Oscillations of recoil particles against mixed states

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ABSTRACT: Some consequences of the oscillations of neutral kaons and neutrinos are discussed, in particular, the possibility of oscillations of particles recoiling against kaons or neutrinos from the production process. We show that there are no stationary oscillations of these recoil particles in any order, and that the apparent long-wavelength oscillations, which might appear to result when an earlier treatment of ours was taken to higher order, are spurious. We show that the recoil particles may show a travelling interference pattern. It may be possible to observe this pattern for $\Lambda$s produced in a reaction, but there seems to be little hope of observing this for the case of neutrinos from muon decay.

I. Introduction

The subject of the oscillations of particles produced in a mixture of mass eigenstates has been discussed for many years. An early example is that of the neutral kaons. Hadronic reactions pro-
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duce these in strangeness eigenstates, \( K^0 \) and \( \bar{K}^0 \), which are mixtures of the mass eigenstates, \( K_L \) and \( K_S \). The possibility has also been discussed that neutrinos might show similar characteristics. It may be that the familiar flavour eigenstates, \( \nu_e, \nu_\mu \) and \( \nu_\tau \) are mixtures of mass eigenstates, \( \nu_1, \nu_2 \) and \( \nu_3 \). If so, they should show a similar behaviour to that of neutral kaons.

The quantum mechanics of the oscillations of such a system have been treated in many standard texts[1]. If a kaon is produced initially in a pure \( K^0 \) or \( \bar{K}^0 \) state, the system oscillates between a \( K^0 \) and a \( \bar{K}^0 \), approaching the equal mixture of a pure \( K_L \) state. This is the well-known phenomenon of strangeness oscillations[1]. Similar oscillations may occur for neutrinos produced in one of the flavour eigenstates, \( \nu_e, \nu_\mu \) or \( \nu_\tau \).

For some years, the oscillations of a neutral kaon or a neutrino were treated in isolation, without regard for the details of the reaction in which the particles are produced. Recently, a series of papers was published by Srivastava, Widom and Sassaroli[2,3,4] in which the kinematics of the production process are considered in detail. They point out that, in the usual experimental situations, the mass eigenstates (\( K_L \) and \( K_S \) for kaons, \( \nu_1, \nu_2 \) and \( \nu_3 \) for neutrinos) have different momenta and different energies, and they investigated the consequences of this for both the neutral particle and the other particles in the reaction.

In examining the calculation of Srivastava et al., we found[5] some errors in the treatment of the relation between coordinates in the various frames. On correcting these, we found results for the oscillation pattern that are different from those of ref. [2], and are, in fact, consistent with the conventional description of kaon or neutrino oscillations. This is surprising, because we incorporated the detailed treatment of the kinematics of the production process, pointed out by Srivastava et al. and by Goldman[6], and earlier by Boehm and Vogel[7]. It seems that this does not change the observed oscillation pattern. Our results have since been confirmed by other
One of the important differences between the results of ref. [2] and those of refs. [5,8,9] is that, in the former, oscillations in the strangeness of the neutral kaon are predicted to give rise to oscillations in the Λs produced in association with the kaon. No such effect was predicted in refs. [5,8,9] or in any earlier work. The possible existence of such oscillations in the neutrino case would be of immense practical importance; the experimentally challenging problem of detection of neutrino oscillations could be replaced by the much simpler problem of observation of, for example, muons from the decay $\pi \rightarrow \mu \nu$. Partly because of this, we have extended our earlier calculation, and in the present paper, we examine two aspects of the problem:

(a) In ref. [5], in common with all treatments so far published, the calculations are carried out only to first order in the mass difference $\delta m = m_L - m_S$. In fact, if our calculation is continued to higher orders, oscillations of the recoil particle appear. However, these are spurious; in section II we prove that in a full calculation there are no oscillations, and we show how these spurious oscillations appear.

(b) Because of the substantial importance of neutrino oscillation experiments, we searched for a possible experimental configuration in which neutrino oscillations could be inferred from measurements on the recoiling muon. There is no stationary oscillation pattern in the lab frame. However, a travelling oscillation pattern may exist. In section III we discuss some aspects of the wave packet solutions for these systems, and in section IV, we discuss the possibility of observing the travelling oscillation pattern. We conclude that, although it may be possible to observe this pattern for Λs produced in a reaction, it is not at present possible in the case of neutrino oscillations.

