Thermal Conductivity Tensor in YBa$_2$Cu$_3$O$_{7-x}$: Effects of a Planar Magnetic Field

Roberto Ocaña and Pablo Esquinazi

Abteilung Supraleitung und Magnetismus, Institut für Experimentelle Physik II,
Universität Leipzig, Linnéstr. 5, D-04103 Leipzig, Germany

We have measured the thermal conductivity tensor of a twinned YBa$_2$Cu$_3$O$_{7-x}$ single crystal as a function of angle $\theta$ between the magnetic field applied parallel to the CuO$_2$ planes and the heat current direction, at different magnetic fields and at $T=13.8$ K. Clear fourfold and twofold variations in the field-angle dependence of $\kappa_{xx}$ and $\kappa_{xy}$ were respectively recorded in accordance with the $d$-wave pairing symmetry of the order parameter. The oscillation amplitude of the transverse thermal conductivity $\kappa_{xy}^0$ was found to be larger than the longitudinal one $\kappa_{xx}^0$ in the range of magnetic field studied here ($0 \leq B \leq 9$ T). From our data we obtain quantities that are free from non-electronic contributions and they allow us a comparison of the experimental results with current models for the quasiparticle transport in the mixed state.

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I. INTRODUCTION

The processes which influence the thermal transport of the quasiparticles (QP) under the application of a magnetic field and below the superconducting critical temperature are a matter of current research in the physics of high-temperature superconductors (HTS). One of the difficulties to obtain clear evidence for one or other mechanism from thermal conductivity measurements that involves the QP, is basically related to the separation of the QP contribution from the measured thermal conductivity. Due to the fact that the experiments measure the total thermal conductivity given by the sum of the contributions due to phonons, $\kappa^{ph}$, and to the QP, $\kappa^{el}$, special methods are necessary to separate the relatively large phonon contribution to the thermal transport in HTS. Basically two methods for this separation have been treated in the literature. One method is based on a phenomenological description of the field dependence of the electronic contribution of the longitudinal thermal conductivity $\kappa_{xx}^{el}(T,B)$, introduced first by Vinen et al.[4] and used in Refs. [5,6] to estimate $\kappa_{xx}^{el}(T)$. The other method, presented by Zeini et al.[7], is based on simultaneous measurements of the longitudinal and transverse thermal conductivity and the assumption of a field independent Hall relaxation time.

To study thermal transport properties which depend on the QP contribution we have chosen to measure the thermal conductivity tensor as a function of angle $\theta$ between heat current direction and the magnetic field applied parallel to the CuO$_2$ planes at different magnetic fields. Because the oscillation amplitude of the thermal conductivities as a function of angle is related only to the QP contribution, most of our results are basically free from phonon contributions and can help to test scattering models for the QP. We discuss our results below in terms of two mechanisms that influence the behavior of QP in the mixed state of HTS: Andreev scattering (AS) is currently discussed in the literature as possible scattering mechanism of QP and the Doppler shift (DS) as the main effect that changes the energy spectrum of the QP with field.

The influence of the scattering mechanism on the thermal properties depends also on the symmetry of the order parameter. In the last years a large number of measurements has confirmed the $d$-wave pairing symmetry for most of the HTS. Due to the $d$-wave symmetry of the superconducting order parameter in HTS, extended quasiparticle states in the mixed state - associated with a gapless structure at the nodes - are expected to contribute to the thermal transport. These measurements have been described in terms of a DS in the energy spectrum of QP and the scattering of QP by vortices (AS) taking into account the presence of line nodes on the Fermi surface, i.e., within a pure $d_{x^2-y^2}$-pairing symmetry.

Try-crystal phase-sensitive measurements have determined that the pairing symmetry of the order parameter is $d_{x^2-y^2}$, which predominates with a small, if any, imaginary component ($< 5\%$) $id_{xy}$ at $T < T_c$. Although symmetry experiments should not be significantly affected by the development of a small $id_{xy}$ contribution, the quasiparticle transport might show (specialy at $T \ll T_c$) quantiative differences from the single $d_{x^2-y^2}$-pairing state due to the absence of line nodes in the resulting superconducting $d_{x^2-y^2} + id_{xy}$-gap. This new state proposed originally by Laughlin could arise either by decreasing temperature or increasing magnetic field as argued in Refs.[8,9]. Up to now, however, there is not enough experimental evidence for such a state although results on the magnetic field dependence of the longitudinal thermal conductivity in BSCCO and temperature dependence of the Hall angle could be explained taking into account the violation of both parity and time-reversal symmetry. In this article we recover this idea and attempt to establish the highest value, if any, for a component $id_{xy}$ of the superconducting order parameter within the current models for the thermal transport in the HTS cuprates. Our results agree with the $d_{x^2-y^2}$-pairing symmetry of the order parameter and provide a maximum value of $\sim 35\%$ for the component $id_{xy}$.

