Microscopic study of low-lying spectra of $\Lambda$ hypernuclei based on a beyond-mean-field approach with covariant energy density functional

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We present a detailed formalism of the microscopic particle-rotor model for hypernuclear low-lying states based on a covariant density functional theory. In this method, the hypernuclear states are constructed by coupling a hyperon to low-lying states of the core nucleus, which are described by the generator coordinate method (GCM) with the particle number and angular momentum projections. We apply this method to study in detail the low-lying spectrum of $^{15}_\Lambda$C and $^{21}_\Lambda$Ne hypernuclei. We also briefly discuss the structure of $^{15}_\Lambda$Sm as an example of heavy deformed hypernuclei. It is shown that the low-lying excitation spectrum with positive parity states of the hypernuclei, which are dominated by $\Lambda$ hyperon in s-orbital coupled to the core states, are similar to that for the corresponding core states, while the electric quadrupole transition strength, $B(E2)$, from the $2_1^+$ state to the ground state is reduced according to the mass number of the hypernuclei. Our study indicates that the energy splitting between the first $1/2^-$ and $3/2^-$ hypernuclear states is generally small for all the hypernuclei which we study. However, their configurations depend much on the properties of a core nucleus, in particular on the sign of deformation parameter. That is, the first $1/2^-$ and $3/2^-$ states in $^{13}_\Lambda$C are dominated by a single configuration with $\Lambda$ particle in the $p$-wave orbitals and thus providing good candidates for a study of the $\Lambda$ spin-orbit splitting. On the other hand, those states in the other hypernuclei exhibit a large configuration mixing and thus their energy difference cannot be interpreted as the spin-orbit splitting for the $p$-orbitals.

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I. INTRODUCTION

The development in $\Lambda$-hypernuclear spectroscopy has enabled one to explore several aspects of hypernuclear structure [1, 2]. Since hyperon-nucleon and hyperon-hyperon scattering experiments are difficult to perform, the information on the $\Lambda N$ and $\Lambda\Lambda$ interactions have been extracted from such studies. Moreover, since a $\Lambda$ hyperon is free from the Pauli principle from other nucleons, it can go deeply inside a nucleus, which can also be used as a sensitive probe in order to study the structure of normal nuclei. Theoretically, many methods have been developed to investigate the spectroscopy of hypernuclei, such as the cluster model [3–9], the shell model [10–12], the ab-initio method [13], the antisymmetrized molecular dynamics (AMD) [14–17], and self-consistent mean-field models [18–26]. Among them, the self-consistent mean-field approach is the only microscopic method which can be globally applied from light to heavy hypernuclei.

One of the characteristic features of atomic nuclei is deformation in the body-fixed frame. In the mean-field approach, the optimized deformation is automatically obtained by minimizing the total energy of a system in the mean-field approximation. It was shown, however, that the potential energy surface of a hypernucleus is generally softer against deformation than that of the corresponding core nucleus [21]. This implies that the shape fluctuation effect, which is not included in the pure mean-field approximation, will be more important in hypernuclei than in normal nuclei. Furthermore, in order to connect mean-field results to spectroscopic observables, such as $B(E2)$ values, one has to transform the results to the laboratory frame. To this end, one has to rely on additional assumptions such as the rigid rotor model, which however would not work for, e.g., nuclei with small deformation or with shape coexistence. To quantify the impurity effect of a $\Lambda$ particle on nuclear structure, one thus has to go beyond the pure mean-field approximation.

In our previous publication [27], we have proposed a new approach using a microscopic particle-rotor model (PRM) for the low-lying states of single-$\Lambda$ hypernuclei. In this method, the $\Lambda$ particle is coupled to the core nucleus states while the $\Lambda$ hyperon interacts with the nucleons inside the nuclear core. For the core nucleus, a beyond-relativistic-mean-field approach is applied for low-lying states by carrying out the angular momentum and the particle number projections as well as the configuration mixing with the generator coordinate method (GCM). We have successfully applied this method to the spectrum of $^{13}_\Lambda$Be.

The motivation of the present work is to introduce a detailed formalism of this method. At the same time, we also apply it systematically in order to study the low-lying states of single-$\Lambda$ hypernuclei in the mass region from light to heavy. For this purpose, we will first discuss the $^{13}_\Lambda$C hypernucleus, which is an ideal hypernucleus in order to discuss the spin-orbit splitting. We will
we present as wave function of the whole Λ hypernucleus is constructed (CDFT). In Sec. PRM based on the covariant density functional theory a detailed introduction to the formalism of microscopic particle-rotor model for Λ hypernucleus, in which Λ is considered as an example of heavy hypernuclei.

FIG. 1: (Color online) A schematic picture of the microscopic particle-rotor model for Λ hypernucleus, in which r denotes the coordinate of the Λ hyperon. In this approach, the nuclear core states are described microscopically with the multi-reference density functional theory.

then discuss $^{21}_{\Lambda}$Ne as an example of hypernuclei in the sd-shell region with a prolate deformation. The $^{155}_{\Lambda}$Sm hypernucleus, which is a well-deformed system and has a well-developed ground-state rotational band, will also be considered as an example of heavy hypernuclei.

The paper is organized as follows. In Sec. II we present a detailed introduction to the formalism of microscopic PRM based on the covariant density functional theory (CDFT). In Sec. III, the results for the low-lying states of hypernuclei and the corresponding core nuclei will be presented. Finally, we will summarize the conclusions of this paper in Sec. IV.

II. MICROSCOPIC PARTICLE-ROTOR MODEL FOR Λ HYPERNUCLEI

A. Coupled-channels equations

The basic idea of the microscopic PRM for a single-Λ hypernucleus is that the valence Λ hyperon couples to the low-lying states of nuclear core, as is illustrated in a schematic picture of Fig. 1. In this approach, a hypernucleus is described in the laboratory frame and the wave function of the whole Λ hypernucleus is constructed as

$$\Psi_{JM}(r, \{r_N\}) = \sum_{n,j,I,I} \mathcal{R}_{njl}(r) \mathcal{F}_{jlm}(\hat{r}, \{r_N\}),$$  

(1)

where

$$\mathcal{F}_{jlm}(\hat{r}, \{r_N\}) = [\Psi_{jlm}(\hat{r}) \otimes \Phi_{ln}(\{r_N\})]^{JM}.$$  

(2)

with r and $r_N$ being the coordinates of the Λ hyperon and the nucleons, respectively. J is the angular momentum for the whole system while M is its projection onto the z-axis. $\Psi_{jlm}(\hat{r})$ is the spin-angular wave function for the Λ hyperon. $\{\Phi_{ln}(\{r_N\})\}$ is the wave functions of the low-lying states of nuclear core, where I represents the angular momentum of the core state and n = 1, 2, ... distinguish different core states with the same angular momentum I. For convenience, hereafter we introduce the shorthand notation $k = \{jlI_n\}$ to represent different channels.

In the relativistic approach, $\mathcal{R}_k(r)$ is the radial wave function of a four-component Dirac spinor and it can be written in the following form

$$\mathcal{R}_k(r) = \begin{pmatrix} f_k(r) \\ ig_k(r) \sigma \cdot \hat{r} \end{pmatrix}. $$  

(3)

The Hamiltonian $\hat{H}$ for the whole Λ hypernucleus can be written as

$$\hat{H} = \hat{T}_\Lambda + \sum_{i=1}^{A_c} \left[ \hat{V}^{(NA)}(r, r_N) + \hat{V}^{(NA)}(r, r_N) \right] + \hat{H}_c, $$  

(4)

where $A_c$ is the mass number of the core nucleus. The first term in Eq. (4), $\hat{T}_\Lambda$, is the kinetic energy of Λ hyperon,

$$\hat{T}_\Lambda = -i \alpha \cdot \nabla_\Lambda + \gamma^0 m_\Lambda.$$  

(5)

Here, $m_\Lambda$ is the mass of the Λ hyperon and α and $\gamma^0$ are Dirac matrices. The second term in Eq. (4) represents the NA interaction term between the valence Λ and the nucleons in the core nucleus. It is composed of both a repulsive vector-type term $\hat{V}^{(NA)}_V$ and an attractive scalar-type term $\hat{V}^{(NA)}_S$, for which we take the following contact coupling forms,

$$\hat{V}^{(NA)}_V(r, r_N) = \alpha^{NA}_V \delta(r - r_N)$$  

(6)

$$\hat{V}^{(NA)}_S(r, r_N) = \alpha^{NA}_S \gamma^0 \delta(r - r_N)\gamma^0_N.$$  

(7)

