Deep Q-learning of global optimizer of multiply model parameters from nonconvex function $f$

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Abstract—Objective: Estimation of the global optima of multiple model parameters is valuable in imaging to form a reliable diagnostic image. Given non convexity of the objective function, it is challenging to avoid from different local minima. Methods: We first formulate the global searching of multiply parameters to be a k-D move in the parametric space, and convert parameters updating to be state-action decision-making problem. We proposed a novel Deep Q-learning of Model Parameters (DQMP) method for global optimization of model parameters by updating the parameter configurations through actions that maximize a Q-value, which employs a Deep Reward Network designed to learn global reward values from both visible curve fitting errors and hidden parameter errors. Results: The DQMP method was evaluated by viscoelastic imaging on soft matter by Kelvin-Voigt fractional derivative (KVFD) modeling. In comparison to other methods, imaging of parameters by DQMP yielded the smallest errors (< 2%) to the ground truth images. DQMP was applied to viscoelastic imaging on biological tissues, which indicated a great potential of imaging on physical parameters in diagnostic applications. Conclusions: DQMP method is able to achieve global optima, yielding accurate model parameter estimates in viscoelastic imaging. Assessment of DQMP by simulation imaging and ultrasound breast imaging demonstrated the consistency, reliability of the imaged parameters, and powerful global searching ability of DQMP. Significance: DQMP method is promising for imaging of multiple parameters, and can be generalized to global optimization for many other complex nonconvex functions and imaging of physical parameters.

Index Terms—Deep Q-learning; Q-value; Viscoelastic imaging; Parameters optimization; Global optima.

I. INTRODUCTION

MODEL fitting is a branch of nonlinear regression problems that simultaneously extracts multiple model parameters by fitting experiment data to a specific model. Estimating multiple model parameters from response curve are of great importance in many measurement and imaging applications [1]-[3]. When the fitting function $f$ is nonconvex and there are few known constraints, achieving global convergence for parameter estimation is challenging.

There is no general algorithm for solving these problems, and the theoretical guarantees regarding convergence to global optima for common algorithms are weak or nonexistent. The established field of optimization is extensive, consisting of basic methods such as Gauss-Newton and gradient descent [4], [5], as well as combinations of these methods, such as Levenberg-Marquardt (LM) [6], [7]. Each of these methods have their strengths and weaknesses.

The simulated annealing algorithm is a stochastic scheme for searching global optimum by minimizing the system of energy through an annealing schedule [8]. Genetic algorithm is bio-inspired heuristic search through heredity and mutation [9],[10]. Particle swarm is another bio-inspired stochastic approach based on the best positions experienced so far by each particle and the whole swarm [11].

Current stochastic and evolutionary schemes can, in theory, jump out of local extrema and increase the probability of finding global solutions. However, jumping from current local extrema may introducing other local ones, ultimately yielding inconsistent solutions unless it can be guided by any valid prior knowledge that might be available and by relevant past experiences.

Deep learning (DL) methods have the capacity to learn from prior knowledge. Deep neural networks can establish maps from input to output and they may be scaled to model arbitrary mappings. The adaptive and nonlinear response of their neural networks can be trained to model highly complex systems. With the successful application of DL in AlphaGo [12], [13], the power of DL has been validated in a variety of applications [14]-[20].

In recent years, DL applied to regression task was reportedly capable of solving multi-parameter optimization and curve fitting problems [19]-[22]. However, learning a large number of model parameters and network weights is a complex optimization problem itself due to its network-like nature.
Convergence may be difficult to achieve when applying DL to learn multiple model parameters [19], [22]. Feedback from past activity is important to human learning. Reinforcement Learning (RL) is an agent-based powerful AI algorithm in which the agents learn the optimal set of actions through their interaction with the environment. RL is able to make appropriate responses because of reinforcing events. These events can include human feedback to responses that includes rewards and punishments as quantified by a value function. In this way, past experiences can guide RL in learning similar new experiences. RL is a promising branch of AI. The essence of RL is to take actions that maximize the value function. [12], [13], [23]-[27].

Q-learning [28] is a model free agent-based RL method that can adapt to environment, including prior knowledge gleaned from past experiences. The key idea of Q-Learning is its value function. The algorithm seeks to be rewarded and to avoid punishment for its current and next action in the form of an increasing value function. Due to a cumulative feedback mechanism [28], the agent learns to associate the optimal action for each state [29] in pursuit of increasing its value function. Q-learning is widely used in decision-making, gambling, and random event processing problems [30]-[32]. Combinations of DL and RL/Q-learning have been successfully implemented to solve complex human activities, for example, AlphaGo [12], [13]. A deep Q-Network can learn from prior human experience and predict the value function through training.

