Graph-based Robust Sequential Localization in Obstructed LOS Situations

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Abstract—This paper presents a factor graph formulation and particle-based sum-product algorithm (SPA) for robust sequential localization in multipath-prone environments. The proposed algorithm jointly performs data association, sequential estimation of a mobile agent position, and adapts all relevant model parameters. We derive a novel non-uniform false alarm (FA) model that captures the delay and amplitude statistics of the multipath radio channel. This model enables the algorithm to indirectly exploit position-related information contained in the MPCs for the estimation of the agent position. Using simulated and real measurements, we demonstrate that the algorithm can provide high-accuracy position estimates even in fully obstructed line-of-sight (OLOS) situations, significantly outperforming the conventional amplitude-information probabilistic data association (AIPDA) filter. We show that the performance of our algorithm constantly attains the posterior Cramér-Rao lower bound (PCRLB), or even succeeds it, due to the additional information contained in the presented FA model.

Index Terms—Obstructed line-of-sight, multipath, sum-product algorithm, probabilistic data association, message passing, belief propagation

I. INTRODUCTION

Localization of mobile agents using radio signals in environments such as indoor or urban territories is still a challenging task [1]–[4]. These environments are characterized by strong multipath propagation and frequent obstructed line-of-sight (LOS) situations, which can prevent the correct extraction of the line-of-sight (LOS) component (see Fig. 1). There exist many safety- and security-critical applications, such as autonomous driving [5], medical services [6], or keyless entry systems [7], where robustness of the position estimate is of critical importance.

A. State-of-the-Art Methods

New localization and tracking approaches that take advantage of large measurement apertures as ultra-wideband (UWB) systems [8] or mmWave multiple input multiple output (MIMO) systems [9] seek to mitigate the effect of multipath propagation (commonly referred to as “NLOS propagation”) and OLOS situations [10], or even take advantage of multipath components (MPCs) by exploiting inherent position information turning multipath from impairment to an asset [2], [11]–[14]. Prominent examples of such approaches are multipath-based methods that take advantage of multipath by estimating MPCs for localization and associating them to virtual anchors (VAs) representing the location of the mirror image of an anchor on a reflecting surface. The locations of VAs are assumed to be known a priori [15] or estimated jointly with the position of agents using simultaneous localization and mapping (SLAM) [12], [13], [16]. Other methods exploit cooperation among individual agents [4], [17], [18], or perform robust signal processing against multipath propagation and clutter measurements in general. The latter comprise heuristics [8], machine learning-based approaches [10], [19]–[21] as well as Bayesian methods [22]–[24], and hybrids thereof [25], [26]. Heuristic methods, such as searching for the first amplitude to exceed a threshold value, are fast and easily implementable but suffer from low accuracy as well as a high probability of outage in low signal-to-noise-ratio (SNR) regions [8]. In recent years, machine learning methods have grown increasingly popular. Early approaches [10], [20] extract specific features from the radio channel applying model-agnostic supervised regression methods on these features. While these approaches potentially provide high accuracy estimates at low computational demand (after training), they suffer from their dependence on a large representative measurement database and can fail in scenarios that are not sufficiently represented by the training data. This is why recent algorithms facilitate deep learning and auto-encoding based methods to directly operate on the received radio signal and reduce the dependence on training data [21], [27], [28]. Multipath-based localization [12], [13], [16], [23], [24], [29], [30], multiobject-tracking [31]–[33], and parametric channel tracking [34], [35] pose common challenges — uncertainties beyond Gaussian noise, like missed detections and clutter, an uncertain origin of

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We define robustness as the percentage of cases in which a system can achieve its given potential accuracy.

Fig. 1. A mobile agent is moving alongside the anchors on an example trajectory. Due to an obstacle, the LOS to all anchors is not always available. There occur partial and full OLOS situations. Multipath propagation may occur, but there is no prior information about the surrounding environment.
measurements, and unknown and time-varying number of objects to be localized and tracked — that can be well addressed by Bayesian inference that leverages graphical models, performing joint detection and estimation. Similarly, the probabilistic data association (PDA) algorithm [31], [36] represents a low-complexity Bayesian method for robust localization and tracking with extension to multiple-sensors PDA [37] and amplitude-information probabilistic data association (AIPDA) [30], [38]. All these methods can be categorized as "two-step approaches", in the sense that they do not operate on the received sampled radio signal, but use extracted measurements provided by a preprocessing step, providing a high level of flexibility and a significant reduction of computational complexity. In contrast, "direct positioning approaches" such as [11], [39], [40] directly exploit the received sampled signal, which can lead to a better detectability of low-SNR features, yet, they are very computational demanding.

B. Contributions

In this paper, we propose a particle-based sum-product algorithm (SPA) that sequentially estimates the position of a mobile agent by utilizing the position-related information contained in the LOS component as well as in MPCs. The proposed algorithm jointly performs probabilistic data association and estimation of the mobile agent state [13], [32] together with all relevant model parameters, employing the SPA on a factor graph [41]. Similar to other two-step approaches, it uses MPC delays and corresponding amplitude measurements provided by a snapshot-based parametric channel estimation and detection algorithm (CEDA). The proposed algorithm is able to operate without any prior information (no floorplan information or training data are needed) and is demonstrated to provide robust estimates for resolved, specular multipath as well as non-resolvable, dense multipath offering real-time capable computational complexity in realistic scenarios. It adapts in an online manner the time-varying component SNR and the detection probability of the LOS [29], [30], [42]. We propose a novel detection probability model that allows for both an exhaustive representation of the detection space and a smooth estimate of the SNR. The algorithm exploits a non-uniform "false alarm (FA) model" that, in contrast to conventional data association schemes, comprises FAs as well as measurements originating from MPCs. We refer to this part of the model using the more appropriate term "non-line-of-sight (NLOS) model" throughout the paper. This model enables the algorithm to utilize the position-related information contained in the MPCs to support the estimation of the mobile agent state without specific map information, which can increase the accuracy and robustness of the agent positioning in challenging environments, characterized by strong multipath propagation and temporary OLOS situations. More specifically, the introduced NLOS model represents the delay-dependent amplitude distribution and resulting delay measurement density caused by MPCs. Additionally, it couples NLOS measurements to the LOS measurement by a jointly inferred bias state allowing to exploit position-related information from MPCs. The contributions of this paper are as follows.

- We derive a novel non-uniform NLOS model that is adapted to the distribution of the MPC delays and amplitudes corresponding to a stochastic radio signal model [1], [43] and verify its potential in a numerical study.
- We present a new factor graph and corresponding SPA in order to efficiently infer the marginal posterior distributions of all state variables. Additionally, we present a modified, "decoupled" algorithm, which we show to offer better convergence and stability behavior than the optimum SPA for a low number of particles.
- We show that the proposed algorithm is capable of even overcoming fully OLOS situations and providing Cramér-Rao lower bound (CRLB)-level position accuracy using both synthetic and real radio signal measurements.
- We analyze the influence of the individual features of our algorithm and compare it to a particle-based variant of the multiple-sensors AIPDA algorithm and to the posterior Cramér-Rao lower bound (PCRLB) [44].

This work advances over the preliminary account of our conference publication [24] (and that of the related work [23]) by (i) applying an accurate model for the joint distribution of delay and amplitude measurements instead of using heuristical models, (ii) sequentially inferring all parameters of the NLOS model together with the agent instead of using predetermined constants, (iii) improving the convergence behavior using a modified, "decoupled" SPA, (iv) demonstrating the performance of the proposed algorithm using simulated radio signals as well as real radio measurements obtained by (v) applying a real CEDA and (vi) comparing to the PCRLB.

Notations and definitions: Column vectors and matrices are denoted by boldface lowercase and uppercase letters. Random variables (RVs) are displayed in san serif, upright font, e.g., $x$ and $\mathbf{x}$ and their realizations in serif, italic font, e.g., $x$ and $\mathbf{x}$; $\hat{x}$ denotes the true value of $x$. The same notation applies for stochastic processes $x(t)$ and their realizations $x(t)$. $f(x)$ and $p(x)$ denote, respectively, the probability density function (PDF) or probability mass function (PMF) of a continuous or discrete RV $x$. $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ denote matrix transpose, complex conjugation and Hermitian transpose, respectively. $\|\cdot\|$ is the Euclidean norm. $\cdot^i$ represents the cardinality of a set. $\text{diag}(\{ x \})$ denotes a diagonal matrix with entries in $x$. $I_{[\cdot]}$ is an identity matrix of dimension given in the subscript. $[\mathbf{X}]_{n,n}$ denotes the $n$th diagonal entry of $\mathbf{X}$. Furthermore, $1_A(x)$ denotes the indicator function that is $1$ if $x \in A$ and $0$ otherwise, for $A$ being an arbitrary set and $\mathbb{R}^+$ is the set of positive real numbers. We predefine the following PDFs with respect to $x$: The truncated Gaussian PDF is

$$f_{\text{TN}}(x; \mu, \sigma, \lambda) = \frac{1}{Q(\frac{\lambda - \mu}{\sigma})\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} 1_{[\lambda]}(x-\lambda)$$

with mean $\mu$, standard deviation $\sigma$, truncation threshold $\lambda$ and $Q(\cdot)$ denoting the Q-function [45]. Accordingly, the Gaussian PDF is $f_{\text{G}}(x; \mu, \sigma) = f_{\text{TN}}(x; \mu, \sigma, \infty)$. The truncated Rician PDF is [46, Ch. 1.6.7]

$$f_{\text{TRice}}(x; s, u, \lambda) = \frac{1}{Q_1(\frac{u}{s})} e^{-\frac{(x-\lambda)^2}{2s^2}} I_0\left(\frac{x u}{s^2}\right) 1_{[\lambda]}(x-\lambda)$$

(2)
with non-centrality parameter \(u\), scale parameter \(s\) and truncation threshold \(\lambda\). \(I_0(\cdot)\) is the 0th-order modified first-kind Bessel function and \(Q_1(\cdot, \cdot)\) denotes the Marcum Q-function [45]. The truncated Rayleigh PDF is [46, Ch. 1.6.7]

\[
f_{\text{Rayl}}(x; s, \lambda) = \frac{x e^{-\frac{x^2}{2s^2}}}{s^2} 1_{\mathbb{R}^+}(x - \lambda)
\]

with scale parameter \(s\) and truncation threshold \(\lambda\). This formula corresponds to the so-called Swirling I model [46]. Finally, we define the uniform PDF \(f_U(x, a, b) = 1/(b - a) 1_{[a, b]}(x)\) and the uniform PMF \(f_{\text{UD}}(x, \lambda) = 1/|\lambda| 1_{|\lambda|}(x)\).