II. Exact treatment of kaon oscillations

We start by summarising the results of our earlier paper [5]. As in that paper, we consider the
case of a neutral kaon produced in the reaction

\[ \pi^- p \rightarrow \Lambda K^0. \]

The equations for neutrinos produced in a flavour eigenstate by a weak process require the treatment of at least three mass eigenstates. We do not consider the additional complications here, but we treat the case of two coupled mass eigenstates for which the equations are analogous. The kaon state at the moment of production, therefore, is the strangeness 1 meson \( K^0 \), which is a mixture of the mass eigenstates

\[
| K^0 \rangle = \sqrt{\frac{1+|\epsilon|^2}{2(1+\epsilon)^2}} (| K_S \rangle + | K_L \rangle)
\]

where \( \epsilon \) is the usual CP-violation parameter. Thus the \( \Lambda K \) state at the moment of production, \( t = 0 \), is

\[
| \Lambda K(t = 0) \rangle = \sqrt{\frac{1+|\epsilon|^2}{2(1+\epsilon)^2}} \{ | \Lambda_S K_S \rangle + | \Lambda_L K_L \rangle \}.
\]

The essential point here is that \( K_L \) and \( K_S \) are two different particles with different masses and different momenta. Thus the recoiling \( \Lambda \)s produced in association with them have different momenta. We denote these \( \Lambda \) states by \( \Lambda_L \) and \( \Lambda_S \) respectively; they are, of course, the same particle. Denoting the total center-of-mass energy by \( \sqrt{s} \), the center-of-mass momenta of the \( K_L \), \( K_S \), \( \Lambda_L \) and \( \Lambda_S \) are given by

\[
p_i^2 = \frac{(s - m_i^2 - m_\Lambda^2)^2 - 4m_i^2m_\Lambda^2}{4s}
\]

where \( i = L \) or \( S \).
The state (2) develops in time according to

\[ | \Lambda K(t) \rangle = \sqrt{\frac{1+|\epsilon|^2}{2(1+\epsilon)^2}} \{ a_S(\tau_{\Lambda_S}, \tau_{K_S}) | \Lambda_S K_S \rangle + a_L(\tau_{\Lambda_L}, \tau_{K_L}) | \Lambda_L K_L \rangle \} \]  (3)

where

\[ a_i(\tau_{\Lambda_i}, \tau_{K_i}) = \exp\{-i(m_{K_i} \tau_{K_i} + m_{\Lambda_i} \tau_{\Lambda_i}) - \frac{1}{2}(\Gamma_{K_i} \tau_{K_i} + \Gamma_{\Lambda} \tau_{\Lambda_i})\} \]  (4)

with \( i = S \) or \( L \). The four proper times, \( \tau_{KL}, \tau_{KS}, \tau_{\Lambda L} \) and \( \tau_{\Lambda S} \), are related to the time in the overall center-of-mass frame, \( t \), by the appropriate Lorentz transformations relating a point \((\xi_i, \tau_i)\) in the frame \( i \) to the point \((x, t)\) in the overall center-of-mass frame:

\[ \xi_i = \gamma_i(x - \beta_i t) \]  (5)

\[ \tau_i = \gamma_i(t - \beta_i x). \]  (6)

A crucial point in our paper[5] is that the two times \( \tau_{KL} \) and \( \tau_{KS} \) are related to a single space-time point, \((x_K, t_K)\), in the overall c.m. system, and similarly for the times \( \tau_{\Lambda L} \) and \( \tau_{\Lambda S} \) for the \( \Lambda \).

Although the analysis is carried out in plane-wave formalism, a more realistic solution in a practical case involves wave packets constructed from these plane waves. Construction of wave packets is discussed in more detail below; for now, we need only assume that the wave packets will be large enough that the \( K_L \) and \( K_S \) packets do not separate significantly, but are nevertheless small compared with the dimensions of the apparatus. Similar statements hold for the \( \Lambda \) momentum components, \( \Lambda_L \) and \( \Lambda_S \). Further, the packet will be centered approximately on
the position of the classical particle. This condition specifies a relation between the time and position of observation of a particle. If we choose to observe at a space point $x$, then the time of observation must be $t = x/\beta$, where $\beta$ is the classical particle velocity. The velocity $\beta$ could be chosen to be $\beta_L$ or $\beta_S$ or some average of these. In our previous work[5], we used $\overline{\beta}$, the average of $\beta_L$ and $\beta_S$. Here, we leave the choice of $\beta$ until later; it will turn out to be important, but for now, we merely require that it does not differ appreciably from $\beta_L$ or $\beta_S$.

