CHAO TICITY AND DISSIPATION OF
NUCLEAR COLLECTIVE MOTION
IN A CLASSICAL MODEL

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Abstract

We analyze the behavior of a gas of classical particles moving in a two-
dimensional "nuclear" billiard whose multipole-deformed walls undergo pe-
riodic shape oscillations. We demonstrate that a single particle Hamil-
tonian containing coupling terms between the particles’ motion and the
collective coordinate induces a chaotic dynamics for any multipolarity, in-
dependently on the geometry of the billiard. The absence of coupling terms
allows us to recover qualitatively the "wall formula" predictions. We also
discuss the dissipative behavior of the wall motion and its relation with
the order-to-chaos transition in the dynamics of the microscopic degrees of
freedom.

1 Introduction

In the last twenty years the dissipation of collective motion in nuclei has been
widely observed [1] in low energy particle and heavy ion collisions and it still
represents a theoretically unsolved problem. It is commonly believed that both
one-body processes, i.e. collisions of nucleons with the nuclear wall generated by
the common selfconsistent mean field, and two-body collisions produce dissipa-
tion, although their interplay is not well known.

In this work we discuss only one-body processes. In this framework, Blocki et al. [2]
analyzed the behavior of a gas of classical non-interacting particles enclosed
in a multipole-deformed container which undergoes periodic shape oscillations.
Particles move on linear trajectories and collide elastically against the walls. In
this model wall and particles’ motion is uncoupled and therefore the wall oscillates
at the same frequency pumping energy into the gas. For this system, the authors
study the increase of the particles’ kinetic energy as a function of time. They
find that for the quadrupole case the gas does not heat up, whereas for higher multipolarities the gas kinetic energy increases with time, in agreement with the "wall formula" predictions [3]. They attribute the different behavior to the fact that for low deformations the particles’ motion is regular and corresponds to an integrable situation, whereas for higher multipolarities the strong shape irregularities leads to divergence between trajectories and therefore to chaotic motion. Although their results look very interesting, their application to the nuclear case is not straightforward because i) the selfconsistent mean field is absent, ii) the total energy is not conserved.

A step forward in this direction has been performed by Bauer et al. in ref. [4]. In this work the authors study the damping of collective motion in nuclei within the semiclassical Vlasov equation. Here selfconsistency is taken into account and the total energy is conserved. A multipole-multipole interaction of the Bohr-Mottelson type is adopted for quadrupole and octupole deformations. In both cases the dynamical evolution shows a regular undamped collective motion which coexists with a weakly chaotic single-particle dynamics.

In order to clarify the relationship between chaos at the microscopic level and damping of the collective motion, we consider a gas of classical non-interacting particles moving in a two-dimensional billiard. Particles collide with the oscillating walls and transfer energy to it, therefore collisions are always inelastic. We consider the gas + billiard as an Hamiltonian system, therefore the total energy conservation imposes that the walls can give back energy to the gas. Therefore the main feature of the model is that particles and walls are strongly coupled, and this has far reaching consequences. The most important one is a) chaos shows up in the single particle motion for any surface deformation and b) the dissipation of the collective motion does not depend on the geometry of the billiard. We describe our model in section 2, and the results in section 3. In section 4 we draw some conclusions.

2 The Model

In ref. [5] we considered a classical version of the vibrating potential model for finite nuclei (see e.g. ref. [6]). In this model several non-interacting classical particles move in a two-dimensional deep potential well and hit the oscillating surface. Using polar coordinates, the Hamiltonian depends on a set of \( \{ r_i, \theta_i \} \) variables, describing the motion of the particles, and the collective coordinate \( \alpha \). The Hamiltonian reads

\[
H(r_i, \theta_i, \alpha) = \sum_{i=1}^{A} \left( \frac{p_{r_i}^2}{2m} + \frac{p_{\theta_i}^2}{2mr_i^2} + V(r_i, R(\theta_i)) \right) + \frac{p_{\alpha}^2}{2M} + \frac{1}{2} M \Omega^2 \alpha^2
\]

