A Review of the Magnitudes of the CKM Matrix Elements

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Flavour mixing is described within the Standard Model by the Cabibbo–Kobayashi–Maskawa matrix elements. With the increasingly higher statistics collected by many experiments, the matrix elements are measured with improved precision, allowing for more stringent tests of the Standard Model. In this paper, a review of the current status of the absolute values of the CKM matrix elements is presented, with particular attention to the latest measurements.

1. INTRODUCTION

The Standard Model (SM) accounts for flavour–changing quark transitions in terms of a V–A charged weak current operator $\mathcal{J}^\mu$ that couples to the $W$ boson according to the interaction Lagrangian: $L_{\text{int}} = -\frac{g}{\sqrt{2}}(\mathcal{J}^\mu W_\mu^+ + \mathcal{J}^{\mu+} W_\mu^-)$, where for quark transitions $\mathcal{J}^\mu = \Sigma_{i,j} V_{ij} J^\mu_{ij} = \Sigma_{i,j} \pi_i \gamma^\mu \frac{1}{2} (1 - \gamma_5) V_{ij} d_j$ ($i, j$ run over the three quark generations). The $V_{ij}$ are the CKM matrix elements. The field operator $u_i (d_j)$ annihilates the $u, c, t$ ($d, s, b$) quarks. The operator $W_{\mu}^+$ annihilates a $W^+$ or creates a $W^-$. The reverse is true for $W_{\mu}^-$. Thus, the CKM matrix $V$ can be regarded as a rotation of the quark mass eigenstates $d, s, b$ to a new set $d', s', b'$ with diagonal coupling to $u, c, t$. The standard notation to represent it is:

$$
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix} =
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
$$

which is an almost unitarity matrix. However, none of the off-diagonal elements are exactly zero, so generation changing transitions between quarks are possible. The values of the CKM matrix elements are fundamental parameters of the SM and cannot be predicted. In the following, a review of the current values of the absolute values of the CKM matrix elements is presented. For each matrix element, all measurements which lead to the measurement of that matrix element are presented. A detailed description of the measurements is only given for the most recent results.

2. FIRST ROW OF THE CKM MATRIX

2.1. $|V_{ud}|$

Three main techniques lead to the measurement of $|V_{ud}|$, with differing precision. The most precise determination comes from superallowed $0^+ \rightarrow 0^+$ nuclear $\beta$ transitions. The intensity of any $\beta$ transition is expressed as an $ft$ value, which depends on the transition energy ($Q_{EC}$), the half life of the $\beta$ emitter, and the branching ratio for the particular transition of interest. Thirteen superallowed transition have an $ft$ value measured to a precision between 0.03% and 0.3% [1]. From the experimentally determined $ft$ value, applying calculated transition correction terms in the SM, the corrected quantity $Ft$ [1], should be identical for all cases. From $Ft$, $|V_{ud}|$ can be extracted. An improved measurement of the $Q_{EC}$ values of $^{50}$Mn and $^{54}$Co [2], which removes the discrepancy between the $Ft$ values of those two elements and the averages $\langle Ft \rangle_{\text{MC}}$ of the most precise thirteen superallowed transitions [1], leads to a value of $|V_{ud}| = 0.97408(26)$. It has a precisions of 0.03% and its error is dominated by the theoretical uncertainties on the radiative and isospin–symmetry–breaking corrections.

Moreover, from the knowledge of the neutron lifetime, $\tau_n$, and the axial–vector/vector couplings, $g_A \equiv G_A/G_V$, another measurement of $|V_{ud}|$ can be performed, leading to $|V_{ud}| = 0.9746(4)\tau_n(18)g_A(2)_{RC}$ [3], where the errors are due to $\tau_n$, $g_A$ and $RC$, where $RC$ denotes the entire effects of electroweak radiative corrections, nuclear structure,
and isospin violating nuclear effects. The error is dominated by the uncertainty on $g_A$. Another measurement of $\tau_\alpha [4]$ leads to a higher value of $|V_{ub}|$. Future studies are expected to resolve this inconsistency.

