Topological and non-topological solutions in the 3-phase model of hybrid chiral bag

K. Sveshnikov\textsuperscript{a,b,*}, Il. Malakhov\textsuperscript{a}, M. Khalili\textsuperscript{a}, S. Fedorov\textsuperscript{a}
\textsuperscript{a}Physics Department and \textsuperscript{b}Institute of Theoretical MicroPhysics, Moscow State University, Moscow 119899, Russia

The 3-phase version of the hybrid chiral bag model, containing the phase of asymptotic freedom, the hadronization phase as well as the intermediate phase of constituent quarks, is proposed. For this model the self-consistent solutions, which take into account the fermion vacuum polarization effects, are found in (1+1) D. The renormalized total energy of the bag is studied as a function of its geometry and topological (baryon) number. It is shown that in the case of non-zero topological charge there exists a set of configurations being the local minima of the total energy of the bag and containing all the three phases, while in the non-topological case the minimum of the total energy of the bag corresponds to vanishing size of the phase of asymptotic freedom.

Keywords: Hybrid Chiral Bag Models, Solitons, Dirac Sea Polarization Effects

*e-mail address: costa@bog.msu.su
1 Introduction

At present the most perspective approach to the description of the low-energy structure of baryons is realized by the hybrid chiral models (HCM) of quark bags [1-3]. The initial formulation of HCM is given in terms of the little bag with massless quarks and gluons confined in the phase of asymptotic freedom, surrounded by the colorless purely mesonic phase, described by some nonlinear theory like the Skyrme model [3-5], while boundary conditions between the phases represent the "chiral confinement" [6]. In (1+1) D these conditions coincide with the exact bosonisation relations, what is the origin of assumption, that the fermion theory inside the bag and the meson one outside are actually equivalent [7]. As a consequence, all the physical properties of the bag should be independent of the choice of the boundary surface, what is the essence of the Cheshire Cat Principle (CCP) [7,8]. However, both the exact bosonisation and so the CCP can be rigorously proved only in (1+1) D, whereas in (3+1) D even the formulation of such a problem is ambiguous. As a result, in the (3+1) dimensional HCM based on the CCP there exists a rather small set of observables (e.g. the topological charge), which really do not depend on the bag radius [3,8].

At the same time, the phenomenology of strong interactions predicts unambiguously the existence of the characteristic confinement scale about 0.5 fm, and so in the realistic (3+1) D models the CCP should be strongly violated regardless on the proof of bosonisation. Moreover, in such 2-phase HCM there is no place for massive constituent quarks, whose concept has been shown to be very efficient in the hadronic spectroscopy [9]. From this point of view the most attractive situation is the one, where the initially free, almost massless current quarks transmute firstly into “dressed” due to interaction massive constituent quarks carrying the same quantum numbers of color, flavor and spin, and only afterwards there emerges the purely mesonic colorless phase. The first step to such version of the bag is given by the 3-phase chiral model, where the additional intermediate phase of interacting quarks and mesons with non-zero radial size is introduced [10,11]. This model allows to take self-consistently into account: i) the phase of asymptotic freedom with free massless quarks; ii) the phase of constituent quarks, which acquire an effective mass due to the chirally invariant interaction with the meson fields in the intermediate region of finite size; iii) the hadronization phase, where the quark degrees of freedom are completely suppressed, while the nonlinear dynamics of meson fields leads to the appearance of the c-number boson condensate in the form of a classical soliton solution, which provides the topological nature of the model as well as the corresponding quantum numbers.

It is worth-while to note, that the direct quark-meson interaction is also considered in a number of other approaches to the description of low-energy hadron structure, in particular, in the cloudy bag models [12-14], as well as in various versions of the chiral quark-soliton models [15-19]. However, the role of this interaction in each of these models is substantionally different. In the cloudy bag models such $\pi\bar{q}q$-coupling is considered only perturbatively, while in quark-soliton models it is the main nonlinear mechanism of dynamical generation of the quark bag in the whole space. In the case under consideration an intermediate variant is realized, where the contribution of the direct chiral quark-meson coupling to the properties of the system is nonlinear, but the confinement of quarks is provided by some additional procedure. Such an approach allows to realize the nonlinear mechanism of dynamical mass generation in the intermediate region, but unlike the quark-soliton models doesn’t have problems related to the absence of the total confinement.

In the present paper a toy (1+1) D model of this kind is considered, in which in the intermediate region the one-flavor fermion field is coupled in a chirally invariant way to the real scalar field, which possesses a nonlinear soliton solution in the exterior region. For this model the self-consistent solutions with different values of topological charge are found by taking into account the effects of fermion vacuum polarization. Within these solutions the renormalized total energy of the bag is studied as the function of its geometry and the topological charge of the solution. It is shown that for non-zero topological charge there exists a set of configurations being the local minima of the total energy of the bag and containing all the three phases, while in the nontopological case the minimum of the bag’s energy corresponds to vanishing size of the phase of asymptotic freedom.
2 Lagrangian and equations of motion

The division of space into separate phases is performed by means of the system of subsidiary fields \( \theta(x) \). To describe the underlying machinery, let us consider the Lagrangian of the form

\[
\mathcal{L}_0 = \frac{1}{2} (\partial_{\mu} \phi)^2 - \theta V(\phi) + \frac{1}{2} (\partial_{\mu} \theta)^2 - g_0^2 W(\theta),
\]

where the coupling constant \( g_0 \) of self-interaction of the field \( \theta \) is assumed to be large enough to neglect the matter fields \( \phi \) in the dynamics of \( \theta \) to the leading order, and thereafter to use \( \theta \) as background fields for the dynamics of \( \phi \)'s [10,20]. One can obviously insert in (2.1) as many fields \( \theta(x) \) as needed with the appropriate self-interaction which will determine (almost) rectangular division of space into regions, corresponding to different phases, while the Lorentz-covariance will be broken only spontaneously, on the level of solutions of equations of motion. Therefore in order to restore the covariance one can use freely the framework of covariant group variables [21]. Assuming further that subsidiary fields \( \theta(x) \) have already formed the required bag configuration, let us start with the following Lagrangian:

\[
\mathcal{L} = \bar{\psi} i \not{\partial} \psi + \frac{1}{2} (\partial_{\mu} \phi)^2 - \frac{1}{2} m_0^2 \phi^2 \theta_I - \frac{M}{2} \left[ \bar{\psi}, e^{ig\gamma_5 \phi} \psi \right]_{\theta_I} - \left( \frac{M_0}{2} \left[ \bar{\psi}, e^{ig\gamma_5 \phi} \psi \right] + V(\phi) \right) \theta_{III},
\]

with \( \theta_I = \theta(|x| < x_1) \), \( \theta_{II} = \theta(x_1 \leq |x| \leq x_2) \), \( \theta_{III} = \theta(|x| > x_2) \) being the step functions, selecting the corresponding regions of the bag. The commutator over fermion fields in terms, corresponding to the chiral fermion-boson coupling, ensures the charge conjugation symmetry of the model. Note that in this model the vacuum pressure term appears to be redundant, since due to the existence of the intermediate phase the Dirac's sea polarization behaves very specifically and itself yields the required "inward pressure". Moreover, there is no special need in "valence" quarks, since it is the boson condensate in the form of the topological soliton which accounts now for the topological quantum numbers of the bag.

So in initial stage we have the theory of two fields, the fermion \( \psi \) and the boson \( \varphi \). In the region \( I \) the bosons have the mass \( m_0 \), in the region \( II \) the field \( \varphi \) is massless and interacts with fermions in a chirally invariant way, what results in emergence of the effective fermion mass \( M \). In the region \( III \) the effective fermion mass becomes equal to \( M_0 \) and the self-interaction of \( \varphi \) switches on, providing the appearance of a soliton solution for the boson field in this region. To form the bag we assume the mass \( M_0 \) to be very large, what leads to the dynamical suppression of the fermion field in the exterior region. Proceeding further this way, we take the mass \( m_0 \) infinitely large, then for convergence of the bag's energy the boson field should vanish in the region \( I \). We'll assume also that the self-interaction potential \( V(\varphi) \) is a an even function. Then the solution of equations of motion for the boson field could be either odd (the topological charge is by convention equal to one), or even (the topological charge is zero) function.

Let us consider now the behavior of fields in more detail. According to the general approach accepted in hybrid models, we consider the boson field in the mean-field approximation, i.e. it is assumed to be a c-number field. Neglecting temporarily the explicitly Lorentz-covariant description, we will consider the center of mass system of the bag, where \( \varphi(x) \) should be a stationary classical field being a background for evolution of fermions. In the region \( I \) \( \varphi(x) = 0 \), while in the region \( III \) it decouples from fermions due to the infinite effective mass of the latters and is formed uniquely by the self-interaction \( V(\varphi) \).

