Symmetric Nash equilibrium of political polarization in a two-party system

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Abstract

The median-voter hypothesis (MVH) predicts convergence of two party platforms across a one-dimensional political spectrum during majoritarian elections. From the viewpoint of the MVH, an explanation of polarization is that each election has a different median voter so that a party cannot please all the median voters at the same time. We consider two parties competing to win voters along a one-dimensional spectrum and assume that each party nominates one candidate out of two in the primary election, for which the electorates represent only one side of the whole population. We argue that all the four candidates will come to the same distance from the median of the total population through best-response dynamics.

Keywords: Political polarization, Median-voter hypothesis, Best-response dynamics

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1. Introduction

Statistical physics deals with macroscopic patterns emerging from microscopic interactions. Collective decision-making is an example of such emergent phenomena arising from individual-level interactions [1, 2, 3, 4]. Broadly speaking, the group of activities associated with collective decision-making in society can be referred to as politics, and it is why statistical physicists have viewed the interplay between individual choices and political changes within the framework of complex systems [5, 6, 7, 8]. One of widely accepted political values is democracy, according to which the people have the right to rule. A democratic government thus has an excellent incentive to meet the people’s demands, but the price is political instability: the governmental policies may suddenly change if the power is transferred to the opposing party by election, and the change can be especially drastic when political opinions are polarized to a great degree.

Whether political polarization is an inevitable part of democracy is not clear. As will be explained below, in a two-party system, the median-voter hypothesis (MVH) predicts that the two parties along a one-dimensional political spectrum will converge to the median voter’s position through majoritarian elections [9]. The assumptions of the MVH might look too restrictive, but it has been reported in congressional voting that very often the issue reduces to a one-dimensional matter [10]. In such a one-dimensional political landscape, a two-party system is robust, as assumed by the MVH, because a centrist party cannot easily find a political “niche” between two existing parties [11]. Thus, the question still remains: Why are they so polarized?

It may be attributed to social homophily or echo chambers [12, 13], and temporary polarization may be consistent with Bayesian updating [14]. Many of the existing approaches assume that people update their political positions through mechanistic interaction with neighbors such as homophily, assimilation, and differentiation [15, 16, 17, 18]. A recent study answers this question by pointing out that voters are not as rational as assumed in the MVH [19]: According to this idea, voters will rather satisfice than maximize their utility functions. By adapting to such voting behavior with a simple gradient ascent method, the parties can develop symmetric polarization, maintaining an equal distance from the median voter.

In this work, we wish to examine an alternative explanation [20], which argues that the median voter’s position experienced by a party may be different from election to election: For example, a party has to nominate a candidate
before a presidential election. To become the nominee, candidates in the
primary should take opinions from their own supporters seriously, even if the
supporters’ overall position differs from the general public opinion, because
their votes are needed to win the primary. By considering the nomination
process as a part of election, this approach explains permanent polarization
without resorting to cognitive biases and bounded rationality. In addition,
our work provides a testable prediction that polarization will only increase
further if the payoff for the loser of the final election also becomes valuable
enough, which has not been proposed by other models.

In the next section, we will formulate our model as a game of two-round
competition among four players, whose strategies are their political positions.
The Nash equilibria will be identified. We then propose how to update the
players’ positions to reach one of the Nash equilibria. Such dynamic con-
sideration is important in several aspects: First, the existence of plausible
dynamics ensures the feasibility of the discovered equilibrium. Second, the
time scale to reach the equilibrium can be estimated: If it diverges, devi-
ation from the equilibrium can prove more pervasive than predicted by a
static analysis [21, 22]. Third, when multiple equilibria exist, the distribu-
tion of convergence points will generally require dynamical consideration. As
a model of dynamics, one could consider an evolutionary process which works
in a large population of simple-minded agents, but we believe that a variety
of strategic moves in the course of election can be better described by best-
response dynamics among the candidates, according to which each candidate
deliberately seeks a position to be the final winner given that the other com-
petitors’ positions are held fixed. The whole dynamics is thus projected onto
four candidates, and the electorate react to the candidates instantaneously
with fixed political positions, which is an assumption that we have borrowed
from the MVH. The key point is that everyone in our model society seeks
the best response to the given configuration, and this is one of common ap-
proaches to model human behavior in evolutionary game theory [23]. In
other words, each player tries to maximize his or her own objective function,
as is not uncommon in physics, but an important difference of game theory
from physics is that such individual optimization may drive the total system
away from an optimal point.
2. Model

