Generalized total least squares for identification of electromagnetic parameters of an induction motor

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Abstract. This paper proposes an algorithm for electromagnetic parameters with errors in variables. The estimates obtained by the ordinary least squares (OLS) are biased due to errors in the variables. It is shown that even if there are errors in all variables with the same variance, the problem is reduced to generalized total least squares (GTLS). To find a solution to the generalized total least squares problem, an augmented system of equations is used that is equivalent to a biased normal system. This approach improves the conditionality of the system of equations being solved in comparison with the biased normal system and requires less memory compared to the solution based on the right singular vector. The simulation results show that the GTLS estimates are much more accurate than OLS estimates. Based on the proposed approach, recursive algorithms can be created for evaluating the parameters of an asynchronous electric motor online.

1. Introduction

The parameters of the induction motor can vary greatly over time, the active rotor and stator resistance in different modes can change by half [1]. The reasons for changing the parameters may vary: environmental conditions, change in operating modes, aging, and wear of the electric motor, in addition, the parameters of specific specimens differ within the error from the parameters given in the passport.

Changes in the parameters of an induction motor can significantly degrade the quality of control. In this regard, the problem of experimental determination of the characteristics of an asynchronous electric motor is urgent. There are two ways to improve the accuracy of the estimated parameters:

1. The use of measuring instruments of higher accuracy, as well as the protection of measuring instruments from external noise, leads to a significant increase in cost and increases the complexity of the implementation of the induction motor control system.

2. Application of noise-proof algorithms for identifying the parameters of induction motors.

Today, methods for identifying the parameters of induction motor based on equivalent circuits are being actively developed. Estimations of induction motor parameters based on ordinary least squares and their recursive modifications are considered in [2], [3]. The application of the total least squares method and its recursive modifications was considered in [3], [4], [5], [6].

In [7], a method for estimating electromagnetic parameters based on the generalized total least squares method is proposed. In this paper, left-hand differences are used to approximate the derivatives. This approach can introduce a significant error in the estimation of derivatives and require a very small
sampling step. Also, the estimation of \( K \) parameters of an asynchronous motor is associated with poor conditionality of the problem (Problem K2 [2], [3], [5]). The minimization of the generalized Rayleigh ratio has low numerical stability.

In [4], a method of generalized total least squares is proposed for estimating the parameters of an induction motor, but the parameterization of the model of an induction motor has a higher dimension than \( K \) parameters.

This paper proposes a generalized total least squares method for estimating \( K \) parameters of an induction motor, as well as its effective numerical implementation based on an augmented system of equations for an equivalent biased normal system.

2. Problem Statement

The mathematical model for the asynchronous traction motor can be expressed in the fixed coordinates \((\alpha, \beta, 0)\) as the following equations:

\[
\frac{d i^{(\alpha)}}{dt} = \frac{1}{\sigma L_s} U^{(\alpha)} - \frac{\beta}{T_r} \Psi^{(\alpha)} + p \beta \omega \Psi^{(\beta)},
\]

\[
\frac{d i^{(\beta)}}{dt} = \frac{1}{\sigma L_s} U^{(\beta)} - \frac{\beta}{T_r} \Psi^{(\beta)} + p \beta \omega \Psi^{(\alpha)},
\]

\[
\frac{d \Psi^{(\alpha)}}{dt} = \frac{L_m}{T_r} i^{(\alpha)} - \Psi^{(\alpha)} - p \omega \Psi^{(\beta)},
\]

\[
\frac{d \Psi^{(\beta)}}{dt} = \frac{L_m}{T_r} i^{(\beta)} - \Psi^{(\beta)} - p \omega \Psi^{(\alpha)},
\]

\[
M = \frac{3}{2} \frac{L_m}{\sigma L_s} (\Psi^{(\alpha)} i^{(\beta)} - \Psi^{(\beta)} i^{(\alpha)}),
\]

\[
\frac{d \omega}{dt} = \frac{p}{J} (M - M_c),
\]

