Autler-Townes splitting and acoustically induced transparency based on Love waves interacting with pillared meta-surface

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The interaction of Love waves with two-line pillared meta-surface is numerically investigated by Finite Element Method. Acoustic analogue of Autler-Townes Splitting (ATS) and cavity modes are first demonstrated in two lines of identical pillars by varying the distance between the pillar lines. ATS appears when the distance is smaller than the half wavelength and a strong coupling is aroused between the pillar lines. Fabry-Perot resonance exists at the positions where the distance between the pillar lines is a multiple of half wavelength. The proximity of Fabry-Perot resonance with pillar intrinsic mode gives rise to the cavity modes. Then, the radius of one line of pillars is modified to detune the pillar resonant frequencies. In the pillar coupling region, the coupling effect decreases with the increase of radius mismatch. When the distance between the pillar lines is a multiple of half wavelength, Fabry-Perot resonance along with the two different pillar resonances give rise to the acoustic analogue of Electromagnetically Induced Transparency (AIT). ATS and AIT formula models are used to fit the transmission spectra, showing good agreements with numerical results. The quality of the fit models is quantitatively evaluated by resorting to the Akaike information criterion (AIC). ATS and AIT are theoretically and analytically discriminated.

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I. INTRODUCTION

Electromagnetically Induced Transparency (EIT) and Autler-Townes Splitting (ATS) are quantum optical phenomena resulting from the interaction between electromagnetic fields and a three-level atomic system. Both of them are characterized by a transparency window in the absorption or transmission spectrum and have shown a wide variety of applications in light control. While the attenuation of absorption in EIT originates from the Fano interference among different transition pathways, in ATS it is the result of a strong coupling-field induced energy level splitting. EIT and ATS were first observed in quantum/atom systems. In recent years, classic analogues of EIT and ATS have attracted increasing interest in platforms such as photonics, optomechanics, plasmonics, and metamaterials. Many discussions have been devoted to their easily confused absorption or transmission spectra. Besides their differences in the physical mechanisms, the Akaike Information Criterion (AIC) has been proposed to quantitatively discern EIT from ATS and the transition from ATS to EIT is thereby carried out. A crossover from EIT to ATS has been shown to exist in hot molecules, and in open ladder systems. In acoustic, the analogue of EIT, also referred to as AIT, has been investigated in different structures. Additionally, the distinction and transition between acoustic analogue of ATS and AIT has not been quantitatively investigated before.

In the last two decades, phononic crystals have received increasing attention. They exhibit Bragg and/or hybridization band gaps to achieve control of elastic waves propagation, and apply to various aspects such as RF communications, acoustic isolators, sensors, thermolectric materials and meta-materials. Phononic pillared meta-surface is a recently proposed structure stems from the pillared phononic crystals. It consists of a single or a line of pillars on top of a slab, with which one can thoroughly investigate the pillar resonant properties. Several studies have been devoted to the interaction of pillared meta-surface with Rayleigh waves and Lamb waves, but no work has been done on Love waves, which are shear horizontal (SH) polarized surface acoustic waves (SAW). Love waves propagate in a thin guiding layer deposited on the surface of a semi-infinite substrate, and is therefore considered a compromise between Rayleigh waves and Lamb waves. They exhibit features such as surface waves confinement (compared with Rayleigh waves) and device toughness (compared with Lamb wave devices). In addition to the above qualities, their compatibility with liquid environment makes Love wave an ideal candidate for sensors, especially for bio-sensor applications.

In this work, the interaction of Love waves with a two-line Ni pillar based meta-surface is investigated on a SiO₂/ST quartz structure. Firstly, torsional mode in one line of cylindrical Ni pillars is demonstrated to be well excited by Love waves and gives rise to a sharp transmission dip due to a destructive interference. Secondly, acoustic analogue of ATS and cavity modes for

