Thermodynamics of baryonic matter with strangeness within non-relativistic energy density functional models

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The phase diagram of dense baryonic matter is investigated in the non-relativistic mean-field framework including the full baryonic octet. It is shown that, depending on the thermodynamic conditions, up to three strangeness-driven phase transitions may occur, such that a huge fraction of the total baryonic density domain corresponds to phase coexistence. The phase transitions are associated to the onset of the different hyperonic species or hyperonic families. We demonstrate that, due to a moderate component of the order parameter along the direction of charge density, phase coexistence persists if the Coulomb coupling to the electrons is accounted for. This makes the phase transition potentially relevant for neutron star and supernova evolution. The sensitivity of the results on the hyperonic coupling constants is explored, both for purely phenomenological energy functionals and for functionals adjusted to microscopic BHF calculations. We show that the presence of a phase transition is compatible both with the observational constraint on the maximal neutron star mass, and with the present hypernuclei experimental information. We also show that two solar mass neutron stars are compatible with important hyperon content. Both the $Y-N$ channel and the $Y-Y$ channel contribute to the phase transition. Still, the parameter space is too large to give a definitive conclusion of the possible occurrence of the phase transition, and further constraints from multiple-hyperon nuclei and/or hyperon diffusion data are needed.

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I. INTRODUCTION

In the effort of building more realistic equations of state (EoS) on which the understanding of astrophysical issues as the structure and evolution of neutron stars (NS) or core-collapsing supernovae (CCSN) relies, special attention is presently paid to the behavior of baryonic matter at densities above nuclear matter saturation density. The subject is challenging as experimental data are too scarce to satisfactorily constrain the respective interactions, in particular if non-nucleonic degrees of freedom are involved.

Though, simple energetic arguments show that no reliable description can be conceived without considering strangeness [1]. As such it is hoped that astrophysical observations can eventually supplement the missing knowledge so far attained in terrestrial laboratories. An example in this sense is the present debate about the measurement of very massive neutron stars, and the associated core composition. The early conclusions ruling out hyperons from the NS core seem to be refuted by recent relativistic and non-relativistic mean-field models showing that a sufficiently repulsive hyperon-nucleon ($Y-N$) and hyperon-hyperon ($Y-Y$) interaction at high densities is able to reconcile the two solar mass measurements corresponding to PSR J1614-2230 [2] and PSR J0348+0432 [3] with the onset of strangeness [4-8] without necessarily a very early deconfinement transition circumventing the hyperon puzzle [9]. The presence of hyperons in dense stellar matter is expected to have important astrophysical consequences. We can recall for instance the modification of the neutron star cooling rate due to hyperonic Urca processes [10-11] leading to very fast cooling for stars with a mass high enough to allow for the onset of hyperons, a result, however, very sensitive to hyperonic pairing [12], and thus subject to large uncertainties. By allowing for weak non-leptonic reactions ($N + N \leftrightarrow N + Y$, $N + Y \leftrightarrow Y + Y$), direct and modified hyperonic Urca and strong interactions ($Y + Y \leftrightarrow N + Y$, $Y + Y \leftrightarrow Y + Y$) hyperons are also shown to impact on bulk viscosity and, thus, damp r-mode instabilities [13].

Most of the predictions are done within mean-field models. However, because of generic attractive and repulsive couplings between the different baryonic species, phase transitions could, in principle, be faced. An example is the liquid-gas phase transition occurring in nuclear matter. If there is a phase transition the mean-field solutions should be replaced by the Gibbs construction in the phase coexisting domains, thus modifying the equation of state. The occurrence of a phase transition in strange compressed baryonic matter has already been discussed in Ref. [14], where a new family of neutron stars characterized by much smaller radii than usually considered was predicted. However, very attractive hyperon-hyperon couplings were considered in that study, which presently appear ruled out by the experimental information on the ground state energy of double-lambda hypernuclei.

A detailed study of the phase diagram of dense baryonic matter was recently undertaked in Ref. [15, 16] within a non-relativistic mean-field model based on phenomenological functionals. The models in Refs. [15, 16] considered a simplified setup, taking only $(n,\Lambda)$ [18] and $(n, p, \Lambda)$ [16] baryon mixtures into account. It was shown that under these assumptions first- and second- order phase transitions exist, and are expected to be explored under the strangeness equilibrium condition characteristic of stellar matter. Two astrophysically relevant consequences have been worked out. In Ref. [16] it
has been demonstrated that in the vicinity of critical points the neutrino mean-free path is dramatically reduced, such that the neutrino transport can be considerably affected. Ref. [17] shows within a spherical simulation that, if during the proto-neutron star contraction after bounce the phase coexistence region is reached, a mini-collapse is induced, leading to pronounced oscillations of the proto-neutron star.

This simplified setup, with \( n_\Lambda \) and \( np\Lambda \)-mixtures is, however, not sufficient to give a complete answer since it is well known that, depending on the hyperonic couplings, \( \Sigma^- \) or \( \Xi^- \) can be even more abundant than \( \Lambda \)'s in the neutron star core. The aim of the present paper is to extend the investigation of Refs. [15, 16] to the whole baryonic octet \( N, \Lambda, \Sigma, \Xi \), and to investigate the dependence of the phase diagram on the hyperonic couplings.

The paper is organized as follows. Section II briefly presents the model and the phenomenology of the phase transition. The phase diagram of uncharged, as well as charge-neutral baryonic matter are spotted in Sections III and IV. The model dependence of the results is analyzed in Section V by considering alternative density functionals for the \( N-Y \) and \( Y-Y \)-interactions and various values for the coupling constants. Conclusions are drawn in Section VII.

