Dirac surface states in superconductors: a dual topological proximity effect

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In this paper we present scanning tunneling microscopy of Bi2Se3 with superconducting Nb deposited on the surface. We find that the topologically protected surface states of the Bi2Se3 leak into the superconducting over-layer, suggesting a dual topological proximity effect. Coupling between theses states and the Nb states leads to an effective pairing mechanism for the surface states, leading to a modified model for a topological superconductor in these systems. This model is confirmed by fits between the experimental data and the theory.

I. INTRODUCTION

The contact of materials with different long-range ordering modifies their properties near the interface. The superconducting proximity effect (PE), whereby superconducting correlations leak into neighboring materials, is a particular example of such general phenomena that has been comprehensively studied over the years (see Refs.1–4 for reviews and references therein). It has been recently used as the basis for topological superconductors where Majorana bound states (MBS) are predicted to be found by placing a conventional superconductor (S) on top of a topological insulator (TI)5. This leads to the appropriate p-wave pairing, which is needed for a topological superconductor, for example Majorana bound states can nucleate at vortices in such hybrid proximity systems6. This scenario was successfully experimentally demonstrated to occur in a field-induced vortex of a topological insulator-superconductor Bi2Te3-NbSe2 heterostructure7. Here we report on an overlooked dual effect whereby the topologically protected surface states (TPSS) of the TI leak into the superconducting material when the two are in contact. This opens up new possibilities for observing MBS.

In this paper we investigate the effect of the TPSS in Bi2Se3 on the superconductor Nb which is placed on top. We demonstrate that, contrary to previous expectations, the TPSS are not confined to the boundary between the materials but spread also into the superconductor. Thus to find Majorana fermions in these hybrid systems there is a delicate range of thicknesses of the superconductor layer which are thick enough to be bulk superconductors, but not so thick as to destroy the two-dimensional (2D) nature and Dirac cone of the TPSS, such that the MBS will exist. Inside the superconducting over-layer there is a superconducting pairing effect for the TPSS mediated by their coupling to the native superconductor states. Thus inside the bulk superconductor one has an effective p-wave like topological superconductor, provided the superconducting layer is of the appropriate thickness. The combination of the TPSS and the standard Bardeen-Cooper-Schrieffer (BCS) superconductivity gives rise to a striking density of states profile which can be measured using a scanning tunneling microscope (STM). The induced superconductivity of the TPSS is nonetheless not equivalent to the proximity induced pairing which gives rise to the topological superconductor in Ref. 5.

After the prediction of p-wave pairing present in hybrid S-TI systems, there have been many other theory proposals and a wide range of experiments aiming to reveal the symmetry implications of this state in various observables e.g. Josephson current-phase relationship, tunneling conductance, current noise spectra. In particular, some experiments have focused on thin films of Bi2Se3 in proximity to an s-wave8,9 or d-wave10 superconductor, or on other TI thin films11. Naturally such a system can only exhibit true 2D TPSS when it is thick enough to be approximately considered as a bulk 3D material, but signatures of px +ipy pairing induced by the superconducting proximity effect and Majorana bound states are thought to be present. Here we present results on a superconducting layer either on the surface of a bulk TI, Bi2Se3, or on the surface of an insulator, SiO2; see figure 1 for an illustration of our devices. The primary experiment tests contact with the sub-layer TI (Bi2Se3); the AlOx sub-layer stripes allow for a control measurement done during the same data run, using the same tip, and all other identical testing conditions. The surface of the TI becomes superconducting due to the proximity effect, which may show signs of p-wave superconductivity12–16. It was found that a long range proximity effect is present for the TPSS16. Furthermore the density of states displays oscillations in space and energy which are reminiscent of, though strictly speaking distinct from, Tomash and Friedel oscillations17. Signatures of topological superconductivity and MBS have also been seen in point contact18 and transport experiments19–27.
II. SAMPLES AND MEASUREMENTS

The TI used in this experiment is Bi$_2$Se$_3$. Five atomic planes with atomic order Se$_1$Bi$_1$Se$_2$-Bi$_1$Se$_3$ form a quintuple layer (QL); the QLs are weakly bound to each other, making it possible to readily expose a pristine surface for study. The exposed QL supports the existence of the TPSS, which features a single Dirac cone.

