Babel on the Petaplex site: On Rival Calculational methods in SO(10) MSGUTs

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We compare and contrast the computations that lead to the NMSGUT spectra and Yukawa couplings that appeared in 2006 and a recent recalculation of the same. We argue that an explicit component based method of computation jeopardizes the power of SO(10) and its subgroups to organize, in a unified and automatically phase correlated way, computations of dynamics beyond the basic mass matrix computation. The correct (one line) prescription for generating MSSM Yukawas from SO(10) ones was given in [3] and requires no computation beyond the identification of null vectors of the Higgs doublet mass matrix and the Clebsches given in [3, 4]. It was already used to derive all fermion Yukawas and Majorana masses in [1, 3]. We thus urge the adoption of a uniform notation and methodology based on descent from SO(10) to the SM through the Pati-Salam maximal subgroup of SO(10) to avoid Babel in this rapidly developing and highly promising subject.

I. INTRODUCTION

Since the discovery of neutrino mass in the late nineties, SO(10) GUTs and particularly Supersymmetric SO(10) GUTs have become the leading contenders for unification. We had constructed the (renormalizable and non-renormalizable) Minimal Supersymmetric Left-Right Models (MSLRMs), which have generically high scale B-L symmetry breaking, automatic R-parity as a part of the gauge group, and naturally accommodate both Type I and Type II seesaw mechanisms for neutrino mass. We thus received the news of neutrino oscillations from SuperKamiokande with some glee since the Seesaw estimate for the $B-L$ breaking scale $M_{B-L}$ corresponding to the neutrino masses indicated by $\geq 10^{14}$ GeV. Thus the construction of Supersymmetric SO(10) GUTs incorporating the insights from our study of MSLRMs was naturally high on our agenda and a model based on the $45 \oplus 54 \oplus 126 \oplus \overline{126}$ Higgs system was duly constructed by us. Nevertheless having long been aware of the formidable calculational problems in handling SO(10) group theory, particularly the translation from orthogonal group spinor labels to unitary group labels, we commenced, in 2000, development of a systematic decomposition of SO(10) labels and invariants into those of its ‘Pati Salam’ maximal subgroup: $SU(4) \times SU(2)_L \times SU(2)_R$. The other maximal subgroup, $SU(5) \times U(1)$, not accidentally, also received attention contemporaneously for essentially the same purpose. The results from these calculations, have furnished manuals for handling any conceivable invariant decomposition in SO(10) into unitary subgroup invariants that is available to all workers in the field. Our intent was that the uniformized methods and notations should make communication and comparison of the complex expressions obtained when decomposing SO(10) labels into Unitary group labels easy, and to provide ‘Clebsches’ that were otherwise hard to compute such as those for the spinorial 16-plet representation. Unfortunately, just as in the biblical story, life has turned out to be more complex and less innocent than we might, in our naivete, have believed earlier.

Firstly just as we completed the ‘SO(10) a la Pati Salam’ decomposition methodology, having thoroughly appreciated the complexity we had uncovered, we searched for a model even simpler than the one we had analyzed. We soon realized that the old model (which we named as the Minimal Supersymmetric Grand Unified Theory (MSGUT)) studied right at the beginning of the Susy GUT era was the best and simplest or minimal home for the ideas on R-parity and Susy LR subsuming GUTs that we had developed. A very

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significant breakthrough was achieved in that the spontaneous symmetry breaking equations could be reduced to the study of a single cubic equation with a single coupling parameter making the problem completely soluble. We therefore immediately applied our newly developed methods to analyze both the symmetry breaking at high scale and the resultant spectra. Indeed already the second version, in August 2003 of our manual on SO(10) decomposition (which we delayed sending to press till 2005 in order that it retain as few calculational mistakes and typos as possible) contained the Clebsches of the matter fermion couplings to the 3 Fermion Mass(FM) Higgs representations in SO(10) (the possible yukawa couplings are just \(16.16.10 \oplus 120 \oplus 126\)), as well as the mass matrices of the two most important superheavy particle types: the MSSM Higgs type doublets and the Higgs triplets \([3,1,\pm\frac{1}{2}]\) responsible for proton decay. We also used these to derive the effective potential for \(d = 5\) operator mediated proton decay.

Clearly the calculation of the complete spectrum and the computation of GUT exotic effects was the next logical step since that was just the purpose of the whole development of calculational methods. We duly proceeded with these calculations, unfortunately delayed somewhat by personal circumstances, and little aware that our leisurely pace was about be forced, completed it finally(along with RG analysis of threshold corrections) only in May 2004. Meanwhile the excitement and attention raised by our revival of the old model of \([9,10]\) motivated two groups, one composed of our own collaborators\([13]\) and another comprising one of the developers of an old method for computing mass matrices in GUTs\([17]\) and their newly attracted collaborators. Both groups adopted the method of\([15]\) in preference to our systematic and general method (which, although complete and general requires considerable patience and effort, while the other method can be implemented on a computer algebra system and thus allows more rapid computation of mass matrices: at the price of abandoning the standard field theoretic and tensorial methods which can also handle more general questions such as the decomposition of all SO(10) interactions into unitary(PS or SM) labels). Moreover the necessity of making lists of SO(10) representation component-wise phase choices that were near impossible to code in simple generative rules that could be communicated easily made it impossible for us to agree with such shortsighted adoption of a limited and opaque technology in preference to the almost transparently luminous(for us!) symmetries of the PS decomposition of SO(10) that we had so laboriously developed: but which, unfortunately, appear to be less than ‘penetrable’ to some. On the other hand perhaps these proprietary conventions grew for the same strong reasons that Babel and private property always threaten human cooperation. In any case with the adoption of two methods(and three sets of phase conventions) the rosy dream of a uniform notation rapidly faded in the face of the harsh reality of a struggle for citation priority and allegations of incorrect results in preference to cooperation to pin down the inevitable errors by cross checking. These issues are discussed at length in \([16]\).

Due to the great interest in whether GUTs can account for the fermion mass and mixing data and the promise of SO(10) in this regard, MSGUTs attracted a great deal of attention focused on these questions\([17]\). Since then, after very promising developments as regards the generic fermion fitting properties, there have been some dramatic reversals\([19,20]\) and then further development\([1,21,21,28]\) of the ability to fit all fermion data using formulae specific to the MSGUTs. MSGUTs have thus matured to the point of becoming quite fully specified falsifiable theories facing challenges (which so far they have, in one avatar or another, duly overcome) which promise to put the surviving version to the stress tests that will either certify its health or consign it to the dustbin of history.

