Criteria for 2D kinematics in an interacting Fermi gas

P. Dyke\textsuperscript{1}, K. Fenech\textsuperscript{1}, T. Peppler\textsuperscript{1}, M. G. Lingham\textsuperscript{1}, S. Hoinka\textsuperscript{1},
W. Zhang\textsuperscript{2}, B. Mulkerin\textsuperscript{1}, H. Hu\textsuperscript{1}, X.-J. Liu\textsuperscript{1}, and C. J. Vale\textsuperscript{1,*}

\textsuperscript{1}Centre for Quantum and Optical Sciences, Swinburne University of Technology, Melbourne 3122, Australia
\textsuperscript{2}Department of Physics and Beijing Key Laboratory of Opto-electronic Functional Materials and Micro-nano Devices, Renmin University of China, Beijing 100872, China

*To whom correspondence should be addressed; E-mail: cvale@swin.edu.au

(Dated: November 19, 2014)

Ultracold Fermi gases subject to tight transverse confinement offer a highly controllable setting to study the two-dimensional (2D) BCS to Berezinskii-Kosterlitz-Thouless superfluid crossover. Achieving the 2D regime requires confining particles to the transverse ground state which presents subtle challenges in interacting systems. Here, we establish the conditions for an interacting Fermi gas to behave kinematically 2D. Transverse excitations are detected by measuring the transverse release energy which displays a sudden increase when the atom number exceeds a critical value $N_{2D}$ signifying a departure from 2D kinematics. For weak interactions $N_{2D}$ is set by the aspect ratio of the trap. Close to a Feshbach resonance, however, the stronger interactions serve to reduce $N_{2D}$ with increasing attraction.

PACS numbers: 03.75.Ss, 03.75.Hh, 05.30.Fk, 67.85.Lm

Fermions confined to move in two-dimensional (2D) planes represent an important paradigm in many-body physics in settings ranging from thin films of superfluid helium-3 \textsuperscript{1,2} to the superconducting planes in high-$T_c$ cuprates \textsuperscript{3}. Ultracold atomic gases confined in oblate potentials also offer access to the 2D regime \textsuperscript{4-15} where interactions between particles can be controlled using a Feshbach resonance \textsuperscript{16}. In 2D Fermi gases, one can realize the BCS to Berezinskii-Kosterlitz-Thouless (BKT) superfluid crossover \textsuperscript{17-25} by tuning the effective attractive interaction between particles in different spin states. Of particular interest is the enhanced pairing due to the transverse confinement \textsuperscript{26-30} and the consequences this has for the phase diagram of the crossover \textsuperscript{15,31,32}.

All atomic gases exist in three dimensional environments, however, lower dimensions can be realized by eliminating dynamics along one or more directions. For atoms in a harmonic potential, with characteristic frequencies $\omega_x, \omega_y$ and $\omega_z$, the 2D regime is achieved when the confinement (taken here along $z$) is strong enough that occupation of transverse excited states is energetically forbidden. When a gas becomes frozen in the transverse ground state, dynamics in the $x$-$y$ plane are decoupled from $z$ and the gas is kinematically 2D. In an ideal gas this requires the thermal energy and chemical potential to be much smaller than the transverse confinement energy $k_BT, \mu \ll \hbar \omega_z$, where $k_B$ is Boltzmann’s constant, $T$ is the temperature and $\mu$ the chemical potential. When interactions are present however, these provide another avenue for generating transverse excitations, especially near a Feshbach resonance where there is enhanced coupling to a bound molecular state \textsuperscript{33,34}.

In this letter, we examine the criteria for an interacting Fermi gas to behave kinematically 2D. We prepare isolated 2D Fermi gases in a blue-detuned TEM\textsubscript{01} mode laser beam combined with a curved magnetic field which allows us to achieve high trap aspect ratios without the use of an optical lattice. By measuring the transverse cloud width after time of flight we obtain the transverse release energy which displays a rapid growth with the presence of transverse excitations. We observe both a geometric and interaction driven departure from the 2D regime as the atom number and interaction strength are varied, allowing us to identify the phase where interacting systems are kinematically 2D. A two-body calculation provides qualitative insight into the observed features.

