On possible origin of an anisotropy in the speed of light in vacuum

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We assume the spacetime foam picture in which vacuum is filled with virtual wormholes. In the presence of an external field the distribution of wormholes changes. We consider an anisotropic distribution of wormholes and analyze its relation to the speed of light. We show that speed of light acquires an anisotropic character and save the normal dispersion a gas of virtual wormholes may possess also an anomalous dispersion, i.e., when the light velocity exceeds that in the vacuum.

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I. INTRODUCTION

An actual wormhole requires the violation of averaged null energy condition (ANEC) \cite{1} and is commonly supposed to be forbidden in classical physics. However, the situation changes when we consider a virtual wormhole. It represents a quantum topology fluctuation (tunnelling between different topologies) which takes place at very small (Planckian) scales and lasts for a very short period of time (the so-called spacetime foam, e.g., see \cite{2, 3}). It does not obey to the Einstein equations and, therefore, ANEC cannot forbid the origin of such an object. In other words, virtual wormholes violate readily ANEC.

Virtual wormholes may be described by Euclidean wormhole configurations which were first suggested in Refs. \cite{4} (see also ref. \cite{5}) where it was proposed that they can lead to loss of quantum coherence. It was latter shown though \cite{6} that the effects of such wormholes can be absorbed into a redefinition of coupling constants of the low energy theory and, therefore, quantum coherence is not lost. However, as it was recently shown \cite{7, 8} virtual wormholes may play an important role in particle physics at very high energies, since they may introduce in a natural way the cutoff at very small scales and may remove divergencies in quantum field theory. Moreover, the existence of virtual wormholes may be important for the possibility to form wormhole-like objects at laboratory scales \cite{9}. Indeed, a coherent set of virtual wormholes may work as an actual wormhole \cite{8} (see also ref. \cite{10}). In particular, by applying an external classical field one may govern the intensity of such fluctuations and try to organize an artificial wormhole \cite{9}.

In the present paper we consider the situation when additional wormholes are incoherent but simply have an anisotropic distribution that leads to an anisotropy in the propagation of light in vacuum and analyze the resulting dispersion relations. We explicitly demonstrate that virtual wormholes may possess both, the normal and anomalous dispersion, i.e., when the resulting speed of light is smaller or bigger than the speed of light in vacuum.

II. VIRTUAL WORMHOLE

In what follows we shall use some of results of Ref. \cite{7}. A virtual wormhole can be described by the Euclidean wormhole configuration as follows (for the Euclidean approach see also Refs. \cite{3, 11}). The simplest virtual wormhole may be constructed by the gluing procedure. Consider the metric ($\alpha = 1, 2, 3, 4$)

$$ds^2 = h^2 (r) \delta_{\alpha \beta} dx^\alpha dx^\beta,$$

(1)

where

$$h (r) = 1 + \theta (a - r) \left( \frac{a^2}{r^2} - 1 \right)$$

(2)

and $\theta (x)$ is the step function. Such a wormhole has vanishing throat length. Indeed, in the region $r > a$, $h = 1$ and the metric is flat, while the region $r < a$, with the obvious transformation $y^\alpha = \frac{a^2}{r^2} x^\alpha$, is also flat for $y > a$. Therefore, the regions $r > a$ and $r < a$ represent two Euclidean spaces glued at the surface of a sphere $S^3$ with the center at the origin $r = 0$ and radius $r = a$. Such a space can be described with the ordinary double-valued flat metric in the region $r_+ > a$ by

$$ds^2 = \delta_{\alpha \beta} dx_+^\alpha dx_+^\beta,$$

(3)

where the coordinates $x^\alpha_\pm$ describe two different sheets of space. We point out that in the quasi-classical region a virtual wormhole may be taken as a solution of the Euclidean Einstein equations and the function $h$ should be smooth.
In particular, the choice \( h(r) = (r^2 + a^2)/r^2 \) corresponds to the so-called Bronnikov-Ellis type metric

\[
ds^2 = dR^2 + (R^2 + 4a^2) d\Omega^2,
\]

where \( d\Omega^2 \) is the angle part of the metric, \( R = r - a^2/r \) and \(-\infty < R < \infty\).

Let identify the inner and outer regions of the sphere \( S^3 \) and construct a wormhole which connects regions in the same space (instead of two independent spaces). This is achieved by gluing the two spaces in (3) by motions of the Euclidean space. If \( R_{\pm} \) is the position of the sphere in coordinates \( x_\pm^\mu \), then the gluing is the rule

\[
x_+^\mu = R^\mu_{\pm} + \Lambda^\mu_\nu (x_-^\nu - R^-_\nu),
\]

where \( \Lambda^\mu_\nu \in O(4) \), which represents the composition of a translation and a rotation of the Euclidean space. In terms of common coordinates such a wormhole represents the standard flat space in which the two spheres \( S^3 \) (with centers at positions \( R_{\pm} \)) are glued by the rule (4). We point out that the physical region is the outer region of the two spheres. Thus, in general, the wormhole is described by a set of parameters: the throat radius \( a \), positions of throats \( R_{\pm} \), and rotation matrix \( \Lambda_\nu^\mu \in O(4) \).

### III. GREEN FUNCTION FOR A SINGLE WORMHOLE CONNECTING TWO SPACES

In the Euclidean field theory the Green function for a scalar field obeys the Laplace equation

\[
(-\Delta + m^2) G(x, x') = \delta(x - x') \tag{5}
\]

where \( m \) is the mass, \( \Delta = \frac{1}{\sqrt{g}} \partial_\alpha (\sqrt{g} g^{\alpha\beta} \partial_\beta) \), and \( g_{\alpha\beta} \) is the metric. In the Euclidean space the metric is flat \( g_{\alpha\beta} = \delta_{\alpha\beta} \) and the above equation has the well-known solution

\[
G_0(x, x') = \frac{m^2 K_1(2ma)}{4\pi^2 m |x - x'|},
\]

where \( K_1 \) is the modified Bessel function.

When considering a space with a wormhole the metric cannot be chosen everywhere flat and the exact form of the Green function depends on the specific structure of the wormhole. In the present section we construct the Green function for the simplest wormhole connecting two Euclidean spaces which is described by the \( O(4) \) invariant metric (4). In this case the equation (5) admits the exact solution, e.g., see details in Ref. [12].

Indeed, consider four-dimensional spherical coordinates \( r, \chi, \theta, \phi \) in which the square of the element of length is

\[
ds^2 = h^2(r) \left( dr^2 + r^2 \left[ d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\phi^2) \right] \right).
\]

Then the angular part of the Green function can be decomposed in terms of four-dimensional spherical harmonics as

\[
G(x, x') = \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} \sum_{m=-l}^{l} Q_{nlm}(\Omega) g_n(r, r') Q_{nlm}(\Omega),
\]

where the four-dimensional harmonics \( Q_{nlm} \) obey to

\[
-\Delta_{\Omega} Q_{nlm} = (n^2 - 1) Q_{nlm},
\]

\((n = 1, 2, \ldots \text{ and } \Delta_{\Omega} \text{ is the angular part of the Laplace operator}) \) and which are (e.g., see Refs. [12])

\[
Q_{nlm}(\Omega) = q^{(n)}_{lm} Y_{lm}(\theta, \phi) \Pi_{nl}(\chi),
\]

where \( q^{(n)}_{lm} \) are normalization constants, \( Y_{lm}(\theta, \phi) \) are the usual three-dimensional spherical harmonics, and

\[
\Pi_{nl}(\chi) = \sin^l \chi \frac{d^{l+1} \cos n \chi}{d(\cos \chi)^{l+1}}, \quad (l = 0, 1, \ldots, n - 1).
\]