Indeed, using the very spectrum, Clebsch and Yukawa couplings that we had computed in \([3,4]\) we were able to show\([19,20]\) that the generic scenarios of fermion spectrum fits that had been shown to be feasible\([17]\) were in fact not feasible in the fully specified MSGUT since the neutrino masses attained, whether by the Type I or Type II seesaw mechanisms, were far too small. Interestingly a group, including the author of \([2]\) working at Trieste, confirmed our results(announced at PLANCK05, held in Trieste in June 2005, and again in December 2005\([20]\)) in April 2006\([22]\).

In \([20]\) we also pinpointed the reasons for this failure as the necessary largeness of the \(126\) couplings (due to their dual function: they generate neutrino Majorana masses and make CKM mixing in the effective MSSM Yukawa couplings possible). So we proposed\([1,20]\) a new version (which we called the Next or New MSGUT (NMSGUT)) in which the theory was completed by inclusion of the remaining possible FM Higgs, namely the \(120\)–plet (which has couplings antisymmetric in family indices which are well adapted to generating CKM mixing angles) and a re-assignment of roles in which the \(126\) couplings, become very small and thus play little or no role in charged fermion masses, but can then boost the Type I seesaw mass since small \(126\) couplings lower the righthanded
neutrino masses. To implement these ideas we made some perturbative studies \cite{23, 24} and more importantly already in \cite{20} used our SO(10) decomposition technology \cite{4} to compute the same information for the NMSGUT: fermion Yukawa couplings, superheavy mass matrices, Baryon violation effective superpotentials and superheavy threshold effects in the RG flow that we had earlier computed for the MSGUT \cite{3, 4, 12}. This paper was released in December 2006 and by now we are already using its results in large scale computer studies of realistic fitting of all spectra in NMSGUTs \cite{21, 25, 28} which show that indeed our proposal is effective in generating viable and NMSGUT specific fits to all fermion data, neatly evading all constraints found earlier. For this they need to invoke the participation of threshold corrections at $M_S$ and then also furnish information on the sfermion spectra compatible with fermion masses, and thus opened up the entire spectrum for theoretical investigation via a vis falsifiability.

Very recently, just as we released \cite{28} at a conference at ICTP, Trieste \cite{29} we encountered the author of \cite{2}, who shortly thereafter released his recalculation of the results of the appendices A and C of \cite{1}. While, as we emphasized above, we have always welcomed the opportunity to cross check our results against independent computations to uncover any discrepancies and errors of detail, it is unfortunately true, as explained in detail in \cite{16}, that it is difficult to compare computations which use the component based approach of \cite{13, 14} and our own Lagrangian oriented SO(10)/PS tensorial approach up to the point where phase differences can be cross checked. The reasons for this are fully explained in \cite{16}.

The author of \cite{2} opined that the phase structure of our analysis was ‘impenetrable’. While, in the absence of detailed reasons, such a judgement is entirely subjective it sufficiently motivated us to study the paper in order to evaluate the validity of the claims. We found that the author’s method shared the non-generality of the component based result as used earlier and even made errors in the actual phase specification. Due to the importance of the basic framework for clarity in this complex subject, we thought it behooved us to point out once again the virtues of our method and the simple and correct prescription to determine the fermion Yukawas. We do this in the hope of damping down the Babel that has developed in the SO(10) PetGeV-tower construction that we have spent much time and effort to lay the foundations of \cite{1, 3, 4, 5, 8, 9, 12, 19, 20, 21, 23, 24, 25, 28}.

In this comment we therefore first discuss in Section 2 the contents of the two papers \cite{1, 2} to evaluate their relative overlap. In Section 3. we compare and evaluate the component method for fixing phases adopted in works of this type \cite{2, 13, 14} relative to our own tensorial method. We then evaluate the ‘penetrable’, ‘optically’ optimal etc method \cite{2} of obtaining MSSM Yukawa couplings from NMSGUT ones and point out some errors it arrives at. We feel these might have been easier to detect in a compact tensor notation like ours rather than an extended explicit naming of fields and components that inhibits conceptual clarity and hides the elementary properties of the basis changes.

II. COMPARISON OF PAPER CONTENTS

Before proceeding let us first see the relation to our previous work, and the notice he had of it, that author himself gives in a Note Added at the end of his paper \cite{2}.

“A day before finishing this manuscript the author’s attention was drawn to the preprint \cite{1} where the relevant part of the next-to minimal SUSY SO(10) model has been previously studied from a similar perspective. As far as one can see through the jungle of different notation, normalization and conventions the results therein seem to agree with those given in this study, but especially the phase conventions that we have spent so much time on arguing about their importance here are virtually impenetrable in \cite{1}. Moreover, since the method we employed was completely different I believe that the current study is worth and does indeed provide a valuable and entirely independent survey of many of the crucial and technically rather demanding prerequisites of any numerical analysis of the NMSGUT. Despite from that, there could still be a good case for even a further check of ours as well as Aulakh & Garg’s results, in particular when it comes to the phases and matching(s). Apart from all this, the current study is certainly much more detailed and we pushed hard to make it maximally self-contained so that a careful and patient reader should be able to read and potentially reproduce all the results with just the ingredients given here and in the ‘canonical’ MSGUT reference \cite{13}. On top of that, a lot of extra information provided in Sections (IV) Higgs Sector Mass Matrices, (VI) NMSGUT Yukawa sector and in particular in Sections V) Goldstones and in Appendix does not have any counterpart in \cite{1}.”
The two principal emphases to note—which we have highlighted—here are

- The author feels that his phase conventions in comparison to ours are ‘penetrable’ because he has spent so much time arguing about their importance. This is like saying that the way to the the front door is long indeed because you went via your window and the moon! As we shall see in detail our phase conventions for every level of descent down from SO(10) to the SM gauge group are specified at one shot when we decompose\[4\] the fundamental vector(10-plet) and spinor(16-plet) irreps (from which two all irreps of Spin(10) can be obtained—with fixed phase and normalization conventions—by tensor products) through the PS maximal subgroup down to the standard model (with the standard embedding of the SM in the PS group). \textit{Once this is done there is simply no phase ambiguity left!} Thus we provide\[4\] the technology for translating SO(10) tensors to PS and SM ones and explicitly provide decompositions\(\text{i.e.}\)all Clebsches but in a field theoretic notation adapted to working with SO(10) lagrangians of all the interactions in the theory that are not trivial to write down. This \textit{generative and prescriptive method} is to our mind the only efficient and modern method of proceeding: not\(\text{see below}\) a list of phases for more than 100 individual states occupying no less than 11 pages appended with the facile assertion that anyone with sufficient determination and time can check and generate the rest for himself by applying colour and electroweak gauge transformations on the explicit states! Why would one set oneself such a headache if a tensor method is available to resolve all group theoretic invariants? Moreover although the author of\[2\] “spent so much time on arguing about their importance” his phase conventions in fact contained an manifest error (the relation between GUT and MSSM fields did not maintain holomorphicity) which terminated our comparison.

