Solving incomplete fuzzy pairwise comparison matrix using fuzzy DEMATEL

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Abstract. Analytic Hierarchy Process (AHP) is one of the multi-criteria decision-making methods that utilize pair-wise comparison in the evaluation process. The method is later extended to the fuzzy environment to cater and solve the problem of ambiguity and imprecise information known as fuzzy AHP. In the evaluation process, the issue of incomplete information has become a prevalent problem which may lead to invalid and biased results. The incomplete information occurs commonly due to inefficiency or negligence in handling the information and also the unavailability of the information for particular instances and situations. In this paper, a fuzzy DEMATEL based method of estimating incomplete values in fuzzy AHP is proposed. An imputation approach is used in the proposed method where the incomplete information is replaced with plausible and reasonable fuzzy values in the fuzzy AHP decision matrix. A step-by-step illustration of approximating the incomplete information will be given through a numerical example. It is found out that the estimation of the incomplete information in fuzzy AHP using fuzzy DEMATEL preserved the consistency of evaluation which is a vital validity instrument of evaluation for the fuzzy AHP.

1. Introduction

Analytic Hierarchy Process (AHP) is a well-known decision-making method that utilizes the pairwise comparison evaluation between two objects in the process of obtaining the outcome. It was introduced by Saaty [1] and has been widely applied in solving selection, ranking and categorization problems. The AHP method has the advantage of able to break down a complex and unstructured problem into a systematic hierarchical order. It will synthesize the judgment from the pairwise comparison process to prioritize the criteria or the alternative effectively. Several extensive reviews have been offered to indicate the wide applications of AHP, which can be found in [2] and [3].

Due to lacking precise information, fuzzy AHP was introduced by [4] to cater for such situations with uncertainty in human preference which is vague, subjective and imprecise. Consequently, many approaches have been introduced that are based on a fuzzy environment [5]. This approach permits a better description of the situation that mimics human perception and thinking. Furthermore, its applications have been extended significantly in solving many real-life problems such as in supply chain management [6], risk assessment [7], service quality [8] and many more.

The issue of incomplete information in the evaluation of AHP and Fuzzy AHP is not new. The incomplete information may occur due to lack of knowledge of the experts or due to incompetency in handling the data itself. To solve this problem, [9] came up with a proposal using the rough set in an
incomplete pairwise matrix to accommodate and approximate the missing values where knowledge reduction is used to eliminate information that is not significant and make some assumptions on the missing values for certain attributes. In [10], a Monte Carlo simulation is presented to investigate the incomplete pairwise comparison matrices in AHP where the complete and incomplete pairwise comparison matrices were compared and the data were obtained from a known structure. In [11], the incomplete values in AHP were calculated by applying a backpropagation multi-layer perception. A new method for completion method was proposed in [12] based on graph theory by using through iteration to evaluate the missing values and later [13] proposed another method using the eigenvector method together with the logarithmic least squares method to obtain the values for the incomplete pairwise comparison matrices in AHP. Their method was then applied to rank professional tennis players which the players in which some of them never played each other. This will incur incomplete pairwise comparison matrices.

Some limitations are occurring through many of the methods for completing the incomplete matrix in AHP [14]. The common problem that exists is the inability to restore the consistency of the matrix besides having more complex and expensive computational costs. To overcome these problems, the method of Decision-Making Trial and Evaluation Laboratory (DEMATEL) has been implemented to estimate the missing values. This method is found to have a low computational cost and can derive a total relation matrix from the direct relation matrix. As there are still not many methods to cater to an incomplete pair-wise comparison matrix under a fuzzy environment, this paper presents a method to facilitate the incomplete FAHP by using the fuzzy DEMATEL. An imputation approach is used to approximate the missing values in the matrix. A step-by-step procedure is given on a numerical example and finally, the consistency of the completed matrix is examined.

