Design and terrainability analysis of a novel mobile robot with variable-diameter wheels

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Abstract. Mobile robots are usually demanded to enter into unstructured terrain. Thus, terrainability of a robot is specially important in cluttered environments. A novel mobile robot with variable-diameter wheels is designed here. This robot can adjust its attitude by variable wheel diameters to improve its terrainability. For terrainability, stability and obstacle-surmounting capability of the robot are studied. Furthermore, variable wheel diameters’ effects on these performances were researched. For lateral and longitudinal stabilities, parametric relationships between the maximum slope that the robot can climb and stability requirements were obtained. To avoid stability failures, appropriate failure criterions were proposed. Stability of the robot can be enhanced by changing wheel diameters. The obstacle-surmounting performances of front and rear wheels were analyzed by building two quasi-static models. The results showed that increased wheel diameters enhanced the obstacle-surmounting height. The analytical calculations and the simulations were compared. The simulations verified the reliability of the theoretical calculations. Based on tip-over failure criterions, appropriate strategies can be used to enhance the stability. These analysis presented also can provide theoretical basis for performance optimization design of the robot.

1. Introduction

Mobile robots have been applied to various applications such as planetary explorations, agriculture, mining industries, and reconnaissance in dangerous situations [1-3]. Such tasks may result in terrainability failures, which mainly includes loss of obstacle-surmounting capability, loss of stability and even tipover. We define terrainability as the ability to negotiate uneven terrain without losing stability and to navigate through obstacles in complicated environments. Loss of terrainability may bring about robot damage and total mission failure [4]. Therefore, terrainability in rugged terrain is a critical criterion for mobile robot design. Excellent terrainability requires the design of innovative locomotion systems. Because reconfigurable mobile robots can adjust their attitudes to improve terrainability [5-7], there has been a growing interest among researchers in these robots.

By controlling the articulated suspension of a rover, its stability can be adjusted [4]. A wheeled-legged hybrid mobile robot was developed to optimize stability [8]. An actively articulated rover with rocker-bogie suspension can increase rollover stability and adhesion by a posture control method [9]. By controlling the attitude of a planetary rover with active suspensions, lateral slips of the wheels are prevented [10]. A reconfigurable rover is proposed to traverse slopes with less slippage by properly changing its configuration [11]. Above researches indicate that reconfigurable mobile robots with active suspension system be capable of enhancing their terrainability by adjusting their configurations.
However, trafficability of these robots may be insufficient. By increasing wheel diameter, a mobile robot can enhance both the tractive performance on soft terrain and terrainability [12]. Hence, an innovative mobile robot with four variable-diameter wheels is introduced here. The robot can actively adjust its posture by four variable-diameter wheels. Configuration of the robot and concept of variable-diameter wheels are presented. Then, lateral stability, longitudinal stability, and obstacle-surmounting capability of this robot are analysed. Analytical stability failure criterions are introduced here. Corresponding simulations are used to verify these criterions. Furthermore, the static equilibrium equations describing obstacle-surmounting capability of the front and rear wheels are established, respectively. The quasi-static obstacle-surmounting simulation of the robot was performed.

2. Design of mobile robot with variable-diameter wheels

2.1. Configuration of mobile robot

As shown in figure 1, a new mobile robot is designed. This robot consists of four variable-diameter wheels, a rocker-bogie suspension and a body frame. Moreover, the robot is equipped with four-wheel drive/four-wheel steering (4WD/4WS) actuators. The total mass of the robot is 200 kg.

![Figure 1. Configuration of mobile robot prototype.](image)

2.2. Concept of variable-diameter wheels

For a variable-diameter wheel, a compliant variable-diameter mechanism allows the wheel to transform its structure using expansion-retraction motion and to possess excellent terrainability. This compliant mechanism consists of helical torsion springs [13]. According to reference [13], when the two wheel hubs rotate at different speeds, which causes the angular deflections of the springs, the changes in the wheel diameter occur, as shown in figure 2. The change range of each wheel diameter is 240-400mm.

![Figure 2. Two configurations of mobile robot with variable-diameter wheels.](image)
3. Trafficability analysis

3.1. Stability

3.1.1. Lateral stability. As shown in figure 3, a mobile robot laterally traverses a slope of angle \( \alpha \). Assume that the traversing speed is low, and its motion remains in a steady state. The forces acting on the front and rear wheels are assumed to be equivalent. When the angle \( \alpha \) beyond the limit value, the tip-over instability of the rover occurs. Generally, the contact point between the downhill wheels and the slope is regarded as the rotation point of tip-over. Hence, a mechanical model of the robot on a side slope is proposed here.

