Almeida-Thouless transition below six dimensions

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The existence of an Almeida-Thouless (AT) instability surface below the upper critical dimension 6 is demonstrated in the generic replica symmetric field theory. Renormalization flows from around the zero-field fixed point are investigated. By introducing the temperature and magnetic field dependence of the bare parameters, the fate of the AT line can be followed from mean field \((d = \infty)\) down to \(d = 6 - \epsilon\).

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Notwithstanding the relative simplicity of the relevant model postulated by Edwards and Anderson [1], the Ising spin glass problem has resisted a thorough understanding for decades. Severe frustration makes numerical simulations extremely hard and computer-time consuming, whereas analytical methods must handle the inhomogeneities caused by the quenched disorder. The model was later extended and studied on the fully connected lattice by Sherrington and Kirkpatrick (SK) [2], the characterization of the spin glass phase by the solution of Parisi (see Ref. [3] for a list of references) is now unanimously accepted as the true mean field theory. This complex phase space structure survives in an external magnetic field up to a phase boundary called the Almeida-Thouless phase space structure. This complex phase breaking develops.

An alternative theory — the so called droplet picture — emerged, however, and continues questioning the relevance of mean field ideas in finite dimensional systems [5]. In this theory the glassy phase is much simpler, and is limited to zero field: a convincing conclusion about the existence or lack of an AT line may resolve a decades long debate about the structure of the spin glass phase in the physical dimensions. Recent numerical simulations [6, 7] in three dimensions essentially excluded the possibility of a transition in a field, whereas the four dimensional case remains somewhat ambiguous (see [8] for references to earlier works). On the analytical side, we must mention the scaling considerations in [9] and renormalization group (RG) calculations [10, 11], whereas a leading order field theoretical computation [12] provided an AT line above 6 dimensions. This letter tries to dissolve the misbelief that the AT line disappears below the upper critical dimension, by explicitly calculating it close to, but below \(d = 6\).

Ising spin glass transition in an external magnetic field can be studied in the generic replica symmetric field theoretical model [13] defined by the Lagrangean \(\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{1}\), where

\[
\mathcal{L}^{(2)} = \frac{1}{2} \sum_{\mathbf{p}} \left[ \left( \frac{1}{2} \eta^{2} + m_{1} \right) \sum_{\alpha \beta} \phi_{\mathbf{p} \alpha} \phi_{\mathbf{p} \alpha}^{\beta} + m_{2} \sum_{\alpha \gamma} \phi_{\mathbf{p} \gamma} \phi_{\mathbf{p} \gamma}^{\beta} + m_{3} \sum_{\alpha \beta \gamma \delta} \phi_{\mathbf{p} \alpha} \phi_{\mathbf{p} \beta} \phi_{\mathbf{p} \gamma} \phi_{\mathbf{p} \delta} \right],
\]

