Production process dependence of neutrino flavor conversion

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Abstract

We perform a covariant wave-packet analysis of neutrino oscillations taking into account the lifetime of the neutrino production process. We find that flavor oscillations in space are washed out when the neutrinos are produced from long lived resonances - and what may be observed in appearance/disappearance experiments is a uniform conversion probability independent of distance. The lifetime of the resonance which produces the neutrinos acts as the effective baseline of the experiment. For this reason the LSND experiment where neutrinos are produced from muon decay has two orders of magnitude more sensitivity to neutrino mass square difference than other experiments where the neutrinos are produced from pion or kaon decays.
There are two different formulas which describe flavor oscillations in spacetime; the one applied to neutrino oscillations [1] is derived in the high energy regime and the other applied to kaon oscillations [2] is valid in the non-relativistic limit. From the derivations of these formulas it is not clear what formula is applicable at intermediate energies (for example for kaon oscillations from stopped protons where \((m^2/E^2) \sim 25\%\)). A covariant derivation of a flavor oscillation formula which would be valid at both high and low energies would be of interest from the conceptual as well as the experimental point of view.

Kayser and Stodolsky [3] have advocated a covariant generalization of the non-relativistic phase factor \(\exp(-imt)\) by replacing the absolute time \(t\) by the Lorenz invariant proper time \(s = (t^2 - x^2)^{1/2}\). In the lab frame the covariant expression for the phase factor is \(\exp(-im_is_i)\). The phase difference between two mass eigenstates is \(\Delta(m^2/E) \simeq (\Delta m^2/E)\) which is twice the phase difference of the standard formula [1]. This result has prompted the claim [4] that kaon oscillations in \(\Phi\) factory will have oscillation length which is half of what is given by the standard formula. This claim has been refuted by [5] who take the view that the interference phase difference should not be evaluated at different space-time points but should be evaluated at the average spacetime interval and taking the phase difference to be \((m_1 - m_2)(s_1 + s_2)/2\) instead of \((m_1s_1 - m_2s_2)\) they recover the standard formula. Other methods of showing that an extra factor of two does not appear in the kaon oscillations formula have been discussed in [6]. A covariant derivation of the neutrino oscillation formula has been given by Grimus and Stockinger [7] who treat the entire process of neutrino production, propagation and detection as a single Feynman diagram. They show that on taking the large distance limit of the neutrino propagator the scattering cross section shows a space-time oscillatory behaviour and the oscillation length is identical to that given by the standard formula [1]. G-S assume the initial states to be plane waves therefore the concept of a coherence length [8] does not emerge in their formulation.

In this paper we derive the oscillation amplitude by evaluating the Feynman propagator of the neutrinos in the large time-like asymptotic limit. The asymptotic propagator of a position eigenstate has the form \(K(x,t; m_i) \simeq (m_i/2\pi s_i)^{3/2} \exp(-im_is_i)\) where \(s_i\) is the spacetime interval propagated by the \(m_i\) mass eigenstate. This is a field theoretic derivation of the Kayser-Stodolsky [3] phase factor. If the initial wave-functions of the propagating particles were strictly delta functions in spacetime then no interference between different mass eigenstates can take place. We therefore generalize the delta function propagators to propagators of Gaussian wave-packets. The interference term as a function of distance is obtained by taking the time-overlap of different mass eigenstate propagators. The expression for flavor conversion probability (for say two flavors with mixing angle \(\theta\)) as a function of distance \(X\) turns out to be,

\[
P(\alpha \rightarrow \beta; X) = \frac{1}{2} \sin^2 2\theta \left(1 - \cos\left(\frac{\Delta m^2}{2P}X\right) e^{-A}\right)
\]

Therefore extra factor of two which came from naively subtracting the phases of plane wave propagators [4] goes away in the wave-packet averaging and the standard expression for the oscillations length is recovered. This is in agreement with [5]-[6]. The covariant wave-packet treatment introduces a new contribution to the exponential factor \(A\) when particles are produced from long lived resonances (for example for neutrinos from muon decay as opposed to neutrinos from \(Z\) decay). At distances smaller than the coherence length, the exponential suppression factor is,
\[ A = \left( \frac{\Delta m^2}{2\sqrt{2}E} \tau \right)^2 \]  

