Theory of enhanced proximity effect by mid gap Andreev resonant state in diffusive normal metal / triplet superconductor junctions

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Abstract

Enhanced proximity effect by mid gap Andreev resonant state (MARS) in a diffusive normal metal / insulator / triplet superconductor (DN/TS) junction is studied based on the Keldysh-Nambu quasiclassical Green’s function formalism. By choosing a $p$-wave superconductor as a typical example of TS, conductance of the junction and the spatial variation of quasiparticle local density of states (LDOS) in DN are calculated as the function of the magnitudes of the resistance $R_d$, Thouless energy in DN and the transparency of the insulating barrier. The resulting conductance spectrum has a zero bias conductance peak (ZBCP) and LDOS has a zero energy peak (ZEP) except for $\alpha = \pi/2$ (0 $\leq$ $\alpha$ $\leq$ $\pi/2$), where $\alpha$ denotes the angle between the lobe direction of the $p$-wave pair potential to the normal to the interface. The widths of the ZBCP and ZEP are reduced with the increase of $R_d$ while their heights are drastically enhanced. These peaks are revealed to be suppressed by applying a magnetic field. When the magnitude of $R_d/R_0$ is sufficiently large, the total zero voltage resistance of the junction is almost independent of the $R_d$ for $\alpha \neq \pi/2$. The extreme case is $\alpha = 0$, where total zero voltage resistance is always $R_0/2$. We also studied the charge transport in $p_x + ip_y$-wave junctions, where only the quasiparticle with perpendicular injection feel MARS. Even in this case, the resulting LDOS in DN has a ZEP. Thus, the existence of the ZEP in LDOS of DN region is a remarkable feature for DN/TS junctions which have never been expected for DN/ singlet superconductor junctions where MARS and proximity effect compete with each other. Based on these results, a crucial test to identify triplet pairing superconductors based on tunneling experiments is proposed.
I. INTRODUCTION

Nowadays, enormous numbers of unconventional superconductors have been discovered where pair potential changes sign on the Fermi surface \[1, 2, 3\]. It is known that reflecting on the sign change of the pair potential \[4, 5, 6, 7, 8\], tunneling spectroscopy of unconventional superconductor is essentially phase sensitive \[7, 9\]. The most dramatic effect is the appearance of zero bias conductance peak (ZBCP) in normal metal / insulator / unconventional superconductor junctions due to the formation of mid gap Andreev resonant states (MARS) \[10, 11\]. The ZBCP are observed in the actual many experiments \[7, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26\].

The MARS is a unique resonant state expected for the interface of unconventional superconductor assisted by the Andreev reflection \[3, 10, 28\]. The origin of this MARS is due to the anomalous interference effect of quasiparticles at the interface where injected and reflected quasiparticles from the unconventional superconductor side feel different sign of the pair potentials \[7, 9\]. It is well known that MARS influences several charge transport properties \[28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47\]. Since MARS is expected both for triplet and singlet superconductor junctions \[48, 49\], it is a challenging issue to present a new idea to discriminate triplet superconducting states \[48, 49, 50, 51, 52, 53\] from singlet ones through tunneling spectroscopy via MARS \[54\].

Although there are several studies about unconventional superconductor junctions \[55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80\] almost all of them are restricted to ballistic regime. If we take into account impurity scattering, we can expect various interesting features even for conventional superconductor junctions \[81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93\]. As regards diffusive normal metal / insulator / conventional singlet s-wave superconductor(DN/CSS) junctions, there has been a remarkable progress in theories of proximity effect \[93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113\]. However, proximity effect in unconventional superconductors \[114\] has been yet to be clarified. Recently, we have developed a theory of charge transport in diffusive normal metal / insulator / unconventional singlet superconductor (DN/USS) junctions \[117, 118\] extending Nazarov’s theory of matrix current \[101, 112\] within quasiclassical treatment \[98, 99, 115, 116\]. First, this theory was applied to \(d\)-wave superconductor junctions \[118\]. Unfortunately, however, it is revealed
that the proximity effect and the MARS compete with each other in DN/USS junctions. Although the interface resistance is reduced by the MARS irrespective of the magnitude of the transparency at the interface, the resulting total resistance of the junction $R$ is always larger than $R_0/2 + R_d$, where $R_d$ and $R_0$ is the resistance in DN and Sharvin resistance at the interface, respectively. This is because that the angular average of many channels at the DN/USS interface destruct the phase coherence of the MARS and the proximity effect (see Fig. 1. of ref. [54]). This destructive angular average is due to the sign change of the pair potentials felt by quasiparticles with injection angle $\phi$ and those with $-\phi$, where the angle $\phi$ is measured from the direction normal to the junction interface. However, in diffusive normal metal / insulator / triplet superconductor (DN/TS) junctions, we can escape from the above destructive average. In the last paper, we have presented a theory which is available for DN/TS junctions [54]. It is revealed that charge transport in diffusive normal metal / triplet superconductor (DN/TS) junctions is significantly unusual. We can expect enhanced proximity effect by the MARS. The total zero voltage resistance $R$ in the DN/TS junctions is significantly reduced by the enhanced proximity effect in the presence of the MARS. It is remarkable that when the resistance in DN, $R_d$, is sufficiently larger than the Sharvin resistance $R_0$, $R$ is given by $R = R_0/C_{-}$, which can become much smaller than the preexisting lower limit value of $R$, i.e., $R_0/2 + R_d$. In the above, $C_{-}$ is a constant completely independent of both $R_d$ and $R_0$, where $R_b$ denotes the interface resistance in the normal state. When all quasiparticles injected at the interface feel the MARS, $R$ is reduced to be $R = R_0/2$ irrespective of the magnitude of $R_d$ and $R_b$. At the same time, local density of states (LDOS) in the DN region has zero energy peak (ZEP) due to the penetration of the MARS into the DN region from the triplet superconductor (TS) side of the DN/TS interface. These outstanding features have never been expected either in DN/CSS junctions or DN/USS junctions. However, the reference [54] does not contain the necessary technical details of the matrix current derivation and the obtained results are limited. In order to understand this remarkable enhanced proximity effect by MARS peculiar to the DN/TS junctions much more in detail, one has to evaluate the conductance spectrum and quasiparticle density of states in wide range of several parameters.

In this paper, we present a detailed derivation of the matrix current in (DN/TS) junctions. We express compact formula of matrix current and Keldysh-Nambu (KN) Green’s functions relevant to the actual boundary condition in comparison with those of DN/USS junctions.
Here, we restrict our attention to triplet superconductors with $S_z = 0$, where $S_z$ denotes the $z$ component of the total spin of a Cooper pair. In order to show up the anomalous charge transport in DN/TS junctions due to the coexistence of the MARS and the proximity effect, we present detailed numerical calculations of the conductance spectra of DN/TS junctions for $p$-wave superconductors. We investigate the dependence of the spectra of the bias voltage conductance on various parameters: the height of the barrier at the interface, resistance $R_d$ in DN, the Thouless energy $E_{Th}$ in DN and the angle $\alpha$ between the normal to the interface and the lobe direction of $p$-wave superconductor. $E_{Th}$ can be expressed by $E_{Th} = \hbar D/L^2$ with diffusion constant $D$ in DN and the length of DN. We normalize the voltage-dependent conductance $\sigma_S(eV)$ by its value in the normal state, $\sigma_N$, so that $\sigma_T(eV) = \sigma_S(eV)/\sigma_N$. At the same time, we focus on the spatial dependence of LDOS in DN for DN/TS junctions. We also studied about several related physical quantities which identify the anomalous charge transport. Our main results are as follows:

1. The ZBCP is always seen in the shape of $\sigma_T(eV)$ for $\alpha \neq \pi/2$ with $0 \leq \alpha \leq \pi/2$. The magnitude of $\sigma_T(0)$ is drastically enhanced with the increase of $R_d/R_b$. The half width of the ZBCP, which is proportional to $\exp(-C_cR_d/R_0)$ for $C_c$, is almost independent of the transparency of the DN/TS interface and $E_{Th}$. On the other hand, $\sigma_T(eV)$ for DN/USS junctions with $d_{xy}$-wave superconductor shows a very different behavior. The magnitude of $\sigma_T(eV)$ is reduced with the increase of $R_d/R_b$ due to the absence of the proximity effect.

2. The LDOS in DN has a ZEP except for the case with $\alpha = \pi/2$ where MARS is absent. The existence of the ZEP in DN means that the penetration of the MARS into DN. The height of the ZEP is significantly enhanced with the increase of $R_d/R_0$. The half width of the ZEP is proportional to $\exp(-C_\rho R_d/R_0)$ where $C_\rho$ is a constant almost independent of $Z$.

3. We can express the LDOS in the DN region using the proximity parameter $\theta$ as $\text{Real}[\cos \theta]$. If we denote the $\theta$ at DN/TS interface as $\theta_0$, $\theta_0 = 2iR_d\cos \alpha/R_0$ is satisfied at $\varepsilon = 0$ where quasiparticle energy $\varepsilon$ is measured from the Fermi energy. Since $\theta_0$ is a pure imaginary number for $\varepsilon = 0$, we can expect a ZEP in LDOS. This unique feature has never been expected for DN/USS of DN/CSS cases, where $\theta_0$ at $\varepsilon = 0$ is always a real number.

4. The total zero voltage resistance $R$ in the DN/TS junctions is significantly reduced by the enhanced proximity effect in the presence of the MARS. It is remarkable that when $R_d$ is sufficiently larger than the Sharvin resistance $R_0$, $R$ is reduced to be $R = R_0/(2\cos \alpha)$, which
can become much smaller than the preexisting lower limit value of \( R \), i.e., \( R_0/2 + R_d \). For low transparent junctions, \( R \) is also reduced to be \( R = R_0/(2 \cos \alpha) \). When all quasiparticles injected at the interface feel the MARS, \( R \) is reduced to be \( R = R_0/2 \) irrespective of the magnitude of \( R_d \) and \( R_b \). This drastic situation is realized for \( \alpha = 0 \).

6. The sharp ZBCP or ZEP in LDOS due to the enhanced proximity effect is sensitive to the applied magnetic field \( H \). The height of ZBCP is significantly reduced by \( H \). The threshold value of the magnetic field is \( H_{Th} \simeq 8\hbar/(eS_{DN}) \) where \( S_{DN} \) denotes the magnitude of the area of DN region.

7. As a candidate of TS (triplet superconductor), we also choose \( p_x + ip_y \)-wave superconductor. This superconducting state is so called chiral superconducting state where broken time reversal symmetry state (BTRSS) is realized. Although only quasiparticles with perpendicular injection feel MARS, both LDOS and \( \sigma_T(eV) \) has ZEP and ZBCP due to the enhanced proximity effect by MARS.

