A comparative study of one parameter lifetime distributions

Abstract

In this paper, a comparative study on some selected one parameter distributions has been carried out. The important properties of distributions have been compared using various datasets from engineering, biological sciences and other fields. The lifetime data have been taken from various fields of studies. Various proposed models have been applied on data to check goodness of fit and their behavior have been discussed with graphically.

Keywords: pranav distribution, akash distribution, shanker distribution, lindley distribution, exponential distribution, statistical properties, estimation of parameter, goodness of fit

Introduction

In the new era, uses of different life time distributions have been becoming more important because of increasing varieties of products and their survivors. Especially in reliability analysis, one can know failure rate as well time to survive of products, which can be calculated using different models. One parameter distribution can be applied easily way for any dataset, and its characteristics and mathematical properties can be calculated. Its applications are crucial in biostatistics as well as actuarial sciences and related field. The event may be failure of a piece of equipment, death of a person, development (or remission) of symptoms of disease, health code violation (or compliance). The modeling and statistical analysis of lifetime data are crucial for statisticians, research workers and policy makers in almost all applied sciences including engineering, medical science/biological science, insurance and finance, amongst others. Many statisticians have been proposed many distributions of one parameter and two parameters, but in this study, specially focused on some selected one parameter, most of them have been proposed recently. In this paper, author is tried to compare statistics of one parameter lifetime distributions using different lifetime data-sets from Engineering, medical sciences and social sciences. Different distributions have been proposed by different statisticians. Names of distributions of one parameter and their introducers are given in Table 1.

Table 1 Pdf of distributions and their introducers

| Name of distribution | Probability distribution function (pdf) | Introducers |
|----------------------|-----------------------------------------|-------------|
| Exponential distribution | \( f(x) = \theta e^{-\theta x} \) | |
| Lindley distribution | \( f(x) = \frac{\theta^2}{\theta + 1} (1 + x) e^{-\theta x} \) | Lindley¹ |
| Akash distribution | \( f(x) = \frac{\theta^2}{\theta^2 + 2} (1 + x^2) e^{-\theta x} \) | Shanker² |
| Pranav Distribution | \( f(x) = \frac{\theta^3}{\theta^2 + 2} (1 + x^3) e^{-\theta x} \) | Shukla³ |
| Ishita Distribution | \( f(x) = \frac{\theta^3}{\theta^2 + 2} (\theta + x^2) e^{-\theta x} \) | Shanker & Shukla⁴ |
| Ram Awadh distribution | \( f(x) = \frac{\theta^6}{\theta^6 + 120} (\theta + x^3) e^{-\theta x} \) | Shukla⁵ |
| Prakaamy distribution | \( f(x) = \frac{\theta^6}{\theta^6 + 120} (1 + x^5) e^{-\theta x} \) | Shukla⁶ |
| Sujatha distribution | \( f(x) = \frac{\theta^3}{\theta^2 + \theta + 2} (1 + x + x^2) e^{-\theta x} \) | Shanker⁷ |

Characteristics of distributions

In this section, different distributions have been compared according to their behavior, moments and dispersion numerically as well as graphically. This section covers behavior of distributions (pdf), coefficient of variation, and coefficient of skewness, kurtosis and index of distribution respectively. Basically these distributions are continuous and known as lifetime distributions can be applied for biological, engineering and agricultural studies. Detailed studies including behavior, moments, stress & strength reliability, parameter estimation and etc. about above mentioned distributions can be shown in their paper. In statistical literature, exponential distribution was first studied by Epstein (1940) and widely used as lifetime model in different fields. The main reason for its wide use and applicability.
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as lifetime model because it is simple to apply on any datasets, and
another important use of this distribution is in the reliability field.
Lindley distribution is introduced by Lindley\(\textsuperscript{1-7}\) and further it’s studied
by Ghitani et al.\(\textsuperscript{1}\) where nature and behavior of Lindley distribution
including mathematical properties can be shown in their paper. They
applied Lindley distribution in waiting time of customer in Bank
and showed that its suitability over other distributions. Ram Awadh
distribution has been introduced and studied by Shukla (2018b),\(\textsuperscript{5}\)
and he showed its superiority over other one parameter life time
distribution in his paper. Similarly other one parameter distributions as
above mentioned are also studied in detailed by different researchers
and shows their superiority over other one parameter distributions.

Let \(X\) be a continuous random variable representing the lifetimes
of individuals in some population. The expressions for probability
density function, \(f(x)\), cumulative distribution function, \(F(x)\),
have been presented in Figure 1 and 2. From the Figure 1, it is
clear that pdf of almost all distributions are increasing as increased
value of parameter. Especially pdf of exponential distribution is
increasing more in comparison to other distributions as increased
value of parameter. Pattern of almost all distributions are same except
exponential distribution.
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Figure 2 Cdf plots of different distributions for varying value of parameter.

Mathematical constants

Coefficient of variation (C.V.), coefficient of skewness \( \sqrt{\beta} \), coefficient of kurtosis \( 2 \beta \), and index of dispersion \( \gamma \) of above mentioned distributions in the Table 1 have been compared. The graphs of C.V., \( \sqrt{\beta} \), \( \beta \), and \( \gamma \) of distributions for varying values of the parameter \( \theta \) are shown in Figure 2.

