Abstract

We use the language of squeezed states to give a systematic description of two issues in cosmological particle creation:

a) Dependence of particle creation on the initial state specified. We consider in particular the number state, the coherent and the squeezed state. b) The relation of spontaneous and stimulated particle creation and their dependence on the initial state. We also present results for the fluctuations in particle number in anticipation of its relevance to defining noise in quantum fields and the vacuum susceptibility of spacetime.
1 Introduction

Cosmological particle creation \cite{1, 2, 3, 4, 5} is a physical process of basic theoretical interest in quantum field theory in curved spacetime \cite{6}, and important applied interest in the quantum dynamics of the early universe \cite{4, 7, 8}. In this paper we use the language of squeezed states \cite{9} to give a systematic description of two interrelated issues:

a) Dependence of particle creation on the initial state. We consider in particular the number state, the coherent and the squeezed state.

b) The relation of spontaneous and stimulated particle creation and their dependence on the initial state.

We also present the result for the fluctuations in particle number in anticipation of its relevance to defining noise in quantum fields.

Both of these issues have been explored before, albeit in a restricted context. The use of Bogolubov transformation relating the canonical operators between the \textit{in} and \textit{out} states was introduced by Parker \cite{1} in cosmological particle creation. He also derived the evolutionary operator based on earlier work of Kamefuchi and Umezawa \cite{10} and briefly discussed induced particle creation. Zel’dovich \cite{3} first pointed out that cosmological particle creation is the quantum version of parametric amplification of classical waves. The connection of cosmological particle creation with processes in quantum optics was thus noticed more than 20 years ago \cite{3, 4, 5, 11, 12, 13}.

Cosmological particle creation in coherent states was discussed by Hu \cite{5} and Berger \cite{13}. The former used a close analogy with a model in quantum optics based on the quantum statistics of coupled harmonic oscillators (e.g., \cite{14, 15}). Entropy generation associated with thermal particle creation in an exponentially expanding universe was discussed by Parker \cite{16}. The distinction of the number state and the coherent state (in the so-called \textit{N} and \textit{P} representation) in the question of entropy generation in particle creation was first noticed by Hu and Pavon \cite{17} in their search for an intrinsic measure of field entropy. They found that the variance of particle number is a monotonically increasing function of time in the \textit{P} representation, and that an increase of particles in the \textit{N} representation is related to the fact that one has chosen an initial state which is an eigenstate of the number operator, as is often the case in most discussions of cosmological particle creation. The phase-particle number uncertainty relation implies that this choice amounts to assuming an initial state with random phase. Kandrup \cite{18} has further clarified these points. Entropy generation in particle creation with interactions was investigated by Hu and Kandrup \cite{19}, who discussed both spontaneous and stimulated production of particles.

Since the concept of squeezed state was introduced to quantum optics in the seventies \cite{9}, there has been much progress in its experimental realizations and theoretical implications. The adoption of the language of squeezed states to cosmological particle creation was introduced recently by Grishchuk and Sidorov \cite{20}. Although the physics is not new (this was
also pointed out by Albrecht et al. [21] in the inflationary cosmology context) and the results are largely known, the use of rotation and squeeze operators give an alternative description which allows one to explore new avenues based on interesting ideas developed in quantum optics. More recent work on entropy generation in cosmological perturbations by Brandenberger and coworkers [22] and Gasperini and Giovannini [23] make use of coarse-graining via a random phase approximation using the language of squeezed states. Matacz [24] has used the squeezed state formalism as a starting point for the study of decoherence of cosmological inhomogeneities in the coherent-state representation.

The issues of initial states and entropy generation have been discussed in restricted conditions, and the issue of spontaneous and stimulated production has only been touched upon before. For the sake of completeness, in this work we will address these issues under a common framework, and present the results for different initial states (the number state, the coherent state and the squeezed state). In Sec. 2 we give a short summary of particle creation described both in the old language of Bogolubov transformations and in the new language of squeezed states, mainly to present the basic concepts and introduce the terminology. Readers familiar with cosmological particle creation and the squeezed state description can go directly to Sec. 3. In Sec. 3 we give the result of spontaneous and stimulated production for different initial states for bosons. In Sec. 4 we work out the change in the fluctuations in particle number. This is in anticipation of its relation to defining noise in quantum fields and vacuum susceptibility in quantum processes in curved spacetimes. We aim in this simple paper only to show how the old and the new language can be used together to describe the quantum statistics of particle creation. Exploration of the implications of these processes will be described with more detail in later works.

2 Particle Creation and Squeezed State

2.1 Particle Creation via Bogolubov Transformation

Readers familiar with the process of cosmological particle creation can skip this subsection. For the original work, see [1]. Our summary here is adopted from [19] with slight modifications in the discussion of spontaneous versus stimulated creation.

Consider a massive (m) scalar field $\Phi$ coupled arbitrarily ($\xi$) with a background spacetime with metric $g_{\mu\nu}$ and scalar curvature $R$. Its dynamics is described by the Lagrangian density

$$L(x) = -\frac{1}{2} \sqrt{-g} g^{\mu\nu}(x) \nabla_\mu \Phi \nabla_\nu \Phi - \left[ m^2 + (1 - \xi) \frac{R}{6} \right] \Phi^2(x).$$

(2.1)

Here $\xi = 0$ and 1 denotes, respectively, conformal and minimal coupling. The scalar field satisfies the wave equation

$$\left[ \Box + m^2 + (1 - \xi) \frac{R}{6} \right] \Phi(\vec{x}, t) = 0,$$

(2.2)
where \( \Box = g^{\mu\nu} \nabla_\mu \nabla_\nu \) is the Laplace-Beltrami operator defined on the background spacetime.