The derivation now proceeds as in ref. [5]. Since $t = x/\beta$, the Lorentz transformations (6) give

$$\tau_i = \gamma_i \left( \frac{x}{\beta} - \beta_i x \right) = \gamma_i x \left( \frac{1}{\beta} - \beta_i \right).$$

Then (4) becomes

$$a_i(t) = \exp \left[ -i \left( m_{\Lambda} \gamma_{\Lambda_i} \left( \frac{1}{\beta_\Lambda} - \beta_{\Lambda_i} \right) x_\Lambda + m_{K_i} \gamma_{K_i} \left( \frac{1}{\beta_K} - \beta_{K_i} \right) x_K \right) 
- \frac{1}{2} \left( \Gamma_{\Lambda} \gamma_{\Lambda_i} \left( \frac{1}{\beta_\Lambda} - \beta_{\Lambda_i} \right) x_\Lambda + \Gamma_{K_i} \gamma_{K_i} \left( \frac{1}{\beta_K} - \beta_{K_i} \right) x_K \right) \right].$$

and the state vector at center-of-mass time $t$ is

$$| \Lambda K(t) \rangle = \sqrt{\frac{1+|\epsilon|^2}{2(1+\epsilon)^2}} \{ a_S(t) \langle \Lambda_S K_S | + a_L(t) \langle \Lambda_L K_L | \}.$$  

As in ref.[5], we project out a specific strangeness for the kaon. If we choose strangeness 1, we get

$$\psi_{\Lambda K^0}(x_{K^0}, x_\Lambda) = \sqrt{\frac{1+|\epsilon|^2}{2(1+\epsilon)^2}} \{ a_S(t) \langle \Lambda K^0 | \Lambda_S K_S \rangle + a_L(t) \langle \Lambda K^0 | \Lambda_L K_L \rangle \}$$

$$= \frac{1}{2} \{ a_S(t) + a_L(t) \}.$$
and the $K^0$ part has the opposite sign in the bracket, $\{a_S(t) - a_L(t)\}$. Writing $a_i(t)$ as $a_i(t) = \exp(-ib_i - c_i)$, the joint probability distribution for detection of a $K^0$ and a $\Lambda$ is given by

$$P(x_\Lambda, x_{K^0}) = \frac{1}{4} \left| a_S(t) + a_L(t) \right|^2$$

$$= \frac{1}{4} \left\{ |a_S(t)|^2 + |a_L(t)|^2 + 2e^{-c_{S+L}}\cos(b_L - b_S) \right\}$$

$$= \frac{1}{4} \left\{ e^{-2c_L} + e^{-2c_S} + 2e^{-(c_{S+L})}\cos(b_L - b_S) \right\}. \quad (11)$$

The oscillations arise from the cosine term, $\cos(b_L - b_S)$, where

$$b_i = m_{\Lambda}\gamma_{\Lambda_i} \left( \frac{1}{\beta} - \beta_{\Lambda_i} \right)x_{\Lambda} + m_{K_i}\gamma_{K_i} \left( \frac{1}{\beta} - \beta_{K_i} \right)x_{K}. \quad (12)$$

In ref. [5], the quantity $(b_L - b_S)$ was evaluated to first order in $\delta m$ from the relation $b_L - b_S = \langle \partial b/\partial m \rangle \delta m$. The result was that $(b_L - b_S)$ is a function of $x_K$ but not of $x_\Lambda$, showing that in spite of a more detailed treatment of the kinematics than in conventional derivations, there are no oscillations of the $\Lambda$ to first order. We also found that the kaon oscillations have the same wavelength as in the usual derivation, not as in ref.[2]. The possibility that oscillations exist in higher order is of considerable experimental importance for the case of neutrino oscillations. If their wavelength were not prohibitively long, then detection of oscillations of a recoil particle, for example muons from the $\pi \rightarrow \mu \nu$ decay, might provide an indirect, relatively simple method for studying neutrino oscillations. However, we shall show that such oscillations cannot exist in any order.

