MIMO radar array synthesis using QPSO with normal distributed contraction-expansion factor

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Abstract

An array synthesis based on Quantum particle swarm optimization (QPSO) is proposed for the orthogonal MIMO radar. To decrease the sidelobe level of the pattern, the elements’ positions is presented by Quantum-behaved particles. Each particle searches the solution space with a normal distributed contraction-expansion factor, and updates its position with respect to the potential attraction at a Quantum probability. When compared with the traditional particle swarm optimization (PSO) and genetic algorithm (GA), the proposed method simulates the Quantum-behave, and balances the exploration and exploitation with a normal distributed contraction-expansion factor, which results in better convergence and static-state performances.

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1. Introduction

Recently, by using space-diversity and waveform-diversity, multiple-input and multiple-output (MIMO) radar has attracted more and more attentions of the researchers[1-7]. Different from the regular sparse array pattern synthesis, MIMO radar may have separated transmit array and receive array, its pattern

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synthesis is a non-linear high-dimensional optimization problem. The traditional Chebyshev-Dolph synthesis and Taylor synthesis only aimed at the co-shared array of transmitter and receiver, not considering the above features, is not adequate for MIMO radar array synthesis[8]. Recently, some intelligent algorithms, such as genetic algorithm (GA)[9, 10] and particle swarm optimization (PSO)[11, 12] have been applied to MIMO radar array synthesis. However, the above algorithms’ convergence performance and static-state performance are still needed to be improved. This paper proposed an array synthesis by using the Quantum-behaved particle swarm optimization (QPSO)[13-15], a contraction-expansion factor with normal distribution is generated to control the balance between exploration and exploitation. The analysis and simulation results show that the proposed method has better convergence performance and static-state performance than GA, PSO and QPSO.

2. Pattern Analysis

Considering a MIMO radar with separated transmit array and receive array, the transmit array has \( N \) antennas, the receive array has \( M \) antennas. Supposing the different antenna element transmit orthogonal poly-phase signal, the signal received at the received array is

\[
X = a_r(\theta)\beta(\theta)a_t^T(\theta)S + Z
\]

(1)

where \( \beta(\theta) \in C \) is the complex amplitude of the target echo from direction \( \theta \), \( a_r(\theta) \) and \( a_t(\theta) \) are steering vectors of transmitter and receiver, respectively, \( Z \) is white Gaussian noise, \( S \) is \( N \times L \) orthogonal poly-phase signals. After pulse compression, the received signal can be vectored as

\[
Y = \text{vec}\left[ \frac{1}{L} X S^H \right] = \beta(\theta)a(\theta) + \text{vec}\left[ \frac{1}{L} Z S^H \right]
\]

(2)

where \( a(\theta) = a_r(\theta) \otimes a_t(\theta) \) is the synthesized steering vector, \( \otimes \) is Kronecker product. When the maximum system freedom is achieved, the MIMO radar is equal to a synthesized array with \( NM \) elements, and there exists \( N+M \) non-overlap elements for the MIMO radar with separated transmit array and receive array. Supposing the transmit array and receive array are positioned linearly. The aperture of the transmit array is \( L_t \), which satisfies \( L_t > (N-1)\lambda / 2 \), the element distance of the transmit array is \( d_t = L_t / (N-1) \). The aperture of the receive array is \( L_r \), the element distance of the receive array is \( d_r = L_r / (M-1) \), which satisfies \( d_r \geq Nd_t \). The positions of transmit elements and receive elements can be represented as \( x=[x_1,x_2,...,x_N]^T \) and \( y=[y_1,y_2,...,y_M]^T \). Supposing the target and the transmit-receive array are in the same plane, the direction of the target is \( \theta_0 \), then the pattern of the synthesis array is

\[
P_r(\theta) = w^H(\theta)a(\theta_0) = \sum_{n=1}^{N} \sum_{m=1}^{M} \exp\left( j \frac{2\pi}{\lambda} (x_n + y_m) \cos \theta - \cos \theta_0 \right)
\]

(3)

To improve the space resolution of MIMO radar, the peak sidelobe level of the pattern should be decrease as much as possible, the optimization function is the maximum relative sidelobe level (Max. Rsll)

\[
\text{Max. Rsll} = 20 \log \left| \frac{P_{\text{max},sl}}{P_{\text{max}}} \right| \text{dB}
\]

(4)

where \( P_{\text{max},sl} \) is maximum sidelobe level, \( P_{\text{max}} \) is the major lobe. To guarantee the maximum array aperture, the allowed positions’ perturbation constraints are

\[
\delta_{t,\text{max}} < d_t \\
\delta_{r,\text{max}} < d_r - L_t
\]

(5)(6)

3. Array Synthesis Based on QPSO
The Quantum-behaved particle swarm optimization (QPSO) permits all particles to have a quantum behavior instead of the classical Newtonian dynamics that was assumed so far in all versions of the PSO [19-22]. Each particle moves toward the center of the potential field and QPSO achieves convergence. The QPSO updates the position of each particle through the following equations

\[ p_j(t) = c \cdot p_{k,j} + (1 - c) \cdot p_{g,j} \]  
\[ M_{best_j}(t) = \frac{1}{K} \sum_{k=1}^{K} p_{k,j}(t) \]  
\[ x_{k,j}(t+1) = p_j(t) \pm \beta |M_{best_j}(t) - x_{k,j}(t)| \ln(1/u) \]

where \( K \) is the swarm size, \( J \) is dimension number of particle, \( p_j \) is the attract point, \( p_{k,j} \) is the \( j \)-th dimension of the \( k \)-th particle’s individual best position so far, \( p_{g,j} \) is \( j \)-th dimension of the swarm’s best position so far, \( M_{best} \) is the mean value of all particles’ individual best positions, \( c \) and \( u \) is the random number generated uniformly from \([0, 1]\), \( \beta \) is the contraction-expansion factor, which is the parameter only needed to be tuned in QPSO. Normally, \( \beta \) is set to be a fixed value.

At the prerequisite of the max system freedom and subjects to the constraints of fixed number of transmit/receive elements and fixed aperture of the array, the goal is to minimize the Max. Rsll, the fitness function can be set as

\[ f_{fitness} = -\text{Max. Rsll} = -20 \log \left( \frac{P_r}{P_{r,\text{max}}} \right) \text{dB} \]  

Generates the elements positions vector \( X = [x, y] \) in the allowed perturbation range, and sends it to QPSO for optimization. Before using QPSO for optimization, the contraction-expansion factor \( \beta \) should be determined. In fact, the contraction-expansion factor \( \beta \) actually determined the balance between exploration and exploitation capabilities, which will affect the convergence performance and static-state performance. Considering the elements’ positions vary in the allowed perturbation range, there exists a current potential attract point \( p \) in each iteration of the algorithm. Before the algorithm comes to convergence, the current potential attract point is not coincident with the potential center of the solution space (the optimum). The contraction-expansion factor should have a value which can not only make the algorithm search the neighborhood of local optimum, but also can make the algorithm divergent from the local optimum. Considering to the above analysis, this paper proposed to generate the contraction-expansion factor according to the normal distribution, and the algorithm can effectively balance the exploration and exploitation through setting mean and variance of the normal distribution. In the following, the QPSO with normal distributed contraction-expansion factor will be abbreviated as N-QPSO for simplicity. The steps of array synthesis for MIMO radar based on N-QPSO are listed as the following

1) Set the iteration index \( t=0 \)
2) Initializes the swarm: generates the structure of the particle, sets the swarm size \( K \), generates random positions \( X_k \) in the allowed perturbation range.
3) Evaluates the fitness of each particle according to equation (15), determined individual’s best \( p_k = X_k \) and swarm’s best \( p_g = \arg \max_{p_i} \{ f_{fitness}(p_i) \} \).
4) While termination condition not satisfied do
   \[ t = t + 1; \]
   \[ \text{for } j = 1 : J \]
calculates the attract point $p_j$ according to equation (7);
calculates the $M_{best}$ according to equation (8);
generate contraction-expansion factor according to a given normal distribution;
updates positions of particles according to equation (9);
end
calculates the fitness of each particle according to equation (10), updates individual’s best position and swarm’s best position.

End

After the algorithm terminates, the swarm’s best position $p_g$ is the optimized elements’ position.

3. Array Synthesis Based on QPSO

The performances of N-QPSO based array synthesis for MIMO radar are investigated by simulations. In the simulation, the transmit array and receive array are located linearly, the number of transmit antenna element is \(N=15\), the number of receive antenna element is \(M=16\). Set the transmit array aperture \(L_t=(N-1)\lambda\), the receive array aperture \(L_r=N(M-1)\lambda\). The contraction-expansion factor \(\beta\) is generated according a normal distribution with mean 0.4 and variance 0.4.

Fig. 1.(a) gives out the synthesis pattern when the target direction is 90°. It can be seen from Fig. 1 that the Max. Rsll has been suppressed effectively after the optimizations of GA, PSO, QPSO and N-QPSO. When compared with GA, PSO and QPSO, the proposed N-QPSO achieves the lowest Max. Rsll.

Fig. 1.(b) gives out the corresponding convergence performance when the target direction is 90°. As can be seen from Fig. 1.(b), GA reaches Max. Rsll of -26.0912dB, PSO reaches Max. Rsll of -29.8312dB, QPSO reaches Max. Rsll of -31.1287dB, N-QPSO reaches -32.3802dB, which is the lowest Max. Rsll achieved by algorithms. Though QPSO converged faster than N-QPSO, QPSO didn’t search the local optimum enough, which resulted to local convergence of QPSO. N-QPSO scarified some convergence speed, but achieved the lower Max. Rsll than QPSO. The elements’ positions achieved by QPSO at the target direction 90° is given out in Table. 1.

![Fig. 1. (a) Synthesis pattern (target angle is 90°) ; (b) Convergence performance](image-url)
Table 1  The elements’ positions obtained by N-QPSO

| Transmit Array | y=[0.9392, 1.8002, 2.7515, 3.6740, 4.5063, 5.5123, 8.3396, 6.3221, 9.1509, 9.9856, 10.7694, 11.5205, 13.3938, 14.0000]’ |
|----------------|--------------------------------------------------------------------------------|
| Receive Array  | x=[0, 14.9382, 29.8399, 45.3971, 59.7602, 74.8413, 90.4978, 105.5953, 119.8147, 134.4141, 149.9179, 164.6444, 180.1480, 194.6615, 210.2129, 225.0000]’ |

5. Conclusions

This paper proposed an N-QPSO based array synthesis for orthogonal MIMO radar. Subjecting to the constraints of fixed array aperture and fixed number of elements, in order to achieve the maximum system freedom, the particles simulate the Quantum-behave in the solution space, and control the exploration and exploitation through the normal distributed contraction-expansion factor. The analysis and simulations show that the proposed N-QPSO not only can reduce the peak sidelobe level of MIMO radar pattern effectively, but also has better convergence performance and static-state performance than GA, PSO and QPSO.

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