Abstract

The angles of the CKM unitarity triangle are now known well enough to allow a determination of its sides from global fits, with good accuracy. Assuming that new physics does not affect the angles, UTfit and CKMfitter find $|V_{ub}| = (3.50 \pm 0.18) \times 10^{-3}$. Using this result, and new high-precision data on the spectrum of $B \rightarrow \pi \ell \nu$ decays from BaBar, we find $|V_{ub}|f_+(0) = (9.1 \pm 0.7) \times 10^{-4}$ and $f_+(0) = 0.26 \pm 0.02$, with $f_+$ the $B \rightarrow \pi$ weak transition form factor. These results are completely model-independent. We compare them to theoretical calculations from lattice and QCD sum rules on the light-cone.

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Ever since the first experimental observation of $b \to u$ transitions by the ARGUS collaboration in 1989 [1], the determination of $|V_{ub}|$ has been one of the major challenges for both experimental and theoretical B physics. While initially exclusive transitions, in particular $B \to \pi e \nu$, were considered as the most promising ones, the realisation that inclusive decays can be calculated using heavy quark expansion [2], with (seemingly) controlled theoretical uncertainties, has spurred a number of very impressive theoretical works, culminating in the calculation of decay spectra based solely on first principles (dressed gluon exponentiation [DGE]) [3] or using additional information from other inclusive decays ($b \to s \gamma$, $b \to c \ell \nu$) in order to extract the relevant non-perturbative quantities (BLNP) [4]. At the same time, the experimental measurement of inclusive $b \to u \ell \nu$ transitions made major progress since the first measurements at ARGUS and is now in a mature state. The results for $|V_{ub}|$ determined in this way are collected and averaged by the Heavy Flavour Average Group (HFAG) [5] and currently (November 06) read

$$|V_{ub}|_{HFAG}^{incl,DGE} = (4.46 \pm 0.20(exp) \pm 0.20(ext)) \times 10^{-3},$$

$$|V_{ub}|_{HFAG}^{incl,BLNP} = (4.49 \pm 0.19(exp) \pm 0.27(ext)) \times 10^{-3},$$

where the first error is experimental (statistical and systematic) and the second external (theoretical and parameter uncertainties). Both results are in perfect agreement.

At the same time, $|V_{ub}|$ can also be determined in a more indirect way, based on global fits of the unitarity triangle (UT), using only input from various CP violating observables which are sensitive to the angles of the UT. Following the UTfit collaboration, we call the corresponding fit of UT parameters UTangles. To be precise, the information entering UTangles comes from the following non-leptonic decays: $B \to \pi\pi$, $B \to \pi\rho$ and $B \to \rho\rho$ which yield the angle $\alpha$ [3]; $B \to D^{(*)}K^{(*)}$ decays yielding $\gamma$ [7]; $2\beta + \gamma$ comes from time-dependent asymmetries in $B \to D^{(*)}\pi(\rho)$ decays [8] and $\cos 2\beta$ from $B_d^0 \to J/\psi K^0_S$ [9]; $\beta$ is determined from $B \to D^0\pi^0$ [10] and, finally, $\sin 2\beta$ from the “golden mode” $B^0_d \to J/\psi K S$ [11]. Both the UTfit [12] and the CKMfitter collaboration [13, 14] find

$$|V_{ub}|_{UTangles}^{UTfit,CKMfitter} = (3.50 \pm 0.18) \times 10^{-3}. \quad (2)$$

The discrepancy between (1) and (2) starts to become significant. One interpretation of this result is that there is new physics (NP) in $B_d$ mixing which impacts the value of $\sin 2\beta$ from $b \to c c s$ transitions, the angle measurement with the smallest uncertainty. The value of $|V_{ub}|$ in (1) implies

$$\beta|_{V_{ub}}^{HFAG} = (26.9 \pm 2.0)^(c) \quad \leftrightarrow \quad \sin 2\beta = 0.81 \pm 0.04, \quad (3)$$

