Testing Grumiller’s modified gravity at galactic scales

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Abstract

Using galactic rotation curves, we test a -quantum motivated- gravity model that at large distances modifies the Newtonian potential when spherical symmetry is considered. In this model one adds a Rindler acceleration term to the rotation curves of disk galaxies. Here we consider a standard and a power-law generalization of the Rindler modified Newtonian potential that are hypothesized to play the role of dark matter in galaxies. The new, universal acceleration has to be -phenomenologically- determined. Our galactic model includes the mass of the integrated gas and stars for which we consider a free mass model. We test the model by fitting rotation curves of thirty galaxies that has been employed to test other alternative gravity models. We find that the Rindler parameters do not perform a suitable fit to the rotation curves in comparison to the Burkert dark matter profile, but the models achieve a similar fit as the NFW’s profile does. However, the computed parameters of the Rindler gravity show some spread, posing the model to be unable to consistently explain the observed rotation curves.

1. Introduction

It is well known that General Relativity is a theory well tested within the solar system and scales below, inasmuch as no deviations to it have been found since many years [1, 2]. However, new theories/models of gravitation have been recently proposed motivated by different theoretical and observational reasons, see Ref. [3] for a review. One of the motivations is to test gravity theories beyond the solar system, and to understand what constraints could be drawn at different length scales. On the one hand, at cosmological scales different corrections apply to the standard theory of large scale structure alone from General Relativity [4, 5, 6] and, in addition, new approaches have been put forward to understand the possible deviations of data to the theory [7, 8, 9, 10]. On the other hand, at galactic scales rotation curves provide a unique laboratory to test kinematical deviations from theoretical expectations and in fact rotation curves are one of the reasons why dark matter has been hypothesized. Although cold dark matter is the most popular candidate, there are other possibilities, e.g. bound dark matter [11, 12, 13], or other theoretical approaches that modify gravity or kinematical laws such as MOND [14, 15] (see however [16]) or f(R)-gravity that apart from playing the role of dark energy also intends to replace dark matter [17, 18].

Recently, a model of gravity has been put forward that stems from quantum gravity corrections to General Relativity and when one applies it to spherical symmetry and local (galactic) scales an extra Rindler acceleration appears in addition to the standard Newtonian formula for rotation curves [18, 19]. The new Rindler term is hypothesized to play the role of dark matter in galaxies. This idea has been tested already in a very recent work [21], where a fit is made to eight galaxies of the The HI Nearby Galaxy Survey (THINGS) [22]. They found that six of the galaxies tend to fit well to the data and that there is a preferred Rindler acceleration parameter of around $a \approx 3.0 \times 10^{-9} \text{ cm/s}^2 (= 926 \text{ km/s}^2\text{kpc})$; they later fixed this acceleration parameter and found acceptable fits for five galaxies, and furthermore, an additional free parameter let them to fit two more galaxies. We have revised this idea using a greater sample (seventeen) of THINGS galaxies, and for the eight original galaxies we find similar conclusions on the fits and to a convergence to a similar Rindler acceleration within 1σ confidence level. But when one adds more galaxies to the analysis the spread in the acceleration blows up, and therefore we concluded that the model is not tenable [23]. However, THINGS rotation curves are based on gas kinematics, whereas there are claims pointing out that complex gas dynamics could not be a good tracer of gravity in spirals, and gas and stellar motion do not exactly coincide in all the cases [24].

This work is organized as follows: In Section 2 we briefly review Grumiller’s model of gravity at large distances, in Section 3 we explain the rotation curve mod-
els, in Section 4 we present our results and compare the fits to results from standard dark matter profiles such as Navarro-Frenk-White (NFW) [25, 26] and Burkert [27]. Finally, Section 5 is devoted to conclusions. Supplementary material is included to support our conclusions.

2. Grumiller’s gravity model at large distances

In order to have a self-consistent description of this work we briefly review the main ideas behind Grumiller’s model, for details see [19]. The model starts with spherical symmetry in four dimensions split in the following way:

\[ ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta + \Phi^2(d\theta^2 + \sin^2\theta d\phi^2), \]

where \( g_{\alpha\beta}(x^\gamma) \) is a 2-dimensional metric and the surface radius \( \Phi(x^\gamma) \) depend upon \( x^\gamma = \{t, r\} \). The idea is to describe these fields in two dimensions since the gravitational potentials \( g_{\alpha\beta} \) and \( \Phi \) that are intrinsically two-dimensional, and their solutions can be mapped into the 4-dimensional world through Eq. (1).

