Study on Constitutive Equation of Cumulative Damage of HTPB Propellant

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Abstract. In order to analyze the interaction of creep and fatigue of composite solid propellant, and provide the basis for calculation of cumulative damage and reliability analysis of the solid rocket motor during the period of storage, the constant stress loading destruction experiment, constant stress amplitude reciprocating tensile experiment and interaction experiments are carried out, then the damage cumulative equation is fitted by the least squares method and also be verified. The results show that there is a logarithmic linear relationship between the loading stress and the creep failure time and the repeated tensile failure times. The verification illustrates that the damage equation parameters can reflect the cumulative damage of propellant well, and the error is less than 10%.

1. Introduction

Some solid rocket engine is in vertical storage state for a long time. During the storage period, it not only experiences its own gravity, but also vibrates every now and then under the influence of the marine environment. The cumulative damage caused by concentrated stress and alternating load of the grain is unrecoverable. As the main component of solid engine, the grain will cause damage and even structural break under stress loading due to its viscoelastic properties. In general, tensile strength is used to represent the ultimate failure capacity of propellant, but the ultimate characteristics cannot be reached during the storage, transportation and duty of the motor. The coupling damage under long-term load becomes the primary reason of engine’s debonding, cracking and final failure.

Currently, some researchers\cite{1-4} have studied the creep characteristic of solid propellant by creep experiments under different temperature and stress level, including double base propellant and HTPB propellant, and some kind of models are used for fitting the strain-time curve. The Norton creep constitutive equation is also used for simulating the creep effect of solid motor \cite{5}. And some literature has carried out the fatigue experiment to study the damage of solid propellant, the conclusion of the interaction of creep and fatigue should be paid attention to is obtained \cite{6-8}.

Based on the actual storage environment of some solid motor, in this paper, the creep experiment, fatigue experiment and their interaction experiments of HTPB propellant derived from a certain type of solid motor were carried out in succession, then the damage equation is fitted and verified, which can provide a basis for the subsequent calculation of cumulative damage of this motor.

2. Specimen preparation and experiment analysis

2.1. Specimen preparation and determination of stress range

The experiment material is HTPB composite solid propellant used in some solid motor, and the material is made into standard dumbbell specimens. Due to the error of manual cutting, the specimens
with the same sectional area are selected as far as possible for the experiment. All experiments are carried out in constant temperature and humidity.

Before the creep and fatigue experiments, it is necessary to determine the range of the stress level, five groups of specimens are selected for uniaxial tensile experiment. The results are in good consistency and the maximum tensile strength is 1.05MPa. In order to reduce the experiment time, the stress level should not be too low. The stress range is controlled between 0.4 and 0.9MPa finally.

2.2. Creep experiment

In order to ensure the veracity of the creep data, two methods are used to carry out the creep experiment of propellant and verify each other.

2.2.1. The timer method. The specimen is hung on the fixed bracket through the mold, and farmers of different qualities are added below to adjust the stress level. In order to reduce the experiment error and obtain the relationship of stress level $\sigma$ and creep time $t_f$ accurately, 30 specimens are selected evenly from 0.4 to 0.9MPa; the creep time of each specimen will be recorded. The $t_f$ of high stress level is between tens and thousands of seconds, so it can be recorded by a stopwatch; for low stress level, the $t_f$ can range from a few hours to days or even tens of days, it need to be recorded by a timer. Different groups of experiments can be carried out simultaneously.

The specific method is as follows: when the specimen breaks, the cord connected to the specimen and the timer pulls out from the batteries, then the timer begins to work, $t_f$ can be obtained by the time difference.

The moment when creep begins is $t_c$, the initial moment of timer is $t_0$, when the specimen breaks after a period of time, the time displayed on the timer is $t_1$ at moment $t_e$, so

$$t_f = (t_e - t_0) - (t_1 - t_0)$$

(1)

2.2.2. The tensile machine method. In order to verify the accuracy of the fitting equation and obtain the creep law, the uniaxial tensile machine is used for the creep experiment under the specified stress level. Due to the working time of the tensile machine, only 0.75MPa and 0.8MPa are selected. The experiment is carried out in two steps, the stress amplitude is loaded at a certain loading rate at first, and then the load is maintained until the specimen fracture. Three groups of experiments are conducted at each stress level, and the result that the fracture time is closest to the fitting equation is final selected.

