Information in spinning sound fields*

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Abstract

The information content of a spinning sound field is analyzed using a combination of exact and asymptotic results, in order to set limits on how accurately source identification can be carried out. Using a transformation of the circular source to an exactly equivalent set of line source modes, given by Chebyshev polynomials, it is found that the line source modes of order greater than the source wavenumber generate exponentially small fields. Asymptotic analysis shows that the remaining, lower order, modes radiate efficiently only into a region around the source plane, with this region shrinking as the mode order is increased. The results explain the ill-conditioning of source identification methods; the successful use of low order models in active noise control; and the low radiation efficiency of subsonic jets.

1 Introduction

Source identification, the problem of determining an acoustic source from field measurements, has been attempted using a number of approaches in various different technologies. This paper examines the problem of identifying the source which generates a spinning acoustic field. The source which generates such fields can be represented as a set of modes which vary with azimuth on a circular disk, whether or not the source includes a spinning element. Examples include rotating systems such as cooling fans [1, 2], helicopter rotors [3, 4] duct terminations such as aircraft engine intakes [5, 6, 7, 8, 9, 10] and jets [11] if a jet is modelled as a distribution of disk-shaped sources.

There are two broad categories of problem where source identification is required, corresponding to ‘forward’ and ‘backward’ projection of the field. In the forward problem, the aim is to estimate the source distribution accurately enough to allow the field to be predicted at positions other than the original measurement points. This has been done, for example, in extracting source parameters from near-field measurements of propeller noise, with the parameters then being used to calculate far-field noise [12].

In backward projection the problem is determination of the source proper from field measurements. This might be done in order to decide on noise control measures or because the acoustic source corresponds to some other physical variable of interest. In the first case, the aim is usually to find the acoustic source strength distributed over the source region, such as a rotor disk [1, 2] or the termination of a duct [7, 8, 9, 10]. An example of the second application is the study of noise generation by turbulent jets [11], where the aim is to determine the fluid-dynamical mechanisms which give rise to the acoustic source.

In any case, it is well known that the problem of source identification is poorly-conditioned, meaning that small measurement errors can give rise to very large changes in the estimated source. Previous studies of the structure of spinning fields [13, 14, 15, 16] have shown that the field decays exponentially away from the source. This means that in the forward projection problem, errors in the estimated acoustic source will decay and the predicted acoustic field may well be quite accurate, even if the source is not well recovered. On the other hand, in the backward projection problem, the exponential decay moving away from the source corresponds to exponential growth moving towards it, leading to large errors in the estimated source.

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In this paper, the radiated field from a disk source is analyzed to examine how much information about the source can actually be detected in the field. In a previous paper on a possible method for source identification [17], it was shown that the far-field noise is band-limited Fourier transform of a line source which is exactly equivalent to the disk source. In this paper, without recourse to far field approximations, it is possible to establish fundamental limits on the number of degrees of freedom of a field, limits which are determined by the source frequency. The implications of the analysis are discussed with respect to some real source identification and radiation prediction problems.

2 Analysis

The problem considered is that of the field generated by an azimuthally varying distribution of monopoles with strength \( s(r_1, \theta_1) \) given by the Rayleigh integral [18, 13]

\[
p(r, \theta, z, \omega) = \int_0^1 \int_0^{2\pi} s(r_1, \theta_1) \frac{e^{jkR}}{4\pi R} d\theta_1 r_1 dr_1,
\]

where the source is distributed over the unit disk in the plane \( z = 0 \), variables of integration have subscript 1 and the coordinate system is shown in Figure 1. The wavenumber \( k = \omega/c \), and \( c \) is the speed of sound.

Taking one azimuthal mode of the source distribution, \( s(r_1, \theta_1) = s_n(r_1) \exp jn\theta_1 \), the radiated field for one mode can be written \( p = p_n \exp jn\theta \):

\[
p_n(r, z) = \int_0^1 \int_0^{2\pi} s_n(r_1) \frac{e^{j(kR - n\theta_1)}}{4\pi R} d\theta_1 r_1 dr_1,
\]

where the integral of Equation 2 has been extensively studied due to its relevance to rotor acoustics and, under suitable conditions, as a good approximation to radiation from ducts. Many problems in source identification [1, 2, 5, 6, 7, 8, 9, 10] can be viewed as attempts to recover the source term \( s_n(r_1) \) from measurements of \( p_n \).

