Spin-isospin Kondo effects for $\Sigma_c$ and $\Sigma^*_c$ baryons and $\bar{D}$ and $\bar{D}^*$ mesons

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We study the Kondo effect for a $\Sigma_c$ ($\Sigma^*_c$) baryon in nuclear matter. In terms of the spin and isospin symmetry ($\text{SU}(2)_{\text{spin}} \times \text{SU}(2)_{\text{isospin}}$), the heavy-quark spin symmetry and the S-wave interaction, we provide the general form of the Lagrangian for a $\Sigma_c$ ($\Sigma^*_c$) baryon and a nucleon. We analyze the renormalization equation at one-loop level, and find that the coexistence of spin exchange and isospin exchange magnifies the Kondo effect in comparison with the case where the spin-exchange interaction and the isospin-exchange interaction exist separately. We demonstrate that the solution exists for the ideal sets of the coupling constants, including the SU(4) symmetry as an extension of the SU(2)$_{\text{spin}} \times \text{SU}(2)_{\text{isospin}}$ symmetry. We also conduct a similar analysis for the Kondo effect of a $\bar{D}$ ($\bar{D}^*$) meson in nuclear matter. On the basis of the obtained result, we conjecture that there could exist a “mapping” from the heavy meson (baryon) in vacuum onto the heavy baryon (meson) in nuclear matter.

I. INTRODUCTION

In 1964, J. Kondo explained why the electrical resistance in the metal which contains some impurity atoms with a non-zero spin increases logarithmically at low temperatures $^1$. The logarithmic increase of the electrical resistance with the heavy impurity occurs when the following conditions are satisfied: (i) Fermi surface (degenerate state), (ii) particle-hole creation (loop effect), and (iii) non-Abelian interaction (e.g. the spin-exchange interaction) $^2$$^4$. It is understood that under these three conditions, the coupling constant for the interaction becomes stronger, and the Landau pole appears. Since his work was recognized, the Kondo effect has had wider implications for theoretical approaches in quantum systems: the renormalization group method $^5$, the numerical renormalization group $^6$, the Bethe ansatz $^7$$^9$, the boundary conformal field theory $^{10}$$^{19}$, the bosonization method $^{17}$$^{21}$, the mean-field approximation (the large $N$ limit) $^{22}$$^{37}$, and so on.

The Kondo effect is not simply studied in condensed matter physics, but applicable to the nuclear physics where the strong interaction plays a role of the main fundamental force.$^1$ For example, we consider the case where the heavy hadrons involving charm or bottom flavor are brought into the nuclear matter (see Refs. $^{39}$, $^{40}$ for reviews). They can be regarded as the heavy impurity particles, because their masses are much heavier than the light (up, down, and

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$^1$At an early stage, the Kondo effect was studied for deformed nuclei, where the itinerant fermion is a nucleon and the impurity is played by the deformed nucleus $^{38}$. The non-Abelian interaction is provided by the spin exchange through the Coriolis force. However, it leads to the suppression of the Kondo effect due to the sign of the coupling constant.
strange) quarks. Several heavy hadrons have been used in the previous studies: a $\bar{D}$ meson ($D^-$ or $\bar{D}^0$ meson) and a $\bar{D}^*$ meson ($D^{*-}$ or $\bar{D}^{*0}$ meson) [41, 42], or a $D_s^-$ meson and a $D_s^{*-}$ meson [43], in charm flavor. It is certainly true that the heavy hadrons are not stable, because they can decay into the light hadrons via the weak interaction. Nevertheless, it is worth considering the heavy hadrons in the nuclear matter when we only consider the strong interaction or the electromagnetic interaction. The heavy hadrons may be produced in atomic nuclei experimentally at the high-energy accelerator facilities. Clearly, the conditions (i) and (ii) for the Kondo effect are met; the Fermi surface and the particle-hole creations exist in the nuclear matter at the low temperatures. When it comes to the condition (iii), the non-Abelian interaction is provided by the spin-exchange interaction and/or by the isospin-exchange interaction, both of which obey the SU(2)$_{\text{spin}}$ symmetry and/or the SU(2)$_{\text{isospin}}$ symmetry, respectively. The research on the Kondo effect for the $\bar{D}$ and $\bar{D}^*$ mesons and the $D_s^-$ and $D_s^{*-}$ mesons in nuclear matter was conducted by using the perturbative calculation [41] and the mean-field approximation [43]. The Kondo effect for the heavy hadron in atomic nuclei was studied in terms of the mean-field approximation in the Lipkin model, in which the fluctuation effect was also considered [42].

From a QCD perspective, it is noteworthy that the Kondo effect was also studied for a charm or bottom quark in quark matter, where the non-Abelian interaction between the heavy quark and the itinerant light quark is provided by the color-exchange interaction in accordance with the SU(3)$_{\text{color}}$ symmetry [41, 44]. This is called the QCD Kondo effect [44]. The QCD Kondo effect was studied in various theoretical methods: the simple perturbation [41], the (perturbative) renormalization group with gluon exchange [44], the mean-field approximation [45–48], the conformal boundary theory [49, 50]. The competition between the QCD Kondo effect and the color superconductivity or the chiral condensate was analyzed [51, 52]. In addition, the transport properties such as the electric conductivity and the shear viscosity were studied [48]. It is important to mention that the QCD Kondo effect in the quark matter with the light flavor $N_f \geq 2$ serves the overscreened Kondo effect instead of the normal Kondo effect with an exact screening, and it leads to the non-Fermi liquid behavior [49–51]. The heavy quark in strong magnetic field induces the QCD Kondo effect at the vanishing density (the magnetically-induced QCD Kondo effect), where the light quarks are confined with degeneracy in the lowest Landau level [53]. It was recently argued that the QCD Kondo effect occurs even in the absence of the heavy quark in quark matter: the color non-singlet gap in the two-flavor superconductivity (2SC) plays the role of the “heavy impurity”, and it leads to the non-Abelian interaction with the light quarks which do not participate to form the 2SC gap [54].

The purpose of the present paper is to study the Kondo effect for a $\Sigma_c$ ($\Sigma_c^*$) baryon in nuclear matter. The $\Sigma_c$ ($\Sigma_c^*$) baryon has spin 1/2 (3/2) and isospin 1, and it can provide the non-Abelian interaction by the spin and isospin-exchange with a nucleon. We consider the heavy mass limit for the heavy quark (a charm quark) [55–57], where the spin-flip and isospin-flip interactions work on the light component in the $\Sigma_c$ ($\Sigma_c^*$) baryon, i.e., the light diquark ($qq$). Indeed, the spin-flip process for the heavy quark is suppressed by the factor $\Lambda_{\text{QCD}}/m_Q$ with $\Lambda_{\text{QCD}}$ being the low-energy scale of the QCD and $m_Q$ being the mass of the heavy quark. Thus, the spin of a heavy quark can be regarded as the conserved quantity in the heavy-quark mass limit. This is called the heavy-quark spin (HQS) symmetry [55–57] (see also Refs. [58]). In the present study, we consider only the leading-order term in the heavy-quark mass limit, and neglect the corrections at $\mathcal{O}(\Lambda_{\text{QCD}}/m_Q)$. For example, the heavy quark symmetry is seen approximately in the small mass splitting between a $\Sigma_c$ baryon and a $\Sigma_c^*$ baryon (about 65 MeV) which is much smaller than the baryon masses (2286 MeV and 2520 MeV). The HQS will provide us with a good approximation as the first step to investigate the
Kondo effect for the $\Sigma_c\, (\Sigma^*_c)$ baryon. The effective theory of the $\Sigma_c\, (\Sigma^*_c)$ baryon can be constructed in a general form when we follow the HQS symmetry [59–64] (see also Refs. [58, 65] for reviews), and this formalism will be applied to the interaction between a $\Sigma_c\, (\Sigma^*_c)$ baryon and a nucleon. Given the fact that the $\Sigma_c\, (\Sigma^*_c)$ baryon has two different non-Abelian interactions of spin and isospin, i.e., the SU(2)$_{\text{spin}} \times$ SU(2)$_{\text{isospin}}$ symmetry, we will see that those two symmetries induce rich structures of the Kondo effect. As an ideal situation, for example, the SU(2)$_{\text{spin}} \times$ SU(2)$_{\text{isospin}}$ symmetry will provide the SU(4) symmetry by tuning the coupling constants in the interaction term appropriately. Throughout the present study, we will perform the analysis based on the renormalization group (RG) equation, namely the poor man’s scaling method, as the simple perturbative method [5].

Several comments are in order. In the literature, the binding of a $\Sigma_c\, (\Sigma^*_c)$ baryon in nuclear matter was estimated by the QCD sum rules [66, 67]. The present discussion about the Kondo effect will be useful for further investigation on the binding energy. We notice that a $\Lambda_c$ baryon is not relevant to the Kondo effect in contrast to the $\Sigma_c\, (\Sigma^*_c)$ baryon, because the light diquark ($qq$) in the $\Lambda_c$ baryon has spin 0 and isospin 0, and there is no exchange interaction of spin and isospin between the baryon and a nucleon, as it was analyzed in Ref. [68] (see also the recent work [69, 70]).

Bottom hadrons, which are in general heavier than charm hadrons, could be more suitable for studying the Kondo effect; however, we will not repeat the same discussion for the bottom hadrons. Replacing a $\Sigma_c\, (\Sigma^*_c)$ baryon by a $\Sigma_b\, (\Sigma^*_b)$ baryon is a straightforward task, although it would provide more favorable conditions for greater accuracy of the HQS symmetry. The greater accuracy is seen directly in the mass splitting between a $\Sigma_b$ baryon and a $\Sigma^*_b$ baryon (about 20 MeV) in comparison to their masses (5810 MeV and 5830 MeV, respectively).

The paper is organized as follows. In Sec. II, we introduce the Lagrangian for a $\Sigma_c\, (\Sigma^*_c)$ baryon and a nucleon. We begin by considering the nuclear matter in which a $\Sigma_c\, (\Sigma^*_c)$ baryon exists as an impurity particle, assuming that the nuclear matter is approximately regarded as the free fermion gas where the nucleon is described by the nonrelativistic two-component spinor field $\varphi(x)$. We follow the procedures for the construction of the field of the heavy hadron based on the HQS symmetry [59–64] (see also Refs. [58, 65] for reviews), and apply this formalism to the interaction between a $\Sigma_c\, (\Sigma^*_c)$ baryon and a nucleon. In this framework, the field of the $\Sigma_c\, (\Sigma^*_c)$ baryon can be decomposed to the diquark part ($qq$) and the heavy quark part ($Q$), where the quantum number of the diquark is spin one and isospin one. We introduce the vector field $A^\mu(x)$ ($\mu = 0, 1, 2, 3$), which satisfies $v_\mu A^\mu = 0$, for the diquark part. We also introduce the effective heavy-quark field $u_v(x)$, which satisfies $v_\nu \gamma^\nu u_v = u_v$, for the heavy quark part.