In an ideal Fermi gas the Pauli exclusion principle sets an upper limit on the allowed size of a 2D system (as $T \to 0$). For finite $\omega_z$ there exists a finite number of single particle states with energy below the first transverse excited state. In the cylindrically symmetric case ($\omega_x = \omega_y = \omega_z$) this critical atom number is given by $N_{2D} \approx (\omega_z/\omega_z)^2$ [14]. Above this, the Fermi energy $E_F$ exceeds $\hbar \omega_z$ and the gas enters a confinement dominated 3D regime where it is no longer restricted to a single transverse state and excited states play a visible role in the properties of the gas [14,26,35,36].

A more complex scenario arises for an interacting gas. Neutral atoms interact via the van der Waals potential which has a range $r_0$ much smaller than the typical transverse quantization length, $\ell_z = \sqrt{\hbar/(m\omega_z)}$, where $m$ is the atomic mass, achievable in experiments. At length scales $r_0 < r < \ell_z$ atomic scattering is barely modified by the transverse confinement and the relative wave function has the same form as in 3D, differing only in the normalization [26]. Low energy three-dimensional scattering is characterized by the $s$-wave scattering length $a$ which can be tuned using a Feshbach resonance. In 3D, stable two-body dimers only exist for $a > 0$. In quasi-2D, however, the tight transverse confinement gives rise to a two-body bound state for all $a \neq 0$ and the magnitude of
the molecular binding energy $E_b$ is always greater than in the 3D case [26, 27]. The 2D scattering length can be defined as $a_{2D} = \sqrt{\hbar^2/(mE_b)}$ which is positive for all $a_{2D}$.

$E_b$ sets the size and character of the dimer state. When $E_b \ll \hbar \omega_z$ the transverse confinement dominates and atoms remain primarily in the transverse ground state. When $E_b \gg \hbar \omega_z$ molecules are tightly bound with a size small compared to $\ell_z$ and will be essentially identical to 3D molecules with equal atomic motion in all three dimensions. Between these limits the discrete transverse levels become increasingly visible when $E_b \sim \hbar \omega_z$ [26]. Even in the $T \to 0$ dilute limit when $E_b < \hbar \omega_z$ the coupling to a molecular state can lead to significant transverse excitations [34]. In interacting systems, with finite $\mu$ and $T$, transverse excitations are expected to play a strong role in quasi-2D gases [24, 32, 40].

Whether an interacting Fermi gas is kinematically two-dimensional therefore depends on the trap geometry, temperature, chemical potential and interactions. Here, we address this question using a highly degenerate Fermi gas of $^6$Li atoms confined in an oblate potential with interactions tuned via a broad Feshbach resonance. At $T \ll T_F$ where $T_F = E_F/k_B$ is the Fermi temperature, $E_F = \sqrt{N\hbar \omega_z}$ in a harmonic trap and $N$ is the total number of atoms, thermal excitations barely affect the energies of colliding particles which are dominated by the Fermi energy and can be varied in experiments.

Figure 1(a) shows a schematic of the experimental setup. The 2D trap is formed between the antinodes of a cylindrically focused, 532 nm (blue detuned), TEM$_{01}$ mode laser beam [6, 11], with $1/e^2$ radii of $w_z \approx 10 \, \mu$m and $w_r \approx 1.4 \, \mu$m, which produces the tight confinement in the $z$ direction and very weak anti-confinement in the $x$-$y$ plane. Residual magnetic field curvature from the Feshbach coils provides highly harmonic and cylindrically

Figure 1. (a) Experimental setup for producing 2D clouds. Atoms are confined between the antinodes of a 532 nm blue-detuned TEM$_{01}$ mode laser beam. Harmonic radial confinement is produced by the residual curvature in the magnetic field generated by the Feshbach coils. (b) Transverse profile of the TEM$_{01}$ mode which defines $V_z$ and (c) harmonic radial trapping potential.

symmetric confinement in the $x$-$y$ plane which completely dominates the anti-trapping of the optical potential. The combined optical and magnetic potentials are plotted in Fig. 1(b) and (c), respectively. The measured trapping frequencies are $\omega_z/2\pi = 5.15$ kHz and $\omega_r/2\pi = 26.4$ Hz at $B = 972$ G. The trap aspect ratio here is approximately 200 which yields $N_{2D} = 3.9 \times 10^4$ atoms. Note that $\omega_r \propto \sqrt{B}$ while $\omega_z$ is independent of $B$ meaning $N_{2D}$ increases at lower magnetic fields.

