Study of Large Amplitude Solitons in Multispecies plasma with non-Maxwellian electrons

Harvinder Kaur\textsuperscript{1} and Tarsem Singh Gill\textsuperscript{2}
\textsuperscript{1}Department of Physics, Khalsa College, Amritsar-143002, India
\textsuperscript{2}Department of Physics, Guru Nanak Dev University, Amritsar-143005, India
E-mail:\textsuperscript{1}hkaur_21@yahoo.com, \textsuperscript{2}nspst99@yahoo.com

Abstract. In this investigation, authors study the propagation of ion-acoustic solitons in a plasma consisting of multispecies with different concentration of masses, charged states, and non-Maxwellian electrons. In the small amplitude approach the reductive perturbation method is used to derive the evolution equation such as Korteweg de-Vries (KdV). The present method, Sagdeev pseudopotential deals with the full nonlinearity of plasma equations. The potential so obtained is general in nature and valid for the large amplitude waves is characterized for solitons as a function of Mach number, temperature of different species, distribution of electrons, different concentration of species over a wide range of parameter space. In the small amplitude approach, we may also obtained KdV solitons. The compressive and rarefactive solitons are observed for the certain set of parameters.

1. Introduction

Nonlinear wave structures are amazing and beautiful manifestations of nature, and they comprise one of the most important aspects of modern plasma physics research. Amongst such structures, solitons present characteristics of interaction between waves and plasmas. Most investigated among the solitons are the ion-acoustic solitons /solitary waves. For linear wave study, we have ion-acoustic wave as a fundamental mode of the plasma. Similarly, ion-acoustic soliton arises in nonlinear theory and its origin is due to the delicate balance between nonlinearity and dispersion property of the plasma. Space environment constitutes a magnificent laboratory for the plasma phenomena, and nonlinear wave structures have been observed through on board spacecraft instrumentation for the last many years. Moreover, space plasmas are of multispecies type and provide a rich source for studying nonlinear waves and solitons. Multispecies plasma have different compositions systems, for example, Ar\textsuperscript{+} plasma with F\textsuperscript{-} negative ions, H\textsuperscript{+} plasma with O\textsuperscript{2}\textsuperscript{-} negative ions, a H\textsuperscript{+} plasma with O\textsuperscript{2}\textsuperscript{-} negative ions etc.[1]. Such plasma systems are occur in D region of the ionosphere.

Some nonlinear partial differential equations possess special solutions. These solutions are called solitons, only belong a limited class of evolution equations, such as KdV, mKdV, nonlinear Schrodinger (NLS) equations etc. Some form of reductive perturbation method is used to derive such equations. Although the methods are elegant and mathematically rigorous, they are acceptable only for weak nonlinear phenomena. Theoretical study by Davies etal.[2]is probably the first paper in the plasma physics where explicit use of quasipotential $\phi(e, b)$ was made.

The multispecies plasmas consisting of cold or warm positive and negative ions with usual Boltzmann's electrons have been the focus of research for the past few years. The most of the research work had been based on the study of ion-acoustic solitons. KdV/mKdV equations were derived in such plasmas using reductive perturbation method [3] It was predicted that with the introduction of negative ions, there exists a critical ion concentration of negative ions below which compressive solitons exist and above which rarefactive solitons exist.
However, it has recently been found that electron and ion distributions play a crucial role in characterizing the physics of these nonlinear waves [1,4,5,6,7,8,9]. They offer considerable increase in richness and variety of wave motion which can exist in plasmas and further influence the conditions required for the formation of these structures. Moreover, it is also well known that electron and ion distributions can be significantly modified in the presence of large amplitude waves [10,11,12]. Besides, the inclusion of thermal effects also affects the nature of wave particle interaction and possibility of having nonisothermal electrons distribution in the potential well. The simultaneous presence of trapped and free electrons can significantly modify the wave propagation characteristics in collisionless plasmas. It may be noted that the onset of electron trapping has been observed under different conditions, for example, when streaming particles are injected in the plasma [13] in the formation of double layer [14,15] as well as in computer simulation [16]. Soliton solutions have reported in number of investigations containing negative ions and different distributions, where plasma is considered as inhomogeneous immersed in externally applied magnetized field [[17]-[20]].