From the Figure 3, it is observed that coefficient of variation, coefficient of skewness and coefficient of kurtosis of Ram Awadh, Prakaamy and Pranav distributions are increasing vastly up to certain points then decreasing as increased value of parameter. As we know that coefficient of variation, coefficient of skewness and coefficient of kurtosis of exponential distribution are independent from theta whereas value of index of dispersion is decreasing for all distributions as increased value of theta. The conditions under which Akash, Shanker and Lindley distributions are over-dispersed \( \mu < \sigma^2 \), equi-dispersed \( \mu = \sigma^2 \), and under-dispersed \( \mu = \sigma^2 \) are summarized in Table 2.

Parameter estimation

In this section, estimation of parameter using maximum likelihood method for Prakaamy, Sujatha, Ram Awadh, Pranav, Akash, Ishita, Lindley and Exponential distributions have been given respectively.

| Distribution  | Over-dispersion \( \mu < \sigma^2 \) | Equi-dispersion \( \mu = \sigma^2 \) | Under-dispersion \( \mu > \sigma^2 \) |
|---------------|-----------------------------------|---------------------------------|-----------------------------------|
| Ram Awadh     | \( \theta < 1.044533 \)           | \( \theta = 1.044533 \)         | \( \theta > 1.044533 \)           |
| Prakaamy      | \( \theta = 1.0421856 \)          | \( \theta = 1.0421856 \)        | \( \theta > 1.0421856 \)          |
| Sujatha       | \( \theta < 1.364271 \)           | \( \theta = 1.364271 \)         | \( \theta > 1.364271 \)           |
| Pranav        | \( \theta < 1.9853197 \)          | \( \theta = 1.9853197 \)        | \( \theta > 1.9853197 \)          |
| Ishita        | \( \theta < 1.53565315 \)         | \( \theta = 1.53565315 \)       | \( \theta > 1.53565315 \)         |
| Akash         | \( \theta < 1.515400063 \)        | \( \theta = 1.515400063 \)      | \( \theta > 1.515400063 \)        |
| Lindley       | \( \theta < 1.170086487 \)        | \( \theta = 1.170086487 \)      | \( \theta > 1.170086487 \)        |
| Exponential   | \( \theta < 1 \)                  | \( \theta = 1 \)                | \( \theta > 1 \)                  |

Prakaamy distribution

Let \( (t_1, t_2, \ldots, t_n) \) be a random sample of size \( n \) from Prakaamy distribution. The maximum likelihood function, \( L \) of Prakaamy is given by

\[
L = \left( \frac{\theta^n}{\theta^3 + 120} \right) \prod_{i=1}^{n} (1 + t_i) e^{-n\theta} 
\]

\[
\ln L = n \left( \ln \theta - \ln(\theta^3 + 120) \right) + \sum_{i=1}^{n} (1 + t_i) - n\theta
\]

Sujatha distribution

Let \( (t_1, t_2, \ldots, t_n) \) be a random sample of size \( n \) from Sujatha distribution. The maximum likelihood function, \( L \) of Sujatha is given by

\[
L = \left( \frac{\theta^n}{\theta^3 + 120} \right) \prod_{i=1}^{n} (1 + t_i) e^{-n\theta} 
\]

\[
\ln L = n \left( \ln \theta - \ln(\theta^3 + 120) \right) + \sum_{i=1}^{n} (1 + t_i) - n\theta
\]

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The natural log likelihood function can be obtained as

\[ \ln L = n \left( \ln \theta^2 - \ln(\theta^2 + 120) \right) + \sum_{i=1}^{n} (\theta + t^2) - n\theta \tau \]

\[ \ln L = n \left( \ln \theta^2 - \ln(\theta^2 + 6) \right) + \sum_{i=1}^{n} (\theta + t^2) - n\theta \tau \]

\[ \ln L = n \left( \ln \theta^2 - \ln(\theta^2 + 1) \right) + \sum_{i=1}^{n} (\theta + t^2) - n\theta \tau \]

\[ \ln L = n \left( \ln \theta^2 - \ln(\theta^2 + 2) \right) + \sum_{i=1}^{n} (\theta + t^2) - n\theta \tau \]

Pranav distribution

Let \( \{t_1, t_2, t_3, ..., t_n\} \) be a random sample of size \( n \) from Pranav distribution. The maximum likelihood function, \( L \) of Pranav is given by

\[ L = \left( \frac{\theta^2}{(\theta^2 + 6)} \right)^n \prod_{i=1}^{n} (\theta + t^2) e^{-n\theta \tau} \]

Akash distribution

Let \( \{t_1, t_2, t_3, ..., t_n\} \) be a random sample of size \( n \) from Akash distribution. The maximum likelihood function, \( L \) of Akash is given by

\[ L = \left( \frac{\theta^2}{(\theta^2 + 1)} \right)^n \prod_{i=1}^{n} (1 + t^2) e^{-n\theta \tau} \]

Ishita distribution

Let \( \{t_1, t_2, t_3, ..., t_n\} \) be a random sample of size \( n \) from Ishita distribution. The maximum likelihood function, \( L \) of Ishita is given by

\[ L = \left( \frac{\theta^3}{(\theta^3 + 2)} \right)^n \prod_{i=1}^{n} (\theta + t^2) e^{-n\theta \tau} \]

\[ \ln L = n \left( \ln \theta^3 - \ln(\theta^3 + 2) \right) + \sum_{i=1}^{n} (\theta + t^2) - n\theta \tau \]

Lindley distribution

Let \( \{t_1, t_2, t_3, ..., t_n\} \) be a random sample of size \( n \) from Lindley distribution. The maximum likelihood function, \( L \) of Lindley is given by
The natural log likelihood function can be obtained as
\[
\ln L = n \ln \theta^2 - n \ln(\theta + 1) + \sum_{i = 1}^{n} (1 + t_i e^{-\theta t_i})
\]

**Applications and goodness of Fit**

In this section the goodness of fit test of Prakaamy, Sujatha, Ram Awadh, Pranav, Akash, Ishita, Lindley and exponential distributions for following eighteen real lifetime data-sets using maximum likelihood estimate have been discussed.

