Optimal Strategies for Search and Rescue Operations with Robot Swarms

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Abstract—Motivated by the modern availability of drones and unmanned surface vessels as well as other low cost search agents, we consider the problem of a swarm of robots searching for a target on the high seas. Coordinating such a search in an effective manner is a non trivial task. In this paper we fully address this problem by first developing an abstract model which allows us to understand searches at sea under ideal conditions, and then apply the abstract model under actual search conditions such as differential search speeds, arrival times to the search area and low probability of detection under poor visibility conditions. We show that the theoretical model still governs the search with suitable adaptations. Lastly we give several search scenarios showing the cost effectiveness of such searches, deriving from lower cost and higher precision in the search.

I. INTRODUCTION

Historically, searches were conducted using a limited number (at most a handful) of vessels and aircrafts. This placed heavy constraints in the type of solutions that could be considered, and this is duly reflected in the modern search and rescue literature\textsuperscript{[6], [7], [8], [9]}. However, the comparably low cost of surface or underwater unmanned vessels allows for searches using hundreds, if not thousands of vessels. For example, the cost of an unmanned search vehicle is in the order of tens of thousands of dollars\textsuperscript{[4]} which can be amortized over hundreds of searches, while the cost of conventional searches range from the low hundred thousands of dollars up to sixty million dollars for high profile searches such as Malaysia Airlines MH370 and Air France 447. This suggests that somewhere in the order of a few hundred to a few tens of thousands of robots can be realistically brought to bear in such a search. Motivated by this consideration we propose search and rescue strategies for the high seas using a large number of agents in an intelligent coordinated swarm fashion.

Coordinating such a search in an effective manner is referred to as a “difficult task” in the search and rescue literature\textsuperscript{[5], [13], [14]}. In this paper we develop (1) an abstract model which allows us to understand searches at sea under ideal conditions, and (2) we progressively incorporate realistic assumptions in the model, specifically different search speeds, different arrival times to the search target and poor visibility conditions. We show that the initial key idea still governs the search under these conditions subject to a few minor adaptations. Lastly we give several search scenarios showing the cost effectiveness of such searches, deriving from lower cost of robot search hardware and higher precision in the search.

We begin with the theoretical model for two and four robots of López-Ortiz and Sweet\textsuperscript{[12]} that abstracts out issues of visibility and differing speeds of searchers. Searching for an object on the plane with limited visibility is commonly modelled by a search on a lattice. Under this setting, visual contact on the plane corresponds to identifying the target upon contact on the grid. This model has been historically used for search and rescue operations in the high seas where a grid pattern is established and search vessels are dispatched in predetermined patterns to search for the target\textsuperscript{[5], [14]}.

In our theoretical model we analyse the efficiency of the search strategies using the competitive ratio measure for online algorithms. The competitive ratio is defined as the ratio between the distance traversed by the searcher in its quest for the target and the length of the shortest path between the starting position of the searchers and the target, as measured in the $L_1$ metric induced by the axis parallel lattice on the plane. In other words, the competitive ratio measures the detour of the search strategy as compared to the optimal shortest route.

In 1989, Baeza-Yates et al.\textsuperscript{[1], [2], [3]} proposed a strategy for searching on a lattice with a single searcher with a competitive ratio of $2n + 5 + \Theta(1/n)$ to find a point at an unknown distance $n$ from the origin. This is shown to be optimal. The strategy follows a spiral pattern exploring $n$-balls in increasing order, for all integer $n$. See URL in\textsuperscript{[11]} for an animation of a single robot search pattern. Not surprisingly this strategy is similar to those used in search and rescue operations\textsuperscript{[5]}.

However, in real life a search strategy occurs in the presence of multiple agents, which join the search at different times (and often at different speeds) and under varying visibility conditions.

A. Summary of Results and Structure of the Paper

We construct a theoretical model and give an optimal strategy for searching with $k$ robots with differing speeds. We then enrich this model with practical parameters and show that the principles from the theoretical solution also govern the more realistic search scenario with suitable modifications. Specifically we deal with cases with a varying probability of location as well as probability of detection (POD).