and

\[
\mathcal{L}^{1} = -N^{\frac{1}{2}} \hbar \sum_{\alpha \beta} \phi_{\mathbf{p} \alpha \beta}^{\alpha} - \frac{1}{6 \sqrt{N}} \sum_{\mathbf{p}_{1} \mathbf{p}_{2} \mathbf{p}_{3}} \left[ w_{1} \sum_{\alpha \beta \gamma} \phi_{\mathbf{p}_{1} \alpha} \phi_{\mathbf{p}_{2} \beta} \phi_{\mathbf{p}_{3} \gamma} \phi_{\mathbf{p}_{3} \gamma}^{\alpha} + w_{2} \sum_{\alpha \beta \gamma} \phi_{\mathbf{p}_{1} \alpha} \phi_{\mathbf{p}_{2} \beta} \phi_{\mathbf{p}_{3} \gamma} \phi_{\mathbf{p}_{3} \gamma}^{\alpha} + w_{3} \sum_{\alpha \beta \gamma} \phi_{\mathbf{p}_{1} \alpha} \phi_{\mathbf{p}_{2} \beta} \phi_{\mathbf{p}_{3} \gamma} \phi_{\mathbf{p}_{3} \gamma}^{\alpha} \right] + \sum_{\alpha \beta \gamma \delta} \phi_{\mathbf{p}_{1} \alpha} \phi_{\mathbf{p}_{2} \beta} \phi_{\mathbf{p}_{3} \gamma} \phi_{\mathbf{p}_{3} \gamma}^{\alpha} + w_{5} \sum_{\alpha \beta \gamma \delta} \phi_{\mathbf{p}_{1} \alpha} \phi_{\mathbf{p}_{2} \beta} \phi_{\mathbf{p}_{3} \gamma} \phi_{\mathbf{p}_{3} \gamma}^{\alpha} + w_{6} \sum_{\alpha \beta \gamma \delta} \phi_{\mathbf{p}_{1} \alpha} \phi_{\mathbf{p}_{2} \beta} \phi_{\mathbf{p}_{3} \gamma} \phi_{\mathbf{p}_{3} \gamma}^{\alpha} + w_{7} \sum_{\alpha \beta \gamma \delta} \phi_{\mathbf{p}_{1} \alpha} \phi_{\mathbf{p}_{2} \beta} \phi_{\mathbf{p}_{3} \gamma} \phi_{\mathbf{p}_{3} \gamma}^{\alpha} + w_{8} \sum_{\alpha \beta \gamma \delta} \phi_{\mathbf{p}_{1} \alpha} \phi_{\mathbf{p}_{2} \beta} \phi_{\mathbf{p}_{3} \gamma} \phi_{\mathbf{p}_{3} \gamma}^{\alpha}.
\]

The \(n(n-1)/2\) component fields are symmetric in the replica indices and \(\phi^{aa} = 0\); the spin glass limit requires...
The terms of the one-loop self-energy (which is computable at that leading order RG calculation. Coupling-like scaling fields are the fixed point \[\Gamma_0\], \[\Gamma_A\] and \[\Gamma_L\], thus generating — while replica symmetry is still preserved — a kind of quadratic symmetry breaking. The crossover region is best studied by the introduction of the nonlinear scaling fields \[g_i\] satisfying the exact renormalization flows \[g_i = \lambda_i g_i\]. [The \[\lambda_i\]'s of the mass sector \((i = 1, 2, 3)\) were computed from the renormalization flow equations, in leading order, in Ref. [17].] The RG equations provide a way to express the bare parameters of the Lagrangean in terms of the scaling fields, hence the masses can be computed as functions of the \(g_i\)'s:

\[
\begin{align*}
\Gamma_R &= g_1 + 2g_2 + g_3 + O(\epsilon), \\
\Gamma_A &= g_1 - (n - 4)g_2 - (n - 3)g_3 + O(\epsilon), \\
\Gamma_L &= g_1 - 2(n - 2)g_2 + \frac{(n - 2)(n - 3)}{2} g_3 + O(\epsilon).
\end{align*}
\]

The \(O(\epsilon)\) terms neglected above have two contributions: the one-loop self-energy (which is computable at that order) and corrections to the bare masses expressed in terms of the \(g_i\)’s — this, however, is not available in a leading order RG calculation. Coupling-like scaling fields \(g_i\)’s with \(i > 3\) enter also at this \(O(\epsilon)\) level. Eq. (3) can be derived by fixing the bare parameters such that \(\langle \Phi^3 \rangle \equiv 0\); this condition determines \(g_0\) unambiguously in terms of the other \(g_i\)’s. Two critical surfaces can be found from Eqs. (3) in the low temperature \((g_1 < 0)\) regime:

- \(\Gamma_R = 0\), \(\Gamma_A\) and \(\Gamma_L\) both positive — i.e. an Almeida-Thouless instability — for \(g_2 < -g_1\);

- \(\Gamma_A = 0\), \(\Gamma_L\) and \(\Gamma_R\) positive \((n > 0)\) for \(g_2 > -g_1\).

The common boundary of these two manifolds (which are two-dimensional now, but allowing for coupling-like scaling fields \(g_i\), \(i > 3\), they will have a complicated higher dimensional structure) for \(g_2 = -g_1\) is massive only in the longitudinal sector [11].