In the presence of the wormhole the Green function \( G \) contains \( G_0 \) as an additive part. The function \( G_0(x, x') \) admits the decomposition as

\[
G_0(x, x') = \sum_{n=1}^{\infty} g_n(r, r') Q_n(\chi) = \frac{m^2 K_1(|m x - x'|)}{4\pi^2 m |x - x'|},
\]
where
\[
g_0^0(r, r') = \frac{nm^2 K_n(mr_a) I_n(mr_<)}{2\pi^2 mr_a mr_<},
\]
\[
Q_n(\chi) = \frac{2\pi^2}{n} \sum_{i=0}^{n-1} \sum_{m=-l} l Q_{nlm}'(\Omega') Q_{nlm}(\Omega),
\]
\(\chi\) denotes the angle between \(r\) and \(r'\), and \(r_>, r_<\) denote the biggest and smallest value of \(r\) and \(r'\) respectively. In the massless case we find
\[
g_0(x, x') = \frac{1}{4\pi^2 (x - x')^2} = \frac{1}{4\pi^2} \sum_{n=1}^{\infty} \frac{r_<^{n-1}}{r_<^{n+1}} Q_n(\chi)
\]
which can be used as the generating function for polynomials \(Q_n\).

The Green function \(G = G_0 + \delta G\) upon the decomposition gives for the region \(r > a\)
\[
G(x, x') = \sum_{n=1}^{\infty} \left( g_0^0(r, r') + A_n \frac{K_n(mr)}{mr} \right) Q_n(\chi)
\]
and for the region \(r < a\) \((\bar{r} = a^2/r > a)\)
\[
G(x, x') = \sum_{n=1}^{\infty} Q_n(\chi) B_n \frac{K_n(m\bar{r})}{m\bar{r}}.
\]

Now to get the solution to \([5]\) we have to match these solutions at \(r = a\), i.e., to require continuity \(\frac{\partial g_n}{\partial r}\) and \(g_n\), which gives the solution for the region \(r > a\)
\[
G(x, x') = \sum_{n=1}^{\infty} \left( g_0^0(r, r') - \mu_n(a, r') g_0^0(r, a) \right) Q_n(\chi)
\]
and as \(r < a\)
\[
G(x, x') = \sum_{n=1}^{\infty} \nu_n(a, r') g_0^0(\bar{r}, a) Q_n(\chi).
\]
The coefficients \(\mu\) and \(\nu\) are given by
\[
\mu_n(a, r') = \frac{\left( \ln \frac{I_n(ma)}{ma} \frac{K_n(ma)}{ma} \right)'}{2 \left( \ln \frac{K_n(ma)}{ma} \right)'} \frac{K_n(mr')}{mr'},
\]
\[
\nu_n(a, r') = \frac{\left( \ln \frac{K_n(ma)}{ma} \right)'}{2 \left( \ln \frac{K_n(ma)}{ma} \right)'} \frac{K_n(mr')}{mr'},
\]
where \(\frac{K_n(x)}{x} = \frac{\partial}{\partial x} \frac{K_n(x)}{x}\). We expect that virtual wormholes have the Planckian size and therefore \(ma = x \to 0\) which gives asymptotically \(I_n(x) x \to \frac{1}{2m} \left( \frac{x}{2} \right)^{n-1}\), and \(\frac{K_n(x)}{x} \to \frac{(n-1)!}{4} \left( \frac{2}{x} \right)^{n+1}\). Therefore, the above coefficients are
\[
\mu_n(a, r') = \frac{1}{(n+1)(n-1)!} \frac{4 (ma)^{n+1}}{2} K_n(mr') mr',
\]
\[
\nu_n(a, r') = \frac{n}{(n+1)(n-1)!} \frac{4 (ma)^{n+1}}{2} K_n(mr') mr',
\]
We expect that virtual wormholes have throats and as $r < a = a/X = a$ which connects two regions in the same space is a couple of conjugated spheres can be accounted for by adding the bias of the source the spheres is removed and surfaces are glued together. Then the proper boundary conditions (the actual topology) which gives for the region $r > a$

$$G(x, x') = \sum_{n=1}^{\infty} \left( g_n^0 (r, r') - \frac{(\frac{ma}{\pi})^{2n}(m^2 K_n(mr') K_n(mr))}{(n+1)((n-1)!)^2} \frac{a^2}{mr'} \right) Q_n(\chi)$$

and as $r < a$

$$G(x, x') = \sum_{n=1}^{\infty} \frac{n (\frac{ma}{\pi})^{2n}}{(n+1)((n-1)!)^2} \frac{m^2 K_n(mr') K_n(m\tilde{r})}{mr'} Q_n(\chi)$$

