A NEW TEST OF THE STATISTICAL NATURE OF THE BRIGHTEST CLUSTER GALAXIES

YEN-TING LIN1,2,3, JEREMIAH P. OSTRIKER1, AND CHRISTOPHER J. MILLER4

ABSTRACT

A novel statistic is proposed to examine the hypothesis that all cluster galaxies are drawn from the same luminosity distribution (LD). In such a "statistical model" of galaxy LD, the brightest cluster galaxies (BCGs) are simply the statistical extreme of the galaxy population. Using a large sample of nearby clusters, we show that BCGs in high luminosity clusters (e.g., \(L_\text{tot} \gtrsim 4 \times 10^{11} h^{-2} L_\odot\)) are unlikely (probability \(\leq 3 \times 10^{-4}\)) to be drawn from the LD defined by all red cluster galaxies more luminous than \(M_r \leq -20\). On the other hand, BCGs in less luminous clusters are consistent with being the statistical extreme. Applying our method to the second brightest galaxies, we show that they are consistent with being the statistical extreme, which implies that the BCGs are also distinct from non-BCG luminous, red, cluster galaxies. We point out some issues with the interpretation of the classical tests proposed by Tremaine & Richstone (1977, TR) that are designed to examine the statistical nature of BCGs, investigate the robustness of both our statistical test and those of TR against difficulties in photometry of galaxies of large angular size, and discuss the implication of our findings on surveys that use the luminous red galaxies to measure the baryon acoustic oscillation features in the galaxy power spectrum.

Subject headings: galaxies: clusters: general – galaxies: elliptical and lenticular, cD – galaxies: luminosity function, mass function

1. INTRODUCTION

Over the past decade, the features in the cosmic microwave background angular power spectrum and galaxy clustering power spectrum due to the baryon acoustic oscillations (BAO) have emerged to be a premier standard ruler, and strong cosmological constraints have been derived using this technique (e.g., Spergel et al. 2003; Eisenstein et al. 2005; Cole et al. 2005; Padmanabhan et al. 2007). In the galaxy BAO measurements, the tracer population of the large scale matter distribution often employed is the so-called luminous red galaxies (LRGs). These are massive elliptical galaxies characterized by an old and passively evolving stellar population (Eisenstein et al. 2001), and tend to be the dominant, central galaxies in group or cluster scale dark matter halos [e.g., Zheng et al. 2008] throughout this paper we do not refer to the most luminous galaxy in clusters and groups differently, but simply use the term brightest cluster galaxies (BCGs)].

Recognizing the constraining power of BAO measurements, an impressive array of on-going and planned cosmological experiments have adopted this method as the main survey component (e.g., BOSS, WiggleZ, HETDEX, ATLAS, PAU, ADEPT, WFMOS). An important issue faced by these surveys is the control of systematics, including e.g., the corrections for the galaxy bias and the redshift distortion. In practical terms, it is necessary to know the statistical properties of the BAO tracer population to extricate detail (such as the way they "populate" their host halos, and their luminosity function). As the LRGs are used as the tracer in most of the aforementioned experiments, it is critical to test for the homogeneity of the LRG population. In this paper we address two key questions: Are BCGs different from other cluster galaxies? Are BCGs distinct from non-BCG luminous, red galaxies in clusters and groups?

Because of their brightness and uniformity in luminosity, the BCGs have long been regarded as an ideal standard candle (e.g., Humason et al. 1956, Scott 1957). Through the systematic investigations separately lead by Sandage and Gunn (e.g., Sandage 1972; Sandage & Hardy 1973; Gunn & Oke 1975, Kristian et al. 1978; Hoessel et al. 1980; Schneider et al. 1983), however, it was realized that various corrections [such as dependences of the BCG luminosity on the cluster richness class (Abel 1958) and Bautz-Morgan type (Bautz & Morgan 1970), and the effect of galactic cannibalism (Ostriker & Tremaine 1975)] have to be applied before any cosmological implication from the BCG Hubble diagram can be extracted.

The small dispersion in BCG luminosity (e.g., \(\sim 0.3\) mag as determined by Sandage 1972) also stimulated the discussion on their origin (Scott 1957, Peebles 1968; Geller & Peebles 1976; Tremaine & Richstone 1977; Hauvman & Ostriker 1978; Geller & Postman 1983; Merritt 1985; Bhavsar & Barrow 1985; Lin & Mohr 2004; Loh & Strauss 2006; von der Linden et al. 2007; Bernardi et al. 2007; Vale & Ostriker 2008) to name a few: whether they are simply "the brightest of the bright" or a different population from other cluster galaxies all together. If the former hypothesis were true, the BCG luminosity distribution (LD) simply results from sampling of an universal LD of all cluster galaxies, and its small dispersion in magnitude reflects the steepness of the bright end of the universal LD. To test for such a "statistical nature" of the BCGs, Tremaine & Richstone (1977, hereafter TR) devised a couple of statistics based on the mean and dispersion of the magnitude difference between the first- and second-ranked...
galaxies ($\Delta$ and $\sigma_{\Delta}$, respectively), and the dispersion of the BCG magnitude ($\sigma_1$). The basic idea is that, if the BCGs are simply the statistical extreme of the cluster galaxy population, both $\sigma_1$ and $\sigma_{\Delta}$ would be greater or comparable to $\Delta$. More specifically, under the assumptions that (1) the numbers of galaxies in non-overlapping magnitude intervals are independent random variables, and (2) the LD drops to zero rapidly at the extreme bright end\(^{10}\), the following conditions need to be satisfied: $T_1 \equiv \frac{\sigma_1}{\Delta} \geq 1$ and $T_2 \equiv \frac{\sigma_{\Delta}}{\Delta} \geq 0.82$ (see also Loh & Strauss 2006).

Using LRGs to identify dense environments such as groups and clusters, Loh & Strauss (2006) found a large magnitude gap ($\Delta \sim 0.8$ mag) between the first- and second-ranked galaxies in $\sim 12,000$ LRG-selected groups and clusters (for which the majority of the LRGs are the BCGs). They concluded that the LRGs are inconsistent with being the statistical extreme of the universal galaxy LD, based on the tests proposed by TR.

One fundamental issue faced by studies of very luminous galaxies, such as BCGs, concerns with their luminosity. On the practical side, for nearby BCGs, measuring their total light is nontrivial. The majority of BCGs are giant elliptical or cD galaxies, whose surface brightness profile is typically flatter than that of normal elliptical galaxies, and can extend to tens or hundreds of kpc (Tonry 1987). To measure the “total” magnitude requires careful subtraction of the sky background. On the physical side, the cD envelope may well extend into the intracluster space, and it is sometimes difficult to separate the luminosity of the BCGs from that of the intracluster stars (e.g., Gonzalez et al. 2005). In this study we will use data from the Sloan Digital Sky Survey (SDSS, York et al. 2000), which is not deep enough to detect contributions from the intracluster stars. However, it is known that the current SDSS pipeline has difficulty handling photometry of galaxies with large angular extents, and therefore may seriously underestimate the luminosity of BCGs (e.g., Lauer et al. 2007; von der Linden et al. 2007; Abazajian et al. 2008). In such cases, $\Delta$ will be biased low. Although corrections to BCG photometry (if at all possible) may decrease the value of $T_1$, it may have the opposite effect on $T_2$, making the interpretation of the TR tests more difficult (see \textsection 4).