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Love waves are demonstrated in two lines of identical pillars by varying the distance between the pillar lines. ATS appears when the distance is smaller than the half wavelength and a strong coupling is aroused between the pillar lines. Fabry-Perot resonance exists at the positions where the distance between the pillar lines is a multiple of half wavelength. The proximity of Fabry-Perot resonance with pillar torsional mode gives rise to the cavity modes with transmission enhancement at the edges of the dip induced by the pillars. Thirdly, the radius of one line of pillar is modified to detune the pillar resonant frequency. In the pillar coupling region, the coupling effect decreases with the increase of radius mismatch. When the distance between the pillar lines is a multiple of half wavelength, Fabry-Perot resonance along with the two different pillar resonances give rise to the AIT resonance. ATS and AIT phenomena are then fitted respectively with corresponding formula models, showing good agreements. The Akaike Information Criterion (AIC) is then used to quantitatively evaluate the quality of the fit models, showing consistent results for the ATS and AIT cases, and the transition from ATS to AIT by increasing the distance between the pillars is presented. The band structures and the transmission spectra are calculated with the finite element method (FEM, COMSOL Multiphysics®).

II. UNIT CELL MODEL

![Unit cell model](image)

FIG. 1: Unit cell of two lines of cylindrical Ni pillars on the SiO₂ film deposited on a 90ST-cut quartz. The period is a along the y direction. Love waves propagate along the x-axis. \( r_1 = r_2 = 0.2a \), \( h = 0.6a \), \( H = 2.4 \mu m \), \( L = 20a \), \( a = 2 \mu m \).

The acoustic meta-surface is constituted of two pillar lines deposited on a bi-layered matrix, of which the unit cell is shown in Fig 1. The matrix is a 2.4μm-height silica guiding layer \((\rho = 2200\text{kg/m}^3, E = 70\text{GPa}, \nu = 0.17)\) that covers a 40μm-height 90ST-cut quartz substrate (Euler angles = (0°, 47.25°, 90°), LH 1978 IEEE) which has been rotated 90 degrees around the z-axis from the ST-cut quartz, to be able to generate fast SH waves (5000m/s) by the electric field. The shear wave velocity in the silica film is 3438m/s, less than that in the 90ST-cut quartz substrate, indicating the existence of Love waves. The period along the y-axis is \( a = 2 \mu m \).

Two cylindrical Ni pillars on the silica film have a radius of \( r_1 = r_2 = 0.2a \) and a height of \( h = 0.6a \). The distance between the centers of the two pillars is denoted by \( d \). The length of the unit cell \( L \) is \( 20a \) to insure a decoupling between the unit cells. The inclusions are chosen because of their strong contrast in density and elastic constants with regard to the matrix. Floquet periodic boundary conditions are applied along the \( x \) and \( y \) directions. The bottom of the substrate is assumed fixed. Love waves propagate along the \( x \)-axis (i.e. the \( y \)-axis of the ST-cut quartz), where Rayleigh waves cannot be generated due to a zero electromechanical coupling factor in the substrate. The surfaces of the pillars coincide with the plane \( z = 0 \). The wavelength normalized energy depth (NED) is calculated to select the surface modes:

\[
\text{NED} = \frac{\iint_D \frac{1}{2} T_{ij}S_{ij}^*(\cdot \cdot \cdot) dxdydz}{\lambda \iint_D \frac{1}{2} T_{ij}S_{ij} dxdydz}. \tag{1}
\]

where \( T_{ij} \) is the stress and \( S_{ij} \) the strain. The star symbol (*) means the complex conjugate. \(-z\) denotes that we integrate in the whole domain of the unit cell \( D \). \( \lambda \) is the wavelength. This formula works well for a relatively large \( k \) where the wave speed is less than the SH waves velocity of the substrate. For a relatively small \( k \) (with wave speed greater than the SH waves velocity of the substrate), which corresponds to a large \( \lambda \), we fix \( \lambda \) to \( 2a \). Surface modes have a NED < 2. The NED can filter out the bulk modes as well as the plate modes that appear in our finite-depth substrate which is supposed to be semi-infinite for Love waves.

