Reducing the shear

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Abstract

As gravitational lensing measurements become increasingly precise, it becomes necessary to include ever higher order effects in the theoretical calculations. Here we show how the difference between the shear and the reduced shear manifest themselves in a number of commonly used measures of shear power. If we are to reap the science rewards of future, high precision measurements of cosmic shear we will need to include this effect in our theoretical predictions.

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1 Introduction

Weak gravitational lensing by large-scale structure (Mellier, 1999; Bartelmann & Schneider, 2001) is becoming a central means of constraining our cosmological model (see e.g. Hoekstra, Yee, & Gladders, 2002; van Waerbeke, Mellier & Hoekstra, 2005, for the current status). Continuing effort has yielded increasingly stringent control of systematic errors and ever larger surveys are decreasing the statistical errors rapidly. As the precision with which the measurements are made increases, one demands ever higher fidelity in the data-analysis, modeling and theoretical interpretation.

One such area is the computation of the theoretical predictions for various statistics involving the measurable ‘reduced shear’ (Mellier, 1999; Bartelmann & Schneider, 2001)

\[ g \equiv \frac{\gamma}{1 - \kappa} \text{ for } |g| < 1 \quad (1) \]

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where $\gamma$ is the ‘plain’ shear and $\kappa$ is the convergence which is related to the projected mass along the line of sight. Both $\kappa$ and $\gamma$ are defined through the Jacobian of the mapping between the source and image planes. If the lensing is weak then $|\kappa| \ll 1$, and to lowest order $g = \gamma$ and the power spectra of the shear and convergence are equal: $C_{\ell}^\gamma = C_{\ell}^\kappa$. Thus measurements are usually compared to predictions of $C_{\ell}^\kappa$ derived from models of the non-linear clustering of matter. This relation has corrections however, which need to be taken into account if precise comparisons with data are to be made in the future. While there has been previous analytic work on this subject (Schneider et al., 1998; Dodelson & Zhang, 2005) and some numerical work on magnification statistics (Takada & Hamana, 2003; Barber & Taylor, 2003; Menard et al., 2003), there has been no comprehensive investigation of this effect in simulations. In this short paper we present some results, derived from simulations similar to those described in (White & Vale, 2004), on the size of the corrections for a variety of commonly used statistics on the angular scales where most current lensing work has focused.

2 Simulation

The results are derived from 7 N-body simulations of a $\Lambda$CDM cosmology run with the TreePM code (White, 2002). Each simulation employed $384^3$ equal mass dark matter particles in a periodic cubical box of side $200\, h^{-1}\text{Mpc}$ run to $z = 0$ with phase space data dumped every $50\, h^{-1}\text{Mpc}$ starting at $z = 4$. The cosmology was the same for each simulation ($\Omega_{\text{mat}} = 0.28$, $\Omega_B h^2 = 0.024$, $h = 0.7$, $n = 1$ and $\sigma_8 = 0.9$) but different random number seeds were used to generate the initial conditions. The purpose of these runs was to allow a study of the statistical distributions of lensing observables. Each box was used to generate 16 approximately independent lensing maps, each $3^\circ \times 3^\circ$, using the multi-plane ray-tracing code described in (Vale & White, 2003). Each map was produced with $2048^2$ pixels and then downsampled to $1024^2$ pixels. Thus a total of 112 maps or 1008 square degrees were simulated, all with the same cosmology. We report on the results for sources at $z \approx 1$ (specifically all at a comoving distance of $2400\, h^{-1}\text{Mpc}$) here. Other data can be obtained from http://mwhite.berkeley.edu/Lensing.

\footnote{Tests with maps downsampled to $512^2$ pixels indicate that our results are converged at the several percent level in the ratios that we quote, with better agreement at larger scales as expected.}
3 Results

3.1 Two point statistics

The lowest order information on large-scale structure comes from studying the 2-point statistics of the shear field. A number of different 2-point statistics have been used in the literature, each with their own strengths and weaknesses. We survey the most commonly used statistics here.

