Oscillating and static universes from a single barotropic fluid

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Abstract. We consider cosmological solutions to general relativity with a single barotropic fluid, where the pressure is a general function of the density, $p = f(\rho)$. We derive conditions for static and oscillating solutions and provide examples, extending earlier work to these simpler and more general single-fluid cosmologies. Generically we expect such solutions to suffer from instabilities, through effects such as quantum fluctuations or tunneling to zero size. We also find a classical instability ("no-go" theorem) for oscillating solutions of a single barotropic perfect fluid due to a necessarily negative squared sound speed.

Keywords: initial conditions and eternal universe, quantum cosmology, dark energy theory

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1 Introduction

The broad dynamics of the universe can be categorized generally as expanding, contracting, or static. The current state, as well as we understand it, is that of accelerating expansion. However, this does not preclude such dynamics being a part of an overall cyclical behavior, or having an origin in a static phase. Such cosmological solutions to General Relativity (GR) have been known and studied for many years, and are still a current research interest.

Besides the novelty of eternal static and oscillating universes, there are several cosmological (and perhaps metaphysical) questions they attempt to answer. For any (past) eternal universe, the question of the origin of time is no longer relevant as there is no “beginning” (see, e.g. [1]). On the other hand, one is instead faced with the question of how a static universe becomes the dynamical universe we observe. Oscillating universes are a framework to answer both the history and future of the universe, and often aim to repeat forever. Both of these types of universes seek to evade an initial singularity, such as the Big Bang.

However, general results of singularity theorems [2–7] in GR prove that most cosmologies that have been studied unavoidably have some sort of singularity (i.e. geodesics necessarily have a starting point). As with many no-go type theorems, it is the assumptions and loopholes that spur further creativity on the part of theorists and model builders.

One such exception is the Einstein static universe, which is the starting point for the “emergent universe” inflationary scenario [8, 9]. This solution evades the singularity theorems by being a closed (curvature constant $k = +1$), static (Hubble parameter $H = 0$) universe. While this purports to be a (classical) solution with no beginning of time, it is classically unstable to homogeneous linear perturbations (necessary for a transition to inflation) and at best neutrally stable against inhomogeneous perturbations [10–14], as well as suffering from other quantum instabilities. Two such problems are in the precise tuning needed for the static solution, which will be broken quantum mechanically [15], as well as the possibility of tunneling to zero size (“tunneling to nothing” [16]). As such scenarios are enticing to many because of their eternal nature, even the tiniest probability of a such behavior is disastrous.

Oscillating universes (see, e.g. [17]) are another type of interesting model that can avoid the singularity theorems. A study of such solutions has been done systematically in the past [18], given various assumptions and ingredients. More recently, Graham et al. [20, 21] have explored a new class of oscillating models, which they have dubbed the “simple harmonic universe”. In these models, a classically stable oscillating solution is achieved through a
combination of positive curvature, a negative cosmological term, and a fluid with an equation of state parameter satisfying $-1 < w < -1/3$, where the equation of state parameter $w$ for a fluid with density $\rho$ and pressure $p$ is defined as

$$w = \frac{p}{\rho}. \quad (1.1)$$

Note, however, that this class of models is also unstable to tunneling to zero size [16] (there may also be an instability due to quantum particle production, as in oscillations of an Einstein static solution [22]).

With such a wide range of inputs, such as exotic matter, not all of the possibilities have been exhausted, even in standard GR. In this work we explore static and oscillating solutions (with positive curvature) for a very general equation of state, the so-called “barotropic fluid”, which encapsulates many different models. We analyze both the general conditions for these types of solutions, as well as toy examples, simplifying and generalizing previous studies. In the next section, we examine barotropic models and derive the conditions necessary to achieve either a stable static solution or an oscillating universe. While we find (classically) stable static solutions, there is a classical instability for oscillating solutions with a single barotropic perfect fluid. We also show how the simple harmonic universe can be mapped onto our model. In section 3, we examine quantum instabilities in our model. We show that these instabilities generically also apply to the barotropic models, further extending the analysis that one cannot easily construct such infinitely long-lived cosmologies. Our conclusions are summarized in section 4.