We are now interested in the RG flows along the AT instability surface when starting in the crossover region. The first order RG equations were all presented in Ref. [17], their structure is best displayed by the following (temporary) redefinition of the couplings: \(w_i/\sqrt{\epsilon} \rightarrow w_i\), which are now, like the masses, order unity. With the scaling factor \(\epsilon^{dl}\), and \(t \equiv \epsilon\):

\[
\begin{align*}
\frac{d m_i}{dt} &= 2m_i - \epsilon M_i(m_1, m_2, m_3; w_1, \ldots, w_8), \quad i = 1, 2, 3; \\
\frac{d w_i}{dt} &= \frac{1}{2} w_i + W_i(m_1, m_2, m_3; w_1, \ldots, w_8), \quad i = 1, \ldots, 8.
\end{align*}
\]

The \(M_i\) and \(W_i\) functions are quadratic and cubic, respectively, in the couplings. The most important feature of the RG equations above is that the flow parameter \(l\) in the mass sector, Eq. (4), is much larger for \(\epsilon \ll 1\) than \(t\) of the couplings, Eq. (5). Thus the masses renormalize in the background of the adiabatically slow couplings: the anomalous (A) and longitudinal (L) components, as they are \(O(1)\) on the AT surface, blow up, whereas the replicon (R) one, \(2m_1 = O(\epsilon)\), evolves into its adiabatic fixed point determined by the initial values of the couplings \(w_1^{\pm}\) and \(w_2^{\pm}\). While \(w_1^{\pm} = w_1(t = 0) = w_1^*\), we must carefully follow the development of \(w_2\) in the transient regime from \(w_2(t = 0) = 0\) to \(w_2^{\pm} \equiv w_2(t)\) with \(\epsilon \ll t \ll 1\) for the following reason: For \(t \gg 1\), i.e. \(t > \epsilon\), \(w_1\) and \(w_2\) decouple from the other bare parameters, and their flow can be put into the pair of equations:

\[
\begin{align*}
\frac{d w_1}{dt} &= \frac{1}{2} w_1 + g_n(r) w_1^3, \\
\frac{dr}{dt} &= -h_n(r) w_1^2,
\end{align*}
\]

where \(g_n(r)\) and \(h_n(r)\) are cubic and quartic polynomials of \(r\) with coefficients which are simple polynomials of \(n\), and \(r \equiv w_2/w_1\). For the case \(n = 0\), these equations were derived and discussed in Ref. [10]. We are now interested in the more generic case \(0 \leq n \lesssim \epsilon\), and observe that the qualitative behaviour of the renormalization flow changes drastically when the initial value of \(r(0) = w_2^{\pm}/w_1^{\pm}\) passes through \(r_1^* = \frac{1}{2} n + O(n^2)\), the unstable fixed point \(r_1^*\) being the solution of the equation \(h_n(r) = 0\). For \((r(0)) > r_1^*\), we have runaway trajectories already noticed in Ref. [10] with \(w_1 \rightarrow \infty\) and \(r \rightarrow r_2^* \approx 14.4 + O(n)\); whereas for \((r(0)) < r_1^*\), \(w_2\) immediately becomes negative, which is physically nonsense.

To get \(r(0)\), we must integrate Eq. (5) for \(i = 2\) in the transient regime from \(t = 0\) to \(\epsilon \ll t \ll 1\), thereby eliminating nonreplicon modes in the process of hardening anomalous and longitudinal masses. This is feasible using Eq. (65) of Ref. [17] together with the table between the different sets of couplings in Eq. (49) of Ref. [13], resulting in \(r(0) < r_1^*\) for \(0 < n \ll 1\) and starting close enough to the zero-field fixed point, whereas the spin

\[1\] More precisely, \(w_2(t = 0) \ll \epsilon\) in the physically relevant part of the AT surface.
glass case is exceptional with the condition \( r(0) > r^*_4 \) always fulfilled.

