Multisensor Data Fusion in Testability Evaluation of Equipment

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The multisensor data fusion method has been extensively utilized in many practical applications involving testability evaluation. Due to the flexibility and effectiveness of Dempster–Shafer evidence theory in modeling and processing uncertain information, this theory has been widely used in various fields of multisensor data fusion method. However, it may lead to wrong results when fusing conflicting multisensor data. In order to deal with this problem, a testability evaluation method of equipment based on multisensor data fusion method is proposed. First, a novel multisensor data fusion method, based on the improvement of Dempster–Shafer evidence theory via the Lance distance and the belief entropy, is proposed. Next, based on the analysis of testability multisensor data, such as testability virtual test data, testability test data of replaceable unit, and testability growth test data, the corresponding prior distribution conversion schemes of testability multisensor data are formulated according to their different characteristics. Finally, the testability evaluation method of equipment based on the multisensor data fusion method is proposed. The result of experiment illustrated that the proposed method is feasible and effective in handling the conflicting evidence; besides, the accuracy of fusion of the proposed method is higher and the result of evaluation is more reliable than other testability evaluation methods, which shows that the basic probability assignment of the true target is 94.71%.

1. Introduction

Testability evaluation, as an important part of testability design of equipment, is often used to test and evaluate whether the equipment meets the development requirements [1]. Through the evaluation of testability level, the defects of testability design can be found, which is related to the progress of finalization process of the equipment design. Since the 21st century, testability, as one of the general quality characteristics of equipment, has been paid more and more attention by equipment manufacturers and users. With the development of equipment testability engineering, testability evaluation method has become one of the hot topics in testability field.

At present, many scholars have studied the testability evaluation method of equipment. Lin et al. [2, 3], Yang et al. [4], Liu et al. [5], and Zhou and Liu [6] proposed a method based on correlation and multisignal model for testability analysis and research by using the method of model prediction. However, in the actual testability evaluation work, it is difficult to carry out a comprehensive testability test, so the accuracy of the evaluation results obtained by model prediction method is low when the test data is small.

To solve this problem, Li et al. [7–9], Zhou et al. [10, 11], and Chang et al. [12, 13] proposed a testability evaluation method based on Bayes theory. Although it can effectively use prior information and expand the available data volume of testability assessment, it uses less information sources, and it is difficult to conduct testability assessment comprehensively and accurately.

In order to make full use of testability prior information, Liang and Zhang [14] proposed a testability evaluation method in the development stage based on information fusion. The classical Dempster–Shafer evidence theory was used to fuse multisource prior information, and the testability level was evaluated by analyzing the fusion results. Deng et al. [15] used evidence discount combination method to evaluate the current testability level of equipment by modifying the weight of prior information in Dempster–Shafer evidence fusion. The above two methods make full use of testable data from different sources, but considering that prior information such as expert experience...
information has certain subjectivity and uncertainty, it is easy to cause conflicts in the process of data fusion. Therefore, how to solve the conflict fusion problem of testability data becomes the focus of this paper.

To summarize all the different reviewed methods, testability evaluation method based on mathematical model prediction can effectively describe the relationship between test and fault of equipment, but the simplification of logical relationship between them will reduce the accuracy of prediction. Testability evaluation method based on Bayes theory can effectively use prior information and expand the available data of testability assessment, but it uses fewer information sources, so it is difficult to evaluate comprehensively and accurately. Although the testability evaluation method based on Dempster–Shafer evidence theory can make full use of testable information, increase the sources of evaluation data, and improve the confidence of testability assessment to a certain extent, due to the uncertainty and subjectivity of testability information and the conflict between data, it may have adverse effects on the final evaluation results. Therefore, this paper intends to solve the conflict problem in the process of testable information fusion and carry out the next research.

The technology of multisensor data fusion has been extensively utilized in many practical applications, such as the risk analysis [16, 17], decision-making [18, 19], fault diagnosis [20–23], wireless sensor networks [24–28], health prognosis [29], image processing [30], target tracking [31, 32], surveillance [33, 34], and so forth [35–38]. Multisensor data fusion can automatically analyze and synthesize the information and data from multiple sensors or multiple sources under certain criteria, so as to complete the information processing process for the required decision-making and estimation. In addition, the technology of data fusion based on the Dempster–Shafer evidence theory and maximum-entropy methods has been applied in presence of uncertainty [39–41]. Hence, the technology of multisensor data fusion is widely applied in many fields of real applications [42, 43]. Multisensor data fusion methods are based on many mathematical approaches: the rough sets theory [44, 45], fuzzy sets theory [46–53], Dempster–Shafer evidence theory [54–58], Z numbers [59, 60], D numbers theory [61–65], evidential reasoning [66–68], and so on [69–72].

Dempster–Shafer evidence theory is a powerful tool to represent and deal with uncertain information. It has been widely used in practical problems related to uncertainty modeling and reasoning, such as information fusion [49, 73–75], fault diagnosis [22, 76–81], decision-making [65, 82–85], risk assessment [86–90], multicriteria decision-making [91, 92], and pattern recognition [95–96]. However, when Dempster–Shafer evidence theory is used for fusion, due to the uncertainty of evidence, it is difficult to avoid the problem of evidence conflict.

