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Combinatorics of Bousquet-Mélou-Schaeffer numbers in the light of topological recursion. (English) [Zbl 1458.05262]

Eur. J. Comb. 90, Article ID 103184, 34 p. (2020).

Summary: In this paper we prove, in a purely combinatorial-algebraic way, a structural quasi-polynomiality property for the Bousquet-Mélou-Schaeffer numbers. Conjecturally, this property should follow from the Chekhov-Eynard-Orantin topological recursion for these numbers (or, to be more precise, the Bouchard-Eynard version of the topological recursion for higher order critical points), which we derive in this paper from the recent result of A. Alexandrov et al. [Commun. Math. Phys. 375, No. 1, 237–305 (2020; Zbl 1472.37078)]. To this end, the missing ingredient is a generalization to the case of higher order critical points on the underlying spectral curve of the existing correspondence between the topological recursion and Givental’s theory for cohomological field theories.

MSC:

05E14 Combinatorial aspects of algebraic geometry
05A05 Permutations, words, matrices
05A15 Exact enumeration problems, generating functions
14N10 Enumerative problems (combinatorial problems) in algebraic geometry
37K10 Completely integrable infinite-dimensional Hamiltonian and Lagrangian systems, integration methods, integrability tests, integrable hierarchies (KdV, KP, Toda, etc.)

Keywords:
Givental’s theory for cohomological field theories

Full Text: DOI arXiv

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