Runaway flows along critical surfaces have been associated with first order transitions in some common situations with crossover phenomena \[13\], and although this scenario cannot be ruled out completely for the spin glass either, we will argue that renormalization of the bare couplings on the AT surface towards their low-temperature limit may cause the runaway trajectories in this renormalization scheme. To see this, we recall the derivation of the microscopic Lagrangean in Ref. \[13\], and the necessity to redefine the fields as \( c \phi^{\alpha \beta}_p \rightarrow \phi^{\alpha \beta}_p \), with \( c \sim T \), to ensure the proper normalization of the kinetic term in \( \mathcal{L}^{(2)} \). This will cause the couplings diverge even if they disappeared for \( T \rightarrow 0 \) otherwise. That kind of normalization was essential in the derivation of Eqs. \[4\] and \[5\], manifested in the flowing \( \eta \) exponents of the three different mass modes. As our approximate RG equations are valid only for \( w_i = O(1) \), one probably needs to modify the RG scheme for detecting the proper zero-temperature behavior on the AT surface in this small \( \epsilon \) regime. This is, however, out of the scope of the present work.

In the remaining part of this letter we want to locate the AT-line of the original Edwards-Anderson spin glass model on the AT-surface of the field theory above. For this reason, we must find out the dependence of the bare parameters in Eqs. \[11\] and \[2\] on temperature \( (T) \) and magnetic field \( (H) \). The criterium which is adopted here is that the tree approximation of the field theory (i.e. neglecting loops) be equivalent with the accepted mean field theory of the Ising spin glass, the SK model, whose replicated partition function has the form \[2\]:

\[
\mathcal{Z}^n \sim \int \mathcal{D}q \exp \left\{ -N \left[ \frac{(kT)^2}{2J^2} \sum_{\alpha < \beta} q_{\alpha \beta}^2 - \ln \zeta \right] \right\},
\]

where

\[
\zeta = \text{Tr} \exp \left( \sum_{\alpha < \beta} q_{\alpha \beta} S^\alpha S^\beta + \frac{H}{kT} \sum_\alpha S^\alpha \right),
\]

\[
\int \mathcal{D}q \equiv \prod_{\alpha < \beta} \left( \int N q_{\alpha \beta} \frac{kT}{2\pi e^2} d\phi_{\alpha \beta} \right) \quad \text{and} \quad J^2, \quad \text{the variation of the Gaussian distribution of the random Ising interactions, sets the energy scale. In the tree approximation fluctuations are omitted, which can be achieved by setting} \quad \phi_{\alpha \beta}^{p=0} = \sqrt{N} q_{\alpha \beta} \quad \text{and zero for} \quad \phi_{\alpha \beta}^{p \neq 0} \quad \text{in} \quad \[11\] \text{and} \quad \[2\], \quad \text{and comparing it with} \quad \[4\] \text{and} \quad \[5\]. \quad \text{Not forgetting that the bare parameters of the field theory are finally tuned by the transformation} \quad \phi^{\alpha \beta}_p - \sqrt{N} q \delta_{p=0} - \phi^{\alpha \beta}_p \quad \text{— rendering the one-point function to zero} \quad \text{—, where} \quad q \quad \text{is the} \quad \text{exact replica symmetric order parameter, they are expressed by} \quad T \quad \text{and} \quad H \quad \text{in the vicinity of the mean field critical point} \quad \text{as} \quad kT^c = J \quad \text{as follows:}
\]

\[
wh = \frac{1}{2} (H/kT)^2 - (m_{1c} - \tau)(wq) + \frac{1}{2} (n-2)(wq)^2 + \ldots
\]

\[
m_1 = (m_{1c} - \tau) + (wq) + (H/kT)^2 - \frac{1}{2} q^2 \left[ u_{01} + u_{02} + \frac{1}{3} (n-1) u_{03} + \frac{1}{3} n (n-1) u_{04} \right] \ldots
\]

\[
m_2 = - (wq) - (H/kT)^2 - \frac{1}{3} q^2 \left[ (n-3) u_{01} + u_{03} \right] \ldots
\]

\[
m_3 = - \frac{1}{6} q^2 \left[ u_{01} + 2u_{04} \right] + \ldots
\]

\[
\gamma_1 = w + O(q, H^2), \quad \text{and} \quad \gamma_i = O(q, H^2) \quad i = 2 \ldots 8.
\]

