NON-GAUSSIAN RADIO-WAVE SCATTERING IN THE INTERSTELLAR MEDIUM

Stanislav Boldyrev and Arieh Königl

Department of Astronomy and Astrophysics, and Enrico Fermi Institute, University of Chicago, 5640 South Ellis Avenue, Chicago, IL 60637; boldyrev@uchicago.edu, arieh@jets.uchicago.edu

Received 2003 January 25; accepted 2003 October 10

ABSTRACT

It was recently suggested by Boldyrev & Gwinn that the characteristics of radio scintillations from distant pulsars are best understood if the interstellar electron density fluctuations that cause the time broadening of the radio pulses obey non-Gaussian statistics. In this picture the density fluctuations are inferred to be strong on very small scales ($\sim 10^8$–$10^{10}$ cm). We argue that such density structures could correspond to the ionized boundaries of molecular regions (clouds) and demonstrate that the power-law distribution of scattering angles that is required to match the observations arises naturally from the expected intersections of our line of sight with randomly distributed, thin, approximately spherical ionized shells of this type. We show that the observed change in the time-broadening behavior for pulsar dispersion measures $\lesssim 30$ pc cm$^{-3}$ is consistent with the expected effect of the general ISM turbulence, which should dominate the scattering for nearby pulsars. We also point out that if the clouds are ionized by nearby stars, then their boundaries may become turbulent on account of an ionization front instability. This turbulence could be an alternative cause of the inferred density structures. An additional effect that might contribute to the strength of the small-scale fluctuations in this case is the expected flattening of the turbulent density spectrum when the eddy sizes approach the proton gyroscale.

Subject headings: ISM: general — MHD — pulsars: general — scattering — turbulence

1. INTRODUCTION

Radio signals received from distant pulsars fluctuate in time and space due to inhomogeneities in the ionized component of the interstellar medium. Observations of pulsar scintillations have thus served as a valuable tool for reconstructing the statistics of interstellar electron density fluctuations (e.g., Rickett 1990; Armstrong et al. 1995; Scalo & Elmegreen 2004). Among a variety of observational quantities, special attention has been given to the time shape of the pulsar signal $I(\tau)$ (the signal intensity as a function of time). This function arises from the interference of waves propagating along different paths (similar to the Feynman interpretation of the propagation of a quantum particle). The deviation of the wave paths from a straight line causes a time broadening of the arriving signal. The intensity function $I(\tau)$ can be interpreted as the probability density function (PDF) of the time delays caused by different paths of propagation. Since the statistics of the path deviations are directly related to the statistics of electron density fluctuations, the function $I(\tau)$ provides information on the latter.

Boldyrev & Gwinn (2003a, 2003b, 2005) recently used the observed time shapes of the measured pulses to probe the shape of the distribution function of the underlying density fluctuations. Based on a comparison of analytical results and observational data, they proposed that the density fluctuations responsible for the pulse broadening are non-Gaussian, with the distribution function of the resulting angular path deviations having a slowly declining, power-law asymptotic. They further deduced that the density fluctuations should be strong on very small scales (estimated in $\S$ 3 to be $\sim 10^8$–$10^{10}$ cm).

After being suitably rescaled by the pulsar distance and the observation wavelength, the observed pulse time shapes exhibit a somewhat “universal” behavior (e.g., Bhat et al. 2004; Löhmer et al. 2004), which indicates that similar processes govern the density fluctuations along different lines of sight. Two intriguing questions then arise regarding the physics of interstellar plasma fluctuations. First, how are strong small-scale density fluctuations generated? And second, how can density fluctuations produce a non-Gaussian, power-law distribution of scattering angles?

In addressing these questions one can consider two distinct (although not mutually exclusive) possibilities. In one interpretation the observed universality is attributed to a general statistical property of density fluctuations. For example, in Kolmogorov-type turbulence, small-scale fluctuations arise as a result of an energy cascade from large to small scales. The properties of the observed signals might then be linked to the universal properties of such a cascade. The second possibility is that density fluctuations are strongly nonuniform and spatially intermittent (e.g., clumps, filaments, and shocks). In this case, the universality could reflect a certain inherent property of the density structures. This property should be fairly robust in view of the fact that different mechanisms (not necessarily related to turbulence) could in principle give rise to a particular structure. For example, scattering by shocks may be universal if the only property that matters is a sharp density discontinuity (see, e.g., Boldyrev & Gwinn 2005).

Let us briefly discuss the first possibility. We assume that interstellar scattering occurs in turbulent ionized regions (H ii regions) of mean density $n_e \sim 10^2$ cm$^{-3}$. Such regions appear when bright stars turn on within or in the vicinity of cold molecular clouds (e.g., Dyson & Williams 1997). Large-scale density fluctuations in a cold molecular cloud of mean density $n_0 \sim 10^2$ cm$^{-3}$ are of the order $\Delta n_0 \sim 10^2$ cm$^{-3}$, since such clouds are usually turbulent with Mach numbers greater than 1 (Larson 1981). When cold neutral gas in a molecular cloud is ionized by a newborn massive star emitting $\sim 10^{49}$ ionizing photons s$^{-1}$, the radius of the ionized sphere (the Strömgren radius) is $R_\text{S} \sim 10^{19}$ cm. This ionization happens very fast, during the first $10^3$ yr after the radiation turns on. This time interval is much shorter than the sound crossing time in the ionized gas within the Strömgren sphere, where the sound speed is $C_2 \sim 10^6$ cm s$^{-1}$. Subsequently, the initial density inhomogeneities relax due to pressure gradients, which drive turbulent internal motions in the H ii region. The longest relaxation time corresponds to the largest spatial scale,
and therefore the turbulence that survives over the lifetime ($\sim$10$^6$ yr) of the star is effectively stirred on the scale $l_{\text{out}} \sim R_\odot$, where the velocity and density fluctuations are of the order of $C_2$ and $\Delta n_\odot$, respectively.

The interstellar medium is magnetized, and the magnetic field is likely to play a role in any turbulent energy cascade. As discussed by several authors (e.g., Higdon 1984; Goldreich & Sridhar 1997; Lithwick & Goldreich 2001), density perturbations in an MHD turbulence are associated with compressible and entropy modes and are passively advected by the Alfvénic cascade toward the smallest (dissipative) spatial scales. The cascade proceeds predominantly in the direction perpendicular to the magnetic field and continues until the fluctuations reach the scale of the ion gyroradius $\rho_i$. A typical strength of the magnetic field in the Galaxy is a few $\mu$G (Zweibel & Heiles 1997). For an ion temperature $T_i \sim 8000$ K and magnetic field amplitude $B \sim 3 \mu$G, the gyroradius is $\rho_i \sim 3 \times 10^7$ cm, so the density fluctuations can reach very small scales.

