Low-energy constraints from unification of matter multiplets

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Abstract

We study the low-energy consequences of the mass and mixing angle relations in grand unified theories (GUT), which follow from an assumption that some quarks and leptons are placed in the same GUT multiplets. This assumption is a simple extension of that for the well-known bottom/tau mass ratio, which is one of the most successful predictions of grand unification of matter multiplets. We show that imposing the GUT relations leads naturally to a limited parameter space from the large lepton mixing between the second and third generations.

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1 Introduction

Exploring the origin of the observed fermion masses and mixing angles has been one of the most important issues in particle physics. Despite the fact that there is apparent hierarchical structure among the masses and mixing angles, no completely definite answer to this problem has been found. Moreover, the recent neutrino experiments indicate that neutrinos have non-zero masses and a rather different mixing pattern than the quark part. Motivated by these facts, many models have been proposed which try to account for the large lepton mixing angles in the frameworks beyond the Standard Model.

In these attempts, the desired masses and mixing angles are usually fixed by taking parameters in the models as suitable values. In some cases, however, dynamical assumptions or symmetry arguments can lead relations among the observables. For example, with the $SU(3)$ flavor symmetry of the light quarks, the well-known Gell-Mann–Okubo mass formula is derived between the octet baryon masses $[1]$. The $SU(3)$ symmetry is approximately well valid below the QCD scale and the mass formula is in good agreement with the experimental values. Another example is the bottom-tau mass equality in $SU(5)$ grand unified theories (GUT). The $SU(5)$ gauge symmetry connects the bottom quark and tau lepton masses and reproduces the correct low-energy value, taking into account the renormalization-group (RG) running effect from the GUT scale down to the weak scale $[2, 3]$. In this way, the predictions of such experimentally well-working relations have convinced us of the relevance of new symmetries and conjectures with which the relations hold.

Recently, such a kind of new relation between the quark and lepton mixing angles has been proposed in grand unified theories $[4]$. This mixing angle relation is derived from an assumption for multiplet structure of quarks and leptons, and naturally incorporated in unification scenarios. It has been shown (and as we will see below) that the relation can also be regarded as a straightforward extension of the bottom-tau mass equality, and it predicts large lepton mixing between the second and third generations at the GUT scale.

In this paper, we analyze the low-energy consequences of the above GUT relations: the bottom-tau mass equality and the mixing angle relation. We assume the minimal supersymmetric standard model (MSSM) with right-handed neutrinos below the GUT scale. With the relations as GUT boundary conditions for the MSSM parameters, we will find new constraints on the parameter space and the intermediate scale where the right-handed neutrinos are decoupled.
In the next section, we first review the derivation of the mixing angle relation proposed in [4] and discuss several its implications for unification scenarios. We perform the RG analyses in section 3. It is found that the low-energy value of the bottom-tau mass ratio becomes more reasonable than the MSSM, while new constraints generally appear in the case of large Yukawa couplings. In section 4, we summarize our results and comment on some possible modifications of the relations.

2 Mixing angle relations

Unification hypotheses for quark and lepton multiplets can lead to some relations among their Yukawa couplings, that is, masses and mixing angles. A well-known example of this approach is the down-quark and charged-lepton mass ratios in GUTs. In this section, we introduce a relation between the quark and lepton mixing angles and discuss its implication in GUT frameworks.

