Analytical results for a parity-time symmetric two-level system under synchronous combined modulations

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We propose a simple method of combined synchronous modulations to generate the analytically exact solutions for a parity-time symmetric two-level system. Such exact solutions are expressible in terms of simple elementary functions and helpful for illuminating some generalizations of appealing concepts originating in the Hermitian system. Some intriguing physical phenomena, such as stabilization of a non-Hermitian system by periodic driving, non-Hermitian analogs of coherent destruction of tunneling (CDT) and complete population inversion (CPI), are demonstrated analytically and confirmed numerically. In addition, by using these exact solutions we derive a pulse area theorem for such non-Hermitian CPI in the parity-time symmetric two-level system. Our results may provide an additional possibility for pulse manipulation and coherent control of the parity-time symmetric two-level system.

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I. INTRODUCTION

Two-level system (TLS) has been an important paradigmatic model in many branches of contemporary physics ranging from radiation-matter interactions to collision physics [1-3]. It lies at the heart of modern applications such as quantum control and quantum information processing as well as constitutes the foundation for fundamental studies of quantum mechanics. Coherent manipulation of two-level systems can be achieved and optimized by introducing external fields, e.g., the most commonly used types of periodic [4] and pulse-shaped [5] driving fields. In the past few decades, the two-level quantum system (or a qubit) involving single and/or combined modulations has proven a fertile ground for a variety of intriguing phenomena, including self-induced transparency [6], complete population inversion (CPI) [5], dynamical stabilization [7], and coherent destruction of tunneling (CDT) [5], to name only a few.

In certain situations, analytically exact solutions of driven two-level problems exist, offering many advantages in developing analytical approaches to the design of qubit control operations. Unfortunately, it is usually extremely difficult to acquire exactly soluble two-level evolutions so far. Started with the famous examples like the Landau-Zener model [4], Rabi problem [4], Rose-Zener model [5], the problems of finding analytic solutions of driven TLSs have been underway for a long time [9-20]. However, most of the known exact solutions of TLSs are expressed in terms of complicated special functions, such as the Gauss hypergeometric function [10-15], Weber function [10-16], associated Legendre function [17], Jacobi elliptic function [18], and Heun function [19-22]. Up to now, only a very few examples of analytically solvable two-level evolutions described only by several of the simplest solutions [4-20], which renders the control strategies more transparent, have been reported.

In recent years, parity-time (PT)-symmetric two-level system have attracted much attention from both theoretical and experimental studies [27-29]. A surprising finding of such PT-symmetric non-Hermitian Hamiltonian systems is phase transition, where the spectrum changes from all real to complex when the gain-loss parameter exceeds a certain threshold. It has been widely demonstrated that external driving fields can be effectively used for implementing coherent controls of PT symmetry, and extension of driven two-level systems to non-Hermitian case is currently of broad interest [30-36]. For the proper control of the PT-symmetric two-level system, different analytic methods are required. For instance, approximate solutions to the PT-symmetric two-level system, different analytic methods are required. For instance, approximate solutions to the PT-symmetric Rabi model have recently been proposed based on perturbation theory [32]. Given the vast literatures about analytical results for Hermitian two-level systems, it would be possible to find a few simple analytical exact solutions to the driven PT-symmetric two-level problems, which will create the possibility of convenient controls of the non-Hermitian systems.

In this paper, we have constructed a set of simple exact analytical solutions for the PT-symmetric two-level systems under synchronous combined modulations, which helps illuminate some generalizations of the appealing concepts which originate in their Hermitian counterparts. In the case of monochromatically synchronous combined modulations, as it is shown, for the nonzero constant
modulation (a periodic oscillating driving with a nonzero static component), the driven $\mathcal{PT}$-symmetric two-level system behaves like the non-driven one with a sudden $\mathcal{PT}$ symmetry breaking, whereas for the zero constant modulation (a purely oscillating driving), the driven $\mathcal{PT}$-symmetric two-level system features all-real spectra and a type of “generalized Rabi oscillations”. Besides, a non-Hermitian analog of coherent destruction of tunneling (CDT), in which the tunneling can be brought to a standstill, has been observed. For a square sech-shaped synchronous modulation, the exact analytical solutions of simple forms enables us to precisely produce complete population inversion (CPI) between two states in $\mathcal{PT}$-symmetric systems. The pulse area theorem for such CPI has also been derived analytically and confirmed numerically. The central purpose of this article is to develop analytical approaches to the design of coherent control of $\mathcal{PT}$-symmetric two-level systems by constructing exact solutions of simple forms through synchronous combined modulations.