**Data set 1**: The data set represents the strength of 1.5cm glass fibers measured at the National Physical Laboratory, England. Unfortunately, the units of measurements are not given in the paper, and they are taken from Smith and Naylor. The data set is as follows:

0.55 0.93 1.25 1.36 1.49 1.52 1.58 1.61 1.64 1.68 1.73 1.81 2.00 0.74 1.04 1.27 1.39 1.49 1.53 1.59 1.61 1.66 1.68 1.76 1.82 2.01 0.77 1.11

**Data set 2**: The data is given by Birnbaum and Saunders on the fatigue life of 6061 – T6 aluminum coupons cut parallel to the direction of rolling and oscillated at 18 cycles per second. The data set consists of 101 observations with maximum stress per cycle 31,000 psi. The data \((5 \times 10^3)\) are presented below (after subtracting 65).

5 25 31 32 34 35 38 39 39 40 42 43 43
43 44 44 47 47 48 49 49 49 51 54 55 55
55 56 56 56 58 59 59 59 59 63 63 64
64 65 65 65 66 66 66 66 66 67 67 67 68
69 69 69 69 71 71 72 73 73 74 74 76
76 77 77 77 77 77 77 79 79 80 81 83 83
84 86 86 87 90 91 92 92 92 93 94 97
98 98 99 101 103 109 136 147

**Data Set 3**: The data set is from Lawless. The data given arose in tests on endurance of deep groove ball bearings. The data are the number of millions revolutions before failure for each of the 23 ball bearings in the life tests and they are:

17.88 28.92 33.00 41.52 42.12 45.60 48.80 51.84 51.96 54.12 55.56 67.80
68.44 68.64 68.88 84.12 93.12 98.64 105.12 105.84 127.92 128.04 173.40

**Data set 4**: The data is from Picciotto and arose in test on the cycle at which the Yarn failed. The data are the number of cycles until failure of the yarn and they are:

8 66 146 251 653 98 249 400 292 131 169 175 176 76
264 15 364 195 262 88 264 157 220 42 321 180 198
38 20 61 121 282 224 149 180 325 250 196 90 229
166 38 337 65 151 341 40 40 135 597 246 211 180
93 315 353 571 124 279 81 186 497 182 423 185 229
400 338 290 398 71 246 185 188 568 55 55 61 244
20 284 393 396 203 829 239 236 286 194 277 143 198
264 105 203 124 137 135 350 193 188

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Data set 5: This data represents the survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli, observed and reported by Bjerkedal.\textsuperscript{13}

12 15 22 24 24 32 32 33 34 38 38 43 44
48 52 53 54 54 55 56 57 58 58 59 60 60
60 60 61 62 63 65 65 67 68 70 70 72 73
75 76 76 81 83 84 85 87 91 95 96 98 99
109 110 121 127 129 131 143 146 146 175 175 211 233
258 258 263 297 341 341 341 376

Data set 6: This data is related with behavioral sciences, collected by N. Balakrishnan, Victor Leiva and Antonio Sanhueza.\textsuperscript{14} The scale “General Rating of Affective Symptoms for Preschoolers (GRASP)” measures behavioral and emotional problems of children, which can be classified with depressive condition or not according to this scale. A study conducted by the authors in a city located at the south part of Chile has allowed collecting real data corresponding to the scores of the GRASP scale of children with frequency in parenthesis, which are:

19(16) 20(15) 21(14) 22(9) 23(12) 24(10) 25(6) 26(9) 27(8) 28(5) 29(6) 30(4)
31(3) 32(4) 33 34 35(4) 36(2) 37(2) 39 42 44

Data set 7: The data set reported by Efron\textsuperscript{15} represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using radiotherapy (RT)

5.63 7 10.42 14.48 16.10 22.70 34 41.55 42 45.28 49.40 53.62 63
64 83 84 91 108 112 129 133 133 139 140 140 146
149 154 157 160 160 165 149 154 157 160 160 165
173 176 218 225 241 248 273 277 297 405 417 420 440
523 583 594 1101 1146 1417

Data set 8: The data set reported by Efron\textsuperscript{15} represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using a combination of radiotherapy and chemotherapy (RT+CT).

