Modeling and solving the dynamic problem of a porous rotational shell based by modified Boit’s theory

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Abstract. Combined the Boit’s theory and the classical elastic theory of thin shell, a new model for analyzing the vibrations of a thin fluid-saturated porous rotational shell is proposed and solved by the extended homogeneous capacity precision integration method and the precise element method. The present model is precise and closer to the real materials as it takes the fluid-solid coupling effect into account and doesn’t make any hypothesis for the fluid displacement. At the end of the paper, the accuracy of the model had been verified by experiment and FEM methods.

1. Introduction
In this article, taking account for the relative action between the fluid and solid phase, we derive the dynamics governing equations of the porous rotational shell based on the classical thin shell theory and Boit’s theory. The equation is written as a first-order differential matrix and solved by the precise integrate method. The experiment is given to verify the accuracy of the present method.

Porous materials have been widely used for passive absorption and noise control in many fields, such as the vehicle, aerospace, geophysics and civil engineering. In the last years, many researchers have paid attention to the vibrations and acoustics characteristics of the poroelastic structures. In 1956, Boit established the three-dimensional theory of the wave propagation in fluid-saturated porous elastic solid [1, 2]. With some hypothesis, Boit’s theory has been applied to porous plates and shells. Manuel et al derived a mix displacement-pressure formulation for the bending vibration of porous plates by the thin plate theory and Boit’s theory [3]. This rigid frame is useful for the material saturated by air. In order to evaluate the energy dissipation by viscous friction, Leclaire and Horoshenkov proposed a analytical description of a fluid-saturated plate by considering the fluid loading terms and relative transverse fluid/solid displacements [4]. However, it is pointed out in the article that the model is only valid in the low frequency since the thickness of the plate is taken to be smaller than any acoustic wavelength and the variation of amplitude of the fluid transverse displacement is small. Ref [5] assumed the in-plane fluid flow relative to the motion of the solid is negligible to the transverse one, and the governing equation written in terms of the three solid displacement and fluid transverse displacement is derived to characteristics the flexural plate dynamics in the frequency domain. Based on the above hypothesis, some works about the porous cylindrical shell were published. Ahmed Shah et al present the frequency equation of axially symmetric vibrations for poroelastic circular cylindrical shells of infinite extent with different boundary conditions [6]. Recently, employing the first-order shear deformation theory and 3D Boit theory, Julien et al derived an analytical model of an infinite sandwich cylindrical shell to demonstrated the influence of the structural damping [7]. Those infinite cylindrical shells are quite far
away from real industrial problem. Owe to the difficulty in the analytical modeling, the finite element method is implemented for the calculation of finite porous cylindrical shells [8]. The finite element method is convenient and widely used, but the finite element model of the porous structure is usually large and complex, in conjunction with time-consuming, so this method can only be applied in low frequency. Boily and Charroin found the finite element model of a sandwich cylindrical shell [9]. In the model, the poroelastic core is simplified by the equivalent fluid model, and the modal reduction method is used to decrease the size of the system. To the knowledge of the authors, the vibration analyses for the porous rotational porous shell are not available in the literature.

2. Governing equations

2.1. Constitutive equations of the porous rotational shell

The stresses-strain relations of a homogeneous, isotropic poroelastic material are

\[
\sigma_{ij} = 2Ne_{ij} + (Ae + Qe)\delta_{ij} (i, j = 1, 2, 3),
\]

\[
-\phi p = Qe + R\varepsilon.
\]  

(1)

The first formula describes the constitutive relation of the solid phase, and the second formula describes the constitutive relation of the fluid phase. The coefficients A and N correspond to the familiar Lamé coefficients in the theory of elasticity. The coefficient R is a measure of the pressure required on the fluid to force a certain volume of the fluid into the aggregate while the total volume remains constant. The coefficient Q represents the coupling between the volume changes of the solid to that of liquid. \(\delta_{ij}\) is the Kronecker’s delta. p is the fluid press. The coefficient \(\phi\) is the porosity. \(\sigma_{ij}\) and \(e_{ij}\) are the solid stresses and strains tensor respectively. \(e\) and \(\varepsilon\) are the dilations of solid and fluid respectively, given by

\[
e = e_{kk} = u_{kk}, \quad \varepsilon = \varepsilon_{kk} = U_{kk}, \quad k = 1, 2, 3,
\]  

(2)

where u is the solid displacement tensor, and U is the fluid displacement tensor; the repetition of the indices k imply summation; commas indicate differentiation with respect to space; \(\varepsilon_{ij}\) denotes the fluid strain tensor.

