Efficient Aeroelastic Design Optimization Based on the Discrete Adjoint Method

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An approach for the aerodynamic optimization design of elastic configurations is implemented and tested. Aeroelastic analysis was carried out by combining an Euler equations solver and finite-element structural solver. Two important techniques used in high-fidelity aerostructural design optimization are discussed: the grid deformation method and finite-element mesh (FEM) update. An improved grid deformation methodology based on transfinite interpolation (TFI) and the radial basis function (RBF) method is presented. It adapts to complex configurations very well. The technique to update the FEM is based on a bilinear interpolation method and RBF method, and it adapts to any grid of finite-element model. The discrete adjoint method is used to get the gradient of the objective function with respect to design variables. Optimizations of a wing and a more realistic wing-body configuration are done to demonstrate the effectiveness of the proposed approach. Results show that the lift-to-drag ratio can be improved with constraints through optimization, which indicates that the present methodology can be successfully applied to design optimization of jigg shapes of aircraft.

Key Words: CFD/CSD Coupling, Aeroelastic, RBF-based Finite-element Model Update, Aerodynamic Design Optimization, Discrete Adjoint

1. Introduction

The span and aspect ratio of large military transport, civil transport and high-altitude, long endurance unmanned aerial vehicle (UAV) are very large. The wings of these aircraft are bent and twisted by the air load during flight, and the performance of these aircraft decreases. To alleviate the influence of aeroelastic distortion, static aeroelastic correction is needed after aerodynamic design. Usually only the twist angles of the wing sections and dihedral angle are adjusted according to the aerodynamic load of cruise condition. The configuration of aircraft in cruise condition is slightly different from the ideal configuration as a result of the aerodynamic and structural analysis method as well as the correction method. So performance decreases in cruise condition, and it may drop more in other states. The structural deformation has to be considered with great care during the design of some special components, such as winglet, etc. In order to improve the quality of aircraft design further, it is very important to take account of the interaction of structure and aerodynamic load in design optimization of the aerodynamic configuration, and multidisciplinary design optimization is necessary.

Much work has been done in aerodynamic configuration design optimization considering aeroelastic deformation. Compared with the traditional aerodynamic design optimization of single discipline, multidisciplinary design optimization has not progressed currently. The optimization method can be divided into two categories, which are gradient-based design optimization and modern intelligent design optimization. The typical modern intelligent design optimization methods include optimization based on generic algorithms (GA) and the surrogate-assisted method. A typical example of them is Nikbay’s work, which presented a practical methodology for aeroelastic optimization via the coupling of high-fidelity commercial codes. This type of method takes a lot of time and expense to obtain the optimum solution because computational fluid dynamics (CFD) and computational structure dynamics (CSD) are both computationally-intensive methodologies. So the gradient-based optimization algorithm is needed to reduce the computational expense and much work has been done. In these studies, the sensitivities of coupled systems are computed by semi-analytical methods, such as using global sensitivity equations, which is a combined version of the direct method, or the coupled-adjoint method. The coupled-adjoint equations of the coupled-adjoint method are very large sparse systems, and are sometimes hard to solve. So only relatively simple configurations are optimized using gradient-based optimization algorithm and most of these approaches are based on inviscid Euler equations. Much work has to be done to solve coupled-adjoint equations.

To avoid the problem, another approach can be adopted. The aeroelastic deformation is considered in calculating the objective functions, and the deformed configuration is supposed to be rigid when we calculate the sensitivity derivatives. In this way, optimizations considering aeroelastic deformation are simplified and maybe applied to complex configuration optimizations considering aeroelastic deformation.
2. A Static Aeroelastic Simulation Method Based on the Loose Coupling of CFD/CSD

2.1. Flow governing equations

The three-dimensional compressible Euler equations in Cartesian coordinates can be written in the conservation form as

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial G}{\partial z} = 0 \quad (1)$$

where

$$Q = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho e \end{bmatrix} \quad E = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uw \\ (\rho e + p)u \end{bmatrix} \quad F = \begin{bmatrix} \rho v \\ \rho vw \\ \rho v^2 + p \\ \rho vw \\ (\rho e + p)v \end{bmatrix} \quad G = \begin{bmatrix} \rho w \\ \rho uw \\ \rho w^2 + p \\ \rho uw \\ (\rho e + p)w \end{bmatrix}$$

and $\rho$, $u$, $v$, $w$, $p$ and $e$ are the density, velocity components for $x$, $y$ and $z$ directions, pressure and total energy, respectively. These equations are discretized with the cell-centered, finite-volume method, and Van Leer’s flux vector splitting is adopted for space discretization. For the time integration, the LU-SGS implicit method is adopted.

