Stable two–brane models with bulk tachyon matter

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Abstract

We explore the possibility of constructing stable, warped two–brane models which solve the hierarchy problem, with a bulk non–canonical scalar field (tachyon matter) as the source term in the action. Among our examples are two models–one with a warp factor (denoted as $e^{-2f(\sigma)}$) which differs from that of the standard Randall–Sundrum by the addition of a quadratic piece in the $f(\sigma)$ and another, where the warping is super-exponential. We investigate the issue of resolution of hierarchy and perform a stability analysis by obtaining the effective inter-brane potentials, in each case. Our analysis reveals that there does exist stable values of the modulus consistent with hierarchy resolution in both the models. Thus, these models, in which the bulk scalar field generates the geometry and also ensures stability, provide viable alternatives to the standard Randall–Sundrum two-brane scenario.

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I. INTRODUCTION

It is almost a decade since Randall and Sundrum (RS) [1], following earlier work [2, 3, 4, 5], proposed a five dimensional model with a warped extra dimension, in order to achieve a resolution of the hierarchy problem in high energy physics and also to suggest an alternative to the largely ambiguous procedure of compactification, necessary in any theory in a spacetime with more than four dimensions. The RS two–brane model had a positive and a negative tension brane and we are supposed to live on the one with negative tension. Such negative tension branes, in general, are not stable. Moreover, it was realized that the modulus (representing the separation between the two 3–branes) in the RS model is not stable and it is easy to show that the branes would collapse onto each other due to this instability. In order to rescue this otherwise elegant model, Goldberger and Wise [6] came up with a solution using bulk fields, which rendered the model stable. The visible brane tension however continued to be negative. Different aspects of the Goldberger–Wise mechanism have been discussed in numerous articles [7, 8, 9, 10, 11, 12, 13, 14, 15]. The stability of Horava–Witten spacetimes (the work of Horava and Witten [16] is the string theoretic motivation for RS models) was discussed in [17]. The Goldberger–Wise (GW) resolution however assumed a negligible back reaction of the bulk scalar on the metric. However, it was realised that one could actually solve for the warp factor in the presence of the cosmological constant and the scalar. The back-reacted warp factor had a linear and an exponentially decaying piece and by adjusting parameters one could justify the exclusion of this term and work with the RS one. Later, it was also realised that bulk non–canonical bulk scalars (sometimes motivated from string theory) could rescue RS and provide stability. Examples along these lines were suggested and worked out in [7, 8].

In this article, we provide new examples of models with a non-canonical bulk scalar and without a cosmological constant. More precisely the bulk action is the tachyon matter (or scalar Born-Infeld) action [18]. With this assumption for the matter in the bulk, it was shown in [19] that one could construct a viable braneworld model. In Section II, we discuss our models in detail. Section III discusses the hierarchy problem and its resolution in both the models. In Section IV, we investigate the stability issue and obtain the inter-brane potentials for each model. We also make a comparison between the two models in this section. Finally, in Section V, we conclude with some remarks on future directions.
II. THE MODELS

The braneworld models we consider in this section are, generically, two brane models. However, unlike the RS1 two-brane scenario where there is only a bulk cosmological constant, we have, in our examples, bulk matter. In particular, as mentioned in the Introduction, we consider situations where the bulk matter is generated by a non-canonical action, namely that of tachyon matter. One of these models have been partially analysed in [19]. The main features of the model construction process is, as given below.

We begin with the action for the bulk scalar field given by

$$S_T = \alpha_T \int d^5\xi \sqrt{-g} V(T) \sqrt{1 + g^{MN} \partial_M T \partial_N T}$$  \hspace{1cm} (2.1)

where $\alpha_T$ is an arbitrary constant, $g_{MN}$ being the five dimensional metric. The scalar field is represented by $T$ and $V(T)$ corresponds to its potential. The constant $\alpha_T$ can take either positive or negative values.

The full action for our model is

$$S = S_G + S_T + S_B$$  \hspace{1cm} (2.2)

$$S_G = 2M^3 \int \sqrt{-g} Rd^5\xi$$  \hspace{1cm} (2.3)

$$S_B = - \int \sum_j \tau(j) \sqrt{-\tilde{g}^{(j)}d^4\xi}$$  \hspace{1cm} (2.4)

where $\tilde{g}_{\mu\nu}^{(j)}$ is the induced metric on the brane and $\tau(j)$ is the vacuum energy density (brane tension) of the $j$th brane. There are two branes in a RS-I set-up and a single brane in RS-II set-up. The $S_B$ terms incorporate the branes and the $S_G$ term takes care of five dimensional gravity. In addition, there is the matter part $S_T$. Variation of the total action w.r.t. $g_{MN}$ leads to the Einstein–scalar equations and that w.r.t. $T$ gives rise to the scalar field equation.