The aim of this paper is to develop a novel Deep Q-learning of Model Parameters (DQMP) algorithm that finds a global optima for estimating multiply model parameters when the objective function is nonconvex. We first convert the search for global parameters into a decision-making problem in parameters space based on both visible (curve fitting) and hidden (parameters fitting) states. For a \( k \)-parameter optimization problem, we subdivide the next action into \( 2^k \) directions in the parameters space. Actions involve selecting a next candidate move in parameter space based on the state of the current model fit, including both the visible state and the hidden state feedbacks. In this way, we formulate model parameter optimization as an action selection problem in parameter space.

To combine data with prior knowledge, a Deep Reward Network (DRN) is proposed to learn the global reward function. This process integrates both visible and hidden state feedbacks. Then a novel DQMP scheme is proposed to maximize the Q-value function. This strategy guides the DQMP search towards the global solution.

DQMP is applied to viscoelastic measurement from soft tissues with the goal of estimating Kelvin-Voigt fractional derivative (KVFD) model parameters. The assessment and validation of DQMP is tested using simulated data, nanoindentation data, and in vivo ultrasonic data with KVFD modeling.

II. METHODS

Let \( \theta = [\theta_1, \theta_2, ..., \theta_k] \) be a \( k \)-dimensional parameter vector for a time-dependent material model that predicts the measurements, \( Y = f(t; \theta) \). If \( \hat{Y} \) is a vector of measurement data from an experiment, then \( \theta \) represents properties of the material if \( \hat{Y} = Y = f(t; \theta) \). Our ultimate goal is to estimate \( \theta \).

Let \( \hat{\theta} \) be an estimate of \( \theta \), and \( \hat{Y} = f(t; \hat{\theta}) \) is a model of the measurement data that is based on that estimate. Let \( \beta_1 \) and \( \beta_2 \) be weights that balance the contributions of two fitting error terms describing model parameters and measurement curves, respectively. The global optimum is estimated by minimizing the following objective function:

\[
\hat{\theta} = \arg\min_\theta \left( \beta_1 \| \theta - \hat{\theta} \|_2^2 + \beta_2 \| f(t; \theta) - \hat{Y} \|_2^2 \right)
\]  

In (1), the visible term involves measurable data that depends on the parameters, whereas the hidden term evaluates the parameters.

We convert the parameter optimizing to a decision-making problems elegantly by minimize (1) through a set of state-action decisions in parameter space. In the fitting problem, the state are referred to \( s = \{ \hat{\theta}, f(t; \hat{\theta}) \} \) including (1) visible state \( f(t; \hat{\theta}) \), where the difference between current curve and the desired curve is measured by \( \| \hat{Y}(t, \theta) - Y(t, \hat{\theta}) \| \); 2) hidden state \( \hat{\theta} \) on which the errors between the current parameter configuration \( \hat{\theta} \) and the true parameter configuration \( \theta \) is measured by \( \| \theta - \hat{\theta} \| \).

There are \( n = 2^k \) candidate actions possible at the current state in \( k \)-D parameter space. To help understand the problem setting, take \( k = 3 \) for example. As listed in Fig. 1, the current parameter state (yellow dot) \( \hat{\theta} \) can move in one of eight possible directions (red dots) \( \hat{\theta}_{a_1}, ..., \hat{\theta}_{a_8} \) in the next step by taking actions \( a_1, ..., a_8 \) respectively. In this way, updating of \( \hat{\theta} \) in the parametric space resembles selection move in the 3D board for game, which is a novel approach for optimizing model parameters.

The optimal actions may be taken to increase a value function as \( \hat{\theta} \to \theta \). Q-value is the expected discounted reward for executing action \( a \) at state \( s \) and the next step optimal action \( a' \) at state \( s' \) by episodes thereafter. Q-Learning is a powerful scheme for agents to learn to act optimally by experiencing the

Fig. 1. Schematic illustration of a global search of state-actions in a 3-D parameter space. The yellow center point denotes the current parameter configuration \( \hat{\theta} \), and the 8 edge points are the candidate actions. The decision to select an action is based on maximizing the Q-value policy. The orange star point denotes the next move through action \( a_8 \) that may maximize the Q-value function.
consequences of actions judged by long-term discounted reward, in which selecting every action can obtain the maximum benefits-Q value.