II. RADIO SIGNAL MODEL

At each discrete time \(n \in \{1, \ldots, N\}\), the mobile agent at position \(p_n\) transmits a signal \(s(t)\) and each anchor \(j \in \{1, \ldots, J\}\) at anchor position \(p_{n,k}^j\) acts as a receiver. The complex baseband signal received at the \(j\)th anchor is modeled as

\[
r_n^{(j)}(t) = \tilde{\alpha}_{n,k}^{(j)} s(t - \tau_{n,k}^{(j)}) + \sum_{k=1}^{K_{n,k}^{(j)}} \tilde{\alpha}_{n,k}^{(j)} s(t - \tau_{n,k}^{(j)}) + \nu_{n}^{(j)}(t).
\]  

(4)

The first and second term describe the LOS component and the sum of \(K_{n,k}^{(j)}\) specular MPCs with their corresponding complex amplitudes \(\tilde{\alpha}_{n,k}^{(j)}\) and delays \(\tau_{n,k}^{(j)}\), respectively. The delays are related to respective distances via \(\tau_{n,k}^{(j)} = \tilde{d}_{n,k}^{(j)}/c\) with \(c\) being the speed of light. The third term represents an additive white Gaussian noise (AWGN) process \(\nu_{n}^{(j)}(t)\) with double-sided power spectral density \(N_0^{(j)}/2\). The LOS distance is geometrically related to the agent position via \(d_{n,0}^{(j)} \triangleq \Delta_{\text{LOS}}(p_n)\) with \(d_{\text{LOS}}^{(j)}(p_n) = \|p_n - p_{n,k}^{(j)}\|\). We assume time synchronization between all anchors and the mobile agent. However, our algorithm can be extended to an unsynchronized system along the lines of [2], [12], [47].

The received signal in (4) is sampled with frequency \(f_s = 1/T_s\) and observed over a duration \(T = N_s T_s\) yielding \(N_s\) samples at a maximum observable distance \(d_{\text{max}} = c T\). By stacking the samples, we obtain the discrete-time signal vector

\[
r_n^{(j)} = \tilde{\alpha}_{n,0}^{(j)} s(\tau_{n,0}^{(j)}) + \sum_{k=1}^{K_{n,k}^{(j)}} \tilde{\alpha}_{n,k}^{(j)} s(\tau_{n,k}^{(j)}) + \nu_{n}^{(j)}
\]

(5)

where \(s(t) \triangleq [s(0 - T_s - \tau) \cdots s(N_s - 1 - T_s - \tau)]^T \in \mathbb{C}^{N_s \times 1}\) is the stacked signal vector. The measurement noise vector \(\nu_{n}^{(j)} \in \mathbb{C}^{N_s \times 1}\) is a zero-mean, circularly-symmetric complex Gaussian random vector with covariance matrix \(\hat{\sigma}(j)^2 I_N\) and noise variance \(\hat{\sigma}(j)^2 = N_0^{(j)}/T_s\). The MPCs arise from reflection or scattering by unknown objects, since we assume that no map information is available.

For a very large number of MPCs \(K_{n,k}^{(j)}\) and limited bandwidth of \(s(t)\), the MPCs cannot be resolved anymore. Hence, the MPCs are described by a zero-mean, circularly-symmetric complex Gaussian stochastic process \(v_{\text{DMC}}^{(j)}(\tau)\) [48], [50]. The corresponding discrete time signal vector reads

\[
r_n^{(j)} = \tilde{\alpha}_{n,0}^{(j)} s(\tau_{n,0}^{(j)}) + \sum_{k=1}^{K_{n,k}^{(j)}} \tilde{\alpha}_{n,k}^{(j)} s(\tau_{n,k}^{(j)}) + \nu_{n}^{(j)}
\]

(6)

with the second term denoting the dense multipath component (DMC) [43], [51]. Assuming uncorrelated scattering for \(v_{\text{DMC}}^{(j)}(\tau)\) [48], [51], the noise covariance matrix of \(r_n^{(j)}\) is given by

\[
C_{Nn}^{(j)} = \int s_{\text{DMC}}^{(j)}(\tau) s(\tau) \, d\tau + \hat{\sigma}^{(j)^2} I_N
\]

(7)

where \(s_{\text{DMC}}^{(j)}(\tau)\) is the delay power spectrum (DPS). Using (7), the SNR of the LOS component is defined as \(\text{SNR}_{n}^{(j)} = \|\bar{\alpha}_{n,0}^{(j)} s(\tau_{n,0}^{(j)}) C_{Nn}^{(j)-1} s(\tau_{n,0})\| \) and the according normalized amplitude is \(\tilde{u}_{n}^{(j)} \triangleq \text{SNR}_{n}^{(j)} \hat{\sigma}(j)^2\).

A. Delay Power Spectrum (DPS) Model

We choose to model the DPS \(s_{\text{DMC}}^{(j)}(\tau)\) as [51]

\[
S_D(\tau; \tilde{\rho}_n, \tilde{\rho}_n^{(j)}) = \tilde{S}_0^{(j)}(1 - e^{-\Delta_0^{(j)}/\tilde{\gamma}_0^{(j)}}) e^{-\Delta_0^{(j)}/\tilde{\gamma}_0^{(j)}} 1_{[\Delta_0^{(j)}, \infty)}(\tau)
\]

(8)

which is a double exponential function with \(\tilde{\gamma}_0^{(j)}\) being the DPS power. The rise time \(\tau_0^{(j)}\) and fall time \(\tilde{\gamma}_0^{(j)}\) are shape parameters. The distance difference \(\Delta_0^{(j)}\) is given by

\[
\Delta_0^{(j)}(\tilde{\rho}_n, \tilde{\gamma}_0^{(j)}) = c \tau - d_{\text{LOS}}^{(j)}(\tilde{\rho}_n) - \tilde{\gamma}_0^{(j)}
\]

(9)

where \(\tilde{\gamma}_0^{(j)}\) is the NLOS bias, which denotes the difference between the LOS distance \(d_{\text{LOS}}^{(j)}(\tilde{\rho}_n)\) and the "onset distance". Experimental evidence motivates this selection: The DPS typically exhibits an exponentially decaying tail [43], [51] and a smooth onset [51], [52]. In particular, when excluding the LOS power, as done in (6). Note that \(\tilde{\gamma}_0^{(j)}\) is mainly determined by the signal bandwidth and onset-density of MPCs. For homogeneous deployment environments the on-set density is well modeled as being invariant. Therefore, \(\tilde{\gamma}_0^{(j)}\) is assumed to be the same for all anchors.

For inference, we additionally define the normalized DPS \(\tilde{S}_D(d/c; \tilde{\rho}_n, \tilde{\gamma}_0^{(j)}) = S_D(d/c; \tilde{\rho}_n, \tilde{\gamma}_0^{(j)}) / \tilde{S}_0^{(j)}\) with \(\tilde{\gamma}_0^{(j)} = [\tilde{\gamma}_0^{(j)}(\tilde{\gamma}_0^{(j)})]^{\tau_0^{(j)}}\), which collects the NLOS shape parameters for each time \(n\) and anchor \(j\). The dense-multipath-to-noise-ratio (DNR) \(\tilde{\omega}_n^{(j)} = \|s(\tau)\| \tilde{S}_0^{(j)} / \tilde{N}_0^{(j)}\) denotes the square-root power ratio between the DMC and AWGN.

The proposed algorithm utilizes the position information contained in \(S_D^{(j)}(\tau)\) to improve the position estimate without explicitly exploiting map information.

B. Parametric Channel Estimation

By applying a suitable snapshot-based channel estimation and detection algorithm (CEDA) [43], [50], [53] to the observed discrete signal vector \(r_{n}^{(j)}\), one obtains at each time \(n\) and anchor \(j\), a number of \(M_n^{(j)}\) measurements denoted by \(z_{n,m}^{(j)}\) with \(m \in M_n^{(j)} = \{1, \ldots, M_n^{(j)}\}\). Each \(z_{n,m}^{(j)} = \left[z_{n,m}^{(j)}, z_{n,m,0}^{(j)}\right]^{T}\) contains a distance measurement \(z_{n,m,0}^{(j)}\) and a normalized amplitude measurement \(z_{n,m}^{(j)}\). The CEDA decomposes the discrete signal vector \(r_{n}^{(j)}\) into individual, decorrelated components according to (5), reducing the number of dimensions (as \(M_n^{(j)}\) is usually much smaller than \(N\)). It thus can be said to compress the information contained.

Note that in the absence of a DMC this reduces to the familiar SNR \(\|\bar{\alpha}_{n,0}^{(j)} s(\tau_{n,0})\|^2 / \hat{\sigma}(j)^2\).
in \( r_n^{(j)} \) into \( z_n^{(j)} = [z_{n,1}^{(j)} \ldots z_{n,M_n^{(j)}}^{(j)}]^T \). See the supplementary material [54, Sec. V] for further details. The stacked vector \( z_n = [z_1^{(j)} \ldots z_{M_n^{(j)}}^{(j)}]^T \) is used as noisy measurement by the proposed algorithm.

### III. System Model

We consider a mobile agent to be moving along an unknown trajectory as depicted in Fig. 1. The current state of the agent is described by the state vector \( x_n = [p_n^T, v_n^T] \), which is composed of the agent’s position \( p_n \) and velocity \( v_n = [v_{x_n}, v_{y_n}]^T \).

We also introduce the following additional state variables, which represent all RV inferred along with \( x_n \). First, we define the extended agent state \( \hat{x}_n = [x_n, y_{in}] \), which collects all RV that are common for all anchors. Second, we define the anchor state \( y_n^{(j)} = [u_{n}^{(j)}, \omega_{n}^{(j)} b_{n}^{(j)}] \) collecting all continuous RVs, which are modeled separately for each anchor. Third, there are two discrete RVs \( q_n^{(j)} \) and \( a_n^{(j)} \), which denote the LOS probability and association variable, respectively, and are modeled separately for all anchors. The role of these RVs is discussed in what follows.

For notational brevity, we also define the vectors \( \xi^{(j)}_{SN} = [b_n^{(j)}, \gamma_n^{(j)}]^T \), \( \xi_n^{(j)} = [\omega_n^{(j)}, \xi_n^{(j)}_{SN}] \), and \( \xi_n^{(j)} = [p_n, u_n^{(j)}, s_n, \xi_n^{(j)}] \).

#### A. Measurement Model

At each time \( n \) and for each anchor \( j \) the CEDA provides the currently observed measurement vector \( z_n^{(j)} \), with fixed \( M_n^{(j)} \), according to Sec. II-B. Before the measurements are observed, they are random and represented by the vector \( z_n^{(j,m)} = [z_{n,1}^{(j,m)} \ldots z_{n,M_n^{(j,m)}}^{(j,m)}]^T \). In line with Sec. II-B we define the nested random vectors \( z_n^{(j)} = [z_{n,1}^{(j)} \ldots z_{n,M_n^{(j,m)}}^{(j,m)}]^T \) and \( z_n = [z_1^{(j)} \ldots z_{M_n^{(j)}}^{(j,m)}]^T \). Also the number of measurements \( M_n^{(j,m)} \) is a RV. The vector containing all measurement numbers is defined as \( M_n = [M_1^{(j)} \ldots M_n^{(j,m)}]^T \).

Each measurement \( z_n^{(j,m)} \) either originates from the LOS or is due to an MPC. It is also possible that a measurement \( z_n^{(j,m)} \) did not originate from any physical component, but from FAs of the CEDA. The presented model only distinguishes between “LOS measurements” originating from the LOS and “NLOS measurements”, i.e., measurements due to MPCs or FAs.

