Time-reversal violating rotation of polarization plane of light in gas placed in electric field

V. G. Baryshevsky*, D. N. Matsukevich†

Institute of Nuclear Problems, Belarusian State University
St. Bobryiskaya 11, 220050, Minsk, Republic of Belarus

(October 30, 2018)

I. INTRODUCTION

Violation of time reversal symmetry was observed in $K_0$ meson and $B_0$ meson decay and remains one of the greatest unsolved problems in the elementary particle physics. A lot of attempts have been undertaken to observe time-reversal violating phenomena in different processes experimentally. However, those experiments have not been successful. Among them there are, for example, measurements of electric dipole moment (EDM) of neutrons, atoms and molecules. No EDM was found but these experiments impose strong restrictions on the theory. At EDM of heavy atoms search, in particular, tight limits for parameters of electron-nucleon $P$-, $T$- violating interactions and value of electron EDM are set.

It is well known that essential progress in measurements of $P$-odd interaction constants was achieved at study of the optical activity of atomic gases. High precision of optical measurements allow us to expect that investigation of time reversal invariance in photon interactions with atoms will provide new limits for constants of $T$-noninvariant weak interaction.

As it was shown in $T$ noninvariant interactions induce several new optical phenomena. They are: $T$ noninvariant rotation of polarization plane of light in electric field, $T$ noninvariant birefringence and $T$ noninvariant rotation of light polarization plane in diffraction grating with non-centrosymmetrical elementary cell.

To observe the rotation of polarization plane of light in electric field experiments are under preparation now. Therefore, it is important to attract attention to

Rotation of polarization plane of light in gas placed in electric field is considered. Different factors causing this phenomenon are investigated. Angle of polarization plane rotation for transition $6S_{1/2} \rightarrow 7S_{1/2}$ in cesium ($\lambda = 539$nm) is estimated. The possibility to observe this effect experimentally is discussed.

The present paper is organized as follows. In Sec. II the general theory describing $PT$ noninvariant rotation of polarization plane of light in atomic gas is examined. Polarization plane rotation caused by interference of $P$-, $T$- odd and Stark-induced transitions is considered in details in Sec. III. Polarization plane rotation caused by nonzero atomic EDM is considered in Sec. IV. Estimates of effects magnitude for different kinds of transitions are given in Sec. V. Sec. VI examines possible sources of $P$-, $T$- violating interactions in atom and Sec VII gives estimates of angle of $P$-, $T$- odd polarization plane rotation for $6S_{1/2} \rightarrow 7S_{1/2}$ transition in cesium. Section VIII briefly discusses possibilities to observe this phenomenon experimentally and summarizes general conclusions.

II. PT-ODD ROTATION OF POLARIZATION PLANE OF LIGHT

To illustrate the mechanism of polarization plane rotation due to interference of $PT$ - odd and Stark-induced transition amplitudes we consider a simple model at first. Let us take an atom in the $s_{1/2}$ state and place it to an electric field. Taking into account the admixture of the nearest $p_{1/2}$ state due to $P$- and $T$- odd interactions and interaction with electric field one can represent the wave function of an atom in the form:

$$|\tilde{s}_{1/2}| = \frac{1}{\sqrt{4\pi}}(R_0(r) - R_1(r)(\tilde{\sigma}\tilde{n})\eta - R_1(r)(\tilde{\sigma}\tilde{n})(\tilde{E}\delta)|\chi_{1/2}|\) \tag{1}

Here $\tilde{\sigma}$ are the Pauli matrices, $\tilde{n} = \tilde{r}/r$ is the unit vector along $\tilde{r}$, $\tilde{E}$ is the electric field strength, $R_0$ and $R_1$ are radial parts of $s_{1/2}$ and $p_{1/2}$ wave functions respectively, $|\chi_{1/2}|$ is the spin part of wave function, $\eta$ and $\delta$ are the mixing coefficients describing $P$ and $T$ noninvariant interactions and electric field respectively, $\tilde{E}$ is the electric field strength.
Interference of Stark and PT - odd terms changes electron spin direction as follows:

\[ \Delta \tilde{s}(\vec{r}) = \frac{\gamma \delta}{8\pi} R_1^2(r) (\chi_{1/2}/(\tilde{\sigma} n) \tilde{\sigma}(\tilde{\sigma} n)(\tilde{\sigma} \hat{E}) + (\tilde{\sigma} \hat{E}) (\tilde{\sigma} n) \tilde{\sigma}(\tilde{\sigma} n)(\chi_{1/2}) \]

\[ = \frac{\gamma \delta R_1^2(r)}{8\pi} (4\tilde{n}(\tilde{n} \hat{E}) - 2\hat{E}) \]  

(2)

Vector field \(4\tilde{n}(\tilde{n} \hat{E}) - 2\hat{E}\) is shown in Fig. 1. Since \(\Delta \tilde{s}\) does not depend on initial direction of atomic spin, this spin structure appears even in nonpolarized atom. Spin vector averaged over spatial variables differs from zero and is directed along \(\hat{E}\). Photons with angular momentum parallel and antiparallel to \(\hat{E}\) differently interact with such gas. It causes rotation of polarization plane of photons.

Let us also note that according to [14] magnetization of gas in electric field induces magnetic field \(\vec{H}_{\text{mld}}(\hat{E})\). This magnetic field interacts with magnetic moment of an atom giving additional contribution to rotation of polarization plane of light [12].

