Signals for the QCD phase transition and critical point in a Langevin dynamical model

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Abstract. The search for the critical point is one of the central issues that will be investigated in the upcoming FAIR project. For a profound theoretical understanding of the expected signals we go beyond thermodynamic studies and present a fully dynamical model for the chiral and deconfinement phase transition in heavy ion collisions. The corresponding order parameters are propagated by Langevin equations of motions on a thermal background provided by a fluid dynamically expanding plasma of quarks. By that we are able to describe nonequilibrium effects occurring during the rapid expansion of a hot fireball. For an evolution through the phase transition the formation of a supercooled phase and its subsequent decay crucially influence the trajectories in the phase diagram and lead to a significant reheating of the quark medium at highest baryon densities. Furthermore, we find inhomogeneous structures with high density domains along the first order transition line within single events.

1. Introduction

What do we know about the phase diagram of QCD and especially the transition between the confined and deconfined or chirally broken and restored phases? For vanishing baryochemical potential $\mu_B$, lattice QCD data confirm a smooth crossover \cite{1}. Studies of effective models indicate a first order phase transition at large values of $\mu_B$ \cite{2, 3}, which necessarily leads to the existence of a critical end point (CEP). In a thermalized system, the correlation length at the CEP diverges, which lead to the suggested search for an enhancement of event-by-event fluctuations in heavy-ion collisions \cite{4, 5}. Nevertheless, the finite size of such systems together with the divergent relaxation times near the CEP will at least weaken the expected signal \cite{6}. On the other hand, nonequilibrium effects like spinodal instabilities \cite{7} may enhance fluctuations at a dynamical first order phase transition.

We investigate these effects within the setup of nonequilibrium chiral fluid dynamics \cite{8, 9, 10, 11, 12} with explicit propagation of the Polyakov loop, taking into account effects of both the chiral and the (de-)confinement phase transition. We present results of fluid dynamic expansions with finite baryon number densities through the crossover, the CEP and the first order phase transition.
2. Nonequilibrium chiral fluid dynamics with propagation of the Polyakov loop

The starting point for our studies is the Lagrangian of the Polyakov-quark-meson model [13]

\[ \mathcal{L} = \bar{q} \left[ i \gamma^\mu \partial_\mu - ig_s \gamma^0 A_0 \right] - g \left[ \sigma + i \gamma_5 \vec{\pi} \cdot \vec{\pi} \right] q + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \vec{\pi})^2 - U(\sigma, \vec{\pi}) - U(\ell, \bar{\ell}) \]  

(1)

with the mesonic potential and a polynomial form of the Polyakov loop potential [14]

\[ U(\sigma) = \frac{\lambda^2}{4} (\sigma^2 - \nu^2)^2 - h_s \sigma - U_0 , \]

(2)

\[ \frac{\mathcal{U}}{T^4} (\ell, \bar{\ell}) = -\frac{b_2(T)}{4} (|\ell|^2 + |\bar{\ell}|^2) - \frac{b_3}{6} (\ell^3 + \bar{\ell}^3) + \frac{b_4}{16} (|\ell|^2 + |\bar{\ell}|^2)^2 . \]

(3)

We set the pion fields to zero, so from now on \( \pi = (\pi) = 0 \). The relevant degrees of freedom are the light quarks \( q = (u, d) \), the sigma field \( \sigma \) the Polyakov loop \( \ell \) as the order parameters of the chiral and deconfinement phase transition respectively. The relation between \( \ell \) and the temporal component of the color gauge field \( A_0 \) is

\[ \ell = \frac{1}{N_c} \left\{ \text{tr}_c P \exp \left( ig_s \int_0^\beta \text{d}\tau A_0 \right) \right\}_\beta , \]

(4)

with \( P \) denoting the path ordering operator. We assume fast local equilibration of the quarks and antiquarks, treating them as an ideal fluid. Their local thermodynamic properties are given by the grand canonical potential which in mean field approximation reads

\[ \Omega_{q\bar{q}} = -2N_f T \int \frac{\text{d}^3 p}{(2\pi)^3} \left\{ \ln \left[ 1 + 3(\ell + \bar{\ell} e^{-\beta(E-\mu)})e^{-\beta(E-\mu)} + e^{-3\beta(E-\mu)} \right] + \ln \left[ 1 + 3(\ell + \bar{\ell} e^{-\beta(E+\mu)})e^{-\beta(E+\mu)} + e^{-3\beta(E+\mu)} \right] \} . \]

(5)

(6)

(7)

Here \( E = \sqrt{p^2 + g^2 \sigma^2} \) is the quark energy and \( \mu = \mu_B/3 \) the quark chemical potential. We have the thermodynamic relations

\[ p(\sigma, \ell, T) = -\Omega_{q\bar{q}}(\sigma, \ell, T, \mu) , \]

(8)

\[ e(\sigma, \ell, T) = T \frac{\partial p(\sigma, \ell, T, \mu)}{\partial T} + \mu \frac{\partial p(\sigma, \ell, T, \mu)}{\partial \mu} - p(\sigma, \ell, T, \mu) , \]

(9)

\[ n(\sigma, \ell, T) = \frac{\partial p(\sigma, \ell, T, \mu)}{\partial \mu} . \]

(10)

together with the conservation laws for energy-momentum and baryon number

\[ \partial_\mu \left( T^{\mu\nu}_q + T^{\mu\nu}_\sigma + T^{\mu\nu}_\ell \right) = 0 \]

(11)

\[ \partial_\mu n^\mu = 0 . \]