In this paper, multisensor data fusion method is based on the improved Dempster–Shafer evidence theory which is first proposed by measuring the Lance distance between the bodies of the evidence and the belief entropy of the evidence. Based on that, the testability evaluation method of equipment is proposed by the multisensor data fusion method. The proposed method considers both of the reliability degree among the multisensor data and the uncertainty measure of the multisensor data on the weight, so that it can measure the degree of conflict of the multisensor data and fuse the conflicting multisensor data effectively. Consequently, the proposed method consists of the following procedures. Firstly, an improvement of Dempster–Shafer evidence theory is proposed, which is improved by the Lance distance function and the belief theory function, and the results of simulation illustrate that the fusion method has priority in the fusion of conflicting evidence. After that, according to the characteristics of testability multisensor data, the corresponding conversion schemes are proposed, which can be used directly in the framework of Dempster–Shafer evidence theory. Finally, the process steps of the testability evaluation of equipment based on the multisensor data fusion method are proposed. According to the analysis of the experimental results, it is found that the proposed method can effectively reduce the adverse effects of conflicting multisensor data fusion compared with other testability evaluation methods; in addition, the fusion accuracy of the proposed method is higher and the evaluation result is more reliable.

The rest of this paper is organized as follows. Section 2 briefly introduces the preliminaries of this paper. In Section 3, the multisensor data fusion method is proposed, which is based on the improvement of the Dempster–Shafer evidence theory. The focus of Section 4 is to show the conversion schemes of testability multisensor data. In Section 5, the testability evaluation model of equipment based on the multisensor data fusion method is proposed. Section 6 concludes this paper.

The contributions of this research are summarized as follows:

1. Aiming at different types of testability data, the corresponding conversion scheme of prior distribution is proposed
2. Aiming at the conflict phenomenon in the process of evidence fusion, the multisensor data fusion method is proposed, which is based on the Dempster–Shafer evidence theory and is improved by introducing Lance distance and belief entropy
3. This paper proposes a new multisensor data fusion method in testability evaluation of equipment: it can effectively reduce the conflict in the process of evidence fusion and obviously improve the reliability and accuracy of testability evaluation results

2. Preliminaries

2.1. Dempster–Shafer Evidence Theory

Evidence theory was first put forward by Professor A. P. Dempster in 1967 and then perfected by G. Shafer and became a complete and independent data system, so it is also called Dempster–Shafer evidence theory. With the continuous improvement of P. Smets and T. Denoeux, evidence theory has developed into the theory of reliability function, which can model the
uncertainty of data and deal with the uncertainty and imprecision of data well. Therefore, it is widely used to solve the problem of information fusion.

In order to explain the basic principles of evidence theory in detail, this section will introduce the four core parts of evidence theory in detail: the frame of discernment, mass function, belief function and plausibility function, and Dempster’s rule of combination.

**Definition 1.** (the frame of discernment).
Let $\Theta$ be a set of mutually exclusive and collectively exhaustive events, indicated by

$$\Theta = \{E_1, E_2, \ldots, E_N\}. \quad (1)$$

The set $\Theta$ is defined as a frame of discernment. A power set $2^\Theta$ is the set of all possible subsets of $\Theta$.

$$2^\Theta = \{\emptyset, \{E_1\}, \{E_2\}, \ldots, \{E_N\}, \{E_1, E_2\}, \ldots, \{E_1, E_2, \ldots, E_N\}, \ldots, \Theta\}, \quad (2)$$

where $\emptyset$ is an empty set.

If $H \in 2^\Theta$, $A$ is called a proposition or hypothesis.

**Definition 2.** (mass function).
Suppose that $\Theta$ is a frame of discernment; $A$ denotes any subset in the frame of discernment $\Theta$, $\forall A \in \Theta$. Mass function is a mapping $m$ from $2^\Theta$ to $[0, 1]$, defined as

$$m(\emptyset) = 0, \quad \sum_{A \in \Theta} m(A) = 1. \quad (3)$$

In Dempster–Shafer evidence theory, a mass function can be also called a basic probability assignment (BPA). If $m(A) > 0$, $A$ is a focal element, and the set of all focal elements is called the core of evidence.

**Definition 3.** (belief function and plausibility function).
Let $A$ be a hypothesis in the frame of discernment $\Theta$. Belief function $\text{Bel}: 2^\Theta \rightarrow [0, 1]$ is defined as

$$\text{Bel}(A) = \sum_{B \subseteq A} m(B). \quad (4)$$

Plausibility function $\text{Pl}: 2^\Theta \rightarrow [0, 1]$ is defined as

$$\text{Pl}(A) = 1 - \text{Bel}(\overline{A}) = \sum_{B \cap A = \emptyset} m(B), \quad (5)$$

where $\overline{A} = \Theta - A$.

Bel($A$) and Pl($A$), respectively, represent the lowest and highest trust degree of proposition $A$, corresponding to the lower bound and upper bound of probability. Pl($A$) ≥ Bel($A$), the D-value of Pl($A$) – Bel($A$) reflects the uncertainty of proposition $A$.

