Precision Studies of Relativity in Electrodynamics

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In this contribution to the proceedings of the 2002 Workshop for Fundamental Physics in Space, a discussion of recent work on astrophysical and laboratory tests of Lorentz symmetry in electrodynamics is presented. Stringent constraints are placed on birefringence of light emitted from galactic and extragalactic sources. The prospect of precision clock-comparison experiments utilizing resonant cavities are considered.

1. Introduction

In the past, high-precision tests of the properties of light have played an important role in the search for new physics. Historically, testing the Lorentz invariance of light has confirmed special relativity to a high degree of precision \[1, 2, 3\]. Many of the traditional experiments fit into one of two categories. Michelson-Morley experiments are designed to test rotational invariance by searching for anisotropy in the speed of light. Kennedy-Thorndike experiments test boost invariance by searching for variations in the speed of light due to changes in the velocity of the laboratory. In this work, I review a recent study of extremely precise tests of Lorentz symmetry in electrodynamics. This research was done in collaboration with Alan Kostelecký. A detailed discussion can be found in Ref. \[4\].

In recent years, the possibility that Planck scale physics may reveal itself at low energies as small Lorentz violations has lead to the development of a general Lorentz-violating standard-model extension \[5, 6, 7\]. It consists of the minimal standard model plus small Lorentz- and CPT-violating terms. The small violations may originate from nonzero vacuum expectation values of Lorentz tensors in the underlying theory \[8\]. Lorentz violations of this type also arise from noncommutative field theories \[9\].

The extension has provided a theoretical framework for a number of high precision tests of Lorentz symmetry. To date, experiments involving hadrons \[10, 11, 12, 13, 14, 15\], protons and neutrons \[16, 17, 18, 19, 20\], electrons \[21, 22\], photons \[4, 23\], and muons \[24\] have been performed.

A Lorentz-violating extended electrodynamics can be extracted from the standard-model extension \[5\]. In this work, we consider some experimental consequences of the extended electrodynamics. The theory predicts novel features which lead to sensitive tests of Lorentz symmetry. One unconventional property is the birefringence of light. The observed absence of birefringence of light emitted from distant sources leads to tight bounds on some of the coefficients for Lorentz violation \[4, 23\]. Some of these bounds are discussed in Sec. 3.

Another observable consequence of Lorentz violation is an orientation and velocity dependence in the frequencies of resonant cavities. This dependence provides the basis for future clock-comparison experiments sensitive to the photon-sector of the standard-model

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extension. Past clock-comparison experiments have been used to place constraints on the fermion sector [16, 17, 18, 19, 20]. Space-based versions of these experiments have recently been considered for precision tests of Lorentz symmetry on board the International Space Station (ISS) and other spacecraft [25]. Tests for Lorentz violation using resonant cavities are considered in Sec. 4.

2. Extended Electrodynamics

The photon sector of the standard-model extension yields a Lorentz-violating electrodynamics. It maintains the usual gauge invariance and is covariant under observer Lorentz transformations. The Lorentz-violating electrodynamics includes both CPT-even and -odd terms. However, the CPT-odd terms are theoretically undesirable since they may lead to instabilities [5, 26]. Furthermore, these terms have been bounded experimentally to extremely high precision using polarization measurements of distant radio galaxies [23]. Neglecting the CPT-odd terms, we are left with a CPT-conserving electrodynamics including small Lorentz violations.

The CPT-even lagrangian associated with the Lorentz-violating electrodynamics is [4]

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} (k_F)_{\kappa\lambda\mu\nu} F^{\kappa\lambda} F^{\mu\nu}, \] (1)

where \( F_{\mu\nu} \) is the field strength, \( F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \). The first term is the usual Maxwell lagrangian. The second is an unconventional Lorentz-violating term. The coefficient for Lorentz violation, \( (k_F)_{\kappa\lambda\mu\nu} \), is real and comprised of 19 independent components. The absence of observed Lorentz violation requires \( (k_F)_{\kappa\lambda\mu\nu} \) to be small.

It is often convenient to work with the electric and magnetic fields, \( \vec{E} \) and \( \vec{B} \), rather than the vector potential \( A^\mu \). In terms of the usual electric and magnetic fields, the lagrangian takes the form

\[ \mathcal{L} = \frac{1}{2} (\vec{E}^2 - \vec{B}^2) + \frac{1}{2} \vec{E} \cdot (\kappa_{DE}) \cdot \vec{E} - \frac{1}{2} \vec{B} \cdot (\kappa_{HB}) \cdot \vec{B} + \vec{E} \cdot (\kappa_{DB}) \cdot \vec{B}. \] (2)

The real \( 3 \times 3 \) matrices \( \kappa_{DE}, \kappa_{HB} \) and \( \kappa_{DB} \) contain the same information as \( (k_F)_{\kappa\lambda\mu\nu} \). The relationship between the two notations can be found in Ref. [4]. Taking \( \kappa_{DE} = \kappa_{HB} = \kappa_{DB} = 0 \) in Eq. (2) results in the usual Maxwell lagrangian in terms of \( \vec{E} \) and \( \vec{B} \). The parity-even matrices, \( \kappa_{DE} \) and \( \kappa_{HB} \), are symmetric, while the parity-odd matrix, \( \kappa_{DB} \), has both symmetric and antisymmetric parts. The matrices \( (\kappa_{DE} + \kappa_{HB}) \) and \( \kappa_{DB} \) are traceless. These symmetries leave 11 parity-even and 8 parity-odd independent components.

