Condensates as components of dark matter and dark energy

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Abstract. We report on recent results according to which the vacuum condensate characterizing many physical systems can give contributions to dark energy and dark matter of the universe. In particular, it is shown that thermal states of the intercluster medium, the vacuum energy of fields in curved space-time and of mixed neutrinos contribute to the dark matter. The vacuum condensates generated by the mixing of axions and photons and the one produced by superpartners of neutrinos may represent a component of the dark energy.

1. Introduction
According to many experimental data, the universe is made up approximately of 68% of dark energy [1]-[5] and about of 27% of dark matter. The rest is ordinary matter. Although different models have been proposed to explain the dark energy [6]-[15] and the dark matter problem [16]-[19], however, the solution to such a problem represents still a very big challenge.

Here it is reported the recent result [20] according to which the vacuum condensate energy characterizing many physical systems [21]-[40], can give contributions to the dark sector of the universe. It is shown that non trivial contributions to the dark matter can be originated by the thermal vacuum of the hot plasma present at the center of a galaxy cluster (intracluster medium), by the vacuum fluctuations of fields in curved space [41] and by the flavor neutrino vacuum [20]. Moreover, it is shown that the condensate of mixed boson [32, 33], such as axions and axion like particles (ALPs) in their interaction with photons, and the condensate of superpartners of mixed neutrinos, can give contributions to the dark energy with the state equation of the cosmological constant [20]. The mixing of unstable particles like kaons, $B^0$, $D^0$ mesons and $\eta - \eta'$ is expected do not contribute to the energy on large scale.

The origin of the non trivial vacuum energy contributions of the systems described above is due to the fact that the physical vacuum of these systems is a condensate of couples of particles and antiparticles which generate a positive value of the zero point energy. The formal analogy among the phenomena generating condensates could allow to simulate the systems here analyzed by means of phenomena reproducible in table top experiments such as the superconductivity, the Casimir effect and the Schwinger effect.

In Sec.II, the Bogoliubov transformations in QFT are introduced and the condensate structure of the transformed vacuum is presented. The general form of the energy momentum tensor of the vacuum condensates is presented in Sec.III. In Secs.IV, V, VI, the contributions given to the
energy of the universe by thermal states, by fields in curved space and by particle mixing are analyzed. Sec.VII is devoted to the conclusions.

2. Vacuum condensate induced by Bogoliubov transformations

Many systems such as the Hawking-Unruh [21, 22] and the Schwinger effects [23], the BCS theory of superconductivity [24], thermal fields in finite temperature QFT [26], the particle mixing [31]-[36], quantum fields in curved spacetimes [40] are described by the Bogoliubov transformations in the QFT context [27]. For bosons, general Bogoliubov transformations are

\[ \hat{a}_k(\xi, t) = U_B^\xi(\xi) a_k(t) - V_B^\xi(\xi) \hat{a}^\dagger_{-k}(t), \]
\[ \hat{a}^\dagger_{-k}(\xi, t) = U_B^{\dagger \xi}(\xi) \hat{a}^\dagger_{-k}(t) - V_B^{\dagger \xi}(\xi) a_k(t), \]

with \( a_k(t) = a_k e^{-i\omega_k t} \), such that \( a_k|0\rangle_B = 0 \) and \( \omega_k = \sqrt{k^2 + m^2} \). The coefficients for bosons satisfy the conditions \( U_k = U_{-k}^\dagger, V_k = V_{-k}^\dagger \), \( |U_k|^2 - |V_k|^2 = 1 \), and similar relations hold for fermions. The parameter \( \xi \) depends on the particular system one considers. For example, \( \xi \) is related to the acceleration of the observer in the Unruh effect case. The Bogoliubov transformations (1) can be written in terms of the generator \( J(\xi, t) \) as: \( a_k(\xi, t) = J^{-1}(\xi, t) a_k(t) J(\xi, t) \), with \( J^{-1}(\xi) = J(-\xi) \), and similar transformations for fermions. The vacua \( |0(\xi, t)\rangle \) annihilated by \( a_k(\xi, t) \) are related to the original ones \( |0\rangle \) by \( |0(\xi, t)\rangle = J^{-1}(\xi, t)|0\rangle \).