SAW include SH type SAW and Rayleigh type SAW. The SH ratio is calculated to distinguish the displacement components:

\[
\text{SH ratio} = \frac{\iint_D u_{SH}u_{SH}^* dxdydz}{\iint_D (u_x u_x^* + u_y u_y^* + u_z u_z^*) dxdydz}. \tag{2}
\]

where \( u_x, u_y \) and \( u_z \) are respectively the displacement along the \( x, y, z \) directions. \( u_{SH} \) is the SH displacement component that can be expressed as \( u_{x} \cos \theta - u_{y} \sin \theta \), which is perpendicular to the wave vector \( k \). \( \theta \) is the angle between \( k \) and the \( y \)-axis such that \( \tan \theta = \frac{k_y}{k_y} \). In our case, \( k = k_x \) and \( u_{SH} = u_y \).

III. RESONANT PROPERTIES OF SINGLE PILLAR LINE

Firstly, we study the resonant properties of the unit cell with only one pillar, which corresponds to a meta-surface with a single pillar line. The dispersion curves or band structure of SH modes (SH ratio>0.5) in the \( x \) direction is shown in Fig 2(b). The X points are the irreducible Brillouin zone limit of the unit cell in the \( x \) direction. The modes colors are determined by their NED values. The modes in red are well confined to the surface, and can therefore be excited by Love waves. Certain modes
become pink as they are less confined to the surface. Two hybridization band gaps are observed respectively in the frequency range [177.3, 183.1] MHz and [501.8, 503.3] MHz, indicated in blue, which originates from the coupling of local resonant pillar modes and the SH SAW.

![Diagram](image)

**FIG. 2**: (a) Transmission spectrum of Love waves propagating through a single pillar line. Inset is the zoom of the torsional mode induced dip. (b) Band structure of SH modes in ΓX direction for the unit cell of a single pillar line. The red-white colors denote the normalized energy depth. A mode in red can be excited by Love waves. (c) \( u_y \) component of the displacement fields for two local resonant pillar modes. The amplitudes in the pillar are normalized to the maximum amplitude in the SiO\(_2\) film. \( r = 0.2a, \ h = 0.6a, \ a = 2\mu m. \)

The transmission spectra is calculated by simulating the same dispersive SAW device introduced in our preceding works \cite{67, 68}. The model consists of two parts of aluminum inter-digital transducers (IDTs) with the unit cell located between IDTs. The input IDT is given a \( V_0 = 1V \) harmonic voltage signal. The output is measured by averaging the voltage difference between odd and even electrodes. The width of the electrodes of IDTs \( L_{IDT} \) is updated for every frequency in the spectrum according to the relation \( L_{IDT} = \frac{v}{f} = \frac{v}{47}. \) \( v \) is the velocity of Love waves for \( H = 2.4\mu m, \) resulting from the basic dispersion relation of Love waves. That is, each frequency corresponds to a single wavelength. The frequency responses are then normalized by that of the matrix, referred to as normalized or relative transmissions \( \Delta S_{21}. \) Fig 2(a) shows the normalized transmission spectrum of the single pillar line. It can be seen that the transmission spectrum corroborates well with the band structure prediction. The displacement fields and the deformations at the two dips are shown in Fig 2(c). Due to their large SH component ratio, as well as the exclusive generation of SH waves by the electric field, we only show the transverse component \( u_y. \) The amplitude in pillar is normalized to that in the matrix and is indicated beside the pillar. The two dips correspond to the pillars’ intrinsic bending and torsional modes, respectively. The torsional mode induced transmission dip is better attenuated, since the excited torsional motion leads to, on the side opposite to the incident wave, a wave of identical amplitude and opposite phase, which is responsible for a destructive interference. To further confirm this mechanism, the emitted wave \( Em \) is calculated by subtracting the incident waves \( I \) (transmitted waves on the bare matrix without pillar) from the totally transmitted waves \( T: Em = T - I, \) \( Em \) is then normalized by \( I. \) Fig 3(a) is the complex plot of the normalized emitted wave in the frequency range [500, 505] MHz around the torsional mode. The rose dot corresponds to the dip at resonant frequency of 502.1 MHz. This mode falls at point (-1,0), which refers to the same amplitude with a phase shift of 180° with respect to the incident waves. This results in a destructive interference and a strong attenuation in transmission. The phase shift of full transmitted waves is shown in Fig 3(b). Red dotted lines represent the incident waves. The dip corresponds to a \( \pi \) change in phase with respect to the incident waves. The phase shift is 0 (2\( \pi \)) before (after) the resonant frequency, meaning that the transmitted waves are in phase with the incident waves when they deviate from the resonant frequency. The corresponding phase derivative or group delay time \( \tau_g \) of the transmitted waves is shown in Fig 3(c), with \( \tau_g = \frac{d\phi}{df}, \) where \( \phi \) denotes the phase and
\[ \omega = 2\pi f \] is the angular frequency. The waves are delayed at the resonant frequency. It can be seen that the resonance is characterized by a rapid variation of phase and amplitude.

