Analysis of student’s geometry reasoning ability at senior high school

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Abstract Each student possesses different levels of reasoning. This difference depends on students' reasoning ability such as observing patterns, formulating conjecture, testing conjecture, constructing arguments, and describing logical conclusions. This study aims to analyze students' reasoning ability. The students' reasoning analysis ability play significant role assisting the teachers to improve students' reasoning, especially in geometric reasoning ability. This study is a qualitative descriptive research on students of high school level. The subjects used in this study were 30 students of grade XI. The results showed that students' geometrical reasoning abilities were still relatively low. Students remains having difficulties in understanding of how to formulate guesses, establish and assess mathematical arguments, and draw logical conclusions about some ideas and their relevance. The average student has difficulty in determining the size of the ribs based on properties of the geometry. Students also find it difficult to provide explanations / arguments regarding the related relationships between the constructs.

1. Introduction
The Regulation issued by the Indonesian Minister of National Education No. 22 of 2006 concerning the Content Standards for Primary and Secondary Education Units reveals that one of the objectives of mathematics is to enable students encouraging themselves to achieve the reasoning ability in terms of the patterns and qualities, to do mathematical manipulation in generalizing, compiling evidence, or explaining the ideas and math statements [1]. Therefore, reasoning is one of the basic mathematical competencies that should be mastered by students. Reasoning is a special kind of thinking in which inference takes place, in which conclusions are drawn from premise [2]. There are two kinds of known reasoning, namely induction (drawing conclusions based on observations) and deduction (drawing conclusions based on the rules set) [3]. Bjuland emphasizes that reasoning was defined as the five interrelated processes of mathematical thinking, categorised as sense-making, conjecturing, convincing, reflecting, and generalising [4]. In line with this, Rohana argues that reasoning is main characteristic of mathematics which cannot be separated from activity of learning and developing mathematics or solve the mathematical problem [5]. Reasoning ability is very important to understand mathematics. This reasoning
The ability is useful for someone in process of building and comparing ideas from various situations faced, so he/she can take appropriate decision in solving the problem of life. This is reinforced by Lithner's opinion that reasoning is the line of thought adopted to produce assertions and reach conclusions in task solving, reasoning can be seen as thinking processes, as the product of these processes, or both [6]. The reasoning ability can help students improving their mathematical abilities, one of the practical abilities is to memorize. Therefore, students will be more flexible in understanding a mathematical concept if they can effectively use their reasoning skills [7].

In fact, students' reasoning ability in the field remains quite low. Of course, it is not blind argument, it was as revealed by Rosnawati in Sumartini argues that the lowest average percentage achieved by Indonesian students was in the cognitive domain at the level of reasoning, namely 17% [8]. Based on data obtained from the Trends International Mathematics and Science Study (TIMSS), it is stated that Indonesian students' mathematical reasoning ability is still below the average of 20% compared with an international average score of 44% [9]. Based on such information, special attention is needed for students' reasoning ability. One of the causes leading students having low mathematical reasoning ability is that the learning process carried out by teacher(s) in the classroom does not involve students active-role in a learning process [10]. Therefore, mathematics learning that take place at classroom also needs to be considered in order that students' learning atmosphere can encourage their reasoning ability. This is reinforced by English’s opinion that teachers should understand the reasoning and know how to use it effectively, teacher's knowledge in this case is quite important, since the reasoning in mathematics is different from that of other sciences [11]. NCTM in Rohana identified several indicators for reasoning ability which include: 1) observe pattern or regularity, 2) formulate generalization and conjecture related with regularity observed, 3) assess/test the conjecture, 4) construct and assess mathematical argument, and 5) describe/validate logical conclusion about some ideas and its relatedness [5].

The mathematics in high school level, geometry is one of underlying fields of discipline being taught. In solving the geometry problems it takes sort of thinking pattern in applying both the concepts and skills in solving problems [12]. That is why, geometry not only develops students' cognitive abilities but also helps them establishing memory, ie about concrete objects into abstract [13]. Studying geometry contributes to students's skills of visual development, critical thinking, intuition, perspective, problem solving, making conjectures, deductive reasoning, logical arguments and proofing [14]. However, in reality, students remains confronting difficulties in learning and solving the geometry problems. The students' potential ability regarding with the available objects in their surroundings have not been maximally observed [15]. Having identified difficulties that students faced in learning geometry subject certainly has an impact on their mastery on geometrical abilities, many of them are less perfect so that it can inhibit further geometry learning [16].

The ability to use reasoning in solving geometry problems is directly related to the level of students' mathematical abilities. In general, students who have a high ability in mathematics, they have a better ability to solve geometric problems. In part of solving geometry problems, it is required the ability to reason both to understand mathematical concepts and strategies used to solve geometry problems. This means that the process of students’ reasoning in solving geometry problems depends on their level of ability in mathematics [17].