From the phenomenological side our work provides interesting details. Namely, we show explicitly how the
transverse thermal conductivity $\kappa_{xy}$ can be calculated from the experimental transverse temperature gradient $\nabla_y T$ and the longitudinal thermal conductivity $\kappa_{xx}$ for magnetic fields applied parallel to the CuO$_2$ planes. As in the case of a perpendicular magnetic field, the resulting $\kappa_{xy}$ depends on the ratio of the transverse temperature gradient to the longitudinal one and represents a physical property of the crystal involving only contributions from the QP property of the crystal involving only contributions from the QP.

We also show that the oscillation amplitude of the transverse thermal conductivity, $\kappa_{xy}^0$, is larger than the longitudinal one, $\kappa_{xx}^0$, in the whole magnetic field range studied ($0 \leq B \leq 9$ T). From the extracted electronic contribution of $\kappa_{xx}$ we attempt also to calculate a Hall-like angle and show that it increases with magnetic field in qualitatively agreement with the AS model.

Our article is organized as follows. In the next section we describe some experimental and sample details; in Sec. II we describe and discuss our results and is divided in four subsections. A summary is presented in Sec. IV.

II. EXPERIMENTAL AND SAMPLE DETAILS

For all the measurements presented here we used a twinned YBa$_2$Cu$_3$O$_{7-x}$ single crystal with dimensions (length $(l)$× width $(w)$× thickness $(d)$) $0.83 \times 0.6 \times 0.045$ mm$^3$ and critical temperature $T_c = 93.4$ K, studied previously. With polarized light microscopy we have determined the positions of the twinning planes. Accordingly, the heat current direction was adjusted along the $a/b$ axes which were parallel to the width or length of the sample. The use of a twinned crystal rules out the influence of orthorhombicity on the thermal transport properties studied in this work. In particular we can assume that $\kappa_{xx}(T, B) = \kappa_{yy}(T, B)$. We note that the influence of the orthorhombicity can be observed in untwinned crystals only if the impurity scattering rate is small enough. As shown in Refs., untwinned crystals with relatively large impurity scattering show similar angle dependences as twinned ones.

The longitudinal ($\nabla_x T$) and transverse ($\nabla_y T$) temperature gradients have been measured at $T = 13.8 K$ with previously calibrated chromel-constantan (type E) thermocouples and a dc picovoltmeter. Our definition of sample axes, temperature gradients and field direction is similar to that used in Ref. If the heat current $Q$ is along $+\hat{x}$ and a positive 90° angle $\theta$ on the $(x, y)$-plane is along $+\hat{y}$, then there is a positive transverse thermal gradient for the magnetic field $B$ at $+45^\circ$, see Fig. 1.

The magnetic field was applied perpendicular to the $c$-axis. Special efforts were made in order to minimize the misalignment of the plane of rotation with the CuO$_2$ planes. We estimate the misalignment angle to be smaller than $\alpha \approx 0.5^\circ$. An in-situ rotation system enabled us the measurement of the thermal conductivity as a function of the angle $\theta$ defined between the applied field and the heat flow direction along $+\hat{x}$, see Fig. 1. We used an initial temperature gradient $\nabla_x T \simeq 241$ K/m ($\Delta_x T \simeq 200$ mK for our crystal) at $\theta = 90^\circ$, i.e. with the magnetic field perpendicular to the heat flow, and we recorded angle scans and magnetic field dependence with a field-cooled procedure to avoid pinning effects. That means that the angle and magnetic field scans at constant temperature were done in such a way that after taking a reading the sample was driven to its normal state by heating up to a few Kelvin above the superconducting critical temperature and immediately was cooled down again to 13.8 K at constant field. We will show in Sec. III C experimental evidence that demonstrates how pinning of vortices influences the field dependence of the measured oscillations, even for the case of parallel field used in this work.

