{MinProbReach}^P_1(F_1) = \text{MinProbReach}^P_2(F_2) \).

**Definition 7** PTA\(_1\) \( \overset{\text{PTA}}{\equiv}_{\phi_1, \phi_2} \) PTA\(_2\) when \( [[\text{PTA}\(_1\)]] \equiv_{\mathbb{F}_1, \mathbb{F}_2} [[\text{PTA}\(_2\)]] \). The criterion for marking target states is that \( F_1 \) corresponds to the target states in the reachability problem for the PCTL formula \( \phi_1 \), while \( F_2 \) corresponds to the target states for the PCTL formula \( \phi_2 \).

4. Probabilistic Models of the 802.11 Protocol

In this section, we present scalable probabilistic models of the 802.11 basic access MAC protocol assuming no hidden nodes\(^1\). The model for the multi-station 802.11 problem consists of the station model and a shared channel, shown in Figures [4] and [9] respectively. We assume familiarity with conventions used in graphical representation of timed automata. In particular, the states marked with a “u” are urgent states while that marked by concentric circles is the start state. The station models are replicated to represent multiple sender-destination pairs. Some critical state variables are: \( bc \) that holds the current backoff counter value, \( tx\_len \) that holds the chosen transmission length and backoff that represents the current remaining time in backoff. The function RANDOM(bc) is a modeling abstraction that assigns a random number in the current contention window.

Similarly, \( NON\_DET(TX\_MIN, TX\_MAX) \) assigns a non-deterministic packet length between \( TX\_MIN \) and \( TX\_MAX \), which are the minimum and maximum allowable packet transmission times respectively. The values used for verification are from the Frequency Hopping Spread Spectrum (FHSS) physical layer [9]. The transmission rate for the data payload is 2 Mbps.

\(^1\) In the absence of hidden nodes [1], the channel is a shared medium visible to all the stations.
The station automaton shown in Figure 4 begins with a data packet whose transmission time it selects non-deterministically in the range from 258µs to 15750µs. On sensing the channel free for a Distributed InterFrame Space (DIFS = 128µs), it enters the Vulnerable state, where it switches its transceiver to transmit mode and begins transmitting the signal. The Vulnerable state also accounts for propagation delay. It moves to the Transmit state after a time $VULN = 48\mu s$ with a synchronization on send. After completing transmission, the station moves to Test_channel via one of the two synchronizations, finish_correct on a successful transmission and finish_garbled on an unsuccessful transmission. The channel keeps track of the status of transmissions, going into a garbled state whenever more than one transmission occurs simultaneously. The station incorporates the behavior of the destination and diverges depending on whether the transmission was successful, or not. If the transmission was successful, the portion of the station corresponding to the destination waits for a Short InterFrame Space (SIFS = 28µs) amount of time before transmitting an ack, which takes $ACK = 183\mu s$ amount of time.

On an unsuccessful transmission, the station waits for the acknowledgment timeout of $ACK_{TO} = 300\mu s$. It then enters a backoff phase, where it probabilistically selects a random backoff period $backoff = RANDOM(bc)$. $RANDOM(bc)$ is a function that selects with uniform probability, a value from the contention window given by the range $[0, (C+1)2^{bc} - 1]$, where $C$ is the minimum contention window (15µs for the FHSS physical layer). The backoff counter (bc) is incremented each time the station enters backoff. The backoff counter is frozen when a station detects a transmission on the medium while in backoff.

The station and channel models are different from those in [15]. The station now fixes a packet transmission length non-deterministically and remembers it rather than allow it to vary on every retransmission. The channel of [15] assumes a fixed topology of two stations, while the channel depicted in Figure 2 is generalized for an arbitrary number of stations. It follows a different design from that in [15], which if generalized would have states exponential in the number of stations. Ours is only linear. Since the models are generalized to an arbitrary number of stations, the synchronization labels have subscripts indicating the station number. However, in the rest of the paper we drop subscripts whenever the station number is clear from the context.