2.2. Structural equations

For a multi-degree-of-freedom system, the differential equation of motion ignoring the structural damping of forced vibration is formulated as

$$[M][\ddot{\mathbf{x}}] + [K][\mathbf{x}] = [f(t)] \quad (2)$$

where $[M]$ and $[K]$ respectively mean the structural mass matrix and stiffness matrix not influenced by aerodynamic results, $[\mathbf{x}(t)]$ means the unknown vector of the structural deformation, and $[f(t)]$ means the force vector transferred from the aerodynamic force and acting on the structure. It is the aerodynamic force that connects the CFD system and CSD system. For aeroelastic simulation, the structural deformation has nothing to do with the time step. Therefore, Eq. (2) can be expressed as

$$[\mathbf{x}] = [K]^{-1}[f(t)] = [C][f] \quad (3)$$

where $[C]$ is the structural flexibility matrix. It will be discretized and solved using the finite element method.

2.3. Aeroelastic simulation method

The loosely coupled aeroelastic simulation method is adopted in this paper. The aerodynamic load is calculated using a CFD solver and then transferred to the nodes of the CSD model, which can produce the structural deformation of the aircraft. The deformed configuration predicted by the finite-element solver will change the aerodynamic performances and produce new pressure distribution, which can affect the configuration again. This procedure continues until some criteria are satisfied. Here, the displacement criteria are used as the convergence criterion. Displacement criteria are as follows:

$$\varepsilon = \max_i \left| \frac{U_i^n - U_i^{n-1}}{U_i^n} \right| \leq 1.0 \times 10^{-4} \quad (4)$$

where $U_i^n$ represents displacement of node $i$ in the $n$th iteration.

Because the CFD and CSD models used in the simulation are independent, the data transfer between these two kinds of solvers is one of the key technologies. While in the process of data transfer, it is very important to guarantee energy conservation with respect to virtual work principle. Many algorithms to transfer data between CFD and CSD models have been developed. These algorithms include Infinite-plate Spline (IPS), Boundary Element Method (BEM), Constant-Volume Tetrahedron (CVT), RBF$^{15-17}$ and so on. In this paper, the RBF method is used to convert the pressure and displacements in the interface.

2.4. Grid deformation

When the structural deformation is transformed to the surface grid of the CFD grid or the configuration of an aircraft is revised during optimization, the interior CFD grid of the aircraft should also be modified accordingly. Hounjet and Meijer’s method is utilized to preserve the original topology of the grid.$^{18}$ This method is purely algebraic and non-iter-
The coefficient $a_i$ formed to point $a_0$ space point ~ block grid can be calculated using the RBF interpolation in the block.

The displacements of all the vertices and edges in a multi-block grid can be calculated using the RBF interpolation method as follows.

We know the displacements of vertex $\tilde{x}_i$ on the surface grid, which is assumed to be $d_i$ (i.e., $i = 1, N, N$ is the number of vertices on surface grid). Then, the displacement of any point $\tilde{x}$ can be expressed as:

$$\tilde{d}(\tilde{x}) = \tilde{d}_a + \sum_{i=1}^{N} \tilde{a}_i R_i(\tilde{x})$$  \hspace{1cm} (5)

where $R_i(\tilde{x}) = \|\tilde{x} - \tilde{x}_i\|$. $R_i(\tilde{x})$ is the distance between space point $\tilde{x}$ and vertices on the surface grid of number $i$. The coefficient $a_i$ (i.e., $i = 0, N$) can be achieved according to

$$\tilde{d}(\tilde{x}) = \tilde{d}_i$$

$$\sum_{i=1}^{N} \tilde{a}_i = 0$$  \hspace{1cm} (6)

After we get the displacements of the vertices and the edges of the blocks, the displacements of the remaining grid points of the multi-block grid are computed using TFI method. Then, the grid of the new configuration can be computed by summing the displacements of all the grid points and the initial grid. More details can be found in Hounjet and Meijer.\(^{(18)}\)

RBF-based TFI methodology fails sometimes because of negative Jacobian near the configuration wall, which sometimes leads to a decline in grid quality near the configuration wall. The reason is found to be the tolerance in solving Eq. (6) for RBF interpolation. The iterative method is adopted to enhance the efficiency of solving these equations. The number of vertices may be large for complex configurations, so the dimension of Eq. (6) is large. The diagonal elements of the coefficient matrix are all zeros, so the coefficient matrix is not diagonally dominant and sometimes the equations can’t converge very well. As a result, vertex $a_0$ of the original surface grid in Fig. 1(a), which should be transformed to point $d’$ as shown in Fig. 1(b) after deformation, is transformed to point $a’$ as shown in Fig. 1(b). Additionally, point b of original grid is transformed to point b’.