12.20 23.56 23.74 25.87 31.98 37 41.35 47.38 55.46 58.36 63.47 68.46 78.26
74.47 81.43 84 92 94 92 110 112 129 133 133 139 140 140 146
155 159 173 179 194 195 209 249 281 319 339 342 469
519 633 725 817 1776

Data set 9: This data set represents remission times (in months) of a random sample of 128 bladder cancer patients reported in Lee and Wang.\textsuperscript{16}

0.08 2.09 3.48 4.87 6.94 8.66 13.11 23.63 0.20 2.23 3.52 4.98 6.97
9.02 13.29 0.40 2.26 3.57 5.06 7.09 9.22 13.80 25.74 0.50 2.46 3.64
5.09 7.26 9.47 14.24 25.82 0.51 2.54 3.70 5.17 7.28 9.74 14.76 6.31
0.81 2.62 3.82 5.32 7.32 10.06 14.77 32.15 2.64 3.88 5.32 7.39 10.34
14.83 34.26 0.90 2.69 4.18 5.34 7.59 10.66 15.96 36.66 1.05 2.69 4.23
5.41 7.62 10.75 16.62 43.01 1.19 2.75 4.26 5.41 7.63 17.12 46.12 1.26
2.83 4.33 5.49 7.66 11.25 17.14 79.05 1.35 2.87 5.62 7.87 11.64 17.36
1.40 3.02 4.34 5.71 7.93 11.79 18.10 1.46 4.40 5.85 8.26 11.98 19.13
1.76 3.25 4.50 6.25 8.37 12.02 2.02 3.31 4.51 6.54 8.53 12.03 20.28
2.02 3.36 6.76 12.07 21.73 2.07 3.36 6.93 8.65 12.63 22.69

Data set 10: This data set is given by Linhart and Zucchini,\textsuperscript{17} which represents the failure times of the air conditioning system of an airplane:

23 261 87 7 120 14 62 47 225 71 246 21 42
20 5 12 120 11 3 14 71 11 14 11 16 90 1
16 52 95

Citation: Shukla KK. A comparative study of one parameter lifetime distributions. Biom Biostat Int J. 2019;8(4):111–123. DOI: 10.15406/bbij.2019.08.00280
Data set 11: This data set used by Bhaumik et al., is vinyl chloride data obtained from clean upgradient monitoring wells in mg/l:

| Value |
|-------|
| 5.1   |
| 1.2   |
| 1.3   |
| 0.6   |
| 0.5   |
| 2.4   |
| 0.5   |
| 1.1   |
| 8     |
| 0.8   |
| 0.4   |
| 0.6   |
| 0.9   |
| 0.4   |
| 2     |
| 4     |
| 6.8   |
| 1.2   |
| 0.4   |
| 0.2   |

Data set 12: This data set represents the waiting times (in minutes) before service of 100 Bank customers and examined and analyzed by Ghitany et al., for fitting the Lindley (1958) distribution.

| Value |
|-------|
| 0.8   |
| 0.8   |
| 1.3   |
| 1.5   |
| 1.8   |
| 1.9   |
| 2.1   |
| 2.6   |
| 2.7   |
| 2.9   |
| 3.1   |
| 3.2   |
| 3.3   |
| 3.5   |
| 4.0   |
| 4.1   |
| 4.2   |
| 4.3   |
| 4.4   |
| 4.6   |
| 4.7   |
| 4.7   |
| 4.8   |
| 4.9   |
| 5.0   |
| 5.3   |
| 5.5   |
| 5.7   |
| 6.1   |
| 6.2   |
| 6.2   |
| 6.3   |
| 6.7   |
| 6.9   |
| 7.1   |
| 7.1   |
| 7.1   |
| 7.4   |
| 7.6   |
| 7.7   |
| 8.0   |
| 8.2   |
| 8.6   |
| 8.6   |
| 8.8   |
| 8.8   |
| 8.9   |
| 8.9   |
| 9.5   |
| 9.6   |
| 9.7   |
| 9.8   |
| 10.7  |
| 10.9  |
| 11.0  |
| 11.0  |
| 11.1  |
| 11.2  |
| 11.2  |
| 11.5  |
| 11.9  |
| 12.4  |
| 12.5  |
| 12.9  |
| 13.0  |
| 13.1  |
| 13.3  |
| 13.6  |
| 13.7  |
| 13.9  |
| 14.1  |
| 15.4  |
| 15.4  |
| 17.3  |
| 17.3  |
| 18.1  |
| 18.2  |
| 18.4  |
| 18.9  |
| 19.0  |
| 19.9  |
| 20.6  |
| 21.3  |
| 21.4  |
| 21.9  |
| 23.0  |
| 27.0  |
| 31.6  |
| 33.1  |
| 38.5  |

Data set 13: This data is for the times between successive failures of air conditioning equipment in a Boeing 720 airplane, Proschan:

| Value |
|-------|
| 74    |
| 57    |
| 48    |
| 29    |
| 502   |
| 12    |
| 70    |
| 21    |
| 29    |
| 386   |
| 59    |
| 27    |
| 153   |
| 26    |
| 326   |

Data set 14: This data set represents the lifetime’s data relating to relief times (in minutes) of 20 patients receiving an analgesic and reported by Gross and Clark:

| Value |
|-------|
| 1.1   |
| 1.4   |
| 1.3   |
| 1.7   |
| 1.9   |
| 1.8   |
| 1.6   |
| 2.2   |
| 1.7   |
| 2.7   |
| 4.1   |
| 1.8   |
| 1.5   |
| 1.2   |
| 1.4   |
| 3     |
| 1.7   |
| 2.3   |
| 1.6   |