Another distribution which recently used by most of the investigators is nonthermal particle distribution. Use of the nonthermal distribution accounted for observed nonlinear structures of Freja [21]. The model of velocity distribution function of nonthermal electrons was used first time to study the ion acoustic solitary structures in the presence of a population of fast energetic electrons together with a population of Maxwellian distributed electrons. Such distribution arises in space plasma due to force field present there. So this type of distribution is called as nonthermal distribution and considered by many authors in the various studies of different collective processes in plasmas and dusty plasmas [[22] - [25], [9], [26] - [35]]

In the present investigation, we have studied the effect of nonthermal electrons on ion-acoustic solitons in a plasma with warm positive and negative ions. We have used quasipotential approach to study the characteristics of ion-acoustic solitons in such plasma. In section 2 basic equations of plasma dynamics are used to derive an equation for energy integral. The last section is devoted for discussion and conclusion of numerical results.

2. Basic Equations:
We consider a collisionless unmagnetized plasma consisting of non-Boltzmann electrons with a constant temperature $T_e$, which also contains warm positive and negative ion species having temperature $T_1$ and $T_2$. We assume that low frequency electrostatic waves propagate in plasma. The number density of the electron fluid, which is assumed to be in a nonthermal state and which was first considered in [21], is given by

$$n_e = \left(1 - \beta_1 \phi + \beta_1 \phi^2 \right) e^\phi$$

(1)

with

$$\beta_1 = \frac{4 \gamma}{1 + 3 \gamma}$$

(2)

where $n_e$, $e$ and $\phi$ are the electron density, the magnitude of electron charge and the electrostatic potential respectively. Further, $\gamma$ is a parameter determining the number of nonthermal electrons present in our nonthermal plasma model. This is a non-Maxwellian distribution function which contains high energy electrons component. The above distribution approaches Boltzmann’s distribution when $\beta \rightarrow 0$.

The dynamics of nonlinear behaviour of ion-acoustic waves is governed by the following equations:

Equation of continuity for positive ions:

$$\frac{\partial n_{i1}}{\partial t} + \frac{\partial (n_{i1} v_{i1})}{\partial x} = 0$$

(3)
Equation of motion of positive ions:
\[
\frac{\partial v_1}{\partial t} + v_1 \frac{\partial v_1}{\partial x} = - \frac{Z_1 e}{m_1} \frac{\partial \phi}{\partial x} - \frac{1}{m_1 n_1} \frac{\partial p_1}{\partial x}
\]  
(4)

Equation of continuity for negative ions:
\[
\frac{\partial n_2}{\partial t} + n_2 \frac{\partial (n_2 v_2)}{\partial x} = 0
\]  
(5)

Equation of motion of negative ions:
\[
\frac{\partial v_2}{\partial t} + v_2 \frac{\partial v_2}{\partial x} = \frac{Z_2 e}{m_2} \frac{\partial \phi}{\partial x} - \frac{1}{m_2 n_2} \frac{\partial p_2}{\partial x}
\]  
(6)

Poisson's equation:
\[
\frac{\partial^2 \phi}{\partial x^2} = 4\pi e (n_e + Z_2 n_2 - Z_1 n_1)
\]  
(7)

In the above equations, \(n_1, v_1, m_1\) and \(n_2, v_2, m_2\) are the density, velocity and mass of the positive ion and negative ion species respectively.

The above equations, in normalized form, appear as follow:
\[
\frac{\partial n_1}{\partial t} + n_1 \frac{\partial (n_1 v_1)}{\partial x} = 0
\]  
(8)

\[
\frac{\partial v_1}{\partial t} + v_1 \frac{\partial v_1}{\partial x} = - \frac{1}{b} \frac{\partial \phi}{\partial x} - \frac{\sigma_1}{b Z_1 n_1} \frac{\partial p_1}{\partial x}
\]  
(9)

\[
\frac{\partial n_2}{\partial t} + n_2 \frac{\partial (n_2 v_2)}{\partial x} = 0
\]  
(10)

\[
\frac{\partial v_2}{\partial t} + v_2 \frac{\partial v_2}{\partial x} = \frac{\epsilon_z}{\mu b} \frac{\partial \phi}{\partial x} - \frac{\sigma_2}{\mu b Z_2 n_2} \frac{\partial p_2}{\partial x}
\]  
(11)

\[
\frac{\partial^2 \phi}{\partial x^2} = 4\pi e (n_e + \epsilon_z \alpha_2 n_2 - \frac{n_1}{1 - \epsilon_z \alpha})
\]  
(12)

Where

\[
b = \frac{\mu + \alpha \epsilon_z^2}{\mu (1 - \epsilon_z \alpha)}, \epsilon_z = \frac{Z_2}{Z_1}
\]

\[
\mu = \frac{m_2}{m_1}, \alpha = \frac{n_2^0}{n_1^0}
\]

\[
\sigma_1 = \frac{T_1}{T_e}, \sigma_2 = \frac{T_2}{T_e}
\]  
(13)