The equations of motion for this lagrangian are

\[ \partial_\alpha F^{\alpha}_{\mu} + (k_F)_{\mu_0\beta\gamma} \partial^{\alpha} F^{\beta\gamma} = 0. \] (3)

These constitute modified source-free inhomogeneous Maxwell equations. The homogeneous Maxwell equations,

\[ \partial_\mu \tilde{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\kappa\lambda} \partial_\mu F_{\kappa\lambda} = 0, \] (4)

remain unchanged.
An interesting analogy exists between this theory and the usual situation in anisotropic media. Define fields \( \vec{D} \) and \( \vec{H} \) by the six-dimensional matrix equation
\[
\begin{pmatrix}
\vec{D} \\
\vec{H}
\end{pmatrix} = \begin{pmatrix}
1 + \kappa_{DE} & \kappa_{DB} \\
\kappa_{HE} & 1 + \kappa_{HB}
\end{pmatrix}
\begin{pmatrix}
\vec{E} \\
\vec{B}
\end{pmatrix},
\]
with \( \kappa_{HE} = -(\kappa_{DB})^T \). Then the modified Maxwell equations take the familiar form
\[
\nabla \times \vec{H} - \partial_0 \vec{D} = 0, \quad \nabla \cdot \vec{D} = 0,
\]
\[
\nabla \times \vec{E} + \partial_0 \vec{B} = 0, \quad \nabla \cdot \vec{B} = 0.
\]
As a result, the behavior of electromagnetic fields in the extended electrodynamics is very similar to that of conventional fields in anisotropic media.

For the purpose of comparing the sensitivity of various experiments, it is convenient to make the decomposition into four \( 3 \times 3 \) traceless matrices
\[
\begin{align*}
(\tilde{\kappa}_{e+})^{jk} &= \frac{1}{2}(\kappa_{DE} + \kappa_{HB})^{jk}, \\
(\tilde{\kappa}_{e-})^{jk} &= \frac{1}{2}(\kappa_{DE} - \kappa_{HB})^{jk} - \frac{1}{3} \delta^{jk}(\kappa_{DE})^{ll}, \\
(\tilde{\kappa}_{o+})^{jk} &= \frac{1}{2}(\kappa_{DB} + \kappa_{HE})^{jk}, \\
(\tilde{\kappa}_{o-})^{jk} &= \frac{1}{2}(\kappa_{DB} - \kappa_{HE})^{jk},
\end{align*}
\]
and a single rotationally symmetric trace component
\[
\tilde{\kappa}_{tr} = \frac{1}{3}(\kappa_{DE})^{ll}.
\]
The matrices \( \tilde{\kappa}_{e+} \) and \( \tilde{\kappa}_{e-} \) and the single coefficient \( \tilde{\kappa}_{tr} \) contain the parity-even coefficients, while the matrices \( \tilde{\kappa}_{o+} \) and \( \tilde{\kappa}_{o-} \) contain the parity-odd. The matrix \( \tilde{\kappa}_{o+} \) is antisymmetric while the other three are symmetric.

In terms of this decomposition, the lagrangian is
\[
\mathcal{L} = \frac{1}{2} [ (1 + \tilde{\kappa}_{tr}) \vec{E}^2 - (1 - \tilde{\kappa}_{tr}) \vec{B}^2 ] + \frac{1}{2} \vec{E} \cdot (\tilde{\kappa}_{e+} + \tilde{\kappa}_{e-}) \cdot \vec{E} + \frac{1}{2} \vec{B} \cdot (\tilde{\kappa}_{o+} + \tilde{\kappa}_{o-}) \cdot \vec{B}.
\]
From the form of Eq. (9), it is evident that the component \( \tilde{\kappa}_{tr} \) corresponds to shift in the effective permittivity \( \epsilon \) and effective permeability \( \mu \) by \( (\epsilon - 1) = -(\mu^{-1} - 1) = \tilde{\kappa}_{tr} \). Therefore, the effect of a nonzero \( \tilde{\kappa}_{tr} \) is an overall shift in the speed of light. This result generalizes to the nine independent coefficients in \( \tilde{\kappa}_{tr}, \tilde{\kappa}_{e+} \) and \( \tilde{\kappa}_{o+} \). To leading order, these can be viewed as a distortion of the spacetime metric of the form \( \eta^{\mu \nu} \rightarrow \eta^{\mu \nu} + k^{\mu \nu} \), where \( k^{\mu \nu} \) is small, real and symmetric.

Small distortions of this type are normally unphysical, since they can be eliminated through coordinate transformations and field redefinitions. However, in the context of the full standard-model extension, eliminating these terms from the photon sector will alter other sectors and the effects of such terms can not be removed. Thus, in experiments where the properties of light are compared to the properties of other matter, these terms are relevant. While in experiments sensitive to the properties of light only, these nine coefficients are
not expected to appear. The resonant-cavity based experiments discussed in Sec. 4 fall into the first category, while the astrophysical tests of Sec. 3 belong to the second. The tests discussed discussed in Sec. 3 rely on measurements of birefringence, which in essence compares the properties of light with different polarizations. Therefore, these tests are only sensitive to the ten independent components of $\bar{\kappa}_e$ and $\bar{\kappa}_o$. When reporting bounds on birefringence it is convenient to express them in terms of a ten-dimensional vector $k^a$ containing the ten independent components of $\bar{\kappa}_e$ and $\bar{\kappa}_o$. The relationship between $\bar{\kappa}_e$, $\bar{\kappa}_o$ and $k^a$ is given by

$$\begin{align*}
(\bar{\kappa}_e)^{jk} &= -\begin{pmatrix}
-2(k^3 + k^4) & k^5 & k^6 \\
k^5 & k^3 & k^7 \\
k^6 & k^7 & k^4
\end{pmatrix}, \\
(\bar{\kappa}_o)^{jk} &= \begin{pmatrix}
2k^2 & -k^9 & k^8 \\
-k^9 & -2k^4 & k^10 \\
k^8 & k^10 & 2(k^1 - k^2)
\end{pmatrix}.
\end{align*}$$

(10)

Bounds can then be expressed in terms of $|k^a| \equiv \sqrt{k^a k_a}$, the magnitude of the vector $k^a$.

