Internal clocks induced by generating functions

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Abstract

We introduce a nonlinear model that defines a complex system possessing diverse mediums shaped by generating functions. Our model is implemented on systems with internal clocks.

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1. Introduction

Order and disorder are fundamental concepts in science and technology. In many cases, order and disorder are related to the difference between inanimate and living systems (for examples, see [1, 2, 3, 4]).

Order processing reduces entropy. Thus, to avoid violating the second law of thermodynamics, systems that increase order, such as systems in which a phase transition from liquid to solid occurs, will release heat to an external thermal bath. Consequently, the entropy of the external reservoir increases such that the total entropy (including the system and the thermal bath) increases [5, 6, 7, 8]. This process is included within the self-organization model, a nonlinear system in which only small parts (the system) of an initially disordered system become ordered [9].

For many systems, the thermal reservoir is identified as an external dissipative environment, such as the cooling of a hot cup of tea in cold weather. However, there are scenarios in which the system itself contains dissipative media. Thus, it can be concluded that the internal parts of a living organism are surrounded by an aqueous solution to compensate for their decreasing entropy. Indeed, it has been numerically
shown, using the thermodynamic path integral technique, that DNA molecules, evolving towards an ordered state, are accompanied by an entropy increase in the surrounding aqueous solution environment [10, 11, 12].

Order is present in internal clock systems, which are systems containing oscillating parts that, by determining a time scale, regulate the operations of the system. Computers are regulated by an internal clock produced by a microchip that regulates the timing and speed of all the functions of the computer. Within this chip is a quartz crystal that oscillates at a specific frequency. This oscillation can be modeled by using a nonlinear dipolar method [13]. A review of this subject can be found in [14]. Clearly, such a process involves releasing heat into an external environment, thereby increasing the global entropy.

Living systems are much more complex. By possessing internal clocks, they also determine timescales that monitor activities, such as body part regeneration. However, in contrast to computer systems, living systems can increase the entropy of an internal environment, as described in the example of the DNA molecule [10, 11, 12]. Another example of the role played by an internal clock in a living system can be found in [15], which reviews the concept of time perception.

If an abstract scale that ranks the animativeness of a system is considered, it becomes clear that internal clock systems are at a higher place than systems that generate ordered shapes, such as a snowflake. We therefore find it important to construct a mathematical framework to describe all internal clock systems and, in particular, ones with self-dissipative environments.

We suggest a mathematical model: The Internal Clock Ordering (ICO) model. Following from the self-organization models, our model consists of nonlinear equations. In particular, we demonstrate our approach with nonlinear classical mechanical systems, such as the damped, driven pendulum [16] or the Duffing equation [17]. In our model, the system is divided into sectors: each sector is associated with a nonlinear equation. For each sector, we solve its corresponding nonlinear equation to obtain two major types of sectors: the internal clock sector, i.e., oscillating bulks with constant global frequencies; and chaotic sectors, which induce ergodic environments.

2. Model

We find it convenient to describe the ICO system by means of operators (not necessarily Hermitian or unitary) operating on states. We emphasize that although we implement mathematical techniques from quantum mechanics, our system belongs to the domain of classical mechanics, as we demonstrate in part 2.1.
2.1. The parts’ states

We divide our system into $N$ parts, where each part behaves similarly to a classical particle. By the term “classical particle,” we mean a body or some other segment that, as a whole, obeys some physical laws. For example, as a response to a linear force, the whole body or segment oscillates at the same frequency.

The fundamental basis for spanning the space of the parts is the labeling basis, $|i\rangle$, which associates each part with consecutive integers, $i = 1, 2, 3..., N$. It should be noted that the parts are distinguishable, namely $\langle i | j \rangle = \delta_{i,j}$.

After a part is labeled, it is associated with physical phenomena. We demonstrate how part $i$ is associated with a classical location quantity. The unit operator is defined by $I_{\vec{R}} = |x \rangle \langle x | + |y \rangle \langle y | + |z \rangle \langle z |$, where $x, y, z$ represent the coordinates in space of the particle.

