Methods for removing uncertainty in multi-criteria problems of effectiveness evaluation of decision-making

V V Moiseev¹, A V Terentyev², V R Rogov² and E A Karelina³

¹Belgorod State Technological University named after V.G. Shukhov, 46 Kostyukova St., Belgorod, 308012, Russia
²Moscow Automobile and Road State Technical University (MADI), 64 Leningradsky Avenue, Moscow, 125319, Russia
³Moscow State University of Technology “STANKIN”, 1 Vadkovsky lane, Moscow, 127055, Russia

E-mail: din_prof@mail.ru

Abstract. A scientific approach to methods of technical support, economic and organizational management, as well as planning the activities of enterprises often dictates the need to make decisions not on one but on several performance indicators simultaneously. The presence of a number of situations with one or another degree of uncertainty in modern difficult conditions of forecasting requires the involvement of a certain mathematical apparatus for its description. The task of removing uncertainty is becoming complicated if it is necessary to evaluate the effectiveness of control actions in the system according to several performance criteria. Therefore, at present, the development of mathematical methods is actively underway, which allows solving applied problems in a multi-criterial formulation. The article describes the approach to solving multi-criteria problems based on the principle of observing the hierarchical ratio of probabilities of possible states of the external environment of the study.

1. Introduction

At present one of the promising trends of enhancing a set of service properties of such bronzes is alloying them with superdispersed powders (SDP). Introduction of their small amount into the melt before a crystallization process allows increasing strength properties of castings [1, 2]. But the mechanism of interaction with lead-tin-based bronzes, as well as the process regularities of such modification, is not studied profoundly. However, such modification of copper alloys is promising from several points of view.

This paper presents the investigation of the influence of different content of additives of the pre-treated aluminium oxide powder on the structure of lead-tin-base bronze under formation.

Uncertainty is not only a condition that has to be taken into account when solving various kinds of problems, but also a factor inherent in any research. The uncertainty of information that one encounters when conducting research has a very different nature and significance. It may be:
- uncertainty caused by the lack of information and its reliability due to technical, social and other reasons;
- uncertainty caused by the behavior of the external environment;
- uncertainty generated by a large number of objects or elements included in a situation;
- "fundamental" uncertainty caused by the lack of material on the topic of research.
The presence of a number of situations that have a certain degree of uncertainty in the research requires a certain mathematical apparatus to describe this process. The task of removing uncertainty is complicated if it is necessary to evaluate the effectiveness of control actions in the system according to several performance criteria. [1]. Therefore, at present, the development of mathematical methods is actively underway, which allows solving applied problems in a multi-criteria formulation [2, 3].

2. Problem Statement
The solution of this task is complicated not only by the fact of multi-criteriality, but by a sufficiently large number of criteria and possible solutions. For example, in the work [4] a method for assessing the quality of a car according to individual groups of indicators is given, and the parameters are selected that ensure the selection and classification of trucks, which contains 22 points. The authors of this study propose the following solution sequence:
1. Choice of indicators of the car, which are the most important from the point of view of the consumer.
2. Definition of hierarchical classification of selected indicators.
3. Determination of complex indicators of the group quality using the "method of profiles".
4. Determination of weight coefficients of each group of indicators using the “method of analyzing hierarchies” [5, 6].
5. Summation of works of complex indicators of the quality of groups and their weight coefficients, obtaining an integral criterion for the quality of a car.

This approach removes the ambiguity of the problem and allows choosing the solution from a variety of alternatives of various types based on criteria that are expressed in both quantitative and qualitative characteristics [16]. The method consists in hierarchical decomposition of the system into simpler components and further processing of the sequence of judgments by a decision maker (DM) by pairwise comparisons. Moreover, the criteria for evaluating experts are formalized and do not require the use of additional computational procedures. Nevertheless, the application of this approach has a significant share of subjectivity, characteristic of any kind of expert assessments.

In addition to the described ones, there is a large number of methods used today in order to differentially evaluate the individual properties of a process or subject of research to compare them. Conventionally, they can be divided into two groups:
1. Methods that reduce the solution of a multi-criteria problem to obtaining a single integral quality criterion. As a rule, they contain a significant drawback for practical application, namely, a number of criteria may have mutually exclusive values or have certain limitations from a regulatory and legal point of view.
2. Methods to determine the weight coefficients of possible solutions for each of the criteria, based on the subjective expert assessment of the significance of each criterion. The disadvantage in this case is the need to give a quantitative relationship between the individual decisions for each criterion. At the same time, with a significant number of possible decisions, according to a number of criteria, the degree of uncertainty of the final decision increases significantly.