Here \(\alpha\) is the equilibrium density ratio of the negative ion species to positive ion species, and \(\epsilon_z\) is the charge multiplicity ratio of negative and positive ion species. \(\mu\) is the mass ratio of the negative ion species to the positive ion species. We normalize the physical quantities \(v, \phi, t\) and \(x\) with respect to modified ion-acoustic wave speed \(c_s = \sqrt{T_e Z b / m}\), potential \(T_e / e\), inverse of
ion-plasma frequency $\omega_{pi} = \sqrt{4\pi n_i e^2 Z_i b / m_i}$ and the Debye length $\lambda_D = \sqrt{T_e / (4\pi n_i e^2)}$ respectively. Also $n_1$, $n_2$ and $n_e$ densities of positive ion, negative ion species and electrons are normalized by $n_0^1$, $n_0^2$ and $n_0^e$ respectively.

To investigate the properties of large amplitude stationary ion-acoustic solitons, we assume that the dependent variables in nonlinear equations [1, 8-12] depend only on a single variable

$$\chi = x - Mt$$

(14)

where $M$ is a solitary wave velocity. Equations (8), (9), (10) and (11) in the stationary frame can be integrated using the boundary conditions for the localized distribution, viz. $|\xi| \to \infty$, $\phi \to 0$, $v_1 \to 0$, $v_2 \to 0$, $n_e \to 1$, $n_2 \to ((1-e z \alpha)/e z \alpha)$, $n_1 \to 2(1-e z \alpha)$ to give

$$n_1 = \frac{2M(1-e z \alpha)}{(M-v_1)}$$

(15)

$$n_2 = \frac{M(1-e z \alpha)}{(M-v_2)e z \alpha}$$

(16)

$$\phi = b \left( M^2 - (v_1 - M)^2 - \frac{3\sigma_1}{bZ_1} n_1^2 + \frac{12\sigma_1}{bZ_1} (1-e z \alpha)^2 \right)$$

(17)

$$\phi = \frac{\mu b}{2e z} \left( (v_2 - M)^2 - M^2 + \frac{3\sigma_2}{\mu bZ_1} n_1^2 + \frac{3\sigma_2}{bZ_1 \mu} \left( \frac{1-e z \alpha}{e z \alpha} \right)^2 \right)$$

(18)

In the above equations, we use the equation of state as

$$p = n^\gamma p_0$$

(19)

where $\gamma$ is ratio of specific heats.

Using equations (15) and (16) to eliminate $v_1$ and $v_2$ respectively from (17) and (18), we obtain the following equations

$$3\sigma_1 n_1^4 - \left(-2Z_1 \phi + bZ_1 M^2 + 12\sigma_1 (1-e z \alpha)^2 \right) n_1^2 + 4bZ_1 M^2 (1-e z \alpha)^2 = 0$$

(20)

$$3\sigma_1 n_2^4 - \left(2Z_2 \phi e z + bZ_2 \mu M^2 + 3\sigma_2 \left( \frac{1-e z \alpha}{e z \alpha} \right)^2 \right) n_2^2 + \mu bZ_2 M^2 \left( \frac{1-e z \alpha}{e z \alpha} \right)^2 = 0$$

(21)

These equations are quadratic in $n_i$ (i = 1, 2) which can be solved by quadratic method to get expressions for $n_i^2$ and. In order to get an easily integrable form for $n_i$ without incorporating any sort of approximations, we assume

$$n_i = \sqrt{p \pm \sqrt{q}}$$

(22)

Standard algebraic manipulations lead to the following expressions for $n_1$ and $n_2$. 


\[
    n_1 = \frac{1}{\sqrt{12\sigma_1}} \left[ \left( \sqrt{bZ_i M + \sqrt{12\sigma_1 (1 - \epsilon, \alpha)}} - 2Z_i \phi \right)^2 - \left( \sqrt{bZ_i M - \sqrt{12\sigma_1 (1 - \epsilon, \alpha)}} - 2Z_i \phi \right)^2 \right]^{1/2}
\]

\[
    n_2 = \frac{1}{\sqrt{12\sigma_2}} \left[ \left( \sqrt{bZ_i \mu M + \sqrt{3\sigma_2 \left( \frac{1 - \epsilon, \alpha}{\epsilon, \alpha} \right)^2} + 2Z_i \epsilon, \phi \right)^2 - \left( \sqrt{bZ_i \mu M - \sqrt{3\sigma_2 \left( \frac{1 - \epsilon, \alpha}{\epsilon, \alpha} \right)^2} + 2Z_i \epsilon, \phi \right)^2 \right]^{1/2}
\]