IV. TWO LINES OF IDENTICAL PILLARS: AUTLER-TOWNES SPLITTING & CAVITY MODE

Transmission spectra are then calculated around the torsional mode frequency for two lines of identical pillars. In addition to our intuitively predicted transmission dip, different phenomena appear when we gradually increase the distance \( d \) between the pillar lines. When \( d < 2a \) (Fig 4(a)), a coupling effect arises between the pillar lines, causing a lifting of frequency degeneracy of the pillar torsional mode, and the original transmission dip splits into two dips with a transparency window in the middle, referred to as an acoustic analogue of Autler-Townes Splitting (ATS). The coupling becomes stronger when \( d \) gets smaller, as shown in Fig 4(a). Note that the pillar coupling also depends on the pillar mass. Therefore, the distance limit 1.4\( a \) can be detuned by changing the pillar size (radius or height). The displacement fields \( u_y \) at the dips and peaks for \( d=0.5a \) and 2.4\( a \) are shown aside in Fig 4(d). It is found that the largest amplitude is located in the pillars for dip1 at 499.8 MHz. The two pillars are in opposite phase at dip1 frequency and in-phase at dip2 frequency, which is a feature of the ATS resonance that can be confirmed by calculating the phase difference between the two pillars. Since the pillars are in torsional mode in the range of measurement, all the point on the side of the pillar that faces the incident waves are in phase. The phases of \( u_y \) on the wave-facing sides of the two pillars are probed. The cases of \( d = 0.5a \) and \( d = a \) are shown as examples in Fig 5(a) and (b). Rose and green dots correspond to transmission dips and peaks, respectively. \( r_1=r_2=0.2a, a=2\mu m \).

FIG. 4: Normalized transmission spectrum of Love waves propagating through the two identical pillar lines around the resonant frequency of torsional mode at 502.1 MHz with different central distance \( d \): (a) presents the lifting of degeneracy and the apparition of Autler-Townes Splitting with the decrease of \( d \); (b) shows the red-shift of cavity mode by increasing \( d \) when \( d < 2a \) (c) presents the appearance of cavity mode peak when \( d > 2a \). (d) and (e) are the displacement fields \( u_y \) at the dips and peaks for \( d=0.5a \) and 2.4\( a \), respectively. \( r_1=r_2=0.2a, a=2\mu m \).

FIG. 5: Pillar phase differences for ATS in the case of (a) \( d = 0.5a \) and (b) \( d = a \). Rose and green dots correspond to transmission dips and peaks, respectively. \( r_1=r_2=0.2a, a=2\mu m \).
Im($T / I$) -0.5 0 0.5 1 0.5 -1
Re($T / I$)

FIG. 6: (a) Complex plots of the normalized transmission in the frequency range [500, 505] MHz for $d$ varies from 1.8$a$ to 2.2$a$. The rose and green marks correspond to the dip and cavity mode frequencies, respectively. $r_1=r_2=0.2a$. (b) Complex plots of the normalized transmissions for $d = 2a \approx \lambda/2$ when all the pillars’ radius vary from 0.196$a$ to 0.204$a$. Inset shows the transmission spectra of corresponding curves.

