Electromagnetically Interacting Massive Spin-2 Field: Intrinsic Cutoff and Pathologies in External Fields

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By employing the St"uckelberg formalism, we argue that the theory of massive spin-2 field coupled to electromagnetism in flat space must have an intrinsic, model independent, finite UV cutoff. We show how the very existence of a cutoff has connection to other pathologies of the system, such as superluminal propagation. We comment on the generalization of the results to arbitrary spin, and to gravitational interaction.

1. INTRODUCTION

Massive charged high-spin particles do exist in nature. Particle colliders have produced unstable composite high-spin resonances like \( \pi_2(1670) \), \( \rho_3(1690) \) or \( a_4(2040) \), whose inverse size is of the same order as their mass. String theory also gives massive high-spin particles, with masses at least as large as the string scale, that may very well couple to a U(1) gauge field. In both cases the electrodynamics can be described by an effective field theory, which makes sense only below a finite cutoff not higher than the mass scale itself. One may wonder whether this is a generic feature of massive charged high-spin fields, or just particular to the above two examples.

On the other hand, massive charged fields with \( s > \frac{1}{2} \) are known to exhibit a variety of pathological features in a constant electromagnetic background in flat space [1]. The high-spin field may suffer from the so-called Velo-Zwanziger acausality [1] with modes that propagate superluminally, or the system even ceases to be hyperbolic, or the number of propagating degrees of freedom may be different from that of the free theory, or the Cauchy problem may become ill-posed. One would like to know if the possible existence of a cutoff is related to all these pathologies.

The coupling of massless high-spin particles to electromagnetism or gravity is plagued by grave inconsistencies [2], because of absence of conserved current invariant under the high-spin gauge symmetry. For gravitational and electromagnetic interactions this problem respectively starts from spin-5/2 [2], and spin-3/2 (in flat space). In fact, there are strong constraints coming from no-go theorems [3, 4] that forbid the existence of interacting massless particles with \( s > 2 \) in Minkowski space. The fact that charged high-spin resonances exist then implies that any local Lagrangian describing electromagnetically interacting massive high-spin fields must be singular in the \( m \to 0 \) limit.

In the unitary gauge this massless singularity is not obvious at all. For example, if one starts with the Singh-Hagen Lagrangian [5] for massive fields of arbitrary spin and introduces electromagnetic coupling by the minimal prescription \( \partial_{\mu} \to D_{\mu} \), one only gets positive powers of the mass in the resulting Lagrangian. Propagators of the massive theory become singular in the massless limit because of high-spin gauge invariance, so that scattering amplitudes diverge. The best way to understand the mass singularity is to employ the St"uckelberg formalism, because it focuses precisely on the gauge modes responsible for bad high energy behavior. Here one renders the massive free theory gauge invariant through the addition of auxiliary fields, which can always be set to vanish using the resulting gauge invariance. When the theory is coupled to a U(1) gauge field, one can make a judicious covariant gauge fixing to obtain diagonal kinetic operators. In this case, there exist non-renormalizable interaction terms involving auxiliary fields, which explicitly depend on inverse powers of the mass.

In this article we consider the simple but nontrivial case of massive spin-2 field coupled to electromagnetism in flat space. In Section 2 we describe massive spin-2 field à la St"uckelberg, and follow the prescription outlined in [6] to show that the massless singularity cannot be cured by any means. We quantify the essential degree of this singularity by finding an expression for the UV cutoff of the effective action in terms of the particle’s mass and electric charge. In Section 3 we argue that, for a judicious choice of non-minimal interaction terms, the system may show pathologies only when one extrapolates the local effective description beyond its regime of validity. We draw concluding remarks in Section 4, and comment on the generalization of the results to arbitrary spin and to gravitational coupling.
2. ELECTROMAGNETIC COUPLING OF MASSIVE SPIN-2 FIELD

Massive spin-2 electromagnetic interaction has been studied by various authors \[1, 2, 3, 4\]. Here we will always work in flat space. We start with the Pauli-Fierz Lagrangian \[10\] for massive spin-2 field.