2 Those studies rely on the $\Lambda_cN$ interaction strength estimated by the lattice QCD simulations [71] and the chiral extrapolations [72]. The obtained binding energy for a $\Lambda_c$ baryon is consistent with the results by the QCD sum rules [73].
We define $u_v(x)$ by $u_v(x) = \frac{1}{2} (1 + \gamma_\mu v^\mu) e^{imq vz} u(x)$ in the $v$-frame with the four-velocity $v^\mu \ (v^0 > 0$ and $v_\mu v^\mu = 1)$ and the heavy quark mass $m_Q$, where $u(x)$ is the original four-spinor heavy-quark field at $x$ in the 4-dimensional coordinate system. We consider that the sum is taken over the repeated indices. The condition $v_\nu \gamma^\nu u_v = u_v$ stems from the requirement to project the field $u(x)$ to the positive-energy part. It is supposed that the heavy quark is at rest in the coordinate frame with the four-velocity $v^\mu$. In the following discussion, we consider the static frame by setting $v^\mu = (1, 0)$. With this setup, we define the composite field for the $\Sigma_c (\Sigma^*_c)$ baryon:

$$\Psi^\mu_v(x) = A^\mu(x) u_v(x).$$

(1)

Notice that $\Psi^\mu_v$ has only the off-mass-shell (residual) energy-momentum component with the energy scale smaller than the heavy-baryon mass, because the $\Sigma_c (\Sigma^*_c)$ baryon is supposed to be at rest in the $v$-frame. We also notice that $\Psi^\mu_v$ satisfies $v_\nu \gamma^\nu \Psi^\mu_v = \Psi^\mu_v$ and $v_\mu \Psi^\mu_v = 0$. The former and latter properties are induced by $v_\nu \gamma^\nu u_v = u_v$ and $v_\mu A^\mu = 0$, respectively. With those two conditions, the number of degrees of freedom in $\Psi^\mu_v$ is $3 \times 2 = 6$.

In the above construction, $\Psi^\mu_v$ is a superposed state of the $\Sigma_c$ baryon (spin $1/2$) and the $\Sigma^*_c$ baryon (spin $3/2$). This reflects the concept that the spin of the diquark and the spin of the heavy quark are good quantum numbers in the heavy-quark symmetry, and that the $\Sigma_c$ baryon and the $\Sigma^*_c$ baryon can be superposed. In the physical space, it is convenient to introduce the fields of $\Sigma_c$ baryon and $\Sigma^*_c$ baryon by projecting $\Psi^\mu_v$ to the $\Sigma_c$ baryon component and the $\Sigma^*_c$ baryon component:

$$\Psi_{v1/2} = \frac{1}{\sqrt{3}} \gamma_5 \gamma_\mu \Psi^\mu_v,$$

(2)

for the $\Sigma_c$ baryon and

$$\Psi_{v3/2} = \Psi^\mu_v - \frac{1}{3} (\gamma^\mu + v^\mu) \gamma_\nu \Psi^\nu_v,$$

(3)

for the $\Sigma^*_c$ baryon. Equivalently, $\Psi^\mu_v$ is expressed as a sum of $\Psi_{v1/2}$ and $\Psi_{v3/2}$,

$$\Psi^\mu_v = \frac{1}{\sqrt{3}} (\gamma^\mu + v^\mu) \gamma_5 \Psi_{v1/2} + \Psi_{v3/2}.$$

(4)

In the HQS formalism, the $\Sigma_c$ baryon and the $\Sigma^*_c$ baryon are degenerate in mass and are interchangeable to each other by the HQS symmetry. For this reason, it is essential to consider a $\Sigma_c$ baryon and a $\Sigma^*_c$ baryon to be the effective degrees of freedom. We will see that the heavy-quark-spin symmetry induces the mixing between the $\Sigma_c N$ state and the $\Sigma^*_c N$ state ($N$ for a nucleon) in the nuclear matter.

With the above setup, we consider the Lagrangian in the case where a nucleon and a $\Sigma_c (\Sigma^*_c)$ baryon interact with each other through the $S$-wave interaction on low-energy scale. The $\Sigma_c N (\Sigma^*_c N)$ interaction was considered in the one-boson exchange model with a non-zero range \[74, 75\]. In contrast to them, we suppose that the $\Sigma_c N (\Sigma^*_c N)$ interaction is provided by the contact-type with a zero range. The contact-type interaction and the HQS symmetry allow us to have the most general form of the Lagrangian:

$$\mathcal{L}[\varphi, \Psi^i_v] = \mathcal{L}_{\text{kin}}[\varphi, \Psi^i_v] + \mathcal{L}_{\text{int}}[\varphi, \Psi^i_v],$$

(5)

with the kinetic term

$$\mathcal{L}_{\text{kin}}[\varphi, \Psi^i_v] = \varphi \frac{D}{Dt} \varphi + \varphi^i \frac{C}{2m} \varphi + \bar{\Psi}^i_v \frac{D}{Dt} \Psi^i_v + \mathcal{O}(1/M),$$

(6)
and interaction term

\[ \mathcal{L}_{\text{int}}[\varphi, \Psi_v] = C_1 \varphi^I (1_2 \otimes 1_2) \varphi \bar{\Psi}_v^I (\delta^{ij} \otimes 1_2 \otimes 1_3) \Psi_v^j + C_2 \varphi^I (\sigma^\ell \otimes 1_2_2) \varphi \bar{\Psi}_v^I (i \varepsilon^{ij\ell} \otimes 1_2 \otimes 1_3) \Psi_v^j + C_3 \varphi^I (1_2 \otimes \tau^d) \varphi \bar{\Psi}_v^I (\delta^{ij} \otimes 1_2 \otimes t^d) \Psi_v^j + C_4 \varphi^I (\sigma^\ell \otimes \tau^d) \varphi \bar{\Psi}_v^I (i \varepsilon^{ij\ell} \otimes 1_2 \otimes t^d) \Psi_v^j + \mathcal{O}(1/M), \]

(7)

with the coupling constants \( C_A \) \((A = 1, 2, 3, 4)\). We notice that the index \( \mu \) in \( \Psi_v^\mu \) is restricted to \( i = 1, 2, 3 \) in the rest frame. The above Lagrangian is invariant under the spin symmetry and the isospin symmetry, \( \text{SU}(2)_{\text{spin}} \times \text{SU}(2)_{\text{isospin}} \).

In the operator \( A \otimes B \) acting on the nucleon (\( \varphi \)), \( A \) and \( B \) are the operators for the spin and the isospin of a nucleon. Similarly, in the operator \( A \otimes B \otimes C \) acting on the \( \Sigma_c \) \((\Sigma_c^+\) baryon (\( \Psi_v^I \)), \( A \) and \( B \) are the operators for the spin of the light component \((q\bar{q})\) and the spin of the heavy quark \((Q)\), respectively, and \( C \) is the operator for the isospin of the light component \((q\bar{q})\). \( 1_2 \) is the 2-by-2 identity matrix for spin or isospin, and \( 1_3 \) is the 3-by-3 identity matrix for isospin. We also use the notations \( \sigma^\ell \) \((\ell = 1, 2, 3)\) and \( \tau^d \) \((d = 1, 2, 3)\) for the Pauli matrices acting on the spin of a nucleon and the isospin of a nucleon or a \( \Sigma_c \) \((\Sigma_c^+)\) baryon, respectively. We define \( \varepsilon^{ij\ell} \) \((\varepsilon^{123} = 1; i, j, \ell = 1, 2, 3)\) as the anti-symmetric tensor for the spin of a \( \Sigma_c \) baryon or a \( \Sigma_c^+ \) baryon, and \( t^d \) \((d = 1, 2, 3)\) as the operator for the isospin of a \( \Sigma_c \) \((\Sigma_c^+)\) baryon, whose explicit forms are given by

\[
\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & -i \\
i & 0 & 0
\end{pmatrix},
\begin{pmatrix}
0 & 0 & i \\
0 & 0 & 0 \\
i & 0 & 0
\end{pmatrix},
\begin{pmatrix}
0 & -i & 0 \\
i & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}.
\]

(8)

They satisfy the following relations:

\[
\sum_{\mu=1,2,3} (t^d)_{\mu\rho} (t^e)_{\rho\nu} = \delta^d_{\mu} \delta^e_{\nu} - \delta^d_{\nu} \delta^e_{\mu},
\]

and this will be used in later calculations. With the basis in the isospin operator \( t^\alpha \), the isospin components in \( \Psi_v^{\nu1/2} \) and \( \Psi_v^{\mu3/2} \) are expressed as

\[
\Psi_v^{\nu1/2} = \begin{pmatrix}
\frac{-1}{\sqrt{2}} (\Sigma_c^{++} + \Sigma_c^0) \\
\frac{1}{\sqrt{2}} (\Sigma_c^{++} - \Sigma_c^0) \\
-i \Sigma_c^{++}
\end{pmatrix},
\Psi_v^{\mu3/2} = \begin{pmatrix}
\frac{-1}{\sqrt{2}} (\Sigma_c^{++} + \Sigma_c^0) \\
\frac{1}{\sqrt{2}} (\Sigma_c^{++} - \Sigma_c^0) \\
-i \Sigma_c^{++}
\end{pmatrix}.
\]

(10)

We notice that this representation is not diagonal in the charge basis. The transformation to the diagonal form by the unitary transformation is shown in Appendix B. It is apparent that the Lagrangian in Appendix B has the spin symmetry and the isospin symmetry, \( \text{SU}(2)_{\text{spin}} \times \text{SU}(2)_{\text{isospin}} \) for both a nucleon and for a \( \Sigma_c \) \((\Sigma_c^+)\) baryon. Although the numerical values of the coupling constants \( C_A \) \((A = 1, 2, 3, 4)\) have not been known, the discussion about the Kondo effect can proceed without the information about the specific value of \( C_A \) as it will be presented later.

For later convenience, we rewrite the interaction term of Eq. (5) in a compact form as

\[
\mathcal{L}_{\text{int}}[\varphi, \Psi_v] = C_1 \varphi^I \Gamma \varphi \bar{\Psi}_v^I \Gamma^{ij} \Psi_v^j + C_2 \varphi^I \Gamma^{\ell} \varphi \bar{\Psi}_v^I \Gamma^{ij} \Psi_v^j + C_3 \varphi^I \Gamma^{d} \varphi \bar{\Psi}_v^I \Gamma^{ij} \Psi_v^j + C_4 \varphi^I \Gamma^{d\ell} \varphi \bar{\Psi}_v^I \Gamma^{ij} \Psi_v^j + \mathcal{O}(1/M),
\]

(11)

where we introduce the following operators:

\[
\Gamma \equiv 1_2 \otimes 1_2, \quad \Gamma^{\ell} \equiv \sigma^\ell \otimes 1_2, \quad \Gamma^{d} \equiv 1_2 \otimes \tau^d, \quad \Gamma^{d\ell} \equiv \sigma^\ell \otimes \tau^d.
\]

(12)

3 Notice the relation \((t^\alpha)_{\mu\nu} = -i \varepsilon^{\alpha\mu\nu} \).
for a nucleon ($\varphi$) and

$$\tilde{\Gamma}_{ij} \equiv \delta^{ij} \otimes 1_2 \otimes 1_3, \quad \tilde{\Gamma}_{ij}^t \equiv i\epsilon^{tij} \otimes 1_2 \otimes 1_3, \quad \tilde{\Gamma}_{ij}^d \equiv \delta^{ij} \otimes 1_2 \otimes t^d, \quad \tilde{\Gamma}_{ij}^{td} \equiv i\epsilon^{tij} \otimes 1_2 \otimes t^d,$$

for a $\Sigma_c (\Sigma^*_c)$ baryon ($\Psi_c^i$). The sum is taken over the repeated indices.

Several comments are in order. Firstly, the heavy-quark-spin does not flip by the interaction with a nucleon in the HQS symmetry, and hence we have only the identity operator ($1_2$) for the heavy quark. This is because the spin for the heavy quark ($c$ quark) in the $\Sigma_c (\Sigma^*_c)$ is independent of the spin for the light diquark ($qq$). Thus, to be precise, the total symmetry should be given by $SU(2)_{\text{light spin}} \times SU(2)_{\text{heavy spin}} \times SU(2)_{\text{isospin}}$ including $SU(2)_{\text{heavy spin}}$ for the spin symmetry of the heavy quark.

