An analysis of unsteady thermal convection of paramagnetic fluid in cubical enclosure under strong magnetic field gradient

L Pyrda¹, S Kenjeres², E Fornalik-Wajs¹ and J S Szmyd¹

¹ Department of Fundamental Research in Energy Engineering, Faculty of Energy and Fuels, AGH University of Science and Technology, Krakow, Poland
² Faculty of Applied Sciences and J.M. Burgerscentre for Fluid Dynamics, Delft University of Technology, Delft, The Netherlands

E-mail: pyrda@agh.edu.pl

Abstract. The experimental studies of the flow and heat transfer of a paramagnetic fluid inside a cubical enclosure in a configuration heated from below and subjected to various strong non-uniform magnetic field gradients are presented. In contrast to the previously reported studies in literature, which observed solely laminar flow regimes, the appearance and sustenance of the periodic- and fully transient - flow motions for the very first time were investigated. This was consequence of using significantly stronger magnetic field gradient (up to 900 T²/m) than those used in previous studies (up to 200 T²/m). The fluid flow was studied at two Rayleigh numbers: 7.89·10⁵ and 1.86·10⁶. Non-monotonic behaviour of flow with increase of imposed magnetic field gradients was observed for both cases. Detailed analysis (Fast Fourier Transform) of temperature time series has been reported.

1. Introduction
First reported quantitative studies on the natural convection for a configuration heated from below (also called Rayleigh-Bénard configuration) is dated on 19th century. The natural convection in such a configuration is widely used in the countless industrial and environmental applications. For many of them a controlling of convection becomes important matter. Application of the external strong magnetic field to paramagnetic, non-conducting working fluid gives such an opportunity. In such system, besides the gravitational force the additional magnetic force occurs. By changing the strength and orientations of imposed magnetic field in reference to gravitational vector the different results could be observed. The potentials of thermal convection of paramagnetic fluid in the presence of a strong stationary magnetic field were firstly presented by [1]. Since then it was investigated both experimentally and numerically (for different geometries and various configurations of magnetic field to gravitation vector), e.g. [2],[3],[4],[5],[6]. All these studies addressed steady laminar flow regimes. The goal of presented investigation was an analysis of unsteady flow regimes in the case of thermal convection. This was achieved, by employing significantly larger magnetic field gradients generated by the state-of-art superconducting magnet (up to 900 T²/m, for magnetic field strength in the centre of superconducting magnet bore |b₀max|=10 T, in contrast to 200 T²/m, for magnetic field strength of |b₀max|=5 T in previous studies). Note that throughout the paper the magnetic field strength conditions were used to identify different flow regimes since it was a simple and easily controllable parameter in experimental investigations, although the magnetic field gradient was the main driving mechanism behind the magnetisation force.
2. Experimental investigations

2.1. Experimental setup

Strong stationary magnetic field gradients was generated by the superconducting magnet shown in figure 1-left. Calculated distributions of the magnetic field and resulting gradients for the upper limit of working conditions with added pointed position of experimental geometry are shown in figure 2.

![Figure 1](image1.png)

**Figure 1.** Left - sketch of experimental setup and location of the cubical enclosure with paramagnetic working fluid. The superconducting magnet (model HF10-100-VHT-B, Sumitomo Heavy Industries, Ltd., Japan) can generate up to 10 T in the magnet bore which has a diameter of 0.1 m and length of 0.5 m; Right - cubical enclosure \(L=D=H=0.032\) m with copper plates and coolant inlet/outlet.

