An Optimization-Accelerated Electromagnetic Time Reversal-Based Fault Location Method for Power Lines With Branches

Guanbo Wang, Chijie Zhuang, Member, IEEE, and Rong Zeng, Senior Member, IEEE

Abstract — It is very important to locate the short-circuit fault in a power system quickly and accurately. Electromagnetic time reversal (EMTR) has drawn increasing attention because of its clear physical background and excellent performance. This article studies the EMTR method for locating the short-circuit fault of transmission and distribution lines with or without branches, and introduces a simulated annealing algorithm to accelerate the calculation of an EMTR fault location. This algorithm is different from the traditional exhaustive method in that it solves the corresponding optimization problem, thus improving the location speed by up to an order of magnitude. With the help of graph theory, a method is proposed that automatically splits a complex line topology with branches into several one-dimensional (1-D) lines. The problem of short-circuit fault location in the branching lines is then transformed into several 1-D optimization problems, which are then solved by the optimization algorithm. This solves the problem of realizing rapid location in a power network with branches. Numerical experiments are carried out in a distribution network model to demonstrate the effectiveness of the method. Results under different conditions show the method works reliably and efficiently.

Index Terms — Branched transmission line, electromagnetic time reversal (EMTR), fault location, graph theory, optimization algorithm.

I. INTRODUCTION

Electric energy plays a vital role in the normal operation of society and people’s lives. It is very important to locate the fault of a short-circuit in a power system quickly and accurately for the continuous and reliable supply of electric energy so that the fault can be eliminated in time, thereby avoiding further disruption and loss.

Existing fault location methods can be divided into at least three categories: impedance-based [3], [4], traveling wave-based [5], [6], [7], and artificial intelligence-based [8], [9], [10]. The impedance method is widely used because of its simplicity and low cost. However, it is greatly affected by the fault impedance and the network structure. The traveling-wave method has better accuracy but requires high-frequency signal acquisition equipment, which results in more cost. The artificial intelligence method needs a lot of historical data to train the model, and it is still in the developmental stage with limited practical application.

In recent years, electromagnetic time reversal (EMTR) has been extensively studied due to its clear physical background and promising performance. The time-reversal method was proposed by Bogert of Bell Laboratories and applied to the field of electronics [2], and later to more fields such as bio-medicine [16], imaging [17], [18], [19], seismic monitoring [20], lightning [34], [35] and partial discharge location [36], [37].

The EMTR method was used to locate faults in transmission lines around 2012 [11]. Razzaghi et al. [12] have since conducted a series of studies on the practicability of EMTR fault location. A fault location test was carried out on a complex T-network of components with different wave impedances, and the locating accuracy was analyzed. Because the telegraph equation of a transmission line, which is the physical basis of EMTR fault location, does not satisfy time-reversal invariance when the transmission line loss caused by line and ground resistance is considered, the loss was considered to test its effect on the locating accuracy [13], [14], [15], which showed that the loss has little influence on the accuracy of fault location in practice. In addition, experiments of EMTR fault location have been carried out in recent years. Wang et al. [16], [17] performed fault location experiments in medium-voltage distribution networks in China and Switzerland. The results showed that the fault locations were correctly identified via EMTR with a high accuracy, e.g., within 10 m. Overall, promising results for EMTR fault location have been obtained both in numerical simulation and real experiments.

Although the principles of different EMTR-based fault location methods are not the same, e.g., finding the maximum short-circuit current energy [11], finding the minimum of the voltage energy [16], and finding the maximum of a correlation [31]. These methods have one characteristic in common: locating the fault is equivalent to solving an optimization problem. Therefore, it is intuitive to use an optimization method like descend method to solve the above optimization problems.
On the other hand, almost all EMTR methods thus far have used exhaustive search (namely, brute-force search), resulting in a large computational load.