Finally, the pion decay $\pi^+ \to \pi^0 e^+ \nu$ can be used to extract $|V_{ud}|$, as its rate depends on $|V_{ud}|^2$. This approach is theoretically very clean, as it is free from nuclear structure uncertainties, but the decay is disadvantaged by a low branching ratio. The most precise measurement is from PIBETA [5] and gives $|V_{ud}| = 0.9728(30)$, with an error of 0.3%, and in agreement with the above values.

### 2.2. $|V_{us}|$

The most precise way to measure $|V_{us}|$ comes from semileptonic kaon decays, whose rate is proportional to $|V_{us}|^2 f_+(0)^2$, where $f_+(q^2)$ at $q^2 = 0$ is the $K^0 \to \pi^+$ transition at zero moment transfer $[q^2 = (p_K - p_\pi)^2 = 0]$, in the limit $m_u = m_d$ and $a_{em} \to 0$. Using worldwide averages of lifetimes, branching ratios, phase space integrals, and the radiative and SU(2) breaking corrections for $K_L \to \pi e\nu$, $K_L \to \pi \mu\nu$, $K_L \to \pi e\nu$, $K^\pm \to e\nu\mu\nu$, globally indicated as $K_{\ell 3}$, from KTeV, NA48, KLOE and Istra+, the FlaviaNet Kaon group [6] extracts $|V_{us}|/f_+(0) = 0.21664(48)$, which has a precision of $\sim 0.2\%$. Differences between the current measurements of the branching fractions and the past ones depend on a proper treatment of the radiative effects. Choosing $f_+(0) = 0.964(5)$ from UKQCD-RBC [8], the value of $|V_{us}|$ is $|V_{us}| = 0.2246 \pm 0.0012$, where the dominant error is experimental. Note that indirect tests of the form factor $f_+(0)$ from the $K_{\ell 3}$ decays requires a further understanding of the disagreement between the NA48 result [7] and the other measurements.

Moreover, using the ratio of kaon and pion leptonic decays $K^+ \to \mu^+\nu/\pi \to \mu^+\nu$, whose kaon rate is dominated by KLOE, FlaviaNet quotes $|V_{us}|/|V_{ud}|f_K/f_\pi = 0.2760 \pm 0.0006$ [6], where $f_K/f_\pi$ is the ratio of the kaon and pion decay constants in the limit $m_u = m_d$ and $a_{em} \to 0$. Using the form factor ratio from the MILC-HPQCD collaboration $f_K/f_\pi = 1.189(7)$ [9], the value $|V_{us}|/|V_{ud}| = 0.2321 \pm 0.0015$ is obtained, where the accuracy is limited by the knowledge of the decay constants.

These results together with the measured value of $|V_{ud}|$ from superallowed $\beta$ decays, can be used to test the unitarity of the first row of the CKM matrix [6], leading to 0.9999(9), neglecting $|V_{ub}|$ with $\chi^2/ndof = 0.65/1$.

Measurements of hadronic tau decays provide a new test of the value of the CKM matrix element $|V_{us}|$, although affected by a higher statistical error with respect to the kaon decays. The extraction of $|V_{us}|$ from hadronic tau decays involves moments of the invariant mass distributions of the final states hadrons [10]. The averaged value of $|V_{us}|$ is $|V_{us}| = 0.2159 \pm 0.0030$ [11], where the dominant contribution error is due to the experimental uncertainty. The average does not include correlations between different measurement as they are not available yet. A recent discussion on the size of the theoretical errors can be found in Ref. [12]. The $|V_{us}|$ value from tau decays is about 3\sigma lower than the value extracted from kaon decays. A new result presented by BABAR on the ratio of the branching fractions for $\tau^- \to K^-\nu$ and $\tau^- \to \pi^-\nu$ decays gives $|V_{us}| = 0.2256 \pm 0.0023$ [11], using $f_K/f_\pi$ from Ref. [9] and $|V_{ud}|$ from Ref. [2], whose error is dominated by uncertainties in particle identification. Although, this value of $|V_{us}|$ is consistent with the one from kaon decays, individually both branching fractions are lower than the universality predictions. More understanding both of the experimental (e.g. correlations in the averages) and theoretical issues is needed to solve the current discrepancy.