Secondly, we remark that the propagator of the nucleon with an energy $p_0$ and a three-dimensional momentum $p$ in nuclear matter with the chemical potential $\mu$ is given by

$$\frac{i}{p_0 - (E_p - \mu) + i\varepsilon} = \frac{i\theta(E_p - \mu)}{p_0 - (E_p - \mu) + i\varepsilon} + \frac{i\theta(\mu - E_p)}{p_0 - (E_p - \mu) - i\varepsilon},$$

with $\varepsilon > 0$ an infinitely small number. $E_p = p^2/(2m)$ is the energy of the nucleon with a mass $m$, and $\mu$ is the chemical potential for the nucleon. Notice the difference in the pole positions between the particle component ($E_p > \mu$) and the hole component ($E_p < \mu$). The propagators of the $\Sigma_c$ and $\Sigma^*_c$ baryons with an energy $p_0$ are given by

$$\frac{i\delta_{\alpha\beta}}{p_0 + i\varepsilon}, \quad \frac{i\delta_{\alpha\beta} \delta^{ij}}{p_0 + i\varepsilon},$$

in rest frame. Notice that the energy in the denominator ($p_0$) describes the residual momentum of the $\Sigma_c$ and $\Sigma^*_c$ baryons.

Thirdly, we remark that the $\Sigma_c$ baryon and the $\Sigma^*_c$ baryon can decay via $\Sigma_c \rightarrow \Lambda_c \pi$ and $\Sigma^*_c \rightarrow \Lambda_c \pi$, whose decay widths are around 2 MeV and 15 MeV, respectively [76]. In the present study, we consider that the $\Sigma_c$ and $\Sigma^*_c$ baryons are in the quasi-stable states whose lifetimes are long enough. We also neglect the coupling between the $\Sigma_c N$ ($\Sigma^*_c N$) and the $\Lambda_c N$ state. Those subjects are left for future work.

III. RENORMALIZATION GROUP EQUATION

A. Derivation of the renormalization group equation

In the Kondo effect, the coupling constants in the medium are enhanced logarithmically in the low-energy region, and the system becomes a strongly-coupled one. In this situation, the coupling constants are not the constant values literally, but they should be regarded as the effective coupling constants whose property is dependent on the relevant energy scale in the medium. We therefore study how the coupling constant $C_A$ ($A = 1, 2, 3, 4$) in Eq. (11) is changed into the effective coupling constants in terms of the Kondo effect. We use the renormalization group (RG) equation. Here we introduce the energy scale $\Lambda$, which is measured from the Fermi energy, and see the effective coupling constants for the small change of $\Lambda$. We estimate the coupling constants on the lower-energy scale $\Lambda - d\Lambda$ by including the loop effect of the particle-hole creations with the energy shell between $\Lambda - d\Lambda$ and $\Lambda$. The initial value of the coupling constant starting in the RG equation is assigned to the bare coupling constants in vacuum, i.e., the coupling constants
in Eq. (11), where the relevant energy is denoted by \( \Lambda_0 \). At the one-loop order, we find that the RG equation reads

\[
\sum_{A=1}^{4} iC_A(\Lambda - d\Lambda)(\Gamma^A)_{\alpha\beta}^{ab}(\tilde{\Gamma}^A)_{\mu\nu}^{ij} = \sum_{A=1}^{4} iC_A(\Lambda)(\Gamma^A)_{\alpha\beta}^{ab}(\tilde{\Gamma}^A)_{\mu\nu}^{ij} + \sum_{A,B=1}^{4} iC_A(\Lambda)iC_B(\Lambda)(\Gamma^A)_{\alpha\gamma}^{ac}(\tilde{\Gamma}^A)_{\mu\rho}^{ik}(\Gamma^B)_{\beta\delta}^{cb}(\tilde{\Gamma}^B)_{\nu\sigma}^{kj} \int_{\text{shell}} \frac{dp_0}{2\pi} \frac{d^3p}{(2\pi)^3} \left( \frac{i}{p_0 - (E_p - \mu) + i\varepsilon} - \frac{i}{p_0 + i\varepsilon} \right),
\]

where the term on the left-hand side denotes the effective coupling-constants on the energy scale \( \Lambda - d\Lambda \), and, on the right-hand side, the first term denotes the effective coupling-constant on the energy scale \( \Lambda \), and the second (third) term denotes the loop-integrals with particle (hole) creation in the energy-shell between \( \Lambda - d\Lambda \) and \( \Lambda \) (Fig. 1). In the above equation, the indices in the operator \( \Gamma^A \) and \( \tilde{\Gamma}^A \) \((A = 1, 2, 3, 4)\) are shown as

\[
(\Gamma^1)_{\alpha\beta}^{ab} = \delta^{ab}\delta_{\alpha\beta}, \quad (\Gamma^2)_{\alpha\beta}^{ab} = (\sigma^\gamma)_{\alpha\beta}^{ab}, \quad (\Gamma^3)_{\alpha\beta}^{ab} = \delta^{ab}(\tau^d)_{\alpha\beta}, \quad (\Gamma^4)_{\alpha\beta}^{ab} = (\sigma^\gamma)_{\alpha\beta}^{ab}(\tau^d)_{\alpha\beta},
\]

for the nucleon part, and

\[
(\tilde{\Gamma}^1)_{\mu\nu}^{ij} = \delta^{ij}\delta_{\mu\nu}, \quad (\tilde{\Gamma}^2)_{\mu\nu}^{ij} = i\varepsilon^{ij}\delta_{\mu\nu}, \quad (\tilde{\Gamma}^3)_{\mu\nu}^{ij} = \delta^{ij}(\tau^d)_{\mu\nu}, \quad (\tilde{\Gamma}^4)_{\mu\nu}^{ij} = i\varepsilon^{ij}(\tau^d)_{\mu\nu},
\]

for the \( \Sigma_c (\Sigma^*_c) \) baryon part. Here \( a, b = 1, 2 \) and \( \alpha, \beta = 1, 2 \) are for the spin and the isospin of a nucleon, respectively, and \( i, j = 1, 2, 3 \) and \( \mu, \nu = 1, 2, 3 \) are for the spin and for the isospin of a dichot component \((q\bar{q})\) in a \( \Sigma_c (\Sigma^*_c) \) baryon, respectively. We consider that the sum over the spin direction \((\ell = 1, 2, 3)\) and the isospin direction \((d = 1, 2, 3)\) is included if necessary. Taking into account that the integral region for the momentum is limited to the energy-shell, \(|E - \mu| \leq \Lambda - d\Lambda, \Lambda\) with \( E = p^2/(2m) \), we obtain the following approximated terms:

\[
\int_{\text{shell}} \frac{dp_0}{2\pi} \frac{d^3p}{(2\pi)^3} \left( \frac{i}{p_0 - (E_p - \mu) + i\varepsilon} - \frac{i}{p_0 + i\varepsilon} \right) \simeq -i\rho_0 \frac{d\Lambda}{\Lambda},
\]

and

\[
\int_{\text{shell}} \frac{dp_0}{2\pi} \frac{d^3p}{(2\pi)^3} \left( \frac{i}{p_0 - (E_p - \mu) + i\varepsilon} - \frac{i}{p_0 + i\varepsilon} \right) \simeq i\rho_0 \frac{d\Lambda}{\Lambda}.
\]
where we leave only the leading terms for a small \( d\Lambda/\Lambda \ll 1 \). We introduce \( \rho_0 \equiv m^{3/2} \sqrt{\mu} \) for the state-number-density at the Fermi surface. Then, we rewrite the RG equation (16) as

\[
\frac{d}{d\Lambda} C_1(\lambda) = 0, \\
\frac{d}{d\Lambda} C_2(\lambda) = \rho_0(4C_2(\lambda)^2 + 8C_4(\lambda)^2), \\
\frac{d}{d\Lambda} C_3(\lambda) = \rho_0(-4C_3(\lambda)^2 - 8C_4(\lambda)^2), \\
\frac{d}{d\Lambda} C_4(\lambda) = \rho_0(8C_2(\lambda)C_4(\lambda) - 8C_3(\lambda)C_4(\lambda)),
\]

for each channel \( A = 1, 2, 3, 4 \). Here, we introduce the new variable \( \lambda \equiv -\ln(\Lambda/\Lambda_0) \) instead of the energy scale \( \Lambda \). The high-energy scale \( \Lambda_0 \) for which the RG equation starts is set to be equal to the chemical potential of the nuclear matter \( \mu \) or the cutoff energy-scale \( D \) in the point-like interaction in Eq. (11). In the present discussion, however, there is no necessity to specify the value of \( \Lambda_0 \) explicitly. We notice that the variable \( \lambda \) changes from \( \lambda = 0 \) to \( \lambda \to \infty \) in correspondence to the change from the high-energy scale to the low-energy scale. As seen in Eq. (21), \( C_1(\lambda) \) is not affected by the change of \( \lambda \), and hence the spin and isospin-independent channel is not subject to the medium effect. Thus, we will consider only \( C_2(\lambda), C_3(\lambda), \) and \( C_4(\lambda) \) in the following discussions. For convenience, we use the following dimensionless effective coupling constants

\[
\tilde{C}_2(\lambda) \equiv 4\rho_0C_2(\lambda), \quad \tilde{C}_3(\lambda) \equiv -4\rho_0C_3(\lambda), \quad \tilde{C}_4(\lambda) \equiv 4\rho_0C_4(\lambda),
\]

instead of \( C_2(\lambda), C_3(\lambda), \) and \( C_4(\lambda) \), and rewrite the RG equation (21) as

\[
\frac{d}{d\lambda} \tilde{C}_2(\lambda) = \tilde{C}_2(\lambda)^2 + 2\tilde{C}_4(\lambda)^2, \\
\frac{d}{d\lambda} \tilde{C}_3(\lambda) = \tilde{C}_3(\lambda)^2 + 2\tilde{C}_4(\lambda)^2, \\
\frac{d}{d\lambda} \tilde{C}_4(\lambda) = 2(\tilde{C}_2(\lambda) + \tilde{C}_3(\lambda))\tilde{C}_4(\lambda).
\]

Those are the basic equations used in the following discussions. Notice that we have added the minus sign for \( C_3(\lambda) \) in Eq. (22) simply for the appearance of the equations. The initial conditions are given as \( \tilde{C}_2(0) = 4\rho_0C_2, \tilde{C}_3(0) = -4\rho_0C_3, \) and \( \tilde{C}_4(0) = 4\rho_0C_4 \) with \( C_2, C_3, \) and \( C_4 \) being the coupling constants in the interaction Lagrangian (11). We plot the right-hand side of Eq. (23), i.e., the vector \( (\tilde{C}_2(\lambda)^2 + 2\tilde{C}_4(\lambda)^2, \tilde{C}_3(\lambda)^2 + 2\tilde{C}_4(\lambda)^2, 2(\tilde{C}_2(\lambda) + \tilde{C}_3(\lambda))\tilde{C}_4(\lambda)) \) in the three-dimensional parameter space \( (\tilde{C}_2(\lambda), \tilde{C}_3(\lambda), \tilde{C}_4(\lambda)) \), and also show the stream lines for \( (\tilde{C}_2(\lambda), \tilde{C}_3(\lambda), \tilde{C}_4(\lambda)) \) varying with \( \lambda \) and the several initial conditions \( (\tilde{C}_2, \tilde{C}_3, \tilde{C}_4) \) at \( \lambda = 0 \) as the solutions of Eq. (23). The initial conditions are plotted by the dots in the figure. We notice that, for the increasing \( \lambda \), there are some initial conditions giving the stream lines convergent to zero and the other initial conditions giving the stream lines divergent. In the following subsections, we will investigate the solutions of Eq. (23) in detail. We will find that the \( C_4 \) term, i.e., the spin and isospin-dependent term in Eq. (11) plays an important role to extend the parameter region of the coupling constants in which the Kondo effect occurs.

