Planning of various forms of transportation routes in the automotive regional transportation network

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Abstract. The problem of transport routing in a regional transport network is considered. The solution method is the Minimum Cost Flow Problem, and the Column Generation Method. For the conditions of degenerate cases, the paper proposes a description of a transport system in the form of directed graphs. As a result, various forms of routes are obtained, as is customary in the transport logistics of road transport.

Introduction

Product delivery in regional networks is an important logistical task. Transportation of farm products, delivery of goods for construction purposes, removal of garbage to disposal places and performing inspection trips with control functions are common routine operations. As a rule, the transport complex of the regions consists only of the automobile component (roads and vehicles). In general, the development of the automobile network is determined by economic and geographical characteristics and usually looks very modest. Mathematical models describe the tasks of scheduling the work of vehicles. It always starts with representing the transport network in the form of a graph G = (V, A), where V is a set of vertices and A is a set of ribs. For each edge (i, j), the cost c_{ij} is set (in particular, the cost means the distance). For the graphs of regional transport systems, one can check if they are Hamiltonian. Checking with the Ore, Posch and Dirac criteria shows that their check conditions are met. However, in many cases there is a Hamiltonian cycle. This is in contrast to urban transport systems where the density of the road network is much higher.

Vehicle routing problem (VRP) has been considered as a mathematical programming task in a very large number of western scientific works. Some of the results are discussed in the article [1]. Many review papers include classification schemes of various vehicle routing tasks and their solution methods. The task has several formulations. The main formulation is the formulation about the flow of the minimum value with binary variables. The work [2] includes the formulation of VRP as a problem of a set decomposition. There are significantly fewer scientific publications of Russian studies. In most publications, the routing problem is limited to the traveling salesman problem with additional constraints, and is solved based on the methods used in the traveling salesman problem. The application of the column generation method in the routing problem for a regional transport network was analyzed in the work [3]. The use of this approach allows us to consistently solve linear programming problems, which can be done by attracting non-commercial solvers of these problems that are part of the office software at the first stage. For large regional systems, high-performance commercial solvers are indispensable.
1. Mathematical formulation

The method of generating columns involves solving a bounded master problem, a dual problem, and finding a new improving solution as a solution to the coordinating subproblem. The master problem as the set covering problem is formulated as follows. You want to find

$$\min \sum_{i \in MT} d_i x_i$$  \hspace{1cm} (1)$$

subject to

$$\sum_{i \in MT} a_{it} x_i \geq 1, \quad \forall i \in N, x_i \in \{0,1\}, \quad \forall t \in MT,$$  \hspace{1cm} (2)$$

where $x_i$ are the tours given on the set of admissible tours MT, $d_i$ is the cost (distance) of the tour $x_i$, $N$ is the set of visiting points (clients), $i$ is the client's index, $a_{it} = 1$ if client $i$ is visited on tour $t$, and $a_{it} = 0$, otherwise. The dual problem has the form

$$\max \sum_{i=1}^N u_i ,$$  \hspace{1cm} (3)$$

subject to

$$\sum_{i=1}^{MT} a_{it}^T u_t \leq d_t .$$  \hspace{1cm} (4)$$

Based on the works [4, 5], a coordinating subproblem is formulated to determine improving tours in the form

$$\max \left( \sum_{i=1}^N u_i y_i - \sum_{j=1}^N \sum_{i=1}^N c_{ij} z_{ij} \right)$$  \hspace{1cm} (5)$$

where the binary variables $y_i$ determine the generated tour for the new iteration of the master problem, its cost $d_{i+1}$ is determined by the second component in the maximization criterion, the binary variables $z_{ij}$ are determined on the set A of edges of the graph G and set the sequence of visiting clients (graph vertices).

The constraints in the generation subproblem are

$$\sum_{j=2}^N z_{1j} = 1$$  \hspace{1cm} (6)$$

$$\sum_{i=2}^N z_{ij} = 1$$  \hspace{1cm} (7)$$

$$\sum_{i=2}^N z_{ij} - \sum_{j=2}^N z_{ji} = 0, \quad \forall i = j \in \{2, N\}$$  \hspace{1cm} (8)$$

$$\sum_{i=1}^N z(i, j) \leq 1, \quad \forall j \in N$$  \hspace{1cm} (9)$$

$$y_i = \sum_{j=1}^N z(i, j), \quad \forall i \in N$$  \hspace{1cm} (10)$$

$$\sum_{i \in S} \sum_{j \in S} z_{ij} \leq |S|-1, \quad \forall S \in V, |S| \geq 2$$  \hspace{1cm} (11)$$

$$z(i, j), y(i) \in \{0,1\}$$  \hspace{1cm} (12)$$

Constraints (6), (7) define the depot as the initial and final vertex, (8) set the balance equations for the vertices, (9) indicate that the vertex is visited by one vehicle and exclude disconnected subcycles (11).
The number of variables in this subproblem is equal to the number of graph vertices and twice the number of edges. The number of variables in the dual problem is equal to the number of vertices. In the master problem, there is the number of rounds in the starting solution plus the number of rounds obtained when generating new improving solutions. The criterion for stopping the iterative process is to obtain the criterion value \( (5) \leq 0 \).