where \( i^{(\alpha)} \) and \( i^{(\beta)} \) are projections of stator current on the axes \( \alpha \) and \( \beta \), respectively; \( U^{(\alpha)} \) and \( U^{(\beta)} \) are projections of stator voltage on the axes \( \alpha \) and \( \beta \), respectively; \( \Psi^{(\alpha)} \) and \( \Psi^{(\beta)} \) are projections of rotor flux linkages on the axes \( \alpha \) and \( \beta \), respectively; \( \sigma = 1 - L_s^2/(L_s L_r) \) is a total dispersion coefficient; \( L_m, L_s, L_r \) are mutual induction, stator induction, and rotor induction, respectively; \( R_s \) and \( R_r \) represent active stator and rotor resistance; \( T_r \) is the time constant for the rotor; \( p \) is the number of pole pairs; \( \omega \) is the rotor rotation speed; \( J \) is the equivalent moment of inertia for the motor; \( M \) is an electromagnetic moment; \( M_c \) is a motion resistance moment; and \( \gamma \) and \( \beta \) are certain coefficients that depend on the induction and active resistance of the motor.

It is noteworthy that the equation parameters are dependent, and to unequivocally identify the asynchronous motor, one only needs to determine \( R_s, L_s, \sigma, \) and \( T_r \).

The identification of parameters requires excluding the unmeasurable projections of rotor flux linkages from the equations. Assuming that \( \frac{d \omega}{dt} = 0 \) after the transformations, we obtain [5]:

\[
- \left( \frac{d^2 i^{(\alpha)}}{dt^2} + p \omega \frac{d i^{(\beta)}}{dt} \right) = K_1 \frac{d i^{(\alpha)}}{dt} + K_2 i^{(\alpha)} + K_3 \rho \omega \psi^{(\beta)} - K_4 \left( \frac{d U^{(\alpha)}}{dt} + p \omega U^{(\beta)} \right) - K_5 U^{(\alpha)},
\]

\[
\frac{d^2 i^{(\beta)}}{dt^2} - p \omega \frac{d i^{(\alpha)}}{dt} = - K_1 \frac{d i^{(\beta)}}{dt} - K_2 i^{(\beta)} + K_3 \rho \omega \psi^{(\alpha)} + K_4 \left( \frac{d U^{(\beta)}}{dt} + p \omega U^{(\alpha)} \right) + K_5 U^{(\beta)}. \]
Using coefficients $K_1, K_2, K_3, K_4,$ and $K_5$, one can find $R_s, L_s, \sigma$ and $T_s$:

$$T_s = \frac{K_4}{K_5}, \quad R_s = \frac{K_3}{K_4}, \quad L_s = \frac{K_1 - K_5}{K_3}, \quad \sigma = \frac{K_5}{K_4 (K_1 - K_5)}.$$ 

As the parameters are identified digitally, it is convenient to move from differential equations to difference equations.

$$D^{i(\alpha)}_k + \rho_0 D^{i(\beta)}_k = -K_i D^{i(\alpha)}_k - K_i j^{i(\alpha)}_k - K_i \rho_0 j^{i(\beta)}_k + K_i \left(D U^{i(\alpha)}_k + \rho_0 j^{i(\beta)}_k\right) + K_i U^{i(\alpha)}_k,$$  \hspace{1cm} (9)

$$D^{i(\beta)}_k - \rho_0 D^{i(\alpha)}_k = -K_i D^{i(\beta)}_k - K_i j^{i(\beta)}_k + K_i \rho_0 j^{i(\alpha)}_k + K_i \left(D U^{i(\beta)}_k - \rho_0 j^{i(\alpha)}_k\right) + K_i U^{i(\beta)}_k,$$  \hspace{1cm} (10)

where $D^{i(\alpha)}_k, D^{i(\beta)}_k, D U^{i(\alpha)}_k, D U^{i(\beta)}_k, D^{i(\alpha)}_k, D^{i(\beta)}_k$ are values of derivatives at time $k$.

Let us note that in identification systems, voltage and current values and their derivatives are measured with errors; that is,

$$\tilde{i}^{(\alpha)}_k = i^{(\alpha)}_k + \xi^{(\alpha)}_k, \quad \tilde{i}^{(\beta)}_k = i^{(\beta)}_k + \xi^{(\beta)}_k,$$  \hspace{1cm} (11)

$$\tilde{U}^{(\alpha)}_k = U^{(\alpha)}_k + \xi^{(U\alpha)}_k, \quad \tilde{U}^{(\beta)}_k = U^{(\beta)}_k + \xi^{(U\beta)}_k,$$  \hspace{1cm} (12)

