Casimir interaction from magnetically coupled eddy currents

Francesco Intravaia and Carsten Henkel
Institut für Physik und Astronomie, Universität Potsdam, 14476 Potsdam, Germany
(Dated: 06 Sep 2009)

We study the quantum and thermal fluctuations of eddy (Foucault) currents in thick metallic plates. A Casimir interaction between two plates arises from the coupling via quasi-static magnetic fields. As a function of distance, the relevant eddy current modes cross over from a quantum to a thermal regime. These modes alone reproduce previously discussed thermal anomalies of the electromagnetic Casimir interaction between good conductors. In particular, they provide a physical picture for the Casimir entropy whose nonzero value at zero temperature arises from a correlated, glassy state.

PACS numbers: 03.70.+K 05.40.-a 42.50.Bq 42.50.Nn

Spatially diffusive transport is a basic physical phenomenon that has been studied with a wealth of methods. For example, the equation for heat conduction was solved by J. Fourier by his transformation that provided later the framework for quantum field theory. Remarkably, the Fourier modes of the diffusion equation itself,

$$\partial_t A = D \nabla^2 A, \quad (1)$$

do not fit into a simple quantum field theory because they have purely imaginary frequencies. The quantization of (over)damped modes can be done, however, with an alternative approach, the ‘system+bath’ paradigm of dissipative quantum dynamics. In this picture, the observables relevant to a mode (specified, e.g., in Fourier space by its wavevector $\mathbf{q}$) are damped in time because they strongly couple to a system with infinitely many degrees of freedom that are not directly accessible (‘bath’)\textsuperscript{1}. The additional fluctuations the bath couples into the system, compensate at equilibrium for dissipative losses\textsuperscript{2}, even at zero temperature. This implies that a quantity like the zero-point energy of a damped mode is no longer given by the usual $\frac{h}{2} \omega_\mathbf{q}$ and must be redefined and reinterpreted. Indeed, the ground state of the combined system+bath is, in general, entangled: the corresponding interaction Hamiltonian is responsible for the change in the zero-point energy relative to the decoupled system\textsuperscript{3, 4}. Clearly this has an impact on all phenomena connected with fluctuation energy, for which the paradigmatic example is the Casimir effect\textsuperscript{5}.

In this letter we discuss a specific example of ‘diffusive modes’: eddy (Foucault) currents in two metallic plates\textsuperscript{6}. We address their quantum and thermal fluctuations and their role in the electromagnetic Casimir interaction\textsuperscript{5}. Similar to the approach of Ref.\textsuperscript{7}, we isolate the contribution of eddy currents among all other modes. We show that these modes quantitatively reproduce the ‘unusual features’ of the Casimir force between good conductors at finite temperature (as predicted by Lifshitz theory), that have been intensely debated for several years\textsuperscript{8-9}.

To begin with, consider a metallic bulk described by a local dielectric function in Drude form: $\varepsilon(\omega) = 1 - \Omega^2/|\omega(\omega + i\gamma)|$ ($\Omega$: plasma frequency, $\gamma$: damping rate). This system allows for a continuum of damped, chargeless modes with a (transverse) current density and (dominantly) magnetic fields at frequencies of order $\gamma$ or below: they are called ‘eddy’ or ‘Foucault currents’ and are at the base of phenomena like magnetic braking or the familiar induction over\textsuperscript{1, 15}. The ‘bath’ for these modes is provided by the phonons in the metal or by impurity scattering. On time scales slower than $1/\gamma$, the vector potential satisfies Eq.\textsuperscript{11} with an electromagnetic diffusion constant $D = \gamma\lambda^2$ ($\lambda \equiv c/\Omega$: plasma penetration depth, $\lambda \approx 20$ nm for gold). A mode analysis in Fourier space yields imaginary frequencies $\omega_\mathbf{q} = -i\xi_\mathbf{q} \approx -iDq^2$ for $q \ll 1/\lambda$: due to the coupling to the electromagnetic field, the damping of eddy currents becomes weaker (compared to $\gamma$) and dispersive.