2 Barotropic cosmology

In this section we will introduce the basics of barotropic fluids and their solutions in GR. By a “barotropic fluid” we mean some matter (or fields) with pressure $p$ and density $\rho$, with the equation of state given by

$$p = f(\rho). \quad (2.1)$$

Barotropic fluids have long been studied in the context of dark energy. Specific examples include the well-known Chaplygin gas [23, 24] and the generalized Chaplygin gas [25], the linear (affine) equation of state [26–29], the quadratic equation of state [29, 30], and the Van der Waals equation of state [31, 32] (see also the review [33]). A general study of the properties of barotropic models for dark energy was undertaken in refs. [34, 35]. It is often convenient to define an equation of state parameter $w$, given by eq. (1.1). So, for instance, nonrelativistic matter is characterized by $w = 0$, while a pure cosmological constant has $w = -1$.

Assuming an isotropic and homogeneous universe, we have the Friedmann equations,

$$\frac{\dot{a}}{a} = -\frac{1}{6} (\rho + 3p), \quad (2.2)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{k}{a^2} + \frac{1}{3}\rho, \quad (2.3)$$

where $a$ is the scale factor, $k = \pm 1, 0$ is the curvature parameter, and we work in units where $\hbar = c = k_B = 8\pi G = 1$. For a barotropic fluid with equation of state parameter $w$, we can rewrite eq. (2.2) as

$$\frac{\dot{a}}{a} = -\frac{1}{6} \rho (1 + 3w). \quad (2.4)$$
We also have the equation for the evolution of the energy density,
\[
\frac{d \ln \rho}{d \ln a} = -3(1 + w).
\] (2.5)

### 2.1 Static solutions

Consider first the case of static, stable solutions to the Friedman equations for a universe containing a single barotropic fluid with equation of state given by eq. (2.1). A static solution requires \( \dot{a} = \ddot{a} = 0 \) at some fixed density \( \rho = \rho_\ast \) and scale factor \( a = a_\ast \). Then eq. (2.4) immediately tells us that \( \ddot{a} = 0 \) requires \( w = -1/3 \). To achieve \( \dot{a} = 0 \), we need (from eq. (2.3))
\[
\rho_\ast = \frac{3k}{a_\ast^2},
\] (2.6)

which requires a universe with positive curvature (\( k = +1 \)).

While these conditions are sufficient for a static solution, they do not insure stability. For a stable solution, we require that a small increase in \( a \) away from the stable solution yields \( \ddot{a} < 0 \), while a small decrease gives \( \ddot{a} > 0 \). From eq. (2.4), this stability condition will be satisfied as long as
\[
\frac{dw}{da} > 0.
\] (2.7)

How does this stability condition translate into a constraint on the equation of state \( f(\rho) \)? For a barotropic fluid, it is easy to verify that [34]
\[
a \frac{dw}{da} = -3(1 + w) \left( \frac{dp}{d\rho} - w \right).
\] (2.8)

Stability requires the right-hand side of this expression to be positive, so, with \( w(a_\ast) = -1/3 \),
\[
\frac{dp}{d\rho} < -1/3.
\] (2.9)

As an example, consider the generalized Chaplygin gas. The equation of state is given by [25]
\[
p = -\frac{A}{\rho^\alpha},
\] (2.10)

corresponding to a density evolution
\[
\rho/\rho_0 = \left[ A_s + (1 - A_s)(a/a_0)^{-3(1+\alpha)} \right]^{1/(1+\alpha)},
\] (2.11)

where the 0 subscript denotes quantities evaluated at an arbitrary fiducial value of the scale factor, and \( A_s \) is given by
\[
A_s = A/\rho_0^{1+\alpha}.
\] (2.12)

The equation of state parameter is given by
\[
w = -\frac{A_s}{A_s + (1 - A_s)(a/a_0)^{-3(1+\alpha)}}.
\] (2.13)

The original version of this model assumed that \( \alpha > -1 \) and \( A_s < 1 \), causing the Chaplygin gas to behave like nonrelativistic matter at early times and a cosmological constant.
at late times. However, Sen and Scherrer [36] extended this model to \( \alpha < -1 \) and \( A_s > 1 \). Setting \( w = -1/3 \) in eq. (2.13) we obtain the condition

\[
A_s = \frac{1}{1 + 2(a_s/a_0)^{1+\alpha}},
\]

which implies that \( 0 < A_s < 1 \). Then from eq. (2.13), we see that the stability condition, \( dw/da > 0 \), gives \( \alpha < -1 \). These parameters for the generalized Chaplygin gas allow for a stable, static solution.

### 2.2 Oscillating solutions

The existence of a stable, static solution automatically implies the existence of oscillating solutions, since one can simply perturb around the stable scale factor. However, we can derive a more general set of oscillating solutions that do not require small perturbations.