The most general 2-dimensional gravitational theory that is renormalizable, that yields a standard Newtonian potential, and that avoids curvature singularities at large \( \Phi \) is:

\[ S = -\int \sqrt{-g} \Phi^2 R + 2\Phi \Phi^2 - 6\Lambda \Phi^2 + 8a\Phi + 2|d^2x|, \]

that depends on two fundamental constants, \( \Lambda \) and \( a \), the cosmological constant and a Rindler acceleration, respectively. The solutions to this action will describe the original line element, Eq. (1), that will model gravity in the infrared. The solutions are:

\[ g_{\alpha\beta}dx^\alpha dx^\beta = -K^2 dt^2 + \frac{dr^2}{K^2}, \]

\[ K^2 = 1 - \frac{2M}{r} - \Lambda r^2 + 2ar, \]

with \( K \) being the norm the Killing vector \( \partial_t \) and \( M \) a constant of motion. Of course, if \( \Lambda = a = 0 \), one recovers the Schwarzschild solution. If \( M = \Lambda = 0 \), it yields the 2-dimensional Rindler metric. Therefore, the resulting gravity theory differs from General Relativity only by the addition of a Rindler acceleration, see also [28].

A geodesics study of time-like test particles moving in a 4-dimensional spherical, symmetric background, according to Eqs. (1), (3), and (4), results in the following eqs:

\[ E = \frac{\ell^2}{2} + V_{\text{eff}} \]

\[ V_{\text{eff}} = -\frac{M}{r} + \frac{\ell^2}{2r^2} + ar \left(1 + \frac{\ell^2}{r^2}\right), \]

where \( E = \text{const.} \) and \( V_{\text{eff}} \) is the effective potential.

When we apply the previous solution to a galactic arena, we set the cosmological constant equal to zero (\( \Lambda = 0 \)) since the mean energy density of a galaxy is much larger than the cosmic inferred \( \Lambda \). We set also \( l = 0 \), to avoid an additional angular momentum to the system that in fact shall not account for the kinematical deficit of rotational curves.

Considering now the effects on rotation curves, the Rindler acceleration yields an additional term in the rotation’s speed \( (v_T) \):

\[ v_T(r) = \sqrt{\frac{\ell^2}{2r^2} + ar}, \]

where \( \phi_T \) is the total gravitational potential that test particles (stars and gas) feel. This is the original Grumiller’s model of gravity at large distances [19]. The new Rinder acceleration term should account for the kinematical difference of the observed and predicted rotation curves. Notice that Eq. (7) diverges asymptotically, at large radius. This is not an observed behaviour in typical rotation curves, but on the contrary they tend to slowly decrease after a few optical radii [22]. Therefore, as a generalization of the previous model one may intend to determine a power-law dependence in the Rindler term, as suggested in Ref. [19]. The new term should not diverge at large distances. Accordingly, we will consider the following generalized Grumiller model:

\[ v_{\text{eff}}(r) = \sqrt{\frac{\ell^2}{2r^2} + a r^n}, \]

where there are two undetermined Rindler parameters \((a, n)\). The case \( n = 1 \) yields acceleration units to \( a \), but a different \( n \) implies \( \text{length}^{n-1} \) units; one could extract an acceleration parameter here if one defines \( a r^n \equiv a_{\text{new}} (r/r_{\text{new}})^{n-1} \), but we would only add an extra parameter \((r_{\text{new}})\) that is completely degenerated with \( a_{\text{new}} \). This could be done a posteriori, if needed.

3. Rotation curve model

In this section we closely follow the model presented in Ref. [22], but for the sake of completeness we present it here again. The galaxy model consists of gas and stars orbiting on a disk plane, and instead of dark matter we include the Rindler acceleration, explained in the previous section. The contribution of gas is computed by integrating the surface brightness as in the standard Newtonian case by assuming an infinitely thin disk. One directly integrates its contribution to the rotation curve \((v_T)\).

For stars we take a standard Freeman disk [30, 31]:

\[ \rho_*(r) = \frac{M_d}{2\pi v_d^2} e^{-r/r_d}, \]

where \( M_d \) is the mass of the disk and \( r_d \) its radius. The rotation curve contribution from stars within standard New-
tonian dynamics, yields \[v^2(r) = \frac{GM_d}{2r_d} \left( \frac{r}{r_d} \right)^2 \left[ I_0 \left( \frac{r}{2r_d} \right) K_0 \left( \frac{r}{2r_d} \right) - I_1 \left( \frac{r}{2r_d} \right) K_1 \left( \frac{r}{2r_d} \right) \right], \tag{10}\]
where \(I\) and \(K\) are the modified Bessel functions.

