On the Joint Effect of Rain and Beam Misalignment in Terahertz Wireless Systems

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ABSTRACT This contribution focuses on extracting the theoretical framework for the assessment and evaluation of the joint effect of rain, beam misalignment and hardware imperfections at long-range outdoor terahertz (THz) wireless systems. In this direction, we first report an appropriate system model for outdoor THz wireless systems that take into account the impact of different design parameters, including antenna gain and transceivers hardware imperfections, atmospheric conditions, such as rain, and parameters, like temperature, humidity and pressure, as well as stochastic beam misalignment that can be caused by thermal expansion, dynamic wind loads and/or weak earthquakes. For this model, we extract novel closed-form expressions for the probability density and cumulative distribution functions of the THz wireless channel that captures the impact of geometric loss, beam misalignment and rain attenuation. We capitalized the aforementioned expressions by presenting closed-form formulas for the outage probability and achievable throughput of the system. Finally, we document an analytical policy that returns the optimal transmission spectral efficiency that maximizes the achievable throughput.

INDEX TERMS Hardware imperfections, misalignment fading, outage probability, performance analysis, radio frequency chain imperfections, rain attenuation, statistical characterization, throughput, terahertz wireless systems, wireless fiber extender.

I. INTRODUCTION The terahertz (THz) band has been identified as a catalyst of the next generation communication era, since it offers comparable to the fiber experience and, as a consequence, enables a number of data rate hungry application scenarios, including wireless backhauling of remote areas and cost-efficient midhauling connectivity [1]–[7]. However, THz signals experience inherent high path-loss, due to their high...
transmission frequency as well as due to molecular absorption [8]–[13]. To counterbalance the severe path loss, high-directional antennas are employed at both the transmitter (TX) and the receiver (RX) [14], [15]. Unfortunately, directionality further complicates the establishment of the link, since perfect alignment between the transmission and reception beam is required [16], [17]. In practical implementations this requirement can be met very rarely. The main reasons are physical phenomena, such as thermal expansion, dynamic wind loads, and weak earthquakes, that can cause a jitter at TX and RX antennas placed in high buildings [18], [19].

Recognizing this drawback, a great amount of research effort was put on analyzing, quantifying, and countermeasuring the impact of beam misalignment in THz wireless systems [4], [5], [20]–[26]. In more detail, in [20], the authors performed ray-tracing simulations in a realistic indoor environment in order to characterize the impact of beam misalignment, while, in [21], the antenna gain degradation due to stochastic misalignment in an indoor THz wireless system was quantified, assuming that the azimuth and elevation misalignment follow independent and identical Gaussian distributions. In [22], the authors experimentally evaluated the impact of deterministic beam misalignment in high-directional indoor THz wireless systems. In [23] and in [24], the impact of beam misalignment in indoor THz wireless systems operating at 100, 300, 400, and 500 GHz in terms of channel coefficient degradation were calculated by means of experimental measurements.

From the theoretical point of view, in [4], the impact of stochastic beam misalignment on the outage, error, and ergodic capacity performance of outdoor wireless THz systems that experience small and/or large scale fading was presented. Similarly, a closed-form expression for the error probability of mixed THz-radio frequency (RF) wireless systems, in which the THz link suffer from beam misalignment, was extracted in [5]. In [25], the authors documented a closed-form expression for the outage probability (OP) of wireless THz systems that experience fading, beam misalignment and phase noise. Finally, in [26], the authors presented novel closed-form expressions for the OP, error probability and ergodic capacity of THz wireless systems in the presence of stochastic beam misalignment and fog.

Another challenge, which outdoor THz wireless systems are facing, is received signal fluctuation under different weather conditions, i.e., weather-induced fading or, as it is called by both the THz the optical technology community scintillation [27], [28]. This phenomenon is generated as a consequence of the variation in the refraction index due to inhomogeneities in atmospheric temperature and pressure as well as water molecular density across the propagation path [29]. Weather-induced fading can dramatically affect the performance of THz wireless systems [30]. Motivated by this, several researchers have recently published a number of contributions concerning characterization, modeling, and analysis of the impact of different weather conditions on the performance of THz wireless systems [31]–[41]. Specifically, in [31], the authors employed a weather emulate chamber in order to perform in-lab characterization of the impact of fog in both THz wireless systems and free space optics. Similarly, in [32], the propagation efficiency of broadband THz pulses through a dense fog by means of received signal strength was experimentally presented. In [33], closed-form expressions for the probability density function (PDF) and cumulative distribution function (CDF), in terms of I-function, which characterize the joint impact of fog and beam misalignment, were presented. Building upon these expressions, the OP, average symbol error rate (SER), and ergodic channel capacity were derived.

In [34], the authors presented an experimental comparison of the impact of laboratory-controlled rain on the received signal strength and error rate performance of THz and infrared (IR) wireless systems. Their analysis concluded in the very interesting result that THz are slightly more robust to the impact of rain compared to the corresponding IR free space optics (FSO) systems. In [35], a rain attenuation characterization for THz wireless systems was conducted. Likewise, in [36], an analytical assessment on physical layer security of a point-to-point THz wireless link in rain and snow with a potential eavesdropper locating outside of the legitimate link path was performed. In the aforementioned paper, the authors employed the Gunn-Marshall and Sekhon-Srivastava to model the snowdrop size, and the Marshall-Palmer as well as Weibull distributions to model the raindrop size.

In [37], an experimental comparison of the impact of turbulence in THz and IR wireless systems was documented. In [38], an analytical model that assess the overall impact of turbulence on THz systems was presented. Building upon this model, in [39], the authors extracted closed-form expression for the average error rate and channel capacity of THz wireless systems in the presence of turbulence, while, in [40], the authors statistically characterized the end-to-end channel of multi-RIS assisted THz wireless systems that experience turbulence. Finally, in [41], the joint impact of snow, turbulence, and beam misalignment on the bit error rate and channel capacity of THz wireless systems were quantified.

We also recall that another factor that needs to be taken into account in the modeling and assessment of THz wireless systems performance is the impact of transceivers hardware imperfections [42]–[44]. In general, the effect of transceivers hardware imperfections was evaluated in several contributions, including [45]–[54]. However, their detrimental effect on THz wireless systems were only recently examined [55]–[57]. In particular, in [55], the authors revealed the detrimental impact of transceivers hardware imperfections in the error performance of THz wireless fiber extenders. Moreover, in [56], the performance degradation due to transceiver hardware imperfections that operate at the 300 GHz band, were documented. Finally, in [57], the authors evaluated the impact of transceivers hardware imperfections in THz wireless systems that employ spatial modulation.

Despite of the paramount importance that the weather
conditions, beam misalignment and transceivers hardware imperfections play an important role in the performance of THz wireless systems, to the best of the authors knowledge only the joint impact of turbulence, beam misalignment and hardware imperfections in terms of OP has been so far investigated [40]. Towards filling this gap, in this paper, we investigate the joint impact of rain, beam misalignment and transceiver hardware imperfections in THz wireless systems. In more detail, the technical contribution of the paper can be summarized as follows:

- We present a suitable system model for outdoor THz wireless systems, which accounts the different design parameters and their interactions as well as the presence and absence of rain. The aforementioned parameters include the transmission distance, the transceivers antenna gains, the level of beam misalignment, the transmission power, the atmospheric temperature, pressure and relative humidity, the expected value and variance of rain attenuation as well as the probability of rain.