It should be remarked that these remarkable features have never been expected either in DN/CSS or DN/USS junctions. The structure of the paper is as follows. We formulate the model in use in section 2. We also present there the detailed derivation of the matrix current and end up with the expression for the normalized conductance. We focus on \( p \)-wave superconductor junctions in section 3 and evaluate \( \sigma_T(eV) \), \( \rho(\varepsilon) \), zero voltage resistance and the measure of the proximity effect \( \theta_0 \) for various cases. The last part of this section, we discuss \( p_x + ip_y \)-wave junctions. We summarize the results in section 4.

II. FORMULATION

In this section we introduce the model and the formalism. We consider a junction consisting of normal and superconducting reservoirs connected by a quasi-one-dimensional diffusive conductor (DN) with a length \( L \) much larger than the mean free path as in our previous paper. The interface between the DN conductor and the TS (triplet superconductor) has a resistance \( R_b \) while the DN/N interface has zero resistance. A schematic illustration of the model we use are shown in Fig. 1. The positions of the DN/N interface and the DN/TS interface are denoted as \( x = -L \) and \( x = 0 \), respectively. According to the circuit theory \[112\], the constriction area \((-L_1 < x < -L_2)\) between DN and TS is considered as composed of the diffusive isotropization zone \((-L_1 < x < -L_2)\), the left side ballistic zone
The scattering zone is modeled as an insulating delta-function barrier with the transparency
\[ T = T(\phi) = 4 \cos^2 \phi / (4 \cos^2 \phi + Z^2), \]
where \( Z \) is a dimensionless constant, \( \phi \) is the injection angle measured from the interface normal to the junction. Here, we express insulating barrier as a delta function model \( H_b\delta(x) \), where \( Z \) is given by
\[ Z = 2 m H_b / (\hbar^2 k_F) \]
with Fermi momentum \( k_F \) and effective mass \( m \). In order to clarify charge transport in DN/TS junctions, we must obtain Keldysh-Nambu (KN) Green's function, which has indices of transport channels and the direction of motion along \( x \) axis taking into account the proper boundary conditions. We restrict our attention to triplet superconductors with \( S_z = 0 \) that preserves time reversal symmetry. \( S_z \) denotes the \( z \) component of the total spin of a Cooper pair. It is by no means easy to formulate a charge transport of DN/TS junctions since the quasiparticle Green's function has no angular dependence by the impurity scattering in the DN. However, as shown in our previous paper \[117, 118\], if we concentrate on the matrix currents \[101, 112\] via the TS to or from the DN, we can make a boundary condition of the KN Green's function. The sizes of the ballistic zone in the DN and the scattering zone in the current flow direction are much shorter than the coherence length \[112, 117, 118\]. Since we assume the flat interface, the momentum parallel to the interface is conserved. Thus it is possible to construct a matrix current \[101, 112\] based on the asymptotic Green's function in TS.

FIG. 1: Schematic illustration of the model of constriction area.
In this section, we will show how to derive the matrix current in DN/TS junctions. Then we will derive the retarded and Keldysh components of the matrix current. Finally, we will show how to calculate the conductance of the junctions.

A. Usadel equation in DN

In order to clarify the charge transport in DN/TS junctions, we first concentrate on the KN Green’s function in DN. We define KN Green’s function in DN as

\[
\hat{G}_N(x) = \begin{pmatrix} \hat{R}_N(x) & \hat{K}_N(x) \\ 0 & \hat{A}_N(x) \end{pmatrix}.
\] (1)

Here, we are using quasiclassical approximation. Then, we can express \(\hat{R}_N(x)\) as

\[
\hat{R}_N(x) = \cos \psi \sin \theta \hat{\tau}_1 + \sin \psi \sin \theta \hat{\tau}_2 + \cos \theta \hat{\tau}_3,
\] (2)

with Pauli matrix in the electron hole space, \(\hat{\tau}_1, \hat{\tau}_2, \) and \(\hat{\tau}_3\). Since \(\hat{R}_N(x)\) obeys Usadel equation, following equations are satisfied,

\[
\hbar D \left[ \frac{\partial^2}{\partial x^2} \theta - \left( \frac{\partial \psi}{\partial x} \right)^2 \cos \theta \sin \theta \right] + 2i\varepsilon \sin \theta = 0,
\] (3)

\[
\frac{\partial}{\partial x} \left[ \sin^2 \theta \left( \frac{\partial \psi}{\partial x} \right) \right] = 0.
\]

In the present case, since no supercurrent is flowing in the junctions, \(\frac{\partial \psi}{\partial x} = 0\) is satisfied. We can choose \(\psi = \psi_0\) where \(\psi_0\) is a constant independent of \(x\). The boundary condition of \(\hat{G}_N(x)\) at DN/TS interface is given by

\[
\frac{L}{R_d} \left[ \frac{\partial \hat{G}_N(x)}{\partial x} \right]_{x=0_-} = -\frac{<I(\phi)>}{R_b}.
\] (4)

\[
\hat{I}(\phi) = 2[\hat{G}_1, \hat{B}]
\]

\[
\hat{B} = (-T_1[\hat{G}_1, \hat{H}_-] + \hat{H}_- \hat{H}_+ - T_2^2 \hat{G}_1 \hat{H}_- \hat{H}_+ \hat{G}_1)^{-1} (T_1(1 - \hat{H}_-^{-1}) + T_2^2 \hat{G}_1 \hat{H}_-^{-1} \hat{H}_+) \]

with \(\hat{G}_1 = \hat{G}_N(x = 0_-), \hat{H}_\pm = (\hat{G}_{2_+} \pm \hat{G}_{2_-})/2, \) and \(T_1 = T/(2 - T + 2\sqrt{1-T})\) where \(\hat{G}_{2_\pm}\) is the asymptotic Green’s function in TS as defined in our previous papers. In the above, the average over the various angles of injected particles at the interface is defined as

\[
<\hat{I}(\phi)> = \int_{-\pi/2}^{\pi/2} d\phi \cos \phi \hat{I}(\phi) / \int_{-\pi/2}^{\pi/2} d\phi T(\phi) \cos \phi
\] (6)
with \( \bar{I}(\phi) = \bar{I} \) and \( T(\phi) = T \). The resistance of the interface \( R_b \) is given by

\[
R_b = \frac{2R_0}{\int_{-\pi/2}^{\pi/2} d\phi T(\phi) \cos \phi}
\]  

(7)

with Sharvin resistance at the interface, \( R_0 \). In the above, \( \tilde{G}_1 \) and \( \tilde{G}_{2\pm} \) can be given by

\[
\tilde{G}_1 = \begin{pmatrix}
\hat{R}_1 & \hat{K}_1 \\
0 & \hat{A}_1
\end{pmatrix}, \quad \tilde{G}_{2\pm} = \begin{pmatrix}
\hat{R}_{2\pm} & \hat{K}_{2\pm} \\
0 & \hat{A}_{2\pm}
\end{pmatrix},
\]

(8)

where the Keldysh component \( \hat{K}_{1,2\pm} \) is given by \( \hat{K}_{1(2\pm)} = \hat{R}_{1(2\pm)}\hat{f}_{1(2)}(0) - \hat{f}_{1(2)}(0)\hat{A}_{1(2\pm)} \) with the retarded component \( \hat{R}_{1,2\pm} \) and the advanced component \( \hat{A}_{1,2\pm} \) using distribution function \( \hat{f}_{1(2)}(0) \). In the above, \( \hat{R}_{2\pm} \) is expressed by

\[
\hat{R}_{2\pm} = (g_{\pm}\hat{\tau}_3 + f_{\pm}\hat{\tau}_2)
\]

with \( g_{\pm} = \varepsilon/\sqrt{\varepsilon^2 - \Delta_{\pm}^2} \), \( f_{\pm} = \Delta_{\pm}/\sqrt{\Delta_{\pm}^2 - \varepsilon^2} \), and \( \hat{A}_{2\pm} = -\hat{\tau}_3\hat{R}_{2\pm}^\dagger\hat{\tau}_3 \) where \( \varepsilon \) denotes the quasiparticle energy measured from the Fermi energy. \( \Delta_+ (\Delta_-) \) is the effective pair potential felt by quasiparticles with an injection angle \( \phi (\pi - \phi) \) as shown in Fig. \( \text{[1]} \). \( \hat{f}_2(0) = f_{0S}(0) = \tanh[\varepsilon/(2k_B T)] \) in thermal equilibrium with temperature \( T \). Here, we put the electrical potential zero in the TS. We also denote \( \hat{H}_+, \hat{H}_-, \hat{B}, \hat{I} = \bar{I}(\phi) \) as follows,

\[
\hat{H}_+ = \begin{pmatrix}
\hat{R}_p & \hat{K}_p \\
0 & \hat{A}_p
\end{pmatrix}, \quad \hat{H}_- = \begin{pmatrix}
\hat{R}_m & \hat{K}_m \\
0 & \hat{A}_m
\end{pmatrix},
\]

(9)

\[
\hat{B} = \begin{pmatrix}
\hat{B}_R & \hat{B}_K \\
0 & \hat{B}_A
\end{pmatrix}, \quad \hat{I} = \begin{pmatrix}
\hat{I}_R & \hat{I}_K \\
0 & \hat{I}_A
\end{pmatrix}.
\]

B. Calculation of the retarded part of the matrix current

First, we pay attention to the boundary condition of the retarded part of KN Green’s function at DN/TS interface in order to determine the value of \( \psi_0 \). The left side of the boundary condition of eq. \( \text{[1]} \) can be expressed by

\[
\frac{L}{R_d} \hat{R}_N(x) \frac{\partial}{\partial x} \hat{R}_N(x)|_{x=0} = \frac{L\hat{i}}{R_d} \left[-\sin \psi_0 \hat{\tau}_1 + \cos \psi_0 \hat{\tau}_2 \right] \left[\frac{\partial \theta}{\partial x}\right]|_{x=0}
\]

(10)

In the above, the most remarkable point is that the \( \hat{\tau}_3 \) component is vanishing. In order to calculate the right side of eq. \( \text{[1]} \), we must study the parity of \( \hat{I}_R \) as a function of \( \phi \). In
general, \( \hat{I}_R \) can be expressed by using several spectral vectors,

\[
\hat{I}_R = 4i T_1 (d_R \cdot d_R)^{-1} \{-\frac{1}{2}(1 + T_1^2)(s_{2+} - s_{2-})^2 [s_1 \times (s_{2+} + s_{2-})] \cdot \hat{\tau}
\]

\[
+ 2T_1 s_1 \cdot (s_{2+} \times s_{2-})[s_1 \times (s_{2+} \times s_{2-})] \cdot \hat{\tau}
\]

\[
+ 2T_1 s_1 \cdot (s_{2+} - s_{2-})[s_1 \times (s_{2+} - s_{2-})] \cdot \hat{\tau}
\]

\[
- i(1 + T_1^2)(1 - s_{2+} \cdot s_{2-})[s_1 \times (s_{2+} \times s_{2-})] \cdot \hat{\tau}
\]

\[
+ 2i T_1 (1 - s_{2+} \cdot s_{2-})[s_1 \cdot (s_{2+} - s_{2-})s_1 - (s_{2+} - s_{2-})] \cdot \hat{\tau}\}
\]

\[
d_R = (1 + T_1^2)(s_{2+} \times s_{2-}) - 2T_1 s_1 \cdot (s_{2+} - s_{2-}) - 2T_1^2 s_1 \cdot (s_{2+} \times s_{2-})s_1
\]

with \( \hat{R}_1 = s_1 \cdot \hat{\tau} \) and \( \hat{R}_{2\pm} = s_{2\pm} \cdot \hat{\tau} \).

