Refinement and testing of the mathematical model of heat and moisture transfer in envelope structures of profiled insulated timber with connectors

N A Tsvetkov¹, S Boldyryev², A V Tolstykh¹, D N Tsvetkov¹ and Ju N Doroshenko¹
¹Tomsk State University of Architecture and Building
²Tomsk Polytechnic University
E-mail: tolstbu@yandex.ru

Abstract. The account of dependence of moisture conductivity coefficient, thermal conductivity coefficient and heat capacity on temperature and moisture is the main peculiarity of the new physico-mathematical model of combined heat and moisture transfer in walls of low-rise wooden insulated timber buildings. This study is aimed at refining and testing such model, including preliminary evaluation of influence of moisture transfer processes on moisture accumulation in insulated timber structures with connectors at low outside temperatures. The study provides approximation dependences that illustrate variation in thermal characteristics of wooden structures made of timber, plywood and insulation. The results of numerical simulation of heat and mass transfer processes occurring in a representative wall fragment made of high-strength profiled timber are given. Temperature and moisture fields are calculated in a selected inhomogeneous fragment. Maximum humidification in insulated profiled timber walls is located on the external surface of the wall. It is found that the use of additional internal longitudinal lamellae in timber with vertical connectors leads to significantly uneven moisture distribution. Based on the calculation results one may conclude that the wall fragment under study has no areas where free moisture can be possibly accumulated.

1. Introduction
Wooden structures are widely used in buildings construction due to their properties of clean and renewable raw material for construction materials and products. Profiled insulated timber with an additional insulation layer made of modern polymer materials [1] is a more advanced structure for a middle-latitude climate zone [2] compared to glued laminated timber. The use of insulated timber reduces the amount of material and financial resources required for construction and operation of wooden buildings.
To date, some research on robustness of timber walls has been published, for instance [3], which notes that the account of moisture transfer and possible moisture accumulation during operation is necessary when forecasting long-term operation of wooden structures or diagnosing current problems. The authors of studies [4, 5] believe that understanding of reasons of humidification occurring in an envelope structure...
is highly important, since this phenomenon belongs to the main factors that cause variation in strength properties of wooden elements of envelope structures. The research [6, 7] study partly insulated wooden parts of a building that are critically assessed with regard to possible moisture condensation.

It should be noted that the most adequate models of heat and moisture transfer in drying process include empirical dependences based on experimental data on moisture sorption isotherms [8]. On the basis of the data [8] and absolute potential of moisture transfer [9] the studies [10, 11] offer a one-dimensional model of heat and moisture transfer in external envelope wooden structure. Its main feature is the account of dependence of moisture transfer coefficient of timber on temperature, which causes specific manner of moisture transfer in external envelope structures. As a result of further advances in the models [10, 11] that proved to be reliable when calculating hydrothermal characteristics of external structures a three-dimensional model of heat and moisture transfer was built [12] for external walls of buildings made of high-strength insulated timber [13] with plywood connectors.

2. Research Methods

Modeling of heat and mass transfer was performed for a design fragment of a timber wall (Fig. 1) with the size of 0.225 × 0.3 × 0.21 m. The thickness of lamellae used in high-strength insulated timber was 0.012 m on internal and external surfaces of the wall. The plywood patch adjacent to the external lamellae had the same thickness. Additional longitudinal lamella was 0.025 m thick. Lamellae were made of pine with thermal conductivity coefficient dependent on both moisture content and temperature. Total thickness of insulation layer with thermal conductivity of 0.033 W/(m·K) varied from 0.11 m to 0.187 m. Values characteristic of the coldest month (January) in Tomsk were used as temperature and outdoor air humidity parameters. These parameters for the plane I (Fig. 1) were taken equal to –39 °C and 0.8; the temperature and indoor air humidity (plane II, Fig. 1) were assumed equal to + 20 °C and 0.5. The value of surface heat transfer coefficient for plane I was 23 W/(m²·K), and for plane II – 8.7 W/(m²·K). Steam coefficient [14] for plane I was assumed equal to 2.09·10⁻⁸ kg/(m²·s·Pa); whereas steam coefficient of indoor air for plane II was equal to 1.04·10⁻⁸ kg/(m²·s·Pa).

