Light hadron production in $B_c \to J/\psi + X$ decays

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Decays of ground state $B_c$-meson $B_c \to J/\psi + n\pi$ are considered. Using existing parametrizations for $B_c \to J/\psi$ form-factors and $W^* \to n\pi$ spectral functions we calculate branching fractions and transferred momentum distributions of $B_c \to J/\psi + n\pi$ decays for $n = 1, 2, 3, 4$. Inclusive decays $B_c \to J/\psi + ud$ and polarization asymmetries of final charmonium are also investigated. Presented in our article results can be used to study form-factors of $B_c \to J/\psi$ transitions, $\pi$-meson system spectral functions and give the opportunity to check the factorization theorem.

I. INTRODUCTION

Recent measurements of $B_c$-meson mass and lifetime in CDF [1] and D0 [2] experiments allow us to hope that more detailed investigation of this particle on LHC collider, where about $10^{10}$ $B_c$-events per year are expected, would clarify mechanisms of $B_c$ production and decay modes. Currently only products of $B_c$-meson production cross section and branching fractions of decays $B_c \to J/\psi \pi$, $J/\psi \ell \nu$ are known experimentally. For example, the following ratios are measured [3]:

$$
\frac{\sigma_{B_c} Br (B_c \to J/\psi e^+ \nu_e)}{\sigma_B Br (B_c \to J/\psi K)} = 0.282 \pm 0.038 \pm 0.074
$$

for positron in the final state and

$$
\frac{\sigma_{B_c} Br (B_c \to J/\psi \mu^+ \nu_\mu)}{\sigma_B Br (B_c \to J/\psi K)} = 0.249 \pm 0.045 \pm 0.107
$$

for muon. These ratios are about an order of magnitude higher than the theoretical predictions based on current estimates of $B_c$-meson production cross section and branching fraction $Br(B_c \to J/\psi \ell \nu) \approx 2\%$ [4]. The mode $B_c \to J/\psi \pi$ was used mainly to determine precisely $B_c$-meson mass. No information on production cross section, decay branching fraction, and even the product of these quantities was determined in this experiment.

Investigation of other $B_c$-meson decay channels and determination of their branching fractions will be one of interesting tasks of future experiments on LHC. Weak $B_c$ decays can be caused by decays of both constituent quarks. Dominant are $c$-quark decay modes, which amount to $\sim 70\%$ of all $B_c$-meson decays. Unfortunately, none of such reactions were observed, although large branching fractions are expected for some of these decay modes (for example, for $B_c \to B_s \rho$ we have approximately $16\%$ branching fraction). Mentioned above decays $B_c \to J/\psi \ell \nu$ and $B_c \to J/\psi \pi$ are examples of other class, caused by $b$-quark decay. Total branching fraction of this process is about $20\%$.

In the present paper we will fill the gap in existing theoretical predictions of $B_c$-meson decay branching fractions [4] and consider multi-particle processes $B_c \to J/\psi + n\pi$ with $n = 1, 2, 3, 4$. These reactions are caused by weak $b$-quark decay $b \to c W^* \to cud$ and clean analogy with similar $\tau$-lepton decays ($\tau \to \nu_\tau + n\pi$) can be easily seen. This analogy allows us to use existing experimental data on $\tau$-lepton decays and give reliable predictions of $B_c \to J/\psi + n\pi$ branching fractions.

In the next section we give analytical expressions for distributions of $B_c \to J/\psi + n\pi$ decays branching fractions over invariant mass of the light hadron system and study different asymmetries of final $J/\psi$-meson polarization as a function of this kinematic variable. In section III we use existing experimental data on $\tau$-lepton decays calculate branching fractions of $B_c \to J/\psi + n\pi$ decays for $n = 1, 2, 3, 4$. In section IV inclusive reaction $B_c \to J/\psi ud$ is considered in connection with duality relation. Short results of our work are given in the final section.

