First Order Superfluid to Bose Metal Transition in Systems with Resonant Pairing

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Systems showing resonant superfluidity, driven by an exchange coupling of strength \(g\) between uncorrelated pairs of itinerant fermions and tightly bound ones, undergo a first order phase transition as \(g\) increases beyond some critical value \(g_c\). The superfluid phase for \(g \leq g_c\) is characterized by a gap in the fermionic single particle spectrum and an acoustic sound-wave like collective mode of the bosonic resonating fermion pairs inside this gap. For \(g > g_c\) this state gives way to a phase uncorrelated bosonic liquid with a \(g^2\) spectrum.

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Introduction. The issue of the crossover between a BCS type superfluid and a Bose Einstein condensation (BEC) of tightly bound fermion pairs has received increasing attention ever since it became feasible to experimentally control such a crossover in fermionic atomic gases in optical traps and lattices\(^1\). From the theoretical side, such a crossover had until then been mainly studied for single component fermionic systems with static, short-range attractive inter-particle interaction\(^2\). It became clear that in these systems one is confronted with a unitarity limit problem, where the scattering length abruptly changes from positive to negative infinity. Numerical as well as analytical results\(^3\) suggest that around such a unitarity point, one should expect, effectively, two kinds of quasi-particles coexisting with each other: itinerant fermions and tightly bound fermion pairs behaving as bosons.

Such a situation had been anticipated in connexion with the physics in electron-lattice coupled systems in a regime between weak and strong coupling and resulted in the proposition of the boson-fermion model (BFM)\(^4\). The essence of the BFM scenario is to assume itinerant fermions and localized bosons (made up of two tightly bound fermions) - the two species which represent the physics of the extreme weak and strong coupling limits of such systems. The crossover behavior between those two limiting regimes is then parametrized by an exchange coupling of strength \(g\) between the two species. This scenario can be justified for: (i) fermionic atomic gases, where pairs of fermions exist in different electron-nuclear spin configurations, favoring or disfavoring binding between them (magnetic field tunable Feshbach resonance)\(^5\), (ii) hole pairing in strongly correlated systems, showing plaquette RVB states\(^6\) and (iii) resonating bipolarons in moderately strongly coupled electron-lattice systems\(^7\), for which this scenario was anticipated in the first place\(^4\).

Contrary to fermionic systems with attractive inter-particle interaction, where the ground state is always superfluid, the BFM scenario shows a superfluid (amplitude driven) BCS-like state for weak coupling, while for strong coupling it represents a liquid of spatially phase uncorrelated bonding pairs: 

\[
\frac{1}{\sqrt{2}}(u_i c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger + v_i b_i^\dagger).
\]

\(u_i = |u_i|e^{i\phi_i}/\sqrt{2}, v_i = |v_i|e^{-i\phi_i}/\sqrt{2}\) (with \(|u_i|^2 + |v_i|^2 = 1\)) denote the local amplitudes and phases of the two components, and \(c_{i\sigma}^\dagger\), respectively \(b_i^\dagger\), the creation operators of the itinerant fermions (with spin \(\sigma\)) and of the localized bosonic tightly bound fermion pairs at site \(i\). The emergence of a macroscopic superfluid state requires the onset of long-range phase locking of such bonding pairs. This needs a concomitant weakening of the local phase correlation of the bonding pairs\(^8\). It is this competition between local and global phase locking which is at the origin of a phase transition between a superfluid state for \(g \leq g_c\) and a state of spatially phase uncorrelated bonding pairs for \(g > g_c\). In this Letter, we argue that this transition is of first order.

The exact nature of that phase uncorrelated state is presently not yet fully understood. For very large values of \(g\), it eventually must be an insulator. Here, we show that close to \(g_c\) it constitutes a Bose Metal made out of such bonding pairs and present the first study of the excitation spectra expected for such a BFM scenario, which treats the fermions and bosons on equal footing as we vary the exchange coupling \(g\). This will be done on the basis of a renormalization procedure for Hamiltonians\(^9, 10\) designed in such a way as to optimally provide a fixed point Hamiltonian with single-particle spectra for effective fermionic and bosonic quasiparticles and reducing the interactions between them to higher order correction terms such that, in a first step, they can be neglected.

