Quark and Lepton Mass Matrices in the SO(10) Grand Unified Theory with Generation Flipping

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Abstract

We investigate the SO(10) grand unified model with generation flipping. The model contains one extra matter multiplet $\psi(10)$ and it mixes with the usual matter multiplets $\psi_i(16)$ when the SO(10) is broken down to SU(5). We find the parameter region of the model in which the observed quark masses and mixings are well reproduced. The resulting parameter region is consistent with the observation that only $\psi_i(16)$ have a source of hierarchies and indicates that the mixing between second and third generations tends to be large in the lepton sector, which is consistent with the observed maximal mixing of the atmospheric neutrino oscillation. We also show that the model can accommodate MSW and vacuum oscillation solutions to the solar neutrino deficit depending on the form of the Majorana mass matrix for the right-handed neutrinos.
1 Introduction

The grand unified theory (GUT) [1] is a very attractive idea which unifies all gauge interactions of the standard model (SM) and explains otherwise peculiar U(1) hypercharge assignments. In particular, its supersymmetric (SUSY) version [2] has achieved great successes of gauge coupling [3] and $b - \tau$ Yukawa coupling [4] unifications, so that various SUSY GUT models have been proposed so far.

Recently, Super-Kamiokande Collaboration has reported convincing evidence for atmospheric neutrino oscillation with mass squared difference $\delta m_{\text{atm}}^2 \simeq 5 \times 10^{-3} \text{eV}^2$ and nearly maximal mixing angle $\sin^2 2 \theta_{\text{atm}} \gtrsim 0.8$ [5]. In the SUSY SM, neutrino masses come from the effective superpotential

$$W_{\text{eff}} = \sum_{i,j=1}^{3} \frac{\kappa_{ij}}{M_R} l_i l_j H_u H_u,$$

where $l_i, H_u$ are SU(2)$_L$-doublet lepton chiral multiplets and the Higgs doublet chiral multiplet giving masses for the up-type quarks, respectively. Here, $i,j = 1, \cdots, 3$ are generation indices, $\kappa_{ij}$ denote dimensionless coupling constants and $M_R$ is the scale at which this operator is generated. The observed mass squared difference $\delta m_{\text{atm}}^2$ implies that the scale $M_R$ is substantially lower than the gravitational scale $M_G \simeq 2.4 \times 10^{18} \text{GeV}$. Thus, it is natural to think that these effective operators arise from virtual exchange of some heavy fields of masses $M_R$ through see-saw mechanism [6]. This motivates SO(10) GUT’s [7], which attain complete unification of all quarks and leptons in a single generation together with the right-handed neutrinos.

The minimal version of SUSY SO(10) GUT contains three generations of quarks and leptons $\psi_i(16)$ belonging to $16$ of the SO(10)$_{\text{GUT}}$ and one Higgs $H(10)$. This minimal model is known to yield a mass degeneracy of up-type and down-type quarks and vanishing quark flavor mixing (CKM [8] mixing) [9], so that it should obviously be extended in order to be reconciled with the observed pattern of quark masses and mixings. However, these predictions are not altogether ridiculous as a zero-th order approximation (though not fully realistic) in the quark sector, since both up-type and down-type quarks have hierarchical mass patterns and the CKM mixings are all small in nature. What requires large deviation from these predictions seems only the large
mixing observed in the atmospheric neutrino oscillation. Thus, we would like to extend the minimal SO(10) model to be able to accommodate the large mixing in the lepton sector, keeping their successful approximate relations in the quark sector. One way to achieve this extension is to slightly modify SU(5)-5* components of $\psi_i(16)$ (nonparallel family structure \[10\]). Since 5* of the standard SU(5)$_{\text{GUT}}$ contains right-handed (down-type) quarks and left-handed leptons, this modification is directly transmitted only to the lepton flavor mixing matrix (MNS \[12\] matrix). As for the quark sector, the effect of the modification is transmitted to the CKM matrix through a diagonalization of the down-type quark mass matrix, inducing small CKM mixings.

In a previous paper, Yanagida and one of the authors (Y.N.) have proposed a SO(10) GUT model which can accommodate realistic quark and lepton mass matrices along the above line of reasoning \[13\]. The model contains one extra matter multiplet $\psi(10)$ belonging to 10 of the SO(10)$_{\text{GUT}}$ and it mixes with three $\psi_i(16)$ when the SO(10)$_{\text{GUT}}$ is broken down to SU(5) \[14\]. As a result, the low-energy quarks and leptons are three linear combinations of $\psi_i(16)$ and $\psi(10)$, and realistic masses and mixings are obtained with the right-handed neutrino Majorana mass matrix $M_N$ proportional to the unit one.

In this paper, we extend the previous model \[13\] and investigate whether it can really accommodate the observed values of quark masses and mixings quantitatively. We find that the model reproduces well the observed quark masses and the CKM matrix elements. The resulting parameter region is consistent with the observation that only $\psi_i(16)$ have a source of hierarchies such as U(1) flavor symmetry charges \[13\]. Furthermore, we discuss flavor mixings in the lepton sector under certain simplifying assumptions. We find that the model can naturally explain the observed atmospheric neutrino deficit \[16, 5\] and solar neutrino deficit \[17, 18\] by the neutrino oscillation \[19, 12\]. The resulting mass squared differences and mixing angles depend on the form of the Majorana mass matrix $M_N$. We consider two cases that $M_N$ is proportional to the unit matrix and that it has a hierarchy consistent with the above observation that only $\psi_i(16)$ have a source of hierarchies.

This paper is organized as follows. In section 2, we briefly review the model in Ref. \[13\] and slightly extend it removing unwanted mass degeneracy between down-type

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1 For other attempts to realize realistic quark and lepton mass matrices in SO(10) GUT, see Ref. \[1\].
quarks and charged leptons. In section 3, we investigate the model and find a parameter region where it is consistent with all the observed quark masses and mixings including a CP-violating phase. We discuss masses and mixings in the lepton sector, particularly solutions to the solar neutrino deficit, in section 4. Section 5 is devoted to our conclusions. We also add an appendix, in which we give a detailed derivation of quark and lepton mass matrices in the model.

2 The model

In this section, we explain the model proposed in Ref. [13] and set up the framework for the phenomenological analyses given in the next section.

The model has three generations of $\psi_i(16)$ and one $\psi(10)$ as matter multiplets. We take a basis where the original Yukawa coupling matrix of the Higgs field $H(10)$ to the matter fields $\psi_i(16)$ is real and diagonal:

$$ W = \frac{1}{2} \sum_{i=1}^{3} h_i \psi_i(16) \psi_i(16) H(10). $$

This leads to a diagonal mass matrix for the up-type quarks as

$$ M_u = m_t \begin{pmatrix} \hat{m}_u & 0 & 0 \\ 0 & \hat{m}_c & 0 \\ 0 & 0 & 1 \end{pmatrix}, $$

where $\hat{m}_u$ and $\hat{m}_c$ are defined as

$$ \hat{m}_u = m_u / m_t, \quad \hat{m}_c = m_c / m_t, $$

and $m_u, m_c$ and $m_t$ are given by

$$ m_u = h_1 \langle 5_H \rangle, \quad m_c = h_2 \langle 5_H \rangle, \quad m_t = h_3 \langle 5_H \rangle. $$

Here, $5_H$ is a SU(5)-5 component of $H(10)$. At this stage, the down-type quark mass matrix $M_{d/l}$ and neutrino Dirac mass matrix $M_{\nu D}$ are completely the same with $M_u$ except for the difference between the vacuum expectation values of the Higgs fields.