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II. ANALYTIC RESULTS

$B_c$-meson decays into light hadrons with vector charmonium $J/\psi$ production are caused by $b$-quark decay $b \to W^* \to c\bar{u}d$ (see diagram shown in fig.1). The effective lagrangian of the latter process reads

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{2\sqrt{2}} V_{cb} V_{ud}^* \left[ C_+(\mu) O_+ + C_-(\mu) O_- \right],$$

where $G_F$ is Fermi coupling constant, $V_{ij}$ are the elements of CKM mixing matrix, $C_{\pm}(\mu)$ are Wilson coefficients, that take into account higher QCD corrections and operators $O_{\pm}$ are defined according to

$$O_{\pm} = (\bar{d}_i u_j) V_{-A}(\bar{c}_i b_j) V_{-A} \pm (\bar{d}_j u_i) V_{-A}(\bar{c}_i b_j) V_{-A}.$$  

In this expression $i,j$ are color indexes of quarks and $\langle \bar{q}_1 q_2 \rangle_{V-A} = \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$. Since in our decays light quark pair should be in color-singlet state, the amplitude of the considered here processes is proportional to

$$a_1(\mu) = \frac{1}{2N_c} [(N_c - 1) C_+(\mu) + (N_c - 1) C_-(\mu)].$$

If QCD corrections are neglected, one should set $a_1(\mu) = 1$. Leading logarithmic strong corrections lead to dependence of this coefficient on the renormalization scale $\mu$ [8], and on $\mu \sim m_b$ it is equal to

$$a_1(m_b) = 1.17.$$  

The matrix element of the decay $B_c \to J/\psi + \mathcal{R}$, where $\mathcal{R}$ is some set of light hadrons, has the form

$$\mathcal{M} [B_c \to W^* J/\psi \to \mathcal{R} J/\psi] = \frac{G_F V_{cb}}{\sqrt{2}} a_1 \mathcal{H}_\mu \epsilon_\mathcal{R}^\mu.$$  

In this expression $\epsilon_\mathcal{R}$ is the effective polarization vector of virtual $W$-boson and

$$\mathcal{H}_\mu = \langle J/\psi | \bar{c} \gamma_\mu (1 - \gamma_5) b | B_c \rangle = \mathcal{V}_\mu - \mathcal{A}_\mu.$$  

Vector and axial currents are equal to

$$\mathcal{V}_\mu = \langle J/\psi | \bar{c} \gamma_\mu b | B_c \rangle = i\epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^\psi (p + k)_\alpha q_\beta F_V (q^2),$$

$$\mathcal{A}_\mu = \langle J/\psi | \bar{c} \gamma_\mu \gamma_5 b | B_c \rangle = \epsilon_\mu^\psi F_0^A (q^2) + (\epsilon_\nu^\psi) (p + k)_\mu F_+^A (q^2) + (\epsilon_\nu^\psi) q_\mu F_-^A (q^2).$$

Figure 1: $B_c \to J/\psi + \mathcal{R}$
where \( p \) and \( k \) are the momenta of \( B_c^- \) and \( J/\psi \)-mesons, \( q = p - k \) is the momentum of virtual \( W \)-boson, and \( F_V(q^2) \), \( F_{V,0}^A(q^2) \) are form-factors of \( B_c \rightarrow J/\psi W^* \) decay. Due to vector current conservation and partial axial current conservation the contribution of the form-factor \( F^A_\perp \) are suppressed by small factor \( \sim (m_u + m_d)^2/M_{B_c}^2 \), so we will neglect it in the following.

One can use different approaches when deriving the form of the form-factors \( F(q^2) \). First of all, it is clear, that quark velocity in heavy quarkonia is small in comparison with \( c \), so one can describe heavy quarkonia in the terms of non-relativistic wave-functions. This fact was used on the so called Quark Models [4, 5, 9–14]. In the following we will refer to this set of form-factors as \( QM \). The speed of the final charmonium in \( B_c \)-meson rest frame, on the other hand, is large, so one can expand the amplitude of the considered here process in the powers of small parameter \( M_{J/\psi}/M_{B_c} \), as it was done in papers [15–20]. In what follows, we will refer to this set of form-factors as \( LC \). One can also use 3-point QCD sum rules to obtain the information on \( B_c \rightarrow J/\psi W^* \) form-factors [4, 11, 21, 22] (SR).

In our paper we use the following simple parametrization of form-factors

\[
F(q^2) = \frac{F(0)}{1 - q^2/M_{pole}^2},
\]

where numerical values of parameters \( F_i(0) \) and \( M_{pole} \) are presented in Table I.

