The modulus of elasticity in the theory of degradation

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Abstract. The theory of degradation appeared on the basis of the analysis of the interaction energy in time. In the presented work used the derived solutions of the theory of degradation. Examines the issues of behavior of the potential degradation and the alter power of the object in time. Used General solution of the theory of degradation. On the basis analysis of General solutions to take the solution in the form of rectangular and triangular distributions of power in time. Justified the presence of an elastic zone at the beginning coordinate of power distribution of the object in time. The article defines the zone of the elastic work of concrete, the concept of the modulus of elasticity of concrete from the point of view of the theory of the degradation and change of properties of the object in time. The analysis of changes of the modulus of elasticity of concrete in time. An example of the calculation of the elastic zone of concrete. The interaction of the modulus of elasticity of concrete with a total duration of testing.

1. Introduction

All studies on the relationship of force and deformation are laid on the components of the instantaneous deformation and creep deformation [1]. The elastic limit is considered as a notional value [2,3]. Elastic deformation is manifest as the material properties of the structure to resist perturbing influences [4]. Currently, it is assumed that the elastic performance of the material depends on the interaction at the nuclear level [5]. However, the analysis of the deformation of the metal [6], showed that processes at the nuclear level, as a rule, does not play a big role in the real deformation and the fracture of the material. At the macroscopic level, the material description is maintained for regions whose dimensions are so large that they can be considered homogeneous. A linear relationship between stress and deformation is the basis for a well-developed linear theory of elasticity. The rejection of assumptions about the smallness and the same order of deformations, displacements and angles of rotation, the rejection of Hooke's law, leads to the nonlinear theory of elasticity [7].

According to the existing Rules for the initial modulus of elasticity of concrete is taken of the value of the secant modulus of elastic-ductility $E_\sigma$ mm along the loading branch of the curve $\sigma$- $\varepsilon$, obtained from standard tests. A. V. Yashin proposed the elastic deformation to be determined at the stage of discharge where there is dissipation (dissipation) of energy [8]. In accordance with modern view elastic deformation of the concrete to depend to the structural features of this multicomponent materials [9,10]. The dynamic modulus determines the elastic deformation of the concrete without the in-
fluence of external loads and deformations of the sample [11]. Experiments show that with increasing loading rate the deformation modulus of concrete increases [12].

2. Theoretical justification for the existence of the modulus of elasticity

Based on the theory of the degradation of the received value of the potential energy of the sample at the current time

\[ B^* = \frac{1}{x} \int_1^x B(t) \, dt \]  

(1)

where \( x \) is the current time, \( L \) - time from beginning of loading of the sample before its destruction, \( B(t) \) is the distribution function of the acceleration energy in time. The value of \( B^*dt \) is an elementary power of resistance the energy connection of inside the sample. The power distribution of the studied sample in time from the beginning of exposure to \( a \) current time \( t \) gets the form:

\[ P(t) = \int_a^x \left( \frac{1}{x} \int_1^x B(t) \, dt \right) \]

(2)

The beginning of interaction is assumed from the moment of time \( t = 0 \). However, in this case, the capacity of the facility is impossible to determine. Hence, it was concluded about the existence of a certain period of time \( a \) during which no breakdown occurs in the sample. Therefore, according to (2) we consider a time interval from \( a \) to \( L \), provided \( 0 < a < x < L \). In this case, the graphic \( P(t) \) is always positive, and for \( x \) tending to \( a \) capacity tending to \( 0 \). However, this raises the question, what is the significance to be have \( a \)? The adoption of \( a \) certain constant significance \( a \) creates a lot of questions that have no logical explanation.

Beginning of the influence exposure \( a \) in this case, we propose to takes how as a value what determined from the dependence \( a = P_{el}/B_m \), where \( P_{el} \) is the current value of the elastic capacity, and \( B_m \) is the modulus of elasticity of the material in the axes of "P - t". Then the line \( P = B_m \, t \) on plane "P - t" separates the working area of the material without the aging and fracture of a material (plastic deformation, pseudo-plastic deformation). A derivative of this line on the plane " \( B - t \) " is a horizontal line, it shows that at the movement along this line does not change the "power" of an object in time, and it never gets old – not destroyed. If we assume the existence of an elastic region, the form her may be possible to describe not only linear dependence.