II. EXACT SOLUTIONS AND THEIR APPLICATIONS

A. Model system

We consider a non-Hermitian version of driven two-level system, in which the equation of motion for the probability amplitudes $C_1$ and $C_2$ for the two states $|1\rangle$ and $|2\rangle$ is given as

$$\frac{i}{\hbar} \frac{d}{dt} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = H(t) \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \tag{1}$$

with the following Hamiltonian $H(t)$

$$H(t) = i \gamma(t) \sigma_z - \nu(t) \sigma_x. \tag{2}$$

Here $\sigma_x$ and $\sigma_z$ are the usual Pauli matrices, the real Rabi frequency $\nu(t)$ describes the coupling between the two levels, and $\gamma(t)$ represents a pair of real gain-loss coefficients. The system can be realized in many physical contexts, such as coupled waveguides with a complex refractive index, dissipative two-state system of cold atoms, and laser-driven atomic or molecular transitions with decay. In this work, we focus on how to design simple synchronous combined modulations to give analytic demonstrations of some intriguing and unexpected physics of non-Hermitian two-level systems. Here synchronous modulations means that the driving terms $\gamma(t)$ and $\nu(t)$ have the same time dependence, in other words, the modulation functions $\gamma(t)$ and $\nu(t)$ satisfy the relation $\gamma(t) = R \nu(t)$ with $R$ being a proportional constant. As we can easily demonstrate, a $\mathcal{PT}$-symmetric two-state problem is obtained if $\gamma(t)$ and $\nu(t)$ (e.g., the cosine and square sech forms) is temporally symmetric with respect to a certain point of time.

To obtain the exact solutions of model, we eliminate $C_2(t)$ and arrive at a second-order linear differential equation for $C_1(t)$ from Eq. (1),

$$\ddot{C}_1(t) - \frac{i \nu(t)}{\nu(t)} C_1(t) + [\nu(t)^2 - \gamma(t)^2]C_1(t) = 0. \tag{3}$$

In the case of synchronous modulations, $\gamma(t) = R \nu(t)$, by introducing a new time scale $\tau = \tau(t) = \int \nu(t) dt$, we can express the second-order differential equation of $C_1(t)$ in a simple form,

$$\frac{d^2 C_1}{d\tau^2} + (1 - R^2) C_1 = 0. \tag{4}$$

The general solution of Eq. (4), mathematically well known, falls into three cases as follows:

Case 1: when $R < 1$, letting $R = \sin \theta$, $\sqrt{1 - R^2} = \cos \theta$, the solution of Eq. (4) is given by

$$C_1(t) = D_1 e^{i \tau \cos \theta} + D_2 e^{-i \tau \cos \theta}. \tag{5}$$

with $D_1$ and $D_2$ being the undetermined constants determined by the initial conditions. According to Eq. (1), $C_2(t)$ is easily obtained as

$$C_2(t) = -D_1 e^{i \theta - i \tau \cos \theta} + D_2 e^{i \theta + i \tau \cos \theta}. \tag{6}$$

