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To cite this article: Sachin Kaothekar and R K Chhajlani 2014 J. Phys.: Conf. Ser. 534 012065

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Jeans instability of self-gravitating rotating radiative plasma with finite Larmor radius corrections

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Abstract. The problem of Jeans instability is investigated for a self-gravitating, rotating, radiative plasma with finite Larmor radius corrections, which has connection in astrophysical condensation. A general dispersion relation is derived using normal mode analysis method with the help of relevant linearized perturbation equations of the problem. We find that the classical Jeans result regarding the size of initial break-up is considerably modified due to rotation, finite Larmor radius corrections and radiative heat-loss function. Numerical calculations have been performed to discuss the effect of various physical parameters on the growth rate of the Jeans gravitational instability.

1. Introduction
In the process of formation of astrophysical objects, the most outstanding phenomenon is interstellar gas fragmentation. These fragments of interstellar gaseous clouds are gravitationally contracted and as a final result the astrophysical objects are formed. In this context, Khan and Bhatia [1] have carried out the effects of rotation, Hall current and finite electrical conductivity on Jeans gravitational instability of plasma. Vyas and Chhajlani [2] have investigated the problem of gravitational instability with rotation, magnetic field, viscosity, finite electrical resistivity and thermal conductivity. Recently Chakrabarti et al. [3] have discussed the problem of nonlinear Jeans instability in an uniformly rotating gas. In addition to this, it is a well-known fact that thermal and radiative processes play a significant role in the instability investigation in gravitating plasma medium. Aggarwal and Talwar [4] have pointed out the role of radiative heat-loss function in the study of magnetothermohal instability in a rotating gravitating fluid. Prajapati et al. [5] have investigated the self-gravitational instability of rotating viscous Hall plasma with arbitrary radiative heat-loss function and electron inertia. Recently Kaothekar et al. [6] have carried out the effect of radiative heat-loss function on self-gravitational instability of viscous thermally conducting partially-ionized plasma. Along with this, in several plasma situations such as in solar corona, interplanetary and interstellar plasmas the FLR plays an important role in stability investigations. Sharma [7] has investigated the stabilizing role of FLR with rotation and magnetic field on gravitational instability of plasma. Kaothekar and Chhajlani [8] have investigated the effect of radiative heat-loss function and finite Larmor radius corrections on Jeans instability of viscous thermally conducting self-gravitating astrophysical plasma. Recently Sharma and Chhajlani [9] have carried out the effect of FLR corrections on Jeans instability of quantum plasma. Thus in the present work, we have investigated the effect of rotation, radiative heat-loss function, FLR...
corrections, thermal conductivity and finite electrical resistivity on Jeans instability of self-gravitating plasma. The above work is helpful in understanding the structure formation in interstellar medium.

2. Linearized Perturbation equations

Let us consider an infinite homogeneous, self-gravitating, thermally conducting, radiating plasma of finite electrical resistivity in the presence of magnetic field $H$ (0, 0, H). The linearized perturbation equations of motion of the problem with these effects are written as:

$$\rho \frac{\partial \tilde{u}}{\partial t} = -\nabla \tilde{p} - \nabla \cdot P + \rho \nabla \delta U + \frac{1}{4\pi} (\nabla \times h) \times H + 2 \rho (u \times \Omega),$$  

(1)

$$\frac{1}{\gamma - 1} \frac{\partial \tilde{p}}{\partial t} - \frac{\gamma}{\gamma - 1} \frac{\partial \rho}{\partial t} + \rho \left[ \delta \left( \frac{\partial L}{\partial \rho} \right) + \delta T \left( \frac{\partial L}{\partial T} \right) \right] - \lambda \nabla^2 \delta T = 0,$$  

(2)

$$\frac{\partial \delta h}{\partial t} = \nabla \times (u \times H) + \eta \nabla^2 h,$$  

(3)

$$\frac{\partial \tilde{p}}{\partial t} \left( \frac{T}{\rho} \right) + \frac{\partial \tilde{p}}{\partial \rho} = -\rho \nabla \tilde{u},$$  

(4)

$$\frac{\partial \tilde{p}}{\partial t} = -4\pi G \rho \tilde{\delta},$$  

(5)

$$\nabla \cdot \delta U = 0.$$  

(6)

$$\nabla \cdot h = 0.$$  

(7)

where $\rho$, $\rho$, $T$, $u(t, u_x, u_y, u_z)$, $\Omega(0, 0, \Omega_z)$, $\eta$, $\lambda$, $U$, G, R, and $\gamma$ denote the fluid density, pressure, temperature, velocity, rotation, electrical resistivity, thermal conductivity, gravitational potential, gravitational constant, gas constant and ratio of two specific heats respectively. Also here $\delta$ is for perturbed quantities and $h$ is perturbed magnetic field. The components of pressure tensor $P$, considering the finite ion gyration radius for the magnetic field along z-axis as given by Roberts and Taylor [10] are

$$P_{xx} = \rho \nu \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_x}{\partial y} \right),$$  

$$P_{yy} = \rho \nu \left( \frac{\partial u_y}{\partial x} + \frac{\partial u_y}{\partial y} \right),$$  

$$P_{zz} = 0, \quad P_{x \gamma} = 2 \rho \nu \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_x}{\partial y} \right),$$  

$$P_{\gamma \gamma} = 2 \rho \nu \left( \frac{\partial u_y}{\partial x} + \frac{\partial u_y}{\partial y} \right).$$  

(8)

The parameter $\nu$ has the dimensions of the kinematics viscosity and defined as $\nu = \Omega_L R_i^2/4$, where $R_i$ is the ion-Larmor radius and $\Omega_L$ is the ion gyration frequency. We assume the variation of perturbed quantities as $e^{i(k_x x + \omega t)}$, (9), where $k_x$ is the wave number of the perturbations along x-axis, and $\sigma$ is the frequency of harmonic disturbance.