We point out here that we start with an abstracted station model, which incorporates the deterministic destination. That this is a valid abstraction has already been shown for the two station case in [15]. The extension to the multi station case does not represent any significant new result and hence has been omitted.

5. Compression of Deterministic Paths: A Technique for State Space Reduction

In the 802.11 protocol, there are numerous cases where the component automata representing the system simply count time or where different resolutions of non-determinism lead to same state but through different paths. If we are verifying an untimed property then such fine grained analysis increases state space without any contribution to probabilistic reachability. We discovered on studying these models that it is possible to derive alternative optimized probabilistic timed automata that avoid the cost of such unnecessary deterministic behavior by compressing these deterministic paths into equivalent but shorter paths. The problem is the lack of a suitable formalism to support our optimizations. This section provides a framework that can be used to justify the equivalence of our optimized models to the original ones.

For purposes of comparison, we assume that the state space is a subset of an implicit global set of states. This allows operations such as intersection and union between the set of states of two different automata. In particular, for this paper we consistently name states across the automata we consider.

Our objective is to formalize “deterministic” behavior of interest. The key relationship used in this formalization is a specialization of dominators as defined in [5]. We refer to this restricted version of dominators as “deterministic dominators” in the rest of this paper.

Definition 8 For a distribution $\pi$ over the finite elementary event set $X$, define the support of the distribution as $\text{supp}(\pi) = \{ x \in X \mid \pi(x) > 0 \}$

Definition 9 Given a probabilistic system consisting of the set of states $S$, define $\prec_D$ as the smallest relation in $S \times S$ satisfying the following: $\forall s \in S$

$s \prec_D s$ and

$\exists t \in S \left[ \forall (a, \pi) \in \text{Steps}(s) : \exists x \left( \text{supp}(\pi) = \{ x \} \land (x \prec_D t) \right) \Rightarrow s \prec_D t \right]$

If the relation $s \prec_D t$ holds then we say that $t$ is the deterministic dominator of $s$.

An example of a deterministic dominator is shown in the probabilistic systems of Figure 4 where $S \prec_D T$.

Definition 10 Given distributions $P_1$ over $S_1$ and $P_2$ over $S_2$, define $P_1 \equiv dist P_2$ when $\text{supp}(P_1) = \text{supp}(P_2) = S$ and $\forall s \in S$ we have $P_1(s) = P_2(s)$.

Based on the notion of equivalence of distributions, we define the notion of equivalence of sets of distributions. Let
Steps$_1$ be a set of labeled distributions over $S_1$ and Steps$_2$ be a set of labeled distributions over $S_2$.

Definition 11 Steps$_1 \equiv_{\text{dist}}$ Steps$_2$ whenever

$$\forall (a, \mu_1) \in \text{Steps}_1 \exists (b, \mu_2) \in \text{Steps}_2 \text{ such that } \mu_1 \equiv \mu_2 \text{ and } \forall (a, \mu_2) \in \text{Steps}_2 \exists (b, \mu_1) \in \text{Steps}_1 \text{ with } \mu_2 \equiv \mu_1.$$ 

Define a path in a probabilistic system as follows:

Definition 12 A path in the probabilistic system $P = (S, \pi, \Sigma, \text{Steps})$ is a sequence of state-action pairs $(s_1, a_1), (s_2, a_2), \ldots (s_n+1)$ such that $\forall i \in \{1..n\}$ we have $\exists (a_i, \mu) \in \text{Steps}(s_i)$ such that $\mu(s_{i+1}) > 0$.

5.1. Deterministic Path Compression in Probabilistic Systems

Consider the two probabilistic systems of Figure 1 each of which has the start state $U$. It should be clear that each of MaxProbReach($X$) and MinProbReach($X$) takes the same value in both the systems since we have only removed (compressed) the deterministic segment $B \rightarrow C$. We formalize this notion of deterministic path compression at the level of probabilistic systems in theorem 1.