This study aims to analyze students' geometrical reasoning ability on the basis of the NCTM reasoning indicators. The aspect of geometrical reasoning which are going to be assessed in this study concerns Van Hiele’s opinion in Petrus [15], specifically, identifying construct based on its apearance form in the entirety, naming the construct based on the properties embedded/associates with it, determining geometric properties based on a direct observation, describing a construct on the basis of its characteristics, conducting problem solving according to the recognizable caracteristics associate with the construct,
providing explanations concerning both the related relationships between constructs, in addition to the inter-relationships between them.

2. Method

This study was conducted in a certain high school in Surakarta using the data collection method of purposive sampling. This study is designed with descriptive-qualitative research. This method was applied to analyze students' geometric reasoning abilities. The subjects of this study were 30 students with heterogeneous ability. The instrument being used was a test of geometric reasoning ability. The test is an essay test range from 45 minutes of working time. The material scope a solid. The procedure in this study among others firstly, namely researchers conducting learning observations and interviews with teachers. Secondly, researchers collected data using test instruments and interviews with students. Thirdly, the researcher conducted an analysis by reducing and presenting data. Fourthly, researchers draw conclusions.

In this study, the reasoning indicator is based on the opinion of NCTM [5]. The aspects used are based on Van Hiele's opinion [15]. These indicators and aspects can be seen in Table 1.

| No | Reasoning Indicators | Aspects of Geometric Reasoning |
|----|----------------------|--------------------------------|
| 1  | Observe pattern or regularity. | 1. Identifying construct based on the form in its entirety. |
|    |                      | 2. Naming construct based on its characteristics. |
| 2  | Formulate generalization and conjecture related with regularity observed. | 1. Determining the geometric characteristics based on the direct observation |
|    |                      | 2. Describing construct based on its characteristics. |
| 3  | Assess/test the conjecture. | Performing problem-solving which engages recognizing characteristics of the construct |
| 4  | Construct and assess mathematical argument. | Providing explanation about the interrelated relationships between constructs. |
| 5  | Describe/validate logical conclusion about some ideas and its relatedness. | Providing explanation about the relationship between constructs. |

The category of geometric reasoning ability applied the purposed categories developed by Maya in Suprihatin [19] as in Table 3.

| Score | Criteria |
|-------|----------|
| 4     | Perfect answer, response (accomplishment) is given completely and correctly. |
| 3     | The answer is correct, however, the given response (accomplishment) leaves a single significant error. |
| 2     | The answer partially correct, however, the given response (accomplishment) leaves one or more significant error/deficiency. |
| 1     | Wrong answers, responses (accomplishment) are not entirely resolved, however, it contains at least one correct argument. |
| 0     | The wrong answer, the response (accomplishment) is based on a wrong process or argument, or no response at all. |

The category of geometric reasoning ability applied the purposed categories developed by Maya in Suprihatin [19] as in Table 3.
3. Result and Discussion
Based on the essay tests conducted on 30 students, the results of the study were realized in the form of scores and percentages on each indicator. The results of this study can be seen in Table 4.

| No | Reasoning Indicators                                                                 | Score | Percentage |
|----|-------------------------------------------------------------------------------------|-------|------------|
| 1  | Observe pattern or regularity.                                                      | 86    | 71.67%     |
| 2  | Formulate generalization and conjecture related with regularity observed.           | 60    | 50.00%     |
| 3  | Assess/test the conjecture.                                                         | 73    | 60.83%     |
| 4  | Construct and assess mathematical argument.                                         | 54    | 45.00%     |
| 5  | Describe/validate logical conclusion about some ideas and its relatedness.          | 43    | 35.83%     |

Based on Table 4 it can be seen that the highest percentage was found in indicators observing patterns or regularities as much as 71.67%. While the lowest percentage is in the indicator describing/validating the logical conclusions about some ideas and their relationship is 35.83%. For all indicators, the average is 52.67%, this means that the students' geometric reasoning ability remains low. On the first indicator, namely observing patterns or regularities, it is obtained as much as 71.67%, which is categorized as high. Among the 30 students who took the test, more than half of the students achieved a good score, which enable them to reach this indicator in a very successful way. Students have been able to identify what construct was meant in the problem. Students are also able to describe the building space by giving the name and size correctly. On the second indicator, namely formulating generalization and conjectures related to the observed regularities, about 50.00% was obtained and categorized as low. Half of the students who took the test only obtained enough scores, due to this indicator the students' achievement is, therefore, considered less optimal. Only a small number of students have been able to determine the size of the construct space based on recognized characteristics. The following is the students' answer to the second indicator.
Figure 1 shows it seems that students have not understood the problem well. With a less precise interpretation, the accomplishment was not appropriately done. Student writes down what is known without specifying the size of other ribs based on the flatness characteristic of a construct. This is reinforced by the student's statement that he only wrote down what was known about the problem and was confused about writing down the size of the other ribs. In Figure 2 the student has written down what is known and determined the size of other ribs on the upright side of the pyramid even though for the pyramid base students have not determined the size of the other ribs. This is reinforced by the student's statement that he only thought about the size of the other ribs on the upright side of the pyramid, while the other ribs on the pyramid were unthought. In this indicator, students have not been able to determine the geometrical characteristics or properties and describe the construct based on its nature.