These NA interactions correspond to the leading-order four-fermion coupling terms in the effective interaction proposed in Ref. [28]. For simplicity, the possible higher-order derivative and tensor coupling terms are neglected in the present study. We note here that the spin-orbit interaction of hyperon is automatically taken into account in the relativistic framework without introducing an additional parameter. The last term in Eq. (4) is the many-body Hamiltonian for the core nucleus satisfying the equation $\hat{H}_c|\Phi_{ln}\rangle = E_{ln} |\Phi_{ln}\rangle$. In this paper, we use $\hat{H}_c$ that corresponds to the following point-coupling energy density functional (EDF) [29].
\[
E_c[\{\rho_i\}, \{j_{\mu}^i\}] = \text{Tr}[\alpha \cdot p + \beta m] \rho_V + \int dr \left( \frac{\alpha S}{2} \rho_S^2 + \frac{\beta S}{3} \rho_S^3 + \frac{\gamma S}{4} \rho_S^4 + \frac{\delta S}{2} \rho_S \Delta \rho_S + \frac{\alpha V}{2} j_{\mu} j^\mu + \frac{\gamma V}{4} (j_{\mu} j^\mu)^2 \right) + \frac{\delta V}{2} j_{\mu} \Delta j^\mu + \frac{\alpha_{TV}}{2} j_{\mu}^T(j_{TV})_\mu + \frac{\delta_{TV}}{2} j_{\mu}^T \Delta(j_{TV})_\mu + \frac{e}{2} j_{\mu}^T A_\mu , \tag{8}
\]

where \( A^\mu \) is the four-component electromagnetic field, and the densities \( \rho_i \) and currents \( j_{\mu}^i \) are bi-linear combinations of Dirac spinors, namely \( \psi \Gamma_i \psi \) with \( i = S, V, TV \) representing the symmetry of the coupling. The subscript \( S \) stands for isoscalar-scalar (\( \Gamma_S = 1 \)), \( V \) for isoscalar-vector (\( \Gamma_V = \gamma^\mu \)), and \( TV \) for isovector-vector (\( \Gamma_{TV} = \gamma^\mu t_3 \)) type of coupling characterized by their transformation properties in isospin and in space-time.

In order to obtain the radial wave function given by Eq. (3) and the energy of hypernuclear low-lying states, we multiply \( \langle \mathcal{F}_k^M \rangle \) to the total Schrödinger equation, \( \hat{H} |\Psi_{JM}\rangle = E_J |\Psi_{JM}\rangle \), and integrate it over \( \hat{r} \) and \( r \). This leads to the following coupled-channels equations,

\[
\begin{align*}
\left( \frac{d}{dr} - \frac{\kappa - 1}{r} \right) g_k(r) + & (E_{I_n} - E_J) f_k(r) + \sum_{k'} \left[ U_{VV}^{kk'}(r) + U_{SS}^{kk'}(r) \right] f_{k'}(r) = 0, \tag{9a} \\
\left( \frac{d}{dr} + \frac{\kappa + 1}{r} \right) f_k(r) - & (E_{I_n} - 2m_\Lambda - E_J) g_k(r) - \sum_{k'} \left[ U_{VV}^{kk'}(r) - U_{SS}^{kk'}(r) \right] g_{k'}(r) = 0, \tag{9b}
\end{align*}
\]

where \( \kappa \) is defined as \( \kappa = (-1)^{j + \ell + 1/2} (j + 1/2) \). With the multipole expansion for the \( \delta \) function in coordinate space,

\[
\delta(r - r') = \frac{\delta(r - r')}{rr'} \sum_{\lambda, \mu} Y_{\lambda \mu}(\hat{r}) Y_{\lambda \mu}^*(\hat{r'}), \tag{10}
\]

the vector and scalar coupling potentials in Eqs. (9a) and (9b) read

\[
U_{VV}^{kk'}(r) = \langle \mathcal{F}_k^M | \alpha_V^{NA} \sum_{i=1}^{A_c} \delta(r - r_{Ni}) | \mathcal{F}_{k'}^M \rangle \\
= (-1)^{j + \ell + j + \ell'} \sum_{\lambda} \langle j'l' | Y_{\lambda} | j' \rangle \left\{ J \begin{array}{c} I \end{array} J \begin{array}{c} I' \end{array} \right\} \times \alpha_V^{NA} \rho_{\lambda, V'}^{I_n, I_{n'}}(r), \tag{11}
\]

and

\[
U_{SS}^{kk'}(r) = \langle \mathcal{F}_k^M | \alpha_S^{NA} \sum_{i=1}^{A_c} \delta(r - r_{Ni}) | \mathcal{F}_{k'}^M \rangle \\
= (-1)^{j + \ell + j + \ell'} \sum_{\lambda} \langle j'l' | Y_{\lambda} | j' \rangle \left\{ J \begin{array}{c} I \end{array} J \begin{array}{c} I' \end{array} \right\} \times \alpha_S^{NA} \rho_{\lambda, S'}^{I_n, I_{n'}}(r), \tag{12}
\]

where \( \rho_{\lambda, V'}^{I_n, I_{n'}}(r) \) and \( \rho_{\lambda, S'}^{I_n, I_{n'}}(r) \) are the reduced vector and scalar transition densities defined, respectively, as

\[
\rho_{\lambda, V'}^{I_n, I_{n'}}(r) = \langle \Phi_I_{n'} || \sum_{i=1}^{A_c} \delta(r - r_{Ni}) Y_{\lambda}(\hat{r}_{Ni}) || \Phi_{I_n'} \rangle, \tag{13a}
\]

\[
\rho_{\lambda, S'}^{I_n, I_{n'}}(r) = \langle \Phi_I_{n'} || \sum_{i=1}^{A_c} \delta(r - r_{Ni}) Y_{\lambda}(\hat{r}_{Ni}) || \Phi_{I_n'} \rangle. \tag{13b}
\]

between the nuclear initial state \( I_{n'} \) and the final state \( I_n \). The detailed expression for the transition densities in the non-relativistic multi-reference DFT framework has been derived in Ref. [30] by one of the present authors. The formalism has also been generalized to the relativistic case within a multi-reference CDFT (MR-CDFT) to study a “bubble” structure in light nuclei [31, 32]. In this work, we extend this formalism to study low-lying states of hypernuclei.

With the radial wave function \( \mathcal{R}_k(r) \) in the coupled-channels equations (9a) and (9b), one can compute the probability \( P_k \) of the channel \( k \) in the total wave function \( \Psi_{JM} \) as,

\[
P_k = \int r^2 dr |\mathcal{R}_k(r)|^2 = \int r^2 dr \left[ |f_k(r)|^2 + |g_k(r)|^2 \right]. \tag{14}
\]

The wave function is normalized as \( \sum_k P_k = 1 \).
B. Projected potential energy surface

In order to apply the formalism presented in the previous subsection, one has to specify the core states \( \Phi_{I_n} \). A simple choice for this is to construct them as the projected mean-field states with the intrinsic deformation \( \beta \)

\[
|\Phi_{IM_1}(\beta)\rangle = \hat{P}^N_{MI,K} \hat{P}^Z_{M_1,K} |\varphi(\beta)\rangle,
\]

where the particle number projector \( \hat{P}^N_{\tau} \) has the form,

\[
\hat{P}^N_{\tau} = \frac{1}{2\pi} \int_0^{2\pi} d\varphi_{\tau} e^{i\varphi_{\tau}(N_{\tau} - N_{\tau})}, \quad (\tau = n, p)
\]

and the operator \( \hat{P}^I_{MI,K} \) is the three-dimensional angular momentum projection operator given by

\[
\hat{P}^I_{MI,K} = \frac{2I + 1}{8\pi^2} \int d\Omega D^*_M(I,K)(\Omega) \hat{R}(\Omega).
\]

Here, \( \Omega \) represents a set of Euler angles \((\phi, \theta, \psi)\), and the measure is \( d\Omega = d\phi \sin \theta d\theta d\psi. \) \( \hat{R}(\Omega) \) and \( D^*_M(I,K)(\Omega) \) are the rotation operator and the Wigner D-function, respectively [33]. In Eq. (15), the wave function \(|\varphi(\beta)\rangle\) is a Slater determinant of quasi-particle states with quadrupole deformation \( \beta \) generated with the constrained relativistic mean-field (RMF) calculation. For simplicity, in this paper we consider only the axial deformation for the nuclear core and thus the \( K \) quantum number is zero in Eq. (15).

In this approach, the hypernuclear states \( \Psi_{JM} \) are given for each deformation \( \beta \). This allows one to construct the projected energy surface for hypernuclei. That is, one can define the total energy \( E_J(\beta) \) by taking \( \langle \Psi_{JM}|\hat{H}|\Psi_{JM}\rangle \) at each deformation \( \beta \) and for each spin-parity, \( J^+ \). Notice that the coupled-channels equations (9a) and (9b) are solved at each deformation, and thus the effect of core excitations is taken into account in the projected energy surface so obtained.

