Critical temperature for quenching of pair correlations

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The level density at low spin in the $^{161,162}$Dy and $^{171,172}$Yb nuclei has been extracted from primary $\gamma$ rays. The nuclear heat capacity is deduced within the framework of the canonical ensemble. The heat capacity exhibits an S-formed shape as a function of temperature, which is interpreted as a fingerprint of the phase transition from a strongly correlated to an uncorrelated phase. The critical temperature for the quenching of pair correlations is found at $T_c = 0.50(4)$ MeV.

The thermodynamical properties of nuclei deviate from infinite systems. While the quenching of pairing in superconductors is well described as a function of temperature, the nucleus represents a finite many body system characterized by large fluctuations in the thermodynamic observables. A long-standing problem in experimental nuclear physics has been to observe the transition from strongly paired states, at around $T = 0$, to unpaired states at higher temperatures.

In nuclear theory, the pairing gap parameter $\Delta$ can be studied as function of temperature using the BCS gap equations [1-3]. From this simple model the gap decreases monotonically to zero at a critical temperature $T_c \sim 0.5 \Delta$. However, if particle number is projected out the decrease is significantly delayed. The predicted decrease of pair correlations takes place over several MeV in infinite systems. While the quenching of pairing in strongly correlated to an uncorrelated phase. The critical temperature for the quenching of pair correlations is found at $T_c = 0.50(4)$ MeV.

Experimental data on the quenching of pair correlations are important as a test for nuclear theories. Within finite temperature BCS and RPA models, level density and specific heat are calculated for e.g. $^{58}$Ni [4]; within the shell model Monte Carlo method (SMMC) [5,6] one is now able to estimate level densities [7] in heavy nuclei [8] up to high excitation energies.

The subject of this Letter is to report on the observation of the gradual transition from strongly paired states to unpaired states in rare earth nuclei at low spin. The canonical heat capacity is used as a thermometer. Since the level density and $\gamma$ strength function from measured $\gamma$ ray spectra. Since the $\gamma$ decay half lives are long, typically $10^{-12} - 10^{-19}$ s, the method should essentially give observables from a thermalized system [10]. The spin window is typically 2-8 $h$ and the excitation energy resolution is 0.3 MeV.

The experiments were carried out with 45 MeV $^3$He projectiles from the MC-35 cyclotron at the University of Oslo. The experimental data were recorded with the CACTUS multidetector array [11] using the ($^3$He,$\alpha\gamma$) reaction on $^{162,163}$Dy and $^{172,173}$Yb self-supporting targets. The beam time was two weeks for each target. The charged ejectiles were detected with eight particle telescopes placed at an angle of 45° relative to the beam direction. Each telescope comprises one Si $\Delta E$ front and one Si(Li) $E$ end detector with thicknesses of 140 and 3000 $\mu$m, respectively. An array of 28 $5'' \times 5''$ NaI(Tl) $\gamma$ detectors with a total efficiency of $\sim 15\%$ surrounded the target and particle detectors.

From the reaction kinematics the measured $\alpha$ particle energy can be transformed to excitation energy $E$. Thus, each coincident $\gamma$ ray can be assigned a $\gamma$ cascade originating from a specific excitation energy. The data are sorted into a matrix of $(E, E_\gamma)$ energy pairs. At each excitation energy $E$ the NaI $\gamma$ ray spectra are unfolded [12], and this matrix is used to extract the primary $\gamma$ ray matrix, with the well established subtraction technique of Ref. [13].

The resulting matrix $P(E, E_\gamma)$, which describes the primary $\gamma$ spectra obtained at an initial excitation energy $E$, is factorized according to the Brink-Axel hypothesis [14] by $P(E, E_\gamma) \propto \rho(E - E_\gamma) \sigma(E_\gamma)$. The level density $\rho(E)$ and the $\gamma$ strength function $\sigma(E_\gamma)$ are determined by a least $\chi^2$ fit to $P$. Since the fit yields an infinitely large number of equally good solutions, which can be obtained by transforming one arbitrary solution by

$$\tilde{\rho}(E - E_\gamma) = A \exp[\alpha(E - E_\gamma)] \rho(E - E_\gamma), \quad (1)$$

$$\tilde{\sigma}(E_\gamma) = B \exp[\alpha E_\gamma] \sigma(E_\gamma), \quad (2)$$

we have to determine the parameters $A$, $B$ and $\alpha$ by comparing the $\rho$ and $\sigma$ functions to known data. The $A$ and $\alpha$ parameters are fitted to reproduce the number of known levels in the vicinity of the ground state [20] and the neutron resonance spacing at the neutron binding energy [21-23].

