On Effective Secrecy Throughput of Underlay Spectrum Sharing $\alpha - \mu$ Málaga Hybrid Model Under Interference-and-Transmit Power Constraints

Md. Ibrahim $\odot$, A. S. M. Badrudduza $\odot$, Graduate Student Member, IEEE, Md. Shakhawat Hossen $\odot$, M. K. Kundu $\odot$, Graduate Student Member, IEEE, Imran Shafique Ansari $\odot$, Senior Member, IEEE, and Imtiaz Ahmed $\odot$, Member, IEEE

Abstract—The underlay cognitive radio-based hybrid radio frequency/free-space optical (RF/FSO) systems have emerged as a promising technology due to their ability to eliminate spectrum scarcity and spectrum under-utilization problems. The physical layer security of such a network with a primary user, a secondary source, a secondary receiver, and an eavesdropper is therefore examined in this work. In this network, secret communication occurs between two reliable secondary peers over the RF and FSO links simultaneously, and the eavesdropper can only overhear the RF link. In particular, the maximum transmits power limitation at the secondary user as well as the permissible interference power restriction at the primary user are also taken into consideration. All the RF and FSO links are modeled with $\alpha-\mu$ fading and Málaga turbulence with link blocking and pointing error impairments. At the receiver, the selection combining diversity technique is utilized to select the signal with the best electrical signal-to-ratio (SNR). Furthermore, to examine the secrecy performance taking into account the effects of each system parameter, closed-form expressions for the secrecy outage probability and effective secrecy throughput are derived. The resultant expressions are finally verified by Monte-Carlo simulations.

Index Terms—Cognitive underlay network, effective secrecy throughput, hybrid RF/FSO system, Málaga turbulence, pointing error, secure outage probability.

I. INTRODUCTION

A. Background and Related Works

With the rapid increase in wireless devices, the shortage of spectrum owing to the data traffic reflects a significant threat in contemporary technology [1], [2]. In order to meet the requirements, several optimistic technologies have been developed recently. Among these, both cognitive radio network (CRN) and free-space optical (FSO) systems have received considerable attention from the research communities [3], [4]. Although FSO communication appears with specific merits, such as high bandwidth, inherent security, and cost-effective operation, several issues like atmospheric turbulence and pointing error can affect the secure transmission greatly [5]. Furthermore, the probability of transmission blockage due to extreme line-of-site (LOS) necessity in FSO link leads to the system failure [6]. Hence, utilizing radio frequency (RF) technology as an alternative option along with the FSO system assures successful transmission since the FSO link delivers high data speed while the RF link is independent of the atmospheric turbulence and weather conditions [7].

Due to the increased demand for faster data rates in wireless communication, hybrid networks, which consist of RF and FSO systems, are deployed utilizing a variety of techniques. An adaptive coding method in order to enhance the system performance of RF/FSO hybrid network was considered in [8], [9]. Nonetheless, the performance of these systems is highly dependent on the existence of feedback information at the receiver [9]. Another attractive feature is the switching mechanism that uses RF link when FSO link is down [10]. Since one link is underutilized at a time, there is a significant amount of bandwidth wasted [11]. To extend the communication range, several existing works like [12], [13], [14], [15] utilize relaying protocols in mixed RF-FSO models. However, weather conditions (i.e. fog) can influence the performance of dual-hop networks badly. Unlike [10], [11], the considered hybrid RF/FSO model dispatches identical information via two separate links simultaneously and combines the received signal using the diversity combining method. Again, the mixed RF-FSO system differs completely from the hybrid RF/FSO model in terms of system configuration. Recently, a comprehensive analysis on hybrid RF/FSO model was performed in [16], [17] where the hybrid system exhibits better link availability compared to the individual links.

In CRN, an unlicensed user gets the permission of accessing the same spectrum of the licensed user through different spectrum sharing methods [18]. More particularly, in the underlay approach, the secondary user (SU) makes use of the...
primary user’s (PU) shared spectrum as long as the interference occurring at SU is less than the permitted threshold value [4]. Considering CR technology in the RF links, the system performance of mixed RF-FSO systems are investigated in [4, 18], [19], [20], [21], [22]. In [20], the performance analysis was accomplished by developing analytical expressions of outage probability (OP) where RF-FSO links were subjected to Nakagami-m fading and double generalized Gamma (DGG) turbulence, respectively. However, this model was extended in [18], [21] where the impact of antenna diversity in RF links was observed. In [22], the OP analysis was performed for amplify-and-forward (AF) relaying-based dual-hop network while multi-users and multi-destination were taken into consideration. The error probability (EP) expression was derived in [4] where the influence of RF and FSO parameters on the system performance was inspected.

Security has become a widespread concern in next-generation wireless communications (5G and beyond) due to its high risk of information leakage [23], [24], [25]. In this domain, physical layer security (PLS) has been recognized as a complementary choice to the traditional cryptography strategy [26], [27]. According to the comprehensive literature analysis, current PLS research has mainly been limited to hybrid RF/FSO models [26], [28], [29] and mixed RF-FSO networks [30], [31], [32]. In [26], the PLS of a hybrid FSO/RF network was examined and it was found that secrecy performance is highly dependent on air turbulence, pointing error, and RF fading parameters. In [28], authors obtained the secrecy diversity gain for the RF backhaul system along with parallel FSO link whereas the impact of power allocation on secrecy performance was investigated in [29] for the parallel mmWave and FSO links. Including link blockage (LB) probability in the FSO link due to the sudden presence of moving objects (e.g. clouds, birds, insects, etc.). [33] introduced the strategy of enhancing secrecy behavior for CR-based hybrid RF/FSO system.

B. Motivation and Contributions

While a CR-based hybrid RF/FSO system configuration is more robust and practical than a single RF or FSO link [28], there have been only a few studies focused on its secrecy analysis. The authors of [26], [28], [34], [35] evaluated the security of hybrid RF-FSO models but did not take into account CR technology with power constraints. As a result, the investigation of PLS in a CR-based hybrid model remains an open concept. To the best of our knowledge, there are no studies that have looked into the secrecy analysis of a hybrid CR RF/FSO system while considering both the transmit power constraints at the SU and the maximum allowable interference power at the PU. To address this research gap, we investigate the security performance of an underlay CR-based hybrid RF/FSO fading channel in the presence of an eavesdropper. We consider the generalized $\alpha$-$\mu$ fading channel for RF links and the Málaga ($\mathcal{M}$) distribution for the FSO link. Special consideration was made while choosing these channel models. In a hybrid RF/FSO system, small-scale fading can have a significant impact on communication performance and the alpha-mu fading channel is a generalized model that can describe small-scale fading perfectly [36]. On the other hand, pointing error and atmospheric turbulence are two very important parameters in the FSO channel and these two parameters can be perfectly represented by $\mathcal{M}$ distribution [37]. Both these channel links are flexible, as they can describe a variety of fading scenarios by adjusting their parameters. Our study also includes both the single power constraint and the double power constraint. The contributions of this work are highlighted as follows:

1) Firstly, the expressions of cumulative density function (CDF) for selection combining (SC) based hybrid cognitive spectrum-sharing RF/FSO network assuming two different scenarios (e.g. single and double power constraints) are derived in closed form. It is noteworthy that most of the works in the literature explored the secrecy behavior of the CR RF/FSO schemes considering maximum interference power constraint at PU only. However, this research takes into account the power limits at both SU and PU, which distinguishes this model from those previously investigated models and makes it more reliable. From this perspective, a hybrid model of this kind is undoubtedly new and more useful. Moreover, this expression of CDF also incorporates some other existing works [16], [17], [26], [28], [33] as special cases.