Consider a thin porous rotational shell, whose middle plane coincides with the natural coordinates, as shown in Fig.1. In the figure, \(\varphi\) and \(\theta\) are meridian and circumferential coordinates, whose relevant curvature radius are \(R_{\varphi}\) and \(R_{\theta}\) respectively. The mid-plane displacement in the meridian, circumferential, and normal directions are u, v, w respectively. The mid-plane Lamé coefficients in meridian and circumferential directions are \(R_{\varphi}\) and \(R_{\theta}\) respectively, which are both the function of \(\varphi\).

For a thin shell, the normal line (z direction) of the middle plane is assumed as a straight line, and whose angles between the vertical line of the meridian and circumferential direction are unchanged. Furthermore, the effect of the total normal stress can be neglect. These mean that
\[ e_{\phi z} = 0, \quad e_{\theta z} = 0, \quad \sigma_z - \phi p \eta = 0. \]  
(3)

The fluid dilation can be evaluated from the second formula in Eq.(1) as
\[ \varepsilon = -\frac{\phi p - Q e}{R}. \]  
(4)

Substituting Eq.(4) into the first formula in Eq.(1) yields
\[
\begin{align*}
\sigma_{\phi} &= 2Ne_{\phi} + \left( A - \frac{Q^2}{R} \right) e - \frac{Q}{R} \phi p, \\
\sigma_{\theta} &= 2Ne_{\theta} + \left( A - \frac{Q^2}{R} \right) e - \frac{Q}{R} \phi p, \\
\sigma_z &= 2Ne_z + \left( A - \frac{Q^2}{R} \right) e - \frac{Q}{R} \phi p,
\end{align*}
\]

\[ \tau_{\phi \theta} = Ne_{\phi \theta}. \]  
(5)

By eliminating \( e_z \) with the aid of Eq. (3) and Eq. (5), we can obtained the solid dilation from the second formula in Eq.(1)
\[ e = \left(1 - \tilde{B}\right)\left(e_{\phi} + e_{\theta}\right) + \tilde{A}\phi p, \]  
(6)

Where \( \tilde{A} \) and \( \tilde{B} \) are constants, with
\[ \tilde{A} = \frac{Q + R}{2NR + AR - Q^2}, \quad \tilde{B} = \frac{AR - Q^2}{2NR + AR - Q^2}. \]

Combing Eq.(5) and Eq.(6), the solid constitutive equations of a porous rotational shell can take the form
\[
\begin{align*}
\sigma_{\phi} &= \tilde{N}e_{\phi} + \tilde{A}e_{\theta} + \tilde{B}\phi p, \\
\sigma_{\theta} &= \tilde{N}e_{\theta} + \tilde{A}e_{\phi} + \tilde{B}\phi p, \\
\tau_{\phi \theta} &= \tilde{N}e_{\phi \theta},
\end{align*}
\]

(7)

Where \( \tilde{A} \), \( \tilde{B} \) and \( \tilde{N} \) are constants, with
\[
\begin{align*}
\tilde{A} &= \left( A - \frac{Q^2}{R} \right)(1 - \tilde{B}), \\
\tilde{B} &= \left( A - \frac{Q^2}{R} \right)\frac{1}{1 - \tilde{B}}, \\
\tilde{N} &= 2N + \left( A - \frac{Q^2}{R} \right)\frac{1}{1 - \tilde{B}}.
\end{align*}
\]

Inserting Eq.(6) into Eq.(4), the fluid constitutive equations of a porous rotational shell can take the form
\[ \varepsilon = \tilde{A}' \phi p + \tilde{B}' \left(e_{\phi} + e_{\theta}\right), \]  
(8)