Here we propose a new way to examine the statistical nature of the BCGs that is less dependent on a robust measurement of the BCG luminosity and $\Delta$, and apply it to a large sample of nearby clusters. Our method is both conceptually and operationally very simple: with a sample of $N$ clusters, whose total galaxy luminosities are $L_{\text{tot},i}$, where $i = 1, 2, \ldots, N$, one can combine all member galaxies to form a composite cluster. By drawing galaxies in random from the composite cluster to create $N$ mock clusters whose total luminosities are matched to the observed ones (see Fig. 1), the most luminous mock galaxies constitute the “statistical” BCG sample. One can then examine the distribution of the BCG luminosities ($L_{\text{BCG}}$) for the observed and mock clusters. In particular, we look for deviations from the statistical expectation from the correlation between $L_{\text{BCG}}$ and $L_{\text{tot}}$. As cluster luminosity is correlated with its mass (e.g., Lin et al. 2004), a comparison of the $L_{\text{BCG}}-L_{\text{tot}}$ correlation between the observed and mock clusters may provide some insights into possible cluster mass dependence (if any) in the BCG formation mechanism(s).

The great virtue of the TR tests is that they only rely on a minimal set of assumptions concerning the nature of the LD, and is immune to any cluster-to-cluster variation. Our proposed test, on the other hand, although relying on the universality of the LD, is independent of the shape of the LD. In this sense the two approaches are highly complementary.

In \textsection 2 we describe our cluster sample and the galaxy data. We show the results of our tests in \textsection 3 and present a comparison with the TR tests in \textsection 4.3. We apply our test to the second brightest cluster galaxies (hereafter G2s) in \textsection 4.4. Extensive tests have been carried out in order to show the robustness of our results (\textsection 4). We conclude by discussing the implications of our findings on the BCG formation scenarios and LRG-based BAO surveys in \textsection 5.

Throughout this paper we adopt a flat $\Lambda$CDM cosmological model where $\Omega_M = 1 - \Omega_{\Lambda} = 0.3$ and $H_0 = 70h_{70}$ km s$^{-1}$ Mpc$^{-1}$.

\textsection 2. CLUSTER SAMPLE AND GALAXY DATA

Our clusters are drawn from an updated version of the C4 cluster catalog (Miller et al. 2005), which uses data from the fifth data release of the SDSS (Adelman-McCarthy et al. 2007). Of the 2037 clusters in the sample, we restrict ourselves to 494 that lie within the redshift range $z = 0.030 - 0.077$, with velocity dispersion $\sigma > 200$ km/s, and contain at least 2 luminous galaxies (see below). While the lower limit in redshift helps to alleviate problems in photometric measurements of bright galaxies with large angular extents in SDSS, the upper limit is chosen to ensure that all galaxies with $M_1 \leq -20$ have redshifts measured by SDSS (i.e., extinction-corrected petrosian magnitude $r_p < 17.77$, among other selection criteria; Strauss et al. 2002). Note that the characteristic magnitude for cluster red galaxies is $M_{r,cp} = -21.70$ (see Ta-
The luminosity function is needed to obtain complete to the SDSS spectroscopic survey for the main galaxy sample is $L_{\text{bcg}}$ (including the photometric members) more luminous than the sum of luminosities from all cluster red member galaxies for the C4 clusters. The total cluster luminosity is estimated upper limit in cluster redshift ($z_{\text{up}} = 0.77$) ensures that the SDSS spectroscopic survey for the main galaxy sample is complete to $M_r = -20$, and therefore no extrapolation in the luminosity function is needed to obtain $L_{\text{tot}}$.

Our first task is to assign cluster membership to galaxies in the SDSS main sample. For every cluster, we include only red galaxies with redshift $|z - z_c| \leq 3\sigma/c$, with $r$-band absolute petrosian magnitude $M_r \leq -20$, and are projected within $0.8h_{70}^{-1}$ Mpc of the cluster center (defined as the peak of bright galaxy density distribution; [Miller et al. 2005]). Here $z_c$ and $c$ are the cluster redshift and speed of light, respectively. A galaxy is considered red if the model color $u - r \geq 2.2$ (e.g., [Strateva et al. 2001]). To calculate the absolute magnitudes, we use the $k$-correction based on the NYU value-added galaxy catalog ([Blanton et al. 2005]). In addition, a correction is made to convert the petrosian magnitudes into “total” magnitudes based on the surface brightness profile of the galaxies, following [Graham et al. 2005].

On average, about 94% of galaxies in the SDSS main sample have fibers assigned to them ([Strauss et al. 2002]). This fraction becomes smaller in crowded fields, such as the clusters (see [Miller et al. 2003] and below), due to the size of the fiber plugs. It is therefore critical to correct for such an incompleteness when trying to include all cluster galaxies using the SDSS main sample. For every cluster we first calculate the mean $g - r$ and $r - i$ colors $c_{gr}$ and $c_{ri}$, of the spectroscopically confirmed member galaxies, then assume the galaxies that (1) satisfy the above selection criteria ($u - r \geq 2.2$, $r_p \leq 17.7$, projected distance $\leq 0.8h_{70}^{-1}$ Mpc), (2) were targeted for spectroscopy but were not assigned a fiber, and (3) have $|g - r - c_{gr}| \leq 0.15$ and $|r - i - c_{ri}| \leq 0.15$ as probable cluster members (hereafter referred to as the photometric members). We treat these galaxies exactly the same way as those with spectroscopic redshift measurements, and derive the absolute magnitudes by assuming they are at the cluster redshift.

Combining the lists of the spectroscopic and photometric members, we record the absolute magnitudes of the galaxies for each cluster. The 5980 galaxies with $M_r \leq -20$ from all 494 clusters form our composite cluster, or a “galaxy pool” that by definition has the statistical galaxy LD. Our goal is to test if the BCGs in real clusters are drawn from the same LD as other cluster members, by comparisons to mock clusters and BCGs generated by the statistical LD.

Finally, we note that 871 of the galaxies are photometric members, of which 97 are BCGs.