The equations of motion read:

in the region \( I \)

\[
i \not{\partial} \psi = 0 ,
\]

\[
\varphi = 0 ,
\]

in the region \( II \)

\[
\left( i \not{\partial} - M e^{ig\gamma_5 \varphi} \right) \psi = 0 ,
\]

\[
\varphi'' = ig \frac{M}{2} \left( \left[ \bar{\psi}, \gamma_5 e^{ig\gamma_5 \varphi} \psi \right] \right) ,
\]

and in the region \( III \)

\[
\left( i \not{\partial} - M_0 e^{ig\gamma_5 \varphi} \right) \psi = 0 ,
\]

\[
-\varphi'' + V'(\varphi) = 0 ,
\]
where \( \langle \cdot \rangle \) in Eq (2.4b) stands for the expectation value with respect to the fermionic state of the bag. To simplify calculations, we put further \( g = 1 \), because the dependence on it can be easily restored by means of the substitution \( \varphi \rightarrow \varphi/g \). Then the spectral problem for fermionic wavefunctions \( \psi_\omega \) with definite energy \( \omega \) reads

\[
\omega \psi_\omega = -i\alpha \psi'_\omega + \beta e^{i\gamma_5 \varphi} [M \theta_{II} + M_0 \theta_{III}] \psi_\omega .
\]  

(2.6)

Upon taking \( M_0 \rightarrow \infty \), we get that \( \psi_\omega \rightarrow 0 \) in the region \( III \) in such a way, that the term \( M_0 \psi_\omega \) in eq.(2.6) vanishes, and the following boundary conditions at the points \( \pm x_2 \) appear

\[
\pm i\gamma^1 \psi_\omega (\pm x_2) + e^{i\gamma_5 \varphi(\pm x_2)} \psi_\omega (\pm x_2) = 0 .
\]

(2.7)

Supplied by the condition of continuity for \( \psi(x) \) on boundaries between the regions \( I \) and \( II \), the eqs.(2.7) provide the correct formulation of the spectral problem for fermions, confined in the bag. Note, that the boundary conditions (2.7) are actually the standard chiral boundary conditions for the hybrid models \([1-8]\). However, they arise now as a direct consequence of an infinite mass of fermions in the region \( III \), rather than from a local surface action, what is not completely correct \([10,22]\). In the region \( I \) the Eq.(2.6) is the equation for free massless fermions

\[
\omega \psi_I = -i\alpha \psi'_I ,
\]

(2.8)

while in the intermediate region \( II \) one has

\[
\omega \psi_{II} = -i\alpha \psi'_{II} + \beta M e^{i\gamma_5 \varphi} \psi_{II} .
\]

(2.9)

Conditions of wavefunction’s continuity on the boundary between \( I \) and \( II \) read

\[
\psi_I (\pm x_1) = \psi_{II} (\pm x_1) ,
\]

(2.10)

while at points \( |x| = x_2 \) the wave functions satisfy the boundary conditions (2.7). Meanwhile the field \( \varphi \) in the Eq.(2.9) is not arbitrary but has to be determined self-consistently from Eq.(2.4) with corresponding continuity conditions at points \( |x| = x_{1,2} \).

3 Solutions with non-zero topological charge

The essential feature of this bag configuration is the fact that the coupled equations (2.4) in the closed intermediate region \( II \) of finite size \( d = x_2 - x_1 \) possess simple and physically meaningful solution which would be unacceptable if these equations were considered in the infinite space. In order to obtain this solution in the most consistent way, we perform firstly in the region \( II \) the chiral Skyrme rotation

\[
\psi_\omega = \exp(-i\gamma_5 \varphi/2) \chi_\omega ,
\]

(3.1)

whatafter the Eq. (2.9) and the boundary conditions (2.7) transform correspondingly into

\[
(\omega - \frac{1}{2} \varphi') \chi_\omega = -i\alpha \chi'_\omega + \beta M \chi_\omega ,
\]

(3.2)

\[
\pm i\gamma^1 \chi_\omega (\pm x_2) + \chi_\omega (\pm x_2) = 0 .
\]

(3.3)

It follows from Eq.(3.2), that if we assume the linear behavior for the field \( \varphi(x) \) in the region \( II \), namely

\[
\varphi' = \text{const} = 2\lambda ,
\]

(3.4)

then it becomes the equation for free massive fermions

\[
\nu \chi = -i\alpha \chi' + \beta M \chi ,
\]

(3.5)

with eigenvalues \( \nu = \omega - \lambda \). So the fermions being massless in the region \( I \), acquire the mass \( M \) in the region \( II \) due to the coupling to the field \( \varphi \), whence the intermediate phase emerges describing massive quasifree ”constituent quarks”.

At this moment, the most important feature of Eq.(3.5) is that it reveals the sign symmetry \( \nu \rightarrow -\nu \), which corresponds to the unitary transformation of fermionic wavefunction

\[
\chi \rightarrow \tilde{\chi} = i\gamma_1 \chi ,
\]

(3.6)
while the chiral currents
\[ j_5 = i\bar{\psi}\gamma_5 e^{i\gamma_5 \phi} \psi = i\chi^+ \gamma_1 \chi \] (3.7)
coincide for these sign-symmetric states:
\[ j_5 = i\chi^+ \gamma_1 \chi = i\bar{\chi}^+ \gamma_1 \chi = \tilde{j}_5. \] (3.8)

However, the sign symmetry of Eq.(3.5) itself cannot provide the corresponding one for the fermion spectrum, since it takes place in the region \( II \) only, while the latter has to be determined from the Dirac equation on the unification of the regions \( I + II \). Meanwhile in the region \( I \) one has the Eq.(2.8), which possesses another symmetry, namely \( \omega \leftrightarrow -\omega \). That means that the sign symmetry \( \nu \leftrightarrow -\nu \) of the fermionic spectrum could hold only for discrete values of the derivative \( \varphi' \) in the region \( II \). These values should be determined from the transcendental algebraic equation for fermionic energy levels, which is obtained from the straightforward solution of Eqs. (2.8) and (2.9) with account of boundary conditions (2.7) and the constraint (2.10), and reads
\[ \exp(4i\nu \chi_1) = \frac{1 - e^{-2ikd M - i(\nu - k)}}{1 - e^{-2ikd M + i(\nu - k)}} \frac{1 - e^{-2ikd M + i(\nu - k)}}{1 - e^{-2ikd M - i(\nu - k)}}, \] (3.9)
where \( \nu^2 = k^2 + M^2 \). Analysing the Eq. (3.9), one easily finds, that the fermionic spectrum reveals the symmetry \( \nu \leftrightarrow -\nu \), if
\[ 4\lambda x_1 = \pi s, \] (3.10)
where \( s \) is integer, since for such values of \( \varphi'(x) \) in the region \( II \) the l.h.s. of Eq. (3.9) reduces to \((-1)^s \exp(4i\nu x_1)\).

Assuming the condition (3.10) to hold, the following consequence of arguments becomes reasonable. In the r.h.s. of Eq.(2.4b), which determines \( \varphi''(x) \) in the region \( II \), we have the v.e.v. of the \( C \)-odd chiral current
\[ J_5 = \frac{1}{2} \left[ \bar{\psi}, i\gamma_5 e^{i\gamma_5 \phi} \psi \right]_\pm = \frac{1}{2} \left[ \chi^+, i\gamma_1 \chi \right]_\pm, \] (3.11)
with \( \chi \) being now the secondary quantized Dirac field in the chiral representation (3.1)
\[ \chi(x,t) = \sum_n b_n \chi_n(x) e^{-i\omega_n t}, \] (3.12)
where \( \chi_n(x) \) are the normalized solutions of the corresponding Dirac equation, \( b_n, b_n^+ \) are fermionic creation-annihilation operators which obey the canonical anticommutation relations
\[ \{b_n, b_{n'}^+\}_+ = \delta_{nn'}, \quad \{b_n, b_{n'}\}_+ = 0. \] (3.13)
The average over the given bag’s state includes, by definition, the average over the filled sea of negative energy states \( \omega_n < 0 \) plus possible filled valence fermion states with \( \omega_n > 0 \) which are dropped for the moment because their status is discussed specially below. Finally,
\[ \langle J_5 \rangle = \langle J_5 \rangle_{\text{sea}} = \frac{1}{2} \left( \sum_{\omega_n < 0} - \sum_{\omega_n > 0} \right) \chi_n^+ i\gamma_1 \chi_n. \] (3.14)
Let us emphasize here, that in Eq. (3.14) the division of fermions into sea and valence ones is made in correspondence with the sign of their eigen-frequencies \( \omega_n \), which differ from sign-symmetric \( \nu_n \) by the shift in \( \lambda \)
\[ \omega_n = \nu_n + \lambda, \] (3.15)
and so do not possess the sign symmetry \( \omega \leftrightarrow -\omega \). However, if we suppose additionally, that \( \nu_n \) and \( \lambda \) are such that for all \( n \) the signs of \( \nu_n \) and \( \omega_n \) coincide, i.e. after shifting by \( \lambda \) none of \( \nu_n \)‘s changes its sign, then the condition \( \omega_n \leq 0 \) in Eq. (3.14) will be equivalent to the condition \( \nu_n \leq 0 \). Hence
\[ \langle J_5 \rangle_{\text{sea}} = \frac{1}{2} \left( \sum_{\nu_n < 0} - \sum_{\nu_n > 0} \right) \chi_n^+ i\gamma_1 \chi_n = 0 \] (3.16)
by virtue of relation (3.8). In turn, it means that (2.4b) in the region II reduces to \( \varphi'' = 0 \), what is in complete agreement with our initial assumption that \( \varphi'(x) = \text{const} \) in the region II. In other words, we obtain the solution of the coupled Eqs. (2.4) in the region II in the form

\[
\varphi(x) = \begin{cases} 
2\lambda(x - x_1), & x_1 \leq x \leq x_2, \\
2\lambda(x + x_1), & -x_2 \leq x \leq -x_1,
\end{cases}
\]

(3.17)

where \( \lambda \) takes discrete values from (3.10) and the fermion energy spectrum is determined from the relation (3.15), while \( \nu_n \) is defined from Eq. (3.9) after replacing the l.h.s. to \((-1)^s \exp(4i\nu x_1)\).