3. Astrophysical Tests

In order to understand the effects of Lorentz violation on the propagation of light, we begin by considering plane-wave solutions. Adopting the ansatz $F_{\mu\nu}(x) = F_{\mu\nu} e^{-ip_\alpha x^\alpha}$, the modified Maxwell equations yield an Ampère law given by the linear equation

$$(-\delta^{jk} p^2 - p^j p^k - 2(k_F)^{j\beta\gamma} k_\beta p_\gamma) E^k = 0 \ .$$

(11)

Solving this equation determines the dispersion relation

$$p^0_\pm = (1 + \rho \pm \sigma)|\vec{p}|, \quad (12)$$

and electric field

$$\vec{E}_\pm \propto (\sin \xi, \pm 1 - \cos \xi, 0) + O(k_F) \ .$$

(13)

To leading order, the quantities $\rho$, $\sigma \sin \xi$ and $\sigma \cos \xi$ are linear combinations of $(k_F)^{\kappa,\lambda,\mu,\nu}$ and depend on $\hat{v}$, the direction of propagation. One unconventional feature of these solutions is the birefringence of light in the absence of matter. In the conventional case, this behavior is commonly found in the presence of anisotropic media.

The general vacuum solution is a linear combination of the $\vec{E}_+$ and $\vec{E}_-$. For nonzero $\sigma$, these solutions obey different dispersion relations. As a result, they propagate at slightly different velocities. At leading order, the difference in the velocities is given by

$$\Delta v \equiv v_+ - v_- = 2\sigma \ .$$

(14)

For light propagating over astrophysical distances, this tiny difference may become apparent.

As can be seen from the above solutions, birefringence depends on the linear combination $\sigma \sin \xi$ and $\sigma \cos \xi$. As expected, these only contain the ten independent coefficients which
appear in $\tilde{\kappa}_{e+}$ and $\tilde{\kappa}_{o-}$. Expressions for $\sigma \sin \xi$ and $\sigma \cos \xi$ in terms of these ten independent coefficients and the direction of propagation can be found in the literature \cite{4}.

Next, we consider two observable effects stemming from the birefringence. The first is the difference in arrival time of two modes in unpolarized light. The second is the change in polarization of polarized light emitted from distant sources.

### 3.1. Pulse Dispersion

For a source producing relatively unpolarized light, the components $\vec{E}_\pm$ associated with each mode will be comparable. For light produced at a given instant, the difference in velocity will induce a difference in the observed arrival time of the two modes given by $\Delta t \simeq \Delta v L$, where $L$ is the distance to the source.

To make use of this idea, we consider sources that produce radiation with rapidly changing time structure. Sources producing narrow pulses of radiation such as pulsars and gamma-ray bursts are ideal. For sources of this type, the pulse can be regarded as the superposition of two independent pulses, one for each mode. As the pulse propagates, the difference in velocity will cause the two pulses to separate. A signal for Lorentz violation would manifest itself as the observation of two sequential pulses of similar structure. The pulses would be linearly polarized at mutually orthogonal polarization angles. This type of double pulse has not yet been observed.

Single pulse measurements can be used to place bounds on Lorentz violation. Suppose a source produces a pulse with a characteristic width $w_s$. As the pulse propagates, the

| Source         | $L$     | $w_o$ |
|----------------|---------|-------|
| GRB971214 [27, 28] | 2.2 Gpc | 50 s  |
| GRB990123 [28, 29] | 1.9 Gpc | 100 s |
| GRB980329 [28, 30] | 2.3 Gpc | 50 s  |
| GRB990510 [28, 31] | 1.9 Gpc | 100 s |
| GRB000301C [32, 33] | 2.0 Gpc | 10 s  |
| PSR.J1959+2048 [34] | 1.5 kpc | 64 $\mu$s |
| PSR.J1939+2134 [34] | 3.6 kpc | 190 $\mu$s |
| PSR.J1824-2452 [34] | 5.5 kpc | 300 $\mu$s |
| PSR.J2129+1210E [34] | 10.0 kpc | 1.4 ms |
| PSR.J1748-2446A [34] | 7.1 kpc | 1.3 ms |
| PSR.J1312+1810 [34] | 19.0 kpc | 4.4 ms |
| PSR.J0613-0200 [34] | 2.2 kpc | 1.4 ms |
| PSR.J1045-4509 [34] | 3.2 kpc | 2.2 ms |
| PSR.J0534+2200 [34, 35] | 2.0 kpc | 10 $\mu$s |
| PSR.J1939+2134 [34, 30] | 3.6 kpc | 5 $\mu$s |

Table 1: Source data for velocity constraints.
two modes spread apart and the width of the pulse will increase. The observed width can
be estimated as \( w_o \simeq w_s + \Delta t \). Therefore, observations of \( w_o \) place conservative bounds
on \( \Delta t \simeq \Delta v L \simeq 2\sigma L \). The resulting bound on \( \sigma \) constrains the ten-dimensional parameter
space of \( \tilde{\kappa}_{+} \) and \( \tilde{\kappa}_{-} \). Since a single source constrains only one degree of freedom, a minimum
of ten sources located at different positions on the sky are required to fully constrain the ten
coefficients.

