Efficient Cosmological Analysis of the SDSS/BOSS data from the Effective Field Theory of Large-Scale Structure

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Abstract

The precision of the cosmological data allows us to accurately approximate the predictions for cosmological observables by Taylor expanding up to a low order the dependence on the cosmological parameters around a reference cosmology. By applying this observation to the redshift-space one-loop galaxy power spectrum of the Effective Field Theory of Large-Scale Structure, we analyze the BOSS DR12 data by scanning over all the parameters of $\Lambda$CDM cosmology with massive neutrinos. We impose several sets of priors, the widest of which is a Big Bang Nucleosynthesis prior on the current fractional energy density of baryons, $\Omega_b h^2$, and a 20\% flat prior on $n_s$. In this case we measure the primordial amplitude of the power spectrum, $A_s$, the abundance of matter, $\Omega_m$, and the Hubble parameter, $H_0$, to about 15\%, 5.0\% and 1.9\% respectively, obtaining $\ln(10^{10} A_s) = 2.87 \pm 0.15$, $\Omega_m = 0.316 \pm 0.016$, $H_0 = 69.0 \pm 1.3 \text{ km}/(\text{s Mpc})$ at 68\% confidence level. A public code is released with this preprint.
1 Introduction

In [1], the Effective Field Theory of Large-Scale Structure (EFTofLSS) has been applied to
the analysis of the BOSS/SDSS data to extract the cosmological parameters. This represents
an important milestone for this research approach, whose history we briefly highlight later in
this introduction. In the analysis of [1], two parameters of the $\Lambda$CDM model, the tilt of the
primordial power spectrum, $n_s$, and the ratio of the baryon versus dark matter abundance,
$f_{bc} \equiv \Omega_b/\Omega_c$, were fixed to the preferred value of the Planck2018 data release [2]. Alternatively,
when neutrinos were allowed to vary their mass, all parameters were fixed to the preferred
value from Planck2018. The purpose of this brief companion paper to [1] is to eliminate these
limitations.

In fact, there are two main reasons why some cosmological parameters of the analysis
of [1] have been fixed. First, on a more general level, the analysis of [1] had the purpose
of originally establishing the usefulness of the EFTofLSS to analyze Large-Scale Structure
data, and also to show how the same data could be powerfully used to measure some of the
cosmological parameters in a way that is competitive with other probes. For example, within
the aforementioned restrictions, Ref. [1] measured $\Omega_m$ with error bars comparable to those of
Planck2018 [2], and the present-day value of the Hubble rate, $H_0$, with error bars competitive
with those from Supernovae surveys [3]. Given the size of the error bars on $f_{bc}$ and $n_s$ from
Planck2018, it is clear that this general conclusions of [1] are not significantly affected by the
approximate treatment of these priors.

The second reason why the analysis of [1] does not scan over the whole parameter space
of $\nu\Lambda$CDM (i.e. $\Lambda$CDM plus massive neutrinos) is instead technical. The evaluation time
of the prediction of the EFTofLSS for a given set of cosmological parameters is around 50 seconds on a normal CPU. This relative slowness is mainly due to the implementation of the IR-resummation, which is the original one developed in [4, 5], that has the benefit of high accuracy but at the cost of a relatively long evaluation time. In order to efficiently evaluate the Likelihood of the cosmological parameters by running a Monte Carlo Markov Chain (MCMC), the authors of [1] pre-computed the power spectra from the EFTofLSS on a grid. Size limitations forbid the creation of too large a grid.

In this companion paper, we eliminate this technical limitation of [1] by exploiting the following fact. Cosmological data are by now precise enough to strongly limit the uncertainty range for all the cosmological parameters of interest, and for some of them the allowed range is extremely small. Since the cosmological predictions from the EFTofLSS (but this is true for any method of prediction) are smooth functions of the cosmological parameters, it is expected, and indeed we will verify that this is true, that the whole range spanned by an observable as we scan over the allowed range of the cosmological parameters is well approximated by a low-order Taylor expansion of the EFTofLSS prediction around a reference cosmology. This approach is not new in the context of the EFTofLSS [6], but was never used beyond the prediction of the dark matter power spectrum nor applied to data.

