Complex Dependencies in Large Software Systems

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Abstract

Two large, open source software systems are analyzed from the vantage point of complex adaptive systems theory. For both systems, the full dependency graphs are constructed and their properties are shown to be consistent with the assumption of stochastic growth. In particular, the afferent links are distributed according to Zipf’s law for both systems. Using the Small-World criterion for directed graphs, it is shown that contrary to claims in the literature, these software systems do not possess Small-World properties. Furthermore, it is argued that the Small-World property is not of any particular advantage in a standard layered architecture. Finally, it is suggested that the eigenvector centrality can play an important role in deciding which open source software packages to use in mission critical applications. This comes about because knowing the absolute number of afferent links alone is insufficient to decide how important a package is to the system as a whole, instead the importance of the linking package plays a major role as well.

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1 Introduction

Scientist and engineers are increasingly turning to the metaphor of the complex adaptive system (CAS) [1, ] in order to understand the behavior of very large software systems involving large numbers of components [2, 3, 4, 5, 6, 7]. A central tenet within the CAS paradigm is that the network of interactions between the constituent entities, together with their intrinsic properties, largely determines a system’s emergent behavior [8, 9, 10]. If the network of interactions is scale-free then it will combine the robustness properties of a random
network with respect to the loss of an arbitrary node, together with the fragility properties of a binary network with respect to the loss of one of its hubs. If a network exhibits small-world characteristics [11], then information should flow efficiently at both global and local levels.

For software systems the network of interactions takes the form of a dependency graph between the components. At the system level, static dependency graphs play an important role in the build and package management processes; hence understanding their properties has important practical consequences for system administration. Dependency graphs also provide a finer grained view of the system than is usually obtained by looking at high level architectural diagrams.

In this paper, the static dependency graphs for two relatively large software systems, Debian [12] and Maven [13] are examined. Debian is an open source Linux distribution comprising over 22,000 separate software packages totaling more than 20 Gigabytes of source code. It is one of the oldest Linux distributions and is still employed as a server OS, not to mention its immense popularity among Linux enthusiasts. Maven is, in the first place, a software project management tool, but it is also a collection of repositories for Java software. It is these repositories which give Maven, as a project management tool, its strength, because they offer all Java developers a convenient mechanism for tracking package dependencies with minimal effort.

Approaching these systems from the CAS viewpoint means attempting to identify and understand their emergent properties. Previous studies of the Debian distribution and collections of Java software have concentrated upon the degree distributions of the afferent and efferent links in the dependency graph [14, 2, 3, 5, 6, 15]. All studies agree that the afferent degree distribution follows a power law, but they obtain significantly different exponents for Debian and Java software, leading one to wonder if the differences are due to some fundamental differences in how the software is constructed, or if they simply reflect the small sample size used in previous studies of Java software. The main motivation for using Maven is to have a well defined, standard set of Java software, representing a much larger collection of packages than used in previous studies. Like the Debian distribution, the Maven repositories represent reproducible data sets upon which other researchers can cross-check the results presented here.

The present work has two aims, first it examines the issue of growth models for describing the distributions seen in the afferent and efferent degrees in an effort to determine whether or not the different distributions seen in these quantities can be explained in terms of a single model. Using completely separate models for describing the distribution of afferent and efferent links as was the case in previous studies is not satisfactory as the links arise from within the same software development process and should therefore be correlated.

The second aim of this paper is to look beyond the degree distribution to examine small world and centrality issues. An agglomeration of all of these properties gives indications regarding the stability, maintainability and maturity of the various software packages.


1.1 Previous Work

Maillart et al. [6] measured the afferent degree distribution for the Debian system. Valverde et al. [14] examined Java software and measured the total degree distribution by treating the dependency graph as an undirected graph. Baxter et al. [16, 15] and Wheeldon and Counsell [4] measured both the afferent and efferent degree distribution for different Java packages, whereby they made a more fine-grained distinction between the different types of dependencies than is being made in this paper. Concas et al. [5] also studied fine-grained afferent and efferent degree distributions related to object-oriented software quality metrics for Smalltalk software, while Myers [2] examined these quantities for several open source software packages. In all of these studies, the afferent degree distribution was found to obey a power law, while the efferent degree distribution, the extent it was measured, was found to follow either a power law [2, 4], or some other distribution [16, 15, 5]. Even though a power law was obtained for the afferent degree distribution in all studies, there was no consensus on the exponent, with various estimates ranging from a low of 1.4 to a high of 2.5.