\[
L = -\frac{1}{2} \left( \partial_{\mu} h_{\nu\rho} \right)^2 + \left( \partial_{\mu} h^{\mu\nu} \right)^2 + \frac{1}{2} \left( \partial_{\mu} h \right)^2 - \partial_{\mu} h^{\mu\nu} \partial_{\nu} h - \frac{m^2}{2} \left[ h^{\mu\nu} - h^2 \right],
\]

where \( h = h^\mu_\mu \). This Lagrangian does not possess any manifest gauge invariance. Now, by the field redefinition

\[
h_{\mu\nu} - \hat{h}_{\mu\nu} = h_{\mu\nu} + \frac{1}{m} \partial_\mu \left( B_\nu - \frac{1}{2m} \partial_\nu \phi \right) + \frac{1}{m} \partial_\nu \left( B_\mu - \frac{1}{2m} \partial_\mu \phi \right),
\]

we introduce new (Stückelberg) fields \( B_\mu \) and \( \phi \) to obtain the following gauge invariance (Stückelberg symmetry):

\[
\delta h_{\mu\nu} = \partial_\mu \lambda_\nu + \partial_\nu \lambda_\mu,
\]

\[
\delta B_\mu = \partial_\mu \lambda - m \lambda_\mu,
\]

\[
\delta \phi = 2m \lambda.
\]

The Stückelberg fields are redundant, in that they can always be gauged away. Yet they are useful since they allow us to unveil the dangerous degrees of freedom and interactions hidden inside the spin-2 action. In Lagrangian \[11\] all the higher dimensional operators cancel by the commutativity of partial derivatives. In fact one can obtain the Stückelberg version of the Pauli-Fierz Lagrangian just by Kaluza-Klein reducing the (4+1)D linearized Einstein-Hilbert action to (3+1)D. All the fields \( h_{\mu\nu}, B_\mu \) and \( \phi \) then originate from a single higher dimensional massless spin-2 field, and the higher dimensional gauge invariance gives rise to the Stückelberg symmetry \[3, 4\] in lower dimension.

Electromagnetic coupling is introduced by complexifying the fields and replacing ordinary derivatives with covariant ones.

\[
L = - |D_\mu \hat{h}_{\nu\rho}|^2 + 2 |D_\mu \hat{h}^{\mu\nu}|^2 + |D_\mu \hat{h}|^2 - [D_\mu \hat{h}^{*\mu\nu} D_\nu \hat{h} + c.c.] - m^2 [\hat{h}^{*\mu\nu} \hat{h}^{\mu\nu} - \hat{h}^{*} \hat{h}] - \frac{1}{4} F_{\mu\nu}^2,
\]

where \( \hat{h}_{\mu\nu} \), the covariant counterpart of (2),

\[
\hat{h}_{\mu\nu} = h_{\mu\nu} + \frac{1}{m} D_\mu \left( B_\nu - \frac{1}{2m} D_\nu \phi \right) + \frac{1}{m} D_\nu \left( B_\mu - \frac{1}{2m} D_\mu \phi \right),
\]

is trivially invariant under the gauged version of the symmetry \[3, 5\]. The authors in Ref. \[9\] also considered a gauge invariant description to investigate consistent interactions of massive high-spin fields. In case of spin-2, say, they introduce Stückelberg fields only in the mass term. This procedure already breaks Stückelberg invariance at tree level, but our approach by construction guarantees that Stückelberg symmetry is intact by the minimal substitution.