To prove this, observe that the coefficient of $x_\Lambda$ in the expression for $b_L - b_S$ is

$$m_{\Lambda}\gamma_{\Lambda_L} \left( \frac{1}{\beta} - \beta_{\Lambda_L} \right) - m_{\Lambda}\gamma_{\Lambda_S} \left( \frac{1}{\beta} - \beta_{\Lambda_S} \right).$$
Setting this equal to zero, we can solve for \( \beta \). We denote the resulting value by \( \beta^* \), and we find

\[
\beta^*_\Lambda = \frac{E_{\Lambda_L} - E_{\Lambda_S}}{p_{\Lambda_L} - p_{\Lambda_S}} = \frac{p_{\Lambda_L} + p_{\Lambda_S}}{E_{\Lambda_L} + E_{\Lambda_S}}. \tag{13}
\]

We denote by \( S^* \) the frame defined by the velocity \( \beta^* \). The \( S^* \) frame is the c.m. frame of the two components of the \( \Lambda \), i.e. the frame in which \( \Lambda_L \) and \( \Lambda_S \) have equal energy and opposite momenta. Thus, to the extent that one may choose \( \beta^* \) rather than \( \beta \) to define the time of observation, one may prove that the coefficient of \( x_\Lambda \) vanishes exactly, and hence there can be no stationary oscillations of the \( \Lambda_s \) from \( \pi^- p \to \Lambda K^0 \) nor, by analogy, of muons from \( \pi \to \mu \nu \).

For the kaon, we can again define a frame in which the two components have equal and opposite momenta, by

\[
\beta^*_K = \frac{p_{K_L} + p_{K_S}}{E_{K_L} + E_{K_S}}. \tag{14}
\]

In this frame, the \( S^* \) frame for the kaon, the coefficient of \( x_K \) in the expression for \( (b_L - b_S) \) is exactly

\[
\frac{m_L^2 - m_S^2}{p_L + p_S}.
\]

To first order in \( \delta m \), this is \( m\delta m/p \), in agreement with our previous work[5] and with the standard result[1].

The origin of the apparent long-wavelength \( \Lambda \) oscillations that appear if the treatment of ref.[5] is taken to higher orders is now clear. In their corresponding \( S^* \) frames, \( p^*_{\Lambda_S} = -p^*_{\Lambda_L} \) and \( p^*_K = -p^*_L \). The energies are equal for the \( \Lambda \), i.e. \( E^*_{\Lambda_S} = E^*_{\Lambda_L} \), but for the kaon, \( E^*_K = E^*_L + m_K \delta m/E^*_S \). Also \( \Gamma^*_\Lambda \) is the same for \( \Lambda_L \) and \( \Lambda_S \) in the \( S^* \) frame of the \( \Lambda \), since the \( \Lambda_L \) and
Λs move at the same speed in this frame. In $S^*$ frame variables, then, the wave function, from (10), is

$$\psi(x_\Lambda, x_K) = \frac{1}{2} \left[ \exp\{i(p_{\Lambda s}^* x_\Lambda^* - E_{\Lambda s}^* t_\Lambda^*) - \frac{1}{2}\Gamma_{\Lambda s}^* t_\Lambda^* + i(p_{KS}^* x_K^* - E_{KS}^* t_K^*) - \frac{1}{2}\Gamma_{KS}^* t_K^*\} + \\ \exp\{i(p_{KL}^* x_\Lambda^* - E_{KL}^* t_\Lambda^*) - \frac{1}{2}\Gamma_{KL}^* t_\Lambda^* + i(p_{Ks}^* x_K^* - E_{Ks}^* t_K^*) - \frac{1}{2}\Gamma_{Ks}^* t_K^*\} \right]$$

where $(x_\Lambda^*, t_\Lambda^*)$ and $(x_K^*, t_K^*)$ are coordinates in the $S^*$ frame of each particle. The probability distribution is

$$P(x_\Lambda^*, x_K^*) = \frac{1}{4} \exp(-\Gamma_{Ks}^* t_K^*) \left[ \left( \exp(-\Gamma_{KL}^* t_K^*) + \exp(-\Gamma_{Ks}^* t_K^*) \right) + \right.$$  
$$\left. 2 \exp\left(-\frac{1}{2}(\Gamma_{KS}^* + \Gamma_{KL}^*) t_K^* \right) \cos \left(2p_{\Lambda s}^* x_\Lambda^* + 2p_{Ks}^* x_K^* + \frac{\delta(m^2)}{2E_K^*} t_K^* \right) \right]$$