We believe the results give evidence towards the fact that a robot-swarm-driven search-and-rescue operation outperforms traditional searches in terms of costs and ability to locate the target. As such the difficulty of a search-and-rescue effort moves from a slow laborious search to a task of coordination of many searchers and management of probabilities as the search progresses.
We first consider the case where all searchers start from a common point which we term the origin, and second, when they start from arbitrary points on the lattice.

Initially we consider the case where all searchers move at the same speed and give a strategy for finding a target with $k = 4r$ robots with a competitive ratio of $2n/k + 5/k$ as well as a lower bound for $k$ searchers of $2n/k + 5/k$ for general $k$, which matches the upper bound up to an $o(1/k)$ term.

This is then generalized to any number of robots (not just multiples of four) and using the same ideas show that the techniques developed also generalize to searchers with various speeds. Lastly, we show that the proposed theoretical strategy also governs a search under actual weather conditions, in which there is a non-negligible probability of the target being missed in a search. We use tables from the extensive literature on SAR (Search-and-Rescue) operations to conduct simulations and give scenarios in which the proposed strategy can greatly aid in the quest for a missing person or object in a SAR setting [6], [7], [8].

II. PARALLEL SEARCHING

López-Ortiz and Sweet [12] consider the case of searches using two and four robots whose search path is shown in Figs. 1 and 2. In this case the robots move in symmetric paths around the origin and prove the following theorem.

Theorem 1: [12] Searching in parallel with $k = 2, 4$ robots for a point at an unknown distance $n$ in the lattice is $(2n + 4 + 4/(3n))k + O(1/n^2)$ competitive. This is in fact optimal for the two robot case, as the next theorem shows.

Theorem 2: Searching in parallel with $k$ robots for a point at an unknown distance $n$ in the lattice requires at least $(2n^2 + 4n + 4/3)/k + O(1/n)$ steps, which implies a competitive ratio of at least $(2n + 4 + 4/(3n))k + O(1/n^2)$.

Proof: Following the notation of [12], let $A(n)$ be the combined total distance traversed by all robots up and until the last point at distance $n$ is visited. We claim that in the worst case $A(n) \geq 2n^2 + 5n + 3/2$ for some $n > 1$. Define $g(n)$ as the number of points visited on the $(n+1)$-ball before the last visit to a point on the $n$-ball.

First note that there are $2n^2 + 2n + 1$ points within in the interior of the closed ball of radius $n$ and that visiting any $m$ points requires at least $m - 1$ steps. Hence $A(n) = 2n^2 + 2n + g(n)$. If $g(n)$ points have already been visited this means that after the last point at distance $n$ is visited there remain $4(n+1) - g(n)$ points to visit in the $n$-ball. Now, visiting $m$ points in a ball requires at least $2m - 1$ steps with one robot, and $2m - k$ with $k$ robots. Thus visiting the remaining points requires at least $2(4(n+1) - g(n)) - k$ steps. Hence, $A(n+1) = A(n)^2 + 2(4(n+1) - g(n)) - k$ as claimed. Now we consider the competitive ratio at distance $n$ and $n + 1$ for each of the robots as they visit the last point at such distance in their described path. We denote by $A_i(n)$ the portion of the points $A(n)$ visited by the $i$th robot. Hence the competitive ratio for robot $i$ at distances $n$ and $n + 1$ is given by $A_i(n)/n$ and $A_i(n+1)/(n+1)$. Observe that $\sum_{i=1}^k A_i(n) = A(n)$ for any $n$ and hence there exist $i$ and $j$ such that $A_i(n) \geq A(n)/k$ and $A_j(n+1) \geq A(n+1)/k$. Lastly the competitive ratio, as a worst case measure is minimized when $A_i(n)/n = A_i(n+1)/(n+1)$ or equivalently when $A(n)/n = A(n+1)/(n+1)$ with solution $g(n) = 2n + 4k$. Substituting in the expression for $A(n)$ we obtain $A(n) = 2n^2 + 4n + (4-k)n(3n+1) = 2n^2 + 4n + 4/3 + O(1/n)$ with a robot searching, in the worst case at least $A(n)/k$ steps for a competitive ratio of $2n + 4 + 4/(3n) + O(1/n^2)$.