- The author also claims that \textit{“the current study is certainly much more detailed ….. on top of that, a lot of extra information provided in Sections IV)Higgs Sector Mass Matrices, VI)NMSGUT Yukawasector and in particular in Sections V) Goldstones and in Appendix does not have any counterpart in”}\[1].

These statements are quite unfounded. Let us therefore list for comparison what our 61 page paper\[1\] actually contains besides the obvious common starting points:

- Section 2.1.1: A discussion of the characteristics of the GUT SSB solutions in terms of the three branches of the cubic.

- Section 2.2 and 3: Group decomposition of the 120 and a \textit{decomposition of the complete additional superpotential into PS sub-invariants i.e the explicit clebsches for SO(10) to PS} (the MSGUT part was already decomposed in\[3\]). Using these decompositions it is trivial to read off not only the mass matrices but also the superpotential couplings between any SM fields in the theory. The latter has no counterpart in\[2\] and cannot even be sensibly stated in that notation.

- Section 3.1, 3.2, 3.3, Appendix A: The explicit SM field multiplets which acquire Unmixed chiral, Mixed pure chiral and Mixed Chiral-Gauge (including a description of the goldstone substructure in terms of null eigenvectors) mass terms and the explicit list of mass matrices (Appendix A). It is worth emphasizing that our mass matrices have rows and columns labelled by SM \textit{tensor multiplets} so that they directly give the mass terms in the Superpotential with correct phases and contractions for all fields without any need to refer to extensive pages of individual component phase assignments (not to mention gauge transformations of the individual states to generate the rest of the (500 ) states and their phases).

- Section 4. This is an extensive analysis of the Renormalization Group threshold corrections due precisely to the spectra calculated and their physically highly significant implications (such as a generically raised unification scale over the viable regions of the parameter space). This is a major physics component of\[1\] with no counterpart in\[2\].

- Section 5.1: MSSM Fermion Mass Formulae and Yukawa couplings in terms of GUT couplings. This set of formulae which includes all Fermion mass relevant clebsches\[4\] and the neutrino masses was first derived correctly for both the MSGUT\[3, 19\] and the NMSGUT\[1, 20\] by us in terms of the null eigenvectors\(\text{after fine tuning}\) of the Higgs mass matrices computed by us in\[4, 20\]. We emphasize that since the Clebsches
of the coupling of FM Higgs to spinors could not be computed without a method for decomposition of the cubic invariants $16 \otimes 16 \otimes 120 \otimes 120$ it is necessary to choose and assign phases and normalizations in a coordinated way and this was indeed the motivation for our decomposition of SO(10) a la Pati salam\cite{4} where simple, complete and generative rules (not lists of phases and normalizations for individual components of large irreps) for fixing phases and normalizations were given. Indeed the these very Clebsches were in fact used by subsequent papers on MSGUTs e.g \cite{18} and others; not always with correct and proper attribution.

These formulae were then used to show that neutrino masses calculated using MSGUT formulae were too small\cite{19}. These results were reported already in May 2005 at PLANCK05 where the author of \cite{2} was in the audience. In fact he wrote a paper\cite{22} checking our result of too small neutrino masses in the MSGUT a year later after we had already proposed the NMSGUT structure\cite{20}. As we shall explain below the confusion in \cite{2} concerning the correct procedure to connect NMSGUT to MSSM Yukawas can be understood in terms of the fact that he was persistently using legacy, but incorrect, unitary transformation formulae (between the original GUT basis and the mass diagonal basis) that violate the basic structure of the theory. We gave the correct simple one line rule for calculating these coefficients as long ago as 2004\cite{3, 19, 20}. In our opinion it is because the structure of the transformation rules was obscured by the unwieldy explicit and obscure notation that the violation of holomorphicity by incorrectly set up transformation formulae was missed earlier by both authors and readers.

- Sections 5.1, 5.2, 5.3, 5.4: Discussed CP violation and the vital question of the actual fits of fermion data as it stood at that time and the indications and contradictions between different approaches: which have recently all been resolved and unified\cite{21, 28}. No counterpart in \cite{2}.

- Section 6: In this section we completed the Baryon decay effective superpotential (dimension 4) that we derived for the MSGUT already in\cite{3, 4} by adding the new terms due to the 120−plet and discussed the unitary transformation to MSSM basis sets. No counterpart in \cite{2}.

- Appendix B: In Appendix B we did a SU(5) assembly recheck of our PS decompositions and showed that indeed the MSSM multiplets and their mass terms could be reorganized according to the other maximal subgroup when the superheavy vevs were such that the unbroken symmetry included SU(5). No counterpart in \cite{2}.

- Appendix C: Finally and very importantly for later calculations\cite{28} we gave explicit expressions for the null eigenvectors of the Higgs doublets mass matrix which are the crucial ingredient in defining the effective MSSM and its Yukawa couplings. While the ‘weights’ given in \cite{2} may turn out to be the same if the relation between the long lists of component-wise phase conventions in \cite{2} and the rules given in \cite{4} is worked out, we shall show that their interpretation is facilitated by our compact notation.

Thus the computation of \cite{2} covers only the repetition of the mass matrix calculations in an idiosyncratic phase convention i.e our Sections 3.1, 3.2, 3.3 and Appendix A, and an attempt at reproducing our Section 5 which seems flawed. We fail to see in what sense it could be judged more complete or what new relevant information was added. From the above listing the reader may judge for herself which computation is more complete; not to speak of correct.