We first consider the mass matrices only for the second and third generations. For simplicity, we go into the basis in which the up-quark mass matrix is diagonalized. The quark mass matrices \( M_u \) and \( M_d \) at the GUT scale are generally written as follows;

\[
M_u = \begin{pmatrix} m_c & m_t \end{pmatrix}, \\
M_d = \begin{pmatrix} V_{cs} & V_{cb} \\ V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} m_s & m_b \end{pmatrix} \begin{pmatrix} V_{\mu 2} & V_{\mu 3} \\ V_{\tau 2} & V_{\tau 3} \end{pmatrix},
\]

(2.1)

where \( m_i \) denote the quark mass eigenvalues and \( V_{ij} \) the mixing matrix elements of quarks and leptons. We have assumed that the lepton mixing is mainly controlled by the charged-lepton mass matrix, that is, neutrinos have small mixing angles (and probably hierarchical mass eigenvalues). It is consequently found that the down-quark mass matrix \( M_d \) is diagonalized by the quark and lepton mixing matrices as above, since at tree level the charged-lepton mass matrix is the transpose of \( M_d \) in GUT frameworks. For the lighter families, some modification of tree-level mass predictions may be required to be consistent with the experimental values. However it does not affect the following discussion because of their tiny Yukawa couplings.

We first impose a natural assumption that the up-quark mass matrix has a hierarchical form, which is commonly seen in the literature. This assumption combined with Yukawa unification may also ensure the hierarchical structure of couplings in the neutrino sector as stated above. Now we suppose that the right-handed up and down quarks of the third generation are contained in a single representation of some (GUT) symmetry (and have
same couplings). Once this assumption is realized, it is easily found that the mass matrix element $M_{d23}$ becomes zero because the right-handed up quarks are rotated such that $M_{u23}$ is set to be zero (i.e., $M_u$ is diagonalized). We are thus led to a following equation from $M_{d23} = 0$ in (2.1),

$$\tan \theta_{\mu\tau} = \frac{m_b}{m_s} V_{cb},$$

(2.2)

where $\theta_{\mu\tau}$ is defined by $\tan \theta_{\mu\tau} = V_{\mu 3}/V_{\tau 3}$. With this relation at hand, the lepton 2-3 mixing angle is expressed in terms of the quark 2-3 mixing angle and mass ratio. It is shown in [4] that with the experimental values of the quark masses and mixing angles, the equation (2.2) is really consistent with the large lepton 2-3 mixing. One of the purposes of this paper is to see whether the equation is well valid even at low energies. If one includes the first generation, the above assumption gives another mixing angle relation involving the Cabibbo angle, which is also in roughly agreement with the large lepton mixing [4]. It is interesting that a single assumption for matter multiplet structure leads to several relations consistent with experiments. In this paper, however, we focus only on the heavier families because the inclusion of the first generation generally involves some ambiguities both experimentally and theoretically.

It is interesting to notice that the relation (2.2) can be understood as a generalization of the mass relation between the bottom quark and tau lepton. The bottom-tau mass relation holds when one requires of the Standard Model that the down quark and charged lepton of the third generation belong to a single multiplet. Note that this requirement is very similar to the one which leads to the mixing angle relation (2.2). As for the bottom-tau mass equality, the requirement results in the $SU(5)$ GUT symmetry beyond the Standard Model. The fact that the mass relation is experimentally well-supported at low energies is one of the great motivations to investigate $SU(5)$ models. In the present case, the assumption that the right-handed up and down quarks come from a single multiplet clearly requires some symmetries beyond the $SU(5)$, such as the left-right symmetry $SU(2)_R \times SO(10)$, and $E_6$ gauge symmetries. As mentioned above, the mixing angle relation is consistent with the experimental results at the GUT scale. This fact may also be one of the convincing proofs of the existence of larger symmetries at high energies.