Assume that a solution exists for which the universe is oscillating between \( a_{\text{min}} \) and \( a_{\text{max}} \). Then \( \ddot{a} = 0 \) at both \( a_{\text{min}} \) and \( a_{\text{max}} \), which will be satisfied as long as the density at these two scale factors is given by

\[
\rho_{\text{min, max}} = \frac{3k}{a_{\text{min, max}}^2}.
\]  

(2.15)

The oscillating solution also requires \( \ddot{a} < 0 \) at \( a = a_{\text{max}} \) and \( \ddot{a} > 0 \) at \( a = a_{\text{min}} \). This will be achieved when the equation of state parameter for the barotropic fluid obeys

\[
w(a_{\text{max}}) > -1/3,
\]

(2.16)

\[
w(a_{\text{min}}) < -1/3.
\]

(2.17)

If \( w \) is a monotonic function of \( a \), then these two equations imply that

\[
\frac{dw}{da} > 0,
\]

(2.18)

during both the expanding and contracting phases, just as in the case of the static solution.

Now we can translate this into a constraint on the pressure as a function of the density. From equation (2.8), the requirement that \( dw/da > 0 \) is equivalent to the condition

\[
\frac{dp}{d\rho} < \frac{p}{\rho}.
\]

(2.19)

For the special case where \( w \) always remains negative, this condition reduces to the particularly simple form:

\[
\frac{d\ln p}{d\ln \rho} > 1.
\]

(2.20)

We can again take the generalized Chaplygin gas as a specific example. For the Chaplygin gas equation of state given by eq. (2.10), our condition in eq. (2.20) will be satisfied as long at \( \alpha < -1 \). The general behavior of such models was examined in ref. [36]. At small \( a \), the equation of state parameter approaches \( w = -1 \), while at large \( a \) it asymptotically behaves like pressureless dust \( (w \to 0) \). Hence, these models were dubbed “transient generalized Chaplygin gas” models. However, in the presence of a positive curvature, this type of fluid can generate oscillating solutions.
As was the case for the oscillating solutions discussed in ref. [20, 21], these models suffer from a classical instability. If we assume that our barotropic fluid is a perfect fluid, then perturbation growth will be unstable whenever the sound speed \( c_s^2 \equiv dp/d\rho < 0 \) (see, e.g., [34]). However, the upper bound on \( c_s^2 \) given by eq. (2.19), combined with the requirement the \( w < -1/3 \) at \( a_{\text{min}} \) automatically produces a negative \( c_s^2 \). This can be taken as a “no-go” theorem: no barotropic perfect fluid can combine with positive curvature to produce an oscillating universe. However, as noted in ref. [20, 21], it is possible to find non-perfect fluid models that mimic a particular equation of state, but with a different sound speed.

2.3 Comparison with the simple harmonic universe

Here we show that the background (homogeneous) evolution of the simple harmonic universe proposed in ref. [20, 21] can be put into the context of a single barotropic fluid, giving a simpler harmonic universe. This section will also serve as a guideline to show how multi-component models can be put into the context of a model with a single barotropic fluid.

The ingredients proposed in the simple harmonic universe are positive curvature, a positive cosmological constant, and fluid with \( w = -2/3 \). From eq. (2.5), the density of the fluid component scales as \( \rho \propto a^{-1} \), so the total density and total pressure in this model are

\[
\rho = \rho_0 \left( \frac{a_0}{a} \right) + \rho_\Lambda, \tag{2.21}
\]

and

\[
p = -\frac{2}{3}\rho_0 \left( \frac{a_0}{a} \right) - \rho_\Lambda, \tag{2.22}
\]

where \( \rho_\Lambda \) and \( \rho_0 \) are constant. These expressions for pressure and density can then be combined to yield the corresponding single-fluid equation of state

\[
p = -\frac{2}{3} \rho - \frac{1}{3} \rho_\Lambda. \tag{2.23}
\]

This is a form of the linear/affine equation of state previously studied in refs. [26–29]. Note that it satisfies (as it must) eq. (2.19) as long as \( \rho_\Lambda > 0 \).

3 Quantum instabilities

While such static (and perhaps oscillating) solutions may be classically stable (or at least not unstable), the situation is more complicated once we include quantum mechanical effects.