2) Secondly, the secrecy characteristics are derived for the proposed system model utilizing the underlay approach over the RF links and developing the analytical expressions for secrecy outage probability (SOP), probability of strictly positive secrecy capacity (SPSC), and effective secrecy throughout (EST) for the two scenarios. Since the considered $\alpha$-$\mu$ and Málaga models are generalized, and combination of the two in such type of hybrid model has been considered for the first time in the literature, according to the authors’ knowledge, the derived expressions are also novel and generalized.

3) The derived expressions are further utilized to realize some numerical outcomes with some specific figures demonstrating the impacts of each system parameter i.e. fading, primary-secondary interference, LB probability, pointing error, detection techniques, and various atmospheric turbulence conditions, etc. It is also notable that both detection techniques, i.e. heterodyne detection (HD) and intensity modulation/direct detection (IM/DD) techniques, have not been considered yet in such type of hybrid system model except in this work. It should be highlighted that as atmospheric turbulence, pointing error, and link blockage probability rise, the security of the systems is compromised. By adopting the HD approach over IM/DD and carefully adjusting the system’s fading parameters, this issue can be mitigated. Finally, the accuracy of the deduced analytical expressions is corroborated via Monte-Carlo (MC) simulations.

C. Organization

The outline of this paper is arranged as follows: In Sections-II, III, and IV, the proposed framework along with the channel models is described. The novel analytical expressions of the performance metrics (i.e. SOP, probability of SPSC, and EST analysis for the two scenarios) are demonstrated in Section-V.
Section-VI incorporates some exclusive numerical results followed by insightful discussions. Finally, the concluding remarks of this work are presented in Section-VII.

II. SYSTEM MODEL

A hybrid cognitive underlay network (CUN) is illustrated in Fig. 1, which incorporates two parallel links: cognitive RF and FSO sub-systems, respectively in order to transmit confidential information from a secondary user transmitter \( S \) to a receiver \( R \) in the presence of an eavesdropper \( E \). It is essential to mention that FSO communication allows the benefits of using a license-free spectrum with high data rates and low latency [12]. The FSO subsystem, however, experiences substantial interruptions in signal transmission due to atmospheric turbulence, aiming inaccuracies, and obstructions along the link, making the communication system dependent on weather conditions. Hence, the RF sub-system is utilized to transmit the same information concurrently with the FSO sub-system ensuring a constant flow of data. To prevent any data loss, \( R \) exploits selection combining diversity and always picks the best signal, i.e., the signal with the highest (signal-to-noise ratio) SNR, among the two sub-links. The proposed underlay technique involves the use of the licensed spectrum of the primary user (\( P \)) by \( S \), and therefore, the system mandates that there must not be any harmful interference to \( P \).

1) Cognitive RF Sub-Link: Based on the underlay approach, \( S \) transmits its concealed information to \( R \) under the following constraints:

- The peak tolerable interference power impinged by \( S \) on \( P \) cannot exceed a predefined value \( P_Q \). In such a case, \( S \) is not considered as a power limited terminal and it has the full freedom to utilize its power depending on a single power constraint, i.e., interference severity at \( P \). This particular case is denoted as Scenario-I in the rest of this work.
- The CUN in Scenario-I sometimes may not be practical, specifically when the channel coefficient of \( S - P \) link rapidly fluctuates. This may lead to a feedback burden since the instantaneous feedback gain is difficult to track. To mitigate these types of feedback burden and feedback errors, mean value power allocation strategy can be adopted [38]. Hence, besides only interference/single power constraint case, in the double power constraint case, \( S \) is assumed to be a power-limited terminal and allowed to exploit maximum transmit power \( P_T \). This particular case is denoted as Scenario-II in the remaining manuscript.

For Scenario-II, utilizing mean value power allocation, the transmit power of \( S \) is denoted as

\[
P_{T,II} = \min \left( \frac{P_Q}{|g_p|^2}, P_T \right),
\]

where \( g_p \) denotes the channel gain of \( S - P \) interference link. Therefore, the SNR at \( R \) is expressed as

\[
\gamma_{r,II} = \min \left( \frac{\Psi_Q}{|g_p|^2}, \Psi_T \right) |g_r|^2,
\]

where \( \Psi_Q = \frac{P_Q}{N_r} \), \( \Psi_T = \frac{P_T}{N_r} \), \( N_r \) represents the noise power imposed on \( R \), and \( g_r \) is the channel gain of \( S - R \) RF sub-link. However, when \( S \) is not a power-limited terminal (Scenario-I), the transmit power at \( S \) is represented as

\[
P_{t,I} = \frac{P_Q}{|g_p|^2}.
\]

In this case, the SNR of \( S - R \) RF sub-link is expressed as

\[
\gamma_{r,I} = \frac{\Psi_Q |g_r|^2}{|g_p|^2}.
\]

2) FSO Sub-Link: The FSO sub-system comprises of \( S \) with a transmit aperture, FSO sub-link, and \( R \) with a receive photodetector. The transmitter dispatches information in optical form that is further converted into electrical one utilizing the photodetector at \( R \). Thereby, instantaneous SNR for \( S - R \) FSO sub-link is written as

\[
\gamma_o = \frac{P_f}{N_o} ||g_o||^2,
\]

where \( g_o \) signifies the channel gain of \( S - R \) FSO sub-link, \( N_o \) denotes the optical noise power at \( R \), and \( P_f \) is the transmit optical power at \( S \).
Finally, after selection combining at $R$, the received SNR is written as [26, Eq. (11)]
\[ \gamma_{f,j} = \max(\gamma_{r,j}, \gamma_o), \] (6)
where $j \in \{I, II\}$.

3) Eavesdropper Link: In the proposed system, an eavesdropper ($E$) targets the RF sub-link to intercept the confidential data stream. The instantaneous SNR at $E$ is expressed as
\[ \gamma_{e,j} = \frac{P_{t,j}}{N_e} \|g_e\|^2, \] (7)
where $g_e$ denotes the channel gain of $S - E$ link and $N_e$ is the noise power at $E$.