Neglected terms above are higher orders in \( q \) and \( H^2 \). The quartic couplings are not included here, although their calculation is similarly straightforward, and they may be important above 8 dimensions. \( \tau > 0 \) measures the distance from the critical temperature of the field theory \( (T_c) \), whereas \( m_{1c} = - \frac{\partial^2}{\partial T^2} (T^c_{\text{rep}} - T^c_{\text{rep}}) \) gives the shift in the critical temperature, and is therefore one-loop order. The field theory is defined by \( \tau, (H/kT)^2 \) and by the bare parameters of the symmetrical theory (zero magnetic field paramagnet): \( w \) (cubic coupling), \( u_{01}, u_{02}, u_{03}, u_{04} \) (quartic couplings), \ldots etc. (see \[19\] for the classification of the quartic couplings). To reproduce the SK model results in the tree approximation, we must put \( w = 1, u_{01} = 3, u_{02} = 2, u_{03} = -6 \) and \( u_{04} = 0 \).

The condition \( \langle \phi \phi \rangle = 0 \) provides us the equation of state, i.e. the order parameter \( q \) around \( T_c \); it is used here to eliminate \( \tau \) from our results, replacing it by \( q \). The calculation of the one-loop contribution to the equation of state, and to the replicon mass is somewhat lengthy due to the complicated replica structure even in the case of replica symmetry. Nevertheless, it is still feasible by the methods of Ref. \[13\]. The result valid for \( d > 8 \), including the SK model by simply taking \( d = \infty \), can be put into the scaling form

\[
\Gamma_R = (wq)^2 \tilde{\Gamma}_R(x, y), \quad x \equiv \frac{(H/kT)^2}{(wq)^2} \quad \text{and} \quad y \equiv \frac{n}{(wq)}.
\]

The scaling function \( \tilde{\Gamma}_R \) has the simple linear form

\[
\tilde{\Gamma}_R(x, y) = ax + by + c, \quad d > 6,
\]

with \( a \) and \( b \) analytical down to 4 and 6 dimensions, respectively, and having their loop-expansions in terms of \( I_k \equiv \frac{1}{N} \sum_\Lambda \frac{1}{p^k} \):

\[
a = 1 - 2w^2 I_4 \quad \text{and} \quad b = 1 - 2w^2 I_6 + (-u_{10} + u_{30} + 4u_{40}) I_4; \quad d > 6.
\]

c however blows up at 8 dimensions, due to the infrared divergence developing in the first order contribution be-
hind the mean field term:
\[ c = -\frac{2}{3} w_{20} w^{-2} - 16 w^2 I_8 + \text{terms with } I_6 \text{ and } I_4, \quad d > 8. \] (12)

As a result, scaling of the replicon mass turns to the following form when \(6 < d < 8\):
\[ \Gamma_R = (wq)^{d/2 - 2} \bar{\Gamma}_R(x, y), \] (13)

with \( x = \frac{(H/kT)^2}{(wq)^{d/2 - 1}} \) and \( y = \frac{n}{(wq)^{d/2 - 3}} \).

The scaling function preserves the form \( \bar{\Gamma}_R \) with \( a \) and \( b \) in \( \bar{\Gamma}_R \), the constant \( c \) however becomes, instead of \( c \):
\[ c = -16 w^2 \int_{2}^{+\infty} \frac{d^d p}{(2\pi)^d} \frac{1}{p^4(p^2 + 2)^2}, \quad 6 < d < 8. \] (14)

The zeros of the scaling function provide the AT transitions, and two important cases can be studied for \( d > 6 \):

- \( H = 0 \), i.e. \( x = 0 \) and \( y = -c/b \). This case has been discussed in [19].

