Preheating constraints in $\alpha$-attractor inflation and Gravitational Waves production

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We propose a scenario where preheating occurs for a specific duration that is parametrized by an e-folds number $N_{\text{pre}}$, our results suggest a direct correlation between the preheating duration and the density of gravitational waves (GWs) produced during this phase. Moreover, we investigate the consequences of the inflationary parameters on the $\alpha$-attractor E model in the small $\alpha$ limits. In this framework, we perform investigations on the preheating parameters involving the number of e-folds $N_{\text{pre}}$, and the temperature of reheating $T_{\text{re}}$, then we show that the parameter $n_s$ associated with the E model of $\alpha$-attractor inflation has a negligible effect on the preheating duration, and we demonstrate that gravitational wave generation during preheating satisfies the restrictions from Planck’s recent data.

I. INTRODUCTION

Among the most exciting theories that explain the origins of the Universe is the inflation scenario [1–3], this modern theory described through various models gives a satisfying solution to the standard Big Bang problems. Accurate measurements of the cosmic microwave background (CMB) and the large-scale structure (LSS) of galaxies provided helpful constraints on inflation models [4], taking into consideration the recent observations of the $\alpha$-attractor-type models, which were suggested in a unified approach combining the Higgs and Starobinsky inflation [5, 6] show good compatibility with data [21, 22]. Reheating is a phenomenon that was proposed as a technique for achieving the hot big bang Universe. At reheating, the energy of the inflaton field is transformed to thermal radiation through processes that may involve particle production and non-equilibrium events, many investigations have been conducted on reheating mechanisms [7–10, 37]. However, recent studies have examined new formalism to constrain the reheating temperature and e-folding number [11–13, 27]. Preheating on the other hand is a phase that occurs at the end of inflation when the periodical oscillations of the inflaton lead to the parametric resonance, which results in an abundant production of light particles whose mode momenta are in the resonant bands [14, 15]. Preheating can accelerate the thermalization of our Universe since the inflaton energy can be transferred rapidly into matter. Due to the fact that only modes with their momenta in the resonant bands grow exponentially, the matter distribution has large time-dependent inhomogeneities in the position space. Thus, the matter produced during preheating has time-dependent quadruple moments and for this reason, it could be an effective source of gravitational waves (GWs) [18–20, 39]. The detection of GWs produced during preheating can contribute to the investigation of inflation and study of the reheating process.

One of the biggest difficulties confronting observational cosmology in the coming decade is the detection of primordial tensor fluctuations, both in the form of CMB B-mode polarization on large angular scales and a spectrum of relic gravitational waves. Because of the direct relationship of primordial GWs to the inflaton potential in the case of the minimum inflaton coupling to gravity, the GW primordial power spectrum at large scales provides vital information on the nature of the inflaton field [23]. Equally significant is the fact that the GWs spectrum $\Omega_{\text{GW}}$ may act as an important probe to physical processes happening after inflation. In this paper, we investigate the preheating epoch from the $\alpha$-attractor inflation focusing on the E-model class of $\alpha$-attractors. Our approach to constrain preheating and the GWs produced at this stage differs from the previous studies since we consider a preheating epoch characterized by an e-folding number $N_{\text{pre}}$, in this way we will extend the formalism developed in Ref. [30] where they include the nonperturbative effects in the reheating scenario, in this way inflation is ended by an initial nonperturbative stage followed by a perturbative one. In our work, the reheating duration is determined through the final reheating temperature, this leads to two phases scenarios where preheating and reheating occur for specific durations. On another hand, we prove that the gravitational wave spectrum $\Omega_{\text{GW}}$ is sensitive to the e-folding number $N_{\text{pre}}$, which makes it possible to set a correlation of gravitational wave energy density with the spectral index $n_s$ detected by the CMB experiments in order to satisfy the constraints from Planck’s data.

The paper is organized as follows. In Sect. II we compute the relations between inflationary and preheating parameters in the $\alpha$-attractor E-model and discuss the constraints from preheating on the inflationary param-
eters, we also briefly describe our proposed two phases scenario following inflation. In Sect. [V] we convert the GWs spectra into physical variables and describe gravitational waves from Planck’s measurements perspective. In Sect. [IV] we present our conclusions.