The spectral vectors \( s_1 \) and \( s_{2\pm} \) are given by

\[
s_1 = \begin{pmatrix}
\sin \theta_0 \cos \psi_0 \\
\sin \theta_0 \sin \psi_0 \\
\cos \theta_0
\end{pmatrix}; \quad s_{2\pm} = \begin{pmatrix}
0 \\
f_{\pm}(\phi) \\
g_{\pm}(\phi)
\end{pmatrix}, \quad \text{(13)}
\]

where \( \theta_0 \) denotes the \( \theta \) at \( x = 0_- \). We postulate that \( \hat{\tau}_3 \) component of \( < \hat{I}_R > \) should be zero. Here, we focus on the parity of \( \hat{I}_R \) since there is an angular average over \( \phi \) in the actual calculation. As a comparison, we also look at the case where TS (triplet superconductor) is substituted with USS (unconventional singlet superconductor). For DN/TS junctions, \( g_\pm(-\phi) = g_\mp(\phi) \) and \( f_\pm(-\phi) = -f_\mp(\phi) \) are satisfied, while for DN/USS junctions, \( g_\pm(-\phi) = g_\mp(\phi) \) and \( f_\pm(-\phi) = f_\mp(\phi) \) are satisfied. \( (d_R \cdot d_R)^{-1} \) can be given by

\[
(d_R \cdot d_R)^{-1} = \begin{cases}
d_e^{(s)}(\phi) + d_o^{(t)}(\phi) \sin \psi_0 & \text{DN/TS} \\
d_e^{(s)}(\phi) & \text{DN/USS}
\end{cases}
\]

\[
\text{(14)}
\]

for DN/TS and DN/USS junctions, where \( d_e^{(s)}(\phi) \) and \( d_o^{(t)}(\phi) \) are even functions with \( \phi \), while \( d_o^{(t)}(\phi) \) is an odd function with \( \phi \). We apply similar discussions for other terms in Eq. (14). For the convenience, we define

\[
s_z = \begin{pmatrix}
0 \\
0 \\
1
\end{pmatrix}
\]

\[
\text{(15)}
\]
After simple manipulations, we can show the following relations

\[-\frac{1}{2}(1 + T_1^2)(s_{2+} - s_{2-})^2[s_1 \times (s_{2+} + s_{2-})] \cdot s_z\]  \hspace{1cm} (16)

\[= \begin{cases} F_{1o}^{(t)}(\phi) \cos \psi_0 & \text{DN/TS} \\ F_{1e}^{(s)}(\phi) \cos \psi_0 & \text{DN/USS} \end{cases} \]

\[2T_1 s_1 \cdot (s_{2+} \times s_{2-})[s_1 \times (s_{2+} \times s_{2-})] \cdot s_z\]  \hspace{1cm} (17)

\[= \begin{cases} F_{2e}^{(t)}(\phi) \cos \psi_0 \sin \psi_0 & \text{DN/TS} \\ F_{2e}^{(s)}(\phi) \cos \psi_0 \sin \psi_0 & \text{DN/USS} \end{cases} \]

\[2T_1 s_1 \cdot (s_{2+} - s_{2-})[s_1 \times (s_{2+} - s_{2-})] \cdot s_z\]  \hspace{1cm} (18)

\[= \begin{cases} (F_{3e}^{(t)}(\phi) \sin \psi_0 + F_{3o}^{(t)}(\phi) \cos \psi_0) & \text{DN/TS} \\ F_{3e}^{(s)}(\phi) \cos \psi_0 & \text{DN/USS} \end{cases} \]

\[-i(1 + T_1^2)(1 - s_{2+} \cdot s_{2-})[s_1 \times (s_{2+} \times s_{2-})] \cdot s_z\]  \hspace{1cm} (19)

\[= \begin{cases} F_{4e}^{(t)}(\phi) \sin \psi_0 & \text{DN/TS} \\ F_{4o}^{(s)}(\phi) \sin \psi_0 & \text{DN/USS} \end{cases} \]

\[2iT_1(1 - s_{2+} \cdot s_{2-})[s_1 \cdot (s_{2+} - s_{2-})s_1 - (s_{2+} - s_{2-})] \cdot s_z\]  \hspace{1cm} (20)

\[= \begin{cases} F_{5e}^{(t)}(\phi) \sin \psi_0 + F_{5o}^{(t)}(\phi) & \text{DN/TS} \\ F_{5o}^{(s)}(\phi) \sin \psi_0 & \text{DN/USS} \end{cases} \]

In the above, $F_{2e}^{(t)}(\phi), F_{3e}^{(t)}(\phi), F_{4e}^{(t)}(\phi), F_{5e}^{(t)}(\phi), F_{1e}^{(s)}(\phi), F_{2e}^{(s)}(\phi)$ and $F_{3e}^{(s)}(\phi)$, are even functions of $\phi$, while $F_{1o}^{(t)}(\phi), F_{3o}^{(t)}(\phi), F_{4o}^{(s)}(\phi), F_{5o}^{(t)}(\phi)$, and $F_{5o}^{(s)}(\phi)$ are odd functions of $\phi$, respectively. By postulating that the $\hat{r}_3$ component of $\hat{I}_R$ should vanish after the angular average over $\phi$, we can show that $\sin \psi_0 = 0$ for DN/TS junctions and $\cos \psi_0 = 0$ for DN/USS junctions.

The resulting retarded part of the boundary condition is given by

\[\frac{L}{R_d} \left( \frac{\partial \theta}{\partial x} \right)_{x=0} = \frac{< F >}{R_b} \]  \hspace{1cm} (21)

\[F = \frac{2T(f_S \cos \theta_0 - g_S \sin \theta_0)}{2 - T + T(\cos \theta_0 g_S + \sin \theta_0 f_S)} \]  \hspace{1cm} (22)
with \(g_S = (g_+ + g_-)/(1 + g_+g_- + f_+f_-)\), \(f_S = i(f_+g_- - g_+f_-)/(1 + g_+g_- + f_+f_-)\) for DN/TS junctions and \(g_S = (g_+ + g_-)/(1 + g_+g_- + f_+f_-)\), \(f_S = (f_+ + f_-)/(1 + g_+g_- + f_+f_-)\) for DN/USS junctions, where \(g_\pm = g_\pm(\phi)\) and \(f_\pm = f_\pm(\phi)\). This is one of the central results of this paper.

C. Calculation of the Keldysh part of the matrix current

Next, we focus on the Keldysh component. We define \(I_b\) as

\[
I_b = \frac{1}{4} \text{Tr}[\hat{\tau}_3 \hat{I}_K] \quad \hat{I}_K = 2(\hat{R}_1 \hat{B}_K + \hat{K}_1 \hat{B}_A - \hat{B}_R \hat{K}_1 - \hat{B}_K \hat{A}_1) \tag{23}
\]

with \(\hat{K}_1 = \hat{R}_1 \hat{f}_1(0) - \hat{f}_1(0) \hat{A}_1 \hat{f}_1(0) = f_{0N}(0) + f_{3N}(0) \hat{\tau}_3\). \(\hat{B}_R\) is given by

\[
\hat{B}_R = \begin{cases} 
  b_1^{(t)} \hat{\tau}_1 + b_2^{(t)} \hat{\tau}_2 + b_3^{(t)} \hat{\tau}_3 & \text{DN/TS} \\
  b_1^{(s)} \hat{\tau}_1 + b_2^{(s)} \hat{\tau}_2 + b_3^{(s)} \hat{\tau}_3 & \text{DN/USS}
\end{cases} \tag{24}
\]

\[
b_1^{(t)} = -\frac{T_1 (T_1 \sin \theta_0 + f_S)}{\Lambda}, \quad b_2^{(t)} = -\frac{T_1 \hat{f}_S}{\Lambda}, \quad b_3^{(t)} = -\frac{T_1 (T_1 \cos \theta_0 + g_S)}{\Lambda},
\]

\[
b_1^{(s)} = -\frac{T_1 \hat{f}_S}{\Lambda}, \quad b_2^{(s)} = -\frac{T_1 (T_1 \sin \theta_0 + f_S)}{\Lambda}, \quad b_3^{(s)} = -\frac{T_1 (T_1 \cos \theta_0 + g_S)}{\Lambda},
\]

\(\Lambda = (1 + T_1^2) + 2T_1 (g_S \cos \theta_0 + f_S \sin \theta_0)\)

In the above \(g_S\), \(f_S\) and \(\hat{f}_S\) are defined by

\[
g_S = \begin{cases} 
  (g_+ + g_-)/(1 + g_+g_- + f_+f_-) & \text{DN/TS} \\
  (g_+ + g_-)/(1 + g_+g_- + f_+f_-) & \text{DN/USS}
\end{cases} \tag{25}
\]

\[
f_S = \begin{cases} 
  i(f_+g_- - f_-g_+)/\left(1 + g_+g_- + f_+f_-\right) & \text{DN/TS} \\
  (f_+ + f_-)/(1 + g_+g_- + f_+f_-) & \text{DN/USS}
\end{cases} \tag{26}
\]

\[
\hat{f}_S = \begin{cases} 
  (f_+ + f_-)/(1 + g_+g_- + f_+f_-) & \text{DN/TS} \\
  i(f_+g_- - g_-f_+)/\left(1 + g_+g_- + f_+f_-\right) & \text{DN/USS}
\end{cases} \tag{27}
\]