Let us now split the metal in two half-spaces, inserting a vacuum layer of thickness $L$ (“cavity”). While the eddy currents cannot leave the medium, the associated electromagnetic fields do; they cross the cavity in the form of evanescent waves. The corresponding magnetic fields provide Johnson noise that dominates, at low frequencies, over the fluctuations of other modes confined within the cavity\textsuperscript{10-12}. The characteristic frequency is the so-called Thouless energy\textsuperscript{13}

$$\hbar\xi_L \equiv \hbar D/L^2 \quad (2)$$

and the scattering rate $\gamma$ provides a strict upper limit to the $\xi_L$. Typical numbers for gold at room temperature are $\hbar\xi_L \sim 20$ K and $\hbar\gamma \sim 500$ K at $L = 100$ nm. We focus in the following on the polarization with the electric field parallel to the metal surface (TE). Indeed, the orthogonal case (TM) is associated with a surface charge that effectively decouples medium and vacuum.

For an overdamped mode, the ‘system+bath’ approach predicts a zero-point energy\textsuperscript{3, 14}

$$E_\mathbf{q} = -\frac{\hbar\xi_\mathbf{q}}{2\pi} \log(\xi_\mathbf{q}/\Lambda) \quad (3)$$

where $\Lambda$ is a cutoff frequency related to the bath response time\textsuperscript{11, 13}. The corresponding Casimir energy captures...
the shift in the (imaginary) mode frequencies $\xi_q = \xi_q(L)$ due to the magnetic cavity field correlating the eddy currents in the two plates. We therefore sum this expression over the eddy current mode continuum and subtract the reference system of two isolated half-spaces. The difference of mode density per area, $\tilde{\rho}(\xi; L)$ is related to the relevant mode continuum. The pressure vanishes for thermally excited at quite low temperatures because of the finite bandwidth $\gamma$ of the zero-point fluctuations in the two plates. We therefore sum this expression over the eddy current mode continuum and subtract the reference system of two isolated half-spaces.

$$E_D(L) = \int \frac{d^2k}{(2\pi)^2} \text{Im} \log \left[ 1 - r_k^2 (-i\xi + 0)e^{-2\kappa L} \right]$$

where $\kappa = \sqrt{k^2 + (\xi/c)^2}$ and $r_k$ the TE reflection coefficient. This shows power laws in $\xi$ below and above the Thouless frequency $\xi_L$, providing the asymptotic behaviours:

$$E_D(L) \sim \begin{cases} \text{const.} - \hbar \gamma L^3 \log(\lambda/\gamma), & L \ll \lambda, \\ h(\xi L/c) (L^2 - L^{-2} \log(\Lambda L/c)), & L \gg c/\gamma. \end{cases} \tag{5}$$

The associated pressure $-dE_D/dL$ shows a repulsive Casimir effect, provided the cutoff is sufficiently large (a few times $\gamma$). The repulsive character can be understood from the Lenz rule for current-current (diamagnetic) interactions. Eq. (5) does not show the scaling $\hbar c L^3$ characteristic of perfect reflectors and is much weaker because of the finite bandwidth $\lesssim \gamma$ of the relevant mode continuum. The pressure vanishes for $\gamma \to 0$ because dynamical fluctuations induced by the bath are suppressed. At small distances, $E_D(L)$ essentially scales with the ‘missing volume’ $L \times (\text{plate area})$ and the bulk mode density $\sim \gamma^{1/2} D^{-3/2}$ (at $\xi \sim \gamma$) of the eddy current continuum. Their zero-point fluctuations exert, in this regime, a constant kinetic and magnetic pressure.

Let us now raise the temperature. Eddy currents are thermally excited at quite low temperatures because of their low characteristic frequencies: for good conductors at room temperature we have $\hbar \xi_L \ll k_B T < h \gamma$ already at $L = 100$ nm. We find indeed that they give a significant contribution to the Casimir interaction. Consider first the Casimir entropy. The contribution due to eddy currents can be calculated in terms of a real frequency density (‘DOS’)$\rho(\omega; L)$ [Fig. 2] using the following Kramers-Kronig like relation:

$$\rho(\omega; L) = \int_0^\infty \frac{d\xi}{\pi} \tilde{\rho}(\xi; L) \frac{\xi}{\xi^2 + \omega^2}. \tag{6}$$

Fig. 2 shows that the DOS is significant in the frequency range $\omega \sim \xi_L \ldots \gamma$. One also sees that in this range, the DOS of the electromagnetic Casimir effect between Drude metals (gray line) is entirely given by Foucault currents. The zero frequency limit is

$$\rho(\omega \to 0; L) \approx -\frac{2 \ln 2 - 1}{8\pi^2 D} < 0. \tag{7}$$

Given this DOS, we get the free energy $F(T, L)$ by multiplying with $k_B T \log(2 \sinh(h \omega/(2k_B T)))$ and integrating over $\omega$ (see Appendix). The entropy, $S(T, L) = -dF/dT$, is cutoff independent and becomes [see also Fig. 2]

$$S = \frac{k_B}{L} = \begin{cases} \frac{\pi^2 k_B T}{3} \rho(0; L), & k_B T \ll \hbar \xi_L, \\ \frac{\hbar^2}{16\pi^2 L^2} f(L/\lambda) = S_{\text{fin}}(L), & k_B T \gg \hbar \xi_L, \end{cases} \tag{8}$$

where the dimensionless function $f(L/\lambda) \to 1$ for $L \gg \lambda$. The characteristic temperature scale $\hbar \xi_L/k_B$ has been noticed previously from the Lifshitz theory with a temperature-independent scattering rate. A negative entropy is not surprising here since we are considering a difference, with the entropy for infinitely separated plates subtracted. The sign means that one plate is acquiring information (entanglement) about the configuration in the other one through the electromagnetic interaction. The entropy vanishes linearly as $T \to 0$, in
agreement with the Nernst heat theorem (third law of thermodynamics). In this limit, modes with frequencies above $k_B T/h$ ‘freeze’ to their ground state, do not contribute to the entropy, and a unique ground state for the system remains at $T = 0$. We have checked that the free energies at low $T$ due to eddy currents alone and in the full Lifshitz theory\cite{16,17} coincide in their first two terms ($T^2$ and $T^{5/2}$).

Another scenario emerges with a $T$-dependent scattering rate when the ratio $\xi(k)/T$ vanishes as $T \to 0$. This happens for example in a ‘perfect crystal’\cite{18} where $\gamma(T) \sim T^n$ ($n \geq 2$). In this case the Foucault modes stay in the high-temperature regime and give a universal contribution $S_{\infty}(L) \sim -f(L/\lambda)/L^2$ to the entropy. The same ‘entropy defect’ has been found in previous work on the Lifshitz theory with the Drude model\cite{18} and traced back to evanescent TE modes\cite{12}. Nevertheless this does not conflict with thermodynamics because, in this limit, the ground state becomes infinitely degenerate: for $D \to 0$, the DOS $\rho(0; L) \to \infty$ in Eq.(7), and the spectrum collapses to zero frequency because the characteristic frequency scale $\xi_L \to 0$. The diffusive modes then become static current loops interlocking with magnetic field lines that permeate the metallic bulk, similar to the frozen magnetization in an ideally conducting medium. This is a glassy state with a nonzero entropy (‘Foucault glass’). As a check of this idea we have formulated a Hamiltonian field theory for a glass of static currents and have calculated the change in entropy per area for two magnetically coupled half-spaces, recovering a result that perfectly coincides with $S_{\infty}(L)$. It is clear, of course, that the Foucault glass is the result of an over-idealized characterization (‘perfect crystal’). Indeed, the Lifshitz entropy vanishes with $T \to 0$ when additional, even small, relaxation mechanisms like Landau damping are included\cite{19}.

