MODELLING AND CONTROL OF THE QBALL X4 QUADROTOR SYSTEM BASED ON PID AND FUZZY LOGIC STRUCTURE

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Abstract. This work focuses on a quadrocopter model, which was developed by Quanser™ and named as Qball X4. First, mathematical model of the Qball X4 is obtained. Then, a conventional PID control technique is presented. This PID control parameters come from Qball user manual. After the presentation of conventional PID control, as an extension of the conventional PID control theory, a different fuzzy controller structure is given. The proposed fuzzy controller structure is based on fuzzy logic and its name is PID type fuzzy controller. All of the simulations are done in MATLAB™ environment.

1. Introduction
Unmanned Aerial Vehicles(UAV) have gained a high level of popularity during the last decade in civilian, military and engineering applications because of the recent advances in sensing, communicating, computing and controlling technologies. UAVs have some advantages than the manned systems like longer endurance, lower cost, maneuverability and less risk to humanity. As an example of UAV systems, the quadrocopter is a relatively simple, affordable and easy to fly system thus it has been widely used to develop, implement and test-fly methods in control. A quadrotor is an aircraft that becomes airborne due to the lift force provided by four rotors usually mounted in cross configuration, hence its name. In this study, Qball-X4 which is constructed and developed by the Quanser is used. This system is a test platform for several unmanned aerial vehicle research applications.

In the literature, there are numerous studies discussing Qball X4 system or fuzzy controller structure. Some of them are given as following: Sadeghzadeh, Chamseddine, Mehta and Zhang [1] proposed a Gain-Scheduled PID control technique which was developed from Active Fault-Tolerant Control (AFTC) and applied to Qball X4 quadrotor system. In this paper, a Fault Detection and Diagnosis (FDD) block is implemented to find and identify the actuator fault. Chamseddine, Zhang, Rabbath, Fulford and Apkarian [2] worked on actuator fault-tolerant control (FTC) for Qball-X4. Their strategy is based on Model Reference Adaptive Control (MRAC). Three different MRAC techniques have been implemented and compared with Linear Quadratic Regulator (LQR) controller. These three different methods are the MIT rule MRAC, the conventional MRAC and the modified MRAC. Nia Maharani Raharja, Iswanto, Muhammad Faris and Adha Imam Cahyadi [3] worked on hover position quadrotor control with fuzzy logic. In
In this work, fuzzy logic controller algorithm has been proposed for hovering mode. The input of the controller is height and change of height. Membership function of the inputs are triangular. Demet Canpolat Tosun, Yasemin Isik and Hakan Korul works on a Qball X4 system with both PID and LQR controller. The LQR and PID controls applied on height, x and y positions and roll, pitch, yaw angles. [4]Ridvan Ozdemir, Mustafa Kaya, Monier Elfarra, Mehmet Onder Efe[5] work on a Qball X4 through analyzing the object following. Regional surveillance for object tracking is done using micro cameras. Communication between camera and computer are done by using the RF system. Indoor autonomous flight of the aerial vehicle is provided by software which is written in MATLAB\textsuperscript{TM} Simulink\textsuperscript{TM} environment. Moghadam[6] addressed the problem of Fault-Tolerant Control (FTC) of the Qball-X4 quadrotor in his thesis. Both actuator loss of effectiveness and sensor bias faults and their impacts on system response were considered. After Fault Detection and Diagnosis (FDD) of actuators, a leader-follower controller with dynamic reference input was given to counteract the actuator faults effects. Sensor FDD was followed by a new approach of Active Fault-Tolerant Control (AFTC) to correct faulty measured values and feed them back to the controller. Also Two-Stage Kalman Filter (TSKF) is utilized to estimate noisy and unmeasured states and realize the actuator FDD and sensor AFTC.

In our study, first, dynamic model of the Qball-X4 system has been revealed. Then the PID control technique has been discussed for the Qball-X4 system. This PID control parameters adopted from Qball user manual in to the simulink environment. Later, PID type fuzzy controller has been proposed. Finally, results of the simulations of classical PID controller, PID type fuzzy controller are shown and compared to each other.

2. Dynamical Model of Qball X4
In this part of the paper, the dynamic model of the Qball-X4 is described. Nonlinear model of Qball X4 is discussed so that different control algorithms can be applied on it. Roll, pitch and yaw angles of the system are defined as the rotation about x, y and z axis.