This is a unitary operation at finite volume, but in QFT, in which \( k \) assumes a continuous infinity of values, is not a unitary transformation any more [28]. Then the vacuum \( |0(\xi, t)\rangle \) cannot be expressed as a superposition of vectors in the Fock space built over \( |0\rangle \) and the two Fock spaces are unitarily inequivalent [26]-[27]. This fact leads to the problem of the right choice of the Fock space and of the physical vacua associated with the particles which appear in observations. For systems characterized by Bogoliubov transformations, the physical vacua are the transformed ones \( |0(\xi, t)\rangle \) [28]. Such vacua have a condensate structure, indeed one has \( \langle 0(\xi, t)|a_k^\dagger a_k|0(\xi, t)\rangle = |V_k|^2 \), and similar for fermions. Such a property generate a non zero energy momentum tensor for \( |0(\xi, t)\rangle \) which is responsible of the contributions to the dark sector of the universe which will be analyzed in the following and produces the spontaneous supersymmetry breaking in all the systems characterized by the presence of condensates [42]-[46].

3. Energy-momentum tensor of vacuum condensate

One computes the expectation value of the energy momentum tensor densities for scalar and for Majorana fields \( T_{\mu\nu}^\lambda(x), (\lambda = B, F, \text{with } B = \text{bosons and } F = \text{fermions}) \) on the transformed vacuum \( |0(\xi, t)\rangle_\lambda \),

\[ \Xi^\lambda_{\mu\nu}(x) \equiv \langle 0(\xi, t) : T^\lambda_{\mu\nu}(x) : |0(\xi, t)\rangle_\lambda = \lambda \langle 0(\xi, t) : T^\lambda_{\mu\nu}(x) |0(\xi, t)\rangle_\lambda - \lambda \langle 0 | T^\lambda_{\mu\nu}(x) |0\rangle_\lambda. \]

Here, \( : \ldots : \) is the normal ordering with respect to the original vacuum \( |0\rangle_\lambda \). Since \( \langle 0(\xi, t) : T^\lambda_{\mu\nu}(x) : |0(\xi, t)\rangle = 0 \), for \( i \neq j \), the condensates induced by Bogoliubov transformations behave as a perfect fluid and the energy density and pressure of boson and fermion condensates are

\[ \rho^\lambda = \langle 0(\xi, t) : T^\lambda_0(x) : |0(\xi, t)\rangle, \]
\[ p^\lambda = \langle 0(\xi, t) : T^\lambda_\phi(x) : |0(\xi, t)\rangle, \]

respectively.

For bosons, one has

\[ \rho_B = \frac{1}{2} \langle 0(\xi, t) : \left[ \pi^2(x) + \left( \hat{\nabla} \phi(x) \right)^2 + m^2 \phi^2(x) \right] : |0(\xi, t)\rangle; \]
\[ p_B = \langle 0(\xi, t) : \left[ \partial_\phi^2(x) \right]^2 + \frac{1}{2} \left[ \pi^2(x) - \left( \hat{\nabla} \phi(x) \right)^2 - m^2 \phi^2(x) \right] : |0(\xi, t)\rangle. \]
In the particular case of the isotropy of the momenta, $k_1 = k_2 = k_3$, one has, $[\partial_j \phi(x)]^2 = \frac{1}{3} \left[ \nabla \phi(x) \right]^2$, then the pressure can be written as

$$p_B = \frac{1}{2} \langle 0(\xi,t) | : \pi^2(x) - \frac{1}{3} \left( \nabla \phi(x) \right)^2 - m^2 \phi^2(x) \rangle : |0(\xi,t) \rangle,$$

and the energy density and pressure can be expressed as [20]

$$\rho_B = \frac{1}{2\pi^2} \int_0^\infty dk k^2 \omega_k |V_k^B|^2,$$  \hspace{1cm} (7)

$$p_B = \frac{1}{6\pi^2} \int_0^\infty dk k^2 \left[ \frac{k^2}{\omega_k} |V_k^B|^2 - \left( \frac{k^2}{\omega_k} + \frac{3m^2}{2\omega_k} \right) |U_k^B|^2 \right] |V_k^B|^2 \cos(\omega_k t) \] .$$  \hspace{1cm} (8)

The state equation is

$$w_B = \frac{1}{3} \int d^3 k \frac{k^2}{\omega_k} |V_k^B|^2 - \frac{1}{3} \int d^4 k \frac{k^2}{\omega_k} |V_k^B|^2 - \frac{1}{3} \int d^3 k \frac{k^2}{\omega_k} |V_k^B|^2 \cos(\omega_k t) .$$  \hspace{1cm} (9)

