Large scale directional anomalies in the WMAP 5yr ILC map

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ABSTRACT: We study the alignments of the low multipoles of CMB anisotropies with specific directions in the sky (i.e. the dipole, the north Ecliptic pole, the north Galactic pole and the north Super Galactic pole). Performing $10^5$ random extractions we have found that: 1) separately quadrupole and octupole are mildly orthogonal to the dipole but when they are considered together, in analogy to [7], we find an unlikely orthogonality at the level of 0.8% C.L.; 2) the multipole vectors associated to $\ell = 4$ are unlikely aligned with the dipole at 99.1% C.L.; 3) the multipole vectors associated to $\ell = 5$ are mildly orthogonal to the dipole but when we consider only maps that show exactly the same correlation among the multipoles as in the observed WMAP 5yr ILC, these multipole vectors are unlikely orthogonal to the dipole at 99.7% C.L..

KEYWORDS: CMBR theory, CMBR experiments.
1. Introduction

The anisotropy pattern of the cosmic microwave background (CMB), obtained by Wilkinson Microwave Anisotropy Probe (WMAP), probes cosmological models with unprecedented precision (see [1] and references therein). WMAP data are largely consistent with the concordance $\Lambda$ cold dark matter ($\Lambda$CDM) model, but there are some interesting deviations from it, in particular on the largest angular scales. Among these deviations, we focus on the alignments of low multipoles. Unlikely (for a statistically isotropic random field) alignment of the quadrupole and the octupole is described in references [2, 3, 4, 5, 6]. Moreover, both quadrupole and octupole align with the CMB dipole [5, 7]. Other unlikely alignments are described in [8].

We study directionality anomalies considering estimators defined for each multipole $\ell$. In the present paper we limit our analysis to the range $\Delta \ell = 2 - 7$. We take into account two different Monte-Carlo (henceforth MC) simulations. In the first one, we study directional anomalies considering the coefficients of the spherical harmonics expansions $a_{\ell m}$ randomly extracted from a Gaussian distribution. In the second one we consider maps in which the relative alignments of the low multipoles are fixed and given by the WMAP 5yr ILC map (henceforth called ILC map for sake of brevity). Both MCs are performed taking into account WMAP's anisotropic pixel noise (see below for details). Through the first MC, we obtain that quadrupole and octupole are mildly unlikely orthogonal to the dipole (this anomaly become stronger when they are combined in analogy to what has been performed in [7]). Moreover we find that $\ell = 4$ is unlikely aligned with the dipole and $\ell = 5$ is very mildly unlikely orthogonal to the dipole. Through the second MC, we find that quadrupole
and octupole are (mildly) unlikely orthogonal to the dipole and combining them together
 stil in analogy to [7] we do not find any increase of this anomaly. In this case \( \ell = 4 \) is
 unlikely aligned with the dipole and \( \ell = 5 \) is unlikely orthogonal to the dipole. We do not
 obtain unlikely alignments with the dipole for \( \ell = 6 \) and \( \ell = 7 \) for both MCs.

The paper is organized as follows: in Section 2 we present the Multipole Vectors
 expansion and the tools that have been used to perform the statistical analysis; such
 analysis is given in Section 3 where we present also our main results; conclusions are
 drawn in Section 4. Some technicalities about the performed simulations can be found in
 Appendix A.

2. Tools and estimators

The alignment of multipoles can be defined using a new representation of CMB anisotropy
 maps where the \( a_{\ell m} \) (coefficients of the expansion over the basis of spherical harmonics)
 are replaced by vectors [2, 3]. In particular, each multipole order \( \ell \) is represented by \( \ell \) unit
 vectors \( \hat{u}_i \) and one amplitude \( A \)

\[
a_{\ell m} \leftrightarrow A^{(\ell)}, \hat{u}_1, \cdots, \hat{u}_\ell.
\]

Note that the number of independent objects is the same in the l.h.s and r.h.s. of equation
 (2.1): \( 2\ell + 1 \) for \( a_{\ell m} \) equals \( 3\ell \) (numbers of components of the vectors) +1 (given by \( A^{(\ell)} \))
 − \( \ell \) (because there are \( \ell \) constraints due to the normalization conditions of the vectors). For a more detailed explanation of equation (2.1) and of the properties of that association
 see for example references [2, 3, 8, 21].