We now assume that the SO(10)$_{\text{GUT}}$ is broken down to SU(5) by condensation of Higgs $\langle \chi(16) \rangle = \langle \chi(16^*) \rangle = V$ with $V$ being $\sim 10^{16}$ GeV. This GUT breaking also
induces a mass term for the matter multiplets through the following superpotential:

\[ W = \sum_{i=1}^{3} f_i e^{i \eta_i} \psi_i(16) \psi(10) \langle \chi(16) \rangle . \] 

(6)

Namely, a linear combination \( \tilde{5}^* \propto \sum_{i=1}^{3} f_i e^{i \eta_i} 5^{s'}_i \) in \( \psi_i(16) \) receives a GUT scale mass together with \( 5_\psi \) in \( \psi(10) \), and the other two linear combinations, \( 5^*_1 \) and \( 5^*_3 \), in \( \psi_i(16) \) remain as massless fields. The relation between these fields can be parametrized as

\[
\begin{pmatrix} 5^{s'}_1 \\ 5^{s'}_2 \\ 5^{s'}_3 \end{pmatrix} = \begin{pmatrix} e^{-i \delta_1 - i \delta_2} & 0 & 0 \\ 0 & e^{-i \delta_1 + i \delta_2} & 0 \\ 0 & 0 & e^{2i \delta_1} \end{pmatrix} \begin{pmatrix} \cos \alpha_1 & -\sin \alpha_1 & 0 \\ \sin \alpha_1 & \cos \alpha_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 5^*_1 \\ 5^*_2 \\ 5^*_3 \end{pmatrix},
\]

(7)

without a loss of generality (see appendix). Here, \( \alpha_1, \alpha_2 \) and \( \delta_i \) are functions of \( f_i \) and \( \eta_i \). Thus, the low-energy quark and lepton fields belonging to \( 5^* \) of SU(5) are \( 5^*_1, 5^*_2 \) and SU(5)-\( 5^* \) of \( \psi(10) \) (i.e. \( 5^*_3 \)), so that the relations \( M_u = M_{\nu D} \propto M_{d/l} \) are removed.

The down-type quark mass matrix is, however, incomplete, since \( 5^*_\psi \) does not have any Yukawa coupling to \( H(10) \). To solve this problem we introduce a pair of Higgs \( H(16) \) and \( \bar{H}(16^*) \) and consider a superpotential\(^2\)

\[ W = \lambda H(10) \bar{H}(16^*) \tilde{\chi}(16^*) + M H(16) \bar{H}(16^*). \]

(8)

U(1) R-symmetry may be useful to have this form of superpotential. The U(1)\(_R\) charges are given in Table [4]. The GUT condensation \( \langle \tilde{\chi}(16^*) \rangle \neq 0 \) induces a mass mixing between \( 5^* \)'s of \( H(10) \) and \( H(16) \) (i.e. \( 5^*_{H(10)} \) and \( 5^*_{H(16)} \)). Thus, a linear combination

\[
5^*_{H} = \cos \theta 5^*_{H(10)} + \sin \theta 5^*_{H(16)};
\]

(9)

\[
\tan \theta = -\frac{\lambda \langle \tilde{\chi}(16^*) \rangle}{M},
\]

(10)

\(^2\) We can take \( \lambda \) and \( M \) to be real using phase degrees of freedom in \( H(16) \) and \( \bar{H}(16^*) \). Strictly speaking, \( \phi_i \) in Eq. (13) are defined in this basis.
remains as a massless Higgs in the standard SU(5)_{GUT} and contributes to the quark and lepton mass matrices. Then, $5^*_H$ can couple to $5^*_\psi$ through the superpotential

$$W = \sum_{i=1}^{3} g_i e^{i\phi_i} \psi_1(16) \psi(10) H(16),$$

and the down-type quark and charged-lepton mass matrix is given by

$$M_{d/l} = m_t \begin{pmatrix} \cos \theta & \sin \theta \tan \beta \\ \sin \theta & \cos \theta \tan \beta \end{pmatrix} \begin{pmatrix} -\hat{m}_u \sin \alpha_1 & x e^{i\phi_x} & \hat{m}_u \cos \alpha_1 \sin \alpha_2 \\ \hat{m}_c \cos \alpha_1 & y e^{i\phi_y} & \hat{m}_c \sin \alpha_1 \sin \alpha_2 \\ 0 & z & \cos \alpha_2 \end{pmatrix}.$$  \tag{12}

Here, $\tan \beta \equiv \langle 5_H \rangle / \langle 5^*_H \rangle$ and

$$x = \frac{g_1}{h_3} \tan \theta, \quad y = \frac{g_2}{h_3} \tan \theta, \quad z = \frac{g_3}{h_3} \tan \theta,$$  \tag{13}

$$\phi_x = \phi_1 - \phi_3 + 3\delta_1 + \delta_2 + \delta_3, \quad \phi_y = \phi_2 - \phi_3 + 3\delta_1 - \delta_2 + \delta_3,$$  \tag{14}

(for a detailed derivation, see appendix). Note that in the present model $\tan \beta$ can take a value in a wide range due to the presence of $\cos \theta$ in Eq. (12).

The Dirac mass matrix $M_{\nu D}$ for neutrinos is also incomplete, since $5^*_\psi$ never couples to SU(5)-singlets of $\psi_1(16)$ (i.e. $1_i$). However, the following nonrenormalizable interactions give desired couplings:

$$W = \sum_{i=1}^{3} k_i e^{i\phi_i} \psi_1(16) \psi(10) \bar{\chi}(16^*) \frac{M_G}{M}. \tag{15}$$

Together with the original couplings in Eq. (4), the nonrenormalizable interactions Eq. (15) yield

$$M_{\nu D} = m_t \begin{pmatrix} -\hat{m}_u \sin \alpha_1 & \delta_x e^{i\phi_x} & \hat{m}_u \cos \alpha_1 \sin \alpha_2 \\ \hat{m}_c \cos \alpha_1 \sin \alpha_2 & \delta_y e^{i\phi_y} & \hat{m}_c \sin \alpha_1 \sin \alpha_2 \\ 0 & \delta_z e^{i\phi_z} & \cos \alpha_2 \end{pmatrix},$$  \tag{16}

Here,

$$\delta_x = \frac{k_1}{h_3} \frac{\langle \bar{\chi}(16^*) \rangle}{M_G}, \quad \delta_y = \frac{k_2}{h_3} \frac{\langle \bar{\chi}(16^*) \rangle}{M_G}, \quad \delta_z = \frac{k_3}{h_3} \frac{\langle \bar{\chi}(16^*) \rangle}{M_G},$$  \tag{17}

$$\varphi_x = \varphi_1 + \delta_1 - \delta_2, \quad \varphi_y = \varphi_2 + \delta_1 - \delta_2, \quad \varphi_z = \varphi_3 + \delta_1 - \delta_2,$$

$$\varphi_t = 2\delta_2, \quad \varphi_u = -3\delta_1 - \delta_2 - 2\delta_3, \quad \varphi_v = -3\delta_1 + \delta_2 - 2\delta_3, \quad \varphi_w = -\delta_3,$$  \tag{18}
(see appendix). Notice that $\delta_x, \delta_y, \delta_z \simeq O(10^{-2})$ as long as $k_i/h_3 = O(1)$. The Majorana masses for the right-handed neutrinos $1_i$ are given by the following nonrenormalizable superpotential:

$$W = \frac{1}{2} \sum_{i,j=1}^{3} j_{ij} \psi_i(16) \psi_j(16) \frac{\bar{\psi}(16^*) \bar{\psi}(16^*)}{M_G}.$$  \hspace{1cm} (19)

After the SO(10)$_{\text{GUT}}$ breaking we obtain the Majorana mass matrix for the right-handed neutrinos,

$$(M_N)_{ij} = \frac{j_{ij}}{|j_{33}|} M_R,$$  \hspace{1cm} (20)

where $M_R = |j_{33}| V^2/M_G$. Then, the mass matrix $M_\nu$ for the light neutrinos is given by $M_\nu = M_{\nu D}^T M_N^{-1} M_{\nu D}$ through see-saw mechanism [6].

