Low Energy Theorem for SUSY Breaking with Gauge Supermultiplets

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Abstract

Low energy theorems of Nambu-Goldstone fermion associated with spontaneously broken supersymmetry are studied for gauge supermultiplets. Two possible terms in the effective Lagrangian are needed to deal with massless gaugino and/or massless gauge boson. As an illustrative example, a concrete model is worked out which can interpolate massless as well as massive gaugino and/or gauge boson to examine the low energy effective interaction of NG-fermion.
1 Introduction

Supersymmetry (SUSY) is one of the most attractive ideas for unified model building [1]. When the SUSY is spontaneously broken, a massless particle appears which is called the Nambu-Goldstone fermion [2], [3]. Its interaction is characterized by the broken SUSY and is typically summarized as low energy theorems [4]–[9]. The purpose of this paper is to study the low energy theorems for gauge supermultiplets, especially in the case of massless gaugino or gauge boson. We find that a new term should be used as the effective Lagrangian for massless gaugino. We also examine the effective interaction terms of NG fermion in an explicit model which can interpolate massless and massive particles in the gauge supermultiplet. This result should be useful even for supergravity theories provided the SUSY breaking scale is small enough, since gravitino behaves almost as an NG fermion for such a case [10].

If the supersymmetry (SUSY) is spontaneously broken, the boson-fermion mass-splitting is induced. There exists a Nambu-Goldstone (NG) fermion $\psi_{NG}$ which shows up in the supercurrent $J_\alpha^\mu(x)$

$$J_\alpha^\mu = \sqrt{2}if (\gamma^\mu \psi_{NG})_\alpha + J_{\text{matter},\alpha}^\mu + \cdots,$$

where $f$ is the order parameter of the SUSY breaking and $J_{\text{matter},\alpha}^\mu(x)$ is the supercurrent for matter fields suitably dressed by the NG fermion. The low energy theorems have been worked out for chiral scalar supermultiplets. The effective coupling of the NG fermion is typically related to the mass-splitting of boson and fermion in the supermultiplet. In the case of gauge supermultiplet consisting of gauge boson $v_\mu$ and gaugino $\lambda$ for $U(1)$ gauge symmetry, the supercurrent $J_{\text{matter},\alpha}^\mu(x)$ becomes

$$J_{\text{matter},\alpha}^\mu = -iv_{\nu\rho}(\sigma^{\nu\rho})_{\alpha\dot{\alpha}} \bar{\chi}^\dot{\alpha} + \cdots,$$

where $v_{\mu\nu}$ is the gauge field strength. It has been shown that the single NG fermion interactions with a gauge supermultiplet can be expressed by the following effective Lagrangian [7] for a massless gauge boson

$$\mathcal{L}_{\text{int}} = \h_{\text{eff}}\psi_{NG}\sigma^{\mu\nu}\lambda v_{\mu\nu} + \text{h.c.}$$

Here the effective coupling constant $h_{\text{eff}}$ is related to the gaugino mass $m_\lambda$ and the order
parameter $f$ by
\[ h_{\text{eff}} = \frac{m_F}{\sqrt{2}f}. \] (4)

This is the SUSY analog of the Goldberger-Treiman relation.

In Ref.\[9\] we have extended this relation not only for a massless gauge boson but also for a massive one. In the case of $m_F \neq 0$ the effective coupling can be expressed in terms of mass-splitting of the gauge boson and the gaugino as
\[
\mathcal{L}_{\text{int}}^{(1)} = h_{\text{eff}}^{(1)} \bar{\psi}_{\text{NG}} \sigma^{\mu\nu} \lambda_{\mu\nu} + \text{h.c.} + \cdots, \\
h_{\text{eff}}^{(1)} = -\frac{1}{\sqrt{2}f} \left( \frac{m_B^2 - m_F^2}{m_F} \right). 
\] (5)

Evidently this effective action is inadequate for massless gaugino $m_F = 0$. We will show that the above effective Lagrangian (3) does not contribute to the low energy theorem to the leading order of the Nambu-Goldstone fermion momentum. We will find that the following interaction term should be used instead as the effective Lagrangian for the case of massless gaugino $m_F = 0$ to describe the low energy theorem correctly
\[
\mathcal{L}_{\text{int}}^{(2)} = i h_{\text{eff}}^{(2)} \bar{\psi}_{\text{NG}} \sigma^\tau \lambda_{\tau\nu} + \text{h.c.}. 
\] (7)

For massive gaugino and gauge boson, we will find that both terms are allowed and are equivalent. Therefore a linear combination of these terms is constrained by the low energy theorem
\[ h_{\text{eff}}^{(2)} + m_F h_{\text{eff}}^{(1)} = -\frac{\Delta m^2}{\sqrt{2}f}, \quad \Delta m^2 = m_B^2 - m_F^2. \] (8)

For massless gauge boson $m_B = 0$, the new term (7) is forbidden because of unbroken gauge invariance and one has to use the known effective Lagrangian (3).

In the next section, we work out the effective Lagrangian and low energy theorem. In sect. 3, we present a concrete model, which interpolate massive and massless gaugino and gauge boson, in order to examine the NG fermion interactions.