On applying $I_{\vec{R}}$ to a labeled state, $|i\rangle$, we get

$$|i\rangle_{\vec{R}} \equiv \sqrt{I_{\vec{R}}} |i\rangle = \hat{x}_i |x\rangle + \hat{y}_i |y\rangle + \hat{z}_i |z\rangle \equiv \vec{R}_i \tag{1}$$

with the unit vector components

$$\hat{x}_i \equiv \langle x | i \rangle, \quad \hat{y}_i \equiv \langle y | i \rangle, \quad \hat{z}_i \equiv \langle z | i \rangle. \tag{2}$$

By knowing the length of the vector, $R_i$ for the $i$ part, we can define the location vector:

$$\vec{R}_i \equiv R_i \hat{R}_i \equiv R_i |i\rangle_{\vec{R}}. \tag{3}$$

2.2. Operators

On implementing standard theories, we suggest the following operators:

- The observable operator $M$
  
  Each basis of states $\{|i\rangle\}$ corresponds with a measurement of a physical quantity. By following quantum mechanical techniques, we associate each basis with the observable operator $M$

  $$M = \sum_i a_i |i \rangle \langle i |. \tag{4}$$

  where $a_i$ is the physical quantity associated with the states $|i\rangle$. There is no evidence for a quantum collapse in our system. Therefore we can say that we implement our formalism for diagonal observables only. If there was
some kind of a collapse, it happened a long time before our system was shaped to today’s current situation.

- The generating operator \( G \)
  This operator divides the system into sectors (indexed by the letter \( \sigma \)), where each sector contains many parts of \(|i\rangle\) to define a medium. Each sector is driven by a unique generating function. A \( \sigma \)-sector operator is represented by partial projection operators \( P_{\sigma} \) of the form \( \sum |i\rangle \langle i| \). The generating term is created by multiplying each \( P_{\sigma} \) by a unique function, \( F_{\sigma}(t) \), such as \( F_{\sigma} = A_{\sigma} \cos \Omega_{\sigma} t \). Thus, we present the generating operator:

\[
G = \sum_{\sigma} F_{\sigma}(t) \sum_{i \in \{\sigma\}} |i\rangle \langle i|.
\]

\[
\text{(5)}
\]

- The linear oscillator operator \( O \).
  This operator creates a basic oscillation frequency to be modified by a nonlinear term. It is presented as

\[
O = \mu \frac{d^2}{dt^2} + \kappa, \quad \mu, k = \text{constants} > 0.
\]

\[
\text{(6)}
\]

- The dissipation term \(-\beta \frac{d}{dt}\) with \( \beta = \text{constant} > 0 \).
  This operator represents losses to either the external or the internal environments.

- The feedback operator \( F \) defined through the operation

\[
F : a \rightarrow \frac{f(a)}{a} - \kappa,
\]

\[
\text{(7)}
\]

meaning that, by applying \( F \) to a function \( a(t) \), it transforms it into the nonlinear functional \( \frac{f(a)}{a} \). \( f(a) \) is a nonlinear term, such as a pendulum term \( f(a) \propto \sin(a) \) [16] or Duffing term, \( f = ga + \lambda a^3 \) [17].

In contrast to a linear oscillator, which can oscillate only with a single or discrete set of resonance frequencies, adding the nonlinear term \( F \) allows fine-tuning the frequencies.

Note that in order to simplify our model and be consistent with known numerical models, we assumed the following:

- The magnitudes of the coefficients, \( \mu, \kappa, \) and \( \beta, \) and the mathematical form of the functional \( f \) are the same for all \( i \)-parts. This means that, except for the generating function, all parts are the same and are subject to similar conditions. Consequently, the nature of each sector is exclusively determined by the generating function.

- Our model has no interaction terms, such as \( f_{i,j}(a_i, a_j) \). In other words, we assume that a basis that eliminates any interaction terms can always be selected.
Figure 1. An example of a bifurcation diagram for a pendulum [18].

We emphasize that these assumptions are not mandatory in our description. All models that follow the universal bifurcation diagram, as described in Figure 1 [19], are appropriate.