II. THE MODEL

In non-relativistic mean-field models, the total baryonic energy density of homogeneous matter is given by the sum of mass and kinetic energy densities of different particle species and the potential energy density:

\[
e_B(\{n_i\}) = \sum_{i=n,p,\Lambda,\Sigma,\Xi} (n_im_ic^2 + \frac{\hbar^2}{2m_i}\tau_i) + e_{pot}(\{n_i\})
= e_B(n_B, n_S, n_Q),
\]

where \( n_B = \sum n_iB_i; n_S = \sum n_iS_i; n_Q = \sum n_Q \) represent the baryon, strangeness and charge number densities, respectively, corresponding to the three conserved charges assuming equilibrium with respect to strong interaction. The particle and kinetic energy densities can be expressed in terms of Fermi-Dirac integrals,

\[
n_i = \frac{1}{2\pi^2\hbar^3} \left( \frac{2m_i}{\beta} \right)^{\frac{3}{2}} F_\frac{1}{2}(\beta \tilde{\mu}_i); \tau_i = \frac{1}{2\pi^2\hbar^3} \left( \frac{2m_i}{\beta} \right)^{\frac{3}{2}} F_\frac{3}{2}(\beta \tilde{\mu}_i),
\]

with \( F_\nu(\eta) = \int_0^\infty dx \frac{x^\nu}{1-\exp(-x\eta)} \), \( \beta = T^{-1} \), \( m_i \) and \( \tilde{\mu}_i \) denote, respectively, the inverse temperature, the effective \( i \)-particle mass and the effective chemical potential of the \( i \)-species self-defined by the single-particle density. The effective chemical potentials are related to the chemical potentials

\[
\mu_i \equiv \frac{\partial e_B}{\partial n_i}
\]

via

\[
\tilde{\mu}_i = \mu_i - U_i - m_ic^2,
\]

where \( U_i = \left. \frac{\partial e_{pot}}{\partial n_{j\neq i}} \right|_{n_j, f \neq i} \) are the self-consistent mean field single-particle potentials.

The potential energy density should in principle account for all possible couplings between nucleonic and hyperonic species, \( N-N \), \( N-Y \) and \( Y-Y \). The nuclear structure data constrain satisfactorily the \( N-N \)-interaction up to densities close to the normal nuclear saturation density and moderate isospin asymmetries, such that well constrained and reliable expressions for this functional, including isospin dependent effective masses and currents are available. The situation is much less clear for higher densities, strong isospin asymmetries as well as for channels containing hyperons. The most general expression of these potential energies can be expanded in a polynomial form

\[
e_{pot}(nc, n_{C'}) = \sum_{k,m} a_{CC'}^{(k,m)} n_k^m n_{C'}^m
\]

As a guideline to characterize the couplings, the single particle potentials of baryon \( C \) in pure \( C' \)-matter are employed:

\[
U^{(CC')}_{C'}(n_{C'}) = \partial e_{pot}(n_c, n_{C'}) / \partial n_{C'}|_{n_c = 0}.
\]

The coupling constants \( a^{(k,m)}_{CC'} \) can then be adjusted to reproduce standard values of these potentials at some reference density, obtained within a (model dependent) analysis of the available experimental data.

A Skyrme-like expression has been frequently employed for the energy density, where the contribution of channel \( CC' \) to the potential energy density is given by

\[
e^{CC'}_{pot}(n_c, n_{C'}) = a^{CC'}_{CC'} n_c n_{C'} + c^{CC'} n_c n_{C'}^{\kappa_{C'}+1} ;
\]

\[
a^{CC'} - c^{CC'} > 0; \quad c^{CC'} > 0; \quad \kappa_{C'} > 0.
\]

This form, which depends on only three parameters for each channel, is the simplest expression which corresponds to a controlled compressibility and fulfills the condition that \( U^{(CC')}_{C'}(n_{C'}) \) vanishes at vanishing \( C' \)-density \( \lim_{n_{C'} \to 0} U^{(CC')}_{C'}(n_{C'}) \to 0 \), and becomes highly repulsive at \( C' \)-high density \( \lim_{n_{C'} \to 0} U^{(CC')}_{C'}(n_{C'}) \to \infty \).

Concerning the channels including strangeness, the available experimental information is particularly scarce. Hypernuclei experiments only provide information on \( \Lambda^- \), \( \Sigma^- \) and \( \Xi^- \) potential well depths in symmetric nuclear matter at saturation densities and, to a less accurate extent, on the \( \Lambda - \Lambda \) interaction potential. Based on a wealth of \( \Lambda \)-hypernuclei data produced in \((\pi^+, K^+)\) reactions, the presently accepted value of \( U^{(N)}_{\Lambda}(n_0) \) is considered to be \( \approx -30 \text{ MeV} \) [18]. \( U^{(N)}_{\Sigma}(n_0) \) is known to be attractive, too, \( \approx -14 \text{ MeV} \), based on missing mass measurements in the \((K^-, K^+)\) reaction on carbon [19].

The situation of \( U^{(N)}_{\Sigma}(n_0) \) is ambiguous. On the one hand \((\pi^-, K^+)\) reactions on medium-to-heavy nuclei point to a repulsive potential of the order of 100 MeV or less [20]. On the other hand, the observation of a \( ^4\text{He}(K^-, \pi^-) \) reaction [21] pleads in favor of an attractive potential. Very few multi-hyperon exotic nuclei data exist so far and all of them correspond to double-\( \Lambda \) light nuclei. The \( \Lambda-\Lambda \)
bond energy can be estimated from the binding energy difference between double-$\Lambda$ and single-$\Lambda$ hypernuclei,

$$\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda}(\Lambda\Lambda) - 2B_{\Lambda}(\Lambda^{-1}Z),$$

(7)

where

$$B_{\Lambda\Lambda}(\Lambda\Lambda) = B(\Lambda\Lambda) - B(\Lambda^{-2}Z).$$

(8)