Consider two finite probabilistic systems $PS_1(S_1, \pi, \text{Act}, \text{Steps}_1)$ and $PS_2(S_2, \pi, \text{Act}, \text{Steps}_2)$ with an identical set of actions. All transitions in Steps$_1$ and Steps$_2$ are simple transitions of the form $(s, a, \mu)$ where $s$ is the originating state, $a \in \text{Act}$ and $\mu$ is a probability distribution over the state space. Note that the $S_1$ and $S_2$ are necessarily not disjoint because of the common start state $s$.

Definition 13 If, for some $s \in S_1 \cap S_2$, $\text{Steps}_1(s) \equiv_{\text{dist}} \text{Steps}_2(s)$ does not hold then $s$ is a point of disagreement between the two probabilistic systems.

Theorem 1 (Equivalence in Probabilistic Systems)
Given two probabilistic systems $PS_1(S_1, \pi, \text{Act}, \text{Steps}_1)$ and $PS_2(S_2, \pi, \text{Act}, \text{Steps}_2)$ satisfying the following conditions:

1. For any state $s \in S_1 \cap S_2$, if $s$ is a point of disagreement then $\exists t \in S_1 \cap S_2$ such that, $t$ is not a point of disagreement and in each of the systems, $s \prec_D t$.

2. Let $F_1 \subseteq S_1$ and $F_2 \subseteq S_2$ be sets of target states we are model checking for. We impose the condition $S_1 \cap S_2 \cap F_1 = S_1 \cap S_2 \cap F_2$. For every $s \in S_1 \cap S_2$, which is a point of disagreement we have the following: For the postulated deterministic dominator $t$ and for every state $u$ on any path in $PS_1$ between $s$ and $t$, $u \in F_1 \Rightarrow (s \in F_1) \lor (t \in F_1)$. Similarly, for every state $u$ on any path in $PS_2$ between $s$ and $t$, $u \in F_2 \Rightarrow (s \in F_2) \lor (t \in F_2)$.

Under these conditions, $PS_1 \equiv_{\text{PS}} F_1, F_2 \equiv_{\text{PS}} PS_2$.

The proof follows from first principles by setting up a bijective mapping between paths in the two probabilistic systems. The complete proof is available in [1].

5.2. Equivalence of Probabilistic Timed Automata

Given two Probabilistic Timed Automata $PTA_1$ and $PTA_2$ and their respective restricted PCTL requirements $\phi_1$ and $\phi_2$, we need a set of conditions under which we may claim $PTA_1 \equiv_{\text{PTA}} PTA_2$. By Definition 14 this is equivalent to showing that $[[PTA_1]] \equiv_{F_1, F_2} [[PTA_2]]$, where $F_1$ and $F_2$ are the corresponding target states of $\phi_1$ and $\phi_2$ respectively. Our optimizations are based on deterministic path compression as outlined in Section 5. Hence, we impose requirements on $PTA_1$ and $PTA_2$ under which we can apply theorem 1 to $[[PTA_1]]$ and $[[PTA_2]]$ to deduce $[[PTA_1]] \equiv_{F_1, F_2} [[PTA_2]]$. The following lemmas have the objective of establishing these requirements.

Consider two Probabilistic Timed Automata with an identical set of clocks and events: $PTA_1 = (L_1, \bar{F}_1, \chi, \Sigma, I_1, P_1)$ and $PTA_2 = (L_2, \bar{F}_2, \chi, \Sigma, I_2, P_2)$. We assume that the automata have the same set of urgent events, $\Sigma^u$.

Definition 14 A state $s \in L_1 \cap L_2$ is a point of disagreement between the two probabilistic timed automata if either they differ on the invariant or they differ in the set of outgoing transitions. Taking a transition out of a state $s$ as the tuple $(s, z, \sigma, P(2^\Sigma \times L))$, call two transitions different if they disagree on either the guard $z$, or the event label on the transition $\sigma$, or the distribution $P(2^\Sigma \times L)$.

The semantic probabilistic systems are $[[PTA_1]]$ and $[[PTA_2]]$ respectively. Let $States([[PTA_1]])$ and
States([PTA₁]) denote states of the semantic probabilistic systems for PTA₁ and PTA₂ respectively. The states in the semantic PS are tuples (s, v) where s is a state of the PTA and v is a clock valuation.