FIG. 7: Dip and peak frequencies as functions of the distance $d$ between two identical pillar lines. Blue dotted line is the dip frequency of a single pillar line. ATS appears in the coupling region of $d < 1.4a$. The 1$^{st}$ and 2$^{nd}$ Fabry-Perot resonances fall at $d=\lambda/2$ and $\lambda$, respectively. $r_1=r_2=0.2a$, $a = 2 \mu m$.

mission spectrum when it matches perfectly the pillar torsion mode, since the waves are totally reflected. However, when we change the distance between pillars around $\lambda/2$, the proximity of FP resonance with the pillar mode gives rise to the cavity modes at the two edges of the dip, as shown in Fig. 5(b) and (c). In these cases, the two pillars act like partial reflectors, and the normally incident waves are multiply reflected to produce multiple transmitted waves with path difference equal to $n\lambda$, where $n$ is an integer. In this way, constructive interference occurs, leading to a resonant enhancement. The two pillars along with the guiding layer between them become a cavity to confine the waves. Nevertheless, in the transmission spectra, the behaviors for $d < 2a$ (Fig. 4(b)) are less marked than that for $d > 2a$ (Fig. 4(c)), where peaks rise at the lower edge of the dip, and give rise to Fano-like resonance line-shapes. However, we have verified that it can not be fitted by a Fano type formula. The displacement field $u_y$ at the peak and dip for $d = 2.4a$ are shown aside in Fig. 4(e). This behavior is different from that of ATS, the largest amplitude in the pillars occurs at the peak where the waves in the guiding layer are highly confined in the cavity.

To show more clearly the transition of the cavity mode with respect to the dip when $d$ changes, we draw the complex plots of the normalized transmissions in Fig. 6(a). The cavity mode is manifested as an additional perturbation on the original ellipse. Since the phase changes clockwise, it can be seen that the cavity mode passes the dip as $d$ increases. When the cavity mode approaches to the dip, the perturbation decreases. For $d = 2a$, the cavity mode coincides with the FP resonance and becomes invisible. Additionally, we can see that the behaviors for $d < 2a$ and for $d > 2a$ are quite similar, with the perturbation frequency either larger or smaller than the dip frequency.

Fig. 6(b) shows the complex plots of the normalized transmissions of Love waves for $d = 2a$ when we change the radius of both pillars from 0.196$a$ to 0.204$a$, indicating a shift in pillar resonant frequency. It is found that the cavity mode remains invisible, i.e. still coincides with the FP resonance. That is because the distance between pillars corresponding to the FP resonance is almost unchanged in our range of measurement, i.e. at $d = 2a$. Therefore when we change the pillar vibration frequency, we obtain the parallel cavity modes for the same value of $d$.

To give an overview of the resonance behaviors, the dip and peak frequencies for different $d$ are shown in Fig. 7. Blue dotted line is the dip frequency of the single pillar line. It can be seen that the pillar coupling induced ATS is in the region $d < 1.4a$, where the two dips are mismatched with the single pillar resonant frequency. This coupling disappears when $d$ exceeds 1.4$a$, and only one zero remains. This dip matches the single pillar resonant frequency except when the cavity modes appear below (upon), the dip frequency shifts slightly upward (downward). Note that the 1$^{st}$ and 2$^{nd}$ FP resonance exists only in the very closed regions around $d = 2a$ and 4$a$, respectively. In these two cases, the cavity mode is a FP resonance. In the other regions, one should avoid to mix up the cavity mode with the FP resonance.

V. TWO LINES OF DISSIMILAR PillARS: AUTLER-TOWNES SPLITTING & ACOUSTICALLY INDUCED TRANSPARENCY

Since an increase of pillar radius or height will induce a decrease in the torsional mode frequency, one can gradually tune the position of the dip by modifying the pillar size. Here we modify the pillar radius as example since the radius is more easy to be tuned in experimental process.