Table 1 lists a sample of 16 sources consisting of gamma-ray bursts and pulsars, as well as
their distances and pulse widths. Each width places a bound on \( \sigma \) for that particular source.
Combining these bounds using a method described in Ref. [4] constrains the ten-dimensional
parameter space. At the 90% confidence level, we obtain a bound of \(|k^a| < 3 \times 10^{-16}\) on the
coefficients for Lorentz violation.

3.2. Spectropolarimetry

In this subsection, we consider the effect of Lorentz violation on polarized light. Decomposing
a general electric field into its birefringent components, we write

\[
\vec{E}(x) = (\vec{E}_+ e^{-i\varphi_0 t} + \vec{E}_- e^{-i\varphi_0 t}) e^{i\vec{p} \cdot \vec{x}}.
\]  

(15)

Each of the components propagates with different phase velocity. A change in the relative
phase results from this difference. The shift in relative phase is given by

\[
\Delta \varphi = (p_+^0 - p_-^0)t \simeq 4\pi \sigma L/\lambda ,
\]  

(16)

where \( L \) is the distance to the source and \( \lambda \) is the wavelength of the light. The change in
relative phase results in a change in the polarization as the radiation propagates.

The \( L/\lambda \) dependence suggests the effect is larger for more distant sources and shorter
wavelengths. Recent spectropolarimetry of distant galaxies at wavelengths ranging from
infrared to ultraviolet has made it possible to achieve values of \( L/\lambda \) greater than \( 10^{31} \). Mea-
sured polarization parameters are typically order 1. Therefore, we expect an experimental
sensitivity of \( 10^{-31} \) or better to components of \((k_F)_{\kappa\lambda\mu\nu}\).

In general, plane waves are elliptically polarized. The polarization ellipse can be pa-
parameterized with angles \( \psi \), which characterizes the orientation of the ellipse, and \( \chi = \pm \arctan \frac{\text{minor axis}}{\text{major axis}} \), which describes the shape of the ellipse and helicity of the wave. The
phase change, \( \Delta \phi \), results in a change in both \( \psi \) and \( \chi \). However, measurements of \( \chi \) are
not commonly found in the literature. Focusing our attention on \( \psi \), we seek an expression
for \( \delta \psi = \psi - \psi_0 \), the difference between \( \psi \) at two wavelengths, \( \lambda \) and \( \lambda_0 \). We find [4]

\[
\delta \psi = \frac{1}{2} \tan^{-1} \frac{\sin \tilde{\xi} \cos \zeta_0 + \cos \tilde{\xi} \sin \zeta_0 \cos(\delta \phi - \phi_0)}{\cos \tilde{\xi} \cos \zeta_0 - \sin \tilde{\xi} \sin \zeta_0 \cos(\delta \phi - \phi_0)},
\]  

(17)

where we have defined \( \delta \phi = 4\pi \sigma (L/\lambda - L/\lambda_0), \tilde{\xi} = \xi - 2\psi_0 \) and \( \phi_0 \equiv \tan^{-1}(\tan 2\chi_0/\sin \tilde{\xi}), \zeta_0 \equiv \cos^{-1}(\cos 2\chi_0 \cos \tilde{\xi}) \). The polarization at \( \lambda_0 \) is given by the polarization angles \( \psi_0 \) and
\( \chi_0 \).
Table 2: Source data for polarization constraints.

| Source                  | $L_{\text{eff}}$ (Gpc) | $10^{30} L_{\text{eff}}/\lambda$ | $\log_{10} \sigma$ |
|-------------------------|-------------------------|-----------------------------------|--------------------|
| IC 5063 [37]            | 0.04                    | 0.56 - 2.8                        | -30.8              |
| 3A 0557-383 [38]        | 0.12                    | 2.2 - 8.5                         | -31.2              |
| IRAS 18325-5925 [38]    | 0.07                    | 1.0 - 4.9                         | -31.0              |
| IRAS 19580-1818 [38]    | 0.14                    | 1.8 - 9.3                         | -31.0              |
| 3C 324 [39]             | 2.44                    | 82 - 180                          | -32.3              |
| 3C 256 [40]             | 3.04                    | 110 - 220                         | -32.4              |
| 3C 356 [41]             | 2.30                    | 78 - 170                          | -32.3              |
| F J084044.5+363328 [42] | 2.49                    | 88 - 170                          | -32.4              |
| F J155633.8+351758 [42] | 2.75                    | 99 - 160                          | -32.4              |
| 3CR 68.1 [43]           | 2.48                    | 84 - 180                          | -32.4              |
| QSO J2359-1241 [44]     | 2.01                    | 110 - 120                         | -31.2              |
| 3C 234 [45]             | 0.61                    | 55 - 81                           | -31.7              |
| 4C 40.36 [46]           | 3.35                    | 120 - 260                         | -32.4              |
| 4C 48.48 [46]           | 3.40                    | 120 - 260                         | -32.4              |
| IAU 0211-122 [46]       | 3.40                    | 120 - 260                         | -32.4              |
| IAU 0828+193 [46]       | 3.53                    | 130 - 270                         | -32.4              |

The idea is to fit existing spectropolarimetric data to Eq. (17). Under the reasonable assumption that the polarization of the light when emitted is relatively constant over the relevant wavelengths, any measured wavelength dependence in the polarization is due to Lorentz violation.