III. COMPARISON OF METHODS AND RESULTS

The crux of the method used in \cite{2} and antecedent papers can be appreciated by first quoting from \cite{2} what he sees as the determining rationale of his method (our italics and our text-compactifying interpolations in square brackets): “For sake of illustration let us remark that there is in total 13,321,010 terms in the sums in [the NMSGUT Superpotential] (out of which 2,111 thousand terms come from the new piece $W_1^{120}$), but fortunately ‘only’ 1,190,170 of them are non-zero by antisymmetry of the tensors under consideration ($W_1^{120}$ then accounts for 338,400 out of this number). Thus, perhaps the only reasonable strategy of handling all these contributions is to work with the antisymmetrized combinations rather than with the very components of the antisymmetric tensors ....”
“In what follows we shall pass through the whole plethora of the Higgs sector states and write down the corre-
sponding mass (fermionic) matrix for each subspace corresponding to a set of fixed values of [its Casimirs and SM
quantum numbers] choosing a single representative configuration of the Cartan eigenvalues for each
value of the relevant Casimir,.......

The mass matrices for all the other components with the same

Casimirs and hypercharge but different colour or electroweak weights can be (if desired) obtained in a straightforward
manner by the relevant $SU(3)_c$ and/or $SU(2)_L$ transformations.”

“.......

For each [set of row and column labels] we shall also display a chunk of the map of the SM components
of [SO(10) multiplets] (i.e. the submultiplets with definite SM quantum numbers) onto the defining basis states
$H$, $\Sigma_{ijklm}$, $\sum_{ijklm}$, $\Phi_{ijkl}$ and $\Psi_{ijk}$ (typically we present only the “lowest” relevant permutation of indices and defer
an interested reader to Appendix or to [13] for further details) in order to provide an information about the
phase convention used in derivation of the mass matrix under consideration. (Note that for sake of simplicity we always choose our phase convention in such a way there are no pending imaginary units in the mass
matrices.) For sake of a simple bookkeeping the top-left box of each table shall indicate the full dimensionality of
the sector under consideration.”

After passing through a list of the component combinations and phase choices of more than 100 separate SM
states the author concludes :

.....

“.......

We have adopted the rule that any submultiplet of an SO(10) field is always denoted by the same symbol as its
parent field, its identity being established by the indices it carries or by supplementary indices, if necessary. Our
notation for indices is as follows : The indices of the vector representation of SO(10) (sometimes also SO(2N))
are denoted by $i,j = 1..10(2N)$. The real vector index of the upper left block embedding (i.e. the embedding
specified by the breakup of the vector multiplet $10 = 6 + 4$ of SO(6) in SO(10) are denoted $a,b = 1,2..6$ and of the
lower right block embedding of SO(4) in SO(10) by $\hat{a},\hat{b} = 7,8,9,10$. These indices are complexified via a Unitary
transformation and denoted by $\hat{a},\hat{b}= 1,2,3,4,5,6 \equiv \overline{7},\overline{8},\overline{9},\overline{10}$. Similarly
we denote the complexified versions of $\hat{a},\hat{b}$ by $\hat{\alpha},\hat{\beta}$ and thus satisfy the SO(6) algebra (square brackets around i

dices denote antisymmetrization)

\[(J_{cd})_{ef} = -i\delta_{c[e}\delta_{f]d} \] (2)

and thus satisfy the SO(6) algebra (square brackets around indices denote antisymmetrization)

\[[J_{cd}, J_{ef}] = i\delta_{c[e}J_{d]f} - i\delta_{f]e}J_{d]c} \] (3)

It is useful to introduce complex indices $\hat{a},\hat{b} = \overline{1}...\overline{6}$ by the unitary change of basis

\[V_{\hat{a}} = U_{a\hat{a}}V_a \quad U = U_2 \times I_3 \] (4)

\[U_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix} \]
so that $V_a W_a = V_{a\bar{a}} W_{\bar{a}}$. The decomposition of the fundamental 4-plet of SU(4) w.r.t. SU(3)×U(1)$_{B-L}$ is 4 = (3, 1/3) ⊕ (1, −1). The index for the 4 of SU(4) is denoted by $\mu, \nu = 1, 2, 3, 4$ while $\bar{\mu}, \bar{\nu} = 1, 2, 3, 4$ label its SU(3) subgroup. In SU(4) labels, the 6 of SO(6) is the 2 index antisymmetric $V_{\mu\nu}$ and decomposes as $V_{\mu\nu} = V_{\bar{\mu}\bar{\nu}} = \epsilon_{\mu\nu\bar{\lambda}} V_{\bar{\lambda}}$. In other words, if one defines $V_{\mu\nu} = \Theta_{\mu\nu}^a V_a$ with $\Theta_{\bar{\mu}4}^a = \delta_{\bar{\mu}}^a$ then since $\Theta_{\mu\nu}^a \Theta_{\lambda\sigma}^b = \epsilon_{\mu\nu\lambda\sigma}$ it follows that the translation of SO(6) vector index contraction is $(\hat{V}_{\mu\nu} = (1/2) \epsilon^{\mu\nu\lambda\sigma} V_{\lambda\sigma})$

$$V_a W_a = \frac{1}{4} \epsilon^{\mu\nu\lambda\sigma} V_{\mu\nu} W_{\lambda\sigma} \equiv \frac{1}{2} \hat{V}_{\mu\nu} W_{\mu\nu}$$

while

$$V_a W_a^* = \frac{1}{2} V_{\mu\nu} (W_{\mu\nu})^*$$

Representations carrying vector indices $a, b, ...$ are then translated by replacing by each vector index by an antisymmetrized pair of SU(4) indices $\mu_1\nu_1, \mu_2\nu_2, ...$. For example

$$A_{ab} B_{ab} = 2^{-4} \epsilon^{\mu_1\nu_1\mu_2\nu_2 \mu_3\nu_3 \mu_4\nu_4} A_{\mu_1\nu_1, \mu_2\nu_2} B_{\mu_3\nu_3, \mu_4\nu_4}$$

while

$$A_{ab} B_{ab}^* = 2^{-2} A_{\mu_1\nu_1, \mu_2\nu_2} B_{\mu_1\nu_1, \mu_2\nu_2}^*$$

C. SO(4) ↔ SU(2)$_L$ × SU(2)$_R$

**Vector/Bidoublet**

We use early greek indices $\tilde{\alpha}, \tilde{\beta} = 7, 8, 9, 10$ for the vector of SO(4) corresponding to $i, j = 7, ..., 10$ of the 10-plet of SO(10). The Hermitian generators of SO(4) have the usual SO(2N) vector representation form: $(J_{\tilde{\alpha} \tilde{\beta}})_{\tilde{\gamma} \tilde{\delta}} = -i \delta_{\tilde{\alpha}}^{\tilde{\gamma}} \delta_{\tilde{\beta}}^{\tilde{\delta}}$.