The width of the \( B_c \rightarrow J/\psi R \) decay is

\[
d\Gamma(B_c \rightarrow J/\psi R) = \frac{1}{2M} \frac{G_F^2 V_{cb}^2}{2} \frac{u_i^2}{u_1^2} H^{\mu\nu} H^{\ast \mu\nu} \epsilon_\mu^* \epsilon_\nu^* d\Phi (B_c \rightarrow J/\psi R),
\]

where Lorentz-invariant phase space is defined according to

\[
d\Phi (Q \rightarrow p_1 \ldots p_n) = (2\pi)^4 \delta^4 (Q - \sum p_i) \prod \frac{d^3 p_i}{2E_i (2\pi)^3}.
\]

It is well known, that the following recurrent expression holds for this phase space:

\[
d\Phi (B_c \rightarrow J/\psi R) = \frac{d q_R^2}{2\pi} d\Phi (B_c \rightarrow J/\psi W^*) d\Phi (W^* \rightarrow R).
\]

Using this expression one can perform the integration over phase space of the final state \( R \):

\[
\frac{1}{2\pi} \int d\Phi (W^* \rightarrow R) \epsilon_\mu^R \epsilon_\nu^{R*} = (q_\mu q_\nu - q^2 g_{\mu\nu}) \rho^R_T (q^2) + q_\mu q_\nu \rho^R_L (q^2),
\]

where spectral functions \( \rho^R_T, L (q^2) \) are universal and can be determined from theoretical and experimental analysis of some other processes, for example \( \tau \rightarrow \nu R \) decay or electron-positron annihilation \( e^+ e^- \rightarrow R \). Due to vector current conservation and partial axial current conservation spectral function \( \rho^R_T \) is negligible on almost whole kinematical region, so we will neglect is in our paper. Explicit expressions for spectral function \( \rho^R_T \) for different final states \( R \) are given in the next section.
Differential distributions of longitudinally and transversely polarized $J/\psi$-meson in $B_c \rightarrow J/\psi + R$ decays can easily be obtained from presented above expressions. In the case of longitudinal polarization the polarization vector $\epsilon_\psi$ is equal to

$$e_\psi^\lambda (\lambda = 0) = \frac{M}{2M_V} \left\{ \beta, 0, 0, \frac{M^2 + M_V^2 - q^2}{M^2} \right\},$$

where $z$-axes is chosen in the direction of $J/\psi$ movement, $M$ and $M_V$ are $B_c$- and $J/\psi$-meson masses and

$$\beta = \sqrt{\frac{(M + M_V)^2 - q^2}{M^2}} \sqrt{\frac{(M - M_V)^2 - q^2}{M^2}}.$$

Differential distribution has the form

$$\frac{d\Gamma [B_c \rightarrow J/\psi^{\lambda=0} + R]}{dq^2} = \frac{G_F^2 M^3 V_{cb}^2 a_0^2}{128\pi M_V^2} \rho^R (q^2) \frac{M^4}{4M_V^2} \left\{ \left( \beta^2 + \frac{4M_V^2 q^2}{M^4} \right) |F_0^A|^2 + M^4 \beta^4 |F_+^A|^2 \right. + \left. 2\beta^2 (M^2 - M_V^2 - q^2) F_0^A F_+^A \right\}.$$ 

In the case of transversely polarized vector meson $e_\psi^\lambda$ has the form

$$e_\psi^\lambda (\lambda = \pm 1) = \left\{ 0, \frac{1}{\sqrt{2}}, \frac{\pm i}{\sqrt{2}}, 0 \right\},$$

and the corresponding differential distribution is

$$\frac{d\Gamma [B_c \rightarrow J/\psi^{\lambda=\pm 1} + R]}{dq^2} = \frac{G_F^2 V_{cb}^2 a_0^2}{32\pi M^2} \beta q^2 \rho^R (q^2) \left\{ |F_0^A|^2 + M^4 \beta^2 |F_V|^2 + \frac{2\beta M_V^2}{M_V^2} \Re (F_0^A F_+^A) \right\}.$$ 

It should be stressed, that the above expressions are universal and spectral function $\rho^R (s)$ depends on the final state $R$.

