Novel String Field Theory and Bound State, Projective Line, and sharply 3-transitive group

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Abstract

Using ideas from our long studied Novel String Field Theory we consider in this article a bound state of infinitely many constituents, something that at least very approximately could mean a hadron, since hadrons have typically very many constituents. Our main point, so far, is to speculate that there should be a very high degree of symmetry between the many constituents, since the constituents behave similarly at different places in the bound state. We assume speculatively that there is a group represented sharply 3-transitively as permutation of the constituents; one can namely only have finite number of elements sharply n-transitively permuted for n larger than 3. The scattering of such bound states will in the zero Bjorken $x$ limit (which is suggested) only occur by exchange of parts of the system of constituents, quite like in our Novel String field Theory the “objects” are exchanged in bunches. The cyclically ordered chain of objects in this Novel String Field Theory are identified as a projective line structure. Also a p-adic field is a natural possibility.

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Figure 1: **Scattering by Exchange of Constituents** This figure illustrates the scattering of two bound states with (infinitely) many constituents marked in the one as blue and in the other bound state as red. After the scattering there appears again two bound states, but now notice that both of them have partly blue-marked and red-marked constituents. This we may call an “exchange of parts of the constituents”-scattering.

1 Introduction

1.1 Bound State with Infinitely Many Constituents

For a bound state of infinitely many constituents you would at first expect that the momentum of such a bound state would be shared evenly, so that each constituent would have a negligible part of the total momentum of the bound state. This is of course not safe, since a small part of the constituents might carry the bulk of the momentum, but then the majority of constituents would carry even less. One usually talks about a Bjorken-$x$ defined for each constituent and denoting the (average) fraction of the bound state momentum carried by that constituent.

Scattering of Constituent on Constituent Not Important for Many Constituents

If the single constituents carry only infinitesimally small fraction of the momentum of the bound state, the scattering of one constituent in one bound state with one in another bound state would not be much connected to the scattering of the two bound states.

Rather scattering of bound states on each other would be dominated by one bound state exchanging a bunch of constituents with the other bound state.

Scattering by Exchange of Constituents
A Motivation for People Interested in making Higher Dimensional Theories: It is wellknown: You do not have genuine renormalizable quantum field theories in higher than 3+1 dimensions. However Exception: A very bad scalar theory with $\phi^3$-interaction in up to 5+1, and completely free theories.

1.2 Novel String Field Theory of Ours

According to the Novel String Field Theory of ours[3, 6], which we shall go a bit into later, (super)string theory can be considered a completely free theory!

“exchange of parts of constituents”-scattering known from our Novel String Field Theory

In our novel string field theory, on which we worked much earlier, the strings are - somewhat similarly, but differently, to/from C. Thorn’s [5] string bit resolution of the string into “bits” (constituents) - described by means of “objects”. After scattering of a couple of strings (almost, except for a null-set) all the objects from the initial strings are either refound in the final state strings or recognized as having been annihilated. Although we can consider the “objects” constituents, they do not scatter on each other, but rather do not interact at all.

2 Motivation

2.1 Motivation and Plan

- Interpret the great feature of string theory to be that it is indeed - in our Novel String field theory - a basically free and therefore solvable theory, so that even no divergence problems appear.

- Ask if we can generalize such a string theory, still clinging the idea that the “constituents” (identified with our “objects”) do not interact under the scattering of the strings (identified as “bound states”).

- Thereby getting e.g. meaningful (renormalizable) theories in higher than 3+1 dimensions.

2.2 This work especially: Generalize Möbius Transformations

Our series of objects making up so to say an open string in our novel string field theory are organized in what we call a cyclically ordered chain, which is topologically a circle. It has in fact a “natural” symmetry under a Möbius group, as we shall explain, and can also be considered a projective line (meaning a line as in projective geometry, in which one adds to the lines an extra “point at infinity”, so that the line topologically becomes a circle rather than a usual line).

As a major part of the presentation we like to seek to go back from a very general group being analogous to the Möbius group to see to what extend we can...
reconstruct projective line for some field (in the sense of the algebraic structure
with unit element and invertibility for both a multiplication and an addition).

