Estimation of the measurement uncertainty in the tests of electric distribution transformers filled with insulating liquid - low and high voltage winding resistance

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Abstract. Considering the lack of a method for comparison and evaluation of the tests of electric distribution transformers filled with insulating liquid, we propose a mathematical modeling of the measurement uncertainty on the resistance measurements of the low and high voltage windings. The mathematical modeling was developed using as theoretical references the Guide to Expression of Uncertainty of Measurement - ISO GUM, the ABNT NBR 5356:2007 standard and Regulatory Ordinances issued by the National Institute of Metrology, Quality and Technology - Inmetro. The models were applied to the test data of transformers as a way to assess their consistency. The measured uncertainties (k = 2) obtained from the tests for resistance measurement of the low and high voltage windings were, respectively, \( U(R_{LV}) = 2.4 \times 10^{-5} \Omega \) and \( U(R_{HV}) = 0.05 \Omega \).

1. Introduction
Studies show that 1/3 of the world's electrical energy produced is lost. 70% of this loss occurs in electric power distribution transformers. It is therefore imperative that the design of these equipments, its use and reform as well as maintenance processes are carried out with the best practices available and supported by reliable metrological processes [1].

Like any other electric transformers, electric power distribution transformers present two main sources of energy losses: copper losses and iron losses. Copper losses occur in the conductors used to manufacture the windings, whereas iron losses occur in the magnetic core [2].

Despite of known standardized methods for testing electric distribution transformers filled with insulating liquid, there is no method for comparing the results of such tests. Hence, there is no way to compare and evaluate them.

In this paper we present the mathematical modeling of the measurement uncertainty in the tests of electric distribution transformers filled with insulating liquid, regarding the resistance measurements of the low and high voltage windings. Electrical resistance is one of the factors for calculating the copper loss, and it is also used to determine the maximum temperature rise of the transformer when operating at full load.
2. Tests for winding resistance measurement
In this test the resistance of the low voltage side winding and the resistance of the high voltage side winding shall be measured.

2.1. Low voltage winding resistance measurement test
The resistance of the winding on the low voltage side is measured according to the voltage drop method, which is one of the measurement methods described in ABNT NBR 5356-1:2007 [3]. This method uses a DC voltage source, a shunt resistor and two voltmeters: one to measure the voltage drop at the low voltage winding terminals and a second voltmeter to measure the voltage drop at the resistor terminals, according to Figure 1. Due to the high current value on the low voltage side, it is convenient to use this method because it offers better measurement accuracy. ABNT NBR 5356-1:2007, Annex E - item E.2.3 requires that the current in this measurement does not exceed 15% of the nominal winding current. In this way it is necessary to use a variable resistor for the correct adjustment of the measuring current. The resistance value of the low voltage winding will be obtained by Ohm's law according to equation (1):

\[ R_{LV} = \frac{V_{LV}}{V_{Ref}} \]  

where:
- \( R_{LV} \) represents the low voltage winding resistance;
- \( V_{LV} \) represents the potential difference (voltage drop) measured at the low voltage winding taps;
- \( V_{Ref} \) represents the potential difference (voltage drop) measured at the reference shunt resistor;
- \( R \) represents the shunt resistor.

![Figure 1 – Low voltage side winding resistance measurement circuit](image)

where:
- \( R_V \) represents the current limiter variable resistor;
- \( I \) represents the current flowing through the circuit.
2.2. High voltage winding resistance measurement test

The winding resistance of the high voltage side is also measured by the voltage drop method. However, since the currents in the high voltage side are smaller, there is no need to use a shunt resistor. For the level of current to be measured, it is easy to find ammeters with good enough class of accuracy available on the market. In this case the current circulating in the circuit is measured directly using an ammeter, according to figure 2. In this way the resistance value of the high voltage winding will be obtained by equation (2):

$$ R_{HV} = \frac{V_{HV}}{I} \quad (2) $$

where:
- $R_{HV}$ represents the high voltage winding resistance, which needs to be determined;
- $V_{HV}$ represents the potential difference (voltage drop) measured at the high voltage winding taps;
- $I$ represents the current flowing through the electric circuit.