For given $x_K^*$ and $t_K^*$, this distribution oscillates as a function of $x_\Lambda^*$, and therefore is an interference pattern that is stationary in the $S^*$ frame of the Λ. Now the frames $S_L$, $S_S$ and $\overline{S}$, defined by the velocities $\beta_L$, $\beta_S$ and $\overline{\beta}$, are almost the same as the $S^*$ frame, since $\delta(m^2)$ is small, but are not quite identical. Thus the frames $S_L$, $S_S$ and $\overline{S}$ move slowly in the frame $S^*$. As the Λ moves out from the reaction point, each of these frames moves slowly across the interference pattern (16), giving the appearance of slow, long-wavelength oscillations. These oscillations are not real; they appear to be present if one looks only at the origin of a frame moving with the particle, and not at the full picture. In fact, eqn. (16) shows that there can be no oscillations of the Λ or muon; the antinode in the $S^*$ frame, defined by the reaction event, passes through all points on the trajectory of the Λ or muon at some time. Thus there can be no stationary (in the c.m. frame) node at any point on the particle’s path.
Just as the above treatment confirms the absence of $\Lambda$ oscillations, as derived in ref.[5], it also confirms the result of ref.[5] that the wavelength of kaon oscillations is given by the usual formula. Two expressions have been published that give different versions. Lipkin[10] showed that if the two mass eigenstates of the $K^0$ are regarded as having equal momentum, the oscillation wavelength may change by a factor of 2. However, the assumption of equal momentum is not correct for either of the situations discussed here. Srivastava et al.[2] treat the kinematics correctly but their error in the wavelength, which is greater than a factor of 2, especially near threshold, results from incorrect treatment of the transformations between the various rest frames (see ref.[5]). The origins of these two factors have sometimes been confused in the literature (see [11,12,13] and sect. V); we believe that our calculation (ref.[5] and the present paper) is the first to treat both the kinematics and the transformations between frames correctly.

III. Wave packeting

As in any scattering process, it is implicitly assumed that the plane-wave solutions discussed above will be used as a basis for the construction of wave packets in order to correspond with a realistic experimental situation (see, e.g., ref. [14]). In the case of oscillations, the use of wave packets is vital to the development of the interference pattern, and this section discusses the packeting in some detail. We assume here that the size of the wave packets will be much larger than the separation of the centers of the $\Lambda_L$ and $\Lambda_S$ packets. If this is not the case, then the two packets will separate and coherence will be lost.

We start with the 2-particle wave function for the final state ($\Lambda K$ or $\mu\nu$) from eqn. (10). The quantities $a_L$ and $a_S$ are given by eqn. (4), but we write the phases in the exponents in the c.m. frame:
\[ \psi_{\Lambda K^0}(x_\Lambda, x_{K^0}) = \]
\[ \frac{1}{2} \left[ \exp \left( i(p_{\Lambda L} x_\Lambda - E_{\Lambda L} t_\Lambda) - \frac{1}{2} \Gamma_{\Lambda} \tau_{\Lambda L} + i(p_{K_L} x_K - E_{K_L} t_K) - \frac{1}{2} \Gamma_{K_L} \tau_{K_L} \right) \right. \]
\[ + \exp \left( i(p_{\Lambda S} x_\Lambda - E_{\Lambda S} t_\Lambda) - \frac{1}{2} \Gamma_{\Lambda} \tau_{\Lambda S} + i(p_{K_S} x_K - E_{K_S} t_K) - \frac{1}{2} \Gamma_{K_S} \tau_{K_S} \right) \left. \right] . \]  
(17)

Following the usual procedure, we replace each sharp momentum, \( p \), by \( p + q \) where \( q \) has a Gaussian distribution:

\[ \phi(q_\Lambda) = e^{-q_\Lambda^2/2\sigma_\Lambda^2}; \quad \phi(q_K) = e^{-q_K^2/2\sigma_K^2}. \]  
(18)

The origin of the spread in the final-state momentum is discussed in sect. IV. Normally, it will result from a measurement on another particle in the system, possibly in the initial state, but that doesn’t affect the argument.