(12)

which determine the dynamics of the quark fluid. The interaction of this locally thermalized background with the order parameter fields is incorporated into Langevin equations of motion. For the sigma field this could be derived self-consistently within the two-particle irreducible effective action [10],

\[ \partial_\mu \partial^\mu \sigma + \eta_\sigma(T) \partial_\ell \sigma + \frac{\partial V_{\text{eff}}}{\partial \sigma} = \xi_\sigma , \]

(13)
where the damping coefficient $\eta_\sigma$ is temperature dependent and vanishes only around the CEP allowing for critical fluctuations. For the Polyakov loop we deploy the idea that the dynamics of the order parameter in the vicinity of the phase transition can be described by a relaxation equation

$$\eta_\ell \partial_t \ell T^2 + \frac{\partial V_{\text{eff}}}{\partial \ell} = \xi_\ell .$$

(14)

Note that the Polyakov loop $\ell$ is originally defined as a static object in Euclidean space which makes this ansatz purely phenomenological, similar to work done in [15, 16]. The stochastic noise fields $\xi_\sigma$ and $\xi_\ell$ are connected to the respective damping coefficients via dissipation-fluctuation relations that ensure relaxation to the proper equilibrium state.

$$\langle \xi_\sigma(t, \vec{x}) \xi_\sigma(t', \vec{x}') \rangle = \frac{1}{V} \delta(t - t') \delta(|\vec{x} - \vec{x}'|) m_\sigma \eta_\sigma \coth\left(\frac{m_\sigma}{2T}\right),$$

(15)

$$\langle \xi_\ell(t, \vec{x}) \xi_\ell(t', \vec{x}') \rangle T^2 = \frac{1}{V} \delta(t - t') \delta(|\vec{x} - \vec{x}'|) 2 \eta_\ell T .$$

(16)

3. Nonequilibrium trajectories

We initialize a spherical droplet with the temperature $T_{\text{ini}}$ and quark chemical potential $\mu_{\text{ini}}$, the edges smoothed by a Woods-Saxon function

$$T(t = 0, \vec{x}) = \frac{T_{\text{ini}}}{1 + \exp\left([|\vec{x}| - r]/a\right)}, \quad \mu(t = 0, \vec{x}) = \frac{\mu_{\text{ini}}}{1 + \exp\left([|\vec{x}| - r]/a\right)} .$$

(17)

The system is propagated by full (3+1)d fluid dynamics. This resembles the situation after the collision of two heavy nuclei. We choose different initial conditions and extract the temperature and chemical potential averaged over a central volume during the evolution. The resulting trajectories in the $T$-$\mu$-plane are shown in fig. 1 and compared to the isentropes of the Polyakov-quark-meson model, fig. 2. We see that the trajectories follow the tendency of the isentropes to bend in the direction of positive chemical potential when they hit the crossover or phase transition line. This is a result of the rapid change of the dynamically generated quark mass $m_q = g \sigma$ at the phase boundary making this effect more pronounced with growing strength of the transition at larger $\mu$.

Furthermore we observe that the trajectories overshoot the first order phase transition line before this bending happens. This is a result of the nonequilibrium evolution that leads to
Figure 3. (Color online) Quark number density at $z = 0$ after $t = 6$ fm (left) and $t = 12$ fm (right) for an evolution through the CEP. The system evolves homogeneously, no strong fluctuations occur.

Figure 4. (Color online) Quark number density at $z = 0$ after $t = 6$ fm (left) $t = 12$ fm (right) for a first order transition. High density clusters develop.

supercooling during the evolution. Below $T_c$ the system gets trapped in the metastable high temperature phase until the spinodal line is reached and the barrier separating the minima vanishes. At values of highest $\mu$, where this effect is strongest, it leads to a reheating of the quark fluid.

4. Density fluctuations
To gain better insight into the processes during the evolution it is instructive to study the structure of fluctuations in a single event. The spatial delta functions in eq. 15 and 16 have the effect of averaging the noise as explained in [12]. We now allow for the formation of structures and domains by correlating the noise fields over a spatial volume of $1\text{fm}^3$ in each time step. The relative baryon number density $n/\langle n \rangle$ in the $z = 0$ plane is shown in fig. 3 for the evolution through the CEP and in fig. 4 for an evolution through the first order transition at high values of $\mu$. For each transition we show an intermediate state at $t = 6$ fm and a late one at $t = 12$ fm. We define $\langle n \rangle$ as the volume average of the baryon number density over all cells with $n$ bigger than a cutoff value.
We observe significant differences between the two transitions. Through the CEP it evolves rather homogeneously, with no strong fluctuations in the quark number density preserving a smooth shape. On the other hand, we observe the formation of irregular structures when the system evolves through the first order phase transition. Here, clusters of high density are embedded in regions with lower density. This is presumably a result of the nonequilibrium evolution through the first order phase transition. The system does not trespass the phase boundary smoothly but by forming domains of the high- and low-temperature phase that coexist for a certain amount of time until all domains of the chirally symmetric and deconfined phase have vanished. Such a behavior leads to large pressure gradients and further on to inhomogeneous structures in the fluid dynamically propagated quantities. In experiment, this should manifest itself e. g. in non-statistical fluctuations of hadron multiplicities [17].

5. Conclusions
We presented a model that allows a dynamical description of the QCD phase transition. A fluid dynamically expanding heat bath of quarks is coupled to the order parameters of the chiral and deconfinement transition. We saw that the nonequilibrium evolution strongly influences the behavior of an expanding system at the phase boundary. Supercooling can be observed in the spinodal region, followed by a reheating at highest baryochemical potential. Within a single event we observe that clusters of high baryon density build during the evolution through the first order phase transition. This is in contrast to the evolution through the CEP where the density remains uniformly distributed. These results can help to improve our understanding of signatures of the phase transition or the CEP at FAIR energies.

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