Measured bond energies are affected by huge error bars. Double-Lambda $\Lambda\Lambda$Be and $\Lambda\Lambda$B data suggest $\Delta B_{\Lambda\Lambda} \approx 5$ MeV while $\delta$-He data point toward a lower value $\Delta B_{\Lambda\Lambda} = 0.67 \pm 0.17$ MeV. In this work we shall consider $\Delta B_{\Lambda\Lambda} \approx 5$ MeV. The bond energy can be interpreted as a rough estimation of the $U_{\Lambda}^{(A)}$ potential at the average $\Lambda$ density $<n_{\Lambda}>$ inside the hypernucleus $^{15}$]. Considering that for such light nuclei $<n_{\Lambda}> \approx n_{0}/5$ $^{15}$, note that this value is compatible with the previously proposed well depth of $\Lambda$ in saturated symmetric nuclear matter, $U_{\Lambda}^{(A)}(n_{0}) \approx -20$ MeV $^{15}$.

For a generic $Y-N$-interaction described by Eq. (6), $U_{Y}^{(N)}(n_{Y} = 0, n_{\Lambda} = n_{0}) = a_{YN} n_{0}$ meaning that the standard values of the potentials only constrain one parameter out of three. This clearly shows that any phenomenological mean-field parameterization is subject to large uncertainties.

In our previous studies $^{15}$ we have employed the functional by Balberg and Gal $^{15}$,

$$e_{pot}^{(BG)}(\{n_{j}\}) = \sum_{i,j} e_{ij}^{(BG)}(n_{i},n_{j});$$

$$e_{ij}^{(BG)}(n_{i},n_{j}) = \left( 1 - \delta_{ij} \right) \left( a_{ij} n_{i} n_{j} + b_{ij} n_{i} n_{j} n_{3} n_{j3} \right)$$

$$+ \left. \left. c_{ij} \right| \frac{\gamma_{i}^{(Y)} + \gamma_{j}^{(Y)} + \gamma_{j}^{(Y)} + \gamma_{j}^{(Y)} - \gamma_{i}^{(Y)}}{n_{i} + n_{j}} \right|, \right)$$

(9)

where $n_{i}, n_{j}$ are the isodensities for nucleons, and $\Lambda$, $\Sigma$, and $\Xi$-hyperons. $n_{3}$ stands for the respective iso-vector densities and the values of $\gamma_{i}$: $\gamma$ are chosen identical for any $(i,j)$ for simplicity. As one may notice, the same functional form is employed in all channels and the potential energy proposed by Eq. (9) deviates from the simple polynomial form truncated at low order Eq. (6) because of the $1/(n_{i} + n_{j})$-dependence of the short-range term. Ref. $^{15}$ proposes three sets of parameters corresponding to different stiffnesses $\gamma = 2.5, 3.4, 4.3$. For the sake of simplicity, a unique value is assumed for $a_{YY}$ and $c_{YY}$.

To illustrate the qualitative features of the phase diagram with strangeness, we will consider the stiffest interaction of Ref. $^{15}$ (BG1), characterized by the values $\gamma, \delta = 2$, $a_{NN} = 784$ MeV fm$^{3}$, $a_{NN} = 214.2$ MeV fm$^{3}$, $c_{NN} = 1936$ MeV fm$^{3}$, $a_{NN} = 340$ MeV fm$^{3}$, $c_{NN} = 1087.5$ MeV fm$^{3}$, $a_{NN} = 340$ MeV fm$^{3}$, $b_{NN} = 214.2$ MeV fm$^{3}$, $c_{NN} = 1087.5$ MeV fm$^{3}$, $a_{NN} = 291.5$ MeV fm$^{3}$, $b_{NN} = 0$, $c_{NN} = 932.5$ MeV fm$^{3}$, $a_{YY} = 486.2$ MeV fm$^{3}$, $b_{YY} = 0$, $c_{YY} = 428.4$ MeV fm$^{3}$, $c_{YY} = 1553.6$ MeV fm$^{3}$ leading to the following values of the different interaction potential depths in symmetric matter at normal nuclear saturation density: $U_{\Lambda\Lambda}^{(N)}(n_{0}) = -26.6$ MeV, $U_{\Lambda}^{(N)}(n_{0}) = -22.8$ MeV, $U_{Y}^{(Y)}(n_{0}) = -19.4$ MeV and $U_{YY}^{(Y)}(n_{0}) = -38$ MeV.

This parameter set is the one producing the highest neutron star maximum mass among the ones proposed in Ref. $^{25}$: the maximum mass exceeds $2M_{\odot}$ if only $\Lambda$’s are considered. However, if the full octet is accounted for, the maximum neutron star mass becomes too low, partly due to a not sufficiently repulsive $Y-Y$ interaction at high densities. To see to what extent the existence of a strangeness driven phase transition is conditioned by the poorly-constrained $Y-Y$-interaction, in section VII we will return to the simple case of a $(N,\Lambda)$-mixture. We will compare the Balberg and Gal parameterization with an energy density functional, where the $N-N$ and $\Lambda-N$ interactions have been fitted to a microscopic Brueckner-Hartree-Fock calculation $^{29}$ and vary the parameters of the $\Lambda-\Lambda$ interaction in both models.