- In order to characterize the stochastic behavior of the THz wireless channel, we extract novel closed-form expressions for the PDF and CDF that captures the joint impact of geometric-loss, rain attenuation and beam-misalignment.

- Building upon the channel model, we present closed-form expressions for the OP and the system throughput. The derived expressions accommodate the impact of stochastic beam misalignment, rain as well as transceiver hardware imperfections. As a benchmark, simplified expressions for the case in which the THz wireless system suffers only from beam misalignment and transceiver hardware imperfections are also presented. Another theoretical benchmark that is reported is the special case in which, although it rains, the THz wireless system does not experience beam-misalignment. Note that this is an ideal non-realistic scenario, which allow us to quantify the severity of beam-misalignment.

- Finally, we present an analytical policy for selecting the optimum transmission spectral efficiency that maximizes the achievable throughput, for given transmission power, levels of transceiver hardware imperfection and rain attenuation.

The rest of this paper is organized as follows. Section II is devoted to the presentation of the system and channel models. The statistical characterization of the THz wireless channel is reported in Section III. Section IV focuses on documenting the performance metrics, namely signal-to-distortion-plus-noise-ratio (SDNR), OP, and throughput, as well as the throughput maximization policy. Numerical results and discussions are given in Section V. Finally, concluded remarks and observations are highlighted in Section VI. The organization of the paper in a glance is illustrated in Fig. 1.

Notations: The absolute value and the square root of $t$ are respectively denoted by $|t|$ and $\sqrt{t}$. Moreover, $t^n$ stands for the $x$ in the power of $n$. The natural logarithm and the base-$e$ logarithm of $t$ are respectively represented by $\ln (t)$ and $\log_n (t)$. The exponential and the tangent of $t$ are respectively given by $\exp (t)$ and $\tan (t)$. The error function is denoted by $erf (\cdot)$, while $erfc (\cdot)$ stands for the complementary error function. Likewise, $t \sim \text{CN} (\mu_t, \sigma_t^2)$ indicates that $t$ is a random variable (RV) that follows complex Gaussian distribution with mean $\mu_t$ and variance $\sigma_t^2$. Finally, $\Pr (\mathcal{A})$ stands for the probability for the event $\mathcal{A}$ to be valid.

II. SYSTEM & CHANNEL MODELS

This section focuses on presenting the system and channel models. In particular, Section II-A reports the THz wireless fiber extender system model and lists all the transmission and reception parameters. Section II-B documents the THz wireless channel model and presents the interconnection between the channel coefficient and the communication parameters as well as the atmospheric and weather conditions.

A. SYSTEM MODEL

As depicted in Fig. 2, we consider a directional point-to-point THz wireless system, in which both the TX and RX
TABLE 1. Parameters definition.

| Parameters | Description |
|------------|-------------|
| T          | atmospheric temperature |
| P          | atmospheric pressure |
| φ          | relative humidity |
| c          | speed of light |
| f          | transmission frequency |
| d          | transmission distance |
| Gt         | transmission antenna gain |
| Θ3dB       | half-power beam-width of the TX antenna |
| Gr         | reception antenna gain |
| Pa         | average transmission signal power |
| αr         | radius of the effective area of the RX antenna |
| Ao         | the portion of the collected power at the RX in the absence of beam misalignment |
| w,d        | beam waist at distance d |
| w,e        | equivalent beam-width |
| σs         | spatial jitter standard deviation |
| μr         | mean value of channel attenuation due to rain |
| σr²        | variance of channel attenuation due to rain |
| Pr         | probability of rain |
| A          | rain attenuation lower-threshold |
| R          | rainfall rate |
| κr         | rain-related frequency- and polarization-dependent parameter |
| κt         | rain-related frequency- and polarization-dependent parameter |
| s          | transmission signal |
| h          | THz wireless channel coefficient |
| h0         | geometric loss coefficient |
| h1         | beam misalignment fading coefficient |
| h2         | rain attenuation coefficient |
| h,f        | free space loss coefficient |
| hg         | molecular absorption loss coefficient |
| n          | additive white Gaussian noise |
| N,         | variance of n |
| ηt         | TX distortion noise |
| ηr         | RX distortion noise |
| ηg         | RX EVM |
| z0         | transmission SNR |
| γ0         | SNR threshold |

with \( P_o \) being the probability of rain. Notice that \( P_o \) depends on the local climate at a specific observation period [58] and it can be generally estimated by the following procedure described in the Recommendation ITU-R P.837 [59].

Finally, the parameters \( \eta_t \) and \( \eta_r \) are the TX and RX distortion noises, due to the RF chain hardware imperfections. Note that, according to [43], [60], [61],

\[
\eta_t \sim CN \left(0, \kappa_t^2 P_s\right), \quad \eta_r \sim CN \left(0, \kappa_r^2 h^2 P_s\right),
\]

with \( \kappa_t \) and \( \kappa_r \) being the error vector magnitudes (EVMs) of the TX and RX, respectively [50], [52], [62]–[64]. Of note, EVM is a common quality metric of the RF transceiver’s quality and is defined as the ratio of the average distortion magnitude to the average signal magnitude. As reported in [4], in THz wireless systems \( \kappa_t \) and \( \kappa_r \) are in the range of \([0, 0.4]\). Notice that \( \kappa_t = \kappa_r = 0 \) corresponds to the ideal RF front-end case. Finally, \( P_s \) denotes the average transmission power.

B. CHANNEL MODEL

The objective of this section is to present the channel model. Towards this direction, the geometric loss model is discussed in Section II-B1, whereas, the beam misalignment model is reported in Section II-B2. Finally, Section II-B3 provides the model for the rain attenuation coefficient.

1) Geometric loss coefficient

The geometric loss coefficient can be further analyzed as

\[
h_l = h_f h_g,
\]

where \( h_f \) models the free space losses and can be evaluated, based on the Friis equation, as

\[
h_f = \frac{c\sqrt{G_t G_r}}{4\pi f d},
\]

where \( G_t \) and \( G_r \) are the transmission and reception antenna gains, respectively, while \( c \) and \( f \) are respectively the speed of light and the transmission frequency. Finally, \( d \) stands for the transmission distance.

