On the philosophical, cognitive and mathematical foundations of symbiotic autonomous systems

Yingxu Wang¹, Fakhri Karray², Sam Kwong³, Konstantinos N. Plataniotis⁴, Henry Leung¹, Ming Hou⁵, Edward Tunstel⁶, Imre J. Rudas⁷, Ljiljana Trajkovic⁸, Okyay Kaynak⁹, Janusz Kaćprzyk¹⁰, Mengchu Zhou¹¹, Michael H. Smith¹², Philip Chen¹³ and Shushma Patel¹⁴

¹FIEE, Department of Electrical and Software Engineering, Schulich School of Engineering and Hotchkiss Brain Institute, Int’l Institute of Cognitive Informatics and Cognitive Computing (ICICC), University of Calgary, Calgary, Canada
²FIEE, Department of Electrical and Computer Engineering, University of Waterloo, Ontario, Canada
³FIEE, Department of Computer Science, City University of Hong Kong, Kowloon, Hong Kong
⁴FIEE, Department of Electrical and Computer Engineering, University of Toronto, Ontario, Canada
⁵SMIEEE, Toronto Research Centre, DRDC, Toronto, ON, Canada
⁶FIEE, Autonomous and Intelligent Systems Department, Raytheon Technologies Research Center, East Hartford, CT, USA
⁷FIEE, University Research and Innovation Center (EKIK), Óbuda University, Budapest, Hungary
⁸FIEE, School of Engineering Science, Simon Fraser University, Burnaby, British Columbia, Canada
⁹FIEE, Bogazici University, Bebek, Istanbul, Turkey
¹⁰IEEE, Systems Research Institute, Polish Academy of Sciences, Warsaw, Poland
¹¹FIEE, Helen and John C. Hartmann Department of Electrical and Computer Engineering, New Jersey Institute of Technology, NJ, USA
¹²SMIEEE, Furaxa, Inc., Orinda, CA, USA

© 2021 The Author(s) Published by the Royal Society. All rights reserved.
Symbiotic autonomous systems (SAS) are advanced intelligent and cognitive systems that exhibit autonomous collective intelligence enabled by coherent symbiosis of human–machine interactions in hybrid societies. Basic research in the emerging field of SAS has triggered advanced general-AI technologies that either function without human intervention or synergize humans and intelligent machines in coherent cognitive systems. This work presents a theoretical framework of SAS underpinned by the latest advances in intelligence, cognition, computer, and system sciences. SAS are characterized by the composition of autonomous and symbiotic systems that adopt bio-brain-social-inspired and heterogeneously synergized structures and autonomous behaviours. This paper explores the cognitive and mathematical foundations of SAS. The challenges to seamless human–machine interactions in a hybrid environment are addressed. SAS-based collective intelligence is explored in order to augment human capability by autonomous machine intelligence towards the next generation of general AI, cognitive computers, and trustworthy mission-critical intelligent systems. Emerging paradigms and engineering applications of SAS are elaborated via autonomous knowledge learning systems that symbiotically work between humans and cognitive robots.

This article is part of the theme issue ‘Towards symbiotic autonomous systems’.

1. Introduction

Philosophy is a formal means of thought for abstraction and inference that not only enables inductive theories of knowledge to be developed based on real-world observations, but also constitutes deductive inferences for rigorous thinking [1–3]. Human intelligence and wisdom are overarchingly represented by the cognitive and scientific philosophy. Basic research and engineering demands on autonomous and symbiotic systems embody a contemporary philosophy of the extended understanding towards intelligence and knowledge sciences in the era from information to intelligence revolution. Symbiotic autonomous systems (SAS) are an emerging field of general AI methodology underpinned by the latest advances in intelligence, cognition, computer, and system sciences. The term symbiosis indicates mutually coherent and heterogeneously intelligent systems for enabling collective intelligence by autonomous human–machine interactions in the emerging hybrid society where humans and smart machines work symbiotically [4–9].

A symbiotic worldview, as illustrated in figure 1, is an overarching abstraction of the universe of discourse of nature. In figure 1, the double arrows denote bi-directional relations between the essences in the dual world where known relations are denoted by solid lines and relations yet to be revealed are denoted by dashed lines not in bold.

**Definition 1.1.** The universe of discourse of the symbiotic worldview is a dual denoted by the information-matter-energy-intelligence (IME-I) model of the natural world (NW). One facet of NW is the physical world (PW) that is modelled by matter (M) and energy (E); while the other facet is the abstract world (AW) that is represented by information (I) as a generic model of human perceptions and abstractions. In the IME-I model, intelligence (I) plays a central role for the transformation among I, M, and E.

The latest development in SAS is underpinned by the deep explorations in intelligence science and the fast growing demand for symbiotic societies where collective intelligence of humans and machines may coherently work together. It is recognized that the universe of discourse of sciences
may be categorized into natural and abstract sciences [10] as shown in figure 2 that indicates the extended horizon of human knowledge, the deepened understanding of the NW, and their symbiotic interactions.

The universe of discourse of contemporary sciences indicates that human abstraction and reasoning power have been advancing from concrete to abstract sciences. The concrete sciences are scientific disciplines about natural and material entities including physics, chemistry, biology, neurology, physiology, brain, and engineering sciences. However, the abstract sciences are scientific disciplines about the abstract entities including data, information, knowledge, intelligence, mathematics, philosophy, and system sciences. This leads to the findings in symbiotic
cognitive cybernetics where intelligence may not be directly aggregated from data no matter how big they are, because there are multiple inductive layers from data to intelligence [10].

Definition 1.2. Intelligence $\tilde{I}$ is a human, animal, or system mechanism that autonomously transfers a piece of information $I$ into a behaviour $B$ or an item of knowledge $K$:

\[
\tilde{I} = f_{\text{to-do}} : I \rightarrow B \tag{1.1a}
\]

\[
\parallel f_{\text{to-be}} : I \rightarrow K \tag{1.1b}
\]

where equation (1.1a) denotes the narrow sense of intelligence, while both equations (1.1a) and (1.1b) in parallel ($\parallel$) represent the broad sense of intelligence. Intelligence-related mathematical models of information, knowledge, and behaviour are to be introduced later.

A series of white papers on SAS have been released by the IEEE Symbiotic Autonomous Systems Initiative (SASI) [11], which indicates the importance of the topic recognized by IEEE, a worldwide opinion making society. IEEE SASI has recognized a wide range of SAS applications including ambient technologies, intelligent interactions, adaption to environment, robotized objects, smart cities, industrial robots, augmented humans, and ethical challenges. One of the perspectives by SASI is the crucial need for developing a formal approach to SAS rooted in and based on various fields of science and technology exemplified by cognitive science, computer science, robotics, autonomous decision-making systems [3,6,8,9,12–14], and run-time intelligent behavioural generation [15]. It is recognized that such a formally sound theory would accelerate the development of the field and facilitate its practical applications.

The advances towards SAS are driven by the theoretical and technological development across intelligence science, brain science, cognitive science, robotics, and computational intelligence. Basic studies of the fundamental constraints and challenges of intelligence science and AI have led to the breakthroughs towards SAS beyond adaptive systems by conventional deterministic computing technologies. SAS are expected to address the persistent challenges of non-deterministic decision-making, autonomous intelligence generation, and run-time machine behavioural generation beyond stored-program-based computing [16]. SAS that involves human collaborations (or interventions) to complete some highly intelligent tasks will become the essence of future AI technologies and applications [13].