Concas et al. [5], Maillart et al. [6] and Baxter et al. [15] proposed various growth models to explain the origins of the observed phenomena. Concas et al. discussed random process involving both independent and proportional growth models as well as the Yule type process [17] suggested by Newman [18]. They found good agreement between the proportional growth model and the efferent degree distribution, as well as good agreement between the predictions of the Yule model and the distribution of afferent degrees. Maillart et al. and Baxter et al. also discuss Yule type process, wherein the latter model is a discrete growth model while the former take a continuum approximation. The continuous model of Maillart et al. was applied to the afferent degree distribution and showed good agreement with the measurements. The discrete mode of Baxter et al. was applied to both the afferent and efferent degrees and while the agreement with measurements for the afferent degree was quite good, the agreement with the measurements for the efferent degree was not as satisfactory.

The small-world properties have been previously measured for several open source software packages by Moura et al. [3] and Valverde and Solé [19]. Both of these studies found evidence for small-world behaviour when using the original Watts and Strogatz’s [11] definition of a small-world graph. To apply this definition, both studies first converted the directed dependency graphs to undirected graphs.

To the best of our knowledge, eigenvector centrality has not previously been studied in the context of dependency graphs for large software systems. Other methods for ranking software based upon multiple factors are discussed by Tsatsaronis et al. [20].

1.2 Scope of Present Work

This paper addresses shortcomings in the previous work by demonstrating that the afferent and efferent links in both the Maven and Debian software systems
have the same distributions and that these distributions are shown to arise from
the same simple growth model. Towards this end, the next section discusses
the construction of the dependency graphs for both systems, while Section 3,
discusses node centrality and the observed distribution ofafferent and efferent
links in more detail.

Section 4 discusses the small world properties of the two systems and shows
why the small-world effects seen in previous studies are an artifact of the con-
version from directed to undirected graphs.

Eigenvector centrality, another global measure of centrality better known
as the Google rank, is examined in Section 5. The final section summarizes
the results of this study discusses how the CAS viewpoint can be applied to
problems in software engineering.

2 Constructing the Dependency Graph

A dependency graph, $G$ is defined as a pair of sets $G = (\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ is
a set of nodes and $\mathcal{L}$ is a set of directed links. If, $n, m \in \mathcal{N}$, then $(n, m)$
denotes a directed link from $n$ to $m$; i.e., a dependency of $n$ on $m$. The nodes of
interest here are either packages in the Debian case, or jar files (Java Archives)
or classes in the Maven case. The directed links symbolize the dependencies
between the packages, jar files or classes. Define the set of afferent links for a
given node $n$ as: $D_A(n) = \{(m, n) | (m, n) \in \mathcal{L}\}$ and the set of efferent links as
$D_E(n) = \{(n, m) | (n, m) \in \mathcal{L}\}$. For convenience, denote the number of afferent
links for $n$ by $q_A(n) = |D_A(n)|$ (or $q_A$ when speaking of an arbitrary node) and
the number of efferent links by $q_E(n) = |D_E(n)|$. $n$’s total degree is then
$q(n) = q_A(n) + q_E(n)$.

There are three graphs of interest here: the Debian dependency graph, $G_D$, the
Maven jars dependency graph, $G_J$ and the Maven class dependency graph
$G_C$. Each of these graphs is constructed from the binary packages without
recourse to the source code.

Determining $G_D$ is straightforward since the Debian package management
system requires the dependency information in order to install the packages,
the information is readily available in the form of a control file accompanying
each package and collated into a single Package file for each directory of the
repository. For each package, its control file list the packages upon which it de-

dpends. There are five types of dependencies defined in the Debian policy manual:
Depends, Recommends, Suggests, Enhances, Pre-Depends. For the present pur-
poses no distinction is made between the different types of dependencies, rather
they are all used to build the edges of the $G_D$.