There are the following keypoints that make this solution meaningful. The first is the finiteness of the intermediate region size \( d \), because for an infinite region II the solution (3.17) should be unacceptable. In our case, however, the size of the intermediate region is always finite by construction, while the boson field \( \varphi(x) \) acquires the solitonic behavior in the region III due to self-interaction \( V(\varphi) \). Here the following circumstance manifests again: in (1+1)D the chiral coupling \( \bar{\psi}e^{i\gamma_5\varphi}\psi \) itself cannot cause the solitonic behavior of the scalar field by virtue of the effects of fermion-vacuum polarization only, i.e. without additional self-interaction of bosons [23]. The second point is the discreteness and the \( \nu \leftrightarrow -\nu \) symmetry of the fermionic spectrum, what leads in turn to a reasonable method of calculation for the average of the chiral current \( J_5 \) over filled Dirac’s sea (3.16), as well as for other C-odd observables like the total fermion number. After all, in the case we consider the boson field is continuous everywhere and so is topologically equivalent to that odd soliton, which would take place in absence of fermions due to the self-interaction \( V(\varphi) \) only. That’s why the topological number of the boson field doesn’t depend on the existence and sizes of the spatial regions containing fermions (the regions I and II). On the other hand, the baryon number of the hybrid bag is, by definition, the sum of the topological charge of the boson soliton and the fermion number of the bag interior. In our case the latter is zero, hence the baryon number of the bag is determined by the topological charge of the boson field only and doesn’t depend on the sizes of the regions I and II containing fermions, what is in agreement with the general ideology of hybrid models. More detailed discussion of this solution of Eq.(2.4) and the arguments in favor of its uniqueness are given in Ref.[10].

It is also worth-while noticing, that although the (topological) quantum numbers of such a bag are determined by its solitonic component, it doesn’t mean that the filled fermion levels with positive energy shouldn’t exist at all. This could take place for small enough values of the parameter \( \lambda \) only. If \( \lambda \) increases, the negative levels \( \omega_n = -|\nu_n| + \lambda \) will inevitably move into the positive part of the spectrum. The change of sign of each such level will decrease \( \langle Q \rangle_{\text{sea}} \) by one unit of charge, but if we fill the emerging positive level with the valence fermion, then the sum \( Q_{\text{val}} + Q_{\text{sea}} \) remains unchanged. Analogously, the total axial current will be equal to \( J_{\text{val}} + J_{\text{sea}} \) and won’t change either, what ensures the vanishing r.h.s. of Eq. (2.4b) and so preserves the status of linear function (3.17) as the self-consistent solution of the field equations. Therefore, the existence or absence of valence fermions in such construction of the ground state of the bag depends actually on the relation between \( \lambda \) and \( |\nu|_{\text{min}} \) and so appears to be a dynamical quantity like the other parameters (the size and mass) which are determined from the total energy minimization procedure.

Another essential feature of this bag configuration is that (3.17) provides the self-consistent solution of Eqs.(2.4) for even values \( s = 2r \) in (3.10) only. The reason is that for odd values \( s = 2r + 1 \) the fermionic spectrum obtained from the solution of Eqs.(2.7-10) under condition \( \varphi' = 2\lambda \) will always contain the nondegenerate energy level \( \chi_0(x) \) with zero frequency \( \nu_0 = 0 \), while for even values \( s = 2r \) all the \( \nu_n \neq 0 \). According to the general theory [24], such a zero mode yields fractionalization, what means, that its contribution to all C-odd observables is given by the operator \( \frac{1}{2} \left( b_0^\dagger b_0 - b_0b_0^\dagger \right) \) with the eigenvalues \( \pm 1/2 \) and the numeric coefficient determined by \( \chi_0(x) \). Now let us note, that in Eq.(2.4b) the chiral current should be averaged over its eigenvector in order to provide the vanishing dispersion of the r.h.s., otherwise the system of Eqs.(2.4) would be ill-defined. So the operator part of the zero mode contribution to the r.h.s. of (2.4b) turns unavoidably into the factor \( \pm 1/2 \), meanwhile the wavefunction \( \chi_0(x) \) appears to be such that the corresponding chiral current in the region II does not vanish (it is proportional to \( \exp(-2M|x|) \)). Hence the r.h.s. of Eq. (2.4b) doesn’t vanish for odd values \( s = 2r + 1 \), and the function (3.17) is no longer the self-consistent solution of Eqs. (2.4).

It’s not difficult, however, to find the way of constructing analogous bags, where the odd values \( s = 2r + 1 \) are allowed. This method is based on a specific for such two-dimensional bag models possibility to choose in the model Lagrangian independently the signs of the chiral fermionic masses \( M, M_0 \), both to the right
and to the left of the region of asymptotic freedom. More specifically, the considered configurations with even $s$ appear from the Lagrangian (2.2), which is symmetric in signs of fermion masses to the left and to the right. Now let us consider the situation, when in the Lagrangian (2.2) all the chiral masses to the left of the central region change their signs. As a result, we obtain the following Lagrangian

$$
\mathcal{L} = \bar{\psi} i\partial \psi + \frac{1}{2} (\partial_{\mu} \varphi)^2 - \frac{1}{2} m_{0}^2 \varphi^2 \theta I - \frac{M}{2} \left[ \bar{\psi}, e^{i\gamma_{5} \varphi} \psi \right]_+ \left( \theta_{II}^+ - \theta_{II}^- \right) - \frac{M_{0}}{2} \left[ \bar{\psi}, e^{i\gamma_{5} \varphi} \psi \right]_- \left( \theta_{III}^+ - \theta_{III}^- \right) - V(\varphi) \left( \theta_{III}^+ + \theta_{III}^- \right),
$$

(3.18)

where $\theta_{I} = \theta(|x| < x_{1})$, $\theta_{II}^{(\pm)} = \theta(x_{1} \leq \pm x \leq x_{2})$, $\theta_{III}^{(\pm)} = \theta(\pm x > x_{2})$, which is antisymmetric in signs of the fermionic masses to the left/right. Qualitively their behaviour for these two cases is shown on the Fig. 1.

It’s easy to see, that the change of the sign of $M_{0}$ to the left leads merely to the change of the sign in the corresponding boundary condition. Namely, in this case we will have at the point $x = -x_{2}$

$$
i\gamma_{1} \psi_{\omega}(-x_{2}) + e^{i\gamma_{5} \varphi(-x_{2})} \psi_{\omega}(-x_{2}) = 0,
$$

(3.19)

instead of (2.7). In the intermediate region after the Skyrme rotation the Dirac equation (3.5) turns into

$$
\nu \chi = -i\alpha \chi' \pm \beta M \chi,
$$

(3.20)

where in the r.h.s. of (3.20) the sign $\pm$ corresponds to the sign of the chiral interaction to the right/left. In this case again the $\nu \rightarrow -\nu$ symmetry of the spectrum takes place for the transformations $\chi \rightarrow \tilde{\chi} = \sigma_{2} \chi$, which do not alter the chiral current $j_{5} = \tilde{j}_{5}$, and so the solution based on the linear Ansatz (3.4) and

---

**Fig.1.** The behaviour of the chiral fermionic masses for the symmetric (up) and antisymmetric (down) cases.
\[ \omega = \nu + \lambda \] remains valid, while the spectral equation for fermionic eigenfrequencies appears firstly in the form

\[
\exp(4i\omega x_1) = - \frac{1 - e^{-2ikd M^{-i(\nu+k)}}}{1 - e^{-2ikd M^{i(\nu-k)}}} 1 - e^{2ikd M^{i(\nu+k)}} \frac{1 - e^{-2ikd M^{-i(\nu-k)}}}{1 - e^{-2ikd M^{i(\nu-k)}}} ,
\]

(3.21)

and differs from the Eq. (3.9) by the sign in the r.h.s. The origin of this additional sign is the antisymmetry of the Lagrangian (3.18) in signs of the fermionic mass terms. The sign symmetry of the spectrum \( \nu \leftrightarrow -\nu \) is still ensured by the condition (3.10), but now the zero mode emerges for even values of \( s \), hence the allowed for the validity of the linear solution (3.17) values of \( s \) are the odd ones, for which \( \exp(4i\omega x_1) = - \exp(4i\nu x_1) \). As a result, both in even and odd cases the final spectral equation for fermions is the same one, namely

\[
\exp(4i\nu x_1) = - \frac{1 - e^{-2ikd M^{-i(\nu+k)}}}{1 - e^{-2ikd M^{i(\nu-k)}}} 1 - e^{2ikd M^{i(\nu+k)}} \frac{1 - e^{-2ikd M^{-i(\nu-k)}}}{1 - e^{-2ikd M^{i(\nu-k)}}} .
\]

(3.22)

It might seem, that the odd case is physically unacceptable, since the Lagrangian (3.18) is not invariant with respect to spatial reflection. However, it is caused by that circumstance, that the considered models contain the minimal set of terms just to describe a single isolated bag. It’s easy to see that the model, containing two odd bags, which are the mirror copies of each other, should be P-invariant. Therefore, both even and odd bags possess actually the same physical status, and together cover all possible values of \( s \) in Eq.(3.10) with the same spectral eq.(3.22) for the eigenfrequencies \( \nu_n \). In other words, there exists a specific discrete symmetry in such systems, since the most essential properties of the bag, e.g. the confinement of fermions, properties of the intermediate region, the spectrum of \( \nu_n \)’s, etc., are invariant with respect to this change of signs in the Lagrangian, which predicts merely the allowed values of \( s \) in Eq.(3.10).

Note also, that the Fig.1 does not present all the possible types of the bags which could be obtained combining the signs of the chiral masses in all possible ways. In particular, there exist such combinations for which the fermions should possess an energy spectrum, which is different from that of Eq.(3.9), and the corresponding bags — reveal a set of different physical properties. The study of this question will be presented separately [25].

4 The total energy of the bag for the non-zero topological charge

Without loss of generality we’ll consider the bag configurations with even values \( s = 2r \), which take place in the P-invariant model (2.2). The resulting configuration of the boson field reveals the structure shown on Fig.2.