Finally, hyperon decays can lead to a further determination of $|V_{us}|$. The semileptonic decay of a spin $1/2$ hyperon involves the hadronic matrix elements of the vector and axial–vector currents. Supported by the fact that there are no first–order corrections to the vector form factor [13], and using an experimental measurement of the axial and vector form factor ratio, thus avoiding SU(3) breaking effects, a value of $|V_{us}| = 0.2250 \pm 0.0027$ [14] is obtained from four hyperon beta decays, where the quoted uncertainty is only experimental. The central value is consistent with the one from kaon decays. Contributions due to second order SU(3) breaking are not taken into account.
Figure 1: $B \rightarrow \pi \ell \nu$ measured branching fractions and their average [19] (left plot) and extracted value of $|V_{ub}|$ [19] (right plot) for different computations: Ball–Zwicky [17], which stands for the QCD light cone sum rule, HPQCD [15] and FNAL [16], which stands for Fermilab/MILC.

2.3. $|V_{ub}|$

Exclusive and inclusive semileptonic decays (e.g. $B \rightarrow \pi \ell \nu$ and $B \rightarrow X_u \ell \nu$, where $X_u$ indicates the fragmentation products from the $u$ quark, respectively) rely on different experimental and theoretical approaches, thus providing a complementary way to extract $|V_{ub}|$.

$|V_{ub}|$ can be extracted from exclusive charmless semileptonic decays, $B \rightarrow \pi \ell \nu$, where the corresponding rate is related to $|V_{ub}|$ by the form factor $f_+(q^2)$, where $q^2$ is the momentum transfer squared to the lepton pair. Non perturbative methods for the calculation of the form factors include unquenched lattice QCD, where we use the HPQCD [15] and Fermilab/MILC [16] calculations, which differ for the treatment of the $b$ quark, and QCD light cone sum rules [17]. Consistent results are obtained using earlier quenched QCD calculations [18]. Measurements of the $B \rightarrow \pi \ell \nu$ decays have been performed by CLEO, BABAR and Belle, exploiting different analysis techniques, where results are presented for the full $q^2$, $q^2 > 16$ GeV and $q^2 < 16$ GeV ranges. The last two phase space regions correspond to regions where the lattice and QCD light cone sum rule calculations of the form factors are restricted to, respectively. The measurement techniques fall into two broad classes: untagged and tagged, depending on whether the $B$ in the event that does not decay into the $\pi \ell \nu$ final state is tagged or not. Higher statistics and higher background discriminate the first method from the second. The corresponding measurements of the total branching ratio for all the collaborations and their average is shown in Fig. 1 (left plot). From the average, and using both lattice QCD and QCD light cone sum rules, the value of $|V_{ub}|$ is shown in Fig. 1 (right plot). The $|V_{ub}|$ results coming from different theoretical calculations are consistent among themselves. The dominant systematic in the $|V_{ub}|$ extraction is due to the theoretical error. An improved treatment of the QCD light cone sum rule calculation was recently presented [20, 21], eventually giving consistent results for the mean value and the error on $|V_{ub}|$. Concerning lattice QCD, an effort is underway to perform a simultaneous fit to the experimental and lattice data using the model independent $z$ parametrization for the form factor [24]. In particular, a 12 bin $q^2$ spectrum was measured by BABAR [25]. More high precision $q^2$ measured spectra are foreseen in the future.

Moreover, still regarding exclusive semileptonic decays, experimental measurements of the $B \rightarrow \rho \ell \nu$ branching ratio have been performed by BABAR, Belle and CLEO and will provide a test of the $|V_{ub}|$ extraction from $B \rightarrow \pi \ell \nu$.
decays, once the corresponding form factors are computed.