Table 2 lists 16 sources with published polarimetric data with wavelengths ranging from 400 to 2200 nm. In this table, the effective distance $L_{\text{eff}}$ is listed which takes cosmological redshift of the light into account. Using a fitting procedure described in Ref. [4], we obtain a bound on $\sigma$ for each source. Combining these bounds in the same manner as in the pulse-dispersion case, a constraint on the ten-dimensional parameter space is found. At the 90% confidence level, we obtain a bound of $|k^a| < 2 \times 10^{-32}$ on the coefficients for Lorentz violation responsible for birefringence.

4. Resonant Cavities

Clock-comparison experiments have proved to be some of the most sensitive tests of Lorentz symmetry [16, 17, 18, 19, 20]. The frequencies of these clocks are typically atomic Zeeman transitions. Lorentz violation causes these frequencies to vary with changes in orientation or velocity of the clock. Experiments searching for a variation due to the rotational motion of the Earth have placed stringent bounds on Lorentz violation in the fermion sectors of the standard-model extension.

Modern versions of the Michelson-Morley and Kennedy-Thorndike experiments utilize
resonating electromagnetic cavities [17, 18, 19]. Resonant cavities serve as clocks in clock-comparison experiments which are sensitive to Lorentz violation in the photon sector. These experiments search for a variation in the resonant frequency of a cavity as its orientation or velocity changes. For a typical Earth-based experiment, the variation in resonant frequency occurs at harmonics of the Earth’s sidereal frequency, \( \omega_\oplus \simeq \frac{2\pi}{23 \text{ hr } 56 \text{ min}} \). Due to the orbital motion of the Earth, the variation may also contain annual components.

The ISS and other spacecraft provide interesting platforms for future clock-comparison experiments. The orbital properties of the spacecraft may result in radically different behavior. For example, the orbital period of the ISS is about 92 min. The comparable period for an Earth-based experiment is the Earth’s sidereal period. This suggests a significant reduction in data-acquisition time for a space-based experiment compared to its Earth-based counterpart.

We begin our discussion by considering the effects of Lorentz violation on the resonant frequency of cavities. We then consider two classes of cavities, optical and microwave, which are currently under development for precision tests of relativity.

### 4.1 General Considerations

The quantity of interest is the fractional resonant-frequency shift \( \frac{\delta \nu}{\nu} \). Consider a harmonic system satisfying the Maxwell equations (6). Suppose \( \vec{E}_0, \vec{B}_0, \vec{D}_0 \) and \( \vec{H}_0 \) are the conventional solutions with resonant angular frequency \( \omega_0 \). Let \( \vec{E}, \vec{B}, \vec{D} \) and \( \vec{H} \) be solutions for nonzero \((k_F)_{\kappa\lambda\mu\nu}\) with angular frequency \( \omega \). Manipulating the Maxwell equations for both sets of fields, we obtain the expression

\[
\frac{\delta \nu}{\nu} = \frac{\omega - \omega_0}{\omega_0} = -\left( \int_V d^3x (\vec{E}_0^* \cdot \vec{D} + \vec{H}_0^* \cdot \vec{B}) \right)^{-1} \times \int_V d^3x \left( \vec{E}_0^* \cdot \vec{D} - \vec{D}_0^* \cdot \vec{E} - \vec{B}_0^* \cdot \vec{H} + \vec{H}_0^* \cdot \vec{B} \right.
\]

\[
\left. - i \omega_0^{-1} \vec{\nabla} \cdot (\vec{H}_0^* \times \vec{E} - \vec{E}_0^* \times \vec{H}) \right),
\]

where the integrals are over the volume \( V \) of the cavity.

Note that the divergence term in Eq. (18) results in a surface integral over the boundary of \( V \). In many situations, we can neglect such boundary terms. For example, neglecting Lorentz violations in other sectors, the fields vanish inside a perfect conductor, by usual arguments. Idealizing the walls of the cavity as a perfect conductor, the Faraday equation \( \vec{\nabla} \times \vec{E} + \partial_0 \vec{B} = 0 \), implies the tangential component of \( \vec{E} \) vanishes at the surface. In this scenario, the divergence term in Eq. (18) is zero.

Using Eq. (18), we can find the frequency shift perturbatively in terms of the conventional solutions. For a cavity void of matter, we have \( \vec{E}_0 = \vec{B}_0 \) and \( \vec{B}_0 = \vec{H}_0 \). The constitutive relations (5) give the approximate equalities

\[
\vec{D} - \vec{E} \simeq \kappa_{DE} \cdot \vec{E}_0 + \kappa_{DB} \cdot \vec{B}_0 , \quad \vec{H} - \vec{B} \simeq \kappa_{HE} \cdot \vec{E}_0 + \kappa_{HB} \cdot \vec{B}_0 .
\]

With these relations and the vanishing of the boundary term, the leading order fractional
The frequency shift is
\[
\frac{\delta \nu}{\nu} = -\frac{1}{4 \langle U \rangle} \int_V d^3x \left( \vec{E}_0^* \cdot \kappa_{DE} \cdot \vec{E}_0 - \vec{B}_0^* \cdot \kappa_{HB} \cdot \vec{B}_0 + 2 \text{Re} (\vec{E}_0^* \cdot \kappa_{DB} \cdot \vec{B}_0) \right),
\]
where \( \langle U \rangle = \int_V d^3x \left( |\vec{E}_0|^2 + |\vec{B}_0|^2 \right)/4 \) is the time-averaged energy stored in the unperturbed cavity. Note that \( \delta \nu/\nu \) is real, indicating that the Q factor of the cavity remains unaffected by Lorentz violation at leading order.