We apply (18) to each term in the wave function (17), giving

\[ \psi_{\Lambda K^0}(x_\Lambda, x_{K^0}) = \]
\[ \frac{1}{2} \int \left\{ \exp \left[ i \left( (p_{\Lambda L} + q_\Lambda) x_\Lambda - (E_{\Lambda L} + q_\Lambda \frac{\partial E}{\partial p_{\Lambda L}}) t_\Lambda \right) - \frac{1}{2} \Gamma_{\Lambda} \tau_{\Lambda L} \right] \times \exp [\Lambda \rightarrow K] \right. \]
\[ + \exp \left[ i \left( (p_{\Lambda S} + q_\Lambda) x_\Lambda - (E_{\Lambda S} + q_\Lambda \frac{\partial E}{\partial p_{\Lambda S}}) t_\Lambda \right) - \frac{1}{2} \Gamma_{\Lambda} \tau_{\Lambda S} \right] \times \exp [\Lambda \rightarrow K] \left. \right\} \]
\[ \exp \left( -q_\Lambda^2/2\sigma_\Lambda^2 - q_K^2/2\sigma_K^2 \right) \ dq_\Lambda \ dq_K \]
\[ = \frac{1}{2} \exp \left( i(p_{\Lambda L} x_\Lambda - E_{\Lambda L} t_\Lambda) - \frac{1}{2} \Gamma_{\Lambda} \tau_{\Lambda L} \right) \exp \left( -\frac{\sigma_\Lambda^2}{2} (x_\Lambda - \beta_{\Lambda L} t_\Lambda)^2 \right) \exp [\Lambda \rightarrow K] \]
\[ + \exp \left( i(p_{\Lambda S} x_\Lambda - E_{\Lambda S} t_\Lambda) - \frac{1}{2} \Gamma_{\Lambda} \tau_{\Lambda S} \right) \exp \left( -\frac{\sigma_\Lambda^2}{2} (x_\Lambda - \beta_{\Lambda S} t_\Lambda)^2 \right) \exp [\Lambda \rightarrow K] \], \]  
(19)

where \([\Lambda \rightarrow K]\) denotes similar factors with \( \Lambda \) replaced by \( K \). The two Gaussian factors for \( L \) and \( S \) are indistinguishable in practical terms. Thus the \( \Lambda \) and \( K^0 \) probability distribution that
follows from this is the same as eqn. (11) for the plane-wave case except that it is multiplied by \( \exp \left( -\sigma^2 x^2 - \sigma^2 x^2 \right) \). In the \( S^* \) frame, the probability distribution (16) becomes

\[
P(x^*_\Lambda, x^*_K) = \frac{1}{4} \exp \left( -\sigma^2 x^2 - \sigma^2 x^2 \right) \exp \left( -\Gamma^* t^*_\Lambda \right) \left[ \exp(-\Gamma^* t^*_K) + \exp(-\Gamma^* t^*_K) \right] + 2 \exp \left( -\frac{1}{2}(\Gamma^*_K + \Gamma^*_K) t^*_K \right) \cos \left( 2p^*_\Lambda x^*_\Lambda + 2p^*_K x^*_K + \frac{\delta(m^2)}{2p_K x_K} \right).
\] (20)

Here, \( \sigma^* = \sigma/\gamma^* \) is the Gaussian momentum spread in the \( S^* \) frames. Eqn. (20) then gives the probability, as a function of variables in the \( S^* \) frames, for observation of a \( \Lambda \) in conjunction with a neutral kaon with \( S = 1 \). As one might anticipate, the wave packet provides an envelope for the travelling oscillation patterns, but does not otherwise affect the analysis. As in eqn. (16), therefore, there are oscillatory patterns in \( x^*_\Lambda \) and \( x^*_K \), and the patterns are stationary in their respective \( S^* \) frames. They can exist only if the widths of the wave packets, \( 1/\sigma_\Lambda \) and \( 1/\sigma_K \), are large enough that the pattern is not heavily damped away from the points \( x^* = 0 \). If the wave packets are narrow, so that \( P(x^*_\Lambda, x^*_K) \) is appreciable only when \( p^*_\Lambda, p^*_K \sim 0 \), then the particles will only be observed essentially at their classical points. In this case, the argument of the cosine term reduces to the value \( (\delta(m^2)/2p_K) x_K \), which is familiar from the standard treatment of strangeness oscillations[1].

### IV. Observability of travelling oscillations

The discussion of section II shows that there is no stationary oscillation pattern in the overall c.m. system for either of these cases, to any order in \( \delta(m^2) \). However, there is a pattern (eqn. (20)) that is stationary in the \( S^* \) frame for the \( \Lambda \) and we now examine the possibility of observing this pattern. Of course, the most important potential application of this would be the study of neutrino oscillations by measurements on the muon from the \( \pi \rightarrow \mu \nu \) decay, but we also discuss
the case of Λs produced in the $\pi^- p \to \Lambda K^0$ reaction.