Within a Debian control file it is also possible to define so-called virtual
packages, which are logically existing packages whose functionality is provided
by some concrete package. Virtual packages are defined through the Provides
field in the control file. For purposes of this study, virtual packages are treated
as concrete packages.

Determining $G_J$ and $G_C$ is more involved. The main repositories are housed
at maven.org, codehouse.org and javax.org. Together these repositories contain some 22,000 jar files from more than 4,000 separate projects. Although Maven defines a project management file (POM) in XML format which can be used to construct $G_J$ in a manner similar to how $G_D$ was constructed, it is more instructive to first construct $G_C$ directly, then use the information therein, combined with the POM file to construct $G_J$. Towards this end, the jar files for each project are opened and the binary class files are parsed to determine their dependencies on other classes. Again, in software engineering one would make a distinction between those dependencies which appear on the class’ public interface, those which are private to the class and those which are local to a class method \[16, 22, 5\]. For the purposes of the present study the technical differences between these different types of dependencies are ignored.

For readers interested in confirming the results presented here, the Debian distribution used in this study was version 5.0.0, also known as Lenny. Its first official release was on February 14, 2009. (Actually, this study was started using a beta version of the Lenny release, but the results were reconfirmed following the first official release.) The Maven repositories are in a more continual state of flux and the data used here was taken during the first week of January 2009.

### 3 Degree Centrality

The degree centrality of node $n$ is defined as the size of the set of links starting or ending at $n$. It plays an important role in the theory of random networks \[9\]. (Throughout this paper the terms “graph” and “network” will be used interchangeably.) When the distribution of $q(n)$ over all $n \in G$ follows a power law, then network’s tolerance to random failures is increased. For directed networks, the distribution of afferent and efferent needs to be treated separately.

#### 3.1 Afferent Links

The distribution of afferent links found in this study is depicted in Fig. 1 for Debian and Fig. 2 for Maven. Throughout this paper the complementary cumulative distribution function, $P(X > x)$, is measured in lieu of the probability mass distribution, $p(x)$; as the former provides more accurate estimates of the distribution’s parameters when the data is noisy \[23, 18, 24\]. The first point to notice is that for all $G$, $p(q_A) \sim q_A^{-\alpha}$ with $\alpha \approx 2 \ (P(Q_A > q_A) \sim q_A^{-\alpha+1})$. Such a distribution generally goes by the name of Zipf’s law and has been previously found to hold for a number of different natural and anthropological phenomena \[18\]. In fact Maillart \emph{et al.} \[6\] have previously demonstrated that this relationship holds for the Debian distribution.

What is new in these results is that Zipf’s law holds for Maven as well. Using the MLE (maximum likelihood estimate)\[23, 24\], an accurate estimate of $\alpha$ can be obtained yielding $\alpha = 2.0 \pm 0.1$, which holds for both the Debian and Maven systems. Previous studies on a number of different Java software have found exponents ranging from a low of $\alpha \approx 1.4$ to a high of $\alpha \approx 2.5 \ [14, 2, 4,$
The discrepancy between past results and the current experiments are due to two factors: 1) previous studies limited the types of dependencies they investigated, and 2) previous studies examined only single Java projects, not the broad collection available under Maven. Limiting the dependency counting to those on the public interfaces suppresses the distribution at large values of \( q_A \) since the majority of dependencies are hidden as local variables inside the body of class’ methods. Finding the same exponent for Maven and Debian is reassuring as it indicates that similar mechanisms are involved in the engineering processes through which the dependency graphs arise.

3.2 A Yule Process

The exact mechanisms for generating Zipf’s law behavior are still under debate[18, 25, 26]. Some suggestions include various Yule processes [17, 27], entropy maximization [28] and highly optimized tolerance [29]. For Graphs, Barabási and Albert [8] defined a variant of Simon’s Yule process [17], called preferential attachment, which leads to power law behavior in the distribution of the afferent and efferent links, and Bollobás et al. [30] extended their model to directed graphs. Although preferential attachment is the most popular model, other discrete growth models similarly lead to power law behavior in the afferent link distribution [31, 32]. On the other hand, Maillart et al. [6] previously demonstrated that a continuous formulation of the Yule process offered a good explanation for describing the distribution \( p(q_A) \). Here, the aim is to provide further support for the latter hypothesis by showing that it can explain the distribution of \( p(q_E) \) as well.

