The rationality about the assumption that the signal and decoy states are indistinguishable in decoy-state quantum key distribution

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Decoy-state quantum key distribution (QKD) has become the most efficient method to resist the photon-number-splitting (PNS) attack and estimate the secure key rate. The decoy-state method has many assumptions, among which a critical one is that an eavesdropper (Eve) cannot distinguish between the signal and decoy states. However, a rigorous proof of the rationality about this assumption is not yet available so far. In fact, due to the difference of photon-number probability distribution between the signal and decoy states, Eve is able to distinguish the two states with a certain probability. In this work, we adopt the Bayesian decision to distinguish the signal and decoy states in one-decoy-state QKD, and perform different PNS attack strategies for the two states according to the previous decision. The numerical simulations indicate that the attack effect is not obvious or even failed. Thus, it is reasonable to assume that the signal and decoy states are indistinguishable in decoy-state QKD. In addition, we also provide the method to set the intensities of signal and decoy states properly, which can not only reduce the preparation cost and improve the communication efficiency, but also avoid the attack from Eve using the intensity difference between the signal and decoy states.

I. INTRODUCTION

Quantum key distribution (QKD) [1–7] is a technique which can provide the information-theoretical security for two remote communication parties (Alice and Bob). The information-theoretical security is guaranteed by the law of quantum mechanics [8–12]. The original BB84-QKD protocol [1] requires a perfect single-photon source. The WCS source sometimes emits pulses that contain more than one photon and an eavesdropper (Eve) can obtain the key through multiphoton pulses without noticed by Alice and Bob, which is called the photon-number-splitting (PNS) attack [13–18]. The PNS attack results in an extremely low secure key rate and the maximum secure transmission distance is less than 10 km [14]. Fortunately, the decoy-state method [19–21] proposed later can resist the PNS attack very well and significantly improve the secure key rate and the maximum secure transmission distance.

In decoy-state QKD, Alice randomly sends either signal state or decoy state to Bob, which is from the WCS source with different intensity. Most importantly, Alice and Bob assume that Eve cannot distinguish between the signal and decoy states, which is a crucial assumption in decoy-state method. Meanwhile, this assumption is also applied to estimate the secure key rate in decoy-state QKD [21]. However, a rigorous proof about this assumption is not yet available so far. In fact, just as Hwang who first proposed the decoy-state method said, “The larger the number of photons of a given pulse is, the more probable it is that the pulse is from the decoy source, by the Bayes’s law and the property of the Poissonian distribution” [19]. By the way, the intensity of decoy state is greater than that of signal state in Ref. [19]. In addition, Wang also mentioned that, “Eve has nonnegligibly small probability to treat the pulses from different classes a little bit differently, even though the pulses have the same state” [20]. Indeed, there is a difference between signal state and decoy state for Eve. Specifically, due to the intensity difference between the signal and decoy states, the photon-number probability distributions of the two states are different. Based on this, Eve is able to distinguish between the signal and decoy states with a certain probability. Once Eve has distinguished the signal and decoy states, she can perform different PNS attack strategies for the two states. Furthermore, if Eve can preserve the measurement statistics of signal and decoy states, she can obtain the key without noticed by Alice and Bob.

In this article, we proved the assumption that the signal and decoy states in decoy-state QKD are indistinguishable is reasonable. We first adopt the Bayesian decision to distinguish the signal and decoy states based on the difference of photon-number probability distribution caused by different intensities. Then we perform different PNS attack strategies for the signal and decoy states according to the previous decision. Note that the attack result can be used to judge the rationality of the assumption. Specifically, if the attack effect is obvious, it means that the assumption is unreasonable. Otherwise, it means that the assumption is reasonable. Finally, the numerical simulations indicate that the attack effect is not obvious or even failed. That is, it is reasonable to assume that the signal and decoy states are indistinguish-
able in decoy-state QKD. In addition, we also provide the method to set the intensities of signal and decoy states properly, which can not only reduce the preparation cost and improve the communication efficiency, but also avoid the attack using the intensity difference between the signal and decoy states.