**Definition 4.** (Dempster’s rule of combination).
Assume that the system’s frame is $\Theta = \{A_1, A_2, \ldots, A_M\}$ and two independent mass functions $m_1$ and $m_2$ are in the frame of discernment $\Theta$. Dempster’s rule of combination, represented in the form $m = m_1 \oplus m_2$, is defined as follows:

$$m(A) = \begin{cases} \frac{\sum_{A \cap A_i = \emptyset} m_1(A_i) \cdot m_2(A_j)}{1 - K}, & A \neq \emptyset, \\ 0, & A = \emptyset, \end{cases} \quad (6)$$

with

$$K = \sum_{A \cap \emptyset = \emptyset} m_1(A_i) \cdot m_2(A_j), \quad (7)$$

where $A_i, A_j \in 2^\Theta$ and $K$ is the conflict coefficient, which is used to measure the degree of conflict between BPAs $m_1$ and $m_2$. The greater the conflict between two bodies of evidence is, the greater the value of conflict coefficient $K$ will be. Note that Dempster’s rule of combination is feasible with the condition that $K < 1$.

Dempster’s rule of combination can be regarded as the operation of orthogonal sum of two bodies of evidence, which meanwhile satisfies the law of exchange and combination, and can be extended to the combination of $N$ bodies of evidence.

2.2. Lance Distance. With the increasing application of Dempster–Shafer evidence theory, the research based on evidence distance is favored by scholars at home and abroad. The distance measurement between the bodies of evidence indicates the mutual support between them. Therefore, there are many methods to measure the distance between the bodies of evidence, including Minkowski distance and Mahalanobis distance. However, Minkowski distance function requires matrix covariance calculation, which is not suitable for large-scale data processing. Therefore, this paper introduces a more ideal distance measurement as the Lance distance function.

Lance distance is a common method used to determine the distance between samples in cluster analysis, which was first proposed by Lance and Williams. Lance distance is a dimensionless quantity, which overcomes the shortcoming that Minkowski distance is related to the dimension of each index, and it is not sensitive to large singular values, which makes it particularly suitable for height offset and data. The main concepts are introduced as blow.

**Definition 5.** (Lance distance function).
Assuming that the system’s frame is $\Theta = \{A_1, A_2, \ldots, A_M\}$ and there are $N$ pieces of evidence $E_1, E_2, \ldots, E_N$ in system, the mass functions of evidence are $m_1, m_2, \ldots, m_N$, where $m_i = [m_1(A_1), m_1(A_2), \ldots, m_1(A_M)]$. Suppose that there are two pieces of evidence $E_i, E_j$ and the Lance distance between the pieces of evidence $E_i, E_j$ is

$$d_{ij} = d(E_i, E_j) = \frac{1}{M} \sum_{x=1}^{M} \left| m_x(A_i) - m_x(A_j) \right|, \quad (8)$$

where $i, j = 1, 2, \ldots, N$ and $x = 1, 2, \ldots, M$. 
The Lance distance function is very sensitive to small changes close to \( m_i(A_x) = 0 \) and \( m_i(A_x) = 0 \). It is generally considered to be a generalization of binary data dissimilarity measure [97]. According to (8), \( d_{ij} \) is a dimensionless attribute and is not sensitive to large singular values. Therefore, \( d_{ij} \) overcomes the defects of Minkowski distance function and Mahalanobis distance function and is more suitable for evidence revision.

2.3. Belief Entropy. Information entropy is defined as the amount of information needed to describe the state of random variables. Intuitively, the wider the distribution range of random variables, the more the information needed to describe them, because the values of random variables need to be specified in a larger range. Information entropy can also be used as the discrete degree of variables or as a measure of uncertainty. In Dempster–Shafer evidence theory, Shannon proposed Shannon entropy to measure the degree of uncertainty of evidence on the basis of information entropy, while Deng Yong’s belief entropy is a general improvement of Shannon entropy [98]. Therefore, this paper introduces the belief entropy to measure the uncertainty of evidence. The basic concepts are introduced as follows.

**Definition 6.** (belief entropy function).

Assume that the system’s frame is \( \Theta = \{ A_1, A_2, \ldots, A_M \} \), \( A_x \) is a hypothesis of the BPA \( m \), and \( |A_x| \) is the cardinality of set \( A_x \). Belief entropy \( E_d \) of set \( A_x \) is defined as follows:

\[
E_d = - \sum_{A_x \in \Theta} m(A_x) \log \frac{m(A_x)}{|A_x| - 1}.
\]

(9)

When the belief value is only assigned to a single element, that is, \( |A_x| = 1 \), the belief entropy degenerates to Shannon entropy [99], i.e.,

\[
E_d^* = - \sum_{A_x \notin \Theta} m(A_x) \log m(A_x).
\]

(10)

It indicates that the greater the belief entropy is, the more information the evidence contains, and the greater the uncertainty of evidence is. When the evidence has large belief entropy, it is supposed to have more support from other evidence, which indicates that this evidence will play an important role in the final evidence combination.