Now, we have SU(5)-invariant mass matrices $M_u, M_d/l$ and $M_\nu$. However, they yield wrong SU(5)$_{\text{GUT}}$ mass relations, $m_\mu = m_\tau = m_\nu$ and $m_e = m_d$, so that we have to remove these unwanted mass relations in order to obtain realistic quark and lepton masses. It can be done by introducing SU(5) breaking effects into $M_{d/l}$ [21]. We assume that the SU(5) is broken down to the SM gauge group by a Higgs multiplet $\Sigma(45)$ belonging to 45 of the SO(10)$_{\text{GUT}}$. Then, if $\Sigma(45)$ has nonrenormalizable interactions to the matter and Higgs fields, the SU(5) breaking effect of order $\langle \Sigma(45) \rangle / M_G$ can be transmitted to the quark and lepton mass matrices [21]. For this purpose, we introduce the following nonrenormalizable superpotential:

$$W = \sum_{i=1}^{3} g_i^e e^{i \phi_i} \psi_i(16) \psi(10) H(16) \frac{\langle \Sigma(45) \rangle}{M_G}.$$  \hspace{1cm} (21)

We consider that only $(1, 2)$ and $(2, 2)$ components of the down-type quark mass matrix $M_d$ and the charged-lepton mass matrix $M_l$ are modified from those of $M_{d/l}$ for simplicity, since the value of $z$ which gives realistic masses and mixings is relatively larger than those of $x$ and $y$ as we shall see later. We represent these modified components with the subscript $d$ and $l$ (i.e. $x_d, y_d, \phi_{xd}, \phi_{yd}, x_l, y_l, \phi_{xl}$ and $\phi_{yl}$).

With the mass matrices $M_u, M_d, M_l$ and $M_\nu$, we can reproduce well the observed quark and lepton flavor structure. In the next section, we search a parameter region of $M_u$ and $M_d$ which gives the observed quark masses and mixings including a CP-violating phase.
The masses and mixings in the quark sector

The quark masses are estimated using various methods such as QCD sum rules. However, the estimated quark masses still have some uncertainties. In our analysis, we adopt the running quark and lepton masses evaluated at the energy of $Z$-boson mass $m_Z$ [22, 23, 24] and the GUT scale $M_{GUT} \simeq 2 \times 10^{16}$ GeV [23].

Table 2: The running quark and lepton masses evaluated at the energy of $Z$-boson mass $m_Z$ [22, 23, 24] and the GUT scale $M_{GUT} \simeq 2 \times 10^{16}$ GeV [23].

| Mass (µ) | $\mu = m_Z$ | $\mu = M_{GUT}$ |
|---------|-------------|-----------------|
| $m_u(µ)$ | $2.2 \pm 0.7$ MeV | $0.98 \pm 0.31$ MeV |
| $m_c(µ)$ | $626 \pm 106$ MeV | $279 \pm 47$ MeV |
| $m_t(µ)$ | $175 \pm 6$ GeV | $110 \pm 19$ GeV |
| $m_d(µ)$ | $4.1 \pm 1.1$ MeV | $1.2 \pm 0.3$ MeV |
| $m_s(µ)$ | $85 \pm 19$ MeV | $24 \pm 5$ MeV |
| $m_b(µ)$ | $3.02 \pm 0.19$ GeV | $1.01 \pm 0.06$ GeV |
| $m_e(µ)$ | $0.487$ MeV | $0.325$ MeV |
| $m_µ(µ)$ | $102.7$ MeV | $68.6$ MeV |
| $m_τ(µ)$ | $1.747$ GeV | $1.171$ GeV |

Table 3: The observed CKM parameters (at $\mu = m_Z$) [22, 25] and their RG-evolved values at the GUT scale $M_{GUT} \simeq 2 \times 10^{16}$ GeV [23].

| Parameter | $\mu = m_Z$ | $\mu = M_{GUT}$ |
|-----------|-------------|-----------------|
| $(V_{CKM})_{us}(µ)$ | $0.215 \sim 0.224$ | $0.215 \sim 0.224$ |
| $(V_{CKM})_{cb}(µ)$ | $0.036 \sim 0.043$ | $0.031 \sim 0.037$ |
| $(V_{CKM})_{ub}/(V_{CKM})_{cb}(µ)$ | $0.060 \sim 0.12$ | $0.061 \sim 0.12$ |
| $J_{CKM}(µ)$ | $(1.5 \sim 4.4) \times 10^{-5}$ | $(1.1 \sim 3.3) \times 10^{-5}$ |

In the following, we search a parameter region of...
the model consistent with these observed masses and mixings.

To compare with the mass matrices obtained in section 2, we have to use renormalization group (RG)-evolved values of the masses and mixings at the GUT scale. We assume particle content of the minimal SUSY SM between the weak scale and the GUT scale. Then, the GUT scale values are not sensitive to the precise mass spectrum of the SUSY particles and also to \( \tan \beta \) as long as it has a moderate value \( 5 \lesssim \tan \beta \lesssim 30 \) \cite{27}. Thus, we fix a value \( \tan \beta = 10 \) for simplicity. The obtained masses and mixings at the GUT scale using two-loop RG equations for the Yukawa couplings are given in Table 2 and 3 \cite{23}. We will use these values to constrain the parameter space of the model.

Now, we search values of \( x_d, y_d, z, \alpha_1, \alpha_2, \phi_{xd}, \phi_{yd} \) which reproduce down-type quark mass ratios in Table 2, \( 32.8 < m_b/m_s < 56.3 \) and \( 12.7 < m_s/m_d < 32.2 \), and all the CKM parameters given in Table 3. We take \( \hat{m}_c \) to be \( \hat{m}_c^{-1}(M_{\text{GUT}}) = 394 \) (central value) and 279 (lowest value) in order to show the dependence of the results on up-type quark masses at the GUT scale. The results hardly depend on \( \hat{m}_u \) due to its smallness, so that we fix \( \hat{m}_u^{-1}(M_{\text{GUT}}) \) to be its central value 11244.

We find a parameter space which reproduces all quark masses and mixings including a CP-violating phase. The parameter region of \( x_d \) and \( y_d \) is shown in Fig. 1. As is readily seen, the slope of this \( x_d-y_d \) graph roughly gives the reciprocal of the Cabibbo angle \( \sim (0.22)^{-1} \). Since ratios of \( x_d, y_d \) and \( z \) are related to the CKM mixing angles, we find that \( x_d, y_d \) and \( z \) also have hierarchical structure. Indeed, the region of \( z \) is around \( 0.02 < z < 0.12 \). The required hierarchy is milder than that of \( h_1, h_2 \) and \( h_3 \), and can be roughly parametrized as

\[
\frac{h_1}{h_2}:\frac{h_3}{} \simeq \frac{\epsilon^4}{\epsilon^2}:1, \tag{22}
\]

\[
\frac{x_d}{y_d}:\frac{z}{} \simeq \frac{g_1}{g_2}:\frac{g_3}{} \simeq \frac{\epsilon^2}{\epsilon}:1,
\]

with \( \epsilon = O(0.1) \). This may indicate that only \( \psi_i(16) \) have a source of hierarchies such as \( U(1) \) flavor symmetry charges (see Eqs. \( \{4, 11\} \)). To reproduce the observed CP violation (the value of \( J_{\text{CKM}} \) given in Table \( \{2\} \) \( \phi_{yd} \) has to be around \( 3\pi/2 \) (\( 10\pi/8 \lesssim \phi_{yd} \lesssim 15\pi/8 \) in both cases of \( \hat{m}_c^{-1} = 394 \) and \( \hat{m}_c^{-1} = 279 \)). The results hardly depend on \( \phi_{xd} \), since

\footnote{The GUT scale value of \( m_t \) is highly dependent on the input value of \( m_t(m_Z) \), especially in the case that \( m_t(m_Z) \gtrsim 180 \) GeV. As we shall see, however, our qualitative conclusions hardly depend on the value of \( m_t(M_{\text{GUT}}) \).}
it can be transferred into the phase of $\hat{m}_u$ by an appropriate phase rotation of the quark doublet $(u, d)$ and $\hat{m}_u$ is very small.

The parameter region of $\alpha_1$ and $\alpha_2$ ($0 \leq \alpha_1, \alpha_2 \leq \pi/2$) is shown in Fig. 2. We find that both $\alpha_1$ and $\alpha_2$ are close to $\pi/2$ ($\cos \alpha_1 \sim \cos \alpha_2 = O(0.1)$). This implies that the extra matter multiplet $\psi(10)$ is dominantly coupled with $\psi_3(16)$ in the superpotential Eq. (6). The hierarchy of the couplings $f_i$ is roughly

$$\cos \alpha_1 \cos \alpha_2 : \sin \alpha_1 \cos \alpha_2 : \sin \alpha_2 = f_1 : f_2 : f_3 \simeq \epsilon^2 : \epsilon : 1,$$

which also is consistent with the above observation that only $\psi_i(16)$ have a source of hierarchies (see Eq. (13)).