The integrals in Eq. (20) are most readily carried out in a frame at rest with respect to the laboratory. Since the laboratory frame is not inertial in general, the laboratory-frame coefficients \( (\kappa_{DE})^{jk}_{\text{lab}}, (\kappa_{DB})^{jk}_{\text{lab}} \) and \( (\kappa_{HB})^{jk}_{\text{lab}} \) are not constant. However, using observer covariance, the laboratory-frame coefficients can be related to the coefficients in an inertial frame through observer Lorentz transformations.

There are many logical candidates for an inertial frame. For our purposes, a Sun-centered celestial equatorial frame will suffice. The coefficients in this frame \( (\kappa_{DE})^{JK}_{\text{Sun}}, (\kappa_{DB})^{JK}_{\text{Sun}} \) and \( (\kappa_{HB})^{JK}_{\text{Sun}} \) can be regarded as constant. The relative smallness of the velocity of the Earth in this frame, \( \beta_\oplus \approx 10^{-4} \), implies it is usually sufficient to expand the transformation in powers of the velocity \( \beta \). To order \( \beta \), the relation between the laboratory-frame coefficients and the Sun-frame coefficients is given by
\[
(\kappa_{DE})^{jk}_{\text{lab}} = T^{jk}_{0} (\kappa_{DE})^{JK}_{\text{Sun}} - T^{(jk) JK}_{1} (\kappa_{DB})^{JK}_{\text{Sun}},
\]
\[
(\kappa_{HB})^{jk}_{\text{lab}} = T^{jk}_{0} (\kappa_{HB})^{JK}_{\text{Sun}} - T^{(jk) KJ}_{1} (\kappa_{DB})^{JK}_{\text{Sun}},
\]
\[
(\kappa_{DB})^{jk}_{\text{lab}} = T^{jk}_{0} (\kappa_{DB})^{JK}_{\text{Sun}} + T^{kj}_{1} (\kappa_{DE})^{JK}_{\text{Sun}} + T^{jk}_{1} (\kappa_{HB})^{JK}_{\text{Sun}},
\]
with \( T^{jk}_{0} \equiv R^{j}_{R} R^{k} \) and \( T^{jk}_{1} \equiv R^{i}_{P} R^{kJ} e^{KPQ} \beta^{Q} \), where \( R^{j}_{R} \) is the rotation from the Sun frame to the laboratory frame, and \( \beta^{Q} \) is the velocity of the laboratory in the Sun frame. The tensor \( T_{0} \) is a rotation, while \( T_{1} \) is a leading-order boost contribution. Although the terms involving \( T_{1} \) are suppressed by \( \beta \), they access distinct combinations of coefficients and can introduce different time dependence, which may lead to fundamentally different tests.

### 4.2 Optical Cavities

Recent examples of modern Michelson-Morley and Kennedy-Thorndike experiments based on optical cavities include Refs. [47, 48, 49]. The basic setup of these experiments consists of a pair of stabilized lasers. One laser is stabilized by an optical cavity. The second laser is stabilized by a molecular transition which in the classical analysis is assumed to be insensitive to Lorentz violations. This laser serves as a reference frequency. The beat frequency of the combined signal is analyzed for a variation due to a change in the orientation or velocity of the cavity.

The sensitivities achieved in these experiments are typically on the order of \( 10^{-13} \) to \( \delta \nu/\nu \). Analyzing these experiments in the context of the extended electrodynamics should therefore yield bounds on components of \( (\kappa_{F})_{\kappa \lambda \mu \nu} \) at the level of \( 10^{-13} \).

Regarding these cavities as two parallel planar reflecting surface, the usual solutions can be approximated as standing waves. In a reference frame at rest in the laboratory, we take
\[
\vec{E}_0(x) = \vec{E}_0 \cos(\omega_0 \vec{N} \cdot \vec{x} + \phi) e^{-i \omega_0 t},
\]
\[ \vec{B}_0(x) = i\vec{N} \times \vec{E}_0 \sin(\omega_0 \vec{N} \cdot \vec{x} + \phi) e^{-i\omega_0 t}, \]  

where \( \vec{N} \) is a unit vector pointing along the length of the cavity, \( \phi \) is a phase, and \( \vec{E}_0 \) is a vector perpendicular to \( \vec{N} \) that specifies the polarization. The resonant frequencies of the conventional solutions are given by \( \omega_0 = \pi m/l \), where \( m \) is an integer and \( l \) is the separation of the reflecting surfaces.

Substituting this solution into Eq. (20) yields the fractional frequency shift:

\[ \frac{\delta \nu}{\nu} = -\frac{1}{2|\vec{E}_0|^2} [\vec{E}_0^* \cdot (\kappa_{DE})_{lab} \cdot \vec{E}_0 - (\vec{N} \times \vec{E}_0^*) \cdot (\kappa_{HB})_{lab} \cdot (\vec{N} \times \vec{E}_0)] . \]  

This result depends on the orientation of the cavity in the laboratory and the polarization of the light. Transforming the laboratory-frame coefficients to the Sun-frame, using Eq. (21), introduces variations in the frequency shift due to the motion of the lab.