However, a problem arises when we address the statistics of density fluctuations that are advected as a passive scalar in MHD turbulence. Numerical results show that passive scalar fluctuations in an incompressible turbulence have an exponential (or stretched exponential) distribution (e.g., Warhaft 2000), not the power-law distribution that we are trying to explain. Another (or stretched exponential) distribution (e.g., Warhaft 2000), not MHD turbulence. Numerical results show that passive scalar fluctuations can be diagnosed by observations of the time shapes of pulsar signals. To this end, we assume that the scattering (refraction) of a radio wave coming from a distant pulsar occurs in separate planar phase screens, uniformly placed along the line of sight to the pulsar. We denote the line-of-sight axis by $z$ and the coordinates in the perpendicular plane by a two-dimensional vector $y$. We also assume that the scattered wave is planar and propagates close to the line of sight. To analyze this model one can make use of the so-called parabolic approximation, which assumes that the wave amplitude changes slowly compared to its phase. The details of this analysis can be found in a number of references (e.g., Tatarskii 1961; Uscinski 1974; Williamson 1972; Lee & Jokipii 1975; Rickett 1990; Boldyrev & Gwinn 2005); here we only give a qualitative discussion of the results.

Each scattering screen generates phase fluctuations that cause the transmitted wave to contain many different angular components in its spectrum rather than follow a single direction. The spatial scale of the fluctuations that contribute to an angular deviation $\Delta \theta$ (expressed in radians) is $\lambda \approx \lambda / \Delta \theta$, where $\lambda$ is the wavelength. The difference of phases accumulated in a single screen along two lines of sight separated by the distance $y$ is given by $\Delta \Phi = i y \Delta N(y)$. It is proportional to the line-of-sight integrated density difference

$$\Delta N(y) = \int_0^y \left[ n(y_1, z) - n(y_2, z) \right] dz,$$

where $y = y_1 - y_2$, and $r_0 \equiv c^2/m_e c^2 = 2.82 \times 10^{-13}$ cm is the classical electron radius. The integration length $l_0$ should exceed
the characteristic size of the scattering region. To produce an angular deviation \(\Delta \theta\), the phase difference should be \(\Delta \Phi \sim 2\pi\) on the scale \(y \sim 1/\Delta \Phi\) (where here, as in the rest of this paper, angles are measured in radians).

The signal emitted by a pulsar has a rather short duration (several percent of the pulsar period). The received signal is broader, with its intensity \(I(\tau)\) exhibiting a sharp rise and a slow decline. This happens because waves propagating along different paths from the pulsar to Earth experience different angular deviations by the electron density prisms that intercept them. This, in turn, leads to different time delays relative to straight (undeflected) propagation. The signal intensity corresponding to a time delay \(\tau\) is proportional to the probability for a path to be delayed by \(\tau\) (e.g., Boldyrev & Gwinn 2005).

Assume that the number of scattering screens is \(N\) and the distance between two neighboring screens along the line of sight is \(z_0\), so that \(d = N z_0\) is the distance to the pulsar. The angle of the path in the segment between screens \(m - 1\) and \(m\) is a sum of the angular increments accumulated at the preceding segments, \(\theta_m = z_0/c (1 - \cos \theta_m) \sim (z_0/2c) \theta_m^2\), and the total time delay is the sum of the individual delays,

\[
\tau = \sum_{m=1}^{N} \Delta \tau_m = \frac{z_0}{2c} \sum_{m=1}^{N} \left[ \sum_{k=1}^{m} \Delta \theta_k \right]^2. \tag{2}
\]

In this equation the individual angular increments \(\Delta \theta_k\) are independent and identically distributed two-dimensional vectors. The intensity \(I(\tau)\) of the received signal, which is effectively the PDF of \(\tau\), can be found numerically using equation (2) once the distribution function for the vectors \(\Delta \theta_k\) is known.

In the classical theory of scintillations the vectors \(\Delta \theta_k\) are assumed to have an independent and identical Gaussian distribution, in which case an exact analytic solution for the \(\tau\)-distribution function \(I(\tau)\) can be derived (e.g., Uscinski 1974; Williamson 1972). However, the scaling and shapes of pulse profiles predicted by this theory seem to disagree with the observational results for distant pulsars, as first noted by Sutton (1971) and Williamson (1974). In a recently proposed alternative model, Boldyrev & Gwinn (2003a) demonstrated that the observed profiles can be matched if the individual fluctuations \(\Delta \theta_k\) have a non-Gaussian, power-law-like declining distribution. In this case the wave path angle \(\theta_m\) traces a so-called Lévy flight rather than the standard random walk.

The Lévy distribution function is a general distribution function whose convolution with itself produces the same function again (appropriately rescaled). The Gaussian distribution is a particular case of a Lévy distribution. A detailed discussion of Lévy distributions can be found in Klafter et al. (1995); the application to wave propagation in the interstellar medium is considered in Boldyrev & Gwinn (2003a, 2003b, 2005). For our present purposes, the main relevant property of the Lévy distribution is its slow (power law) asymptotic decrease. Whereas \(\theta_0^2 \propto m\) for a Gaussian random walk, the scaling produced by a Lévy flight is \(\theta_0^2 \propto m^{2/\beta}\), where \(\beta\) (which satisfies \(0 < \beta \leq 2\)) is the parameter of the Lévy distribution. For \(\beta < 2\), the Lévy PDF exhibits a power-law decrease, \(P_\beta(\Delta \theta) \propto |\Delta \theta|^{-1-\beta}\) for \(|\Delta \theta| \gg \Delta \theta_0\), where \(\Delta \theta_0\) is a typical angular fluctuation value.

Pulsar signals are observed to satisfy the scaling \(\tau \propto N^4\) (more precisely, the observationally inferred scaling is \(\tau \propto DM^4\), where the pulsar dispersion measure, \(DM\), is proportional to the distance to the pulsar and, therefore, to the number of scattering events, \(N\), which motivated Boldyrev & Gwinn (2003a, 2003b, 2005) to propose that radio scintillations are described by a Lévy model with \(\beta \sim 2/3\). A Gaussian distribution corresponds to \(\beta = 2\) and implies a linear scaling of \(\theta_0^2\) with \(m\), which is not consistent with the observational data. In §3 we demonstrate that the difference in the implied scaling between these two models corresponds to a profound difference in the physics of the respective scattering mechanisms.

3. LÉVY STATISTICS OF SCINTILLATIONS

We assume that the PDF of \(\Delta \theta\) has a characteristic width \(\Delta \theta_0\) (corresponding, say, to the 1/e amplitude level), and we denote the width of the resulting PDF of \(\tau\) by \(\tau_0\). Equation (2) and the scaling \(\theta_0^2 \propto m\) imply that \(\tau_0\) scales as \((\Delta \theta_0)^2 N^2\) in a Gaussian model. On the other hand, if we consider \(\tau_0\) to arise from the action of individual fluctuations, each of magnitude \(\Delta \theta_0\), we find (using eq. [2] again) that \(\tau_0 \propto (\Delta \theta_0)^{2} N^{2/\beta}\). By comparing these two expressions we obtain, as expected, \(\Delta \theta \sim \Delta \theta_0 N^{1/\beta}\), which is consistent with the notion that the typical time delay may be produced by many small deflections.