Finally, we plot the down-type quark mass ratios $m_b/m_s$ and $m_s/m_d$ as functions of $z/\cos \alpha_2$ in Fig. 3 and 4, respectively. We have ascertained that $m_b/m_s$ and $m_s/m_d$ take any value in a range $32.8 < m_b/m_s < 56.3$ and $12.7 < m_s/m_d < 32.2$, so that there is no relation between them. Note that the allowed region of $z/\cos \alpha_2$ is hardly dependent on $\hat{m}_c^{-1}$, since $z$ and $\cos \alpha_2$ respond to the change of $\hat{m}_c^{-1}$ in the same way. Thus, our qualitative discussion below is almost independent of $\hat{m}_c^{-1}(M_{\text{GUT}})$. The quantity $z/\cos \alpha_2$ almost corresponds to the tangent of the MNS mixing angle, $\tan \theta_{\mu 3}$, between second and third generations, since it dictates the mixing between left-handed charged leptons of second and third generations. Actual mixing angle is the sum of it and an additional contribution from neutrino mass matrix $M_\nu$. Considering that $\delta_z$ is given by Eq. (17) and $h_3 = O(1)$, the additional contribution can be of order the Cabibbo angle. Thus, if two contributions are added up constructively, the region $z/\cos \alpha_2 \gtrsim 0.4$ is consistent with the observed near maximal mixing of the atmospheric neutrino oscillation ($\nu_\mu \leftrightarrow \nu_\tau$) \cite{10, 11, 12} We find that significant region is consistent with the large angle between $\nu_\mu$ and $\nu_\tau$.

\footnote{If some components of the down-type quark mass matrix other than (1,2) and (2,2) ones are modified by nonrenormalizable interactions, the parameter region of $(\alpha_1, \alpha_2)$ is not necessarily very close to $(\pi/2, \pi/2)$. This possibility will be considered elsewhere.}

\footnote{Even if we have no contribution from neutrino sector, the region $z/\cos \alpha_2 \gtrsim 0.6$ is consistent with the observed near maximal mixing of the atmospheric neutrino oscillation, $\sin^2 2\theta_{\mu 3} \gtrsim 0.8$. In this case, relatively large value of $m_b/m_s$ is favorable.}
4 The masses and mixings in the lepton sector

In this section, we discuss the lepton sector of the model taking \( \hat{m}_u^{-1} \) and \( \hat{m}_c^{-1} \) to be their central values for simplicity. First, we begin with the charged-lepton masses whose precise values are known experimentally as in Table 2. The mass matrix for the charged leptons is different from that for the down-type quarks due to the presence of nonrenormalizable interactions Eq. (21). We assume that the nonrenormalizable interactions Eq. (21) modify only \((1,2)\) and \((2,2)\) components of the mass matrices. Thus, we search a parameter region of \(x_l\) and \(y_l\) which can well reproduce the known lepton mass ratios at the GUT scale, \(m_\tau/m_\mu = 17.07\) and \(m_\mu/m_e = 211.1\), using the values of \(z, \alpha_1\) and \(\alpha_2\) obtained in section 3 which reproduce the quark mass ratios and the CKM parameters.

The resulting region of \(x_l\) and \(y_l\) is given in Fig. 5 together with that of \(x_d\) and \(y_d\) obtained in Fig. [1]. We find that there is significant region consistent with the observed charged-lepton masses. The preferred region is \(x_l \sim x_d\) and \(y_l \sim 3y_d\), which agrees with the earlier observation [20]. Thus, we conclude that the presence of nonrenormalizable interactions Eq. (21) with \(g_i' = O(1)\) is sufficient to remove unwanted SU(5)GUT mass relations, \(m_\mu = m_s\) and \(m_e = m_d\), also in the present model.

Next, we discuss the neutrino masses and mixings qualitatively. We call the mass eigenstates for three neutrinos \(\nu_1, \nu_2\) and \(\nu_3\) such that \(m_{\nu_1} < m_{\nu_2} < m_{\nu_3}\). In the present model, we have a mass hierarchy \(m_{\nu_2} \ll m_{\nu_3}\) due to the fact that \(\delta_x, \delta_y, \delta_z \lesssim 10^{-2}\). Then, the data of atmospheric neutrino oscillation (\(\nu_\mu \leftrightarrow \nu_\tau\)) from Super-Kamiokande implies that \(m_{\nu_3} \simeq 7 \times 10^{-2}\) eV. The neutrino mass matrix \(M_\nu\) is given by \(M_\nu = M_{\nu D}^TM_{N}^{-1}M_{\nu D}\). Thus, the scale \(M_R\) of the Majorana masses is determined as \(M_R \simeq 10^{12} - 10^{13}\) GeV, which is close to the natural scale derived from Eq. (19) with \(j_{33} = O(1)\). The observed maximal mixing is also naturally obtained as a result of fitting quark masses and mixings. In the following, we discuss the implications of the model on the solar neutrino deficit. For simplicity, we ignore CP-violating effects in the lepton sector and take all values appear in the neutrino mass matrix to be real, \(\varphi_x = \varphi_y = \varphi_z = \varphi_t = \varphi_u = \varphi_v = \varphi_w = 0\) or \(\pi\).

The observed solar neutrino deficit is explained by either matter-enhanced neutrino oscillation (MSW [28]) solution or the vacuum oscillation solution [29] (\(\nu_e \leftrightarrow \nu_\mu, \nu_\tau\)).
Table 4: The allowed regions of mass squared differences and mixing angles which reproduce the observed solar neutrino deficit in terms of the neutrino oscillation \cite{30, 31}. We have also shown mass ratios $m_{\nu_2}/m_{\nu_3}$ under the mass hierarchy $m_{\nu_2} \ll m_{\nu_3}$.

| solutions            | $\delta m^2_{sol}$          | $\sin^2 2\theta^\text{sol}$ | $m_{\nu_2}/m_{\nu_3}$ |
|----------------------|-----------------------------|-----------------------------|------------------------|
| small angle MSW      | $(4 - 12) \times 10^{-6} \text{ eV}^2$ | $(2 - 12) \times 10^{-5}$ | $\sim 0.03$            |
| large angle MSW      | $(8 - 25) \times 10^{-6} \text{ eV}^2$ | $0.5 - 0.8$ | $\sim 0.05$            |
| vacuum oscillation   | $(6 - 7) \times 10^{-10} \text{ eV}^2$ | $0.8 - 1.0$ | $\sim 10^{-4}$         |
|                      | $(4 - 5) \times 10^{-10} \text{ eV}^2$ | $0.7 - 1.0$ | $\sim 10^{-4}$         |
|                      | $(5 - 9) \times 10^{-11} \text{ eV}^2$ | $0.6 - 0.9$ | $\sim 10^{-4}$         |

Then, the allowed regions of mass squared differences and mixing angles are as shown in Table 4 \cite{30, 31}. The MSW solution has two distinct regions, that is, the small and the large angle ones. Below, we investigate which solutions the model can realize in two cases that the neutrino Majorana mass matrix $M_N$ is proportional to the unit matrix or that it has a hierarchy consistent with the observation that $\psi_i(16)$ have a source of hierarchies. The RG effects between the GUT scale and the weak scale are negligible for our qualitative argument, so that we evaluate the masses and mixing angles at the GUT scale and compare them with the observed values given in Table 4.

