Discover potential of the next-to-minimal supergravity-motivated model

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Applying a likelihood analysis to the next-to-minimal supergravity-motivated model, we identify parameter space regions preferred by present experimental limits from collider, astrophysical, and low energy measurements. We then show that favored regions are amenable to detection by a combination of the CERN Large Hadron Collider and an upgraded Cryogenic Dark Matter Search, provided that the more than three sigma discrepancy in the difference of the experimental and the standard theoretical values of the anomalous magnetic moment of the muon prevails in the future.

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I. INTRODUCTION

Supersymmetry is very successful in solving outstanding problems of the standard model (SM) of elementary particles. The theory naturally explains the dynamics of electroweak symmetry breaking while preserving the hierarchy of fundamental energy scales, it incorporates dark matter and the asymmetry between baryons and antibaryons, it reconciles the unification of gauge forces and accommodates gravity, and more [1]. Thus, it is important to examine the prospects of the CERN Large Hadron Collider (LHC) finding supersymmetry.

One of the main motivations for supersymmetry is that it can naturally bridge the hierarchy between the weak and Planck scales. Unfortunately, the presence of the superpotential $\mu$ term in the minimal supersymmetric extension of the standard model (MSSM) undermines this very aim [1]. Experimental data have also squeezed the MSSM into fine-tuned regions, creating the supersymmetry little hierarchy problem [18]. Extensions of the MSSM by gauge singlet superfields not only resolve the $\mu$ problem, but can also reduce the little hierarchy [2, 3, 4]. In the next-to-minimal MSSM (NMSSM), the $\mu$ term is naturally generated and no dimensionful parameters are introduced in the superpotential (other than the vacuum expectation values that are all naturally weak scale), making the NMSSM a truly natural model [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22].

Over the last two decades, due to its simplicity and elegance, the constrained MSSM (CMSSM) and the minimal supergravity-motivated (mSuGra) model became a standard in supersymmetry phenomenology. Guided by this, within the NMSSM, we impose the universality of sparticle masses, gaugino masses, and trilinear couplings at the grand unification theory (GUT) scale, thereby defining the next-to-minimal supergravity-motivated (NmSuGra) model. This approach ensures that all dimensionful parameters of the NMSSM scalar potential also naturally arise from supersymmetry breaking in a minimal fashion. NmSuGra also reduces the electroweak and dark matter fine-tunings of mSuGra.

Using a simple likelihood analysis, first we identify the parameter regions of the NmSuGra model that are preferred by the present experimental limits from collider, astrophysical, and various low energy measurements. We combine theoretical exclusions with limits from the CERN Large Electron-Positron (LEP) collider, the Fermilab Tevatron, NASA’s Wilkinson Microwave Anisotropy Probe (WMAP) satellite (and other related astrophysical measurements), the Soudan Cryogenic Dark Matter Search (CDMS), the Brookhaven Muon g–2 Experiment, and various b-physics measurements including the rare decay branching fractions $b \to s\gamma$ and $B_s \to l^+l^-$. Next we show that the favored parameter space can be detected by a combination of the LHC and an upgraded CDMS, provided that the $3\sigma$ discrepancy between the experimental and the standard theoretical values of the anomalous magnetic moment of the muon ($\Delta a_\mu$) persists in the future.

II. THE NMSUGRA MODEL

In this work, we adopt the superpotential

$$W = W_Y + \lambda \tilde{S} H_u \cdot \tilde{H}_d + \frac{\kappa}{3} \tilde{S}^3,$$

where $W_Y$ is the MSSM Yukawa superpotential [23], $\tilde{S} (\tilde{H}_{u,d})$ is a standard gauge singlet (SU(2)$_L$ doublet) chiral superfield, $\lambda$ and $\kappa$ are dimensionless couplings, and $H_u \cdot H_d = \epsilon_{\alpha\bar{\beta}} H_u^\alpha H_d^\beta$ with the fully antisymmetric tensor normalized as $\epsilon_{11} = 1$. The corresponding soft supersymmetry breaking terms are

$$L^{soft} = L^{soft}_{MSSM} + m_S^2 |S|^2 + (\lambda A_S H_u \cdot H_d + \frac{\kappa A}{3} \tilde{S}^3 + h.c.),$$