Ref. [1], and this paper which should be understood as a companion, are the completion of a long journey for the EFTofLSS, which started with the initial development of the theory [7, 8, 9] and underwent many important steps in order for it to be compared to observational data. Before starting, we mention a few of these results to provide a rough orientation to the interested reader. The dark matter power spectrum has been computed at one-, two- and three-loop orders in [8, 10, 11, 12, 13, 14, 15, 16, 6, 17, 18]. Some additional theoretical developments of the EFTofLSS that accompanied these calculations were a careful understanding of renormalization [8, 19, 20] (including rather-subtle aspects such as lattice-running [8] and a better understanding of the velocity field [10, 21]), of the several ways for extracting the value of the counterterms from simulations [8, 22], and of the non-locality in time of the EFTofLSS [10, 12, 23]. These theoretical explorations also include an instructive study in 1+1 dimensions [22]. In order to correctly describe the Baryon Acoustic Oscillation (BAO) peak, an IR-resummation of the long displacement fields had to be performed. This has led to the so-called IR-resummed EFTofLSS [4, 24, 25, 5, 26]. A method to account for baryonic effects was presented in [27]. The dark-matter bispectrum has been computed at one-loop in [28, 29], the one-loop trispectrum in [30], the displacement field in [31]. The lensing power spectrum has been computed at two loops in [32]. Biased tracers, such as halos and galaxies, have been studied in the context of the EFTofLSS in [23, 33, 34, 35, 36, 37] (see also [38]), the halo and matter power spectra and bispectra (including all cross correlations) in [23, 34]. Redshift space distortions have been developed in [4, 39, 36]. Clustering dark energy has been included in the formalism in [40, 17, 41, 42], primordial non-Gaussianities in [34, 43, 44, 45, 39, 46], and neutrinos in [47, 48]. Faster evaluation schemes for evaluation for some of the loop integrals have been developed in [49].

The EFTofLSS became ready to be compared with observational data of galaxy clustering
with the completion of [36], as only at that point the IR-resummed one-loop power spectrum of biased tracers in redshift space in ΛCDM cosmology had been computed. This is where the journey of Ref. [1] and of this paper begins. Since this paper represents, as mentioned, a small but important extension of the analysis pipeline of [1], we refer the reader to [1] for all the introductory and overview material about the EFTofLSS, as well as for some physical explanation, and for some comparisons of the performance of the EFTofLSS in terms of measurement of cosmological parameters against numerical simulations. Here we focus only on the non-trivial results that are novel in this paper.

With this publication, we release a C++ code that computes the IR-resummed one-loop power spectrum multipoles and tree-level bispectrum monopole of biased tracers in the EFTofLSS, called CBiRd, and includes a Python-based construction of the approximation by Taylor expansion: Code for Biased tracers in Redshift space at CBiRd GitHub (see also for a repository of all EFTofLSS codes: EFTofLSS repository).

Note Added: Ref. [50], which has just appeared, has some significant overlap with this work.

2 Taylor-Expanded Functional Dependence

The cosmological parameters affect the predictions of the EFTofLSS in two main ways: through the linear power spectrum, and through the EFT-coefficients that encode the effect of short-distance non-linearities at long distances (for example the so-called speed of sound or bias coefficients). Exploiting the cosmology dependence of the EFT parameters requires to be able to predict the physics within the non-linear regime as a function of the different cosmologies. Though this is a promising avenue to follow (as, for example, it would improve the constraining power of the EFTofLSS), in this paper, as in [1], we decide to be completely agnostic about such a dependence.

Instead, clearly, the cosmological dependence of the linear and one-loop parts of the power spectrum, or of the IR-resummation cannot be neglected. However, since observational data force us to explore a limited range of cosmological parameter space, where the observables are allowed to change very little around a reference cosmology, we can Taylor expand the EFT prediction around this reference cosmology to a low order in the deviation of the cosmological parameters from the reference cosmology. In formulas, if \( \vec{\Omega} \) represents the set of cosmological parameters, with \( \vec{\Omega}_{\text{ref}} \) the reference cosmology, and \( \vec{b} \) the EFT-parameters, we can write the EFT prediction for any observable in the following way. Here for simplicity we will focus on the power spectrum, and we take the model to be the same as in [1], to which we refer the
reader for a detailed description. If \( P_{\text{EFT}}(k, \vec{b}, \vec{\Omega}) \) is the EFT prediction, we can write:

\[
P_{\text{EFT}}(k, \vec{b}, \vec{\Omega}) = P_{\text{EFT}}(k, \vec{b}, \vec{\Omega}_{\text{ref}}) + \frac{\partial}{\partial \vec{\Omega}_i} P_{\text{EFT}}(k, \vec{b}, \vec{\Omega}) \bigg|_{\vec{\Omega}_{\text{ref}}} \left( \vec{\Omega} - \vec{\Omega}_{\text{ref}} \right)_i + \frac{1}{2} \frac{\partial^2}{\partial \vec{\Omega}_i \partial \vec{\Omega}_j} P_{\text{EFT}}(k, \vec{b}, \vec{\Omega}) \bigg|_{\vec{\Omega}_{\text{ref}}} \left( \vec{\Omega} - \vec{\Omega}_{\text{ref}} \right)_i \left( \vec{\Omega} - \vec{\Omega}_{\text{ref}} \right)_j + \ldots, \tag{1}
\]

where \( \ldots \) represent higher order terms in \( \left( \vec{\Omega} - \vec{\Omega}_{\text{ref}} \right) \). We can then decide the order of the Taylor expansion and then evaluate the required derivatives with some finite difference method to the desired accuracy.