1. **LOS Measurement Model:** The LOS likelihood function (LHF) of the distance measurement \( z_n^{(j)} \) is given by

\[
 f_L(z_n^{(j)}|p_n, u_n^{(j)}) \triangleq f_N(z_n^{(j)}; d_{LOS}(p_n), \sigma(a_n^{(j)}))
\]

with mean \( d_{LOS}(p_n) \) and variance \( \sigma^2(a_n^{(j)}) \). The variance is determined based on the Fisher information given by

\[
 \sigma^2(a_n^{(j)}) = \frac{c^2}{(8\pi^2 \beta_{bw}^2 u_n^{(j)} \gamma^2)},
\]

where \( \beta_{bw} \) is the root mean squared bandwidth [1], [2] and \( u_n^{(j)} \) is the normalized amplitude at anchor \( j \). The LOS LHF of the normalized amplitude measurement \( z_n^{(j)} \) is modeled as\(^3\) [30], [34]

\[
 f_L(z_n^{(j)}|s_n, \gamma_{n,m}) \triangleq f_{TRice}(z_n^{(j)}; s_n, u_n^{(j)}, \gamma_{n,m})
\]

where \( f_{TRice}(\cdot) \) is a truncated Ricean PDF [3] and

\[
 z_n^{(j)} = \frac{1}{2}(u_n^{(j)} + s_n^{(j)}), \quad s_n^{(j)} = \frac{1}{2}(u_n^{(j)} - s_n^{(j)}) + 1
\]

is the NLOS scale function. See the supplementary material [54, Sec. I] for details about its derivation.

2. **NLOS Measurement Model:** The NLOS LHF of the normalized amplitude measurement \( z_n^{(j,m)} \) is given as

\[
 f_{NL}(z_n^{(j,m)}; d_{n,m}, p_n, \xi_{n,m}^{(j)}) \triangleq f_{TRayl}(z_n^{(j,m)}; s_n^{(j)}, p_n, \xi_{n,m}^{(j)}, \gamma_{n,m})
\]

where \( f_{TRayl}(\cdot) \) is a truncated Rayleigh PDF [3] and

\[
 z_n^{(j,m)} = \frac{1}{2}(u_n^{(j,m)} + s_n^{(j,m)}), \quad s_n^{(j,m)} = \frac{1}{2}(u_n^{(j,m)} - s_n^{(j,m)}) + 1
\]

is the NLOS scale function. See the supplementary material [54, Sec. I] for details about its derivation. The shape of (12) with respect to \( z_n^{(j)} \) and \( z_n^{(j,m)} \) is shown in Fig. 2a. The NLOS LHF of the distance measurement \( z_n^{(j,m)} \) is given by

\[
 f_{NL}(z_n^{(j,m)}; d_{n,m}, p_n, \xi_{n,m}^{(j)}) = Q(0; p_n, \xi_{n,m}^{(j)})^{-1} \int_{\gamma} f_{TRayl}(u; s_n^{(j)}, p_n, \xi_{n,m}^{(j)}, \gamma) \, du
\]

The presented model describes the distribution of the amplitude estimates of a complex baseband signal in AWGN obtained using maximum likelihood estimation and generalized likelihood ratio test detection [38], [45], [55].
values of $\gamma$ is shown in Fig. 2b. Note that (14) approaches a uniform PDF when $\gamma$ or $\omega_{u(j)}$ approach 0.

The presented NLOS measurement model is valid independently of the DPS model chosen in (8). However, (8) is a reasonable choice as it is physically motivated [51] and is of moderate computational complexity.

B. Data Association Model

At each time $n$ and for each anchor $j$, the measurements, i.e., the components of $z_{n,m}^{(j)}$ are subject to data association uncertainty. Thus, it is not known which measurement $z_{n,m}^{(j)}$ originated from the LOS, or which one is due to an “NLOS measurement”, i.e., measurements originating from MPCs or FAs. Based on the concept of PDA [36], we define the association variable $a_{n}^{(j)}$ as

$$a_{n}^{(j)} = \begin{cases} m \in \mathcal{M}_{n}, & z_{n,m}^{(j)} \text{ is the LOS meas. in } z_{n}^{(j)} \text{.} \\ 0, & \text{no LOS meas. in } z_{n}^{(j)} \text{.} \end{cases} \quad (15)$$

Assuming the number of NLOS measurements to follow a uniform distribution (so-called “non-parametric model”), the joint PMF of $a_{n}^{(j)}$ and $M_{n}^{(j)}$ can be shown to be [36]

$$p(a_{n}^{(j)}, M_{n}^{(j)}|u_{n}^{(j)}, q_{n}^{(j)}) = \begin{cases} \frac{p_{u}(u_{n}^{(j)}, q_{n}^{(j)})}{M_{n}^{(j)} \cdot M_{\text{max}}}, & a_{n}^{(j)} \in \mathcal{M}_{n}^{(j)} \\ \frac{1-p_{u}(u_{n}^{(j)}, q_{n}^{(j)})}{M_{n}^{(j)} \cdot M_{\text{max}}}, & a_{n}^{(j)} = 0 \end{cases} \quad (16)$$

where $p_{u}(u_{n}^{(j)}, q_{n}^{(j)})$ is the probability that there is a LOS measurement for the current set of measurements defined in Sec. III-C and $M_{\text{max}}$ is an irrelevant constant. Incorporating $a_{n}^{(j)}$ into the model, we define the overall distance LHF as

$$f(z_{d,n,m}^{(j)}|s_{En}^{(j)}) = \begin{cases} f_{L}(z_{d,n,m}^{(j)}|p_{n}, u_{n}^{(j)}), & a_{n}^{(j)} = m \\ f_{NL}(z_{d,n,m}^{(j)}|p_{n}, s_{En}^{(j)}), & a_{n}^{(j)} \neq m \end{cases} \quad (17)$$

The shape of (17) is depicted in Fig. 3a. Further, the overall amplitude LHF is given by

$$f(z_{a,n,m}^{(j)}|s_{En}^{(j)}) = \begin{cases} f_{L}(z_{a,n,m}^{(j)}|u_{n}^{(j)}), & a_{n}^{(j)} = m \\ f_{NL}(z_{a,n,m}^{(j)}|z_{d,n,m}^{(j)}, p_{n}, s_{En}^{(j)}), & a_{n}^{(j)} \neq m \end{cases} \quad (18)$$

which is shown in Fig. 3b. Using the common assumption of the measurements to be independent for different values of $m$ [32], the joint LHF for all measurements per anchor $j$ and time $n$ is

$$f(z_{n}^{(j)}|s_{En}^{(j)}) = \prod_{m=1}^{M_{d,n}^{(j)}} f(z_{a,n,m}^{(j)}|s_{En}^{(j)}) f(z_{d,n,m}^{(j)}|s_{En}^{(j)}). \quad (19)$$

C. LOS Existence Probability Model

We model the LOS existence probability given in (16) as $p_{E}(u_{n}^{(j)}, q_{n}^{(j)}) = p_{D}(u_{n}^{(j)}) q_{n}^{(j)}$. The probability of detection $p_{D}(u_{n}^{(j)})$ is the probability that at time step $n$ and anchor $j$ the agent generates a radio signal component whose amplitude is high enough so that it leads to an LOS measurement. It is modeled by the counter probability of a Rician cumulative distribution function (CDF) given as

$$p_{D}(u_{n}^{(j)}) = Q_{1}\left(\frac{u_{n}^{(j)} - \gamma}{\sigma_{u}(u_{n}^{(j)})}\right). \quad (20)$$

by assuming that the proposed algorithm is applied after a generalized likelihood ratio test detector. $q_{n}^{(j)}$ is the probability of the event that the LOS is not obstructed, which is referred to as LOS probability in the following, and acts as a prior probability to the detection event. According to [29], [42], [56], we model $q_{n}^{(j)}$ as discrete RV that takes its values from a finite set $Q = \{\lambda_{1}, ..., \lambda_{Q}\}$, where $\lambda_{1} \in (0, 1]$. The temporal evolution of $q_{n}^{(j)}$ is modeled by a first-order Markov process, which results in a conventional Markov chain, with $[Q_{n+1}^{(j)}]_{i,k} = q_{n}^{(j)} = \lambda_{i} q_{n-1}^{(j)} = \lambda_{k}$ being the elements of the transition matrix. The LOS probabilities for different sensors $j$ are assumed to be independent. The proposed LOS existence probability model $p_{E}(u_{n}^{(j)}, q_{n}^{(j)})$ correctly incorporates the detection process into the system model via $p_{D}(u_{n}^{(j)})$ excluding a detection of measurements with $z_{a,n,m}^{(j)}$ below $\gamma$ and it can cope with amplitude model mismatch by correcting the amplitude-related probability of detection with $q_{n}^{(j)}$. With respect to implementation (see Sec. V-B1) this means that our model allows for smooth sequential inference of slow amplitude variations (e.g., due to path loss) via $p_{D}(u_{n}^{(j)})$, while $q_{n}^{(j)}$ ensures a complete representation of the probability space, covering rapid amplitude variations (e.g., due to OLOS).

D. State transition model

We model the evolution of $\bar{x}_{n}$ and $y_{n}^{(j)}$ and $q_{n}^{(j)}$ over time $n$ as independent first-order Markov processes, which are defined by the joint state transition PDF

$$f(\bar{x}_{n}, y_{n}^{(j)}, q_{n}^{(j)})|\bar{x}_{n-1}, y_{n-1}, q_{n-1}) = f(\bar{x}_{n}|\bar{x}_{n-1}) \prod_{j=1}^{J} f(y_{j}^{(j)}|y_{j-1}^{(j)} a_{n}^{(j)} \neq m \quad (21)$$

IV. PROBLEM FORMULATION AND FACTOR GRAPH

In this section we formulate the sequential estimation problem of interest and present the joint posterior and the factor graph underlying the proposed algorithm.

A. Problem Statement

The problem considered is the sequential estimation of the agent state $x_{n}$. This is done in a Bayesian sense by calculating
the minimum mean-square error (MMSE) \cite{55} of the extended agent state
\[ \hat{x}_{\text{MMSE}}^{\text{MMSE}} \triangleq \int \hat{x}_n f(\hat{x}_n | z) d\hat{x}_n, \] (22)
with \( \hat{x}_{\text{MMSE}}^{\text{MMSE}} = [\hat{x}_{\text{MMSE}}^{\text{MMSE}} \hat{z}_{\text{MMSE}}^{\text{MMSE}}] \) and \( \hat{z}_{\text{MMSE}}^{\text{MMSE}} = [\hat{p}_{\text{MMSE}}^{\text{MMSE}} T \hat{v}_{\text{MMSE}}^{\text{MMSE}} T] \). Furthermore, we also calculate
\[ \hat{y}_{\text{MMSE}}^{(j)MMSE} \triangleq \int \hat{y}_n^{(j)} f(y_n^{(j)} | z) dy_n^{(j)}, \] (23)
\[ \hat{q}_{\text{MMSE}}^{(j)MMSE} \triangleq \sum_{\lambda_i \in \Omega} \lambda_i p(q_n^{(j)} = \lambda_i | z) \] (24)
with \( \hat{y}_{\text{MMSE}}^{(j)MMSE} = [\hat{y}_{\text{MMSE}}^{(j)MMSE} \hat{\omega}_{\text{MMSE}}^{(j)MMSE} \hat{\beta}_{\text{MMSE}}^{(j)MMSE}] \). In order to obtain (22), (23), and (24), the respective marginal posterior PDFs need to be calculated. Since direct marginalization of the joint posterior PDF is computationally infeasible \cite{32}, we perform message passing by means of the SPA rules on the factor graph that represents a factorized version of the joint posterior of our statistical model discussed in Sec. III.