Q-learning consists of a set of states $S$, a set of actions $A$, and a reward function $r: S \times A \rightarrow R^+$. It uses the Q-value to feedback the actions of every step. The policy $\pi$ maps states to actions as $\pi: S \rightarrow A$. The policy is maximizing Q-value by [28]

$$Q(s, a) \leftarrow Q(s, a) + \xi \left[ r(s, a) + \gamma \max_{a'} Q(s', a') - Q(s, a) \right],$$  

(2)

$r(s, a)$ is the immediate reward of selecting action $a$ at distinct state $s$. The reward can be any positive value for rewarding an action. The reward function should be a decreasing function of the fitting errors in the fitting problems. $Q(s', a')$ is the Q value found by selecting the next state-action pairs $(s', a')$. $\xi \in [0,1]$ is the learning rate and $\gamma \in [0,1]$ is a discount factor. In this work, $\xi$ is 0.6 and $\gamma$ is 0.5.

In this way, we formulate the parameter search to be a state-action decision-making in the parameter space by increasing the Q-value function policy. The parameter update is formulated to be $a = \arg\max Q(s, a_i)$ at every step, then parameters are updated by $\hat{\theta} \leftarrow \hat{\theta} + \Delta \hat{\theta}$ as indicated in the left table of Fig. 1. The adaptive steps for updating parameter are the 1% of the current parameter.

To guide the global parameter search, the Q-value function should integrate both the curve fitting (visible) and parameter fitting (hidden) feedbacks.

In order to learn the prior rewards from hidden states, a Deep Reward Network (DRN) is proposed to learn the reward function whose global constraints are absorbed from both visible and hidden states. In this way, a novel DQMP algorithm is proposed, where a deep initial network (DIN) applies experimental data to initialize $\hat{\theta}$, while a DRN is to predict global reward value consisting of both the curve fitting (visible reward) and parameters fitting (hidden reward) feedbacks. DQMP iteratively updates the state through convergence such that a global solution can be found following the maximizing Q-value policy.

A. Deep Q-learning of Model Parameters (DQMP) from nonconvex function

1) Q-value integrating rewards from both hidden and visible states

Q-value is inherently weighted sum of the immediate reward, cumulative reward and the future reward. Learning of reward function $r(s, a)$ that rewards both visible and hidden state feedbacks is crucial to guide the global search. Let $R_{\text{curve}}$ denote the curve fitting reward (visible) and $R_\theta$ the parameters fitting reward (hidden). The reward function $r(s, a)$ is formulated as (3), where the global reward $R_g$ combines both the hidden reward $R_\theta$ and visible reward $R_{\text{curve}}$ as expressed by (4).

$$r(s, a) = \beta_g R_g + (1 - \beta_g) R_{\text{curve}}$$  

(3)

$$R_g = \beta_\theta R_\theta + \beta_c R_{\text{curve}}$$  

(4-1)

$$R_\theta = 1 - \frac{||\theta - \hat{\theta}||_2}{\sqrt{k}}$$  

(4-2)

$$R_{\text{curve}} = \begin{cases} 0, & |\Delta| > e_{\text{max}} \\ g(|\Delta|), & e_{\text{min}} \leq |\Delta| \leq e_{\text{max}} \\ 1, & |\Delta| < e_{\text{min}} \end{cases}$$  

(4-3)

$\beta_\theta \in [0,1]$ is the weight to balance the global reward and the curve fitting reward, $\beta_\theta, \beta_c \in [0,1]$ are adjustable weights for $R_\theta$ and $R_{\text{curve}}$, respectively. $k$ is the number of parameters.

$g(\cdot)$ is a decreasing function, $e_{\text{min}}$ and $e_{\text{max}}$ are the two thresholds such that $g(e_{\text{min}}) = 1$ and $g(e_{\text{max}}) = 0$, and $|\Delta| = E(|Y - \hat{Y}|)$ is expectation of Mean Absolute Error (MAE) between the current curve and desired curve. In this work, $\beta_\theta$ is 0.02, $\beta_c$ is 0.6, $\beta_\theta$ is 0.4, $e_{\text{min}}$ is $10^{-10}$ and $e_{\text{max}}$ is 1.

Applying (3) and (4) to (2), a novel Deep Q-learning method is then proposed, which updates the Q-value as expressed by

$$Q(s, a) \leftarrow Q(s, a) + \xi \left[ \beta_g (\beta_c R_\theta + \beta_c R_{\text{curve}}) + (1 - \beta_g) R_{\text{curve}} + \right.$$

$$\left. \gamma \max_{a'} Q(s', a') - Q(s, a) \right]$$  

(5)

In the immediate fitting environment, $R_g$ is the global rewards that can be learned and predicted by the DRN.

2) Deep Reward Network

The Deep Reward Network (DRN) was designed to predict $R_g$ that consists of reward from both visible and hidden states. The schematic illustration of Deep Reward Network is shown in the Fig. 2.

