Analysis of cracked RC beams under vibration

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Abstract. Among the methods of monitoring of integrity, vibration analysis is more convenient as non-destructive testing (NDT) method. Many aspects regarding the vibration monitoring of the structural integrity of damaged RC elements have not been completely analysed in literature. The correlation between the development of the crack pattern on concrete surface under bending loadings, as well as the width and depth of cracks, and the variation of dynamic parameters on a structural element is an important aspects that has to be more investigated. This paper deals with cracked RC beams controlled by NDT based on natural vibration, which may be correlated to damage degree due to cracking of concrete under severe state of loading. An experimental investigation on the assessment of RC beams in different scale under loading has been done through dynamic tests in different constraint conditions of edges measuring frequency values and frequency variation. Envelope of Frequency Response Functions (FRFs) are shown and the changes of natural frequency values are related to the damage degree of RC beams subjected to static tests. Finally, a comparison between data obtained by finite element analysis and experimental results is shown.

1. Introduction

Damage of reinforced structures may be the result of many problems, some of them grown during their life as insufficient reinforcement, large deflections, poor concrete quality, corrosion of steel reinforcement or insufficient capacity. All these issues cause the cracking of structural elements; cracking and depth of cracks in reinforced concrete (RC) beams depict damage degree. Among the methods of monitoring of integrity and availability of beams there are obviously static tests; on the other hand, vibration analysis is more convenient as non-destructive testing (NDT) method [1-7]. Tests of vibration of undamaged RC beams and damaged permit the evaluation of variation of frequency values correlated to the damage of RC beams due to increased bending moment [8]. Although damage is generally a local phenomenon, whereas vibration response is a global characteristic, the dynamic characteristics of damaged structures have been associated to the location and size of damage [2]. The basic concept behind vibration monitoring is that dynamic characteristics are functions of structures’ physical properties, therefore, any change caused by damage results as a change in dynamic response [3]. There are several scientific reports in literature regarding the use of vibration data for locating damage in controlled conditions or in numerical simulations of elements having mainly homogeneous material. Though available results may be obtained with NDT based on the variation of dynamic parameters of RC beams [8-10], there are few results in scientific literature so that extensive investigation is yet necessary to develop convenient method of analysis. In this paper it is described an
investigation of RC vibration with different constraints at the ends and with different scale. Envelope of Frequency Response Functions (FRFs) obtained through dynamic testing are shown and the changes of natural frequency values are related to the damage degree of RC beams represented by the cracking on the concrete due to static bending tests of loading. At the end, the results are discussed comparing experimental and numerical results.

2. Free vibration of beam

Experimental values of frequency are compared below with theoretical data obtained considering as first approximation the behavior of undamaged RC beam as uniform homogeneous Euler-Bernoulli (EB) uniform slender beam. The natural frequencies of EB beam are considered neglecting gravity forces, the effects of rotary inertia, shear deformation and damping. For a beam in flexure only the component of displacement \( v \) is significant for us and \( v \) is function of \( x \) and \( t \). In the case of natural vibration of a uniform beam with homogeneous material, the expression of circular natural frequency \( \omega_i^f \) for generic \( r \) mode of vibration in the case of both free ends, is obtained [11]:

\[
\omega_i^f = \left( \frac{\xi_i \cdot r \cdot \pi}{L} \right)^2 \cdot \sqrt{\frac{EI}{\rho A}}
\]  

being the eigenvalue \( \lambda_i = r \cdot \pi/L \) for a simply supported beam. The eigenvalue for a free end beam at the \( r \) mode \( \lambda_i = r \cdot \pi/L \) may be correlated to the value \( \lambda_i^f \) for a simply supported beam: \( \lambda_i^f = \xi_i \cdot \lambda_i \), with \( \xi_i \) = coefficient that depends on the different \( r \) mode of vibration, equal to 1.506, 1.25, 1.167 and 1.125, respectively, for the first four modes. Circular frequency values of uniform beam with free-free edge expressed by Eq. (1) may be as it follows:

\[
\omega_i^f = \left( \frac{\xi_i \cdot r \cdot \pi}{L} \right)^2 \cdot \frac{\psi}{L \cdot \kappa}
\]  

