A fault diagnosis and quality prediction method of ball valves based on state transition probability matrix in Markov chain

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Abstract. Product quality is very important because many faults have a real impact on users directly. It is significative that Markov chain theory is applied in product quality prediction. A new method of state transition probability matrix in Markov chain is used to provide fault diagnosis and quality prediction in this paper. The aim of this study is to deliver a quality prediction method for mechanical and electrical product by using the data collection and analysis techniques. Firstly, a new approach of fault diagnosis was presented by using the theory of state transition probability matrix. Then, Markov chain theory is used to build the mathematical patterns in the quality prediction with the state transition probability matrix. Finally, we illustrate the veracity rationality and science of the approved method using a numerical example of ball valves.

1. Introduction

Product quality work is of vital significance to the prosperity of international market. But how to improve the products quality is a difficult problem for manufacturing enterprises. The quantitative forecasting approaches of statistical or mathematical technique play an important role in research of product quality analysis and prediction. Especially, these methods are suitable for the quality prediction in the manufacturing of product [1]. These technical innovation and prediction method innovation is instrumental in improving the prediction accuracy. Moreover, how to reduce product costs and forecast product quality is an urgent problem of technical and managerial personnel [2]. The quality prediction and quality analysis problem of mechanical and electrical product are presented by using Markov chain theory in this paper. Based on the prediction theory, the product quality prediction system was developed and predicted successfully [3]. In this study, the survey of industry and academia identified state transition probability matrix in Markov chain for fault diagnosis and quality prediction of ball valves. The aim of this paper is to provide a simple and effective way to apply state transition probability matrix in Markov chain when a data analysis is applied together with the data consolidation pattern.

On the other hand, it is difficult to conduct product assembly process control due to its complex assembly process. To ensure quality and reduce failures, workers operate strictly according to regulations during the process of product machining and assembly [4]. Numerical methods offer one approach for automatic forecasting rules in intelligent fault diagnosis in machining process [5]. It is very important in theory and practice that the product assembly process of and key influence facts are researched. The basic product assembly process has important application opportunity for the
development of quality prediction model. This paper goes some way toward helping to make the application of state transition probability matrix in Markov chain. Moreover, many articles provide a new method for product quality analysis and fault diagnosis, which includes neural network, grey theory, Markov model, and so on [6]. Many studies have demonstrated that Markov chain theory of fault diagnosis approach is correct and effective. This paper studies the fault diagnosis and prediction rules methodology based on state transition probability matrix in Markov chain.

The content of this paper is arranged as follows: The first chapter is an introduction. Section 2 discusses a new approach of fault diagnosis and quality prediction. Markov chain theory and state transition probability matrix are introduced at various fields. A numerical example of fault diagnosis and quality prediction method of ball valves are presented based on state transition probability matrix in Markov chain in Section 3. Finally, some useful conclusions have been obtained.

2. A new approach of fault diagnosis and quality prediction

A new approach of fault diagnosis and quality prediction was presented to predict the product quality in later chapters. Let us suppose we are predicting the product quality by using these observations can be immediately applied to improve fault diagnosis performance. Note that the choice of numerical quantities including the range of years, data for assembly failure rate and the quality prediction method used below are chosen for illustrative purposes. The product assembly process model based on Markov chain method was presented to solve the problem that information was expressed incompletely in state transition probability matrix [7]. However, it is necessary to use a more complicated approach to quality prediction without an adequate approximating model. The technique of condition monitoring and fault diagnosis used to analysis the cause of the high failure rate had been found out in manufacturing processes. There are many data extraction tools to extract failure data from tables, lists and other data structures in the product assembly process. In this study, the executive process of calculation process of fault diagnosis and quality prediction method based on state transition probability matrix in Markov chain is shown in figure 1.