Let us now consider the eddy current contribution to the Casimir pressure at $T > 0$. The debate around the electromagnetic Casimir interaction\cite{8,9} has revealed that the large-distance pressure between Drude metals is effectively halved compared to perfectly reflecting mirrors, even for an infinitesimally small dissipation rate. This is illustrated in Fig.4 where the temperature-dependent part of the Casimir pressure is plotted for the Drude model (gray lines vanishing as $L \gg \lambda_T \equiv h\xi_L/(2k_B T)$). The Drude model thus not describe the limit of a perfect reflector (dashed line in Fig.4), even if we take $\gamma \to 0$ or $\lambda \to 0$ (‘ideal conductor’). Models that go over to the perfect reflector (as $L \gg \lambda_T$) are superconductors\cite{20,21} and the so-called ‘plasma model’ (lossless dielectric function with $\gamma = 0$ right from the start, solid line in Fig.4). The thermal pressure from eddy currents accounts nearly completely for the difference between the Drude and the previous models\cite{10,11,12}: they thus represent very precisely the object under debate. In particular eddy currents provide a simple physical explanation why in the Drude model, the thermal Casimir pressure sets in at unusually short distances (compared to the thermal wavelength, arrows in Fig.4): this happens indeed when the dissipative energy scale crosses over from the quantum into the classical regime, $\hbar D/L^2 \sim k_B T$\cite{17}. The absence of eddy currents in the plasma model is actually not surprising since it behaves like a superconductor and expels low-frequency magnetic fields\cite{22}. The previously discussed ‘perfect crystal’ limit ($D \to 0$ of the Drude model), on the contrary, corresponds to the ideal conductor that does not show the Meißner effect\cite{22}.

Recent measurements of the Casimir interaction between gold-coated bodies (a plate and a sphere) at room temperature have been argued to favor a theory within the (lossless) plasma model\cite{23}, but these results do not appear to be commonly accepted yet. In view of our analysis, a similar prediction could equally emerge within the Drude model if the contribution of eddy currents were somehow reduced, keeping only the other (photonic) modes. How this should be implemented physically, remains to be understood in detail. Potential mechanisms

![](image)

**FIG. 3:** Eddy current Casimir entropy vs temperature $T$. Dashed: Eq. (5) for $T \to 0$. We normalize to the perfect reflector entropy at high temperatures, $\alpha/L^2 = \zeta(3)k_B/(16\pi L^2)$. Arrows: crossover temperature $k_B T = h\xi_L = \hbar D/L^2$.

**FIG. 4:** Thermal Casimir pressures (TE polarization only) vs. distance $L$, normalized to the perfect reflector $\left[\zeta(3)k_B T/(8\pi L^3)\right]$. In the Drude model (gray, $\gamma = 0.08\Omega$), the pressure is repulsive and vanishes for $L \gg \hbar c/(2k_B T)$ (black arrows). Blue dots: Drude model with eddy current pressure subtracted. Black: plasma model. Gray arrows: electromagnetic diffusion length $[hD/(k_B T)]^{1/2}$. 
should take into account, however, that in other contexts, thermally excited eddy currents have been observed, in agreement with the usual thermodynamic theory.\textsuperscript{24}

To reduce the eddy current Casimir interaction, we have considered a calculation at constant entropy, noting that also the thermalization rate of low-frequency modes is very small. This approach leads, however, to a significant overshooting of the Casimir pressure at large distances (very low specific heat according to Eq.\textsuperscript{[8]})\textsuperscript{.} Any mechanism that gives a nonzero real part to the frequencies of eddy current modes (see e.g. Ref.\textsuperscript{[11]}) would reduce their contribution at low temperatures because they then ‘freeze out’ at $k_B T \ll \text{Re} \hbar \omega_k$. This is not likely for current room temperature experiments, however, as estimates based on finite-size effects show.