The essence of all Yule processes is the Gibrat principle [18, 33]: the probability that an entity will experience an increase in the value of one of its properties in the next time step is proportional to the current value of that property. In other words, the rich get richer. This principle leads to a stochastic formulation for the growth, the details of which vary with the particular model. To start with, let \( X = q_A \), then the growth in \( X \) varies over time according to the stochastic equation:

\[
dX = \mu X \, dt + \sigma X \, dW_t,
\]

where \( \mu \) and \( \sigma \) are constants and \( W_t \) is a standard Wiener process, meaning \( W_0 = 0 \), \( W_t \) is continuous, and \( W_{t+\tau} - W_t \sim N(0, \tau), \forall t, \tau > 0 \). Although \( X \) is in the present case discrete, eq. 1 is nevertheless a good approximation when \( X \) is large. With the help of the Ito Lemma, eq. 1 can be readily integrated to obtain the probability mass distribution:

\[
p(x; T) = \frac{1}{x \sqrt{2 \pi \sigma^2 T}} e^{-\frac{\ln(x)+(1-2\mu/\sigma^2)x^2 T/2\sigma^2}{2\sigma^2 T}},
\]

where \( T \) is the time period over which the stochastic growth occurs and the initial value of \( X(t) \) is taken to be \( X(0) = 1 \).
Note that individual sets of afferent links, \( D_A(n) \), have disparate histories, i.e., they have been growing for different lengths of time, with the oldest packages more than 15 years old and the youngest not more than a year; hence, to compare with the data, one needs to average over the distribution of \( T \) [8, 34, 33]. Although it is possible in principle to track down the lifetime of each package, and thereby obtain the true distribution of \( T \), the effort to do so would be prodigious. Instead one can make progress by assuming all possible values of \( T \) up to some large value, \( T = T \), which is the lifetime of the software systems themselves, are equally likely:

\[
p(x) = \int_0^T \frac{1}{T} dT \, p(x; T)
\]

\[
\approx \frac{4\sigma^2}{T(1 - \frac{2\mu}{\sigma^2})} \begin{cases} 
  x^{-1} & \text{if } x < 1, \\
  x^{-2 + \frac{2\mu}{\sigma^2}} & \text{if } x \geq 1,
\end{cases}
\]

\[- O \left( e^{-2\sigma^2[1 - \frac{2\mu}{\sigma^2}]^2 T} \right) \]

Eq. 3 holds for \( \mu < \sigma^2/2 \) and large \( T \). Under the latter condition, the first term in eq. 3 dominates and all higher order terms in \( T \) can be neglected.

When \( \mu \ll \sigma^2/2 \), the growth is dominated by the stochastic fluctuations and \( p(x) \sim x^{-2} \), in good agreement with the measurements.

### 3.3 Efferent Links

The distribution of efferent links, \( p(q_E) \), is depicted in Fig. 3 for Debian and Fig. 4 for Maven. Figs. 1-4 demonstrate clearly that the distribution of the number of afferent links is not commensurate with the distribution of the number of efferent links. The number of efferent links follows a lognormal distribution in agreement with [5, 15].

At first glance this might cast doubt on the explanation of stochastic growth presented in the previous section; however, an examination of the assumptions leading from eq. 2 to eq. 3 reveals a difference between the two. The set of afferent links, \( D_A(n) \), for a given node, \( n \), grows when another node, \( m \), links to it. In software engineering, this linkage occurs if the package or class represented by \( n \) has services required by the package or class represented by \( m \). Over time, as the size of Debian or Maven collection increases, \( D_A(n) \) will continue to grow as new packages are built using the services of existing packages. The set of efferent links, \( D_E(n) \), on the other hand, expands only when the responsibilities of package \( n \) expand. In software engineering, it is considered poor practice to extensively change a class’s responsibilities once it has become well established in the community. The preferred approach is to create a new class which extends the older class. This “open to extension, closed to change” philosophy [35]
implies that the set $D_E(n)$ will grow only for the relatively short time needed for $n$ to become mature and widely accepted. Assuming that this settling time, to be denoted by, $T_s$, is constant, independent of any particular node, and relatively short compared with time the newer nodes have been in existence, then the distribution, $p(qe)$, would expect to have the form given in eq. 2 with $T_s$ replacing $T$.