This work explores the philosophical, cognitive, and mathematical foundations of SAS. It investigates the technical bottlenecks of SAS and indispensable theories underpinned by the latest advances in intelligence science, computational intelligence, and intelligent mathematics (IM). It addresses the challenges to seamlessly incorporate human and machine interactions in SAS theoretically and pragmatically. This may enhance SAS-based machine intelligence towards autonomous and cognitive intelligence in order to augment human collective capability by advanced machine intelligence. Sections 2 and 3 formally elaborate on symbiotic and autonomous systems, respectively, which lead to the theoretical framework of SAS in section 4. Emerging SAS paradigms including machine knowledge learning systems, brain-inspired systems, and cognitive robots are demonstrated. The breakthroughs in SAS are expected to trigger a wide range of novel applications towards the next generation of general AI, cognitive computers, and mission-critical intelligent systems.

2. Symbiotic systems

Symbiosis is a widely observable phenomenon in biological, mental, and social systems where mutual dependences exist among plants, animals, and human societies as a necessary condition for them to coevolve [4–7]. Symbiosis is particularly important to human societies because of the fundamental need for extending individuals’ physical, intellectual, and/or resource limits. Therefore, it becomes a fundamental principle of system science and the universal context of modern sciences and engineering.
(a) Conceptual model of symbiotic systems

The conceptual model of symbiotic systems is formally introduced and elaborated in this subsection.

**Definition 2.1.** A symbiotic system (SS) is a bio-brain-social-inspired system characterized by heterogeneously coherent autonomous structures and behaviours for embodying collective intelligence.

Recent basic research and theoretical breakthroughs [6] have led to the systematic revelations of the parallel and recursive framework of contemporary sciences and human-nature symbiosis. In a broad sense, all subsystems of a complex system are symbiotic in the forms of positive and negative coherency where the former is for structural or functional augmentation of the system, while the latter is for system stabilization and autonomous control [15].

An example of a natural SS with negative feedback is the predator-prey system (PPS) where the populations of lions and gazelles in a territory are modelled as a dynamic SS by a system of ordinary differential equations:

\[
\begin{align*}
\frac{dN_L}{dt} &= b_L N_L N_G - d_L N_L \\
\frac{dN_G}{dt} &= b_G N_G - d_G N_G N_L
\end{align*}
\]  

(2.1)

where \(N_L\) and \(N_G\) represent the populations of lions and gazelles, \(b_L, b_G, d_L,\) and \(d_G,\) their birth and death rates, respectively.

The simulation of this natural SS in MATLAB is shown in figure 3 where the predator and prey are mutually dependent for food and health, respectively. It is interesting in this natural symbiotic eco-system that the population of the predators is eventually controlled by the preys. From a similar point of view, the COVID-19 pandemic is an SS phenomenon where the viruses depend on human population. Therefore, in philosophy, natural viruses never overall defeat mankind because that may contradict their fatalistic purpose.

**Figure 3.** Simulation of a predator-prey symbiotic system. (Online version in colour.)
**Figure 4.** The hyperstructural model of a general symbiotic system. (Online version in colour.)

**Definition 2.2.** The mathematical model of a general SS § is an 8-tuple:

\[
\text{§} \triangleq (C, B, R_c, R_b, R_f, R_i, R_o, \Theta) \tag{2.2}
\]

where \( C \) is a finite set of components of §; \( B \) a finite set of behaviours; \( R_c = C \times C \) a set of component relations; \( R_b = B \times B \) a set of behavioural relations; \( R_f = B \times C \) a set of functional relations; \( \Theta \) the environment of §; \( R_i = \Theta \cdot B \times \text{§} \) a set of input relations between the B dimension of \( \Theta \) and § (i.e., \( \Theta \cdot B \) and §·B); and \( R_o = \text{§} \cdot B \times \Theta \) a set of output relations.

Definition (2.2) provides a formal model of general SS that constitutes the properties of arbitrary SS. The hyperstructure of the abstract SS is illustrated in figure 4 where \( C, B, \) and \( R = \{R_c, R_b, R_f, R_i, R_o, \Theta\} \) denote the components, behaviours, and their structural/behavioural/functional/input/output relations, respectively. The environment \( \Theta \) represents the external space of the given SS embodied by other external SS'.

According to definition (2.2), human group coordination theory, sociology, and management science are an applied SS domain. It is discovered that coherency is the essence of SS [3,17–19] in SS theory, social networks, and Internet of Things (IoT). For instance, in computer science and complex software engineering, a mechanism is found where the number of people in a coordinated group may be traded with time or project duration under certain symbiotic conditions [20].

**(b) Topological properties of symbiotic systems**

SS is pervasively indispensable in the natural world in general and human societies in particular. The general topology of an SS or a system of SS' is a hierarchical and recursive structure rather than a flat one [3,18].

**Definition 2.3.** The general topology \( S \) of SS is a recursively hierarchical structure where each \( k \)th layer \( S^k \) in the \( n \)-layer system hierarchy is embedded into the adjacent upper layer \( S^{k+1} \):

\[
\overline{S} \triangleq \bigcup_{k=1}^{n} R_k S^k(S^{k-1}), \quad S^0 = (C_0, B_0, R_{c0}, R_{b0}, R_{f0}, R_{i0}, R_{o0}, \Theta_0)
\]

\[
= S^n(S^{n-1}(\ldots(S^1(S^0)))) \tag{2.3}
\]

where the *big-R notation* [20] is a general recursive operator that denotes a set of recurring structures or a set of iterative/embedded behaviours.
Theorem 1. The general topology \( \tilde{S} \) of SS is constituted by the following properties:

(a) \( \tilde{S} \) is necessarily constrained by bottom-up inductions;
(b) \( \tilde{S} \) is sufficiently constrained by top-down deductions.

Proof. \( \forall \ S^0 = (C_0, B^0_0, R^0, \Theta_0) = (C_0, B_0, R_0^b, R_0^l, R_0^i, R_0^o, \Theta_0) \) as a primitive SS according to definition (2.3) where all attributes are known and concrete:

(a) The necessary property of \( \tilde{S} \) holds by a series of bottom-up inductions if \( S^0 \) is known:

\[
S^1 = \prod_{i_1=1}^{k_1} \mathcal{R}_{i_1} S^0_{i_1} = \prod_{i_1=1}^{k_1} \mathcal{R}_{i_1} (C^0_{i_1}, B^0_{i_1}, R^0_{i_1}, \Theta^0_{i_1}) \\
S^2 = \prod_{i_2=1}^{k_2} \mathcal{R}_{i_2} S^1_{i_2} = \prod_{i_2=1}^{k_2} \mathcal{R}_{i_2} \prod_{i_1=1}^{k_1} \mathcal{R}_{i_1} (C^0_{i_1}, B^0_{i_1}, R^0_{i_1}, \Theta^0_{i_1}) \\
\vdots \\
S^n = \prod_{i_n=1}^{k_n} \mathcal{R}_{i_n} \ldots \mathcal{R}_{i_2} \mathcal{R}_{i_1} S^0_{i_1 \ldots i_n} = \prod_{i_n=1}^{k_n} \mathcal{R}_{i_n} \ldots \mathcal{R}_{i_1} (C^0_{i_1 \ldots i_n}, B^0_{i_1 \ldots i_n}, R^0_{i_1 \ldots i_n}, \Theta^0_{i_1 \ldots i_n}) \\
= S^n(S^{n-1}(\ldots(S^1(S^0)))) = S 
\]

(b) The sufficient property of \( \tilde{S} \) holds by a series of top-down deductions by an inverse process of equation (2.4) that reduce the SS from top-down until it is realized by \( S^0 \).

