We present theoretical predictions for the spectral, temporal and intensity signatures of the electromagnetic radiation emitted during the process of the gravitational collapse of a stellar core to a black hole, during which electromagnetic field strengths rise over the critical value for $e^+e^-$ pair creation. The last phases of this gravitational collapse are studied, leading to the formation of a black hole with a subcritical electromagnetic field, likely with zero charge, and an outgoing pulse of initially optically thick $e^+e^-$-photon plasma. Such a pulse reaches transparency at Lorentz gamma factors of $10^2$–$10^4$. We find a clear signature in the outgoing electromagnetic signal, drifting from a soft to a hard spectrum, on very precise time-scales and with a very specific intensity modulation. The relevance of these theoretical results for the understanding of short gamma-ray bursts is outlined.

Keywords: EMBH — electron-positron plasma — gravitational collapse — gamma-ray bursts

1. Introduction

The discovery in 1997 of the afterglows of Gamma-Ray Bursts (GRBs) \(^1\) has evidenced the cosmological nature of these sources. By the analysis of the first and second BATSE catalogs\(^a\) Tavani in 1998 \(^2\) confirmed the existence of two families of
GRBs: the so-called “long-bursts” with a soft spectrum and duration $\Delta t > 2.5\,\text{sec}$ and the “short-bursts” with harder spectrum and duration $\Delta t < 2.5\,\text{sec}$. In 2001 the theory was advanced \(^3\) that both short-bursts and long-bursts originate from the same underlying physical process due to the vacuum polarization of electromagnetic overcritical gravitational collapse leading to the creation of $e^+ - e^-$ pairs at the expenses of the extractable energy of a black hole \(^4\). The difference between the short-bursts and long-bursts in this theory is mainly due to the amount of baryonic matter encountered by the $e^+ e^-$ pairs in their relativistic expansion. A support of such a theory was given by Schmidt \(^5\) showing that short-bursts and long-bursts have the same isotropic-equivalent characteristic peak luminosity.

In recent work we have systematically developed the theoretical background of a process of gravitational collapse of matter involving an electromagnetic field with field strength higher than the critical value for $e^+ e^-$ pair creation \(^6,7,8,9,10,11\). The goal has been to clarify the physical nature of the process of extracting the mass energy of a black hole by the creation of $e^+ e^-$ matter pairs \(^4\) and to analyze the electromagnetic radiation emission process during the transient dynamical phases of the gravitational collapse leading to the final formation of the black hole.

In this letter we conclude this analysis by making precise predictions for the spectra, the energy fluxes and characteristic time-scales of the radiation for short-bursts. If the precise luminosity variation and spectral hardening of the radiation we have predicted will be confirmed by observations of short-bursts, these systems will play a major role as standard candles in cosmology.

These considerations will also be relevant for the analysis of the long-bursts when the baryonic matter contribution will be taken into account.

2. The model

The idea that the origin of GRBs is related to the energy extractable from a black hole \(^4\) by process of vacuum polarization \(^12,13,14\) and the creation of $e^+ e^-$ plasma was advanced in 1974 by Damour and Ruffini \(^16\). The basic considerations on the dynamics of the $e^+ e^-$ plasma in the context of GRBs were outlined in 1978 by Cavallo and Rees \(^15\), without addressing the issue of the origin of this plasma. In 1998 \(^17\) these concepts were further evolved by the identification of the region around an already formed black hole in which such $e^+ e^-$ plasma can be created and the concept of “dyadosphere” was introduced.

In this letter for the first time we present progress in describing the expected radiation from the dynamical formation of the dyadosphere in the process of gravitational collapse.

The dynamics of the collapse of an electrically-charged stellar core, separating itself from an oppositely charged remnant in an initially neutral star, was first modelled by an exact solution of the Einstein-Maxwell equations corresponding to a shell of charged matter in Ref. \(^6\). The fundamental dynamical equations and their analytic solutions were obtained, revealing the amplification of the electromagnetic...
field strength during the process of collapse and the asymptotic approach to the final static configuration. The results, which properly account for general relativistic effects, are summarized in Fig. 1 and Fig. 2 of Ref. 6.