Next, we diagonalize the kinetic operators to make sure that the propagators in the theory have good high energy behavior, i.e., that all propagators are proportional to \( 1/p^2 \) for momenta \( p^2 \gg m^2 \). The field redefinition \( h_{\mu\nu} \rightarrow h_{\mu\nu} - (1/2) \eta_{\mu\nu} \phi \) eliminates some kinetic mixings, and also generates a kinetic term for \( \phi \) with the correct sign. Adding the gauge fixing terms: \( L_{gf1} = -2 |\partial_{\mu} h^{\mu\nu} - (1/2) \partial^\nu h + m B^\nu|^2 \), and \( L_{gf2} = -2 |\partial_{\mu} B^\mu + (m/2)(h - 3\phi)|^2 \), we exhaust all gauge freedom to obtain diagonal kinetic terms. We are left with

\[
L = \hat{h}^{*\mu\nu} (\Box - m^2) h_{\mu\nu} - \frac{1}{2} \hat{h}^{*} (\Box - m^2) h + 2 B_\mu^* (\Box - m^2) B^\mu + \frac{3}{2} \phi^* (\Box - m^2) \phi - \frac{1}{4} F_{\mu\nu}^2 + L_{\text{int}}.
\]

Here \( L_{\text{int}} \) contains all interaction terms, among which there exist higher dimensional operators, thanks to the non-commutativity of covariant derivatives. The most potentially dangerous non-renormalizable operators in the high energy limit \( m \rightarrow 0 \) are the ones with the highest dimensionality. Parametrically in \( e \ll 1 \), for any given operator dimensionality, the \( O(e) \)-terms are more dangerous than the others. We have

\[
L_{\text{int}} = \frac{e}{m^2} \partial_\mu F^{\mu\nu} [i(2 \partial_\mu B_\rho^* \partial^\rho \partial_\nu \phi - \partial_\mu B_\rho^* \Box \phi) + c.c.] + (...) + \frac{e}{m^3} F^{\mu\nu} [i(2 \partial_\mu B_\rho^* \partial^\rho \partial_\nu \phi - \partial_\mu B_\rho^* \Box \phi) + c.c. + (...) .
\]
where (...) stands for less divergent terms. The $O(\varepsilon)$ dimension-8 operator is proportional the Maxwell equations; it can be eliminated by a local field redefinition: $A_\mu \to A_\mu - (\varepsilon/m^3)J_\mu$, where $J_\mu \equiv ([i/2]\partial_\mu \phi^* \partial^\mu \phi + \text{c.c.}]$. This introduces $O(\varepsilon^2)$-terms: $(e^2/4m^8)[\partial_\mu J_\nu - \partial_\nu J_\mu]_2$, which must be canceled as well. Indeed, adding the local function: $L_{\text{add}} = (e^2/4)[\tilde{h}^*_{\mu \nu} \tilde{h}^\rho_\nu - \tilde{h}^*_{\mu \nu} \tilde{h}^\rho_\mu]^2$ to the Lagrangian serves the purpose. But now we will have terms proportional to $e^2/m^7$, coming both from $L_{\text{add}}$, and from the shift of $A_\mu$ acting on the dimension-7 operator in (9). The latter gives

$$L_{11} = \frac{e^2}{m^7} \{ \partial_\mu \partial_\nu \phi^* \partial^\rho \partial^\sigma \phi - (\mu \leftrightarrow \nu) \} \{ 2\partial_\mu B^*_{\rho \nu} \partial^\rho \partial^\sigma \phi - \partial_\mu B^*_{\rho \nu} \boxdot \phi \} + \text{c.c.}$$

Notice that any local function of the spin-2 field $\tilde{h}_{\mu \nu}$ is fully invariant under the special Stückelberg symmetry:

$$B_\mu \to B_\mu + b_\mu + (b_{\mu \nu} - b_{\nu \mu}) x^{\nu}, \quad \phi \to \phi + c + c_\mu x^{\mu},$$

where $b_\mu, b_{\mu \nu}, c, c_\mu$ are constant tensors. Now the operators in Eq. (10), being invariant only up to a nontrivial total derivative, cannot be eliminated by adding local functions of $\tilde{h}_{\mu \nu}$. We must have terms proportional to $e^2/m^7$.

