A finite element algorithm for solution of the natural vibration problem of electroelastic bodies with passive external electric circuits, interacting with a quiescent fluid

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Abstract. In this paper we consider a mathematical statement of the problem on natural vibrations of piecewise-homogeneous electroelastic bodies with passive external electric circuits (shunting circuits) of arbitrary configuration and interacting with a quiescent fluid. The behavior of the piezoelectric body is described using the equations of electrodynamics of deformable electroelastic media in the quasi-static approximation. The motion of an ideal fluid in the case of small perturbations is considered within the framework of the acoustic approximation. Small strains in a thin plate are determined using the Reissner – Mindlin theory. The numerical solution is developed using the finite element method. The proposed algorithm is based on the approach, in which the global stiffness matrix generated with the aid of the ANSYS software package is decomposed into required constituents. The system of governing equations is constructed using the developed algorithm, which is realized in the FORTRAN language. Complex eigenvalues of the examined system are defined from the solution of the non-classic modal problem using the Mueller method. A thin plate with piezoelectric element located on the free surface of a layer of a quiescent fluid of finite size is considered as an example.

1. Introduction
Nowadays, devices made of piezoelectric materials and connected to external electric circuits are widely used to control the dynamic behavior of different structures including structures interacting with a fluid. A detailed review of studies concerning their practical applications to submerged structures is given in [1]. The authors of the paper have briefly outlined and discussed the basic results related to such topics as modal analysis, active sound and vibration control, energy harvesting and atomic force microscopes. The obtained results are used in different areas from the military and oil industries to biology and medicine.

In a number of papers the authors consider a possibility of using piezoelectric elements for sound absorption of submerged structures [2–5]. The idea of the method is in piezoelectric coating or attaching a piezoelectric patch to the acoustic surface and connecting the piezoelectric layer to a passive electric RLC-circuit. In paper [2], different kinds of shunting circuits are investigated using the proposed one-dimensional electro-acoustic model. It has been shown that the use of resistors and inductances allows achieving good absorption of sound only for a narrow
bandwidth of frequency. However, to achieve broadband reduction of sound, a series-connected RC-elements should be used [2, 3]. In papers [4, 5] in order to expand a bandwidth of controlling frequencies, a multimodal vibration damping technique is applied. For this purpose, in [4] a multi-layered piezoelectric coating is used. Each layer is connected to a passive RL-circuit that provides sound absorption at a specified frequency. A one-dimensional electro-acoustic model based on Mason equivalent circuit theory and an iteration algorithm has been developed by the authors. Among other things, the sound absorption coefficient of a four-layered specimen was determined experimentally. The results obtained from the experiment qualitatively agree with the results of the finite element simulation performed in the ANSYS software package. However the difference between the experimental results and those obtained in the framework of one-dimensional model is quite noticeable. The use of several piezoelectric elements, each of them being located at a specified site and shunted to its own circuit, is a rather wide-spread practice in the multimodal vibration damping strategy. Instead of this option, in paper [5] only one piezoelectric patch is used in combination with a blocked shunt circuit, which leads to a significant reduction in transmitted noise for a multiple modes of the panel.

Although the above mentioned papers consider the interaction with an acoustic media the models used there are formulated in terms of electric impedance with the aid of simplified (equivalent) one-dimensional equations. It means that the majority of physical processes in elastic bodies, liquid or gas media are not taken into account. When analyzing behavior of structures interacting with fluid, it is often necessary to define the response of a damped system to external harmonic or arbitrary impacts. With this in mind, it is reasonable to suggest that this can be done only in the case of an adequate description of the hydroelastic and electroelastic interactions.

A control of dynamic deformation of structures and damping of dangerous vibrations of various nature can be accomplished using either active or passive techniques. In the first case, the electric potential produced by a piezoelectric element, acting as a sensor, is further processed and amplified, and then the control action is transferred to another piezoelectric element, which serves as an actuator. In the case when the passive technique is used, it is necessary to select the parameters and configuration of the passive external electric circuits such, that could provide maximum damping of undesirable vibrations, or to perform a frequency shift, so that the resonant frequencies of the structure do not fall into the range of dangerous frequencies.

