Dark Matter and The Anthropic Principle

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Abstract

We evaluate the problem of galaxy formation in the landscape approach to phenomenology of the axion sector. With other parameters of standard ΛCDM cosmology held fixed, the density of cold dark matter is bounded below relative to the density of baryonic matter by the requirement that structure should form before the era of cosmological constant domination of the universe. Galaxies comparable to the Milky Way can only form if the ratio also satisfies an upper bound. The resulting constraint on the density of dark matter is too loose to select a low axion decay constant or small initial displacement angle on anthropic grounds.
INTRODUCTION

In the “landscape” approach to string phenomenology [1] - [4], one starts with an assumption of a large number, $\sim 10^{several \ hundred}$, of discrete vacua of the fundamental theory, a picture which is supported by several estimates based on counting solutions to the effective action of string theory on Calabi-Yau manifolds. The goal of the program is to explain low energy phenomena not as a unique and direct consequence of Planck scale physics, but as one realization among a rich set of possibilities. An important input in this approach is the “anthropic cut”: The only potentially phenomenologically relevant parts of the landscape are the vacua whose effective physics allows for the occurrence of biologically complex life.

One fruitful application of the anthropic approach has been Weinberg’s reasoning [5] leading to a bound on the cosmological constant from the minimal condition that galaxies must have enough time to form. From this apparent success we can derive a general lesson about the application of anthropic reasoning: the simplest anthropic constraints are those related to physics which is almost decoupled from phenomena at ordinary scales. Attempts to extract anthropic predictions for other (particle or cosmological) standard model parameters is hampered by our ignorance of the distribution of vacua, but also by the intricacy and interconnectedness of the effects of those parameters on the development of life, and on each other at different scales via RG evolution.

One highly decoupled sector where anthropic reasoning might usefully be applied is that of the Peccei-Quinn axion, $a$. Postulated as a mechanism for solving the strong CP problem, the axion is a pseudoscalar field with an approximate shift symmetry $a \rightarrow a + \theta$ which is broken only by the coupling $\Delta \mathcal{L} = antr(F \wedge F)_{SU(3)}$. If $a$ is dimensionless, its kinetic term, $\Delta \mathcal{L}_{\text{kinetic}} = f^2(\partial a)^2$ contains a dimensionful parameter $f$, known as the axion decay constant. QCD instantons then induce an axion potential of the form

$$V(a) = c\Lambda_{\text{QCD}}^4 \sin^2(na)$$

where $c$ is a numerical coefficient which can be set to 1 by a choice of definition of the dynamical scale $\Lambda_{\text{QCD}}$. An integer rescaling of $f$ can also set $n$ to 1. So the physical mass of the axion is given by $\Lambda_{\text{QCD}}^2/f$ at tree level. The massive parameter $f$ is determined by physics at scales above the standard model. In a generic high-scale model, such as any GUT or string model, the expectation would be that $f$ should have its magnitude set by the scale at which new physics enters—say $10^{16}$ GeV or so. This expectation is borne out.
specifically in superstring models such as the heterotic string [6], as well as in many of the more recently studied perturbative superstring vacua [7], [25].

As has been realized for a long time [8], such a value of $f$ appears in conflict with the cosmological standard model, as relic axions produced from initial vacuum displacement in the early universe make a contribution to the dark matter density that exceeds the observed value by orders of magnitude. Linde has pioneered the approach [9] of using anthropic ideas to loosen this bound in inflationary cosmology, where the homogeneous initial misalignment angle, $\theta_0$, of the axion is a free parameter, putatively “environmental”, in the sense of varying from region to region. If $f$ is fixed at its natural value, however, inflationary fluctuations are too large to make this work [10], a conclusion which possibly can be avoided in models of hybrid inflation [11]. More recently, Banks and Dine [12], and Banks, Dine, and Graesser [13] have emphasized that the cosmological axion problem is dominated in the supersymmetric context by the Saxion (and other moduli), and that a satisfactory resolution might require a much more drastic modification of the history of the universe between inflation and decoupling.