In addition, \( h_g \) is the molecular absorption channel coefficient, which can be calculated as

\[
h_g = \exp\left(-\frac{1}{2}\kappa_a(T, \phi, P) d\right),
\]

with \( \kappa_a(T, P, \phi) \) being the molecular absorption coefficient. The molecular absorption coefficient depends on the atmospheric temperature, \( T \), and pressure, \( P \), as well as the relative humidity, \( \phi \), and can be evaluated by leveraging the radiative transfer theory and the measured data provided in the high resolution transmission molecular absorption (HITRAN) database [65], as described in [8], [66], [67]. Alternative approaches to extract the molecular absorption coefficient by means of analytical formulas are provided in [68]–[70].

are equipped with high directional antennas. The baseband equivalent received signal can be expressed as

\[
r = h \left(s + \eta_t + \eta_r + n\right),
\]

where \( s \) stands for the transmission signal, while \( n \) represents the additional additive white Gaussian noise (AWGN), which can be modeled as a zero-mean Gaussian RV of variance \( N_o \). Moreover, \( h \) denotes the channel coefficient and can be expressed as

\[
h = h_l h_m \left((1 - \delta) + \delta h_r\right),
\]

where \( h_l \) and \( h_m \) respectively stand for the geometric loss and the beam misalignment fading coefficients, while \( h_r \) is the rain channel attenuation coefficients. Moreover, \( \delta \) is a binary RV that is defined as

\[
\delta = \begin{cases} 
1, & \text{if it rains} \\
0, & \text{otherwise} 
\end{cases}
\]

for which

\[
\Pr(\delta = 1) = P_o, \quad \text{and} \quad \Pr(\delta = 0) = 1 - P_o,
\]
2) Beam misalignment channel coefficient

The beam misalignment channel coefficient is a RV. The PDF and CDF of $h_m^2$ can be respectively expressed as

$$f_{h_m^2}(x) = \begin{cases} \frac{x}{A_o^2} x^{\frac{x^2}{A_o^2} - 1}, & 0 \leq x \leq A_o \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

and

$$F_{h_m^2}(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{A_o^2}, & 0 \leq x \leq A_o \\ 1, & x > A_o \end{cases} \quad (10)$$

where $A_o$ is the portion of the collected power by the RX in the absence of beam misalignment fading, and is given by

$$A_o = (\text{erf}(v))^2, \quad (11)$$

with

$$v = \sqrt{\frac{\pi}{2} \alpha_r \omega_d}. \quad (12)$$

In (12), $\alpha_r$ is the radius of the effective area of the reception antenna, which, according to [71, eqs. (2-117) and (2-119)] can be expressed as

$$\alpha_r = \frac{c \sqrt{\sigma_r}}{2\pi f}. \quad (13)$$

Likewise, $\omega_d$ stands for the beam waist at distance $d$ and can be evaluated as

$$\omega_d = d \tan \left( \frac{\Theta_{3\text{dB}}}{2} \right), \quad (14)$$

with $\Theta_{3\text{dB}}$ being the half-power beamwidth of the transmission antenna. Note that based on the antenna transmission type $\Theta_{3\text{dB}}$ is connected to the antenna gain with a one-to-one relationship. In particular, for the case in which the TX employs a Cassegrain antenna, the following formula is satisfied [71]:

$$\Theta_{3\text{dB}} = \sqrt{\frac{4\pi}{G_t}}. \quad (15)$$

Moreover, in (9) and (10)

$$\xi = \frac{1}{4} \frac{w_e^2}{\sigma_s^2}, \quad (16)$$

where $\sigma_s$ stands for the spatial jitter standard deviation, and $w_e$ is the equivalent beamwidth that can be obtained as

$$w_e^2 = \sqrt{\pi} \omega_d \text{erf}(v) \exp(v^2) / 2v. \quad (17)$$

3) Channel attenuation due to rain

According to [72], the channel attenuation due to rain, $h_r^2$, can be accurately modeled as a log-normal distribution with mean and variance $\mu_r$ and $\sigma_r^2$, respectively. As a consequence, the PDF and CDF of $h_r^2$ can be respectively expressed as

$$f_{h_r^2}(x) = \frac{1}{\sqrt{2\pi \sigma_r}} x^{-1} \exp \left( -\frac{(\ln(x) - \mu_r)^2}{2\sigma_r^2} \right) \quad (18)$$

and

$$F_{h_r^2}(x) = \frac{1}{2} \text{erfc} \left( \frac{\mu_r - \ln(x)}{\sqrt{2}\sigma_r} \right). \quad (19)$$

Notice that in this work, we use a log-normal distribution to model the attenuation conditioned to the presence of rain. Moreover, note that $\mu_r$ and $\sigma_r$ depend on the local climate and the link characteristics, such as transmission distance, operation frequency, as well as polarization of the electromagnetic wave. Ideally, $\mu_r$ and $\sigma_r$ should be obtained as a result of a fitting problem to rain attenuation measurements. However, since experimental rain attenuation datasets are not usually available for THz frequencies, as described in [73] and in [74], we employ the following procedure in order to evaluate $\mu_r$ and $\sigma_r$.

By employing the Dirac-lognormal model and leveraging (19), the probability that rain attenuation exceeds a lower-threshold, $A$, can be expressed as in [74]

$$P_A = \text{Pr} \left( h_r^2 \geq A \right) = \frac{\alpha_r}{2} \text{erfc} \left( \frac{\ln(A) - \mu_r}{\sqrt{2}\sigma_r} \right). \quad (20)$$

In relatively-short distance (less than some decades of km) terrestrial THz wireless systems, it can be assumed that rainfall rate is homogeneous in the path. As a consequence, it can be estimated by evaluating the rain specific attenuation, $\rho$ that is measured in dB/km, for a corresponding rainfall rate, $R$, measured in mm/h, and multiply by the transmission distance, $d$, measured in km, i.e.

$$A = \rho d, \quad (21)$$

which based on the ITU-R Recommendation P.838-3 [75], can be rewritten as

$$A = k_r R^{a_r} d. \quad (22)$$

In (22), $k_r$, $\rho$, and $a_r$ are frequency- and polarization-dependent parameters provided in [75] for frequencies between 1 and 1000 GHz.

The procedure that we followed in this contribution in order to derive $P_o$, $\mu_r$, and $\sigma_r$ for the desired frequency and site, starts by obtaining the complementary CDF (CCDF) of rainfall rate $P_o$ for the site using the model of ITU-R Rec. P.837-7 [76], which provides rainfall rate distributions for every location in the world for propagation modeling purposes. The probability of rain presence $P_{0,r}$ is also provided. Assuming the links are relatively short, i.e., less than 1 km, (22) is used to convert each $R$ value associated to a probability into an $A$ value associated to the same probability, and the probability of having rain attenuation present is made equal to the probability of rain presence, i.e. $P_o = P_{0,r}$. Then, the parameters $\mu_r$ and $\sigma_r$ of the log-normal distribution are calculated by fitting the distribution $P_A$ obtained from $\text{Pr}(x \leq R)$ to a distribution with the form of (20), using the procedure described in Annex 2 of ITU-R Rec. P.1057-6 [77].
III. STATISTICAL CHARACTERIZATION OF THE THZ WIRELESS CHANNEL

The following Lemmas presents closed-form expressions for the PDF and CDF of \( h^2 \) in the absence and presence of rain.