![Fig.2. The configuration of the boson field for an isolated bag with non-zero topological charge.](image)
In the region $I\; \varphi \equiv 0$, in the region $II$ the boson field is the linear function (3.17), which after restoring the $g$-dependence is sewn together with the solution of Eq.(2.5b), which is responsible for the dynamics of bosons in the bag’s exterior. To avoid the dependence on the structure of self-interaction $V(\varphi)$, we’ll suppose, that in the region $III$ the asymptotic expansion of the soliton solution of Eq.(2.5b) for large $|x|$ can be used, namely
\[ \varphi_{sol}(x) = \frac{\pi}{g} \left( 1 - Ae^{-mx} \right), \quad x > x_2, \quad (4.1) \]
with $m$ being the (nonzero) meson mass in the bag’s exterior, while for $x < -x_2 \varphi_{sol}(x)$ is determined by oddness. The chiral symmetry in the exterior region of the bag is obviously lost. It should be noticed, however, that it occurs due to specific features of (1+1) D field theories, which make the presence of the meson mass the necessary condition for the required soliton profile to be formed.

The condition $d \geq 0$ yields then an additional restriction for the size of the confinement region, i.e. for the external size $x_2$ of the bag
\[ mx_2 \geq r, \]
which shows, that $r$ could be naturally interpreted as the index enumerating the excited states of the bag, whose sizes increase with $r$.

Proceeding further, we find the relation between the parameter $\lambda$ and the bag’s size:
\[ 2\lambda = \pi m \frac{r + 1}{mx_2 + 1}, \quad (4.3) \]
whence the total energy of the bosonic soliton can be represented as
\[ E_\varphi = m \frac{\pi^2 r + 1}{g^2 mx_2 + 1}. \quad (4.4) \]
The total energy of the bag is the sum of $E_\varphi$ and of the fermionic contribution $E_\psi$
\[ E_{bag} = E_\varphi + E_\psi. \quad (4.5) \]
As it follows from (4.4), the boson field energy decreases smoothly for increasing $x_2$ without producing any vacuum pressure, despite of the fact that in the region $II$ the gradient of $\varphi$ gives rise to the constant positive contribution to the energy density $\frac{d}{dx} \varphi^2 = 2\lambda^2$, what could be identified with the vacuum pressure $B$ in the standard HCM. Actually, it is an artifact of one spatial dimension in our problem: when the bag’s size increases, the gradient of $\varphi$ in the region $II$ decreases equally in any number of spatial dimensions, while the volume of the region $II$ in one space dimension increases only linearly and so cannot compensate the decrease of $\lambda$, what would take place in 2- and 3- (space) D. Thus, in (1+1)D all the non-trivial dependence of the total bag energy $E_{bag}$ on the model parameters could originate from the fermionic contribution $E_\psi$ only, which is the sum of the filled Dirac’s sea of negative energy states and positive energy valence fermions
\[ E_\psi = E_{val} + E_{sea}. \quad (4.6) \]
For the ground state of the bag described above, the sum (4.6) can be reduced to a single universal expression by taking into account, that the charge conjugation symmetry dictates the following definition of the Dirac’s sea energy [23, 26]
\[ E_{sea} = \frac{1}{2} \sum_{\omega_n < 0} \omega_n - \frac{1}{2} \sum_{\omega_n > 0} \omega_n. \quad (4.7) \]
If the transformation from $\omega_n$ to $\nu_n$ is sign preserving for all $n$ and so there are no valence fermions in the ground state of the bag (to provide the vanishing v.e.v for the charge and chiral current), one finds from (4.7):
\[ E_\psi = E_{sea} = \frac{1}{2} \sum_{\nu_n > 0} (-\nu_n + \lambda) - \frac{1}{2} \sum_{\nu_n > 0} (\nu_n + \lambda) = - \sum_{\nu_n > 0} \nu_n. \quad (4.8) \]
Note, that in (4.8) the inequality is strict because there are no levels with \( \nu = 0 \).

If the parameter \( \lambda \) appears to be large enough, so that the initially negative level \( \omega_n = -|\nu_n| + \lambda \) changes its sign, it turns into the filled valence state. The latter is again necessary to provide the vanishing v.e.v for the charge and axial current. In this case it is convenient to calculate \( E'_\psi \) in two steps. First, we consider the contribution from all states with \( |\nu_n| > \lambda \) to \( E_{\text{sea}} \), which in analogy to (4.8) reads

\[
E'_{\text{sea}} = - \sum_{\nu_n > \lambda} \nu_n .
\]

To this expression the energy of emerging valence fermions \( E_{\text{val}} = -|\nu_n| + \lambda \) and the contribution of the positive levels with \( \omega_n = \pm |\nu_n| + \lambda \) to the Dirac’s sea energy should be added, what yields

\[
E_\psi = -|\nu_n| + \lambda - \frac{1}{2} \left( (-|\nu_n| + \lambda) + (|\nu_n| + \lambda) \right) + E'_{\text{sea}} = - \sum_{\nu_n > 0} \nu_n ,
\]

i.e. the same expression (4.8) as we have got for the energy of fermions without filled valence states.

For what follows it is convenient to introduce a set of new parameters, in terms of which the total energy of the bag will be expressed in the most appropriate form. First, we introduce the dimensionless quantities

\[
\alpha = 2M x_1 , \quad \beta = 2Md , \quad \rho = 2M x_2 ,
\]

and consider in more detail the Eq.(3.9), which determines the energy levels \( \nu_n \). This equation has two branches of roots. The first one corresponds to real \( k \) and in terms of parameters \( \alpha \) and \( \beta \) can be transformed into the following form

\[
\tan \left( \alpha \sqrt{1 + x^2} \right) = \frac{x \cos \beta x + \sin \beta x}{\sqrt{x^2 + 1} - \cos \beta x + x \sin \beta x} .
\]

where the unknown quantity is the dimensionless \( x \) defined through \( k = Mx \), so that \( \nu = M \sqrt{1 + x^2} \). These real roots \( x_n \) belong to the half axis \( 0 \leq x_n < \infty \), since the fermionic wavefunctions are in fact the standing waves in a finite spatial box with degeneracy in the sign of \( k \), while the corresponding frequencies \( \nu_n \) lie in the interval \( M \leq \nu_n < \infty \). The second branch corresponds to imaginary \( k = iMx \), \( \nu = M \sqrt{1 - x^2} \), \( 0 \leq x \leq 1 \) and can be derived from (4.12) by means of the analytical continuation

\[
\tan \left( \alpha \sqrt{1 - x^2} \right) = \frac{x \cosh \beta x + \sinh \beta x}{\sqrt{1 - x^2} \cosh \beta x + x \sinh \beta x - 1} .
\]

For this branch \( 0 < \nu_n \leq M \).

Therefore, \( \nu_n \) and so \( E_\psi \) appear to be functions of two dimensionless parameters \( \alpha \) and \( \beta \), which are not independent, however, but give in the sum the dimensionless total size of the confinement domain \( \rho \):

\[
\alpha + \beta = \rho .
\]

Proceeding further, it is convenient to extract the mass of the ”constituent quark” \( M \) from the sea energy and fermionic frequencies as a dimensional factor:

\[
\varepsilon_n = \nu_n / M = \sqrt{1 + x_n^2} ,
\]

hence \( E_\psi = -M \sum_n \varepsilon_n \). Upon introducing the dimensionless ratio of the two mass parameters of the model

\[
\mu = m / 2M ,
\]

the dimensionless energy of fermions \( E_\psi = E_\psi / M \) and analogously the dimensionless total energy \( E_{\text{bag}} = E_{\text{bag}} / M \), for the latter one finds:

\[
E_{\text{bag}} = E_\psi (\alpha, \beta) + 2\mu \frac{\pi^2}{g^2} \frac{r + 1}{\mu + 1} ,
\]

where the dimensionless parameters \( \alpha, \beta \) are defined directly from \( \mu \) and \( \rho \)

\[
\alpha = \frac{r}{r + 1} \left( \rho + 1 / \mu \right) , \quad \beta = \frac{\rho - r / \mu}{r + 1} .
\]
So the total energy of the bag depends ultimately on two dimensionless parameters — $\mu$ and $\rho$, where the parameter $\mu$ is fixed by the ratio of the masses $m$ and $M$, while the optimal value of the bag’s size should be found from the condition of minimum of the total energy $E_{bag}(\rho)$ for given $\mu$.

To study the behaviour of $E_{bag}(\rho)$, first of all we have to renormalize the fermion sea energy $E_\psi$, which obviously diverges in the upper limit. Let us start with the asymptotics of roots of Eq.(4.12) in the UV domain, when $x_n \gg 1$. Representing the Eq.(4.12) as

$$\sin \alpha \sqrt{1 + x^2} = \frac{1}{2} (\sqrt{1 + x^2} + x) \sin \left( \alpha \sqrt{1 + x^2} + \beta x + \delta \right) + \frac{1}{2} (\sqrt{1 + x^2} - x) \sin \left( \alpha \sqrt{1 + x^2} - \beta x - \delta \right),$$

where $\delta = \arctan x$, one finds that

$$\varepsilon_n(\alpha, \beta) = \frac{\pi \gamma/\rho}{\rho} + \frac{(-1)^{n/2} \sin \left( \pi/2 + \pi n \right) \alpha/\rho + 1 + \beta/\rho}{\pi/2 + \pi n} + O(1/n^2).$$ (4.20)

In the expression (4.20) the first term yields the quadratic and linear divergences in $\sum_n \varepsilon_n$ and the second one produces the logarithmic one, while the term with the sine doesn’t yield any divergence at all. To compensate the contribution of the first term, the energy of the sea of free fermions contained in the same “volume” $\rho$ should be subtracted, while the logarithmic divergence proportional to $\beta/\rho$, is compensated by the relevant one-loop counterterm of the boson self-energy $[10]$. The remaining logarithmic divergence corresponding to the term $1/(\pi/2 + \pi n)$ doesn’t depend on the bag parameters, and originates from the fermion confinement inside the bag, rather than by some local interaction. Actually, it is the (infinite) energy of interaction between fermions and the confining potential (bag boundaries). The appearance of such diverging surface energy in $E_\psi$ is a specific feature of fermion vacuum polarization in all the bag models $[3,8,27-32]$.