C. Multi-reference covariant density functional for nuclear core states and transition densities

One can improve the calculations in the previous subsection by using \( \Phi_{I_n} \) from the MR-CDF calculation in the context of generator coordinate method (GCM) [34–36], that is,

\[
|\Phi_{I_nM_1}\rangle = \sum_{\beta} F^I_{\beta}(\beta) \hat{P}^I_{MI,K} \hat{P}^N_{M_1,K} \hat{P}^Z |\varphi(\beta)\rangle
\]

In this wave function, a set of Slater determinants with different quadrupole deformation \( \beta \) is superposed according to the idea of GCM. The weight function \( F^I_{\beta}(\beta) \) in the wave function (18) is determined by requiring that the energy expectation value is stationary with respect to an arbitrary variation of \( F^I_{n}(\beta) \), which leads to the Hill-Wheeler-Griffin (HWG) equation [37].

\[
\sum_{\beta'} \left[ \mathcal{H}^I(\beta, \beta') - E_{II} \mathcal{A}^I(\beta, \beta') \right] F_{\beta}(\beta') = 0.
\]

Here, \( \mathcal{A}^I(\beta, \beta') = \langle \varphi(\beta)|\hat{P}^I_{M_1,K} \hat{P}^N \hat{P}^Z|\varphi(\beta')\rangle \) and \( \mathcal{H}^I(\beta, \beta') = \langle \varphi(\beta)|\hat{H}|\hat{P}^I_{M_1,K} \hat{P}^N \hat{P}^Z|\varphi(\beta')\rangle \) are the norm and the energy kernels, respectively. In the calculations, the energy overlap in the energy kernel is taken to be the same functional form as in the nuclear mean-field energy, but replacing the densities and currents with mixed ones, that is, off-diagonal components of the density and current matrices [34–36].

Since the projected mean-field states do not form an orthogonal basis and the weights \( F^I_{\beta}(\beta) \) in Eq. (18) are not orthogonal functions, it is convenient to construct a set of orthonormal collective wave functions \( g^I_{\beta} \) as [38]

\[
g^I_{\beta}(\beta) = \sum_{\beta'} \left[ \mathcal{A}^I \right]^{1/2}(\beta, \beta') F^I_{\beta}(\beta').
\]

Notice that the modulus square of \( g^I_{\beta}(\beta) \) does not represent the probability to find the deformation \( \beta \) in the GCM state. For the axial symmetric case, however, \( g^I_{\beta}(\beta) \) provides a good indication about the dominant configuration in the collective states.

With the GCM wave functions for \( |\Phi_{I_n}\rangle \), one can calculate the reduced vector transition density in Eq. (13a) as follows [30],

\[
\rho^I_{\lambda \nu J'}(r) = (-1)^{J - J'} \frac{2}{I}\sum_{\beta, \beta'} |F^I_{\beta}(\beta) F^I_{\beta'}(\beta')\rangle \\
\times \sum_{K} \langle J0\lambda\nu|I'0\rangle \\
\times \int d\vec{r} Y_{\lambda\nu}(\hat{r}) \langle \varphi(\beta)|\hat{\rho}_{\nu}(\vec{r}) \hat{P}^I_{M_1,K} \hat{P}^N \hat{P}^Z|\varphi(\beta')\rangle,
\]

(21)

where the notation \( \hat{I} = \sqrt{2I + 1} \) is introduced for simplicity, and the vector density operator is defined as follows

\[
\hat{\rho}_{\nu}(\vec{r}) = \sum_{i=1}^{A_F} \delta(\vec{r} - \vec{r}_{Ni}).
\]

The scalar reduced density in Eq. (13b) can also be expressed in a similar way.

D. Electric quadrupole transition strengths between hypernuclear states

The electric quadrupole \((E2)\) transition strength from an initial state \( |J_i\rangle \) to a final state \( |J_f\rangle \) in \( \Lambda \) hypernuclei is defined as

\[
B(E2; J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \left| \langle J_f||\hat{Q}_z||J_i\rangle \right|^2.
\]

(23)
Here, the $E2$ operator reads $\hat{Q}_{2\mu} = \sum_{i \in p} r_i^2 Y_{2\mu}(\vec{r}_i)$. Substituting the wave function for the hypernuclear states, Eq. (1), to this equation, one finds the reduced matrix element to be

$$\langle J_f | \hat{Q}_2 | J_i \rangle = \sum_{k_i,k_f} \int dr r^2 \mathcal{R}_{k_f}(r) \mathcal{R}_{k_i}(r) \langle \mathcal{F}_{k_f} | \hat{Q}_2 | \mathcal{F}_{k_i} \rangle,$$

(24)

with

$$\langle \mathcal{F}_{k_f} | \hat{Q}_2 | \mathcal{F}_{k_i} \rangle = \delta_{j_j,j_f} \delta_{\ell_{j_j},\ell_{j_f}} (-1)^{I_f + j_{i} + j_{f}} \langle I_f | J_{i} J_{f} \rangle \langle I_i | I_f \rangle \langle \nu_{i} | \nu_{f} \rangle,$$

(25)

(see Eq. (7.1.8) in Ref. [33]). Here, $\langle \nu_{i} | \nu_{f} \rangle$ is the reduced $E2$ transition matrix element between the nuclear core states $| I_f, \nu_f \rangle$ and $| I_i, \nu_i \rangle$ and it is related to the proton vector transition density in Eq. (21) as

$$\langle \nu_{i} | \nu_{f} \rangle = \hat{P}_{2} \int drr^4 \rho_{2, V}^{I_{i}, I_{f}}(r).$$

(26)

III. RESULTS AND DISCUSSIONS

We now numerically solve the coupled-channels equations and discuss low-lying spectrum of hypernuclei. The procedure of the calculations and the numerical details are listed as follows.

(i) Self-consistent deformation constrained RMF+BCS calculation for the nuclear core states: This step is to generate a set of deformed states $| \varphi(\beta) \rangle$ with different quadrupole deformation $\beta$. The Dirac equation for nucleons is solved with the basis of a three-dimensional harmonic oscillator (3DHO) with $N_{sh} = 10$ major shells. The oscillator length parameter in the 3DHO is chosen as $b_x = b_y = b_z = \sqrt{\hbar/m\omega_0}$, where $m$ is the nucleon mass and the oscillator frequency is determined to be $\hbar\omega_0 = 41 A^{-1/3}$ MeV. In the most of calculations shown below, we employ the non-linear point-coupling EDF with the PC-F1 [29] set for the particle-hole channel, although we also use the PC-PK1 set [30] in order to study the parameter set dependence. In these energy density functionals, a density independent $\delta$ force is used for the particle-particle channel, supplemented with an energy-dependent cutoff for the pairing active space [40].

(ii) MR-CDFT calculation for the low-lying states of nuclear core: This step is to obtain the wave functions $\Phi_{I_n}$ for the core state $I_n$. The mean-field wave functions are projected onto good particle numbers $(N, Z)$ and angular momentum $I$, which form a set of non-orthogonal basis. The Gauss-Legendre quadrature is used for integrals over the Euler angle $\theta$ in the calculations of the norm and Hamiltonian kernels. For the $^{13}_{\Lambda}$C nucleus, the number of mesh points in the interval $[0, \pi]$ for $\theta$ is chosen to be 14. The number of gauge angle $\varphi$ for the particle number projection is chosen to be 7. For $^{20}_{\text{Ne}}$ and $^{154}_{\text{Sm}}$, we use the mesh points of 16 and 9 for the angular momentum and the particle number projections, respectively. The energy and wave function of the core state $I_n$ are determined by solving the HWG equation, Eq. (19) [34-38]. With the wave functions of nuclear core states, one can calculate the transition densities which are used to determine the coupling potentials in the coupled-channels equations.

(iii) Coupled-channels calculation for the low-lying states of $\Lambda$ hypernuclei: With the coupling potentials so obtained, the coupled-channels equations are solved by expanding the radial wave function $\mathcal{R}_{I, \nu}(r)$ on the basis of eigenfunctions of a spherical harmonic oscillator with 18 major shells. From the solutions of the coupled-channels equations, we construct the spectrum of hypernucleus and calculate the $B(E2)$ transition strengths.

A. Application to $^{13}_{\Lambda}$C

1. Properties of the core states

We first apply the method to $^{13}_{\Lambda}$C. Figure 2 shows the energy curves for the core nucleus, $^{12}_{\text{C}}$. The dotted line is obtained in the mean-field approximation, while the other lines show the projected energy surface for each angular momentum $I$. The mean-field energy curve exhibits a pronounced minimum at the spherical configuration with a steep rise with deformation $\beta$, as expected for a nucleus with large neutron and proton shell gaps. The energy gained from restoration of rotational symmetry increases with deformation $\beta$ and together with the particle number projection the location of energy minimum is shifted on the curve. The minimum of the energy curve with $0^+$ is found on the oblate side with $\beta = -0.3$. Besides, the second minimum appears around $\beta = 2.4$ in the mean-field energy curve, which is shifted to $\beta = 2.7$ in the projected energy curve for $I^* = 0^+$. It has been shown that the configuration for this minimum has a 3$\alpha$ linear-chain structure [41].