3. The Improvement of Dempster–Shafer Evidence Theory

To improve the accuracy, validity, and reliability of multi-sensor data fusion, the proposed method adopts the Lance distance function and the belief entropy to improve the Dempster–Shafer evidence theory. In addition, the algorithm of the proposed method is presented in this section.

3.1. Process Steps

**Step 1.** Measure the reliability degree of the evidence.

**Step 2.** Measure the uncertainty degree of the bodies of evidence.

In order to avoid entrusting zero weight to the evidence in some cases, this paper determines the uncertainty degree \( U_i \) of the body of evidence \( E_i \) by calculating the exponential form of belief entropy as follows:

\[
U_i = e^{E_d} = e^{-\sum_{A_x \in m_i(A_x) \log m(A_x)}/|A_x| - 1}.
\]

(14)

The uncertainty of evidence, which is denoted as \( \text{Unc}_i \), is normalized as follows:

\[
\text{Unc}_i = \frac{U_i}{\sum_i U_i}.
\]

(15)

**Step 3.** Generate and fuse the final weight of evidence based on the reliability degree and uncertainty degree.

Based on the reliability degree \( \text{Re}_i \), the uncertainty degree \( U_i \) of the evidence \( m_i \), denoted as \( W_i \), will be adjusted:

\[
W_i = \text{Re}_i \times \text{Unc}_i.
\]

(16)

The weight \( W_i \) that is considered as the final weight in terms of each evidence \( m_i \) is normalized:

\[
\overline{W_i} = \frac{W_i}{\sum_i W_i}.
\]

(17)

**Step 4.** Determine the revised evidence.

On the basis of the weighting factor of mass function \( m_i \) of the evidence \( E_i \), mass function \( m_{\text{rev}} \) of the revised evidence can be obtained as follows:
$m_{Re} = \sum_{i=1}^{N} (\mathbf{W}_i \cdot m_i)$. \hspace{1cm} (18)

**Step 5. Evidence fusion.**

According to Dempster’s rule of combination, the mass function is synthesized $N-1$ times ($N-1$ is the number of pieces of evidence); $\oplus$ follows Dempster’s rule of combination. The result $m_{Fus}$ of evidence fusion is as follows:

$$m_{Fus} = ((m_{Re} \oplus m_{Re} \oplus \cdots) \oplus m_{Re})_{N-1} \cdot ... \oplus m_{Re}$$ \hspace{1cm} (19)

3.2. Algorithm. When there are $N$ pieces of evidence $E_1, \ldots, E_n, \ldots, E_N$ collected, the masses of evidence are $m_1, \ldots, m_n, \ldots, m_N$. After collecting $N$ pieces of evidence, a fusion result is expected to be generated for the later research. The pseudocode for the proposed method of evidence fusion is presented in Algorithm 1.

3.3. Simulation and Result. In order to verify the efficiency and effectiveness of the improved evidence fusion method based on Lance distance and belief entropy proposed in this paper, the complete example of [100] is used for experiment, the analysis and comparison with Dempster’s combination method and other combination methods are carried out, and the final fusion results are analyzed in detail under the conditions of consistent evidence and conflicting evidence.

3.3.1. Evidence Fusion with Consistent Evidence. Assume that the frame of evidence system is $\Theta = \{A_1, A_2, A_3\}$, the evidence set is $E = \{E_1, E_2, E_3, E_4, E_5\}$, and the mass function set of evidence is $m = \{m_1, m_2, m_3, m_4, m_5\}$. Suppose that $A_1$ is the true target; the mass function of consistent evidence is shown in Table 1.

It is evident in Table 1 that all evidence is relatively consistent in identifying target $A_1$. Thus, the target $A_1$ should be given the biggest support by the effective fusion method.

Then, the fusion results of different methods are obtained in Table 2.

According the fusion results of different methods, BPAs of different methods with consistent evidence are shown in Figure 1.

According to Table 2 and Figure 1, we can get the following analyses about the existing modified methods:

1. In the fusion results of Dempster’s combination method, although the BPA for target $A_1$ is the biggest, the BPAs for targets $A_2$ and $A_3$ are 0, which do not match the original evidence source.

2. In the fusion results of Deng et al.’s method, the BPA for target $\Theta$ is 0.2760, which means that the probability assigned to unknown items is too high in evidence fusion, leading to low accuracy of fusion.

3. Compared with Ye et al.’s combination method, the BPA for target $A_1$ of proposed method is higher. Thus, the fusion result of the proposed method gets higher accuracy.

3.3.2. Evidence Fusion with Conflicting Evidence. In order to verify the effectiveness and reliability of the proposed method, conflict evidence is modified and generated based on the original evidence source in Section 3.3.1. The mass function of conflicting evidence is shown in Table 3.

Analysis of the data in Table 3 shows that the bodies of evidence $E_1, E_2, E_3, E_4, E_5$ support the target $A_1$, while the body of evidence $E_2$ supports the target $A_2$. Obviously, evidence $E_2$ is the conflicting evidence. After that, the fusion results of the different methods are exhibited in Table 4.

According the fusion results of different methods, BPAs of different methods with conflicting evidence are shown in Figure 2.

