Anomalously light mesons in a (1+1)-dimensional supersymmetric theory with fundamental matter

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Abstract

We consider $\mathcal{N} = 1$ supersymmetric Yang–Mills theory with fundamental matter in the large-$N_c$ approximation in 1+1 dimensions. We add a Chern–Simons term to give the adjoint partons a mass and solve for the meson bound states. Here mesons are color-singlet states with two partons in the fundamental representation but are not necessarily bosons. We find that this theory has anomalously light meson bound states at intermediate and strong coupling. We also examine the structure functions for these states and find that they prefer to have as many partons as possible at low longitudinal momentum fraction.
1 Introduction.

Supersymmetry is a property of some quantum field theories that provides a beautiful solution to a host of theoretical problems [1]. It is a pressing experimental issue to see if nature takes advantage of this elegant option. Of course we already know that supersymmetry is rather badly broken since we do not see any superpartners for the particles of the standard model. It is assumed that all of the superpartners are in fact very heavy and that we will see them as we go to higher energies in accelerators. The question then becomes, what are our expectations for the lightest superparticles that we might first see? As we know from QCD, symmetries can give rise to very light bound states. There are indications that the same thing can happen with supersymmetry [2, 3].

Long ago ’t Hooft [4] showed that two-dimensional models can be powerful laboratories for the study of the bound-state problem. These models remain popular to this day because they are easy to solve and share many properties with their four-dimensional cousins, most notably stable bound states. Supersymmetric two-dimensional models are particularly attractive since they are also super-renormalizable. Given that the dynamics of the gauge field is responsible for the strong interaction and for the formation of bound states, it comes as no surprise that a great deal of effort has gone into the investigation of bound states of pure glue in supersymmetric models [5, 6]. While such theories capture the essential properties of the mass spectrum, and some of them are relevant for the string theory [7], the wave functions are quite different from the ones for mesons and baryons. Extensive study of the meson spectrum of non-supersymmetric theories has been done (see [8] for a review). Recently an initial study addressed some of these states in the context of supersymmetric models [9].

Throughout this paper we use the word “meson” to indicate the group structure of the state. Namely we define a meson as a bound state whose wave function can be written as a linear combination of parton chains, each chain starting and ending with a creation operator in the fundamental representation. In supersymmetric theories, the states defined this way can have either bosonic or fermionic statistics.

Previously we saw that the lightest bound states in $N = 1$ supersymmetric theories are very interesting [2, 3]. In supersymmetric Yang–Mills (SYM) theories in 1+1 and 2+1 dimensions, the lightest bound states in the spectrum are massless Bogomol'nyi–Prasad–Sommerfield (BPS) bound states [5]. These states are exactly massless at all couplings. When we add a Chern–Simons (CS) term to the (1+1)-dimensional SYM theory, which gives a mass to the constituents, we find approximate BPS states. The masses of these states are approximately independent of the coupling, and at strong coupling these states are the lightest bound states in the theory [2]. In (2+1)-dimensional SYM-CS theory we find that at strong coupling there are anomalously light bound states [3]. In both 2+1 and 1+1 dimensions, these interesting states appear because of the exact BPS symmetry in the underlying SYM theory.

Here we will look at the lightest bound states of SYM–CS theory with fundamental matter. The properties of the entire spectrum will be presented elsewhere [10]. Again we will see that the lightest bound states are significantly lighter than one would have naively expected. We will see that the lightest state is nearly massless compared to the threshold for the bound states at one unit of adjoint-parton mass.

There are several approaches to the solution of this problem. For QCD-like theories, lattice gauge theory is probably the most popular approach since the lattice
approximation does not break the most important symmetry, gauge symmetry. Similarly for supersymmetric theories, supersymmetric discrete light-cone quantization (SDLCQ) \cite{8, 11, 5, 12} is probably the most powerful approach since the discretization does not break the most important symmetry, supersymmetry. In this paper we consider supersymmetric theories and follow this latter approach. To simplify the calculation we will consider only the large-$N_c$ limit \cite{4}, which has proven to be a powerful approximation for bound-state calculations. While baryons can be constructed in this limit \cite{13}, they have an infinite number of partons and thus practical calculations for such states are complicated. Note that throughout this paper we completely ignore the zero-mode problem \cite{14, 15}; however, it is clear that considerable progress on this issue could be made following our earlier work on the zero modes of the two-dimensional supersymmetric model with only adjoint fields \cite{16}.