While this debate is by no means settled, it shows that if the strong CP problem is solved by a Peccei-Quinn axion in our universe, realizing observational results on dark matter density will most likely require some degree of fine adjustments of parameters and/or initial conditions. In the absence of a mechanism, but in the background of the landscape, we may ask if such adjustments can perhaps be justified anthropically, as has been done in [9] and elsewhere (see e.g., [14]). While not as severe a case as the cosmological constant, it seems a sensible sector to apply anthropism, since dark matter is essentially decoupled from everyday low-energy physics just as dark energy is.

We will here evaluate the extent to which the requirement that habitable structures form bounds the ratio of dark matter to baryonic matter in the universe. It is well accepted that in order for galaxies and similar astrophysical objects to form out of the primordial density perturbations seeded during inflation, the matter density $\rho_{\text{matter}}$ should contain a predominant dark matter component, $\rho_{\text{DM}}$, which drives the growth of structure between equality and decoupling. It is also clear that without any baryons, $\rho_b \rightarrow 0$, all structure would remain dark and uninhabited. A combination that works well is when the ratio $\zeta = \rho_{\text{DM}}/\rho_b \approx 5$ as in our universe. It thus being clear that a universe with $\zeta = 5$ is habitable, and a universe with $\zeta^{-1} = 0$ is not, one wonders what range of values of $\zeta$ life
can actually tolerate. Our chief interest here is to understand what general form such an
anthropic bound may take and where in the allowed region our universe is situated, as well
as what detailed astrophysics the tightness of the bound depends on.

To keep control, we will fix all other cosmological parameters, such as baryon to photon
ratio which we call $\eta \equiv n_b/n_\gamma$, scale and spectrum of initial perturbations, etc., to the values
we have observed today. Sometimes, it will be convenient to keep the cosmological constant
term $\rho_\Lambda$ as a free parameter in the discussion. As in [5], it is $\rho_\Lambda$ which threatens life by
halting the ultimate global formation of structure at a later stage in the evolution of the
universe.

AN ELEMENTARY BOUND

At equality of matter and radiation, $\rho_\gamma = \rho_{\text{matter}} = \rho_b + \rho_{\text{DM}} \approx \rho_{\text{DM}} = \zeta \rho_b$. Using
$\rho_\gamma = T_{\text{eq}}^4 = T_{\text{eq}} n_\gamma$, and $\rho_b = \mu n_b$, where $\mu = 1 \text{GeV}$ is the mass of a baryon, this gives for the
temperature at equality [26]

$$T_{\text{eq}} = \mu \eta \zeta$$

(1)

The density perturbations, which we assume to be of inflationary origin, can be divided
roughly into two classes, depending on their mass scale, $M$. Since perturbations only grow
logarithmically in the radiation dominated era, all density perturbations whose physical size
is smaller than the horizon size at equality have their primordial strength $\delta \approx \delta_0 = 10^{-5}$.
Perturbations which are superhorizon at equality will reach their scale-invariant amplitude
when they enter the horizon and can be ascribed a strength $\delta \approx \delta_0 (\lambda/H_{\text{eq}}^{-1})^{-2}$ at equality.
Here, $\lambda = (M/\rho_{\text{eq}})^{1/3}$ gives the relation between the size, $\lambda$, of a perturbation and its mass
scale, $\rho_{\text{eq}} = T_{\text{eq}}^4 = (\mu \eta \zeta)^4$ is the energy density at equality, and $H_{\text{eq}}^{-1} = (G \rho_{\text{eq}})^{-1/2}$ is the
horizon size. Thus,

$$\delta_{M,\text{eq}} \approx \begin{cases} \delta_0 & \lambda_{M,\text{eq}} < H_{\text{eq}}^{-1} \\ \delta_0 (M^{1/3}(\mu \eta \zeta)^{2/3}/M_{\text{Pl}})^{-2} & \lambda_{M,\text{eq}} > H_{\text{eq}}^{-1} \end{cases}$$

(2)