In the massless case $m = 0$ this solution transforms to (as $r > a$)

$$G(x, x') = G_0 + \Delta G = \frac{1}{4\pi^2} \sum_{n=1}^{\infty} \left( \frac{r^{n-1}}{r^{n+1}} - \frac{1}{n+1} \frac{a^2}{r^r} \right) Q_n(\chi),$$

and as $r < a$

$$G(x, x') = \Delta G = \frac{1}{4\pi^2} \sum_{n=1}^{\infty} \frac{n}{n+1} \frac{a^2}{r^r} Q_n(\chi).$$

In what follows we shall use the dilute gas approximation and, therefore, it is sufficient to retain the lowest (monopole $n = 1$) term only. Then we find

$$\Delta G_\pm (x, x') = \mp 2\pi^2 a^2 G_0 (r') G_0 (r \pm)$$

where $r_\pm > a$ and the sign $\pm$ corresponds to the regions $r_+ = r > a$ and $r_- = a^2/r$ ($r < a$) respectively.

**IV. GREEN FUNCTION IN A GAS OF VIRTUAL WORMHOLES**

Consider now the Green function in the presence of a gas of wormholes. In the previous section we have shown that in the presence of a single wormhole which connects two Euclidean spaces the source $\delta(x - x')$ generates a set of multipoles placed in the center of sphere (i.e., on the throat). In the present paper we shall consider a dilute gas approximation and, therefore, it is sufficient to retain the lowest (monopole $n = 1$) term only. A single wormhole which connects two regions in the same space is a couple of conjugated spheres $S^3_\theta$ of the radius $a$ with a distance $\tilde{X} = \tilde{R}_+ - \tilde{R}_-$ between centers of spheres. So the parameters of the wormhole are $[\text{20}] \xi = (a, \tilde{R}_+, \tilde{R}_-)$. The interior of the spheres is removed and surfaces are glued together. Then the proper boundary conditions (the actual topology) can be accounted for by adding the bias of the source

$$\delta(x - x') \to N(x, x') = \delta(x - x') + b(x, x').$$

In the approximation $a/X \ll 1$ (e.g., see also [14]) the bias takes the form

$$b_0 (x, x', \xi) = 2\pi^2 a^2 (G_0 (R_+ - x') - G_0 (R_+ - x')) \times$$

$$\times \left[ \delta^4 (x - R_+) - \delta^4 (x - R_-) \right]$$

We expect that virtual wormholes have throats $a \sim \ell_{pl}$ of the Planckian size, while in the present paper we are interested in much larger scales. Therefore, the form [8] is sufficient for our aims. However this form is not acceptable in considering the short-wave behavior and vacuum polarization effects (e.g., the stress energy tensor). In the last case one should account for the finite value of the throat size and replace in [8] the point-like source with the surface density (induced on the throat) e.g., see for details [7], $\delta^4 (x - R_+) \to \frac{1}{\sqrt{\pi \ell_{pl}}} \delta(x - R_+) - a$.

