Light Goldstone boson and domain walls in the $K^0$-condensed phase of high density quark matter

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Abstract

It is pointed out that $K^0$ condensation in high density matter gives rise to an extremely light Goldstone boson whose mass comes entirely from weak interactions. This implies the existence of metastable non-topological domain walls with a long lifetime. We comment on the mass of the superfluid mode if baryon number is violated.

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Recently, many authors [1–3] emphasize the likelihood of kaon condensation in the color-flavor locked (CFL) [4] phase of QCD at high baryon densities. The crucial observation is that kaons have small masses at high densities [5]. A relatively small strangeness chemical potential is thus sufficient to drive kaon condensation. Moreover, it was argued that the mass of the strange quark also works in favor of kaon condensation [2].

In contrast to the conventional charge kaon condensation in nuclear matter [6], in the CFL phase it is the neutral kaons which are more likely to condense. This is due to the inverse mass ordering [5] of mesons in the CFL phase, which makes the neutral kaons lighter than the charge kaons, at least at very high densities. The fact that neutral kaons rather than charge kaons condense might have important astrophysical consequences, since it implies that the kaon-condensed phase does not require electrons to be electrically neutral, which is one of the properties of the pure CFL phase without kaon condensation [7]. In this paper, we show that the $K^0$-condensed phase possesses another distinct feature: it has in its spectrum an extremely light bosonic particle, whose presence implies the existence of non-topological metastable domain walls.

Let us first review the symmetry arguments underlying this feature. Like most Bose-Einstein condensates, the $K^0$ condensate spontaneously breaks a global U(1) symmetry. The choice of the broken generator is not unique: one can always add an unbroken generator to a broken one. The simplest operator is strangeness,

$$ S = \int dx \bar{s} \gamma^0 s. $$

Since $K^0$ carries a strange charge, its condensation breaks the corresponding symmetry. From Goldstone’s theorem, one expects a Goldstone boson to appear in the spectrum\(^1\). This boson is the U(1)\(_S\) phase of the condensate, which will be denoted as $\varphi$. In addition, the system inherits the spontaneous breaking of the baryon U(1) symmetry from the CFL quark pairings. It might appear that the $K^0$-condensed phase is a two-component superfluid, the dynamics of which is determined by two U(1) phases.

A closer examination reveals that the particle arising from the U(1)\(_S\) breaking is in fact only a pseudo-Goldstone boson. Although strangeness is an exact symmetry of QCD, it is violated by weak processes, hence the would-be Goldstone boson acquires a mass. Since this mass comes entirely from weak interactions, it must be very small, much smaller than other hadronic masses in the theory. It is proportional to the square root of the Fermi constant $G_F$, in the same way as the pion mass (in vacuum) is proportional to the square roots of the quark masses which violate the conservation of the axial currents.

The existence of a light Goldstone boson in the spectrum leads to the appearance of metastable domain walls. Indeed, at very low energies, the system can be described in term

\(^1\)Here and after, by “kaons” we mean the quasiparticle excitations of the CFL phase which carry the same quantum numbers as the kaons in vacuum.

\(^2\)If the isospin symmetry was exact, there would be two Goldstone bosons, one with a linear dispersion relation and another with a quadratic dispersion relation [8,9]. In this paper we will consider the realistic case when the isospin symmetry is not exact.
of the variable \( \varphi \) alone [if one freezes the U(1) baryon phase]. The effective Lagrangian for \( \varphi \) must have the form

\[
L = \frac{f^2}{2} \left[ (\partial_0 \varphi)^2 - u^2 (\partial_i \varphi)^2 \right] - V(\varphi),
\]

where \( f \) is the decay constant of the boson (which of order \( \mu \)), \( u \) is its velocity, and \( V(\varphi) \) comes entirely from the explicit violation of strangeness by weak interactions. Due to the nature of \( \varphi \) as a phase variable, \( V(\varphi) \) is required to be a periodic function of \( \varphi \). Moreover, to leading order in \( G_F \), \( V(\varphi) \sim \cos \varphi \). The simplest way to see that is to express the superfluid ground state with a definite value of \( \varphi \) as a superposition of states with definite values of strangeness,

\[
|\varphi\rangle = \sum e^{iS\varphi} |S\rangle.
\]

To leading order in perturbation theory, the energy shift of the state \( |\varphi\rangle \) caused by an interaction Hamiltonian \( H_\text{int} \) is \( \langle \varphi | H_\text{int} | \varphi \rangle \). The Hamiltonian of weak interactions has only \( \Delta S = 1 \) matrix elements, so \( V(\varphi) \) is proportional to \( \cos \varphi \), without higher harmonics.