Where \( \tilde{A}' \) and \( \tilde{B}' \) are constants, with
\[ \tilde{A}' = -\left(1 + \frac{Q\tilde{A}}{R}\right), \quad \tilde{B}' = -\frac{Q}{R} \left(1 - \tilde{B}\right). \]

2.2. Governing equations of the solid frame
According to the Reissner model in the classical thin shell theory, the solid displacements can be written as
\[ \hat{u}_s = u_s + z\theta_p, \quad \hat{v}_s = v_s + \frac{z}{R_0} \left( v_s \sin \varphi - \frac{\partial w_s}{\partial \theta} \right), \quad \hat{w}_s = w_s, \quad (9) \]

where \( \hat{u}_s = u_s, \quad \hat{v}_s = v_s, \quad \hat{w}_s = w_s \) are the solid displacements at any point, and \( u_s, \quad v_s, \quad w_s \) are the corresponding solid displacements in the middle plane, in which the mid-plane rotation \( \theta_p \) being defined as

\[ \theta_p = \frac{1}{R_p} \left( u_s - \frac{\partial w_s}{\partial \varphi} \right). \quad (10) \]

The solid strains at any point can be represented by the mid-plane solid displacements as

\[
\begin{align*}
\varepsilon_{\varphi} &= \left( \frac{1}{R_0} \right) \frac{\partial u_s}{\partial \varphi} + \frac{1}{R_0} \frac{w_s}{\partial \varphi} + \frac{z}{R_0} \frac{\partial \theta_p}{\partial \varphi}, \\
\varepsilon_{\theta} &= \left( \frac{\cos \varphi}{R_0} \right) u_s + \frac{1}{R_0} \frac{\partial v_s}{\partial \theta} + \left( \frac{\sin \varphi}{R_0} \right) w_s + \frac{z}{R_0} \frac{\partial v_s}{\partial \theta} + \frac{z}{R_0} \frac{\partial w_s}{\partial \theta} + \left( \frac{\cos \varphi}{R_0} \right) \frac{\partial \theta_p}{\partial \varphi}, \\
\varepsilon_{\varphi\theta} &= \frac{1}{R_0} \frac{\partial u_s}{\partial \theta} + \frac{1}{R_0} \frac{\partial v_s}{\partial \varphi} - \left( \frac{\cos \varphi}{R_0} \right) v_s + \frac{2z}{R_0} \frac{\partial \theta_p}{\partial \varphi}.
\end{align*}
\]

It is assumed that the dynamic excitation is harmonic in time and hence the shell response is also harmonic. In the other hands, the variables in the rotational shell can be expanded with the Fourier series along the circumferential direction, so that that the function \( f(\varphi, \theta, t) \) has the form

\[ f(\varphi, \theta, t) = \left( \sum_n f_n^{(n)}(\varphi) e^{i\omega t} \right) e^{jne}, \quad (12) \]

Where \( f_n^{(n)} \) is the amplitude of \( f, \quad j = \sqrt{-1} \) and \( \omega \) is the circular frequency. In the following, the factors \( e^{j\omega t}, \quad e^{j\omega t}, \) and the superscript \( n \) is omitted in the follows for simplicity conveniences.

By combing Eq.(7) and Eq.(11), and implement the Fourier series expansion, the Internal force – displacement relations of the solid frame are derived in frequency domain. Based on the thin shell theory, and taking into the dynamics equations of the solid frame, the solid dynamics equations of the fluid-saturated porous rotational shell can be derived in the frequency domain as