Two distinct sample growth methods were employed. Samples of type A consist of 40 QL of Bi$_2$Se$_3$ grown via molecular beam epitaxy on c-plane sapphire. Via mechanical masking and Ar+ ion milling, stripes of Bi$_2$Se$_3$ were removed, exposing bare Al$_2$O$_3$. In another deposition step, the samples were gently milled before evaporating Nb of 40, 60, or 200 nm on the entire sample. A capping layer of 5 nm of Au was evaporated in-situ with the Nb layer to prevent the formation of NbOx. This process was used both for the experimental stripes (Nb on Bi$_2$Se$_3$) and for the control stripes (Nb on Al$_2$O$_3$).

Samples of type B are uniform with respect to the x-y plane. They were grown by slowly cooling a stoichiometric mixture of Bi and Se from a temperature of 850 °C. The surface of the crystal was then cleaved in a nitrogen gas environment. Testing done in our lab shows cleaving Bi$_2$Se$_3$ in a nitrogen gas environment shifts the Dirac cone near the Fermi level. A paper on this effect is forthcoming, but a similar effect has been seen with water vapor using ARPES and has been predicted for other gases. Subsequently, 30 nm of Nb were dc sputtered on the surface at room temperature. Samples were then transferred to our custom-designed Besocke-style STM system for measurement. All the previous steps were done in a vacuum, nitrogen, or helium environment, so the sample is minimally exposed to air. To measure the tunneling spectra, the dc voltage applied between the tip and sample was summed with a 100 Hz sinusoidal voltage of 0.3 mV rms for most of the data shown in this paper. However, the data shown in figures 2(a) and 5 utilized an ac amplitude of 4.0 mV rms. The tip was positioned over the area of interest and the STM feedback turned off; the dc voltage was then slowly ramped allowing $dI/dV$-versus-$V$ to be measured using a standard current amplifier and lock-in amplifier. All STM spectra presented were taken at 4.2 K.

III. THEORETICAL MODEL

To model our experimental system we consider a superconductor deposited on top of a 3D topological insulator, as in figure 1. The superconductor is a metal with s-wave pairing, but the surface states from the TI will also spread into the metal, a process which is inevitable based on generic considerations. We seek for the simplest possible model that captures this physics. Therefore the minimal Hamiltonian has four terms

$$H = H_M + H_\Delta + H_{TPSS} + H_C,$$  \hspace{1cm} (1)

where $H_M$ is the Hamiltonian for a simple clean 2D metal, $H_\Delta$ is s-wave pairing for the metallic states, $H_{TPSS}$ is for the TI surface states (TPSS), which have spread throughout the Nb over-layer, and $H_C$ is a two-particle local coupling between the surface states and metallic states.

Firstly, the superconductor is described by

$$H_{M,\Delta} = \int d^2r \Psi^\dagger \mathcal{H}_{M,\Delta} \Psi,$$

where $\mathcal{H}_M$ and $\mathcal{H}_\Delta$ are the 2D spatial coordinate, with

$$\mathcal{H}_M + \mathcal{H}_\Delta = \hat{\xi}\tau^x + \Delta \tau^3. \hspace{1cm} (2)$$

We use the Nambu basis, with $\Psi^\dagger = \{c_{\tau_1}^\dagger, c_{\tau_1}^\dagger, c_{\tau_1}^\dagger, c_{\tau_1}^\dagger\}$, and a wavefunction $\psi^\dagger_{\tau_1} = \{u_{\tau_1}, u_{\tau_1}, u_{\tau_1}, u_{\tau_1}\}$. Here $c_{\tau_1}^\dagger$ creates a particle of spin $\sigma$ at position $r$. We will also use $\vec{\sigma}$ as the Pauli matrices in the particle-hole subspace and $\vec{\sigma}$ as the Pauli matrices operating in the spin subspace. The band operator is