The reasons for this situation come from at least two difficulties. The first is that the objective functions of the above optimization problems are not explicit after considering the short-circuit processes, which makes the gradient-based optimization methods not easy to apply. A procedure based on generic algorithm (GA), instead of the brute-force search, was introduced in [33] to solve the associated optimization problem. However, it did not overcome the second difficulty. For power lines with branches, it is difficult to traverse all segments in a network using a single variable at the same time. In [33], the network needs to be divided into many segments, therefore, many optimization problems need to be solved.

In this article, we propose a global optimization algorithm to accelerate fault location via EMTR, with special focus on power lines with branches. The main contribution of this article includes the following.

1) We overcome the first difficulty by using the simulated annealing (SA) method. Here, SA rather than GA is specifically selected, which will be discussed at the end of Section III-A.
2) We overcome the second difficulty using the graph theory. We not only rigorously prove the minimal number of paths that a network needs to be decomposed into, namely, the minimal number of optimization problems that requires to be solved, but also give a practical method to do such decompositions.

We wish to emphasize that in this article, we only focus on accelerating the EMTR-based fault location methods, not improving their accuracy. The location accuracy is determined by the EMTR-based methods that are incorporated with the proposed optimization-based algorithm, which will be discussed in Section V.

The rest of the article is organized as follows: Section II briefly reviews the existing EMTR fault location methods; Section III introduces a one-dimensional (1-D) optimization algorithm instead of the exhaustive process for non-branched power lines to greatly improve the calculation speed of classical EMTR fault location; in Section IV, we show a way to decompose a complex network into one-dimensional (1-D) paths, thereby transforming the whole problem into a superposition of problems finding the global maxima of the unary functions. An example of faults in a distribution network is given in Section V to demonstrate the effectiveness and efficiency of the method. Finally, some conclusion is drawn in Section VI.

II. BASIC PRINCIPLE OF THE EXISTING EMTR FAULT LOCATION METHOD

The basic principle of the classical EMTR fault location method is given in a series of papers [11], [12], [13], [14], [15]. A short-circuit fault occurs at \( x = x_f \) in Fig. 1(a) and is represented by a voltage source \( U_f \).

A transient electromagnetic signal generated by the fault is collected at one end of the line (in this case \( U_0 \) is measured). The expression for \( U_0 \) in the frequency domain is

\[
U_0(\omega) = \frac{(1 + \rho_0)e^{\gamma x_f}}{1 + \rho_0 e^{-2\gamma x_f}} U_f(\omega) \tag{1}
\]

where \( \gamma \) is the line propagation constant, \( \gamma = j \beta = j \frac{c}{\gamma x_f} \) (c is the propagation speed) without considering loss, and \( \rho_0 \) is the reflection coefficient given by

\[
\rho_0 = \frac{Z_0 - Z_C}{Z_0 + Z_C} \tag{2}
\]

The signal is reinjected into the original system using the Norton equivalent from the same point after the time-reversal operation as shown in Fig. 1(b). A short-circuit branch is set at \( x = x'_f \) along the line as the guessed fault location. The expression for the short-circuit current in the frequency domain is

\[
I_f(x'_f, \omega) = \frac{(1 + \rho_0)e^{-\gamma(x'_f - x_f)}}{Z_0(1 + \rho_0 e^{-2\gamma x'_f})(1 + \rho_0 e^{2\gamma x_f})} U_f^*(\omega) \cdot \tag{3}
\]

Parseval’s theorem is used to calculate the short-circuit current energy, which reaches the maximum when \( x'_f = x_f \). It is worth noting that the time domain signal can be directly used to calculate the energy in practical application

\[
E = E(x'_f) = \sum_{j=1}^{n} I_f^2(x'_f, j) \Delta t, \quad n = \frac{T}{\Delta t}, \tag{4}
\]
where $T$ is the total duration of the short-circuit current signal, and $\Delta t$ is the simulation time step size. The position of the short-circuit branch corresponding to $E_{\text{max}}$ is the true fault location

$$x_f = \arg \max_{0 < x_f \leq L} \{E(x_f)\}.$$  

(5)

Another EMTR fault location method is discussed in [16] and [27] and involves calculating the voltage energy along the line. The minimum energy occurs at the mirror point of the fault. However, this method is not suitable for power lines with branches because there is no mirror point for the fault generally.