3. THE $L_{\text{bcg}}$–$L_{\text{tot}}$ CORRELATION AND THE STATISTICAL NATURE OF BCGS

Correlations between the luminosity of the BCGs and the properties of host clusters such as mass or total luminosity have been noted by previous studies (e.g., [Lin & Mohr 2004; Yang et al. 2005; Hansen et al. 2007]). Here we utilize the BCG–cluster luminosity correlation ($L_{\text{bcg}} - L_{\text{tot}}$) to show that BCGs are indeed special—at least in luminous/massive clusters.

In Fig. 2 (top panel) we show the observed $L_{\text{bcg}}$–$L_{\text{tot}}$ correlation for the C4 clusters. The total cluster luminosity is the sum of luminosities from all cluster red member galaxies (including the photometric members) more luminous than $M_r = -20$, projected within a radius of $0.8h_{70}^{-1}$ Mpc. Our chosen upper limit in cluster redshift ($z_{\text{up}} = 0.77$) ensures that the SDSS spectroscopic survey for the main galaxy sample is complete to $M_r = -20$, and therefore no extrapolation in the luminosity function is needed to obtain $L_{\text{tot}}$.

The magenta squares in the top panel are the mean of BCG luminosity $L_{\text{bcg,obs}}$, as a function of $L_{\text{tot}}$. The meaning of the cyan crosses will be described below.

3.1. The Mock Clusters and BCGs

To check for the statistical nature of BCGs, we proceed as follows. Based on the observed LD, we generate many realizations of the $L_{\text{bcg}}$–$L_{\text{tot}}$ correlation, from which we derive the mean $L_{\text{bcg}}$–$L_{\text{tot}}$ relation and determine if it is consistent with the mean from the mock data.

We show in the middle panel of Fig. 2 one such realization. Basically, for each cluster in our sample, we create a corresponding mock cluster by randomly drawing galaxies from the galaxy pool, until the total luminosity of the mock cluster matches that of the observed one (see also Fig. 1). Specifically, suppose the first $N$ galaxies give a total lumi-
nosity of $L_N < L_{\text{tot}}$, and the next mock galaxy brings the total luminosity to $L_{N+1} > L_{\text{tot}}$. We keep the $N + 1$-th galaxy if $L_{N+1} - L_{\text{tot}} < |L_N - L_{\text{tot}}|$, otherwise we only use the first $N$ galaxies. Because of this procedure, although the total luminosity for massive (luminous) clusters can be matched fairly well, for low luminosity clusters the difference in $L_{\text{tot}}$ between the mock and real clusters can be large.

The black squares in the middle panels of Fig. 2 show the mean in BCG luminosity, $\langle L_{\text{bcg}} \rangle_{\text{sim}}$ in this particular ensemble of simulated clusters. The cyan crosses in both the top and middle panels are the mean of $L_{\text{bcg,sim}}$ from 200 Monte Carlo realizations of the mock cluster ensemble, which we denote as $\langle L_{\text{bcg,sim}} \rangle$. It is found that for luminous clusters (e.g., $L_{\text{tot}} > 3.7 \times 10^{11} h_7^{-2} L_\odot$), the observed $L_{\text{bcg,obs}}$ is higher than $\langle L_{\text{bcg,sim}} \rangle$ from mock clusters (e.g., compare the magenta squares with the cyan crosses in the top panel). For low luminosity clusters, the observed and mock values are comparable.

3.2. Statistical Significance

By comparing the observed and mock $L_{\text{bcg}} - L_{\text{tot}}$ correlations we infer that BCGs in real clusters have systematically higher luminosities than the mock BCGs. One might easily imagine a systematic measurement error that systematically gives too large a luminosity for bright galaxies. But that would not produce the effect we find. One would need a systematic error that increased the brightness of BCGs but had a less dramatic effect on second brightest galaxies in other (more luminous) clusters of the same intrinsic luminosity. Here we quantify the significance of the difference between the two populations.

Using 200 realizations of the mock cluster ensemble (each containing 494 clusters), we calculate $\langle L_{\text{bcg,sim}} \rangle$, as well as the difference between logarithms of the mean BCG luminosity $L_{\text{bcg,sim}}$ from individual realizations and $\langle L_{\text{bcg,sim}} \rangle$. $d_{\text{sim}} = \log L_{\text{bcg,sim}} - \log L_{\text{bcg,obs}}$.

By dividing the cluster sample into 15 bins in $L_{\text{tot}}$, each containing roughly equal number of clusters, for each mock cluster ensemble we have 15 evaluations of the statistic $d_{\text{sim}}$. One such example is shown in the lower panel of Fig. 2 (open red symbols). We see that $d_{\text{sim}}$ roughly scatters about zero. We expect that for mock clusters, the distribution of $d_{\text{sim}}$ should follow a Gaussian, which is shown as the blue histogram in the top panel of Fig. 3. The green curve is a Gaussian fit to the histogram.

We can similarly calculate $d_{\text{obs}} = \log L_{\text{bcg,obs}} - \log L_{\text{bcg,sim}}$ for the real clusters. We show $d_{\text{obs}}$ as a function of $L_{\text{tot}}$ in the lower panel of Fig. 2 as the solid blue points; it is apparent that $d_{\text{obs}}$ correlates positively with cluster luminosity. The distribution of $d_{\text{obs}}$ is further shown as the red dashed histogram in the top panel of Fig. 3 and is clearly different from a Gaussian. A Kolmogorov-Smirnov (KS) test shows that the probability of $d_{\text{obs}}$ to be drawn from the same distribution of $d_{\text{sim}}$ is $P = 0.79\%$.

We examine in more detail the distribution of $d$ in high and low luminosity clusters in the other two panels of Fig. 3. Setting a division luminosity of $L_{\text{div}} = 3.7 \times 10^{11} h_7^{-2} L_\odot$ separates our clusters into two subsamples each containing roughly equal number of member galaxies. The distribution of $d$ for the high (low) luminosity subsample is shown in the middle (bottom) panel. As we note above, in high luminosity clusters the BCGs are on average more luminous than the mock ones, resulting positive $d_{\text{obs}}$, and there is only a 0.028% probability that they are drawn from the same LD, based on a KS test. On the other hand, $L_{\text{bcg,obs}}$ in low luminosity clusters scatter about $\langle L_{\text{bcg,sim}} \rangle$, and $d_{\text{obs}}$ and $d_{\text{sim}}$ have a much higher probability ($P = 54.5\%$) to be drawn from the same distribution. These results are recorded in Table 1 (the column under “galaxy pool”; for meaning of the other columns, see §4.2).