In the case of rectangular plots of acceleration of energies for the potentials, having a constant value to can record the value of the lifetime using the relative time \( \gamma = L/x \). The relationship of modulus of elasticity with acceleration energy is written as:

\[ B^* = B_1 \frac{L-x}{x} = B_1 \frac{yx-x^2}{x} = B_1(\gamma - 1) = E_{el}. \]

In the case of the triangular plot \( B^* = B_0 \frac{y^2x^2-x^4}{2y^2} = B_0 \frac{y^2-1}{2y} = E_{el}. \)

The analysis of the curves of degradation of the material allowed us to offer for the analysis the behavior of the material simple rectangular energy distribution form. Analysis of this form does not give exact values behavior of the material, but allows to visualize its behavior in time.

Let us write the expression for the potential in a series for the case \( L=1 \):

\[ B = b_1 \left( \frac{1}{x} - 1 \right) + b_2 \left( \frac{1}{x} - x \right) + b_3 \left( \frac{1}{x} - x^2 \right) + b_4 \left( \frac{1}{x} - x^3 \right) + \ldots \]

(3)

Consider the case: \( \int_1^x B(t) \, dt = const = G = b_0 L + \frac{b_2 x^7}{2} + \frac{b_3 x^9}{3} + \frac{b_4 x^{11}}{4} + \ldots \),

if \( L=1 \), to \( G_1 = b_0 + \frac{b_2}{2} + \frac{b_3}{3} + \frac{b_4}{4} + \ldots \).

(4)

We rewrite the expression for the potential, using (4):

\[ B = \frac{1}{x} \left( \frac{b_2}{2} + \frac{b_3}{3} + \frac{b_4}{4} + \ldots \right) - \frac{b_0}{x} - x \left( \frac{b_2}{2} + \frac{b_3}{3} x + \frac{b_4}{4} x^2 + \ldots \right) \]

or \( B = \frac{1}{x} G_1 - \frac{b_0}{x} - x \left( \frac{b_2}{2} + \frac{b_3}{3} x + \frac{b_4}{4} x^2 + \ldots \right). \)

(5)

The current capacity of the facility in this case is written as:
\[ P(t) = \int_0^x B' \, dt = \int_0^x \left[ \frac{1}{x} G_1 - \frac{b_0}{1} - x \left( \frac{b_1}{2} + \frac{b_2}{3} x + \frac{b_3}{4} x^2 + \cdots \right) \right] \, dt. \]

Or \[ P(t) = \frac{b_0}{1} \ln x - \frac{b_0^2}{4} x^2 - \frac{b_0 b_1}{9} x^3 - \frac{b_0 b_2}{16} x^4 - \cdots - G \ln a + \frac{b_0^2}{1} + \frac{b_1 b_1}{4} a^2 + \frac{b_2 b_2}{9} a^3 + \frac{b_3 b_3}{16} a^4 + \cdots \]

Or \[ P(t) = G \ln \frac{x}{a} + \frac{b_0 (a-x)}{1} + \frac{b_1}{4} (a^2-x^2) + \frac{b_2}{9} (a^3-x^3) + \frac{b_3}{16} (a^4-x^4) + \cdots \quad (6) \]

If you use only the first term, we obtain the dependences for potential and power for a rectangular energy distribution at \( L=1 \):

\[ B^* = B_0 \left( \frac{L}{x} - 1 \right) \quad \text{and} \quad P(t) = B_0 \ln \frac{x}{a} + B_0 (a-x). \quad (7) \]

If you use two, the first member of the series, we get the dependence off the total amount of rectangular and triangular distribution of acceleration energy when \( L=1 \):

\[ B^* = B_1 \frac{L-x}{x} + B_0 \frac{L^2-x^2}{2x} \quad \text{and} \quad P(t) = \ln \frac{x}{a} \left( b_0 + \frac{b_1}{2} \right) + B_0 (a-x) + \frac{b_1}{4} (a^2-x^2). \quad (8) \]

If we consider the resulting expressions to determine their values at \( x \to 0 \) will get an invalid element functions. Therefore, determine the modulus of elasticity of material at this point makes no sense. Have in mind that at the point with abscissa \( x = 0 \) no defects. If you determine the value of these functions at the point \( x = 0 \), then the value \( B^* \) will be equal to the theoretical modulus of elasticity at this point, and the value of \( P(t) = 0 \).