Consequently, this article uses the method for calculating short-circuit current energy.

III. EMTR FAULT LOCATION METHOD FOR POWER LINES WITHOUT BRANCHES

A. EMTR Fault Location Method Using an Optimization Algorithm

As discussed above, the fault location can be realized by calculating the maximum value of the short-circuit current energy. In the classical method, this is realized by exhausting the position of the short-circuit branch on the power line, which requires a huge amount of calculation. The fault location is equivalent to solving the following optimization problem:

$$E_{\text{max}} = \max_{0 < x_f \leq L} \{E(x_f)\}$$

(6)

where $L$ is the total length of the line.

The objective function in the optimization problem expressed in (6) has the following characteristics. First, it is difficult to find its analytical expression. Second, the number of extremum points of the function is unknown. The iteration process may fall into a local extremum when we use a traditional optimization algorithm, such as the steepest-descent method, and the fault point cannot be accurately located.

This article uses SA because it has better global convergence and does not require the analytical expression of the function or its derivative [26]. SA was proposed by Kirkpatrick et al. [23] to solve the problem of optimization falling into a local extremum in 1983. The idea of SA originates from the process of solid annealing. ‘Annealing’ refers to a process of heating an object to a certain temperature and then cooling it.

SA mainly includes two parts, the cooling process and the Metropolis criterion [24]. The algorithm parameters include the starting temperature $T_0$, termination temperature $T_1$, cooling coefficient $k$ ($0 < k < 1$), and termination parameter $N$.

The iterative process starts at a high temperature $T = T_0$. The new solution is generated by randomly disturbing the selected initial solution, which is accepted if it is higher (lower for minimization problems) than the present solution; otherwise, it is accepted with the following probability

$$P = \begin{cases} 1, & E_{\text{new}} < E_{\text{old}} \\ \frac{1}{e^{E_{\text{new}} - E_{\text{old}}}}, & E_{\text{new}} \geq E_{\text{old}} \end{cases}.$$  

(7)

Equation (7) is the Metropolis criterion. $T$ is the current temperature, which drops by $T^\prime = kT$ as the iterative process carries on. Because the starting temperature is high, the algorithm accepts most of the new solutions in the initial stage of the search. As the temperature decreases, the probability of accepting worse solutions gradually decreases, and the searching direction stabilizes accordingly.

In classical SA, temperature drops only after the algorithm converges at the given temperature, which is achieved when no new solution is generated after several iterations, and the final solution at the termination temperature is output as the optimal solution. For the problems and required accuracy in this article, numerical experiments show it is not necessary to converge at every temperature.

Therefore, we make some adjustments to the algorithm. First, temperature drops once a new solution is accepted. Second, if no new solution is generated after $N$ iterations, it is determined that the algorithm has converged and the current solution is output as the optimal solution.

The SA flow chart is shown in Fig. 2.

The Metropolis criterion causes the objective function to accept a worse solution with a certain probability, which retains the possibility of jumping out of a local optimum and continuing to search for the global optimum without remaining trapped in the local extremum. Therefore, the method performs better at searching for the global optimum. It has been proved that the SA algorithm asymptotically converges to the global optimal solution with a probability of 1 under certain conditions [26]. It is not trivial to rigorously prove that SA always converges to global optimum for general cases. Fortunately, in our practice in the following examples, SA always succeeds in finding the global optimum.

Fig. 2. Flow chart of the SA method.
We wish to comment that we select SA, instead of GA, for special consideration. Each iteration of GA requires enough size of population to ensure the evolution during the iterative process, which requires many calls to the objective functions in each iteration. Therefore, for relatively medium accuracy requirement like fault location, it is even less efficient than the brute-force search which also requires calls to the objective functions.