The group element is $R = \exp \frac{i}{2} \omega^{\tilde{\alpha} \tilde{\beta} \tilde{\gamma}} J_{\tilde{\alpha} \tilde{\beta}}$. The generators of SO(4) separate neatly into self-dual and anti-self-dual sets of $3, J_{\tilde{\alpha} \tilde{\beta}} = \frac{1}{2} (J_{\tilde{\alpha} \tilde{\beta}} \pm J_{\tilde{\beta} \tilde{\alpha}})$. Then if $\tilde{\alpha}, \tilde{\beta} = 1, 2, 3$ the generators and parameters of the SU(2)$_{\pm}$ subgroups of SO(4) are defined to be

$$J_{\tilde{\pm}} = \frac{1}{2} \epsilon_{\tilde{\alpha} \tilde{\beta} \tilde{\gamma}} J_{\tilde{\alpha} \tilde{\beta}} J_{\tilde{\gamma} \tilde{\delta}} ; \quad \omega_{\tilde{\pm}} = \frac{1}{2} \epsilon_{\tilde{\alpha} \tilde{\beta} \tilde{\gamma}} \omega_{\tilde{\alpha} \tilde{\beta} \tilde{\gamma} \tilde{\delta}} \pm \omega_{\tilde{\alpha} \tilde{\beta} \tilde{\gamma} \tilde{\delta}}$$

The SU(2)$_{\pm}$ group elements are $\exp(i \omega_{\tilde{\pm}} \cdot \tilde{J}^{\tilde{\pm}})$. The vector 4-plet of SO(4) is a bi-doublet (2, 2) w.r.t. to SU(2)$_{-} \otimes$ SU(2)$_{+}$. We denote the indices of the doublet of SU(2)$_{L} = SU(2)_{-}$ (SU(2)$_{R} = SU(2)_{+}$) by undotted early greek indices $\alpha, \beta = 1, 2$ (dotted early greek indices $\tilde{\alpha}, \tilde{\beta} = 1, 2$). Then one has

$$V_7 = V_4 = \frac{V_7 + i V_8}{\sqrt{2}} = V_{22} \quad V_9 = V_5 = \frac{V_9 + i V_{10}}{\sqrt{2}} = V_{12}$$

$$V_8 = V_1 = \frac{V_7 - i V_8}{\sqrt{2}} = -V_{11} \quad V_{10} = V_6 = V_5 = \frac{V_9 - i V_{10}}{\sqrt{2}} = V_{21}$$

SU(2)$_{L}$ (SU(2)$_{R}$) indices are raised and lowered with $\epsilon^{\alpha \beta}, \epsilon_{\alpha \beta} (\epsilon^{\tilde{\alpha} \tilde{\beta}}, \epsilon_{\tilde{\alpha} \tilde{\beta}})$ with $\epsilon^{12} = + \epsilon_{21} = 1$ etc. The SO(4) vector index contraction translates as

$$V_{\alpha} W_{\bar{\alpha}} = -V_{\alpha \bar{\alpha}} W_{\beta \bar{\beta}} \epsilon^{\alpha \beta} \epsilon_{\alpha \beta} = -V^{\alpha \bar{\alpha}} W_{\alpha \bar{\alpha}}$$

while

$$V_8 W_{\alpha} = V_{\alpha \bar{\alpha}} W_{\alpha \bar{\alpha}}$$

**Thus the above rules enable the decomposition of any SO(10) tensor invariant not involving spinors into PS invariants (and then trivially into SM invariants)** by decomposing the SU(4) quartet w.r.t. SU(3)×U(1)$_{B-L}$ as 4 = (3, 1/3) ⊕ (1, −1) and relating the SM hypercharge to $T_{3R}, B - L$ in the standard way 20: $Y/2 = T_{3R} + (B - L)/2$.

It remains to quote the same for the spinor indices:

The Clifford algebra of SO(2N) acts on a $2^N$ dimensional space which is given the convenient basis of eigenvectors $| \epsilon \rangle = \pm 1 >$ of $\gamma_3$:

$$| \epsilon_1, ..., \epsilon_n > = | \epsilon_1 > \otimes ... \otimes | \epsilon_n >$$

In this basis $\gamma_F = \prod_{i=1}^n \epsilon_i$. So the basis spinors of SO(2N) decompose into odd and even subspaces w.r.t. $\gamma_F$.

$$2^n = 2^{n-1} + 2^{n-1}$$

(15)
D. SO(6) Spinors

The $4(\psi_\mu)$ and $\bar{4}(\bar{\psi}^\mu)$ of SU(4) may be consistently identified with the $4_-, 4_+$ chiral spinor multiplets of SO(6) by identifying components $\psi_\mu$ of the 4 with the coefficients of the states $|e_1 e_2 e_3 >$ in $4_- = |\psi >$ as

$$|\psi > = \psi_1 |--- > + \psi_2 |--- > + \psi_3 |--- > + \psi_4 |--- >$$

and also $\bar{\psi}^\mu$ in the $4_+ = |\bar{\psi} >$ as

$$|\bar{\psi} > = -\bar{\psi}_1 |--- > + \bar{\psi}_2 |--- > - \bar{\psi}_3 |--- > + \bar{\psi}_4 |--- >$$

The reason for the extra minus signs is that then the charge conjugation matrix $C_2^{(3)}$ correctly combines the 4, $\bar{4}$ components in the 2$^3$-plet spinors of SO(6) to make SU(4) singlets and covariants. For example (we take $\psi, \chi$ to be non-chiral $8 = 4_+ + 4_-$ spinors to preserve generality)

In this basis one has in the 8 dimensional spinor rep. of SO(6)

$$exp\left(\frac{i\omega_{\alpha\beta}}{2} J_{\alpha\beta}\right) = Diag(exp\left(\frac{i\theta^A}{2} \lambda^A\right),exp\left(-\frac{i\theta^A}{2} \lambda^A\right))$$