2 Low energy theorem for gauge supermultiplets

We will consider interaction of NG fermion with momentum $q_{\text{NG}}$ with gaugino of momentum $p_F$ and gauge boson of momentum $p_B$ and polarization $\epsilon(p_B)$ which satisfies $p_B \cdot \epsilon(p_B) = 0$. In
the effective Lagrangians. We see that these Lagrangians in Eqs.(5) and (7) contribute to the same matrix element between in-coming state of NG fermion and gaugino $|q_{NG}; p_F\rangle_{in}$ and out-going state of gauge boson $|p_B\rangle_{out}$. Using gaugino equation of motion, we obtain

$$
\langle p_B | L^{(1)}_{int} + L^{(2)}_{int} | q_{NG}; p_F \rangle
= i h^{(1)}_{eff} \chi_{NG}(\epsilon^* \cdot \sigma) (\bar{\sigma} \cdot p_B) \chi_F + i h^{(2)}_{eff} \chi_{NG}(\epsilon^* \cdot \sigma) \bar{\chi}_F
= i \left( \frac{h^{(1)}_{eff}}{m_F} + \frac{h^{(2)}_{eff}}{m_F} \right) \chi_{NG}(\epsilon^* \cdot \sigma) (\bar{\sigma} \cdot p_B) \chi_F - q_{NG} i \left[ \frac{h^{(2)}_{eff}}{m_F} \chi_{NG}(\epsilon^* \cdot \sigma) \bar{\sigma}^\mu \chi_F \right] + \text{h.c.}
= i \left( h^{(2)}_{eff} + m_F h^{(1)}_{eff} \right) \chi_{NG}(\epsilon^* \cdot \sigma) \bar{\chi}_F + q_{NG} i \left[ h^{(1)}_{eff} \chi_{NG}(\epsilon^* \cdot \sigma) \bar{\sigma}^\mu \chi_F \right] + \text{h.c.}.
$$

(9)

We see that these Lagrangians in Eqs.(5) and (7) contribute to the same matrix element ignoring higher $q^\mu_{NG}$ terms. In this sense, we can use both interaction terms $L^{(1)}_{int}$ and $L^{(2)}_{int}$ as the effective Lagrangians.

To derive the relation between effective coupling constants $h^{(1)}_{eff}, h^{(2)}_{eff}$ and the gauge boson mass $m_B$ and the gaugino mass $m_F$, we introduce form factors $A_i(q^2)$, $i = 1, \cdots, 12$ of the supercurrent $J_\mu^\alpha(x)$ between one-particle states of the gaugino $|p_F\rangle$ and the gauge boson $|p_B\rangle$ following our previous work.

$$
\langle p_B | J_\alpha^\mu(0) | p_F \rangle
= \epsilon^\nu_{\nu}(p_B) \left[ A_1(q^2) q^{\nu} q^\mu + A_2(q^2) q^{\nu} k^\mu + A_3(q^2) q^{\nu} \sigma^\mu \sigma^\rho q_\rho + A_4(q^2) \eta^\mu^\nu + A_5(q^2) \sigma^\nu \bar{\sigma}^\mu \right]_{\alpha} \chi_{F\beta}(p_F)
+ \epsilon^\nu_{\nu}(p_B) \left[ A_6(q^2) q^{\nu} \sigma^\mu + A_7(q^2) q^{\nu} \sigma^\mu \sigma^\rho q_\rho + A_8(q^2) \eta^\mu^\nu \sigma^\rho q_\rho + A_9(q^2) \eta^\mu^\nu \sigma^\rho q_\rho
+ A_{10}(q^2) q^{\nu} \sigma^\nu + A_{11}(q^2) \sigma^\nu \bar{\sigma}^\mu k^\rho + A_{12}(q^2) \sigma^\nu \bar{\sigma}^\mu \sigma^\rho q_\rho \right]_{\alpha\beta} \bar{\chi}_F(p_F),
$$

(10)

where $q^\mu = p_B^\mu - p_F^\mu$, $k^\mu = p_B^\mu + p_F^\mu$. The supercurrent conservation $\partial_\mu J_\mu^\alpha = 0$ gives a relation among the form factors in terms of mass-splitting $\Delta m^2 \equiv m_B^2 - m_F^2$ between the gauge boson and the gaugino

$$
q^2 \left[ A_{10}(q^2) + A_{11}(q^2) - A_{12}(q^2) \right] = -2 \Delta m^2 A_{11}(q^2).
$$

(11)

As is usual in current algebra, the supercurrent matrix element can have a massless NG fermion pole. So some of form factors in Eq.(10) are singular in the limit of $q^2 \to 0$. Eq.(11)
implies a non-vanishing matrix element of the supercurrent between the vacuum and the single NG fermion state \(|q, \psi_{\text{NG}}\rangle\) with momentum \(q^\mu\)

\[
\langle 0 | J_\alpha^\mu(0) | q, \psi_{\text{NG}} \rangle = \sqrt{2i} f \sigma^\mu_{\alpha\dot{\alpha}} \bar{\chi}_{\text{NG}},
\]

where \(f\) is the order parameter of the SUSY breaking. Since the combination \(J_\alpha^\mu - \sqrt{2i} f \sigma^\mu_{\alpha\dot{\alpha}} \bar{\psi}_{\text{NG}}\) has vanishing matrix element between the vacuum and the single NG fermion state, its matrix element between gauge boson and gaugino states becomes non-singular in the limit \(q^2 \to 0\). Now we define an NG fermion source \(j^\text{NG}_\alpha(x)\) by using the NG fermion field \(\psi_{\text{NG}}(x)\)

\[
j^\text{NG}_\alpha(x) \equiv -i \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu \bar{\psi}_{\text{NG}}(x).
\]