Before proceeding to low-energy RG analyses, we here discuss some implications of the assumption in GUT multiplet structures. In the following, we use the $SU(5)$ GUT
language, for simplicity. In $SU(5)$ GUTs, all quarks and leptons are assigned into 10 and 5* representations of $SU(5)$. The left-handed quarks belong to 10-dimensional representation fields and the left-handed lepton are in 5* fields. As a result, the quark mixing angles are determined only by the structure of 10-dimensional fields, and on the other hand, the lepton mixing only by that of 5*-representation fields. One of the simple ways to achieve the small quark mixing is that hierarchical suppression factors are attached to 10-dimensional fields. This can be realized in various dynamical ways [3], for example, with a charge assignment of $U(1)$ flavor symmetries. One of the assumptions we adopt in the above, that is, a hierarchy in the up-quark masses, is thus established. Interestingly, that also explains the larger hierarchy among the observed up-quark masses than those of the down quarks and the charged leptons. On the other hand, it has been experimentally confirmed that there exists large generation mixing in the lepton side [6]. An interesting idea to explain this observation within GUT frameworks is to mix the 5*-representation fields. That is, by rotating 5*'s, one can obtain large lepton mixing while the quark mixing angles remain small. It has been shown that this mechanism is indeed incorporated naturally in the GUT frameworks based on $SU(5)$ [7], $SO(10)$ [8], and $E_6$ [9] gauge groups.

To have proper values of the mixing angles, we need an implicit requirement that $M_{d_{32}}$ is of the same order of $M_{d_{33}}$. This is just a condition which associates the quark mixing angle with the lepton one. However it should be noted that the equation (2.2) itself holds without any such additional assumptions, besides that for matter multiplet structure. Since we suppose a hierarchical form of 10-dimensional fields structure, one might wonder that the 5* fields also have a hierarchy and the large mixing angle condition, $M_{d_{32}} \sim M_{d_{33}}$, may not be realized in GUT frameworks. The 5*-rotating mechanism now gives a simple solution to this problem. That is, one can satisfy the condition by suitably twisting the second and/or third generation 5* fields, preserving the small quark mixing angles. In the $SO(10)$ model [8], the original second generation 5* is replaced by another 5* in the extra matter fields so that the large mixing condition is fulfilled. Moreover, no particular assumption is imposed on the third-generation fields and thereby they have the third-generation 10_3 and 5*_3 coming from a single field 16_3 in $SO(10)$. As a result, the mixing angle relation (2.2) certainly follows. For another example, in the $E_6$ model [9] which utilizes a similar 5*-rotating mechanism, the situation is a bit different. In a fundamental representation 27 of $E_6$, the 5* representation of $SU(5)$ appears twice and hence one naturally has a possibility to choose the low-energy 5* candidates without any extra matter fields. This is an interesting feature of $E_6$ GUT models. In particular, in Ref. [9], the third generation 5*_3
is exchanged with the inherent $5_3'$ in the same 27-dimensional field. Since $10_3$ and $5_3'$ have the same couplings connected by the $E_6$ gauge symmetry, the relation (2.2) also holds in this case.

After all, we find that the mixing angle relation is one of the significant predictions of GUT models with $5^*$-rotations, which ensure the lepton large mixing. But we would like to emphasize that the relation generally holds under only one assumption for the matter multiplet structure and is independent of any details of models. Moreover it has much more generality and may be valid without $5^*$-rotations. For example, the relation also follows in an $SO(10)$ GUT model [10] and with a special ansatz of mass texture [11]. To check the validity of the relation at the electroweak scale will be an important probe for internal family structure at high energies.

3 RG evolution of the GUT relations

We have shown in the previous section that the simple hypotheses for matter multiplet structure, which imply the beyond the Standard Model, lead to two GUT relations: the bottom-tau mass equality and the mixing angle relation (2.2). In this section, we study the low-energy consequences of the relations by assuming that the low-energy effective theory below the GUT scale is the MSSM with right-handed neutrinos. We are not concerned with particular mechanisms in GUT models which actually induce the relations. Instead, the relations are treated as the boundary conditions for Yukawa couplings of the MSSM. According to the results in the previous section, an appropriate choice for the boundary values of the second and third generation Yukawa couplings is

$$Y_u|_{\text{GUT}} = c_u \begin{pmatrix} f \\ 1 \end{pmatrix}, \quad Y_d|_{\text{GUT}} = c_d \begin{pmatrix} f \\ h \end{pmatrix}, \quad Y_e|_{\text{GUT}} = c_d \begin{pmatrix} h \\ 1 \end{pmatrix}, \quad (3.1)$$