where \( \tau \) is the lifetime of the resonance which produces the particle which undergo flavor oscillations. If the uncertainty in position of the initial particle \( v \tau \) is larger than the detector distance \( X \), the exponential term washes out the oscillations in (1). In neutrino experiments where the source is pions, kaons, muons or nuclear fission, the spatial oscillations of the conversion probability cannot be observed. What can be observed in these experiments is a constant (distance independent) conversion probability. The conversion probability is sensitive to values of \( \Delta m^2 \simeq (2\sqrt{2}E/\tau) \), which means that the experimental bound on \( \Delta m^2 \) is lower with longer-lifetime sources. For this reason the LSND experiment [9] which uses neutrinos from muon decay (\( \tau_{\mu} = 2.19 \times 10^{-6} \)s) is sensitive to two orders of magnitude lower neutrino mass square difference compared to other accelerator experiments like BNL-E776 [10], Karmen [11] and CCFR [12] which use neutrinos from pion and kaon decays (\( \tau \sim 10^{-8} \)sec). We fit the experimental data from LSND along with BNL-E776, Karmen, CCFR and Bugey [13] experiments with the conversion probability formula (1) and plot the allowed range for the mass difference and mixing angle. The lower bound on allowed \( \Delta m^2 \) is two orders lower than what is obtained by fitting the LSND muon decay data with the standard oscillation formula (equation (1) with \( A \) set to zero).

Field theoretic derivation of phase factor: The phenomenon of flavor oscillations takes place because particles are produced and detected as weak interaction eigenstates (the neutrino states \( \nu_e, \nu_\mu, \nu_\tau \) or the Kaon states \( K^0, \bar{K}^0 \) etc) but the propagators are diagonal in the mass eigenstates (the neutrino mass eigenstates \( \nu_i, i = 1 - 3 \) or the Kaon mass eigenstates \( K_L, K_S \)). The probability amplitude for oscillations of a gauge eigenstate \( |\alpha > \) to another \( |\beta > \) is a linear superposition of the propagation amplitude of the mass eigenstates \( < i | e^{-iHt} | i > \).

\[ A(\alpha \rightarrow \beta; t) = \sum_i < \beta | i > < i | e^{-iHt} | i > < i | \alpha > \]  

(3)

In relativistic field theory the propagation amplitude of the mass eigenstates \( < i | e^{-iHt} | i > \) can be identified with the Feynman propagator \( S_F(x_f - x_i, m_i) = < T \nu_i(x_i) \bar{\nu}_i(x_f) > \). The Feynman propagator in position space is,

\[ K(x, m_i) = -i \int \frac{d^4p}{(2\pi)^4} \frac{(i \beta + m_i)}{p^2 - m_i^2 - i\epsilon} e^{-ip \cdot x}. \]  

(4)

This integration can be done by expressing the denominator as an exponential,

\[ \frac{-i}{p^2 - m_i^2 - i\epsilon} = \int_0^\infty d\alpha \ exp \ \{i\alpha (p^2 - m_i^2 - i\epsilon)\} \]  

(5)

and integrating over the resulting Gaussian in \( p \). The remaining integral over \( \alpha \),

\[ \int_0^\infty d\alpha \ \alpha^{-2} \ exp \ \{i\alpha m_i^2 + \frac{i(t^2 - |x|^2)}{4\alpha}\} \]  

(6)

can be performed by substituting \( s = (t^2 - |x|^2)^{1/2} \), \( z = ims \) and \( \eta = 2(\alpha m/s) \) and making use of the integral formula [14] for the Bessel function.
\[
\frac{1}{2} \int_0^\infty d\eta \, \eta^{-(\nu+1)} \exp - \frac{z}{2} (\eta + \frac{1}{\eta}) = K_\nu(z). \tag{7}
\]

The resulting expression for the Feynman propagator (4) is

\[
K(x, m_i) = \left( \frac{i}{4\pi^2} (i \not{\partial} + m_i) \frac{m_i}{s_i} \right) K_1(i m_i s_i) \tag{8}
\]

where \( s_i = (t^2 - x^2)^{1/2} \) is the invariant spacetime interval propagated by the \( \nu_i \) mass eigenstate. If this interval is large (\( s >> m_i^{-1} \)) and time-like (\( t \geq |x| \)) then we can use the asymptotic expansion of the Bessel function

\[
K_1(ims) \simeq \left( \frac{2}{\pi i m} \right)^{1/2} e^{-ims} \tag{9}
\]

to obtain from (8) the expression for the propagation amplitude at large time-like separation

\[
K(x, t; m_i) = \left( \frac{m}{2\pi i \sqrt{(t_f - t)^2 - (x_f - x_i)^2}} \right)^{3/2} \exp \left\{ -im_i \sqrt{(t_f - t)^2 - (x_f - x_i)^2} \right\} \tag{10}
\]