The refraction index of gas is given by

\[ n = 1 + \frac{2\pi N}{k^2} f(0) \]  

(3)

where \(N\) is the number of atoms per \(\text{cm}^3\), \(k\) is the photon wave number, \(f(0) = f_{ik} e^*_i e_k\) is the amplitude of elastic coherent forward scattering of photons by atoms. Here \(e^*\) and \(e^\prime\) are the polarization vectors of initial and scattered photons respectively. Repeated indices imply summation. In dipole approximation

\[ f_{ik} = \omega^2 \alpha_{ik}/c^2 \]  

(4)

where \(\alpha_{ik}\) is the tensor of dynamical polarizability of an atom, \(\omega\) is the frequency of incident light. According to \([11,14,17]\) the amplitude of light scattering by nonpolarized atomic gas in electric field is expressed by

\[ f_{ik} = f_{ik}^{\text{sc}} + \omega^2 (i\beta^P_{ik} e_{ikl} n_{\gamma l} + i\beta^PT_{ik} e_{ikl} n_{el} + \beta_{ik}^T (\tilde{n} \gamma \cdot \tilde{n} \hat{E}) \delta_{ik}) \]  

(5)

where \(n_+\) and \(n_-\) are the refraction indices for left and right circularly polarized photons. Vectors \(e^+_i\) and \(e^-_i\) describe left and right circularly polarized photons respectively. \(e^+_k = \mp (e^+_i \pm i e^-_i)/\sqrt{2}\). Using (3) and (5) we can express polarization plane rotation as follows

\[ \phi = \frac{2\pi N \omega}{c} (\beta^P_s + \beta^PT_s (\tilde{n} \hat{E} \tilde{n}_G))^l \]  

(6)

Term proportional to \(\beta^P_s\) describes the well known phenomenon of \(P\)-odd but \(T\)-even rotation of polarization plane of light. Term proportional to \(\beta^PT_s\) corresponds to \(P\) - and \(T\)- noninvariant light polarization plane rotation about the direction of electric field [14].

In contrast to \(P\)-odd, \(T\)-even rotation the reversion of electric field direction changes the sign of \(P\), \(T\)-odd rotation of light polarization plane. This allows to distinguish \(P\)-, \(T\)-odd effects from the other possible effects of polarization plane rotation.

According to \([11,12]\) tensor of dynamical polarizability of an atom (molecule) in the ground state \(|\tilde{g}_n\rangle\) has the form

\[ \alpha_{ik}^n = \sum_m \left\{ \frac{|\tilde{g}_n| d_i |\tilde{e}_m\rangle \langle \tilde{e}_m| d_k |\tilde{g}_n\rangle}{E_{em} - E_{gn} - \gamma \omega} \right\} \]

\[ + \frac{|\tilde{g}_n| d_i |\tilde{e}_m\rangle \langle \tilde{e}_m| d_k |\tilde{g}_n\rangle}{E_{em} - E_{gn} + \gamma \omega} \]  

(8)

where \(|\tilde{g}_n\rangle\) and \(|\tilde{e}_m\rangle\) are the wave functions of an atom in the ground and excited states perturbed by electric field and \(P\)-, \(T\)- noninvariant interactions, \(\gamma\) is the operator of dipole transition, \(E_{em}\) and \(E_{gn}\) are the energies of atom states \(|\tilde{g}_n\rangle\) and \(|\tilde{e}_m\rangle\), respectively.

In general case atoms are distributed over the magnetic sub-levels of ground state \(g_n\) with the probability \(P(n)\). Therefore \(\alpha_{ik}^n\) should be averaged over all states \(n\). As a result, the polarizability can be written as

\[ \alpha_{ik} = \sum_n P(n) \alpha_{ik}^n \]  

(9)

In the present paper we discuss the nonpolarized atomic gas. In this case \(P(n) = 1/2j_g + 1\) where \(j_g\) is the total moment of atom in the ground state \(g\).

Tensor \(\alpha_{ik}\) can be decomposed into irreducible parts as

\[ \alpha_{ik} = \alpha_0 \delta_{ik} + \alpha^a_{ik} + \alpha^s_{ik}. \]  

(10)

Here \(\alpha_0 = \frac{1}{2j_g + 1} \sum_i \alpha_{ii}\) is the scalar, \(\alpha^a_{ik} = \frac{1}{2}(\alpha_{ik} + \alpha_{ki}) - \alpha_0 \delta_{ik}\) is the symmetric tensor of rank two, \(\alpha^s_{ik} = \frac{1}{2}(\alpha_{ik} - \alpha_{ki})\) is the antisymmetric tensor of rank two:

\[ \alpha^a_{ik} = \frac{\omega}{(2j_g + 1)\hbar} \sum_{m,n} \left\{ \frac{|\tilde{g}_n| d_i |\tilde{e}_m\rangle \langle \tilde{e}_m| d_k |\tilde{g}_n\rangle - |\tilde{g}_n| d_i |\tilde{e}_m\rangle \langle \tilde{e}_m| d_k |\tilde{g}_n\rangle}{\omega_{em, gn}^2 - \omega^2} \right\} \]

(11)
where \( \omega_{em,gn} = (E_{em} - E_{gn})/\hbar \).

If atoms are nonpolarized then in the absence of P- and T-odd interactions the antisymmetric part of polarizability is equal to zero. Therefore comparison of (7) and (3) yields

\[
\alpha^a_{ik} = i \epsilon_{ikl}(\beta^P_{n\gamma} + \beta^T_{E} n_{EI}).
\]

(12)

According to [12,14,15] correct consideration of P-odd but T-even interactions requires to take into account both E1 and M1 transition amplitudes. If only E1 transition operators are considered in (8) the P-odd but T-even polarizability \( \beta^P_s \) becomes equal to zero.

Evaluation of expression (12) for left (or right) circular polarization of incident light at \( \vec{n}_E \parallel \vec{n}_E \) yields \( \alpha^a_{ik} = \beta^P_{E} \). As a result we can represent P-, T-odd scalar polarizability of an atom as follows:

\[
\beta^P_{E} = \frac{\omega}{(2j_g + 1)\hbar} \sum_{n,m} \left\{ \frac{\langle \vec{g}_n | d_- | \vec{e}_m \rangle \langle \vec{e}_m | d_+ | g_n \rangle}{\omega_{em,gn}^2 - \omega^2} - \frac{\langle \vec{g}_n | d_+ | \vec{e}_m \rangle \langle \vec{e}_m | d_- | g_n \rangle}{\omega_{em,gn}^2 - \omega^2} \right\}
\] (13)

where \( d_{\pm} = \mp (d_x \pm id_y)/\sqrt{2} \).

