Fast Reliability Assessment Algorithm of Distribution Network based on Topological Similarity Analysis

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Abstract: In this paper, the concept of topological similarity is introduced to realize a fast and accurate reliability assessment of distribution network. Based on the singular value sequence as the similarity index, the mathematical relationship between similarity of network topology and power supply reliability of distribution network is developed. Firstly, the matrix representation, similarity index and its determination method of distribution network topology are presented, while the compression and reconstruction of matrix are analyzed to ensure the topological characteristics of adjacent matrix. Then, a fast reliability assessment algorithm of distribution network based on topological similarity is proposed, which uses root mean square method to determine similarity according to topology database and updates reliability indexes, together with the calculation flow. Lastly, case study of IEEE 33-node distribution network is undertaken, which shows that the calculation efficiency of the proposed algorithm is seven times higher than that of other conventional algorithms under the same calculation accuracy.

1. Introduction

Distribution network is located at the end of power systems, which directly supplies and distributes energy to electricity users [1]. The reliability of distribution network contains the contingencies of all electricity users, protection equipment, and distribution feeders [2, 3]. Therefore, reliability assessment of distribution network is an important task of electric industry. Several approaches have been developed to assess reliability of distribution networks, which are classified into analytical methods [4] and analogical methods [5]. The former one mainly includes state space method, network method and system state enumeration method [6], which represent the system property according to the system structure, state of components, and logical relationship between them [7]. However, with the increase of components, the computation load boosts exponentially [8]. On the other hand, Monte Carlo method [9] is the typical approach of the latter one, which estimates the reliability by simulating the actual process and the random behavior of system, then the reliability index can be calculated from a large number of simulation results. However, the calculation accuracy of Monte Carlo method is closely related to the calculation time, e.g., it often consumes a long period of time to obtain a satisfactory calculation accuracy.

In recent years, topology modelling of distribution networks has attracted a large number of research interests. In reference [10], a new topology algorithm was proposed for remote real-time monitoring of the distribution network. Based on the distribution network structure and operation, as well as the optimal planning of distribution network, work [11] presented a conceptual clustering method of distribution network topology combining with fuzzy logic. Besides, literature [12] calculated the topological characteristics parameters of the distribution network, which investigates the
tolerance of distribution network against random failures and targeted attacks, as well as proposes a methodology for the relationship between small-world effects and the reliability of power grid. Nevertheless, the current research on topology of distribution network is still at the early stage, in which no studies have been undertaken to apply topological similarity to assess the reliability of distribution networks.

In this paper, the topology of distribution networks is developed by adjacency matrix, which employs the singular value of adjacency matrix as the similarity index. Then, a novel concept called topological similarity is introduced, which constructs the mathematical relationship between topological similarity and reliability of distribution network. Moreover, reliability index of original distribution network can be rapidly calculated by the new distribution network with known reliability and topological similarity.

2. Fast Reliability Assessment Algorithm of Distribution Network based on Topological Similarity Analysis

2.1 Standard topology database development

A well-developed database can improve calculation accuracy, and the standard database framework structure is shown in Figure 1. For each determined number of node \( n \), corresponding to \( n \) nodes and \( n-1 \) branch topology information in standard database; for any topology in standard database, it contains adjacency matrix describing the topology, singular value sequences corresponding to adjacency matrix as well as reliability index of distribution network.

The aim of developing standard database is to exhaustively find the singular value sequences of all topologies of \( n \) node and \( n-1 \) branch. Figure 2 shows the exhaustive results of singular value sequences of all topologies when the number of node is 5–8.

![Figure 1. Frame structure of standard database.](image)

![Figure 2. Exhaustive results of singular value (5–8 nodes).](image)
2.2 Similarity determination and reliability index update

For two singular value sequences, similarity can be determined by the root mean square (rms) method. Set the values of two singular sequences are \( \sigma_A = \{\sigma_1, \sigma_2, \ldots, \sigma_n\} \) and \( \tau_B = \{\tau_1, \tau_2, \ldots, \tau_m\} \), respectively. Then, the rms of two singular value sequences can be given as

\[
X_{\text{rms}} = \sqrt{\frac{1}{k} \sum_{i=1}^{k} \mu_i (\sigma_i - \tau_i)^2}
\]  

where \( \mu_i \) is the corresponding weight coefficient; \( k \) is the length of singular value sequence. According to the singular value theory, the larger singular value can better reflect the characteristics of the topology. Therefore, the larger value of singular value, the greater impact on similarity, and the larger value of \( \mu_i \). In the actual calculation, the topology with the smallest rms is taken as the most similar topology.