where: \( \psi = \sqrt{E/\rho} \) is the dimensional length on time coefficient of beam; \( \kappa = L/i \) the slenderness and \( i \) the radius of inertia of section. The frequency ratio values for beam in different scale, B0 full scale, and BM0 (or BM1) in reduced scale, may be expressed by:

\[
\frac{f_{r,B0}}{f_{r,BM0}} = \frac{\omega_{r,B0}^f}{\omega_{r,BM0}^f} = \frac{\kappa_{BM0} \cdot L_{BM0}}{\kappa_{B0} \cdot L_{B0}}
\]  

considering beams characterized by the same uniform material. The experimental concrete beams tested by static and dynamic analysis are represented by beams: B0 and BM0-BM1 with different scale. The ratio of slenderness between the experimental beams BM0-BM1 and B0 adopted in the experimental vibration analysis is: \( \mu = \frac{i_{B0} \cdot L_{BM0}}{i_{BM0} \cdot L_{B0}} = \frac{\kappa_{BM0}}{\kappa_{B0}} \approx 0.9 \).

3. Experimental tests

Static and dynamic tests were planned for three RC beams strengthened with steel bars, beam B0 in full scale, and BM0 and BM1 beams, in reduced scale.

3.1. Static and dynamic tests on full scale RC beam B0

The dimensions of beam sections are 150mm·220mm and 1700mm in length. The steel reinforcement used was 4 bars measuring 10mm and stirrups at interval of 60mm having a diameter of 6mm. The beam was characterized by concrete having average cylinder strength equal to \( f_{c,av} \approx 44.3\,\text{N/mm}^2 \) and steel bars with an average yielding stress equal to \( f_{y,av} \approx 500\,\text{N/mm}^2 \). Set-up of test for full scale beam B0 is shown in Figure 1; bending tests were carried out on beam, increasing the load \( P \) applied in two points measuring 300mm (Figure 1). The instruments were vertical jacks for load \( P/2 \) in two points; strain gauges on steel bars at midspan section; LVDTs used to record deformation on the compressed edge.
concrete; LVDTs to measure the deflection at midspan section and close to the support. Beam was subjected to cyclic loading paths and, successively, increasing load till failure. Every cyclic loading corresponds to a damage degree $D_i$ with $i=1,…,5$. The main experimental results for beam B0 are summarized in Table 1. The failure of beam B0 was reached at $P\approx 47.7\text{kN}$ with crash of compressive concrete. The dynamic test was carried out with a mobile accelerometer measures the acceleration due to impact of hammer in a fixed point at 45mm from edge of beam (Figure 2a).

![Figure 1. Exp. static tests - set up of test for full scale RC beam B0.](image)

| Damage degrees | Load (kN) | Moment (kN·m) | $\delta$ deflection (mm) | $\varepsilon_{\text{c,amp}}$ concrete strain ($10^{-3}$) | $\varepsilon_{\text{s}}$ steel strain ($10^{-3}$) | $\chi$ curvature ($10^{-5}$·1/mm) |
|----------------|-----------|---------------|-------------------------|-------------------------------------------------|---------------------------------|-----------------------------|
| $D_1$          | 7.20      | 2.16          | 0.55                    | 0.20                                            | 0.66                            | 0.53                        |
| $D_2$          | 14.28     | 4.28          | 1.79                    | 0.49                                            | 1.09                            | 0.84                        |
| $D_3$ (yield of steel bars) | 31.54 | 9.46          | 5.07                    | 1.01                                            | 2.39                            | 1.84                        |
| $D_4$          | 38.68     | 11.60         | 8.61                    | 1.79                                            | 4.67                            | 3.22                        |
| $D_5$ (failure) | 47.76     | 14.32         | 28.02                   | -                                               | -                               | -                           |

The frequencies provided by 14 measuring points placed at regular distances had to be mediated so as not to coincide with the structure’s nodal or null points (Figure 2a) with an average of 10 beats per location. Given the impossibility of achieving a true free-free condition, testing proceeds using a suspension system with springs with constant $k\sim 0.01\text{ N/m}$. A FFT two-channel analyzer, and PULSE Labshop software were used for the dynamic tests and for experimental data acquisition. In order to evaluate the reliability of the measurements, a function referred to as coherence was considered. A comparison between the theoretical and experimental dynamic characterization of the beams without damage, $D_0$, is shown in Table 3 assuming parameters for RC beam B0 in Table 2.