A first step toward the understanding of the process of extracting energy from a black hole was obtained in Ref. 7, where it was shown how the extractable electromagnetic energy is not stored behind the horizon but is actually distributed all around the black hole. Such a stored energy is in principle extractable, very efficiently, on time-scales $\sim h/m_e c^2$, by a vacuum polarization process à la Sauter-Heisenberg-Euler-Schwinger. 12,13,14. Such a process occurs if the electromagnetic field becomes larger than the critical field strength $E_c$ for $\pm\text{pair creation}}$.

In Ref. 7 we followed the approach of Damour and Ruffini 16 in order to evaluate the energy density and the temperature of the created $\pm\text{-photon plasma}}$. As a byproduct, a formula for the irreducible mass of a black hole was also derived solely in terms of the gravitational, kinetic and rest mass energies of the collapsing core. This surprising result allowed us in Ref. 8 to obtain a deeper understanding of the maximum limit for the extractable energy during the process of gravitational collapse, namely 50% of the initial energy of the star: the well known result of a 50% maximum efficiency for energy extraction in the case of a Reissner-Nordström black hole 4 then becomes a particular case of a process of much more general validity.

The crucial issue of the survival of the electric charge of the collapsing core in the presence of a copious process of $\pm\text{-pair creation}}$ was addressed in Refs. 9,10. By using theoretical techniques borrowed from plasma physics and statistical mechanics 19,20,21,22,23,24,25 based on a generalized Vlasov equation, it was possible to show that while the core keeps collapsing, the created $\pm\text{-pairs are entangled in the overcritical electric field. The electric field itself, due to the back reaction of the created $\pm\text{-pairs, undergoes damped oscillations in sign finally settling down to the critical value $E_c$. The pairs fully thermalize to an $\pm\text{-photon plasma on time-scales typically of the order of $10^2-10^4 h/m_e c^2$. During this characteristic damping time, which we recall is much larger than the pair creation time-scale $h/m_e c^2$, the core moves inwards, collapsing with a speed $0.2-0.8c$, further amplifying the electric field strength at its surface and enhancing the pair creation process.}

Turning now to the dynamical evolution of such an $\pm\text{ plasma we recall that, after some original attempt to consider a steady state emis sion}}$, 26,27, the crucial progress was represented by the understanding that during the optically thick phase such a plasma expands as a thin shell. There exists a fundamental relation between the width of the expanding shell and the Lorentz gamma factor. The shell expands, but the Lorentz contraction is such that its width in laboratory frame appears to be constant. Such a result was found in 28 on the basis of a numerical approach, further analyzed in Bisnovatyi-Kogan and Murzina 29 on the basis of an analytic approach. Attention to the role of the rate equations governing the $\pm\text{-annihilation were given in 30, where approximations to the full equation were introduced. These results were improved in two important respects in 1999 and 2000 31,32: the initial conditions were made more accurate by the considerations
of the dyadosphere as well as the dynamics of the shell was improved by the self-
consistent solution of the hydrodynamical equation and the rate equation for the
\(e^+e^-\) plasma following both an analytic and numerical approach.

We are now ready to report in this letter the result of using the approach in \(^{31,32}\) in this general framework describing the dynamical formation of the dyadosphere.

The first attempt to analyze the expansion of the newly generated and thermalized \(e^+e^-\)-photon plasma was made in Ref.\(^{11}\). The initial dynamical phases of the expansion were analyzed, using the general relativistic equations of Ref.\(^{6}\) for the gravitational collapse of the core. A \emph{separatrix} was found in the motion of the plasma at a critical radius \(\bar{R}\): the plasma created at radii larger than \(\bar{R}\) expands to infinity, while the one created at radii smaller than \(\bar{R}\) is trapped by the gravitational field of the collapsing core and implodes towards the black hole. The value of \(\bar{R}\) was found in Ref.\(^{11}\) to be \(\bar{R} = 2GM/c^2[1 + (1 - 3Q^2/(4GM^2)]^{1/2}\), where \(M\) and \(Q\) are the mass and the charge of the core, respectively.