The Lagrangian (2) now becomes that of the sine-Gordon model,

\[
L = \frac{f^2}{2} \left[ (\partial_0 \varphi)^2 - u^2 (\partial_i \varphi)^2 \right] + f^2 m^2 \cos \varphi,
\]

where the coefficient in front of the \( \cos \varphi \) term was written in such a way that \( m \) is the mass of the Goldstone boson. It is well known that the sine-Gordon theory possesses a domain wall solution, which interpolates between \( \varphi = 0 \) and \( \varphi = 2\pi \),

\[
\varphi(z) = 4 \arctan e^{mz/u}.
\]

Since the two values \( \varphi(z = -\infty) = 0 \) and \( \varphi(z = +\infty) = 2\pi \) actually refer to the same ground state, this domain wall is non-topological and, in principle, can decay, but its lifetime may be very long. The domain wall here is an exact replication, under different physical circumstances, of the axion domain wall [10], the U(1)\(_A\) domain wall of high-density QCD [11], and the domain wall of two-component Bose-Einstein condensates of atomic gases [12]. Like in all other cases, the appearance of the domain wall is deeply rooted in the spontaneous breaking of an approximate U(1) symmetry.

The whole discussion above is based solely on symmetry arguments and is independent of all details about the dynamics. Therefore, we should expect the light boson and the domain wall to be very robust consequences of the \( K^0 \) condensation. We now turn to the calculation of the mass of the Goldstone boson in the kaon condensed phase, from which we can extract the tension of the domain wall and its lifetime.

The strangeness-violating piece of the four-fermion effective Lagrangian for weak interactions is

\[
L = \frac{4G_F}{\sqrt{2}} \cos \theta_c \sin \theta_c (\bar{s}_L \gamma^\mu u_L)(\bar{u}_L \gamma_\mu d_L) + \text{H.c.}
\]

In order to generate a mass for the Goldstone boson we need an effective interaction of the type \( \bar{s}_R \bar{u}_R u_L d_L \) and \( \bar{s}_L \bar{u}_L u_R d_R \) (and their complex conjugates): these terms, when averaged
over the kaon-condensed state, lifts the degeneracy of states with different \( \varphi \) (see below). Thus one needs to transform two left-handed quarks into right-handed ones. This can be done using two mass insertions. One can put the mass insertions in two different ways as in Fig. 1. When all external lines are on mass shells, each of the mass insertions introduces a factor of \( \gamma_0 m/2E \), where \( E \) is the energy flowing along the line where the mass insertion is made. Since we will be interested in the situation where all external momenta near the Fermi surface, we can replace \( E \) by \( \mu \). Thus we arrive to the following effective interaction,

\[
L_{\text{int}} = \frac{G_F}{\sqrt{2} \mu^2} \cos \theta_c \sin \theta_c \left[ m_u m_s (\bar{s}_R \gamma^0 \gamma^\mu u_L)(\bar{u}_R \gamma^0 \gamma_\mu d_L)
+ m_u m_d (\bar{s}_L \gamma^0 \gamma^\mu u_R)(\bar{u}_L \gamma^0 \gamma_\mu d_R) \right] + \text{H.c.} \tag{7}
\]

![FIG. 1. The effective vertices responsible for the mass of the Goldstone boson](image)