To produce a 2D Fermi gas we begin with a 3D cloud of approximately $N/2 = N_o = 2 \times 10^5$ $^6$Li atoms in each of the lowest two spin states $|F = 1/2, m_F = \pm 1/2\rangle$ in a 1075 nm single beam (3D) optical dipole trap. The cloud is evaporatively cooled to $T \sim 0.1 T_F$ at 780 G on the BEC side of the broad Feshbach resonance. We then ramp on the 2D optical potential in 350 ms and ramp off the single beam optical trap in a further 350 ms. The TEM$_{01}$ laser beam is initially turned on at low power, and $B$ is set to 832.2 G where the 3D scattering length diverges [32]. A magnetic field gradient is then applied over 2 seconds to achieve further evaporative cooling of the quasi-2D cloud which also allows the atom number to be controlled in a precise manner. Next, we simultaneously ramp up the TEM$_{01}$ beam to full power and ramp down the magnetic field gradient in 750 ms. Finally, we sweep the magnetic field to the desired value in 250 ms and hold the cloud for 50 ms before taking an absorption image either in situ or after time of flight. Imaging is achieved by illuminating the cloud with a 10 $\mu$s pulse of resonant laser light at an intensity of approximately half of the saturation intensity. We can image from the top to directly obtain the 2D density $n(x,y)$ or from the side after a short expansion time (600 $\mu$s) to obtain the transverse size of an expanded cloud, Fig. 1(a). We produce clouds with $N_o$ between $2 \sim 60 \times 10^4$ atoms per spin state. We estimate the temperature of the 2D clouds to be $0.1 T_F$ by fitting a 2D Thomas-Fermi profile to a weakly interacting cloud containing with $N_o = 12 \times 10^4 (= 0.6 N_{2D})$ at a field of 972 G where $E_b/E_F \approx 0.001$.

In a first series of experiments we measured the transverse cloud size $\langle z^2 \rangle^{1/2}$, at time $\tau = 600 \, \mu$s after switching off the TEM$_{01}$ laser as the magnetic field was varied across the Feshbach resonance. Results for $N_o = 0.6 N_{2D}$ are shown in Fig. 2(a). By suddenly removing the transverse confinement the gas is placed out of equilibrium with an excess of kinetic energy in the transverse direction that drives rapid expansion along $z$. The rate of expansion depends on the transverse energy and interactions. Measuring this provides a basis for identifying the 2D regime where all atoms occupy the transverse ground state. The expansion time is short enough ($\tau \ll 1/\omega_r$) that no changes are detected in the radial cloud profile.

For an ideal gas we would expect the expanded cloud to have a Gaussian transverse profile with a width set by the energy of the zero-point motion in the trap for all $N <
The dotted line in Fig. 2(a) indicates this width including the finite resolution of our imaging system. For fields above 870 G, where the scattering length is small and negative, the cloud width is roughly constant but lies below the ideal case due to the attractive interactions still present during expansion. In this (BCS) regime a shallow quasi-2D bound state exists with \( E_b \ll E_F \) and the gas behaves similarly to the ideal case.

At lower magnetic fields, near 850 G, the transverse width begins increasing rapidly, peaking on the BEC side of the Feshbach resonance just above 800 G. Such a sharp feature is somewhat surprising given the width of the resonance (\( \sim 300 \) G) but provides a clear signature of the departure from 2D kinematics. These transverse excitations occur at densities for which the local 2D Fermi energy \( E_F = \frac{\pi \hbar^2 n}{m} \) lies well below \( \hbar \omega_z \). Near the resonance, colliding atoms or dimers are strongly coupled to a bound molecular state which enhances the exchange of excitations between the radial and transverse directions, breaking the criteria for 2D kinematics [31, 40].