![Figure 2. Set-up for (a) dynamic test on free-free edge - B0 and (b) hinge-hinge edge – BM1.](image)

In Table 3 the frequency values in damaged condition, $D_i$, with $i=1,…,4$ are indicated with values determined for beam B0 in un-damaged, $D_0$ and the frequency variations in percent with reference to the different damage degree are shown. The FRF envelopes, for the different damage degree, for every
position point of the accelerometer, are shown in Figure 3 for B0. In Figure 3 it is possible to observe the shift of the diagrams at various steps $D_i$ with reduction of the frequency values due to increasing concrete cracking damage.

### Table 2. Parameters of RC beam B0 to evaluate theoretical frequency values.

| Concrete Beam | Length of beam L [m] | Section of beam A [m$^2$] | Young’s Modulus $E_c$ [kN/m$^2$10$^5$] | Moment of inertia I [m$^4$10$^{-3}$] | Density $\rho$ [kg/m$^3$] |
|---------------|----------------------|---------------------------|------------------------------------------|-------------------------------------|-------------------|
| B0            | 1.7                  | 0.03483                   | 359.60                                   | 0.14357                             | 2376.12           |

### Table 3. Average frequency values for vibration of beam B0 with free-free edges.

| Damage degrees | $f_1$ (Hz) | $\Delta f_1/f_{D0}$ (%) | $f_2$ (Hz) | $\Delta f_2/f_{D0}$ (%) | $f_3$ (Hz) | $\Delta f_3/f_{D0}$ (%) | $f_4$ (Hz) | $\Delta f_4/f_{D0}$ (%) |
|----------------|------------|-------------------------|------------|-------------------------|------------|-------------------------|------------|-------------------------|
| Theor. EB beam | 307.73     | -                       | 848.24     | -                       | 1663.10    | -                       | 2749.01    | -                       |
| $D_0$          | 275.07     | -                       | 706        | -                       | 1274       | -                       | 1878       | -                       |
| $D_1$          | 249.93     | 9.14                    | 685        | 3.04                    | 1234       | 3.16                    | 1822       | 2.99                    |
| $D_2$          | 227.36     | 17.35                   | 635        | 10.06                   | 1190       | 6.59                    | 1753       | 6.65                    |
| $D_3$          | 215.79     | 21.55                   | 580        | 17.80                   | 1078       | 15.38                   | 1661       | 11.54                   |
| $D_4$          | 180        | 34.56                   | 520        | 26.28                   | 1000       | 21.54                   | -          | -                       |

**Figure 3.** Envelope exp. diagrams of FRFs for B0 at damage degrees $D_i$, $i=1,\ldots,4$.

Comparison between experimental and theoretical diagrams moment versus curvature and variation in percent of frequency values is shown in Figure 4. The comparison between static and dynamic values is a convenient method to correlate dynamic response to static damage degree. In particular, we can see the influence of the RC beam nonlinear behavior due to concrete cracking under loading on the beams in the different vibration modes considered. As shown in Figure 4, the variation percent of the frequency values as compared to the undamaged state, $D_0$, is always increasing from the elastic–linear uncracked phase (I-II), to the elastic-linear cracked phase (I-II), to the inelastic and plastic phase (I-III). In any case, the natural frequency values tend to describe the beam’s global response relative to loss of bending stiffness and they are less sensitive to local stiffness variations. In general, the percent of frequency reduction increases more markedly in the elastic–linear cracked phase (I-II) as compared to phase (0-I) non-cracked; the frequencies’ percent of variations increases considerably in phase (II-III), beyond flexing and, hence, when damage is permanent.

### 3.2. Static and dynamic tests of beams in scale BM0–BM1

Main parameters of RC beam in scale, BM0 and BM1, are shown in Table 4. BM0-BM1 were reinforced with 4 steel bars and stirrups with diameter of 6mm, with average yielding stress $f_{y,av} \sim 500$N/mm$^2$ and $E_s \sim 2.1 \cdot 105$N/mm$^2$. 

