TOPOLOGICAL DEFECTS AND THE FORMATION OF
STRUCTURE IN THE UNIVERSE

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Abstract

Topological defects, in particular cosmic strings, give rise to an interesting mechanism for generating the primordial perturbations in the early Universe which are required to explain the present structure. An overview of the cosmic string model will be given, focusing on the predictions of the theory for the large-scale structure of the Universe and for cosmic microwave background anisotropies.

I. Introduction

Over the past fifteen years, we have witnessed the development of the first theories based on causal microphysics which attempt to explain the observed structure in the Universe and make predictions for observations on even larger scales than those for which we now have information. The class of models which is receiving most attention is based on inflation. According to the inflationary Universe scenario, quantum fluctuations during a period of exponential expansion in the very early Universe generate a random phase and approximately scale-invariant spectrum of density fluctuations on scales which are cosmological today.

Inflation, however, is not without its own problems, and thus it is of great interest to have an alternate class of theories based on physics more accessible to experiment. Topological defect models provide such a class of models. In defect models, topological defects which form during a phase transition in the very early Universe provide the seeds about which galaxies and even larger structures form by gravitational clustering. The most promising of the defect models is the cosmic string theory.

In this lecture I will give an overview of the cosmic string theory of structure formation, focusing on the predictions of the model for large-scale structure.
and cosmic microwave background (CMB) anisotropy observations. For detailed expositions of the theory, the reader is referred to several recent publications [2], [1].

Any cosmological model requires (at least) two inputs, namely both the specification of the source of perturbations and of the dark matter which makes up the bulk of the matter in the Universe. In inflation-based theories, most of the dark matter must be cold (i.e. the corresponding particles are nonrelativistic at the time of equal matter and radiation $t_{eq}$, the time when density fluctuations begin to increase in amplitude) whereas in the cosmic string theory it is possible (as will be discussed below) for the dark matter to be hot (i.e. the corresponding particles are relativistic at $t_{eq}$).

II. Defects and their Classification

The topological defects of interest in cosmology are those which arise in relativistic field theories. Due to the high temperature and density of the early Universe, relativistic field theory is believed to give the correct description of matter. Based on particle physics experiments it is also believed that the Universe underwent a series of phase transitions in the early Universe during which internal symmetries of matter fields were spontaneously broken. This symmetry breaking is described by an order parameter $\phi$ (which does not necessarily have to be a scalar field but which - as in condensed matter physics - could represent a condensate of fermionic fields) with a potential of the form

$$V(\phi) = \frac{1}{4} \lambda (\phi^2 - \eta^2)^2$$

(1)

The phase transition will take place on a short time scale $\tau < H^{-1}$, and will lead to correlation regions of radius $\xi < t$ inside of which $\phi$ is approximately constant, but outside of which $\phi$ ranges randomly over the vacuum manifold $\mathcal{M}$, the set of values of $\phi$ which minimize $V(\phi)$ - in our example $|\phi| = \eta$. The correlation regions are separated by regions in space where $\phi$ leaves the vacuum manifold $\mathcal{M}$ and where, therefore, potential energy is localized. Via the usual gravitational force, this energy density can act as a seed for structure.

Topological defects are familiar from solid state and condensed matter systems. Crystal defects, for example, form when water freezes or when a metal crystallizes. Point defects, line defects and planar defects are possible. Defects are also common in liquid crystals. They arise in a temperature quench from the disordered to the ordered phase. Vortices in $^4$He are analogs of global cosmic strings. Vortices and other defects are also produced during a quench below the critical temperature in $^3$He. Finally, vortex lines also play an important role in the theory of superconductivity.
The analogies between defects in particle physics and condensed matter physics are quite deep. Defects form for the same reason: the vacuum manifold is topologically nontrivial. The arguments which say that in a theory which admits defects, such defects will inevitably form, are applicable both in cosmology and in condensed matter physics. Different, however, is the defect dynamics. The motion of defects in condensed matter systems is friction-dominated, whereas the defects in cosmology obey relativistic equations, second order in time, since they come from a relativistic field theory.