using the recent Belle result $\gamma = (53 \pm 20)^{(c)}$ from the Dalitz-plot analysis of the tree-level process $B^+ \to D^{(*)}K^{(*)+}$ [15].\footnote{We use the Belle measurement rather than that from BaBar, $\gamma = (92 \pm 44)^{(c)}$ [16], because the uncertainty of the latter is too large to allow any meaningful statement. At present, HFAG does not provide an average of the BaBar and Belle measurements.} This value disagrees by more than $2\sigma$ with the HFAG average for $\beta$ from $b \to c c s$ transitions, $\beta = (21.2 \pm 1.0)^{(c)}$ ($\sin 2\beta = 0.675 \pm 0.026$). The
difference of these two results indicates the possible presence of a NP phase in $B_d$ mixing, $\phi_d^{NP} \approx -10^3$. This interpretation of the experimental situation is in line with that of Ref. [17]. An alternative interpretation is that there is actually no or no significant NP in the mixing phase of $B_d$ mixing, a scenario compatible with the MFV hypothesis [18], but that the uncertainties in either UTangles or inclusive $b \to u\ell\nu$ transitions (experimental and theoretical) or both are underestimated and that (1) and (2) actually do agree. In either case, the present situation calls for a critical re-assessment of both UTangles and the inclusive analysis and for an independent determination of $|V_{ub}|$ from other sources. The aim of this letter is to provide such a determination from exclusive $B \to \pi\ell\nu$ decays, based on theoretical (lattice and QCD sum rule) calculations and recent new data published by BaBar.

As for $B \to \pi\ell\nu$, the primary observable is the branching ratio, for which HFAG quotes, combining charged and neutral $B$ decays using isospin symmetry [5],

$$B(B^0 \to \pi^+\ell^-\bar{\nu}_\ell) = (1.37 \pm 0.06{\text{(stat)}} \pm 0.06{\text{(syst)}}) \times 10^{-4}. \quad (4)$$

The extraction of $|V_{ub}|$ from this measurement requires a theoretical calculation of the hadronic matrix element

$$\langle \pi(p_\pi)|\bar{u}\gamma_\mu b|B(p_\pi + q)\rangle = \left(2p_{\pi\mu} + q_\mu - q_\mu \frac{m_B^2 - m_\pi^2}{q^2}\right)f_+(q^2) + \frac{m_B^2 - m_\pi^2}{q^2} q_\mu f_0(q^2), \quad (5)$$

where $q_\mu$ is the momentum of the lepton pair, with $m_\pi^2 \leq q^2 \leq (m_B - m_\pi)^2 = 26.4\text{ GeV}^2$. $f_+$ is the dominant form factor, whereas $f_0$ enters only at order $m_\pi^2$ and can be neglected for $\ell = e, \mu$. The spectrum in $q^2$ is then given by

$$\frac{d\Gamma}{dq^2}(\bar{B}^0 \to \pi^+\ell^-\bar{\nu}_\ell) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda^{3/2}(q^2) |f_+(q^2)|^2,$$

where $\lambda(q^2) = (m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2m_\pi^2$ is the phase-space factor. The calculation of $f_+$ has been the subject of numerous papers; the current state-of-the-art methods are unquenched lattice simulations [19] [20] and QCD sum rules on the light-cone (LCSRs) [21] [22]. A particular challenge for any theoretical calculation is the prediction of the shape of $f_+(q^2)$ for all physical $q^2$: LCSRs effectively involve the parameter $m_b/(2E_\pi)$ and become less reliable for small $E_\pi$, i.e. large $q^2$. Lattice calculations, on the other hand, are to date most reliable for small $E_\pi$, although this is expected to change in the future with the implementation of “moving NRQCD”, i.e. a non-relativistic description of the $b$ quark in a moving frame of reference (instead of its rest frame) [24]. Hence, until very recently, the prediction of the $B \to \pi\ell\nu$ decay rate necessarily involved an extrapolation of the form factor, either to large or to small $q^2$. If, on the other hand, the $q^2$ spectrum was known from experiment, the shape of $f_+$ could be constrained, allowing an extension of the LCSR and lattice predictions beyond their region of validity. A first study of the

\footnote{This is not to say that LCSRs are a power expansion in $m_b/(2E_\pi)$, which is not a small parameter. Rather, the order parameter $1/m_b$ in the twist expansion of the LCSR for the form factor at $q^2 = 0$, see Refs. [23], becomes $1/(2E_\pi)$ for $q^2 > 0$, for contributions of twist 4 and higher.}
impact of the measurement of the $q^2$ spectrum in 5 bins in $q^2$ by the BaBar collaboration [25] on the shape of $f_+$ was presented in Ref. [26]; in view of the limited accuracy of the data available in 2005 the only firm conclusion that could be drawn in [26] was that the simplest possible parametrisation of the form factor by a simple pole at $q^2 = m_{B^*}^2$, assuming dominance of the $B^*(1^-)$ meson, is disfavoured. The situation has improved dramatically in summer 2006 with the publication of (preliminary) high-precision data of the $q^2$ spectrum by the BaBar collaboration [27], with 12 bins in $q^2$ and full statistical and systematic error correlation matrices. These data allow one to fit the form factor to various parametrisations and determine the value of $|V_{ub}| f_+(0)$. As it turns out, the results from all but the simplest parametrisation agree up to tiny discrepancies which suggests that the resulting value of $|V_{ub}| f_+(0)$ is truly model-independent.