2. Preliminaries
In this section, some fundamental definitions and concepts used in this paper are discussed.

A fuzzy set $A$ [15] is defined as a set of an ordered pair,

$$ A = \{ (x, \mu_A(x)) \mid x \in A, \mu_A(x) \in [0,1] \}, $$

where $\mu_A(x)$ is a membership function denoting the grade to any element $x$ in $A$ in the interval $[0,1]$. A normal fuzzy set refers to a set that contains at least an element $x$ in $A$ such that $\mu_A(x) = 1$ meanwhile the fuzzy set is convex if there exist $x_1$ and $x_2$ in $A$ such that

$$ \mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1),\mu_A(x_2)), \; \lambda \in [0,1]. $$

Consequently, a fuzzy number is a fuzzy set that is convex and normal. The two common types of fuzzy numbers are the triangular fuzzy number and the trapezoidal fuzzy number. In this paper, only the triangular fuzzy number will be defined and discussed. The membership function of a triangular fuzzy number $A = (a,b,c)$ is represented as

$$ \mu_A(x) = \begin{cases} 
\frac{x-a}{b-a} & a \leq x \leq b, \\
\frac{c-x}{c-b} & b \leq x \leq c, \\
0 & \text{elsewhere}.
\end{cases} $$

Given two triangular fuzzy numbers $A = (a_1,a_2,a_3)$ and $B = (b_1,b_2,b_3)$, the arithmetic operations between $A$ and $B$ are as follows:

- **Addition**, $A \oplus B = (a_1,a_2,a_3) \oplus (b_1,b_2,b_3) = (a_1+b_1,a_2+b_2,a_3+b_3)$
- **Subtraction**, $A \ominus B = (a_1,a_2,a_3) \ominus (b_1,b_2,b_3) = (a_1-b_1,a_2-b_2,a_3-b_3)$
- **Multiplication**, $A \otimes B = (a_1,a_2,a_3) \otimes (b_1,b_2,b_3) = (a_1b_1,a_2b_2,a_3b_3)$
- **Division**, $A \oslash B = (a_1,a_2,a_3) \oslash (b_1,b_2,b_3) = (a_1/b_1,a_2/b_2,a_3/b_3)$, provided that $b_i \neq 0$. 
A linguistic variable is an important tool in fuzzy set theory that describes a phenomenon imitating human perception. A linguistic variable $L$ [16] is defined as a quintuple $(H, T(H), U, G, M)$ where

- $H$ is the variable’s name,
- $T(H)$: is the set of linguistic term values of $H$,
- $U$ is a universe of discourse that stated the feature of the variable,
- $G$ is a syntactic rule that takes the grammar form and
- $M$ is a semantic rule to associate the meaning for each $H$.

A simple example of a linguistic variable is the height of a person whose linguistic values are naturally described as very short, short, average, high, and very high. Each linguistic term is represented by fuzzy numbers in the computation.

3. Fuzzy DEMATEL-based completion method for fuzzy pairwise comparison method.
A method of completing missing information in AHP using DEMATEL has been proposed in [14]. In reality, most cases will involve a vague and subjective environment and thus a fuzzy approach will suit most. However, missing information may also occur and the complexity of the problem increases as more parameters need to be considered. By extending the work of [14], a method of fuzzy DEMATEL based is proposed to solve the incomplete information problem in Fuzzy AHP. Some modifications of the approach are made where an imputation approach is used to approximate plausible and reasonable values for the Fuzzy AHP method. We first briefly describe the method of fuzzy AHP and Fuzzy DEMATEL.

3.1 Fuzzy AHP
This pairwise comparison method is an extension of the AHP with the evaluation made under a fuzzy environment. Although there are some variations of steps in the different approach of Fuzzy AHP, in general, the steps involved are

- Identify the problem and objectives that need to be achieved. The criteria and sub-criteria are determined for the problem.
- The problem is decomposed into a hierarchy structure with the goal at the top, followed by the criteria and the sub-criteria. The final level will be the alternatives.
- A comparison matrix is set up from the pairwise comparison evaluation between the criteria, sub-criteria, and alternatives that can be considered as the degree of preference between two objects. The matrix will be first normalized before the vector of priorities is calculated.
- The consistency ratio is calculated to reflect the consistency of evaluation by the decision-makers.
- To find the criteria/sub-criteria weight and the weight of the rating for alternatives, the elements in each row of the matrix will be aggregated.