In this letter we pursue further the evolution of such a system, describing the
dynamical phase of the expansion of the pulse of the optically thick plasma all
the way to the point where the transparency condition is reached. Some pioneering
work in this respect were presented in Goodman in 1986\(^{33}\). In this process the pulse
reaches ultrarelativistic regimes with Lorentz factor \(\gamma \sim 10^2-10^4\). The spectra, the
luminosities and the time-sequences of the electromagnetic signals captured by a
far-away observer are analyzed here in detail for the first time. The relevance of
these theoretical results for short-bursts is then discussed.

3. The expansion of the \(e^+e^-\gamma\) plasma as a discrete set of
elementary slabs

We discretize the gravitational collapse of a spherically symmetric core of mass \(M\)
and charge \(Q\) by considering a set of events along the world line of a point of fixed
angular position on the collapsing core surface. Between each of these events we
consider a spherical shell slab of plasma of constant coordinate thickness \(\Delta r\) so
that:

(1) \(\Delta r\) is assumed to be a constant which is small with respect to the core radius;
(2) \(\Delta r\) is assumed to be large with respect to the mean free path of the particles
so that the statistical description of the \(e^+e^-\gamma\) plasma can be used;
(3) There is no overlap among the slabs and their union describes the entirety of
the process.

We check that the final results are independent of the special value of the chosen
\(\Delta r\).

In order to describe the dynamics of the expanding plasma pulse the energy-
momentum conservation law and the rate equation for the number of pairs in the
Reissner-Nordström geometry external to the collapsing core have to be integrated:

\[
T^{\mu\nu}\rho = 0, \quad (1)
\]
\[(n_{e^+e^-} u^\mu)_{;\mu} = \overline{\sigma} \overline{\nabla} \left[ n_{e^+e^-}^2 (T) - n_{e^+e^-}^2 \right], \quad (2)\]

where \( T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + pg^{\mu\nu} \) is the energy-momentum tensor of the plasma with proper energy density \( \epsilon \) and proper pressure \( p \), \( u^\mu \) is the fluid 4-velocity, \( n_{e^+e^-} \) is the pair number density, \( n_{e^+e^-} (T) \) is the equilibrium pair number density at the temperature \( T \) of the plasma and \( \overline{\sigma} \overline{\nabla} \) is the mean of the product of the \( e^+e^- \) annihilation cross-section and the thermal velocity of the pairs. We use Eqs. (1) and (2) to study the expansion of each slab, following closely the treatment developed in Refs. \(^{31,32}\) where it was shown how a homogeneous slab of plasma expands as a pair-electromagnetic pulse (PEM pulse) of constant thickness in the laboratory frame. Two regimes can be identified in the expansion of the slabs:

1. In the initial phase of expansion the plasma experiences the strong gravitational field of the core and a fully general relativistic description of its motion is needed. The plasma is sufficiently hot in this first phase that the \( e^+e^- \) pairs and the photons remain at thermal equilibrium in it. As shown in Ref. \(^{11}\), under these circumstances, the right hand side of Eqs. (2) is effectively 0 and Eqs. (1) and (2) are equivalent to:

\[
\left( \frac{dr}{c dt} \right)^2 = \alpha^4 \left[ 1 - \left( \frac{n_{e^+e^-}}{n_{e^+e^-}} \right)^2 \left( \frac{\alpha_0}{\alpha} \right)^2 \left( \frac{r}{r_0} \right)^4 \right],
\]

\[
\left( \frac{r}{r_0} \right)^2 = \left( \frac{\epsilon + p}{\epsilon_0} \right) \left( \frac{n_{e^+e^-}}{n_{e^+e^-}} \right)^2 \left( \frac{\alpha}{\alpha_0} \right)^2 - \frac{p}{\epsilon_0} \left( \frac{r}{r_0} \right)^4,
\]

where \( r \) is the radial coordinate of a slab of plasma, \( \alpha = \left( 1 - 2MG/c^2r + Q^2G/c^4r^2 \right)^{1/2} \) is the gravitational redshift factor and the subscript “\( 0 \)” refers to quantities evaluated at the initial time.

