ABSTRACT

In this paper, we would like to disseminate a serendipitous discovery involving Lyapunov exponents of a 1-D time series and their use in serving as a filtering defense tool against a specific kind of deep adversarial perturbation. To this end, we use the state-of-the-art CleverHans library to generate adversarial perturbations against a standard Convolutional Neural Network (CNN) architecture trained on the MNIST as well as the Fashion-MNIST datasets. We empirically demonstrate how the Lyapunov exponents computed on the flattened 1-D vector representations of the images served as highly discriminative features that could be to pre-classify images as adversarial or legitimate before feeding the image into the CNN for classification. We also explore the issue of possible false-alarms when the input images are noisy in a non-adversarial sense.

Keywords

Deep learning, adversarial attacks, transfer learning

1. BACKGROUND ON DEFENSES AGAINST ADVERSARIAL ATTACKS

In the recent past, a plethora of defenses against adversarial attacks have been proposed. These include SafetyNet [13], adversarial training [21], label smoothing [22], defensive distillation [19] and feature-squeezing [24, 25] to name a few. There is also an ongoing Kaggle contest [2] underway for exploring novel defenses against adversarial attacks.

As evinced by the recent spurt in the papers written on this topic, most defenses proposed are quelled by a novel attack that exploits some weakness in the defense. In [10], the authors queried if one could concoct a strong defense by combining multiple defenses and showed that an ensemble of weak defenses was not sufficient in providing strong defense against adversarial examples that they were able to craft.

In the accompanying blog associated with [17], the authors Goodfellow and Papernot posit that Adversarial examples are hard to defend against because it is hard to construct a theoretical model of the adversarial example crafting process and contend with the idea if attacking machine learning easier than defending it?

With this background, we shall now look more closely at a specific type of defense and motivate the relevance of our method within this framework.

1.1 The pre-detector based defenses

One prominent approach that emerges in the literature of adversarial defenses is that of crafting pre-detection and filtering systems that flag inputs that might be potentially adversarial. In [9], the authors posit that adversarial examples are not drawn from the same distribution as the legitimate samples and can thus be detected using statistical tests. In [15, 7], the authors train a separate binary classifier to first classify any input image as legitimate or adversarial and then perform inference on the passed images. In approaches such as [6], the authors assume that DNNs classify accurately only near the small manifold of training data and that the synthetic adversarial samples do not lie on the data manifold. They applying dropout at test time to ascertain the confidence of adversariality of the input image.

In this paper, we would like to disseminate a model-agnostic approach towards adversarial defense that is dependent purely on the quasi-time-series statistics of the input images that was discovered in a rather serendipitous fashion. The goal is to not present the method we propose as a fool-proof adversarial filter, but to instead draw the attention of the DNN and CV communities towards this chance discovery that we feel is worthy of further inquiry. In order to facilitate reproducibility of results and effective criticism, we have duly open-sourced the implementation as a well annotated jupyter-notebook shared at the following location: https://github.com/vinayprabhu/Lyapunov_defense

2. PROCEDURE FOLLOWED

2.1 A quick introduction to Lyapunov exponents

For a given a scalar time series \( \{ x_t; t = 1, ..., n \} \) whose time evolution is assumed to be modeled by a differentiable
dynamical system (in a phase-space of possibly infinite dimensions), we can define Lyapunov exponents corresponding to the large-time behavior of the system. These characteristic exponents numerically quantify the sensitivity to the initial conditions and emanate from the Ergodic theory of differentiable dynamical systems, introduced by Eckmann and Ruelle in [3][4]. Simply put, if the initial state of a time series is slightly perturbed, the characteristic exponent (or Lyapunov exponents) represent the exponential rate at which the perturbation increases (or decreases) with time. For a detailed understanding of these characteristic exponents, we would like to refer the user to [3][4].

Given a finite-length finite-precision time sequence sequence, the 3 stage Embed-Tangent maps-QR decomposition based recipe introduced in [4] can be used to numerically compute the Lyapunov exponents. This numerical recipe is implemented in many time-series analysis packages such as [1]. The background of the serendipitous discovery was that we were investigating usage of metrics from time-series analysis literature to identify adversarial attacks on 1-D axis-wise mobile-phone motion sensor data example, Accelerometer and gyroscope (see [20]) and we realized that this procedure could be easily extended to vectorized images (or flattened images viewed as discrete time series indexed by the pixel location. That is, given a normalized image, \( X \in [0,1]^{n \times n} \), we would have the flattened time-series representation \( x \in [0,1]^n \) as simply,

\[
x_{in+j} = X_{i,j}; \quad i, j = 0, \ldots n - 1;
\]

3. EXPERIMENTS

In this paper, we used the MNIST [11] dataset. In the appendix, we showcase similar results obtained with the Fashion-MNIST dataset [23] as well. The rest of this section details the different stages involved in our experimentation.