Lemma 1 A state (s, v) ∈ States([PTA₁]) ∩ States([PTA₂]) is a point of disagreement (with regard to condition 1 of theorem) between the two PS implies that s is a point of disagreement between PTA₁ and PTA₂.

The condition that labels should also be identical might seem too restrictive considering that we are only interested in probabilistic reachability. However, the next set of lemmas will show that when composing PTAs labels are important.

Most real world systems and the 802.11 protocol in particular are modeled as a composition of PTAs. In a composed system, the above lemma will only tell us whether a particular common state in the PTA can generate a point of disagreement in the semantic PS. This common state represents the composed state of all the PTAs composing the model. The next few lemmas extend lemma 1 to the scenario of composed probabilistic timed automata.

Definition 15 Consider two PTAs formed of compositions, as follows,
PTA₁ = PTA₁¹ || PTA₁² || PTA₁³ || ... || PTA₁ⁿ and
PTA₂ = PTA₂¹ || PTA₂² || PTA₂³ || ... || PTA₂ⁿ.
Define the difference set as the set D ⊆ {1, 2, ..., n} such that ∀i ∈ D : PTA₁ⁱ ≠ PTA₂ⁱ and ∀i /∈ D : PTA₁ⁱ = PTA₂ⁱ. By equality we mean exactly the same automaton in both the compositions (component wise equality of the tuples defining them).

Definition 16 We define the specific difference set for the index i ∈ D as Dᵢ = states(PTAᵢ¹) ∩ states(PTAᵢ²) where Dᵢ is the set of states that disagree across the automata as outlined in Definition. For every i /∈ D set Dᵢ = {}.

Lemma 2 Consider the composed PTA models of Definition. Let S_common be the set of common states between PTA₁ and PTA₂. A composed state in S_common, say (l₁, l₂, ..., lₙ) is a point of disagreement between PTA₁ and PTA₂ implies that at least one automaton is in its specific difference set.

In the composed PTAs of definition. Each state in the semantic PS for a PTA is a combination of states and clock valuations of the individual PTA in the composition. The next lemma combines lemma 1 and lemma 2.

Lemma 3 (PTA level requirements) A state in States([PTA₁]) ∩ States([PTA₂]) = (s₁, s₂, ..., sₙ, v) is a point of disagreement implies that for at least one i ∈ {1, ..., n}, the common state sᵢ of both PTAᵢ¹ and PTAᵢ² is an element of their specific disagreement set.

The purpose of lemma 3 is to identify precisely those states in the component PTA that may cause a disagreement in the PS for the composed system.

5.3. Proof Technique

We will use the framework in this section to prove the correctness of our reduced models. Although our objective is the 802.11 protocol, the concept of deterministic path compression has been developed in a generalized manner anticipating its application to other protocols.

To prove that a reduced PTA model (PTA₂) corresponding to the original PTA model (PTA₁) is correct, we need to prove that PTA₁ ≡ φ₁ ∨ PTA₂. Here φ₁ and φ₂ are the corresponding PCTL formulas in the two models. For our purposes φ₁ = φ₂ since we are interested in proving that we will arrive at the same result for the same particular PCTL formula. We proceed with the proof in the following manner.

1. Identify the difference set (Definition 15). Compute the specific difference set of each component automaton in the difference set using Definition 16. This is easily done by a visual inspection of the automata.

2. Identify composed states where one or more automata are in their specific difference set. At this point we use protocol specific proofs to limit such combinations to a manageable size. From Lemma 2 we know the set of composed states obtained in this step is a superset of the actual difference set across the composed PTA.

