Homogeneous Turbulence Generated by Multi-scale Grids

Per-Åge Krogstad\textsuperscript{1} & Peter Davidson\textsuperscript{2}
\textsuperscript{1}Norwegian University of Science and Technology, N-7491 Trondheim, Norway.
\textsuperscript{2}University of Cambridge, Cambridge CB2 1PZ, U.K.
E-mail: per.a.krogstad@ntnu.no

Abstract. We investigate wind tunnel turbulence generated by a conventional and two multi-scale grids. The conventional and multi-scale grids were all designed to produce turbulence with the same integral scale, so that a direct comparison could be made between the different flows. The decay of the turbulent energy was mapped in detail from a distance from the grid less than one mesh width, down to distances of the order of 200 meshes using a combination of laser doppler and hot wire anemometry tools. The turbulent decay rate behind our multi-scale grids was found to be virtually identical to that behind the equivalent conventional grid after the initial transition had been completed. In particular, all flows exhibit a power-law decay of energy, $u^2 \sim t^{-n}$, where $n$ is very close to the classical Saffman exponent of $n = 6/5$ in the far field. Our results are at odds with some other experiments performed on multi-scale grids, where significantly higher energy decay exponents have been reported.

1. Introduction

Classical theories for grid turbulence predict that the energy decays as $\langle u^2 \rangle \sim t^{-n}$ and predicts that homogeneous turbulence can decay no faster than $n \approx 10/7$. In recent years a number of wind tunnel experiments and numerical simulations have focused on quasi-homogeneous turbulence generated by multi-scale grids, suggesting that they generate quite different turbulence properties than conventional grids. (See, for example, Hurst & Vassilicos (3), and Nagata \textit{et al.} (8).) In some of these experiments the turbulence appears to behave in unexpected ways. For example, Hurst & Vassilicos (3) report considerably higher values for the decay exponents, and also exponential decay rate has been observed for some grid geometries.

However, this suggestion is somewhat at odds with the evidence of direct numerical simulations in periodic cubes, where it is usually found that, after some transient, the behavior of the turbulence is largely independent of the precise form of the initial energy spectrum (see, for example, Ossai & Lesieur (9)). Indeed, it is often argued that, once fully developed, all the turbulence remembers of its initial conditions is the prefactor, $c_m$, in the expression $E(k \to 0) = c_m k^m$, $c_m$ being an invariant for $m \leq 4$ (Ossai & Lesieur, (9), Davidson, (1), Ishida \textit{et al.}, (4), and Davidson, (2)).

In order to test the hypothesis that turbulence behind multi-scale grids will develop in a different way than the flow downstream of conventional grid geometries, a series of wind tunnel experiments using two multi-scale grids were investigated and the results compared with data obtained using a similar conventional grid. In order to ensure that the comparison is meaningful,
the dimensions of the grids were chosen such that the integral scale of the turbulence some short distance downstream of each grid, \( \ell_0 \), is virtually the same in all three cases (to within a percent or so). The experiments were carried out in a large working section, and measurements taken virtually from the grid plane down to \( x \approx 400\ell_0 \) for the conventional grid, which translates to \( x \approx 240m \). The turbulence generated by all three grids is strongly geometry dependent close to the grid, but after the initial decay where the flow is highly non-uniform, the energy level as well as the decay rate of turbulent energy becomes virtually identical for all three grids.

2. The Experimental Set-up

The experiments were performed in the large recirculating wind tunnel described in Krogstad & Davidson (6). The tunnel test-section has transverse dimensions of 2.7m x 1.8m (measured at the start of the test section) and is 12m long. There is an adjustable roof to compensate for the growth of the sidewall boundary layers.

All three grids were produced from 2mm thick sheet metal. The conventional grid (labeled \( cg \)) has square holes 30mm x 30mm punched at 40mm spacing, giving a mesh size of \( M = 40mm \), a bar width of \( t = 10mm \), and a solidity of \( \sigma = 44\% \). The tests on this grid were all performed at a Reynolds number of \( Re_M = UM/\nu = 3.6 \times 10^4 \), where \( U = 13.5m/s \) was the mean speed in the tunnel.