3.3. Tremaine-Richstone Tests

Next we compare our results with those from the TR tests. Based on the whole cluster sample, we find that $\sigma_1 = 0.58$, $\Delta = 0.62$, and $\sigma_\Delta = 0.49$, resulting in $T_1 = \sigma_1 / \Delta = 0.93$ and $T_2 = \sigma_\Delta / \Delta = 0.78$. Recall that if the BCGs are “statistical”, we would have obtained $T_1 \geq T_{1 \lim} = 1$ and $T_2 \geq T_{2 \lim} = 0.82$ (§4 but see below). Note that these lower limits are derived under rather general assumptions about the form of the LD (TR). For example, assuming the underlying LD follows the Schechter (1976) form, the limits are $T_{1 \lim, \text{sch}} \approx 1.16$ and $T_{2 \lim, \text{sch}} \approx 0.88$.

Dividing the clusters into high and low luminosity subsamples (using the same division luminosity $L_{\text{div}}$ as in §4.2,
we find \((\sigma_1, \Delta, \sigma_\Delta, T_1, T_2) = (0.39, 0.55, 0.43, 0.70, 0.79)\) and \((0.54, 0.65, 0.50, 0.84, 0.77)\), respectively. That \(T_1\) for the whole sample is larger than \(T_1\) for both high and low luminosity subsamples is due to the weak, positive correlation between \(L_{bcg}\) and \(L_{tot}\).

Both our test and the TR tests indicate that, using the whole cluster sample, the BCGs are not drawn from the same LD as other cluster galaxies. Separating the low luminosity clusters from the luminous ones, our test suggests that this conclusion is mainly driven by the BCGs in the luminous clusters. The \(T_1\) statistic implies a larger deviation from the statistical expectation for BCGs in high luminosity clusters \((T_{1,lim} - T_1 = 0.30)\), compared to their counterparts in less luminous clusters \((T_{1,lim} - T_1 = 0.16)\).

It is important to quantify how significant are the deviations of the \(T_1\) and \(T_2\) values we obtain from the statistical limits. Without assuming any particular form of the LD, we evaluate the significance empirically, using the “galaxy pool” from all the member galaxies. More specifically, we construct ensembles of mock clusters following the method described in §3.1; we can then derive the mean and dispersion of the two statistics from the distributions of \(T_1\) and \(T_2\) based on the mock clusters. For all clusters in our sample, when the BCGs are generated statistically, the mean and standard deviation of the two statistics are \(\bar{T}_1 = 1.01\), \(\sigma_{T_1} = 0.05\), and \(\bar{T}_2 = 0.90\), \(\sigma_{T_2} = 0.03\). The distributions of the TR statistics are slightly non-Gaussian (see Fig. [4]). We find that about 3.5% of the Monte Carlo realizations have \(T_1\) as low as the observed value (0.93), but only 0.007% have \(T_2\) as low as 0.78, the observed value. We denote these fractions as \(p_1 \equiv p_1(\leq T_1, \text{obs})\) and \(p_2 \equiv p_2(\leq T_{2,\text{obs}})\). For high luminosity clusters, \((\bar{T}_1, \sigma_{T_1}, p_1, \bar{T}_2, \sigma_{T_2}, p_2) = (0.96, 0.08, 0.014%, 0.84, 0.06, 14%)\). For fainter ones, the values are \((0.90, 0.05, 8.8\%, 0.88, 0.04, 0.07\%)\).

For clarity, these results are summarized in Table 2. We present the results for the whole sample, as well as the high and low luminosity subsamples. For each (sub)sample, the numbers under “obs.” are the observed values, and the triplet of numbers under “stat.” denote the mean and standard deviation of the TR statistics based on \(10^5\) ensembles of mock datasets, and \(p_i\) \((i = 1, 2)\). Looking at \(p_2\) for the high and low luminosity clusters, one would conclude that BCGs in the faint clusters are much more likely to be statistical. One would draw the opposite conclusion if considering the \(T_1\) statistic, however. It is therefore not clear if the TR tests give a consistent picture (e.g., dependence on \(L_{tot}\)) for the degree of deviation of BCGs from the galaxy population in clusters.

In their study of LRG-selected groups and clusters, Loh & Strauss (2006) found that \((\sigma_1, \Delta, \sigma_\Delta, T_1, T_2) = (0.30, 0.87, 0.52, 0.35, 0.59)\) at \(z \approx 0.12\). They also examined the richness dependence of the TR statistics. For the richest systems (75%–100% quartile in the richness distribution) they found \((T_1, T_2) = (0.75, 0.71)\); for those with richness in the 25%–50% quartile (with about 2–3 galaxies on the red sequence), \((T_1, T_2) = (0.27, 0.34)\). The results of Loh & Strauss (2006) suggest a dependence of the TR statistics on richness that is opposite to our findings (if their richness estimator correlates well with the total luminosity).

The main cause for the differences seems to be in the magnitude gap, \(\Delta\). Although for the richest systems, Loh & Strauss (2006) obtained \(\Delta \sim 0.55\), a value comparable to ours, their finding of \(\Delta \sim 1\) for the much poorer systems (25%–50% quartile in richness) is much higher than our value. Because Loh & Strauss (2006) looked for galaxy concentrations around LRGs using (primarily) photometric data,

### Table 2: Tremaine–Richstone Statistics

|          | all clusters | luminous clusters | faint clusters |
|----------|--------------|-------------------|----------------|
|          | obs. | stat. | obs. | stat. | obs. | stat. |
| \(T_1\)  | 0.93 (1.01, 0.05, 3.5%) | 0.70 (0.96, 0.08, 0.01%) | 0.84 (0.90, 0.05, 8.8%) |
| \(T_2\)  | 0.78 (0.90, 0.03, 0.007%) | 0.79 (0.85, 0.06, 14%) | 0.77 (0.88, 0.04, 0.07%) |

\(\Delta\) numbers in parenthesis are the mean and standard deviation based on \(10^5\) ensembles, and the proportion \(p_i\) \((i = 1, 2)\) of the Monte Carlo realizations that give TR statistics as low as the observed value.

*Fig. 4.* Distribution of the \(T_1\) and \(T_2\) values obtained from \(10^5\) ensembles of mock datasets. From top to bottom, we show the distributions for all clusters, high and low luminosity clusters, respectively. In each panel the solid/red histograms show the distribution of \(T_1\), while the dotted/blue histogram shows that of \(T_2\). The short arrows denote the mean values of the distributions. The long arrows represent the observed values (see Table 2). The area of the region within the distribution leftwards of the long arrow gives the probability to obtain the TR statistics as low as the observed value by chance. Looking at the middle panel (for high luminosity clusters), one sees that the long pink arrow is at the extreme of the solid red histogram, and would conclude that BCGs are unlikely to be statistical (based on the \(T_1\) statistic). However, the long green arrow is well within the dotted blue histogram (of \(T_2\)), suggesting the BCGs are statistical. The opposite situation happens when one looks at the lower panel (for faint clusters).
we suspect many of their low-richness systems may not be real, but simply chance projections of galaxies with similarly red colors. On the other hand, galaxy systems in the C4 sample were identified in both (spectroscopic) redshift and color space, thus minimizing spurious detections.