The generation of training data set is displayed on the left of Fig. 2. First, two parameter configurations, true parameter $\theta$ and current parameter $\hat{\theta}$ were randomly chosen in the parameter space. The two curves, generated by $\theta$ and $\hat{\theta}$ according to function $f$, exactly simulates the desired experimental curve $f(t; \theta)$ and the immediate curve $f(t; \hat{\theta})$, respectively. The current parameters $\hat{\theta}$ perform actions $a_i (i = 1, ..., \beta)$ to achieve 8 candidate parameter configurations.

Fig. 2. Schematic illustration of Deep Reward Network (DRN) to learn and predict global reward. The left figure shows the flow of generating training data, and the right figure shows the structure of DRN, which has three LSTM layers followed by a fully-connected layer. The last hidden layer applies a sigmoid nonlinearity, and other hidden layers are followed by a rectifier nonlinearity.
\[ \hat{\theta}_k (i = 1, \ldots, 8) \] With that, the corresponding immediate curve is generated by \( f(t; \hat{\theta}_k) \). For each candidate action \( a_i \), using (4-2) and (4-3), we can calculate the \( R_{\text{curve}} \) and \( R_g \), and hence \( R_{\text{g}} \). In this way, how to reward the current fitting by doing actions \( a_i (i = 1, \ldots, 8) \) is recorded in global rewards \( R_g \) as illustrated on the left of Fig. 2.

The structure of DRN is shown on the right of Fig. 2. Long Short-Term Memory (LSTM) neural network is used to construct the DRN. LSTM is a special Recurrent Neural Network (RNN) that is appropriate at dealing with sequence data modeling. Comparing with ordinary RNN, LSTM architecture is good at dealing with long time sequence data, unlimited state numbers, and avoid problems related to vanishing and exploding gradients [33]. The input of the Deep Reward Network (DRN) is the difference between the current curve and desired curve (\( \Delta(s) = \hat{Y} - \hat{Y} \)), followed by three LSTM layers and a fully-connected layer. The output of the DRN is the global rewards (\( \hat{R}_{g1}, \hat{R}_{g2}, \ldots, \hat{R}_{g8} \)) (\( n = 2^k \)) for each action. The last hidden layer is followed by a sigmoid nonlinearity, and the other hidden layers are followed by a rectifier nonlinearity.

The loss function \( L_{\text{Reward}} \) of Deep Reward Network is given by

\[
L_{\text{Reward}}(R_g; \hat{R}_g) = \sum_{i=1}^{N} \hat{R}_{g_i} - \hat{R}_g_i. 
\] (6)

Totally 1,000,000 pair-wise parameters \( \theta \) and \( \hat{\theta} \) are generated in the training set. We randomly split the datasets into training, validation and test sets with proportions 80%, 10% and 10%, respectively. After training, DRN expects to predict global reward \( R_g \) when provided with the current curve and the desired experimental curve in every immediate fitting steps.

3) Deep Initial Network

To speed up the convergence, initial guess of parameters is learned from a Deep Initial Network (DIN). The schematic illustration of DIN is shown in Fig. 3. The input of the DIN is curve \( \hat{Y} \), followed by two LSTM layers and a fully-connected layer. The output of the network is the parameters \( \hat{\theta} \) which roughly approach global solution \( \theta \). The last hidden layer is followed by a sigmoid nonlinearity and other hidden layers are followed by a rectifier nonlinearity. Totally 1,000,000 training data are generated to train DIN.

The loss function integrates the prior knowledge from both curve fitting and parameters fitting are as follows.

\[
L_{\text{initial}}(\theta, \hat{Y}, \hat{\theta}, \hat{Y}) = \beta_{Lg} L_{g} + \beta_{Lc} L_{c}
\]

\[
L_{g} (\theta, \hat{\theta}) = \sum_{i=1}^{N} \hat{\beta}_i | \hat{\theta}_i - \theta_i | = \langle \beta_i | \hat{\theta}_i - \theta_i | \rangle, 
\] (7)

\[
L_{c} (Y, \hat{Y}) = \sum_{i=1}^{N} \frac{(|Y - \hat{Y}| - |(\hat{\theta}(\theta) - \hat{\theta}(\theta))|)}{\max(f(\theta))} + \sum_{i=1}^{N} \frac{|(\hat{\theta}(\theta) - \hat{\theta}(\theta))|}{\max(f(\theta))}
\]

where \( L_{g} \) and \( L_{c} \) are loss functions from the parameters error and curve error respectively, \( \beta_i \) is weight for the \( i \)-th parameter, and \( \hat{\theta}_i \) is the \( i \)-th parameter to be determined, \( \beta_{Lg} \) and \( \beta_{Lc} \) are weight for \( L_{g} \) and \( L_{c} \) respectively. In this work, \( \beta_{Lg} \) and \( \beta_{Lc} \) are both 1, \( \beta_i \) for \( E_0, \alpha, \) and \( \tau \) are 0.09, 0.9 and 0.01 in DIN.

