Is “Heavy Quark Damping Rate Puzzle” in Hot QCD Really the Puzzle?

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Within the framework of perturbative resummation scheme of Pisarski and Braaten, the decay- or damping-rate of a moving heavy quark (muon) to leading order in weak coupling in hot QCD (QED) is examined. Although, as is well known, the conventionally-defined damping rate diverges logarithmically at the infrared limit, shown is that no such divergence appears in the physically measurable decay rate. The cancellation occurs between the contribution from the “real” decay diagram and the contribution from the diagrams with “thermal radiative correction”.

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In hot gauge theories, a consistent perturbation scheme (the hard-thermal-loop (HTL) resummation scheme) has been established \[1, 2\]. This scheme enables consistent evaluations of any thermal reaction rates to leading order in the coupling constant. The HTL resummed propagators screen the infrared singularities, and render otherwise divergent physical quantities finite, if they are not sensitive to a further resummation of the corrections of \(O(g^2T)\). Much interest has been taken \[1, 3, 4\] in the so-called damping rate of a particle moving in a hot quark-gluon (electron-photon) plasma, because this is an example of those which are sensitive to the further resummation mentioned above. In other words, the damping rate of a moving particle is still divergent even within the HTL resummation scheme. It is anticipated \[1, 4\] that the further resummation makes the damping rate finite.

In this Letter, within the HTL resummation scheme \[1, 2\], we analyze the decay- or damping-rate of a moving heavy quark (muon) in a hot quark-gluon (electron-photon) plasma, in terms of a measurable quantity. We shall find that, although the damping rate as defined in a conventional way diverges, no such divergence occurs in the physically measurable decay rate. The mechanism of resulting the finite decay rate is the same as the familiar Bloch-Nordsieck mechanism, which operates to cancel the infrared divergences in reaction rates in vacuum theories \[3\].

We consider the heavy quark with velocity \(v = p/E (O(g) << v)\) injected into the quark-gluon plasma. By the term “heavy quark” we mean so heavy a quark that it is not thermalized, i.e., \(e^{-E/T} << 1\). The heavy muon immersed in the hot
electron-photon plasma may be treated in a parallel manner (see below). We employ
the real-time formalism of thermal field theory \[6\], which is formulated on the time
path \(-\infty \rightarrow +\infty \rightarrow -\infty \rightarrow -\infty - iT^{-1}\), in a complex time plane.

The decay rate of the heavy quark with any one of the color states is obtained as
\[7, 8\]
\[
R_d \equiv -i \frac{M}{E} \frac{1}{2} \sum_s \bar{u}_s(P) \Sigma_{21}(P) u_s(P),
\]
(1)
where \(\Sigma_{21}(P)\) with \(P = (E, \mathbf{p})\) is the (21)-component of the self-energy matrix
\(\Sigma_{ji}(P)\) \((i, j = 1, 2)\) of the heavy quark. Here \(i\) and \(j\) designate the type of heavy
quark-gluon vertex \[3, 8\], the type-1 (type-2) vertex stands for the vertex of physi-
cal or type-1 (thermal-ghost or type-2) fields. [The “Feynman ru-
les” in the above-men-
tioned real-time formalism is equivalent to the circled diagram rules of Kobes and
Semenoff \[7\], provided that the physical (thermal-ghost) field is identi-

ded with the field of “uncircled” (“circled”) type in \[7\].] To one-loop order in the HTL resummation
scheme, \(\Sigma_{21}\) is diagrammed in Fig. 1 with \(i = 1\) and \(j = 2\);
\[
\Sigma_{21}(P) = -i g^2 C_f \int \frac{d^4Q}{(2\pi)^4} \ast \Delta_{21}^{\mu\nu}(Q) \gamma_\mu S_{21}(P') \gamma_\nu,
\]
(2)
where \(C_f = 4/3\) and \(P' = P - Q\). In (2),
\[
S_{21}(P') = -2\pi i \delta(p_0') (\gamma \cdot P' + M) \delta(P'^2 - M^2),
\]
(3)
and \(\ast \Delta_{21}^{\mu\nu}(Q)\) is the (21)-component of the (HTL resummed) effective thermal gluon
propagator. For the heavy muon in the hot electron-photon plasma, in (2) and in the
following, \(g\) should read \(e\) and \(C_f = 1\).
Throughout this Letter, we keep only the terms that yield the leading contribution of \( O(g^2 \ln g) \), which, in fact, diverges logarithmically (cf. (8) and (9) below). It is well known [1, 3, 4] that the relevant terms come from the chromomagnetic part of \( \Delta_{21}^{\mu
u}(Q) \) in the soft \( Q = (q_0, q) \) region, \( |Q_\mu| \leq O(gT) \):