The phase factor of the amplitude (10) is Lorentz-invariant and can be written in terms of the neutrino energy \( E \) and the time of flight \( t \) as measured from the lab frame as

\[
-imp_i (1 - v_i^2)^{1/2} t = -i \frac{m_i^2}{E_i} t.
\]

An extra factor of two appears on subtracting the phases at different spacetime points. A flavor eigenstate neutrino or kaon is observed at a single spacetime point and one should therefore compute the phase difference at the overlap of the two mass eigenstate wave-packets. This averaging over spacetime is done formally by considering the propagators of Gaussian wave-functions as opposed to plane waves, and the standard expression for the oscillation length is recovered.

**Conservation laws for long distance propagators:** A flavor eigenstate propagates as a linear combination of different mass eigenstates which are on-shell. The phase difference is obtained by some authors by assuming that the different mass eigenstates have common energy, and by some by assuming that they have a common momentum [15]. In this section we show that the linear combination of different mass eigenstates have neither the same energy nor the same momentum. We show that particles propagating over large distances (\( X >> P/m^2 \)) are constrained by the conservation laws at the vertex to be on shell. The on-shell condition is all that is needed to fix the phase difference and the oscillation length.

Consider a diagram with a propagator between vertices at spacetime points \((x_1, x_2)\) with a number of external legs at the vertices. The amplitude is proportional to

\[
\int d^4x_1 \, d^4x_2 \, \exp(-i \sum_i q_i \cdot x_1) \, K(x_1, x_2) \, \exp(i \sum_f q_f \cdot x_2) \tag{11}
\]

Where \( q_i \) are the incoming four momenta from the external legs at \( x_1 \) and \( q_f \) are the incoming four momenta from the external legs at \( x_2 \). Substituting the propagator

\[
K(x_1, x_2) = \int d^4p \frac{e^{-ip \cdot (x_1 - x_2)}}{p^2 - m^2 + i\epsilon} \tag{12}
\]
in (11) and integrating over $x_1$ and $x_2$ gives the usual energy momentum conservation
\[
\delta^4(\sum_i q_i - p) \quad \text{and} \quad \delta^4(\sum_f q_f - p)
\]
at the vertices. When the proper distance between the two vertices $s$ is larger than $m^{-1}$, then the propagator used in (11) is the asymptotic form (10), which in momentum space can be written as [16],
\[
K(x_1, x_2; s >> m^{-1}) = \frac{1}{(2ms)^{3/2}} e^{-ims}
\]
\[
= \int \frac{d^4p}{\sqrt{P^2 + m^2}} \exp \{-i(\sqrt{P^2 + m^2})(t_f - t_i) + iP \cdot (x_f - x_i)\} \quad (13)
\]
Substituting (13) in (11) and integrating over $x_1$ we obtain
\[
\int d^4x_1 \exp \{-i(\sum_i E_i - \sqrt{P^2 + m^2})t_1 + i(\sum_i q_i - P) \cdot x_1\}
\]
\[
= \delta(\sum_i E_i - \sqrt{P^2 + m^2}) \quad \delta^3(\sum_i q_i - P) \quad (14)
\]
This implies that at a vertex energy and momentum are conserved and in addition those particles which propagate without freely over distances larger than $(|P|/m^2)$ obey the mass shell constraint, $E = \sqrt{P^2 + m^2}$.