### B. An Application Example of the Optimization Method

Theoretically, the solution domain of SA method could be the entire real domain. However, the traditional EMTR method using exhaustive search has to restrict the solution domain to a set of numbers that are multiples of the required accuracy. Therefore, in order to compare the calculation speed, we restrict the solution domain of SA method to the same set.

The short-circuit fault shown in Fig. 1(a) is considered. We assume that the short-circuit fault occurs at the location \( x_f = 4 \) km. The short-circuit type is set to ideal (short-circuit impedance equals to 0). A 10-kV power-frequency ac voltage is applied to the head end of the line. Both ends are connected with 100-kΩ resistances that serve as the equivalent of power transformers. The circuit parameters are given in Table I.

We use SA to locate the fault. The disturbance is set as a random number within one-fifth of the length of the defined function domain in this article. To accelerate the calculation, the disturbance interval is halved for each iteration if \( n \geq N/2 \).

When \( T_0 \) is set to such a small value that the Metropolis criterion almost only accepts better solutions, the SA method degenerates into a random walk optimization (RWO) [29], and therefore may fall into a local optimum. To test this case, we set \( k = 0.9 \) and \( N = 10 \), and set \( T_0 = 10^{-9} \) K and \( T_0 = 10^{-1} \) K as examples for RWO and SA, respectively.

As shown in Fig. 3, the iterative process starts from the same point \( x = 1500 \) m. SA obtains the global maximum while RWO falls into a local optimum, which demonstrates the effectiveness of SA.

The uncertainty in the search direction and initial value of the algorithm makes the number of iterations random. Therefore, the calculation is repeated ten times with the accuracy ranging from 5 to 20 m. The calculation time and number of iterations are shown in Fig. 4.

The calculation times shown in Fig. 4(a) are proportional to the number of iterations, which indicates that the efficiency can be represented by the number of iterations. When the locating accuracy is set to 5 m, the average number of SA iterations is 93 as shown in Fig. 4(b), while 2000 calculations are required for the exhaustive method. The calculation speed of classical EMTR fault location is more than ten times faster using SA.

Therefore, the optimization algorithm greatly improves the locating efficiency.

As discussed in [31], the method directly calculating the current energy may output bad solutions in some cases, and the transfer function method, which also solves an optimization problem with a different objective function, may provide a better accuracy. However, with the replacement of the objective function, this optimization-accelerated method is also applicable to other EMTR-based fault location methods as long as they seek an optimum.
IV. EMTR FAULT LOCATION METHOD FOR POWER LINES WITH BRANCHES

A distribution network may contain many power lines with branches. It is difficult to use a single variable to traverse all possible paths in the network. Therefore, the fault location problem cannot be transformed into a univariate optimization problem as in (6).

So one has to decompose the network into a series of one-dimensional lines such that a single variable can traverse each possible path. In [33], this was done by separating the segments at each intersection, which requires solving many optimization problems if the network has many branches. Intuitively, two questions are raised: what is the minimal number of segments that a given network must be decomposed into (equivalently, the number of optimization problems we need to solve)? and how can we find such a “optimal” decomposition?

A complex network of power lines with branches can be regarded as a connected graph. In this section, the two above questions are answered using graph theory. We will show that the network can be divided into minimal number of one-dimensional simple lines; then, the fault can be located by solving the optimization problem on each 1-D simple line.

A. Method of Decomposing Complex Connected Graphs

First, we give some theorems on how to decompose a complex connected graph, with the knowledge of definitions in [28] (i.e., connected graph and path) and in the Appendix.

**Theorem I:** The number of odd nodes (defined in Appendix) in the graph \( G = (V, E) \) must be even. \( V \) and \( E \) represent for the nodes and edges in the graph, respectively.

