1. Introduction

Traditional amusement ride related textbook problems include free-fall, circular motion, pendula and energy conservation in roller coasters, where the moving bodies are typically considered point-like. However, an amusement park can offer many more examples that are useful in physics and engineering education, many of them with strong mathematical content. This paper analyses forces on riders in a large rotating pendulum ride, where the Coriolis effect is sufficiently large to be visible in accelerometer data from the rides and leads to different ride experiences in different positions.

2. Collecting and analysing data

Using apps such as Physics Toolbox Roller Coaster [5] or PhyPhox [6], today’s students can
easily measure acceleration and rotation. It is important that students are acquainted with the data collection tools before embarking on data collection in an amusement park. Many problems can be avoided if the students have the opportunities to test the equipment in familiar and simple situations, such as elevators or playground swings, where the motion can be easily understood and related to the data collected. Students need to realise that accelerometers do not measure acceleration but the components of the vector \((a - g)/|g|\) in a comoving coordinate system. This insight is needed to understand the accelerometer data collected in pendula or roller coasters. The interpretation of the data from smartphones offers good practice in identifying coordinate axes \([7–9]\), useful also in interpreting data from other acceleration and rotation sensors commonly used in high school physics classes \([10, 11]\). Figure 2 shows the coordinate system used in the data collection as well as in the analysis of forces on the rider \([12]\).

3. Pendulum rides

We consider first the purely 2D motion in the Screamin’ Swing pendulum ride, shown in figure 3. The forces on a rider with mass \(m\) during different parts of the motion of this ride are illustrated in figure 4. At the maximum angle \(\theta_0\) the force on the rider is \(mg \cos \theta_0\) towards the rotation axis of the pendulum and the acceleration is purely tangential, \(a = g \sin \theta_0\). For smaller angles, the circular motion also requires a centripetal acceleration, \(v^2/L\), towards the rotation axis. To evaluate the speed for different
Figure 2. Definition of the rider-fixed coordinate systems commonly used in analysing biomechanical effects on riders. These are also the axes of an accelerometer carried along on a ride, e.g. in a data vest. The $x$, $y$ and $z$ axes in this comoving system are often referred to as ‘longitudinal’, ‘lateral’ and ‘vertical’, respectively. The biomechanical effects on the body are determined by the forces along these axes [12].

Figure 3. The Screamin’ Swing pendulum ride Uppswinget at Liseberg.

Figure 4. Forces on the rider at the turning point and at a smaller angle for the pendulum ride shown in figure 5.
angles, the conversion between potential and kinetic energy can be used: For a pendulum of length $L$, the centre of mass elevation differs by $L(1 - \cos \theta_0)$ between the highest and lowest points, giving an expression $v^2 = 2gL(1 - \cos \theta_0)$ for the speed $v$ at the lowest point. The force from the swing at the lowest point is then obtained as $mg + mv^2/L = mg(3 - 2\cos \theta_0)$, independent of the pendulum length. For an arbitrary angle, the centripetal acceleration is given by $v^2/L = 2g(\cos \theta - \cos \theta_0)$ and the force from the swing on the rider is $mg(3\cos \theta - 2\cos \theta_0)$.

Even a relatively simple motion as a pendulum brings surprises when accelerometer data are collected; the data differ significantly from zero only in the axis in the direction of the chain.

Figure 5. Accelerometer data for the pendulum ride Uppswinget (figure 3), showing the vertical (z) component of the force from the ride on the rider divided by $mg$. The largest values are obtained when the pendulum passes the lowest point, where the size of force from the ride on the rider is expected to be $mg(3 - 2\cos \theta_0)$. For the turning points, the value should be $mg \cos \theta_0$. Note that the deviation from a value $mg$ for the normal force is twice as large at the bottom of the ride (and with opposite sign) compared to the values at the turning points.

Figure 6. Accelerometer data from riders at four different positions during the ride in the Loke Gyro Swing ride. The positions are shown in figure 8. The red and blue graphs are for riders at opposite sides of the circle.
Pendulum rides, rotations and the Coriolis effect

(Bringing along a soft mug with a small quantity of liquid in a swing leads to analogous surprises [9, 13–15]). This can be understood by noting that the tangential acceleration corresponds exactly to the tangential component of the acceleration of gravity, causing the tangential component of $a - g$ to vanish. Figure 5 shows the radial component of the accelerometer data for the pendulum ride shown in figure 3. The corresponding data for Loke are shown in figure 6.

For a physical pendulum, where the mass is not concentrated in a point, the moment of inertia must be taken into account. However, the expressions for acceleration and velocities hold for a distance $L^2 = I_{\text{axis}}/m$, where $I_{\text{axis}}$ is the moment of inertia with respect to the rotation axis of the pendulum and $\sqrt{I_{\text{axis}}/m}$ is the ‘radius of gyration’. For the rides considered here, the accelerometer values at the bottom of the ride indicate that the distance to the main rotation axis is close to the radius of gyration.