$$D^{i(\alpha)}_k = D_i^{(\alpha)} + \xi^{(D\alpha)}_k, \quad D^{i(\beta)}_k = D_i^{(\beta)} + \xi^{(D\beta)}_k,$$  \hspace{1cm} (13)

$$D^{\tilde{U}^{(\alpha)}_k} = D U^{(\alpha)}_k + \xi^{(Du\alpha)}_k, \quad D^{\tilde{U}^{(\beta)}_k} = D U^{(\beta)}_k + \xi^{(Du\beta)}_k,$$  \hspace{1cm} (14)

$$D^{\tilde{i}^{(\alpha)}_k} = D^{i(\alpha)}_k + \xi^{(Di\alpha)}_k, \quad D^{\tilde{i}^{(\beta)}_k} = D^{i(\beta)}_k + \xi^{(Di\beta)}_k,$$  \hspace{1cm} (15)

where $i^{(\alpha)}_k, i^{(\beta)}_k, D_i^{(\alpha)}_k; D_i^{(\beta)}_k; D^{i(\alpha)}_k; D^{i(\beta)}_k$ are true values for stator-current projections, $\tilde{i}^{(\alpha)}_k, \tilde{i}^{(\beta)}_k, D^{\tilde{i}^{(\alpha)}_k} ; D^{\tilde{i}^{(\beta)}_k}; D^{\tilde{i}^{(\alpha)}_k}; D^{\tilde{i}^{(\beta)}_k}$ are noisy values for stator-current projections, respectively; $U^{(\alpha)}_k, U^{(\beta)}_k, D U^{(\alpha)}_k, D U^{(\beta)}_k$ are true values for stator-voltage projections, respectively; $\tilde{U}^{(\alpha)}_k; \tilde{U}^{(\beta)}_k, D^{\tilde{U}^{(\alpha)}_k}; D^{\tilde{U}^{(\beta)}_k}$ are noisy values for stator-voltage projections, respectively; $\xi^{(\alpha)}_k; \xi^{(\beta)}_k; \xi^{(U\alpha)}_k; \xi^{(U\beta)}_k; \xi^{(D\alpha)}_k; \xi^{(D\beta)}_k; \xi^{(Du\alpha)}_k; \xi^{(Du\beta)}_k$ are measurement errors for the corresponding values.

Thus, the problem of identifying parameters can be formulated as one of finding coefficient estimates for $K_1, K_2, K_3, K_4, \text{and} K_5$ in equations (9), (10) according to noisy observations (10)-(15).

3. Identification Criterion

Let us assume that the following conditions are satisfied:

1. $\zeta^{(u\alpha)}_k, \zeta^{(u\beta)}_k, \zeta^{(D\alpha)}_k, \zeta^{(D\beta)}_k, \zeta^{(Du\alpha)}_k, \zeta^{(Du\beta)}_k, \zeta^{(Di\alpha)}_k, \zeta^{(Di\beta)}_k$ are sequences with

$$E \left( \zeta^{(u\alpha)}_k \right) = E \left( \zeta^{(u\beta)}_k \right) = E \left( \zeta^{(Du\alpha)}_k \right) = E \left( \zeta^{(Du\beta)}_k \right) = E \left( \zeta^{(Di\alpha)}_k \right) = E \left( \zeta^{(Di\beta)}_k \right) = 0,$$

where $E$ is an expectation operator.

2. Noise sequences have limited dispersion

$$E \left( \left( \zeta^{(u\alpha)}_k \right)^2 \right) = E \left( \left( \zeta^{(u\beta)}_k \right)^2 \right) = \sigma^2_u,$$

$$E \left( \left( \zeta^{(Du\alpha)}_k \right)^2 \right) = E \left( \left( \zeta^{(Du\beta)}_k \right)^2 \right) = \sigma^2_{Du},$$

$$E \left( \left( \zeta^{(Di\alpha)}_k \right)^2 \right) = E \left( \left( \zeta^{(Di\beta)}_k \right)^2 \right) = \sigma^2_{Di}.$$