III. THERMODYNAMIC ANALYSIS OF THE PHASE DIAGRAM

The phase diagram of a $\mathcal{N}$-component system is, at constant temperature, a $\mathcal{N}$-dimensional volume. The frontiers of the phase coexistence domain(s), $n_{i}^{(N)}$: $i = 1,...,\mathcal{N}$; $j = 1,...,\mathcal{M}$, are determined by the $(\mathcal{N} + 1)(\mathcal{M} - 1)$ conditions of thermodynamic equilibrium between $\mathcal{M}$ different phases,

$$\left( \frac{\partial f}{\partial n_{i}} \right)_{\mathcal{P}_{i}} = \cdots \left( \frac{\partial f}{\partial n_{i}} \right)_{\mathcal{P}_{M}} = \mu_{i}; \ i = 1,...,\mathcal{N};$$

$$\left( -f + \sum_{i} n_{i} \frac{\partial f}{\partial n_{i}} \right)_{\mathcal{P}_{i}} = \cdots \left( -f + \sum_{i} n_{i} \frac{\partial f}{\partial n_{i}} \right)_{\mathcal{P}_{M}} = P.$$

(10)

where $f = e - Ts$, $s$ and $P$ stand for the free energy density, entropy density and, respectively, pressure. For a system to present a phase coexistence, its mean-field solutions should be more expensive in terms of free energy than the state mixing given by Eqs. (10). Mathematically, this is equivalent to the presence of a convexity anomaly of the thermodynamic potential in the density hyperspace. The number of coexisting phases is determined by the number of order parameters or, in terms of local properties, the number of spinodal instability directions. The last quantity is equal to the number of negative eigenvalues $\mathcal{N}_{neg}$ of the free energy curvature matrix $C_{ij} = \partial^{2} f / \partial n_{i} \partial n_{j}$, such that $\mathcal{M} = \mathcal{N}_{neg} + 1$.

The problem of phase coexistence in a $\mathcal{N}$-component system can be reduced to a problem of phase coexistence in a one-component system by Legendre transforming the thermodynamical potential $f$ with respect to the remaining $(\mathcal{N} - 1)$-chemical potentials $^{31}$.

Under the condition of equilibrium with respect to the strong interaction, baryonic matter is a three-component system with the densities $(n_{B}, n_{Q}, n_{S})$. To reduce the dimensionality for studying phase coexistence, one may then perform the Legendre transformation with respect to any set $(\mu_{B}, \mu_{B})$, $(\mu_{S}, \mu_{Q})$ and $(\mu_{S}, \mu_{Q})$. Formally the three choices should be considered in order to investigate all possible phase separation directions, as required by a complete study. Within the simpler $(n, p, \Lambda)$ system studied in Ref. $^{15}$, which has the same
dimensionality of the full octet, we found that the order parameter is always one dimensional. This means that a single Legendre transformation is enough to spot the thermodynamics provided that the order parameter is not orthogonal to the controlled density. The most convenient framework to easily access the physical trajectories is the one controlling the \( n_B \)-density:

\[
\hat{f}(n_B, \mu_S, \mu_Q) = f(n_B, n_S, n_Q) - \mu_S n_S - \mu_Q n_Q.
\]

The coexisting phases, if any, will then be characterized by equal values of \( \mu_B = 0 \) \( \mu_B = \frac{\partial \hat{f}}{\partial n_B} \) and \( P \) and the phase instability regions will be characterized by a back-bending behavior of \( \mu_B(n_B)|_{\mu_S, \mu_Q} \).

### IV. THE PHASE DIAGRAM OF THE \((n, p, Y)\)-SYSTEM

Within this section we will analyze the phase diagram of pure baryonic matter with strangeness following the lines exposed in the previous section. We employ the Balberg and Gal energy density \([25]\), parameterization BGI, see Section II. The upper panel of Fig. 1 illustrates the evolution of the baryonic chemical potential as a function of baryonic density at constant values of \( \mu_S = 0, \mu_Q = 0 \) and \( T=1 \) MeV \([36]\) while the bottom panel depicts the abundances of different nucleonic and hyperonic species. Three back-bending regions in \( \mu_B(n_B) \) exist. We can see that each back-bending is strictly correlated with the onset of new species, and a decrease of abundances of species already present. Upon choosing \( \mu_S = 0, \mu_Q = 0 \), the hyperonic production thresholds are exclusively determined by the particle’s rest mass and the interaction potentials. Since within the BGI parameterization, the \( Y-Y \) and \( Y-Y' \) interaction depends only very weakly on the particular channel, the rest mass effects dominate. The first strange particle to appear, at about 2.6\( n_0 \) is therefore the less massive one, \( \Lambda \). The three quasi-degenerate \( \Sigma \)-particles whose masses are 74 MeV higher than the \( \Lambda \)-mass, are produced starting from about 3.6\( n_0 \). The most massive hyperons, the \( \Sigma \)-particles, are the last to be created, at about 5.5\( n_0 \). At high densities hyperons become more abundant than nucleons. This shows that having accurate \( Y-Y', Y-Y'' \) and \( N-Y \) interactions is as important as having reliable nucleonic ones. Let us emphasize that although hypernuclear data, constraining the value of \( U^{(\Lambda)}(n_0/5) \), are very useful at low densities, additional information is needed. This is especially true at supra-saturation densities where hypernuclear data have no constraining power.

Investigation of \( P(\mu_B) \) and \( \hat{f}(n_B) \) (not shown) confirms that any back-bending can be cured by a Maxwell construction and that the linear combination of stable phases has a lower value for \( \hat{f} \) than the mean-field solution, and corresponds thus to the energetically favored solution. This means that three distinct phase coexistence regions exist, induced by the onset of each hyperonic family.

Different thermodynamical conditions, i.e. different values of \( \mu_S, \mu_Q \) and \( T \), will obviously change particle production thresholds, abundances, and the location of phase coexistence regions. By correspondingly changing the values of \( \mu_S, \mu_Q \), the whole 3-dimensional phase diagram for a given temperature can be explored. Considering that in astrophysically relevant situations the system is in equilibrium with respect to weak strangeness changing interactions, the most physically relevant part of the phase diagram is the cut corresponding to \( \mu_S = 0 \), which will be the only one considered within this work. The projections of the phase diagram of the \((n, p, Y)\) mixture at the arbitrary temperature of 1 MeV to the \( n_B-n_Q \) (panel (a)) and the \( n_S-n_Q \)-plane (panel (b)) are represented in Fig. 2. The arrows indicate the directions of phase separation. Roughly speaking, two large phase coexistence domains exist: the first one corresponds to the appearance of \( \Lambda \)- and \( \Sigma \)-hyperons, while the second one is due to Cascades. They are well separated and extend over a significant total baryonic density range.