Case 2: when $R = 1$, Eq. (4) gives the analytical solution of Eq. (1) as

$$C_1(t) = D_1 \tau + D_2 \tag{7}$$

$$C_2(t) = i D_1 \tau - i D_1 + i D_2. \tag{8}$$

Case 3: when $R > 1$, letting $R = \cosh \phi$, $\sqrt{R^2 - 1} = \sinh \phi$, and combining Eq. (4) with Eq. (1), the exact solutions of $C_1$ and $C_2$ are obtained as

$$C_1(t) = D_1 e^{\tau \sinh \phi} + D_2 e^{-\tau \sinh \phi} \tag{9}$$

$$C_2(t) = i D_1 e^{\phi + \tau \sinh \phi} + i D_2 e^{\phi - \tau \sinh \phi}. \tag{10}$$

Any of the synchronous modulation functions $\nu(t)$ and $\gamma(t)$ will produce an analytical solution to the Schrödinger equation, which thus enables us to generate an infinite variety of analytically solvable $\mathcal{PT}$-symmetric two-state problems along with their explicit solutions. Here we will display some interesting physics by taking several specific cases as examples.

B. Synchronous modulation of a monochromatically periodic form

We present firstly the case of usual periodic modulation,

$$\nu(t) = \nu_0 + \nu_1 \cos \omega t, \tag{11}$$

where the new time scale is given by $\tau(t) = \nu_0 t + \frac{\phi}{\omega} \sin \omega t$. If the particle is initially populated at level $|1\rangle$, we have
the initial condition $C_1(0) = 1, C_2(0) = 0$. Applying them to Eqs. (5)-(10) yields the undetermined constants $D_j (j = 1, 2)$ and the exact population dynamics as follows

(i) when $R < 1$,

\[|C_1(t)|^2 = \frac{1}{2 \cos^2 \theta} \left( 1 + \cos(2\tau \cos \theta - 2\theta) \right), \quad (12)\]

\[|C_2(t)|^2 = \frac{1}{2 \cos^2 \theta} - \frac{\cos(2\tau \cos \theta)}{2 \cos^2 \theta}, \quad (13)\]

(ii) when $R = 1$,

\[|C_1(t)|^2 = \tau^2 + 2\tau + 1, \quad (14)\]

\[|C_2(t)|^2 = \tau^2, \quad (15)\]

and (iii) when $R > 1$,

\[|C_1(t)|^2 = \frac{\sinh^2(\tau \sinh \phi + \phi)}{\sinh^2 \phi}, \quad (16)\]

\[|C_2(t)|^2 = \frac{\sinh^2(\tau \sinh \phi)}{\sinh^2 \phi}, \quad (17)\]

From Eqs. (12)-(17), we immediately have the following observations: (1) for zero constant modulation ($v_0 = 0$), the solution should be periodic irrespective of the value of proportional constant $R$. (2) for the nonzero constant modulation ($v_0 \neq 0$), the solution does not have to be bounded. Indeed, as long as the proportional constant $R$ is smaller than the critical value, $R = 1$, the system will exhibit oscillations with bound, while as $R$ increases above the critical value, it will exhibit exponential growth. At the critical value, $R = 1$, the time-averaged population shows a parabolic growth. A naive intuition might suggest that apart from a distinct time scale, periodic synchronous driving will bring no new physics to the non-Hermitian system. Indeed, for the nonzero constant modulation ($v_0 \neq 0$), the driven $\mathcal{PT}$-symmetric two-level system acts like the undriven one, with a phase transition from full to broken $\mathcal{PT}$ symmetry. However, for the zero constant modulation ($v_0 = 0$), stabilization of the non-Hermitian system can be achieved by the periodic synchronous driving in the full range of system parameters, which is contrary to our naive intuition. This is confirmed by numerical simulations in Fig. 1.