3.2 Fuzzy DEMATEL.
Fuzzy DEMATEL is known to be a pairwise comparison method that shows causal relations between factors. The general steps of the method are given as follows:

- Define the goal, establish the scale and evaluation list of criteria.
- Perform the pairwise comparison to assess the effect between factors using suitable linguistic scales. The usual scales used are No Influence, Very Low Influence, Low Influence, High Influence and Very High Influence.
- A fuzzy direct relation matrix $D$ is developed from the judgment. This matrix is then transformed into a normalized direct relation matrix $N$.
- The total relation matrix $T$ is computed where $T = N(I - N)^{-1}$. The sum of row (D) and the sum of column (R) are then determined.
- The causal diagram is generated with $(D+R)$ and $(D-R)$ as its horizontal and vertical axis respectively. The $(D+R)$ values indicate the degree of importance factor meanwhile the $(D-R)$
shows the extension of the influence. If \( (D-R) \) is positive, the factor is in the causal category and when it is negative, the factor is in the effect category.

3.3 The proposed method.

Suppose that the evaluation of the incomplete pairwise comparison matrix of Fuzzy AHP of size \( n \times n \) is given as

\[
C = [x_{ij}]_{n \times n}, \ i = 1, ..., n \text{ and } j = 1, ..., n
\]

such that \( x_{ji} = \frac{1}{x_{ij}} \) for some missing elements in the matrix. The elements \( x_{ij} \) is in the form of triangular fuzzy numbers where \( x_{ij} = (x_{ijl}, x_{ijm}, x_{iju}) \) with \( l, m \) and \( u \) as the lower, middle and upper values of the triangular fuzzy numbers respectively. Besides, it is noted that

\[
\frac{1}{x_{ij}} = \frac{1}{(x_{ijl}, x_{ijm}, x_{iju})} = \left( \frac{1}{x_{iju}, \frac{1}{x_{ijm}, \frac{1}{x_{ijl}}}} \right).
\]

Step 1. Separate the corresponding elements of the triangular fuzzy number in the matrix into three different matrices with crisp values of the lower \((l)\), the middle \((m)\) and the upper \((u)\) values of the triangular fuzzy numbers. Thus we have

\[
C_l = [x_{ij1}]_{n \times n}, \ C_m = [x_{ij2}]_{n \times n}, \ C_u = [x_{ij3}]_{n \times n}.
\]

Each of the matrices has some missing values at the same corresponding cells.

Step 2. The missing values in each matrix of \( C_l, C_m \) and \( C_u \) will be substituted with 0. This will transform the matrices into direct relation matrices denoted as \( D_l, D_m \) and \( D_u \) respectively. Note that each matrix is now presented in the DEMATEL form.

Step 3. The direct relation matrices will then be normalized using the equation

\[
N_k = \frac{D_k}{\max \{|D_{ij}|_{j=1}^{n} \}} \quad \text{where } k = l, u, m. \tag{1}
\]

Step 4. The normalized direct relation matrices will be converted into total relation matrices \( T_k \) using the equation

\[
T_k = \lim_{p \to \infty} (N_k^p) = N_k(l - N_k)^{-1} \quad \text{where } k = l, u, m. \tag{2}
\]

At this stage, the 0s assigned to the missing values in the matrices will be replaced by certain non-zero values accordingly.

Step 5. The total relation matrices are transformed into pairwise comparison matrices as required for the Fuzzy AHP scheme with values between 0 and 1. Since one of the requirements of the Fuzzy AHP is to have value 1 along the diagonal, thus to ensure this happens, the equation

\[
m_{ij}^k = \sqrt{t_{ij}^k/t_{ji}^k} \tag{3}
\]

is applied to the total relation matrices \( T_k \).
Here, we already have the complete matrix $T_k$ with approximated values of missing elements. Thus the complete matrix with fuzzy number entries is given as

$$M = [m_{ij}]_{n \times n} = [(m^l_{ij}, m^m_{ij}, m^u_{ij})]_{n \times n}.$$ 

Step 6. Approximation of the fuzzy numbers of missing values is carried out where the closest fuzzy number among the designated linguistic term used in the evaluation is selected. As an example, let the linguistic terms and their corresponding triangular fuzzy numbers are as in table 1 and their illustrations are in figure 1.