After mixing the projected mean-field configurations, one obtains the energies of low-lying states (see the filled squares in Fig. 2). The wave function of these states is displayed in Fig. 3 for the lowest three states with $I = 0, 2, 4$, and 6. The ground state of $^{12}_{\text{C}}$ is dominated by the spherical configuration. The collective wave functions and the energy spectrum indicate that there is a coexistence of an anharmonic spherical vibrator and an oblate deformed band at low excitation energies of $^{12}_{\text{C}}$. Both structures are not pure and distorted by their strong mixing. The high-lying $2^+_1$, $2^+_3$ and $4^+_1$ states seem to form a rotational band dominated by the 3$\alpha$-linear configuration, in which the collective wave functions are much extended to a large deformation region. Similar rotational band corresponding to a 4$\alpha$-linear configuration
FIG. 2: (Color online) The energy curve of the mean-field state and the particle-number and angular momentum projected \( \{ I^m \} = 0^+, 2^+, \) and \( 4^+ \) states for \( ^{12}\text{C} \) as a function of the intrinsic quadrupole deformation \( \beta \). The filled squares indicate the lowest three GCM solutions for each \( I^m \), which are plotted at their average deformation \( \bar{\beta} \equiv \sum_{\beta} |g^i_{\beta} (\beta)|^2 \beta \), where \( g^i_{\beta} (\beta) \) is the collective wave functions defined by Eq. (20). The results are obtained with the PC-F1 force.

FIG. 3: (Color online) The collective wave functions \( g^i_{\beta} \) given by Eq. (20), for the first three states in \( ^{12}\text{C} \) with spin-parity of \( 0^+, 2^+, 4^+ \) and \( 6^+ \).

The convolution of the proton vector transition densities in Fig. 4 with a Gaussian form factor for a finite proton size yields the charge transition densities, \( \rho_{L,ch}^{I^m I^{m'} \lambda} (r) \), which are related to the form factor \( F_L(q) \) for electron scattering with an angular momentum transfer \( L \) by the following relation [30],

\[
F_L(q) = \frac{\sqrt{4\pi}}{Z} \int_0^\infty d\rho r^2 \rho_{L,ch}^{I^m I^{m'} \lambda} (r) j_L(qr),
\]

where \( j_L(qr) \) is the spherical Bessel function. The coefficient \( \sqrt{4\pi}/Z \) is chosen so that the elastic part of the form factor \( F_0(q) \) is unity at \( q = 0 \), as in the experimental data. A comparison of our results with the data is shown in Fig. 5. One can see that the form factors \( F_L(q) \) are in rather good agreement with the data except for the underestimation of the elastic form factor after the first minimum, as was found also in the recent studies for \( ^{12}\text{C} \) [43] and \( ^{24}\text{Mg} \) [30] based on the Skyrme forces. This may be because the spreading of the collective wave function in quadrupole deformation space is somewhat overestimated in the calculations, decreasing the weights of the large-\( q \) components of the transition density [30, 43]. In fact, the charge radius of \( ^{12}\text{C} \) by the present GCM calculation is 2.57 fm, which is larger than the empirical value of 2.47 fm [44].

Figure 6 shows the charge form factors for the interband transitions between the two bands with \( n = 1 \) and \( n = 2 \) in \( ^{12}\text{C} \). Since the \( 0^+_2 \) state is the Holye state with dilute \( 3\alpha \) structure, which is beyond the model space of the present calculation, the inelastic form factor \( F_0(q) \) corresponding to the transition from the \( 0^+_1 \) to the \( 0^+_2 \) states is significantly underestimated in the high-\( q \) region beyond the first minimum. It is worthwhile to mention that the calculated electric monopole transition matrix element \( |M(E0 : 0^+_2 \rightarrow 0^+_1)| = 4.1 \text{ e fm}^2 \) and the charge radius of \( 0^+_2 \) (2.73 fm) are in good agreement with the results (4.5 \pm 0.2 \text{ e fm}^2 and 2.73 \pm 0.02 \text{ fm}, respectively) of the recent configuration mixing calculation based on a Skyrme force [43]. These values should be compared with the data \( |M(E0 : 0^+_2 \rightarrow 0^+_1)| = 5.4(2) \text{ e fm}^2 \) and the charge radius from other calculations for the Holye state, such as 3.27 fm by the antisymmetrized molecular dynamics [45], 3.38 fm by the fermionic molecular dynamics [46] and 3.83 fm by the alpha-condensation model [47].

The low-lying spectrum of \( ^{12}\text{C} \) with MR-CDFT calculation is shown in Figure 7, in which the results are compared with the experiment data [48, 49] as well as with other model calculations[43, 45, 46]. One can see that the low-lying spectrum is reproduced rather well by the present calculation, although the excitation energies are systematically overestimated. The electric monopole transition matrix element \( |M(E0)| \) and the quadrupole transition strength \( B(E2) \) are also in good agreement with the data and the other model calculations.
show the resultant en-
energy as a function of

The core states obtained in the previous subsection are used as inputs for the coupled-channels calculations for the hypernucleus $^{13}\Lambda C$. To this end, the parameters $\alpha_{\Lambda N}^S$ and $\alpha_{\Lambda N}^V$ in the $N\Lambda$ interaction, Eqs. (6) and (7), are fitted with the microscopic particle-rotor model to the experimental $\Lambda$ binding energy of $^{13}\Lambda C$, that is, $B_{\Lambda}\text{(exp.)} = 11.38 \pm 0.05$ MeV [1]. Figure 8(a) shows a contour plot of the absolute value of the difference between the theoretical and the experimental hyperon binding energies as a function of $\alpha_{\Lambda N}^S$ and $\alpha_{\Lambda N}^V$. This is obtained by including in Eq. (1) the core states up to $n_{\text{cut}} = 2$ and $I_{\text{cut}} = 4$. Obviously, the two strength parameters cannot be uniquely determined by fitting only to $B_{\Lambda}$ and are linearly correlated as illustrated in Fig.8(a).

Taking a few sets of the parameters along the valley with $B_{\Lambda}^{\text{th}} = B_{\Lambda}^{\text{exp.}}$ in Fig. 8(a), we calculate the energy of each of the low-lying excited states of $3/2^+$, $3/2^-$ and $1/2^-$ in $^{13}\Lambda C$ (see Fig.8(b)). One can see that the excitation energies of $3/2^+$, $3/2^-$ depend on the choice of the parameters only weakly. The energy of $1/2^-$ state slightly decreases with the decrease of the absolute value of the coupling strengths. For all the sets of the parameters ($\alpha_{\Lambda N}^S$, $\alpha_{\Lambda N}^V$) in the region of concerned, the energy splitting between the first $1/2^-$ and $3/2^-$ states is in agreement with the data $152 \pm 54(\text{stat}) \pm 36(\text{syst})$ keV [53, 54]. Therefore, as one of the choices, we first fix the value of $\alpha_{\Lambda N}^S$ to be $-4.2377 \times 10^{-5}$ MeV$^{-2}$, which is the same value as in the PCY-S2 set [28], and determine $\alpha_{\Lambda N}^V$ to be $1.969 \times 10^{-5}$ MeV$^{-2}$. With this parameter set for the $\Lambda N$ interaction, the energy splitting between the $1/2^-$ and $3/2^-$ states is 198.9 keV.

### 3. Projected potential energy surface

With the $N\Lambda$ interaction determined in the previous subsection, let us first investigate the projected potential energy surface for $^{13}\Lambda C$. Figure 9 shows the resultant energy $E_j(\beta)$ for the $J^\pi = 1/2^+$, $1/2^-$ and $3/2^-$ states in $^{13}\Lambda C$ as a function of the deformation $\beta$ of the core nucleus. These are obtained by solving the coupled-channels equations for each $\beta$. For comparison, the figure also shows the potential energy curve in the single-channel calculations without taking into account the core excitations (the dot-dashed lines) as well as those for the core nucleus (the dashed lines). The energy surfaces obtained with the coupled-channel calculations are systematically lower than that with the single-channel calculation due to the additional configuration mixing effect. One can see that the hypernuclear energy curve with spin-parity of $1/2^+$ has an oblate minimum with $|\beta|$ significantly smaller than that of $^{12}C$ with $0^+$, indicating a smaller
root-mean-square charge radius of with the available data [46].