In summary, we have discussed an example of quantum thermodynamics for an overdamped field, calculating the Casimir energy due to (suitably generalized) zero-point energies and its dependence on temperature. This applies to a simple physical system of two metallic plates that has been controversially discussed for several years by now. We have considered diffusive (eddy or Foucault) currents in normally conducting plates and their correlations via magnetic fields that lead to a repulsive pressure. These modes have a range of characteristic frequencies that is very different from photon modes. They cross over from a quantum to a thermal regime in a range of parameters that is accessible with current experiments, namely when the energy scale of diffusive transport (Thouless energy $\hbar D/L^2$) exceeds $k_B T$. The finite-temperature quantum field theory of these modes gives significant entropic corrections to the Casimir pressure that set in at distances shorter than the thermal photon wavelength and explain quantitatively the large thermal corrections to the Casimir pressure in the Lifshitz theory for Drude metals. We reproduce the nonzero Casimir entropy for this system and suggest a physical understanding of its unusual behaviour. In the particular case that the damping rate becomes infinitesimally small at low $T$, an entropy defect remains that has been interpreted in terms of a glassy state of quasi-static Foucault currents, the Nernst theorem being not applicable. This limit does not recover the lossless case (plasma model) because there, magnetic fields are expelled from the bulk, as in the London description of a superconductor. Similar conclusions could also apply to other classes of modes (like overdamped coupled surface plasmons\textsuperscript{[6]}) providing insight into similar controversies and perhaps a route to engineer the Casimir force, similar to the suggestion in\textsuperscript{[7]}. From a theoretical point of view, our analysis illustrates that the unusual behavior of quantum field theory for normally conducting mirrors finds a clear physical justification in the framework of open quantum systems, glassy dynamics, and superconductivity, and paves the way for further theoretical and experimental investigations.

Acknowledgments. — The work of FI was supported by the Alexander von Humboldt Foundation and in part by the US National Science Foundation (Grant No. PHYS05-51164). We acknowledge funding from the ESF research network programme “New Trends and Applications of the Casimir Effect” and the Deutsche Forschungsgemeinschaft. We thank G. Bimonte, D. Dalvit, S. Ellingsen, J.-J. Greffet, H. Haakh, B. Horovitz, G.-L. Ingold, G. Klimchitskaya, A. Lambrecht, S. K. Lamoreaux, V. M. Mostepanenko, L. Pitaevskii, S. Reynaud, F. da Rosa, and F. Sols for instructive comments.

Appendix: eddy current mode density

For the density of modes at real frequencies, $\rho(\omega; L)$, of the electromagnetic Casimir effect we use the following prescription: the free energy (per unit area) is written in the form

$$ F = \int_0^\infty d\omega \, \rho(\omega; L) f(\omega) $$

where $f(\omega) = k_B T \log[2 \sinh(\hbar \omega / 2 k_B T)]$ is the free energy per mode. For two parallel, identical plates, this gives

$$ \rho(\omega; L) = -\frac{1}{\pi} \partial_\omega \text{Im} \sum_\mu \int_0^\infty \frac{d^2 k}{(2 \pi)^2} \log D_\mu(k, \omega + i0) $$

$$ D_\mu(k, \omega) = 1 - \nu_\mu^2(k, \omega) e^{-2 \kappa L} $$

The integration is over the wavevector $k$ parallel to the plates, and the two polarizations $\mu = TE, TM$ are summed over. The $\nu_\mu(k, \omega)$ are the reflection coefficients for the electromagnetic field from a single half-space, and $\kappa$ is defined after Eq.\textsuperscript{[13]} below. Passivity does not allow for poles and zeros of the dispersion function $D_\mu(k, \omega)$ in the upper half of the complex plane. Its analytical continuation into the lower half plane does show, however, poles, zeros and branch cut singularities: the zeros (poles) actually define the complex mode frequencies of the two mirror system (of two isolated mirrors, respectively). Branch cuts arise by taking a difference of mode densities in the continuous part of the spectrum. The expansion of the mode density $\rho(\omega; L)$ over these singularities, displaying explicitly the contribution of a branch cut along the negative imaginary axis, reads\textsuperscript{[11]}

$$ \rho(\omega; L) = -\int d\xi \text{Im} \frac{\tilde{\rho}(\xi; L)}{\omega + i \xi} + \rho_{\text{other modes}}(\omega; L) $$

where we have introduced the (real-valued) mode density along the branch cut $\tilde{\rho}(\xi; L)$. The first term is Eq.(6) of the main paper.