III. SEARCH STRATEGY

A. Even-work strategy for parallel search with $k = 4r$ robots

A natural generalization of the $k = 2$ and $k = 4$ robot cases, as shown in Figs. 1 and 2 suggest a spiral strategy consisting of $k$ nested spirals searching in an outward fashion. However, because the pattern must replicate or echo the shape of inner paths, all attempts lead to an unbalanced distribution of the last search levels and thus a suboptimal strategy. A better competitive ratio gives us the strategy described in this section that we call even-work strategy. Each of the $r$ robots covers an equal region of a quadrant using the pattern in Fig. 3. The entire strategy consists of
four rotations of this pattern, one for each quadrant in the plane.

![First quadrant of a parallel search with k = 4 robots, where r = 7.](image)

**Theorem 3:** Searching in parallel with \( k = 4 \) \( r \) robots for a point at an unknown distance \( n \) in the lattice has the asymptotic competitive ratio of at most \( \frac{2n + 7.42}{7/4} \).

**Proof:** We know the lower bound for asymptotic competitive ratio is \( \frac{2n}{k} + \frac{5}{k} \). We want to describe the upper bound of even-work strategy of \( \frac{2n}{k} + \frac{7}{4} \). From \((a, b)\), we look at the last \( 4n \) points (on the \( n \) ball), for each of the \( 4n \) points, we have \( 3n \) work. Thus, \( 7/4 \) amount of work per point (lower bound). From where we get the relation: \( \frac{n/k(1+8/5)}{n/k(1+3/4)} = \frac{13/5}{7/4} = 1.486. \) and \( 5 \cdot 1.486 = 7.428 \).

Let \( k = 4r \) be the number of robots searching in parallel starting from a common origin. The following pseudo-code describes the algorithm of parallel search using \( r \) robots for the first quadrant. A simple rotation applied to the code gives the search strategy for the other three quadrants, which we omit for reasons of clarity.

### Algorithm 1 Strategy\((r, n)\)

**Input:** Let \( k = 4r \) the number of robots, and let \( n \) be the covered distance.

**Output:** parallel search strategy of \( r \) robots in a quadrant.

```plaintext
Robot-1\((n)\).
for \( i = 2 \) to \( r - 1 \) do
    Middle-robots\((i, n)\).
end for
Robot-r\((n)\).
```

### Algorithm 2 Stairs\((n, d, direction)\)

**Input:** Let \( n \) be the number of steps in the stair, \( d \) - the initial horizontal or vertical step and \( direction \) either NW for North-West or SE for South-East.

**Output:** The stairs in direction \( direction \) starting with the first step \( d \).

```plaintext
if \( direction = NW \) then
    1 up.
    Init-Stair\((n, d, NW)\).
    1 up.
else
    1 right.
    Init-Stair\((n, d, SE)\).
    1 right.
end if
```
Algorithm 3 Initialization$(x, y)$

**Input:** Let $(x, y)$ be the initial starting point.

**Output:** Constructs the initial pattern for a robot

1. for $v = 1$ to $n$
   1. for $j = 1$ to $2(r - i)$
      1. Stairs($3 + 8(v - 1), horizontal, NW$).
      1. Stairs($5 + 8(v - 1), horizontal, SE$).
   end for
   1. Stairs($4 + 8(v - 1), horizontal, NW$).
   1. Stairs($6 + 8(v - 1), vertical, SE$).
   1. Stairs($8 + 8(v - 1), horizontal, NW$).
   end for
   1. Stairs($7 + 8(v - 1), vertical, SE$).
   1. Stairs($9 + 8(v - 1), vertical, NW$).
end for

end for

Algorithm 4 Middle-robots$(i, n)$

**Input:** Let $k = 4r$ the total number of robots, $i$ - the number of the current robot and let $n$ be the covered distance.

