Eddy-wave range of geostrophic turbulence

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Abstract. New information on the nature of the ocean's mesoscale variability is presented at [1] based on the analysis of altimetric observations accumulated over almost thirty years. Processing of remote sensing data presented in this paper made it possible to test general conclusions about the properties of mesoscale variability obtained earlier including theoretical assumptions concerning eddy structure and kinematics. Analysis of altimetry measurements presented in [1] is applied in this paper to test major conclusions made earlier based on restricted POLYMODE observations and theoretical models. Altimetry confirms the rare location of eddies through space and universal eddy structure. Comparison of the meridional displacements of eddies from theory and observations gives indirect confirmation of radiation of Rossby waves by eddies, the possibility of transferring energy from waves to eddies, as well as estimations of the impact of large-scale currents on the eddy propagation. Speculations about possible baroclinic instability impact and spectral energy cascading to mesoscale range permit to build a bridge between observations and the concept of the eddy-wave range of geostrophic turbulence.

1. Introduction
Fifty years ago, in the experiment "Polygon - 70" Soviet oceanographers discovered mesoscale eddies of the open ocean. These formations are similar to cyclones and anticyclones observed in the atmosphere in terms of their characteristics. Mesoscale eddies in the ocean were then intensively studied in large-scale oceanic experiments such as the U.S. MODE program and the joint Soviet-USA POLYMODE program. The POLYMODE program, which was started in 1977, continuously monitored the hydrological characteristics of the marine environment during a year at 500x500 km test site with the resolution of mesoscale processes. Successive research vessels performed regular measurements of the temperature and salinity of seawater according to the typical samples scheme. Besides, continuous observations of currents were carried out on the network of anchored buoys. Collected observations allowed describing in sufficient detail the properties of the ocean's mesoscale variability.

Analysis of observations obtained in large-scale ocean experiments showed that the density of kinetic energy of mesoscale processes is almost everywhere significantly higher than that of average currents. Therefore, the current state of the oceans is significantly different from the climatic one. It is determined by mesoscale processes and cannot be taken from climatic atlases.

The discovery of eddies in the ocean stimulated the organization of continuous ocean observations under the Global Ocean Observing System (GOOS) program. An essential component of the ocean operational observations is satellite altimetry, which allows observing mesoscale eddies throughout the World Ocean. Satellite altimetry observations of the ocean have been carried out since the early 1990s.
During this time, a large amount of information about mesoscale processes in the ocean has been accumulated. The methods of processing these observations, developed in the last 10 to 15 years [2], allow carrying out analysis of mesoscale eddies dynamics. The altimetry observations analysis and theoretic results about the so-called beta-effect influence on the dynamics of mesoscale eddies allow substantiating the idea of an eddy-wave range of geostrophic turbulence in the ocean. The second section describes the characteristic properties of ocean mesoscale variability based on observations at the POLYMODE test site and satellite altimetric observations and highlights the main properties of the evolution of the mesoscale eddies on the beta plane. The third section consider kinematics of mesoscale eddies according the theory and observations. The fourth section presents the discussion of an eddy-wave range of geostrophic turbulence in the ocean.

2. Mesoscale eddies structure according to theory and observations

The rather complete description of the various characteristics of the mesoscale eddies in the ocean is given in [1]. The paper analyses long-lived mesoscale eddies, namely, such eddies, which are continuously observed for at least sixteen weeks. A 16-year interval of observations has been selected for analysis since 1992. During this time, about 36,000 eddies have been located throughout the ocean. The results of the analysis of such a vast array allow comparing and expanding the understanding of the mesoscale eddies properties, compiled based on the processing of observations at the POLYMODE.

Altimetry observations show that in temperate latitudes intense eddies in the ocean occur almost everywhere. At the same time, altimetry confirms the important property of the mesoscale eddies field revealed at the POLYMODE test site. In most parts of the World Ocean, the location of eddies in space is far from dense packaging. According to hydrological surveys at the POLYMODE test site, on average, a little less than two eddies were accounted for one-degree square of the ocean surface per year. According to the analysis of altimetric observations in a one-degree square in relatively quiet areas, 2-3 eddies appear per year. In energy-active areas of the World Ocean, the number of eddies observations increases and reaches 4-6 eddies in a one-degree square per year. Altimetric observations show that the horizontal scale of most of the mesoscale eddies lies within 50 to 150 km. This is well in line with observations at the POLYMODE test site, where the size of the observed eddies varied from 60 to 160 km. However, altimetry observations show that, on average, the size of eddies decreases monotonously with the growth of latitude. The characteristic spatial size of eddies according to altimetry data varies from 250 km in the tropics to 75 km at a latitude of 60°. Despite the low density of vortex packaging, they contain a significant amount of energy. Eddies at the POLYMODE test site contained up to 80% of the available potential energy. The global estimate across the World Ocean based on altimetric measurements gives an average estimate of 40%. However, it should be taken into account that the intensity of eddies decreases when moving to the “desert” areas.

