Probing $WW\gamma$ Couplings from $\sigma_{\text{tot}}(W)/\sigma_{\text{tot}}(Z)$ in High Energy $ep$ Collisions*

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Abstract

We investigate the sensitivity of total cross sections of $e + p \rightarrow W, Z$ to CP-conserving non-standard $WW\gamma$ couplings. We include all the important production mechanisms and study the dependence of the total $W$ cross sections on the anomalous $WW\gamma$ couplings, $\kappa$ and $\lambda$. We argue that the ratio of $W$ and $Z$ production cross sections is particularly well suited, being relatively insensitive to uncertainties in the theoretical and experimental parameters.

*Talk given by C.S. Kim at the 2nd International Workshop on future $e^+e^-$ colliders at Waikoloa, Hawaii in April 1993
In order to parametrize non-standard $WW\gamma$ couplings, it is important to know what sort of additional couplings can arise once the restrictions due to gauge invariance are lifted. As has been previously shown\(^1\), there can be 14 or more non-standard couplings in the most general case. To keep the analysis manageable, we restrict ourselves to CP and $U(1)$ conserving couplings. This restriction leads to just two anomalous form-factors traditionally denoted by $\lambda$ and $\kappa$, which can be related\(^2\) to the anomalous electric quadrupole and the anomalous magnetic dipole moment of the $W$. In the Standard Model (SM) at tree level $\lambda = 0$ and $\kappa = 1$. At present the best experimental limits $-3.6 < \lambda < 3.5$ and $-3.5 < \kappa < 5.9$ are from a recent analysis by UA(2) collaboration\(^3\). The total production of $W$ boson at $ep$ colliders will provide a very precise test of the structure of SM triple-boson $WW\gamma$ vertex. A measurement of the anomalous coupling in the $WW\gamma$ vertex at $ep$ colliders can best be achieved by considering the ratio of the $W$ and $Z$ production cross sections. The advantage of using a cross section ratio is that uncertainties from the luminosity, structure functions, higher-order corrections, QCD scale, etc. tend to cancel\(^4\). We include both the lowest-order resolved processes and the dominant direct photoproduction processes. Care must be taken to avoid double counting those phase-space regions of the direct processes which are already included in the resolved processes.

We first focus on the total production of $W$ and $Z$ in $ep$ collisions. In the short term these processes will be studied at HERA($E_e = 30$ GeV, $E_p = 820$ GeV, $\mathcal{L} = 200$ pb\(^{-1}\) yr\(^{-1}\)), while in the long term availability of LEP $\times$ LHC($E_e = 50$ GeV, $E_p = 8000$ GeV, $\mathcal{L} = 1000$ pb\(^{-1}\) yr\(^{-1}\)) collider will give collision energies in excess of 1 TeV. We calculate the total cross sections for the five different processes which contribute to single $W$ and $Z$ production at $ep$ colliders. From the sum of these contributions we then calculate the ratio $\sigma(W)/\sigma(Z)$ as a function of the anomalous $WW\gamma$ coupling parameters $\kappa$ and $\lambda$. The processes are

\begin{align*}
e^- + p &\to e^- + W^\pm + X, \quad (1a) \\
&\to \nu + W^- + X, \quad (1b) \\
&\to e^- + Z + X \ (Z \text{ from hadronic vertex}), \quad (1c)
\end{align*}
→ $e^- + Z + X$ ($Z$ from leptonic vertex), \hspace{1cm} (1d)

→ $\nu + Z + X$. \hspace{1cm} (1e)