**Reminder of: Möbius transformations in Veneziano model and string theory**

From very early times in string theory and Veneziano model theory the Möbius transformations has shown up. In fact physicists were so kind as to call the variable in the formulation of the Veneziano model with some extra variables so that the formulation became precisely invariant under Möbius transformations Koba-Nielsen variables.

**What is Möbius transformations?** A priori the Möbius transformations are defined as transformations of the extended complex number set \( \mathbb{C} \cup \{\infty\} \) of the complex numbers with a number \( \infty \) added, a set equivalent to the complex projective line \( \mathbb{C}P^1 \) given by the transformation function

\[
z \rightarrow f(z) = \frac{az + b}{cz + d},
\]

but shall in the present article be more interested just in the real number version transforming only \( \mathbb{R} \cup \{\infty\} \), and with the constants \( a, b, c, d \) being real numbers.

### 3 Novel SFT

**How we thought in our Novel String Field Theory in the present Articles:** We used the splitting of the position variable field on the string into left and right-moving parts

\[
X^\mu(\sigma, \tau) = X^\mu_R(\tau - \sigma) + X^\mu_L(\tau + \sigma),
\]

where \( \sigma \) is the “spatial” coordinate enumerating the points along the string and \( \tau \) a “time” for the single string, both arranged in the conformal gauge, meaning they have been partially gauge chosen so that the Lagrangian simplified to a usual 1+1 dimensional massless scalar for each value of the external index \( \mu \) enumerating the imbedding space dimensions 25+1.

#### 3.1 Crucial Feature of Our Novel String Field Theory, Use \( X_R \) and \( X_L \).

Our approach was to discretize into small pieces - analogous to the string bits by Charles Thorn, who used the full \( X \) - in the variables on which these \( X_R \) and \( X_L \) only depends, namely \( \tau - \sigma \) and \( \tau + \sigma \) respectively. This means that we contrary to C. Thorn discretize into pieces variables which are not a priori physically enumerating the material of which the string consists, but a priori could be just formal parameters enumerating some degrees of freedom of the system (=the string). Therefore a priori we could not be sure if the “objects” corresponding to the small pieces in variables \( \tau - \sigma \) or \( \tau + \sigma \) can be considered “constituents”.

**By Changing Physical Interpretation a bit the “Objects” may be Constituents**, though.

A priori the “objects” are associated only with half the degrees of freedom of a string bit - namely only the right or the left moving d.o.f. - are not genuine constituents. If you speculate that the string is just a smart way of
looking at it, but not necessarily the only way, then we may speculate physically to split up a string bit (as by C. Thorn) into two physically separate objects, a right and a left. Since the two, when interpreted as the “objects” do not interact, are not really needed to be considered the same constituent, we can then make the physical speculation, or interpretation rather, that the two “objects” for same string bit are two quite independent constituents.

So we are allowed to take it that the “objects” are constituents.

3.2 Objects describing strings

The figure shall illustrate how two kinds of “objects” denoted by red and blue colored lines telling their path through space (respectively R and L), when flowing through space in long series - infinitesimally close to the neighbors - can represent/look like a string moving with lower velocity.

The “objects” move with velocity of light - actually they are free so they never change even direction -, but the string seemingly there move typically slower. The string at one moment is just where the objects meet at that moment.

The strings are just some way of seeing the objects.

3.3 Philosophy of Looking at String Theory in this Talk:

String Theory is a successful theory in higher dimensions because it is actually - according to our Novel String Field Theory - a free theory, so that it is, one can say, renomalizable even in higher dimensions. The “objects” are free massless particles.

String Versus Novel Object Chains?, Is there a truth?
We claim that there seemingly are two different ways of imagining the strings in string theory:

- 1. The **strings** are the true physical objects.
- 2. The **chains of “objects”** are the true physical objects.

You may of course claim, that if we are right that the two ways of looking at it are equivalent, then both are right!

But you could also begin to find argument, that one viewpoint is better or more true than the other one:

In a moment we shall give a couple of weak arguments, that the **chains of objects are more true!**

**Mysterious in String Theory: Cross sections for End and String crossing are same order of magnitude?**

You expect End hitting much more unlikely than hitting of proper string bulk

- For two sticks or strings you expect the cross section for that they hit to be of the order of the product of their length.
- But two genuine point particles will have in principle zero crosssection for hitting each other.