The five dimensional bulk metric with four dimensional Poincare invariance is taken to be of the form:

$$ds^2 = e^{-2f(\sigma)} \eta_{\mu\nu} d\xi^\mu d\xi^\nu + d\sigma^2$$  \hspace{1cm} (2.5)

Here $\sigma$ denotes the extra dimensional coordinate and $\xi$ are the coordinates on a $\sigma =$constant hypersurface. The warp factor ($f$) and the scalar field ($T$) are considered to be functions of $\sigma$ only. The extra dimension is compactified on an $S^1/Z_2$ orbifold and we assume the two branes are located at the fixed points of the orbifold, $\sigma = 0, \sigma = \pi r$. With these ansatze, the
Einstein-scalar equation without a cosmological constant reduces to the following system of coupled, nonlinear ordinary differential equations:

\[
f'' = \frac{a}{2} \frac{V(T)}{\sqrt{1 + T'^2}}
\]

\[
f'' = -aV(T) \frac{T'^2}{\sqrt{1 + T'^2}} + \frac{1}{12M^2} \sum_j \tau_j \delta(\sigma - \sigma_j)
\]

where \(a = \frac{\alpha T}{12M^3}\) and a prime denotes derivative with respect to the coordinate \(\sigma\). We choose \(\alpha T = 1\) henceforth. In general, we do not expect to obtain a general solution of the above equations. However, we will discuss two special solutions of these equations below. A look at the equation will immediately suggest that, in the bulk (i.e. excluding the brane terms) the relation \(f' \cdot f'' = -2T'^2\) holds. Thus, postulating \(T\) one can obtain \(f\). Further, we can multiply the LHS and RHS of the two equations and obtain an expression for \([V(T)]^2\), which depends on \(f, T\) and their derivatives. We follow this approach while obtaining solutions. Note however that a \(f(\sigma) \sim |\sigma|\) would result in a constant \(T\) in the bulk (away from the branes), since \(f''\) would be equal to zero everywhere in the bulk. This will imply a triviality because the tachyon matter action would then become equivalent to having a cosmological constant contribution in the bulk.

In addition to the bulk solution, one also needs to look at the equation of motion for the scalar field. This, for bulk tachyon matter, turns out to be:

\[
\frac{V(T)}{(1 + T'^2)} T'' + 4 \frac{f'V(T)}{\sqrt{1 + T'^2}} T' - \sqrt{1 + T'^2} \frac{\partial V}{\partial T} + \sum_i \frac{\partial \tau_i}{\partial T} \delta(\sigma - \sigma_i) = 0
\]

Integrating this equation once would give the boundary (jump) conditions on \(T'\), given as

\[
[T']_i = -\left(\frac{1 + T'^2}{V(T)}\right)_i \left\{ \frac{\partial \tau_i}{\partial T} \right\}_i
\]

where \(i\) denotes the location of the \(i\)-th brane.

**A. Model 1:**

It is possible to construct exact solutions of the above equations and boundary conditions. For example, one may choose:

\[
f(\sigma) = k_1|\sigma| - k|\sigma| \left( \frac{|\sigma|}{|\sigma_0|} - 1 \right)
\]
Here the warp factor at $\sigma = 0$ has a value equal to one while at $\sigma = \sigma_0 = \pi r$ its value is the same as that in the RS model. Notice that we have a parameter $k_1$ which, as we shall see below, may be assumed to depend on the modulus $r$.

The full solution of the Einstein equations, i.e. $T(\sigma)$ and $V(T)$ can be found easily. These turn out to be:

$$T(\sigma) = -\frac{1}{2} \sqrt{\frac{\sigma_0}{k}} \ln \left( (k_1 + k)\sigma_0 - 2k|\sigma| \right)$$ \hspace{1cm} (2.11)

$$V(T) = \frac{2}{a\sigma_0^2} \left[ e^{-4\sqrt{T_0}T} + k\sigma_0 \right]^{\frac{1}{2}} e^{-2\sqrt{T_0}T}$$ \hspace{1cm} (2.12)

The brane tensions are:

$$\tau(1) = 24M^3 (k_1 + k)$$ \hspace{1cm} (2.13)

$$\tau(2) = -24M^3 (k_1 - k)$$ \hspace{1cm} (2.14)

Interestingly, here both the tensions can be positive if $\frac{k_1}{k} \leq 1$ whereas if $\frac{k_1}{k} \geq 1$ one of them is negative. We shall see later that requirements of stability and hierarchy resolution imply that $\frac{k_1}{k} > 1$. The tachyon field here behaves logarithmically with $\sigma$ and the tachyon potential has a complicated, but primarily exponential relation with $T$ (which, in a sense, is somewhat reminiscent of the nature of the tachyon potential in the context of String theory).