The first important point is that this pattern, stationary in the $S^*$ frame of the recoiling muon or Λ, exists only if the neutrino flavour or the kaon strangeness is measured. This can be seen from eqn. (11) for the probability distribution of a Λ recoiling against a $K^0$. If we had selected a $\bar{K}^0$ rather than a $K^0$, then the cosine term in eqn. (11) would have a minus sign. If we don’t observe the strangeness of the kaon, then we must add these two probability distributions and the cosine term drops out.

A further basic requirement is that it is necessary to measure coordinates for both particles in the final state. As can be seen in eqn. (20), both $x^*_\Lambda$ and $x^*_K$ occur in the argument of the cosine term; if either particle is unobserved, we must integrate eqn. (20) over it, and the oscillatory term in the $S^*$ frame vanishes. This is in contrast to the situation described by eqn. (11), where particles are observed at their classical points, so that $x^*_\Lambda \sim x^*_K \sim 0$. Then, the argument of the cosine is a function of $x^*_K$ but not of $x^*_\Lambda$, so that integration over $x^*_\Lambda$ does not change the kaon oscillation pattern.

To summarise, there are four requirements to observe the travelling oscillation pattern (20) in the $S^*$ frame:

(i) A detector to determine the kaon strangeness or the neutrino flavor.

(ii) Detectors to measure both the time and position of each particle with appropriate resolution (see below). It is important to measure both time and position because the $S^*$ frame is moving, and it is necessary to know the position in the $S^*$ frame at which the particle is detected.

(iii) Some method to determine the position of the $S^*$ frame, since the observations must be transferred to that frame.

(iv) The Λ oscillation pattern, eqn. (20) is centered on the production antinode which is stationary
the $S^*$ frame, i.e. the classical particle position, and extends on either side of this by a distance determined by the spacial width of the wave packet. It is therefore necessary to prepare the state in a wave packet which is broad enough to cover a sufficient width of the oscillation pattern.

We can estimate the dimensions of the travelling interference pattern from eqn. (20). For the $\pi^- p \rightarrow \Lambda K^0$ reaction, we assume that the $\Lambda$s are produced in the at a center-of-mass energy 0.2 GeV above threshold. Using the standard value of $\delta m = 3.5 \times 10^{-15}$ GeV, the wavelength of $\Lambda$ oscillations in the $S^*$ frame is predicted to be 40 cm. For muons from the $\pi \rightarrow \mu \nu$ decay, we must make an assumption about the neutrino masses. If we assume $m_{\nu_\mu} \sim 10$ eV and $m_{\nu_e} \sim 0$ eV, then the predicted neutrino oscillation wavelength is about 50 cm. Smaller values for $m_{\nu_\mu}$ predict longer oscillation wavelengths.

The determination of the location of the $S^*$ frame is rather different for the two cases. For the $\pi p \rightarrow \Lambda K^0$ reaction, the necessary information could be provided by a detector in the pion beam, since the group velocities of all wave packets are known. This relies on the fact that the reaction time is negligible, so that the time of arrival of the pion at the proton is essentially the same as that when the $S^*$ frame coincides with the overall c.m. frame. It would be necessary to measure positions to about 1 cm and times to about 1 ns or better, for both the kaon and the $\Lambda$.

Alternatively, a measurement of the kaon coordinates can also be used to locate the position of the antinode in the $S^*$ frame of the $\Lambda$, since this antinode is at the classical $\Lambda$ position which can be determined from the kaon coordinates. This measurement is required in any case for point (ii) above. In this case, measurement of the kaon narrows the wave packets for both the kaon and the $\Lambda$ to widths determined by the time resolution. Since the kaon wave packet is centered on the point of observation, it follows that $x^*_K = 0$. The position of the $S^*$ frame of the $\Lambda$ can readily be calculated from the kaon detection coordinates and the known classical velocities.
The first of these methods will not work for the $\pi \to \mu \nu$ decay, since the pion lifetime is not negligibly small. However, the information could again be provided by the neutrino detector since we can be sure that the neutrino and muon start from the pion decay point at the same time, and their velocities are known.