2.2.3. Results analysis. Creep time $t_f$ increases exponentially with the linear decreasing of the stress level $\sigma$, the fracture time of 0.7MPa, 0.6MPa and 0.5MPa is about 20000s, 150000s and 1000000s, the linear equation is $\sigma = -0.1124 \times \lg t + 1.1817$, as shown in Fig.1.

![Figure 1. Logarithmic linear data points and fitting relationship.](image)

The strain-time curves of the specimens at 0.75MPa and 0.8MPa are obtained by stretching machine, as shown in Fig.2.
2.3. Fatigue experiment

2.3.1. Experiment method. In order to find out the influence of stress amplitude on the fatigue damage of propellant specimens in the reciprocating tensile process, the fatigue experiments under six stress amplitude between 0.55MPa and 0.9MPa are carried out and the numbers of repeated tensile failure are recorded, the experiment load curve is shown in Fig.3. The tensile rate is 100mm/min, the corresponding tensile force is calculated by the sectional area and the given stress amplitude. After stretching to the predetermined stress, the direction of motion will be changed to zero stress and then stretch again, so as to realize the reciprocating stretching until the failure of the specimen.

2.3.2. Results analysis. The results are similar to those of creep experiment, that is, $\sigma$ is linear with $\lg N_f$. The $N_f$ of 0.8MPa, 0.7MPa and 0.6MPa is 65,381 and 2812, and the linear equation is $\sigma = -0.1282 \times \lg N + 1.0350$, as shown in Fig.4.

In order to understand the stress and strain change process in the reciprocating tensile process, the stress-strain curve and the displacement-time curve are drawn with 0.75MPa as an example, as shown in Fig.5.
Figure 5. The reciprocating tensile results of 0.75MPa.

As can be seen from Fig.5 (a), each cycle is represented by a shape of tip at both ends and drum in the middle. That is, when the stress is close to the zero, the strain changes obviously under a smaller increasing of stress. When approaching the stress amplitude point, it is to the opposite that the stress changes obviously in the smaller strain range.

As can be seen from Fig.5 (b), with the increasing in the numbers of reciprocating stretching, the overall unrecoverable strain of the specimen increases gradually, while the single unrecoverable strain decreases continuously, resulting in fatigue damage of the specimen.

3. Interaction experiment and fitting verification

3.1. Interaction damage theory
The damage model of low cycle fatigue damage and creep damage is used to analyze the interaction between fatigue and creep [9]. If \( D_f \) is fatigue damage, \( D_{cr} \) is creep damage, then the incremental expressions of the two kinds of damage are

\[
\begin{align*}
\frac{dD_f}{dN} &= f_f (\Delta P, D_f, D_{cr}) \quad \text{(2)} \\
\frac{dD_{cr}}{dt} &= f_{cr} (\sigma_{eq}, D_{cr}, D_f) 
\end{align*}
\]

Where,

\[
D_f = \sum_{i=1}^{n} \frac{N_i}{N_{fi}} 
\]

\[
D_{cr} = \sum_{i=1}^{n} \frac{t_i}{t_{fi}} 
\]

Where, \( N_i \) and \( N_{fi} \) express the number of reciprocating tensile and the number of reciprocating tensile failure under the stress amplitude \( \sigma_i \), \( t_i \) and \( t_{fi} \) express the loading time and loading failure time under the stress \( \sigma_i \), \( \sigma_{eq} \) is the equivalent stress, and \( \Delta P \) is the cumulative plastic strain.

The literature [10] and [11] describe the nonlinear interaction between creep and fatigue by introducing the fatigue-creep interaction term, as formulas (5) and (6) below.

\[
\begin{align*}
D_{cr} + A(D_{cr} D_f)^{0.5} + D_f &= 1 \\
D_{cr} + AD_{cr}^{m} D_f^{1-m} + D_f &= 1 
\end{align*}
\]

The two equations both consider the interaction; the former considers that the interaction has symmetry, while the latter considers the inhomogeneity of the two effects.
3.2. Interaction damage theory

The interactive experiment is carried out on the basis of creep experiment and fatigue experiment data. The experiment is divided into two parts, namely, creep-fatigue(C-F) experiment and fatigue-creep (F-C) experiment. The stress loading processes in the experiment are shown in Fig.6.

