Effects of Various Inductances on the Dynamic Models of the Z-pinch Implosion of Nested Wire Arrays

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Abstract. The Z-pinch implosion dynamics of nested wire array have been described by its equivalent electric circuit. The currents flowing through the outer and inner array in the initial stage of implosion depend on the array dimensions, i.e. the currents depend on the inductances of the arrays. The arrays inductance varies can result in four dynamic models of the Z-pinch implosion of nested wire arrays. Fast and inexpensive 0D modeling predicts the implosion time and the rate of thermalization of the kinetic energy of the nested-wire array at Qingguang-I facility.

Keywords: Z-pinch implosion plasma, nested wire array load, inductance, dynamic model
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INTRODUCTION

In 1997 Davis firstly has suggested a concept of double wire array/nested concentric wire array,1 next year, the experiment of a nested wire array has been made successful on the Z machine.2 After then, many middle or small facilities one after the other carried out the nested-wire-array Z-pinch experiments. Z-pinch experiments have demonstrated that the soft x-ray power increases 40% with a nested-wire array compared with a single-layered wire array.2 In 2000 Lebedev reported3 that the two different modes of nested wire array implosion driven by MAGPIE current pulse were observed, determined by the fraction of total current induced in the inner array through skillfully altering the high of the inner array. In 2001 Chinttenden proposed three theoretical modes4 of the nested-wire-array Z-pinch implosion. Based on analyzing the RT instability, he given two interpretations as a result that first mode is called “hydrodynamic collision” mode and second is “transparent inner” mode. He also presumed that a third possibility is that the current flowing initially in the inner array generates a small seed magnetic flux between the arrays which is compressed by the imploding outer array. As the outer array approaches the inner, the amplitude of the compressed flux becomes large enough to push the inner array towards the axis ahead of the outer. Thus, a magnetic flux buffer prevents collision of the two arrays until both reach the axis almost simultaneously. Despite the fact that the optimal wire array parameters for each mode are different, there is as yet no conclusive evidence as to which elements of these three theoretical modes are in operation in experiments on Z. The development of the technique obtaining higher x-ray power using nested wire array configurations has been largely empirical. This has, in part, been due to the absence of adequate theoretical models to describe interaction of the two arrays. In addition, 2D MHD modeling of nested wire array implosions is so difficulty, and is very inconvenience to optimize the design of nested-wire arrays.

The nested-wire-array Z-pinch experiment carried out on the Qingguang-I facility in China. However the “hydrodynamic collision” mode cannot make clear phenomenon observed from the nested-wire-array experiments. Furthermore, we think that it is very important to determine an implosion model of a nested-wire array Z-pinch for designing optimal wire-array loads.
AN “INDUCTANCE” MODEL OF A NESTED-WIRE-ARRAY Z-PINCH IMPLOSION DYNAMICS

The kinetic energy of wire-array plasma changing into the internal energy and X-ray radiation energy is a key issue of Z-pinch implosion physical mechanisms. However, during the electromagnetism energy provided by a pulse power driver transferring into the plasma kinetic energy of a nested-wire-array load, the change of the load inductance severity affects the dynamic states of the Z-pinch plasma implosion. In order to understand and clarify the physical pictures of the nested-wire-array Z-pinch implosion, an “inductance” model of its dynamic has been presented for a variety of pulsed power systems. We have modeled nested-wire-array Z pinch experiments using point-mass (0-D) simulations methods, and just consider the dynamic processes of the wire-array Z-pinch implosion. The momentum equations of the inner array and outer array are coupled to an external circuit package.

For a Z-pinch facility, we know the drive voltage rather than the current. Then we describe the nested array by its equivalent electric circuit (see Fig. 1) and solve the corresponding circuit equation. If we model the driving circuit as a voltage source, $V(t)$, in series with a resistor, $Z_0$, an external inductor, $L_0$, and the nested-wire-array imploding plasma, then the total current can be found from the solution a circuit equation as follow

$$\left(L_0 + L(t)\right) \frac{dI}{dt} + Z_0 I + I \frac{dL(t)}{dt} = V(t)$$

(1)

Here, $L(t)$ is the total inductance of a nested-wire-array load, $L(t) = L_1(t) + L_2(t) + M$, where

$$L_1(t) = \frac{\mu_0}{2\pi} h_1 \ln \left( \frac{R_t}{r_1(t)} \right) + \frac{\mu_0}{2\pi} \frac{1}{N_1} h_1 \ln \left( \frac{r_1(t)}{N_1 r_{1c}} \right),$$

(2)

$$L_2(t) = \frac{\mu_0}{2\pi} h_2 \ln \left( \frac{R_t}{r_2(t)} \right) + \frac{\mu_0}{2\pi} \frac{1}{N_2} h_2 \ln \left( \frac{r_2(t)}{N_2 r_{2c}} \right).$$

(3)