There are four parametrisations of $f_+$ which are frequently used in the literature. All but one of them include the essential feature that $f_+$ has a pole at $q^2 = m_{B^*}^2$; as $B^*(1^-)$ is a narrow resonance with $m_{B^*} = 5.325$ GeV $< m_B + m_\pi$, it is expected to have a distinctive impact on the form factor. The parametrisations are:

(i) Becirevic/Kaidalov (BK) [29]:

$$ f_+(q^2) = \frac{f_+(0)}{1 - q^2/m_{B^*}^2} \left(1 - \frac{\alpha_{BK} q^2}{m_{B^*}^2}\right), $$

where $\alpha_{BK}$ determines the shape of $f_+$ and $f_+(0)$ the normalisation;

(ii) Ball/Zwicky (BZ) [22]:

$$ f_+(q^2) = f_+(0) \left(1 - q^2/m_{B^*}^2\right) \left(1 - \frac{r q^2}{m_{B^*}^2} \left(1 - \frac{\alpha_{BZ} q^2}{m_{B^*}^2}\right)\right), $$

with the two shape parameters $\alpha_{BZ}$, $r$ and the normalisation $f_+(0)$; BK is a variant of BZ with $\alpha_{BK} := \alpha_{BZ} = r$;

(iii) the AFHNV parametrisation of Ref. [30], based on an $(n+1)$-subtracted Omnes representation of $f_+$:

$$ f_+(q^2) \stackrel{n \geq 1}{=} \frac{1}{s_{th} - q^2} \prod_{i=0}^{n} \left[f_+(q_i) s_{th} - q_i^2\right]^{\alpha_i(q^2)} , $$

with

$$ \alpha_i(s) = \prod_{j=0,j \neq i}^{n} \frac{s - s_j}{s_i - s_j} , \quad s_{th} = (m_B + m_\pi)^2 ; $$

this parametrisation assumes that $f_+$ has no poles for $q^2 < s_{th}$; the shape parameters are $f_+(q_i^2)/f_+(q_0^2)$ with $q_0^2,...,q_n^2$ the subtraction points; following [30], we choose evenly spaced $q_i^2 = q_{\text{max}}^2 i/n$; again the normalisation is given by $f_+(0)$; the assumption of no $B^*$ pole is likely to mostly impact the form factor at large $q^2$.

The spectrum has been measured previously, by BaBar, CLEO and Belle [25, 28], in a smaller number of $q^2$ bins. As the new BaBar data are more precise, and the correlation of uncertainties is unknown for the earlier measurements, we do not include them in our analysis.
(iv) the BGL parametrisation based on analyticity of \( f_+ \) [31]:

\[
f_+(q^2) = \frac{1}{P(t)\phi(q^2, q_0^2)} \sum_{k=0}^{\infty} a_k(q_0^2)[z(q^2, q_0^2)]^k, \tag{11}
\]

with

\[
z(q^2, q_0^2) = \frac{((m_B + m_\pi)^2 - q^2)^{1/2} - ((m_B + m_\pi)^2 - q_0^2)^{1/2}}{((m_B + m_\pi)^2 - q^2)^{1/2} + ((m_B + m_\pi)^2 - q_0^2)^{1/2}} \tag{12}
\]