B. Joint Posterior and Factor Graph

For each \( n \), let \( y_n = [y_n^{(1)} ... y_n^{(J)}]^T \), \( a_n = [a_n^{(1)} ... a_n^{(J)}]^T \), and \( q_n = [q_n^{(1)} ... q_n^{(J)}]^T \). Furthermore, let \( z = [z_1^T ... z_J^T]^T \), \( \bar{x} = [\bar{x}_0^T ... \bar{x}_J^T]^T \), \( y = [y_0^T ... y_J^T]^T \), \( q = [q_0^T ... q_J^T]^T \), and \( M = [M_1^T ... M_J^T]^T \). We now assume that the measurements \( z \) are observed and thus fixed. Applying Bayes’ rule as well as some commonly used independence assumptions \cite{15, 32} the joint posterior for all states up to time \( n \) and all \( J \) anchors can be derived up to a constant factor as
\[ f(\bar{x}, a, y, q, M | z) \]
\[ \propto f(z | \bar{x}, a, y, q) f(\bar{x}, a, y, q) f(a | y, q) f(y) \]
\[ = f(z | \bar{x}, a, y, q) f(\bar{x}, a, y, q) f(a | y, q) f(y) \]
\[ \propto f(\bar{x}_0) \prod_{j=1}^J \int \prod_{n'=1}^n \mathcal{T}(\bar{x}_{n'} | \bar{x}_{n'-1}) \Phi(\hat{y}_n^{(j)} | y_{n'-1}^{(j)}) \]
\[ \times \Psi(q_n^{(j)} | q_{n'-1}^{(j)}) \hat{g}(z_n^{(j)} | p_n^{(j)}, y_n^{(j)}, a_n^{(j)}, q_n^{(j)}) \] (25)
where we introduced the state-transition functions \( \mathcal{T}(\bar{x}_{n'} | \bar{x}_{n'-1}) \triangleq f(\bar{x}_{n'} | \bar{x}_{n'-1}) \), \( \Phi(\hat{y}_n^{(j)} | y_{n'-1}^{(j)}) \triangleq f(y_n^{(j)} | y_{n'-1}^{(j)}) \), and \( \Psi(q_n^{(j)} | q_{n'-1}^{(j)}) \triangleq p(q_n^{(j)} | q_{n'-1}^{(j)}) \), as well as the pseudo LHF \( \hat{g}(z_n^{(j)} | p_n^{(j)}, y_n^{(j)}, a_n^{(j)}, q_n^{(j)}) \triangleq h(a_n^{(j)}; y_n^{(j)}, q_n^{(j)}) g(z_n^{(j)} | p_n^{(j)}, y_n^{(j)}, a_n^{(j)}) \) with its shorthand notation \( \hat{g}_n^{(j)} \). Finally, we define \( g(z_n^{(j)} | q_n^{(j)}) \triangleq f(x_n^{(j)} | q_n^{(j)}) \) and
\[ h(a_n^{(j)}; y_n^{(j)}, q_n^{(j)}) \propto p(a_n^{(j)}; M_n^{(j)}; u_n^{(j)}, q_n^{(j)}) \]
\[ \begin{cases}
\frac{p_n(u_n^{(j)}, q_n^{(j)})}{M_n^{(j)}}, & a_n^{(j)} \in M_n^{(j)} \\
1 - p_n(u_n^{(j)}, q_n^{(j)}), & a_n^{(j)} = 0
\end{cases} \] (26)


V. SUM-PRODUCT ALGORITHM

A. Marginal Posterior and Sum-Product Algorithm (SPA)

The marginal posterior can be calculated efficiently by passing messages on the factor graph according to the SPA \cite{41}. For the proposed algorithm, we specify not to send messages backward in time. This makes the factor graph in Fig. 4 an acyclic graph. For acyclic graphs the SPA yields exact results for the marginal posteriors \cite{41}. At time \( n \), the following calculations are performed for all \( J \) anchors. The prediction messages are given as
\[ \eta(\bar{x}_n) = \int \mathcal{T}(\bar{x}_n | \bar{x}_{n-1}) \hat{f}_x(\bar{x}_{n-1}) d\bar{x}_{n-1} \] (27)
\[ \phi(y_n^{(j)}) = \int \Phi(y_n^{(j)} | y_{n-1}^{(j)}) \hat{f}_y(y_{n-1}^{(j)}) dy_{n-1}^{(j)} \] (28)
\[ \psi(q_n^{(j)}) = \sum_{q_{n-1}^{(j)} = 1}^{N_q} \Psi(q_n^{(j)} | q_{n-1}^{(j)}) \hat{p}_q(q_{n-1}^{(j)}) \] (29)

where \( \hat{f}_x(\bar{x}_{n-1}), \hat{f}_y(y_{n-1}^{(j)}) \) and \( \hat{p}_q(q_{n-1}^{(j)}) \) are messages of the previous time \( n - 1 \). The measurement update messages are given by
\[ \xi^{(j)}(\bar{x}_n) = \int \phi(y_n^{(j)}) \sum_{q_n^{(j)} = 1}^{M_n^{(j)}} \psi(q_n^{(j)}) \hat{g}_n^{(j)}(\cdot) dy_n^{(j)} \] (30)
\[ \chi^{(j)}(\bar{x}_n) = \eta(\bar{x}_n) \prod_{j'=1}^J \xi^{(j')}(\bar{x}_n)/\xi^{(j)}(\bar{x}_n) \] (31)
\[ \nu(y_n^{(j)}) = \sum_{q_n^{(j)} = 1}^{M_n^{(j)}} \psi(q_n^{(j)}) \int \chi^{(j)}(\bar{x}_n) \hat{g}_n^{(j)}(\cdot) d\bar{x}_n \] (32)
\[ \beta(q_n^{(j)}) = \int \phi(y_n^{(j)}) \chi^{(j)}(\bar{x}_n) \sum_{a_n^{(j)} = 1}^{M_n^{(j)}} \hat{g}_n^{(j)}(\cdot) d\bar{x}_n dy_n^{(j)} \] (33)
Finally, we calculate the posterior distributions as \( f(\bar{x}_n | z) \propto \hat{f}_s(\bar{x}_n) = \eta(\bar{x}_n) \prod_{j=1}^J \xi(j)(\bar{x}_n) \), \( f(y^{(j)}_{n,u} | z) \propto \hat{f}_j(y^{(j)}_{n,u}) = \phi(y^{(j)}_{n,u}) \nu(y^{(j)}_{n,u}) \) and \( p(\bar{q}^{(j)}_{n,u} | z) \propto \hat{p}_q(\bar{q}^{(j)}_{n,u}) = \psi(\bar{q}^{(j)}_{n,u}) \beta(\bar{q}^{(j)}_{n,u}) \).

We additionally compare the performance of the above optimum SPA to that of a suboptimal message passing algorithm, which we refer to as "decoupled SPA". Inspired by [29], we replace (31) by \( \chi^{(j)}(\bar{x}_n) = \eta(\bar{x}_n) \) neglecting the mutual dependency of the uncertainties of individual anchor states \( y^{(j)}_{n,u} \). We demonstrate this modified algorithm to lead to improved numerical stability for a low number of particles.

**B. Implementation Aspects**

1. **Particle-Based Implementation**: Since the integrals involved in the calculations of the messages and beliefs (27)-(33) cannot be obtained analytically, we use a computationally efficient sequential particle-based message passing implementation that provides approximate computation. Our implementation uses a "stacked state" [57], comprising the agent state as well as the anchor states of all \( J \) anchors. The resulting problem complexity scales only linearly in the number of particles. See [13], [29], [32], [42], [58] for further details. For computational efficiency of the particle-based implementation the LOS LHF of the normalized amplitude measurement (11) is approximated by a truncated Gaussian PDF, i.e.,

\[
\hat{f}_s(z^{(j)}_{n,m,n}, a^{(j)}_{n}) = f_{\text{TN}}(z^{(j)}_{n,m,n}; \sigma_a^{(j)}(n), \gamma^{(j)}_a).
\]

2. **Initial State Distributions**: The initial distributions are factorized as \( \hat{f}_s(x_0) = f_b(p_0) f_o(v_0) f_{\gamma_0}(\gamma_0) \) and \( \hat{f}_s(y^{(j)}_0) = \hat{f}_s(u^{(j)}_0) f_\Psi(\omega^{(j)}_0 f_k(b^{(j)}_0) f_{\gamma^{(j)}_a}(\gamma^{(j)}_0)) \). We propose to initialize the NLOS shape parameters as \( f_{\gamma_0}(\gamma_0) = f_U(\gamma_0, 0, d_{\text{max}}) \), \( f_k(b^{(j)}_0) = f_U(b^{(j)}_0, 0, d_{\text{max}}) \) and \( f_{\gamma^{(j)}_a}(\gamma^{(j)}_0) = f_U(\gamma^{(j)}_0, 0, d_{\text{max}}) \).

The LOS PMFs are initialized as a discrete uniform PMF \( \hat{f}^{(j)}(\omega^{(j)}_0, Q_0) = f_{\text{UD}}(Q_0, Q) \) taking all values of \( Q \) with equal probability. We assume the velocity vector \( v_0 \) to be zero mean, Gaussian, with covariance matrix \( \sigma_v^2 I_2 \) and \( \sigma_v = 6 \text{ m/s} \), as we do not know in which direction we are moving.

The remainder of the states are initialized heuristically, assuming an initial measurement vector \( z_0 \) containing \( M_0 \) measurements to be available. The normalized amplitude PDFs are initialized as \( \hat{f}_s^{(0)}(u^{(0)}_0, z^{(0)}|u^{(0)}_0) = f_{\text{SNR}}(u^{(0)}_0; z^{(0)}|u^{(0)}_{\text{max}}, 0.05 z^{(0)}|u^{(0)}_{\text{max}}, \gamma) \) with \( z^{(0)}|u^{(0)}_{\text{max}} \) is the maximum normalized amplitude measurement in \( z^{(0)}_0 \). The position state is initialized as \( f(p_0) \sim \prod_{j=1}^J \prod_{m=1}^{M_j} f_{\text{LHF}}(z^{(j)}|p^{(j)}_{\text{init}}, z^{(j)}|p^{(j)}_{\text{max}}) \), where the proposal distribution \( f_p(p^{(j)}_{\text{max}}) \) is drawn uniformly on two-dimensional discs around each anchor \( j \), which are bounded by the maximum possible distance \( d_{\text{max}} \) and a sample is drawn from each of the \( J \) discs with equal probability. The DNR PDFs are initialized as \( \hat{f}_s(\omega^{(j)}_n) = f_{\text{SNR}}(\omega^{(j)}_n; \omega^{(j)}_{\text{init}}, 0.05 \omega^{(j)}_{\text{init}}, \gamma) \), where \( \omega^{(j)}_{\text{init}} \) is determined as described in the supplementary material [54, Sec. II].