The possible influence of demagnetization and flux pin-
III. RESULTS AND DISCUSSION

A. Angle dependence of the longitudinal $\kappa_{xx}$ and transverse $\kappa_{xy}$ thermal conductivities

Figure 2 shows the field-angle dependence of the temperature gradient $(\Delta_T - 200)/l$ at 13.8 K for a magnetic field strength of 8 T (solid circles) and 3 T (open circles). The heat flow was constant during the whole scan and chosen to produce 200 mK of temperature difference at the initial scan angle $\theta = 90^\circ$. Dashed lines are fits to the function $a(\cos(4\theta) - 1)$ with $a(T,B)$ as free parameter.

The curves in Fig. 2 can be fitted satisfactorily with the simple function $a(\cos(4\theta) - 1)$ with only $a$ as fit parameter which depends on temperature and magnetic field. The fact that minima are found in the field-angle profile of the temperature gradient $(\Delta_T - 200)/l$ at the nodal directions could be explained in terms of Andreev scattering of QP by vortices. According to this effect at any point the quasiparticle energy, as viewed from the superfluid frame, equals the gap then, at that point, Andreev reflection occurs: the quasiparticle is converted to a quasihole reversing its velocity and its contribution to the heat transport. This mechanism of thermal resistance is always induced when spatial variations of the amplitude or phase of the order parameter take place. Since the QP in the mixed state are in the presence of a phase gradient (superfluid flow) they should also have a DS in their energy spectrum given by the scalar product of the momentum and superfluid velocity, $\mathbf{p} \cdot \mathbf{v}_s$ as viewed from the laboratory frame. That phase gradient is then the necessary mechanism that induces the Andreev reflection. Thus, when a quasiparticle with momentum $\mathbf{p}$ is moving parallel to the magnetic field no DS occurs and hence, no Andreev reflection. In other words, the field acts as a filter for that quasiparticle that contributes to reduce the total temperature gradient. For different orientations of the quasiparticle momentum with the field, there is a finite probability for the Andreev reflection and thereby, for an enhancement of the thermal resistance. The directionality of this scattering mechanism is therefore the responsible feature for the identification of the preferential directions of the QP by rotating the magnetic field parallel to the basal planes.

On the other hand, a DS in the energy spectrum of the quasiparticle could also give rise to fourfold symmetry in the longitudinal thermal conductivity by rotating the field parallel to the CuO$_2$ planes, but in principle, with opposite sign. The usual effect would be to produce an excess of quasiparticles in the direction perpendicular to the field, and thereby, reducing the thermal resistance in that direction. However, the DS affects both the carrier density as well as the scattering rate of QP, and since the latter dominates at high enough temperatures the QP may move easier parallel to the field and the sign turns out to the correct one. We note that, although it is not clear which effect (AS or DS) should prevail at low fields, the quasiparticle mean free path shortens with increasing temperature gradient, this fact could be definitive in distinguishing between Andreev reflection and DS. However, the inclusion of an impurity scattering rate in the...
FIG. 3: Angle dependence at two magnetic fields, 8 T (solid circles) and 3 T (open circles), applied parallel to the CuO$_2$ planes of the (a) longitudinal and (b) transverse thermal conductivities. The continuous curves are obtained from Eq. (3) with the parameters given in the text and a pure $d_{x^2-y^2}$-wave order parameter.

For a better comparison with theoretical predictions we need to obtain the transverse thermal conductivity from the measured transverse temperature gradient. As in the case the field is applied normal to the basal planes, we calculate the transverse thermal conductivity, $\kappa_{xy}$ as a function of the field angle $\theta$ from the Onsager relations, using the fact that the net heat current flowing in the $y$-direction is zero. At constant temperature, this can be written as follows:

$$\kappa_{xy}(B, \theta) = \kappa_{xx}(B, \theta) \frac{\nabla_y T(B, \theta)}{\nabla_x T(B, \theta)} \approx -\frac{J \nabla_y T(B, \theta)}{\left(\nabla_x T(B)\right)^2},$$

(1)

where we have used the experimental fact that the total longitudinal temperature gradient ($\sim 241$ K/m) is much larger than its fourfold oscillation ($\sim 3$ K/m) as shown in Fig. 2. This allows us to approximate $(\nabla_y T(B, \theta))^2 \approx (\nabla_x T(B))^2$ in (1) and thereby, the fourfold symmetry does not enter when calculating $\kappa_{xy}$. In other words, the transverse thermal conductivity has the same symmetry as the transverse temperature gradient but opposite sign. We note that (1) ensures us that $\kappa_{xy}$ measures in fact a physical properties of the crystal as in the usual Righi-Leduc effect. Figure 3(b) shows the angle dependence of the transverse thermal conductivity at two different magnetic fields.