II. PREHEATING CONSTRAINTS IN α-ATTRACTOR E-MODEL

A. α-attractor inflation

In this work we focus on a class of single-field inflation models of the α-attractors, This class of inflation models can be generated by spontaneously breaking the conformal symmetry [30], we consider the E-model potential Eq. (1), knowing that after the slow-roll approxima-
tion during inflation, the Klein-Gordon equation obeys $3H\dot{\phi} + V = 0$, while the Friedmann equation is given as $H^2 = V(\phi)/3M_p^2$.

$$V = V_0 \left(1 - e^{-\sqrt{\frac{2\kappa}{3\alpha}}\phi}\right)^{2n},$$

(1)

here, $V_0$, $n$, and $\alpha$ are free parameters.

In the standard inflation the slow-roll parameters are introduced as $\epsilon = (1/2)(M_pV'/V)^2$ and $\eta = M_p^2V''/V$, which could be related to the spectral index and tensor-to-scalar ratio as

$$n_s = 1 - 6\epsilon + 2\eta,$$

(2)

$$r = 16\epsilon,$$

(3)

the energy density during inflation is written as $\rho = (1 + \epsilon/3)V$. At the end of this epoch the energy density of the Universe can be given as $\rho_{\text{end}} = (4/3)V_{\text{end}}$ considering that $\epsilon = 1$.

To begin, we look for the scalar spectral index and tensor-to-scalar ratio in the small $\alpha$ limits. In this case, we may rewrite the E-model potential Eq. (1) as follows:

$$V = V_0 \left[1 - 2n \exp \left(-\sqrt{\frac{2\kappa}{3\alpha}}\phi\right)\right],$$

(4)

one can compute the number of e-foldings between the horizon exit of the pivot scale and the end of inflation as

$$N = \frac{1}{M_p^2} \int_{\phi_{\text{end}}}^{\phi_k} \frac{V}{\sqrt{\epsilon}} \, d\phi$$

(5)

$$= \frac{1}{M_p^2} \sqrt{\frac{3\alpha}{2\kappa}} \left[\frac{1}{2n} \sqrt{\frac{3\alpha}{2\kappa}} e^{\frac{2\kappa}{3\alpha} \phi} \right]_{\phi_{\text{end}}}^{\phi_k},$$

using the approximations $\phi_k >> \phi_{\text{end}}$, and $M_p e^{\frac{2\kappa}{3\alpha} \phi} >> \phi$, the field at the horizon crossing is then given as

$$\phi_k = \sqrt{\frac{3\alpha}{2\kappa^2}} \ln \left(\frac{4n^2 N}{3\alpha}\right).$$

(6)

The above equations considering the small limit of $\alpha$ predicts

$$n_s = 1 - 2\frac{2}{N}, \quad r = \frac{12\alpha}{N^2}.$$

(7)

TABLE I. Testing the compatibility of the Tensor-to-scalar ratio parameter for $0.05 \leq \alpha \leq 0.8$ in the case of $N = 50$ and $N = 60$ inflationary e-folds.

| tensor-to-scalar ratio | $\alpha = 0.8$ | $\alpha = 0.2$ | $\alpha = 0.05$ |
|------------------------|----------------|----------------|----------------|
| $r$ (N = 50)           | 0.0038         | 0.0009         | 0.0002         |
| $r$ (N = 60)           | 0.0026         | 0.0066         | 0.0016         |

The expression of the scalar field at the end of inflation $\phi_{\text{end}}$ is obtained by solving the equation $\epsilon(\phi_{\text{end}}) = 1$, which gives

$$\phi_{\text{end}} = \sqrt{\frac{3\alpha}{8\kappa^2}} \ln \left(\frac{4n^2}{3\alpha}\right).$$

(8)

According [11, 31] we know that $r = 2H_k^2/\pi^2 M_p^2 A_s$, using the expressions above we can derive $H_k$ as a function of $n_s$ and $A_s$

$$H_k = \pi M_p \sqrt{\frac{3}{2}} \alpha A_s (1 - n_s),$$

(9)

we can determine $V_{\text{end}} = V(\phi_{\text{end}})$ using the expression of the scalar field at the end of inflation given by Eq. (5), we should note here that $V_0$ is chosen to take the value $V_0^{1/4} = 10^{15} GeV$ [37]. From Table I the inflationary parameter $r$ presents good consistency for small $\alpha$, and the spectral index $n_s$ in Eq. (7) can best fit observations for $N = 60$. However, the energy density at the end of inflation gives additional constraints on the $\alpha$ parameter in order to have a valid energy $V_{\text{end}} > 0$, according to Eq. (5) the parameter must be bounded as $\alpha \lesssim 0.3$.