On the BCS side of the Feshbach resonance, any quasi-2D dimers will break upon removal of the transverse confinement leaving a cloud of expanding atoms. However, below 832 G stable dimers may exist in 3D and removing the confinement will not necessarily break molecules. Figure 2(a) shows that the transverse width grows rapidly between 832 G to 820 G indicating that any 3D molecules in this range dissociate after removing the confinement. This is not surprising considering the kinetic energies of the atoms will be relatively large compared to the 3D binding energy \( E_{b,3D} / (\hbar \omega_z) = 0.1 \) at 820 G.

At even lower fields (< 800 G), the final binding energy in 3D eventually exceeds the initial transverse confinement energy and molecules remain bound after removing the confinement. At this point we observe an expanding gas of repulsively interacting bosons. As the field is further reduced, the repulsion between molecules becomes weaker and the width after time of flight decreases towards the noninteracting molecule limit (\( \sim 16 \) µm for our imaging system).

We emphasize that the behavior observed in Fig. 2(a) is markedly different from a 3D gas where the release energy in all three dimensions decreases monotonically as one crosses from the BCS to BEC regimes [33]. In the radial direction (not shown) we observe the cloud width to shrink monotonically from the BCS to BEC regimes similar to the 3D case. Figure 2(b) shows the same measurement for a lower atom number \( N_\sigma = 6 \times 10^3 \) \((0.3 N^{(2D)}_{2D})\) which displays similar features to the main panel, but has a smaller and narrower peak near 810 G. At high fields the weaker attractive interactions at lower densities mean the width comes closer to the ideal gas prediction. Also on the BCS side the lower densities mean that stronger interactions are required before a departure from 2D is detected. However, the peak position remains fixed indicating that its location is set by the ratio \( E_b / (\hbar \omega_z) \), while the reduced peak width shows that the nonzero Fermi energy is important for accessing higher transverse states.

Theoretically, we can gain insight into these results by finding exact solutions for the two-body bound states under quasi-2D confinement. As the attractive interactions increase in strength the axial wave function for the relative motion distorts from Gaussian and becomes narrower [33]. This can be observed from the projection onto harmonic oscillator states in which one finds a significant atomic excited state fraction that increases monotonically from below 900 G approaching unity in the BEC limit where the binding energy \( E_b \) dominates, Fig. 2(c) [31]. This is qualitatively consistent with the experimental observations above the Feshbach resonance. As the excited fraction increases, so does the mean transverse kinetic energy, which translates into a more rapid expansion. In the BEC limit where the cloud expansion is dominated by bound molecules, the molecular occupation of excited states can be estimated using a mean-field description of the BEC of fermion pairs, Fig. 2(c). Here, the fraction of excited molecules is always small (apart from right at the resonance where the calculations become unreliable) indicating that other factors, such as the break up of molecules near resonance and repulsive dimer-dimer interactions during expansion, are responsible for the widths observed on the BEC side.

In a complementary study we fixed the magnetic field and measured the transverse cloud width after time of flight as a function of atom number. As shown previously [14] for a weakly interacting gas, one observes an elbow like feature in the transverse width, located at
atom number $N_\sigma$, when new transverse states become energetically accessible. In Fig. 3(a-c) we plot curves demonstrating such features at fields of 832 G, 865 G, and 950 G. We determine $N_{2D}$ from the intersect of two straight line fits to the (filled) data points above and below the elbow. For the weakly interacting clouds (950 G), where the 2D binding energy $E_b = 0.0015 \hbar \omega_z$, one observes a clear elbow at $N_{\sigma} \approx 20 \times 10^3$, consistent with the geometric limit $N_{2D}^{(2D)}$ set by the aspect ratio of the trap. At 865 G ($E_b = 0.04 \hbar \omega_z$) we still observe an elbow however, it is shifted to a significantly lower atom number ($\approx 10 \times 10^3$). Here the departure from 2D is driven by interactions as here $N_{2D}^{(2D)}/2 \approx 21 \times 10^3$ due to the weaker radial confinement. Once we reach the Feshbach resonance at 832 G ($E_b = 0.244 \hbar \omega_z$) the plateau corresponding to single transverse state occupation is no longer visible for the atom numbers we can access and the gas is never in the strictly 2D regime. Note that the binding energies for these three measurements lie well below the transverse oscillator energy highlighting that even moderate interactions lead to significant transverse excitations.