The measurement of the inclusive decays rate for \( B \rightarrow X_u \ell \nu \) decays is affected by a large background of the order \( |V_{ub}/V_{cb}|^2 \sim 1/50 \), due to the look–alike \( B \rightarrow X_c \ell \nu \) decays. To suppress this background stringent kinematic cuts are applied. Thus, a partial branching fraction, \( i.e. \) limited to the particular kinematic region selected, which ranges from \( \sim 20\% \) to \( \sim 60\% \) of the total rate, is measured. This challenges theory. Whilst the total branching fraction can be computed using Heavy Flavour Expansion (HQE) and QCD perturbation theory, the partial rate needs further theoretical tools, which have been the subject of intense theoretical effort, especially in the last years.

The kinematic cuts are applied using the following variables: the lepton energy \( (E_\ell) \), the invariant mass of the hadron final state \( (M_X) \), the light–cone distribution \( (P^+ = E_X - |p_X|, E_X \) and \( p_X \) being the energy and the magnitude of the 3–momentum of the hadronic system) and a two dimensional distribution in the electron energy and \( s^{\max} \), the maximal \( M_X^2 \) at fixed \( q^2 \) and \( E_\ell \). The differential rate needed from theory to extract \( |V_{ub}| \) from the experimental results has been calculated using several different theoretical approaches. In chronological order, they are BLNP [26] (a shape function approach, where the shape function represents the momentum distribution function of the \( b \) quark in the \( B \) meson), DGE [28, 29], (a resummation based approach), GGOU [30] (an HQE based structure function parametrization approach) and ADFR [31, 32] (a soft gluon resummation and analytic time–like QCD coupling approach). Concerning BLNP, recent NNLO corrections [27] were presented. The models depend strongly on the \( b \) quark mass, except for ADFR, so it is very important to use a precise determination of the \( b \) quark mass. The fit performed to obtain the value of the \( b \) quark mass is described in section 3.3. The same value of the mass is used for the four models for consistency, translated to the different mass schemes as needed [19]. The results obtained by these methods and the corresponding averages are shown in Fig. 2.

In inclusive decays, higher values of \( |V_{ub}| \) than from the exclusive decays are found, although consistent within 1\( \sigma \) from the Fermilab/MILC result. The error on \( |V_{ub}| \) from inclusive decays is smaller than from exclusive decays. The predicted value of \( |V_{ub}| \) from \( \sin 2\beta \) favours values closer to the exclusive one [33]. Intense theoretical and experimental activity is undergoing to compare the different methods among themselves and with the experimental results [34]. A slightly higher value of \( |V_{ub}| \) \( (|V_{ub}| = (4.87 \pm 0.24_{-0.38}^{+0.35}) \times 10^{-3}) \) is obtained by another approach (BLL [35]), OPE based, where a combined cut in the \( (M_X, q^2) \) plane is proposed to reduce the theoretical uncertainties. Moreover, a different strategy also has been adopted to overcome the problem of the knowledge of the shape function. As the leading shape function can be measured in \( B \rightarrow X_s \gamma \) decays, there are prescriptions that relate directly the partial rates for \( B \rightarrow X_s \gamma \) and \( B \rightarrow X_u \ell \nu \) decays [36–39], thus avoiding any parametrization of the SF. However, uncertainties due to the sub–leading SF remain. Results with this method have been obtained by \( BaBar \) [40] \( |V_{ub}| = (4.92 \pm 0.32 \pm 0.36) \times 10^{-3} \), and Ref. [41], using the \( BaBar \) results in Ref. [42], \( |V_{ub}| = (4.28 \pm 0.29 \pm 0.26 \pm 0.28) \times 10^{-3} \), \( |V_{ub}| = (4.40 \pm 0.30 \pm 0.41 \pm 0.23) \times 10^{-3} \), using Refs. [37] and [38, 39], respectively.

There are several determinations of \( |V_{ub}| \). Finally, we choose to quote as result for the inclusive decays the one from ADFR \( |V_{ub}| = (3.76 \pm 0.13 \pm 0.22) \times 10^{-3} \).

Very recently, a preliminary result from Belle using a multivariate analysis [43], in which \( \sim 90\% \) of the total rate is measured, has been presented. This experimental measurement is extremely interesting as it will help in a further understanding of \( |V_{ub}| \) from inclusive decays.

