Acoustic radiation controls dynamic friction: Evidence from a spring-block experiment

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Brittle failures of materials and earthquakes generate acoustic/seismic waves which lead to radiation damping feedbacks that should be introduced in the dynamical equations of crack motion. We present direct experimental evidence of the importance of this feedback on the acoustic noise spectrum of well-controlled spring-block sliding experiments performed on a variety of smooth surfaces. The full noise spectrum is quantitatively explained by a simple noisy harmonic oscillator equation with a radiation damping force proportional to the derivative of the acceleration, added to a standard viscous term.

The science of solid friction has a long history, dating back in the western world to the geometrical work of Leonardo di Vinci, continuing with the empirical Amon-tons’ law two centuries later and Coulomb’s investigations of the influence of sliding velocity on friction in the 18’th century. Some 40 years ago, R. Feynmann in his famous lectures stated that “Friction is a very complicated matter... and in view of all the work that has been done it is surprising that more understanding of this phenomenon has not come about.” Only three decades ago was it recognised that friction plays a role in the mechanics of earthquakes and it was proposed that stick-slip as observed in friction was relevant to earthquake dynamics. Numerous laboratory experiments have been carried out to identify the relevant parameters controlling the solid friction and hence the stick-slip behaviour. Low velocity (below ≈ 1 cm/s) experiments have established that solid friction is a function of both the velocity of sliding and of one or several state parameters, characterising the true surface of contact. These so-called Ruina-Dieterich laws now constitute the basic ingredients in most models and numerical elasto-dynamic calculations directed at understanding the process of rupture and earthquake nucleation.

A well-known and serious limitation of these calculations based on laboratory friction experiments is that the friction laws used have been determined under steady-state sliding conditions using velocities of no more than ≈ 1 cm/s (and often much less), i.e., considerably below the sliding velocity of m/s occurring during an earthquake. Thus, one may ask whether it is correct to extrapolate these laws and their velocity weakening dependence to higher velocities relevant to earthquakes? Such considerations become all the more relevant when one examines the underlying physical mechanisms of the friction laws. At low velocity, effects such as hysteretic elastic and plastic deformations at the scale of roughness asperities seem to play a dominant role. At larger velocities, new mechanisms come into play. Collisions between asperities and transfer of momentum between the directions parallel and perpendicular to the motion are potentially important mechanisms. Recently, Tsutsumi and Shimamoto have performed friction measurements on rotating cylindrical samples at velocities up to 1.8 m/s and for slips of several tens of meters. Their results indicate a change of regime from velocity weakening to velocity strengthening at large velocities. This is confirmed by 3-d numerical simulations performed in a regime of velocities of meters to tens of meters per second, reflecting the increasing strength of vibrational damping.

Radiation damping is well-documented in electromagnetism and nuclear physics. Surprisingly, the corresponding mechanism of dynamic friction due to radiation of phonons or seismic waves has received little attention (see however [11]), notwithstanding its large potential impact on the dynamics. For instance, it is well-known that Burridge-Knopoff spring-block models does not recover the correct elasto-dynamic continuous limit but rather lead to a Klein-Gordon equation with a mass term implying finite range interactions [2]. This problem can be addressed by adding a viscous damping accounting for radiation losses with an amplitude finely tuned to the critical damping value [3]. The influence of ultrasound on crack dynamics in brittle materials has been demonstrated by using both the natural sound emitted by the propagating crack and an artificially generated ultrasound burst [4]. Although the acoustic energy is only 5% of the energy needed to propagate the crack, the presence of sound waves in the specimen strongly modifies the fracture dynamics because the sound interacts with the crack tip.
Here, we re-analyse the high-frequency part of the power spectrum from an experimental investigation of stick-slip in dry metallic friction [13]. We focus on the dynamic friction in the high velocity regime, i.e., slip velocities ranging up to \( v_{\text{max}} \approx 0.35 \, \text{m/s} \), which provides a direct demonstration of the role and nature of radiation damping feedback on the dynamics. Other aspects of these experiments have been reported elsewhere [13,16], but a satisfying explanation for the high velocity behaviour could not be proposed at that time.

The elongation of the spring was measured using strain gauges mounted on each side of the spring, see figure 1 and caption. The steel spring is placed inside a metal box (3) kept fixed in the laboratory frame (4). The metal box (3) forms a common shield with the electronic measuring devices and the battery (not shown) powering the Wheatstone bridge containing the strain gauges mounted on each side of the spring, see figure 2, and placed in a Wheatstone bridge. The use of batteries both for the bridge and the pre-amplifier as well as a common shield reduced electromagnetic noise significantly and the overall signal-to-noise ratio was better than 1000:1, corresponding to an elongation of the spring of 5 \( \mu \text{m} \) to 5 mm or more.

