The Mechanical Behavior and Model of the Crossfeed of the Contact Bodies

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Abstract

Interactions occur between the contact bodies under the outside loads. The crossfeed is the dynamic changing process of the contact surface, named crossfeed surface, under the loads, which is an important task in civil engineering. Aiming at the shortcoming of the classical contact mechanics, in which the contact surface is considered smooth, with the help of the shearing and ploughing of the tribology, the mechanical behavior and correspond model of the interacted contact bodies were studied. Based on the fracture mechanics, the cohesion of the crossfeed surface varied with the contact areas was acquired, and the algorithm of the crossfeed mechanical model of the contact bodies was studied. Compared with the photo-elastic experiment of the joint sample, the result shows that the crossfeed mechanical model of the contact bodies is more practical than the traditional model.

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1. Introduction

The interactions between contact bodies are the basic issue of engineering stability research. One of the key assumptions of classical contact mechanics is that contact surface is geometric smooth. However, the solid surface has roughness shown in recent studies. If the surface roughness is much smaller than the size of contact area, classic contact mechanics is still applied. When the surface roughness is close to the size of contact area, the classical contact mechanics would lead to some considerable errors, so a new branch is produced from classical contact mechanics in modern time, that is, rough surface contact

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mechanics. Due to non-full contact caused by the presence of surface roughness, the contact area between contact bodies is only part of the apparent area. According to the contact studies[1], there is a power law relation between the actual contact area $A_r$ and compression load $P$ indicated, that is $P \propto A_r^n$, the reasons for the establishment of the relationship and analysis of value $n$ have been the focus of the study, but has not yet been fully understood. The consideration of Amonton friction law which proposed that $n$ is close to 1, may be due to taking into account the pure plastic deformation or the geometric arrangement of roughness in elastic deformation.

2. Crossfeed mechanical behavior of contact bodies

The shearing and ploughing theory of the tribology is applied to the crossfeed analysis of contact bodies to simulate actual mechanical behavior. According to Binder’s theory of friction, there are direct relationships between shear deformation characteristic and size of the relative deformation of crossfeed surface[2]. Shear stiffness of crossfeed surfaces after closing associated with both shear strain and stress of crossfeed surfaces. Assuming a small region $S'$ near the Gauss point, with same normal compressive stress of crossfeed surfaces, there is an actual contact area $S''$ in this region. And the $S'/S$ is a function of the normal compressive stress $\sigma$, the larger the $\sigma$, the larger the $S'/S$. In the actual contact area $S''$, the relationship between the maximum shear stress $\tau_{\text{max}}$ and peak shear strain $\gamma_p$ is linear with the ratio $G$, named the corresponding secant shear stiffness of peak shear strain of soft material. According to Mohr-Coulomb theory, in which the compressive stress and shear stress are uniform distribution in region $S$. However, the shearing resistance $F$ is obtained by $\tau_{\text{max}}$ through integral in region $S''$, with the same $\tau_{\text{max}}$ for the same material, we can get:

$$\tau = F/S = \tau_{\text{max}} S'/S = G \gamma_p S'/S = G' \gamma_p$$

where, $G'$ is nominal shear stiffness in $S'$, namely:

$$G' = G S'/S$$

3. Crossfeed mechanical model of contact bodies

The Object of finite element analysis is region $S$ rather than region $S''$, and the shear stress obtained is not the maximum shear stress $\tau_{\text{max}}$. Therefore, the nominal shear stiffness $G'$ should be used. In the actual contact area, when the shear stress is less than the maximum shear stress $\tau_{\text{max}}$, namely that has not yet reached the maximum shear force, nominal shear stiffness of the area $S$ should be between $G_0 S'/S$ and $G'$. In order to establish a crossfeed mechanical model, corresponding deformation modes should be clear. According to different states of crossfeed surface, three deformation modes are provided: ① bonded mode of thin-layer unit before sliding; ② when the normal stress is compressive stress, or the tensile stress is less than the tensile strength, and when the shear stress is greater than the shear strength; ③ when the tensile stress is greater than the tensile strength. By the constitutive relation of crossfeed surface and the above deformation mode, when crossfeed surface units are in the elastic stage, there is:

$$\begin{bmatrix} d\sigma_x' \\ d\sigma_y' \\ d\sigma_z' \end{bmatrix} = D'd\varepsilon' = \begin{bmatrix} G S'/S & 0 & 0 \\ 0 & G S'/S & 0 \\ 0 & 0 & E_n \end{bmatrix} d\varepsilon'$$

(3)

When materials come into the plastic state, the constitutive relation is:
The stress-strain relationships at different normal strain are complied with different criteria. Thus, establish $D_p$ in three cases.