To classify the possible topological defects, we consider theories with an $n$-component order parameter $\phi$ and with a potential energy function (free energy density) of the form (1) with $\phi^2$ denoting the sum of the squares of all of the components. There are various types of topological defects (regions of trapped energy density) depending on the number $n$ of components of $\phi$. For a single component real scalar field as order parameter (i.e. $n = 1$), the defects are domain walls, for $n = 2$ the defect configurations are one dimensional strings (cosmic strings), and for $n = 3$ monopoles result. In theories with a global symmetry and $n = 4$ it is possible to have textures, defects in space-time (collapsing lumps of scalar field gradient energy in space).

Theories with domain walls forming above the scale of electroweak symmetry breaking are ruled out since a single domain wall stretching across the present horizon would overclose the Universe. Local monopoles corresponding to a scale of symmetry breaking comparable to the scale of grand unification are also ruled out since they would likewise overclose the Universe. Promising theories from the point of view of cosmology are models with cosmic strings, global monopoles or textures (for a review of the latter see [4]). In the following, I will focus on cosmic strings.

A theory with a complex order parameter ($n = 2$) admits cosmic strings. In this case the vacuum manifold of the model is $M = S^1$, which has nonvanishing first homotopy group $\Pi_1(M) = Z \neq 1$. A cosmic string is a line of trapped energy density which arises whenever the field $\varphi(x)$ circles $M$ along a closed path in space (e.g., along a circle). To construct a field configuration with a string along the $z$ axis, take $\varphi(x)$ to cover $M$ along a circle with radius $r$ about the point $(x, y) = (0, 0)$:

$$\varphi(r, \vartheta) \simeq \eta e^{i\vartheta}, \quad r \gg \eta^{-1}. \tag{2}$$

This configuration has winding number 1, i.e., it covers $M$ exactly once. Maintaining cylindrical symmetry, we can extend (2) to arbitrary $r$

$$\varphi(r, \vartheta) = f(r)e^{i\vartheta}, \tag{3}$$

where $f(0) = 0$ and $f(r)$ tends to $\eta$ for large $r$. The width $w$ of the string is defined as the value of $r$ beyond which $\eta - f(r) < \frac{1}{2}\eta$. It can be estimated by balancing potential and tension energy. The result is

$$w \sim \lambda^{-1/2}\eta^{-1}. \tag{4}$$
For local cosmic strings, i.e., strings arising due to the spontaneous breaking of a gauge symmetry, the energy density decays exponentially for $r \gg w$. In this case, the energy $\mu$ per unit length of a string is finite and depends only on the symmetry breaking scale $\eta$

$$\mu \sim \eta^2$$

(independent of the coupling $\lambda$). The value of $\mu$ is the only free parameter in a cosmic string model.

### III. Formation of Topological Defects

The Kibble mechanism [3] ensures that in theories which admit topological defects, such defects will inevitably be produced during a phase transition in the very early Universe.

Consider a mechanical toy model (see Fig. 1), first introduced by Marenko, Unruh and Wald [5] in the context of inflationary Universe models, which is useful in understanding the scalar field evolution. Consider a lattice of points on a flat table. At each point, a pencil is pivoted. It is free to rotate and oscillate. The tips of nearest neighbor pencils are connected with springs (to mimic the spatial gradient terms in the scalar field Lagrangean). Newtonian gravity creates a potential energy $V(\varphi)$ for each pencil ($\varphi$ is the angle relative to the vertical direction). $V(\varphi)$ is minimized for $|\varphi| = \eta$ (in our toy model $\eta = \pi/2$). Hence, the Lagrangean of this pencil model is analogous to that of a scalar field with symmetry breaking potential (1).

![Figure 1: The pencil model: the potential energy of a single pencil has the same form as that of scalar fields used for spontaneous symmetry breaking. The springs connecting nearest neighbor pencils give rise to contributions to the energy which mimic spatial gradient terms in field theory.](image)

At high temperatures $T \gg T_c$, all pencils undergo large amplitude, high frequency oscillations. However, by causality, the phases of oscillation of pencils with large separation $s$ are uncorrelated. For a system in thermal equilibrium, the length $s$ beyond which phases are random is the correlation length $\xi(t)$. However, since the system is quenched rapidly, there is a causality bound on $\xi$:

$$\xi(t) < t,$$
where \( t \) is the causal horizon.