The other problem lies in the requirement (iv), the preparation of a quantum state with a sufficiently long wave packet. The problem is quite different for the $\Lambda K$ and $\mu \nu$ cases, since the state is prepared differently in these two cases. For the production of a $\Lambda K^0$ pair in the $\pi^- p \to \Lambda K^0$, the size of the final-state wave packets is likely to be determined by the localisation in space-time of the incident pion. This may be determined by a detector in the beam. If so, the pion is localised in time by the time resolution, $t_{\text{res}}$, of the detector, and in space by $\beta_{\pi} c t_{\text{res}}$. Typically, a detector would have a time resolution of about $10^{-9}$ sec, which would give a spatial wave packet of about 30 cm. A wave packet this small would suppress the interference pattern of eqn. (20), since the Gaussian would have fallen off somewhat by the first zero in the oscillatory term. The counter would have to be carefully designed to produce a coherent wave packet over its time resolution, which would have to be significantly longer than $10^{-9}$ sec. If there is no counter in the incident beam, then the wave packeting will probably be produced by the properties of the accelerator. There seems to be little discussion in the literature of the coherence length of accelerator beams, and we know of no experimental measurements that would determine whether a coherence length of several m is feasible. It should be remarked that if the coherence length of the accelerator beam is too short, then the use of a detector on the beam will not produce the desired wave packet, since the packet size will then be driven by the accelerator coherence length rather than the detector time resolution.

In any case, the kaon detector is likely to be the limiting factor that determines the packet size for the $\Lambda$. When the kaon is detected, the system is prepared in a new quantum state in which
the packet widths of both the kaon and the Λ are determined by the time resolution of the kaon detector.

The case of muons from the $\pi \rightarrow \mu \nu$ decay is rather different. The pion lifetime is $2.6 \times 10^{-8}$ sec, and if there are no measurements on the decay products and no limitations on the observation time, this decay time will determine the packet size. Again, however, the neutrino detector is likely to determine the packet width for the muon, by preparing a state in which the muon position is defined to some accuracy, which will ultimately determine the coherence length of the muon.

V. Discussion

We have shown that there are no stationary oscillations in the overall c.m. system of the recoil particle under any circumstances, in any order, and that the oscillations of the mixed particle, the $K^0$ or Λ, have a wavelength given by the conventional expression. This is consistent with several other recent treatments, but there has been some confusion in certain preprints over two possible deviations from this expression for the wavelength. Lipkin[10] pointed out that an error of a factor of 2 may result if the two neutrino or kaon components are regarded (incorrectly) as having the same momentum. A different error, which is always greater than 2, occurs in the work of Srivastava et al.[2,3,4]. As pointed out in section II, this is of quite a different origin, though Kiers and Weiss[11] and Mohanty[12] seem to imply that they are the same. In fact, the treatment of Kiers and Weiss[11] differs from ours in two ways. Firstly, they take the source to be infinitely massive, so the kinematics are less realistic. Also, their detector involves an inverse $\beta$ decay, and is sharply resonant. This can give rise to detection effects which don’t occur with the non-resonant detectors assumed here.

We have also shown that wave-packeting the states of the initial particles produces the expected
results on the final state. This is in agreement with other publications, especially that of Grimus and Stockinger[15] who seem to be the first to discuss this in the recent literature. Their treatment is more detailed and sophisticated than ours, but the result is the same. Wave-packeting is also discussed by Giunti et al.[13], whose results are again generally consistent with ours. In particular, they point out, in agreement with our treatment and with Srivastava et al., that the requirement of exact 4-momentum conservation at the neutrino production point implies a mixture of 3-momenta for the neutrino state.

Although there is no stationary interference pattern for the recoil (unmixed) particle, we have shown that, under the right circumstances, a travelling interference pattern should exist. This pattern is stationary in a very specific frame, that in which the two momentum components of the recoil particle have equal energy. To observe this in the \( \pi p \rightarrow \Lambda K^0 \) case requires the appropriate time and position measurements to determine the location of this frame and also a measurement of the strangeness of the neutral kaon. Further, it is necessary to prepare the initial state with a sufficiently long coherence length. It may be possible to achieve all of these requirements; probably a better understanding of the coherence length of accelerator beams is needed to design an experiment. For the more important case of a muon recoiling against a mixed neutrino, the experimental requirements are much more difficult to achieve. Unfortunately, it seems that the neutrino detector must be able to measure the neutrino flavour and also the time and position of the neutrino detection with the appropriate accuracy. Such a detector would presumably be capable of observing the neutrino oscillations directly; if so, the possibility of inferring neutrino oscillations from measurements on the muon alone, which was one of the motivations for this work, would not be realised.
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