1. Normally cracking of thin-layer units along the crossfeed surface

Since crossfeed surface cannot stand normal tension, the corresponding yield function is:

$$ F = \sigma_n - \sigma_i = 0 \quad \text{namely,} \quad F = \sigma_n = 0 $$

After a series of derivation, $D_p$ is obtained as:

$$ D_p = \frac{D'(\frac{\partial F}{\partial \sigma})^T D'}{\frac{\partial F}{\partial \sigma}} = \frac{1}{H_0} \begin{bmatrix} H_{x'y'}^2 & H_{x'y'} H_{x'z'} & H_{x'} H_{z'y'} \\ H_{y'z'} H_{x'z'} & H_{y'}^2 & H_{y'} H_{z'y'} \\ H_{z'} H_{x'z'} & H_{z'} H_{y'z'} & H_{z'}^2 \end{bmatrix} $$

Where, $H_0 = E S' / S$, $H_{x'} = E S' / S$ and $H_{y'z'} = H_{z'y'} = 0$.

2. Shear yield of thin-layer units along crossfeed surface

Directional Mohr-Coulomb criterion is used as yield criterion, and the yield function is:

$$ F = \tau_{max} - f \sigma_n - c = 0 $$

Where, $c$ and $f$ are cohesion and friction coefficient respectively; $\sigma_n$ and $\tau_{max}$ are normal stress and the tangential maximum shear stress respectively.

After substituting (7) into (6), the plastic matrix is obtained when shear yielding of thin-layer units along the crossfeed surface.

3. shear yield and normal rupture of crossfeed surface simultaneously

The yield function vector is:

$$ F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} \sigma_n - \sigma_i \\ \tau_{max} - f \sigma_n - c \end{bmatrix} = 0 $$

By the yield function vector in (13), corresponding elastic-plastic constitutive relation can be calculated. Detailed derivation of formulas is shown in references [3] and [4]. And use nominal the shear stiffness of equation (7) instead of shear stiffness.

4. Determination of cohesive of crossfeed surface

The change of cohesion $c$ of crossfeed surface should be considered of the contacted bodies, which is mainly determined by the rock bridge of the crossfeed surface. Thus, establish fracture mechanics model of the rock bridge with width of $2a$ as shown in Fig.1. From the reference [5] to [9], stress intensity factor of $II$ type is:

$$ K_{II} = \frac{2\tau W}{\sqrt{\pi a}} $$

where, the meaning of all symbols are shown in Fig.1.
Assume no friction along the fracture surface in equation (10). However, if considering the friction of fracture surfaces, using principle of superposition, it is:

\[ K_{II} = \frac{2(\tau - \sigma_n \tan \varphi)W}{\sqrt{\pi a}} \]  

(10)

where, \( \varphi \) is friction angle of the fracture surfaces.

The solution of equation (10) is not only suitable for a single fracture, but also for several rock bridge with total width of \( 2a \) the bridge are suitable (see Fig.1). Assume that the width of shear crack increased to the maximum when the stress intensity factor of break surface reaches fracture toughness \( K_{Ic} \). From (10) and the above assumptions, shear stress when fracture surface was destroyed is obtained:

\[ \tau = \frac{K_{Ic} \sqrt{\pi a}}{2W} + \sigma_n \tan \varphi \]  

(11)

Thus, the shear stress when fracture surface was destroyed consists of cohesion and friction. And consider that rock bridge cohesion has relations with fracture toughness, size of the rock bridge and the distance between the ends of two side seams of the bridge. Aiming at the crossfeed surface of contact bodies, and calculation formula of cohesion of the crossfeed surface can be obtained:

\[ c = \frac{K_{Ic} \sqrt{\pi a}}{2W} = \frac{K_{Ic} \sqrt{\pi a}}{2\sqrt{W}} = \frac{K_{Ic} \sqrt{\pi S'/S}}{2\sqrt{W}} = c_0 \sqrt{S'/S} \]  

(12)

where, \( S'/S \) is the ratio of the actual contact surface area to the nominal contact area crossfeed surface; \( c_0 \) is initial cohesion.

5. Calculation example

Taking photoelastic experimental data [10] of joint sample as an example, based on optimization theory of genetic algorithm, the actual contact area of the joint sample is inversed.