Thus we are forced to make a **Conclusion:**

Something wrong with string interpretation!
Figure 4: This figure illustrates how one may think, that all the strings continue into being virtually present in the vacuum, so that although we at first think of strings being like particles located to a place and moving around, then in fact these phenomenological strings have indeed tails continuing out as virtual present string-material present virtually in the vacuum (which is of course in all quantum (field) theories a very complicated state). We call this possible phenomenon of virtual string pieces in vacuum “Strings lost in vacuum”.

Can Vacuum Extensions of String Tails Solve Mystery by one string having an end common with another without knowing
Scattering of two circular chains (of “objects”) always goes with two local interactions of the chains

4 Generalizing

It Would be Wonderful to Generalize String Theory, Now we say it is Free

Basically as soon as you calculate scatterings by approximating that all constituents continue without interacting, you are in our present sense generalizing string theory.

So bound states of very many constituents so as each of them having very little momentum share are scattering as a generalization of the strings seen as composed from objects.

5 Symmetry

Symmetry

If a bound state or an almost bound state consists of infinitely many constituents, then one will, unless there are infinitely many types of particles, expect that most of the constituents are in many ways very similar in their way of sitting in the bound state.

One thus expects a large amount of symmetry between the constituents.
Figure 5: Here we illustrate how two cyclically ordered chains of objects (to the left) becomes two other cyclically ordered objects (to the right) via two steps of local modification. First - illustrated by the left-most of the two arrows - the two cyclically ordered chains on the left side by a single local combination becomes the single cyclically ordered chain illustrated in the middle. Next this single cyclically ordered chain touches itself and thereby split into the two cyclically ordered chains illustrated on the right-most.

An idea to implement this expectation is to postulate a group of transformations of the constituents into each other, under which the “structure” of the bound state is invariant.

*Much Transformations / Much Symmetry High n Transitivity*

5.1 3-transitive group action

We say the group $G$ acts on the space $S$ $n$-transitively, when you for every $n$ points $A,\ldots,K$ can find a group element $f \in G$ transforming these $n$ into $n$ prescribed image points\textsuperscript{11} \textsuperscript{12}. We call it “sharply”, when the group element achieving that is unique.

\begin{align*}
\text{Any } f & \in G \quad (3) \\
\text{acts } f & : S \to S \quad (4) \\
\text{For any } n \text{ points } A,\ldots,K \quad (5) \\
\text{and another set of } n \text{ points } A',\ldots,K' \quad \text{there exists } f \quad (6) \\
\text{so that } f(A) & = A' \quad (7) \\
\vdots & \quad (8) \\
f(K) & = K' \quad (9)
\end{align*}
Figure 6: The horizontal line here illustrates the set on which a group $G$ acts sharply 3-transitively meaning that there is a unique (that is what "sharply" means) group element $f$ mapping the three points $A$, $B$, and $C$ into three given points $f(A)$, $f(B)$, and $f(C)$ (that is what means it is 3-transitive).

5.2 Zassenhaus theorem, that is a not quite true theorem on 3-transitive transformations

A mathematical article by Katrin Tent, Advances in Mathematics Volume 286, 2 January 2016, Pages 722-728, begins:

"The finite sharply 2- and 3-transitive groups were classified by Zassenhaus in [16] in the 1930’s and were shown to arise from so-called near-fields. They essentially look like the groups of affine linear transformations $x \rightarrow ax + b$ or Moebius transformations $x \rightarrow \frac{ax+b}{cx+d}$, respectively."

5.3 Our own Zassenhaus-like theorem:

Thinking instead Zassenhaus finite groups on infinite ones:

A set on which transforms a group in a sharply 3-transitive way will be a projective line corresponding to some field $F$ and the transformations under the group will be Möbius transformations $x \rightarrow \frac{ax+b}{cx+d}$ with the variable enumerating the points on the "projective line" $x$ as well as the constants $a, b, c, d$ of the transformation(group element) belong to the field $F$.