We now need to make use of the boundary condition on the scalar field at each brane. Choosing the brane tensions as:

$$\tau_i = \pm \frac{A}{2} \sqrt{\frac{\sigma_0}{k}} \left[ e^{-x_i} - \sqrt{b \arctan \left( \frac{e^{-x_i}}{\sqrt{b}} \right)} + C_i \right]$$ \hspace{1cm} (2.15)

where $x_i = 2\sqrt{\frac{k}{\sigma_0}} T(\sigma = \sigma_i)$, $b = \sigma_0 k$, $A = \frac{2}{\sigma_0^2} \sqrt{\frac{k}{\sigma_0}}$ and $C_i$ are integration constants, which, as we will see, will play a crucial role. The expression with a $+$ sign in 2.15 is for the brane at $\sigma = 0$, while the one with a $-$ sign in 2.15 is for the $\sigma = \sigma_0$ brane. With the above choices, we obtain the following relations by identifying the tensions given above with the ones obtained earlier (i.e. $\tau(1) = 24M^3 (k_1 + k)$ and $\tau(2) = -24M^3 (k_1 - k)$).

$$C_1 = (k_1 + k)\sigma_0 + \sqrt{b \arctan \left( \frac{(k_1 + k)\sigma_0}{\sqrt{b}} \right)}$$ \hspace{1cm} (2.16)

$$C_2 = (k_1 - k)\sigma_0 + \sqrt{b \arctan \left( \frac{(k_1 - k)\sigma_0}{\sqrt{b}} \right)}$$ \hspace{1cm} (2.17)

As we shall see later, the possible value of $k_1\sigma_0$ (denoted as $z$) is dictated by the requirement of hierarchy resolution. The allowed value of $k\sigma_0$ is found from the stability requirements.
These values can then be used to finally evaluate $C_1$ and $C_2$ such that the boundary conditions on the scalar field hold. Note that setting either or both of $C_1$, $C_2$ to zero leads to inconsistencies.

B. Model 2:

One can easily verify that another solution exists with the following expressions for $f(\sigma)$ and $T(\sigma)$

$$f(\sigma) = \frac{a_1}{k} \left( 1 - e^{-k|\sigma|} \right)$$

$$T(\sigma) = \sqrt{\frac{2}{ka_1}} e^{\frac{k}{2}|\sigma|}$$

with the potential $V(T)$ given as:

$$V(T) = \frac{8}{ak^2} \left( 1 + \frac{k^2}{4} T^2 \right) \frac{1}{2}$$

The brane tensions are:

$$\tau_1 = 24M^3 a_1$$

$$\tau_2 = -24M^3 a_1 e^{-k\pi r}$$

Notice that the brane tensions are not equal and, one of them is exponentially smaller (though negative in value) than the other. Also, the warp factor is decaying and has a value equal to 1 on the positive tension brane. Its value on the negative tension brane is $e^{-2\frac{a_1}{k}(1-e^{-k\pi r})}$. The tachyon field grows larger as $\sigma$ increases. The tachyon potential, is nonsingular in the domain in which the tachyon field is defined. Moreover to have appropriate warping from Planck to TeV scale, both $a_1$ and $k$ must be positive.

Let us now analyse the boundary conditions on the scalar field $T$. Following the method outlined for Model 1, we first write down our choices for the brane tensions in terms of $T$. These are given as:

$$\tau_i = \pm \frac{2k}{a} \left[ \frac{1}{2x_i^2} + \ln x_i - \frac{1}{2} \ln(1 + x_i^2) + D_i \right]$$

where $x = \frac{kT}{2}$ and $D_i$ are integration constants (the parallel of the $C_i$ in Model 1). The expression with a $+$ sign in 2.15 is for the brane at $\sigma = 0$, while the one with a $-$ sign in
2.15 is for the \( \sigma = \sigma_0 \) brane. Evaluating the tensions at \( \sigma \) and equating them to the ones obtained earlier in this subsection, we arrive at the following relations.

\[
D_1 = \frac{1}{2} \ln (2y + 1) \tag{2.24}
\]

\[
D_2 = \frac{1}{2} \ln (2ye^{-x} + 1) \tag{2.25}
\]

where \( y = \frac{4k}{k} \) and \( x = \pi kr \).