FIG. 6: (Color online) The charge form factors between the transition from the ground state to (a) the ground state, (b) the excited $2^+_1$ state, and (c) the excited $4^+_1$ state in $^{12}$C calculated with the GCM method with the PC-F1 force, in comparison with the available data [50, 51]. $R$ on the top panel is the root-mean-square charge radius of $^{12}$C.

FIG. 5: (Color online) The charge form factor for the transition from the ground state to (a) the ground state, (b) the $0^+_1 \rightarrow 2^+_1$ transition, and (c) the $0^+_1 \rightarrow 4^+_1$ transition in $^{12}$C calculated with the GCM method with the PC-F1 force, in comparison with the available data from Refs. [50, 51].

FIG. 7: (Color online) The spectrum of $^{12}$C obtained with several methods, that is, the GCM with the PC-F1 interaction (the present calculation), GCM with the Skyrme SLy4 force [43], anti-symmetrized molecular dynamics (AMD) [45] and the alpha cluster model [46]. The excitation energies are given in units of MeV. The solid and the dashed arrows are the quadrupole transition strength $B(E2)$ ($e^2$ fm$^4$) and the electric monopole transition matrix element $|M(E0)|$ ($e$ fm$^2$), respectively. The experimental data are taken from Refs. [48, 49].

FIG. 8: (Color online) (a) A contour plot of the absolute value of the difference between the theoretical and the experimental hyperon binding energies of $^{13}$C hypernucleus as a function of the coupling strength parameters $\alpha_S^{\Lambda}$ and $\alpha_V^{\Lambda}$ in the NA interaction. The calculated energies are obtained with the microscopic particle-rotor model. (b) Energy levels of the $3/2^+$, $3/2^-$ and $1/2^-$ states in $^{13}$C calculated with the strength parameters denoted by the dots in the panel (a).

collectivity. On the other hand, for $J^\pi = 1/2^-$, the deformation around the oblate minimum is similar to that of $^{12}$C, but with a higher barrier at the spherical shape. This leads to a smaller effect of shape mixing between the prolate and oblate configurations and thus a larger average deformation in $^{13}$C. The main component of the $1/2^+$ and $1/2^-$ hypernuclear states are the $\Lambda$ particle in $s_{1/2}$ and $p_{1/2}$ orbits coupled to the ground state of $^{12}$C, respectively. It implies that a $\Lambda$ particle in the $s$ ($p$) orbit decreases (increases) the collectivity of $^{12}$C, which is consistent with the findings in the recent studies [14, 23].
hand, a spectrum is more complex for the case of Λ particle in the $p_{3/2}$ orbital. In this case, the multiplet states with $J \in \{ |I - j|, (I + j) \}$ are ordered according to the properties of the coupling potential in Eq. (28).

For instance, in the case of $\Lambda_{p_{3/2}} \otimes \beta_1^n$, the multiplets are ordered as $5/2^-, 3/2^-, 7/2^-$ and $1/2^-$ (see the column (g) in Fig. 10). In order to understand this, we write the coupling potentials for the configuration $k$ as

$$U^{kk}_{im} = (-1)^{I + I_j} \sum_{\lambda} \langle j\ell|Y_\lambda|j\ell \rangle \left\{ \begin{array}{ccc} j & I & j \\ \lambda & I_j & \lambda \\ j & \ell & j \end{array} \right\} \alpha^N_{\lambda m} \rho^I_{\lambda m},$$

where the indices $m = S$ and $V$ represent the scalar and vector potentials, respectively. The coefficients $C_n$ ($n = 0, 1, 2$) are defined as $C_0 \equiv (-1)^{J + I_j}$,

$$C_1 \equiv \langle j\ell|Y_\lambda|j\ell \rangle = (-1)^{I + I_j} \sqrt{\frac{2}{j+1}} \left( \begin{array}{ccc} j & \lambda & j \\ 1/2 & 0 & -1/2 \end{array} \right) \delta_{\lambda, even},$$

and

$$C_{2\lambda} \equiv \left\{ \begin{array}{ccc} J & I & j \\ \lambda & I_j & \lambda \end{array} \right\}.$$

Table I lists the value of each of the coefficients $C_n$. The transition densities $\alpha^N_{\lambda m} \rho^I_{\lambda m}(r)$ and the potential $U^{kk}_V(r) + U^{kk}_S(r)$ with $k = (j, \ell, I_n) = (\frac{3}{2}, \frac{1}{2}, 2)$ are displayed in Fig. 11 as a function of radial coordinate $r$. It is seen that the potential $U^{kk}_V(r) + U^{kk}_S(r)$ becomes gradually deeper in the order of $J^\pi = 1/2^-, 7/2^-, 3/2^-, 5/2^-$, which is consistent with the distribution of energy levels of these multiplets. It is seen in Table I that the product $C_0 C_1 C_{2\lambda}$ is the same among the multiplets for $\lambda = 0$, and thus the origin for the energy difference among these four hypernuclear states is the non-zero $\lambda = 2$ term in the potential (30). That is, the splitting of $J^\pi = 1/2^-, 7/2^-, 3/2^-$, and $5/2^-$ hypernuclear states is originated from the non-zero transition density $\rho^I_{\lambda m}(r)$ (cf. Fig. 11(a)) due to the reorientation effect (that is, the transition between the same state) of $2^+_1$ state in the deformed shape of $^{12}$C. Since $\alpha^N_{\lambda m} \rho^I_{\lambda m}(r)$ is negative as shown in Fig. 11(a), the potential is most attractive for $J^\pi = 5/2^-$, which has a positive $C_0 C_1 C_{2\lambda}$.

For the configuration with $\Lambda$ in $p_{1/2}$ orbital coupled to the nuclear core $2^+_1$ state, the resultant doublet states $3/2^-$ and $5/2^-$ are degenerate in energy, since the coefficient $C = C_0 C_1 C_{2\lambda}$ is not zero only for $\lambda = 0$, having the same value of $1/\sqrt{20\pi}$ between those two states (see Table I).

5. Coupled-channels calculations

Let us now solve the coupled-channels equations for $^{13}_\Lambda C$. To this end, we first examine the convergence feature of the excitation energies with respect to the cutoff
shows that the low-lying spectra for $^{13}\Lambda$C with the $\Lambda$ particle in the $s_{1/2}$, $p_{1/2}$ and $p_{3/2}$ orbitals are also plotted in the columns (d), (e), (f), and (g).

FIG. 10: (Color online) The low-energy excitation spectra of $^{12}$C ((a)-(c)) and $^{13}$C ((d)-(l)). For $^{13}$C, the full GCM calculations are compared with the experimental data taken from Ref. [52]. The columns (h) and (i) show the positive-parity states in $^{13}$C, while the columns (j) and (k) show the negative-parity states. The experimental data of $^{13}$C are taken from Refs. [53, 54]. For comparison, the results of single-channel calculation for $^{13}$C with the $\Lambda$ particle in the $s_{1/2}$, $p_{1/2}$ and $p_{3/2}$ orbitals are also plotted in the columns (d), (e), (f), and (g).

TABLE I: The coefficients in the potential for the $[\Lambda_{ij}\otimes 2^+](J^p)$ configurations (see Eqs. (31) and (32)).

| $[\Lambda_{ij}\otimes 2^+]$ | $J^p$ | $C_0$ | $C_{20}$ | $C_{12}C_{20}$ | $C_{22}$ | $C_{0}C_{10}C_{20}$ |
|--------------------------|--------|-------|----------|----------------|---------|---------------------|
| $[\Lambda_{p3/2}\otimes 2^+]$ | 1/2$^-$ | 1.00 | $\sqrt{30}$ | $\sqrt{30}$ | $\sqrt{14/5}$ | $\sqrt{20}$ |
| $[\Lambda_{p3/2}\otimes 2^+]$ | 3/2$^-$ | $\sqrt{20}$ | $\sqrt{20}$ | 0.00 | 0.00 | $\sqrt{20}$ |
| $[\Lambda_{p3/2}\otimes 2^+]$ | 5/2$^-$ | 1.00 | $\sqrt{20}$ | $\sqrt{20}$ | $\sqrt{30}$ | $\sqrt{30}$ |
| $[\Lambda_{p3/2}\otimes 2^+]$ | 7/2$^-$ | $\sqrt{20}$ | $\sqrt{20}$ | 0.00 | 0.00 | $\sqrt{30}$ |
| $[\Lambda_{p3/2}\otimes 2^+]$ | 3/2$^-$ | $\sqrt{20}$ | $\sqrt{20}$ | 0.00 | 0.00 | $\sqrt{20}$ |
| $[\Lambda_{p3/2}\otimes 2^+]$ | 5/2$^-$ | $\sqrt{20}$ | $\sqrt{20}$ | 0.00 | 0.00 | $\sqrt{20}$ |

of core states $n_{\text{cut}}$ and the core angular momentum $I_{\text{cut}}$. Fig. 12 shows that $n_{\text{cut}} = 2$ and $I_{\text{cut}} = 4$ yield a good convergence for the low-lying excited states, and we use these cut-offs in the calculations presented below.