3. **Normalization of the NLOS Distance Likelihood**: As discussed in Sec. III-A2, the NLOS LHF in (14) must be normalized by \( Q_0(p_n, \zeta^{(j)}_n) \). However, \( Q_0(p_n, \zeta^{(j)}_n) \) cannot be determined analytically and, being a function of \( p_n \) and \( \zeta^{(j)}_n \), it needs to be calculated for each individual particle (see Sec. V-B1). Thus, we need an efficient numerical approximation. For details see the supplementary material [54, Sec. III].

**VI. RESULTS**

We validate the proposed model and analyze the performance gain caused by the features of the proposed algorithm using both synthetic data obtained using numerical simulation and real radio measurements. The performance is compared with the PCRLB and that of the AIPDA.

**A. Common analysis setup**

The following setup and parameters are commonly used for all analyses presented.

The PDF of the joint agent state \( x_n \) is factorized as \( f(x_n | x_{n-1}) = f(x_n | x_{n-1}) \), \( f(\gamma_{n-1} | \gamma_{n-1}) \), where the agent motion, i.e. the state transition PDF \( f(x_n | x_{n-1}) \) of the agent state \( x_n \), is described by a linear, constant velocity and stochastic acceleration model [59, p. 273], given as \( x_n = A x_{n-1} + B w_n \), with the acceleration process \( w_n \) being i.i.d. across \( n \), zero mean, and Gaussian with covariance matrix \( \sigma_a^2 I_2 \), \( \sigma_a \) is the acceleration standard deviation, and \( A \in \mathbb{R}^{4 \times 4} \) and \( B \in \mathbb{R}^{4 \times 2} \) are defined according to [59, p. 273], with sampling period \( \Delta T \). The state transition of the rise distance \( \gamma_{n-1} \), i.e., the state transition PDF \( f(\gamma_{n-1} | \gamma_{n-1}) \), is \( \gamma_{n-1} = \gamma_{n-1} + \varepsilon_{\gamma_{n}} \), where the noise \( \varepsilon_{\gamma_{n}} \) is i.i.d. across \( n \), zero mean, Gaussian, with variance \( \sigma_{\gamma_{n}}^2 \). Similarly, the state transition model of the joint anchor state \( y^{(j)}_n \), i.e. the state transition PDF \( f(y^{(j)}_n | y^{(j)}_{n-1}) \), is chosen as \( y^{(j)}_n = y^{(j)}_{n-1} + \varepsilon_{y^{(j)}_{n}} \), where the noise vector \( \varepsilon_{y^{(j)}_{n}} \) is i.i.d. across \( n \) and \( j \), zero mean, jointly Gaussian, with covariance matrix \( \Sigma_{\varepsilon_{y^{(j)}_{n}}} = [\sigma_{\varepsilon_{y^{(j)}_{n}}}^2, \sigma_{\varepsilon_{y^{(j)}_{n}}}^2, \sigma_{\varepsilon_{y^{(j)}_{n}}}^2, \sigma_{\varepsilon_{y^{(j)}_{n}}}^2] \) and the individual state-transition variances (STV) \( \sigma_{\varepsilon_{y^{(j)}_{n}}}^2 \), \( \sigma_{\varepsilon_{y^{(j)}_{n}}}^2 \), \( \sigma_{\varepsilon_{y^{(j)}_{n}}}^2 \), \( \sigma_{\varepsilon_{y^{(j)}_{n}}}^2 \). Unless noted differently the STV are set as \( \sigma_a = 2 \text{ m/s}^2 \), \( \sigma_b = 0.05 \text{ m} \), \( \sigma_{\varepsilon_{y^{(j)}_{n}}} = 0.05 \varepsilon_{y^{(j)}_{n}} \cdot \text{MMSE} \), \( \sigma_{\varepsilon_{y^{(j)}_{n}}} = 0.05 \varepsilon_{y^{(j)}_{n}} \cdot \text{MMSE} \). For the STV of all other parameters
we use values relative to the root mean square error (RMSE) estimate of the previous time step $n-1$ as a heuristic. Note that this choice allows 

no tuning of the STV to be required for all experiments presented, even though the propagation environments are considerably different. We used $5 \cdot 10^4$ particles before the first resampling operation and 5000 particles for inference during the track. We set the detection threshold as low as $\gamma = 1.77$ (5 dB) for all simulations, which allows the algorithm to facilitate low-energy MPCs (this choice is further discussed in Sec. VI-B). The set of possible LOS probabilities is chosen as $Q = \{0.01, 0.33, 0.66, 1\}$. The state transition matrix $Q^{(j)}$ is set as follows: $[Q]_{1,1} = 0.9$, $[Q]_{4,4} = 0.95$, $[Q]_{2,1} = 0.1$ and $[Q]_{3,4} = 0.05$. For $2 \leq k \leq 3$, $[Q]_{k,k} = 0.85$, $[Q]_{k-1,k} = 0.05$ and $[Q]_{k+1,k} = 0.1$. For all other tuples $\{i, k\}$, $[Q]_{i,k} = 0$ in order to encourage high LOS probabilities [42]. For the numerical approximation of $Q_0(p_n, \zeta^{(j)}_n)$ as discussed in Sec. V-B3, we used $K_T = 30$.

The results are shown in terms of the RMSE of the estimated agent position $e_n^{RMSE} = \sqrt{E[(p_n - \hat{p}_n)^2]}$, evaluated using a numerical simulation with 500 realizations. For each of the scenarios investigated, we consistently analyze the influence of the individual features of our algorithm according to Table I. It shows the algorithm variants implemented and the corresponding features that are enabled for an algorithm (x) or not ( ). When “$q_n^{(j)}$ tracking” is deactivated, we set $q_n^{(j)} = 0.999$ for all $n, j$. When we use “decoupled SPA”, the suboptimal message passing scheme presented in Sec. V-A is used. Not applying the “non-uniform $f_{NL}$” means (13) is replaced by $s_n^2 = 1/2$, and for AL4’ and AL5’ we use $5 \cdot 10^4$ particles” instead of 5000. Note that AL1 represents a multisensor variant of the conventional AIPDA.

As a performance benchmark, we provide the CRLB for a single position measurement without tracking (SP-CRLB) [60] as well as the PCRLB [44] considering the dynamic model of the agent state.

B. Analysis on Synthetic Measurements

For the synthetic setup, we investigate the scenario shown in Fig. 5. The agent moves along a trajectory, with two distinct direction changes, where the agent velocity is set to vary around a magnitude of 0.8 m/s. It is observed over a continuous measurement time $t' \in [0, 18.9]$ s, with a constant sampling rate of $\Delta T = 100$ ms, resulting in $N = 190$ discrete steps $n \in \{1, \ldots, N\}$. We simulate three anchors, A1-A3, which are placed in close vicinity to each other. Note that the environment setup shown in Fig. 5, i.e., walls and resulting

| TABLE I |

| ALGORITHM VARIANTS INVESTIGATED FOR DIFFERENT SCENARIOS | AL1 | AL2 | AL3 | AL4 | AL5 | AL4' | AL5' |
|--------------------------------------------------------|-----|-----|-----|-----|-----|------|------|
| $q_n^{(j)}$ tracking                                   | x   | x   | x   | x   | x   | x    | x    |
| non-uniform $f_{NL}$                                  | x   | x   | x   | x   | x   | x    | x    |
| decoupled SPA                                          | x   | x   | x   | x   | x   | x    | x    |
| $5 \cdot 10^4$ particles                               | x   | x   | x   | x   | x   | x    | x    |

Fig. 9, Fig. 11, and Fig. 13 | Fig. 9 only
obstructions, are only used in Sec. VI-B2. For all synthetic radio measurements involving the proposed CEDA (see [54, Sec. VI]), we choose the transmitted complex baseband signal $s(t)$ to be of root-raised-cosine shape with a roll-off factor of 0.6 and a duration of 2 ns (bandwidth of 500 MHz). The signal is critically sampled, i.e., $T_S = 1.25$ ns, with a total number of $N_s = 161$ samples, amounting to a maximum distance $d_{\text{max}} = 60$ m.

1) Synthetic Measurements with Stochastic Multipath: In this section we present results using synthetic measurements generated by simulating the MPCs as zero mean stochastic process. More specifically, we compare results obtained by simulating the radio signal according to (6) and applying the CEDA to results obtained using fully synthetic measurements, which are generated according to Sec. III without involving the CEDA. For fully synthetic measurements the average number of NLOS measurements per time $n$ and anchor $j$ prior to the simulated detection process was approximated as $N_s$. Detection further reduces the prior number of NLOS events by the mean NLOS detection probability. We simulate two OLOS situations clearly separated in time, a partial one at $t' \in [7.4,10.3]$ m, where only the LOS to anchor A2 is blocked, and a full one at $t' \in [11.4,14.3]$ m, where the LOS to all anchors is blocked. The following true system parameters are used, which are set constant for all time steps $n$ and anchors $j$: The normalized amplitude is set to $\hat{u}_n = [\sqrt{19.5}\, \text{dB} \, \sqrt{20.0}\, \text{dB} \, \sqrt{20.5}\, \text{dB}]^2$ and the parameters of the DPS are set to $\hat{\omega}_n^{(j)} = \sqrt{25}$ dB, $\hat{\gamma}_n = 0.7$ m, $\hat{\omega}_n^{(j)} = 6$ m, $\hat{b}_n^{(j)} = 0.7$ m.

We start by validating the system model presented in Sec. III. For this experiment the relatively defined STV are set with respect to the true values instead of the RMSE values, given as $\sigma_\omega = 0.05 \hat{\omega}_n^{(j)}$, $\sigma_\gamma = 0.05 \hat{\gamma}_n$, $\sigma_b = 0.05 \hat{b}_n^{(j)}$, $\sigma_{\hat{\omega}_n^{(j)}} = 0.05 \hat{\gamma}_n^{(j)}$, $\sigma_{\hat{\omega}_n^{(j)}} = 0.5 \hat{\gamma}_n$. Fig. 6, 7 and 8 show the results of the performed numerical simulations. Fig. 6 shows MMSE estimates of all state variables as a function of time $t'$ and compares to the respective true values. The MMSE estimates are determined according to (22)-(24) using both fully synthetic measurements and CEDA-based measurements. Fig. 7 compares distance-model-agnostic, bin-based estimates (BBEs) of scale parameter and relative measurement frequency with the presented model functions, i.e., with the NLOS scale function (13) and the NLOS distance LHF (14). Each of the functions is determined both ways, using the MMSE estimates of $\hat{\xi}_{200}$ of the last time step, given as $\hat{\xi}_{200}$ and using the respective true values used for simulation $\hat{\xi}_{(1)}$. The BBEs are determined using all NLOS measurements (the LOS measurements are removed) of the last 20 time steps, given as $\{\hat{b}_n^{(1)}, m \in \mathcal{M}_n^{(1)} \setminus \hat{a}_n^{(1)}, n \in \{180, ..., 200\}$. For details about the BBEs see the supplementary material [54, Sec. IV]. This analysis is complemented by Fig. 8 which shows the position RMSE $e_{\text{pos}}^m$ in two ways. First, as a function of the continuous measurement time $t'$ and, second, as the cumulative frequency of the RMSE evaluated over the whole time span. Fig. 6 demonstrates that using CEDA-based measurements the MMSE estimates of the parameters of the NLOS LHF $\hat{\xi}_n^{(1)}$ are slightly biased, in particular the DNR estimate $\hat{\omega}_n^{(1)}$. This effect is a consequence of the asymptotic bandwidth assumption used in the derivation of the NLOS likelihood model (see [54, Sec. II]). However, as in Fig. 6 the model functions parameterized with the MMSE values accurately fit the BBEs, the MMSE estimate of the
agent position $\hat{p}_n^{\text{MMSE}}$ in Fig. 6 remains unbiased and, thus, the positioning performance in Fig. 8 using "CEDA-based measurements" is identical to the performance using "fully synthetic measurements" up to random deviations.