B. Preheating constraints

At the early stages of the Universe’s evolution, the process of preheating occurs. One of the important parameters which take specific values in each step throughout the evolution of the Universe is the equation of state (EoS) parameter, which is defined as the ratio of the pressure and energy density of the Universe $\omega = p/\rho$. The choice of the right EoS value after inflation is essential, especially during preheating. Since the Universe cools as it expands, there must be a period that follows inflation
that prepares thermally for the reheating step, that we call preheating. To extract information about preheating we need to consider the phase between the time observable CMB scales crossed the horizon to the present time, which can be described by [27]

\[ N_{\text{pre}} = \left[ 61.6 - \frac{1}{4} \ln \left( \frac{V_{\text{end}}}{\gamma H_0^2} \right) - N \right] - \frac{1 - 3 \omega}{4} N_{\text{re}}, \quad (10) \]

the reheating duration \( N_{\text{re}} \) can be obtained as a function of the reheating temperature in the following way [27]

\[ N_{\text{re}} = \frac{1}{3(1 + \omega)} \ln \left( \frac{3^2 \cdot 5 V_{\text{end}}}{\gamma^2 \pi^2 g_* T_{\text{re}}^4} \right). \quad (11) \]

Figs. [12] presents the variations of the preheating e-folds number \( N_{\text{pre}} \) as a function of reheating e-folds for the case of \( \gamma = 10^5 \) considering different values of the (EoS) parameter \( \omega \), each region is determined by a reheating temperature bound, in the black dashed region the temperature is bounded as \( T_{\text{re}} \geq 6 \times 10^{11} \text{GeV} \), the red region is bounded as \( T_{\text{re}} \geq 9 \times 10^{12} \text{GeV} \), the blue region is bounded by \( T_{\text{re}} \geq 2 \times 10^{12} \text{GeV} \), the green one is bounded by \( T_{\text{re}} \geq 1 \times 10^{10} \text{GeV} \), and the purple region is bounded by \( T_{\text{re}} \geq 1 \times 10^6 \text{GeV} \).

FIG. 1. Variation of the preheating duration \( N_{\text{pre}} \) as a function of reheating e-folds for the case of \( \gamma = 10^5 \) considering different values of the (EoS) parameter \( \omega \), each region is determined by a reheating temperature bound, in the black dashed region the temperature is bounded as \( T_{\text{re}} \geq 6 \times 10^{11} \text{GeV} \), the red region is bounded as \( T_{\text{re}} \geq 9 \times 10^{12} \text{GeV} \), the blue region is bounded by \( T_{\text{re}} \geq 2 \times 10^{12} \text{GeV} \), the green one is bounded by \( T_{\text{re}} \geq 1 \times 10^{10} \text{GeV} \), and the purple region is bounded by \( T_{\text{re}} \geq 1 \times 10^6 \text{GeV} \).

This means that a reheating temperature is bounded as \( T_{\text{re}} \lesssim 10^{13} \text{GeV} \), the higher limit from the bound will cause an instant transition of reheating which makes the reheating duration independent of the choice of the (EoS) parameter. However, a lower temperature value can increase the reheating duration, as a consequence \( N_{\text{pre}} \) will directly depend on the parameters \( T_{\text{re}} \), \( \omega \), and \( N_{\text{pre}} \) can be calculated considering the final reheating thermalization temperature. Here, we must mention that in order to have a valid reheating duration \( N_{\text{re}} \geq 0 \) we must consider the bound

\[ T_{\text{re}} \leq \left( \frac{3^2 \cdot 5 V_{\text{end}}}{\gamma^2 \pi^2 g_*} \right)^{1/2}, \quad (12) \]

FIG. 2. Variation of the preheating duration \( N_{\text{pre}} \) as a function of reheating e-folds for the case of \( \gamma = 10^6 \) considering different values of the (EoS) parameter \( \omega \), each region is determined by a reheating temperature bound, in the black dashed region the temperature is bounded as \( T_{\text{re}} \geq 6 \times 10^{11} \text{GeV} \), the red region is bounded as \( T_{\text{re}} \geq 8 \times 10^{11} \text{GeV} \), the blue region is bounded by \( T_{\text{re}} \geq 1 \times 10^0 \text{GeV} \), the green one is bounded by \( T_{\text{re}} \geq 1 \times 10^1 \text{GeV} \), and the purple region is bounded by \( T_{\text{re}} \geq 1 \times 10^{-3} \text{MeV} \).
1. **Preheating and reheating as two phases scenario**