According to Table 4 and Figure 2, we can get the following analyses about the existing modified methods:

1. The fusion result of Dempster’s combination method completely believes that $A_1$ is the true target, which is contrary to intuition judgement. Obviously, Dempster’s combination method cannot effectively handle the fusion of conflicting evidence.

2. In the fusion result of Deng et al.’s combination method, the BPA for target $\Theta$ is 0.3863, which illustrates that the uncertainty in the fusion result is still high. Thus, Deng et al.’s combination method may lead to the error in the fusion of conflicting evidence.

3. Compared with Ye et al.’s combination method, the BPA for target $A_1$ of proposed method is higher. Thus, the fusion result of the proposed method gets higher accuracy.

3. Compared to other methods, the proposed method has a better convergence effect that can deal with conflicting situations more effectively. Thus, it has priority in the fusion of conflicting evidence.

The fusion times of all methods are the same, all of which are $n - 1$. In order to obtain the value of mass function for fusion, the proposed method in this paper calculates Lance distance and belief entropy, and it has more steps than other methods, but this work is to measure the conflict degree of evidence, so the complexity of calculation is inevitable and acceptable.

4. Conversion Schemes of Testability Multisensor Data

There are many kinds of testability data types in equipment design, production, and use, such as testability virtual test data, testability test data of replaceable unit, testability growth test data, and other multisensor data. In order to ensure that the testability multisensor data can be used in the framework of Dempster–Shafer evidence theory, the various

These four methods all get the reasonable fusion results and recognize the true target $A_1$ precisely, but the fusion accuracy of the proposed method is higher and more reliable.
testability data is converted into a unified prior distribution form, that is, to solve the prior distribution parameters $a, b$.

4.1. Testability Virtual Test Data. With the development of testable modeling and simulation technology, the application of virtual test technology has been paid attention to.

| Evidence | Targets |
|----------|---------|
| $E_1$: $m_1(\cdot)$ | $A_1$ | $A_2$ | $A_3$ |
| $E_2$: $m_2(\cdot)$ | 0.90 | 0 | 0.10 |
| $E_3$: $m_3(\cdot)$ | 0.88 | 0.01 | 0.11 |
| $E_4$: $m_4(\cdot)$ | 0.50 | 0.20 | 0.30 |
| $E_5$: $m_5(\cdot)$ | 0.98 | 0.01 | 0.01 |

Table 1: Mass function of consistent evidence.

Some achievements have been made in the construction of testable virtual prototype and the implementation of virtual test. In the process of testable virtual test, considering that there are different types and levels of units in the system, the difficulty of modeling is different, so there are corresponding testable virtual tests.

In the current research, the data type of testable virtual test is unit level data, and the data type is success or failure.
According to the characteristics of this kind of data, this paper introduces the information entropy theory to solve the conversion problem from unit level data to system level data.

Suppose a virtual test system is composed of $M$ independent units. According to the theory of information entropy, each unit corresponds to a source. Suppose the test data expression form of the $i$-th unit is $(n_i, c_i)(i = 1, 2, \ldots, m)$, where $n_i$ is the total number of tests of the $i$-th unit, and $c_i$ is the number of test failures in the $i$-th unit. Assuming that the probability of success in each test in the $i$-th unit is $p_i$, the probability of failure is $1 - p_i$, and the average amount of information provided by the $i$-th unit in a test is

$$H_i = -\left[p_i \ln p_i + (1 - p_i) \ln (1 - p_i)\right].$$  \hfill (20)

Then the amount of information provided by the $i$-th unit is $I_i = n_i H_i$, and for a system composed of $m$ units, the total amount of information provided by it is

$$I = \sum_{i=1}^{m} I_i = \sum_{i=1}^{m} n_i H_i = -\sum_{i=1}^{m} n_i \left[p_i \ln p_i + (1 - p_i) \ln (1 - p_i)\right].$$  \hfill (21)

Assuming that $a$ is the number of unit tests after equivalent reduction, $b$ is the number of failures, $p$ is the probability of the unit succeeding in the test, and $1 - p$ is the probability of failure, then the equivalent amount of information of the unit in $a$ tests is

$$I' = -a[p \ln p + (1 - p) \ln (1 - p)].$$  \hfill (22)

According to the principle of information equivalence, the total information provided by each unit test data before equivalent conversion is equal to the total information provided by system data after equivalent conversion, i.e., $I = I'$. For the success probability before and after the data conversion, take the corresponding maximum likelihood estimation value; that is, for each unit, take $\bar{p}_i = n_i - c_i/n_i$, and for the system, take $\bar{p} = a - b/a$; then, we can get the formula of conversion from each unit level to the system level:

| Evidence | Targets | $A_1$ | $A_2$ | $A_3$ |
|----------|---------|-------|-------|-------|
| $E_1$: $m_1(\cdot)$ | 0.90 | 0 | 0.10 |
| $E_2$: $m_2(\cdot)$ | 0 | 0.01 | 0.99 |
| $E_3$: $m_3(\cdot)$ | 0.50 | 0.20 | 0.30 |
| $E_4$: $m_4(\cdot)$ | 0.98 | 0.01 | 0.01 |
| $E_5$: $m_5(\cdot)$ | 0.90 | 0.05 | 0.05 |

| Evidence | Targets | $A_1$ | $A_2$ | $A_3$ |
|----------|---------|-------|-------|-------|
| $E_1$: $m_1(\cdot)$ | 0.90 | 0 | 0.10 |
| $E_2$: $m_2(\cdot)$ | 0 | 0.01 | 0.99 |
| $E_3$: $m_3(\cdot)$ | 0.50 | 0.20 | 0.30 |
| $E_4$: $m_4(\cdot)$ | 0.98 | 0.01 | 0.01 |
| $E_5$: $m_5(\cdot)$ | 0.90 | 0.05 | 0.05 |