**Table 1.** Linguistic terms and corresponding fuzzy numbers for evaluation in FAHP.

| Linguistic Terms | Triangular Fuzzy Numbers |
|------------------|--------------------------|
| Just Equal (JE)  | (1, 1, 1)                |
| Equally Important (EI) | (1, 3, 5)          |
| Weakly Important (WI) | (3, 5, 7)        |
| Moderately Important (MI) | (5, 7, 9)    |
| Strongly Important (SI) | (7, 9, 11)    |

**Figure 1.** Graphical representation of linguistic terms for evaluation in FAHP.

The vertex method of [16] will be employed to calculate the distance between two fuzzy numbers and the fuzzy number with the least will be chosen to represent the suitable linguistic terms for the missing value, and thus will be used as the entry for the missing cell.

Let $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ be two fuzzy numbers. The distance between $A$ and $B$ using the vertex method is given as

$$d(A, B) = \frac{1}{\sqrt{3}} \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}.$$ 

4. **Numerical example**

As an illustration of the proposed method suppose that we have an evaluation of Fuzzy AHP using triangular fuzzy numbers with linguistic terms as in table 1. Let there be some missing evaluations denoted by $\alpha_{ij} = (\alpha^l_{ij}, \alpha^m_{ij}, \alpha^u_{ij})$ and is given as in the matrix below.
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\[ C = \begin{bmatrix}
(1,1,1) & (1,3,5) & (3,5,7) & (\alpha_{14l}, \alpha_{34m}, \alpha_{14u}) & (5,7,9) \\
(1/5, 1/3, 1) & (1,1) & (1,3,5) & (3,5,7) & (5,7,9) \\
(1/7, 1/5, 1/3) & (1/5, 1/3, 1) & (1,1) & (1,3,5) & (5,7,9) \\
(\alpha_{41l}, \alpha_{41m}, \alpha_{41u}) & (1/7, 1/5, 1/3) & (1/5, 1/3, 1) & (1,1) & (3,5,7) \\
(1/9, 1/7, 1/5) & (1/9, 1/7, 1/5) & (\alpha_{53l}, \alpha_{53m}, \alpha_{53u}) & (1/7, 1/5, 1/3) & (1,1) \\
\end{bmatrix} \]

where

\[(\alpha_{41l}, \alpha_{41m}, \alpha_{41u})^{-1} = (\alpha_{14l}, \alpha_{14m}, \alpha_{14u}) \text{ and } (\alpha_{53l}, \alpha_{53m}, \alpha_{53u})^{-1} = (\alpha_{35l}, \alpha_{35m}, \alpha_{35u}).\]

Separating the matrix into low, medium and upper matrices, we obtain

\[ C_l = \begin{bmatrix}
1 & 1 & 3 & \alpha_{14l} & 5 \\
1 & 1 & 1 & \alpha_{35l} & 3 \\
1/3 & 1 & 1 & 1 & \alpha_{53l} \\
1/5 & 1/5 & \alpha_{53l} & 1/3 & 1 \\
\end{bmatrix}, \quad C_m = \begin{bmatrix}
1 & 3 & 5 & \alpha_{14m} & 7 \\
1/5 & 1/3 & 1 & 3 & \alpha_{35m} \\
1/7 & 1/7 & \alpha_{35m} & 1/5 & 1 \\
\end{bmatrix}, \quad C_u = \begin{bmatrix}
1 & 5 & 7 & \alpha_{14u} & 9 \\
1/5 & 1/5 & 1 & 5 & \alpha_{35u} \\
1/7 & 1/7 & 1 & 5 & \alpha_{53u} \\
1/9 & 1/9 & \alpha_{53u} & 1/7 & 1 \\
\end{bmatrix} \]

