Analysis of nanofluid based natural convection in a square cavity applying corner heating and cooling

Abhinav Saha, Koushik Ghosh and Nirmal K. Manna
Department of Mechanical Engineering, Jadavpur University, Kolkata–700032, India
e-mail: saha.abhinav.2010@gmail.com

Abstract. The present investigation aims at the numerical analysis of natural convection in a square cavity. The cavity is filled with nanofluid using water as the base fluid and Al₂O₃ as nanoparticles. The investigation is carried out in a square cavity with corner heating and cooling. The numerical simulations are performed using a finite volume method. The average Nusselt number is calculated for a wide range of Rayleigh number from $10^4$ to $10^6$ keeping solid volume fraction of nanoparticle fixed at 0.04. The details of flow and heat transfer for various cases are analyzed using the contour of streamlines and isotherms.

1. Introduction
Natural convection flow - also known as the buoyancy-driven flow is caused by the variation in fluid density in presence of the gravitational field. A thorough understanding of flow and heat transfer characteristics in such situations is important due to its application in the cooling of electronic equipment. Conventionally, fluid such as water, oil, ethylene glycol etc. used in cooling applications achieves low heat transfer due to lower thermal conductivity. The thermal conductivity is enhanced by adding nano-metallic particles to base fluid known as nanofluids.

Putra et al. [1] performed an experimental investigation of natural convection of nanofluid. They studied the effect of Rayleigh number, aspect ratio and volume fraction for different nanoparticles suspended in the base fluid. Ternik et al. [2] studied natural convection in differentially heated side walls while the other two walls are kept adiabatic. They studied the effect of different particles for a wide range of Rayleigh number and volume fraction. There have been many excellent review papers on natural convection of a nanofluid in an enclosure. Oztop et al [3] presented an excellent review paper of natural convection of nanofluid under partial heating of an enclosure. In the recent years, and Bhuiyana et al. [4] and Mendu et al. [5] studied the natural convection in a square cavity filled with nanofluid with a discrete heater placed at the bottom of an enclosure. They also studied the Rayleigh number, Prandtl number, solid volume fraction of nanoparticle and heater position.

Literature review reveals that natural convection flow of a nanofluid in an enclosure depends on the Rayleigh number, Prandtl number, aspect ratio, solid volume fraction of nanoparticle as well as the end wall boundary conditions. The dependence on such a large number of parameters, however, compounds the unpredictability in flow and heat transfer characteristics in such problems.

To the best of the author’s knowledge, no studies have been reported on natural convection of nanofluid with corner heating and cooling. Experimental investigation of details of flow and temperature fields is also fraught with uncertainty as the velocity and temperature differences may be too small sometimes to be resolved accurately. The scope of the present work is to employ CFD
methodology for natural convection flow and heat transfer in a square cavity with corner heating and cooling.

2. Problem definition
Figure 1 shows the computational domain of square cavity filled with nanofluid. In the figure, discrete heaters are embedded at the corner of a cavity. The hot heater and cold heater are kept at a constant temperature of $T_h$ and $T_c$ respectively while other walls are assumed to be adiabatic. The hot heaters and cold heaters are embedded at diagonal of square enclosure. The hot heaters are located at the top-right corner and bottom-left corner while cold heaters are located at top-left corner and bottom right corner.

![Figure 1. Schematic diagram of a square enclosure filled nanofluid along with boundary condition](image)

A two-dimensional, steady, laminar, incompressible and Newtonian fluid is considered. The properties of the working fluid are fixed, except the variation of fluid density with temperature which is accounted for in the formulation of buoyancy term using the Boussinesq approximation. The thermal equilibrium exists between the base fluid and nanoparticle.

3. Mathematical Formulation
The flow is governed by the conservation laws of mass, momentum and energy. The following governing equations are written in the non-dimensional form under the above-mentioned assumptions.

