Adaptive fuzzy PID control for a quadrotor stabilisation

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Abstract. This paper deals with the design of an adaptive fuzzy PID control law for attitude and altitude stabilization of a quadrotor despite the system model uncertainties and disturbances. To this end, a PID control with adaptive gains is used in order to approximate a virtual ideal control previously designed to achieve the predefined objective. Specifically, the control gains are estimated and adjusted by mean of a fuzzy system whose parameters are adjusted online via a gradient descent-based adaptation law. The analysis of the closed-loop system is given by the Lyapunov approach. The simulation results are presented to illustrate the efficiency of the proposed approach.

1. Introduction
Nowadays, the most famous and successful unmanned aerial vehicles (UAVs) is the quadrotor. The reasons behind this success return to its simple and useful configuration, its low cost, and its huge domain of application.

The challenge now is the development of control algorithms that ensure a reliable, safe and optimal tasks’ achievement by quadrotors. Particularly, the quadrotor attitude and/or altitude stabilization is essential for many applications; it can be sufficient for some missions or a part of a control system for other ones. In fact, many classical and advanced control techniques have been applied to this end; adaptive sliding mode control [1], adaptive backstepping control [2], Lyapunov-based control [3], state feedback control [4], fuzzy PID control [5, 6] and the classical PID control[7-11].

The utilization of classical PID control is argued by its simple universal structure familiar to operators, its low cost, ease of implementation and its efficiency with a large class of systems. Unfortunately, this technique gives poor performances when the system is subject to disturbances and/or exhibits some changing characteristics. In order to deal with such circumstances, a PID control needs to be equipped with an additional mechanism for the gains online adaptation according to system and the control objective.

In this paper, one proposes an adaptive PID control scheme for the attitude and altitude stabilization of a quadrotor. Whithin this scheme, an adaptive fuzzy system for online estimation of the PID control gains such that desired performances are fulfilled.

The paper is organized as follows. In section 2 one presents the dynamical model of the quadrotor. Section 3 presents the proposed fuzzy adaptive PID control scheme. In section 4, one shows some simulation results. Section 5 concludes the paper.

2. Quadrotor model
The quadrotor is a small Unmanned Aerial Vehicle (UAV) with four propellers actuated by DC motors mounted on the end of two perpendicular arms. A basic diagram is shown in Figure 1. Each
The rotors pair of the same arm rotates in same direction; one pair rotates clockwise, while the other rotates counter clockwise. The quadrotor moves by adjusting the angular velocity of each rotor.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{quadrotor_config.png}
\caption{The quadrotor configuration [5]}
\end{figure}

One consider the dynamical model of quadrotor in [10] given by

\[
\begin{align*}
\dot{\phi} &= \theta \dot{\psi} \frac{1}{l_x} - \phi \dot{\Omega}_r + \frac{1}{l_x} u_1 \\
\dot{\theta} &= \phi \dot{\psi} \frac{1}{l_y} + \theta \frac{1}{l_y} \dot{\Omega}_r + \frac{1}{l_y} u_2 \\
\dot{\psi} &= \phi \dot{\psi} \frac{1}{l_z} + \frac{1}{l_z} u_3 \\
\ddot{X} &= \sin(\theta) \cos(\phi) \frac{1}{m} u_4 \\
\ddot{Y} &= -\sin(\phi) \frac{1}{m} u_4 \\
\ddot{Z} &= -g + \cos(\theta) \cos(\phi) \frac{1}{m} u_4
\end{align*}
\]  

(1)

Where \( \phi \) is the roll angle, \( \theta \) is the pitch angle and \( \psi \) is the yaw angle. \( X, Y, \) and \( Z \) are the Cartesian position coordinates. \( \Omega_r = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4 \) is the sum of rotors angular velocities. The relation between the control signal and the rotors angular velocities is given by

\[
\begin{bmatrix}
u_1 \\
u_2 \\
u_3 \\
u_4
\end{bmatrix} = \begin{bmatrix}
0 & -b & 0 & b \\
b & 0 & -b & 0 \\
-d & d & -d & d \\
b & b & b & b
\end{bmatrix} \begin{bmatrix}
\Omega_1 \\
\Omega_2 \\
\Omega_3 \\
\Omega_4
\end{bmatrix}
\]