Figure 1: BPAs of different methods with consistent evidence: (a) BPAs for different objects; (b) BPAs for target $A_1$.
4.2. Testability Test Data of Replaceable Unit. The system level testability test of equipment needs the cooperation and integration of each replaceable unit, sharing the model and parameters of testability design, and considering the interaction of environment, which is often difficult to achieve. Therefore, in the design and development stage, there is a lack of corresponding test means to evaluate the FDR/FIR of replaceable units in a system, if the testability index of the $i$-th replaceable unit is $p_i$, then its prior distribution $\pi_R(p)$ can be expressed as follows:

$$
\pi_R(p_i) = \text{Beta}(p_i; a_i, b_i) = \frac{p_i^{a_i-1} (1-p_i)^{b_i-1}}{B(a_i, b_i)}.
$$

(24)

We use a small amount of testability test data $(n_i, f_i)$ collected from the replaceable unit, where $n_i$ is the number of fault samples injected by the $i$-th replaceable unit and $f_i$ is the corresponding failure times of fault detection/isolation. By substituting the data into Bayes formula, the posterior distribution Beta $(p_i; a_i + n_i - f_i, b_i + f_i)$ of $p_i$ can be obtained. Then, the posterior expectation and variance of $p_i$ are

$$
\begin{align*}
E(p_i) &= \frac{a_i + n_i - f_i}{a_i + b_i + n_i}, \\
\text{Var}(p_i) &= \frac{(a_i + n_i - f_i)(a_i + n_i - f_i + 1)}{(a_i + b_i + n_i)(a_i + b_i + n_i + 1)}.
\end{align*}
$$

(25)

By introducing the failure rate of the replaceable unit, the testability index value of the unit level can be converted into the system level. Let $\lambda_i$ denote the failure rate of the $i$-th replaceable unit; then, the testability index $p$ of the system level is
A mathematical model is used to describe the growth process. It can not only assess the reliability of the design, which can be summarized as the process of "Test-detect defects, detecting feedback problems, and improving design measures of a type of special test. Testability growth mainly includes three processes: identifying failure or failure of testability index to achieve the goal, and then improve the testability design and verify the improvement measures. For example, in reliability, it is necessary to choose different methods to calculate the value according to the structure model, reliability, and maintainability of equipment. In this paper, we do not consider the influence of the coupling relationship of each unit on the testability index of the system level, but we use the weighted method based on the unit failure rate to get the testability index of the system level by weighting the testability index of the unit level.

The expectation and variance of testability index $p$ at system level are obtained as follows:

$$
E(p) = \frac{\sum_{i=1}^{m} \lambda_i E(p_i)}{\sum_{i=1}^{m} \lambda_i} = \frac{a}{a + b},
$$

$$
\text{Var}(p) = \frac{\sum_{i=1}^{m} \lambda_i \text{Var}(p_i)}{\sum_{i=1}^{m} \lambda_i} = \frac{a(a + 1)}{(a + b + 1)(a + b)}. \quad (27)
$$

According to (25) and (27), the testability prior distribution parameters $a, b$ of replaceable units before data fusion can be solved.

4.3. Testability Growth Test Data. Testability growth test is to inject fault into equipment through testability design to make it run under specified environmental stress, observe and count the performance of fault detection/isolation in the test system, find out the cause of failure detection/isolation failure or failure of testability index to achieve the goal, and then improve the testability design and verify the improvement measures of a type of special test. Testability growth test mainly includes three processes: identifying design defects, detecting feedback problems, and improving design, which can be summarized as the process of "Test-Analysis-Improvement-Test."

The Gompertz model is often used to describe the growth process. It can not only assess the reliability of products, but also be used to solve test growth curves. Its mathematical model is [102]

$$
p(i) = uv^w, \quad (28)
$$

where $0 < u < 1, 0 < v < 1, 0 < w < 1$, and $i$ indicates that the equipment is in the $i$-th testability growth test in the development stage.