Mass conservation equation

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$$

Momentum conservation equation in x-direction:

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\mu_f}{\rho_f \alpha_l} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)$$

Momentum conservation equation in y-direction:

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\mu_f}{\rho_f \alpha_l} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \left( \frac{\rho \beta}{\rho_f \beta_l} \right) R a P r \frac{\partial \theta}{\partial Y}$$

Energy conservation equation:
The boundary condition used in non-dimensional form is shown in figure 1. The closure relations to calculate properties of nanofluid is given by Eqs. (5) to (9).

\[
\rho_{af} = (1-\phi)\rho_f + \phi\rho_p
\]

(5)

\[
(p_C)_f = (1-\phi)(p_C)_f + \phi(p_C)_p
\]

(6)

\[
(p\beta)_f = (1-\phi)(p\beta)_f + \phi(p\beta)_p
\]

(7)

\[
\mu_{af} = \mu_f / (1-\phi)^{2.5}
\]

(8)

The properties of base fluid (H\textsubscript{2}O) and nanoparticle (Al\textsubscript{2}O\textsubscript{3}) used in present study is given in Table 1.

Table 1: The properties of base fluid (H\textsubscript{2}O) and nanoparticle (Al\textsubscript{2}O\textsubscript{3}) [5].

|               | \(\rho\) (kg / m\textsuperscript{3}) | \(C_p\) (J / kg - K) | \(k\) (w / m - K) | \(\mu\) (kg / m - s) | \(\beta\) (1 / K) |
|---------------|-------------------------------------|----------------------|-------------------|---------------------|------------------|
| Base fluid (H\textsubscript{2}O) | 997.5                              | 4178                 | 0.6129            | 0.0008295          | 0.00028          |
| Nanoparticle (Al\textsubscript{2}O\textsubscript{3}) | 3970                              | 765                  | 40                | 0.85E-5            |                  |

The governing equations are given in Eqs. (1) to (4) subjected to boundary condition mentioned in figure 1. The geometry of the square cavity is created in Gambit software. The numerical solution is solved in ANSYS FLUENT based on finite volume approach. The convergence criteria for all the governing equations are set at residuals less than \(10^{-6}\).

The convective heat transfer is determined by Nusselt number. The local Nusselt number \(\text{Nu}\) along the hot wall can be determined using the expression:

\[
\text{Nu} = \frac{K_{af}}{\kappa_f} \frac{\partial \theta}{\partial n}
\]

(10)

where \(\mathbf{n}\) is the outward normal vector.

The average Nusselt number \(\overline{\text{Nu}}\) can be obtained by:

\[
\overline{\text{Nu}} = \frac{1}{L} \int_0^L \text{Nu} \, dl
\]

(11)

where \(L\) is the total length of the hot wall.

4. Results and Discussion

The natural convection flow and heat transfer is carried in a square cavity. The cavity filled with nanofluid where base fluid is H\textsubscript{2}O and nanoparticle is Al\textsubscript{2}O\textsubscript{3}. The numerical analysis is carried out for varying \(Ra\) in the range of \(10^4\) to \(10^6\) while keeping solid volume fraction \(\phi = 0.04\). The contours of isotherms and stream functions help to understand flow physics and heat transfer. The scope of the present investigation is to analyze the effect of \(Ra\) on flow physics and heat transfer in a square cavity with corner heating and cooling.

4.1 Grid Independence Test


The grid independent study is carried out for pure water ($\phi=0$) at $Ra = 10^4$ and $10^6$. The grid study is carried out using three different grid namely 50×50, 100×100 and 200×200 shown in Table 2. The results of $\bar{Nu}$ obtained with different grids are very close to each other. The intermediate grid count 100×100 is adopted for further computations in this work as a good compromise between accuracy and computational effort.