III. CHANNELS REALIZATION

In this section, the channel models of the $S - R$ (RF and FSO sub-links), $S - E$, and $S - P$ links are realized individually for further utilization in the mathematical modeling of the proposed CUN system.

A. PDF and CDF of SNRs for the RF Links

The $S - R$ RF sub-link, $S - P$, and $S - E$ links are assumed to follow $\alpha - \mu$ distribution, where $\alpha$ denotes the non-linearity parameter of propagation environment ($\alpha > 0$) and $\mu$ represents the number of multi-path clusters ($\mu > 0$). Since $\alpha - \mu$ model exhibits generalized characteristics [39], several multi-path models [40], e.g. exponential, Rayleigh, Weibull, Gaussian, Gamma, and Nakagami-m, etc., can be obtained by setting different values of $\alpha$ and $\mu$. The PDF of $\gamma_i$, $i \in \{r, p, e\}$, where $r, p,$ and $e$ correspond to the $S - R$ (RF), $S - P$, and $S - E$ links, respectively, is given as [41, Eq. (2)]
\[ f_{\gamma_i}(\gamma) = \frac{\alpha_{\gamma_i}^{\mu_{\gamma_i}}}{\Gamma(\mu_{\gamma_i})} e^{-\delta_i \gamma^{\alpha_{\gamma_i}}} \gamma^{\mu_{\gamma_i}-1} \Theta_i. \] (8)
Here $\delta_i = \Phi_i^{-1}$, $\Theta_i = \alpha_i \mu_i - 1$, $\alpha_i = \frac{1}{\alpha_i}$, $\Phi_i$ is the average SNR of the RF links, and $\Gamma(\cdot)$ denotes the Gamma operator. The CDF of $\gamma_i$ is defined as
\[ F_{\gamma_i}(\gamma) = \int_0^\gamma f_{\gamma_i}(\gamma) d\gamma. \] (9)
Plugging (8) into (9), and after performing integration upon utilizing [42, Eq. (3.381.8)], the CDF leads to
\[ F_{\gamma_i}(\gamma) = \frac{\gamma^{\mu_{\gamma_i}} (\delta_i \gamma^{\alpha_{\gamma_i}})}{\Gamma(\mu_{\gamma_i})}, \] (10)
where $\gamma(\ldots)$ denotes the lower incomplete Gamma function. Finally, with the help of [42, Eq. (8.352.6)], the CDF of $\gamma_i$ can be expressed in an alternative form as
\[ F_{\gamma_i}(\gamma) = 1 - e^{-\delta_i \gamma^{\alpha_{\gamma_i}}} \sum_{m_i=0}^{\mu_{\gamma_i}-1} \frac{(\delta_i \gamma^{\alpha_{\gamma_i}})^{m_i}}{m_i!}. \] (11)

B. PDF and CDF of SNR for FSO SUB-Link

The $S - R$ FSO sub-link is considered to follow Málaga ($M$) turbulence. Since Málaga turbulence is a generalized FSO model, it incorporates some other popular models, such as Gamma-Gamma, lognormal, Rice-Nakagami, $K$ distribution and Gamma as special cases [43, 44]. Hence, the PDF of $\gamma_o$ is expressed as [45, Eq. (9)]
\[ f_{\gamma_o}(\gamma) = \frac{e^2 \chi_o}{2\pi_{\gamma_o}} \sum_{m_o=0}^{\beta_o} \varphi_{nG(1,3)} \left[ \varphi(\mu_{\gamma_o}) \left( \frac{\gamma}{\mu_{\gamma_o}} \right)^{\frac{1}{2}} \frac{e^{2 + 1}}{\gamma^{\beta_o + \mu_{\gamma_o}}} \right], \] (12)
where
\[ \chi_o = \frac{2\alpha_o \mu_{\gamma_o}}{\gamma^{1 + \frac{1}{\beta_o}} \Gamma(\mu_{\gamma_o})} \left( \frac{g \beta_o}{\beta_o + \Omega_o} \right)^{\beta_o + \mu_{\gamma_o}}, \]
\[ \varphi = \frac{e^2 \alpha_o \beta_o (g + \Omega_o)}{(e^2 + 1)(g \beta_o + \Omega_o)}, \]
\[ \varphi_n = v_n \left( \frac{\alpha_o \beta_o}{g \beta_o + \Omega_o} \right)^{\frac{\alpha_o + \mu_{\gamma_o}}{2}}, \]
\[ v_n = \left( \frac{\beta_o}{\mu_{\gamma_o}} \right)^{\frac{\mu_{\gamma_o}}{2}} \left( \frac{1}{(m_o - 1)!} \right)^{\frac{\alpha_o}{g}} \left( \frac{\alpha_o + \mu_{\gamma_o}}{\beta_o} \right)^{\mu_{\gamma_o}}, \]
$\alpha_o$ and $\beta_o$ denote the fading parameters, $g = 2b_o(1 - \rho)$ represents the average power of scattering components received through off-axis eddies, $2b_o$ denotes the total average power of scattering components, $\epsilon (0 \leq \rho \leq 1)$ signifies the amount of scattering power coupled to the LOS component, the average power realized from coherent contributions is symbolized by $\Omega_o = \Omega_o + 2b_o + 2 \sqrt{2b_o} \Omega_o \cos(\theta_A - \theta_B)$, $\Omega_o$ is termed as the average power of LOS components, $\theta_A$ and $\theta_B$ represent the deterministic phases of LOS, $\mu_1 = \Phi_o$, $\mu_2 = (\alpha_o + 1)(2g \beta_o + \Omega_o) + (1 + 1/\beta_o)$, $s$ denotes the detection technique (i.e. $s = 1$ refers to HD technique and $s = 2$ refers to IM/DD technique), $\Phi_o$ denotes the average SNR of FSO sub-link that is related to electrical SNR $\mu_{\gamma_o}$, and $G[\cdot]$ represents the Meijer’s G function [46]. Plugging (12) into (9), the CDF of $\gamma_o$ is written as [45, Eq. 11]
\[ F_{\gamma_o}(\gamma) = K \sum_{m_o=0}^{\beta_o} \gamma^{3.3} \left[ \frac{V}{\mu_{\gamma_o}} \right] \left[ q_1 + q_2, 0 \right], \] (13)
where
\[ K = \frac{e^2 \chi_o}{2\pi_{\gamma_o}}, \quad \gamma = \varphi \left( n \right), \quad V = \frac{\alpha_o}{\gamma}, \quad q_1 = \left\{ (e^{2} + 1), \ldots, (e^{2} + s) \right\} \text{ including } s \text{ number of terms, and} \]
\[ q_2 = \left\{ (e^{2} + 1), \ldots, (e^{2} + s) \right\} \text{ including } s \text{ number of terms.} \]
The FSO link can be temporarily blocked due to its extreme LOS requirements. Hence, including link blocking probability ($P_o$) in FSO and assuming SNR of the $S - R$ FSO sub-link as $\gamma_o'$, the PDF of $\gamma_o'$ is expressed finally as [6, Eq. (10)]
\[ f_{\gamma_o'}(\gamma) = P_o \delta(\gamma) + (1 - P_o) f_{\gamma_o}(\gamma), \] (14)
where $\delta(\cdot)$ denotes a Dirac delta function [47, Eq. (14.03.02.0001.01)]. Now, the CDF of $\gamma_o'$ is expressed as
\[ F_{\gamma_o'}(\gamma) = \int_0^\gamma f_{\gamma_o'}(\gamma) d\gamma = P_o + (1 - P_o) F_{\gamma_o}(\gamma). \] (15)
Substituting (13) into (15), we obtain