3. For each composed state, we argue about the possible evolution of the untimed model obtained through Definition 3. We show that in each case

i) There is the same deterministic dominator in each of [[PTA₁]] and [[PTA₂]]. This is the hardest part of the proof. However, we use the fact that the deterministic dominator state in the PS is expressible as the combination of a composed state and clock valuation in the PTA. Hence the proofs are in terms of the PTA rather than the PS. We generally show that each component automaton reaches the state in the composition and progress can only be made when the entire model is in the composed state.

ii) Final states in [[PTA₁]] and [[PTA₂]], corresponding to the PCTL formulas φ₁ and φ₂ respectively, are distributed as specified in condition 2 of Theorem 4. From Lemma 2 we know that this is sufficient for Theorem 4 to hold. Hence we conclude that at the level of PTAs PTA₁ ≡ φ₁ ∨ PTA₂.

Deterministic Path Compression, at the level of Probabilistic Systems bears similarity to weak bisimulation that can abstract away internal actions. However, a notable difference in our approach from weak bisimulation is that we are able to change invariants on states in the Probabilistic Timed Automata. This corresponds to removing time
steps (Definition 3) in the corresponding semantic probabilistic system. These time steps are not internal actions because composed probabilistic systems must synchronize on time steps to maintain the semantics of PTA composition. A possibility would be to apply weak bisimulation to the final composed model but this would mean fixing the number of stations in the composition. The reduced models would no longer be valid for the general multi station problem.

6. Reducing the 802.11 Station Automaton

For the 802.11 problem, we optimize the station automaton, in multiple steps, starting from the original abstract station model of Figure 4. In each case, the set of final states correspond to the PCTL formula \( \phi = P_{<\lambda}[\Diamond(bc = k)] \). For every reduction from PTA1 to PTA2, we prove the correctness of our optimizations by showing that PTA1 \( \equiv PTA2 \). Due to space constraints, we omit the complete proofs (they are available in [1]) and only motivate the key ideas. Our proofs are driven by behavior exhibited by the 802.11 PTA models. For example, a key aspect of many of our proofs is the fact that 802.11 backoff counters are frozen when a busy channel is detected. This allows us to essentially ignore stations in backoff when the channel is busy.

6.1. Removing the SIFS Wait

Our first optimization removes the SIFS wait following a successful transmission. The original model is \( AbsLAN = AbsStn_1 \parallel AbsStn_2 \parallel \cdots \parallel AbsStn_n \parallel Chan \) and the reduced model is \( IntLAN = IntStn_1 \parallel IntStn_2 \parallel \cdots \parallel IntStn_n \parallel Chan \). The intermediate station model IntStn with the SIFS wait removed is shown in Figure 5. The difference set (see Definition 15) includes all the stations and does not include the channel, which is unchanged. The specific difference set is only the Test_Channel urgent state immediately after asserting finish_correct. The key idea of the proof is as follows: All the other stations will detect the busy channel and move into the Wait_until_free or Wait_until_free II state. The successfully completing station will move into the Done state while the rest of the stations will move either into Wait_for_DIFS or Wait_for_DIFS II states, which gives us a deterministic dominator in both the automata (AbsLAN and IntLAN). In the proof, we exploit the fact that in the 802.11 protocol, the backoff counters are frozen when a transmission is detected on the channel. This is modeled by the station in Backoff moving into the Wait_until_free II state. The key idea of the proof, in an example for three stations, is shown in Figure 5.

6.2. Removing the DIFS Wait

In the final reduced station model RedStn of Figure 7 the DIFS wait has been removed. The model is given by the composition \( RedLAN = RedStn_1 \parallel RedStn_2 \parallel \cdots \parallel RedStn_n \parallel Chan \). Proving the deterministic dominator relationship is a little more complicated here because we need to consider both collision and successful transmission cases. In each case however, all stations detect the busy channel and move to Wait_until_free or Wait_until_free II. The specific difference set consists of Wait_until_free, Wait_until_free II and Wait_for_ACK_TO. In the semantic probabilistic system corresponding to the composed model we can always prove that for any point of disagreement and for any adversary, there is always a deterministic dominator, which is the state of the system after the DIFS wait is over. The key idea for a three station example is shown in Figure 2.