For further analysis more detailed expressions for wave functions of an atom are necessary. Since constants of P-, T- noninvariant interactions are very small we can use perturbation theory. Let \( |\vec{g} \rangle \) and \( |\vec{e} \rangle \) be the wave functions of ground and excited states of atom (molecule) in electric field \( \vec{E} \) in the absence of P-, T- odd interactions. Switch on P-, T- noninvariant interaction \( (H_T \neq 0) \). According to the perturbation theory the wave functions \( |\vec{g} \rangle \) and \( |\vec{e} \rangle \) take the form

\[
|\vec{g} \rangle = |g \rangle + \sum_n |n\rangle \langle n| H_T |\vec{g} \rangle \frac{1}{E_g - E_n}
\]

\[
|\vec{e} \rangle = |e \rangle + \sum_n |n\rangle \langle n| H_T |\vec{e} \rangle \frac{1}{E_e - E_n}
\]

(14)

where \( H_T \) is Hamiltonian of P-, T- noninvariant interactions.

It should be reminded that denominator of (13) contains shifts caused both by interaction of electric dipole moment of an atom with electric field \( \vec{E} \) and magnetic moment of an atom with T-odd induced magnetic field \( H_{ind}(\vec{E}) \) [13]. If \( H_T \) is small, one can represent total polarizability \( \beta^P_{E} \) as the sum of two terms

\[
\beta^P_{E} = \beta_{mix} + \beta_{split}.
\]

Here

\[
\beta_{mix} = \frac{\omega}{(2j_g + 1)\hbar} \sum_{n,m} \left\{ \frac{\langle \vec{g}_n | d_- | \vec{e}_m \rangle \langle \vec{e}_m | d_+ | g_n \rangle}{\omega_{em,gn}^2 - \omega^2} - \frac{\langle \vec{g}_n | d_+ | \vec{e}_m \rangle \langle \vec{e}_m | d_- | g_n \rangle}{\omega_{em,gn}^2 - \omega^2} \right\}
\]

(16)

\[
\beta_{split} = \frac{\omega}{(2j_g + 1)\hbar} \sum_{n,m} \left\{ \frac{\langle \vec{g}_n | d_- | \vec{e}_m \rangle \langle \vec{e}_m | d_+ | g_n \rangle}{\omega_{em,gn}^2 - \omega^2} - \frac{\langle \vec{g}_n | d_+ | \vec{e}_m \rangle \langle \vec{e}_m | d_- | g_n \rangle}{\omega_{em,gn}^2 - \omega^2} \right\}
\]

(17)

where energy levels \( E_{e,m}(\vec{E}) \) and \( E_{g,n}(\vec{E}) \) contain shifts caused by interaction of electric dipole moment of an atom with electric field \( \vec{E} \) and magnetic moment of an atom with T-odd induced magnetic field \( H_{ind}(\vec{E}) \).

Below we consider small detuning of radiation frequency from resonance frequency of atomic transition. Therefore (13) and (17) can be written as follows

\[
\beta_{mix} = \frac{1}{2\hbar(2j_g + 1)} \sum_{n,m} \left\{ \frac{\langle \vec{g}_n | d_- | \vec{e}_m \rangle \langle \vec{e}_m | d_+ | g_n \rangle}{\omega_{em,gn}^2 - \omega^2} - \frac{\langle \vec{g}_n | d_+ | \vec{e}_m \rangle \langle \vec{e}_m | d_- | g_n \rangle}{\omega_{em,gn}^2 - \omega^2} \right\}
\]

(18)

\[
\beta_{split} = \frac{1}{2\hbar(2j_g + 1)} \sum_{n,m} \left\{ \frac{\langle \vec{g}_n | d_- | \vec{e}_m \rangle \langle \vec{e}_m | d_+ | g_n \rangle}{\omega_{em,gn}^2 - \omega^2} - \frac{\langle \vec{g}_n | d_+ | \vec{e}_m \rangle \langle \vec{e}_m | d_- | g_n \rangle}{\omega_{em,gn}^2 - \omega^2} \right\}
\]

(19)

### III. Interference of PT-odd and Stark Induced Amplitudes

In this section we consider effects associated with \( \beta_{mix} \). Rotation associated with \( \beta_{split} \) is studied in Section IV.

Let us assume that electric field is small enough. For atoms in the ground state the energy of Stark interactions is usually less than the difference of energies of levels mixed by electric field. In this case we can use first order of perturbation theory. Perturbed states \( |\vec{g} \rangle \) and \( |\vec{e} \rangle \) have the form

\[
|\vec{g} \rangle = |g \rangle + \sum_n |n\rangle \langle n| H_T |\vec{g} \rangle \frac{1}{E_g - E_n}
\]

\[
|\vec{e} \rangle = |e \rangle + \sum_n |n\rangle \langle n| H_T |\vec{e} \rangle \frac{1}{E_e - E_n}
\]
Here \( H_T \) is the Hamiltonian of \( P^- \), \( T^- \) noninvariant interactions, \(|g⟩\) and \(|e⟩\) are the unperturbed ground and excited states of an atom, \( \vec{E} \) is the external electric field. We assume that electric field is directed along \( z \) axis.

Using (20) we can rewrite \( \beta_{mix} \) as follows:

\[
\beta_{mix} = -\frac{2}{3\hbar(2j_g + 1)} \frac{Re(⟨e|d^{PT}|g⟩)}{\sqrt{2(ω_{zm,zn} - ω)}}
\]

where

\[
⟨g|d^{PT}|e⟩ = \sum_{m_e, m_e} \frac{⟨g|H_T|m⟩⟨m|d^z_e|e⟩}{E_m - E_g} + \frac{⟨g|d^z_e|m⟩⟨m|H_T|e⟩}{E_m - E_e}
\]

and

\[
⟨g|d^{PT}|e⟩ = \Lambda_{ik} e_i E_k,
\]

is the Stark-induced amplitude of transition between states \( g \) and \( e \) in the constant electric field \( \vec{E}, \vec{e} \) is the polarization vector of photon.

\[
\Lambda_{ik} = \sum_{m} \frac{⟨g|d_k|m⟩⟨m|d^z_e|e⟩}{E_m - E_g} + \frac{⟨g|d^z_e|m⟩⟨m|H_T|e⟩}{E_m - E_e}
\]