![Figure 1. Qball-X4 axes and sign convention.][7]

2.1. Actuator Dynamics
Thrust force is modeled using the following first-order[5] system. That thrust force generated by each of the propeller.

\[ F = K \frac{\omega}{S + \omega} u \]  \hspace{1cm} (1)

\( u \): PWM input to the actuator
\( \omega \): Actuator Bandwidth
\( K \): Positive Gain
\( v \) is the state variable and will be used for the representation of the actuator dynamics. And it is defined as follows:

\[
v = \frac{\omega}{s + \omega}
\]  

(2)

2.2. Roll/Pitch Model

When we assume the rotation about x and y axes are decoupled, we can model roll/pitch motion as follows:

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{roll_pitch_axis.png}
\caption{A model of the roll/pitch axis.[7]}
\end{figure}

Also, roll/pitch angle(\( \theta \)), can be found from the following equations:

\[
J \ddot{\theta} = \Delta FL
\]  

(3)

Where \( J \) equals to \( J_{roll} = J_{pitch} \). These are the rotational inertia of the device for both roll and pitch axes.

L: distance between the propeller and the center of gravity

\[
\Delta F = \Delta F_1 - \Delta F_2
\]  

(4)

2.3. Height Model

Motion of the Qball-X4 along Z axis is affected by all of the propellers. Thus, the dynamic model of the height could be written as follows:

\[
M \ddot{Z} = 4F \cos r \cos p - Mg
\]  

(5)

\( F \): Thrust which is generated by each propeller

\( M \): Total mass of the device

\( Z \): Height

\( r \) and \( p \): roll and pitch angles

2.4. X-Y Position Model

Total thrust and the changing roll/pitch angles are the main reason for the motion of the Qball-X4 along the X and Y axes. If the yaw angle is zero then the dynamics of the motion in X and Y axes can be obtain as follows [7]:

\[
M \ddot{X} = 4F \sin p
\]  

(6)

\[
M \ddot{Y} = -4F \sin r
\]  

(7)
Height, x and y axis model can be connected like that:

![Diagram of Height, x and y axis model connection](image)

**Figure 3.** Height, x and y axis model connection.[8]

### 2.5. Yaw Model

The torque, $\tau$, has the following equation with respect to the PWM input, $u$

$$\tau = K_y u$$  \hspace{1cm} (8)

where $K_y$ is a positive gain. The difference between the torques exerted by the two clockwise and the two counter-clockwise rotating propellers results in the motion of the yaw axis.[7] Motion in the yaw axis could be described using the following equation:

$$J_y \ddot{\theta}_y = \Delta \tau$$  \hspace{1cm} (9)

$\theta_y$ : yaw angle

$J_y$ rotational inertia about the z axis.

Total torque of the motors, $\Delta \tau$ can be calculated from:

$$\Delta \tau = \tau_1 + \tau_2 - \tau_3 - \tau_4$$  \hspace{1cm} (10)
Table 1. System Parameters.

| Parameter | Value             |
|-----------|-------------------|
| $K$       | 120 N             |
| $\omega$ | 15 rad/sec        |
| $J_{roll}$| 0.03 kg.m$^2$     |
| $J_{pitch}$| 0.03 kg.m$^2$   |
| $M$       | 1.4 kg            |
| $K_y$     | 4 N.m             |
| $J_y$     | 0.032 kg.m$^2$    |
| $L$       | 0.2 m             |

3. PID Control

This classical PID control and how to apply the dynamical model of the Qball-X4 quadrotor system are shown in [8,9,10]. All of the parameters are adopted from the Qball-X4 user manual (Table 1) [7]. For example, for height model: $K_p = 0.011$, $K_d = 0.009$, $K_i = 0.007$. For x and y axis model: $K_p = 0.366$, $K_d = 0.382$, $K_i = 0.045$, for roll and pitch model: $K_p = 0.062$, $K_d = 0.013$, $K_i = 0.018$ and for yaw axis: $K_p = 0.013$, $K_d = 0.009$, $K_i = 0.010$. Although, system is analyzed for different values of parameters such as $K$, $J$, $M$ and $K_y$, $J_y$ and their responses, to be able to understand the system behavior if the parameters are uncertain.