The structure of this paper is organized as follows. In Sec. II, we propose a scheme to justify the assumption that the signal and decoy states are indistinguishable in decoy-state QKD. We first briefly review the one-decoy-state QKD protocol and its crucial assumptions. Then we describe Bayesian decision to distinguish between signal and decoy states, and the PNS attack based on the previous decision. In Sec. III, numerical simulations and the method to set the intensities of signal and decoy states properly are shown. We conclude the paper in Sec. IV.

II. THE SCHEME

The overall idea of our scheme mainly includes three phases, as shown in Fig. 1. For the first phase, Alice and Bob perform the decoy-state QKD protocols together. For simplicity, we adopt one-decoy-state QKD to illustrate the rationality of the assumption that the signal and decoy states are indistinguishable. For the second phase, Eve intercepts the quantum state sent by Alice to Bob and uses quantum nondemolition (QND) [22–28] measurement to obtain the photon number of the intercepted state. Then Eve adopts Bayesian decision to distinguish between the signal and decoy states. In the last phase, Eve performs different PNS attack strategies for the signal and decoy states respectively based on the previous decision. Finally, we can determine the rationality of the assumption according to the attack result.

![FIG. 1. Schematic diagram of the QND + Bayesian decision + PNS attack on a decoy-state QKD system.](Image)

A. One-decoy-state QKD

In one-decoy-state QKD. Alice randomly (e.g. with equal probability) emits either signal state (s) or decoy state (d). The intensities of the signal and decoy states are \( \mu \) and \( \nu \), respectively. Without loss of generality, we assume \( 1 > \mu > \nu > 0 \). Due to using the WCS sources, the photon numbers of the two states both follow the Poisson distribution. The probabilities of \( i \)-photon state from signal state and decoy state are, respectively,

\[
P_s^i = \frac{e^{-\mu}\mu^i}{i!}, \quad P_d^i = \frac{e^{-\nu}\nu^i}{i!}.
\]

The total gains and quantum bit error rates (QBERs) of signal state and decoy state can be written as, respectively,

\[
Q_\mu = \sum_{i=0}^{\infty} P_s^i Y_s^i, \quad Q_\nu = \sum_{i=0}^{\infty} P_d^i Y_d^i.
\]

\[
Q_\mu E_\mu = \sum_{i=0}^{\infty} P_s^i Y_s^i e_s^i, \quad Q_\nu E_\nu = \sum_{i=0}^{\infty} P_d^i Y_d^i e_d^i.
\]

Here \( Y_s^i \) and \( Y_d^i \) are the yields of \( i \)-photon state from signal state and decoy state, respectively. \( e_s^i \) and \( e_d^i \) are the error rates of \( i \)-photon state from signal state and decoy state, respectively.

In decoy-state QKD [30] with the WCS source, the lower bound of the key rate can be written as

\[
R^l = q \left( -Q_\mu f(E_\mu)H_2(E_\mu) + Y_1^s \mu e^{-\mu} [1 - H_2(e_1^s)] \right).
\]

Here \( q = 1/2 \) represents the efficiency of basis alignment for the BB84 protocol (if one uses the efficient BB84 protocol [31], \( q \approx 1 \)). \( f(x) \) is the bidirectional error correction efficiency, normally \( f(x) \geq 1 \) with Shannon limit \( f(x) = 1 \). \( H_2(x) = -x\log_2(x) - (1-x)\log_2(1-x) \) is the binary Shannon information entropy. \( Y_1^s \) and \( e_1^s \) are the yield and error rate of single-photon signal state, respectively, which can be estimated by decoy-state method. According to the postprocessing scheme presented in the one-decoy-state QKD [30], we can obtain the lower bound of \( Y_1^s \) and the upper bound of \( e_1^s \), respectively,

\[
Y_1^s \geq \frac{\mu}{\mu \nu - \nu^2} (Q_\nu e^{-\nu} - Q_\mu e^{-\mu} \frac{\nu^2}{\mu^2}) - E_\mu Q_\mu e^{\mu} \mu^2 - \nu^2 ) / e_0 \mu^2,
\]

\[
e_1^s \leq \frac{E_\mu Q_\mu e^{\mu}}{Y_1^s}.
\]