Representation of the second-rank tensors \( \Lambda_{ik} \) and \( e_i E_k \) in terms of their irreducible spherical components yields [10]:

\[
⟨e|d^{PT}|g⟩ = \sum_{q, q'} (-1)^q q' \Lambda_{q, q'}^K E_{q}^{−q} E_{q'}^{−q'}
\]

\[
= \sum_{K Q} (-1)^Q \Lambda^K_Q (E \otimes e)^K_{−Q},
\]

where subscripts \( q \) and \( q' \) refer to the spherical vector components, \( \Lambda^K_Q \) and \((E \otimes e)^K_{−Q}\) are the components of irreducible spherical tensors. Using Wigner - Ekhard theorem we can represent \( \Lambda^K_Q \) as follows:

\[
\Lambda^K_Q = (-1)^j_e - m_e \begin{pmatrix} j_e & K \\ -j_e & m_g \end{pmatrix} \Lambda^K_{Q}
\]

Reduced matrix elements \( \Lambda^K \) \((K = 0, 1, 2)\) are proportional to the scalar, vector and tensor transition polarizability respectively. Substituting (20), (23) into (22), representing the matrix element of \( PT^- \) odd \( E1 \) transition (22) in terms of reduced matrix element \( ⟨e|d^{PT}|g⟩ \) and performing summation over \( m_e, m_e \) one obtains the following expression for \( \beta_{mix} \)

\[
β_{mix} = -\frac{2}{3b(2j_g + 1)} \frac{Re(⟨e|d^{PT}|g⟩)}{√2(ω_{zm,zn} - ω)}
\]

We assume here that electric field \( \vec{E} \) is parallel to the direction of light propagation and use the expression \( ⟨E \otimes e^2⟩_Z = E/√2 \). Due to orthogonality of 3j-symbols only terms proportional to the vector part of transition polarizability remains in (27) after summation over magnetic sub-levels.

Equations (23) and (1) give the angle of polarization plane rotation without considering of the Doppler broadening in gas. Because of Doppler shift the resonance frequency of transition for a single atom depends on atom velocity. In order to obtain expression for angle of polarization plane rotation in this case we should average (27) over Maxwell distribution of atom velocity.

If nucleus has a nonzero spin we should take into account the hyperfine structure. After routine calculations the angle of \( P^- \),\( T^- \) odd rotation of polarization plane can be expressed as:

\[
φ = 4πN_F \frac{ω}{hcΔ_D} g(u, v) \frac{1}{3(2F_g + 1)} K^2
\]

\[
\times Re(⟨g|d^{PT}|e⟩) \Lambda^1 E \frac{1}{√2}.
\]

For completeness we give here the expression for absorption length of light in atomic gas [13]

\[
L^{-1} = 4πN_F \frac{ω}{hcΔ_D} f(u, v) \frac{1}{3(2F_g + 1)} K^2
\]

\[
\times |⟨g|A|e⟩|^2
\]

Here \( F_g, F_e \) are the total angular moments of an atom in ground and exited states respectively, \( j_g \) and \( j_e \) are the total electron moments in these states, \( i \) is the nuclear spin.

\[
N_F = N - \frac{2F_g + 1}{(2i + 1)(2j_g + 1)}
\]

is the density of atoms with total moment \( F_g \),

\[
K^2 = (2F_g + 1)(2F_e + 1) \begin{pmatrix} i & j_g & F_g \\ 1 & F_e & j_e \end{pmatrix}
\]

\[
Δ_D = ω_0√2kT/Mc^2 \] is the Doppler line width,

\[
g(u, v) = \frac{Im}{Re} \left( e^{-iu} - e^{-iv} \right)
\]

\[
f(u, v) = \sqrt{1 - \Phi(−iw)}
\]

where \( w = u + iv, \Phi(z) = 2π∫_0^∞ dc e^{-ct^2}, u = (ω-ω_0)/Δ_D, v = Γ/2Δ_D, Γ \) is the recoil line width, \( ⟨g|A|e⟩ \) is the reduced matrix element of dipole transition between states \(|e⟩\) and \(|g⟩\).

IV. ROTATION OF POLARIZATION PLANE DUE TO ATOMIC EDM

Presence of EDM in ground or excited state of atom also causes rotation of polarization plane of light. We can
derive the expression for the angle of polarization plane rotation performing the calculations similar to those described in Sec. III, but using $\beta_{\text{split}}$ instead of $\beta_{\text{mix}}$. But in this case the calculations can be appreciably simplified if we note that $P$, $T$- noninvariant rotation caused by atomic EDM is similar to Faraday rotation of photon polarization plane in a weak magnetic field. Indeed, according to [15,18] a weak magnetic field affects the refractive index of atomic gas in two ways: through the change of the magnetic sub-levels energies and through the mixing of hyperfine states.

If we consider only terms proportional to the magnetic field strength $H$ and neglect the terms of higher orders then the level shift is [18]:

$$\Delta E_i = -H \langle i | \mu_z | i \rangle$$

The magnetic field $H$ mixes states of the same $F_z$ but different $F$, so the state $|j\rangle$ becomes

$$|\bar{j}\rangle = |j\rangle - \sum_{k \neq j} H |k\rangle \frac{\langle k | \mu_z | j \rangle}{E_k - E_j}$$

If atom has an EDM then electric field similarly affects the refraction index (see [17]) and leads to atomic levels shift:

$$\Delta E_i = -E \langle i | d_z | i \rangle$$

It also mixes the hyperfine states of atom with the same $F_z$ but different $F$

$$|\bar{j}\rangle = |j\rangle - \sum_{k \neq j} E |k\rangle \frac{\langle k | d_z | j \rangle}{E_k - E_j}$$

As a result we can use the expression for rotation of polarization plane of light in a weak magnetic field [15,18] for calculations of effect of polarization plane rotation in electric field. For this we must substitute $H \rightarrow E$, $\mu_g \rightarrow d_g$, $\mu_e \rightarrow d_e$ where $d_e, d_g$ are EDM of atom in ground and excited states, $\mu_i$ is the magnetic moment of state $i$.

