The space observation systems' imagers computer modeling

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Abstract. The computer modeling of widely used in the space observation systems charge coupled devices matrix imagers with the time delay and integration regime is presented here. To suppress the aliasing problem of output images distortion, caused by the matrix discrete structure, the algorithm of every pixel conversion to 9 subpixels has been modeled. This algorithm performs the digital interpolation of output images and presents an approximation of the Kotel'nikov two dimensional low-pass filter. To study the effect of the imager's output signals processing the Modulation transfer function with a sinusoidal law of the matrix illumination had been used. The image motion influence on the imager's resolution had been modeled to estimate the Modulation transfer function anisotropy in the time delay and integration regime. The paper results open new ways to improve the space observation systems' resolution.

1. Introduction
A wide utilization of photosensitive charge coupled devices (CCD) matrix imagers in the space objects and Earth surface observation systems produced the revolution in astrophysics, astronomy and in the Earth surface sciences. As a recognition of the outstanding contribution in science and technology, the CCD inventors Boyle and Smith [1] had been awarded in 2009 by the Nobel prize in physics. A wide utilization of CCD matrices in smartphones and in web and CCTV cameras is favorable for a further perfection of the CCD technology. A steady progress in micro- and nano-electronics technology results in a constant growth of the matrix picture elements (pixels) number and the matrices' resolution and sensitivity improvements, thus providing improvement of the space observation systems.

The space observation systems now use not only CCD, but also photosensitive CMOS matrices [2]. Both types of matrices possess output signals distortions, resulting from the matrices' discrete structure. These distortions become essential at the image detail near the pixel dimension Δ. This phenomenon had been named as the "aliasing". For many observation systems just the image details at the resolution limit are of the greatest interest. For this reason, the methods directed towards the unwanted effects of discretization elimination are of a top importance.

1.1. The Kotel'nikov theorem for image processing
Great soviet scientist Kotel'nikov in 1933 formulated the theorem [3] about discrete temporal sampling of continuous signals with a limited spectrum, where he showed that the conversion with the sampling frequency two and more times greater than the processed signal maximal frequency provides the signal recovery with an absolute precision. It was supposed that both input and output signals pass through the low-pass filters with their cut-off frequencies equal to the maximal frequency of the signal and transfer coefficients of both filters are ideal. The filter's transfer coefficient ideality means: at frequencies under the cut-off frequency it is constant and at frequencies over the cut-off frequency it is zero.
This theorem in the World science is named after Nyquist, but he in the paper [4] from the year of 1928 discussed only the bandwidth, needed to transfer the pulsed signal, but not the continuous signal form recovery. 16 years after Kotel'nikov and independently on him Shannon came to a similar theorem [5]. The Kotel'nikov theorem is applicable not only to temporal discrete samplings, but also to a discrete two-Dimensional transfer of continuous images with a limited range of spatial frequencies. Petersen and Middleton extended this approach to N-Dimensional Euclidean Spaces [6].

1.2. The aliasing suppression problem
In the space observation systems the spatial spectrum of image is limited by diffraction and other image blurring factors in the optical system, but this blurring does not reproduce the ideal low-pass spatial filter characteristics. It leads to the first type of the aliasing. The absence or no ideality of the low-pass filter at the imager's output leads to another kind of the aliasing. Currently a large attention is paid to the aliasing suppression in smartphones, but their limited computational possibilities do not support complex anti-aliasing algorithms.

In this paper to approximate the two-Dimensional spatial low-pass filter the algorithm of nine subpixels generation for every original pixel, $A_{i}^{9}$, is considered. The complicated $A_{i}^{9}$ algorithm can be realized in the ground-based digital processing complex of the space observation system. Its computational power essentially overcomes the productivity of the on-board computers or smartphones. To estimate the final image distortion from the original one the computer modelling [7] of the image processing for total pass from the imager to the ground-based processor may be used.

1.3. The moving images processing problem
For space objects the photosensitive matrices produce signals frame-by-frame [8], but for detailed Earth surface observation the CCD matrices performing charges accumulation synchronously with the image movement are preferred. This regime had been named as the Time Delay and Integration (TDI) [9]. Here one more problem of the image discretization takes place: the image moves continuously, but the accumulated charges jump time-by-time. It results in some blurring of the output charges distribution at the moving imagers receiver (MIR) output. In this paper the effect of this blurring on the MIR Modulation transfer function (MTF) had been modeled.

2. The anti-aliasing $A_{i}^{9}$ algorithm modeling
The $A_{i}^{9}$ algorithm fulfills the next operations:
- Formation of the illumination distribution at the matrix surface;
- Formation of the pixel charges with an account of image blurring;
- Generation from every pixel charge nine subpixel signals basing on the supposed filter core;
- Formation of the total subpixels based output digital signal;
- Fitting of the filter core coefficients to come to an acceptable form of the total signal;
- Estimation of the Monomer transfer function (MTF) for sinusoidal gratings.

To realize the low-pass spatial frequencies filter for the $A_{i}^{9}$ algorithm the pixel charges distribution at the CCD matrix output undergoes convolution [10] with the filter core, shown at the figure 1. This operation multiplies the number of digital signals for 9 times: 3 by 3 subpixels for every pixel. By this operation the discrete interpolation [11] of the raw digital data from the space observation system may be performed to approximate the two dimensional low-pass filter. The filter core looks like a wavelet, but the sum of its coefficients is not equal to zero, as it is typical for a wavelet [12].

