Josephson vortex Cherenkov radiation

R.G. Mints

School of Physics and Astronomy,
Raymond and Beverly Sacler Faculty of Exact Sciences,
Tel Aviv University,
Tel Aviv 69978, Israel

I.B. Snapiro

Physics Department,
Technion–Israel Institute of Technology,
Haifa 32000, Israel

Abstract

We predict the Josephson vortex Cherenkov radiation of an electromagnetic wave. We treat a long one–dimensional Josephson junction. We consider the wave length of the radiated electromagnetic wave to be much less than the Josephson penetration depth. We use for calculations the nonlocal Josephson electrodynamics. We find the expression for the radiated power and for the radiation friction force acting on a Josephson vortex and arising due to the Cherenkov radiation. We calculate the relation between the density of the bias current and the Josephson vortex velocity.

74.60. Ec, 74.60. Ge

Typeset using REVTEX
The Josephson vortex is a well-known and an important example of a sinh–Gordon soliton in solid state physics. This soliton is a propagating non-linear wave describing the phase difference between two weakly coupled superconductors and the dynamics of the fluxons residing in this contact. Results concerning the general features of the motion of a Josephson vortex are interesting for different systems in solid state physics where the sinh–Gordon soliton exists.

Detailed knowledge of the Josephson vortex dynamics is important for the flux dynamics and related phenomena in superconductors, e.g., flux creep, flux flow, magnetization relaxation, current–voltage characteristics, etc. Specific features of a Josephson vortex motion are currently under thorough experimental and theoretical study. In particular, very fast moving Josephson vortices are observed and treated in annular Josephson tunnel junctions.

Josephson vortex dynamics is very important in the novel layered high-temperature superconductors due to their crystalline structure. In particular, the most prominent Bi and Tl based copper oxide compounds consist of a periodic stack of weakly coupled two-dimensional CuO layers where the superconductivity presumably resides. In this case a variety of linear crystalline structure defects result from the crossing of the superconducting layers with planar crystalline structure defects, e.g., the grain boundaries, twins, etc. These linear crystalline structure defects can be treated as Josephson junctions. The critical current density for the Josephson junctions in the superconducting layers is relatively high, especially for the coherent crystalline structure defects, e.g., for low–angle grain boundaries and twins. The Josephson penetration length is decreasing if the Josephson critical current density is increasing. As a result, the effect of the nonlocal Josephson electrodynamics becomes important while treating the Josephson vortices in the superconducting layers with the Josephson penetration depth being of the order or less than the London penetration depth.

In this paper we consider the Josephson vortex Cherenkov radiation of an electromagnetic wave. We treat a long one-dimensional Josephson junction. We consider the wave length of
the radiated electromagnetic wave to be much less than the Josephson penetration depth. We show that the Josephson vortex velocity can be equal to the phase velocity of this electromagnetic wave, which is the necessary condition for the Cherenkov radiation. We use for calculations the nonlocal Josephson electrodynamics. We find the amplitude and power of the radiated wave and the radiation friction force acting on a Josephson vortex and arising due to the Cherenkov radiation. We calculate the relation between the density of the bias current across the Josephson junction and the stationary Josephson vortex velocity. We consider the case of a Josephson junction with a very high electrical resistivity, i.e., with a very low damping constant.

The dynamics of a Josephson vortex in a one–dimensional Josephson junction parallel to the $x$ axis is described by the sine–Gordon equation for the space and time dependent phase difference $\varphi(x, t)$. Taking into account the damping resulting from the resistance of the junction it reads

$$\varphi_{\tau\tau} - \varphi_{\xi\xi} + \eta \varphi_\tau + \sin \varphi = \beta. \tag{1}$$