In the canonical quantization approach, one assumes a foliation of spacetime into dynamically evolving, time-ordered, spacelike hypersurfaces \( \Sigma \), expands the field on \( \Sigma \) in normal modes, imposes canonical commutation relations on the time-dependent expansion functions now regarded as creation and annihilation operators, defines the vacuum state, and then constructs the Fock space. In flat space, Poincaré invariance guarantees the existence of a unique global Killing vector \( \partial_t \) orthogonal to all constant-time spacelike hypersurfaces, an unambiguous separation of the positive-and negative-frequency modes, and a unique and well-defined vacuum. In curved spacetime, general covariance precludes any such privileged choice of time and slicing. There is no natural mode decomposition and no unique vacuum. At any constant-time slice, one can expand the field \( \Phi \) in terms of a complete set of (spatial) orthonormal modes \( u_\vec{k}(\vec{x}) \) [1]

\[
\Phi(x) = \sum_\vec{k} [\psi_\vec{k}(t)u_\vec{k}(\vec{x}) + \psi_\vec{k}^\dagger(t)u_\vec{k}^\ast(\vec{x})].
\] (2.3)

After second quantization, the fields \( \Phi \) and their amplitudes \( \psi_\vec{k} \) become operator-valued functions. Write

\[
\psi_\vec{k}(t) = a_\vec{k}(t)\phi_\vec{k}(t),
\] (2.4)

where \( a_\vec{k} \) are the annihilation operators and the (c-number) functions \( \phi_\vec{k}(t) \) obey the wave equation derived from (2.2). The canonical commutation rules on \( \Phi \) imply these conditions on \( a_\vec{k} \) and \( a_\vec{j}^\dagger \), i.e.,

\[
[a_\vec{k}, a_\vec{j}] = [a_\vec{k}^\dagger, a_\vec{j}^\dagger] = 0 \quad \text{and} \quad [a_\vec{k}, a_\vec{j}^\dagger] = \delta_{\vec{k}\vec{j}}.
\] (2.5)

Assume that initially a vacuum state \( | 0 \rangle \) at \( t_0 \) can be defined by

\[
a_\vec{k} | 0 \rangle_{t_0} = 0,
\] (2.6)

and a Fock space can be constructed from the \( n \)-particle states by the action of the creation operators. At a later time, say \( t_1 = t_0 + \Delta t \), the vacuum state defined at \( t_0 \) will no longer be vacuous, since the annihilation operator \( b_\vec{k}(t_1) \) at \( t_1 \) is not equal to \( a_\vec{k}(t_0) \). In general, they are related by a set of Bogolubov transformations

\[
b_j(t_1) = \sum_{\vec{k}} [\alpha_{\vec{j}\vec{k}}(t) a_\vec{k} + \beta_{\vec{j}\vec{k}}^\ast(t) a_\vec{k}^\dagger].
\] (2.7)

A new vacuum state \( | 0 \rangle \) at \( t_1 \) can be defined by

\[
b_j | 0 \rangle_{t_1} = 0
\] (2.8)

and from this a new Fock space can be constructed. One can easily see that \( a_\vec{k} | 0 \rangle \neq 0 \). The two vacua are different by the coefficients \( \alpha, \beta \) whose time dependence are determined by the amplitude functions \( \phi_\vec{k}(t) \). In particular, any \( \phi_\vec{k} \) with only a positive-frequency component
initially at $t_0$ will acquire a negative-frequency component at $t_1$. The new vacuum at $t_1$ now contains

$$s_j = (0 \mid N_j \mid 0) = \sum_{\vec{k}} \mid \beta_{\vec{j}\vec{k}} \mid^2$$  \hspace{1cm} (2.9)$$

particles, where

$$N_j \equiv b_{\vec{j}}^b b_\vec{j}$$  \hspace{1cm} (2.10)$$
is the particle number operator. From (2.7) one sees that $\beta_{\vec{j}\vec{k}}$ measures the negative-frequency component generated by dynamics. In curved space the inequivalence of Fock representation due to the lack of a global timelike Killing vector makes the constant separation of positive-and negative-frequency modes in general impossible. The mixing of positive- and negative-frequency modes in second-quantized form leads to vacuum particle creation. Particle creation may arise from topological, geometrical, or dynamical causes. In cosmological spacetimes the inequivalence of vacua appears at different times of evolution, and thus cosmological particle creation is by nature a dynamically induced effect. Note that we are dealing here with a free-field: particles are not produced from interactions, but rather from the excitation (parametric amplification \cite{3}) of vacuum fluctuations by the changing background gravitational field.

### 2.1.1 Spontaneous Production

For spacetimes with certain symmetries, some natural mode decomposition may present itself. For example, in the class of conformally static spacetimes (e.g., Robertson-Walker universe), where the metric is conformally related to a static spacetime (e.g., the Minkowski metric),

$$g_{\mu\nu}(x) = a^2(\eta) \eta_{\mu\nu},$$  \hspace{1cm} (2.11)$$

where $a$ is the conformal factor, there exists a global conformal Killing vector $\partial_\eta$, where $\eta = \int dt/a(t)$ is the conformal time. Thus the vacuum defined by the mode decomposition with respect to $\partial_\eta$ is a globally well-defined one, known as the conformal vacuum. For conformally-invariant fields [e.g., a massless, scalar field with $\xi = 0$ in (2.1)] in conformally-static spacetimes, it is easy to see that there is no particle creation \cite{1}. Thus any small deviation from these conditions, e.g., small $m, \xi$, can be treated perturbatively from these states. Consider the spatially-flat Robertson-Walker metric with line element

$$ds^2 = a^2(\eta)(d\eta^2 - d\vec{x}^2).$$  \hspace{1cm} (2.12)$$
The scalar fields can be separated into modes

$$\Phi(\eta, \vec{x}) = \sum_{\vec{k}} \phi_{\vec{k}}(\eta)e^{i\vec{k} \cdot \vec{x}},$$  \hspace{1cm} (2.13)$$

where $\phi_{\vec{k}}$ are the amplitude functions of the $\vec{k}$th mode. Define new field variables $a(\eta)\phi_{\vec{k}}(\eta) = \chi_{\vec{k}}(\eta)$. From the wave equation (2.2) for the $\vec{k}$th mode $\chi_{\vec{k}}(\eta)$ satisfies

$$\chi_{\vec{k}}''(\eta) + [k^2 + (m^2 - \xi R/6)a^2]\chi_{\vec{k}}(\eta) = 0.$$  \hspace{1cm} (2.14)$$
where \( k \equiv |\vec{k}| \). One sees that, for massless \((m = 0)\) conformally coupled \((\xi = 0)\) fields, \( \chi_{\vec{k}} \) admits solutions
\[
\chi_{\vec{k}}(\eta) = Ae^{i\Omega_{\vec{k}}\eta} + Be^{-i\Omega_{\vec{k}}\eta},
\]
which are of the same form as travelling waves in flat space. Since \( \Omega_{\vec{k}} = k = \text{const} \), the positive- and negative-frequency components remain separated and there is no particle production \([1]\). More generally, the wave equation for each mode has a time-dependent natural frequency given by
\[
\Omega_{\vec{k}}^2(\eta) = k^2 + (m^2 - \xi R/6)a^2 \equiv \omega_{\vec{k}}^2a^2.
\]
The negative-frequency modes can thus be excited by the dynamics of the background through \( a(\eta) \) and \( R(\eta) = 6a''/a^3 \) (a prime denotes \( d/d\eta \)). In analogy with the time-dependent Schrödinger equation, one can view the \((m^2 - \xi R/6)a^2\) term in (2.14) as a time-dependent potential \( V(\eta) \) which can induce backscattering of waves \([3, 5]\). The number of created particles in the \( \vec{k} \)th mode is given in terms of \( \chi_{\vec{k}}' \) and \( \chi_{\vec{k}} \) by
\[
s_{\vec{k}} = |\beta_{\vec{k}}|^2 = \frac{1}{2\Omega_{\vec{k}}^2}( |\chi_{\vec{k}}'|^2 + \Omega_{\vec{k}}^2 |\chi_{\vec{k}}|^2 ) - \frac{1}{2}. \tag{2.17}
\]
The energy density associated with these particles is given by the expectation value of the \(00\) component of the conformal energy-momentum tensor with respect to the conformal vacuum:
\[
\rho_0 = \langle 0 | \Lambda_0^0 | 0 \rangle = \frac{1}{a^4} \int \frac{d^3k}{2(2\pi)^3} ( |\chi_{\vec{k}}'|^2 + \Omega_{\vec{k}}^2 |\chi_{\vec{k}}|^2 )
\]
\[
= \frac{1}{a^4} \int \frac{d^3k}{(2\pi)^3} (2s_{\vec{k}} + 1) \frac{\Omega_{\vec{k}}^2}{2}. \tag{2.18}
\]
In a Hamiltonian description of the dynamics of a finite system of parametric oscillators, the Hamiltonian is simply
\[
H_0(t) = \frac{1}{2} \sum_k (\pi_{\vec{k}}^2 + \Omega_{\vec{k}}^2 q_{\vec{k}}^2) = \sum_k (N_{\vec{k}} + \frac{1}{2})\Omega_{\vec{k}}, \tag{2.19}
\]
Comparing this with (2.18) one can identify \(|\chi_{\vec{k}}|^2\) and \(|\chi_{\vec{k}}'|^2\) with the canonical coordinates \( q_{\vec{k}}^2 \) and moment \( \pi_{\vec{k}}^2 \), the eigenvalue of \( H_0 \) being the energy \( E_{\vec{k}} = (N_{\vec{k}} + \frac{1}{2})\Omega_{\vec{k}} \). The analogy of particle creation with parametric amplification is formally clear: (2.17) defines the number operator
\[
N_{\vec{k}} = \frac{1}{2\Omega_{\vec{k}}^2}(\pi_{\vec{k}}^2 + \Omega_{\vec{k}}^2 q_{\vec{k}}^2) - \frac{1}{2}, \tag{2.20}
\]
and (2.18) says that the energy density of vacuum particle creation comes from the amplification of vacuum fluctuations \( \hbar \Omega_{\vec{k}}^2/2 \) by the factor \( A_{\vec{k}}^2 = 2s_{\vec{k}} + 1 \).
2.1.2 Stimulated Production

Equation (2.18) gives the vacuum energy density of particles produced from an initial vacuum, a pure state. If the initial state at $t_0$ is a statistical mixture of pure states, each of which contains a definite number of particles, then an additional mechanism of particle creation enters. This is categorically known as induced creation. In particular, as already pointed out in the original paper of Parker [1], if the statistical density matrix $\mu$ is diagonal in the representation whose basis consists of the eigenstates of the number operators $a^\dagger_k a_k$ at time $t_0$, then for bosons, this process increases the average number of particles (in mode $\vec{k}$ in a unit volume) at a later time $t_1$ over the initial amount:

$$N_{\vec{k}}(t) = Tr[\mu b^\dagger_{\vec{k}}(t)b_{\vec{k}}(t)]$$

$$= \langle N_{\vec{k}}(t_0) \rangle + |\beta_{\vec{k}}(t) |^2 [1 + 2\langle N_{\vec{k}}(t_0) \rangle], \quad (2.21)$$

where $\langle N_{\vec{k}}(t_0) \rangle = Tr[\mu a^\dagger_{\vec{k}} a_{\vec{k}}]$. For fermions it decreases the initial number.

The above result can be understood in the parametric oscillator description as the amplification by a factor $A_{\vec{k}} = 2s_{\vec{k}} + 1$, of a) the vacuum fluctuation, yielding $|\beta_{\vec{k}}(t) |^2$, and of b) the particles already present $N_{\vec{k}}(t_0)$, i.e.,

$$\langle N_{\vec{k}}(t) \rangle = |\beta_{\vec{k}}(t) |^2 + A_{\vec{k}} \langle N_{\vec{k}}(t_0) \rangle \quad (2.22)$$

where $s_{\vec{k}} = |\beta_{\vec{k}}(t) |^2$. The second part is called stimulated production. It yields an energy density $\rho_n$ given by

$$\rho_n = \langle n | \Lambda_0^0 | n \rangle$$

$$= \frac{1}{a^4} \int \frac{d^3k}{(2\pi)^3} (| \chi'_{\vec{k}} |^2 + \Omega_{\vec{k}}^2 | \chi_{\vec{k}} |^2) \langle a^\dagger_{\vec{k}} a_{\vec{k}} \rangle,$$

$$= \frac{1}{a^4} \int \frac{d^3k}{(2\pi)^3} (2s_{\vec{k}} + 1)\Omega_{\vec{k}} \langle N_{\vec{k}}(t_0) \rangle \quad (2.23)$$

where $\langle a^\dagger_{\vec{k}} a_{\vec{k}} \rangle = \langle N_{\vec{k}}(t_0) \rangle = Tr[\mu a^\dagger_{\vec{k}} a_{\vec{k}}]$. Combining (18) and (23), for a density matrix diagonal in the number state, the total energy density of particles created from the vacuum and from those already present in the $n$-particle state is given by

$$\rho(t) = \rho_0 + \rho_n = \frac{1}{a^4} \int \frac{d^3k}{(2\pi)^3} (| \chi'_{\vec{k}} |^2 + \Omega_{\vec{k}}^2 | \chi_{\vec{k}} |^2)(\frac{1}{2} + \langle a^\dagger_{\vec{k}} a_{\vec{k}} \rangle)$$

$$= \frac{1}{a^4} \int \frac{d^3k}{(2\pi)^3} A_{\vec{k}} \Omega_{\vec{k}} (\frac{1}{2} + \langle N_{\vec{k}}(t_0) \rangle). \quad (2.24)$$

This can be understood as the result of parametric amplification by the factor $A$ of the energy density of vacuum fluctuations $\hbar\Omega/2$ and that of the particles originally present in the $k$th mode at $t_0$, i.e., $\langle N_{\vec{k}}(t_0) \rangle = \frac{1}{2} \Omega_{\vec{k}}$. For the special but important case where $\mu$ is thermal at temperature $T = \beta^{-1}$, $\langle N_{\vec{k}} \rangle$ obeys the Bose-Einstein distribution function for scalar fields. The magnification of the
n-particle thermal state gives the finite-temperature contribution of particle creation, with energy density

\[ \rho_T = \frac{1}{a^4} \int \frac{d^3k}{(2\pi)^3} (2s_\vec{k} + 1)\Omega_{\vec{k}}/(e^{\beta\Omega_{\vec{k}}} - 1). \] (2.25)

For a massless conformal field, this yields the familiar Stefan-Boltzmann relation

\[ \rho_T = \frac{\pi^2}{30} T^4. \] (2.26)

Finite-temperature particle creation and the related entropy generation problem have been discussed in [25, 26]. For a more general density matrix the behavior of the induced or stimulated part of particle creation could increase or decrease, depending on the correlation and phase relation of the initial state, even though the spontaneous creation part always give an increase in particle number. Both are important factors in the consideration of entropy generation processes [19].

2.2 Evolutionary Operator, Squeezing and Rotation

An equivalent description of particle creation is by means of the evolutionary operator \( U \) defined by

\[ b_{\pm\vec{k}}(t) = U(t)a_{\pm\vec{k}}U^\dagger(t) \] (2.1)

where \( UU^\dagger = 1 \). The form of \( U \) was deduced by Parker [1] following Kamufuchi and Umezawa [10]. In the modern language of squeezed states [9], one can write \( U = RS \) as a product of two unitary operators, the rotation operator

\[ R(\theta) = \exp[-i\theta(a_+^\dagger a_+ + a_-^\dagger a_-)] \] (2.2)

and the two mode squeeze operator

\[ S_2(r, \phi) = \exp[r(a_+ a_- e^{-2i\phi} - a_-^\dagger a_+^\dagger e^{2i\phi})] \] (2.3)

where \( r \) is the squeeze parameter with range \( 0 \leq r < \infty \) and \( \phi, \theta \) are the rotation parameters with ranges \(-\pi/2 < \phi \leq \pi/2\), \( 0 \leq \theta < 2\pi \). (These parameters and \( U, R, S \) should all carry the label \( \vec{k} \). The \( \pm \) on \( a \) refer to the \( \pm \vec{k} \) modes.) Note that

\[ S_2^\dagger(r, \phi) = S_2^{-1}(r, \phi) = S_2(r, \phi + \pi/2) \] (2.4)

The three real functions \((\theta_{\vec{k}}, \phi_{\vec{k}}, r_{\vec{k}})\) are related to the two complex functions \((\alpha_{\vec{k}}, \beta_{\vec{k}})\) by

\[ \alpha_{\vec{k}} = e^{i\theta_{\vec{k}}} \cosh r_{\vec{k}}, \quad \beta_{\vec{k}} = e^{i(\theta_{\vec{k}} - 2\phi_{\vec{k}})} \sinh r_{\vec{k}}. \] (2.5)

For mode-decompositions in spatially-homogeneous spacetimes leading to no mode-couplings, the Bogolubov transformation connecting the \( a_{\vec{k}} \) and the \( b_{\vec{k}} \) operators is given by (for more general situations, see [3]):

\[ b_{\pm\vec{k}} = \alpha_{\vec{k}} a_{\pm\vec{k}} + \beta_{\vec{k}}^* a_{\mp\vec{k}}^\dagger. \] (2.6)
We see that because of the linear dependence of $b_{+k}$ on $a_{+k}$ and $a^\dagger_{-k}$ (but not $a^\dagger_{+k}$) a two-mode squeeze operator is needed to describe particle pairs in states $\pm k$.

The physical meaning of ‘rotation’ and ‘squeezing’ can be seen from the result of applying these operators for a single-mode harmonic oscillator as follows: ($\hat{k}$th mode label is omitted below)

The Hamiltonian is

$$H_0 = \omega (a^\dagger a + \frac{1}{2})$$

(2.7)

Under rotation,

$$R|0\rangle = |0\rangle, \quad RaR^\dagger = e^{i\theta} a.$$  

(2.8)

Also,

$$R(\theta)R(\theta') = R(\theta + \theta').$$

(2.9)

This implies that

$$R\hat{x}R^\dagger = \cos \theta \hat{x} - \sin \theta \hat{p}$$
$$R\hat{p}R^\dagger = \sin \theta \hat{x} + \cos \theta \hat{p}.$$  

(2.10)

where

$$a = \frac{1}{\sqrt{2}}(\sqrt{\omega} \hat{x} + i \frac{\hat{p}}{\sqrt{\omega}}).$$

(2.11)

Thus the name rotation. Let $\Delta a = a - \langle a \rangle$, (where $\langle \rangle$ denotes the expectation value with respect to any state) then the second-order ‘noise moments’ of $a$ are defined as [9]:

$$\langle (\Delta a)^2 \rangle = \langle a^2 \rangle - \langle a \rangle^2 = \langle (\Delta a^\dagger)^2 \rangle^*$$

$$= \frac{1}{2}[\langle (\Delta x)^2 \rangle - \langle (\Delta p)^2 \rangle] + i \langle (\Delta x \Delta p)_{\text{sym}} \rangle$$

$$\langle |\Delta a|^2 \rangle = \frac{1}{2} \langle \Delta a \Delta a^\dagger + \Delta a^\dagger \Delta a \rangle = \frac{1}{2}[\langle (\Delta x)^2 \rangle + \langle (\Delta p)^2 \rangle].$$

(2.12)

The first quantity is the variance of $a$, a complex second moment, while the second is the correlation, a real second moment, which, as seen in the more familiar $x, p$ representation, measures the mean-square uncertainty (called total noise in [9]). Rotation preserves the number operator

$$Ra^\dagger a R^\dagger = a^\dagger a.$$  

(2.13)

It rotates the moment

$$\langle R(\Delta a)^2 R^\dagger \rangle = e^{2i\theta} \langle (\Delta a)^2 \rangle$$

(2.14)

corresponding to a redistribution between $\hat{x}, \hat{p}$, but preserves the uncertainty

$$\langle R|\Delta a|^2 R^\dagger \rangle = \langle |\Delta a|^2 \rangle.$$  

(2.15)
One can define a displacement operator as

\[ D(\mu) = \exp[\mu a^\dagger - \mu^* a]. \]  

(2.16)

Note that \( D^{-1}(\mu) = D^\dagger(\mu) = D(-\mu). \) The coherent state can be defined as

\[ |\mu\rangle = D(\mu)|0\rangle. \]  

(2.17)

Thus

\[ a|\mu\rangle = \mu|\mu\rangle, \]  

(2.18)

and

\[ Da^\dagger D^\dagger = a^\dagger a - (\mu a^\dagger + \mu^* a) + |\mu|^2. \]  

(2.19)

Under displacement,

\[ D(\mu)aD^\dagger(\mu) = a - \mu. \]  

(2.20)

The displacement operation also preserves the uncertainty

\[ \langle D|\Delta a|^2D^\dagger \rangle = \langle |\Delta a|^2 \rangle. \]  

(2.21)

The single-mode squeeze operator is defined as

\[ S_1(r, \phi) = \exp \left[ \frac{r}{2} (a^2 e^{-2i\phi} - a^\dagger e^{2i\phi}) \right]. \]  

(2.22)

A squeezed state is formed by squeezing a coherent state,

\[ |\sigma\rangle_\mu = S_1(r, \phi)|\mu\rangle \]  

(2.23)

or,

\[ |\sigma\rangle_\mu = |r, \phi, \mu\rangle = S_1(r, \phi)D(\mu)|0\rangle. \]  

(2.24)

Call \( b = S_1 a S_1^\dagger, \) then

\[ b|\sigma\rangle = \mu|\sigma\rangle \]  

(2.25)

and

\[ b = S_1 a S_1^\dagger = a \cosh r + e^{2i\phi} a^\dagger \sinh r. \]  

(2.26)

Thus a squeezed state in the Fock space of \( a \) becomes a coherent state in the Fock space of \( b \) with the same eigenvalue. From this we see the result of \( S_1 \) acting on \( \hat{x} \) and \( \hat{p} \):

\[
S_1 \hat{x} S_1^\dagger = (\cosh r + \cos 2\phi \sinh r)\hat{x} + (\sin 2\phi \sinh r)\hat{p}
\]

\[
S_1 \hat{p} S_1^\dagger = (\cosh r - \cos 2\phi \sinh r)\hat{p} + (\sin 2\phi \sinh r)\hat{x}. \]  

(2.27)

For \( \phi = \pi/2 \), these give

\[ S_1 \hat{x} S_1^\dagger = e^{-r} \hat{x}, \quad S_1 \hat{p} S_1^\dagger = e^r \hat{p}. \]  

(2.28)
Hence the name ‘squeezing’. Two successive squeezes, with the same rotation parameter, result in one squeeze with the squeeze parameter as the sum of the two parameters:

\[ S_1(r, \phi)S_1(r', \phi) = S_1(r + r', \phi). \]  

(2.29)

The expectation value of squeezing the number operator is

\[ \langle S_1^\dagger a^\dagger aS_1 \rangle = \sinh^2 r + (1 + 2 \sinh^2 r) \langle a^\dagger a \rangle + \sinh 2r Re[e^{-2i\phi} \langle a^2 \rangle] \]  

(2.30)

and that of the correlation is

\[ \langle S_1^\dagger |\Delta a|^2 S_1 \rangle = \cosh 2r \langle |\Delta a|^2 \rangle + \sinh 2r Re[e^{-2i\phi} \langle (\Delta a)^2 \rangle] \]  

(2.31)

which for the vacuum and coherent states is always greater than or equal to the original value.

The two-mode squeeze operator defined before is more suitable for the description of cosmological particle creation. One can show that the out state is generated from the in state by including contributions from all \( \vec{k} \) modes,

\[ |\text{out}\rangle = RS|\text{in}\rangle, \quad \text{or} \quad |\rangle = RS|\rangle \]

(2.32)

where

\[ S = \Pi_{k=0}^\infty S_2(r_{\vec{k}}, \phi_{\vec{k}}) \]

(2.33)

In general

\[ \langle \text{out}|F(b_\pm, b_\pm^\dagger)|\text{out}\rangle = \langle \text{in}|F(a_\pm, a_\pm^\dagger)|\text{in}\rangle. \]

(2.34)

The \( |\text{in}\rangle \) state can be a number state, a coherent state or a squeezed state. If the initial state is a vacuum state, \( |\text{in}\rangle = |0\rangle \), then

\[ |0\text{out}\rangle = S(r, \phi - \theta)|0\text{in}\rangle \]

(2.35)

where

\[ S(r, \phi - \theta) = \exp\left\{ \Sigma_{\vec{k}} r_{\vec{k}}^2 [e^{-2i(\phi - \theta_{\vec{k}})} a_{\vec{k}} a_{-\vec{k}} - e^{2i(\phi - \theta_{\vec{k}})} a_{\vec{k}}^\dagger a_{-\vec{k}}^\dagger] \right\} \]

(2.36)

The squeeze parameter \( \sinh^2 r_{\vec{k}} = |\beta_{\vec{k}}|^2 \) measures the number of particles created. Rotation does not play a role. Thus, as observed by Grishchuk and Sidorov [20], cosmological particle creation amounts to squeezing the vacuum. The same can be said about Hawking radiation [27]. For a massless scalar field in an eternal black hole, call \( e^{iJ} \) the unitary operator which connects the Kruskal vacuum with the Schwarzschild vacuum (see e.g., [8])

\[ |0\rangle_S = e^{iJ}|0\rangle_K \]

(2.37)

where

\[ iJ = \Sigma_{\vec{k}} \tanh^{-1}(e^{-\pi\omega/\kappa}) (b_{\vec{k}}^{(1)} b_{\vec{k}}^{(2)} - b_{-\vec{k}}^{(1)} b_{-\vec{k}}^{(2)}). \]

(2.38)

Then the squeeze and rotation parameters can be identified as

\[ r_{\vec{k}} = \tanh^{-1}(e^{-\pi\omega/\kappa}), \quad \phi_{\vec{k}} = \theta_{\vec{k}}. \]

(2.39)

where \( \kappa \) is the surface gravity of the black hole. This is the well-known expression for Hawking radiation. We see that for low-momentum modes in a black hole of high temperature, the squeezing is strong.
3  Number, Coherence and Initial States

3.1 Number does not always increase

We will show in this section that the number of particles produced depends very much on the initial state chosen. The common impression of a net number increase associated with cosmological particle creation is premised upon the assumption that the initial state is an eigenstate of the number operator (called ‘number state’ for short here), and an implicitly invoked random-phase approximation. For states other than this, or for fermions, this is not necessarily true. This was already pointed out in [17, 18, 19]. We shall show this explicitly for the coherent state and the squeezed states.

The number operator for a particle pair in mode $k$ is given by

$$N = a_+^\dagger a_+ + a_-^\dagger a_-. \quad (3.1)$$

The expectation value of the number operator with respect to the $|\text{out}\rangle$ vacuum for a general initial state is

$$(N) = \langle S_2^\dagger R^\dagger N R S_2 \rangle = 2|\beta|^2 + (1 + 2|\beta|^2)\langle N \rangle - 2|\alpha||\beta|(e^{2i\phi}\langle a_+^\dagger a_\dagger^\dagger \rangle + e^{-2i\phi}\langle a_- a_\dagger \rangle). \quad (3.2)$$

Comparing this expression with (2.22), the factor of two for the first $|\beta|^2$ term comes from the spontaneous creation of particles in the $\pm \vec{k}$ modes. The net change in the particle number from the initial to the final state is

$$\Delta N \equiv (N) - \langle N \rangle = 2|\beta|^2[1 + \langle N \rangle] - 2|\beta||\alpha|\{e^{2i\phi}\langle a_+^\dagger a_\dagger^\dagger \rangle + e^{-2i\phi}\langle a_- a_\dagger \rangle\}. \quad (3.3)$$

Here, the first two terms in the square brackets are respectively the spontaneous and stimulated emissions and the last term in the curly brackets is the interference term. The difference between spontaneous and stimulated creation of particles in cosmology was explained first by Parker [1] and explored in more detail by Hu and Kandrup [19]. Note that since there is no $\theta$ dependence, rotation has no effect. If $r_\vec{k} \neq 0$ for some $\vec{k}$ both spontaneous and stimulated contributions are positive. The interference term can be negative for states which give nonzero $\langle a_+ a_- \rangle$. Only when this term is non-zero can $\Delta N$ be negative.

We will calculate the change in particle number for some specific initial states.

a. number state

For an initial number state $|n\rangle = |n_+, n_\rangle$

$$\Delta N = 2|\beta|^2(1 + n_+ + n_-). \quad (3.4)$$

We see that the number of particles will always increase.

b. coherent state

For an initial coherent state

$$|\mu\rangle = D(\mu_+)D(\mu_-)|0, 0\rangle \quad (3.5)$$
we find that
\begin{equation}
\Delta N = 2|\beta|^2[1 + \langle N_+ \rangle + \langle N_- \rangle] - 4|\beta||\alpha|\sqrt{\langle N_+ \rangle\langle N_- \rangle} \cos(2\phi - \zeta_+ - \zeta_-),
\end{equation}

where
\begin{equation}
\mu_+ = \sqrt{\langle N_+ \rangle} e^{i\kappa_+}, \quad \mu_- = \sqrt{\langle N_- \rangle} e^{i\kappa_-}.
\end{equation}

Note the existence of the interference term which can give a negative contribution. It depends not only on the squeeze parameters $|\beta|$ and $\phi$, but also on the particles present and the phase of the initial coherent state. Conditions favorable to a decrease in $\Delta N$ are $\cos(2\phi - \zeta_+ - \zeta_-) = 1$ and $\langle N_+ \rangle = \langle N_- \rangle = \langle N \rangle/2$. In this case we find $\Delta N$ is negative if
\begin{equation}
\langle N \rangle > \frac{|\beta|}{|\alpha| - |\beta|}.
\end{equation}

c. single-mode squeezed vacuum state
For an initial one-mode squeezed state
\begin{equation}
|\sigma \rangle_1 = S_{1+}(r_+, \phi_+)S_{1-}(r_-, \phi_-)|0, 0\rangle
\end{equation}
generated by squeezing the vacuum with $S_{1\pm}$ for the $\pm \vec{k}$ modes, we get
\begin{equation}
\Delta N = 2|\beta|^2(1 + \langle N_+ \rangle + \langle N_- \rangle).
\end{equation}

Once again particle number will always increase.
d. two-mode squeezed vacuum state
For an initial two-mode squeezed vacuum
\begin{equation}
|\sigma \rangle_2 = S_2(r_0, \phi_0)|0, 0\rangle
\end{equation}
where $S_2$ is defined earlier,
\begin{equation}
\Delta N = 2|\beta|^2[1 + \langle N \rangle] + 2|\beta||\alpha|\sqrt{\langle N \rangle(2 + \langle N \rangle)} \cos 2(\phi - \phi_0).
\end{equation}

The cosine factor shows that particle number can decrease given the right phase relations. It can be shown that for $\cos 2(\phi - \phi_0) = -1$ particle number would decrease ($\Delta N \leq 0$) if $r_0 \geq r/2$. If the phase information is randomized the cosine factor averages to zero and there is a net increase in particle number. Since a squeezed state is the end result of squeezing a vacuum via particle creation, one might naively expect to see a monotonic increase in number. Our result shows that this is true only if the phase information is lost in the squeezed state to begin with.

In summary we can make the following observations:
a) Rotation $R$ in the evolution operator $U = RS$ does not influence particle creation.
b) For an initial number state or single mode squeezed vacuum we find a net increase in the number of particles.
c) For an initial coherent state and two mode squeezed vacuum, particle number can increase or decrease. A net increase can nevertheless be obtained by suitable choices of $S_2(r, \phi)$ and $S_2(r_0, \phi_0)$.
d) If random phase is assumed for the initial state the interference term can be averaged out to zero and there will be a net increase in number of particles.
3.2 Coherence can persist

A measure of the coherence of the system is given by the uncertainty (called variance in [3, 14 and 17])

$$|\Delta a|^2 = \frac{1}{2} (\Delta a \Delta a + \Delta a + \Delta a)$$ (3.13)

where $\Delta a = a - \langle a \rangle$. The expectation value of the uncertainty with respect to a state $|\psi\rangle$ is thus,

$$\langle \psi | |\Delta a|^2 |\psi\rangle = \langle \psi | a a |\psi\rangle - |\langle \psi | a |\psi\rangle|^2 + \frac{1}{2}.$$ (3.14)

The expectation value of the uncertainty with respect to a transformed state $|\psi\rangle \equiv RS|\psi\rangle$ is given by

$$\langle \sigma | |\Delta a|^2 |\sigma\rangle = \cosh 2r \langle \sigma | |\Delta a|^2 |\sigma\rangle - 2 \sinh 2r Re[e^{-2i\phi} \langle \sigma | \Delta a + \Delta a - |\sigma\rangle]$$ (3.15)

where $|\Delta a|^2 = |\Delta a +|^2 + |\Delta a -|^2$. For an initial number state, $|\psi\rangle = |n\rangle$,

$$\langle n | |\Delta a|^2 |n\rangle = 2(\frac{1}{2} + |\beta|^2) \langle n | |\Delta a|^2 |n\rangle \geq \langle n | |\Delta a|^2 |n\rangle$$ (3.16)

For a coherent state, $|\psi\rangle = |\mu\rangle$

$$\langle \mu | |\Delta a|^2 |\mu\rangle = 2(\frac{1}{2} + |\beta|^2) \langle \mu | |\Delta a|^2 |\mu\rangle \geq \langle \mu | |\Delta a|^2 |\mu\rangle$$ (3.17)

where the first term corresponds to the vacuum fluctuation and the second term (whose sum over all modes is equivalent to $tr(v_k^+ v_k)$ in [3, 17]) measures the mixing of the positive and negative frequency components of different modes. This result was first derived in [3], and discussed further in [17]. Notice that it is always greater than the original value $\langle |\Delta a|^2 |\mu\rangle$.

For a squeezed state, $|\psi\rangle = |\sigma\rangle = S_2(r_0, \phi_0)|\mu\rangle$

$$\langle \sigma | |\Delta a|^2 |\sigma\rangle = \cosh 2r \langle \sigma | |\Delta a|^2 |\sigma\rangle - 2 \sinh 2r Re[e^{-2i\phi} \langle \sigma | \Delta a + \Delta a - |\sigma\rangle],$$ (3.18)

which can be smaller than the initial value.

Notice that of the three states we discussed, only the squeezed state can allow for a decrease in the uncertainty, i.e., an increase in the coherence as the system evolves. In addition, even though the total number and the total uncertainty of the initial state of the two modes change with particle creation, their difference remains a constant. This is because cosmological particle creation is described by the two mode squeezed operator which satisfies the relations:

$$\langle \psi | S^\dagger (a_+^\dagger a_+ - a_-^\dagger a_-) S |\psi\rangle = \langle \psi | a_+^\dagger a_+ - a_-^\dagger a_- |\psi\rangle,$$

$$\langle \psi | S^\dagger (|\Delta a_+|^2 - |\Delta a_-|^2) S |\psi\rangle = \langle \psi | (|\Delta a_+|^2 - |\Delta a_-|^2) |\psi\rangle.$$ (3.19)
4 Fluctuations in Number

Spontaneous particle creation can be viewed as the parametric amplification of vacuum fluctuations (or squeezing the vacuum). Particle number is an interesting quantity as it measures the degree the vacuum is excited. The fluctuation in particle number is another interesting quantity, as it can be related to the noise of the quantum field and the susceptibility of the vacuum. This is similar in nature to the energy fluctuation (measured by the heat capacity at constant volume) of a system being related to the thermodynamic stability of a canonical system, or the number fluctuation (measured by the compressibility at constant pressure) of a system being related to the thermodynamic stability of a grand canonical system. In gravity, we know that the number fluctuation of a self-gravitating system can be used as a measure of its heat capacity (negative) \cite{28}; and those associated with particle creation from a black hole can be used in a linear-response theory description as a measure of the susceptibility of spacetime \cite{29, 30}. We expect that this quantity associated with cosmological particle creation may provide some important information about quantum noise and vacuum instability \cite{34, 35}.

Define $\delta_i O \equiv [\langle O^2 \rangle - \langle O \rangle^2]$ as the variance or mean-square fluctuations of the variable $O$ with respect to the initial state $| \rangle$, and the corresponding quantity $\delta_f O$ as that with respect to the final state $| \rangle$. Consider the difference between the final and the initial number fluctuation of both the $\pm$ kinds,

$$\delta N = (\delta_f N_+ + \delta_f N_-) - (\delta_i N_+ + \delta_i N_-).$$  \hspace{1cm} (4.1)

Using the expressions given in Sec. 2, we obtain

$$\delta N = 2|\alpha|^2|\beta|^2[\delta N_+ + \delta N_- + \delta L + \partial(N_+ N_-)]_i$$

$$- (|\alpha|^3|\beta| + |\alpha||\beta|^3)[\partial(N_+ L) + \partial(N_- L)]_i$$  \hspace{1cm} (4.2)

where the subscript $i$ refers to the fact that the expectation values are taken with respect to the initial states $| \rangle$, the symbol $\partial$ denotes

$$\partial(PQ) \equiv [\langle PQ \rangle + \langle QP \rangle - 2\langle P \rangle \langle Q \rangle]$$  \hspace{1cm} (4.3)

and

$$L = e^{2i\phi} a^+_+ a^-_- + e^{-2i\phi} a_- a^+_+.$$  \hspace{1cm} (4.4)

Now for an initial number state $|n\rangle = |n_+, n_-\rangle$,

$$\delta N = 2|\alpha|^2|\beta|^2(1 + n_+ + n_- + 2n_+ n_-).$$  \hspace{1cm} (4.5)

we see that the number fluctuations will always increase. For an initial coherent state $|\mu\rangle = D(\mu_+) D(\mu_-)|0, 0\rangle$, where $\mu_\pm = \sqrt{\langle N_\pm \rangle} e^{i\zeta_\pm}$,

$$\delta N = 2|\alpha|^2|\beta|^2[1 + 2(\langle N_+ \rangle + \langle N_- \rangle)]$$

$$- 4\sqrt{\langle N_+ \rangle \langle N_- \rangle}(|\alpha|^3|\beta| + |\alpha||\beta|^3) \cos(2\phi - \zeta_+ - \zeta_-).$$  \hspace{1cm} (4.6)
We find that under the conditions \(\cos(2\phi - \zeta_+ - \zeta_-) = 1\) and \(\langle N_+ \rangle = \langle N_- \rangle = \langle N \rangle / 2\)

\[
\langle N \rangle > \frac{|\beta||\alpha|}{|\alpha|^2 + |\beta|^2 - |\beta||\alpha|}
\] (4.7)

\(\delta N\) can be negative. In the weak particle creation limit \(|\beta| \to 0, |\alpha| \to 1\) we find that (4.7) is equivalent to (3.8). Clearly conditions for a decrease in number fluctuations are not the same as those for a decrease in the number.

For a single-mode squeezed state \(|\sigma\rangle_1 = S_1+(r_+, \phi_+)S_1-(r_-, \phi_-)|0, 0\rangle\)

\[
\delta N = 2|\alpha|^2|\beta|^2[(1 + \langle N_+ \rangle + \langle N_- \rangle)^2 + \langle N_+ \rangle(1 + \langle N_+ \rangle) + \langle N_- \rangle(1 + \langle N_- \rangle) - 2 \sqrt{\langle N_+ \rangle(1 + \langle N_+ \rangle)\langle N_- \rangle(1 + \langle N_- \rangle)\cos 2(2\phi - \phi_+ - \phi_-)].
\] (4.8)

From this it can be shown that, like the change in number, the change in the number fluctuations will always be positive for an initial single mode squeezed vacuum.

For a two-mode squeezed state \(|\sigma\rangle_2 = S_2(r_0, \phi_0)|0, 0\rangle\)

\[
\delta N = |\alpha|^2|\beta|^2\{2(1 + \langle N \rangle)^2 + \langle N \rangle(2 + \langle N \rangle)\} - 2(1 + \langle N \rangle)^2 \sqrt{\langle N \rangle(2 + \langle N \rangle)\cos 2(\phi - \phi_0)}.
\] (4.9)

Note that there is no definite relation between \(\Delta N\) and \(\delta N\). For large \(N >> 1\) or small \(|\beta| << 1\), \(\delta N \leq 0\). The relevance of the number fluctuations in cosmological particle creation in defining the susceptibility of the vacuum and the noise of quantum fields have been hinted upon earlier [26, 31, 32]. The result obtained here will be useful in relating to issues of noise and fluctuation of quantum fields, and dissipation and instability of spacetime in semiclassical gravity and quantum cosmology [33, 34, 35, 36].

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**References**

[1] L. Parker, Ph. D. Thesis, Harvard University, 1966; Phys. Rev. Lett. 21, 562 (1968); Phys. Rev. 183, 1057 (1969); D3, 346 (1971)

[2] R. U. Sexl and H. K. Urbantke, Phys. Rev., 179, 1247 (1969)

[3] Ya. B. Zel’dovich, Pis’ma Zh. Eksp. Teor. Fiz, 12, 443 (1970) [JETP Lett. 12, 307(1970)]; Ya. B. Zel’dovich, in *Physics of the Expanding Universe*, ed. M. Demiansky (Springer, Berlin, 1979)
[4] Ya. B. Zel’dovich and A. A. Starobinsky, Zh. Teor. Eksp. Fiz. 61, 2161 (1971) [Sov. Phys. JETP 34, 1159 (1972)]; Ya. B. Zel’dovich and A. A. Starobinsky, Pis’ma Zh. Eksp. Teor. Fiz. 26, 373 (1977) [Sov. Phys. JETP Lett. 26, 252 (1977)]

[5] B. L. Hu, Ph. D. Thesis, Princeton University, 1972.

[6] N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space, (Cambridge University Press, Cambridge, 1982).

[7] B. L. Hu and L. Parker, Phys. Lett. 63A, 217 (1977); Phys. Rev. D17, 933 (1978)

[8] M. V. Fischetti, J. B. Hartle and B. L. Hu, Phys. Rev. D20, 1757 (1979) J. B. Hartle and B. L. Hu, Phys. Rev. D20, 1772 (1979); D21, 2756 (1980).

[9] C. M. Caves and B. L. Schumacher, Phys. Rev. A31, 3068, 3093 (1985); B. L. Schumacher, Phys. Rep. 135, 317 (1986).

[10] S. Kamefuchi and H. Umezawa, Nuovo Cimento 31, 429 (1964) Appendix A.

[11] B. L. Hu, Phys. Rev. D9, 3263 (1974)

[12] L. Grishchuk, Zh. Eksp. Teor. Fiz. 67, 825 (1974) [Sov. Phys. JETP 40, 409 (1975)].

[13] B. K. Berger, Phys. Rev. D11, 2770 (1975)

[14] B. R. Mollow, Phys. Rev. 162, 1256 (1967); L. S. Brown and L. J. Carson, Phys. Rev. A20, 2486 (1979).

[15] B. L. Hu and A. Matacz, “Quantum Brownian Motion in a Bath of Parametric Oscillators”, Univ. Maryland preprint 93-210 (1993)

[16] L. Parker, Nature 261, 20 (1976); and in Asymptotic Structure of Spacetime pp. 107-227 eds F. P. Esposito and L. Witten (Plenum Press, N. Y. 1977)

[17] B. L. Hu and D. Pavon, Phys. Lett. B180, 329 (1986)

[18] H. E. Kandrup, Phys. Rev. D37, 3505 (1988)

[19] B. L. Hu and H. E. Kandrup, Phys. Rev. D35, 1776 (1987)

[20] L. Grishchuk and Y. V. Sidorov, Phys. Rev. D42, 3414 (1990)

[21] A. Albrecht et al, Imperial College preprint TP/92-93/21 (1992)
[22] R. H. Brandenberger, V. Mukhanov and T. Prokopec, Phys. Rev. Lett. 69, 3606 (1992); Phys. Rev. D48, 2443 (1993)

[23] M. Gasperini and M. Giovanni, Phys. Lett. B301, 334 (1993); M. Gasperini and M. Giovanni and Veneziano, Phys. Rev. D48, R439 (1993).

[24] A. Matacz, Univ. of Adelaide preprint ADP-92-198/M13 (1993)

[25] B. L. Hu, Phys. Lett. 108B, 19 (1982); 123B, 189 (1983)

[26] B. L. Hu, Phys. Lett. A90, 375 (1982); B. L. Hu, in Cosmology of the Early Universe, ed. L. Z. Fang and R. Ruffini (World Scientific, Singapore, 1984)

[27] S. W. Hawking, Comm. Math. Phys. 43, 199 (1985)

[28] D. Lynden-Bell and R. M. Lynden-Bell, MNRAS 181, 405 (1977)

[29] P. Candelas and D. W. Sciama, Phys. Rev. Lett. 38, 1372 (1977)

[30] E. Mottola, Phys. Rev. D33, 2136 (1986)

[31] B. L. Hu, Phys. Lett. A97, 368 (1983); B. L. Hu, Vistas in Astronomy 37, 391 (1993)

[32] B. L. Hu, Physica 158, 399 (1989)

[33] B. L. Hu and S. Sinha, “Fluctuation-Dissipation Relation in Cosmology”, Univ. Maryland preprint 93-164 (1993)

[34] E. Calzetta and B. L. Hu, “Noise and Fluctuations in Semiclassical Gravity”, Univ. Maryland preprint 93-216 (1993)

[35] B. L. Hu and A. Matacz, “Quantum Noise in Gravitation and Cosmology” Invited Talk at the Workshop on Fluctuations and Order, ed. M. Millonas (MIT Press, Cambridge, 1994). Univ. Maryland preprint 94-44 (1994)

[36] B. L. Hu and A. Matacz, “Einstein-Langevin Equation for Backreactions in Semiclassical Cosmology”, Univ. Maryland preprint 94-31 (1993)