Modeling of heat transfer in the design fragment under study was performed with three-dimensional non-stationary equations of heat and moisture transfer in Cartesian coordinates with the use of a model [12]. The closing relations for the proposed model of heat and moisture transfer that are only partially presented in [12] shall be complemented by approximating dependences that enable calculation of variable thermal physical parameters of timber elements made of wood, plywood and insulation.

Data from the study [15] were used for calculating variation in thermal conductivity of plywood elements $\lambda_p$ in the process of moisture accumulation (Fig. 2), based on which the following approximation equation was obtained:

for positive temperatures

$$\lambda_p = 0.132169 w_p + 0.388456;$$

(1)

for negative temperatures

$$\lambda_p = 0.131992 w_p + 1.104732,$$

(2)

where $w_p$ – moisture content of plywood in unit fractions (kg/kg).
Figure 1. Design scheme of a wall fragment of high-strength insulated glued laminated timber with vertical H-shaped connectors and central lamella: 1, 2 – profiled longitudinal lamellae, 3, 4 – layers of plywood; 5 – layers of insulation, 6 – internal longitudinal lamella, 7 – H-shaped connector; I – external surface of the wall, II – internal surface of the wall.

Thermal conductivity coefficient of insulation (expanded polystyrene) $\lambda_{ins}$ was calculated from the dependence below

$$\lambda_{ins} = \lambda_{ins, dr}(1 + 6w_{ins}),$$

presented in [16] ($w_{ins}$ – moisture content of insulation in unit fractions (kg/kg), $\lambda_{ins, dr}$ – thermal conductivity coefficient of dry expanded polystyrene).

For calculation of partial pressure values for water steam in the design fragment of a wall with moisture content one shall use the equations of sorption isotherms [8]. Along with the increase in the relative humidity $\varphi$ from zero, first, the increase in adsorbed moisture occurs and then, when $\varphi > 0.7$ filling of micro-capillaries occurs. When $\varphi = 0.99$ the amount of moisture bound in the wood reaches the maximum value of $w_s$, which is called fibre saturation point. This part of sorption isotherm is not related to the properties of various wood species and for $\varphi \geq 0.45$ it can be approximated by the dependence from [18]

$$w = 0.512 [0.217 - ((273 + r)/1000)^2] / (1.22 - \varphi).$$

With further increase of $\varphi$ from 0.99 to 1.0 and the maximum filling of micro-capillaries moisture content reaches the maximum value of $w_{max}$. The value $w_{max}$ unlike $w_s$ is not dependent on the temperature; however it is related to the properties of the definite wood species. This part of the sorption diagram is usually presented as a linear dependence:

$$w = w_s + 100(w_{max} - w_s)(\varphi - 0.99).$$
At negative temperatures part of the bound moisture freezes; and therefore (4) is complemented by the dependence equation of the amount of bound water not turned into ice $w_{mb}$ on the temperature. According to the data [19] the value $w_{mb}$ (fibre saturation point at $t < 0 \, ^\circ\text{C}$) can be determined using the formula:

$$w_{mb} = (w_i - 0.195) + 0.195\exp(0.055t).$$

(6)

Moisture content of waterproof plywood made of veneer sheet treated with phenol formaldehyde resin and insulation can be determined with the use of sorption isotherms (Fig. 3, 4) presented in [15, 20]. The following ratios were used for approximation of isotherms of waterproof plywood:

made of veneer sheet treated with phenol-formaldehyde resin

$$w_p = 0.008115 + 0.250231 \phi_p + 0.308411 \phi_p^2 - 1.248699 \phi_p^3 + 0.965118 \phi_p^4;$$

(7)

made of veneer sheet treated with alcohol-soluble phenol-formaldehyde resin

$$w_p = 0.004546 - 0.047302 \phi_p + 0.977389 \phi_p^2 - 2.498734 \phi_p^3 + 1.738939 \phi_p^4.$$

(8)

**Figure 3.** Equilibrium moisture content curves for plywood at 20 °C: 1 – plywood made of veneer sheet treated with phenol-formaldehyde resin; 2 – plywood made of veneer sheet treated with alcohol-soluble phenol-formaldehyde resin.