The stars’ contribution to the rotation curves is normally multiplied by the mass-to-light ratio \((\Upsilon_*\), that is an additional free parameter in the mass model, introduced because we generally can only measure the distribution of the light instead of the mass. When we estimate the Rindler parameters \((a, n)\), \(\Upsilon_*\) is an important source of uncertainty, because these parameters are degenerate through Eq. \((11)\), see below. However, since stars have a major contribution near the center of the galaxy and the Rindler acceleration contribute most at large distance, \(\Upsilon_*\) does not significantly affect the uncertainties of the Rindler parameters, as we have shown in Ref. \cite{23}.

The \(\Upsilon_*\) has been modeled, e.g. in Salpeter \cite{33}, Kroupa \cite{34}, and Bottema \cite{35}, but the precise value for an individual galaxy is not well known and depends on extinction, star formation history, initial mass function, among others. Some assumptions have to be made respect to \(\Upsilon_*\) in order to reduce the number of free parameters in the model. In a previous work \cite{23}, one of us (JLCC) has studied the Kroupa, diet-Salpeter, and \(\Upsilon_*\) as a model-independent free parameter. It was shown that the different stellar mass models do not significantly change the determined value of the Rindler parameters for most of the galaxies, and from the three models, the free \(\Upsilon_*\) yields the best fits to the rotation curves. Thus, for the purpose of this letter, we adopt the \(\Upsilon_*\) free mass model.

Gathering all contributions to the total \((T)\) rotation curve and including a generalized Rindler (GR) term \[v^2_T(r) = \Upsilon_* v^2_* + v^2_G + v^2_{GR}(r), \tag{11}\]
where we explicitly use \(\Upsilon_*\) and therefore assume \(M_d\) with a solar mass-to-light in Eqs. \((11),(10)\), the power-law generalized Rindler term is \[v^2_{GR}(r) \equiv a |r|^n. \tag{12}\]

The case \(n = 1\) is the original model of modified gravity at large distances \cite{11,23}, as the Rindler contribution in Eq. \((7)\). The new free parameters of the model of galactic rotation curves are \(a\) and \(n\), and they have to be determined by observations. In standard dark matter profiles such as NFW \cite{25,26}, Burkert \cite{27}, pseudo-isothermal, or alternative Bound Dark Matter \cite{11} one also uses two free parameters, and therefore the number of degrees of freedom to fit is same as in the generalized Rindler model; for a comparison of these profiles see Refs. \cite{12,13}. To extract information for the Rindler parameters, as an input we will need the observational rotation curve and the computed gas contribution.

4. Rotation curve fits and results

To perform the fits we employed the same method as in Ref. \cite{23}, but here it is applied to a set of thirty galaxies that has been used in the past to test alternative gravity models \cite{18}. Most of galaxies possess wanted properties such as smoothness in the data, symmetry, and they are extended to large radii. We fit the observational velocity curve to the theoretical model \((11)\) using the \(\chi^2\) goodness-of-fit test \((\chi^2\) test), that computes the parameters’ best fits. In general the \(\chi^2\) test statistics are of the form:

\[\chi^2 = \sum_{i=1}^{m} \frac{(v_{obs, i} - v_{model, i}(r, a, n))^2}{\sigma_i}, \tag{13}\]
where \(\sigma\) is the standard deviation, and \(m\) is the number of observations. One defines the reduced \(\chi^2_{red} \equiv \chi^2/(m - p - 1)\), in which \(m\) is the number of observations and \(p\) is the number of fitted parameters. The total velocity \((11)\) defines our model - \(v_{model}\) in Eq. \((13)\) - and depends on the three parameters: \(\Upsilon_*\) (or alternatively the mass of the disk \(M_d\) that we actually fit), and the two Rindler parameters \((a, n)\).

We firstly analyze the original Rindler model \((n = 1)\) and proceed to fit the parameters \(a\) and \(M_d\). Their physical units are \(\text{km}^2/\text{s}^2\text{kpc}\) and \(M_\odot\), respectively. We assume a flat prior for these parameters in the following intervals: \(10^7 < M_d < 10^{12}\) and \(0 < a < 10000\). Given the large space that would require to show all rotation curve fits and parameters’ contour plots, we include this information as supplementary material. We gather our results in Table \(1\). The uncertainties in the rotation velocity are reflected in the uncertainties in the model parameters. One observes some spread in the values for \(a\), ranging from \(341.26^{+64.84}_{-64.46}\) for M31 to \(2891.25^{+293.75}_{-287.25}\) for NGC7339 to account for a difference of an order of magnitude, but the uncertainties are small to account for such a difference. In addition to this discrepancy, the fits to some of the galaxies present very high \(\chi^2_{red}\) values that result in poor fittings. Only thirteen (of thirty) galaxies had \(\chi^2_{red} \leq 1\), and for these later galaxies we have included a distribution plot (as supplementary material) that shows a big spread in the values of the acceleration parameter. We also plot the B-band mass-to-light ratios that are similar to others reported in the literature \cite{36,18}.