Other models for the distribution of links in a dependency graph, such as some form of preferential attachment [31, 32, 30], highly optimized tolerance [29], local optimization [14] and entropy maximization [28] predict the same distribution for both the afferent and efferent links. Hence, at least in the field of software engineering, the model presented here appears to be the most suitable in the sense that it is able to accommodate different distributions in the afferent and efferent links starting from a common mechanism for their growth.

4 Small Worlds

$G_D$, $G_J$ and $G_C$ are all sparse graphs. If they were completely random, then one would expect to observe a low degree of local clustering coupled with a small, average path length between nodes. A so-called small world graph [11] on the other hand, combines a high degree of local clustering with a small, average path length between nodes. Watts and Strogatz's original definition of a small world network considered only connected graphs, with undirected links; whereas all the graphs studied here contain directed links. Furthermore, the graphs are disconnected in the sense that is not possible to start at an arbitrary node and visit any other arbitrary node while transversing the links in their proper direction. (If the links were undirected, then $G_J$ and $G_C$ would be connected. Directed graphs fulfilling this condition are often referred to as weakly connected.)

An adequate definition for a small world signature in the case of directed graphs was given by Latora and Marchiori [36]. In their paper they first defined the efficiency of a graph as:

$$E(G) = \frac{1}{|V|(|V| - 1)} \sum_{(n,m) \in E} \frac{1}{d_{nm}}$$  (4)

where $d_{nm}$ is the directed distance from node $n$ to node $m$ and $d_{mn} \neq d_{nm}$. As links in the networks studied here are not weighted, $d_{nm}$ is simply the total number of links transversed in their proper direction while walking from $n$ to $m$. If there is no directed path from $n$ to $m$, then by convention $d_{nm} = \infty$. (Note that $G$ is directed acyclic graph, meaning it does not contain any circular dependencies, including self-dependencies.) $E(G)$ is normalized, $E(G) \in [0, 1]$, with values near 1 indicate highly efficient information flow, while values near 0 indicate information spreads slowly through the network.

Having defined $E(G)$ to measure how efficiently information flows through the entire graph, a similar quantity to measure the efficiency with which information flows locally within subgraphs can be defined as $E(G(n))$, where $G(n)$
is the subgraph of $G$ consisting of all the nearest neighbors of $n$ together with their mutual links, but not $n$ itself, nor any links to $n$.

Small world graphs are then defined as those having large values of both global, $E(G)$, and local efficiency, $\langle E(G(n)) \rangle$. A random graph on the other hand would be expected to exhibit relatively large values of $E(G)$ but relatively small values of $\langle E(G(n)) \rangle$; while a well ordered graph should exhibit small values of $E(G)$ and large values of $\langle E(G(n)) \rangle$. Note: just as in the original Watts and Strogatz definition, the Latora and Marchiori definition does not provide exact definitions for “small” and “large”. Generally, their paper consider values less than 0.1 as “small” and those greater than than 0.25 as “large”.

Table 1 lists the values of the local and global efficiency found for the dependency graphs, $G_D$, $G_J$ and $G_C$. As can be seen, all of the graphs have large local efficiency, combined with small global efficiencies; hence, these graphs do not fulfill the small world conditions. This result contrasts with previous studies claiming to have uncovered the small world signature in software dependency graphs.[3, 19]

The problem arises in that previous studies relied on the original Watts-Strogatz definition of a small world in terms of undirected graphs rather than the Latora-Marchiori definition for directed graphs. Simply treating directed links as undirected links gives a false impression of the efficiency with which information flows through the network.

