Method of Probe Radius Compensation for Optical Complex Surface Measurement

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Abstract. When surface measuring with a contact method, the data obtained by the LM50 laser interferometer is the data of the center of the probe actually, not the real measurement point. Therefore, based on the theory of NURBS and the theory of Delaunay triangulation, a method of "enveloping surface - triangulation" is proposed, and the normal vector of the measuring points of the surface is derived so that the radius of the probe is compensated accurately and quickly. After triangular segmentation, the normal vector of each triangle around a measuring point has an influence on the normal vector of the vertex, so it is necessary to calculate the weight function of each triangulation. In this paper, the influence of the edge length and the angle factor on the vertex normal vector is considered, and the calculation of the vertex normal vector is improved in the triangulation, and the programming and calculation are simplified on the premise of guaranteeing the accuracy.

Key words: contact measurement; the theory of NURBS; the theory of Delaunay triangulation; probe radius compensation; vertex normal vector

1. Introduction
A complex surface can be defined as a rotating invariant surface without an axis, with arbitrary shape, regular or irregular surface structure[1]. Optical complex surfaces are widely used in new and traditional fields, such as communication and new energy automotive engineering, high-end equipment manufacturing, biomedicine and so on, so the surface shape accuracy of surfaces must be guaranteed. Compared with non-contact assessment, the measurement efficiency is not very high, but its evaluation accuracy can reach a high standard, which is widely used in the detection of various complex curved parts. The measuring device adopts the LM50 laser interferometer of German company, which can realize the measurement resolution of nanometer level in the range of 50mm, and the contact measurement method is used to evaluate the measurement. This type of measuring instrument is composed of probe, electrical box and data processing software, The instrument has its own measurement program. In order to better apply to the machine tool, the instrument also has a dynamic link library. C program can be used for secondary development to complete the measurement and evaluation of various complex parts. The surface is measured by contact measuring method[2-5], The coordinate value of the real measuring point of the surface obtained by LM50 laser interferometer is not the coordinate value of the real measuring point of the surface, but the value of the center of the probe is the actual measured data, so the radius of the probe should be compensated. In the case of complex curved surface, the method of measuring the radius of the measuring head has the micro-plane method, the micro-sphere method, the surface quasi-legal and the Delaunay triangulation.
method, and the method has to be further improved in terms of the accuracy of the radius compensation and the calculation efficiency. In this paper, the radius compensation method of probe based on surface fitting and Delaunay triangulation is proposed by using surface fitting method and Delaunay triangulation method, which can improve the surface measurement.

2. Radius compensation based on NURBS Surface Theory

2.1. Mathematical Model and partial derivative Vector of complex Surface

A NURBS surface is a double variable vector value function as shown below[6-10]:

\[ S(u, v) = \sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) \omega_{i,j} P_{i,j}, \quad 0 \leq u,v \leq 1 \]  

(1)

The control grid is formed by \( \{P_{i,j}\} \), \( \{\omega_{i,j}\} \) is a weighted factor, \( \{N_{i,p}(u)\} \) and \( \{N_{j,q}(v)\} \) is the NURBS basis function specified in the node vector \( U \) and \( V \), the definition \( \{N_{i,p}(u)\} \) is as follows:

\[
N_{i,p}(u) = \begin{cases} 
1, & u_i \leq u \leq u_{i+1} \\
0, & \text{else}
\end{cases}
\]

\[
N_{i,p}(u) = \frac{u-u_i}{u_{i+p}-u_i} N_{i,p-1}(u) + \frac{u_{i+p+1}-u}{u_{i+p+1}-u_{i+1}} N_{i+1,p-1}(u)
\]

(2)

\[
\text{stipulate} \; \frac{0}{0} = 0
\]

The same principle is available \( \{N_{j,q}(v)\} \).

where

\[ U = \left\{ a, \cdots, a, u_{p+1}, \cdots, u_{r-p-1}, b, \cdots, b \right\}_{p+1} \]

\[ V = \left\{ a, \cdots, a, v_{q+1}, \cdots, v_{s-q-1}, b, \cdots, b \right\}_{q+1} \]

here \( r = n + p + 1, s = m + q + 1 \).