Planning to ""derive"" this our own theorem, like Zassenhaus

We should at least reconstruct the field of real numbers $\mathbb{R}$ in the case we consider the Möbius transformations of the real projective line $\mathbb{R} \cup \infty$ as the sharply 3-transitive group of transformations.

First step in Reconstructing the Field $F$ from the Group of sharply
3-transitive transformations

Choose a point in the set $S$ being transformed sharply 3-transitive under the group $G$ and call it $\infty$. Then look for the subgroup $G_1$ of the group $G$ consisting of the elements in $G$ with only one fixed point in $S$, being $\infty$, (and of course also the unit element in $G$)

The idea is to identify the subgroup $G_1$ having $\infty$ as the only fixed point with the additive group of the field $F$ to be found. The group multiplication in $G_1$ inherited from $G$ of course shall be written with $+$.  
(Say $y, z \in (G_1, \ast)$, then $y \ast x = y + z$).  
(Here $\ast$ is the group multiplication in $G$.)

Second Step in Derivation, Identify Scalings from Two Fix-point Transformations

Next we notice that by requiring just one more fixed point than the $\infty$ we get (at least in the true Möbius case) a group of scalings of the “numbers” (the elements in $G_1$) around a certain number. We might call the second fix-point 0 and a similarity transformation of $G_1$ (the subgroup with one fix-point) by one $m$ in the group leaving 0 and $\infty$ say $G_2$, (so $m \in G_2$) say the similarity transformation

\[ y \in G_1 \rightarrow m \ast y \ast m^{-1} \in G_1 \]  
(10)

would be called $y \rightarrow m \cdot y$.  
(11)

This would first be a multiplication with an $m \in G_2$.

Third step, Get Identification of $G_2$ with $G_1$ by Selecting Point in $S$ to call 1

A priori the subgroup $G_1$ leaving $\infty$ and no other points in $S$ invariant, is of course different from the subgroup $G_2$ of elements in the 3-transitive transformation group of $S$, which we called $G$ having only two invariant points $\infty$ and $0 \in S$.

We may, however, choose a third point $1 \in S$ (just in a few lines we call it explicitly $1 \in S$, because we want to use the notation 1 also for an element in $G_1$, which we must then call $1_{n_1 G_1}$ to distinguish) different from the two points $\infty$ and $0 \in S$ and define a correspondence:

\[ y \in G_1 \sim m_y = m \text{ so that } y = m \cdot 1 = m \ast 1 \ast m^{-1} \]  
(12)

(here we needed a $1 \in G_1$, but we can make a corresponding 1 in $S$ as $1_{n_1 S} = 1_{n_1 G_1}(0)$. Remember that $1_{n_1 G_1}$ is indeed an element in subgroup $G_1$ of $G$ and thus a map $1_{n_1 G_1} : S \rightarrow S$ so that it makes sense to take the image of an element in $S$, namely $0 \in S$).

6 Why just sharply 3-transitive?, A good question.

Above we went to consider just sharply 3-transitively acting symmetries as the symmetry required for the system of constituents of the bound state with very many constituents and very much symmetry. It was done in the spirit that 3 was a high number, when we talk about sharp transitivity. But is it a high number, and why just 3?
The answer is that this 3 is indeed the highest transitivity one can have, if one wants the set on which the group acts to be infinite.

Jordan\cite{11} classified, that finite quadruply transitive group in which only the identity fixes four letters must be one of the following groups: the symmetric group of four or five letters, the alternating group of six letters or the Mathieu group\cite{12} on eleven letters. In a work by Marshall Hall\cite{12}, we find this work slightly extended and especially implying that there is no infinite set on which a group acts sharply 4-transitively.

Thus in our search for a bound state consisting of infinitely many or at least more than 11 constituents (the Mathieu group case; we shall not here tell exactly what a Mathieu group is) we simply cannot find any sharply 4-transitive group acting on it. We must be satisfied at most with a 3-transitive.

7 Wavefunction?

No Interaction: What determines the Wavefunction?

If we either assume or approximate away the interaction between the constituents, then what can determine the wave function?

