The modification of generalized uncertainty principle applied in the detection technique of femtosecond laser

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Abstract. Generalized uncertainty principle (GUP), also known as the generalized uncertainty relationship, is the modified form of the classical Heisenberg's Uncertainty Principle in special cases. When we apply quantum gravity theories such as the string theory, the theoretical results suggested that there should be a "minimum length of observation", which is about the size of the Planck-scale (10^{-35}m). Taking into account the basic scale of existence, we need to fix a new common form of Heisenberg's uncertainty principle in the thermodynamic system and make effective corrections to statistical physical questions concerning about the quantum density of states. Especially for the condition at high temperature and high energy levels, generalized uncertainty calculations have a disruptive impact on classical statistical physical theories but the present theory of Femtosecond laser is still established on the classical Heisenberg's Uncertainty Principle. In order to improve the detective accuracy and temporal resolution of the Femtosecond laser, we applied the modified form of generalized uncertainty principle to the wavelength, energy and pulse time of Femtosecond laser in our work. And we designed three typical systems from micro to macro size to estimate the feasibility of our theoretical model and method, respectively in the chemical solution condition, crystal lattice condition and nuclear fission reactor condition.

1. Introduction
With the rapid development of the modern detection technology, the accuracy of the experimental apparatus has been dramatically increased these years. However, the errors between the theoretical calculations and the results from experiments still exist among some special cases. In order to apply the uncertainty principle to multiple special systems and reduce the errors between the theoretical calculations and the results from experiments, scientists decided to modify the original uncertainty principle equation by appending more parameters and it is now known as the Generalized Uncertainty Principal [1]. In the classical Heisenberg's Uncertainty Principle, the position and momentum of a particle cannot have a definite value at the same time. If one of the quantity is very determined, another one will be relatively uncertain. But this theory does not provide for a minimum length of observation, meaning that the minimum length can be infinitely tends to 0. Especially for some systems of ultra-high temperature and ultra large energy for which the classical statistical physics is no longer suitable, some implements of GUP (Generalized Uncertainty Principal) such as the modification of the Quantum Transition Energy Spectrum under the GUP [2], the thermodynamic
properties of Fermi gas under the GUP [3], and the entanglements for identical particles under the GUP [4] all provide us an entirely new perspective of how do things work under the quantum level and in which make the theoretical calculations more rational for the experimental results. In detail, under the circumstances of GUP, the heat capacity will tend to 0 when temperature tends to infinity. But the traditional statistical physics suggests that the heat capacity is a result of equipartition at high temperatures, which is a temperature-independent constant. In low temperature conditions, GUP statistical physics cannot subvert traditional concepts of statistical physics, but the theoretical results make differences in the thermodynamic system calculations concerning about the state density, which leads to some of the traditional physical phenomena that cannot be answered by classical physics, such as the thermodynamic properties of ultra cold Fermi gas [5].

As one of the detection techniques with highest time resolution for material characterizations, the femtosecond laser is a kind of laser which emits optical pulses with duration between a few femtosecond and hundreds of femtoseconds. This type of laser can provide us huge amount energy in a very short time period, which enables us to accurately detect the ultra-fast physical or chemical reaction and observe the events at a single molecule level. According to the HUP (Heisenberg Uncertainty Principle), the product of $\Delta E$ and $\Delta T$ are greater than a certain value $h/4\pi$, meaning that they are inversely proportional towards each other. It means that we can obtain a laser impulse of ultra-short $\Delta T$ through widening the range of wave length of the light which determines the range of energy. And since the $\Delta T$ is directly proportional with $\Delta X$ (optical path difference), two femtosecond lasers could be controlled by changing the wavelength deliberately at the same time, which help us obtain and analyze the information of the testing system. Thus, we can apply this laser to some special circumstances we have interests in.

In this article, we want to formalize a new form of Uncertainty Principle equation under the femtosecond technological detection. The reliability of this technological detection will be evaluated by three different systems, including the vibration of covalent bonds during the chemical reactions, the manipulation of crystal texture in nanomaterial, and the laser-induced acceleration for the nuclear fission. Each individual system will have one individual expression form. Also, we will control the form of the parameters used in the individual system and try to represent all the equations used in the three different systems through one general term.