Wave-packet analysis: The propagator $K(x_f, x_i)$ given in (10) is the amplitude for the propagation of a particle localized at $x_i$ (a delta function initial wave function) to be detected at $x_f$. The propagator of some general initial wave-packet $\Psi_{in}(x - x_i)$ is obtained by using the expressions for the delta function propagator given in (10) and the superposition principle,
\[
\tilde{K}(x_f - x_i) = \int d^4x \ K(x_f, x) \ \Psi_{in}(x - x_i) \quad (15)
\]
If the initial wave function is a Gaussian,
\[
\Psi_{in}(x - x_i) = N \exp\{-iE_a(t - t_i) + iP_a \cdot (x - x_i)\} \ \exp\{-\frac{(x - x_i)^2}{4\sigma_x^2} - \frac{(t - t_i)^2}{4\sigma_t^2}\} \quad (16)
\]
where $\sigma_x$ and $\sigma_t$ are the uncertainties is the initial position and time of production of the particle respectively and $N$ is the normalization constant. In the earlier wave-packet analyses of oscillation problem [8], the uncertainty in the time of production was neglected. In a covariant treatment both should be included. In the next section we show that the time uncertainty gives rise to a novel phenomenon of conversion without oscillations in many experimentally relevant situations. Substituting the expression for $\tilde{K}(x_f, x)$ given in (10) and the Gaussian initial wave-packet (16) in (15) and evaluating the integral by the stationary phase method [17] we obtain the expression for the propagation amplitude of a Gaussian wave-packet,
\[
\tilde{K}(X, T; m_a) = (\frac{1}{4\pi|X|^2})^{1/2}\left(\frac{\pi}{\sigma_x^2 + v_a^2\sigma_t^2}\right)^{-1/4} \exp\{-i\frac{m_a^2 T}{E_a} \quad + iP_a \cdot (X - v_a T)\}
\]
\[
\times \exp\{-\frac{(X - v_a T)^2}{4(\sigma_x^2 + v_a^2\sigma_t^2)}\} \quad (17)
\]
where \( \mathbf{X} = \mathbf{x}_f - \mathbf{x}_i \) and \( T = t_f - t_i \) are the space and time intervals propagated by the center of the Gaussian wave-packet and \( \mathbf{v}_a \equiv \mathbf{P}_a/E_a \). The normalization constant has been fixed such that \( \int dT d^3\mathbf{X} \tilde{K}(\mathbf{X}, T; m_a) \tilde{K}^\dagger(\mathbf{X}, T; m_b) = 1 \). The factor of \((1/4\pi|\mathbf{X}|^2)^{1/2}\) in (17) shows that the particle flux decreases inversely with the square of distance. Neutrino disappearance experiments look for evidence of depletion of a certain neutrino species over and above the expected inverse square decrease in flux. In the following we will not display this factor \((1/4\pi|\mathbf{X}|^2)\) in the probability expressions. The amplitude for the oscillation of one type of neutrino flavor to another is obtained by substituting the propagation amplitudes for mass eigenstates (17) in ,

\[
\mathcal{A}(\alpha \rightarrow \beta; X, T) = \sum_a^3 U_{\alpha a} \tilde{K}(m_a; X, T) U^*_{\alpha\beta} \tag{18}
\]

where \( \alpha, \beta \) denote the flavor eigenstates \((\nu_e, \nu_\mu, \nu_\tau \text{ or } \bar{K}^0, \bar{K}^0)\) and the summation index \( a \) denotes a mass eigenstates \((\nu_1, \nu_2, \nu_3 \text{ or } \bar{K}_L, \bar{K}_S)\) . \( U_{\alpha a} = < \alpha | a > \) and \( U^*_{\beta a} = < a | \beta > \) are the elements of the mixing matrix which relate the flavor eigenstates with the mass eigenstates. The probability of flavor oscillation as a function of space-time is the modulus squared of the amplitude (27)

\[
P(\alpha \rightarrow \beta; X, T) = | \sum_a^3 U_{\alpha a} \tilde{K}(m_a; X, T) U^*_{\alpha\beta} |^2 \tag{19}
\]

In interference experiments the time of flight of the particle is not measured and only the distance between the source and the detector is accurately known [15] . The probability for the flavor conversion as a function of distance is given by the time integral of (19) ,

\[
P(\alpha \rightarrow \beta; X) = \int dT | \sum_a^3 U_{\alpha a} \tilde{K}(m_a; X, T) U^*_{\alpha\beta} |^2 \tag{20}
\]