The critical temperature \( T_c \) is the temperature at which the free energy of a pencil is equal in the vertical and horizontal positions. For \( T < T_c \), it is energetically preferable for a pencil to lie flat on the table. However, the orientations of the pencils are random beyond a distance of \( \xi(t) \) determined by equating the free energy gained by symmetry breaking (a volume effect) with the gradient energy lost (a surface effect). As expected, \( \xi(T) \) diverges at \( T_c \). Very close to \( T_c \), the thermal energy \( T \) is larger than the volume energy gain \( E_{corr} \) in a correlation volume. Hence, these domains are unstable to thermal fluctuations. As \( T \) decreases, the thermal energy decreases more rapidly than \( E_{corr} \). Below the Ginsburg temperature \( T_G \), there is insufficient thermal energy to excite a correlation volume into the state \( \varphi = 0 \). Domains of size \( \xi(t_G) \sim \lambda^{-1} \eta^{-1} \) freeze out \([3],[6]\). The boundaries between these domains become topological defects. An improved version of this argument has recently been given by Zurek \([7]\) (see also \([8]\)).

We conclude that in a theory in which a symmetry breaking phase transition satisfies the topological criteria for the existence of a fixed type of defect, a network of such defects will form during the phase transition and will freeze out at the Ginsburg temperature. The causality bound implies that the initial correlation length obeys \( \xi(t_G) < t_G \). For times \( t > t_G \) the evolution of the network of defects may be complicated (as for cosmic strings) or trivial (as for textures). In any case, the causality bound persists at late times and states that even at late times, the mean separation and length scale of defects is bounded by \( \xi(t) \leq t \).

Applied to cosmic strings, the Kibble mechanism implies that at the time of the phase transition, a network of cosmic strings with typical step length \( \xi(t_G) \) will form. According to numerical simulations \([9]\), about 80% of the initial energy is in infinite strings (strings with curvature radius larger than the Hubble radius) and 20% in closed loops.

The evolution of the cosmic string network for \( t > t_G \) is complicated. The key processes are loop production by intersections of infinite strings and loop shrinking by gravitational radiation. These two processes combine to create a mechanism by which the infinite string network loses energy (and length as measured in comoving coordinates). As a consequence, the correlation length of the string network is always proportional to its causality limit

\[
\xi(t) \sim t .
\]  

Hence, the energy density \( \rho_\infty(t) \) in long strings is a fixed fraction of the background energy density \( \rho_c(t) \)

\[
\frac{\rho_\infty(t)}{\rho_c(t)} \sim G\mu .
\]

We conclude that the cosmic string network approaches a “scaling solution” in which the statistical properties of the network are time independent if all distances are scaled to the horizon distance. Although the qualitative characteristics of the
cosmic string scaling solution are well established, the quantitative details are not. The main reason for this is the fact that the Nambu action, the action which describes the evolution of cosmic strings, breaks down at kinks and cusps. However, kinks and cusps inevitably form and are responsible for the small-scale structure on strings. Hence, neither the exact number of segments of the long string network which on average cross a Hubble volume, nor the amount of small-scale structure on strings is known. This is at the present time the main obstacle to developing the cosmic string theory in greater quantitative detail.

IV. Cosmic Strings and Structure Formation

The starting point of the structure formation scenario in the cosmic string theory is the scaling solution for the cosmic string network, according to which at all times $t$ (in particular at $t_{eq}$, the time when perturbations can start to grow) there will be a few long strings crossing each Hubble volume, plus a distribution of loops of radius $R \ll t$ (see Fig. 2).

![Figure 2](image)

**Figure 2.** Sketch of the scaling solution for the cosmic string network. The box corresponds to one Hubble volume at arbitrary time $t$.

The cosmic string model admits three mechanisms for structure formation: loops, filaments, and wakes. Cosmic string loops have the same time averaged field as a point source with mass. Hence, loops will be seeds for spherical accretion of dust and radiation. However, according to the recent cosmic string evolution simulations, most of the mass in strings is in the long string network, and hence the loop mechanism is a subdominant mechanism of structure formation.

![Figure 3](image)

**Figure 3.** Sketch of the mechanism by which a long straight cosmic string $S$ moving with velocity $v$ in transverse direction through a plasma induces a velocity perturbation $\Delta v$ towards the wake. Shown on the left is the deficit angle, in the
center is a sketch of the string moving in the plasma, and on the right is the sketch of how the plasma moves in the frame in which the string is at rest.