For fermions, the energy density and the pressure are

$$\rho_F = \frac{1}{2} \langle 0(\xi,t) | : -i \bar{\psi} \gamma_j \partial^j \psi + m \bar{\psi} \psi \rangle : |0(\xi,t) \rangle ;$$  \hspace{1cm} (10)

$$p_F = \langle 0(\xi,t) | : \left( \frac{i}{2} \bar{\psi} \gamma_j \vec{\partial} \gamma_j \psi \right) : |0(\xi,t) \rangle .$$  \hspace{1cm} (11)

and explicitly one has [20]

$$\rho_F = \frac{1}{\pi^2} \int_0^\infty dk k^2 \omega_k |V_k^F|^2,$$  \hspace{1cm} (12)

$$p_F = \frac{1}{3\pi^2} \int_0^\infty dk \frac{k^4}{\omega_k} |V_k^F|^2,$$  \hspace{1cm} (13)

$$w_F = \frac{1}{3} \int d^3 k \frac{k^2}{\omega_k} |V_k^F|^2 .$$  \hspace{1cm} (14)

The non-zero values of $\rho_\lambda$ and $p_\lambda$ are due to the condensate structure of the physical vacuum. Notice also that, being $J^{-1}(\xi,t) = J^!(\xi,t) = J(\xi,t)$, one can write

$$\lambda \langle 0(\xi,t) | : T^\lambda_{\mu\nu}(x) : |0(\xi,t) \rangle \lambda = \lambda \langle 0|J^{-1}_\lambda(-\xi,t) : T^\lambda_{\mu\nu}(x) : J_\lambda(-\xi,t) |0 \rangle \lambda .$$

In the following we use the notation $\Theta(-\xi,x) = J^{-1}(\xi,t) \Theta(x) J(\xi,t)$ to denote the operators transformed by the generator $J(-\xi,t)$. Moreover, it will be used the condition, $\nabla J(\xi,t) = 0$, satisfied by the generators $J$'s of the thermal states and of the particle mixing phenomenon.

Eqs. (7), (8) and (12), (13), hold for many phenomena. The particular system is specified by the explicit form of the Bogoliubov coefficients.
4. Contributions given by thermal states, Hawking and Unruh effects
In the framework of the Thermo Field Dynamics (TFD) [26]-[27], the physical vacuum of systems at non-zero temperature is the thermal vacuum state $|0(\xi(\beta))\rangle_\lambda$, with $\lambda = B, F$, $\beta \equiv 1/(k_B T)$ and $k_B$ the Boltzmann constant. In such a formalism, the thermal statistical average is given by $\langle N_{\chi k} (\xi) = \lambda \langle 0(\xi(\beta))|N_{\chi k}|0(\xi(\beta))\rangle_\lambda$, with $N_{\chi k} = \lambda a_{\chi k}$, ($\chi = a, \alpha$) the number operator [26]-[27]. The Bogoliubov coefficients are given by $U_k^T = \sqrt{\frac{e^{\beta \omega_k}}{e^{\beta \omega_{k+1}}}}$ and $V_k^T = \sqrt{\frac{1}{e^{\beta \omega_{k+1}}}}$, with $-\imath$ for bosons and $+\imath$ for fermions, $\beta = 1/k_B T$ and $\omega_k = \sqrt{k^2 + m^2}$ [26]-[27]. Such coefficients, used in Eqs.(7), (8) and (12), (13) give the contributions of the thermal vacuum states to the energy and pressure. The results one finds are the following: for the cosmic microwave background temperature, $T = 2.72 K$, only photons and particles with masses of order of $(10^{-3} - 10^{-4}) eV$ contribute to the energy radiation with $\rho \approx 10^{-51} GeV^4$ and state equations, $w = 1/3$ [47]. Negligible contributions to vacuum energy are given by the thermal states describing the Unruh and of the Hawking effects [47, 48]. On the other hand, the thermal vacuum of the hot plasma filling the center of galaxy clusters, which has temperatures of order of $(10^{10} - 10^{10}) K$, has an energy of $(10^{-48} - 10^{-47}) GeV^4$ and a state equation $w = 0.01$. Such values are in agreement with the ones on the dark matter [20].