3.1 Generating targeted adversarial perturbations

We targeted 10 random samples from the MNIST dataset (1 belong to each class/digit) using the Carlini-Wagner-L2 attack [3] in the targeted mode implemented in the CleverHans library [17] with the following parameterization:

\[
cw\_params = \{'binary\_search\_steps': 1, \\
'y\_target': adv\_ys, \\
'max\_iterations': attack\_iterations, \\
'learning\_rate': 0.1, \\
'batch\_size': source\_samples \* nb\_classes, \\
'initial\_const': 10\} \\
\]

The resultant image grid containing the images along with the adversarially perturbed counterparts are as shown in fig 1(a). Fig 1(b) showcases the norm-distance between the images and their adversarial examples.

3.2 Computing Lyapunov exponents of the flattened images

The lyapunov exponents of the flattened images showcased in fig 1(a) were numerically computed with the `lyap_e()` method implemented in [1] with the following parameterization:

\[
\text{PARAM_MNIST} = \{\text{emb\_dim}=10, \text{matrix\_dim}=4, \\
\text{min}\_nb=\text{min}(2 \* \text{matrix\_dim}, \text{matrix\_dim} + 4), \\
\text{min}\_tsep=0, \text{tau}=1\} \\
\]

An example images along with its flattened quasi-time-series representation and the computed Lyapunov exponents is shown in fig 1(a). Notice the change of sign of the lyapunov exponents between the true image and the adversarial images. In time-series analysis, the existence of at least one positive Lyapunov exponent is interpreted as a strong indicator for chaos. As we introduce more adversarial noise, the prevalence of positive Lyapunov exponents becomes more prevalent. Fig 4 contains an example of an image (digit-5) and all its 9 associated adversarially perturbed counterparts (each targeting a different class) and their lyapunov exponents.

3.3 Unsupervised clustering of input images based on Lyapunov exponents

In fig 4(b) where we have showcased the scatter-plot of the first 2 lyapunov exponents, we see clear clustering between the legitimate examples and the adversarial examples. We see that the only adversarial examples that are inside the cluster of legitimate examples are those where the target class is the same as the true class. (These are images whose every element is just a fixed \( \varepsilon \) offset from the true image)

3.4 Supervised classification

Detection can also be performing from the viewpoint of outlier detection where we train a one-class classifier (ex: Isolation-Forest [12]) with just the inlier (legitimate) images’ lyapunov exponents used during training. Fig 3(a) shows the heatmap of the features in the train and test subsets (200: Training samples 300 Test samples of which 200 are legitimate and 100 are adversarial). We received 100% attacker rejection rate and a 83.5% true acceptance rate (or a 16.5% False-alarm rate) with no feature engineering or hyper-parameter tuning. Fig 3(b) showcases the scatter-plot of the train and test samples along with the decision boundaries found by the isolation forest algorithm.

3.5 Effect of non-adversarial noise on the detection performance

In order to ascertain the effect of non-adversarial noise on the detection performance, we chose the following noise models:

\[
[\text{'gaussian'}, \text{'pepper'}, \text{'poisson'}, \text{'salt'}, \\
\text{'Salt and Pepper'}, \text{'Speckle'}, \text{'local-variance Gaussian'}] \\
\]

(Here ‘local-variance Gaussian’ refers to Gaussian-distributed additive noise with specified local variance at each pixel of the image).

The effect of these noise-models on the image and its lyapunov exponents are shown in fig 2. As seen, the ‘local-variance Gaussian’ noise has the most discernible effect on the lyapunov exponents and hence is chosen as the model of choice to perturb the legitimate images with.

3.6 Augmenting the dataset with non-adversarial perturbed images

\[\text{http://scikit-image.org/docs/dev/api/skimage.util.html}\]
(a) Image grid with legitimate MNIST images and their adversarial counterparts

(b) Norm distances for the images in the grid (between legitimate F-MNIST images and their adversarial counterparts)

Figure 1: In the left sub-figure above, the \([i, j]^{th}\) grid image is an image belong to the \(j^{th}\) class adversarially perturbed to be classified as belonging to the \(i^{th}\) class.