In the normal quantum channel, the gains and error rates of signal state and decoy state are, respectively,

\[
Q_\mu = 1 - e^{-\mu}, \quad Q_\nu = 1 - e^{-\nu},
\]

and

\[
E_\mu Q_\mu = e_0^d Y_0^s + e_d (1 - e^{-\mu}),
\]

\[
E_\nu Q_\nu = e_0^d Y_0^d + e_d (1 - e^{-\nu}).
\]
Here $c_0^e = c_0^d = 1/2$ is the dark count rate. $c_d$ is the probability that a photon hits the erroneous detector, characterizing the alignment and stability of the optical system. $\eta$ represents the overall efficiency between Alice and Bob, and can be written as

$$\eta = \eta_{Bob} 10^{-\alpha L/10},$$

where $\eta_{Bob}$ is the transmittance on Bob’s side including the internal transmittance of optical components and detector efficiency, and $\alpha$ is the loss coefficient measured in dB/km and $L$ is the length of the channel in km.

Most importantly, the decoy-state method assumes that Eve cannot distinguish signal state and decoy state, and has the following equations,

$$Y_i^* = Y_i^d, \quad e_i^* = e_i^d.$$

In fact, due to the intensity difference between the signal and decoy states, the photon-number probability distributions of the two states are different. Based on this, Eve is able to distinguish between the signal and decoy states with a certain probability.

### B. Bayesian decision

Suppose that a sample can only come from two classes, and some characteristics of the sample are known. Under this condition, one determines that the sample comes from the class whose conditional probability is larger, which is called the Bayesian decision or Maximum a posteriori (MAP) estimation.

In our scheme, Alice equally sends signal state ($s$) or decoy state ($d$) to Bob. Eve intercepts the quantum state and uses a QND measurement to get the photon number of the intercepted state. According to the Bayesian decision theory, given the photon number of the intercepted state, we determine that it comes from the state whose conditional probability is larger between signal state and decoy state. The random variable $M$ denotes the source of the intercepted state, $M \in \{s,d\}$. The random variable $N$ denotes the photon number of the intercepted state, $N \in \{0,1,2,3,\ldots\}$. Given the $i$-photon state, the conditional probability of the intercepted state from signal state and decoy state are, respectively,

$$P(M = s | N = i) = \frac{P(M = s, N = i)}{P(N = i)} = \frac{P(M = s)P(N = i | M = s)}{P(N = i)},$$

$$P(M = d | N = i) = \frac{P(M = d, N = i)}{P(N = i)} = \frac{P(M = d)P(N = i | M = d)}{P(N = i)}.$$  \hspace{1cm} (10)

Because the state emitted by Alice comes from signal state or decoy state equally, we have $P(M = s) = P(M = d) = 1/2$. In addition, the denominators of Eqs. (10) and (11) are the same. Thus, if we want to compare $P(M = s | N = i)$ and $P(M = d | N = i)$, we only need to compare $P(N = i | M = s)$ and $P(N = i | M = d)$. In fact, $P(N = i | M = s)$ is just $P_i^s$, and $P(N = i | M = d)$ is just $P_i^d$. Given the photon number $i$, the intensity of signal state $\mu$ and the intensity of decoy state $\nu$, $P_i^s$ and $P_i^d$ are easy to calculate by Eq. (1). Therefore, Eve only needs to know the photon number of the intercepted quantum state and the intensities of signal and decoy states. Then Eve is able to determine whether the quantum state is from signal state or decoy state according to the Bayesian decision.