![Figure 2 – High voltage side winding resistance measurement circuit](image)

where $R_V$ represents the current limiter variable resistor.

3. Mathematical modeling of the measurement uncertainty in low voltage winding resistance measurement test

The low voltage winding resistance value is obtained by Ohm’s law, according to the simplified mathematical model described in equation (1) and according to the test circuit of figure 1.

Other effects are present in the measurement circuit. From eq. (1), these effects can be properly represented by the mathematical model of equation (3):

$$ R_{LV} = \frac{V_{LV} + rV_{LV} + TDV_{LV}}{V_{Ref} + V_{Ref} + TDV_{Ref}} \quad (3) $$

where:
- $R_{LV}$ represents the low voltage winding resistance;
- $V_{LV}$ represents the potential difference (voltage drop) measured at the low voltage winding taps;
- $rV_{LV}$ represents the correction due to the voltmeter (related to $V_{LV}$) resolution;
- $TDV_{LV}$ represents the voltmeter (related to $V_{LV}$) time drift;
- $V_{Ref}$ represents the potential difference (voltage drop) measured at the reference shunt resistor;
Starting from equation (3), following the methodology described in the Guide to Expression of Uncertainty of Measurement - ISO GUM [4], one can get mathematical modeling of the low voltage winding resistance measurement uncertainty, represented in equation (4) and detailed in Table 1:

\[
U(R_{LV}) = k_4 \times \left[ \left( \frac{u^2(V_{LV})}{k_1} + \frac{u^2(rV_{LV})}{\sqrt{12}} + \frac{u^2(TDV_{LV})}{k_2} + \frac{u^2(V_{Ref})}{\sqrt{3}} \right) + \left( \frac{u^2(TDV_{Ref})}{k_2} + \frac{u^2(R)}{k_3} + \frac{u^2(TC_{sh})}{\sqrt{12}} \right) \right]^{1/2} \tag{4}
\]

where:

\[
c_i_1 = \frac{R + TC_{sh}}{V_{Ref} + rV_{Ref} + TDV_{Ref}} \tag{5}
\]

\[
c_i_2 = - \frac{(V_{LV} + rV_{LV} + TDV_{LV}) \times (R + TC_{sh})}{(V_{Ref} + rV_{Ref} + TC_{Ref})^2} \tag{6}
\]

\[
c_i_3 = \frac{V_{LV} + rV_{LV} + TDV_{LV}}{V_{Ref} + rV_{Ref} + TDV_{Ref}} \tag{7}
\]

The standard uncertainties \((u_i)\), the combined uncertainty, \(u_c(R_{LV})\), the effective degrees of freedom \(\nu_{eff}\), the coverage factor \(k\), and the expanded uncertainty related to the low voltage winding resistance test value estimation, \(U(R_{LV})\), are calculated according to [4].
Table 1 - Low voltage winding resistance test uncertainty budget