B. Separation of the phonon and QP contributions

In order to separate the phonon and quasiparticle contributions from the longitudinal thermal conductivity, needed for a quantitative comparison with the AS model, we have measured the field dependence up to 9 T with calculations of the DS effect accounts for the sign of both fourfold and twofold oscillations as well. Furthermore, in the case of BSCCO it has been argued that at the range of magnetic field and temperature studied here, both effects are taking place, and depending on the values of the parameters governing the gap at the nodes, i.e., $v_F/v_\parallel$ and $k_F$, along with an impurity scattering rate $\Gamma_0$, different regimes in which DS dominates or Andreev scattering or vice versa, could be identified. Unfortunately, there is no yet theory for both longitudinal and transverse thermal conductivities which takes into account both effects when the field is oriented parallel to the CuO$_2$ planes. Also, the theory for the thermal transport for our case (i.e. parallel field, low temperatures and finite impurity scattering) under the influence of DS is still not yet developed. Therefore, in most of this paper we restrict ourselves to a quantitative comparison of our experimental results to the predictions obtained from the AS mechanism; deviations from the predictions may well indicate that the other mechanism could influence the thermal transport, as suggested in Ref. 21.

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the field oriented along both antinodes and nodes directions. As shown in Fig. 3, the field profile can be rather well fitted with the empirical expression

\[ \kappa_{xx}(T, B) = \kappa_{xx}^{ph}(T) + \frac{\kappa_{xx}^{el}(T)}{1 + \beta(T)B^n}, \]  

where, in general, \( \beta(T) \) is proportional to the zero-field electronic mean-free-path of the QP and \( n \) can be related to the nature of the quasiparticle scattering.

It is clear that the phononic and electronic contributions at zero field, \( \kappa_{xx}^{ph} \) and \( \kappa_{xx}^{el} \), are independent of the angle \( \theta \). Further, we assume that \( \beta(T) \) remains unchanged by rotating the field parallel to the CuO\(_2\) planes at constant temperature. In this case the exponent \( n \) might be field-angle dependent due to the different types of effective scattering which the QP experience at different orientations of the magnetic field. This assumption remains thereby valid either when the main effect is the DS or the AS. As in Ref. 3 we used the value \( n = 1 \), when the field was aligned parallel to the direction of the measured temperature gradient (parallel to the heat current, \( \theta = 0^\circ \)) and it was allowed to take other values for different orientations of the field. As shown in Fig. 3, we get satisfactory fits to the experimental points with \( \kappa_{xx}^{ph} = 5.29 \text{ W/Km}, \kappa_{xx}^{el} = 0.89\text{ W/Km}, \) and \( n \simeq 0.82 \), the latter when the field is oriented along the nodal direction \( \theta = 45^\circ \). To the best of our knowledge, it appears that this is the first time such a dependence of the exponent \( n \) in [3] with angle \( \theta \) is reported.

The microscopic model for the scattering rate of the QP by vortices using Andreev reflection \( \Gamma_v(B, p) \) was formulated by Yu et al. The thermal conductivity can be calculated using a 2D version of the Bardeen-Richayzen-Tewordt model

\[ \kappa_{\alpha\beta}^{el} = \frac{1}{2\pi^2ck_BT^2h^2} \int_{p_F}^{\infty} d^2p v_{\alpha\beta}E_p^2 \Gamma(B, p, T) \text{sech}^2 \left( \frac{E_p}{2k_BT} \right), \]  