In the case below, the second pillar radius $r_2$ is tuned from 0.195$a$ to 0.205$a$, while the first pillar radius $r_1$ being fixed to 0.2$a$. Fig. 8 shows the transmission spectra of Love waves propagating through the two dissimilar pillar lines for different $d$ that remains unchanged for each case. Dotted blue and rose lines denotes the dip positions for
a single line of pillars with radius equals to $r_1$ and $r_2$, respectively. Fig 8(a) corresponds to the case $d = 0.5a$, where the coupling between the pillars is so strong that the two dips stay all the way mismatched with their corresponding single pillar dip positions. When $d$ gets larger, the coupling becomes less strong. Fig 8(b) corresponds to the case $d = a$. It is found that this coupling decreases with the increase of radius mismatch. In the case of $r_2 = 0.195a$ and $0.205a$, each dip almost coincides with the corresponding resonant frequency of one single pillar. In order to show the anti-crossing lines of ATS for $d < 1.4a$, we plotted in Fig 9 the dip frequencies for different $d$ as functions of $r_2$. It can be seen that with the increase of pillar distance, the anti-crossing lines get closer to the crossing line (for $d = 1.4a$), and that each anti-crossing line will rejoin its individual pillar resonant frequency when the radius mismatch is sufficiently large. This means the pillar coupling effect decreases when their distance of/and their radius difference increases.

For $d = 2a$ as shown in Fig 8(c), two pillars with different radius give rise to two dips with a transparency window in the middle. These peaks have a narrower line-shapes compared with the cases of ATS (see also Fig 11). Each dip is consistent with the corresponding dip frequency of one single pillar since no more coupling exists, which means each of them originates from individual pillars torsional mode. The displacement fields $u_y$ for $r_2 = 0.202a$ are presented in Fig 10. It can be seen that each dip corresponds to a large amplitude in a single pillar, indicating the attenuation of transmission at each pillar’s resonant frequency. The peak correspond to a Fabry-Perot resonance since $d$ is close to $\lambda/2$, and large amplitudes are observed for the three parts of the cavity (two pillars and the guiding layer in between). This three-resonance system induced transparency window is referred to as the Acoustic analogue of Electromagnetically Induced Transparency, also called Acoustically Induced Transparency (AIT). The peak rises and gets wider with the increase of radius difference. As for $d = 2.4a$ shown in Fig 8(d), where a cavity mode peak is observed for the two identical pillars, it is found that with the increase of pillar radius mismatch, the peak confinement decreases and the dip1 becomes evident for $r_2 = 0.198a$ and $0.202a$. Compared with the case of $d = 2a$ (Fig 8(d)), the peak between two dips is less confined due to the red shift of cavity mode with respect to the pillar resonant frequency. Note that AIT requires a well excited resonance between the two dips. Therefore in the case of $d = 2.4a$, the transparency window is only two dips stem from the different resonant frequencies of two pillars.

The transmission spectra of AIT are similar with those
of ATS, however, they originate from different mechanisms. ATS appears only when the two pillars are coupled to each other, and exists even when the two pillars are identical. AIT appears when $d$ is out of the pillar coupling region and only when the two pillars are different. The pillars and the cavity interact at the peak. Moreover, AIT requires a clearly identified 3-level resonant system, which is not the case for ATS.

Besides the different mechanisms related to ATS and AIT as presented above, corresponding analytical formulas of ATS and AIT for transmission spectra can be used to fit the numerical data to better distinguish these different transparency windows. The transmission curves for ATS can be written as the sum of two separate inverse Lorentzian profiles representing the two dips, while the transmission for AIT is expressed as the difference of a broad Lorentzian profiles and a narrow one with a similar central frequency [6–20]:

$$T_{ATS} = 1 - \frac{C_1 (\Gamma_1/2)^2}{(f - \delta_1)^2 + (\Gamma_1/2)^2} - \frac{C_2 (\Gamma_2/2)^2}{(f - \delta_2)^2 + (\Gamma_2/2)^2},$$

(3)

$$T_{AIT} = 1 - \frac{C_+ (\Gamma_+/2)^2}{(f - \delta_+ - \epsilon)^2 + (\Gamma_+/2)^2} + \frac{C_- (\Gamma_-/2)^2}{(f - \delta_-)^2 + (\Gamma_-/2)^2},$$

(4)

where $C_1$, $C_2$, $C_+$, $C_-$ are the amplitudes of the Lorentzian profiles, $\Gamma_1$, $\Gamma_2$, $\Gamma_+$, $\Gamma_-$ are their full width at half maximum (FWHM). $\delta_1$, $\delta_2$, $\delta_+$, $\delta_-$ are the central frequencies with $\epsilon$ denoting a possible slight shift on $\delta_-$. $\Gamma_1$, $\Gamma_2$, $\delta_1$, $\delta_2$ and $\delta_-$ can be directly taken from the transmission spectra.