Actually, we use d’ instead of a’ to calculate the displacements of the edges in space, which leads to a negative Jacobian. Sometimes it does not lead to a negative Jacobian, but it reduces the grid quality as shown in Fig. 2(b). Initial spacing normal to the walls (the first layer) changes after deformation, while the spacing of the second layer almost remains the same. This phenomenon leads to the reduction of grid quality. This is even more so for a viscous grid. The spacing normal to walls is very small for viscous grid, and sometimes the spacing of the first layer changes dramatically after deformation, so the growth rate of the grid normal to wall is much greater or less than 1 and the grid quality drops.

To solve this problem, the displacements of grid points at vertices and edges not lying on the configuration surface or not linking to the configuration surface should be computed by RBF, and the displacements of edges linking to the configuration surface should be computed by linear interpolation. By this means, the negative Jacobian and grid quality reduction are avoided.

To demonstrate this problem, the grid deformation of a very complex configuration is conducted. The configuration and framework of the grid are shown in Fig. 3(a), and the negative Jacobian appears in the region of the rectangular after a rotation of 10 deg around the nose of the aircraft. Figure 3(b) shows the figure enlarged. Point a in Fig. 3(b) is on the surface of the aircraft, and point b, c and d are the points away from the wall of the aircraft. As can be seen, point b is below the wall of the aircraft after deformation. It’s not strange that the negative Jacobian appears near the wall. Figure 3(c) shows the grid generated by the improved grid deformation methodology. As can be seen, points b, c and d are now outside the aircraft, and the negative Jacobian disappears.

3. Optimization Method Considering Structural Deformation

3.1. Discrete adjoint method

The objective of multidisciplinary aerodynamic design optimization is to maximize the lift-to-drag ratio of configurations. The discrete adjoint method is used to analyze the sensitivity of objective functions. Only the aerodynamic sensitivity is considered during the sensitivity analysis, and the aircraft is treated as being rigid.

The discrete residual of the nonlinear, multidimensional steady-state governing equations of the fluid and boundary conditions are approximated as a large system of coupled nonlinear algebraic equations as
where $Q$ is the vector of converged steady field variables, $X$ represents the computational grid, and $b$ is the vector of independent input (design) variables. The aerodynamic characteristics of the aircraft are achieved by the integration of pressure over the surface grids. Similarly, the vector of aerodynamic output functions (objective functions) $J$ depends on $Q$, $X$, and $b$. So, $J$ can be written symbolically as

$$ J = J(Q(b), X(b), b) $$

Differentiate Eqs. (7) and (8) with respect to $b$, and we get

$$ J' = \frac{\partial J}{\partial Q} Q' + \frac{\partial J}{\partial X} X' + \frac{\partial J}{\partial b} $$

$$ R' = \frac{\partial R}{\partial Q} Q' + \frac{\partial R}{\partial X} X' + \frac{\partial R}{\partial b} = 0 $$

where $J' = \frac{dJ}{db}$, $R' = \frac{dR}{db}$, $Q' = \frac{dQ}{db}$, $X' = \frac{dX}{db}$.

$$ \frac{dJ}{db} = \left[ \frac{\partial J}{\partial b_1}, \frac{\partial J}{\partial b_2}, \ldots, \frac{\partial J}{\partial b_m} \right] $$

and $m$ is the number of design variables.

The governing equation, Eq. (7), is added to the objective function, Eq. (8), as equality constraints and we get the Lagrange equation

$$ L(b) = J(Q(b), X(b), b) + \lambda^T R(Q(b), Z(b), b) $$

where $\lambda$ is the Lagrange multiplier, and it can be any constant matrix. Differentiate $L(b)$ with respect to $b$ and then:

$$ J' = L'(b) = \frac{\partial J}{\partial X} X' + \frac{\partial J}{\partial b} + \lambda^T \left( \frac{\partial R}{\partial X} X' + \frac{\partial R}{\partial b} \right) $$

$$ + \left( \frac{\partial J}{\partial Q} + \lambda^T \frac{\partial R}{\partial Q} \right) Q' \tag{11} $$