4. Fuzzy Control

In this part, a different method has been applied to our Qball-X4 system. As an extension of the conventional PID control theory, a different fuzzy controller structure is applied. The name of the control structure is PID type fuzzy controller. Fuzzy controller structure is based on a fuzzy logic. Fuzzy logic is a logic, in which the truth variables can take any real number between 0 and 1. It is different than the boolean logic, because in boolean structure truth variables can take only 0 or 1. This control structure has just two inputs, and the rules base is two dimensions. Also, compared to fuzzy PI and fuzzy PD in general it is possible to enhance system performance using fuzzy PID. Fuzzy control design includes three important aspects. First one is, knowledge based design, second is control tuning parameters and the last one is membership functions. [8,9,10]

4.1. PID Type Fuzzy Controller Structure

A PID type fuzzy controller structure includes both PD type and PI type fuzzy controllers. And it is known that fuzzy PI type control is more practical than the fuzzy PD type. [8,9,10,11] Removing the steady state error is difficult for the fuzzy PD, and it is the main reason why PI type is more practical. Although, fuzzy PI type control does not provide adequate performance in transient response for high order process because of the internal integration operation. PID type fuzzy controller structure is using the error and the rate of change of error as it inputs. [8,9,10] PI and PD type fuzzy controllers connect in parallel and it shows in the Figure 4. And the rules are expressed like that, if $e$ is _ and $\dot{e}$ is _ then $u$ is __[6,7].
Figure 4. The PID Type fuzzy control structure.

Also, the output of the PID type fuzzy controller as follows:

\[ u_c = \alpha u + \beta \int u \, dt \]  \hspace{1cm} (11)

\[ u_c = \alpha \left( A + PK_e e + DK_d \dot{e} \right) + \beta \int \left( A + PK_e e + DK_d \dot{e} \right) dt \]  \hspace{1cm} (12)

\[ u_c = \alpha A + \beta At + (\alpha K_e P + \beta K_d D) e + \beta K_e P \int edt + \alpha DK_d \dot{e} \]  \hspace{1cm} (13)

And the control components are like[9,10]:
- Proportional: \( \alpha K_e P + \beta K_d D \)
- Integral: \( \beta K_e \)
- Derivative: \( \alpha DK_d \)

These control components show that how difficult and complex of the fuzzy logic control structure. For example \( \alpha \) parameter can be in both proportional and derivative terms. Parameter of \( \alpha, \beta, K_e \) and \( K_d \) are obtained as follows. First, using the genetic algorithm to find the parameter of \( \alpha, \beta, K_e \) and \( K_d \), provides a starting point for these parameters. In this method, MATLAB\textsuperscript{TM} genetic algorithm toolbox has been used. ISE function is minimized with using this toolbox. Then using the trial and error method, much better values of the parameters have been found. Membership functions of error, change rate of the error and u are shown in Figure 5.

Figure 5. Membership functions of \( e, \dot{e} \) and \( u \).
The fuzzy PID type control rule is shown in Figure 6[8,9,10]:

| E/E | NL | NM | NS | ZR | PS | PM | PL |
|-----|----|----|----|----|----|----|----|
| PL  | ZR | ps | pm | pl | pl | pl | pl |
| PM  | ns | ZR | ps | pm | pl | pl | pl |
| PS  | nm | ns | ZR | ps | pm | pl | pl |
| ZR  | nl | nm | ns | ZR | ps | pm | pl |
| NS  | nl | nl | nm | ns | ZR | ps | pm |
| NM  | nl | nl | nl | nm | ns | ZR | ps |
| NL  | nl | nl | nl | nl | nm | ns | ZR |

**Figure 6.** General fuzzy PID type rule base.

The surface of the error, change rate of the error and u are shown in Figure 7.

**Figure 7.** Surface of the $e$, $\dot{e}$ and $u$.

The output $u$ is determined by using the method of the center of gravity(in Figure 7).