![Fig. 6. Interactive experiment stress loading mode.](image)

(a) C-F experiment

(b) F-C experiment

In order to reduce the experiment time, 0.7MPa is taken as the stress value of creep and fatigue experiments. The creep failure time corresponding to 0.7MPa is 19301s, and the repeated tensile failure times is 381. Due to the specimen preparation error, the stress value oscillates around 0.7MPa. During the experiment, different creep and fatigue life fractions are selected, and then another set of experiments is conducted. Three specimens are selected for each life fraction.

The data points of the two groups of experiments are listed in Fig.7. All data points in the figure are arranged in the lower left of line $D_{cr}+D_f=1$, which verifies the interaction between creep and fatigue. After a period of creep, the creep damage had been produced accelerated the fatigue failure. At the same time, after a certain number of repeated stretching, the fatigue damage had been produced accelerated the creep failure, so there is a relationship that the sum of damages is less than 1.

![Fig. 7. Interaction experiment results.](image)

(a) C-F experiment results

(b) F-C experiment results

3.3. Coupled damage equation fitting

12 groups of $D_f$ and $D_{cr}$ are obtained by the average of experiment data under each life fraction above, then the least square method is used for looking for $A$ and $m$ to establish the formula (7).

$$D' = \min \left( \frac{1}{2} \sum_{i=1}^{12} (D_{cr} + AD_{cr}^mD_f^{1-m} + D_f - 1)^2 \right)$$

The fitting equation is

$$D_{cr} + 1.3382D_{cr}^{0.5136}D_f^{0.4864} + D_f = 1$$

The correlation coefficient is 0.9485, and the 3D fitting results are shown in Fig.8.
3.4. Equation verification

The damage equation is verified by two kinds of experiment schemes. One is that the stress value is constant and the life fraction is changed. Second, both the stress value and the life fraction are changed. 8 groups of specimens are selected for verification. The verification results are shown in Table 1 and Table 2.

| $D_{cr}$ | $\sigma_c$ | $t_{fi}$ | $t_i$ | $\sigma_f$ | $N_{fi}$ | $N_i$ | $D_f$ |error%/ |
|----------|-----------|---------|------|-----------|--------|------|------|-------|
| 0.5532   | 0.699     | 18077   | 10000| 0.7       | 381    | 35   | 0.0919| 3.57   |
| 0.3191   | 0.601     | 146680  | 46800| 0.7       | 381    | 125  | 0.3281| 9.42   |
| 0.1951   | 0.599     | 152810  | 29810| 0.6       | 2812   | 1220 | 0.4339| 2.67   |
| 0.0520   | 0.523     | 724970  | 40800| 0.6       | 2812   | 2027 | 0.7208| 4.6    |

| $\sigma_f$ | $N_{fi}$ | $N_i$ | $D_f$ | $\sigma_c$ | $t_{fi}$ | $t_i$ | $D_{cr}$ |error%/ |
|------------|---------|------|------|------------|--------|------|--------|-------|
| 0.7        | 381     | 100  | 0.2625| 0.702      | 18527  | 4800 | 0.2591 | 11.81 |
| 0.7        | 381     | 200  | 0.5250| 0.602      | 143710 | 18040| 0.1255 | 0.16  |
| 0.65       | 984     | 600  | 0.6098| 0.550      | 416970 | 39800| 0.0954 | 3.04  |
| 0.6        | 2812    | 200  | 0.0711| 0.501      | 1137800| 59600| 0.5238 | 13.09 |

The average error of the 8 groups of experiments is 6.045%, which verifies the feasibility of the fitting equation. The F-C experiment has a larger fitting error when the fatigue damage is smaller, which may be caused by the cumulative damage of specimen cutting and the inaccurate measurement of specimen sectional area. The experimental data points are drawn in Fig.9.

![Figure 8. 3D fitting results.](image)

![Figure 9. Verification results of interaction experiment.](image)
4. Conclusion
From the experiments and fitting analysis, some conclusions can be obtained as follows.

The creep experiment shows that the loading stress has a logarithmic linear relationship with the creep failure time; the fatigue experiment shows that the loading stress amplitude has a logarithmic linear relationship with the number of cyclic failures; the interaction experiment of propellant shows that there is an asymmetric positive interaction between creep and fatigue, and fatigue-creeep damage can aggravate creep-fatigue failure. The interactive damage equation is obtained, and it is verified by the experiment that the error is within 10%. This equation can be used to describe the cumulative damage of solid engine during storage.

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