\[
(*) \Delta_{21}^{\mu
u}(Q)\text{mag} = \left( \delta^{\mu\nu} - \tilde{\delta}^{\mu\rho} \tilde{\delta}^{\nu\sigma} \hat{q}_\rho \hat{q}_\sigma \right) \left[ \theta(q_0) + n_B(|q_0|) \right] \\
\times \left[ \frac{1}{Q^2 - \Pi_t(q_0 + i\epsilon, q)} - \text{c.c.} \right],
\]

where \( \tilde{\delta}_{\mu\nu} = (0, 1, 1, 1) \), \( \hat{q} \equiv q/q = |q| \), and “c.c.” stands for complex conjugate.

In (4), \( n_B(|q_0|) \) is the Bose distribution function and \([1, 2] \)

\[
\Pi_t(q_0, q) = \frac{3}{2} m_T^2 \left[ \left( \frac{q_0}{q} \right)^2 + \frac{q_0 (q^2 - q_0^2)}{2 q^3} \ln \frac{q_0 + q}{q_0 - q} \right],
\]

\[
m_T^2 = \begin{cases} 
\frac{1}{9} \left( 3 + \frac{N_f}{2} \right) (gT)^2 & \text{for QCD} \\
\frac{1}{9} (\epsilon T)^2 & \text{for QED}
\end{cases},
\]

with \( N_f \) the number of quark flavors that constitutes the quark-gluon plasma. Here we introduce the spectral density

\[
\rho_t(q_0, q) = -\frac{1}{\pi} \text{Im} \frac{1}{Q^2 - \Pi_t(q_0 + i\epsilon, q)}.
\]

By noticing that the factor \( \theta(q_0) \) in (4) can be ignored, since \( n_B(|q_0|) \approx T/|q_0| \geq O(1/g) \gg 1 \), we can reduce (4) to

\[
(*) \Delta_{21}^{\mu
u}(Q)\text{mag} \simeq -2\pi i \left( \tilde{\delta}^{\mu\nu} - \tilde{\delta}^{\mu\rho} \tilde{\delta}^{\nu\sigma} \hat{q}_\rho \hat{q}_\sigma \right) \frac{T}{q_0} \rho_t(q_0, q).
\]

Substituting (5) for \( (*) \Delta_{21}^{\mu
u}(Q) \) in (2) and using (3) with \( \delta(P_\perp^2 - M^2) \simeq \delta(\mathbf{v} \cdot \mathbf{q} -
\[
\Sigma_{21}(P) \simeq i \, g^2 \, C_f \, T \int \frac{d^4Q}{(2\pi)^2} \, \frac{1}{q_0} \, \delta(v \cdot q - q_0) \, \{E\gamma_0 - (p \cdot \hat{q}) (\gamma \cdot \hat{q}) - M\} \, \rho_t(q_0, q).
\]

It is also well known \[1, 3, 4\] that the dominant contribution to \(\Sigma_{21}(P)\) or \(\mathcal{R}_d\) comes from the region \(v \cdot q = q_0 \ll vq\), where

\[
\rho_t(q_0, q) \simeq \frac{3 \, m_T^2}{2} \, \frac{q \, q_0}{q^6 + \left(\frac{3\pi}{4} \, m_T^2\right)^2 \, q_0^2}.
\]

Using this approximation for \(\rho_t(q_0, q)\), we obtain

\[
\Sigma_{21}(P) \simeq \frac{3\pi}{2} \, i \, g^2 \, C_f \, T \, m_T^2 \, (E\gamma_0 - M) \times \int \frac{d^3q}{(2\pi)^3} \, \frac{q}{q^6 + \left(\frac{3\pi}{4} \, m_T^2\right)^2 \, (v \cdot q)^2}.
\] (6)