By introducing form factors \(B_i(q^2), i = 1, \cdots, 4\) of the NG fermion source, we obtain

\[
\langle p_B | j^\text{NG}_\alpha(0) | p_F \rangle = \epsilon^*_\nu(p_B) \left[ B_1(q^2) q^\nu + B_2(q^2) q_\rho \sigma^\rho \sigma^\nu \right] \chi_{F\beta}(p_F)
+ \epsilon^*_\nu(p_B) \left[ B_3(q^2) q^\nu \sigma^\rho q_\rho + B_4(q^2) \sigma^\nu \right] \bar{\chi}_F(p_F),
\]

\[
\langle p_B | \bar{\psi}^\dot{\alpha}_{\text{NG}}(0) | p_F \rangle = \epsilon^*_\nu(p_B) \left[ \frac{B_1(q^2)}{q^2} q^\nu \sigma^\rho q_\rho + B_2(q^2) \sigma^\nu \right] \chi_{F\beta}(p_F)
+ \epsilon^*_\nu(p_B) \left[ B_3(q^2) q^\nu - \frac{B_4(q^2)}{q^2} q_\rho \sigma^\rho \sigma^\nu \right] \bar{\chi}_F(p_F).
\]

Form factors in the combination \(J_\alpha^\mu - \sqrt{2i} f \sigma^\mu_{\alpha\dot{\alpha}} \bar{\psi}_{\text{NG}}\) must be regular in the limit \(q^2 \to 0\)

\[
\langle p_B | J_\alpha^\mu(0) - \sqrt{2i} f \sigma^\mu_{\alpha\dot{\alpha}} \bar{\psi}_{\text{NG}} | p_F \rangle
= \epsilon^*_\nu(p_B) \left[ A_1(q^2) q^\nu q^\mu + A_2(q^2) q^\nu k^\mu + \left( A_3(q^2) + \sqrt{2i} f \frac{B_1(q^2)}{q^2} \right) q^\nu \sigma^\mu \sigma^\rho q_\rho \right.
+ \left( A_4(q^2) + 2 \sqrt{2i} f B_2(q^2) \right) \eta^\nu \nu + \left( A_5(q^2) + \sqrt{2i} f B_2(q^2) \right) \sigma^\nu \sigma^\mu \left] \chi_{F\beta}(p_F) \right. \\
+ \left. \epsilon^*_\nu(p_B) \left[ A_6(q^2) - \sqrt{2i} f B_3(q^2) \right) q^\nu \sigma^\mu + A_7(q^2) q^\nu \sigma^\rho q_\rho + A_8(q^2) q^\nu k^\mu \sigma^\rho q_\rho \\
+ A_9(q^2) \eta^\mu \nu \sigma^\rho q_\rho + A_{10}(q^2) q^\nu \sigma^\mu \nu + A_{11}(q^2) \sigma^\nu \sigma^\rho \sigma^\mu k^\rho \right.
+ \left. \left( A_{12}(q^2) + \sqrt{2i} f \frac{B_4(q^2)}{q^2} \right) \sigma^\mu \sigma^\rho \sigma^\nu q_\rho \right] \bar{\chi}_F(p_F).
\]
Therefore we obtain the singularity of the form factors $A_3(q^2)$ and $A_{12}(q^2)$ at $q^2 = 0$ unless $B_1(0)$ and $B_4(0)$ vanish respectively

$$\lim_{q^2 \to 0} q^2 A_3(q^2) = -\sqrt{2} i f B_1(0), \quad \lim_{q^2 \to 0} q^2 A_{12}(q^2) = -\sqrt{2} i f B_4(0).$$

(17)

Combining Eq. (17) with Eq. (11) in the limit $q^2 \to 0$ gives

$$\sqrt{2} i f B_4(0) = -2 \Delta m^2 A_{11}(0).$$

(18)

To relate the form factor $B_4(0)$ to an effective coupling constant of the NG fermion with the gauge boson and the gaugino, we evaluate a transition amplitude between the in-state $|q; p_F\rangle_{\text{in}}$ and the out-state $|p_B\rangle_{\text{out}}$. This S-matrix element can be expressed by using an effective interaction Lagrangian $\mathcal{L}_{\text{int}}$ as

$$\langle p_B| q; p_F \rangle_{\text{in}} = i \langle p_B| e^{i S_{\text{int}}(x)} |q; p_F\rangle_{\text{int}}
\simeq i \int d^4 x \langle p_B| e^{-i p_F x} \mathcal{L}_{\text{int}}(0) e^{i p_F x} |q; p_F\rangle_{\text{int}}
= i(2\pi)^4 \delta^4(p_B - p_F - q) \langle p_B| \mathcal{L}_{\text{int}}(0) |q; p_F\rangle_{\text{int}},$$