At moderate values of \( \mu_Q \), where particle production is mainly dictated by the rest mass, the thresholds for \( \Lambda \)- and \( \Sigma \)-hyperons are pulled apart, and the phase coexistence regions corresponding to the their respective onsets actually split up, as previously observed for \( \mu_Q = 0 \), see Fig. 1. At more important and negative \( \mu_Q \)-values negatively charged particles are favored and consequently the \( \Sigma^- \)-threshold is shifted to
lower densities and that for $\Sigma^+$ to higher ones. Upon increasing the absolute value of $\mu_Q$ finally the phase coexistence region triggered by the onset of $\Sigma$-hyperons merges with that for $\Lambda$-hyperons. The same happens for positive values of $\mu_Q$, but with the roles of $\Sigma^-$ and $\Sigma^+$ interchanged.

The direction of the order parameter is not constant over the phase coexistence region. The phase transition induced by $\Lambda$-hyperons is always characterized by a very small component of the order parameter along $n_Q$, as the transition is mainly triggered by neutral $\Lambda$-hyperons, as already emphasized in Ref. [16]. The $\Sigma$-induced phase transition has a small component along $n_Q$ when the global $\Sigma$-charge is small, that is at low $\mu_Q$-values, and a significant component at high $\mu_Q$-values, i.e. for a high total $\Sigma$-charge. The $\Xi$-induced phase transition has an order parameter with important contribution along $n_Q$ whenever both $\Xi^0$ and $\Xi^-$ are created as their total charge can not vanish. At high-$\mu_Q$-values the $\Xi^-$ production threshold is beyond the density domain considered for this study such that only $\Xi^0$ exist and consequently the charge dependence of the order parameter becomes very weak.

V. THE PHASE DIAGRAM OF $(n,p,Y,e)$-SYSTEM

The phenomenology of baryonic matter, as the one considered above, is purely academic. What is pertinent from the physical point of view is the phenomenology of electrically neutral matter, where the baryonic charge is compensated by leptonic charge. The net charge neutrality is a prerequisite condition for the thermodynamic limit to exist and corresponds to matter that constitutes compact objects where baryons exist together with leptons and photons. It is commonly accepted that the different sectors are in thermal and chemical equilibrium with respect to strong and electromagnetic interactions. Chemical equilibrium with respect to weak interactions can be satisfied or not depending on how fast the considered astrophysical system evolves compared with weak interaction rates. As such, $\beta$-equilibrium is reached in neutron stars while core-collapsing supernovae typically evolve out of $\beta$-equilibrium. To be as general as possible for the moment we shall not assume $\beta$-equilibrium. As mentioned before, we will, however, assume equilibrium with respect to strangeness changing weak interactions.

In the mean-field approximation, the total thermodynamic potential can be written as the sum of a baryonic, leptonic and photonic contribution, $f = f_B + f_L + f_Y$. Leptons and photons are well described by fermionic and, respectively, bosonic ideal gases [32]. The introduction of lepton degrees of freedom does not increase the dimensionality of the problem because the strict charge neutrality condition $n_Q = n_L$, imposed by thermodynamics, fixes $n_Q$ in terms of leptonic density. Thus the charge degree of freedom is removed and the associated chemical potential, $\mu_Q$, becomes ill-defined. Within this work, $n_L = n_e - n_{\mu}$. The effect of other leptons, in particular muons, is considered beyond the scope of the present work and disregarded. Technically, the only modification with respect to the analysis in the case of pure baryonic matter discussed in the previous section is the replacement of the charge density with the (electron) leptonic one.

Adding an ideal gas contribution to the free energy might change the convexity, i.e. the stability of the system. Indeed, the thermodynamics of charge neutral matter can deeply differ from that of pure baryonic matter. As an example, the liquid-gas (LG) phase transition taking place in nuclear matter at sub-saturation densities is strongly quenched [33, 34] by the presence of electrons via the charge neutrality condition. This is due to the fact that a first order (LG) transition is associated with a macroscopic density fluctuation in direction of the order parameter. In the case of the nuclear LG transition, the order parameter has a strong component in charge direction, implying a macroscopic charge density fluctuation. This fluctuation is, however, strongly suppressed by the high incompressibility of the electron gas.

Ref. [16] shows that, for the $(n,p,\Lambda)$ system, the strangeness-driven phase transition is essentially not affected by the electrons. This is not surprising because $\Lambda$ fluctuations are poorly correlated to the electric charge, see previous section, too. The situation is different here, because of the presence of charged hyperons. As discussed before, we can see in Fig. [2] that the order parameter has a significant component
of the order parameter along the charge density especially for the \( \Sigma \)-induced transition at low \( \mu_Q \) values, and it is in this domain that we expect the most dramatic alteration of the phase coexistence region.

The phase diagram of the \((n, p, Y, e)\)-system under strangeness equilibrium at \( T=1 \) MeV is displayed in Fig. 3 in the plane \( n_B-n_L \). As before, the arrows mark the directions of the order parameter. The pattern of the phase diagram is roughly the same as for pure baryonic matter: depending on \( \mu_L \), \( \Lambda \)- and \( \Sigma \)-hyperons are responsible for one or two phase transitions which extend over \( 0.3 \leq n_B \leq 0.6 \text{ fm}^{-3} \) and a \( \Sigma \)-induced phase transition occurs at higher baryonic densities. The most important shrinking of the phase coexistence is obtained at low \( n_L \) values. The direction of phase separation is rotated in the sense that its component along \( n_L \) gets smaller, which is expected since large electron density fluctuations are effectively suppressed.