In the literature, there are a lot of studies related to active structural vibration control of thin plates [6–8] and shells [9–11] interacting with a fluid. A finite element formulation of the dynamic problem for a laminated plate, incorporating both the piezoelectric and viscoelastic layers is presented in [6]. Here, it is assumed that the structure performs harmonic vibrations in an acoustic half-space. The influence of the half-space is taken into account using the Rayleigh surface integral. The possibility of vibration control with the aid of negative velocity feedback control algorithm was demonstrated by solving a number of example problems. In paper [7], the same author offers a somewhat different algorithm of feedback between the piezoelectric sensor and the actuator. This algorithm is based on the mode shapes obtained for a structure in vacuum and for a structure interacting with a fluid. The results of experimental and theoretical studies for a partially submerged vertical cantilever plate with piezoceramic sensors and actuators are presented in [8]. The vibration control is executed by applying MIMO (Multiple-Input Multiple-Output) positive position feedback controller. The frequency response curves and the values of natural vibration frequencies obtained numerically by the Rayleigh – Ritz method have been verified and have proved to provide a qualitatively correct description of experimental data.

In the problems of passive control of structural dynamic behavior with the aid of piezoelectric elements and passive electric circuits, a special attention is paid to the analysis of the influence of external electric circuits on the damping properties of the object under study. A level of decrease in the amplitude of forced vibrations or in the rate of damping of transient processes can be
considered as such properties. These data can be gained from the solution of the corresponding dynamic problems.

The analysis of the literature shows that comprehensive studies on the effective use of piezoelectric materials for control of dynamic behavior of complex multi-component objects and conclusions, summarizing the results of these studies, are impossible without mathematical modeling. In many papers, the finite element method is used for this purpose [12–14]. Some commercial software packages contain the relevant models and algorithms, which however are insufficient for covering all the requirements of researchers. As it was noted in [15], the modal analysis is the most efficient tool for constructing optimization algorithms due to rather low calculation costs. In the present paper, a previously developed approach [15] will be used to analyze the dynamic behavior of electroelastic structures connected to passive external electric circuits and interacting with a quiescent fluid. The distinguishing feature of this approach is solving the non-classic eigenvalue problem by the Mueller method programmatically realized in the FORTRAN language. Here the possibilities of ANSYS finite element software package are used for assembling the global matrices.

2. Mathematical statement of the problem

As an example, let us consider a structure, which is thin plate of length $L_s$, width $W_s$ and thickness $h_s$. This plate is located on the free surface of a layer of a quiescent fluid of finite dimensions $(L_f \times W_f \times H_f)$ (see Figure 1). The variational equation of motion of the combined body of volume $V = V_s + V_p$, consisting of the elastic plate of volume $V_s$ and the attached piezoelectric element of volume $V_p$, is formulated in terms of the relations of Reissner – Mindlin theory, linear theory of elasticity and quasi-static Maxwell equations [15–18]. The electroded coatings applied to the top ($S_{top}$) and the bottom ($S_{bot}$) parts of the surface of piezoelectric body are assumed to be ideal conductors with negligible mass. The final governing equation in the absence of body forces and free surface charges in the matrix form can be written as

$$
\int_{V_p} \left( \{\delta \varepsilon_p\}^T [D_p] \{\varepsilon_p\} - \{\delta \varepsilon_p\}^T [e] \{E\} - \{\delta E\}^T [e]^T \{\varepsilon_p\} - \{\delta E\}^T [\beta]^T \{E\} \right) dV + \text{(1)}
$$

Figure 1. Computational scheme for cantilever plate located on the free surface of quiescent fluid with attached piezoelectric element and series $RL$-circuit.
The following notations are accepted: column vectors of strain tensors and are defined according to the well-known relations [16, 17]; \( \{ \varepsilon \} \), \( \{ \beta \} \) are the matrices of piezoelectric and dielectric coefficients; \( \{ E \} \) is the electric field intensity vector; \( \{ u_p \} \) is the vector of piezoelectric element displacements; \( \{ u_s \} \) is the generalized displacement vector for the plate, which includes rotation angles; \( \rho \) is the material density; \( \{ t_s \} \) is the vector of the surface traction caused by the interaction with the fluid on the part of the plate surface \( S_s \).

The potentiality condition is fulfilled for an electric field

\[ -\{ E \} = \nabla \varphi, \tag{2} \]

where \( \varphi \) is the electric potential.

