Heterogeneity and Disorder: Contributions of Rolf Landauer

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Abstract

Rolf Landauer made important contributions to many branches of science. Within the broad area of transport in disordered media, he wrote seminal papers on electrical conduction in macroscopically inhomogeneous materials, as well as fundamental analyses of electron transport in quantum mechanical systems with disorder on the atomic scale. We review here some of these contributions.

Key words: transport, disorder, conductance, heterogeneous materials

1. Introduction

Rolf Landauer was recognized for outstanding accomplishments in many branches of science. In addition to his work on transport in inhomogeneous systems, which will be the focus of the present article, Landauer wrote papers on noise and fluctuations, on nonlinear wave propagation and soliton formation, on ferroelectric instabilities and displacive soft modes, entropy production in systems out of equilibrium, philosophical principles of science and technology, and above all, on the physical limits to computation. His influence in this last area was of such a magnitude that he was the subject of a “Profile” article by Gary Stix in the September 1998 issue of \textit{Scientific American} \cite{1}. The article was titled “Riding the Back of Electrons”, and subtitled “Theoretician Rolf Landauer remains a defining figure in the physics of information.”

During his lifetime, Landauer received many awards for his work including the Ballantine Medal of the Franklin Institute in 1992, the 1995 Buckley Prize of the American Physical Society, the LVMH, Inc. Science for Art Prize in 1997, the 1998 IEEE Edison Medal, an honorary doctorate from the Technion in 1991, and a Centennial Medal from Harvard University in 1993. He was elected to the National Academy of Science, the National Academy of Engineering, and the American Academy of Arts and Sciences in the US, and to the European Academy of Arts.

Landauer’s influence on science and technology was not limited to the importance of his scientific discoveries and research. He had much to say about the conduct of research and about philosophical issues of how science should be interpreted. He also delighted in challenging entrenched ideas and in forcing people to think more carefully about the foundations of their work—see Fig.\textsuperscript{1} for a typical appearance of Rolf Landauer in this mode. There is perhaps no better way to illustrate this aspect of his character than to cite some of the titles of articles that Landauer wrote in the last decade of his life: “Light faster than light” \cite{2}, “Is quantum mechanics useful?” \cite{3}, “Mesoscopic noise: Common sense view” \cite{4}, “Zig-zag path to understanding” \cite{5}, “Conductance is transmission” \cite{6}, “The physical nature of information” \cite{7}, and “Fashions in science and technology” \cite{8}.

Due to limitations of space, our discussion of Landauer’s scientific contributions will be restricted to his...
work on transport in inhomogeneous systems, and some closely related work on quantum mechanical effects in mesoscopic systems. However, we include at the end of the article a brief summary of Landauer’s personal history.

2. Landauer and inhomogeneous systems

Although major advances in the understanding of electrical conductivity of disordered and heterogeneous media were made by a number of his contemporaries, Landauer’s approach was unique in this field: On one hand, people like William Fuller Brown, Jr. [9] or Zvi Hashin and Shmuel Shtrikman [10] confined themselves to a discussion of systems where a classical physics approach is valid, and described the electrical response in terms of a position dependent conductivity. This lead, eventually, to concepts like percolation threshold, which determines the macroscopic response of a metal/insulator mixture [11].

On the other hand, people like Philip W. Anderson [12] and Neville F. Mott [13, 14] focussed upon the effects of microscopic disorder on the detailed form of the quantum mechanical wave function. This lead, eventually, to concepts like Anderson localization and Mott transition as governing the macroscopic response of such a system. By contrast, Landauer often tried to combine quantum considerations with classical physics considerations. This is clearly evident in his work on electromigration [13, 16], but also in his classic paper which derived the famous “Landauer Formula” [17], where he used the classical physics Einstein relation in order to derive the macroscopic conductivity of a one dimensional disordered system from the diffusion coefficient of a single electron. A similar combination of quantum and classical approaches can also be found in Landauer’s work on conductivity of cold-worked metals [18] and on Lorentz corrections to electrical conductivity [19].