In the matter dominated era, the strength of the perturbations grows linearly with the scale
factor of the universe, $\delta \propto a$. The non-linear regime is reached when $a/a_{\text{eq}} \approx 1/\delta_{M,\text{eq}}$, after
which the structure breaks away from the overall expansion of the universe. The celebrated
Weinberg bound expresses the fact that this should happen before the universe is dominated
by vacuum energy, $\rho_\Lambda$, lest acceleration disrupt the forming structure. So, structures of scale $M$ have time to form between equality and cosmological constant domination if and only if

$$\rho_\Lambda \lesssim \rho_{eq} (\delta_{M,eq})^3 \quad (3)$$

Now, we have to decide what scale of structure is required for life, and how this depends on $\zeta$. Beyond its usual murkiness, this question is even more delicate in the universes that we are envisaging, because the structures may look quite different from those that form with our value of $\zeta$, as we will describe in more detail below. As an example, we can consider the fate of a perturbation that has a chance of evolving to a galaxy like ours. This will give us our strongest bound on $\zeta$, and useful expectations for a more careful study (see Fig. 1).

The Milky Way contains about $M_{\text{gal}} = 10^{11} M_\odot$ worth of baryons and so corresponds to a total mass scale $M = \zeta M_{\text{gal}}$. Inserting this into (3) implies the bounds,

$$\rho_\Lambda \lesssim \begin{cases} (\mu \eta)^4 \delta_0^3 \zeta^4 & \zeta M_{\text{gal}}^{1/3} (\mu \eta)^{2/3} / M_{\text{Pl}} < 1 \\ M_{\text{Pl}}^2 M_{\text{gal}}^{-2} \delta_0^3 \zeta^{-2} & \zeta M_{\text{gal}}^{1/3} (\mu \eta)^{2/3} / M_{\text{Pl}} > 1 \end{cases} \quad (4)$$

In other words, if the perturbation giving rise to our galaxy is subhorizon at equality, it enters the non-linear regime after the energy density has dropped by a fixed amount. Since the energy density at equality scales with the fourth power of $\zeta$, the cosmological constant can be correspondingly larger. If our galaxy is superhorizon size at equality, the strength of the corresponding perturbation is down by a factor of $\zeta^{-2}$, and it takes correspondingly longer to grow to non-linearity. The cosmological constant cannot be too large.

Besides its simplicity, the interest of this derivation is that, for fixed $\Lambda$, it yields both a lower and an upper bound on $\zeta$. Numerically, in our universe, $\eta = 10^{-9}$, $\zeta = 5$, $\delta_0 = 10^{-5}$, $\lambda_{\text{gal}} / H_{eq}^{-1} \approx 5 \times 10^{-2}$, while the vacuum energy density is comparable to its upper bound. (We are using numbers from, e.g., [15].) Therefore, if we increased $\zeta$ by a factor of 20, the perturbation corresponding to our galaxy would have extended up to the horizon at equality, and $\Lambda$ could have been $\sim 10^5$ times larger. Increasing $\zeta$ by another factor of 400 brings back the bound on $\Lambda$ to the familiar value. Thus, for fixed $\Lambda$, the existence of our Milky Way can tolerate a value of $\zeta$ in the range $5 \lesssim \zeta \lesssim 8 \times 10^4$.

The simplest version of the upper bound is to say that for fixed $\Lambda$, the maximum mass of any structure which can form is given by $M^{(\text{max})} \lesssim M_{\text{Pl}}^3 \delta_0^{3/2} \rho_\Lambda^{-1/2}$, and the largest number of baryons which can exist in a gravitationally bound structure is $M_b^{(\text{max})} = \zeta^{-1} M^{(\text{max})}$.
REFINEMENTS

Nonlinear analysis of structure formation

In this derivation, we have used a linear analysis of the growth of density perturbations into gravitationally bound structures. The motivation for this heuristic treatment is that
local self-gravity will only be important when $\delta \rho / \rho$ is of order 1; and due to the universal attractiveness of gravity, the growth of structures can only be hastened when the perturbation strength is large, rather than retarded. This suggests that the time scale for the development of a bound structure should not be significantly longer or shorter than the time scale for the perturbation to reach unit strength.

We point out that a more careful linear treatment alters our bound only by a factor of order 1. A formula due to Weinberg [5] states that the minimum density perturbation which can collapse gravitationally must obey the bound

$$\rho_\Lambda \leq \frac{500}{729} \rho_{eq} \delta_{M,eq}^3,$$

which gives a version of equation (3) with slightly better numerical precision. The effect is merely to change the upper bound on $\zeta$ by a factor of $(500/729)^{1/2} \approx 0.91$, or $(500/729)^{-1/2} \approx 1.21$, respectively, in the two regimes. As these factors are smaller than several other numerical imprecisions in our argument, we are justified in neglecting them before and hereafter.