In [7], for identification with uncorrelated observational noises, a criterion was proposed for finding an estimate of the vector of parameters $K$.
\[
\min_k \left\| \mathbf{Y} - \Phi \mathbf{K} \right\|_2^2
\]

where \( \mathbf{K} = \begin{pmatrix} K_1 & K_2 & K_3 & K_4 & K_5 \end{pmatrix}^T \),

\[
\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \quad \mathbf{Y} = \begin{pmatrix} \frac{\mathbf{Y}}{\mathbf{Y}} \end{pmatrix}, \quad h_0 = \sigma_{\mathbf{d}^2}^2 + p^2 \sigma_{\mathbf{d}^2} \mathbf{K}^T \mathbf{H} \mathbf{K}.
\]

\[
\Phi_1 = \begin{pmatrix} -D_{11}^{\alpha \alpha} & -p \omega_{1}^{\alpha \beta} & D_{11}^{\alpha \alpha} + p \omega_{1}^{\alpha \beta} & U_{1}^{\alpha \alpha} \\ -D_{12}^{\alpha \alpha} & -p \omega_{1}^{\alpha \beta} & D_{12}^{\alpha \alpha} + p \omega_{1}^{\alpha \beta} & U_{2}^{\alpha \alpha} \\ \vdots & \vdots & \vdots & \vdots \\ -D_{N1}^{\alpha \alpha} & -p \omega_{N}^{\alpha \beta} & D_{N1}^{\alpha \alpha} + p \omega_{N}^{\alpha \beta} & U_{N}^{\alpha \alpha} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} -D_{11}^{\beta \beta} & -p \omega_{1}^{\beta \alpha} & D_{11}^{\beta \beta} - p \omega_{1}^{\beta \alpha} & U_{1}^{\beta \beta} \\ -D_{12}^{\beta \beta} & -p \omega_{1}^{\beta \alpha} & D_{12}^{\beta \beta} - p \omega_{1}^{\beta \alpha} & U_{2}^{\beta \beta} \\ \vdots & \vdots & \vdots & \vdots \\ -D_{N1}^{\beta \beta} & -p \omega_{N}^{\beta \alpha} & D_{N1}^{\beta \beta} - p \omega_{N}^{\beta \alpha} & U_{N}^{\beta \beta} \end{pmatrix}, \quad \mathbf{Y}_1 = \begin{pmatrix} D_{11}^{\alpha \alpha} + p \omega_{1}^{\alpha \beta} \\ D_{12}^{\alpha \alpha} + p \omega_{1}^{\alpha \beta} \\ \vdots \\ D_{N1}^{\alpha \alpha} + p \omega_{N}^{\alpha \beta} \end{pmatrix}, \quad \mathbf{Y}_2 = \begin{pmatrix} D_{11}^{\beta \beta} - p \omega_{1}^{\beta \alpha} \\ D_{12}^{\beta \beta} - p \omega_{1}^{\beta \alpha} \\ \vdots \\ D_{N1}^{\beta \beta} - p \omega_{N}^{\beta \alpha} \end{pmatrix}.
\]

The Cholesky decomposition for the matrix is

\[
\mathbf{H} = \mathbf{L}^T \mathbf{L},
\]

We introduce new vectors of variables

\[
\mathbf{K}' = \mathbf{L} \mathbf{K}, \quad \mathbf{K} = \mathbf{L}^{-1} \mathbf{K}'.
\]

Criterion (16) using a new vector of variables is

\[
\min_k \left\| \mathbf{Y} - \Phi \mathbf{L}^{-1} \mathbf{K}' \right\|_2^2
\]

Minimum (20) can be found as a solution to the augmented system of equations [8], [9]:

\[
\begin{pmatrix} \sigma_{\mathbf{d}^2}^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_i^2 & 0 & 0 & 0 \\ 0 & 0 & p^2 \omega^2 \sigma_i^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\mathbf{d}^2}^2 + p^2 \omega^2 \sigma_i & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\mathbf{d}^2}^2 \end{pmatrix}
\]
\[
\begin{pmatrix}
I_{2N} & 0 & m\Phi L^{-1} \\
0 & I_3 & jm\sigma_{\min} I_5 \\
(m\Phi L^{-1})^T & jm\sigma_{\min} I_5 & 0
\end{pmatrix}
\begin{pmatrix}
m\tilde{e} \\
m\tilde{Y} \\
0
\end{pmatrix}
= \begin{pmatrix}
I_0 \\
0 \\
0
\end{pmatrix},
\]

(21)

where \( m \) is an arbitrary positive factor, the choice of factor is considered in [9]; \( \sigma_{\min} \) is minimal singular values of a matrix \((\Phi L^{-1}, Y)\). \( I_1, I_3, I_{2N} \) are unit matrices of corresponding dimensions, \( j = \sqrt{-1} \).