After obtain the most similar topology, reliability index of this new topology can be roughly considered as the reliability index of original topology which is required to solve. However, reliability index need to be updated to improve the accuracy of results.

Set the threshold \( t^* \), assuming that there are \( m \) singular value sequences in database which conform to the screening condition: \( X_{\text{rms}} \leq t^* \). Then, the rms values set of \( m \) singular value sequences are recorded as \( \{X_{\text{rms}}^{(1)}, X_{\text{rms}}^{(2)}, \ldots, X_{\text{rms}}^{(m)}\} \). And rms values of the most similar topology can be given as

\[
X^{(n)} = \min \left\{ X_{\text{rms}}^{(1)}, X_{\text{rms}}^{(2)}, \ldots, X_{\text{rms}}^{(m)} \right\}
\]  

Accordingly, the results of further screening for \( \{X_{\text{rms}}^{(1)}, X_{\text{rms}}^{(2)}, \ldots, X_{\text{rms}}^{(m)}\} \) are recorded as \( \{X^{(1)}_{\text{rms}}, X^{(2)}_{\text{rms}}, \ldots, X^{(m)}_{\text{rms}}\} \). And the screening criterion is \( X^{(j)}_{\text{rms}} \leq 100X^{(n)}_{\text{rms}}, j \in [1, m] \). In other words, when the deviation is too large, the rms should be discarded even if it satisfies threshold condition.

Finally, reliability indexes are updated by the secondary screening results \( \{X^{(1)}_{\text{rms}}, X^{(2)}_{\text{rms}}, \ldots, X^{(m)}_{\text{rms}}\} \). Each rms value corresponds to the singular value sequence in standard database, and the singular value sequence corresponds to the adjacency matrix and its reliability index. So the reliability index corresponds to the rms of secondary screening results can be defined as \( \{R^{(1)}, R^{(2)}, \ldots, R^{(j)}\} \), and \( R^{(n)} \) is reliability index corresponding to \( X^{(n)}_{\text{rms}} \). Then, the expression of reliability index for distribution network which is required to solve can be written by

\[
R = \frac{\frac{1}{X^{(n)}_{\text{rms}}} \sum_{i=1}^{k} \frac{1}{X^{(i)}_{\text{rms}}} R^{(i)}}{\frac{1}{X^{(n)}_{\text{rms}}} \sum_{i=1}^{k} \frac{1}{X^{(i)}_{\text{rms}}}}
\]  

2.3 Overall execution procedure

To this end, the overall execution procedure of the proposed fast reliability assessment algorithm is illustrated in Figure 3.

In the fast calculation of reliability, it is just need to acquire singular value sequence of original distribution network topology, and compare it with the exhaustive result in database to find the new distribution network topology which has the most similar singular value sequence. After updating the reliability index of the new distribution network topology, the reliability index of the original distribution network topology can be rapidly obtained.
2.4 Comparisons
In order to evaluate the calculation speed of proposed algorithm, this section carries out three cases, e.g., distribution networks of 5-node, 9-node and 26-node. And the other comparison algorithms are: Monte Carlo method [13] and Failure-mode-effect analysis [14]. For each case, each algorithm is repeatedly calculated 100×10 times, and the average value of consuming time is recorded in Table 1 (unit: s/hund).

| Node | Calculation time /s |
|------|---------------------|
|      | Monte Carlo method | Failure-mode-effect analysis | Proposed algorithm |
| 5    | 27.71062            | 0.708454                   | 0.090826            |
| 9    | 27.1299             | 3.831871                   | 0.223855            |
| 26   | 3357.6329           | 19.149210                  | 0.380121            |

As shown in Table 1, the calculation speed of proposed algorithm is obviously faster than other algorithms, and the calculation amount does not increase significantly with the increase of matrix order. Especially, for Monte Carlo method, when the matrix order is over 50, the time required for complete operation of the program is more than 30 seconds, which is much larger than that of the proposed algorithm. Therefore, with the increasing complexity of distribution network, the advantage of proposed algorithm in computation speed is more prominent.