The first observations starting from the Polygon-70 experiment showed that the synoptic eddies on average move westwards. It is well known that the western movement of the phase is a characteristic feature of the planetary Rossby waves. Therefore, many researchers tried to interpret intense eddies as a superposition of Rossby waves. However, observations at the POLYMODE test site have clearly shown that the orbital rotation rate of fluid particles in intense eddies significantly exceeds the speed of their propagation [3]. As a result, eddies capture and carry large amounts of fluid over long distances (Figure 1). The altimetric observations definitely confirmed this conclusion.

The processing of altimetry revealed a certain similarity of the eddy structure. Normalization of horizontal coordinates on the characteristic size of the vortex, and scaling the sea level to its maximum deviation allows building a universal dependence of the sea level as a function of distance from the center of the vortex. The mode vortex profile is axisymmetric with an area of solid-body rotation near its center. This area is matched with a ring of almost constant sea-level value [1] (Figure 1). The theory presented at [4] gives explanation of the observed eddy structure.

The dynamics of intense eddies is described by the equation of potential vorticity conservation in a quasi-geostrophic approximation, which qualitatively takes into account their major features.
\[ \frac{\partial}{\partial t} \left( \Delta \psi - \frac{1}{R_d^2} \psi \right) + \frac{\partial (\psi \Delta \psi)}{\partial (x, y)} + \beta \frac{\partial \psi}{\partial x} = 0 \] (2.1)

In the equation (1) \( \psi \) is the stream function, through which the components of the current velocity are expressed, \( x, y \) are the horizontal coordinates, \( t \) is the time, \( \Delta \) is the symbol of the Laplacian operator. The second summand in equation (1) means Jacobian. The X and Y coordinates are oriented to the east and north respectively. The parameter \( \beta \) describes the impact of the spherical Earth. Here \( R_d \) is the Rossby deformation radius of the first baroclinic mode. It takes into account the vertical density stratification of the ocean.

In the extreme case of small amplitudes, the equation (1) is linearized. It describes planetary wave oscillations known as Rossby waves. In another extreme case of high-intense motion with a spatial scale of a much smaller Rossby radius, we get an equation describing two-dimensional turbulence. At the same time, the typical for two-dimensional turbulence flux of energy to large scales stops when approaching the wave number \( R_d^{-1} \) [5].

Theoretical consideration shows that the intense cyclonic vortex on a beta-plane moves north, and anticyclonic – to the south. According to [4] an intense vortex over some time tends to a quasi-equilibrium state. In this state, the vortex moves predominantly westwards at a speed slightly lower than the phase speed of long Rossby waves of the first baroclinic mode, equal to \( \beta R_d^{-2} \).

High-intensity vortex, when the speed of orbital rotation significantly exceeds its propagation speed, is almost symmetric. With high accuracy, its structure is described by the equation

\[ \Delta \psi - \frac{1}{R_d^2} \psi + \beta \bar{y} = \Delta \psi_0 - \frac{1}{R_d^2} \psi_0 \] (2.2)

where \( \psi_0(r) \) is an axisymmetric vortex profile at the initial position, \( r \) is the distance from the center of a vortex, and \( \bar{y} \) is the distance of the meridian displacement of the vortex center from its initial position. The equation (2) is solved with boundary conditions

\[ \psi = \frac{\partial \psi}{\partial r} = 0 \text{ when } r = R \] (2.3)

where \( R \) is the trap zone radius, within which fluid particles are involved in orbital motion and transported by a vortex. Extra boundary conditions allow finding the radius of the trap zone depending on the displacement \( \bar{y} \). Let us mention that the universal form of the mesoscale eddy found in [1] exactly fits the equation (2) (see Figure1). Note first that within the model (1) the sea level is
proportional to the stream function. Then the best fit of empirical approximation of the sea level depending on the distance from the eddy center, proposed in [1], has the form

\[
\psi = \begin{cases} 
-\frac{1}{4} \omega (R_0^2 - r^2) + d, & r \leq R_0 \\
 d, & r > R_0 
\end{cases}
\]  

(2.4)

Expression for \(\psi_0(r)\) follows from (4) if assuming \(d = 0\). Moreover, it follows from (2) that \(d = \beta R_d^2 \bar{y}\). Good consistency of the theory and observations permits to use model presented at [4] for further interpreting of observations.