**Lemma 1.** In the absence of rain, the PDF and CDF of \( h^2 \) can be respectively expressed as

\[
f_{h^2|\delta=0}(x) = \begin{cases} \frac{x}{h_t^2} A^2_x, & 0 \leq x \leq \frac{h_t^2}{A_x} \\ 0, & \text{otherwise} \end{cases} \tag{23}
\]

and

\[
F_{h^2|\delta=0}(x) = \begin{cases} 1, & x < 0 \\ \frac{x}{h_t^2} A^2_x, & 0 \leq x \leq \frac{h_t^2}{A_x} \\ 0, & \text{otherwise} \end{cases} \tag{24}
\]

Proof: For brevity, the proof of Lemma 1 is provided in Appendix A.

**Lemma 2.** In the presence of rain, the PDF and CDF of \( h^2 \) can be respectively expressed as

\[
f_{h^2|\delta=1}(x) = \frac{1}{2} \frac{x}{h_t^2} A^2_x \exp \left( \frac{\sigma_r^2 x^2}{2} - \frac{\xi \mu_r}{\sigma_r} \right) \left( 1 + \frac{\xi \mu_r}{\sigma_r} \right) x^{-1} \tag{25}
\]

\[
\times \text{erfc} \left( \frac{\sigma_r}{\sqrt{2}} \left( \frac{x}{h_t^2} - \frac{\xi \mu_r}{\sigma_r} \right) + \frac{1}{\sqrt{2} \sigma_r} \ln \left( \frac{x}{h_t^2} \right) \right) \tag{26}
\]

and (26), given at the top of the next page.

Proof: For brevity, the proof of Proposition 3 is provided in Appendix B.

**Proposition 1.** The PDF and CDF of \( h^2 \) can be respectively expressed as

\[
f_{h^2}(x) = (1 - \rho_o) f_{h^2|\delta=0}(x) + \rho_o f_{h^2|\delta=1}(x) \tag{27}
\]

and

\[
F_{h^2}(x) = (1 - \rho_o) F_{h^2|\delta=0}(x) + \rho_o F_{h^2|\delta=1}(x) \tag{28}
\]

Proof: Since \( \delta \) is a discrete RV and independent from \( h_r \) and \( h_m \), by employing the Bayes theorem [78], we can respectively express the PDF and CDF of \( h^2 \) as

\[
f_{h^2}(x) = \Pr(\delta = 0) f_{h^2|\delta=0}(x) + \Pr(\delta = 1) f_{h^2|\delta=1}(x) \tag{29}
\]

and

\[
F_{h^2}(x) = \Pr(\delta = 0) F_{h^2|\delta=0}(x) + \Pr(\delta = 1) F_{h^2|\delta=1}(x) \tag{30}
\]

By applying (4) into (29) and (30), we respectively obtain (27) and (28), respectively.

IV. PERFORMANCE ANALYSIS

Building upon the statistical characterization framework presented in III, in this section, we present novel closed-form expressions for the OP and the achievable throughput. Specifically, the rest of the section is organized as: in Section IV-A, the instantaneous SDNR is presented, while, in Section IV-B, the OP is derived. Finally, the achievable throughput is given in Section IV-C.

A. SDNR

From (1), the instantaneous SDNR at the receiver can be evaluated as

\[
\gamma = \frac{h^2 P_s}{(\kappa_t^2 + \kappa_r^2) h^2 P_s + N_o}. \tag{31}
\]

Note that in the ideal RF-case, where \( \kappa_t = \kappa_r = 0 \), (31) can be simplified to

\[
\gamma_{id} = \frac{h^2 P_s}{N_o}. \tag{32}
\]

B. OP

The following proposition returns a closed-form expression for the OP.

**Proposition 2.** The OP can be obtained as in (33), given at the top of the next page. In (33),

\[
\tilde{\gamma} = \frac{P_s}{N_o} \tag{34}
\]

and \( \gamma_{th} \) stands for the SNR threshold.

Proof: For brevity, the proof of Proposition 3 is provided in Appendix C.

**Special case 1:** In the absence of rain, but in the presence of beam misalignment, the OP can be evaluated from (33), by setting \( P_o = 1 \), as

\[
P_{out}(\gamma_{th}) = \begin{cases} F_{h^2|\delta=0} \left( \frac{1}{1 - \gamma_{th}(\kappa_t^2 + \kappa_r^2)} \right) \frac{2\gamma_{th}}{\gamma}, & \text{for } \gamma_{th} < \frac{1}{\kappa_t^2 + \kappa_r^2} \\ 1, & \text{otherwise} \end{cases} \tag{35}
\]

**Special case 2:** In the presence of both rain and beam misalignment, the OP can be evaluated from (33), by setting \( P_o = 0 \), as

\[
P_{out}(\gamma_{th}) = \begin{cases} F_{h^2|\delta=1} \left( \frac{1}{1 - \gamma_{th}(\kappa_t^2 + \kappa_r^2)} \right) \frac{2\gamma_{th}}{\gamma}, & \text{for } \gamma_{th} < \frac{1}{\kappa_t^2 + \kappa_r^2} \\ 1, & \text{otherwise} \end{cases} \tag{36}
\]

**Special case 3 (Ideal RF front):** In the special and ideal case in which both the TX and RX are equipped with ideal RF front-ends, i.e., \( \kappa_t = \kappa_r = 0 \), (33) can be simplified as

\[
P_{out}^{id}(\gamma_{th}) = (1 - P_o) F_{h^2|\delta=0} \left( \frac{\gamma_{th}}{\gamma} \right) + P_o F_{h^2|\delta=1} \left( \frac{\gamma_{th}}{\gamma} \right) \tag{37}
\]

By comparing (33) and (37) and taking into account that the spectral efficiency and the SNR threshold are connected through

\[
\eta_{th} = \log_2 (1 + \gamma_{th}), \tag{38}
\]

it becomes evident that the hardware imperfections creates a spectral efficiency upper bound beyond which the OP becomes equal to 1.
The transmission frequency and distance are set to simulations. In this directions, unless otherwise stated, lines This section focuses on verifying the analytical framework RF front-ends, i.e., \( \gamma_{\text{id}} \), where both the TX and RX are equipped with ideal \( \gamma_{\text{th}} \), that maximizes the achieved throughput.

Proposition 3. The optimal SNR threshold, \( \gamma_{\text{th}} \), that maximizes the achievable throughput is the solution of (40), given at the top of the next page.

Proof: For brevity, the proof of Proposition 3 is provided in Appendix D.

Note that (40) is very difficult or even impossible to be analytically solved. However, \( \gamma_{\text{th}} \) can be obtained by applying low-complexity numerical approaches, such as the Newton’s method [79]. Finally, note that by applying (38) to the solution of (40), the optimal spectral efficiency of the transmission scheme can be extracted.

Special case 1: In the absence of rain, but in the presence of beam misalignment, (40) can be simplified as in (41), given at the top of the next page.

Special case 2: In the presence of both rain and beam misalignment, (40) can be simplified as in (42), given at the top of the next page.