**Figure 8.** Output $u$. 

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5. Comparison
In this part of paper, there is a detailed comparison of the controller structures. Firstly, height response of the system is given.[8,9]

![Figure 9](image1.png)

**Figure 9.** Response of Height for different controllers using K=120 gain.

| Table 2. Height Performance Comparison |
|----------------------------------------|
| Time Response                      | PID | PID type Fuzzy |
|-------------------------------------|-----|----------------|
| Overshoot                          | 57.937 % | 0.5 %       |
| Rising Time                        | 514.484 ms | 2.203 sec |
| Settling Time                      | 15.447 sec | 6.711 sec |

According to figure(9) and the table(2), PID type fuzzy controller structure better than classical PID structure. Because, with the PID type Fuzzy Controller, there is a negligible overshoot and the settling time is shortened(Figure 9)[8,9].

X and Y positions responses are given as follows. [8,9]

![Figure 10](image2.png)

**Figure 10.** Response of X position for different controllers using K=120 gain.
Table 3. X axis Performance Comparison

| Time Response | PID  | PID type Fuzzy |
|---------------|------|----------------|
| Overshoot     | 0.505 % | 0.497 %       |
| Rising Time   | 2.114 sec | 2.036 sec    |
| Settling Time | 7.862 sec | 6.726 sec    |

Figure 11. Response of Y position for different controllers using K=120 gain.

Table 4. Y axis Performance Comparison

| Time Response | PID  | PID type Fuzzy |
|---------------|------|----------------|
| Overshoot     | 0.505 % | 0.497 %       |
| Rising Time   | 2.093 sec | 2.036 sec    |
| Settling Time | 6.649 sec | 6.336 sec    |

With these figures(10,11) and tables(3,4), we can say that PID type fuzzy controller structure better than classical PID structure since with the PID type Fuzzy Controller, both overshoot and settling time are reduced(Figure 10, 11)[8,9].
Pitch and Roll angle responses are given as follows:

![Pitch Response Graph](image1)

**Figure 12.** Response of Pitch for different controllers using K=120 gain.

| Time Response   | PID     | PID type Fuzzy |
|-----------------|---------|----------------|
| Overshoot       | 11.798% | 0.5%           |
| Rising Time     | 242.04 ms | 2.143 sec     |
| Settling Time   | 15.731 sec | 8.810 sec     |

**Table 5.** Pitch Performance Comparison

![Roll Response Graph](image2)

**Figure 13.** Response of Roll for different controllers using K=120 gain.
Table 6. Roll Performance Comparison

| Time Response | PID | PID type Fuzzy |
|---------------|-----|----------------|
| Overshoot     | 11.798 % | 0.5 %          |
| Rising Time   | 257.211 ms | 2.143 sec     |
| Settling Time | 16.213 sec | 8.779 sec      |

According to these figures(12,13) and tables(5,6), if the PID type fuzzy controller structure is used, there is a negligible overshoot in the system. Moreover, the settling time is shortened(Figure 12, 13)[8,9].

Figure 14. Response of yaw for different Controllers using $J_y=0.1$ gain.

Table 7. Yaw Performance Comparison

| Time Response | PID      | PID type Fuzzy |
|---------------|----------|----------------|
| Overshoot     | 0.505 %  | 0.486 %        |
| Rising Time   | 856.946 ms | 1.046 sec     |
| Settling Time | 5.292 sec | 4.569 sec      |

With this figure(14) and table(7), we can say that PID type fuzzy controller structure better than classical PID structure. Because, with the PID type Fuzzy Controller, both overshoot and settling time are reduced.(Figure 13)[8,9]
6. Conclusion and Future Work
In this paper, firstly the dynamical model of the Qball-X4 system has been described. Non-linear model of the system have been given. After the examination of the dynamical model, PID control has been described. After the PID control part, fuzzy control part is presented. In this part, as an extension of the conventional PID control theory, a different fuzzy controller structure is described. The name of control structure is PID type fuzzy controller. Both of the controllers simulation results’ are shown for the position controls along x,y,z axis and roll, pitch yaw angles.

To improve further the performance of classical PID control structure, we proposed a fuzzy controller structure which is based on fuzzy logic and its name is PID type fuzzy controller. With this PID type fuzzy controller x,y,z positions and roll,pitch, yaw angles are controlled in MATLAB environment. The simulation results show that with the PID type fuzzy controller, there is not any overshoot and the settling time is shortened.[8,9]

In the future a different kind of structure will be suggested in which the proposed PID type fuzzy controller is developed. Also, a real time performance of the PID type fuzzy controller would be examined or fault tolerant control could be applied on Qball X4 quadrotor system.

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