being \( \{ p_{r_i}, p_{\theta_i}, p_{\alpha} \} \) the conjugate momenta of \( \{ r_i, \theta_i, \alpha \} \). \( m = 938 \) MeV is the nucleon mass, and \( M = \eta m A R_0^2 \) is the Inglis mass, chosen proportional to the
total number of particles $A$ and to the circular billiard radius $R_o$. The factor $\eta$ ensures that during the dynamical evolution the equilibrium fluctuations are small. In our case $\eta = 1$ for the monopole oscillation, whereas $\eta = 10$ for quadrupole and octupole. Therefore in the $L = 0$ case, collisions of particles against the walls are more inelastic. $\Omega$ is the oscillation frequency of the collective variable $\alpha$. The potential $V(r, R(\theta))$ is zero inside the billiard and a very steeply rising function on the surface, $V(r, R(\theta)) = \frac{V_o}{1+\exp\left(\frac{R(\theta)-r}{a}\right)}$, with $V_o = 1500$ MeV and $a = 0.01$ fm. The surface is described by $R(\theta) = R_o(1+\alpha P_L(\cos \theta))$, where $P_L$ is the Legendre polynomial with multipolarity $L$. Therefore this potential couples the collective variable motion to the particles’ dynamics and prevents particles from escaping. The numerical simulation is based on the the Hamilton’s equations, which are solved with an algorithm of fourth-order Runge-Kutta type with typical time step sizes of 1 fm/c. All calculations were performed with a number of particles $A = 30$. The total energy was conserved with high accuracy.

As far as the initial conditions are concerning, we assign random positions to the particles inside the billiard and random initial momenta according to a two-dimensional Maxwell-Boltzmann distribution with a temperature $T = 36$ MeV. We consider the wall oscillation taking place close to adiabatic conditions. For this purpose we impose a wall frequency smaller than the single particle one and choose $\Omega = 0.05$ c/fm, which corresponds to a oscillation period $\tau_w = \frac{2\pi}{\Omega} \sim 125.66$ fm/c. The single particle oscillation period is equal to $\tau_p = \frac{2R_o}{v}$, being $v$ the most probable particle speed, $v = \sqrt{\frac{2T}{m}}$. We choose $R_o = 6$ fm and this gives a single particle period $\tau_p \sim 43.3$ fm/c.

Since in the realistic nuclear case the collective motion takes place around equilibrium, the initial wall coordinate has been chosen equal to $\alpha_0 = \bar{\alpha} + \delta \alpha$, being $\bar{\alpha}$ the equilibrium value and $\delta \alpha$ a small deviation. The equilibrium value $\bar{\alpha}$ corresponds of course to the thermodinamic limit, which is actually reached when considering an ensemble of copies of the system, all of them with an initial value $\alpha = \bar{\alpha}$ and differing from each other in the initial microscopic distribution of particles’ positions and momenta. It can be shown that the equilibrium value actually depends on the considered multipolarity [7]. Moreover at time $t = 0$ we put $p_\alpha = 0$, the wall having only potential energy. After we checked that the numerical simulation produces the good equilibrium properties, we slightly perturbed the equilibrium collective coordinate $\bar{\alpha}$ by an amount $\delta \alpha = 0.3$, and let both the billiard and the particles evolve in time.

3 Results

3.1 Scatter plots

One possible way in order to investigate the role played by coupling and see whether it can induce a chaotic dynamics, is drawing Poincare’s surface of sections
for the single particle coordinate, which is impossible to perform in our case because of the large number of degrees of freedom. An alternative way to visualize a chaotic behavior is to draw scatter plots, see Fig.1. There we display the final radial coordinate at a time $t$ of one chosen particle vs. the one at $t = 0$ for the monopole, quadrupole and octupole deformations at times $t = 50, 200, 300$ and $500 \text{fm}/c$. These plots are very useful and are commonly used in transient irregular situations like chaotic scattering [8] and nuclear multifragmentation [9]. The idea is that if the dynamics is regular, two initially close points in space stay close even at later times, but if the dynamics is chaotic the two points will soon separate due to the exponential divergence induced by chaos. In the first case this plot will show a regular curve, whereas in the other one a diffused pattern appears. We note that for all multipolarities the initially regular curves change into scatter plots, which clearly show that the coupling to the wall oscillation randomizes the single particle motion. This is at variance with what was discussed in ref.[2], where chaos is supposed to appear only for multipolarities $L > 2$. In our model the coupling between wall and particles’ motion produces a chaotic dynamics even for $L \leq 2$. In addition, the higher the multipolarity the earlier chaos starts because of the increased shape irregularity.