where $f$ and $h$ are the small ($O(10^{-1-2})$) and $O(1)$ constants, respectively. We have explicitly denoted only the relevant matrix elements which are responsible for the mass and mixing angle relations. The other elements are negligibly small in each matrix and do not affect the RG analyses in what follows. There are two essential points in the above form of boundary conditions. The one is that the second column of $Y_d$ is the same as the second row of $Y_e$. The other is the second rows of $Y_u$ and $Y_d$ are proportional to each other. The former condition gives the bottom-tau mass equality, and the latter one, together with the former, produces the mixing angle relation. In fact, this is a weaker assumption than that we have argued in section 2 in a sense that the proportionality constants $c_u$ and $c_d$
generally take different values. However it is certainly a sufficient condition to obtain the mixing angle relation. The constants $c_u$ and $c_d$ are model-dependent and fix the ratio of two vacuum expectation values of the Higgs doublets in the MSSM. For example, in the $E_6$ model with $5^*$ rotations discussed before, a hierarchy between the two constants, $c_u \gg c_d$, is preferred and actually realized assuming relevant Higgs couplings at the GUT scale.

To estimate the evolution of the relations down to low energies, it is useful to define the following two quantities;

\[
R \equiv \frac{m_b}{m_\tau} = \left( \frac{Y_{d_{32}}^2 + Y_{d_{33}}^2}{Y_{e_{23}}^2 + Y_{e_{33}}^2} \right)^{1/2}, \tag{3.2}
\]

\[
X \equiv \frac{V_{cb} m_b}{\tan \theta_{W} m_s} = \frac{Y_{d_{32}}^2 + Y_{d_{33}}^2}{Y_{d_{22}} Y_{d_{33}} - Y_{d_{32}} Y_{d_{23}}} \frac{Y_{e_{33}}}{Y_{e_{23}}} \left( \frac{Y_{d_{22}} Y_{d_{32}} + Y_{d_{23}} Y_{d_{33}}}{Y_{d_{32}}^2 + Y_{d_{33}}^2} - Y_{u_{23}} \right). \tag{3.3}
\]

The subscripts 2, 3 are the generation indices. At the GUT scale, the boundary conditions are $R|_{\text{GUT}} = X|_{\text{GUT}} = 1$ due to the GUT relations we adopt. It is interesting to note that the boundary value of $X$ is independent of that of the small Yukawa couplings such as $Y_{d_{22}}|_{\text{GUT}}$, though the expression of $X$ partly contains these Yukawa couplings. The relevant couplings for $X|_{\text{GUT}}$ are only those we have described in the matrices (3.1). With this freedom at hand, we can tune the mass eigenvalues of the second generation, for example, by use of the Georgi-Jarlskog factor \[12\] coming from non-minimal representations for the Higgs sector, and supersymmetric loop corrections \[13\]. Such a detail of small Yukawa couplings, however, has nothing to do with the RG analyses and we will safely neglect their effects.

The 1-loop RG equations for $R$ and $X$ read

\[
\frac{dR}{dt} = \frac{R}{16\pi^2} \left[ Y_{u_{33}}^2 - \left( \frac{Y_{e_{33}}^2}{Y_{e_{23}}^2 + Y_{e_{33}}^2} \right) Y_{e_{33}}^2 + 3(Y_{d_{32}}^2 + Y_{d_{33}}^2 - Y_{e_{23}}^2 - Y_{e_{33}}^2) \right. \\
\left. - \frac{16}{3} g_3^2 + \frac{4}{3} g_1^2 \right], \tag{3.4}
\]

\[
\frac{dX}{dt} = \frac{X}{16\pi^2} \left[ 2(Y_{d_{32}}^2 + Y_{d_{33}}^2) + Y_{e_{33}}^2 \right], \tag{3.5}
\]