When there are no forces from ground acting on the uphill wheels, the moment equilibrium equation of the robot with normal wheels is

\[
G \cos \alpha (B/2) = Gh \sin \alpha
\]

where \( \alpha \) is the slope angle; \( B \) (1090mm) is the distance between the downhill and uphill wheels along the slope surface; \( h \) (360mm) denotes the distance between the center of gravity (COG) of the robot and the slope surface; \( r_i \) is the \( i \)th wheel radius; \( G \) is the gravity of the robot.

Thus, the condition that a lateral tip-over does not occur is

\[
\alpha \leq \tan^{-1} \left( \frac{B}{2h} \right)
\]

The mobile robot with variable-diameter wheels can change the downhill wheel diameter to rotate its body, as shown in figure 3 (b). The moment equilibrium equation is established by

\[
G \cos \alpha' \left[ \frac{(B/2) \cos \theta + h' \sin \theta}{h' \cos \theta - (B/2) \sin \theta} \right] = G \sin \alpha' \left[ h' \cos \theta - (B/2) \sin \theta \right]
\]

where \( \alpha' \) is the slope angle; \( \phi \) is the imaginary slope angle; \( h' \) denotes the distance between the COG of the robot and the bottom of the downhill wheels; \( r_{2i} \) is the adjusted wheel radius.

With the change in downhill wheel diameter, the maximum slope angle becomes

\[
\alpha' = \tan^{-1} \left[ \frac{(B/2) + h' \tan \theta}{h' - (B/2) \tan \theta} \right]
\]

Note that

\[
\theta = \tan^{-1} \left[ \frac{(r_{2i} - r_i)}{B} \right] = \tan^{-1} \left[ \frac{(h' - h)}{B} \right]
\]

Thus,
\[
\alpha' = \tan^{-1}\left[ \frac{B^2 + 2h'(h' - h)}{B(h' + h)} \right]
\]  

(6)

When \( r_2' = nr_1 \), there is

\[
h' - h = (n - 1)r_i
\]

(7)

Equation (6) becomes

\[
\alpha' = \tan^{-1}\left\{ \frac{B^2 + 2(n - 1)^2r_i^2 + 2h(n - 1)r_i}{B\left[ (n - 1)r_i + 2h \right]} \right\}
\]

(8)

According to figure 3 (b), there is

\[
\alpha' - \theta = \tan^{-1}\left( \frac{B}{2h} \right) - \beta
\]

(9)

That is,

\[
\alpha' - \tan^{-1}\left( \frac{B}{2h} \right) = \theta - \beta
\]

(10)

If \( \theta > \beta \), it is indicated that when the downhill wheel diameter increases, the maximum slope angle that the robot can traverse will enlarge.

\[
\beta = \tan^{-1}\left( \frac{B}{2h'} \right) - \tan^{-1}\left( \frac{B}{2h} \right) = \tan^{-1}\left( \frac{(B/2h') - (B/2h)}{1 + B^2/(4h'h)} \right)
\]

(11)

Obviously,

\[
\left( h' - h \right) / B - \frac{(B/2h') - (B/2h)}{1 + B^2/(4h'h)} = \frac{(h' - h)[4h'h + 3B^2]}{B(4h'h + B^2)} > 0
\]

(12)

Hence,

\[
\alpha' > \tan^{-1}\left( \frac{B}{2h} \right)
\]

(13)

(a) Lateral stability  
(b) Longitudinal stability

Figure 4. Simulation analysis of stability.
Therefore, the increase in downhill wheel diameter can improve the lateral stability of the robot. Moreover, the simulation result also illustrates the improvement in figure 4 (a). Based on the simulation, the robot can traverse a slope at a greater limit angle, compared with a robot with normal wheels.

3.1.2. Longitudinal stability. As presented in figure 5, a mobile robot goes up or down a slope of angle $\beta$. When the angle $\beta$ beyond a limit value, the tip-over instability of the robot occurs. The forces acting on the left and right wheels are assumed to be equivalent. The contact point between the downhill wheels and the slope is regarded as the rotation point of tip-over. Hence, a mechanical model of a mobile robot on a slope is proposed in Figure 5.

![Figure 5. Mechanical models of longitudinal stability.](image)

The moment equilibrium equation of the robot with normal wheels is established by

$$N_i L = G b \cos \beta - G h \sin \beta$$  \hspace{1cm} (14)

where $\beta$ is the slope angle; $L$ (1055mm) is the distance between the downhill and uphill wheels along the longitudinal slope surface; the COG of the mobile robot is located at distances $b$ (650mm) and $a$ (340mm) from the downhill and uphill wheels along the slope surface; $h$ (360mm) denotes the distance between the COG of the robot and the slope surface; $N_i$ is the supporting force acting on the underside of the uphill wheels; $r_i$ is the $i$th wheel radius.