(19)

where $|p_B\rangle_{\text{int}}$ and $|q; p_F\rangle_{\text{int}}$ denote states in the interaction picture. On the other hand, using the LSZ reduction formula in four dimensions, it can also be written as

$$\langle p_B| q; p_F \rangle_{\text{in}} = -i \int d^4 x \epsilon^{ij} \bar{\nu}_{\text{NG}} i \gamma^i \partial_{\mu j} \langle p_B| \Psi_{\text{NG}}(x) |p_F\rangle_{\text{int}}
= -i(2\pi)^4 \delta^4(p_B - p_F - q) \chi_{\text{NG}}(q) q_\mu \sigma^\mu_1 \langle p_B| \bar{\psi}_{\text{NG}}(0) |p_F\rangle_{\text{int}}
= -i(2\pi)^4 \delta^4(p_B - p_F - q) \bar{\chi}_{\text{NG}}(q) q_\mu \bar{\sigma}^\mu_1 \langle p_B| \psi_{\text{NG}}(0) |p_F\rangle_{\text{int}},$$

(20)

where Dirac spinors $\Psi_{\text{NG}}$ and $\bar{\nu}_{\text{NG}}$ are decomposed into Weyl spinors as

$$\Psi_{\text{NG}} \equiv \begin{pmatrix} \psi_{\text{NG}} \\ \bar{\psi}_{\text{NG}} \end{pmatrix}, \quad \bar{\nu}_{\text{NG}} \equiv (\chi_{\text{NG}}, \bar{\chi}_{\text{NG}}).$$

(21)

Since we do not need to distinguish the interaction picture and the Heisenberg picture for one-particle states, we drop the subscript $I$ for one-particle states in the following. Therefore Eqs. (13) and (20) lead to a relation between matrix elements of the interaction Lagrangian and NG fermion source

$$\langle p_B| \mathcal{L}_{\text{int}}(0) |q; p_F\rangle = -\chi_{\text{NG}}(q) q_\mu \sigma \langle p_B| \bar{\psi}_{\text{NG}}(0) |p_F\rangle - \bar{\chi}_{\text{NG}}(q) q_\mu \bar{\sigma} \langle p_B| \psi_{\text{NG}}(0) |p_F\rangle
= -\chi_{\text{NG}}(q) \langle p_B| j^{\text{NG}}(0) |p_F\rangle - \bar{\chi}_{\text{NG}}(q) \langle p_B| \bar{j}^{\text{NG}}(0) |p_F\rangle.$$

(22)
Comparing Eqs. (9) and (22) with Eq. (14), we obtain
\[ \chi_{NG}(q)\epsilon^*(p_B) \left[ \{ B_1(0) - 2B_2(0) \} q^\nu + \{ B_2(0) + ih^{(1)}_{\text{eff}} \} \sigma^\nu \cdot q \right] \hat{\alpha} \beta \chi_{F\beta}(p_F) \\
+ \chi_{NG}(q)\epsilon^*(p_B) \left[ B_3(0)q^\nu q \cdot \sigma + \{ B_4(0) + ih^{(2)}_{\text{eff}} \} \sigma^\nu \right] \hat{\alpha} \beta \tilde{\chi}_{F\beta}(p_F) = 0. \tag{23} \]

For the case of \( m_F \neq 0 \), we can use the gaugino equation of motion
\[ \tilde{\chi}_F = \frac{1}{m_F} \hat{\sigma} \cdot p_F \chi_F = \frac{1}{2m_F} \hat{\sigma} \cdot (k - q) \chi_F, \tag{24} \]
which gives relations among the form factors and the couplings \( h^{(1)}_{\text{eff}}, h^{(2)}_{\text{eff}} \)
\[ B_4(0) = -i \left( h^{(2)}_{\text{eff}} + m_F h^{(1)}_{\text{eff}} \right), \quad \frac{1}{2} B_1(0) = B_2(0) = - \frac{B_4(0) + ih^{(2)}_{\text{eff}}}{m_F}, \quad B_3(0) = 0. \tag{25} \]
Thus Eq. (17) becomes
\[ -\sqrt{2}(h^{(2)}_{\text{eff}} + m_F h^{(1)}_{\text{eff}}) f = 2\Delta m^2 A_{11}(0). \tag{26} \]

Let us consider the case of \( m_F = 0 \). Eq. (9) shows that the Lagrangian \( \mathcal{L}_{\text{int}}^{(1)} \) does not contribute to the matrix element in leading orders of \( q^\mu_{NG} \) in this case. For \( m_F = 0 \), Eq. (23) leads to
\[ B_4(0) = -ih^{(2)}_{\text{eff}}, \quad \frac{1}{2} B_1(0) = B_2(0) = -ih^{(1)}_{\text{eff}}, \quad B_3(0) = 0. \tag{27} \]
Therefore the effective Lagrangian consists solely of \( \mathcal{L}_{\text{int}}^{(2)} \) and its coupling \( h^{(2)}_{\text{eff}} \) is given in terms of the mass-splitting \( \Delta m^2 \) for the case of massless gaugino \( m_F = 0 \)
\[ -\sqrt{2}h^{(2)}_{\text{eff}} f = 2\Delta m^2 A_{11}(0). \tag{28} \]