In an intermediate state, the transmission spectra can be fitted by a transition formula that considers both the features of ATS and AIT:

$$T_{ATS/AIT} = 1 - \frac{C_a (\Gamma_a/2)^2}{(f - \delta_1)^2 + (\Gamma_a/2)^2} - \frac{C_b (\Gamma_b/2)^2}{(f - \delta_2)^2 + (\Gamma_b/2)^2} - \frac{(f - \delta_1)C_d (\Gamma_d/2)^2}{(f - \delta_1)^2 + (\Gamma_d/2)^2} + \frac{(f - \delta_2)C_e (\Gamma_e/2)^2}{(f - \delta_2)^2 + (\Gamma_e/2)^2},$$

(5)

where $C_a$, $C_b$, $C_d$, $C_e$, $\Gamma_a$, $\Gamma_b$, $\Gamma_d$, $\Gamma_e$ are parameters to be determined. Note that this formula can also be used to fit the ATS and AIT cases.

FIG. 11: Transmission spectra and model fits of ATS and AIT, for $r_1 = 0.2a$, $r_2 = 0.202a$ and $a = 2\mu$m. Numerical data (black dots) are presented together with the best fits of functions $T_{ATS}$ (red lines) and $T_{AIT}$ (blue lines). For (a) $d=0.5a$ and (b) $d = a$, $T_{ATS}$ fits the numerical data better than $T_{AIT}$. (c) For $d=2a$, $T_{AIT}$ fits the numerical data better than $T_{ATS}$. (d) For $d=2.4a$, $T_{ATS}/AIT$ (green line) can be used to fit the numerical data whereas $T_{ATS}$ and $T_{AIT}$ do not fit well.

where $N$ is the number of values calculated by FEM. $F_i$ is the value of $T_{ATS}$ or $T_{AIT}$ corresponding to each data frequency. It can be seen that, as expected, for $d = 0.5a$ and $a$ where coupling exists between the pillars, $T_{ATS}$ fits the numerical data much better than $T_{AIT}$. Whereas for $d = 2a$, $T_{AIT}$ fits the numerical data better than $T_{ATS}$. For $d = 2.4a$, both $T_{ATS}$ and $T_{AIT}$ do not fit the numerical data well. In this case, $T_{ATS/AIT}$ (green line) fits the data better.

By fitting the numerical data to the model fits of $T_{ATS}$ and $T_{AIT}$ for different $r_2$ ($r_1$ fixed), the relation between the fits parameters and $r_2$ can be obtained as shown in Fig. 12. Where Fig. 12a presents the fit parameters of the ATS model for $d = 0.5a$ and Fig. 12b are those of the AIT model for $d = 2a$. The parameters of ATS model describing each Lorentzian curve are independent of each other. It can be seen that $\Gamma_1$ increases with the increase of radius mismatch while $\Gamma_2$ increases with the increase of $r_2$. We think this is resulting from the symmetrical/asymmetrical vibration of the two pillars at dip1/dip2 in the ATS cases. $C_1$ is relatively stable while $C_2$ presents a slight tendency to increase. The parameters of AIT model are related to each other. As can be seen in Fig. 12b), these four parameters all increase as the radius mismatch increases and are almost symmetrical in our range of measurement. Despite a larger values of $\Gamma_+$ compared with $\Gamma_-$, $C_+$ and $C_-$ are almost the same for different $r_2$.

With the increase of distance between pillars, the transition from ATS to AIT can be quantitatively studied by...
The Akaike information criterion (AIC) is used to discern AIT from ATS, which provides a method to select the best model from a set of models. This criterion quantifies the amount of information lost, i.e. the degree of unfitness, and is given as $I_j = 2k - 2ln(L_j)$, where $k = 4$ is the number of unknown parameters and $L_j$ the maximum likelihood.

