The $\eta$-$\eta'$ mixture is discussed in the chiral bag model to calculate the pseudoscalar octet-singlet mixing angle consistent with the experimental data. The color anomaly is taken into account with the modified boundary conditions, which shows the relation between the $\eta'$ mass and the gluon condensate inside the chiral bag. We show, however, that $\eta$-$\eta'$ mixing angle can follow the Cheshire Cat Principle, i.e., insensitivity to the bag radius.
1 Introduction

Nearly all known mesons can be understood as bound states of a quark \( q \) and antiquark \( \bar{q} \) (the flavors of \( q \) and \( \bar{q} \) may be different). The nine possible \( \bar{q}q \) combinations containing \( u, d \) and \( s \) quarks group themselves into an octet and a singlet:

\[
\mathbf{3} \otimes \mathbf{\bar{3}} = \mathbf{8} \oplus \mathbf{1}.
\]

States with the same \( IJ^P \) and additive quantum numbers can mix (if they are eigenstates of charge conjugation \( C \), they must also have the same value of \( C \)). Thus the \( I = 0 \) member of the ground state pseudoscalar octet mixes with the corresponding pseudoscalar singlet to yield the \( \eta \) and \( \eta' \). These appear as members of a nonet.

For the pseudoscalar mesons the Gell-Mann-Okubo formula is given by

\[
m_\eta^2 = \frac{4}{3} m_K^2 - \frac{1}{3} m_\pi^2
\]

assuming no octet-singlet mixing, which is extremely sensitive to SU(3) symmetry breaking.

The value of the \( \eta-\eta' \) mixing angle has been the subject of discussion almost from the time that SU(3) flavor symmetry was proposed. In the simplest possible situation where one assumes the presence of only an octet and a singlet, the quadratic Gell-Mann-Okubo mass formula yields a pseudoscalar mixing angle of \( \theta = -10^\circ \). With the same assumption a Gell-Mann-Okubo mass formula which is linear in the masses gives \( \theta = -23^\circ \). For reasons that have to do with both theory and experiment at a given time over the years most authors\[1, 2\] have taken \( \theta = -10^\circ \).

Nowadays there has been significant discussion concerning the strangeness in the nucleon structure. Especially the measurement of the spin structure function of the proton given by European Muon Collaboration (EMC) experiment on deep inelastic muon scattering\[3\] has suggested a lingering question touched on by physicists that the effect of strange quarks on nucleon structure is not small. The EMC result has been interpreted as the possibility of a strange quark sea strongly polarized opposite to the proton spin. Similarly such interpretation of the strangeness has been brought to other analyses of low energy elastic neutrino-proton scattering\[4\] and the kaon condensation\[5, 6\] in the neutron star matter.

On the other hand, the chiral bag model (CBM)\[7\] couples fundamental hadron constituents inside to the pseudoscalar meson fields obeying nonlinear
chiral lagrangian outside the chiral bag through the boundary term on the bag surface introduced to restore the chiral invariance.

The CBM has enjoyed considerable success in predictions\cite{8, 9} of the baryon static properties such as the EMC experiments and the magnetic moments of baryon octet. After the discovery of the Cheshire Cat Principle \cite{10} the CBM has been also regarded as a candidate which unifies the MIT bag and Skyrme models and gives model-independent relations insensitive to the bag radius. Moreover Brown et al.\cite{11} have calculated the pion-cloud contributions to the baryon magnetic moments by using the SU(2) CBM as an effective nonrelativistic quark model. This scheme has been generalized \cite{12, 13} to SU(3) CBM to yield the minimal multi-quark structure \cite{14} so that the meson-cloud could be generated inside the chiral bag in terms of the nonperturbative higher representation mixing in the wave functions of the baryons.

In this paper we will introduce the SU(3) symmetry breaking terms to produce the $\eta$-$\eta'$ mixture and to estimate the octet-singlet mixing angle, under the assumption that the physical states are orthogonal, namely, the mixing is independent of energy. We will also investigate the color anomaly in terms of the gluon condensate and $\eta'$ mass. The quark condensate will be discussed to yield the pseudoscalar octet masses. Then we will show that the $\eta$-$\eta'$ mixing angle is insensitive to the bag radius in accordance with the Cheshire Cat Principle.

In Section 2, the CBM lagrangian with extended boundary condition will be introduced.