$$F_{\gamma_{r}}(\gamma) = P_{o} + (1 - P_{o}) K \sum_{m_{r}=0}^{\beta_{o}} \sum_{m_{s}=0}^{\beta_{s}} G_{s+1,3,3+1}^{3,1} \left[ \frac{V_{\gamma}}{\mu_{s}} \right]_{1,q_{1}}^{1,q_{2},0} \left[ \frac{V_{\gamma}}{\mu_{s}} \right]_{1,q_{1}}^{1,q_{2},0}.$$  

(16)

IV. COGNITIVE UNDERLAY NETWORK

This section demonstrates the CDFs of SNRs for the hybrid RF/FSO CUN considering both scenarios (Scenarios-I & II).

A. Scenario I

Considering the interference of $S - P$ link and denoting SNR of the $S - R$ RF sub-link as $\gamma_{r,I}$, the CDF of $\gamma_{r,I}$ is written as [4, Eq. (4)]

$$F_{\gamma_{r,I}}(\gamma) = \int_{0}^{\infty} F_{\gamma_{r}} \left( \frac{\gamma}{\Psi_{Q}} \right) f_{\gamma_{r}}(x) dx.$$  

(17)

Now substituting (11) and (8) into (17), the CDF is expressed further as

$$F_{\gamma_{r,I}}(\gamma) = 1 - \sum_{m_{r}=0}^{\mu_{r}-1} \frac{\alpha_{r} \delta_{r}^{m_{r}} \delta_{p}^{m_{r}}}{2\Gamma(\mu_{p})\mu_{r}!} \left( \frac{\gamma}{\Psi_{Q}} \right)^{\alpha_{r} m_{r}} 
\times \int_{0}^{\infty} e^{-\left(\delta_{r} x^{m_{r}} + \Xi_{r}^{\alpha_{r}} \delta_{r}^{m_{r}} x^{m_{r}} + \delta_{p} x^{m_{r}} \right)} x^{\mu_{p}} dx,$$

(18)

where $\Xi_{r}^{\alpha_{r}} = \delta_{r} + \alpha_{r} \delta_{p}^{m_{r}}$. Now, following some algebraic manipulations, assuming $\alpha_{p} = \alpha_{r}$, and performing integration utilizing [42, Eq. (3.326.2)], the CDF of $S - R$ RF sub-link is finally obtained as

$$F_{\gamma_{r,I}}(\gamma) = 1 - \sum_{m_{r}=0}^{\mu_{r}-1} \frac{\Gamma(\Xi_{r}^{\alpha_{r}})}{\alpha_{r} \Gamma(\mu_{p})\mu_{r}!} \left( \frac{\gamma}{\Psi_{Q}} \right)^{\alpha_{r} m_{r}} 
\times \left( \Xi_{r}^{\alpha_{r}} \gamma^{\alpha_{r}} \delta_{p} \right)^{m_{r}} = \Xi_{r}^{\alpha_{r}} \gamma^{\alpha_{r}},$$

(19)

where $\Xi_{r}^{\alpha_{r}} = \alpha_{r} \delta_{p} \delta_{r}^{m_{r}} + m_{r} \delta_{p}^{m_{r}}$. According to (6), the CDF of end-to-end SNR at $R$ is written as

$$F_{\gamma_{r,I}}(\gamma) = F_{\gamma_{r,I}}(\gamma) \times F_{\gamma_{o}}(\gamma).$$

(20)

By replacing (19) and (16) into (20), the CDF of $\gamma_{r,I}$ is derived in (21) shown at the bottom of this page.

B. Scenario II

Assuming double power constraints in the RF sub-link, the CDF of $\gamma_{r,I}$ is written as

$$F_{\gamma_{r,I}}(\gamma) = Pr \left\{ |g_{r}|^{2} \leq \gamma_{r} \right\}.$$  

(22)

Since $g_{r}$ and $g_{p}$ are completely independent to one other, hence, $\Lambda_{1}$ is expressed in an alternative form as

$$\Lambda_{1} = Pr \left\{ |g_{r}|^{2} \leq \frac{\Psi_{Q}}{\Psi_{T}} \right\} \times Pr \left\{ |g_{r}|^{2} \leq \frac{\Psi_{Q}}{\Psi_{T}} \right\}.$$  

(23)

Substituting (11) into (23), $\Lambda_{1}$ can be finally written as

$$\Lambda_{1} = 1 - \sum_{m_{p}=0}^{\mu_{r}-1} \sum_{m_{r}=0}^{\mu_{r}-1} \Xi_{1,I}^{\mu_{p} \delta_{p} \delta_{r}^{m_{r}}} e^{-\delta_{r} \Psi_{T}^{\alpha_{r}} \gamma_{I}} + \sum_{m_{p}=0}^{\mu_{r}-1} \sum_{m_{r}=0}^{\mu_{r}-1} \Xi_{1,I}^{\mu_{p} \delta_{p} \delta_{r}^{m_{r}}} e^{-\delta_{r} \Psi_{T}^{\alpha_{r}} \gamma_{I}}.$$  

(24)

Replacing (8) and (11) into (25), $\Lambda_{2}$ is obtained as

$$\Lambda_{2} = \Xi_{1,I}^{\mu_{p} \delta_{p} \delta_{r}^{m_{r}}} e^{-\delta_{r} \Psi_{T}^{\alpha_{r}} \gamma_{I}}.$$  

(25)

Finally, utilizing (24) and (26) in (22) results in

$$F_{\gamma_{r,I}}(\gamma) = Pr \left\{ |g_{r}|^{2} \leq \frac{\Psi_{Q}}{\Psi_{T}} \right\} \times Pr \left\{ |g_{r}|^{2} \leq \frac{\Psi_{Q}}{\Psi_{T}} \right\}.$$  

(26)
Now, substituting (27) and (16) into (28), the CDF of \( \gamma_{f,II} \) is obtained as shown in (29) shown at the bottom of this page, where \( X = 1 + \Xi_{II} = -\sum_{m_{s}=0}^{\infty} \Xi_{1}^II \).

V. PERFORMANCE ANALYSIS

In this section, we demonstrate the novel analytical expressions for SOP, probability of SPSC, and EST utilizing (8), (21), and (29).