Special case 3 (Ideal RF front-end): In the special and ideal case in which both the TX and RX are equipped with ideal RF front-ends, i.e., \( \kappa_t = \kappa_r = 0 \), (40) can be simplified as

\[
P_{\text{out}} \left( \gamma_{\text{th}} \right) = \begin{cases} 
  \left( 1 - P_o \right) F_{h^2|d=0} \left( \frac{1}{1 - \gamma_{\text{th}}(\kappa_t + \kappa_r)} \frac{\gamma_{\text{th}}}{\gamma} \right) + P_o F_{h^2|d=1} \left( \frac{1}{1 - \gamma_{\text{th}}(\kappa_t + \kappa_r)} \frac{\gamma_{\text{th}}}{\gamma} \right), & \text{for } \gamma_{\text{th}} \leq \frac{1}{\kappa_t + \kappa_r}, \\
  1, & \text{otherwise}
\end{cases}
\]

\[ \ln (2) \left( 1 + \gamma_{\text{th}} \right) \]

where \( \gamma_{\text{th}} \) stands for the optimal SNR threshold for the ideal RF front-end case.

V. RESULTS & DISCUSSION

This section focuses on verifying the analytical framework that was documented in Section IV through Monte Carlo simulations. In this directions, unless otherwise stated, lines and markers are used for analytical results and simulations, respectively. The following scenario is considered. The transmission frequency and distance are set to 120 GHz and 100 m, respectively. The relative humidity, atmospheric pressure, and temperature are 50%, 101325 Pa, and 296 °K. Finally, both the TX and RX antenna gains are 55 dB.

The rest of the section is organized as: In Section V-A, the joint impact of rain and misalignment is quantified assuming that both the TX and RX are equipped with ideal RF front-ends, i.e., \( \kappa_t = \kappa_r = 0 \). Section V-B documents the joint impact of transceivers hardware imperfections, rain, and beam misalignment on the outage and throughput performance of THz wireless systems.

A. IDEAL RF FRONT-END

Figure 3 quantifies the impact of beam misalignment on the outage performance of a THz wireless system that operates under rain. In more detail, the OP is plotted as a function of the transmission SNR, \( \gamma_{\text{th}} \), for different values of \( \gamma_{\text{th}} \) and \( \sigma_m \), assuming that \( \mu_r = -2.04 \) and \( \sigma_r = 0.86 \). Note that the aforementioned values correspond on raining conditions in Athens, Greece, for transmission distance that equals 100 m. Moreover, we assume that \( P_o = 1 \). This indicates that we assess the THz wireless system performance for an observation period that is raining. As a benchmark, in this figure, we also presented the outage performance in the absence of misalignment. We observe that, for fixed \( \sigma_m \) and \( \gamma_{\text{th}} \), as \( \gamma_{\text{th}} \) increases, the OP decreases. For example, for \( \sigma_m = 0.05 \), \( \gamma_{\text{th}} = 0 \), \( \gamma_{\text{th}} = 30 \) dB, an approximately two orders of magnitude OP improvement is observed. Moreover, for given \( \gamma_{\text{th}} \) and \( \sigma_m \), as \( \gamma_{\text{th}} \) increases, i.e., as the spectral efficiency of the transmission scheme increases, the OP also increases. For instance, for \( \gamma_{\text{th}} = 25 \) dB and \( \sigma_m = 0.05 \), the OP increases for one order of magnitude, as the \( \gamma_{\text{th}} \) changes from 0 to 5 dB. From this figure, it also becomes apparent that, for fixed \( \gamma_{\text{th}} \), the outage performance degrades as the level of beam misalignment, i.e., \( \sigma_m \), increases. For example, for \( \gamma_{\text{th}} = 25 \) dB and \( \gamma_{\text{th}} = 0 \), the OP increases from \( 5.5 \times 10^{-3} \) to \( 1.16 \times 10^{-2} \), as \( \sigma_m \) increases from 0.05 to 0.1. Therefore, the importance of accounting the impact of misalignment when assessing the performance of THz wireless systems is highlighted. For instance, for \( \gamma_{\text{th}} = 30 \) dB, \( \gamma_{\text{th}} = 0 \) dB and \( \sigma_m = 0.1 \), an outage performance evaluation error that is greater than two orders of magnitude occurs, if the impact of beam-misalignment is not taken into consideration. This indicates that neglecting the impact of beam misalignment...
when evaluating the OP of THz wireless systems may lead to important assessment errors.

Figure 4 presents the OP as a function of $P_o$, for different values of $\gamma_s/\gamma_{th}$ and $\sigma_m$, assuming $\mu_r = -2.04$ and $\sigma_r = 0.86$. As described in Section II-B3, the parameters $\mu_r$, $\sigma_r$, and $P_o$ are interdependent climate parameters that are affected by the observation period. However, in order to present the generality of the presented analysis and to derive some useful insights that connects the outage performance with the probability of rain, in this figure, we fix the parameters $\mu_r$ and $\sigma_r$, and quantify the OP for different values of $P_o$. Furthermore, notice that $P_o = 0$ refers to the case in which the THz wireless system suffers only from beam misalignment. As expected, for given $P_o$ and $\gamma_s/\gamma_{th}$, the OP increases, as $\sigma_m$ increases. For example, for $P_o = 0.5$ and $\gamma_s/\gamma_{th} = 30$ dB, the OP increases from $2.62 \times 10^{-5}$ to $1.17 \times 10^{-4}$, as $\sigma_m$ increases from 0.05 to 0.1. Likewise, for fixed $P_o$ and $\sigma_m$, as $\gamma_s/\gamma_{th}$ increases, the outage performance improves. For instance, for $P_o = 0.5$ and $\sigma_m = 0.05$, the OP decreases from $4.6 \times 10^{-4}$ to $5.6 \times 10^{-2}$ as $\gamma_s/\gamma_{th}$ increases from 10 to 20 dB. Finally, we observe that, for given $\sigma_m$ and $\gamma_s/\gamma_{th}$, the OP increases as $P_o$ increases. For example, for $\gamma_s/\gamma_{th} = 30$ dB and $\sigma_m = 0.05$, the OP increases from $5.24 \times 10^{-8}$ to $5.24 \times 10^{-5}$, as $P_o$ increases from $10^{-3}$ to 1. This reveals the severity of the joint impact of rain and beam misalignment.

Figure 5 illustrates the throughput as a function of $\gamma_s$, for different values of $\gamma_{th}$ and $\sigma_m$, assuming $P_o = 0$, $\mu_r = -2.04$, and $\sigma_r = 0.86$. As a benchmark, the throughput in the absence of beam misalignment is also plotted. As expected, for given $\gamma_{th}$ and $\sigma_m$, as $\gamma_s$ increases, the OP decreases; thus, the throughput increases. For instance, for $\gamma_{th} = 0$ dB and $\sigma_m = 0.05$, the throughput increases from $7.16 \times 10^{-2}$ to 0.89, as $\gamma_s$ increases from 10 to 20 dB. Moreover, for a fixed $\sigma_m$, the selection of the appropriate transmission scheme that maximizes the achievable throughput depends from the available transmission SNR. For example, for $\sigma_m = 0$, to achieve the maximum possible throughput, a transmission
that in the low $\gamma_s/\gamma_{th}$ regime, where $\gamma_s/\gamma_{th} \leq \frac{1}{\kappa_r h^2}$, i.e., the transmission and reception beams are fully misaligned and thus $F(h|s=0) (\gamma_s/\gamma_{th}) = 1$, the throughput increases, as $P_o$ increases. For instance, for $\gamma_{th} = \gamma_s = 0$, the throughput increases by about 10 times as $P_o$ changes from 0.2 to 1. On the other hand, for $\gamma_s/\gamma_{th} > \frac{1}{\kappa_r h^2}$, as $P_o$ increases, the throughput decreases.