The paper has the following organization. In Section 2 we consider three-dimensional supersymmetric QCD (SQCD) and dimensionally reduce it to 1+1 dimensions. We perform the light-cone quantization of the resulting theory by applying canonical commutation relations at fixed $x^+ \equiv (x^0 + x^1)/\sqrt{2}$ and choosing the light-cone gauge ($A^+ = 0$) for the vector field. After solving the constraint equations, we obtain a model containing 4 dynamical fields. We construct the supercharge for this dimensionally reduced theory. In Section 3 we discuss the structure of the lighter meson bound states. In Section 4 we discuss the addition of a Chern-Simons (CS) term to the supercharge \cite{17, 18, 19}. We explain that, in the context of this model, this is equivalent to adding a mass to the partons in the adjoint representation. In particular, in Section 5 we discuss bound-state solutions that we find for the lowest mass states. We also present an analysis of the convergence in the longitudinal resolution. Finally, in this same section we display the structure function for the lowest state. In Section 6 we discuss our results and the future directions that are indicated by this research.

2 Supersymmetric systems with fundamental matter.

We consider the supersymmetric models in two dimensions which can be obtained as the result of dimensional reduction of SQCD$_{2+1}$. Our starting point is the three-dimensional action

$$S = \int d^3x \text{tr} \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \bar{\Lambda} \Gamma^\mu D_\mu \Lambda + D_\mu \xi \gamma^\mu \xi + i \bar{\Psi} D_\mu \Gamma^\mu \Psi \ight.$$  

$$\quad - g \left[ \bar{\Psi} \Lambda \xi + \xi \gamma^1 \Lambda \bar{\Psi} \right] \right).$$  

(1)

This action describes a system of a gauge field $A_\mu$, representing gluons, and its superpartner $\Lambda$, representing gluinos, both taking values in the adjoint representation of $SU(N_c)$, and two complex fields, a scalar $\xi$ representing squarks and a Dirac fermion $\Psi$ representing quarks, transforming according to the fundamental representation of the same group. In matrix notation the covariant derivatives are given by

$$D_\mu \Lambda = \partial_\mu \Lambda + ig[A_\mu, \Lambda], \quad D_\mu \xi = \partial_\mu \xi + igA_\mu \xi, \quad D_\mu \Psi = \partial_\mu \Psi + igA_\mu \Psi.$$

(2)
The action (1) is invariant under the following supersymmetry transformations, which are parameterized by a two-component Majorana fermion $\varepsilon$:

$$
\delta A_\mu = \frac{i}{2} \varepsilon \Gamma_\mu \Lambda ,
\delta \Lambda = \frac{1}{4} F_{\mu \nu} \Gamma^{\mu \nu} \varepsilon ,
\delta \xi = \frac{i}{2} \varepsilon \Psi ,
\delta \tilde{\Psi} = -\frac{1}{2} \Gamma^{\mu \varepsilon} D_\mu \xi .
$$

(3)

Using standard techniques one can construct the Noether current corresponding to these transformations as

$$
\bar{\varepsilon} q^\mu = \frac{i}{4} \varepsilon \Gamma^{\alpha \beta} \Gamma^\mu \text{tr} (\Lambda F_{\alpha \beta}) + \frac{i}{2} \Gamma^{\mu} \xi^\dagger \bar{\varepsilon} \Psi + \frac{i}{2} \Gamma^{\mu \varepsilon} D_\mu \Psi
$$

$$
- \frac{i}{2} \bar{\Psi} \varepsilon D^\mu \xi + \frac{i}{2} D_\nu \bar{\Psi} \Gamma^{\mu \varepsilon} \xi .
$$

(4)