Fig.1. The final radial coordinate for one particle is drawn as a function of the initial one at different times $t = 50, 200, 300, 500 \text{fm}/c$. Calculations are performed for multipolarities $L = 0, 2, 3$.

For this purpose, a more quantitative analysis can be performed because scat-
ter plots of Fig.1 have a typical fractal structure. As already done in ref. [9], a fractal correlation dimension $D_2$ can be calculated from the correlation integral $C(r)$ [10]. One first counts how many points have a smaller distance than some given distance $r$. As $r$ varies, so does $C(r)$, defined as

$$C(r) = \frac{1}{M^2} \sum_{i,j} \Theta(r - |z_i - z_j|)$$

being $\Theta$ the Heaviside step function and $z_i$ a vector whose two components $(x_i, y_i)$ are the points coordinates. $M$ is the total number of points. The fractal correlation dimension $D_2$ is then defined by

$$D_2 = \lim_{r \to 0} \frac{\log C(r)}{\log r}$$

In Fig.2 we display $D_2$ vs. time for each multipolarity. At very beginning $D_2$ is equal to one, showing that the motion is regular. As time goes on, regularity is lost and the motion becomes chaotic until a complete randomness is reached, in which case $D_2 = 2$ as expected [9]. This result confirms the one published in ref. [5], where a different method of calculation was however employed.

Fig.2 : The time evolution of the fractal correlation dimension $D_2$ is plotted for the multiplicities $L = 0, 2, 3$. The dashed lines are to guide the eye.

### 3.2 Chaos and dissipation

Now let us discuss the dissipative properties of our system. On the left-hand side of Fig.3 we plot the evolution of the collective variable $\alpha$ vs. time for one event
only. Each panel corresponds to a fixed multipolarity. We note that the amplitude of the collective motion shows an irregular oscillation, at variance with the results found in ref.[4]. A slight damping can be observed, at variance with the results published in ref.[5] where calculations with a smaller number of particles were performed. Please note that $\alpha$ keeps on oscillating around the equilibrium value $\bar{\alpha}$ which is equal to zero for $L = 2, 3$, whereas in the monopole case it differs from zero [5] and depends on the gas temperature and on the wall frequency. On the right-hand side of Fig.3 we plot the time evolution of the excitation energy of the gas, defined as the relative variation of the total energy of the gas $E$ with respect to its initial value $E_0$. In all three cases the gas has been heated up, although a monotonically increasing trend shows up for the quadrupole and octupole modes, while an irregular oscillating pattern is visible for the monopole case. Moreover, in the $L = 0$ oscillation the energy gained by the gas is lower than in the $L = 2, 3$ cases. Therefore some dissipation is present for all multipolarities and is larger for increased $L$.

Fig.3 : On the left-hand side the time evolution of the collective variable is shown for the multipolarities $L = 0, 2, 3$. On the right-hand side the time evolution of the gas excitation energy is shown for the same $L$.

In order to have a good global picture of the macroscopic system, many events are needed and average observables should be considered. In Fig.4 we display the behavior of an ensemble of 1000 different events, each obtained by assigning random initial conditions to the particles both in coordinate and momentum
space. Average quantities are reported.

The collective variable $\alpha$, plotted on the left-hand side of Fig.4, shows a damping much stronger than the one observed in the single event. This is due to the fact that, once chaos has developed, the particles hit the wall essentially at random, and this produces a stochastic dephasing of the collective motion between different events. The overlap of many of those events produces then a strong damping. We note that the motion of the collective variable is completely damped out for times which depend on the multipolarity and develops around equilibrium. The reader should keep in mind that for $L = 0$ collisions are more inelastic (see paragraph 2) due to the lighter mass of the wall, therefore the monopole oscillation damps out earlier than the $L = 2$ mode. The same mass was used in the calculations published in ref.\[5\]. If we put $\eta = 10$ even for $L = 0$, the damping time is longer \[7\].

Fig.4 : Same as Fig.3, but for an ensemble of 1000 events.

