Supernova neutrino scattering on the $^{56}$Fe nucleus at finite temperatures

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Abstract. Thermal effects on the neutral-current inelastic neutrino-nucleus scattering in a supernova environment are examined by using the thermal quasi-particle random phase approximation (TQRPA). We concentrate on the total cross section of neutrino scattering on the nucleus $^{56}$Fe, which plays a significant role in core-collapse supernova dynamics. The calculations are performed for several nuclear temperatures relevant for supernova physics and for incoming neutrino energies up to 60 MeV. Our results show that finite temperature effects cause a significant enhancement in the cross section for low-energy neutrinos. These findings are in agreement with previous large-scale shell-model calculations where such an increase is closely related to Gamow-Teller transitions stemming from thermally populated nuclear states.

1. Introduction
Supernovae, the luminous stellar explosions, are among the most energetic phenomena in the known universe. Since most of the supernova explosion energy is radiated in neutrinos, an essential part of supernova simulations is a detailed treatment of neutrino transport including the interactions of neutrinos with the supernova environment. One of these interaction processes is neutral-current inelastic neutrino-nucleus scattering (INNS). This process contributes to the thermalization of neutrinos with the supernova matter and increases noticeably the opacity for high-energy neutrinos after the bounce [1].

Neutrino-induced reactions in a supernova environment occur at finite temperatures, for which the thermal population of nuclear excited states becomes possible. As it was first realized in Ref. [2], transitions from these states can contribute to a significant enhancement in the INNS cross section for low-energy neutrinos.

Recently, thermal effects on the inelastic neutrino-nucleus scattering cross section were studied for iron-group nuclei within a large-scale shell-model (LSSM) diagonalization approach [3]. This approach provides a detailed strength distribution of Gamow-Teller (GT) transitions that strongly dominate the INNS cross section at low neutrino energies. Nevertheless, this method has its own shortcoming mainly due to the fact that the LSSM partially employs the Brink hypothesis when treating GT transitions from nuclear excited states. Here, we present an alternative approach to treat thermal effects on supernova neutrino-nucleus scattering cross sections.

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2. Description of the method

Our approach is based on the quasi-particle random phase approximation (for zero temperature) and extended to finite temperatures (TQRPA) by the thermofield dynamics formalism [4] which treats a many-particle system in thermal equilibrium with a heat bath and a particle reservoir in the grand canonical ensemble. In Ref. [5] TQRPA was applied to study thermal effects on electron capture in a supernova environment. The generalization of the method to study neutrino-nucleus scattering at finite temperature was introduced in Ref. [6]. TQRPA does not rely on Brink’s hypothesis and allows to calculate the strength distribution for various multipole transitions as a function of the nuclear temperature. In our study we use a phenomenological nuclear Hamiltonian of the quasiparticle-phonon nuclear model (QPM) [7]. It consists of a spherically symmetric Woods-Saxon mean-field potential for protons and neutrons, BCS pairing interaction and residual separable multipole and spin-multipole particle-hole interactions in the isoscalar and isovector channels. All parameters in the QPM Hamiltonian are fitted to reproduce experimental data at zero temperature. The cross section calculations are performed with the exact momentum-dependent multipole transition operators [8].

3. The results

We have employed the TQRPA framework in modeling the neutral-current neutrino scattering on the nucleus $^{56}$Fe. The respective ground-state cross section and its multipole composition are displayed in Fig. 1. As seen from the figure, at low neutrino energies the cross section is dominated by $1^+$ transitions. However, as the energy increases to 60 MeV, the role of other multipoles becomes important, in particular the contribution of $1^-$ and $2^\pm$ transitions. Inelastic neutrino scattering on $^{56}$Fe have been also studied in resent works based on QRPA [9, 10]. For comparison, the cross section obtained by using realistic two-body forces (Bonn CD potential) [9] is also shown in Fig. 1. Although for $E_\nu > 10$ MeV the cross sections from the two models are in a good quantitative agreement, for low-energy neutrinos the cross section from the present study appears up to an order of magnitude smaller than the one from [9]. This discrepancy originates from different parametrization of single particle energies and residual interaction.

During the collapse, inelastic neutrino-nucleus scattering is most important in the thermalization phase when neutrinos are produced mainly by electron capture on free protons and hence have energies of order $8-15$ MeV [1]. At these neutrino energies, we can replace the exact $1^+$ transition operator by the Gamow-Teller operator $\vec{\sigma}t_0$. In Fig. 2 we present the temperature evolution of GT$_0$ strength for $^{56}$Fe. At zero temperature the GT$_0$ strength is mainly concentrated in the resonance region around $8-10$ MeV. Moreover, our QRPA calculations predict an energy gap about 4 MeV between the ground state and first excited $1^+$ state in $^{56}$Fe, which translates...
into an energy threshold for inelastic neutrino scattering at zero temperature. With increasing temperature two effects occur in our model that influence the $\text{GT}_0$ strength:

(i) At low temperatures, due to pairing, GT transitions involve the breaking of a Cooper pair associated with some energy cost. This extra energy is removed at temperatures higher the critical one ($T_{\text{cr}} \approx 0.8\text{ MeV}$). As a result the position of $\text{GT}_0$ resonance depends on $T$ and with temperature increase it moves to lower excitation energies. This indicates a violation of the Brink hypothesis within the present approach.