![Figure 2](image2.png)

**Figure 2.** Calculated distributions of: left - magnetic field (contours) and magnetic flux (lines), right - the magnitude of the magnetic square gradients in the central vertical plane \((y-z)\) of the experimental setup shown in figure 1 for the upper limit of working conditions, i.e. \(b_{0\text{max}} = 10\) T.
The cubical enclosure could be positioned at different locations along the vertical axis of the superconducting magnet and combined effects (subtractive or additive) of gravitational and magnetisation forces could be observed. The suppression of the flow and heat transfer could be obtained by locating cubical enclosure in the lower part of the bore, i.e. $y=0, z\leq0 \text{m}$ - in figure 2. In the present study, the focus of the investigations was placed on possible flow and heat transfer enhancements, i.e. situations where the generated magnetisation force would support its gravitational counterpart. To achieve this, the cubical enclosure with dimensions $L=H=D=0.032 \text{ m}$ was located in the upper part of the bore with its centre at $y=0$ and $z=0.11 \text{ m}$ (figure 2). The experimental enclosure was constructed to provide an easy and controllable heating or cooling of the thermally active horizontal walls that were made from copper plates (figure 1-right). The sidewalls were made from plexiglass and six k-type thermocouples were placed inside one of them at depth of 6mm at different positions to provide detailed insight into analysed phenomena (their positions are shown in figure 4 right). Another six thermocouples were placed in heated and cooled plates for integral heat transfer (the Nusselt number) measurements. The heating was done by nichrome wire placed just below the heated plate. The electric voltage and electric current for the nichrome wire were recorded by multimeters. The cooling was imposed by running water through a thermo-stating bath.

2.2. Working fluid
As a working fluid for experimental studies 50% volume aqueous solution of glycerol with 0.8 mol/(kg of solution) concentration of gadolinium nitrate hexahydrate (Gd(NO$_3$)$_3$·x6H$_2$O) was chosen. Mass magnetic susceptibility was measured with the magnetic susceptibility balance utilizing Evans’ method. Dynamic viscosity is a function of temperature ($\theta$) and the expression describing it was obtained from experimental data shown in figure 3-left and took the form of following equation:

$$\mu(\theta) = 1.057 \cdot 6.503 \cdot 10^{-3} \theta + 1.007 \cdot 10^{-5} \theta^2.$$  

The Rayleigh number was defined as: 

$$Ra = \beta g \Delta \theta D^3 / \nu \alpha,$$

where $g$ is gravitational vector, $\Delta \theta$ is temperature difference between horizontal thermally active walls, the rest of symbols used in this chapter are described in table 1. For 5°C of temperature difference between thermally active horizontal walls the Rayleigh number was $Ra=7.89\cdot10^5$, while for 11°C of temperature difference was $Ra=1.86\cdot10^6$.

The Prandtl number was defined as: 

$$Pr = \nu / \alpha = 101.4.$$  

Detailed information of fluid properties are listed in table 1. All properties of the working fluid were obtained experimentally.

### Table 1 Properties of the working fluid at 298 K.

| Properties                      | Symbol | Value    | Unit  |
|---------------------------------|--------|----------|-------|
| Heat capacity                   | $C_p$  | 2.92·10$^3$ | J/kgK |
| Thermal diffusivity             | $\alpha$ | 9.13·10$^{-8}$ | m$^2$/s |
| Density                         | $\rho$ | 1411     | kg/m$^3$ |
| Dynamic viscosity               | $\mu$  | 1.30·10$^{-2}$ | kg/m·s |
| Kinematic viscosity             | $\nu$  | 9.25·10$^{-6}$ | m$^2$/s |
| Thermal conductivity            | $\lambda$ | 0.376    | W/m·K |
| Thermal expansion coefficient   | $\beta$ | 4.78·10$^{-4}$ | 1/K   |
| Mass magnetic susceptibility    | $\chi_m$ | 2.39·10$^{-4}$ | m$^3$/kg |
| Prandtl number                  | Pr     | 101.4    | –     |
2.3. Integral heat transfer and heat loss

The integral heat transfer coefficient (the Nusselt number) was calculated as a ratio between the net convective heat transfer rate \( \dot{Q}_{\text{net, conv}} \) and the net pure conduction contribution \( \dot{Q}_{\text{net, cond}} \):

\[
\text{Nu} = \frac{\dot{Q}_{\text{net, conv}}}{\dot{Q}_{\text{net, cond}}} \tag{1}
\]