2. At asymptotically late times the temperature of the plasma drops below an equivalent energy of 0.5 MeV and the \( e^+e^- \) pairs and the photons can no longer be considered to be in equilibrium: the full rate equation for pair annihilation needs to be used. However, the plasma is so far from the central core that gravitational effects can be neglected. In this new regime, as shown in Ref. \(^{31}\), Eqs. (1) and (2) reduce to:

\[
\frac{\delta N_{e^+e^-}}{\delta t} = \frac{\gamma^2 \gamma_{V}}{v_{0} \gamma_{V}} \Gamma,
\]

\[
\sqrt{\frac{\gamma}{\gamma_{0}}} = \sqrt{\frac{\gamma_{V}^{2}}{v_{0} \gamma_{V}}},
\]

\[
\frac{\partial}{\partial t} N_{e^+e^-} = -N_{e^+e^-} \frac{1}{V} \frac{\partial V}{\partial t} + \overline{\sigma} \overline{\nabla} \left[ N_{e^+e^-}^2 (T) - N_{e^+e^-}^2 \right],
\]

where \( \Gamma = 1 + p/\epsilon \), \( V \) is the volume of a single slab as measured in the laboratory frame by an observer at rest with the black hole, \( N_{e^+e^-} = \gamma n_{e^+e^-} \) is the pair number density as measured in the laboratory frame by an observer at rest with the black hole, and \( N_{e^+e^-} (T) \) is the equilibrium laboratory pair number density.
4. The reaching of transparency and the signature of the outgoing gamma ray signal

Eqs. (3) and (4) must be separately integrated and the solutions matched at the transition between the two regimes. The integration stops when each slab of plasma reaches the optical transparency condition given by

$$\int_{0}^{\Delta r} \sigma_T n_{e^+e^-} \, dr \sim 1,$$

where $\sigma_T$ is the Thomson cross-section and the integral extends over the radial thickness $\Delta r$ of the slab. The evolution of each slab occurs without any collision.
or interaction with the other slabs; see the upper diagram in Fig. 1. The outer layers are colder than the inner ones and therefore reach transparency earlier; see the lower diagram in Fig. 1. In Fig. 1, Eqs. (3) and (4) have been integrated for a core with

\[ M = 10M_\odot, \quad Q = 0.1\sqrt{GM}; \]

the upper diagram represents the world lines of the plasma as functions of the radius, while the lower diagram shows the corresponding Lorentz \( \gamma \) factors. The overall independence of the result of the dynamics on the number \( N \) of the slabs adopted in the discretization process or analogously on the value of \( \Delta r \) has also been checked. We have repeated the integration for \( N = 10, N = 100 \) reaching the same result to extremely good accuracy. The results in Fig. 1 correspond to the case \( N = 10 \).

We now turn to the results in Fig. 2, where we plot both the theoretically predicted luminosity \( L \) and the spectral hardness of the signal reaching a far-away observer as functions of the arrival time \( t_a \). Since all three of these quantities depend in an essential way on the cosmological redshift factor \( z \), see Refs. 35, 36, we have adopted a cosmological redshift \( z = 1 \) for this figure.

As the plasma becomes transparent, gamma ray photons are emitted. The energy \( \hbar \omega \) of the observed photon is \( \hbar \omega = k\gamma T/(1 + z) \), where \( k \) is the Boltzmann constant, \( T \) is the temperature in the comoving frame of the pulse and \( \gamma \) is the Lorentz factor of the plasma at the transparency time. We also recall that if the initial zero of time is chosen as the time when the first photon is observed, then the arrival time \( t_a \) of a photon at the detector in spherical coordinates centered on the black hole is given by \( t_a \):

\[ t_a = (1 + z) \left[ t + \frac{r_0}{c} \cos \theta - \frac{r(t)}{c} \cos \theta \right] \]