In this paper, we focus on the analysis of the power spectra monopole and quadrupole of the BOSS D12 data (see for example [51, 52, 53]). As we will show, to perform such an analysis it is enough to expand \( P_{\text{EFT}}(k, \vec{b}, \vec{\Omega}) \) just to second order in all the cosmological parameters except \( A_s \). For this parameter, due to the larger error bars, we also include the term that goes as \( \frac{\partial^3}{\partial A_s^3} P_{\text{EFT}}(k, \vec{b}, \vec{\Omega}) \bigg|_{\vec{\Omega}_{\text{ref}}} (A_s - A_{s,\text{ref}})^3 \). We summarize the specifications of the Taylor expansion we use for this analysis in Table 1. We however stress that our code is already set up to include up to the fifth derivative. Notice that the Taylor expansion is performed directly at the level of the observable quantity that is being compared to data, that is after the application of the Alcock-Paczynski effect [54], of the window function and of the fiber collision correction [55], all of which are applied as in [1].

The cosmological parameters we choose for the Taylor expansion are \( A_s, h, \omega_c, \omega_b, n_s, \sum_i m_{\nu_i} \), where \( h \) is the present value of the Hubble constant in units of \( 100 \text{ km}/(\text{s Mpc}) \), \( \sum_i m_{\nu_i} \) is the sum of the neutrino masses, that for simplicity we take to follow the normal hierarchy, and \( \omega_b = \Omega_b h^2 \), with \( \Omega_b \) being the baryon abundance. Since we evaluate the derivatives with a finite difference method with second order accuracy, we need a minimum number of \( 2^n \) evaluations of the power spectra, where \( n \) is the number of parameters. To this we add the reference cosmology evaluation, and additional 2 evaluations to calculate the third derivative w.r.t. \( A_s \). Such a very small number of evaluations does not seem to represent a numerical challenge \(^1\).

We perform the following two tests to validate the accuracy of the Taylor expansion with respect to the EFT prediction for the same cosmology. In the first test, we compare the relative difference of our prediction with several points that in the cosmological parameter space differ with respect to the reference cosmology by more than one standard deviation, \( \sigma_{\text{stat}} \). We take \( \sigma_{\text{stat}} \) from the value of the CMASS error bars of [1]. For \( f_{bc} \) and \( n_s \) we take either 5\% and 10\% respectively or 3\% and 6\%. This last range is the largest range we explore as when we impose the so-called wide priors. In Fig. 1, we plot the relative difference for both the monopole and quadrupole for 100 cosmologies randomly chosen on the surface of a hypercube

\(^1\)Furthermore, we did not make any significant effort in optimization. For example, one could consider a different number of derivatives for each cosmological parameter. For instance, we found that already truncating the Taylor expansion to linear order would have actually been sufficient for the analysis of the CMASS NGC sample of the BOSS data.
Table 1: Technical specifications of the Taylor expansion for the data analysis. The size of the step is expressed as percentage of reference value.

| Cosmological Parameter | Order of Derivative | Size of Step | Reference value |
|------------------------|---------------------|--------------|-----------------|
| $A_s$                  | 3                   | 10 %         | $1.518 \times 10^{-9}$ |
| $h$                    | 2                   | 2 %          | 0.697           |
| $\omega_c$             | 2                   | 4 %          | 0.1262          |
| $\omega_b$             | 2                   | 4 %          | 0.02339         |
| $n_s$                  | 2                   | 2 %          | 0.9649          |
| $\sum_i m_{\nu_i}$, [eV] (NH) | 2                   | 25 %         | 0.3             |

where each of the faces lie at $0.9\sigma_{\text{stat}}$ from the reference cosmology. Since neutrinos have not been measured, it is unclear how to define the statistical error, we take the two faces of the hypercube to lie, on the top, at 0.06eV and 0.5eV, as this is the range of interest for the data, or, on the bottom, 0.2eV and 0.4eV, which is a more realistic range given bounds from CMB. To further clarify the effect of neutrinos, we choose to show in Fig. 2 the relative deviation on the reference cosmology, varying the sum of neutrino masses from 0.1 to 0.8 eV. In Fig. 3 and 4, we show the relative deviation on the reference cosmology varying the ratio $f_{bc} = \omega_b/\omega_c$ and $n_s$ over a range much wider than its Planck2018 prior, respectively by 40% and 30%. On the same figures, we also plot the $1\sigma$ error bars of the data. We see that the deviations between the Taylor expansion and the actual calculation is safely smaller than the error bars and so negligible.