In Section 3, the $\eta$ degrees of freedom will be discussed both inside and outside the chiral bag to incorporate the $\eta$-$\eta'$ mixing, so that one can have nontrivial FSAC and estimate the pseudoscalar mixing angle. The color anomaly will be also discussed together with the gluon and quark condensates. Our conclusions will be found in Section 4.

## 2 Model with Extended Boundary Condition

Now in order to introduce the $\eta$-$\eta'$ mixing effects in the CBM we start with the lagrangian of the form

\[
\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_M(= \mathcal{L}_{CS} + \mathcal{L}_{CSB} + \mathcal{L}_{FSB}) + \mathcal{L}_I
\]  

(2.1)
\[ \mathcal{L}_{QCD} = (\bar{\psi} i \gamma^\mu D_\mu \psi - \bar{\psi} M \psi - \frac{1}{2} \text{tr} F_{\mu \nu} F^{\mu \nu}) \Theta_B \]  
\[ \mathcal{L}_{CS} = \left( -\frac{1}{4} f_\pi \text{tr}(l_\mu l^\mu) + \frac{1}{32\epsilon^2} [l_\mu, l_\nu]^2 + \mathcal{L}_{WZ} \right) \bar{\Theta}_B \]  
\[ \mathcal{L}_{CSB} = \frac{1}{4} f_\pi^2 m_\pi^2 (\text{tr}(U + U^\dagger - 2) - \frac{1}{3} \epsilon (-\text{ln} \det U)^2) \bar{\Theta}_B \]  
\[ \mathcal{L}_{FSB} = \frac{1}{6} f_\pi^2 (\chi^2 m_K^2 - m_\pi^2) \text{tr}((1 - \sqrt{3} \lambda_8)(U + U^\dagger - 2)) \bar{\Theta}_B \]  
\[ \mathcal{L}_{FSB} = -\frac{1}{12} f_\pi^2 (\chi^2 - 1) \text{tr}((1 - \sqrt{3} \lambda_8)(U l_\mu l^\mu + l_\mu l^\mu U^\dagger)) \bar{\Theta}_B \]  
\[ \mathcal{L}_I = \frac{1}{2} \bar{\psi}_5 \psi \Delta_B \]  
\[ (2.2) \]
\[ (2.3) \]
\[ (2.4) \]
\[ (2.5) \]
\[ (2.6) \]

where one has the quark field \( \psi \) with SU(3) degrees of freedom and the gluon field strength tensor \( F_{\mu \nu} = \partial_\mu G_\nu - \partial_\nu G_\mu + ig [G_\mu, G_\nu] \) inside the bag. Here \( D_\mu = \partial_\mu + ig G_\mu^a \lambda_a^2 \) is the covariant derivative with the effective strong coupling constant \( g \). And Gell-Mann matrices \( \lambda_a \) are normalized to satisfy \( \lambda_a \lambda_b = \frac{2}{3} \delta_{ab} + (i f_{abc} + d_{abc}) \lambda_c \) and \( \Theta_B(= 1 - \bar{\Theta}_B) \) is the bag theta function (one inside the bag and zero outside the bag).

In the ref. \[12\] the authors have considered the case SU(3) × SU(3) chiral theory without \( \eta \) degrees of freedom in the Skyrme phase. Also it was suggested that inside the bag the quark-antiquark annihilation channel could be induced via anomalous gluon effect in the U(1) flavor sector. Now we extend to the chiral bag with U(3) × U(3) group structure so that we can incorporate gluons and the \( \eta \) fields consistently. To do this we modify the chiral field \( U \) as follows

\[ U = \exp(i(\lambda_0 \pi_0 + \lambda_a \pi_a)/f_\pi), \quad U_5 = \exp(i \gamma_5(\lambda_0 \pi_0 + \lambda_a \pi_a)/f_\pi) \]  
\[ (2.7) \]

where \( \pi_0 \) is the zeroth eta field and \( \lambda_0 = \sqrt{2/3} I (I : 3 \times 3 \text{ unit matrix}) \). Thus outside the bag the chiral field \( U \) is described by the pseudoscalar meson fields \( \pi_0 \) and \( \pi_a (a = 1, \ldots, 8) \). Here \( l_\mu = U^\dagger \partial_\mu U \) and \( \mathcal{L}_{WZ} \) stands for the topological Wess-Zumino-Witten term. In the numerical calculation we will use the parameter fixing \( \epsilon = 4.75 \), \( f_\pi = 93 \text{ MeV} \) and \( f_K = 114 \text{ MeV} \). The surface interaction term \( \mathcal{L}_I \) typical in the CBM plays crucial role in the restoration of the chiral symmetry by coupling the pseudoscalar meson fields to the quarks on the bag boundary.