where $t = \ln \mu$ is the renormalization scale, $Y_\nu$ is the neutrino Yukawa coupling, and $g_{1,3}$ are the gauge coupling constants of the $U(1)$ hypercharge and $SU(3)$ color gauge groups, respectively. We have assumed that the neutrino Yukawa matrix has a hierarchical form similar to the up-quark matrix, at least between the second and third generations. This assumption is usually expected in GUT frameworks.
We first discuss bottom-tau unification, $R|_{\text{GUT}} = 1$. It is known that in the MSSM with right-handed neutrinos, bottom-tau unification casts severe constraints on the parameter space [14]. In particular, there are two parameters which are restricted in the presence of the tau-neutrino Yukawa coupling. One is $\tan \beta$ which is defined by the ratio of vacuum expectation values of the up- and down-type Higgs doublets. The other is $M_R$ where the (third-generation) right-handed neutrinos are decoupled. The essential point they argue in [14] is that in the RG equation for the bottom-tau ratio, the effect of tau-neutrino Yukawa coupling cancels that of the top-quark one. The latter plays a significant role to produce the proper low-energy value of the bottom-tau ratio in the case with no neutrino Yukawa couplings. As a consequence, a small value of $\tan \beta$ (i.e., small bottom and tau Yukawa couplings) is disfavored, and in addition, a lower scale of $M_R$ is excluded because the contribution of tau neutrino becomes larger. Several models have been discussed to ameliorate this problem with extra matter and gauge dynamics [15].

In the present situation, the matter contents and couplings are just the same as that in Ref. [14], but the important difference is in the boundary condition of the Yukawa couplings. We do impose bottom-tau unification at the GUT scale but some additional $O(1)$ couplings are included into the analysis, that is relevant for the lepton large mixing. It is seen from (3.4) that with our boundary condition, the cancellation between the top and tau-neutrino Yukawa couplings is rather reduced even close to the GUT scale, for $Y_{e23}^2/(Y_{e22}^2 + Y_{e33}^2) \simeq 1/2$. In Fig. 1, we illustrate typical lower bounds on $\tan \beta$ and $M_R$ from bottom-tau mass unification. In this figure, we take the top Yukawa coupling and an upper bound for the physical pole mass of the bottom quark as $Y_{u33}|_{\text{GUT}} = 2.5$ and $m_{b}^\text{pole} = 5.2$ GeV, which may be conservative values in estimating allowed parameter space. The dashed lines correspond to the case of the usual MSSM boundary condition (i.e., that of Ref. [14]) and the solid ones for the present case. For a smaller value of $Y_{u33}$, the low-energy prediction for $R$ is similar to that in the usual MSSM case. This is because the net effect of $Y_d$ and $Y_e$ in the RG evolution is same in both cases, given the low-energy values of fermion masses. We find from the figure that the constraint from bottom-tau unification is weakened and in particular, for small gauge coupling constants, physically meaningful bounds on $M_R$ disappear even in case of small $\tan \beta$. One of the interesting influences of this result is on the lepton flavor violation phenomenon. It is known that the lepton flavor violation processes induced by the right-handed neutrinos are largely enhanced with $\tan \beta$. That combined with the Yukawa matrix form like eq. (3.1) is shown to already exclude a large value of $\tan \beta$ [16]. However we find in the above analysis that the small
The \( \tan \beta \) region is still available even with bottom-tau unification, unlike the usual MSSM case. It is interesting that the boundary condition, which leads to the GUT relations for large lepton mixing, gives at the same time a simple solution to the bottom-tau unification problem.