$$\hat{\xi} = -1/(2m)\nabla^2 - \mu$$

and

$$\mathcal{H}_{TPSS} = (-i\nabla \cdot \vec{\sigma} - \mu_{TPSS}) \tau^z. \hspace{1cm} (3)$$
FIG. 2. [Color online] Panel (a): Wide range $dI/dV$ measurement done on 200 nm Nb on Bi$_2$Se$_3$. The local minimum at -250 mV (inset) is attributed to the Dirac point of the underlying Bi$_2$Se$_3$. The gap-like feature at 0 V is too large to be superconductivity and likely arises from the band structure of electrons near the conduction band edge of the underlying Bi$_2$Se$_3$. Panel (c): $dI/dV$ curves taken around 0 V on 40 nm Nb on Bi$_2$Se$_3$. Two gaps are apparent. The larger gap is consistent with the aforementioned gap-like feature in figure 2(a). The interior gap appears to be of superconducting origin and is fitted with a BCS $s$-wave superconducting energy gap fit. The fitting parameters are: $T = 4.2$ K, $\Delta = 1.50$ meV (3 mV peak to peak), mean free path $\ell = 32$ nm. Panel (c): The black curve is experimental data taken on Nb with AlO$_x$ sub-layer. The dashed red curve is the same BCS fitting function used in figure 2(b), with the following parameters: $T = 4.2$ K, $\Delta = 1.50$ meV, $\ell = 200$ nm. The slight background slope points to a tunneling barrier on the scale of a few eV, consistent with the vacuum tunneling barrier formed by Nb (4 eV work function) and PtIr (5 eV work function).

with $\gamma^i = \{a^i_{\alpha\tau}, a^{i\dagger}_{\alpha\tau}, a_{\alpha\tau}, -a^{i\dagger}_{\alpha\tau}\}$ and $H_{\text{TPSS}} = \int d^2 r \gamma^i \mathcal{H}_{\text{TPSS}} \chi^i$, where $a^i_{\sigma \tau}$ creates a particle of spin $\sigma$ at position $r$ at a TI surface.

We consider the simplest uniform coupling mechanism

$$H_C = \gamma \int d^2 r \gamma^i \tau^\dagger \chi^i + \text{H.c.},$$

namely a local spin independent hybridization of strength $\gamma$. After a Fourier transform and a spin rotation (see appendix A for more details), which diagonalizes $H_{\text{TPSS}}$ but leaves $H_M + H_\Delta$ unaffected, the rotated Hamiltonian density for the TPSS is $H_{\text{TPSS}} = \int (v_F k \sigma^x - \mu_{\text{eff}}) \tau^x$, and $H_M + H_\Delta = \sum_k \xi_k \tau^z + \Delta \tau^x$, with $\xi_k = (k_x^2 + k_y^2)/2m - \mu$. The Hamiltonian Eq. (1) can then be directly decoupled into two Hamiltonians:

$$H^\pm = \begin{pmatrix} -\xi_k & \Delta & 0 & -\gamma \\ \Delta & \xi_k & 0 & \gamma \\ 0 & \gamma & \xi_k & 0 \\ -\gamma & 0 & 0 & -\xi_k \end{pmatrix},$$

with $\xi_k = v_F k - \mu_{\text{eff}}$. Diagonal blocks describe S and TI states, whereas off-diagonal terms describe their mutual coupling.

From Eq. (5) it is simple to find the dispersion of the eight energy bands:

$$\varepsilon_k^{abc} = \frac{a}{\sqrt{2}} \left[ 2\gamma^2 + \xi_k^2 + c\xi_k^2 + b \sqrt{\left(\xi_k^2 - c\xi_k^2\right)^2 + 4\gamma^2 \left(\Delta^2 + (\xi_k + c\xi_k)^2\right)} \right]^{1/2},$$

where $\{a, b, c\} = \{\pm 1, \pm 1, \pm 1\}$ and $\xi_k = \sqrt{\Delta^2 + \xi_k^2}$. In the limit $\gamma \rightarrow 0$ we recover the BCS and TPSS dispersions as required. For large coupling $\gamma$ there is still a full gap in the spectrum. However for small $\gamma$ there are states inside the BCS gap caused by the TPSS which will have only a weak superconducting pairing effect. We use this model to compare directly with the results of the STM measurements.