4) DQMP algorithm

Schematic illustration of DQMP is shown in Fig. 4, where a DIN is used to initialize the parameter vector \( \theta \), and a DRN is used to predict the global rewards function, \( R_g \). The pseudo-code of DQMP is provided in Appendix A.

![Fig. 4. Schematic illustration of the Deep Q-learning of Model Parameters (DQMP) algorithm. DIN is used to predict initial guess of parameters, and DRN is used to predict global rewards.](https://example.com/fig4.png)

B. KVFD model and its solution

The KVFD model provides mathematically simple and experimentally flexible descriptions of viscoelastic properties of soft biological materials with physically interpretable model parameters [34]-[39]. The KVFD constitutive equation consists of a spring in parallel with a fractional-order dashpot element.
The constitutive equation relating stress $\sigma(t)$ to strain $\varepsilon(t)$ for the KVFD model is expressed as [39],

$$
\sigma(t) = E_0\varepsilon(t) + E_0\tau^\alpha \frac{d\varepsilon(t)}{dt},
$$

where $E_0$ is an elastic modulus (Pa), $\tau$ is a time constant, and $\alpha$ is a unitless real number between (0,1) that defines the derivative order. The three model parameters, $E_0$, $\alpha$, and $\tau$, characterize respectively the elasticity, the fluidity, and the viscous time constant of the soft matter.

KVFD solutions to a variety of experimental protocols [39] collected using a spherical indenter are the model functions applied during parameter estimation. These solutions are listed in Table I.

In (9-1)-(9-3), $T_r$ is the ramp time. $h_m$ and $P_m$ are the maximum displacement and force loadings. $v = h_m/T_r$, $k = P_m/T_r$. $B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$ (with $\text{Re}(x) > 0$, $\text{Re}(y) > 0$) is a complete beta function and $B(\alpha;x,y) = \int_0^1 t^{\alpha-1}(1-t)^{y-1}dt$ for $\alpha \in [0,1]$ is an incomplete beta function. In addition, $E_{a,\beta}(z) = \sum_{k=0}^{\infty} \frac{k^a}{\Gamma(\alpha k+\beta)}$ (with $\alpha > 0$, $\beta > 0$) is the Mittag-Leffler (M-L) function, for $\beta = 1$. It is written as the M-L function in one parameter, i.e. $E_{a,1}(z) \equiv E_a(z)$. The standard test setting is provided in Appendix B.

In the following, the DQMP algorithm is validated by fitting raw experimental data to the KVFD model equations.

### III. RESULT

The DQMP algorithm was applied to extract viscoelastic parameters [$E_0, \alpha, \tau$] by fitting force mapping curves to KVFD solutions under different protocols (hereinafter referred to as KVFD equations, in which the fitting function $f$ is provided in (9-1)-(9-3) according to corresponding experimental protocols) and capability of DQMP algorithm was evaluated. The training data was generated by the KVFD modeling. The input of the curve sampled providing the time steps in the experiments.

All instances were set as follows: the experimental curves were sampled to $m$=250 points according to actual time-steps. To balance the importance of every input features, all parameters were normalized by the min-max normalization method [40]. For simulation, the KVFD parameter space range was $E_0 \in [10,1000000]$ , $\alpha \in [0.01,0.99]$ and $\tau \in [1,10000]$ . The training data (1,000,000 curves) is randomly generated in parameters space. The detailed experimental settings is provided in Appendix C.

#### A. Validation and evaluation of DQMP algorithm

1) KVFD curve fitting

The simulation data was generated with the parameter $\theta = [20000, 0.2, 50]$ by KVFD equations (9-1)-(9-3). The random

![Fig. 5. KVFD fittings of the noisy simulation data generated with parameters $\theta = [E_0, \alpha, \tau]= [20000, 0.2, 50]$](image)

The curves in 1st-3rd columns are relaxation, load-unload, creep curves respectively, and curves in 1st-5th rows are the curve without noise, with Gaussian noise, Uniform noise, Rayleigh noise, Exponential noise respectively. The corresponding standard deviations of noise level are $10^{-2}$, $10^{-6}$, $10^{-10}$, $10^{-13}$ and $10^{-7}$, respectively. The simulated data are drawn with blue circles and the predicted fitting curves by DQMP are drawn with red lines.
noise was then added to the data. As shown in Fig. 5, the proposed method yielded a precise fit between the simulated data (blue circles) and the prediction curves (red lines) across all noise levels (rows) under all three loading protocols (columns), as the fitting is precise with goodness of fit $R^2 \geq 0.9912$, confirming that DQMP can fit the experimental data accurately. The estimated parameters listed in Table II are close to its global solutions as the relative errors of the estimated parameters are lower than 3% for $E_0$, $\alpha$ and $\tau$ respectively in all cases, indicating a great consistency of the estimated parameters and the true ones.