To reduce the negative impact of the deficiencies inherent in the abovementioned methods for evaluating processes, a mathematical apparatus is needed that allows meeting certain criteria:
1. To allow various groups of indicators to objectively and differentially take into account the state of quality of an object, taking into account the numerous environmental factors of its operation
2. To determine the pattern of transition of an object from one state to another in conditions of instability (dynamism) of quality indicators over time.

3. Research Questions
Traditionally, the conditions of operation of control systems are considered taking into account the situation in which one has to make control decisions. They are divided into deterministic and stochastic ones. Deterministic tasks are characterized by complete and reliable information on all input parameters. In stochastic tasks, all or some of the input parameters are undefined or random. There are two types of situations:
1) parameters are random, but their probabilistic characteristics are known (tasks with risk); 2) parameters are random, and there are no grounds for assessing their probabilistic characteristics (tasks with an uncertain situation).

In these situations, any problem of optimizing the decision is characterized by three main concepts:
- set of possible solutions (actions);
- many types of environment or state of the environment (SE);
- efficiency of any decision for each type of situation.

Below the authors will use the following notation:
- \( m \) – the number of possible options for an action;
- \( n \) – the number of possible SEs;
- \( a_{ij} \) – efficiency of the \( i \)-th action for the \( j \)-th state of the SE, \( = \frac{1}{m}, j = 1, \ldots, n \),

The matrix of efficiencies of various actions at different SEs is:

\[
\begin{pmatrix}
    a_{11} & a_{12} & \cdots & a_{1n} \\
    a_{21} & a_{22} & \cdots & a_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{m1} & a_{m2} & \cdots & a_{mn}
\end{pmatrix}
\]

(1)

If the SE is known (deterministic task), then the optimal action \( i^* \) is determined from the condition

\[
a_{i,j^*} = \max_i a_{ij}.
\]

(2)

In some cases, when choosing a solution, the concept of risk is introduced [7], defining it as the difference between the maximum efficiency and efficiency during given activity associated with the same SE, i.e.

\[
b_{ij} = \max_i a_{ij} - a_{ij}.
\]

(3)

Obviously, when choosing the optimal action, the risk is equivalent to zero, and for any other action it is greater than zero. It is also obvious that for any fixed SE

\[
b_{ij} + a_{ij} = Const = \max_i a_{ij}
\]

(4)

If the role of the initial data on the SE is played by the probability distribution (a stochastic task with risk), then the measure of the efficiency of an action is the mathematical expectation of the efficiency

\[
\overline{a}_i = \sum_{j=1}^{n} P_j a_{ij},
\]

(5)

where \( P_j \) is the probability of the \( j \)-th SE occurring.

\[
\sum_{j=1}^{n} P_j = 1, P_j \geq 0, j = 1, 2, \ldots n
\]

(6)

and the optimal should be considered the action for which the expectation of efficiency will be maximum, i.e.

\[
\overline{a}_i = \max_i \overline{a}_i = \max_i \sum_{j=1}^{n} P_j a_{ij}
\]

(7)

Sometimes they introduce the concept of mathematical expectation of risk (average expected risk) when choosing the \( i \)-th action.

\[
\overline{b}_i = \sum_{j=1}^{n} P_j b_{ij}
\]

(8)

In these cases, the optimum should be considered the action, which corresponds to the minimum value of the expectation of risk, i.e.
\[ \bar{b}_i = \min_i \bar{b}_i = \min_i \sum_{j=1}^{n} P_i b_{ij}. \] (9)

It is easy to show that \( \bar{b}_i + \bar{a}_i = Const \), and the choice of the optimal action can be carried out with equal success either from the requirement of appeal to a maximum of \( \bar{a}_i \), or from the requirement of appeal to a minimum of \( \bar{b}_i \). Both conclusions always lead to the same action, called a priori Bayes action [8–10]. At the same time, there remains the risk of choosing a solution that does not correspond to the real state of SE or the “nature” of factors. It is possible to single out a number of information situations (certainty, lack of information and uncertainty) and the main methods used to remove uncertainty, namely: methods based on expert assessments; methods for obtaining an a priori probability distribution; methods of providing guaranteed levels of solution values; zoning methods.