In this work we are interested by the stabilization of the attitude (angles) and the height of the quadrotor, accordingly, the quadrotor can be seen as an interconnection of four subsystems of following model form

\[
\dot{y}_i = f_i(x) + g_i(x)u_i, \ i = 1, \ldots, 4
\]

(2)

Where \( x = [\phi \ \dot{\phi} \ \theta \ \dot{\theta} \ \psi \ \dot{\psi} \ X \ Y \ Z \ \dot{Z}]^T \) is the state vector, which is assumed available for measurement. \( u = [u_1 \ u_2 \ u_3 \ u_4]^T \) is the control input vector and \( y = [y_1 \ y_2 \ y_3 \ y_4]^T = [\phi \ \dot{\phi} \ \dot{\theta} \ \dot{\psi}] \) is the output vector. The nonlinear functions \( f_i(x) \) and \( g_i(x) \) are supposed unknown in this work.

3. Adaptive fuzzy PID control

The tracking errors are defined as

\[
e_i(t) = y(t) - y_i(t), \ i = 1, \ldots, 4
\]

and the corresponding filtered errors as

\[
s_i(t) = \hat{e}_i + \lambda_i e_i, \ \lambda_i \geq 0, \ i = 1, \ldots, 4
\]

(3)

Since the equality \( s_i(t) = 0 \) means that the tracking error \( e_i(t) \) and all its derivatives decrease exponentially to zero [12], therefore, it is sufficient to design a control law that ensures \( s_i(t) \to 0, \ i = 1, \ldots, 4 \).

The time derivative of the filtered tracking errors can be written as

\[
\frac{\dot{s}_i}{\dot{t}} = \frac{\dot{e}_i}{\dot{t}} + \lambda_i \frac{\dot{e}_i}{\dot{t}}
\]
\[ \dot{s}_i = \lambda_i \dot{e}_i - f_i(x) - g_i(x)u_i \]

We can prove easily that the ideal control laws \( u_i^* \) given by (5) achieve the control goal

\[ u_i^* = g_i(x)^{-1} \left( -f_i(x) + \lambda_i \dot{e}_i + +k_1s_i + k_0 \text{sign} \left( \frac{s_i}{\epsilon_0} \right) \right) \]

Where \( k_1 \) and \( k_0 \) are positive constants and \( \epsilon_0 \) is a small positive constant.

Since \( f_i(x) \), \( g_i(x) \) are unknown, the ideal controller (5) cannot be implemented. To overcome this issue and to achieve the control objective, in this paper one proposes an adaptive fuzzy PID control scheme to approximate the ideal control.

In fact, each ideal control \( u_i^*, i = 1, \ldots, 4 \), is approximated by an adaptive fuzzy PID control law \( u_i \) as

\[ u_i = K_{P1}e_i(t) + K_{I1} \int_0^t e_i(r) \, dr + K_{D1} \frac{de_i(t)}{dt} = E_i^T K_{PIDi} \]

Where \( K_{P1}, K_{I1} \) and \( K_{D1} \) are the proportional, the integral and the derivative gains respectively.

\[ E_i = [E_{1i}, E_{2i}, E_{3i}]^T = [e_i, \int_0^t e_i(r) \, dr, \frac{de_i(t)}{dt}] \text{ and } K_{PIDi} = [K_{P1}, K_{I1}, K_{D1}]^T. \]

Where each gains vector \( K_{PIDi} \) is estimated online using a zero order Takagi-Sugeno fuzzy system which has as inputs the vector \( E_i \). For each input \( E_{ij} (j = 1, 2, 3) \), we define \( m_k \) fuzzy sets \( F^k_{ij}, k = 1, \ldots, m_{ij} \). The system fuzzy rule base is the collection of \( If \rightarrow Then \) rules of the form

\[ R^k_{ij} : If \ E_{ij} \text{ is } G^k_{ij} \text{ and } E_{i2} \text{ is } G^k_{i2} \text{ and } E_{i3} \text{ is } G^k_{i3} \text{ THEN } K_{pi} \text{ is } c^k_{pi} \text{ and } K_{ii} \text{ is } c^k_{ii} \text{ and } K_{di} \text{ is } c^k_{di} \]

where \( G^k_{ij} \) is a fuzzy sets of \( F^k_{ij} \). For each input \( E_{ij} \), we define fuzzy sets \( \mu_{G_{ij}}, c^k_{pi}, c^k_{ii} \) and \( c^k_{di} \) as the crisp outputs of the \( l \)-th rule.