**Figure 4.** Dependence of equilibrium sorption moisture of expanded polystyrene on relative humidity.

The value of moisture content $w_{ins}$ in expanded polystyrene layers used in the timber structures under study as insulation was found from the following approximating dependence with the account of relative humidity $\phi_{ins}$ (Fig. 4)

$$w_{ins} = 8.716160 - 2.120306 \cdot 10^3 \phi_{ins} + 1.904561 \cdot 10^5 \phi_{ins}^2 - 7.083122 \cdot 10^6 \phi_{ins}^3 + 9.4437258 \cdot 10^7 \phi_{ins}^4.$$

(9)
Formula (9) was obtained as a result of data processing on sorption isotherms for expanded polystyrene presented in (20) for the relative humidity variation interval $0.4 \leq \phi_{ins} < 1$.

3. Results and Discussion
Calculations were carried out under the following basic parameters defining the processes of heat and moisture transfer in timber structural elements: thermal conductivity coefficient, vapor conductivity coefficient of insulation and plywood were accepted as 0.033 W/(m·K), 0.012 mg/(m·h·Pa) and 0.02 mg/(m·h·Pa), respectively; heat capacity of insulation and plywood – 1,300 J/(kg·K) and 2,300 J/(kg·K), respectively; the initial density of lamellae wood – 500 kg/m³; density of insulation and plywood – 30 kg/m³ and 650 kg/m³, respectively. Initial moisture content values of materials were accepted equilibrium at the indoor air temperature of +20 °С and relative humidity of 0.5.

As a result of computer simulation of heat and mass transfer processes were obtained fields of temperature, moisture content, relative humidity presented in Fig. 5 – 7 for typical sections of insulated profiled timber at the end of calculation period (more than 100 hours).

Figure 5. Temperature field in the design fragment: a – temperature distribution in the form of isolines in the vertical symmetry plane of connectors; b – temperature distribution in the form of isolines on the internal surface of the wall.
Figure 6. Moisture content in the design fragment: a – moisture content distribution in the form of isolines in the vertical symmetry plane of connectors; b – moisture content distribution in the form of isolines on the internal surface of the wall.

Figure 7. Relative humidity in the design fragment: a – relative humidity distribution in the form of isolines in the vertical symmetry plane of connectors; b – relative humidity distribution in the form of isolines on the internal surface of the wall.

As a result of data processing it was found that as soon as stable conditions were set the value of maximum difference between temperatures on internal and external surfaces was 56.83 °C (Fig. 5), which almost equals the value of 56.7 °C calculated for a similar structure without accounting moisture transfer in [21]. Averaged maximum difference of internal and external surface temperatures
of timber related to the impact of connectors (thermal bridges) on heat transfer is the same for modeling with (Fig. 5) and without the account of moisture transfer [21] and equals 0.1 °C.

As can be seen from Fig. 6, the maximum humidification ($w \approx 0.12$ kg/kg) in timber walls the structure of which provides use of vertical connectors with internal longitudinal lamella is located on the external surface that contacts with the outside air. Besides, moisture content distribution (Fig. 6) demonstrates that in spots where connectors are fixed to lamellae areas of excessive humidification appear where the values of moisture content are as high as 0.11 kg/kg.

Connectors cause uneven moisture distribution in lamellae material. In the timber structure under study significant unevenness in moisture distribution is found on the external surface (Fig. 6b) where the maximum moisture content difference is 0.0037 kg/kg.

The data on relative humidity presented in Fig. 7 demonstrate that the design wall has no so called “wet” zones in which relative humidity equals 1.

4. Conclusion
Based on the presented modeling results for heat and moisture transfer processes in envelope structures made of insulated timber with connectors the following conclusions can be made:

- the physico-mathematical model of combined heat and moisture transfer in the insulated timber walls with connectors is refined and numerically tested; its principal peculiarity is the account of dependence of coefficients of heat and moisture transfer and heat capacity of lamella wood on temperature (also negative) and humidity. The dependence of coefficients of thermal conductivity of plywood and insulation on moisture content is also accounted;
- non-stationary fields of temperature, moisture content and relative humidity are calculated for a representative fragment of a timber wall;
- averaged maximum differences between temperatures (that occur because of connectors) on internal and external surfaces of a timber structure, as well as on internal surface of timber, calculated with the account of moisture transfer appear to be almost identical to the similar values obtained when modeling heat transfer without the account of humidity variation;
- maximum humidification in insulated profiled timber walls is located on the external surface of the wall;
- in timber structures with vertical connectors (in case of internal longitudinal lamella) zones of increased humidity may appear in proximity to the areas where connectors are fixed to lamellae;
- the use of additional internal lamella in timber with vertical connectors causes significantly uneven moisture distribution;
- in all variants of performed calculations, no areas of free moisture with relative humidity equal to 1 are found in the selected timber wall fragment.

References
[1] Gao S, Liu J and Gao G 2018 Experimental study on structure and property of chemical building materials based on sem analysis technology Chemical Engineering Transactions 66 pp 1135–1140 doi: 10.3303/CET1866190
[2] Titunin A A and Zaitseva K V 2007 Solution of the problem of optimization of parameters of resource-saving enclosing structures of wooden buildings Actual problems of the forest complex 20 pp 137–140
[3] Soudek P 2016 Moisture Monitoring of Built-In Wooden Elements Applied Mechanics and Materials 861 pp 303–310 doi: 10.4028/www.scientific.net/AMM.861.303
[4] Hans G 2011 Coordination of Performance Standards on Moisture Control in Buildings Moisture Migration in Buildings, ed. M. Lieff and H. Trechsel (West Conshohocken, PA: ASTM
International) pp 141–147 doi: DOI: 10.1520/STP38691S

[5] Saft S and Kaliske M 2011 Numerical simulation of the ductile failure of mechanically and moisture loaded wooden structures Computers & Structures 89 pp 2460–2470 doi: 10.1016/j.compstruc.2011.06.004

[6] Wegerer P and Bednar T 2017 Hygrothermal performance of wooden beam heads in inside insulated walls considering air flows Energy Procedia 132 pp 652–657 doi: 10.1016/j.egypro.2017.09.710

[7] Wang L Ge H 2018 Stochastic modelling of hygrothermal performance of highly insulated wood framed walls Building and Environment 146 pp 12–28

[8] Zhukov A V 2008 Engineering formulas for calculating heat and humidity properties of wood materials News of higher educational institutions. Construction 5 pp 81–84

[9] Perekhozhentsev A G 1993 Questions of the theory and calculation of the moisture state of heterogeneous sections of building envelopes p 273

[10] Kuzin A Ya, Tsveetkov N A and Draganov V A 2003 Non-Stationary heat and moisture transfer in multi-layer external fencing Thermophysics and Aeromechanics 10(4) pp 599–609

[11] Kuzin A Ya, Miroshnichenko T A and Tsveetkov D N 2007 Non-Stationary heat and moisture transfer in multi-layer external fencing Vestnik of Tomsk State University of Architecture and Building 2 pp 186–194

[12] Tsveetkov N A, Khutornoy A N, Tolstykh A V and Kolesnikova A V 2017 Physical and mathematical model of heat and moisture transfer in enclosing structures made of profiled thermal timber News of higher educational institutions. Construction 2 pp 12–29

[13] Tsveetkov D N 2012 Thermal engineering justification of external building fences made of glued wooden energy-efficient grades Vestnik of Tomsk State University of Architecture and Building 2 pp 81–90

[14] Timoshenko A T 1996 Building envelope with wet operation in extreme conditions of the Far North Federal Research Centre p 200

[15] Sterlin D M 1977 Drying in the production of plywood and chipboards p 384

[16] Pavlov V A 1973 Styrofoam p 240

[17] Shubin G S 1990 Thermal processing of wood p 336

[18] Kretchetov I V 1980 Wood drying p 432

[19] Sergovsky P S, Rassev A I 1987 Hydrothermal processing and preservation of wood p 360

[20] Fokin K F 2006 Construction heat engineering of enclosing parts of p 256

[21] Tsveetkov N A, Khutornoy A N, Tolstykh A V and Doroshenko Ju N 2018 Comparative analysis of heat-insulating properties of walls made of profiled beams Vestnik of Tomsk State University of Architecture and Building 20(2) pp 124–136