(3)

where \( \Gamma(B, p, T) \) may be taken as a relaxation rate given by the sum of the following scattering mechanisms acting in series: scattering of QP by impurities \( \Gamma_{imp}(p) \), by phonons \( \Gamma_{ph}(p, T) \), by QP \( \Gamma_{QP}(B, p, T) \) and by vortex supercurrents \( \Gamma_v(B, p, T) \). The parameter \( E_p \) is the QP energy, \( \alpha \) and \( \beta \) denote the \( x \) or \( y \) directions on the plane of the sample, \( v_{\alpha\beta} \) is the group velocity along the \( \alpha \) direction and \( c \) is the c-axis lattice parameter. In Ref. 3 an approximate expression for the effective rate of AS by vortex current was obtained which is basically given by

\[ \Gamma_v \propto a_v^{-1} \exp(-a_v^2f(E_p)\phi/\ln(a_v/\xi)). \]  

(4)

In this expression the effective flux-line-lattice parameter is given by \( a_v^2 \propto B_{c2}/B_\gamma \), where \( B_{c2} \) is the upper critical field parallel to the planes and an effective mass factor \( \gamma \sim (m^*/m)^2 \). The function \( f \) depends on the energy of the QP, and consequently on the gap \( \Delta(p) \), and the angle \( \phi \) between the direction of the momentum of the QP and the magnetic field. For more details see Ref. 3.

As we mentioned above, we would like to know how large, if any, can be the contribution of a term \( id_{xy} \) in the order parameter compatible with our transport measurements. Therefore we use the following gap

\[ \Delta(p, T) = \Delta(T)\left(\Delta_{d_{xy}} + i\Delta_{d_{xy}}\right), \]  

(5)

which can be explicitly written taking into account the momentum components as

\[ \Delta(p, T) = \Delta(T) \left( \frac{\cos(\frac{p_x^0}{\pi}) - \cos(\frac{p_y^0}{\pi})}{1 - \cos(\frac{p_x^0}{\pi})} + i \frac{2 \sin(\frac{p_x^0}{\pi}) \sin(\frac{p_y^0}{\pi})}{1 - \cos(\frac{p_x^0}{\pi})} \right). \]  

(6)

The percentage values indicated below for the \( id_{xy} \) part means its weighted contribution relative to the total gap.

Our measurements are done at constant temperature, therefore if we make the simplifying assumption that the scattering rates \( \Gamma_{imp}, \Gamma_{ph} \) and \( \Gamma_{qp} \) do not depend on the momentum of the excitations, we can write \( \Gamma_{imp} + \Gamma_{ph} + \)
\[ \Gamma_{qp} \equiv \Gamma_0 \simeq cte. \] At higher temperatures it would be necessary, however, to include a model for the energy dependence of the inelastic scattering rate \( \Gamma_{qp} \), as argued by many authors [29,30]. Nowadays there is consent that the peak observed in the longitudinal thermal conductivity as a function of the temperature is due to the competition between a decreasing inelastic scattering rate and the decrease in the number of quasiparticles with decreasing temperature.

From [22] we obtain \( \kappa_{xx}^{id} \Gamma_0 = 7.2 \times 10^{12} \text{(W/Kms)} \) at zero field assuming \( d_{x^2-y^2} \)-pairing symmetry and maximum gap \( \Delta(T = 0) = \Delta_0 = 20 \text{ meV} \). Then, if we use the quasiparticle contribution \( \kappa_{xx}^{id} \) obtained from the fits using [22], we get \( \Gamma_0^{-1} \simeq 0.12 \text{ ps} \). Using this value and the parameters from Ref. [25], i.e. \( B_{c2} = 500 \text{ T}, \text{ Ginzburg-Landau parameter } \kappa = 100, \gamma = 4 \text{ and } \Gamma_0^{-1} (B = B_{c2}) \approx \frac{3}{2} \text{ at } 8 \text{ T} \) (chosen to reproduce the experimental amplitude of the oscillation at 8 T) it is not possible to fit satisfactorily the thermal conductivity \( \kappa_{xx}(B, \theta) \) of Fig. 3(a) for both curves at 3T and 8T simultaneously. However, if we increase strongly the upper critical field to an unrealistically high \( B_{c2} \approx 9,500 \text{ T} \) or increase \( \gamma \) by a factor \( \sim 5 \), Eq. 3 accounts for the fourfold oscillation observed in \( \kappa_{xx}(B) \) (and hence, for the normalized temperature gradient \( \Delta_x T = 200 / l \)) when the field is rotated from \( \theta = +90^\circ \) to \( \theta = -90^\circ \) as shown in Fig. 3(a).