The first of the multi-scale grids (labeled \( msg_1 \)) is similar to the cross-grid-type (a) of Hurst & Vassilicos (3) and is shown in Figure 1(a). It has bar widths ranging from \( t_1 = 8mm \) down to \( t_3 = 2mm \), and mesh sizes ranging from \( M_1 = 64mm \) to \( M_3 = 15mm \). The solidity of \( msg_1 \) is also \( \sigma = 44\% \). These measurements were taken at \( U = 14.0m/s \). The second multi-scale grid (\( msg_2 \)) is shown in Figure 1(b). As for \( msg_1 \), the bar widths vary from \( t_1 = 8mm \) down to \( t_3 = 2mm \), though the mesh sizes are larger, with \( M_1 = 88mm \) to \( M_3 = 21mm \). This reduces the solidity of \( msg_2 \) to \( \sigma = 33\% \). This grid was tested at \( U = 15.5m/s \).

The pressure drop across the grids are shown in Figure 2. Our conventional grid has the same \( \sigma \) as the conventional grid used by Hurst & Vassilicos, which was also a monoplane grid. As expected, the two pressure coefficients are almost identical. Our multi-scale grids, which were not designed in the strict fractal way as those used by Hurst & Vassilicos, but still uses bars and mesh sizes formed by three different scales, form a natural extension of their three least solid fractal grids. However, the pressure drop formed by our most solid multi-scale grid is considerably lower than both the \( \sigma = 0.40 \) grid of Hurst & Vassilicos and the conventional grids.
Figure 2. Pressure drop coefficients measured for the three grids investigated (solid symbols). (Open symbols are data from Hurst & Vassilicos (3).)

Hence our data does not support Hurst & Vassilicos’ claim that the multi-scale grids generate higher pressure drops and therefore better mixing at equal $\sigma$ in this case. Unfortunately we only had access to one conventional grid, so we are unable to say if the same applies to the lower solidity grid.

As will be shown in the next section, the turbulence produced by these grids becomes homogeneous and fully developed somewhere between $x = 1$ and $2m$. At $x = 2m$ the Kolmogorov microscale depends on the grid geometry and was found to be $\eta \approx 0.22mm \rightarrow 0.26mm$. On the other hand, the integral scales at $x = 2m$, $\ell_0 = \ell(x = 2m)$, turned out to be grid independent at $\ell_0 = 23.9mm$ (for $cg$), $\ell_0 = 23.6mm$ (for $msg1$), and $\ell_0 = 23.4mm$ (for $msg2$), respectively. Note that the geometric length-scales associated with the two multi-scale grids almost span the range of dynamic scales associated with the turbulence, from around $9\eta$ up to several integral scales. Finally we note that, in terms of $\ell_0$, the tunnel cross-section is approximately $115\ell_0 \times 80\ell_0$, thus ensuring that there is minimal influence of the side-wall boundary layers.

Two sets of measurements were performed, one mapping the near grid development and another investigating the decay in the far field. The turbulence levels in these two investigations were expected to be very different and therefore two different measurement techniques were applied.

For the field immediately downstream of the grid, where the turbulence level may be extremely high and even reversed flows were detected, two component LDA was mainly applied. Additional measurements were taken using single hot wire anemometry, which turned out to produce identical results except for distances of the order of a few mesh sizes from the grid. For the far field the data was primarily obtained using single and two component hot-wire anemometry which was expected to have lower system noise than the LDA and benefits from the possibility of obtaining better spectral measurements, but additional measurements were also made using LDA. Further information about the instrumentation and data analysis may be found in Krogstad & Davidson (6).
Figure 3. Spanwise distributions of mean velocity in the near field. a) $x=50\text{mm}$, b) $x=300$ and 1200mm.

3. Near field development

Because of the differences in geometry it is expected that the near field of the two grids behave very differently. The multi-scale grids generate a large number of wakes at various scales which interact at different distances from the grid. The smaller scale wakes which are located relatively close are expected to interact rather quickly and therefore loose much of their identity relatively quickly, while the largest scales might develop more in the way of a conventional monoplane grid exposed to background turbulence.