In order to calculate the mode density $\tilde{\rho}(\xi; L)$, we specialize to a specific form for the reflection coefficients:
namely the Fresnel form with the Drude model

\[ \varepsilon(\omega) = 1 - \frac{\Omega^2}{\omega(\omega + i\gamma)} \]  

(13)

involving the plasma frequency \( \Omega \) and \( \gamma \) the scattering rate for the current density. In the TE and TM polarization, respectively [6]

\[ r_{TE}(k, \omega) = \frac{\kappa - \kappa_m}{\kappa + \kappa_m}, \quad r_{TM}(k, \omega) = \frac{\varepsilon(\omega)\kappa - \kappa_m}{\varepsilon(\omega)\kappa + \kappa_m} \]  

(14)

where \( \kappa = \sqrt{k^2 - (\omega/c)^2} \) and \( \kappa_m = \sqrt{k^2 - (\omega/c)^2\varepsilon(\omega)} \).

The branch cut at negative imaginary frequencies \( \omega = -i\xi \) is associated with the square root \( \kappa_m \); it connects the solutions of the equations

\[ k^2 - \varepsilon(\omega)(\omega/c)^2 = 0, \quad -\infty \]  

(15)

The second equation easily gives \( \omega = -i\gamma \); the first equation is a third order equation of which we pick the purely imaginary root \( \omega = -i\kappa_k \). (The other two correspond to the plasma edge above which electromagnetic waves penetrate into the bulk of the mirrors.) In the limit \( k \ll \Omega/c \), we get

\[ \omega = -i\kappa_k \approx -iDk^2 \]  

(16)

where \( D = \gamma c^2/\Omega^2 \) is the diffusion coefficient. Both reflection coefficients and hence the dispersion function \( \{11\} \) show a branch cut for \( \omega \in [-i\kappa_k, -i\gamma] \). The mode density \( \tilde{\rho}(\xi; L) \) along this cut is found from the argument principle, taking an integration contour around the negative imaginary axis. This gives

\[ \tilde{\rho}(\xi; L) = -\partial_\xi \text{Im} \sum_\mu \int \frac{d^2 k}{(2\pi)^2} \text{Im} \log D_\mu(k, -i\xi + 0) \]  

(17)

where the +0 prescribes the side of the branch cut in the right half plane (setting the sign of the wave vector \( \kappa_m \)).

The same result can also be obtained using a scattering approach. In this case, we begin with the dispersion relation in a bulk Drude metal,

\[ 0 = \frac{c^2q^2 - \omega^2\varepsilon(\omega)}{c^2\varepsilon^2 - \omega^2} = 1 + \frac{\Omega^2}{c^2q^2 - \omega^2} \frac{\omega}{\omega + i\gamma} \]  

(18)

where \( \varepsilon(\omega) \) is again the Drude model. Both dispersion functions in the numerator and denominator of this expression are a simple re-writing of the wave equations after Fourier transformation. The poles of Eq.\{18\} illustrate the subtraction operated in the bulk dispersion energy: the free space photon continuum and locally damped currents (not coupled to the long-range electromagnetic field). Looking for the solutions to Eq.\{18\}, three modes are found, of which one always occurs at imaginary frequencies \( \omega_q = -i\kappa_q \). Its limits for small and large \( q \) are

\[ q \ll \Omega/c: \quad \omega_q \approx -iDq^2 \]  

(19)

\[ q \gg \Omega/c: \quad \omega_q \approx -i\gamma \]  

(20)

where \( D \) is again the electromagnetic diffusion constant. In view of planar surfaces for the metal, we label the modes by the wavevector \( k \) projected onto the plane \( z = 0 \) and the frequency \( -i\gamma \). The perpendicular wavevector \( k_z \) is then fixed by the dispersion relation \( \{18\} \).

The electric current density associated with diffusive waves cannot escape into vacuum (except for the displacement current). Hence we get total internal reflection with a coefficient of modulus unity. In the vacuum gap, the (transverse) electromagnetic field varies exponentially with a decay constant \( k^2 = k^2 + \varepsilon^2/c^2 \). The single-interface reflection coefficient is obtained applying the usual boundary conditions for the electromagnetic field. In the TE-polarization, we find

\[ r_D(k, -i\xi) = -\frac{\kappa + ik_z}{\kappa - ik_z} = -r_{TE}(k, -i\xi + 0) \]  

(21)