![Figure 1: An illustration of the lower bounds on \( M_R \) and \( \tan \beta \) from bottom-tau unification, with our boundary condition (thick) and the MSSM ones (thin) for \( \alpha_3(M_Z) \equiv g_3^2(M_Z)/4\pi = 0.115 \) (dotted), 0.12 (dashed), and 0.125 (solid). The thick dotted curve is on the outside of this region. In all cases, \( Y_{u33}|_{\text{GUT}} = 2.5 \) and an upper bound of the bottom-quark mass \( m_b^{\text{pole}} = 5.2 \text{ GeV} \) are used.](image)

Another GUT relation, the mixing angle relation (2.2), has rather different consequences for the low-energy physics. The RG evolution of \( X \) is governed only by the down-quark and neutrino Yukawa couplings at one-loop level. The low-energy value of \( X \) hence depends on the two parameters, \( M_R \) and \( \tan \beta \). The intermediate scale \( M_R \) determines the size of the contribution of neutrino Yukawa couplings, and on the other hand, \( \tan \beta \) corresponds to the down-quark Yukawa contribution, given a fixed value of the top quark mass. From the RG equation (3.3), we find that if the right-handed tau neutrino mass \( M_R \) is close to the GUT scale and \( \tan \beta \) is small, the mixing angle relation, i.e., \( X \), is effectively RG-invariant. The dependence of the low-energy \( X \) on these two parameters is shown in Fig. 2. We see from this figure that the low-energy value of \( X \) has a relatively mild \( M_R \)-dependence but is a monotonically decreasing function of the down-quark Yukawa couplings. Since the gauge coupling contributions to the RG evolution is very small even at higher-loop level, the maximal value of \( X \) turns out to be 1. As we discussed in section 2, at the GUT
Figure 2: Typical values of the mixing angle ratio $X$ at the weak scale. The horizontal axes denotes the value of down-quark Yukawa couplings at the GUT scale: $Y_d \equiv Y_{d32}|_{\text{GUT}} = Y_{d33}|_{\text{GUT}}$. The solid, dashed, and dotted curves correspond to $M_R = 10^{10}$, $10^{13}$, and $10^{16}$ GeV, respectively.

scale, the mixing angle relation is in good agreement with the experimentally observed large lepton mixing. A deviation from the exact relation $X = 1$, therefore, could exclude a part of the parameter space in the model, in particular, the large tan $\beta$ (i.e., large $Y_d$) region. We perform a numerical analysis with the 2-loop RG equations and show in Fig. 3 the parameter region allowed by the constraints from two GUT relations. We have taken $X(M_Z) > 0.8$ as an experimental lower bound of the low-energy value. This is roughly translated into a lower bound of the lepton mixing angle, $\sin^2 2\theta_{\mu\tau} > 0.9$ (see Fig. 4), which is indicated by the Superkamiokande experiment. In the whole parameter space, the small tan $\beta$ region is excluded from the first relation, the bottom-tau mass equality. For a smaller value of tan $\beta$, the bottom quark becomes heavier beyond the experimental limit (or the tau lepton is too light). On the other hand, the large tan $\beta$ region is eliminated by the second relation, the mixing angle relation, which is one of the characteristics of the present models. A large value of the bottom Yukawa coupling reduces the lepton mixing angle during the RG evolution down to low energies. In this way, the two GUT relations play complementary roles in obtaining the limits for the model parameters. In the viewpoint of the top Yukawa coupling, the complementarity is more clearly seen. That is, with a smaller $Y_{u33}$, the constraint from bottom-tau unification becomes severer, but that from the mixing angle relation is less important in the case of top-neutrino Yukawa unification.

We find that only the intermediate tan $\beta$ region is typically left in this analysis. There
Figure 3: The parameter regions for \((\tan \beta, M_R)\) experimentally allowed with the constraints from two GUT relation: \(R = X = 1\). Three different regions are shown for the values of \(Y_{33} = 1, 2,\) and \(3\). The averaged value of \(\alpha_3(M_Z) = 0.118\) is used.

has been a discussion in the MSSM that this region of \(\tan \beta\) is disfavored by the observed value of the top quark mass, under the bottom-tau unification assumption \([3]\). Their arguments, however, depend on a particular choice of parameters such as \(\alpha_3, m_t,\) and the boundary conditions for Yukawa and supersymmetry breaking terms. In the present models with the boundary conditions \([3,4]\), different results should be expected. We leave these analyses, including possible corrections to the relations, to future investigations.