and substituting the missing values with 0 gives

\[ D_l = \begin{bmatrix}
1 & 1 & 3 & 0 & 5 \\
1 & 1 & 1 & 3 & 5 \\
1/3 & 1 & 1 & 1 & 0 \\
0 & 1/3 & 1 & 1 & 3 \\
1/5 & 1/5 & 0 & 1/3 & 1 \\
\end{bmatrix}, \quad D_m = \begin{bmatrix}
1 & 3 & 5 & 0 & 7 \\
1/5 & 1/3 & 1 & 3 & 0 \\
1/7 & 1/7 & 0 & 1/5 & 1 \\
\end{bmatrix}, \quad D_u = \begin{bmatrix}
1 & 5 & 7 & 0 & 9 \\
1/5 & 1/5 & 1 & 5 & 0 \\
1/7 & 1/5 & 1 & 5 & 0 \\
0 & 1/7 & 1/5 & 1 & 7 \\
1/9 & 1/9 & 0 & 1/7 & 1 \\
\end{bmatrix} \]

which are now known as the direct relation matrices. These matrices are then being normalized using equation (1) to become the normalized direct relation matrices as follows:

\[ N_l = \begin{bmatrix}
0.071 & 0.071 & 0.214 & 0.000 & 0.357 \\
0.071 & 0.071 & 0.071 & 0.214 & 0.357 \\
0.023 & 0.071 & 0.071 & 0.071 & 0.214 \\
0.000 & 0.024 & 0.071 & 0.071 & 0.214 \\
0.014 & 0.014 & 0.000 & 0.024 & 0.071 \\
\end{bmatrix}, \quad N_m = \begin{bmatrix}
0.050 & 0.150 & 0.250 & 0.000 & 0.350 \\
0.017 & 0.050 & 0.150 & 0.250 & 0.350 \\
0.010 & 0.017 & 0.050 & 0.150 & 0.000 \\
0.000 & 0.010 & 0.017 & 0.050 & 0.250 \\
0.007 & 0.007 & 0.000 & 0.010 & 0.050 \\
\end{bmatrix} \]
Using equation (2), the above matrices will be converted into the total relation matrices as in the following.

\[ T_l = \begin{bmatrix}
0.099 & 0.114 & 0.267 & 0.0592 & 0.481 \\
0.097 & 0.110 & 0.129 & 0.2795 & 0.528 \\
0.036 & 0.091 & 0.101 & 0.1076 & 0.074 \\
0.010 & 0.040 & 0.090 & 0.1000 & 0.273 \\
0.019 & 0.020 & 0.008 & 0.0334 & 0.099 \\
\end{bmatrix} \quad T_m = \begin{bmatrix}
0.063 & 0.178 & 0.310 & 0.101 & 0.484 \\
0.024 & 0.067 & 0.180 & 0.314 & 0.485 \\
0.012 & 0.023 & 0.063 & 0.174 & 0.059 \\
0.003 & 0.014 & 0.022 & 0.063 & 0.286 \\
0.008 & 0.009 & 0.004 & 0.014 & 0.063 \\
\end{bmatrix} \quad T_u = \begin{bmatrix}
0.046 & 0.215 & 0.337 & 0.130 & 0.490 \\
0.012 & 0.048 & 0.216 & 0.339 & 0.477 \\
0.006 & 0.011 & 0.046 & 0.213 & 0.066 \\
0.001 & 0.008 & 0.010 & 0.046 & 0.296 \\
0.005 & 0.006 & 0.003 & 0.008 & 0.046 \\
\end{bmatrix} \]

Since the fuzzy AHP matrix needs to have the diagonal to be of value 1, equation (3) will accommodate this requirement. Hence, by applying equation (3) to \( T_l, T_m, T_u \) in particular, for the missing values, we have

\[
(m_{14}^l,m_{14}^m,m_{14}^u) = \left( \frac{0.060}{0.010}, \frac{0.101}{0.003}, \frac{0.130}{0.001} \right) = (2.449, 5.802, 11.402) \]

\[
(m_{35}^l,m_{35}^m,m_{35}^u) = \left( \frac{0.074}{0.009}, \frac{0.059}{0.004}, \frac{0.066}{0.003} \right) = (2.867, 3.841, 4.690) \]