When $N_i=0$, the condition that a longitudinal tip-over does not occur is

$$\beta \leq \tan^{-1} \left( \frac{b}{h} \right)$$  \hspace{1cm} (15)

According to equation (15), when the distance $b$ increases, the maximum slope angle will enlarge. Thus, this change can improve the longitudinal stability of the robot.

With the change in wheel downhill diameter, when $N_i=0$, the moment balance equation of the robot is established by

$$-G \cos \beta' \left( b \cos \phi + h' \sin \phi \right) + G \sin \beta' \left( h' \cos \phi - b \sin \phi \right) = 0$$  \hspace{1cm} (16)

where $\beta'$ is the slope angle; $\phi$ is the imaginary slope angle; $h'$ denotes the distance between the COG of the robot and the bottom of the downhill wheels; $r_2$ is the adjusted wheel radius.

Therefore,

$$\beta' = \tan^{-1} \left( \frac{b + h' \tan \phi}{h' - b \tan \phi} \right)$$  \hspace{1cm} (17)

Note that

$$\theta = \tan^{-1} \left[ \frac{(r_2 - r_i)}{L} \right] = \tan^{-1} \left[ \frac{(h' - h)}{L} \right]$$  \hspace{1cm} (18)

Thus,
\[ \beta' = \tan^{-1} \left[ \frac{bL + h'(h' - h)}{h'L - b(h' - h)} \right] \]  

When \( r_2' = nr_1 \), there is \( h' - h = (n - 1)r_1 \)  

Equation (19) becomes
\[ \beta' = \tan^{-1} \left[ \frac{bL + (n - 1)^2 r_1^2 + h(n - 1)r_1}{hL + (n - 1)r_1 (L - b)} \right] \]  

According to Figure 4 (b), there is \( \beta' - \theta = \tan^{-1} \left( \frac{b}{h} \right) - \gamma \)  

That is, \( \beta' - \tan^{-1} \left( \frac{b}{h} \right) = \theta - \gamma \)  

If \( \theta > \gamma \), it is indicated that when the downhill wheel diameter increases, the maximum slope angle that the robot can climb will enlarge.

\[ \gamma = \tan^{-1} \left( \frac{b}{h'} \right) - \tan^{-1} \left( \frac{b}{h} \right) = \tan^{-1} \left( \frac{(b/h') - (b/h)}{1 + b^2 / (h'h)} \right) \]  

Obviously, \( (h' - h) / L - (b/h') - (b/h) = \frac{(h' - h)(h'h + b^2 + bL)}{L(h'h + b^2)} > 0 \)  

Hence, \( \beta' > \tan^{-1} \left( \frac{b}{h} \right) \)  

The adjustment in downhill wheel diameter can enhance the longitudinal stability of the robot. Moreover, the simulation result also illustrates the improvement in figure 4 (b). Based on the simulation, the robot can climb a slope at a greater limit angle without tip-over, compared with a robot with normal wheels.

### 3.2. Obstacle-surmounting capability

The obstacle-surmounting capability of a mobile robot is elevated by the vertical height of an obstacle or step. Assume that the obstacle-surmounting speed of the robot is low, and its movement remains in a steady state. The forces acting on the left and right wheels are assumed to be equivalent. The quasi-static models that a mobile robot climbs a step are presented here. The first case is that the front wheels of the robot climb a step. Static equilibrium equations in the case are as follows.
where $F_1$ is the radial force (from the step) acting on the front wheel; $F_2$ is the radial force (from the ground) acting on the rear wheels; the static friction coefficient, $\phi$, is 0.7; $R_i$ is the $i$th wheel radius; $G$ is the total weight of the robot.

By eliminating $F_1$, $G$, and $F_2$, a dimensionless equation is obtained by

\[
\left( \frac{\varphi R_i}{L \cos \beta} - 1 \right) \sin \alpha - \left( \frac{R_i}{L \cos \beta} - \frac{1 + \frac{a + h_b \tan \beta}{L \varphi} + \frac{\varphi (a + h_b \tan \beta)}{L}} \right) \cos \alpha = \frac{\varphi R_i}{L \cos \beta}
\]  

(30)

When $R_1 = R_2$, $\beta$ is zero. The two cases correspond to an obstacle-surmounting mobile robot with normal wheels.