Here one notes that, together with the gluon and quark mass terms in \[ (2.2) \], the terms in \( \mathcal{L}_{CSB} \) break the chiral symmetry and the pion mass term...
in $\mathcal{L}_{CSB}$ is chosen such that it will vanish for $U = 1$. Also the SU(3) flavor symmetry breaking with $m_K/m_\pi \neq 1$ and $\chi = f_K/f_\pi \neq 1$ is included in $\mathcal{L}_{FSB}$.

Even though the mass terms in $\mathcal{L}_{QCD}$, $\mathcal{L}_{CSB}$ and $\mathcal{L}_{FSB}$ break both the SU(3)$\times$SU(3) and the diagonal SU(3) symmetry so that chiral symmetry cannot be conserved, these terms without derivatives yield no explicit contribution on the flavor singlet axial currents (FSAC) and at least in the adjoint representation of the SU(3) group the FSAC are conserved and of the same form as the chiral limit result. However the kinetic term in $\mathcal{L}_{FSB}$ gives rise to the chiral symmetry broken FSAC.

3 $\eta$-$\eta'$ Mixture

In the chiral symmetric case we have seen that the FSAC vanishes at leading order in $1/N_c$ since for example the $\pi_0$ decouples from the other mesons in the Skyrmion limit, namely $g_{\pi_0 NN} = 0$. In order to obtain nontrivial contribution to the FSAC we need to include the $\eta$-$\eta'$ mixing by introducing to the lagrangian (2.1) the symmetry breaking terms $\mathcal{L}_{CSB} + \mathcal{L}_{FSB}$ as mentioned above.

Adding the kinetic term in $\mathcal{L}_{CS}$ in (2.3) to $\mathcal{L}_{FSB}$ in (2.5) we obtain the bilinear kinetic terms in the weak field approximation

$$\mathcal{L}_{\text{kin}} = \frac{1}{2} \partial_\mu \pi_i \partial^\mu \pi_i + \frac{1}{2} \partial_\mu \pi_M \partial^\mu \pi_M + \frac{1}{2} \partial_\mu \tilde{\pi}^\dagger \mathcal{M} \partial^\mu \tilde{\pi}$$

where $\pi_i$ ($i = 1, 2, 3$) and $\pi_M$ ($M = 4, 5, 6, 7$) are the pion and kaon fields and $\tilde{\pi}^\dagger = (\pi_0, \pi_8)$. The matrix elements of $\mathcal{M}$ are then given by

$$\begin{align*}
\mathcal{M}_{11} &= (1 + 2\delta) \frac{f_\pi^2}{f_{\pi_0}} \\
\mathcal{M}_{22} &= (1 + 4\delta) \frac{f_\pi^2}{f_\pi^8} \\
\mathcal{M}_{12} &= \mathcal{M}_{21} = -2\sqrt{2}\delta \frac{f_\pi^2}{f_{\pi_0} f_\pi^8}
\end{align*}$$

where $\delta = \frac{1}{3}(\chi^2 - 1)$. In the $\pi_0$-$\pi_8$ channel we diagonalize the matrix $\mathcal{M}$ with
the physical fields defined as
\[
\eta = (1 + 2\delta) \frac{f_\pi}{f_{\pi_8}} \pi_8 + a\delta \frac{f_\pi}{f_{\pi_0}} \pi_0 = \pi_8 \cos \theta - \pi_0 \sin \theta
\]
\[
\eta' = b\delta \frac{f_\pi}{f_{\pi_8}} \pi_8 + (1 + \delta) \frac{f_\pi}{f_{\pi_0}} \pi_0 = \pi_8 \sin \theta + \pi_0 \cos \theta
\]
(3.3)

where \( \theta \) is the \( \eta-\eta' \) mixing angle.