For realistic quark masses, the second term in the square bracket of Eq. (7) can be neglected compared to the first term. To compute the average of \( L_{\text{int}} \) we need to be more specific about the ground state. The order parameters of the CFL phase of QCD are two \( 3 \times 3 \) matrices \( X \) and \( Y \), defined as \[13,5\],

\[
\langle q_{ai}^{\alpha \gamma} q_{bj}^{\beta \gamma} \rangle^* = \epsilon_{\alpha \beta} \epsilon^{abc} \epsilon_{ijk} X_{ai}^{\gamma}, \quad \langle q_{ai}^{\alpha \gamma} q_{bj}^{\beta \gamma} \rangle^* = \epsilon_{\dot{\alpha} \dot{\beta}} \epsilon^{abc} \epsilon_{ijk} Y_{ai}^{\gamma}, \tag{8}
\]

where \( a, b, c = 1, 2, 3 \) are color indices, \( i, i, k = u, d, s \) is flavor indices, and \( \alpha, \beta, \dot{\alpha}, \dot{\beta} = 1, 2 \) are Dirac spinor indices. The color-neutral order parameter describing the breaking of chiral symmetry is the unitary matrix \( \Sigma \sim X^1 Y \). The CFL ground state without kaon condensate corresponds to \( \Sigma = 1 \), while the \( K^0 \) condensate state corresponds to

\[
\Sigma = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta & \sin \theta e^{i \varphi} \\
0 & -\sin \theta e^{-i \varphi} & \cos \theta
\end{pmatrix} \tag{9}
\]

In Eq. (9), \( \theta \) is the parameter characterizing the strength of kaon condensation (relative to chiral symmetry breaking), which is fixed by the strangeness chemical potential and the kaon mass, and \( \varphi \) is our Goldstone phase variable which remains undetermined if only strong interactions are taken into account. We will limit ourselves to the case of maximal \( K^0 \) condensation where \( \theta = \pi/2 \). This is achieved, for example, when the strangeness chemical potential (or the effective chemical potential induced by the strange quark mass
is much larger than the kaon mass. The knowledge of $\Sigma$ does not fix $X$ and $Y$ due to the freedom in performing a color rotation. Since physical results should not depend on the particular choice of $X$ and $Y$, we can choose, for the simplicity of calculations, $X^{ai} = \delta^{ai}|X|$, $Y^{ai} = \Sigma^{ai}|X|$. With this choice, the $u_L d_L$ diquark and the $d_R s_R$ diquark are of the third color,

$$
\langle u_L^a d_{Lb}^b \rangle = \epsilon_{\alpha\beta} \epsilon^{ab\beta}|X|, \quad \langle u_R^a s_{R\dot{\beta}}^\dot{\beta} \rangle = \epsilon_{\dot{\alpha}\dot{\beta}} \epsilon^{a\dot{\alpha}\dot{\beta}} e^{i\phi}|X|.
$$

Taking the average of $L_{\text{int}}$ over the kaon-condensed state, we find the potential term in the effective Lagrangian (2),

$$
V(\varphi) = -16\sqrt{2}G_F \cos \theta_c \sin \theta_c \frac{m_u m_s}{\mu^2} |X|^2 \cos \varphi.
$$

In the CFL phase at asymptotically high densities [11],

$$
|X| = \frac{3}{2\sqrt{2}} \frac{\mu^2 \Delta}{g},
$$

where $\mu$ is the chemical potential, $\Delta$ is the Bardeen-Cooper-Schrieffer gap (more precisely, the smaller one) of color superconductivity. Substituting Eq. (12) to Eq. (11), we find

$$
V(\varphi) = -\frac{18\sqrt{2}}{g^2} G_F \cos \theta_c \sin \theta_c m_u m_s \mu^2 \Delta^2 \cos \varphi.
$$

By comparing the kinetic term of Eq. (2) with that of the non-linear Lagrangian for $\Sigma$, one finds that, for maximal $K^0$ condensation, $f$ is equal to the decay constant of the pseudoscalar mesons, which has been computed in Ref. [5],

$$
f^2 = \frac{21 - 8 \ln 2}{18} \frac{\mu^2}{2\pi^2}.
$$

The velocity $u$ is also equal to that of pseudoscalar mesons in the pure CFL phase, which is $1/\sqrt{3}$ [9]. From Eqs. (13) and (14), one find the mass of the Goldstone boson,

$$
m^2 = \frac{162\sqrt{2} \pi}{21 - 8 \ln 2} \frac{G_F}{\alpha_s} \cos \theta_c \sin \theta_c m_u m_s \Delta^2.
$$

If one takes into account the second diagram (Fig. [1b]), the factor $m_s$ in Eq. (13) will be replaced by $(m_s + m_d)$.