The interference term given by the time-overlap of the propagation amplitudes (wavefunctions) of different mass eigenstates can be evaluated for the Gaussian propagator (17) and is given by,

\[
Re \int dT \tilde{K}(X, T; m_a) \tilde{K}^\dagger(X, T; m_b) = \frac{2}{v} \cos\{ (\mathbf{P}_a - \mathbf{P}_b) \cdot \mathbf{X} - (E_a - E_b) \frac{(\mathbf{v}_a + \mathbf{v}_b)}{2v^2} \cdot \mathbf{X} \} \\
\times \exp\{ - (E_a - E_b)^2 \frac{(\sigma_x^2 + v^2\sigma_t^2)}{2v^2} - \frac{(\mathbf{v}_a - \mathbf{v}_b)^2}{8(\sigma_x^2 + v^2\sigma_t^2)} \frac{X^2}{v^2} \} \tag{21}
\]

where \( v \equiv \sqrt{(v^2_a + v^2_b)/2} \). In terms of the average momentum \( \mathbf{P} \) and energy \( E \) and their respective differences \( \Delta \mathbf{P} \) and \( \Delta E \) the interference term (21) ,to the leading order in \((\Delta \mathbf{P}/\mathbf{P})\) and \((\Delta E/E)\), reduces to the form,
\[ \frac{2}{v} \cos\left( \frac{X}{P} (E\Delta E - \mathbf{P} \cdot \Delta \mathbf{P}) \right) \exp\{-A\} \]  

(22)

where the exponential damping factor,

\[ A = (\Delta E)^2 \frac{\bar{\sigma}^2}{2} \left( \frac{E^2}{P^2} \right) + \left( \frac{\Delta P}{P} \right)^2 \frac{X^2}{4\bar{\sigma}^2} \]  

(23)

and \( \bar{\sigma}^2 \equiv (\sigma_x^2 + (P/E)^2 \sigma_t^2) \).

In general neither \( \Delta E \) nor \( \Delta P \) is zero and they depend upon how the state is prepared. For example if the mommentum of the initial and the associated final state is measured then \( \Delta P = 0 \). In the last section we have shown that for long distance propagation \( s > m^{-1} \) \( \mathbf{P} \) and \( E \) are not independent and are related by the mass shell constraint. The particular combination that appears in the phase difference turns out to be independent of the preparation and is fixed by the condition that each of the mass eigenstates be on shell,

\[ E_i^2 - P_i^2 = m_i^2 \quad i = a, b \]

\[ \Rightarrow \quad 2E\Delta E - 2\mathbf{P} \cdot \Delta \mathbf{P} = \Delta m^2 \]  

(24)

Substituting (24) in (22) we see that the interference term of (21) is given by,

\[ \frac{2}{v} \cos\left( \frac{\Delta m^2}{2P} X \right) \exp\{-A\} \]  

(25)

The probability for the flavor conversion as a function of distance (20) is therefore,

\[ P(\alpha \rightarrow \beta; X) = \sum_a \frac{1}{v_a} |U_{\beta a}|^2 |U_{\alpha a}|^2 \]

\[ + \sum_{a \neq b} \frac{1}{v} |U_{\beta a} U_{\alpha a}^* U_{\beta b} U_{\alpha b}^*| \cos\left( \frac{(m_a^2 - m_b^2)}{2P} X + \delta \right) \exp\{-A\} \]  

(26)

where \( \delta = \arg(U_{\beta a} U_{\alpha a}^* U_{\beta b} U_{\alpha b}) \).

We see that the standard oscillation formula which was obtained for relativistic particles [1] is actually valid at all energies. In other words although the formula (26) is usually derived by taking the leading term in a \( (m^2/P^2) \) series, our covariant calculation shows that there are actually no \( O(m^2/P^2) \) corrections to (26).

In the non-relativistic regime where \( P = (m_a + m_b)v/2 \), the standard kaon oscillation formula \( \cos(m_a - m_b)T \) is recovered.

There is no extra factor of two in the relativistic kaon oscillation formula as would have appeared without the wavepacket averaging. We see that although the two mass eigenstates have different proper times the wave packet overlap results in the average proper time appearing in the interference term. Therefore the interference phase is actually \( (m_a - m_b)\bar{s} = (m_a - m_b)(m_a + m_b)T/(E_a + E_b) = (m_a^2 - m_b^2)T/2E \). The average time prescription of [5] can therefore be justified using the covariant propagator method. Other methods of showing that an extra factor of two does not appear in the flavor oscillations formula have been discussed in [6].
Conversion without oscillation: The suppression factor $A$ which goes with the oscillation term has some interesting new implications. If the parameters of the experiment are such that $A$ becomes large then no oscillations is spacetime can be observed. What can be observed is a constant conversion probability.