Using the above eta fields (3.3) we can obtain the desired quadratic kinetic terms as follows
\[
\frac{1}{2} \partial_\mu \eta \partial^\mu \eta + \frac{1}{2} \partial_\mu \eta' \partial^\mu \eta' = \frac{1}{2} (1 + 4\delta) \frac{f_\pi^2}{f_{\pi_8}^2} \partial_\mu \pi_8 \partial^\mu \pi_8 + \frac{1}{2} (1 + 2\delta) \frac{f_\pi^2}{f_{\pi_0}^2} \partial_\mu \pi_0 \partial^\mu \pi_0
\]
\[
+ (a + b)\delta \frac{f_\pi^2}{f_{\pi_0} f_{\pi_8}} \partial_\mu \pi_8 \partial^\mu \pi_0
\]
(3.4)
to yield the relations
\[
f_{\pi_8}^2 = (1 + 4\delta) f_\pi^2 \approx f_\pi^2
\]
(3.5)
\[
f_{\pi_0}^2 = (1 + 2\delta) f_\pi^2 \approx f_\pi^2
\]
(3.6)
\[
a + b = -2\sqrt{2}
\]
(3.7)

Here one notes that there is a residual ambiguity in the choice of \((a, b)\) which corresponds to the freedom of performing an orthogonal transformation on (3.4). That ambiguity can be removed by considering the mass quadratic terms
\[
\mathcal{L}_{mass} = -\frac{1}{2} m_{\pi_8}^2 \pi_8^2 - \frac{1}{2} m_{\pi_0}^2 \pi_0^2 - \frac{1}{2} m_\pi^2 \pi_{8}\pi_{0}\tilde{\pi}^i \mathcal{N}_{i}
\]
(3.8)

where the mass matrix elements are given by
\[
\mathcal{N}_{11} = \frac{2}{3} m_K^2 \frac{f_K^2}{f_{\pi_0}^2} + \left( \frac{1}{3} + \epsilon \right) m_\pi^2 \frac{f_\pi^2}{f_{\pi_0}^2}
\]
\[
\mathcal{N}_{22} = \frac{4}{3} m_K^2 \frac{f_K^2}{f_{\pi_8}^2} - \frac{1}{3} m_\pi^2 \frac{f_\pi^2}{f_{\pi_8}^2}
\]
\[
\mathcal{N}_{12} = \mathcal{N}_{21} = -\frac{2\sqrt{2}}{3} \left( m_K^2 \frac{f_K^2}{f_{\pi_0} f_{\pi_8}} - m_\pi^2 \frac{f_\pi^2}{f_{\pi_0} f_{\pi_8}} \right).
\]
(3.9)
Again we diagonalize the matrix $\mathcal{N}$ with the transformation (3.3) to yield the quadratic mass terms

$$\begin{align*}
-\frac{1}{2}m_\eta^2\eta^2 - \frac{1}{2}m_\eta'\eta'^2 &= -\frac{1}{2}m_\eta^2((1 + 4\delta)\frac{f_\pi^2}{f_{\pi_0}}\pi_8^2 + 2a\delta\frac{f_\pi^2}{f_{\pi_0}}\pi_8\pi_0) \\
&\quad - \frac{1}{2}m_\eta'^2(2b\delta\frac{f_\pi^2}{f_{\pi_0}}\pi_0\pi_8 + (1 + 2\delta)\frac{f_\pi^2}{f_{\pi_0}}\pi_0\pi')
\end{align*}$$

so that we obtain the other relations

$$\begin{align*}
m_{\eta'}^2 &= \frac{21 + 3\delta}{31 + 2\delta}m_K^2 + \frac{11 + 3\epsilon}{31 + 2\delta}m_\pi^2 \\
m_\eta^2 &= \frac{41 + 3\delta}{31 + 4\delta}m_K^2 - \frac{1}{31 + 4\delta}m_\pi^2 \\
am_\eta^2 + bm_{\eta'}^2 &= -\frac{2\sqrt{2}}{3\delta}((1 + 3\delta)m_K^2 - m_\pi^2 - \frac{2}{3}(1 + 3\delta)^2m_\pi^2 - \frac{1}{3})
\end{align*}$$

Here one notes that in the limit $\delta = 0$ where $f_K = f_\pi$ the relation (3.12) reproduces the Gell-Mann-Okubo formula (1.1). Also the relation (3.11) determines the constant $\epsilon$ as below

$$\epsilon = (1 + 2\delta)\frac{m_{\eta'}^2}{m_\pi^2} - \frac{2}{3}(1 + 3\delta)\frac{m_K^2}{m_\pi^2} - \frac{1}{3}$$

and the combination of (3.7) and (3.13) yields

$$a = \frac{1}{m_{\eta'}^2 - m_\eta^2}\frac{2\sqrt{2}}{3\delta}((1 + 3\delta)m_K^2 - m_\pi^2 - 3\delta m_{\eta'}^2).$$