![Figure 1. Executive process of the calculation process of fault diagnosis and quality prediction.](image-url)
2.1. Markov chain theory
Based on Markov chain theory, a process matrix for the change of failure state in assembly process is set up in this paper. Let us assume that \( \{X(t), t \in T\} \) is a random variables, \( E \) is the state space. Note that \( t_1 p t_2 p K p t_n p t \), and \( x_1, x_2, K, x_n, x \in E \), random variable \( X(t) \) belong to \( X(t_1) = x_1, K, X(t_n) = x_n \) are in contact with the conditional probability distribution of \( X(t_0) = x_0 \), has nothing to do with \( X(t_1) = x_1, K, X(t_{n-1}) = x_{n-1} \).

That is, some conditions for the above uniformly relationship are obtained the following form:

\[
F\left(x, t \mid x_n, x_{n-1}, K, x_2, x_1, t_n, t_{n-1}, K, t_2, t_1\right) = F\left(x, t \mid x_n, t_n\right) \tag{1}
\]

And the following conditions must be met:

\[
P\left\{X(t) \leq x \mid X(t_n) = x_n, K, X(t_1) = x_1\right\} = P\left\{X(t) \leq x \mid X(t_n) \leq x_n\right\} \tag{2}
\]

The fault diagnosis model was established based on the state transfer of the quality prediction system by using Markov process [8].

It is important to choose an appropriate discrete type random variable of \( x(i) \) to satisfy Markov property in the following constrained equations:

\[
P\left\{X(t) \leq x \mid X(t_n) = x_n, K, X(t_1) = x_1\right\} = P\left\{X(t) \leq x \mid X(t_n) \leq x_n\right\} \tag{3}
\]

Where \( \{X(t), t \in T\} \) satisfies the needs of Markov property, the process that can be described in this manner is a Markov process [9].

Then, the Markov process and Markov chain of stochastic process theory was presented in the section.

The most common approach to estimate the parameters is to compute the given state \( x_1, x_2, K, x_{n-1} \) and the current state of \( x_n \), the future state of \( x_{n+1} \) for the conditional distribution laws of the state of past states. A probability integral equation was used for constraint condition of conditional distribution by the probability basic equation, which can be calculated by equation of \( P\left(x_{n+1} / X_n\right) \).

Next, we present the method of state transition probability matrix in Markov chain in the following.

2.2. State transition probability matrix
State transition probability matrix in Markov chain has been incorporated by many organizations around the world as a primary tool to predict future problems [10]. In this article, the fault diagnosis and quality prediction method were studied based on probability analysis and the method of vector Markov process [11].

The time variable is referred to as random process of \( X(n) \), the state within a certain time of \( X(k) \), these dynamic tags relate to the state of the time variable and can be used to Markov process. Moreover, the state within a certain time of \( (k+1) \) was related to the state within the certain time of \( k \), and has nothing to do with the certain time of \( (k-1), (k-2) \) and so on. Thus, this process is known as Markov chain.

The method can estimate the posterior probability distribution of each location by using the Markov chain method by equation (4)

\[
P_{ij}(s,n) = p\left\{x_n = a_j \mid x_s = a_i\right\} \tag{4}
\]

When the \( x_s = a_i \), the transition probability of network states of \( x_n = a_j \) was the basic principle of Markov chain under the condition of \( a_i \) within a certain time of \( s \).

Next, a calculation equation based on the transfer probability matrix of the Markov link is given in the following:
\[ P(s, n) = \begin{bmatrix} P_1(s, n) & P_{1N}(s, n) \\ M & O & M \\ P_{N1}(s, n) & P_{NN}(s, n) \end{bmatrix} \]  

(5)

Where the transition probability of \( P_i(s, n) \) was concerned with \( i, j \) and time span of \( (n-s) \), the transition probability of \( P_{i,j}(k) \) for parameters \( k \) is

\[ P_i(s, n) = P_{i,j}(k) \]  

(6)

The transfer probability matrix includes stationarity of data with main factors influencing precision of dimension computation. When \( k = 1 \), the one-step transition probability is recorded as \( P(1) \).