**B. Analytical solutions in special cases**

Although Eq. (23) may look simple, it is difficult to obtain the analytic solution due to the nonlinearity of the equation. Therefore, we have to perform the numerical calculation. In order to understand roughly the properties of
FIG. 2. Left: the plot of the vector $(\tilde{C}_2(\lambda)^2 + 2\tilde{C}_4(\lambda)^2, \tilde{C}_3(\lambda)^2 + 2\tilde{C}_4(\lambda)^2, 2(\tilde{C}_2(\lambda) + \tilde{C}_3(\lambda))\tilde{C}_4(\lambda))$, i.e., the right-hand side of Eq. (23) in the three-dimensional parameter space $(\tilde{C}_2(\lambda), \tilde{C}_3(\lambda), \tilde{C}_4(\lambda))$. Right: the stream lines of $(\tilde{C}_2(\lambda), \tilde{C}_3(\lambda), \tilde{C}_4(\lambda))$ as the solutions of Eq. (23). The initial conditions $(\tilde{C}_2, \tilde{C}_3, \tilde{C}_4)$ are expressed by dots. The solid and dashed lines with gray indicate the manifold in the SU(4) limit (cf. Sec. III B 4).

the solutions before the numerical computing, we seek to obtain analytic solutions by restricting the parameter space to simpler subspaces and focusing on special cases: (i) $\tilde{C}_4(\lambda) = 0$, (ii) $\tilde{C}_3(\lambda) = 0$ (or $\tilde{C}_2(\lambda) = 0$), (iii) $\tilde{C}_2(\lambda) = \tilde{C}_3(\lambda)$ with $|\tilde{C}_4(\lambda)| \ll 1$, and (iv) $\tilde{C}_2(\lambda) = \tilde{C}_3(\lambda) = \pm \sqrt{2/3}\tilde{C}_4(\lambda)$. We will show that, in the last case, the SU(4) symmetry is realized as an extension from the SU(2)$_\text{spin} \times$ SU(2)$_\text{isospin}$ symmetry in the Lagrangian. The simple settings from (i) to (iv) will provide us with fresh insights about the Kondo effect for the $\Sigma^c_c (\Sigma^*_c)$ baryon in the nuclear matter.

1. Conventional case

We consider the case of $\tilde{C}_4(\lambda) = 0 (C_4 = 0)$, i.e., neglecting the spin and isospin-dependent term in the interaction. Then, the RG equation (23) is transformed to

$$
\frac{d}{d\lambda} \tilde{C}_2(\lambda) = \tilde{C}_2(\lambda)^2,
\frac{d}{d\lambda} \tilde{C}_3(\lambda) = \tilde{C}_3(\lambda)^2,
\frac{d}{d\lambda} \tilde{C}_4(\lambda) = 0.
$$

We find that $\tilde{C}_4(\lambda)$ is constant, while $\tilde{C}_2(\lambda)$ and $\tilde{C}_3(\lambda)$ change according to the change of the energy scale. Because $\tilde{C}_2(\lambda)$ and $\tilde{C}_3(\lambda)$ are decoupled, each of the spin-dependent term and the isospin-dependent term obeys the usual Kondo effect with a single non-Abelian symmetry. The Kondo effect of the single non-Abelian symmetry is summarized in the appendix A. The solutions of $\tilde{C}_2(\lambda)$ and $\tilde{C}_3(\lambda)$ are found to be

$$
\tilde{C}_2(\lambda) = \frac{\tilde{C}_2}{1 - \tilde{C}_2\lambda},
\tilde{C}_3(\lambda) = \frac{\tilde{C}_3}{1 - \tilde{C}_3\lambda}.
$$
with $\tilde{C}_2 = 4\rho_0 C_2$ and $\tilde{C}_3 = -4\rho_0 C_3$ as the initial condition at $\lambda = 0$. Thus, the three-dimensional parameter space is essentially reduced to the one-dimensional one. Let us consider the behavior of the solution $\tilde{C}_2(\lambda)$ in detail in the energy scales from $\lambda = 0$ (high energy) to a larger value (low energy). For the positive value of $C_2$ ($C_2 > 0$), we notice that $\tilde{C}_2(\lambda)$ becomes divergent at the end of the energy scale $\Lambda = \Lambda_K$ with $\Lambda_K = \Lambda_0 e^{-1/(4\rho_0 C_2)}$. $\Lambda_K$ is called the Kondo scale (the Landau pole) which is quantity smaller than $\Lambda_0$ due to the exponential factor. At the Kondo scale, the coupling constant becomes sufficiently large. Thus, the system becomes a strongly coupled one and the non-perturbative analysis should be adopted. For the negative value of $C_2$ ($C_2 < 0$), the effective coupling constant becomes zero without divergence in the low-energy limit ($\lambda \to \infty$), and hence such interaction disappears in the ground state.

A similar analysis is applied to the case of $\tilde{C}_3(\lambda)$. We find that the effective coupling constant becomes divergent at the Kondo scale $\Lambda_K' = \Lambda_0 e^{1/(4\rho_0 C_3)}$ for the negative value of $C_3$ ($C_3 < 0$), while it disappears for the positive value of $C_3$ ($C_3 > 0$). Notice that the sign of $C_3$ for the Kondo effect is different from that of $C_2$ due to the definition in Eq. (22) and that the values of $\Lambda_K$ and $\Lambda_K'$ can be different in general.

So far we have set $\tilde{C}_4(\lambda) = 0$ ($C_4 = 0$) by neglecting the spin and isospin-dependent term in Eq. (11), and have seen that $C_2 < 0$ and $C_3 > 0$ lead to the absence of the Kondo effect. However, this is the case only for $\tilde{C}_4(\lambda) = 0$ ($C_4 = 0$). In the following cases, we will demonstrate that the Kondo effect can occur even for $C_2 < 0$ and $C_3 > 0$ when a non-zero value of $\tilde{C}_4(\lambda)$ is considered.

2. Two-dimensional case I

By setting $\tilde{C}_3(\lambda) = 0$ in Eq. (23), we consider the two-dimensional parameter space spanned by $(\tilde{C}_2(\lambda), \tilde{C}_4(\lambda))$. We present the case of $\tilde{C}_3(\lambda) = 0$ for the demonstration. The similar conclusion is reached also for $(\tilde{C}_3(\lambda), \tilde{C}_4(\lambda))$ by setting $\tilde{C}_2(\lambda) = 0$. By setting $\tilde{C}_3(\lambda) = 0$, the RG equation (23) is reduced to

$$\frac{d}{d\lambda} \tilde{C}_2(\lambda) = \tilde{C}_2(\lambda)^2 + 2\tilde{C}_4(\lambda)^2, \quad \frac{d}{d\lambda} \tilde{C}_4(\lambda) = 2\tilde{C}_2(\lambda)\tilde{C}_4(\lambda).$$

(26)

To find the solution, we eliminate $\tilde{C}_2(\lambda)$ in the above equations, and obtain the equation for $\tilde{C}_4(\lambda)$,

$$\tilde{C}_4(\lambda) \frac{d^2}{d\lambda^2} \tilde{C}_4(\lambda) - \frac{3}{2} \left( \frac{d}{d\lambda} \tilde{C}_4(\lambda) \right)^2 - 4\tilde{C}_4(\lambda)^4 = 0.$$  

(27)

Interestingly, this nonlinear differential equation has the simple analytical solution. As a result we obtain the solutions

$$\tilde{C}_2(\lambda) = \frac{(-\tilde{C}_2^2 + 2\tilde{C}_4^2)\lambda + \tilde{C}_2}{1 - 2\tilde{C}_2\lambda + (\tilde{C}_2^2 - 2\tilde{C}_4^2)\lambda^2}, \quad \tilde{C}_4(\lambda) = \frac{\tilde{C}_4}{1 - 2\tilde{C}_2\lambda + (\tilde{C}_2^2 - 2\tilde{C}_4^2)\lambda^2},$$

(28)

with $\tilde{C}_2 = 4\rho_0 C_2$ and $\tilde{C}_4 = 4\rho_0 C_4$ as the initial condition. The Kondo effect occurs, when $\tilde{C}_2(\lambda)$ and $\tilde{C}_4(\lambda)$ becomes divergent at a large value of $\lambda$ as the Kondo scale. To find the Kondo scale, we solve $(\tilde{C}_2^2 - 2\tilde{C}_4^2)\lambda^2 - 2\tilde{C}_2\lambda + 1 = 0$, and we obtain $\lambda = \lambda_{\pm}$ with $\lambda_{\pm} = 1/(\tilde{C}_2 \pm \sqrt{2}\tilde{C}_4)$. In order for that either $\lambda_+ > 0$ or $\lambda_- > 0$ is satisfied, the values of $\tilde{C}_2$ and $\tilde{C}_4$ should satisfy $\tilde{C}_4 > -\tilde{C}_2/\sqrt{2}$ or $\tilde{C}_4 < \tilde{C}_2/\sqrt{2}$ in the two-dimensional parameter space $(\tilde{C}_2, \tilde{C}_4)$. The
Kondo scale $\Lambda_K$ is obtained as

$$\Lambda_K = \Lambda_0 \exp \left( -\frac{1}{4\rho_0 \max(C_2 + \sqrt{2}C_4, C_2 - \sqrt{2}C_4)} \right) \text{ for } \hat{C}_4 > -\hat{C}_2/\sqrt{2} \text{ and } \hat{C}_4 < \hat{C}_2/\sqrt{2},$$

$$\Lambda_K = \Lambda_0 \exp \left( -\frac{1}{4\rho_0 (C_2 + \sqrt{2}C_4)} \right) \text{ for } \hat{C}_4 > -\hat{C}_2/\sqrt{2} \text{ and } \hat{C}_4 < \hat{C}_2/\sqrt{2},$$

$$\Lambda_K = \Lambda_0 \exp \left( -\frac{1}{4\rho_0 (C_2 - \sqrt{2}C_4)} \right) \text{ for } \hat{C}_4 < -\hat{C}_2/\sqrt{2} \text{ and } \hat{C}_4 < \hat{C}_2/\sqrt{2},$$

with $\Lambda_0$ being the high-energy scale ($\mu$ or $D$) as the initial condition. The equation forms of the Kondo scale is dependent on the region of $(\hat{C}_2, \hat{C}_4)$.

In Fig. 3, we plot the region where the Kondo effect occurs. In the left panel, we show the two-dimensional vector $(\hat{C}_2(\lambda)^2 + 2\hat{C}_4(\lambda)^2, 2\hat{C}_2(\lambda)\hat{C}_4(\lambda))$, i.e., the right-hand side in Eq. (26). The gray region is the area of $\hat{C}_4(\lambda) < -\hat{C}_2(\lambda)/\sqrt{2}$ and $\hat{C}_4(\lambda) > \hat{C}_2(\lambda)/\sqrt{2}$. In the right panel, the solution Eq. (28) is shown by the streaming red lines. The initial values of $(\hat{C}_2(\lambda), \hat{C}_4(\lambda))$ are denoted by the points. When the initial points are in the gray region (the left panel), the effective coupling constants become zero at the end of the low-energy scale, which indicates that the Kondo effect does not occur. On the other hand, when the initial points are outside the gray region (the left panel), the effective coupling constants become infinity, and accordingly the Kondo effect occurs. Here the existence of the $C_4$-term is important. In section III.B.1, we showed that the negative value of $\hat{C}_2(\lambda)$ has not led to the Kondo effect, when the $C_4$-term is absent ($C_4 = 0$). However, when the $C_4$-term is present ($C_4 \neq 0$), the negative value of $\hat{C}_2(\lambda)$ can produce the Kondo effect as long as $\hat{C}_4(\lambda) > -\hat{C}_2(\lambda)/\sqrt{2}$ or $\hat{C}_4(\lambda) < \hat{C}_2(\lambda)/\sqrt{2}$ is satisfied. Therefore, we conclude that the non-zero value of $|\hat{C}_4(\lambda)|$ is important to enhance the parameter region of $\hat{C}_2(\lambda)$ to realize the Kondo effect.

