Comparative Performance Study of Tuned Liquid Column Ball Damper for Excessive Liquid Displacement on Response Reduction of Structure

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1. INTRODUCTION

Vibration control techniques [1, 2] for the safe performance of structures against dynamic load are widely acclaimed amongst researchers as a viable alternative to traditional designs. In contrast to these tradition means, the vibration control approaches substantially decrease the structural responses to confirm negligible damage to structural systems. The TLCD is more efficient than the other kind of damping devices and can be easily installed with low maintenance cost. The liquid-filled in U-shaped TLCD having a uniform cross-sectional area is generally adopted to suppress the structural response generated due to seismic loading [3], wind [4] and wave motion [5]. TLCD usually includes an immovable orifice in the middle of the horizontal portion of the column. The nonlinear mathematical modeling of such a device was reported by Sakai [6]. Following their work, various types of modification have been proposed in the literature to improve the seismic protection of the building as introducing the combination of spring and viscous damper with the structure [7], hybrid TLCD with variable orifice opening condition [8] and use of magneto-rheological fluid in place of normal fluid [9].

In recent times, Al-Saïf et al. [10] introduced a metal ball instead of the orifice in TLCD, which is named TLCBD. The TLCBD showed an improvement in response reduction [10] after experimentally studied which was subjected to harmonic loading. The effectiveness of TLCBD over TLCD subjected to wave-induced vibration had been analytically assessed by Chatterjee and Chakraborty [11]. The robustness of TLCBD turns out to be confirmed further from the work of Gur et al. [12]. They considered various structural and damper properties for performing a comparative study of the optimized response of the structure under stochastic and real earthquake loading. The stochastic structural

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optimization (SSO) [13] was used for minimizing the desired objective function. SSO is an optimization technique, where, the optimum parameters are obtained by searching an appropriate series of design variables over a possible permissible domain. In continuation, various researches are still going on for improving the properties of TLCBD considering unconstrained optimization only.

Recently, Pandey and Mishra [14] examined the performance of a modified circular-shaped TLCBD by using both experimental and analytical methods. This modified device is found capable of controlling the root mean square displacement (RMSD) of the structure subjected to wind-induced torsionally coupled vibration. Tanveer et al. [15] studied the effectiveness of TLCBD for a multi-storeyed building and compared it with TLCD, both analytically and experimentally. It is found to be suitable for mitigating the response of the structure with more than one degree of freedom (DOF). Although unconstrained optimization of TLCBD has been widely discussed; limited studies on TLCBD have been performed.

Generally, liquid dampers are used to mitigate the vibration caused by strong earthquakes. It is quite obvious in this scenario the maximum liquid movement can exceed the vertical portion of the pipe in course of vibrational response mitigation of the structure. In fact, Gao et al. [16] have mentioned in his study in strong-motion earthquake case liquid can vacate one of the vertical columns of TLCBD whereby the liquid will lose U-shape characterization and becomes L-shaped. As the liquid occupies only the horizontal part of the container it behaves like sloshing type dampers and this changes the mathematical expression and physical behavior of TLCBD. Ultimately, for this, the damper systems become less effective.

Won et al. [17] have highlighted in his paper if liquid oscillation is too high then the inertia coupling of the combined governing equation of TLCBD structure decrease. It should be worth mentioned here that liquid dampers are effective in case the strong motion of earthquake. Moreover, in case of higher magnitude of earthquake, the length of the vertical tube should be excessive long enough to accommodate the high amplitude of liquid movement which will create serious headroom problems for the installation of TLCBD. This problem can be solved by restricting the liquid movement in the vertical part of the column considered in the case of constrained optimization. Though the development of TLCBD was introduced in 2011 [10], but very limited studies have been performed on this area considering the earthquake loading [12, 14] to the best of knowledge of the authors. However, the above studies are only considered the unconstrained optimization to evaluate the performance of TLCBD for response mitigation of structure.