In addition, in Fig. 8 we compare to fully synthetic measurements with (i) known initial state distributions, slightly lowering the RMSE at $t' = 0$, and (ii) assuming the parameters of $\xi_n^{(j)}$ to be known constants, leading to a significant increase of performance at the end of the full OLOS situation as the bias information does not vanish over time $t'$. With CEDA-based measurements we also compare to results where (i) we calculate the relatively defined STV using the RMSE values of the respective last time step $n - 1$ according to Sec. VI-A and where (ii) we use a uniform delay intensity function $f_{nL}(\xi_n^{(j)}) = 1/d_{\text{max}}$ showing no significant degradation of performance. The latter result suggests that for low values of $\gamma$, the information provided in (14) is insignificant (c.f. Fig. 2b). Therefore, in what follows, we keep the uniform delay intensity function leading to a considerable reduction of runtime since $Q_0(p_n, \xi_n^{(j)})$ does not need to be calculated (see also Sec. V-B3 and Sec. VI-D).

Next, we investigate the influence of the individual features of our algorithm as described in Sec. VI-A and Table I. Fig. 9 shows the RMSE of this experiment as a function of $t'$ as well as the cumulative frequency of the RMSE. The RMSE of the multi-sensor AIPDA (AL1) mostly attains the PCRLB during LOS and partial OLOS situations. A reason for that is that the angle, which the remaining anchors A1 and A3 span with respect to the agent is sufficiently large to provide a reasonable position estimate. However, AL1 shows a slightly increased RMSE around $t' = 8 \text{ s}$ due to the agent direction change and significantly deviates from the very beginning of the full OLOS situation, losing the track in every single realization. Comparing the curves of AL2-AL5, one can conclude that every single algorithm feature

investigated lowers the RMSE significantly when activated. The RMSE of the proposed algorithm AL5 constantly attains the PCRLB, which indicates no lost track, even falling below the PCRLB in full OLOS situations. This is possible as it leverages the additional position information contained inside the MPCs via the non-uniform NLOS LHF, which is not considered by the PCRLB model. In contrast, AL2 loses a large percentage of tracks after the full OLOS situation, because NLOS measurements significantly contribute to the LOS based position hypotheses due to the insufficient representation of the existence probability by the amplitude state particles (see Sec. III-C). While AL3 constantly attains the PCRLB during the LOS situation as well the partial OLOS situation, it loses the track for every realization in full OLOS; After a short amount of time in which AL3 can maintain the agent position through the agent state transition model and the decreasing LOS probability, it identifies MPCs as the LOS component due to their coherent appearance and large amplitude, which is not covered by the uniform NLOS model, and loses the track. AL4 shows a seemingly random performance degradation, which is due to the insufficient representation of the high dimensional joint state by the particle filter and some resulting lost tracks, which AL5 overcomes by decoupling the anchor states (see Sec. V). However, the discrepancy between AL4 and AL5 can be dissolved by using a sufficiently high number of particles (see AL4’ and AL5’), at the cost of significantly increasing the runtime (see Sec. VI-D).

2) Synthetic Measurements with Geometry-related Multi-path: In this section, we discuss results using synthetic measurements based on the simple floorplan shown in Fig. 5. The measurements are obtained by simulating a radio signal according to (5), consisting of the LOS component and
specular MPCs, and using the proposed CEDA. The MPC delays are calculated out of the floorplan (i.e., W1-W5) using the image source model up to the third order \cite{61, 62}. The SNR of the LOS component as well as the MPCs \cite{34} are set to 20 dB at a distance of 1 m and are assumed to follow free-space path loss. The SNR of the individual MPCs are additionally attenuated by 3 dB after each reflection (e.g., 6 dB for a second-order reflection). As depicted in Fig. 5, for this experiment the anchors are obstructed by an obstacle (W5), which leads to partial and full OLOS situations in the center of the investigated trajectory. Figs. 10 and 11 show results of the performed numerical simulation. Fig. 10 provides a graphical representation of the measurement space, showing a single measurement realization together with the corresponding MMSE estimates according to (22)-(24) of the proposed algorithm (AL5). In particular, Fig. 10a shows that the estimate of the LOS delay $d_{\text{LOS}}^{(1)}$ remains stable over the whole OLOS situation and the maximum of the NLOS LHF follows the first NLOS component available. Fig. 10b shows that the DNR $\omega_{\text{NLOS}}^{(i)}$ accurately represents the dynamic behavior of the multipath energy, deceasing rapidly when the strongest, first MPC is covered. Fig. 10c shows the LOS existence probability $q_{\text{LOS}}^{(1)}$ well representing the OLOS situation. Fig. 11 shows the RMSE as a function of the continuous measurement time $t'$ as well as the cumulative frequency of the RMSE. Again, we investigate the influence of the individual features of our algorithm according to Sec. VI-A and Table I. Comparing the presented curves, we again observe AL5 to significantly outperform the other algorithm variants, with the qualitative performance differences being almost identical to those of Sec. VI-B1. The only significant dissimilarity is the seemingly smaller deviation between AL4 and AL5. This is because AL4 does not lose any tracks during initialization, as the average energy and distance to the LOS component of the measurements of the first time step $z^{(1)}$ are significantly lower in this scenario, leading to a better coverage of the state space by the particle filter. Thus, we only observe a slightly more unstable local behavior of AL4.

C. Performance for Real Radio Measurements

For further validation of the proposed algorithm, we use real radio measurements collected in a laboratory hall of NXP Semiconductors, Gratkorn, Austria. The hall, shown in Fig. 12a, features a wide, open space and includes a demonstration car (Lancia Thema 2011), furniture, and metallic surfaces, thereby representing a typical multipath-prone industrial environment. An agent is assumed to move along a pseudo-random trajectory (selected out of a grid of agent positions), obtained in a static measurement setup. We selected $N=195$ measurements, assuming a sampling rate of $\Delta T = 170 \text{ms}$. The agent velocity is set to vary around a magnitude $\Delta v=33\text{m/s}$. At each selected position, a radio signal was transmitted from the assumed agent position, which was received by 4 anchors. The agent was represented by a polystyrene build, while the anchor antennas were mounted on the demonstration car. The agent as well as the anchors were equipped with a dipole antenna with an approximately uniform radiation pattern in the azimuth plane and zeros in
the floor and ceiling directions. The radio signal was recorded by an M-sequence correlative channel sounder with frequency range 3 – 10 GHz. Within the measured band, the actual signal band was selected by a filter with raised-cosine impulse response \( s(t) \), with a roll-off factor of 0.6, a two-sided 3-dB bandwidth of 499.2 MHz and a center frequency of 7.9872 GHz, corresponding to channel 9 of IEEE 802.15.4a. We created two full OLOS situations at \( t' \in [13.5, 15.6] \) and \( t' \in [27.6, 29] \) using an obstacle consisting of a metal plate covered with attenuators as shown in Fig. 12b. A floor plan showing the track, the environment (i.e., the car, other reflecting objects and walls), the antenna positions, and OLOS conditions with respect to all antennas is shown in Fig. 12c. The metal surface of the car strongly reflected the radio signal, leading to a radiation pattern of 180\(^\circ\) for A1 and A2 and 270\(^\circ\) for A3 and A4. Thus, during large parts of the trajectory the LOS of 2 or 3 out of 4 anchors is not available. Moreover, the pulse reflected by the car surface strongly interferes with the LOS pulse, leading to significant fluctuations of the amplitudes. In addition, this leads to the channel estimator being prone to produce a high SNR component just after the LOS component. As this violates our signal model, we processed the CEDA measurements attenuating all components, where \( z_{n,m}^{(j)} \in \tilde{d}_{n,m}^{(j)} + [0, 2cT_p] \), except for the highest component. As only two antennas (A1 and A2) are visible at the track starting point, the position estimate obtained by trilateration is ambiguous. In the scenario presented, the relative antenna position with respect to the car can be assumed to be known. Thus, for this experiment, we used the antenna pattern as prior information for initialization of the position state. For the numerical evaluation presented, we added AWGN to the real measurements and showed that the additional information provided by the NLOS model can support the estimation of the agent position. The performance of our algorithm significantly outperformed the conventional AIPDA filter, consistently attained the PCRLB in partial OLOS situations (i.e., no lost tracks), and even exceeded it in fully OLOS situations.

A possible direction for future research includes extending the model to multiple biases with respect to several MPCs using joint probabilistic data association and dynamic MPC initialization [32], [34] or to several MPC clusters by using data association with extended objects [63].

### VII. Conclusion

We have presented a particle-based sum-product algorithm (SPA) that sequentially estimates the position of a mobile agent using range and amplitude measurements provided by a snapshot-based channel estimation and detection algorithm (CEDA). We introduced a novel false alarm (FA) model that is adapted to the delay power spectrum (DPS) of the multipath radio channel. We analyzed the performance of the proposed algorithm using both numerically simulated and real measurements and showed that the additional information provided by the NLOS model can support the estimation of the agent position. The performance of our algorithm significantly outperformed the conventional AIPDA filter, consistently attained the PCRLB in partial OLOS situations (i.e., no lost tracks), and even exceeded it in fully OLOS situations.

### Table II

| Scenario          | Sec. VI-B1 | Sec. VI-B2 | Sec. VI-C |
|-------------------|------------|------------|-----------|
| \( |M|_{|avg.} | \times J \) | 28 \times 3 | 12 \times 3 | 7 \times 4 |
| AL5 (proposed)    | 53 ms      | 34 ms      | 30 ms     |
| AL1 (AIPDA)       | 40 ms      | 27 ms      | 23 ms     |

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Graph-based Robust Sequential Localization in Obstructed LOS Situations: Supplementary Material

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This manuscript provides additional analysis for the publication “Graph-based Robust Sequential Localization in Multipath-prone Environments” by the same authors [1].

I. DERIVATION OF THE NLOS LIKELIHOOD FUNCTION

Previous work [2], [3] presents non-uniform non-line-of-sight (NLOS) models for delay/distance measurements consisting of a weighted mixture of two distribution functions: First, a uniform distribution modeling false-alarm measurements, and, second, different types of exponentially decaying functions modeling multipath components (MPCs). Thereby, the MPCs are represented by the typical exponential path loss of expected received power as observable in radio propagation channels [4]. More specifically, [2] uses an approximate convolutive model for delay measurements only, where a single exponential kernel is convolved with a Gaussian function, while [3] approximates the distribution of delay measurements using a double exponential function and a conventional additive white Gaussian noise (AWGN) Rayleigh model for the amplitude measurements. Both of these models are heuristically motivated and lack of an accurate description of the joint NLOS likelihood function (LHF) for delay and amplitude measurements of a channel estimation and detection algorithm (CEDA). Hence, we seek to find an accurate model for the NLOS LHF, which captures the MPC-related statistic of the multipath radio channel.