With the same parameters mentioned above, \( \kappa_{xy} \) can be calculated from 3 as shown in the Fig. 3(b). We find agreement between the experiment and theory if, as for \( \kappa_{xx} \), we increase the intervortex spacing \( \sim 5 \) times the value defined in Ref. [25]. This result indicates that the AS rate needed to fit the experimental data is much smaller than the one obtained from the proposed microscopic model and with the parameters used in Ref. [25].

We can also extend the AS model including in 3 an additional component \( id_{xy} \) to the pure \( d_{x^2-y^2} \)-gap and compare it to the experimental angle dependence of \( \kappa_{xx}(\theta) \), for example. The results shown in Fig. 6 indicate that the fourfold oscillation is no longer observable with a component larger than \( \sim 35\% \) at 13.8 K. Because we compare the symmetry of both theoretical and experimental curves taking the normalized \( \kappa_{xx} \) (see Fig. 6), this upper limit is not affected using other parameters in Eq. 3.

### C. Field dependence of the oscillation amplitudes \( \kappa_{xx}^0 \) and \( \kappa_{xy} \): Doppler shift and the effect of pinning of vortices

Other way to compare theory and experiment that does not need any empirical relation to separate the QP contribution is based on the measurement of the field strength dependence of both fourfold and twofold oscillation amplitudes, which are in fact pure electronic effects. It is convenient to define the quantities

\[ \kappa_{xy}^0 \equiv \kappa_{xy}(\theta = 0^\circ) - \kappa_{xy}(\theta = 45^\circ), \]

\[ \kappa_{xx}^0 \equiv \kappa_{xx}(\theta = 45^\circ) - \kappa_{xx}(\theta = 0^\circ), \]

which can be directly compared with any model that accounts for the contribution of the QP. In Fig. 7 we show the magnetic field dependence of \( \kappa_{xy}^0 \) and \( \kappa_{xx}^0 \). As shown, we find that the resolution of \( \kappa_{xy}^0 \) is smaller than that of \( \kappa_{xx}^0 \). The fourfold oscillation observed in Fig. 2 has a thermal background of \( \sim 200 \text{ mK} \), which makes more

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**FIG. 6.** Angle dependence of the normalized longitudinal conductivity at a field of 8 T \( \kappa_{xx}(\theta)/\kappa_{xx}(0) \). The data points are taken from Fig. 2(a) normalized by an arbitrary factor in order to compare with the theoretical predictions given by the continuous, dashed and dotted lines. These lines were calculated with (3) assuming a pure \( d_{x^2-y^2} \) symmetry, plus a 35% and 50% \( id_{xy} \) contribution added to the gap (see Eq. 3), respectively, and with the same parameters used to fit the data of Fig. 3.

**FIG. 7.** Magnetic field dependence of \( \kappa_{xx}^0 \) (open circles) and \( \kappa_{xy}^0 \) (solid circles). Solid lines are fits using Eq. (3). Dashed lines are fits using the DS model following Eqs. (6). The inset shows the amplitude \( \kappa_{xy}^0 \) recorded with a non-field cooled procedure (+) and with a field-cooled procedure (solid circles, the same data as in the main figure).
difficult the observation of any small superposed signal. The thermal background in the transverse thermocouple due to a small misalignment in the x-direction was \( \sim 8.5 \text{mK} \).

We note that the oscillation amplitude of the transverse thermal conductivity \( \kappa_{xy}^0 \) is larger than the one of the longitudinal thermal conductivity \( \kappa_{zz}^0 \). Note the fact, that Eq. (3) with the parameters described above (including an intervortex spacing \( \sim 5 \) times larger) accounts for this behavior, see Fig. 7. One would tend to believe that changing the effective mass factor \( \gamma \) we may compensate the too high \( B_{c2} \)-value needed to fit the experimental data in Fig. 7. Note that \( \gamma \) enters not only in \( a \nu \) but in other parts of Eqs. (3) and (4). Our numerical results show no agreement with the experimental data of Fig. 7.