The situation is quite different in the Lévy case with \(\beta = 2/3 < 1\). Equation (2) implies \(\tau_0 \propto (\Delta \theta_0)^{2} N^{4/3}\), and if we compare this expression with our estimate in terms of individual fluctuations, \(\tau_0 \propto (\Delta \theta_0)^{5/3} N^{4}\), we infer that such fluctuations must be large, \(\Delta \theta \sim N^{1/2} \Delta \theta_0\). But such large individual fluctuations have a small probability, \(P \propto 1/(\Delta \theta_0)^{2-5}\). The situation is quite different in the Lévy case with \(\beta = 2/3 < 1\). Equation (2) implies \(\tau_0 \propto (\Delta \theta_0)^{2} N^{4/3}\), and if we compare this expression with our estimate in terms of individual fluctuations, \(\tau_0 \propto (\Delta \theta_0)^{5/3} N^{4}\), we infer that such fluctuations must be large, \(\Delta \theta \sim N^{1/2} \Delta \theta_0\). But such large individual fluctuations have a small probability, \(P \propto 1/(\Delta \theta_0)^{2-5}\). It is therefore highly unlikely that all \(N\) individual fluctuations are of that order. Instead, the most probable situation is that one of the fluctuations is extremely large, \(\Delta \theta = \Delta \theta_{\text{max}} \sim N^{3/2} \Delta \theta_0\), whereas all the others are small (\(\lesssim \Delta \theta_0\)). Thus, in particular realization, the total scattering is most probably produced by only one, randomly chosen, screen. This behavior is a general property of sums of Lévy-distributed variables with \(\beta < 1\) (Feller 1971). When the pulse shape is averaged over a certain time interval (or over a statistical ensemble), these screens can be different in different realizations, since the interstellar medium fluctuates and both the pulsar and the Earth move through it.

We can now estimate the width of the pulse as \(\tau_0 \sim z_0 N (\Delta \theta_{\text{max}})^2/c \sim d (\Delta \theta_{\text{max}})^2/c\), where \(d = N z_0\) is again the pulsar distance. This expression has a geometric meaning, namely, that only one refractive event dominates the path deviation from a straight line. For the typical parameters of distant pulsars (e.g., Bhat et al. 2004), \(d \sim 3 \times 10^{22}\) cm, \(\tau_0 \sim 0.01\) s, and \(\lambda \sim 30\) cm (corresponding to a frequency of 1 GHz), we obtain \(\Delta \theta_{\text{max}} \approx 10^{-7}\) rad and a density fluctuation scale \(y \sim \lambda/\Delta \theta_{\text{max}} \approx 3 \times 10^8\) cm. In view of the scaling \(\tau \propto d^4\), closer pulsars (characterized by \(d \sim 3 \times 10^{21}\) cm and \(\tau_0 \sim 10^{-8}\) s) correspond to \(\Delta \theta_{\text{max}} \approx 3 \times 10^{-9}\) rad and \(y \approx 10^{10}\) cm.

What regions in the interstellar medium might be responsible for the indicated strong-scattering events? To address this question, we first consider whether the maximal angular deviation estimated above could be produced by regions of uniform and homogeneous turbulence. In the following discussion we therefore neglect the contribution of the boundaries of the turbulent regions or of any other structures.

The amplitude of the density fluctuations that give rise to the large inferred angular deviations can be estimated from the results presented after equation (1), which imply

\[
\Delta N(y)/y \sim \Delta \theta_{\text{max}}/(\lambda^2 \tau_0) \sim 10^3\ \text{cm}^{-3}.\tag{3}
\]
This defines the value that the density fluctuations must be able to reach on the scale $r$. Suppose that the density fluctuations are associated with a turbulent cascade. If the density is passively advected by the fluid turbulence (e.g., Lithwick & Goldreich 2001), then the density scaling follows that of the velocity field, and for a spatially homogeneous turbulence with an outer scale $l_{\text{out}}$ we have $\Delta n(y)/\Delta n_0 \sim \delta x(l)/\Delta l_0 \sim (l/l_{\text{out}})^{\delta/2}$. Here $\Delta l_0$ and $\Delta n_0$ are, respectively, the rms values of the velocity and density fluctuations on the scale $l_{\text{out}}$ and $\alpha$ is the turbulence scaling exponent, which ranges between 2/3 (the Kolmogorov case) and 2. This estimate neglects damping effects, which can be different for $\delta \dot{x}$ and $\delta \dot{n}$; we discuss damping further on in this section.

As we pointed out in §1, the distribution function of density fluctuation amplitudes in Kolmogorov turbulence declines faster than a power law. The central limit theorem then implies that the distribution of line-of-sight integrated density fluctuations is Gaussian. Under these conditions, the $y$-dependence of the integrated density difference is

$$\Delta N(y) \sim \Delta N(0) (y/l_{\text{out}})^{\delta/2},$$  

where $\delta = (1 + \alpha)/2$ for $\alpha < 1$ and $\delta = 1$ for $\alpha \geq 1$. Equation (4) follows directly from the expression for the correlator $\langle (\Delta N(y))^2 \rangle$ in a homogeneous and isotropic turbulence. Note that this correlator is proportional to the integration distance $l_{\text{out}}$, as expected for Gaussian fluctuations.

In the standard picture of scintillations (the first scenario discussed in §1), turbulent HII regions around bright stars are considered to be the likely sites of radio-wave scattering. In this case the outer scale of the turbulence, $l_{\text{out}} \sim 10^{14}$ cm, is of the order of the size $l_0$ of the Strömgren sphere. Setting $l_0 = l_{\text{out}}$ in equation (4) and comparing with equation (3), we obtain the following condition on the large-scale density fluctuations:

$$\Delta n_0 \sim 10^3 (y/l_{\text{out}})^{(1-\alpha)/2} \text{ cm}^{-3}. \quad (5)$$

Taking the velocity difference at $l_{\text{out}}$ to be of the order of the sound speed in the ionized region ($C_2 \sim 10^8$ cm s$^{-1}$) and assuming (conservatively) that the turbulence in the H II region indeed obeys a Kolmogorov scaling ($\alpha = 2/3$) and that $y \sim 10^8$ cm, we find that equation (5) is satisfied on this scale for electron density fluctuations $\Delta n_0 \gtrsim 30$ cm$^{-3}$. Although this value is consistent with the typical densities of molecular clouds, in reality the required value of $\Delta n_0$ should be much larger (and therefore implausible) in view of the fact that the compressible and entropy MHD modes associated with the density fluctuations are damped when the wave cascade passes through the cooling scale (see Lithwick & Goldreich 2001), which is about $10^{14}$ cm in this case (see §1).

The cooling length constraint limits the ability of the standard picture to explain the strong, small-scale density fluctuations that are required in the Lévy flight scenario. Furthermore, as discussed in §1, the distribution of passive scalar fluctuations in the incompressible MHD model is not power law—like. These two difficulties suggest that non-Gaussian scattering cannot be explained within the standard framework. In §4 we demonstrate that such scattering may be explained if it is produced in thin shells—the ionized boundaries of molecular regions (clouds).

4. NON-GAUSSIAN SCATTERING BY IONIZED BOUNDARIES OF MOLECULAR CLOUDS

A typical value for the photon flux of ionizing interstellar radiation ($\nu > 13.6$ eV) through a unit surface can be estimated by converting the standard Habing flux (Habing 1968) of dissociating photons ($6$ eV $< \nu < 13.6$ eV) using a blackbody spectrum of temperature $T \sim 3 \times 10^4$ K (see Yusef-Zadeh et al. 1994). This gives $J_0 \sim 2 \times 10^7$ cm$^{-2}$ s$^{-1}$. In the vicinity of a bright star the ionizing flux can be several orders of magnitude higher. For example, at a distance of 1 pc from a star emitting $S = 10^{49}$ ionizing photons s$^{-1}$, this flux is $J \sim 10^{14}$ cm$^{-2}$ s$^{-1}$.