The subscripts $\tau$ and $\xi$ are to denote the derivatives over the dimensionless time $\tau = t\omega_J$, and coordinate $\xi = x/\lambda_J$,

$$\omega_J = \sqrt{\frac{2e j_c}{\hbar C}} \tag{2}$$

is the Josephson plasma frequency, $C$ is the specific capacitance of the junction, $j_c$ is the critical current density of the Josephson junction,

$$\lambda_J = \sqrt{\frac{c\Phi_0}{16\pi^2\lambda j_c}} \tag{3}$$

is the Josephson penetration length, $\Phi_0$ is the flux quantum, $\lambda$ is the London penetration depth,

$$\eta = \frac{1}{\omega_J RC} \tag{4}$$

is the damping constant, $R$ is the specific resistance of the junction, and $\beta = j/j_c$ is the dimensionless density of the bias current across the junction.
The well-known solution of Eq. (1)

\[ \varphi_0(x, t) = 4 \tan^{-1}\left[ \frac{x - vt}{\lambda J \sqrt{1 - v^2/c_s^2}} \right] \]

(5)
describes the uniform motion of a Josephson vortex with a certain velocity \( v \) in the case of zero dissipation and zero driving force (\( \gamma = \beta = 0 \)). It follows from Eq. (5) that in a long Josephson junction a Josephson vortex moves similar to a relativistic particle with the highest possible velocity \( c_s = \lambda J \omega_J \) (Swihart velocity)\[\text{.}\]

An electromagnetic wave with a specific dispersion relation exists in a long one-dimensional Josephson junction\[\text{.}\] The solution of Eq. (1) in the form of a plain wave with a small amplitude

\[ \varphi(x, t) = \varphi_a \exp(-i\omega t + ikx), \quad |\varphi_a| \ll 1 \]

(6)
describes this electromagnetic wave.

Let us consider the case of zero dissipation (\( \eta = 0 \)). Then, the relation between the frequency \( \omega \) and the wave vector \( k \) is given by the formula\[\text{.}\]

\[ \omega = \omega_J \sqrt{1 + \lambda_J^2 k^2}, \]

(7)
and the phase velocity of this electromagnetic wave \( v_\varphi \) is equal to

\[ v_\varphi = \frac{\omega}{k} = c_s \sqrt{1 + \frac{1}{k^2 \lambda_J^2}}, \]

(8)

We can determine the phase difference \( \varphi(x, t) \) in the mainframe of local Josephson electrodynamics, \textit{i.e.}, by the sin–Gordon equation, as long as \( \lambda \ll l_\varphi \), where \( l_\varphi \) is the characteristic space scale of \( \varphi(x, t) \). It means, in particular, that Eqs. (5), (7) and (8) are valid if \( \lambda \ll \lambda_J \) and \( k\lambda \ll 1 \). We show the dependence \( v_\varphi(k) \) calculated by means of Eq. (8) by a solid line in Fig. 1.

In the range of applicability of Eq. (8) the phase velocity of the electromagnetic wave in a long Josephson junction is a monotonically decreasing function of the wave vector. It follows from Eq. (8) that for \( k\lambda \ll 1 \) the value of \( v_\varphi \) is higher than \( c_s \). Since \( c_s \) is
the highest possible velocity of a Josephson vortex it means that there is no Josephson vortex Cherenkov radiation[6] of an electromagnetic wave in the approximation of the local Josephson electrodynamics.

Let us now consider the general case, i.e., the case when the only restriction on the length scale $l_\varphi$ is given by the inequality $\xi \ll l_\varphi$, where $\xi$ is the correlation length. The relation between the phase difference $\varphi(x,t)$ and the magnetic field in the banks of the Josephson junction is nonlocal if $l_\varphi < \lambda$. It results in the nonlocal Josephson electrodynamics[8-10]. In the general form the equation generalizing Eq. (1) reads:

$$\varphi_{\tau\tau} + \eta \varphi_\tau = \frac{\lambda J^2}{\pi \lambda} \int_{-\infty}^{\infty} K_0\left(\frac{|x-u|}{\lambda}\right) \frac{\partial^2 \varphi}{\partial u^2} du - \sin \varphi + \beta,$$

(9)

where $K_0(x)$ is the zero order modified Bessel function. The integro–differential equation (9) is valid as long as $\xi \ll l_\varphi$.