Probabilities

In Weinberg’s original derivation [5], the upper bound on the cosmological constant turned out 2–3 orders of magnitude bigger than the then valid observational upper bound (which is the now measured value). The conceptual input into the derivation is that the earliest galaxies should form before the era of $\Lambda$-domination, and indeed we can recover Weinberg’s bound by recalling that galaxies at high redshift $z \approx 4$ are seeded by density perturbations of strength $10^{-4}$. Such density perturbations lie on the tail of a distribution with mean strength $10^{-5}$, as determined by precision measurements of the CMB.

One might try to improve the bound by asking whether a set of cosmological parameters which leave more time for galaxies to form might be in some sense more anthropically favorable than parameters which leave just the minimal amount of time [16, 17]. A really scrupulous attempt to answer this question is hampered by the lack of an a priori notion of relative anthropic favorability of two acceptable universes. Weinberg et al. [18] improve the bound on $\rho_\Lambda$ by assigning universes an anthropic probability based on the number density of observers. Since most galaxies form from the peak of the distribution, it is no surprise that the most likely value of $\Lambda$ is equal to its upper bound with the amplitude of fluctuations set to their mean value [18].
The logical foundation of such a weighting is unclear. Indeed the notion of number density of observers is gauge dependent, and is defined in [18] in an ad hoc way relative to FRW time-slices. However one does not wish to argue with success; the approach of [18] predicts a vacuum energy equal to $o(1)$ of the critical density, matching the value of $\rho_\Lambda \simeq 0.71\rho_{\text{crit}}$. Perhaps something valuable can be learned by putting this approach on firmer logical ground.

For the distribution of both parameters ($\zeta, \Lambda$), we would consider it premature to do a similar computation, not least because it is difficult to guess an appropriate a priori probability distribution without specifying a microscopic model for dark matter. (The cosmological constant is slightly more favorable in this regard because one can argue [18] that any a priori probability distribution should be flat in the anthropically allowed region, which is narrow and far away from the natural value.) If we assume, however, that dark matter is an axion relic originating in string theory, it seems that the peak of the a priori distribution should lie at large values of $\zeta$, with at least a power-law cutoff for smaller values. It appears unlikely that weighting with the number of observers can reverse this trend and produce a peak close to the values we observe (the star in Fig. 1)—unless one of the astrophysical effects discussed below serves to eliminate observers for large values of $\zeta$ altogether.

**LIFE IN A BARYON-POOR GALAXY?**

As we have seen, the requirement of gravitationally bound structures containing a certain fixed number $N$ of baryons imposes a sharp cutoff on the allowed values of $\zeta$, the actual value $\zeta(N)$ of the cutoff being $N$-dependent. For purposes of obtaining a first impression, we have taken $N = 10^{11} M_\odot/\mu$. This is a very strong requirement, and is essentially biased by the fact that the only life we know of is the one on Earth. The choice ignores the possibility that observers could evolve in a galaxy with far fewer baryons — $N \sim 10^6 M_\odot/\mu$, for example.

Moreover, our discussion so far has ignored the stages of structure evolution which happen after the inhomogeneities break away from the overall expansion of the universe. The cosmological constant is to a good degree irrelevant after this stage because its gravitational pull is so weak. The dark matter to baryon ratio, however, controls the type and succession of structures that do manage to form, and thereby has a significant influence on both the top-down (first stars to quasars to galaxies to clusters) as well as the bottom-up (galaxies
to stars to planets) branches of the subsequent evolution.

This impact is of course very difficult to evaluate, given that the details of non-linear formation of structure are not under complete control even in our universe! Nevertheless, the relevant qualitative features are reasonably well understood, and extrapolating to extreme values of $\zeta$ will allow us to at least address some of the issues.

Cooling

The first thing that has to happen after the density perturbations have become non-linear is that the baryonic gas that has fallen into the collapsed dark matter halos and shock-heated to the virial temperature must be able to cool efficiently in order to contract beyond virialization, fragment, and ignite stars after reaching nuclear densities [19].