Of course, the results of Bayesian decision are sometimes correct and sometimes wrong. Define the conditional probability $P(s | s)$ is the probability that Eve guesses the state is a signal state when Alice sends a signal state, the conditional probability $P(d | s)$ is the probability that Eve guesses the state is a decoy state when Alice sends a signal state, the conditional probability $P(d | d)$ is the probability that Eve guesses the state is a decoy state when Alice sends a decoy state and the conditional probability $P(s | d)$ is the probability that Eve guesses the state is a signal state when Alice sends a decoy state. According to the Bayesian decision theory, we have

$$P(s | s) = \sum_{i \in N, P_i^s > P_i^d} P_i^s, \quad P(d | s) = \sum_{i \in N, P_i^s > P_i^d} P_i^d,$$

$$P(d | d) = \sum_{i \in N, P_i^d > P_i^s} P_i^d, \quad P(s | d) = \sum_{i \in N, P_i^d > P_i^s} P_i^s.$$ \hspace{1cm} (12)

Since $1 > \mu > \nu > 0$, then

$$P(d | s) = P_i^s, \quad P(s | s) = P_i^s + P_2^s + P_3^s + \ldots,$$

$$P(d | d) = P_0^d, \quad P(s | d) = P_1^d + P_2^d + P_3^d + \ldots.$$ \hspace{1cm} (13)

The relationship between $P_i^s$ and $P_i^d$ is presented in Appendix A.

### C. PNS attack

In the previous Sec. II B, Eve determines whether the intercepted quantum state is a signal state or a decoy state through Bayesian decision. According to the previous decision, Eve executes different PNS attack strategies for signal state and decoy state respectively. Define $Z_i^\mu$ and $Z_i^\nu$ to be the yields of the $i$-photon state Eve sets, conditioned on the Bayesian decision, as listed in the last row in Table I. Specifically, $Z_0^\mu = Z_0^\nu = 0$ which we assume for the remaining analysis. This is because an vacuum pulse contains no information to Eve. If Eve forwards any photon to Bob when she gets a vacuum pulse, she may introduce errors. Thus, the yields $Y_i^s$ and $Y_i^d$, from Bob’s point of view, are composed of two parts as listed in Table I,

$$Y_i^s = P(s | s) Z_i^\mu + P(d | s) Z_i^\nu,$$

$$Y_i^d = P(s | d) Z_i^\mu + P(d | d) Z_i^\nu.$$ \hspace{1cm} (14)
TABLE I. Probabilities of Eve’s guess, conditioned on different intensity states sent by Alice, and the yields for Eve’s decision and photon numbers $i$.

|       | Bayesian decision |       |       |
|-------|-------------------|-------|-------|
|       | Signal state      | Decoy state |
| Signal state | $P(s|s)$          | $P(d|s)$ |
| Decoy state | $P(s|d)$          | $P(d|d)$ |
| Yields  | $Z_i^\mu$         | $Z_i^\nu$ |

Then taking Eqs. (1) and (14) into Eq. (2), we have

$$Q_\mu = \sum_{i=1}^{\infty} [P(s|s)Z_i^\mu + P(d|s)Z_i^\nu]e^{-\mu \frac{H_i}{t!}},$$

$$Q_\nu = \sum_{i=1}^{\infty} [P(s|d)Z_i^\mu + P(d|d)Z_i^\nu]e^{-\nu \frac{H_i}{t!}}.$$  \hspace{1cm} (15)

In order to avoid being noticed by Alice and Bob, Eve needs to preserve the detection statistics. In other words, Eq. (6) and Eq. (15) are satisfied simultaneously,

$$Q_\mu = 1 - e^{-\eta_\mu} = \sum_{i=1}^{\infty} [P(s|s)Z_i^\mu + P(d|s)Z_i^\nu]e^{-\mu \frac{H_i}{t!}},$$

$$Q_\nu = 1 - e^{-\eta_\nu} = \sum_{i=1}^{\infty} [P(s|d)Z_i^\mu + P(d|d)Z_i^\nu]e^{-\nu \frac{H_i}{t!}}.$$ \hspace{1cm} (16)

In the decoy-state postprocessing [30], the secure key only can be derived from the single-photon component. The upper bound of the key rate under PNS attack [32] is given by

$$R^u = Y_1^s e^{-\eta\mu} = [P(s|s)Z_1^\mu + P(d|s)Z_1^\nu]e^{-\mu \frac{H_1}{t!}}.$$ \hspace{1cm} (17)