Here, $N_1$ and $N_2$ is the number of wires in the inner and outer array, $\mu_0$ is the permeability of free space, $R_t$ is the radius where return conductors are located, $r_{1c}$ and $r_{2c}$ is the plasma-corona radius of wires on the inner- and outer-array. Obviously, the currents flowing through the inner and outer arrays ($I_1$ and $I_2$, respectively) in this stage of implosion should depend on the array dimensions (the radii $r_1$ and $r_2$), namely the array inductances $L_1$ and $L_2$. In order to estimate the fraction of the total current, $I$, flowing through the inner array in the implosion processes, we describe the nested array by its equivalent electric circuit (see Fig.1) and solve the corresponding circuit equation. From the total current $I = I_1 + I_2$ and Eq.(1), we find...
\[
\frac{I_1}{I} = \frac{L_2 - M}{L_1 + L_2 - 2M}.
\]

(4)

\(M\) is the coefficient of mutual inductance of the arrays,
\[
M(t) = \frac{\mu_0}{2\pi} \min(h_1, h_2) \ln \left( \frac{R_r}{\max(r_1(t), r_2(t))} \right).
\]

(5)

In a cylindrical array, all wires carry the same current \(I/N\), have the same radial coordinate \(r\), and their acceleration in the radial direction is defined by the formula from Ref. 5. Therefore, for the inner- and outer-array, the momentum equations are as follows
\[
m_1 \frac{dv_1}{dt} = -\frac{N_1 - 1}{N_1} \frac{\mu_0}{4\pi} \frac{I_1^2(t)}{r_1(t)}, \quad \frac{dr_1}{dt} = v_1,
\]

(6)

\[
m_2 \frac{dv_2}{dt} = -\frac{N_2 - 1}{N_2} \frac{\mu_0}{4\pi} \frac{I_2^2(t)}{r_2(t)}, \quad \frac{dr_2}{dt} = v_2.
\]

(7)

Given initial conditions and known driver parameters \(V(t)\), \(Z_0\) and \(L_0\), a set of Eq.(1), (6) and (7) can be solved and obtain the implosion trajectories of the inner \(r_1\) and outer \(r_2\) arrays, the kinetic energy of the inner- and outer-array, and the total current \(I\) divided through the inner \(I_1\) and outer \(I_2\) arrays represent the predictions by this conventional 0D model.

**ANALYSIS FOR NESTED-WIRE-ARRAY IMPLOSION DYNAMIC MODES**

Using mentioned above the “inductance” model, we worked out ZPI-0D program to describe nested wire array Z-pinch implosion. By changing the height \((h_1, h_2)\), initial radius \((r_{10}, r_{20})\), wire number \((N_1, N_2)\) and diameter, arrays mass \((m_1, m_2)\), and the return conductor radius \((R_r)\), the four dynamic modes of nested-wire-array implosion have been obtained shown as Fig. 2. The first mode of nested-wire-array implosion dynamic presents the inner and outer array individually motion and then at the same time pinch to the center axis of an array load. The second mode is that the outer array impacts the inner array and then both array move to axis together. The third mode is that the inner array through enough current pinch to the center before the outer array move to axis. The last mode is that the inner array wires remain discrete until the outer array material has passed through their gaps. The full current is then transferred rapidly to what was initially the inner, imploding it rapidly onto the outer array material on axis.

**FIGURE.2** Schematic showing the different trajectories of the four dynamical modes of nested array implosion.

We simulated the nested-wire-array Z-pinch experiment on the Qiangguang-I facility. In 07014 shot the inner- and outer-array has same wire number 12, using 3µm tungsten wire and 20 µm aluminous wire in the outer and inner respectively. The calculated load current shown in the Fig.3a is consistent with the experimental measurement basically. The implosion trajectory, the current partitions in the inner- and outer-array, and the total
current through the loads of 07014 shot are shown in Fig.3b and Fig.3c.

FIGURE.3. (a) The voltage and load current waveform of 07014 shot in the Qiangguang-I, and the simulated load current. (b) The implosion trajectory and (c) the current partitions in the inner- and outer-array, and the total current through the loads of 07014 shot.

Comparing with a wire dynamics model (WDM) presented by Esaulov and his results using it simulated the nested wire array implosion experiments at the 1 MA Zebra facility, we find that on both low-current facilities the nested wire array Z-pinch implosion dynamic exhibits the “transparent” mode. In addition, either “inductance” model or WDM, these fast and inexpensive 0D modeling can predict the array implosion time and the rate of thermalization of the kinetic energy, and can estimate the timing of the x-ray pulse.

CONCLUSIONS

A 0D analysis method coupling an equivalent circuit with point-mass model for nested wire array Z-pinch implosion was established in this paper. Our study indicated that the inner- and outer-array inductance distribution and alteration critically affect the implosion dynamic mode of nested wire array Z-pinch. The analysis and calculations showed that the initial inductances of a nested-wire-array load determined the initial current partition of the both arrays, and dynamic inductances direct affect the dynamic modes of nested-wire-array Z-pinch implosion. The four modes of nested-wire-array implosion dynamic were obtained. By analyzing the nested wire array Z-pinch experiments on the Qiangguang-I facility and comparing the experimental results on the Zebra facility, we found that in the low-current Z-pinch facility, if have no special management for loads, the nested-wire-array implosion dynamic exhibits the fourth mode also is “transparent” mode observed on the MAGPIE and Angara-5-1’s nested-wire-array Z-pinch experiments. This analysis method helps us to understand the implosion dynamic of nested wire array Z-pinches.

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