With two interfaces at \( z = \pm L/2 \), odd and even modes with respect to the mid-plane of the cavity vary as \( \sinh \kappa z \) and \( \cosh \kappa z \) for \( |z| \leq L/2 \). In the medium, we find that the reflection coefficient \( r_D \) is modified by a phase shift \( e^{-i\delta} \), and this gives access to the change in the mode density (i.e., the DOS \( \tilde{\rho}(k, \xi) \)) of the diffusive wave continuum \{20\}. The phase shifts are measured with respect to the configuration where the mirrors are placed an infinite distance apart. Alternatively, we can compute the transmission coefficient across the vacuum gap and get the DOS from its phase \{27\}. Summing over even and odd modes we get a phase shift

\[ e^{-2i\delta} = \frac{1 - (r_D^*)^2 e^{-2\kappa L}}{1 - r_D^* e^{-2\kappa L}} \]  

(22)

The mode density along imaginary frequencies, for k-vector and polarization, is thus

\[ \tilde{\rho}(k, \xi) = -\frac{1}{\pi i \xi} \text{Im} \log [1 - r_D^* (k, -i\xi) e^{-2\kappa L}] \]  

(23)

Integrating over the parallel wavevector and summing over the polarizations, we recover Eq.\{17\}.
[7] C. Henkel et al., Phys. Rev. A 69, 023808 (2004); F. Intravaia and A. Lambrecht, Phys. Rev. Lett. 94, 110404 (2005); F. Intravaia, C. Henkel, and A. Lambrecht, Phys. Rev. A 76, 033820 (2007).
[8] K. A. Milton, J. Phys. A 37, R209 (2004).
[9] V. M. Mostepanenko et al., J. Phys. A 39, 6589 (2006).
[10] J. R. Torgerson and S. K. Lamoreaux, Phys. Rev. E 70, 047102 (2004).
[11] G. Bimonte, New J. Phys. 9, 281 (2007).
[12] V. B. Svetovoy, Phys. Rev. A 76, 062102 (2007).
[13] D. J. Thouless, Phys. Rev. Lett. 39, 1167 (1977).
[14] F. Intravaia and C. Henkel, J. Phys. A 41, 164018 (2008).
[15] F. Intravaia, S. Maniscalco, and A. Messina, Phys. Rev. A 67, 042108 (2003).
[16] J. S. Høye et al., Phys. Rev. E 75, 051127 (2007), comment: G. L. Klimchitskaya and V. M. Mostepanenko, Phys. Rev. E 77, 023101 (2008); reply ibid. 023102
[17] G.-L. Ingold, A. Lambrecht, and S. Reynaud, arXiv:0905.3608 (2009).
[18] V. B. Bezerra et al., Phys. Rev. A 69, 022119 (2004).
[19] B. E. Sernelius, Phys. Rev. B 71, 235114 (2005); V. B. Svetovoy and R. Esquivel, J. Phys. A 39, 6777 (2006).
[20] G. Bimonte, Phys. Rev. A 78, 062101 (2008).
[21] H. Haakh, F. Intravaia, and C. Henkel, in preparation.
[22] F. London and H. London, Proc. Roy. Soc. (London) A 149, 71 (1935).
[23] R. Decca et al. Phys. Rev. D 75, 077101 (2007); Eur. Phys. J. C 51, 963 (2007).
[24] A. Kittel et al., Phys. Rev. Lett. 95, 224301 (2005); P.-O. Chapuis et al., Phys. Rev. B 77, 035431 (2008); T. Varpula and T. Poutanen, J. Appl. Phys. 55, 4015 (1984); M. P. A. Jones et al., Phys. Rev. Lett. 91, 080401 (2003).
[25] F. Chen et al., Opt. Express 15, 4823 (2007); G. L. Klimchitskaya and B. Geyer, J. Phys. A 41, 164032 (2008). V. B. Svetovoy, Phys. Rev. Lett. 101, 163603 (2008).
[26] G. Barton, Rep. Prog. Phys. 42, 963 (1979).
[27] M. Bordag and U. Mohideen and V. M. Mostepanenko, Phys. Rep. 353, 1 (2001).