5. Contribution of fields in curved background
A vacuum condensed structure characterizes also fields in curved spaces [40]. In these cases, the energy momentum structure depends on the particular metric considered. Here one analyzes the spatially flat Friedmann Robertson-Walker metric, $ds^2 = dt^2 - a^2(t)dx^2 = a^2(\eta)(d\eta^2 - dx^2)$, where $a$ is the scale factor, $t$ is the comoving time, $\eta$ is the conformal time, $\eta(t) = \frac{t}{M_{Pl}}$, with $t_0$ arbitrary constant and considers the boson case. Assuming a cutoff on the momenta much smaller than the comoving mass of the field, $K \ll ma$ and assuming that $m \gg H$, with $H$ Hubble constant, the energy density and pressure of the vacuum condensate, become [41]

$$\rho_{\text{curv}} = \frac{1}{8\pi^2} \int_0^K dk k^2 \left( \frac{2m}{a^3} + \frac{9H^2}{4ma^3} + \frac{k^2}{ma^5} \right),$$
$$p_{\text{curv}} = \frac{1}{8\pi^2} \int_0^K dk k^2 \left( \frac{9H^2}{4ma^3} - \frac{k^2}{3ma^5} \right).$$

The state equation, in the infrared regime, is then the one of the dark matter, $w_{\text{curv}} \approx 0$. Numerical values of $\rho_{\text{curv}}$, compatible with the ones of dark matter can be found when, $\frac{mK^3}{a^3} \sim 10^{-45} GeV^4$.

6. Contributions given by particle mixing
The phenomenon of particle mixing is represented by the neutrino and quark mixing in fermion sector and by the axion-photon mixing and the mixing of kaons, $B^0$, $D^0$, and $\eta - \eta'$ systems, in boson sector. For two fields, it is expressed, both for fermions and bosons, as

$$\varphi_1(\theta, x) = \varphi_1(x) \cos(\theta) + \varphi_2(x) \sin(\theta),$$
$$\varphi_2(\theta, x) = -\varphi_1(x) \sin(\theta) + \varphi_2(x) \cos(\theta),$$

where, $\theta$ is the mixing angle, $\varphi_i(\theta, x)$ are the mixed fields and $\varphi_i(x)$ are the free fields, with $i = 1, 2$.

The mixing transformations (16) can be written as $\varphi_i(\theta, t) \equiv J^{(-)}(\theta, t) \varphi_i(x) J(\theta, t)$, where $i = 1, 2$, and $J(\theta, t)$ is the transformation generator [31, 32].
The physical vacuum where particle oscillations appears is $|0(\theta,t)\rangle \equiv J^{-1}(\theta,t) |0\rangle_{1,2}$. It has a condensate structure [31, 32]:

$$
\langle 0(\theta,t)|\chi_{k,i}^\dagger \chi_{k,i}^{}|0(\theta,t)\rangle = \sin^2 \theta |\Upsilon^\lambda_k|^2 ,
$$

(17)

where $\lambda = B, F$, $i = 1, 2$. The Bogoliubov coefficient $\Upsilon^\lambda_k$ is for boson and fermion [31, 32]

$$
|\Upsilon^B_k| = \frac{1}{2} \left( \frac{\Omega_{k,1}}{\Omega_{k,2}} - \frac{\Omega_{k,2}}{\Omega_{k,1}} \right) ,
$$

(18)

$$
|\Upsilon^F_k| = \frac{(\Omega_{k,1} + m_1) - (\Omega_{k,2} + m_2)}{2\sqrt{\Omega_{k,1}\Omega_{k,2}(\Omega_{k,1} + m_1)(\Omega_{k,2} + m_2)}} |k| ,
$$

(19)

respectively, with $|\Sigma^B_k|^2 - |\Upsilon^B_k|^2 = 1$ and $|\Sigma^F_k|^2 + |\Upsilon^F_k|^2 = 1$, $\Omega_{k,i}$ energies of the free fields, $i = 1, 2$.

6.1. Boson mixing

For mixed bosons, the kinetic and gradient terms of mixed vacuum are equal to zero [20], then the energy density and pressure become

$$
\rho^B_{\text{mix}} = \langle 0| : \sum_i m_i^2 \phi^2_i(-\theta,x) : |0\rangle ,
$$

(20)

$$
\rho^B_{\text{mix}} = -\langle 0| : \sum_i m_i^2 \phi^2_i(-\theta,x) : |0\rangle ,
$$

(21)

and the state equation coincides with the one of the the cosmological constant, $w^B_{\text{mix}} = -1$, independently on the choice of the cut-off on the momenta. By setting $\Delta m^2 = |m_2^2 - m_1^2|$, the energy density of the boson mixed vacuum is

$$
\rho^B_{\text{mix}} = \frac{\Delta m^2 \sin^2 \theta}{8\pi^2} \int_0^K dk k^2 \left( \frac{1}{\omega_{k,1}} - \frac{1}{\omega_{k,2}} \right) ,
$$

(22)

where $K$ is the cut-off on the momenta.