We will consider the reduction of this system to two dimensions, which means that the field configurations are assumed to be independent of the transverse coordinate $x^2$. In the resulting two-dimensional system we will implement light-cone quantization, where the initial conditions as well as canonical commutation relations are imposed on a light-like surface $x^+ = \text{const}$. In particular, we construct the supercharge by integrating the current (4) over the light-like surface to obtain

$$
\bar{\varepsilon} Q = \int dx^- dx^+ \left( \frac{i}{4} \varepsilon \Gamma^{\alpha \beta} \Gamma^+ \text{tr} (\Lambda F_{\alpha \beta}) + \frac{i}{2} \xi D^- \xi^\dagger \bar{\varepsilon} \Psi + \frac{i}{2} \xi^\dagger \varepsilon \Gamma^{+ \nu} D_\nu \Psi
$$

$$
- \frac{i}{2} \bar{\Psi} \varepsilon D^+ \xi + \frac{i}{2} D_\nu \bar{\Psi} \Gamma^{+ \nu} \varepsilon \xi \right) .
$$

(5)

Since all fields are assumed to be independent of $x^2$, the integration over this coordinate gives just a constant factor, which we absorb by a field redefinition.

If we use the following specific representation for the Dirac matrices in three dimensions:

$$
\Gamma^0 = \sigma_2 , \quad \Gamma^1 = i\sigma_1 , \quad \Gamma^2 = i\sigma_3 ,
$$

(6)

the Majorana fermion $\Lambda$ can be chosen to be real. It is also convenient to write the fermion fields and the supercharge in component form as

$$
\Lambda = (\lambda, \tilde{\lambda})^T , \quad \Psi = (\psi, \tilde{\psi})^T , \quad Q = (Q^+, Q^-)^T .
$$

(7)

In terms of this decomposition the superalgebra has an explicit $(1,1)$ form

$$
\{Q^+, Q^+\} = 2\sqrt{2} P^+ , \quad \{Q^-, Q^-\} = 2\sqrt{2} P^- , \quad \{Q^+, Q^-\} = 0 .
$$

(8)

The SDLCQ method exploits this superalgebra by constructing $P^-$ from a discrete approximation to $Q^-$ [11], rather than directly discretizing $P^-$, as is done in ordinary DLCQ [8].

To begin to eliminate nondynamical fields, we impose the light-cone gauge ($A^+ = 0$). Then the supercharges are given by

$$
Q^+ = 2 \int dx^- \left( \lambda \partial_- A^2 + \frac{i}{2} \partial_- \xi^\dagger \psi - \frac{i}{2} \psi^\dagger \partial_- \xi - \frac{i}{2} \xi^\dagger \partial_- \psi + \frac{i}{2} \partial_- \psi^\dagger \xi \right) ,
$$

$$
Q^- = -2 \int dx^- \left( -\lambda \partial_- A^- + i \xi^\dagger D_2 \psi - i D_2 \psi^\dagger \xi + \frac{i}{\sqrt{2}} \partial_- (\bar{\psi}^\dagger \xi - \xi^\dagger \tilde{\psi}) \right) .
$$

(9)

(10)
Note that apart from a total derivative these expressions involve only left-moving components of the fermions ($\lambda$ and $\psi$). In fact, in the light-cone formulation only these components are dynamical. To see this we consider the equations of motion that follow from the action (1), in light-cone gauge. Three of them serve as constraints rather than as dynamical statements; they are

$$
\partial_- \tilde{\lambda} = -\frac{ig}{\sqrt{2}} \left( [A^2, \lambda] + \sqrt{2} \left( \frac{\sqrt{2}}{g} \left( \partial_- A \right) \right) - i\sqrt{2} \partial_- \xi^\dagger \right),
$$

(11)

$$
\partial_- \tilde{\psi} = -\frac{ig}{\sqrt{2}} A^2 \psi + \frac{g}{\sqrt{2}} \lambda \xi,
$$

(12)

and

$$
\partial_2 A^+ = gJ,
$$

(13)

with

$$
J \equiv i[A^2, \partial_- A^2] + \frac{1}{\sqrt{2}} \left\{ \lambda, \lambda \right\} - i\hbar \partial_- \xi^\dagger + i\xi \partial_- \xi^\dagger + \sqrt{2} \psi \psi^\dagger.
$$

(14)

Apart from the zero-mode problem [14], one can invert the last constraint to write the auxiliary field $A^+$ in terms of physical degrees of freedom. Substituting the result into the expression for the supercharge and omitting the boundary term, we get

$$
Q^- = Q^-_s + Q^-_1 + Q^-_2 + Q^-_3,
$$

(15)

where $Q^-_s$ is the supercharge of pure adjoint matter [11] and

$$
Q^-_1 = -\frac{g}{\sqrt{2}} \int dx^- \left( i\sqrt{2} \xi \partial_- \xi^\dagger - i\sqrt{2} \partial_- \xi \xi^\dagger \right) \frac{1}{\partial_-} \lambda,
$$