We first show the results on the variance of the shear, smoothed on a range of angular scales. Though it has numerous drawbacks in interpretation, this was one of the first statistics computed from shear maps due to its ease of computation. The left panel of Figure 1 shows the (reduced) shear variance as a function of smoothing scale. For each point we compute a map of the amplitude of the reduced shear, smooth the map with a 2D boxcar of side length $R$ and then compute the variance of the resulting map. The points are highly correlated. Since we have full information about the shear and convergence from the simulation we are able to compare this with the plain shear results which are usually computed. The right panel of Figure 1 shows the difference of this statistic computed for the reduced shear ($g$) to that for plain shear ($\gamma$). We see that the reduced shear variance is larger than the usually predicted shear variance by a non-trivial amount. The bias very gradually drops as we go to larger scales (not shown here) but we are unable...
to follow it to convergence due to the finite size of the fields simulated.

Naively one might imagine that the corrections would be smaller than shown in Figure 1 because $\kappa$ is small ‘on large scales’. However $\kappa$ is only small when averaged over a sizeable region of sky, and this smoothing does not commute with the division in Eq. (1). This fact also means that perturbative calculations must be used with care, because the convergence or shear amplitude can be quite large on small scales leading to a breakdown of the approximation.

Though the smoothed shear variance has the worst behavior of the two point functions we consider, we will see qualitatively similar behavior below. In general this tendency for the small-scale power to be increased over the theoretical prediction for plain shear needs to be considered before attributing “excess small-scale power” to additional physical effects e.g. intrinsic galaxy alignments. A signature of the effect might be the difference in the sensitivity of the various statistics that we compute here.

Another popular measure of the power is the aperture mass (Schneider et al., 1998). This is a scalar quantity which can be derived from an integral over the shear

$$M_{ap}(\theta_0; R) \equiv \int d^2\theta \, Q(\vec{\theta}; R) \gamma_T(\vec{\theta} + \vec{\theta}_0)$$

(2)

where $\gamma_T$ is the tangential shear as measured from $\theta_0$ and $Q$ is a kernel. We have chosen the $\ell = 1$ form for definiteness, so

$$Q(r = x/R; R) = \frac{6}{\pi R^2} r^2(1 - r^2) \text{ for } r \leq 1$$

(3)

and $Q$ vanishes for $r > 1$. The results for the aperture mass variance are shown in Figure 2. Note that the correction is quite large on small scales, but drops rapidly on scales above a few arcminutes.

Finally we show results for the 2-point correlation function of the shear in Figure 3. This is the best behaved 2-point statistic that we consider, since it explicitly eliminates the contribution from small angular scales\(^3\). We define $\xi = \langle \gamma_+\gamma_+ \rangle$ where $\gamma_+$ is (minus) the component $\gamma_1$ of the shear in the rotated frame whose $\hat{x}$-axis is the separation vector between the two points being correlated. The component at $45^\circ$ is called $\gamma_x$. We find that the correction to the correlation function is quite small on the scales shown here. The correction to the other correlation function, $\langle \gamma_x\gamma_x \rangle$, is even smaller than for $\xi$.

It is not too surprising that $\xi$ receives smaller corrections than $\text{Var}[M_{ap}]$ when we recall that $M_{ap}$ probes smaller scales than $R$ by a factor of roughly 3 (Schneider et al., 1998). A smaller piece of the difference is that $M_{ap}$ can be

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\(^3\) For this reason the 2-point correlation function is less sensitive to small-scale observational systematics than other quantities such as $\text{Var}[M_{ap}]$.\(^4\)
Fig. 2. The relative difference of the $M_{ap}$ variance computed for plain shear and reduced shear. The points show the mean difference, averaged over 112 maps, each $3^\circ \times 3^\circ$, while the error bars indicate the standard deviation of the ratio.

expressed as an integral over $\xi$ extending all the way to zero lag\textsuperscript{4}, making $M_{ap}$ slightly more sensitive to the difference between $g$ and $\gamma$ on small scales. A similar argument holds for $\text{Var}[|g|_R]$, with the effect being more pronounced. A reduction in the sensitivity to small scale power and to shot-noise could be obtained from differencing these measures between scales, with an associated loss of power.

These are all of the 2-point functions commonly derived from shear data – the power spectrum has only been derived by two groups (Pen et al., 2003; Brown et al., 2003) despite being almost ubiquitous in forecasts of the potential of future lensing experiments (e.g. Hui, 1999; Huterer, 2001; Benabed & Bernardeau, 2001; Hu, 2001; Weinberg & Kamionkowski, 2002; Munshi & Wang, 2003; Refregier et al., 2004; Takada & White, 2004; Huterer & Takada, 2005).