According to theorem (1), a set of properties of system symbiosis may be derived including: (a) the general topological structure of SS is a recursive hierarchical structure that links the embedded structures and relations between two arbitrarily adjacent layers of SS; (b) the abstraction principle of systems states that any SS may be inductively integrated or composed with decreasing details at different layers \( 0 \leq k \leq n \) from the bottom up; and (c) the refinement principle of systems states that any SS may be deductively specified or analysed with increasing details at different layers \( 0 \leq k \leq n \) from the top down.

(c) Symbiotic mechanisms of symbiotic systems

The principle of symbiosis for systems may be revealed by the mechanism of system fusions formally expressed by a novel mathematical model of incremental union of sets of relations [18]. In this approach, the symbiotic system gaining from simple standalone or lower-level systems may be rigorously explained and quantitatively determined.

Definition 2.4. A symbiotic fusion \( \sqcup \) is an incremental union of a pair of sets of relations, \( R_1 \) and \( R_2 \), between two systems \( S_1 \) and \( S_2 \):

\[
R(S_1, S_2) \triangleq R_1 \sqcup R_2 = R_1 \cup R_2 \cup \Delta R_{12}(C_1, C_2) 
\]

where \( C_1 \) and \( C_2 \) are sets of components (entities) in \( S_1 \) and \( S_2 \) respectively, and \( \Delta R_{12} \) is the set of symbiotic relations generated between \( S_1 \) and \( S_2 \).

Based on definition (2.4), an important SS principle known as a symbiotic mechanism is introduced to explain the nature of system symbiosis.
Figure 5. The symbiosis of general systems and the SAS gains. (Online version in colour.)

Theorem 2. Given a pair of arbitrary systems $S_1 = (C_1, B_1, R_1, \Theta_1)$ and $S_2 = (C_2, B_2, R_2, \Theta_2)$, system symbiosis $\Omega_s(S_1, S_2)$ in an SS is generated by a set of symbiotic gains $\Delta R_{12}$ during a symbiotic fusion $\oplus$:

$$\Omega_s(S_1, S_2) = |\Delta R_{12}(C_1, C_2)|$$

$$= 2|C_1| \cdot |C_2|, \forall \Delta R_{12} \subseteq R_1 \oplus R_2 = R_1 \cup R_2 \cup \Delta R_{12}$$  \hspace{1cm} (2.6)

where the newly created symbiotic gain $\Delta R_{12}$ is an element of $R_1 \oplus R_2$ but $\Delta R_{12} \not\subseteq R_1 \wedge \Delta R_{12} \not\subseteq R_2$.

Proof. $\forall S_1 = (C_1, B_1, R_1, \Theta_1)$ and $S_2 = (C_2, B_2, R_2, \Theta_2)$, the symbiotic result $\Delta R_{12}$ are obtained as the difference between the size of the entire relations $|R|$ of the SS and the sum of the individual subsystems $|R_1|$ and $|R_2|$. Therefore, theorem (2.6) holds because:

$$\forall \Delta R_{12} \subseteq SS, R_1 \supseteq S_1, \text{and } R_2 \supseteq S_2,$$

$$\Omega_s(S_1, S_2) = |\Delta R_{12}(C_1, C_2)|$$

$$= |R| - (|R_1| + |R_2|)$$

$$= (n_{c_1} + n_{c_2})^2 - (n_{c_1}^2 + n_{c_2}^2) = (n_{c_1}^2 + 2n_{c_1}n_{c_2} + n_{c_2}^2) - (n_{c_1}^2 + n_{c_2}^2)$$

$$= 2n_{c_1}n_{c_2}$$

$$= 2|C_1||C_2|$$  \hspace{1cm} (2.7)

Theorem (2) formally elaborates on the natural mechanisms of symbiosis in SS and how it is rigorously determined. The symbiotic principle of SS as derived in theorem (2) explains that the composition $\oplus$ of two systems, $SS = S_1 \oplus S_2$, results in the generation of new functions $\Delta R_{12}$ that solely belong to the SS but do not exist in any of the individual systems $S_1$ or $S_2$ when they stand alone.

For instance, given a pair of simple systems $S_1$ and $S_2$ as shown in figure 5, the symbiotic system generated by their composition results in a new set of relations or functions, as well as complexities, which may be rigorously determined according to theorem (2.6) as $\Omega_{12} \cdot |\Delta R_{12}| = 2|C_1| \cdot |C_2| = 2(3 \cdot 2) = 12$.

Another symbiotic principle of group gains may be formally described based on the random nature of human-system error-making mechanisms in group task performance [21]. A statistical property of SS is that the occurrences of an identical error by different individuals may most likely happen at different times. This human error-making mechanism in symbiotic groups provides a foundation for fault-tolerance towards symbiotic trustworthiness. It indicates that human/system errors may be prevented from happening or be collectively cancelled in a symbiotic architecture of peers and/or human–machine systems. The following result states the principle of symbiotic error cancellation.
Theorem 3. Symbiotic error cancellation in an SS is a mechanism that the collective reliability $\mathcal{R}(SS)$, of an SS, as a complement of its error rate $\mathcal{E}(SS)$, is always greater than the sum of its $n$ individual subsystems $\mathcal{R}(\sum_{k=1}^{n} S(k))$:

$$\mathcal{R}(SS) \gg \mathcal{R}(\sum_{k=1}^{n} S(k)) \quad (2.8)$$

where $\mathcal{R}(SS)$ is the symbiotic trustworthiness gained by the SS.

Proof. Let the symbiotic error cancellation $\Delta_r(SS) = \mathcal{R}(SS) - \mathcal{R}(\sum_{k=1}^{n} S(k))$ be the difference between the reliabilities of the SS and the sum of its subsystems $S(k)$.

$$\forall \mathcal{R} = 1 - \mathcal{E} \text{ and } \prod_{k=1}^{n} (r_e(k) < 1.0),$$

$$\Delta_r(SS) = \mathcal{R}(SS) - \mathcal{R}(\sum_{k=1}^{n} S(k))$$

$$= \left(1 - \prod_{k=1}^{n} r_e(k)\right) - \left(1 - \sum_{k=1}^{n} r_e(k)\right)$$

$$= \sum_{k=1}^{n} r_e(k) - \prod_{k=1}^{n} r_e(k), \quad R \left( r_e(k) < 1.0 \right)$$

$$\gg 0$$

$$\Rightarrow \mathcal{R}(SS) \gg \mathcal{R}(\sum_{k=1}^{n} S(k)) \quad (2.9)$$

Empirically, the symbiotic reliability of a system may be gained by intensive rechecking based on the random nature of error distributions and the independent nature of error patterns among individuals or components in an SS. The principle of symbiotic error cancellation may be applied in SS and hybrid intelligent systems where humans and machines work together, particularly in mission-critical and safety-critical SS.

It is recognized that humans are the most dynamic and active part of SS. Because the most matured SS paradigm is the brain, advanced SS are naturally open to incorporate human intelligence as indicated in figure 6. According to theorems (2) and (3), a hybrid SS with humans in the loop gains strengths towards the implementation of cognitive intelligent systems. The cognitive SS sufficiently enables a powerful intelligent system with the strengths of both human and machine intelligence. This is why intelligence and system sciences may inspire SS designers to develop fully autonomous intelligent systems in highly demanded engineering applications. More general SS gains have been recognized in this work including: (a) extend physical capability of individuals; (b) improve group productivity and efficiency; (c) enhance quality and reliability; (d) enable information/skills/knowledge sharing; (e) gain exponential learning power; and (f) enable collective intelligence and wisdom.

