Longitudinal Polarization in $K_L \rightarrow \mu^+\mu^-$ in MSSM with large $\tan\beta$

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Abstract

A complete experiment on decay $K_L \rightarrow l^+l^-$ will not only consist of measurement of the decay rates but also lepton polarization etc. These additional observations will yield tests of CP invariance in these decays. In $K_L$ and $K_S$ decays, the $e$ mode is slower than the $\mu$ mode by roughly $(m_e/m_\mu)^2$ [4]. As well discussed in literature [5] the Standard Model contribution to the lepton polarization is of order $2 \times \sim 10^{-3}$. We show that in MSSM with large $\tan\beta$ and light higgs masses ($\sim 2 M_W$), the longitudinal lepton polarization in $K_L \rightarrow \mu^+\mu^-$ can be enhanced to a higher value, of about $10^{-2}$.

The Flavor Changing Neutral Current (FCNC) decays of the K-meson are forbidden in the lowest order in the standard Electroweak theory but can occur through loop diagrams in higher order. In

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effect these processes thus are a deeper probe into the underlying field theory. The amplitude for such processes can be divided into a Short-Distance (SD) part where the quarks involved interact over a range $\sim M_W^{-2}$ and a Long Distance (LD) part which may be thought of as proceeding via low lying intermediate states and particularly through resonances. Theoretically reasonable techniques have been developed for estimating the SD - part through Operator Product Expansion (OPE) [1] and this is quite successful in analyzing decays like $b \to s\gamma$ or $b \to s \ell^+\ell^-$ in regions of phase space where no resonances are involved [2,3]. For K-meson decays, resonances are nearby and the LD part is important. Unfortunately the LD-part is quite model dependent and thus theoretical results are relatively more uncertain compared to the ones for B-decay.

![Figure 1: The 2$\gamma$ intermediate state contribution to the LD part of $K_L \to \mu^+\mu^-$](image)

The decay of $K_L$ into $\mu^+\mu^-$, a FCNC process, is a somewhat special one amongst all rare decays. Amongst the intermediate states which contribute to the LD-part, the two photon state (Fig -1) stands as the most important one [4]. The absorptive part of the amplitude, which can be computed from the known decay rate $K_L \to 2\gamma$ and the QED amplitude $\gamma\gamma \to \mu^+\mu^-$, by itself leads to a decay rate almost equal to experimental decay rate of $K_L \to \mu^+\mu^-$ [5]. The short-distance contributions are real [6] and since the rate of $K_L \to \mu^+\mu^-$ depend on the sum squares of the absorptive and the real parts, the SD parts become somewhat insignificant for $K_L \to \mu^+\mu^-$decay rate.

A second experimentally accessible quantity in $K_L \to \mu^+\mu^-$decay is the longitudinal polarization of
the leptons $P_L$. The state $K_L$ is a Superposition of the CP-eigenstates $K_1^0$ and $K_2^0$.

$$K_L = (1 + |\varepsilon|^2)^{1/2} \left( K_2^0 + \varepsilon K_1^0 \right)$$  \hspace{1cm} (1)

where we have followed the Wu-Yang phase convention and $\varepsilon$ is the CP-violating $K_1^0 - K_2^0$ mixing parameter given by:

$$\varepsilon \simeq (2 \times 10^{-3}) \exp(i\pi/4)$$  \hspace{1cm} (2)

It has been known for a long time \cite{7} that $P_L$ would be zero unless P and CP are both violated in the decay. For the $K_L$-decay, a finite $P_L$ thus can arise (i) directly from CP-violating decay of $K_2^0$ and (ii) from CP-conserving the decay of $K_1^0 \rightarrow \mu^+\mu^-$, because of the presence of $\varepsilon K_1^0$ component of $K_L$.