**Output:** The parallel search strategy of $r - 2$ (middle) robots in a quadrant.

Initialization$(r - i + 1, 5 * (i - 1))$.

for $v = 1$ to $n$
  for $j = 1$ to $2(r - i)$
    Stairs($3 + 8(v - 1), horizontal, NW$).
    Stairs($5 + 8(v - 1), horizontal, SE$).
  end for
  Stairs($4 + 8(v - 1), horizontal, NW$).
  Stairs($6 + 8(v - 1), vertical, SE$).
  Stairs($8 + 8(v - 1), horizontal, NW$).
  for $j = 1$ to $2(r - 1)$
    Stairs($8(v - 1) + 4k - 2, vertical, SE$).
    Stairs($8(v - 1) + 4k, horizontal, NW$).
  end for
end for

Algorithm 5 Robot-$r(n)$

**Input:** Let $k = 4r$ the total number of robots and let $n$ be the covered distance.

**Output:** The parallel search strategy of the $r$th robot in a quadrant.

Initialization$(1, 5(r - 1))$;

Stairs($8(v - 1) + 4, horizontal, NW$).

Stairs($8(v - 1) + 6, vertical, SE$).

Stairs($8(v - 1) + 8, horizontal, NW$).

for $j = 1$ to $2(r - 1)$
  Stairs($8(v - 1) + 7, vertical, SE$).
  Stairs($8(v - 1) + 9, vertical, NW$).
end for

Algorithm 6 Robot-1($n$)

**Input:** Let $k = 4r$ the total number of robots and let $n$ be the covered distance.

**Output:** The parallel search strategy of the first robot in a quadrant.

Initialization$(r, 0)$;

for $v = 1$ to $n$
  for $j = 1$ to $2(r - 1)$
    Stairs($8(v - 1) + 3, horizontal, NW$).
    Stairs($8(v - 1) + 5, horizontal, SE$).
  end for
end for

Algorithm 7 Init-Stair($n, d, direction$)

**Input:** Let $n$ be the number of steps in the stair, $d$ - the initial horizontal or vertical step and $direction$ either NW for North-West or SE for South-East.

**Output:** The $n$ stairs in direction $direction$ starting with the first step $d$.

if $n > 1$ then
  if $d = horizontal$ then
    if $direction = NW$ then
      2 left.
    else
      2 right.
    end if
  else
    if $direction = NW$ then
      2 up.
    else
      2 down.
    end if
  end if
end if

B. Parallel search with any number of robots

This case illustrates how the abstract search strategy for a number of robots multiple of four can readily adapted to an arbitrary number of robots. Let $k$ be the number of robots, where $k$ is not necessarily divisible by 4. We first design the strategy for $4k$ robots obtaining 4 times as many regions as robots. We then assign to every robot 4 consecutive regions as shown in Fig. 6 for the case $k = 7$. Observe that now some of the regions span more than one quadrant and how the search path for each robot transitions from region to region while exploring the same ball of radius $n$ in all four regions assigned to it. Observe that from Theorems 2 and 3 it follows that this strategy searches the plane optimally as well.
IV. FROM THEORY TO PRACTICE

A. The Search Strategy

In Figs. 7 and 8 we show the search strategy with $k = 4r$ robots as it progresses in time. The snapshots are taken at search times $t = 40, 80, 160$ and 260. Since the robots traverse at unit speed, the total distance explored by each robot is $t$ and $kt$ for the combination of all robots.

B. Adding robots to the search

While we envision the swarm of robots being usually deployed from a single vessel and as such all of them starting from the same original position, for certain searches additional resources are brought to bear as more searchers join the search-and-rescue effort. In this setting we must consider an agent or agents joining a search effort already under way. In this case we can compute the exact time at which the additional searcher will meet up with the explored area and have the search agents switch from a $k$ robot search pattern to a $k+1$ search pattern. The net cost of this transition effort is bounded by the diameter of the $n$ ball at which the extra searcher joins, with no ill effect over the competitive ratio. Hence, the search is asymptotically optimal.

