Pentaquark states with hidden charm

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Abstract. I develop an extension of the usual three-flavor quark model to four flavors (u, d, s and c), and discuss the classification of pentaquark states with hidden charm. This work is motivated by the recent observation of such states by the LHCb Collaboration at CERN.

1. Introduction
The recent observation of candidates for pentaquark states by the LHCb Collaboration [1, 2, 3] has generated an enormous renewed interest in these multiquark states with almost weekly publications in the arXive and the scientific literature. The first evidence was found in the $\Lambda_b^0 \rightarrow J/\psi pK^-$ decay [1]. An analysis of the $J/\psi$ invariant mass spectrum showed the need to introduce two hidden charm pentaquark states, $P_c^+(4380)$ and $P_c^+(4450)$, with opposite parity. The evidence for the existence of these pentaquark states was confirmed later in a model-independent analysis [2], as well as in the $\Lambda_b^0 \rightarrow J/\psi p\pi^-$ decay [3]. So far, the angular momentum and parity could not be determined uniquely. The most likely values are $J^P = 3/2^-$ for $P_c^+(4380)$, and $5/2^+$ for $P_c^+(4450)$, although other combinations, like $(3/2^+, 5/2^-)$ or $(5/2^+, 3/2^-)$, are not excluded. There are proposals to study the $P_c^+(4380)$ and $P_c^+(4450)$ states in photoproduction of $J/\psi$ on the proton to verify their existence and to determine their spin and parity [4]-[9].

Theoretically, several narrow $N^*$ and $\Lambda^*$ resonances with hidden charm were predicted above 4 GeV in a coupled-channel unitary approach [10, 11] several years before the LHCb data, as well as [12, 13]. After the discovery of the $P_c^+$ pentaquark states many different explanations have been explored, ranging from weakly bound molecular states of charmed baryons and mesons [13]-[19], and interpretations in terms of multiquark degrees of freedom [20]-[26], to kinematic effects [27, 28, 29]. More information on the experimental and theoretical aspects of pentaquark states, as well as a more complete list of references, can be found in the reviews [30, 31, 32].

It is the aim of the present contribution to present a classification of all possible $uudc\bar{c}$ hidden-charm pentaquark states in an extension of the three- to the four-flavor $SU(4)$ quark model, and to calculate the corresponding magnetic moments of these configurations. This work is an extension of earlier studies of $uudd\bar{s}$ pentaquarks [33, 34].

2. Four-flavor SU(4) quark model
In this section, I review the four-flavor $SU(4)$ quark model which is based on the flavors up, down, strange and charm [35]-[38]. Even though the $SU(4)$ flavor symmetry is expected to be
much more broken than the three-flavor $SU(3)$ flavor symmetry due to the large mass of the charm quark, it nevertheless provides a useful classification scheme for baryons and mesons.

The spin-flavor states of the $udsc$ quark model can be decomposed into their flavor and spin parts as

$$SU_{sf}(8) \supset SU_f(4) \otimes SU_s(2) S \rangle,$$

followed by a reduction to the three-flavor $uds$ states

$$SU_f(4) \supset SU_f(3) \otimes U_Z(1) \langle g, h, Z \rangle,$$

and finally to their isospin and hypercharge contents

$$SU_f(3) \supset SU_I(2) \otimes U_Y(1) \langle h, I, Y \rangle.$$

The quarks transform as the fundamental representation $[1]$ under $SU(n)$, where $n = 2$ for the spin, $n = 3$ for the color, $n = 4$ for the flavor, and $n = 8$ for the combined spin-flavor degrees of freedom, whereas the antiquarks transform as the conjugate representation $[1^*]$. The hypercharges $Y$ and $Z$ are related to the baryon number $B$, the strangeness $S$ and charm $C$ by $Y = B + S - C/3$, $Z = 3B - C/4$, and the electric charge $Q$ is given by the generalized Gell-Mann-Nishijima relation $Q = I_3 + B + S + C/2$.