The second test that we perform to check the accuracy of the Taylor expansion is to analyze the North Galactic Cap (NGC) of the CMASS sample and also the whole CMASS sample by fixing $n_s$ and $\Omega_b/\Omega_c$ to the Planck2018 preferred values, and compare with the results of [1]. Here we choose $\Omega_{\text{ref}}$ to be close to the peak of the distribution. The results are plotted in Fig. 5, where we see that the difference of the results is negligibly small ($i.e.$ almost within the truncation error). This test does not check for the accuracy of the Taylor expansion in the direction of varying $n_s$ or $\Omega_b/\Omega_c$. However, the priors we will impose on them are so stringent, together with the results of Fig. 1, 3, and 4 to assure that the dependence on these parameters, over the range we are going to consider, is very well reproduced.

We end this section with the following observation. The former results check the accuracy of the Taylor expansion only for a large but finite range of departure from the reference cosmology. Of course, if the posterior distribution of some cosmological parameter is peaked very far from the reference cosmology, the accuracy of the Taylor expansion at some point might become too low. We have found that the results for the cosmological parameters are accurate even when the peak of the distribution are further than what is represented in Fig. 1. However we point out that the construction of the Taylor expansion and the analysis through the MCMC is so fast, that one can iterate the construction of the Taylor expansion by adjusting the parameters of the reference cosmology, or alternatively carry the Taylor expansion to an higher order.

6
3 Cosmological Analysis of the BOSS data

3.1 Minimal-Mass Neutrinos

We are now ready to analyze the BOSS DR12 data. We focus on the monopole and quadrupole and analyze the data up to \( k_{\text{max}} = 0.2 h \text{ Mpc}^{-1} \) for the CMASS sample and up to \( k_{\text{max}} = 0.18 h \text{ Mpc}^{-1} \) for the LOWZ sample. First, following what Planck2018 did and also what was done in [1], we fix the neutrino spectrum to one single massive neutrino with mass equal to 0.06 eV \(^2\). We impose a Planck2018 prior on \( f_{\text{bc}} \): a Gaussian prior with \( f_{\text{bc}, \text{center}} = 0.1860 \) and \( \sigma_{f_{\text{bc}}, \text{prior}} = 0.0031 \); and on \( n_s \): a Gaussian prior with \( n_{s, \text{center}} = 0.9649 \) and \( \sigma_{n_s, \text{prior}} = 0.0044 \). The results of the analysis are shown in Fig. 6 and Table 2. We find that the central values

\(^2\)We actually use a normal hierarchy with sum of masses equal to 0.06 eV.
Figure 2: Relative difference between the computation of a EFTofLSS power spectrum by the direct evaluation or by approximation with the Taylor expansion, for cosmologies that lie on the fiducial cosmology, in which we vary the sum of neutrino masses from 0.1 to 0.8 eV. We also plot the error bars of the data as the solid blue lines. On the left we plot the monopole, on the right the quadrupole. We see that the disagreement is negligibly small when compared to the error bars of the data.

Figure 3: Relative difference between the computation of a EFTofLSS power spectrum by the direct evaluation or by approximation with the Taylor expansion, for cosmologies that lie on the fiducial cosmology, in which we vary the ratio $f_{bc}$ by about 40% from 0.15 to 0.22, as indicated in the legend. We also plot the error bars of the data as the solid blue lines. On the left we plot the monopole, on the right the quadrupole. We see that the disagreement is negligibly small when compared to the error bars of the data.

and error bars for $A_s, \Omega_m$ and $h$ are extremely similar to the ones of [1] (see Table 4 there, the difference is less than $\sigma_{\text{stat}}/7$). This shows that having fixed $f_{bc}$ and $n_s$ to the Planck2018 data and $\sum_i m_{\nu_i}$ to 0.06 eV was a good approximation in place for imposing Planck2018 priors on the same parameters: the actual Planck2018 posteriors for these parameters are sufficiently narrow with respect to what could be measured with our data. This result has also the consequence that we do not need to test against simulations the presence of a significant theoretical systematic error in the case under consideration. In [1] a careful comparison with several sets of simulations was performed by fixing $f_{bc}$ and $n_s$ to the Planck2018 preferred values, and it was found that the theoretical systematic error of the EFTofLSS is negligible.
Figure 4: Relative difference between the computation of a EFTofLSS power spectrum by the direct evaluation or by approximation with the Taylor expansion, for cosmologies that lie on the fiducial cosmology, in which we vary the tilt of the power spectrum, $n_s$, by about 30% from 0.8 to 1.1, as indicated in the legend. We also plot the error bars of the data as the solid blue lines. On the left we plot the monopole, on the right the quadrupole. We see that the disagreement is negligibly small when compared to the error bars of the data.

Table 2: 68% confidence interval for the cosmological parameters from the individual analyses over CMASS and LOWZ NGC samples of the BOSS data up to $k_{\text{max}} = 0.2h\text{Mpc}^{-1}$ for CMASS and $k_{\text{max}} = 0.18h\text{Mpc}^{-1}$ for LOWZ NGZ, apart for $k_{\text{max}} = 0.23h\text{Mpc}^{-1}$ for CMASS $k_{\text{max}} = 0.20h\text{Mpc}^{-1}$ for LOWZ NGZ in the first and last lines. The entry ‘prior’ allows us to identify on which parameters the prior is being imposed. Unless the confidence interval is given, the error is dominated by the prior.