In RedStn we continue to keep the Wait_for_DIFS state. The reason for this is as follows. It is possible for a station to leave Wait_for_ACK_TO and wait for DIFS amount of time while all other stations which have not transmitted are sitting in Backoff. Since the amount of time spent in backoff is unpredictable, there is no deterministic dominator. Consequently, we cannot simply remove the DIFS wait after Wait_for_ACK_TO. However, we may always remove the transition into this state due to the DIFS wait on detecting a busy channel after transmission. Again, a key component of the proof is the fact that 802.11 backoff counters are frozen on detecting a busy channel. This allows us to essentially ignore the stations in backoff during transmission.

6.3. Restricting the allowed transmission length

The major contributor of state space in the protocol is the large range of allowed transmission lengths. The range is from 315\( \mu s \) to 15717\( \mu s \) and this proves to be a significant impediment.

We make a minor change in our PTA models, with the objective of making the proofs of equivalence more direct. Rather than having a non-deterministic edge that selects packet lengths, which are subsequently held constant, we parameterize the models by a packet length and remove the non-deterministic choice. Hence, we now have a series of PTA models depending on the choice of parameterizations. The allowable assignment of packet (transmission) lengths is from \( Par^{full} \), the set of all possible parameterizations. Each of \( tx_{len_1}, \ldots, tx_{len_n} \) is assigned a value from the interval \( [TX_{MIN}, TX_{MAX}] \). Formally, \( Par^{full} = [TX_{MIN}, TX_{MAX}]^n \).
Consider the reduced set of parameterizations $\text{Par}^{\text{reduced}} \subset \text{Par}^{\text{full}}$ where $tx_{\text{len}}_1 = TX\_MIN$ and $tx_{\text{len}}_{i+1} - tx_{\text{len}}_i \leq VULN, 1 \leq i < n$. Here we restrict the maximum allowable increase in transmission length of one station over its immediate predecessor. This eliminates many parameterizations that would have assigned transmission lengths close to maximum resulting in a large state space. We have shown using the framework of Section 6 that it is sufficient to consider only this limited range of transmission lengths. The key objective is to show that for every model $\text{PTA}_1$ whose parameters are selected from $\text{Par}^{\text{full}}$, there exists a model $\text{PTA}_2$ whose parameters are contained in $\text{Par}^{\text{reduced}}$ such that $\text{PTA}_1 \equiv_{\phi,\phi} \text{PTA}_2$. Here the specific difference set is only the Transmit state whose invariant is different in the two models (due to differing transmission lengths). Again, we use the fact that 802.11 backoff counters are frozen during transmission. This means that changing the transmission length has no effect on stations that were in backoff when the channel became busy. The hardest part is to select a proper model from $\text{Par}^{\text{reduced}}$ such that any $m$ stations in a generalized n-station scenario, that collide by transmitting simultaneously, complete transmission in the same order in both the models. This is necessary because an inspection of the station automaton shows that during a collision, any station that finishes while some other station is still occupying the channel, would detect the busy channel and behave differently from the station that finished last. Hence ensuring that stations finish in the same order leads to the same deterministic dominator in both the models.

7. Soft Deadline Verification

The probability of meeting soft deadlines, which is the minimum probability of a station delivering a packet within a certain deadline, is a real time property that can be formulated as a probabilistic reachability problem. For example, in an 802.11 topology of three senders and three receivers, we are interested in the probability that every station successfully transmits its packet within a given deadline. The reductions presented in this paper, which depend on deterministic path compression, do not preserve total time elapsed since certain states in the probabilistic timed automata where the composite model can count have been removed. As a result, paths are replaced with shorter (time wise) versions.

However, one key aspect of our reductions is that they affect deterministic and well-defined segments of the automata. The intuition is that it should be possible to "compensate" for the reductions by using additional available information. For example, removing the acknowledgment protocol has the effect of subtracting a $\text{SIFS} + \text{ACK}$ period for every successful transmission made. On the other hand removing $\text{DIFS}$ wait results in subtracting $\text{DIFS}$ from the elapsed time for any transmission made.