In the considered 3-phase bag model this effect acquires some additional features. First, it takes place for nonzero size $d \neq 0$ of the intermediate phase only, while the emerging surface energy is negative and diverges as $|\sum_n 1/(\pi/2 + \pi n)|$. More concretely, if $\alpha \to \rho$ then $(-1)^{n/2} \sin \left( \pi/2 + \pi n \right) \alpha/\rho \to -1$, hence there remains only the logarithmic term $\beta/(\pi + 2\pi n)$ in the asymptotics (4.20). Therefore in this limit $E_\psi$ becomes finite just after subtraction of the energy of perturbative vacuum and adding the one-loop counterterm. On the other hand, the limit $\alpha \to \rho$ is equivalent to $\beta/\alpha \to 0$, and so the infinite interaction energy between fermions and bag boundaries takes place for $d \neq 0$ and the finite size of the central region (of the phase of asymptotic freedom) of the bag only.

So the considered 3-phase bag model doesn’t actually reveal the ability to the smooth transition into a 2-phase configuration for $d \to 0$, although such an opportunity exists formally on the level of the initial Lagrangian (2.2). In fact, in the case of the two-phase bag ($d \equiv 0$) the exact values of $\varepsilon_n$ are $\varepsilon_n = (\pi/2 + \pi n)/\rho$, and so $E_\psi$ becomes finite after a single subtraction of the energy of the perturbative vacuum. Therefore the transition between 2- and 3-phase bag configurations requires an infinite amount of energy, what is a specific feature of such many-phase systems. Note also, that the ultimate role of the intermediate phase is the dynamical generation of the fermion mass, whereas in the case of the two-phase bag ($d \equiv 0$) the massless fermions are reflected directly from the bag boundaries. From this point of view, there is an intimate connection between the infinite surface energy of the bag and the circumstance, that for $d \neq 0$ the boundaries of the bag reflect massive fermions.

Within the considered 3-phase bag models we have an opportunity to demonstrate this effect in a even more apparent way. For these purposes let us consider a P-invariant model describing the 1+1-dimensional analog of a "dibaryon", i.e. the configuration with the topological charge 2. Such an object consists of two identical topological bags of the type described above, which are placed so close to each other that their neighbouring intermediate regions overlap. Upon dropping the $g$-dependence for simplicity, the corresponding Lagrangian reads

$$\mathcal{L} = \bar{\psi} i \partial \psi + \frac{1}{2} (\partial_\mu \varphi)^2 - \frac{M}{2} \left[ \bar{\psi}, e^{ig\gamma_5 \varphi} \psi \right]_-(\theta_I + \theta_{III}) -$$

$$- \frac{1}{2} m_0^2 \left( (\varphi - \pi)^2 \theta_I^{(+)} + (\varphi + \pi)^2 \theta_I^{(-)} \right) - \left( \frac{M_0}{2} \left[ \bar{\psi}, e^{ig\gamma_5 \varphi} \psi \right]_+ + V(\varphi) \right) \theta_{IV},$$ (4.21)

where $\theta_I = \theta(\ |x| \leq x_0 \), \theta_I^{(\pm)} = \theta(x_0 < \pm x < x_1) \), \theta_{III} = \theta(x_1 \leq |x| \leq x_2) \), \theta_{IV} = \theta(|x| > x_2)$. The behaviour of the chiral fermionic masses for this case is shown on the Fig.3.
Fig. 3. The behaviour of the chiral fermionic masses for the dibaryon configuration.

Again, we seek for the self-consistent solution of the model (4.21) corresponding to such "dibaryon" configuration, assuming the linear behavior (3.4) for the boson field in the intermediate regions and taking account of the sign-symmetry $\nu \leftrightarrow -\nu$ as well as of the conservation of the chiral current $j_5 = \tilde{j}_5$ for the transformations $\chi \to \tilde{\chi} = \sigma_2\chi$. Omitting some straightforward, but lengthy calculations, let us present the main results.

The profile of the boson field corresponding to the dibaryon configuration is shown on the Fig.4.

For the intermediate regions of this dibaryon one obtains $\varphi' = const = 2\lambda$, where $\lambda$ satisfies the condition

$$2\lambda a = \pi s, \quad a = x_1 - x_0$$

(4.22)

The latter condition is quite analogous to Eq.(3.10) for a single isolated bag, because the parameter $2x_1$ in (3.10), as well as $a$ in (4.22), has the sense of the size of the region of asymptotic freedom of a single bag. Note also, that on the contrary to the single bag models considered above, in the case of the dibaryon there are no zero modes in the fermionic spectrum for any values of $s$. So there are no additional restrictions imposed on the integer $s$ in Eq.(4.22).

It should be clear, that the 1+1-dimensional model (4.21) cannot be considered as a realistic model of the dibaryon to any extent. However, being simple and non-trivial simultaneously, it turns out to be a very fruitful illustration for the study of the origin of additional logarithmically divergent terms $1/(\pi/2 + \pi n)$ in the UV-asymptotics of the fermionic spectrum in such 3-phase bag models. The latter is again obtained from the corresponding transcendent equation for fermion levels, written in the trigonometric
form analogous to that of Eq.(4.12):

$$\sin \left(2\alpha \sqrt{1 + x^2} \right) \left(x \sqrt{1 + x^2} \cos \left(\left(\beta + \gamma \right)x + \delta \right) - x \cos \gamma x \right) +$$

$$+ \cos \left(2\alpha \sqrt{1 + x^2} \right) \left(\sin \left(\left(\beta + \gamma \right)x + \delta \right) - \sqrt{1 + x^2} \sin \gamma x + \sin \gamma x \sin \left(\beta x + \gamma \right) \right) +$$

$$+ \left(\sqrt{1 + x^2} - \cos \left(\beta x + \gamma \right) \right) \sin \gamma x = 0,$$

(4.23)

where $\alpha = Ma$ , $\beta = 2Md$ , $d = x_2 - x_1$ , $\gamma = 2mx_0$ , $\delta = \arctan x$. The parameter $d = x_2 - x_1$ is the size of the external intermediate regions for each of the single bags forming the dibaryon, while $2x_0$ is the size of their common internal intermediate region, i.e. the region of their mutual interaction. In this case the UV-asymptotics of $\varepsilon_n$’s has the following form

$$\varepsilon_n(\alpha, \beta, \gamma) = \frac{\pi/2 + \pi n}{\rho} +$$

$$+ \frac{(-1)^{n+1} \left(\sin \left[\left(\pi/2 + \pi n \right)\left(2\alpha + \gamma \right)/\rho \right] - \sin \left[\left(\pi/2 + \pi n \right)\gamma/\rho \right] \right) + 1 + (\beta + \gamma)/2}{\pi/2 + \pi n} + O(1/n^2),$$

(4.24)

where

$$\rho = 2\alpha + \beta + \gamma = M(2a + d + 2x_0) = 2Mx_2$$

(4.25)

is the total dimensionless bag’s size. As in the case of a single isolated bag, the main divergent term in the asymptotics (4.24) corresponds to the sea energy of free fermions in the "volume" $\rho$, while the logarithmic term, proportional to $(\beta + \gamma)/2$, is exactly compensated by one-loop self-energy counterterm. The change of the coefficient in this term compared to (4.20) is caused by the fact, that in the considered case the interaction between fermions and boson field takes place in the region of the size $2d+2x_0$. Besides this, there remains again a logarithmically divergent term $1/(\pi/2 + \pi n)$, that corresponds to the (infinite) energy of the interaction between fermions and the confining potential (bag boundaries), and it follows from Eq.(4.24), that on the level of divergent terms the surface energy of the dibaryon coincides exactly with that of a single isolated bag. So we are led unambiguously to the conclusion, that it is indeed the effect of confinement of fermions in a simply connected region, that yields the term $1/(\pi/2 + \pi n)$ in Eqs.(4.20) and (4.24), since in the dibaryon configuration the number of boundary points is just the same as in the case of one isolated bag. Note also, that the direct consequence of this statement is that in (1+1) D the dibaryon configuration can’t be obtained as a result of continuous fusion of two isolated bags, since when they are separated enough from each other, the sum of their surface energies is twice larger than that of the dibaryon. In other words, in (1+1)D the reconstruction of the bag’s surface in the fusion-fission processes requires an infinite amount of energy.

After all, it follows from (4.24), that for $\beta \to 0$, i.e. for vanishing external intermediate regions of the dibaryon one gets $(-1)^{n+1} \sin \left[\left(\pi/2 + \pi n \right)\left(2\alpha + \gamma \right)/\rho \right] \to -1$, what compensates the term $1/(\pi/2 + \pi n)$, and the infinite interaction energy between fermions and bag boundaries disappears too. So we are again led to the conclusion, made for a single bag by analysis of the asymptotics (4.20), that the infinite surface energy appears only then fermions pass through the intermediate phase just before reflection from the bag boundaries.