3. Classical inhomogeneous systems

Rolf Landauer became interested in inhomogeneous systems early on in his career. In 1952 he published a paper entitled “The electrical resistance of binary metallic mixtures” [20], where he developed a simple approximation for calculating that resistance, which is a macroscopic property of such mixtures. This approximation yields the following equation for the macroscopic scalar conductivity \(\sigma_{\text{eff}}\) of a multi-constituent, \(d\)-dimensional, isotropic composite medium in terms of the constituent scalar conductivities \(\sigma_i\) and constituent volume fractions \(p_i\), with \(\sum p_i = 1\):

\[
0 = \sum p_i \frac{\sigma_{\text{eff}} - \sigma_i}{\sigma_{\text{eff}} + (d-1)\sigma_i}.
\]

In the case where there are just two components, this becomes a quadratic equation, which has the explicit solution

\[
2 \frac{d-1}{d} \sigma_{\text{eff}} = \sigma_1 \left( p_1 - \frac{1}{d} \right) + \sigma_2 \left( p_2 - \frac{1}{d} \right)
+ \left\{ \left( \sigma_1 \left( p_1 - \frac{1}{d} \right) + \sigma_2 \left( p_2 - \frac{1}{d} \right) \right)^2
+ 4 \frac{d-1}{d^2} \sigma_1 \sigma_2 \right\}^{1/2}.
\]

This approximation is still widely used, especially in the context of a disordered microstructure, and is generally known as the self-consistent (or symmetric) effective medium approximation (SEMA). This approximation had actually already been discovered in 1935 by D. A. G. Bruggeman [21], who was then a high school teacher in the Netherlands. Landauer did not know about this, (nor did the reviewer of his manuscript at J. Appl. Phys.) and he therefore achieved this breakthrough independently. This approximation can be contrasted with an earlier approximation, known as the Clausius-Mossotti (CM) or Maxwell Garnett approximation. In the latter approximation, \(\sigma_{\text{eff}}\) satisfies a linear algebraic equation which can be expressed in the following concise form:

\[
\frac{\sigma_{\text{eff}} - \sigma_2}{\sigma_{\text{eff}} + (d-1)\sigma_2} = p_1 \frac{\sigma_1 - \sigma_2}{\sigma_1 + (d-1)\sigma_2}.
\]

The CM result is non-symmetric in the two constituents: The \(\sigma_2\) constituent plays the role of host while the \(\sigma_1\) constituent plays the role of inclusions. It is easy to generalize the CM result to any number of different inclusion constituents that are embedded in one common host constituent. This is achieved by rewriting \(\sigma_2\) as \(\sigma_{\text{host}}\), \(\sigma_1, p_1\) as \(\sigma_i, p_i\), and summing the right hand side of the resulting equation over the different types of inclusions \(i\). This leads to an equation that is still equivalent to a linear algebraic equation for \(\sigma_{\text{eff}}\).

In contrast with CM, the SEMA result is symmetric in all of the constituents. When SEMA is extended to more than two constituents, the result for \(\sigma_{\text{eff}}\) then becomes the solution of a polynomial equation with order equal to the number of constituents.

Interestingly, neither Bruggeman nor Landauer realized, at first, that SEMA predicts the existence of
a conductivity threshold in the case where one of the constituents is a perfect insulator. This crucial consequence, which does not follow from the CM approximation, was first appreciated by others [22]. Somewhat later it was realized that the conductivity threshold in this system is associated with a percolation threshold of the conducting constituent [23], which is a geometric property of the microstructure. This threshold is a critical point, i.e., a singular point in the physical response of the system as function of the physical parameters. This point is characterized by the “percolation threshold” \( p_c = 1/d \): When the volume fraction of the conducting constituent \( p_M \) is greater than \( p_c \), the macroscopic conductivity \( \sigma_{\text{eff}} \) is nonzero, but it vanishes when \( p_M \leq p_c \). For \( p_M \geq p_c \), \( \sigma_{\text{eff}} \) increases linearly with increasing \( p_M \), starting from 0:

\[
\frac{\sigma_{\text{eff}}}{\sigma_M} = \frac{d}{d-1} \left( p_M - \frac{1}{d} \right), \quad p_M \geq p_c = \frac{1}{d},
\]

