Towards a fully consistent Milky Way disc model – II. The local disc model and SDSS data of the NGP region

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Accepted 2010 October 13. Received 2010 August 27

ABSTRACT

We have used the self-consistent vertical disc models of the solar neighbourhood presented in Paper I, which are based on different star formation histories (SFRs) and fit the local kinematics of main-sequence stars equally well, to predict star counts towards the North Galactic Pole (NGP). We combined these four different models with the local main sequence in the filter system of the Sloan Digital Sky Survey (SDSS) and predicted the star counts in the NGP field with $b > 80^\circ$. All models fit the Hess diagrams in the F–K dwarf regime better than ±20 per cent and the star number densities in the solar neighbourhood are consistent with the observed values. The $\chi^2$ analysis shows that model A is clearly preferred with systematic deviations of a few per cent only. The SFR of model A is characterized by a maximum at an age of 10 Gyr and a decline by a factor of 4 to the present-day value of $1.4 \, M_\odot \, pc^{-2} \, Gyr^{-1}$. The thick disc can be modelled very well by an old isothermal simple stellar population. The density profile can be approximated by a $\sech^2$ function. We found a power-law index $\alpha_t = 1.16$ and a scaleheight $h_t = 800 \, pc$ corresponding to a vertical velocity dispersion of $\sigma_t = 45.3 \, km \, s^{-1}$. About 6 per cent of the stars in the solar neighbourhood are thick-disc stars.

Key words: Galaxy: disc – Galaxy: evolution – Galaxy: kinematics and dynamics – solar neighbourhood – Galaxy: stellar content – Galaxy: structure.

1 INTRODUCTION

In Just & Jahreiss (2010, hereafter Paper I) a self-consistent model of the vertical structure of the Milky Way disc in the solar neighbourhood was presented. The model is based on the star formation history (SFR), the age velocity dispersion relation (AVR) and a simple chemical enrichment model. The vertical density profiles of the stellar subpopulations are self-consistently calculated in the total gravitational potential of stars including the contribution of the gas component and the dark matter halo. The input parameters are selected and optimized to reproduce the velocity distribution functions $f_j(W)$ for the vertical velocity component $W$ along the main sequence (MS). It turned out that for each SFR($t$) the range of fitting AVRs is very small. On the other hand, the SFR is not well determined by the local kinematics only. Therefore, we discussed in Paper I four different SFRs with similar $\chi^2$ values to cover the range of possible functional shapes (models A, B, C and D). The overall range of best-fitting AVRs is still well restricted. Due to the different age distributions and consequently different relative contributions of the subpopulations as function of scaleheight, the vertical density profiles of MS stars differ significantly at large distances $z$ above the Galactic mid-plane.

A combination of the local model with number densities of MS stars in the solar neighbourhood allows predictions of star counts at high Galactic latitude. Therefore, we can use star counts of large surveys for an independent test of the local model. As a new additional result, we expect to find restrictions on the SFR. As local normalization along the MS, we use Hipparcos stars complemented by the Fourth Catalogue of Nearby Stars (CNS4) at the faint end (Jahreis & Wielen 1997). The model predictions will be compared to the North Galactic Pole (NGP) data of the Sloan Digital Sky Survey (SDSS), which has collected at present the largest and most homogeneous data base comprising about 106 stellar objects in the Milky Way (Gunn et al. 1998; Abazajian et al. 2009). The SDSS photometry is in the $ugriz$ filter system. Despite the enormous wealth of information that the SDSS data base provides in the $ugriz$ filter system, the majority of current observational and theoretical knowledge of resolved stellar populations is based largely on the Johnson–Kron–Cousins $UBVR_C$ photometric system and some other systems such as the Strömgren, DDO, Vilnius and Geneva systems. To overcome this difficulty, we use an empirical transformation of the nearby MS star photometry into the $ugriz$ system (Just & Jahreiss 2008). For each theoretical model, we derive a best-fitting solution for the full Hess diagrams in $(g-r, g)$, which

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quantify the number density distributions in the colour–magnitude diagram (CMD).

The paper is organized as follows. In Section 2 we describe the data selection, in Section 3 important properties of the theoretical models are presented, in Section 4 the local normalization is discussed, in Section 5 we present the best-fitting procedure, in Section 6 we discuss the results and in Section 7 we draw some conclusions.

2 SDSS DATA

The SDSS is an imaging and spectroscopic survey (York et al. 2000) that has since 1998 with its 2.5-m dedicated telescope mapped more than a quarter of the sky (Gunn et al. 1998). Photometric sky coverage of the SDSS Data Release 7 (DR7) amounts to 11 663 deg$^2$, including a 7646 deg$^2$ large contiguous area around the NGP (Adelman-McCarthy et al. 2008; Abazajian et al. 2009).

Smith et al. (2002) defined the $ugriz$ photometric system on 158 standard stars, a subset of $UBVR_c$; $I_c$ standard stars from Landolt (1992), using the USNO-1.0-m telescope. Unfortunately, the photometric system of the 2.5-m SDSS telescope, denoted as $ugriz$, slightly differs from the $ugriz$ one (Abazajian et al. 2003). Moreover, the SDSS standards are too bright and saturate in the 2.5-m SDSS telescope during its normal operational mode. These inconveniences have been resolved by using fainter secondary standards scattered throughout the SDSS survey area and by using simple linear transformation equations between the two photometric sets (Tucker et al. 2006). The nightly photometry obtained by the SDSS 2.5-m telescope can be thus calibrated to the native $ugriz$ system with magnitude zero-points accurate on the AB system to within a few per cent.

Imaging data are produced simultaneously in the five photometric bands, namely $u, g, r, i$ and $z$ (Fukugita et al. 1996; Hogg et al. 2001; Smith et al. 2002; Gunn et al. 2006). The images are automatically processed through specialized pipelines (Lupton, Gunn & Szalay 1999; Lupton et al. 2001; Stoughton et al. 2002; Pier et al. 2003; Tucker et al. 2006) producing corrected images, object catalogues, astrometric solutions, calibrated fluxes and many other data products. SDSS photometry is homogeneous and deep ($r < 22.5$), repeatable to 0.02 mag (Ivezić et al. 2003) and with a zero-point uncertainty of $\sim$0.01–0.02 (Abazajian et al. 2004; Ivezić et al. 2004). In DR7 a homogeneous photometry over the full sky coverage was established at the 1 per cent level in $griz$ and 2 per cent in $u$ in a process called ubercalibration (Padmanabhan et al. 2008).

We have limited our analysis to the stars in the magnitude range $14 \leq g \leq 20.5$. For brighter stars the CCD camera of the SDSS telescope saturates. At the faint end, we wanted to completely avoid the problems in the galaxy–star separation and we set a conservative -magnitude limit for this purpose. The SDSS photometric data contain also quality flags for each object to aid in the selection of ‘good’ measurements. We have carefully analysed the appearance of these photometric flags with respect to the object brightness. We concluded that the problematic flags relate mostly to the stars fainter than our faint magnitude limit. Many of these flags are also tightly correlated with the magnitude measurement error. To make our stellar samples as complete as possible we have finally applied only one ‘cleaning’ criterion, i.e. we have rejected all the measurements with reported magnitude error larger than 0.2 in $g$ or in $r$ filter, which typically accounted for 1 per cent or even less of the total star counts. It is also necessary to mention that the typical magnitude error in the chosen magnitude range is much smaller than the applied magnitude error limit.

We selected the colour range $-0.2 < g - r < 1.2$ with a symmetric bin size of $\pm 0.025$ mag. This range covers the MS down to K dwarfs, where the local normalization is reliable. In order to test the predictions from the vertical structure of the local disc model (Paper I) with the available SDSS photometric data, we have selected a field at the NGP. The field should not reach too low Galactic latitude in order to avoid projection effects and a dependence of the star counts on the radial properties of the disc model. At the same time, the field should be as large as possible in order to reduce the Poisson noise of the star counts. We have chosen a NGP field with Galactic latitudes $b > 80^\circ$ and an area of $A_{\text{NGP}} = 313.36 \text{deg}^2$. In $g$ magnitude, we use a resolution of $\Delta g = 0.01$ mag, but for the fitting procedure we applied a car-box smoothing of $\Delta g = 0.1, 0.2$ and 0.5 mag, where the last value is our standard case. The resolution in colour is $\Delta (g - r) = 0.05$ mag. The Hess diagram of the data is shown in the bottom panel of Figure 4.

In order to avoid confusion of observational errors in the SDSS data and uncertainties of the model predictions, we compare dereddened number counts $Y(g - r, g)$ in the colour – apparent magnitude plane.

3 THE MILKY WAY MODEL

We use the self-consistent local model of the thin and thick discs described in detail in Paper I and add a simple stellar halo component. In this section, we report the necessary features to understand the procedure applied here. The disc model relies on the kinematics of MS stars in the solar neighbourhood measured by volume complete local samples of stars combining Hipparcos stars and faint stars of the CNS4. The properties of the model depend on the total stellar mass density in the solar neighbourhood and only weakly on the adopted initial mass function (IMF). It is independent of star counts at large distances.

The thin-disc model at the solar radius is based on a pair (SFR, AVR) as input functions, which determines the age distribution and the velocity distribution functions of the stellar subpopulations. The vertical density profiles are calculated self-consistently in dynamical equilibrium in the total gravitational potential of stars, gas and dark matter halo. The chemical enrichment is also included for the determination of MS lifetimes, colour indices and luminosities from Padova population synthesis models. For each pair of functions (SFR, AVR), the local disc model provides a unique connection of local star counts and the number density of stars above the Galactic plane with no additional free parameters. In Paper I we compared four different models A, B, C and D which all yield similar best-fitting $\chi^2$ values. The SFR and AVR of these models are plotted in Fig. 1. The density profile of MS stars at a given colour index is characterized by the MS lifetime of the stars. It is composed of a series of density profiles of coeval stars according to the SFR up to the lifetime. The top panel of Fig. 2 shows the normalized density profiles of model A for the relevant colour index range in $g - r$. Bluer thin-disc stars are too bright to be visible in SDSS at high Galactic latitude.

The lower panels of Fig. 2 show the density profiles of models A–D divided by a sech$^2(\chi/2\chi_0)$ profile with exponential scaleheight...
z_t for two colour (MS lifetime) bins. For both lifetimes, we use the corresponding exponential scaleheight of model A, i.e., z_t = 270, 108 pc, respectively. The middle panel is in linear scale, whereas the lower panel for stars with shorter lifetime is in log-scale. The deviations from a sech^2 profile of model A are significant for all lifetimes, because the populations are not isothermal and their spatial distribution is influenced by the gravitational potential of all other components. For younger populations, the deviations are much stronger than for older populations. Between the different models, the differences exceed a factor of 2. Models B–D with a lower fraction of stars older than 10 Gyr require larger velocity dispersions of the old thin-disc populations (see Fig. 1). This leads to shallower density profiles at z > 1 kpc. All profiles are based on a Scalo IMF. The differences at the mid-plane z = 0 can be corrected by adjusting the IMF, which will be done implicitly by fitting the local normalization (see below). In any case, large differences of star counts as function of distance (apparent magnitude) remain.

In each g – r colour bin, the density profile is characterized by the MS lifetime. According to Paper I and Just & Jahreiss (2008), we use for the bins g – r = 0.15, 0.2 and 0.25 the lifetime 1.2, 1.4 and 1.6 Gyr, respectively. In the colour bins g – r = 0.3, 0.35 and 0.4, we include the contribution of turn-off stars and use lifetime ranges 2.4–3.2, 4.0–6.2 and 8.2–12 Gyr, respectively. For all colours g – r ≥ 0.45, the lifetime is larger than 12 Gyr leading to the same density profile.

An isothermal thick-disc component is self-consistently included in the local model. We have shown in Paper I that it can be well fitted by a

\[ \rho_t(z) = \rho_{t,0}\text{sech}^2[z/(\alpha t z_t)] \] (1)

Figure 1. The upper panel shows the SFRs as function of time and the lower panel gives the AVRs of models A–D as function of age (age is running backwards).

Figure 2. The top panel shows normalized density profiles of model A for different colour bins (with the corresponding MS lifetime in parenthesis). The middle panel shows the deviations of the density profiles for models A–D from a \( \rho_0\text{sech}^2(z/2 z_t) \) profile with \( z_t = 270 \) pc for stars with lifetime larger than 12 Gyr \( (g – r > 0.4) \). The lower panel is in log-scale and shows the same for a lifetime of 3 Gyr using \( z_t = 108 \) pc.

The upper panel shows the SFRs as function of time and the middle panel is in linear scale, whereas the deviations from the MS lifetime are both proportional to \( z_t > 0 \) for two colour (MS lifetime) bins. For both lifetimes, we use for the bins g – r = 0.15, 0.2 and 0.25 the lifetime 1.2, 1.4 and 1.6 Gyr, respectively. In the colour bins g – r = 0.3, 0.35 and 0.4, we include the contribution of turn-off stars and use lifetime ranges 2.4–3.2, 4.0–6.2 and 8.2–12 Gyr, respectively. For all colours g – r ≥ 0.45, the lifetime is larger than 12 Gyr leading to the same density profile.

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where \( \alpha_t z_t \) and exponential scaleheight \( z_t \), Since the influence of the thick disc on the total gravitational potential \( \Phi(z) \) is very small, the thick-disc parameters can be varied in a large range with negligible influence on \( \Phi(z) \) and the thin-disc structure. From the Jeans equation, it follows that \( z_t \) and \( \alpha_t^{-1} \) are both proportional to the square of the velocity dispersion \( \sigma_t \) of the thick disc in a given gravitational potential. As a consequence, we find

\[ \alpha_t z_t \approx \text{constant}, \] (2)

which we use to maintain dynamical equilibrium, when changing the thick-disc parameters. For the stellar population of the thick disc, we adopt a simple population with an age of 12 Gyr and metallicity [Fe/H] = −0.7.
We include a simple stellar halo described by a flattened power-law distribution
\[ \rho_h = \rho_{h,0} \left( \frac{R^2 + z^2/b^2}{R_0^2} \right)^{-\alpha_h/2}, \]
with flattening \( b \) and local normalization \( \rho_{h,0} \). For the distance of the Sun to the Galactic Centre, we adopt \( R_0 = 8.0 \) kpc. Since we are looking only to one line of sight and to distances not large compared to \( R_0 \), the parameters \( \alpha_h \) and \( b \) are strongly degenerated. A stronger flattening requires a shallower slope in order to reproduce similar stellar densities at a distance of \( z = 5\text{--}10 \) kpc. One choice to get minimum \( \chi^2 \) values is \( \alpha_h = -3.0 \) and \( b = 0.7 \), which we fix for further investigations.

For the calculation of star counts, we use the density profiles of each component \( v \) at distance \( s \) along the line of sight pointing to the Galactic coordinate position \((R, z)\) determined at the solar position \((R_0, 0)\),
\[ \rho_v(s) = \frac{\rho(R, z)}{\rho_0} \quad \text{with} \quad \rho_0 = \rho(R_0, 0), \]
\[ z = s \sin b, \quad R = \sqrt{R_0^2 - 2R_0 s \cos b \cos l + s^2 \cos^2 b}. \]
For the thin disc, the density profile depends also on the colour \( g - r \). The vertical offset \( z_0 \approx 20 \) pc of the solar position can be easily included but is negligible for high Galactic latitude fields. The density distributions of thin and thick discs can be extended in the radial direction by an exponential profile, if necessary. The normalized density profiles of each component are transformed to number density profiles by multiplying with the local number densities \( n_{v,0}(s,g-r,M_v) \) determined from the Hertzsprung–Russell diagram (HRD) in the solar neighbourhood.

4 LOCAL NORMALIZATION

The Galaxy model described in the last section must be complemented by the local stellar number density \( n_{v,0}(g-r,M_v) \) of each component \( v \). In general, the full HRD of a volume complete sample should be used. Since most of the modelled CMD is dominated to more than 95 per cent by MS stars, we restrict the star count predictions to MS stars including a correction for turn-off stars. Only at the bright red corner of the CMD the K giants of thick disc and halo contribute significantly.

The stellar content in the solar neighbourhood is quantified by the number of MS stars \( N_{25} \) with
\[ N_{25} = \sum_v N_{25,\nu}(g-r,M_v) \quad \text{with} \quad \nu = s, t, h \]
in a sphere of 25-pc radius, with volume \( V_{25} = 65,450 \text{ pc}^3 \). Here the contributions of thin and thick discs and halo are denoted by the indices \( s, t \) and \( h \), respectively. In Just & Jahreiss (2008), absolute magnitudes and colours of the mean MS in the \( ugriz \) filter system were determined based on different transformation formulae available in the literature. We use that mean MS for the thin disc, because the influence of thick disc and halo stars in the solar neighbourhood is negligible.

In Fig. 3 we show \( M_v(g-r) \) for the thin disc based on the transformation J06 (Jordi, Grebel & Ammon 2006). The black crosses denote the MS stars of the CNS4 which are the basis for the determination of the mean MS. Each red full circle along the MS denotes a data point, where \( N_{25,\nu}(g-r,M_v) \) is a free fitting parameter. The locus of the MS corresponds to a slightly subsolar mean metallicity and is consistent with the fiducial sequences of the open clusters M 67 ([Fe/H]=0) and NGC 2420 ([Fe/H]=-0.37) of An et al. (2008). Since the mean metallicity of the thin-disc population decreases with increasing distance from the mid-plane, a correction to the mean luminosity in each colour bin is necessary. We use a simple analytic approximation
\[ \Delta M_v([\text{Fe/H}](z)) = 0.4 \left( \frac{z}{z+600 \text{ pc}} \right)^2 \text{ mag}, \]
which is consistent with the determination of Ivezić et al. (2008) for the \( M_v \) luminosity of thin-disc MS stars.

For the thick disc, we use an interpolated MS with \( N_{25,t}(g-r,M_v) \) corresponding to an old Population I with metallicity \([\text{Fe/H}]=-0.7\) (green open circles in Fig. 3), where we used the fiducial sequence of M13 (An et al. 2008) with an extrapolation to the faint end.

In the turnover regime of F stars, we split the MS luminosity into two or three points with different luminosities in order to include the brighter turn-off stars. The total number of thin and thick discs and halo should add up to the observed total number of MS stars \( N_{25}(g-r) \), which were also determined in Just & Jahreiss (2008) – see histogram in Fig. 8 below. We treat the local number densities of the components as free fitting parameters and then compare the results of the best-fitting model to the observed stellar content in the solar neighbourhood.

The predicted Hess diagrams, i.e. the star counts in the CMD \( N(g-r,g) \) per \( d(g-r) \) mag bin in a cone with cross-section \( d\ell d\cos b \) pointing to the Galactic coordinate position \((l,b)\) are calculated by adding up the contributions of each component along the line of sight. The contribution of each component \( v \) by a volume element at distance \( s \) using \( ds/s = 0.2 \log(10) \text{ d}g \), is given by
\[ \Delta N_v(g-r,g) = N_{25,\nu}(g-r,M_v) \rho_v(s)^2 ds d\ell d\cos b \]
\[ = \frac{N_{25,\nu}(g-r,M_v) \rho_v(s)^2}{2 V_{25}} \frac{4\pi}{41 253 \text{ deg}^{-2}} \frac{4\pi}{41 253 \text{ deg}^{-2}} \]
\[ g = M_v + \Delta M_v + 5 \log_{10} s - 5. \]
Adding up the contributions of all components along the line of sight yields the predicted number \( N(g-r,g) \) in each bin of the Hess diagram. In the next step, the values are smoothed in the same way as the SDSS data. The star numbers in the Hess diagrams are normalized to 1 deg\(^2\) at the sky (second line of equation 8). In
(g − r, g) we need to distinguish between the resolution d (g − r) dg, the smoothing Δg and the normalization. All Hess diagrams are normalized to 0.1 mag in (g − r) and 1.0 mag in g.

5 FITTING PROCEDURE

The best-fitting procedures in the colour bins are independent of each other. In each colour bin the contribution of thin and thick discs and stellar halo to the local star counts in V25 are quantified by the free parameters N25,t, N25,h, and N25,b, respectively. The (g − r) arguments are dropped here for simplicity. We use the non-linear Levenberg–Marquard algorithm (Press et al. 1992) to minimize in each colour bin i,

\[ \chi^2_i = \sum_{s_i} \frac{(\log Y_{ij} - \log N_i)^2}{\sigma_{ij}^2} \quad \text{with} \quad \sigma_{ij}^2 = \frac{1}{Y_{ij}} \]

in log-scale with Poisson noise for the statistical weights \( \sigma_{ij} \). In linear scale, the regions with maximum density would dominate the \( \chi^2 \) value resulting in an unsatisfactory overall fit. The \( \chi^2 \) are derived from normalized values and have to be corrected by the area \( A_{\text{NGP}} \) in units of deg\(^2\) and the bin size \( d(g − r)\Delta g \) in units of 0.1 mag \( \times \) 1 mag. Due to the smoothing in g over n data points, \( \chi^2/n \) is the average over all subsets of independent bins with \( \chi^2/k \), shifted by \( n \times d g \). In each colour bin, the degrees of freedom \( \text{dof} \) depend on the fit regime in g, the bin size \( \Delta g \) and the number of fitting parameters. We derive the mean reduced \( \chi^2 \) by adding up the normalized \( \chi^2 \) values,

\[ \chi^2 = \frac{A_{\text{NGP}} d(g − r) \Delta g}{1 \text{deg}^2 \times 0.1 \text{mag} \times 1 \text{mag}} \sum_{i} \chi^2_{ij} \frac{n \times \text{dof}_i}{n \times \text{dof}}. \]

In order to get reliable fits, we restrict the colour range of the fitting regime. For the thin disc, we fix \( N_{25,t}(g − r, M_g) \) for \( g − r < 0.35 \), since the main contribution falls outside the bright limit \( g = 14 \) mag. For the halo, we extrapolate \( N_{25,h}(g − r, M_g) \) for \( g − r > 1.0 \), since the main contribution falls outside the faint limit \( g = 20.5 \) mag. Additionally, the part of the CMD, which we use to minimize \( \chi^2 \), is restricted dependent on the aspect of investigation.

6 RESULTS

We discuss first the construction of the Hess diagrams and compare the best-fitting results of models A–D. Then we investigate the dependence of the fitting result on the fitting regime and the MS properties for model A.

6.1 Hess diagram fitting

The bottom panel of Fig. 4 shows the Hess diagram of the NGP data with a total of 276 180 stellar objects. Star number densities normalized to 0.1 mag \( \times \) 1.0 mag in \( (g − r, g) \) and 1 deg\(^2\) are colour coded in log-scale ranging from 1 \( \ldots \) 100 (purple \ldots red). The plots are smoothed in colour and magnitude in boxes in steps of 0.01 mag with box size \( \Delta(g − r) \times \Delta g = 0.05 \times 0.5 \) mag (in the fitting procedure, the colour bins are not smoothed). Most of the Hess diagram is strongly dominated by MS stars. Exceptions are turn-off stars in the colour range \( g − r \approx 0.3–0.4 \) and red giants of thick disc and/or halo in the upper right corner. The nature of the faint very blue objects with \( g − r < 0.15 \) is unclear (misidentified extragalactic sources, White Dwarfs, halo BHB stars, \ldots).

The three top panels of Fig. 4 show the contributions of the different components to the full Hess diagram for model A. There is a significant overlap of thin/thick disc, as well as thick disc/halo.

Therefore the stellar halo must be included to determine the thick-disc properties and the thick disc is needed to fix the thin-disc parameters. The second last plot shows the sum of all components for model A.

Figure 4. Top to bottom: contributions of thin and thick discs and stellar halo, and full Hess diagram of model A-R06-05 (see Table 1) followed by the NGP data. Colour coding ranges from 1 (purple) to 100 deg\(^{-2}\) (red) in log-scale.
Table 1. Parameters of the best-fitting solutions.

| Model       | Δg [mag] | χ² | N_{25} | z₁ [pc] | α₁ | σ₁ [km s⁻¹] |
|-------------|----------|----|--------|---------|----|-------------|
| A-J06-05-r  | 0.5      | 2.59 | 739   | 800     | 1.16 | 45.3        |
| B-J06-05-r  | 0.5      | 3.75 | 748   | 880     | 1.07 | 47.4        |
| C-J06-05-r  | 0.5      | 3.84 | 811   | 885     | 1.23 | 51.2        |
| D-J06-05-r  | 0.5      | 5.64 | 709   | 930     | 0.99 | 48.0        |
| A-R06-05-s  | 0.5      | 2.61 | 715   | 800     | 1.16 | 45.3        |
| A-C08-05-r  | 0.5      | 2.60 | 733   | 800     | 1.16 | 45.3        |
| A-J06-05    | 0.5      | 4.50 | 762   | 800     | 1.16 | 45.3        |
| A-R06-05    | 0.5      | 4.31 | 710   | 800     | 1.16 | 45.3        |
| A-C08-05    | 0.5      | 5.09 | 752   | 800     | 1.16 | 45.3        |
| A-C08-02    | 0.2      | 2.53 | 750   | 800     | 1.16 | 45.3        |
| A-C08-01    | 0.1      | 1.84 | 753   | 800     | 1.16 | 45.3        |

Note: the models are named by the disc models A–D followed by the local normalization and the smoothing Δg. For the first block ‘r’ is added for the reduced fit regime. The observed number of stars ranges from N_{25} = 726 (with R06) to N_{25} = 770 (with C08).

For the comparison of disc models A–D, we restrict the colour range to 0.5 ≤ g − r ≤ 1.2 mag in order to avoid the F turn-off regime, which may be improperly modelled and thus dominate the total χ² value. The magnitude range is 14.5 ≤ g ≤ 20.5 mag safely excluding a contribution of saturated stars brighter than 14.25 mag in the brightest bins and a significant contamination by misidentified extragalactic sources at the faint end. In all models, the thick-disc parameters are varied according to equation (2) to find the minimum χ² value. The parameters of the best fits are collected in the first four rows of Table 1. In the model names, there are tokens for the local normalization (-J06 for Jordi et al. 2006, -R06 for Rodgers et al. 2006, -C08 for Chonis & Gaskell 2008) and the smoothing Δg (−01, −02, −05, respectively) attached. For the first block ‘r’ is attached for the reduced fit regime.

### 6.2 The star formation history

The thin-disc models A–D have very different SFRs leading to different star count predictions. In each model, we optimized the thick-disc scaleheight and power-law index according to equation (2) starting from a self-consistent isothermal model. In Fig. 5, the Hess diagrams of the models are presented in the left-hand panels (A–D, top to bottom). The middle panels show the relative differences (data-model)/model, where the coloured region covers the range −20 per cent (purple) to +20 per cent (red) in linear scale. Lower values are represented by black colour and higher values by white colour. The right-hand panels show the contribution of each bin to χ² (equation 9) in log-scale.

All models fit most of the Hess diagram better than ±20 per cent. The local normalization of the blue part with g − r < 0.5, which is not included in the best fit, is adapted by hand to model A and not varied for the other models. In the bright red triangular region, there is a significant fraction of stars missing in the models as expected, since the SDSS data contain red giants of thick disc and halo, which cover that colour–magnitude regime.

In Fig. 6 the contributions of thin and thick discs and stellar halo for model A are shown in the colour bin g − r = 0.7 as a typical example. The maximum contribution of the thin disc is at g = 16.5 mag corresponding to z = 900 pc and of the thick disc at g = 19 mag corresponding to z = 2500 pc. These large distances demonstrate that it is very important to construct models, which hold also for the outer profiles of the components.

Model A-J06-05-r is the best model with a reduced χ² = 2.59 (see Table 1). In model A the deviations of data and model are less than ±10 per cent over the full colour range and in the magnitude regime, where the MS stars of thin and thick discs or halo dominate. The χ² distribution in Fig. 5 shows a strong noise component and the contribution by systematic deviations of the predicted density profiles from the real profiles. In the top panel of Fig. 7 the reduced χ² values in each colour bin are quantified. The contributions to the total χ² are fairly uniform.

The main feature of the systematic discrepancies is the shallow valley in the transition of thin and thick discs in the data, which is not reproduced by the model. This is a sign of too many spurious stars in the models, which is possible for the thin disc. A continuous transition between the thin- and thick-disc density at large heights leading to a deficit closer to the plane. A more detailed look at the χ² distributions shows that model A is superior in a wide colour range (Fig. 7).

Since we used the local number densities of all components as free fitting parameters, a comparison of the fitting results and the observed star counts in the solar neighbourhood is a crucial test of the models. In Fig. 8 the histogram with error bars shows the data from the CNS4. The full (blue) circles are the results of model A–D (from top to bottom). In all cases, there is a reasonable match and also the total number of MS stars agree (see Table 1). The observed number of MS stars range from N_{25} = 726 (with R06) to N_{25} = 770 (with C08). Additionally the contribution of thin and thick discs and halo is plotted separately in Fig. 8 (enhanced by some factor to make it visible) demonstrating that the local number densities of all components are smooth functions of (g − r). A more detailed discussion is given in Section 6.3.

As a bottom line, we find that the local disc model A is fully consistent with the NGP star count data of SDSS. Model A with a relatively large fraction of stars older than 8 Gyr and a correspondingly small maximum vertical velocity dispersion of σ_e = 25 km s⁻¹ in the thin disc and with σ_i = 45.3 km s⁻¹ can be excluded, because it would add more stars to the transition regime.

In models B, C and D, the fits in the magnitude range of dominating thin-disc contribution are significantly worse, whereas the thick-disc parameters can be adjusted to reach a similar good fit at fainter magnitudes as in model A. This is also quantified in the χ² values (see Table 1). Model D with the small disc age and strongly declining SFR is the worst model with χ² = 5.64. Models B and C are comparable with χ² = 3.75 and 3.84, respectively. All three models overestimate the thin-disc density at large heights leading to a deficit closer to the plane. A more detailed look on the χ² distributions shows that model A is superior in a wide colour range (Fig. 7).

### 6.3 Filter transformations

In Fig. 9 the local MS is shown for three different transformations (J06 for Jordi et al. 2006, R06 for Rodgers et al. 2006 and C08 for Chonis & Gaskell 2008). There are small systematic deviations in M_z (g − r) which may influence the best-fitting parameters of the model. The histograms in Fig. 11 show the local star counts for R06 and C08. Additionally to the differences in the absolute magnitudes for thin and thick discs, the fraction of thin disc turn-off stars is reduced in C08 and is set to zero in R06 in order to test their influence on the χ² values and number counts.

We present the comparison using the different local MS for model A. The effects in models B, C and D are very similar. First we derived...
Figure 5. Comparison of models A, B, C, D-J06-05-r (top to bottom). From left to right: Hess diagram (same colour coding as in Fig. 4), relative difference to SDSS data ranging from $-0.2$ (purple) to $+0.2$ (red) in linear scale, $\chi^2_{ij}$ distribution in log-scale.

Figure 6. Vertical cut of the Hess diagram at $g-r = 0.7$ mag. The SDSS data are compared to the predictions of model A-J06-05-r. $N$ is the number of stars per bin $d(g-r)\Delta g = 0.05 \times 0.5$ mag for the full NGP field. The contributions of thin and thick discs and stellar halo are shown separately.

The best-fitting models for the reduced colour–magnitude regime as before in A-J06-05-r for models A-R06-05-r and A-C08-05-r (see Table 1). The total $\chi^2$ values and local normalizations of the three models are very similar.

Then we extended the fit regime to the full CMD and recalculated the local normalizations and the $\chi^2$ values. Fig. 10 shows the relative deviations of data and models. The patterns of the deviations differ slightly in the regime, where the thin disc dominates. The corresponding $\chi^2_{ij}$ values in the colour bins are shown in the lower panel of Fig. 7. For the extended fit regime, there are small differences arising mainly from the turn-off stars. In the colour bin $g-r = 0.4$, the fraction of turn-off stars is 80 per cent (J06), 0 per cent (R06) and 60 per cent (C08), respectively. Model A-R06-05 without turn-off stars gives the best result. A consequence is that a larger fraction of stars are allocated to the thick disc instead of the thin disc in model A-R06-05 (see Fig. 11). At $g-r = 0.35$, model A-J06-05 with 55 per cent turn-off stars gives the best result and for $g-r \leq 0.3$, the turn-off stars are too bright to contribute to the fit. At the red end, $\chi^2_{ij}$ is influenced by the missing giants. Therefore, it is not useful to discuss here the differences of the fits in more detail.
The local normalizations do not differ significantly (see Fig. 11). The differences in the histograms for the transformations R06 and C08 show the possible variations in the local star counts due to noise and the effect of a different slope in the MS leading to a shift of stars in colour. All transformations are consistent with the local star count data. Only in the transition of K to M dwarfs the large number of thin disc stars in \( g - r = 1.2 \) may be a hint that the MS turning point is relatively blue as in C08.

6.4 Smoothing

Smoothing of the data has two main effects. On the one hand statistical noise is reduced. On the other hand, physical features in the data are smeared out and may be shifted systematically. Fig. 12 shows the NGP data and corresponding models A-C08-05, 02, 01 for different smoothing lengths \( \Delta g = 0.5, 0.2, 0.1 \) mag, respectively. The \( \chi^2 \) values are decreasing due to the increasing degrees of freedom (see Table 1 and Fig. 7). The local normalizations are statistically indistinguishable. Only in the colour bin \( g - r = 0.4 \), a few percent of thin-disc stars are shifted to the thick disc at higher resolution reducing the contribution to \( \chi^2 \) by the bright magnitude bins dramatically.

7 CONCLUSIONS

We have used four different models with different SFRs of the thin disc, which fit the local kinematics of MS stars (Paper I) and compared the star count predictions for the NGP field with \( b > 80^\circ \) of the SDSS. For the thin disc, we applied the absolute magnitudes of the local MS as determined in Just & Jahreiss (2008). A self-consistent isothermal thick disc and a simple stellar halo model was added to complete the contributions of MS stars to the star counts. We used the local normalizations of thin and thick discs and stellar halo (×50) separately.

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Monthly Notices of the Royal Astronomical Society © 2010 RAS

Figure 7. Reduced \( \chi^2 \) values in each colour bin. The top panel shows the comparison of disc models A–D. The bottom panel shows the dependence on the filter transformation and on the smoothing in \( g \).

Figure 8. Local normalizations for disc models A–D (top to bottom). The histogram shows the data from the CNS4 and the full blue circles are the best-fitting values. The contributions of thin and thick discs (×5) and stellar halo (×50) are shown separately.
the self-consistent disc models in the range of $|z| < 1$ kpc. The derived local normalizations are consistent with the star count data of the CNS4 in the solar neighbourhood. The $\chi^2$ analysis shows that model A is clearly preferred with systematic deviations of a few per cent only. The SFR of model A is characterized by a maximum at an age of 10 Gyr and a decline by a factor of 4 to the present-day value of $1.4 M_\odot$ pc$^{-2}$ Gyr$^{-1}$. In the thin disc, the present-day fraction of stars older than 8 Gyr is with 54 per cent significantly higher than for models B, C and D (Paper I). Especially model C with a constant SFR, and model D with a disc age of 10 Gyr can be ruled out.

The thick disc can be modelled very well by an isothermal simple stellar population. The density profile can be approximated by a sech$^2(\alpha z/h)$ function. For model A we find a power-law index of $\alpha_t = 1.16$ in between an exponential profile and a sech$^2$ profile (the latter corresponds to an isolated isothermal disc). The exponential scaleheight is $h_t = 800$ pc corresponding to a vertical velocity dispersion of $\sigma_z = 45.3$ km s$^{-1}$. About 6 per cent of the stars in the solar neighbourhood are thick-disc stars. In Jurić et al. (2008), the stellar density distribution in the Milky Way based on SDSS star counts was fitted by exponential thin-disc and thick-disc profiles. The result is a much larger thick-disc scaleheight, which balances the flattening of the profile at low $z$.

The results do not depend significantly on the filter transformations used for the local MS and on the smoothing of the data in luminosity. For the future, an extension of the model to include turn-off stars in more detail and the contribution of giants as well as a higher resolution in colour at the blue end would be very useful.

We also plan to apply the model to lower Galactic latitudes in order to determine radial scalelengths of thin and thick discs as well as radial gradients in the stellar populations.

**ACKNOWLEDGMENTS**

SG is supported by a grant of the China Scholarship Council (CSC). SV was supported by an Alexander von Humboldt Fellowship (website: http://www.humboldt-foundation.de).

Funding for the SDSS and SDSS-II has been provided by the Alfred P. Sloan Foundation, the Participating Institutions, the National Science Foundation, the U.S. Department of Energy, the National Aeronautics and Space Administration, the Japanese Monbukagakusho, the Max Planck Society and the Higher Education Funding Council for England. The SDSS website is http://www.sdss.org/.
Figure 12. Decreasing smoothing $\Delta g = 0.5, 0.2, 0.1$ (top to bottom). NGP data (left-hand panel), models A-C08-05, 02, 01 (middle panel) and relative differences (right-hand panel). Same colour coding as in Fig. 5.

The SDSS is managed by the Astrophysical Research Consortium for the Participating Institutions. The Participating Institutions are the American Museum of Natural History, Astrophysical Institute Potsdam, University of Basel, University of Cambridge, Case Western Reserve University, University of Chicago, Drexel University, Fermilab, the Institute for Advanced Study, the Japan Participation Group, Johns Hopkins University, the Joint Institute for Nuclear Astrophysics, the Kavli Institute for Particle Astrophysics and Cosmology, the Korean Scientist Group, the Chinese Academy of Sciences (LAMOST), Los Alamos National Laboratory, the Max-Planck-Institute for Astronomy (MPIA), the Max-Planck-Institute for Astrophysics (MPA), New Mexico State University, Ohio State University, University of Pittsburgh, University of Portsmouth, Princeton University, the United States Naval Observatory and the University of Washington.

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