It has been noted earlier [8] one condition $A$ must be small which implies that $X$ must be smaller than the coherence length $L_{coh} = (2\bar{\sigma}P/\Delta P)$. We shall show below that the constraint $L_{osc} < L_{coh}$ is not a sufficient condition to ensure the occurrence of flavor oscillations in space. In the analysis of refs [8] only the uncertainty in the initial position $\sigma_x$ was considered, in our analysis we have included the contribution of $\sigma_t$, the uncertainty in time at which the neutrino was produced, which as it turns out, makes the larger contribution to the suppression term $A$. This happens when the neutrinos are produced from long lived resonances. The neutrino wavepacket which is produced has a spread in time with width $\sigma_t$ which cannot be smaller than the lifetime of the resonance whose decay produces it. The dominant contribution in that case arises from the first term of $A$,

$$A \simeq (\Delta E)^2 \frac{\bar{\sigma}^2}{2} \simeq \left( \frac{\Delta m^2 \tau}{2\sqrt{2}E} \right)^2$$

(27)

where we have equated the initial time uncertainty $\sigma_t$ with $\tau$ - the lifetime of the resonance that produces the neutrino, and we have ignored the spatial spread of the wave-packet $\sigma_x$ which in most experiments is many orders of magnitude smaller than $\sigma_t$. In terms of the oscillation length $L_{osc} = 4\pi E/(\Delta m^2)$ the suppression factor $A = (\sqrt{2}\pi \tau/L_{osc})^2$. In order to observe oscillations the source-detector distance $L$ must be larger than $L_{osc}$. Which that the minimum source-detector distance in order that neutrino oscillations be observed is given by condition, $L_{min} = (\pi)\sqrt{2}\tau$. We shall show below that most neutrino experiments do not satisfy this criterion for observability of space oscillations. On the other hand spatial oscillations of flavor can be observed in the $\Phi$ and $B$ factories owing to the large width of $\phi$ and $\Upsilon$ resonances.

For relativistic particles produced from long lived resonances the formula for the conversion probability which should be fitted with the experiments is,

$$P(\nu_\alpha \rightarrow \nu_\beta) = \frac{1}{2} \sin^2 2\theta \left( 1 - \cos \left( \frac{2.53 \Delta m^2 L}{E} \right) \exp \left\{ -\left( \frac{1.79 \Delta m^2 \tau}{E} \right)^2 \right\} \right)$$

(28)

where $\Delta m^2$ is the mass square difference in $eV^2$, $L$ is the detector distance in $m \ (km)$, $\tau$ is the lifetime of the parent particle in the lab frame in $m \ (km)$ and $E$ is the energy in $MeV \ (GeV)$. The limits on the values of $\Delta m^2$ and $\sin^2 2\theta$ obtained by fitting the results of different experiments with the oscillation formula (28) are listed in Table I.
TABLE I. The asymptotic limits on $\Delta m^2$ and $\sin^2 2\theta$ from different experiments according to the oscillation formula (28). $\tau$ is the lifetime of the neutrino source in the lab frame, $<E_\nu>$ is the average $\nu$ energy, $L$ is the detector distance, $L_{\text{min}}$ is the minimum detector distance for observing oscillations in space and $P$ is the experimental value of the conversion probability.