We begin with the traditional “decoration” of a PTA in order to verify real time properties, as exemplified in [16]. Assume the existence of a composed state $\text{Done}$, which is the composition of the state $\text{Done}$ across the components the model. Decorating the PTA involves adding a global clock (say $y$) to the system that counts total time elapsed and a state $\text{Deadline\_exceeded}$. Edges are added from each state other than $\text{Done}$, with guard $y \geq \text{deadline}$ to $\text{Deadline\_exceeded}$. Every invariant except at $\text{Done}$ and $\text{Deadline\_exceeded}$ is taken in conjunction with $y \leq \text{deadline}$. The objective is to model check for the PCTL formula $P_{\geq \lambda}[\Diamond \text{Done}]$, which expresses the soft deadline property.

We depart from the traditional model by decorating the PTA as follows: We define a non-decreasing linear function $\phi(y, X)$ on the global clock and numerical system variables (which does not include the clock valuation). The global clock $y$ and state $\text{Deadline\_exceeded}$ are added. Edges are added to $\text{Deadline\_exceeded}$ with guard $\phi(y, X) \geq \text{deadline}$. Each invariant is taken in conjunction with $\phi(y, X) \leq \text{deadline}$. Since the dependence on $X$ may be represented as different functions depending on the current state, we do not depart from the traditional definition of a PTA. The idea is that while $y$ represents absolute system time, $\phi(y, X)$ represents a corrected version that takes into account deterministic path compression.

In order to compute real time properties, we annotate the channel with the extra variables $\text{transmissions}$ and $\text{successes}$, where each is initialized to zero in the start state. The former is incremented on every synchronization on $\text{finish\_correct}$ or $\text{finish\_garbled}$ while the latter is incremented only on a synchronization on $\text{finish\_correct}$. Their semantics, hence, follow their nomenclature. In the $\text{RedLAN}$ model, without parameter restrictions, set $\phi(y, X) = y + \text{successes} \times (\text{SIFS} + \text{ACK}) + \text{transmissions} \times \text{DIFS}$. This function compensates for ack protocol removal by adding $\text{SIFS} + \text{ACK}$ for each successful transmission and for $\text{DIFS}$ removal by adding $\text{DIFS}$ for every transmission. For the $\text{AbsLAN}$ model, we set $\phi(y, X) = y$, reflecting the standard construction. Due to space constraints we omit the proof of correctness of our construction here. We essentially need to repeat the proofs referred to in Section 6 taking into account the fact that a clock value assigned to $y$ in the original model will be mapped to $\phi(y, X)$ in the changed model and we are now model checking for $P_{\geq \lambda}[\Diamond \text{Done}]$.

We intend to extend our technique for retaining soft deadline properties to cover parameter restrictions in future
8. Verification Results

Our verification platform is a 1.2 GHz Pentium III server with 1.5 GB of ECC memory and running Linux 2.4. Our experiments used the Multi-Terminal Binary Decision Diagram (MTBDD) engine of PRISM and all properties were checked with an accuracy of $10^{-6}$.

The largest constant in the model, even after the optimizations, is 354. This is still prohibitively large. Hence, before translating into actual PRISM models, we perform a time scaling operation [15]. For time scaling, we used the backoff contention slot length of $50\mu s$ and divided all guards and invariants by the chosen unit, rounding upper bounds on the values of clocks up and lower bounds on the values of clocks down. This is the only transformation where we loosen the maximal and minimal probabilities to bounds rather than exact values. We also removed the states corresponding to acknowledgments from the channel, since we no longer model them.

### 8.1. State Space Growth

The growth in state space for the multi station problem is shown in Tables 1 and 2. We report the number of states and transitions in the model. We also report the number of choices, which is total number of nondeterministic choices summed across all the states of the model. In Table 1 we compare our optimized generalized model for the base case of two station with the models of [15]. We show a significant improvement in model size. However, when we consider models of three and four stations in Table 2, the unoptimized models obtained by extending those of [15] cannot even be built by the model checker within the resources provided. Hence, we only report the state space for our own optimized models.