where \( \sigma_M \) is the scalar conductivity of the conducting constituent. There is thus no discontinuity in the function \( \sigma_{\text{eff}}(p_M) \), only a discontinuous slope at \( p_M = p_c \). The value of \( p_c = 1/d \) predicted by SEMA depends only on the dimensionality. That and the linear dependence of \( \sigma_{\text{eff}} \) upon \( p_M \) are leading characteristics of SEMA. In practice, both characteristics are rather inaccurate: Experiments on real continuum composites show that the value of \( p_c \) depends on details of the microstructure [24]. Only in the case of a two-dimensional (\( d = 2 \)) disordered composite medium where the microstructure is symmetric in the two constituents is the value \( p_c = 0.5 \), predicted by SEMA, correct. The linear form of \( \sigma_{\text{eff}}(p_M) \), predicted by SEMA for \( p_M \geq p_c \), is also usually contradicted by experiments on real composites and discrete network models, although the SEMA prediction that \( \sigma_{\text{eff}}(p_M) \) is continuous at \( p_c \) is verified. In reality, the behavior of \( \sigma_{\text{eff}}(p_M) \) for small positive values of \( p_M - p_c \) is well described by a power law \( \sigma_{\text{eff}}(p_M) \propto \sigma_M(p_M - p_c)^t \), where the “critical exponent” \( t \) has values that depend on general properties of the microstructure but not on minute details—that is known as “universality”. For example, in discrete network models with finite-range-correlated randomness, it is found that \( t \approx 1.3 \) when \( d = 2 \), \( t \approx 2.0 \) when \( d = 3 \), \( t = 0.5 \) when \( d \geq 6 \) [25]. In continuum composites, the value of \( t \) also sometimes depends on details of the microstructure, e.g., in the cases of “swiss cheese” and “inverse swiss cheese” models of a conductor/insulator mixture [26], and in the case of a singular distribution of conductances in a random resistor network [27]. In any case, in contrast with the SEMA prediction, \( t \) is never equal to 1.

These failures of SEMA are related to the fact that it is an uncontrolled approximation which cannot be improved in any systematic fashion: SEMA is based on a simple, intuitive physical idea, namely, that when trying to calculate the electric field and current in and near a single spherical inclusion with conductivity \( \sigma_1 \) or \( \sigma_2 \) one can replace the rest of the heterogeneous system by a fictitious, uniform host with \( \sigma_{\text{eff}} \) as its uniform conductivity. The value of \( \sigma_{\text{eff}} \) is then found by imposing the self-consistency requirement that the dipole current source, which is excited when an external uniform electric field is imposed on any isolated spherical inclusion in this fictitious uniform host, yields zero when summed over all the different inclusions in the system. While this approximation becomes exact when \( \sigma_1 \rightarrow \sigma_2 \) or when the system is dilute, i.e., when either \( p_1 \ll 1 \) or \( p_2 \ll 1 \), it is impossible to estimate the error when neither of these conditions is satisfied. The failure of SEMA in predicting correct values for critical exponents, like \( t \), and its inability to include relevant details of the microstructure, as in the case of the above mentioned swiss cheese model, have lead many scientists to abandon this approximation. Instead, they chose to use techniques like renormalization group transformation or brute force simulation of discrete, random network models in order to study the macroscopic response near a percolation threshold. (A review of the different approaches to calculations of the macroscopic conductivity of a composite medium, including cases where the system is near a percolation threshold, can be found in Ref. [25].) However, extension of SEMA to the case where a strong magnetic field is applied to a macroscopic mixture of two conductors with different but comparable resistivities [28]–[36], or to a mixture of three constituents where one is a normal conductor while the other two are a perfect insulator and a perfect conductor [37]–[39], have resulted in the discovery of some new critical points which are unrelated to the geometric percolation threshold. This demonstrates a great advantage of SEMA and its extensions: Because they lead to closed form expressions for the elements of the macroscopic resistivity tensor, or at least to closed form (though complicated) coupled equations which determine those elements, it is much easier to identify a mathematical singularity in those moduli, which signals the existence of a critical point.