4. Research Methods

The most difficult is the situation if information about the probabilistic characteristics of the SE is missing. There are two types of situations:

1. The choice of the type of situation is carried out by the “rational adversary”. If the parties pursue opposing interests, then a conflict is created, which is studied by the mathematical game theory. The choice of the optimal solution (optimal strategy) in conflict situations is relatively not difficult precisely because of the antagonism of the parties involved in the game. Indeed, a “reasonable opponent” will not "play at a giveaway," but will try to make the best decision for himself. This circumstance allows the choice of the solution of each of the parties to use the principles of minimax and maximin. To determine optimal strategies in conflict situations, game theory has a number of powerful and very effective methods, widely reported in the literature.

2. “Nature” acts as an adversary. As already noted, the mathematical theory of games with nature deals with the study of this type of situation. Unlike the first type of situations with uncertainty, there is no antagonism in games with nature, and the principles of minimax degenerate into principles of extreme caution. But a SE exists objectively. It does not help a decision maker, but does not prevent him from doing it. Consequently, the choice of a solution based on the minimax principle in such situations may not be the most successful [11].

In the second case, it is possible to apply methods based on the use of subjective criteria. Let us consider some of them.

According to the “principle of insufficient foundation” proposed by Pierre-Simon Laplace, the action should be considered as optimal that corresponds to

\[ \max_i \frac{1}{n} = \sum_{j=1}^{n} a_{ij}. \] (10)

This criterion is subjective because the probabilities of SE are derived by the subject from ignorance of the true state. However, the law of uniform distribution can be inferred from the knowledge that some outcomes do not have a greater objective possibility of appearance than others, but not out of ignorance of whether some outcomes have a greater objective possibility of appearance in comparison with others or not.

According to the “principle of extreme caution” (A. Wald's maximin criterion), the action for which the performance indicator takes the greatest value for the most unfavorable SE should be considered optimal. Thus,

\[ \max_j \min_i a_{ij}. \] (11)

The minimax risk criterion proposed by L. Savage is also based on the extremely careful choice of the solution. According to this criterion, it is necessary to choose an action for which the risk value takes the smallest value in the most unfavorable situation, i.e.

\[ \min_i \max_j b_{ij}. \] (12)
Both of these criteria (by A. Wald and L. Savage) are subjective, because they obviously set up the most unfavorable situation, that is, they are applicable only to idealized practical solutions. But the SE exists objectively, irrespective of the solutions chosen.

Therefore, in the general case, there are no grounds for extreme pessimism in choosing a solution. In order to occupy a more balanced position, the criterion proposed by A. Hurwitz is applied, the evaluation function of which lies between the points of view of ultimate optimism and extreme pessimism. This criterion is derived from the classical criteria. According to the criterion of A. Hurwitz (the criterion of pessimism-optimism) it is unwise, taking into account the smallest gain, not to take into account the biggest gain; it is necessary to subjectively introduce a certain coefficient $\alpha$, and consider that action as optimal for which the following condition is fulfilled

$$\max_i \left[ \alpha \cdot \min_j a_{ij} + (1-\alpha) \max_j a_{ij} \right], \text{ where } 0 \leq \alpha \leq 1.$$  \hspace{1cm} (13)

It is easy to see that with $\alpha = 0$ this criterion turns into a criterion of an absolute optimist, with $\alpha = 1$ it coincides with A. Wald’s maximin criterion, and with it forms a mixture of pessimistic and extremely optimistic success estimates in forthcoming actions.

There is a number of “derived criteria” that are widely represented in specific literature [12, 13]: Hodges-Lehmann criterion, Hermeyer criterion, BL (MM) criterion, etc. The choice of any of the four criteria considered cannot be objectively justified, and actions recommended by these subjective criteria often do not match.

5. Results

Suppose that the SE probability distribution $P_j$ is unknown, but there is a basis (from qualitative considerations) for placing them in the priority series according to the principle of “weak dominance”.

$$P_1 > P_2 > \ldots > P_n$$  \hspace{1cm} (14)

The authors present one of the possible approaches for determining the numerical values of $P_1$ in this situation:

for $n = 3$: $P_1 = \frac{3}{6}, P_2 = \frac{2}{6}, P_3 = \frac{1}{6}$

for $n = 4$: $P_1 = \frac{4}{10}, P_2 = \frac{3}{10}, P_3 = \frac{2}{10}, P_4 = \frac{1}{10}$

for $n = 5$: $P_1 = \frac{5}{15}, P_2 = \frac{4}{15}, P_3 = \frac{3}{15}, P_4 = \frac{2}{15}, P_5 = \frac{1}{15}$