- In the case of the axion-photon mixing, one has a value compatible with the estimated upper bound on the dark energy, $\rho_{\text{mix}}^{\text{axion}} = 2.3 \times 10^{-47} GeV^4$, for magnetic field strength $B \in [10^6 - 10^{17}] G$, axion mass $m_a \simeq 2 \times 10^{-4} eV$, $\sin^2 \theta \sim 10^{-2}$ and a Planck scale cut-off, $K \sim 10^{19} GeV$.

- In the case superpartners of the neutrinos, considering masses $m_1 = 10^{-3} eV$ and $m_2 = 9 \times 10^{-3} eV$, such that $\Delta m^2 = 8 \times 10^{-5} eV^2$ and $\sin^2 \theta = 0.3$, one obtains, $\rho^B_{\text{mix}} = 7 \times 10^{-47} GeV^4$ for a cut-off on the momenta $K = 10 eV$, and $\rho^B_{\text{mix}} = 6.9 \times 10^{-40} GeV^4$ for a cut-off of order of the Planck scale, $10^{19} GeV$. Smaller values of the mixing angle lead to values which are compatible with the estimated value of the dark energy also in the case in which the cut-off is $K = 10^{19} GeV$, indeed $\rho^B_{\text{mix}}$ depends linearly by $\sin^2 \theta$ [20].

6.2. Fermion mixing

For fermion mixing, energy density and pressure become

$$
\rho^F_{\text{mix}} = -\langle 0| : \sum_i \left[ \psi_i^\dagger(-\theta,x)\gamma_0\gamma^j \partial_j \psi_i(-\theta,x) + m\psi_i^\dagger(-\theta,x)\gamma_0\psi_i(-\theta,x) \right] : |0\rangle ,
$$

(23)

$$
p^F_{\text{mix}} = i\langle 0| : \sum_i \left[ \psi_i^\dagger(-\theta,x)\gamma_0\gamma^j \partial_j \psi_i(-\theta,x) \right] : |0\rangle ,
$$

(24)
with $\psi_i(-\theta, x)$ flavor neutrino fields or the quark fields. Being

$$\langle 0 | : \sum_i \bar{\psi}_i(-\theta, x) \gamma^0 \gamma_j \partial_j \psi_i(-\theta, x) : | 0 \rangle = 0,$$

one has

$$\rho_{mix}^F = -(0) | : \sum_i [m_i \bar{\psi}_i(-\theta, x) \gamma^0 \psi_i(-\theta, x) : | 0 \rangle, \quad (25)$$

$$p_{mix}^F = 0. \quad (26)$$

The state equation is then $w_{mix}^F = 0$, which is the one of the dark matter. The energy density of the fermion mixed vacuum is

$$\rho_{mix}^F = \frac{\Delta m \sin^2 \theta}{2 \pi^2} \int_0^K dk k^2 \left( \frac{m_2}{\omega_{k,2}} - \frac{m_1}{\omega_{k,1}} \right).$$

For masses of order of $10^{-3} eV$, such that $\Delta m^2 \simeq 8 \times 10^{-5} eV^2$, one has $\rho_{mix}^F = 4 \times 10^{-47} GeV^4$ for a cut-off on the momenta $K = m_1 + m_2$. Such a value is in agreement with the estimated upper bound of the dark matter. For $K$ of order of the Plank scale one has $\rho_{mix}^F \sim 10^{-46} GeV^4$ [20]. Notice that for quarks, the confinement inside the hadrons inhibits the gravitational interaction of the quark vacuum condensate, thus we suppose that hadronic quark condensate should not contribute to the dark matter of the universe.

7. Conclusions

The vacuum condensates generated by many phenomena can give contributions to the dark energy and to the dark matter of the universe. In particular, the thermal states of the intercluster medium, the vacuum of fields in curved space-time and the flavor vacuum of neutrinos have a behavior similar to one of dark matter, while the axion-photon mixing reproduces the behavior and the estimated value of the dark energy.

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