We have performed a series of measurements like those shown in Fig. 3(a-c) at a range of magnetic fields and used these to construct a map of the transverse cloud width as a function of atom number and interactions. This is plotted in false color in Fig. 3(d) versus the 2D Fermi energy $E_F = \sqrt{N} \hbar \omega_r$ and interaction strength in units of $\ell_z/a$. For each set of width measurements (open circles) we determined the critical atom number $N_{2D}$, as for Figs. 3(a-c), above which the cloud has clearly detectable transverse excitations (light grey circles). In the region below the dashed line the gas behaves kinematically 2D while above this, transverse excitations are present and dynamics in the z-direction is no longer decoupled from the radial directions. This plot highlights that the gas is only kinematically 2D over a relatively small region of low atom numbers and weak attractive interactions corresponding to the 2D BCS limit. Even at a magnetic field of 860 G and $N_{\sigma} = 5 \times 10^3$ which lies just in the 2D regime we find $E_b/E_F \approx 0.1$. The geometric upper limit for 2D kinematics ($E_F = \hbar \omega_z$), indicated by the white dotted line only applies in the far BCS regime. We note that increasing the aspect ratio of the trap may not dramatically alter the size of the 2D region as the binding energy also increases with the confinement.

In summary, we have established a phase diagram for an interacting quasi-2D Fermi gas, showing the range of interactions and particle numbers for which the gas can be considered kinematically 2D. Strict 2D behavior only takes place in the lower right portion of the phase diagram where $E_b \ll E_F$ and outside of this range we detect rapid growth of transverse excitations. These observations can be qualitatively understood using a two-body bound state calculation but this cannot model the density dependence. Future studies of the equation of state of a 2D Fermi gas will need to account for these transverse excitations in the strongly interacting regime.

We thank P. Hannaford for helpful discussions. C. J. V. acknowledges financial support from the Australian Research Council programs FT120100034 and DP130101807.

[1] J. C. Davis, A. Amar, J. P. Pekola, and R. E. Packard, Phys. Rev. Lett. 60, 302 (1988).
[2] L. V. Levitin, R. G. Bennett, A. Casey, B. Cowan, J. Saunders, D. Drung, Th. Schurig, and J. M. Parpia, Science 340, 841 (2013).
[3] M. Tinkham, Introduction to Superconductivity, Dover, Mineola, New York, 2nd Ed. (2004).
[4] A. Göröitz, J.M. Vogels, A.E. Leanhardt, C. Raman, T.L. Gustavson, J.R. Abo-Shaeer, A.P. Chikkatur, S. Gupta, S. Inouye, T.P. Rosenband, and W. Ketterle, Phys. Rev. Lett. 87, 130402 (2001).
[5] G. Modugno, F. Ferlaino, R. Heidemann, G. Roati, and M. Inguscio, Phys. Rev. A 68, 011601 (2003).
[6] N. L. Smith, W. H. Heathcote, G. Hechenblaikner, E. Nugent and C. J. Foot, J. Phys. B 38, 223 (2005).
[7] K. Günter, T. Stöferle, H. Moritz, M. Köhl, and T.
Esslinger, Phys. Rev. Lett. 95, 230401, (2005).

[8] S. Stock, Z. Hadzibabic, B. Battelier, M. Cheneau, and J. Dalibard, Phys. Rev. Lett. 95, 190403, (2005).

[9] X. Du, Y. Zhang, and J. E. Thomas, Phys. Rev. Lett. 102, 250402 (2009).

[10] P. Cladé, C. Ryu, A. Ramanathan, K. Helmerson, and W. D. Phillips, Phys. Rev. Lett. 102, 170401 (2009).

[11] J. I. Gillen, W. S. Bakr, A. Peng, P. Unterwaditzer, S. Fölling, and M. Greiner, Phys. Rev. A 80, 021602 (2009).

[12] K. Martiyanov, V. Makhalov, and A. Turlapov, Phys. Rev. Lett. 105, 030404 (2010).

[13] B. Fröhlich, M. Feld, E. Vogt, M. Koschorreck, W. Zwerg, and M. Köhl et al., Phys. Rev. Lett. 106, 105301 (2011).