SS paradigms include simulated natural intelligence systems, social computing systems, human–machine systems, cognitive systems, cognitive robots, bioinformatics systems, brain-inspired systems, self-driving automobiles, unmanned systems, and intelligent IoT. Some of them are analysed in section 4.

3. Autonomous systems

The transdisciplinary advances in intelligence, cognition, computer, cybernetic, and systems sciences have led to the emerging field of autonomous systems [22–30]. The ultimate goal of autonomous systems is to implement a brain-inspired system that may think and act as a human.
Definition 3.1. Autonomous systems (AS) are advanced intelligent systems that function without human intervention for implementing complex cognitive abilities aggregating from reflexive, imperative, and adaptive intelligence to autonomous and cognitive intelligence.

(a) Intelligence science foundations of autonomous systems

Intelligence is the paramount cognitive ability of humans that may be mimicked by AS implemented using computational intelligence. A classification of intelligent systems may be derived according to the forms of system inputs and outputs as shown in table 1. The low-level reflexive and imperative systems are able to process deterministic stimuli by deterministic or indeterministic algorithms. The adaptive systems are designed to deal with indeterministic stimuli by deterministic behaviours predefined at the design time. However, AS is characterized by both indeterministic stimuli and indeterministic behaviours pending for run-time contexts.

Definition 3.2. Intelligence science is a contemporary discipline that studies the mechanisms and properties of intelligence, formal principles, mathematical means for intelligence manipulations, generation of intelligence through the neural, cognitive, functional and mathematical levels, as well as their engineering and computational implementations.
The hierarchical intelligence model (HIM) introduced in figure 6 reveals the levels of intelligence and their increasing complexities and difficulties for implementation in computational intelligence based on the abstract intelligence (αI) theory [16]. The levels of intelligence in HIM are aggregated from reflexive, imperative, adaptive, autonomous, and cognitive intelligence. Types of system intelligence across the HIM layers may be formally described by using the pattern of stimulus/event-driven behaviours that are formally modelled in definition (3.3).

The field of AS studies the properties of intelligence for realizing brain-inspired systems by high-level machine intelligence beyond those of imperative and adaptive systems with deterministic or prescriptive behaviours. In the past 60 years of AI and systems engineering, few actual AS’s were developed because the theoretical foundations for autonomous intelligence and systems have not sufficiently matured [16,31]. Many current AI systems are still bounded by the intelligence bottleneck of adaptive mechanisms where machine intelligence is constrained by the lower-level reflexive, imperative, and deterministic intelligent abilities. The findings indicate that, to extend the intelligence power of traditional AI systems via AS, a general AI (GAI) system needs not only to mimic the imperative and iterative intelligence but also to realize more powerful human equivalent intelligence according to the HIM theory.

(b) System science foundations of autonomous systems

The structural properties of AS naturally well fit the principle of recursive systems [3,9,17,18]. According to the HIM model, the AS framework provides an explanation of how the advanced AS intelligence is generated from the bottom up based on the transformability among the five-level hierarchy of system intelligence. The HIM framework of AS outlined in figure 6 may be refined by 16 intelligent behaviours at five levels listed in table 2. Each of the intelligent behaviours may be formally described according to the event dispatching operation [16].

**Definition 3.3.** The mathematical model of AS is a formalization of HIM by a unified event-driven dispatching pattern that denotes 16 intelligent behaviours at five levels:

\[
AS \subseteq \bigcap_{i=1}^{5} \left( \bigcap_{j=1}^{m_i} \left( e_j \mid \mathbb{T} \mathbb{\rightarrow} B_j \mid \mathbb{PM} \right) \right) \tag{3.1}
\]

where \( \bigcap_{i=1}^{5} m_i = \{1, 3, 3, 5, 4\} \), \( \mathbb{\rightarrow} \) represents the event dispatching operator, \( e_j \mid \mathbb{T} \) the \( j \)-th event prespecified by a type suffix \( \mathbb{T} \) at the \( i \)-th level, and \( B_j \mid \mathbb{PM} \) the corresponding behaviour triggered by a certain event as a process model (PM).

AS exhibit non-deterministic, context-dependent, and adaptive behaviours of machine intelligence. AS are nonlinear systems that not only depend on current stimuli or demands to the system, but are also modulated by internal status and perceptions formed by historical events and current rational goals.

**Theorem 4.** AS are characterized by (a) a hierarchical architecture and (b) a series of recursively inclusive behaviours:

\[
AS \mathcal{\triangleq} \left\{ \begin{array}{l}
\bigcap_{k=1}^{4} \left( B^k \left( B^{k-1} \right) \right), B^0 = \bigcap_{i=1}^{n_{ref}} e_i \mid \mathbb{REF} \mathbb{\rightarrow} B_{ref} (i) \mid \mathbb{PM} \\
B_{Cog} \supseteq B_{Aut} \supseteq B_{Ada} \supseteq B_{Imp} \supseteq B_{Ref}
\end{array} \right. \tag{3.2a}
\]

**Proof.** (a) The behavioural architecture \( \bigcap_{k=1}^{4} B^k \left( B^{k-1} \right) \) of arbitrary AS in equation (3.2a) aggregates \( B^0 \) through \( B^4 \) hierarchically from the bottom up if \( B^0 \) is deterministic. (b) Because the five-level behaviours of AS in equation (3.2b) form a partial order across all layers, AS are recursively inclusive from the bottom up.

\[
\tag{3.2b}
\]
Table 2. The framework of autonomous intelligence embodied in AS.

| level | category          | type              | symbol | description                                                                 |
|-------|-------------------|-------------------|--------|----------------------------------------------------------------------------|
| 1     | reflective intelligence | reflexive        | $\text{ref}$ | a wired behaviour directly driven by specifically coupled external stimuli or triggering event |
| 2     | imperative intelligence | event-driven    | $\text{emp}$ | a predefined imperative behaviour driven by an event                      |
|       |                   | time-driven      | $\text{emp}$ | a predefined imperative behaviour driven by a point of time               |
|       |                   | interrupt-driven | $\text{emp}$ | a predefined imperative behaviour driven by a system-triggered interrupt event |
| 3     | adaptive intelligence | analogy-based    | $\text{ab}$ | an adaptive behaviour that operates by seeking an equivalent solution for a given analogue request |
|       |                   | feedback-modulated | $\text{ab}$ | an adaptive behaviour rectified by the feedback of temporal system output   |
|       |                   | environment-aware | $\text{ab}$ | an adaptive behaviour where multiple prototype behaviours are modulated by the change of external environment |
| 4     | autonomous intelligence | perceptive        | $\text{aut}$ | an autonomous behaviour based on the selection of a perceptive inference    |
|       |                   | problem-driven   | $\text{aut}$ | an autonomous behaviour that seeks a rational solution for a given problem |
|       |                   | inference-driven | $\text{aut}$ | an autonomous behaviour seeking an optimal path towards the given goal      |
|       |                   | decision-driven  | $\text{aut}$ | an autonomous behaviour embodied by the outcome of a decision process      |
|       |                   | deductive        | $\text{aut}$ | an autonomous behaviour driven by a deductive process based on known principles |
| 5     | cognitive intelligence | knowledge-based | $\text{cog}$ | a cognitive behaviour generated by introspection of acquired knowledge     |
|       |                   | learning-driven  | $\text{cog}$ | a cognitive behaviour generated by both internal introspection and external acquisition |
|       |                   | goal-driven      | $\text{cog}$ | a cognitive behaviour that creates a causal chain from a problem to a rational solution |
|       |                   | inductive        | $\text{cog}$ | a cognitive behaviour that draws a general rule based on multiple observations or common properties |
Theorem (4) indicates that any lower layer AS behaviour is a subset of a higher layer. In other words, any higher layer AS behaviour is a natural aggregation of those of lower layers as shown in equation (3.2) and figure 6.