3.4. Are G2s Special?

We have applied a new method to confirm that the BCGs are special. This approach can be used to check if G2s are statistical, that is, are their LD consistent with that of the overall cluster galaxy population. The results can then be used to answer the second question we set out to address: Are BCGs different from other luminous, red, cluster galaxies?

We proceed in a similar fashion as in the previous sections. The main difference is that we are now comparing the observed $L_u - L_{tot}$ correlation with the corresponding mean relation from mock clusters. Requiring that mock clusters must have at least two members causes a small modification to the way mock clusters are constructed, which mainly affects the low luminosity clusters. Therefore we will focus on high luminosity clusters in this section. We find that $P = 67\%$ for G2s, implying that they are consistent to be the statistical extreme of the galaxy LD.

This result immediately suggests that BCGs are distinct from G2s, and very likely, other luminous member galaxies.

This simple exercise also demonstrates one advantage of our method over the TR tests: while the latter by definition could not determine the statistical nature of G2s, our method can in principle be applied to even the third- or lower-ranked member galaxies.

4. SYSTEMATICS

In this section we examine the robustness of our findings on various aspects in the analysis, including the selection of cluster red galaxies in SDSS, the way mock cluster and galaxy samples are constructed, and the issue of photometry of galaxies with large angular extent.

4.1. Sensitivity on Galaxy and Cluster Sample Selection

We first check the sensitivity of our results on the galaxy selection criteria, by repeating our test using a much more stringent set of conditions to assign cluster memberships for red galaxies. In addition to the basic requirements ($u - r \geq 2.2$, $r_p \leq 17.7$, projected within $0.8h_{70}^{-1}$ Mpc), for the spectroscopic members, we only include galaxies whose (1) redshifts satisfy $\epsilon|z - z_{cl}| \leq v$, where $v \equiv \min(2\sigma, 1500 \text{ km/s})$, (2) $g - r$ and $r - i$ colors fall in the range $\bar{c} - 1.5\sigma_c$ to $\bar{c} + 2.5\sigma_c$, where $\bar{c}$ and $\sigma_c$ are the mean and dispersion of the $g - r$ and $r - i$ colors from the galaxies at redshifts broadly consistent with the cluster, (3) “concentration parameter” $c_{\text{cm}} \equiv r_{90}/r_{50} \geq 2.6$, where $r_{90}$ and $r_{50}$ are radii that enclose 90% and 50% of the Petrosian flux, respectively; this last condition aims to select galaxies with early type morphology (Strateva et al. 2001). As for the photometric members, along with the filters in the r-band flux, $u - r$ color, spatial distribution, and morphology, we require that their $g - r$ and $r - i$ colors to lie between $\bar{c} - \sigma_c$ and $\bar{c} + 1.5\sigma_c$, where $\bar{c}$ and $\sigma_c$ are now derived from the spectroscopic members.

These criteria reduce our cluster and galaxy sample; only 344 clusters containing 2819 galaxies are included. We find that the probability $P$ that $d_{\text{cm}}$ and $d_{\text{cm}}$ are drawn from the same distribution is 1.5%, 0.15%, and 67% for the whole, high luminosity, and low luminosity (sub)samples. These values confirm the results found in §3.2.

We note in §2 that 20% ($\approx 97/494$) of the BCGs are photometrically confirmed members. Some of these may be foreground/background galaxies. To evaluate the effect of possible contamination due to these photometric BCGs, we repeat our analysis using the 397 clusters whose BCGs are spectroscopically confirmed members (with the membership assignment criteria of §2). Based on the galaxy pool constructed from the 3661 galaxies in these clusters, we find that $P = 1.7\%$, 0.28%, and 27%, for the whole, high luminosity, and low luminosity (sub)samples.

Although the absolute values of $P$ changes somewhat with respect to our nominal values recorded in Table 1, the trend remains clear that the BCGs in high luminosity clusters are much less likely to be drawn from the same LD as other cluster members when compared to their counterparts in lower luminosity clusters.

Let us comment next on one effect our cluster selection may have on the results. The requirement that the clusters need to host at least two galaxies with $M_r \leq -20$ potentially excludes systems dominated by a single $\sim M_\odot$ galaxy, such as the “fossil groups” (which are defined to have $\Delta \geq 2$ and extended X-ray emissions). These systems are believed to evolve in isolation, with last major merger with other galactic systems being long enough in the past that a dominant central galaxy can result from the dynamic friction (Ponman et al. 1994). The BCGs in these groups would then deviate significantly from the statistical extreme, which makes them distinct from the other BCGs in low luminosity groups included in our sample. One might be concerned that the exclusion of fossil groups from our cluster sample may have contributed to our conclusion that, for the lower luminosity systems, there is no statistically significant deviation of BCGs from the statistical distribution of all cluster galaxies. However, given the rarity of the fossils (number density $\approx 2 \times 10^{-9} h_{70}^{-3}$ Mpc$^{-3}$; e.g., La Barbera et al. 2009, Voevodkin et al. 2009), we expect there to be at most $\sim 20$ such systems in the volume we sample ($z = 0.03-0.077, \approx 5700 \text{ deg}^2$), and therefore our conclusion should be robust.

4.2. Construction of Mock Clusters

Instead of utilizing the galaxy pool, we can construct mock clusters using fits to the observed LD of cluster galaxies. The LDs for the whole cluster sample, and for the high and low luminosity subsamples, are measured by the method developed in Lin & Mohr (2004), to which we refer the reader for more details. The results are shown in Fig. 5. From top to bottom are the LDs for the high luminosity clusters, the whole sample, and the low luminosity clusters.

When BCGs are included, the Schechter function is not a good description of the LDs. Instead, the bright end may be more appropriately described by a log-normal distribution. Therefore, in fitting the LDs we consider both a bright end-truncated Schechter function plus a Gaussian,

$$
\phi(M) dM = \begin{cases} 
\frac{\ln 10}{\Delta \sigma_c} \phi_s M^{\alpha+1} \exp(-M/M_c) dM & \text{if } M \geq M_c \\
\phi_s \exp\left(-\frac{(M-M_c)^2}{2\sigma_c^2}\right) dM & \text{otherwise}
\end{cases}
$$

where $\phi(M) = 10^{-0.4(M-M_c)}$, and a single Schechter function. The best-fit parameters are given in Table 3.