\( \{Q_{l,j}\} \) is \( (n+1) \times (m+1) \) data points given \( k = 0,1,\ldots,n \) and \( l=0,1,\ldots,m \). Create a \( \{p,q\} \) degree surface which interpolated to these given data points.

\[ Q_{l,j} = S(u_l, v_l) = \sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(u_k) N_{j,q}(v_l) p_{i,j} \]

(3)

firstly, \( U \) and \( V \) can be respectively solved by a centripetal parameter method.

\[ U = \left\{ 0, \cdots, 0, u_{p+1}, \cdots, u_{r-p-1}, 1, \cdots, 1 \right\}_{p+1} \]

\[ V = \left\{ 0, \cdots, 0, v_{q+1}, \cdots, v_{s-q-1}, 0, \cdots, 0 \right\}_{q+1} \]
The second is the reverse seeking control point. (3) is a set of equations composed of \((n + 1) \times (m + 1)\) equations. \(S(u, v)\) is a tensor product surface, \(P_{i,j}\) can be operations more conveniently and more quickly by a series of interpolation operations. For fixed \(l\), (3) is written as

\[
Q_{k,l} = \sum_{j=0}^{m} N_{l,p}(v) \left( \sum_{j=0}^{m} N_{j,q}(v) P_{l,j} \right) = \sum_{j=0}^{m} N_{l,p}(v) R_{l,j}
\]

(4) is Interpolation operation for data point. \(Q_{k,j}\) \((k = 0, 1, \ldots, n)\). \(R_{l,j}\) is the control point on the surface for \(S(u, v)\) when \(v = v_l\). Now order \(i\) no change, let \(l\) change, (5) is equations of data points performing curve interpolation. \(P_{i,0}, L, P_{i,m}\) is the control vertex to calculate. The form of its matrix can be written as

\[
Q = AP
\]

where

\[
\begin{bmatrix}
Q_0 \\
Q_1 \\
\vdots \\
Q_n
\end{bmatrix} = \begin{bmatrix}
1 & N_{0,0} & N_{0,0} & N_{0,0} \\
Q_0 \\
N_{0,0} & N_{0,0} & N_{0,0} \\
O & O & O
\end{bmatrix} \begin{bmatrix}
P_{0} \\
P_{1} \\
\vdots \\
P_{n}
\end{bmatrix}
\]

(6)

Using Gaussian elimination method to solve the system of equations. So find all the control points \(P_{i,j}\). The algorithm of:

1. With \(U\) and parameters \(u_k\) do interpolation operation of \(m + 1\) times: \(l = 0, 1, 2, \ldots, m\), The curve equation interpolated at the point \(Q_{0,l}, L, Q_{n,l}\) is formed in this order. As a result, \(R_{i,j}\) can be obtained;

2. With \(V\) and parameters \(v_l\) do interpolation operation of \(n + 1\) times: \(i = 0, 1, 2, \ldots, n\), The curve equation interpolated at the point \(R_{i,0}, L, R_{i,m}\) is formed in this order. As a result, \(P_{i,j}\) can be obtained;

What needs to be required is the partial derivative vector of the NURBS surface. order

\[
S(u, v) = \frac{\omega(u, v) S(u, v)}{\omega(u, v)} = A(u, v)
\]

where

\[
A(u, v) = \sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) \omega_{i,j} P_{i,j}
\]

(9)

thus

\[
S_\alpha(u, v) = \frac{A_\alpha(u, v) - \omega_{\alpha}(u, v) S(u, v)}{\omega(u, v)}
\]

(10)

The first-order partial guide vector is formed by the complex curved surface, In the equation. \(\alpha\) Representative \(u\) or \(v\).