3. The relationship of modulus of elasticity with defects material

In fact, the actual material has the initial defects, which determine the angle of inclination of the tangent at the start point (continue) destruction. In this case, the modulus of elasticity of the material is characteristic of the "defectiveness" of the material. Initially, the "defectiveness" - the "imperfections" of its structure and then "defectiveness" – the development of defects in its structure. So we know that the influence of the thermal motion of the molecules on the elastic constants of the material is sufficiently small: at room temperature, the values of the elastic constants of only on 10 to 20 percent less than at absolute zero.

Consider the point of the beginning of life \( t_0 = 0 \) (undefined y coordinate). Really very difficult to determine the initial value of the energy at this point. So the Federal law of the Russian Federation of 30 December 2009 N 384-FZ "Technical regulations on safety of buildings and structures" into concepts, defines the life cycle of a building or structure as a period during which the carried out engineering survey, design, construction (including conservation), maintenance (including running repairs), reconstruction, repair, demolition of buildings or structures. In fact, engineering studies are not the beginning of the life cycle of the building, as there are the original Rules for the design, development design and calculations, books are written, etc. Every stage is imperfect, at each step energy is lost. Therefore, at the time of the start of the building of the initial power of the building depends on those defects that are inherent in this building the entire previous history of technology, those defects that arose during survey, design and construction of (probably not only in technology, but also the entire history).

Consider elastic zone of the work object. As already shown above, if the external energy which acting on the object does not exceed some threshold – the destruction and of new defects in the object no occurs.

Characteristics the initial "defectiveness" of the object to appoint a tangent at the origin to the curve of power distribution of the object in time, which collectively defines a "defective" condition of the object at the beginning of the test. With increasing defectiveness of the object must decrease the angle of this line relative to the y-axis.

The defectiveness, for example, of the specimen concrete is due not only to destruction of the sample, but also with defectiveness of minerals, its components, the defects of molecules forming minerals, macro defects, etc. Therefore, the destruction of a concrete sample does not fully characterize the growth of defects in material. After the destruction of the sample, its individual components are still "concrete" and change their properties after material cease to meet the definition of "concrete." Therefore, we can assume that the modulus of elasticity of concrete characterizes the particular material property and
varies within a certain variation in its properties. Of course, with the increase in the number of defects in time the destruction of the sample, its elastic properties change, but also to a certain limit

The initial modulus of elasticity, this to be line $P_a = \alpha t g \beta x$. On the graph of the potential of this line becomes the horizontal straight line $B = t g \beta = E$, crossing the line potential at the point with abscissa $t = t_{min}$. This point marks the beginning of the destruction of the material and means that from this point the modulus of elasticity of the material starts to decrease, since there are additional defects in the material. In some materials the possible non-linear modulus that characterizes the material’s, ability to recover the defects after the tension relief.

For Body without defects, the modulus of elasticity is no determined, and tend to infinity. For example, defects in the concrete specimen arise from defects of structure and defects of individual components of the concrete (including defects of crystal lattices, etc.), defects that are acquired by the concrete during production and operation. Therefore, in the graphs occurs area characterizing the initial defects of concrete what to allow determine the initial modulus of elasticity of the concrete at a certain age and setting conditions. This characteristic of concrete has which be not been subjected to the loading zone - the area of elastic work a. Within this zone, the defects do not grow and do not occur. For concrete which subjected to loads beyond this zone - to be result destruction concrete, happen plastic deformation and pseudo plasticity deformation (appearance and growth of cracks). Area of elastic work is increased. The initial modulus of elasticity of concrete to decrease (tangent to the graph at the end of the elastic zone operation). The initial modulus of elasticity of concrete close to "dynamic" the modulus of elasticity.