- The spin glass limit \( n = 0 \), i.e. \( y = 0 \) and \( x = x_0 = -c/a \). The AT line close to \( T_c \) in the two regimes is:
\[
\begin{cases}
(H/kT)^2 = x_0 (wq)^3 & \text{for } 8 < d, \\
(H/kT)^2 = x_0 (wq)^{d/2 - 1} & \text{for } 6 < d < 8.
\end{cases}
\] (15)

From (11) and (12), with \( w_{20} = 2 \), the SK value \( x_0 = 4/3 \) is reproduced, whereas \( x_0 \) becomes one-loop order for \( 6 < d < 8 \), see (14).

Below 6 dimensions, the leading scaling behaviour can be obtained by using fixed point values in [13], and also neglecting correction terms providing:
\[
w^* h = \frac{1}{2} (H/kT)^2 - (m_1^* - \tau) (w^* q) + \frac{1}{2} (n - 2) (w^* q)^2, \\
m_1 = m_1^* - \tau + (w^* q), \quad m_2 = -(w^* q), \quad m_3 = 0, \\
w_1 = w^*, \quad w_i = 0, \quad i = 2 \ldots 8.
\]

After eliminating \( \tau \) by the equation of state, we are left with a two-parameter theory, with the simple RG flows close to the fixed point:
\[
\dot{q} \simeq (2 - \epsilon/2 + \eta^*/2) q \quad \text{and} \quad (H/kT)^2 \simeq \lambda_0 (H/kT)^2,
\] (16)

with \( \lambda_0 = 4 - \epsilon/2 - \eta^*/2 \) and \( \eta^* = -\epsilon/3 \). The scaling fields can now be expressed as
\[
g_i \simeq (w^* q)^{\delta_i} \tilde{g}_i(x), \\
x = \frac{(H/kT)^2}{(w^* q)^\delta} \quad \text{and} \quad \delta = \frac{4 - \epsilon/2 - \eta^*/2}{2 - \epsilon/2 + \eta^*/2}.
\]

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**FIG. 1:** AT lines on the two sides of the upper critical dimension 6. See [13], [15] and footnote 2

From (14) follows that \( x \) is invariant under renormalization, and \( x_1 \) must be \((2 - \epsilon/2 + \eta^*/2)^{-1} \lambda \) for \( g_i = \lambda_i g_i \) to be satisfied. Any observable \( O \) satisfying the approximate RG flows \( O \simeq k \bar{O} \) around the fixed point can now be written as \( O \sim (w^* q)^{\delta_i (2 - \epsilon/2 + \eta^*/2)} \) times a function of \( x \). For a mass \( k = 2 - \eta^* = (\delta - 1) (2 - \epsilon/2 + \eta^*/2) \), and therefore the replicon mass takes the scaling form
\[
\Gamma_R \simeq (w^* q)^{\delta - 1} \bar{\Gamma}_R(x), \quad x = \frac{(H/kT)^2}{(w^* q)^\delta}.
\] (17)

The most important new feature of (17) when compared with the \( d > 6 \) cases, Eqs. (11) and (13), is the lack of the second scaling variable, which is proportional to \( n \). The AT line ends now in the zero field critical point \(^2\) even for \( n \) small but nonzero:
\[
(H/kT)^2 = x_0 (w^* q)^\delta, \quad x_0 = -n + [2 + O(n)] \epsilon + O(\epsilon^2),
\] (18)

and it disappears completely for \( n > 2 \epsilon \).

To conclude, we followed the fate of the AT line from mean field down to \( d = 6 - \epsilon \). An exceptional feature of the spin glass case \( n = 0 \) is that the runaway flows towards zero-temperature behavior — found below \( d = 6 \) — originate in the close vicinity of the zero-field fixed point. Our results do not exclude a possible lack of the AT surface in \( d = 3 \) — as suggested by recent numerical works [6, 7] —, a scenario, with some lower critical dimension to explain this, has been suggested in [13].

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\(^2\) When \( 6 < d < 8 \) the AT line takes the form \((H/kT)^2 \sim n^{1/2} (q - q_c)\) for \( n \geq 0 \), where \( q_c \sim n^{-2/\epsilon} \). For \( d \geq 8 \) the mean field phase diagram restores [3, 13, 19].
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