We first investigate the spanwise distribution of the mean velocity in the near field measured at three streamwise stations is shown in Figure 3. The spanwise distance has been normalised with the large mesh scale for each grid ($M_1$ in Figure 1). At $x=50\text{mm}$ (Figure 3(a)) the flow behind the conventional grid (corresponding to $x/M_1=1.25$) is seen to be dominated by single large scale jets, while the flow behind the two multi-scale grids are, as expected, less regular. $x/M_1=0.78$ for $msg_1$ and 0.57 for $msg_2$ so the profiles may not be directly comparable. However, it is obvious that the initial flows are different. As the flow develops downstream (Figure 3(b)), the characteristics of the initial condition is gradually lost as the spanwise distributions gradually become sinusoidal for both types of grids and at $x=1200\text{mm}$ spanwise variations only persist for the largest multi-scale grid, $msg_2$, which has the largest $M_1$ scale and therefore need the longest distance for the spanwise diffusion process to be completed.

Next, streamwise traverses were made starting from the centre of the mesh. For the conventional grid this means starting from the centre of the hole where the high velocity jet exits (Figure 3(a)). However, for the two multi-scale grids, the centre of the largest mesh coincides with the cross formed by the second scale bars (see Figure 1.) Hence the initial development along the centre line is quite different, as seen in Figure 4. From $x \approx 2000\text{mm}$ the flow is independent of the streamwise position.

From the observations about the mean field it is obvious that grid-specific development of the turbulence characteristics in the near field must be expected. The interesting question is however how far this effect extends and if the decay further downstream is grid dependent. Figure 5 shows the energy decay as expressed by $\langle u^2 \rangle / U^2$ as function of distance from the grid. The flow downstream of the conventional grid starts from a low turbulence chore region of the jet which quickly picks up energy from the wakes formed by the bars. Peak energy is found at $x \approx 150\text{mm}$ and the energy then drops rapidly. The kinetic energy in the two multi-scale grids is initially two orders of magnitude higher, but immediately starts to decay as $\langle u^2 \rangle / U^2 \sim x^{-n}$. Next a transition region is apparent for both grids before the energy again decreases at the same rate. The transition region starts first for the smallest scale grid, $msg_1$, but the shape is the
Figure 4. Streamwise development of the mean velocity along the centre line. Circles: Conventional grid, Squares: Multi-scale grid 1, Diamonds: Multi-scale grid 2.

Figure 5. Streamwise development of the streamwise turbulent energy, $\langle u^2 \rangle / U^2$. Circles: Conventional grid, Squares: Multi-scale grid 1, Diamonds: Multi-scale grid 2.

same for both grids. It is therefore reasonable to assume that the transition region is linked to the position where the wakes from the largest bars have diffused to the centre line, thereby increasing the energy in this region.

Finally we show the ratios between the streamwise and lateral stresses, $\langle u^2 \rangle / \langle v^2 \rangle$ (Figure 6). For the three first positions behind the conventional grid the ratio is identical to 1, but in general the ratio is very grid dependent. It may however be seen that for all grids, the ratio seem to settle on a level which is somewhat higher than 1 for $x > 2000\text{mm}$.

After the initial transition is completed, the energy levels at a certain $x$ position (or equivalently at a given $x/\ell_0$ position), and the decay rates, are virtually the same for all grids. Hence it appears that there is no benefit in going to the more complicated multi-scale grid unless
the purpose is to generate increased mixing very near the grid.

4. Far field development

Because the stress ratio $\langle u^2 \rangle / \langle v^2 \rangle$ appeared to settle at a ratio that was different from 1, it was decided to move the grids upstream into the test section contraction to improve isotropy when studying the far field decay. From the location of the grid to the entrance of the test section, the area contraction ratio was 1.48 and the test section starts $x = 1200\text{mm}$ downstream of the grid.

In this way the first development was under the influence of a favourable pressure gradient, but for $x > 2\text{m}$, $U$ was constant to within $\pm 0.5\%$ (see Figure 7), with $Re = U l_0 / \nu = 2.1 \times 10^4$, $2.2 \times 10^4$ and $2.4 \times 10^4$, respectively, for the three grids. Figure 7 also shows the streamwise variation of $Re_\lambda = (u_x^2)^{1/2} \lambda / \nu$ for all three grids and there is no significant difference between the conventional and multi-scale grids.