Substituting \(n_1, n_2\) and \(n_3\) in equation (12) and following quasipotential method along with appropriate boundary conditions, we obtain

\[
    \frac{1}{2} \left( \frac{\partial^2 \phi}{\partial \xi^2} \right) + V(\phi) = 0
\]

Where

\[
    V(\phi) = \left\{ (1 - 3\beta_1^2) - (1 + 3\beta_1 - 3\beta_1^2) \phi \right\}
\]

\[
    + \frac{1}{6\sqrt{3\sigma_2 Z_i}} \left[ \alpha \left( \sqrt{bZ_i \mu M + \sqrt{3\sigma_2 \left( \frac{1 - \epsilon, \alpha}{\epsilon, \alpha} \right)^2} - \left( \sqrt{bZ_i \mu M - \sqrt{3\sigma_2 \left( \frac{1 - \epsilon, \alpha}{\epsilon, \alpha} \right)^2} \right)^2 \right) \right]^{1/2}
\]

\[
    + \frac{1}{6Z_i (1 - \epsilon, \alpha) \sqrt{3\sigma_1}} \left[ \left( \sqrt{bZ_i M + \sqrt{12\sigma_1 (1 - \epsilon, \alpha)}} - \left( \sqrt{bZ_i M - \sqrt{12\sigma_1 (1 - \epsilon, \alpha)}} \right)^2 \right) \right]^{1/2}
\]

\[
    \left( \sqrt{bZ_i M + \sqrt{12\sigma_1 (1 - \epsilon, \alpha)}} - 2Z_i \phi \right)^{1/2} + \left( \sqrt{bZ_i M - \sqrt{12\sigma_1 (1 - \epsilon, \alpha)}} - 2Z_i \phi \right)^{1/2}
\]

3. Results and Discussion:

When working with the reductive perturbation theory, we derive the KdV equation, which is nonlinear partial differential equation in one dimension. Such equations are solved by use of similarity transformation e.g. \(\xi = x - vt\). The purpose is to reduce the nonlinear partial differential equation to ordinary partial differential equation, which can possess special solutions like solitons, double-layers, shock waves etc. The role of the velocity \(v\) is different as dependence from two parameters reduces to one parameter. However, when pseudopotential approach is used for large amplitude waves, the potential function is highly nonlinear and contains a very large number of parameters including \(M\). So, there is certain range of \(M\)'s value which implicitly affects the other parameters. They restrict the range of possible soliton solutions.
It is in this context that the value of ‘M’ plays crucial role on characterization of the solitons [12], [36]. An other pseudopotential approach is in vogue these days, which involve total plasma pressure and energy function both of which are of Bernoule-type, put limit on sonic character longitudinal flow for the existence of solitary structures [37].

It is well known that the equation (25) is similar in form to “energy integral” of a classical particle in potential. The conditions for the existence of localized solution of (25) are as follow (i) $V(\phi) = 0$, $dV(\phi)/d\phi = 0$ and $d^2V(\phi)/d\phi^2 < 0$ for $\phi = 0$, i.e. the fixed point at the origin is unstable, (ii) there exists a nonzero $\phi_m$, the maximum (or minimum) value of $\phi$ at which $V(\phi_m) = 0$, and (iii) $V(\phi) < 0$, when $\phi$ lies between 0 and $\phi_m$.

Apparently pseudopotential $V(\phi)$ is observed to depend upon the number of parameters such as nonthermal $\beta_1$, charge ratio $e_1$ of the ions ($Z_2/Z_1$), density ratio $\alpha$, mass ratio $\mu$ and temperature ratios $\sigma_1$ and $\sigma_2$ of positive and negative ions respectively. All these parameters enter in the expression in a complicated way. The characterization of solitons obviously depends on these parameters. To study the effect of these parameters on the $V(\phi)$ we numerically solve $V(\phi)$ as function of $\phi$ for the different set of the parameters. For one set of parameters corresponding to (H$,\overline{\text{H}}$) plasma, we have:

$$\alpha = 0.1, \sigma_1 = 0.1, \sigma_2 = 0.001, Z_1 = Z_2 = 1, \mu = 1$$

We plot the graphs for the above set of parameter and $\beta_1 = 0$ which is the special case for the nonthermal distribution. It is noteworthy to observe that this set of parameters, we have Maxwellian. Graph for $V(\phi)$ as a function of $\phi$ for different values of M as shown in Figure 1. We find there exist both type of solitons (i.e. compressive and rarefactive) for $M = 1.79$ and 1.80 and rarefactive for $M = 1.85$. However, when the nonthermal parameter $\beta_1 = 0.05$ is introduced, it is observed that M plays very crucial role for the formation of solitons. On the other hand as shown in Figure 2 where $M = 1.79$, we find only compressive soliton, but when $M = 1.80$ both type of solitons coexist. Further increase in the value of $M = 1.85$ leads to disappearance of compressive solitons and only rarefactive solitons exist. Thus existence of type of solitons is crucially dependent on M and $\beta_1$. It is also noticed that depth of potential well and maximum $\phi$ decrease. It implies that fast energetic electrons or the electrons distribution plays important role in the characterization of solitons. In other words the electron contributions dominate over the ion contribution even for small change in M. This is due to interaction of nonthermal electrons with the wave during its excitation [19].

It is further observed that the fast energetic electrons as well as Mach number control the behaviour of the solitons. When we increase the value of M, there exist the rarefactive solitons for $M \geq 2.2$ as shown in Figure 3, which is not obtained for distribution in case of Maxwellian and nonisothermal distribution [cf [12]]. Thus large limiting value of M is observed for obtaining solitons which is contrary to case of Maxwellian/ nonisothermal distribution.

In order to study the effect of different parameter space, we have chosen the following set of parameters:

$$M = 2.1, \alpha = 0.1, \sigma_1 = \sigma_2 = 0.001, Z_1 = Z_2 = 1, \mu = 0$$

This is corresponds to (H$,\overline{\text{H}}$, O$^-$) plasma. Numerically, when $V(\phi)$ is solved for $\phi$ no compressive solitons are obtained. $V(\phi)$ vs $\phi$ for three different values of $\beta_1 = 0.1, 0.3$ and 0.5 as shown in Figure 4. Increasing $\beta_1$ result in decrease in $\phi_m$. Rarefactive solitons are observed, which disappeared with the increase of nonthermal parameter. Depth of potential well also decreases with the increase of the nonthermal parameter. This is also the contrast to results to the nonisothermal distribution of electrons [12]. It is further observed from the figures that depth of potential well is also decreases with the decrease of the temperature of the positive species. It implies that the depth of the potential well is controlled by the temperature of the species and the increase of the temperature leads to the increase of the amplitude.

To study the effect of mass ratio, we consider (H$,\overline{\text{H}}$, O$^-$) plasma where we have chosen the following set of parameters:

$$M = 2.1, \alpha = 0.1, \sigma_1 = 0.1, \sigma_2 = 0.001, Z_1 = Z_2 = 1, \mu = 32$$
It is observed that for the certain values of nonthermal parameter in the range $\beta_1 = 0.41$ to 0.48 rarefactive solitons exist as shown in Figure 5. Above and below this values there no soliton solution is permissible. It is also observed that the masses of the different species also play the important role in the formation of the solitons. Furthermore, in case of nonisothermal electrons only compressive solitons exist [cf [12]] .

In general, it is observed that the electron distribution play the crucial role in the formation of the solitons.

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Figure 1. Variation of quasipotential $V(\phi)$ with $\phi$ for different values of $M$ with the following set of parameters: $\beta_1=0$, $\alpha=0.1$, $\sigma_1=0.1$, $\sigma_2=0.001$, $Z_1=Z_2=1$, $\mu=1$
Figure 2. Variation of quasipotential $V(\phi)$ with $\phi$ for different values of M with the following set of parameters: $\beta_1=0.05$, $\alpha=0.1$, $\sigma_1=0.1$, $\sigma_2=0.001$, $Z_1=Z_2=1$, $\mu=1$
Figure 3. Variation of quasipotential V(\(\phi\)) with \(\phi\) for different values of M with the following set of parameters: \(\beta_1=0.1\), \(\alpha=0.1\), \(\sigma_1=0.1\), \(\sigma_2=0.001\), \(Z_1=Z_2=1\), \(\mu=1\)
Figure 4. Variation of quasipotential $V(\phi)$ with $\phi$ for different values of $\beta_1$ with the following set of parameters: $M=2.1$, $\alpha=0.1$, $\sigma_1=\sigma_2=0.001$, $Z_1=Z_2=1$, $\mu=1$.
\textbf{Figure 5.} Variation of quasipotential $V(\phi)$ with $\phi$ for different values of $\beta_1$ with the following set of parameters: $M = 2.1, \alpha = 0.1, \sigma_1 = 0.1, \sigma_2 = 0.001, Z_1 = Z_2 = 1, \mu = 32$