VI. MODEL AND PARAMETER DEPENDENCE

The predictions of a phenomenological density-functional model depend dramatically on the functional form assumed for the interaction potentials and the employed values of the coupling constants. As discussed in Section 11, the functional form of the energy density in a non-relativistic phenomenological model is subject to large arbitrariness. The same is true for the coupling constants as the experimental data (a) correspond exclusively to low matter density, (b) are insufficient to constrain all the parameters of the potential energy functional and (c) are often subject to large uncertainties, especially for the \( Y-Y' \) channels. As a consequence, instead of one particular functional with one parameter set, one should rather consider different parameter sets and functional forms, satisfying the experimental conditions.

For this reason, we will first examine the correlation between the existence of the phase transition and the parameters of the Balberg and Gal \[25\] energy density functional. To avoid proliferation of unconstrained parameters, the issue is considered in the simple case of a \((n, p, \Lambda)\) mixture, which nevertheless satisfies the basic requirement of accounting for all relevant degrees of freedom, \( B, S, \) and \( Q \) \[37\]. The two interaction channels which can be responsible of the phase transition are the \( N-\Lambda \) and the \( \Lambda-\Lambda \) one. Since the \( Y-Y' \) interactions are poorly known, we first consider the extreme situation where the \( \Lambda-\Lambda \) coupling is completely absent.

A. The \( N-\Lambda \)-interaction

The parameters of the \( N-\Lambda \)-channel, \( \gamma, a_{AN} \) and \( c_{AN} \), are considered as free variables which have to satisfy the unique condition \( U_A^{(N)}(n_0) = -26.6 \text{ MeV} \), keeping for simplicity the reference value of BGI. We consider \( 1.1 \leq \gamma \leq 3 \), \(-1000 \text{ MeV} \text{ fm}^3 \leq a_{AN} \leq -100 \text{ MeV} \text{ fm}^3 \) and, in each case, calculate \( c_{YN} = (U_A^{(N)}(n_0) - a_{NA} n_0)/(\gamma n_0^2) \). The nuclear part remains the same as for BGI.

The upper panel of Fig. 4 plots, as a function of the stiffness parameter \( \gamma \), the maximum values of the coupling constant \( a_{NA} \) for which symmetric \((N, \Lambda)\)-matter manifests phase coexistence along \( \mu_S = 0 \). As one may note, irrespective of \( \gamma \), there is a wide range of values for the attractive \( Y-N \) coupling meaning that, in this model, phase coexistence in hyperonic matter is not conditioned by the \( Y-Y' \)-interaction.

The behavior of the \( \Lambda \)-potential in symmetric nuclear matter, \( U_A^{(N)}(n_B) = \partial \epsilon_{\text{pot}}(n_N, n_\Lambda)/\partial n_\Lambda |_{n_\Lambda=0} \), as a function of nucleonic density is illustrated in the panel (b) of Fig. 4 for few representative \( \gamma \)-values \((\gamma = 1.72, 2 \) and \( 3 \)) and coupling constants situated at the extremities of the considered range \((a_{NA} = -900 \) and \(-300 \text{ MeV} \text{ fm}^3 \)), both inside and outside the domain compatible with phase coexistence, as indicated on the figure. We can see that a wide variety of density behaviors are compatible with the presence of a phase transition.

The new very precise astrophysical measurements of neutron star masses close to two solar masses \[2, 3\] represent a validity test for any astrophysical equation of state. As the rich recent literature testifies, this supplementary piece of information can neither confirm nor rule out the presence of hyperons in neutron stars. Indeed, while it is true that in principle any extra degree of freedom softens the EOS and, thus, lowers the maximum mass of the star, various models \[4, 8\] prove that hyperons are compatible with the two solar mass constraint. The predictions of the \( \beta \)-equilibrium EOS at zero temperature for the neutron star mass as a function of central density is obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) \[35\] equations for hydrostatic equilibrium of a spheri-
We then consider different parameter sets keeping the $\Lambda$-potential in uniform matter at 1/5 of nuclear saturation density fixed, $U_\Lambda^{(N)}(n_0) = -5$ MeV which leads to $c_{\Lambda\Lambda} = (U_\Lambda^{(N)}(n_0/5) - a_{\Lambda\Lambda}n_0/5) \cdot 2/(\gamma + 1)/(n_0/5)$. Then we consider different parameter sets $(\gamma, a_{Y\Lambda}, a_{\Lambda\Lambda})$ in the ranges $1.2 \leq \gamma \leq 3$, $-1000$ MeV fm$^3 \leq a_{Y\Lambda} \leq -200$ MeV fm$^3$ and $-10000$ MeV fm$^3 \leq a_{\Lambda\Lambda} \leq -100$ MeV fm$^3$.

Adding the $\Lambda$-A long-range attractive - short-range repulsive interaction, the $(n,p,\Lambda)$ toy system manifests phase coexistence in a much broader parameter range. More precisely,
Fig. 5: (Color online) $\Lambda$-potential in uniform $\Lambda$-matter as a function of density corresponding to BGI parameterization with modified $\Lambda$-$\Lambda$ interaction: $a_{\Lambda\Lambda}=-900$, -480, -300 and -200 MeV $fm^3$ and $\gamma=2$.

Fig. 6: (Color online) (a) Neutron star mass as a function of central density and (b) Relative $\Lambda$ abundances as a function of baryonic density along the beta-equilibrium trajectory. In all cases, for the $N$-$\Lambda$-channel the BGI parameter values have been employed. The relative ordering of the various curves is easily understandable for high central densities where the short range repulsion is effective: the stronger is the $\Lambda$-$\Lambda$ repulsion, the smaller is the relative $\Lambda$ density and the larger is the obtained NS mass. Equally predictable is the fact that the small differences in the low density attractive part of the potential result in minor modifications of the $\Lambda$-production threshold, to a large extend dictated by the $N$-$Y$ interaction. Indeed, for the most attractive considered potential the $\Lambda$s spring off at a baryonic density only $3 \cdot 10^{-3} fm^{-3}$ lower than the one corresponding to the least attractive potential.