Keeping the above shortcomings in the background, the present study investigates the impact of constraining the allowable displacement of liquid movement on the optimum performance of TLCBD parameters and RMSD of the structure considering stochastic earthquake. Here it can be noted that the unconstrained optimization of TLCBD refers to search of design variables such as ball tube diameter ratio and tuning ratio considering the objective function as RMSD of the structure. Whereas unconstrained optimization talks about the same but constrained applied on maximum liquid displacement in the vertical portion of the tube to certain allowable liquid displacement. For efficient controlling, the maximum value of liquid displacement needs to keep within some allowable limits. This is for the fact that large liquid movement can overflow the liquid from the tube and makes the TLCBD less effective and change the hydraulic system of TLCBD. By restraining the liquid movement, the optimum design parameters of TLCBD are found which will ensure the performance of vibration mitigation as well as the constrained condition of not lowering the liquid column movement beyond a certain limit. Here, the performance index for optimization is considered the RMSD of the primary structure constraining the maximum allowable liquid displacement beyond a certain limit.

2. STOCHASTIC RESPONSE ANALYSIS OF STRUCTURE- TLCBD SYSTEM

In the present investigation, the primary structure is considered as a single DOF system. A TLCBD is assumed to be attached to that. The vertical liquid height, $h$, is the height from the middle of the horizontal part of the column. The overall length ($L$) and horizontal length ($B_h$) of the liquid column are assumed for the formulation, where, overall length $L$ is expressed as $L = B_h + 2h$. The scheme of the mechanical model of the structure-TLCBD system is presented in Figure 1.

Lagrangian formulation [12] is used here to formulate the motion equation of the ball (Equation (1)) exist in the horizontal part of the TLCBD as shown in Figure 1.

$$
\left( m_{1b} + \frac{l_{1b}}{R_{1b}^2} \right) \ddot{x}_1 + d_v \dot{x}_1 + \frac{2m_{1b} \nu R_{1b}^2}{L} \ddot{x}_2 + d_v \ddot{x}_2 + \frac{l_{1b}}{R_{1b}^2} \dddot{x}_1 = \dddot{x}_3 + \dddot{x}_g
$$

(1)

The equivalent viscous damping of the ball (Figure 1) is denoted by $d_v = 6\pi \nu R_{1b}$. Where, mass moment of inertia of the same is represented as $I_{1b} = \frac{2m_{1b} R_{1b}^2}{5}$. $m_{1b}$ and $m_{2l}$ signifies the mass of the ball and liquid, respectively. $\nu$ denotes the kinematic viscosity of the liquid. $R_{1b}$ denotes the radius of the ball and $R_{12}$ represents the ratio between the diameter of the ball and the tube.
The equation of motion of the structure attached with TLCBD can be expressed as follows:

\[
\left( m_{3s} + m_{2l} + \frac{1}{n_{2l}} \right) \ddot{x}_3 + 2m_{3s} \xi_{3s} \omega_{3s} x_3 + m_{3s} \omega_{3s}^2 x_3 = \left( \frac{1}{n_{2l}} \right) \ddot{x}_1 - p m_{2l} \ddot{x}_2 - \left( m_{3s} + m_{2l} + \frac{1}{n_{2l}} \right) \ddot{g}
\]  \hspace{1cm} (2)

Here, the natural frequency and damping ratio of the structure is denoted by \( \omega_{3s} = \sqrt{\frac{k_{3s}}{m_{3s}}} \) and \( \xi_{3s} = \frac{c_{3s}}{2m_{3s} \omega_{3s}} \) respectively. Where, \( m_{3s}, c_{3s}, k_{3s} \) denotes mass, damping, and stiffness of the primary structure. \( p = \frac{m_{2l}}{m_{3s}} \) defines the mass ratio. \( \ddot{g} \) is the seismic acceleration applied at the base of the structure. Gravitational acceleration has been expressed by \( g \). The motion of the liquid present in the column is expressed by Equation (3):

\[
m_{2l} \ddot{x}_2 + \left( \frac{2m_{2l} \omega_{2l}}{L} \right) \ddot{x}_2 + \left( 2m_{2l} \xi_{2l} \omega_{2l} \right) x_2 = -p m_{2l} \left( \ddot{x}_3 + \ddot{g} \right)
\]  \hspace{1cm} (3)

The natural frequency (\( \omega_{2l} \)) of the liquid present in TLCBD is expressed as \( \sqrt{\frac{2g}{L}} \) and the tuning ratio (\( \gamma \)) is introduced as \( \gamma = \frac{\omega_{2l}}{\omega_{3s}} \). \( \ddot{x}_j \) represents the damping in terms of the head loss coefficient and the numerical values of \( \xi_{2l} \) has been considered from the literature [10].

Combining Equations (1), (2) and (3) and expressing them in matrix form:

\[
\begin{bmatrix}
1 & 0 & -p_1 & -\mu_1 \\
0 & 1 & 0 & \mu_1 (1 + \mu + \mu_1) \\
-c_1 & -c_1 & 0 & 0 \\
2\gamma \xi_{3s} \omega_{3s} & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_1 \\
\ddot{x}_2 \\
\ddot{x}_3 \\
\ddot{g}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]  \hspace{1cm} (4)

\( [r] \) is the influence coefficient vector, expressed as \( [r] = [0 \ 0 \ 1]^T \). The other abbreviation is used as follows, \( p_1 = \frac{2}{\gamma} \), \( \mu_1 = \frac{8 \rho_{1b} R_{1b}^2 \lambda_1^2 L}{15} \), \( C_1 = \frac{45 \nu}{14 R_{1b}^2 \rho_{1b}^2} \), \( K_1 = \frac{15 \rho_{1b} R_{1b}^2}{14 R_{1b}^2 \rho_{1b}} \). Where \( \rho_{1b} \) is the density of the ball. Equation (4) can be written in a concise form as follows:

\[
[M]\ddot{Z} + [C]\dot{Z} + [K]Z = -[M][r]\ddot{g}
\]  \hspace{1cm} (5)

The [M], [C], and [K] represents the combined mass, damping and stiffness matrix.

2.1. Stochastic Structural Response Analysis of Structure-Damper System

The response of the structure-TLCBD system is assessed by considering the primary structure is subjected under stochastic ground motion at the base. To represent the stochastic excitation for a wide-ranging scope of pragmatic circumstances, a broadly accepted model for the stationary ground movement known as Kanai-Tajimi model [18, 19] has been used. Here, white noise which is acting at the bed rock portion is filtering through filter which is signifying as a soil. The filter equations [18, 19] are stated as follows:

\[
\ddot{x}_f + 2 \xi g \omega_f \dot{x}_f + \omega_f^2 x_f = -\ddot{w}
\]  \hspace{1cm} (6)

\[
\ddot{x}_g = -2 \xi_g \omega_g \dot{x}_f - \omega_g^2 x_f
\]  \hspace{1cm} (7)

where, \( \ddot{w} \) denotes white noise intensity having power spectral density (PSD)\( \delta_0 \). \( x_f \) represents the displacement of the ground and single and double dot over it signifies velocity and acceleration. Substituting \( \ddot{x}_g \) from Equation (7) and incorporating it with Equation (4). By introducing the state variables, augment state vector can be expressed as follows:

\[
\begin{bmatrix}
Y \\
X
\end{bmatrix}
= \begin{bmatrix}
[X] \ \dot{x}_f \ \ddot{x}_f \ \ddot{x}_g
\end{bmatrix}^T
\]  \hspace{1cm} (8)

where \( [X] = [x_1, x_2, x_3] \).

This dynamic equation can be converted to state space equation as mentioned below:

\[
\ddot{\hat{X}} = [A][\hat{X}] + [r]\ddot{g}
\]  \hspace{1cm} (9)
Here, \([A]\) is an augmented matrix of size \((8 \times 8)\) and can be written as follows:
\[
[A] = \begin{bmatrix}
[M]^{-1}[K] & [M]^{-1}[C]
\end{bmatrix}
\]
In the stochastic analysis, the covariance of responses is normally evaluated rather than the direct responses. Assuming the stochastic structural process to be Markovian, the response covariance matrix can be obtained stated as follows [20]:
\[
[A][R_{yy}]^T + [R_{yy}][A]^T + [S_{ww}] = \frac{\Delta}{\delta t}[R_{yy}]
\]  
(10)
The matrix \([S_{ww}]\) is a matrix of size \((8 \times 8)\) and can be expressed as all the terms zero in the matrix except the last diagonal term \(2\pi S_0\). Here \([R_{yy}]\) is the covariance matrix of size \((8 \times 8)\). By solving Equation (9), the RMSD of the structure-TLCBD system can be found from the covariance response matrix, using Runge-Kutta method of 4th order.

Expression of RMSD of primary structure and liquid are as follows:
\[
\sigma_x = \sqrt{R_{yy}(3,3)}, \sigma_z = \sqrt{R_{yy}(2,2)}
\]  
(11)
The response statistics (acceleration) of the derivative process can be obtained from the equation given below:
\[
[R_{yy}] = [A][R_{yy}][A]^T + [S_{ww}]
\]  
(12)
The root mean square (RMS) acceleration of liquid can be obtained from the following equation:
\[
\sigma_x = \sqrt{R_{yy}(6,6)}
\]  
(13)

3. OPTIMIZATION OF STRUCTURE-TLCBD SYSTEM

To search the appropriate set of design variables for a system subjected under random excitation, SSO [21, 22] of the system parameters need to be done over an admissible domain. In this kind of optimization, normally the objective functions are considered as RMS response (displacement, velocity, and acceleration), total building life cycle cost, etc. Generally, the conventional SSO problem is converted into a standard nonlinear design problem, expressed by the protected structure response, counted as the objective function.

3. 1. Unconstrained Stochastic Optimisation of Structure-TLCBD System

RMS has been considered as an objective function to obtain the response of the primary structure following the work of Gur et al. [12]. The optimum \(r\) and \(R_{12}\) was needed to determine the optimized TLCBD system. The design vectors have been considered here \(b_0 = (r, R_{12})\). The objective function thus can be expressed as follows:

Find \(b_0\) to minimize \(f_0 = \sigma_x\)
\]  
(14)

The above unconstrained optimization equation is solved in MATLAB toolbox considering gradient-based standard nonlinear optimization.

3. 2. Constrained Stochastic Optimisation of Structure - TLCBD System

Generally unconstrained SSO procedure only determines the optimum value of the system response which does not incorporate any limitations on the liquid displacement. Whereas, the displacement plays a very important role in design of the TLCBD as well as determining the response mitigation. Therefore, the constrained non-linear optimization is used to evaluate the function of TLCBD system. The peak value of the liquid displacement is related to \(\sigma_x\) (denoted as \(D_x\)) [20], \(D_x = \beta \sigma_x\).

In which, the peak factor \(\beta\) is expressed as follows:
\[
\beta = \sqrt{2 \ln(\eta)} + \frac{0.577}{\sqrt{\ln(\eta)}}
\]  
(15)
where, the term \(\eta\) is defined as \(\frac{\sigma_x}{\sigma_x}\); also \(\sigma_x\) can be found in Equation (13) and the duration of the ground motion is denoted by \(T\). By incorporating the peak liquid displacement constrained optimization procedure can be described like so,

Find \(b_0\) for minimizing \(f_0 = \sigma_x\) such that \(D_x \leq d\)
\]  
(16)
where \(d\) is the maximum permissible displacement allowed for the liquid.

4. NUMERICAL STUDY

The structure-TLCBD arrangement shown in Figure 1 has been considered to evaluate the behavior of the TLCBD under constrained liquid movement subjected to stochastic earthquake excitation. In this numerical study, the particular focus is to observe the constrained criteria effect on TLCBD performance and the optimum parameters responsible for that.

The different allowable displacements ranging from 500 mm to 1100 mm are considered here for comparing constrained and unconstrained response cases of structure-TLCBD system. For the present numerical study the following values are considered as follows, time period of the structure = 1.3 sec, \(\mu = 3\%\), \(\xi_s = 2\%\), \(\rho_{21} = 1000\) Kg/m\(^3\), \(\rho_{12} = 7500\) Kg/m\(^3\), \(\nu = 0.001\) Nm/sec, \(\omega_g = 9\pi\) rad/sec, \(T = 20\) sec, \(\xi_g = 0.6\), \(p = 0.75\), \(S_0 = 0.03\) m\(^2\)/sec\(^3\). The primary structure is having RMSD of 15.03 cm without any TLCBD attached.

The variation of optimized \(r\) and \(R_{12}\) with the changing mass ratio for various allowable displacement considered have been plotted in Figures 2 and 3, respectively.

From the figure it can be seen, for higher constraining effect the changing in values of optimum parameters is
The deviation of optimized tuning ratio, optimized ball tube diameter ratio, and corresponding RMSD has been plotted in Figures 5 to 7 for various $\xi_{3s}$ considering the various allowable displacements. The same pattern for optimum $\gamma$ and $R_{12}$ is followed as the mass ratio case.

It is clearly visible that the displacement of the structure decreases with an increasing damping ratio and for higher damping ratio lesser displacement of liquid is needed to achieve the same efficiency.

