Study on the Transverse Elastic Vibration Equation and the Generalized Aerodynamic Force of Space Vehicle

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Abstract. In order to avoid divergence of space vehicle attitude, structural elastic movements should be taken into consideration in the control system design. Space vehicle structure generally has a large length-diameter ratio, the corresponding rolling and longitudinal modal with higher natural frequency has a slight impact on the design of the control system, while the transverse elastic modal with lower natural frequency including pitching and yawing should be paid more attention in the design of space vehicle control system. In this paper, the influence of elastic vibration on the design of attitude control system of space vehicle is analyzed firstly. Then the transverse elastic vibration equation of spacecraft and the calculation method of generalized aerodynamic force are studied. Furthermore, the equation of elastic vibration considering the coupling effect of aerodynamic force between pitching and yawing control channels is proposed and the engineering calculation method of generalized aerodynamic force for typical stations in spacecraft is improved. The research results can provide significant reference for the design of the new generation of spacecraft control system.

1. Introduction

The attitude control system, which is one of the most important parts in spacecraft flight control system, is mainly responsible for stabilizing and controlling the pitching, yawing and rolling motion of spacecraft so as to guarantee the spacecraft to flight according to the predetermined trajectory. In the flight process, the combination effects of the aerodynamic force, the jet engine thrust and the servo driving force will lead to the structural deformation and elastic vibration, which will affect the motion of spacecraft around the center of mass and feedback to the control loop through inertial and gyro perception, and eventually have impact on the movements of aerial rudder, swing jet engines and gas rudder. In order to avoid divergence of space vehicle attitude, structural elastic movements should be taken into consideration in the control system design. However, the space vehicle structure generally has a large length-diameter ratio, the corresponding rolling and longitudinal modal with higher natural frequency has a slight impact on the design of the control system. Therefore, the transverse elastic modal with lower natural frequency including pitching and yawing should be paid more attention in the design of space vehicle control system.

The bending deformation of space vehicle caused by elastic vibration will introduce additional thrust components and additional attitude angle, which affect the space vehicle control system significantly. Since the late 1960s, extensive researches have been conducted by scholars from all over the world to study the influence of spacecraft structural flexibility on control system design and stability. Reis and Sundberg [1] studied the elastic bending of the rocket during launch and free flight by using two-rigid-body model and continuum model respectively. Cochran et al [2] analyzed the
influence of elastic vibration on the attitude of free flight rocket, and concluded that the lateral bending would cause large transverse angular velocity. Choi and Bang [3] investigated the control system of KSR(Korea Sounding Rocket)-II sonic velocity rocket and found that the elastic vibration would be superimposed on the output of the controller through the feedback channel, which would affect the stable operation of the control system and even cause the system instability. So the adaptive notch technology was used to filter out this influence in the feedback channel so as to ensure the system stability. Wu and He [4] proposed the elastic vibration equation and the design method of attitude control system of large and medium-sized bundled rocket based on space modal, which took the coupling effects of rigid movements, fluid sloshing and elastic vibration into account. Wei et al[5] developed a full-component dynamic model, designed the self-adaptive attitude controller and variable gain scheduling controller based on frequency identification and realized the high-precision control of the elastic launch vehicle.

The traditional spacecraft, represented by launch vehicles and missile weapons, has symmetric aerodynamic shapes and structures and the corresponding pitching, yawing and rolling channels are independent. Thus, tri-channel control system with decoupled strategy is always employed [6]. However, with the development of new types of aircrafts and extensive applications of new types of flight trajectory, the coupling effect of the aerodynamic forces on the structure surface in the transverse pitching and yawing channels becomes serious, so it is necessary to fully consider the coupling effect and establish the corresponding transverse elastic vibration equations.

In this paper, in order to consider the coupling effect of spacecraft aerodynamic forces in pitching and yawing channels, the influence of spacecraft elastic vibration on the design of attitude control system of space vehicle is analyzed firstly. Then the lateral elastic vibration equation of spacecraft and the calculation method of generalized aerodynamic force are studied. Furthermore, the equation of elastic vibration considering the influence of aerodynamic coupling between pitching and yawing control channels is proposed and the engineering calculation method of generalized aerodynamic force for typical stations in spacecraft is improved. The research results can provide significant reference for the design of the new generation of spacecraft control system.