3. $qqqq$ Pentaquark states

The pentaquark wave functions depend both on the spatial degrees of freedom and the internal degrees of freedom of color, flavor and spin

$$\psi = \psi^o \phi^f \chi^s \psi^c.$$

In order to classify the corresponding states, we shall make use as much as possible of symmetry principles. In the construction of the classification scheme we are guided by two conditions: (i) the pentaquark wave function should be antisymmetric under any permutation of the four quarks, and (ii) as all physical states, it should be a color singlet. In addition, we restrict ourselves to flavor multiplets containing $uudc \bar{c}$ configurations without orbital excitations and with spin and parity $J^P = S^P = 3/2^-$. The permutation symmetry among the four quarks is characterized by the $S_4$ Young tableaux $[4]$, $[31]$, $[22]$, $[211]$ and $[1111]$ or, equivalently, by the irreducible representations of the tetrahedral group $T_d$ (which is isomorphic to the permutation group $T_d \sim S_4$) as $A_1$, $F_2$, $E$, $F_1$ and $A_2$, respectively.

The total pentaquark wave function has to be a $[222]$ color-singlet state. Since the color wave function of the antiquark is a $[11]$ anti-triplet, the color wave function of the four-quark
Table 1. Spin-flavor decomposition of $q^4$ states. The subindices denote the dimension of the multiplet.

| $SU_{ud}(8)$ | $SU_f(4)$ | $SU_s(2)$ | $\psi_{F_2}$ |
|-------------|----------|------------|-------------|
| $[f]$       | $[g]$    | $[g']$     |             |
| $[31]_{630}$ | $[4]_{35} \otimes [31]_{3}$ | $[\phi A_1 \times \chi F_2]_{F_2}$ |
| $[31]_{45} \otimes [4]_{5}$ | $[\phi F_2 \times \chi A_1]_{F_2}$ |
| $[31]_{3} \otimes [\phi F_2 \times \chi F_2]_{F_2}$ |
| $[22]_{1}$ | $[31]_{3} \otimes [\phi E \times \chi F_2]_{F_2}$ |
| $[22]_{20} \otimes [31]_{3}$ | $[\phi F_1 \times \chi F_2]_{F_2}$ |
| $[211]_{15} \otimes [31]_{3}$ | $[\phi F_1 \times \chi F_2]_{F_2}$ |
| $[22]_{1}$ | $[\phi F_1 \times \chi F_2]_{F_2}$ |

Table 2. $SU_f(4) \supset SU_f(3) \otimes U_Z(1)$ flavor classification of four-quark states (here $q = u, d, s$).

| $SU_f(4)$ | $SU_f(3)$ | $U_Z(1)$ |
|-----------|------------|-----------|
| $[g]$     | $[h]$      |           |
| $[4]_{35} \supset [4]_{15} \oplus [3]_{10} \oplus [2]_{6} \oplus [1]_{3} \oplus [0]_{1}$ |
| $[31]_{45} \supset [31]_{15} \oplus [3]_{10} \oplus [21]_{8} \oplus [2]_{6} \oplus [11]_{3} \oplus [1]_{3}$ |
| $[22]_{20} \supset [22]_{8} \oplus [21]_{8} \oplus [2]_{6}$ |
| $[211]_{15} \supset [211]_{3} \oplus [21]_{8} \oplus [111]_{1} \oplus [11]_{3}$ |

Configuration has to be a $[211]$ triplet with $F_1$ symmetry under $T_d$. The total $q^4$ wave function is antisymmetric ($A_2$), hence the orbital-spin-flavor part is a $[31]$ state with $F_2$ symmetry

$$\psi = \left[ \psi_{F_1}^c \times \psi_{F_2}^{osf} \right]_{A_2}$$

where the subindices refer to the symmetry properties of the four-quark system under permutation. Moreover, it is assumed that the orbital part of the pentaquark wave function corresponds to the ground state (without orbital excitations) with $A_1$ symmetry. As a consequence the spin-flavor part of the four-quark wave function has $F_2$ symmetry

$$\psi_{F_2}^{osf} = \left[ \psi_{A_1}^o \times \psi_{F_2}^{sf} \right]_{F_2}$$

The flavor and spin content of the only allowed spin-flavor configuration, $[31]$ with $F_2$ symmetry, is given in Table 1. There are four different flavor multiplets $[g] = [4], [31], [22]$ and $[211]$. For pentaquark states with $J^{P} = 3/2^-$ without orbital excitations the allowed values of the spin of the four-quark system are $S = (g_1' - g_2')/2 = 1$ and $2$. In the last column, we show the allowed spin-flavor configurations, where $\phi$ and $\chi$ denote the flavor and spin wave functions, respectively.