Using the Gaussian+Schechter function fit to the whole sample as the LD, we find that for the whole, high, and low luminosity samples, $P = 0.47\%$, 0.013%, and 39%, respectively. If using the Schechter function fit to the LD of the whole sample, $P = 0.57\%$, 0.015%, and 44% for the three
As our first approach, we adopt the methodology of von der Linden et al. (2007) to apply corrections to the magnitudes of large galaxies by the official SDSS pipeline. Using an independent photometric reduction software, Hyde & Bernardi (2009) quantified the degree of underestimation of the magnitudes of large galaxies by the official SDSS pipeline. Their tests suggest that, for early type galaxies with effective radius $\approx 10^7$, the mean magnitude deficit is about 0.25 mag, with a 68% scatter about 0.04 mag. They provided a fitting function that gives the mean deficit $\Delta m$ as a function of the angular size $\theta$, which we use as the basis of our second method for recovering the true galaxy magnitudes. For every galaxy in our sample, we assume its magnitude was underestimated by the SDSS pipeline by $\Delta m(\theta) + \delta m$, where $\delta m$ is a Gaussian random variate with dispersion of 0.04 mag, and $\theta$ is determined from the de Vaucouleur fits to the galaxy surface brightness profile (see Hyde & Bernardi 2009 for details).

For the galaxy and cluster catalogs thus modified, we find that $(T_1, T_2) = (0.99, 0.77)$ for the whole sample, and the probability to obtain $T_1$ and $T_2$ from the galaxy pool as low as the “observed” values is 11.3% and $8 \times 10^{-5}$, respectively. On the other hand, the probability that $d_{\text{obs}}$ and $d_{\text{amp}}$ are drawn from the same distribution is $P = 1.2\%$. Breaking the sample into high and low luminosity subsamples, the results are $(T_1, p_1, T_2, p_2) = (0.68, 5 \times 10^{-5}, 0.78, 2.2\%)$ for the luminous clusters, and $(0.84, 7.5\%, 0.77, 2 \times 10^{-4})$ for the faint clusters. From our test we find that BCGs in high and low luminosity clusters have probabilities of $P = 0.012\%$ and $P = 49\%$ to be statistical.

These results suggest that both our test and the TR tests are insensitive to the uncertainties in galaxy photometry due to sky subtraction.

It is also important to check if the low luminosity clusters are systematically at lower redshifts than the luminous ones. If this were true, the issue with sky subtraction of the SDSS pipeline may affect the photometry of BCGs in low luminosity clusters (hereafter LLCBCG) more strongly than that of BCGs in high luminosity clusters (hereafter HLCBCG), causing the photometry of LLCBCG to be underestimated more than the case for HLCBCG. This would in turn affect our finding that LLCBCG are consistent with being drawn from the same LD as other cluster galaxies (§3.2). It turns out in our cluster sample, the high and low luminosity clusters have nearly identical redshift distribution, and therefore we do not think the photometry of LLCBCG and HLCBCG is treated differently.

5. DISCUSSIONS AND SUMMARY

5.1. Implications on BCG Formation

The main result of the present study is that BCGs as a whole have a LD that is distinct from that of the majority of red cluster galaxies (those with $M_r \leq -20$). This conclusion is primarily due to the high luminosities of HLCBCG, which has only 0.03% of probability to be drawn from the LD of all galaxies. On the contrary, LLCBCG are more likely to be simply the statistical extreme of the LD of all galaxies.

With the help of mock clusters, and the elements that come into the calculation of TR tests, one gains some insight into the BCG formation process. For high luminosity clusters, we...
find that \((\sigma_1, \sigma_2, \overline{\Delta}) = (0.39, 0.43, 0.55)\) from the real data, while the corresponding values from the mock dataset are \((0.42, 0.37, 0.44)\). Therefore, although \(\sigma_1\) in real and mock clusters are comparable, in real clusters \(\overline{\Delta}\) is higher that that in mock ones (suggesting that real BCGs are on average 0.11 mag brighter than the statistical extreme). Physics of BCG formation drives the magnitude gap to be larger than the statistical expectation.

To explain the origin of HLCBCG, one therefore needs to invoke galactic mergers (e.g., Ostriker & Tremaine 1975, Hausman & Ostriker 1978, Lin & Mohr 2004, Cooray & Milosavljević 2005, Vale & Ostriker 2008), which is naturally expected within the hierarchical structure formation paradigm (e.g., Dubinski 1998; De Lucia & Blaizot 2007). However, the details of the mergers (e.g., major mergers between BCG and very luminous, minor mergers between BCG and \(M_\star\)-type galaxies, or the "galactic cannibalism"\(^{11}\)) remain to be understood. For example, while Lin & Mohr (2004) suggested that major mergers are a viable route for forming HLCBCG, Vale & Ostriker (2008) were more in favor of minor mergers.

In principle, the extent of late mergers can be constrained by the \(L_{\text{bcg}}-L_{\text{tot}}\) correlation, or statistics related to the magnitude gap between BCG and second-ranked galaxy (e.g., Milosavljević et al. 2006, Yang et al. 2008). We have developed a simple merger model for cluster galaxies, and will present constraints on the importance of mergers based on the observations obtained in this paper in a future publication (Lin & Ostriker 2009, in preparation).

After confirming that BCGs are indeed different from the bulk of cluster galaxy population, it is natural to ask if BCGs are different from other luminous/massive cluster galaxies (e.g., \(M \lesssim M_\star - 1\)). It has long been known that BCGs follow different scaling relations from other early type galaxies (e.g., Oegerle & Hoessel 1991, Lauer et al. 2007, Destroches et al. 2007, Bernardi et al. 2007), in the sense that the BCGs are larger, less dense, and have lower velocity dispersion, compared to other early type galaxies of the same luminosity. However, at the present it is not clear if the structure of the BCGs (in terms of Sersic fits) is indeed different from non-BCG early type galaxies of comparable color and stellar mass (see Guo et al. 2009).

We address this question somewhat indirectly; in §4.3 it is shown that the LD of G2s in high luminosity clusters are consistent with that of the whole cluster galaxy population. (Our method could not be robustly applied to examine the statistical nature of G2s in low luminosity clusters, unfortunately.) Although we do not check the third-ranked brightest members, we believe a similar result will emerge for them. Effectively, we suggest that among the most luminous, red, cluster members, only BCGs in high luminosity clusters show significant deviations from the statistical extreme, and therefore these galaxies are a distinct population.

Our results suggest both BCGs in low luminosity/mass clusters and G2s in luminous/massive clusters are the statistical extreme of the galaxy LD. As the latter are very likely BCGs (in clusters or groups that merge with the current host clusters) themselves at earlier stages of their lives, this finding seems to give a self-consistent picture of the luminous galaxy evolution within the hierarchical cluster evolution scenario.