where \( t, r(t), \theta, \phi \) labels the laboratory emission event along the world line of the emitting slab and \( r_0 \) is the initial position of the slab. The projection of the plot in Fig. 2 onto the \( t_a-L \) plane gives the total luminosity as the sum of the partial luminosities of the single slabs. The sudden decrease of the intensity at the time \( t = 0.040466 \) s corresponds to the creation of the \( separatrix \) introduced in Ref. 10. We find that the duration of the electromagnetic signal emitted by the relativistically expanding pulse is given in arrival time by

\[ \Delta t_a \sim 5 \times 10^{-2} \text{s}. \]

The projection of the plot in Fig. 2 onto the \( kT_{\text{obs}} \), \( t_a \) plane describes the temporal evolution of the spectral hardness. We observe a precise soft-to-hard evolution of the spectrum of the gamma ray signal from \( \sim 10^2 \) KeV monotonically increasing to \( \sim 1 \) MeV. We recall that \( kT_{\text{obs}} = k\gamma T/(1 + z) \).

The above quantities are clearly functions of the cosmological redshift \( z \), of the charge \( Q \) and the mass \( M \) of the collapsing core. We present in Fig. 3 the arrival
time interval for $M$ ranging from $M \sim 10M_\odot$ to $10^3M_\odot$, keeping $Q = 0.1\sqrt{GM}$. The arrival time interval is very sensitive to the mass of the black hole:

$$\Delta t_a \sim 10^{-2} - 10^{-1}\text{s}. \quad (9)$$

Similarly the spectral hardness of the signal is sensitive to the ratio $Q/\sqrt{GM}$ \cite{37}. Moreover the duration, the spectral hardness and luminosity are all sensitive to the cosmological redshift $z$ (see Ref. \cite{37}). All the above quantities can also be sensitive to a possible baryonic contamination of the plasma due to the remnant of the progenitor star which has undergone the process of gravitational collapse.
5. Conclusions

The above results were obtained considering $e^+e^-$ plasma without any baryonic contamination and are therefore directly relevant for short-bursts. The characteristic spectra, time variabilities and luminosities of the electromagnetic signals from collapsing overcritical stellar cores, here derived from first principles, agrees very closely with the observations of short-bursts. New space missions must be planned, with temporal resolution down to fractions of µs and higher collecting area and spectral resolution than at present, in order to verify the detailed agreement between our model and the observations. It is now clear that if our theoretical predictions will be confirmed, we would have a very powerful tool for cosmological observations: the independent information about luminosity, time-scale and spectrum can uniquely determine the mass, the electromagnetic structure and the distance from the observer of the collapsing core, see e.g. Fig. 3 and Ref. 37. In that case short-bursts may become the best example of standard candles in cosmology. The introduction we are currently analysing is the introduction of baryonic matter in the optically thick phase of the expansion of the $e^+e^-$ plasma which can affect the structure of the Proper-GRB (P-GRB) as well as the structure of the long-bursts.

An interesting proposal was advanced in 2002 that the $e^+e^-$ plasma may have a fundamental role as well in the physical process generating jets in the extragalactic radio sources. The concept of dyadosphere originally introduced in Reissner-Nordström black hole in order to create the $e^+e^-$ plasma relevant for GRBs can also be generalized to the process of vacuum polarization originating in a Kerr-Newman black hole due to magneto-hydrodynamical process of energy extraction.
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(see e.g. 43 and references therein). The concept therefore introduced in this letter becomes relevant for both the extraction of rotational and electromagnetic energy from the most general black hole 4.

After the submission of this letter we have become aware that Ghirlanda et al. 44 have given evidence for the existence of an exponential cut off at high energies in the spectra of short bursts. We are currently comparing and contrasting these observational results with the predicted cut off in Fig. 2 which results from the existence of the separatrix introduced in 9. The observational confirmation of the results presented in Fig. 2 would lead for the first time to the identification of a process of gravitational collapse and its general relativistic self-closure as seen from an asymptotic observer.

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