The research by Landauer described above was done while he worked at the NACA Lewis Laboratory, but by the time Ref. [20] appeared in print, he had moved to IBM. Shortly afterwards, Landauer got interested in the problem of magneto-transport in a macroscopically het-
Landauer in Ref. [32], it is not even necessary for the appearance of a 1955 IBM Technical Report [41], published in J. Appl. Phys. in 1956 [40] shortly after the original work had almost vanished into oblivion. This article, which appeared in the first volume of the IBM Journal of Research and Development [15], was not properly appreciated until it transpired that the insights developed in it are extremely relevant for understanding the phenomenon of electromigration—see Ref. [53] for a detailed list of relevant references on this topic. Because the availability of Ref. [15] was so limited, the unusual step was taken of re-publishing it as an article in the Journal of Mathematical Physics nearly 40 years after the original publication—see Ref. [54]. This is just one example of how far ahead of most other physicists Landauer was in his scientific thinking and insight: Until others caught up with his 1957 results, the original work had almost vanished into oblivion.

The phenomenon of electromigration was actually a major interest of Landauer for much of his life. The subject was of great practical importance to IBM, as a principal mechanism for the failure of integrated circuits is deterioration caused by electromigration of defects and impurities near junctions in the circuit. Landauer’s focus was on the microscopic understanding of forces responsible for the motion of defects. An important early contribution to this field was the paper by Landauer and Woo, “Driving force in electromigration”, published in 1974 [55].

The central idea of this paper was that the inhomogeneity in the electron density near a defect or impurity carries with it a change in the local conductivity. When an electric current is applied, this leads to formation of electric dipoles, which can exert a force on the defect, in addition to forces resulting from the direct transfer of momentum from an electron to the impurity during a scattering process. The issue confronting Landauer and Woo was how to properly take this force into account. Landauer wrote a number of subsequent papers on the driving forces for electromigration, which we will not have room to summarize here. However, the interested reader can find a review of Landauer’s contributions to the subject in an article by R. Sorbello, enti...
tled “Landauer fields in electron transport and electromigration”, published in 1998 [56].

When the first ETOPIM conference was convened, in Columbus, OH during 7–9 September 1977 [57], Rolf Landauer was asked to deliver the opening keynote address. In that talk, he presented an exhaustive review of the development of theoretical treatments for the physical properties of a composite medium up to that time. The article which summarizes that talk in the conference proceedings volume [58] is an invaluable historical review, which also lists and discusses all the important articles in that field which were known at that time—altogether 163 references.

4. Quantum systems

At an early stage of his career, Landauer became interested in the study of systems of non-interacting electrons in a one-dimensional disordered potential, which could be studied analytically or numerically with the computers of the time, and could shed light on more realistic three-dimensional systems which were then not tractable. The work of Landauer and Helland, in 1953, was a pioneering work in this area [59]. However, the most influential paper that Landauer wrote based on the analysis of one-dimensional systems was his 1970 paper, “Electrical resistance of disordered one-dimensional lattices” [17]. The 1970 paper was important because of its contribution to our understanding of the phenomenon of localization in one-dimensional systems, but even more significantly, it established a connection between electrical conductance and transmission probabilities, that has been the basis for much future work on mesoscopic systems, often referred to as the Landauer formalism.

What Landauer did in the 1970 paper was to study statistical properties of the transmission matrix through a one-dimensional region with a sequence of partially reflecting barriers randomly spaced. As Landauer noted, if the disordered region (let us call it the “sample”) is connected on either end by smooth wires to reservoirs at different chemical potentials, there will be a net current through the sample determined by the potential difference of the reservoirs and the transmission probability $T$ for an electron with energy close to the Fermi energy, incident on the sample from either side. (It is a consequence of the principle of detailed balance that the transmission probability will be the same whether the electron is incident from left or right.) Landauer used this result to define a conductance for the system, which he found to be

$$G = \frac{e^2}{h} \frac{T}{1 - T}. \quad (6)$$

The formula is for spinless non-interacting electrons, in the limit of zero temperature. The transmission probability is, in turn, related to the complex transmission amplitude $t$, by $T = |t|^2$.

A decade after Landauer’s formulation, an alternative relation between conductance and transmission probability was proposed by Economou and Soukoulis [60] and others. (See the discussion in Ref. [61], particularly pages 93–103.) For spinless electrons in one dimension, this relation is simply

$$\Gamma = \frac{e^2}{h} \Gamma. \quad (7)$$

This formula was also generalized to the case where there can be several transverse channels for electrons in the wires connected to the sample. In this case we have [62]

$$\Gamma = \frac{e^2}{h} \sum_{ij} |t_{ij}|^2, \quad (8)$$

where $i$ and $j$ label the channels in the left and right leads respectively, and $t_{ij}$ is the matrix of transmission amplitudes.