The first of the above relations defines \( D_1 \) for given \( y \). The second defines \( D_2 \) for given \( x \) and \( y \). As shown later, we get \( y \) from hierarchy resolution and \( x \) from stability requirements.

III. THE RESOLUTION OF HIERARCHY IN THESE MODELS

We now turn towards analysing the question of hierarchy resolution in these models.

A. Model 1

In this model, the mass scales on the two branes are related as

\[
\frac{m}{m_0} = e^{-\pi k_1 r} \tag{3.1}
\]

Using the definitions, \( z = \pi k_1 r \), the resolution of hierarchy requires, \( e^{-z} \sim e^{-40} \) or \( z \sim 40 \) (or \( k_1 r \sim 13 \), similar to the RS model).

Furthermore, the relation between \( M_{Pl}^2 \) and \( M^3 \) turns out to be:

\[
M_{Pl}^2 = \frac{M^3}{2k} \frac{\sqrt{\pi}}{p} e^{-\frac{4k}{2p}(p+2)} \left[ \text{Erf} \left( \frac{1}{2} \sqrt{\frac{z}{p}}(p+2) \right) - \text{Erf} \left( \frac{1}{2} \sqrt{\frac{z}{p}}(p-2) \right) \right] \tag{3.2}
\]

where \( p = \frac{k}{k} \) and \( \text{Erf} \) denotes the error function.

If \( z \sim 40 \) and \( p \sim 40 \), we find that \( M_{Pl}^2 = 0.024M^3/k \). Therefore assuming, \( M \sim M_{Pl} \) leads to \( k \sim 0.24M_{Pl} \), unlike \( M_{Pl} \sim M \sim k \) in the RS model (based on the relation \( M_{Pl}^2 \sim \frac{M^3}{k} \left( 1 - e^{-2k\pi r_c} \right) \) in RS and for moderate \( kr_c \sim 12 \)). Note however, that the \( k \) in our model is not the same as the RS ‘\( k \)’–which is attached to the linear part in the RS warp factor. In our model, the parallel of the RS ‘\( k \)’ is \( k_1 \). Since \( k_1 = kp \sim 40k \) we get \( k_1 \sim 0.96M_{Pl} \). Thus, one can set \( k_1 \), \( M \) and \( M_{Pl} \) of the order of \( M_{Pl} \) while \( k \) needs to be set at an order of \( 10^{-2}M_{Pl} \). Therefore, in this model, an extra small hierarchy, somewhat similar to the ADD scenario exists and cannot be avoided.
B. Model 2

We note that the relation between mass scales on the branes at \( \sigma = 0 \) and \( \sigma = \pi r \) is

\[
m = m_0 e^{-f(\pi r)} = e^{-y(x)(1-e^{-x})}
\]  

(3.3)

To solve the hierarchy problem one needs \( \frac{m}{m_0} \sim e^{-40} \). Thus, \( \frac{\pi}{k} \) must be around 40.

In addition, the relation between \( M_{Pl}^2 \) and \( M^3 \) turns out to be:

\[
M_{Pl}^2 = \frac{2M^3}{k} e^{-2y} [Ei[2y] - Ei[2ye^{-x}] ]
\]  

(3.4)

where \( Ei \) denotes the exponential integral function.

With \( y = 40 \) and \( x \) moderate (about 3 or larger, as we will see from the stability analysis), we obtain, for \( M_{Pl} \sim M, k \sim 0.025M_{Pl} \). As in the Model 1, we have the parameter \( a_1 \) as the parallel of the RS ‘k’ and this takes on the value \( a_1 \sim 40k \sim M_{Pl} \). Similar to Model 1, an extra small hierarchy seems to exist here too (through the value of the k in our model) and the resolution of hierarchy seems to have an ADD-like flavour.

IV. STABILITY ANALYSIS: INTER-BRANE POTENTIALS

We shall now investigate the stability of our two-brane models.

Let us first look at the original RS-I model (with only a bulk negative cosmological constant) and construct the interbrane potential. This turns out to be (omitting the \( \int d^4\xi \))

\[
V(x) = -18M^3k (1-e^{-4x})
\]  

(4.1)

where \( x = \pi kr \) and the warp factor is just \( e^{-k|\sigma|} \). Note that there is only one free parameter here (i.e. \( x \)) and the potential does not have any minimum.