The density of states, where the delta function peaks have been broadened into Gaussians of width $\Gamma$, is given by

$$\nu(\omega) = \int \frac{d^2 k}{(2\pi)^2} \frac{1}{\sqrt{4\Gamma}} \exp \left[-(\omega - \varepsilon_k)^2/\Gamma^2\right].$$

This can be calculated numerically with Eq. (6).

If one integrates out the BCS states then one is left with the effective model:

$$H_{\text{eff}} = \int \frac{d^2 k}{(2\pi)^2} \frac{1}{\sqrt{4\Gamma}} \exp \left[-(\omega - \varepsilon_k)^2/\Gamma^2\right].$$

If we can neglect the momentum dependence of the effective pairing, $\Delta_{\text{eff}}^x$, and chemical potential, $\mu_{\text{eff}}$, then the model is that of Ref. 5, which has a density of states

$$\nu_{\text{TPSS}}(\omega) = \frac{|\mu|}{\pi v_F} \Theta (\Omega^2 - \mu^2) \left[ 1 + \frac{\mu |\Theta (\Omega^2)|}{\sqrt{\Omega^2}} \right],$$

where $\Omega^2 = \omega^2 - \Delta^2$. More information is in appendix B. We note that neither this nor the BCS density of states are sufficient for explaining the experimental results, which requires the fully coupled model.

IV. RESULTS AND DISCUSSION

Samples of both types were studied. Samples of type A have a pristine sub-layer of Bi$_2$Se$_3$ that was prepared in vacuum, resulting in a Dirac point around -250 mV, as expected.
The superconducting energy gap of the over-layer Nb is always centered on the Fermi level, 0 mV, well away from the Dirac point. This asymmetry between the salient features, the Dirac point and the superconducting energy gap, lets us resolve both effects separately on the same measurement. Scanning tunneling spectroscopy (STS) was performed first over a stripe of Nb on AlO$_x$. A superconducting energy gap with a peak-to-peak width of 3 mV was measured and fit using an $s$-wave BCS fitting function [see figure 2(b)]. This measurement served as a quality check for our PtIr STM tip before moving on to a stripe of Nb with underlying Bi$_2$Se$_3$. At this location, both wide and narrow voltage ranges were explored. Figure 2(a) shows a wide voltage range study of 200 nm of Nb on top of Bi$_2$Se$_3$ (40 and 60 nm samples shown in appendix C). Comparison of these density of states (DOS) measurements to density of states measurements in the literature$^{31,32}$ and done in our lab$^{33}$ for bare Bi$_2$Se$_3$ highlight two features. First, a local minima [figure 2(a) inset] is observed at ~250 mV, which we interpret as the Dirac point of the underlying Bi$_2$Se$_3$. A second gap-like feature at 0 V is consistent with the band structure of electrons near the conduction band edge seen in bare Bi$_2$Se$_3$. The gap at 0 V is not the superconducting energy gap, as this gap is approximately 45 mV in width.

Narrow voltage range measurements were then carried out around 0 mV near the Fermi level [figure 2(b)]. Two gap-like features are evident. The smaller of the gaps appears to be of superconducting origin. To explore this, we fit the smaller gap with the same BCS fitting function used in figure 2(c). The fitting parameters for figure 2(b) employ more scattering, but the same temperature and energy gap as the fit for the superconductor with no underlying TI. The larger of the gaps is approximately 45 mV wide, and is consistent with the conduction band structure dip of the sub-layer Bi$_2$Se$_3$ observed in the long range DOS [figure 2(a)]. Data on type A samples support the notion of the TPSS of the TI leaking through the superconductor, but cannot ultimately be fit with the theoretical model, as the model requires symmetry about 0 V.

Samples of type B are symmetric with respect to the Dirac cone and superconducting energy gap about zero, and serve as ideal candidates for testing the theoretical model. We compare the differential conductance measured experimentally on three areas of 30 nm Nb on Bi$_2$Se$_3$ to the theory. At low temperature the differential conductance measured in the experiments is then $dI/dV \approx \nu(eV)$. In figure 3 we show several fits using $\mu$TPSS = 0, and $\nu$, $\Delta$, $\gamma$, and $\Gamma$ as fitting parameters. The overall magnitude of the density of states is also a fitting parameter. We find very good fits to both the gap structure and the Dirac cones. From these fits one has $O(\Delta) = O(\nu)$, so the induced superconductivity for the TPSS is still reasonable. However although the magnitude of $\gamma$ is found to be of the order of meV, the fits are not sufficient to pinpoint it precisely, and a certain range of possible values could still be consistent with the data. Also shown are fits to the standard BCS theory, which naturally can not fit the Dirac cone like features which we see. We also considered fits to equation (9), not shown, but they do not capture the gap feature.