The columns (h), (i), (j), and (k) in Figure 10 show the calculated low-energy excitation spectra of $^{13}$C, in comparison with the corresponding data. One can see that the low-lying spectra for $^{13}$C are reproduced rather well, although the excitation energies are slightly overestimated.

In the coupled-channels calculation, the doublets $(5/2^+, 3/2^+)$ and $(9/2^+, 7/2^+)$ in the column (h) mainly consist of the configuration of $\Lambda s_{1/2}\otimes 2^+_1$ and $\Lambda s_{1/2}\otimes 4^+_1$, respectively. See Tab. II for the probabilities for the dominant components in each state. These doublets are degenerate in the single-channel calculation, as already shown in the column (d) in Fig. 10. The states of $3/2^+$ and $5/2^+$ are different from each other by 10 keV due to the weak mixing of other configurations. The levels in the column (i) correspond to the configuration of $\Lambda s_{1/2}$ coupled to the second band ($n = 2$) in $^{12}$C. These levels share similar features as those in the column (h).

FIG. 11: (a) The transition densities $\alpha_{\Lambda}^{2\lambda}\rho_{\Lambda\lambda} + \alpha_{S}^{2\lambda}\rho_{S\lambda}^{2\lambda}$ ($\lambda = 0$ and $\lambda = 2$) and (b) the potential $U^{\Lambda}_{kk}(r) + U^{S}_{kk}(r)$ in Eq. (28) for the hypernuclear states $J = 1^{2-}, 3^{2-}, 5^{2-}, 7^{2-}$ with the $\Lambda_{p3/2}\otimes 2^+_1$ configuration as a function of the radial coordinate $r$.

The negative-parity states are shown in the columns (j) and (k) in Fig. 10. The energy splitting between the $3/2^-$
shows the calculated hyperon in the single-channel calculation the energy difference between 1/2− and 5/2− states is as small as 199 keV. Notice that in the single-channel calculation the energy difference between the pure configurations of Λp3/2 ⊗ 0^+ and Λp1/2 ⊗ 0^+ is 180 keV. That is, the energy splitting of 3/2− and 1/2− states reflects mainly the spin-orbit splitting of Λ hyperon in the p_{3/2} and p_{1/2} states. A small splitting between the 3/2− and 1/2− states has been shown also in our previous calculation for ^9Be [27], although it does not reflect the Lambda spin-orbit splitting because of a strong mixing between the Λp_{1/2} ⊗ 0^+ and the Λp_{3/2} ⊗ 2^+ configurations in the 1/2− state.

For the second 1/2− and 3/2− states, one can see a large configuration mixing (see Table II). This is because there are two states with unperturbed energy in the single-channel calculations, E_{I_{ch}}^{(0)} is close to one another. These two states are strongly coupled due to the off-diagonal components of the coupling potentials in the coupled-channels equations. Notice that, in ^9Be, this happens already in the first 1/2− state [27], because the reorientation effect discussed in the previous subsection brings the Λp_{3/2} ⊗ 2^+ configuration close to the Λp_{1/2} ⊗ 0^+ configuration in energy due to the prolate nature of the 2^+ state of ^8Be.

According to our calculation, the experimentally observed level at excitation energy of 11.8 MeV has the spin-parity of 1/2^+, dominated by the configuration Λ1/2 ⊗ 0^+_2 (c.f. Fig. 10(c)) or the first radial excitation state of the configuration Λ1/2 ⊗ 0^+_1 (c.f. Fig. 10(d)).

Figure 13 shows a comparison of low-energy excitation spectra of ^\Lambda^4He obtained with the present microscopic particle-rotor model (MPRM), the multi-channel algebraic scattering (MCAS) approach [55], and the 3α + Λ cluster model [56], together with the experimental data [1]. The basic idea of MCAS approach for Λ hypernuclei[55] is similar to the microscopic PRM model in a sense that the hypernuclear wave function is given by the Λ hyperon coupled to the low-lying states of nuclear core. In contrast to our full microscopic models, in which all the inputs are from the multi-reference CDFT calculation, the MCAS approach adopts the experimental data for the energies of nuclear core states with an assumption of a pure collective rotational states and a phenomenological deformed Woods-Saxon potential for the coupling potentials. In contrast to the MCAS approach and cluster model calculation, the ordering of the first degenerate 3/2^+ and 5/2^+ states are not reproduced in the microscopic PRM calculation, since we do not include a spin-spin interaction in our calculation. Except for this, the ordering of low-lying states and the structure of spectrum are the same between the microscopic PRM and the MCAS approach. The main components of each state obtained with the microscopic PRM calculation are similar to those in the cluster model calculation.

Table III shows the calculated E2 transition strengths for low-lying positive parity states of the hypernucleus and the corresponding core nucleus. In order to remove the trivial factor due to the angular momentum coupling for s_{1/2} for the Λ particle and see more clearly the impulsivity effect of Λ particle on nuclear collectivity, we define the cB(E2) value (that is, the B(E2) value for the core
TABLE III: The calculated $E_1(2)$ transition strengths (in units of $e^6$ fm$^3$) for low-lying positive parity states of $^{12}$C and $^{13}$C. The $cB(E2)$ values are calculated according to Eq. (33). The changes in the $B(E2)$ is indicated with the quantity defined by $\Delta \equiv (cB(E2) - B(E2; ^{12}$C))/B(E2; $^{12}$C). The value in the parenthesis for $^{13}$C is the experimental data taken from Ref. [48].

| $I^+_2 \rightarrow I^+_1$ | $B(E2)$ | $J^+_2 \rightarrow J^+_1$ | $I^+_2 \rightarrow I^+_1$ | $B(E2)$ | $cB(E2)$ | $\Delta$ (%) |
|--------------------------|---------|--------------------------|--------------------------|---------|----------|--------------|
| $2_1^+ \rightarrow 0_1^+$ | 6.62    | $3/2^- \rightarrow 1/2^+$ | 6.58                    | 5.68    | 5.68     | -14.17      |
| (7.6 ± 0.4)               |         | $5/2^- \rightarrow 1/2^+$ | 5.68                    | 5.68    | 5.68     | -14.17      |
| $4_1^+ \rightarrow 2_1^+$ | 14.60   | $3/2^- \rightarrow 3/2^+$ | 10.34                   | 11.48   | 11.48    | -21.36      |
|                          |         | $5/2^- \rightarrow 3/2^+$ | 1.15                    | 11.49   | 11.49    | -21.35      |
|                          |         | $5/2^- \rightarrow 5/2^+$ | 11.48                   | 11.48   | 11.48    | -21.36      |

part) as,

$$cB(E2 : I_i \rightarrow I_f)$$

$$\equiv I_i^{-2} J_f^{-2} \left\{ \begin{array}{c} J_i \\ J_i \end{array} \right\}^{-2} B(E2 : J_i \rightarrow J_f), (33)$$

where $j_i$ is the value for the main channel in the initial state. The impurity effect of $\Lambda$ particle can be discussed by comparing the $B(E2)$ values for the core nucleus and the $cB(E2)$ values for the corresponding hypernucleus. One can see that the $E2$ transition strength for $2_1^+ \rightarrow 0_1^+$ in $^{12}$C is significantly reduced, by a factor of $\sim 14\%$, due to the addition of a $\Lambda$ particle.

B. Low-energy spectroscopy of $^{21}_\Lambda$Ne

We next consider an application to hypernuclei in the sd-shell region. For this purpose, we discuss the $^{21}_\Lambda$Ne hypernucleus. Since the $\Lambda$ binding energy in $^{21}_\Lambda$Ne has not yet been measured, we fit the value of coupling strength parameters $\alpha^{N\Lambda}N$ and $\alpha^{N\Lambda}N$ to the $\Lambda$ binding energy estimated with a deformed relativistic mean field calculation [23]. With the PC-F1 and PCY-S1 forces for $NN$ and $N\Lambda$ interactions, respectively, $B_\Lambda$ is estimated to be 14.35 MeV for the lowest $\Lambda$ hyperon state. With the same process as in Sec. III A-2, we obtain a parameter set of $\alpha^{N\Lambda}N = -4.2377 \times 10^{-5}$ MeV$^{-2}$ and $\alpha^{N\Lambda}N = 1.6694 \times 10^{-5}$ MeV$^{-2}$.