Now we consider the case of massless gauge boson \( m_B = 0 \). In this case, we have an unbroken gauge symmetry, which requires the vanishing matrix element of Eq. (22), if we replace \( \epsilon^*(p_B) \) by \( p_{B\mu} \). Using Eq. (14), we obtain relations among the form factors \( B_i(0) \)
\[ \frac{1}{2} B_1(0) = B_2(0) = - \frac{B_4(0)}{m_F}, \quad B_3(0) = 0. \tag{29} \]
These relations combined with Eq. (27) imply \( h^{(2)}_{\text{eff}} = 0 \). Therefore gauge invariance forbids \( \mathcal{L}_{\text{int}}^{(2)} \) as a piece of effective Lagrangian for \( m_B = 0 \).
We can determine the form factor $A_{11}(0)$ by substituting the following expression of the supercurrent into Eq.(10) in the limit $q^2 \rightarrow 0$

$$J_\alpha^\mu = \sqrt{2} if \sigma_\alpha^\mu \tilde{\psi}_\text{NG} = -iv_{\nu \rho} (\sigma^\nu \sigma^\mu)_{\alpha \bar{\alpha}} \tilde{\lambda}_\alpha + \cdots,$$

(30)

where $\cdots$ denotes higher order terms with NG fermion and possible interaction terms. Neglecting possible “renormalization effects” due to interactions and higher order terms, we find

$$A_{11}(0) = \frac{1}{2},$$

(31)

which leads to the SUSY Goldberger-Treiman relation

$$h_{\text{eff}}^{(2)} + m_F h_{\text{eff}}^{(1)} = -\frac{\Delta m^2}{\sqrt{2} f}.$$ 

(32)

Now we consider the case where fermion mass-eigenstates $\{\tilde{\lambda}_i\}$ are mixtures of gauginos $\{\lambda_i\}$ which are superpartners of gauge bosons $\{(v_i)_\mu\}$ as

$$\lambda_i(x) = V_{i,j} \tilde{\lambda}_j(x),$$

(33)

where $V_{i,j}$ is the unitary mixing matrix. In this case, the effective Lagrangian should be written in terms of mass-eigenstates as follows

$$\mathcal{L}_{\text{int}} = \sum_{i,j} h_{\text{eff},i,j}^{(1)} \psi_\text{NG} \sigma_{\mu \nu} \tilde{\lambda}_j (v_i)_{\mu \nu} + \sum_{i,j} h_{\text{eff},i,j}^{(2)} i \psi_\text{NG} \sigma_\nu \tilde{\lambda}_j (v_i)_{\nu} + \text{h.c.},$$

(34)

and thus Eq.(28) becomes

$$-\sqrt{2} (h_{\text{eff},i,j}^{(2)} + m_F h_{\text{eff},i,j}^{(1)}) f = 2 \Delta m_{i,j}^2 A_{11}(0)_{i,j},$$

(35)

where $m_{i,j}^2 = m_{R,i}^2 - m_{F,j}^2$. Taking account of the mixing Eq.(33), the supercurrent becomes

$$J_\alpha^\mu = \sqrt{2} if \sigma_\alpha^\mu \tilde{\psi}_\text{NG} = i \sum_{k,l} V_{k,l} (v_k)_{\nu \rho} (\sigma^\nu \sigma^\rho)_{\alpha \bar{\alpha}} \tilde{\lambda}_{\bar{\alpha}} + \cdots,$$

(36)

which determines the value of the form factor $A_{11}(0)_{i,j}$ as

$$A_{11}(0)_{i,j} = \frac{1}{2} V_{i,j},$$

(37)

and so Eq.(32) can be written as

$$h_{\text{eff},i,j}^{(2)} + m_F h_{\text{eff},i,j}^{(1)} = -\frac{\Delta m_{i,j}^2}{\sqrt{2} f} V_{i,j}.$$ 

(38)
In summary, we should use the following effective Lagrangians

\[ m_F \neq 0, m_B = 0 : \quad \mathcal{L}_{\text{int}} = h_{\text{eff}}^{(1)} \psi_{NG} \sigma^\mu \lambda v_{\mu\nu} + \text{h.c.}, \quad (39) \]

\[ m_B = 0, m_B \neq 0 : \quad \mathcal{L}_{\text{int}} = i h_{\text{eff}}^{(2)} \psi_{NG} \sigma^\nu \bar{\lambda} v_{\nu} + \text{h.c.}, \quad (40) \]

\[ m_F \neq 0, m_B \neq 0 : \quad \mathcal{L}_{\text{int}} = h_{\text{eff}}^{(1)} \psi_{NG} \sigma^\mu \lambda v_{\mu\nu} + i h_{\text{eff}}^{(2)} \psi_{NG} \sigma^\nu \bar{\lambda} v_{\nu} + \text{h.c.}. \quad (41) \]

The corresponding effective couplings are given by the SUSY Goldberger-Treiman relations

\[ m_F \neq 0, m_B = 0 : \quad h_{\text{eff}}^{(1)} = \frac{m_F}{\sqrt{2} f}, \quad (42) \]

\[ m_F = 0, m_B \neq 0 : \quad h_{\text{eff}}^{(2)} = -\frac{m_B^2}{\sqrt{2} f}, \quad (43) \]

\[ m_F \neq 0, m_B \neq 0 : \quad h_{\text{eff}}^{(2)} + m_F h_{\text{eff}}^{(1)} = -\frac{\Delta m^2}{\sqrt{2} f}, \quad \Delta m^2 = m_B^2 - m_F^2. \quad (44) \]

If there is a mixing for gaugino, we only need to multiply them by the mixing matrix \( V_{i,j} \) as in Eq.\((38)\).