The largest contributions for $W$ and $Z$ productions come from the processes (1a) and (1c) which are dominated by the real photon exchange Feynman diagrams with a photon emitted from the incoming electron, $e^- + p \to \gamma/e + p \to V + X$. The dominant subprocesses for $\gamma + p \to V + X$ would appear to be the lowest order $q_{/\gamma}^{(l)} + q \to V$, where $q_{/\gamma}$ is a quark inside the photon. However this may not be strictly true, even at very high energies, since quarks inside the photon $q_{/\gamma}$ exist mainly through the evolution $\gamma \to q\bar{q}$. Hence the direct process $\gamma + q \to q^{(l)} + V$ could be competitive with the lowest order contribution $q_{/\gamma}^{(l)} + q \to V$. This raises the subtle question of double counting\(^4\). Certain kinematic regions of the direct processes contribute to the evolution of $q_{/\gamma}$ which is already included in the lowest order process. Both double counting and the mass singularities are removed\(^5\) if we subtract the contribution of $\gamma + q \to q^{(l)} + V$ in which the $\hat{t}$-channel-exchanged quark is on-shell and collinear with the parent photon. Thus the singularity subtracted lowest order contribution from the subprocesses $q_{/\gamma}^{(l)} + q \to V$ is

$$
\sigma^L(e^- + p \to V + X) = \frac{C^L_V}{s} \int_{m^2_V/s}^{1} \frac{dx_1}{x_1} \left[ \sum_{qq'} (f_{q/e} - \tilde{f}_{q/e})(x_1) f_{q'/p}(\frac{m^2_V}{x_1s}) + (q \leftrightarrow q') \right],
$$

(2)

where $C^L_V = W, Z$ are defined in Ref. 4. The electron structure functions $f_{q/e}$ (and $\tilde{f}_{q/e}$) are obtained as usual by convoluting the photon structure functions $f_{q/\gamma}$ with the Weizäcker-Williams approximation of (quasi-real) photon radiation. The part of photon structure function, $\tilde{f}_{q/\gamma}$, results from photon splitting at large $x$ (with large momentum transfer)\(^4\). To obtain the total cross section of processes (1a) and (1c), we add to the cross section in (2) the contribution from the direct subprocesses, $\gamma + q \to q^{(l)} + V$,

$$
\sigma^D(e^- + p \to V + X) = \frac{C^D_V}{s} \int_{m^2_V/s}^{1} \frac{dx_1}{x_1} \int_{m^2_V/x_1s}^{1} \frac{dx_2}{x_2} \left[ \sum_q f_{q/e}(x_1) f_{q/p}(x_2) \right] \eta_V(\hat{s}),
$$

(3a)

where the integrated hard scattering cross sections $\eta_V$ are

$$
\eta_{V=Z}(\hat{s}, m^2_Z, \Lambda^2) = (1 - 2z + 2z^2) \log \left( \frac{\hat{s} - m^2_Z}{\Lambda^2} \right) + \frac{1}{2}(1 + 2z - 3z^2),
$$

2
\[ \eta_{\nu=W}(\hat{s}, m_{W}^2, \Lambda^2, Q = |e_q|, \kappa, \lambda) = (Q - 1)^2(1 - 2z + 2z^2) \log \left( \frac{\hat{s} - m_{W}^2}{\Lambda^2} \right) \]

\[ - \left[ (1 - 2z + 2z^2) - 2Q(1 + \kappa + 2z^2) + \frac{(1 - \kappa)^2}{4z} - \frac{(1 + \kappa)^2}{4} \right] \log z \]

\[ + \left[ \left( 2\kappa + \frac{(1 - \kappa)^2}{16} \right) \frac{1}{z} + \left( \frac{1}{2} + \frac{3(1 + Q^2)}{2} \right) \frac{Q}{z} - \frac{(1 + \kappa)^2}{16} + \frac{Q^2}{2} \right] (1 - z) \]

\[ - \frac{\lambda^2}{4z^2}(z^2 - 2z \log z - 1) + \frac{\lambda}{16z}(2\kappa + \lambda - 2) [(z - 1)(z - 9) + 4(z + 1) \log z] , \]

and \( C_{V=W,Z}^D \) are defined in Ref. 4.