3. Dispersion relation and discussion

The substitution of equation (9) into equations (1)-(7) using equation (8), gives the dispersion relation for transverse mode having axis of rotation along magnetic field (i.e. $k_x = k$ and $\Omega_z = \Omega$) is

$$\omega^3 \left( \omega + \frac{\nu k_x}{\omega + \eta k^2} \right) + \left( k_x^2 - 2 \Omega \right)^2 \frac{\lambda^2 T}{\rho} + TL - \rho \lambda \frac{\partial \rho}{\partial t} - 4\pi G \rho \left( \gamma - 1 \right) \frac{\lambda^2 T^2}{p} + T p L_x + \omega \left( c^2 k^2 - 4\pi G \rho \right) \right) = 0.$$  

(10)

Equation (10) has two independent factors. The first factor of equation (10) gives $\omega^3 = 0$, which is a marginal stable mode. The second factor gives the fourth degree dispersion relation in $\omega$, which is modified by the presence of radiative heat-loss function, self-gravitation, rotation, FLR correction, thermal conductivity, finite electrical resistivity and magnetic field. The condition of instability obtained from the constant term is given as
From equation (11) we conclude that the fundamental Jeans criterion of gravitational instability is modified due to rotation, FLR correction, thermal conductivity and radiative heat-loss function.

For an infinitely conducting medium in absence of FLR corrections \((\eta = \nu_0 = 0)\), the condition of instability is given as

\[
\left\{k^2 \left( \frac{\lambda k^2 T}{\rho} + TL_T - \rho L_p \right) - \frac{T \rho L_T}{p} \left(4\pi G \rho + 4 \Omega \nu_0 k^2 - 4 \Omega^2 - \nu_0^2 k^4 \right) \right\} < 0,
\]

From equation (11) we conclude that the fundamental Jeans criterion of gravitational instability is modified due to rotation, FLR correction, thermal conductivity and radiative heat-loss function.

For an infinitely conducting medium in absence of FLR corrections \((\eta = \nu_0 = 0)\), the condition of instability is given as

\[
\left\{k^2 (\gamma - 1) \left( \frac{\lambda k^2 T}{\rho} + TL_T - \rho L_p \right) - (\gamma - 1) \frac{T \rho L_T}{p} \left(4\pi G \rho - V^2 k^2 - 4 \Omega^2 \right) \right\} < 0,
\]

From equation (12) we conclude that rotation and magnetic field stabilizes the radiative instability. The above inequality (12) can be solved to yield the following critical wave number.

\[
k_{j_1} = \frac{1}{2} \left\{ \left[ \frac{4\pi G \rho}{c^2} + \frac{\rho L_T}{\lambda T} - \frac{4 \Omega^2}{c^2} \right] \left(1 + \frac{V^2}{c^2}\right)^{-1} - \frac{T L_T}{\lambda} \right\} \pm \left\{ \left[ \frac{4\pi G \rho}{c^2} + \frac{\rho L_T}{\lambda T} - \frac{4 \Omega^2}{c^2} \right] \left(1 + \frac{V^2}{c^2}\right)^{-1} \right\}^{1/2}
\]

In absence of rotation the above modified critical Jeans wave number reduces to that of Aggrawal and Talwar [4]. Numerical calculations were performed for equation (10) by making it dimensionless in terms of self-gravitation to determine the roots of \(\omega^*\) as a function of wave number \(k^*\) taking \(\gamma = 5/3\).

Out of the four modes only one mode is unstable for which the calculations are presented in figures 1-4, where the growth rate \(\omega^*\) (positive real value of \(\omega\)) has been plotted against the wave number \(k^*\) to show the dependence of the growth rate on the different physical parameters.

**Figure 1.** The normalized growth rate v/s normalized wave number with variation in FLR \(\nu_0^*\) keeping the other parameters fixed.

**Figure 2.** The normalized growth rate v/s normalized wave number with variation in rotation \(\Omega^*\) keeping the other parameters fixed.

It is clear from figure 1 that growth rate decreases with increasing the value of FLR correction. Thus the effect of FLR correction is stabilizing. From figure 2 we conclude that the growth rate decreases on increasing the value of rotation. Thus the effect of rotation is stabilizing. From figure 3 we conclude that growth rate increases with increasing density dependent heat-loss function. Thus the effect of density dependent heat-loss function is destabilizing. One can observe from figure 4 that the...
growth rate decreases with increase in temperature dependent heat-loss function. Thus the effect of temperature dependent heat-loss function is stabilizing.

Figure 3. The normalized growth rate v/s normalized wave number with variation in density dependent heat-loss function $L_p$ keeping the other parameters fixed.

Figure 4. The normalized growth rate v/s normalized wave number with variation in temperature dependent heat-loss function $L_T$ keeping the other parameters fixed.

4. Conclusion
Thus in the present paper, the effects of rotation, FLR corrections and radiative heat-loss function on the Jeans gravitational instability is investigated in the presence of thermal conductivity and finite electrical resistivity. The instability conditions are obtained on the basis of Jeans criterion. We find that Jeans criterion of instability gets modified due to the presence of rotation, FLR corrections, thermal conductivity and radiative heat-loss function.

Acknowledgments
The authors are indebted to Dr. S.K. Ghosh, Prof. and Head, S.S. in Physics Vikram University, Ujjain; for his constant encouragements. One author (SK) is grateful to Er. Praveen Vashishtha, Chairman of Mahakal Institute of Technology and Science, Dr. V.S. Ubboveja, Advisor MITS, and Dr. A.C. Shukla Director, MITS Ujjain; for continuous support and guidance.

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