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Figure 4. Comparison of exp. and theor. diagrams of moment-curvature vs variation frequency - B0.

Table 4. Experimental geometric and mechanical parameters of RC beams in scale BM0-BM1.

| Concrete Beams | Width b [mm] | Height h [mm] | Length L [m] | Young’s modulus E_c [N/mm^2] | Density ρ [Ns^2/mm^4] | Moment of inertia I_g [mm^4] |
|----------------|--------------|---------------|--------------|----------------------------|------------------------|-----------------------------|
| BM0 – BM1      | 80           | 120           | 1.1          | 35.310                     | 2.547·10^9             | 11.5·10^6                   |

Beam BM1 (similar to BM0) was also subjected to the same cycle bending path with damage degrees D_1,...,D_5. In Table 5, main experimental results recorded for beam BM1 subjected to bending tests are shown. In Figures 5 view of cracking damage on beam BM1 are shown for damage degree D_5 being P~16kN. Beam BM0 and BM1 were tested dynamically after every cycle of loading: the beam BM0 was analysed by a suspension system while the BM1 with hinge-hinge end constraints (Figure 2(b)).

Table 5. Exp. results from static cycle tests for beam in scale BM1.

| Damage degrees | Load (kN) | Moment (kNm) | δ deflection (mm) | ε_{concrete} strain (10^{-3}) | χ curvature (10^{-5}·1/mm) |
|----------------|-----------|--------------|-------------------|-------------------------------|-----------------------------|
| D_1            | 4.00      | 0.70         | 0.67              | 0.15                          | 0.53                        |
| D_2            | 8.00      | 1.40         | 2.08              | 0.56                          | 2.30                        |
| D_3            | 12.42     | 2.17         | 3.61              | 0.85                          | 1.90                        |
| D_4            | 14.00     | 2.45         | 4.41              | 0.91                          | 3.20                        |
| D_5 (yield of steel bars) | 16.13 | 2.82         | 6.54              | 0.95                          | 3.74                        |

In Figure 6, variation in percent of experimental frequency values Δf/f_D0 (%) in comparing with condition D_0 for beam BM0 is shown at different damage degree D_i for first three modes. As in the case of full scale beam B0, it is possible to see how the cracking of the concrete influenced the dynamic behavior of in scale beam. The variation in percent of frequency values presents main variation in two phases: the first, between the undamaged state and cracking phase; the second, at the beginning of plastic phase for steel bars. BM1 was tested under vibration with hinge-hinge end condition made by a specific steel support system (Figure 2(b)).
Figure 5. Exp. cracking of BM1 under static test at damage degree D5.

Figure 6. Variation of exp. frequency values BM0 at different damage degree at first three r modes.

Experimental and theoretical frequency values respect to D0 for beam BM1 are shown in Tables 6 together with recorded frequency values at different damage degree D_i with i = 1, ..., 5 for first four r-modes 1, ..., 4 considering position of accelerometer no. 1.

The frequency variations in percent with reference to the undamaged condition was evaluated for each damage degree. The envelope diagrams of FRFs for BM1 are shown in Figure 7; it is possible to observe the shift of the diagrams of FRFs at various steps D_i with reduction of the frequency values due to increasing concrete cracking damage.

| Damage degrees | f_1  | Δf_1/f_D0   | f_2  | Δf_2/f_D0   | f_3  | Δf_3/f_D0   | f_4  | Δf_4/f_D0   |
|----------------|------|-------------|------|-------------|------|-------------|------|-------------|
| Theor. EB beam | 175.75 | -           | 703.00 | -           | 1581.75 | -           | 2812.00 | -           |
| D_0            | 180  | -703.00     | -    | -703.00     | -1234 | -703.00     | -1794 | -703.00     |
| D_1            | 164  | 8.89        | 686  | -2.08       | 1244 | -0.81       | 1774 | 1.11        |
| D_2            | 178  | 1.11        | 658  | 2.08        | 1218 | 1.29        | 1746 | 2.67        |
| D_3            | 176  | 2.22        | 650  | 3.27        | 1194 | 3.24        | 1734 | 3.34        |
| D_4            | 176  | 2.22        | 644  | 4.17        | 1180 | 4.37        | 1722 | 4.01        |
| D_5            | 166  | 7.78        | 492  | 26.78       | 1110 | 10.04       | 1634 | 8.91        |

By experimental tests we can conclude that the influence of cracking on the natural frequency values for increment of bending may be summarized as following: the variation in percent of frequency values is always growing respect the values at undamaged D_0 degree although it presents main variation in two phases: the first, between the undamaged state and cracking phase; the second, at the beginning of plastic phase for steel bars.