One possibility is that the universe can tunnel to another state. Specifically, we will be concerned with tunneling to zero size, \( a \to 0 \), or “tunneling to nothing”. To calculate this process we will use the Wheeler-DeWitt equation (we will follow [16]; see also [37] for a review). We start with the classical Hamiltonian for the Friedmann equation, eq. (2.3),

\[
H = -\frac{1}{24\pi^2a} \left( p_a^2 + U(a) \right), \tag{3.1}
\]

with the momentum conjugate,

\[
p_a \equiv -12\pi^2a\dot{a}, \tag{3.2}
\]

and potential,

\[
U(a) = (12\pi^2a)^2 \left( k - \frac{1}{3} \alpha^2 \rho(a) \right). \tag{3.3}
\]
With the canonical quantization of $p_a \to -i \frac{d}{da}$, the Hamiltonian becomes the operator $\mathcal{H}$ acting on $\psi$, the so-called wavefunction of the universe,

$$\mathcal{H}\psi = 0.$$  \hfill (3.4)

We will use the generalized Chaplygin gas as our prototypical example, namely the density evolution of eq. (2.11) (with $k = 1$). The potential then has the form

$$U(a) = (12\pi^2 a)^2 \left(1 - \frac{1}{3} a^2 \rho_0 \left[A_s + (1 - A_s) \left(\frac{a}{a_0}\right)^{-3(1+\alpha)}\right]^{1/(1+\alpha)}\right).$$  \hfill (3.5)

Immediately we see that, with $\alpha < -1$, generically there is a minimum at $a = 0$.\footnote{To avoid a minimum at $a = 0$ the density would need a negative component that goes to zero faster than $a^{-2}$.} A second minimum at finite, nonzero $a$ must be computed given the parameters $A_s$ and $\alpha$. We can compute a probability to collapse to $a = 0$ using the WKB action,

$$S_{\text{WKB}} = \int_0^{a_-} da \sqrt{U(a)},$$  \hfill (3.6)

with $U(a_-) = 0$ the turning point closest to $a = 0$. The probability is proportional to $\exp(-2S_{\text{WKB}})$. These calculations can be performed numerically, but it is clear that even a tiny probability for tunneling to zero size is disastrous if one wants an eternal universe.

One may wonder how general such a problem might be and possible ways to escape such a fate (e.g. using the Casimir energy [21, 38]). One approach would be to impose a boundary condition at $a = 0$ such that the wavefunction vanishes. However, there is no such freedom once the boundary condition that the wavefunction $\psi \to 0$ as $a \to \infty$ is required [38]. Thus, in general one cannot evade such a tunneling process, but in special circumstances the wavefunction may nonetheless be zero at $a = 0$ [38].

However, such an application for the Wheeler-DeWitt equation is perhaps overreaching in its applicability. We would expect that quantum gravity effects (from a complete theory of quantum gravity) to become important as the size of the universe approaches the Planck length, let alone smaller. So while tunneling to nothing appears to be legitimate concern for these types of models, we cannot say definitively that such a calculation is valid.

There are also other quantum instabilities that may arise in these models due to quantum fluctuations. Consider the static solution, which requires a specific value for the energy density, given the scale factor, eq. (2.6). Quantum fluctuations may upset such a balance (e.g. for an analysis for such a problem with the emergent universe scenario, see [15]). Furthermore, the universe cannot truly be static forever, as eventually there needs to be some sort of evolution, such as inflation. There is then a general conflict between something that is stable indefinitely and yet eventually “does something”. Thus eternal universes with a static “beginning” are contradictory by their nature.

4 Discussion and conclusions

Our results show that both static and oscillating behavior can be achieved in the context of a universe with positive curvature containing a single barotropic fluid, and we have derived the corresponding conditions on such a fluid. Note that there is a simple mapping from
barotropic fluids to purely kinetic $k$-essence models [34], so our results are easily generalized to the latter class of models. Unfortunately, the quantum instabilities that plague previous models of this kind also apply to our model as well.

In this work we have focused on models which classically have no singularities. It is possible to avoid the barotropic “no-go” theorem by loosening this restriction, e.g. a singularity in $H$ as in the “sneezing universe” [39]. Of course, in this case one loses one of the most appealing features of static and oscillating models (lack of singularities) while also introducing further difficulties and assumptions in the model.

The static and oscillating solutions with a single barotropic fluid are a generalization and extension of previous studies of these types of solutions in GR. Our exploration of barotropic oscillating cosmologies found a new difficulty in constructing such models, in addition to previous quantum instabilities. Thus we have also found that it is difficult to find truly eternal models. How this may fit in to answering broad questions, such as if the universe necessarily had a beginning, is an open question.

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