2.2. Radius compensation Operation in Surface Measurement

The tangent vector of any point on a complex surface are all contained in the plane of \(S_u(u, v), S_v(u, v)\) that is open. The normal vector at each point on the surface is the normal vector for the plane to be opened, The expression of the unit normal vector is[11]
\[ n = \frac{S_u(u,v) \times S_v(u,v)}{|S_u(u,v) \times S_v(u,v)|} \]  

(11)

Radius compensation using NURBS interpolation method, that is the process by which the interpolation surface moves in the direction of the normal vector. \( r \) is the radius of the interferometer probe. Hypothesis \( P_i(x, y, z) \) is a measuring point, then the contact point \( P'_i(x, y, z) \) is

\[ P'_i(x, y, z) = P_i(x, y, z) \pm rn \]  

(12)

3. Delaunay Compensation of Segmentation Measurement Theory

3.1. Delaunay Triangulation based on Bowyer-Watson [1 2]
In order to segment the surface by Delaunay triangulation, the following two conditions must be satisfied: maximizing the minimum angle and empty circle, maximizing the minimum angle to ensure that the smallest angle in the triangle formed by triangulation is the largest, and empty circle ensuring the uniqueness of triangulation. The specific subdivision steps are as follows:

Step1: Form a maximum triangle so that it includes all the measuring points;
Step2: The points measured by each interferometer are inserted step by step, and each point is inserted into to find the triangle containing the points contained in the outer circle of the triangle.
Step3: Remove the searched triangles to form a cavity of polygons;
Step4: A new triangle is built by connecting the polygons cavity with the inserted measurement points, which are vertexed by the inserted measurement points.

3.2. Calculate the normal vector of each triangulation triangle
Calculate the normal vector of \( \Delta a_1a_2a_3 \), set the coordinates of each vertex \( a_1(x_1, y_1, z_1) \), \( a_2(x_2, y_2, z_2) \), \( a_3(x_3, y_3, z_3) \). The formula for calculating the normal vector \( \overrightarrow{n}(n_i, n_j, n_k) \) of standard \( \Delta a_1a_2a_3 \) is as follows:

\[ \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} i \\ j \\ k \end{bmatrix} \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{bmatrix} \]  

(13)

3.3. Calculation of improved Vertex normal Vector
As shown in figure 1, After triangulation of the surface, the normal vector of each triangle around the measuring point \( a \) has an influence on the normal vector of the vertex, so it is necessary to calculate the weight function of each triangle surface. In this paper, the influence of edge length and angle on the normal vector of the vertex is considered synthetically. The larger the edge length is, the farther away the other vertices are from the center point. The smaller the influence of the triangular patch on the normal vector of the vertex, the greater the angle of the triangle at the apex of the triangle, and the greater the influence of the triangle on the normal vector of the vertex.
The formula for calculating the weighted average of the normal vector $\vec{n}$ of the Vertex is as follows:[13-14]:

$$\vec{n} = \frac{\sum_{i=1}^{k} \overrightarrow{N_i} R_i}{\sum_{i=1}^{k} R_i}$$  \hspace{1cm} (14)

where $k$ is the number of triangular slices around the point $a$, $\overrightarrow{N_i}$ is the normal vector of the $i$ triangular patch around $a$ point, $R_i$ is the radius of the outer circle of the triangular patch. Compared with the previous model, it is simplified in calculation and programming.

4. Envelope Surface-triangulation Radius compensation method

The method of NURBS surface fitting is used to measure the accuracy and the compensation error is small, but the data processing is more complicated and the calculation amount is large, which is not conducive to the rapid measurement. Maximum error of triangulation The minimum error is close to the surface fitting method, and the average error is large, but the data processing is relatively simple and the calculation is small. Through analysis, the smaller the curvature of the center trajectory surface of the triangulation probe is, that is, the larger the curvature radius is, the higher the fitting accuracy is, the higher the calculated normal vector direction is to the normal vector direction of the real surface, and the higher the compensation accuracy is. On the contrary, the larger the curvature of the probe center trajectory surface, the lower the fitting accuracy, the greater the difference between the normal vector direction of the calculated measuring point and the real normal vector direction, and the lower the compensation accuracy.