A. Secure Outage Probability Analysis

In order to investigate the secrecy behavior of the proposed hybrid system, SOP is one of the fundamental performance metrics. SOP particularly depends on whether the target secrecy rate \( (Y_{e}) \) is greater than secrecy capacity \( (C_{s}) \), where \( C_{s} = \lfloor \log_{2}(1 + \gamma_{f,j}) - \log_{2}(1 + \gamma_{e,s}) \rfloor^{+} \) and \( \lfloor z \rfloor^{+} = \max \{ z, 0 \} \). So, mathematically, SOP can be described as [49, Eq. (14)]

\[
SOP = \Pr \{ C_{s}(\gamma_{f,j},\gamma_{e,s}) \leq Y_{e} \}
= \int_{0}^{\infty} F_{\gamma_{f,j}}(\sigma \gamma + \sigma - 1)f_{\gamma_{e,s}}(\gamma)d\gamma,
\]

where \( \sigma = 2^{T_{e}} \) and \( Y_{e} > 0 \). Since Meijer’s \( G \) function exists in (21) and (29), derivation of SOP utilizing (30) is almost impossible. Hence, the lower bound of SOP can be derived as [50], [51]

\[
SOP \geq SOP_{L} = \int_{0}^{\infty} F_{\gamma_{f,j}}(\sigma \gamma)f_{\gamma_{e,s}}(\gamma)d\gamma.
\]
where \( \zeta_1 = \Delta (\alpha_r, -\Theta_r - q_2) \), \( \zeta_2 = \Delta (\alpha_r, -\Theta_r-1) \), \( \zeta_3 = \Delta (\alpha_r, -\Theta_r-q_1) \), and \( \Delta (c, r) = \frac{r + 1}{e^r + 1}, \ldots, \frac{r + e-1}{e} \) as defined in [52, Eq. (22)].

**Derivation of \( \mathcal{F}_3 \):** \( \mathcal{F}_3 \) is expressed as

\[
\mathcal{F}_3 = \int_0^\infty (\Xi^l_1 e^{-\delta_r \sigma_r} + \Phi_0 e^{-\delta_r}) - \Xi^l_1 e^{-\delta_r \sigma_r} \sigma_r e^{-\delta_r \gamma R} d\gamma.
\]

Utilizing binomial expansion and [42, Eq. (3.326.2)], \( \mathcal{F}_3 \) is expressed in alternative form as

\[
\mathcal{F}_3 = \sum_{m_2=0}^{\infty} \int_0^\infty \Xi^l_1 e^{-\delta_r \sigma_r} \sigma_r e^{-\delta_r \gamma R} d\gamma = \sum_{m_2=0}^{\infty} \frac{\Gamma(\Xi^l_1)}{\alpha_r^m} \Xi_{\alpha_r}^l d\gamma,
\]

where \( \Xi_{\alpha_r}^l \) = \( \Theta_r + \alpha_r m_r + \alpha_p m_r \) and \( \Xi^l_1 = \frac{\Xi_{\alpha_r}^l + 1}{\alpha_r} \).

**Derivation of \( \mathcal{F}_4 \):** Following the similar approach of \( \mathcal{F}_2 \) and \( \mathcal{F}_3 \), \( \mathcal{F}_4 \) is expressed as

\[
\mathcal{F}_4 = \int_0^\infty \gamma \sigma_r (m_r + m_s + m_m) + \Theta_r e^{-\delta_r \psi_0 e^{-\delta_r \gamma R} \sigma_r e^{-\delta_r \gamma R}} d\gamma = \int_0^\infty \gamma \sigma_r (m_r + m_s + m_m) + \Theta_r e^{-\delta_r \psi_0 e^{-\delta_r \gamma R} \sigma_r e^{-\delta_r \gamma R}} d\gamma.
\]

Following some mathematical manipulations defined in [53] and utilizing identities [46, Eqs. (8.4.3.1) and (2.24.1.1)], \( R_2 \) and \( R_3 \) are further derived as

\[
R_2 = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma.
\]

where \( \Xi_{\alpha_r}^l = \Xi_{\alpha_r}^l + \Xi^l_1 \).

**Derivation of \( \mathcal{R}_4 \):** By following the similar approach as of \( R_2 \) and \( R_3 \), \( \mathcal{R}_4 \) is expressed as

\[
\mathcal{R}_4 = \int_0^\infty \gamma \sigma_r (m_r + m_s + m_m) + \Theta_r e^{-\delta_r \psi_0 e^{-\delta_r \gamma R} \sigma_r e^{-\delta_r \gamma R}} d\gamma = \int_0^\infty \gamma \sigma_r (m_r + m_s + m_m) + \Theta_r e^{-\delta_r \psi_0 e^{-\delta_r \gamma R} \sigma_r e^{-\delta_r \gamma R}} d\gamma = \int_0^\infty \gamma \sigma_r (m_r + m_s + m_m) + \Theta_r e^{-\delta_r \psi_0 e^{-\delta_r \gamma R} \sigma_r e^{-\delta_r \gamma R}} d\gamma = \int_0^\infty \gamma \sigma_r (m_r + m_s + m_m) + \Theta_r e^{-\delta_r \psi_0 e^{-\delta_r \gamma R} \sigma_r e^{-\delta_r \gamma R}} d\gamma = \int_0^\infty \gamma \sigma_r (m_r + m_s + m_m) + \Theta_r e^{-\delta_r \psi_0 e^{-\delta_r \gamma R} \sigma_r e^{-\delta_r \gamma R}} d\gamma.
\]

where \( \Xi_{\alpha_r}^l = \Xi_{\alpha_r}^l + \Xi^l_1 \).

**Derivation of \( \mathcal{R}_5 \):** With some mathematical manipulations and following identities [46, Eqs. (8.4.3.1) and (2.24.1.1)], \( \mathcal{R}_5 \) is expressed as

\[
\mathcal{R}_5 = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma.
\]

where \( \Xi_{\alpha_r}^l = \Xi_{\alpha_r}^l + \Xi^l_1 \).

The SOP for the \( \mathcal{R}_4 \) in the \( m_1 \) stage is given by

\[
\frac{\alpha_r^2 d_P}{2 \Gamma(\mu_r)} \left[ \Phi_0 \left( \mathcal{R}_1 - \sum_{m_1=0}^{\mu_r-1} \Xi_{\alpha_r}^l \sigma_r m_r R_2 + \sum_{m_1=0}^{\mu_r-1} \Xi_{\alpha_r}^l \sigma_r m_r e^{-\delta_r \psi_0 e^{-\delta_r \gamma R} \sigma_r e^{-\delta_r \gamma R}} R_3 - \sum_{m_1=0}^{\mu_r-1} \sum_{m_2=0}^{m_3} \Omega \sum_{m_3=0}^{m_4} \Xi_{\alpha_r}^l \sigma_r m_r e^{-\delta_r \psi_0 e^{-\delta_r \gamma R} \sigma_r e^{-\delta_r \gamma R}} e^{-\delta_r \gamma R} \right) \right] + (1 - P_r) K \sum_{m_2=0}^{\mu_r-1} \Xi_{\alpha_r}^l \sigma_r m_r e^{-\delta_r \psi_0 e^{-\delta_r \gamma R} \sigma_r e^{-\delta_r \gamma R}} e^{-\delta_r \gamma R} \right]
\]

where \( \Xi_{\alpha_r}^l = \Xi_{\alpha_r}^l + \Xi^l_1 \).