For the generalized model \((n \neq 1)\) we consider a flat prior in the interval \(0 < n < 10\), and the same other conditions as previous model. We determine now the two Rindler parameters \((a, n)\) and the stellar disk’s mass. The
results are shown in Table 2. Again a spread is observed in the acceleration parameter, ranging from 0.26 ± 0.10 for M31 to 11605 ± 564.60 for UGC 10981 resulting in a difference of four orders of magnitude. Since the \( a \) value for M31 is very small and is the only one with a value much less than one hundred, and since we did not include gas data to the analysis, we may exclude it for this analysis. The second lowest value of \( a \) is 113.39 ± 85.89 for UGC 6917. Still there is a difference of two orders of magnitude between the smallest and biggest values for the Rindler acceleration. On the other hand, the power-law exponent ranges from 0.16 ± 0.02 for NGC 6503 to 3.31 ± 0.54 for M31 or, again excluding M31, 1.60 ± 0.03 for IC 2574, which is an order of magnitude difference. For both parameters the uncertainties are too small to account for the encountered differences. Similarly as the previous Rindler model, one only has thirteen (of thirty) galaxies with \( \chi^2_{\text{red}} \leq 1 \), and again, for these later galaxies we have included distribution plots (as supplementary material) that show a big spread in both parameters of the generalized Rindler model.

By comparing both fits (\( n = 1 \) vs \( n \neq 1 \)), the goodness of fits are better in the generalized model for sixteen galaxies, and for fourteen both models are equally well fitted. The Rindler acceleration varied for the standard Rindler model one order of magnitude and for the generalized model two orders of magnitude. In our previous work, when one of us analyzed the THINGS’ galaxies [23], both the \( \chi^2_{\text{red}} \) and Rindler acceleration values changed more substantially: two and three orders of magnitude, respectively. With respect to the power-law exponent of the generalized model, the present analysis results in one order of magnitude difference, whereas for the THINGS galaxies resulted in two orders of magnitude. The reason to have smaller differences in the computed parameters is that the present set of galaxies, while it is a larger collection, its uncertainties in data are bigger. Although the present analysis soften the difference in the parameter computation, still such differences are not justified. On the other hand, the mass-to-light ratios found do not present systematic differences.

From the set of galaxies considered in the present work and that in Ref. [23] there is a common galaxy, NGC 2403. The data considered are different and subject to different systematics. However, we may expect some similar parameter estimation. The computed parameters in Ref. [23] were, for the \( n = 1 \) Rindler model, \( a = 797.22^{+57.65}_{-30.32} \) and \( M_D = 10^{10.2^{+5.8}_{-4.8}} M_\odot \), whereas for the present computation table 1 shows \( a = 900 \pm 20 \) and \( M_D = 1.4^{+0.4}_{-0.3} \times 10^{10} M_\odot \). Clearly, both Rindler parameters and stellar disk masses are within 1 \( \sigma \). For the generalized model the previous work gives \( a = 3070 \pm 16, \) \( n = 0.59 \pm 0.002, \) and \( M_D = 10^{9.9^{+1.8}_{-1.8}} M_\odot \), whereas in the present work we have \( a = 3600 \pm 200, \) \( n = 0.54 \pm 0.02, \) and \( M_D = 8 \pm 0.4 \times 10^9 M_\odot \). In this case, the Rindler parameters are not quite different, but given the uncertainties, they are a few \( \sigma \) away from each other; stellar disk masses are within 1 \( \sigma \).

The discrepancy in Rindler parameters’ uncertainties may be due to the fact that the THINGS sample include more data and are more precise. On the other hand, we do not expect a big influence of a small bulge, that was taken into account in Ref. [23], on the computed values of the Rindler parameters, since the main influence of the modified gravity is in the outer parts of the galaxies, where the bulge, or even the disk, counts less. In our present work, we have not taken into account bulge contributions, since it is known the present set has negligible bulges [18].