with \( \phi(q^2, q_0^2) \) as given in [31]. The “Blaschke” factor \( P(q^2) = z(q^2, m_{B^*}^2) \) accounts for the \( B^* \) pole. The expansion parameters \( a_k \) are constrained by unitarity to fulfill \( \sum_k a_k^2 \leq 1 \). \( q_0^2 \) is a free parameter that can be chosen to attain the tightest possible bounds, and it defines \( z(q_0^2, q_0^2) = 0; \ |z| < 1 \) for \( q_0^2 < (m_B + m_\pi)^2 \). The series in (11) provides a systematic expansion in the small parameter \( z \), which for practical purposes has to be truncated at order \( k_{\text{max}} \). We let data decide where to truncate and do a \( \chi^2 \) analysis for increasing \( k_{\text{max}} \) till an absolute minimum of \( \chi^2_{\text{min}} \) is reached. The shape parameters are then given by \( \{a_k\} \equiv \{\tilde{a}_k\} \times \text{const.} \) and we choose \( (\text{const.})^2 = \sum_{k=0}^{k_{\text{max}}} a_k^2 \), which implies \( \sum_{k=0}^{k_{\text{max}}} \tilde{a}_k^2 = 1 \), so that the \( \tilde{a}_k \) can be parametrised by generalised \( k_{\text{max}} + 1 \) dimensional spherical polar angles. For \( k_{\text{max}} = 2 \) we choose

\[
\tilde{a}_0 = \cos \theta_1, \quad \tilde{a}_1 = \sin \theta_1 \cos \theta_2, \quad \tilde{a}_2 = \sin \theta_1 \sin \theta_2,
\]

and for \( k_{\text{max}} = 3 \)

\[
\tilde{a}_0 = \cos \theta_1, \quad \tilde{a}_1 = \sin \theta_1 \cos \theta_2, \quad \tilde{a}_2 = \sin \theta_1 \sin \theta_2 \cos \theta_3, \quad \tilde{a}_3 = \sin \theta_1 \sin \theta_2 \sin \theta_3.
\]

We then minimize \( \chi^2 \) in \( \theta_i \) for the shape of \( f_+ \), for two choices of \( q_0^2 \):

(a) \( q_0^2 = (m_B + m_\pi)(\sqrt{m_B^2} - \sqrt{m_\pi^2})^2 = 20.062 \text{ GeV}^2 \), which minimizes the possible values of \( z, \ |z| < 0.28 \), and hence also minimizes the truncation error of the series in (11) across all \( q^2 \); the minimum \( \chi^2 \) is reached for \( k_{\text{max}} = 2 \);

(b) \( q_0^2 = 0 \text{ GeV}^2 \) with \( z(0, 0) = 0 \) and \( z(q_{\text{max}}^2, 0) = -0.52 \), which minimizes the truncation error for small and moderate \( q^2 \) where the data are most constraining; the minimum \( \chi^2 \) is reached for \( k_{\text{max}} = 3 \).

The advantage of BK and BZ is that they are both intuitive and simple; they are obtained from the dispersion relation for \( f_+ \),

\[
f_+(q^2) = \frac{\text{Res}_{q^2=m_{B^*}^2} f_+(q^2)}{q^2 - m_{B^*}^2} + \frac{1}{\pi} \int_{(m_B + m_\pi)^2}^{\infty} dt \frac{\text{Im} f_+(t)}{t - q^2 - i\epsilon}, \tag{15}
\]

by replacing the second term on the right-hand side by an effective pole. However, they cannot easily be extended to include more parameters. AFHNV, on the other hand, is based on a completely different approach, so it is interesting to compare the best-fit results with those from the other approaches; its shortcoming is the failure to include the \( B^* \) pole, which is possible, but difficult, see Ref. [30]. Finally, BGL offers a systematic
expansion whose accuracy can be adapted to that of the data to be fitted, so we choose it as our default parametrisation.

We determine the best-fit parameters for all four parametrisations from a minimum-\(\chi^2\) analysis. Our results are given in Tabs. 1 and 2. In Tab. 1 we give the results for \(|V_{ub}|f_+(0)\) obtained from fitting the various parametrisations to the BaBar data for the normalised partial branching fractions in 12 bins of \(q^2\): \(q^2 \in \{[0,2], [2,4], [4,6], [6,8], [8,10], [10,12], [12,14], [14,16], [16,18], [18,20], [20,22], [22,26.4]\} \text{GeV}^2\); the absolute normalisation is given by the HFAG average of the semileptonic branching ratio, Eq. (4). It is evident that good values of \(\chi^2_{\text{min}}\) are obtained for all parametrisations. We have also determined \(\chi^2\) for the (parameter-free) simple-pole/vector-dominance parametrisation \(f_+ \propto 1/(1 - q^2/m_B^2)\) and find \(\chi^2 = 45.3\) implying that this shape is largely incompatible with data, which confirms the result of Ref. [26]. The central values of \(|V_{ub}|f_+(0)\) agree for all parametrisations with more than one shape parameter, i.e. all parametrisations except the simplest one, BK. The uncertainty induced by the shape parameters is largest for the BGL parametrisation. As our final result we quote

\[|V_{ub}|f_+(0) = (9.1 \pm 0.6(\text{shape}) \pm 0.3(\text{branching ratio})) \times 10^{-4}\]  