Next, matrix that incorporates both \( k \) transition probability and combination matrix was given as:

\[ P_i(k) = \begin{bmatrix} \mu_{i1}(k) & \mu_{i2}(k) & K & \mu_{1N}(k) \\ \mu_{21}(k) & \mu_{22}(k) & K & \mu_{2N}(k) \\ M & M & O & M \\ \mu_{N1}(k) & \mu_{N2}(k) & K & \mu_{NN}(k) \end{bmatrix} \]  

(7)

If the transfer is random, \( k \) transition probability would be matrix one-step transition probability times \( k \), which can be calculated

\[ P(k) = (P_0(k))_{N^+N} = P(1)^k \]  

(8)

The future state of things can be forecasted by using transfer probability among differ stages of Markov chain method. Therefore, for every object of transition probability of \( P(k) \), there can only be one \( k \) object connected to it.

\[ P(k) = P(k-1)/P \]  

(9)

where \( P_0(k) \) of \( k \) transition probability is a subset with the following two properties, that is

\[ P_i(k) \geq 0, i, j = 1, 2, K, N \]  

(10)

Next, according to ergodic properties of the discrete parameter Markov chain, there was a transfer probability limit in the following:

\[ \lim_{k \to \infty} P_i(k) = P_j, i, j \in E \]  

(12)

Then there exists a positive integer \( k \), the following conditions must be met

\[ P_i(k) \neq 0, i, j = 1, 2, K, N \]  

(13)

The statistic on testing equality and its limit distribution of \( \{P_j, j = 1, 2, K, N\} \), which was given as:

\[ p_j = \sum_{i=1}^{N} P_j P_{i,j}, j = 1, 2, K, N \]  

(14)

With conditions, the variable until the condition is met:

\[ p_j \neq 0 \{j = 1, 2, K, N\}, \sum_{j=1}^{N} p_j = 1 \]  

(15)
3. A numerical example

In this section, we applied the approved method to the use of fault diagnosis and quality prediction of ball valves. Then, an example where increased prediction accuracy of a Chinese manufacturing firm was illustrated in this paper. The new method was validated by using state transition probability matrix in Markov chain with few recent data. The original data are used to fault diagnosis and there is a need to quantify the uncertainty on quality prediction when limited amounts of data are collected. Therefore, transforming the data to a normal distribution before assuming a Markov chain model is a reasonable method. For the principle of maximum probability, we chose it as best we could predict the assembly quality is less flexible and limits how well we could match the probability of failure. For two data scenarios of ball valves, we apply state transition probability matrix in Markov chain to update the fault diagnosis and quality prediction.

Moreover, we are particularly interested in failure state because that is the uncertainty bound we will use for quality prediction. In the first scenario, we observe the statistics of weekly failure rate in 8 months. In the second scenario, we observe data tests of 47 weeks with no fails like the first scenario, and in addition tests are performed in one week. Next, the trouble rate of ball valves and the quality characteristics of assembly stages with eight months of data in 2017. Then, the fault diagnosis technology of ball valves was divided into the efficient normal state, inefficient normal state, simple repair state, complex repair state, scrapped state, which were shown as 1, 2, 3, 4 and 5 respectively. The failure data on gathering analysis of 48 weeks a week in a Chinese manufacturing was used as firm covariance when a trouble occurred in time and reason. Then, the statistics data of failure state of ball valves with 200 ball valves was shown in table 1, table 2, table 3 and table 4.

| Week | Failure state |
|------|---------------|
| 1    | 2             | 1 | 1 | 2 | 3 | 4 | 3 | 3 | 2 | 2 | 1 | 1 |

Table 1. The statistics data of failure state of ball valves of 1-12 weeks.

| Week | Failure state |
|------|---------------|
| 13   | 14            | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |

Table 2. The statistics data of failure state of ball valves of 13-24 weeks.

| Week | Failure state |
|------|---------------|
| 25   | 26            | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 |

Table 3. The statistics data of failure state of ball valves of 25-36 weeks.

| Week | Failure state |
|------|---------------|
| 37   | 38            | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 |

Table 4. The statistics data of failure state of ball valves of 37-48 weeks.

We can see from these tables, the first 47 weeks of the fault status and data information was used to calculate failure rate, and the data of 48 week was applied to contrast with failure forecasting
models for the data set. The assembly quality was study from the pose errors of the mating part and base part of ball valves.