The thermal background in the transverse thermocouple enters not only in \( a \nu \) but also in the density of QP and their mass. Both expressions provide the correct sign for the tensor components. From these equations and as shown in Ref. 26 we obtain for the field dependence of the oscillation amplitudes \( \kappa_{xx}^0 = aB \ln(1/bB) \) with \( a \) and \( b \) fit parameters, and \( \kappa_{xy}^0 = aB \). These dependences also agree well with the experimental data, see Fig. 7. The fit of \( \kappa_{xy}^0 \) gives \( a \approx 0.015 \) (W/TKm) and \( b = 0.048 \text{G}^{-1} \); from the expression proposed for \( \kappa_{xx}^0 \) we get \( a \approx 0.010 \) (W/TKm). This discrepancy in \( a \) is not too large taking into account that neither the superclean limit used in developing this model nor the field-temperature range where the Eqs. (3) and (4) are valid, reflect the situation studied here. Therefore, a more accurate treatment of the impurity scattering is necessary to check if the DS picture can fit quantitatively the experimental data. It is striking that the DS description appears also to provide the experimental field dependences of the oscillation amplitudes.

In the inset, we show the effect of the pinning of vortices on the oscillation amplitude \( \kappa_{xy}^0 \). While the points taken with a field-cooled procedure show a monotonic behavior in the range of fields studied here, the points taken with a non-field-cooled procedure, i.e. in this case the field was increased from zero at constant temperature and always at the same initial angle, show a saturation above \( \sim 3 \) T. This effect could have a different origin from the DS, and would indicate that vortex scattering becomes more important with increasing field. We note that the pinning of the Josephson-like vortices parallel to the planes is strongly affected by vortices perpendicular to the planes due to the misalignment of the crystal respect to the applied field. The results in the inset of Fig. 7 clearly stress the necessity of field-cooled experiments to get the true magnetic field dependence of thermal transport parameters.

### D. Hall-like angle and quasiparticles-dependent ratios

The angle dependent conductivities we have measured allow us the definition of quantities which just involve contributions from the QP as in the usual Rigoli-Leduc effect. As noted above we do not have to assume strictly any method to separate electronic and phononic contributions from the total thermal conductivity to obtain quantities which can be directly compared with the theory of QP transport. As shown in the last section, some of these quantities are themselves the field dependence of both \( \kappa_{xy}^0 \) and \( \kappa_{xx}^0 \). However, the ratio \( \kappa_{xy}^0/\kappa_{xx}^0 \) might provide also an interesting result for comparing with theory. In fact, it describes an intrinsic property of the crystal studied, free of the experimental uncertainty to determine the total power given to the sample.

As shown in Fig. 7, two regimes in the oscillation ratio \( \kappa_{xy}^0/\kappa_{xx}^0 \) can be distinguished: From zero to \( \sim 2 \) T the ratio decreases stronger with field than at fields above \( \sim 2 \) T. An abrupt decrease of this ratio is predicted by Eq. (3) but it fits well the experimental data only above...
\[ \tan \alpha_{\parallel} = \frac{\langle|\kappa_{xy}(B)|\rangle_{\theta}}{\langle|\kappa_{xx}^\parallel(B)|\rangle_{\theta}}, \quad (9) \]

where \( \langle \cdot \rangle_{\theta} \) means the integration over \( \theta \) in the range mentioned above.

In Fig. 8 we show the field dependence of \( \tan \alpha_{\parallel} \) in which we used the phononic contribution to the thermal conductivity \( \kappa_{xx}^{ph} \) extracted using (2) to calculate the total quasiparticle contribution \( \kappa_{xx}^\parallel(B, \theta) \). Note that the field dependence of \( \tan \alpha_{\parallel} \) is affected by the value of \( \kappa_{xx}^{ph} \) obtained from (4), in particular the linear dependence with the magnetic field observed above \( \sim 2 \, \text{T} \). Decreasing 5\% the obtained \( \kappa_{xx}^{ph} \) the computed \( \tan \alpha_{\parallel} \) decreases substantially at high fields, see Fig. 9. This result emphasizes the difficulty to obtain true field dependences of thermal transport parameters by using phenomenological separation methods.