Usually the wave function of a bound state in non-relativistic physics is given as an eigenfunction of a Hamiltonian

\[ H\psi(\text{"constituent-positions"}) = E\psi(\text{"constituent-positions"}) \] (13)

But relativistically one has to use the Nambu-Bethe-Salpeter equation\cite{1}, which is analogous to this eigenvalue equation for a bound state, where \( E \) is the energy of the bound state. But if no interaction the Hamiltonian \( H \) is just trivial (essentially 0) and of no help!

\textbf{Nambu-Bethe-Salpeter-equation}

Alternatives to make Wavefunction (meaningful)

- Postulate a new law of nature on initial and even final conditions specifying the nature of allowed wave functions (to have e.g. the smooth chain character giving the string topologically)

- Say that you do not take non-interaction 100% serious but allow very short scale or high energy interactions.

This would be very realistic in higher dimensions than 3+1.

In fact couplings of a certain energy scale order of magnitude are very weak in higher dimensions (at low energy). So one might at low energy in higher dimensions only see the interactions as left over influence on the wave functions, but that would be negligible in the scatterings (at low energy).

\textbf{Speculation of mainly Short Distance Interacting Particles in High Dimensions}

- From dimensional arguments the interaction between particles in an effective quantum field theory in high dimension goes to zero for small energy scales. So the particles have only short range interactions.
Figure 7: Formal graphic writing of the Bethe-Salpeter equation with two constituents. The propagators with $S$ are the propagators for the constituents, the black sphere $\Gamma$ is the wave function for the bound state, and the block $K$ symbolizes the interaction between the constituents. You may consider the Bethe-Salpeter-equation as analogous to the eigenvalue equation in the case of non-relativistic constituents. It is worth calling attention to, that while we for non-relativistic binding are accustomed to only describing the constituents by say their momenta (and the energy is only a consequence of that and it is positive energy for the constituent,) then here in the Nambu-Bethe-Salpeter equation the energy (or equivalently its conjugate $x^0$) is also formally a degree of freedom for the constituent. And the energy of the constituents can also be negative.

- It is very easy that bound states can exist bound by the short range forces; but they will typically have interactions only at short distances too and high masses unless mass protected.

- If some bound states or “fundamental” particle, e.g. from being chiral fermions, are mass protected they will of course be massless until their symmetry protection somehow gets spoiled.

Speculation of mainly Short Distance Interacting Particles in High Dimensions (continued)

- But if they are of small extension - like fundamental scale - they will effectively not interact from a low energy scale point of view.

- Only if we have spatially largely extended bound states, can there be appreciable interactions at low energy scales.

8 Hamiltonian?

There is a little not quite acceptable point in our “novel string field theory” consisting in, that we stress that there is no development or at least that “objects” behave freely - in fact the position moves in a trivial way if we identify a conjugate variable to what is essentially the momentum (called in our papers $J$) - but nevertheless we had also a paper\cite{17} in which a Hamiltonian appeared,
Figure 8: Here we have drawn a presumably very realistic picture of a bound state of many particles. Analysing perturbative quantum field theory you will typically find that each dressed particle consists of at first two constituents - so that the dressed particles are really bound states - but since when even further analysed, these will again turn out to consist of more constituents each, even the dressed particles come to consist of more and more constituents. So a picture with pairs forming again pairs and so on is very realistic for such perturbatively treated states. Such a picture with pairs forming pairs etc. is also very similar to what gets when the field we arrive at happens to be the 2-adic field.
which gave the usual string spectrum. That of course cannot both be right: Either we have trivial or no development at all or we have the not completely trivial Hamiltonian formulated in our variables for the “objects”.

Here we must remind of an a bit technical detail in our novel string field theory:

Because variables the $X_R$ and $X_L$, which we used to take out the degrees of freedom for the “objects” do not commute with themselves taken at neighboring points of the variable such as $\tau - \sigma$ we had to after having divided the variable $\tau - \sigma$ into small pieces to include in the true representing objects only every second one of the pieces. That we formulated by saying we only take as the true objects the small pieces with an even number in the series along in the variable $\tau - \sigma$. Then the idea was to instead represent the odd numbered pieces as being proportional to the difference of the conjugate momenta to the variables on the even places.