3 Low energy theorem in a concrete model

So far we derived the SUSY Goldberger-Treiman relation for gauge supermultiplet not only with massless gauge boson as in Ref.\([7]\) but also with massive one. We will examine and check the result in Eqs.\((42) - (44)\) by a spontaneously broken SUSY model interpolating \( m_F = 0 \) and \( m_F \neq 0 \).

The model considered here is the spontaneously broken \( U(1) \) gauge theory in four dimensions which gives mass for the gauge boson and the gaugino. In order to have NG fermion without containing gaugino component, SUSY is broken by the O’Raifeartaigh mechanism\([3]\). We introduce the following Lagrangian with chiral superfields \( \Phi^{(i)} = (A^{(i)}, \psi^{(i)}, F^{(i)}) \), \( i = 0, 1, 2 \) neutral under \( U(1) \) group, and chiral superfields \( \Phi^+, \Phi^- \) with \( U(1) \) charge \( \pm e \), respectively

\[
\mathcal{L} = \left[ \sum_{i=0}^{2} \bar{\Phi}^{(i)} \Phi^{(i)} + \bar{\Phi}^+ e^V \Phi^+ + \bar{\Phi}^- e^{-V} \Phi^- + \alpha \bar{\Phi}^{(0)} \Phi^+ \Phi^- + \alpha \bar{\Phi}^{(0)} \bar{\Phi}^+ \bar{\Phi}^- \right]_{\bar{\theta}^2 \bar{\theta}^2} \\
+ \left[ \frac{1}{4} W^\alpha W_\alpha + \frac{\Phi}{M} W^\alpha W_\alpha + P(\Phi^{(i)}, \Phi^{\pm}) \right]_{\bar{\theta}^2} + \text{h.c.}. \quad (45)
\]
The gauge kinetic function has a non-minimal piece with the parameter $1/M$ and is chosen to be proportional to the NG fermion superfield $\Phi = (A, \psi, F)$, $\psi = \psi_{NG}$, $\langle F \rangle = -f$, which is determined later. This non-minimal kinetic term is added to give mass term for gaugino bilinear. The Kähler potential has a non-minimal piece with the parameter $\alpha$ of mass dimension $-1$. This term is added to allow a possibility of massless fermion containing gaugino component as we show below. The superpotential $P$ is given in terms of parameters $\ell$ of mass dimension two, $m, k$ and $h$ of mass dimension one and dimensionless $g$

$$P(\Phi^{(i)}, \Phi^{\pm}) = \ell \Phi^{(2)} + m \Phi^{(0)} \Phi^{(1)} + g \Phi^{(1)} \Phi^{(1)} \Phi^{(2)} - \frac{k^2}{h} \Phi^{+} \Phi^{-} + \frac{1}{2h} (\Phi^{+} \Phi^{-})^2.$$  \hspace{1cm} (46)

Now we consider the minimum of the potential for scalar fields

$$V = \frac{1}{2} D^2 + \sum_{i,j} g_{ij} \frac{\partial P}{\partial A^i} \frac{\partial \bar{P}}{\partial A^j},$$ \hspace{1cm} (47)

where $g_{ij}$ is Kähler metric in field space. For the case of $g \ell < 0$ and $2|g \ell| \geq m^2/(1 - \beta^2)$ where $\beta \equiv \sqrt{2} \alpha k$, we obtain an absolute minimum with a relation

$$\langle A^{(2)} \rangle = -\frac{m}{2g \langle A^{(1)} \rangle} \langle A^{(0)} \rangle, \quad \langle A^{+} \rangle = \langle A^{-} \rangle$$  \hspace{1cm} (48)

In order to simplify the determination of the explicit value of the fields, we choose to impose the following fine tuning of parameters of the model

$$h = \frac{\alpha mv}{2(1 - \beta^2)k^2}, \quad v \equiv \langle A^{(1)} \rangle.$$  \hspace{1cm} (49)

We obtain the minimum of the scalar potential at the following value for fields

$$\langle A^{(1)} \rangle = \sqrt{\frac{2|g \ell| - m^2/(1 - \beta^2)}{2g^2}}, \quad \langle A^{+} \rangle = \langle A^{-} \rangle = k, \hspace{1cm} (50)$$

We obtain a flat direction along $\langle A^{(0)} \rangle$ and choose $\langle A^{(0)} \rangle = 0$ in the following. In this vacuum, both SUSY and $U(1)$ gauge symmetry are broken. So the gauge boson becomes massive $m_B = ek$, and there is non-zero vacuum energy caused by $\langle F^{(0)} \rangle, \langle F^{(2)} \rangle, \langle F^{(1)} \rangle \neq 0$.

To find an NG fermion, we focus on fermion mass terms. Since $\tau \equiv (\psi^{+} - \psi^{-})/i \sqrt{2}$ and gaugino $\lambda$ mixes together and do not contain NG fermion component, we consider first possible massless fermions arising from a mixing of $\xi \equiv (\psi^{+} + \psi^{-})/\sqrt{2}$, and $\psi^{(0)}, \psi^{(1)}, \psi^{(2)}$. 