This expression is valid for \(q \leq O(gT)\) and \(v \cdot \hat{q} \ll 1\). The integral in (6) diverges \[1, 3, 4\] logarithmically at the infrared limit \(q \to 0\);

\[
\Sigma_{21}(P) \simeq \frac{i}{\pi^2} \, g^2 \, C_f \, \frac{T}{p} \, (E\gamma_0 - M) \times \int_{0^+}^{O(m_T)} dq \frac{1}{q} \, \arctan \left(\frac{vm_T^2}{q^2}\right) \] (7)

\[
\simeq \frac{i}{2\pi} \, g^2 \, C_f \, \frac{T}{p} \, (E\gamma_0 - M) \left[\ln \frac{m_T}{0^+} + O(1)\right].
\] (8)

Inserting (8) into (1), we obtain

\[
\mathcal{R}_d \simeq \frac{1}{2\pi} \, g^2 \, C_f \, vT \left[\ln \frac{m_T}{0^+} + O(1)\right].
\] (9)

The damping rate, \(\gamma\), is defined \[1, 3, 4\] to be \(i\) times the imaginary part of the pole with \(\text{Re} \, p_0 > 0\) of the quasiparticle propagator \(\{P \cdot \gamma - M - \tilde{\Sigma}_F(P)\}^{-1}\), where
\[F(P) = -\Sigma_{22}(p_0(1+i\epsilon), p) - \Sigma_{21}(p_0(1+i\epsilon), p)\] (cf. Chapt. 3 of \[3\]). From (8) and (12)-(14) below, we obtain, for \(p_0 \approx E\),

\[
Im \Sigma_{21}(P) \approx -2 Im \Sigma_{22}(P) \\
\approx \frac{E}{p^2} (E \gamma_0 - M) \mathcal{R}_d.
\]

Then we see that, to the present approximation,

\[
\gamma = \frac{1}{2} \mathcal{R}_d. \tag{10}
\]

Thus, within the HTL resummation scheme, the damping rate \(\gamma\) of the moving heavy quark is logarithmically divergent, the well-known result which became to be an opening of a variety of work \([1, 3, 4]\) under the name of “damping-rate puzzle”.

In an experiment, we “detect” the quark with momentum \(p' = p - q\) (Fig. 1). Then, the independent variables are \(p'\) and \(Z \equiv \hat{p} \cdot \hat{p}'\). Instead of using these variables, we use \(q = |p - p'|\) and \(z \equiv \hat{p} \cdot \hat{q}\); \(p'Z = p - qz\) and \(p'^2 = p^2 + q^2 - 2pqz \approx p^2 - 2pqz\).

As mentioned above, the dominant contribution to \(\mathcal{R}_d\) comes from the small region \(z = q_0/(vq) << 1\). Then, we consider the differential decay rate \(d\mathcal{R}_d/dq\), where the integration over \(z\) has been carried out. As is seen in (7), at \(q \approx 0\), \(\mathcal{R}_d\) exhibits a \(dq/q\) spectrum, causing the logarithmic divergence. It should be recalled here that a detector has a finite resolution with a typical resolution \(\Delta q\). Then, what we measure as \(d\mathcal{R}_d/dq\) \(q < \Delta q\) in an experiment is

\[
\left[ \frac{d\mathcal{R}_d}{dq} (\Delta q) \right]_{q < \Delta q} \equiv \int_{0^+}^{\Delta q} \frac{d\mathcal{R}_d}{dq} dq
\]
\[
\zeta \approx \frac{1}{2\pi} g^2 C_f v T \left[ \ln \frac{\Delta q}{0^+} + O(1) \right].
\]

(11)

We will see below that the diverging decay rate (11) is compensated by the analogous term in the “elastic process”.

Now we consider the contribution of the diagrams with “HTL-resummed virtual” gluon, Fig. 1 with \(i = j = 1\) and with \(i = j = 2\);

\[
\mathcal{R}_{\text{virtual}} \equiv -i \frac{M}{E} \frac{1}{2} \sum_s \bar{u}_s(P) \left[ \Sigma_{11}(P) + \Sigma_{22}(P) \right] u_s(P)
\]

\[
= g^2 \frac{M}{2E} C_f \int \frac{d^4Q}{(2\pi)^4} \sum_{\ell=1}^{2} \sum_s \bar{u}_s(P) \left[ \gamma_\mu S_{\ell\ell}(P') \gamma_\nu \ast \Delta^{\mu\nu}_{\ell\ell}(Q) \right] u_s(P), \quad (12)
\]

where

\[
S_{\ell\ell}(P') = (-)^{\ell-1} \frac{\gamma \cdot P' + M}{P'^2 - M^2 - i(-)^{\ell} \epsilon} \quad (\ell = 1, 2).
\]