**B. NON-IDEAL RF FRONT-END**

Figure 7 illustrates the OP as a function of $\kappa_t$ and $\kappa_r$ for different values of $\sigma_m$, assuming that $\mu_r = -2.04$, $\sigma_r = 0.86$, $P_o = 0$, $\gamma_s = 30$ dB, and $\gamma_{th} = 1$ dB. Note that $\kappa_t = \kappa_r = 0$ corresponds to the ideal case in which both the TX and RX are equipped with ideal RF front-end. As expected, for fixed $\sigma_m$ and $\kappa_t$, as $\kappa_r$ increases, the OP increases. For example, for $\sigma_m = 0$ and $\kappa_t = 0.22$, the OP increases from $2.59 \times 10^{-6}$ to $6.96 \times 10^{-6}$, as $\kappa_r$ increases from 0.2 to 0.4. Similarly, for given $\sigma_m$ and $\kappa_r$, as $\kappa_t$ increases, the OP also increases. Likewise, it becomes evident that system A with $\kappa_t = v_1$ and $\kappa_r = v_2$ achieve the same outage performance as system B with $\kappa_t = v_2$ and $\kappa_r = v_1$, under the same $\sigma_m$. Likewise, from this figure, it becomes apparent that for a fixed $\sigma_m$ and a constant $\kappa_t + \kappa_r$, the worst outage performance is achievable for $\kappa_t = \kappa_r$. For instance, for $\sigma_m = 0$, and $\kappa_t + \kappa_r = 0.2$, we observe that the OP that is achieved for $\kappa_t = 0$ and $\kappa_r = 0.2$, which is equal to $2.19 \times 10^{-6}$, is lower than the OP achieved for $\kappa_t = \kappa_r = 0.1$ (i.e., $3.19 \times 10^{-6}$). In addition, we verify that systems with the same $\kappa_t^2 + \kappa_r^2$ achieve the same OP. Finally, for given $\kappa_t^2 + \kappa_r^2$, the OP increases, as $\sigma_m$ increases. For example, for $\kappa_t^2 + \kappa_r^2 = 0.02$, the OP decreases from $3.19 \times 10^{-6}$ to $2.54 \times 10^{-4}$, as $\sigma_m$ changes from 0 to 0.1. This highlights the importance of accurately modeling the level of beam misalignment.

Figure 8 depicts the OP as a function of $P_o$ for different values of $\kappa_t$, $\kappa_r$, and $\gamma_{th}$, assuming $\mu_r = -2.04$, $\sigma_r = 0.86$, and $\sigma_m = 0.05$. Interestingly, for fixed $\kappa_t$, $\kappa_r$, and $\gamma_{th}$, as $P_o$ increases, the OP also increases. Moreover, for given $P_o$ and $\gamma_{th}$, as the level of hardware imperfections increases, the OP also increases. For example, for $P_o = 0.5$ and $\gamma_{th} = 5$ dB, the OP increases from $3.8 \times 10^{-3}$ to $7.7 \times 10^{-3}$, as $\kappa_t = \kappa_r$ changes from 0.1 to 0.2. This reveals the importance of accurately modeling the level of hardware imperfections, when assessing the outage performance of wireless THz systems in the presence of misalignment and rain. Finally from this figure, it becomes evident that for a given $P_o$, as $\gamma_{th}$ increases, the impact of hardware imperfections on the system’s outage performance becomes more detrimental.

Figure 9 presents the throughput as a function of $\kappa_t$ and $\kappa_r$ for different values of $\sigma_m$, assuming that $\mu_r = -2.04$, $\sigma_r = 0.86$, $P_o = 1$, $\gamma_s = 30$ dB, and $\gamma_{th} = 5$ dB. For given $\sigma_m$ and $\kappa_t$, as $\kappa_r$ increases, the OP increases; as the result, the achievable throughput decreases. For example, for $\sigma_m = 0$ and $\kappa_t = 0.2$, the achievable throughput decreases from 2.06 to 2.02 bits/s/Hz, as $\kappa_r$ changes from 0 to 0.4, while, for $\sigma_m = 0.1$, the same $\kappa_t$ and $\kappa_r$ change...
FIGURE 8. OP vs $P_o$ for different values of $\kappa_t$, $\kappa_r$, and $\gamma_{th}$.

results to an achievable throughput variation from 2.02 to 1.81 bits/s/Hz. This example indicates that as the level of beam misalignment increases, the impact of hardware imperfections on the throughput performance becomes more severe. Moreover, for fixed $\sigma_m$ and $\kappa_t + \kappa_r$, the minimum achievable throughput is achieved for $\kappa_t = \kappa_r$. Likewise, for a given $\sigma_m$, the same throughput is achieved for every $\kappa_t$ and $\kappa_r$ combination for which $\kappa_t^2 + \kappa_r^2$ is constant. Finally, for a given $\kappa_t^2 + \kappa_r^2$, the throughput decreases, as $\sigma_m$ increases.

Figure 10 presents the throughput as a function of $\gamma_{th}$ for different values of $\gamma_s$ and $\kappa_t = \kappa_r$, assuming that $\mu_r = -2.04$, $\sigma_r = 0.86$ and $P_o = 1$. From this figure, it becomes evident that an optimal $\gamma_{th}$ exists that maximizes the achievable throughput. The optimal $\gamma_{th}$ depends on the transmission SNR $\gamma_s$ as well as the level of transceivers hardware imperfections. In more detail, for a given $\gamma_s$, as the level of hardware imperfections increases, the optimal $\gamma_{th}$ shifts to lower values. For instance, for $\gamma_s = 40$ dB, as $\kappa_t$, $\kappa_r$ changes from 0 to 0.2, the optimal $\gamma_{th}$ shifts from 22 to 10.3 dB. Furthermore, for fixed $\gamma_s$ and $\gamma_{th}$, as $\kappa_t = \kappa_r$ increases, the achievable throughput decreases. For example, for $\gamma_s = 30$ dB and $\gamma_{th} = 30$ dB, the achievable throughput decreases from 3.46 to 1.8 bits/s/Hz, as $\kappa_t = \kappa_r$ increases from 0 to 0.2. Moreover, for a fixed level of transceiver hardware imperfections, the optimal $\gamma_{th}$ takes higher values, as $\gamma_s$ increases. Finally, from this figure, we verify that for $\gamma_{th} \geq \frac{1}{\kappa_t^2 + \kappa_r^2}$, the achievable throughput is equal to 0.