Original image

\([-0.15204129 -1.18814337 -1.70479167 -1.75485373]\)

gaussian:
\([0.28242487 0.07765591 -0.06113432 -0.38037845]\)

pepper:
\([0.02092013 -0.96793717 -1.33508384 -1.61110401]\)

poisson:
\([-0.14193536 -1.17601728 -0.98012441 -2.26296401]\)

salt:
\([0.09110281 -0.44300380 -1.15924442 -1.60302079]\)

s&p:
\([0.05007739 -0.78992772 -1.35534716 -1.78148997]\)

speckle:
\([-0.27073824 -1.11521339 -1.52682781 -2.10146165]\)

localvar:
\([0.28221309 0.07588112 -0.07270029 -0.25669238]\)

Figure 2: The images of an example image sample (5) along with its noisy counterparts and their Lyapunov coefficients.

We added the training datasets with 100 noisy images (with Gaussian perturbations) and re-train the isoforest classifier. The results are as shown in fig 3.5. We see that the false alarm rate drops to 15% while the attacker rejection rate is almost unchanged (only 1 attacker image is accepted in).

We further observe that when the magnitude of random noise is selected from the distribution of norm distances among adversarial perturbations, the classifier retains its ability to distinguish between adversarially perturbed and randomly perturbed images.

3.7 Testing the defense with other attacks

We consider whether the Lyapunov method can be used
3.7 Testing the effect of a new attack

Table 1: True positive rates for several targeted and untargeted attacks.

| Attack          | Targeted/Untargeted | 2D | 4D |
|-----------------|---------------------|----|----|
| Carlini Wagner  | Targeted            | 0.90| 0.91|
|                 | Untargeted          | 1.0 | 1.0 |
| FGSM            | Targeted            | 0.0 | 0.62|
|                 | Untargeted          | 0.0 | 0.8 |
| JSMA            | Targeted            | 0.03| 0.04|
|                 | Untargeted          | 0.03| 0.04|
| Madry et al.    | Targeted            | 0.0 | 1.0 |
|                 | Untargeted          | 0.0 | 1.0 |
| DeepFool        | Targeted            | NA  | NA |
|                 | Untargeted          | 1.0 | 1.0 |

Table 1: True positive rates for several targeted and untargeted attacks.

4. CONCLUSION AND FUTURE WORK
Via this paper, we have sought to disseminate a serendipitous discovery entailing usage of lyapunov exponents as a model-agnostic tool that can be used to pre-filter input images as potentially adversarially perturbed. We have shown the validity of the idea defending against images that were adversarially perturbed using the Carlini-Wagner-\$l_2\$ attack procedure across 2 datasets, namely MNIST and fashion-MNIST.

We have used the latest version of CleverHans library (version: 2.0.0-7f7f9b18a1988bf6d37d5c4deabae6) and have open-sourced the code to ensure repeatability of the results presented here.

We are currently investigating its potential across various other datasets and attacks.

![Image](image1.png)

(a) An image (class-5- MNIST), its flattened quasi-time-series representation obtained by flattening

![Image](image2.png)

(b) 2-D PCA embedding of the lyapunov exponents for the 100 images from the MNIST dataset shown in Fig 1(a)

Figure 4: Quasi-time-series representations of an example image and their Lyapunov exponents