Now we use the experimental data\[16\] $f_K = 114$ MeV, $m_\pi = 140$ MeV, $m_K = 496$ MeV, $m_{\eta'} = 960$ MeV and $m_\eta = 550$ MeV to estimate the above parameters as follows

$$a = -1.037, \quad b = -1.790, \quad \delta = 0.168$$

and the $\eta-\eta'$ mixing angle

$$\theta = -12.7^o$$

which is qualitatively consistent with phenomenological analyses\[17\].
On the other hand inside the bag surface we need to have the mechanism consistent with the pseudoscalar octet-singlet mixing in the Skyrme phase if we assume that the Cheshire Cat Principle holds in the CBM. In ref.\[12\] it has been shown that the inside-mesons originate from the minimal multi-quark Fock space $qqq+qqqqq$ whose possible SU(3) representations are constrained by the Clebsch-Gordan series $8 \oplus 10 \oplus 27$. Thus one could have the meson-cloud, namely $qq$ content, inside the bag through the channel of $qqqqqq$ multi-quark Fock space. Here the mesonic $qq$ contents refer to all the possible flavor combinations to construct the pseudoscalar mesons inside the bag.

Until now there is no explicit known mechanism to explain the meson cloud inside the chiral bag. Presumably the mechanism seems closely related to the pseudoscalar composite operators $\bar{\psi}_i \gamma_5 \lambda_0 \psi \sim \pi_0$ and $\bar{\psi}_i \gamma_a \lambda_0 \psi \sim \pi_a$ ($a = 1, ..., 8$) and to the quark-antiquark annihilation channel\[18\] through the anomalous gluon effect. In the U(1) flavor sector the gluons are thus supposed to mediate the pseudoscalar meson via the $qq$ pair creation and annihilation mechanism. In this mechanism the composite operators $\bar{\psi}_i \gamma_5 \lambda_0 \psi$ and $\bar{\psi}_i \gamma_5 \lambda_8 \psi$ corresponding to $\pi_0$ and $\pi_8$ respectively could mix to yield the pseudoscalar octet-singlet mixture

$$\eta \sim \bar{\psi}i\gamma_5(\lambda_8 \cos \theta - \lambda_0 \sin \theta)\psi$$
$$\eta' \sim \bar{\psi}i\gamma_5(\lambda_8 \sin \theta + \lambda_0 \cos \theta)\psi$$

so that the Cheshire Cat Principle could be satisfied and one could have the same $\eta$-$\eta'$ mixing angle both inside and outside the chiral bag.

Moreover the presence of the $\eta'$ field at the boundary induces a color electric field normal to the bag surface in the form $\eta' \hat{n} \cdot \vec{B}/f_{\eta'}$. As a result color will be pulled out of the bag at a rate proportional to $i\xi \hat{n} \cdot \vec{B}/f_{\eta'}$. This phenomenon is essentially the color anomaly\[19, 20\]. At the quantum level a counter term thus has to be added at the bag boundary which exactly cancels the induced color electric field. The most general form of the surface charge counter term is given by\[19\]

$$L_{CT} = \frac{ig^2}{32\pi^2} \oint_{\Sigma} d\beta K_5^\mu n_\mu (\text{trln}U^\dagger - \text{trln}U)$$

where $\beta$ is a point on the bag surface $\Sigma$ and $K_5^\mu = e^{\mu\nu\rho\sigma}(G_\nu F_\rho^\sigma - \frac{2}{3} f^{abc} g G_\nu G^a G^b G^c)$ is the properly regularized Chern-Simons current. Also the counter term induces changes in the boundary conditions for the color electric and magnetic
fields in the quasi-abelian approximation as below

\[ \hat{n} \cdot \vec{E}^a = -\frac{N_f g^2}{8\pi^2 f_{\eta'}} \hat{n} \cdot \vec{B}^a \eta', \]

\[ \hat{n} \times \vec{B}^a = \frac{N_f g^2}{8\pi^2 f_{\eta'}} \hat{n} \times \vec{E}^a \eta'. \]  \hspace{1cm} (3.18)

In the weak field approximation, using the above modified boundary conditions one can obtain the following relation inside the chiral bag[20]

\[ f_{\eta'}^2 m_{\eta'}^2 = 4N_f \frac{g^2}{16\pi^2} \frac{g^2}{16\pi^2} \langle G^2 \rangle_V \]  \hspace{1cm} (3.19)

where \( \langle G^2 \rangle_V \) is the amount of gluon condensate averaged over the volume \( V \). Here one notes that in the scaling argument presented by Witten[21] \( m_{\eta'}^2 \) scales as \( 1/N_c \) since \( \langle G^2 \rangle \) scales as \( N_c^2 \), \( g^2 \) as \( 1/N_c \) and \( f_{\eta'}^2 \) as \( \sqrt{N_c} \).