**Figure 2.** The variation of optimized tuning ratio for different mass ratios with varying allowable displacement

**Figure 3.** The variation of optimized ball tube diameter ratio for different mass ratios with varying allowable displacement

significant as constraining make the optimizing case more inflexible. Also, it is noticeable that the effect of the optimum $R_{12}$ effect is more compared to the optimum $\gamma$. With the variation of $\mu$ the response of the structure in terms of RMSD is presented in Figure 4. The higher the mass ratio the more is the overlapping tendency of RMSD for lower values of allowable liquid displacement. For any particular mass ratio higher response of the structure is noticeable when the allowable displacement is less and, on the other hand, as the allowable displacement increases the effect for the constrained optimization declines showing the lesser amount of response of the primary structure.

**Figure 4.** The RMSD with varying allowable displacement for various mass ratios

**Figure 5.** The variation of optimized tuning ratio for different damping ratios of structure with varying allowable displacement

**Figure 6.** The variation of optimized ball tube diameter ratio for different damping ratios of structure with varying allowable displacement

**Figure 7.** The RMSD with varying allowable displacement for different $\xi_{3s}$
In the case of a structure with a lower damping ratio, subjected to a strong magnitude earthquake the response generated cannot get mitigated by the structures by its own damping properties. A properly designed external damping device is needed for this purpose. TLCBD with appropriate constrained liquid height can be a better option, applicable to mitigate such responses for flexible structures with low damping property.

For the variation of allowable displacement with the different length ratios, the associated RMSD is shown in Figure 8. With an increase in length ratio, higher efficiency can be achieved for TLCBD as the mass participating in the reduction of the response gets improved.

In the case of designing liquid dampers, it is already established that the more liquid mass involved, the more the response mitigation property enhanced. By restricting the liquid movement in the vertical column, the mass involved for effective damping properties can be increased. The effect of various $S_0$ on the primary structure represented as RMSD is plotted in Figure 9. RMSD of the structure is obviously less for the lower seismic excitation and the overlapping effect of constraint case with that of unconstraint case increases with a higher level of seismic excitation.

In this context, it is important to restrict the liquid movement, as prior knowledge of the intensity of the earthquake cannot be accurately predicted in which the liquid movement can exceed the total height of the tube and also the unlimited height of the liquid column cannot be practically provided. Therefore, for the proper design of TLCBD the information of RMSD for constraining liquid movement in higher intensity of an earthquake is indeed a well-required criterion.

5. CONCLUSION

In the present work, an investigation has been done to evaluate the optimum performance of structure-TLCBD system for vibration mitigation considering constraint put by maximum liquid displacement in the tube, less than the allowable displacement. Here the allowable displacement is considered up to 1100 mm. By restraining the allowable liquid displacement in the vertical column of the TLCBD, the effect of constraint in the optimization procedure has shown particularly. The response reduction of the structure considering the constrained movement of the liquid is then compared with the response reduction achieved by the unconstrained optimization procedure i.e by SSO method. Various TLCBD, structure and excitation parameters have been considered to establish this investigation like length ratio, mass ratio, damping ratio of structure, PSD of the earthquake considered. The proposed constrained optimization of TLCBD results is in resemblance with the unconstrained optimization results. However, there is a slight variation in the optimum result when the maximum liquid displacement is considered in the optimization process. The maximum change in response is nearly 25% whereas the variation of optimized $\gamma$ and $R_{g2}$ are almost 5 and 8% respectively for limiting the average allowable displacement in the range of 800 mm which is quite high with respect to allowing the vertical tube. The response comparison value (almost 50%) is quite alarming with respect to the high seismic intensity (0.05 m²/s³) level. For lower values of allowable displacement, the optimum solution show higher variation as lower values make the constrained more inflexible. The optimal solution attempt to increase the $\gamma$ of the liquid for constraint condition whereas to pay off a loss in high tuning the increase in $R_{g2}$ is wanted in the optimization process. However, the constrained and unconstrained results get overlapped with the higher allowable displacement of the TLCBD as the constrained allowable displacement reaches to the unconstrained level. So, in general, it clearly indicates that constrained optimal results are found to be compromised than the relevant unconstrained values owing to the constraint effect tends to shrink the admissible search field of the

**Figure 8.** The RMSD with varying allowable displacement for different length ratios of TLCBD

**Figure 9.** The RMSD with varying allowable displacement for different $S_0$
design variables up to a certain allowable displacement of liquid. However, the effectiveness is not wholly reduced. But, it should ensure that the maximum liquid displacement cannot exceed beyond an allowable limit whereas the efficiency TLCBD system is risking by allowing air entering into the horizontal portion of the tube.

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