The bottom electroded surface of the piezoelectric element \( S_{bot} \) is grounded and has a zero-value electric potential. The corresponding boundary condition can be written as \( \varphi = 0 \). On the electrode-free parts of the surface of piezoelectric body there are no free electric charges. When an external voltage supply is absent, the other parts of the electrode surface are considered free. In this case, the open circuit mode \( (o/c) \) is realized. On the other hand, it can be treated as having zero-value electric potential, in which case the short circuit mode is realized \( (s/c) \).

By using the electroded surfaces, the external electric circuits of an arbitrary configuration, including resistive \( (R) \), capacitive \( (C) \) or inductive \( (L) \) elements, can be attached to the surface of the system under consideration. If these circuits are not supplied by an external power, they are classed as the internal elements of the system and then the following term should be added to equation (1) [15]:

\[
\sum_{p=1}^{nL} \delta A_{L_p} + \sum_{q=1}^{nR} \delta A_{R_q} + \sum_{r=1}^{nC} \delta A_{C_r} = \sum_{p=1}^{nL} \frac{1}{L_p} \int \int \left( \varphi_1^{L_p} - \varphi_2^{L_p} \right) \delta \varphi \, dt \, dt + \sum_{r=1}^{nC} \frac{1}{C_r} \int \int \left( \varphi_1^{C_r} - \varphi_2^{C_r} \right) \delta \varphi \, dt \, dt + \sum_{r=1}^{nC} \frac{1}{C_r} \int \int \left( \varphi_1^{C_r} - \varphi_2^{C_r} \right) \delta \varphi, \tag{3} \]

where \( n_L, n_R, n_C \) are the quantities of the inductive, resistive and capacitive elements, respectively; \( L_p, R_q, C_r \) are the values of the inductance, resistance and capacitance of the corresponding circuit elements; \( \varphi_1^{cir} - \varphi_2^{cir} \) is the potential difference in the corresponding circuit element \( (cir = L_p, R_q, C_r) \).

The small amplitude vibrations of an ideal compressible fluid in the region \( V_f \) is described by the well-known Euler equations, continuity equation and state equation. The elimination of velocity from them gives the Helmholtz equation governing the hydrodynamic pressure \( p \)

\[ \nabla^2 p = \ddot{p}/c^2, \tag{4} \]

where \( c \) is the speed of sound in the fluid.

In the proposed mathematical model, we assume that sloshing of the free surface of the fluid \( (S_{free}) \) is absent. The corresponding boundary condition is written as \( p = 0 \). At the interface between the liquid and rigid wall \( S_g \), the normal velocity of the fluid is equal zero.

At the fluid-structure interface the pressure must satisfy the boundary condition

\[ \frac{\partial p}{\partial n} = -\rho_f \ddot{w}, \tag{5} \]
where \( n \) denoting the normal to the fluid region, \( w \) is the normal displacement of the plate, \( \rho_f \) is the density of the fluid.

Equation (4) together with the boundary conditions and the impermeability condition (5) are converted to a weak form using the Bubnov – Galerkin method [19]. Finally, we have

\[
\int_{V_f} \delta p \left( \frac{1}{c^2} \dddot{p} + \nabla^2 p \right) dV + \rho_f \int_{S_o} \delta \dot{w} dS = 0. \tag{6}
\]

To take into account the fluid-structure interaction, the traction integral in equation (1) can be expressed as

\[
\int_{S} \sigma \{ \delta u_s \}^T \{ t_s \} dS = \int_{S} \sigma \{ \delta u_s \}^T \{ n_f \} p dS, \tag{7}
\]

where \( \{ n_f \} \) is the vector of outward normal to the fluid region. We suppose that positive pressure is defined in compression and \( \{ n_s \} = -\{ n_f \}. \)

Let us consider a perturbed motion of the fluid and the plate with the attached piezoelectric element defined as

\[
U(x, t) = \{ u_p(x, t), \varphi(x, t), u_s(x, t), p(x, t) \} = \tilde{U}(x) e^{i\lambda t}, \quad x = \{ x, y, z \}, \tag{8}
\]

where \( \tilde{U}(x) \) is the function of coordinates, \( \lambda = \omega + i\gamma \) the characteristic parameter, \( \omega \) corresponds to the circular natural frequency of vibrations and \( \gamma \) is the rate of its damping and \( i = \sqrt{-1}. \)