If the polarization if final vector meson is not observed, the $q^2$-distribution is, obviously,

$$\frac{d\Gamma [B_c \rightarrow J/\psi + R]}{dq^2} = \sum_{\lambda=0,\pm 1} \frac{d\Gamma [B_c \rightarrow J/\psi^{\lambda} + R]}{dq^2}. \tag{2}$$

It is also useful to study some polarization asymmetries. For example, polarization degree $\alpha$ is defined according to

$$\alpha = \frac{d\Gamma_{\lambda=+1} + d\Gamma_{\lambda=-1} - 2d\Gamma_{\lambda=0}}{d\Gamma_{\lambda=+1} + d\Gamma_{\lambda=-1} + 2d\Gamma_{\lambda=0}}.$$
Production of transversely polarized, longitudinally polarized and unpolarized \( J/\psi \)-meson corresponds to \( \alpha = 1 \), \( \alpha = -1 \) and \( \alpha = 0 \) respectively. We would like to note, that in the framework of factorization model this asymmetry does not depend on final state \( R \). So, experimental investigation of this asymmetry can be used for determination of \( B_c \)-meson form factors and test of QCD factorization. In fig.2 we show \( q^2 \)-dependence of this asymmetry for different sets of \( B_c \)-meson form-factors. One can easily explain qualitatively the behavior of these curves. Let us consider \( q^2 \)-dependence of asymmetry \( \alpha \) in \( B_c \to J/\psi \bar{u}d \) decay. At low \( q^2 \) the direction of \( \bar{u} \)- and \( d \)-quarks momenta in \( B_c \)-meson rest frame will be close to each other and opposite to the direction of the momentum of \( J/\psi \)-meson. The spin of light \( \bar{u} \)-antiquark (\( d \)-quark) is directed along (opposite to) its momentum (see fig.3a), so quark-antiquark pair has \( \lambda = 0 \) projection on \( O_z \) axis. From angular momentum conservation it follows, that \( J/\psi \)-meson should also be longitudinally polarized. This can be observed in figure 2, where at low \( q^2 \) we have \( \alpha = -1 \) for all sets of \( B_c \)-meson form-factors. In high \( q^2 \)-region, on the contrary, direction of quark and antiquark momenta are opposite to each other and \( J/\psi \)-meson stay at rest in \( B_c \)-meson rest frame (see fig.3b). As a result, final \( J/\psi \)-meson is unpolarized in this region and \( \alpha = 0 \).

Another example is transverse asymmetry

\[
\alpha_T = \frac{d\Gamma_{\lambda = 1} - d\Gamma_{\lambda = -1}}{d\Gamma}
\]

This asymmetry also depends only on \( B_c \)-meson form-factors and its dependence on squared transferred momentum is shown in fig.4.

### III. EXCLUSIVE DECAYS

In this section we present differential widths and branching fractions of the decays \( B_c \to J/\psi + n\pi \) using presented above universal formula (2) and specific expressions for spectral function \( \rho^R_T (q^2) \).
A. $B_c \to J/\psi \pi$

Let us first of all consider two-particle decays $B_c \to J/\psi \pi$ and $B_c \to J/\psi \rho$.

In the case of $B_c \to J/\psi \pi$ decay the $W^* \to \pi$ transition is expressed through leptonic constant $f_\pi$:

$$\langle \pi | \bar{u} \gamma_\mu \gamma_5 d | 0 \rangle = \sqrt{2} f_\pi \eta_\mu.$$  \hspace{1cm} (3)

The numerical value of this constant can be determined from $\pi \to \mu \nu$ decay width: $f_\pi \approx 140 \text{ MeV}$. The spectral function, that corresponds to vertex (3) is

$$\rho_T^\pi (q^2) = 2 f_\pi^2 \delta (q^2).$$

Using this spectral function it is easy to obtain the following values of $B_c \to J/\psi \pi$ decay branching fractions for different sets of form-factors:

- $\text{Br}_{LC} (B_c \to J/\psi \pi) = 0.13\%$
- $\text{Br}_{QM} (B_c \to J/\psi \pi) = 0.17\%$
- $\text{Br}_{SR} (B_c \to J/\psi \pi) = 0.17\%$

B. $B_c \to J/\psi + 2\pi$

The $2\pi$ channel is saturated mainly by $B_c \to J/\psi \rho$ decay. The $W^* \to \rho$ transition vertex is also expressed through $\rho$-meson leptonic constant

$$\langle \rho | \bar{u} \gamma_\mu d | 0 \rangle = \sqrt{2} f_\rho M_\rho \epsilon_\mu$$

where $f_\rho \approx 150 \text{ MeV}$. If one neglects the width of $\rho$-meson, the corresponding spectral function has the form

$$\rho_T^\rho (q^2) = 2 f_\rho^2 \delta (q^2 - m_\rho^2).$$  \hspace{1cm} (4)