(16)

$$
Q^-_2 = -\frac{g}{\sqrt{2}} \int dx^- \left( 2 \psi \psi^\dagger \right) \frac{1}{\partial_-} \lambda,
$$

(17)

$$
Q^-_3 = -2g \int dx^- \left( \xi^\dagger A^2 \psi + \psi^\dagger A^2 \xi \right).
$$

(18)

In order to solve the bound-state problem $2P^+ P^- |M\rangle = M^2 |M\rangle$, we apply the methods of SDLCQ. Namely we compactify the two-dimensional theory on a light-like circle ($-L < x^- < L$), and impose periodic boundary conditions on all physical fields. This leads to the following mode expansions:

$$
A^2_{ij}(0, x^-) = \frac{1}{\sqrt{4\pi}} \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}} \left( a_{ij}(k)e^{-ik\pi x^-/L} + a_{ji}(k)e^{ik\pi x^-/L} \right),
$$

(19)

$$
\lambda_{ij}(0, x^-) = \frac{1}{2\sqrt{2L}} \sum_{k=1}^{\infty} \left( b_{ij}(k)e^{-ik\pi x^-/L} + b_{ji}(k)e^{ik\pi x^-/L} \right),
$$

(20)

$$
\xi_i(0, x^-) = \frac{1}{\sqrt{2\pi}} \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}} \left( c_i(k)e^{-ik\pi x^-/L} + c_i^\dagger(k)e^{ik\pi x^-/L} \right),
$$

(21)

$$
\psi_i(0, x^-) = \frac{1}{2\sqrt{2L}} \sum_{k=1}^{\infty} \left( d_i(k)e^{-ik\pi x^-/L} + d_i^\dagger(k)e^{ik\pi x^-/L} \right).
$$

(22)

We drop the zero modes of the fields; including them could lead to new and interesting effects (see [16], for example), but this is beyond the scope of this work. In the light-cone formalism one treats $x^+$ as the time direction, thus the commutation relations
between fields and their momenta are imposed on the surface \( x^+ = 0 \). For the system under consideration this means that

\[
\left[A^2_{ij}(0, x^-), \partial_+ A^2_{kl}(0, y^-) \right] = i \left( \delta_{il} \delta_{kj} - \frac{1}{N} \delta_{ij} \delta_{kl} \right) \delta(x^--y^-),
\]

(23)

\[
\{\lambda_{ij}(0, x^-), \lambda_{kl}(0, y^-) \} = \sqrt{2} \left( \delta_{il} \delta_{kj} - \frac{1}{N} \delta_{ij} \delta_{kl} \right) \delta(x^--y^-),
\]

(24)

\[
[\xi_i(0, x^-), \partial_+ \xi_j(0, y^-)] = i \delta_{ij} \delta(x^--y^-),
\]

(25)

\[
\{\bar{\psi}_i(0, x^-), \psi_j(0, y^-) \} = \sqrt{2} \delta_{ij} \delta(x^--y^-).
\]

(26)

These relations can be rewritten in terms of creation and annihilation operators as

\[
[a_{ij}, a^\dagger_{kl}] = \left( \delta_{il} \delta_{kj} - \frac{1}{N} \delta_{ij} \delta_{kl} \right), \quad \{b_{ij}, b^\dagger_{kl}\} = \left( \delta_{il} \delta_{kj} - \frac{1}{N} \delta_{ij} \delta_{kl} \right),
\]

(27)

\[
[c_i, c^\dagger_j] = \delta_{ij}, \quad [\bar{c}_i, c^\dagger_j] = \delta_{ij}, \quad \{d_i, d^\dagger_j\} = \delta_{ij} \quad \{\bar{d}_i, \bar{d}^\dagger_j\} = \delta_{ij}.
\]

(28)

In this paper we will discuss numerical results obtained in the large-\( N_c \) limit, i.e. we neglect \( 1/N_c \) terms in the above expressions. Although \( 1/N_c \) corrections may have interesting consequences, they are beyond the scope of this work.

