Primordial Nucleosynthesis: Accurate Predictions

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Big Bang Nucleosynthesis (BBN) represents a key subject of modern cosmology since it is a powerful tool to study fundamental interactions. In the recent years, the improvement on the measurement accuracy of light primordial nuclei abundances allowed BBN to enter in a sort of precision era. In view of this, a great theoretical effort has been devoted to make theoretical predictions comparably accurate. Unfortunately, as far as $D$, $^3He$ and $^7Li$ are concerned, the theoretical uncertainty on their primordial abundances is greatly dominated by a poor knowledge of many nuclear reactions involved in their production. On the contrary, the $^4He$ abundance results into a robust prediction and thus an effort to reduce at less than 1% its theoretical uncertainty is meaningful.

To improve the accuracy on the prediction of $^4He$ abundance in Ref. [1] we performed a thorough analysis of all corrections to the proton/neutron conversion rates, $\nu_e + n \leftrightarrow e^- + p, e^+ + n \leftrightarrow \overline{\nu}_e + p, n \leftrightarrow e^- + \overline{\nu}_e + p$ which fix at the freeze out temperature $\sim 1 MeV$ the neutron to proton density ratio. The Born rates, obtained in the tree level $V - A$ limit and in the limit of infinite nucleon mass ($Born$ approximation), one has

$$\omega_B = \frac{G_F^2 (C_v^2 + 3 C_A^2)}{2\pi^3} \int_0^\infty d|p'| |p'|^2 q_0^2 \times \Theta(q_0) [1 - F_\nu(q_0)] [1 - F_e(p'_0)], \quad (1)$$

$p'$ and $p'_0$ are the electron momentum and energy, and $q_0$ the neutrino energy. The functions $F_\nu$ and $F_e$ denote the neutrino (antineutrino) and electron (positron) Fermi distributions, respectively.

The accuracy of Born approximation can be tested by comparing, for example, the prediction for neutron lifetime obtained from (1) in the limit of vanishing temperature, $\tau_n \simeq 961$ s, with the experimental value $\tau_n^{exp} = (886.7\pm1.9)$ s [3].

To recover the experimental value, a correction of about 8% is expected to come from radiative and/or finite nucleon mass effects. In Figure 1 the Born rates for $n \rightarrow p$ processes are reported. In Figure 2 the total corrections, listed above, to Born rates are shown. They are essentially dominated by radiative and kinetic corrections which at low temperature amount to 8% of the total rates.

2. The set of equations for BBN

The BBN equations [2] can be transformed in a set of $N_{nuc} + 1$ differential equations for

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*$^*$Talk given by G. Miele at the International Workshop on Particles in Astrophysics and Cosmology: From Theory to Observation, Valencia 1999.
\[ \hat{h} \equiv n_B/T^3 \] and the nuclide relative abundances \( X_i \) with \( z = m_e/T \) as the evolution parameter. In terms of these new variables the BBN set of equations becomes

\[ \frac{dh}{dz} = \left[ 1 - \hat{H} G \right] \frac{3\hat{h}}{z}, \quad (2) \]

\[ \frac{dX_i}{dz} = G \frac{\hat{\Gamma}_i}{z}, \quad i = 1, \ldots, N_{\text{nuc}}, \quad (3) \]

where \( (\Theta = \Theta(z_D - z)) \)

\[ G = \left[ \sum_{\alpha} (4\hat{\rho}_\alpha - z \frac{\partial\hat{\rho}_\alpha}{\partial z}) + 4\Theta\hat{\rho}_\nu + \frac{3}{2}\hat{h} \sum_j X_j \right] \]

\[ \times \left\{ 3 \left[ \sum_{\alpha} (\hat{\rho}_\alpha + \hat{\rho}_\alpha) + \frac{4}{3}\Theta\hat{\rho}_\nu + \hat{h} \sum_j X_j \right] \hat{H} \right. \]

\[ + \hat{h} \sum_j \left( z\Delta\hat{M}_j + \frac{3}{2}\hat{\Gamma}_j \right) \right\}^{-1} \quad (4) \]