In the rarefied gas approximation the bias function for the gas of wormholes is additive, i.e.,

$$b_{\text{total}} (x, x') = \sum b_0 (x, x', \xi_i) = \int b_0 (x, x', \xi) F(\xi) d\xi,$$
where \( F(\xi) \) is the density of virtual wormholes in the configuration space (i.e., in the space of wormhole parameters) which is given by

\[
F(\xi) = \sum_{i=1}^{N} \delta(\xi - \xi_i).
\]  

(10)

In the vacuum case we may expect a homogeneous distribution \( \rho(\xi) = \langle 0|F(\xi)|0 \rangle = \rho(a, X) \) \(^{21}\), then for the mean bias we find

\[
\bar{b}_{total}(x - x') = \int 4\pi^2 a^2 (G_0(R_-) - G_0(R_+)) \delta^4(x - x' - R_+) \rho(a, X) \, d\xi
\]

(11)

Consider the Fourier transform \( \rho(a, X) = \int \rho(a, k) e^{-ikX} \frac{d^4k}{(2\pi)^4} \) then we find for \( b(k) = \int b(x) e^{ikx} d^4x \) the expression

\[
\bar{b}_{total}(k) = \frac{4\pi^2}{k^2 + m^2} \int a^2 (\rho(a, k) - \rho(a, 0)) \, da,
\]

(12)

which forms the background cutoff function \( \overline{N}(k) = 1 + \bar{b}_{total}(k) \), so that the regularized vacuum Green function \( G_{reg}(k) \) has the form

\[
G_{reg}(k) = \frac{1}{k^2 + m^2 \overline{N}(k)}.
\]

(13)

General properties of the cutoff is that \( \overline{N}(k) \to 0 \) as \( k \gg k_{pl} \). We point out that in the case of a dense gas \(^{12}\) one has to account for the multiple scattering \( \overline{N}(k) = 1 + \bar{b}(k) + \bar{b}^2(k) + ... \) (and in general one has also to add the contribution from higher order multipoles) which gives \( \overline{N}(k) = 1/(1 - \bar{b}(k)) \). Then in the low energy limit \( k \ll k_{pl} \) one finds \( \rho(a, k) \approx \rho(a, 0) + \frac{\pi}{2} \rho''(a, 0) k^2 \) and \( G_{reg}(k) \) reduces to the renormalization of coupling constants

\[
G_{reg}(k) = \frac{Z}{k^2 + m^2 Z}
\]

where \( Z = 1/(1 + \beta) \) and \( \beta = -2\pi^2 \int a^2 \rho''(a, 0) \, da \) (\( \beta > 0 \), e.g., see Refs. \[^{7, 14}\]) which is in agreement with the statement in Refs. \[^{8}\].

**V. COHERENT SET OF VIRTUAL WORMHOLES**

As it was demonstrated in Ref. \[^{8}\] a coherent set of virtual wormholes (e.g., beads of virtual wormholes) may work as an actual wormhole. In the present section we clarify this relation and give simplest examples. Consider first the distribution function which corresponds to a particular wormhole. Such a function is given simply by

\[
F(\xi) = \delta(a - a_0) \delta^4(R_+ - y) \delta^4(R_- - y')
\]

which corresponds to the single wormhole with \( R_+ = y, R_- = y' \), and \( a = a_0 \). Let us fix the vector \( X = y - y' = const \) and add the obvious symmetry for the replacement \( R_+ \leftrightarrow R_- \) which gives

\[
F(\xi, y) = \delta(a - a_0) \frac{1}{2} \left( \delta^4(R_+ - y) \delta^4(R_- - y + X) + \delta^4(R_- - y) \delta^4(R_+ - y + X) \right).
\]

This allows us to define coherent sets of wormholes by integrating the above distribution over some portion of space with a density \( n(y) \) as

\[
F(\xi, n) = \int F(\xi, y) n(y) d^4 y.
\]

(14)

In the limit \( a_0 \to 0 \) the virtual wormhole degenerates into a point and the above expression for \( n(y) = \delta(f(y)) \) defines merely the gluing of the hypersurface \( S = \{ f(y) = 0 \} \) and the shifted surface \( S' \) by the rule that every point on \( S \), i.e., \( y \in S \) is glued to a point on \( S' \), i.e., \( y' = y - X \in S' \). In a more general case when \( a_0 = const \neq 0 \) the density \( n(y) < 1 \) (we have the obvious restriction that every wormhole cuts the volume \( 2\pi^2 a_0^4 \) in space) and the above density corresponds to a coherent set of wormholes which glue roughly the domains \( D = \{ n(y) \neq 0 \} \) and the analogous
domain \( D' \) shifted in space on the vector \( X \). The density \( n(y) \) here defines the degree of gluing (the transparency coefficient for the resulting wormhole).