The streamwise development of the anisotropy of the turbulence in the far field, as measured by the ratio $\langle q^2 \rangle / 3 \langle u_x^2 \rangle$, where $\langle q^2 \rangle = \langle u_x^2 + u_y^2 + u_z^2 \rangle$, is shown in Figure 8. For all grids $\langle u_x^2 \rangle / \langle u_2^2 \rangle$, $\langle u_2^2 \rangle / \langle u_2^2 \rangle$ and $\langle q^2 \rangle / 3 \langle u_x^2 \rangle$ were close to unity at $x = 2\text{m}$, but $\langle u_x^2 \rangle / \langle u_2^2 \rangle$ and $\langle u_2^2 \rangle / \langle u_2^2 \rangle$ were found to divert slowly with increasing $x$. The largest departure from isotropy was observed at the exit of the test section, where $\langle u_x^2 \rangle / \langle u_2^2 \rangle$ reached values in the range of 0.8 to 0.9. There was a corresponding growth in $\langle u_x^2 \rangle / \langle u_2^2 \rangle$, to produce the almost constant $\langle q^2 \rangle / 3 \langle u_x^2 \rangle$, ratios shown in the figure.

In most of what follows we shall restrict the analysis of our data to the region $x > 2\text{m}$. This is to avoid any acceleration effects caused by the streamwise variations in $U$ and also to be sure not to include data suffering from the initial spanwise inhomogeneities. Also the data for $x > 8\text{m}$ will be excluded, because of the increased levels of noise for large $x$ as the turbulence level drops well below 1%. From the data presented above we see that, in this range, the turbulence is reasonably homogeneous and that, although there is some anisotropy, the levels are not excessive and are comparable to, if not better, than in most other experiments.
4.1. Comparison With Classical Energy Decay Models.

As noted in Davidson (1) and Krogstad & Davidson (6), there are two classical predictions for $n$. One arises when $E(k \to 0) \sim k^2$, a situation called Saffman turbulence, and the other when $E(k \to 0) \sim k^4$, so-called Batchelor turbulence.

The decay of turbulent kinetic energy in homogeneous turbulence is given by

$$\frac{du^2}{dt} = -A \frac{u^3}{\ell}, \quad A = \text{constant},$$

which is combined with the assumption of a power law decay

$$\frac{\langle u_x^2 \rangle}{U^2} = a \left[ \frac{x - x_0}{\ell_0} \right]^{-n} \Rightarrow \ln \left[ \frac{\langle u_x^2 \rangle}{U^2} \right] = \ln a - n \ln \left[ \frac{x - x_0}{\ell_0} \right],$$

Figure 7. Streamwise distributions of $Re_{\lambda}$ (left axis) and the test section centre-line speed, $U$, (right axis). $U$ is normalized by $U$ at $x = 2m$.

Figure 8. Streamwise distributions of $\langle q^2 \rangle / 3 \langle u_x^2 \rangle$. 
where, as before, $\ell_0$ is the integral scale at $x = 2m$.

This produces a decay exponent of $n = 6/5$ (Saffman’s exponent; see Saffman (10)) in $E(k) \sim k^2$ turbulence, and $n = 10/7$ (Kolmogorov’s exponent; see Kolmogorov (5)) in $E(k) \sim k^4$ turbulence. The first of these exponents was observed in the experiments on the decay behind the classical grid by Krogstad & Davidson (6) and the second was observed in the numerical simulations of Ishida et al. (4). In the following we will study which of the two classical predictions is the most likely candidate for the multi-scale grids. However, there is a slight complication which arises when comparing these predictions with experiments: in wind tunnel data it is well known that the coefficient $A$ can vary slowly along the test section, and this causes slight departures from the ideal values of $n = 6/5$ or $n = 10/7$. Indeed, we shall see shortly that just such a slow variation of $A$ occurs in all our experiments.

It is notoriously difficult to obtain reliable estimates of $n$. First, we do not know in advance where $x_0$ will lie. Second, if the range of $x/\ell_0$ is too short, the decay exponent becomes very sensitive to the choice of the unknown $x_0$. Third, if data from the inhomogeneous region close to the grid is included in the fit, then higher (and misleading) values of $n$ may be obtained, as seen from the rapid initial decay for the conventional grid in the region $150 < x < 400mm$ (Figure 5). Fourth, for large $x/\ell_0$ the turbulence intensity is low, so that noise starts to become problematic and therefore the very far field data should be ignored.