Data Set 15: This data set is the strength data of glass of the aircraft window reported by Fuller et al:

| Value |
|-------|
| 18.83 |
| 20.8  |
| 21.657|
| 23.03 |
| 23.23 |
| 24.05 |
| 24.321|
| 25.5  |
| 25.52 |
| 25.8  |
| 26.69 |
| 26.77 |
| 26.78 |
| 27.05 |
| 27.67 |
| 29.9  |
| 31.11 |
| 33.2  |
| 33.73 |
| 33.76 |
| 33.89 |
| 34.76 |
| 35.75 |
| 35.91 |
| 36.98 |
| 37.08 |
| 37.09 |
| 39.58 |
| 44.045|
| 45.29 |
| 45.381|

Data set 16: The following data represent the tensile strength, measured in GPa, of 69 carbon fibers tested under tension at gauge lengths of 20mm (Bader and Priest:

| Value |
|-------|
| 1.312 |
| 1.314 |
| 1.479 |
| 1.552 |
| 1.700 |
| 1.803 |
| 1.861 |
| 1.865 |
| 1.944 |
| 1.958 |
| 1.966 |
| 1.997 |
| 2.006 |
| 2.021 |
| 2.027 |
| 2.055 |
| 2.063 |
| 2.098 |
| 2.140 |
| 2.179 |
| 2.224 |
| 2.240 |
| 2.253 |
| 2.270 |
| 2.272 |
| 2.274 |
| 2.301 |
| 2.301 |
| 2.359 |
| 2.382 |
| 2.382 |
| 2.426 |
| 2.434 |
| 2.435 |
| 2.478 |
| 2.490 |
| 2.511 |
| 2.514 |
| 2.535 |
| 2.554 |
| 2.566 |
| 2.570 |
| 2.586 |
| 2.629 |
| 2.633 |
| 2.642 |
| 2.648 |
| 2.684 |
| 2.697 |
| 2.726 |
| 2.770 |
| 2.773 |
| 2.800 |
| 2.809 |
| 2.818 |
| 2.821 |
| 2.848 |
| 2.880 |
| 2.954 |
| 3.012 |
| 3.067 |
| 3.084 |
| 3.090 |
| 3.096 |
| 3.128 |
| 3.233 |
| 3.433 |
| 3.585 |
| 3.858 |

Data set 17: The first set of data represents the failure times (in minutes) for a sample of 15 electronic components in an accelerated life test Lawless and the data are

| Value |
|-------|
| 1.4   |
| 5.1   |
| 6.3   |
| 10.8  |
| 12.1  |
| 18.5  |
| 19.7  |
| 22.2  |
| 23.0  |
| 30.6  |
| 37.3  |
| 46.3  |
| 53.9  |
| 59.8  |
| 66.2  |

Data set 18: The following data set represents the number of cycles to failure for 25 100-cm specimens of yarn, tested at a particular strain level.

| Value |
|-------|
| 15    |
| 20    |
| 38    |
| 42    |
| 61    |
| 76    |
| 86    |
| 98    |
| 121   |
| 146   |
| 149   |
| 157   |
| 175   |
| 180   |
| 180   |
| 198   |
| 220   |
| 224   |
| 251   |
| 264   |
| 282   |
| 321   |
| 325   |
| 653   |

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Goodness of Fit

In order to compare the goodness of fit of all distributions, \(-2\ln L\), AIC (Akaike Information Criterion), BIC (Bayesian Information Criterion), K-S Statistics (Kolmogorov-Smirnov Statistics) for all eighteen real lifetime data- sets have been computed and presented in Table 3. The formulae for computing AIC, BIC, and K-S Statistics are as follows:

\[
AIC = -2\ln L + 2k, \quad BIC = -2\ln L + k \ln n \quad \text{and} \quad D = \sup_x \left| F_n(x) - F_0(x) \right|
\]

Where \( k = \) the number of parameters, \( n = \) the sample size and \( F_n(x) \) is the empirical distribution function. The best distribution is the distribution which corresponds to lower values of \(-2\ln L\), AIC, BIC, and K-S statistics.

Table 3 MLEs, -2ln L, AIC, BIC, K-S Statistics of the fitted distributions of datasets 1-18