The method invented by Ozoe and Churchill [7] was used to estimate the net convection and the net conduction heat fluxes:

\[
\begin{align*}
\dot{Q}_{\text{net, conv}} &= \dot{Q}_{\text{conv}} - \dot{Q}_{\text{loss}} \tag{2} \\
\dot{Q}_{\text{net, cond}} &= \dot{Q}_{\text{cond}} - \dot{Q}_{\text{loss}} \tag{3}
\end{align*}
\]

It was assumed that the heat loss \( \dot{Q}_{\text{loss}} \) depends only on the temperature of heated wall and not depends on the heat transfer mode inside the enclosure. To estimate heat loss, the conduction experiment was carried out with reversely placed thermally active horizontal walls of experimental enclosure at investigated position. At first a specific temperature difference between horizontal walls was selected and then after thermal stabilisation of experimental enclosure the heating power was measured, which together with theoretical heat flux \( \dot{Q}_{\text{theor, cond}} \) obtained from Fourier’s law allow to estimate heat loss:

\[
\dot{Q}_{\text{loss}} = \dot{Q}_{\text{cond}} - \dot{Q}_{\text{theor, cond}} \tag{4}
\]

\[
\dot{Q}_{\text{theor, cond}} = \lambda A (\theta_h - \theta_c) / l \tag{5}
\]

where \( A = l^2 \) is conduction area of cross-section, \( \theta_h, \theta_c \) - are temperature of hot and cold walls respectively.

Heat loss presented in figure 3-right was calculated according to eq.(4) and could be linearly approximated for various heating rates \( \Delta \theta = (\theta_h - \theta_c) = 2.5 \div 15 \text{K} \), according to:

\[
\dot{Q}_{\text{loss}} = a_{\text{loss}} \Delta \theta \tag{6}
\]

The temperature of cold wall was kept at the temperature of air inside magnet.

Assuming that convection heat flux was equal to heater heat flux \( \dot{Q}_{\text{net, conv}} = \dot{Q}_{\text{heater}} \) and applying the equations (2), (4), (5), to equation (1) it can be rewritten in the form:

\[
\text{Nu} = \frac{\dot{Q}_{\text{heater}} - \dot{Q}_{\text{loss}}}{\dot{Q}_{\text{theor, cond}}} = \frac{IU - \dot{Q}_{\text{loss}}}{\lambda (\theta_h - \theta_c)} \tag{7}
\]

![Figure 3](image-url) 

Figure 3 Left – dynamic viscosity of working fluid. Right – Heat loss at several temperature difference between thermally active horizontal walls
2.4. Experimental procedures
At first, paramagnetic working fluid was injected into experimental apparatus. Then enclosure was placed in a bore of super-conducting magnet at selected position. The temperature of cooling water was set up at constant level (18°C) equal to the air temperature inside magnet. The heater power was controlled to obtain constant temperature difference (5°C and 11°C) between thermally active horizontal walls. After all preparations experimental setup was left for 10-12 hours to acquire thermal stabilisation. Then thermal measurements were carried out and the temperature together with the values of voltage and current were stored. After the analysis of natural convection the superconducting magnet was turned on and magnetic induction was set up to 1 T. Supplied to heater voltage and current were corrected to keep constant difference (5°C and 11°C) between thermally active horizontal walls. From this point, the experimental setup was left for 2 hours to achieve thermal stabilisation. Then all measurements were carried out. Magnetic induction was gradually raised again and all procedure was repeated.

3. Results and discussion
The Nusselt number plotted versus the magnetic induction in the centre of magnet ($|b_{0\text{max}}|$) for 5°C and 11°C of temperature difference between horizontal thermally active walls are presented in figure 4 left. Measurements errors were calculated with combined uncertainty analysis. Note that in comparison with the neutral state, a significant heat transfer increase was obtained for magnetically active cases. The integral heat transfer enhancement was more than 300% (for $|b_{0\text{max}}|=10$ T) for both cases.