Since the choice of the Lagrange multiplier is arbitrary, term $Q'$ can be eliminated by solving the following equation

$$ \frac{\partial J}{\partial Q} + \lambda^T \frac{\partial R}{\partial Q} = 0 \tag{12} $$

hence,

$$ J' = \frac{\partial J}{\partial X} X' + \frac{\partial J}{\partial b} + \lambda^T \left( \frac{\partial R}{\partial X} X' + \frac{\partial R}{\partial b} \right) \tag{13} $$

Equation (12) is a linear algebraic equation that represents the discrete adjoint equation. A pseudo time term is added to Eq. (12) to enhance the diagonal dominance and the solution is obtained by marching in time, just like the flow solver.

$$ \left( \frac{V}{\Delta t} + \left( \frac{\partial R}{\partial Q} \right)^T \right) \Delta \lambda^n = - \left( \frac{\partial R}{\partial Q} \right)^T \lambda^n + \left( \frac{\partial J}{\partial Q} \right)^T \tag{14} $$

$$ \Delta \lambda^n = \lambda^{n+1} - \lambda^n $$

where $\partial R/\partial Q$ is an approximation of the exact Jacobian matrix $\partial R/\partial Q$. The above equation can be solved using the LU-SGS method.\(^{(19)}\)

### 3.2. Updating of the wing-tip grid of the CFD surface grid

The upper and lower surfaces of the wing in the CFD grid can be generated using a geometric representation method. The wing-tip surface grid (as shown in Fig. 4) has to be updated according to the edges of the wing-tip during design optimization. A simple method to update the wing-tip surface grid is to interpolate the displacements of surface grid using a TFI algorithm. This method is sometimes difficult to accomplish if the topology is complex as shown in Fig. 4. Here, a method based on RBF is proposed to process generation of the wing-tip surface grid. Since we know the dis-

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**Fig. 3.** Grid deformation of complex configuration.

**Fig. 4.** Surface grid of the wing-tip.
placements of the edges of the wing-tip surface grid, all the displacements of the surface grid points on the wing-tip surface can be generated using the RBF method, as done in Section 2.4. The advantage of this method is that it adapts to any grid topology and very simple to accomplish. Figure 5 shows the grid before and after deformation. The sectional geometry of the wing-tip section is revised and rotated downward by 5 deg around the leading edge. As can be seen, this method works very well.

3.3. Structural model update

The FEM must be updated after we update the design variables during optimization. Usually, the points on the surface grid of the CFD grid do not coincide with that of the FEM. We can’t add the surface grid displacement of the CFD grid to the surface of the FEM directly, so interpolation is needed. The internal points of FEM have to be updated if the surface mesh is updated. Since the methodology to update the FEM is seldom discussed, it is necessary to discuss it in detail.

Here, a method based on bilinear interpolation and RBF is proposed to update the FEM. This method is divided into two steps. In the first step, we construct a mapping of the points from the CFD surface grid to the CSD surface grid. Then, the surface grid displacement in the FEM can be decided using bilinear interpolation method according to the surface grid displacement in CFD. In the second step, the internal grid displacement in the FEM is decided by the RBF method, which is very similar to the grid deformation of the CFD grid.

Suppose structural node \( \tilde{x}_d \) of the initial FEM is surrounded by the four closest non-coplanar points, \( \tilde{x}_{ai,j-1} \), \( \tilde{x}_{ai,j} \), \( \tilde{x}_{ai-1,j} \) and \( \tilde{x}_{ai-1,j-1} \) of the initial CFD surface grid as shown in Fig. 6. Firstly, assume that the coordinate of point \( \tilde{x}_d \) in the quadrilateral can be expressed as (s, t) and 0 ≤ s, t ≤ 1. Then, point \( \tilde{x}_d \) can be expressed as:

\[
\tilde{x}_d = (1 - s)(1 - t)\tilde{x}_{ai-1,j-1} + (1 - s)s\tilde{x}_{ai,j-1} + (1 - t)t\tilde{x}_{ai-1,j} + st\tilde{x}_{ai,j}
\]