(ii) Thermal smearing of proton and neutron Fermi surfaces unblocks GT transitions which are Pauli blocked at $T = 0$ and makes possible transitions from thermally occupied orbitals to low-lying orbitals. This results in the appearance of low- and negative-energy $\text{GT}_0$ strength. As a consequence there is no reaction threshold for INNS at finite temperatures and, as we will see, the cross section significantly increases for low neutrino energies.

Negative energy transitions correspond to transitions from thermally populated nuclear states to states at lower excitation energies. Note that the strength at around $\omega \approx -9\text{ MeV}$ corresponds to de-excitation of the thermally excited $\text{GT}_0$ resonance. Negative energy transitions contribute to up-scattering of neutrinos, as their energy in the final state is larger than in the initial state.

The thermal INNS cross section for $^{56}\text{Fe}$ is calculated taking into account the exact momentum dependence of multipole transition operators ($J^\pi \leq 6^+$) [8]. Our calculations do not include neutrino blocking in the final state. In the left panel of Fig. 3 the thermal cross section is shown for the same temperatures as those in Fig. 2. The right panel of Fig. 3 demonstrates the contribution of $1^+$ transitions to the cross section. Comparing the left and right panels, we conclude that for all relevant temperatures the $1^+$ contribution dominates the cross section for neutrinos with $E_\nu \leq 30\text{ MeV}$.

As it follows from our calculations, as well as from shell-model study [3], finite temperature effects are unimportant for $E_\nu \geq 15\text{ MeV}$ where excitation of the $\text{GT}_0$ resonance becomes possible and dominates the cross section. At lower neutrino energies, however, the cross section significantly depends on temperature. Thermal effects enhance the low energy cross section by up two orders of magnitude as the temperature rises from 0.86 MeV to 1.72 MeV. Again, comparing the left and right panels of Fig. 3, we conclude that the obtained enhancement is merely due to thermally unblocked GT transitions.

The relative importance of negative energy GT transitions can be demonstrated by introducing

Figure 2. $\text{GT}_0$ strength built on the ground-state of $^{56}\text{Fe}$ and for three different nuclear temperatures relevant for supernova physics ($0.1\text{MeV} \approx 1.2 \times 10^9\text{ K}$). All strength distributions are plotted as a function of energy transfer to the nucleus. The $\text{GT}_0$ resonance at around 10 MeV is mainly formed by the proton and neutron single-particle transitions $1f_{7/2} \rightarrow 1f_{5/2}$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{GT$_0$ strength built on the ground-state of $^{56}\text{Fe}$ and for three different nuclear temperatures relevant for supernova physics ($0.1\text{MeV} \approx 1.2 \times 10^9\text{ K}$). All strength distributions are plotted as a function of energy transfer to the nucleus. The GT$_0$ resonance at around 10 MeV is mainly formed by the proton and neutron single-particle transitions $1f_{7/2} \rightarrow 1f_{5/2}$.}
\end{figure}
the ratio $\sigma_u/\sigma$, where $\sigma_u$ is the up-scattering contribution to the cross section. In the inset of Fig. 3 this ratio is plotted as a function of the initial neutrino energy for selected temperatures. As expected, the ratio $\sigma_u/\sigma$ is small for $E_\nu > 15$ MeV where neutrino down-scattering is a dominating process and temperature effects are unimportant. Contrary, at low neutrino energies, the up-scattering process dominates the cross section. We also see that the relative contribution of $\sigma_u$ increases with temperature, since a larger fraction of nuclear excited states can be populated.

4. Conclusion

We have presented the approach that allows calculations of inelastic neutrino-nucleus scattering cross section at finite temperature under supernova conditions. Our calculations for $^{56}$Fe (as well as for $^{54}$Fe [6]) have revealed the same thermal effects as were found within LSSM studies[3]: (i) the temperature increase enhances the neutrino scattering cross section for low-energy neutrinos; (ii) the increase in the cross section is related to de-excitation of thermally populated nuclear states. What is more, the calculated INNS cross sections are close to the LSSM values. Thus, the results of our study show that the present approach provides a valuable tool for the evolution of the INNS cross section under supernova conditions.

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References

[1] Bruenn S W and Haxton W S 1991 Astrophys. J. 376 678
[2] Fuller G M and Meyer B S 1991 Astrophys. J. 376 701
[3] Juodagalvis A, Langanke K, Martínez-Pinedo G, Hix, W R, Dean D J, Sampaio J M 2005 Nucl. Phys. A 747 87
[4] Takahashi Y, Umezawa H 1975 Collect. Phenom. 2 55
[5] Dzhioev A A, Vdovin A I, Ponomarev V Y, Wambach J, Langanke K, and Martínez-Pinedo G 2010 Phys. Rev. C 81 015804
[6] Dzhioev A A, Vdovin A I, Ponomarev V Y, Wambach J 2011 Phys. At. Nucl. 74 1162
[7] Soloviev V G 1992 Theory of atomic nuclei: quasiparticles and phonons (Bristol: Institute of Physics)
[8] Walecka J D 1972 Semi-leptonic weak interactions in nuclei Muon Physics ed V W Hughes and C S Wu (New York: Academic Press)
[9] Tsakstara V, Kosmas T S, Dzhioev A A, Vdovin A I Journal of Physics: Conference Series, submitted
[10] Dapo H and Paar N 2012 Phys. Rev. C 86 035804