The remainder of this paper consists of an analysis of the integral of Equation 2 which will establish limits on the information about the source which is radiated into the acoustic field, thereby fixing how accurately a source can be identified. The results are also applicable to the question of the detail with which a source need be specified in order to accurately predict the acoustic field, and to that of the radiation efficiency of jets.
2.1 Equivalent line source

The first step in the analysis is to transform the disk source into a line source which generates (exactly) the same acoustic field. This is a transformation which has been used, in the axisymmetric case, in studies of transient radiation from pistons\[19, 20\], and, with azimuthal variation, in studies of rotor acoustics\[13, 14, 15, 16\]. The first stage is to switch from the source-centred cylindrical coordinates \((r, \theta, z)\) of Figure 1 to the observer-centred coordinates \((r_2, \theta_2, z)\) of Figure 2. Under this transformation, Equation 2 becomes, for \(r > 1\):

\[
p_n(k, r, z) = \int_{r-1}^{r+1} \frac{e^{ikR}}{R} K(r, r_2) r_2 \, dr_2, \tag{3}
\]

\[
R = (r_2^2 + z^2)^{1/2},
\]

\[
K(r, r_2) = \frac{1}{4\pi} \int_{\theta_2^{(0)}}^{2\pi - \theta_2^{(0)}} e^{-jn\theta_1} s_n(r_1) \, d\theta_2, \tag{4}
\]

where the source function \(K(r, r_2)\) depends on \(r\), the observer lateral separation, but is independent of \(z\), the axial displacement. The coordinate systems are related by:

\[
r_1^2 = r^2 + r_2^2 + 2rr_2 \cos \theta_2, \tag{5a}
\]

\[
\theta_1 = \tan^{-1} \frac{r_2 \sin \theta_2}{r + r_2 \cos \theta_2}. \tag{5b}
\]

and the limits of integration in Equation 4 are given by setting \(r_1 = 1\):

\[
\theta_2^{(0)} = \cos^{-1} \frac{1 - r^2 - r_2^2}{2rr_2}. \tag{6}
\]

The function \(K(r, r_2)\) has square-root behavior at its end-points\[17\], \(r_2 = r \pm 1\), and can be expanded:

\[
K(r, r_2) = \sum_{m=0}^{\infty} u_m(r) U_m(s)(1 - s^2)^{1/2} \tag{7}
\]

with \(s = r_2 - r\) and \(U_m(s)\) the Chebyshev polynomial of the second kind.
Inserting this expansion into Equation [\(\sum\)]

\[ p_n(k, r, z) = \sum_{m=0}^{\infty} u_m(r) I_m(k, r, z), \quad (8) \]

\[ I_m(k, r, z) = \int_{-1}^{1} e^{j k R / R} U_m(s)(r + s)(1 - s^2)^{1/2} \, ds, \quad (9) \]

\[ = \int_{0}^{\pi} e^{j k R / R} (r + \cos \beta) \sin(m + 1) \beta \, \sin \beta \, d\beta, \quad (10) \]

with the transformation \(s = \cos \beta\) and use of the definition of the Chebyshev polynomial [21], \(U_m(s) = \sin[(m + 1)\beta] / \sin \beta\).

3 Radiated field

The analysis of the previous section gives us a model of a spinning acoustic field expressed in terms of an exactly equivalent line source composed of a superposition of modes given as Chebyshev polynomials, with the modal coefficients functions of observer radius \(r\), but not of axial displacement \(z\). In this section, we use the model to draw basic conclusions about the acoustic information which is available for source identification.

3.1 Cut-off modes

The first conclusion we can draw from the integral expression for \(I_m\) is that modes with \(m > k\) are exponentially small and can be considered ‘cut-off’. For \(z = 0\), \(I_m(k, r, 0)\) can be evaluated exactly using Equation [10] and standard relations for Bessel functions [21]:

\[ I_m(k, r, 0) = j^m \pi (m + 1) J_{m+1}(k) / k \, e^{j kr}. \quad (11) \]

For \(m + 1\) large and \(k < m + 1\), the Bessel function \(J_{m+1}(k)\) decays exponentially, i.e. the higher modes are ‘cut off’. Since \(|I_m(r, z)|\) has its maximum in the plane \(z = 0\), we can further conclude that modes with \(m > k\) are cut off everywhere and cannot be detected in the field. This is an exact result which places a first limit on the information radiated.