| Experiment(Source) | $\tau$ (m) | $<E_\nu>$ (MeV) | $L$ (m) | $L_{\text{min}}$ (m) | $P$ | $\Delta m^2$ (eV$^2$) | $\sin^2 2\theta$ |
|--------------------|-------------|-----------------|---------|----------------------|-----|----------------------|-----------------|
| LSND ($\mu$) [9]   | 658.6       | 30              | 30      | 2927                 | (0.16 - 0.47) $\times 10^{-3}$ | (2.0 - 3.0) $\times 10^{-3}$ | 0.003 - 0.009    |
| LSND ($\pi$) [18]  | 17          | 130             | 30      | 76                   | (0.26 $\pm$ 0.15) $\times 10^{-2}$ | 0.8 - 1.6         | 0.002 - 0.0082   |
| Karmen ($\pi$) [11]| 7.8         | 29.8            | 17.5    | 34.6                 | $<0.3 \times 10^{-2}$                | $<0.16$            | $<0.6 \times 10^{-2}$         |
| E776 ($\pi$) [10]  | 578         | $5 \times 10^3$ | $10^3$  | $2.6 \times 10^4$    | $<0.15 \times 10^{-2}$               | $<0.2$            | $<0.3 \times 10^{-2}$         |
| CCFR (K) [12]      | $5.41 \times 10^3$ | $140 \times 10^3$ | $1.4 \times 10^4$ | $24 \times 10^4$ | $<0.9 \times 10^{-3}$          | $<1.2$          | $<0.18 \times 10^{-2}$         |
| Bugey(U,Pu) [13]   | $3 \times 10^{10}$ | 5          | 95      | $10^{14}$            | $<0.75 \times 10^{-1}$               | $<10^{-9}$         | $<0.15$ |
FIG. 1. Lsnd allowed regions with the standard (between the dotted lines) and the covariant oscillation formulas (between the continuous line). The conversion probability formulae are averaged over a Gaussian distribution of neutrino energy with mean $30\text{MeV}$ and width $10\text{MeV}$. 
FIG. 2. Karmen allowed regions with the standard (dotted curve) and the covariant oscillation formulas (dashed curve). The conversion probability formulae are averaged over a Gaussian distribution of neutrino energy with mean $30\text{MeV}$ and width $10\text{MeV}$.
FIG. 3. The combined fit of the covariant oscillation formula with all experiments. The region between the continuous lines is allowed by the LSND $\mu$ experiment [9]. The region between the dotted lines is allowed by the LSND $\pi$ experiment [?] . Region ruled out by E776 [10] is above the dashed-dotted curve and by Karmen [11] is above dashed curve. The region above the top-most dashed curved is ruled out by CCFR [12].
In the standard oscillation formula the oscillatory term averages to zero when \( \Delta m^2 > (4\pi E/L) \). Using the wavepacket formula (28) however we see that the oscillatory term is exponentially damped at much lower values of \( \Delta m^2 \) (provided \( \tau > (\sqrt{2}\pi L/4\pi) \)). If neutrino sources have a large decay time which is the case in most experiments, the conversion probability is sensitive to mass differences \( \Delta m^2 > (\sqrt{2}\pi E/\tau) \). For this reason the LSND experiment which uses neutrinos from muon decay has two orders of magnitude more sensitive than KARMEN, BNL-E776 etc where the neutrinos are from \( \pi \) and \( K \) decay. This is evident by comparing the fit of LSND \( \mu \) DAR allowed region Fig1 with the Karmen allowed region Fig 2. In experiments where the source is in flight w.r.t the detector the gain in the source lifetime by the Lorentz factor \( \gamma = (E_\pi/m_\pi) \) is somewhat compensated by the larger \( E_\nu \) in the denominator of \( A \) (27), so there is no gain in sensitivity with in flight neutrino sources (as in CCFR and BNL-E776 experiments). The reactor experiments where the neutrinos arise from the \( \beta \)-decay of \( ^{235}\text{U} \) and \( ^{229}\text{Pu} \) nuclei, \( \tau \sim 10^2 \text{s} \) and therefore these experiments are in fact sensitive to values of \( \Delta m^2 \) as low as \( 10^{-9}\text{eV}^2 \). But the probability measurement in reactor experiments is poor compared to the accelerator experiments which is why they rule out only a small region of parameter space. If the recent LSND \( \pi \) decay in flight result \[18\] is taken as an actual observation of \( \nu_\mu - \nu_e \) conversion then it rules out most of the low \( \Delta m^2 \) region allowed by the LSND \( \mu \) decay experiment.

**Conclusions**

The covariant formulation gives a very different result for the oscillation probability formula compared to the standard treatment when the neutrino source has a lifetime larger than the baseline of the experiment. We use the condition that the minimum spread of the neutrino wave function along the time axis is given by the lifetime of the particle whose decay produces the neutrino. When neutrinos are produced from long lived particles like muons, the interference term in the conversion probability formula vanishes for much smaller values of the neutrino mass square difference compared with the standard formula. The reason this happens can be understood with the following picture. When the spread of the neutrino is large along the time axis, its spread along the energy axis is small. The neutrino wave-functions for different masses are then energy eigenstates which are mutually orthogonal. The interference term which is the overlap of the wave-functions of different mass states therefore vanishes when these states become orthogonal.

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