Figure 9 shows $\langle u_x^2 \rangle /U^2$ and $\langle q^2 \rangle /3U^2$, obtained using two different alignments of the $x$-wires, all plotted as a function of $x$. This plot demonstrates that, for all three grids, there is a clear power-law relationship between $u^2$ and $x$. It also shows that $\langle u_x^2 \rangle$ and $\langle q^2 \rangle$ follow essentially the same power law. Three different fitting procedures were used and their results compared. All three methods are described in detail in Krogstad & Davidson (6) and the data obtained from the fits are shown in Table 4.1. The essence of method M1 is to do a regression analysis of the data using the logarithmic form in Eq. 2 by sorting out the range of the data points which produces the lowest variance in the fit. This is a conventional Regression method. In method M2 we use the fact that a large number of data was obtained with very small streamwise increments. This allowed the exponent to be obtained using a 3 point discretization scheme directly on the data. This was called the Local exponent method. Finally a method called the Maximum decay range method was used. In this procedure a range of $x_0$ was tested in Eq. 2 and compared to the measurements. The value of $x_0$ which produced the widest range of constant decay exponent, $n$, was deemed to be the correct one and the corresponding exponent is shown in Table 4.1. Full details of the fit methods are given in Krogstad & Davidson (6).

| Method: | $x_0$ | $n$ | $n$ | $n$ |
|---------|-------|-----|-----|-----|
| Conv. grid | 0.26m | 1.13 | 1.14 | 1.17 |
| Grid 1 | 0.43m | 1.12 | 1.17 | 1.19 |
| Grid 2 | 0.30m | 1.25 | 1.25 | 1.23 |

**Table 1.** Estimates of the energy decay exponent $n$ obtained using three different methods, M1, M2 and M3 (see description in text).

There are some striking features of Table 1. First, all three grids yield decay exponents very close to the classical Saffman value of $n = 6/5 = 1.2$. Second, the conventional and multi-scale grids produce almost identical results (to within experimental uncertainty). It therefore appears that the decay rate is closer to the Saffman prediction than the Batchelor model.

Finally we consider the dimensionless dissipation coefficient $A$ in Eq. (1), which is normally taken to be constant during the decay of isotropic turbulence. Assuming isotropy at the small
scales, the viscous dissipation rate, $\epsilon$, can be written as

$$\epsilon = \frac{3}{2} \frac{A u^3}{\ell} = 15 \nu \left\langle \left( \frac{\partial u_x}{\partial x} \right)^2 \right\rangle,$$  \hspace{1cm} (3)$$

which allows us to estimate $A$ from measurements of $\left\langle (\partial u_x/\partial x)^2 \right\rangle$, $u$ and $\ell$. The corresponding values of $A$ are plotted in Figure 10 for all three grids as a function of $(x - x_0)/\ell_0$. Again, there is some scatter, largely due to the difficulty in estimating $\ell$. Never-the-less, once the turbulence is fully developed, say for $(x - x_0)/\ell_0 > 100$, there is a slow but steady decline in $A$ which is consistent across all three grids. As noted earlier, this slow variation in $A$ means that, even if we have Saffman turbulence, with $u^2\ell^3 = \text{constant}$, we need not recover $n = 6/5$, as this value of $n$ relies on $A$ being strictly constant. How the spatial dependence of $A$ affects the observed decay exponent, $n$, is described in Krogstad & Davidson (6).
5. Conclusions

Our primary findings are two-fold. First, it seems that Saffman’s decay law is reasonably robust, since the energy decay exponents for all three grids are close to Saffman’s classical prediction of \( n = 6/5 \). Second, the multi-scale grids used here produce almost identical results to the equivalent classical grid. In particular, all three flows exhibit remarkably similar streamwise distributions of \( Re_\lambda \) (Figure 7) and dimensionless decay coefficient \( A \) (Figure 10). It is also worth noting that the spectra (not shown here, but may be found in Krogstad & Davidson (7)) for the multi-scale grids exhibit classical Kolmogorov scaling, with \( E(k) \) collapsing on \( \ell \) and \( u \) at low \( k \), and on \( \eta \) and \( v \) at high \( k \).

Our findings contradict those of some previous studies which report unusual behavior behind similar multi-scale grids, in particular, a very high energy decay exponent of around \( n \sim 2.0 \) and unusually high values of \( Re_\lambda \). A decay exponent of \( n \sim 2.0 \) is particularly worrying as the theoretical maximum for \( n \) (assuming the dimensionless decay coefficient, \( A \), is constant) is \( n = 10/7 \). However, these earlier measurements were taken much closer to the grid where the flow exhibits initial grid-dependent inhomogeneities; inhomogeneities which, according to the present data, disappear further downstream.

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