| QUANTITY                             | SOURCE                          | INPUT ESTIMATION | PROBDIST | v_i | DIV FACTOR | c_i | STANDARD UNCERT. | COMBINED UNCERT. | EXPANDED UNCERT. |
|--------------------------------------|---------------------------------|------------------|----------|-----|------------|-----|------------------|------------------|------------------|
| V_LV calibration uncertainty         | U(V_LV) = 6.2x10^-4 V          | Normal           | 2        | k_1 | 2.00       | c_i | u(I(R)) = 4.17x10^-7 Ω | u(R) = 1.19x10^-4 Ω | u(R) = 2.4x10^-4 Ω |
| V_LV resolution                      | U(R) = 1x10^-4 V               | Rectangular      | ∞        | 12  | 3          | c_i | u(R) = 3.85x10^-5 Ω |                   |                  |
| V_LV time drift                      | U(TDV_LV) = 1.6x10^-4 V        | Rectangular      | ∞        | 3   | 1.23x10^-4 Ω |   |                  |                   |                  |
| V_nr calibration uncertainty        | U(V_nr) = 4.72x10^-2 Ω         | Normal           | 2        | k_2 | 2.00       | c_i | u(R) = 8.76x10^-5 Ω |                   |                  |
| V_nr resolution                      | U(R) = 1x10^-4 V               | Rectangular      | ∞        | 12  | 1.07x10^-4 Ω |   |                  |                   |                  |
| V_nr time drift                      | U(TDV_nr) = 3.8x10^-4 Ω        | Rectangular      | ∞        | 3   | 8.15x10^-4 Ω |   |                  |                   |                  |
| Shunt calibration uncertainty       | U(R) = 2.35x10^-7 Ω            | Normal           | 2        | k_3 | 2.00       | c_i | u(R) = 3.27x10^-7 Ω |                   |                  |
| Shunt thermal coefficient           | U(R) = 1.0x10^-4 Ω             | Rectangular      | ∞        | 12  | 8.03x10^-4Ω |    |                  |                   |                  |

where:

\[ c_i = 0.133438199 \text{ A}^{-1}; \]
\[ c_i = -3.713266199 \text{ A}^{-1}; \]
\[ c_i = 27.82827636 \text{ Ω} / \text{ Ω}. \]

4. Mathematical modeling of the measurement uncertainty in high voltage winding resistance measurement test

The high voltage winding resistance value is obtained by Ohm's law, according to the simplified mathematical model described in equation (2) and according to the test circuit of figure 2.

Other effects are present in the measurement circuit. From eq. (2), these effects can be properly represented by the mathematical model of equation (8):

\[
R_{HV} = \frac{V_{HV} + rV_{HV} + TDV_{HV}}{I + rA + TDA}
\]

where:

\[ R_{HV} \] represents the high voltage winding resistance;
\[ V_{HV} \] represents the potential difference (voltage drop) measured at the high voltage winding taps;
\[ rV_{HV} \] represents the correction due to the voltmeter (related to \( V_{HV} \)) resolution;
\[ TDV_{HV} \] represents the voltmeter (related to \( V_{HV} \)) time drift;
\[ I \] represents ammeter A measurements;
\[ rA \] represents the correction due to the ammeter A resolution;
\[ DTA \] represents the ammeter A time drift.

Similarly, to the previous section, starting from equation (8), following the methodology described in the Guide to Expression of Uncertainty of Measurement - ISO GUM [4], one can get mathematical modeling of the low voltage winding resistance measurement uncertainty, represented in equation (9) and detailed in Table 2:
\[ U(R_{HV}) = k_7 \times \left[ \left( \frac{u^2(V_{HV})ci_4}{k_5} + \frac{u^2(rV_{HV})ci_4}{\sqrt{12}} + \frac{u^2(TDV_{HV})ci_4}{\sqrt{3}} + \frac{u^2(A)ci_5}{k_6} + \frac{u^2(rA)ci_5}{\sqrt{12}} \right) \right]^{\frac{1}{2}} \]

where:

\[ cl_4 = \frac{1}{1 + rA + TDA} \]

\[ cl_5 = -\frac{(V_{HV} + rV_{HV} + TDV_{HV})}{(1 + rA + TDA)^2} \]