Using Eqs. (6) and (9) we find the dispersion relation $\omega(k)$ for an electromagnetic wave in a long Josephson junction in the mainframe of the nonlocal Josephson electrodynamics. In the case of zero dissipation ($\eta = 0$) it has the form

$$\omega = \omega_J \sqrt{1 + \frac{k^2 \lambda^2}{\sqrt{1 + k^2 \lambda^2}}},$$

(10)

and thus the phase velocity of this electromagnetic wave is equal to

$$v_\varphi = \frac{\omega}{k} = c_s \sqrt{\frac{1}{\sqrt{1 + k^2 \lambda^2}} + \frac{1}{k^2 \lambda^2}}.$$

(11)

Note, that the expressions given by Eqs. (8) and (11) for $v_\varphi$ coincide if $k \lambda \ll 1$, i.e., in the range of validity of the local Josephson electrodynamics.

It follows from Eq. (11) that the electromagnetic wave phase velocity $v_\varphi$ given by Eq. (11) is a monotonically decreasing function of $k$. The value of $v_\varphi$ tends to zero when the wave vector $k$ tends to infinity. In particular, in the limiting case $k \lambda \gg 1$ we have[3]:

$$v_\varphi \approx \frac{c_s}{\sqrt{k \lambda}} \ll c_s, \quad k \lambda \gg 1.$$

(12)

We show the dependence $v_\varphi(k)$ given by Eq. (11) by the dashed line in Fig. 1. We use for this plot the value $\lambda_J = 5 \lambda$, it i.e., $\lambda_J \gg \lambda$. 5
Thus, there exist a certain region \( k \geq k_c \), where the phase velocity of an electromagnetic wave in a long Josephson junction is lower than the highest possible velocity of a Josephson vortex. We have the equation \( v_\varphi(k_c) = c_s \) to find the value of \( k_c \). In case when \( \lambda \ll \lambda_J \) the solution of this equation is given by an approximate formula

\[
k_c \approx \frac{1}{\lambda_J} \sqrt{\frac{2\lambda_J}{\lambda} + \frac{3}{4}}.
\]

(13)

The existence of an electromagnetic wave with the phase velocity lower than \( c_s \) results in the Josephson vortex Cherenkov radiation. This dissipation mechanism is especially effective when the Josephson vortex velocity is approaching the highest possible velocity \( c_s \).

The Josephson vortex Cherenkov radiation results, in particular, in a friction force acting on the radiating vortex. In order to find this radiation friction force we solve the following problem. Let us consider the uniform motion of a Josephson vortex in a long Josephson junction. We treat the velocity of this motion \( v \) as a given constant value. We use for calculations the perturbation theory, \( i.e. \), we neglect the dissipation arising due to the resistance of the junction while considering the Josephson vortex Cherenkov radiation of an electromagnetic wave.

In order to find the amplitude of the radiated wave we look for a solution of Eq. (9) with \( \eta = 0 \) and \( \beta = 0 \) in the form

\[
\varphi(x, t) = \varphi_0(x - vt) + f(x, t),
\]

(14)

where \( \varphi_0(x - vt) \) is the phase difference given by Eq. (5) for a single uniformly moving Josephson vortex and \( |f(x, t)| \ll 1 \).

A straightforward calculation shows that in the region behind the front of the nonlinear wave \( \varphi_0(x - vt) \), \( i.e. \), in the region \( (vt - x) \gg \lambda_J \sqrt{1 - v^2/c_s^2} \), the function \( f(x, t) \) is a plain wave taking the form

\[
f(x, t) = f_0(k_p) \theta(vt - x) \exp[ik_p(x - vt)],
\]

(15)

where \( \theta(x) \) is the \( \theta \)–function. The amplitude of this plain wave \( f_0(k_p) \) is given by the following formula
\[ f_0(k_p) = \pi \frac{v}{v_p - v} \frac{k_p^2 c_s^2 - \omega_p^2 + \omega_J^2}{\omega_p^2} \cosh(0.5 \pi k_p \lambda_J \sqrt{1 - v^2/c_s^2}) \], \quad (16)

where the wave vector \( k_p \) is the root of the equation

\[ \omega(k_p) = k_p v, \] \quad (17)

the dispersion relation \( \omega(k) \) is given by Eq. (10), the frequency of the radiated wave \( \omega_p = \omega(k_p) \), and \( v_p \) is the group velocity for a wave with \( k = k_p \)

\[ v_p = \frac{\partial \omega}{\partial k} \bigg|_{k_p}. \] \quad (18)