In particular, there is a criteria for the success ability of PNS attack in Ref. [29], that is,

$$R^l > R^u,$$ \hspace{1cm} (18)

where $R^l$ is the lower bound of key rate Alice and Bob estimate through the decoy-state postprocessing, and $R^u$ is the upper bound of key rate taking Eve’s attack into consideration. Because $R^l$ represents the key rate at which Alice and Bob generate that they think is secure, if the rate is higher than the key rate under Eve’s attack, it means that some of the key Alice and Bob generate are insecure. In other words, Eve obtains some information about the final key. Thus, one can believe the attack is successful. In our scheme, we use this criterion and extend it to judge the rationality of the assumption that the signal and decoy states are indistinguishable in the decoy-state method. Specifically, if $R^l$ is significantly greater than $R^u$, it means that the attack is successful using our scheme and the assumption is unreasonable. On the contrary, it means that the attack is failed and the assumption is reasonable. In other words, we can believe that the signal and decoy states are indeed indistinguishable for the eavesdroppers in decoy-state QKD.

Note that the value of $Y_1^s$ in $R^l$ and the value of $Y_1^s$ in $R^u$ are different in calculation. The value of $Y_1^s$ in $R^l$ is estimated by Alice and Bob through the decoy-state postprocessing, that is, Eq. (5). The value of $Y_1^s$ in $R^u$ is optimized by Eve in the PNS attack. In Eq. (17), given $\mu$ and $\nu$, $P(s|s)$ and $P(s|d)$ can be obtained by Eq. (12). Therefore, we only need to optimize $Z_i^\mu$ and $Z_i^\nu$ and make $Y_1^s$ minimum to obtain the supremum of $R^u$. That is, we have a optimization problem subject to preserving the gain statistics. Specifically, the optimization problem can be stated as follows,

$$\min_{\{Z_i^\mu, Z_i^\nu\}} Y_1^s$$  \hspace{1cm} (19)

subject to

$$Q_\mu = 1 - e^{-\eta_\mu} = \sum_{i=1}^{\infty} [P(s|s)Z_i^\mu + P(d|s)Z_i^\nu]e^{-\mu \frac{H_i}{t!}},$$

$$Q_\nu = 1 - e^{-\eta_\nu} = \sum_{i=1}^{\infty} [P(s|d)Z_i^\mu + P(d|d)Z_i^\nu]e^{-\nu \frac{H_i}{t!}},$$

$$Z_i^\mu, Z_i^\nu \in [0,1].$$ \hspace{1cm} (20)

Here $\mu$ and $\nu$ are the intensities of the signal and decoy states Alice selects, respectively. $P(s|s)$, $P(d|s)$, $P(s|d)$ and $P(d|d)$ can be obtained by Bayesian decision, see Eqs. (1), (12) and (13). For a given overall efficiency between Alice and Bob, we can calculate $Y_1^s$ and further get the upper bound of key rate $R^u$.

Note that we do not constrain the error statistics here. In fact, when we preserve the gain and error statistics simultaneously using our scheme, the optimization problem has no solution. In other words, Eve cannot preserve $Q_\mu$, $Q_\nu$, $E_\mu$, $E_\nu$, simultaneously and the attack is failed. This also shows that it is reasonable to assume that signal state and decoy state are indistinguishable in decoy-state method. To get more details of this discussion, please see Appendix B.

### III. NUMERICAL SIMULATION

In this section, the experimental data in Gobby-Yuan-Shields (GYS) [33] / Huang et al. [34], Tang et al. [29], and Zhao et al. [35] are respectively used for numerical simulation. The key parameters in details are shown in Table II.