This potential will be modified, if we also take into account the five-dimensional gravitational part of the action and integrate it over the extra dimension. In summary, the gravity part is given as \( \int \sqrt{g^5} R d^5\xi \) which has a piece which would give 4D gravity upon integrating over extra dimensions (provided we have a curved brane) and another piece (proportional to \( \int e^{-4f} (8f'' - 20f'^2) d\sigma d^4\xi \)). It is a debatable issue as to whether we should include this piece in the potential or not. We, however remain non-committal on this and discuss both scenarios (i.e. with and without the gravitational piece). Including this so-called gravity piece...
part into the inter-brane potential for RS-I, we find that

$$V(x) = 4M^3k \left(1 - e^{-4x}\right)$$  \hspace{1cm} (4.2)

The potential is now positive definite but, as before, has no minimum.

The above analysis shows that, at least for RS-I, by including the gravity part one cannot make the branes stable. Goldberger and Wise \[6\] therefore included an extra bulk scalar field which was crucial in order to achieve stability.

In our models, we have bulk tachyon matter which generates the spacetime geometry. In both the models discussed above we have two parameters( $z$, $p$ in Model 1 and $x$, $y$ in Model 2). Following GW, we consider the matter action (tachyon–matter) + the two brane actions and insert the solutions for $T(\sigma)$ and $f(\sigma)$ in them. Thereafter, we integrate over the extra dimension to obtain the effective inter-brane potential as a function of $r$.

As mentioned before, in either model, we have two independent parameters and, further, a pair of constants which depend on these parameters. We adopt the following strategy to find the allowed values of the parameters and the constants. Let us assume that in either model the parameters are related functionally. We do not provide any compelling reason for doing so–however there is no reason either why such a relationship may be disallowed. In other words, in Model 1, we have $p(z)$ and in Model 2, $y(x)$. We can now choose simple forms of these functions (preferably trigonometric) in order to have a minimum in either potential. Let us now focus on the two models separately.

A. Model 1:

Firstly, let us follow the procedure outlined above to obtain the potentials. Define $p = k_1/k$ as before, so that

$$f(\sigma) = k \left[p|\sigma| - |\sigma| \left(\frac{|\sigma|}{|\sigma_0|} - 1\right)\right]$$  \hspace{1cm} (4.3)

Now let $z = \pi k_1 r$ and $p = p(z)$. Then, the expressions for the potentials, including and excluding the Ricci scalar contribution in the action are, respectively (omitting the integration over $d^4x$),
Without gravity term contribution:

\[
V(z) = -24 M^3 k(p + 1) + 24 M^3 k(p - 1)e^{-4z} + \int_0^1 dx\ 24 M^3 ke^{-4z(x+x/p-x^2/p)}
+ \int_0^1 dx\ 24 M^3 kze^{-4z(x+x/p-x^2/p)}(1 + 1/p - 2x/p)^2
\] (4.4)

With gravity term contribution:

\[
V_{\text{mod}}(z) = 8 M^3 k(p + 1) - 8 M^3 k(p - 1)e^{-4z} - \int_0^1 dx\ 8 M^3 ke^{-4z(x+x/p-x^2/p)}
- \int_0^1 dx\ 16 M^3 zke^{-4z(x+x/p-x^2/p)}(1 + \frac{1}{p} - \frac{2}{p} x)^2
\] (4.5)

where, in the above integrals \( x = \frac{\sigma}{\sigma_0} \). Note, both the integrals above can be done—we do not write them out here because the expressions are complicated and it is more useful to plot the potentials directly. It should also be realised that the dominant contributions to the potentials essentially come from the first two terms in each of them.

The strategy to find possible stable values of \( z \) will be as follows. Note that the hierarchy resolution requires \( z = 40 \). Let us choose \( p(z) = p_0 \cos^2 z \). This restricts \( p \) to have values between 0 and \( p_0 \). Further it ensures the presence of minima in \( V(z) \) since the linear in \( p \) terms will contribute to their existence. Using this form of \( p(z) \) (with \( p_0 = 40 \), say) in either of the above expressions for the potentials we find that there is a minimum around \( z = 40 \) (the value required for hierarchy resolution). We show the potentials in Fig. 1 and Fig. 2. Since \( z \) and \( p \) values are now known and we can easily find out \( C_1 \) and \( C_2 \) by substituting these values in the expressions for the \( C_i \) given earlier.