3. SECOND ROW OF THE CKM MATRIX

3.1. \( |V_{cd}| \)

The most precise measurement of the \( |V_{cd}| \) matrix element comes from neutrino production of charm at high energy, where the underlie process is a neutrino interacting with a \( d \) quark, producing a charm quark that fragments into a charmed hadron. The extracted value of \( |V_{cd}| \) is \( |V_{cd}| = 0.230 \pm 0.011 \) [3] from the average of several experiments. The dominant error comes from the mean semi–muonic branching ratio for charmed hadrons produced in neutrino anti–neutrino scattering, followed by the QCD scale uncertainty.
of the q factor, which measures the probability to form the final state hadron, has to be input. The form factor is a function confined to the hadronic current. To extract the CKM matrix element from the semileptonic decay rate the form factor, where semileptonic decays are a preferred way to determine the matrix elements as strong interaction effects are consistent value of new tagged analysis [45], which is partially overlapping with the untagged analysis in Ref. [44], and it presents a

experimental and theoretical, respectively. The theory error is dominant. Very recently, CLEO-c has published a new tagged analysis [45], which is partially overlapping with the untagged analysis in Ref. [44], and it presents a consistent value of |V_{cd}|.

The CKM matrix element |V_{cd}| can be determined also through the study of semileptonic decays of the D meson, where semileptonic decays are a preferred way to determine the matrix elements as strong interaction effects are confined to the hadronic current. To extract the CKM matrix element from the semileptonic decay rate the form factor, which measures the probability to form the final state hadron, has to be input. The form factor is a function of the q^2, the square of the transfer momentum to the lepton neutrino pair. There are a variety of model dependent calculations of the form factor. Here we will adopt the lattice QCD results [22]. Decay rates have been measured by Belle [23] and CLEO-c [44]. Their average leads to the value |V_{cd}| = 0.218 ± 0.007 ± 0.023, where the errors are experimental and theoretical, respectively. The theory error is dominant. Very recently, CLEO-c has published a new tagged analysis [45], which is partially overlapping with the untagged analysis in Ref. [44], and it presents a consistent value of |V_{cd}|.
3.2. \( |V_{cs}| \)

The most precise measurement of the CKM matrix element \( |V_{cs}| \) comes from the study of semileptonic \( D \) decays. The average of the \( BaBar \) [47], Belle [23] and CLEO-c [44] results is \( |V_{cs}| = 0.99 \pm 0.01 \pm 0.10 \), where the errors are experimental and theoretical, respectively, and the form factors from Ref. [22] are adopted. Results from the CLEO-c tagged analysis [45], very recently published, are consistent with the one reported in Ref. [44].

The leptonic \( D \) decays, since no hadronic interactions are present in the leptonic final state \( \ell \nu \), provide a very clean environment to determine \( |V_{cs}| \). In these decays, strong interaction effects can be parametrized by the pseudoscalar decay constant \( f_D \), which describes the amplitude for the \( c \) and \( \bar{d} \) quarks within the \( D^+_c \) to have zero separation, a condition necessary to annihilate into the virtual \( W^+ \) boson that produces the \( \ell \nu \) pair. Results from \( BaBar \) [48], Belle [49, 50] and CLEO-c [51], assuming \( f_{D_s} \) computed by lattice QCD [9] give: \( |V_{cs}| = 1.07 \pm 0.08 \) [3], where the error is dominated by the determination of \( f_{D_s} \). This is the best measurement of \( |V_{cs}| \), without assuming unitarity of the CKM matrix.

Decays of the \( W \) boson at LEP2 have been used to determine \( |V_{cs}| \) by Delphi, tagging the decay \( W \to cs \). The result is \( |V_{cs}| = 0.94^{+0.32}_{-0.26} \pm 0.13 \) [52], where the errors are statistical and systematic, respectively. Moreover, using hadronic \( W \) decay and assuming the unitarity of the CKM matrix, from averages of the LEP experiments, \( |V_{cs}| = 0.977 \pm 0.014 \) [53] is obtained, where the total error depends on the other CKM matrix elements, but it is dominated by the experimental uncertainty.