The calculations presented so far were all obtained with a phenomenological Skyrme-like functional, that proposed by Balberg and Gal [25], both for the $N$-$\Lambda$ and the $\Lambda$-$\Lambda$ channel. One can therefore wonder if the observed phase transition is not a pathology of the assumed and largely arbitrary functional form of the energy density.

C. BHF $N$-$Y$ interaction potentials

For more than fifteen years different microscopically motivated $N$-$N$ and $N$-$Y$ interaction potentials have been proposed. These functionals have all been adjusted to Brueckner-Hartree-Fock calculations of hypernuclei [24, 26-28] and, more recently, hyper-nuclear matter [29, 30] based on different bare $N$-$N$ and $N$-$Y$ interactions. In all these works, $Y$-$Y$ interactions have been disregarded because of missing experimental constraints for the basic two-particle $Y$-$Y$ interaction.

The two parametrizations designed for hyper-nuclear matter, Refs. [29, 30], rely on the same energy density functional,

$$
\epsilon_{NL}^{(BHF)} = \left( a_{N}^0 + a_{N}^1 x + a_{N}^2 x^2 \right) n_N n_L + \left( b_{N}^0 + b_{N}^1 x + b_{N}^2 x^2 \right) n_N^c n_L
$$

where $x = n_p/n_N$. They differ in the coupling constants values as the BHF calculations correspond to various treatments of the three-body forces and $Y$-$N$-potentials and predict significantly different $\Lambda$- and $\Sigma^-$-abundances [30]. For our application we have chosen to use the Burgio-Schulze-Li parameterization [29] because of its stiffer $U_{\Lambda}^{(N)}(n_N)$ dependence.

Despite the functional dissimilarity between Eqs. (13) and (9), the two parameterizations can be bridged via the $\Lambda$-.
potential in uniform symmetric nuclear matter,

\[ U^{(N)}(n_B = n_N) = \frac{\partial U^{(BLS)}_{\text{pot}}(n_N, n_A)}{\partial n_A} \bigg|_{n_A=0} = \frac{\partial U^{(BLS)}_{\text{pot}}(n_N, n_A)}{\partial n_A} \bigg|_{n_A=0} = \left( a_0^n + \frac{a_1^n}{2} + \frac{a_2^n}{4} \right) n_N + \left( b_0^n + \frac{b_1^n}{2} + \frac{b_2^n}{4} \right) n_N^{c_0^n}, \tag{14} \]

which, in both cases, is a polynomial of the baryonic density. More precisely, Eq. (14) can be mapped onto \( \frac{\partial U^{(BLS)}_{\text{pot}}}{\partial n_A} \bigg|_{n_A=0} \) provided that \( \gamma = c_\Lambda = 1.72, a_{NL} = a_0^n + a_1^n/2 + a_2^n/4 = -294.75, c_{NL} = (b_0^n = b_1^n/2 + b_2^n/4) = 462.75. \) The (1.72,-294.75) point is represented in Fig. 4 (a) by a star and sits outside the phase coexistence domain of a symmetric \((N,\Lambda)\) mixture at strangeness equilibrium. We note that these values are very close to \( (\gamma = 1.72, a_{NL} = -300 \text{ MeV fm}^3) \) for which \( U^{(N)}(n_N) \) is depicted in Fig. 7.

We have, however, to keep in mind that this functional gives an EoS which is much too soft and fails to reproduce 2\( M_\odot \) maximum neutron star mass [30]. This is shown in the bottom panel of Fig. 7 which depicts the NS mass as a function of central baryon number density. This is due to the lacking repulsion in the high density domain, meaning that probably the functional is not very reliable at the densities relevant for the phase transition. It is thus important at this point to stress that no firm conclusion can be drawn. It is certainly true that the phenomenological BG form is largely arbitrary; however the description of the nucleon-hyperon interaction in the BHF theory cannot be complete, neither.

As we have already stressed, the \( Y-Y \) interaction cannot be neglected in hyperonic matter. It could well be the source of missing repulsion in microscopic models. Due to the lack of information on this channel within microscopic calculations, for this channel we will adopt the simple polynomial form of BG, and supplement the BSL functional, Eq. (13), with it. The upper panel of Fig. 7 illustrates the maximum values of the coupling constant \( a_{\Lambda\Lambda} \) for which phase coexistence occurs in symmetric \((N,\Lambda)\) matter at various values of the stiffness parameter \( \gamma \). The considered domains are \( 1.1 \leq \gamma \leq 3 \) and \(-1500 \leq a_{\Lambda\Lambda} \leq -100 \text{ MeV fm}^3\). As before, \( c_{\Lambda\Lambda} \) is obtained form the condition \( U^{(A)}(n_0/5) = -5 \text{ MeV} \). One can see that, in case of moderate \( N-Y \)-repulsion as it is the case for BSL, a phase coexistence can still be obtained, but it requires a considerable attraction in the \( Y-Y \) channel.