| Model      | Parameter estimate | -2ln L | AIC  | BIC  | K-S statistic |
|------------|--------------------|--------|------|------|--------------|
| Data 1     | Prakaamy           | 2.49738| 186.05| 188.05| 190.29       | 0.467       |
|            | Sujatha            | 1.3500 | 154.80| 156.80| 158.72       | 0.430       |
|            | Ram Awadh          | 2.0794 | 212.42| 214.42| 214.62       | 0.468       |
| Pranav     | 1.56071            | 180.96 | 182.96| 184.87| 0.355        | 0.488       |
| Akash      | 1.35544            | 163.73 | 165.73| 169.93| 0.355        | 0.468       |
| Ishita     | 1.25202            | 168.28 | 170.28| 172.19| 0.355        | 0.468       |
| Lindley    | 0.99611            | 162.56 | 164.56| 166.70| 0.355        | 0.468       |
| Exponential| 0.66364            | 177.66 | 179.66| 181.80| 0.355        | 0.468       |
| Prakaamy   | 0.087813           | 918.72 | 920.72| 922.95| 0.355        | 0.468       |
| Sujatha    | 0.04357            | 951.78 | 953.78| 955.69| 0.355        | 0.468       |
| Ram Awadh  | 0.087811           | 918.70 | 920.70| 920.91| 0.355        | 0.468       |
| Data 2     | Pranav             | 0.05854| 934.06| 936.06| 937.97       | 0.355       |
| Akash      | 0.048376           | 950.97 | 952.97| 955.58| 0.355        | 0.468       |
| Ishita     | 0.043906           | 950.90 | 952.90| 954.83| 0.355        | 0.468       |
| Lindley    | 0.028859           | 983.11 | 985.11| 987.71| 0.355        | 0.468       |
| Exponential| 0.014635           | 1044.87| 1046.87| 1049.48| 0.355        | 0.468       |
| Prakaamy   | 0.083070           | 228.29 | 230.29| 232.53| 0.355        | 0.468       |
| Sujatha    | 0.041229           | 227.17 | 229.17| 231.08| 0.355        | 0.468       |
| Ram Awadh  | 0.083070           | 228.29 | 230.29| 230.49| 0.355        | 0.468       |
| Data 3     | Pranav             | 0.055384| 226.05| 228.05| 229.96       | 0.355       |
| Akash      | 0.041510           | 227.06 | 229.06| 230.20| 0.355        | 0.468       |
| Ishita     | 0.041533           | 227.03 | 229.03| 230.95| 0.355        | 0.468       |
| Lindley    | 0.027321           | 231.47 | 233.47| 234.61| 0.355        | 0.468       |
| Exponential| 0.013845           | 242.87 | 244.87| 246.01| 0.355        | 0.468       |
| Prakaamy   | 0.027162           | 1329.86| 1331.86| 1334.10| 0.355        | 0.468       |
| Sujatha    | 0.013491           | 1255.53| 1257.53| 1259.44| 0.355        | 0.468       |
| Ram Awadh  | 0.027041           | 1327.50| 1329.50| 1329.76| 0.355        | 0.468       |
| Data 4     | Pranav             | 0.01803| 1273.63| 1275.63| 1277.54       | 0.355       |
| Akash      | 0.013514           | 1255.83| 1257.83| 1260.43| 0.355        | 0.468       |
| Ishita     | 0.013514           | 1255.84| 1257.84| 1259.75| 0.355        | 0.468       |
| Lindley    | 0.008970           | 1251.34| 1253.34| 1255.95| 0.355        | 0.468       |

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### Table Continued

|        |        |        |        |        |        |
|--------|--------|--------|--------|--------|--------|
| **Exponential** | 0.004505 | 1280.52 | 1282.52 | 1285.12 | 0.190 |
| **Data 5**<br>Prakaamy | 0.033935 | 873.53 | 875.53 | 877.77 | 0.168 |
|        | **Sujatha** | 0.016919 | 851.58 | 853.58 | 855.49 | 0.096 |
|        | Ram Awadh | 0.033935 | 873.54 | 875.54 | 875.73 | 0.168 |
| **Akash** | 0.016966 | 851.62 | 853.62 | 855.53 | 0.095 |
| **Ishita** | 0.016985 | 851.63 | 853.63 | 855.54 | 0.095 |
| **Lindley** | 0.01127 | 858.55 | 860.55 | 862.46 | 0.162 |
|        | **Exponential** | 0.005684 | 889.22 | 891.22 | 893.13 | 0.296 |
|        | Prakaamy | 0.24035 | 899.93 | 901.93 | 904.53 | 0.308 |
|        | **Sujatha** | 0.11745 | 985.69 | 987.69 | 989.60 | 0.403 |
|        | **Ram Awadh** | 0.240359 | 899.92 | 901.92 | 902.12 | 0.308 |
| **Data 6**<br>Pranav | 0.16022 | 945.03 | 947.03 | 948.94 | 0.362 |
|        | Akash | 0.119610 | 981.28 | 983.28 | 986.18 | 0.393 |
|        | Ishita | 0.120089 | 980.02 | 982.02 | 983.93 | 0.399 |
|        | Lindley | 0.077247 | 1041.64 | 1043.64 | 1046.54 | 0.448 |
|        | **Exponential** | 0.040060 | 1130.26 | 1132.26 | 1135.16 | 0.525 |
|        | Prakaamy | 0.026533 | 955.97 | 957.97 | 960.20 | 0.400 |
|        | Sujatha | 0.013257 | 802.84 | 804.84 | 806.75 | 0.297 |
|        | **Ram Awadh** | 0.026534 | 955.97 | 957.97 | 958.16 | 0.400 |
| **Data 7**<br>Pranav | 0.017704 | 851.06 | 853.16 | 855.07 | 0.339 |
|        | Akash | 0.013263 | 803.96 | 805.96 | 810.01 | 0.298 |
|        | Ishita | 0.013269 | 804.08 | 806.08 | 807.99 | 0.298 |
|        | Lindley | 0.008804 | 763.75 | 765.75 | 767.81 | 0.245 |
|        | **Exponential** | 0.004421 | 744.87 | 746.87 | 748.93 | 0.166 |
|        | Prakaamy | 0.026860 | 726.69 | 728.69 | 730.93 | 0.393 |
|        | Sujatha | 0.013415 | 609.38 | 611.38 | 613.29 | 0.278 |
|        | **Ram Awadh** | 0.026860 | 726.69 | 728.69 | 728.89 | 0.393 |
| **Data 8**<br>Pranav | 0.01791 | 646.17 | 648.17 | 650.08 | 0.327 |
|        | Akash | 0.013423 | 609.93 | 611.93 | 613.71 | 0.280 |
|        | Ishita | 0.013448 | 609.95 | 611.95 | 613.86 | 0.279 |
|        | Lindley | 0.008910 | 579.16 | 581.16 | 582.95 | 0.219 |