3.2 Cut-on modes

A second limit on the information available in the acoustic field can be found by asymptotic analysis of \(I_m\) which can be rewritten:

\[ I_m = (Q_{m+2}(k, r, z) + Q_{m-2}(k, r, z) \]
\[ - Q_m(k, r, z) - Q_{-m}(k, r, z)) / 4, \]

with:

\[ Q_m(k, r, z) = \int_{0}^{\pi} e^{j k \psi(\beta)} (r + \cos \beta) / R \, d\beta, \quad (12) \]

\[ \psi(\beta) = R + \gamma \beta, \]
\[ \gamma = m / k. \]
Figure 3: Cut-off lines for $\gamma = m/8$, $m = 1, 2, \ldots, 6, 7$; exact solution solid; $z_c = \alpha r/\gamma$ dashed.

The integral $Q_m$ is in a suitable form for stationary phase analysis[22], which depends on finding the stationary points of $\psi$, i.e. values of $\beta$ where $d\psi/d\beta = 0$ with $0 \leq \beta \leq \pi$. Upon differentiation and rearrangement, the condition $d\psi/d\beta = 0$ takes the form of a quartic equation:

$$ (\alpha^2 - C^2)(r + C)^2 - \gamma^2 z^2 = 0, \quad (13) $$

where $C = \cos \beta$ and $\alpha^2 = 1 - \gamma^2$. To lie in the domain of integration, the stationary phase points must be real with $|C| < 1$. This leads to the requirement that $0 < \gamma < 1$ and $|z| < z_c$, a ‘cut-off’ value of observer axial displacement beyond which the phase function $\psi$ has no valid stationary points. The two values of $C$, denoted $C_+ = C_+(z_c)$ and $C_- = C_-(z_c)$, at the limits of $|z|$ are:

$$ C_\pm(z) = \begin{cases} \pm \alpha, & z = 0, \\ -r/4 + (r^2 + 8\alpha^2)^{1/2}/4, & |z| = z_c. \end{cases} \quad (14) $$

Denoting $C_c = C_+(z_c) = C_-(z_c)$, the cut-off value of $z$ is:

$$ z_c = (\alpha^2 - C_c^2)^{1/2}(r + C_c)/\gamma, \quad (15) $$

$$ \rightarrow \alpha r/\gamma, \quad \alpha/r \rightarrow 0. $$

Written in spherical coordinates, the asymptotic cut-off lines $z_c = \alpha r/\gamma$ are rays with polar angle $\phi = \sin^{-1} \gamma$. For completeness, we note that for $\gamma = 0$, there is no cutoff and the line source mode radiates into the whole field with amplitude proportional to $k^{-1/2}$.

Figure 3 shows the cut-off lines, exact and asymptotic, for $0 < \gamma < 1$. The accuracy of the asymptotic approximation for the cut-off line is confirmed and the plot shows which radiated modes are detectable in a given part of the acoustic field.

Using the previous results, the asymptotic behavior of the basic integral is given by:

$$ Q_m \sim j^{3/2} \frac{e^{jk\psi_+}}{(kR_+)^{1/2}} \left[ \frac{2\pi}{C_+(r + 2C_+) - \alpha^2} \right]^{1/2} (r + C_+) $$

$$ + j^{1/2} \frac{e^{jk\psi_-}}{(kR_-)^{1/2}} \left[ \frac{2\pi}{C_-(r + 2C_-) - \alpha^2} \right]^{1/2} (r + C_-), \quad k \rightarrow \infty, \quad (16) $$

$$ R_\pm^2 = (r + C_\pm)^2 + z^2, \quad \psi_\pm = R_\pm + \gamma \cos^{-1} C_\pm, $$

and:

$$ I_m \sim (Q_{m+2}(k, r, z) - Q_m(k, r, z))/4, \quad (17) $$

where $Q_{-m}$ and $Q_{-m-2}$ are neglected since they have no stationary phase points and decay much faster than $Q_m$ and $Q_{m+2}$ with increasing $k$. 