Inflation must end when the value of $\omega$ became larger than $-1/3$, in order to satisfy the condition of energy density dominance and preserve the causality $\omega$ must be smaller than 1 [12]. Considering the results shown in Fig. 1, the choice of a specific (EoS) value enables a reheating temperature bound, our purpose is to constrain the preheating e-fold considering various cases of $T_r$ and $\omega$ provided by the previous findings. It is well known that the mechanism of nonperturbative effect which occurs before reheating is what we call preheating. This process of parametric resonance does not completely decay inflaton into the radiation field. Therefore, a subsequent thermalization must take place and the perturbative decay will be necessary to complete the reheating process. Our objective here is to consider a two phases model to describe both preheating and reheating durations concurrently. Figure 3 describes our methodology of previous analysis which presents good consistency according to Planck’s results [4,24]. Figures 6 and 7 demonstrate the evolution of the GW density spectrum with respect to the spectral index $n_s$ for different values of the current GW spectra $\Omega_{gw}$. We plot $\Omega_{gw,0} h^2$ considering the expansion of the Universe from the end of inflation up to later times of preheating, because of the bound of the final temperature of reheating $T_r \lesssim 10^{13} \text{GeV}$, the duration $N_{pre}$ from Eq. (11) could be considered as instantaneous, which make preheating duration minimally dependent on the (EoS) parameter $\omega$ as observed in Figs. 11. We choose the values of $\gamma = 10^3$, $10^6$ from the previous analysis which presents good consistency according to Planck’s results, the curves decrease as the GW energy density became very negligible $\Omega_{gw} \rightarrow 0$. For both cases $\gamma = 10^3$ and $\gamma = 10^6$ with $\Omega_{gw,0} h^2 \leq 1.85 \times 10^{-6}$ all the lines with different $\Omega_{gw}$ tends towards the central value of the spectral index $n_s$.

**III. PRIMORDIAL GRAVITATIONAL WAVES FROM PREHEATING**

Because we are interested in the connection of gravity-wave energy density spectrum with Planck data, we must convert the GW spectrum into actual physical values. The present scale factor in comparison to the one when GW production stops can be expressed as [28]

$$\Omega_{gw,0} h^2 = \Omega_{gw}(f) \left( \frac{\bar{g}_s}{g_0} \right)^{-1/3} \Omega_{r,0} h^2 \left( \frac{a_{end}}{a_{pre}} \right)^4. \quad (13)$$

The number of e-folds between the end of inflation to the time when preheating completed can be written as $a_{end}/a_{pre} = e^{-N_{pre}}$, which makes the current gravity-wave energy density spectrum related to the preheating e-folds considering $\omega = 1/3$ given as

$$\Omega_{gw,0} h^2 = \Omega_{gw}(f) \left( \frac{\bar{g}_s}{g_0} \right)^{-1/3} \Omega_{r,0} h^2 \exp \left( -4 \left[ 6.16 - \frac{1}{4} \ln \left( \frac{V_{end}}{\gamma H_{rf}^2} \right) - N \right] \right). \quad (14)$$

From Fig. 5 the variation of $\Omega_{gw,0} h^2$ as a function of $N_{pre}$ for some fixed values of $\Omega_{gw}$ are presented, we plotted the energy density spectrum $\Omega_{gw,0} h^2$ as a function of the preheating duration $N_{pre}$, taking the present GW spectra to be $8 \times 10^{-4} \leq \Omega_{gw} \leq 4 \times 10^{-3}$, when $N_{pre} \rightarrow 0$ the GW density spectrum takes an initial value for all the cases with different $\Omega_{gw}$. When we increase the present GW energy density, the initial values that correspond to $\Omega_{gw,0} h^2(N_{pre} = 0)$ increases as well, knowing that values of preheating with GW density spectrum bounded as $\Omega_{gw,0} h^2 \leq 1.85 \times 10^{-6}$ are most compatible according to Planck’s results [4,24]. Figures 6 and 7 demonstrate the evolution of the GW density spectrum with respect to the spectral index $n_s$ for different values of the current GW spectra $\Omega_{gw}$, we plot $\Omega_{gw,0} h^2$ considering the expansion of the Universe from the end of inflation up to later times of preheating, because of the bound of the final temperature of reheating $T_r \lesssim 10^{13} \text{GeV}$, the duration $N_{pre}$ from Eq. (11) could be considered as instantaneous, which make preheating duration minimally dependent on the (EoS) parameter $\omega$ as observed in Figs. 11. We choose the values of $\gamma = 10^3$, $10^6$ from the previous analysis which presents good consistency according to Planck’s results, the curves decrease as the GW energy density became very negligible $\Omega_{gw} \rightarrow 0$. For both cases $\gamma = 10^3$ and $\gamma = 10^6$ with $\Omega_{gw,0} h^2 \leq 1.85 \times 10^{-6}$ all the lines with different $\Omega_{gw}$ tends towards the central value of the spectral index $n_s$.