The model. The Hamiltonian for the BFM is divided into a free and a boson-fermion exchange coupling interacting part, i.e., \(H = H_0 + H_{\text{int}}\):

\[
H_0 = \sum_{k,\sigma} (\epsilon_k, - \mu) c_{k,\sigma}^\dagger c_{k,\sigma} + \sum_q (E_q - 2\mu) b_q^\dagger b_q
\]

\[
H_{\text{int}} = \frac{1}{\sqrt{N}} \sum_{k, p} \left( g_{k, p} b_k^\dagger b_{k + p}^\dagger c_{k + p, \uparrow} c_{k, \downarrow} + g_{k, p}^* b_{k + p}^\dagger b_k c_{k + p, \downarrow} c_{k, \uparrow} \right)
\]

Important for this model is to remember that the bosons are composed of two fermions. This is taken care
FIG. 1: (Color online) Phase diagram of the BFM. The thin solid line marks the transition from a Fermi liquid to a BCS-like superfluid phase which merges into a crossover (shaded) with the pseudogap phase. The thick solid (dashed) line mark the first (second) order transition from a superfluid of locally phase correlated boson-fermion bonding pairs to an itinerant bosonic metal (Bonding Pair Bose Metal). Diamonds represent the results of the present calculation.

of by requiring that both types of particles have the same chemical potential $\mu$, fixed such that the total particle density $n_{\text{tot}} = n_{F1} + n_{F1} + 2n_{B}$ is constant. We shall in the present study consider the case of low overall density and choose for that purpose $n_{F1} \simeq n_{B} \simeq 0.25$.

Fig. 1 shows the generic phase diagram of the BFM at low densities. For small values of exchange couplings $g$, the thin solid line indicates the transition from a Fermi liquid (FL) to a BCS type superfluid phase, controlled by amplitude fluctuations and characterized by the opening of a gap in the single particle fermion spectrum. This transition line merges, with increasing $g$, into the pseudogap crossover (shaded) which separates a canonical FL from a fermionic system with strong pair correlations (pseudogap phase). This crossover was analyzed in detail in Refs. 11, 12. In the present work, we focus on the large exchange coupling regime and the transition from a superfluid to a Bose Metal phase of resonating fermion pairs.

The method. The flow equation approach consists in performing continuous unitary transformations upon the initial Hamiltonian until the Hamiltonian is (block) diagonal. This is done in a systematic way by first decoupling states with high energy differences and, in the end, decoupling almost degenerate states. Previous applications of continuous unitary transformations considered the weak coupling regime of the BFM, where the zero momentum Cooper pair channel is the dominant interaction and the bosonic dispersion can be approximated by its $q = 0$-value[11]. In order to describe the evolution of the Hamiltonian in the strong-coupling phase with and without global phase coherence, it is crucial to treat the bosonic dispersion in the whole Brillouin zone.

Flow equations are formulated in terms of an anti-Hermitean generator $\eta(\ell)$ and read $\partial_\ell H(\ell) = [\eta(\ell), H(\ell)]$, with $\ell$ being the flow parameter. In this work, we shall choose the generator canonically, i. e., $\eta(\ell) = [H_0(\ell), H(\ell)]$, implying $\partial_\ell \text{Tr} [H(\ell) - H_0(\ell)]^2 \leq 0$ and hence $g_{k,p} \to 0$ for $\ell \to \infty$[10]. The infinitesimal transformations induce a flow on the fermion as well as boson dispersion and exchange coupling constants $g_{k,p}$:

$\partial_\ell g_{k,p} = -\alpha_{k,p}^2 g_{k,p}, \quad \alpha_{k,p} = \epsilon_k^1 + \epsilon_p^1 - E_{k+p}$

$\partial_\ell \epsilon_k^1 = \frac{2}{N} \sum_p \left( \alpha_{p,k}|g_{p,k}|^2 \delta_{\sigma,\downarrow} + \alpha_{k,p}|g_{k,p}|^2 \delta_{\sigma,\uparrow} \right) n_{k+p}^{(BE)}$

$\partial_\ell E_q = \frac{2}{N} \sum_p \left( \alpha_{q-p,p}|g_{q-p,p}|^2 [-1 + n^{(FD)}_{\uparrow,q-p}] + \alpha_{p,q-p}|g_{p,q-p}|^2 n^{(FD)}_{\downarrow,q-p} \right)$

For $\ell = \infty$, the renormalized fermions and bosons are decoupled and characterized by the fixed point dispersions $\epsilon_k^* = \epsilon_k(\ell = \infty)$ and $E_q^* = E_q(\ell = \infty)$.