The gap $\Delta$ used in the fits is slightly smaller than the known value for bulk Nb, which is 3.05meV. This is to be expected as the thin film can only just be considered a bulk sample, and additionally the presence of the TPSS will have an effect in reducing the strength of the pairing. The Fermi velocity of the TPSS is also reduced from its value on the Bi$_2$Se$_3$ surface, in this case drastically. The velocity on the clean surface is $5 \cdot 10^5$ms$^{-1}$. According to the fits in figure 3 it is reduced by three orders of magnitude, caused by the states widening throughout the Nb layer. A smaller Fermi velocity for the TPSS in turn means that their coherence length will be smaller, and hence any MBS will be confined very sharply near the vortex cores. Curiously, strong localization of MBS was recently found in a different system of ferromagnetic atomic chains deposited on a surface of a superconductor$^{34}$. It was understood that short coherence length of MBS results from the strong Fermi velocity renormalization caused by a quasiparticle weight shift of the electrons’ spectral weight from the adatom-wire into the SC via local hybridization mechanism. We reveal that a similar effect of strong velocity renormalization takes place in our proximity system.

![Figure 3](image-url)
V. SUMMARY

We have investigated the density of states on the surface of superconducting Nb deposited on a large Bi₂Se₃ substrate, finding some striking and unforeseen behavior. A Dirac cone is quite clearly visible, despite not being native to the Nb. This is due to the TPSS of the underlying Bi₂Se₃ leaking into the Nb, and hybridizing with its states. Our theoretical model fits very well to the experimental data, strengthening this proposed explanation. While such a setup does still contain effective p-wave like pairing, resulting in a topological superconductor which could host MBS there are several caveats to the Fu-Kane model which arise. As the TPSS leak into the superconductor, a thick superconducting layer would destroy the Fu-Kane model. While such a setup does still contain effective Fermi like pairing, resulting in a topological superconductor which could host MBS there are several caveats to the TPSS in the TI surface layers. We also note that the effective Fermi velocity for the TPSS is reduced by several orders of magnitude due to spreading into the superconductor, which changes the parameter space in which MBS could be found.

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Appendix A: Mediated pairing for the topologically protected surface states

As stated in the main text both the standard Bardeen-Cooper-Schrieffer (BCS), \( \mathcal{H}_M + \mathcal{H}_\Delta \), and the topologically protected surface state (TPSS), \( \mathcal{H}_{TPSS} \), parts of the Hamiltonian equation (1) can be simultaneously diagonalized upon a Fourier transform and a proper rotation. Let \( \Psi_k = U_k^\dagger \psi_k \) and \( \mathcal{H}_M + \mathcal{H}_\Delta = U_k^\dagger \mathcal{V}_k (\mathcal{H}_M + \mathcal{H}_\Delta) \mathcal{V}_k U_k \) with \( U_k \) and \( \mathcal{V}_k \) two rotation matrices. Firstly we have

\[
U_k = \frac{1}{\sqrt{2}v} \begin{pmatrix} 0 & \frac{\Delta}{\alpha v} & 0 & -\frac{\Delta}{\alpha v} \\ \frac{\Delta}{\alpha v} & 0 & -\frac{\Delta}{\alpha v} & 0 \\ 0 & \frac{\Delta}{\alpha v} & 0 & \frac{\Delta}{\alpha v} \\ \frac{\Delta}{\alpha v} & 0 & \frac{\Delta}{\alpha v} & 0 \end{pmatrix},
\]

where \( \epsilon_k = \sqrt{\Delta^2 + \xi^2_k} \) and \( \alpha_k^\pm = \frac{\xi^2_k \pm \epsilon_k}{\epsilon_k} \). Secondly \( \mathcal{V}_k \) is a spin rotation which commutes with \( \mathcal{H}_M + \mathcal{H}_\Delta \). Then we find

\[
\bar{\mathcal{H}}_M + \bar{\mathcal{H}}_\Delta = \mathcal{V}_k \mathcal{H}_M \mathcal{V}_k^\dagger + \mathcal{H}_\Delta.
\]