In Fig. 1, we show the time evolutions of the populations for both $v_0 \neq 0$ and $v_0 = 0$ cases with the initial state $C_1(0) = 1, C_2(0) = 0$. In our simulation, the numerical solutions of the equation (14) are obtained through the use of the Runge-Kutta method. As expected, for $v_0 \neq 0$, like the non-driving situation, the system experiences a transition from a state with quasi-periodic oscillation ($\mathcal{PT}$-symmetric state, $R < 1$, see Fig. 1(a)) to a state with unbounded growth (broken $\mathcal{PT}$-symmetric state, $R > 1$, see Fig. 1(c)), whereas for $v_0 = 0$, stable and coherent but non-norm-preserving population oscillations, referred to as generalized Rabi oscillation in Ref. [34], are observed in Fig. 1(d)-(f) for all values of the proportional constant $R$. We have compared the analytical (circles) with the numerical (red solid lines) results in Fig. 1.
\( \varepsilon_{1,2} = \pm \nu_0 \cos \theta \) if \( R < 1 \) and broken \( \mathcal{PT} \) symmetry (both eigenvalues are imaginary, \( \varepsilon_{1,2} = \pm i\nu_0 \sinh \phi \) if \( R > 1 \)), however, for the zero constant modulation (\( \nu_0 = 0 \)), the two eigenvalues (quasienergies) are degenerate and always equal to zero. Now we take a closer look at what happens at the transition point, \( R = 1 \).

At this point, for \( \nu_0 \neq 0 \), the two Floquet modes coalesce to \( u_{1,2} = (1, i)^T \) and the system has only one linearly independent eigenstate, leading to an exceptional point. Nevertheless, for \( \nu_0 = 0 \), at the point \( R = 1 \), the two linearly independent Floquet modes are given by \( u_1 = (1, i)^T \) and \( u_2 = (\nu_1 \sin \omega t/\omega, \nu_1 \sin \omega t - i)^T \) respectively, with corresponding degenerate eigenvalues equal to zero. It should be emphasised that for \( \nu_0 = 0 \), the general solution (5)-(10) itself is also a kind of Floquet state with a zero quasienergy, due to the fact that any linear combination of two degenerate Floquet eigenstates (the coefficients of linear combination \( D_j (j = 1, 2) \)) are adjusted by the initial condition) is also a Floquet eigenstate of the Hamiltonian operator corresponding to the same eigenvalue (quasienergy).

![Fig. 2](image)

**FIG. 2:** (color online) Real (left) and imaginary (right) parts of the quasienergies as a function of the proportional constant \( R \) for the system (1) subject to a periodic modulation (11) with (a)-(b) \( \nu_0 = 0.5 \) (upper row) and (c)-(d) \( \nu_0 = 0 \) (lower row). The other parameters are \( \nu_1 = 1, \omega = 3 \).

In Fig. 2 we exhibit the numerically obtained quasienergies \( \varepsilon \) of the system (1) for both \( \nu_0 \neq 0 \) and \( \nu_0 = 0 \) cases. The quasienergies are numerically computed through direct diagonalization of the time evolution operator over one period of the driving, \( U(\Lambda, 0) = \mathcal{T} \exp[-i \int_0^A H(t)dt] \), where \( \mathcal{T} \) is the time-ordering operator. As we can see in Figs. 2 (a) and (b), for \( \nu_0 \neq 0 \), the numerical eigenvalue (quasienergy) spectrum indeed changes from being real to complex, indicating a spontaneous \( \mathcal{PT} \)-symmetry-breaking transition. However, for \( \nu_0 = 0 \), the resulting periodically driven two-level system, although being non-Hermitian, processes a pair of degenerate all-real quasienergies (\( \varepsilon_1 = \varepsilon_2 = 0 \)) for the whole range of tuning parameters, as illustrated in Figs. 2 (c) and (d). These numerical results are completely consistent with our theoretical analysis.

Coherent destruction of tunneling (CDT) is one of the notable effects of the periodically driven system, a quantum interference effect discovered originally in the periodically driven Hermitian two-level system, upon the occurrence of which the tunneling dynamics can be brought to a complete standstill. The CDT is connected to the degeneracy of the quasienergies. In fact, CDT requires in addition that the degenerate Floquet states do not show appreciable amplitude oscillations within one oscillation cycle, which can be realized in the high frequency limit \( (\omega \gg \nu) \). Interestingly, in the non-Hermitian case, CDT still exists if the quasienergies of the underlying time-periodic system are degenerate.