In order to close the infinite hierarchy of newly generated interaction terms, higher order interaction terms in their normal ordered form with respect to the fixed point Hamiltonian $H_0$ are neglected. To lowest order, this introduces bilinear expectation values of the fermionic and bosonic operators. We will choose the expectation values explicitly $\ell$-dependent, i. e.,

$n_{k\sigma}^{(FD)}(\ell) = \langle c_{k\sigma}^\dagger c_{k\sigma}(\ell) \rangle = \left( e^{(\epsilon_k^*(\ell) - \mu(\ell))/k_B T} + 1 \right)^{-1}$

$n_q^{(BE)}(\ell) = \langle b_q^\dagger b_q(\ell) \rangle = \left( e^{(E_q^*(\ell) - \mu(\ell))/k_B T} - 1 \right)^{-1}$

The chemical potential $\mu(\ell)$ is determined such that the total number of particles $N_{\text{tot}} = \sum_{k\sigma} n_{k\sigma}^{(FD)}(\ell) + \sum_q n_q^{(BE)}(\ell)$ is conserved, which induces a flow of $\mu(\ell)$ with a fixed point value $\mu^* = \mu(\ell = \infty)$.

The justification for assuming fermion and boson distribution functions, Eq. (3), in form of the distribution functions for non-interacting quasi-particles is the following: For small $\ell$, states with high energy differences are decoupled, which corresponds to the perturbative regime and thus to well defined fermionic and bosonic quasi-particles. For large $\ell$, the exchange interaction $g_{k,p}$ has basically dropped to zero, according to how this renormalization procedure was constructed. It is this asymptotic regime $\ell \to \infty$ which determines the critical behavior of the system [13]. Since the main effect of the exchange coupling $g$ is not to induce a significant change in the relative number of fermions or bosons but to introduce phase coherence between the Cooper pairs and bosons, Eq. (3) represents a good approximation also in the strong coupling regime [14]. Hence, even though the truncation scheme is perturbative in the exchange coupling, the strong coupling regime can be discussed within
the flow equation approach (as has been shown also for the Hubbard model, see e.g. Ref. 16).

Numerical results. As initial conditions for the renormalization flow, we choose a dispersionless bosonic band with \( E_q = \Delta_B = -0.6 \) and a fermionic tight-binding dispersion \( \epsilon_{k,\sigma} = \epsilon_k = -2t \cos(ka) \) where we set the lattice constant \( a = 1 \) and use the bandwidth \( D = 4t \) as energy unit. We further assume a local exchange interaction \( g_{k,p} = g \). For the numerical integration of the flow equations, Eq. (2), we choose a one-dimensional system with \( N = 100 \) lattice sites which already resembles the thermodynamic limit.[15] As we shall discuss below, the phase transition is determined by the chemical potential moving out of the renormalized fermionic band of the fixed point Hamiltonian. Since the divergencies of the density-of-states at the bare band edges is smeared out in the course of the renormalization flow, the basic results are expected to remain qualitatively valid also in higher dimensions.

The present study, concentrating on the strong coupling regime, deals with the nature of transition from the superfluid state of resonating fermion pairs to their phase uncorrelated Bose Metal state. The critical interaction strength \( g_c \) as function of temperature \( T \) is indicated by diamonds (△) in Fig. 1. Such a Bose Metal consists of current carrying states being composed of bonding pairs without any contributions from single particle fermion states. The transition is characterized by \( \mu^* \) moving out of the fermionic band \( \epsilon_k^* \). It is discontinuous for \( T \leq T_0 \approx 0.75 \) (thick solid line in Fig. 1) and changes into a continuous transition for \( T > T_0 \) (dashed line).

The nature of the transition can be deduced from the renormalized fermionic dispersion \( \epsilon_k^* \) which is shown in the upper panels of Fig. 2 for various exchange interactions \( g \) and temperatures \( T = 0.01 \) and \( T = 0.1 \).