**Theorem II:** Deleting all edges in a path reduces the number of odd nodes in \( G = (V, E) \) by a maximum of 2.

The proofs for Theorems I and II can be found in the Appendix.

**Theorem III:** A graph with \( 2k \) \((k \in \mathbb{N}^+)\) odd nodes cannot be decomposed into less than \( k \) paths.

**Proof:** According to Theorem II, deleting edges in a path can reduce the number of odd nodes by a maximum of 2. Therefore, the number of odd nodes can be reduced by no more than \( 2(k-1) \) with less than \( k \) paths. Therefore, for a graph with \( 2k \) \((k \in \mathbb{N}^+)\) odd nodes, there are still odd nodes left in the graph, which means the graph is not completely decomposed.

**Theorem IV:** A graph with \( 2k \) \((k \in \mathbb{N}^+)\) odd nodes can be decomposed into \( k \) paths.

**Proof:** (Mathematical Induction)

1) For \( k = 1 \), the graph is a semi-Euler graph; therefore, the problem degenerates into finding the Euler path [25], which was solved already.

2) We assume the theorem holds for \( k \) and consider the case \( k+1 \).

A path that both starts and ends with an odd node is selected, and the edges in it are deleted. We know from Theorem II that the number of odd nodes decreases by 2 [from \( 2(k+1) \) to \( 2k \)]. According to the inductive hypothesis, the remaining part of the graph can be decomposed into \( k \) paths. Therefore, the entire graph can be decomposed into \( k+1 \) paths.

According to Theorems III and IV, a graph with \( 2k \) odd nodes can be decomposed into \( k \) paths, and \( k \) is optimal.

We now present a practical decomposing method based on Theorem IV and the Fleury Algorithm (see the Appendix).

The adjacency matrix \( A \) of a graph is defined as an \( n \times n \) matrix whose elements are defined as

\[
 a_{ij} = \begin{cases} 
 1, & e_{ij} = (v_i, v_j) \in E \\
 0, & \text{else}
\end{cases}
\]

It is easy to conclude that \( A \) is symmetric, and the degree of node \( i \) equals \( \sum_{j=1}^{n} a_{ij} \).

A path both starting and ending with an odd node is selected randomly from the graph. Then the edges in the path are deleted. The elements \( a_{ij} \) and \( a_{ji} \) in the adjacency matrix of the graph are zeroed to target each edge in the path accordingly. The path is regarded as the first path of decomposition.

We perform the above operation \( k-1 \) times and obtain \( k-1 \) decomposition paths. Meanwhile, the number of odd nodes in the graph \( G = (V, E) \) is reduced by \( 2(k-1) \) according to Theorem II. Hence, there will be only 2 odd nodes left in the graph, making the remaining graph a semi-Euler graph, which can be traversed using one Euler path.

An efficient algorithm to obtain the Euler path was proposed by Fleury in 1921 [28], which we refer to as Theorem V as is shown in Appendix.

The above algorithm establishes a practical way to execute the decomposition. It is worth mentioning that the decomposition is not unique because the selection of the initial nodes and the path is not unique. However, as stated above, the algorithm is optimal in the sense that a graph with \( 2k \) odd nodes can always be decomposed into \( k \) paths, and \( k \) is optimal.

We give two examples to illustrate the correctness of the decomposing method. The Chinese character “Chuan” is shown in Fig. 5(a). There are only two odd nodes, nodes 1 and 14, so the graph is a semi-Euler graph, which can be decomposed into a single path. A possible way given by the Fleury algorithm is illustrated in Fig. 5(b).

Here, we wish to emphasize one of the key differences between this article and [33]. For the network shown in Fig. 5(a), [33] must solve nine optimization problems (1-3, 3-2-5-6, 3-6,
Fig. 6. Connected graph of the Königsberg Seven Bridges and its decomposition into two paths.