To further understand the previous point, calculate the Pearson correlation coefficient between the number of afferent and efferent links at the end points of each link in the network:

$$r_{\alpha\beta}(G) = \frac{1}{|L| - 1} \sum_{(n,m) \in L} \left( \frac{q_\alpha(n) - \langle q_\alpha \rangle}{\sigma_{q_\alpha}} \right) \left( \frac{q_\beta(m) - \langle q_\beta \rangle}{\sigma_{q_\beta}} \right)$$  \hspace{1cm} (5)

where $\sigma_{q_\alpha}$ is the standard deviation of $q_\alpha$ and $\alpha, \beta \in \{A, E\}$. Eq. 5 reduces to Newman’s assortative mixing coefficient [37] when $q = q_A + q_E$. $r(G) \in [-1, 1]$ with $r(G) = 1$ indicating a perfect positive correlation (assortative mixing [37]), i.e., nodes with a given value of $q(n)$ connect only to other nodes with the same value $q(n)$, and $r(G) = -1$ indicating perfect negative correlation (disassortative mixing [37]) i.e., nodes with small $q(n)$ connect only to nodes with large $q(n)$ or vice versa.

Table 1 lists the four mixing coefficients for the networks studied here. The main trend to notice in the data is the uniformly, relative large values of $r_{E,A}(G)$ and the uniformly, relative small values of $r_{A,A}$ and $r_{A,E}$. These results indicate that the directed links have a disassortative preference with nodes of low efferent degree connecting to nodes of high afferent degree. In such networks information flow is inhibited since nodes with high afferent degree, tend to have low efferent degree thus inhibiting the efficient spread of information at the global level.
5 Google Rank

The degree centrality discussed in section 3 provides clues about how important a given node is to the network in that it measures how many other nodes depend upon it. However, this is not the full picture, since a node with a small $q_A(n)$ can be equally important to the network as a whole if nodes with large $q_A(n)$ depend upon it. To understand this phenomena one needs to quantify the importance of a node in a graph based on the importance of the nodes linking to it. This problem was famously solved by Brin and Page. Their solution rests on a variation of the eigenvector centrality measure, which is defined in terms of the principal eigenvector of the adjacency matrix, $A$.

The entries of the adjacency matrix in normalized form are defined as $A_{nm} = 1/qE(n)$ if $(n, m) \in L$ and 0 otherwise. In the present case some nodes have no efferent links, while others have no afferent links, implying $A$ is singular; therefore there is no guarantee that all the components of the principal eigenvector will be non-negative, which is a necessary prerequisite for using components of the eigenvector as a centrality measure. To overcome the problem of non-existent efferent links add to $A$, the matrix $B$, defined as: $B_{nm} = 1/|N|$ if $qE(n) = 0$, and 0 otherwise. To overcome the problem of no afferent links, add to $A$ the matrix $C$ defined as: $C_{nm} = 1/|N|$ for all $n, m$. Since all the $G(N, E)$ constructed here are sparse, $1/|N| << 1/qE(n)$, $\forall n$; thus, the contributions from $B$ and $C$ to $A$ are small and should not significantly distort the centrality measure. The new matrix is:

$$P = \gamma(A + B) + (1 - \gamma)C,$$

and the eigenvalue equation now reads:

$$R = P^TR$$

In this form, eqs. 6 and 7, define the Google page rank. In order to understand the meaning of eq. 7, one can expand it:

$$R_n = \frac{1 - \gamma}{|N|} + \sum_{(m,n) \in D_A(n)} \frac{\gamma R_m}{qE(m)} + \sum_{qE(m)=0} \frac{\gamma R_m}{|N|},$$