Since the current interest is in four-quark states with one charm quark, it is convenient to make a decomposition of four into three flavors according to $SU_f(4) \supset SU_f(3) \otimes U_Z(1)$. Table 2 shows that the states with one charm quark have $Z = 0$ and belong to either an $SU(3)$ decuplet ($[h] = [3]$), octet ($[21]$) or singlet ($[111]$). Since the flavor singlet corresponds to a $udsc$
Figure 1. Pentaquark decuplet and octet

configuration, the only $SU(3)$ flavor multiplets that contain a $uud$ state are the decuplet and the octet.

Finally, the $uudc\bar{c}$ pentaquark states belong to $SU(3)$ multiplets that are obtained by combining the four-quark $[3]$ decuplet and $[21]$ octet with the $[111]$ singlet for the antiquark with charm $\bar{c}$

$$[3] \otimes [111] = [411],$$
$$[21] \otimes [111] = [321].$$

(9)

In summary, the $uudc\bar{c}$ pentaquark states belong to a $SU(3)$ flavor decuplet with $[h] = [411] \equiv [3]$ or an octet $[321] \equiv [21]$ (see Fig. 1). The corresponding flavor wave functions can be expressed in terms of the quantum numbers

$$|\phi_{A_1}\rangle = |[g], [h]_{qqqq}, [h]_{q\bar{q}}, [h], I, I_3, Y, Z\rangle,$$
$$= |[4], [3], [111]; [411], \frac{3}{2}, \frac{1}{2}, 1, \frac{3}{4}\rangle,$$

$$|\phi_{F_2}\rangle = |[31], [3], [111]; [411], \frac{3}{2}, \frac{1}{2}, 1, \frac{3}{4}\rangle,$$

(10)

for the decuplet, and

$$|\phi_{F_2}\rangle = |[31], [21], [111]; [321], \frac{1}{2}, \frac{1}{2}, 1, \frac{3}{4}\rangle,$$

$$|\phi_{E}\rangle = |[22], [21], [111]; [321], \frac{1}{2}, \frac{1}{2}, 1, \frac{3}{4}\rangle,$$

$$|\phi_{F_1}\rangle = |[211], [21], [111]; [321], \frac{1}{2}, \frac{1}{2}, 1, \frac{3}{4}\rangle,$$

(11)

for the octet.

4. Magnetic moments
In this section, we study the magnetic moments of the $uudc\bar{c}$ pentaquark states with spin and parity $J^{P} = \frac{3}{2}^{-}$ discussed in the previous section. The magnetic moment is a crucial ingredient in calculation of the photoproduction cross sections of pentaquarks. In Ref. [22], the magnetic moments were calculated for a molecular model, a diquark-triquark model and a
The magnetic moments of the up, down and strange quarks were fitted to the magnetic moments of the proton, neutron and Λ hyperon. The value of the magnetic moment of the charm quark is consistent with a constituent mass of 1.550 GeV.

The numerical values are obtained by using

$$\sum_i \mu_i (2s_i^2 + l_i^2),$$

where $\mu_i$ denotes the magnetic moment of the $i$–th quark or antiquark.

Since at present, we do not consider radial or orbital excitations, the magnetic moment only depends on the spin part. The results for the different $uudc\bar{c}$ configurations are given in Table 3.

The magnetic moments of the up, down and strange quarks were fitted to the magnetic moments of the proton, neutron and Λ hyperon. The value of the magnetic moment of the charm quark is consistent with a constituent mass of 1.550 GeV.

### 5. Summary and conclusions

In conclusion, in this contribution we discussed the classification of $uudc\bar{c}$ pentaquark states with angular momentum and parity $J^P = 3/2^-$. At present we did not include orbital excitations. The coupling between the four-quark states $uudc$ and the antiquark $\bar{c}$ was carried out at the level of the $SU(3)$ flavor group which results in a pure $uudc\bar{c}$ state. Couplings at the level of the $SU(8)$ spin-flavor group or the $SU(4)$ flavor group will lead to admixtures with $uu\bar{du}$, $uudd\bar{d}$ and $uudd\bar{s}s$ configurations [39].