Table: 68% confidence interval for the cosmological parameters from the individual analyses over CMASS and LOWZ NGC samples of the BOSS data up to $k_{\text{max}} = 0.20h\text{Mpc}^{-1}$ for CMASS and $k_{\text{max}} = 0.18h\text{Mpc}^{-1}$ for LOWZ NGZ, apart for $k_{\text{max}} = 0.23h\text{Mpc}^{-1}$ for CMASS $k_{\text{max}} = 0.20h\text{Mpc}^{-1}$ for LOWZ NGZ in the first and last lines. The entry ‘prior’ allows us to identify on which parameters the prior is being imposed. Unless the confidence interval is given, the error is dominated by the prior.
\[
\ln(10^{10} A_s) = 2.57^{+0.16}_{-0.17}
\]
\[
\Omega_m = 0.307^{+0.013}_{-0.012}
\]
\[
h = 0.729^{+0.030}_{-0.026}
\]

Figure 5: Left: Posterior distributions of \(\ln(10^{10} A_s)\), \(\Omega_m\) and \(h\) obtained from the analysis of the CMASS NGC sample, up to \(k_{\text{max}} = 0.23 h \text{ Mpc}^{-1}\) using the Taylor expansion approximation for the EFT power spectrum. This figure should reproduce exactly Fig. 14 [1], where the same data were analyzed using a grid (see also Table 4 there). In vertical dashed we plot the expectation value from [1]. We find that the disagreement is negligible for all 3 parameters, showing the great accuracy of the Taylor expansion for this dataset. Right: Same but for the CMASS sample. Now one should compare with Fig. 15 [1]. The agreement is again remarkably good, though one should keep in mind that here we used a slightly different \(\sum m_{\nu_i} = 0.075 \text{ eV}\) instead of \(\sum m_{\nu_i} = 0.06 \text{ eV}\).

3.2 Free-varying Neutrino spectrum

3.2.1 Planck prior on \(f_{bc}\) and \(n_s\)

We now analyze the same data by letting the sum of the neutrino masses vary within the range [0.06, 1] eV, while imposing always a normal hierarchy. We work up to \(k_{\text{max}} = 0.20 h \text{ Mpc}^{-1}\). The effect of neutrinos are approximated in exactly the same way, and with the same caveats, as discussed in sec. 5.2 of [1], to which we refer the reader for details. This method should correctly account for the leading effects. The results of the analysis are given in Fig. 7 and in Table 2. We see that as anticipated in [1], \(A_s\) is moved to higher values, closer to the central value from Planck2018, but also \(\Omega_m\) and \(h\) are slightly shifted to higher values, as all of these parameters are positively correlated with neutrino masses. Overall, neutrinos are bounded to be lighter than 0.72 eV at 95% C.L., which is consistent with the upper bound from Planck2018 [2].

On the right of Fig. 7, we plot the same analysis but by adding a flat prior on the sum of neutrino masses: \(0.06 \text{ eV} \leq \sum_i m_{\nu_i} \leq 0.25 \text{ eV}\), motivated by Planck2018 [2]. One notices the slight shift in the cosmological parameters implied by this analysis.
Figure 6: Posterior distributions for the cosmological parameters obtained from the analysis of the CMASS and LOWZ NGC samples using the Taylor expansion approximation for the EFT power spectrum up to $k_{\text{max}} = 0.20h\,\text{Mpc}^{-1}$ for CMASS and $k_{\text{max}} = 0.18h\,\text{Mpc}^{-1}$ for LOWZ NGC. We put a Planck2018 prior on $f_{\text{bc}}$ and on $n_s$ and we fix the neutrino spectrum to one single massive neutrino with mass equal 0.06eV. In vertical dashed we plot the expectation value from [1].