(16)

and choose BGLa as default parametrisation with best \(\chi^2_{\text{min}}\) for a minimum number of parameters. We would like to stress that this result is completely model-independent, and also independent of the value of \(|V_{ub}|\); it relies solely on the experimental data for \(B \to \pi\ell\nu\) from BaBar for the spectrum [27] and the HFAG average of the branching ratio, Eq. (4). Using the two competing results for \(|V_{ub}|\), [11] and [2], [16] implies

\[f_+(0)|_{\text{incl}} = 0.20 \pm 0.02, \quad f_+(0)|_{\text{UTangles}} = 0.26 \pm 0.02.\]  

(17)

We also give the result for \(|V_{ub}|f_+(0)\) obtained from non-leptonic \(B \to \pi\pi\) decays using SCET [32]. The method used in [32] to obtain a constraint on \(|V_{ub}|f_+(0)\) from the decay rates for \(B^\pm \to \pi^\pm\pi^0\) and \(B^0 \to \pi^+\pi^-\) and the CP asymmetries of the latter decay is only valid at tree-level and to leading order in \(1/m_b\); corrections to this relation are of order \(\alpha_s\) and \(1/m_b\) and apparently, according to the BaBar data, are of the order of 15%. It remains to be seen whether a calculation of these corrections in SCET is feasible.

In Fig. 1 we show the best fit curves for all parametrisations together with the experimental data and error bars. All fit curves basically coincide except for the BK parametrisation which has a slightly worse \(\chi^2_{\text{min}}\). This can be easily understood because BK has only one shape parameter which is not sufficient to describe the whole spectrum. Still, BK gives, within errors, the same result for \(|V_{ub}|f_+(0)\) as the other parametrisations. The situation becomes more complicated, however, if one wants to fit lattice data obtained at large \(q^2\) to BK, as done, for instance, in Ref. [19]; we will come back to that point below when discussing lattice data. In Fig. 2 we show the best-fit form factors themselves. The curve in the left panel is an overlay of all five parametrisations; noticeable differences only occur for large \(q^2\), which is due to the fact that these points are phase-space suppressed in the spectrum and hence cannot be fitted with high accuracy. In the right panel we graphically enhance the differences between the best fits by normalising all parametrisations to
| $|V_{ub}|f_+(0)$ | Remarks |
|-----------------|---------|
| $9.3 \pm 0.3 \pm 0.3 \times 10^{-4}$ | $\chi^2_{\text{min}} = 8.74/11 \text{ dof}$ $\alpha_{\text{BK}} = 0.53 \pm 0.06$ |
| $9.1 \pm 0.5 \pm 0.3 \times 10^{-4}$ | $\chi^2_{\text{min}} = 8.66/10 \text{ dof}$ $\alpha_{\text{BZ}} = 0.40_{-0.22}^{+0.15}, r = 0.64_{-0.13}^{+0.14}$ |
| $9.1 \pm 0.6 \pm 0.3 \times 10^{-4}$ | $\chi^2_{\text{min}} = 8.64/10 \text{ dof}$ $q_0^2 = 20.062 \text{ GeV}^2$ $\theta_1 = 1.12_{-0.04}^{+0.03}, \theta_2 = 4.45 \pm 0.06$ |
| $9.1 \pm 0.6 \pm 0.3 \times 10^{-4}$ | $\chi^2_{\text{min}} = 8.64/9 \text{ dof}$ $q_0^2 = 0 \text{ GeV}^2$ $\theta_1 = 1.41_{-0.03}^{+0.02}, \theta_2 = 3.97 \pm 0.10, \theta_3 = 5.11_{-0.39}^{+0.67}$ |
| $9.1 \pm 0.3 \pm 0.3 \times 10^{-4}$ | $\chi^2_{\text{min}} = 8.64/8 \text{ dof}$ $f_+(q_0^2 \cdot \{1/4, 2/4, 3/4, 4/4\})/f_+(0)$ $= \{1.54 \pm 0.07, 1.54 \pm 0.11, 5.4 \pm 0.4, 26 \pm 11\}$ |
| $8.0 \pm 0.4 \times 10^{-4}$ | using the method of Ref. [32] |