One finds the following useful identities hold

$$\psi^T C_2^{(3)} \chi = \psi_\mu \bar{\chi}^\mu + \psi^\mu \chi_\mu = \psi_\mu \bar{\chi}^\mu + \psi \cdot \chi$$
$$\psi^T C_2^{(3)} \gamma_{\mu\nu} \chi = \sqrt{2} [\psi_\mu \chi_\nu + \bar{\psi}^\lambda \bar{\chi}^\nu_\lambda \epsilon_{\mu\nu\lambda\sigma}]$$
$$\psi^T C_2^{(3)} \gamma_{\mu\nu} \lambda\sigma \chi = -2 \{ \bar{\psi}^\rho \chi_{[\lambda\sigma]_\rho\mu\nu} + \psi_\mu \chi_{[\lambda\sigma]_\rho\mu\nu} \}$$
$$\psi^T C_2^{(3)} \gamma_{\mu\nu} \lambda\sigma \gamma^\rho \delta \chi = (\sqrt{2})^3 \{ \psi_\mu \chi_{[\lambda\sigma]_\rho\delta\mu} + \bar{\psi}^\rho \bar{\chi}^\mu_\rho \epsilon_{\mu\nu\lambda\sigma} \}$$

The results when $\psi^T C_2^{(3)} \rightarrow \psi^\dagger$ are obtained by the replacements $\psi_\mu \rightarrow \bar{\psi}_\mu^* \quad \text{and} \quad \bar{\psi}^\mu \rightarrow \psi_\mu^*$ on the R.H.S of all the identities in [8]. The square root factors arise because the antisymmetric pair labels for the gamma matrices correspond to complex indices $\bar{a}, \bar{b}$. Note that one does not need the identities for more than 3 gamma matrices.

See the appendix of [4] for useful translations of SO(6) spinor-tensor invariants calculable from these identities.

E. SO(4) Spinors

In the case of SO(4) the spinor representation is 4 dimensional and splits into $2_+ \oplus 2_-$. It is not hard to see that with the definitions adopted for the generators of SU(2)$_{\pm}$ the chiral spinors $2_{\pm}$ may be identified with the doublets $\psi_\alpha, \psi_\dot{\alpha}$ of SU(2)$_{-} = SU(2)_L$, and SU(2)$_{+} = SU(2)_R$ as

$$|2 > = |\psi > = |\psi_1 > | - + > + |\psi_2 > | - > , \quad |2 > = |\psi > = |\psi_1 > | + + > - |\psi_2 > | - >$$

As in the SO(6) case one transforms to the unitary basis where $4 = 2_+ \oplus 2_-$ has components $(\psi_\alpha, \psi_\dot{\alpha})$. Then in that basis

$$C_2 = \begin{pmatrix} \epsilon^{\alpha\beta} & 0_2 \\ 0_2 & -\epsilon^{\dot{\alpha}\dot{\beta}} \end{pmatrix} , \quad C_1 = - \begin{pmatrix} \epsilon^{\alpha\beta} & 0_2 \\ 0_2 & \epsilon^{\dot{\alpha}\dot{\beta}} \end{pmatrix} , \quad [\gamma_{\rho\dot{\rho}}] = \sqrt{2} \begin{pmatrix} 0_2 & \epsilon_{\rho\dot{\alpha}} \epsilon_{\dot{\beta}}^\dot{\rho} \\ \epsilon_{\rho\dot{\alpha}} \epsilon_{\dot{\beta}}^\dot{\rho} & 0_2 \end{pmatrix}$$

The following expressions for spinor covariants then follow

$$\psi^T C_2^{(2)} \chi = \psi_\alpha \chi_\alpha - \psi_\dot{\alpha} \chi_\dot{\alpha}$$
$$\psi^T C_1^{(2)} \chi = \psi_\dot{\alpha} \chi_\alpha + \psi_\alpha \chi_\dot{\alpha}$$
$$\psi^T C_2^{(2)} \gamma_{\alpha\dot{\alpha}} \chi = \sqrt{2} (\psi_\alpha \chi_\dot{\alpha} - \psi_\dot{\alpha} \chi_\alpha)$$
$$\psi^T C_1^{(2)} \gamma_{\alpha\dot{\alpha}} \chi = \sqrt{2} (\psi_\dot{\alpha} \chi_\alpha + \psi_\alpha \chi_\dot{\alpha})$$
$$\psi^T C_2^{(2)} \gamma_{\alpha\dot{\alpha}} \gamma_{\beta\dot{\beta}} \chi = 2 \epsilon_{\alpha\dot{\beta}} \psi_\alpha \chi_\beta - 2 \epsilon_{\alpha\dot{\beta}} \psi_\dot{\alpha} \chi_\beta$$
$$\psi^T C_1^{(2)} \gamma_{\alpha\dot{\alpha}} \gamma_{\beta\dot{\beta}} \chi = -2 \epsilon_{\dot{\alpha}\dot{\beta}} \psi_\alpha \chi_\beta - 2 \epsilon_{\dot{\alpha}\dot{\beta}} \psi_\dot{\alpha} \chi_\beta$$
Furthermore
\[
\begin{align*}
\psi^\dagger \chi &= \psi_\alpha^\dagger \chi_\alpha + \psi_\dot{\alpha}^\dagger \chi_{\dot{\alpha}} \\
\psi^\dagger \gamma_{\alpha\dot{\alpha}} \chi &= -\sqrt{2}(\psi_{\alpha}^* \chi_\dot{\alpha} + \psi_{\dot{\alpha}}^* \chi_\alpha) \\
\psi^\dagger \gamma_{\alpha\beta} \chi &= 2\epsilon_{\alpha\beta\dot{\gamma}} \psi^* \chi_\dot{\gamma} + 2\epsilon_{\alpha\dot{\gamma}\beta} \psi^* \chi_\gamma
\end{align*}
\]

(22)

Note that these can be obtained from the corresponding identities involving \(C_1^{(2)}\) by the replacements \(\psi^\dagger \rightarrow \psi_\alpha^\dagger, \psi^\alpha \rightarrow \psi_\alpha^\dagger\) or from the \(C_2\) identities by \(\psi^\dagger \rightarrow \psi_\alpha^\dagger, \psi^\alpha \rightarrow -\psi_\alpha^\dagger\).