Unfortunately, an explicit analytical expression for the association given in equation
 (2.1), is possible only for \( \ell = 1 \) and for \( \ell \neq 1 \) numerical method are needed\(^1\). The Copi et
 al.’s algorithm (which use is acknowledged here) for constructing multipole vectors from a
 standard spherical harmonic decomposition is described in [2] and the implementation of
 it is publicly available\(^2\). Other methods exist but, as far as we know, their implementation
 is not publicly available on a standard platform (see for example [3, 9] where the problem
 of finding \( \ell \) vectors is translated into the problem of finding the zeros of a polynomial of
 degree \( 2\ell \)).

In order to investigate whether a map presents unlikely directions with respect to
 random CMB sky extractions\(^3\), we define the following estimators (scalars quantities) for
 each multipole order \( \ell \):

\[
S_{\ell 1} = \sum_{i=1}^{\ell} |\hat{u}_i \cdot \hat{d}| / \ell,
\]  

where \( \hat{u}_i \) represent the multipole vectors associated to a multipole order \( \ell \) of a map, \( \hat{d} \) is a
 fixed direction in the sky and where the absolute value is taken into account since multipole

\(^1\)Indeed, for \( \ell = 2 \) it is possible to obtain the multipole vectors computing the eigenvectors of a symmetric
 and traceless tensor representing the quadrupole, see [11, 12].

\(^2\)http://www.phys.cwru.edu/projects/mpvectors/

\(^3\)See Section 3.1 for a detailed description of the performed extractions.
vectors are vectors without “head” (this is the so called reflection symmetry). The division by $\ell$ is considered in order to define the estimators (2.2) between 0 and 1.

For sake of comparison, in this paragraph we present the estimator used in [5, 7] (where it is shown that quadrupole and octupole are unlikely aligned with the dipole). The definition of that estimator is the following

$$S_{321} = \frac{1}{4} \left( |\vec{q} \cdot \hat{d}| + \sum_{i=1}^{3} |\vec{o}_i \cdot \hat{d}| \right),$$  \hspace{1cm} (2.3)

where $\vec{q}$ and $\vec{o}_i$ are the area vectors built from the quadrupole and octupole multipole vectors respectively. More explicitly:

$$\vec{q} = \hat{q}_{21} \times \hat{q}_{22},$$  \hspace{1cm} (2.4)

and

$$\vec{o}_1 = \hat{o}_{32} \times \hat{o}_{33},$$  \hspace{1cm} (2.5)

$$\vec{o}_2 = \hat{o}_{33} \times \hat{o}_{31},$$  \hspace{1cm} (2.6)

$$\vec{o}_3 = \hat{o}_{31} \times \hat{o}_{32},$$  \hspace{1cm} (2.7)

where $\hat{q}_{2i}$ with $i = 1, 2$ are the two multipole vectors associated to the quadrupole and $\hat{o}_{3j}$ with $j = 1, 2, 3$ are the three multipole vectors associated to the octupole. In eq. (2.3) the division by 4 has been taken into account to define the estimator $S_{321}$ in the range $[0, 1]$ and the absolute values are considered to fulfill the reflection symmetry.

3. The Analysis

3.1 Description of the Random Extractions

As already stated in Section 1 we perform two kinds of MCs in order to study how likely the ILC map $^4$ is aligned to some specific direction.

For the first MC we extract $10^5$ random maps from a Gaussian distribution from an angular power spectrum corresponding to the best fit of WMAP 5yr (see footnote 4). For each random extraction, the estimators $S_{\ell 1}$ (with $\ell = 2, 3, 4, 5, 6, 7$) and $S_{321}$ are computed for some direction of interest such as the Dipole (DIP), the north Ecliptic pole (NEP), the north Galactic pole (NGP) and the north Super Galactic pole (NSGP).

For the second MC, as done in [5], we freeze the relative alignments of the low multipoles such as those that are observed in the ILC map and perform $10^5$ random rotations of that map. For each random rotation the estimators $S_{\ell 1}$ (with $\ell = 2, 3, 4, 5, 6, 7$) and $S_{321}$ are computed for the aforementioned directions (DIP, NEP, NGP and NSGP namely).

Both MCs are performed taking into account WMAP’s anisotropic pixel noise$^5$. The resolution that has been considered for the analysis is given by the HEALP$\text{IX}^6$ [10] param-

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$^4$The 5 years WMAP ILC map, as well as other CMB data products, is publicly available at the Lambda web site: http://lambda.gsfc.nasa.gov/.

$^5$We consider the WMAP V band noise and hits files available at Lambda web site. See footnote 4.