For the TPSS we make the spin rotation \( \mathcal{V}_k = \chi_k \mathcal{V}_k^\dagger \) with \( \mathcal{V}_k = \frac{\mathcal{V}_k}{\epsilon_k} \). We can also write \( \tan^{-1}[k_y/k_x] = \pi/2 - \phi_k \) with \( \phi_k \) the polar angle. The rotated Hamiltonian density for the TPSS is

\[
\bar{\mathcal{H}}_{TPSS} = \mathcal{V}_k^\dagger \mathcal{H}_{TPSS} \mathcal{V}_k = (v_F k \sigma^z - \mu_{TPSS}) \tau^z.
\]

The coupling becomes

\[
\bar{\mathcal{H}}_C = \int d^2 \bar{k}_F \gamma \tau^z \mathcal{U}_k \Psi_k + \text{H.c.}
\]

which all together reduces to equation (5).

If we are interested in the properties of the TPSS we can find an effective model by integrating out the BCS states. In the functional integral representation this amounts to calculating

\[
S' = -\ln \left\{ e^{-\tau \sum_n \int d^2 \bar{k} \Gamma_n \bar{\mathcal{V}}_{\bar{k}} \bar{\mathcal{V}}_{\bar{k}}^\dagger \mathcal{U}_k \Psi_{\bar{k}} + \text{H.c.} \right\}_{\mathcal{M}+\mathcal{A}}.
\]

We use the Matsubara formalism, with frequencies \( \omega_n = 2\pi n / T \) for \( n \in \mathbb{Z} \), and \( T \) is temperature. As the action we need to integrate over is quadratic this is a standard integral and we find

\[
S' = \frac{1}{2} T \sum_n \int d^2 \bar{k} \bar{\mathcal{V}}_{\bar{k}} \mathcal{U}_k g_{\bar{n}k} \bar{\mathcal{V}}_{\bar{k}} \bar{\mathcal{V}}_{\bar{k}}^\dagger \mathcal{V}_k \bar{\mathcal{V}}_{\bar{k}} + \text{H.c.},
\]

where \( g_{\bar{n}k} = \langle \bar{\Psi}_{\bar{n}k} \bar{\Psi}_{\bar{k}}^\dagger \rangle \) is the Green’s function for the superconducting states. Therefore we find, with \( S = S' + S_{\mathcal{M}+\mathcal{A}} \rightarrow S' = T \sum_n \int d^2 \bar{k} \bar{\mathcal{V}}_{\bar{k}} \mathcal{H}_{\mathcal{M}+\mathcal{A}} \bar{\mathcal{V}}_{\bar{k}} + \text{H.c.} \)

\[
\mathcal{H}_{\mathcal{M}} = -\epsilon_{\bar{k}} \text{e}^{-\tau \mathcal{U}_k} \mathcal{U}_k \tau^z \mathcal{U}_k = \Delta_{\mathcal{M}}^{\text{eff}} \mathcal{V}_k \tau^z,
\]

\[
\Delta_{\mathcal{M}}^{\text{eff}} = \frac{\gamma^2 \Delta}{\omega_n^2 + \epsilon_k^2}, \quad \text{and} \quad \mu_{\mathcal{M}} = \frac{\gamma^2 \xi_k}{\omega_n^2 + \epsilon_k^2}.
\]

The effective action is then over the new Hamiltonian \( \mathcal{H}_{TPSS} = \mathcal{H}_{\mathcal{M}} + \mathcal{H}_{\mathcal{A}} \).
FIG. 4. [Color online] Magnetic field spectroscopic measurements acquired on 60 nm Nb on SiO$_2$. The coherence peaks and gap depth diminish as the field increases, with superconductivity ultimately suppressed at 3.0 T. These critical field values are in agreement with transport measurements performed on similar Nb films.