Table 2: Grid independent study for $\bar{Nu}$ at $Ra = 10^4$ and $10^6$

| Number of Grid Counts | \text{Ra} = 10^4 | \text{Ra} = 10^6 |
|-----------------------|-----------------|-----------------|
| \text{N}_1 (50×50)   | 2.285815        | 9.673208        |
| \text{N}_2 (100×100) | 2.276351        | 9.334908        |
| \text{N}_3 (200×200) | 2.273612        | 9.247624        |

4.2 Comparison of Results
To the best of author’s knowledge, no work is available in the literature considering a corner heating and cooling of nanofluids. Therefore, the numerical model is validated for natural convection in a differentially heated cavity filled with pure water ($\phi=0$). In the cavity side walls are kept at different temperature while the other two walls are kept adiabatic.

Table 3: The comparison of $\bar{Nu}$ for differentially heated square enclosure

| Ra | Mendu et al. [5] | Present study | % Error |
|----|-----------------|---------------|---------|
| $10^4$ | 2.272 | 2.276 | 0.18 |

4.3 Effect of Rayleigh number on isotherms and streamlines
Figure 2 represents isotherms and streamlines for $Ra$ in the range of $10^4$ to $10^6$ while keeping $\phi=0.04$. The streamlines are presented in figure 2 (b). It is observed from the contour of streamlines that two convective cells are formed which is counter-rotating in nature for all values of $Ra$. The counterclockwise and clockwise rotations of flow are represented by the positive and negative sign of streamlines respectively. The convective cells formed are symmetric about horizontal mid-line at $Y=0.5$. The line of $\psi=0$ acts like a virtual wall across which no flow takes place. It is further observed that strength of convective cells increases with an increase in $Ra$. The isotherms are presented in figure 2 (b). The isotherms are also symmetric about horizontal mid-line at $Y=0.5$. The isotherms are crowded near the hot heater which is located at top-right corner and bottom-left corner of cavity. It is further observed that isotherms gradually becomes more overpopulated with an increase in $Ra$. The increase in density of isotherms represent rise in temperature gradient near the corner of a cavity. The rise in temperature gradient results in an enhancement in heat transfer.

![Isotherms](image-url)
Figure 2. Contours of (a) Isotherms and (b) Streamlines for $Ra = 10^5$ to $10^6$ at $\phi = 0.4$

The graph of $\psi_{\max}$ vs $Ra$ presented in figure 3 (a) shows that maximum value of stream function increases with an increase in $Ra$. The graph of $\overline{Nu}$ vs $Ra$ presented in figure 3 (b) shows that convective heat transfer monotonically increases with an increase in $Ra$.

Figure 3. (a) Variation of $\psi_{\max}$ with $Ra$ (b) Variation of $\overline{Nu}$ with $Ra$ at the heated wall

5 Conclusions
The salient observations from the study can be summarized as follows:
- The maximum value of stream function increases with an increase in the Rayleigh number.
- The enhancement in convective heat transfer is observed with an increases in Rayleigh number.

References
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Nomenclature

| Greek symbols | Nomenclature |
|---------------|--------------|
| $g$           | Acceleration due to gravity |
| $\rho$        | Density      |
| Symbol | Description                        | Unit               |
|--------|------------------------------------|--------------------|
| L      | Dimension of a square enclosure    | m                  |
| P      | Non-dimensional pressure           |                    |
| U,V    | Dimensionless velocity in X and Y |                    |
| Ra     | Rayleigh number                    |                    |
| Pr     | Prandtl number                     |                    |
| Nu     | Local Nusselt number               |                    |
| Nu     | Average Nusselt number             |                    |
| v      | Kinematic viscosity                | m²/s               |
| θ      | Non-dimensional temperature        |                    |
| α      | Thermal diffusivity                |                    |
| β      | Thermal expansion coefficient      |                    |
| ψ      | Non-dimensional stream function    |                    |

**Subscripts**
- h, c: Hot wall, Cold wall
- f: Fluid
- nf, np: Nanofluid, Nanoparticle