Figure 1 shows the extracted level densities and $\gamma$ strength functions for the $^{161,162}$Dy and $^{171,172}$Yb nuclei. The data for the even nuclei are published recently [6], and are included to be compared to odd nuclei. Also the
The heat capacity is then given by
\[ C = \frac{\partial (E)}{\partial T}, \]
and the averaging made in Eq. (6) gives a smooth temperature dependence of \( C(T) \). A corresponding \( C((E)) \) may also be derived in the canonical ensemble.

The deduced heat capacities for the \( ^{161,162}\text{Dy} \) and \( ^{171,172}\text{Yb} \) nuclei are shown in Fig. 2. All four nuclei exhibit similarly S-shaped \( C(T) \)-curves with a local maximum relative to the Fermi gas estimate at \( T_c = 0.5 \text{ MeV} \). The S-shaped curve is interpreted as a fingerprint of a phase transition in a finite system from a phase with strong pairing correlations to a phase without such correlations. Due to the strong smoothing introduced by the transformation to the canonical ensemble, we do not expect to see discrete transitions between the various quasiparticle regimes, but only the transition where all pairing correlations are quenched as a whole. In the right panels of Fig. 3 we see that \( C((E)) \) has an excess in the heat capacity distributed over a broad region of excitation energy and is not giving a clear signal for quenching of pairing correlations at a certain energy.

In order to extract a critical temperature for the quenching of pairing correlations from our data, we have to be careful, not to depend too much on the extrapolation of \( \rho \). An inspection of Fig. 3 shows that the level density is roughly composed of two components as proposed by Gilbert and Cameron [27]: (i) a low energetic part; approximately a straight line in the log plot, and (ii) a high energetic part including the theoretical Fermi gas extrapolation; a slower growing function. For illustration, we construct a simple level density formula composed of a constant temperature level density part with \( \tau \) as temperature parameter, and a Fermi gas expression
\[ \rho(E) \propto \frac{\eta \exp(E/\tau)}{E^{3/2} \exp(2\sqrt{E})} \quad \text{for } E \leq \varepsilon \]
\[ \rho(E) \propto \frac{E^{-3/2} \exp(2\sqrt{E})}{E^{3/2} \exp(2\sqrt{E})} \quad \text{for } E > \varepsilon, \]
where \( \eta = \varepsilon^{-3/2} \exp(2\sqrt{\varepsilon}) \) accounts for continuity at the energy \( E = \varepsilon \). If we also require the slopes to be equal at \( \varepsilon \), the level density parameter \( a \) is restricted to
\[ a = \left( \frac{\sqrt{\varepsilon}}{\tau} + \frac{3}{2\sqrt{\varepsilon}} \right)^2. \]
Figure 3 shows the heat capacity evaluated in the canonical ensemble with the level density function of Eq. (7) and \( \tau^{-1} = 1.7 \text{ MeV}^{-1} \). The left hand part simulates a pure Fermi gas description, i.e. the case \( \varepsilon = 0 \), assuming a level density parameter \( a = 20 \text{ MeV}^{-1} \). One can see that a pure Fermi gas does not give rise to the characteristic S-shape of the heat capacity as in Fig. 3. The right hand part simulates the experiments, where \( \varepsilon = 5 \text{ MeV} \) and \( a \) fulfills Eq. (7), i.e. again \( a = 20 \text{ MeV}^{-1} \). The characteristic S-shape emerges.

Therefore, our method to find \( T_c \) relies on the assumption that the lower energetic part of the level density can be approximately described by a constant temperature level density. Calculating \( \langle E(T) \rangle \) and \( C(T) \) within the canonical ensemble for an exponential level density gives
\[ T^{-1} = \langle E(T) \rangle^{-1} + \tau^{-1} \]
Thus, plotting $T^{-1}$ as function of $(E(T))^{-1}$ one can determine $\tau$ from Fig. 4. The quantity $\tau$ is then identified with the critical temperature $T_c$, since $C_V(T)$ according to Eq. (3) exhibits a pole at $\tau$ and the analogy with the definition of $T_\lambda$ in the theory of superfluids becomes evident. The $C_V(T)$ curve of Eq. (3) with $T_c = \tau$ using the extracted critical temperatures for the four nuclei is shown as dashed-dotted lines in Fig. 2. This simple analytical expression with only one parameter $T_c$ fits the experimental data up to temperatures of $\sim 0.4$ MeV. The critical temperature itself is marked by the vertical lines. The extracted $T_c$ is rather close to the estimate of $T_c$ represented by the arrows. The extracted values are $T_c = 0.52, 0.49, 0.50$ and $0.49$ MeV for the $^{161,162}\text{Dy}$ and $^{171,172}\text{Yb}$ nuclei respectively, which are somehow delayed compared to a degenerated BCS model with $T_c = 0.5 \Delta$ yielding $T_c \sim 0.48, 0.46, 0.41$ and $0.38$ MeV for the respective nuclei, where $\Delta$ is calculated from neutron separation energies [22].