3-4-7-6, 6-9, 9-8-11-12, 12-14, 9-10-13-12, 9-12), while our method only needs to solve 1.

Fig. 6(a) shows an abstracted graph derived from the Königsberg seven bridges problem. There are four odd nodes, namely 1, 4, 7, and 8 in the graph. A decomposition is shown in Fig. 6(b). The graph is decomposed into two paths accordingly. In spite of the complexity of the graphs in Figs. 5 and 6, they can still be decomposed in an optimal way, which proves the decomposing method is correct.

B. EMTR Fault Location Method for Power Lines With Branches

The topology of power lines with branches can be abstracted into a connected graph with 2k odd nodes. The decomposing method described above is used. The k paths obtained from the decomposition are actually k 1-D power lines. Only one variable is needed to traverse all the positions on every single line. Therefore, (6) can be applied to each line i using the optimization algorithm

$$E_{i, \text{max}} = \max_{0 < x'_f, L_i \leq L_i} \left( E_i \left( x'_f \right) \right), \quad i = 1, 2, \ldots, k, \quad (9)$$

where $L_i$ is the length of the one-dimensional power line i. The maximum value of all $E_{i, \text{max}}$’s in (9) corresponds to the location $x_f$ of the real fault point

$$x_f = \arg \left[ \max_{1 \leq i \leq k} \left( E_{i, \text{max}} \right) \right]. \quad (10)$$

We clarify a common misunderstanding to the proposed algorithm. For each subproblem (9), we limit the solution domain to the corresponding path obtained from the decomposition, while the EMTR process are still performed using the original power network. Table II gives the process of EMTR fault location for power lines with branches.

We give an example to illustrate the EMTR fault location method. The abstract connected graph of a T-type network is shown in Fig. 7. The transient voltage generated by the fault is collected at node 1 and reinjected into the original network after time reversal. All nodes in Fig. 7 are odd, so the graph shall be decomposed into at least two paths. One way to do this is: $1 \rightarrow 2 \rightarrow 3$, $3 \rightarrow 4$. Therefore, two optimization problems need to be solved as shown in Fig. 7.

We emphasize that for each optimization problem, the EMTR process is always performed using the original power network, not in the decomposed paths.

V. APPLICATION EXAMPLES

We consider a distribution network of power lines containing 11 nodes totally, as shown in Fig. 8. Part of the network above node 4 is cable-connected, which is bold in Fig. 8. The overhead line and cable parameters are given in Tables I and III.

A 10-kV power-frequency ac voltage is applied to node 1. The terminals of the network are connected to power transformers,
which are equivalent to a large impedance. We adopt 100 kΩ in the example [3]. A short-circuit fault is set along the line. The transient voltage signal generated by the fault is obtained at node 1, and then injected into the system after time reversal.

There are ten odd nodes in the graph, which indicates the network should be decomposed into five one-dimensional lines. One way to realize this is

\[ 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5, \quad 2 \rightarrow 6, \quad 3 \rightarrow 7, \]
\[ 4 \rightarrow 8 \rightarrow 9 \rightarrow 10, \quad 5 \rightarrow 9 \rightarrow 11. \]

Therefore, the guessed short-circuit branch is set on five 1-D power lines. Then, SA is used to solve the corresponding optimization problem with the short-circuit current energy as the objective function. We set \( N = 10 \) in Fig. 2. The starting temperature is 1 K and the cooling coefficient is 0.8.

Different fault conditions are adopted to analyze the effectiveness and efficiency of EMTR fault location compared with the classical exhaustive search.

All the faults were simulated in MATLAB using Bergeron models with a time step of 0.1 μs.

### A. Fault Position

As shown in Fig. 8, a series of fault positions (100 in total) are considered along the line, each separated from its direct neighbors with a spatial step of 0.2 km. The fault angle is set to 90°, and the fault impedance is 1 Ω. The impedance of the short-circuit branch that is the guessed fault position in the EMTR method is set to 20 Ω.