Solve the inconsistent equations and we get s and t. This mapping is fixed during optimization. Assume that the displacements of \( \tilde{x}_{ai-1,j-1}, \tilde{x}_{ai,j-1}, \tilde{x}_{ai-1,j} \) and \( \tilde{x}_{ai,j} \) are \( \tilde{d}_{ai-1,j-1}, \tilde{d}_{ai,j-1}, \tilde{d}_{ai-1,j} \) and \( \tilde{d}_{ai,j} \), respectively, during optimization. Then, the displacement of \( \tilde{x}_d \) can be determined according to the vertices displacement of the quadrilateral as:

\[
\tilde{d}(\tilde{x}) = \tilde{d}_0 + \sum_{j=1}^{M} \tilde{a}_j R_j(\tilde{x}_d)
\]

where \( R_j(\tilde{x}_d) = ||\tilde{x} - \tilde{x}_j|| \). \( R_j(\tilde{x}_d) \) represents the distance between space point \( \tilde{x} \) and vertices on the surface grid of number \( j \). The coefficient \( a_j \) can be achieved according to

\[
\tilde{a}_j = \frac{1}{\sum_{j=1}^{M} \tilde{a}_j}
\]

Then, the new FEM can be achieved by summing the initial FEM and corresponding displacements of the FEM grid points.

No matter how large the deformation of the CFD surface grid is, the CFD surface grid displacement can be interpolated to construct the displacement of the surface grid in the CSD mesh. The advantage of the method proposed is that the FEM is free to construct, and it adapts to structural or unstructured FEM. The FEM needn’t have a relation with the CFD surface grid. This method is purely algebraic and non-iterative as well, which is similar to CFD grid deformation. The computational expense is very low. It is adapted to complex configurations. If the displacement of the CFD surface grid is zero, then the deformed and initial CSD mesh will be identical. We can utilize some codes of the CFD grid deformation to accomplish the methodology, so it is easy to code it.

A FEM surface grid is tested to validate the method first. A wing is rotated around the x coordinate axes by 5 deg. The
FEM before and after rotation is shown in Fig. 7(a), and the CFD surface grid is shown in Fig. 7(b). The rotated surface grid of the FEM generated by the proposed methodology almost coincides with the surface grid of the FEM generated by rotation directly. The root-mean-square error of all the corresponding nodes is $1.27 \times 10^{-5}$ m. The semispan of the wing is 16.75 m. So the error is very small compared with the size of the wing. The FEM update method of surface grid is reliable and it satisfies the requirement of optimization.

Secondly, an internal mesh of the FEM is used to validate the method proposed. A wing is deformed and the wing-tip section is rotated downward 5 deg. The FEM of the wing before and after deformation is compared in Fig. 8. As can be seen, the internal grid points are updated, which indicates the effectiveness of the method proposed.

### 3.4 Optimization procedure

The objective is to maximize the lift-to-drag ratio of elastic aircraft with lift and geometric constraints. The loosely coupled aeroelastic simulation in Section 2.3 is adopted.

The Sequential Quadratic Programming (SQP) method is utilized to maximize the objective function. During the sensitivity calculation, the aircraft is treated as being rigid and aeroelasticity is ignored. The optimization process is shown in Fig. 9.

### 4. Results

To demonstrate the approach developed in this paper, wing and wing-body configurations are considered. The structural parameters, such as thickness of shell element for the skin, are fixed during optimization.

#### 4.1 Case 1: ONERA M6 optimization

The ONERA M6 wing is used to demonstrate the proposed optimization method. The structural model is formed using the triangular shell elements. The thickness of shell element for the skin is 0.002 m, and the thickness of shell element for the ribs is 0.004 m. Aluminium alloy is chosen as the material. Its modulus of elasticity is 70 GPa and the Poisson’s ratio is 0.33 for the CSD solver. The skins, ribs and spars are shown in Fig. 10.

The freestream Mach number is 0.84. The grid size is 0.24 million. The lift coefficient of the rigid ONERA M6 wing at an angle of 3.06 deg is 0.274, and it is reduced to be 0.265 after aeroelastic deformation. The comparison of the wing before and after deformation, as well as surface grid, is shown in Fig. 11.

Hicks-Henne functions are used to parameterize the upper and lower surface of the wing sections. Four sections are parameterized along the wing span, and the locations are depicted in Fig. 11 (A–D). The number of design variables is 10 for the upper surface and lower surface of every section. The angle of attack and the twist angle of these sections, except for the root section, are also chosen to be design variables. So the total number of design variables is 84. The optimization model is
Max \( f(X) = K \)
\[ g_1(X) = \bar{c}_{1, \text{max}} > 0.098 \]
\[ g_2(X) = \bar{c}_{2, \text{max}} > 0.098 \]
\[ g_3(X) = \bar{c}_{3, \text{max}} > 0.098 \]
\[ g_4(X) = \bar{c}_{4, \text{max}} > 0.098 \]
\[ g_5(X) = |C_L - 0.265| < 0.003 \]

where \( \bar{c}_{1, \text{max}} - \bar{c}_{4, \text{max}} \) represents the maximum relative thickness of the wing sections.