Finally, the vertex method will be used to determine the suitable corresponding linguistic term to be used. For instance, if the linguistic set in table 1 is used, the two closest linguistic term that is close to \( m_{35} \) would be EI = (1,3,5) and WI = (3,5,7). We have

\[
d(m_{14}, EI) = \sqrt{\frac{1}{3}(2.867 - 1)^2 + (3.841 - 3)^2 + (4.69 - 5)^2} = 1.196
\]

\[
d(m_{14}, WI) = \sqrt{\frac{1}{3}(2.867 - 3)^2 + (3.841 - 5)^2 + (4.69 - 7)^2} = 1.494
\]

and thus, the suitable linguistic term for \( m_{35} \) is Equally Important (EI) and consequently for \( m_{53} \) is its reciprocal EI. Utilizing the same process, it is found that the suitable linguistic term for \( m_{14} \) is MI. The
complete matrix that will be used for the Fuzzy AHP with approximated values for the incomplete cells is
\[
\begin{pmatrix}
(1,1,1) & (1,3,5) & (3,5,7) & (5,7,9) & (5,7,9) \\
(1/5,1/3,1) & (1,1,1) & (1,3,5) & (3,5,7) & (5,7,9) \\
(1/7,1/5,1/3) & (1/5,1/3,1) & (1,1,1) & (1,3,5) & (1,3,5) \\
(1/9,1/7,1/5) & (1/7,1/5,1/3) & (1/5,1/3,1) & (1,1,1) & (3,5,7) \\
(1/9,1/7,1/5) & (1/9,1/7,1/5) & (1/5,1/3,1) & (1/7,1/5,1/3) & (1,1,1)
\end{pmatrix}
\]

4.1. Consistency Ratio of FAHP
The consistency of the evaluation using the approximated values is important to ensure its validity of the FAHP. Thus the consistency ratio of an \( n \times n \) matrix may be calculated [17] as
\[ CR = \frac{CI}{RI} \]
with \( CI = \frac{\lambda_{\text{max}} - n}{n-1} \) where \( \lambda_{\text{max}} \) is the largest eigenvalue of the matrix and the \( RI \) (random indices) is determined from table 2 [1].

| Size of matrix \((n)\) | 1-2 | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|------------------------|-----|-----|-----|-----|-----|-----|-----|-----|
| Random Indices \((RI)\) | 0.0 | 0.58| 0.9 | 1.12| 1.24| 1.32| 1.41| 1.45|

Thus, we have
\[ CI = \frac{\lambda_{\text{max}} - n}{n-1} = \frac{5.29 - 5}{5 - 1} = 0.0725 \]
and
\[ CR = \frac{CI}{RI} = \frac{0.0725}{1.12} = 0.06 \]
which is less than 0.1. This indicates that the evaluation is consistent [1].

5. Conclusion
This paper proposed a method to solve the issue of missing or incomplete information in the pairwise comparison matrix under a fuzzy environment. The method of FAHP is used as a case study the approximation of the incomplete is obtained using the amputation approach through Fuzzy DEMATEL. The proposed method is proven to be effective and simple. By checking the consistency ratio of the completed matrix, it is also found to be consistent with reasonable approximated values.

By referring to the result of the ranking and the comparison with the existing completion method that has been introduced by other researchers, it is proven that the proposed completion method is more effective and simple. This is because the proposed completion method has a low computational cost and does not require much time to apply to solve the problem in an incomplete fuzzy pairwise comparison. It can estimate the values of the missing elements simultaneously in the fuzzy pairwise comparison matrix in fuzzy AHP. Hence, following the procedure of the proposed method, the completion of the incomplete fuzzy pairwise comparison matrix becomes straightforward and efficient. Since the main concern in dealing with FAHP with larger \( n \) is its consistency, this work may be extended to specifically looking into the consistency issue of the proposed method with higher dimensions.
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