showing that 7 has the desired property of weighting the rank of each node by the rank of the nodes which link to it. The last term in eq. 8 adds a small weight from all nodes with no efferent links, while the first term gives a small weight to nodes with no afferent links. The properties of $P$ are well known [39]: it is a large, sparse, column stochastic matrix whose dominant eigenvalue is equal to 1; the eigenvector corresponding to the dominant eigenvalue has only non-negative elements; and the second largest eigenvalue is $\gamma$. As long as $\gamma >> 1/|N|$, the exact value does not influence the results, however, if a power method is used to solve for the principal eigenvector, the rate of convergence is equal to $\gamma$; meaning the power method will fail to converge as $\gamma \rightarrow 1$. In this study the value $\gamma = 0.9$ is used.
Figs. 5 depicts the average rank as a function of the number of afferent links for the various graphs considered here. As can be seen there is a clear trend, with large numbers of afferent links correlating with high average rank. However, the average values glosses over significant details of the graph structure as can be seen by examining Table 2. The number third ranked Debian package has a mere 8 afferent links, while the 7th ranked package has 5,620. And the number 6th ranked jar file in the Maven repositories has a only 2 afferent links while the file ranked 7th has 1,092. The reason a node with a very small number of afferent links can be ranked so highly is that other, more highly rank nodes depend upon it.

6 Summary and Conclusions

This study has demonstrated three points. Firstly, the distribution of afferent and efferent links for dependency graphs of large software systems are similar independent of the details of how the system is constructed. For the afferent links, this distribution obeys Zipf’s Law. For the efferent links, the distribution is lognormal. Both of these distributions can be explained in terms of the same stochastic growth process, with the differences in the final form of the distribution explainable in terms of the different time scales over which the growth takes place. From these results one can hypothesize, that the dependency graph of any sufficiently large software system, will, from a complexity viewpoint, have the same properties.

Secondly, this study has shown that the small world metaphor does not apply to large software systems, because the global efficiency is too small. This result contrasts with previous work which used the small world definition for directed networks, but can be understood by considering about the high level architecture of the Debian system. From the software engineering point of view, the Debian distribution is well structured, employing a layered architecture with the Linux operating system on the bottom, and Gnome/GTK+ (or KDE/Qt) applications on the top. In principle, developers should target their applications at a particular layer, building upon services in the layer below, while offering services to the layer above. In practice, applications in a given layer will often use services from any or all of the lower lying layers, but never from a higher layer. The long-range links typical of small-world networks would, in a layered architecture, proceed from higher layers down to the lower layers. Without the reverse couplings information flows upward in the stack, but not downward; thus stifling the global efficiency and the classical small-world effects. It would be an interesting extension to develop a signature for layered networks similar in spirit to the signature for small world networks.

Thirdly, the importance of a node in a dependency graph depends not only on the number of afferent links, but also on the importance of the node from which the afferent link arises. For example, the glibc-doc package has only 8 afferent links; however, it is ranked 3rd amongst all Debian packages because the number one ranked package, libc6, links to it.
To further understand how CAS considerations can play a more active role in software engineering consider the questions of stability, maintainability and maturity raised in the introduction. In the context of object oriented software development, an important software quality metric is the instability, which is defined as the ratio of the number of efferent links to the sum of efferent and afferent links for a given object [40]. Within the CAS paradigm, one would apply this metric on the system, as a whole using the methodology for determining the link distribution outlined above. In the notation used here, the instability of node \( n \), would be written as: 
\[
I(n) = \frac{q_E}{q_E + q_A}.
\]
This would lead to a measure of instability for each open source package and provide important clues about the risk of using any given package in one’s own project. Large values of \( I(n) \), indicate an instable package which is likely to change more often than a package with a relatively smaller value of \( I(n) \).

The maintainability and maturity of a software package is not just a question of its age or the number of previous releases, but also a question of its acceptance. If it is not being used by other projects, then it will sooner or later fade away. Determining which packages are more likely to be maintained and updated over time, is a key factor in deciding whether or not to use open source software in mission-critical settings. Tsatsaronis et al. [20] tackle this question by creating a model for open source software repositories containing 19 common parameters for each package. While these parameters are very good at judging the current health of an open source project, they do not provide enough insight into question of whether or not a package will likely be maintained in the long run. Successful projects will always look good according to the criterion of Tsatsaronis et al. as long as the project’s originator is still running the project; however, what happens when the project’s originator, for whatever reason, is no longer able to coordinate the project? How likely is it that the software will be maintained? This results shown here suggests that the answer to this question, is to expand Tsatsaronis et al. parameter list to include the Google Rank of the package in the network of all open-source software. The more central a given package is to the system as a whole, the more likely it will be that a talented developer will step forward to maintain a package once its originator has left.