When the block moves relatively to the surface of the table, there are many collisions between the asperities of the two surfaces. Now, the asperities typically have a spacing of 10 – 100 microns, which means that the block (on average) will experience thousands of collisions between asperities per second. The motion of the block can thus be modeled as a “noisy” damped harmonic oscillator with the following equation of motion for the position \( x(t) \) of the block

\[
\ddot{x} + c_{\text{rad}} \dot{x} + c_{\text{vis}} \dot{x} + \omega_0^2 x = \eta(t),
\]

where \( \eta(t) \) is a “white” noise term, i.e., \( \langle \eta(t) \rangle = 0 \) and \( \langle \eta(t) \eta(t') \rangle = 2 \delta(t - t') \) accounting for the stochastic motion of the asperities. We introduce two damping terms. The non-standard \( c_{\text{rad}} \dot{x} \) term is the Abraham-Lorenz expression for the first-approximation of the direct reaction force due to radiational damping. A viscous friction \( c_{\text{vis}} \dot{x} \) must also be present in order to produce a friction force at constant velocities. It includes the effect of all other friction mechanisms, including a renormalisation of the Abraham-Lorenz term at the scale of individual asperities that undergo acceleration/deceleration even under constant block velocity. As we shall see below, the radiative term dominates completely above the characteristic frequency \( \omega_0 \).

It is useful to recall the derivation of the radiation damping reactive force \( F_{\text{rad}} = c_{\text{rad}} \dot{x} \). We start from the general expression for the power \( P(t) \) radiated by this element. By Galilean invariance, \( P(t) \) must be zero if the velocity \( \dot{x} \) is constant and becomes non-zero when the acceleration \( \ddot{x} \) is non-zero. Assuming analyticity and symmetry under \( \dot{x} \to -\dot{x} \) and performing a Taylor expansion in powers of \( \dot{x} \), we get the leading term as

\[
P(t) = m_c \tau \ddot{x}^2.
\]

The quadratic dependence of the radiated power by a small acceleration element is so general that it applies to any physical problem involving an accelerating body coupled to a wave. \( m_c \) is the mass of the element and \( \tau \) is...
a characteristic time proportional to $R/c$ where $R$ is the
typical linear size of the element and $c$ is a wave velocity.
Expression (2) recovers the Larmor power formula for
the electromagnetic radiation of an electric charge. It
also describes the acoustic radiation from an accelerating
volume element, the fluid gravity waves radiated from an
accelerating surface distortion or even the gravitational
waves from an accelerating black hole.

In order to obtain the expression of the reaction force
$F_{\text{rad}}$ due to radiation, we follow Jackson [3] and view,
by the requirement of energy conservation, the radiated
power $P(t)$ as minus the work per unit time of $F_{\text{rad}}$
\[ \int_{t_1}^{t_2} dt F_{\text{rad}} \dot{x} = -\int_{t_1}^{t_2} dt P(t). \]
Integrating the r.h.s. by part and neglecting the boundary term (which are zero
for periodic motion), we get the Abraham-Lorenz expression
for the radiative force
\[ F_{\text{rad}} = m_c \tau \ddot{x}. \] (3)

As a first-order approximation, it must be replaced by
an integro-differential equation when radiation damping
becomes the dominant term in the dynamics.

The consequence of this result is dramatic. For a given
oscillatory amplitude, the radiative damping is propor-
tional to $\omega^3$ compared to the usual $\omega$ for viscous damping.
This corresponds to a weaker damping at low frequency
and a more efficient effect at large frequencies (but still
sufficiently small so that the wavelength remains larger
than the source size).

In (4), a renormalised coefficient $c_{\text{rad}}$ is used instead of
$m_c \tau / m$ to account for the collective behaviour summed
over all asperities. We can estimate it theoretically by
transforming expression (3) in the Fourier domain as
\[ P_\omega = c_{\text{rad}} m a^2 V^2, \]
where $m$ is the block mass and $V$ the particle velocity of the sound wave generated by the
moving mass. We then equate this expression to the
radiated power of $N$ coherent asperities of radius $R$
and contact pressure $p$ given by $1.2N^2 \omega^2 (\pi R^2 p)^2 / \rho c^2$ [17],
where $\rho \approx 7.8$g/cm$^3$, $c \approx 5800$m/s for steel and the coefficient 1.2 is for Poisson ratio of 1/4 and varies slowly.
Notice that $N R^2 p \approx mg$, assuming that the contacts
are represented by discs of radius $R$, where $g = 9.8$ m/s$^2$
is the earth acceleration. This approximation assumes
that there is no significant additional inertial pressure
due to vertical motion of the block other than its static
weight on the contact asperities. This leads to the following
simple expression:
\[ c_{\text{rad}} \approx \frac{1.2}{\rho m} \left( \frac{mg}{Vc} \right)^2. \] (4)

All parameters in (4) are known except the particle wave
velocity $V$, which can be determined from our fit to the
spectrum.