### 8.2. Backoff Counter

We solve the probabilistic model checking problem of computing the upper bound on the probability of the backoff counter on any station reaching a specified value. As a starting point, we show that our generalized models are capable of reproducing the results of the specialized two station models of [15]. In Table 3, we show state space cost and in Table 4, we show verification costs. Our results are the same as in [15] but the verification costs are lower.

The same results in the case of a three station network is shown in Table 5. The probabilities are higher than the two

| Backoff Counter | Time original (secs) | Time optimized (secs) | Maximum probability |
|-----------------|----------------------|-----------------------|---------------------|
| 1               | 0.69                 | 0.09                  | 1.0                 |
| 2               | 8.95                 | 1.15                  | 0.18359375          |
| 3               | 37.37                | 6.29                  | 0.01703262          |
| 4               | 113.25               | 29.12                 | 7.9424586e-4       |
| 5               | 327.04               | 120.5                 | 1.8566660e-5       |
| 6               | 970.38               | 508.26                | 2.1729427e-7       |

Table 3: Probability of the backoff counter reaching a specified value in the two station case - our model vs. Kwiatkowska et al. [15] (original)
station case. This is to be expected since three stations represents more contention for the channel than the two station case. It has been mentioned that the 3 station problem using the original unoptimized station models are beyond the reach of PRISM on our platform.

8.3. Voice over 802.11: A Real Time Case Study

An example of soft deadlines for probabilistic verification is given by the following scenario: An area serviced by a single 100 Mbps 802.3 Local Area Network is occupied by three overlapping but independent wireless networks, each consisting of an access point and mobile devices. All of these are equipped with 802.11 capabilities and the access point is distributing voice data to each of the other n stations in its network. We consider the specific case where n = 7 and we use one of two variants of the voice encoding schemes. In the case of the voice variant, the frame size is 64 bytes and bandwidth requirement is 33.6 Kbps, resulting in a soft deadline of 2196 µs (rounding down to get a stricter integer deadline). On the other hand, in G.729(2) with a frame rate of 74 bytes and bandwidth requirement of 19.2 Kbps, we have a soft deadline of 4044 µs. For soft deadline verification, we start with a model parameterized by the frame size. Subsequently, we use the construction of Section 7 on RedLAN for verification.

The results for the real time voice delivery problem that translates into soft deadlines for a three station topology, are reported in Table 5. They indicate that in the worst case G.729(1) cannot meet the soft deadline requirements while G.729(2) has only a 1% probability of doing so.

9. Conclusion

In this paper, we have introduced deterministic path compression, a new technique to remove protocol redundancies. We have been successful in tackling the state space problem for the 802.11 wireless LAN protocol. We have also shown that it is possible to compute the minimum probability of meeting soft deadlines in spite of the optimizations. This is surprising because our optimizations, at first sight, do not seem amenable to soft deadline verification.

We are yet to reach a solution that can make verifying models with four or more stations feasible. One option is to use the optimized models as input to a tool like RAPTURE, which can identify dominators at the Probabilistic System level, in a manner similar to our approach at the Probabilistic Timed Automata level. Our work is still essential because it brings the model within reach of a tool like RAPTURE. It remains to be seen whether significant improvements at the Probabilistic System level are possible. There are also a number of extensions to the basic access protocol that we have considered. Modeling these would justify application of probabilistic verification, which is extremely expensive compared to simulation, to real world problems.

Acknowledgments: We thank Deepak D’ Souza for his constructive feedback and Marta Kwiatkowska with her colleagues for making the code of the models used in available to us.

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Figure 5: PTA model for an Intermediate Abstracted and Reduced Station - The ACK protocol has been removed

Figure 6: PTA model for the Channel - Generalized for the multiple station case

Figure 7: PTA model for the Final Abstracted and Reduced Station
Waiting

Ack

finish_correct(i)

y:=0

start_ack(i)

y:=0

end_ack(i)

y==ACK

SIFS

SIFS

finish_garbled(i)

y==ACK

end_ack(i)