If we neglect quadrupole transition amplitudes (it is possible for example for $6S_{1/2} \rightarrow 7S_{1/2}$ transition in cesium), then the angle of polarization plane rotation has the form

$$\phi = \frac{2\pi N l}{(2i + 1)(2j_g + 1)} \frac{\omega}{\hbar \Delta_D} \frac{E_z}{\hbar \Delta_D} \left| \langle g || A || e \rangle \right|^2 \times \left( \frac{\partial g(u, v)}{\partial u} \delta_1 + 2g(u, v) \gamma_1 \right).$$

(31)

The expressions for parameters $\gamma_1$ and $\delta_1$ are given below
\[ \gamma_1 = \frac{(2F_g + 1)(2F_e + 1)}{\sqrt{6}} \cdot (-1)^i \left\{ \begin{array}{ccc} i & j_g & F_g \\ 1 & F_e & j_e \end{array} \right\} [d_e(-1)^j_e + F_g] \sqrt{(j_e + 1)(2j_e + 1)} \]

\[ \frac{\Delta_D}{\Delta_{hf}(F_e, F_e - 1)} (2F_e - 1) \left\{ \begin{array}{ccc} i & j_g & F_g \\ 1 & F_e - 1 & j_e \end{array} \right\} \left\{ \begin{array}{ccc} i & j_e & F_e \\ 1 & F_e - 1 & j_e \end{array} \right\} \left\{ \begin{array}{ccc} F_g & 1 & F_e \\ 1 & F_e - 1 & 1 \end{array} \right\} \\
+ \frac{\Delta_D}{\Delta_{hf}(F_e, F_e + 1)} (2F_e + 3) \left\{ \begin{array}{ccc} i & j_g & F_g \\ 1 & F_e + 1 & j_e \end{array} \right\} \left\{ \begin{array}{ccc} i & j_e & F_e \\ 1 & F_e + 1 & j_e \end{array} \right\} \left\{ \begin{array}{ccc} F_g & 1 & F_e \\ 1 & F_e + 1 & 1 \end{array} \right\} \\
- \left( j_e \leftrightarrow j_g, F_e \leftrightarrow F_g, d_e \leftrightarrow d_g, \right) \]

and

\[ \delta_1 = \frac{(2F_g + 1)(2F_e + 1)}{\sqrt{6}} \cdot (-1)^i \left\{ \begin{array}{ccc} i & j_g & F_g \\ 1 & F_e & j_e \end{array} \right\} [d_e(-1)^j_e + F_g] \sqrt{(j_e + 1)(2j_e + 1)} \]

\[ (2F_e + 1) \left\{ \begin{array}{ccc} i & j_e & F_e \\ 1 & F_e & j_e \end{array} \right\} \left\{ \begin{array}{ccc} F_g & 1 & F_e \\ 1 & F_e & 1 \end{array} \right\} + \left( j_e \leftrightarrow j_g, F_e \leftrightarrow F_g, d_e \leftrightarrow d_g, \right) \]
Here $\Delta_{hf}$ is the hyperfine level splitting. First term in \([31]\) arises from the level splitting in electric field. It describes effect similar to Macaluso - Corbino rotation of photon polarization plane in magnetic field. Second term is caused by mixing of hyperfine levels with the different total moment $F$ but the same $F_z$ in electric field. It describes the $T$ noninvariant analog of polarization plane rotation due to Van-Vleck mechanism.

V. ESTIMATES

Let us compare the angle of $P- , T$-odd polarization plane rotation for different kinds of transitions. Angle of rotation of polarization plane per absorption length due to interference of $P- , T$-odd and Stark - induced transition amplitudes according to \([28]\) is expressed by

$$\phi(L_{abs}) = \frac{g(u,v) \text{ Re} \langle g||d^{PT}||e\rangle}{f(u,v) \sqrt{2} |\langle g||A||e\rangle|^2} \Lambda^1 E$$

(32)

If detuning $\Delta \sim \Delta_D$ then $g \sim f \sim 1$.

The value of transition matrix element depends on kind of transition. For allowed $E1$ transition $\langle g||A||e\rangle \sim (d) \sim e a_0$, for allowed $M1$ transition $\langle g||A||e\rangle \sim (\mu) \sim \alpha (d)$. For strongly forbidden $M1$ transition in electric field dominant contribution to the angle of rotation gives the Stark-induced $E1$ transition. Its amplitude can be estimated as $\langle g||A||e\rangle \sim \langle d \rangle^2 E_z/\Delta E$ where $\Delta E$ is the typical difference between the opposite parity levels in atom. For transition $6S_{1/2} \rightarrow 7S_{1/2}$ in Cs and experimentally accessible strength of electric field $E \sim 10^3 - 10^4$ V/cm, value of $\langle d \rangle E_z/\Delta E \sim 10^{-3} - 10^{-4}$. Here $E_z$ is the electric field strength, $e$ is the electron charge, $a_0$ is the Bohr radius, $\Delta E \sim R_y$ is the typical difference between the energy levels of the opposite parity states, $R_y = 13.6$eV is the Rydberg energy constant, $\alpha \simeq 1/137$ is the fine structure constant.

The numerator of \([32]\) has the same order of magnitude for all kinds of transitions considered above: $\langle g||d^{PT}||e\rangle \sim \langle d \rangle^2 \langle H_T \rangle \langle d \rangle E_z/\Delta E)^2$. Now we can estimate the angle of polarization plane rotation per absorption length.

For allowed $E1$ transition

$$\phi(L_{abs}) \sim \frac{\langle H_T \rangle \langle d \rangle E_z}{\Delta E}\frac{E_z}{\Delta E}$$

(33)

where $\langle H_T \rangle$ is the typical value of matrix element of $PT$- odd Hamiltonian.