2) Evaluation of the estimated parameters by statistical analysis

Ten thousand randomly curves (by adding Gaussian noise variance $= 10^{-7}$ to the ideal curves to simulate experimental noisy data) were generated, and the parameters $[E_0, \alpha, \tau]$ were extracted by DQMP. The curve fitting was accurate with $R^2 \geq 0.952$. Pair-wise difference analysis and Pearson correlation analysis were conducted between the estimated parameters and the actual parameters. The results show that $E_0$ and $\alpha$ were close to their true values within 5% and 1.5% ($p < 0.05$). The estimated fluidity $\alpha$ ($\tau = 0.999, p < 0.001$) and elastic modulus $E_0$ ($\tau = 0.983, p < 0.001$) were close to and highly correlated to their global solutions. To a lesser extent, the estimated viscosity $\tau$ ($\tau = 0.866, p < 0.001$) correlated to its global solutions, as $\tau$ is sensitive to noise.

3) Evaluation of KVFD parameters imaging

As shown in Fig. 6(a), the simulation image was generated with four sets of KVFD parameters [20000, 0.7, 800], [40000, 0.5, 600], [60000, 0.3, 400] and [80000, 0.1, 200] in the four 8 x 8 sub regions respectively. The corresponding four ideal curves generated and degraded by adding random Gaussian noise (variance $= 10^{-6}$) to simulate the 256 experimental noisy curves. The initial guess of the parameters is provided by the proposed DIN.

The representative fitting of the noisy data by the proposed DQMP algorithm is shown in the first column of Fig. 6(b). All 256 noisy curves were fitted with $R^2 \geq 0.9643$. The estimated parameters were imaged as shown in the 2nd, 4th column, from which we can see the extracted parameters were close to the ideal values and robust to Gaussian noise. The images of elastic modulus $E_0$ and fluidity $\alpha$ are almost uniform. The viscosity image of $\tau$ is with only slight noise fluctuation. All three imaged parameters can depict the true parameters well, which confirmed the accuracy and robustness of DQMP, indicating its potential of finding global parameters close to the true ones.

The results of DQMP (as displayed in Fig. 6(b)) is also compared with Q-learning (QL) and Least Square Method (LSM) optimization algorithms, respectively. As shown in Fig. 6(c), the Q-learning algorithm could also fit well as $R^2 \geq 0.9641$. However, with only curve fitting errors constraints, there is no parameter constraints introduced, therefore, it equals to $R_\alpha = 0$. The images of $E_0, \alpha, \tau$ are deviated to their ideal values, especially for $\tau$. As shown in Fig. 6(d), the LSM

| Load protocol | Noise | $E_{E0}\%$ | $E_{\alpha}\%$ | $E_{\tau}\%$ | $R^2$ (mean) |
|---------------|-------|------------|------------|------------|-------------|
| Relaxation    | Non   | 0.6±0.4    | 0.3±0.2    | 1.6±1.2    | 0.9999      |
|               | Gaussian | 0.5±0.4    | 0.4±0.3    | 1.5±1.4    | 0.9958      |
|               | Uniform | 0.5±0.4    | 0.3±0.2    | 1.4±1.4    | 0.9996      |
|               | Rayleigh | 0.5±0.4    | 0.4±0.3    | 1.5±1.3    | 0.9982      |
|               | Exponential | 0.6±0.4    | 0.4±0.3    | 1.6±1.4    | 0.9955      |
| Load-unload   | Non   | 0.4±0.3    | 0.8±1.0    | 1.1±0.9    | 0.9999      |
|               | Gaussian | 0.6±0.5    | 2.8±2.7    | 1.0±0.8    | 0.9929      |
|               | Uniform | 0.5±0.4    | 2.0±2.0    | 1.1±0.8    | 0.9993      |
|               | Rayleigh | 0.6±0.5    | 2.6±2.3    | 1.2±0.9    | 0.9969      |
|               | Exponential | 0.7±0.6    | 3.0±2.6    | 1.2±0.9    | 0.9924      |
| Creep         | Non   | 0.9±0.7    | 0.5±1.1    | 2.7±3.0    | 0.9999      |
|               | Gaussian | 0.9±0.7    | 0.8±1.3    | 2.6±2.6    | 0.9945      |
|               | Uniform | 0.8±0.9    | 0.8±1.8    | 2.6±2.8    | 0.9993      |
|               | Rayleigh | 0.9±0.7    | 0.6±1.2    | 2.6±2.4    | 0.9979      |
|               | Exponential | 0.9±0.9    | 0.7±1.9    | 2.5±2.3    | 0.9948      |
algorithm for extracting viscoelastic parameters show the
difference in the montage interface.