4 Summary and discussion

In summary, we have studied the low-energy consequences and validity of the GUT-scale relations among the masses and mixing angles. These relations follow from the simple hypotheses for multiplet structure of quarks and leptons. The first relation is the celebrated bottom-tau mass equality. It is derived with an assumption that the third-generation down quark and charged lepton belong to a single multiplet of some symmetry beyond the Standard Model. This requirement results in the \(SU(5)\) grand unification. The second relation we have discussed is a straightforward extension of the first one. The assumption in this case is that the right-handed up and down quarks in the third generation come from a single multiplet. We thus have the mixing angle relation, which connects the lepton 2-3 mixing angle with the quark mixing angle and mass ratio. This assumption clearly suggests the existence of some dynamics beyond the \(SU(5)\) gauge symmetry, for example,
Figure 4: The dependence of the lepton mixing angle $\sin^2 2\theta_{\mu\tau}$ on $X$ with the experimental values of the quark masses and mixing angle as an input. $X = 1$ corresponds to the exact GUT relation (or the case that $X$ is RG-invariant). The weak-scale value $m_s = 100$ MeV is used, but a smaller value of $m_s$ tends to give a severer bound on $\sin^2 2\theta_{\mu\tau}$.

higher unification of $SO(10)$ or $E_6$. The fact that the mixing angle relation is indeed experimentally well-supported may convince us of such new physics at high energies. We have also discussed that to rotate $5^*$ representation fields is one of the simplest ways to achieve the large lepton 2-3 mixing while the quark mixing angle remains small.

To see the low-energy consequences of the GUT-scale relations, we have performed the renormalization-group analyses in the MSSM with right-handed neutrinos. We have adopted a particular choice of the boundary conditions for Yukawa couplings suggested by the relations. We first shown that the low-energy prediction for the bottom-tau ratio can be fitted to the experimental value better than the usual MSSM case. That gives a simple resolution to the unwilling situation argued in Refs. [14]. The analyses are deeply concerned with two model-parameters: the intermediate scale $M_R$ and $\tan\beta$. From the experimental bounds on the GUT-scale relations, we have plotted the allowed range for these two parameters. There each GUT relation eliminates each side (small or large value) of $\tan\beta$, and the two GUT relations thus play complementary roles. As a result, only the intermediate range of $\tan\beta$ is still left available.

There may be some sources of deviations from the exact GUT-scale relations. First, one could have non-negligible lepton mixing from the neutrino side. Even if the up-neutrino
Yukawa unification is assumed, a large mixing angle can be predicted depending on the forms of right-handed neutrino Majorana mass terms (for example, [17]). That, however, generally requires a hierarchy in the mass matrices and hence some tuning of parameters. The threshold corrections at the GUT and supersymmetry breaking scales also give possible modifications of the relations. They could become considerably large, in particular, for the bottom quark mass via the 1-loop diagrams involving supersymmetric particles, gluino and chargino [18]. However these corrections have large ambiguities depending on the field contents and symmetries in the models and are less under our control. Once supersymmetry is discovered in future experiments, we can definitely evaluate the corrections for the first time. Note that these complicated and potentially large supersymmetry contributions do not modify the mixing angle relation. Deviations from the exactness $X = 1$ is predominated only by the RG evolution discussed in this paper. Thus the mixing angle relation has an advantage for finding a clue to high-energy physics in that it is not spoiled by any threshold effects at intermediate scales. Combined with these factors, more precise measurements of the quark and lepton mixing angles would probe higher-scale matter multiplet structure via the GUT-scale relations as its remnants.

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