In GYS [33] / Huang et al. [34], $\mu$ and $\nu$ are set to be 0.6 and 0.2, respectively. In Fig. 2, we can see that $R^u$ is always and significantly greater than $R^l$, and the attack fails. It means that the assumption that the signal and decoy states are indistinguishable in decoy-state method is reasonable. Fix $\nu = 0.2$, change the value of $\mu$ and select the transmission distances 40 km and 120 km, respectively. In Fig. 3, we find that $R^u$ is still significantly greater than $R^l$, which again verifies that the assumption is believable.
Bob's share will be no longer secure. On the other hand, the decoy state is so large, it will lead to if the intensity difference between the signal state and intensity of signal state is as larger as possible. However, generation efficiency is. Therefore, we hope that the in-

assumption that the signal and decoy states are indistin-

erly, the attack may always fail. This also shows that the

the intensities of signal and decoy states are chosen prop-

\begin{table}[h]
\centering
\caption{Key parameters for QKD experiments.}
\begin{tabular}{llllllll}
\hline
Experiment & $\mu$ & $\nu$ & $q$ & $Y_0$ & $\epsilon_0$ & $\epsilon_d(\%)$ & $\alpha$ & $f(E_{\mu})$ & $\eta_{\text{obs}}(\%)$ \\
\hline
GYS [33] / Huang et al. [34] & 0.6 & 0.2 & 1 & $1.7 \times 10^{-6}$ & 0.5 & 3.3 & 0.21 & 1.22 & 4.5 \\
Tang et al. [29] & 0.5 & 0.1 & 1 & $10^{-7}$ & 0.5 & 2 & 0.21 & 1 & 5 \\
Zhao et al. [35] & 0.8 & 0.1 & 0.4478 & $2.11 \times 10^{-5}$ & 0.5 & 0.827 & 0.21 & 1.22 & 2.27 \\
\hline
\end{tabular}
\end{table}

In Tang et al. [29], $\mu$ and $\nu$ are set to be 0.5 and 0.1, respectively. In Fig. 4, $R^l$ is slightly larger than $R^u$, the attack is successful but the attack effect is not obvious. Fix $\nu = 0.1$, change the value of $\mu$ and select the transmission distances 50 km and 150 km, respectively. In Fig. 5, when $\mu > 0.4$, $R^l > R^u$ and the attack is successful and when $\mu < 0.4$, $R^l < R^u$ and the attack fails. This is because when $\mu/\nu$ is large, the difference of photon-number probability distribution between the signal and decoy states is also large, and the two states are easy to distinguish. Thus, the attack can be successful. On the contrary, when $\mu/\nu$ is little, the difference between the signal and decoy states is small and the two states are hard to distinguish resulting in the failed attack. If the intensities of signal and decoy states are chosen properly, the attack may always fail. This also shows that the assumption that the signal and decoy states are indistinguishable is reasonable.

Note that, on the one hand, the higher the intensity is, the lower the preparation cost is and the higher the state generation efficiency is. Therefore, we hope that the intensity of signal state is as larger as possible. However, if the intensity difference between the signal state and the decoy state is so large, it will lead to $R^l > R^u$, that is, Eve’s attack will be successful and the key Alice and Bob share will be no longer secure. On the other hand, when the intensity of decoy state is fixed (e.g. $\nu = 0.1$), the closer the intensity of signal state is to that of decoy state, the more difficult it is to distinguish the two states, and the more likely Eve is to fail, that is, $R^l < R^u$. Thus, from this point of view, we want the intensity of signal state to be as small as possible (of course, it should always be greater than the intensity of decoy state). However, the preparation and time cost will increase correspondingly. Therefore, the best way is to select the intensity of signal state when $R^l = R^u$. Specifically, when $\nu = 0.1$, we consider $\mu = 0.4$ is better than $\mu = 0.5$ which is set by Tang et al. [29], as shown in Fig. 5.