3. Intense eddies kinematics

Altimetric observations show that the propagation speed of intense mesoscale eddies is close to the phase speed of long baroclinic Rossby waves of the first mode. This fact fits well with prediction of theory [4]. Note also that the proximity of the propagation speed of intense mesoscale eddies to the phase speed of long baroclinic waves of the first mode indirectly indicates the predominance of the first baroclinic mode in the oceans. Let us mention that at the POLYMODE test site, where hydrological samplings were conducted, the fact of the first baroclinic mode predominance was established directly based on temperature fluctuations decomposition on the empirical orthogonal functions [3].

Altimetric observations have shown also another important feature of eddy kinematics. Analysis of a large number of trajectories of eddies propagation showed that cyclonic vortices predominantly move to the northwest, while anticyclonic ones move to the southwest. The evidence of mesoscale eddies movement and their meridional displacement from observations is consistent with theoretical results about Rossby wave radiation by mesoscale eddies [4]. The moving eddy contaminates the ambient fluid and radiates Rossby waves. The wave number of most of the radiated waves is roughly equal to \(\sqrt{\frac{\beta}{c} - \frac{1}{R_d^2}}\), where \(c\) is an eddy propagation speed. An eddy moves at a speed slightly less than the phase velocity of long baroclinic waves according the theory. Therefore it should radiate rather long waves.

Figure 2. The scheme of moving eddies trajectories. Solid lines restrict angles where trajectory of cyclonic eddies concentrates according the theory if mean currents are absent. Dot lines show an angle where trajectory of cyclonic eddies concentrates according the theory if mean currents are directed to the South – West. Area marked by ellipse approximately corresponds to the trajectory cloud from altimetric observations [1].

The field of the radiated waves in the assumption that the eddy moves at a quasi-stationary mode was found first in [3]. The expression for the wave drag force also was found in this paper. The interpretation of the pulse and energy fluxes associated with the wave radiation through the wave
action is presented in [6]. Moving eddy loses energy due to the work against the wave drag force. It is easy to find from the equation (2), that when the energy is lost, an eddy must be displaced along the meridian.

The theory of moving eddies explains the displacement of cyclonic eddies poleward and anticyclonic one equatorward if do not consider climatic currents (Figure 2). Observations partially confirm this deduction as about 55% cyclonic eddies moves poleward and 70% anticyclonic eddies move equatorward. However residual part of eddies move in opposite direction along the meridian [1]. It is assumed there that the movement of eddies is influenced by large-scale baroclinic currents. The study of the impact of mean currents on the eddy trajectory based on equation (1) is presented in [4]. It is shown that large-scale baroclinic currents affect an eddy trajectory in two ways. One obvious effect is the simple transport of eddy by currents. However, there is another mechanism of influencing kinematics of large-scale baroclinic currents. Let us mention first that large-scale currents are supported by the slope of the main thermocline due to geostrophic balance. The slope of the thermocline acts on eddies in much the same way as the bottom slopes, modifying the beta effect [4]. Thus, large-scale baroclinic currents can significantly change the direction of eddy propagation. In particular, under a special selection of the intensity and direction of large-scale currents, the cyclonic eddy can move south (Figure 2) that explains observations (Figure 2).

4. Discussion. Eddy-wave turbulence range

An analysis of the intensity of the mesoscale eddies presented in [1] shows that in temperate latitudes north of the equator, relatively narrow areas adjacent to the frontal zones of jet currents, and the rest of the ocean should be considered separately. Near the frontal zones, eddies are formed when the current meanders pitch off. A well-known example is a process of forming cold and warm rings of the Gulf Stream. Frontal eddies have a high intensity, but their further evolution depends on the geographical location of frontal zones. The eddies formed by the meandering of the western border currents, due to the predominance of the tendency to move to the west, usually after a while are again merging by the current. As noted in [1], the meandering of the eastern boundary currents also forms eddies, which then moves to the open ocean. Thus, the eastern border of the basin is an area of formation of a significant portion of mesoscale eddies. According to the theoretical analysis presented above, such eddies may move westwards at distances up to several thousand kilometers. The moving eddies radiate the planetary Rossby waves, filling the space outside relatively rare spaced eddies. Losing energy on the wave radiation, eddies acquire a meridional component of propagation velocity. Moving along the meridian, eddies should decrease their size and intensity. It follows from the equation (2) that each eddy moves up to the “rest” latitude, reaching which it disappears dissolving in the wave background.