Obviously, for each of these options $\sum_{j=1}^{n} P_j = 1$

It is also obvious that in the general case (with the number of possible SEs = $n$) it is obtained

$$P_1 = \frac{n}{x}, P_2 = \frac{n-1}{x}, P_3 = \frac{n-2}{x}, \ldots, P_n = \frac{1}{x}.$$  \hspace{1cm} (15)

Consequently

$$\frac{n + (n-1) + (n-2) + \cdots + 1}{x} = 1,$$  \hspace{1cm} (16)

In the numerator of the left side of this equation is an arithmetic progression, then

$$x = \frac{n(n+1)}{2}.$$  \hspace{1cm} (17)

Thus

$$P_j = \frac{2(n-j+1)}{n(n+1)}, \quad j = 1, 2, \ldots n.$$  \hspace{1cm} (18)
6. Conclusion
In general, it is impossible to say which is better: minimize the maximum risk (Savage's criterion) or maximize minimum efficiency (Wald criterion), accept states of nature equally probable (Laplace criterion) or maximize a mixture of minimum and maximum efficiency by subjectively introducing the coefficient of optimism (Hurwitz criterion) [14]. Each of the four subjective criteria prescribes to take as its optimal decision, and it is not clear which of the criteria to give preference, that is, what solution to choose. Therefore, the developed method, which allows obtaining an analytical solution to a multi-criteria task in conditions of an uncertain state with minimal information about the state of research environment (availability of priority criteria), is an objective method of finding a subjective solution [15].

References
[1] Podinovskiy V V 1982 Pareto optimal solutions for multicriteria problems Science 9–64
[2] Terentyev A V 2015 Methods for solving motor transportation problems Current problems of science and education [in Russian – Sovremennye problemy nauki i obrazovaniya] I Available at: http://www.science-education.ru/125-19863
[3] Kolesov Yu B and Sinichenkov Yu B 2013 Component technology of mathematical modeling: a tutorial (St Petersburg: St Petersburg State University) p 233
[4] Krakhalova A V and Faskhiev F A 2005 Methods for assessing the quality of cars Marketing in Russia and abroad [in Russian – Marketing v Rossii i za rubezhom] 4
[5] Saaty T and Kearnes K 1991 Analytical Planning. The Organization of Systems Radio and communication 224
[6] Saaty T L 2008 Decision Making with Dependencies and Feedbacks: Analytical Networks (LKI Publishing house) p 360
[7] Antonova A S and Aksenov K A 2012 Multi-criteria decision making under risk based on the integration of multi-agent, simulation, evolutionary modeling and numerical methods Engineering Journal of Don 4(2) Available at: http://www.ivdon.ru/magazine/archive/n4p2y2012/1466
[8] Muschik E and Mueller P 1990 Methods for making technical decisions MIR: Modernization. Innovation. Development [in Russian – Mir. Modernizatsija. Innovatsija. Razvitie] 208
[9] Taha H A 2005 Introduction to Operations Research (Publishing house “Williams”) p 912
[10] Chernorutzkiy I G 2005 Methods of decision making (BHV-Peterburg) p 416
[11] Prudovskiy B D and Terentyev A V 2014 Vector optimization Proc. of the 2nd int. sci. and practical conf. “Innovative planning and management systems in transport and engineering” 1 64
[12] Prudovskiy B D 2015 Methods for solving multi-criteria motor transport problems Civil Engineering Bulletin [in Russian – Vestnik grazhdanskih inzhenerov] 2(49) 154–159
[13] Afanas'ev A S, Egoshin A M and Alekseev S V 2018 Justification of logistical approach application in road safety management IOP Conference Series: Earth and Environmental Science 194(7)
[14] Terentyev A V, Yefimenko D B and Kareлина M Yu 2017 Zoning methods as methods for optimizing road transport processes Civil Engineering Bulletin [in Russian – Vestnik grazhdanskih inzhenerov] 6(65) 291–294
[15] Terentyev A V 2019 Scientific and methodical approach to multi-criteria assessment of the life of the vehicle, Dissertation abstract, p 43
[16] Nitsevich V F, Moiseev V V and Sudorgin O A 2017 To the question of effectiveness of government management European Proceedings of Social and Behavioural Sciences 35 933–944