Table 1: Model-independent results for $|V_{ub}|f_+(0)$ using the BaBar data for the spectrum [27] and the HFAG average for the total branching ratio $B(B \to \pi \nu) = (1.37 \pm 0.08) \times 10^{-4}$ [5]. The results are obtained using different parametrisations of the form factor $f_+(q^2)$: Becirevic/Kaidalov (BK) [29], Ball/Zwicky (BZ) [22], Boyd/Grinstein/Lebed (BGL) [31] and the Omnes representation of Ref. [30] (AFHNV). The first error is induced by the uncertainties of the parameters determining the shape of $f_+$; these parameters are given in the right column (our result for $\alpha_{\text{BK}}$ coincides with that obtained in [27]). The second error comes from the uncertainty of the branching ratio. We also give the corresponding result obtained from $B \to \pi\pi$ decays using SCET [32] (with $\gamma = (53 \pm 20)^\circ$); the error is purely experimental.
Figure 1: Experimental data for the normalised branching ratio $\delta B/B$ per $q^2$ bin, $\sum \delta B/B = 1$, and best fits. We have added statistical and systematic errors in quadrature. The lines are the best fit results for the five different parametrisations listed in Tab. We have added statistical and systematic errors in quadrature. The increase in the last bin is due to the fact that it is wider than the others (4.4 GeV$^2$ vs. 2 GeV$^2$).

Figure 2: Left panel: best-fit form factors $f_+$ as a function of $q^2$. The line is an overlay of all five parametrisations. Right panel: best-fit form factors normalised to BGLa. Solid line: BK, long dashes: BZ, short dashes: BGLb, short dashes with long spaces: AFHNV. Our preferred choice BGLa; for $q^2 < 25$ GeV$^2$, all best-fit form factors agree within 2%. It is evident that BZ and AFHNV yield too slow an increase for $q^2 > 25$ GeV$^2$, that is in close proximity of the $B^*$ pole at 28.4 GeV$^2$. For AFHNV this is expected, as it features the pole at a slightly larger $q^2$, $(m_B + m_\pi)^2 = 29.4$ GeV$^2$. In Tab. we give explicit values for the best-fit form factors for various $q^2$.

As mentioned above, theoretical predictions for $f_+$ are available from lattice calculations and LCSR and are collected in Tab. The LCSR calculations include twist 2 and 3 contributions to $O(\alpha_s)$ accuracy and twist-4 contributions at tree-level. The lattice calculations are unquenched with $N_f = 2 + 1$ dynamical flavours, i.e. mass-degenerate $u$ and $d$ quarks and a heavier $s$ quark. These quarks are described by an improved staggered quark action, which allows a simulation much closer to the (physical) chiral limit than with alternative actions. The two calculations differ in the treatment of the $b$ quark: whereas HPQCD simulates it in nonrelativistic QCD, FNAL employs a tadpole-improved
| $q^2$ [GeV]$^2$ | LCRSs [22] | $q^2$ [GeV]$^2$ | HPQCD [20] | $q^2$ [GeV]$^2$ | FNAL [32] |
|----------------|------------|----------------|---------|----------------|--------|
| 0              | 0.26 ± 0.01| 0.26 ± 0.01   | 0.26 ± 0.02 | 0.26 ± 0.02   | 0.26 ± 0.01 |
| 2              | 0.35 ± 0.01| 0.35 ± 0.01   | 0.36 ± 0.01 | 0.36 ± 0.01   | 0.36 ± 0.02 |
| 4              | 0.50 ± 0.01| 0.51 ± 0.01   | 0.51 ± 0.01 | 0.51 ± 0.01   | 0.51 ± 0.02 |
| 6              | 0.80 ± 0.01| 0.81 ± 0.02   | 0.81 ± 0.02 | 0.81 ± 0.02   | 0.80 ± 0.03 |
| 8              | 0.86 ± 0.01| 0.86 ± 0.02   | 0.86 ± 0.02 | 0.86 ± 0.02   | 0.86 ± 0.04 |
| 10             | 0.90 ± 0.02| 0.90 ± 0.02   | 0.90 ± 0.03 | 0.90 ± 0.03   | 0.90 ± 0.04 |
|                | 1.01 ± 0.02| 1.02 ± 0.03   | 1.01 ± 0.04 | 1.01 ± 0.04   | 1.01 ± 0.04 |
|                | 1.15 ± 0.03| 1.15 ± 0.05   | 1.15 ± 0.05 | 1.15 ± 0.06   | 1.15 ± 0.06 |
|                | 1.18 ± 0.04| 1.18 ± 0.05   | 1.18 ± 0.05 | 1.18 ± 0.06   | 1.18 ± 0.06 |
|                | 1.33 ± 0.05| 1.33 ± 0.07   | 1.32 ± 0.06 | 1.32 ± 0.08   | 1.33 ± 0.08 |
|                | 1.56 ± 0.06| 1.55 ± 0.09   | 1.55 ± 0.08 | 1.54 ± 0.11   | 1.55 ± 0.10 |
|                | 1.86 ± 0.09| 1.84 ± 0.12   | 1.84 ± 0.10 | 1.84 ± 0.15   | 1.85 ± 0.15 |
|                | 3.21 ± 0.21| 3.13 ± 0.29   | 3.17 ± 0.23 | 3.17 ± 0.39   | 3.16 ± 0.53 |