We will now discuss how sensitive the extracted critical temperature is with respect to the extrapolation, and we will give an estimate of the uncertainty of the extracted critical temperatures. In the fit in Fig. 4, we use only energies from $(E) \sim 0.5–2$ MeV. This corresponds to energies in the level density curves up to $E_n \sim 6$ MeV according to Eq. (4). Also, in the interval where Eq. (4) fits the experimental data is $T \sim 0.4$ MeV. This corresponds to energies in the level density curves up to $E_n \sim 8$ MeV. Thus, the extracted critical temperature does only depend weakly on the actual extrapolation of $\rho$ curve. Also the S-shape of the $C_V(T)$ curve depends only on the fact, that the nuclear level density develops somehow as a Fermi gas expression at energies somewhere above the neutron binding energy. However, the actual values of the $C_V(T)$ curve above $T \sim 0.5$ MeV do depend on the specific extrapolation chosen.

With respect to $T_c$, the extrapolation is only important for determining the parameters $A$ and especially $\alpha$ of Eq. (3). We can therefore estimate the error of $T_c$ by

$$
\left( \frac{\Delta E}{\Delta T_c} \right)^2 = \left( \frac{\Delta \alpha}{2 \sqrt{\alpha U_n}} \right)^2 + \left( \frac{\Delta D}{D} \right)^2 + 2(0.05)^2,
$$

(10)

where $\Delta E$ is the energy difference between the upper and lower energy where $A$ and $\alpha$ are determined. $\Delta \alpha$ is the uncertainty of the level density parameter, $\Delta U$ is the energy difference between the neutron binding energy and the upper point where $A$ and $\alpha$ are determined, $U_n$ is the (shifted) neutron binding energy. $D$ and $\Delta D$ are the neutron resonance spacing and its error. The errors of $5\%$ are added in order to account for the two fitting procedures, one fitting $A$ and $\alpha$ the other fitting $T_c$, both with an uncertainty of some $5\%$. Using a very conservative estimate of $a \approx 17.5(6.0)$ MeV$^{-1}$, $U_n < 8$ MeV, $\Delta U < 3$ MeV, $D/\Delta D < 0.2$ and $\Delta E > 4.5$ MeV, we obtain $\Delta T_c < 0.04$ MeV. This yields a maximum error of $T_c$ of some $8\%$. It is also important to notice, that due to the strong smoothing in Eq. (4), the errors of the experimental level density curves are negligible in our calculation.

In conclusion, we have seen a fingerprint of a phase transition in a finite system for the quenching of pairing correlations as a whole, given by the S-shape of the canonical heat capacity curves in rare earth nuclei. For the first time the critical temperature $T_c$ at which pair correlations in rare earth nuclei are quenched, has been extracted from experimental data. The reminiscence of the quenching process is distributed over a 6 MeV broad excitation energy region, which is difficult to observe and interpret in the microcanonical ensemble. Simple arguments show that the peak in the heat capacity arises from two components in the level density; a constant temperature like part and a Fermi gas like part. It would be very interesting to compare our results with SMMC calculations performed for a narrow spin window.

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FIG. 1. Experimental level density (points in left panels) and $\gamma$ strength function (right panels) for $^{161,162}$Dy (upper part) and $^{171,172}$Yb (lower part). The error bars show the statistical uncertainties. The solid lines are extrapolations based on a shifted Fermi gas model (see text). The isolated point at the neutron binding energy is obtained from neutron resonance spacing data.
FIG. 2. Semi-experimental heat capacity as function of temperature (left panels) and energy $\langle E \rangle$ (right panels) in the canonical ensemble for $^{161,162}$Dy and $^{171,172}$Yb. The dashed lines describe the approximate Fermi gas heat capacity. The arrows indicate the first local maxima of the experimental curve relative to the Fermi gas estimates. The dashed dotted lines describe estimates according to Eq. (9) where $\tau$ is set equal to the critical temperature $T_c$. $T_c$ is indicated by the vertical lines.
FIG. 3. A pure Fermi gas model cannot give rise to the characteristic S-shape of the canonical heat capacity curve $C_V(T)$ (left panel). A simple composite level density can simulate our experimental findings (right panel).
FIG. 4. The critical temperature is deduced by fitting a straight line with slope 1 to the data points between the arrows.