Table IV gives the calculated maximum short-circuit current energy value along each path for the case of fault 73. It shows that the SA method gives the same result as the exhaustive search, including the same maximum energy value and the associate positions.

This optimization-accelerated method is also applicable to other EMTR-based fault location methods that seek an optimum, such as the high-impedance fault location method [30] and the transfer function method [31].

On the other hand, as shown in Fig. 9, the SA iteration numbers required are about 250 for different fault positions, compared with 2010 iterations in the traditional exhaustion method, namely, the optimization algorithm improves the calculation efficiency by nearly one order of magnitude. The efficiency (represented by the number of iterations) is not sensitive to the fault position.

### B. Fault Angle and Fault Impedance

The fault angle and fault impedance affect the amplitude of the transient electromagnetic signal. A small fault angle or high fault impedance produces a small amplitude, which results in less short-circuit current energy.

In this case, the angle is set to 30°, 60°, and 90°, respectively. An accuracy of 10 m can be achieved at these fault angles using EMTR. Fault position 16 is chosen as an example, and the number of iterations is shown in Fig. 10 under the same SA parameters.
To test the influence of fault impedances, fault position 16 is again chosen as an example, and the fault angle is set to 90°. The fault impedance ranges from 0 to 50 Ω. The SA parameters remain the same.

Fig. 11 shows the number of iterations under different fault impedances. We see that the calculation slows down as the fault impedance increases.

C. Discussion

Two problems are discussed in this section.

1) Choice of the Starting Temperature in SA: The number of iterations increases with decreasing fault angle and increasing fault impedance. According to (7), when the amplitude of the short-circuit current energy decreases, there is a higher probability of accepting worse solutions in SA, which increases the time to achieve convergence. Therefore, the selected starting temperature is related to the amplitude of the objective function, and has a direct impact on the calculation speed.

In real applications, the following choice of the starting temperature has been proved to be effective. According to (3), the fault current at the real fault position is

\[ I_f(x_f, \omega) = \frac{(1 + \rho_0) e^{-j\gamma x_f}}{Z_0 (1 + \rho_0 e^{-2\gamma x_f})} U_0^* (\omega) \]

\[ = \frac{U_0 (\omega) U_0^* (\omega)}{Z_0 U_f (\omega)}. \]  

The fault signal \( U_f(\omega) \) can be approximated by \( 1/j\omega \) as in [12], and \( U_0(\omega) \) is measured at the end of the line. Therefore, the starting temperature can be estimated by

\[ T_0 = \alpha \int_{\omega_0}^{+\infty} \frac{1}{2\pi} |I_f(x_f, \omega)|^2 d\omega. \]  

In order to maintain a high calculating efficiency, \( \alpha \) is an additional coefficient that is close to 1 to make \( \log T_0 \) an integer. Noting that the frequency spectrum of the fault-generated transients does not exceed a few hundreds of kilohertz, the interval of integration in (12) can be limited to 1 Hz–1MHz.

Equation (12) defines \( T_0 \) as a type of energy, which makes the term \( E_{T_0} \) in (7) always be O(1) for different problems at the start point of SA algorithm. According to (7), the temperature should be roughly proportional to the value of the objective function to maintain the calculation efficiency. The transient amplitude becomes smaller when the fault impedance grows, resulting in a smaller current energy at all guessed fault locations. Therefore, a higher fault impedance requires a lower temperature to maintain the calculation speed. Fig. 12 shows the number of iterations when the starting temperature is changed from 0.01 to 1 K. In this case, the calculation speed is nearly ten times faster compared with the exhaustive search when the starting temperature is 0.01 K, which is close to the maximum current energy.

2) Possible Failure of EMTR-Based Methods: The fault location experiments are carried out in the network for 100 times shown in Fig. 8. Fig. 13 shows, three cases fail out of 100 cases. Location failures were also reported and discussed in [14] and [31]. We also checked and confirmed that for all the three failed cases, SA returns the same results as the exhaustive search, therefore these failures should result from the defects of the EMTR-based method described in Section II.