We now compare the Rindler models with standard dark matter profiles, such as NFW [25, 26] and Burkert [27]. The former is an example of a cuspy dark matter profile, whereas the latter is shallow. The explicit computations are as those done in our previous works [13, 23] and are included as supplementary material. To compare among the different models we constructed Table 3 with the \( \chi^2_{\text{red}} \) values for NFW, Burkert, standard Rindler with \( n = 1 \), and generalized Rindler (\( n \)-free), for the free stellar mass model. The results are as follows:

- As already mentioned, the Rindler model with two free parameters (\( a, n \)) fits equally well or better than the model with a single parameter (\( a, n = 1 \)) for all galaxies.

- The standard Rindler model (\( n = 1 \)) fits worst than Burkert’s profile, but similarly well as NFW. The standard Rindler model achieves an equally well or a better fit than both NFW and Burkert only for six galaxies (M 31, NGC 3949, NGC 3953, NGC 4183, NGC 4217, and UGC 6917) and, in addition, it fits better than Burkert for one galaxy (M 33), and it fits equally well or better than NFW for eleven galaxies (DDD 47, ESO 287-G13, IC 2574, NGC 3877, NGC 3917, NGC 3972, NGC 4085, NGC 7339, UGC 6399, UGC 6983, and UGC 11455). In summary, the NFW’s profile fits equally well or better for 16 galaxies (out of 30) and Burkert’s profile achieves an equally well or a better fit for 26 galaxies (out of 30) than the standard Rindler model.

- The power-law generalized Rindler model (\( n \)-free) fits worst than Burkert’s profile, but slightly better than NFW. This Rinder model fits equally well or better than both NFW and Burkert models for six galaxies (M 31, M 33, NGC 3953, NGC 4183, NGC 6503, and UGC 6917) and, in addition, it fits equally well or better than NFW for fourteen galaxies (DDD 47, ESO 287-G13, IC 2574, NGC 3877, NGC 3917, NGC 3949, NGC 3972, NGC 4085, NGC 4217, NGC 7339, UGC 128, UGC 6399, UGC 6983, and UGC 11455). In summary, the NFW profile fits equally well or better for 14 galaxies (out of 30) and Burkert’s profile achieves an equally well or better fit for 25 galaxies (out of 30) than the generalized Rindler model. The fact that Burkert’s shallow profile fits
Table 1: Best fits for the standard Rindler model (n = 1). It is shown in column (2) the galactic type and in (3) its disk radius, in (4) the acceleration parameter, in (5) the galactic disk mass, (6) the B-band mass-to-light ratio in solar units, and in (7) the $\chi^2_{\text{red}}$.