\[
\begin{align*}
R_0 \cos \varphi \dddot{\varphi} + R_0 \frac{\partial \dddot{\varphi}}{\partial \varphi} + R_0 \frac{\partial \dddot{\varphi}}{\partial \theta} - R_0 \cos \varphi \dddot{\varphi} + R_0 \dddot{\varphi} + R_0 \dddot{\varphi} - R_0 \dddot{\varphi} - R_0 h = 0, \\
R_0 \frac{\partial \dddot{\varphi}}{\partial \theta} + 2R_0 \cos \varphi \dddot{\varphi} + R_0 \frac{\partial \dddot{\varphi}}{\partial \varphi} + R_0 \frac{\partial \dddot{\varphi}}{\partial \theta} + R_0 \dddot{\varphi} + R_0 \dddot{\varphi} - R_0 \dddot{\varphi} - R_0 h = 0, \\
R_0 \cos \varphi \dddot{\varphi} + R_0 \frac{\partial \dddot{\varphi}}{\partial \varphi} + R_0 \frac{\partial \dddot{\varphi}}{\partial \theta} - R_0 \dddot{\varphi} + R_0 \dddot{\varphi} - R_0 \dddot{\varphi} - R_0 \dddot{\varphi} = 0, \\
R_0 \frac{\partial \dddot{\varphi}}{\partial \theta} + R_0 \cos \varphi \dddot{\varphi} + R_0 \frac{\partial \dddot{\varphi}}{\partial \varphi} + R_0 \cos \varphi \dddot{\varphi} - R_0 \dddot{\varphi} = 0, \\
R_0 \cos \varphi \dddot{\varphi} + R_0 \frac{\partial \dddot{\varphi}}{\partial \varphi} + R_0 \frac{\partial \dddot{\varphi}}{\partial \theta} - R_0 \dddot{\varphi} \cos \varphi \dddot{\varphi} - R_0 \dddot{\varphi} = 0, \quad (13)
\end{align*}
\]
Where \( \Omega_{11} = b \omega j - \rho_{11} \omega^2 \) and \( \Omega_{12} = \rho_{12} \omega^2 \tilde{u}_j + b \omega j \) are frequency parameters. \( \tilde{u}_j = U_1 \), \( \tilde{v}_j = \tilde{U}_2 \) and \( \tilde{w}_j = \tilde{U}_3 \) are the amplitude of fluid mid-plane displacements. \( Q_\varphi \) and \( \tilde{Q}_\varphi \) are the amplitude of shear forces per unit length along \( \varphi \) and \( \theta \) direction respectively, which can be transformed to the Kelvin—Kirchhoff equivalent shear force
\[
\tilde{S}_\varphi = N_{\varphi\theta} + \frac{\sin \varphi}{R_0} M_{\varphi\theta},
\]
\[
\tilde{V}_\varphi = \tilde{Q}_\varphi + \left( \frac{n_j}{R_0} \right) \tilde{M}_{\varphi\theta}.
\]  

2.3. Governing equations of the fluid

Combining the dynamics equations of the fluid of the Boit theory and thin shell theory, we inferred the governing equations of the fluid.

\[
-R_0 (\Omega_{21} - \Omega_{22} \tilde{B}) \frac{d \tilde{\varphi}}{d \varphi} + \left[ \frac{R_0 \cos \varphi}{R_0} \right] \frac{d \tilde{P}}{d \varphi} + \frac{d^2 \tilde{P}}{d \varphi^2} = \]
\[
\left( \frac{R_0^2 \cos \varphi}{R_0} \right) \Omega_{21} - \Omega_{22} \tilde{B} \tilde{\varphi} + \left( \frac{R_0^2 n_j}{R_0} \right) \left( \Omega_{21} - \Omega_{22} \tilde{B} \right) \tilde{\varphi} + \frac{R_0^2}{R_0} \left( \frac{\sin \varphi}{R_0} + \frac{1}{R_0} \right) \left( \Omega_{21} - \Omega_{22} \tilde{B} \right) \tilde{\varphi} - \]
\[
R_0^2 \left( \frac{n_j^2}{R_0^2} + \Omega_{22} \tilde{A} \right) \tilde{\varphi} + \left( \frac{\sin \varphi}{R_0} + \frac{1}{R_0} \right) \left( \frac{R_0^2}{h} \right) \tilde{\phi} (p_1 - p_2) - \]
\[
\left( \frac{1}{R_0^2} \right) \frac{d \tilde{\varphi}}{d \varphi} - \left[ \frac{n_j^2}{R_0^2} + \Omega_{22} \tilde{A} \right] \tilde{\varphi} + \left( \frac{R_0^2}{R_0^2} \right) \frac{d \tilde{\varphi}}{d \varphi} + \frac{\Omega_{21} - \Omega_{22} \tilde{B}}{R_0} \frac{\cos \varphi}{R_0} \tilde{\theta} = 0
\]  

(15)

Where \( \tilde{\varphi} \) and \( \tilde{\varphi} \) are the amplitude of the press on the surfaces of the shell, and \( R_0' = \frac{\partial R_0}{\partial \varphi} \).