For the considered models, i.e. $j = \text{ATS or AIT}$. Since we already found the best fit functions of $T_{ATS}$ and $T_{AIT}$, it is sufficient to calculate the likelihood of these two functions. Then, the AIC weight $W_j = e^{-I_j/2n} / \sum_k e^{-I_k/2n}$ can give the relative likelihood of a candidate model. $N$ is the number of considered model. In our case, $N = 2$ as only two models are involved. Since we have more than one calculated data for each model, we utilize the AIC mean per-point weight $\bar{w}_j = e^{-I_j}/2n / \sum_k e^{-I_k}/2n$ to calculate the statistically synthesized likelihood of the candidate model. $n$ is the calculated data number. The AIC mean weight can be rewritten as:

$$\bar{w}_{ATS} = \frac{e^{-I_{ATS}/2n}}{e^{-I_{ATS}/2n} + e^{-I_{AIT}/2n}}$$

FIG. 14: Normalized transmission spectra of Love waves propagating through the two lines of pillars with different heights, $h_1 = h + \Delta h$ and $h_2 = h - \Delta h$, in the case of $d = 2a$. AIT peak rises and becomes wider with the increase of pillar height mismatch. $h = 0.6a$, $r_{1,2} = 0.2a$ and $a = 2\mu m$.
The pillar height. The heights of the two pillars are in-
the AIT model and therefore cannot be ascribed to
also be obtained by detuning the pillar height. The heights of the two pillars are in-
creased/reduced from the original height \( h = 0.6a \) by the
same amount \( \Delta h \). That is, \( h_1 = h + \Delta h \) and \( h_2 = h - \Delta h \).
The radius of both the two pillars \( r_1 \) and \( r_2 \) are fixed at 0.2a. Fig [4] shows the transmission spectra of Love
waves through the two lines of pillars when \( d = 2a \), i.e.
in the region of AIT. It can be seen that by decreasing
\( \Delta h \), the two dips approach to each other and the peak
with almost unchanged frequency decreases and becomes
invisible for identical pillars.

VI. CONCLUSION

In this work, the interaction of Love waves with two
lines of cylindrical Ni pillars are investigated on a silica
film deposited on a 90ST quartz substrate. Firstly, pil-
lar intrinsic torsional mode is demonstrated to be well
excited by Love waves. One line of pillars can give rise
to a sharp transmission dip due to a destructive inter-
ference. Secondly, acoustic analogue of Autler-Townes
Splitting (ATS) and Fabry-Perot resonance of Love waves
are demonstrated in two lines of identical pillars by vary-
ing the distance between the pillar lines. ATS appears
when the distance is smaller than the half wavelength
and a strong coupling is aroused between the pillar lines,
cau~ng the pillar mode induced transmission dip to split
into two dips with a transparency window in the mid-
dle. This coupling decreases with the increase of pillar
distance. Fabry-Perot resonance exists at the positions
where the distance between the pillar lines is a multiple
of half wavelength. The proximity of Fabry-Perot reso-
nance with pillar intrinsic mode gives rise to the cavity
modes with transmission enhancement on the two edges
of the single dip. Thirdly, the radius of one line of pillar
is modified to detune the pillar resonant frequency. In
the pillar coupling region, the coupling effect decreases
with the increase of radius mismatch, and the two dips
will rejoin their individual pillar mode frequencies. When
the distance between the pillar lines is a multiple of half
wavelength, Fabry-Perot resonance along with the two
different pillars’ resonances give rise to the Acoustically
Induced Transparency (AIT). Same phenomena can also
be obtained by detuning the pillar height. With similar
transparency window in the transmission spectra, ATS
and AIT phenomena are then fitted respectively with
the corresponding formula models, showing good agree-
ments. The fit parameters are demonstrated as functions
of the geometrical parameter. The Akaike information
criterion (AIC) is then used to quantitatively evaluate the
quality of the fit models, which illustrates the transition
from ATS to AIT by increasing the distance between the
pillar lines. The theoretical and analytical differentiation
of ATS and AIT should be used together to discriminate
the assignment of the observed spectrum to one or the
other physical mechanism. The results presented in this
study could be used to potential acoustic applications
such as wave control, meta-materials and bio-sensors.

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