**B. Model 2:**

For the analysis of the stability of branes in Model 2, recall, the dimensionless variables \( y = \frac{2a}{k} \) and \( x = \pi kr \). Omitting, as before, the explicit writing of the integration over \( d^4\xi \) we obtain the following effective potentials:

Without gravity term contribution:

\[
V(x) = M^3 k \left[ \frac{3}{2} e^{-4y} e^{4y e^{-x}} \ (12 y e^{-x} - 1) - 18y + \frac{3}{2} \right]
\] (4.6)

With gravity term contribution:
The dominant contribution in each potential comes from the linear in $y$ term. In either expression, we now choose $y(x) = y_0 \cos^2 x$ (with $y_0 \sim 40$). Once again, this choice restricts $y$ to values between 0 and 40. With this choice, we now plot the potentials and see whether there are minima at finite values of $x$. 

$$V_{\text{mod}}(x) = 4M^3ky \left[ 1 - e^{-4y}e^{-x}e^{4y}e^{-x} \right]$$ 

FIG. 1: Potential $V(z)/M^3k$ vs. $z$ (without gravity term contribution) for Model 1. Note a minimum at $z \sim 41$.

FIG. 2: Potential $V_{\text{mod}}(z)/M^3k$ vs. $z$ (with gravity term contribution) for Model 1. Note a minimum around $z \sim 39.5$. 

The dominant contribution in each potential comes from the linear in $y$ term. In either expression, we now choose $y(x) = y_0 \cos^2 x$ (with $y_0 \sim 40$). Once again, this choice restricts $y$ to values between 0 and 40. With this choice, we now plot the potentials and see whether there are minima at finite values of $x$. 

$$V_{\text{mod}}(x) = 4M^3ky \left[ 1 - e^{-4y}e^{-x}e^{4y}e^{-x} \right]$$ 

The dominant contribution in each potential comes from the linear in $y$ term. In either expression, we now choose $y(x) = y_0 \cos^2 x$ (with $y_0 \sim 40$). Once again, this choice restricts $y$ to values between 0 and 40. With this choice, we now plot the potentials and see whether there are minima at finite values of $x$. 

$$V_{\text{mod}}(x) = 4M^3ky \left[ 1 - e^{-4y}e^{-x}e^{4y}e^{-x} \right]$$
The figures (Fig. 3 and Fig. 4) below demonstrate that we do indeed have minima in the potentials such that \( y \) is close to 40 at some finite positive \( x \).

![Graph](image1.png)

**FIG. 3**: Plot for \( V(x)/M^3 k \) vs. \( x \) (without gravity term contribution) for Model 2. Note a minimum around \( x \sim 3 \).

![Graph](image2.png)

**FIG. 4**: Plot of \( V_{mod}(x)/M^3 k \) vs. \( x \) (with gravity term contribution) for Model 2. Note a minimum around \( x \sim 4.7 \).

Thus, as in Model 1, knowing \( x \) and \( y \) we can easily find out \( D_1 \) and \( D_2 \) from the abovementioned expressions.

### C. Comparison between the two models

It is reasonable to ask–which among the above two is a better model. A very clear answer to this question does not seem to exist, at this stage. We list below a few points which might shed light on this aspect.
• The second model is very different from RS in terms of the functional form of the warp factor. The first one can, in a way, be thought of as a perturbation.

• At the level of solutions (without invoking any constraints that arise from hierarchy resolution or stability), Model 1 seems to allow the existence of two positive tension branes, while Model 2 does not. This feature of Model 1 differs from what we obtain in standard RS where there is no scope of having two positive tension branes even at the level of solutions.

• In the first model the scaling of masses occurs in a way different (super–exponential) from RS whereas in the second model the scaling is the same (exponential) as for RS.

• The nature of the tachyon potential in the first model is far removed from whatever string theorists write down from varied considerations (which, in some sense may not be applicable here because there arguments are based on cosmological considerations–rolling tachyon etc.). In the second model, by virtue of being exponential (not exactly, but in some way related) the tachyon potential is perhaps a bit closer to the stringy ones.

• The stability issue is solved in both the models by incorporating a similar logic (i.e. a functional realtionship between the two dimensionless parameters). Neither model can be called as preferred, from this perspective.

• For hierarchy resolution, we have seen that Model 2 is different in nature than Model 1, which is closer to RS. This is because in Model 1, the value of $p$ does not affect hierarchy resolution as long as the choice of $p(z)$ results in a minimum in $V(z)$ at the right value of $z$ (required for the solution of the hierarchy problem). On the other hand, in Model 2, a proper value of $y$ is required for hierarchy resolution and the function $y(x)$ should attain that value at a minimum in the potential.

• Further, in the resolution of hierarchy in either model we seem to be introducing an element of hierarchy through the values of the extra parameters (the $k$ in Models 1 and 2). This aspect seems to be closer to the hierarchy resolution in the ADD model [5], though it is not, in any way dependent on the number of extra dimensions, as in ADD where it happens through the factor associated with the volume of the extra dimensions.