4.1 The case of unit Majorana mass matrix

We first consider the unit Majorana mass matrix case;

$$M_N = M_R \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (24)$$

Here, we simply assume that $M_N$ is proportional to the unit matrix for some reasons in the basis that the original Yukawa couplings are diagonal, and regard $\delta_x$, $\delta_y$ and $\delta_z$ as free parameters. Then, the neutrino mass matrix, $M_\nu = M^T_{\nu D} M_N^{-1} M_{\nu D}$, is given as

$$M_\nu = \frac{m_t^2}{M_R} \begin{pmatrix} m_u s_1 & m_u c_1 \\ \delta_x & \delta_y \\ \hat{m}_u s_1 s_2 & \hat{m}_c s_1 s_2 \end{pmatrix} \begin{pmatrix} -\hat{m}_u s_1 & \hat{m}_u c_1 s_2 \\ \hat{m}_c c_1 & \hat{m}_c c_1 s_2 \\ \hat{m}_c s_1 s_2 \end{pmatrix} \begin{pmatrix} -\hat{m}_u s_1 & \hat{m}_u c_1 s_2 \\ \hat{m}_c c_1 & \hat{m}_c c_1 s_2 \\ \hat{m}_c s_1 s_2 \end{pmatrix}$$

$$\approx \frac{m_t^2}{M_R} \begin{pmatrix} m_u^2 c_1^2 + m_u^2 \delta_x + m_u \delta_x c_1 & m_u^2 c_2^2 + m_u^2 \delta_y + m_u \delta_y c_2 \\ m_u^2 c_1 \delta_x + m_u \delta_x c_1 & m_u^2 c_2 \delta_x + m_u \delta_x c_2 \end{pmatrix} \quad (25)$$
where $c_i \equiv \cos \alpha_i$ and $s_i \equiv \sin \alpha_i$ ($i = 1, 2$).

The contribution to the mixing angle between second and third generations from the neutrino mass matrix Eq. (25) is given by $\theta_{\nu}^{\mu_3} \sim \delta_z/c_2$. In view of $c_2 = O(0.1)$, we choose $\delta_z \sim \epsilon^2$ to give naturally the near maximal mixing in the atmospheric neutrino oscillation (see discussion at the end of section 3). Then, the relevant mass ratio and mixing to the solar neutrino oscillation are given as

$$\frac{m_{\nu_2}}{m_{\nu_3}} \sim \frac{\delta_y^2}{c_2^2} \sim \frac{\delta_y^2}{\epsilon^2},$$

$$\theta_{\nu_2}^\nu \sim \frac{\hat{m}_e c_1}{\delta_y} \sim \frac{\epsilon^3}{\delta_y},$$

as long as $\delta_x \lesssim \delta_y$ and $\delta_y$ is not very small compared with $\delta_z$.

From Eq. (25), we find that if we choose $\delta_y \sim \epsilon^2$ we obtain the mass ratio consistent with the MSW solutions. Then, the mixing angle $\theta_{\nu_2}^\nu$ is of order of $\epsilon$ from Eq. (27). The actual mixing angle is the sum of $\theta_{\nu_2}^\nu$ and the corresponding contribution from charged-lepton mass matrix $\theta_{\ell_2}^\nu$. Since analyses of the previous section indicate $\theta_{\ell_2}^\nu \simeq 0.04 - 0.19$, the resulting mixing angle is consistent with the small angle MSW solution if two contributions are added up destructively, while it cannot reach that of the large angle MSW solution even if two contributions are added up constructively. The required cancellation is rather mild such that the reduction of factor two or three is sufficient. We also mention that the mixing angle $\theta_{e_3}$ is small, so that it is not conflict with the result of long baseline reactor neutrino oscillation experiment (CHOOZ), $\theta_{e_3} \lesssim 0.22$.

If we choose $\delta_y \sim \epsilon^3$, on the other hand, both mass ratio and mixing angle are consistent with vacuum oscillation solution. The reason for this coincidence can be traced back to the fact that the required neutrino mass ratio $m_{\nu_2}/m_{\nu_3} \sim 10^{-4}$ is approximately equal to $(m_c/m_t)^2$ as predicted by a simple (unit $M_N$) see-saw model in the SO(10) GUT. That is, if we intend to obtain large solar-neutrino mixing angle we have to choose $\delta_y \simeq \hat{m}_e c_1$, and then we necessarily have the mass ratio $m_{\nu_2}/m_{\nu_3} \sim \delta_y^2/c_2^2 \sim \hat{m}_e^2$. The resulting MNS matrix has the so-called bi-maximal form and evades the constraint from CHOOZ experiment. We note that this vacuum oscillation solution is realized with $k_1 : k_2 : k_3 \simeq \epsilon^2 : \epsilon : 1$ which is implied by the hierarchy Eqs. (22, 23) (see Eq. (15)).
4.2 The case of hierarchical Majorana mass matrix

If $\psi_i(16)$ have a source of hierarchies indicated by Eqs. (22, 23), it is natural to think that the Majorana mass matrix $M_N$ also has hierarchy consistent with it. Thus, we consider the case where the Majorana mass matrix has the hierarchical form:

$$
M_N \simeq M_R \begin{pmatrix}
\epsilon^4 & \epsilon^3 & \epsilon^2 \\
\epsilon^3 & \epsilon^2 & \epsilon \\
\epsilon^2 & \epsilon & 1
\end{pmatrix},
$$

up to order-one coefficients.

In this subsection, we stick to the possibility that $\psi_i(16)$ have a source of hierarchies indicated by Eqs. (22, 23), so that we assume that $k_1 : k_2 : k_3 \simeq \epsilon^2 : \epsilon : 1$. Considering that $\delta_z \sim \epsilon^2$ in order to give naturally the near maximal mixing between $\nu_\mu$ and $\nu_\tau$, elements of the $M_\nu^D$ have the following order of magnitude:

$$
\delta_x \simeq \epsilon^4, \quad \delta_y \simeq \epsilon^3, \quad \delta_z \simeq \epsilon^2.
$$

Then, the neutrino mass matrix, $M_\nu = M_\nu^T M_N^{-1} M_\nu^D$, is given as

$$
M_\nu \simeq \frac{m_i^2}{M_R} \begin{pmatrix}
\epsilon^{-4}\tilde{m}_u^2 & \tilde{m}_u & \epsilon^{-3}\tilde{m}_u\tilde{m}_c \\
\tilde{m}_u & \epsilon^4 & \epsilon\tilde{m}_c \\
\epsilon^{-3}\tilde{m}_u\tilde{m}_c & \epsilon\tilde{m}_c & \epsilon^{-2}\tilde{m}_c^2
\end{pmatrix},
$$

where $\tilde{m}_u \equiv \hat{m}_u + \epsilon\hat{m}_c c_1$ and $\tilde{m}_c \equiv \hat{m}_c + \epsilon c_2$. This gives the relevant mass ratio and mixing to the solar neutrino oscillation as

$$
\frac{m_{\nu_2}}{m_{\nu_3}} \sim \frac{\epsilon^6}{\tilde{m}_c^2} = \epsilon^6 (\tilde{m}_c + \epsilon c_2)^2,
$$

and

$$
\theta^\nu_{\nu_2/\nu_3} \sim \frac{\tilde{m}_u}{\epsilon^4} = \frac{\hat{m}_u + \epsilon\hat{m}_c c_1}{\epsilon^4}.
$$

From Eq. (31), we obtain $m_{\nu_2}/m_{\nu_3} \simeq 10^{-2} - 10^{-1}$, which is consistent with both small and large angle MSW solutions. Also, the angle is given as $\theta^\nu_{\nu_2} \simeq 0.1 - 1$ from Eq. (32), so that we can reproduce the large angle MSW solution naturally. To reproduce the small angle solution, $\theta^\nu_{\nu_2}$ and $\theta^\nu_{\nu_3}$ have to be added up destructively, but the required cancellation is mild. (The reduction of factor three to five is sufficient.) Thus, we conclude that the model can accommodate both large and small angle MSW solutions in the case of the hierarchical Majorana mass matrix.
5 Conclusions

In this paper, we have investigated the SO(10) GUT model with generation flipping. The model contains one extra matter multiplet $\psi(10)$ in addition to the usual matter multiplets $\psi_i(16)$, and it mixes with $\psi_i(16)$ when the SO(10)$_{\text{GUT}}$ is broken down to SU(5) by the vacuum expectation values of $\chi(16)$ and $\bar{\chi}(16^*)$ fields. The low-energy quarks and leptons are three linear combinations of $\psi_i(16)$ and $\psi(10)$.