The determination of \( |V_{cs}| \) from neutrino and antineutrino interactions suffers from the uncertainty of the \( s \)-quark sea content leading to \( |V_{cs}| > 0.59 \) [46].

3.3. \( |V_{cb}| \)

The \( |V_{cb}| \) matrix element is determined from semileptonic exclusive and inclusive \( b \to c \ell \nu \) decays, which rely on different theoretical calculations. Several results were presented recently by \( BaBar \) and Belle. The determination of \( |V_{cb}| \) from exclusive \( b \to c \ell \nu \) decays is based on the \( B \to D^{(*)} \ell \nu \) decays, for which, in the assumption of infinite \( b \) and \( c \) quark masses, the form factors describing the \( B \to D^{(*)} \) transitions depend only on the product, \( w \), of the initial, \( v \), and final, \( v' \), state hadron four–velocities, \( w \equiv v \times v' \), and relies on a parametrization of the form factors using the Heavy Quark Symmetry (HQS) [54–56] and a non–perturbative calculation of the form factor normalization at \( w = 1 \), which corresponds to the maximum momentum transfer to the leptons. We adopt the parametrization from Ref. [57], and lattice QCD to correct the normalization of the form factor at \( w = 1 \), due to the finite quark masses. Experimentally, the \( w \) spectrum is measured and \( |V_{cb}| \) is obtained from an extrapolation of the measured \( w \) spectrum to 1. Several analyses from \( BaBar \) [58, 59, 61, 62], and Belle [63], which adopt different experimental techniques, were recently presented. In particular, in Ref. [61] \( B \to D^{(*)} \ell \nu \) are selected applying for the first time a global fit to \( D^{(*)} \ell \) reconstructed combinations in a three dimensional space of kinematic variables to determine their branching fractions and the form factor parameters. The form factors for the \( B \to D \ell \nu \) and \( B \to D^* \ell \nu \) decays are \( G(w) \) and \( F(w) \), respectively. The bi–dimensional plots of the form factor at \( w = 1 \) times \( |V_{cb}| \) versus the slope parameter for the form factors \( \rho \) is shown in Fig. 3, whose fitted values are \( G(1)|V_{cb}| = (42.4 \pm 1.6) \times 10^{-3} \), \( \rho^2 = 1.16 \pm 0.05 \) and \( F(1)|V_{cb}| = (35.41 \pm 0.52) \times 10^{-3} \), \( \rho^2 = 1.19 \pm 0.06 \), where the two slopes are two different parameters. Assuming \( G(1) = 1.074 \pm 0.018 \pm 0.016 \) [16] and \( F(1) = 0.924 \pm 0.012 \pm 0.019 \) [64], where the errors are statistical and systematical, respectively, once corrected by a factor 1.007 for QCD effects, \( |V_{cb}| = (39.2 \pm 1.5 \pm 0.9) \times 10^{-3} \) and \( |V_{cb}| = (38.2 \pm 0.6 \pm 1.0) \times 10^{-3} \) [19] are obtained, for \( B \to D \ell \nu \) and \( B \to D^* \ell \nu \) decays, respectively, where the errors are \( \sim 10\% \), which means an improvement of \( \sim 50\% \) with respect to the previous results mainly thanks to Ref. [61], and \( \sim 3 \% \). The two results are completely consistent.