Figure 14 shows the calculated yrast rotational states of $^{20}$Ne. In order to see the parameter set dependence for the $NN$ interaction, we use both PC-F1 and PC-PK1 parameter sets. For a comparison, the figure also shows the results of $o+^{16}$O cluster model [57] and the AMD model [14]. One sees that all these models reproduce the rotational character of the yrast states, although they tend to overestimate the moment of inertia.

Figure 15 shows the obtained energy curve $E_{\beta}(\beta)$ for the $J^\pi = 1/2^+$ and $1/2^-$ states in $^{21}_\Lambda$Ne as a function of the deformation $\beta$ of the core nucleus. The left and the right panels show the result with PC-F1 and PC-PK1 forces, respectively. For the latter, we use the same $N\Lambda$ interaction as in the former calculation, even though the parameters are determined with PC-F1. We have confirmed that this yields the $B_\Lambda$ value of 14.33 MeV with PC-PK1, which is similar to the value with the PC-F1 set, that is, 14.35 MeV. For PC-F1, the hypernuclear energy curve with spin-parity of $1/2^+$ and $1/2^-$ has a prolate minimum with a smaller $\beta$ than that of $^{20}$Ne with $0^+$. For PC-PK1, on the other hand, the value of $\beta$ at the energy minimum remains almost the same for the $1/2^+$ configuration while that for the $1/2^-$ con-
shows the spectra of both III A 4. Moreover, it is seen that the spectra of E P (d). Even though PC-F1 and PC-PK1 forces predict the energy at the absolute minimum coincides with that for (the right panel) forces. In order to make a comparison easy, these are obtained with PC-F1 (the left panel) and PC-PK1 (the right panel) forces. In order to make a comparison easy, each hypernuclear curve is shifted by a constant value so that the energy at the absolute minimum coincides with that for 20Ne with $I^+ = 0^+$. The difference between the energy curve of $^{21}\Lambda$Ne and that of 20Ne is shown in the panels (c) and (d) for the PC-F1 and PC-PK1 forces, respectively.

The configuration increases as compared to the deformation for 20Ne with 0$^+$. Notice that the energy surface for 1/2$^-$ has a higher barrier at the spherical shape than the barrier for 20Ne for both the interactions. This indicates that $^{21}\Lambda$Ne with 1/2$^+$ and 1/2$^-$ has a smaller and a larger collectivity than that of 20Ne, as in $^{13}$C. The energy differences between the 1/2$^+$ state in $^{21}\Lambda$Ne and the ground state of 20Ne, as well as the 1/2$^-$ state in $^{21}\Lambda$Ne and the ground state of 20Ne, are shown in Figs. 15 (c) and 15 (d). Even though PC-F1 and PC-PK1 forces predict somewhat different energy curves, those energy curves are qualitatively similar to each other, especially when those are plotted with respect to the energy curve for 20Ne.

Figure 16 shows the spectra of both 20Ne and $^{21}\Lambda$Ne. To examine the channel-coupling effect on hypernuclear states, we also include the results from single-channel calculations. The figure only shows the results with PC-F1, since the results with PC-PK1 are similar (see also Fig. 18 below). The probability of the main components is summarized in Table IV. It is shown that the hypernuclear states with positive parity plotted in the columns (i) and (j) from the full coupled-channels calculation are close to the results of single-channel calculation shown in the columns (e) and (f). The analysis of hypernuclear wave functions demonstrates that these states are dominated by the configuration of $\Lambda s_{1/2}$ coupled to the state $I$ in the first ($n = 1$) and the second ($n = 2$) bands in 20Ne, respectively, with the weight between 98% and 99%. It is seen that the hypernuclear doublet states $(2^{-} \frac{1}{2}, 2^{-} \frac{3}{2})^+$ with configuration $\Lambda f_{1/2} \otimes I^+$ are degenerate, as discussed in Sec. III A 4. Moreover, it is seen that the spectra of positive-parity states in $^{21}\Lambda$Ne is close to that of 20Ne with similar excitation energies to each other. In other words, the presence of a $\Lambda s_{1/2}$ does not change significantly the low-energy structure of the core nucleus 20Ne.

The negative-parity states in $^{21}\Lambda$Ne are shown in the columns (k) and (l). One can see that the channel-coupling effect plays an important role in their excitation energies. Moreover, we note that the energy difference between the first 1/2$^-$ and 3/2$^-$ states is less than 40 keV. Notice that the 1/2$^-$ state is a strong admixture of the configurations $\Lambda p_{1/2} \otimes 0^+_1$ and $\Lambda p_{3/2} \otimes 2^+$. On the other hand, the 3/2$^-$ state is a strong admixture of the configurations $\Lambda p_{3/2} \otimes 0^+_1$, $\Lambda p_{3/2} \otimes 2^+_1$ and $\Lambda p_{1/2} \otimes 2^+_1$. Therefore, the splitting of the 1/2$^-$ and 3/2$^-$ levels in $^{21}\Lambda$Ne does not reflect the strength of $\Lambda$ spin-orbit interaction, which is in marked difference from the case in $^{13}$C. From yet another point of view, it is interesting to point out that a typical rotational band having $L=1^-$, $3^-$, $5^-$, $\cdots$ is realized as seen in the column (k) of Fig. 16, apart from the spin of the hyperon. This group can be characterized by the $K = 0^-$ band based on the strong coupling between the nuclear rotation and the hyperon in the p-state, and thus this band manifests a genuinely hypernuclear state with the $[5](90)$ symmetry which is
The underlying reason for the difference between $^{13}\Lambda$C and $^{21}\Lambda$Ne is due to the different properties of the core nuclei. $^{20}$Ne is well-deformed with a much larger transition density $\rho^2(r)$ than that in $^{12}$C. Notice also that the ordering of the $\Lambda_{p3/2} \otimes 2^+_1$ multiplet states is opposite to that in $^{13}\Lambda$C, reflecting the fact that the sign of quadrupole moment is opposite (that is, prolate deformation for $^{20}$Ne and oblate deformation for $^{12}$C). In Figure 17, we plot the transition density for the $^{21}\Lambda$Ne hypernucleus. One can see that the $\lambda=2$ component has the opposite sign as compared to the transition density for $^{13}\Lambda$C shown in Fig. 11. Similar to the $^9\Lambda$Be case, these result in several $1/2^-$ and $3/2^-$ states close in energy in the single-channel calculations, which are strongly mixed in the full coupled-channels treatment. Similarly to the $1/2^-$ and $3/2^-$ states, the $5/2^-$ and $7/2^-$ states also show the strong configuration mixing between $\Lambda$-hyperon in $p_1/2$ and $p_3/2$ orbits. We have found that this feature of strong mixing found in $^9\Lambda$Be and $^{21}\Lambda$Ne persists also in heavier systems, such as $^{41}\Lambda$Si and $^{65}\Lambda$Sm.

Notice that the $^{20}$Ne nucleus has prominent negative-parity bands originated from the $\alpha+^{16}$O structure. For simplicity, in the present calculations, we have assumed reflection symmetry for $^{20}$Ne. The inclusion of these negative-parity states in the coupled-channels calculations is thus beyond the scope of the present paper. It would be an interesting future work to include them and study how the negative parity states in $^{21}\Lambda$Ne are perturbed.

Figure 18 shows a comparison of low-energy excitation spectra of $^{21}\Lambda$Ne obtained with the cluster model [57], the AMD [14], and the present microscopic PRM calculations based on the PC-F1 and PC-PK1 interactions. The positive-parity band in the microscopic PRM is closer to the result of cluster model as compared to the result of AMD, which has a slightly larger moment of inertia. The negative-parity states are similar to the AMD results but with lower excitation energies, which might be due to the large channel-coupling effect taken explicitly into account in the present work, see Fig. 16 and Table IV.

Table V lists the $E2$ transition strengths for low-lying states of $^{21}\Lambda$Ne with PC-F1. For comparison, the table also shows the change in $B(E2)$ from $^{20}$Ne to $^{21}\Lambda$Ne obtained with the PC-PK1 force, the cluster model [57], and...
the AMD [14]. The $B(E2)$ value decreases by adding a $\Lambda$ hyperon in $s$-orbit in these calculations. However, the cluster model and AMD model predict more reduction compared to the microscopic PRM. A further study would be necessary in order to reconcile this difference.

We have also applied the microscopic particle-rotor model to study another $sd$-shell hypernucleus, $^{31}_A\Lambda$Si. We have found that the impurity effect of $\Lambda$ hyperon in $^{31}_A\Lambda$Si is qualitatively the same as that in $^{21}_A\Lambda$Ne.