The power spectrum corresponding to (5) is
\[ S(\omega) = \frac{2b}{(\omega^2 - \omega_0^2)^2 + \omega^2 (c_{\text{rad}} \omega^2 - c_{\text{vis}})^2}, \]
leading to an $1/\omega^6$-decay of the power spectrum for fre-
quencies larger than the natural frequency $\omega_0 \approx 28$ s$^{-1}$.

Figure 3 shows the power spectrum measured experimen-
tially in the regime where the block was constantly sliding.
Note the corner-frequency (or shoulder) at $\omega_0 \approx 28$ s$^{-1}$.
There appears to be a $1/\omega^2$ background below this corner
frequency. This is most likely an artifact of making a fi-
nite time measurement thus creating an illusion of a slow
constant drift in the signal. Indeed, if $x(t) \rightarrow x(t) + at$
then $\tilde{x}(\omega) \rightarrow \tilde{x}(\omega) + a/\omega$ (for periodic boundary condi-
tions) resulting in the addition of a $a^2/\omega^2$-term to equa-
tion (3).

In figure 3 is also shown a fit to the data with equation
(5) with this correction term added and with only viscous
damping, i.e., ($c_{\text{rad}} = 0$). It emphasises the importance
of the radiation term to account correctly for high fre-
cuency tail of the spectrum. In figure 3 we also show a
fit of the high frequency tail of the spectrum with the pre-
dicted $1/\omega^6$-decay from the radiation term. The agree-
ment between the data and the prediction is excellent.
The fit of the entire spectrum with both damping terms
non-zero in equation (5) is found to be highly unstable
due to the complete dominance of the radiative damping
compared to the viscous damping indicating that most
of the useful high-frequency spectrum shown in figure 3
is controlled by the novel radiation damping term.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{spectrum.png}
\caption{The power spectrum of the motion of the block in the experi-
mental setup shown in figures 1 and 2. The fit is equation (6) with a $a/\omega^2$-term added to account for the
apparent drift and with $c_{\text{rad}} = 0$ and $\omega_0 = 28 \text{s}^{-1}$. The high
frequency tail of the spectrum has been fitted with a pure
$\omega^{-6}$-decay.}
\end{figure}

From the fit of the high frequency tail of the spectrum
we get $c_{\text{rad}} \approx 0.02$ s. Using (4) we obtain an estimation
$V \approx 25$ µm/s for the particle velocity giving an
acoustic pressure $\rho c V \approx 1100$ Pa corresponding to very
energetic local sources.
This finding may illuminate the so-called “cut-off” frequency problem \([\omega_0]\), on the source acceleration spectra of moderate to strong Californian earthquakes. The unexpected result is that the seismic spectrum falls off very fast beyond a frequency attributed to a characteristic size of cohesive fault size or to scale-length of heterogeneities of the fault plane. Our results may quantify this phenomenon by confirming the role of the radiation by accelerating asperities and specifies the quantitative shape of the spectral fall-off. However, earthquakes faults have a thick layer of fault gouge between the sliding rock blocks which might affect the magnitude of the radiation term. In order to clarify this issue, further experimental work with such an intermediate layer should be performed.

In conclusion, we have presented a new analysis of high sensitivity measurements of noise spectra in spring-block experiments \([15]\) with sliding velocities in the range \(\approx 0.35 \text{ m/s}\). We have shown that the spectrum exhibit an approximate \(1/\omega^6\)-decay for frequencies larger than the natural corner value \(\omega_0 = \sqrt{k/m}\). This \(\omega\)-dependence can be rationalised simply by the generic Abraham-Lorenz radiation damping law, with a reactive force proportional to \(\dot{x}\), generated by the radiation of sound waves due to collisions of asperities. We expect that this finding extends to the slip-stick regime for which measurements of the two-point correlations of successive slip characteristics, slip distance and time of slip were found to be very weak \([16]\) indicating highly nonlinear dynamics. The implication of this finding for rupture and earthquake modelling is of great potential impact. Our results suggest that the Ruina-Dieterich friction laws cannot be extended to the high velocity regime relevant for earthquakes and many cases of rupture. Future works include the generalisation of the acoustic Abraham-Lorenz radiation damping beyond the first-order approximation and the derivation of a generalised Fluctuation-Dissipation theorem relating the noise amplitude \(b\) to the damping coefficients \(c_{\text{rad}}\) and \(c_{\text{vis}}\).