The above conclusion applies also to the case for the two-dimensional space $(\hat{C}_3(\lambda), \hat{C}_4(\lambda))$ with $\hat{C}_2(\lambda) = 0$.

### 3. Two-dimensional case II

We consider the solutions in the case of $\hat{C}_2(\lambda) = \hat{C}_3(\lambda)$. In addition, we suppose a small value of $|\hat{C}_4(\lambda)|$. For convenience, we introduce a function $\hat{C}_{23}(\lambda) \equiv \hat{C}_2(\lambda) = \hat{C}_3(\lambda)$, expressing the RG equations for $\hat{C}_{23}(\lambda)$ and $\hat{C}_4(\lambda)$ as:

$$\frac{d}{d\lambda} \hat{C}_{23}(\lambda) = \hat{C}_{23}(\lambda)^2 + 2\hat{C}_4(\lambda)^2,$$

$$\frac{d}{d\lambda} \hat{C}_4(\lambda) = 4\hat{C}_{23}(\lambda)\hat{C}_4(\lambda).$$

By eliminating $\hat{C}_{23}(\lambda)$ in the two equations, we find the simple equation for $\hat{C}_4(\lambda)$:

$$\hat{C}_4(\lambda) \frac{d^2}{d\lambda^2} \hat{C}_4(\lambda) - \frac{5}{4} \left( \frac{d}{d\lambda} \hat{C}_4(\lambda) \right)^2 - 8\hat{C}_4(\lambda)^4 = 0.$$  \hspace{1cm} (31)

For a further simplification, we introduce the function $F(\lambda)$ defined by $\hat{C}_4(\lambda) = 1/(F(\lambda))^4$ with $F(\lambda) > 0$. Then, the equation for $F(\lambda)$ reads

$$\frac{d^2}{d\lambda^2} F(\lambda) + \frac{2}{F(\lambda)^3} = 0,$$  \hspace{1cm} (32)

which looks much simpler than Eq. (31). However, it is still difficult in general to find an analytical solution of $F(\lambda)$. Here we try to find an approximate solution, and for this purpose we restrict our attention to a small value of
\[ |\tilde{C}_4(\lambda)|, \text{ i.e., } |\tilde{C}_4(\lambda)| \ll 1 \text{ or } F(\lambda) \gg 1, \text{ where the perturbation can be used. Then, the equation for } F(\lambda) \text{ is reduced to } \frac{d^2F(\lambda)}{d\lambda^2} \approx 0, \text{ and the solution is found to be } F(\lambda) \simeq c_1\lambda + c_2 \text{ with the appropriate constants } c_1 \text{ and } c_2. \] 

The values of \( c_1 \) and \( c_2 \) should be fixed by the initial condition of \( \tilde{C}_{23}(\lambda) \) and \( \tilde{C}_4(\lambda) \) at \( \lambda = 0 \). Finally, we obtain the approximate solution

\[
\tilde{C}_{23}(\lambda) \simeq \frac{2\tilde{C}_{23}}{2 - \tilde{C}_{23}\lambda}, \quad \tilde{C}_4(\lambda) \simeq \frac{16\tilde{C}_4}{(2 - \tilde{C}_{23}\lambda)^4}.
\] (33)

with \( \tilde{C}_{23} = 4\rho_0C_2 = -4\rho_0C_3 \) and \( \tilde{C}_4 = 4\rho_0C_4 \) as the initial condition. The perturbative approach involving the above solution requires that \( \tilde{C}_{23}(\lambda) \) and \( \tilde{C}_4(\lambda) \) should not be divergent, and the denominators in \( \tilde{C}_{23}(\lambda) \) and \( \tilde{C}_4(\lambda) \) should satisfy \( 2 - \tilde{C}_{23}\lambda > 0 \) for any \( \lambda > 0 \). It indicates that the range of the value of \( \tilde{C}_{23}(\lambda) \) should be restricted to \( \tilde{C}_{23}(\lambda) < 0 \) as long as the value of \( |\tilde{C}_4(\lambda)| \) is small \( (|\tilde{C}_4(\lambda)| \ll 1) \).

In Fig. 4 we plot the two-dimensional vector field \( (\tilde{C}_{23}(\lambda)^2 + 2\tilde{C}_4(\lambda)^2, 4\tilde{C}_{23}(\lambda)\tilde{C}_4(\lambda)) \), i.e., the right-hand side of Eq. (30). We also plot the solutions \( (\tilde{C}_{23}(\lambda), \tilde{C}_4(\lambda)) \) starting from \( \lambda = 0 \) by the streaming lines. It is shown that the solutions from the initial points with the negative value of \( \tilde{C}_{23}(\lambda) \) \( (\tilde{C}_{23}(\lambda) < 0) \) and the small value of \( |\tilde{C}_4(\lambda)| \) \( (|\tilde{C}_4(\lambda)| \ll 1) \) become convergent to zero for \( \lambda \to \infty \). From the numerical calculation, we find that the initial points in the gray region defined by \( \tilde{C}_{23}(\lambda) > \tilde{C}_4(\lambda) \) and \( \tilde{C}_{23}(\lambda) < -\tilde{C}_4(\lambda) \) do not lead to the divergence. The initial points outside this gray region can lead to the divergence and therefore can produce the Kondo effect. From the above analysis, we conclude that the non-zero value of \( \tilde{C}_4(\lambda) \) extends the parameter region of \( (\tilde{C}_{23}(\lambda), \tilde{C}_4(\lambda)) \) for the Kondo effect.
FIG. 4. The flow diagram on the $\tilde{C}_2(\lambda)$-$\tilde{C}_4(\lambda)$ plane. Left: the plot of the vector $(\tilde{C}_2(\lambda)^2 + 2\tilde{C}_4(\lambda)^2, 4\tilde{C}_2(\lambda)\tilde{C}_4(\lambda))$ on the right-hand side of Eq. (30). The gray region is the region where the effective coupling constants become zero in the low-energy limit and the Kondo effect does not occur. The Kondo effect can occur outside this gray region. Right: the solution $(\tilde{C}_2(\lambda), \tilde{C}_4(\lambda))$ in Eq. (30) is shown by the red lines, where the initial conditions in each line are expressed by the points. The initial points in the gray region in the left converge into zero in the low-energy limit, while the other initial points become divergent.

4. One-dimensional case—the SU(4) limit

Finally, we consider the one-dimensional case that the parameter $(\tilde{C}_2(\lambda), \tilde{C}_3(\lambda), \tilde{C}_4(\lambda))$ is restricted to the one-dimensional space $\tilde{C}_2(\lambda) = \tilde{C}_3(\lambda) = \pm \sqrt{2/3} \tilde{C}_4(\lambda)$. In this case, we introduce the function $\tilde{C}(\lambda)$ defined by $\tilde{C}(\lambda) \equiv \tilde{C}_2(\lambda) = \tilde{C}_3(\lambda) = \pm \sqrt{2/3} \tilde{C}_4(\lambda)$ for the short notation. Here, $\text{SU}(2)_{\text{spin}} \times \text{SU}(2)_{\text{boson}}$ in the Lagrangian (5) happens to be extended to the SU(4) symmetry according to the implication relation $\text{SU}(2) \times \text{SU}(2) \subset \text{SU}(4)$. We call this one-dimensional case the SU(4) limit. The SU(4) symmetry is made explicit by introducing the operators of the fifteen generators in the SU(4) symmetry, $\lambda^a/2$ or $\rho^a$ ($a = 1, 2, \ldots, 15$), where the operators $\lambda^a$ and $\rho^a$ are defined by

$$
\lambda^1 = \sigma^1 \otimes 1_2, \quad \lambda^2 = \sigma^2 \otimes 1_2, \quad \lambda^3 = \sigma^3 \otimes 1_2,
\lambda^4 = 1_2 \otimes \tau^1, \quad \lambda^5 = 1_2 \otimes \tau^2, \quad \lambda^6 = 1_2 \otimes \tau^3,
\lambda^7 = \pm \sigma^1 \otimes \tau^1, \quad \lambda^8 = \pm \sigma^1 \otimes \tau^2, \quad \lambda^9 = \pm \sigma^1 \otimes \tau^3,
\lambda^{10} = \pm \sigma^2 \otimes \tau^1, \quad \lambda^{11} = \pm \sigma^2 \otimes \tau^2, \quad \lambda^{12} = \pm \sigma^2 \otimes \tau^3,
\lambda^{13} = \pm \sigma^3 \otimes \tau^1, \quad \lambda^{14} = \pm \sigma^3 \otimes \tau^2, \quad \lambda^{15} = \pm \sigma^3 \otimes \tau^3,
$$

and

$$
\rho^1 = s^1 \otimes 1_2 \otimes 1_3, \quad \rho^2 = s^2 \otimes 1_2 \otimes 1_3, \quad \rho^3 = s^3 \otimes 1_2 \otimes 1_3,
\rho^4 = 1_3 \otimes 1_2 \otimes t^1, \quad \rho^5 = 1_3 \otimes 1_2 \otimes t^2, \quad \rho^6 = 1_3 \otimes 1_2 \otimes t^3,
\rho^7 = \sqrt{\frac{3}{2}} s^1 \otimes 1_2 \otimes t^1, \quad \rho^8 = \sqrt{\frac{3}{2}} s^1 \otimes 1_2 \otimes t^2, \quad \rho^9 = \sqrt{\frac{3}{2}} s^1 \otimes 1_2 \otimes t^3,
\rho^{10} = \sqrt{\frac{3}{2}} s^2 \otimes 1_2 \otimes t^1, \quad \rho^{11} = \sqrt{\frac{3}{2}} s^2 \otimes 1_2 \otimes t^2, \quad \rho^{12} = \sqrt{\frac{3}{2}} s^2 \otimes 1_2 \otimes t^3,
$$
\[ \rho^{13} = \sqrt{\frac{3}{2}} s^3 \otimes \mathbf{1}_2 \otimes t^1, \quad \rho^{14} = \sqrt{\frac{3}{2}} s^3 \otimes \mathbf{1}_2 \otimes t^2, \quad \rho^{15} = \sqrt{\frac{3}{2}} s^3 \otimes \mathbf{1}_2 \otimes t^3. \] (35)

