Is Dark Matter really necessary?

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Abstract. The impossibility of a gravitational equilibrium of clusters of galaxies was the main argument supporting the existence of Dark Matter. However, the expansion of Universe is also a main element of evolution of clusters and it leads to a dispersion of these large structures at a rate that is slower than the expansion itself: the presence of Dark Matter becomes useless…

1. Classical elements of the Newtonian n-body problem

Let us consider n punctual masses m₁, m₂, … mₙ. Their positions in a Euclidian three dimensional space will be given by the n vectors r₁, r₂,… rₙ, and, with G = gravitational constant = 6.674x10⁻¹¹m³/kg.s²,
the well-known “Virial Theorem” is given by the following: we use the axis of the center of mass

\[ \sum m_j \mathbf{r}_j = 0 \]  

with \( j = 1, 2, \ldots, n \); the semi-moment of inertia

\[ \frac{1}{2} \sum m_j r_j^2 = I \]  

with \( j = 1, 2, \ldots, n \); the kinetic energy

\[ \frac{1}{2} \sum m_j (d\mathbf{r}_j/dt)^2 = K \]  

with \( j = 1, 2, \ldots, n \); the gravitational potential

\[ G \sum m_i m_j / r_{ij} = U \]  

when \( 1 \leq i < j \leq n \); \( r_{ij} = |\mathbf{r}_j - \mathbf{r}_i| \); the total energy

\[ h = K - U \]  

The virial theorem, also called Lagrange-Jacobi identity is then:

\[ d^2I/dt^2 = 2K - U = U + 2h \]  

According to the sign of the total energy \( h \), this identity leads to the following classical discussion:

\( h > 0 \), that is \( K > U \).

The second derivative \( d^2I/dt^2 \) is always larger than the positive quantity \( 2h \), hence the semi-moment of inertia \( I \) goes to infinity at least as \( ht^2 \) and the n-body system cannot be stable; \( h < 0 \), that is \( K < U \).

The n-body system can be stable, but the virial theorem presented in (6) shows that the ratio \( K/U \)
must remain in the vicinity of 1/2 or have fluctuations below and above 1/2.

For instance a stable two-body system of eccentricity e has a ratio \( K/U \) equal to \( (1 + e \cos E)/2 \), where \( E \) is the eccentric anomaly.

### 2. The galaxy clusters and the virial theorem

Most galaxies belong to a cluster with hundreds or thousands of galaxies. However, the existence of these giant formations seemed paradoxical to the astronomers of the twentieth century: their ratio \( K/U \) was not in the vicinity of 1/2 but usually above 10, if not above 25...

How is it possible to be stable with such a large kinetic energy and such a small gravitational potential? A first possible answer was given by Fritz Zwicky in 1933 [1].

A sufficiently large amount of attracting and invisible “Dark Matter” at the suitable place will stabilize the clusters. Much later Mordehai Milgrom will present another possible explanation: a small modification of Newton’s law [2].

Let us understand that the answer of Zwicky was very logical: the Newtonian Mechanics was triumphant, it had led to the discovery of planet Neptune and the known “relativistic effects” were extremely small: less than 1’ per century for the perihelion advance of Mercury, less than 2” for the deviation of light beams. Even as late as 2009, the book “Galaxies et Cosmologie” uses the virial theorem for the introduction of the notion of Dark Matter [3], page 117.

However, for galaxy clusters the relativistic effects are dominant and the application of the virial theorem is misleading. So, we will study here a third assumption: no dark matter, no modification of the law of Newton, but a careful analysis of the motion of galaxies and clusters in an expanding Universe, with the following question: **Is the dispersion of clusters slower or faster than the expansion?**

If the dispersion of clusters is not faster than the expansion, the clusters will keep their individuality and the notion of “Dark Matter” will become useless.

The ratio \( K/U \) being very large for most clusters of galaxies, the influence of ordinary gravitational attractions remains very small and the galaxies move almost as in an empty Universe, but we will see that the expansion is decisive.

