The surface tension effect on flexoelectric energy harvesting based on isogeometric analysis

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Abstract. Surface tension can cause important deformations in fluids but is usually overlooked in solids. Recent work has shown that surface tension can become significant in the deformations of soft solids. Here, we present an isogeometric-analysis-based numerical example of cantilever beams under a point load for modeling the influences of surface tension on the flexoelectric energy harvesting system. The interface of the fluid/solid is implicitly represented using level set method. These results show that the maximum electric potential decreases when the number of liquid inclusions increases. However, the electric potential near the solid/liquid interface increases when considering the surface tension effect.

1. Introduction

Recently, energy harvesters based on flexoelectricity have attracted intensive research interests from both fundamental science and technological applications [1-8]. The flexoelectric effect is an electromechanical effect which describes the coupling between strain gradient and electrical polarization. Flexoelectricity exist in all dielectrics, including biological membranes, polymers, liquid crystals, and piezoelectric materials. Researchers observed an unexpected increase of few orders of magnitude in voltage and energy conversion efficiency when the size of flexoelectric material was reduced to the micro- or nano-scale [9-12]. The size-dependent phenomenon indicates that the flexoelectric effect is dominant relative to the piezoelectric effect at the micro-/nano-scale. Recent works have suggested that surface tension need to be taken into account in soft materials [13]. Recent experiments which have shown that surface tension (denoted by $\gamma$) can also significantly affect soft solids at the micron scale [13-15]. The purpose of this work is to demonstrate the surface tension effect on the mechanical and electrical response of fluid inclusions in a soft flexoelectric solid. The general formulation for flexoelectricity considering the surface-tension effects is presented. Then, we perform simulations of cantilever beams embedded droplets with different size under bending to study how the surface-tension effect influences the energy harvester based on flexoelectricity.

2. Theory

The weak form of the governing equation accounting for flexoelectricity of a linear dielectric solid can be written as [6, 16-18]

$$\int_{\Omega} \left( C_{ijkl} \delta e_{ij} \delta e_{kl} - \epsilon_{ijkl} \delta e_{ij} - \kappa_{ijkl} \delta \varepsilon_{ij} \delta \varepsilon_{kl} - \mu_{ijkl} \delta \varepsilon_{ij} \delta \varepsilon_{kl} - \mu_{ijkl} \delta \varepsilon_{ij} \delta \varepsilon_{kl} - \mu_{ijkl} \delta \varepsilon_{ij} \delta \varepsilon_{kl} \right) d\Omega - \int_{\Gamma} \tau \delta u_{i} dS + \int_{\Gamma_{T}} \sigma_{ij} \delta \theta dS = 0$$  \hspace{1cm} (1)

In Eq. (1), $u_{i}$ and $\theta$ are the mechanical displacements and electric potential and $C_{ijkl}$ represents the fourth-order tensor of elastic moduli, $\epsilon_{ijkl}$ denotes the third-order piezoelectric tensor, $\kappa_{ijkl}$ and $E_{i}$ are the mechanical strain and electric field, $\mu_{ijkl}$ denotes the fourth-order flexoelectric tensor.
and \( \kappa \) denotes the second-order dielectric tensor. Using B-spline basis functions based on isogeometric analysis (IGA), \( N_u \) and \( N_\theta \), we approximate \( u \) and \( \theta \) fields as

\[
u(x, y) = \sum_{m=1}^{m_{\text{cp}}} \sum_{n=1}^{n_{\text{cp}}} N_u^{m,n}(\xi, \eta) u^m \]

\[
\theta(x, y) = \sum_{m=1}^{m_{\text{cp}}} \sum_{n=1}^{n_{\text{cp}}} N_\theta^{m,n}(\xi, \eta) \theta^m
\]

Then, Eq. (1) can be eventually expressed as

\[
\begin{bmatrix}
K_{uu} & K_{u\theta} \\
K_{u\theta} & K_{\theta\theta}
\end{bmatrix}
\begin{bmatrix}
u \\
\theta
\end{bmatrix} =
\begin{bmatrix}
f_u \\
f_\theta
\end{bmatrix}
\]

where

\[
K_{uu} = \sum_{\Omega} \int (B_u)^T C (B_u) d\Omega
\]

\[
K_{u\theta} = \sum_{\Omega} \int (B_u)^T (B_\theta) d\Omega
\]

\[
K_{\theta\theta} = \sum_{\Omega} \int (B_\theta)^T (B_\theta) d\Omega
\]

\[
f_u = \sum_{\Gamma_u} \int_{\Gamma_u} N_u^{m,n} t_m ds
\]

\[
f_\theta = \sum_{\Gamma_\theta} \int_{\Gamma_\theta} N_\theta^{m,n} \sigma ds
\]