The surface pressure contour of the elastic M6 wing before and after optimization is presented in Fig. 12, and the sectional pressure distribution comparison of original and optimized elastic configurations is presented in Fig. 13. The locations of the selected wing sections are depicted in Fig. 11 (A–D). Table 1 summarizes the design results.

The lift-to-drag ratio is increased by 56.1% compared to the original results.

### 4.2. Case 2: Transport plane optimization

To further demonstrate the ability of the optimization system, a transport plane is optimized. The freestream Mach number is 0.785 and \( \alpha = 2.0 \) deg. The grid size is 1.8 million. Figure 14 displays the surface grid. The deformation of the body is not large, and it has little influence on the aerodynamic characteristics of the wing-body configuration, so the finite-element analysis is performed only on the wing.

When constructing the structural model of the wing, the main load-carrying components of the wing box are considered, including skins, ribs and wing spars. The front and rear spars are defined at 15% and 65% along the chord, respectively. The spars and ribs are assumed to be ‘T-beams’ with two rectangular cap models with beams, connected by a web of shell elements. The skins, ribs and spars are shown in Fig. 15. Aluminium alloy is chosen as the material for the
state aerodynamic numerical simulation, so its modulus of elasticity is 70 GPa and the Poisson’s ratio is 0.33 for the CSD solver. The ultimate strength of the material is 412 MPa and its density is $2.7 \times 10^3$ kg/m$^3$.

Static aeroelastic analysis is conducted for the initial configuration. The lift coefficients before and after deformation are 0.5565 and 0.502, respectively. The configurations before and after deformation are shown in Fig. 16. As can be seen, deformation is very obvious.

Six wing sections are parameterized by Hicks-Henne functions along the wing span and the locations are depicted in Fig. 16 (A–F). The number of design variables is 10 for each the upper surface and lower surface of every section. The twist angel of each section is also chosen as a design variable and the total number of design variables is 126. The optimization model can be described as:

Max $f(X) = K$

$g_1(X) = \tilde{e}_1 \max > 0.117$

$g_2(X) = \tilde{e}_2 \max > 0.115$

St. $g_3(X) = \tilde{e}_3 \max > 0.107$

$g_4(X) = \tilde{e}_4 \max > 0.107$

$g_5(X) = \tilde{e}_5 \max > 0.105$

$g_6(X) = \tilde{e}_6 \max > 0.105$

$g_7(X) = |C_L - 0.502| < 0.01$

where $\tilde{e}_1 \max$ represents the maximum relative thickness of the selected wing sections.

The characteristic is improved after optimization. The results are shown in Table 2. The lift-to-drag ratio increased by 4.06% after optimization. The surface pressure contours of the initial and final configurations are presented in Fig. 17. Figure 18 shows the comparison of $C_p$ distributions on selected wing sections, and the location of the selected wing sections are depicted in Fig. 16 (A–F). As can be seen, the shock is weakened after optimization.

The semispan of this configuration is 16.75 m. The maximum deformation of the wing-tip before and after optimization is 0.80 m and 0.796 m, respectively, and the difference is negligible. The von Mises stress contours are presented in Fig. 19. It shows that the von Mises stress of the
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The comparison of the two stress distributions illustrates that the optimized wing reduced the maximum stress and expanded the stress concentration range.

5. Conclusions

We considered the problem of optimizing the static aeroelastic system by varying aerodynamic parameters. A general aerodynamic design optimization framework considering aeroelastics has been presented based on the discrete adjoint method. It is a very efficient methodology for the aerodynamic design of jig shapes. Within this framework, a FEM update method is proposed, and test shows that it is very efficient. The ONERA M6 wing and a wing-body configuration are successfully optimized using the proposed methodology, and results show the effectiveness of the method.

All of this work lays a very solid foundation for future research, in which the optimization of structural parameters should be considered in a new framework. The new framework is also based on the discrete adjoint method. The aerodynamic and structural performance will be predicted by jig shape correction, which is very efficient. The optimization efficiency will be improved by several times compared to general gradient-based multidisciplinary design optimization because the aerodynamic and structural analyses are truly decoupled and time-consuming aeroelastic analysis is avoided.

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