$^6$http://healpix.jpl.nasa.gov
eter \( N_{side} = 16 \) that corresponds to 3072 pixels\(^7\). See appendix A.2 for details about the degradation issue.

### 3.2 Results

In this Section we provide the results for both MCs.

Results for the first MC are summarized in Table 1 where the probabilities to obtain a smaller value for the analyzed directions are listed for the ILC map.

**Table 1:** First MC: Probabilities (in percentage) to obtain a smaller value for the considered directions and estimators

| Estimator | DIP | NEP | NGP | NSGP |
|-----------|-----|-----|-----|-----|
| \( S_{21} \) | 6.7 | 42.3 | 9.6 | 41.5 |
| \( S_{31} \) | 3.6 | 86.0 | 7.2 | 56.2 |
| \( S_{321}^{\text{new}} \) | 0.8 | 73.6 | 2.6 | 49.4 |
| \( S_{41} \) | 99.1 | 2.6 | 48.2 | 74.5 |
| \( S_{51} \) | 9.9 | 30.9 | 56.5 | 87.7 |
| \( S_{61} \) | 76.2 | 58.8 | 37.5 | 95.0 |
| \( S_{71} \) | 54.6 | 4.0 | 41.4 | 88.6 |

Table 1 shows that multipole vectors associated to quadrupole, octupole and \( \ell = 5 \) are mildly unlikely orthogonal to the dipole. Multipole vectors associated to \( \ell = 4 \) are unlikely aligned to the dipole at the 99.1\% of C.L..

In analogy to what performed in \([5, 7]\), when \( S_{21} \) and \( S_{31} \) are combined as follows

\[
S_{321}^{\text{new}} = (2 S_{21} + 3 S_{31})/5
\]  

we find that the percentages to obtain a smaller value for the considered directions are 0.8, 73.6, 2.6, 49.4 for DIP, NEP, NGP and NSGP respectively. This confirms that when quadrupole and octupole are considered together as in eq. (3.1) then the anomaly related to the dipole direction becomes more severe. Note that the estimator defined in eq. (3.1) differs from eq. (2.3) even if both are of course related in information content. Considering eq. (2.3), it is shown in \([7]\) that the anomaly is as low as 0.3\%. Therefore both estimators (i.e. \( S_{321}^{\text{new}} \) and \( S_{321} \)) provide the same level of anomaly\(^8\).

Results for the second MC are reported in Figures 1, 2, 3, 4, 5, 6, 7 and 8 where we show the probability distribution function (pdf) for each \( S_{\ell 1} \), for \( S_{321} \) and for \( S_{321}^{\text{new}} \), and the value of the same estimator for the ILC map with various directions, represented by vertical lines in all the plots. We have chosen the following conventions: black line for the dipole, green line for north Ecliptic pole, blue line for the north Galactic pole, yellow line for the north Super Galactic pole. In Fig. 1 we recover the pdf of \( S_{321} \), given in \([7]\). See appendix A.1 for comments about its particular shape. In Fig. 2 we show the pdf for \( S_{21} \).

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\(^{7}\)The Healpix parameter \( N_{side} \) is related to the number of pixels \( N_{pix} \) through \( N_{pix} = 12 N_{side}^2 \).

\(^{8}\)We should remind that the ILC map used in \([7]\) was the WMAP 3 yr ILC map and not the last one available now (analyzed in this paper).
and in Fig. 3 for $S_{31}$. Since the DIP-values stand in left tails of the pdfs of both estimators, it is possible to state that the multipole vectors associated to quadrupole and octupole are nearly orthogonal to the dipole. The probability to obtain a smaller value is 6.9% and 7.8% for $S_{21}$ and $S_{31}$ respectively. When these estimators are combined as in eq. (3.1) than we obtain the pdf shown in Fig. 8. In this joint case the probability to obtain a smaller value than the DIP value is 6.6%. In Fig. 4 it is shown the $S_{41}$ estimator. Since the DIP-value of this estimator is in the right tail of its pdf, the multipole vectors associated to $\ell = 4$ are unlikely aligned to the dipole. The probability to obtain a larger value is 0.9%. In Fig. 5 we show the $S_{51}$ estimator. The DIP-value for this estimator is in the left tail of the corresponding pdf. Therefore we conclude that the multipole vectors associated to $\ell = 5$ are unlikely orthogonal to the dipole. The probability to obtain a smaller value is 0.3%. In Figs. 6 and 7 we show the pdfs for $S_{61}$ and $S_{71}$ respectively. In these cases, since the DIP-values fall in the middle of the two pdfs we do not find any anomalies. For sake of completeness, the probability to obtain a smaller value than the one that is observed is 68.9% for the $S_{61}$ and 55.1% for the $S_{71}$. In Table 2 the probabilities for each estimator and for each considered direction are summarized. The most unlikely percentages have
In Table 3 we report the multipole vectors associated to the low multipoles of the ILC 5yr map. For each multipole order $\ell$ we give the $\ell$ multipole vectors writing their colatitude and the longitude (in degrees).
Figure 6: $S_{61}$. Same conventions as Fig. 2. The plot presents the counts (y-axis) versus the statistic (x-axis).