3.1. Angular velocities

The mathematical treatment of a pendulum typically focuses on the angle $\theta$, angular velocity, $\Omega = \dot{\theta}$ and angular acceleration, $\ddot{\Omega} = \ddot{\theta}$, giving the relation

$$\ddot{\theta} = -\frac{g}{L} \sin \theta$$

(1)

where $L$ is the length of the mathematical pendulum, or the radius of gyration for a physical pendulum. For small angles $\sin \theta \approx \theta$ and the period for the pendulum is then given by $T = 2\pi \sqrt{L/g}$. Large angles lead to longer pendulum periods [16]. This was also examined in a recent paper using smartphones [17]. For the Gyro Swing Loke (figure 1), with a length $L = 24$ m, the expected period for small angles
becomes $T = 9.8$ s. For $120^\circ$, the theoretical expressions give a 34% increase in the pendulum period, giving $T(120^\circ) = 13.2$ s for Loke.

For a maximum angle of $120^\circ$, we expect a normal force from the ride $N = -mg/2$. (The minus sign indicates that the force acts on the shoulders of the rider.) At the lowest point we expect a force $N = 4mg$, essentially consistent with the accelerometer data in figure 6. With a centripetal acceleration of $a_c = 3g$ at the bottom and a pendulum length $L = 24$ m, the velocity of the centre of the circle of Loke can be estimated to $v = \sqrt{3Lg} \approx 27$ m s$^{-1}$.

The angular velocity due to pendulum motion is $\Omega = v/L$ with a largest value at lowest points. For Loke we expect $|\Omega| \leq 1.1$ rad s$^{-1}$, in good agreement with the data shown in figure 7, collected using a smartphone. Not long ago, measuring rotation was a specialised task, requiring equipment not generally available [18], whereas today, students can collect their own data, also for rotation, using apps such as Physics Toolbox Roller Coaster [5] or PhyPhoX [6].

During the amusement park physics day at Liseberg 2017, four high school students, each carrying a wireless dynamic sensor system [10] in a data vest, rode the Loke ride in different positions of the circle. The resulting data (figure 6) are in relatively good agreement with the expressions for a mathematical pendulum, but show small and position-dependent deviations, to be discussed below. Their positions in the ride are shown in figure 8.

4. A rotating circle at the end of the pendulum

In the Gyro Swing ride Loke (figure 1), the riders are seated in a large circle around the end of the pendulum arm. Curious students may be interested in understanding the small force corrections required for the motion of the rider relative to the centre of the circle and visible in the accelerometer data.

Since the riders look out from the ride, the $x$ axis for the rider points away from the centre of the circle. The $y$ axis points to the left of the rider, along the tangent of the circle. The $z$ axis points from the seat to the head and is in the same direction for all riders at any given time. When the circle is in the lowest point, the $z$ axis coincides with the $Z$ axis in the fixed coordinate system, where we let the $X$ be the pendulum axis, as shown in figure 9.

The riders in the circle with radius $r \approx 4.05$ m rotating 5 times per minute, move with a tangential
velocity $v' = v'e_x$, where $v' \approx \pm 2.1 \text{ m s}^{-1}$ relative to the centre of the circle. (The direction of rotation changes between different rides.) The pendulum swings back and forth around the fixed $X$ axis, with a time-dependent angular velocity $\Omega = \Omega e_X$, with a maximum value as the pendulum passes the lowest point $\Omega_{\text{max}} = 1.1 \text{ rad s}^{-1}$.

The direction of the fixed $X$ axis can be written as a combination of the $x$ and $y$ axes, which rotate around the $z$ axis, as discussed in an earlier work on the family ride Rockin’ Tug [7]. The measured angular velocities are thus more complicated than for a simple pendulum as shown in figure 7.

Figure 10. Forces at the highest points. The arrows in cyan indicate the additional acceleration due to the rotation of the circle as the pendulum starts moving down.

which is orthogonal both to the velocity relative to the centre of the star, and to the main rotation axis. This Coriolis effect can reach $a_{\text{Cor}} = 2d\Omega \omega \approx 0.48g$ depending on the position of the rider. When the circle is at the lowest point, this additional acceleration is upwards on one side and downwards on the other, requiring a larger (or smaller) ‘vertical force’ from the ride.

4.1. Forces at the lowest point

A comparison between the different accelerometer graphs in figure 6 shows that for some positions, the forces at the bottom of the Loke ride is $4mg$, as expected, whereas in other cases, the force is smaller or larger, and the deviation may be nearly $mg/2$. These corrections can be understood by considering how the motion of the circle of riders combines with the pendulum motion.