As in the case of \(\mathcal{R}_d\) above, the magnetic parts of \(\ast \Delta^{\mu\nu}_{\ell\ell}(Q) \quad (\ell = 1, 2)\) in the region \(|q_0| \ll q \leq O(gT)\) dominates the integral in (12), where

\[
(\ast \Delta^{11}_{11}(Q))^{\text{mag}} = - (\ast \Delta^{11}_{22}(Q))^{\text{mag}} \ast
\]

\[
= \left( \delta^{\mu\nu} - \tilde{\delta}^{\mu\rho} \tilde{\delta}^{\nu\sigma} q_\rho \tilde{q}_\sigma \right)
\]

\[
\times \left[ 1 + \frac{1}{Q^2 - \Pi_i(q_0 + iq_0\epsilon, q)} - \frac{n_B(|q_0|)}{Q^2 - \Pi_i(q_0 - iq_0\epsilon, q)} \right]
\]

\[
\approx (\ast \Delta^{11}_{21}(Q))^{\text{mag}}. \quad (13)
\]

Substituting (13) with (5) for \(\ast \Delta^{\mu\nu}_{\ell\ell}(Q) \quad (\ell = 1, 2)\) in (12) and using \(S_{11}(P') + S_{22}(P') = -2\pi i (\gamma \cdot P' + M) \delta(P'^2 - M^2)\), we obtain

\[
\Sigma_{11}(P) + \Sigma_{22}(P) \simeq -\Sigma_{21}(P), \quad (14)
\]
and then
\[ R_{\text{virtual}} = -R_d. \] (15)

It is to be noted that the physically measurable quantity “at \( q < \Delta q \),

\[ [(dR/dq)(\Delta q)]_{q<\Delta q}, \] is

\[ \left[ \frac{dR}{dq}(\Delta q) \right]_{q<\Delta q} = \left[ 1 + R_{\text{virtual}} \right] \]

\[ + \left[ \frac{dR_d}{dq}(\Delta q) \right]_{q<\Delta q}. \] (16)

The factor 1 on the r.h.s. of (16) is the zeroth-order contribution. Substituting (11) and (15) with (9) into (16), we see that the cancellation of divergences occurs between “virtual”- and “real”-contributions, and find

\[ \left[ \frac{dR}{dq}(\Delta q) \right]_{q<\Delta q} \simeq 1 - \frac{1}{2\pi} g^2 C_f v T \left[ \ln \frac{m_T}{\Delta q} + O(1) \right]. \]

Thus the measurable quantity, although sensitive to the resolution \( \Delta q \), is free from divergence. The mechanism of cancelling divergences in (16) is exactly the same as in the cancellation of infrared divergences in vacuum theory (Bloch-Nordsieck mechanism) [4]. In some reaction rate in QED at \( T = 0 \), both the diagram with electron (soft) bremsstrahlung and the diagram with radiative correction diverge, but they cancel each other: The former is the counterpart to \([(dR_d/dq)(\Delta q)]_{q<\Delta q} \) (Eq. (11)), while the latter is the counterpart to \( R_{\text{virtual}} \) (Eq. (15)). It is worth pointing out, in passing, that we obtain

\[ \left[ \frac{dR}{dq}(\Delta q) \right]_{q<\Delta q} + \int_{\Delta q}^{O(m_T)} \frac{dR_d}{dq} dq = 1, \]
as it should be.

Now we are in a position to discuss the physical content of (16) in terms of the processes taking place in the quark-gluon plasma. For any given diagram in the real-time formalism, general rules of identifying the physical processes are available \cite{8}: Given a (thermal) diagram, like Fig. 1, representing some thermal reaction rate, one may divide it into two parts; the one is the reaction’s $S$-matrix element in \textit{vacuum theory} and the other is the $S^*$-matrix element. The $S$- and $S^*$-matrix elements represent the reactions between the considered particle(s) (the heavy quark in the present case) and the particles in the quark-gluon plasma. The type-1 (type-2) vertices in the thermal diagram go to the vertices in the $S$- ($S^*$-) matrix element. A thermal propagator with momentum $K$ from a type-1 vertex to a type-2 vertex is “cut”. When $k_0 > 0$ ($k_0 < 0$) the “cut” is the “final-state cut” (“initial-state cut”). For a thermal propagator from a type-2 vertex to a type-1 vertex, the opposite rules apply. For more details, we refer to \cite{8}.