We can calculate \(\hat{B}_A\) similar to the case of \(\hat{B}_R\). After simple manipulation, we can show that \(\hat{B}_A = -\hat{\tau}_3 \hat{B}_R^\dagger \hat{\tau}_3\) and the resulting \(I_b\) is given by

\[
I_b = \text{Trace}\{\hat{\tau}_3 (\hat{R}_1 \hat{B}_K + \hat{R}_1^\dagger \hat{B}_K) - [\hat{\tau}_3 (\hat{R}_1 \hat{B}_R^\dagger + \hat{B}_R \hat{R}_1 + \hat{B}_R^\dagger \hat{R}_1 + \hat{R}_1^\dagger \hat{B}_R)] f_{0N}(0) - [(\hat{R}_1 + \hat{R}_1^\dagger)(\hat{B}_R + \hat{B}_R^\dagger)] f_{3N}(0)\}/2, \tag{28}
\]
After angular average over $\phi$ and the operation of the trace, it is easily shown that the second term which is proportional to $f_{0N}(0)$ does not contribute to $< I_b >$. It is necessary to obtain $\hat{B}_K$ which is given by

$$\hat{B}_K = \hat{D}_R^{-1} \hat{N}_K - \hat{D}_R^{-1} \hat{D}_K \hat{B}_A$$

with $\hat{B}_A = \hat{D}_A^{-1} \hat{N}_A$ and

$$\hat{D} = -T_1[\hat{G}_1, \hat{H}_-^{-1}] + \hat{H}_-^{-1} \hat{H}_+ - T_1^2 \hat{G}_1 \hat{H}_-^{-1} \hat{H}_+ \hat{G}_1, \quad \hat{D} = \begin{pmatrix} \hat{D}_R & \hat{D}_K \\ 0 & \hat{D}_A \end{pmatrix},$$

where $\hat{N}_K$ and $\hat{N}_A$ is the Keldysh and advanced part of $\hat{N}$ given by

$$\hat{N} = T_1 - T_1 \hat{H}_-^{-1} + T_1^2 \hat{G}_1 \hat{H}_-^{-1} \hat{H}_+ \quad \hat{N} = \begin{pmatrix} \hat{N}_R & \hat{N}_K \\ 0 & \hat{N}_A \end{pmatrix}.$$ 

We can express $\hat{N}_K$ and $\hat{D}_K$ as linear combinations of distribution functions $f_{0S}(0)$, $f_{0N}(0)$, and $f_{3N}(0)$ as follows both for DN/TS and DN/US (j)unctions,

$$\hat{N}_K = \begin{cases} \hat{C}_1^{(t)} f_{0S}(0) + \hat{C}_2^{(t)} f_{0N}(0) + \hat{C}_3^{(t)} f_{3N}(0), & \text{DN/TS} \\ \hat{C}_4^{(s)} f_{0S}(0) + \hat{C}_5^{(s)} f_{0N}(0) + \hat{C}_6^{(s)} f_{3N}(0), & \text{DN/US} \end{cases}$$

$$\hat{D}_K = \begin{cases} \hat{C}_7^{(t)} f_{0S}(0) + \hat{C}_8^{(t)} f_{0N}(0) + \hat{C}_9^{(t)} f_{3N}(0), & \text{DN/TS} \\ \hat{C}_{10}^{(s)} f_{0S}(0) + \hat{C}_{11}^{(s)} f_{0N}(0) + \hat{C}_{12}^{(s)} f_{3N}(0), & \text{DN/US} \end{cases}$$

$$\hat{D}_R^{-1} = \begin{cases} \hat{C}_7^{(t)} + \hat{C}_8^{(t)} & \text{DN/TS} \\ \hat{C}_{10}^{(s)} & \text{DN/US} \end{cases}$$

with $\hat{C}_1^{(t)} = \hat{C}_1^{(t)} + \hat{C}_1^{(t)}$, $\hat{C}_2^{(t)} = \hat{C}_2^{(t)} + \hat{C}_2^{(t)}$, $\hat{C}_4^{(t)} = \hat{C}_4^{(t)} + \hat{C}_4^{(t)}$, $\hat{C}_5^{(t)} = \hat{C}_5^{(t)} + \hat{C}_5^{(t)}$, and $\hat{C}_6^{(t)} = \hat{C}_6^{(t)} + \hat{C}_6^{(t)}$, by $2 \times 2$ matrix $\hat{C}_r^{(r)}$ with $r = t, s$, $j = 1, \ldots, 7$ and $k = e, o$. When the suffix $k$ is $e$ ($o$), $\hat{C}_r^{(r)}$ is an even (odd) function of $\phi$. $\hat{C}_{1e}^{(t)}$, $\hat{C}_{2e}^{(t)}$, $\hat{C}_{3o}^{(t)}$, $\hat{C}_{5o}^{(t)}$, $\hat{C}_{6e}^{(t)}$, and $\hat{C}_{7e}^{(t)}$ are linear combinations of $\hat{1}$ and $\hat{1}_2$, while $\hat{C}_{1o}^{(t)}$, $\hat{C}_{2o}^{(t)}$, $\hat{C}_{3e}^{(t)}$, $\hat{C}_{4o}^{(t)}$, $\hat{C}_{5e}^{(t)}$, $\hat{C}_{6o}^{(t)}$, and $\hat{C}_{7e}^{(t)}$ are linear combinations of $\hat{1}$ and $\hat{1}_3$. $\hat{1}$ is a unit matrix in the electron hole space. On the other hand, $\hat{C}_{1o}^{(t)}$, $\hat{C}_{2o}^{(t)}$, $\hat{C}_{3o}^{(t)}$, and $\hat{C}_{6o}^{(t)}$ are linear combinations of $\hat{1}_2$ and $\hat{1}_3$, while $\hat{C}_{3o}^{(s)}$, $\hat{C}_{4o}^{(s)}$, $\hat{C}_{5o}^{(s)}$, and $\hat{C}_{7o}^{(s)}$ are linear combinations of $\hat{1}_1$. Taking account of these facts, after angular average over $\phi$, $< I_b >$ can be given by

$$< I_b > = < \text{Trace} \{[\hat{R}_1 + \hat{R}_1^t][\hat{B}_{KE}\hat{\tau}_3 - (\hat{B}_R + \hat{B}_R^t)]f_{3N}(0) \} > /2,$$
\[ \hat{B}_{KE} = \hat{D}_R^{-1}[\hat{C}_3 - \hat{C}_6\hat{D}_A^{-1}\hat{N}_A], \]
\[ \hat{C}_3 = T_1^2(\hat{R}_1\hat{\tau}_3 - \hat{\tau}_3\hat{A}_1)\hat{A}_m^{-1}\hat{A}_p, \]
\[ \hat{C}_6 = T_1[-(\hat{R}_1\hat{\tau}_3 - \hat{\tau}_3\hat{A}_1)\hat{A}_m^{-1} + \hat{R}_m^{-1}(\hat{R}_1\hat{\tau}_3 - \hat{\tau}_3\hat{A}_1)] - T_1(\hat{R}_1\hat{\tau}_3 - \hat{\tau}_3\hat{A}_1)\hat{A}_m^{-1}\hat{A}_p\hat{A}_1 - T_1\hat{R}_1\hat{R}_m^{-1}\hat{R}_p(\hat{R}_1\hat{\tau}_3 - \hat{\tau}_3\hat{A}_1). \]  

With \( \hat{C}_3 = \hat{C}_{3e}^{(t)} (\hat{C}_3 = \hat{C}_{3o}^{(s)}) \) and \( \hat{C}_6 = \hat{C}_6^{(t)} (\hat{C}_6 = \hat{C}_{6o}^{(s)}) \) for DN/TS (DN/USS) junctions.

Since following equations are satisfied,
\[ \hat{D}_A^{-1}\hat{N}_A = -\hat{\tau}_3\hat{B}_R^\dagger\hat{\tau}_3, \quad \hat{A}_m(p) = -\hat{\tau}_3\hat{R}_m(p)\hat{\tau}_3, \]  

\[ \hat{D}_R^{-1}(T_1 - T_1\hat{R}_m^{-1} + T_1^2\hat{R}_1\hat{R}_m^{-1}\hat{R}_p) = \hat{B}_R, \]  

\[ \hat{B}_R(1 + \hat{R}_m^{-1} + T_1\hat{R}_1\hat{R}_p\hat{R}_m^{-1}) = T_1\hat{R}_m^{-1}\hat{R}_p \]

\( < I_b > \) is given by
\[ < I_b > = \frac{1}{2} \frac{\text{Trace}\{-(\hat{R}_1 + \hat{R}_1^\dagger)\hat{B}_R(\hat{R}_1 + \hat{R}_1^\dagger)\hat{B}_R^\dagger - (\hat{R}_1 + \hat{R}_1^\dagger)(\hat{B}_R + \hat{B}_R^\dagger)\} > f_{3N}(0) \]

For the later convenience, we define \( < I_{b0} > \) with \( < I_{b0} >= < I_b > / f_{3N}(0) \). Then the final resulting expression of \( < I_{b0} > \) is given by the following equation,
\[ < I_{b0} >= \frac{T}{2} > \frac{C_0}{(2 - T) + T(\cos \theta_0 g_S + \sin \theta_0 f_S)^2} > \]  

\[ C_0 = T(1 + |\cos \theta_0|^2 + |\sin \theta_0|^2)[|g_S|^2 + |f_S|^2 + 1 + |\bar{f}_S|^2] \]
\[ + 4(2 - T)[\text{Real}(g_S)\text{Real}(\cos \theta_0) + \text{Real}(f_S)\text{Real}(\sin \theta_0)] \]
\[ + 4T[\text{Imag}(\cos \theta_0 \sin \theta_0^*\text{Imag}(f_S g_S^*))] \]

In the above, the definition of \( g_S, f_S \) and \( \bar{f}_S \) are given in eqs. (25), (26) and (27). By choosing \( \theta_0 = 0 \), we can reproduce well known results in ballistic limit [9, 48, 119].
D. Calculation of the conductance

The electric current is expressed using $\hat{G}_N(x)$ as

$$I_{el} = \frac{-L}{4eR_d} \int_0^\infty d\varepsilon \text{Tr} [\hat{\tau}_3 (\hat{G}_N(x) \frac{\partial \hat{G}_N(x)}{\partial x})^K],$$  \hspace{1cm} (38)

where $(\hat{G}_N(x) \frac{\partial \hat{G}_N(x)}{\partial x})^K$ denotes the Keldysh component of $(\hat{G}_N(x) \frac{\partial \hat{G}_N(x)}{\partial x})$. In the actual calculation, we introduce a parameter $\theta = \theta(x)$ which is a measure of the proximity effect in DN as described in the previous subsections, where we denoted $\theta(0) = \theta_0$. Using $\theta(x)$, $\hat{R}_N(x)$ can be denoted as

$$\hat{R}_N(x) = \hat{\tau}_3 \cos \theta(x) + \hat{\tau}_2 \sin \theta(x).$$  \hspace{1cm} (39)