The mean free path of an ionizing photon that penetrates a neutral medium of density $n_0$ is $\lambda_{ph} \sim 1/(\alpha_0 n_0)$, where $\alpha_0 = 6.8 \times 10^{-18}$ cm$^2$ is the hydrogen ionization cross section at the threshold energy $\nu = 13.6$ eV (e.g., Spitzer 1978).

When a homogeneous molecular cloud is irradiated by an ionizing photon flux $J$ it develops an ionized “skin” whose width $\Delta r_i$ can be found from the ionization balance condition $J = \beta_2 n_0^2 \Delta r_i$, where $\beta_2$ is the recombination coefficient ($\beta_2 = 2 \times 10^{-11}$ cm$^3$ s$^{-1}$ for $T \sim 10^4$ K; e.g., Dyson & Williams 1997). For $n_0 \sim 10^2$ cm$^{-3}$ and $J = J_0$ one finds $\Delta r_i \sim 10^4$ cm, which can be taken as the thickness of the ionized boundary. The width of the transition layer between the ionized skin and the neutral interior of the cloud is of the order of $\lambda_{ph} \approx 10^{15}$ cm for the adopted value of $n_0$.

We now demonstrate that thin scattering shells (representing the ionized boundaries of molecular clouds) can produce non-Gaussian scintillations even if they are not turbulent. In particular, we show that scattering by thin curved layers (which we represent for simplicity as thin spherical shells) produces a power-law distribution of the scattering angles. Most remarkably, the derived power-law index coincides with the value originally inferred by Boldyrev & Gwinn (2003a) from an interpretation of the scintillation data in terms of the Lévy theory (see §§2 and 3). The agreement with the observations suggests that a shell-like structure is a general feature of the ionized scattering regions in the interstellar medium.

To derive these results, we assume that each of the electron-scattering regions has the form of a thin spherical shell of radius $R$ and thickness $\Delta R \ll R$ (see Fig. 1). We denote by $\Delta n$ the electron density excess in the shell relative to its surroundings.
and employ geometric optics to calculate the angular deviation $\Delta \theta$ experienced by a ray that intercepts the shell at a distance $b$ from the center. For a typical density excess $\Delta n \sim 10^2 \, \text{cm}^{-3}$ this deviation is very small, $\Delta \theta \ll 1$, so the trajectory of the ray through the scattering region can be approximated by a straight line.

Scattering by a shell may be viewed as a superposition of scatterings by two spheres of radii $R$ and $R - \Delta R$, respectively. The total angular deviation produced by a sphere of radius $r$ is readily obtained using Snell’s law and is given as a function of the impact parameter $b$ by $\theta _i (b) = \Delta \theta_2 (b/R) (1 - b^2 / R^2)^{-1/2}$. In this expression $\Delta \theta_2 = l^2 r_0 \Delta n / \pi$, where $l$ and $r_0$ are again the wavelength and the classical electron radius, respectively.

The angular deviation produced by the shell is $\Delta \theta (b) = \theta (b) - \theta _i - \Delta \theta_2 (b)$. It is given, up to first order in the small parameter $\Delta R/R$, by $\Delta \theta (b) = \Delta \theta_2 (b R / R^2) (1 - b^2 / R^2)^{-3/2}$. Large angular deviations, $\Delta \theta (b) \gg \Delta \theta_2 / R$, correspond to $b \approx R$, and in this limit we obtain

$$\Delta \theta (b) \approx \Delta \theta_2 (\Delta R / R) (2 \Delta b / R)^{-3/2},$$

where $\Delta b \equiv R - b$. When $b$ is not close to $R$, the angular deviations are small, $\Delta \theta \approx \Delta \theta_2 (\Delta R / R)$.

The probability density for the ray to pass at a distance $b$ from the center is estimated using $P(b) \, db = 2 \pi b \, db / (\pi R^2) \sim (2 / R) b \, db$, where we again assume $b \approx R$. The corresponding PDF of the deviation angle $\Delta \theta$ can be found using equation (6), which gives

$$P(\Delta \theta) \sim (2/3) (\Delta \theta_2 R / \Delta R)^{2/3} (\Delta \theta)^{-5/3}. \tag{7}$$

This expression describes the asymptotic behavior of the PDF for $\Delta \theta \gg \Delta \theta_2 R / \Delta R$. The derived exponent ($-5/3$) of the power-law decline of $P(\Delta \theta)$ coincides with the exponent predicted in the Lévy model (see § 2). Overall, the PDF $P(\Delta \theta)$ has a typical width of order $\Delta \theta_2 / \Delta \theta_2 (\Delta R / R)$ (corresponding to impact parameters $b$ not close to $R$). At larger angular deviations the PDF develops the power-law shape obtained above, which extends up to the cutoff value $\Delta \theta_2 / \Delta \theta_2 (\Delta R / R)^{1/2}$ (attained when $\Delta b$ decreases below $\Delta R$). For even larger angular deviations (i.e., for $\Delta b \ll \Delta R$) the scattering is produced only by the outer sphere in Figure 1 and the PDF exhibits a fast decline, $P(\Delta \theta) \propto (\Delta \theta)^{-3}$. This PDF is sketched in Figure 2.

For our fiducial parameter values $\Delta \theta_2 \sim 10^{-8} \, \text{rad}$ and $\Delta \theta_2 \sim 10^{-7} \, \text{rad}$ (see § 3). Based on equation (6), the inferred value of the maximal angular deviation is attained when $(\Delta R / R) (2 \Delta b / R)^{3/2} \sim \Delta \theta_{\text{max}} / \Delta \theta_2 \sim 10$. Given that $\Delta b$ cannot be smaller than the shell thickness (i.e., $\Delta b \approx \Delta R$), the shell thickness must satisfy $\Delta R / R < 10^{-2}$. If the shell’s radius of curvature is $R \sim 10 \, \text{pc}$, we have $\Delta R < 3 \times 10^{16} \, \text{cm}$, which is in excellent agreement with our estimate of $\Delta R$ in typical molecular clouds. We note, however, that to calculate the total probability distribution function of the angular deviation $\Delta \theta$ one needs to convolve the distribution $P(\Delta \theta)$ (which was obtained for a boundary of given curvature and density) with the curvature radii and density distributions of real clouds. This could in principle change the value of the coefficient $(\Delta \theta / \Delta R / R)^{2/3}$ in equation (7), as well as the range of angular deviations over which the derived power-law asymptotic applies. The curvature radii of a cloud’s boundary are typically smaller than the cloud size, so we may use the cloud size as an upper bound on the curvature $R$. Now, $\Delta \theta / \Delta R / R$ scales approximately as $1 / (\rho R)$. To the extent that the column density of molecular clouds is approximately a constant (e.g., Larson 1981; McKee 1989), the coefficient in equation (7) would change little from cloud to cloud. In a similar vein, the cutoff angular deviation $\Delta \theta_2$ scales approximately as $R^{-3/2}$. Therefore, our choice of fiducial cloud radius ($R = 10 \, \text{pc}$), which lies near the upper end of the observed distribution (e.g., Spitzer 1978), is conservative in that it leads to a lower bound on $\Delta \theta_2$. All in all, we expect that integrating the expression (7) over a cloud distribution would have little effect on the derived asymptotic form of $P(\Delta \theta)$ and, in particular, would not change the inferred value of its power-law exponent.