Eq.(15) are the governing equation of the fluid in the fluid-satured porous rotational shell. It should be emphasized that, in the above derivation, we have not make any assumption for the displacement, and consider the interactions between the fluid and solid phases fully.

2.4. The integrated governing equations of a porous rotational shell saturated in fluids

Combining the solid governing equations Eq.(13) and the fluid governing equations Eq.(15), we can obtain the integrated governing equations of a porous rotational shell by eliminating some dependant variables. Then we implant the dimensionless procedure, letting

\[
\tilde{u}_i = L \tilde{u}_i, \tilde{v}_i = L \tilde{v}_i, \tilde{w}_i = L \tilde{w}_i, \tilde{\varphi}_i = \tilde{\varphi}_i, \tilde{N}_\varphi = \left( h \tilde{\varphi} \right) \tilde{N}_\varphi, \tilde{S}_\varphi = \left( h \tilde{\varphi} \right) \tilde{S}_\varphi, \tilde{V}_\varphi = \left( h \tilde{\varphi} \right) \tilde{V}_\varphi,
\]
\[
\tilde{M}_\varphi = \left( \frac{\tilde{N}_\varphi}{12L} \right) \tilde{M}_\varphi, \tilde{P} = \tilde{N} \tilde{P}, \tilde{M}_\varphi \cdot q_\varphi = \left( \frac{\tilde{N}}{BL} \right) \tilde{q}_\varphi, q_\varphi = \left( \frac{\tilde{N}}{BL} \right) \tilde{q}_\varphi, q_\varphi = \left( \frac{\tilde{N}}{BL} \right) \tilde{q}_\varphi.
\]  

(16)

Where \( \tilde{q}_\varphi, \tilde{q}_\varphi, \tilde{q}_\varphi \) are the amplitudes of the load along \( \varphi, \theta \) and \( z \) directions. Finally, the integrated governing equations can be expressed as the first-order differential matrix equation as
Eq. (17) can be expressed as:

\[ \frac{dZ}{d\varphi} = BZ + F, \]

Where \( Z = \begin{bmatrix} \bar{u} & \bar{v} & \bar{w} & \bar{\bar{u}} & \bar{\bar{v}} & \bar{\bar{w}} & \bar{\bar{\bar{u}}} & \bar{\bar{\bar{v}}} & \bar{\bar{\bar{w}}} & \bar{\bar{\bar{\bar{u}}}} & \bar{\bar{\bar{\bar{v}}}} & \bar{\bar{\bar{\bar{w}}}} & \bar{\bar{\bar{\bar{\bar{u}}}}} & \bar{\bar{\bar{\bar{\bar{v}}}}} & \bar{\bar{\bar{\bar{\bar{w}}}}} & \bar{\bar{\bar{\bar{\bar{\bar{u}}}}} & \bar{\bar{\bar{\bar{\bar{\bar{v}}}}} & \bar{\bar{\bar{\bar{\bar{\bar{w}}}}} & \bar{\bar{\bar{\bar{\bar{\bar{\bar{u}}}}} & \bar{\bar{\bar{\bar{\bar{\bar{\bar{v}}}}} & \bar{\bar{\bar{\bar{\bar{\bar{\bar{w}}}}} & \bar{\bar{\bar{\bar{\bar{\bar{\bar{\bar{u}}}}} & \bar{\bar{\bar{\bar{\bar{\bar{\bar{\bar{v}}}}} & \bar{\bar{\bar{\bar{\bar{\bar{\bar{\bar{w}}}})))) \end{bmatrix} \]

is the integrated state vector, \( A \), \( B \) are coefficient matrices and \( F \) is the integrated load vector respectively. Their non-zero elements are listed in Appendix.