In order to facilitate a comparison we tabulate the essential results of Models 1 and 2 in Table I.
| Feature                  | Model 1                                                                 | Model 2                                                                 |
|-------------------------|-------------------------------------------------------------------------|-------------------------------------------------------------------------|
| **Warp factor**         | \( f(\sigma) = k_1|\sigma| - k|\sigma| \left(\frac{|\sigma|}{|\sigma_0|} - 1\right) \) | \( f(\sigma) = \frac{a_1}{k} \left(1 - e^{-k|\sigma|}\right) \)       |
| **Tachyon field**       | \( T(\sigma) = -\frac{1}{2} \sqrt{\frac{a}{k}} \ln \left((k_1 + k)|\sigma_0| - 2k|\sigma|\right) \) | \( T(\sigma) = \sqrt{\frac{2}{ka_1}} e^{\frac{k}{2}|\sigma|} \)       |
| **Tachyon Potential**   | \( V(T) = \frac{2}{\alpha_G} \left[e^{-4\sqrt{\frac{T}{\sigma_0}} + k|\sigma_0|} \right]^{-\frac{1}{2}} e^{-2\sqrt{\frac{T}{\sigma_0}}} \) | \( V(T) = \frac{8}{\alpha_G} \frac{a_1}{k} \left(1 + \frac{k^2T^2}{4}\right)^{\frac{1}{2}} \) |
| **Parameters**          | \( k, k_1, r \)                                                         | \( a_1, k, r \)                                                         |
| **Brane tensions**      | \( \tau_1 = 24M^3(k_1 + k) \)                                         | \( \tau_1 = 24M^3a_1 \)                                               |
|                         | \( \tau_2 = -24M^3(k_1 - k) \)                                         | \( \tau_2 = -24M^3a_1 e^{-k\pi r} \)                                   |
| **Reduced parameters**  | \( z = \pi k_1r, p = \frac{k_1}{k} \)                                  | \( y = \frac{a_1}{k}, x = \pi kr \)                                   |
| **B.C. and constants**  | \( C_{1,2} \) (Eq. 2.16, 2.17)                                         | \( D_{1,2} \) (Eq. 2.24, 2.25)                                         |
| **Relations**           | \( p = p_0 \cos^2 z \)                                                 | \( y = y_0 \cos^2 x \)                                                |
| **Hierarchy resolution**| \( m = m_0 e^{-z} \)                                                  | \( m = m_0 e^{-y(1-e^{-z})} \)                                       |
|                         | \( m \sim \text{TeV}, m_0 \sim M_{Pl}, z \sim 40 \)                    | \( m \sim \text{TeV}, m_0 \sim M_{Pl}, y \sim 40 \)                    |
|                         | \( x \sim 3 \text{ to } 5 \)                                           | \( x \sim 3 \text{ to } 5 \)                                           |
| **M_{Pl}, M relation**  | \( M \sim M_{Pl} \sim k_1, k \sim 0.024M_{Pl} \)                      | \( M \sim M_{Pl} \sim a_1, k \sim 0.025M_{Pl} \)                      |
| **Inter-brane potentials** | \( \text{Eq. 4.4, Eq. 4.5} \)                                      | \( \text{Eq. 4.6, Eq. 4.7} \)                                      |
| **Stability**           | Pot. minimum around \( z \sim 40 \)                                   | Pot. minimum around \( x \sim 3 \text{ to } 5 \)                       |
|                         | \( p = p_0 \sim 40, \cos^2 z \sim 1 \)                                | \( y = y_0 \sim 40, \cos^2 x \sim 1 \)                                |
| **Deficiency**          | Additional small hierarchy in \( k \)                                  | Additional small hierarchy in \( k \)                                  |

Table 1: Summary and comparison of the two models.

V. REMARKS AND CONCLUSION

In this article, we have tried to build two-brane models with bulk matter, which solve the stability and hierarchy problems.

How different are our models from the standard Randall-Sundrum? From Table 1, we can write down the warp factors for the two models using the values of the parameters quoted there. These turn out to be:
Model 1:

\[ f(\sigma) = M_{Pl} \left[ |\sigma| - 0.024|\sigma| \left( \frac{|\sigma|}{40} M_{Pl} - 1 \right) \right] \]  \hspace{1cm} (5.1)

Model 2:

\[ f(\sigma) = 40 \left( 1 - e^{-0.025M_{Pl}|\sigma|} \right) \]  \hspace{1cm} (5.2)

If we plot the above two warp factors along with the linear-in-\( \sigma \) RS one we will see that the deviations are very small and the overall decaying nature is maintained. So, where does this small difference make a crucial contribution? To see this let us go back to the expressions for the inter-brane potentials. In either model, the inter-brane potentials are dominated by the linear terms (\( z \) in Model 1 and \( y \) in Model 2), both of which are essentially the ratio of each of two parameters (\( k_1, k \) in Model 1 and \( a_1, k \) in Model 2). The existence of these linear terms therefore depends on the existence of more than one parameter (apart from \( r \)) in either model, which, in turn, is made possible through the presence of bulk matter. Postulating a relationship between the parameters (i.e. \( y(x) \) or \( p(z) \)) thus becomes a possibility (unlike the RS case) and hence shows a way of achieving stability.