5.1. Introduction of photoelastic experiment [1]

Six samples are made of polycarbonate sheet, and photoelastic method is applied to simulate meshing type joints, shown in Fig.2. A common parallel light photoelasticimeter is used in experiment. A total of four sets of loading experiments [1] [10] are simulated. The first group is an axial compression experiment which are nine grade of normal pressures (from 0.3kN to 2.7kN), and the other three are pressure-shear coupling experiments under normal pressures maintained at 0.7kN, 1.3kN and 1.9kN respectively. Shear stress is determined by eight kinds of loads from 0.2kN to 1.6kN respectively.
Using dial indicator to measure normal and lateral displacement during loading process, size of the loads can be measured by pressure sensor. When shear slide of the upper and lower joints occurs, tangential displacement $u$ of the joints will be produced along the horizontal direction, as well as the normal displacement $v$ along the vertical direction. Normal displacement $v$ of the joint specimen during the process of normal pressure is shown in Table 1.

### Table 1 Experiment data and inversed ratio of crossfeed surface area of joint sample

| Normal Force/kN | 0.30 | 0.60 | 0.90 | 1.20 | 1.50 | 1.80 | 2.10 | 2.40 |
|-----------------|------|------|------|------|------|------|------|------|
| Normal Stress/MPa | 0.49 | 0.98 | 1.47 | 1.96 | 2.45 | 2.94 | 3.43 | 3.92 |
| Normal Displacement /mm | 0.191 | 0.239 | 0.300 | 0.343 | 0.375 | 0.424 | 0.449 | 0.472 |
| Maximum Shear Stress /MPa | 6.390 | 10.650 | 13.845 | 14.910 | 18.105 | 19.170 | 21.300 | 23.430 |
| Area Ration of Inversion | 0.250 | 0.385 | 0.520 | 0.660 | 0.730 | 0.782 | 0.796 | 0.811 |

### 5.2. Inversion of actual contact area of joint surface

Axial compression experiment is taken as an example to inverse the joint surfaces actual contact area, and joint direction is simulated by finite element meshes. Boundary conditions of the model consist of two parts. One is the top surface loads applied by jack, the other is to fix the points on a boundary, which is between polycarbonate sheet and bottom slide in loaded photo elastic experiment system. Polycarbonate sheets on both sides of joints specimen are isoparametric element unit of eight-nodes hexahedral, and joints is improved thin-layer element, with a total of 1004 units (see Fig.3). Elastic modulus of polycarbonate sheet is 2.25GPa, with both compressive strength and tensile strength of 65.00MPa, and Poisson's ratio of 0.38; the elastic modulus and Poisson's ratio of joints are 1.315GPa and 0.38, respectively.

### 5.3. Summary

By use of the finite element model above, stress field and displacement field joint specimen can be calculated. The data of axial compression experimental are shown in Table 1. Relationship curve of the corresponding standard load (ration of normal stress to the maximum normal stress) and area ratio is shown in Fig.4. Table 1 and Fig.4 show that: (1) the increase in normal loads leads to the increases in the contact area ratio. (2) Ratio of contact area of crossfeed surface when normal force of 2.4kN to that when normal force of 0.3kN is more than 3.0. Therefore, considering the actual contact area of crossfeed surface under loads is meaningful. (3) When the ratio of crossfeed surface area increases to a certain
extent, with increase in same load, the increase in area ratio gets smaller. This is because new contact points of contact surface increased rapidly during initially loading, but after a certain extent, plastic flow of asperities near some contacts points has occurred, which leads to merger of contact points and make the contact area tend to stable.

![Normalized load VS. inversed ratio of crossfeed surface area](Fig.4)

The actual contact area of crossfeed surface is obtained by inversion above. And analysis and calculation of the stress field of joint sample under the normal stress of 1.8kN by finite element model (Fig.3) is shown in Fig.5. The stipulation of symbols is as follows: stress in tension is positive, and it in compression is negative. Fig.5 shows that: (1) nephogram of stress field and photoelastic stress fringe pattern in reference [1] are very similar, especially at the joints of specimen. It presents that inversion of actual contact area of crossfeed surface by measured data is viable. (2) The maximum vertical displacement of the specimen obtained by calculation is 0.4244mm, which fits to the normal displacement of 0.424mm by experimental observation. The result shows the method is accurate.

![Stress field of joint sample](Fig.5)
6. Conclusion

Based on the Hertz contact elasticity theory and elastic-plastic theory, the crossfeed mechanical model, in which the constitutive relation under the influence of the real contact area is considered, is created. Based on the fracture mechanics model, the cohesion of the crossfeed surface varied with the contact areas is acquired. Compared with the photo-elastic experiment of the joint sample, the result shows that the crossfeed mechanical model of the contact bodies is more practical than the traditional model.

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