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Figure 4 compares the asymptotic approximation for \( I_{1}(k, r, z) \) to a numerical evaluation of the integral. The cut-off value of \( z \) for \( \gamma = 3/k \) is indicated and the real and imaginary parts of the integrals are plotted separately. As \( z \to z_c \), the stationary phase approximation to \( Q_{m+2} \) breaks down and there is a resulting loss of accuracy. Away from this point, however, the approximation to \( I_m \) is accurate on both sides of \( z_c \).

The asymptotic analysis shows that the cut-on line source modes, those with \( m < k \), radiate efficiently into a region bounded by \( \pm z_c(r, \gamma) \). This gives a second limit on the information available in the acoustic field.

### 3.3 Far field approximation

For completeness, we give a far field approximation of the line source radiation integral, valid outside the region covered by the asymptotic expansions of §3.2. Using the standard approximation, \( R \approx R_0 + (r - r_2) \sin \phi \), \( 1/R \approx 1/R_0 \) with \( R_0^2 = r^2 + z^2 \):

\[
I_m \approx j^m \pi \frac{e^{jkR_0}}{R_0} m + 1 \frac{k \sin \phi}{k \sin \phi} \left[ \left( r + j \frac{m + 2}{k \sin \phi} \right) J_m(k \sin \phi) \right. \\
\left. - jJ_m(k \sin \phi) \right]
\]

so that in the far field, \( I_m \) decays as \( k^{-1} \).

### 4 Information in spinning sound fields

Summarizing the results of the previous section, the nature of a spinning sound field is seen to be determined by its wavenumber \( k \) and the relation of \( k \) to the set of modes contained in the equivalent line source. The first result, that line modes with \( m > k \) generate exponentially small fields, means that the acoustic field contains a limited amount of information about the source. Such a result has been derived previously by showing that the far field pressure is a band-limited Fourier transform of the line source strength [17], but this new result establishes an exact limit on the information in the field, without needing a far-field approximation.

The second result, from the stationary phase analysis, shows that the modes which do radiate, those with \( m < k \), are more efficient in some parts of the field than in others. The higher order radiating modes are detectable only near the source plane, with a lower radiation efficiency at larger \( z \). The only mode which radiates efficiently over the whole field is the ‘plane’ mode \( m = 0 \).

The following sections discuss some implications of these findings for different problems.
4.1 Low speed rotors

One result which is of immediate interest is that, in some sense, low speed rotors have the same acoustic field. Given that for a system of radius $a$ rotating at angular velocity $\Omega$ the non-dimensional wavenumber of the $n$th harmonic of the radiated field $k = n\Omega a/c = nM_t$, with $M_t$ the source tip Mach number, low speed rotors will have $k < 1$ over the first few harmonics of the signal. This means that the sound field is dominated by the zero order line mode and any set of rotors of a given blade number operating at the same speed, whatever their blade geometry, will have the same acoustic field, to within a scaling factor. Indeed, experiments in active noise control of noise from cooling fans\cite{23} have found good results by discretizing the inverse model of the fan into three sections, i.e. using three degrees of freedom, for a value of $ka \approx 0.8$. The same group, in an earlier study of the conditioning of the inversion problem, using a hemispherical arrangement of microphones\cite{1} found that the condition number reduced as frequency was increased, a finding they ascribe to “insufficient spatial resolution of the source for frequency below 200Hz.” In the light of the analysis above, an alternative viewpoint might be that as frequency is increased, more line source modes are cut on and the information available in the acoustic field becomes more nearly sufficient for source reconstruction.

4.2 Source identification

The original motivation for this work was the problem of identifying a rotating source. The results of §2 and §3 can be used to help show why this is a hard problem and to indicate how it might best be approached.

The first obvious consequence of the result of §3.1 is that the acoustic field has a limited number of degrees of freedom. For a field of given wavenumber $k$, no more than $k$ modes can be detected in the field, i.e. the field has $M$ degrees of freedom, with $M$ the largest integer $M < k$. Attempting to identify sources using more than $M$ degrees of freedom is inherently ill-conditioned because the modes with $m > k$ generate exponentially small fields.