A magnitude free of the separation model is given by the ratio between the averaged change with field of the longitudinal thermal conductivity, \( \langle|\kappa_{xx}(B = 0) - \kappa_{xx}(B, \theta)|\rangle_{\theta} \) and the averaged transverse thermal conductivity \( \langle|\kappa_{xy}(B)|\rangle_{\theta} \), which only contains information of the quasiparticle contributions. We define then,

\[ \beta = \frac{\langle|\kappa_{xx}(B = 0) - \kappa_{xx}(B, \theta)|\rangle_{\theta}}{\langle|\kappa_{xy}(B)|\rangle_{\theta}} = \frac{\pi}{2} \left[ \frac{\kappa_{xx}^{el}(0, 0) - \kappa_{xx}^{el}(0, B)}{\kappa_{xy}^{0}} - \frac{\kappa_{xx}^{0}}{2\kappa_{xy}^{0}(B)} \right]. \quad (10) \]

As shown in Fig. 9 (right axis), the ratio \( \beta \) increases roughly proportional with field. Note first that the simple subtraction \( \langle|\kappa_{xx}(B = 0) - \kappa_{xx}(B, \theta)|\rangle_{\theta} \) cancels the phononic contribution of the thermal conductivity which we assume it does not change with field. Second, \( \beta \) does not provide the same information as the ratio \( \kappa_{xx}^{ph}/\kappa_{xy}^{0} \). Therefore, \( \beta \) can be used for comparing with...
any model for the quasiparticle transport. We note that the observed linear regime of $\beta$ should disappear with increasing field. When the field is increased, the vortices are moving closer together and hence the scattering probability of QP by vortices increases. Thus, at higher fields than those used in the present work, either a saturation or a non-monotonic behavior in the field dependence of $\kappa_{0y}$ as well as $\kappa_{0x}$ is expected.

With Eq. (3) and with the same parameters that fit the oscillation amplitudes (see Fig. 5) we have calculated both parameters $\tan(\alpha(q))$ and $\beta(q)$ see Fig. (b)). We found that the field dependence of both parameters is qualitatively reproduced by the AS mechanism. However, both their absolute values and their slopes with field are not given by Eq. (3) correctly. A disagreement is also found if we want to fit the curves given in Fig. 5 using the same parameters as in Fig. 4. Clearly, the actual AS model is not enough to understand the experimental data quantitatively.

IV. SUMMARY

In summary, we have measured the components of the thermal conductivity tensor at 13.8 K in an optimally doped twinned single crystal of YBCO as a function of the magnetic field and the angle $\theta$ between magnetic field and heat current. The field-angle dependence measurements show the predominance of the $d_{x^2-y^2}$-wave pairing symmetry over a complex component $i d_{xy}$, which is estimated to be less than $\sim 35\%$ of the resulting total gap, within the AS picture for the interaction of QP with vortices.

The magnetic field dependence of the amplitude coefficients of the fourfold and twofold oscillations observed in $\kappa_{0x}$ and $\kappa_{0y}$ respectively, allows us to define magnitudes free of contributions from the phonons and, hence, suitable for comparison with theory. In particular, the dependence of the amplitude coefficients $\kappa_{0y}$ and $\kappa_{0x}$, their ratio, a Hall-like angle $\alpha(q)$ and the ratio $\beta$ between the decrease of the longitudinal thermal conductivity and the transverse thermal conductivity with field were obtained. Assuming only the AS mechanism along with an impurity scattering rate, we get a good quantitative description of both field dependences of the oscillation amplitude of the conductivities only if we increase the intervortex spacing five times (or increase the parallel critical field to 9500 T) the value defined in Ref. 9. This implies a much smaller scattering rate due to AS than the one obtained from the original theory. Also, the AS model does not fit with the same parameters all the experimental data, although it provides a good qualitative description of the field and angle dependences of all the parameters.

The less developed DS model does not allow a rigorous quantitative comparison. It is worth to mention, however, that the DS model alone also provides field dependences of the oscillation amplitudes similar to those obtained experimentally. An impurity scattering rate should be introduced in the DS theory within a more realistic limit than the superclean limit to perform a detailed discussion of the quantities involved in the equations. The overall results suggest that both AS and DS effects are responsible for the observed magnetic field dependences but with different regimes of predominance. As suggested in Ref. 9 it is more likely that the DS plays an important role at low fields providing a slower magnetic field dependence in the components of the thermal conductivity tensor than the AS. This can be understood taking into account that while the QP always suffer a DS in the mixed state, the probability for Andreev reflection increases with increasing field. This might explain the less abrupt decrease with field of the ratio $\kappa_{0y}/\kappa_{0x}$ below $\sim 2$ T and the observed quantitative discrepancies between the Hall-like angle $\alpha(q)$, the parameter $\beta$ and the AS model. Further development of the theory involving both mechanisms is necessary to confirm this hypothesis.

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