3. Validation of model correctness

3.1. Comparison with the experiment result

To verify the present model, a porous cylindrical shell is considered which is degenerated by the porous rotational shell. In this case, the geometric parameters \( R_p \to \infty, R_b = R, \varphi = \frac{\pi}{2} \). The coefficients and experimental set-up and are shown in table 1 and figure 2 respectively. A cantilever porous cylindrical shell is excited by a point force \( 2e^{\nu t} N \) at the bottom of the free end \((\vartheta = 0)\), and the deflection of the bottom point and top point of the free end are both measured and compared with the present method. The results are shown in figure 3 and figure 4 respectively, in which the response function is \( \text{FRF} = \log_{10}(|\bar{W}|) \).

It is seen from figure 3 and figure 4 that the deflections curves given by the present method and experiment are almost the same trend. Moreover, we can know from figure 3 that, the first order natural frequencies of the numerical and experimental results are both 116Hz, while the numerical and experiment results are 116Hz and 115Hz respectively in figure 4. The above comparisons have indicated the reliability of the present method.

| Table 1. Coefficients of the porous cylinder shell |
|-----------------------------------------------|
| \( l \) (mm) | \( R \) (mm) | \( h \) (mm) | \( \rho_i \) (kg/m³) | \( \rho_f \) (kg/m³) | \( \mu \) (GPa) | \( E \) (GPa) | \( \phi \) | \( q \) (m²) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 5×10³ | 100 | 10 | 2700 | 1.213 | 0.3 | 0.38 | 0.7 | 2.7×10⁻²⁰ |

Fig. 2 Vibration experiment of porous cylinder shell
3.2. Comparison with the equivalent models

In existing work of modeling porous materials with saturated fluid, the equivalent models are widely used, and the effective medium method and equivalent density fluid method are the most popular. The effective medium method is applied to the porous materials with rigid solid frames. It assumes that the micro elements of porous materials can be seen as an effective elastic element [10]. The equivalent density fluid method is used to the porous materials with soft solid frames. In this model, it is assumed that the frame does not resist to external excitations, the frame of the porous medium is ignored, and the micro elements of porous materials can be seemed as an effective limp element. Both of the two models describe the porous mediums by equivalent mass density and ignore the solid-fluid coupling.

In this section, we further compare the results with the effective medium method and the equivalent density fluid method to show the utility of the present method in the case of vibration of performance. A porous rotational shell with major axis $a = 210 mm$, minor axis $b = 105 mm$, start angle $\varphi = \frac{\pi}{6}$, end angle $\varphi = \frac{5\pi}{6}$ and layer thicknesses $h = 10 mm$ is considered. The material parameters and boundary condition are the same as section 3.1. According to the effective medium method, the equivalent elastic modulus $E_e$ is equal to $0.04E$ and the equivalent density $\rho_e$ is equal to $0.3\rho_s + 0.7\rho_f$ in the finite element method. The results of the natural frequency of the shell are listed in Table 2, and some FEM results are shown in figure 5. In which, $m$ denotes the half wave numbers of the meridian direction and $n$ denotes that of the circumferential direction. The close agreement between the present solution and FEM solution indicates the correction of the proposed method again.

$$m = 1 \text{ n} = 3 \text{ Freq}= 234.92 Hz$$

$$m = 1 \text{ n} = 4 \text{ Freq} = 375.60 Hz$$

Fig. 5 several typical FEM results of porous rotational shell
Table 2. Comparison of natural frequency

| order (m,n) | model (Hz) | FEM (Hz) |
|------------|------------|----------|
| (1,1)      | 76.37      | 74.724   |
| (1,2)      | 181.93     | 180.92   |
| (1,3)      | 233.78     | 234.92   |
| (1,4)      | 369        | 375.60   |
| (1,5)      | 610.5      | 621.37   |

4. Conclusion

In this paper, a new theoretical model of thin porous rotational shell has been established based on the Biot’s theory and the classical elastic theory of thin shell. The accuracy of the present model had been verified by comparing with the results of experiment and finite element method. When comparing with the effective density fluid model, the two methods give almost identical results in the low and medium frequency range, but the resonance amplitudes of the two models have a discrepancy in higher frequency range.

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