We explicitly use in the above calculations the phase difference \( \varphi_0(x, t) \) in the form given by Eq. (5). It is valid as long as \( l_\varphi = \lambda_J \sqrt{1 - v^2/c_s^2} \gg \lambda \), which means that Eq. (16) is valid if

\[ \left( \frac{\lambda}{\lambda_J} \right)^2 \ll 1 - \left( \frac{v}{c_s} \right)^2. \] \quad (19)

The amplitude \( f_0(k_p) \) is increasing when the velocity of the Josephson vortex is approaching the highest possible velocity \( c_s \). In the vicinity of \( c_s \), i.e., in the region \( c_s - v \ll c_s \), Eq. (16) can be simplified and the values of \( f_0(k_p) \), \( k_p \) and \( \omega_p \) can be found in an explicit analytical form. The result of these calculations is as follows

\[ f_0(k_p) \approx 2\pi \exp \left[ -\pi \sqrt{2} \frac{\lambda_J}{\lambda} \left( 1 - \frac{v}{c_s} \right) \right], \quad f_0(k_p) \ll 1 \]

\[ k_p \approx \frac{2}{\lambda} \sqrt{1 - v/c_s}, \quad \frac{1}{\lambda_J} \ll k_p \ll \frac{1}{\lambda}, \] \quad (20)

\[ \omega_p \approx 2\omega_J \frac{\lambda}{\lambda_J} \sqrt{1 - v/c_s}, \quad \omega_p \gg \omega_J. \] \quad (21)

Let us now find the radiation friction force \( f_r \) acting on a unit length of a uniformly moving Josephson vortex. To do it we use the energy conservation law equating \( f_r v \) and \( E_w v \), where \( E_w \) is the electromagnetic wave energy per unit area of the junction. It leads to the relation \( f_r = E_w \). Using the free energy functional corresponding to the nonlocal Josephson electrodynamics we find that

\[ f_0(k_p) \approx 2\pi \exp \left[ -\pi \sqrt{2} \frac{\lambda_J}{\lambda} \left( 1 - \frac{v}{c_s} \right) \right], \quad f_0(k_p) \ll 1 \]
\[ E_w = \frac{\Phi_0^2}{64\pi^3\lambda} \frac{\omega_p^2}{c_s^2} f_0^2(k_p). \]  
\text{(23)}

Thus, the Josephson vortex Cherenkov radiation of an electromagnetic wave results in a radiation friction force

\[ f_r = \frac{\Phi_0^2}{4\pi\lambda^3} \left(1 - \frac{v}{c_s}\right) \exp\left[-2\sqrt{2}\pi \frac{\lambda J}{\lambda} \left(1 - \frac{v}{c_s}\right)\right]. \]  
\text{(24)}

The expression given by Eq. (24) is valid until the absolute value of the exponent is bigger than one.

Let us now consider a uniform motion of a Josephson vortex in a Josephson junction with a bias current. In this case the vortex is subjected to the Lorentz force \( f_L \) and the radiation friction \( f_r \) forces. The value of \( f_L \) acting per unit length of the vortex is equal to \( \Phi_0 j/c \).

Equating \( f_L \) and \( f_r \) we obtain the following relation between the bias current density \( j \) and the velocity of a uniform motion of the Josephson vortex \( v \)

\[ \frac{j}{j_c} = 4\pi \frac{\lambda J^2}{\lambda^2} \left(1 - \frac{v}{c_s}\right) \exp\left[-2\sqrt{2}\pi \frac{\lambda J}{\lambda} \left(1 - \frac{v}{c_s}\right)\right]. \]  
\text{(25)}

A relation analogous to the one given by Eq. (25) and taking into account only the damping due to the resistance of the junction reads

\[ \frac{j}{j_c} = \frac{4\pi}{\eta} \frac{v}{\sqrt{c_s^2 - v^2}}. \]  
\text{(26)}

The dependence \( j(v) \) given by Eq. (26) is shown in Fig. 2 by the solid line. We use for this plot the value \( \eta = 0.05 \).