The effect of the \( \Lambda-\Lambda \)-interaction on the NS mass-central density relation and, respectively, the \( \Lambda \)-hyperon abundances in \( \beta \) equilibrium is represented in the bottom and middle panels of Fig. 7. The two considered \( \Lambda-\Lambda \)-interactions correspond, respectively, to phase coexistence \( (a_{\Lambda\Lambda} = 1200 \text{ MeV fm}^3) \) and stability with respect to phase separation \( (a_{\Lambda\Lambda} = 200 \text{ MeV fm}^3) \). For the sake of the argument, also the case of a purely repulsive and very strong \( \Lambda \)-interaction characterized by \( (\gamma = 2, a_{\Lambda\Lambda} = 0, c_{\Lambda\Lambda} = 3 \cdot 10^4) \) is considered. We can see that employing a strongly attractive coupling at low densities

![FIG. 7: (Color online) BSL parameterization + \( \Lambda-\Lambda \)-interaction: (a) Limiting values of the coupling constant \( a_{\Lambda\Lambda} \) for which, at different \( \gamma \), the symmetric \((n, p, \Lambda)\)-system at \( T=0 \) manifests strangeness driven phase transition along \( \mu_S = 0 \); \( c_{\Lambda\Lambda} \) is fixed via the condition \( U^{(A)}(n_0/5) = -5 \text{ MeV} \); (b) \( \Lambda \)-density along the \( \beta \)-equilibrium path at \( T=0 \) for the original BSL potential and the cases in which BSL is supplemented with a \( \Lambda-\Lambda \)-interaction following the functional form proposed by Balberg and Gal [28] (see eq. (9)) and obeying the condition \( U^{(A)}(n_0/5) = -5 \text{ MeV} \), with \( \gamma=2 \) and \( a_{\Lambda\Lambda} = -1200 \), -200 \( \text{MeV fm}^3 \); and, respectively, \( \gamma=2 \) and \( a_{\Lambda\Lambda} = 0 \) and \( c_{\Lambda\Lambda} = 30000 \text{ MeV fm}^3 \). (c) Gravitational mass as solution of the TOV equations as a function of central baryon number density for the cases considered at (b).]

does only slightly shift the density threshold for $\Lambda$-production with respect to a weakly attractive coupling, but strongly enhances the equilibrium abundances just above threshold. As we have already noted several times, a strong attraction at low density is correlated to a strong repulsion at high densities, leading to a relative decrease of the $\Lambda$ abundances at higher densities with the smaller $a_{\Lambda\Lambda}$. As a consequence, too, decreasing $a_{\Lambda\Lambda}$ leads to an increase of the maximum NS mass. Though, the 2 solar mass neutron star limit is not reached. In conclusion, we can say that the ad-hoc inclusion of an extra term in the BSL functional effectively accounting for the missing $Y-Y$ interaction does not solve the well-known neutron star maximal mass problem of the BHF theory.

VII. CONCLUSIONS

In this work, we have presented a complete study of the low temperature phase diagram of baryonic matter including hyperonic degrees of freedom within the phenomenological non-relativistic Balberg and Gal model \[25\]. We have shown that the hyperon production thresholds are systematically associated with thermodynamic instabilities, leading to distinct first order phase transitions. These transitions can merge into a wide coexistence zone if the production thresholds of different hyperonic species are sufficiently close. As a consequence, a huge part of the phase diagram corresponds to phase coexistence between low-strangeness and high-strangeness phases.

In contrast to the nuclear liquid-gas phase transition which is strongly quenched, this result is only slightly affected by adding electrons and positrons to fulfill the charge neutrality constraint. The only effect is a rotation of the direction of phase separation which reduces the electric charge density component of the order parameter. The reason is that this phase transition is driven mainly by the strangeness degree of freedom, such that the electric charge plays only a minor role. We thus recover the finding for the $(n, p, \Lambda, e)$-system of Ref. \[16\] including the full baryon octet and in particular charged hyperons.

Along the beta-equilibrium trajectory with $\mu_L = 0$ the phase coexistence region corresponding to the pop up of $\Lambda$- and $\Sigma$-hyperons, as predicted by the parameterization BG1, extends over $0.3 \leq n_B \leq 0.4$ fm$^{-3}$. Physically this path is explored by neutron stars with untrapped neutrinos. Following the study in the simple $(n, p, \Lambda, e)$-model in Ref. \[16\], we expect that this phase coexistence region remains at higher temperatures and extends over density and lepton fraction $Y_L = n_L/n_B$ domains explored by warm proto-neutron star matter. A more quantitative analysis is left for future work.

The possible existence of such a phase transition is strongly conditioned by the $N-Y$ and $Y-Y$ interaction. In the second part of the paper we have thus investigated on the one hand the dependence of the phase diagram on the interaction parameters within the Balberg and Gal energy density functional and on the other hand, we have compared the results with an energy density functional based on microscopic BHF calculations, that by Burgio, Schulze and Li \[29\]. A complete parameter study of the energy functional would be very cumbersome and not very illuminating, because of the huge number of insufficiently constrained couplings. We have therefore considered the simplified situation of symmetric nuclear matter with $\Lambda$-hyperons. The phase diagram of this simple model is very similar to the one obtained upon including with the full baryonic octet at densities below the threshold of appearance of more massive hyperons. We therefore believe that this simple model can give correct qualitative results for the full problem.

Both, $N-Y$ and $Y-Y$-couplings are seen to play a role in determining the existence of an instability. Within the BG model it is shown that an instability exists over a very large parameter domain and the two solar mass limit of NS is compatible with important hyperonic abundances. At variance, BSL is stable with respect to phase separation. Though, phase instability can be reached when the original interaction potential is supplemented with a phenomenological $Y-Y$ interaction. We have considered both pure repulsive and attractive-repulsive potentials who fit the experimental data, the measurement of a positive bond energy in double-$\Lambda$ hypernuclei. The results show that an extra $Y-Y$ interaction always results in an enhanced maximum NS mass. Though even the most repulsive potentials failed to reach the two solar mass limit.

More precise measurements on a higher number of double-$\Lambda$ hypernuclei and/or the measurement of $\Lambda-\Lambda$ phase shifts in diffusion experiments are needed to determine the $N-Y$ and $Y-Y$ potentials over a broad density domain upon which the definite answer on the presence on strangeness-driven instabilities in the core of neutron stars depends.

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