5. REFERENCES

[1] https://github.com/cschoel/nolds. 2017.
[2] https://www.kaggle.com/c/nips-2017-defense-against-adversarial-attack 2017.
[3] N. Carlini and D. Wagner. Towards evaluating the robustness of neural networks. In Security and Privacy (SP), 2017 IEEE Symposium on, pages 39–57. IEEE, 2017.
[4] J.-P. Eckmann, S. O. Kamphorst, D. Ruelle, and S. Ciliberto. Liapunov exponents from time series. Physical Review A, 34(6):4971, 1986.
[5] J.-P. Eckmann and D. Ruelle. Ergodic theory of chaos and strange attractors. Reviews of modern physics, 57(3):617, 1985.
[6] R. Feinman, R. R. Curtin, S. Shintre, and A. B. Gardner. Detecting adversarial samples from artifacts. arXiv preprint arXiv:1703.00410, 2017.
[7] Z. Gong, W. Wang, and W.-S. Ku. Adversarial and clean data are not twins. arXiv preprint arXiv:1704.04960, 2017.
[8] I. J. Goodfellow, J. Shlens, and C. Szegedy. Explaining and Harnessing Adversarial Examples. ArXiv e-prints, Dec. 2014.
[9] K. Grosse, P. Manoharan, N. Papernot, M. Backes, and P. McDaniel. On the (statistical) detection of adversarial examples. arXiv preprint arXiv:1702.06280, 2017.
[10] W. He, J. Wei, X. Chen, N. Carlini, and D. Song. Adversarial example defenses: Ensembles of weak defenses are not strong. arXiv preprint arXiv:1706.04704, 2017.
[11] Y. LeCun. The mnist database of handwritten digits. http://yann.lecun.com/exdb/mnist/, 1998.
[12] F. T. Liu, K. M. Ting, and Z.-H. Zhou. Isolation forest. In Data Mining, 2008. ICDM’08. Eighth IEEE International Conference on, pages 413–422. IEEE, 2008.
[13] J. Lu, T. Issaranon, and D. Forsyth. Safetynet: Detecting and rejecting adversarial examples robustly. arXiv preprint arXiv:1704.00103, 2017.
[14] A. Madry, A. Makelov, L. Schmidt, D. Tsipras, and A. Vladu. Towards Deep Learning Models Resistant to Adversarial Attacks. ArXiv e-prints, June 2017.
[15] J. H. Metzen, T. Genewein, V. Fischer, and B. Bischoff. On detecting adversarial perturbations. arXiv preprint arXiv:1702.04267, 2017.
[16] S.-M. Moosavi-Dezfooli, A. Fawzi, and P. Frossard. DeepFool: a simple and accurate method to fool deep neural networks. ArXiv e-prints, Nov. 2015.
[17] I. G. e. a. Nicolas Papernot, Nicholas Carlini, cleverhans v2.0.0: an adversarial machine learning library. arXiv preprint arXiv:1610.00768, 2017.
[18] N. Papernot, P. McDaniel, S. Jha, M. Fredrikson, Z. Berekay Celik, and A. Swami. The Limitations of Deep Learning in Adversarial Settings. ArXiv e-prints, Nov. 2015.
[19] N. Papernot, P. McDaniel, X. Wu, S. Jha, and A. Swami. Distillation as a defense to adversarial perturbations against deep neural networks. In Security and Privacy (SP), 2016 IEEE Symposium on, pages 582–597. IEEE, 2016.
[20] V. U. Prabhu and J. Whaley. Vulnerability of deep learning-based gait biometric recognition to adversarial perturbations. In CVPR Workshop on The Bright and Dark Sides of Computer Vision: Challenges and Opportunities for Privacy and Security (CV-COPS 2017), 2017.

[21] C. Szegedy, W. Zaremba, I. Sutskever, J. Bruna, D. Erhan, I. Goodfellow, and R. Fergus. Intriguing properties of neural networks. arXiv preprint arXiv:1312.6199, 2013.

[22] D. Warde-Farley and I. Goodfellow. 11 adversarial perturbations of deep neural networks. Perturbations, Optimization, and Statistics, page 311.

[23] H. Xiao, K. Rasul, and R. Vollgraf. Fashion-mnist: a novel image dataset for benchmarking machine learning algorithms. 2017.

[24] W. Xu, D. Evans, and Y. Qi. Feature squeezing: Detecting adversarial examples in deep neural networks. arXiv preprint arXiv:1704.01155, 2017.

[25] W. Xu, D. Evans, and Y. Qi. Feature squeezing mitigates and detects carlini/wagner adversarial examples. arXiv preprint arXiv:1705.10686, 2017.
Figure 5: Example of an image (Number 5) along with its ensemble of adversarial counterparts targeting different classes. The Lyapunov exponents are as shown in the titles of the constituent images.
Figure 6: 2-D PCA embedding of the lyapunov exponents for the 100 images from the Fashion-MNIST dataset.
Figure 7: 2-D PCA embedding of the lyapunov exponents for the 100 images from the Fashion-MNIST dataset
Figure 8: Example of an image (Shoe) along with its ensemble of adversarial counterparts targeting different classes. The Lyapunov exponents are as shown in the titles of the constituent images.
Figure 9: ROC curves for logistic models trained on all but one attack and tested on the left out attack.