$\hat{A}_N(x)$ and $\hat{K}_N(x)$ satisfy the following equations, $\hat{A}_N(x) = -\hat{\tau}_3 \hat{R}_N(x) \hat{\tau}_3$, and $\hat{K}_N(x) = \hat{R}_N(x) \hat{f}_1(x) - \hat{f}_1(x) \hat{A}_N(x)$ with the distribution function $\hat{f}_1(x)$ which is given by $\hat{f}_1(x) = f_{3N}(x) + \hat{\tau}_3 f_{3N}(x)$. In the above, $f_{3N}(x)$ is the relevant distribution function which determines the conductance of the junction we are now concentrating on. From the retarded or advanced component of the Usadel equation, the spatial dependence of $\theta(x)$ is determined by the following equation

$$\hbar D \frac{\partial^2}{\partial x^2} \theta(x) + 2i\varepsilon \sin[\theta(x)] = 0,$$  \hspace{1cm} (40)

while for the Keldysh component we obtain

$$D \frac{\partial}{\partial x} [\frac{\partial f_{3N}(x)}{\partial x} \cosh^2 \theta_{\text{imag}}(x)] = 0.$$  \hspace{1cm} (41)

with $\theta_{\text{imag}}(x) = \text{Imag}[\theta(x)]$. At $x = -L$, since DN is attached to the normal electrode, $\theta(-L) = 0$ and $f_{3N}(-L) = f_{t0}$ are satisfied with

$$f_{t0} = \frac{1}{2} \{ \tanh[(\varepsilon + eV)/(2k_B T)] - \tanh[(\varepsilon - eV)/(2k_B T)] \},$$

where $V$ is the applied bias voltage. As shown in our previous paper, from the Keldysh part of Eq. (41), we obtain

$$\frac{L}{R_d} \left( \frac{\partial f_{3N}}{\partial x} \right) \cosh^2 \text{Imag}(\theta_0) \bigg|_{x=0} = -\frac{<I_b>}{R_b}.$$  \hspace{1cm} (42)

After a simple manipulation, we can obtain $f_{3N}(0_-)$

$$f_{3N}(0_-) = \frac{R_b f_{t0}}{R_b + \frac{R_d <f_{t0}>}{L} \int_0^L \cosh^2 \theta_{\text{imag}}(x) \, dx}.$$
Since the electric current $I_{el}$ can be expressed via $\theta_0$ in the following form

$$I_{el} = \frac{L}{eR_d} \int_0^\infty \left( \frac{\partial f_{3N}}{\partial x} \right) |_{x=0} \cosh^2[\text{Imag}(\theta_0)]d\varepsilon,$$

we obtain the following final result for the current

$$I_{el} = \frac{1}{e} \int_0^\infty d\varepsilon \frac{-R_b}{<I_{b0}>} + \frac{R_d}{L} \int_0^L \frac{dx}{\cosh^2 \theta_{\text{imag}}(x)}.$$  \hspace{1cm} (43)

Then the total resistance $R$ at zero temperature is given by

$$R = \frac{R_b}{<I_{b0}>} + \frac{R_d}{L} \int_{-L}^L \frac{dx}{\cosh^2 \theta_{\text{imag}}(x)}.$$ \hspace{1cm} (44)

In the following section, we will discuss the normalized conductance $\sigma_T(eV) = \sigma_S(eV)/\sigma_N(eV)$ where $\sigma_S(N)(eV)$ is the voltage-dependent conductance in the superconducting (normal) state given by $\sigma_S(eV) = 1/R$ and $\sigma_N(eV) = \sigma_N = 1/(R_d + R_b)$, respectively.

It should be remarked that in the present theory, $R_d/R_b$ can be varied independently of $T$, i.e. of $Z$, since we can change the magnitude of the constriction area independently. In the other words, $R_d/R_b$ is no more proportional to $T_{av} (L/l)$, where $T_{av}$ is the averaged transmissivity and $l$ is the mean free path in the diffusive region, respectively. Based on this fact, we can choose $R_d/R_b$ and $Z$ as independent parameters.

\section*{III. RESULTS}

In this section, we focus on the line shapes of the conductance of DN/TS junctions where $p$-wave symmetry is chosen as a pairing symmetry of triplet superconductor (TS). The pair potentials $\Delta_\pm$ are given by $\Delta_\pm = \pm \Delta_0 \cos[(\phi \mp \alpha)]$ where $\alpha$ denotes the angle between the normal to the interface and the lobe direction of the $p$-wave pair potential and $\Delta_0$ is the maximum amplitude of the pair potential. In the above, $\phi$ denotes the injection angle of the quasiparticle measured from the $x$-axis. In the following, we choose $0 \leq \alpha \leq \pi/2$. It is known that quasiparticles with injection angle $\phi$ with $-\pi/2 + \alpha < \phi < \pi/2 - \alpha$ feel the MARS at the interface and induce ZBCP.

\subsection*{A. Line shapes of the conductance}

Let us first choose $\alpha = 0$ where all quasiparticles feel MARS. For $\alpha = 0$, we also call $p_x$-wave case in the following. In this case, at $eV = 0$, the total resistance of the junction
$R$ is $R_0/2$ and completely independent of $R_d$ as shown in our previous paper [54]. Thus the resulting $\sigma_T(0)$ is $\sigma_T(0) = 2(R_d + R_b)/R_0$. In Figs. 2 and 3, voltage dependent conductance $\sigma_T(eV)$ is plotted for low (Fig. 2) and high transparent (Fig. 3) interface. For $Z = 3$ (Fig. 2), we always expect ZBCP independent of the detailed value of $R_d/R_b$. With the increase of the magnitude of $R_d/R_b$, the width of the ZBCP becomes narrow (see curves b and c in the left panel), while its height is drastically enhanced. The width of the ZBCP is rather reduced by choosing small magnitude of $E_{Th}$ (see curves b and c in the right panels of Fig. 2).

![Normalized conductance $\sigma_T(eV)$ for $Z=3$, and $\alpha = 0$. $E_{Th} = \Delta_0$ for the left panel and $E_{Th} = 0.02\Delta_0$ for the right panel, respectively. a, $R_d/R_b = 0$; b, $R_d/R_b = 0.1$; and c, $R_d/R_b = 1$.](image)

FIG. 2: Normalized conductance $\sigma_T(eV)$ for $Z=3$, and $\alpha = 0$. $E_{Th} = \Delta_0$ for the left panel and $E_{Th} = 0.02\Delta_0$ for the right panel, respectively. a, $R_d/R_b = 0$; b, $R_d/R_b = 0.1$; and c, $R_d/R_b = 1$.

Similar plots for $Z = 0$ is shown in Fig. 3. In this case, $R_b = R_0$ is satisfied due to the absence of the insulating barrier at the interface. For $R_d/R_b = 0$, $\sigma_T(eV)$ has a broad ZBCP due to the absence of the normal reflection at the interface [6]. With the increase of the magnitude of $R_d/R_b$, the width of the ZBCP is reduced (see curves b and c both in the left and right panels), while its height is enhanced. As seen from the right panel of Fig. 2, the width of the ZBCP is reduced drastically with the decrease of the magnitude of $E_{Th}$.

In Fig. 4, the corresponding plots for DN/USS (diffusive normal metal / insulator / unconventional singlet superconductor )junctions are shown with $d$-wave superconductor, where the misorientation angle between the normal to the interface and the crystal axis is chosen to be $\beta = \pi/4$. For $\beta = \pi/4$, we also call $d_{xy}$-wave case in the following. $\Delta_{\pm}$ is given by $\Delta_{\pm} = \pm\Delta_0\sin(2\theta)$. Although quasiparticles feel MARS independent of their injection angles, as shown in our previous papers [118], the proximity effect and MARS strongly compete with each other and the proximity effect is completely absent. $\sigma_T(eV)$ is independent of the value of $E_{Th}$. Both for $Z = 0$ and $Z = 3$, the magnitude of $\sigma_T(eV)$ is
reduced monotonically with the increase of $R_d/R_b$. This feature is completely different from that in DN/TS junctions as shown in Figs. 2 and 3.

Next, we look at $\alpha$ dependence of $\sigma_T(eV)$ for DN/TS junctions. In this case, an injected quasiparticle with the injection angle $\phi$ with $-\pi/2 + \alpha < \phi < \pi/2 - \alpha$ feel the MARS. It is shown in our previous paper, that only the quasiparticle channel with this injection angle can contribute to the proximity effect. Only at $\alpha = \pi/2$, neither proximity effect nor MARS exist. As shown in Fig. 5 ZBCP is absent for $\pi/2$. In other cases, $\sigma_T(eV)$ has a ZBCP.

In order to clarify the proximity effect in DN, it is necessary to focus on the local density of states (LDOS) of the quasiparticles in DN region. In Fig. 6 LDOS of DN/TS junctions is plotted for $\alpha = 0$ with $E_{Th} = \Delta_0$ and $E_{Th} = 0.02\Delta_0$.

In Fig. 7 the corresponding plot with various $\alpha$ for $E_{Th} = \Delta_0$ is shown. $\rho(\varepsilon)$ is the normalized LDOS by its value in the normal state, where $\varepsilon$ denotes the quasiparticle energy measured from the Fermi surface. The curves $a$, $b$ and $c$ denote the LDOS at $x = -L/4$, $-L/2$ and $-L$, respectively. Since DN is connected to the normal electrode at $x = -L$, $\rho(\varepsilon) = 1$ is satisfied independent of $\varepsilon$ as shown in curves $c$ in Figs. 6 and 7. As shown in Fig. 8 the $\rho(\varepsilon)$ has a zero energy peak (ZEP) in DN (curves $a$ and $b$ in left and right panels). Even if $\alpha$ deviates from $0$, ZEP in LDOS does not vanish (curves $a$ and $b$ in the left and middle panels of Fig. 7). This is because that quasiparticles with injection angle $\phi$ with $-\pi/2 + \alpha < \phi < \pi/2 - \alpha$ feel MARS and can contribute to the proximity effect. Exceptional case is $\alpha = \pi/2$, where proximity effect is absent. Then the resulting $\rho(\varepsilon) = 1$ independent of $\varepsilon$.

In order to compare with DN/USS cases, we choose $d$-wave pair potential with $\Delta_\pm =$

![FIG. 3: Normalized conductance $\sigma_T(eV)$ for Z=0, and $\alpha = 0$. $E_{Th} = \Delta_0$ for the left panel and $E_{Th} = 0.02\Delta_0$ for the right panel, respectively. a, $R_d/R_b = 0$; b, $R_d/R_b = 0.1$; and c, $R_d/R_b = 1.$](image)
FIG. 4: Normalized conductance $\sigma_T(eV)$ for DN/USS junctions with $d_{xy}$-wave superconductor. $Z = 3$ for the left panel and $Z = 0$ for the right panel. a, $R_d/R_b = 0$; b, $R_d/R_b = 0.1$; and c, $R_d/R_b = 1$.