It is often stated that to measure the efficiency of cooling, one should compare the dominant cooling rate $\tau_{\text{cool}}^{-1}$ with the dynamical time scale $\tau_{\text{dyn}} = (G\rho_{\text{vir}})^{-1/2}$. If $\tau_{\text{cool}} \gg \tau_{\text{dyn}}$, cooling is considered inefficient. It is obvious that in our universe, structures with cooling times much longer than the age of the universe simply have not had the time to evolve until today. In some other scenarios, one can also imagine that structures of some given size should cool significantly before the next bigger structures collapse onto them, since otherwise the smaller structures would not survive as distinct entities. In alternate universes of the type we are considering here, however, the last structures to form before cosmological constant domination will not suffer from this, and it is conceivable that given sufficient time, they will cool and can possibly develop life. (We must, of course, assume that the cooling time is not competitive with the lifetime of the proton!)

In any event, by following the standard treatments, discussed for example in [20] in the context of anthropic constraints on the amplitude of the primordial perturbations, we can estimate how the efficiency of the cooling varies with $\zeta$.

The dominant mechanisms which have contributed to the cooling of the structures in our universe are atomic and molecular line cooling of hydrogen and heavier elements as well as bremsstrahlung resulting from collisions of constituents of the charged plasma in the potential well of the dark matter halo. Both these mechanisms depend on the density, temperature, and the ionization level of the gas, and hence on $\zeta$.

The virialization density $\rho_{\text{vir}}$ is proportional to the total matter density at the time of collapse, $\rho_{\text{coll}} \approx \rho_{\text{eq}} \delta_{\text{M,eq}}^3$ and the virial temperature $T_{\text{vir}} = GM\mu/r = M_{\text{Pl}}^{-2} \mu M^{2/3} \rho_{\text{vir}}^{1/3}$. As we
have seen, the density at collapse increases independent of $M$ as $\zeta^4$ until $M \sim M_{Pl}^3 (\mu \eta \zeta)^{-2}$, and thereafter drops as $M^{-2}$, independent of $\zeta$. As a result, $T_{\text{vir}} \propto M^{2/3} \zeta^{4/3}$ and $T_{\text{vir}} \approx \mu \delta_0$ in the two regimes, respectively. [27] Therefore, if we increase $\zeta$, the virial temperature will soon exceed $10^4 K$, and line cooling will cease to be relevant for most structures. Cooling by bremsstrahlung will dominate, and we find

$$\frac{\tau_{\text{brems}}}{\tau_{\text{dyn}}} \propto \begin{cases} \zeta^{-1/3} M^{1/3} & \text{M subhorizon at equality} \\ \zeta M & \text{M superhorizon at equality} \end{cases}$$

(6)

We note that for fixed mass $M$, this has a maximum as a function of $\zeta$ with different power laws in the two regimes, very much as we found in (4). An interesting point is that for fixed $M_{\text{gal}} = M/\zeta$, the efficiency of bremscooling is at first independent of $\zeta$, so that a baryonic structure like the Milky Way might indeed have a very similar cooling history in universes with quite different values of $\zeta$.

The main effect if we increase $\zeta$, however, will be that the structures will soon form so early that the dominant cooling is from Compton scattering off the cosmic microwave background. Compton cooling is (in some regime) independent of the temperature and density of the baryons, but depends quite sensitively on the temperature of the CMB at the time when the structures have formed. Quantitatively, $\tau_{\text{comp}} \propto T^{-4}_\gamma$, and hence

$$\frac{\tau_{\text{comp}}}{\tau_{\text{dyn}}} \propto \begin{cases} \zeta^{-2} & \zeta^{4/3} M^{5/3} \end{cases}$$

(7)

again with a characteristic kinked power law behavior.

In the regime of dominant Compton cooling, the effect of the CMB is to act as friction for the charged components in the plasma. In contrast to other cooling mechanisms, it is quite efficient in absorbing angular momentum. The baryons should therefore lose their angular momentum and radial kinetic energy in typical time $\tau_{\text{comp}}$ and slide down into the minimum of the potential in a radially symmetric way. [28] At the bottom of the potential, they will have little angular momentum support, and fragmentation into stars is also likely to be inhibited if the collapse is sufficiently isothermal so that the Jeans mass does not decrease too rapidly.