The invariant amplitude for the decay $K_1^0 (i = 1, 2)$ into $\mu^+(p_+)\mu^-(p_-)$ can be written as \cite{8}:

$$\mathcal{M}_i = \bar{u}(p_-) [a_i \gamma_5 + ib_i] v(p_+)$$  \hspace{1cm} (3)

where $a_2, b_1$ are CP-conserving and $a_1, b_2$ violate CP-invariance. The amplitude for $K_L^0$ decay into $\mu^+\mu^-$ is then given by an expression similar to the above one with $a_i, b_i$ replaced by $a_L, b_L$:

$$a_L = a_2 + \varepsilon a_1 \hspace{1cm} b_L = b_2 + \varepsilon b_1$$  \hspace{1cm} (4)

The longitudinal polarization $P_L$ can be expressed in terms of $a_2, b_2$ as upto $O(\varepsilon)$

$$P_L = \frac{m_K r^2 Im(a_2^* b_2 + a_2^* \varepsilon b_1)}{4\pi \Gamma}$$  \hspace{1cm} (5)

where $r = (1 - 4m_\mu^2/m_K^2)^{1/2}$, and $\Gamma$ is the total decay width:

$$\Gamma = \frac{m_K r}{8\pi} (|a_L|^2 + r^2 |b_L|^2)$$  \hspace{1cm} (6)
In the Standard Model (SM) the direct contribution proportional to $Im(a_2^s b_2)$ in eqn.(5), is small. The leading contribution comes from the induce $\bar{s}d - \text{Higgs}(H)$ vertex. This could potentially be large if the Higgs mass $m_H$ is close to $m_K$ but in view of the current limits $m_H > 77.5 GeV$ [10] the direct contribution to $P_L$ would be smaller than $10^{-4}$ [9]. The indirect contribution to $P_L$ represented by the term $Im(a_2^s e b_1)$ numerator of eqn.(5), has been investigated in detail by Herczeg [5] assuming $a_2$ to be dominantly imaginary. A more complete treatment without this assumption has been given by Ecker and Pich [11]. They obtain a value $|P_L| \simeq 2 \times 10^{-3}$ but observe that in view of the uncertainties of chiral perturbation theory employed for the estimate, experimental value of $|P_L| > 5 \times 10^{-3}$ would be evidence for new physics beyond SM.

The purpose of this note is to reexamine the direct contribution to $P_L$ in the context of the minimal supersymmetric extension of the standard model (MSSM) [12]. Compared to SM, the parameter space of MSSM is much bigger and the number of neutral Higgs jumps from one in SM to three, two of which are CP - even and one CP - odd.

The immediate consequence of the minimum SUSY extension of the standard model in respect of $\Delta S = 1$ neutral current operators have been examined by Cho et.al. and Bertolini et.al.[13,14]. The basic structure of the effective Hamiltonian for $\Delta S = 1$ obviously remains unchanged since the superpartner particles are all heavy and as with other heavy particles do not make their appearance in the low-dimensional operators responsible for low energy $\Delta S = 1$ processes. The effect of superpartner particles are felt through their modifying the values of the various Wilson co-efficients in the effective Hamiltonian:

$$\mathcal{H}_{eff} = \frac{\alpha G_F}{\sqrt{2} \pi} \lambda \left[ C_8^{eff} (\bar{s} \gamma^\mu P_L d)(\bar{l} \gamma^\mu l) + C_{10} (\bar{s} \gamma^\mu P_L d)(\bar{l} \gamma^\mu \gamma_5 l) + 2 \frac{C_7 m_b}{p^2} (\bar{s} \not p \gamma^\mu P_R d)(\bar{l} \gamma^\mu \gamma_5 l) \right]$$

with $p$ being the sum of the lepton momenta. The structure in eqn. (7) is obtained by taking account of box and $Z^0$-penguin diagrams together with their superpartner counterparts. Using the constraints
of the MSSM parameter space forced by experimentally observed $b \to s\gamma$ decay [15], the changes in
the Wilson co-efficients from their SM values have been found to be mild. In any case the Hamiltonian
eqn.(7) results in a CP - invariant $K_L \to \mu^+\mu^-$ amplitude and thus does not contribute to $P_L$. However,
if the parameter $\tan\beta$ in MSSM is large, of the order of 25 or more, the contribution of neutral higgs
bosons (NHBs) exchange amplitude (which is not included in the effective hamiltonian eqn. (7)) can
become significant. The purpose of this note is to investigate this aspect.

The dominant NHB exchange contributions to the effective Hamiltonian for the process $K_L \to \mu^+\mu^-$ are
shown in Figure 2. The effective Hamiltonian from NHB has the structure [18,19]:

$$H_{eff}^{NHB} = \frac{\alpha G_F}{2\sqrt{2}\pi} \lambda \left[ C_{Q_1} \bar{s}(1 + \gamma_5)d \bar{l}l + C_{Q_2} \bar{s}(1 + \gamma_5)d \bar{l}\gamma_5l + h.c. \right]$$

(8)

$C_{Q_1}, C_{Q_2}$ are Wilson co-efficients at scale $\mu$, which for our case will be $\sim 1GeV$. The $C_{Q_1}$ term above
contributes a CP - violating piece to the invariant amplitude for $K_L \to \mu^+\mu^-:

$$M^{NHB} = \frac{\alpha G_F}{2\sqrt{2}\pi} \frac{C_{Q_1}}{\sqrt{2}} 2 (i \text{ Im} \lambda) \langle 0 \mid \bar{s}\gamma_5d \mid K_0 \rangle \bar{u}(p_-)v(p_+)$$

(9)

where $p_+, p_-$ are the momentum of $l^+$ and $l^-$ respectively. We write the invariant amplitude following
the convention of Herczeg [5]:

$$M = a_2 \bar{u}(p_-)\gamma_5v(p_+) + ib_2 \bar{u}(p_-)v(p_+)$$

(10)

where the phases of $K_2$ has been chosen such that $a_2$ and $b_2$ are real except for unitary phases. In (10) we have taken the $K_L$ amplitude to be the same as the CP - odd $K_2$, since we are interested in
the contribution to $P_L$ arising out of the direct part. With this convention for the $K_L$ amplitude, we
can relate the matrix element of $\bar{s}\gamma_5d$ between vacuum and $K^0$ via the kaon-decay constant $(f_K)$ as
follows:

$$\langle 0 \mid \bar{s}\gamma_5d \mid K_0 \rangle = \frac{f_K m_k^2}{(m_s + m_d)c}$$

(11)
where the suffix c indicates that the masses are current quark masses. The NHB contributions to $b_2$ is:

$$b_{2}^{NHB} = \frac{\alpha G_F}{2\pi} C_{Q_1}(\mu) \left(Im\lambda\right) \frac{f_k m_K^2}{(m_s + m_d)c}$$

(12)
The amplitude $a_2$ is almost totally saturated by the $\gamma\gamma$ intermediate state where contribution has been estimated by Herczeg [5]:

$$Im(a_2^{\gamma\gamma}) \approx 2 \times 10^{-12}$$

(13)
We shall use this value as the total $Ima_2$. We now then have all the ingredients for estimating the NHB exchange graphs contribution to the direct parts of the contribution to $P_L$:

$$P_L = \frac{2r Im(b_2a_2^*)}{|a_2|^2 + (1 - \frac{4m_c^2}{m_k^2}) |b_2|^2}$$

(14)
For numerical estimation, we use the value of $Im\lambda$ in terms of Wolfenstein parameterization:

$$Im\lambda = A^2 \lambda^5 \eta$$

(15)
Using the input parameters as given in Appendix we get:

$$P_L^{dir} = 0.4 C_{Q_1}$$

(16)
The numerical value expected thus is directly proportional to the unknown Wilson co-efficient $C_{Q_1}(\mu)$ at scale $\sim 1$ GeV. For this corresponding co-efficient in $b \rightarrow s$ transition, this co-efficient has been calculated in terms of MSSM parameters by Dai et.al [18]. This was done in standard way, by calculating the penguin terms perturbatively at scale $\sim M_W$ and then using Renormalization Group equations (RGE) to evolve down to much lower scales. The RGE evolution involves no operator mixing and so this is a multiplicative correction in coming down from $M_W$ to $m_b$. For us, the RGE is identical & so
the only difference will be in the perturbative estimate of $C_{Q_1}(M_W)$. The mass of the quark enters the calculation of this since the Higgs coupling to quarks is directly proportional to the quark mass. Thus the $C_{Q_1}(\mu)$ for the $s \to d$ transition will effectively be a factor $(m_s/m_b)$ down from its value for the $b \to s$ transition. From a purely field theoretical point of view, the masses above would be the masses in the SM - langrangian, namely the 'current' quark masses, where values for the light quark are determined through low-energy chiral symmetry breaking analysis (Cheng & Li [17] Section 5.5). We will use the masses and Wolfenstein parameters of CKM as given in appendix.