As a result, for a 3-phase bag with $d \neq 0$ the extraction of the finite part from $E_\psi$ consists actually of two separate procedures. The first one is the standard renormalization onto perturbative vacuum with account of the one-loop counterterm, caused by virtual fermion pairs [10]. The second one is the compensation of the surface energy by means of an appropriate subtraction, and both procedures suffer from an ambiguity in the choice of subtraction point. In the "classical" renormalization scheme, the uncertainty in the choice of subtraction point is cancelled by fixing the physical values for a corresponding number of parameters. For obvious reasons, we avoid doing that in our "toy" (1+1)D model, but instead consider the most straightforward approach to the compensation of divergences in the sum (4.8), which preserves the continuous dependence of the result of substraction on the model parameters. The essence of this approach is that we subtract from $\sum_n \varepsilon_n$ another sum with the same summation index $n$, whose common term coincides exactly with the divergent part of asymptotics (4.20). The result is the finite quantity

$$\tilde{E}_\psi = -\sum_n \left[\varepsilon_n - \left(\frac{\pi/2 + \pi n}{\rho} + \frac{1 + \beta/2}{\pi/2 + \pi n} \right) \right].$$

(4.26)
This method requires no counterterms because all the divergences are already cancelled by the subtracted sum. Of course, to some extent the physical meaning of such procedure is lost. It should be emphasized however, that it is only the (1+1)D case when the theory with coupling $\mathcal{L}_I = G \bar{\psi}(\sigma + i\gamma_5\pi)\psi$ is (super)renormalizable and any counterterm has explicit physical meaning. For higher space dimensions this is already not true and so the procedure of compensation of divergences in the energy based on (4.26) should not be considered as having no motivation. For more detailed discussion on the extraction of the finite part from the divergent Dirac’s sea energy in 3+1 D HCM see Refs. [30-32].

Proceeding further, let us turn to the study of the total bag energy

$$\mathcal{E}_{bag} = \hat{\mathcal{E}}_\psi(\alpha, \beta) + 2\mu\pi^2 \frac{r + 1}{\mu\rho + 1}$$

(4.27)

as a function of the parameters $\mu, \rho$. The analysis of the contribution of convergent logarithmic part from the sine-term in the asymptotic expression (4.20) to $\hat{\mathcal{E}}_\psi$ yields the first feature of $\mathcal{E}_{bag}$. Let us transform this contribution to the form

$$\left(\hat{\mathcal{E}}_\psi\right)_{log} (\alpha, \beta) = \frac{1}{\pi} \sum_{n > 1} (-1)^n \sin \frac{(\pi\alpha/\rho)(n + 1/2)}{n + 1/2}$$

(4.28)

and then use the well-known relation

$$\sum_{n=0}^{\infty} (-1)^n \sin \frac{z(n + 1/2)}{n + 1/2} = \ln \tan(\pi/4 + z/4), \quad |z| < \pi.$$  

(4.29)

It is easy to see, that the sums (4.28) and (4.29) possess the similar common term, while the sum (4.28) diverges as $(-\ln(\pi - z))$ when $z \to \pi$. Hence, for $\pi\alpha/\rho \to \pi$, what means $\beta \to 0$, the sum (4.29) will show the similar behavior, namely:

$$\left(\hat{\mathcal{E}}_\psi\right)_{log} (\alpha, \beta) \to -\frac{1}{\pi} \ln \beta, \quad \beta \to 0.$$  

(4.30)

Therefore, both the regularized fermion energy (4.26) and the total bag’s energy reveal the logarithmic singularity for $\beta \to 0$, i.e. for $\rho \to r/\mu$, what is in complete agreement with the qualitative analysis of the vacuum polarization effects in the 3-phase bag, performed above. In particular, for $\beta \to 0$ both the intermediate phase and the infinite energy of coupling between fermions and bag boundaries disappear, what in the present case shows up in the unlimited increase of fermion energy, since after the renormalization of $\mathcal{E}_\psi$ by means of subtraction (4.26) the divergent part of the surface bag energy is included in the onset of energy.

$\mathcal{E}_{bag}$ will also grow for $\rho \to \infty$. In this case $\alpha/\rho \to r/(r + 1)$, so the logarithmic term (4.28) remains finite, what implies, that we have to deal now with the whole sum (4.26). However, the leading order behaviour of $\hat{\mathcal{E}}_\psi$ could be evaluated from (4.26) quite effectively by using the fact that for $\rho \to \infty$ the fermionic spectrum becomes quasicontinuous, what allows to transform the sum over $x_n$ into integral over $dx$. In particular, the analysis of distribution of the roots of Eq. (4.12) shows, that for this limit $\sum_n \varepsilon_n$ is approximated by the following (divergent) integral:

$$\sum_n \varepsilon_n \to \frac{1}{\pi} \int dx \sqrt{1 + x^2} \left[\beta + \frac{1}{1 + x^2} + \frac{x^2}{x^2 + \sin^2 (\alpha\sqrt{1 + x^2})} - \sin \left(\frac{\alpha\sqrt{1 + x^2}}{\alpha\sqrt{1 + x^2}}\right) \cos \left(\frac{\alpha\sqrt{1 + x^2}}{\alpha\sqrt{1 + x^2}}\right)\right].$$

(4.31)

For the subtracted sum in (4.26) one can easily find

$$\sum_n \left(\frac{\sin \pi/2 + \pi n}{\rho} + \frac{1 + \beta/2}{\pi/2 + \pi n}\right) \to \frac{\rho}{\pi} \int dx \left( -\frac{1 + \beta/2}{\rho x}\right).$$  

(4.32)

The integrals (4.31) and (4.32) have obviously the same divergent part

$$\frac{1}{\pi} \int dx (\rho x + 1/x + \beta/2x),$$

and so their difference yields a converging integral, in agreement with the subtraction procedure. The leading term of the integrand in this difference, taken with the (correct) inverse sign, is $\beta/8\pi x^3$. Since
\[ \frac{\beta}{\rho} \to \frac{1}{(r + 1)} \text{ for } \rho \to \infty, \] this finally leads to the emergence of the positive, proportional to \( \rho \), contribution to \( E_\psi \), and correspondingly to \( E_{bag} \).

The numerical calculation confirms completely such qualitative predictions for the behavior of \( E_\psi(\rho) \) and \( E_{bag}(\rho) \). In the present paper such calculation has been performed for \( \mu = 0.25 \), what corresponds approximately to the ratio \( m_\pi/2M_Q \), where the constituent quark mass \( M_Q \) is assumed to be equal to 300 MeV, and for \( g = 1 \), because the energy of boson soliton doesn’t have any significant influence on the main properties of \( E_{bag}(\rho) \). The results of \( E_{bag}(\rho) \) calculation for \( r = 1, 2, 3, 4, 5 \) are depicted on the Fig. 5 and show, that the size and energy of the solution, determined from the minimum of \( E_{bag}(\rho) \), continuously grow for increasing \( r \), whereas the curvature of \( E_{bag}(\rho) \) in the minimum decreases, what provides an unambiguous interpretation of configurations with \( r > 1 \) as excited states of the bag.

### 5 Bags with zero topological charge

Now let us consider the bag with vanishing topological charge, what for the relevant configuration of the boson field should be an even one \( \varphi(x) = \varphi(-x) \). The principal difference between this case and the previous one is that for even \( \varphi(x) \) the sign symmetry \( \omega \leftrightarrow -\omega \) is an inmanent feature of the spectral problem for fermions (2.7-10), what can be easily justified by means of the following transformation of fermionic wavefunctions

\[ \psi_\omega(x) \to \psi_{-\omega}(x) = \pm \gamma_5 \psi_\omega(-x). \] (5.1)

However, the corresponding axial currents are related now in the following way

\[ j^5_{-\omega}(x) = -j^5_\omega(-x), \] (5.2)

so there is no automatic compensation between positive- and negative-frequency terms in the v.e.v. of \( J_5(x) \). From (5.2) one can derive only the relation

\[ \langle J_5(x) \rangle_{\text{sea}} = \langle J_5(-x) \rangle_{\text{sea}}, \] (5.3)

what guarantees the consistence of Eq. (2.4b) with respect to parity. The direct consequence of such fermion properties is that the even configuration of the boson field, similar to (3.17),

\[ \varphi(x) = \begin{cases} +2\lambda(x - x_1), & x_1 \leq x \leq x_2, \\ -2\lambda(x + x_1), & -x_2 \leq x \leq -x_1, \end{cases} \] (5.4)

is no longer an exact solution of Eqs. (2.4), since in this case \( \langle J_5(x) \rangle_{\text{sea}} \neq 0 \) in the region \( II \).

Nevertheless, the configuration (5.4) plays an important role in the study of the non-topological case. First of all, for small values of \( g \) it turns out to be a rather good approximation to the precise solution. To argue this statement, let us note firstly, that the substitution \( \varphi = \tilde{\varphi}/g \) removes \( g \) from the Eq.(2.4a), while Eq.(2.4b) will contain \( g \) only as a coefficient in the r.h.s, namely

\[ \varphi'' = ig^2 \frac{M}{2} \left[ \bar{\psi}, \gamma_5 e^{i\gamma_5 \varphi} \psi \right] \] (5.5)

Proceeding further, it would be natural to assume that the potential \( V(\varphi) \) depends on \( g \) as

\[ V(\varphi) = W(g\varphi)/g^2, \] (5.6)

where \( W(f) \) should be an even polynom to preserve the (anti)symmetry of soliton solutions. Therefore, for small \( g \) there emerges quite naturally an expansion in powers of \( g^2 \) in the problem. Within this expansion, the zero-order approximation for the rescaled boson field \( \tilde{\varphi}(x) \) is the configuration (5.4) in the region \( II \), while \( \tilde{\varphi}(x) \equiv 0 \) in the region \( I \), and in the region \( III \) it is assumed to merge with the asymptotics of an even soliton solution

\[ \tilde{\varphi}_{sol}(x) = \pi \left( 1 - Ae^{-m|x|} \right), \quad |x| > x_2, \] (5.7)

quite similar to the topological case. Sewing together (5.4) and (5.7) by means of the continuity conditions for \( \varphi \) and \( \varphi' \) yields the following relation

\[ 2\lambda = \frac{\pi m}{md + 1}, \] (5.8)
whence for the energy of the boson field one finds
\[ E_{\tilde{\varphi}} = \frac{\pi^2 m}{md + 1}. \] (5.9)

Note, that in the initial variables \( \varphi \) the dependence on \( g \) in \( E_{\varphi} \) is restored by adding the coefficient \( 1/g^2 \).