2. Transverse elastic vibration equation

The main structure of space vehicle is always axial symmetric and has a relatively large length-diameter ratio. Its elastic vibration modal shape mainly includes motions in longitudinal, pitching, yawing and rolling directions with pitching axis perpendicular to trajectory plane and yawing axis in the trajectory plane perpendicular to structural longitudinal axis. When transverse elastic vibrations in pitching and yawing directions are analyzed as shown in Figure 1, both computational fluid method and finite element method should be employed to obtain characteristics of the aerodynamic distribution and modal shapes respectively.

![Figure 1. Elastic deformation of space vehicle structure](image)

In modal analysis, the finite element method is always adopted and the elastic vibration equation of the spacecraft can be expressed by [7]:

\[ \text{Equation} \]
\[ M\ddot{X}(t) + C\dot{X}(t) + KX(t) = F(t) \] (1)

where \( M, C \) and \( K \) are mass matrix, damping matrix and stiffness matrix of the structural system respectively; \( X(t), \dot{X}(t) \) and \( \ddot{X}(t) \) are the finite element node displacement, velocity and acceleration vector respectively; \( F(t) \) is the finite element node loading vector. Mode-superposition method is exploited to solve the equation, and it is assumed that

\[ X(t) = \sum_{i=1}^{n} \Phi_i q_i(t) \] (2)

where \( \Phi_i \) is the \( i \)-th order modal shape of the structural system, then Equation (1) can be decoupled as:

\[ \ddot{q}_i(t) + 2\zeta_i\omega_i \dot{q}_i(t) + \omega_i^2 q_i(t) = F_i(t)/M_i \quad i = 1, 2, ..., n \] (3)

where \( q_i(t), \dot{q}_i(t) \) and \( \ddot{q}_i(t) \) are the \( i \)-th order modal displacement, velocity and acceleration in generalized coordinates respectively; \( \omega_i, \zeta_i \) \( F_i(t) \) and \( M_i \) are the \( i \)-th order modal natural frequency, damping coefficient, generalized force and generalized mass of the structural system. \( M_i \) and \( F_i(t) \) can be expressed as:

\[ M_i = \Phi_i^T M \Phi_i \] (4)

\[ F_i(t) = \Phi_i^T F(t) \] (5)

\[ \omega_i = \sqrt{\Phi_i^T K \Phi_i / \Phi_i^T M \Phi_i} \] (6)

\[ 2\zeta_i \omega_i = \sqrt{\Phi_i^T C \Phi_i / \Phi_i^T M \Phi_i} \] (7)

In the control system of traditional launch vehicles and missile weapons, the influences of attack angle on lateral aerodynamic force and sideslip angle on normal aerodynamic force are usually ignored, and the elastic vibration equation in the direction of pitching and yawing can be obtained as [7]:

\[ \ddot{q}_i(t) + 2\zeta_i\omega_i \dot{q}_i(t) + \omega_i^2 q_i(t) = D_{1i} \Delta \phi + D_{2i} \Delta \alpha + D_{3i} \Delta \delta_\phi + D_{3ii} \Delta \ddot{\phi} + Q_{ly} \] (8)

\[ D_{1i} = -\frac{180qS_M}{\pi M_V} \sum_{k=1}^{K} C_{nn}^{\alpha} \Phi_i(X)(X_T - X_S)dX \] (9)

\[ D_{2i} = \frac{180qS_M}{\pi M_V} \sum_{k=1}^{K} C_{nn}^{\alpha} \Phi_i(X)dX \] (10)

\[ D_{3i} = \frac{NK_K}{M_i} [F_{nn}^{K} \Phi_i(X_K) + m_K \alpha_X l_K \Phi_i'(X_K)] \] (11)

\[ D_{3ii} = \frac{NK_K}{M_i} [m_K \alpha_X l_K \Phi_i(X_K) + j_K \Phi_i'(X_K)] \] (12)