### 3 Mesons

We will consider here only meson-like states. In the large-\( N_c \) approximation these are color-singlet states with exactly two partons in the fundamental representation. The boson bound states will have either two bosons in the fundamental representation or two fermions in the fundamental representation. In general a boson bound state will have a combination of these types of contributions. Because this theory and the numerical formalism are exactly supersymmetric, for each boson bound state there will be a degenerate bound state that is a fermion. The fermion bound state will have one fermion in the fundamental representation and one boson in the fundamental representation. In the string interpretation of these theories, such states would correspond to open strings with freely moving endpoints. In the language of QCD, the model corresponds to a system of interacting gluons and gluinos which bind dynamical \((s)\)quarks and \((anti-)\)(s)quark pair. Thus the Fock space is constructed from states of the following type:

\[
\tilde{f}^\dagger_i(k_1)a^\dagger_{112}(k_2)\ldots b^\dagger_{n-1n+1}(k_{n-1})\ldots f^\dagger_i(k_n)|0\rangle.
\]

(29)

Here \( \tilde{f}^\dagger_i \) and \( f^\dagger_i \) each create one of the fundamental partons, and \( |0\rangle \) is the vacuum annihilated by \( a_{ij}, b_{ij}, c_i, \bar{c}_i, d_i, \bar{d}_i \). In this basis \( P^+ \) is automatically diagonal.

The three supercharges that govern the behavior of the fundamental matter in these states are \( Q_1^- \), \( Q_2^- \), and \( Q_3^- \). For example, after substituting the expansions (20) and (22) one gets the mode decomposition of \( Q_2^- \)

\[
Q_2^- = \frac{i2^{-1/4}g\sqrt{L}}{\pi} \sum_{k_1,k_2=1}^{\infty} \frac{1}{k_1} [d^\dagger_i(k_2)b^\dagger_{ij}(k_1)\tilde{d}_j(k_1+k_2) + \tilde{d}^\dagger_j(k_1+k_2)d_i(k_2)b_{ij}(k_1)]
\]

\[+ \quad d^\dagger_i(k_2)b^\dagger_{ij}(k_1)d_j(k_1+k_2) + d^\dagger_j(k_1+k_2)\bar{d}_i(k_2)b_{ij}(k_1)].
\]

(30)
The other color-singlet bound states in this theory are states that are composed of traces of only adjoint mesons. These can be thought of as loops. At finite $N_c$ this theory has interactions that break these loops and insert a pair of fundamental partons, making an open-string state. This type of interaction can of course also form loops from open strings and break open strings into two. In principle, a calculation of the spectrum of such a finite-$N_c$ theory is within the reach of SDLCQ. The only significant change is to include states in the basis with more than one color trace.

4 Supersymmetric Chern–Simons theory

The CS term we use in this calculation is obtained by starting with a CS term in 2+1 dimensions and reducing it to 1+1 dimensions. This term has the effect of adding a mass for the adjoint partons. In this calculation we are including fundamental matter because we are interested in QCD-like meson bound states. Without a mass for the adjoint matter, this theory is known to produce very long light chains of adjoint partons. In a finite-$N_c$ calculation we would not have these very long chains because they would break, but in the large-$N_c$ approximation they do not. While SDLCQ can be used to do finite-$N_c$ calculations, it is much easier to add a mass to restrict the number of adjoint partons in our bound states. We choose the CS mechanism to give the adjoint partons a mass because we can do this without breaking the supersymmetry.

The Lagrangian of this theory is

$$\mathcal{L} = \mathcal{L}_{\text{SQCD}} + \frac{\kappa}{2} \mathcal{L}_{\text{CS}},$$ (31)

where $\mathcal{L}_{\text{SQCD}}$ is the SQCD Lagrangian we discussed earlier, $\kappa$ is the CS coupling, and

$$\mathcal{L}_{\text{CS}} = \epsilon^{\mu
u\lambda} \left( A_\mu \partial_\nu A_\lambda + \frac{2i}{3} g A_\mu A_\nu A_\lambda \right) + 2 \bar{\Psi} \Psi.$$ (32)