On the other hand the pseudoscalar octet mesons inside the chiral bag obtain their masses via the quark condensate \( 2\sigma = \langle \bar{u}u \rangle = \langle \bar{d}d \rangle \approx \langle \bar{s}s \rangle \) \[22\]

\[ f_{\pi}^2 m_{\pi}^2 \approx -2\sigma (m_u + m_d) \]

\[ f_{\pi}^2 m_{K}^2 \approx -2\sigma m_s \]  \hspace{1cm} (3.20)

and the relation \( (3.12) \).

According to the Cheshire Cat Principle, the total masses of the pseudoscalar octet and singlet mesons come from the contributions from the mesons both inside (for example Eq. \( (3.19) \) for \( \eta' \)) and outside chiral bag. Here for simplicity we assume that in accordance with the Cheshire Cat Principle all the masses of the mesons inside the chiral bag increase at the same rate as the bag radius increases. For instance let \( p \) be the ratio of the meson masses outside to those inside the chiral bag then in \( (3.15) \) the numerator and denominator have the same factor \( p^2 \), which can be cancelled out. This leads us to the conclusion that the contributions from the right hand side of Eq. \( (3.19) \) and Eq. \( (3.20) \) should have the identical dependence on \( p^2 \). In other words, the parameter \( a \) should have the same value regardless of the amount of the gluon and quark condensates. One can thus have the same pseudoscalar octet-singlet mixing angle insensitive to the chiral bag radius upon the Cheshire Cat Principle.

\[ ^1 \text{Here one notes that in the } U_A(1) \text{ channel we have used } \eta' \text{ field instead of } \pi_0, \text{ ignoring the possible } \eta-\eta' \text{ mixing which is the negligible secondary effect in the color anomaly.} \]

8
4 Conclusions

In this paper we have introduced the $\eta$ degrees of freedom outside the chiral bag to include the physical fact that the $I = 0$ member of the ground state pseudoscalar octet mixes with the corresponding pseudoscalar singlet. To do this, we have modified the chiral field $U$ to have $U(3) \times U(3)$ group structure, and also we have included the SU(3) chiral and flavor symmetry breaking terms in the lagrangian as shown in (2.4) and (2.5).

From the full CBM lagrangian we have the bilinear kinetic and mass terms (3.1) and (3.8) with matrices $\mathcal{M}$ and $\mathcal{N}$ respectively in the weak field approximation. In the $\pi_0-\pi_8$ channel we have diagonalized the matrices $\mathcal{M}$ and $\mathcal{N}$ under the transformation (3.3) to yield the $\eta-\eta'$ mixing angle $\theta = -12.7^\circ$ which is in good agreement with the experimental data $\theta_{\text{exp}} = -10^\circ \sim -23^\circ$ [16]. Here one notes that depending on what assumptions are made, the bilinear kinetic and mass terms are consistent with both $\theta = -10^\circ$ and $\theta = -23^\circ$ and is unable to discriminate decisively between them [16, 17].

Also we have proposed the mechanism to incorporate inside the bag surface the $\eta-\eta'$ mixture consistent with the pseudoscalar octet-singlet mixing angle by introducing the pseudoscalar composite operators originated from the minimal multi-quark Fock space. In this mechanism the composite operators $\bar{\psi}i\gamma_5\lambda_0\psi \sim \pi_0$ and $\bar{\psi}i\gamma_5\lambda_8\psi \sim \pi_8$ could mix to produce the pseudoscalar octet-singlet mixture so that one could have the same $\eta-\eta'$ mixing angle both inside and outside the chiral bag, in accordance with the Cheshire Cat Principle.

Finally the color anomaly has been discussed in terms of the $\eta'$ mass and the gluon condensate inside the chiral bag. Moreover the pseudoscalar octet meson masses inside the chiral bag have been described via the quark condensate. Assuming that the meson masses inside the chiral bag increase at the same rate as the bag radius increases, we have obtained the same $\eta-\eta'$ mixing angle regardless of the amount of the gluon and quark condensates, in accordance with the Cheshire Cat Principle.

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