Constraints upon the CKM angle $\phi_2 (\alpha)$ from Belle

A. J. Schwartz

Physics Department, University of Cincinnati, P.O. Box 210011, Cincinnati, Ohio 45221 USA

(For the Belle Collaboration)

The Belle experiment has measured branching fractions and $CP$ asymmetries for the charmless decays $B^0 \to \pi^+\pi^-$ and $B^0 \to \rho^\pm\pi^\mp$. From these measurements, constraints upon the CKM angle $\phi_2$ can be obtained. These constraints indicate that $\phi_2$ is around $100^\circ$.

1. Overview

The Standard Model predicts $CP$ violation to occur in $B^0$ meson decays owing to a complex phase in the $3 \times 3$ Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. This phase is illustrated by plotting the unitarity condition $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$ as vectors in the complex plane: the phase results in a triangle of nonzero height. One interior angle of the triangle, denoted $\phi_1$ or $\beta$, is determined from $B^0 \to J/\psi K^0$ decays. Another interior angle, $\phi_2$ or $\alpha$, is determined from charmless decays such as $B^0 \to \pi^+\pi^-$ and $B^0 \to \rho^\pm\pi^\mp$. Here we present measurements of these charmless decays from Belle$^\dag$ and the resulting constraints upon $\phi_2$.

To select $B^0 \to \pi^+\pi^-/\rho^\pm\pi^\mp$ decays, we require two opposite-charge pion-candidate tracks originating from the interaction region. For $B^0 \to \rho^\pm\pi^\mp$, one of these tracks is combined with a $\pi_0$ candidate. The charged pion identification criteria are based on information from time-of-flight counters, aerogel cherenkov counters, and $dE/dx$ information from the central tracker$^\ddag$ $B$ decays are identified via the “beam-constrained” mass $m_{bc} \equiv \sqrt{E_b^2 - p_B^2}$ and the energy difference $\Delta E \equiv E_B - E_b$, where $p_B$ is the reconstructed $B$ momentum, $E_B$ is the reconstructed $B$ energy, and $E_b$ is the beam energy, all evaluated in the $e^+e^-$ center-of-mass (CM) frame. The $m_{bc}$ and $\Delta E$ distributions are jointly fit for the signal event yields.

A tagging algorithm is used to identify the flavor of the $B$ decay, i.e., $B^0$ or $\bar{B^0}$. This algorithm examines tracks not associated with the signal decay to identify the flavor of the non-signal $B$. The signal-side tracks are fit for a decay vertex, and the tag-side tracks are fit for a separate decay vertex; the distance $\Delta z$ between vertices is (to good approximation) proportional to the time difference between the $B$ decays: $\Delta z \approx (\beta\gamma c)\Delta t$, where $\beta\gamma$ is the Lorentz boost of the $e^+e^-$ system.

$^\dag$Charge-conjugate modes are included throughout this paper unless noted otherwise.
The dominant background for both modes is $e^+e^- \rightarrow q\bar{q}$ continuum events, where $q = u, d, s, c$. In the CM frame such events tend to be collimated along the beam directions, whereas $B\overline{B}$ events tend to be spherical. The “shape” of an event is quantified via Fox-Wolfram moments of the form $h_\ell = \sum_{i,j} p_i p_j P_\ell(\cos \theta_{ij})$, where $i$ runs over all tracks on the tagging side and $j$ runs over all tracks on either the tagging side or the signal side. The function $P_\ell$ is the $\ell$th Legendre polynomial and $\theta_{ij}$ is the angle between momenta $\vec{p}_i$ and $\vec{p}_j$ in the CM frame. These moments are combined into a Fisher discriminant, and this is combined with the probability density function for the cosine of the angle between the $B$ direction and the electron beam direction. This yields an overall likelihood $L$. Continuum events are rejected by requiring that the ratio $L_{\pi\pi}/(L_{\pi\pi} + L_{qq})$ be greater than a minimum value.