A trace of the color matrices is understood. The discrete version of the CS part of the supercharge in 1+1 dimensions is

$$Q_{\text{CS}} = \left( \frac{2^{-1/4} \sqrt{L}}{\sqrt{\pi}} \right) \sum_n \frac{\kappa}{\sqrt{n}} \left( A^\dagger(n) B(n) + B^\dagger(n) A(n) \right),$$ (33)

where $A$ and $B$ are rescaled discrete field operators

$$A(n) \equiv \sqrt{\frac{\pi}{L}} a_{ij}(n\pi/L), \quad B(n) \equiv \sqrt{\frac{\pi}{L}} b_{ij}(n\pi/L).$$ (34)

It is important to compare $Q_{\text{CS}}$ with the supercharge for $\mathcal{N} = 1$ SYM in 2+1 dimensions [20], which has a contribution of the form

$$Q_{\perp} = i \left( \frac{g 2^{-1/4} \sqrt{L}}{\sqrt{\pi}} \right) \sum_{n,n_{\perp}} \frac{k_{\perp}}{\sqrt{n}} \left( A^\dagger(n,n_{\perp}) B(n,n_{\perp}) - B^\dagger(n,n_{\perp}) A(n,n_{\perp}) \right),$$ (35)

where $k_{\perp} = 2\pi n_{\perp}/L_{\perp}$ is the discrete transverse momentum. Notice that $k_{\perp}$ and $\kappa$ enter the supercharge in very similar ways. Because the light-cone energy is of the form $(k^2 + m^2)/k^+$, $k_{\perp}$ behaves like a mass, and therefore $\kappa$ also behaves in many ways like a mass for the adjoint particles.

The partons in the fundamental representation in this theory will remain massless. Of course, in a more physical theory the supersymmetry would be badly broken; the squark would acquire a large mass, and only the quarks would remain nearly massless.
5 Numerical results

This SYM-CS theory with fundamental matter has two dimensionful parameters with dimension of a mass squared, the YM coupling squared $g^2 N_c/\pi$ and the CS coupling squared $\kappa^2$. The latter is also the mass squared of the partons in the adjoint representation. Furthermore, we are only considering meson bound states. These are states of the form shown in Eq. (29) with two fundamental partons linked by partons in the adjoint representation. Since we are working in the large-$N_c$ approximation, this class of states is disconnected from the other allowed class of pure adjoint matter bound states and multiparticle states.

This theory also has a $Z_2$ symmetry [21] which is very useful in labeling the states and reducing the dimension of the Fock basis that one has to consider in any one diagonalization step. For this theory the $Z_2$ symmetry divides the basis into states with an even or odd number of gluons. Here we will focus on the lowest mass states in the sector with an odd number of adjoint bosons. The lowest mass state in the sector with an even number of bosons has a mass that converges to about $0.2\kappa^2$; at strong coupling this state converges significantly slower than the state we consider here, and it will be presented in detail in a future publication, where we will discuss the entire spectrum of this theory [10].

We find that the $Z_2$-odd spectrum of meson bound states divides into two bands of states, a very light band and a heavy band, as can be seen in Fig. 1. This is easy to understand if we start by considering large $\kappa$. At large $\kappa$, the light band is primarily composed of two fundamental partons and a small mixture of adjoint partons, and the heavy band is composed of bound states that have at least one adjoint parton. In the limit of very large $\kappa$, all the particles in the low-mass band become massless. Here we will focus on the low-mass band but keep $\kappa$ at or below $g\sqrt{N_c/\pi}$. These moderate values of $\kappa$ will allow a significant mixture of adjoint matter in the bound states. The lowest mass state at $\kappa = g\sqrt{N_c/\pi}$ has an average particle count of about three, two fundamental partons and one adjoint parton.

If this state has on average one adjoint parton, which has mass $\kappa^2$, and the fundamental partons are massless, one would expect the lowest bound state in the spectrum to have a mass of order $\kappa^2$. We actually see that the lowest mass state is nearly massless. We appear to have found a state similar to those found in $N = 1$ SYM-CS theory [2, 3]. The lowest mass state is anomalously light and in fact nearly massless.