Table 2 – High voltage winding resistance test uncertainty budget

| QUANTITY | SOURCE | INPUT ESTIMATION | PROB DIST | ci | STANDARD UNCERT. | COMBINED UNCERT. | EXPANDED UNCERT. |
|----------|--------|------------------|-----------|----|------------------|-------------------|------------------|
| \( V_{HV} \) calibration uncertainty | U(V_{HV}) = 1.805\times10^{-2} V | Normal | 2 | \( k_4 = 2.00 \) | \( u_1(R_{HV}) = 4.198\times10^{-4} \) \( \Omega \) | \( U(R_{HV}) = 0.026 \) \( \Omega \) | \( \) |
| \( V_{HV} \) resolution | U(rV_{HV}) = 1\times10^{-4} V | Rectangular | \( \infty \) | \( k_4 \) | \( u_2(R_{HV}) = 1.343\times10^{-4} \) \( \Omega \) | \( u_3(R_{HV}) = 6.998\times10^{-4} \) \( \Omega \) | \( \) |
| \( V_{HV} \) time drift | U(TDV_{HV}) = 2.606\times10^{-7} V | Rectangular | \( \infty \) | \( k_4 \) | \( u_4(R_{HV}) = 1.118\times10^{-2} \) \( \Omega \) | \( u_5(R_{HV}) = -3.759\times10^{-2} \) \( \Omega \) | \( \) |
| \( A \) calibration uncertainty | U(A) = 1.717\times10^{-4} A | Normal | 2 | \( k_6 = 2.00 \) | \( u_1(R_{HV}) = -1.339\times10^{-5} \) \( \Omega \) | \( u_2(R_{HV}) = -2.366\times10^{-2} \) \( \Omega \) | \( \) |
| \( A \) resolution | U(rA) = 1\times10^{-4} A | Rectangular | \( \infty \) | \( k_6 \) | \( u_3(R_{HV}) = -1.343\times10^{-4} \) \( \Omega \) | \( u_4(R_{HV}) = -2.366\times10^{-2} \) \( \Omega \) | \( \) |
| \( A \) time drift | U(TDA) = 3.147\times10^{-7} A | Rectangular | \( \infty \) | \( k_6 \) | \( u_5(R_{HV}) = -1.339\times10^{-5} \) \( \Omega \) | \( u_6(R_{HV}) = -2.366\times10^{-2} \) \( \Omega \) | \( \) |

where:

\( ci_4 = 4.651400769 \) A\(^{-1} \);

\( ci_5 = -130.2198491 \) V.A\(^{-2} \);

5. Considerations

The critical analysis of the uncertainty components for a given measurement can be considered as a diagnosis. The information contained therein may provide clear indication regarding the following aspects: whether the measurement is affected by instability of the instrument or measurement system; if the finite resolution of the meter prevents the presentation of the coherent value of the measured quantity; if the temporal drift of the instrument or measuring system indicates that it must be calibrated at smaller intervals (in order to provide more reliable measurements) etc. It is therefore imperative that the metrological processes are conducted with rigor. Also, it is important the coherent estimation of measurement uncertainty is stated, based on the understanding of the theoretical principles and the practical experience of the performance of the test method as quoted in the standard ISO / IEC 17025: 2017, subsection 7.6.3 [5].
6. Conclusion
According to the family of standards ABNT NBR 5356 and with the Brazilian Regulation, by means of the Ordinances Inmetro 378/2010 [6] and 510/2016 [7], there are several tests to be performed in transformers of electrical distribution, to check their operational condition. However, there is no usual method for comparison of the results. In this paper, we proposed a mathematical modeling of the measurement uncertainty on the resistance measurements of the low and high voltage windings based on the worldwide accepted ISO GUM [4]. The models were applied to the test data of transformers as a way to assess their consistency. The measured uncertainties (k = 2) obtained from the tests for resistance measurement of the low and high voltage windings were, respectively $(R_{LV}) = 2.4 \times 10^{-5} \Omega$ and $U(R_{HV}) = 0.05 \Omega$.

The practical application of mathematical modeling using real test data was useful for the evaluation of mathematical modeling, allowing the presentation of metrological valid results. Also, through analysis, it was possible to identify improvement points in the measurement process, which should be performed by the metrologist based on the measurement uncertainty calculation budget.

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