Since for future studies of pentaquark states in photoproduction reactions it is important to have information on the electromagnetic couplings, we presented a first step by calculating the magnetic moments of all allowed $uudc\bar{c}$ states. The numerical values are positive and in the range from $1 - 3 \mu_N$.

Finally, if indeed the pentaquark exists, there should be an entire multiplet of pentaquark states. The study of the structure of these multiplets and the mass spectrum is the subject of work in progress [39].
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References
[1] Aaij R et al. (LHCb Collaboration) 2015 Phys. Rev. Lett. 115 072001
[2] Aaij R et al. (LHCb Collaboration) 2016 Phys. Rev. Lett. 117 082002
[3] Aaij R et al. (LHCb Collaboration) 2016 Phys. Rev. Lett. 117 082003
[4] Kubarovskv V and Voloshin M B 2015 Phys. Rev. D 92 031502(R)
[5] Wang Q, Liu X H and Zhao Q 2015 Phys. Rev. D 92 034022
[6] Karliner M and Rosner J L 2016 Phys. Lett. B 752 329
[7] Hiller Blin A N, Fernández-Ramírez C, Jackura A, Mathieu V, Mokeev V I, Pilloni A and Szczepaniak A P 2016 Phys. Rev. D 94 034002
[8] Fernández-Ramírez C, Hiller Blin A N and Pilloni A 2017 arXiv:1703.06928
[9] Meziani Z E et al. 2016 arXiv:1609.09676
[10] Wu J J, Molina R, Oset E and Zou B S 2010 Phys. Rev. Lett. 105 232001
[11] Wu J J, Molina R, Oset E and Zou B S 2011 Phys. Rev. C 84 015202
[12] Yang Z C, Sun Z F, He J, Liu X and Zhu S L 2012 Chin. Phys. C 36 6
[13] Karliner M and Rosner J L 2015 Phys. Rev. Lett. 115 122001
[14] Chen R, Liu X, Li X Q and Zhu S L 2015 Phys. Rev. Lett. 115 172001
[15] Chen H X, Chen W, Liu X, Steele T G and Zhu S L 2015 Phys. Rev. Lett. 115 172001
[16] Foca L, Nieves J and Oset E 2015 Phys. Rev. D 92 094003
[17] He J 2016 Phys. Lett. B 753 547
[18] Eides M I, Petrov V Yu, Polyakov M V 2016 Phys. Rev. D 93 054039
[19] Yamasuichi Y and Santopinto E 2016 arXiv:1606.08330
[20] Maiani L, Polosa A D and Riquer V 2015 Phys. Lett. B 749 289
[21] Lebed R F 2015 Phys. Lett. B 749 454
[22] Wang G J, Chen R, Ma L, Liu X and Zhu S L 2016 Phys. Rev. D 94 094018
[23] Yang G, Ping J and Wang F 2017 Phys. Rev. D 95 014010
[24] Deng C, Ping J, Huang H and Wang F 2017 Phys. Rev. D 95 014031
[25] Takeuchi S and Takizawa M 2017 Phys. Lett. B 764 254
[26] Santopinto E and Giachino A 2017 arXiv:1604.03769v2
[27] Guo F K, Meissner U G, Wang W and Yang Z 2015 Phys. Rev. D 92 071502
[28] Liu X H, Wang Q and Zhao Q 2016 Phys. Lett. B 757 231
[29] Mikhasenko M 2015 arXiv:1507.06552
[30] Burns T J 2015 Eur. Phys. J. A 51 152
[31] Chen H X, Chen W, Liu X and Zhu S L 2016 Phys. Rep. 639 1
[32] Patrignani C et al. (Particle Data Group) 2016 Chin. Phys. C 40 100001
[33] Bijker R, Giannini M M and Santopinto E 2004 Eur. Phys. J. A 22 319
[34] Bijker R, Giannini M M and Santopinto E 2004 Phys. Lett. B 595 260
[35] Bjorken J J and Glashow S L 1964 Phys. Lett. 11 255
[36] Glashow S L, Iliopoulos J and Maiani L 1970 Phys. Rev. D 2 1285
[37] Gaillass M K, Lee B W and Rosner J L 1975 Rev. Mod. Phys. 47 277
[38] De Rújula A, Georgi H and Glashow S L 1976 Phys. Rev. D 12 147
[39] Bijker R and Fernández-Ramírez C 2017, work in progress