That the Goldstone boson becomes massless if $m_d = m_s = 0$ can be seen from a symmetry argument: the $K^0$ condensate is not neutral under the following charge,

$$
Q_{s-d}^{R} = \int d\mathbf{x} (\bar{s}_R \gamma^0 s_R - \bar{d}_R \gamma^0 d_R),
$$

which is an exact symmetry of both strong and weak interactions if $m_d = m_s = 0$. Goldstone’s theorem now guarantees the vanishing mass.

The origin of the factor $m_u$ in the mass formula is less clear: there is no symmetry which implies that the Goldstone boson should become massless when $m_u = 0$. Indeed, if one
takes non-perturbative effects into account, there is an instanton-induced effective vertex \( \bar{u}_R d_R u_L d_L \) which can replace the two mass insertions in Fig. 1b. This instanton contribution is proportional to \( m_s \) and does not vanish when \( m_u \) goes to 0. However, since the density of instantons quickly drops as the baryon chemical potential is increased, their effects are likely to be small.

None of the uncertainties in our calculations should change the qualitative conclusion that the mass of the boson is very small, which is due to the presence of the Fermi constant \( G_F \) in the mass formula (15). Quantitatively, if we substitute to Eq. (15) the numerical values \( \alpha_s = 0.3 \), \( m_u = 4 \) MeV, \( m_s = 150 \) MeV, and \( \Delta = 100 \) MeV, we obtain the estimate \( m \sim 50 \) keV. It is instructive to compare \( m \) with the kaon masses found in Ref. [5] for the pure CFL phase without kaon condensation. The ratio between the Goldstone boson mass to the kaon masses in the pure CFL phase is

\[
\frac{m^2}{m_K^2} = \mathcal{O}\left(\frac{G_F \mu^2}{\alpha_s}\right) \ll 1. \tag{17}
\]

The large separation of mass scales between the Goldstone boson and the other mesons in the theory is responsible for the existence and metastability of the domain wall. The situation is mathematically similar to the cases previously considered in Ref. [10,11] so we will present here only the final formulas, and refer the reader to the literature for the details of the calculations. The domain wall tension can be computed from its profile (5),

\[
\sigma = 8u f^2 m. \tag{18}
\]

The spontaneous decay of the domain wall occurs via hole nucleation, with a rate suppressed by the exponent of the corresponding bounce solution [10,11]

\[
\Gamma \sim \exp\left(-\frac{16\pi}{3} \frac{\nu^3}{u \sigma^2}\right), \tag{19}
\]

where \( \nu \) is the tension of the vortex line which is the boundary of the nucleated hole. Its value is

\[
\nu = \pi u^2 f^2 \ln \frac{m_K}{m}, \tag{20}
\]

where by \( m_K \) we denote the mass scale of other pseudoscalar mesons in the theory. Thus

\[
\Gamma \sim \exp\left(-\frac{\pi^4 u^3}{12} \frac{f^2}{m^2} \ln^3 \frac{m_K}{m}\right). \tag{21}
\]

Due to the large ratio \( f^2/m^2 \sim \mu^2/m^2 \), the exponent in \( \Gamma \) is huge, so the domain wall is practically stable at zero temperature with respect to hole nucleation.

In conclusion, we have shown that the \( K^0 \)-condensed state of high density matter possesses a unique type of Goldstone boson which owes its existence to the dynamics of strong interactions, but its mass to weak interactions. The tiny mass of such a boson gives rise to non-topological domain walls. A network of such domain walls, in principle, can be formed when the core of a neutron star (if it is to form a \( K^0 \) condensate) cools to temperatures below the kaon condensation temperature. It would be interesting to explore the implications of the presence of these domain walls in the cores of neutron stars.
As a consequence of the nonzero mass of the Goldstone boson considered in this paper, the only true gapless mode in the $K^0$-condensed phase is the baryon $U(1)$ phase. Thus, as far as the macroscopic behavior is concerned, both the CFL phases with and without $K^0$ condensate are one-component superfluids at zero temperature. Only at distance scales smaller than the $m^{-1}$ scale ($\sim 10^{-9}$ cm which is large compared to the inter-quark spacing but still microscopic) does the $K^0$-condensed system behave like a two-component superfluid.