4. The modulus of elasticity as the energy feature

The modulus of elasticity is characterized by the tangent to the power curve (deformation) or the angle $\beta$. On the curve of the potential him corresponds to a horizontal line $t g \beta$ and the quantity of potential energy $B_{a, max} = t g \beta$. The initial value quite small. Thus, the elastic modulus in this formulation is the energy characteristics of defects of concrete. In the zone of elasticity, concrete is not destroyed, but crossing the line of elasticity are within the zone of destruction. In this formulation, the elastic zone of the concrete increases and the initial modulus with the increase in the number of defects is reduced. If the concrete had crossed the line of elasticity and had new defects, when re-loading the module size should be reduced. But the concrete begin to show already features of the concrete structure. In compression concrete is not only receiving defects, but compacted. Therefore, the visible reduction of the modulus of elasticity according to the diagram of repeated load cases begins to emerge after the load which is equal approximately to half from the destruction load. For the multiple-re-load this property of the curve deformations is more pronounced [13]. This pattern of changes of the modulus of elasticity is evident for metal. In the diagram the concrete can be show the line of elasticity characterizing the initial modulus of elasticity of concrete $E_a$ which not subjected to loading by (4) and the conditional module of elasticity $E_b$. The conditional module will be in 2...3 times lower than theoretical. In addition, the measurements of deformations without pre-loading, due to the appearance of zones a, may to be cause distortion in the beginning of the chart.

The decrease lines of the modulus of elasticity of concrete, perhaps to the point of destruction (the line of maximum elasticity) – from the starting point of loading to the point of destruction $E_L$ will not be a straight line (depends on rate of loading and number of repetitions), but close to a straight line. Therefore, the curve of the concrete can be represented as a horizontal line with the ordinate equal to $R_b l_{br}$.

This line crosses the line capacity in the simple case of a rectangular distribution of the acceleration energy (7) in the point with abscissa

$$\varepsilon = k \varepsilon_{rb}^2 B_0 / (R_b + \varepsilon_{rb} B_0).$$

where $k$ -coefficient of scaling.

This point characterizes the largest number of defects perceived by the concrete without destroying the integrity and is a constraint on the area of elastic deformation ($a = L$) and, in fact, shows the largest deformation, which are able to perceive the concrete specimen before destruction. As comments can be noted, that the above reasoning does not concern the change of the ultimate strain under repeated and long-term load cases.
5. Conclusions

1. In the general setting deformation of the sample consists of elastic deformation and deformation which cause fracture or plastic deformations (if not to divide the physical nature of deformations that cause destruction). Because of the potential curves differ little from each other, the curve behavior of the concrete can be approximately describe a logarithmic curve. The largest deformation in, this case, which can show a sample at a given level of loading is described by a straight line a maximum of elasticity. Then the horizontal distance between the curve of the works concrete and the line of maximum elasticity in the beginning increases, reaches a maximum and then decreases. Maximum distance between Lines will characterize in this case the beginning of the growth of the main crack.

2. If you take a simple logarithmic diagram to the work concrete, then diagram will be characterized by just two points. This the end point of the elastic work of concrete and any point on the schedule. For comfort as a second point for the diagram take the top point of destruction diagram. If you want to save a sample (e.g., extracted from the body of the object) to draw the graph for concrete can you to take any intermediate point. For the energy distribution according to the schedule it is necessary to use a breakpoint or a different known point of the chart. In many cases it is sufficient to analyze behavior charts concrete use pure logarithmic graph.

3. For the transformation of the work schedule of the concrete sample in time it is necessary to adjust only two points of his own work. During operation of the sample initial modulus of deformation of concrete increases, and the top point of the diagram is transferred according to the duration of loading. To estimate the change of initial modulus need to add regulatory or experimental point from the graph.

4. The initial modulus of elasticity, zone of the initial elastic behavior of the sample "a" and the last plastic deformation associated to the fact that these values are characterized by one value – the number of defects formed during deformation of the sample. The more plastic deformation, the lower value the initial modulus of elasticity and more size elastic zone work of the concrete sample.