**Derivation of \( \mathcal{R}_5 \):** With some mathematical manipulations and following identities [46, Eqs. (8.4.3.1) and (2.24.1.1)], \( \mathcal{R}_5 \) is expressed as

\[
\mathcal{R}_5 = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma = \frac{1}{\alpha_r^2} \int_0^\infty \gamma \Xi_{\alpha_r}^l d\gamma.
\]

where \( \Xi_{\alpha_r}^l = \Xi_{\alpha_r}^l + \Xi^l_1 \).
is derived as

\[
\mathcal{R}_5 = \int_0^\infty \gamma e^{-\alpha \gamma} G_{3+s,1}^{3,s,1}(z_{\mu,s}) \left[ \frac{V \sigma \gamma}{\mu_s} \right] d\gamma
\]

\[
= \int_0^\infty \gamma e^{-\alpha \gamma} G_{3,s+1,1,3,s+1}^{1,0} \left[ \frac{V \sigma \gamma}{\mu_s} \right] d\gamma
\]

\[
= \frac{\alpha e^{(2\theta_e+1)\left( \frac{V \sigma \gamma}{\mu_s} \right) \mu_s - 1, q_1, q_2, 0}}{2\pi (\mu_s - 1, q_1, q_2, 0)}
\]

\[
\times G_{1+\alpha e^{2s\alpha e}}^{1+\alpha e,3+s\alpha e} \left[ \frac{V \sigma \gamma}{\mu_s} \right] \left( \xi_1, \Delta(\alpha e, -\theta_e) \right) \left( 0, \xi_2, \xi_3 \right).
\]

(44)

**Derivation of \( \mathcal{R}_6 \) and \( \mathcal{R}_7 \):** \( \mathcal{R}_6 \) and \( \mathcal{R}_7 \) are derived by following [46, Eq. (8.4.3.1)] as

\[
\mathcal{R}_6 = \mathcal{R}_7 = \int_0^\infty \gamma e^{-\alpha \gamma} m_x + \Theta_x e^{-\delta x \gamma} \Psi_T^{-\alpha x} \Psi_\alpha^{-\gamma} e^{-\delta x \gamma} \]

\[
\times G_{3,s+1,1,3,s+1}^{3,s,1}(z_{\mu,s}) \left[ \frac{V \sigma \gamma}{\mu_s} \right] d\gamma
\]

\[
= \int_0^\infty \gamma e^{-\alpha \gamma} m_x + \Theta_x G_{1,0}^{1,0,1,0,0,1,0,1,1} \left[ \frac{V \sigma \gamma}{\mu_s} \right] d\gamma
\]

\[
\times G_{3,s+1,1,3,s+1}^{3,s,1}(z_{\mu,s}) \left[ \frac{V \sigma \gamma}{\mu_s} \right] d\gamma
\]

\[
= \frac{\alpha e^{(2\theta_e+1)\left( \frac{V \sigma \gamma}{\mu_s} \right) \mu_s - 1, q_1, q_2, 0}}{2\pi (\mu_s - 1, q_1, q_2, 0)}
\]

\[
\times G_{1+\alpha e^{2s\alpha e}}^{1+\alpha e,3+s\alpha e} \left[ \frac{V \sigma \gamma}{\mu_s} \right] \left( \xi_1, \Delta(\alpha e, -\theta_e) \right) \left( 0, \xi_2, \xi_3 \right).
\]

(45)

Now, for tractable analysis, transforming Meijer’s \( G \) functions into Fox’s \( H \) functions via utilizing [54, Eqs. (6.2.3) and (6.2.8)] and performing integration by means of [55, Eq. (2.3)] and [39, Eq. (3)], \( \mathcal{R}_6 \) and \( \mathcal{R}_7 \) are obtained as

\[
\mathcal{R}_6 = \mathcal{R}_7 = \int_0^\infty \gamma e^{-\alpha \gamma} m_x + \Theta_x H_{3,s+1,1,3,s+1}^{3,s,1}(z_{\mu,s}) \left[ \frac{V \sigma \gamma}{\mu_s} \right] d\gamma
\]

\[
= \int_0^\infty \gamma e^{-\alpha \gamma} m_x + \Theta_x H_{1,0,1,0,1,0,1,1}^{1,0,0,1,0,1,1,1}(z_{\mu,s}) \left[ \frac{V \sigma \gamma}{\mu_s} \right] d\gamma
\]

\[
= \frac{\alpha e^{(2\theta_e+1)\left( \frac{V \sigma \gamma}{\mu_s} \right) \mu_s - 1, q_1, q_2, 0}}{2\pi (\mu_s - 1, q_1, q_2, 0)}
\]

\[
\times G_{1+\alpha e^{2s\alpha e}}^{1+\alpha e,3+s\alpha e} \left[ \frac{V \sigma \gamma}{\mu_s} \right] \left( \xi_1, \Delta(\alpha e, -\theta_e) \right) \left( 0, \xi_2, \xi_3 \right).
\]

(46)

**Derivation of \( \mathcal{R}_8 \):** \( \mathcal{R}_8 \) is derived by following the similar identities from \( \mathcal{R}_6 \) and \( \mathcal{R}_7 \) as

\[
\mathcal{R}_8 = \int_0^\infty \gamma e^{-\alpha \gamma} m_x + \Theta_x e^{-\delta x \gamma} \Psi_T^{-\alpha x} \Psi_\alpha^{-\gamma} e^{-\delta x \gamma} \]

\[
\times G_{3,s+1,1,3,s+1}^{3,s,1}(z_{\mu,s}) \left[ \frac{V \sigma \gamma}{\mu_s} \right] d\gamma
\]

\[
= \int_0^\infty \gamma e^{-\alpha \gamma} m_x + \Theta_x G_{1,0}^{1,0,1,0,1,1,1,1}(z_{\mu,s}) \left[ \frac{V \sigma \gamma}{\mu_s} \right] d\gamma
\]

\[
\times G_{3,s+1,1,3,s+1}^{3,s,1}(z_{\mu,s}) \left[ \frac{V \sigma \gamma}{\mu_s} \right] d\gamma
\]

\[
= \frac{\alpha e^{(2\theta_e+1)\left( \frac{V \sigma \gamma}{\mu_s} \right) \mu_s - 1, q_1, q_2, 0}}{2\pi (\mu_s - 1, q_1, q_2, 0)}
\]

\[
\times G_{1+\alpha e^{2s\alpha e}}^{1+\alpha e,3+s\alpha e} \left[ \frac{V \sigma \gamma}{\mu_s} \right] \left( \xi_1, \Delta(\alpha e, -\theta_e) \right) \left( 0, \xi_2, \xi_3 \right).
\]