$\Delta_0 \cos[2(\theta \mp \beta)]$, where $\beta$ denotes the angle between the normal to the interface and the crystal axis of $d$-wave superconductor [118]. The corresponding $\rho(\varepsilon)$ is shown in Fig. 8. $\rho(\varepsilon)$ is nearly constant as a function of $\varepsilon$ and does not have a sharp ZEP (Figs. 6 and 7). In this case, although quasiparticles with injection angle $\pi/4 - \beta < | \phi | < \pi/4 + \beta$ feel MARS, they can not contribute to the proximity effect. For $\beta = 0$, MARS is absent and proximity effect becomes conventional one (see also Fig. 1 of [118]). $\rho(\varepsilon)$ at $x = -L/4$ has a gap like structure (curve a in the left panel of Fig. 8). Although $\rho(\varepsilon)$ at $x = -L/4$ has a broad peak like structure for $\beta = \pi/8$, $\rho(0) \leq 1$ is satisfied contrary to the DN/TS junction’s case. For $\beta = \pi/4$, although $\sigma_T(eV)$ has a ZBCP, due to the absence of the proximity effect, $\rho(\varepsilon) = 1$ for any case. Thus we can conclude that line shapes of $\rho(\varepsilon)$ in DN region of DN/TS junctions are significantly different from those of DN/USS junctions.

FIG. 5: Normalized conductance $\sigma_T(eV)$ for $Z=3$, and $R_d/R_b = 1$. $E_{Th} = \Delta_0$ for the left panel and $E_{Th} = 0.02\Delta_0$ for the right panel, respectively. a, $\alpha = 0$; b, $\alpha = \pi/4$; and c, $\alpha = \pi/2$. 

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FIG. 6: Normalized local density of states $\rho(\varepsilon)$ in DN for $Z=3$, $R_d/R_b = 1$ and $\alpha = 0$. $E_{Th} = \Delta_0$ (left panel) and $E_{Th} = 0.02\Delta_0$ (right panel). a, $x = -L/4$; b, $x = -L/2$; and c, $x = -L$.

FIG. 7: Normalized local density of states $\rho(\varepsilon)$ in DN for $Z=3$, $R_d/R_b = 1$ and $E_{Th} = \Delta_0$. $\alpha = \pi/4$ (left panel), $\alpha = 3\pi/8$ (middle panel), and $\alpha = \pi/2$ (right panel). a, $x = -L/4$; b, $x = -L/2$; and c, $x = -L$.

In Figs. 9 and 10, the proximity parameter $\theta_0 = \theta(x = 0_{-})$ is plotted as a function of $\varepsilon$. In Fig. 11, $\alpha$ dependence of the $\theta_0$ is plotted. The magnitude of $\text{Real}(\text{Imag})(\theta_0)$ is increased with the increase of $R_d/R_b$. Contrary to the case of DN/USS junctions, $\text{Real}(\theta_0)$ vanishes at $\varepsilon = 0$ while $\text{Imag}(\theta_0)$ remains nonzero at $\varepsilon = 0$. For fully transparent case (Fig. 9), $\text{Real}(\theta_0)$ and $\text{Imag}(\theta_0)$ decrease with the increase of $\varepsilon/\Delta_0$ for $E_{Th} = \Delta_0$. For $E_{Th} = 0.02\Delta_0$, $\text{Real}(\theta_0)$ decreases from 0 and has a minimum at about $\varepsilon \simeq 0.02\Delta_0$. $\text{Imag}(\theta_0)$ decreases rather rapidly as compared to the case with $E_{Th} = \Delta_0$. For low transparent case with $Z = 3$ (Fig. 10), $\text{Real}(\theta_0)$ has a sudden change around $\varepsilon \simeq 0$. This sudden change is remarkable for the large magnitude of $R_d/R_b$ (see curves b and c). With the increase of the magnitude of $\alpha$, the resulting magnitude of $\theta_0$ is reduced as shown in Fig. 11. For $\alpha = \pi/2$, the resulting $\theta_0$ is zero.

As seen from the sudden change of $\theta_0$ near zero energy for low transparent junction, we
FIG. 9: Real(upper panels) and imaginary parts(lower panels) of $\theta_0$ are plotted as a function of $\varepsilon$ for $\alpha = 0$ and $Z = 0$ with $E_{Th} = \Delta_0$ (left panels) and $E_{Th} = 0.02\Delta_0$ (right panels). a, $x = -L/4$; b, $x = -L/2$; and c, $x = -L$.

FIG. 8: Normalized local density of states $\rho(\varepsilon)$ in DN for DN/USS junction with $Z = 3$, $R_d/R_b = 1$ and $E_{Th} = \Delta_0$. We have chosen $d$-wave superconductor with $\Delta_{\pm} = \Delta_0 \cos[2(\theta \mp \beta)]$. $\beta = 0$ (left panel), $\beta = \pi/8$ (middle panel), and $\beta = \pi/4$ (right panel). a, $x = -L/4$; b, $x = -L/2$; and c, $x = -L$.

can imagine that there is a very small energy scale. Actually, as seen from Fig. 2, the width of $\sigma_T(eV)$ is significantly reduced for low transparent junction with the increase of the magnitude of $R_d$. The sudden reduction of the magnitude of $\sigma_T(eV)$ around $eV \sim 0$ is related to the sudden change of $\theta_0$ around $\varepsilon = 0$.

In order to understand this small energy scale, we focus on the half width of the zero bias conductance peak of $\sigma_T(eV)$. We define the half width $E_C$ as $\sigma_T(E_C) = \frac{1}{2}\sigma_T(0)$ and $\rho(E_\rho) = \frac{1}{2}\rho(0)$, respectively. In Fig. 12, $E_C$ is plotted as a function of $R_d/R_0$ for various $Z$. 
FIG. 10: Real(upper panels) and imaginary parts(lower panels) of $\theta_0$ are plotted as a function of $\varepsilon$ for $\alpha = 0$ and $Z = 3$. $E_{Th} = \Delta_0$ (left panels) and $E_{Th} = 0.02\Delta_0$ (right panels). a, $R_d/R_b = 0.1$; b, $R_d/R_b = 0.5$; and c, $R_d/R_b = 1$.

FIG. 11: Real(upper panels) and imaginary parts(lower panels) of $\theta_0$ are plotted as a function of $\varepsilon$ for $R_d/R_b = 1$ and $Z = 3$. $E_{Th} = \Delta_0$ (left panels) and $E_{Th} = 0.02\Delta_0$ (right panels). a, $\alpha = 0$; b, $\alpha = 0.25\pi$; and c, $\alpha = 0.375\pi$.

If the magnitude of $R_d/R_0$ is large, $E_C$ can be expressed by $E_C/\Delta_0 \sim E_{C1} \exp(-C_C R_d/R_0)$, where $E_{C1}$ and $C_C$ are independent of $R_d$. It is interesting that the magnitude of $C_C$ is almost independent of $Z$. As seen from left and right panels, the magnitude of $C_C$ is almost
constant by the change of $E_{Th}$. It is a very unique feature of the present proximity effect that $E_C$ has an exponential dependence of $R_d$ and the magnitude of $E_C$ is drastically reduced with the increase of $R_d$. The corresponding $\rho(\varepsilon)$ also has a sharp ZEP as shown in Fig. 6.

In order to understand this small energy scale, we define $E_\rho$ as $\rho(E_\rho) = \frac{1}{2}\rho(0)$. In Fig. 13, $E_\rho$ is plotted as a function of $R_d/R_0$. $E_\rho$ can be expressed by $E_\rho/\Delta_0 \sim E_{\rho 1} \exp(-C_\rho R_d/R_0)$ for the large magnitude of $R_d/R_0$ where $E_{\rho 1}$ and $C_\rho$ are independent of $R_d$. Comparing the curvature of curves from $a$ to $c$, the magnitude of $C_\rho$ is almost independent of $Z$. As seen from left and right panels, the magnitude of $C_\rho$ is almost constant by the change of $E_{Th}$. The magnitude of $E_\rho$ is drastically suppressed with the increase of $R_d$ as in the case of $E_C$.

**B. Properties at zero voltage**

In this subsection, we focus on the zero voltage properties of DN/TS junctions with $p$-wave superconductor, where we can obtain several analytical results without solving Usadel
equation numerically. The $\theta_0$ is given by $2iR_d \cos \alpha / R_0$ by solving eq. (4). By using eqs. (37) and (44), the zero voltage resistance $R$ is given by

$$R = R_0 \left\{ \frac{\tanh \theta_{0i}}{I_1} + \frac{2}{[1 + \exp(2 | \theta_{0i} |)]I_1 + \cosh^2 \theta_{0i} I_2} \right\}$$

(45)

$$I_1 = \int_{-\pi/2+\alpha}^{\pi/2-\alpha} \cos \phi d\phi$$

$$I_2 = \int_{-\pi/2}^{-\pi/2+\alpha} \frac{2T^2(\phi) \cos \phi}{[2 - T(\phi)]^2} d\phi + \int_{\pi/2-\alpha}^{\pi/2} \frac{2T^2(\phi) \cos \phi}{[2 - T(\phi)]^2} d\phi$$

with $\theta_{0i} = 2R_d \cos \alpha / R_0$. First, we calculate the zero-voltage resistance ($\varepsilon \rightarrow 0$) at different values of $\alpha$ as a function of $R_b / R_0$ for the DN/TS junctions with two extreme cases (curves $a$ and $b$ of Fig. 14). For $\alpha = \pi/2$, namely the $p_y$-wave case, $R$ increases linearly as a function of $R_d$, where no proximity effect appears (curve $a$ of Fig. 14). For $\alpha = 0$, $p_x$-wave case, where $R = R_0/2$ is satisfied independent of $R_d$ (curve $b$ of Fig. 14). This anomalous $R$ dependence is a most striking feature by the enhanced proximity effect by the MARS. The corresponding result for the DN/CSS junctions with $s$-wave (curve $c$) and DN/USS junctions with $d_{xy}$-wave case (curve $d$) is also plotted as a reference. For $s$-wave case, it is well known [96] that $R$ has a reentrant behavior $\partial R / \partial R_d |_{R_d=0} < 0$ as shown in curve $c$ of Fig. 14. In $p$-wave cases, this reentrant behavior of $R$ does not appear. Actually, from the relation

$$\frac{\partial R}{\partial R_D} \bigg|_{R_D=0} = 1 - \frac{\cos^2 \alpha}{\cos \alpha + \int_{\pi/2-\alpha}^{\pi/2} \frac{T^2(\phi) \cos \phi}{[2 - T(\phi)]^2} d\phi}^2,$$

(46)

we can show that $dR / dR_D |_{R_D=0} > 0$ is always satisfied. For $d_{xy}$-wave case, due to the formation of the MARS for all $\phi$ as in the case of $p_x$-wave junction, $R/R_0 = R_0/(2R_b)$ at $R_d = 0$, which is identical to that for $p_x$-wave junction (curve $d$ of Fig. 14). However, for nonzero $R_d$, $R/R_b = R_0/(2R_b) + R_d/R_b$ is satisfied, due to the absence of the proximity effect as discussed in our previous papers [117, 118].