We have found the parameter region of the model in which the observed quark masses and mixings are well reproduced. The required hierarchies of the coupling constants are consistent with the observation that only $\psi_i(16)$ have a source of hierarchies such as U(1) flavor symmetry charges. As for the lepton sector, the model can reproduce the charged-lepton masses if there are suitable nonrenormalizable interactions which introduce SU(5) breaking effects into $M_{d/l}$. The obtained charged-lepton mass matrix indicates that the mixing between second and third generations tends to be large in the lepton sector, which is consistent with the observed maximal mixing of the atmospheric neutrino oscillation.

We have also discussed neutrino masses and mixings qualitatively. We have considered two cases that the Majorana mass matrix $M_N$ for the right-handed neutrinos is proportional to the unit matrix and it has the hierarchical form. In the former case, the model can accommodate small angle MSW solution and vacuum oscillation solution to the solar neutrino deficit, depending on the values of the coupling constants $k_i$. The vacuum oscillation solution is realized when $k_i$ have the hierarchy indicated in the quark sector, and the resulting MNS mixing matrix has the form of so-called bi-maximal mixing.

In the latter case, the model can accommodate both small and large angle MSW solutions and the parameter region is consistent with the hierarchy required in the quark sector. It is remarkable that the single anomalous U(1)$_X$ flavor symmetry with the $\phi$ field which has the vacuum expectation value of order $10^{17}$ GeV realizes this possibility. Then, the operators in the superpotential have suppression factors of appropriate powers

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6 This structure may be realized in a framework of E$_6$ GUT [34].

7 The nonrenormalizable operators in Eq. (21) might be generated via exchanges of some heavy fields, so that they are suppressed by the mass scale of the heavy fields instead of $M_G$. We think that the masses for these heavy fields, the GUT scale $V$ and the mass $M$ are not related to the anomalous U(1)$_X$ breaking scale, $\langle \phi \rangle$. 

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Table 5: $U(1)_X$ charges. The other fields have vanishing charges.

| $U(1)_X$ | $\psi_1(16)$ | $\psi_2(16)$ | $\psi_3(16)$ | $\psi(10)$ | $H(10)$ | $\chi(16)$ | $\bar{\chi}(16^*)$ | $\phi$ |
|----------|-------------|-------------|-------------|-----------|---------|------------|----------------|------|
|          | 2           | 1           | 0           | 0         | 0       | 0          | 0              | −1   |

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A The mass matrices in the model

In this appendix, we give a detailed derivation of the mass matrices in the model, clarifying a phase convention we have used.

The model contains matter fields $\psi_1(16), \psi(10)$ and Higgs fields $H(10), H(16)$. After

\[8 \text{ Under the charge assignment of Table 5, the mass } M \text{ in Eq. (8) should be } 10^{17} \text{ GeV. On the other hand, if } \psi(10) \text{ has the } U(1)_X \text{ charge } 1, M \text{ should be the GUT scale, } 10^{16} \text{ GeV.} \]
the spontaneous breakdown of SO(10) to SU(5), these fields decompose as

\[
\begin{align*}
10_1 + 5_1' + 11 &= \psi_1(16), \\
10_2 + 5_2' + 12 &= \psi_2(16), \\
10_3 + 5_3' + 13 &= \psi_3(16), \\
5_\psi + 5_\psi' &= \psi(10),
\end{align*}
\]

(33)

\[
\begin{align*}
5_H + 5_{H(16)}' &= H(10), \\
10_{H(16)} + 5_{H(16)}' &= H(16).
\end{align*}
\]

(34)

The original Yukawa couplings are given by the superpotential Eq. (2) and the Majorana masses for the right-handed neutrinos by Eq. (19). At this stage, the quark and lepton masses are written as

\[
W = \frac{1}{2} (10_1 10_2 10_3) \begin{pmatrix} h_1 \langle 5_H \rangle & 0 & 0 \\ 0 & h_2 \langle 5_H \rangle & 0 \\ 0 & 0 & h_3 \langle 5_H \rangle \end{pmatrix} \begin{pmatrix} 10_1 \\ 10_2 \\ 10_3 \end{pmatrix}
\]

\[
+ (10_1 10_2 10_3) \begin{pmatrix} h_1 \langle 5_{H(10)}' \rangle & 0 & 0 \\ 0 & h_2 \langle 5_{H(10)}' \rangle & 0 \\ 0 & 0 & h_3 \langle 5_{H(10)}' \rangle \end{pmatrix} \begin{pmatrix} 5_1' \\ 5_2' \\ 5_3' \end{pmatrix}
\]

\[
+ (11_1 11_2 11_3) \begin{pmatrix} h_1 \langle 5_H \rangle & 0 & 0 \\ 0 & h_2 \langle 5_H \rangle & 0 \\ 0 & 0 & h_3 \langle 5_H \rangle \end{pmatrix} \begin{pmatrix} 5_1' \\ 5_2' \\ 5_3' \end{pmatrix}
\]

\[
+ \frac{1}{2} (11_1 11_2 11_3) \begin{pmatrix} M_R \tilde{j}_{11} & M_R \tilde{j}_{12} & M_R \tilde{j}_{13} \\ M_R \tilde{j}_{21} & M_R \tilde{j}_{22} & M_R \tilde{j}_{23} \\ M_R \tilde{j}_{31} & M_R \tilde{j}_{32} & M_R \tilde{j}_{33} \end{pmatrix} \begin{pmatrix} 11_1 \\ 12_1 \\ 13_1 \end{pmatrix},
\]

(35)

in terms of SU(5) language. Here, \(\tilde{j}_{ij} = j_{ij} / |j_{33}|\) and \(M_R\) is given as \(M_R = |j_{33}| V^2 / M_G\). If we assume that the Majorana mass matrix is proportional to the unit matrix, \(j_{ij} = j \delta_{ij}\), in the basis that the original Yukawa couplings are diagonal, \(\tilde{j}_{ij}\) and \(M_R\) are reduced to \(\tilde{j}_{ij} = \delta_{ij}\) and \(M_R = j V^2 / M_G\).

The superpotential Eq. (3) mixes \(\psi(10)\) with \(\psi(16)\). This term can be written in terms of SU(5) language as

\[
W = \sqrt{|f_1|^2 + |f_2|^2 + |f_3|^2 \langle \chi(16) \rangle} \tilde{5}_s^* \tilde{5}_\psi,
\]

(36)

where \(\tilde{5}_s^*\) is a linear combination of \(5_1'\), \(\tilde{5}_s^* = \sum_{i=1}^3 f_i e^{i\eta} 5_i' / \sqrt{|f_1|^2 + |f_2|^2 + |f_3|^2}\). We call remaining two linear combinations \(5_1^*\) and \(5_3^*\). The fields \(\tilde{5}_s^*, 5_1^*\) and \(5_3^*\) are related
to $\tilde{5}^{s'}$ by a unitary transformation. Thus, using arbitrariness of defining $\tilde{5}^*_1$ and $\tilde{5}^*_3$ and phases of the fields $\tilde{5}^*_1, \tilde{5}^*_2$ and $\tilde{5}^*_3$, we can write this relation as

\[
\begin{pmatrix}
\tilde{5}^{s'}_1 \\
\tilde{5}^{s'}_2 \\
\tilde{5}^{s'}_3
\end{pmatrix} = \begin{pmatrix}
e^{-i\delta_1-i\delta_2} & 0 & 0 \\
0 & e^{-i\delta_1+i\delta_2} & 0 \\
0 & 0 & e^{2i\delta_1}
\end{pmatrix} \begin{pmatrix}
\cos \alpha_1 & -\sin \alpha_1 & 0 \\
\sin \alpha_1 & \cos \alpha_1 & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
\tilde{5}^*_1 \\
\tilde{5}^*_2 \\
\tilde{5}^*_3
\end{pmatrix},
\]

(37)

without a loss of generality. Here, $\alpha_1, \alpha_2$ and $\delta_i$ are functions of $f_i$ and $\eta_i$. Note that $\tilde{5}^*$ receives a GUT scale mass together with $\tilde{5}_\psi$ by Eq. (36), so that the low-energy quark and lepton fields belonging to $\tilde{5}^*$ of SU(5) are $\tilde{5}^*_1, \tilde{5}^*_3$ and $\tilde{5}^*_\psi$.