Inclusive \( b \to c \ell \nu \) decays, \( B \to X_c \ell \nu \), where \( X_c \) indicates the fragmentation products from the \( c \) quark, can be used to determine \( |V_{cb}| \) from their branching fraction and with parameters that describe the motion of the \( b \) quark in the \( B \) meson. These parameters, within the framework of the Heavy Quark Expansion (HQE), include the \( b \) quark mass, \( m_b \), \( |V_{cb}| \), \( m_b \) and other parameters can be extracted simultaneously from a global fit to the measured moment of the leptonic energy and hadronic mass spectra. Moreover, also moments from \( b \to s \gamma \) decays are included.
in the global fit, as at Leading Order they can be represented by the same shape function. Higher order corrections are expected to be small [67]. To note however that just a fit to the $b \rightarrow s\gamma$ moments tends to give a value of $m_b$ about 1σ lower than the one from $b \rightarrow s\gamma$ and $b \rightarrow c\ell\nu$ moments combined. However, the latest is consistent with the measurement coming from the study of bottomium resonances [68]. The fitted value of $|V_{cb}|$, using expressions in the kinetic scheme [65, 66], is: $|V_{cb}| = (41.67 \pm 0.43 \pm 0.08 \pm 0.58) \times 10^{-3}$ [19], where the errors are due to the global fit, the $B$ lifetime and theory , respectively. This value is more than 2σ higher than the corresponding values from the $B \rightarrow D^{(*)}\ell\nu$ decays, but twice more precise, especially with respect to the $B \rightarrow D\ell\nu$ decays, dominated by the experimental uncertainty. Whilst $B \rightarrow D\ell\nu$ decays account for ~ 70% of the total $b \rightarrow c\ell\nu$ rate, the contribution of resonant and non resonant decays to other charm states is not very well measured and may help to explain the difference in the $|V_{cb}|$ determination [69], whereas the current difference between the total exclusive and inclusive final states is ~ 10%.

4. THIRD ROW OF THE CKM MATRIX

4.1. $|V_{td}|$ AND $|V_{ts}|$

The top quark is expected to decay almost completely to a $W$ boson and a $b$ quark (the corresponding rate is > 99.8% at 90% CL), so the CKM matrix elements $|V_{td}|$ and $|V_{ts}|$ are measured indirectly from the $B\rightarrow \overline{B}$ oscillations mediated by box diagrams with top quarks, or from the loop–mediated rate of $K$ and $B$ decays. The major uncertainty in the extraction of the parameters comes from the theoretical hadronic uncertainties. The time–integrated measurements of $B^0\overline{B}^0$ mixing have been performed for both the $B_d$ and more recently also the $B_s$ mesons. The measured mass differences $\Delta m_d$ for the neutral $B_d$ meson mass eigenstates was measured by many collaborations, using a variety of different techniques. A high precision was achieved, and the corresponding average [3], assuming a decay width difference $\Delta \Gamma_d = 0$ and no $CP$ violation in mixing, is $\Delta m_d = (0.507 \pm 0.005) \text{ps}^{-1}$, dominated by the $B$–factories $\text{BABAR}$ and Belle. The statistical and systematic errors equally contribute to the final uncertainty. The squared mass difference $\Delta m_d^2$ is related to $|V_{td}|$ through the product of the $B_d$ decay constant and the bag factor: $f_{B_d} \sqrt{\Delta m_d^2}$. Using the unquenched lattice QCD calculation, $f_{B_d} \sqrt{\Delta m_d^2} = (225 \pm 25) \text{MeV}$ [70], the value

- $\Delta m_d = (0.507 \pm 0.005) \text{ps}^{-1}$
- $f_{B_d} \sqrt{\Delta m_d^2} = (225 \pm 25) \text{MeV}$
\[|V_{td}| = (8.0 \pm 0.9) \times 10^{-3}\] is obtained, whereas the theory uncertainty dominates.

The \(B_s^0 \rightarrow \rho^0\) oscillation has been observed for the first time in 2006 by CDF \[71\]. The measured mass difference is \(\Delta m_s = (17.77 \pm 0.10 \pm 0.07)\text{ps}^{-1}\), where the errors are statistical and systematic, respectively. Similarly, there are studies by D0 \[72\], which also hint to oscillations in \(B_s\) mixing at \(\sim 2\sigma\) level. Using the measured value of \(\Delta m_s\), and the product \(f_{B_s}\sqrt{B_{B_s}} = (270 \pm 30)\text{ MeV} \[70\] from unquenched lattice QCD calculations, the value \(\Delta m_s = (39.4 \pm 4.4) \times 10^{-3}\) is obtained, where again the dominant uncertainty is due the lattice QCD.

However, if the ratio \(\Delta m_d/\Delta m_s\) is calculated, the corresponding theoretical uncertainties decrease. Being \(f_{B_s}\sqrt{\Gamma_{B_s}}/f_{B_d}\sqrt{\Gamma_{B_d}} = 1.21 \pm 0.04 \[70\], the value of the ratio of the two mass differences is: \(|V_{td}|/|V_{ts}| = (0.206 \pm 0.001 \pm 0.007) \times 10^{-3}\), whose error is greatly reduced with respect to the individual computations.