For the typical amplitudes of eddies reported in [1], radiating eddies may drift along the meridian by several degrees. On the spaghetti diagram of the eddy trajectories presented in [1], displacements of eddies up to 10 to 12 degrees were observed along the meridian. It seems that the radiation of Rossby waves explains a significant part but not all of the observed meridional displacement. Observed meridional displacement of eddies on the larger distances, than it was predicted by the wave radiation model, as well as the propagation of cyclonic eddies to the south, and anticyclonic – to the north [1], can be explained by the influence of large-scale currents in the open ocean. Let us consider eddies which are located on the southern periphery of the anticyclonic gyre, where the climatic currents are directed to the west. The main thermocline is deepening to the north in this region according to the direction of the currents. It follows from the theory [4] that if the speed of the western current is greater than \( \beta R^2 \), cyclonic eddies should move south. At the same time cyclonic eddies located at the area where climatic currents directed to the east should be displaced more north than it is prescribed by only Rossby wave radiation.

Due to the large scale of the climatic currents, their influence explains only regular shifts in the trajectories of eddies. However, a small scale fluctuation of eddy trajectories also visible in [1]. Let us mention that a vortex not only loses energy radiating waves. It may also get energy from the Rossby wave [4]. The necessary condition of the reverse energy flux is that the wave should be stationary in
the coordinate system moving together with a vortex. Besides, the wave should induce near the vortex current velocity with a meridional component directed to the south in case of the cyclonic vortex and the north for the anticyclonic one.

It has been shown in [3] that the space between eddies was filled with Rossby waves at the POLYMODE test site. The paper [1] also suggests that ridges and troughs in the ocean level, observed by altimetry in the space between eddies, may be related to Rossby waves. Thus, the fluctuations on eddy trajectories, most likely appear due to the interaction of eddies with Rossby waves.

Rossby waves radiated by eddies should have rather long wavelengths. The group velocity of such waves is directed to the west and therefore the energy of the background wave field should increase up to the western borders of the basin. Thus, the density of the total energy of eddies and waves should roughly persist. At the same time, if the only source of eddies is the meandering of currents near the eastern borders of the basin, the percentage of the energy of eddies in the total energy of synoptic variability in the western part of the basin would be lower than in its eastern part. Besides, the amplitude and the size of eddies should decrease on the way from the east to the west. However, the results presented in [1] show that there are no zonal trends in the distributions of the above-mentioned parameters. Observed uniform distributions of the size and intensity of eddies along the circle of the latitude, as well as the proportions of the energy of eddies and background are consistent with the observation of eddies origin everywhere in the ocean.

The most likely source of energy for eddies formed away from jet streams is the baroclinic instability of currents that form ocean gyres. A possible option is that initially, baroclinic instability generates well-packed relatively small-scale structures. Further turbulence is cascading energy in the direction of the Rossby scale. The size of eddies increases due to their merging. Accordingly, over some time, the density of eddies decreases, and they are located at a considerable distance from each other. The fluid motion outside eddies corresponds to a turbulent mode until the beta effect begins to affect. To move to a wave mode in the areas outside of intense eddies, it is necessary to have rather a low wave energy density. Particularly, the border between turbulence and waves, which is \( E'_{\omega} B^{-0.5} \) according to Rhines [7], should be smaller than the Rossby radius. This condition requires a small fraction of the wave background in the energy density of the mesoscale variability and a large proportion of the area occupied by the wave background. These conditions were met in particular at the POLYMODE test site. Waves occupied about 70% of the ocean surface but contained only 20% of the total energy of mesoscale variability [3]. Observations presented in [1] do not provide an opportunity to confirm or disprove the assumption about the formation of mesoscale eddies from a small-scale background particularly because available altimetric products allow resolving structures that have a scale of more than 40 km.

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