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Because of non-minimal Kähler potential, the kinetic terms of the fields $\Phi$, $\Phi^+$ and $\Phi^-$ become non-canonical. We define the following fermions to normalize the kinetic terms

$$\zeta \equiv \sqrt{\frac{1+\beta^2}{2}} (\psi^{(0)} + \xi) \quad \eta \equiv \sqrt{\frac{1-\beta^2}{2}} (\psi^{(0)} - \xi) \quad \Rightarrow \quad \psi^{(0)} = \frac{1}{\sqrt{2(1+\beta)}} \zeta + \frac{1}{\sqrt{2(1-\beta)}} \eta \quad \xi = \frac{1}{\sqrt{2(1+\beta)}} \zeta - \frac{1}{\sqrt{2(1-\beta)}} \eta \quad .$$

The mass matrix of these canonically normalized fermions becomes

$$-\frac{1}{2}(\psi^{(1)}, \psi^{(2)}, \zeta, \eta) \left( \begin{array}{cccc} 0 & 2gv & m & m \\ 2gv & 0 & m & 0 \\ m & m & 0 & 0 \\ m & m & 0 & 0 \end{array} \right) \frac{1}{\sqrt{2(1+\beta)}} \left( \begin{array}{c} \psi^{(1)} \\ \psi^{(2)} \\ \zeta \\ \eta \end{array} \right) .$$

We find two zero-eigenvalues for the mass matrix and the corresponding zero-eigenmodes as

$$\tilde{\psi}_{01} = \frac{1}{\sqrt{m^2 + 4g^2v^2(1-\beta^2)}} \left( -m\psi^{(2)} + 2gv(1-\beta) \sqrt{\frac{1+\beta^2}{2}} \zeta + 2gv(1+\beta) \sqrt{\frac{1-\beta^2}{2}} \eta \right),$$

$$\tilde{\psi}_{02} = \frac{1}{\sqrt{2}} \zeta - \sqrt{\frac{1-\beta^2}{2}} \eta .$$

To identify the NG fermion from a linear combination of these zero-modes, we consider their SUSY transformation

$$\delta_e \tilde{\psi}_{01} = -\sqrt{2} f e, \quad \delta_e \tilde{\psi}_{02} = 0 ,$$

where $f$ is the order parameter of SUSY breaking and is given by the square root of the vacuum energy

$$f^2 = g^{00} \frac{\partial P}{\partial A^{(0)}} \frac{\partial \bar{P}}{\partial A^{(0)}} + \frac{\partial P}{\partial A^{(2)}} \frac{\partial \bar{P}}{\partial A^{(2)}} = \frac{m^2}{4g^2(1-\beta^2)^2} \{m^2 + 4g^2v^2(1-\beta^2)\} .$$

Since one of zero-modes $\tilde{\psi}_{02}$ has no contribution to SUSY breaking, we identify $\tilde{\psi}_{01}$ as the NG fermion $\psi_{NG}$ which can be rewritten using (51) as