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[1] W. R. Brace and J.D. Byerlee, Science 153, N3739, 990 (1966).
[2] F.P. Bowden and L. Leben, Proc. Roy. Soc. (London) A 169, 371 (1939).
[3] B.N.J. Persson and E. Tosatti, eds., Physics of sliding friction, NATO ASI Series, Kluwer Academic Publishers, Dordrecht (1996).
[4] See for instance: W.F. Brace, Tectonophysics 14, 189-200, (1972). J.H. Dieterich, J. Geophys. Res. 77, 3690-3697 (1972); Pure and Applied Geophysics 116, 790-806 (1978); J. Geophys. Res. 84, 2161-2168, (1979); Tectonophysics 211, 115-134 (1992). A. Ruina, J. Geophys. Res. 88, 10359-10370 (1983). S.J.D Cox in Deformation Mechanisms, Rheology and Tectonics, Knipe et Rutter, eds., vol. 54, 63-70, Geological Society Special Publication (1990). N.M. Beeler et al., J. Geophys. Res. 101, 8697-8715 (1996). N.M. Beeler et al., Geophys. Res. Lett. 21, 1987-1990 (1994). C.H. Scholz, Nature 391, N6662, 37-42 (1998). G. Zheng and J. R. Rice, Bull. Seism. Society Am., Vol 88, No. 6 1466-1483 (1998).
[5] See for instance: F.P. Bowlen and D. Tabor, The Friction and Lubrication of Solids, Oxford University Press (1954) H.J Jensen et al., J. Phys. I France 3, 611-623 (1993). J. Dieterich and B.D. Kilgore, Pure and Applied Geophysics 143, 283-302 (1994). C. Caroli and P. Nozières in Physics of Sliding Friction, Persson et Tosatti, eds., Kluwer Academic Publishers (1996). A. Tanguy, and P. Nozières, J. Phys. I France 6, 1251-1270 (1996). A. Tanguy and S. Roux, Phys. Rev. E 55, 2166-2173 (1997). C. Caroli and B. Velicky, J. Phys. I France 7, 1391-1416 (1997). L. Bocquet and H.J. Jensen, J. Phys. I France 7, 1603-1625 (1997).
[6] See for instance: J.A.C. Martin and J.T. Oden, Comp. Meth. Appl. Mech. Eng. 52, 527 (1985); J.A.C. Martin, J.T. Oden and F.M.F Simões, Int. J. Eng. Sci. 28, 29 (1990). J. Lomnitz-Adler, J. Geophys. Res. 96, 6121 (1991). D. Pisarenko and P. Mora, Pure and Applied Geophysics 142, 447 (1994).
[7] A. Tsutsumi and T. Shimamoto, J. Geol. Soc. Japan 102, 240-248 (1996). Geophys. Res. Lett. 24, 699-702 (1997).
[8] C. Maveyraud et al., J. Geophys. Res. Cond-mat/9809213.
[9] J.D. Jackson, Classical Electrodynamics, 2nd ed. (John Wiley and Sons, New York, 1975), p. 783.
[10] F.V. Hartemann and N.C. Luhmann, Phys. Rev. Lett. 74, 1107 (1995).
[11] J.B. Sokoloff, Phys. Rev. Lett. 71, 3450 (1993). M.S. Tomassone et al., Phys. Rev. Lett. 79, 4798 (1997).
[12] J.M. Carlson et al., Rev. Mod. Phys. 66, 657-670 (1994).
[13] H.J. Xu and L. Knopoff, Phys. Rev. E 50, 3577 (1994).
[14] J.F. Boudet and S. Ciliberto, Phys. Rev. Lett. 80, 341 (1998).
[15] A. Johansen, Dynamics and Statistics of a Stick-Slip Experiment. Master Thesis, Niels Bohr Inst. (Mar. 1993).
[16] A. Johansen, P. Dimon, C. Ellegaard, J.S. Larsen and H.H. Rugh, Phys. Rev. E 48, 4779 (1993). A. Johansen, P. Dimon and C. Ellegaard, Wear 172, 93 (1994).
[17] G.F. Miller and H. Pursey, Proc. R. Soc. London, Ser. A 233, 55 (1955); A.M. Stoneham and A.H. Harker, Wear 80, 377 (1982).
[18] A. Papageorgiou and K. Aki, Bull. Seism. Soc. Am. 78, 609 (1988).