Based on the theory of state transition probability matrix in Markov chain, the above tables show us that status rate was divided into different states, \( N=47, n_1=15, n_2=18, n_3=11, n_4=2, n_5=1 \). We can calculate the initial probability distribution by using the following formula:

\[
P = \left( \frac{n_1}{N}, \frac{n_2}{N}, \frac{n_3}{N}, \frac{n_4}{N}, \frac{n_5}{N} \right) = \left( \frac{15}{47}, \frac{18}{47}, \frac{11}{47}, \frac{2}{47}, \frac{1}{47} \right).
\]

Then, the state transition table can be shown in table 5.

| State | The future state |
|-------|-----------------|
| 1     | 8 5 1 0 0       |
| 2     | 2 7 5 6 0       |
| 3     | 0 7 2 1 0       |
| 4     | 0 0 1 0 1       |
| 5     | 0 0 1 0 0       |

So, the state transition probability matrix in Markov chain can be calculate as:

\[
P = \begin{bmatrix}
E_{11} & E_{12} & E_{13} & E_{14} & E_{15} \\
N_1 & n_1 & n_1 & n_1 & n_1 \\
E_{21} & E_{22} & E_{23} & E_{24} & E_{25} \\
N_2 & n_2 & n_2 & n_2 & n_2 \\
E_{31} & E_{32} & E_{33} & E_{34} & E_{35} \\
N_3 & n_3 & n_3 & n_3 & n_3 \\
E_{41} & E_{42} & E_{43} & E_{44} & E_{45} \\
N_4 & n_4 & n_4 & n_4 & n_4 \\
E_{51} & E_{52} & E_{53} & E_{54} & E_{55} \\
N_5 & n_5 & n_5 & n_5 & n_5
\end{bmatrix}
\begin{bmatrix}
8 & 5 & 1 & 0 & 0 \\
15 & 15 & 15 & 0 & 0 \\
7 & 5 & 6 & 0 & 0 \\
18 & 18 & 18 & 0 & 0 \\
0 & 7 & 2 & 1 & 0 \\
11 & 11 & 11 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 2 & 0 & 2 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0.533 & 0.333 & 0.067 & 0 & 0 \\
0.389 & 0.278 & 0.333 & 0 & 0 \\
0 & 0.636 & 0.182 & 0.091 & 0 \\
0 & 0 & 0.500 & 0 & 0.500 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}.
\]

Next, the failure state of week 47 is the efficient normal state of 1, after three moves of failure state, the state probability can be calculated as follows.

\( E_{11} = 0.533, E_{12} = 0.333, E_{13} = 0.067, E_{14} = 0, E_{15} = 0 \).

An ordered sequence of the state transition probability matrix was created by setting maximum values for the attributes of Markov chain. We use these values as maximum value by using the principle of the maximum probability in the following:

\[ \text{Max} \{ E_{11}, E_{12}, E_{13}, E_{14}, E_{15} \} = E_{11} = 0.533. \]

Thus, the results show that failure state would be efficient normal state of 1at 48 weeks. The runtime data collected from the process of fault diagnosis and quality prediction to be measured against the simulation results. Additionally, a same test was constructed to provide a comparison to the results obtained from traditional methods, it was found that the results have more accurate and credible by using the theory of state transition probability matrix in Markov chain.
4. Conclusions
In this paper, we presented a fault diagnosis and quality prediction method based on state transition probability matrix in Markov chain. For the numerical example of ball valves in a manufacturing firm, we have assumed that the data are normally distributed with a reasonable assumption. The use of the approved method is motivated by state transition probability matrix in Markov chain of potential mechanisms that may initiate at any time. We can make an accurate decision on the fault diagnosis in accordance with the result of quality prediction and actual situations.

Importantly, this paper provides accurate quality prediction with an easily applied and objective means of Markov chain theory. In this study, we present the new method in terms of their assumptions and properties of state transition probability matrix. Based on Markov Chain theory, the state transition probability matrix was established and then the prediction value was calculated by using the original state probability and the condition transition probability matrix. Moreover, the numerical example illustrates that the method can be applied in many fields that are related to other mechanical and electrical products.

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