The theories and technologies of AS explain the evolution of human and system intelligence as an inductive process of intelligence generation. They also reveal the properties of system autonomy at five levels implemented by computational intelligence and system engineering technologies. Advances in AS are expected to lead towards highly intelligent machines for augmenting human capabilities. Typical emerging AS include unsupervised computational intelligence, cognitive systems, brain-inspired systems, general AI, cognitive robots, unmanned systems, human intelligence augmentation systems, and intelligent IoTs.

4. Symbiotic autonomous systems

The synergy between AS and SS has logically led to the emergence of SAS [4–7]. As advanced intelligent system theories and technologies, they enable AS, SS, and human groups to coherently work in a hybrid society towards the generation of collective intelligence. It leads to a unified universe of discourse of cybernetics underpinned by the emergence of abstract sciences and their counterparts of classical concrete sciences.

It is recognized that epistemology plays an indispensable role in the kernel of philosophy for human intelligence expression and advancement [2,32]. A key approach to system intelligence augmentation by SAS is via symbiotic knowledge learning for autonomous problem solving. Knowledge learning helps to extend the state space of mind as the foundation for creativity and wisdom generation, while autonomous problem solving enables an SAS to generate intelligent behaviours at run-time as a counterpart of humans in the loop [16,23,33,34].

**Definition 4.1.** SAS are advanced intelligent and cognitive systems embodied by computational intelligence in order to facilitate collective intelligence among human–machine interactions in a hybrid society.

**(a) Conceptual model of SAS**

Recent basic research in SAS has revealed that novel solutions to fundamental AI problems are deeply rooted in the understanding of both natural intelligence and maturity of suitable mathematical tools for rigorously modelling the brain in machine understandable forms [10]. It leads to the emergence of IM [35,36] as a category of the denotational mathematics as a supplement to the inadequate power of classic analytic and numerical mathematics. IM is demanded by SAS based on the observation that the domain of the cognitive entities in SAS and AI has been out of the traditional domain of real numbers (\(\mathbb{R}\)). As a result, new problems demand new mathematics recognized as IM for dealing with hyperstructures (\(\mathbb{H}\)) beyond classic mathematics of logic, numerical methods, probability, and analytics.

**Definition 4.2.** The hierarchy of cognitive objects \(CO\) represented in the brain as typical \(\mathbb{H}\) is a 4-tuple in the categories of data (\(D\)), information (\(I\)), knowledge (\(K\)), and intelligence (\(\dot{I}\)) from the bottom up according to their levels of abstraction:

\[
CO \triangleq (D, I, K, \dot{I}) = \begin{cases} 
D = f_d : O \to Q \\
I = f_i : D \to S \\
K = f_k : I \to C \\
\dot{I} = f_{\dot{i}} : I \to B
\end{cases}
\]

where the symbols denote the cognitive entities object (\(O\)), quantity (\(Q\)), semantics (\(S\)), concept (\(C\)), and behaviour (\(B\)), respectively.

The relationship among the cognitive entities in the brain may be formally described by a hierarchical model according to the SS theory of theorem (1). It derives the following principle for explaining intelligence generation in the brain from low-level entities.
Corollary 1. Let $\kappa^0$ through $\kappa^4$ be the hierarchical layers of human cognition objects of data ($\mathbb{D}$), information ($\mathbb{I}$), knowledge ($\mathbb{K}$), and intelligence ($\mathbb{I}$). The transformability among the cognitive entities is determined by a recursive structure $S_{\Xi}$ where any higher-layer system or entity is represented by that of its lower layers deductively, or vice versa inductively:

$$S_{\Xi} = \mathcal{R}^4 \kappa^4(\kappa^3(\kappa^2(\kappa^1(\kappa^0))))$$

Corollary (1) provides a general system science theory. Each cognitive layer ($\kappa^k$) in equation (4.2) is specified by the adjacent lower layer ($\kappa^{k-1}$) until the cognitive hierarchy is terminated at the bottom sensorial layer ($\kappa^0$), which is embodied by a set of $n + 1$ dimensional data, $\mathcal{R}^n d_i | T_i = (d_0 | T_0, d_1 | T_1, \ldots, d_n | T_n)$ as acquired by abstraction and quantification of the brain where a type suffix convention $| T$ is adopted to denote a variable $x| T$ in SAS.

According to corollary (1), the philosophy of current machine learning would be questionable, because it is implemented by two phases known as training (domain calibration) and reflexive regression [37]. The former is implemented by large-scale data-driven regressions; while the latter is a reflexive classification. None of them are autonomous because the former is supervised and intensively dependent on data labelling and preprocessing. However, the latter is constrained by the pretrained domain and norms of samples as special solutions within a particular scope of sample data. In generic mathematical theory, there are often infinitive special solutions for the entire domain of a category of targets such as facial images and road conditions [37–40]. Therefore, there are no generic solutions yet discovered for all-purpose image recognition. Some systems for facial recognition would claim 80% accuracy in North America. However, the same system may not work well in Africa or Asia. Therefore, it is not a surprise that any best-trained facial recognition neural network may fail when a feeding image is upside-down. It also explains these phenomena explain why so many researchers work on the same problem for facial recognitions because there is not only a lack of general methodology in-line with the SAS principles, but also a missing of underpinning knowledge about how the brain processes visual information [38,41,42].

(b) Machine knowledge learning as a key paradigm of symbiotic autonomous systems

Human and machine learning as well as knowledge representation for both individuals and collective intelligence are a key SAS paradigm. Machine learning systems as a popular technology may have inherited fundamental immaturity and lack of rigorous theories from the SAS point of view. It is recognized that learning is a cognitive process of knowledge and behaviour acquisition [43]. However, collective knowledge learning as the most important machine learning demand still remains a theoretical and technical challenge to GAI and SAS.

Definition 4.3. Machine learning as an SAS paradigm is classified into six categories encompassing: (1) object identification; (2) cluster classification; (3) pattern recognition; (4) functional regression; (5) behavioural generation; and (6) knowledge acquisition, as formally described:

$$
\begin{align*}
& L_d(x, P|x \in X) \triangleq x = P \cdot x & // \text{object identification} \\
& L_c(x, P) \triangleq X \subset P & // \text{cluster classification} \\
& L_r(x, P) \triangleq X = P & // \text{pattern recognition} \\
& L_f(x, P) \triangleq X \Rightarrow P(X) & // \text{functional regression} \\
& L_b(x, P) \triangleq X \Rightarrow b(P(X)) & // \text{behaviour generation} \\
& L_k(x, K) \triangleq X \Rightarrow c(X) \cup K & // \text{knowledge acquisition}
\end{align*}
$$
where $X$ denotes a vector of objects $x, x \in X$, $P$ a target set of patterns, $b$ a behaviour determined by $P$, $c$ a formal concept, $K$ a set of knowledge, and $\uplus$ an operator of knowledge composition.