where \( q \) is the dynamic pressure; \( S_M \) is the reference area; \( V \) is the flight velocity; \( M_i \) is general mass of the \( i \)-th order modal; \( \Phi_i \) and \( \Phi_i' \) are the modal shape and corresponding slope respectively; \( \Delta \phi \) is the pitching (or yawing) rotational angular velocity; \( \Delta \alpha \) is the attack angle (or sideslip angle); \( \Delta \delta_\phi \) is the angle of aerial rudder; \( \Delta \ddot{\phi} \) is the angular acceleration of aerial rudder; \( Q_{ly} \) is the general force corresponding to the disturbing force in the direction of main shaft vibration; \( C_{nn}^{\alpha} \) is the coefficient derivative of transverse(in the direction of pitching and yawing) distribution of aerodynamic force; \( F_{nn}^{K} \) is the coefficient of the transverse(in the direction of the pitching and the yawing) aerodynamic force of aerial rudder; \( X_T \) is the longitudinal coordinate of space vehicle’s mass center; \( X_S, X_R \) are the longitudinal coordinates of all rocket(missile) body stations and connection location of the aerial rudder; \( \alpha_X \) is the longitudinal acceleration; \( m_K \) is the mass of aerial rudder; \( l_K \) is the distance between
mass center of the aerial rudder and corresponding spinning shaft; \( J_K \) is the moment of inertia of aerial rudder; \( N_K \) is the number of aerial rudders; \( H_K \) is the layout coefficient of aerial rudders.

For the new generation of space vehicles, especially for those with lifting body shape, the change of sideslip angle will inevitably cause the significant change of lifting force, which means that the influence of the derivative of normal aerodynamic force to sideslip angle on the control system cannot be ignored. In addition, in the large sideslip condition, the change of attack angle, to some extent, will also cause the change of lateral force. Namely, the influence of the derivative of the lateral aerodynamic force to the attack angle on the control system should also be considered in the control system design.

Aerodynamic computational fluid simulation is conducted on some cabin parts of the American x-51 aircraft shown in Figure 2 when rudder angle are 0° and 25°, and the results of derivative of normal aerodynamic force to attack angle (\( \alpha_f \)) and sideslip angle (\( \beta \)) are shown in Figures 3 and 4. It can be indicated from Figures 3(b) and 3(e) that when the aircraft flies in the state of zero sideslip and zero rudder deflection, the change of sideslip angle has slight impact on the normal aerodynamic force \( C_{y, \beta} \), while the change of pitching angle has a greater impact on the normal aerodynamic force \( C_{y, \alpha_f} \), as shown in Equation (8). However, it can be seen from Figures 3 and 4 that when the aircraft flies with a large sideslip angle or rudder deflection, there are many states where \( |C_{y, \beta}| \geq |C_{y, \alpha_f}| \). Therefore, the change of sideslip angle is more important than the change of pitching angle and should not be ignored in these cases.

![Figure 2. American X-51 Aircraft](image)

![Figure 3. Normal aerodynamic force derivatives to attack angle \( \alpha_f \) and sideslip angle \( \beta \) when the rudder angle is 0°](image)
Figure 4. Normal aerodynamic force derivatives to attack angle $\alpha_f$ and sideslip angle $\beta$ when the rudder angle is 25°

Consequently, both the influence of attack angle and sideslip angle should be taken into consideration in the design of the pitching and yawing channel control system of space vehicles. Thus, the elastic vibration equation in the direction of the pitching and the yawing should be modified as follows.

$$\ddot{q}_i(t) + 2\zeta_i\omega_i \dot{q}_i(t) + \omega_i^2 q_i(t) = D_{2i}^1 \Delta \phi + D_{2i}^2 \Delta \alpha + D_{2i}^3 \Delta \beta + D_{2i}^4 \Delta \delta \phi + D_{2i}^5 \ddot{\delta} \phi + Q_{iy}$$  \hspace{1cm} (13)

$$D_{2i}^* \delta \beta$$ is the additional influential component caused by sideslip (or attack) angle to directional general force; \(C_{nn}^\beta\) is the coefficient of transverse (in the direction of the pitching and the yawing) aerodynamic force distribution.

3. Computation method of general aerodynamic derivative in traditional station

For the generalized force of aerodynamic distribution force derivative of traditional launch vehicle and missile weapon, the rocket or missile structure is always divided into several stations in engineering, and the generalized force is obtained by calculating the aerodynamic force derivative and the vibration mode on all stations.