We want to determine the statistic of the stochastic, NLOS fraction of our signal model in [1, Eq. (6)]. To this end, we define the NLOS-only model

\[ r_{Nn}^{(j)} = \int s(\tau) v_{Dn}^{(j)}(\tau) d\tau + w_n^{(j)} \]  

(1)

with \( s(\tau) \), \( v_{Dn}^{(j)}(\tau) \) and \( w_n^{(j)} \) defined in accordance to the main text (see [1, Sec. II]). Thus, \( r_{Nn}^{(j)} \) is a zero-mean circularly-symmetric complex Gaussian random vector, with covariance matrix corresponding to [1, Eq. (7)].

Since we only consider a single radio signal snapshot in the current analysis, in the following we omit the indices for time \( n \) and anchor \( j \) for brevity of notation.

We base our analysis on the optimum estimation and detection method of a single signal component in AWGN1:

\[ \text{The maximum likelihood (ML) estimator for the normalized amplitude reads [6]} \]

\[ \hat{u}_{\text{ML}}(r) = \arg \max_{u, \tau, \sigma} f_{CN}(u; r^H s(\tau), \sigma) \]

\[ = \max_{\tau, \sigma} \frac{|r^H s(\tau)|}{\parallel s(\tau) \parallel} \]  

(2)

with \( f_{CN}(x; \mu, \sigma) \) being a circular-symmetric complex Gaussian distribution with mean \( \mu \) and standard deviation \( \sigma \). Accordingly, the generalized likelihood ratio test (GLRT) [7] can be defined using (2) as

\[ u_{\text{ML}}(r) \overset{H_1}{\sim} \gamma \]

(3)

with the test statistic being equivalent to \( u_{\text{ML}}(r) \).

We are interested in the statistic of the normalized amplitude estimate when the GLRT decides \( H_1 \), i.e., we consider

\[ f(u_N; H_1) \quad \text{with} \quad u_N = u_{\text{ML}}(r_N). \]

(4)

A joint CEDA in the sense of [1, Sec. II-B] attempts to decompose the received signal into a finite number of individual, decorrelated components. Consider that (4) implies the assumption that the CEDA is not able to decompose elements of the convolution in (1), i.e., it is not able to decorrelate the elements of \( r_N \) and to reduce the cross-terms of the noise covariance matrix [1, Eq. (7)]. This assumption holds in good approximation, as the integral in (1) equivalently models a sum of a non-countable infinity of signal components with infinitesimal spacing and, thus, the influence of the decomposition procedure onto the resulting statistic is negligible.

Now we assume that delay \( \tau \) and noise standard deviation \( \sigma \) are known constants neglecting the influence of jointly estimating these parameters along with the normalized amplitude \( u \) in (2). Then \( u_N; H_1 \) follows a truncated Rayleigh distribution [1, Eq. (3)] cut off at the detection threshold \( \gamma \), given as

\[ f(u_N; H_1; \tau, \sigma) = f_{\text{Rayl}}(u_N; s_N(\tau, \sigma), \gamma) \]

(5)

with the Rayleigh scale parameter being

\[ s_N^2(\tau, \sigma) = \frac{1}{2} \left( \frac{C_\sigma(\tau)}{\sigma^2 \parallel s(\tau) \parallel^2 + 1} \right). \]

(6)

\( C_\sigma(\tau) \) denotes the covariance function of the zero-mean, Gaussian inner product \( r_N^H s(\tau) \), given as

\[ C_\sigma(\tau) \triangleq \mathbb{E} \{ |r_N^H s(\tau)|^2 \} \]

\[ = \int s(\tau)^H s(\tau') s(\tau')^H S_D(\tau') d\tau' + \parallel s(\tau) \parallel^2 \sigma^2 \]  

(7)

1Note that the amplitude model of the commonly used amplitude-information probabilistic data association (AIPDA), which is the basis for the line-of-sight (LOS) model presented in [1, Sec. III-A1], is derived using the same approach [5].
with the delay power spectrum (DPS) $S_D(\tau)$, defined according to [1, Eq. (8)] (see the main text, [1, Sec. II-A] for further discussion). Assuming bandwidth and respective sampling time to approach infinity, the inner product $s(\tau)\bar{s}(\tau')$ in (7) approaches $||s(\tau)||^2 \delta(\tau' - \tau)$, where $\delta(\cdot)$, denotes the Dirac delta distribution. Inserting into (6) yields

$$s^2_N(\tau, \sigma) = \frac{1}{2} \left( \frac{||s(\tau)||^2 S_D(\tau)}{\sigma^2} + 1 \right)$$

(8)

which is equivalent to [1, Eq. (13)] and, thus, (5) becomes equivalent to [1, Eq. (12)].

While the effect of estimating $\sigma$ becomes negligible for a large number of samples $N$, which is true for wideband ranging applications in general [9], estimating $\tau$ leads to a small but visible bias in the scale parameter [10]. Following the steps of [10], this bias, which is usually ignored in amplitude models [5], [11]–[13], can be represented by replacing the truncated Rayleigh distribution of (5) with a truncated Rician distribution [1, Eq. (2)] with non-centrality parameter of $\sqrt{0.5}$. However, different from [10] the expected power of the stochastic process in (1) is not a constant with respect to the elements of $r_0$, and, thus, neither is the corresponding scale parameter (6) with respect to $\tau$. To take this behavior into account, we model the expected amplitude of the stochastic process to be constant in the local environment, i.e., we modify the non-centrality parameter of $\sqrt{0.5}$ by the ratio of the current scale parameter to the scale parameter for AWGN, given as $s_N(\tau, \sigma)/\sqrt{0.5}$. We get

$$f(\eta_N; H_1, \tau, \sigma) = f_{\text{TRice}}(\eta_N; s_N(\tau, \sigma), f_N(\tau, \sigma), \gamma).$$

(9)

We validate the above model by a numerical simulation study. The results are provided in Fig. 1. We show (6) and (8) together with estimates obtained by applying the proposed CEDA (see Sec. V) to simulated radio signals. The simulated radio signals are obtained using numerical simulation according to (1), i.e., they consist of a stochastic process only. The DPS parameters are set to constant values, chosen in line with [1, Sec. VI-B1] for simulation. We used $\gamma = 0$ dB in order accept all CEDA estimates. For simplicity, the distance to the LOS component was assumed to be equal to zero. For this experiment, we use two versions of the CEDA that solve the optimization problem in Sec. V, (23) in two ways: First, we use the normal variant as suggested in Sec. V involving continuous unconstrained optimization [14] (BBE Rayleigh opt, BBE Rice opt.) and, second, we use grid-based optimization only, where the grid values correspond to the sampling grid of the radio signal (BBE Rayleigh grid). Out of the estimates provided by the CEDA we then estimate the scale parameter by grouping the estimates into delay/distance bins and applying, respectively, the maximum likelihood estimator for a truncated Rayleigh distribution (BBE Rayleigh opt, BBE Rayleigh grid), or truncated Rice distribution (BBE Rice opt.); see Sec. IV for details about the bin-based estimation process.

2For unknown $\sigma$, the statistic of two times the squared normalized amplitude $2u^2_N$ is described by a Fisher distribution [8, Ch. 15.10.3] with numerator degrees of freedom equal to 2 and denominator degrees of freedom equal to $2N$. For large $N$ the statistic of $u^2_N$ can be well approximated by a $\chi^2$ distribution [7, Ch. 2.2] and, therefore, the statistic of $u_N$ by the Rayleigh distribution described in the main text.

3In terms of the root mean squared error (RMSE) of the agent position estimate.

4Indeed, preliminary simulations using (5) even showed significantly worse performance of the proposed algorithm, due to local minima introduced by the oscillating nature of $s(t)$ leading to unstable behavior of the particle-based implementation.

We simulated 600 signals amounting to approximately 500 samples per estimation bin (empirically determined value) at the signal parameters and detection threshold configured.

Analyzing Fig. 1 one can observe that the simplified, asymptotic model (8) significantly underestimates the spread parameter as it neglects correlations in $C_S(\tau)$ occurring due to finite signal bandwidth. The correlation-aware model in (5) accurately represents the spread parameter for grid-based estimates of $\tau$ (BBE Rayleigh grid). However, continuous optimization of $\tau$ (BBE Rayleigh opt) leads to an offset, which can be considered using the Rician model (9) instead (BBE Rice opt.). However, as we assumed the noise level to be constant in the local environment, (9) cannot represent the influence of finite signal bandwidth with respect to estimation of $\tau$. This effect is visible at lag 0, due to the rapid change of variance in the adjacent region.

The above analysis showed that the approximate model consisting of (5) and (8) is insufficient as a model for generating measurements. However, in [1, Sec. VI-B1] we demonstrate it to suffice as a model for inference: We obtain no loss in performance of the proposed algorithm when comparing results with data generated according to (8) to results using the stochastic radio signal model [1, Eq. (6)] and the proposed CEDA (see Sec. V) for generating measurements. In contrast, the runtime of the overall algorithm using the approximate model is orders of magnitude lower than using the full model consisting of (9) and (6), especially since the latter requires numerical approximation of the convolutions in (7). See [1, Sec. VI-B1] for further details.

II. DENSE-MULTIPATH-TO-NOISE RATIO (DNR) INITIALIZATION

First we determine the ML estimator for a set of i.i.d. samples $X = \{x_1, ..., x_N\}$ following a truncated Rayleigh distribution, i.e. we solve $\arg\max_s \prod_{x \in X} f_{\text{Trayl}}(x; s; \lambda)$. This
can be done in a straightforward manner by calculating the first derivative of [1, Eq. (3)] and equating to zero. We find
\[ s_{\text{ML}}^2(\mathcal{X}, \lambda) = \frac{1}{2|\mathcal{A}_{\lambda}>^2} \sum_{x \in \mathcal{A}_{\lambda}} x^2 - \lambda^2, \]
(10)
where \( \lambda > \lambda \) \( \{ x \in \mathcal{X} \mid x > \lambda \} \).