| Galaxy | Type | $R_d$ (kpc) | $\alpha$ (km/s^2/kpc) | $M_d$ ($M_\odot$) | $\Gamma_B^0$ | $\chi^2_{\text{red}}$ |
|--------|------|-------------|------------------------|------------------|-------------|-------------------|
| DDO 47 | IB   | 0.50        | $820^{+104}_{-100}$    | $1.0^{+2.4}_{-0.1} \times 10^9$ | 0.1         | 4.9               |
| ESO 116-G12 | SBcd | 1.70        | $1010 \pm 50$         | $3.0 \pm 0.4 \times 10^9$ | 0.6         | 3.8               |
| ESO 287-G13 | Sb   | 3.28        | $920 \pm 30$          | $3.9^{+3.0}_{-0.1} \times 10^{10}$ | 1.3         | 1.8               |
| IC 1027 | S}Bm | 1.78        | $400^{+30}_{-100}$    | $1.0^{+2.6}_{-0.1} \times 10^9$ | <0.1       | 38.9              |
| M 31   | Sb   | 4.50        | $340 \pm 60$          | $1.68^{+0.04}_{-0.01} \times 10^{11}$ | 8.4         | 1.6               |
| M 33   | Sc   | 1.42        | $900 \pm 30$          | $5.0 \pm 0.1 \times 10^9$ | 9.0         | 4.3               |
| NGC 55 | SBm  | 1.60        | $870 \pm 50$          | $5.9 \pm 2.3 \times 10^8$ | 0.2         | 3.7               |
| NGC 300 | Sd   | 1.70        | $790^{+100}_{-90}$    | $2.7 \pm 0.3 \times 10^9$ | 1.2         | 0.7               |
| NGC 1099 | Sbc  | 3.40        | $580 \pm 20$          | $5.0 \pm 0.2 \times 10^9$ | 1.3         | 3.8               |
| NGC 2403* | SABm | 2.08        | $900 \pm 20$          | $1.4^{+1.5}_{-0.3} \times 10^9$ | 1.7         | 4.0               |
| NGC 3877 | Sc   | 2.80        | $1500 \pm 400$        | $2.7 \pm 0.6 \times 10^9$ | 1.0         | 0.9               |
| NGC 3917 | Scd  | 3.10        | $1000 \pm 100$        | $1.3 \pm 0.2 \times 10^9$ | 1.2         | 3.6               |
| NGC 3949 | Sbc  | 1.70        | $2300 \pm 1200$       | $1.5 \pm 0.5 \times 10^9$ | 0.8         | 0.4               |
| NGC 3953 | SBBc | 3.80        | $1300 \pm 400$        | $8.7 \pm 1.2 \times 10^9$ | 2.1         | 0.4               |
| NGC 3972 | Sbc  | 2.00        | $1800 \pm 300$        | $3.8^{+1.0}_{-0.0} \times 10^9$ | 0.6         | 0.5               |
| NGC 4085 | Sc   | 1.60        | $2700 \pm 600$        | $3.4^{+3.0}_{-2.7} \times 10^9$ | 0.5         | 2.0               |
| NGC 4100 | Sbc  | 3.57        | $500 \pm 100$         | $6.7 \pm 0.4 \times 10^9$ | 2.7         | 1.0               |
| NGC 4157 | Sb   | 2.60        | $940 \pm 100$         | $5.2 \pm 0.5 \times 10^9$ | 1.7         | 0.8               |
| NGC 4183 | Scd  | 3.20        | $370 \pm 70$          | $1.6 \pm 0.2 \times 10^9$ | 1.7         | 0.1               |
| NGC 4217 | Sb   | 2.90        | $1100 \pm 200$        | $4.6^{+0.5}_{-0.6} \times 10^9$ | 2.2         | 0.7               |
| NGC 5585 | SABc | 1.26        | $870 \pm 30$          | $1.3 \pm 0.1 \times 10^9$ | 0.9         | 7.3               |
| NGC 6597* | Sc   | 1.74        | $610 \pm 10$          | $1.37^{+0.03}_{-0.1} \times 10^9$ | 2.7         | 6.8               |
| NGC 7339* | SABb | 1.50        | $2900 \pm 300$        | $1.2 \pm 0.1 \times 10^9$ | 1.6         | 1.4               |
| UGC 128  | Sd   | 6.40        | $350 \pm 70$          | $2.6^{+0.7}_{-0.7} \times 10^9$ | 3.0         | 0.2               |
| UGC 6399 | Sm   | 2.40        | $660 \pm 260$         | $3.6 \pm 2.2 \times 10^9$ | 2.3         | 0.3               |
| UGC 6917 | SBd  | 2.90        | $590 \pm 170$         | $1.0 \pm 0.2 \times 10^9$ | 2.3         | 0.2               |
| UGC 6983 | SBcd | 2.70        | $510 \pm 100$         | $1.1 \pm 0.2 \times 10^9$ | 2.7         | 0.7               |
| UGC 8017* | Sb   | 2.10        | $2000 \pm 150$        | $1.58 \pm 0.04 \times 10^{11}$ | 4.0         | 6.3               |
| UGC 10981* | Sbc  | 5.40        | $430 \pm 40$          | $2.10 \pm 0.02 \times 10^{11}$ | 1.8         | 16.0              |
| UGC 11455* | Sc  | 5.30        | $2800 \pm 200$        | $1.2 \pm 0.1 \times 10^{11}$ | 2.6         | 17.7              |

* For these galaxies we have no gas data.

better than the cuspy NFW profile to some of these galaxies has been reported in the literature \[57\] as well as further analysis on the NFW fits in Ref. \[38\].

5. Conclusions

Using a collection of rotation curves of thirty galaxies, we have tested the standard (n = 1) and power-law generalized (n-free) Grunmiller’s model of modified gravity at large distances. The corresponding gravitation potential implies a new (Rindler) acceleration constant in nature that affects the rotation curve as $v_f^2(r) = \mathcal{F} v^2 + v_d^2 + a_r r^n$, where the last term would replace the contribution of the dark matter profile.

The results of the fits are in Tables \[1\] and \[2\] and a comparison of the goodness-of-fit to NFW’s and Burkert’s profiles is presented in Table \[3\]. Our results show that: i) the standard Rindler model (n = 1) does not achieve good fits since only thirteen (of thirty) galaxies had $\chi^2_{\text{red}} \leq 1$, and these best-fitted galaxies also show a big spread in the acceleration parameter; ii) the power-law generalized model (n ≠ 1) does achieve an equally well (14/30) or a better fit (16/30) than the standard Rindler’s model for all galaxies, but again only thirteen (of 30) galaxies had $\chi^2_{\text{red}} \leq 1$, and also these best-fitted galaxies show big spreads in the Rindler parameters; iii) the comparison of these modified gravity models with standard dark matter profiles yields that the standard Rindler model (n = 1) fits worst Burkert’s profiles, but it fits equally well as NFW. The generalized model achieves better fits than NFW’s profile, but much poorer fits than Burkert’s profile.