For further analysis only signal from one thermocouple placed at the central lowest position ($y=0$ m, $z=-0.012$ m – figure 4 right) was selected to obtain detailed information on flow phenomena occurring in the enclosure presented in figures 5 and 6.

Temperature time series of investigated fluid for 5°C temperature difference between thermally active horizontal walls show transition from oscillatory state for the neutral case ($|b_{0\text{max}}|=0$ T – figure 5a) to stable state at presence of weak magnetic field ($|b_{0\text{max}}|=2$ T – figure 5b). Maximum amplitude of oscillations was reached at $|b_{0\text{max}}|=4$ T (figure 5c). Then with further increase of magnetic field induction ($|b_{0\text{max}}|>4$ T) decreasing of oscillation amplitude could be observed (figure 5d-e).

Temperature time series of investigated fluid for 11°C temperature difference between thermally active horizontal walls shows transition from stable state for the neutral case ($|b_{0\text{max}}|=0$ T – figure 6a) to oscillatory state at presence of weak magnetic field ($|b_{0\text{max}}|=1$ T – figure 6b). Maximum amplitude of oscillations was reached at $|b_{0\text{max}}|=3$ T (figure 6c). Then with further increase of magnetic field induction ($|b_{0\text{max}}|>3$ T) decreasing of oscillation amplitude could be observed (figure 6d-e).

![Figure 4](image-url)  
*Figure 4* Left – The Nusselt number plotted versus the magnetic induction at the centre of magnet ($|b_{0\text{max}}|$) for 5°C and 11°C of temperature difference between horizontal thermally active walls. Right – position of thermocouples in one of the sidewalls.
The Fast Fourier Transform (FFT) analysis was conducted for all cases and the results are presented as the power spectrum density (PSD) versus frequency in figures 5 and 6. For 5°C of temperature difference between thermally active horizontal walls for oscillatory regime without magnetic field (figure 5a) they indicate the characteristic frequencies at 0.01176 Hz. The FFT analysis of temperature time series for most oscillating results obtained at $|b_{0_{max}}|=5$ T shows peak at 0.05176 Hz.

![Figure 5](image)

**Figure 5** Temperature time series and FFT PSD versus frequency for 5°C difference between thermally active horizontal walls case at various values of $|b_{0_{max}}|$: a) 0 T, b) 3 T, c) 5 T, d) 7 T, e) 10 T.
For 11°C of temperature difference between thermally active horizontal walls for oscillatory regime at $|b_0|_{\text{max}} = 1 \text{T}$ (figure 6b) the results indicate the characteristic frequencies at 0.02941 Hz. The FFT analysis of temperature time series for most oscillating result obtained at $|b_0|_{\text{max}} = 3 \text{T}$ shows peak at 0.03176 Hz.

**Figure 6** Temperature time series and FFT PSD versus frequency for 11°C difference between thermally active horizontal walls case at various values of $|b_0|_{\text{max}}$: a) 0 T, b) 1 T, c) 3 T, d) 6 T, e) 10 T.
4. Conclusions
The analysis of unsteady flow in the case of thermal convection was studied experimentally. An influence of Rayleigh number and magnetic field strength was examined. Due to the application of steady magnetic field at chosen temperature difference three various flow types were observed – steady, oscillatory and turbulent. The non-monotonic behaviour of flow with increase of imposed magnetic field gradients (initially stabilising and then de-stabilising effects) for thermo-magnetic convection of paramagnetic fluid was presented. It was observed in the range $0 \leq |b_{b_{\text{max}}}| \leq 4$ T of magnetic induction for analysed cases. With an application of Fast Fourier Transform it was possible to determine the characteristic frequency for temperature time series. The FFT analysis clearly indicates the type of the flow.

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