### 3.2.2 Planck prior on $\omega_b$ and $n_s$

So far, we have always put a prior on $f_{\text{bc}} = \omega_b/\omega_c$. If instead we impose a Planck2018 prior on $\Omega_b h^2$, we find the constraints on $A_s$, $\Omega_m$ and $h$ given in Fig. 8. One sees that while the error bar on $\Omega_m$ is marginally increased and the one on $A_s$ is also approximately unaffected, the one for $h$ is reduced by about a factor of two. Since $\Omega_b h^2$ is very well measured both by Planck2018 and also from Big Bang Nucleosynthesis, this suggests that imposing such a prior could be a very interesting way to analyse the BOSS data. Given that the error bars on $h$ are so significantly reduced, in this case we cannot trust the analysis of the systematic errors of [1], because the minimally detectable error in that analysis is too large in this case. We therefore do the analysis of the Challenge simulations and of Patchy mocks, as done in [1], to verify that there is no large systematic theory error. Our results are shown in appendix A, where we verify that the theory systematic error is under control even with smaller statistical errors on $h$, by using the data up to $k_{\text{max}} = 0.20h\,\text{Mpc}^{-1}$ for the CMASS sample (and, following the procedure described in [1], up to $k_{\text{max}} = 0.18h\,\text{Mpc}^{-1}$ for LOWZ), as we do.
Figure 7: Left: Posterior distributions for the cosmological parameters obtained from the analysis of the CMASSxLOWZ sample using the Taylor expansion approximation for the EFT power spectrum up to $k_{\text{max}} = 0.20h\,\text{Mpc}^{-1}$ for CMASS and $k_{\text{max}} = 0.18h\,\text{Mpc}^{-1}$ for LOWZ NGC. We put a Planck2018 prior on $f_{\text{bc}}$ and $n_s$. Right: Same as the left plot, but with an additional flat prior for the neutrino masses: $0.06\,\text{eV} \leq \sum_i m_{\nu_i} \leq 0.25\,\text{eV}$.

3.2.3 BBN prior on $\omega_b$ and loose prior on $n_s$

If we want to constrain the cosmological parameters using only galaxy clustering and no CMB information, we can put a loose prior on both $\omega_b$ and $n_s$. Namely, we choose a Gaussian prior on $\omega_b$ with $\sigma = 0.00745$ and central value 0.00225, which is dictated by Big Bang Nucleosynthesis (BBN) constraints [56], and a flat prior on $n_s$, $0.87 < n_s < 1.07$. In Fig. 9 we use this prior to analyze the combination of CMASS sample on the right, and the CMASS x LOWZ NGC on the left. We see that, even without the stringent CMB priors, the cosmological parameters are recovered to a remarkable precision. In App. B we show the results of the analysis of the CMASS sample, showing that the data set are compatible. In the same appendix, we also present the posterior for $n_s$, showing that its value is measured, though at a marginal level. As we show in App. A, because of the slightly larger statistical errors, the theoretical systematic error is negligible even at $k_{\text{max}} = 0.23h\,\text{Mpc}^{-1}$ for CMASS (and, following the procedure described in [1], up to $k_{\text{max}} = 0.20h\,\text{Mpc}^{-1}$ for LOWZ), which is the $k_{\text{max}}$’s at which we perform the analysis in this case.
We have measured the cosmological parameters from the CMASS and LOWZ NGC samples of the BOSS DR12 data. We have done this by matching the cosmology-dependent predictions for the monopole and quadrupole power spectra of galaxies in redshift space from the Effective Field Theory of Large-Scale Structure (EFTofLSS) against the measured data up to $k_{\text{max}} = 0.2h \, \text{Mpc}^{-1}$ and $k_{\text{max}} = 0.23h \, \text{Mpc}^{-1}$. We have allowed the amplitude of the primordial power spectrum, $A_s$, the abundance of dark matter, $\Omega_m$, the present value of the Hubble rate, $H_0$, to vary freely, while we have imposed several sets of priors on the other cosmological parameters: either Planck2018 priors on the ratio of baryons with respect to dark matter, $f_{bc}$, and on the tilt of the primordial power spectrum, $n_s$; or Planck priors on the baryon fraction of the energy density, $\Omega_b h^2$, and on the tilt of the primordial power spectrum, $n_s$; or a BBN prior on $\Omega_b h^2$ and a 20% flat prior on $n_s$, that we identify as ‘wide’ priors. For the neutrinos, by assuming a normal hierarchy, we have either allowed their total mass to vary freely, or fixed it.

Figure 8: Posterior distributions for the cosmological parameters obtained from the analysis of the CMASS sample combined with the LOWZ NGC sample, using the Taylor expansion approximation for the EFT power spectrum. We put a Planck2018 prior on $\omega_b = \Omega_b h^2$ and $n_s$.

4 Conclusions

We have measured the cosmological parameters from the CMASS and LOWZ NGC samples of the BOSS DR12 data. We have done this by matching the cosmology-dependent predictions for the monopole and quadrupole power spectra of galaxies in redshift space from the Effective Field Theory of Large-Scale Structure (EFTofLSS) against the measured data up to $k_{\text{max}} = 0.2h \, \text{Mpc}^{-1}$ and $k_{\text{max}} = 0.23h \, \text{Mpc}^{-1}$. We have allowed the amplitude of the primordial power spectrum, $A_s$, the abundance of dark matter, $\Omega_m$, the present value of the Hubble rate, $H_0$, to vary freely, while we have imposed several sets of priors on the other cosmological parameters: either Planck2018 priors on the ratio of baryons with respect to dark matter, $f_{bc}$, and on the tilt of the primordial power spectrum, $n_s$; or Planck priors on the baryon fraction of the energy density, $\Omega_b h^2$, and on the tilt of the primordial power spectrum, $n_s$; or a BBN prior on $\Omega_b h^2$ and a 20% flat prior on $n_s$, that we identify as ‘wide’ priors. For the neutrinos, by assuming a normal hierarchy, we have either allowed their total mass to vary freely, or fixed it.
to 0.06eV, or even imposed a Planck-motivated flat prior for this quantity to be smaller than 0.25eV. Though we have stressed that the modeling of neutrinos is not very accurate, our implementation is expected to account for the leading effects. We have either used the results of [1] or directly compared with several sets of simulations to conclude that the theoretical systematic error induced by the EFTofLSS modeling is negligibly small.