For \( T = 0.01 \) and \( g < g_c \approx 0.8 \) (solid lines of the upper left panel of Fig. 2), the fermionic excitations show a single band separated by a gap which is reminiscent of a lower and upper Bogoliubov band, when keeping only the parts with maximal spectral weight. As \( g \) increases, the gap gets bigger and the fraction of coherent fermions which are separated from incoherent excitations by the superconducting gap becomes smaller. For \( g > g_c \) (dashed lines), the lower Bogoliubov band has disappeared and the upper Bogoliubov band now plays the role of our effective fermionic band \( \epsilon_k^* \) which has abruptly moved up in energy and now lies above \( \mu^* \). The fermionic spectral function now consists of purely incoherent contributions characterized by breaking up the strongly bound fermion pairs in the purely local-phase correlated liquid. The total number of coherent and incoherent fermions is approximately constant since the exchange coupling \( g \) does not significantly change the relative number of bosons and fermions. For \( T = 0.1 \), the change of the fermionic dispersion is continuous for increasing \( g \) when \( \mu^* \) comes to lie below \( \epsilon_{k=0}^* \), as shown on the upper right hand side of Fig. 2. This yields \( g_c \approx 0.35 \).

At low temperatures and close to the phase transition, we notice a qualitative change in \( \mu^* \) as a function of \( g \) for different temperatures. Fig. 3 shows \( \mu^* \) as a monotonically decreasing function of \( g \) for \( T = 0.1 \), changing to a non-monotonic behavior at \( T = 0.5 \). This is indicative for the continuous transition for \( T > T_0 \approx 0.75 \) changing into a first order transition for \( T \leq T_0 \) where we find the coexistence of two phases with different relative densities of bosons and fermions. This feature is accompanied by a smooth, respectively abrupt, change of the position of \( \mu^* \) from inside the fermionic band \( \epsilon_k^* \) for \( g \leq g_c \), to outside of that band for \( g > g_c \), as seen in the insert of Fig. 3, where \( \epsilon_{k=0}^* - \mu^* \) is plotted as function of \( g \).

For \( T < T_0 \), also the bosonic dispersion changes abruptly from a linear to a quadratic spectrum at low \( g \) as \( g \) increases beyond \( g_c \) (lower left panel of Fig. 2). The initially dispersionless bosons thus first become superfluid and then itinerant as \( g \) increases from weak to strong coupling. For \( T_0 < T < 0.1 \) (lower right panel), the transition is continuous, nevertheless the width of the occupation number \( n_k^{BE}(\ell = \infty) \) around \( q = 0 \) of the renormalized dispersion changes rapidly, indicating the transition from superfluidity to a Bose metal at finite temperatures.

Summary and discussion. We have discussed the
critical values

E

to quadratic in momentum behavior, i.e., in the

tive strength of the hopping matrix element

both phases show a gap. But, depending on the rela-
tive temperatures \( T > T_0 \), \( T \approx T_0 \) and \( T < T_0 \) where \( T_0 \) defines the transition from second to first-order phase transition.

FIG. 3: (Color online) The renormalized chemical potential \( \mu^* \) as function of the coupling strength \( g \) with respect to the critical values \( \mu_c \) and \( q_c \) of the phase transition from superflu-
dity to a Bose Metal. The curves correspond to three different

tion is seen in the bosonic dispersion as a change from

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\]

Since \( E^*_q \propto q^2 \), it represents a Bose Metal composed of the above mentioned bonding pairs.

The first order transition has been deduced from the dynamical properties of the system. In order to discuss the transition within thermodynamic quantities, one has to study the flow of the operators in addition to the one of the Hamiltonian. This is a separate problem which will be addressed in the future. The present study dealt with spatially homogeneous systems, but should have its bearing on inhomogeneous systems, such as fermionic gases in optical traps.