![Fig. 3](image)

**FIG. 3:** (color online) Dynamics of the system (1) subject to a periodic modulation (11) with a zero static component \( \nu_0 = 0 \), starting from the initial state \( C_1(0) = 1, C_2(0) = 0 \). (a) Time evolution of the probability \( P_1 = |C_1|^2 \), with \( \nu_1 = 1, R = 0.5 \) and different values of the driving frequency; (b) Localization, defined as the minimum value of \( P_1 / P_0 \), as a function of \( \omega \), with the same parameters as in (a); (c) Time evolution of the probability \( P_1 = |C_1|^2 \), with \( \nu_1 = 1, \omega = 3 \) and different values of the proportional constant; (d) Localization as a function of \( R \), with the same parameters as in (c).

Next, we study further the dynamics of the system (1) for the case of \( \nu_0 = 0 \), where the two quasienergies are degenerate and equal to zero. The results of the numerical simulation of the probability distribution \( P_1 = |C_1|^2 \) are presented in Fig. 3 (a) for different values of driving frequencies. For the low-frequency driving, the occupation of level \( |1 \rangle \) oscillates and at certain times drops to zero, indicating no suppression of tunneling. As the driving frequency \( \omega \) is increased, the degree of localization is enhanced and the amplitude of oscillations of \( P_1 \) is smaller. If \( \omega \) is increased further, \( P_1 \) remains near unity and the tunneling dynamics is frozen, which is a direct signature of CDT. To quantify the stable but non-unitary dynamics more precisely, we have used localization, which is defined as the minimum value of \( P_1 / P \), to measure the suppression of tunneling. When CDT occurs, the localization is
close to 1; when there is no suppression, localization is zero. In Fig. 3(b) we plot the minimum value of $P_1/P$, or the localization, as a function of the driving frequency. For small values of $\omega$, the minimum value of $P_1/P$ equals zero, and thus no CDT occurs. As $\omega$ is increased, however, the localization is close to one, and the oscillations in $P_1$ are quenched, producing CDT. We have also examined the effects of the proportional constant $R$ on the localization as shown in Figs. 3(c) and (d), where with the increase of $R$ the amplitude of oscillations of $P_1$ is larger and the localization is decreased accordingly.

Since the time scale $\tau$ of the driven system is inversely proportional to the driving frequency, it follows from Eqs. (15)-(21) that the amplitude oscillations of the degenerate Floquet doublet would be vanishingly small in the high frequency limit. So far, we have demonstrated the existence of CDT in the non-Hermitian system, for which the basic two conditions, degeneracy of quasienergies and absence of appreciable amplitude oscillations of the two degenerate Floquet states, must be simultaneously satisfied. This is in full analogy to the Hermitian case.

C. Synchronous modulation of a square sech-shaped form

We now consider the synchronous modulation of a square sech-shaped (a pulse-shaped) form. Let

$$\nu(t) = \text{Asech}^2 t, \quad (22)$$

such that the new time variable becomes

$$\tau(t) = A \tanh t. \quad (23)$$

When $R = 0$ (hence $\gamma(t) = R\nu(t) = 0$), the system reduces to the famous Rose-Zener model without detuning [5]. We first briefly overview the main results of the Rose-Zener model on resonance.

The case $R \leq 1$. Inserting the initial condition (25) into Eqs. (5)-(6) and (23) gives

$$D_1 = -\frac{e^{-A\cos \theta}}{2\cos \theta}, \quad D_2 = \frac{e^{A\cos \theta}}{2\cos \theta}. \quad (26)$$

The final level populations at $t = +\infty$, according to the exact solutions (15)-(20), are thus given by

$$|C_1(\pm \infty)|^2 = \frac{\sin^2(2A\cos \theta)}{\cos^2 \theta}, \quad (27)$$

$$|C_2(\pm \infty)|^2 = \frac{\cos^2(2A\cos \theta + \phi)}{\cos^2 \phi}. \quad (28)$$

From Eqs. (27) and (28), we readily observe the following circumstances:

1. If $2A \cos \theta = n\pi, n = 1, 2, 3, ..., (29)$ then the population restores to its initial state at $t = +\infty$.