2. Methods and models

In classic statistical physics, a phonon is a collective excitation in a periodic, elastic arrangement of atoms or molecules in condensed matter, which is a way to describe the harmonic vibration between the atoms. It is well known that the speed of the phonon is around $300\text{m/s}$, and the difference between two atoms is about one angstrom (ten to the power negative ten). So it’s not hard for us to deduce that the time for a phonon to travel between two atoms is about $10^{-12}\text{second}$, which is a perfect size for the femtosecond laser to detect. Also, the time to analyze of the variation in molecular bond during the chemical reaction is also on the same magnitude as femtosecond. According to the electronic mobility in graphene whose electrons are nearly ideal Dirac Fermions [6], we can assume the electrons which has a travel speed approximately $10^{-12}\text{m/s}$ to form the bonds in the chemical reaction, which means that the time for the electrons to complete the formation of the bond is around $10^{-16}\text{second}$ because $10^{-10}\text{(the distance between two molecules)}$ divided by $10^{-6}\text{(the speed of electron)}$ is about this order of magnitude. This is also a perfect size for the femtosecond laser to detect. And we list the basic physical models in Figure 1 for the former two systems in our paper. At last, we are trying to use the femtosecond laser to induce the nuclear fusion, which needs enormous amount of energy. The energy that femtosecond laser can produce is about $10^{10}\text{joule per second}$ which is enough to induce the nuclear fusion. And it is important for us to accurately control the time of output to achieve the induction of nuclear fusion.

It is well known that in quantum mechanics, the standard HUP equation is often given by $\Delta X \cdot \Delta P \geq \frac{\hbar}{4\pi}$ which has been widely used in statistical physics. Obviously, it doesn’t consider the
effect of the general theory of relativity, in which the $\Delta X$ has a minimum value to observe due to the gravitational force. In order to satisfy this precondition, we must reformulate the equation on the right side of the HUP with a factor $\gamma$, of which yet, we don’t know the value.

First of all, we aim to cancel the $\Delta P$ on the left side of the equation because that’s the common way [7] to formulize an equation with a minimum value of $\Delta X$ regardless to the value of $\Delta P$. Then, we may give a minimum value for $\Delta X$, which could temporarily be $\frac{h}{4\pi}$ times “$\sqrt{A}$”, in which A is a GUP parameter. Conversely, we could deduce the form for the factor $\gamma$ in a backward way. According to the inequality of arithmetic and geometric means, $(a + b) \geq 2\sqrt{ab} \geq 0$, it’s not hard for us to discover that we could rewrite the $\sqrt{A}$ into another form, which is $2\times \frac{1}{\sqrt{\Delta P}} \cdot (A \cdot \Delta P)$. Our purpose here is to try to reform the equation with the same value on the left-hand side as the older one and deduce it from the minimum value of $\Delta X$ as we mentioned above. After moving the $\Delta P$ to the left-hand side, we successfully rebuild the entire Uncertainty Principle with a factor $\gamma$ of which the value is $(1 + A \cdot \Delta P^2)$. It’s obvious that when the value of the parameter $A$ equals zero, the equation will formalize to the ordinary HUP. Precisely, the factor $\gamma$ is not modifying the entire system, but the value of Planck constant “h”.

For three-dimensional solid materials in the free electron model uses $g(E)=\frac{2}{\pi}\frac{g_0}{V(2m)^{3/2}E^{1/2}/h^3}$ to express the density of states. Considering the modification in the GUP, the value of $h$ is this equation can be corrected by the factor $\gamma$ which we have been used in the modification of HUP. Based on the Taylor series expansion [8], which is $f(z) = \sum_{n=0}^{\infty} f^{(n)}(a) \frac{(z-a)^n}{n!}$, we could easily find more precise value of the density of states $g(E)=\frac{2}{\pi}\frac{g_0}{V(2m)^{3/2}E^{1/2}/h^3 \cdot (1+A\Delta P^2)^{3/2}}$.

3. Modification and evaluation
To be honest, the chemical reaction in most cases happens in a transient moment, meaning that it needs a very precise detection method to observe. Therefore, the femtosecond laser is the best detective equipment for the process of chemical system since it possesses the highest temporal resolution and weak interaction interfere. And we illustrate the structure of our ideal equipment [9] in Figure 2. During the chemical reaction, electrons whose travel speed is around $10^8\ m/s$ will break the bonds and then reform it in order to produce new compounds or substances. The distance between two atoms is approximately $10^{-10}\ m$. By using the most fundamental equation $t = \frac{\xi}{v}$, we could easily estimate the time for the chemical reaction which is about $10^{16}s$. Femtosecond laser is definitely capable of detecting the chemical reactions due to its $\Delta t$. In chemistry, the internal coordinate is a way to represent a system build of atoms. It provides a description of each atom in a molecule in terms of its bond length, bond angle, and dihedral angle. The bond length could be seen as the frequency of the of the electrons since it’s the only way for electrons to travel horizontally. During different chemical reaction, we may have various bond length which is the frequency of the electrons due to the bond characteristic itself. The goal here is to use the intrinsic velocity of electrons to detect the variation of the internal coordinate. A graph of frequency vs. intensity of light could be easily formulized by using the femtosecond laser detection. The correction can be examined by calculating the peaks of graph which are the bonds who have resonance with the light frequency. (If the bond has the same frequency as the incident light, it will absorb energy which could enhance its intensity of reflection light). Comparing the results produced by femtosecond laser and the results obtained by common systems in Figure 3, it is easy for us to find out the superior one. The right shift of absorption angle suggests that the IR spectroscopy without GUP modification underestimates the bond strength (length) due to the the limitation of $\Delta t$ for common IR. And the Intensity of each peak is similar, which indicates the modified ID spectroscopy is basically correct and possesses the higher accuracy.