Considering now different densities \( n(y) \) we may get different wormholes. It is supposed that an actual wormhole exists for all times and therefore the density \( n(y) \) does not depend on the time-like part of \( y = (t, \vec{r}) \) (or \( n \neq 0 \) for a sufficiently big interval of time \( t \)). The simplest example is the analog of an astrophysical wormhole which can be modelled by a spherically symmetric function e.g., \( n(y) = n(\vec{r}) = n \theta(r - \vec{b}) \), where \( b \) is related to the radius of the throat and \( r = |\vec{r}| \). Another simple example can be called the Star-Gate when the density \( n(\vec{r}) \) is concentrated on a two-dimensional disk, e.g., \( n(\vec{r}) = n(\rho, \varphi, z) = n \theta(\rho - \vec{b}) \delta(z) \) (where \( \rho, \varphi, z \) are polar coordinates of the vector \( \vec{r} \)). In the homogeneous case \( n(y) = n \) and we get the expression below (13).

Substituting different \( n(y) \) into (14) and (13) into (9) we may define the resulting Green function and define the propagation of particles in the space with such a gluing.

VI. ADDITIONAL DISTRIBUTION OF WORMHOLES AND ANISOTROPY IN THE SPEED OF LIGHT

Here we consider a more simple situation when additional wormholes are incoherent but simply have an anisotropic distribution. Let we get a perturbation in the background (vacuum) distribution of virtual wormholes with a particular distribution function \( f (a, X) = \delta \rho(\xi) \) of the form

\[
    f (a, X) = \delta n \delta (a - a_0) \frac{1}{2} (\delta^4 (X - r_0) + \delta^4 (X + r_0)),
\]

where \( \delta n = \delta N/V \) is the change in the density of wormholes. We point out that in the vacuum case the background density of wormholes is always positive \( n \geq 0 \), while the value \( \delta n \) admits both signs. The above distribution corresponds to a set of wormholes with the throat \( a_0 \), oriented along the same direction \( r_0 \) and with the distance between throats \( r_0 = |R_+ - R_-| \). Then \( f (a, k) \) reduces to

\[
    f (a, k) = \delta n \delta (a - a_0) \cos (kr_0),
\]

where \( (kr_0) = k_\mu r_0^\mu \). Thus from (12) we find

\[
    \delta b (k) = -\delta n a^2 \frac{4\pi^2}{k^2 + m^2} (1 - \cos (kr_0)).
\]

Consider the structure of the bias of the unit source (16) in the coordinate representation. Substituting (14) into (11) we find

\[
    \delta b (x) = 2\pi^2 \delta n a^2 (G_0(x + r_0) + G_0(x - r_0) - 2G_0(x)).
\]

We recall that here \( G_0(x - x') \) is the standard Euclidean Green function which, upon the continuation to the Minkowsky space, transforms to the Feynman propagator which is important in quantum field theory. However when considering the propagation of signals we should use the retarding Green function, e.g., in the massless case

\[
    G_0 \to G_{ret} (x, x') = \frac{1}{R} \delta (t' - t + \frac{1}{c} |\vec{R}|),
\]

while the bias has the same structure (e.g., see [15]). Thus we see that the additional source represents three outgoing spherical waves which originate at positions \( x = 0 \) and \( x = \pm r_0 \). If \( r_0 \) has only spatial direction, the additional source \( b (x) \) will form the wavefront which may overrun the standard wave in the direction \( r_0 \) which may lead to an anomaly shift \( \Delta t = r_0/c \) in the duration of the propagation of signals. The intensity of such an additional signal is described by the portion of the primary signal scattered on additional virtual wormholes which is given by \( b = |\int \delta b (x) d^4 x| \leq 1 \).