Finally, let us make a comment on the possibility of baryon number violation. In this case, the corresponding superfluid Goldstone mode also acquires a mass. Since the superfluid order parameter, in both nuclear matter and quark matter, is a dibaryon ($nn$ in neutron superfluid or $qqqqqq$ in the CFL phase), the mass square of the superfluid Goldstone boson is proportional to the amplitudes of the $\Delta B = 2$ processes, but not the $\Delta B = 1$ ones. This can be seen by making an expansion similar to Eq. (3). The $\Delta B = 2$ interactions lead to neutron-antineutron oscillations [14]. We thus have the following crude estimate for the mass of the superfluid mode,

$$m^2 \sim m_P^3 \tau_{n\leftrightarrow\bar{n}},$$

where $\tau_{n\leftrightarrow\bar{n}}$ is the characteristic time scale for $n\bar{n}$ oscillations, and the proton mass $m_P$ was inserted for dimensionality. Using the experimental bound on $n\bar{n}$ oscillations, $\tau_{n\leftrightarrow\bar{n}} > 10^8$ s [15], we find $m < 10^{-7}$ eV. The thickness of the corresponding domain wall is larger than about 1 m, and still might be less than the radius of neutron stars. However, unless the neutron star under consideration rotates very slowly, a domain wall that thick is unlikely to exist because of the high density of vortices (the mean distance between the vortices is typically $10^{-2}$ cm [14]). In most grand unified theories, the dominant baryon violating processes are $\Delta B = 1$. In these theories, the amplitudes of $\Delta B = 2$ processes are suppressed by $1/M_X^4$, where $M_X$ is at the GUT scale. This makes the mass of the superfluid mode very small, $m^2 \lesssim m_P^6/m_X^4$. For $M_X \sim 10^{16}$ GeV, the Compton wavelength of the superfluid mode is as large as 1 pc, so for all practical purposes it can be considered massless.

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REFERENCES

[1] T. Schäfer, Phys. Rev. Lett. 85, 5531 (2000).
[2] P.F. Bedaque and T. Schäfer, hep-ph/0105150.
[3] D.B. Kaplan and S. Reddy, hep-ph/0107265.
[4] M.G. Alford, K. Rajagopal and F. Wilczek, Nucl. Phys. B 537, 443 (1999).
[5] D.T. Son and M.A. Stephanov, Phys. Rev. D 61, 074012 (2000); ibid. 62, 059902(E) (2000).
[6] D.B. Kaplan and A.E. Nelson, Phys. Lett. B 175, 57 (1986).
[7] K. Rajagopal and F. Wilczek, Phys. Rev. Lett. 86, 3492 (2001).
[8] V.A. Miransky and I.A. Shovkovy, hep-ph/0108178.
[9] T. Schäfer, D.T. Son, M.A. Stephanov, D. Toublan, and J.J.M. Verbaarschot, hep-ph/0108210.
[10] A. Vilenkin and E.P.S. Shellard, Cosmic Strings and Other Topological Defects (Cambridge University Press, Cambridge, UK, 1994).
[11] D.T. Son, M.A. Stephanov, and A.R. Zhitnitsky, Phys. Rev. Lett. 86, 3955 (2001).
[12] D.T. Son and M.A. Stephanov, cond-mat/0103451.
[13] R. Casalbuoni and R. Gatto, Phys. Lett. B 464, 111 (1999).
[14] R.N. Mohapatra and R.E. Marshak, Phys. Rev. Lett. 44, 1316 (1980) [Erratum-ibid. 44, 1643 (1980)]; Phys. Lett. B 94, 183 (1980); L.N. Chang and N.P. Chang, Phys. Lett. B 92, 103 (1980).
[15] D.E. Groom et al. [Particle Data Group Collaboration], Eur. Phys. J. C 15, 1 (2000).
[16] S.L. Shapiro and S.A. Teukolsky, Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects (Wiley, New York, 1983).