3. Case Studies
IEEE 33-node distribution network is taken as a case to implement reliability analysis in this paper. The average service availability index (ASAI) is used as reliability index, and the IEEE 33-node distribution network is shown in Figure 4. According to the 33-node distribution network topology structure diagram, selecting database information of partial 33 nodes, extracting its singular value sequences, and comparing it with the singular value sequences of IEEE 33-node distribution network topology, as shown in Figure 5.
In Figure 5, the red sequence set is a collection of selected singular value sequences in the database, totaling 5815 cases. The black sequence is the singular value sequence of the IEEE 33-node distribution network topology. Figure 5 demonstrates that find the singular value sequence with the smallest rms value in the selected singular value sequence set, and extract reliability index of the topology corresponding to this sequence, then the ASAI of IEEE 33-node distribution network can be obtained through result updating.

![Figure 5](image)

**Figure 5.** Comparison between the singular value of IEEE 33-node distribution network and standard database.

In order to screen the similar singular value sequence, it needs to determine the value of threshold and the length of singular value sequence. For comparisons, the threshold is set to 0.01, 0.02, and 0.03, respectively. And the length of singular value sequence is set to 20, 30, and 33, respectively. Then, number of singular value sequence satisfying the condition is shown in Table 2.

| Threshold | 20 | 30 | 33 |
|-----------|----|----|----|
| 0.01      | 0  | 0  | 0  |
| 0.02      | 52 | 2  | 4  |
| 0.03      | 620| 161| 67 |
| Number of singular value sequence with the smallest error | 2699 | 2134 | 2134 |

As shown in Table 2, if threshold is too harsh, it will unable to screen out the singular value sequences satisfying the condition from the database. And if threshold is too loose, it will obtain too many singular value sequences which would reduce the calculation accuracy. What’s more, if singular value sequence length is too small, the calculation accuracy will also be reduced. For example, when the singular value sequence length is 20, the 2699th singular value sequence has the smallest error. And when the singular value sequence length is 30, the number of singular value sequence has the smallest error is 2134. Moreover, if singular value sequence length is too large, the calculation speed will be reduced. Therefore, choosing the appropriate threshold and singular value sequence length can improve the calculation efficiency and calculation accuracy.

In particular, the singular value sequence length and threshold value are set to be 30 and 0.021, respectively. The calculation shows that three singular value sequences satisfying the condition:
$X_{\text{rms}}^{(1)} = 0.0192, X_{\text{rms}}^{(1)} = 0.0192$ and $X_{\text{rms}}^{(2)} = 0.0207$, and the corresponding reliability indicators ASAI are $R^{(1)} = 99.9950, R^{(1)} = 99.9975$ and $R^{(2)} = 99.9954$, respectively. Then, reliability index through updating, as follows

$$R = \frac{1}{X_{\text{rms}}^{(1)}} + \sum_{i=1}^{n} \frac{1}{X_{\text{rms}}^{(1)}} = 99.9960$$

(4)

4. Conclusions
1) Based on the adjacency matrix characteristics of distribution network, the relationship between the topology and the singular value sequence of the matrix is revealed. It is firstly proved that distribution networks with similar topological structures have similar singular value sequences.
2) The concept of topological similarity is introduced into the calculation of distribution network reliability index. Besides, the similarity of topological structure is determined by singular value sequence and the typical distribution network topology with known reliability in standard database, such that the reliability index of distribution network can be calculated rapidly.
3) At present, the reliability assessment algorithm of distribution network based on topological similarity is only applied to single power supply distribution network, and the fast reliability calculation algorithm for multi-power supply distribution network is one of the important research directions in future.

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