$$\psi_{NG} = \frac{1}{\sqrt{m^2 + 4g^2v^2(1-\beta^2)}} \left( 2gv(1-\beta^2) \psi^{(0)} - m\psi^{(2)} \right) .$$
Thus the NG fermion superfield $\Phi$ in our model (45) can be chosen as
\[
\Phi = \frac{1}{\sqrt{m^2 + 4g^2v^2(1 - \beta^2)}} \left( 2gv(1 - \beta^2)\Phi^{(0)} - m\Phi^{(2)} \right). \tag{58}
\]
In this case, Lagrangian (45) has the following fermion mass terms between $\tau \equiv (\psi_+ - \psi_-)/i\sqrt{2}$ and gaugino $\lambda$
\[
{\mathcal{L}}_{\text{mass}} = -\frac{1}{2} \left( -2k^2h \right) \tau^\alpha \tau_\alpha - \frac{1}{2} \left( \frac{2\langle F \rangle}{M} \right) \lambda^\alpha \lambda_\alpha - (ek)\tau^\alpha \lambda_\alpha + \text{h.c.}, \tag{59}
\]
where $\langle F \rangle = -f$ and the NG fermion interaction terms
\[
{\mathcal{L}}_{\text{int}} = i\sum e \frac{-2gv\beta}{\sqrt{m^2 + 4g^2v^2(1 - \beta^2)}} \psi_{\text{NG}} \sigma^\mu \bar{\tau}_\mu v_\mu - \frac{\sqrt{2}}{M} \psi_{\text{NG}} \sigma^{\mu\nu} v_{\mu\nu} + \text{h.c.}. \tag{60}
\]
The mass matrix between $\lambda$ and $\tau$ becomes
\[
-\frac{1}{2} (\tau^\alpha \lambda^\alpha) \begin{pmatrix} a & m_{B,2}^2 \\ m_{B,2}^2 & b \end{pmatrix} \begin{pmatrix} \tau_\alpha \\ \lambda_\alpha \end{pmatrix}, \quad m_{B,2} \equiv ek, \quad a \equiv -2k^2h, \quad b \equiv -2f/M. \tag{61}
\]
This leads to the following eigenvalue equation
\[
m_F^2 - (a + b)m_F + ab - m_{B,2}^2 = 0, \tag{62}
\]
and so mass-eigenvalues are
\[
m_{F,1} = \frac{a + b}{2} + \sqrt{\left( \frac{a - b}{2} \right)^2 + m_{B,2}^2}, \quad m_{F,2} = \frac{a + b}{2} - \sqrt{\left( \frac{a - b}{2} \right)^2 + m_{B,2}^2}. \tag{63}
\]
By denoting corresponding mass-eigenstates as $\tilde{\psi}_1$ and $\tilde{\psi}_2$, their mixing matrix becomes
\[
\begin{pmatrix} \tau \\ \lambda \end{pmatrix} = \begin{pmatrix} V_{1,1} & V_{1,2} \\ V_{2,1} & V_{2,2} \end{pmatrix} \begin{pmatrix} \tilde{\psi}_1 \\ \tilde{\psi}_2 \end{pmatrix}, \quad V_{1,1} = \frac{m_{F,1} - b}{m_{B,2}} V_{2,1}, \quad V_{1,2} = \frac{m_{F,2} - b}{m_{B,2}} V_{2,2}, \tag{64}
\]
where we denote gauge boson and its mass as $(v_{i=2})_\mu \equiv (v_2)_\mu$ and $m_{B,2}$ instead of $v_\mu$ and $m_B$. Therefore NG fermion interaction Lagrangian Eq.(60) becomes
\[
{\mathcal{L}}_{\text{int}} = \frac{V_{2,1}}{\sqrt{2}f} b \psi_{\text{NG}} \sigma^{\mu\nu} \tilde{\psi}_1(v_2)_\mu - i \frac{V_{2,1}}{\sqrt{2}f} (-am_{F,1} + ab) \psi_{\text{NG}} \sigma^\mu \tilde{\psi}_1(v_2)_\mu + \text{h.c.}, \tag{65}
\]
\[
+ \frac{V_{2,2}}{\sqrt{2}f} b \psi_{\text{NG}} \sigma^{\mu\nu} \tilde{\psi}_2(v_2)_\mu - i \frac{V_{2,2}}{\sqrt{2}f} (-am_{F,2} + ab) \psi_{\text{NG}} \sigma^\mu \tilde{\psi}_2(v_2)_\mu + \text{h.c.}, \tag{65}
\]
and so these coupling constants are
\begin{align}
h^{(1)}_{\text{eff},1} &\equiv \frac{V_{2,1}}{\sqrt{2}f} b, \quad h^{(2)}_{\text{eff},1} \equiv -\frac{V_{2,1}}{\sqrt{2}f} (am_{F,1} + ab), \quad (66) \\
h^{(1)}_{\text{eff},2} &\equiv \frac{V_{2,2}}{\sqrt{2}f} b, \quad h^{(2)}_{\text{eff},2} \equiv -\frac{V_{2,2}}{\sqrt{2}f} (am_{F,2} + ab). \quad (67)
\end{align}

Now we consider the relation among these coupling and mass-splitting
\[
m^2_{B,2} - m^2_{F,i} = -(a + b)m_{F,i} + ab. \quad (68)
\]

First we consider \(m_B = 0\) and \(m_F \neq 0\). In this case, \(m_{B,2} = ek = 0\), \(i.e., a = 0, b = m_{F,2}\) and there is no mixing \(V_{2,2} = V_{1,1} = 1, V_{1,2} = V_{2,1} = 0\). This result agrees with Eq.(12)
\[
h^{(1)}_{\text{eff},2} = \frac{m_{F,2}}{\sqrt{2}f}. \quad (69)
\]

Next for \(m_B \neq 0\) and \(m_F = 0\), this is realized by taking \(ab = m^2_{B,2}\) for \(m_{F,2}\) in Eq.(63) and thus
\[
h^{(2)}_{\text{eff},2} = -\frac{V_{2,2}}{\sqrt{2}f} m^2_{B,2}, \quad (70)
\]
which is the result of Eq.(13) with mixing.

For general cases of \(m_B \neq 0\) and \(m_F \neq 0\), the mass-splitting Eq.(68) can be rewritten as
\[
m^2_{B,2} - m^2_F = -(a + b)m_F + ab = [-am_F + ab] + [-m_F b], \quad (71)
\]
and so the combination of two couplings \(h^{(2)}_{\text{eff}} + m_F h^{(1)}_{\text{eff}}\) leads
\begin{align}
\left( h^{(2)}_{\text{eff}} + m_F h^{(1)}_{\text{eff}} \right)_{2,1} &= -\frac{V_{2,1}}{\sqrt{2}f} \left( m^2_{B,2} - m^2_{F,1} \right), \quad (72) \\
\left( h^{(2)}_{\text{eff}} + m_F h^{(1)}_{\text{eff}} \right)_{2,2} &= -\frac{V_{2,2}}{\sqrt{2}f} \left( m^2_{B,2} - m^2_{F,2} \right), \quad (73)
\end{align}
which agree with the SUSY Goldberger-Treiman relation with mixing in Eq.(14).

Thus the couplings and mass-splittings obtained in a SUSY breaking model discussed in this section all satisfy our results in Eqs.(12)-(14).

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