Physics of suction cups

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We have developed a theory for the contact between suction cups and randomly rough surfaces. The theory predicts the dependency of the pull-off time (lifetime) on the pull-off force, and is tested with measurements performed on suction cups made from a soft polyvinyl chloride (PVC). As substrates we used sandblasted poly(methyl methacrylate) (PMMA). The theory is in good agreement with the experiments, except for surfaces with the root-mean-square (rms) roughness below \( \approx 1 \) \( \mu \)m, where we observed lifetimes much longer than predicted by the theory. We show that this is due to out-diffusion of plasticizer from the soft PVC, which block the critical constrictions along the air flow channels.

\textit{Introduction}–All solids have surface roughness which has a huge influence on a large number of physical phenomena such as adhesion, friction, contact mechanics and the leakage of seals\textsuperscript{[1–13]}. Thus when two solids with nominally flat surfaces are squeezed into contact, unless the applied squeezing pressure is high enough, or the elastic modulus of at least one of the solids low enough, a non-contact region will occur at the interface. If the non-contact region percolate there will be open channels extending from one side of the nominal contact region to the other side. This will allow fluid to flow at the interface from a high fluid pressure region to a low pressure region.

For elastic solids with randomly rough surfaces the contact area percolate when the relative contact area \( A/A_0 \approx 0.42 \) (see \textsuperscript{14}), where \( A_0 \) is the nominal contact area and \( A \) the area of real contact (projected on the \( xy \)-plane). When the contact area percolate there is no open (non-contact) channel at the interface extending across the nominal contact region, and no fluid can flow between the two sides of the nominal contact.

The discussion above is fundamental for the leakage of static seals\textsuperscript{[1–13]}. Here we are interested in rubber suction cups. In this application, the contact between the suction cup and the counter surface (which form an annulus) must be so tight that negligible air can flow from outside the suction cup to inside it.

Suction cups find ubiquitous usage in our everyday activities such as hanging of items to smooth surfaces in our houses and cars, and for technology demanding applications such as lifting fragile and heavy objects safely in a controlled manner using suction cups employing vacuum pumps. Suction cups are increasingly used in robotic applications, such as robots which can climb walls and clean windows. The biomimetic design of suction cups based on octopus vulgaris, remora (sucker fish), and limpets is an area of current scientific investigations that could lead to suction cups exhibiting adhesion under water and on surfaces with varying degree of surface roughness.

\textit{Theory}–The suction cups we study below can be approximated as a truncated cone with the diameter \( 2r_1 \).

![](image1.png)

\textbf{FIG. 1: Model of suction cup used in the present study.}

The angle \( \alpha \) and the upper plate radius \( r_0 \) are defined in Fig. \textsuperscript{1}. When a suction cup is pressed in contact with a flat surface the rubber cone will make apparent contact with the substrate in an annular region, but the contact pressure will be largest in a smaller annular region of width \( l(t) \) formed close to the inner edge of the nominal contact area (see Fig. \textsuperscript{1}). We will assume that the rubber-substrate contact pressure in this region of space is constant, \( p = p(t) \), and zero elsewhere.

If we define \( h(t) = r_0 \tan \alpha \) the volume of gas inside the suction cup is

\[
V = \pi r^2 \frac{1}{3} (h + h_0) - \pi r_0^2 \frac{1}{3} h_0
\]

Since

\[
\frac{r}{r_0} = 1 + \frac{h}{h_0}
\]

we get

\[
V = V_0 \left[ \left( \frac{r}{r_0} \right)^3 - 1 \right]
\]

where \( V_0 = \pi r_0^2 h_0/3 \). The elastic deformation of the rubber film (cone) needed to make contact with the substrate require a normal force \( F_0(h) \), which we will refer to as the cup (non-linear) spring-force. This force result from the bending of the film and to the (in-plane) stretching.
and compression of the film needed to deform (part of) the conical surface into a flat circular disc or annulus. The function $F_0(h)$ can be easily measured experimentally (see below).

We assume that the rubber cup is in repulsive contact with the substrate over a region of width $l(t)$. Since the thickness of the suction cup material decreases as $r$ increases, we expect that $l$ decreases as $r$ increases. From optical pictures of the contact we have found that to a good approximation

$$l = l_0 + l_a \left(1 - \frac{r}{r_0}\right) = l_0 - l_a \frac{h}{h_0} \quad (3)$$

where $l_a = (l_1 - l_0)/(1 - r_1/r_0)$ where $l_1$ is the width of the contact region when $r = r_1$, and $l_0$ the width of the contact region when $r = r_0$. The contact pressure $p = p(t)$ in the circular contact strip is assumed to be constant

$$p \approx \frac{F_0(h)}{2\pi rl} + \beta (p_a - p_b) \quad (4)$$

where $\beta$ is a number between 0 and 1.