5. To describe the work concrete sample in cross-section of eccentrically compressed and bending elements it is proposed to take into account the different rate of loading fibers in cross-section concrete sample and different rate of growth of plastic deformations in different fiber sections. Graphically, it is sufficient for each cross-section to build your work schedule, taking into account the previous paragraph.

6. To result repeatedly re-loads, the increase of deformation in the beginning to come is according to the original schedule, later the deformation is reset by the according modified modulus of elasticity, the following load to changes according is on schedule based on the modified elastic modulus and so on until the module is attain the minimum value.

6. Example

Take, for example, for the original concrete C40 \((R_b=29 \text{ MPa}, \varepsilon_b=200\cdot10^{-5}, E_b=36\cdot10^3 \text{ MPa})\) and write the system of equations for two points: the first is \(\sigma_1 = R_b=29 \text{ MPa}, \varepsilon_1 = \varepsilon_b = 200\cdot10^{-5};\) the second is \(\sigma_2 = 0.2R_b=5.8 \text{ MPa}\) and the corresponding deformation \(\varepsilon_{b,2} = 0.2 \cdot 29/36 \cdot 10^3 = 16\cdot10^{-5}:

\[
\begin{aligned}
29 &= 10^{-5}B \left(200 \ln \frac{200}{a} - 200 + a\right), \\
5.8 &= 10^{-5}B \left(200 \ln \frac{16}{a} - 16 + a\right).
\end{aligned}
\]

The solution of this system: zone of elastic behavior of concrete in relative units \(\varepsilon_a = 10.4 \cdot 10^{-5}\), potential \(B = kB_1 = 7210\) (in units of MPa) and the refined initial modulus of elasticity \(E^* = 5.8/10.4 \cdot 10^{-5} = 55.8 \cdot 10^3 \text{ MPa}\) that, for example, close to the value of the dynamic modulus of elasticity obtained by Le Camus [14]. Taking into account the obtained results for further analysis will take over the original simplified diagram:

\[
\sigma = 7210(200 \cdot 10^{-5} \cdot \ln \frac{x}{10.4 \cdot 10^{-5}} - x + 10.4 \cdot 10^{-5})
\]

View the effect of duration of loading on the chart. Let us assume that \(BL = 7210 \cdot 200 \cdot 10^{-5} = 14.42 = \text{const} = C\). Rewrite the last equation for the point \(\varepsilon_R = x = L\):

\[
\sigma = 7210(200 \cdot 10^{-5} \cdot \ln \frac{L}{10.4 \cdot 10^{-5}} - L + 10.4 \cdot 10^{-5})
\]
\[ R = B \left( L \ln \frac{x}{a} - L + a \right) = C \ln \frac{L}{a} - C + C \frac{a}{L} = 14.42 \left( \ln \frac{L}{a} - 1 + \frac{a}{L} \right) \]

The analysis of the last dependence shows that if the concrete strength with the growth of the strain \( \varepsilon_R = L \) preserve, the elastic work area increases. With the growth of \( L \) grows \( a \). Thus, to take based on known experiments Rasha [15], is visible reduced the initial modulus of elasticity and strength of concrete. However, if the reduction of concrete strength \( R \), then modulus of elasticity decreases less than, while maintaining the strength of concrete, and to a lesser extent increases the zone elastic of work concrete. Here it should be recalled that the last dependence does not determine the elastic properties of the material, but related to the zone of elastic work. The increase in the elastic zone of concrete and the decrease of the initial modulus of elasticity associated with the growth of plastic deformations, therefore, with the increasing number of defects in the concrete.

As can be seen from the constructed dependency (5) zone elastic behavior \( a \) is only in the free member of the equations. The analysis of the value of \( a \) shows that with the increase of strain \( L \), the value of \( a \) increases proportionally. However, the increase in \( a \) in the two times changes all the terms in the equation except the free member of the equation. The free member of the equation reduces the strength of concrete by 10% (due to the appearance of the logarithm of \( a \)). Therefore, the increase of the duration of the test without additional external influences is theoretically slightly reduces the strength of concrete.

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