In view of the above two methods, a "envelope surface triangulation" method based on NURBS theory and Delaunay segmentation theory is proposed. That is, after the design surface is given, the curvature at each interpolation point is obtained. When the maximum curvature of the surface exceeds the given value, the NURBS surface fitting method is adopted, and when the maximum curvature of the surface is less than the given value, the Delaunay segmentation method is adopted, so that the radius error can be compensated quickly and accurately.

5. Simulation and analysis

In order to verify the above method, the simulation of this method is carried out, Let theoretical surface \( \tilde{1} \) be as follows:

$$z(x, y) = 30 - 30\sin\left(\frac{\pi x}{300}\right)\sin\left(\frac{\pi y}{300}\right) \quad 0 \leq x, y \leq 300$$
Distribution points in equal step size according to the surface parameter $u, v$, a series of point sets $P_{i,j}$ are obtained. The maximum curvature of the surface exceeds the curvature threshold, and the normal vector of each point in the point set is obtained. Each point on a complex surface moves upward in the direction of the normal vector 1.5 mm (Probe radius size), the new point set is the measurement point, and then the new point set is inversely calculated by NURBS surface interpolation, and the new surface is obtained, and the normal vector at the top of the new surface is obtained as shown in figure 2. Move 1.5 mm down along the normal vector direction, and get a new point set $P'_{i,j}$ after compensation. The index determined by deviation $e = |P_{i,j} - P'_{i,j}|$ for accuracy is shown in Table 1. Using the micro-plane method to perform the radius compensation calculation, the average surface error average value is $8.63 \times 10^{-3}$ mm.

**Table 1** Enveloping Surface - Triangulation Probe Radius Compensation accuracy analysis

| Surface No | Surface ① | Surface ② |
|------------|-----------|-----------|
| Maximum error $e_{\text{max}}$ (mm) | $2.93 \times 10^{-4}$ | $4.67 \times 10^{-4}$ |
| Least error $e_{\text{min}}$ (mm) | $1.28 \times 10^{-4}$ | $1.65 \times 10^{-4}$ |
| Average error $e$ (mm) | $1.95 \times 10^{-4}$ | $2.14 \times 10^{-4}$ |

**Figure 2** Radius Compensation simulation

The theoretical surface $\overrightarrow{2}$ is:

$$z(x, y) = 15 \sin \left( \frac{\pi x}{300} \right) \sin \left( \frac{\pi y}{300} \right) \quad 0 \leq x, y \leq 300$$

Distribution points in equal step size according to the surface parameter $u, v$, a series of point sets $P_{i,j}$ are obtained. By calculating the maximum curvature of the surface, the normal vector of each point is obtained. Moving 1.5 mm upward along the normal vector at each point on the surface (Probe radius size), The new point is the measuring point, the triangular dividing surface is obtained by triangulating the set of new points, and the normal vector of each triangular dividing surface is obtained as shown in figure 3, the weighted average obtains the normal vector of each triangle vertex as shown in figure 4. Move 1.5 mm down along the normal vector direction, and get a new point set $P'_{i,j}$ after compensation. The index determined by using deviation $e = |P_{i,j} - P'_{i,j}|$ as precision is as shown in Table 1. From the experimental data, it can be seen that this method is feasible.
6. Conclusion
The "envelope surface-triangulation" method to realize the radius compensation of the measuring head is put forward, and the surface radius compensation problem for different size curvature is successfully solved.

The mathematical model of calculating the normal vector of the Vertex after triangulation is established, and the algorithm is improved, and the simplification of calculation and programming is realized.

By means of MATLAB simulation, the "envelope surface-triangulation" method is compared with the micro-plane method, and the precision is greatly improved.

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