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| Exponential | 0.004475 | 0.65098 | 1.11677 | 1.11877 | 1.12100 | 0.957 |
| Data 9       | Prakaamy  | 0.303635 | 0.87322 | 0.87522 | 0.87713 | 0.922 |
|              | Sujatha    | 0.65728  | 1.12319 | 1.12519 | 1.12539 | 0.957 |
|              | Ram Awadh  | 0.43771  | 0.96242 | 0.96442 | 0.96633 | 0.943 |
|              | Pranav     | 0.310500 | 0.88789 | 0.88989 | 0.89274 | 0.198 |
|              | Akash      | 0.326152 | 0.89412 | 0.89612 | 0.89803 | 0.928 |
|              | Ishita     | 0.196045 | 0.83906 | 0.84106 | 0.84391 | 0.116 |
| Exponential  | 0.106773  | 0.82868  | 0.83068 | 0.83354 | 0.077 |
|              | Prakaamy   | 0.10067  | 0.46494 | 0.46694 | 0.46918 | 0.966 |
|              | Sujatha    | 0.04989  | 0.35246 | 0.35446 | 0.35637 | 0.966 |
|              | Ram Awadh  | 0.10072  | 0.46615 | 0.46815 | 0.46834 | 0.966 |
| Data 10      | Pranav     | 0.067146 | 0.39123 | 0.39323 | 0.39515 | 0.966 |
|              | Ishita     | 0.050362 | 0.35652 | 0.35852 | 0.36043 | 0.966 |
|              | Shanker    | 0.033569 | 0.32574 | 0.32774 | 0.32914 | 0.351 |
|              | Lindley    | 0.033021 | 0.32327 | 0.32527 | 0.32667 | 0.345 |
| Exponential  | 0.016779  | 0.30526  | 0.30726 | 0.30866 | 0.213 |
|              | Prakaamy   | 2.28430  | 1.2914  | 1.3114  | 1.3337  | 0.980 |
|              | Sujatha    | 1.14606  | 1.1554  | 1.1754  | 1.1945  | 0.963 |
|              | Ram Awadh  | 2.10944  | 1.2388  | 1.2588  | 1.2608  | 0.975 |
| Data 11      | Pranav     | 1.46645  | 1.1667  | 1.1867  | 1.2058  | 0.965 |
|              | Akash      | 1.165719 | 1.1515  | 1.1715  | 1.1868  | 0.156 |
|              | Ishita     | 1.157035 | 1.1460  | 1.1660  | 1.1851  | 0.961 |
|              | Lindley    | 0.823821 | 1.1261  | 1.1461  | 1.1613  | 0.133 |
| Exponential  | 0.532081  | 0.11091  | 0.11291 | 0.11443 | 0.089 |
|              | Prakaamy   | 0.60712  | 0.72794 | 0.72994 | 0.73217 | 0.221 |
|              | Sujatha    | 0.28461  | 0.63963 | 0.64163 | 0.64355 | 0.088 |
|              | Ram Awadh  | 0.60887  | 0.72970 | 0.73170 | 0.73190 | 0.221 |
| Data 12      | Pranav     | 0.46478  | 0.66591 | 0.66791 | 0.66982 | 0.129 |
|              | Akash      | 0.295277 | 0.64193 | 0.64393 | 0.64593 | 0.100 |
|              | Ishita     | 0.30157  | 0.64369 | 0.64569 | 0.64761 | 0.108 |
|              | Lindley    | 0.186571 | 0.63807 | 0.64007 | 0.64268 | 0.058 |