The $\alpha$ dependence of $R$ is shown in Fig. 15. For $Z = 0$, as shown in curves $b$ and $c$, $R$ increases gradually with $R_d/R_b$. However, for the large magnitude of $R_d/R_b$, the ratio of the increment is saturated and $R$ is almost constant as a function of $R_d/R_b$. For $Z = 3$, $R$ is nearly constant with the increase of $R_d/R_b$. As seen from Eq. (15), for sufficiently large magnitude of $R_d/R_0$, the second term in Eq. (15) becomes negligibly small and $R$ converges to $R_0/(2 \cos \alpha)$ independent of the magnitudes of $Z$ and $R_b$. On the other hand, for small transparent junctions, i.e., the large magnitudes of $Z$, we can neglect the term proportional...
FIG. 14: Total zero voltage resistance of the junctions $R$ is plotted as a function of $R_d/R_b$ for $Z = 1.5$ with a, $\alpha = 0.25\pi$; and b, $\alpha = 0.4\pi$. The curve c and d presents the same dependence for the DN/CSS junctions with $s$-wave superconductor and DN/USS junctions with $d_{xy}$-wave superconductor, respectively.

To $I_2$ in Eq. (45). In such a case, $R$ converges to be $R_0/(2\cos\alpha)$ independent of $R_d$. For $Z = 3$ (see the right panel of Fig. 15), $R$ saturates to be constant much more rapidly with the increase of $R_d/R_b$ as compared to the corresponding case for $Z = 0$ (see the right panel of Fig. 15).

FIG. 15: Total zero voltage resistance of the junctions $R$ is plotted as a function of $R_d/R_b$ for $Z = 0$ (left panel) and $Z = 3$ (right panel). a, $\alpha = 0$; b, $\alpha = \pi/2$; and c, $\alpha = 0.4\pi$.

Before closing this subsection, we pay attention to the zero voltage electrostatic potential distribution at zero temperature $\phi_0(x)$. Due to the enhanced proximity effect by the MARS, the spatial dependence of $\phi_0(x)$ is very unusual. The electrostatic potential in the DN region is expressed by

$$
\Phi(x) = \frac{1}{4} \int_0^\infty d\varepsilon \text{Trace}[\hat{K}_N(x)] = \int_0^\infty d\varepsilon f_{3N}(x) \text{Real}[\cos \theta].
$$

(47)
The zero-bias voltage static potential distribution is defined by (48)

\[ \phi_0(x) = \lim_{V \to 0} \frac{\Phi(x)}{V} \]

Since \( f_{3N}(x) \) is expressed by

\[ f_{3N}(x) = \frac{R_b + \frac{R_d}{L} \int_0^x \frac{dx}{\cosh^2 \theta_{imag}(x)} f_{00}}{R_b + \frac{R_d}{L} \int_{-L}^0 \frac{dx}{\cosh^2 \theta_{imag}(x)}} \]

with \( \theta_{imag}(x) = \text{Imag}[\theta(x)] \), the resulting \( \phi_0(x) \) is given by

\[ \phi_0(x) = \text{Real}[\cos \theta(x)] \frac{R_b + \frac{R_d}{L} \int_0^x \frac{dx}{\cosh^2 \theta_{imag}(x)}}{R_b + \frac{R_d}{L} \int_{-L}^0 \frac{dx}{\cosh^2 \theta_{imag}(x)}} \]

at zero temperature. In Fig. 16, \( x \) dependence of \( \phi_0(x) \) is plotted both DN/USS junction with \( d_{xy} \)-wave superconductor and DN/TS junction with \( p_x \)-wave superconductor. Although quasiparticles always feel MARS both for \( d_{xy} \)-wave and \( p_x \)-wave cases, for \( d_{xy} \)-wave case the proximity effect is completely absent, while for \( p_x \)-wave case, proximity effect is enhanced by MARS. For \( d_{xy} \)-wave case, since \( \theta(x) = 0 \), \( \phi_0(x) \) is given by

\[ \phi_0(x) = \frac{LR_0 - 2xR_d}{L(R_0 + 2R_d)} \] (51)

Actually, as shown in Fig. 16, \( \phi_0(x) \) has a linear \( x \) dependence and decreases with \( x \). This linear dependence corresponds to the ohm’s rule. The magnitude of \( \phi_0(x) \) is much reduced with the increase of \( R_d/R_b \) (see curves c and d). For \( p_x \)-wave case, since \( \theta(x) = 2iR_d/R_0 \) is satisfied, the resulting \( \phi_0(x) \) is

\[ \phi_0(x) = \exp\left(-\frac{2(x + L)}{L} \frac{R_d}{R_0}\right) \] (52)

Due to the enhanced proximity effect by MARS, \( \phi_0(x) \) is reduced drastically with the increase of \( R_d/R_b \) (see curves c and d).

C. Magnetic field dependence of the conductance

From the experimental view point, it is interesting to clarify how ZBCP in DN/TS junctions are influenced by the applied magnetic field \( H \). It is known for conventional superconductor junctions, that coherence of electrons by the proximity effect is broken by the applied magnetic field or the magnetic impurity in DN [99, 102] as shown in the following equation

\[ \hbar D \frac{\partial^2}{\partial x^2} \theta + 2i \varepsilon \sin \theta - \frac{\hbar}{\tau_H} \sin 2\theta = 0 \] (53)
FIG. 16: Zero voltage electrostatic potential distribution at zero temperature $\phi_0(x)$ is plotted as a function of $x$ for $Z = 3$. DN/USS junction with $d_{xy}$-wave superconductor (left panel) and DN/TS junction with $p_x$-wave superconductor (left panel). a: $R_d/R_b = 0$, b: $R_d/R_b = 0.1$, c: $R_d/R_b = 1$, and d: $R_d/R_b = 2$.

with $h/\tau_H = 6w^2D^2H^2$, where $w$ is the width of the DN region and $H$ is the applied magnetic field. With the increase of the magnetic field, the height of the ZBCP is suppressed as shown in Fig. 17. The sharp peak structure around the zero voltage is significantly suppressed when the magnitude of $h/\tau_H$ is almost $10E_{Th}$. This is because the magnitude of $\text{Im}\theta_0$ is significantly reduced by the increase of the magnitude of $h/\tau_H$.

FIG. 17: Normalized conductance $\sigma_T(eV)$ for DN/TS junctions with $Z = 1.5$ and $R_d/R_b = 1$ where $p$-wave superconductor with $\alpha = 0$ is chosen. We choose $E_{Th} = \Delta_0$ for the left panel and $E_{Th} = 0.1\Delta_0$ for the right panel, respectively. a, $h/\tau_H = 0$; b, $h/\tau_H = E_{Th}$; and c, $h/\tau_H = 10E_{Th}$.

In order to understand the suppression of the proximity effect, we also look at $\rho(\varepsilon)$ in the DN region. The height of the ZEPs in $\rho(\varepsilon)$ both in the left ($E_{Th} = \Delta_0$) and right panels ($E_{Th} = 0.1\Delta_0$) is reduced seriously with the increase of the magnitude of $h/\tau_H$. For $h/\tau_H = 10E_{Th}$, $\rho(\varepsilon) \approx 1$ for both cases (see curves c in the left and right panels in Fig. 18). In this case, the proximity effect is almost absent and the ZBCP in $\sigma_T(eV)$ in curves...
c in Fig. 17 originates purely from the formation of the MARS at the DN/TS interface. The corresponding situation is realized in DN/USS junction with $d_{xy}$-wave superconductor (see Fig. 4). Here, we estimate the threshold magnetic field which suppresses the sharp ZBCP or ZEP originating from the enhanced proximity effect. Here, $\hbar/\tau_H$ and $E_{Th}$ are given by $e^2w^2DH^2/(6\hbar)$ and $\hbar D/L^2$, respectively. We can define threshold magnetic field as $10E_{Th} = e^2w^2DH^2_{Th}/(6\hbar)$. We can easily show that $H_{Th} \simeq \frac{8\hbar}{eLw}$. By choosing $L = 1 \times 10^{-6}$[m] $w = 1 \times 10^{-6}$[m], the resulting $H_{Th}$ is $5 \times 10^{-3}$[Tesla].

FIG. 18: Normalized local density of states of quasiparticle $\rho(eV)$ for DN/TS junctions with $Z = 1.5$ and $R_d/R_b = 1$ where $p$-wave superconductor with $\alpha = 0$ is chosen. We choose $x = -L/4$. $E_{Th} = \Delta_0$ for the left panel and $E_{Th} = 0.1\Delta_0$ for the right panel, respectively. a, $\hbar/\tau_H = 0$; b, $\hbar/\tau_H = E_{Th}$; and c, $\hbar/\tau_H = 10E_{Th}$.

D. Conductance for $p+ip$-wave superconductor

In this subsection, we discuss the case where $p_x + ip_y$-wave superconductor is chosen as TS(triplet superconductor). Stimulated by intensive studies about Sr$_2$RuO$_4$ after its discovery [2], $p_x + ip_y$-wave superconductor, which is a $p$-wave superconductor with broken time symmetry, has a wide interest at the present. In this case, one of the important feature is that the MARS is formed only by the quasiparticle with perpendicular injection. The position of the resonance energy strongly depends on the injection angle of the quasiparticles. Also in this case, we can calculate the corresponding conductance $\sigma_T(eV)$ following similar calculations as in section 2. Since the derivation of the matrix current and the relevant $<I_{\delta 0}>$ is rather long, we will only show the final results. The details of the calculations will be shown in elsewhere [120]. First, we look at the case with $R_d = 0$, i.e., ballistic
junctons. As shown in curves a in the left and right panels of Fig. 19, $\sigma_T(eV)$ has a rather broad peak. This is because that the position of the resonance energy deviates with the increase of the magnitude of the quasiparticle energy $\epsilon$. For $E_{Th} = \Delta_0$, the magnitude of $\sigma_T(0)$ is suppressed with the increase of $R_d/R_b$, contrary to the case with DN/TS junctions without BTRSS as shown in Figs. 2, 3, and 5. For $E_{Th} = 0.02\Delta_0$, with the increase of $R_d/R_b$, $\sigma_T(0)$ has a sharp and narrow peak around zero voltage. This feature becomes prominent with the increase of $R_d/R_b$.