By logarithmic transformation of (28), the following results can be obtained:

$$
\ln[p(i)] = \ln u + w^i \ln v. \quad (29)
$$

Suppose that a total of $m$ testability growth tests are carried out for a certain equipment, where $m = 3z, z$ is a positive integer. Since the testability growth test data is a success or failure type, the $i$-th testability growth test data can be recorded as $(n_i, c_i)$, where $i = 1, 2, \ldots, m$, and the estimated test index points of the $i$-th testability growth test are

$$
\hat{p}(i) = \frac{n_i - c_i}{n_i}. \quad (30)
$$

Substituting the test data of $m$ times into (29), we can get the following results:

$$
\begin{align*}
\sum_{i=1}^{z} \ln \left[ \frac{n_i}{p(i)} \right] &= z \ln u^* + \ln v^* \sum_{i=1}^{z} (w^*)^i, \\
\sum_{i=z+1}^{2z} \ln \left[ \frac{n_i}{p(i)} \right] &= z \ln u^* + \ln v^* \sum_{i=z+1}^{2z} (w^*)^i, \\
\sum_{i=2z+1}^{m} \ln \left[ \frac{n_i}{p(i)} \right] &= z \ln u^* + \ln v^* \sum_{i=2z+1}^{m} (w^*)^i.
\end{align*}
$$

From (31), the Gompertz parameter values $u^*, v^*, w^*$ can be obtained, and then, substituting into the Gompertz model, the Gompertz formula for testability growth test can be obtained as

$$
p(i) = u^*v^*\ln(w^*)^i. \quad (32)
$$

Through the $m-1$ testability growth test data, the testability index points of the final testability growth test can be estimated as follows:

$$
\hat{p}(m + 1) = u^*v^*\ln(w^*)^m. \quad (33)
$$

The prior distribution of the testability growth test data can be expressed by $\pi_i(p)$, and the entropy function of the prior distribution determined by the testability growth test is known to be

$$
H[\pi_i(p)] = - \int_0^1 \pi_i(p) \ln \pi_i(p) dp. \quad (34)
$$

Therefore, the optimal solution of the prior distribution parameters $a, b$ determined by the testability growth test can be transformed into the following programming problems:

$$
\begin{align*}
\text{max} \, H[\pi_i(p)] \\
\text{s.t.} \quad \hat{p}(m + 1) = \frac{a}{a + b} \\
a, b \geq 0.
\end{align*}
$$

5. The Testability Evaluation Method of Equipment

In this paper, a multisensor data fusion method in testability evaluation of equipment is proposed. The multisensor data fusion method is based on the Dempster–Shafer evidence theory which is improved by introducing Lance distance and belief entropy, and the testability evaluation model of equipment based on multisensor data fusion is established, which can effectively enhance the reliability and accuracy of testability evaluation results.
5.1. Process Steps. In order to make effective use of the collected testability data of equipment from sensor and formulate a reasonable testability evaluation scheme, this paper proposes a concrete conversion scheme of testability data and uses an improvement of Dempster–Shafer evidence theory based on Lance distance and belief entropy to conduct multisensor data fusion, so as to further carry out testability evaluation of equipment. The flowchart of the proposed method is shown in Figure 3.

Step 1. Collect testability data of equipment from sensor.

Step 2. The conversion of testability multisensor data. According to conversion schemes, the prior distributions of the three testability multisensor data are shown in Table 5.

Step 3. Construct mass functions of prior distributions. The testability evaluation method proposed in this paper is based on Dempster–Shafer evidence theory. Therefore, the identification framework of testability evaluation system can be regarded as three parts: testability virtual test data, testability test data of replaceable unit, and testability growth test data. Each part can be divided into three focal elements.

Assuming that the target value of testability index given by the manufacturer is \( P_0 \) and the minimum acceptable value specified by the user is \( P_1 \), then when testability index \( P > P_0 \), it is the first focal element, defined as \( H_1 \); when \( P_0 > P > P_1 \), it is the second focal element, defined as \( H_2 \); when \( P < P_1 \), it is the third focal element, defined as \( H_3 \).

As for the testability virtual test data, \( \pi_v(P; a_v, b_v) \) can be regarded as evidence on the identification framework \( \Theta \), and the mass function \( m_v \) of the evidence on the identification framework \( \Theta \) is established:

\[
\begin{align*}
  m_v(H_1) &= \int_{P_0}^{1} \pi_v(P; a_v, b_v)dP, \\
  m_v(H_2) &= \int_{P_0}^{P_1} \pi_v(P; a_v, b_v)dP, \\
  m_v(H_3) &= \int_{0}^{P_1} \pi_v(P; a_v, b_v)dP.
\end{align*}
\]

In the same way, the mass functions \( m_r, m_g \) of testability test data of replaceable unit and testability growth test data are as follows:

\[
\begin{align*}
  m_r(H_1) &= \int_{P_0}^{1} \pi_r(P; a_r, b_r)dP, \\
  m_r(H_2) &= \int_{P_0}^{P_1} \pi_r(P; a_r, b_r)dP, \\
  m_r(H_3) &= \int_{0}^{P_1} \pi_r(P; a_r, b_r)dP, \\
  m_g(H_1) &= \int_{P_0}^{1} \pi_g(P; a_g, b_g)dP, \\
  m_g(H_2) &= \int_{P_0}^{P_1} \pi_g(P; a_g, b_g)dP, \\
  m_g(H_3) &= \int_{0}^{P_1} \pi_g(P; a_g, b_g)dP.
\end{align*}
\]

The mass functions of testability prior distributions can be obtained by (36)–(38), as shown in Table 6.