It is interesting to consider some observational implications of the proposed model. As we established in this section, $\Delta R / R < (\Delta \theta_2 / \Delta \theta_{\text{max}})^{2/3}$. On the other hand, from § 3 we know that when the scattering angles have a Lévy distribution with $\beta = 2/3$, the number of scattering structures along the line of sight scales with the maximal deflection angle as $N^{3/2} \sim \Delta \theta_{\text{max}} / \Delta \theta_2 \sim (\Delta \theta_{\text{max}} / \Delta \theta_2) (R / \Delta R)$. Therefore, our considerations suggest that the number of scattering structures out to the characteristic distance $d \sim 3 \times 10^{22} \, \text{cm}$ can be estimated as $N > (\Delta \theta_{\text{max}} / \Delta \theta_2)^2 \sim 100$, or about one scattering structure per 100 pc. This estimate is consistent with the overlap radius of radiatively cooled supernova shells (within which dense clouds are envisioned to form) in the McKee & Ostriker (1977) model of the interstellar medium. The estimated number of scattering regions could in principle be smaller (resulting in a weaker constraint on their separation), since several independent scattering structures (ionized boundaries) might in practice form inside a single scattering region.

The physical basis for the statistical properties of the thin, curved scattering structures in the proposed interpretation is their intermittent spatial distribution. Specifically, the free electrons responsible for the scattering are envisioned to occupy only a small fraction of the interstellar medium—namely, the ionized boundaries of molecular clouds. Furthermore, the molecular clouds themselves are sparsely distributed in space, so the effects of two different clouds on wave propagation are uncorrelated. Thus, a radio signal from a distant pulsar does not experience significant scattering over most of its propagation distance to Earth. However, when its trajectory intersects a curved, ionized cloud boundary, the scattering may be strong. As discussed in § 3, this behavior is characteristic of Lévy flight wave paths. The spatial intermittency in this scenario is distinct from the intermittent nature of turbulence that underlies the deviation angle statistics in the traditional interpretation of interstellar scintillations. In contrast with the traditional picture, the cumulative effect of many independent scatterings by different thin shells is not Gaussian (which can be understood formally from the fact that the central limit theorem does not apply to slowly declining distributions); it is instead a Lévy flight with $\beta \sim 2/3$, as implied by the observations (see § 2).
The foregoing derivation of the statistical properties of scattering by thin shells is based exclusively on geometric considerations. If, however, the shells are turbulent, then the effects of turbulence need to be considered in conjunction with those of geometry. Turbulence in the ionized boundary of a molecular cloud can be a natural consequence of the turbulent structure of the cloud itself or of the surrounding ionized medium. It is, however, difficult to construct a detailed model of how the turbulence is excited in the cloud or penetrates into it, since the theory of compressible turbulence is still in its infancy. However, the spectrum of a homogeneous, compressible turbulence has been investigated in some detail and found to be close to the Kolmogorov one (Boldyrev 2002; Boldyrev et al. 2002a, 2002b). This should suffice for our qualitative estimate.

We denote the outer scale of such a preexisting turbulence by $R_{\text{out}}$. This value should not be confused with the outer scale of turbulence excited inside expanding H II regions, which we analyzed in § 3. We assume that the density fluctuations on this scale are $\delta n_{\text{out}} \sim n_0 \times 10^2 \text{ cm}^{-3}$ (e.g., Larson 1981). To simplify the analysis, we neglect the interplay of turbulent fluctuations with the boundaries of the ionized shells. We consider the contribution of $N$ slabs of homogeneous turbulence, each of width $\Delta R$, that lie along the line of sight. The integrated density fluctuations are given by an expression analogous to equation (4), $\Delta N(y) = n_0(y/R_{\text{out}})^{1+\alpha}R_{\text{out}}^2(N\Delta R)^{1/2}$. When the interstellar ionizing radiation is not very strong ($J \sim J_0$, the boundaries are not fully ionized, and ion-neutral collisions are important. One can assume that turbulent fluctuations are smoothed out below the collisional cutoff scale $R_{\text{mb}} \sim 1.5 \times 10^{15} n_0^{-1/2} \text{ cm}$, which is assumed to be dominated by collisions with helium atoms (e.g., Lithwick & Goldreich 2001). This estimate indicates that we can safely assume that the turbulence is predominantly in the inertial range (with $\alpha \approx 2/3$) on the scales of interest to us.

Observations have revealed that the scaling of pulse broadening with dispersion measures changes from $\tau \propto \text{DM}^3$ for distant pulsars to $\tau \propto \text{DM}^2$ for nearby ones. The transition between the two scaling behaviors occurs at DM $\sim 30 \text{ pc cm}^{-3}$ (see, e.g., Fig. 4 in Bhat et al. 2004) and appears as an “elbow” in the $\tau$ versus DM diagram. This dispersion measure corresponds to a pulsar distance $d \sim 1 \text{ kpc}$, which, by the arguments given in § 3, implies a density fluctuation scale $\gamma \sim 10^{10} \text{ cm}$. We propose that this change in the scaling behavior is a reflection of the fact that, for nearby pulsars, the scattering angle is small and the scattering is dominated by turbulence within the ionized cloud boundaries rather than by the shape of these boundaries. Based on the arguments presented in § 2, the correlation scale $\gamma$ of transmitted waves due to turbulent fluctuations satisfies the condition $\Delta \Phi(y) = 2\gamma R_{\text{out}} 0.5 \pi$. Setting $\lambda = 0.3 \text{ cm}$, $n_0 \sim 10^2 \text{ cm}^{-3}$, and $\gamma \sim 10^{10} \text{ cm}$, we infer a value of $R_{\text{out}} \sim 100 \text{ pc}$ for the outer scale of the turbulence. It is gratifying to note that this estimate agrees with the value commonly attributed (on both theoretical and observational grounds) to the general interstellar medium (ISM) turbulence (e.g., Armstrong et al. 1995).

In the case of a strong turbulence inside the boundaries ($J \gg J_0$) it is conceivable that the properties of the turbulence itself may play a key role in establishing the non-Gaussian nature of the scattering for distant pulsars (the first scenario outlined in § 1). In § 5 we consider the possibility that molecular clouds that are situated in the vicinity of a bright star or near the Galactic center (so that $J \gg J_0$) may naturally develop turbulence that is strong and non-Gaussian on small scales. We caution, however, that the properties of the envisioned turbulence are not yet well established. Furthermore, to fully analyze this case, one needs to specify how the turbulence is generated and how it interacts with the boundaries of the ionized layer. The answer to both of these questions may depend on the particular mechanism of cloud formation, which may not be universal and which, in any case, is beyond the scope of the present discussion. Therefore, in contrast to the robust and general conclusions that we reached in this section on the basis of purely geometric arguments, the results we obtain in § 5 are at this point still tentative.