Figure 7: $S_{71}$. Same conventions as Fig. 2. The plot presents the counts (y-axis) versus the statistic (x-axis).

Figure 8: $S_{121}^{\text{new}}$. Same conventions as Fig. 2. The plot presents the counts (y-axis) versus the statistic (x-axis).

4. Conclusions

Taking into account a new estimator defined in eq. (3.1), we have confirmed the unlikely alignment of quadrupole and octupole with the dipole in the WMAP 5yr ILC map. More-
Table 2: Second MC: probabilities (in percentage) to obtain a smaller value for the considered directions and estimators

| Estimator | DIP  | NEP  | NGP  | NSGP |
|-----------|------|------|------|------|
| $S_{21}$  | 6.9  | 45.5 | 9.6  | 44.5 |
| $S_{31}$  | 7.8  | 90.8 | 12.4 | 45.7 |
| $S_{new}$ | 6.6  | 65.7 | 10.5 | 41.7 |
| $S_{41}$  | 99.1 | 2.2  | 55.6 | 67.5 |
| $S_{51}$  | 0.3  | 16.4 | 69.0 | 96.9 |
| $S_{61}$  | 68.9 | 51.8 | 37.7 | 96.1 |
| $S_{71}$  | 55.1 | 11.3 | 45.3 | 79.4 |

over we have shown that there are new directional anomalies. Multipole vectors associated to $\ell = 4$ are unlikely aligned with the dipole at 99.1% C.L.. Note that this anomaly holds for both the MCs. Furthermore the multipole vectors associated to $\ell = 5$ are unlikely orthogonal to the dipole itself, with a probability at 99.7% C.L. when the MC is performed considering frozen the relative alignments of the low multipoles. This has been shown through MC simulations that properly take into account WMAP’s anisotropic pixel noise level (see footnote 5).

What causes these directional anomalies? It is difficult to answer this question. It is still unknown whether these anomalies come from fundamental physics or whether they are the residual of some not perfectly removed astrophysical foreground or systematic effect [13, 14, 15, 16, 17, 18, 19]. As an example of the latter kind, in references [20, 21] it is presented a study about the impact of the dipole straylight contamination on the low amplitude of the quadrupole and on the low $\ell$ alignments for Planck’s characteristics and capabilities.

Of course these anomalies might come from fundamental physics, e.g. a non trivial topology of the universe [23] or magnetic fields [24]. See also [22] where a Quadratic Maximum Likelihood method has been adopted to study a dipolar modulation.

As far as we know few attempts have been tried to study alignments anomalies in polarization [25] with WMAP data [26]. New data (i.e. Planck data) are awaited with great interest.

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9http://www.rssd.esa.int/planck
| Multipole order | Colatitude (°) | Longitude (°) |
|----------------|---------------|---------------|
| $\ell = 2$     | 102.03        | 305.16        |
| $\ell = 2$     | 106.96        | 184.22        |
| $\ell = 3$     | 77.98         | 313.59        |
| $\ell = 3$     | 130.23        | 272.81        |
| $\ell = 3$     | 97.18         | 203.91        |
| $\ell = 4$     | 25.05         | 203.82        |
| $\ell = 4$     | 118.63        | 154.69        |
| $\ell = 4$     | 90.00         | 68.91         |
| $\ell = 4$     | 128.68        | 37.97         |
| $\ell = 5$     | 57.20         | 288.28        |
| $\ell = 5$     | 52.83         | 39.38         |
| $\ell = 5$     | 125.69        | 279.84        |
| $\ell = 5$     | 37.20         | 228.60        |
| $\ell = 5$     | 92.39         | 355.78        |
| $\ell = 6$     | 37.20         | 34.20         |
| $\ell = 6$     | 54.32         | 77.34         |
| $\ell = 6$     | 73.04         | 288.28        |
| $\ell = 6$     | 81.61         | 222.19        |
| $\ell = 6$     | 32.60         | 239.32        |
| $\ell = 6$     | 105.71        | 151.88        |
| $\ell = 7$     | 55.77         | 345.94        |
| $\ell = 7$     | 69.26         | 92.81         |
| $\ell = 7$     | 97.18         | 203.91        |
| $\ell = 7$     | 48.19         | 7.03          |
| $\ell = 7$     | 62.72         | 174.78        |
| $\ell = 7$     | 157.94        | 45.00         |
| $\ell = 7$     | 69.26         | 286.88        |