In the previous equations \( z_D = m_e/T_D \) (\( T_D \) is the neutrino decoupling temperature), \( \alpha = e, \gamma \), and the dimensionless Hubble parameter \( \hat{H} = H/m_e \) reads

\[ \hat{H} = \sqrt{\frac{8\pi}{3} \frac{m_e}{M_P} \frac{1}{z^2} \left[ \hat{\rho}_\gamma + \hat{\rho}_e + \hat{\rho}_\nu \right]} \]

The quantities \( \hat{M_u} = M_u/m_e, \hat{\Delta M}_j = \Delta M_j/m_e, \) \( \hat{\Gamma}_j = \Gamma_j/m_e \) are the dimensionless atomic mass unit, mass excess and rate, respectively.

The initial value for (4) is provided in terms of the final baryon to photon density ratio \( \eta \) according to the equation

\[ \hat{h}_{\text{in}} = \frac{2\zeta(3)}{\pi^2} \hat{\eta}_{\text{in}} = \frac{11}{4} \frac{2\zeta(3)}{\pi^2} \eta. \]

The condition of Nuclear Statistical Equilibrium (NSE), very well satisfied at the initial temperature \( T_{\text{in}} = 10 \text{ MeV} \), fixes the initial nuclide relative abundances. From NSE one gets

\[ X_i(T_{\text{in}}) = \frac{A_{i-1}}{2} \left( \frac{\zeta(3)}{\pi} \right)^{\frac{A_i}{2}} A_i^{\frac{3}{2}} \eta^{A_i-1} \]

\[ \times \left( \frac{T_{\text{in}}}{M_N} \right)^{\frac{2}{3}(A_i-1)} X_p^{Z_i} X_n^{A_i-Z_i} e^{\frac{B_i}{M_N}}, \quad (7) \]

where \( B_i \) denotes the binding energy.

3. Light Element Abundances

By using in the BBN equations the corrected rates for \( n \leftrightarrow p \) processes one can predict,
high accuracy, the primordial values for $D$, $^3He$, $^4He$ and $^7Li$

$$Y_2 = \frac{X_3}{X_2}, \quad Y_3 = \frac{X_5}{X_2}, \quad Y_4 = \frac{M_6 X_5}{\sum_j M_j X_j}, \quad Y_7 = \frac{X_8}{X_2}. \quad (8)$$

Using the results of [4] to quantify the uncertainties coming from nuclear reaction processes, one can observe that only for $Y_4$ the correction to Born rates affects result on $Y_4$ by an amount larger than the theoretical uncertainties, including nuclear reactions. For $D$, $^3He$ and $^7Li$ the uncertainty, due to the poor knowledge of nuclear reaction rates, is estimated to be of the order of $(10 \div 30)\%$ [4], thus much less than the effects of radiative/thermal correction on $n \leftrightarrow p$ rates.

In Fig. 3 the predictions on $Y_4$ are shown versus $\eta$ for $N_\nu = 2, 3, 4$ and for a $1 \sigma$ variation of $\tau_n^{\\text{ee}}$. The two experimental estimates for the primordial $^4He$ mass fraction, $Y_4^{(l)} = 0.234 \pm 0.002 \pm 0.005$ and $Y_4^{(h)} = 0.243 \pm 0.003$ (see for example [3]) are the horizontal bands. Figs. 4 and 5 show the predictions for $D$ and $^7Li$ abundances. Note that, due to the negligible variation of $Y_2$ and $Y_7$ on small $\tau_n$ changes, no splitting of predictions for $1 \sigma$ variation of $\tau_n^{\\text{ee}}$ is present.

4. Conclusions

A detailed study of the effects on primordial abundances of the radiative, finite nucleon mass, thermal and plasma corrections to Born rates $n \leftrightarrow p$ has been recently carried out [3]. This analysis which has reduced the uncertainty on $Y_4$ to less than $1\%$ has been performed using an update version of the BBN standard code [4].

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Figure 4. The quantity $Y_2$ versus $\eta$ is reported. The same notation of Fig. 3 is used. The horizontal bands dashed and dotted are the experimental values (see for example [5]).

Figure 5. The quantity $Y_7$ versus $\eta$. The same notation of Fig. 3 is used. The horizontal bands dashed and dotted are the experimental values (see for example [5]).