We keep using the notations \( A \otimes B \) and \( A \otimes B \otimes C \) which were introduced in Eq. (7), along with the anti-symmetric tensor \( \varepsilon^{ijk} \). \( \lambda^a \) and \( \rho^a \) are normalized as \( \text{tr} \lambda^a \lambda^b = 4 \delta^{ab} \) and \( \text{tr} \rho^a \rho^b = 12 \delta^{ab} \), respectively. Adopting the restriction of the parameter \( \tilde{C}_2(\lambda) = \tilde{C}_3(\lambda) = \pm \sqrt{2/3} \tilde{C}_4(\lambda) \) and the operators \( \lambda^a \) and \( \rho^a \), we rewrite the Lagrangian (11) as

\[
\mathcal{L}_{\text{int}}[\psi, \bar{\psi}] = C_1 \varphi^\dagger \varphi \bar{\psi}_v \psi_v - C \sum_{a=1}^{15} \varphi^\dagger \lambda^a \varphi \bar{\psi}_v \rho^a \psi_v, \tag{36}
\]

with \( C \equiv C_2 = C_3 = \pm \sqrt{2/3} C_4 \). It is easy to prove that Eq. (36) is invariant under the SU(4) symmetry. We notice that the SU(2)\text{spin} symmetry and the SU(2)\text{isospin} symmetry are unified to the SU(4) symmetry. Thus, it provides the Kondo effect for a single non-Abelian symmetry. Regarding the coupling constant \( C \) as the effective coupling constant dependent on the energy scale \( C(\lambda) \), the RG equations reads

\[
\frac{d}{d\lambda} \tilde{C}(\lambda) = 4 \tilde{C}(\lambda)^2, \tag{37}
\]

with \( \tilde{C}(\lambda) \equiv 4 \rho_0 C(\lambda) \). This is indeed obtained by setting \( \tilde{C}(\lambda) = \tilde{C}_2(\lambda) = \tilde{C}_3(\lambda) = \pm \sqrt{2/3} \tilde{C}_4(\lambda) \) in Eq. (23). The solution is given in a simple equation as

\[
\tilde{C}(\lambda) = \frac{\tilde{C}}{1 - 4C(\lambda)}, \tag{38}
\]

with \( \tilde{C} = 4 \rho_0 C \) as the initial condition at \( \lambda = 0 \). The region of the parameter space for the Kondo effect is limited to \( \tilde{C} > 0 \), i.e., \( \tilde{C}_2 = \tilde{C}_3 > 0 \). From the RG equation, we obtain the Kondo scale \( \Lambda_K = \Lambda_0 \rho_0^{-1/(4 \tilde{C}_i)} (i = 1, 2) \) with \( \Lambda_0 \) being the initial energy scale for the RG flow. We note that the sign of \( \tilde{C}_4 \) is irrelevant to the condition for the Kondo effect, because of the positive and negative signs in \( \tilde{C}(\lambda) = \pm \sqrt{2/3} \tilde{C}_4(\lambda) \). In the right panel in Fig. 2 we plot the line (manifold) which is constrained by \( \tilde{C}_2(\lambda) = \tilde{C}_3(\lambda) = \pm \sqrt{2/3} \tilde{C}_4(\lambda) \) as the SU(4) limit. We observe that the non-zero value of \( |\tilde{C}_4(\lambda)| \) leads to the Kondo effect. The relevant sign of \( \tilde{C}_4(\lambda) \) is either of \( \tilde{C}_4(\lambda) > 0 \) for \( \tilde{C}_2(\lambda) > 0 \) and \( \tilde{C}_3(\lambda) > 0 \) or \( \tilde{C}_4(\lambda) < 0 \) for \( \tilde{C}_2(\lambda) < 0 \) and \( \tilde{C}_3(\lambda) < 0 \), depending on \( \tilde{C}(\lambda) = \pm \sqrt{2/3} \tilde{C}_4(\lambda) \). Thus, the non-zero value of \( |\tilde{C}_4| \) is important to bring about the Kondo effect in the SU(4) limit.

### C. Flow diagrams in general cases

In the previous subsections, we highlighted special cases where the non-zero value of \( |C_4| \), i.e., the spin and isospin-dependent interaction in the Lagrangian (11), extends the parameter region of \( \tilde{C}_2, \tilde{C}_3, \) and \( \tilde{C}_4 \) and allows the Kondo effect to occur. As a summary, we consider the solutions \( (\tilde{C}_2(\lambda), \tilde{C}_3(\lambda)) \) which is projected to the two-dimensional surface with a constant value of \( \tilde{C}_4(\lambda) \). We suppose the initial conditions of \( |\tilde{C}_4| = 0, 0.5, \) and 1 for the numerical demonstration. The results are shown in Fig. 5. For each \( C_4 \), the initial conditions of \( \tilde{C}_2 \) and \( \tilde{C}_3 \) are shown by the dots in the figure.

Under the initial condition of \( \tilde{C}_4 = 0 \), the Kondo effect occurs for \( \tilde{C}_2 > 0 \) or \( \tilde{C}_3 > 0 \) and does not for both \( \tilde{C}_2 < 0 \) and \( \tilde{C}_3 < 0 \). This is confirmed directly in the figure, because the flows in the former is divergent toward large \( \tilde{C}_2(\lambda) \) and \( \tilde{C}_3(\lambda) \), while the flows in the latter stops at \( \tilde{C}_2(\lambda) = \tilde{C}_3(\lambda) = 0 \). In contrast, if \( \tilde{C}_4 \) has a non-zero value for the initial condition, the Kondo effect can occur even for both \( \tilde{C}_2 < 0 \) and \( \tilde{C}_3 < 0 \). For example, let us see the initial points of \( \tilde{C}_2 = -1.0 \) and \( \tilde{C}_3 = -0.2 \) for \( |\tilde{C}_4| = 0.5 \) and the initial points of \( \tilde{C}_2 = -1.0 \) and \( \tilde{C}_3 = -0.4 \) for \( |\tilde{C}_4| = 1 \).
Symmetry is SU(2), because the C when they grow in the three-dimensional H. Ref. [43]. In the present study, we extend their discussions to the case where both of them exist. We introduce H previous studies, only the isospin symmetry was taken in Refs. [41, 42], and only the spin symmetry was taken in the Kondo scales for the SU(2) symmetry in FIG. 5. The solutions $\tilde{C}_2(\lambda), \tilde{C}_3(\lambda)$, and $\tilde{C}_4(\lambda)$ of the RG equation [23] on the two-dimensional $(\tilde{C}_2(\lambda), \tilde{C}_3(\lambda))$ plane. The initial conditions given by $(\tilde{C}_2, \tilde{C}_3)$ at each point and $|\tilde{C}_4| = 0, 0.5, 1$. The solutions are projected to the $(\tilde{C}_2(\lambda), \tilde{C}_3(\lambda))$ plane when they grow in the three-dimensional $(\tilde{C}_2(\lambda), \tilde{C}_3(\lambda), \tilde{C}_4(\lambda))$ space.

Therefore, we understand numerically that the non-zero value of $\tilde{C}_4$ helps to extend the region of the parameter space of $\tilde{C}_2$ and $\tilde{C}_3$ for which the Kondo effect occurs.

Comparison of the Kondo scales allows us to grasp the importance of the $C_4$ term, and to do so we consider the Kondo scales for the SU(2) symmetry in $C_4 = 0$ and for the SU(4) symmetry in $C_4 \neq 0$. In the former case, assuming $\tilde{C}_2 = \tilde{C}_3$, we have obtained the Kondo scale $\Lambda^{SU(2)}_K = \Lambda_0 e^{-1/C_3} (i = 2, 3)$ as shown in section III B 4. The symmetry is SU(2), because the $C_2$ term and the $C_3$ term are completely decoupled. In the latter case, assuming $\tilde{C}_2 = \tilde{C}_3 = \pm \sqrt{3/2} \tilde{C}_4$, we have obtained the Kondo scale $\Lambda^{SU(4)}_K = \Lambda_0 e^{-1/(4C_1)} (i = 2, 3; \tilde{C}_2 = \tilde{C}_3)$ as shown in section III B 4. As the two Kondo scales are strongly influenced by the exponential factors, their magnitudes are quite different: $\Lambda^{SU(2)}_K \ll \Lambda^{SU(4)}_K$. Therefore, keeping the same coupling constants $C_2$ and $C_3$, we find that the $C_4$ term, i.e., the mixing term of both spin and isospin, enhances the Kondo scale. Such enhancement makes the Kondo effect with the non-zero value of $C_4$ occur on higher energy scales than the case of $C_4 = 0$. This conclusion supports the argument that the $C_4$ term is important to magnify the Kondo effect for the $\Sigma_c (\Sigma^*_c)$ baryon in nuclear matter.

IV. REVISITING $\bar{D}$ AND $\bar{D}^*$ MESONS

Now we consider other systems where multiple number of non-Abelian symmetries exist, and here we focus on the Kondo effect for a $\bar{D}$ ($\bar{D}^*$) meson in terms of the SU(2) spin symmetry and the SU(2) isospin symmetry. Although there have been many studies on the $\bar{D}N$ ($\bar{D}^*N$) interaction and the properties of a $\bar{D}$ ($\bar{D}^*$) meson in nuclear systems in the literature, there are only a few studies on the Kondo effect for a $\bar{D}$ ($\bar{D}^*$) meson. In the previous studies, only the isospin symmetry was taken in Refs. [41, 42], and only the spin symmetry was taken in Ref. [43]. In the present study, we extend their discussions to the case where both of them exist. We introduce $H_v$ defined by $H_v = (\gamma^\mu P_{v\mu}^* + i\sigma_\mu P_v) 1_{2\times2}$ with $P_{v\mu}^* \sim (q\bar{Q})_{\text{spin}1} (\mu = 0, 1, 2, 3)$ for the vector meson and $P_v \sim (q\bar{Q})_{\text{spin}0}$ for a pseudoscalar meson. We note that the asterisk (*) denotes the vector field, not the complex conjugate. The vector field satisfies $v^\mu P_{v\mu}^* = 0$ and $\bar{H}_v = \gamma^0 H_v^\dag \gamma^0$. Under the spin and isospin-symmetries and the S-wave interaction

\[ |C_2| = 0, 0.5, 1 \]

\[ |C_3| = 0, 0.5, 1 \]

\[ |C_4| = 0, 0.5, 1 \]

\[ \Lambda^{SU(2)}_K = \Lambda_0 e^{-1/C_3} (i = 2, 3) \]

\[ \Lambda^{SU(4)}_K = \Lambda_0 e^{-1/(4C_1)} (i = 2, 3; \tilde{C}_2 = \tilde{C}_3) \]

\[ \Lambda^{SU(2)}_K \ll \Lambda^{SU(4)}_K \]
at the low energies, we write the interaction Lagrangian as follows:

\[ \mathcal{L}[\psi, H_v] = \mathcal{L}_0[\psi, H_v] + \mathcal{L}_{\text{int}}[\psi, H_v], \] 

(39)

with the kinetic term

\[ \mathcal{L}_0[\psi, H_v] = \varphi^\dagger \frac{\partial}{\partial \theta} \varphi + \varphi^\dagger \left( i\nabla \right)^2 \varphi + \text{tr} H_v \left( -\frac{i}{\gamma} \frac{\partial}{\partial \theta} \right) H_v, \] 

(40)

and the interaction term

\[ \mathcal{L}_{\text{int}}[\psi, H_v] = \sum_{i=1}^{4} \sum_{j=1}^{3} \delta_{ij} P_{v}^\dagger P_{v}^j + P_{t}^\dagger P_{t}, \]

\[ + iD_2 \sum_{k=1}^{3} \varphi^\dagger \sigma^k \varphi \left( \sum_{i,j=1}^{3} \epsilon_{ijk} P_{v}^\dagger P_{v}^j - \left( P_{v}^k P_{v} - P_{v}^\dagger P_{v}^k \right) \right), \]

\[ + D_3 \sum_{a=1}^{3} \varphi^\dagger \tau^a \varphi \left( \sum_{i,j=1}^{3} \delta_{ij} P_{v}^\dagger \tau^a P_{v}^j + P_{v}^\dagger \tau^a P_{v} \right), \]

\[ + iD_4 \sum_{k=1}^{3} \varphi^\dagger \sigma^k \tau^a \varphi \left( \sum_{i,j=1}^{3} \epsilon_{ijk} P_{v}^\dagger \tau^a P_{v}^j - \left( P_{v}^k \tau^a P_{v} - P_{v}^\dagger \tau^a P_{v}^k \right) \right), \] 

(41)

(42)

in the rest frame \( v^\mu = (1,0) \), where we define the new coupling constants by \( D_1 = -(d_1^2 - d_2^2) \), \( D_2 = 2d_3^2 + d_4^2 \), \( D_3 = -(d_1^2 - d_2^2) \), and \( D_4 = 2d_3^2 + d_4^2 \). Eq. (42) is invariant under the flavor symmetry for the light quark and under the spin symmetries for the spin of the light quark and the heavy antiquark. In terms of the spin symmetry, the transformation of \( P_v \) and \( P_{v}^s \) is given by

\[ P_v \mapsto P_v + \delta P_v = -\frac{1}{2} \theta^i P_{v}^s, \quad P_{v}^s \mapsto P_{v}^s + \delta P_{v}^s = P_{v}^s + \left( -\frac{1}{2} \epsilon_{ijk} \theta^j P_{v}^k + \frac{1}{2} \theta^i P_v \right), \]

(43)

for the small rotation angle \( \theta^i \) (\( i = 1, 2, 3 \)).