The mass of the lightest bound state as a function of the resolution is shown in Fig. 2 for $\kappa = g\sqrt{N_c/\pi}$ and $\kappa = 0.1g\sqrt{N_c/\pi}$. The convergence plot shows a rather unusual oscillatory behavior as a function of the resolution $K$. This type of behavior was seen in a DLCQ study of (1+1)-dimensional large-$N_c$ QCD coupled to a massive adjoint Majorana fermion [22], and the explanation there is that the spectrum of two free particles as a function of the resolution oscillates and therefore a bound state that is in some way closely related to a free-particle spectrum might oscillate. The oscillations we see here are in fact much larger than those seen in [22]. As we discussed above, the low-mass band in the spectrum is strongly connected to the free spectrum of two fundamental partons, particularly at large $\kappa$, so some oscillation might be expected.

The oscillatory behavior made this calculation particularly challenging numerically. We were forced to go to very high resolution, $K = 13$, to be certain that the spectrum really converged. This was made even more difficult because we had 4 species of
Figure 1: The mass-squared spectrum, in units of $g^2 N_c / \pi$, as a function of $\kappa$, in units of $g \sqrt{N_c / \pi}$, at a resolution of $K = 6$.

Figure 2: The mass squared of the lowest mass state, in units of $g^2 N_c / \pi$, as a function of $1/K$ for (a) $\kappa = g \sqrt{N_c / \pi}$ and (b) $\kappa = 0.1 g \sqrt{N_c / \pi}$. The solid curve is a fit to the computed points.
Table 1: Properties of the lowest mass boson bound state, including the average numbers of adjoint bosons $aB$, adjoint fermions $aF$, fundamental bosons $fB$, and fundamental fermions $fF$, for different values of the resolution $K$. The mass squared $M^2$ is given in units of $g^2N_c/\pi$. The CS coupling is $\kappa = g\sqrt{N_c/\pi}$.

| $K$ | $M^2$ | $<n>$ | $<naB>$ | $<nfB>$ | $<naF>$ | $<nfF>$ |
|-----|-------|-------|---------|---------|---------|---------|
| 3   | 0.178 | 2.30  | 0.30    | 1.03    | 0.01    | 0.97    |
| 4   | 0.006 | 2.56  | 0.51    | 1.86    | 0.05    | 0.14    |
| 5   | 0.049 | 2.69  | 0.63    | 1.29    | 0.06    | 0.71    |
| 6   | 0.016 | 2.83  | 0.75    | 1.71    | 0.08    | 0.30    |
| 7   | 0.029 | 2.84  | 0.76    | 1.45    | 0.08    | 0.55    |
| 8   | 0.022 | 2.92  | 0.83    | 1.58    | 0.09    | 0.42    |
| 9   | 0.025 | 2.92  | 0.83    | 1.49    | 0.10    | 0.51    |
| 10  | 0.024 | 2.96  | 0.86    | 1.52    | 0.10    | 0.48    |
| 11  | 0.025 | 2.97  | 0.87    | 1.49    | 0.11    | 0.51    |
| 12  | 0.025 | 3.00  | 0.89    | 1.48    | 0.11    | 0.52    |
| 13  | 0.026 | 3.01  | 0.90    | 1.47    | 0.11    | 0.53    |

particles in the problem. To be certain that we were identifying the states properly and isolating the correct state, it was very useful to calculate a number of properties of the states. These are shown in Table 1. There are, of course, degenerate fermion bound states whose properties we do not show. We find that the average number of partons in this state is approximately three and that for the most part the adjoint particle is a boson. We also see that about $2/3$ of the wave function of this state is composed of two fundamental bosons and an adjoint parton and about $1/3$ of the wave function is made of two fundamental fermions and an adjoint parton. Within the context of the standard model, this state is primarily a bound state of two squarks and a gluon.

This state is different than the special state that we saw in pure SYM and SYM-CS theories. Those states had masses that were above threshold. The state that we are considering here has a mass near zero, and threshold is at $\kappa^2$. Thus this is a deeply bound state. In a more realistic theory, the squark would be very heavy. It is conceivable that this mechanism would give a binding well below the threshold even then.