Assume that the pull-force $F_1$ act on the suction cup (see Fig. 1). The sum of $F_1$ and the cup spring-force $F_0$ must equal the force resulting from the pressure difference between outside and inside the suction cup, i.e.

$$F_0 + F_1 = \pi r^2 (p_a - p_b) \quad (5)$$

We assume that the air can be treated as an ideal gas so that

$$p_b V_b = N_b k_B T. \quad (6)$$

The number of molecules per unit time entering the suction cup, $\dot{N}_b(t)$, is given by

$$\dot{N}_b = f(p, p_a, p_b) \frac{L_y}{L_x} \quad (7)$$

Here $L_x$ and $L_y$ are the lengths of the sealing region along and orthogonal to the gas leakage direction, respectively. In the present case

$$\frac{L_y}{L_x} = \frac{2\pi r}{l}$$

The (square-unit) leak-rate function $f(p, p_a, p_b)$ will be discussed below.

The equations (1)-(7) constitute 7 equations from which the following 7 quantities can be obtained: $h(t)$, $r(t)$, $l(t)$, $V(t)$, $p_b(t)$, $p(t)$ and $\dot{N}_b(t)$. The equations (1)-(7) can be easily solved by numerical integration.

The suction cup stiffness force $F_0(h)$ depends on the speed with which the suction cup is compressed (or decompressed). The reason for this is the viscoelastic nature of the suction cup material. To take this effect into account we define the contact time state variable $\phi(t)$ as \[1 \ 19 \ 24\]:

$$\dot{\phi} = 1 - \frac{r \phi}{l} \quad (8)$$

with $\phi(0) = 0$. For stationary contact, $\dot{r} = 0$, this equation gives just the time $t$ of stationary contact, $\phi(t) = t$. When the ratio $\dot{r}/l$ is non-zero but constant (8) gives

$$\phi(t) = \left(1 - e^{-l/\tau}\right) \tau,$$

where $\tau = l/\dot{r}$. Thus for $t \gg \tau$ we get $\phi(t) = \phi_0 = \tau$, which is the time a particular point on the suction cup surface stay in the rubber-substrate contact region of width $l(t)$. It is only in this part of the rubber-substrate nominal contact region where a strong (repulsive) interaction occur between the rubber film and the substrate, and it is region of space which is most important for the gas scaling process.

From dimensional arguments we expect that $F_0(h)$ is proportional to the effective elastic modulus of the cup material. We have measured $F_0(h)$ at a constant indentation speed $\dot{h}$, corresponding to a constant radial velocity $\dot{r} = \dot{h}(r_0/h_0)$ (see (1)). In this case the effective elastic modulus is determined by the relaxation modulus $E_{\text{eff}}(t)$ calculated for the contact time $\phi_0 = l/\dot{r}$. However, in general $\dot{r}$ may be strongly time-dependent. We can take that into account by replacing the measured $F_0(h)$ by the function $F_0(h) E_{\text{eff}}(\phi(t))/E_{\text{eff}}(\phi_0)$.

**Diffusive and ballistic gas leakage**–The gas leakage result from the open (non-contact) channels at the interface between the rubber film and the substrate. Most of the leakage occur in the biggest open flow channels. The most narrow constriction in the biggest open channels are denoted as the critical constrictions. Most of the gas

![FIG. 2: Diffusive (a) and ballistic (b) motion of the gas atoms in the critical junction. In case (a) the gas mean free path $\lambda$ is much smaller than the gap width $u_c$ and the gas molecules makes many collisions with other gas molecules before a collision with the solid walls. In the opposite limit, when $\lambda \gg u_c$ the gas molecules makes many collisions with the solid wall before colliding with another gas molecule. In the first case (a) the gas can be treated as a (compressible) fluid, but this is not the case in (b).](image-url)
and B. show results for two different suction cups, denoted A and B. The suction cups are squeezed against a smooth glass plate with a hole in the center through which the air can leave so the air pressure inside the rubber suction cup is the same as outside (atmospheric pressure). The glass plate is lubricated with soap-water.

The suction cups A and B are both made from similar type of soft PVC and both have the diameter \( \approx 4 \) cm. However, for suction cup B the angle \( \alpha = 21^\circ \) in contrast to \( \alpha = 33^\circ \) for suction cup A, and the PVC film is thicker for the cup A. This difference in the angle \( \alpha \) and the film thickness influence the suction cup stiffness force as shown in Fig. 3. Note that before the strong increase in the \( F_0(h) \) curve which result when the suction cup is squeezed into complete contact with the counter surface the suction cup A has a stiffness nearly twice as high as that of the suction cup B.