In summary, we find that, using the CMASS and LOWZ NGC samples of the BOSS DR12 data up to \( k_{\text{max}} = 0.23h \text{ Mpc}^{-1} \), and by imposing the ‘wide’ prior on \( \Omega_b h^2 \) and \( n_s \), we can measure \( A_s \) to 15%, \( \Omega_m \) to 5.0%, \( h \) to 1.9%, \( n_s \) to 6% accuracy, and bound the neutrinos to be lighter than 0.75 eV at 95% confidence level. The 68% confidence level intervals for \( A_s \), \( \Omega_m \), \( h \) and \( n_s \) read \( \ln(10^{10} A_s) = 2.87 \pm 0.15 \), \( \Omega_m = 0.316 \pm 0.016 \), \( H_0 = 69.0 \pm 1.3 \text{ km/(s Mpc)} \), \( n_s = 0.971 \pm 0.053 \). Similar results hold for different sets of priors.

With respect to the analysis performed in [1], the main generalization of our analysis is the fact that we do not fix \( f_{\text{bc}} \) and \( n_s \) to the best values from Planck2018, but rather impose a consistent prior, and also that we consider the case of a prior on \( \Omega_b h^2 \) instead of \( f_{\text{bc}} \), including a rather wide one that does not rely on the CMB. In all these cases, our Monte Carlo Markov Chain (MCMC) needs to explore a much-higher dimensional cosmological parameter space.

*Notice that posterior for \( n_s \) is still somewhat-marginally affected by the prior, which is why we still mention that we use a wide prior on \( n_s \).*

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Figure 9: **Left:** Posterior distribution for the cosmological parameters obtained from the analysis of the CMASS sample combined with the LOWZ NGC sample, using the Taylor expansion approximation for the EFT power spectrum. We put a BBN prior on \( \omega_\text{s} = \Omega_\text{s} h^2 \) and a wide prior on \( n_s \), as explained in the text, working at \( k_{\text{max}} = 0.23h \text{ Mpc}^{-1} \) for CMASS and \( k_{\text{max}} = 0.18h \text{ Mpc}^{-1} \) for LOWZ NGC. **Right:** Same as the left plot, but with an additional flat prior for the neutrino masses: 0.06 eV \( \leq \sum_i m_{\nu_i} \leq 0.25 \text{ eV} \).
In order to do this in an effective way, following the idea initially developed in [6], we have used the fact that the range of parameter space that is consistent with the observational data is small enough to approximate the dependence of the EFTofLSS power spectrum on cosmological parameters with a Taylor expansion around a reference cosmology. By comparison with direct calculations of the same observable, we have found that, for the data we analyze, it is enough to include all the second order terms plus the third order term in \( A_s \).

The resulting power spectrum and likelihoods, can be evaluated extremely fast. Additionally, the Taylor-expanded prediction can be constructed with the evaluation of a small number of power spectra, making the overall procedure very expedite. We publicly release the whole pipeline of the analysis with the appearance of this preprint.

There are several directions that our findings open up:

- It would be interesting to remove all priors, and see what are the constraints that the BOSS observations can impose per se.

- It should be possible to include additional cosmological parameters and perform an analysis exploring an even larger parameter space. For example, one could include non-vanishing curvature, primordial non-Gaussianities, an equation of state for dark energy, dynamical dark energy, etc., as well as a more accurate implementation of the effect of neutrino masses, as described in [1].

We leave this, and more, to future work. More generally, we believe this work, together with [1], shows the usefulness and versatility of using the EFTofLSS to analyze Large-Scale Structure data, which, in turn, are revealed to be extremely rich in cosmological information.

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A Tests on Challenge simulations and Patchy mocks

In this section, we show the results of the analysis of the 5 Challenge simulation boxes A, B, F, G, D and of the mean of 16 Patchy mocks, which were already used in [1]. In Fig. 10
and 11, we show the posterior distributions of the 3 cosmological parameters $\ln A_s$, $\Omega_m$, $h$ from the analysis of the Challenge boxes done varying $\omega_b$, $n_s$ and the EFT-parameters, with either a wide or a Planck2018 prior on $\omega_b$ and $n_s$. For the case of the Planck priors we work at $k_{\text{max}} = 0.2h$ Mpc$^{-1}$. For the case of the wide priors, we work at $k_{\text{max}} = 0.23h$ Mpc$^{-1}$ because, as we will see, the larger statistical errors allow us to work at higher wavenumber.