The second mechanism involves long strings moving with relativistic speed in their normal plane, giving rise to velocity perturbations in their wake [10]. The mechanism is illustrated in Fig. 3: space normal to the string is a cone with deficit angle \( \alpha = 8\pi G \mu \).

If the string is moving with normal velocity \( v \) through a bath of dark matter, a velocity perturbation
\[
\delta v = 4\pi G \mu v \gamma
\]
[with \( \gamma = (1 - v^2)^{-1/2} \)] towards the plane behind the string results. At times after \( t_{eq} \), this induces planar overdensities, the most prominent (i.e., thickest at the present time) and numerous of which were created at \( t_{eq} \), the time of equal matter and radiation [12], [13], [14]. The corresponding planar dimensions are (in comoving coordinates)
\[
t_{eq} z(t_{eq}) \times t_{eq} z(t_{eq}) v \sim (40 \times 40v) \text{ Mpc}^2.
\]

The thickness \( d \) of these wakes can be calculated using the Zel’dovich approximation [15]. The result is [14]
\[
d \simeq G \mu v \gamma z(t_{eq})^2 t_{eq} \simeq 4v \text{ Mpc}.
\]

Wakes arise if there is little small scale structure on the string. In this case, the string tension equals the mass density, the string moves at relativistic speeds, and there is no local gravitational attraction towards the string.

In contrast, if there is small scale structure on strings, then the string tension \( T \) is smaller [16] than the mass per unit length \( \mu \) and there is a gravitational force towards the string which gives rise to cylindrical accretion, thus producing filaments.

Which of the mechanisms – filaments or wakes – dominates is determined by the competition between the velocity induced by the gravitational potential on the string and the velocity perturbation of the wake.

By the same argument as for wakes, the most numerous and prominent filaments will have the distinguished scale
\[
t_{eq} z(t_{eq}) \times d_f \times d_f
\]
\[
where \( d_f \) can be calculated using the Zel’dovich approximation [17].

The cosmic string model predicts a scale-invariant spectrum of density perturbations, exactly like inflationary Universe models but for a rather different reason. Consider the r.m.s. mass fluctuations on a length scale \( 2\pi k^{-1} \) at the time \( t_H(k) \) when this scale enters the Hubble radius. From the cosmic string scaling solution
it follows that a fixed (i.e., $t_H(k)$ independent) number $\tilde{v}$ of strings of length of the order $t_H(k)$ contribute to the mass excess $\delta M(k, t_H(k))$. Thus

$$\frac{\delta M}{M}(k, t_H(k)) \sim \frac{\tilde{v} \mu t_H(k)}{G^{-1} t_H^{-2}(k) t_H^3(k)} \sim \tilde{v} G \mu. \quad (14)$$

Note that the above argument predicting a scale invariant spectrum will hold for all topological defect models which have a scaling solution, in particular also for global monopoles and textures.

The amplitude of the r.m.s. mass fluctuations (equivalently: of the power spectrum) can be used to normalize $G \mu$. Since today on galaxy cluster scales $\frac{\delta M}{M}(k, t_0) \sim 1$, the linear theory for the growth of fluctuations yields [18], [19]

$$\frac{\delta M}{M}(k, t_{eq}) \sim 10^{-4}, \quad (15)$$

and therefore, using $\tilde{v} \sim 10$,

$$G \mu \sim 10^{-5}. \quad (16)$$

Thus, if cosmic strings are to be relevant for structure formation, they must arise due to a symmetry breaking at energy scale $\eta \simeq 10^{16}$GeV. This scale happens to be the scale of unification (GUT) of weak, strong and electromagnetic interactions. It is tantalizing to speculate that cosmology is telling us that there indeed was new physics at the GUT scale.

A big difference of the cosmic string model compared to inflation-based theories is that HDM is a viable dark matter candidate. The neutrino free streaming length decreases as $a(t)^{-1/2}$. Whereas in inflation-based HDM models, all perturbations on scales smaller than the maximal neutrino free streaming scale are erased before $t_{eq}$, cosmic string seeds survive and can seed structures on small scales. Growth of perturbations on small scales is delayed (compared to models with CDM) but not prevented. Accretion of hot dark matter by string wakes was studied in Ref. [14]. In this case, nonlinear perturbations develop only late. Accretion onto loops and small scale structure on the long strings provide two mechanisms which may lead to high redshift objects such as quasars and high redshift galaxies. The first mechanism has recently been studied in Ref. [20].