For the coupling constants in the Lagrangian (42), we consider the effective coupling constants \( D_a(\lambda) \) \( (a = 1, 2, 3, 4) \) which follows the RG equation, as we have considered for a \( \Sigma_c \) (\( \Sigma_c^* \)) baryon in section III. Referring the similar diagram in Fig. 1 and using the momentum integrals (19) and (20) as well as the identities (A1), we obtain the RG equations

\[ \frac{d}{d\lambda} \bar{D}_1(\lambda) = 0, \]

\[ \frac{d}{d\lambda} \bar{D}_2(\lambda) = 2\bar{D}_2(\lambda)^2 + 6\bar{D}_4(\lambda)^2, \]

\[ \frac{d}{d\lambda} \bar{D}_3(\lambda) = 2\bar{D}_3(\lambda)^2 + 6\bar{D}_4(\lambda)^2, \]

\[ \frac{d}{d\lambda} \bar{D}_4(\lambda) = 4 \left( \bar{D}_2(\lambda) + \bar{D}_3(\lambda) \right) \bar{D}_4(\lambda), \]

(44)
with $\lambda =-\ln (\Lambda /\Lambda _{0})$, where we define the dimensionless quantities by

$$\tilde{D}_{i} (\lambda ) \equiv -\rho _{0} D_{i} (\lambda ) , \quad \tilde{D}_{2} (\lambda ) \equiv -\rho _{0} D_{2} (\lambda ) , \quad \tilde{D}_{3} (\lambda ) \equiv -\rho _{0} D_{3} (\lambda ) , \quad \tilde{D}_{4} (\lambda ) \equiv -\rho _{0} D_{4} (\lambda ) .$$

Here, the minus sign is put by convention. The RG equation [44] is essentially the same as the RG equation for a $\Sigma _{c} (\Sigma _{c}^{*})$ baryon, Eq. (21) or Eq. (23). Thus, we obtain the similar behavior for the Kondo effect which indicates the importance of the spin and isospin-dependent term with $D_{4}$.

As a simple case, we consider the SU(4) limit by setting $D (\lambda ) \equiv D_{2} (\lambda ) = D_{3} (\lambda ) = \pm D_{4} (\lambda )$. The SU(4) symmetry is a large group which includes the SU(2)$_{\text{spin}}$ symmetry and the SU(2)$_{\text{isospin}}$ symmetry as its subgroups. In this limit, the RG equation of $\tilde{D}_{i} (\lambda ) (i = 2, 3, 4)$ is reduced to

$$\frac{d}{d\lambda } \tilde{D} (\lambda ) = 8 \tilde{D} (\lambda )^2 ,$$

with $\tilde{D} (\lambda ) \equiv \tilde{D}_{2} (\lambda ) = \tilde{D}_{3} (\lambda ) = \pm \tilde{D}_{4} (\lambda )$, and we obtain the analytical solution

$$\tilde{D} (\lambda ) = \frac{\tilde{D}}{1 - 8\tilde{D} \lambda } ,$$

with $\tilde{D} = \tilde{D} (0)$ as the initial condition. In the low-energy scale (a large value of $\lambda \gg 1$), this solution indicates the divergence at the Kondo scale, $\Lambda _{K} = \Lambda _{0} e^{\frac{1}{8\tilde{D} \lambda ^{-1}}}$ for $D < 0$, while it leads to the convergence to zero for $D > 0$. We denote $D \equiv D_{2} = D_{3} = \pm D_{4}$ for the (bare) coupling constant $D_{i} (i = 2, 3, 4)$ in the Lagrangian [12].

V. DISCUSSION: MESON-BARYON MAPPING INDUCED BY THE KONDO EFFECT

So far we have demonstrated that the presence of the spin and isospin-exchange term magnifies the Kondo effect, i.e. the increase of the Kondo scale $\Lambda _{K}$. This has been based on the perturbative analysis as we have relied on the RG equation. On the energy scale near or lower than the Kondo scale, however, the perturbative approach is no longer useful due to the enhanced coupling strength, and hence the non-perturbative approach should be adopted. So far there have been several nonperturbative analyses such as the numerical renormalization method [6], the Bethe ansatz [7, 9], the boundary conformal field theory [10, 16], the bosonization method [17, 21], the mean-field approximation (the large $N$ limit) [22, 37], and so on. One of the present authors have conducted the analysis based on the mean-field approximation for $D_{c}$ and $D_{c}^{*}$ mesons in nuclear matter [43] and for a $\tilde{D}$ meson in an atomic nucleus [42]. It is still open to question how we should systematically analyze the non-perturbative properties of the Kondo effect on the low-energy scale for both $\Sigma _{c}$ and $\Sigma _{c}^{*}$ baryons as well as for both $\tilde{D}$ and $\tilde{D}^{*}$ mesons, where the spin symmetry and the isospin symmetry should be taken into account simultaneously. In the following, we discuss the expected non-perturbative properties for those systems in a qualitative manner.

It is known that one of the interesting low-energy properties in the Kondo effect is the formation of the singlet pairing in the ground state [2, 4]. Here the singlet pairing indicates the bound state where an itinerant fermion is bound to an impurity particle and the total spin of the bound state is singlet. In other words, this is the dressed state surrounded by of particles and holes around the impurity site (exact screening). The dressed state is also known as the Kondo cloud. The singlet pairing was studied for $D_{s}^{*}$ and $D_{s}^{*-}$ mesons in nuclear matter [33] and for a $\tilde{D}$ meson in an atomic nucleus [42]. It is also possible that the singlet pairing exists for the $\tilde{D}$ and $\tilde{D}^{*}$ mesons. In such a situation, the singlet pairing should be composed of a nucleon ($N$) and a light quark ($q = u, d$) in the $\tilde{D}$ ($\tilde{D}^{*}$) meson, i.e., the
For a $D_s^- (D_s^{*-})$ meson, the singlet pairing as the Kondo cloud is composed of a nucleon ($N$) and the $s$ quark inside the $D_s^- (D_s^{*-})$ meson, i.e., the composite state ($Ns$) with spin 0 and isospin 1/2. In fact, the singlet condensate composed of a nucleon and a $D_s^- (D_s^{*-})$ meson was studied in the mean-field approximation \cite{43}. Thus, the $D_s^- (D_s^{*-})$ meson in nuclear matter should behave like a $\Xi_c$ baryon.

In contrast, the $\Sigma_c (\Sigma^*_c)$ baryon cannot have the singlet pairing. In fact it is known that the singlet pairing is not formed when the dimensions of the representations (fundamental, adjoint, etc.) in SU($N$) are different in the itinerant fermion and the impurity particle. Let us consider the itinerant fermion with spin 1/2 and the impurity particle with spin $S$. We observe that, for $S > 1/2$, the spin of the impurity particle cannot be screened by the spin of one itinerant fermion, and that there remains an unscreened spin $S^* = S - 1/2$ for the impurity site. This is called the underscreening Kondo effect \cite{112}. A similar situation arises for a $\Sigma_c (\Sigma^*_c)$ baryon in nuclear matter. That is, the spin $S = 1$ and the isospin $I = 1$ of the diquark ($qq$) in the $\Sigma_c (\Sigma^*_c)$ baryon would lead to the unscreened Kondo effect, making the $Nqq$ state with the spin $S^* = 1/2$ and the isospin $I^* = 1/2$ as the dressed state by particles and holes. Furthermore, we argue that it would lead to the composite state of $NqqQ$ with spin 0 or 1 and isospin 1/2, i.e. the same spin and isospin as the $q\bar{Q}$ meson such as a $\bar{D}$ and $\bar{D}^*$ meson. Therefore, it is thought that the $\Sigma_c (\Sigma^*_c)$ meson in nuclear matter should behave as the composite state ($NqqQ$), which has the same spin and isospin as a $\bar{D}$ ($\bar{D}^*$) meson.

The above consideration helps us introduce the concept of the “meson-baryon mapping” induced by the Kondo effect. As we have discussed, a $\bar{D}$ ($\bar{D}^*$) meson or a $D_s^- (D_s^{*-})$ meson in nuclear matter can be regarded as a $\Lambda_c$ baryon or a $\Xi_c$ baryon, and a $\Sigma_c (\Sigma^*_c)$ baryon in nuclear matter can be regarded as a $\bar{D}$ ($\bar{D}^*$) meson (table I). Thus, the heavy meson is “baryonized” and the heavy baryon is “mesonized” due to the Kondo effect. Such a meson-baryon mapping may cast new light on the properties and the dynamics of heavy hadrons in nuclear matter. We comment that the simple correspondence between the composite state ($NqQ$ or $NqqQ$) and the hadron-like state ($\Lambda_c$-like or $\bar{D}(\bar{D}^*)$-like) holds only when both spin and isospin are subject to the Kondo effect. When only spin (isospin) is subject to the Kondo effect and isospin (spin) is not, there should arise an additional degeneracy by isospin (spin) leading to the hadron-like state whose quantum number is not realized in vacuum. A more detailed investigation of these things must await another occasion.

| heavy hadron | dressed state (mapped) | screening type | Ref. |
|--------------|-------------------------|----------------|-----|
| $\bar{D}, \bar{D}^*$ meson | $\Lambda_c$ baryon-like | exact screening | —   |
| $D_s^*, D_s^{*-}$ meson | $\Xi_c$ baryon-like | exact screening | \cite{43} |
| $\Sigma_c, \Sigma^*_c$ baryon | $\bar{D}, \bar{D}^*$ meson-like | underscreening | —   |

TABLE I. The meson-baryon mapping induced by the Kondo effect. See the text for explanation.
VI. CONCLUSION

We have studied the Kondo effect for a $\Sigma_c$ ($\Sigma_c^*$) baryon in nuclear matter. By virtue of the $\text{SU}(2)_{\text{spin}} \times \text{SU}(2)_{\text{isospin}}$ symmetry, the HQS symmetry, and the S-wave interaction, we have provided the spin-exchange (or spin-nonexchange) and isospin-exchange (or isospin-nonexchange) interactions between the $\Sigma_c$ ($\Sigma_c^*$) baryon and the nucleon. By adopting the RG equation at one-loop order, we have found that the coexistence of the spin exchange and the isospin exchange magnifies the Kondo effect. We have extensively investigated the RG equation for several cases in terms of the coupling constants, including the SU(4)-limit case. We have also conducted the analysis for the $\bar{D}$ ($\bar{D}^*$) meson with the $\text{SU}(2)_{\text{spin}} \times \text{SU}(2)_{\text{isospin}}$ symmetry, and have shown the solution in the SU(4)-limit. In addition, we have ventured to develop the concept of the “meson-baryon mapping” for the $\Sigma_c$ ($\Sigma_c^*$) baryon, the $\bar{D}$ ($\bar{D}^*$) meson, and the $D_s$ ($D_s^*$) meson in the Kondo effect. It is straightforward to apply the mapping to other heavy hadrons when the light component in the heavy hadron has the spin interaction with a nucleon which flips the spin and/or the isospin.