For box ABFG, if we put the Planck2018 prior on $\omega_b$ and $n_s$, we detect a small, but still negligible, systematic bias of 0.02 on $\ln A_s$ and of 0.001 on $\Omega_m$ and $h$. For box D, we detect only a small systematic error on $\ln(10^{10} A_s)$ equal to 0.03, which is negligible. On the other hand, with a loose prior on $\omega_b$ and $n_s$, we have a systematic bias of 0.002 on $\Omega_m$, which is negligible, for box ABFG, and, for box D, a systematic error for $\ln(10^{10} A_s)$ equal to 0.03, which is again small.

If we combine the systematic error between the measurements on boxes ABFG and the ones on box D as done in [1], in the case of Planck priors we find only a systematic error on $\ln A_s$ of 0.04, which is safely negligible being $\sim \sigma_{\text{stat, CMASSxLOWZ}}/4$ and an even more negligible systematic error on $\Omega_m$ of 0.002. In the case of wide priors we have a similar systematic error on $\ln A_s$ of 0.03, and a systematic error on $\Omega_m$ of 0.003, which are both tiny. In Fig. 12, we show the analysis on the mean of 16 Patchy mock simulations, from which it is apparent that we detect no systematic error. In particular for the case of wide prior we detect a negligible systematic error in $\ln(10^{10} A_s)$ equal to 0.02, and a still negligible systematic error in $\Omega_m$ equal to 0.04 $\approx \sigma_{\text{stat, CMASSxLOWZ}}/4$. In conclusion, we verify that the systematic error is negligible with respect to the errors on the data at the $k_{\text{max}}$‘s under consideration.

![Figure 10](image_url)

Figure 10: Mean of the posterior distributions for the cosmological parameters obtained from the analysis of the Challenge simulations A, B, F, G, using the Taylor expansion approximation for the EFT power spectrum. **Left:** We put the Planck2018 prior on $\omega_b = \Omega_bh^2$ and $n_s$ at $k_{\text{max}} = 0.20h$ Mpc$^{-1}$. **Right:** We put the BBN prior on $\omega_b = \Omega_bh^2$ and a wide one on $n_s$ at $k_{\text{max}} = 0.23h$ Mpc$^{-1}$.

## B CMASS sub-sample analysis

In this brief appendix, we show in Fig. 13 the result of the analysis with the ‘wide’ priors on the CMASS sample, to show that it gives similar results as the CMASS $\times$ LOWZ analysis, showing the compatibility of the data set.
Figure 11: Mean of the posterior distributions for the cosmological parameters obtained from the analysis of the Challenge simulation D, using the Taylor expansion approximation for the EFT power spectrum. Left: We put the Planck2018 prior on $\omega_b = \Omega_b h^2$ and $n_s$ at $k_{\text{max}} = 0.20h \text{ Mpc}^{-1}$. Right: We put the BBN prior on $\omega_b = \Omega_b h^2$ and a wide one on $n_s$ at $k_{\text{max}} = 0.23h \text{ Mpc}^{-1}$.

Figure 12: Posterior distributions for the cosmological parameters obtained from the analysis of the mean of 16 Patchy mocks using the Taylor expansion approximation for the EFT power spectrum. Left: We put the Planck2018 prior on $\omega_b = \Omega_b h^2$ and $n_s$ at $k_{\text{max}} = 0.20h \text{ Mpc}^{-1}$. In vertical dashed we plot the expectation value from [1]. Right: We instead put the BBN prior on $\omega_b = \Omega_b h^2$ and a loose one on $n_s$ at $k_{\text{max}} = 0.23h \text{ Mpc}^{-1}$.

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Figure 13: Left: Posterior distribution for the cosmological parameters obtained from the analysis of the CMASS sample, using the Taylor expansion approximation for the EFT power spectrum. We put a BBN prior on $\omega_b = \Omega_b h^2$ and a wide prior on $n_s$, as explained in the text, working at $k_{\text{max}} = 0.23 h \text{Mpc}^{-1}$. Comparing with the left of Fig. 9, we can see the compatibility of the datasets. Right: Posterior distribution for the cosmological parameters obtained from the analysis of the CMASS × LOWZ NGC sample, using the Taylor expansion approximation for the EFT power spectrum. We put a BBN prior on $\omega_b = \Omega_b h^2$ and a wide prior on $n_s$, as explained in the text, working at $k_{\text{max}} = 0.23 h \text{Mpc}^{-1}$. Contrary to Fig. 9, here we include the posteriors over $n_s$, rather than over the neutrino mass. We can see that we can measure also $n_s$, though at a marginal level.

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