5.2. Cosmological Implications

The results and reasoning presented in §4.3 & §5.1 suggest that LRGs that are BCGs in high luminosity (massive) clusters are different from other LRGs (e.g., those that are LLCBCG and G2s or lower-ranked galaxies in high luminosity clusters), and call for very careful modeling and selection of the LRGs. Suppose the finding that HLCBCG and LLCBCG are two distinct populations continues to hold towards higher redshifts. In a flux-limited survey, the LRGs at higher redshifts would be intrinsically more luminous than those at lower-\(z\). Therefore, a larger fraction of LRGs at higher-\(z\) would be composed of HLCBCG, when compared to LRGs at lower-\(z\). When accounting for the Malmquist bias present in the LRG sample, one cannot simply assume that BCGs follow the same LD as LRGs as a whole, otherwise the inferred luminosity would be lower than the true value. This potential systematic effect would be larger towards higher-\(z\).

It is thus important to take into account the difference in the LD of HLCBCG and LLCBCG when creating mock LRG catalogs for BAO surveys. Unfortunately the BCGs in the present study are at lower redshifts (\(z < 0.1\)) even compared to SDSS and SDSS-II LRG studies (at \(z \sim 0.3\); e.g., Eisenstein et al. 2005), and therefore the LRs we measure (see the Gaussian fits in Table 4) are not readily applicable. Accordingly our study strongly motivates for a systematic investigation on the \(L_{\text{bcg}}-L_{\text{tot}}\) correlation at \(z > 0.3\), either through direct observations (similar to the methodology used in the present paper, utilizing spectroscopic data from e.g., BOSS or GAMA surveys), through the halo occupation distribution analysis (e.g., Zheng et al. 2008), or the non-parametric method of Vale & Ostriker (2006).

5.3. Summary

The question of whether the luminosity distribution of BCGs is drawn from the same LD as other cluster galaxies has been extensively discussed over the last four decades. Here we propose a simple new test to examine the statistical nature of the BCGs, and to supplement the classical tests proposed by Tremaine & Richstone (1977, TR).

Our basic idea is to shuffle the cluster galaxies and see how

| TABLE 3 | LUMINOSITY DISTRIBUTION PARAMETERS IN I-BAND |
|---------|-----------------------------------------------|
| sample  | single Schechter                                       | Gaussian+Schechter\(^a\) |
|         | \(\alpha\) | \(\phi_0\) | \(M_\star\) | \(\alpha\) | \(\phi_0\) | \(M_\star\) | \(M_{\text{ch}}\) | \(\phi_0\) | \(M_\star\) | \(\sigma_\Delta\) |
| all     | -1.01     | 8.53      | -21.92   | -0.93     | 10.25     | -21.70   | -22.40   | 1.38     | -22.25   | 0.32 |
| high    | -1.17     | 12.62     | -22.13   | -0.99     | 18.30     | -21.73   | -22.60   | 1.90     | -22.57   | 0.25 |
| low     | -0.93     | 6.34      | -21.78   | -0.73     | 8.92      | -21.28   | -22.05   | 1.35     | -21.98   | 0.33 |

\(^a\) see Eq. 1 for the definition of the parameters.
The averaged BCG luminosity from all 200 realizations of mock datasets, \( \langle L_{\text{bcg,sim}} \rangle \), gives the expected value when BCGs are statistical, as a function of cluster luminosity. The differences between the logarithms of mean mock BCG luminosity (\( \langle L_{\text{bcg,sim}} \rangle \)) from each ensemble and (\( \langle L_{\text{bcg,sim}} \rangle \)) represent the degree of scatter expected if BCGs are stochastically selected from a global LD. In Fig. 3 we compare the distribution of (\( d_{\text{sim}} = \log L_{\text{bcg,sim}} - \log \langle L_{\text{bcg,sim}} \rangle \)) with that of (\( d_{\text{obs}} = \log L_{\text{bcg,obs}} - \log \langle L_{\text{bcg,sim}} \rangle \)), and find that the real BCGs are more luminous than the statistical expectation; there is 0.8% of probability for (\( d_{\text{obs}} \)) to be drawn from the same distribution of (\( d_{\text{sim}} \) (see Table 1). Separating the clusters into high and low luminosity subsamples (with a division luminosity of (\( L_{\text{div}} = 3.7 \times 10^{11} L_{\odot} \)), we conclude that the difference is mainly coming from BCGs in high luminosity clusters (only 0.03% chance to be statistical). The luminosities of BCGs in low luminosity clusters are roughly consistent with the global LD.

We extend the analysis to discuss the statistical nature of the second brightest galaxies (G2s), and find strong evidence that (in high luminosity clusters), G2s have a LD similar to that of the whole cluster galaxy population.

We also apply the tests proposed by TR to our cluster sample, and record the results in Table 2. The two statistics (\( T_1 \) and \( T_2 \)) both suggest a small probability for BCGs to be statistical, consistent with our findings. However, examining the TR statistics for the high and low luminosity clusters reveals confusing trends. According to the (\( T_1 \) statistic, BCGs in the high luminosity clusters are much less likely to be statistical than their counterparts in the low luminosity clusters. However, the (\( T_2 \) statistic suggests the opposite conclusion.

Our results confirm previous findings that BCGs in high luminosity clusters are a distinct population from other cluster galaxies and tentatively supports the physical mechanism of cannibalism (Ostriker & Tremaine 1975) which is essentially dynamical friction. We also suggest that BCGs are distinct from other non-BCG LRGs. As the majority of LRGs are the brightest member in groups and clusters, such an effect should be taken into account in selecting a homogeneous sample of LRGs for on-going and future BAO surveys.

We are grateful to Scott Tremaine and Bob Nichol for insightful comments on the manuscript. YTL thanks Michael Strauss, Antonio Vale, and Jim Gunn for helpful discussions, and IH for constant encouragement. YTL acknowledges support from the Princeton-Católica Fellowship, NSF PIRE grant OISE-0530095, FONDAP-Andes, and the World Premier International Research Center Initiative, MEXT, Japan.

Funding for the SDSS and SDSS-II has been provided by the Alfred P. Sloan Foundation, the Participating Institutions, the National Science Foundation, the U.S. Department of Energy, the National Aeronautics and Space Administration, the Japanese Monbukagakusho, and the Max Planck Society, and the Higher Education Funding Council for England. The SDSS Web site is http://www.sdss.org/.

The SDSS is managed by the Astrophysical Research Consortium for the Participating Institutions. The Participating Institutions are the American Museum of Natural History, Astrophysical Institute Potsdam, University of Basel, University of Cambridge, Case Western Reserve University, The University of Chicago, Drexel University, Fermilab, the Institute for Advanced Study, the Japan Participation Group, The Johns Hopkins University, the Joint Institute for Nuclear Astrophysics, the Kavli Institute for Particle Astrophysics and Cosmology, the Korean Scientist Group, the Chinese Academy of Sciences, Los Alamos National Laboratory, the Max-Planck-Institute for Astronomy, the Max-Planck-Institute for Astrophysics, New Mexico State University, Ohio State University, University of Pittsburgh, University of Portsmouth, Princeton University, the United States Naval Observatory, and the University of Washington.