C. Low-energy spectroscopy of $^{155}_A\Lambda$Sm

One of the advantages of the microscopic particle-rotor model is that this method is not limited to light hypernuclei but it can also be applied to medium-heavy and heavy hypernuclei. As an example of application to heavy deformed hypernuclei, we next consider $^{154}_A\Lambda$Sm and $^{155}_A\Lambda$Sm. By fitting to $B_A=24.98$ MeV estimated with the deformed RMF calculation, we obtain a parameter set of $\Lambda N$ interaction as $\alpha_S^{\Lambda N} = -4.2377 \times 10^{-5}$ MeV$^{-2}$ and $\alpha_S^{\Lambda N} = 1.0401 \times 10^{-5}$ MeV$^{-2}$. Figure 19 shows the projected energy curves for $^{154}_A\Lambda$Sm and $^{155}_A\Lambda$Sm obtained with this $\Lambda N$ interaction together with the PC-F1 for the $NN$ interaction. For the $1/2^+$ state, the polarization effect of $\Lambda$ particle in $s$-orbit on the properties of $^{154}_A\Lambda$Sm is much smaller than that on $^{12}_C$ and $^{20}_Ne$ due to the large mass number, although the effect is still large for the negative-parity states due to the strong channel coupling effects.

Figure 20 shows the calculated low-energy spectrum of $^{154}_A\Lambda$Sm and $^{155}_A\Lambda$Sm with the PC-F1 force. The ground-state band and the two $\beta$-bands in $^{154}_A\Lambda$Sm are reasonably reproduced, although the band-head energy of the $\beta$-bands are overestimated. The low-lying positive-parity states $J^+$ in $^{155}_A\Lambda$Sm are dominated by the single-configuration of $\Lambda s_{1/2} \otimes I^+$ with similar excitation energy as that of the nuclear core state with $I^+$. As shown in Tab. VI, the positive-parity states $J^+$, except for $1/2^+$, are nearly two-fold degenerate. These characters are similar to the hypernuclei in the light-mass region. On the other hand, one can see that the negative-parity bands are well separated from the positive-parity ground band in $^{155}_A\Lambda$Sm, which is different from the light hypernuclei. It is because the energy scale of the rotational motion is proportional to $A^{-7/3}$ (see Eq. (1.50) in Ref. [38]), while that of single-$\Lambda$ excitation from $s$ to $p$ orbit is proportional to $A^{-1/3}$. Therefore, with the increase of mass number $A$, the rotational energy spectrum is compressed faster than the single-$\Lambda$ excitation spectrum. Besides, the low-lying negative-parity states $J^-$ are nearly two-fold degenerate, even though there are strong configuration-mixing in these states.

Table VII presents the $E2$ transition strengths in $^{154}_A\Lambda$Sm and $^{155}_A\Lambda$Sm. It is shown that the change in the $B(E2)$ values by adding a $\Lambda$ hyperon in $s$ orbital is less than 1%, which is much smaller than that in the light hypernuclei studied in this paper. This is consistent with the small polarization effect of $\Lambda$ particle discussed in connection to the projected energy surface shown in Fig. 19.

IV. SUMMARY

We have presented the detailed formalism of the microscopic particle rotor model based on a covariant density functional theory for the low-lying states of single-$\Lambda$ hypernuclei. In this formalism, the wave functions for hypernuclei have been constructed by coupling the $\Lambda$ hyperon to the low-lying states of the core nucleus. The radial wave functions are obtained by solving the corresponding coupled-channel equations, in which the coupling potentials are provided in terms of the transition densities of the nuclear core states. For simplicity, in this paper we have adopted only the leading-order four-
fermion coupling terms of scalar and vector types for the \(\Lambda N\) effective interaction. Applying this method to \(^{13}\text{C}\), we have reproduced reasonably well the experimental energy spectrum of this hypernucleus. We have applied this method also to \(^{21}\text{Ne}\), and \(^{155}\Lambda\text{Sm}\), and have achieved a good agreement with other model studies both for the excitation energies and the compositions of wave functions. We mention that our method is the only one which can be applied to such heavy hypernuclei. We have found

\[
\text{TABLE VI: Same as Table II, but for } ^{155}\Lambda\text{Sm hypernuclei.}
\]

| \(J^\pi\) | \(E\) | \((I j) \otimes I^\pi\) | \(P_{11}\) | \(E(0)\) |
|-----|-----|-----------------|------|------|
| \(1/2^+\) | 0.00 | \(s_{1/2} \otimes 0^+\) | 0.98 | 0.00 |
| \(3/2^+\) | 0.11 | \(s_{1/2} \otimes 2^+\) | 0.98 | 0.11 |
| \(5/2^+\) | 0.11 | \(s_{1/2} \otimes 2^+\) | 0.98 | 0.11 |
| \(7/2^+\) | 0.35 | \(s_{1/2} \otimes 4^+\) | 0.98 | 0.35 |
| \(9/2^+\) | 0.35 | \(s_{1/2} \otimes 4^+\) | 0.98 | 0.35 |
| \(1/2^-\) | 1.68 | \(s_{1/2} \otimes 0^-\) | 0.99 | 1.58 |
| \(3/2^-\) | 1.99 | \(s_{1/2} \otimes 2^-\) | 0.99 | 1.90 |
| \(5/2^-\) | 1.99 | \(s_{1/2} \otimes 2^-\) | 0.99 | 1.90 |

| \(5/2^-\) | 6.28 | \(p_{3/2} \otimes 2^-\) | 0.66 | 7.58 |
| \(p_{1/2} \otimes 0^-\) | 0.32 | 8.51 |
| \(p_{3/2} \otimes 2^-\) | 0.32 | 8.42 |
| \(p_{1/2} \otimes 2^-\) | 0.29 | 8.61 |
| \(7/2^-\) | 6.44 | \(p_{3/2} \otimes 4^-\) | 0.53 | 8.08 |
| \(p_{1/2} \otimes 4^-\) | 0.33 | 8.61 |
| \(p_{3/2} \otimes 4^-\) | 0.11 | 8.96 |
| \(p_{1/2} \otimes 4^-\) | 0.29 | 8.61 |

\[
\text{TABLE VII: Same as Table III, but for } ^{154}\text{Sm and } ^{155}\Lambda\text{Sm. The experimental data for } ^{154}\text{Sm, shown in the parenthesis, is taken from Ref. [59].}
\]

| \(^{154}\text{Sm}\) | \(I^\pi\) | \(B(E2)\) | \(J_{1/2}^\pi \rightarrow J_{1/2}^\pi \) | \(\Delta(\%)\) |
|-----|-----|-----|-------------------|-----|
| \(2_{1}^{-} \rightarrow 0_{1}^{-}\) | 9358.49 | \(3/2_{1}^{-} \rightarrow 1/2_{1}^{-}\) | 9284.69 | 9284.69 − 0.79 |
| \((8720 \pm 100)\) | | | | |
| \(4_{1}^{-} \rightarrow 2_{1}^{-}\) | 13512.18 | \(7/2_{1}^{-} \rightarrow 3/2_{1}^{-}\) | 12081.04 | 13423.38 − 0.66 |
| | | | | |

that the \(NN\) interaction with the PC-F1 and PC-PK1 sets lead to similar hypernuclei spectra to each other. For all the hypernuclei, the low-lying excited states with positive parity \(J^\pi\), except for \(1/2^+\), are nearly two-fold degenerate and dominated by the single-configuration of \(A_{1/2} \otimes I^\pi\), where \(I^\pi\) is the spin-parity of the nuclear core states. In contrast, in general there are large configuration mixing in the negative-parity states. We have, however, found an exception for this, that is, for the first \(3/2^-\) and \(1/2^-\) states in \(^{13}\text{C}\) the effect of configuration mixing is rather small, and thus the energy splitting of these states reflects the spin-orbit splitting of \(\Lambda\) hyperon in the \(p_{3/2}\) and \(p_{1/2}\) states. Concerning the electromagnetic transitions, we have found that for all the systems the \(B(E2)\) value from the first \(2^+\) to the ground states in the core nuclei is reduced by adding a \(\Lambda\) particle in the positive-parity states. The reduction factor is about 14\% for \(^{13}\text{C}\), 3.2\% for \(^{21}\text{Ne}\), and 0.7\% for \(^{155}\Lambda\text{Sm}\), and thus the reduction factor is larger for the oblate hypernuclei. For \(^{21}\text{Ne}\) and \(^{31}\text{Si}\), a slightly larger impurity effect was

\[
\text{FIG. 19: (Color online) Same as Figure 9, but for } ^{154}\text{Sm and } ^{155}\Lambda\text{Sm.}
\]

\[
\text{FIG. 20: (Color online) The low-energy excitation spectra of } ^{154}\text{Sm \(((a)-(f))\) and } ^{155}\Lambda\text{Sm \(((g)-(i))\).}
\]
found with the PC-PK1 force as compared to the PC-F1 force.

New measurements of γ-ray spectroscopy of hypernuclei will soon start at the new generation experimental facilities such as J-PARC. It would be interesting if the low-lying spectra predicted in this paper are confirmed in near future.

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