The last category of knowledge acquisition in equation (4.3) [43] is underpinned by knowledge science and IM known as concept and semantic algebras [36]. Machine knowledge learning is indispensable because: a) theoretically, epistemology is in the centre of fundamental philosophy; and b) empirically, knowledge learning is a lifelong endeavour beyond any other types of learning as identified in definition (4.4).

**Definition 4.4.** The basic structure of knowledge $\kappa$ is a conceptual relation between two concepts $c_1$ and $c_2$ that is neurologically embodied by a synaptic connection between a pair of pivot neurons:

$$\kappa = c_1 \times c_2, \quad c_1, c_2 \in C$$

(4.4)

where the formal concept $C$ as the basic cognitive structure of knowledge is a 5-tuple:

$$C \overset{\Delta}{=} (A, O, R^c, R^i, R^o)$$

(4.5)

includes $A$ as a finite set of attributes or intension of $C$; $O$ a finite set of objects or extension of $C$; $R^c$ a non-empty set of internal relations $R^c = O \times A$; $R^i$ a set of input relations $R^i \subseteq C' \times C$ where $C'$ is a distinguished set of concepts in knowledge; and $R^o$ a set of output relations $R^o \subseteq C \times C'$.

Knowledge is one of the few fundamental abstract concepts in contemporary sciences that has yet to be accurately quantified and rigorously measurable. Empirical knowledge measurements have used to be such as ‘erudite,’ ‘plenty of,’ and ‘informative.’ However, none is rigorous measurable because of the immaturity of knowledge science [43]. This is in analogy with the case that people had to use the empirical and unquantifiable unit horsepower for measuring physical power for centuries before Newton formally clarified the unit of force as N (newton) such that the unit of power is $Nm \, s^{-1}$ [1].

Based on definitions (4.5), an important discovery in basic research on the nature of knowledge by Wang [10,44] has revealed that the basic unit of knowledge is a **binary relation** $(bir)$ that is comparable to that of **bit** (binary digit) for data and information introduced by Shannon [45]. This finding is supported by empirical evidence of the synaptic connections in neurology and brain science [32].

**Definition 4.5.** The **basic unit of knowledge** $u(\kappa)$ is a general quantitative metrics for knowledge measurement:

$$u(\kappa) \overset{\Delta}{=} u(c_1 \times c_2) = bir$$

(4.6)

**Theorem 5.** Knowledge symbiosis $\Omega_\kappa \left( \bigcap_{i=1}^{n} c_i, R_i \right)$ between two sets of formal concepts results in new knowledge quantifiable by bir:

$$\Omega_\kappa \left( \bigcap_{i=1}^{n} c_i, R_i \right) \overset{\Delta}{=} \bigcap_{i=1}^{n} R_i (c_i \times c_j) [bir]$$

(4.7)

**Proof.** Theorem (5) holds based on definition (4.5) by proving each of the five dimensions of attributes (intension, $A$), objects (extension, $O$), and internal/input/output relations
\( (R^c, R^i \text{ and } R^o): \)

\[
\forall \ R_{c_i} \text{ and } m \ R_{c_j} \in C = (A, O, R^c, R^i, R^o),
\]

\[
\Omega_k \left( \prod_{i=1}^{n} R_{c_i} \right) = R \ R (c_i \times c_j)
\]

\[
= n \ \prod_{i=1}^{m} R_{c_i} \left( c_i \times c_j \right) \left[ \binom{c_i}{A} \times \binom{c_j}{A} \right]
\]

\[
\left[ \binom{c_i}{O} \times \binom{c_j}{O} \right] \left[ \binom{c_i}{R^i} \times \binom{c_j}{R^i} \right]
\]

\[
\left[ \binom{c_i}{R^c} \times \binom{c_j}{R^c} \right]
\]

\[
(4.8)
\]

Theorem (5) and definition (4.5) may be applied to a wide range of phenomena in knowledge and intelligence sciences, for instance, the derivation of the following results. Based on theorem (5), the general theory of knowledge for both the itemized and entire knowledge may be rigorously described.

**Definition 4.6.** An *itemized knowledge* \( \kappa^0 \) in the universe of discourse of knowledge \( \Omega \) is an \((n+1)-ary\) relation \( r_k \) between a particular concept \( C_0 \) and a set of existing concepts \( \prod_{i=1}^{n} R_i \) in a knowledge base:

\[
\kappa \wedge = r_k: C_0 \rightarrow R_{c_i} \ C_i \in \mathcal{R}
\]

\[
= C_0 \prod_{i=1}^{n+1} C_i \ [bir]
\]

(4.9)

The mathematical model of itemized knowledge may be extended to the entire knowledge as a Cartesian product of all formal concepts in the knowledge base of a person or an SAS.

**Theorem 6.** The entire knowledge \( \mathcal{R} \) is determined by a recursive Cartesian product among all formal concepts \( C_i \subset \mathcal{R} \) at all \( k \)-th layers represented by a hierarchical knowledge base:

\[
\mathcal{R} \subset \prod_{k=1}^{p} \ R^k \ (\mathcal{R}^{k-1}), \mathcal{R}^0 = \kappa = \prod_{i=1}^{n} C_i
\]

\[
= \mathcal{R}^p \left( \mathcal{R}^{p-1} \left( \prod_{i=1}^{n} C_i \right) \left[ bir \right] \right)
\]

(4.10)

**Proof.** Theorem (6) is inductively proven by definitions (4.5) and (4.6) based on theorem (1):

\[
\mathcal{R} = \left\{ \begin{array}{l}
\mathcal{R}^1 = \kappa = \prod_{i=1}^{n} C_i \\
\mathcal{R}^2 = \mathcal{R}^1 \ (\mathcal{R}^0) = \mathcal{R}^1 \left( \prod_{i=1}^{n} C_i \right) \\
\mathcal{R}^3 = \mathcal{R}^2 \ (\mathcal{R}^0) = \mathcal{R}^2 \left( \prod_{i=1}^{n} C_i \right) \\
\mathcal{R}^4 = \mathcal{R}^3 \ (\mathcal{R}^0) = \mathcal{R}^3 \left( \prod_{i=1}^{n} C_i \right) \\
\mathcal{R}^p \left( \mathcal{R}^{p-1} \left( \prod_{i=1}^{n} C_i \right) \left[ bir \right] \right)
\end{array} \right.
\]

(4.11)
The formal model of human knowledge leads to another discovery for rigorously explaining the measurable \( \text{bir} \)s of knowledge that an individual may remember and the maximum potential of human memory capacity according to the SAS theory.

**Theorem 7.** The capacity of human memory \( \Omega(K) \) for knowledge representation and memorization is bounded by \( 10^{8,432} \) \( \text{bir} \) as a result of all potential synaptic connections \( n_\psi \) created by the total number of nerves \( n_\mu \) in the brain:

\[
\Omega(K) \leq 10^{8,432} \text{ bir, } \exists n_\mu = 10^{11} \text{ and } n_\psi = 10^3
\]

**Proof.** Let the total number of neurons and the average synaptic connections among them be \( n_\mu = 10^{11} \) and \( n_\psi = 10^3 \), respectively, based on neurological observations [32]. \( \Omega(K) \) is determined by all potential synaptic connections in the brain:

\[
\Omega(K) = C_{n_\mu}^{n_\psi} = \frac{n_\mu!}{n_\psi!(n_\mu - n_\psi)!} = \frac{10^{11}!}{10^3!(10^{11} - 10^3)!} \\
\lesssim 10^{8,432} \text{ bir} \quad (4.13)
\]

Theorem (7) indicates that human memory capacity as SAS based on the symbiotic neural structures of the brain is much greater than any human-made memory chip and hard disk, even their collected sum. This reveals that the human brain memory may never be exhausted.

The social implication of machine knowledge learning as a brain-inspired SAS is represented by an unpresented advantage of symbiotic learning sharable by humans and machines where acquired knowledge may be directly transferred to peers based on the common platform of knowledge representation and the shared knowledge manipulation engines. In this novel approach, the human learning redundancy where similar knowledge needs to be repetitively learned and relearned by individuals may be avoided as enabled by symbiotic learning. Therefore, SAS-based machine knowledge learning will become an indispensable form of learning between human and machines by mutually sharable knowledge bases in an unprecedented hybrid SAS society. The sixth form of SAS learning for knowledge acquisition will enable collective knowledge acquisition for exponentially extending human learning ability toward a wide range of novel applications.

The theoretical framework of symbiotic knowledge learning may trigger the seventh form of AI learning discovered recently as introspective learning. **Introspective learning (IL)** [46] recursively generates new knowledge based on existing knowledge, which incubates deepened understanding across previously acquired knowledge in recursive and inductive ways. IL occurs in the brains as the most advanced and highly frequent form of knowledge learning. The mechanism of IL is formally explained in definition (4.6) and theorem (6), which reveals that most of our advanced knowledge is created by IL in human brains. According to theorem (6), IL may be implemented by SAS technologies supported by a cognitive knowledge base (CKB) [46] in computational intelligence.

The formal and comprehensive approach described in this paper has revealed the universe of discourse of SAS theories and engineering applications, which may lead to groundbreaking developments in the near future. A proper way towards SAS implementations and applications may be to tailor the theoretical models for specific problems along the line of the human-in-the-loop paradigm. In addition to the example of SAS for symbiotic knowledge learning, many emerging SAS paradigms encompass: (1) brain-inspired systems [25]; (2) cognitive robots [47]; (3) intelligent IoT (IoT) [48,50–52]; (4) social computing systems; (5) brain-machine interfaces; (6) cognitive systems; (7) bioinformatics systems; (8) internet of minds (IoM); (9) self-driving automobiles; (10) human-collaborative robots; (11) unmanned systems; (12) manned-unmanned teaming; (13) robot memetics [49]; and (14) intelligent defence systems. It is noteworthy that the SAS applications towards the next generation of cognitive computers, GAI, and hybrid
symbiotic human–machine societies are not trivial. Related SAS and AS projects undertaken in our laboratories address challenges for abstract intelligence, IM for SAS, the tripartite framework of SAS trustworthiness, autonomous decision making, a transdisciplinary theory for cognitive cybernetics, humanity and systems science, cognitive foundations of knowledge science, and the abstract system theory for SAS. The advances of SAS theories and technologies should lead from information revolution to the era of intelligence revolution for unprecedented breakthroughs for enabling pervasive human–machine symbiotic systems.

5. Conclusion

This work has developed a theoretical framework of SAS as an emerging field of advanced and general AI technologies. The SAS architecture has been deduced to the synergy between symbiotic and autonomous systems. Extended intelligent behaviours of SAS have been formally revealed by the hierarchical intelligence model (HIM). The fundamental challenge to seamlessly enable human and machine interactions in a hybrid environment has been formally addressed. This work has revealed that SAS are: (a) A recursive structure of intelligent systems; (b) A collective intelligent systems for human societies and engineering applications; (c) A general AI technology that leads to emerging hybrid societies of human and intelligent machines; and (d) The most advanced form of AI systems underpinned by IM for enabling epistemology learning by machines. The basic research and methodologies presented in this work have provided a rigorous foundation towards SAS modelling and implementations, which may trigger a wide range of novel applications for augmenting human capabilities and intelligent power. SAS are expected to open a new era from information revolution to intelligence revolution.

Data accessibility. This article has no additional data.

Authors’ contributions. Co-authors, Y.W., F.K., S.K., K.N.P., H.L., M.H., E.T., I.J., L.T., O.K., J.K., M.Z., M.H.S., P.C. and S.P., have contributed evenly to this work through the Canadian Department of National Defence project AutoDefence, NSERC, and the IEEE SMCS Technical Committee on Brain-Inspired Cognitive Systems (TC-BCS). They are confirmed to meet all of the four authorship criteria.

Competing interests. We declare we have no competing interests.

Funding. This work is supported in part by the Canadian Department of National Defence through the AutoDefence project, Natural Sciences and Engineering Research Council (NSERC), and the IEEE SMC Society Technical Committee on Brain-Inspired Cognitive Systems (TC-BCS).

Acknowledgment. The authors thank the anonymous reviewers for valuable suggestions and comments.

References

1. Newton I. 1729 The principia: The mathematical principles of natural philosophy. London, UK: Benjamin Motte.
2. O’Cosnorr T, Robb D. 2013 Philosophy of mind: contemporary readings. London, UK: Routledge.
3. Bunge M. 1978 General systems theory challenge to classical philosophy of science. Int. J. Gen. Systems 4, 3–28.
4. Coradeschi S, Saffiotti A. 2006 Symbiotic robotic systems: humans, robots, and smart environments. IEEE Intell. Syst. 21, 82–84. (doi:10.1109/MIS.2006.59)
5. Crosby ME, Scholtz J, Ward P. 2006 Symbiotic performance between humans and intelligent systems. J. Interacting Comput. 18, 1165–1169. (doi:10.1016/j.jntcom.2006.08.013)
6. Wang Y. 2018 Keynote: The emergence of abstract sciences and brain-inspired symbiotic systems. In IEEE Future Direction Committee Workshop on Symbiotic Autonomous Systems (WSAS’18), Miyazaki, Japan, Oct, p. 2.
7. Doltsinis S, Ferreira P, Lohse N. 2018 A symbiotic human–machine learning approach for production ramp-up. IEEE Trans. Hum. Mach. Syst. 48, 229–240. (doi:10.1109/THMS.2017.2717885)
8. Woolley AW, Chabris CF, Pentland A, Hashmi N, Malone TW. 2010 Evidence of a collective intelligence factor in the performance of human groups. Science 330, 686–688. (doi:10.1126/science.1193147)
9. Tunstel E et al. 2021 Systems science and engineering research in the context of systems, man, and cybernetics: recollection, trends, and future directions. *IEEE Trans. Systems, Man Cybern. Syst.* 51, 5–21. (doi:10.1109/TSMC.2020.3043192)

10. Wang Y, Tunstel E. 2019 Emergence of abstract sciences and transdisciplinary advances in systems, man, and cybernetics. *IEEE System, Man Cybern. Mag.* 5, 12–19. (doi:10.1109/MSMC.2019.2899698)

11. Saracco R. 2017 Looking ahead to 2050 - symbiotic autonomous systems (I-XI). New York, NY: IEEE Future Directions Blog. See https://digitalreality.ieee.org/publications/blog/looking-ahead-to-2050.

12. Wang Y. 2020 A rigorous cognitive theory for autonomous decision making. In *IEEE 2020 Int’l Conf. on Systems, Man, and Cybernetics (SMC’20)*, Toronto, pp. 1021–1026.

13. Kacprzyk J, Yager RR, Merigo JM. 2019 Towards human-centric aggregation via ordered weighted aggregation operators and linguistic data summaries: a new perspective on Zadeh’s Inspirations. *IEEE Comput. Intell. Mag.* 14, 16–30. (doi:10.1109/MCI.2018.2881641)

14. Kacprzyk J, Nurmi H, Zadrozny S. 2017 Reason vs. rationality: from rankings to tournaments in individual choice. *Trans. Comput. Collective Intell.* 27, 28–39. (doi:10.1007/978-3-319-70647-4_2)

15. Wang Y. 2020 Keynote: how will autonomous systems and cognitive robots augment human intelligence? In *Future Technologies Conference (FTC’20)*, Vancouver, Canada, pp. 2.

16. Wang Y, Hou M, Plataniotis KN, Kwong S, Leung H, Tunstel E, Rudas IJ, Trajkovic L. 2021 Towards a theoretical framework of autonomous systems underpinned by intelligence and systems sciences. *IEEE/CAS J. Autom. Sinica* 8, 52–63. (doi:10.1109/JAS.2020.1003432)

17. Klir GJ. 1992 *Facets of systems science*. New York, NY: Plenum.

18. Wang Y. 2015 A denotational mathematical theory of system science: system algebra for formal system modeling and manipulations. *J. Adv. Math. Appl.* 4, 132–157. (doi:10.1166/jama.2015.1082)

19. Zhu H. 2015 Rule-based collaboration and the E-CARGO: revisiting the developments of the last decades. *IEEE System. Man and Cybernetics Magazine* 1, 27–35. (doi:10.1109/MSMC.2015.2460612)

20. Wang Y. 2014 Software science: on general mathematical models and formal properties of software. *J. Adv. Math. Appl.* 3, 130–147. (doi:10.1166/jama.2014.1060)

21. Wang Y. 2008 On cognitive properties of human factors and error models in engineering and socialization. *Int’l J. of Cogn. Infor. and Nat’l Intell.* 2, 70–84. (doi:10.4018/jcini.2008100106)

22. Mnih V et al. 2015 Human-level control through deep reinforcement learning. *Nature* 518, 529–533. (doi:10.1038/nature14236)

23. Hou M, Banbury S, Burns C. 2014 *Intelligent adaptive systems: An interaction-centered design perspective*. Boca Raton, FL: CRC Press.

24. Watson DP, Scheidt DH. 2005 Autonomous systems. *Johns Hopkins Appl. Tech. Digest* 26, 268–376.

25. Wang Y et al. 2020 ‘Brain-inspired systems: a transdisciplinary exploration on cognitive cybernetics, humanity, and systems science towards AI. *IEEE System, Man Cybern. Mag.* 6, 6–13. (doi:10.1109/MSMC.2018.2889502)

26. Wang Y et al. 2020 A Tripartite Framework of Trustworthiness of Autonomous Systems. In *IEEE 2020 Int’l Conf. Systems, Man, and Cybernetics*, pp. 3375–3380.

27. Abbass HA, Leu G, Merrick K. 2016 A review of theoretical and practical challenges of trusted autonomy in big data. *IEEE Access* 4, 2808–2830. (doi:10.1109/ACCESS.2016.2571058)

28. Ma Y, Wang Z, Yang H, Yang L. 2020 Artificial intelligence applications in the development of autonomous vehicles: a survey. *IEEE/CAA J. Autom. Sinica* 7, 315–329. (doi:10.1109/JAS.2020.1003021)

29. Martin M et al. 2020 Learning features while learning to classify: a cognitive model for autonomous systems. *Comp. Math Organ Theory* 26, 23–54. (doi:10.1007/s10588-018-9279-3)

30. Bench-Capon TJM. 2020 Ethical approaches and autonomous systems. *Artif. Intell.* 281, 103239. (doi:10.1016/j.artint.2020.103239)

31. Bender EA. 2000 *Mathematical methods in artificial intelligence*. Los Alamitos, CA: IEEE CS Press.

32. Wilson RA, Frank CK. 2001 *The MIT encyclopedia of the cognitive sciences*. Cambridge, MA: MIT Press.
33. Netzer E, Geva AB. 2020 Human-in-the-loop active learning via brain computer interface. *Ann Math Artif. Intell.* 88, 1191–1205. (doi:10.1007/s10472-020-09689-0)

34. Miriam G, Albert M, Fons J, Pelechano V. 2020 Engineering human-in-the-loop interactions in cyber-physical systems. *Inform. Software Technol.* 126, 106349. (doi:10.1016/j.infsof.2020.106349)

35. Wang Y. 2020 Keynote: intelligent mathematics (IM): indispensable mathematical means for general AI, autonomous systems, deep knowledge learning, cognitive robots, and intelligence science. In *IEEE 19th Int’l Conf. on Cognitive Informatics & Cognitive Computing (ICCI* CC’20), Tsinghua University, Beijing, China*, pp. 5.

36. Wang Y. 2012 In search of denotational mathematics: novel mathematical means for contemporary intelligence, brain, and knowledge sciences. *J. Adv. Math. Appl.* 1, 4–25. (doi:10.1166/jama.2012.1002)

37. Wang Y et al. 2018 A survey and formal analyses on sequence learning methodologies and deep neural networks. In *17th IEEE Int’l Conf. Cognitive Informatics & Cognitive Computing (ICCI* CC’18), University of California, Berkeley*, pp. 6–15.

38. Chen CLP, Liu Z, Feng S. 2020 Universal approximation capability of broad learning system and its structural variations. *IEEE Trans. Neural Netw. Learn. Syst.* 30, 1191–1204. (doi:10.1109/TNNLS.2018.2866622)

39. Yang S, Wen Y, He L, Zhou M. 2020 Sparse common feature representation for undersampled face recognition. *IEEE Internet Things J.* 8, 5607–5618. (doi: 10.1109/JIOT.2020.3031390)

40. Wang Y. 2018 Keynote: cognitive foundations and formal theories of human and robot vision. In *17th IEEE Int’l Conf. Cognitive Informatics & Cognitive Computing (ICCI* CC 2018), UC Berkeley, USA*, pp. 5.

41. Wang Y. 2016 Deep learning and deep reasoning by cognitive robots and computational intelligent systems. In *Proc. 15th IEEE Int. Conf. Cognitive Informatics & Cognitive Computing (ICCI* CC’16)*, July, pp. 3: Stanford University

42. Wang Y. 2017 Keynote: cognitive foundations of knowledge science and deep knowledge learning by cognitive robots. In *16th IEEE Int’l Conf. Cognitive Informatics & Cognitive Computing (ICCI* CC 2017)*, pp. 4. University of Oxford, UK: IEEE CS Press.

43. Shannon CE. 1948 A mathematical theory of communication. *Bell Syst. Tech. J.* 27, 623–656. (doi:10.1002/js.1948.tb00917.x)

44. Wang Y. 2014 On a novel cognitive knowledge base (CKB) for cognitive robots and machine learning. *Int’l J. Software Sci. Comp. Intell.* 6, 42–64. (doi:10.4018/ijssci.2014040103)

45. Fortino G et al. 2021 Towards a theoretical framework of autonomous systems underpinned by intelligence and systems sciences. *IEEE/CAA J. Autom. Sinica* 8, 52–63. (doi:10.1109/JAS.2020.1003432)

46. Fortino G et al. 2020 IoT-enabled autonomous system collaboration for disaster-area management. *IEEE/CAA J. Autom. Sinica* 7, 1249–1262. (doi:10.1109/JAS.2020.1003291)

47. Fortino G et al. 2021 Internet of Things as system of systems: a review of methodologies, frameworks, platforms and tools. *IEEE Trans. Syst. Man Cybern. Syst.* 51, 223–236. (doi:10.1109/TSMC.2020.3042898)