Based on Table 4 on the third indicator, namely assessing / testing conjectures, 60.83% was categorized as moderate. Some students got very good scores on this indicator. In this indicator, students are able to do problem solving engaging the Pythagoras's theorem following the appropriate steps. On the fourth indicator, namely constructing and assessing the mathematical arguments, the result of 45.00% is obtained which is categorized as low. Most students with low categories have not been able to reach this indicator properly. Students have not been able to solve the problem by giving an explanation of the inter-relationships between constructs. The following is the student's answer to the fourth indicator.

Figure 3 shows it appears that students have not fully understood the problem, consequently, they have not been able to provide any explanation of the relationship between constructs. (See in Figure 3) students do not understand what is meant by the problem, this has led them to wrongly answer. This is reinforced
by a student's statement, that she did not even understand how to determine the glass area, it leads her seeking around the entire area of the pyramid. In figure 4, students with high category correctly answer the glass area, however, these students have not verbally connected to how the relationship between the area of a triangle with the lid of a pyramid glass. This is reinforced by the student's statement that she understood what was meant by the problem, unfortunately did not comprehend how to write the relationship in detail.

Based on Table 4 on the fifth indicator, namely describe/validate the logical conclusions concerning some of the ideas and their relationships, as much as 35.83% and is categorized as low. Most students with low categories also have not been able to properly achieve this indicator. They were not able to provide both conclusion and explanation regarding the relationships between constructs. Students were only able to draw conclusions without necessarily associate the relevant situation. Next is the students' answer to the fifth indicator.

Figure 5 shows that it seems they have not understood the problem very well, in addition to be unable to derive conclusion by providing an explanation of the relationship between constructs. In Figure 5, students' answers about the area of glass are wrong and students also do not conclude by connecting the inter-building situation. This is reinforced by the student's statement that he was confused as to how to find the area of the glass, therefore, he looked for the area of the pyramid, and when it came to conclude, he barely concluded what he got at the end of the answer. In Figure 6 the high category, students correctly answer the glass area, however, they did not conclude by associating situation between the constructs. This is reinforced by the student's statement that he could only conclude what is being asked without relating the situation between constructs.

From the several reviews on student answer sheets above, the conclusion is derived. Most students find it difficult to establish conclusions and guesses, build and assess mathematical arguments, and draw logical conclusions about some ideas and their interrelationships. This is in line with research that emphasizes students' abilities in reasoning are quite lacking, students tend to answer questions without providing a guess beforehand [20]. It is also sustained by research that states students' reasoning abilities at the level of qualitative and quantitative thinking are remaining less than optimal in reaching the indicators submitting purposing a guess and drawing conclusions [21]. In addition, there are still many
found students who are confused in making conclusions and examining the approximate answers / arguments leading them in making some errors in completion [18]. In general, students experience obstacles in determining the ribs to establish space. Students only write known ribs without writing down other ribs based on the construct nature. Students also find it difficult to integrate question with the existing statements, in addition to concluding problems by providing sufficient explanations / arguments that have an impact on their resolution.

4. Conclusion

The percentage of geometrical reasoning ability indicators in terms of observing patterns or regularities by 71.67%, formulating conclusions and conjectures related to the observed regularity by 50.00%, assessing / testing conjectures of 60.83%, establishing and assessing the mathematical arguments by 45.00%, and describing / validating the logical conclusions on ideas and their linkages by 35.83%. Overall, students' geometrical reasoning abilities were still relatively low at 52.67%. This was because most students have not been able to determine the ribs size from shape of the space based on the known geometric properties. Students also have not been able to conclude problem by providing sufficient explanation of the interrelationships between constructs. Thus, in formulating conclusion and conjectures, building and assessing the mathematical arguments, and drawing logical conclusions, the indicators have not yet been that optimal. Therefore, students should be often trained with geometry questions to gain better in geometrical understanding.

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