Taking into account the above mentioned form of the solution, instead (1), (3) and (6) we get

\[
\int_{V_p} \left\{ \delta \varepsilon_p \right\}^T \left[ D_p \right] \left\{ \varepsilon_p \right\} dV - \lambda^2 \int_{V_s} \left\{ \delta u_s \right\}^T \rho_s \left\{ \dddot{u}_s \right\} dV + \int_{S_s} \left\{ \delta \varepsilon_s \right\}^T \left[ D_s \right] \left\{ \varepsilon_s \right\} dS - \lambda^2 \int_{V_p} \left\{ \delta u_p \right\}^T \rho_p \left\{ \dddot{u}_p \right\} dV - \int_{S_o} \left\{ \delta u_s \right\}^T \rho_s \left\{ \dddot{u}_s \right\} dS - \lambda^2 \int_{S_o} \delta p \left\{ n_f \right\} dS - \sum_{k=1}^{N_L} \frac{1}{L_k} \left( \varphi_{1k}^L - \varphi_{2k}^L \right) \delta \varphi + \sum_{q=1}^{N_R} \frac{1}{i\lambda R_q} \left( \varphi_{1q}^R - \varphi_{2q}^R \right) \delta \varphi + \sum_{r=1}^{N_C} C_r \left( \varphi_{1r}^C - \varphi_{2r}^C \right) \delta \varphi = 0, \tag{9}
\]

\[
\int_{V_f} \delta p \left( \frac{1}{c^2} \dddot{p} + \nabla^2 p \right) dV - \lambda^2 \rho_f \int_{S_o} \delta \dot{w} dS = 0. \tag{10}
\]

3. Finite element formulation
Using standard procedures of the finite element method we can convert the problem of the natural vibrations of electroelastic bodies with an external circuit interacting with quiescent fluid, which is described by equations (9)–(10), to the following algebraic eigenvalue problem:

\[
\left( -\lambda^2 \left[ M \right] + \left[ C(\lambda) \right] + \left[ K \right] \right) \left\{ \tilde{U} \right\} = 0, \tag{11}
\]
where

\[
M = \begin{bmatrix}
M_p & 0 & 0 & 0 \\
0 & M_p & 0 & 0 \\
0 & 0 & M_s & 0 \\
0 & 0 & \rho f Q^T & M_f
\end{bmatrix}, \quad C(\lambda) = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & C_\varphi(\lambda) & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad K = \begin{bmatrix}
K_p & K_p \varphi & 0 & 0 \\
(K_p \varphi)^T & K_\varphi + K_\varphi^C & 0 & 0 \\
0 & 0 & K_s & -Q \\
0 & 0 & 0 & K_f
\end{bmatrix},
\]

\[
[C_\varphi(\lambda)] = -\sum_{p=1}^{n_f} \frac{1}{\lambda^2 L_p} [C_\varphi^{Lp}] + \sum_{q=1}^{n_p} \frac{1}{i\lambda R_q} [C_\varphi^{Rq}], \quad [K_\varphi^C] = \sum_{r=1}^{n_c} C_r [K_\varphi^{C_r}].
\]

Here, the subscripts “f” and “ϕ” refer to the corresponding variables to the fluid and the electric potential, \([Q]\) is the fluid-structure interaction matrix [19], and all zeroes are matrices also. The matrices \([C_\varphi^{Lp}], [C_\varphi^{Rq}], [K_\varphi^C]\) contain the coefficients “1” and “−1” only in positions that refer to the nodal variables of the corresponding elements of electric circuit. The remaining typical matrices of individual finite elements are determined by the known manner [19, 20]. Discretization of the fluid, piezoelectric element and plate computational domains is based on the spatial 20-node brick elements and the 8-node flat rectangular finite elements with a quadratic approximation of the unknown variables.

Equation (11) essentially differs from the generalized eigenvalue problem due to the presence of the matrix \([C_\varphi(\lambda)]\), which describes the external electric circuit. The solution was performed using the capabilities of the ANSYS software package for generating the finite element matrices. We employ the program written in FORTRAN language for assembling the system of governing equations (11) using the sparse matrix technology for basic operations and computation of complex eigenvalues by the Mueller method [15].