The angle of rotation per absorption length near allowed $M1$ transition is larger than in $E1$ case

$$\phi(L_{abs}) \sim \frac{\langle H_T \rangle \langle d \rangle E_z}{\alpha^2 \Delta E}\frac{E_z}{\Delta E}$$

(34)

The largest value of angle of rotation per absorption length can be observed near strongly forbidden $M1$ transition because the absorption of light is the lowest in this case

It is interesting to compare these estimates with angle of rotation of polarization plane caused by nonzero EDM of atom. In the absence of hyperfine structure the angle of rotation per absorption length can be estimated using \([28]\) and \([31]\) as follows

$$\phi_{EDM}(L_{abs}) = \frac{1}{2(2j_y+1)} \frac{E_z \delta}{\Delta_D \partial u} \frac{d_a E_z}{\Delta_D} \sim \frac{\langle d \rangle E_z \langle H_T \rangle}{\Delta_D \Delta E},$$

(36)

where $d_a \sim \langle d \rangle \langle H_T \rangle/\Delta E$ is the EDM of atom, $\Delta_D \sim (10^{-5} \sim 10^{-6}) \Delta E$ is the Doppler line width. The value of $\phi(L_{abs})$ here does not depend on transition amplitude $\langle g||A||e\rangle$ and has the same order of magnitude for all kinds of transition considered above.

For allowed $E1$ transition $\phi(L_{abs})/\phi_{EDM}(L_{abs}) \sim \Delta_D/\alpha^2 \Delta E \sim 1$ and both mechanisms contribute comparably.

Near allowed $M1$ transition $\phi(L_{abs})/\phi_{EDM}(L_{abs}) \sim \Delta_D/\alpha^2 \Delta E \sim 1$ and both mechanisms contribute comparably.

Near the strongly forbidden $M1$ transition $\phi(L_{abs})/\phi_{EDM}(L_{abs}) \sim \Delta_D/\alpha^2 \Delta E \sim 1$ and the interference of $P- , T$-odd and Stark-induced transition contributes the most to the angle of polarization plane rotation.

VI. P AND T - ODD INTERACTIONS IN ATOM

Several mechanisms can induce the violation of $P$- and $T$- invariance in atom. According to \([15]\) they are: $P-$, $T$-odd weak interactions of electron and nucleon, interaction of electric dipole moment of electron with electric field inside atom, interaction of electrons with electric dipole and magnetic quadrupole moments of nucleus and $P-$, $T$-odd electron-electron interaction.

We consider effects that according to \([15]\) give the dominant contribution in our case: $P-$, $T$-odd electron-nucleon interaction and interaction of electron EDM with electric field inside atom.

According to \([15][19][20]\) Hamiltonian of $T$-violating interaction between electron and hadron has the form

$$H_T = C_v \frac{G}{\sqrt{2}} (\bar{e}i\gamma_5 e) (\bar{n}n) + C_t \frac{G}{\sqrt{2}} (\bar{e}i\gamma_5 \sigma_{\mu\nu} e) (\bar{n}n^{\mu\nu} n)$$

(37)

where $G = 1.055 \cdot 10^{-5} m_p^{-2}$ is the Fermi constant, $m_p$ is the proton mass, $e$ and $n$ are the electron and hadron field operators respectively, $C_v$ and $C_t$ are the dimensionless constants characterizing the strength of $T$-violating interactions relatively to $T$-conserving weak interaction.
The first term in (37) describes scalar hadronic current coupling with pseudoscalar electronic current, and the second one describes tensor hadronic current coupling with the pseudotensor electronic current.

Matrix element of $T$-odd Hamiltonian according to [15] is equal to

$$
\langle s_{1/2}| H_T| p_{1/2} \rangle = \frac{Gm_e^2 \alpha^2 Z^2 R}{2\sqrt{2\pi}} \sqrt{\frac{\nu}{\nu_0}} 2\gamma C_s A
$$

(38)

where $m_e$ is the electron mass, $\nu_i$ is the effective principal quantum number of state $i$, $A$ is the atomic number, $\gamma = \sqrt{(j + 1/2)^2 - Z^2\alpha^2}$ and $j$ is the total angular moment of atom. The “relativistic enhancement factor” $R$ is given by

$$
R = 4 \left(\frac{a_0/2Zr_0}{2^\gamma + 1}\right)^2.
$$

Here $r_0 = A^{1/3} 1.2 \times 10^{-13}$ cm is the approximate nuclear radius. For cesium $R = 2.8$. We neglect tensor part of interaction for simplicity.

Hamiltonian of interaction of electron EDM and electric field inside atom that mix opposite parity atomic states has the form [15]

$$
H_d = \sum_k (\gamma_0 k - 1) \Sigma_k \hat{E}_k
$$

(39)

where

$$
\Sigma_k = -\gamma_5 \gamma_0 \gamma_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix},
$$

(40)

$\sigma_k$ are the Pauli matrices, $E_k$ is the electric field strength acting upon electron $k$. If summation in (39) is performed over one valence electron and electric field strength near the nucleus approximately equals $\hat{E} = Z\alpha r/r^3$, matrix element of operator $H_d$ can be written as follows [16]

$$
\langle j, l = j + 1/2|H_d|j, l' = j - 1/2 \rangle = -\frac{4(Z\alpha)^3}{\gamma(4\gamma^2 - 1)(\nu\omega_0)^{3/2} a_0^5}
$$

(41)

where $l$ and $l'$ are the orbital angular moments.

**VII. ESTIMATES FOR 6S_{1/2} \rightarrow 7S_{1/2} CESIUM TRANSITION**

Let us estimate $P_\pi$, $T$-odd rotation of polarization plane for highly forbidden $M1$ transition $6s_{1/2} \rightarrow 7s_{1/2}$ in cesium. The scheme of cesium energy levels is shown in Fig. 2.