### 3. Free motion of a galaxy in an expanding universe

The \( ds^2 \) of an expanding Universe is not that of special relativity and in such a Universe the free motions have not a uniform velocity even if the small gravitational effects are neglected (and anyway the small gravitational effects are favorable to the stability of clusters).

The \( ds^2 \) of a Friedman-Lemaître metric is well known:

\[
c^2 ds^2 = c^2 dt^2 - R^2 [d\alpha^2 + u^2 (d\varphi^2 + \sin^2 \varphi dL^2)]
\]

(7)

with: \( c = 299,792,458 \) m/s = velocity of light; \( s \) = proper time; \( t \) = “cosmical time”; \( R = R(t) \) = “radius of Universe” \( \alpha \), \( \varphi \), \( L \): radial parameter, colatitude and longitude (from some suitable origin O).

\[
u = u(\alpha):
\]

\[
u = \sinh \alpha, \text{ for an open universe (} k = -1 \);
\]

\[
u = \alpha, \text{ for a flat universe (} k = 0 \);
\]

\[
u = \sin \alpha, \text{ for a closed universe (} k = +1 \).
\]

A Friedman-Lemaître closed universe is an expanding hypersphere with radius \( R(t) \), this justify the name “radius of Universe” also used in the two other cases.

The evolution of the “radius of Universe” \( R(t) \) is given by the Friedman-Lemaître equations:

\[
d^2 R/dt^2 = (\lambda R/3) - (4\pi G \rho_o R_o^3/3R^2)
\]

(8)

\[
(dR/dt)^2 = (\lambda R^2/3) + (8\pi G \rho_o R_o^3/3R) - kc^2
\]

(9)
(9) is an integral of (8) with: \( \lambda = \) cosmological constant; \( \rho_o = \) present average density of Universe; \( R = \) present radius of Universe.

The ratio \( \lambda c^2/8\pi G \) is a density of energy, it is sometimes called “density of energy of vacuum”, but notice that, with the expansion, this would imply an increasing total energy of Universe (if \( \lambda > 0 \)), a property contrary to the conservation of energy.

Friedman in 1922 especially considered the case \( \lambda = 0 \) and Lemaître in 1929 the general case.

They have built the theory of the expanding Universe and the corresponding “Hubble constant” is \( dR/Rdt \), whose present value is about 70km/s per Megaparsec.

The three cases, open, flat and closed are very similar and we will consider the flat case and the following classical transformation:

\[
\begin{align*}
R_o \alpha \sin \phi \cos L &= x \\
R_o \alpha \sin \phi \sin L &= y \\
R_o \alpha \cos \phi &= z
\end{align*}
\]  
\( x,y,z \) are the Cartesian co-mobile coordinates, and the metric (7) becomes:

\[
c^2ds^2 = c^2dt^2 - (R/R_o)^2 \cdot (dx^2 + dy^2 + dz^2)
\]  
Notice, for the present time, the similarity of this metric and the usual Minkowsky metric.

The main question is now that of the free motions in the metric (13), for instance the motion with the maximum proper time \( s_2 - s_1 \) for a motion starting at the space-time point \( x_1, y_1, z_1, t_1 \) and leading to the space-time point \( x_2, y_2, z_2, t_2 \).

This problem is an ordinary optimization problem and we will use the following:

A) The vector \( r = (x, y, z) \), it goes from \( r_1(t_1) \) to \( r_2(t_2) \).

B) The state parameters \( r \) and \( s \), and their corresponding adjoint parameters \( p \) and \( q \).

C) The corresponding Hamiltonian \( H \) and the optimal Hamiltonian \( H^* \).