As far as the excitation energy of the gas is concerned, in the right-hand side of Fig.4 we note that the monopole oscillation behaves quite differently than the ones with $L = 2, 3$. In fact, while in those modes the average among different events only decreases the fluctuations with respect to the single event, in $L = 0$ the average produces a dramatically different trend than in the single event. One could deduce that the stochastic dephasing plays a major role for the monopole oscillation. Moreover for $L = 2, 3$ we note the presence of two different regimes: a first one lasting for about 250 fm/c characterized by a sharp rising in the exci-
tation energy, and a second one at successive times where saturation dominates. As it can be deduced from Fig.1, the first stage is related to the onset of chaos in the single-particle motion. After that chaos has fully developed, and a large part of the total energy has been pumped into the gas, a certain degree of thermalization is reached and saturation shows up. This behavior is completely absent in the monopole case. Therefore it seems that different kinds of dissipation can originate from the same underlying chaotic single-particle motion. It should be stressed that, within the time of chaos development, the wall has dissipated only a fraction of its energy, as it is clearly shown in Fig.5.

A completely different dynamics appears if the coupling terms in the single-particle Hamiltonian (1) are neglected, thus allowing the wall to pump energy into the particles’ gas. On the contrary, the particles cannot transfer energy to the wall, and therefore the whole system is dissipative. In this case the collective variable $\alpha$ is a regularly oscillating curve\[5\]. In Fig.6 we show the time evolution of the excitation energy of the gas. While in the monopole case the gas has been slightly heated up, a strong dissipation characterized by only one regime shows up for $L = 2, 3$, thus recovering qualitatively the trend predicted by the "wall formula" and examined in ref.\[3\]. This is also confirmed by the blow-up shown in Fig.7, where we compare the different trends with coupling terms (left-hand side) and without (right-hand side) for quadrupole and octupole modes. Dashed lines are best-fit curves. In the case with coupling we note that the dissipation rates related to the slopes $b$ are identical, while they strongly depend on the

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**Fig.5:** The wall energy, averaged over an ensemble of 1000 events, is reported vs. time for different multipolarities.
multipolarity in the uncoupled case. This means that when including coupling terms the dynamics plays a major role, independently on the surface irregularity, while in the uncoupled case the geometry of the problem is more important.

Fig. 6: The excitation energy calculated without coupling terms is reported vs. time for different multipolarities.

Fig. 7: A blow-up of fig. 6 during the first 500 fm/c is displayed vs. time for $L=2, 3$. On the left-hand side results for the coupled case are shown, while on the right-hand side coupling terms are off. $b$ is the slope of the best-fit curves (dashed lines).
4 Conclusions

In conclusions, we have presented a novel approach based on the solution of the Hamilton’s equations for several classical particles moving in a classical billiard having nuclear-like dimensions, in order to explain dissipation of the collective motion. We found that the presence of a coupling term in the single particle Hamiltonian induces chaotic motion at microscopic level.

As far as the monopole mode is concerned, we found irregular behavior and a slight damping in the single event. On the other hand, a whole bunch displays dissipation because of incoherence among different events. This incoherence is produced by the chaotic single particle dynamics, which makes all events belonging to the same ensemble strongly different one from each other. The dissipative process looks different for the quadrupole and octupole modes. In fact, while the single event properties are similar to the the monopole case, an ensemble of events shows that two different regimes appear: a) an initial fast dissipative evolution corresponding to the onset of chaos in the single-particle motion and b) a slower dissipative trend towards equilibrium.

A strongly different dynamics appears if the coupling terms are switched off. The excitation energy of the gas monotonically increases and only one dissipative regime shows up. However, the main result of our work is that dissipation is present for any multipolarity, at variance with the prediction of ref. [2].

One should realize that our model as well as the one of Swiatecki et al. imply real particle wall collisions. Such particle wall collisions are absent in mean field calculations [3]. It is unclear at this moment whether this difference can have strong consequences on the damping mechanism of collectivity but this possibility is certainly not excluded. Corresponding studies are planned for the future.

We hope that the results obtained in such a schematic classical model could be of help in understanding the damping of nuclear collective motion.

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See also Rapisarda’s talk at this conference.

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