where $L^{soft}_{MSSM}$ contains the mass and Yukawa terms but not the $B\mu$ term.
The superpotential \( \Pi \) possesses a global \( Z_3 \) symmetry which is broken during the electroweak phase transition in the early universe. The resulting domain walls should disappear before nucleosynthesis; however \( Z_3 \) breaking (via singlet tadpoles) leads to a vacuum expectation value (vev) for the singlet that is much larger than the electroweak scale. Thus the requirement of the fast disappearance of the domain walls appears to destabilize the hierarchy of vevs in the NMSSM. Fortunately, in Ref. [26, 27] it was shown that, by imposing a \( Z_2 \) R-symmetry, both the domain wall and the stability problems can be eliminated. Following [26], we assume that tadpoles are induced, but they are small and their effect on the phenomenology is negligible.

We also assume that the soft masses of the gauginos unify to \( M_{1/2} \), those of the sfermions and Higgses to \( M_0 \), and all the trilinear couplings (including \( A_\kappa \) and \( A_\lambda \)) to \( A_0 \) at the GUT scale. This leaves nine free parameters: \( M_0, M_{1/2}, A_0, \langle H_u \rangle, \langle H_d \rangle, \langle S \rangle \) (the Higgs and singlet vevs), \( m_\Sigma, \lambda, \kappa \). The three minimization equations for the Higgs potential \( \Pi \) and \( \langle H_u \rangle^2 + \langle H_d \rangle^2 = v^2 \) (here \( v = \sqrt{2/g_2} \) is the standard Higgs vev) eliminate four parameters. With the introduction of \( \mu = \lambda_i(S) \), and \( \tan \beta = (\langle H_u \rangle/\langle H_d \rangle) \), our remaining free parameters are

\[
M_0, M_{1/2}, A_0, \tan \beta, \lambda, \kappa, \mu, \text{sign}(\mu).
\]

Constrained versions of the NMSSM have been studied in the recent literature. The most constrained version is the cNMSSM \([28]\) with \( m_S = M_0 \). In other cases the \( A_\kappa = A_\lambda \) relation is relaxed \([28]\), and/or \( \kappa \) is taken as a free parameter \([29, 30]\), or the soft Higgs masses are allowed to deviate from \( M_0 \) \([32]\) giving less constrained models. In the spirit of the CMSSM/mSuGra, we adhere to universality and use only \( \lambda \) to parametrize the singlet sector. This way, we keep all the attractive features of the CMSSM/mSuGra while the minimal extension alleviates problems rooted in the MSSM.

Our goal is to show that the experimentally favored region of the NmSuGra model can be discovered by nascent experiments in the near end. To this end, we use the publicly available computer code NMSPEC \([33]\) to calculate the spectrum of the superpartner masses and their physical couplings from the model parameters given in Eq. (3). Then, we use NMSSMTools 1.2.5, an extensively modified version of ISATools and that of DarkSUSY 4.1 \([34]\) to calculate the relic density of neutralinos (\( \Omega \)), the spin-independent neutralino-proton elastic scattering cross section (\( \sigma_{SI} \)), the NmSuGra contribution to the anomalous magnetic moment of the muon (\( a_\mu^{NnSuGra} \)), and various b-physics related quantities.

For each set of the model parameters, we quantify the experimental preference in terms of

\[
\sqrt{\chi^2} = \left( \sum_{i=1}^{7} \left( \frac{m_i^{\text{experiment}} - m_i^{\text{NmSuGra}}}{\sigma_i} \right)^2 \right)^{1/2}
\]

where \( m_i \) is the central value of a physical quantity measured by an experiment or calculated in the NnSuGra model, and \( \sigma_i \) is the combined experimental and (where available) theoretical uncertainty. The sum includes the experimental upper limits \([50]\) for

1. \( \Omega h^2 = 0.1143 \pm 0.0034 \) \([53]\),
2. \( Br(B_s \rightarrow \mu^+\mu^-) = 5.8 \times 10^{-8} \) (95 % CL) \([54]\), and
3. \( \sigma_{SI} \) by CDMS \([57]\),

the LEP lower limits of the lightest scalar Higgs and chargino masses (which can be approximately stated as) \([38]\):

4. \( m_{h} > 114.4 \text{ GeV} \) for \( \tan \beta \lesssim 10 \)
5. \( m_{h} > 91 \text{ GeV} \) for \( \tan \beta \gtrsim 10 \),
6. \( m_{\tilde{W}_L} > 104 \text{ GeV} \),
7. \( m_{\tilde{t}} > 4214 \text{ GeV} \) and \( m_{\tilde{t}}^{pole} = 171.4 \text{ GeV} \) are used.