Now let us show, that the first-order \( O(g^2) \) correction to the energy of the boson field (5.9) vanishes exactly for any current in the r.h.s. of Eq.(5.5), provided the asymptotics (5.7) for the boson field in the region \( III \) remains valid beyond the perturbation expansion in \( g^2 \), what implies, that the corrections caused by the r.h.s. of (5.5) disturb only the value of the parameter \( A \). To simplify calculations, we will consider further only the positive half-axis. The contribution of the negative one is exactly the same.

From the asymptotics (5.7) we derive
\[ m\tilde{\varphi}(x_2) + \tilde{\varphi}'(x_2) = \pi, \] (5.10)
while in the region \( III \)
\[ \tilde{\varphi}'_{III}(x) = \tilde{\varphi}'(x_2)e^{-m(x-x_2)}, \] (5.11)
which are valid beyond the \( g^2 \)-expansion as well. Using the virial theorem which is also relevant beyond the latter expansion, we obtain the following general expression for the contribution of the region \( III \) to \( E_{\tilde{\varphi}} \)
\[ E_{\tilde{\varphi}_{III}} = \int_{III} dx \tilde{\varphi}'^2 = \frac{(\tilde{\varphi}'(x_2))^2}{2m}. \] (5.12)

Proceeding further, on account of the first-order correction from the non-zero \( \langle J_5(x) \rangle \) one obtains for the boson field in the region \( II \)
\[ \tilde{\varphi}(x) = 2\lambda(x-x_1) + g^2\tilde{\varphi}_1(x). \] (5.13)

At the same time, it follows from the condition \( \tilde{\varphi}(x) \equiv 0 \) in the region \( I \) and the boundary conditions (5.10) that
\[ \tilde{\varphi}_1(x_1) = 0, \quad m\tilde{\varphi}_1(x_2) + \tilde{\varphi}'_1(x_2) = 0. \] (5.14)

Then for the boson field energy in the region \( II \) with the first \( O(g^2) \) correction one finds
\[ E_{\tilde{\varphi}_{II}} = \frac{1}{2} \int_{II} dx \tilde{\varphi}'^2 = 2\lambda \left( \lambda d + g^2\tilde{\varphi}_1(x_2) \right). \] (5.15)

On the other hand, it follows in the same approximation from (5.12) and (5.13), that
\[ E_{\tilde{\varphi}_{III}} = \frac{2\lambda}{m} \left( \lambda + g^2\tilde{\varphi}'_1(x_2) \right). \] (5.16)

Returning to Eq.(5.14), one finds, that in the sum \( E_{\tilde{\varphi}_{II}} + E_{\tilde{\varphi}_{III}} \) the contribution of \( \tilde{\varphi}_1 \) vanishes. In other words, within the \( g^2 \)-expansion the corrections to the leading approximation (5.9) in \( E_{\tilde{\varphi}} \), caused by nonvanishing \( \langle J_5(x) \rangle \), begin from the second order \( O(g^4) \) only.

Next, let us note, that for fermions the \( g^2 \)-expansion starts from the order \( O(g^0) \). In this approximation the spectral problem (2.7-10) leads to the following equation for the fermionic spectrum
\[ \exp(4i\omega x_1) = \left[ \frac{\nu_+ + k_+}{\nu_- + k_-} \right] - \frac{e^{-2ik_+d M-Mi(\nu_+-k_-)}}{M-i(\nu_-+k_-)} \frac{e^{-2ik_-d M-i(\nu_-+k_-)}}{M-i(\nu_-+k_-)}, \] (5.17)
where \( \nu_\pm = \omega \pm \lambda, \quad \nu_\pm^2 = k_\pm^2 + M^2 \). The total energy of the bag is still given by the sum (4.5), where the fermion energy has the following form
\[ E_{\psi} = - \sum_{\omega_n<0} \omega_n. \] (5.18)

In (5.18), as well as in (4.8), the inequality \( \omega_n < 0 \) is strict, because for the configuration (5.4) there are no levels with \( \omega_n = 0 \) for any values of \( x_1, x_2 \).
Finally, after restoring dependence on $g^2$ in $E_\phi$ we obtain the following expression for the total energy of the bag

$$E_{bag} = \frac{\pi^2}{g^2} \frac{m}{md + 1} + E_\psi + O(g^2), \quad (5.19)$$

where the first two leading terms in $E_{bag}$ — the bosonic $O(1/g^2)$ and fermionic $O(g^0)$ — are determined by the zero-order approximation for the boson field (5.4.7) only, while the corrections start with $O(g^2)$ terms, simultaneously in the bosonic and fermionic parts of the total energy.

The numerical calculation completely confirms such behaviour of the system for small $g$. On the Fig.6 a typical profile of the numerical solution for the boson field is shown, obtained by means of minimization procedure of the total energy functional for the values of $g$ from the interval $0.01\div0.5$, which shows clearly that for such values of $g$ the exact solution in the intermediate region is almost indistinguishable from the linear function (5.4).

![Fig.6. The profile of the numerical solution for boson field, obtained by the minimization procedure of the bag's total energy functional for $g = 0.1$ on a spatial lattice with $\simeq 10^3$ nodes. The self-interaction of bosons in the region $III$ is taken in the form $V(\varphi) = (\pi^2 - (g\varphi)^2)^2 / 2g^2$. The dashed line, which merges almost with the solid one, corresponds to the calculation with the doubled number of nodes on the lattice, what indicates the accordance of the numerical solution with the continuous limit.](image)

Moreover, the respective simplicity of Eq. (5.17) makes it possible to analyse the fermionic spectrum in a semi-analytic way, what in turn allows to use the configuration (5.4.7) as a trial one for a qualitative study of the ground state properties for the non-topological bag for even larger values of $g \simeq 1$.

Thus, in further analysis of the main properties of the non-topological bag we will use the first two terms in the total energy (5.19), which can be found directly from the configuration (5.4.7). Recalculating $E_\phi$ to the dimensionless variables, introduced in (4.11,14-16), one obtains

$$E_{bag}(\alpha,\beta) = E_\psi(\alpha,\beta) + \frac{\pi^2}{g^2} \frac{2\mu}{\mu/\beta + 1}. \quad (5.20)$$
Note, that in (5.20) the sum of the parameters $\alpha, \beta$ defines the dimensionless size of the bag $\rho$, but on the contrary to the previous case in all other respects they should be considered as independent ones. So the study of the bag’s energy as a function of its geometry becomes a qualitatively different problem of finding the two-dimensional surface $\mathcal{E}_{\text{bag}}(\alpha, \beta)$.

The extraction of the finite part from $\mathcal{E}_\psi(\alpha, \beta)$ undergoes the same main stages as in the topological case, but reveals some peculiar features, caused by the independence of $\alpha$ and $\beta$. After some transformations it’s not difficult to obtain from (5.17) the UV-asymptotics of the energy levels in the following form

$$\varepsilon_n(\alpha, \beta) = \frac{\pi}{2} + \frac{\pi n}{\rho} + \frac{(-1)^n}{\pi} \cos(2\lambda d) \sin\left(\frac{\pi}{2} + \frac{\pi n}{\rho}\right) + 1 + \frac{\beta}{2} + O(1/n^2). \tag{5.21}$$

It follows immediately from the structure of the logarithmic term in (5.21), that as in the topological case, the renormalized via asymptotics $\mathcal{E}_\psi$, as well as $\mathcal{E}_{\text{bag}}$, show up a logarithmic divergence ($-\ln \beta/\pi$) for $\beta \to 0$. Besides this, $\mathcal{E}_\psi$ and $\mathcal{E}_{\text{bag}}$ increase for $\beta \to \infty$ and finite $\alpha$. Since $\rho$ grows simultaneously with $\beta$, this effect turns out to be quite similar to the increase of $\mathcal{E}_\psi$ and $\mathcal{E}_{\text{bag}}$ for $\rho \to \infty$ in the topological case: in the UV-domain the difference between $\nu_+$ and $\nu_-$ vanishes and Eq.(5.17) turns into (3.9). Thereafter to analyse the renormalized $\mathcal{E}_\psi$ one may use the integral approximation (4.31,32), in which for $\beta \to \infty$ the main term in the integrand of $\mathcal{E}_\psi$ is positive and proportional to $\beta$.