The $\psi(10)$ is coupled with Higgs and $\psi_i(16)$ by superpotentials Eqs. (32, 33). Then, from Eqs. (34, 35) we obtain down-type quark (charged-lepton) mass matrix $M_{10-5^*}$ and neutrino Dirac mass matrix $M_{1-5^*}$ defined by

\[
W = (\begin{pmatrix}
10_1 & 10_2 & 10_3
\end{pmatrix}) M_{10-5^*} \begin{pmatrix}
\tilde{5}^*_1 \\
\tilde{5}^*_\psi \\
\tilde{5}^*_3
\end{pmatrix} + (\begin{pmatrix}
1_1 & 1_2 & 1_3
\end{pmatrix}) M_{1-5^*} \begin{pmatrix}
\tilde{5}^*_1 \\
\tilde{5}^*_\psi \\
\tilde{5}^*_3
\end{pmatrix},
\]

(38)

as

\[
M_{10-5^*} = 
\begin{pmatrix}
-h_1 \langle \tilde{5}^*_H(10) \rangle e^{-i\delta_1-i\delta_2} \sin \alpha_1 & g_1 \langle \tilde{5}^*_H(16) \rangle e^{i\phi_1} & h_1 \langle \tilde{5}^*_H(16) \rangle e^{-i\delta_1-i\delta_2-i\delta_3} \cos \alpha_1 \sin \alpha_2 \\
h_2 \langle \tilde{5}^*_H(16) \rangle e^{-i\delta_1+i\delta_2} \cos \alpha_1 & g_2 \langle \tilde{5}^*_H(16) \rangle e^{i\phi_2} & h_2 \langle \tilde{5}^*_H(16) \rangle e^{-i\delta_1+i\delta_2-i\delta_3} \sin \alpha_1 \sin \alpha_2 \\
0 & g_3 \langle \tilde{5}^*_H(16) \rangle e^{i\phi_3} & h_3 \langle \tilde{5}^*_H(16) \rangle e^{2i\delta_1} \cos \alpha_2
\end{pmatrix},
\]

\[
M_{1-5^*} = 
\begin{pmatrix}
-h_1 \langle \tilde{5}^*_H \rangle e^{-i\delta_1-i\delta_2} \sin \alpha_1 & k_1 \langle \tilde{5}^*_H \rangle \frac{V_{MH}}{M_H} e^{i\phi_1} & h_1 \langle \tilde{5}^*_H \rangle e^{-i\delta_1-i\delta_2-i\delta_3} \cos \alpha_1 \sin \alpha_2 \\
h_2 \langle \tilde{5}^*_H \rangle e^{-i\delta_1+i\delta_2} \cos \alpha_1 & k_2 \langle \tilde{5}^*_H \rangle \frac{V_{MH}}{M_H} e^{i\phi_2} & h_2 \langle \tilde{5}^*_H \rangle e^{-i\delta_1+i\delta_2-i\delta_3} \sin \alpha_1 \sin \alpha_2 \\
0 & k_3 \langle \tilde{5}^*_H \rangle \frac{V_{MH}}{M_H} e^{i\phi_3} & h_3 \langle \tilde{5}^*_H \rangle e^{2i\delta_1} \cos \alpha_2
\end{pmatrix},
\]

(39)

SU(5)-$5^*$ of the Higgs fields $H(10)$ and $H(16)$ are also mixed by the superpotential Eq. (8). One linear combination $\tilde{5}^*_H$ gets a GUT scale mass together with SU(5)-$5$ of $\tilde{H}(16^*)$ and the other combination $\tilde{5}^*_H$ remains as a massless Higgs in the standard SU(5)GUT. We can parametrize these fields as

\[
\begin{pmatrix}
\tilde{5}^*_H(10) \\
\tilde{5}^*_H(16)
\end{pmatrix} = \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
\tilde{5}^*_H \\
\tilde{5}^*_H
\end{pmatrix},
\]

(40)
where \( \tan \theta = -\lambda V/M \). That is,
\[
\langle 5^*_H(10) \rangle = \cos \theta \langle 5^*_H \rangle, \\
\langle 5^*_H(16) \rangle = \sin \theta \langle 5^*_H \rangle.
\] (41)

Substituting Eq. (41) for Eq. (39) and combining with Eq. (35), we obtain complete mass matrices. Defining
\[
m_u \equiv h_1 \langle 5_H \rangle, \quad m_c \equiv h_2 \langle 5_H \rangle, \quad m_t \equiv h_3 \langle 5_H \rangle, \quad \tan \beta \equiv \langle 5_H \rangle / \langle 5^*_H \rangle,
\] (42)

the quark and lepton masses are written as
\[
W = (u c t) \begin{pmatrix} m_u & 0 & 0 \\
0 & m_c & 0 \\
0 & 0 & m_t \end{pmatrix} \begin{pmatrix} \tilde{u} \\
\tilde{c} \\
\tilde{t} \end{pmatrix} + \frac{\cos \theta}{\tan \beta} (d s b) \begin{pmatrix} -m_u e^{-i\delta_1 - i\delta_2} \sin \alpha_1 & m_t x e^{i\phi_1} & m_u e^{-i\delta_1 - i\delta_2 - i\delta_3} \cos \alpha_1 \sin \alpha_2 \\
m_c e^{-i\delta_1 + i\delta_2} \cos \alpha_1 & m_t y e^{i\phi_2} & m_c e^{-i\delta_1 + i\delta_2 - i\delta_3} \sin \alpha_1 \sin \alpha_2 \\
0 & m_t z e^{i\phi_3} & m_t e^{2i\delta_1} \cos \alpha_2 \end{pmatrix} \begin{pmatrix} \tilde{d} \\
\tilde{s} \\
\tilde{b} \end{pmatrix} + \frac{\cos \theta}{\tan \beta} (e \mu \tau) \begin{pmatrix} -m_u e^{-i\delta_1 - i\delta_2} \sin \alpha_1 & m_t x e^{i\phi_1} & m_u e^{-i\delta_1 - i\delta_2 - i\delta_3} \cos \alpha_1 \sin \alpha_2 \\
m_c e^{-i\delta_1 + i\delta_2} \cos \alpha_1 & m_t y e^{i\phi_2} & m_c e^{-i\delta_1 + i\delta_2 - i\delta_3} \sin \alpha_1 \sin \alpha_2 \\
0 & m_t z e^{i\phi_3} & m_t e^{2i\delta_1} \cos \alpha_2 \end{pmatrix} \begin{pmatrix} e \\
\mu \\
\tau \end{pmatrix} + (\tilde{\nu}_1 \tilde{\nu}_2 \tilde{\nu}_3) \begin{pmatrix} M_R \tilde{j}_{11} & M_R \tilde{j}_{12} & M_R \tilde{j}_{13} \\
M_R \tilde{j}_{21} & M_R \tilde{j}_{22} & M_R \tilde{j}_{23} \\
M_R \tilde{j}_{31} & M_R \tilde{j}_{32} & M_R \tilde{j}_{33} \end{pmatrix} \begin{pmatrix} \tilde{\nu}_1 \\
\tilde{\nu}_2 \\
\tilde{\nu}_3 \end{pmatrix},
\] (43)

in terms of the SM fields. Here, we have defined various parameters as
\[
x \equiv \frac{g_1}{h_3} \tan \theta, \quad y \equiv \frac{g_2}{h_3} \tan \theta, \quad z \equiv \frac{g_3}{h_3} \tan \theta,
\] (44)

\[
\delta_x \equiv \frac{k_1}{h_3} V/M_G, \quad \delta_y \equiv \frac{k_2}{h_3} V/M_G, \quad \delta_z \equiv \frac{k_3}{h_3} V/M_G,
\] (45)

and taken the effects of nonrenormalizable superpotential Eq. (21) into account by changing (1, 2) and (2, 2) components of the down-type quark and charged-lepton mass matrices from their original values \( xe^{i\phi_1} \) and \( ye^{i\phi_2} \).
Now, we redefine various SM fields as

\[
\begin{align*}
\begin{cases}
(u, d) &\to (u, d) e^{i\delta_1 + i\delta_2}, \\
(c, s) &\to (c, s) e^{i\delta_1 - i\delta_2}, \\
(t, b) &\to (t, b) e^{-2i\delta_1 - i\delta_3}, \\