A glance at experimental likelihood reveals a significant statistical preference for relatively narrow intervals of \( M_0, M_{1/2} \) and \( A_0 \). This is illustrated in FIG. II which shows \( \sqrt{\chi^2} \) as the function of \( M_0, M_{1/2}, A_0 \) and \( \tan \beta \) for sign(\( \mu > 0 \)) for a randomly selected set of models \([51]\). From FIG. II we conclude that it is enough to examine the low \( M_0, M_{1/2} \) and \( |A_0| \) region, since the rest is disfavored by the above combination of the experimental data at (or more than) 99 % CL. Similarly to the
CMSSM/mSuGra, at high values of $M_0$, $M_{1/2}$ and $|A_0|$, $\chi^2$ is dominated by $\Delta a_\mu$.

Based on the above, we limit our study to the following ranges of the continuous parameters [52]:

$$0 < M_0 < 4 \text{ TeV}, \quad 0 < M_{1/2} < 2 \text{ TeV},$$

$$0 < |A_0| < 5 \text{ TeV}, \quad 1 < \tan \beta < 60, \quad 0.01 < \lambda < 0.7.$$ \hspace{1cm} (5)

The upper limit on $\lambda$ arises from the requirement that its running remains perturbative as it evolves up to the GUT scale. Restricting $M_0$, $M_{1/2}$ and $A_0$ to such values also appears consistent with the electroweak precision data [39] and greatly reduces fine tuning.

### III. DETECTABILITY OF NMSUGRA

Having defined the NmSuGra model and constraining the range of its parameters, we set out to show that this parameter region will be detectable by the LHC and an upgraded CDMS detector. To this end, we randomly select about 20 million models in the range defined in Eq. (5), and for each model point evaluate Eq. (4). Two million theoretically allowed representative model points are projected in FIG. 2 to the $\Omega h^2$ vs. $(N_{1j}^2 + N_{2j}^2)/(1 - N_{11}^2 - N_{22}^2)$ plane [40]. This plane of $\Omega h^2$ and the neutralino mixing matrix elements ($N_{ij}$) is a good indicator of the gaugino, higgsino and singlino admixture of the lightest neutralino. From FIG. 2 it is evident that the WMAP upper limit (green horizontal line) allows models with mostly bino- (red) and higgsino-like (magenta) lightest neutralino, while the fraction of allowed models with singlino-like (blue) dark matter is negligible. As such the NmSuGra model is very similar to the CMSSM/mSuGra.

The similarity is even more evident when we examine the grouping of the WMAP allowed model points. By checking mass relations and couplings, we can easily establish that the various branches (denoted by 1, 2, 3 and 4) belong to models with distinct neutralino (co)annihilation mechanisms well known from the CMSSM/mSuGra [41]. Branch 4 contains only models with dominant neutralino-stop coannihilation, while branch 3 corresponds neutralino-stau coannihilation. Branch 2 represents the Higgs resonance corridors, and branch 1 is the equivalent of the CMSSM/mSuGra focus point region.

To gauge the detectability of the NmSuGra model, first we identify model points that could have been seen at LEP. We require the lighter chargino to be lighter than 103.5 GeV or the lightest scalar Higgs to be lighter than 114.4 GeV. We relax the latter to $m_{H^+_1}$ when the mass of the lightest pseudo-scalar Higgs ($m_{H^+_1}$) approaches that of the lightest scalar ($m_{H^+_1}$). We do not apply the 114.4 GeV LEP Higgs limit when we encounter either a mostly singlet lightest Higgs or when $m_{H^+_1} < m_{H^+_1}$ [52, 42]. Focusing our attention to the lower right corner of FIG. 2 in the top left frame of FIG. 3 we show the same model points in different coloring. Models detectable by LEP are colored green, that is the green points represent the reach of LEP for the NmSuGra model. The red colored models pass the above LEP constraints, i.e. are allowed by LEP. Just as in the CMSSM/mSuGra the neutralino-stop coannihilation region is mostly covered by LEP.