Secondly, the asymptotic analysis shows that much of the information in the field is not detected by microphones. For a given microphone polar angle, only a subset of the $M$ radiating modes will be easily detected. If the microphone is at polar angle $\phi$, modes with $\gamma > \sin \phi$ may not be detected or, alternatively, the microphone only detects radiation from modes of order $m < k/\sin \phi$.

The effect this has on the source identification problem is shown in Figure 5. This shows, as a thick line, $K(r, r+s)$ for $r = 5/4$, $n = 2$ and $s(r_1) \equiv 1$, calculated using previously published closed-form expressions\cite{14}. The thin lines show $K$ reconstructed using the first $M$ terms of the sum in Equation 7 with $M = 0, 1, 2, 3, 4$. The convergence towards the exact value of $K$ is obvious, but it is also obvious that this convergence is so slow that a large number of terms will be needed in order to accurately reconstruct $K$. Given that the number of modes which can be detected depends on wavenumber $k$, it is clear that except at very high frequencies, only low resolution source reconstruction is possible.
4.3 Source resolution

The observations of the previous subsection give a pessimistic outlook for source identification: the information necessary for accurate reconstruction of a source is simply not available, even in theory. On the other hand, if the objective is to reproduce an acoustic field, for noise control, say, only limited knowledge of the source is necessary. Looking again at Figure 5, although there is a large difference between the exact line source and the source produced by summation of lower order modes, the acoustic field generated by the lower order modes with \( m < k \) will be indistinguishable from that generated by the exact source, since the modes with \( m > k \) do not contribute to the radiated sound.

Figure 6 shows the acoustic field on a sideline \( r = 1.25 \) computed using the exact line source of Figure 5 and by summation of the field due to the lower order modes, with \( m \leq 0, 2, 4 \) shown. The wavenumber \( k = 2.5 \), so that cutoff begins at \( m = 3 \). Indeed, the sum over the first three modes is very close to the exact result, and when the first five modes are included, the result is indistinguishable from the exact field.

Figure 7 shows the development of \( p_n \) at \( z = 0 \) as more modes are included in the summation. It is clear that the sum has all but converged after the \( m = 2 \) mode is added and is practically unchanged after the \( m = 4 \) mode is added, confirming the conclusion that the higher order modes are cut-off and do not contribute to the acoustic field, in spite of their quite high amplitudes. The clear conclusion is that the acoustic field depends on the lower order modes and sources which differ only in the higher order terms of their line source decomposition cannot be distinguished by a source identification procedure. The corollary of this statement is that for an accurate prediction of the field, sources need only be specified to a resolution sufficient to correctly identify the amplitudes of the lower order modes.
4.4 Jet noise

In a recent paper\cite{11}, Jordan et al. examine noise production by a turbulent jet using proper orthogonal decomposition (POD) to perform a modal decomposition of the flow and a technique called MOD (‘most observable decomposition’) to decompose the acoustic far field. They find that while 350 flow modes are needed to account for 50% of the turbulent kinetic energy in the flow, only 24 modes are needed to account for 90% of the acoustic energy. If the jet is viewed as a distribution of disk sources along the jet axis, the results of this paper show that we should expect that only a small fraction of the modes will radiate noise and that in a complex source such as a jet, the bulk of the modes will have $m > k$ and will generate exponentially small fields. In a study of noise sources in a jet, Freund\cite{24} filters the source terms to leave “a set of modes capable of radiating to the far field”, based on a wavenumber criterion, but he notes that “additional cancellation may occur due to the radial structure of the source”. The analysis of the previous sections offers an approach for the identification of the radial terms which will give such cancellation.

5 Conclusions

An analysis of the information content of a spinning sound field has been presented. It has been shown, on the basis of an exact analysis, that the acoustic field around a spinning source has at most $M$ degrees of freedom with $M < k$, the acoustic wavenumber. This result arises from the replacement of the disk source with an exactly equivalent line source given by a sum of modes. Most of these modes generate exponentially small acoustic fields, i.e. are cut off, and the remaining, lower order, modes radiate efficiently only into sectors of the acoustic field which become smaller as the modal order increases. The results explain a number of features which have been observed in the literature, including: the possibility of using low order source models for noise control; the ill-conditioning of source identification methods; and the low radiation efficiency of subsonic jets.

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