### 5. The Onset of Turbulence in Strongly Ionized Cloud Boundaries

We consider only the extreme case, when a bright star is turned on very close to (or, alternatively, inside) a molecular cloud. In this case the surrounding gas gets ionized up to the Strömgren radius, $R_S$, which can be evaluated from the ionization balance condition $S = (4/3)\pi R_S^2 n_2^2/\beta_2$ (where, again, $n_2$ is the density of ionized gas, $\beta_2$ is the recombination coefficient, and spherical symmetry is assumed for simplicity). During the initial phase of fast ionization the gas density does not change ($n_2 \sim n_0$), and one obtains $R_S \sim 2 \times 10^{20} n_0^{-2/3} \text{ cm}$ for $S = 10^{49} \text{ s}^{-1}$, which for $n_0 \sim 10^2 \text{ cm}^{-3}$ yields $R_S \sim 10^{19} \text{ cm}$. The duration of the fast-ionization phase is $\sim 10^3 - 10^4 \text{ yr}$ from the turn-on time of the ionization source (e.g., Spitzer 1978; Dyson & Williams 1997). The pressure imbalance between the gas inside the Strömgren sphere (which is rapidly heated to $T \sim 8000 \text{ K}$) and the surrounding cold molecular gas ($T \sim 30 \text{ K}$) then leads to an expansion of the ionized region, which proceeds at approximately the sonic speed $C_2 \sim 10^6 \text{ cm s}^{-1}$ (this “intermediate” or sonic expansion) phase lasts for $\sim 10^6 - 10^7 \text{ yr}$, until pressure balance with the ambient gas is restored at the final phase of the H II region’s evolution.

As the lifetime of the ionizing star is typically $\sim 10^6 \text{ yr}$, only the intermediate phase is relevant in practice. This phase nominally starts when the ionizing flux that reaches the boundary of the H II region drops below the critical value $2n_0 C_2$. After this time a hydrodynamic shock forms ahead of the ionization front (IF). This shock compresses the neutral gas to a density $n_1$ such that a weak D-type IF is set up (e.g., Spitzer 1978). In this configuration (shown schematically in Fig. 3) the speed of the IF with respect to the compressed gas is smaller than the D-critical speed $V_D = C_{D,1}^2 C_2$, where $C_{D,1} \sim 3 \times 10^4 \text{ cm s}^{-1}$ is the sound speed in the neutral gas. This is equivalent to the condition $J / n_1 \leq V_D$, which implies that the neutral gas is compressed to $n_1 \geq JC_{D,1}^2 / C_2^2$, which is $\sim 10^5 \text{ cm}^{-3}$ for $J > J_0$.

In the limit when the relative speed of the IF and the compressed gas equals $V_D$, the ionized gas flows out of the front at the sound speed $C_2$. Since this is also roughly the speed at which the front propagates into the (stationary) ambient gas, one can argue that the front should always be close to the D-critical state (Kahn 1954). Qualitatively, when the ionizing flux reaching the front drops below its D-critical value, the density of the ionized gas behind the moving front drops too. Since the photon absorption is proportional to $n_2^2$, this would lead to an increase in the flux reaching the IF, which would turn it back into a D-critical front. Modeling and numerical simulations confirm that the speed of the ionized gas leaving the IF is indeed near sonic, or possibly smaller by a factor of a few (Dyson & Williams 1997; Williams 2002).

Weak or critical D-type IFs are subject to an instability with a linear growth rate $\sim k C_1$ (for perturbations with wavenumber $k$), so the shortest modes initially have the fastest growth (Kahn 1958; Vandervoort 1962; Axford 1964; Williams 2002). The range of scales of unstable perturbations in real fronts is limited at the lower end by the front’s width $l_f \sim 1 / (\alpha_{n1})$, which is $\lesssim 10^{12} \text{ cm}$ in the case we consider, and at the upper end by the recombination length $l_{\text{rec}} \sim C_2 / (\beta_2 n_2)$, which for our fiducial parameters is
Numerical simulations by Williams (2002) have confirmed that this instability indeed develops and evolves into the nonlinear regime. It was found that an unstable IF becomes rough, with the perturbation wavelength and the perturbation amplitude saturating below the recombination length. We conjecture (following the original speculation by Axford 1964) that the IF instability would generate strong small-scale turbulence. One can, in fact, argue quite generally that, in analogy with a flow through a grid, a large Reynolds number outflow from a rough surface should be turbulent. The numerical simulations of Williams (2002) have indeed revealed the existence of large velocity shears on small scales, but the resolution of these experiments was not high enough to study the details of the indicated fine-scale structures. We note, however, that a compressible flow converging on a scale $l$ will in general create density fluctuations of the order $n^2/V^2$. For near-sonic conditions characterizing the downstream IF flow, the density fluctuations could thus be close to (within a factor of a few of) $n^2$. A schematic picture of the envisioned turbulence is shown in Figure 3.

We propose that the turbulence likely excited behind IFs that propagate into strongly ionized clouds could generate the small-scale density perturbations inferred in the Lévy model of interstellar scintillations. A strong ionizing flux gives rise to a D-type front that produces a significant compression of the upstream neutral gas, and this in turn leads to a sufficiently large value of the ratio $l_{rec}/l_{if} \propto n_1/n_2$ for the turbulence to develop. (As we estimated in § 4, the ratio of the thickness of the ionized skin of a molecular cloud to the mean free path of an ionizing photon is only $\leq 10$ in the case of the typical interstellar flux $J_0$.) The estimated outer scale of this turbulence, $l_{out} \sim 10^{12}-10^{16}$ cm, is much smaller than the $\sim 10$ pc scale that characterizes the turbulent H II regions invoked in the standard interpretation of interstellar scintillations (see § 3). However, if (as can be plausibly expected) the magnitude of the velocity fluctuations on the outer scale is $\Delta V \leq C_2$ also in this case, then the perturbations should be much stronger on the scales $\sim 10^8-10^{10}$ cm of interest. Furthermore, in the present interpretation the turnover time of the largest eddies ($t_{out} \sim 10^6-10^{10}$ s) is typically shorter than the cooling time $t_{cool}$, and since the ratio of the turnover time to the cooling time decreases with the eddy scale, the difficulty associated with the expected damping of the compressible and entropy modes when these two times become comparable (see § 1) would be avoided.

To illustrate the above points, we estimate the expected density fluctuation on the small scale $y_g \sim 10^8$ cm. The density fluctuations at the outer scale are $\Delta n_2 \leq 10^2$ cm$^{-3}$. Assuming a Kolmogorov scaling (as proposed in the Goldreich & Sridhar 1997 turbulent cascade model), $\delta n(y)/n \sim (y/l_{out})^{1/3}$. Adopting an outer scale of $\sim 10^{12}$ cm (for which radiative damping should be unimportant), we infer $\delta n(y) \lesssim 0.1$ cm$^{-3}$. For comparison, the value obtained by adopting $l_{out} \sim 10$ pc is a factor of $\sim 30$ smaller, and, even more significantly, in the latter case the fluctuation might be strongly damped by radiative cooling.