We can also consider the effect on the BCS superconductor caused by the TPSS. In this case the self energy term would be

$$S'' = -\ln \left( e^{-\tau \sum_n \int d^2k \bar{\psi}_n \psi_n + H.c.} \right)_{\text{TPSS}}, \quad \text{(A9)}$$

which gives $S'' = \sum_n \int d^2k \bar{\psi}_n H'_n \psi_n$, and $H'_n = \sum\bar{\psi}_n^+ \bar{\tau}_x \gamma \gamma \bar{\tau}_x \psi_n + H.c.$ This leads to

$$H'_n = \left[ \Sigma_{\text{eff}} - \sigma_{\text{eff}} \bar{k} \sigma \right] \bar{\tau}_x,$$

$$\Sigma_{\text{eff}} = \frac{\gamma^2 \mu_n^2 + \omega_n^2 - v_F^2 k^2}{4 \mu_n^2 \omega_n^2 + [\mu_n^2 - \omega_n^2 - v_F^2 k^2]^2}, \quad \text{(A10)}$$

$$\sigma_{\text{eff}} = \frac{\gamma^2 v_F \mu_n^2 - \omega_n^2 - v_F^2 k^2}{4 \mu_n^2 \omega_n^2 + [\mu_n^2 - \omega_n^2 - v_F^2 k^2]^2}.$$

$\Sigma_{\text{eff}}$ and $\sigma_{\text{eff}}$ describe electronic and magnetic scattering events respectively, which will contribute to the broadening of the BCS density of states. The effective action is then the new Hamiltonian $\mathcal{H}_{\text{BCS}} = \mathcal{H}_M + \mathcal{H}_S + \mathcal{H}_{\text{TPSS}}$.

Appendix B: Density of states for topologically protected surface states with s-wave pairing

We start from Gorkov’s equation:

$$\begin{pmatrix} i \omega_n - H & \Delta \\ -\Delta^\dagger & -i \omega_n - H^* \end{pmatrix} \begin{pmatrix} G_{n,k} \\ \mathcal{F}_{n,k} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \text{(B1)}$$

with

$$H^{(+)} = v_F \begin{pmatrix} 0 & \pm k_x - ik_y \\ \pm k_x + ik_y & 0 \end{pmatrix} - \mu \mathbb{I}_2,$$

which describes the 2D surface states of a 3D topological insulator. $\mathbb{I}_n$ is an $n \times n$ identity matrix. $\Delta = i \sigma^y$ is the s-wave pairing. Note that $\Delta H = H^* \Delta$. Naturally this is in the Matsubara representation with Green’s functions

$$G(r, \tau; r', \tau') = -\langle T_\tau \psi_{\sigma}(r, \tau) \psi_{\sigma}^\dagger(r', \tau') \rangle = T \sum_n e^{-i \omega_n (\tau - \tau')} G_n(r, r'), \quad \text{(B3)}$$

and

$$\mathcal{F}_{\tau}(r, \tau; r', \tau') = -\langle T_\tau \psi_{\sigma}(r, \tau) \psi_{\sigma}'(r', \tau') \rangle = T \sum_n e^{-i \omega_n (\tau - \tau')} \mathcal{F}_{\tau,n}(r, r'), \quad \text{(B4)}$$

where $T_\tau$ is time ordering along the imaginary time axis and $\psi_{\sigma}(r, \tau)$ is a Heisenberg operator. Finally $r = (x, y)$ is the 2D spatial coordinate.

We find as the differential equation for $\mathcal{F}_{\tau,n}$

$$\frac{i \sigma^y}{\Delta} \left[ |\Delta|^2 + \omega_n^2 + (H^*)^2 \right] \mathcal{F}_{\tau,n} = 1.$$  \quad \text{(B5)}$$

The bulk solution is therefore

$$\mathcal{F}_{\tau,n,k} = -\frac{i \Delta \sigma^y}{\epsilon_{nk} - 4 \mu^2 v_F^2 k^2} \left( \frac{\epsilon_{nk}^2}{\epsilon_{nk}^2 - 2 \mu v_F k} \right) \frac{-2 \mu v_F k - \epsilon_{nk}^2}{\epsilon_{nk}^2 - 2 \mu v_F k}.$$  \quad \text{(B6)}$$

where $k_x = k_x \pm i k_y$ and

$$\epsilon_{nk}^2 = \mu^2 + \omega_n^2 + \Delta^2 + v_F^2 k^2.$$  \quad \text{(B7)}$$