The value of $C_{Q_1}$ depends crucially on MSSM parameters. As is well known, MSSM has an undesirably long list of parameters. Most phenomenological analysis in MSSM use unification model in which SUSY is softly broken (at around Planck scale) leading to the 'SUGRA' version of MSSM. Such models are completely specified by a common gaugino mass term, a scalar mass term, trilinear coupling (all specified at Planck scale) together with the higgs sector parameter and $\tan\beta$ in addition to SM parameters. Several authors [21] have analyzed this parameter space and the constraints imposed therein by SM - parameters as well as by the now known $b \to s\gamma$ data as given by CLEO [15]. We in particular, work inside the parameter space as analyzed e.g. by Goto et.al [20] where strict universality of the soft SUSY breaking mass holds separately for squarks and scalars. With such relaxed universality, working within allowed parameter space region consistent with all low energy SM - parameters and $b \to s\gamma$, it is possible [19] to have regions of parameter space where $\tan\beta$ is large but the NHBs are relatively light ($\approx 2M_W$). Such allowed values of MSSM parameters have a wide range wherein the $C_{Q_1}$ for $b \to s$ transition is of the order $O(1)$; the value of $C_{Q_1}$ for $s \to d$ transition would be down by a factor $m_s/m_b \approx 0.025$. Fig. (3) shows typical values of the co-efficient $C_{Q_1}$, for $s \to d$ transition relevant to $K_L \to \mu^+\mu^-$ decay. For value of the CP-odd higgs mass ($m_A$) large (say greater than 200 GeV), $C_{Q_1}$ indeed is too small . However, for somewhat lower values of $m_A$, with $\tan\beta > 25$ (which is within the acceptable range of parameters), $C_{Q_1}$ can be sufficiently large for the NHB - exchange contribution to $P_L$ (16) to overwhelm the SM-estimate. Thus for a typically low value of $m_A = 150 GeV$, we get from eqn.(16) values of $P_L^{dir} = 0.7 \times 10^{-2}, 1.2 \times 10^{-2}, 1.8 \times 10^{-2}$ respectively for $\tan\beta = 25, 30, 35$. 

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In summary, the predictions of MSSM for $P_L$ are as follows. If the parameter $\tan\beta$ is small then MSSM does not change the SM value, which as stated before is dominated by the indirect contribution and estimated at $|P_L| \simeq 2 \times 10^{-3}$ by Ecker and Pich. For large $\tan\beta$ and masses of Higgs bosons exceeding 250 GeV, once again MSSM does not change SM predictions. However if $\tan\beta$ is large ($\sim 25$ or more) and the Higgs masses are in the range of upto $2M_W$, the NHB-exchange contributions to $P_L$ start becoming significant. A typical value for $\tan\beta = 30$ and $m_A = 150$GeV gives $P_L = 1.2 \times 10^{-2}$. When one is able to narrow down the acceptable parameter space of MSSM, experimental measurement of $P_L$ would thus provide a very useful confirmatory test.

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Appendix

Input parameters

Wolfenstein parameters [16] : $A \simeq 0.8$ ; $\lambda = 0.22$ ; $\eta = 0.34$

Current quark masses [17] : $m_d = 7$ MeV ; $m_s = 130$ MeV

$m_{\mu} = 105$ MeV , $G_F = 1.16 \times 10^{-5}$ GeV$^{-2}$ , $\alpha_s(m_Z) = 0.119$ , $m_b \approx 5$ GeV

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Figure 2: The dominant contributions at scale $M_W$, of NHB exchange contribution to the effective Hamiltonian for $K_L \to \mu^+ \mu^-$.
Figure 3: Typical values of $C_{Q_1}$ for $s \to d$ transition at GeV scale. Values of $C_{Q_1}$ have been plotted with pseudoscalar higgs mass ($m_A$). The other MSSM parameters are $m = M = 150 \text{ GeV}$, $A = -0.5$. 

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