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2. D. M. Eagles, Phys. Rev. 186, 456 (1969); A. J. Leggett, in “Modern trends in the theory of Condensed Matter”, ed. A. Pekalski and J. Przystawa (Springer Verlag, Berlin,1980), p14; P. Nozières and S. Schmitt-Rink, J. Low Temp. Phys. 59, 195 (1985): M. Randeria, in “Bose-Einstein Condensation” ed. by A. Griffin, D. W. Snoke and S. Stringari (Cambridge University Press, N.Y., 1995), p.355.
3. A. Bulgac, J. E. Drut and P. Magierski, Phys. Rev. Lett. 96 90404 (2006); P. Nikolic and S. Sachdev, cond-mat/0609106.
4. J. Ranninger and S. Robaszkiewicz, Physica B & C 135B, 468 (1985).
5. E. Timmermans, P. Tommasini, M. Hussein and A. Kerman, Phys. Rep. 315, 199 (1999).
6. E. Altman and A. Auerbach, Phys. Rev. B 65, 104508 (2002).
7. J. Ranninger and A. Romano, Eur. Phys. Lett. 75, 461 (2006).
8. M. Cuoco and J. Ranninger, Phys. Rev. B 70, 104509 (2004); ibid. Phys. Rev. B 74, 94511 (2006).
9. S. D. Glazek and K. G. Wilson, Phys. Rev. D 48, 5863 (1993); Phys. Rev. D 49, 4214 (1994).
10. F. Wegner, Ann. Physik 3, 77 (1994); S. Kehrein, “Flow Equation Approach to Many Particle Systems”, Springer Tracts in Modern Physics, 217 (Springer, New York, 2006).
11. T. Domanski and J. Ranninger, Phys. Rev. B 63, 134505 (2001).
12. T. Domanski and J. Ranninger, Phys. Rev. Lett. 91, 255301 (2003); Phys. Rev. B 70, 184513 (2004).
13. P. Lenz and F. Wegner, Nucl. Phys. B 482 [FS], 693

In the strong-coupling Bose metal phase (\( g > g_c \)), all one-particle fermion states exclusively exist as compo-

nents of the bonding pairs. The single-fermion particle spectrum then exhibits a correlation gap and the strong

local phase coherence, leading to a locally fluctuating

field, destroys all coherence of the itinerant fermions.

The bosonic dispersion, \( E^*_q \), of the renormalized Hamiltonian corresponds, within such a flow equation pro-
cedure, to renormalized bosonic operators of the form

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\sum_k \epsilon_{k,q} c_{k+q,1}^\dagger c_{k,1}^\dagger + \sum_k c_{k,1} b_k^\dagger + \sum_{k,q} \epsilon_{k,q} c_{k+q,1}^\dagger c_{k,1}^\dagger \quad [12]
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Since \( E^*_q \propto q^2 \), it represents a Bose Metal composed of the above mentioned bonding pairs.

In the superfluid phase, the renormalized dispersion for the bosons is linear for wave numbers \( q = 0 \) up to some \( q_{\text{max}} \) which depends on the size of the regions where local phase correlations between the itinerant fermions and the tightly bound boson pairs prevail. They become stronger for \( g \rightarrow g_c \) and render impossible the maintenance of the spatial phase correlations for the Cooper-pairs in the fermionic subsystem over short distances and hence a linear dispersion of the bosons for large momenta. \( q_{\text{max}} \) thus decreases with increasing \( g \) and eventually reaches zero at the phase transition. We note that the slope of the linear dispersion, i.e., the superfluid velocity \( v_{SF} \), hardly changes even for \( q \approx q_c \). This is yet a further manifestation of the first order phase transition (in case of a second order phase transition the slope would eventually tend to zero).

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7. J. Ranninger and A. Romano, Eur. Phys. Lett. 75, 461 (2006).
8. M. Cuoco and J. Ranninger, Phys. Rev. B 70, 104509 (2004); ibid. Phys. Rev. B 74, 94511 (2006).
9. S. D. Glazek and K. G. Wilson, Phys. Rev. D 48, 5863 (1993); Phys. Rev. D 49, 4214 (1994).
10. F. Wegner, Ann. Physik 3, 77 (1994); S. Kehrein, “Flow Equation Approach to Many Particle Systems”, Springer Tracts in Modern Physics, 217 (Springer, New York, 2006).
11. T. Domanski and J. Ranninger, Phys. Rev. B 63, 134505 (2001).
12. T. Domanski and J. Ranninger, Phys. Rev. Lett. 91, 255301 (2003); Phys. Rev. B 70, 184513 (2004).
13. P. Lenz and F. Wegner, Nucl. Phys. B 482 [FS], 693
[14] One can generalize Eq. (3) by multiplying the bosonic density \( n_{BE}^{\ell}(\ell) \) by a parameter \( \gamma(\ell) \), thus changing the relative weight with respect to the non-interacting case due to interaction effects. Setting \( \gamma(\ell) = 1 + \gamma_0 e^{-D_{2\ell}} \) with \( \gamma_0 \in [-0.5, 1] \), this leads to qualitatively the same phase diagram.

[15] A system with \( N = 200 \) yields values within less than 1% compared to the reported ones.

[16] I. Grote, E. Körding, and F. Wegner, J. Low Temp. Phys. 126, 1385 (2002).