Step 4. Evidence fusion. According to Algorithm 1, the final result of evidence fusion is \( m_{\text{Fus}} \).

Step 5. Testability evaluation. According to the mass function \( m_{\text{Fus}}(H_1), m_{\text{Fus}}(H_2), m_{\text{Fus}}(H_3) \) of the final results of evidence fusion, combined with the testability index values agreed on by the manufacturer and the user, the testability evaluation results are analyzed.

5.2. Experiment Analysis. Taking the fault detection rate (FDR) of a certain type of steam turbine generator set as the research object, the analysis of testability evaluation is carried out. After a joint agreement between the contractor and the user, the target value of FDR is \( P_0 = 0.95 \) and the minimum acceptable value is \( P_1 = 0.90 \).

The specific steps of testability evaluation are as follows:

Step 1. Collect three types of testability data from sensor at different stages of equipment.

Step 2. The conversion of testability multisensor data.

(1) Testability Virtual Test Data. Through the establishment of the testability virtual prototype model of the generating set, the testability virtual test is carried out, and the five groups of binary data are collected, in which \( i = 1, 2, 3, 4, 5 \), as shown in Table 7.
According to (23), the prior distribution of testability virtual test data can be calculated,
\[\pi_v(P) = \text{Beta}(54.213, 2.04)\].

(2) Testability Test Data of Replaceable Unit. The testability test is carried out on six replaceable units of this type of steam turbine generator set, and the test data are shown in Table 8.

Substituting the posterior data of each replaceable unit and its fault rate into (26) and (27), the prior distribution of testability test data of replaceable unit can be calculated,
\[\pi_r(P) = \text{Beta}(30.29, 5.84)\].

(3) Testability Growth Test Data. By collecting the data of three stages in the test growth test, that is, (10, 10), (15, 0), (38, 9), the prior distribution of testability growth test data can be calculated,
\[\pi_g(P) = \text{Beta}(118, 19)\].

In summary, the prior distributions of the testability multisensor data are shown in Table 9.

Step 3. Mass function of testability prior information as shown in Table 10.

Step 4. Evidence fusion.

According to the Algorithm 1, the final results of evidence fusion are
\[m_{\text{Fus}}(H_1) = 0.0413, m_{\text{Fus}}(H_2) = 0.0116, \text{ and } m_{\text{Fus}}(H_3) = 0.9471\].

Step 5. Testability evaluation.

The final fusion results are compared with the testability evaluation method based on the traditional Bayes theory [103] and the testability evaluation method based on the Dempster–Shafer evidence theory [50]; the results are shown in Table 11.

It can be seen from Table 11 that when using the testability evaluation method based on the traditional Bayes
| Name of replaceable units | Test data of replaceable units | Hyperparameters of replaceable units | Posterior distribution | Posterior expectation | Posterior variance | Fault rate $\lambda_i$ |
|---------------------------|-------------------------------|-------------------------------------|------------------------|----------------------|-------------------|------------------|
| Stator core               | 112                           | 10                                 | Beta (119, 24)         | 0.83                 | 0.69              | 0.030            |
| Screw M6 x 12             | 108                           | 24                                 | Beta (130, 15)         | 0.90                 | 0.80              | 0.025            |
| Bolt M6 x 16              | 54                            | 30                                 | Beta (79, 10)          | 0.89                 | 0.79              | 0.010            |
| Fan impeller              | 49                            | 8                                  | Beta (54, 19)          | 0.74                 | 0.55              | 0.020            |
| Nut washer                | 156                           | 26                                 | Beta (178, 26)         | 0.87                 | 0.76              | 0.015            |
| Generator coil            | 167                           | 50                                 | Beta (209, 44)         | 0.83                 | 0.68              | 0.025            |

Table 8: Test data of replaceable units of a certain type of steam turbine generator set.
In this paper, a testability evaluation method of equipment based on improved Dempster–Shafer evidence theory is proposed. Firstly, by collecting testability virtual test data, replaceable unit test data, and testability growth test data existing in the process of equipment design, production, and use, the corresponding prior distribution reduction scheme is proposed. Then, through considering the reliability and uncertainty of various types of data, an improved Dempster–Shafer evidence method is proposed. Finally, the testability evaluation of equipment is carried out on this basis. Compared with the traditional testability evaluation method, the proposed method can effectively solve the problem of conflict data fusion in the evaluation process, significantly improve the reliability of testability evaluation results, and provide a new technical way for testability evaluation of equipment.

The research results of this paper provide effective theoretical and technical support for scientific and reasonable testability assessment and have important theoretical and engineering application value. This paper studies the evaluation of fault detection rate and fault isolation rate in testability index of traditional testability design. The verification and evaluation of testability design oriented to prognostic and health management have not been carried out, and further research is needed in the future.

### Data Availability
All data and models generated or used during the study can be obtained from the corresponding author upon request.

### Conflicts of Interest
The authors declare no conflicts of interest.

### Authors’ Contributions
X. W. contributed most of the work in this paper. P. D. contributed to manuscript revising. All authors have read and agreed to the published version of the manuscript.

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