With the cosmical time \( t \) as parameter of description we obtain:

\[
H = p \cdot \frac{dr}{dt} + q \cdot \frac{ds}{dt}
\]

The optimal Hamiltonian \( H^* \) is a function of \( t, r, p, s, q \), it is the maximum of \( H \) when the condition (13) is satisfied. Thus, with \( p = |p| \):

\[
H^* = H^*(t, r, p, s, q) = [q^2 + (p^2c^2R_o^2/R^2)]^{1/2}
\]

and, with the usual equations of motion:

\[
\frac{dr}{dt} = \partial H^*/\partial p; \quad \frac{ds}{dt} = \partial H^*/\partial q; \quad \frac{dp}{dt} = -\partial H^*/\partial r; \quad \frac{dq}{dt} = -\partial H^*/\partial s
\]

we obtain that the adjoint parameters \( p \) and \( q \) are constant and:

\[
\begin{align*}
\frac{dr}{dt} &= \partial H^*/\partial p = p \frac{c^2R_o^2}{R^2H^*} = p \frac{c^2R_o^2}{R^2} [q^2 + (p^2c^2R_o^2/R^2)]^{1/2} \\
\frac{ds}{dt} &= \partial H^*/\partial q = q/H^* = q/[q^2 + (p^2c^2R_o^2/R^2)]^{1/2}
\end{align*}
\]

For instance:

A) For a light beam:

\[
q = 0; \quad \frac{ds}{dt} = 0; \quad \frac{dr}{dt} = (cR_o/R) \frac{p}{|p|}
\]

A photon moves straightforward in a fixed direction with a velocity that is \( (cR_o/R) \) in co-mobile coordinates and that is thus always \( c \) in local coordinates.

B) For a slow motion: the ratio \( pc/q \) is very small, hence \( ds/dt \) is almost 1 and practically:

\[
\frac{dr}{dt} = p \frac{c^2R_o^2}{R^2}q
\]
The expansion implies a very fast decrease of the velocity.

4. The Doppler-Fizeau effect in an expanding Universe

Let us consider an astronomer A receiving a light beam at the present time $t_a$, we will assume that at $t_a$ he is at the origin of coordinates and that his velocity in co-mobile coordinates is $x_{a}', y_{a}', z_{a}'$ (modulus $v_a = (x_{a}'^2 + y_{a}'^2 + z_{a}'^2)^{1/2}$, about 300 km/s).

We can also assume that the light beam was along the positive half of the x-axis and was emitted by the star S at the cosmical time $t_S$ and at the co-mobile position $x_s, 0, 0$; the co-mobile velocity of the star is $x_{s}', y_{s}', z_{s}'$ (modulus $v_s$).

The existence of the light beam implies:

$$\int_{t_S}^{t_a} \frac{cR_0/R(t)}{dt} \, dx_s$$

Equation (21) for this second light beam and its difference with (21) gives:

$$c \delta - [cR_0/R(t)]\epsilon = \epsilon \, x_s' - \delta \, x_s'$$

Equation (22) that is, with $R(t) = R_0$:

$$\frac{\delta}{\epsilon} = \frac{[x_{s}' + (cR_0/R_0)]}{(x_{s}' + c)}$$

Equation (23) gives the redshift $\lambda$ of the star S is given by the following ratio:

$$1 + \lambda = \frac{[x_{s}' - (cR_0/R_0)]}{[1 - (v_s R_0/c^2 R_0^2)]^{1/2}}$$

Equation (24) is almost equal to the ratio $\delta/\epsilon$ given in (23), but notice the following: let us compare two nearby stars, the difference of their redshifts will give the difference of their cosmical radial velocities and we have to multiply this difference by the small ratio $R_0/R_0$ to obtain that velocity difference in local coordinates…

That property must be considered.

5. Application to a cluster of galaxies

Let us consider a classical galaxy cluster such as the Virgo cluster. It contains about 2000 galaxies, its average distance is about 54 million light-years and its transversal diameter is about 14 million light-years.

It is a logical assumption to assume that its radial diameter is also about 14 million of light-years and indeed most of its galaxies have a radial velocity corresponding more or less to these distances. For the few percent that have a larger or a smaller radial velocity let us see, with the example of the Andromeda galaxy, that the corresponding dispersion requires cosmological durations.