The main problem, however, is that both Rindler parameters (a, n) show at least one order of magnitude spread that cannot be explained by the corresponding uncertainties, not pointing to single universal values.

In comparison with previous, similar studies \[22\], where
| Galaxy     | $a$ (km$^2$/s$^2$/kpc$^n$) | $n$ | $M_4$ ($M_\odot$) | $\Gamma_{10}^B$ | $\chi^2_{\text{red}}$ |
|------------|--------------------------|-----|------------------|---------------|---------------------|
| DDO 47     | $420 \pm 60$             | 1.5 | $3.3^{+2.8}_{-2.2} \times 10^7$ | 0.3           | 1.2                 |
| ESO 116-G12| $1100 \pm 200$           | 1.0 | $2.7 \pm 0.4 \times 10^7$   | 0.6           | 3.7                 |
| ESO 287-G13| $320^{+400}_{-70}$       | 1.3 | $4.3 \pm 0.1 \times 10^8$   | 1.5           | 1.6                 |
| IC 2574    | $180 \pm 8$              | 1.6 | $1.1 \pm 0.2 \times 10^6$   | 0.1           | 2.4                 |
| M 31       | $0.3^{+1.0}_{-0.2}$      | 3.3 | $1.8 \pm 0.0 \times 10^{11}$ | 8.8           | 1.1                 |
| M 33       | $1600 \pm 100$           | 0.8 | $3.8 \pm 0.1 \times 10^7$   | 0.7           | 3.3                 |
| NGC 55     | $1000 \pm 100$           | 0.9 | $3.9^{+1.5}_{-1.3} \times 10^8$ | 0.1           | 3.5                 |
| NGC 300    | $630^{+1300}_{-120}$     | 1.1 | $3.0 \pm 0.3 \times 10^9$   | 1.3           | 0.7                 |
| NGC 1090   | $1400 \pm 300$           | 0.8 | $4.5 \pm 0.2 \times 10^{10}$ | 1.2           | 3.4                 |
| NGC 2403   | $3600 \pm 200$           | 0.5 | $8.0 \pm 0.4 \times 10^7$   | 1.0           | 2.3                 |
| NGC 3877   | $550^{+1500}_{-340}$     | 1.4 | $3.2 \pm 0.4 \times 10^8$   | 1.2           | 0.9                 |
| NGC 3917   | $430^{+700}_{-140}$      | 1.3 | $1.7 \pm 0.2 \times 10^{10}$ | 1.5           | 3.4                 |
| NGC 3949   | $800^{+1100}_{-500}$     | 1.5 | $1.8 \pm 0.2 \times 10^{10}$ | 0.9           | 0.4                 |
| NGC 3953   | $7900^{+13000}_{-3100}$  | 0.5 | $6.0^{+2.6}_{-2.2} \times 10^{10}$ | 1.5           | 0.4                 |
| NGC 3972   | $880^{+13000}_{-350}$    | 1.3 | $6.4^{+2.3}_{-2.1} \times 10^{10}$ | 1.0           | 0.5                 |
| NGC 4085   | $1400^{+2000}_{-600}$    | 1.3 | $5.6^{+1.7}_{-1.5} \times 10^{10}$ | 0.8           | 2.0                 |
| NGC 4100   | $770^{+1100}_{-440}$     | 0.9 | $6.6 \pm 0.4 \times 10^{10}$ | 2.6           | 1.0                 |
| NGC 4157   | $2600^{+11000}_{-1100}$  | 0.7 | $4.5 \pm 0.5 \times 10^9$ | 1.5           | 0.7                 |
| NGC 4183   | $830^{+3000}_{-390}$     | 0.8 | $1.4 \pm 0.2 \times 10^9$ | 1.4           | 0.1                 |
| NGC 4217   | $730^{+1000}_{-410}$     | 1.2 | $4.8 \pm 0.4 \times 10^9$ | 2.3           | 0.7                 |
| NGC 5585   | $1000 \pm 100$           | 0.9 | $1.1 \pm 0.1 \times 10^9$ | 0.7           | 6.9                 |
| NGC 6503   | $7900 \pm 400$           | 0.2 | $4.3 \pm 0.3 \times 10^9$ | 0.9           | 1.4                 |
| NGC 7339   | $2000 \pm 300$           | 1.2 | $1.3 \pm 0.1 \times 10^9$ | 1.8           | 1.4                 |
| UGC 128    | $260^{+3000}_{-180}$     | 1.1 | $2.8 \pm 0.6 \times 10^9$ | 3.2           | 0.2                 |
| UGC 6399   | $240^{+700}_{-500}$      | 1.4 | $5.1^{+1.8}_{-1.6} \times 10^9$ | 3.2           | 0.3                 |
| UGC 6917   | $110^{+200}_{-90}$       | 1.6 | $1.3 \pm 0.1 \times 10^9$ | 2.9           | 0.2                 |
| UGC 6983   | $340^{+2000}_{-120}$     | 1.1 | $1.2 \pm 0.1 \times 10^9$ | 2.9           | 0.7                 |
| UGC 8017   | $8500 \pm 500$           | 0.6 | $1.1 \pm 0.0 \times 10^9$ | 2.9           | 4.8                 |
| UGC 10981* | $12000 \pm 600$          | 0.2 | $1.5 \pm 0.0 \times 10^9$ | 1.2           | 10.4                |
| UGC 11455  | $650^{+1000}_{-120}$     | 1.5 | $1.5 \pm 0.0 \times 10^{10}$ | 3.3           | 16.9                |