It is plausible that the final state of such a collapse is one where the bulk of the baryonic matter forms a supermassive black hole (SMBH), possibly with a brief intermediate stage
of life as a supermassive star. Indeed, it is believed that most larger galaxies in our universe have an SMBH at their center. As has recently become clear, for instance in the celebrated $M-\sigma$ relation [21] linking the hole mass to the velocity dispersion of the central region of the galaxy, the history of these black holes is intrinsically linked to the formation of the galaxy itself. The SMBH can grow by accretion or mergers but most models assume a sizable seed black hole whose likely origin is the collapse of gas under conditions with inhibited star formation. (See [22] for a short list of references.) If Compton cooling in addition withdraws angular momentum support, collapse to a black hole is a very likely outcome.

Clearly, if increasing $\zeta$ would confine baryons into black holes, this would be a strong anthropic basis for selecting universes with roughly equal proportions of baryons and dark matter. At this stage, however, it seems that numerical simulations would be needed to confirm whether SMBHs are a reasonable scenario. [29]

**Supernova pressure**

One argument that is often cited to justify the claim that galaxies with fewer that $10^6M_\odot$ of baryons are unlikely to support life is that in our universe, such “galaxies” reside inside of halos of roughly the same size and have a much more shallow gravitational potential. As a consequence, when the first stars are formed, pressure created by supernova explosions are powerful enough to eject gas (as well as the heavy elements produced in the supernovae, which are plausibly necessary for life dependent on an interesting chemistry) from the galaxy, thus reducing the prospects of forming a second generation of stars, with planets around them.

A similar mechanism would probably provide a lower cutoff on the mass of baryon-containing galaxies for larger values of $\zeta$, but the formation of initial baryonic structure is not under good analytic control, so we do not know how to compute the dependence of this effect on $\zeta$. The all-important feedback processes are also likely to differ. This would be another good direction for future study in the subject of anthropic constraints on dark matter. [30]
CONCLUSIONS

In this paper we have described the likely evolution of universes with values of $\zeta$ greater than in our own universe, with other parameters of $\Lambda$CDM cosmology held fixed. Moderately larger values of $\zeta$ allow the formation of structure with astrophysical conditions similar to those in our own galaxy.

Some uncertainties remain. Anthropic constraints on the ratio of dark matter to baryonic matter appear too weak to force $\zeta$ as low as we observe it. Lacking a detailed understanding of the evolution of baryon-poor dark matter halos, one can impose looser or more stringent assumptions on the conditions necessary for life; however no reasonable assumption appears stringent enough to force the upper bound on $\zeta$ lower than $\sim 10^5$.

On the less restrictive end, one can explore the possibility that baryonic structures with arbitrarily low mass—say, $10^6 M_\odot$ of baryons in a $10^{12} M_\odot$ halo—can ultimately cool and ignite stars, though perhaps on a time scale far longer than the age of our current universe. One relevant question is the ability of such low proportions of baryons to form gravitationally bound structures inside the halo in the first place, whether or not they can eventually cool and form stars.

This question has recently been answered in the affirmative by the discovery of a “dark galaxy” in the Virgo cluster [23] —a gravitationally bound structure of $4 \times 10^7 M_\odot$ inside of a $2 \times 10^{10} M_\odot$ dark matter halo. This galaxy, known as VIRGOHI 21, has a density and temperature too low to cool efficiently by the available mechanism of hydrogen-line cooling, but nonetheless the baryons have been able to separate themselves from the ambient halo enough to form a disk whose structure is determined by its own gravity.

On the more restrictive end, we can see that even if we require baryonic structures of $10^{11} M_\odot$, there is little which can impede their formation for values of $\zeta$ up to $10^5$. Certainly we can be confident that halos containing the requisite number of baryons will form in this range. The dynamics of baryonic structure formation inside the halo at higher values of $\zeta$ are not entirely clear, but the inhomogeneities in the baryonic density can only increase with time, and the result of these growing inhomogeneities will likely be hydrogen-fusing stars.