2. Preliminary results.
The research of the VRP problem for regional transport systems was carried out in various directions: which graph will reliably represent the regional transport system, how to set the initial starting values of the tours, and what is the process of convergence to the optimal value. In the examples considered, the same pattern of the convergence process was qualitatively observed. Based on the starting values, a round was generated, which significantly reduced the objective function, changing the structure of the current solution of the master problem. Further, tours were generated many times, which did not change the current solution in any way, but according to the stopping criterion, the process continued. At some iteration, a round is generated that abruptly changes the structure of the solution and gives an optimal solution to the problem.

The optimal solution structure was both one Hamiltonian tour and a set of partial tours. The solution was determined by the initial data for the road network.

It should be noted that the presence of an intermediate node between the depot and other vertices can complicate obtaining the final solution due to a large number of iterations. The coordinating task in its decisions proposes the solutions that can simply enumerate the most diverse options, although formally we are talking about a sequential decrease in the optimal values in the coordinating task.

Experts studying the application of the method of generating columns in various versions note that the convergence to the optimum can also be oscillatory. In this case, it is proposed to apply convergence stabilization methods by stabilizing the values of the dual variables of the master problem. It is also noted that convergence is accelerated in the presence of additional restrictions in the main original problem.

With regard to the routing problem, restrictions on cargo capacity and/or trip time are applied. However, in this case, in practice, there is simplification of routes, degeneration of circular routes into partially or completely rectilinear ones. In the mathematical model, this is not permissible due to the presence of a restriction on the formation of subcycles (11).

3. Suggestion for solution
We propose to consider the graphs of highways as they are in the traffic flow theory. Each road is presented as two directed graphs with certain directions, but without details of its traffic capacity. Junctions are presented in the form of intersections with permitted driving directions. In this case, in the mathematical model of the transport system, the simplest element between points is the path to the object with a U-turn, including four directed arcs: two arcs back and forth, and two arcs that turn in the opposite direction. There is an imitation of traffic on the road: for example, driving to the object on the right side of the road, then returning on the left side. Driving in the opposite direction is prohibited.

Each object of visit in the graph is proposed to be represented by a set of vertices that imitate "corners of the intersection". A visit to any of the vertices included in this set is considered as a visit to an object. An object must have at least two vertices. An object with multiple connecting roads is described by corner vertices. A usual intersection has four vertices.

The initial graph of the transport system \( G = (V, A) \), can be represented as \( G = (V_1, \ldots, V_i, \ldots, V_n, A_1, \ldots, A_i, \ldots, A_n, A) \), where \( V_i \) are the nodes associated with the object \( i \), \( A_i \) are the arcs associated with the object (intersection), \( A \) is a set of oriented arcs between the objects (intersections). For each object, there is a set of incoming and a set of outgoing arcs.

Mathematically, the condition of visiting an object is written in the form:

\[
\sum_{i,j \in A^*} z(i, j) \geq 1, \quad \forall i \in A
\]
The relationship between the variables in the coordinating problem $y(i)$ and $z(i, j)$ is written in the form of systems of two inequalities [6]:

$$y_i \leq \sum z(i, j), \ \forall i \in V$$  \hspace{1cm} (14)

$$y_i \geq z(i, j), \ \forall i \in V, \forall j \in \delta^*$$  \hspace{1cm} (15)

The algorithm for solving the routing problem based on the directed graph of roads will be partially modified in comparison with the classical algorithm.

1. Setting the starting values. Case studies have shown that the number of starting values in the master task should be no less than the number of objects visited. In other words, the dual set covering the problem has a lot of solutions, and the entire algorithm can loop. The shortest paths to each vertex and back are taken as the starting values.

   Unlike the classical shortest path problem as a linear integer programming problem, the shortest path can be defined as the minimum cost flow through a given vertex and back along a modified directed graph. Pointing to the top is performed by specifying the condition (13).

2. The master problem (set covering problem) and the dual problem are performed in the usual way.

3. The coordinating task is performed according to inequalities (14) and (15) that index the visited nodes.

Restrictions (11) are also subject to modifications, which do not allow the formation of subcycles. Circular traffic on the roads connecting the objects of visit and circular traffic on the objects (intersections) are prohibited. The minimal set $S = 4$.

4. Calculation results.

The modified method was used to calculate the numerical examples in Microsoft Excel. The calculation results showed the following:

1. Now it is possible to calculate the routes to single vertices.

2. The calculated routes may differ significantly from the classical circular routes. The vehicles often return from a group of objects along the same roads as they traveled to it, which is more consistent with the practice of road transport.

3. When visiting a group of objects, traffic with right turns at intersections is preferable.

4. A disadvantage in the problem is that the number of variables significantly increases due to the arcs describing the direction of movement at intersections. The number of restrictions also increases.

5. In the examples considered, the calculation time has scarcely changed. The calculation time is largely influenced by the organization of the data arrays, which refers to the internal component of the optimization program.

6. The forms of the resulting routes include pendulum and circular routes, as well as their various combinations, as is customary in classical transport logistics for road transport.

Conclusions.

Linear programming is implemented as a method for generating columns for the routing problem and allows this problem to be solved for regional transport systems accurately and using more reliable input data compared to the scheme for solving the traveling salesman problem. In the course of the solution, improvement occurs in jumps rather than at each iteration, which can be explained by the specifics of the problem with binary variables. In contrast to the classical formulation of the problem with undirected graphs, a modified version with directed graphs is proposed, which makes it possible to take into account additional aspects of road transport.

References

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