Table 2: Results for $f_+(q^2)$ using the best fits collected in Tab. [1] and the UTangles value for $|V_{ub}|$, $(3.5 ± 0.18) × 10^{-3}$. The errors refer to the fit of the various parametrisations to the data; the additional error induced by $|V_{ub}|$ is ±5% and that from the total branching ratio ±3%.

Table 3: Theoretical predictions of $f_+(q^2)$ from LCRSs [22] and lattice [20,32]. The errors quoted for HPQCD are combined statistical and chiral extrapolation errors. The FNAL numbers are quoted from Ref. [32], as we were unable to track down any publication of the FNAL group giving these numbers; the error is statistical.
Table 4: $|V_{ub}|$ and $|V_{ub}|f_+(0)$ from various theoretical methods. The column labelled BK gives the results obtained from a fit of the form factor to the BK parametrisation, and the column labelled BGLa that from a fit of $f_+(0)$ to the best-fit BGLa parametrisation from Tab. 1. The first uncertainty comes from the shape parameters, the second from the experimental branching ratios; the latter are taken from HFAG [5].

clover action with the Fermilab interpretation. The obvious questions are (a) whether these predictions are compatible with the experimentally determined shape of the form factor and (b) what the resulting value of $|V_{ub}|$ is. In order to answer these questions, we fit the lattice and LCSR form factors to the BK parametrisation and extract $|V_{ub}|$, for lattice, from $\mathcal{B}(B \rightarrow \pi \ell \nu)_{q^2 \geq 16 \text{GeV}^2}$, and for LCSR from $\mathcal{B}(B \rightarrow \pi \ell \nu)_{q^2 \leq 16 \text{GeV}^2}$; the cuts in $q^2$ are imposed in order to minimise any uncertainty from extrapolating in $q^2$. In our fits we treat the theory errors given in Tab. 3 as uncorrelated and add another 12% fully correlated systematic error, both for LCSR and lattice predictions, which is the procedure followed by experimental and lattice papers (with the exception of Ref. [20] where BZ is used). The results are shown in the BK column of Tab. 4. Equipped with the experimental information on the form factor shape, i.e. the BGLa parametrisation of Tab. 1 we suggest a different procedure and perform a fit of the theoretical predictions to this shape, with the normalisation as fit parameter. The corresponding results are shown.

| Theory | $f_+(0)$ | $|V_{ub}|$ | $|V_{ub}|f_+(0)$ |
|--------|----------|----------|----------------|
| LCSR   | $0.26 \pm 0.03$ | $(3.5 \pm 0.6 \pm 0.1) \times 10^{-4}$ | $(0.95 \pm 0.07) \times 10^{-4}$ |
| Ref. 22 | $0.26 \pm 0.03$ | $(3.5 \pm 0.6 \pm 0.1) \times 10^{-4}$ | $(0.95 \pm 0.07) \times 10^{-4}$ |
| HPQCD  | $0.26 \pm 0.03$, $\alpha_{\text{BK}} = 0.63^{+0.17}_{-0.11}$ | $(4.3 \pm 0.7 \pm 0.3) \times 10^{-4}$ | $(0.35 \pm 0.04) \times 10^{-4}$ |
| Ref. 20 | $0.26 \pm 0.03$ | $(4.3 \pm 0.5 \pm 0.1) \times 10^{-4}$ | $(0.35 \pm 0.04) \times 10^{-4}$ |
| FNAL   | $0.26 \pm 0.03$, $\alpha_{\text{BK}} = 0.63^{+0.07}_{-0.10}$ | $(3.6 \pm 0.6 \pm 0.2) \times 10^{-4}$ | $(8.2^{+1.0}_{-0.8} \pm 0.3) \times 10^{-4}$ |
| Ref. 32 | $0.26 \pm 0.03$ | $(3.7 \pm 0.4 \pm 0.1) \times 10^{-4}$ | $(8.2^{+1.0}_{-0.8} \pm 0.3) \times 10^{-4}$ |