The fuzzy system output, namely the PID control gains are calculated by

\[ K_{PIDi} = \frac{\sum_{l=1}^{N_{ij}} \mu_{G_{ij}} c_{pi}}{\sum_{l=1}^{N_{ij}} \mu_{G_{ij}}}, K_{Ii} = \frac{\sum_{l=1}^{N_{ij}} \mu_{G_{ij}} c_{ii}}{\sum_{l=1}^{N_{ij}} \mu_{G_{ij}}}, K_{Di} = \frac{\sum_{l=1}^{N_{ij}} \mu_{G_{ij}} c_{di}}{\sum_{l=1}^{N_{ij}} \mu_{G_{ij}}}. \]

Where: \( \mu_{G_{ij}} = \prod_{j=1}^{3} \mu_{G_{ij}}(e_{ij}), \mu_{G_{ij}} \in \{ \mu_{F^k_{ij}}, \ldots, \mu_{M^k_{ij}} \} \) and \( \mu_{G_{ij}}(c_{ij}) \) is the \( F^k_{ij} \) fuzzy membership function.

Denote \( w^i_l = [\mu_{G_{ij}}, \ldots, \mu_{G_{ij}}]^T \), \( l = 1, \ldots, N \) \( \omega_i = [w^i_1, \ldots, w^i_N]^T \). \( c_{pi} = [c_{pi}^1, \ldots, c_{pi}^N]^T \), \( c_{ii} = [c_{ii}^1, \ldots, c_{ii}^N]^T \), \( c_{di} = [c_{di}^1, \ldots, c_{di}^N]^T \). From these notations (7) can be written in compact form as

\[ K_{PIDi} = [K_{pi}, K_{ii}, K_{di}] \text{ and } \omega_i = d \text{tag}[w^i_1, \ldots, w^i_N]. \]

Where \( c_i = [c_{pi}, c_{ii}, c_{di}]^T \), and \( W_i = d \text{tag}[w^i_1, \ldots, w^i_N] \).

Now, we replace the expression of \( K_{PIDi} \), which obtained by a fuzzy system (8) in equation (6) as

\[ u_i = E_i^T W_i^T c_i \]

Accordingly to the universal approximation theorem [13], one assume that the fuzzy PID control laws (9) can approximate the unknown ideal controls \( u_i^* \), which means that the unknown ideal controls \( u_i^* \) can be rewritten as

\[ u_i^* = E_i^T W_i c_i^* \]

where \( c_i^* = [c_{pi}^*, c_{ii}^*, c_{di}^*]^T \) is the unknown optimal vector value of \( c_i \).

Now, let us find an adaptation law for the fuzzy system parameters which guarantee the error (11) between the PID control and the ideal control as small as possible.

\[ \tilde{e}_i = u_i^* - u_i = E_i^T W_i c_i^* - E_i^T W_i c_i = E_i^T W_i \tilde{e}_i \]

Where \( \tilde{e}_i = c_i^* - c_i \) is the vector of the parameter error.

Using (4) and (5), to define the dynamic surface as

\[ s_i = -k_0s_i - k_{qi} \text{sign}(s_i) + g_i(x)u_{id} \]

To generating the adaptation law we minimize the quadratic cost function \( J(c_i) = \frac{g_i(x)}{2} e_i^2 = \frac{g_i(x)}{2} \left( u_i^* - E_i^T W_i c_i \right)^2 \) by applying the gradient descent method on it, and introducing the so called here \( s \)-modification term \( \sigma_i \left| s_i \right| c_i \) to improve the robustness performance and guarantees the stability as

\[ \dot{c}_i = \eta_i W_i^T E_i (s_i + k_0s_i + k_{qi} \text{sign}(s_i)) - \sigma_i \left| s_i \right| c_i \]

Unlike when we used the \( \sigma \)- modification or \( e \)-modification concept, the combination of the \( s \)-modification term and the \( k_{qi} \text{sign}(s_i) \) can assure a convergence of the tracking error to zero.
Theorem 1: Consider the system (1) and assume that all the considered assumptions are verified, the control law (9) and (13) ensures that the errors $e_i(t)$ ($i = 1, ..., 4$) convergence asymptotically and the boundedness of all the closed loop system signals.