\textsuperscript{4} The variance of any convolution of $\kappa$ must have weight at zero lag.
3.2 Three point statistics

Since the lensing maps are non-Gaussian there is information beyond\(^5\) the 2-point statistics. We show in Figure 4 the dominant 3-point function for equilateral triangles as a function of scale. The 3-point function is \(\langle \gamma_+ \gamma_+ \gamma_+ \rangle\) with \(\gamma_+\) defined with respect to a line joining the triangle center to each vertex (Takada & Jain, 2002; Zaldarriaga & Scoccimarro, 2003; Schneider & Lombardi, 2003). In the weakly non-Gaussian limit one can argue that the corrections to the 3-point function from considering reduced shear could be very large. Counting powers of our small parameter, \(\kappa\) or \(\gamma\), the lowest order contribution vanishes in the Gaussian limit. Thus the first non-vanishing term is of order \(\kappa^4\) (Schneider et al., 1998; Dodelson & Zhang, 2005). The first correction from the reduced shear also comes in at order \(\kappa^4\). However we see that the difference is not significantly larger than in the case of the 2-point functions on the scales shown here.

\(^5\) This information is not, however, independent. We find a strong correlation between the amplitude of the 2- and 3-point functions on scales of several arcminutes in our maps.
We also show, in Figure 5, how the configuration dependence of the 3-point function is modified. The 3-point function for triangles with sides $\theta_1 = 2'$ and $\theta_2 = 3'$ is plotted as a function of the cosine of the included angle. We have chosen here a triangle with no special symmetries in order to illustrate a “typical” case. Note that the correction is dependent on angle, showing that the shear and reduced shear have different shape dependence which should be taken into account in cosmological inferences (e.g. Ho & White, 2004; Dolney, Jain & Takada, 2004).

4 Conclusions

Deflection of light rays by gravitational potentials along the line of sight introduces a mapping between the source and image plane. The Jacobian of this mapping defines the shear and convergence as a function of position on the sky. In the absence of size or magnification information neither the shear nor the convergence is observable, rather the combination $g = \gamma/(1 - \kappa)$ is. Since on small scales $\kappa$ can be non-negligible this introduces complications in predicting the observables of weak lensing. We have illustrated specifically the

![Graph showing the relative difference of the 3-point correlation function, $\zeta_{+++}$, for equilateral triangle configurations computed for plain shear and reduced shear. The points show the mean difference, averaged over 32 maps, each $3^\circ \times 3^\circ$, while the error bars indicate the standard deviation. ACDM, $z_s \approx 1$ and Equilateral triangles are indicated. The x-axis represents $\theta$ (arcmin) ranging from 0 to 10, and the y-axis represents $\zeta_{s+++}/\zeta_{\gamma+++} - 1$ ranging from 0 to 0.25.]}
Fig. 5. The relative difference of the 3-point correlation function for one triangle size, $\zeta_{+++}(\theta_1 = 2', \theta_2 = 3')$, computed for plain shear and reduced shear. The points show the mean difference, averaged over 32 maps, each $3^\circ \times 3^\circ$, while the error bars indicate the standard deviation.

The effect of using $g$ rather than $\gamma$ on a number of commonly used statistics of weak lensing. We found that the correlation function is least affected, as expected since it specifically eliminates small-scale information. The variance of the shear amplitude, smoothed on a scale $R$, is the most dramatically affected, and the amplitude of the effect declines very slowly with increasing smoothing scale.

Existing lensing experiments derive most of their cosmological constraints from angular scales of a few arcminutes to a few tens of arcminutes. The corrections described here are below the current statistical errors, and so should not affect existing constraints. The next generation of experiments may have sufficient control of systematic errors, and sufficient statistics, that this effect will need to be included in the analysis. Note that even for the correlation function the difference between the commonly predicted plain shear result and the reduced shear value is comparable to the level of systematic control that we need to achieve to study dark energy! If we are to reap the science rewards of future, high precision measurements of cosmic shear we will need to be able to include this effect in our theoretical predictions.

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