The radiative penguins decays \(b \rightarrow d\gamma\) and \(b \rightarrow s\gamma\) constitute an independent way than \(B\) oscillation to measure the matrix elements \(|V_{td}|\) and \(|V_{ts}|\), respectively, as they are affected by different experimental and theoretical (there are penguin instead of box diagrams) uncertainties. Recent results were presented by both \(\text{BaBar}\) and Belle for exclusive \[73-75\] and inclusive \[76\] decays. The extracted values of \(|V_{td}|/|V_{ts}|\) are \(|V_{td}|/|V_{ts}| = (0.233 \pm 0.025 \pm 0.022) \times 10^{-3}\) and \(|V_{td}|/|V_{ts}| = (0.195 \pm 0.020 \pm 0.015) \times 10^{-3}\) for the \(\text{BaBar}\) and Belle exclusive analyses, respectively, and \(|V_{td}|/|V_{ts}| = (0.177 \pm 0.043 \pm 0.001) \times 10^{-3}\) for the \(\text{BaBar}\) inclusive analysis. They are consistent with the result from \(B\) oscillations, but with an error about \(5\sigma\) larger.

The measurement of the \(K \rightarrow \pi\ell\nu\) branching fraction can lead to the cleanest determination of \(|V_{td}V_{ts}^*|\). Only three events have been observed so far by E949 \[77\], but a statistically more accurate measurement is foreseen in the future by NA62 \[78\].

4.2. \(|V_{tb}|\)

All direct measurements of production and decay of the top quark have been performed by the CDF and D0 collaborations at Fermilab. Measuring the ratio of the top quark branching fractions \(R = (Bt \rightarrow Wb)/(Bt \rightarrow Wq)\), a value of \(|V_{tb}|\) can be extracted assuming unitarity \(R = |V_{tb}|^2/[|V_{tb}|^2 + |V_{ts}|^2 + |V_{td}|^2]\). Both results from CDF \[79\] and D0 \[80\] are available. The limit is \(R = 0.97^{+0.09}_{-0.08}\) with a total uncertainty of about 9%, where the uncertainty is statistical and systematic. The largest uncertainty comes with the limited statistics. The corresponding limit on \(|V_{tb}|\) is \(|V_{tb}| > 0.89\) at 95\% CL.

Single top quark events can be used to study the \(Wtb\) coupling and thus, without any unitarity assumption, directly extracting \(|V_{tb}|\). The first observation of the single top production is presented by D0 \[81\]. Similarly, evidence for single top productions is found in the CDF data \[82\]. Higher statistics was used, but a lower cross section is indicated. D0 measures a single top cross section \(\sigma = (4.9 \pm 1.4)\text{ pb}\), where the uncertainty is statistical and systematic. The largest uncertainty comes with the limited statistics. The extracted value of \(|V_{ub}|\) is \(|V_{ub}| = (1.3 \pm 0.2)\).

5. SUMMARY

A review of the current status of the CKM matrix elements is given, with particular attention to the latest results. For each matrix element, there are different processes which can be used to determine its value. Choosing the determination with the smallest error, the CKM matrix is:

\[
\begin{pmatrix}
|V_{td}| & |V_{us}| & |V_{ub}|
\end{pmatrix}
\begin{pmatrix}
|V_{cd}| & |V_{cs}| & |V_{cb}|
\end{pmatrix} = \begin{pmatrix}
0.97408(26) & 0.2246(12) & 0.00376(26)
0.230(11) & 0.99(10) & 0.04167(72)
0.000206(7) & 1.3(2)
\end{pmatrix}
\]

where only the total error per each matrix element is shown. A significant progress has been made in the past years (e.g. understanding of the \(|V_{us}|\) value from kaon decays, more \(|V_{ub}|\) theoretical calculations, etc.), and more work is needed in the future to achieve a better understand of the current measurements as highlighted in the text.
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