Finally, we discuss the implications for the axion sector. Vilenkin’s “mediocrity principle” [16] can be interpreted as saying that when a parameter has a range of values which would allow life to exist, we should expect the parameter to lie at the point within that range
which is most favored by conventional notions of naturalness, or perhaps by the statistics of discrete vacua in a fundamental theory. Given our anthropic range for $\zeta$, what does Vilenkin’s principle tell us about axion physics?

In a model in which all dark matter is axionic, $\zeta \propto f^{3/2} \theta_0^2$ (see, e.g., [24]), so a bound on $\zeta$ can be interpreted either as a bound on the axion decay constant or on the initial displacement angle. Conventional naturalness and statistical arguments would both seem to favor large values of $f$. [31] Likewise, the statistics of initial values of the axion push $\theta_0$ towards values of $o(1)$.

The key point is that the pressures of mediocrity on $f$ and $\theta_0$ should both push $\zeta$ towards the higher end of its anthropic window, which is apparently high enough to falsify an anthropic explanation for the observed size of $\zeta$. Values of $\zeta$ up to $10^5$ are compatible with the evolution of habitable stars and galaxies, even with the conservative assumption that a galaxy needs $10^{11} M_\odot$ of baryons in order to support life. The observed value $\zeta \sim 5$ is some $20,000$ times lower than the upper end of the anthropic window, meaning that neither a low value of $f$ nor a natural value of $f$ with a low value of $\theta_0$ would make sense anthropically.

How firm is this conclusion? The axion sector is sufficiently decoupled from the standard model that we can map out with confidence much of the cosmological history of universes with large amounts of axionic dark matter. There is nothing obvious in these alternative universes to obstruct the development of life, implying that a non-anthropic explanation of the smallness of $f$ and/or $\theta_0$ is required.

Some aspects of this argument could be tightened. If the baryons in the high-$\zeta$ universes manage to collapse in a sufficiently isotropic way to proceed directly to a SMBH, this might lower the anthropic upper limit on $\zeta$ to below $10^5$. Nor can we yet estimate reliably the time scale for baryons to cool and form stars inside halos for large values of $\zeta$, though the only clearly anthropic time ceiling on this process would be the proton lifetime. Some of these questions could be answered by numerical simulations, similar to those by which we have learned about structure formation in our own universe.

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[25] It was noted in ref. [7] that smaller values of $f$ might also be realized in certain string models. The distribution of axion decay constants in, e.g., the string landscape is not known.

[26] We are here assuming that baryons are non-relativistic at equality. For $T_{eq} > \mu$, the matter to photon ratio $\xi = \rho_{\text{matter}}/n_\gamma = T_{eq}$ is a more useful parameter [20], while $\zeta = \xi/(\eta\mu)$ obtains significance only once the temperature drops below $\mu$. The present parameterization, which is sensible up to $\zeta = \eta^{-1} \approx 10^9$, is more convenient for our later considerations. In order to avoid interfering with big bang nucleosynthesis, one might also wish to keep $T_{eq}$ below 1MeV.

[27] These estimates also imply that the mean density of a collapsed structure has a maximum as a function of the mass for fixed $\zeta$. It also has a maximum as a function of $\zeta$ for fixed mass. The collapse density alone therefore does not seem suited as anthropic gauge as in [9].

[28] Even without Compton cooling, the presence of dark matter inhomogeneities at smaller scales redistributes angular momentum quite efficiently, leading to an “angular momentum problem” in simulations of galaxy formation.

[29] One of the bigger uncertainties is what fraction of baryons would initially collapse to the SMBH. If a significant fraction remains in the halo, it would be subject to the usual accretion and feedback processes.

[30] A second effect that is believed to inhibit efficient formation of galaxies with baryonic mass below $10^6 M_\odot$ is that smaller gas clouds can not shield themselves from the radiation that reionizes the universe at a redshift $z \approx 6$. As a consequence, atomic and molecular line
cooling will be much less efficient and the galaxies would take too long to collapse beyond virialization. As we have explained, cooling is a very sensitive issue which depends on the succession of structure formation. With different values of \( \zeta \), baryon-poor galaxies might still be able to cool efficiently, even after the inter-galactic medium has been reionized.

[31] It has been pointed out [7] that all known weakly coupled regions of string theory have values of \( f \) of at least \( 10^{16} \text{GeV} \), which leads us to suspect a statistical enhancement at large values.