The normal Green’s function is found from

$$G_{n,k} = -i \omega_n + H^* \frac{i \sigma^y}{\Delta} \mathcal{F}_{\tau,n,k}. \quad \text{(B8)}$$

From this we find

$$G_{n,k} = \frac{-i \omega_n + H^*}{\epsilon_{nk} - 4 \mu^2 v_F^2 k^2} \left( \frac{\epsilon_{nk}^2}{\epsilon_{nk}^2 - 2 \mu v_F k} \right) \frac{-2 \mu v_F k - \epsilon_{nk}^2}{\epsilon_{nk}^2 - 2 \mu v_F k}.$$  \quad \text{(B9)}$$

The density of states for the TPSS is

$$\nu_{\text{TPSS}}(\omega) = -\frac{1}{\pi} \text{Im} \int \frac{d^2k}{4 \pi^2} \text{tr} G_{n,k} \bigg|_{\omega_n = -\omega + \text{i}\delta}.$$  \quad \text{(B10)}$$

FIG. 5. [Color online] Wide range $dI/dV$ measurement acquired on three thicknesses of Nb (40, 60, 200 nm) on 40 nm Bi$_2$Se$_3$. A local minima around -250 mV is seen for all three thicknesses, which is evidence for the TPSS of the sub-layer Bi$_2$Se$_3$ leaking to the surface. In addition, the gap-like feature at 0 V previously discussed is present.
We have introduced here \( \Omega \) with the branch cut taken along the negative axis. Thus

\[
\Omega = \sqrt{\omega^2 + \Delta^2}
\]

and can be calculated to give

\[
\Omega(\omega) = \begin{cases} \sqrt{\Delta^2 - \omega^2} & \text{if } |\omega| < \Delta, \text{ and} \\ -i|\text{sgn}(\omega)| \sqrt{\omega^2 - \Delta^2} & \text{otherwise.} \end{cases}
\]

The integral \( I_\omega(\mu) \) is defined as

\[
I_\omega(\mu) = \int_{\mu}^{\infty} dy \frac{\mu}{\Omega^2(\omega) + y^2},
\]

and can be calculated to give

\[
I_\omega(\mu) = -\frac{i\mu}{2\Omega(\omega)} \left[ \ln(\mu + i\Omega(\omega)) - \ln(\mu - i\Omega(\omega)) \right],
\]

with the branch cut taken along the negative axis. Thus

\[
\text{Im} \left[ I_\omega(\mu) + I_\omega(-\mu) \right] = \frac{\mu|\text{sgn}(\omega)|}{\sqrt{\omega^2 - \Delta^2}} \Theta(\mu^2 - \omega^2 + \Delta^2) \Theta(\omega^2 - \Delta^2).
\]

Finally we find

\[
\nu_{\text{TPSS}}(\omega) = \frac{|\omega|}{\pi v_F} \Theta(\omega^2 - \mu^2) \left[ 1 + \frac{|\mu|\Theta(\omega^2 - \Delta^2)}{\sqrt{\omega^2 - \Delta^2}} \right].
\]

\( \Theta \) is the Heaviside theta function.

**Appendix C: Supplemental measurements**

Figures 4 and 5 show additional measurements acquired on samples of type A. The data in figure 4 shows baseline magnetic-field dependence of the superconducting energy gap in a location without an underlying TI. We see that the energy gap is diminished with increasing field, implying a critical field between 2.5 T and 3.0 T. This is consistent with transport measurements performed on similar Nb films.

Figure 5 compares spectroscopy acquired on three thicknesses of Nb: 40 nm, 60 nm and 200 nm. At this wide voltage range the superconducting gap is not resolved. This comparison shows that the relative spectral weight of the Dirac cone and semiconductor gap edge features varies with Nb thickness. Additional measurements will be needed to characterize the length scale of the leakage of Dirac cone states.
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