**A. Rotation of polarization plane of light due to electron nucleon interactions**

The $P$ and $T$ odd electron nucleon interactions mix $s$ and $p$ states of cesium. Since the $E1$ transition amplitudes $6s \rightarrow 6p$ and $7s \rightarrow 7p$ are negligibly small when $n > 6$ and $n' > 7$ [15] we should take into account only admixture of $6p_{1/2}$ and $7p_{1/2}$ states. Using Eq. (14) and matrix element [15] one can represent the wave functions perturbed by $P$ and $T$ noninvariant electron-nucleon interaction as follows:

$$
|\tilde{6s}_{1/2}\rangle = |6s_{1/2}\rangle + 10^{-11} \left( 2\gamma \frac{A}{N} C_s \right) \left( 1.17 |6p_{1/2}\rangle + 0.34 |7p_{1/2}\rangle \right)
$$

$$
|\tilde{7s}_{1/2}\rangle = |7s_{1/2}\rangle + 10^{-11} \left( 2\gamma \frac{A}{N} C_s \right) \left( 0.87 |6p_{1/2}\rangle - 1.33 |7p_{1/2}\rangle \right).
$$

(42)

Where $N$ is the number of neutrons in atomic nucleus. Using Eq. (14) and values of radial integrals [15] $\rho(6s_{1/2},6p_{1/2}) = -5.535$, $\rho(7s_{1/2},6p_{1/2}) = 5.45$, $\rho(7s_{1/2},7p_{1/2}) = -12.30$ we obtain reduced matrix element of $PT$ - odd $E1$ transition

$$
\langle 6s_{1/2}|d^{PT}|7s_{1/2}\rangle = 1.27 \cdot 10^{-10} |e| a_0 C_s
$$

(43)

The matrix element of Stark-induced $6s_{1/2} \rightarrow 7s_{1/2}$ transition in cesium is usually written as [21]:

$$
\langle 6s_{1/2}, m'|d_{\gamma 7}^{ST}|7s_{1/2}, m \rangle = \alpha E_i \delta_{mm'} + i \beta \epsilon_{ijk} E_j (m'|\sigma m| m')
$$

where $m$ and $m'$ are the magnetic quantum numbers of ground and excited states of cesium, $E_i$ is the electric field strength, $\alpha$ and $\beta$ are the scalar and vector transition polarizability (see also [21]). The value of $\Lambda$ introduced in (28) can be expressed for cesium via the vector transition polarizability as follows: $\Lambda = -2\sqrt{3} \beta E$. Value of $\beta$ is well known from theoretical calculations [22] as well as from experiment [23]. According to [22] it is equal to $\beta = 27.0 a_0^3$. Therefore

$$
\Lambda^1 = -1.81 \cdot 10^{-8} |e| a_0 E(V/cm)
$$

(44)

Suppose temperature is $T = 750 K$. Then pressure of Cs vapor is $p = 10$ kPa [24], concentration of atoms is $N = 10^{18}$ cm$^{-3}$ and Doppler line width is $\Delta_D/\omega_0 = 10^{-6}$. For transition between hyperfine levels $F_i = 4 \rightarrow F_e = 4$ coefficient $K^2$ in formula (28) is maximal ($K^2 = 15/8$).

Suppose detuning $\Delta \sim \Delta_D$, then $\nu = \Gamma/2\Delta_D \approx 0.1$ and $f \approx 1$, $g \approx 0.7$. Absorption length in longitudinal electric field $E = 10^4$ V/cm is equal to $L_{abs} = 7\mu m$.

As a result angle of $PT$ noninvariant rotation of polarization plane is

$$
|\phi| = 1.0 \cdot 10^{-13} C_s l E.
$$

The optimal signal to noise ratio is achieved when $l = 2L_{abs}$ [13]. The best limit on the parameters of electron-nucleon interaction $C_S < 4 \cdot 10^{-7}$ was set in [3]. Corresponding limit to the angle of rotation of polarization plane is $|\phi| < 0.5 \cdot 10^{-12}$ rad.
B. Rotation of polarization plane of light due to cesium EDM

Using wave functions (12) we can obtain EDM in $6s_{1/2}$ and $7s_{1/2}$ states of Cs

$$d_{6s_{1/2}} = -1.35 \times 10^{-10} C_s |e| a_0$$

$$d_{7s_{1/2}} = -4.39 \times 10^{-10} C_s |e| a_0.$$ 

As a result, expression (31) yields the angle of polarization plane rotation due to level splitting in electric field

$$|\phi_1| = 1.4 \times 10^{-24} l C_s E_z^3 (\text{V/cm}) < 8 \times 10^{-16}$$

and the angle of rotation due to hyperfine levels mixing

$$|\phi_2| = 2.1 \times 10^{-24} l C_s E_z^3 (\text{V/cm}) < 1.2 \times 10^{-15}.$$ 

(We assume here that for detuning $\Delta \sim \Delta_D$ functions $g(u, v) \simeq 0.7$, $\partial g(u, v)/\partial u \simeq 1.1$). These angles are three orders of magnitude lower than angle of polarization plane rotation arising from interference of Stark-induced and $P$, $T$-noninvariant transition amplitudes.

C. Rotation of polarization plane of light due to electron EDM

If the $PT$ noninvariant interaction in atom is induced by interaction of electron EDM with the electric field of nucleus then using (14) and (41) one can represent the wave functions of $6s$ and $7s$ states of cesium as follows

$$|\bar{6}s_{1/2}\rangle = |6s_{1/2}\rangle$$

$$-35|6p_{1/2}\rangle + 10.5|7p_{1/2}\rangle|d_e/(ea_0)$$

$$|\bar{7}s_{1/2}\rangle = |7s_{1/2}\rangle$$

$$+ (27.7|6p_{1/2}\rangle - 36.2|7p_{1/2}\rangle|d_e/(ea_0)$$

Using Eq. (43) we can obtain reduced matrix element of electric dipole transitions between $6s_{1/2}$ and $7s_{1/2}$ states

$$\langle 6s_{1/2}|d^{PT}|7s_{1/2}\rangle = -72 d_e$$

and electric dipole moment of cesium in ground state $d_{6s_{1/2}}$ and excited state $d_{7s_{1/2}}$

$$d_{6s_{1/2}} = 131 d_e$$

$$d_{7s_{1/2}} = 400 d_e.$$ 

where $d_e$ is the electron EDM.

As we mentioned before, two effects induce $T$-noninvariant rotation of polarization plane in electric field. The first of them is the interference of $PT$-odd and Stark induced transition amplitudes and the second is the interaction of atomic EDM with electric field. After substitution of (46) to (28) one can obtain the angle of rotation arising from interference of amplitudes $|\phi| < 0.6 \times 10^{-12}$ in the same experimental conditions as before.