(47)

**B. Strictly Positive Secrecy Capacity Analysis**

In order to ensure secrecy in a wiretap-prone model, the probability of SPSC acts as one of the crucial performance metrics that ensure uninterrupted data stream conveyance only when the secrecy behavior of the system remains up to scratch \( \gamma_{ij} > \gamma_{e,j} \). Mathematically, the analytical expression of the probability of SPSC can be easily deduced from the SOP that is given as [32, Eq. (25)]

\[
SPSC = \Pr \{ C_s > 0 \}
\]

\[
= 1 - \Pr \{ C_s \leq 0 \}.
\]

Hence, \( \text{SPSC} = 1 - \text{SOP}_L \big|_{\gamma_e = 0} \) (Scenario-I) \( (48) \)

and \( \text{SPSC}^\text{II} = 1 - \text{SOP}_L \big|_{\gamma_e = 0} \) (Scenario-II) \( (49) \)

**C. Effective Secrecy Throughput Analysis**

EST is another secrecy performance metric that ensures achievable secure average throughput measurements. When the reliability issue is considered equally important along with the security issue then this parameter is considered. Mathematically, EST is formulated as the product of target secrecy rate and the probability of successful transmission that is given as [57, Eq. (5)]

\[
\text{EST} = \gamma (1 - \text{SOP}).
\]

Hence, \( \text{EST}^\text{I} = \gamma (1 - \text{SOP}_L) \) (Scenario-I) \( (50) \)

and \( \text{EST}^\text{II} = \gamma (1 - \text{SOP}_L) \) (Scenario-II) \( (51) \)

**VI. Numerical Results**

In this section, the impacts of all the system parameters (e.g., \( \alpha_t, \mu, \theta_e, \Phi, \Psi_T, P_o, \alpha, \beta, \epsilon, \) and \( s \)) on the secrecy behavior of the proposed scenarios are illustrated graphically utilizing the expressions in (32), (39), (48), (49), (50), and (51). Note that the univariate Meijer’s \( G \) and Fox’s \( H \) functions can be easily computed utilizing standard Mathematica packages whereas the EGBFHF is computed utilizing [39, Table I]. Moreover, all the mathematical expressions are also validated via MC simulations averaging 10^6 random samples of \( \alpha - \mu \) and Málaga random variables. It can clearly be seen that MC simulation results are in good agreement with the analytical results that strongly
corroborate the analysis of this paper. According to [43], all
the numerical results due to the FSO link, are demonstrated
assuming \((\alpha_o, \beta_o) = (2.296, 2)\) for strong, \((\alpha_o, \beta_o) = (4.2, 3)\) for moderate, and \((\alpha_o, \beta_o) = (8, 4)\) for weak turbulences, \(r = 1\) for HD and \(r = 2\) for IM/DD techniques. On the other hand, similar
to [5], we assume that RF links have the following parameters:
\(\alpha_r = \alpha_p = \alpha_e, \mu_r = \mu_p = \mu_e,\) and \(\Upsilon_e = 0.05\) bits/sec/Hz, unless specified otherwise. Note that even though the representa-
tion of (27) and (29) are done through an infinite series, it quickly
leads to precise outcomes within the initial hundred terms of the
series.

The impacts of \(\alpha_i\) on the EST and SPSC are demonstrated in
Figs. 2 and 3 for Scenario-I.

It is observed that EST gradually increases with \(\Psi_Q\). Similar
observations are also seen in [58]. Again, as the non-linearity
parameter increases, the communication link improves [5] (i.e.
the link will undergo less amount of fading with higher \(\alpha_i\)),
therefore the EST becomes better for lower \(\alpha_p\) and \(\alpha_e\). On
the other hand, as expected, the SPSC is enhanced as \(\Phi_c\) and \(\Phi_r\)
increase from a lower to a higher value.

The SOP is plotted against \(\Phi_p, \Phi_Q,\) and \(\Phi_r\) considering
Scenario-I in Figs. 4, 5, and 6, respectively, with a view to
observing the impacts of the number of multipath clusters.
It can clearly be noted that the SOP performance is improved
by \(\Psi_Q\) and \(\Phi_r\), and diminished by \(\Phi_p\) as reported in [4]. This is as

![Fig. 2. The EST versus \(\Psi_Q\) for selected values of \(\alpha_o\) and \(\alpha_e\) with \(\alpha_o = 2.296, \beta_o = 2, \Omega_o = 1, g = 2, \epsilon = 1, P_o = 0.5, \alpha_r = 2, \mu_r = \mu_p = \mu_e = 2, \Phi_r = 15 dB, \Phi_p = 10 dB, \Phi_e = \mu_s - 5 dB,\) and \(\Upsilon_e = 0.05\) bits/sec/Hz.](image)

![Fig. 3. The SPSC versus \(\Psi_Q\) for selected values of \(\alpha_o\) and \(s\) with \(\alpha_o = 2.296, \beta_o = 2, \Omega_o = 1, g = 2, \epsilon = 1, P_o = 0.5, \alpha_e = 5, \mu_r = \mu_p = \mu_e = 6, \Phi_r = \Phi_p = 15 dB, \Phi_e = 0 dB,\) and \(\Psi_Q = 15 dB\).](image)

![Fig. 4. The SOP versus \(\Phi_p\) for selected values of \(\mu_c\) and \(\Phi_c\) with \(\alpha_o = 2.296, \beta_o = 2, \Omega_o = 1, g = 2, \epsilon = 1, \alpha_c = \alpha_p = \alpha_e = 2, \mu_c = \mu_p = 2, P_o = 0.1, \Phi_r = 15 dB, \mu_r = 12 dB, \Psi_Q = -5 dB,\) and \(\Upsilon_e = 0.05\) bits/sec/Hz.](image)

![Fig. 5. The SOP versus \(\Psi_Q\) for selected values of \(\mu_p\) with \(\alpha_o = 2.296, \beta_o = 2, \Omega_o = 1, g = 2, \epsilon = 1, \alpha_e = \alpha_p = \alpha_e = 2, \mu_r = \mu_p = \mu_e = 2, P_o = 0.1, \Phi_p = 10 dB, \Phi_r = 15 dB, \Phi_e = 10 dB,\) and \(\Upsilon_e = 0.05\) bits/sec/Hz.](image)
expected because the larger $\Psi_Q$ indicates the higher transmitting power at $S$. On the other hand, $\Phi_r$ improves the $S-R$ RF sub-link while $\Phi_p$ strengthens the $S-P$ link. It is obvious that an increase in $\mu_r$ will increase the channel diversity thereby mitigating the channel fading. Therefore, the SOP decreases with $\mu_r$ and increases with $\mu_p$ and $\mu_e$. Similar results were also demonstrated in [5] that corroborate the outcomes in this work. In Fig. 7, the impact of $\mu_p$ is demonstrated on SOP considering Scenario-II. Similar to Scenario-I, in this particular case, it is also observed that the SOP drastically degrades with $\mu_p$.