VI. CONCLUSIONS

We investigated the joint effect of rain attenuation, stochastic beam misalignment, and transceiver hardware imperfections in outdoor THz wireless systems. Specifically, we presented a general analytical framework for assessing the OP and achievable throughput for both cases of ideal and non-ideal RF front-end as well as in the presence and the absence of rain. Our results highlighted the degradation due to the joint effect of rain attenuation, misalignment fading, as well as hardware imperfections on the outage and throughput per-
performance of THz wireless systems. Moreover, the existence of an optimal transmission spectral efficiency that depends on the characteristics of the links, the weather conditions, the levels of beam misalignment as well as hardware imperfections at the TX and the RX was revealed. A policy to identify the optimal transmission spectral efficiency that maximizes the achievable throughput was reported. Finally, the importance of accurately characterizing the rain attenuation, misalignment fading, and the levels of hardware imperfections when designing and evaluating the outage and throughput performance of outdoor THz wireless systems was documented.

**APPENDICES**

**APPENDIX A**

**PROOF OF LEMMA 1**

In the absence of rain, the CDF of $h^2$ can be obtained as

$$F_{h^2|\delta=0}(x) = \Pr(h^2 \leq x | \delta = 0),$$

which, with the aid of (2), can be rewritten as

$$F_{h^2|\delta=0}(x) = \Pr(h^2_t h^2_m \leq x | \delta = 0),$$

or equivalently

$$F_{h^2|\delta=0}(x) = \Pr(h^2_m \leq \frac{x}{h^2_t} | \delta = 0).$$

By applying (10), we obtain (24).

The PDF of $h^2$ can be obtained as

$$f_{h^2|\delta=0}(x) = \frac{dF_{h^2|\delta=0}(x)}{dx},$$

which, by applying (24) can be rewritten as in (23). This concludes the proof.

**APPENDIX B**

**PROOF OF LEMMA 2**

The PDF of $h^2$ can be evaluated as

$$f_{h^2|\delta=1}(x) = f_A \left( \frac{x}{h^2_t} \right).$$

**FIGURE 9.** Throughput vs $\kappa_t$ and $\kappa_r$ for a) $\sigma_m = 0$, b) $\sigma_m = 0.05$, and c) $\sigma_m = 0.1$.

**FIGURE 10.** Throughput vs $\gamma_{th}$ for different values of $\gamma_s$ and $\kappa_t = \kappa_r$. 

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where \( f_A (x) \) is the PDF of
\[
A = h_r^2 h_m^2.
\] (49)

Since \( h_r^2 \) and \( h_m^2 \) are independent RVs, the PDF of \( A \) can be obtained as [78]
\[
f_A (x) = \int_{x}^{\infty} \frac{1}{\pi} f_h (y) f_h (x/y) \, dy.
\] (50)

which, by applying (9) and (18) can be written as
\[
f_A (x) = \frac{1}{\sqrt{2\pi} \sigma_r} \frac{\xi}{A_o} \left( \frac{x}{A_o} \right)^{-\frac{\xi}{\sigma_r}} \exp \left( -\frac{(\ln (y) - \mu_r)^2}{2\sigma_r^2} \right) \exp \left( -\frac{(x - \mu_r)^2}{2\sigma_r^2} \right) \, dy.
\] (51)

By setting \( z = \ln (y) \), (51) can be rewritten as
\[
f_A (x) = \frac{1}{\sqrt{2\pi} \sigma_r} \frac{\xi}{A_o} \left( \frac{x}{A_o} \right)^{-\frac{\xi}{\sigma_r}} \exp \left( -\frac{(z - \mu_r)^2}{2\sigma_r^2} - \xi z \right) \, dz
\] (52)
or equivalent
\[
f_A (x) = \frac{1}{\sqrt{2\pi} \sigma_r} \frac{\xi}{A_o} \left( \frac{x}{A_o} \right)^{-\frac{\xi}{\sigma_r}} \exp \left( -\frac{\mu_r^2}{2\sigma_r^2} \right) \int_{\ln(\frac{z}{\sigma_r})}^{\infty} \exp \left( -\frac{\xi^2}{2\sigma_r^2} - \xi z \right) \, dz.
\] (53)

By applying [80, eq.(3.322/1)] in (53), we get
\[
f_A (x) = \frac{1}{2} \exp \left( \frac{\sigma_r^2 \xi^2}{2} - \frac{\xi \mu_r}{\sigma_r} \right) x^{-\frac{\xi}{\sigma_r}} \exp \left( -\frac{\xi^2}{2\sigma_r^2} \right) \erfc \left( \frac{\xi}{\sqrt{2}} \right) + \frac{1}{\sqrt{2\sigma_r}} \ln \left( \frac{x}{A_o} \right).
\] (54)

With the aid of (54), (48), can be written as (25).

The CDF of \( h_r^2 \) can be obtained as
\[
F_{h_r^2|a=1} (x) = F_A |_{\xi=1} \left( \frac{x}{h_r^2} \right),
\] (55)

where \( F_A |_{\xi=1} (x) \) is the CDF of \( A \), which can be evaluated as
\[
F_A |_{\xi=1} (x) = \int_{x}^{\infty} f_A (y) \, dy.
\] (56)

By applying (54) in (56), we obtain
\[
F_{A|\xi=1} (x) = \frac{1}{2} \exp \left( \frac{\sigma_r^2 \xi^2}{2} - \frac{\xi \mu_r}{\sigma_r} \right) x^{-\frac{\xi}{\sigma_r}} \exp \left( -\frac{\xi^2}{2\sigma_r^2} \right) \erfc \left( \frac{\xi}{\sqrt{2}} \right) + \frac{1}{\sqrt{2\sigma_r}} \ln \left( \frac{x}{A_o} \right) \, dy.
\] (57)

By setting
\[
z = \frac{\sigma_r}{\sqrt{2}} \left( \xi - \frac{\mu_r}{\sigma_r} \right) - \frac{1}{\sqrt{2\sigma_r}} \ln \left( \frac{x}{A_o} \right) + \frac{1}{\sqrt{2\sigma_r}} \ln (y),
\] (58)

we can rewrite (57) as in (59), given at the top of the next page. After some algebraic manipulations, (59) can be written as (60), given at top of the next page. By employing [81] and after some algebraic manipulations, (60) can be expressed as in (61), given at the top of the next page. Finally, by applying (61) in (55), we obtain (26). This concludes the proof.