Without going too much in detail with this problem of only using every second of the bits into which we cut the variable $\tau - \sigma$ we may tell that we obtained a Hamiltonian $H$ which gives the energy of the string in say infinite momentum frame expressed by means of the momentum variable (which we called $J^\mu(I)$ and defined only for even values of the piece enumerating integer $I$) and the conjugate position variable (which we called $\Pi^\mu(I)$ again using only even $I$).

The crux of this writing down the Hamiltonian, that could give the string energy is that not only does it contain the for a free theory expected terms going with the square of the momenta, but it had also, namely corresponding to the odd pieces in the chain which were of the form

$$\left(\Pi^I(I + 1) - \Pi^I(I - 1)\right)^2 = \left(X^I_R(I + 1) - X^I_R(I - 1)\right)^2 \quad \text{(for } I \text{ odd)} \quad (14)$$

Such terms look like interaction terms between the neighboring even numbered “objects” and should not be allowed if we want to think of the model for the string as being a construction / a bound state made out of non-interacting constituents!

Infact the Hamiltonian giving the usual energy of the string takes the form, for the Mass square $M^2$:

$$M^2 = 2P^+P^- - \sum_{i=1}^{24}(P^i)^2 \quad (15)$$

$$= 2\left(\sum_{I=0}^{N-1}J^+(I)\right)\left(\sum_{I=0}^{N-1}J^-(I)\right)\left(\frac{1}{2\pi\alpha'}\right)^2 \quad (16)$$

$$-\sum_{i=1}^{24}\left(\sum_{I=0}^{N-1}J^i(I)\right)^2 \frac{1}{(2\pi\alpha')^2} \quad (17)$$

in infinite momentum frame, where the summation $I$ runs over the objects as sitting along the cyclically ordered chain (identified with the projective line discretized) and $i$ over the transverse dimensions, while the infinite momentum frame coordinates denoted with index $+$ and the index $-$ take care of the longitudinal momentum and of the energy.

In our works on the novel string field theory we make in order to not have too many degrees of freedom in the description with the “objects” the trick of replacing the odd $I$ objects present at first by an expression in terms of the conjugate momenta for the even $I$ variables $J^\mu(I)$, (for the transverse coordinates
\[ J^i(I) = -\pi\alpha'(\Pi^i(I + 1) - \Pi^i(I - 1)). \] (18)

In this notation with only the even objects being taken as physical, while the odd ones are replaced by the conjugate of the neighbors the mass square may be rather written

\[
M^2 = \frac{1}{(2\pi\alpha')^2} \sum_{i=1}^{24} (N\sum_{I=0}^{N-2} \text{even}(J^i(I))^2 - \sum_{K=0}^{N-2} \text{even} \sum_{I=0}^{N-2} \text{even} J^i(I)J^i(K)) \\
+ \frac{N}{4} \sum_{i=1}^{24} \sum_{I=1}^{N-1} \text{I odd} (\Pi^i(I + 1) - \Pi^i(I - 1))^2
\] (19)

For details on this kind of Hamiltonian expressions we refer to our work [17]. This kind of Hamiltonian expressions obviously has the problem of having the seeming interaction between the neighboring even numbered objects.

But then how can we keep up our claim of free constituents?

The idea to overcome this seeming contradiction in our description of our novel string field theory previously has to do with yet a technical point to which we must allude:

We had to impose a condition that the state of the cyclical chain of objects to make up the description of an open string should obey

\[
J^i(I + 1) \approx -\alpha'(\Pi^i(I + 1) - \Pi^i(I - 1)) \approx J^i(I - 1) \text{ (here } i \text{ odd)} \] (20)

where in our a bit stupid notation the momentum of an even object is denoted by \( J^i(I) \) for \( I \) even and the corresponding position variable in our old notation is \( \Pi^i(I) \) correspondingly, since the \( \Pi \) and the \( J \) are conjugate.

But now the way this approximate relation has to be implemented is by the state of the chain of objects has to be so as to fulfill it.

So this relation relating the vector from one even object to the next in position space to the momenta of the two neighboring objects is a restriction on the state, one could say an initial state condition.