2.1. Median-voter hypothesis

The MVH is based on two main assumptions \[24\]: First, political positions are defined along a one-dimensional spectrum. Second, each voter has single-peaked preferences along the axis. To define single-peakedness, let \( U_i(x) \) be voter \( i \)'s utility function defined over \( x \). The ideal point for \( i \), denoted as \( x^*_i \), is such that \( U(x) < U(x^*_i) \) for any \( x \neq x^*_i \). Choose two points \( y \) and \( z \) along the \( x \) axis so that either \( y, z > x^*_i \) or \( y, z < x^*_i \). Voter \( i \)'s preferences are single-peaked if and only if \( U_i(y) > U_i(z) \) is equivalent to \( |y - x^*_i| < |z - x^*_i| \). That is, when \( y \) and \( z \) are on the same side of \( x^*_i \), voter \( i \) prefers \( y \) to \( z \) if and only if \( y \) is closer to \( x^*_i \) than \( z \) is.

Imagine a set of ideal points of \( N \) voters, \( \{x^*_1, x^*_2, ..., x^*_N\} \). Let \( N_L(x) \) be the number of \( x^*_i \leq x \) for a certain position \( x \), and let \( N_R(x) \) be the number of \( x^*_i \geq x \). Then, a median position is any number \( x_m \) such that \( N_L(x_m) \geq N/2 \) and \( N_R(x_m) \geq N/2 \). When the above two main assumptions hold true, the MVH states that this median position does not lose under majority rule. The reason is the following: For any \( z < x_m \), voters on the right-hand side of \( x_m \) prefer \( x_m \) to \( z \) because of their single-peaked preferences, and the number of such voters is greater than or equal to \( N/2 \) by definition. A similar argument shows that \( x_m \) does not lose to any \( z > x_m \).

The above analysis on the median voter has provided the basis for Hotelling’s law, which says that two party platforms will eventually converge to the median voter’s position. This is better described as a working hypothesis rather than a mathematical statement because the assumptions behind the MVH are only approximate in reality. Some researchers have reported that local governments with two parties tend to adopt more moderate policies than those with single parties, in support of the MVH \[25\], but whether the relationship between the median voter and policy variables is statistically significant remains in question \[26\].

2.2. Semifinalists’ dilemma

Before explaining our model of election, let us first review a strongly related model called semifinalists’ dilemma \[27\]: Imagine a single-elimination tournament among four players, in which only the final winner earns a unit payoff. Four equally strong semifinalists have to decide how much stamina to use in the semifinals, given that only the rest is available in the final [Fig. 1(a)]. The strategy of a player, say, \( i \), is thus the amount of stamina to
Figure 1: Schematic diagrams to show the semifinalists’ dilemma and its equivalence to the two-round election. (a) In the semifinalists’ dilemma, four players begin with equal stamina (bottom). The one who invests more stamina than the opponent will proceed to the final, but one cannot win the final if he or she spent too much stamina in the semifinal. (b) In the two-round election, we have two parties, $\alpha$ (blue) and $\beta$ (red), and each party has two candidates in the primary (bottom). The bluish color box represents the political spectrum among $\alpha$-supporters, $I_\alpha = [-2,0]$, and the reddish one represents its $\beta$-counterpart, $I_\beta = [0,2]$. The candidates’ positions are marked in the color boxes by sliders. The arrows below the color boxes show the party supporters’ median positions, $x^*_\alpha = -1$ and $x^*_\beta = 1$, respectively. Each party organizes a primary election, in which the party supporters nominate one of the candidates for the final by measuring the distances (dotted lines). The final winner is the one closer to the position $x^*$ of the median voter among the whole electorate.
invest in the semifinal, which may be regarded as a continuous variable $x_i$ in the unit interval $[0, 1]$. The simplest rule for deciding the winner of a match between two competitors is that it is the one who invests more stamina in the match. Therefore, player $i$ must spend enough stamina $x_i$ to pass the semifinal, and also keep $1 - x_i$ large enough to win the final. The question is how much is the optimal level of $x_i$.