For some time, there was much discussion about which of the two quantities, $G$ or $\Gamma$, is the “correct” definition of the electrical conductance. We now understand that they are, in some sense, both correct, but refer to different experiments [61, 63]. The quantity $\Gamma$ should be thought of as a two-terminal conductance. If the sample is connected by ideal wires to two large reservoirs, in equilibrium at voltages $V_1$ and $V_2$, and $I$ is the current through the sample, then

$$\Gamma = \frac{I}{V_1 - V_2}. \quad (9)$$

In contrast, $G$ may be thought of as a four terminal conductance. If one could attach an ideal voltage probe to the leads on either side of the sample, which would measure the voltages $V_3$ and $V_4$ in the leads without drawing any current from the leads and without disturbing them in any way, we would have

$$G = \frac{I}{V_3 - V_4}. \quad (10)$$

Unfortunately, it is not entirely clear how one could construct an ideal voltage probe that would not disturb a mesoscopic system [61]. In fact, since the electrons
the leads are not in thermal equilibrium, there may be some question how one should properly define a voltage in the leads. Landauer had in mind that the voltage would be defined by the total density of left and right moving electrons, as well as by the electrostatic potential, which should be determined self-consistently. On the other hand, it has proved relatively easy to fabricate mesoscopic systems with good connections to external reservoirs of known voltage, so the two-terminal conductance $\Gamma$ has proved to be an extremely useful concept. Despite the difference between $\Gamma$ and the conductance $G$ that Landauer originally introduced, Landauer deserves a great deal of credit for introducing the idea that the conductance should be determined by the transmission probabilities.

In Landauer’s original paper, and much of the subsequent work, analysis was restricted to non-interacting electrons, or models where the Coulomb interaction is introduced only in the form of a self-consistent potential. However, we understand that the analysis is also applicable for interacting electron systems, provided the temperature is low and the system sufficiently small so that electrons that enter the sample will leave it before suffering an inelastic collision. Since the time for inelastic collisions increases as the temperature is reduced, studies of these phenomena are generally carried out at very low temperature.

Landauer’s 1970 paper had importance separate from the general question of conductance through mesoscopic systems. The paper shed very important light on the issue of electron localization in one-dimensional systems. In previous work, by Mott and Twose, by Borland, and by others, it had been established that for non-interacting electrons in a disordered potential in one dimension, in the limit of an infinite wire, the electron eigenstates would all be localized, except for a possible set of measure zero [64, 65, 66]. What this meant was that for each eigenstate, there would be a point on the line where the magnitude had a maximum, and on either side of that point, the wave function would decrease exponentially, with a decay length that depended on the energy and the strength of the disorder, but would remain finite in the limit of an infinite system. As a result of this, it was argued that the resistance of a long one-dimensional disordered system of non-interacting electrons should increase exponentially with the length $L$, in contrast to a classical wire, where the resistance is linear in $L$. Landauer was able to explain the exponentially diverging resistance in terms of transmission amplitudes and quantum mechanical interference in the wire.

To understand Landauer’s argument, let us consider his formula, Eq. (5) of Ref. [17], for the inverse of the conductance $G$ of a sample consisting of two barriers in series:

$$\frac{e^2}{\hbar G} = \frac{1 - T}{T} = \frac{1 - T_1 + (1 - T_2) + 2(1 - T_1)^{1/2}(1 - T_2)^{1/2} \cos \phi}{T_1 T_2}.$$  \hspace{1cm} (11)

Here $T$ is the transmission of the system as a whole, $T_1$ and $T_2$ are the transmissions of the individual barriers, and the phase $\phi$ depends on the distance between barriers. That phase arises from the interference of contributions in which the particle is reflected multiple times by the barriers before finally emerging from one side or the other of the system. Formulas for three or more barriers can be obtained by iteration, adding one barrier at a time.