As shown in Fig. 8, the AFM force map of the punctured
breast tissue was acquired under load-unload testing with a
spherical indenter (Ethics number: XJTU1AF2017LSK-46). All
curves fitting were accurate with $R^2 \geq 0.9708$. Images of
viscoelastic parameters $[E_0, \alpha, \tau]$ for benign (upper) and
malignant (lower) tissue samples in $16 \times 16$ pixel regions
($20\mu m \times 20\mu m$) are provided in the second to fourth
column. There were significant differences found between benign tissue
versus malignant tissue based on the elastic modulus $E_0$ ($p < 0.001$),
the fluidity $\alpha$ ($p < 0.001$) and the viscous time constant
$\tau$ ($p < 0.001$). It can be seen that benign tissue is regionally
homogenous in its viscoelastic properties, while malignant
tissue is more heterogeneous.

Furthermore, ultrasound viscoelastic imaging of breast tissue
is validated. The data collected under the condition of creep
loading was fitted to ramp creep solutions with a plate indenter
as follows [39],

$$
e_r(t) = \frac{\sigma_0}{E_0}\left[ 1 - \frac{\alpha}{\tau} \left( \frac{t}{\tau} \right)^\alpha \right], \quad 0 \leq t \leq \tau,
$$

$$
e_r(t) = \frac{\sigma_0}{E_0}\left( \frac{t}{\tau} \right)^\alpha - \frac{\sigma_0}{E_0}\left( \frac{t}{\tau} \right)^\alpha - \frac{\alpha}{\tau} \left( \frac{t}{\tau} \right)^\alpha, \quad \tau \leq t.
$$

where $\sigma_0$ is the maximum loading stress, and $e_r$ is the response
strain.

All curves fitting were accurate with $R^2 \geq 0.9831$. Fig. 9(b)-(d)
and Fig. 9(f)-(h) are in-vivo images of the viscoelastic
parameters $[E_0, \alpha, \tau]$ displayed as color overlays onto in-vivo
ultrasonic strain images of benign and malignant breast tissues,
respectively. The results indicate malignant tissue has higher
elastic modulus and fluidity whereas lower viscosity as
in relative errors. Moreover, DQMP is also robust to different types of noise. Among Gaussian, Uniform, Rayleigh, and Exponential noise, DQMP performs best for curves with Uniform noise.

Second, simulation imaging demonstrated the consistency and reliability of the parameter image. As compared with QL and LSM, the parameters imaged by DQMP were closer to the ground truth image (≤2% in relative error). The proposed DQMP are with the least errors for imaging parameters, showing the powerful global searching ability of DQMP.

Third, DQMP is applied to viscoelastic imaging on a variety of soft tissues under different loading protocols. The results indicate a great potential of DQMP for imaging of physical parameters in diagnosis purpose (i.e., benign vs. malignant). Imaging on breast tissue samples further validates the robustness, applicability, and efficacy of the proposed DQMP in that the increased elastic modulus and fluidity for malignant tissues compared to benign tissue provides with valuable information for disease diagnosis.

V. CONCLUSION

This is the first report to formulate model parameter optimization task to be a state-action decision making in the parameter space. We leveraged Q-learning learning integrated with deep learning to build a model designed to learn global reward values from both visible state (curve fitting) and hidden state (parameters fitting), and proposed DQMP scheme for global parameters optimization on complex nonconvex function. Through DQMP, parameters searching resembles an action selection (k-D move) from 2^k configurations in the k-dimensional parameters space. The proposed DQMP combines the prior knowledge through DRN. Appropriate decision is made by maximizing Q-value, which combines the current reward and future reward functions from both visible states and hidden states, so as to update parameters toward global solution iteratively.

In summary, the novelty of the work is as follows:
1) Model parameter optimization problem is converted to be state-action decision making in the parameters space.
2) To guide global searching, a deep reward network is proposed to learn the global reward from both hidden state and visible state.
3) A novel DQMP method considers not only current reward function but also future reward function, so as to lead global searching iteratively by maximizing Q-value in the parameter space.

The proposed DQMP is demonstrated to be capable of finding global optimal model parameters and show potential for imaging of physical parameters in many applications.

While the proposed DQMP was tested in KVFD model in this study, the proposed frameworks can be generalized to global optimization for other complex nonconvex functions and physical parameters imaging. In particular, in the field of ultrasonic mechanical imaging, the DQMP is expected to obtain reliable viscoelastic imaging. DQMP could also accurately find global model parameters with high accuracy and consistency, which is crucial for the development of imaging algorithm and equipment.