The behaviour of $\mathcal{E}_\psi$ and $\mathcal{E}_{\text{bag}}$ for $\alpha \to \infty$ and finite $\beta$ requires special consideration, for in this case the logarithmic term with the sine in asymptotics (5.21) becomes significant again, but unlike the case of separate branches for imaginary $k_\pm$ in (5.17), where $0 < \nu_+ \leq M$. Directly for the Eq.(5.17) this effect shows up in an intricate enough way due to the presence of separate branches for imaginary $k_+$ and $k_-$ and therefore can be analysed in detail only numerically, but its essence could be understood quite simply, if we neglect for a while the difference between $k_+$ and $k_-$. Then we are left with only one branch with $0 < \nu_n \leq M$ determined from Eq. (4.13), from which it is easily seen that for $\alpha \to \infty$ the spectrum of energy levels belonging to this branch becomes quasicontinuous with the interval between the levels of order $\pi/\alpha$, hence $\sum_n \varepsilon_n$ over this branch can be approximated by the integral

$$\sum_n \varepsilon_n \to -\frac{1}{\pi} \int_0^1 dx \sqrt{1-x^2} \left[ -\frac{\alpha x}{\sqrt{1-x^2}} + \frac{x(\beta + 1/(1-x^2)) - \sinh(\beta x + \gamma)/\sqrt{1-x^2}}{\cosh(\beta x + \gamma) - \sqrt{1-x^2}} \right]. \tag{5.22}$$

Therefore, for $\alpha \to \infty$ the second branch contribution to $\sum_n \varepsilon_n$ takes the form $\alpha/2\pi + \text{finite terms}$ depending on $\beta$ only. Transforming further the subtracted sum to the integral one obtains

$$\sum_n \left( \frac{\pi/2 + \pi n}{\rho} + \frac{1 + \beta/2}{\pi/2 + \pi n}\right) \to \frac{1}{\pi} \int dx \left( \frac{x}{\rho} + \frac{1 + \beta/2}{x} \right), \tag{5.23}$$

whence it follows that for $\alpha \to \infty$ the main terms in the subtracted sum should be

$$\frac{\alpha^2}{2\pi \rho} + \frac{1 + \beta/2}{\pi} \ln \alpha. \tag{5.24}$$

The leading, proportional to $\alpha$ terms in Eqs.(5.22,24) cancel each other, so after subtraction the contribution of the second branch to the renormalized $\mathcal{E}_\psi$ should be $(1/\pi)(1 + \beta/2)\ln \alpha + \text{finite terms}$. For the case of separate branches for $k_+ \ k_-$ the general features of their asymptotic behaviour for $\alpha \to \infty$ remain the same. As a result, after combining this asymptotics with the corresponding input of the logarithmic term in the UV-asymptotics (5.21), for the leading term in $\mathcal{E}_\psi$ for $\alpha \to \infty$ one finds

$$\frac{1}{\pi} (1 + \beta/2 + \cos 2\lambda d) \ln \alpha, \ \alpha \to \infty, \tag{5.25}$$

which is definitely positive for all $\beta$. So in this limit the bag’s energy also grows, but now proportional to $\ln \alpha$. 

---

18
The numerical calculation confirms completely such qualitative behaviour of $\tilde{E}_\psi$ and $E_{bag}$. It has been performed for the same values of $\mu = 0.25$ and $g = 1$ as for the topological bags, shown on Fig. 5. The most specific feature of the non-topological case is that in accordance to (5.9) the boson field energy depends now on $\beta$ only, while the renormalized energy of fermions decreases smoothly for $\alpha \to 0$. A profile of the surface $\tilde{E}_\psi(\alpha, \beta)$ is depicted on Fig. 7, where $\tilde{E}_\psi = 0$ for $\alpha = \beta = 0$ is an artifact of the chosen subtraction method. As a result, there isn’t any non-trivial minimum in the total energy for the non-topological case at all, while the minimal energy is achieved by the configuration with vanishing size of the phase of asymptotic freedom and for finite non-zero $\beta$, what is clearly seen from Figs. 8,9, on which the profiles of the 2D surfaces $E_{bag}(\alpha, \beta)$ are presented in different scales. So for the bags with zero topological (i.e. baryon) charge the considered three-phase model predicts that the main role should be played by the intermediate phase of constituent quarks, what is quite consistent with semi-phenomenological quark models of mesons [9,33].

6 Conclusion

This work was aimed at the construction of a reasonable model of a hybrid chiral bag without special assumptions about the intimate relations between the fermionic and bosonic phases. Our results show that such a model can be actually formulated in a quite consistent fashion, and to certain extent could be more efficient way of description of low-energy hadron physics compared to the standard HCM [1-8]. The main advantages of such an approach include, first of all, a more correct formulation of chiral boundary conditions for which all the components in the Lagrangian possess clear physical meaning, the existence of the intermediate phase, describing quasifree massive "constituent quarks", as well as a quite acceptable physically behaviour of the total bag’s energy as a function of its size, which takes the form of an infinitely deep potential well with a distinct minimum in the topological case, whereas in non-topological case the minimal energy of the bag is achieved by the configuration, where the phase of asymptotic freedom disappears.

Besides this, in this model the condition of fermions confinement, incorporated into it from the very beginning, shows up more explicitly. It manifests, in particular, in the fact that there is no need in the vacuum-pressure term $B$, which in the standard approach is inserted into the model by means of some extra assumptions, since in the considered case the Dirac’s sea polarization itself produces the infinite increase of energy at large distances. Another essential feature is the appearance of infinite interaction energy between fermions and bag boundaries (confining potential) for $d \neq 0$, what implies, that the size of the intermediate region doesn’t actually vanish, although on the level of the initial Lagrangian the formal limit $d \to 0$ exists and yields the standard two-phase model of a hybrid bag. In other words, such a 3-phase model cannot be continously transformed into a 2-phase one, what is the ultimate reason of remarkably different features of this model compared to the standard 2-phase ones.

It is worth-while to mention once more the question of the choice of method of calculation of the Dirac’s sea averages for fermion bags. The method we used is based on the discreteness of the fermionic energy spectrum what by means of quite obvious considerations leads to very simple solution of self-consistent equations of the bag in the intermediate region. Let us remark, however, that despite of arguments in favour of such method of calculation of sea averages, we cannot completely reject alternative methods like the thermal regularization. The question of which one is more adequate to the physics of the problem, should be answered only by means of detailed study of realistic models.

It should be also emphasized, that by constructing such a 3-phase model we have substantially used the condition of lorentz-covariance. The initial formulation of the model is a local field theory and regardless on the variety of classical solutions one needs to deal with, the covariance is broken only spontaneously, and so can be freely restored by means of methods of Refs.[21] based on the covariant group center-of-mass variables for a localized quantum-field system. However, such an explicitly covariant framework requires some essential changes in the calculation techniques, since the invariant dynamics of fields in the c.m.s. acquires a specific finite-difference form [21], and so will be considered separately.

This work was supported in part by RFBR under Grant 00-15-96577 and by Sankt-Petersburg Concurrency Centre of Fundamental Sciences, Grant 00-0-6.2-22.
References

[1] G.E.Brown, M.Rho, Phys.Lett. B 82 (1979) 177;
[2] V.Vento, M.Rho, E.M.Nyman, J.H.Jun, G.E.Brown, Nucl.Phys. A 345 (1980) 413;
[3] H. Hosaka and O. Toki, Phys. Reports 277 (1996) 65;
[4] I. Zahed, G. E. Brown, Phys. Reports 142 (1986) 1;
[5] G.Holzwarth and B.Schwesinger, Rep.Progr.Phys. 49 (1986) 825;
[6] A.Chodos, C.B.Thorn, Phys.Rev. D 11 (1975) 2733; T.Inoue, T.Maskawa, Progr. Theor. Phys. 54 (1975) 1853;
[7] S.Nadkarni, H.B.Nielsen and I.Zahed, Nucl. Phys. B253 (1985) 308; S.Nadkarni and H.B.Nielsen, Nucl. Phys. B263 (1986) 1,23;
[8] M. Rho, Phys. Reports 240 (1994) 1;
[9] S.Gasiorowicz and J.L.Rosner, Amer.J.Phys. 49 (1981) 1954; L.Montanet et al., Phys.Rev. D50 (1994) 1173; B.Povh et al., Particles and Nuclei (Springer,Berlin, 1995); 
[10] K.Sveshnikov, P.Silaev, Theor.Meth.Phys. 117 (1998) 263;
[11] I.Cherednikov, S.Fedorov, M.Khalili, K.Sveshnikov, Nucl.Phys. A 676 (2000) 339;
[12] S. Théberge, A.W. Thomas and G.A. Miller, Phys.Rev. D 22 (1980) 2838; D 24 (1981) 216;
[13] A.W. Thomas, Adv. Nucl. Phys. 13 (1984) 1;
[14] G.A.Miller, Int.Rev. Nucl. Phys. 2 (1984) 190;
[15] S.Kahana, G.Ripka, V.Soni, Nucl.Phys. A 415 (1984) 351;
[16] M.K.Banerjee, W.Broniowski and T.D.Cohen, in Chiral Solitons, ed. by K.F.Liu. (1987, World Scientific, Singapoure);
[17] D.I. Diakonov, V.Yu. Petrov, P.V.Povilitsa, Nucl. Phys. B306 (1988) 809;
[18] M.C.Birse, Progr. Part. Nucl. Phys., 25 (1991) 1;
[19] R.Alkofer, H.Reinhardt, H.Weigel, Phys.Rep. 265 (1996) 139;
[20] M.Creutz, Phys.Rev. D 10 (1974) 1749; M.Creutz, K.Soh, Phys.Rev. D 12 (1975) 443;
[21] K.Sveshnikov, Phys. Lett. A136 (1989) 1; Teor.Mat.Fiz. 82 (1990) 55;
[22] R.Perry and M.Rho, Phys.Rev. D 34 (1986) 1169;
[23] D.K.Campbell, Y.-T. Liao, Phys. Rev. D14 (1976) 2093;
[24] A.J.Niemi, G.W.Semenoff, Phys. Reports 135 (1986) 99;
[25] I.Malakhov, K.Sveshnikov, in print;
[26] R.Daschen, B.Hasslacher, A.Neveu, Phys.Rev. D12 (1975) 2443;
[27] A.D.Jackson, M.Rho, Phys.Lett. B 173 (1986) 217,220; A.D.Jackson, L.Vepstas, Phys.Rep. 187 (1990) 109;
[28] P.J.Mulders, Phys.Rev. D 30 (1984) 1073;
[29] G.Plumien, B. Muller, W. Greiner, Phys. Reports 134 (1986) 87;
Fig. 5. $\mathcal{E}_{bag}(\rho)$ for $r = 1, 2, 3, 4, 5$ in the topological case.
Fig. 7. The profile of the surface $\tilde{\mathcal{E}}_{\psi}(\alpha, \beta)$ for the non-topological bag.
Fig. 8. The profile of the surface $\mathcal{E}_{bag}(\alpha, \beta)$ for the non-topological bag.
Fig. 9. The profile of the surface $E_{bag}(\alpha, \beta)$ for the non-topological bag, rescaled to observe the behavior for small $\beta$. 