We note that $\Delta_{21}^\mu\nu$ in (2) and $\Delta_{\ell\ell}^\mu\nu$ in (12) may be expanded in powers of $\Pi_\ell$ (cf. (4) and (13)), and thus $R_d$ and $R_{virtual}$ turn out to be represented by infinite series; $R_d = \sum_{i=0}^{\infty} R_d^{(i)}$ and $R_{virtual} = \sum_{i=0}^{\infty} R_{virtual}^{(i)}$. From each contribution thus obtained, we can identify the physical processes according to the rules outlined above. An example of the physical process that is involved in $R_d^{(2)}$ ($R_{virtual}^{(2)}$) is depicted in Fig. 2 with the final-state cut line $C_d$ ($C_v$); the left side part of the cut line represents the $S$-matrix element in \textit{vacuum theory}, while the right part represents the $S^*$-matrix.
element. The group of particles on top of Fig. 2 stands for spectators which are constituent particles (quarks, antiquarks and gluons) of the quark-gluon plasma. In Fig. 2, \( Q \) is soft \( \sim O(gT) \), while \( K_1, K_2, P_1 \) and \( P_2 \) are hard \( \sim O(T) \). It is to be noted that, in Fig. 2 with the final-state cut line \( C_v \), the heavy quark in the \( S^* \)-matrix element is simply a spectator particle.

For \( Q \simeq 0, P \simeq P' \), and then the “heavy quark detector” cannot discriminate the final-state heavy quark \( (P') \) in Fig. 2 with the final-state cut line \( C_d \) and the one \( (P) \) in Fig. 2 with \( C_v \). Then, what we measure corresponds to the sum of Fig. 2 with \( C_d \) and Fig. 2 with \( C_v \), as well as of all other diagrams included in \( R_d \), Eq. (1), and \( R_{\text{virtual}} \), Eq. (12), (cf. (16)).

We like to emphasize again that two heavy quarks, one with \( P' = P \ (q = 0) \) and another with \( P' \ (|p' - p| < \Delta q) \), cannot be discriminated experimentally. In other words, one cannot recognize the heavy quark with \( |p' - p| < \Delta q \) as a (thermal) decay product, which is responsible for diverging damping rate \( \gamma \), Eq. (10). On the basis of this observation, we propose to introduce a “observable damping rate” defined as (cf. (10))

\[
\gamma_{\text{ob}}(\Delta q) = \frac{1}{2} \left\{ R_d - \left[ \frac{dR_d}{dq} (\Delta q) \right]_{q<\Delta q} \right\} \\
\simeq \frac{1}{4\pi} g^2 C_f v T \ln \left( \frac{m_T}{\Delta q} \right).
\]

This \( \gamma_{\text{ob}}(\Delta q) \) is the damping rate that originates from the heavy-quark decay into physically distinguishable states.

It is to be noted that, for a heavy quark at rest, the damping rate \( \gamma \) is infrared-safe
and finite quantity of $O(g^2 T)$ \[1, \ldots, 3\], and then the difference between $\gamma$ and $\gamma_{\mathrm{obs}}(\Delta q)$ is negligibly small. The reason for the infrared safety of $\gamma$ is traced back to the fact that the heavy quark at rest couples only to the chromoelectric parts of $\Delta_{21}^{\mu \nu}(Q)$ and $\Delta_{\ell \ell}^{\mu \nu}(Q) (\ell = 1, 2)$, which, in contrast to the chromomagnetic part, develops a (thermal) Debye mass and there emerges the energy gap between the single heavy-quark state and the states with a heavy quark plus a “HTL-resummed gluon” with $Q \approx 0$.

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Figure captions

Fig.1. A thermal self-energy diagram of a heavy quark in real-time thermal field theory. $i$ and $j$ ($i, j = 1, 2$) designate the type of vertex. The blob indicates the (HTL resummed) effective gluon propagator.

Fig.2. Typical processes taking place in the quark-gluon plasma. The diagram with the final-state cut line $C_d$ ($C_v$) is the process that is included in $\mathcal{R}_d$ ($\mathcal{R}_{\text{virtual}}$) or in Fig. 1 with $i = 1$ and $j = 2$ ($i = 1$ and $j = 1$). The left side of the final-state cut line represents the $S$-matrix element, while the right side represents the $S^*$-matrix element in vacuum theory.