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A. Some technicalities about MC simulations

In this Appendix we deal with some technicalities about the performed simulations. In particular we focus on the second aforementioned MC in which the relative alignments of the low multipoles are fixed and given by the ILC map. Subsections A.1 and A.2 refer to such a MC.
A.1 Shape of the estimators

The shape of the pdfs of the estimators is due to the relative alignments of the multipole considered. In this Section we perform a sort of “bootstrap” to the ILC map in order not to change its angular power spectrum while obtaining a new map with different relative alignments among the multipole vectors. As an example, we recompute the pdf of $S_{321}$ for this new map. This is done just to show that the particular (and sometimes puzzling) shape of the pdf of the considered estimators strongly depend on the phases of map taken under analysis. In order to not to change the angular power spectrum, we arbitrarily reshuﬄe the $a_{\ell m}$ for $\ell = 2, 3$ of the ILC map as follows:

$$a_{ILC}^{ILC}_{2,1/2} \rightarrow a_{ILC}^{ILC}_{2,2/1}$$

and

$$a_{ILC}^{ILC}_{3,1/2/3} \rightarrow a_{ILC}^{ILC}_{2,2/3/1}$$

leaving $a_{ILC}^{ILC}_{2,3,0}$ unchanged. In Fig. 9 we show how the shape of the pdf for the estimator $S_{321}$ change.

A.2 Impact of the degradation procedure

In this Section we briefly discuss our MC inputs. The ILC map is provided by the WMAP team at $N_{side} = 512$ and has to be degraded to be handled in a MC. We chose to work at $N_{side} = 16$ and we have to keep under control the impact of this degradation\(^\text{10}\). In order to evaluate the effect of the degradation on our results, we have compared the pdf’s of the considered estimators obtained performing a MC directly at high resolution (i.e. $N_{side} = 512$) to the ones obtained by a (far less expensive) MC at low resolution (i.e. $N_{side} = 16$). Since the high resolution MC is computationally heavy, we have reduced

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\(^{10}\)Since we are interested in quantities that depend on phases rather than norms of $a_{\ell m}$, the impact of the degradation upon angular power spectrum is not what we have to study. However, for sake of completeness, the effect of this degradation on angular power spectrum is of the order of few $\mu K^2$ that is well below the cosmic variance but above the angular power spectrum of the WMAP noise expected at those angular scales.
the number of random (angles) extractions to $10^3$. As an example, in the left panel of Fig. 10 we show the difference between the histogram at high resolution and the histogram at low resolution for the estimator $S_{21}$. This plot of the difference has to be compared to the difference between the histogram at low resolution that include an (anisotropic) noise realization (at the V band WMAP level) and the histogram at low resolution (just degraded, without any noise realization on it). See right panel of Fig. 10 for the noise effect on the estimator $S_{21}$. The level of the fluctuations in the right panel of Fig. 10 is larger than in the left panel of the same Figure, showing that the effect of noise covers the effect of the degradation. In order to quantify degradation and noise effects, we compute the standard deviation of the two histograms of the differences, obtaining $0.353$ for the noise impact, and $0.136$ for the degradation impact. See Table 4 for a summary of the above defined standard deviation for all the considered estimators. Since the effect of V band WMAP (anisotropic) noise is larger than the effect of the degradation for all the considered cases, we conclude that the process of the degradation does not introduce any significant spurious effect on the estimators we used to study anomalous directions.

Table 4: Impact of noise vs degradation

| Estimator | $\sigma_{\text{Noise}}$ | $\sigma_{\text{Degrad}}$ |
|-----------|-------------------------|--------------------------|
| $S_{21}$  | 0.353                   | 0.136                    |
| $S_{31}$  | 0.444                   | 0.066                    |
| $S_{41}$  | 0.403                   | 0.108                    |
| $S_{51}$  | 0.242                   | 0.107                    |
| $S_{61}$  | 0.573                   | 0.191                    |
| $S_{71}$  | 0.512                   | 0.132                    |

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