It is very instructive to look at the structure functions of this bound state. We use a standard definition of the structure functions

$$\hat{g}_A(x) = \sum_q \int_0^1 dx_1 \cdots dx_q \delta \left( \sum_{i=1}^q x_i - 1 \right) \prod_{l=1}^q \delta(x_l - x) \delta^A_{A_l} |\psi(x_1, \ldots x_q)|^2.$$  \hspace{2cm} (36)

Here $A$ stands for the chosen representation (fundamental or adjoint) and statistics (boson or fermion) of the constituent of interest. The sum runs over all parton numbers $q$, and the Kronecker delta $\delta^A_{A_l}$ selects partons with matching representation and statistics $A_l$. The discrete approximation $g_A$ to the structure function $\hat{g}_A$ with harmonic resolution $K$ is

$$g_A(n) = \sum_{q=2}^K \sum_{n_1, \ldots n_q = 1}^{K-q} \delta \left( \sum_{i=1}^q n_i - K \right) \prod_{l=1}^q \delta^A_{A_l} |\psi(n_1, \ldots n_q)|^2.$$  \hspace{2cm} (37)
Figure 3: Structure functions of the lowest mass bound state at resolution $K = 12$ for (a) fundamental bosons $fB$ and (b) adjoint bosons $aB$, with the CS coupling fixed at $\kappa = g\sqrt{N_c/\pi}$. The solid curve is a fit to the computed points.

The functions $g_A(n)$ are normalized so that summation over the argument gives the average number of type A particles; their sum is then the average parton number, and we compute these sums as a check. We plot the structure functions as functions of the longitudinal momentum fraction $x = k/P^+ = n/K$ carried by an individual parton. In this lightest state the bulk of the partons are fundamental or adjoint bosons, and their structure functions are shown in Fig. 3. In the fit shown we have forced the fitting function to vanish at $x = 0$. This is an assumption, and a fitting function which goes to a finite value at $x = 0$ would also work. We see that both of the distributions in Fig. 3 are peaked at small $x$. This reflects strong binding of the fundamental partons, allowing them to be widely separated in momentum, combined with only a small contribution to the momentum from the adjoint boson. We cannot, however, fit this peak at small $x$ with a divergent function, such as $1/x$ or log $x$, since such a structure function would imply that the average number of partons also diverges. As we probe smaller and smaller $x$ by increasing the resolution, we would see this divergence. In Table 1 we see instead that the average number of partons converges at high resolution to about three. Such strong peaks at low momentum are not unique to this state, and we will discuss this further when we consider the full spectrum [10].

6 Discussion.

In this work we studied the lowest mass meson of SYM-CS theory with fundamental matter in 1+1 dimensions. The CS term is included to give masses to the adjoint partons. The calculations were performed at large $N_c$ in the framework of SDLCQ; namely, we compactified the light-like coordinate $x^-$ on a finite circle and calculated the Hamiltonian as the square of a supercharge $Q^-$, which we then diagonalized numerically.

In previous work we have found that the lowest mass states of a number of $\mathcal{N} = 1$ supersymmetric theories solved at large $N_c$, using SDLCQ have very interesting properties. In SYM in 1+1 and 2+1 dimensions we found that there are a number of exactly massless BPS states. In SYM-CS theory in 1+1 dimensions we saw that at strong coupling the lightest states are approximately BPS states whose masses are independent
of the YM coupling. In SYM-CS theories in 2+1 dimensions at strong coupling there is again an anomalously light bound state.

Now in this SYM-CS theory with fundamental matter in 1+1 dimensions we find a very light bound state composed primarily of squarks and gluons, and we find that this persists at intermediate and strong coupling. This state is nearly massless and well below the threshold for the spectrum. The structure function of this bound state shows that the dynamics of this theory tend to maximize the number of small-longitudinal-momentum partons in the bound states. From the SDLCQ numerical perspective the states are interesting because of the oscillatory convergence in the resolution. To be certain of this convergence we pushed our resolution to $K = 13$, the highest we have attained in a (1+1)-dimensional problem without truncating the basis. A study of the entire spectrum of this theory will be presented elsewhere [10].

There remains a considerable amount of work to be done on SYM theories with fundamental matter. The most straightforward extension of the present work is to consider calculations in 2+1 dimensions [23, 24, 20]. The $\mathcal{N} = 1$ theory in 2+1 dimensions is easily within our reach. Beyond that the $\mathcal{N} = 2$ theory [25] in 2+1 dimensions, which is the dimensional reduction of the $\mathcal{N} = 1$ theory 3+1 dimensions, will be very interesting.

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