Consider a Earth-based laboratory. The transformation (21) includes variations at the Earth’s sidereal and orbital frequencies. The orbital frequency components are a result of a boost, and are therefore suppressed relative to the purely rotational contributions. Consequently, the resonant frequency fluctuates at \( \omega_\oplus \) and the second harmonic \( 2\omega_\oplus \), along with suppressed oscillations associated with the annual variation in the Earth’s velocity.

Different experimental configurations result in different sensitivities to the coefficients \( (k_F)_{\kappa\lambda\mu\nu} \), and can result in different frequencies in frequencies in the variations of \( \delta\nu/\nu \).

As an example, consider a cavity positioned horizontally in the laboratory with vertical polarization. Let \( \theta \) be an angle specifying the orientation of the cavity in the horizontal plane. The frequency shift takes the form

\[ \frac{\delta \nu}{\nu} = A + B \sin 2\theta + C \cos 2\theta, \]  

where

\[
\begin{align*}
A &= A_0 + A_1 \sin \omega_\oplus T_\oplus + A_2 \cos \omega_\oplus T_\oplus + A_3 \sin 2\omega_\oplus T_\oplus + A_4 \cos 2\omega_\oplus T_\oplus , \\
B &= B_0 + B_1 \sin \omega_\oplus T_\oplus + B_2 \cos \omega_\oplus T_\oplus + B_3 \sin 2\omega_\oplus T_\oplus + B_4 \cos 2\omega_\oplus T_\oplus , \\
C &= C_0 + C_1 \sin \omega_\oplus T_\oplus + C_2 \cos \omega_\oplus T_\oplus + C_3 \sin 2\omega_\oplus T_\oplus + C_4 \cos 2\omega_\oplus T_\oplus .
\end{align*}
\]  

The quantities \( A_{0,1,2,3,4} \), \( B_{0,1,2,3,4} \), and \( C_{0,1,2,3,4} \) are linear in the coefficients for Lorentz violation and depend on the latitude of the laboratory.

From Eq. (24), we see that one possible strategy for searches for Lorentz-violation would be to rapidly rotate the cavity in the laboratory and search for variations at the harmonics of the cavity rotation frequency. This is the method used in the experiment of Brillet and Hall [47]. It has been estimated that their analysis constrains one combination of coefficients to about a part in \( 10^{15} \) [4].

Hills and Hall performed an experiment with the cavity fixed in the laboratory [48]. A bound is placed on the sidereal variation on the order of \( 10^{-13} \). We see from Eqs. (24) and (25) that this constrains some combination of the coefficients \( A_1, A_2, B_1, B_2, C_1, \) and \( C_2 \).
A similar experiment has recently been performed by Braxmaier et al. \cite{Braxmaier}. Their analysis focuses on variations due to the orbital motion of the Earth. In the present context, this corresponds to $\beta$ suppressed terms arising from the leading order boost contributions in the transformation \cite{Braxmaier}. They achieve fractional-frequency sensitivity of $4.8 \pm 5.3 \times 10^{-12}$, which leads to an estimated constraint on a combination of coefficients on the order of $10^{-8}$.

It should be noted that $\beta$ suppressed terms involve parity-odd coefficients, while the unsuppressed terms are only sensitive to parity-even coefficients. Therefore, consideration of these terms seems worthwhile even at reduced sensitivity.

The above experiments place loose constraint on three combinations of coefficients. It is likely that reanalyzing these experiments in terms of the standard-model extension would place constraints on more combinations at similar levels.

4.3 Microwave Cavities

Microwave-cavity oscillators are among the most stable clocks. Cavities constructed of superconducting niobium have achieved frequency stabilities of $3 \times 10^{-16}$. In an effort to perform improved tests of relativity, superconducting microwave oscillators are being developed by the SUMO project for use on upcoming ISS missions \cite{SUMO}.

Equation \cite{Braxmaier} can be applied to cavities of any geometry and operated in any mode. Here we consider a cylindrical cavity of circular cross section, operated in the fundamental TM$_{010}$ mode. The integrals in Eq. \cite{Braxmaier} are easily carried out in a frame fixed to the cavity with its 3 axis along the symmetry axis. In terms of coefficients in the cavity-fixed frame, the fractional frequency shift is

$$\frac{\delta \nu}{\nu} = -\frac{1}{2} (\kappa_{DE})^{33}_{\text{cav}} + \frac{1}{4} [(\kappa_{HB})^{11}_{\text{cav}} + (\kappa_{HB})^{22}_{\text{cav}}] .$$

(26)

It is not difficult to generalize this expression to an arbitrary laboratory frame in which the cavities symmetry axis points in a direction specified by a unit vector $\hat{N}$. The result is

$$\frac{\delta \nu}{\nu} = \frac{1}{4} (\kappa_{HB})^{ij}_{\text{lab}} - \frac{1}{4} \hat{N}^j \hat{N}^k [2 (\kappa_{DE})^{jk}_{\text{lab}} + (\kappa_{HB})^{jk}_{\text{lab}}] .$$

(27)