In Fig. 8, the probability of link blockage is observed via plotting SOP against the average SNR of FSO link under various levels of pointing error. It is observed that the probability of link blockage imposes a significant impact on the SOP performance, that is, the SOP degrades with the increase in $P_o$ (increased $P_o$ signifies a stronger link blockage). It is obvious because similar to the observations in [6], the atmospheric turbulence degrades with the increase in link blockage, which yields to a degraded secrecy performance.

Fig. 9 depicts the impact of $\Psi_T$ on the EST performance for Scenario-II. It can be noted that an increased $\Psi_T$ is beneficial for EST performance since in that case the increased transmit power yields a better received SNR at the destination. After 15 $dB$, an error floor is observed, as expected. This occurs as the transmit power at $S$ is limited by a threshold value.
The FSO link performance is significantly affected by atmospheric turbulence. Based on the values of $\alpha_o$ and $\beta_o$, three turbulence conditions (e.g., strong, moderate, and weak) are considered in this work. Figs. 10 and 11 demonstrate how the secrecy performance is affected by various turbulence conditions under Scenarios I & II.

As stronger turbulence results in lower SNR at the destination, weaker turbulence leads to improved secrecy performance. This is depicted in Fig. 12 by showing the SOP as a function of the average SNR of the FSO channel experiencing Scenario-I. The result exhibits that a higher pointing error (lower value of $\epsilon$) is more detrimental for better SOP than a lower pointing error (higher value of $\epsilon$). This happens since a lower pointing error designates a better-pointing accuracy that yields a better detection of the received signal. This statement can be easily justified with similar results presented in [57].

The purpose of plotting the EST against $\Upsilon_e$ in Fig. 13 is to examine the influence of changes in $\Upsilon_e$ on the system’s secrecy performance. This graph also showcases the balance between the resources needed to keep the information secure and the rate at which confidential information can be transmitted. It

Besides the atmospheric turbulence, pointing error also plays a vital role in the FSO link performance that is illustrated in Fig. 12 by depicting SOP as a function of the average SNR of the FSO channel experiencing Scenario-I. The result exhibits that a higher pointing error (lower value of $\epsilon$) is more detrimental for better SOP than a lower pointing error (higher value of $\epsilon$). This happens since a lower pointing error designates a better-pointing accuracy that yields a better detection of the received signal. This statement can be easily justified with similar results presented in [57].
is noticed that a lower $T_e$ exhibits in lower resources being required to maintain the desired secrecy level, resulting in larger EST performance. As $T_e$ increases, the desired level of security will need more resources for the increased secrecy threats, hence the EST decreases.

VII. CONCLUSION

In this work, the secrecy performance of a CUN-based hybrid RF/FSO network has been investigated wherein both maximum transmit power and interference power constraints within the CR spectrum are taken into consideration. Assuming all the RF and FSO links are subjected to $\alpha - \mu$ fading and Malaga ($M$) turbulence, respectively, the expressions of SOP, probability of SPSC, and EST are derived in closed-form. The tightness of the simulation and analytical results are also demonstrated via MC simulations. Furthermore, the impacts of the system parameters on the security performance are analyzed by capitalizing on the derived expressions. The results found from our analysis are as follows.

1) A slight increase in fading parameter of the main link can increase the system security significantly.
2) The variation in EST with the fading parameter ($\alpha$) is almost linear until it reaches the maximum value.
3) For a similar amount of fading or atmospheric turbulence, IM/DD performs worse than the HD technique.
4) With the increase of the distance between the transmitter and receiver, the average SNR decreases, thereby diminishing the secrecy of the system under consideration.
5) The high chance of link obstruction and severe pointing error also remarkably deteriorate the secrecy of the system. The validity of our result is further demonstrated by the Monte-Carlo simulation.

APPENDIX

A. Proof of $\Lambda_2$

\[ \Lambda_2 = \int_{\Psi_{\mathbb{Q}}}^{\infty} \frac{\alpha_p \delta p}{2 \Gamma(\mu_p)} y^{\Psi_{\mathbb{Q}}} e^{-\delta_p y^{\Psi_{\mathbb{Q}}}} dy \]

\[ \int_{\Psi_{\mathbb{Q}}}^{\infty} \frac{\mu - 1}{2 \Gamma(\mu_p)} \frac{\delta_p^{\mu_p}}{m_p} y^{\Psi_{\mathbb{Q}}} e^{-\delta_p y^{\Psi_{\mathbb{Q}}}} dy \]

By using [42, Eq. (3.381.9)], $P_1$ is expressed as

\[ P_1 = \frac{\alpha_p \delta p}{2 \Gamma(\mu_p)} \int_{\Psi_{\mathbb{Q}}}^{\infty} y^{\Psi_{\mathbb{Q}}} e^{-\delta_p y^{\Psi_{\mathbb{Q}}}} dy = \Xi \Gamma^\mu \frac{\delta p}{\delta_p} \]

Due to tractable analysis, assuming $\alpha_p = \alpha_r$ and using similar formula as for $P_1$, $P_2$ is expressed as

\[ P_2 = \sum_{m_r=0}^{\mu - 1} \frac{\alpha_p \delta p}{2 \Gamma(\mu_p)} \frac{\delta_m}{m_r} y^{\Psi_{\mathbb{Q}}} e^{-\delta_m y^{\Psi_{\mathbb{Q}}}} \int_{\Psi_{\mathbb{Q}}}^{\infty} y^{\Psi_{\mathbb{Q}}} - \delta_m y^{\Psi_{\mathbb{Q}}} - \delta_m y^{\Psi_{\mathbb{Q}}} = \Xi \Gamma^\mu \frac{\delta p}{\delta_p} \]

Utilizing the integral formula of [42, Eq. (8.352.7)] and performing some mathematical manipulations, $P_2$ is demonstrated in an alternative form as

\[ P_2 = \sum_{m_r=0}^{\mu - 1} \sum_{m_m=0}^{\mu - 1} \sum_{m_0=0}^{\mu - 1} \sum_{m_5=0}^{\mu - 1} \frac{m_3}{m_4} (\frac{\Omega + m_5 - 1}{m_5}) (\frac{\Xi \Gamma^\mu \frac{\delta p}{\delta_p}}{\delta_m y^{\Psi_{\mathbb{Q}}}}) \]

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