**APPENDIX C**

**PROOF OF PROPOSITION 2**

The OP is defined as
\[
P_{out} (\gamma_{th}) = \Pr \left( \gamma \leq \gamma_{th} \right),
\] (62)

which, by applying (31) can be equivalently written as
\[
P_{out} (\gamma_{th}) = \Pr \left( h_r^2 (1 - \gamma_{th}) (\kappa_r^2 + \kappa_m^2) \leq \gamma_{th} N_o \right).
\] (63)

For \( \gamma_{th} (\kappa_r^2 + \kappa_m^2) \geq 1 \), the condition \( 1 - \gamma_{th} (\kappa_r^2 + \kappa_m^2) \leq 0 \leq \gamma_{th} N_o \) is always valid. Thus,
\[
P_{out} (\gamma_{th}) = 1, \text{ for } \gamma_{th} \geq 1 \frac{\kappa_r^2 + \kappa_m^2}{\kappa_r^2 + \kappa_m^2}.
\] (64)

On the other hand, for \( \gamma_{th} < 1 \frac{\kappa_r^2 + \kappa_m^2}{\kappa_r^2 + \kappa_m^2} \), (63) can be rewritten as
\[
P_{out} (\gamma_{th}) = \Pr \left( h_r^2 \leq \frac{1}{(1 - \gamma_{th} (\kappa_r^2 + \kappa_m^2)) \gamma_{th} N_o} \right),
\] for \( \gamma_{th} < 1 \frac{\kappa_r^2 + \kappa_m^2}{\kappa_r^2 + \kappa_m^2} \).
\] (65)

or
\[
P_{out} (\gamma_{th}) = n_{th} \left( \frac{1}{(1 - \gamma_{th} (\kappa_r^2 + \kappa_m^2)) \gamma_{th} N_o} \right),
\] for \( \gamma_{th} < 1 \frac{\kappa_r^2 + \kappa_m^2}{\kappa_r^2 + \kappa_m^2} \).
\] (66)

By combining (64) and (66), we obtain (33). This concludes the proof.

**APPENDIX D**

**PROOF OF PROPOSITION 3**

By applying (33) to (39), the achievable throughput can be rewritten as
\[
D (\gamma_{th}) = \begin{cases} 
\log_2 (1 + \gamma_{th}) (1 - (1 - P_o) F_{h_r^2|a=0} (\zeta_{gamma})) & \text{for } \gamma_{th} < \frac{1}{\kappa_r^2 + \kappa_m^2}, \\
-P_o F_{h_r^2|a=1} (\zeta_{gamma}) & \text{for } \gamma_{th} \geq \frac{1}{\kappa_r^2 + \kappa_m^2},
\end{cases}
\] (67)

where
\[
\zeta = \frac{1}{\gamma_{th}} \frac{1}{1 - \gamma_{th} (\kappa_r^2 + \kappa_m^2)}.
\] (68)

From (67), for \( \gamma_{th} < \frac{1}{\kappa_r^2 + \kappa_m^2} \), it is evident that in the low-\( \gamma_{th} \) regime, where \( F_{h_r^2|a=0} (\zeta_{gamma}) \) and \( F_{h_r^2|a=1} (\zeta_{gamma}) \) tend to 0, the achievable throughput is mainly affected by
\[
f_1 (\gamma_{th}) = \log_2 (1 + \gamma_{th}),
\] (69)
which is an increasing function. On the other hand, in the high-$\gamma_{\text{th}}$ regime, where $F_{h}\mid_{\gamma=0} (\zeta_{\text{th}})$ and $F_{h}\mid_{\gamma=1} (\zeta_{\text{th}})$ tend to 1, the achievable throughput is mainly affected by
\[
f_2 (\gamma_{\text{th}}) = 1 - (1 - P_o) F_{h}\mid_{\gamma=0} (\zeta_{\text{th}}) - P_o F_{h}\mid_{\gamma=1} (\zeta_{\text{th}}),
\]
which is a decreasing function. Since the achievable throughput is a continuous function of $\gamma_{\text{th}}$, the aforementioned observations indicate that (67) is a concave function. In other words, there exists $\gamma_{\text{th}}^o$ that maximizes the achievable throughput. Apparently, for the $\gamma_{\text{th}}$, the following inequality is valid:
\[
\gamma_{\text{th}}^o < \frac{1}{\kappa_t^2 + \kappa_r^2}.
\]
For $\gamma_{\text{th}} < \frac{1}{\kappa_t^2 + \kappa_r^2}$, the first derivative of (67) can be obtained as in (72), given at the top of the next page, or equivalently as in (73), also given at the top of the next page. Notice that
\[
\frac{dF_{h}\mid_{\gamma=0} (\zeta_{\text{th}})}{d(\zeta_{\text{th}})} = f_{2}\mid_{\gamma=0} (\zeta_{\text{th}}) - P_o f_{2}\mid_{\gamma=1} (\zeta_{\text{th}}) + P_o \left( f_{2}\mid_{\gamma=1} (\zeta_{\text{th}}) - f_{2}\mid_{\gamma=0} (\zeta_{\text{th}}) \right),
\]
and
\[
\frac{dF_{h}\mid_{\gamma=1} (\zeta_{\text{th}})}{d(\gamma_{\text{th}})} = (1 - P_o) f_{2}\mid_{\gamma=0} (\zeta_{\text{th}}),
\]
with
\[
\frac{d\zeta}{d\gamma_{\text{th}}} = \frac{\kappa_t^2 + \kappa_r^2}{\gamma (1 - (\kappa_t^2 + \kappa_r^2) \gamma_{\text{th}})}.
\]
\[
\frac{dD(\gamma_{th})}{d\gamma_{th}} = \frac{1}{\ln(2)} \frac{1}{1 + \gamma_{th}} \left( (1 - (1 - P_o) F_{h^2|\delta=0} (\zeta_{\gamma_{th}}) - P_o F_{h^2|\gamma=1} (\zeta_{\gamma_{th}}) \right) \\
- \log_2(1 + \gamma_{th}) \left( 1 - P_o \right) \frac{dF_{h^2|\delta=0} (\zeta_{\gamma_{th}})}{d\gamma_{th}} + P_o \frac{dF_{h^2|\gamma=1} (\zeta_{\gamma_{th}})}{d\gamma_{th}} \right) 
\]

(72)

\[
\frac{dD(\gamma_{th})}{d\gamma_{th}} = \frac{1}{\ln(2)} \frac{1}{1 + \gamma_{th}} \left( (1 - (1 - P_o) F_{h^2|\delta=0} (\zeta_{\gamma_{th}}) - P_o F_{h^2|\gamma=1} (\zeta_{\gamma_{th}}) \right) \\
- \log_2(1 + \gamma_{th}) \left( \zeta + \gamma_{th} \cdot \frac{\kappa_1^2 + \kappa_2^2}{\gamma (1 - (\kappa_1^2 + \kappa_2^2) \gamma_{th})^2} \right) \\
\left( 1 - P_o \right) f_{h^2|\delta=0} (\zeta_{\gamma_{th}}) + P_o f_{h^2|\gamma=1} (\zeta_{\gamma_{th}}) \right) 
\]

(73)

\[
\frac{dD(\gamma_{th})}{d\gamma_{th}} = \frac{1}{\ln(2)} \frac{1}{1 + \gamma_{th}} P_{out} \left( \gamma_{th} \right) - \log_2(1 + \gamma_{th}) \left( \zeta + \gamma_{th} \cdot \frac{\kappa_1^2 + \kappa_2^2}{\gamma (1 - (\kappa_1^2 + \kappa_2^2) \gamma_{th})^2} \right) f_{h^2} (\zeta_{\gamma_{th}}) 
\]

(79)
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