4. Approbation of the algorithm

In this Section using the obtained results we demonstrate the reliability of computing the complex natural vibration frequencies of the examined system on the basis of the developed algorithm. In order to apply the obtained results to future studies, the numerical simulation was performed using the geometry and characteristics of the real experimental setup. The fluid volume was a rectangular prism 200 mm long, 150 mm wide and 195 mm high. In view of the specific features of the excitation device, a light neodymium magnet weighing 0.10 g was glued to the plate surface at a distance of 66 mm from the clamped edge of the plate and 3 mm away from its long side. It was modeled as a mass element, which was defined at a single node with the specified concentrated mass and rotational inertia.

The following characteristics were used in the numerical simulations: plate (duraluminium alloy) — Young’s modulus \(E = 68.5\) GPa, Poisson’s ratio \(\nu = 0.3\), \(\rho_s = 2714\) kg/m³; fluid (water) — \(c = 1500\) m/s, \(\rho_f = 1000\) kg/m³; and piezoelectric element was made of PZT-19 piezo-ceramics (lead zirconate-titanate), polarized along the \(z\)-axis, with the following physical and mechanical properties: \(C_{11} = C_{22} = 10.9 \times 10^{10}\) Pa, \(C_{13} = C_{23} = 5.4 \times 10^{10}\) Pa, \(C_{12} = 6.1 \times 10^{10}\) Pa, \(C_{33} = 9.3 \times 10^{10}\) Pa, \(C_{44} = C_{55} = C_{66} = 2.4 \times 10^{10}\) Pa, \(\beta_{13} = \beta_{23} = -4.9\) C/m², \(\beta_{33} = 14.9\) C/m², \(\beta_{01} = \beta_{02} = 10.6\) C/m², \(\epsilon_{11} = \epsilon_{22} = 8.2 \times 10^{-9}\) F/m, \(\epsilon_{33} = 8.4 \times 10^{-9}\) F/m, \(\rho_s = 7500\) kg/m³.

The parameters of the finite element mesh, which is used for subsequent calculations, are determined from the analysis of asymptotical convergence of the solution depending on the number of nodal unknowns. During this research the maximal relative difference between the first eleven complex eigenvalues was evaluated for different degree of discretization. The option was considered as acceptable, if the obtained results differed from the previous ones by less than 1% with a significant increase in the number of unknowns. In our example, the finite element model was defined by 50623 degrees of freedom.

A series of numerical calculations were carried out to verify the validity of the results obtained with the use of the proposed algorithm. For these computations a series RL-circuit was connected
to the electrodes of piezoelectric element (Figure 1). The case when both parameters of the inductance and resistance are set negligibly small (of the order of magnitude of about $10^{-15}$), is equivalent to the case when the piezoelectric element operates in the short circuit mode. And vice versa, extremely high values of both parameters of inductance and resistance (of the order of magnitude of about $10^{15}$), corresponds to the case when the piezoelectric element operates in the open circuit mode. This peculiarity allows us to make a comparison between the results obtained with the help of the proposed algorithm and the results obtained with the use of the ANSYS software. To determine natural vibration frequencies in the calculations done in the ANSYS package, the open circuit and short circuit modes were realized by setting the corresponding boundary conditions for electric potential.

Table 1. Natural vibration frequencies of the plate with piezoelectric element connected to the series $RL$-circuit and located on the free surface of quiescent fluid.

| $\omega$ | Short circuit mode | Open circuit mode |
|----------|--------------------|-------------------|
|          | ANSYS | Proposed solution | ANSYS | Proposed solution |
| $\omega_1$ | 19.9329 | 19.9329 | 19.8982 | 19.8982 |
| $\omega_2$ | 122.8790 | 122.8793 | 122.8008 | 122.8008 |
| $\omega_3$ | 346.0930 | 346.0931 | 346.0619 | 346.0619 |
| $\omega_4$ | 385.7038 | 385.7038 | 385.7038 | 385.7038 |
| $\omega_5$ | 684.8710 | 684.8710 | 684.8572 | 684.8572 |

Table 1 show the first five natural vibration frequencies for the plate with the attached piezoelectric element connected to the series $RL$-circuit and interacting with a quiescent fluid (Figure 1). The parameters of the electric circuit correspond to the piezoelectric element operating in the open circuit ($L = 10^{15}$ H, $R = 10^{15}$ $\Omega$) and short circuit ($L = 10^{-15}$ H, $R = 10^{-15}$ $\Omega$) modes. In should be noted that the eigenvalues of the system of equations (11) are purely real in both cases, so $\lambda = \omega$. The data presented in Table 1 show that the results obtained in the ANSYS and on the basis of the proposed algorithm differed by less than 0.02%.