Also, we mention that several issues are left unanswered: the corrections at $O(1/m_Q)$ (beyond the heavy-quark mass limit); applying the $\Sigma_c N$ ($\Sigma_c^* N$) interaction to many-body problems [74, 75]; discussing the “meson-baryon mapping” within the non-perturbative framework; the production mechanisms of the heavy hadrons in atomic nuclei; applications to atomic nuclei; and (as a more advanced topic) the continuity of the Kondo effect between the hadronic phase and the quark phase (see the discussions in Ref. [44]). The continuity, which was proposed for the color-flavor locked color superconductivity in Refs. [113, 114], is now studied intensively in view of topological objects [115–120]. It is worthwhile to study how the Kondo cloud changes from the hadronic matter to the quark matter. Simulations of the Kondo effect with SU(3) symmetry in cold atomic gases are also important [121]. It remains unclear as to how the Kondo effect with SU(4) symmetry for a $\Sigma_c$ ($\Sigma_c^*$) baryon is related to the Kondo effect with SU(4) symmetry in condensed matter systems, such as quantum dots, which has been studied theoretically [122–129] and experimentally [130–133]. Those issues need to be addressed in future work.

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Appendix A: Brief review on the Kondo effect with one single non-Abelian interaction

We consider the Kondo effect with one single non-Abelian interaction for an itinerant fermion in the fermi gas and the heavy impurity. We suppose that they belong to the fundamental representation of the SU($N$) symmetry, and the interaction between the itinerant fermion and the heavy impurity is provided by the non-Abelian interaction $\mathcal{L}_{\text{int}} = g(\lambda^a)_{ij}(\lambda^a)_{kl}$ with the coupling constant $g$ and the Gell-Mann matrices $\lambda^a$ ($a = 1, 2, \ldots, N^2 - 1$) in the SU($N$) symmetry. For example, the case of $N = 2$ is the spin-exchange interaction, in which an attraction between the itinerant fermion and the heavy impurity is provided in the spin-antiparallel channel for $g < 0$ and in the spin-parallel channel for $g > 0$. We consider the RG group equation at one-loop level. Utilizing the momentum integrals [19] and
and the relationships for the Gell-Mann matrices

\[ \sum_{a,b} \sum_{i',k'} (\lambda^a_{i'})(\lambda^b_{k'}) (\lambda^b_{i'j}) (\lambda^b_{k'l}) = 4 \left( 1 - \frac{1}{N^2} \right) \delta_{ij} \delta_{kl} + \left( -\frac{4}{N} \right) \sum_{a,b} (\lambda^a_{ij})(\lambda^a_{kl}), \]

\[ \sum_{a,b} \sum_{i' j, k'} (\lambda^a_{i'j})(\lambda^b_{k'}) (\lambda^b_{i'j}) (\lambda^b_{k'l}) = 4 \left( 1 - \frac{1}{N^2} \right) \delta_{ij} \delta_{kl} + \left( 2N - \frac{4}{N} \right) \sum_{a,b} (\lambda^a_{ij})(\lambda^a_{kl}), \]  

we obtain the RG equation

\[ \frac{d}{d\lambda} g(\lambda) = -2\rho_0 Ng(\lambda)^2, \]  

with the energy scale \( \lambda = -\ln(\Lambda/\Lambda_0) \). Here \( \Lambda \) is the energy scale moving from high-energy to the low-energy region, and \( \Lambda_0 \) is the ultraviolet-energy scale as the initial point. The solution of the above RG equation is found to be

\[ g(\lambda) = \frac{g}{1 + 2\rho_0 Ng\lambda}, \]  

with \( g \) being the coupling constant in vacuum or in the interaction Lagrangian. Noting that the energy scale runs from \( \lambda = 0 \) (the high-energy scale) to \( \lambda \to \infty \) (the low-energy scale), we find that negative coupling constant \( g < 0 \) leads to divergence of the coupling constant \( g(\lambda) \) at \( \lambda_K = -1/(2\rho_0 Ng) \) or \( \lambda_K = e^{1/(2\rho_0 Ng)} \) and that the positive coupling constant \( g > 0 \) leads to the vanishing coupling constant \( g(\lambda) \to 0 \). The relevant fixed point in the former case gives the Kondo effect, while the irrelevant fixed point in the latter does not. Thus, the coupling strength in the spin-antiparallel channel is enhanced, while that in the spin-parallel channel is suppressed.

**Appendix B: Spin and charge basis of \( \Psi^i_v \)**

We consider the interaction term for the spin-1/2 field (a \( \Sigma_c \) baryon) and the spin-3/2 field (a \( \Sigma_c^* \) baryon). In the rest frame, from Eqs. (2) and (3), we utilize the expression

\[ \Psi_{v1/2} = \frac{1}{\sqrt{3}} \sum_i \sigma^i \Psi^i_v, \]  

(B1)

and

\[ \Psi^i_{v3/2} = \sum_j (\delta^{ij} - \frac{1}{3} \sigma^{ij} \sigma^j) \Psi^j_v, \]  

(B2)

for the spin-1/2 and the spin-3/2 fields.\(^4\) Then, we rewrite the interaction term in Eq. (7) as

\[ \mathcal{L}_{\text{int}}[\bar{\psi}^i_v, \Psi^i_v] = C_1 \phi^i \Phi \left( \Psi^i_{v3/2} \Psi^i_{v1/2} + \Psi^i_{v1/2} \Psi^i_{v1/2} \right) \]

\[ + C_2 \phi^i \sigma^i \phi \left( i \varepsilon^{ijk} \Psi^j_{v3/2} \Psi^j_{v1/2} - \frac{1}{\sqrt{3}} \Psi^j_{v1/2} \Psi^j_{v1/2} - \frac{1}{\sqrt{3}} \Psi^k_{v3/2} \Psi^k_{v1/2} - \frac{2}{3} \Psi^k_{v1/2} \Psi^k_{v1/2} \right) \]

\[ + C_3 \phi^i \phi^d \phi^d \left( \Psi^i_{v3/2} \Psi^j_{v1/2} + \Psi^i_{v1/2} \Psi^j_{v1/2} \right) \]

\[ + C_4 \phi^i \phi^k \phi^k \phi^d \left( i \varepsilon^{ijk} \Psi^j_{v3/2} \Psi^j_{v1/2} - \frac{1}{\sqrt{3}} \Psi^j_{v1/2} \Psi^j_{v1/2} - \frac{1}{\sqrt{3}} \Psi^k_{v3/2} \Psi^k_{v1/2} - \frac{2}{3} \Psi^k_{v1/2} \Psi^k_{v1/2} \right). \]  

(B3)

\(^4\) We take the summation over the indices \( i, j = 1, 2, 3 \) when they are repeated. Notice the constraint condition \( \sigma^i \Psi^i_v = 0 \).
Here we mention that the $\Sigma_c$ baryon and the $\Sigma'_c$ baryon can be swapped with each other by the HQS symmetry ($\Sigma_c \leftrightarrow \Sigma'_c$). In the HQS symmetry, the heavy quark changes as $u_v \rightarrow e^{i\sigma \cdot \theta/2} u_v \approx (1 + i\sigma \cdot \theta/2) u_v$ with $\theta = (\theta^1, \theta^2, \theta^3)$ for the small $\theta_i$ ($i = 1, 2, 3$). This transformation leads to the change of the fields of $\Sigma_c$ and $\Sigma'_c$ baryons: $\Psi^i_v \rightarrow e^{i\sigma \cdot \theta/2} \Psi^i_v \approx (1 + i\sigma \cdot \theta/2) \Psi^i_v$. Notice that $\Psi^i_v$ ($i = 1, 2, 3$) is in the rest frame. Then, we find that $\Psi_{v1/2}$ and $\Psi_{v3/2}$ change to $\Psi_{v1/2}^i + \delta \Psi_{v1/2}^i$ and $\Psi_{v3/2}^i + \delta \Psi_{v3/2}^i$, respectively, where $\delta \Psi_{v1/2}^i$ and $\delta \Psi_{v3/2}^i$ are given by

$$
\delta \Psi_{v1/2}^i = \frac{1}{\sqrt{3}} \sigma^i \delta \Psi_v^i = -\frac{i}{6} \theta_i^2 \sigma^i \Psi_{v1/2} - \frac{i}{2 \sqrt{3}} \delta^{ijk} \theta_j \sigma^j \Psi_{v3/2},
$$

and

$$
\delta \Psi_{v3/2}^i = \left( \delta^{ij} - \frac{1}{3} \sigma^i \sigma^j \right) \delta \Psi_v^j = -\frac{1}{\sqrt{3}} \sigma^i \Psi_{v1/2} + \frac{1}{2} \theta_i \sigma^i \Psi_{v3/2} + \frac{i}{\sqrt{3}} \delta^{ijk} \theta_j \sigma^j \Psi_{v3/2},
$$

In terms of the isospin operator $t^a$ ($a = 1, 2, 3$) for $\Sigma_c$ and $\Sigma'_c$ baryons, the basis used in Eq. (8) may not be suitable for describing the charged particles such as $\Sigma^+_c$, $\Sigma^0_c$, and $\Sigma^-_c$ because none of $t^1$, $t^2$, and $t^3$ is diagonal. Instead, it can be useful to introduce the following operator for isospin $\hat{t}^a$ ($a = 1, 2, 3$):

$$
\hat{t}^1 = \left( \begin{array}{ccc} 0 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{array} \right), \quad \hat{t}^2 = \left( \begin{array}{ccc} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{array} \right), \quad \hat{t}^3 = \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{array} \right),
$$

which are related to $t^a$ by the unitary transformation $t^a = U \hat{t}^a U^\dagger$ with the unitary matrix

$$
U = \left( \begin{array}{ccc} \frac{i}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & -i & 0 \end{array} \right).
$$

We note that the commutation relation holds: $[\hat{t}^a, \hat{t}^b] = i \varepsilon^{abc} \hat{t}^c$. Then, we obtain the new field $\hat{\Psi}_{v1/2}$ and $\hat{\Psi}_{v3/2}$ expressed by the charge basis:

$$
\hat{\Psi}_{v1/2} = \begin{pmatrix} \Sigma^+_{c} \\ \Sigma^0_{c} \\ \Sigma^-_{c} \end{pmatrix}, \quad \hat{\Psi}_{v3/2} = \begin{pmatrix} \Sigma^+_{c} \\ \Sigma^0_{c} \\ \Sigma^-_{c} \end{pmatrix},
$$

which are related to $\Psi_{v1/2}$ and $\Psi_{v3/2}$ through the unitary transformation $\Psi_{v1/2} = U^\dagger \hat{\Psi}_{v1/2}$ and $\Psi_{v3/2} = U^\dagger \hat{\Psi}_{v3/2}$.

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