It turns out that this game has infinitely many Nash equilibria. That is, as long as everyone chooses the same strategy $x$, no one has reason to deviate from it, regardless of the specific value of $x$: Assume that player $i$ finds everyone else using a certain common strategy $x$. If $x_i < x$, player $i$ will lose the semifinal. If $x_i > x$, player $i$ will lose the final. Only by choosing $x_i = x$, player $i$ can ensure victory with probability $1/4$, which is a natural consequence of symmetry among the four players.

We may also think of variants of this game, e.g., by assigning a positive payoff $u < 1$ to the runner-up as well. This makes the final less important because winning the semifinals already guarantees the second place. If player $i$ spends all the stamina in the semifinal, the expected gain is $u$, whereas the player would earn $(1+u)/4$ on average by conforming to the common strategy $x$. The former expected payoff exceeds the latter when $u > 1/3$. Therefore, if the second-place prize is more valuable than one third of the grand prize, the final will lose attraction, and players will devote all their efforts to the semifinals.

2.3. Two-round election

Let us consider a process of election between two parties, $\alpha$ and $\beta$, as depicted in Fig. 1(b). The election consists of two rounds: In the primaries, each party starts with two candidates and nominates one of them so that the nominees compete in the second round. By assumption, infinitely many voters are uniformly distributed inside a one-dimensional interval $I \equiv [-2, 2]$. Half of them in $I_\alpha \equiv [-2, 0]$ support $\alpha$, whereas the other half in $I_\beta \equiv [0, 2]$ support $\beta$. Each of the $\alpha$-candidates, denoted by $A$ and $a$, chooses a position inside $I_\alpha$, and the same is true for the other party $\beta$ with candidates $B$ and $b$. The four candidates’ positions are their strategies and will be denoted by $x_A$, $x_a$, $x_B$, and $x_b$, respectively. As assumed in the MVH, every voter votes for a candidate with the closest position. The winner of this game is the candidate who wins both the rounds by getting a larger share of voters than the competitor in each round.
In the primaries, the median voter for $\alpha$ is located at position $x^*_\alpha \equiv -1$, whereas the median voter for $\beta$ is at $x^*_\beta \equiv +1$. In the second round, the median voter is found at $x^* = 0$. Therefore, for any $\alpha$-candidate, a position outside $\tilde{I}_\alpha \equiv [-1, 0]$ is unreasonable, and both $x_A$ and $x_a$ must thus belong to $\tilde{I}_\alpha$. By the same token, both $x_B$ and $x_b$ must belong to $\tilde{I}_\beta \equiv [0, 1]$. The connection to the semifinalists’ dilemma is clear: Just as a player’s stamina is divided between the semifinal and the final, a candidate’s political position divides an interval of unit length into two pieces. One of them shows the distance from the median opinion of party supporters, and the other shows the distance from the origin. If the former distance is short, he or she has a high chance to win the primary, but that can be disadvantageous in the final because the median voter at the origin will not choose such a one-sided position.

It is again straightforward to show that any symmetric configuration such that $-x_A = -x_a = x_B = x_b \in [0, 1]$ constitutes a Nash equilibrium: Suppose that $A$ tries moving closer to the origin when the four players have reached the equilibrium. This move will only make the candidate lose the primary because his or her competitor $a$’s position is closer to the median voter’s position $x^*_\alpha = -1$. On the other hand, if $A$ moves away from the origin, $A$ will be defeated by the opponent from $\beta$ in the second round because $\|x_A\| > \|x_B\| = \|x_b\|$.

3. Results

3.1. Numerical simulation of best-response dynamics

To simulate best-response dynamics, we consider the following stochastic dynamics: We have four candidates denoted as $A$, $a$, $B$, and $b$, respectively. The former two, $A$ and $a$, choose their initial positions randomly from $\tilde{I}_\alpha = [-1, 0]$, whereas the latter two, $B$ and $b$, do the same from $\tilde{I}_\beta = [0, 1]$. At each time step, let us randomly pick up a candidate $i \in \{A, a, B, b\}$. Then, if this candidate $i$’s belongs to party $\alpha$, his or her new position is sampled $M$ times within $\tilde{I}_\alpha$. Or, if $i$ belongs to $\beta$, the new position is sampled $M$ times within $\tilde{I}_\beta$. If the new position leads to $i$’s victory under the assumption that the other three positions are fixed, the focal candidate $i$ moves to the new position. If more than one out of the $M$ sampled positions are found to bring victory, the last one will be chosen as $i$’s position at the next time step. If none of the $M$ sampled positions produce victory, he or she maintains the status quo. After trying $M$ new positions for $i$ in this way, the next time step
Figure 2: (a) A simulation of best-response dynamics, in which the candidates’ positions converge to a Nash equilibrium, which has $\|x\| \approx 0.46$ in this particular sample. The convergence point depends on the initial positions, the order of updates, and the $M$ trials for updating each candidate's position. (b) Histograms of the equilibrium distances from the origin over random initial conditions and random sequential updates. The initial positions are drawn either uniformly from the respective intervals for the parties (purple), or according to the truncated normal distribution in $[-1, 1]$ with a mean at $x = 0$ and a scale parameter $\sigma = 0.3$ (green). The dark green rectangles represent the overlap between the two histograms.

starts by picking up another candidate from $\{A, a, B, b\}$. This simulation will reproduce best-response dynamics in the limit of $M \to \infty$, and we set $M$ to be sufficiently large number of $O(10^2)$ in practice. This algorithm is statistically equivalent to choosing the first winning position as long as the payoff is binary, i.e., either victory or defeat. We can also speed up the simulation by noting that one cannot gain victory if his or her competitor in the same party has a larger distance from the origin than the candidates of the other party.

Our simulation shows that this dynamics converges to a symmetric Nash equilibrium within a finite time scale [Fig. 2(a)]. In fact, the convergence slows down as the distances among the candidates approach $1/M$, but we may neglect this later phase of convergence in the limit of $M \to \infty$. Because of the stochasticity of the dynamics, the convergence point depends on the initial positions, the order of updates, and the $M$ trials for updating each candidate's position. If we take independent samples by generating different random number sequences uniformly from $[0, 1]$, the probability density of $\|x\|$ after convergence has a maximum around 0.7 [Fig. 2(b)]. It is intuitively plausible that the candidates will not easily converge to $\|x\| = 0$ or 1, but they tend to exhibit a higher degree of polarization than $1/2$, which is their average distance from the origin at the beginning. The increase of polarization is
a general tendency: We have also tried a unimodal distribution centered at $x = 0$, and the average distance from the origin after reaching a Nash equilibrium is again greater than at the beginning.

3.2. Convergence to the Nash equilibrium

We can prove the convergence property as follows: Let us write the candidates in ascending order of their distances from the true median voter at the origin, regardless of their signs. In Fig. 3(a), we show an example of $abAB$ on the left, which means that $a$ is the closest candidate to the origin whereas $B$ is the farthest. From each candidate’s point of view, the winning condition is to have a larger distance than the intra-party competitor to win the first round, but closer than at least one competitor from the other party to win the second round. If $B$ is chosen to move in this example, therefore, $B$ has to take a position between $b$ and $A$. Or, if it was $a$’s turn, $a$ would move somewhere between $A$ and $B$. Either of these moves shrinks the effective length of the interval occupied by the four candidates, defined by

$$L \equiv \max_{i,j} (\|x_i\| - \|x_j\|),$$  \hspace{1cm} (1)

because $B$ and $a$ have moved inward from the boundary. On the other hand, $A$’s move to a gap between $a$ and $b$ is length-preserving in terms of the interval length because $A$ was already inside the interval. Note that candidate $b$ cannot win no matter where he or she moves, so $abAB$ has a self-loop, which is omitted in the diagram for the sake of brevity.

In general, a winner exists in any arbitrary configuration, and he or she wins by keeping the same position. For this reason, each node must have a self-loop, and we will not draw it explicitly. Although each node has four incoming links and four outgoing ones in principle, therefore, only three incoming and three outgoing links are enough to represent the connection structure around each node. By combining all such elementary transitions, we obtain a graph as shown in Fig. 3(b), where $L$-decreasing and $L$-preserving transitions are colored red and black, respectively. At any node of the graph, it is impossible to increase the effective interval length. If we run a random walk on this graph, it cannot keep following black arrows, implying that $L$ will keep shrinking until everyone has no reason to move further, i.e., arriving at a Nash equilibrium with $L \rightarrow 0$. 

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Figure 3: (a) Elementary transition among states, where a state is represented by a list of candidates in ascending order of their distances from the origin. The first transition shows a case that $B$ is chosen for update, and his or her new position must be between $b$ and $A$ to defeat both of them. Or, if it is $a$’s turn as represented by the second transition, his or her new position must be between $A$ and $B$. These two transitions, marked as the red arrows, make their positions more symmetric with respect to the origin than before by shrinking the effective length $L$ defined in Eq. (1). By contrast, if $A$ is chosen to move, the transition (black arrow) preserves $L$. Note that $b$ has no reason to move because it is impossible to win this game no matter where he or she goes, and we have omitted the resulting self-loop. (b) Combination of elementary transitions among 24 possible states. The red and black arrows represent elementary transitions decreasing and preserving $L$, respectively. In principle, each node has four incoming links and four outgoing ones, but we have simplified the diagram by omitting a self-loop at each node.
4. Summary and Discussion

In summary, we have investigated the origin of persistent polarization in democracy and proposed an answer based on the existence of multiple political centers. We have constructed a minimalist model of primary elections before the final and showed that all the four candidates keep the same distance from the median voter at the Nash equilibrium reached by best-response dynamics. We have found that such a structure tends to intensify political polarization as shown in Fig. 2(b). This conclusion is consistent with an argument that closed primaries tend to perpetuate polarization [28]. Possible reforms include open participation in primaries and a top-two primary system, although their empirical evidence is not decisive yet [29, 30, 31].

As an extension of the MVH, our model inherits its modeling assumptions of one-dimensionality and single-peakedness (Sec. 2.1), along with its strengths and weaknesses. Practically speaking, this model implies an informationally demanding task because all the players must be able to identify candidates’ positions accurately. Although unrealistic, we stick to this assumption because our goal has been to propose a mechanism of polarization among such ideal agents instead of relying on their imperfections. Another assumption that we have added is that each candidate’s position remains fixed between the two rounds of election. Of course, candidates could shift back to more centrist positions after the primaries, but our assumption still describes a part of reality in the sense that no candidates would want their pledges to be taken as matters of political expediency.

We have analyzed the resulting two-round election by using the similarity to the semifinalists’ dilemma [27], and our system can actually be regarded as its limiting case in which only the final winner is rewarded (Sec. 2.2). Based on this observation, some might attribute polarization to the winner-take-all structure in majoritarian elections. However, a counter-intuitive prediction of our theory is that polarization will only increase further if the payoff for the second-place winner, i.e., the loser of the final election, also becomes valuable enough: If it definitely pays to secure the victory in the primary election, candidates will be motivated to appeal only to their supporters whose median is located far from the political center of the whole electorate. Moreover, the resulting polarization may exhibit strong hysteresis because the corresponding strategy profile \((x_1, x_2, x_3, x_4) = (1, 1, 1, 1)\) is a Nash equilibrium in the semifinalists’ dilemma even for \(u = 0\). It would be an intriguing future direction to check this prediction empirically.
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