2.3. The ICO operator equation

We construct the ICO operator equation as follows. First, the physical characteristics of each part \( |i \rangle \) are represented through the observable operator \( \mathbf{M} \). Then, by applying the sum \( \{ \mathbf{O} + \mathbf{F} \} \) to the observable operator \( \mathbf{M} \), we qualify the system parts to oscillate. The exact nature of the oscillations is determined by comparing the product of operators \( \{ \mathbf{O} + \mathbf{F} \} \cdot \mathbf{M} \) to the generation operator \( \mathbf{G} \). We then obtain the ICO operator equation:

\[
\{ \mathbf{O} + \mathbf{F} \} \cdot \mathbf{M} = \mathbf{G}.
\]  

(8)

Substituting eqs. (5), (6), and (7), we obtain

\[
\sum_i \left( \mu \frac{d^2 a_i}{dt^2} - \beta \frac{d a_i}{dt} + f(a_i) \right) |i \rangle \langle i| = \sum_\sigma F_\sigma (t) \sum_{i \in \sigma} |i \rangle \langle i|
\]

(9)

or

\[
\forall i \in \{ \sigma \}, \quad \mu \frac{d^2 a_i}{dt^2} - \beta \frac{d a_i}{dt} + f(a_i) = F_\sigma (t).
\]

(10)

With the appropriate \( f \) and \( F \), these equations belong to a class of nonlinear equations exhibiting universal behavior, as shown in Figure 1 [20].

2.4. Sectors within the ICO model

Through Eq. (10), the following sectors are found:

- The pure dissipative sector \( (\sigma = \delta) \).
It is defined as \( \forall i \in \{ \delta \}, F = 0 \), which yields

\[
\sum_i \left( \mu \frac{d^2 a_i}{dt^2} - \beta \frac{da_i}{dt} + f(a_i) \right) = 0,
\]

with the solution \( \forall i \in \{ \delta \} a_i \to 0 \), representing the dissipation.

The other sectors are obtained by solving the equation

\[
\sum_{i \in \sigma \neq \delta} \left( \mu \frac{d^2 a_i}{dt^2} - \beta \frac{da_i}{dt} + f(a_i) \right) = F_\sigma (t),
\]

where different generating functions, \( F_\sigma \), shape different sectors, as follows:

- The single clock sector.
  This applies for a single frequency solution (\( \sigma = \emptyset \)).
- The few-clock solution.
  Following a bifurcation behavior, the generating function is tuned such that \( a_i \) converges to forking frequency, namely, period doubling \( 2, 4, 8, \ldots \), etc. [21].
- The environment sector, or, the chaotic sector.
  In this sector, \( a_i \) has chaotic behavior, i.e., it has an infinite spectrum of frequencies.
- The windows sector.
  It is known that even in the chaotic regime, there is a narrow regime (islands or windows) in which the system behaves in a regular manner. However, instead of the usual number of \( 1, 2, 4, \ldots \)-periods doubling, there are periods of odd number \( 1, 3, 5, \ldots \) [21].

It should be noted that in order to avoid discontinuities between nearby oscillating sectors, we assume that each internal clock sector is surrounded by a chaotic medium.

3. Discussion and conclusions

This paper has introduced a model describing the physics of internal clock systems. Although our primary motivation was to describe these systems, the proposed model goes beyond this.

Our model exhibits diverse sectors shaped by generating functions. It has been shown that when there are many degrees of freedom, our ICO system behaves as a microcosm with thermal baths (the chaotic sectors), regular sectors determining timescales, and open sectors emitting heat to an external environment. Thus, we can say that in addition to the real physical laws, the generating functions can induce a platform for a new set of rules that shape the permitted activities of the
system. We suggest that these rules shape a living environment in which biological activity occurs. By assuming that our model describes living systems, we can say that biological activities are induced by generating functions.

Internal clocks must measure various time scales: appetite and sleep–wake time scales are determined by cycles of days, subjective time estimation requires a resolution of minutes, sound localization requires a resolution of milliseconds and even below [22]. In our model, each clock and its corresponding medium is determined by a different generating function. In our mathematical presentation, the generating function was introduced through an implicit function $F_\sigma(t)$ which is periodic with the appropriate frequency. It would be interesting to discover explicit forms of $F_\sigma(t)$ for the various clocks. It would be interesting to find out if, regardless of the different time scales, they all have a similar mathematical form.

**Figure 1** shows bifurcation behavior, namely, the generating function is tuned such that we obtain a few frequencies, usually 2, 4, 8, ... frequencies. It is possible that no internal-clock biological systems has such a generating function. However, finding such a bifurcation behavior can support our theory. We may also find internal clocks within the environment sector as chaotic maps may also have periodic windows, usually with an odd number of frequencies.

**Declarations**

**Author contribution statement**

Yehuda G. Roth: Conceived and designed the analysis; Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper.

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**Additional information**

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