Finally, as discussed above, the layered architectures examined here do not exhibit small-world properties for good reasons; however, it is an open question whether or not other architectural patterns might benefit from small-world behavior. In particular when using a Service Oriented Architecture (SOA) to establish an ecosystem of services [41], the small-world property may have advantages and may arise naturally. The important concept to consider is how information should flow in the system and whether information should flow as freely over long distance as it does over short distances.

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Table 1: The global and local efficiencies, $E(G)$ and $\langle E(G(n)) \rangle$ respectively along with the Pearson coefficients for the links, e.g., $r_{E_A}(G)$ measures the correlation of $q_E$ and $q_A$ between connected nodes.

|     | $E(G)$ | $\langle E(G(n)) \rangle$ | $r_{AA}(G)$ | $r_{E_A}(G)$ | $r_{A_E}(G)$ | $r_{E_E}(G)$ |
|-----|--------|-----------------------------|-------------|--------------|--------------|--------------|
| $G_D$       | 0.017  | 0.41                        | 0.055       | -0.17        | -0.056       | 0.21         |
| $G_J$       | 0.0087 | 0.37                        | -0.00043    | -0.10        | 0.00072      | -0.020       |
| $G_C$       | 0.024  | 0.33                        | -0.021      | -0.28        | -0.033       | -0.095       |
Table 2: Rank and number of links.

| rank | $G_D$  | $G_I$  | $G_C$  |
|------|--------|--------|--------|
| 1    | 21,014 | 3,498  | 128,390|
| libc6| rt     | String |
| 2    | 6,068  | 45     | 143,574|
| libgcc1| jce    | Object |
| 3    | 8      | 12     | 30,141 |
| glibc-doc| script-api | Throwable |
| 4    | 200    | 83     | 26,426 |
| locales| tools   | Exception |
| 5    | 12     | 88     | 13,741 |
| libc6-i686| jsse    | IllegalArgumentException |
| 6    | 86     | 2      | 18,868 |
| gcc-4.3-base| charsets | StringBuilder |
| 7    | 1,606  | 1,001  | 40,783 |
| debconf| groovysoap-all | Class |
| 27   | 208    | 123    | 4      |
| netbase| geronimo-spec-j2ee | AbstractStringBuilder |
| 28   | 136    | 81     | 2,408  |
| libgdbm3| org-openide-util | Float |
Figures

Figure 1 Complementary cumulative distribution function, $P(Q_A > q_A)$, for $q_A$ in $G_D$. The solid line, $P(Q_A > q_A) = 1/q_A$, is a guide to the eye.

Figure 2 Complementary cumulative distribution function, $P(Q_A > q_A)$ for $q_A$ in $G_C$ (circles) and $G_J$ (squares). The solid line, $P(Q_A > q_A) = 1/q_A$, is a guide to the eye.

Figure 3 Complementary cumulative distribution function, $P(Q_E > q_E)$, for $q_E$ in $G_D$. The solid line is a best fit to erfc $\left(\frac{\ln(q_E) - 2(1-2\mu/\sigma^2)s^2 T_s}{\sqrt{\sigma^2 T_s}}\right)$, which stems from eq. 2.

Figure 4 Complementary cumulative distribution function, $P(Q_E > q_E)$, for $q_E$ in $G_C$ (circles) and $G_J$ (squares). The solid lines are best fits to erfc $\left(\frac{\ln(q_E) - 2(1-2\mu/\sigma^2)s^2 T_s}{\sqrt{\sigma^2 T_s}}\right)$, which stems from eq. 2.

Figure 5 Average rank versus $q_A$. The circles represent $G_C$, the triangles, $G_J$ and the squares $G_D$. 
Figure 1: $P(Q_A > q_A)$
Figure 2: 

\[ P(Q_A > q_A) \]
Figure 3: Distribution of efferent links $P(J > j)$.
Figure 4: 

\[ P(J > j) \]

- Efferent Links
Figure 5: