On the equivalence between topologically and non-topologically massive abelian gauge theories

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Abstract

We analyse the equivalence between topologically massive gauge theory (TMGT) and different formulations of non-topologically massive gauge theories (NTMGTs) in the canonical approach. The different NTMGTs studied are Stückelberg formulation of (A) a first order formulation involving one and two form fields, (B) Proca theory, and (C) massive Kalb-Ramond theory. We first quantise these reducible gauge systems by using the phase space extension procedure and using it, identify the phase space variables of NTMGTs which are equivalent to the canonical variables of TMGT and show that under this the Hamiltonian also get mapped. Interestingly it is found that the different NTMGTs are equivalent to different formulations of TMGTs which differ only by a total divergence term. We also provide covariant mappings between the fields in TMGT to NTMGTs at the level of correlation function.
Massive gauge theories have been a subject of intense study with popular approach being
the Higgs mechanism. The fact that the Higgs Boson is yet experimentally elusive, forces
an intense study of alternate models displaying gauge invariant massive spin-1 fields. One
such model which is being studied at both abelian and non-abelian level [1] is topologically
massive gauge theory (TMGT), known as $B \wedge F$ theory, where one and two form gauge
fields are coupled in a gauge invariant way. $B \wedge F$ theory has also found applications in
areas like Josephson junction arrreys, black hole physics etc., [2]. There also exist other non-
topologically massive gauge theories (NTMGTs), wherein St"uckelberg fields are added with
a compensating transformation [3], such that the mass term in the Lagrangian is gauge
invariant. The reason for this nomenclature is, as explained below, the equations of motion
for the former case vanishes as an identity whereas for the latter they vanish as a consequence
of other equations of motion. The purpose of this paper is to study the relationship between
these two apparently different formulations. Equivalence of degrees of freedom between
TMGT and Proca theory and their higher dimensional generalizations has been shown in
[4]. The NTMGTs studied here are St"uckelberg formulations of (A) a first order formulation
involving one and two form fields, (B) Proca theory, and (C) massive Kalb-Ramond(K-R)
theory described below ( cf: (29),(45) and (50) respectively). Non-abelian version of (A),
with out the St"uckelberg fields have been discussed in [5]. The massive St"uckelberg form of
K-R theory has been studied in [3] and has also occured in a description of confining string [7].
The equivalence of the first one of the above to $B \wedge F$ theory has been established recently by
phase space path integral approach [8]. Duality equivalence, following Buscher’s procedure
[9] was established between TMGT and NTMGTs (B and C) [10]. But a complete analysis
of the equivalence between TMGT and NTMGTs from the Hamiltonian formulation is still
lacking. Also these systems by themselves are interesting due to their interesting constraint
structure. In this paper we analyse, from the canonical frame work, the equivalence between
$B \wedge F$ theory and NTMGTs by providing a mapping between the phase space variables of
TMGT to free NTMGTs which render the same algebra in the respective reduced phase
spaces. This mapping between the canonical variables also map the corresponding canonical Hamiltonians and the equations of motion. Interestingly it is found that different NTMGTs gets mapped to different formulations of TMGT which differ only by a total divergence. We point out that evaluation of Dirac brackets for these reducible systems following the phase space extension approach developed recently in [11] by itself is a new result. We also provide an equivalence at the level of corelators for the field in configuration space between the TMGT and NTMGTs. This paper is organised in the following way: In section I we study the Hamiltonian formulation of the model given in (1), and quantise it by enlarging the phase space by introducing a pair of new canonical coordinates and obtain the Dirac brackets. In section II we quantize the three different spin-one NTMGTs, taking into account of the reducibility in the same way. In section III, we find the mapping between the phase space variables of $B^\wedge F$ theory to the Stückelberg formulations which generates the same algebra of canonical variables in the reduced phase space. We conclude with discussion in section IV.

We use the metric $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ and $\epsilon_{0123} = 1$.

**Section I**

The Lagrangian for the abelian $B^\wedge F$ theory is given by

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2 \times 3!} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + \frac{m}{4} \epsilon_{\mu\nu\rho\lambda} B^{\mu\nu} F^{\rho\lambda},$$

(1)

where $F_{\mu\nu}$ and $H_{\mu\nu\lambda}$ are the field strengths associated with the fields $A_{\mu}$ and $B_{\mu\nu}$. The equation of motion following from this Lagrangian are

$$\partial_{\mu} F^{\mu\nu} + \frac{m}{3!} \epsilon_{\nu\alpha\beta\lambda} H_{\alpha\beta\lambda} = 0,$$

(2)

and

$$\partial_{\mu} H^{\mu\nu\lambda} - m \epsilon^{\nu\lambda\alpha\beta} \partial_{\alpha} A_{\beta} = 0.$$

(3)

It is easy to see that this system describes a free spin-one theory by solving the linearly coupled equations (2) and (3). Since the divergence of (2) and (3) vanishes as an identity,
the Lagrangian (1) is said to describe TMGT. This should be contrasted with NTMGTs
given in section II, where the divergence of equations of motion vanish only after using other
equations of motion.

The primary constraint following from the Lagrangian are $\Pi_0$ and $\Pi_{0i}$ which are the
momentum conjugates of $A^0$ and $B^{0i}$ respectively. The momenta, from which velocities are
inverted, are given by,

$$\Pi_i = -F_{0i} + \frac{m}{2}\epsilon_{0ijk} B^{jk}, \quad (4)$$

$$\Pi_{ij} = H_{0ij}, \quad (5)$$

The canonical Hamiltonian following from this Lagrangian is,

$$\mathcal{H}_c = -\frac{1}{2}\Pi_i \Pi^i + \frac{1}{4}\Pi_{ij} \Pi^{ij} + \frac{1}{4} F_{ij} F^{ij} - \frac{1}{2 \times 3!} H_{ijk} H^{ijk}$$

$$+ \frac{m^2}{4} B_{lm} B^{lm} + \frac{m}{2} \epsilon_{0ijk} \Pi^i B^{jk} \quad (6)$$

The consistancy of the primary constraints leads to the secondary constraints,

$$\Omega = \partial_i \Pi^i, \quad (7)$$

$$\Lambda_i = -\partial^j \Pi_{ij} + \frac{m}{2} \epsilon_{0ijk} F^{jk} \quad (8)$$

Note that these are first class and (8) is reducible; $\partial^i \Lambda_i = 0$. The gauge-fixing conditions
for these constraint are given by

$$\partial^i A_i \approx 0, \quad (9)$$

$$\omega^i = \partial_j B^{ij} \approx 0. \quad (10)$$

Here the gauge-fixing condition $\partial_j B^{ij} \approx 0$ is also not linearly independent.

In usual Dirac procedure, one first isolate the linearly independent constraints, and then
procede to construct the Dirac brackets [12]. But this elimination can not be done uniquely.
More over, in general, the linearly independent constraints obtained in this way do not
generate all those transformations under which the Lagrangian is invariant. Both these
deffects can be avoided in the phase space extension method developed in [11]. Here one handles the reducibility by modifying the constraints itself. In this method one enlarges the phase space by introducing a pair of canonically conjugate auxiliary variables $p$, and $q$ with

\[ \{ p(x), q(y) \}_PB = -\delta(x - y). \]  

(11)

The modified constraint and the corresponding gauge-fixing condition become,

\[ \tilde{\Lambda}_i = -\partial^j\Pi_{ij} + \frac{m}{2}\epsilon_{0ijk}F^{jk} + \partial_i p \approx 0, \]  

(12)

\[ \tilde{\omega} = \partial_j B^{ij} + \partial^i q \approx 0. \]  

(13)

Note that constraints are no longer reducible. The primary first class constraints have the gauge-fixing conditions,

\[ A_0 \approx 0, \]  

(14)

\[ B_{0i} \approx 0. \]  

(15)

The non-vanishing Dirac brackets, computed using the modified constraints are now given by,

\[ \{ A_i(x), \Pi^l(y) \}^* = \left( \delta_i^l + \frac{\partial^j\partial_{ij}}{\nabla^2} \right) \delta(x - y), \]  

(16)

\[ \{ B_{ij}(x), \Pi^{lm}(y) \}^* = \left[ (\delta_i^l\delta_j^m - \delta_i^m\delta_j^l) + \frac{1}{\nabla^2} \left( \delta_i^l\partial_j\partial^m - \delta_i^m\partial_j\partial^l - \delta_j^m\partial_i\partial^l + \delta_j^l\partial_i\partial^m \right) \right] \delta(x - y), \]  

(17)

\[ \{ \Pi_i(x), \Pi_{lm}(y) \}^* = -\frac{m}{\nabla^2} (\epsilon_{0ij}\partial_m - \epsilon_{0im}\partial_j) \partial^j \delta(x - y). \]  

(18)

(16, 17, and 18) are in agreement with the Dirac brackets obtained by other methods [13].

Note that the Dirac brackets are independent of the new variables $p$ and $q$. Once the Dirac brackets are evaluated, we can implement the constraints strongly. Hence, on the constraint surface, $\partial^i\tilde{\Lambda}_i = -\nabla^2 p = 0$. This sets the auxiliary variable $p = 0$. In the same
way we get \( q = 0 \). Thus the new variables introduced to handle reducibility vanish on the constraint surface and do not appear in the final Dirac brackets.

Next we study in the same manner two other equivalent formulations of \( B \land F \) theory (differing only by a total divergence), where the topological term is \( H \land A \) or \( (B \land F - H \land A) \). We refer them as \( H \land A \) and symmetric \( B \land F \) formulations respectively. The reason for this study is that these different formulations map naturally to the different formulations of NTMGTs.

Instead of \( + \frac{m}{4} \epsilon_{\mu \nu \rho \lambda} B^{\mu \nu} F^{\rho \lambda} \) term in (1), now we consider \( - \frac{m}{2 \times 3!} \epsilon_{\mu \nu \rho \lambda} H^{\mu \nu \rho} A^\lambda \) (\( H \land A \) formulation). The primary constraints remain the same, but the momenta for the unconstrained fields are now,

\[
\Pi_i = - F_{0i},
\]

\[
\Pi_{ij} = H_{0ij} - m \epsilon_{0ijk} A^k.
\]

and the structure of the secondary constraints are changed to

\[
\Omega = \partial_i \Pi^i + \frac{m}{2} \epsilon_{0ijk} \partial^j B^{jk}
\]

\[
\Lambda_i = \partial^j \Pi_{ij}.
\]

The canonical Hamiltonian now reads,

\[
H_c = - \frac{1}{2} \Pi_i \Pi^i + \frac{1}{4} \Pi_{ij} \Pi^{ij} + \frac{1}{4} F^{ij} F_{ij} - \frac{1}{2 \times 3!} H_{ijk} H^{ijk} + \frac{m}{2} \epsilon_{0ijk} \Pi^{ij} A^k - \frac{m^2}{2} A_i A^i
\]

Note again that the second constraint (22) is a reducible one, which can be handled in the same way as in (12) and (13). The Dirac brackets \( \{ A_i, \Pi^i \}^* \) and \( \{ B_{ij}, \Pi_{lm} \}^* \) remains the same as (16) and (17) while that between the momenta now reads,

\[
\{ \Pi_i, \Pi_{lm} \} = \frac{m}{\sqrt{2}} \epsilon_{0lmj} \partial_i \partial^j \delta(x - y).
\]

Next we give the constraint structure of (1), where the mass term is written symmetrically as \( + \frac{m}{8} \epsilon_{\mu \nu \rho \lambda} B^{\mu \nu} F^{\rho \lambda} + - \frac{m}{4 \times 3!} \epsilon_{\mu \nu \rho \lambda} H^{\mu \nu \rho} A^\lambda \). Primary constraints again remain same and the secondary constraints are now,
\[ \Omega = \partial^j \Pi_i + \frac{m}{4} \epsilon_{ijk} \partial^j B^{jk}, \] (25)

\[ \Lambda_i = \partial^j \Pi_{ij} + \frac{m}{4} \epsilon_{ijk} F^{jk}. \] (26)

Here too, one of the constraints (26) is reducible. The canonical Hamiltonian will now read
\[ H_c = -\frac{1}{2}(\Pi^i + m\epsilon^{ijk} B_{jk})(\Pi_i + m\epsilon_{ilm} B^{lm}) + \frac{1}{4}(\Pi_{ij} + m\epsilon_{ijk} A^k)(\Pi^{ij} + m\epsilon^{ijl} A_l) \]
\[ + \frac{1}{4} F^{ij} F_{ij} - \frac{1}{2 \times 3!} H^{ijk} H_{ijk} \] (27)

The non-vanishing Dirac bracket (16) and (17) remains same while that between the momenta will now become
\[ \{\Pi_i, \Pi_{jk}\}^* = \frac{m}{2\nabla^2} \left[ \epsilon_{ijkl} \partial_i \partial^j - (\epsilon_{0jm} \partial_k - \epsilon_{0km} \partial_j) \partial^m \right] \delta(\vec{x} - \vec{y}) \] (28)

**Section II**

In this section we quantise the three different Stückelberg formulations (A, B, and C)

The Lagrangian describing (A) is
\[ L = -\frac{1}{4}(H_{\mu\nu} - C_{\mu\nu})(H^{\mu\nu} - C^{\mu\nu}) + \frac{1}{2}(G_{\mu} + \partial_{\mu} \Theta)(G^\mu + \partial^m \Theta) \]
\[ - \frac{1}{4m} \epsilon_{\mu\nu\lambda\sigma} H^{\mu\nu} \partial^\lambda G^\sigma - \frac{1}{4m} \epsilon_{\mu\nu\lambda\sigma} \partial^m H^{\nu\lambda} G^\sigma, \] (29)

where \( C_{\mu\nu} = (\partial_{\mu} C_{\nu} - \partial_{\nu} C_{\mu}). \) This Lagrangian is invariant under the transformations \( \delta(G_{\mu}) = \partial_{\mu} \Lambda, \delta(\Theta) = -\Lambda, \) and \( \delta(H_{\mu\nu}) = (\partial_{\mu} \Lambda_{\nu} - \partial_{\nu} \Lambda_{\mu}), \delta(C_{\mu}) = \Lambda_{\mu}. \) Note that when \( \Lambda_{\mu} = \partial_{\mu} \omega, \) \( H_{\mu\nu} \) is invariant implying the reducible nature of the constraints.

The primary constraints following from this Lagrangian are
\[ \Pi_0, \quad \Pi_{0i}, \quad \Pi_0, \] (30)
\[ \Omega_i = (\Pi_i - \frac{1}{4m} \epsilon_{ijk} H^{jk}), \] (31)
\[ \Lambda_{ij} = (\Pi_{ij} + \frac{1}{2m} \epsilon_{ijk} G^k). \] (32)

The canonical Hamiltonian and the secondary constraints are
\[ H_c = \frac{1}{4}(H_{ij} - C_{ij})(H^{ij} - C^{ij}) - \frac{1}{2}(G_i + \partial_i \Theta)(G^i + \partial^i \Theta) - \frac{1}{2} \Pi_i \Pi^i + \frac{1}{2}(\Pi_\Theta)^2, \] (33)
\[ \Lambda = \Pi_\Theta + \frac{1}{2m} \epsilon_{0ijk} \partial^i H^{jk}, \quad (34) \]
\[ \Lambda_i = -\tilde{\Pi}_i + \frac{1}{m} \epsilon_{0ijk} \partial^j G^k, \quad (35) \]
\[ \omega = (\partial^i \tilde{\Pi}_i) \quad (36) \]

In the above, \( \tilde{\Pi}_\mu \) are the conjugate momenta corresponding to \( C_\mu \) fields and \( \Pi_\Theta \) is the conjugate momentum of \( \Theta \). Following Fadeev and Jackiw, \[14\] we impose the symplectic conditions (31) and (32) strongly. This results a non-vanishing bracket
\[ \{ G_i, H_{ij} \}_{pb} = -m \epsilon_{0ijk} \delta(x - y). \quad (37) \]

Since \( \partial^i \Lambda_i + \omega = 0 \), this theory is reducible.

As in the previous section, we introduce auxiliary phase space variables \( p \) and \( q \) obeying (11) and modify \( \Lambda_i \) to \( \tilde{\Lambda}_i + \partial_i p \) to handle reducibility.

We choose the gauge fixing conditions to be
\[ \partial^i G_i \approx 0, \quad (38) \]
\[ \partial_i H^{ml} + \partial^m q \approx 0, \quad (39) \]
\[ \partial^i C_i \approx 0. \quad (40) \]

Now the non-vanishing Dirac brackets are
\[ \{ G_i(x), H_{jk}(y) \}^* = -m \left[ \epsilon_{oijk} (\delta_i^l + \frac{\partial_i \partial^j}{\nabla^2}) + \frac{1}{\nabla^2} (\epsilon_{oijl} \partial_k - \epsilon_{0ikt} \partial_j) \partial^l \right] \delta(x - y), \quad (41) \]
\[ \{ G_i(x), C_j(y) \}^* = \frac{m}{\nabla^2} \epsilon_{oijk} \partial^k \delta(x - y), \quad (42) \]
\[ \{ H_{ij}(x), \theta(y) \}^* = \frac{m}{\nabla^2} \epsilon_{oijk} \partial^k \delta(x - y), \quad (43) \]
\[ \{ \tilde{\Pi}_i(x), C^j(y) \}^* = - \left[ \delta_i^l + \frac{\partial_i \partial^j}{\nabla^2} \right] \delta(x - y). \quad (44) \]

Next we consider the Lagrangian describing \( (B) \),
\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (mA_{\mu} - \partial_{\mu} \Phi)(mA^\mu - \partial^\mu \Phi). \] (45)

This is the well studied Stückelberg - Proca Lagrangian and not a reducible system. We give the Dirac brackets, which are rather well known. The Hamiltonian following from this Lagrangian is

\[ H_c = -\frac{1}{2} \Pi_i \Pi^i + \frac{1}{2} \tilde{\Pi} \tilde{\Pi} + \frac{1}{4} F_{ij} F^{ij} - \frac{m^2}{2} A_i A^i + mA_i \partial^i \Phi - \frac{1}{2} \partial_i \Phi \partial^i \Phi, \] (46)

and the constraints \( \Pi_0 \approx 0, \omega = (\partial \Pi_i - m \tilde{\Pi}) \approx 0 \), are first class. The non-vanishing Dirac brackets evaluated in Coloumb gauge are

\[ \{ A_i, \Pi^j \}^* = (\delta^j_i + \frac{\partial \partial^j_i}{\sqrt{2}}) \delta(\vec{x} - \vec{y}), \] (47)

\[ \{ \tilde{\Pi}, \Phi \}^* = -\delta(\vec{x} - \vec{y}), \] (48)

\[ \{ \Pi_i, \Phi \}^* = \frac{m}{\sqrt{2}} \partial \delta(\vec{x} - \vec{y}). \] (49)

The Lagrangian describing (C) is

\[ \mathcal{L} = \frac{1}{2 \times 3!} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - \frac{1}{4} (m B_{\mu\nu} - \Phi_{\mu\nu})(m B^{\mu\nu} - \Phi^{\mu\nu}), \] (50)

where \( \Phi_{\mu\nu} = (\partial_{\mu} \Phi_{\nu} - \partial_{\nu} \Phi_{\mu}) \), and \( \Phi_{\mu} \) is the Stückelberg field. The canonical Hamiltonian following from this Lagrangian is

\[ H_c = \frac{1}{4} \Pi_{ij} \Pi^{ij} - \frac{1}{2} \Pi_i \Pi^i - \frac{1}{2 \times 3!} H_{ijk} H^{ijk} + \frac{1}{4} (m B_{ij} - \phi_{ij})(m B^{ij} - \phi^{ij}), \] (51)

The persistence of the primary constraints \( \Pi_0 \approx 0, \) and \( \Pi_{0i} \approx 0 \) give the Gauss law constraints,

\[ \omega = \partial^i \Pi_i \approx 0, \] (52)

\[ \Lambda_i = - (\partial^i \pi_{ij} + m \pi_i) \approx 0. \] (53)

All these constraints are first class. Here, we have \( \partial^i \Lambda_i + m \omega = 0 \), which implies that \( \Lambda_i \) and \( \omega \) are not linearly independent and thus the Stückelberg formulation given by the Lagrangian (50) describes a reducible gauge system.
Here also we handle reducible constraints and their gauge fixing conditions in the same way as we have done in the other two reducible theories by enlarging the phase space by introducing auxiliary variables $p$ and $q$ obeying (11). The modified constraint and gauge fixing condition are

\begin{align}
\tilde{\Lambda}_i &= -\partial^j \Pi_{ij} - m \Pi_i + \partial_i p, \\
\tilde{\Theta}_i &= \partial^j B_{il} + \partial_i q,
\end{align}

and corresponding to the generator $\omega$, we choose the gauge condition

$$\partial^i \Phi_i \approx 0.$$  

The non-vanishing Dirac brackets are

\begin{align}
\{\Pi_{ij}(x), B^{lm}(y)\}^* &= -\left(\delta_i^l \delta_j^m - \delta_i^m \delta_j^l\right) + \frac{1}{\sqrt{2}}(\partial_j \partial^m \delta_i^l - \partial_i \partial^m \delta_j^l - \partial_i \partial^m \delta_j^l + \partial_i \partial^m \delta_j^l) \delta(\vec{x} - \vec{y}), \\
\{\Pi_i(x), \Phi^j(y)\}^* &= -(\delta_i^j + \partial_i \partial^j) \delta(\vec{x} - \vec{y}), \\
\{\Pi_{ij}(x), \Phi^l(y)\}^* &= \frac{m}{\sqrt{2}}(\partial_i \delta_j^l - \partial_j \delta_i^l) \delta(\vec{x} - \vec{y}).
\end{align}

This completes the Hamiltonian analysis of NTMGTs.

**Section III**

In this section, from the considerations of the algebra of phase space variables in the reduced phase space of the NTMGTs and TMGT, we arrive at the following correspondence between the canonical variables.

The mappings between the phase space variables of symmetric $B\wedge F$ theory and NTMGT (A) is

$$\left(\Pi^l + \frac{m}{4} \epsilon^{0lmn} B_{mn}\right)_{BF} \leftrightarrow \frac{1}{2} \epsilon^{0lmn} (H_{mn} - C_{mn})_A,$$

(60)
\begin{align}
(\Pi^{lm} + \frac{m}{2} \epsilon^{0mn} A_n)_{BF} & \leftrightarrow \epsilon^{0mn} (G_n + \partial_n \Theta)_A, \quad (61) \\
\left(\frac{1}{3!} \epsilon^{0ijk} H_{ijk}\right)_{BF} & \leftrightarrow (\Pi_\Theta)_A, \quad (62) \\
\left(\frac{1}{2} \epsilon^{0ijk} F_{ijk}\right)_{BF} & \leftrightarrow (\tilde{\Pi}_i)_A, \quad (63) \\
(A_i)_{BF} & \leftrightarrow \frac{1}{m} (G_i)_A, \quad (64) \\
(B_{ij})_{BF} & \leftrightarrow \frac{1}{m} (H_{ij})_A. \quad (65)
\end{align}

Under this map the Dirac brackets (41), (42), (43) and (44) and the Hamiltonian (33) of the Stückelberg theory (29) gets mapped to that of symmetric $B \wedge F$ theory.

In the case of the Stückelberg formulation described by (45), the mapping

\begin{align}
(\Pi_{ij})_{BF} & \leftrightarrow (-\epsilon_{aijk} \partial^k \phi)_B, \quad (66) \\
(B_{ij})_{BF} & \leftrightarrow (-\frac{1}{\sqrt{2}} \epsilon_{aijk} \partial^k \tilde{\Pi})_B, \quad (67) \\
(A_i)_{BF} & \leftrightarrow (A_i)_B, \quad (68) \\
(\Pi_i)_{BF} & \leftrightarrow (\Pi_i)_B, \quad (69)
\end{align}

transform the Dirac brackets (47), (48) and (49) and the Hamiltonian of (46) to that of $H \wedge A$ theory (23).

From the Dirac bracket structure of (50) and $B \wedge F$ the following mappings can be obtained,

\begin{align}
(\Pi_i)_{BF} & \leftrightarrow (\epsilon_{aijk} \partial^j \Phi^k)_C, \quad (70) \\
(A_i)_{BF} & \leftrightarrow (\frac{1}{\sqrt{2}} \epsilon_{aijk} \partial^j \Pi^k)_C, \quad (71) \\
(B_{ij})_{BF} & \leftrightarrow (B_{ij})_C, \quad (72) \\
(\Pi_{ij})_{BF} & \leftrightarrow (\Pi_{ij})_C, \quad (73)
\end{align}

under which the Dirac brackets (57), (58) and (59) and the Hamiltonian of Stückelberg formulation (51) gets mapped to that of $B \wedge F$ theory (1).

Thus in the case of the mapping from TMGT to (B), the two form field and its conjugate momentum undergo a canonical transformation and in the case of mapping from TMGT to
(C), the vector field and its momentum transform canonically.

It can be checked that these identifications also map their respective Hamiltonian equations of motions.

Section IV

We have found the mappings between the canonical variables of NTMGTs (A), (B), and (C) to $B \wedge F$, $H \wedge A$ and symmetric $B \wedge F$ formulations respectively. This is consistent with our earlier observation that the $B \wedge F$ and $H \wedge A$ formulations go over to (C) and (B) respectively by Buscher’s duality procedure. This is understandable as Buscher’s procedure is itself is known to be a canonical transformation [15]. The first order Stückelberg theory was also shown to be equivalent to symmetric $B \wedge F$ theory by phase space path integral approach, which is in agreement with what is observed here. Thus this analysis in the canonical approach neatly complements the earlier studies.

The Gauss law constraint (7) and its gauge fixing condition in the case of $B \wedge F$ theory can be solved uniquely on the reduced phase space to get $\Pi_i = \epsilon_{oijk} \partial^j \psi^k$ and $A_i = \epsilon_{oijk} \partial^j \chi^k$. Here $\psi_i$ is an arbitrary field and $\chi_k$ is its conjugate momentum. These solutions, after identifying $\psi_i$ and $\chi_i$ with the Stückelberg field and its momentum (scaled by a factor of $\frac{1}{\sqrt{2}}$) are same as the mappings (70) and (71). In the case of $H \wedge A$ theory also we can solve the Gauss law constraint (22) and its gauge fixing condition uniquely. These solutions, also provide the same mappings (66) and (67). Thus the mapping of TMGTs to NTMGTs depend on the form of the Gauss law constraints.

We can also bring out the identification between the fields of in TMGT and NTMTs in a covariant way in Lagrangian formulation. For this purpose, couple the gauge invariant (dual) field strengths $\tilde{F}_{\mu\nu}$ and $\tilde{H}_{\mu}$ in $B \wedge F$ theory to sources $J^{\mu\nu}$ and $J^{\mu}$ respectively. By following the Buscher’s duality procedure, as in [10], we get the dual Lagrangian to be that of (50), with additional source dependent terms of the form

$$J^{\mu} \tilde{H}_{\mu} - \frac{1}{4} J_{\mu\nu} J^{\mu\nu} + \frac{1}{2} (m B_{\mu\nu} - \phi_{\mu\nu}) J^{\mu\nu}.$$
The sources have to be coupled to field strengths, rather than to the fields, so that there is a global shift symmetry of the fields, needed for applying Buscher’s procedure. Thus we get the identification, at the level of correlation function

$$\langle \tilde{F}_{\mu\nu}(x), \tilde{F}_{\lambda\sigma}(y) \rangle_{BF} = -\langle \tilde{B}_{\mu\nu}(x), \tilde{B}_{\lambda\sigma}(y) \rangle_{C} + (g_{\mu\lambda}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\lambda}) \delta(x - y),$$  \hspace{1cm} (74)

where $\tilde{B}_{\mu\nu} = (mB_{\mu\nu} - \Phi_{\mu\nu})$. Thus, apart from this contact term, we identify $\tilde{F}_{\mu\nu}$ in $B \wedge F$ theory to $(mB_{\mu\nu} - \phi_{\mu\nu})$ of the Stückelberg formulation (C). Similar analysis can be done with the other equivalent formulation of $B \wedge F$ theory (23), leading to the identification $\tilde{H}_{\mu}$ in $H \wedge A$ theory to $(mA_{\mu} - \partial_{\mu}\Phi)$ of the Stückelberg formulation of Proca theory (B). Also by starting with the first-order Lagrangian (29) and using the same procedure we identify $(G_{\mu} + \partial_{\mu}\Theta)$ and $(H_{\mu\nu} - C_{\mu\nu})$ of (29) to $\tilde{H}_{\mu}$ and $\tilde{F}_{\mu\nu}$ respectively, of the symmetric $B \wedge F$ theory. All these identification are modulo non-propagating contact terms. Note that in all cases gauge invariant filed strengths of TMGT get mapped to gauge invariant combination of fields in NTMGTs.

It will be interesting to see whether the equivalence established here for free theories will hold good for the interacting as well as for the non-abelian generalisations of the models studied in this paper. Work along these lines is in progress.

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