2. If $2A \cos \theta + \phi = (n + \frac{1}{2})\pi, n = 0, 1, 2, 3, ..., (30)$ then the population is completely inverted to the level $|1⟩$ at $t = +\infty$.

It is clearly seen that when $R = 0$ (hence $\cos \theta = 1$), Eqs. (29) and (30) recover the characteristic of pulse area for the on-resonant Rose-Zener problem.

The case $R > 1$. At this point, starting with the initial state (25), we have from Eqs. (7)-(8) and (23),

$$|C_1(\pm \infty)|^2 = 4A^2, \quad (31)$$

$$|C_2(\pm \infty)|^2 = 1 - 4A + 4A^2. \quad (32)$$

It is obvious that if

$$A = \frac{1}{2}, \quad (33)$$

we get the complete population inversion (CPI).

The case $R > 1$. In that case, from Eqs. (9)-(10) and (23), the final populations in levels $|1⟩$ and $|2⟩$ corresponding to the initial condition (25) can be easily found to be

$$|C_1(\pm \infty)|^2 = \frac{\sinh^2(2A\sinh \phi)}{\sinh^4 \phi}, \quad (34)$$

$$|C_2(\pm \infty)|^2 = \frac{\sinh^2(\phi - 2A\sinh \phi)}{\sinh^4 \phi}. \quad (35)$$

If

$$2A\sinh \phi = \phi, \quad (36)$$

one finds that $|C_1(\pm \infty)|^2 = 1, |C_2(\pm \infty)|^2 = 0$, such that a complete population inversion (CPI) occurs.
Thus, three distinct modified pulse area conditions for CPI in the non-Hermitian two-level system have been identified, which are dependent on the choice of the proportional constant $R$ and given by Eqs. (30), (33) and respectively.

The above-mentioned CPI conditions in the parametric plane of $(A, R)$ are visualized in Fig. 4(a), which are directly obtained from Eqs. (30) (the $n = 0$ case), (33) and (36). As an example, in Fig. 4(b), we select $R = 1.5$ and the pulse area $2A = \arccosh R / \sqrt{R^2 - 1}$ given by Eq. (36) to show the numerical results for time evolutions of the population distributions with the initial state.

FIG. 4: (color online) (a) Parametric values of $(A, R)$ allowed for CPI in the system subjected to a synchronous modulation of the sech$^2$ form given by Eq. (22). The allowed parameter values of $(A, R)$ are obtained from Eqs. (30) ($n = 0$), (33) and (36). (b) Time evolution of the population distributions for the initial state $C_1(-\infty) = 0$, $C_2(-\infty) = 1$. The system parameters are set as $R = 1.5$ and $2A = \arccosh R / \sqrt{R^2 - 1}$.

Clearly, a complete population inversion (CPI) has been fully confirmed by our numerical simulations.

III. CONCLUSION

In summary, we have presented a class of exact analytical solutions for a $\mathcal{PT}$-symmetric two-level system under synchronous combined modulations. Such exact solutions are expressible in terms of simple elementary functions and are applicable in the coherent control of quantum states. With these analytical solutions, we have demonstrated that for a cosine synchronous driving with a nonzero static component, the system behaves like the undriven one with a phase transition from full to broken $\mathcal{PT}$ symmetry, while for a purely cosine synchronous driving, the system, although being non-Hermitian, feature all-real quasienergy spectra for the whole range of tuning parameters. Surprisingly, we have found that coherent destruction of tunneling (CDT) and complete population inversion (CPI) still exist in the non-Hermitian situation. Furthermore, we have given analytically the modified pulse area conditions for such non-Hermitian analog of CPI, whose mathematical formulations are found to be dependent on the choice of the proportional constant between the two combined synchronous modulations. These simple exact analytical solutions are numerically confirmed and may find applications in any and all physical contexts where the $\mathcal{PT}$-symmetric two-state problems arise.

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