REFERENCES

Abazajian, K., et al. 2008, ApJS, submitted (arXiv:0812.0649)
Abell, G. O. 1958, ApJS, 3, 211
Adelman-McCarthy, J. K., et al. 2007, ApJS, 172, 634
Bautz, L. P. & Morgan, W. W. 1970, ApJ, 162, L149
Bernardi, M., Hyde, J. B., Sheth, R. K., Miller, C. J., & Nichol, R. C. 2007, AJ, 133, 1741
Bhavsar, S. P. & Barrow, J. D. 1985, MNRAS, 213, 857
Blanton, M. R., Schlegel, D. J., Strauss, M. A., Brinkmann, J., Finkbeiner, D., Fukugita, M., Gunn, J. E., Hong, D. W., Ivezić, Z., Knapp, G. R., Lupton, R. H., Munn, J. A., Schneider, D. P., Tegmark, M., & Zehavi, I. 2005, AJ, 129, 2562
Cole, S., et al. 2005, MNRAS, 362, 505
Cooray, A. & Milosavljević, M. 2005, ApJ, 627, L85
De Lucia, G. & Blaizot, J. 2007, MNRAS, 375, 2
Desroches, L.-B., Quataert, E., Ma, C.-P., & West, A. A. 2007, MNRAS, 377, 402
Dubinski, J. 1998, ApJ, 502, 141
Eisenstein, D. J., et al. 2001, AJ, 122, 2267
Eisenstein, D. J., et al. 2005, ApJ, 633, 560
Geller, M. J. & Peebles, P. J. E. 1976, ApJ, 206, 939
Geller, M. J. & Postman, M. 1983, ApJ, 274, 31
Gonzalez, A. H., Zabludoff, A. I., & Zaritsky, D. 2005, ApJ, 618, 195
Graham, A. W., Driver, S. P., Petrosian, V., Conselice, C. J., Bershady, M. A., Crawford, S. M., & Goto, T. 2005, AJ, 130, 1535
Gunn, J. E. & Oke, J. B. 1975, ApJ, 195, 255
Guo, Y., et al. 2009, MNRAS, submitted (arXiv:0901.1150)

Hansen, S. M., Sheldon, E. S., Wechsler, R. H., & Koester, B. P. 2007, ApJ, submitted (arXiv:0710.3780)
Hausman, M. A. & Ostriker, J. P. 1978, ApJ, 224, 320
Hoeessel, J. G., Gunn, J. E., & Thuan, T. X. 1980, ApJ, 241, 486
Humason, M. L., Mayall, N. U., & Sandage, A. R. 1956, AJ, 61, 97
Hyde, J. B., & Bernardi, M. 2009, MNRAS, accepted (arXiv:0810.4922)
Kristian, J., Sandage, A., & Westphal, J. A. 1978, ApJ, 221, 383
La Barbera, F., de Carvalho, R. R., de la Rosa, I. G., Sorrentino, G., Gal, R. R., & Kohl-Moreira, J. L. 2009, AJ, 137, 3942
Lauer, T. R., Faber, S. M., Richstone, D., Gebhardt, K., Tremaine, S., Postman, M., Dressler, A., Aller, M. C., Filippenco, A. V., Green, R., Ho, L. C., Kormendy, J., Magorrian, J., & Filippenko, A. V. 2007, ApJ, 662, 808
Lin, Y.-T. & Mohr, J. J. 2004, ApJ, 617, 879
Lin, Y.-T., Mohr, J. J., & Stanford, S. A. 2004, ApJ, 610, 745
Loh, Y.-S. & Strauss, M. A. 2006, MNRAS, 366, 373
Merritt, D. 1985, ApJ, 289, 18
Miller, C. J., Nichol, R. C., Reichart, D., Wechsler, R. H., Evrard, A. E., Annis, J., McKay, T. A., Bahcall, N. A., Bernardi, M., Boehinger, H., Connolly, A. J., Goto, T., Kniazev, A., Lamb, D., Postman, M., Schneider, D. P., Sheth, R. K., & Voges, W. 2005, AJ, 130, 968
Milosavljević, M., Miller, C. J., Furlanetto, S. R., & Cooray, A. 2006, ApJ, 637, L9
Oegerle, W. R., & Hoessel, J. G. 1991, ApJ, 375, 15
Ostriker, J. P. & Tremaine, S. D. 1975, ApJ, 202, L113
Padmanabhan, N., et al. 2007, MNRAS, 378, 852
Peebles, P. J. E. 1968, ApJ, 153, 13
Ponman, T. J., Allan, D. J., Jones, L. R., Merrifield, M., McHardy, I. M., Lehto, H. J., & Luppino, G. A. 1994, Nature, 369, 462
Sandage, A. 1972, ApJ, 178, 1
—. 1973, ApJ, 183, 731
Sandage, A. & Hardy, E. 1973, ApJ, 183, 743
Schechter, P. 1976, ApJ, 203, 297
Schneider, D. P., Gunn, J. E., & Hoessel, J. G. 1983, ApJ, 264, 337
Scott, E. L. 1957, AJ, 62, 248
Spergel, D. N., Verde, L., Peiris, H. V., Komatsu, E., Nolta, M. R., Bennett, C. L., Halpern, M., Hinshaw, G., Jarosik, N., Kogut, A., Limon, M., Meyer, S. S., Page, L., Tucker, G. S., Weiland, J. L., Wollack, E., & Wright, E. L. 2003, ApJS, 148, 175
Strateva, I., et al. 2001, AJ, 122, 1861
Strauss, M. A., et al. 2002, AJ, 124, 1810
Tonry, J. L. 1987, in IAU Symp. 127: Structure and Dynamics of Elliptical Galaxies, 89–96
Tremaine, S. D. & Richstone, D. O. 1977, ApJ, 212, 311
Vale, A. & Ostriker, J. P. 2006, MNRAS, 371, 1173
—. 2008, MNRAS, 383, 355
Voevodkin, A., Borozdin, K., Heitmann, K., Habib, S., Vikhlinin, A., Mescheryakov, A., & Hornstrup, A. 2009, ApJ, submitted (arXiv:0902.0619)
von der Linden, A., Best, P. N., Kauffmann, G., & White, S. D. M. 2007, MNRAS, 379, 867
Yang, X., Mo, H. J., Jing, Y. P., & van den Bosch, F. C. 2005, MNRAS, 358, 217
Yang, X., Mo, H. J., & van den Bosch, F. C. 2008, ApJ, 676, 248
York, D. G., et al. 2000, AJ, 120, 1579
Zheng, Z., Zehavi, I., Eisenstein, D. J., Weinberg, D. H., & Jing, Y. 2008, ApJ, submitted (arXiv:0809.1868)