The tachyon matter scalar field is responsible for providing the source for the bulk metric and keeping the branes stable. The crucial point in our analysis has been the fact that among the two free parameters in each of our models (\( x, y \) and \( z, p \)), we have invoked relationships (i.e. \( y(x) \), \( p(z) \)). This has facilitated the resolution of the stability and hierarchy problems simultaneously. We agree that we do not have a good reason behind choosing \( y(x) \) or \( p(z) \) although, as shown, trigonometric functions seems to be the preferred choice in order to have minima in the potentials and hence, stability.

The issue of whether to include the contribution of the gravitational part of the action in defining the effective inter-brane potential remains an open issue. We have preferred to be non-committal on this in our paper and that is why we include results for both scenarios. However, we may note that we do not see any major difference in our results after including the gravitational part of the action. The only difference seems to be the fact that in the presence of the gravity part the potential in all cases is positive, while, otherwise, it is negative. If we had seen a more significant difference (i.e. say, the existence of minima with the gravity part and no minima without or vice versa), we might have had a case for
choosing one over the other. At this stage, we do not make any definite statement on this aspect.

We conclude this article by stating that our analysis of stability and our proposal on a functional relationship between the parameters in our models, surely needs more justification at a more basic level. In addition, it is crucial to address field localisation, massive modes, KK graviton effects, corrections to Newtonian gravity and other phenomenological issues using the new warp factors introduced in this article. We hope to address some of these issues in our future investigations.

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[1] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 3370 (1999); ibid. Phys. Rev. Lett. 83, 4690 (1999)
[2] K. Akama, in Proceedings of the Symposium on Gauge Theory and Gravitation, Nara, Japan (Springer-Verlag, 1982) hep-th/0001113
[3] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B 125, 139 (1983)
[4] M. Visser, Phys. Letts. B 159,22 (1985). [hep-th/9910093]
[5] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429, 263-272 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B436 257-263 (1998); N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Rev. D59 086004-21 (1999)
[6] W. D. Goldberger and M. B. Wise, Phys. Rev. D60,107505 (1999) ibid. Phys. Rev. Letts. 83,4922 (1999)
[7] D. Maity, S. SenGupta, S. Sur, Phys. Lett. B643, 348
[8] D. Maity, S. SenGupta, S. Sur, arXiv:hep-th/0609171. A. Dey, D. Maity, S. SenGupta, Phys. Rev. D75, 107901 (2007); S. Das, A. Dey, S. SenGupta, arXiv:0704.3119
[9] K. Ghoruku and A. Nakamura, Phys. Rev. D64, 084028 (2001); K. Ghoruku, arXiv:hep-th/0402102
[10] B. Grzadkowski and J. F. Gunion, Phys. Rev. D 68, 055002 (2003)
[11] D. Choudhury, D.P. Jatkar, U. Mahanta, S Sur, JHEP (09), 021 (2000)
[12] J. Lesgourgues, L. Sorbo, Phys.Rev. D69 084010 (2004)
[13] A. S. Mikhailov, Y. S. Mikhailov, M. N. Smolyakov, I. P. Volobuev, Class.Quant.Grav. 24 231 (2007)
[14] F. Bruemmer, A. Hebecker, E. Trincherini, Nucl.Phys. B738 283 (2006)
[15] A. Lewandowski and R. Sundrum, Phys. Rev. D 65, 044003 (2002)
[16] P. Horava and E. Witten, Nucl. Phys. B 460, 506 (1996); Nucl. Phys. B475, 94 (1996)
[17] J.-L.Lehners, P. Smythe and K. S. Stelle, Class.Quant.Grav. 22 2589 (2005)
[18] A. Sen, JHEP 0204, 048 (2002); A. Sen, JHEP 0207, 065 (2002); A. Sen, Mod.Phys.Lett. A17, 1797 (2002)
[19] R. Koley, S. Kar, Phys.Lett. B623 244 (2005); Erratum-ibid. B631 199 (2005)