To introduce the effects of disorder, Landauer considered a model of $N$ barriers having identical individual transmission probabilities $T_1$, but with random spacings among them. In particular, Landauer assumed that the phases $\phi$ between successive barriers could be treated as independent random variables, uniformly distributed from 0 to $2\pi$. This can be strictly justified when the variation in the distance between barriers is large on the scale of the wavelength of the electrons, but the final results are actually much more general. Landauer showed that the mean value of the resistance $1/G$ for his model is given by the formula

$$\left\langle \frac{e^2}{\hbar G} \rightangle = \frac{1}{2} \left[ (\frac{2 - T_1}{T_1})^N - 1 \right]. \hspace{1cm} (12)$$

It follows that the mean value of the resistance will diverge exponentially with the length of the system, unless $T_1 = 1$, i.e., unless there is perfect transmission for the individual barriers.

When the resistance of the sample is very large, it does not matter whether one considers the four-terminal resistance $1/G$ or the two-terminal resistance $1/\Gamma$. The mean values of both quantities will diverge, at the same exponential rate, as $N$ becomes large. By contrast, the mean values of $G$ and $\Gamma$ are quite different. Though typical values of $G$ will be exponentially small, as expected from the large value of $\langle G^{-1} \rangle$, the mean value of $G$, for a sample of specified length $N$, will actually be infinite, as Landauer noted in his paper:

$$\langle G \rangle = \left\langle \frac{T}{1 - T} \right\rangle = \infty. \hspace{1cm} (13)$$

The reason for this can be seen by inspection of Eq. (11). The formula implies that when $\Delta T \equiv T_1 - T_2$
and \( \Delta \phi \equiv \phi - \pi \) go to zero, the value of \( G \) will diverge as \( [ (\Delta T)^2 + (\Delta \phi)^2]^{-1} \). Let us divide our sequence of \( N \) barriers into two roughly equal halves, let \( T_1 \) and \( T_2 \) be the separate transmission probabilities of the two halves, and let \( \phi \) be the phase accumulation in the space between the two halves. As the probability density for \( \Delta T \) and \( \Delta \phi \) will, in general, be finite when the two variables go to zero, the mean value of \( G \) will diverge logarithmically.

4.1. Applications

An important application of the Landauer-type approach to conductance resulted from the dramatic experimental discovery in 1988 of quantized conductance steps in semiconductor devices with a narrow constriction \([67, 68, 69]\). In these devices, fabricated from two-dimensional electron systems in GaAs, the width of the constriction could be varied continuously by applying a negative bias to a pair of gate electrodes on the surface of the sample—see lower panel of Fig. 3. As the width of the constriction was varied, the conductance was not a linear function of the gate voltage, but was seen to exhibit a series of plateaus, with values \( \Gamma = 2Ne^2/h \), where \( N \) is a positive integer—see upper panel of Fig. 3. These observations could be understood using Landauer’s ideas, if we assume that for electrons in a given channel of transverse motion, as soon as the constriction is wide enough to permit transmission of electrons at the Fermi energy, the transmission probability \( T \) is very close to unity. If the constriction is too narrow, then transmission in the given channel will be close to zero. Thus, the conductance will be close to an integer times \( e^2/h \). The factor 2 appears because of the degeneracy due to electron spin. The sudden increase in \( T \) from complete reflection to complete transmission is very plausible in these systems because the controlling gates are set back from the two-dimensional electron gas by a distance large compared to the Fermi wave length. Thus the potential felt by the electrons should be very smooth, and the the transmission problem reduces to the semiclassical problem of a particle incident upon a barrier, where the transmission probability is either 0 or 1, depending on whether the particle has enough energy to get over the barrier. In the years since 1988, an enormous number of experimental and theoretical investigations have been built on these experiments and their interpretation.