The processes (1b) and (1d) are dominated by configurations where a (quasi-real) photon is emitted (either elastically or quasi-elastically) from the incoming proton and subsequently scatters off the incoming electron, i.e. \( e^- + p \rightarrow e^- + \gamma/p \rightarrow e^- (\text{ or } \nu) + V \). For the elastic photon, the cross section can be computed using the electrical and magnetic form factors of the proton. For the quasi-elastic scattering photon, the experimental information on electromagnetic structure functions \( W_1 \) and \( W_2 \) can be used, following Ref. 7. The hard scattering cross section for \( e^- + \gamma/p \rightarrow e^- (\text{ or } \nu) + V \) is given by

\[ \hat{\sigma}(e^- + \gamma/p \rightarrow e^- (\text{ or } \nu) + V) = \frac{C_{V=W}^D}{s} \eta_{\nu}(Q = |e_q| = 1). \]

Finally for process (1e), which is a pure charged current process, we simply use the results of Bauer et. al. \(^7\) to add to the contributions from (1c) and (1d). The contribution from this process to the total \( Z \) production cross section is almost negligible even at LEP \( \times \) LHC \( ep \) collider energies. With the anticipated luminosities the total \( Z \) production cross section corresponds to 84 events/yr (HERA) and 5400 events/yr (LEP \( \times \) LHC). After including a 6.7 % leptonic branching ratio (i.e. \( Z \rightarrow e^+e^-, \mu^+\mu^- \)), the event numbers become about 6 events/yr(HERA) and 360 events/yr(LEP \( \times \) LHC).

In Fig. 1, we show the ratio of \( \sigma(W^\pm)/\sigma(Z) \) and \( \sigma(W^-)/\sigma(Z) \) as a function of \( \kappa \) and \( \lambda \). Rather than vary both parameters simultaneously, we first set \( \kappa \) to its Standard Model value and then vary \( \lambda \) and vice versa. The error range represents the variation in the cross section by varying the theoretical input parameters as follows : \( m_{\nu}^2/10 \leq Q^2 \leq m_{\nu}^2 \), photon structure functions \( f_{q/g} \) from DG\(^8\) and DO+VMD\(^9\), and proton structure functions \( f_{q/p} \) from
EHLQ$^{10}$ and HMRS(B)$^{11}$. It is important to note that once photoproduction experiments at HERA determine $f_{q/p}$ and $f_{q/\gamma}$ more precisely, we will be able to predict the total cross sections for each process with much greater accuracy. After 5 years of running, HERA will produce about $30\, e+p \rightarrow Z+X \rightarrow l^++l^-+X$ events, and this will enable us to determine$^{12}$ $\kappa$ and $\lambda$ with a precision of order $\Delta\kappa \approx \pm 0.3$ for $\lambda = 0$, $\Delta\lambda \approx \pm 0.8$ for $\kappa = 1$. At LEP $\times$ LHC, one year’s running will give $\Delta\kappa \approx \pm 0.2$ for $\lambda = 0$, $\Delta\lambda \approx \pm 0.3$ for $\kappa = 1$.

**Acknowledgements**

The work was supported in part by the Korea Science and Engineering Foundation and in part by the Korean Ministry of Education. The work of CSK was also supported in part by the Center for Theoretical Physics at Seoul National University and in part by a Yonsei University Faculty Research Grant.
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Figure Captions

Fig. 1. Total cross section ratios of \( \sigma(W^\pm)/\sigma(Z) \) and \( \sigma(W^-)/\sigma(Z) \) as a function of (a) \( \kappa \), and (b) \( \lambda \) at the HERA and LEP×LHC \( ep \) colliders. Rather than vary both parameters simultaneously, we first set \( \kappa \) to its Standard Model value and then vary \( \lambda \) and vice versa. The error range represents the variation in the cross section by varying the theoretical input parameters as follows: \( m_V^2/10 \leq Q^2 \leq m_V^2 \), photon structure functions \( f_{q/\gamma} \) from DG\(^8\) and DO+VMD\(^9\), and proton structure functions \( f_{q/p} \) from EHLQ\(^1\)\(^{10}\) and HMRS(B)\(^1\)\(^{11}\).
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