* The upper limit for $a$ has been extended to find the $\chi^2$ minimum.

seventeen THINGS galaxies were employed, the results here are less conclusive, since the spreads on the computed Rinder parameters are smaller. Nevertheless, our present work points again to inconsistent standard and power-law generalized Rinder models.

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| Galaxy       | $\chi^2_{red}$ | Burkert | NFW | $(n = 1)$ | $(n\text{-free})$ |
|-------------|---------------|---------|-----|----------|-----------------|
| DDO 47      | 1.0           | 5.7     | 4.9 | 1.2      |                 |
| ESO 116-G12 | 0.9           | 2.6     | 3.8 | 3.7      |                 |
| ESO 287-G13 | 1.5           | 2.1     | 1.8 | 1.6      |                 |
| IC 2574     | 2.0           | 43.5    | 38.9| 2.4      |                 |
| M 31        | 1.9           | 1.7     | 1.6 | 1.1      |                 |
| M 33        | 5.5           | 4.1     | 4.3 | 3.3      |                 |
| NGC 55      | 0.3           | 2.9     | 3.7 | 3.5      |                 |
| NGC 300     | 0.4           | 0.6     | 0.7 | 0.7      |                 |
| NGC 1090    | 0.8           | 1.8     | 3.8 | 3.4      |                 |
| NGC 2403    | 1.5           | 1.1     | 4.0 | 2.3      |                 |
| NGC 3877    | 0.3           | 1.1     | 0.9 | 0.9      |                 |
| NGC 3917    | 0.7           | 3.9     | 3.6 | 3.4      |                 |
| NGC 3949    | 0.6           | 0.6     | 0.4 | 0.4      |                 |
| NGC 3953    | 0.5           | 0.6     | 0.4 | 0.4      |                 |
| NGC 3972    | 0.1           | 0.6     | 0.5 | 0.5      |                 |
| NGC 4085    | 0.5           | 2.4     | 2.0 | 2.0      |                 |
| NGC 4100    | 0.6           | 0.9     | 1.0 | 1.0      |                 |
| NGC 4157    | 0.5           | 0.6     | 0.8 | 0.7      |                 |
| NGC 4183    | 0.1           | 0.1     | 0.1 | 0.1      |                 |
| NGC 4217    | 0.3           | 0.7     | 0.7 | 0.7      |                 |
| NGC 5585    | 0.4           | 1.9     | 7.3 | 6.9      |                 |
| NGC 6503    | 2.1           | 5.3     | 6.8 | 1.4      |                 |
| NGC 7339    | 1.2           | 1.5     | 1.4 | 1.4      |                 |
| UGC 128     | 0.0           | 0.2     | 0.2 | 0.2      |                 |
| UGC 6399    | 0.0           | 0.3     | 0.3 | 0.3      |                 |
| UGC 6917    | 0.4           | 0.3     | 0.2 | 0.2      |                 |
| UGC 6983    | 0.6           | 0.7     | 0.7 | 0.7      |                 |
| UGC 8017    | 3.1           | 3.7     | 6.3 | 4.8      |                 |
| UGC 10981   | 7.2           | 6.6     | 16.0| 10.4     |                 |
| UGC 11455   | 13.0          | 19.3    | 17.7| 16.9     |                 |

Table 3: Summary of the $\chi^2_{red}$ values for the different dark matter profiles and gravity models.