Note that we have ignored any forces acting on the surface due to external potentials (e.g. surface charges in an electric field), and have assumed that the surface has no bending rigidity. On the surface of the liquid inclusion, the mechanical stress satisfies the Young-Laplace equation [15],

\[
\sigma \cdot n = -pn + \gamma n
\]

Assuming \( p = 0 \), the surface tension force simplifies to

\[
F_\gamma(x) = \gamma k(x, n(x))
\]

It can be obtained by using the continuum surface force (CSF) model [19]. Using the level set method, the normal vector and curvature can be calculated as follows

\[
n = \frac{\nabla \Phi}{|\nabla \Phi|}
\]

\[
k = \frac{\nabla^2 \Phi}{|\nabla \Phi|^3}
\]

The level set function \( \Phi_i(x) \) can be written as the following subdomains

\[
\Phi_i(x) \begin{cases} 
\Phi_i(x) > 0 & \forall x \in \Omega_i \setminus \partial \Omega_i \\
\Phi_i(x) = 0 & \forall x \in \partial \Omega_i \cap \Omega_i \\
\Phi_i(x) < 0 & \forall x \in \Omega_i \setminus \Omega_i
\end{cases}
\]

where \( \Omega_i \) represents the domain of the \( i \)th level set function.

### 3. Numerical results and discussion

We consider cantilever beams embedded a liquid inclusion under boundary conditions, as depicted in Fig. 1. The aspect ratio of the cantilever beam is set to \( L/h = 5 \). The flexoelectric and piezoelectric effect of the liquid inclusion is ignored and we assume that the liquid is approximately incompressible, using \( \nu = 0.495 \). The dielectric constant of the liquid inclusion is \( 80 \varepsilon_0 \), where \( \varepsilon_0 = 8.8542 \times 10^{-12} \text{ F/m} \) denotes the vacuum dielectric constant. The surface tension coefficient of the liquid/solid interface is \( \gamma = 3.6 \text{ mJ/m}^2 \). In order to simplify the model, we consider the embedded liquid inclusions as elastic inclusions with stiffness.
\[ Y_f = Y \frac{24Y}{10 + 9Y} \] (9)

Table 1. Solid material and load data for the cantilever beam

| $L/h$ | $\nu$ | $Y$ | $\varepsilon_{31}$ | $\mu_{12}$ | $\kappa_{11}$ | $\kappa_{33}$ | $F$ |
|-------|-------|-----|---------------------|------------|---------------|---------------|-----|
| 5     | 0.495 | 1.7kPa | -4.4Cm$^{-2}$ | 1uCm | 11nC/Vm | 12.48nC/Vm | 100uN |

$\nu$: Poisson ratio; $Y$: Young's modulus.

Figure 1. boundary condition of cantilever beam

Figure 2. Electric potential from the computational model with a centre liquid inclusion (a) and 7 liquid inclusions (b).
Figure 3. Displacement in the y-direction as a function of radius R of the droplet
Using the material property and load data in Table 1, the distribution of the electric potential is obtained from the computational model with a center inclusion in Fig. 2(a). And Fig 2(b) for the present model with 7 liquid inclusions evenly distributed along the x-axis. These results are obtained for $h = 8.5 \mu m$. When the number of liquid inclusions increases, the maximum electric potential decreases. The electric potential near the solid/liquid interface increases considering the surface tension effect. Fig 3 presents the results for the displacement in the y-direction as a function of radius R of the droplet considering the height of the beam $h = 8.5R$. The displacement in the y-direction increases when $R > \gamma / E$. Large droplets make a solid more compliant. However, when $R < \gamma / E$, the displacement in the y-direction decreases. The liquid inclusion acts like a rigid body that resists deformation of the beam.

4. Conclusion
In the present research, we proposed a computational simulation to study the impact of surface tension at the fluid/solid interface on the mechanical and electrical responses of liquid inclusions in a soft solid matrix, based on isogeometric analysis (IGA). According to the model, we find the droplet seems to “disappear” when the elasto-capillary number $\gamma / ER = 1$. When $\gamma / ER > 1$, surface tension effect dominates and the droplet stays spherical. When the elastocapillary length L is much smaller than the characteristic radius R of the liquid inclusion, the surface tension increases the stiffness of the structure. Contrary, the surface tension effect can be ignored when $\gamma / ER << 1$. The maximum electric potential decreases when the number of liquid inclusions increases. However, the electric potential increases near the solid/liquid interface when considering the surface tension effect.

Acknowledgments
The authors acknowledge the financial support from the National Natural Science Foundation of China (Grant No. 51579084), Jiangsu Province Key R&D Project (Grant No. BE2017167) and the Fundamental Research Funds for the Central Universities (Grant No. 2018B48514).

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