F. \(\text{SO}(10)\) Spinors

The spinor representation of \(\text{SO}(10)\) is \(2^5\) dimensional and splits into chiral eigenstates with \(\gamma_F = \pm 1\) as
\[
\begin{align*}
2^5 &= 2^4 + 2^4 = 16 + 16_- \\
16 &= 16_+ = (4_+, 2_+) + (4_-, 2_-) = (\mathbf{1}, 1, 2) + (4, 2, 1) \\
\mathbf{16} &= 16_- = (4_+, 2_-) + (4_-, 2_+) = (\mathbf{3}, 2, 1) + (4, 1, 2)
\end{align*}
\]

(23) (24) (25)

Where the first equality follows from the definition of \(\gamma_F\) and the second from the \(\text{SO}(6)\) to \(\text{SU}(4)\) and \(\text{SO}(4)\) to \(\text{SU}(2)_L \times \text{SU}(2)_R\) translations: \(4_- = 4, 2_+ = 2, 2_- = 2_+\). Thus we see that the \(\text{SU}(4)\) and \(\text{SU}(2)_L \times \text{SU}(2)_R\) properties of the submultiplets within the \(16, \mathbf{16}\) are strictly correlated. Use of the \(\text{SO}(6)\) and \(\text{SO}(4)\) spinor covariant identities allows fast construction of \(\text{SO}(10)\) spinor invariants. For example,
\[
\psi^T C_2^{(5)} \gamma^{(5)}_{\mu\nu} \chi = \psi^T (C_2^{(3)} \times C_1^{(2)}) (\gamma^{(3)}_{\mu\nu} \times \tau_3 \times \tau_3) \chi = \psi^T (C_2^{(3)} \gamma^{(3)}_{\mu\nu} \times C_2^{(2)}) \chi
\]

(26)

Next one uses the identities \([18, 21]\) in parallel, keeping in mind that in the \(16\)-plet the dotted \((\text{SU}(2)_R)\) spinors are always 4-plets of \(\text{SU}(4)\) and the undotted ones are 4-plets and vice versa for \(\mathbf{16}\). When \(\psi, \chi\) are both \(16\)-plets one immediately reads off the result
\[
\psi^T C_2^{(5)} \gamma^{(5)}_{\mu\nu} \chi = \sqrt{2}(\psi_{\mu}^\alpha \chi_{\nu})_{\alpha} + \gamma^{\lambda\dot{\alpha}\sigma}_{\gamma\alpha} \chi_{\alpha} \epsilon_{\mu\nu\lambda\sigma}
\]

(27)

In addition to the above rules for the decomposition of the two fundamental irreps of \(\text{Spin}(10)\) we also gave \([4]\) extensive tables of decompositions of \(\text{SO}(10)\) cubic invariants e.g. \(16, 16, (10 \oplus 120 \oplus \mathbf{126})\) and even the matter kinetic terms that are directly usable in the \(\text{SO}(10)\) superpotential. Finally the standard PS embedding of a matter fermion generation in the \(16\)-plet completes the specification of all normalizations and phases from the \(\text{SO}(10)\) down to the SM.

\[
(4, 2, 1) = (Q, L) \\
(\mathbf{1}, 1, 2) = (\bar{Q}, \bar{T})
\]

(28)

with
\[
Q = \begin{pmatrix} U & D \end{pmatrix} \\
L = \begin{pmatrix} \nu \\ e \end{pmatrix} \\
\bar{Q} = \begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix} \\
\bar{T} = \begin{pmatrix} \bar{e} \\ \bar{\nu} \end{pmatrix}
\]

(29)

Note that in \([4]\) and thereafter we only perform unitary basis transformations on the fields and thus always maintain unit norm as defined by unit coefficient of canonical kinetic terms in the Lagrangian.

We conclude with a quote from \([4]\) which summarizes our consistent position on the relative merits of the component wise and systematic decomposition via maximal sub-group approaches: “We emphasize that our method allows computation, not only of spectra but also of the couplings of all the multiplets in the theory (whether they are renormalizable or heavy-exchange induced effective couplings) without any ambiguity. Moreover our results are obtained by an analytic tensorial reprocessing of labels of fields in the Lagrangian. This approach might thus find preference with field theorists in comparison with the more restricted capabilities of the approach of \([15]\), which, so far, has not proved capable of generating all the Clebsches of the \(\text{SO}(10)\) theory and which relies on an explicit multiplet representative and computer based approach which is tedious to connect to the unitary group tensor methods so familiar to particle theorists.”
G. Fermion Yukawa couplings

Since it is so simple let us summarize our prescription for determining the MSSM couplings in terms of GUT ones. When we rewrite the equations of [2] in our notation the non-holomorphic connection problem will become all too obvious.

We call the (6 pairs of) EW type doublets contained in the SO(10) fields \[ H_i[1, 2, -1] \oplus H_i[1, 2, 1]; i = 1...6 \] and their mass matrix due to mass terms in the superpotential \( \mathcal{W}(x, \lambda) \). Here we have exhibited the (holomorphic) dependence of the mass matrix on the generic vevs \( \langle x \rangle \) and superpotential couplings \( \langle \lambda \rangle \). The mass eigenstates of \( \mathcal{H} \) are called \( \{ \bar{H}_i[1, 2, -1] \oplus H_i[1, 2, 1]; i = 1...6 \} \). A bi-unitary transformation connects the two sets :

\[
\begin{align*}
    \bar{h}^{(i)} &= U_{ij}H^{(j)} ; & \bar{h}^{(i)} &= \bar{U}_{ij}H^{(j)}
\end{align*}
\] (30)

The columns of \( U(\bar{U}) \) are the unit normalized right eigenvectors of \( \mathcal{H}^\dagger \mathcal{H} \). When a fine tuning condition \( \det(\mathcal{H}) = 0 \) is imposed [3, 4, 12, 13] one pair of doublets remains light i.e., massless on the scale \( M_X \) which we call \( \{ \bar{H}^{(1)}, H^{(1)} \} \). Then it follows that

\[
\begin{align*}
    \Lambda_{H} &= \text{Diag}(m_{H}^{(1)}, m_{H}^{(2)}, \ldots) = \bar{U}^T H U \\
    W &= \bar{h}^T \mathcal{H} h + \ldots = \bar{H}^T \Lambda_{H} H + \ldots
\end{align*}
\]

where the first eigenvalue \( m_{H}^{(1)} = 0 \) and the rest can always be made positive by a choice of the phases of the eigenvalues. The passage to the renormalizable effective MSSM is simple indeed: simply set all the Superheavy Higgs doublets \( \{ \bar{H}_i[1, 2, -1] \oplus H_i[1, 2, 1]; i = 2...6 \} \) to zero in all SO(10) invariants involving them! In other words

\[
\begin{align*}
    h^{(i)} &\rightarrow \hat{a}_{i}H^{(1)} = \hat{a}_{i}H ; & \bar{h}^{(i)} &\rightarrow \hat{\alpha}_{i}\bar{H}^{(1)} = \hat{\alpha}_{i}\bar{H}
\end{align*}
\] (31)

where the numbers \( \hat{a}_{i} = U_{1i}, \hat{\alpha}_{i} = \bar{U}_{1i} \), which for obvious reasons we call Higgs fractions, are the critical information which is to be extracted by diagonalizing \( \mathcal{H} \). It is clear that the un-normalized Higgs fractions \( \hat{a}_{i}(\hat{\alpha}_{i})[1] \) are the right and left null eigenvectors of \( \mathcal{H} \) and are holomorphic in the vevs and couplings.