| Theory | $f_+(0)$ | $|V_{ub}|$ | $|V_{ub}|f_+(0)$ |
|--------|----------|----------|----------------|
| BK     | $0.26 \pm 0.03$ | $(3.5 \pm 0.4 \pm 0.1) \times 10^{-4}$ | $(0.95 \pm 0.07) \times 10^{-4}$ |
| BGLa   | $0.26 \pm 0.03$ | $(3.5 \pm 0.4 \pm 0.1) \times 10^{-4}$ | $(0.95 \pm 0.07) \times 10^{-4}$ |
in the right column. Comparing these results, we observe the following:

- $|V_{ub}|$ from LCSR and FNAL is in better agreement with the UTangles value (2) than that from inclusive decays (1); $|V_{ub}|$ from HPQCD agrees with (1); the discrepancy between LCSR and (1) is at the 2$\sigma$ level, for FNAL it is slightly smaller;

- the difference in results for the two parametrisations is strongest for FNAL, which is due to the small number of theory input points (3); the “quality” of the BK parametrisation can be measured by the result for $|V_{ub}|f_+(0)$ which for LCSR and HPQCD perfectly agrees with the experimental value (16), whereas the central value for FNAL is a bit low;

- comparing the errors for $|V_{ub}|$ in both columns, it is evident that the main impact of the experimentally fixed shape, i.e. using the BGLa parametrisation of $f_+$, is a reduction of both theory and experimental errors; this is due to the fact that, once the shape is fixed, $|V_{ub}|$ can be determined from the full branching ratio with only 3% experimental uncertainty, whereas the partial branching fractions in the BK column induce 4% and 6% uncertainty, respectively, for $|V_{ub}|$; the theory error becomes smaller because the errors on $f_+$ in Tab. 3 are still rather large, which implies errors on the shape parameter $\alpha_{BK}$ which are larger than those of the experimentally fixed shape parameters.

The main conclusion from this discussion is that both LCSR and FNAL predictions for $f_+$ support the UTangles value for $|V_{ub}|$, and differ at the 2$\sigma$ level from the inclusive $|V_{ub}|$, whereas HPQCD supports the inclusive result. Using the experimentally fixed shape of $f_+$ in the analysis instead of fitting it to the theoretical input points reduces both the theoretical and experimental uncertainty of the extracted $|V_{ub}|$.

To summarize, we have presented a truly model-independent determination of the quantity $|V_{ub}|f_+(0)$ from the experimental data for the spectrum of $B \to \pi \ell \nu$ in the invariant lepton mass provided by the BaBar collaboration [27]; our result is given in (16). We have found that the BZ, BGL and AFHNV parametrisations of the form factor yield, to within 2% accuracy, the same results for $q^2 < 25$ GeV$^2$. We then have used the best-fit BGLa shape of $f_+$ to determine $|V_{ub}|$ using three different theoretical predictions for $f_+$, QCD sum rules on the light-cone [22], and the lattice results of the HPQCD [20] and FNAL collaborations [19, 32]. The advantage of this procedure compared to that employed in previous works, where the shape was determined from the theoretical calculation itself, is a reduction of both experimental and theoretical uncertainties of the resulting value of $|V_{ub}|$. We have found that the LCSR and FNAL form factors yield values for $|V_{ub}|$ which agree with the UTangles result, but differ, at the 2$\sigma$ level, from the HFAG value obtained from inclusive decays. The HPQCD form factor, on the other hand, is compatible with both UTangles and the inclusive $|V_{ub}|$. Our results show a certain preference for the UTangles result for $|V_{ub}|$, disfavouring a new-physics scenario in $B_d$ mixing, and highlight the quite urgent need for a re-analysis the inclusive case.
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