FIG. 19: Normalized conductance $\sigma_T(eV)$ for DN/TS junctions with $p_x + ip_y$ superconductor for $Z = 3$ is plotted. We choose $E_{Th} = \Delta_0$ for the left panel and $E_{Th} = 0.02\Delta_0$ for the right panel, respectively. a, $R_d/R_b = 0$; b, $R_d/R_b = 0.1$; and c, $R_d/R_b = 1$.

The corresponding plot for $\rho(\epsilon)$ is shown in Fig. 20. $\rho(\epsilon)$ is the normalized LDOS by its value in the normal state, where $\epsilon$ denotes the quasiparticle energy measured from the Fermi surface. The curves a, b and c denote the LDOS at $x = -L/4$, $x = -L/2$ and $x = -L$, respectively. Since DN is connected to the normal electrode at $x = -L$, $\rho(\epsilon) = 1$ is satisfied independent of $\epsilon$ as shown in curves c in the left and right panels of Fig. 20. $\rho(\epsilon)$ has a zero energy peak (ZEP) in DN (curves a and b). Although only the quasiparticles with perpendicular injection at the DN/TS interface feel the MARS, the ZEP in LDOS remains as in the case in Figs. 6 and 7. The line shapes of LDOS for $p_x + ip_y$-wave case is significantly different from the corresponding ones for DN/USS junctions with $d$-wave superconductor (see Fig. 8). The existence of the ZEP in LDOS of DN is a remarkable feature peculiar to triplet superconductor. We think that this peculiar property which has never been expected in singlet junctions may serve as a guide to identify the triplet superconducting state.
IV. CONCLUSIONS

In the present paper, detailed theoretical investigation of the voltage-dependent conductance of diffusive normal metal / insulator / triplet superconductor (DN/TS) junctions is presented. We have provided the detailed derivation of the formula of the matrix current presented in our previous paper [54]. For the reader’s convenience, we explicitly present the retarded and the Keldysh parts of the matrix current in the DN/TS (diffusive normal metal / triplet superconductor) junctions. Applying our general formula to DN/insulator/p-wave superconductor junctions, we have obtained the following main results. In the present paper, by changing the barrier parameter at the interface $Z$, the resistance $R_b$ at the DN/TS interface, the resistance $R_d$ in DN, the Thouless energy $E_{Th}$ in DN and the angle $\alpha$ between the normal to the interface and the lobe direction of $p$-wave superconductor, we have studied the charge transport in DN/TS junctions in detail.

1. The zero bias conductance peak (ZBCP) is always seen in the line shape of $\sigma_T(eV)$ except for $\alpha = \pi/2$. The magnitude of $\sigma_T(0)$ is drastically enhanced with the increase of $R_d/R_b$. On the other hand, $\sigma_T(eV)$ for diffusive normal metal / insulator / unconventional singlet superconductor (DN/USS) junctions with $d_{xy}$-wave superconductor shows a very different behavior. The magnitude of $\sigma_T(eV)$ is reduced with the increase of $R_d/R_b$ due to the absence of the proximity effect. For the large magnitude of $R_d/R_0$, the half width of the ZBCP is proportional to $\exp(-C_C R_d/R_0)$, where $R_0$ is the Sharvin resistance at the DN/TS interface. $C_C$ is almost constant and independent of $Z$ and $E_{Th}$.

\[ \text{FIG. 20: Normalized local density of states } \rho(\varepsilon) \text{ for DN/TS junctions with } p_x + ip_y \text{-wave superconductor is plotted for } Z = 3. \text{ We choose } E_{Th} = \Delta_0 \text{ for the left panel and } E_{Th} = 0.02\Delta_0 \text{ for the right panel, respectively. a, } x = -L/4; \text{ b, } x = -L/2; \text{ and c, } x = -L. \]
2. The LDOS has a ZEP except for the case with $\alpha = \pi/2$ where MARS is absent. The height of the ZEP is significantly enhanced with the increase of $R_d/R_0$. For the large magnitude of $R_d/R_0$, the half width of the ZEP is proportional to $\exp(-C_\rho R_d/R_0)$. $C_\rho$ is almost constant and independent of $Z$ and $E_{Th}$.

3. The proximity parameter at the DN/TS interface $\theta_0$ is $2i\cos\alpha R_d/R_0$ at $\varepsilon = 0$ where quasiparticle energy $\varepsilon$ is measured from the Fermi energy. This unique feature has never been expected for DN/USS or diffusive normal metal / insulator / conventional s-wave singlet superconductor (DN/CSS) case, where $\theta_0$ at $\varepsilon = 0$ is always a real number.

4. The total zero voltage resistance $R$ in the DN/TS junctions is significantly reduced by the enhanced proximity effect in the presence of the MARS. It is remarkable that when $R_d$ is sufficiently larger than the Sharvin resistance $R_0$, $R$ is reduced to be $R = R_0/(2 \cos \alpha)$, which can become much smaller than the preexisting lower limit value of $R$, i.e., $R_0/2 + R_d$. For low transparent junctions, $R$ is also reduced to be $R = R_0/(2 \cos \alpha)$ for any $R_d$. When all quasiparticles injected at the interface feel the MARS, $R$ is reduced to be $R = R_0/2$ irrespective of the magnitude of $R_d$ and $R_0$. This dramatic situation is realized for $\alpha = 0$.

5. The sharp ZBCP or ZEP in LDOS due to the enhanced proximity effect is sensitive to the applied magnetic field $H$. The height of ZBCP or ZEP is significantly reduced by $H$. The threshold value of the magnetic field is $H_{Th} \simeq 8\hbar/(eS_{DN})$ where $S_{DN}$ denotes the magnitude of the area of DN region.

6. We also choose $p_x + ip_y$-wave state as a model of chiral superconductor. Although only quasiparticles with perpendicular injection feel MARS, $\sigma_T(eV)$ has a ZBCP and LDOS has a ZEP, due to the proximity effect.

These novel features have never been expected either in DN/CSS or DN/USS junctions. Here, we propose an experimental setup which discriminate triplet superconducting state from singlet one by Scanning tunneling spectroscopy (STS) as shown in Fig. 21. Only for triplet case, LDOS measured by STS has a ZEP. The spatial dependence of the height

![FIG. 21: Schematic illustration of the experimental setup to discriminate triplet superconducting state from singlet one. LDOS measured by STS only have a ZEP for DN/TS junctions.](image-url)
of ZEP can be observed by the proposed STS. According to our calculation discussed in the last section, we can obtain the LDOS for various $x$, where the positions of the DN/TS interface is denoted as $x = 0$. As in our previous section, we assume that DN is attached to the normal electrode (N) at $x = -L$. The spatial dependence of $\rho(\varepsilon)$ for $\varepsilon = 0$ is plotted in Fig. 22 both for $p_x$-wave and $p_x + ip_y$-wave cases. Both for two cases, the magnitude of $\rho(0)$ decreases monotonically with the increase of the magnitude of the absolute value of $x/L$, where the position of the tip of STS changes from the DN/TS interface to the DN/N interface. We hope above spatial dependence of LDOS will be observed near future. As far as we know, the experiment of Knight shift measuring the uniform susceptibility is an only promising way to detect the triplet superconducting state. Our proposal to identify the triplet superconducting state based on the proximity effect in the presence of the MARS is an innovational idea.

In the present paper, we have considered the normal metal / triplet or singlet superconductor junctions, where the mean free path in the normal metal region is much shorter than the length of it. Thus, it is possible to use Usadel equation. In the actual junctions, there is a possibility that the magnitude of the mean free path in normal metal becomes as the same
order as its length. To describe this situation, we have to solve Eilenberger equation where the strength of the impurity scattering can be changed as an input parameter. Very recently, Löfwander \cite{121} calculated the local density of states in normal metal /$d$-wave superconductor junctions based on the Eilenberger equation. His obtained result is consistent with our theory when the impurity scattering effect becomes significant. It is an interesting problem to study the charge transport in triplet junctions based on the Eilenberger equation.

It is also possible to calculate tunneling conductance by the numerical simulation in the lattice model, where impurity potential is introduced in DN, where conductance is calculated for various samples which have different configuration of randomness each other \cite{122, 123, 124, 125}. Although it is difficult to take the large size of the system, the merit of the simulation is to take into account the impurity scattering effect exactly. The prominent property, i.e., $R$ is completely independent of $R_d$ for $p_x$-wave junction, is verified by the numerical simulation \cite{125}.

A direct evidence of the mesoscopic interference effect by the proximity effect in unconventional superconductors has been reported in size dependence effect of high-$T_c$ superconductor junctions. When the junction size becomes smaller, the conductance spectra are shown to be modified from the ballistic features. Since the high sensitivity of the ZBCP to the applied field was observed, the experimental data are consistent with the present results \cite{126}. Another way to detect the phase coherence in DN is to observe the LDOS of normal conductor in the vicinity of unconventional superconductors by tunneling spectroscopy. There have been presented two different types of tunneling results on the LDOS observation of Au coated on YBCO. Experimental results of Ref. \cite{127} show the presence of gap structure both for Au/YBCO(100) and Au/YBCO(110), while Asulin et al. observed ZBCP on Au/YBCO(110) when Au is thinner than 7nm, while the ZBCP suddenly disappears as the Au layer becomes thicker \cite{128}. These experimental reports are consistent with the present theory \cite{118} in the point that MARS can not penetrate into DN in the case of $d$-wave superconductors. However, since both of these measurements were performed on planer bilayer system (not mesoscopic size junctions), and the Au overlayer cannot be regarded as DN, the detailed comparison of these results with the present theory seems to be inadequate. On the other hand, it is a challenging issue to make a junction using Sr$_2$RuO$_4$ where ZBCP is already reported in tunneling spectroscopy \cite{19}. In order to observe the enhanced proximity effect, we think the measurements in extremely small residual magnetic
field environments are necessary. This is because the ZBCP is easily suppressed even by the terrestrial magnetism.

There are several problems which have not been discussed in the present paper. In the present study, we have focused on N/S junctions. The extension of the circuit theory to long diffusive S/N/S junctions has been performed by Bezuglyi et al. [11]. In S/N/S junctions, the mechanism of multiple Andreev reflections produces the subharmonic gap structures on I-V curves [129, 130, 131, 132, 133, 134, 135] and the situation becomes much more complex as compared to N/S junctions. Moreover, in S/N/S junctions with unconventional superconductors, MARS leads to the anomalous current-phase relation and temperature dependence of the Josephson current [37]. An interesting problem is an extension of the circuit theory to S/N/S junctions with unconventional superconductors. Recent numerical simulation indicates the existence of the anomalous current phase relation in triplet junctions [136]. In the present paper, since we follow the quasiclassical Green’s function formalism, the impurity scattering is taken into account within the self-consistent Born approximation. It is a challenging problem to study the weak localization effects.

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