The distance of the Andromeda galaxy is 2.5 million light-years and that distance is presently decreasing with a velocity of 120 km/s. If the transversal velocity is negligible, we will have a collision between our galaxy and the Andromeda galaxy after a duration roughly $2.5 \times 10^{10}$ years. With the decrease of velocities due to the expansion that duration must be increased especially if the cosmological constant $\lambda$ of (8) and (9) is larger than the limit $5.2 \times 10^{-36} \text{s}^{-2}$ the limit above which the radius of Universe $R$ will be always accelerating in the future and not decelerating…
The below figures 1 and 2 [3] p.125, give a histogram of the radial velocities of 466 galaxies that seems to belong to the cluster “Abell 496” (since it is not so easy to determine if a given galaxy belongs really to a given cluster). The radial velocities present a very well-defined peak in the vicinity of 10 000 km/s and few outside results that certainly correspond to nearer galaxies (smaller velocities) or further galaxies (larger velocities).

![Histogram of radial velocities of 466 galaxies of cluster Abell 496.](image)

The cluster Abell 496 has a core with a diameter about 0.8 Megaparsec [4], page 3 and its transversal diameter is about 10 Megaparsecs [5], most of the radial velocities of figures 1 and 2 correspond to the distances of interest and the dispersion of the too slow or too fast galaxies will take cosmological durations… even in the absence of dark matter.

Also notice another factor of stability: If the radial velocity V of a galaxy G corresponds to a distance D in the expansion of Universe and if the real distance of the galaxy G is larger than D, its velocity is smaller than that of the local expansion and in co-mobile coordinates it approaches the distance D. The same approach exists symmetrically for distances smaller than D.

6. Difficulty of the analyses

Let us consider a cluster of galaxies. Our main tools of analysis are (most accurate to the lesser one):

A) The spectroscopy, with the Doppler-Fizeau effect and the chemical analysis.
B) The photography, the angular measurements and the measures of luminosity at various wavelengths.
C) The evaluation of distances with their various techniques: redshifts, Cepheids, RR Lyrae, etc.
D) The lensing effects and the evaluation of masses.
E) Our knowledge of the other clusters and the corresponding statistics.
F) The general information related to standard cosmology.

The difficulties arise especially in the evaluation of the masses of clusters.

Four types methods have been used: 1. Gravitational methods with the help of the virial theorem. 2. Methods using the lensing effect. 3. Analyses of the luminosity at various wavelengths. 4. Methods analyzing the X-ray emissions and the intergalactic clouds of gas.

All these methods are strongly dependent of the evaluation of distances, but we can already write that the evaluations given by the method A and the virial theorem are misleading: they give evaluations that are far larger than the other ones (sometimes hundred times larger…).

The method C gives generally the smallest evaluations and the methods B and D are certainly very good (when they are possible…) since that, in spite of their difference, they give evaluations with the same order of magnitude: their ratio is almost always between 0.5 and 2.
The evaluation of the mass of the Milky Way, our nearest neighbor, emphasizes the difficulty.
In the forties of the twentieth century that mass was one billion solar mass, in the sixties it was 20 billion, in the nineties it was 100 billion and today the mass of the Milky Way is supposed to be about 240 billion solar masses. We are still very far from perfection!

7. Conclusion
The singularity of the motion of galaxy clusters was pointed out by Fritz Zwicky [1] and studied by many astronomers, for instance in [2-8], but the Virial Theorem, this beautiful product of the Newtonian mechanics, was unsuitable and misleading in a domain in which the relativistic effects are dominant.

The galaxy clusters don’t have the gravitational stability, but their dispersion is not faster than the expansion of Universe and so they keep their individuality even in the absence of dark matter…

Nevertheless, the study of galaxy clusters remains a very difficult subject full of surprises, for instance the intergalactic gas represents a large part of their total mass.

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