**IV. CONCLUSION**

After we examine the basic equations that describe gravity waves production using metric perturbation and
FIG. 4. Constraints on the preheating duration $N_{\text{pre}}$ for the $\alpha$-attractor E model potential considering both cases: $\gamma = 10^3$ and $\gamma = 10^6$ with different values of the thermalization temperature and (EoS) parameter, we chose $n = 1, \alpha = 0.1$. Here, the vertical light blue region represents Planck’s bounds on $n_s = 0.9649 \pm 0.0042$.

FIG. 5. The variation of the density spectra of GWs as a function of $N_{\text{pre}}$. For different values of gravity wave energy density $\Omega_{gw}$, the grey region corresponds to the bound $\Omega_{gw,0}h^2 \leq 1.85 \times 10^{-6}$.

Calculate $\Omega_{gw,0}h^2$ that represents the abundance of gravity wave energy density today, we discuss the E model $\alpha$-attractor and calculate the expression of the observational parameters $n_s$ and $r$, we compute these parameters as functions of inflation’s e-folds $N$ and test the compatibility of the parameter $\alpha$ with Planck’s results, our results show that small values of $\alpha$ give good results on the tensor-to-scalar ratio parameter, while the spectral index $n_s$ is best estimated for $N = 60$. We derive the preheating duration as functions of inflationary parameters in Eq. (10), and consider different thermalization temperature to discuss the case when the thermalization temperature is low enough to conduct to two phases scenarios where preheating and reheating occurs for specific durations. we provide additional constraints that estimate the bound on the reheating temperature as $T_{\text{re}} \lesssim 10^{13}\text{GeV}$ in order to have a valid reheating duration $N_{\text{re}} \gtrsim 0$. Our results suggest that when $N_{\text{re}} \to 0$ the preheating e-folds is independent on the choice of the (EoS), the duration of preheating is plotted as a function of the spectral index for the two models we considered previously, we show that $N_{\text{pre}}$ is weakly sensitive to $\gamma$ and independent on the E model inflation parameter $n$. We finally calculate the current gravity wave energy density spectrum as a function of the duration $N_{\text{pre}}$, which is a possible alternative to study GW density spectrum according to recent Planck’s results. Assuming the density parameter $\Omega_{gw}$ to be bounded as $8 \times 10^{-4} \leq \Omega_{gw} \leq 4 \times 10^{-3}$, we chose $n = 1, \alpha = 0.1$, and $\gamma : 10^3, 10^6$, and found that both cases where $\gamma = 10^3, \gamma = 10^6$ show good consistency with observation. We conclude that the E
FIG. 6. The variation of the density spectra of GWs as a function of the spectral index $n_s$. For different values of gravity wave energy density $\Omega_{gw}$, we choose $8 \times 10^{-4} \leq \Omega_{gw} \leq 4 \times 10^{-3}$, and $\gamma = 10^3$. The black line corresponds to $\Omega_{gw} = 4 \times 10^{-3}$, red line corresponds to $\Omega_{gw} = 2 \times 10^{-3}$, blue line corresponds to $\Omega_{gw} = 10^{-3}$, green line corresponds to $\Omega_{gw} = 8 \times 10^{-4}$, the red shaded region represents Planck’s bounds on $n_s$ and the grey region corresponds to the bound $\Omega_{gw,0}h^2 \leq 1.85 \times 10^{-6}$.

FIG. 7. The variation of the density spectra of GWs as a function of the spectral index $n_s$. For different values of gravity wave energy density $\Omega_{gw}$, we choose $8 \times 10^{-4} \leq \Omega_{gw} \leq 4 \times 10^{-3}$, and $\gamma = 10^6$. The black line corresponds to $\Omega_{gw} = 4 \times 10^{-3}$, red line corresponds to $\Omega_{gw} = 2 \times 10^{-3}$, blue line corresponds to $\Omega_{gw} = 10^{-3}$, green line corresponds to $\Omega_{gw} = 8 \times 10^{-4}$, the red shaded region represents Planck’s bounds on $n_s$ and the grey region corresponds to the bound $\Omega_{gw,0}h^2 \leq 1.85 \times 10^{-6}$.

Model $\alpha$-attractor parameters appear to have consistent results considering the preheating duration, regardless of whether the process is instant or takes a certain number of e-folds to complete, once we determine the final temperature of thermalization $T_{re}$, other preheating parameters are determined using the E model $\alpha$-attractor inflation. As a result, it would be interesting to investigate the physics of preheating in the context of Primordial GWs.

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