Figure 5. Station divisions of traditional launch vehicle and missile weapon

As shown in Figure 5, the whole body of traditional launch vehicle and missile can be divided into 18 stations. The cross sections between stations are marked 1* to 17* respectively. Besides, and the
following conditions are required for station division:

1. The variation of aerodynamic force derivative between stations is slight and can be linear;
2. The variation of mode between stations is also slight and can be linear.

Based on the above station division conditions, if the aerodynamic distributed force distribution derivative between two adjacent stations with distance \( L \) is denoted by \( F_X(x) = Ax + B \) in the direction of \( X \) and the vibration modal shape between stations is denoted by \( \Phi_{1,X}(x) = Cx + D \), then the theoretical generalized force \( F_{1,X} \) is

\[
F_{1,X} = \int_0^L (Ax + B)(Cx + D)dx = \frac{1}{3}ACL^3 + \frac{1}{2}ADL^2 + \frac{1}{2}BCL^2 + BDL
\]  

(15)

In the traditional design, the aerodynamic distribution characteristics of the rocket (missile) body surface are usually obtained by engineering calculation or computational fluid simulation, and the aerodynamic force derivatives \( F^{(s)}(s) \) at each station are calculated by equally divided distance to obtain the generalized force [7]. In details, the aerodynamic force derivative on station 1 is the resultant force from station 1 to 1*. Similarly, the one on station 18 is the resultant force from station 17* to 18, and the aerodynamic force derivative on the middle station is the resultant force between its left and right equal sections. Thus, the generalized force obtained by this method is

\[
F_{1,X} - F'_{1,X} = \frac{1}{24}ACL^3
\]  

(17)

Therefore, the generalized force of typical stations based on the equal distance division should divide stations as much as possible so as to weaken the influence of station distance \( L \), change slope of aerodynamic derivative \( A \) and change slope of vibration modal shape \( C \), and eventually to improve the calculation accuracy.

In order to eliminate the influence of station division, aerodynamic characteristics and vibration modal shapes on generalized force, the generalized aerodynamic force derivatives of typical stations can be calculated by the following method of distribution of pressure center.

If \( q_Y(x) \) is the distribution of the transverse aerodynamic force derivative in the direction of \( Y \) and \( \Phi_{1,Y}(x) \) is the distribution of the vibration modal shape in the direction of \( Y \) as shown in Figure 6, aerodynamic force derivatives of right and left stations are derived as follows:

\[
F_{Y, \text{left}}^{(s)} = \int_0^L q_Y(x) \frac{x}{L} dx
\]  

(18)

\[
F_{Y, \text{right}}^{(s)} = \int_0^L q_Y(x) \frac{(L-x)}{L} dx
\]  

(19)

Because both changes of aerodynamic force and vibration modal shape between stations are linear, it can be assumed that
\[
q_Y(x) = Ax + B \quad (20)
\]
\[
\Phi_{i,y}(x) = Cx + D \quad (21)
\]

Then, according to vibration modal shapes and aerodynamic forces on the left and right sides of the station, the following can be obtained:

\[
F_{i*,y} = F_{i,y}^{left} \cdot \Phi_{i,y}^{left} + F_{i,y}^{right} \cdot \Phi_{i,y}^{right} = D \int_0^L (Ax + B) \left( \frac{L - x}{L} \right) dx + (CL + D) \int_0^L (Ax + B) \frac{x}{L} dx
\]

\[
= \frac{1}{3} ACL^3 + \frac{1}{2} ADL^2 + \frac{1}{2} BCL^2 + BDL \quad (22)
\]

\[
F_{i,X} = F_{i,X}^{**} \quad (23)
\]

Therefore, the generalized aerodynamic force of typical station calculated by the method of distribution of pressure center is more accurate than that obtained by traditional engineering method as shown in Equation (16).

4. Conclusions

In this paper, the influence of the transverse elastic vibration equation on the attitude control system of spacecraft is analyzed, and the elastic vibration equation of the spacecraft cabin considering the coupling effect of the aerodynamic forces between pitching and yawing channels is developed according to the characteristics of the aerodynamic forces derivatives to the attack angle and sideslip angle. The improved calculation method of the generalized aerodynamic derivatives of typical stations based on the pressure center distribution is proposed. This computation method can reduce the requirement on the station density and improve the accuracy of calculation results. The research results in this paper can be widely used in the overall design of the new generation of missile weapons, launch vehicles and space vehicles, and can provide important reference for the design of control system and stability analysis.

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