In the proposed picture turbulence is continuously regenerated in the layer (of width $l_{rec}$) behind the IF. The energy comes from the ionizing radiation and is deposited into fluctuating streams of hot gas that leave the rough surface of the front. Without such...
regeneration, turbulence would decay on the relatively short crossing time $t_{rec}/C_2$. It is also worth noting that the outer scale of this turbulence in not the effective Strömgren radius (i.e., the thickness of the ionized cloud boundary), as is sometimes assumed (e.g., Yusef-Zadeh et al. 1994), but rather the recombination length in the ionized gas, which is much shorter.

To account for the inferred non-Gaussian nature of the density fluctuations, we make a connection with the fact that in the Lévy flight interpretation the fluctuations responsible for the observed radio pulse broadening span a narrow range of scales, $10^8$–$10^{10}$ cm (see § 3), which is close to the estimated scale of the proton gyroradius $\rho_i$ in the ionized regions of molecular clouds (see § 1). Now, the turbulent MHD cascade discussed above corresponds to shear Alfvén waves only on scales $\gg \rho_i$. On scales that approach the ion gyroradius the Alfvén waves become kinetic and develop an electric field component that is parallel to the magnetic field. The electrons, as the most mobile particles, rapidly respond to this field by moving along the magnetic field lines in the direction of increasing electric potential. This provides a mechanism for coupling density fluctuations in the plasma to the cascade of shear Alfvén waves. One can show that the coupling is strongest for transverse wavenumbers $k_\perp$ satisfying $k_\perp \rho_i \sim 1$ and that the Fourier amplitude of the induced density fluctuations may be estimated from $\delta n_i/n \sim \phi_0 / k_\perp T \sim (k_\perp \rho_i) \delta B_\perp / B_0$, where $\phi$ is the fluctuating electric potential, $T$ is the temperature, and $k_B$ is Boltzmann’s constant (e.g., Camargo et al. 1996).

The above coupling mechanism provides yet another means of accounting for the inferred presence of strong density fluctuations on small scales. Even more significant, however, is the fact that, as the ion gyroradius scale is approached, the turbulence itself gets modified. This happens because in the presence of relatively strong electron density fluctuations and of the corresponding perpendicular electric field (see the above estimate of $\delta n_i/n$), the ion polarization drift velocity becomes comparable to the velocity of fluid fluctuations, and so ions can drift compressively across the magnetic field lines. This leads to the coupling of density fluctuations to the Alfvénic fluctuations; a similar effect is at work in the so-called electromagnetic drift wave (or drift Alfvén) turbulence, which has been investigated in the context of plasma fusion devices (e.g., Hasegawa & Mima 1978; Hasegawa & Wakatani 1983; Terry & Horton 1982; Hazeltine 1983; Camargo et al. 1996; Krommes 2002). Interestingly, numerical simulations of such turbulence have revealed a rather high level of density fluctuations (in equipartition with kinetic fluctuations) and statistical properties that are clearly non-Gaussian (e.g., Terry et al. 2001; Graddock et al. 1991).1

On the basis of the foregoing results one could plausibly attribute the generation of strong, non-Gaussian density fluctuations in turbulent, ionized molecular gas to the onset of compressive effects in a shear Alfvén wave cascade that approaches the proton gyroradius scale. (As we discussed at the beginning of this section, IF instability at the boundaries of molecular clouds provides a likely mechanism for generating such turbulence that might by itself produce strong fluctuations on small scales.) More definitive statements must, however, await further studies of this turbulence in an explicitly astrophysical context. In the meantime one could perhaps look for a signature of the compressive effects in the density spectrum inferred from the pulsar scintillations data on scales $\sim 10^8$–$10^{10}$ cm. A likely signature would be a flattening of the spectrum on these scales: a flattening of this type was observed in the electron density spectrum of the solar wind (Coles & Harmon 1989), where it was, in fact, attributed to kinetic Alfvén wave effects (Hollweg 1999).

6. CONCLUSION

To conclude our discussion, we recapitulate the main ideas and results of this work. We have been motivated by a recent comparison of radio signals from distant pulsars with an analytical model (Boldyrev & Gwinn 2003a), from which a non-Gaussian power-law distribution of wave-scattering angles was inferred (corresponding to Lévy statistics). It was also deduced that the density fluctuations responsible for this scattering should be rather strong on small scales ($\sim 10^8$–$10^{10}$ cm; see § 3). These conclusions differ from the predictions of the classical theory of scintillations, which are based on Gaussian statistics.

The inferred non-Gaussian behavior of the scattering process indicates that the scattering medium has a different structure than the one envisioned in the standard model, where the angular deviations are attributed to the cumulative effect of a large number of weak-scattering events along the line of sight. In contrast, we propose in this paper that the observed scintillations are dominated by rare scattering events in spatially intermittent density structures. We identify these structures with thin and curved layers of ionized gas, which could plausibly be the ionized boundaries of molecular clouds. In this picture, a ray propagating from a distant pulsar can experience significant scattering only when its trajectory intersects such a boundary. The angular deviation induced during such an encounter can be rather large, so a single scattering event may dominate the total angular deflection. The trajectory of the ray therefore has the character of a Lévy flight.

For a simple calculation, we modeled the ionized cloud boundaries as thin spherical shells. We found that the scattering angle probability density function predicted by this model has the asymptotic form $P(\Delta \theta) \propto (\Delta \theta)^{-5/3}$ for large angular deviations (see § 4). Remarkably, this is precisely the form predicted by Boldyrev & Gwinn (2003a) on the basis of a comparison of the Lévy model with observations. Our analysis therefore strongly suggests that the scattering structures in the interstellar medium are likely to possess a shell-like morphology.

The spatially intermittent nature of the electron density distribution is an important ingredient of our theory. So far, analytical and numerical investigations of interstellar turbulence were mostly restricted to homogeneous settings with periodic (or otherwise simplified) boundary conditions. However, the actual interstellar electron density distribution is not homogeneous, and geometric boundaries of scattering regions can play a dominant role in the line-of-sight integrated density fluctuations. Such boundary effects have been observed in aero-optical experiments, in which an optical beam propagated through a confined turbulent region (e.g., Dimotakis et al. 2001).

The distant-pulsar relationship $\tau \propto DM^2$ between the pulse time delay and the pulsar dispersion measure, for which the above model provides a natural explanation, is observed to change in nearby pulsars to $\tau \propto DM^4$, the expected scaling for Gaussian turbulence. We have interpreted this change as a consequence of the fact that, for nearby pulsars, the scattering is dominated by turbulence within the ionized cloud boundaries rather than by the shape of these boundaries. We have found that the measured value of the transition DM ($\sim 30$ pc cm$^{-3}$) is consistent with the inferred value ($\sim 10^{20}$ cm) of the outer scale of the general ISM turbulence.

Although, as we just summarized, the non-Gaussian, Lévy-type statistics of scattering angles may have a purely geometric origin, we also discussed (see § 5) the situation in which the

---

1 We are grateful to Paul Terry for drawing our attention to these numerical results.
scattering regions are strongly turbulent, so that the contribution from turbulent density fluctuations dominates the scattering. This may happen when a molecular cloud boundary is ionized by radiation from nearby stars or by strong interstellar radiation in the vicinity of the Galactic center. In this case a near-critical D-type ionization front develops and propagates into the cloud.