The inverse transformations are obviously \( H = U^\dagger h, \bar{H} = \bar{U}^\dagger \bar{h} \) so that in particular

\[
\begin{align*}
    H^{(1)} &\equiv H = (U^\dagger)_{1j}h^{(j)} = (U^\dagger)_{1j}\alpha_{j}^* h^{(j)} \\
    \bar{H}^{(1)} &\equiv \bar{H} = (\bar{U}^\dagger)_{1j}\bar{h}^{(j)} = (\bar{U}^\dagger)_{1j}\bar{\alpha}_{j}^* \bar{h}^{(j)}
\end{align*}
\] (32)

So that the coefficients of the substitution rule [31] are the unconjugated Higgs fractions while those in equation [32] for the light Higgs in terms of the GUT doublets are conjugate. Note that the relation between SO(10) chiral multiplets and MSSM ones is holomorphic as it should be. Thus all relevant information is contained in the Higgs fractions \( \{ \hat{a}_{i}, \hat{\alpha}_{i}; i = 1, 6 \} \) alone.

Let us now turn to the corresponding equations in [2] and explain why they are incorrect. We shall quote the necessary equations directly from that paper but also repeat them in our notation but with primes on corresponding quantities to make the correspondence perfectly clear. Corresponding to the equation [32] has :

\[
\begin{align*}
    h_{u} &\propto w_{u10}^u H^u + w_{u120}^u \Sigma^u + w_{u210}^u \Phi^u + w_{u120}^u \Psi_{(1)}^u + w_{u120}^u \Psi_{(2)}^u \\
    h_{d} &\propto w_{d10}^d H^d + w_{d120}^d \Sigma^d + w_{d210}^d \Phi^d + w_{d120}^d \Psi_{(1)}^d + w_{d120}^d \Psi_{(2)}^d
\end{align*}
\] (33)

which in our notation would read

\[
\begin{align*}
    H' &= \alpha'_{j}h^{(j)'} \\
    \bar{H}' &= \bar{\alpha}'_{j}\bar{h}^{(j)'}
\end{align*}
\] (34)

Comparing with [32] the coefficient functions \( w_{i}^{d,n} = \alpha'_{i}, \alpha'_{i} \) which are given as unconjugated in [2] are seen to lack a conjugation and should be rather \( \alpha'_{i}, \bar{\alpha}'_{i} \). Conversely when we examine the correspondents of the substitution rule [31] we find that they are the equations (47,48) of [2]: “Let us define the projections of the electroweak doublet VEVs onto the neutral components of the defining basis doublets \( H^u,d, \Sigma^u,d, \Phi^u,d, \Psi_{(1)}^u,d, \Psi_{(2)}^u,d \) as follows:

\[
\begin{align*}
    \langle H^u \rangle &\equiv u_{10}^u, \quad \langle \Sigma^u \rangle &\equiv u_{120}^u, \quad \langle \Psi_{(1)}^u \rangle &\equiv u_{120}^u, \quad \langle \Psi_{(2)}^u \rangle &\equiv u_{120}^u \\
    \langle H^d \rangle &\equiv u_{10}^d, \quad \langle \Sigma^d \rangle &\equiv u_{120}^d, \quad \langle \Psi_{(1)}^d \rangle &\equiv u_{120}^d, \quad \langle \Psi_{(2)}^d \rangle &\equiv u_{120}^d
\end{align*}
\] (35)
The main virtue of this definition is that these factors are indeed simple functions of the decomposition weights in $\text{SO}(10)$ and the VEVs $v_u$ and $v_d$ of the MSSM light Higgs doublets ($\langle h_{u,d} \rangle \equiv v_{u,d}$), namely:

$$u_{10} = (w_{10}^{u,d} v_{u,d})^*, \quad u_{26} = (w_{26}^{u,d} v_{u,d})^*, \quad u_{120}^{(1)} = (w_{120}^{u,d(1)} v_{u,d})^* \quad \text{and} \quad u_{120}^{(2)} = (w_{120}^{u,d(2)} v_{u,d})^*$$  \(36\)

The equation (35) is simply a name for vevs so it is not clear what significance the emphasis on define has for the author; one could stay with $\langle \rangle$ to indicate vevs just as well. We found the next equation (36) had an error: the field relations were anti-holomorphic and the Higgs fractions were conjugated but in our substitution rule they are not. Thus further comparison was pointless till the discrepancies are corrected.

IV. DISCUSSION

In this comment we have taken the trouble to deconstruct the calculation of [2] to make clear that there are defects in the method of calculation. In our view use of group theoretic methods adequate to conveniently, systematically and unambiguously encode the complexities of the embeddings in SO(10) and the decomposition w.r.t. the SM subgroup in a compact and generative (i.e. tensor index rule based rather than adhoc) are essential. Such methods should not be based on voluminous lists of arbitrary phase choices but on generative rules that can be repeated applied with confidence that they will be consistent globally. We provided one such framework in [4]. Another might perhaps be based on [11, 27]. It would certainly be highly interesting to cross check the conventions, phases and normalizations between the two different maximal subgroup methods. The explicit state based methods used in [2] and its antecedent papers are, in our view, useful only for a limited purpose of checking magnitudes and will never serve as an efficient basis for computation in SO(10) GUTs in all their complex interactive glory. The arbitrary phase conventions used are all but impossible to check between the multiple computations that now exist [12, 13, 14]. Thus they create and amplify Babel on the SO(10) ‘tower’ (or PetaPlex) construction site where we collectively labour, rather than enable its speedy erection.

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