Using transformation \cite{Braxmaier} we express this in terms of the Sun-frame coefficients. We find

$$\frac{\delta \nu}{\nu} = -\frac{1}{4} \hat{N}^j \hat{N}^k R^J R^K (\tilde{\kappa}_{e'})^{JK} - \frac{1}{2} (\delta^{jk} + \hat{N}^j \hat{N}^k) R^J R^K \epsilon^{JPQ} \beta^Q (\tilde{\kappa}_{o'})^{KP} - \tilde{\kappa}_{tr} ,$$

(28)

where for convenience we define the linear combinations

$$\tilde{\kappa}_{e'}^{JK} = 3 (\tilde{\kappa}_{e+})^{JK} + (\tilde{\kappa}_{e-})^{JK} , \quad \tilde{\kappa}_{o'}^{JK} = 3 (\tilde{\kappa}_{o-})^{JK} + (\tilde{\kappa}_{o+})^{JK} .$$

(29)

The matrix combinations $\tilde{\kappa}_{e'}$ and $\tilde{\kappa}_{o'}$ are traceless. The first contains five linearly independent combinations of the 11 parity-even coefficients for Lorentz violation, while $\tilde{\kappa}_{o'}$ contains all eight parity-odd coefficients.

As an example, consider two identical cavities, operated in the above mode, oriented at right angles to each other on the ISS. In general, the resonant frequency of the cavities will
vary at the first and second harmonics of the stations orbital frequency \( \omega_s \). A search for Lorentz violation could be performed by looking for this variation in the beat frequency of the two cavities. The variation takes the general form

\[
\frac{\nu_{\text{beat}}}{\nu} \equiv \frac{\delta \nu_1}{\nu} - \frac{\delta \nu_2}{\nu} = A_s \sin \omega_s T_s + A_c \cos \omega_s T_s + B_s \sin 2\omega_s T_s + B_c \cos 2\omega_s T_s + C, \tag{30}
\]

where \( A_s, A_c, B_s, \) and \( B_c \) are four linear combinations of the coefficients for Lorentz violation. These combinations depend on the orientations of the cavity pair and on the orientation of the orbital plane with respect to the Sun-centered frame. The precession of the ISS orbit slowly changes the four combinations, allowing access to more coefficients. Typically, these are rather cumbersome \([4]\) and are omitted here.

The sensitivity to the coefficients \( (k_f)_{\kappa \lambda \mu \nu} \) strongly depends on the orientations of the cavities. It can be shown that orienting a cavity with \( \hat{N} \) in the orbital plane maximizes the sensitivity to the second harmonics, at leading order in \( \beta \). Orienting a cavity so that \( \hat{N} \) is 45° out of the plane maximizes sensitivity to the first harmonics. A sensible configuration might have one cavity in the orbital plane and one 45° out of it.

There are many variations of the above experiment that could be performed. Earth-based experiments similar to those discussed Sec. 4.2 could also be performed using microwave cavities. Operating in different modes or using cavities filled with matter changes the combinations of coefficients to which the experiment is sensitive. It is also possible to compare the resonant frequency of a cavity to a reference clock other than another cavity oscillator. For example, the reference clock could be a hydrogen maser or atomic clock, which could conveniently be operated on a transition known to be insensitive to Lorentz violation \([23]\).

With current stabilities, it seems likely that microwave-cavity oscillators could access coefficients that are currently unmeasured, at levels comparable to the those of optical-cavity experiments and perhaps at the \(10^{-16}\) level.

5. Summary

In this work, we considered the experimental consequences of a Lorentz-violating electrodynamics which arises from a Lorentz- and CPT-violating standard-model extension. We found

| Coeff. | No. | Velocity | Polarization | Astrophysical Tests | Cavity Tests |
|--------|-----|----------|--------------|---------------------|--------------|
| \( \tilde{\kappa}_{e+} \) | 5 | -16 | -32 | * | - |
| \( \tilde{\kappa}_{e-} \) | 5 | n/a | n/a | * | - |
| \( \tilde{\kappa}_{o+} \) | 3 | n/a | n/a | * | - |
| \( \tilde{\kappa}_{o-} \) | 5 | -16 | -32 | * | - |
| \( \tilde{\kappa}_{tt} \) | 1 | n/a | n/a | * | - |

Table 3: Existing constraints.
that astrophysical bounds on birefringence lead to stringent constraints on ten coefficients for Lorentz violation. Access to the remaining coefficients may be accomplished through clock-comparison tests involving resonant cavities.

We summarize the current constraints in Table 3. The 19 coefficients \((k_F)_{\kappa\lambda\mu\nu}\) are represented by the matrices \(\tilde{\kappa}_{e+}, \tilde{\kappa}_{e-}, \tilde{\kappa}_{o+}, \tilde{\kappa}_{o-}, \tilde{\kappa}_{tr}\) defined in Eq. (7). The number of independent components in each matrix is shown in the second column. The third and fourth column give the order of magnitude of astrophysical bounds. Laboratory experiments with optical and microwave cavities can in principle access all the coefficients. The matrices for which a few components are probably constrained by the optical cavity experiments discussed in Sec. 4.1 are indicated by the symbol \(\star\) in the table. To date, no measurements of Lorentz violation using microwave cavities have been reported.

We conclude by remarking that even though the ten coefficients in \(\tilde{\kappa}_{e+}\) and \(\tilde{\kappa}_{o-}\) are tightly constrained by astrophysical measurements, confirming these in laboratory experiments provides an important check because the systematics in the two types of experiments are significantly different. Furthermore, cavity experiments access currently unexplored regions in parameter space, and they offer the possibility of discovering physics beyond the standard model.

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