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| Data 13 | Prakaamy | 0.049484 | 241.20 | 243.20 | 245.43 | 0.931 |
| Data 13 | Sujatha  | 0.024637 | 193.93 | 195.93 | 197.85 | 0.904 |
| Data 13 | Ram Awadh| 0.04948 | 241.20 | 243.20 | 243.40 | 0.931 |
| Data 13 | Pranav   | 0.03298  | 209.03 | 211.03 | 212.94 | 0.921 |
| Data 13 | Akash    | 0.024734 | 194.30 | 196.30 | 197.01 | 0.456 |
| Data 13 | Ishita   | 0.024745 | 194.32 | 196.32 | 198.23 | 0.905 |
| Exponential | Lindley | 0.016360 | 181.34 | 183.34 | 184.05 | 0.386 |
| Data 13 | Exponential | 0.008246 | 173.94 | 175.94 | 176.65 | 0.277 |
| Data 13 | Prakaamy | 2.27350  | 61.43  | 63.43  | 65.67  | 0.515 |
| Data 13 | Sujatha  | 1.13674  | 57.49  | 59.49  | 61.40  | 0.442 |
| Data 13 | Ram Awadh| 2.04587  | 68.52  | 70.52  | 70.72  | 0.514 |
| Data 13 | Pranav   | 1.401401 | 62.38  | 64.38  | 66.29  | 0.485 |
| Data 13 | Akash    | 1.156923 | 59.52  | 61.52  | 62.51  | 0.320 |
| Data 13 | Ishita   | 1.094847 | 60.16  | 62.16  | 64.07  | 0.325 |
| Data 13 | Lindley  | 0.816118 | 60.50  | 62.50  | 63.49  | 0.341 |
| Exponential | Exponential | 0.526316 | 65.67  | 67.67  | 68.67  | 0.389 |
| Data 13 | Prakaamy | 0.194733 | 223.07 | 225.07 | 227.31 | 0.197 |
| Data 13 | Sujatha  | 0.095613 | 241.50 | 243.50 | 245.41 | 0.302 |
| Data 13 | Ram Awadh| 0.19473  | 223.07 | 225.07 | 225.27 | 0.197 |
| Data 13 | Pranav   | 0.12981  | 232.77 | 234.77 | 236.68 | 0.253 |
| Data 13 | Akash    | 0.097062 | 240.68 | 242.68 | 244.11 | 0.266 |
| Data 13 | Ishita   | 0.097328 | 240.48 | 242.48 | 244.39 | 0.297 |
| Data 13 | Lindley  | 0.062988 | 253.99 | 255.99 | 257.42 | 0.333 |
| Data 13 | Exponential | 0.032455 | 274.53 | 276.53 | 277.96 | 0.426 |
| Data 13 | Prakaamy | 2.00984  | 188.77 | 190.77 | 193.00 | 0.261 |
| Data 13 | Sujatha  | 0.936119 | 221.60 | 223.60 | 225.52 | 0.364 |
| Data 13 | Ram Awadh| 1.84921  | 207.41 | 209.41 | 209.60 | 0.303 |
| Data 13 | Pranav   | 1.225139 | 217.12 | 219.12 | 221.03 | 0.303 |
| Data 13 | Akash    | 0.964726 | 224.28 | 226.28 | 228.51 | 0.348 |
| Data 13 | Ishita   | 0.931565 | 223.14 | 225.14 | 227.05 | 0.330 |
| Data 13 | Lindley  | 0.659000 | 238.38 | 240.38 | 242.61 | 0.390 |
| Exponential | Exponential | 0.407941 | 261.74 | 263.74 | 265.97 | 0.434 |

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The best fitting has been shown by making -2ln L, AIC, BIC, and K-S Statistics in bold.

Data17

| Prakaamy  | 0.21781 | 158.03 | 160.03 | 162.26 | 0.281 |
| Sujatha   | 0.10668 | 132.86 | 134.86 | 136.78 | 0.177 |
| Ram Awadh | 0.21790 | 158.29 | 160.29 | 160.49 | 0.281 |
| Pranav    | 0.145325| 141.44 | 143.44 | 145.35 | 0.231 |
| Akash     | 0.108478| 133.68 | 135.68 | 137.59 | 0.184 |
| Ishita    | 0.10898 | 134.40 | 136.40 | 138.31 | 0.185 |
| Lindley   | 0.070223| 128.81 | 130.81 | 132.72 | 0.110 |
| Exponential | 0.036300| 129.47 | 131.47 | 133.38 | 0.155 |
| Prakaamy  | 0.033657| 336.97 | 338.97 | 341.21 | 0.206 |
| Sujatha   | 0.01677 | 309.23 | 311.23 | 313.14 | 0.124 |
| Ram Awadh | 0.03365 | 336.97 | 338.97 | 339.17 | 0.206 |
| Pranav    | 0.023255| 249.54 | 251.54 | 253.45 | 0.144 |

Data18

| Pranav    | 0.016822| 309.41 | 311.41 | 313.32 | 0.125 |
| Akash     | 0.016839| 309.42 | 311.42 | 313.33 | 0.124 |
| Ishita    | 0.011183| 305.01 | 307.01 | 308.92 | 0.129 |
| Lindley   | 0.005622| 309.17 | 311.17 | 313.09 | 0.199 |
| Exponential | 0.033657| 336.97 | 338.97 | 341.21 | 0.206 |

Conclusion

In this paper an attempt has been made to find the suitability of Prakaamy, Sujatha, Ram Awadh, Pranav, Akash, Ishita, Lindley and exponential distributions for modeling real lifetime data from engineering, medical science and other fields of knowledge. Nature and behavior of distributions have been presented graphically. Coefficient of Variation, Coefficient of Skewness, coefficient of kurtosis and Index of dispersion of distributions have also been presented graphically. The conditions under which different distributions are over-dispersed, equi-dispersed, and under-dispersed have also been given. The goodness of fit has been tested of above mentioned distributions on eighteen real lifetime datasets for their suitability for modeling lifetime data. It is observed that Exponential distribution gives good fits over other distributions for six datasets, whereas three datasets are related to biological fields, and three datasets are related chemical and engineering fields. Lindley distribution give better fit than other considered distributions for three datasets, whereas one of them is related to biological field and two of them are related to engineering fields. Ram Awadh distribution gives closer fit over other considered distributions for three datasets, whereas two of them are related to engineering and one dataset is related to biological field. Pranav distribution gives good fit over other considered distributions for two datasets, which are related to medical science and engineering fields. Prakaamy distribution gives close fit over other considered distributions for three datasets, which are related to medical science and engineering fields. Prakaamy distribution gives good fit over other considered distributions for one dataset which is related to engineering field. From the goodness of fit test, it can be observed that exponential and Lindley distribution can be considered as good model for biological as well as engineering studies. Ram Awadh and Sujatha distribution can also be considered good model for biological and engineering fields whereas Pranav and Prakaamy distribution can been considered good model for engineering filed.

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Conflicts of interest

Author declares that there are no conflicts of interest.

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