2. $B^0 \rightarrow \pi^+\pi^-$

The decay time dependence of $B^0/\overline{B}^0 \rightarrow \pi^+\pi^-$ decays is given by

$$\frac{dN}{dt} \propto e^{-\Delta t/\tau} \left[ 1 - q C_{\pi\pi} \cos(\Delta m \Delta t) + q S_{\pi\pi} \sin(\Delta m \Delta t) \right],$$

where $q = +1$ (-1) corresponds to $B^0$ ($\overline{B}^0$) tags, and $\Delta m$ is the $B^0-\overline{B}^0$ mass difference. The parameters $C_{\pi\pi}$ and $S_{\pi\pi}$ are CP-violating and related to $\phi_2$ via

$$C_{\pi\pi} = \frac{1}{R} \cdot \left( 2 \frac{P}{T} \sin(\phi_1 - \phi_2) \sin \delta \right)$$

$$S_{\pi\pi} = \frac{1}{R} \cdot \left( 2 \frac{P}{T} \sin(\phi_1 - \phi_2) \cos \delta + \sin 2\phi_2 - \left| \frac{P}{T} \right|^2 \sin 2\phi_1 \right)$$

$$R = 1 - 2 \left| \frac{P}{T} \right| \cos(\phi_1 + \phi_2) \cos \delta + \left| \frac{P}{T} \right|^2,$$

where $\phi_1 = (23.2^{+1.6}_{-1.5})^\circ$ \cite{5} $|P/T|$ is the magnitude of a possible penguin amplitude relative to that of the tree-level amplitude, and $\delta$ is the strong phase difference between the two amplitudes. If there were no penguin contribution, $C_{\pi\pi} = 0$ and $S_{\pi\pi} = \sin 2\phi_2$. Since Eqs. (2) and (3) have three unknown parameters, measuring $C_{\pi\pi}$ and $S_{\pi\pi}$ determines a volume in $\phi_2 - \delta - |P/T|$ space. The most recent Belle measurement of $C_{\pi\pi}$ and $S_{\pi\pi}$ is with 140 fb$^{-1}$ of data. \cite{7}

The event sample consists of 224 $\overline{B}^0 \rightarrow \pi^+\pi^-$ candidates and 149 $B^0 \rightarrow \pi^+\pi^-$ candidates after background subtraction. These events are subjected to an unbinned maximum-likelihood (ML) fit in $\Delta t$; the results are $C_{\pi\pi} = -0.58 \pm 0.15$ (stat) $\pm 0.07$ (syst) and $S_{\pi\pi} = -1.00 \pm 0.21$ (stat) $\pm 0.07$ (syst), which indicate large CP violation. The nonzero value for $C_{\pi\pi}$ indicates direct CP violation. Fig. \cite{4} shows the $\Delta t$ distributions for $q = \pm 1$ tagged events; a clear difference is seen between the distributions.

These values determine a 95% C.L. volume in $\phi_2 - \delta - |P/T|$ space. Projecting this volume results in the constraints $90^\circ < \phi_2 < 146^\circ$ for $|P/T| < 0.45$ (as predicted by
QCD factorization and perturbative QCD, and \(|P/T| > 0.17\) for any value of \(\delta\). The dependence upon \(|P/T|\) and \(\delta\) can be removed by making additional theoretical assumptions. A model based on SU(3) symmetry that uses the measured rates for \(B^+ \to K^0 \pi^+\) and \(B^0 \to K^+ \pi^-\) obtains \(\phi_2 = (103 \pm 17)^\circ\). 

3. \(B^0 \to \rho^\pm \pi^\mp\)

For \(B^0 \to \rho^\pm \pi^\mp\) the final state is not a \(CP\) eigenstate, and there are four decays to consider: \(B^0 \to \rho^\pm \pi^\mp\) and \(\bar{B}^0 \to \rho^- \pi^+\). The rates can be parametrized as

\[
dN(B \to \rho^\pm \pi^\mp)/d\Delta t \propto (1 \pm A_{CP}^{\rho\pi}) \times e^{-\Delta t/\tau} \left[ 1 - q(C_{\rho\pi} \pm \Delta C_{\rho\pi}) \cos(\Delta m \Delta t) + q(S_{\rho\pi} \pm \Delta S_{\rho\pi}) \sin(\Delta m \Delta t) \right],
\]

where \(q = +1\) \((-1)\) corresponds to \(B^0\) \((\bar{B}^0)\) tags. The parameters \(C_{\rho\pi}\) and \(S_{\rho\pi}\) are \(CP\)-violating, whereas \(\Delta C_{\rho\pi}\) and \(\Delta S_{\rho\pi}\) are \(CP\)-conserving. \(\Delta C_{\rho\pi}\) characterizes the difference in rates between the "\(W \to \rho^+\) process \(B^0 \to \rho^+ \pi^-\) or \(\bar{B}^0 \to \rho^- \pi^+\) and the "spectator \(\to \rho^\) process \(B^0 \to \rho^- \pi^+\) or \(\bar{B}^0 \to \rho^+ \pi^-\). \(\Delta S_{\rho\pi}\) depends, in addition, on differences in phases between the \(W \to \rho\) and spectator \(\to \rho\) amplitudes. The parameter \(A_{CP}^{\rho\pi}\) is the time and flavor integrated asymmetry \(\Gamma(B^0 \to \rho^+ \pi^-) - \Gamma(\bar{B}^0 \to \rho^+ \pi^-)\) divided by the sum of the four rates. We also define the asymmetries \(A_{\pm\pm} \equiv [N(B \to \rho^+ \pi^-) - N(B \to \rho^\pm \pi^\mp)] / [N(B \to \rho^+ \pi^-) + N(B \to \rho^\pm \pi^\mp)]\). \(A_{\pm\pm}\) depends only on \(W \to \rho\) processes and \(A_{\pm\mp}\) depends only on spectator \(\to \rho\) processes.