As we introduced before, phonon is a quantum mechanical description of an elementary vibrational motion in which a lattice of atoms or molecules uniformly oscillates at a single frequency ($f$), meaning that the frequency of the single oscillation can be regard as a characteristic of the crystal.
Figuratively speaking, if we consider the atoms of the crystal as small balls, the entire crystal structure is composed of many small balls which are linked by a spring between each other. Thus the vibration of each atom will affect the surrounding atoms, which makes the vibration propagate in the form of elastic wave in the crystal. So this kind of vibration can be considered as a series of basic vibration superposition. Phonons play a major role in many of the physical properties of condensed matter, like thermal conductivity and electrical conductivity, because all of these properties are decided by the Vs (speed of phonons) and more importantly, the structure of the crystal. Phonons could carry energy, like thermal and kinetic energy between two atoms, and the speed of phonon is always constant. The distance between two atoms is approximately one angstrom; equivalently, the $f$ of the phonon is about $10^{-12}$/s due to the constant speed of phonon (approximately 300m/s), which is an extremely small number but a neat size for femtosecond laser to detect. By detecting the frequency of the crystal, we could use the lattice structure of the crystal to conceive the entire structure of the crystal. Hence, we could use this model to examine our modification in the Figure 4. Similarly, the right shift of absorption angle suggests that the IR spectroscopy without GUP modification underestimates the lattice parameters and we can obtain more accurate crystal structure according the modified pattern. The last system, which is the most different system from the former two systems, is using the femtosecond laser to induce the nuclear fission. The energy it needed to induce the nuclear fission is enormous, and perfect for femtosecond to produce due to the magnitude of $\Delta t$. The energy that the femtosecond could produce per second can be calculated by the HUP: $\Delta E \cdot \Delta t \geq \frac{\hbar}{4\pi \Delta t} \approx 10^{-19}/s$. Then the energy should be divided by the $\lambda^2$ which will help calculate the energy for one single point of femtosecond laser. Assuming the $\lambda$ is about 700nm, then the $\Delta E = \frac{10^{-19}}{(7\times10^{-7})^2} \approx 10^{-5}/s$. Then, it is not hard for us to calculate the power of the femtosecond laser which could produce such huge amount of energy, for which it is vital to design an accurate controllable output procedure. Even the most powerful voltage on earth can only keep it working for $10^{-4}$s because the energy that the voltage can supply is only about $10^6$/s. In Figure 5, we design an output procedure for the femtosecond laser, on which the graph can be represented by $\Delta E$ vs. $\Delta t$, and the lasting time of impulse is about $10^{-6}$s.

![Figure 1](image-url)
Figure 2. In the figure 1A, the light source structure of the common Nd:YLF pump laser is illustrated, which is also the targeted equipment modified in our work; In the figure 1B, we draw a brief illustrative diagram for the coherence method of 2D ultra-fast IR spectroscopy.

Figure 3. This graph indicates the IR spectroscopy of Sodium Chloride solution before and after the GUP modification.

Figure 4. This graph indicates the IR spectroscopy of LaAlO3 crystal before and after the GUP modification.
Figure 5. This graph represents the pulses procedure of the femtosecond laser dependent on controllable time. And we set up two kinds of output power mechanisms between a 2 second-interval.

4. Summary and outlook
According to the rough calculation and analysis above, we could carefully draw following conclusions: First of all, the femtosecond laser is definitely capable of detecting the structure of super small size and processes of ultra fast speed. Secondly, the femtosecond possesses following unique advantages such as high time resolution, low damage to sample, good energy efficiency and flexible controllability. Moreover, the absorption spectroscopy from femtosecond lasers can be extended to a 2D-IR image, which contains larger amount of information from the testing systems. After the modification of Generalized Uncertainty Principal for the accuracy of femtosecond laser, the resolution of detection can be enhanced extremely under the relatively high temperature which can help femtosecond laser perform well in detecting the step reactions.

The reason why the accuracy of the data is poor is that the measuring tools are not accurate enough, which causes the accuracy of input and output becoming erroneous. Although it will not affect the measurements in macroscopic world, the effects in the quantum level are tremendous. Solving one error is not really a big deal, we have to stand on a higher level to view the entire system and correct the basic inaccuracy from the measurement tools, which causes the entire system imprecise. However, there are lots of systems could be corrected by this method, such as electrons. The electrons could be used as a detector. By using the same method, the results can be more accurate. But the mass of the electron cannot be ignored, meaning that at some degree, it might interfere the intrinsic properties of the system and conduct some physical or chemical reactions when we provide electrons with high energy.

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