We recall that the dilute gas approximation requires \( 2\pi^2 |\delta n|a^4 \ll 1 \), i.e., the portion of the volume cut by virtual wormholes should be sufficiently small, while the ratio \( r_0/a \) may be an arbitrary parameter and therefore in general \( b \) may reach the order of unity.

The above structure of the additional source does not guarantee that actual signals may overrun the basic signal (22). In the first place it shows that the phase velocity may exceed the speed of light. The physical meaning however has the group velocity only which can be found from the dispersion relations, i.e., from poles of the Green function

\[
    \frac{1}{G(k)} \approx (k^2 + m^2) (1 - b(k)) = (k^2 + 4\pi^2 \delta n a^2 (1 - \cos (kr_0)) + m^2) = 0.
\]
which in the long-wave or low energy limit \((kr_0 \ll 1)\) becomes

\[
\frac{1}{G(k)} \approx k^2 + (1 + 2\pi^2\delta n a^2 r_0^2)k^2 + m^2 = 0
\]

where \(k_\perp\) and \(k_\parallel\) denote orthogonal and longitudinal components of the wave vector with respect to the direction \(r_0\).

We recall that in the Minkowsky space one should replace \(k_0 \to \iota \omega\), so the poles become real.

Let \(r_0\) has only spatial direction \((0, r_0)\). Then we see that the above dispersion relation corresponds to a particle with an anisotropic speed of light

\[
c_\perp = 1, \quad c_\parallel^2 = 1 + 2\pi^2\delta n a^2 r_0^2.
\]

Whether the longitudinal light velocity \(c_\parallel\) exceeds or not the vacuum value \((c_\parallel > 1\) or \(c_\parallel < 1\)) depends on the sign of the perturbation in the vacuum density of wormholes \(\delta n\). If \(\delta n > 0\), then \(c_\parallel > 1\) and the speed of light will exceed the vacuum value, i.e., such a medium (gas of virtual wormholes with the perturbation \((15)\)) will possess an anomalous dispersion. We point out that in this case we cannot speak on a superluminality (or subluminality), e.g., as it is discussed in Ref. [16], since the above change holds for all particles and it merely changes the causal structure of space.

Consider the case when \(r_0\) has the time-like direction. Then the speed of light remains isotropic but changes according to

\[
c^2 = 1/(1 + 2\pi^2\delta n a^2 r_0^2) \simeq 1 - 2\pi^2\delta n a^2 r_0^2.
\]

Here we get the inverse situation, the speed of light increases \(c > 1\), if \(\delta n < 0\), and it decreases \(c < 1\) when \(\delta n > 0\). In this case masses of particles also renormalize as \(m^2 \to m^2(1 - 2\pi^2\delta n a^2 r_0^2)\).

The interpretation of the physical mechanism which cause the renormalization in the speed of light is rather simple. Indeed let \(\delta n > 0\), then the spatial vector \(r_0 = (0, \vec{r}_0)\) defines an instant transport of a portion of the basic signal (via virtual wormholes) in the direction \(\vec{r}_0\). Due to homogeneous distribution of virtual wormholes \((15)\) such a mechanism works everywhere in space and this leads to the renormalization of longitudinal speed of light which becomes \(c_\parallel > 1\). On the contrary a time-like vector \(r_\parallel = (T, 0)\) defines a retarding \(T\) (by wormholes) in the propagation of signals which leads to the common decrease of the speed of light \(c < 1\). What kind of situation is realized in the presence of external classical fields \(\varphi_{\text{ext}}\) depends on the exact relation between \(\delta n\) and \(\varphi_{\text{ext}}\).

In conclusion we point out that the external field may have a complex cosmological evolution which gives quite naturally rise to a non-trivial variation of interaction constants \([17]\) (for discussions of the variation of the fine structure constant see also Refs. \([18]\) and, in particular, to the variable speed of light cosmology \([19]\).

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[20] The additional parameter (rotation matrix A) is important only for multipoles of higher orders.

[21] Such a function admits a rigorous definition and we consider it in a separate paper.

[22] I am obligated to V.A. Berezin for pointing out to this fact.