### 8.1 Rewritting the Hamiltonian

The crux of the matter of the idea to solve the just above explained seeming contradiction is to say:

**As long as we are only interested in states of the system of “objects” obeying the equation (20), we should at least approximately be allowed to use this condition (20) to substitute parts of the Hamiltonian as using it as an equation, and then one can easily compute that we can rewrite the wanted hamiltonian to totally free one!**

This then means that indeed we can claim that if we have series of infinitely many genuine particles (scalars in the simple case of the bosonic string) which are free - they do not interact - but are in such a states that they form a chain and further obey our constraint (20), then the free Hamiltonian can for such special states be replaced by the usual string Hamiltonian. So for the states of relevance we indeed get the usual energy spectrum well known for the strings, in spite of the fact that we take the constituent particles to be genuine particles that do not interact.
9 An Idea of a Picture

Let us here present the idea, that we should apply the present infinite constituent bound state picture in a world in which the genuine physics theory is higher dimensional - as e.g. the model by Norma Mankoc Borstnik - and that we in practice only see bound states of very many mass protected particles such as chiral fermions or gauge particles. For example in 4 dimensions a chiral fermion is described by spinor field $\psi(x)$ imposed the restriction

$$\left(\Gamma - 1\right)\psi(x) = 0$$

where $\Gamma$ is the chirality, the analogue of $\gamma_5$. (21)

Then at low energies there will be effectively no interactions, because the dimensionalities of the coupling constants, $\kappa$ say, would be of dimensions mass to negative powers. E.g. an interaction could be of the four fermion type

$$L(x) = \kappa \bar{\psi}(x)\gamma_\mu \psi(x) \ast \bar{\psi}(x)\gamma^\mu \psi(x) + ...$$

$$[\kappa] = \left[GeV^{-n}\right]$$

so $\kappa \approx 0$ at low energies. (24)

So bound states of many such mass protected particles would have in the effective low energy scattering limit no interactions, just as we discussed in the present article.

However, the initial - and even final - conditions for the bound states, which in the no interaction approximation need an extra explanation, could now be understood as a left over from the high energy time, when the bound states were formed. That is to say: Once upon a time in the time just after Big Bang, say, the coming constituents interacted because at high energies, the couplings - like $\kappa$ - with negative power of mass dimensions would not be quite negligible. This interaction at high energy would bring the mass protected particles into bound state structures - once cooling takes place - , which would then survive into the colder times, when only the low energy approximation would be relevant, and the constituents would effective no longer interact.

If it happens that the surviving bound states are of the many constituent types, and they would obtain the symmetry properties speculated in the present article, they would, if the symmetry becomes the Mobius group with real numbers as the field, become real field projective lines, meaning circles topologically. That is to say, we would get the cyclically ordered chains of objects described in our earlier Novel String Field Theory. If one would instead take the p-adic field, one (presumably) would obtain instead the p-adic Veneziano model scheme.

In any case in the here now suggested background model for our infinitely many constituent bound states, there would be a usual quantum field theory picture in higher dimensions behind, and the model should inherit the good physical properties from such a quantum field theory, although it would not be a renormalizable one. Only the low energy limit with only exchange of constituents interactions would be what one might call “renormalizable”. Really it would in the most important case be string theory and that we would rather call “finite” than renormalizable, but the “finite” theories are, one could say, included in the renormalizable ones.
By taking such an at high energy interacting scheme as the model behind our bound states one would get a more solid physical picture and could from this picture better understand how to treat e.g. the Nambu-Bethe-Salpeter-equation technique. In such a philosophy of the high dimensional theory behind one should have a good chance using Weinbergs effective field theory\cite{13} thinking to see that the bound state scattering based on the Nambu-Bethe-Salpeter formalism\cite{11} would lead to amplitudes acceptable from say axiomatic field theory\cite{15} point of view. It thus looks as an outlook that we are close to having a scheme for making models for scattering amplitudes, that are physically acceptable.

9.1 Scale Symmetry

Let us note, that when we in the low energy limit have just the free mass protected, say chiral fermions or free “photons", then the theory is scale invariant. There are no dimensionized parameters left. So if it is as suggested string theory the dimensionized parameter $\alpha'$ in string theory cannot be in the low energy approximation proper, but can only come in via initial and final state conditions. That is to say here, that the Regge trajectory slope parameter $\alpha'$ can only come in by having been inherited from the era, when it was hot and cooled down so that the dimensionized parameters such as $\kappa$ were relevant.