C. Parallel search with $k$ robots with different speeds

This is another case which nicely illustrates how the abstract search strategy for robots with equal speed can be readily adapted to robots of varying speeds. Suppose we are given $k$ robots with varying speeds. Let the speed of the $k$ robots be $s_1, s_2, \ldots, s_k$ respectively. We can consider the speeds to be integral, subject to proper scaling and rounding. Let $s = \sum_{i=1}^{k} s_i$. We use the strategy for $4s$ robots and we assign for each robot respectively: $4s_1, 4s_2, \ldots, 4s_k$ regions. It follows that every robot completes the exploration of its region at the same time as any other robot since the difference in area explored corresponds exactly to the difference in search speed and the search proceeds uniformly and optimally over the entire range as well.

D. Probability of detection

In real life settings there is a substantial probability that the search agent might miss the target even after exploring the immediate vicinity of the target. Indeed in searches on high or stormy seas often multiple passes must be made before a man overboard is located and rescued in stormy seas. In this case the search vessels uses a nautical pattern resembling a clover and known as sector search. (See [10] for a robotics perspective of sector searches and other SAR techniques).

The search-and-rescue literature provides ready tables of probability of detection (POD) under various search conditions [6], [7], [8]. Fig. 9 shows the initial probability map for a typical man overboard event. Fig. 10 shows the probability of detection as a function of the width of the search area spanned. The unit search width magnitude is computed using location, time, target and search-agent specific information such as visibility, lighting conditions, size of target and height of search vessel. We consider then a setting in which a suitable POD distribution has been computed taking into account present visibility conditions and size of target (see Fig. 9). Armed with this information the swarm of robots
must then make a choice between searching an unexplored cell in the lattice or revisiting a previously explored cell.

Consider first a model in which a robot can “teleport” from any given cell to another, ignoring any costs of movement related to this switch. It can be shown formally using a standard greedy technique proof that in this setting the optimal strategy consists of robots moving greedily to the cell with current highest probability of containing the target. Each cell is then searched using the corresponding pattern for the number of robots deployed in the cell.

Observe that the probability of each cell evolves over time. It remains at its initial value so long as it’s still unexplored and it becomes \((1 - p)^m\) times its initial value after \(m\) search passes, where \(1 - p\) is the probability of not detecting a target present in the current cell during a single search pass.

In real life, of course, there is a cost associated with moving from a cell to another. In this case the robot moves to the cells of highest probability and starts a grid pattern search of this cell and all surrounding cells of high probability. Observe that there is a choice as to how many robots to redeploy from the present explored position to the next highest probability cell or cells. The optimal value can be obtained via an iterative argument where we compute the probability of the cells explored if we send one robot to the high probability cell and the rest remain in the present expanding pattern. If the probability of the central area combined exceeds that of the combined expanding search area it follows that the strategy only improves if we send two robots towards the remote high probability area. This process continues, considering 3 robots, then 4 and so on. Optimality is achieved at the point of equilibrium when the probability of the lowest probability cells explored by the robots changing cell are equal to the lowest probability cells explored by the robots continuing on the expanding search pattern of Fig. 9.

V. COST EFFECTIVENESS

There are several parameters of a SAR cost. First is the total cost of the search effort as measured in vessel and personnel hours times the number of hours in the search. The second is the effectiveness of the search in terms of the probability of finding the target. Lastly the time to discovery or speed-to-destination as time is of the essence in most search rescue scenarios. That is to say a multiple robot search is preferable to a single agent search with the same cost and probability of detection as time to discovery is lower. Observe as well that multiple robots allow for higher coverage of the unit search width, dramatically increasing the probability of detection. The strategies presented in this paper suggest that robot swarm searches outperform traditional searches in all three of these parameters.

VI. CONCLUSION

We present optimal strategies for robot swarm searches under realistic considerations. We argue that such searches are also more cost effective, both from a net dollar perspective and in terms of speed of recovery for time critical searches. The search strategies are based on a theoretical search primitive which is then enriched with realistic considerations. Interestingly, the theoretical model is resilient to these assumptions and can be readily adapted to take them into consideration. We give pseudo-code showing that the search primitives are simple and can easily be implemented with minimal computational and navigational capabilities.

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