Next, we determine the integral of the scale parameter \( s_{\text{ML}}^2(z_{d,n,m}, p_n, \zeta^{(j)}_n) \) from [1, Eq. (13)] over \( z_{d,n,m} \) as
\[ \int_0^{d_{\text{max}}} s_{\text{ML}}^2(d) \, dd = \frac{1}{2} \left( \omega_{\text{init}}^2 \delta B(d) \, dd + \int_0^{d_{\text{max}}} \, dd \right) \]
(11)
dropping the dependence on \( p_n, \zeta^{(j)}_n \) and \( \zeta^{(j)}_n \). Evaluating the integrals on the right-hand side and reordering yields
\[ \omega_{\text{init}}^2 = 2 \int_0^{d_{\text{max}}} s_{\text{ML}}^2(d) \, dd - d_{\text{max}}. \]
(12)

Next, we assign all \( M_d^{(j)} \) normalized amplitude measurements of the initial step \( z_{d,n,m} \), except the measurement with the largest normalized amplitude\(^5\) into \( k \in \{ 1, \ldots, N_{w} \} \) equally spaced bins, depending on the value of their corresponding distance measurements \( z_{d,n,m}^{(j)} \) (see also [1, Sec. II-B]). The discussed bins are given as the sets \( U_d^{(j)} = \{ z_{d,n,m} \mid m \in M_d^{(j)} \land m_{\text{max}}, d_{w,k-1} \leq z_{d,n,m} \leq d_{w,k} \} \) with \( m_{\text{max}} = \arg \max_{m \in M_d^{(j)}} z_{d,n,m}, d_{w,k} = d_{\text{max}}/N_d^{(j)} \) and \( N_d^{(j)} = 2 + [M_d^{(j)}/3] \), where the divisor of 2 were set empirically. We numerically approximate the integral in (12) by individually estimating the scale parameter for each bin using (10) and summing the rectangles formed by each bin, i.e.,
\[ \omega_{\text{init}}^2 \approx 2 \frac{d_{\text{max}}}{N_d^{(j)}} \sum_{k=1}^{N_d^{(j)}} s_{\text{ML}}^2(U_d^{(j)}, \lambda) - d_{\text{max}}. \]
(13)

III. NORMALIZATION OF THE NLOS DISTANCE LIKELIHOOD

As discussed in [1, Sec. V-B3], the normalization constant \( Q_0(p_n, \zeta^{(j)}_n) \) in [1, Eq. (14)] cannot be found analytically. For computational efficiency, we approximate the integral using the trapezoid rule [15] as
\[ Q_0(p_n, \zeta^{(j)}_n) \approx \sum_{k=1}^{K_T} f_{\text{int}}(d_{n,k-1}^{(j)}) + f_{\text{int}}(d_{n,k}^{(j)}) \Delta d_{n,k} \]
where \( f_{\text{int}}(d) = \exp(-\gamma^2/(2s_{\text{ML}}^2(d, p_n, \zeta^{(j)}_n))) \) with \( \Delta d_{n,k} = d_{n,k} - d_{n,k-1} \) and supporting points chosen non-uniformly at \( d_{n,0} = 0, d_{n,1} = d_{\text{LOS}}(p_n), d_{n,K_T} = d_{\text{max}} \) and, for \( 2 \leq k \leq K_T - 1, d_{n,k} = \exp\left(\text{ln}(d_{n,K_T}) - \text{ln}(d_{n,k-1}) - \text{ln}(d_{n,k})) \right) \). Fig. 2 visualizes the discussed approximation scheme.

IV. BIN-BASED ESTIMATES (BBE)

This section discusses the bin-based estimation of the statistic of the NLOS process, as needed for evaluation in [1, Sec. VI-B1] (esp. [1, Fig. 7]) and Sec. I (esp. Fig. 1). We assume all selected measurements to be denoted by \( \bar{u}_{\text{BBB}} \).

\(^5\)Note that when the SNR is high the LOS measurement tends to show the largest normalized amplitude, which, if not excluded, biases the DNR estimate significantly. When the SNR is low, its influence is negligible.

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Fig. 2. Trapezoid integral with non-uniformly spaced support points. We set \( K_T = 7 \) for demonstration purposes.

and \( d_{\text{BBB}} \), respectively, with \( m \in \mathcal{M}_{\text{BB}} = \{ 1, \ldots, M_{\text{BB}} \} \) and \( M_{\text{BB}} = |\mathcal{M}_{\text{BB}}| \) being the number of selected measurements. Similar to Sec. II, we assign all selected measurements to \( k \in \mathcal{N}_{\text{BB}} = \{ 1, \ldots, N_{\text{BB}} \} \) equally spaced bins. For the normalized amplitudes, we define \( U_{\text{BB}} = \{ \bar{u}_{\text{BB}} \mid m \in \mathcal{M}_{\text{BB}}, d_{l,k-1} < d_{\text{BBB}} \leq d_{l,k} \} \) and for the distances \( D_{\text{BB}} = \{ d_{\text{BB}} \mid m \in \mathcal{M}_{\text{BB}}, L_{k-1} < d_{\text{BBB}} \leq L_k \} \) with \( d_{l,k} = \Delta d_{\text{BBB}} \) and \( N_{\text{BB}} = 2N, d_{\text{max}} \) is the maximum observable distance from [1, Sec. II] and the factor of 2 is an empirically chosen constant for visualization. Additionally, we define the bin centers as \( d_{ck} = (k + \frac{1}{2}) \Delta d_{\text{BBB}} \). We visualize the bin-based statistics in terms of the Rayleigh scale parameter as
\[ \{ (d_{ck}, s_{\text{ML}}^2(U_{\text{BB}}), \lambda) \mid k \in \mathcal{N}_{\text{BB}} \} \]
(14)
where \((\cdot, \cdot)\) denotes a couple and \( s_{\text{ML}}^2 \) is determined according to (10). The Rician scale parameter \( \lambda \) is visualized in the same manner as
\[ \{ (d_{ck}, s_{\text{ML}}^2(U_{\text{BB}}), \lambda) \mid k \in \mathcal{N}_{\text{BB}} \} \]
(15)
where \( s_{\text{ML}}^2(\mathcal{X}, \lambda) \) is the ML estimator for a set of i.i.d. samples \( \mathcal{X} = \{ x_1, \ldots, x_{|\mathcal{X}|} \} \) following a truncated Rician distribution, given as \( s_{\text{ML}}^2(\mathcal{X}, \lambda) = \arg \max_{s_0} \prod_{x \in \mathcal{F}_{\text{Rice}}(x; u, s, \lambda)} \), with \( u \triangleq 1/2s \) according to (9). As there is no straightforward analytical solution to this optimization problem, we solve numerically using a grid search. Finally, the relative frequency is visualized as
\[ \{ (d_{ck}, \sum_{k=1}^{N_{\text{BB}}} \sum_{d \in D_{\text{BB}}} d^{-1} \sum_{d \in D_{\text{BB}}} \mid k \in \mathcal{N}_{\text{BB}}) \}. \]
(16)

V. CHANNEL ESTIMATION AND DETECTION ALGORITHM (CEDA)

We start by redefining the discrete-time specular signal vector [1, Eq. (5)] for notational convenience as
\[ r_n^{(j)} = S(\tilde{r}_n^{(j)})x_n^{(j)} + w_n^{(j)}, \]
(17)
where \( x_n^{(j)} = [\tilde{x}_n^{(j)}, \ldots, \tilde{x}_n^{(j)}]^{T} \) are the complex amplitudes and \( w_n^{(j)} = [w_n^{(j)}, \ldots, w_n^{(j)}]^{T} \) are the delays of all \( K_n^{(j)} + 1 \) signal components, including the LOS component and \( K_n^{(j)} \) MPCs and \( S(\tilde{r}_n^{(j)}) = [s(\tilde{r}_n^{(j)}), \ldots, s(\tilde{r}_n^{(j)})] \) is the signal matrix. Since the proposed CEDA operates independently on each radio signal snapshot, we omit the indices for time \( n \) and anchor \( j \) in the following for brevity of notation.
Algorithm 1: Snapshot-based CEDA

Initialization:
• \( m = 0 \) and \( \hat{\tau}_0 = [ ] \);

Iterations:
do
\( m \leftarrow m + 1; \)
compute \( \tau_{\text{res}} = r - S(\hat{\tau}_{m-1})\hat{\alpha}(\hat{\tau}_{m-1}) \)
using results of the last iteration if available;
add component using \( \hat{\tau}_m = \arg \max_r \frac{|r_{\text{res}}(\tau)|^2}{s(\tau)\hat{\alpha}^2(\tau)} \);
\( \hat{\tau}_m \leftarrow \text{prepend } \hat{\tau}_m \text{ to } \hat{\tau}_{m-1}; \)
compute \( \hat{\sigma}^2 = \frac{1}{N-1}||r_{\text{res}}||^2; \)
compute \( \hat{\alpha}(\hat{\tau}_m) \) using (20);
while \( u_{\text{ML}}(r_{\text{res}}) < \gamma; \)

Using (17) the model LHF of a single signal snapshot can be written as
\[
f(r; \tau, \alpha, \sigma^2) = \frac{e^{-\frac{(r-S(\tau)\alpha)^2}{2\sigma^2}}}{(2\pi\sigma^2)^N}. \tag{18}
\]

Based on (18) we formulate a deterministic maximum likelihood (ML) estimator for delays of multiple components, with the complex amplitudes and the noise variance as nuisance parameters.

Taking the natural logarithm of (18) enables formulating the maximization problem as
\[
\{\hat{\tau}, \hat{\alpha}, \hat{\sigma}^2\} = \arg \max_{\tau, \alpha, \sigma^2} \left(-N \ln(\sigma^2) - \frac{||r - S(\tau)\alpha||^2}{\sigma^2} \right) \tag{19}
\]
where a “hat” denotes ML parameter estimates. Taking the gradient w.r.t. \( \alpha \) gives a closed form solution for \( \hat{\alpha}(\tau) \) as [6]
\[
\hat{\alpha}(\tau) = (S(\tau)H^2S(\tau))^{-1}S(\tau)H^r \tag{20}
\]
only depending on the delays. Inserting (20) into (19) removes the amplitude dependency from the (log-)likelihood. The maximization problem becomes
\[
\{\hat{\tau}, \hat{\sigma}^2\} = \arg \max_{\tau, \sigma^2} \left(-N \ln(\sigma^2) - \frac{||r||^2}{\sigma^2} - \frac{\sigma^2}{\sigma^2} \right)
\quad + r^H S(\tau)(S(\tau)H^2S(\tau))^{-1}S(\tau)H^r \sigma^2. \tag{21}
\]
To solve for \( \tau \) we simplify (21), by assuming the individual signal components to be uncorrelated, i.e., \( S(\tau)H^2S(\tau) = \text{diag}([|s(\tau_0)|^2 ... |s(\tau_K)|^2]) \). Thus, we can decompose the optimization problem with respect to \( \tau \) into individual terms. Following an expectation maximization scheme similar to [16], we can solve the equation iteratively, in a bottom-up manner. The expectation term reads
\[
r_{\text{res}} = r - S(\hat{\tau}_{m-1})\hat{\alpha}(\hat{\tau}_{m-1}), \tag{22}
\]
and the maximization terms are
\[
\hat{\tau}_m = \arg \max_{\tau_m} \frac{|r_{\text{res}}(\tau_m)|^2}{||s(\tau_m)||^2}; \tag{23}
\]
and
\[
\hat{\sigma}^2 = \frac{1}{N-1}||r_{\text{res}}||^2. \tag{24}
\]

We solve (23) by successively performing grid-based optimization with the grid set to \( T_s/3 \) and applying a continuous unconstrained optimizer [14]. Following [17], we search for components until the GLRT for a single signal component in noise, as given in (2), falls below the detection threshold \( \gamma \), which is a constant to be chosen. See [10] on how to determine \( \gamma \) out of a fixed value for the false alarm probability per signal snapshot.

An overview of the resulting algorithm is shown in Algorithm 1, which represents a search-and-subtract approach in the sense of [18]. Note that the presented scheduling is suboptimal with respect to the joint update of \( \alpha \) in (20) but offers the advantage of millisecond execution time even with a large number of visible components, making the sum-product algorithm (SPA) presented in the main part of the paper the time-critical part of the overall algorithm.

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