Equations (30) and (34) become
\[
\left(\frac{\varphi R_1}{L} - 1\right) \sin \alpha - \left(\frac{R_1}{L} \frac{1}{\varphi} + \frac{a}{L\varphi} + \frac{\varphi a}{L}\right) \cos \alpha = \frac{\varphi R_1}{L} \\
\left(\frac{\varphi R_2}{L} + 1\right) \sin \alpha - \left(\frac{R_2}{L} \frac{1}{\varphi} - \frac{b}{L\varphi} - \frac{\varphi b}{L}\right) \cos \alpha = \frac{\varphi R_2}{L}
\]

(35)

(36)

\[\text{(a) Obstacle-surmounting front wheels} \quad \text{(b) Obstacle-surmounting rear wheels}\]

Figure 7. Relationships between step height and obstacle-surmounting wheel diameter

With the increases in wheel radiuses, obstacle height increases, as represented in figure 7. The obstacle-surmounting front wheels can pass over much higher obstacles than that of the obstacle-surmounting rear wheels. Based on dimension parameters in Section 3.1, quasi-static obstacle-surmounting simulation of the robot was performed. The simulation results showed that when the front wheel radius \( R_1 \) was 120 mm, the step height that the robot can climb was 102 mm; when the front wheel radius \( R_1 \) was 200 mm, the step height was 186 mm; when the rear wheel radius \( R_2 \) was 120 mm, the corresponding step height was 43 mm; when the rear wheel radius \( R_2 \) was 200 mm, the step height was 67 mm. In general, very good agreement is found for the simulation results and the analytical results.

4. Conclusions

Unlike conventional mobile robots, most of the weight of the novel mobile robot is concentrated in the variable-diameter wheels, which reduces the height of COG. This robot with variable-diameter wheels can adjust its body attitude to adapt to rough terrains so that it can cross over obstacles or climb a slope. For the folded limit status, the wheel becomes a circular wheel. This provides good ride performance for the robot on flat terrain. Furthermore, variable-diameter wheels can enhance the trafficability of the robot on soft terrain by increasing the wheel diameters.

Both the analytic models and the simulation results represent that with the increase of the downhill wheel diameters, lateral and longitudinal stabilities of the robot are improved. Failure criterions of stability were introduced. The robot can traverse or climb a slope at a greater limit angle, compared with a robot with normal wheels. Considering variable wheel diameters, the static equilibrium equations describing obstacle-surmounting performances of the robot are proposed here. Diameters of the obstacle-surmounting wheels are linear with the limit obstacle height that the robot can cross over. The obstacle-surmounting front wheels can climb much higher steps than that of the obstacle-surmounting rear wheels. The simulation results show that the predictions made by the analytical model are in good agreement.

Acknowledgments

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References

[1] Bruzzone L and Quaglia G 2012 Locomotion systems for ground mobile robots in unstructured environments Mech. Sci 3 49-62

[2] Tarokh M, Ho D H and Bouloubasis A Jan. 2013 A systematic kinematics analysis and balance control of high mobility rovers over rough terrain Robot. Auton. Syst 61 13-24

[3] Luo Z, Shang J, Wei G and Ren L Oct. 2018 A reconfigurable hybrid wheel-track mobile robot based on Watt II six-bar linkage Mech. Mach. Theory 128 16-32

[4] Iagnemma K, Rzepniewski A, Dubowsky S and Schenker P Jan. 2003 Control of robotic vehicles with actively articulated suspensions in rough terrain Auton. Robot 14 5-16

[5] Aoki T, Murayama Y and Hirose S Feb. 2014 Development of a transformable three-wheeled lunar rover: Tri - Star IV J. Field. Robot 31 206-223

[6] Edwin E L, Denhart D J, Gemmer R T, Ferguson M S and Mazzoleni P May. 2014 A performance analysis and technical feasibility assessment of a transforming roving-rolling explorer rover for mars exploration J. Mech. Design 136 1-11

[7] Zheng J, Gao H, Yuan B, Liu Z, Yu H, Ding L and Deng Z Oct. 2018 Design and terramechanics analysis of a Mars rover utilising active suspension Mech. Mach. Theory 128 125-149

[8] Grand C, Benamar F, Plumet F and Bidaud P Oct. 2004 Stability and traction optimization of a reconfigurable wheel-legged robot Int. J. Robot. Res 23 1041-58

[9] Kubota T and Naiki T 2011 Novel mobility system with active suspension for planetary surface exploration in 2011 Aerospace Conf. (Big Sky Montana American: IEEE) p 1-9

[10] Chen F and Genta G Dec. 2012 Dynamic modeling of wheeled planetary rovers: a model based on the pseudo-coordinates approach Acta. Astronaut. 81 288-305

[11] Inotsume H, Sutoh M, Nagaoka K, Nagatani K and Yoshida K Sep. 2013 Modeling, analysis, and control of an actively reconfigurable planetary rover for traversing slopes covered with loose soil J. Field. Robot 30 875-896

[12] Siddiqi A and de Weck O L Nov. 2009 Reconfigurability in planetary surface vehicles Acta Astronaut. 64 589-601

[13] Zeng W, Gao F, Jiang H, Huang C, Liu J and Li H 2018 Design and analysis of a compliant variable-diameter mechanism used in variable-diameter wheels for lunar rover Mech. Mach. Theory 125 240–258