In Zhao et al. [35], $\mu$ and $\nu$ are set to be 0.8 and 0.1, respectively. In Fig. 6, $R^u$ is always greater than $R^l$, that is, the attack is failed. This verifies again the rationality of the assumption that the signal and decoy states are indistinguishable. Fix $\nu = 0.1$, change the value of $\mu$ and select the transmission distances 20 km and 80 km, respectively. In Fig. 7, for the distance 20 km, when $\mu < 0.8$, $R^l < R^u$ and the attack fails and when $\mu > 0.8$, $R^l > R^u$ and the attack is successful, but the attack effect is not obvious. For the distance 80 km, $R^u$ is always greater than $R^l$, that is, Eve’s attack always fails.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Fig2.pdf}
\caption{The lower bound $R^l$ and optimized upper bound $R^u$ of the key rate under our attack with the experimental parameters from GYS [33] / Huang et al. [34], as listed in the second row in Table II.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Fig3.pdf}
\caption{The lower bound $R^l$ and optimized upper bound $R^u$ of the key rate under our attack with the experimental parameters from GYS [33] / Huang et al. [34], as listed in the second row in Table II. Fix $\nu = 0.2$, change the value of $\mu$ and select the transmission distances 40 km and 120 km, respectively. Note that $R^u$ is still significantly greater than $R^l$.}
\end{figure}
Thus, the assumption that the signal and decoy states are indistinguishable is valid if the intensities are selected properly. By the way, the experimental parameters are set very well by Zhao et al. [35] where the intensities of signal state and decoy state are set to be 0.8 and 0.1, respectively. At this time, $R_l = R_u$. It not only ensures the lower preparation cost and higher efficiency, but also avoids the attack from Eve using the intensity difference between the signal and decoy states.

IV. CONCLUSION

In this paper, we proved the rationality of the assumption that the signal and decoy states are indistinguishable in decoy-state QKD. The key idea of our scheme is using Bayesian decision and the difference of photon-number probability distributions between the signal and decoy states. The numerical simulation results show that the PNS attack combined with the intensity difference fails. In other words, it is reasonable to assume that the sig-
nal and decoy states are indistinguishable in decoy-state QKD protocols. In addition, our method can also properly set the intensities of signal and decoy states. Specifically, fix the intensity of decoy state and select the intensity of signal state which makes $R^d = R^s$. In this way, the preparation cost of quantum state will be reduced and the communication efficiency will be improved, and the attack from Eve using the intensity difference between the signal and decoy states can be avoided to guarantee the security of the final key.

In this paper, we only consider the case of one-decoy-state QKD. Meanwhile, the signal and decoy states are emitted equally by Alice. In fact, one can further adopt Bayesian decision theory to analyze multiple intensities decoy-state QKD protocols [36–39] and the probabilities of Alice sending signal and decoy states are not necessarily equal. Moreover, our scheme is also applicable to other decoy-state QKD protocols which take advantage of the assumption that the signal and decoy states are indistinguishable, such as decoy state measurement-device-independent QKD (Decoy State MDI-QKD) [40–43]. In addition, our scheme can also be combined with the methods of distinguishing between the signal and decoy states from the perspectives of imperfect light source and optical device to improve the success probability of differentiation, such as without phase randomization [29], time domain mismatch or frequency domain mismatch between the signal and decoy states [34].

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**Appendix A: $P_i^s$ VS $P_i^d$**

Taking advantage of Eq. (1), we have

$$\frac{P_i^s}{P_i^d} = e^{-\frac{(\mu - \nu)}{\nu}}.$$  \hspace{1cm} (A1)

Obviously, $P_i^s/P_i^d$ is an increasing function of $i$ due to $1 > \mu > \nu > 0$.

i) For $i = 0$, $P_0^s/P_0^d = e^{-(\mu - \nu)} < 1$, that is, $P_0^s < P_0^d$.

ii) For $i = 1$, $P_1^s/P_1^d = e^{-\frac{\mu}{\nu}}$. Let $f(x) = xe^{-x}$, $(0 < x < 1)$, then $f'(x) = (1 - x)e^{-x} > 0$. That is, $f(x)$ is an increasing function of $x$. Since $\mu > \nu$, we have $f(\mu) > f(\nu)$, that is, $P_1^s > P_1^d$.

iii) For $i > 1$, since $P_i^s > P_i^d$, we get $P_i^s/P_i^d > 1$. And $P_i^s/P_i^d$ is an increasing function of $i$, so we have $P_i^s/P_i^d > P_{i-1}^s/P_{i-1}^d > 1$. That is, $P_i^s > P_i^d$.