Ionization fronts of this type are known to be linearly unstable. In its nonlinear phase, this instability could lead to a strong turbulence on scales below the recombination length $l_{\text{rec}} \sim 10^{16}$ cm and above the ionization front thickness $l_{\text{if}} \sim 10^{12}$ cm (where the numerical values correspond to a cloud of density $\sim 10^2$ cm$^{-3}$). The outer scale of this turbulence is $\sim l_{\text{rec}}$, so that the density fluctuations could be strong enough on small scales to produce the inferred scattering. We contrast this situation with the standard theory, where the outer scale of turbulence is of the order of the Strömgren radius of an H II region ($R_S \sim 10^{19}$ cm). In this case the density is passively advected by the Alfvénic cascade from the large outer scale to the small dissipative one. This cascade has to pass through the radiative cooling scale ($\sim 10^{14}$ cm), where the density fluctuations could be significantly attenuated, so by the time it reaches the much smaller scales implicated in the scattering process the turbulence might be too weak to account for the observed scintillations. In the case of a turbulent ionization front the outer scale of turbulence is smaller than the cooling scale and this problem is avoided.

We also noted that the small scales implied by the non-Gaussian interpretation happen to be close to the ion gyroscale in an ionized molecular cloud. The turbulent cascade would become compressible on these scales because of kinetic effects, with the density fluctuations becoming coupled to the Alfvénic ones. This is expected to produce a flattening of the density spectrum in this range, providing an alternative means of accounting for the relative strength of the fluctuations on small scales. Even more tantalizingly for our contemplated application, there have been indications from numerical simulations for a clearly non-Gaussian behavior of the fluctuations in this case. However, as neither the nonlinear development of the ionization front instability nor the behavior of kinetic Alfvén turbulence in astrophysical contexts have yet been studied in detail, our discussion of the properties of turbulent scattering layers must be regarded as tentative at this stage.

We are grateful to Steven Cowley, William Dorland, Jeremy Goodman, Carl Gwinn, John Krommes, Christopher McKee, Barney Rickett, Paul Terry, Farhad Yusef-Zadeh, and the referee, Anthony Minter, for useful comments and discussions. S. B. acknowledges the hospitality of the Aspen Center for Physics, where part of this work was done. A. K. similarly acknowledges the hospitality of the Kavli Institute for Theoretical Physics at Santa Barbara (where partial support under NSF grant PHY-99-07949 was provided). The work of S. B. was supported by the NSF Center for Magnetic Self-Organization in Laboratory and Astrophysical Plasmas at the University of Chicago. A. K. was supported in part by a NASA Astrophysics Theory Program grant NNG04G178G.

REFERENCES

Armstrong, J. W., Rickett, B. J., & Spangler, S. R. 1995, ApJ, 443, 209
Axford, W. I. 1964, ApJ, 140, 112
Bhat, N. D. R., Cordes, J. M., Camilo, F., Nice, D. J., & Lorimer, D. R. 2004, ApJ, 605, 759
Boldyrev, S. 2002, ApJ, 569, 841
Boldyrev, S., & Gwinn, C. R. 2003a, ApJ, 584, 791
———. 2003b, Phys. Rev. Lett., 91, 131101
———. 2005, ApJ, 624, 213
Boldyrev, S., Nordlund, A., & Padoan, P. 2002a, ApJ, 573, 678
———. 2002b, Phys. Rev. Lett., 89, 031102
Camargo, S. J., Scott, B. D., & Biskamp, D. 1996, Phys. Plasmas, 3, 3912
Coles, W. A., & Harmon, J. K. 1989, ApJ, 337, 1023
Cordes, J. M., & Lazio, T. J. W. 2001, ApJ, 549, 997
Cordes, J. M., Weisberg, J. M., & Boriakoff, V. 1985, ApJ, 288, 221
Dimotakis, P. E., Cattrakis, H. J., & Forgueutte, D. C. 2001, J. Fluid Mech., 433, 105
Dyson, J. E., & Williams, D. A., eds. 1997, The Physics of the Interstellar Medium (2nd ed.; New York: Inst. Phys.)
Feller, W. 1971, An Introduction to Probability Theory and Its Applications (3rd ed.; New York: Wiley)
Goldreich, P., & Sridhar, S. 1997, ApJ, 485, 680
Gradiscak, G. G., Diamond, P. H., & Terry, P. W. 1991, Phys. Fluids B, 3, 304
Habing, H. J. 1968, Bull. Astron. Inst. Netherlands, 19, 421
Hasegawa, A., & Mima, K. 1978, Phys. Fluids, 21, 87
Hasegawa, A., & Wakatani, M. 1983, Phys. Fluids, 26, 2770
Hazeltine, R. D. 1983, Phys. Fluids, 26, 3242
Higdon, J. C. 1984, ApJ, 285, 109
Hill, A. S., Stonebring, D. R., Asplund, C. T., Berwick, D. E., Everett, W. B., & Hinkel, N. R. 2005, ApJ, 619, L171
Hollweg, J. V. 1999, J. Geophys. Res., 104, 14811
Kahn, F. D. 1954, Bull. Astron. Inst. Netherlands, 12, 187
Kahn, F. D. 1958, Rev. Mod. Phys., 30, 1058
Klafter, J., Zumofen, G., & Shlesinger, M. F. 1995, in Lévy Flights and Related Topics in Physics, ed. M. F. Shlesinger et al. (Berlin: Springer), 196
Krommes, J. A. 2002, Phys. Rep., 360, 1
Lambert, H. C., & Rickett, B. J. 2000, ApJ, 531, 883
Larson, R. B. 1981, MNRAS, 194, 809
Lazio, T., Joseph, W., Fey, A. L., & Gaume, R. A. 2001, Ap&SS, 278, 155
Lee, L. C., & Jokipii, J. R. 1975, ApJ, 196, 695
Lithwick, Y., & Goldreich, P. 2001, ApJ, 562, 279
Löhmer, D., Mitra, D., Gupta, Y., Kramer, M., & Ahuja, A. 2004, A&A, 425, 569
McKee, C. F. 1989, ApJ, 345, 782
McKee, C. F., & Ostriker J. P. 1977, ApJ, 218, 418
Rickett, B. J. 1990, ARA&A, 28, 561
Scalo, J., & Elmegreen, B. 2004, ARA&A, 42, 275
Spitzer, L., Jr. 1978, Physical Processes in the Interstellar Medium (New York: Wiley)
Sutton, J. M. 1971, MNRAS, 155, 51
Tatarskii, V. I. 1961, Wave Propagation in a Turbulent Medium (New York: McGraw-Hill)
Terry, P. W., & Horton, W. 1982, Phys. Fluids, 25, 491
Terry, P. W., McKay, C., & Fernandez, E. 2001, Phys. Plasmas, 8, 2707
Vandervoort, P. O. 1962, ApJ, 135, 212
Warhaft, Z. 2000, Annu. Rev. Fluid Mech., 32, 203
Williams, R. J. R. 2002, MNRAS, 331, 693
Williams, J. P. 1972, MNRAS, 157, 55
———. 1974, MNRAS, 166, 499
Yusef-Zadeh, F., Cotton, W., Wardle, M., Melia, F., & Roberts, D. A. 1994, ApJ, 434, L63
Zweibel, E. G., & Heiles, C. 1997, Nature, 385, 131