The most recent Belle results are from 140 fb\(^{-1}\) of data. To remove charge-ambiguous decays and possible interference between \(B^0 \to \rho^+ \pi^-\) and \(B^0 \to \rho^- \pi^+\) amplitudes, we require both \(0.57 < m_{\pi^+ \pi^-} < 0.97\) GeV/c\(^2\) and \(m_{\pi^+ \pi^-} > 1.22\) GeV/c\(^2\). We define a signal region \(m_{bc} > 5.27\) GeV/c\(^2\) and \(-0.10 < \Delta E < 0.08\) GeV; there are 1215 events in this region and 329 \(B^0 \to \rho^+ \pi^\mp\) candidates. Fitting to \(\Delta t\) yields \(A_{CP}^{\rho\pi} = -0.16 \pm 0.10 \pm 0.02\), \(C_{\rho\pi} = 0.25 \pm 0.17^{+0.02}_{-0.06}\), \(S_{\rho\pi} = -0.28 \pm 0.23^{+0.10}_{-0.08}\), \(\Delta C_{\rho\pi} = 0.38 \pm 0.18^{+0.04}_{-0.02}\), and \(\Delta S_{\rho\pi} = -0.30 \pm 0.24 \pm 0.09\). From these values we calculate \(A_{\pm\pm} = -0.02 \pm 0.16^{+0.05}_{-0.02}\) and \(A_{\pm\mp} = -0.53 \pm 0.29^{+0.09}_{-0.04}\). The first errors...
Fig. 2. The $B^0 \rightarrow \rho^\pm \pi^\mp$ $\Delta t$ distribution for $q = +1$ tags (left) and $q = -1$ tags (right), and the resulting $CP$ asymmetry (bottom). The asymmetry is shown separately for high-quality ($r > 0.5$) and low quality ($r \leq 0.5$) tags. The smooth curves are projections of the unbinned ML fit.

listed are statistical and the second systematic. The $\Delta t$ distributions for $q = \pm 1$ tagged events and the resulting $CP$ asymmetry are shown in Fig. 2.

These values can be used to constrain $\phi_2$. However, since the penguin contribution is unknown, additional information is needed. An $SU(3)$-based model that uses the measured rates or limits for $B^0 \rightarrow K^{*\pm} \pi^\mp$, $B^0 \rightarrow \rho^{\mp} K^\pm$, $B^\pm \rightarrow K^{*0} \pi^\pm$, and $B^\pm \rightarrow \rho^\pm K^0$ obtains $\phi_2 = 102 \pm 19^\circ$. This value is surprisingly close to that resulting from the values of $C_{\pi\pi}$ and $S_{\pi\pi}$ measured in $B^0 \rightarrow \pi^+ \pi^-$ decays.

References
1. http://belle.kek.jp/
2. A. Abashian et al., Nucl. Instr. Meth. A479, 117 (2002).
3. G. C. Fox and S. Wolfram, Phys. Rev. Lett. 41, 1581 (1978).
4. M. Gronau, Phys. Rev. Lett. 63, 1451 (1989).
5. M. Gronau and J. L. Rosner, Phys. Rev. D65, 093012 (2002).
6. http://www.slac.stanford.edu/xorg/hfag/
7. K. Abe et al., Phys. Rev. Lett. 93, 021601 (2004).
8. M. Beneke and M. Neubert, Nucl. Phys. B675, 333 (2003).
9. Y. Y. Keum and A. A. Sanda, Phys. Rev. D67, 054009 (2003).
10. M. Gronau and J. L. Rosner, Phys. Lett. B595, 339 (2004).
11. M. Gronau, *Phys. Lett.*, **B233**, 479 (1989).
12. C. C. Wang et al., hep-ex/0408003 (2004).
13. M. Gronau and J. Zupan, hep-ph/0407002 (2004).