Table 2. Complex eigenfrequencies of the plate with piezoelectric element connected to the series $RL$-circuit ($L = 3000$ H, $R = 2000$ $\Omega$) and located on the free surface of quiescent fluid.

| $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ |
|-------------|-------------|-------------|-------------|-------------|
| $\omega$    | 19.2269     | 20.4141     | 122.8814    | 346.0932    | 385.7038    |
| $\gamma$    | 0.0312      | 0.0219      | 0.0000      | 0.0000      | 0.0000      |

The solution of the modal problem was performed using the proposed algorithm with following values of parameters for the external electric circuit: $L = 3000$ H, $R = 2000$ $\Omega$. In this case the natural vibration frequency of the external electric circuit is incorporated into initial spectrum of the system (Table 1). If the values of parameters of external electric circuit is chosen in such a way that incorporated frequency turns out to be equal to the certain mode of vibrations, then this vibration mode can be damped. The values of the first five complex natural vibration frequencies for the examined system are presented in Table 2. Verification of the solution for this case was done by comparing the real parts of the calculated eigenvalues with peaks of the frequency response curve obtained using the module of harmonic analysis of the ANSYS software.
Figure 2. Frequency response curve for cantilever plate located on the free surface of quiescent fluid with attached piezoelectric element and series RL-circuit.

...package. For this purpose the problem of forced harmonic vibrations of the plate with attached piezoelectric element with connected external series RL-circuit and interacting with a quiescent fluid was considered. The excitation of vibrations was realized by applying the harmonic force \( \{F\} = \{0, 0, -1\} \) to the point B of the structure (Figure 1). Figure 2 shows frequency response curve of the normal component \( w \) of the displacement vector of the plate for the point A. The locations of the peaks obtained in the vicinity of a resonance correspond to the values of \( \omega \) presented in Table 2 with the tolerance up to 0.1% for the frequency step equal to 0.01 Hz.

It should be noted that the incorporated frequency is presented on the frequency response curve (Figure 2). The new resonance peak appears at the frequency 19.22 Hz. Herewith the first natural vibration frequency of the system changes its value from 19.9329 Hz to 20.4141 Hz (Table 2). Figure 2 demonstrates that resonance peaks \( \omega_1 \) and \( \omega_2 \) have finite amplitude. This fact evidences the presence of damping in the system. It corresponds to the appearance of imaginary part for these complex eigenvalues when the modal problem is solved (Table 2).

5. Conclusions

A mathematical formulation of the natural vibration problem for a piecewise-homogeneous electroelastic body connected to a shunt external electric circuit and interacting with a quiescent fluid has been proposed. The process of obtaining the problem solution was based on the finite element method. The possibilities of the ANSYS software package were used for generating global matrices. The final system of governing equations was assembled based on the developed algorithm written in FORTRAN language. The sparse matrix technology was used for basic operations. The solution of non-classic eigenvalue problem was found using the Mueller method.

A thin rectangular plate located at the free surface of the fluid with an attached piezoelectric element connected to the external series RL-circuit was considered as an illustrative example. The functionality of the developed program and the reliability of numerical results were verified by comparing the obtained eigenvalues with the ones computed with the aid of the ANSYS package for two extreme cases of the piezoelectric element operation.

A mathematical statement of the modal problem proposed in this paper has no analogues in modern commercial software packages for finite element modeling. This formulation allows us to determine natural vibration frequencies and corresponding rates of their damping regardless of the form of the exciting impact. In this regard, the problem being considered can be viewed as an efficient tool for constructing the optimization algorithms, which can be readily applied to problems of passive damping of hydroelastic vibrations.
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