The branching fractions of $B_c \to J/\psi \rho$ for different sets of form-factors are:

- $\text{Br}_{LC} (B_c \to J/\psi \rho) = 0.38\%$
- $\text{Br}_{QM} (B_c \to J/\psi \rho) = 0.44\%$
- $\text{Br}_{SR} (B_c \to J/\psi \rho) = 0.48\%$

In order to take $\rho$-meson width into account, one can use experimental data on $\tau \to \nu_\tau + 2\pi$ decay. The differential branching ratio of this reaction is equal to

$$\frac{d\Gamma (\tau \to \nu_\tau R)}{dq^2} = \frac{G_F^2}{16\pi m_\tau} \left( \frac{m_\tau^2 - q^2}{m_\tau^2} \right)^2 \left( m_\tau^2 + 2q^2 \right) \rho_T^\pi (q^2).$$

This method was used by ALEPH collaboration to measure the spectral function $\rho_T^{2\pi} (q^2)$ in the kinematically allowed region $q^2 < m_\tau^2$ and can be approximated by the expression (see fig.5)

$$\rho_T^{2\pi} (s) \approx 1.35 \times 10^{-3} \left( \frac{s - 4m_\pi^2}{s} \right)^2 \frac{1 + 0.64s}{(s - 0.57)^2 + 0.013},$$

where $s$ is measured in GeV$^2$. In fig.5 we show corresponding distributions $d\Gamma (B_c \to J/\psi + 2\pi) / dq^2$. Solid, dashed and dash-dotted lines in this figure correspond to form-factors SR, QM, and LC respectively. The branching fractions of the decay $B_c \to J/\psi + 2\pi$ are almost equal to $B_c \to J/\psi \rho$ decay branching fractions:

- $\text{Br}_{LC} (B_c \to J/\psi \pi \pi) = 0.35\%$
- $\text{Br}_{QM} (B_c \to J/\psi \pi \pi) = 0.44\%$
- $\text{Br}_{SR} (B_c \to J/\psi \pi \pi) = 0.48\%$. 
C. $B_c \to J/\psi + 3\pi$

In the case of $B_c \to J/\psi + 3\pi$ decay (where $3\pi$ stands for the sum of $\pi^-\pi^0\pi^0$ and $\pi^-\pi^+\pi^-$ decay modes) the $G$-parity of the final state is negative. So we can expect, that this mode is saturated by axial-vector resonance $a_1$. The width of this state is too large to neglect it, so we cannot use the expression similar to (4) for $W^* \to 3\pi$ transition. The corresponding spectral function can be determined from experimental and theoretical data on $\tau \to \nu_\tau + 3\pi$ decay. In our article we use the following expression to approximate this function (see. fig.6a):

$$
\rho_T^{3\pi}(s) \approx 5.86 \times 10^{-5} \left( \frac{s - 9 m_\pi^2}{s} \right)^4 \frac{1 + 190 s}{(s - 1.06)^2 + 0.48^2}.
$$

Distributions over $q^2$ for different sets of $B_c$-meson form factors are shown in fig.6b. The branching fractions of $B_c \to J/\psi + 3\pi$ decay are

$$
\text{Br}_{LC}(B_c \to J/\psi + 3\pi) = 0.52\%,
\text{Br}_{QM}(B_c \to J/\psi + 3\pi) = 0.64\%,
\text{Br}_{SR}(B_c \to J/\psi + 3\pi) = 0.77\%.
$$

D. $B_c \to J/\psi + 4\pi$

In the decay $B_c \to J/\psi + 4\pi$ both $\pi^-\pi^0\pi^0\pi^0$ and $\pi^-\pi^+\pi^-\pi^-$ modes are possible in the following we consider the sum of these states. The kinematically allowed region in $\tau \to \nu_\tau + 4\pi$ decay is too small to determine the form of spectral function $\rho_T^{4\pi}$, so it is more convenient to use energy dependence of $4\pi$ production cross section in
electron-positron annihilation. It is easy to obtain the following expression for this cross section:

$$\sigma \left( e^+e^- \rightarrow 4\pi \right) = \frac{4\pi\alpha^2}{s}\rho_{4\pi T}^2(s).$$