To estimate the LHC reach, we rely on the similarity between the mSuGra and NmSuGra models. According to Ref. [44] the reach of the LHC for mSuGra can be well approximated by the combined reach for gluinos and squarks. Based on this, if either the gluino mass is below 1.75 TeV, or the geometric mean of the stop masses is below 2 TeV for a given model point, we consider it discoverable at the LHC. While this is an approximate statement that has to be supported by a detailed study of LHC event generation in the given models, in the light of Ref. [44] it is a conservative estimate of the LHC reach.

The top right frame of FIG. 3 shows the model points that can be reached by LEP and the LHC using the above criteria. As in the CMSSM/mSuGra, most of the slepton coannihilation and the bulk of the Higgs resonance branches are covered by the LHC. A good part of the focus point is also within reach of the LHC, with the exception of models with high $M_0$ and/or $M_{1/2}$.

The bottom left frame of FIG. 3 shows the reach of a one ton equivalent of CDMS (CDMS1T). As expected from the CMSSM/mSuGra, the rest of the focus point and most of the remaining Higgs resonances are in the reach of CDMS1T. The small number of models that re-
FIG. 3: As FIG. 2 except green models can be reached by the combination of LEP, the LHC and CDMS1T, while red ones cannot. The last frame dismisses experimentally inaccessible points which have $\chi^2 > 3$.

main inaccessible are all located in regions that have relatively low $M_0$ and high $M_{1/2}$ with dominant neutralino annihilation via s-channel Higgs resonances. The NmSuGra contribution to $\Delta a_\mu$ in these model points is outside the preferred 99% CL region as shown by the last frame.

While we focused our discussion and plotting on the higgsino and gaugino regions, we carefully checked that LEP, the LHC and the upgraded CDMS experiments combined can also detect models with WMAP allowed singlino-like dark matter. The reason for this is the following. The singlino-like lightest neutralinos typically take part in more than one (co)annihilation mechanisms to satisfy the WMAP limit. While being on a Higgs resonance, they also coannihilate with sfermions. In this case, either being in the sfermion coannihilation region ensures the LHC detectability or the Higgs resonance is strong enough to enhance recoil detection.

Furthermore, the universal mass relations ensure that the lightest scalar Higgs never decays to a pair of the pseudo-scalar Higgs bosons in the phenomenologically allowed region of NmSuGra, just as in the cNMSSM. Similarly, the lightest singlinos, allowed by the present experimental constraints are always heavier than about 100 GeV. Assuming that the NmSuGra contribution to the anomalous magnetic moment of the muon is larger than a minute $3.1 \times 10^{-10}$ constrains slepton and chargino masses below 3 and 2.5 TeV, respectively. Since universality restricts the mass hierarchy within NmSuGra, the resulting mass spectrum is typically mSuGra-like. Thus, the cascade decays and their signatures at LHC are not expected to be significantly deviate from that of the mSuGra case. The most typical NmSuGra decay cascade at the LHC would be gluino $\rightarrow$ squark, quark $\rightarrow$ chargino/neutralino, W/Z $\rightarrow$ neutralino, SM particles.

IV. CONCLUSIONS

Analyzing the next-to-minimal supergravity motivated (NmSuGra) model, we found that the LHC and an upgraded CDMS experiment will be able to discover the experimentally favored region of this model, provided that the present deviation between the experimental and standard theoretical values of the muon anomalous magnetic moment prevails. If, due to future experimental input, the $g - 2$ constraint weakens, then certain parameter regions of the NmSuGra model will have to be detected in alternative ways.

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[50] The WMAP limit on the abundance of dark matter, for example, is only used as an upper bound, that is the $\Omega h^2$ contribution to $\chi^2$ vanishes if the calculated relic density is below the experimental central value.
[51] Positive values of the $\mu$ parameter are favored by $\Delta a_\mu$ and $Br(b \rightarrow s\gamma)$ [48].
[52] These limits arising mainly from the anomalous magnetic moment of the muon are very similar to the ones in the CMSSM [39].