The pendulum motion leads also to a rotation $\Omega = \Omega e_X$ of the coordinate system for the circle, resulting in an additional acceleration

$$a_{\text{Cor}} = 2\Omega \times v'$$

which is orthogonal both to the velocity relative to the centre of the star, and to the main rotation axis. This Coriolis effect can reach $a_{\text{Cor}} = 2d\Omega \omega \approx 0.48g$ depending on the position of the rider. When the circle is at the lowest point, this additional acceleration is upwards on one side and downwards on the other, requiring a larger (or smaller) ‘vertical force’ from the ride.

4.2. Forces at the top of Loke

When the pendulum motion in Loke has reached the turning point at the maximum angle, the angular velocity $\Omega$ for the pendulum motion is zero, and the Coriolis effect discussed above gives no contribution. However, the graphs in figure 6 show that even in the turning point the force depends on the location in the ride. Some students may want a closer investigation of these discrepancies, which can be understood by considering the angular acceleration in the pendulum motion: $\ddot{\theta} = \Omega = -g \sin \theta / L$. This angular acceleration influences the whole circle of riders, and size of the effect depends on the position in the circle. For riders in the highest position of the circle this leads to an additional acceleration in the $z$ (‘vertical’) direction with the
size $|\dot{\theta}| = -rg \sin \theta / L$. In the turning points, where $|\theta| = \theta_0 = 120^\circ$, this gives a contribution $(r/L)g \sin \theta_0 \approx g\sqrt{3}/12 \approx 0.14g$. For a rider in the lowest point of the circle, the angular acceleration of the pendulum gives an equally large, but negative, acceleration in the $z$ direction. Figure 10 shows these additional accelerations that depend on the location in the circle and account for the differences in the normal forces on the rider, seen in figure 6.

5. Discussion

For high school students, understanding the forces for a non-rotating pendulum may be sufficiently challenging. The data collected by the four students in different positions of the rotating pendulum illustrate that the ride experience depends on the position, which can be quite confusing. This paper is intended to provide teachers with background to discuss the small deviations when student questions arise. Students may also enjoy discovering the size of the Coriolis effect in the GyroSwing ride.

ORCID iDs

Ann-Marie Pendrill https://orcid.org/0000-0002-1405-6561

References

[1] Intamin, Gyro Swing www.intaminworldwide.com/project/gyro-swing/
[2] Bagge S and Pendrill A-M 2002 Classical physics experiments in the amusement park Phys. Educ. 37 507–11
[3] Pendrill A-M 2008 How do we know that the Earth spins around its axis? Phys. Educ. 43 158–64
[4] Gurri P, Amat D, Espar J, Puig J, Jimenez G, Sendra L and Pardo L C 2017 Pendulum dynamics in an amusement park Eur. J. Phys. 38 035003
[5] Vieyra C Physics Toolbox Roller Coaster http://vieyrasoftware.net/
[6] Staacks S et al 2018 Advanced tools for smartphone-based experiments: phyphox Phys. Educ. 53 045009
[7] Pendrill A-M and Rohlén J 2011 Acceleration and rotation in a pendulum ride, measured using an iPhone 4 Phys. Educ. 46 676–81
[8] Löfström M and Pendrill A-M 2016 Aerodynamics in the amusement park: interpreting sensor data for acceleration and rotation Phys. Educ. 50 055015
[9] Pendrill A-M 2016 Rotating swings—a theme with variations Phys. Educ. 51 15014
[10] Vernier Wireless dynamic sensor system www.vernier.com/products/sensors/wdss/
[11] Pasco Wireless sensors www.pasco.com/wireless/
[12] Eager D, Pendrill A-M and Reistad N 2016 Beyond velocity and acceleration: jerk, snap and higher derivatives Eur. J. Phys. 37 065008
[13] Pendrill A-M and Williams G 2005 Swings and slides Phys. Educ. 40 527
[14] Fägerlind C-O and Pendrill A-M 2015 Liquids in accelerated motion Phys. Educ. 50 648–50
[15] Tornarí F, Monteiro M and Marti A C 2014 Understanding coffee spills using a smartphone Phys. Teach. 52 502
[16] Hyperphysics Large amplitude pendulum http://hyperphysics.phy-astr.gsu.edu/hbase/pendl.html
[17] Fernandes J C, Sebastio P J, Gonalves L N and Ferraz A 2017 Study of large-angle anharmonic oscillations of a physical pendulum using an acceleration sensor Eur. J. Phys. 38 045004
[18] Pendrill A-M and Rödgård H 2005 A roller coaster viewed through motion tracker data Phys. Educ. 40 522–6

Ann-Marie Pendrill is director of the Swedish National Resource Centre for Physics Education. Her research background is computational atomic physics, but her more recent work has focused on various aspects of physics and science education. She has used examples from playgrounds and amusement parks in her teaching in physics, teaching and engineering programmes. She enjoys studying physics in rotating systems.

Conny Modig is high school teacher at Gymnasium Skövde Västerhöjd. In his teaching he has been particularly interested in Amusement Park Physics and computer-based measurement technology in teaching. Conny has also been involved in teacher education in these areas.