Rotation caused by atomic EDM is a sum of two contributions. Using the first term in (11) and (12) one can obtain the angle of rotation induced by splitting of magnetic sub-levels in electric field $|\phi_1| < 1.3 \times 10^{-15}$. The mix of hyperfine components (second term in Eq. (32)) gives the contribution $|\phi_2| < 2 \times 10^{-15}.$

For estimates we use experimental limit on electron EDM from $|d_e| < 4 \times 10^{-27} |e| \text{ cm}$. We should note that estimates of rotation angle for electron-nucleon $T$-noninvariant weak interactions and electron EDM give close values.

VIII. CONCLUSION

In the present article we have considered phenomenon of rotation of polarization plane of light in gas placed in electric field. Calculations of angle of polarization plane rotation are performed for $6S_{1/2} \rightarrow 7S_{1/2}$ transition in atomic cesium. Two mechanisms of effect are considered. They are:

1. Light polarization plane rotation caused by the pseudo-Zeeman splitting of atomic levels in atoms with nonzero EDM in electric field.

2. Light polarization plane rotation due to interference of $PT$-odd and Stark-induced transition amplitudes.

Both of them can be induced by $PT$ noninvariant interaction between electrons and atomic nucleus and by interaction of electron EDM with electric field inside atom.

For the highly forbidden $M1$ transition $6S_{1/2} \rightarrow 7S_{1/2}$ in cesium we can expect the angle of polarization plane rotation per absorption length due to $PT$-odd atomic polarizability $\beta^P_T$ about $|\phi| < 10^{-12}$. The rotation induced by atomic EDM for this transition is three orders of magnitude smaller.

Angle of polarization plane rotation can be significantly greater for other atoms, for example, for rare-earth elements, where additional amplification arises from close levels of opposite parity.

The simplest experimental scheme to observe pseudo-Faraday rotation of polarization plane of light in electric field includes a cell with atomic gas placed in the electric field and sensitive polarimeter. In case of large absorption length one can place this cell in resonator or delay line optical cavity to reduce the size of experimental setup (see e. g. (24)).

Several schemes are proposed to increase the sensitivity of measurements. One of them is based on the nonlinear magneto-optic effect (NMOE) (14). Since the change of rotation angle with the change of applied field in this case is several orders of magnitude larger than in the traditional scheme, sensitivity of this kind of experiment...
can be very high. The authors of [10] expect to achieve the sensitivity for the cesium EDM $|d_{Cs}| < 10^{-26}|e| \text{ cm}$. The corresponding limit for the electron EDM is $|d_e| < 10^{-28}|e| \text{ cm}$.

The method of measurement of polarization plane rotation proposed in [11–13] probably can provide even higher sensitivity. This method is based on observation of evolution of polarization of light in a cell with atomic vapor and amplifying media placed in a resonator. According to [11–13] the compensation of absorption of light in a cell allows to increase the observed angle of polarization plane rotation and, according to estimates [12], allows to increase the sensitivity for electron EDM up to $|d_e| < 10^{-30}|e| \text{ cm}$.

Therefore we can hope that experimental measurement of described phenomenon can provide sensitivity for parameters of $P, T$ noninvariant interactions between electron and nucleus and electron EDM, comparable or even higher than current atomic EDM experiments.

[1] J. H. Christenson, J. W. Cronin, V. L. Fitch and R. Turlay, Phys. Rev. Lett., 3, 1138 (1964).
[2] A. Angelopoulos, et al. Phys. Lett. B444, 43 (1998).
[3] K. Abe et al. Phys. Rev. Lett. 87, 802 (2001), (or LANL e-print archive hep-ex/0107061).
[4] K. F. Smith et al. Phys. Lett. B234, 191 (1990).
[5] E. D. Commins, S. B. Ross, D. DeMille and B. G. Regan, Phys. Rev. A 50, 2960 (1994).
[6] S. A. Murthy, D. Krause, Jr. Z. L. Li and L. R. Hunter, Phys. Rev. Lett. 63, 2559 (1989).
[7] D. Cho, K. Sangster and E. A. Hinds, Phys. Rev. Lett. 63, 2559 (1989).
[8] N. B. Baranova, Yu. V. Bogdanov, B. Ya. Zeldovich, Usp. Fiz. Nauk 123, 349 (1977).
[9] L. M. Barkov, M. S. Zolotorev, D. Melik-Pashaev, JETP Letters, 48, 144 (1988), O. P. Sushkov, F. F. Flaumbaum, Zh. Eksp. Teor. Fiz. 75, 1208 (1978).
[10] V. Yashchuk, D. Budker, and M. Zolotorev in: Trapped Charged Particles and Fundamental Physics, D. H. E. Dubin and D. Schneider, eds. AIP conference proceedings 457, pp. 177-181.
[11] V. G. Baryshevsky, Phys. Lett. A260, 24 (1999).
[12] V. G. Baryshevsky, hep-ph/9912438
[13] V. G. Baryshevsky, hep-ph/9912276
[14] V. G. Baryshevsky, J. Phys. B27, 4421 (1994).
[15] I. B. Khriplovich, Parity nonconservation in atomic phenomena, Gordon and Breach, New York, 1991.
[16] D. DeMille and M. G. Kozlov, physics/9801034
[17] M. G. Kozlov and S. G. Porsev, Phys. Lett. A142, 233 (1989).
[18] G. J. Roberts et al. J. Phys. B 13, 1389 (1980).
[19] S. M. Barr, Phys. Rev. D 45, 4148 (1992).
[20] L. R. Hunter, Science 252, 73 (1991).
[21] M. A. Bouchiat, C. Bouchiat, J. Physique 36, 493 (1975).
[22] S. A. Blundell, J. Sapirstein, W. R. Johnson, Phys. Rev. D 45, 1602 (1991).
[23] R. N. Watts, S. L. Gilbert, G. E. Wiemann, Phys. Rev. A 27, 2769 (1983).
[24] Physical quantities, edited by I. S. Grigor’ev, E. Z. Meilikhov, Moscow, Energoatomizdat, 1991 (in Russian).
[25] R. Cameron et al. Phys. Rev. D 47, 3707 (1993).