The power spectra in the cosmic string models with CDM and HDM are obviously different on scales smaller than the maximal neutrino free streaming length. The power spectrum in a model with cosmic strings and HDM has less power on small scales than an inflationary CDM model. Recent numerical simulations [21], [22] demonstrate show that a COBE normalized cosmic string theory with HDM has a power spectrum in better agreement with the recent APM observational results than a standard inflationary CDM model.

V. Specific Signatures
Topological defect models give rise to some specific signatures and are hence falsifiable - a condition on any good scientific theory. The cleanest specific signatures can be found in the microwave background, although the signatures in the predicted large-scale structure are perhaps more easily detectable.

All theories of structure formation give rise to Sachs-Wolfe type CMB temperature fluctuations. In the cosmic string model there are, in addition, specific signatures which cannot be described in a linear perturbative analysis. As described in the previous section, space perpendicular to a long straight cosmic string is conical with deficit angle given by (9). Consider now CMB radiation approaching an observer in a direction normal to the plane spanned by the string and its velocity vector (see Fig. 4). Photons arriving at the observer having passed on different sides of the string will obtain a relative Doppler shift which translates into a temperature discontinuity of amplitude

$$\frac{\delta T}{T} = 4\pi G\mu \nu \gamma(v),$$  \hspace{1cm} (17)

where \(v\) is the velocity of the string. Thus, the distinctive signature for cosmic strings in the microwave sky are line discontinuities in \(T\) of the above magnitude.

*Figure 4:* Sketch of the Kaiser-Stebbins effect by which cosmic strings produce linear discontinuities in the CMB. Photons \(\gamma\) passing on different sides of a moving string \(S\) (velocity \(v\)) towards the observer \(O\) receive a relative Doppler shift due to the conical nature of space perpendicular to the string (deficit angle \(\alpha\)).

Given ideal maps of the CMB sky it would be easy to detect strings. However, real experiments have finite beam width. Taking into account averaging over a scale corresponding to the beam width will smear out the discontinuity, and it turns out to be surprisingly hard to distinguish the predictions of the cosmic string model from that of inflation-based theories using quantitative statistics which are
easy to evaluate analytically, such as the kurtosis of the spatial gradient map of
the CMB [24].

Other topological defect models predict different distinctive signatures. Text-
tures, for example, produce a distribution of hot and cold spots on the CMB
sky with typical size of several degrees [25]. This signature is much easier to
see in CMB maps. Inflationary models typically predict a Gaussian random field
distribution of temperature fluctuations with a scale invariant spectrum.

As discussed in the previous section, the distinctive prediction of the cosmic
string theory for large-scale structure is that on scales larger than the comoving
Hubble radius at $t_{eq}$, the distribution of galaxies should be dominated by a network
of sheets. The regions between the sheets form voids which should also be quite
empty of dwarf galaxies. These predictions can be quantified using topological
statistics. In contrast, in inflation-based models, the topology of the large-scale
structure should show no difference on the above scales between overdense and
underdense regions.

VI. Conclusions

The cosmic string theory of structure formation appears at the present time to
be a viable alternative to inflation-based models. Note however, that this theory
(in contrast to inflation) does not address other cosmological problems such as
the flatness and horizon problems.

The cosmic string model predicts a scale-invariant primordial spectrum of
density fluctuations. With hot dark matter, the power on scales smaller than the
neutrino free streaming length at $t_{eq}$ is reduced, but not totally wiped out. In
principle, the theory contains only one free parameter, $G\mu$. The normalizations
of $G\mu$ from the COBE measurement of CMB anisotropies and from the power
spectrum on cluster scales is consistent according to the present calculations [26]
(which should be further refined). Recent [27] work indicates that the position
of the “Doppler peaks” of the CMB power spectrum might allow a distinction
between inflation-based and topological defect models. However, more work needs
to be done to improve the calculations.

There are several distinctive predictions of the cosmic string theory: line dis-
continuities in the CMB temperature maps and planar topology of the mass dis-
tribution on large scales, to name just two. Recent observations [28] of structure
of the Universe on scales of greater than $100h^{-1}$Mpc are showing some evidence
that the distribution of galaxies on large scales is indeed planar. This gives further
motivation to tackle the difficult problem of making the predictions of the model
more quantitative.

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