Proof: To analyze the tracking errors convergence, the following Lyapunov function is consider

$$V_i = \frac{1}{2} s_i^2 + \frac{1}{z_i} \hat{c}_i^T \hat{e}_i$$

(14)

Its time derivative is written with using (12) and (13)

$$\dot{V}_i = -k_i s_i^2 - k_{0i} |s_i| + s_i g_i(x) e_{ui} - \hat{c}_i^T W_i^T e_i g_i(x) e_{ui} + z_i |s_i| \hat{c}_i^T \hat{c}_i$$

(15)

By using $\hat{c}_i^T c_i \leq -\frac{1}{2} \| \hat{c}_i \|^2 + \frac{1}{2} \| c_i \|^2$, $g_i(x) s_i e_{ui} \leq \frac{g_i(x)}{4} e_{ui}^2 + g_i(x) s_i^2$ and (11), (15) becomes

$$\dot{V}_i \leq -(k_i - g_i(x)) s_i^2 - (k_{0i} - \frac{z_i \| c_i \|^2}{2}) |s_i| - \frac{3g_i(x)}{4} e_{ui}^2 - \frac{z_i}{2} |s_i| \| \hat{c}_i \|^2$$

(16)

It is clear that for $k_i \geq g_i(x)$ and $k_{0i} \geq \frac{z_i \| c_i \|^2}{2}$, $\dot{V}_i$ can be bounded as follows

$$\dot{V}_i \leq -k_i s_i^2$$

(17)

Where $k_i$ is a positive constant

Equation (17) implies the boundedness of $V_i$ and the parameter error vector $\hat{c}$ ($V_i, \hat{c}_i \in L_\infty$).

By integrating (17) we can establish that

$$\int_{t}^{t+T} \| s_i \|^2 \leq \frac{1}{k_i} (V_i(t) - V_i(t + T))$$

(18)

Since $V_i(t) \in L_\infty$, we find from (18) that $s_i \in L_2$, and we have $\hat{s}_i \in L_\infty$, consequently $s_i \in L_2 \cap L_\infty$ and $\hat{s}_i \in L_\infty$, by Barbalat’s Lemma [14], we have, $\lim_{t \to \infty} \| s_i(t) \| = 0$, which implies $\lim_{t \to \infty} e_i = 0$.

4. Simulation results

In simulation we have take the following parameters: $\lambda_1 = 4, \lambda_2 = 3, \lambda_3 = 3, \lambda_4 = 2, \ k = \text{diag}[5, 7, 7, 7], \ k_0 = \text{diag}[20, 10, 11, 10], \ \sigma = 0.01, \ \eta = 0.01, \ \theta = 8$, the the initial states: $\phi(0) = 0.5 \ rad, \ \theta(0) = -0.5 \ rad, \ \psi(0) = -0.5 \ rad, \ \zeta(0) = 0 \ m$ and the desired angles and height: $\phi_d = 0 \ rad, \ \theta_d = 0 \ rad, \ \psi_d = 0 \ rad$, and $Z_d = 10 \ m$. For the fuzzy system, we have used the same membership functions as [15],with $c_{i1} = 1, \ c_{i2} = 0.5, \ c_{i3} = 1.5, \ i = 1, ..., 4$.

Figures 2-5 show the simulations results without disturbances, while figures 6-9 show the simulation results when a Gaussian noise with variance 0.004 and mean zero is added to all the states measurements.
5. Conclusion
In this paper one has proposed a fuzzy PID control law for a quadrotor UAV attitude and altitude stabilization. Concretely, a PID control law to approximate an ideal control law which ensures the asymptotic stabilization of both attitude and height even with model uncertainties and disturbances. The PID control gains are estimated online by mean of an adaptive fuzzy system based on the gradient descent algorithm. The stability analysis is done using the Lyapunov approach. The simulation results show the efficiency of the proposed control scheme.

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