In summary, when $1 > \mu > \nu > 0$, the relationship between $P_i^s$ and $P_i^d$ is as follows:

$$P_i^s < P_i^d, \quad P_i^s > P_i^d, \quad (i \geq 1).$$  \hspace{1cm} (A2)

**Appendix B: ERROR STATISTICS ANALYSIS**

To maintain the error statistics, Eve should satisfy the following equations,

$$E_\mu Q_\mu = \frac{1}{2} Y_0 + e_d(1 - e^{-\eta \mu})$$

$$= \frac{1}{2} Z_0^\mu + \sum_{i=1}^{\infty}[\epsilon^\mu_i P(s|s)Z_i^\mu + \frac{1}{2} P(d|s)Z_i^\nu e^{-\mu \frac{\eta^i}{\nu^i}}],$$

$$E_\nu Q_\nu = \frac{1}{2} Y_0 + e_d(1 - e^{-\eta \nu})$$

$$= \frac{1}{2} Z_0^\nu + \sum_{i=1}^{\infty}[\epsilon^\nu_i P(d|d)Z_i^\nu + \frac{1}{2} P(s|d)Z_i^\mu e^{-\nu \frac{\eta^i}{\nu^i}}].$$  \hspace{1cm} (B1)

Here $\epsilon^\mu_i$ is the error when Alice sends a signal (decoy) state and Eve guesses it is a signal (decoy) state. $Z_0^\nu$ is the dark count rate Eve sets when there is no photon in the signal (decoy) state Alice sends. Note that we simply set the QBER to be the upper bound $\frac{1}{2}$ when Eve gets the incorrect result through the Bayesian decision.

Similarly to Sec. II C, the optimization problem of minimizing the key rate upper bound $R^e$ can be stated as follows,

$$\min_{\left\{ Z_i^\mu, Z_i^\nu \right\}} Y_i^s$$  \hspace{1cm} (B2)

subject to

$$Q_\mu = 1 - e^{-\eta \mu} = \sum_{i=1}^{\infty}[P(s|s)Z_i^\mu + P(d|s)Z_i^\nu e^{-\mu \frac{\eta^i}{\nu^i}}],$$

$$Q_\nu = 1 - e^{-\eta \nu} = \sum_{i=1}^{\infty}[P(s|d)Z_i^\mu + P(d|d)Z_i^\nu e^{-\nu \frac{\eta^i}{\nu^i}}],$$

$$E_\mu Q_\mu = \frac{1}{2} Y_0 + e_d(1 - e^{-\eta \mu}) \geq \sum_{i=1}^{\infty} \frac{1}{2} P(d|s)Z_i^\mu e^{-\mu \frac{\eta^i}{\nu^i}},$$

$$E_\nu Q_\nu = \frac{1}{2} Y_0 + e_d(1 - e^{-\eta \nu}) \geq \sum_{i=1}^{\infty} \frac{1}{2} P(s|d)Z_i^\mu e^{-\nu \frac{\eta^i}{\nu^i}},$$

$$Z_i^\mu, Z_i^\nu \in [0, 1].$$  \hspace{1cm} (B3)

Here $\mu$ and $\nu$ are the intensities of signal state and decoy state Alice selects, respectively. $P(s|s)$, $P(d|s)$, $P(s|d)$ and $P(d|d)$ can be calculated by the Bayesian decision, see Eqs. (1), (12) and (13). For a given overall efficiency between Alice and Bob, we can calculate $Y_i^s$ and further get the upper bound of key rate $R^e$.

However, when the experimental parameters in Sec. III are taken into Eq. (B3), we find that the optimization problem has no solution. In other words, Eve cannot maintain the observed statistics $(Q_\mu, Q_\nu, E_\mu, E_\nu)$ same as those in the normal case through the difference of photon-number probability distribution between signal state and decoy state, and the attack fails. This also indicates that it is reasonable to assume that signal state and decoy state are indistinguishable in decoy-state QKD.
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