Gas leakage—We have studied how the failure time of a suction cup depends on the pull-off force (vertical load) and the substrate surface roughness. The suction cup was always attached to the lower side of a horizontal surface and a mass-load was attached to the suction cup. We varied the mass-load from 0.25 kg to 8 kg. If full vacuum would prevail inside the suction cup, the maximum possible pull-off force would be \( \pi r_1^2 p_a \). Using \( r_1 = 19 \) mm and \( p_a = 100 \) kPa we get \( F_{\text{max}} = 113 \) N or about 11 kg mass load. However, the maximum load possible in our experiments for a smooth substrate surface is about 9 kg, indicating that no complete vacuum was obtained. This may, in least in part, be due to problems to fully remove the air inside the suction cup in the initial state. In addition we have found that for mass loads above 8 kg the pull-off is very sensitive to instabilities in the macroscopic deformations of the suction cup, probably resulting from small deviations away from the vertical direction of the applied loading force.

The dependency of the failure time on the pull-off force—Fig. 4 shows the dependency of the pull-off time (failure time) on the applied pulling force. The results are for the soft PVC suction cups A (red) and B (green) in contact with a sandblasted PMMA surface with the rms-roughness 1.89 \( \mu \)m. Before the measurement, all surfaces were cleaned with soap water. The solid lines are the theory predictions, using as input the surface roughness power spectrum of the PMMA surface, and the measured stiffness of the suction cup, the latter corrected for viscoelastic time-relaxation as described above. Note the good agreement between the theory and the experiments in spite of the simple nature of the theory.

Fig. 3(a) shows the calculated time dependency of the radius of the non-contact region, and (b) the gas pressure in the suction cup. We show results for several pull-off forces from \( F_1 = 5 \) N in steps of 5 N to 80 N.

The smaller angle \( \alpha \) for suction cup B than for cup

![Diagram](image_url)
FIG. 4: The dependency of the pull-off time (failure time) on the applied (pulling) force. The soft PVC suction cups A and B are in contact with a sandblasted PMMA surface with the rms roughness 1.89 µm. All surfaces were cleaned with soap water before the experiments.

A imply that if the same amount of gas would leak into the suction cups the gas pressure $p_b$ inside the suction cup will be highest for the suction cup B. This will tend to reduce the lifetime of cup B. Similarly, the smaller stiffness of the cup B result in smaller contact pressure $p$, which will increase the leakage rate and reduce the lifetime. Hence both effects will make the lifetime of the suction cup B smaller than that of the cup A.

The dependency of the failure time on the substrate surface roughness—Fig. 6 shows the dependency of the pull-off time (failure time) on the substrate surface roughness. The results are for the type A soft PVC suction cups in contact with sandblasted PMMA surfaces with different rms roughness, and a table surface. Note that for “large” roughness the predicted failure time is in good agreement with the measured data, but for rms roughness below $\approx 1 \mu m$ the measured failure times are much larger than the theory prediction. In addition, the dependency of the radius $r(t)$ of the non-contact region on time is very different in the two cases: For roughness larger than $\approx 1 \mu m$ the radius increases continuously with time as also expected from theory (see Fig. 5(a)). For roughness below $\approx 1 \mu m$ the boundary line $r(t)$ stopped to move a short time after applying the pull-off force, and remained fixed until the detachment occurred by a rapid increase in $r(t)$ (catastrophic event). We attribute this discrepancy between theory and experiments to transfer of plasticizer from the soft PVC to the PVC-PMMA interface; this (high viscosity) fluid will fill-up the critical constrictions and hence stop, or strongly reduce, the flow of air into the suction cup. This is consistent with many studies of the transfer of plasticizer from soft PVC to various contacting materials. These studies show typical transfer rates (at room temperature) corresponding to a
~ 1–10 µm thick film of plasticizer after one week waiting time. Optical pictures of the rough PMMA surface after long contact with the suction cup A also showed darkened (and sticky) annular regions indicating transfer of material from the PVC to the PMMA surface.

Summary—We have studied the leakage of suction cups both experimentally and using a multiscale contact mechanics theory. In the experiments suction cups (made of soft PVC) were pressed against sandblasted PMMA sheets. We found the failure times of suction cups to be in good agreement with the theory, except for surfaces with rms-roughness below ≈ 1µm, where diffusion of plasticizer occurred, from the PVC to the PMMA counterface resulting in blocking of critical contrictions.

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