[14] P. Dyke, E. D. Kuhnle, S. Whitlock, H. Hu, M. Mark, S. Hoinka, M. Lingham, P. Hannaford, and C. J. Vale, Phys. Rev. Lett. 106, 190404 (2011).

[15] M. G. Ries, A. N. Wenz, G. Zürn, L. Bayha, I. Boettcher, D. Kedar, P. A. Murthy, M. Neidig, T. Lompe, and S. Jochim, arXiv:1409.5373 [cond-mat.quant-gas] (2014).

[16] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. 82, 1225 (2010).

[17] V. L. Berezinskii, Sov. Phys. JETP 32, 493 (1971).

[18] J. Kosterlitz, and D. Thouless, J. Phys. C 6, 1181 (1973).

[19] D. S. Petrov, M. A. Baranov, and G. V. Shlyapnikov, Phys. Rev. A 67, 031601(R) (2003).

[20] S. S. Botelho, and C. A. R. Sá de Melo, Phys. Rev. Lett. 96, 040404 (2006).

[21] W. Zhang, G. D. Lin, and L.-M. Duan, Phys. Rev. A 78, 043617 (2008).

[22] M. Iskin, and C. A. R. Sá de Melo, Phys. Rev. Lett. 103, 165301 (2009).

[23] G. Bertaina, and S. Giorgini, Phys. Rev. Lett. 106, 110403 (2011).

[24] A. M. Fischer, and M. M. Parish, Phys. Rev. A 88, 023612 (2013).

[25] M. Bauer, M. M. Parish, and T. Enss, Phys. Rev. Lett. 112, 135302 (2014).

[26] D. S. Petrov, and G. V. Shlyapnikov, Phys. Rev. A 64, 012706 (2001).

[27] I. Bloch, J. Dalibard, and W. Zwerg, Rev. Mod. Phys. 80, 885 (2008).

[28] M. Feld, B. Fröhlich, E. Vogt, M. Koschorreck, and M. Köhl, Nature 480, 75 (2011).

[29] A. T. Sommer, L. W. Cheuk, M. J. H. Ku, W. S. Bakr, and M. W. Zwierlein, Phys. Rev. Lett. 108, 045302 (2012).

[30] Y. Zhang, W. Ong, I. Arakelyan, and J. E. Thomas, Phys. Rev. Lett. 108, 235302 (2012).

[31] W. Zhang, G. D. Lin, and L.-M. Duan, Phys. Rev. A 77, 063613 (2008).

[32] V. Makhalov, K. Martiyanov, and A. Turlapov, Phys. Rev. Lett. 112, 045301 (2014).

[33] Z. Idziaszek, and T. Calarco, Phys. Rev. A 71, 050701(R) (2005).

[34] J. P. Kestner, and L. -M. Duan, Phys. Rev. A 74, 053606 (2006).

[35] J. Schneider, and H. Wallis, Phys. Rev. A 57, 1253 (1998).

[36] P. Vignolo, and A. Minguzzi, Phys. Rev. A 67, 053601 (2003).

[37] E. J. Mueller, Phys. Rev. Lett. 93, 190404 (2004).

[38] H. Hu, Phys. Rev. A 84, 053624 (2011).

[39] K. Merloti, R. Dubessy, L. Longchambon, M. Olshanii, and H. Perrin, Phys. Rev. A 88, 061603(R) (2013).

[40] A. M. Fischer, and M. M. Parish, arXiv:1409.5373v1 (2014).

[41] S. P. Rath, T. Yefsah, K. J. Günter, M. Cheneau, R. Desbuquois, M. Holzmann, W. Krauth, and J. Dalibard, Phys. Rev. A 82, 013609 (2010).

[42] G. Zürn, T. Lompe, A. N. Wenz, S. Jochim, P. S. Julienne, and J. M. Hutson, Phys. Rev. Lett. 110, 135301 (2013).

[43] T. Bourdel, L. Khaykovich, J. Cubizolles, J. Zhang, F. Chevy, M. Teichmann, L. Tarruell, S. J. J. M. F. Kokkelmans, and C. Salomon, Phys. Rev. Lett. 93, 050401 (2004).