The first approximation for the filter core in agreement with the Kotel'nikov theorem was taken as the product of two functions: $\text{sinc}(x)$ and $\text{sinc}(y)$, both set on segments instead of infinite axes to have a limited number of the filter core elements. The limiting of the filter core area forced us to select new values for peripheral elements, figure 2, by computer modeling of the one dimension filter transfer characteristics.
2.1. The CCD matrix response to the sinusoidal illumination distribution

To estimate the Monomer transfer function the illumination waveform is sinusoidal with a half wave measured in a number of pixel sizes Δ. As a criterion to select the peripheral elements' parameters was the filtered signal quality: its proximity to the sinusoidal waveform and an absence of zones with negative values, figure 3. The figure 3 shows that at the grating stroke width over 1,5 pixel the aliasing noises do not exist, but at a lower stroke width they become noticeable, figure 4, though not so large as before filtration.

The residual aliasing noises at a small stroke width appear because here the spatial frequencies filter core width is limited by 4 Δ. Practically unlimited computational power of the ground based digital processing complex permits a wider core filters realization. In this case the multi-matrix structure of the moving images receiver (MIR) should not introduce unwanted image distortions at the edges of matrices. The TDI regime creates the ground based digital data base, which presents the total MIR as one enormous matrix.
2.2. The CCD matrix Modulation transfer function

In the literature there is an opinion that the CCD matrix MTF may be given by the Nyquist formula: 

\[
\text{MTF}(\nu) = \text{sinc}(\pi \nu \Delta).
\]

But for this formula to be correct every maximum and every minimum of \(I(x)\) should coincide with centers of some pixels. This condition may not be fulfilled for all spatial frequencies \(\nu\). When the illumination maximum or minimum shift from the center of pixel, the local charge distribution contrast changes, resulting in the aliasing. It is seen at the figure 4. So, we should estimate an average MTF value.

The limited filter core width influences also on the modeled imager's Modulation transfer function (MTF). To estimate the MTF we analyze the matrix output signal, named here as the charge \(Z\), dependence on the illumination value \(I\). So, basing on data, like used for figures 3 and 4, we build the \(Z(I)\) graph, containing many lines, which belong to separate half waves of \(I(x)\) and \(Z(x)\), figure 5.

The averaged for all half waves line is close to a straight line and its trend 

\[Z = k I + a\]

gives us coefficients \(k\) and \(a\), which permit to find the MTF by the formula

\[
\text{MTF}(2\nu \Delta) = \left(1 + \frac{2a}{k}\right)^{-1}.
\]

By repeating this procedure for different values of \(\Delta\), we can find the averaged MTF dependence on the spatial frequency \(\nu = 1/2\Delta\), figure 6.

![Figure 5](image1.png)

**Figure 5.** The filtered output signal \(Z\) (blue lines) dependence on the CCD matrix sinusoidal illumination \(I\) at the stroke width equal 1,4 \(\Delta\); The trend line \(Z_{av}(I)\) is shown by red.

![Figure 6](image2.png)

**Figure 6.** The averaged Modulation transfer function (blue line) in comparison with the Nyquist MTF \(= \text{sinc}(\pi \nu \Delta)\) (red line).

The figure 6 shows that at \(2 \nu \Delta < 0,4\) MTF \((2 \nu \Delta)\) is close to 1. It means in this spatial frequency domain the model conditions correspond to the Kotel'nikov theorem and the \(Z(x)\) distribution reproduces the \(I(x)\) distribution precisely. But at \(2 \nu \Delta > 0,4\) the MTF deviation from 1 grows. This deviation results from the limited area for the filter core. The MTF computer modeling permits to select optimal parameters of the two-dimensional filter, realized at the ground-based processing.

The figure 6 shows also a large value for the Nyquist MTF at \(2 \nu \Delta\) values close to 1. But this large value is useless because it is accompanied by large aliasing noise, seen at the figure 4.

3. The image movement influence on the TDI matrix imager MTF

The matrix TDI imager MTF differs for directions along and across the image movement. Along the image movement there is an image shift on the one tact electrode width. For the three tact TDI CCD there is the image blurring on one third of the pixel size. For this reason the author in 1974 put the 30% difference in pixel sizes of CCD matrices to be produced for space observation systems. But the computer modeling shows somewhat lower anisotropy of the MTF.

Considering in the \(A_{16}\) algorithm the charge accumulation in the CCD photosensitive element at the image sliding we can compare the TDI CCD MTF with the ordinary CCD MTF, figure 7. The figure 7 shows that the resolution along the image movement falls approximately on 12% for a pixel with a square area. To avoid the MTF anisotropy the pixel dimensions difference along and across the movement direction should be 12 instead of earlier estimated 30%. So, the MTF computer modeling helps to optimize the construction of the TDI CCD matrix.
Figure 7. The three stage TDI matrix MTF along the image movement direction (red line) and in the transversal direction (blue line) for a square form pixel.

The synchronous accumulation of the moving images energy in TDI-matrices opens the way to improve the MIR resolution by utilization of the multi-sectional matrices with a sub-pixel shifts between sections [13]. The algorithm similar to the A19 may be used to improve the final image quality after signals summation. The computer modeling permits to estimate the effect of signals from different sections summation [14].

4. Conclusion
The mathematical modeling of the image digital processing in the space observation system with the TDI CCD matrix imager leads not only to practical recommendations, but also to new theoretical vision of this important technology principles of work, such as:

- the total chain of the image processing improvement by the ground based digital processing of data delivered by the space observation system;
- the MTF and aliasing nature for the matrix CCD imagers;
- the possible ways for the space observation systems’ resolution improvement.

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