APPENDIX

A. DQMP algorithm

The DQMP algorithm is described as follows.

Firstly, initial guess of parameters $\hat{\theta}$ for experimental curve $\hat{Y}$, also referring to desired curve, is predicted by Deep Initial Network (DIN). Given the immediate curve (determined by $f(t; \hat{\theta})$) and the desired curve, global reward value $R_g$ was predicted by DRN. In each DQMP iteration, $r(s, a)$ and $\max_a'Q(s', a')$ for current state were calculated by combining predicted global reward $R_g$ and immediate curve fitting reward $R_{\text{curve}}$. Q-value is updated according to (2), and action of parameters updating is conducted by maximizing Q-value policy. The algorithmic flow of DQMP is provided in Algorithm 1.

B. Standard indentation test

As shown in Fig. A.1, the responses of samples (lower row of curves) to a force or displacement input over time (upperrow of curves) is recorded by the indenter for stress-relaxation, load-unload or creep-recovery experiments. These are the raw data used to estimate stress-strain behavior and ultimately the viscoelastic properties. $T_r$ is the ramp time. $v = h_m/T_r$, $k = P_m/T_r$.

C. Experimental settings of different protocols performed in the experiments

The detailed experimental settings of different protocols performed in the experiments are provided in Table A.1.

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Fig. A.1. Schematic illustration of (a) ramp-hold relaxation for ramp-on time $T_r$, (b) load-unload and (c) ramp-hold creep protocols. The excitation signals are shown in small gray boxes above the response signals, where $t$ is time, $h$ is indentation depth and $P$ is force.
Algorithm 1: Algorithmic flow of DQMP, $\bar{\theta} = \text{DQMP}(\bar{Y})$

Input:
- $\bar{Y}$ – Experimental data; $\bar{Y} \approx Y = f(t; \theta)$; $f$ is the complex, nonconvex mathematical modeling function.
- $\theta$ is global optimal parameters.

Output:
- $\bar{\theta}$ – Estimated global optimal parameters approaching the global optimal solution $\theta$

1: $j = 1$
2: Initialize Q-table
3: $\bar{\theta}^{(1)} = \text{DIN}(\bar{Y})$ // DIN is Deep Initial Network, see section 2.1.3.
4: while (~convergence) do
5: $\bar{Y}^{(j)} = f(t; \bar{\theta}^{(j)})$
6: $R_g = \text{DRN}(\bar{Y}^{(j)} - \bar{Y})$ // DRN is Deep Reward Network, see section 2.1.2, $R_g$ is a vector
7: for all actions $a$: $(a = a_1, ..., a_N)$ do
8: $R_{\text{curve},a} \leftarrow g\left(\mathbb{E}\left(|\bar{Y}^{(j)}_a - \bar{Y}|\right)\right)$ // $\bar{Y}^{(j)}_a$ is the estimation of $Y$ when selecting action $a$
9: $r(s,a) \leftarrow \beta_{g}R_{g,a} + (1 - \beta_{g})R_{\text{curve}, a}$ // $R_{g,a}$ is the global rewards corresponding to action $a$
10: call DRN($\bar{Y}^{(j)}_a - \bar{Y}$) to predict $R_{g,a}$
11: for all actions $a'$ ($a' = a_1', ..., a_N'$) do
12: calculate $r(s',a')$ with $R_{g,a'}$
13: end for
14: $Q(s,a) \leftarrow Q(s,a) + \xi[r(s,a) + \gamma \max_{a'} r(s',a') - Q(s,a)]$
15: end for
16: choose action $a$: $a \leftarrow \text{argmax}_a Q(s,a)$
17: $\bar{\theta}^{(j+1)} \leftarrow \bar{\theta}^{(j)} + \Delta \bar{\theta}_a$
18: $j \leftarrow j + 1$
19: end while
20: return $\bar{\theta} = \bar{\theta}^{(f)}$

Table A.1: Detailed Experimental Settings of Different Protocols Performed in the Experiments

| Experiment category | Protocols     | Probe type | Probe shape | Loading range | Ramp time $T_r$ | Hold time $T_{\text{hold}}$ |
|---------------------|---------------|------------|-------------|---------------|----------------|-----------------------------|
| Relaxation          | Spherical     | Radius (8.5μm) | Depth (5μm) | 2s            | 3s             |
| Simulation data     | Load-unload   | Spherical  | Radius (8.5μm) | Depth (5μm) | 25s           | /                          |
| Nanindentation data | Spherical     | Radius (8.5μm) | Stress (5μN) | 2s            | 3s             |
| Ultrasonic data     | Creep         | Plate      | Area (24 cm$^2$) | Stress (833.3 Pa) | 0.25s | 9.75s |

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