9.2 Momentum Fluctuation in “Unexcited cyclically ordered system of objects/constituents”

At least for the unexcited bound state, but approximately for all in practice occurring bound states in the model, we should have that the momentum distribution for the constituents should be invariant under the Möbius transformations.

In the figure we see a drawing of the cyclically ordered chain equivalent to the real number projective line. The “four" (really we should call it d-momentum, because we consider here more than four dimensions in order to have the effective disappearance of the couplings) momentum $p_1$ and $p_2$ of the collection of the constituents sitting in the projective line in the two small (infinitesimally small) pieces $AB$ and $CD$ respectively have in the “ground state” of the bound state the correlation

\[
\langle p_1, p_2 \rangle = \text{coef.} \cdot (A, C; B, D) \quad (26)
\]

where $(A, C; B, D)$ = “anharmonic raatio” \hspace{1cm} (27)

\[
\frac{AB \cdot AD}{CB \cdot CD} \quad (28)
\]

or more precisely: \hspace{1cm} $\langle p_1', p_2' \rangle = g^{\mu \nu} \cdot \text{coef.} \cdot (A, C; B, D). \quad (29)$

The form given by the anharmonic ratio is specified by the Möbius invariance under real numbers, because only the anharmonic ratios are invariant for four points. Here the pairs of letters like $AB$ means the difference of the field $F$ coordinate for $B$ minus that for $A$. However, the coefficient $\text{coef.}$ is an a priori unspecified constant. What the value shall be is only specified by the initial and final state information on the bound states we had to impose. In the picture of there being a short distance or high energy interaction underneath the $\text{coef.}$ would inherit from such a short distance interaction. In the string theory with
Figure 9: Illustrated are the two small pieces $AC$ and $BD$ with respectively momenta $p_2$ and $p_1$ on the projective line drawn as a circle. We use the symbol $(A, C; B, D)$ for the anharmonic ratio, which in terms of e.g. distances counted with sign on the circle would be $(A, C; B, D) = \frac{AB \cdot CD}{CB \cdot AD} = \frac{AB}{CB} \cdot \frac{AD}{CD}$.

the open strings identified with our bound states this coefficient would be given by the Regge-slope $\alpha'$,

\[
\text{coef. } \propto \frac{1}{\alpha'}. \tag{30}
\]

It should be in mind that the scale symmetry is only broken by these initial and final state informations.

So the $\alpha'$ energy scale in the Veneziano model in our picture comes in via the initial and final state information, only.

10 The Cyclic ordering partly violates the full Møbius symmetry

We have to mention what formally looks like a little problem:

In our novel string field theory we had to impose the condition (20), which is not invariant under shift of orientation along the cyclically ordered chain. Formally such a condition would break the symmetry under half of the Møbius transformations. To have this condition consistent with the symmetry we should only keep those Møbius transformations, which leave the orientation along the chain or in the words of the present article along the projective line intact. This means that formally the group does not act 3-transitively, but only 3-transitively modulo the cyclic orientation.
But from what we could call an estetic point of view this only orientation keeping subgroup is quite nice in as far as it indeed lies inside the full Möbius group as a topologically separate part, a component.

It might be needed in our building of the bound states from a high energy theory to let this one have sufficient breaking of its symmetries so as to deliver such bound states that the orientation gets fixed.

11 Conclusion

We have proposed an approximation applicable hopefully to some bound states: that they have so many constituents with so equally divided momenta - or better Bjorken $x'$s $\approx 0$ [2] - that we can ignore the scattering of the constituents, when the bound states scatter.

(This means the constituents are in the approximation free, and thus the bound state not truly bound)

Conclusion Details

• Requiring High Symmetry in form of 3-transitive symmetry operation we expected - like Zassenhaus - the constituents to form a structure like a projective line $F \cup \{\infty\}$ for a field $F$. The string is the case $F = \mathbb{R}$ i.e. the field is the real number field. (Topologically the projective line is a circle.)

• We suggest that such string theory might be used when the approximation of many constituents with little momentum each becomes good. (of course string theory historically started as attempt to describe hadron physics[18])

• The p-adic theory[8] of Veneziano model is suggestively incorporated.

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