Spectral function $\rho_{4\pi T}^2$, calculated from experimental data \cite{24} is shown in fig.7a and later we use the following parametrization:

$$\rho_{4\pi T}^2(s) \approx 1.8 \times 10^{-4} \left( \frac{s - 16m_\pi^2}{2} \right) \left[ (s - 1.83)^2 + 0.61 \right].$$

The distributions corresponding to this spectral function are shown in fig.7b. The branching fraction for different sets of $B_c$-meson form-factors are

$$\text{Br}_{LC}(B_c \rightarrow J/\psi + 4\pi) = 0.26\%,$$
$$\text{Br}_{QM}(B_c \rightarrow J/\psi + 4\pi) = 0.33\%,$$
$$\text{Br}_{SR}(B_c \rightarrow J/\psi + 4\pi) = 0.40\%.$$
Figure 8: Differential $B_c \rightarrow J/\psi \bar{u}d$ branching fractions for different sets of $B_c$-meson form-factors. Notations are the same as in fig [5].

| $\pi$  | $2\pi$ | $3\pi$ | $4\pi$ | $\bar{u}d$ |
|-------|-------|-------|-------|----------|
| $LC$  | 0.13  | 0.35  | 0.52  | 0.26     | 7        |
| $QM$  | 0.17  | 0.44  | 0.64  | 0.33     | 8.6      |
| $SR$  | 0.17  | 0.48  | 0.77  | 0.40     | 12       |

Table II: $B_c \rightarrow J/\psi R$ decays branching fractions (in %) for different sets of $B_c$-meson form-factors

where $\Delta$ is the duality window. If we restrict ourselves to $n \leq 4$ in the right-hand side of this relation, it is valid for

$$\Delta \approx 0.6 \text{ GeV}.$$ 

It is interesting to note that this value is almost independent on the choice of $B_c$-meson form-factors and close to the value of duality parameter in $gg \rightarrow J/\psi c\bar{c}$ and $\chi_b \rightarrow J/\psi c\bar{c}$ reactions [25, 26].

V. CONCLUSION

In our paper we study exclusive and inclusive decays of $B_c$-meson into light hadrons and vector charmonium $J/\psi$, that is the processes $B_c \rightarrow J/\psi + \bar{u}d$ and $B_c \rightarrow J/\psi + n\pi$ where $n = 1, 2, 3, 4$. According to QCD factorization theorem the amplitude of these processes splits into two independent parts. The first factor describes the decay $B_c \rightarrow J/\psi W^*$ and one can use existing parametrizations of $B_c$-meson form-factors to calculate this amplitude. The second factor describes the fragmentation of virtual $W$-boson. The information about these processes was taken from experimental distributions of multi-pion production in $\tau$-lepton decays and electron-positron annihilation.

Our results are gathered in table [II] where branching fractions of multi-pion production in $B_c \rightarrow J/\psi + n\pi$ for different $B_c$-meson form-factors are presented. The last column of this table contains the branching fraction of the inclusive decay $B_c \rightarrow J/\psi + \bar{u}d$. It is clear that up to $KK$-production threshold only $\pi$-mesons could be produced in $B_c \rightarrow J/\psi + X$ decay, so some duality relation should hold. In our article it is shown, that to satisfy this relation it is sufficient to integrate the inclusive spectrum up to square transferred momentum $q^2 = (2m_K + \Delta)^2$. It turns out, that $\Delta$ is almost independent on the choice of $B_c$-meson form-factors and equals to $\sim 0.6 \text{ GeV}$.

The other interesting point are the polarization asymmetries of final $J/\psi$-meson. In the framework of factorization model these asymmetries do not depend on the final state $R$, so one can use them to investigate form-factors of $B_c$-meson and to test the factorization theorem. In our paper we present the polarization degree $\alpha = (d\Gamma_T/dq^2 - 2d\Gamma_L/dq^2)/(d\Gamma_T/dq^2 + 2d\Gamma_L/dq^2)$ and transverse polarization asymmetry $\alpha_T = (d\Gamma_{\lambda=1}/dq^2 - d\Gamma_{\lambda=-1}/dq^2)/(d\Gamma/dq^2)$ for different sets of form-factors.

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