Landauer’s ideas were also important in the understanding of the resistance oscillations of a microscopic metal ring attached to two leads, as a function of the magnetic flux threading the ring. The key theoretical paper here was the work of Büttiker, Imry, Landauer and...
Pinhas, in 1985 [70]. This work was, in turn, closely tied to experiments carried out at IBM at that time [71], which studied the magnetoresistance of a thin gold ring, approximately 800 nm in diameter, with a thickness of approximately 40 nm. Upon varying the magnetic field, the experiments found oscillations in the resistance corresponding to a fundamental period of the addition of one quantum of magnetic flux, \(\Phi_0 = \hbar/e\), through the hole in the ring. Fourier transform of the data showed strong peaks at frequencies corresponding to this fundamental, and also at the first harmonic, corresponding to addition of half a flux quantum. As explained by Büttiker et al. [70], the fundamental frequency in the transmission amplitude arises from the quantum interference between paths in which an electron may travel from one contact to the other along either side of the ring. The phase of this interference term depends on the precise location of scattering centers in the sample, and would be expected to vary randomly from one sample to another. Thus oscillations at the fundamental period would be expected to vanish, or be greatly reduced, in an experiment where the signal was averaged over many different rings. However, oscillations corresponding to one-half flux quantum would not vanish on averaging, and thus would dominate an averaged measurement. This analysis was consistent with the results of previous experiments and on multiple rings, and on tubular samples, which may be thought of as many rings in parallel [72, 73, 74].

Another geometry that interested Landauer was the case of an isolated metal ring, with no electrical contacts. Classically, the conductance of a closed metal ring can be measured by placing it in a time-varying external magnetic field and measuring the magnetic moment induced in the ring. The induced moment would be proportional to the current flowing around the ring, which in turn would be proportional to the time-derivative of the flux and the conductance of the ring. For a mesoscopic wire at low temperatures, where the discrete quantization of electronic levels becomes important, the situation is more complicated. In this case, there can be a non-zero “persistent current”, in equilibrium in a dc magnetic field, which will be an oscillatory function of the flux through the loop. Such persistent currents are well known in superconductors, but they also occur (with much smaller magnitudes) in normal metal loops. Landauer and his collaborators wrote a number of important papers in the 1980s which discussed both the existence and magnitude of persistent currents in a dc magnetic field, and the behavior to be expected in a time-varying magnetic field [75, 76, 77, 78, 79]. Landauer’s analysis of the latter problem also allowed him to address fundamental issues of the nature of dissipation in small closed loops.

Landauer’s approach to conductance was also the basis for important work on shot noise in mesoscopic systems. The formula for the current noise-power, per unit frequency, in the limit of zero temperature, is given by [80, 81, 82, 83]

\[
S = 2(e^2/h)V \sum_i T_i(1 - T_i),
\]  

where \(V\) is the applied voltage and \(T_i\) is the transmission probability for the \(i\)th channel. [Here, we have made a unitary transformation on the channels in the two leads so that the transmission matrix \(t_{ij}\) is diagonal, and the conductance formula (8) becomes \(G = (e^2/h) \sum_i T_i\).] The noise formula (14) is very widely used, and has been the basis for much subsequent work. Landauer’s own views on shot noise may be found in his article “Mesoscopic Noise: Common Sense View”, published in 1996 [4]. (See also the 1991 article by Landauer and Martin [84].)

Another interest of Landauer, related to mesoscopic systems, was the concept of transit time in tunneling events. The reader is referred to Refs. [85, 86] for Landauer’s views on this subject.

5. Biographical summary

Rolf Landauer was born in Stuttgart, Germany, in 1927. He moved to the United States, with his family, in 1938, several years after the death of his father in 1935. His father, who had fought for Germany in World War I, and had been severely wounded, was very patriotic, and did not want to leave the country. He strongly believed that the Nazi antisemitism would pass. Landauer has said that were it not for the early death of his father, due in part to problems resulting from his war wounds, his family would have undoubtedly remained in Germany until it was too late to leave.

Landauer’s family settled in New York City, where he went to high school, before entering Harvard College at the age of 16. He graduated in two years, and enlisted in the U. S. Navy, where he claimed to have learned as much as he had learned in college. Eventually, he returned to Harvard for graduate studies, where he received his Ph.D. in 1950. After graduate school, Landauer worked for two years at the Lewis Laboratory of the National Advisory Committee for Aeronautics (NACA, later to be renamed NASA, acronym for the National Aeronautics and Space Administration). In 1952, he moved to IBM Laboratories in Poughkeepsie, NY, (later to be renamed the IBM Thomas J. Watson
Research Center), where he continued to work until he passed away in 1999.

At IBM, in addition to his research work, Landauer held important management posts at various times. He was responsible for much of IBM’s early work on large scale integration, and has been given credit for inventing the term. Landauer was awarded the title of “IBM Fellow” in 1969.

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