As we mentioned in the Introduction section, the location accuracy of the method proposed in this article is determined by the methods incorporated with it. This is intuitive because they solve the same optimization problem with same objective functions, e.g., (10) in this article. To cure the possible location failures, one may select the other methods in, e.g., [27], [31], as the incorporating methods.
Given a semi-Euler $G = (V, E)$, the contribution of $m$ edges to all node degrees is $2m$ because the contribution of each edge to the degree of nodes $u$ and $v$ is 1.

We suppose that $V_e$ is the even node set of graph $G$, and $V_o$ is the odd node set of graph $G$. Then we have

$$\sum_{v \in V_e} d(v) + \sum_{v \in V_o} d(v) = 2m. \quad (13)$$

Therefore, $\sum_{v \in V_o} d(v)$ is an even number, which indicates $G$ contains an even number of odd nodes.

**Proof for Theorem II:** A node in a path is either internal or terminal.

For internal nodes, two edges are deleted, which has no effect on the parity of the degree.

For the terminal nodes of the path, only one connected edge is deleted, which changes the parity of the degree. There are three possible parity combinations:

1) The degrees of both terminal nodes are odd.
   The deletion changes the degrees of the two nodes from odd to even, which reduces the number of odd nodes by 2.
2) One terminal node is odd and the other is even.
   The deletion changes the odd degree into even and the even into odd. Consequently, the number of odd nodes keeps unchanged.
3) The degrees of both nodes are even.
   The deletion changes the degrees of the two nodes from even to odd, which increases the number of odd nodes by 2.

Therefore, the deletion reduces the number of odd nodes by a maximum of 2.

**Theorem V (Fleury Algorithm [28]):** Given a semi-Euler graph $G = (V, E)$, the algorithm for finding an Euler path in $G$ is as follows.

1) An odd node $v_0$ is selected such that $P = \{v_0\}$.
2) Assuming the path $P = \{v_0, e_1, v_1, e_2, \cdots e_i, v_i\}$ is determined, $e_{i+1}$ is chosen according to the following principles.
   a) $e_{i+1}$ is connected with $v_i$.
   b) $e_{i+1}$ should not be one of the bridges in $G = (V, E)$ unless there is no other choice.
3) The algorithm ends if (2) cannot proceed anymore.

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Guanbo Wang was born in Changchun, Jilin, China, in 1999. He received the B.Eng. degree from the Department of Electrical Engineering, Tsinghua University, Beijing, China, in 2021, where he is currently working toward the M.Eng. degree, both in electrical engineering. His main research interests include Electromagnetic Time Reversal and power line fault location.

Chijie Zhuang (Member, IEEE) was born in 1983. He received the B.Eng. and Ph.D. degrees in electrical engineering from Tsinghua University, Beijing, China, in 2006 and 2011, respectively. He was an Assistant Professor with the Department of Electrical Engineering, and was an Associate Professor in 2015. He has authored or coauthored about 60 journal papers, and served as the member of several CIGRE working groups and IEEE standard committees. His current research interests include sensors, electrical discharge, and scientific computing for electrical engineering problems.

Dr. Zhuang has been an Editor for IEEE TRANSACTIONS ON POWER DELIVERY since 2021.

Rong Zeng (Senior Member, IEEE) was born in Shaanxi, China, in 1971. He received the B.Eng., M.Eng., and Ph.D. degrees from Tsinghua University Beijing, China, in 1995, 1997, and 1999, respectively, all in electrical engineering. He was a Lecturer with the Department of Electrical Engineering, Tsinghua University in 1999, an Associate Professor and a Professor in 2002 and 2007, respectively. He is the Vice President with Tsinghua University. He is working in the fields of insulation, electromagnetic compatibility in power systems, HVdc systems and direct current circuit breaker.