\end{cases}
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
(\nu_1, e) &\to (\nu_1, e) e^{i\delta_1 + i\delta_2}, \\
(\nu_2, \mu) &\to (\nu_2, \mu) e^{i\delta_1 - i\delta_2}, \\
(\nu_3, \tau) &\to (\nu_3, \tau) e^{-2i\delta_1 - i\delta_3},
\end{cases}
\end{align*}
\]

in order to simplify the form of the quark and lepton mass matrices. Then, from Eq. (43) we obtain the mass matrices defined by

\[
W = (\begin{array}{ccc} u & c \\ t & \end{array}) M_u \begin{pmatrix} \bar{u} \\ \bar{c} \\ \bar{t} \end{pmatrix} + (\begin{array}{ccc} d & s & b \end{array}) M_d \begin{pmatrix} \bar{d} \\ \bar{s} \\ \bar{b} \end{pmatrix} + (\begin{array}{ccc} \bar{e} & \bar{\mu} & \bar{\tau} \end{array}) M_l \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}
\]

\[
+ (\begin{array}{ccc} \bar{\nu}_1 & \bar{\nu}_2 & \bar{\nu}_3 \end{array}) M_{\nu D} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} + \frac{1}{2} (\begin{array}{ccc} \bar{\nu}_1 & \bar{\nu}_2 & \bar{\nu}_3 \end{array}) M_N \begin{pmatrix} \bar{\nu}_1 \\ \bar{\nu}_2 \\ \bar{\nu}_3 \end{pmatrix},
\]

as

\[
M_u = m_t \begin{pmatrix} \hat{m}_u & 0 & 0 \\ 0 & \hat{m}_c & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]

\[
M_d = m_t \begin{pmatrix} \cos \theta \\ \tan \beta \end{pmatrix} \begin{pmatrix} -\hat{m}_u \sin \alpha_1 & x_d e^{i\phi_{xd}} & \hat{m}_u \cos \alpha_1 \sin \alpha_2 \\ \hat{m}_c \cos \alpha_1 & y_d e^{i\phi_{yd}} & \hat{m}_c \sin \alpha_1 \sin \alpha_2 \\ 0 & z & \cos \alpha_2 \end{pmatrix},
\]

\[
M_l = m_t \begin{pmatrix} \cos \theta \\ \tan \beta \end{pmatrix} \begin{pmatrix} -\hat{m}_u \sin \alpha_1 & x_l e^{i\phi_{xl}} & \hat{m}_u \cos \alpha_1 \sin \alpha_2 \\ \hat{m}_c \cos \alpha_1 & y_l e^{i\phi_{yl}} & \hat{m}_c \sin \alpha_1 \sin \alpha_2 \\ 0 & z & \cos \alpha_2 \end{pmatrix},
\]

\[
M_{\nu D} = m_t \begin{pmatrix} \hat{m}_e e^{i\phi_{\nu e}} \cos \alpha_1 & \delta_x e^{i\phi_{\nu x}} & \hat{m}_u e^{i\phi_{\nu u}} \cos \alpha_1 \sin \alpha_2 \\ \delta_y e^{i\phi_{\nu y}} & \hat{m}_c e^{i\phi_{\nu c}} \sin \alpha_1 \sin \alpha_2 \\ 0 & \hat{m}_\tau e^{i\phi_{\nu \tau}} e^{i\phi_{\nu w}} \cos \alpha_2 \end{pmatrix},
\]

\[
M_N = M_R \begin{pmatrix} \tilde{j}_{11} & \tilde{j}_{12} & \tilde{j}_{13} \\ \tilde{j}_{21} & \tilde{j}_{22} & \tilde{j}_{23} \\ \tilde{j}_{31} & \tilde{j}_{32} & \tilde{j}_{33} \end{pmatrix},
\]

where \( \hat{m}_u \equiv m_u/m_t, \hat{m}_c \equiv m_c/m_t \). Here, we have defined various phases as

\[
\phi_{xd} \equiv \phi_{1d} - \phi_3 + 3\delta_1 + \delta_2 + \delta_3, \quad \phi_{yd} \equiv \phi_{2d} - \phi_3 + 3\delta_1 - \delta_2 + \delta_3, \\
\phi_{xl} \equiv \phi_{1l} - \phi_3 + 3\delta_1 + \delta_2 + \delta_3, \quad \phi_{yl} \equiv \phi_{2l} - \phi_3 + 3\delta_1 - \delta_2 + \delta_3,
\]

(50)
\[ \varphi_x \equiv \varphi_1 + \delta_1 - \delta_2, \quad \varphi_y \equiv \varphi_2 + \delta_1 - \delta_2, \quad \varphi_z \equiv \varphi_3 + \delta_1 - \delta_2, \]
\[ \varphi_t \equiv 2\delta_2, \quad \varphi_u \equiv -3\delta_1 - \delta_2 - 2\delta_3, \]
\[ \varphi_v \equiv -3\delta_1 + \delta_2 - 2\delta_3, \quad \varphi_w \equiv -\delta_3. \]  
(51)

Note that we have retained the form of \( M_N \). As a result, not all phases are physically independent. The domains of \( \alpha_1 \) and \( \alpha_2 \) can be restricted to

\[ 0 \leq \alpha_1 \leq \pi/2, \quad 0 \leq \alpha_2 \leq \pi/2, \]  
(52)

without a loss of generality by redefining the phases of the SM fields appropriately.

Finally, the mass matrix for the light neutrinos defined by

\[ W = \frac{1}{2} (\nu_1 \nu_2 \nu_3) M_\nu \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \]  
(53)

is given by

\[ M_\nu = M_{\nu_D}^T M_N^{-1} M_{\nu_D}, \]  
(54)

through see-saw mechanism.
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Figure 1: The parameter region of $x_d$ and $y_d$ where all quark masses and the CKM parameters are reproduced. We have set $\hat{m}_u^{-1}$ to be 112244 and $\hat{m}_c^{-1}$ to be (a)394 (central value) (b)279 (lowest value).
Figure 2: The parameter region of $\alpha_1$ and $\alpha_2$ where all quark masses and the CKM parameters are reproduced. $\alpha_1$ and $\alpha_2$ are represented with radians and their domains are $0 \leq \alpha_1 \leq \pi/2$ and $0 \leq \alpha_2 \leq \pi/2$. Solid lines denote $\alpha_1 = \pi/2$ and $\alpha_2 = \pi/2$. We have set $\tilde{m}_u^{-1}$ to be 112244 and $\tilde{m}_c^{-1}$ to be (a)394 (central value) (b)279 (lowest value).
Figure 3: The mass ratio $m_b/m_s$ as a function of $z/\cos\alpha_2$. The allowed region is between two horizontal lines ($32.8 < m_b/m_s < 56.3$). We have set $\hat{m}_u^{-1}$ to be 112244 and $\hat{m}_c^{-1}$ to be (a)394 (central value) (b)279 (lowest value).
Figure 4: The mass ratio $m_s/m_d$ as a function of $z/\cos\alpha_2$. The allowed region is between two horizontal lines ($12.7 < m_b/m_s < 32.2$). We have set $\hat{m}_u^{-1}$ to be 112244 and $\hat{m}_c^{-1}$ to be (a)394 (central value) (b)279 (lowest value).
Figure 5: The parameter region of $x_l$ and $y_l$ where all lepton mass ratios are reproduced. The hatched region represents that of $x_d$ and $y_d$ given in Fig. 3(a). We have set $\hat{m}_u^{-1} = 112244$ and $\hat{m}_c^{-1} = 394$. 