Let us consider the Josephson vortex Cherenkov radiation for \( \eta \neq 0 \). It follows from the energy conservation law that for \( \eta \ll 1 \) the dependence \( j(v) \) is a sum of the two dependencies given by Eqs. (25) and (26), \( i.e., \)

\[ \frac{j}{j_c} = 4\pi \frac{\lambda J^2}{\lambda^2} \left(1 - \frac{v}{c_s}\right) \exp\left[-2\sqrt{2}\pi \frac{\lambda J}{\lambda} \left(1 - \frac{v}{c_s}\right)\right] + \frac{4\pi}{\eta} \frac{v}{\sqrt{c_s^2 - v^2}} \]  
\text{(27)}

The dependence \( j(v) \) given by Eq. (27) is shown in Fig. 2 by the dashed line. We use for this plot the values \( \eta = 0.05 \) and \( \lambda J = 5\lambda \). It is seen from Fig. 2 that at \( j \sim j_c \) the value of \( v \) can be significantly less than \( c_s \).
The Josephson vortex velocity tends to a certain maximum, $v_m$, when the current density tends to the critical current density. Using Eq. (27) we can estimate $v_m$ as

$$1 - \frac{v_m}{c_s} \sim \frac{1}{2 \sqrt{2\pi}} \frac{\lambda}{\lambda_J}.$$  

(28)

It follows from Eq. (28) that a noticeable difference between $v_m$ and $c_s$ can be observed even if $\lambda_J > \lambda$.

Note that, when the Josephson vortex velocity tends to its maximum value $v_m$ the energy dissipation in the Josephson junction is mainly due to the Josephson vortex Cherenkov radiation, i.e., the power release happens in the form of electromagnetic radiation.

To summarize, we calculate the Josephson vortex Cherenkov radiation of an electromagnetic wave in a long Josephson junction. This dissipation mechanism results in the radiation friction force and is especially effective if the velocity of Josephson vortex is approaching the highest possible velocity $c_s$. We find the relation between the density of the bias current across the junction and the Josephson vortex velocity.

We are grateful to Dr. E. Polturak for useful discussions. This work was supported in part by the Foundation Raschi.
REFERENCES

1 A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982).

2 A.V. Ustivov, T. Doderer, R.P. Huebener, N.F. Pedersen, B. Mayer, and V.A. Oboznov, Phys. Rev. Lett. **69**, 1815 (1992).

3 A.V. Ustivov, T. Doderer, R.P. Huebener, B. Mayer and V.A. Oboznov, Europhys. Lett. **19**, 63 (1992).

4 A. Gurevich, Phys. Rev. B **48**, 12857 (1993).

5 R.G. Mints and I.B. Snapiro, Phys. Rev. B **49**, 6188 (1994).

6 P. Chaudhari *et al.*, Phys. Rev. Lett. **60**, 1653 (1988).

7 D. Dimos *et al.*, Phys. Rev. Lett. **61**, 219 (1988).

8 A. Gurevich, Phys. Rev. B **46**, 3187 (1992).

9 R.G. Mints and I.B. Snapiro, Physica A **200**, 426 (1993).

10 L.D. Landau, E.M. Lifshitz and L.P. Pitaevskii, *Electrodynamics of Continuous Media* (Pergamon Press, Oxford, 1984).

11 D.W. McLaughlin and A.C. Scott, Phys. Rev. A **18**, 1652 (1978).
FIGURES

FIG. 1. The dependence of the phase velocity $v_{ϕ}$ on the wave vector $k$. The solid line represents a plot using Eq. (8), the dashed line represents a plot using Eq. (11).

FIG. 2. The dependence of the bias current density $j$ on the Josephson vortex velocity $v$. The solid line represents a plot using Eq. (26), the dashed line represents a plot using Eq. (27).