4. Simulation Results
The algorithm based on criterion (21) was compared with the ordinary least squares, total least square. The values of currents and voltages were obtained using a Matlab Simulink model of the asynchronous motor. The motor had the following technical characteristics: nominal rated power \( P_n = 37kW \); nominal linear voltage \( U_n = 400V \); nominal rotation frequency \( n = 1480 \) rot/min.

In terms of that motor, parameters of the equivalent circuit were specified:
\( R_s = 0.0823\Omega, R_r = 0.0503\Omega, L_s = 0.0278H, L_m = 0.02711H, \sigma = 0.0513, p = 2, \sigma = 0.0513 \)

The true values of \( K \) parameters are:
\( K_1 = 92.8023, K_2 = 104.1040, K_3 = 57.6070, K_4 = 699.7079, \) and \( K_5 = 1264.5 \).

The sampling frequency is \( f_s = 1000 \).

We used the relative mean-square error (RMSE) of parameter estimation as a quality indicator for the model:
\[
\delta K_m = \sqrt{\frac{\| \hat{K}_m - K_m \|^2}{\| K_m \|^2}} \times 100\%.
\]

It was assumed that currents and voltages were measured with noise. The noise was modeled by using independent Gaussian random variables with zero means.

The noise-to-signal ratio for the standard deviations of currents, voltages, and their derivatives is 0.001.

| Parameters | True values | OLS | RMSE OLS, % | GTLS | RMSE GTLS, % |
|------------|-------------|-----|-------------|------|--------------|
| \( L_s \)  | 0.0278      | 0.292 | 4.9601      | 0.0285 | 2.4651       |
| \( R_s \)  | 0.0823      | 0.0842 | 2.3272      | 0.0826 | 0.2712       |
| \( \sigma \)| 0.0513      | 0.0500 | 5.4469      | 0.0501 | 2.7202       |
| \( T_r \)  | 0.5534      | 0.5835 | 2.5427      | 0.5684 | 2.4528       |

The noise-to-signal ratio for the standard deviations of currents, voltages, and their derivatives is 0.005.
Table 3. Estimates of $K$ parameters and their RMSE

| Parameters | $K_{true}$ | $\hat{K}_{ols}$ | $\delta K_{ols}$ % | $\hat{K}_{gtls}$ % | $\delta K_{gtls}$ % |
|------------|------------|-----------------|-------------------|-------------------|-------------------|
| $K_1$      | 92.8023    | 77.430          | 16.5636           | 94.7501           | 2.0990            |
| $K_2$      | 104.104    | 1122.113        | 977.877           | -21.3507          | 120.5090          |
| $K_3$      | 57.6070    | 55.39183        | 3.8452            | 58.1129           | 0.8782            |
| $K_4$      | 699.708    | 462.378         | 33.9184           | 729.1765          | 4.2226            |
| $K_5$      | 1264.47    | 321.9110        | 74.5419           | 1298.8161         | 2.6643            |

Table 4. Estimates of electromagnetic parameters and their RMSE

| Parameters | True values | OLS   | RMSE OLS,% | GTLS   | RMSE GTLS, % |
|------------|-------------|-------|------------|--------|--------------|
| $I_s$      | 0.0278      | 0.685 | 145.96     | 0.0282 | 1.3955       |
| $R_s$      | 0.0823      | 0.1198| 45.51      | 0.0797 | 3.1986       |
| $\sigma$   | 0.0513      | 0.0316| 38.47      | 0.0486 | 5.3681       |
| $T_s$      | 0.5534      | 1.4364| 159.56     | 0.561  | 1.5071       |

5. Conclusion

The paper proposes a method for estimating the electromagnetic parameters of an induction motor based on generalized total least squares. To find a solution to the generalized total least squares problem, an augmented system of equations is used that is equivalent to a biased normal system. This approach improves the conditionality of the system of equations being solved in comparison with the biased normal system and requires less memory compared to the solution based on the right singular vector. The simulation results show that the GTLS estimates are highly accurate than OLS estimates.

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