Referring to the frequency variations values in comparing to the undamaged condition, it can be noted that, in the case of measurements on beams in free-end condition, the variation is mainly higher. For hinge end constraint conditions at beams’ extremity monitoring the damage state through the vibration method is significant if the damage degree registered is high.

4. Finite element modelling
The FE modeling of RC beam B0 and in scale BM0 and BM1 was carried out with ANSYS code. Three types of models were created to simulate beams with all materials including concrete and longitudinal steel reinforcements and to reproduce the experimental constraint conditions.
Figure 7. Envelope exp. diagrams of FRFs for beam in scale BM1 with hinge-hinge ends at different damage degrees D_i, i=1,...,4.

Figure 8. Typical mesh for FE analysis: (a) item of B0 beam section; (b) item of in scale BM0 and BM1 beam section; (c) beams hang by springs; (d) BM1 beam with hinge-hinge end.

The eight-node solid brick elements - Solid65 - were used to model the concrete. A Link180 element was used to model steel reinforcement. Element Combin14 was inserted in the numerical modelling in addition to the aforementioned elements in order to model the beams’ suspension springs simulating the same free beam conditions in vibration. The element is defined by two nodes and a spring constant k equal to 0.01 N/m. The typical FE modelling of the reinforced beams is shown in Figure 9. A variable Young’s modulus E of concrete was used to take into account damage degree due to cracking; the moment of inertia I and density ρ have been assumed constant. The natural frequency values determined for each damage degree for the free-free in scale beam B0 and for the hinge-hinge in scale beam BM1 are contained in Table 7 with the variation of frequency values respect to D_0.

Table 7. FE frequency values at damage degree D_i for BM0 and BM1.

| Damage degree | f_1 (Hz) | Δf_1/f_D0 (%) | f_2 (Hz) | Δf_2/f_D0 (%) | f_3 (Hz) | Δf_3/f_D0 (%) | f_4 (Hz) | Δf_4/f_D0 (%) |
|---------------|---------|---------------|---------|---------------|---------|---------------|---------|---------------|
| D_0           | 355.72  | -             | 924.84  | -             | 1696.2  | -             | 2605.3  | -             |
| D_1           | 310.59  | 12.69         | 806.17  | 12.83         | 1475.76 | 13.00         | 2262.2  | 13.17         |
| D_2           | 268.63  | 24.48         | 695.47  | 24.80         | 1434.2  | 15.45         | 1939.9  | 25.54         |
| D_3           | 269.22  | 24.32         | 697.03  | 24.63         | 1437.7  | 15.24         | 1944.5  | 25.36         |
| D_4           | 270.85  | 23.86         | 701.36  | 24.16         | 1447.4  | 14.67         | 1957.1  | 24.88         |
| D_5           | 242.4   | 31.86         | 625.95  | 32.32         | 1139.1  | 32.84         | 1735.6  | 33.38         |

BM1 (hinge-hinge end condition)
Considering the variations of frequency values in particular for the first mode, we may note that the theoretical results obtained with FE modelling for free-free and hinge-hinge conditions are confrontable with experimental results by dynamic tests. The comparison between experimental and theoretical data confirms the suitability of experimental apparatus for dynamic tests both in the case of hinge-hinge end and free-free ends of beams.

5. Conclusions

The monitoring of RC beams during its life may be done through the dynamic response as NDT method based on the measure of frequency values of vibration. This paper describes experimental and theoretical results obtained analyzing the response of RC beams under vibration both in different undamaged and damaged degrees and in different constraint conditions. Dynamic analysis of full and in scale RC beams allows the evaluation of the drop in frequency measured values due to damage. Variation of frequency values are influenced both damage degree due to cracking and different constraint condition.

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