On the other hand, mere addition of a dipole term leaves us only with terms proportional to $e/m^3$, which is already an improvement over field redefinition plus addition of local term. Indeed, a dipole term $2ie\alpha F^{\mu \nu} \tilde{h}^*_{\mu \nu} \tilde{h}^\rho_\nu$ gives

$$L_{\text{dipole}} = \frac{e}{m^3} \partial_\mu F^{\mu \nu}[-i\alpha \partial_\nu \phi + c + \text{c.c.}] + \frac{e}{m^3} F^{\mu \nu}[-2i\alpha \partial_\mu B_\rho^* \partial^\rho \partial_\nu \phi + \text{c.c.}] + (\ldots)$$

If we choose $\alpha = 1/2$, the dimension-8 operators at $O(\varepsilon)$ all cancel leaving us only with dimension-7 operators. In the scaling limit: $m \to 0, e \to 0$, such that $e/m^3=\text{constant}$, the non-minimal Lagrangian reduces to:

$$L = L_{\text{kin}} + \frac{ie}{2m^4} F^{\mu \nu} \left\{ 2\partial_\mu B^*_{\rho \nu} \partial^\mu \partial_\nu \phi - \partial_\mu B^*_{\rho \nu} \boxdot \phi \right\} + \text{c.c.}$$

(13)

These operators contain pieces that are not proportional to the equations of motion. Therefore the degree of divergence could not have been improved further. Thus our theory has an intrinsic model-independent cutoff $\Lambda$:

$$\Lambda = \frac{m}{e^{\gamma/3}}.$$

(14)

Here we notice the curious fact that the Lagrangian (13) has acquired a $U(1)$ gauge invariance for the vector Stückelberg $B_\mu$. This is because an appropriately chosen dipole term cancels not only the $O(\varepsilon)$ dimension-8 operator, but also any $O(\varepsilon)$ dimension-7 operators that get generated from a transformation $B_\mu \to B_\mu + \partial_\mu \theta$.

3. SUPERLUMINAL PROPAGATION AND ALL THAT

All the pathologies of interacting high-spin system originate from a gauge invariance in the kinetic part of the free Lagrangian, which implies the existence of zero modes. In electromagnetic backgrounds the zero modes acquire non-vanishing but non-canonical kinetic term, which may cause some of them to propagate faster than light, or even lose hyperbolicity. The Stückelberg formalism is tailored to single out the dynamics of precisely these gauge modes, which are nothing but the Stückelberg fields. For the case of massive spin-2, to simplify our analysis we set the $B_\mu$ field on shell. By doing this we can only check if the scalar $\phi$ exhibits non-standard dynamics. In a constant electromagnetic background $\phi$ will experience a new effective background metric, different from Minkowski, because of some $O(\varepsilon^2)$ dimension-8 operators [3]:

$$\tilde{\eta}^{\mu \nu} = \left\{ \frac{3}{2} + \frac{e^2}{4m^4} (5 - 4\alpha) F^2_{\mu \nu} \right\} \eta^{\mu \nu} + \frac{e^2}{4m^4} (2 + 8\alpha) F^{\rho \sigma} F^\rho_\mu F^\sigma_\nu,$$

(15)

where $\alpha$ is a generic dipole coefficient. $\tilde{\eta}^{\mu \nu}$ is proportional to the Minkowski metric for $\alpha = -1/4$, so that $\phi$ propagates at the speed of light. But even for this value of $\alpha$ the system ceases to be hyperbolic in a strong field; precisely when $e^2F^2_{\mu \nu}/m^4 = -1$. By computing the characteristic speeds one finds that the choice $\alpha < -1/4$ is downright pathological [6], because it gives superluminal propagation even for infinitesimally small values of the background fields. Other values of $\alpha$ are safe for sufficiently weak external fields, and pathologies exist only in strong external fields: $E^2, B^2 \gtrsim (m/\sqrt{\varepsilon})^4 > \Lambda^4$. But this is already beyond the regime of validity of our effective field theory. In that dynamical regime, pathologies may be cured by new degrees of freedom or strong coupling phenomena.
4. CONCLUSION

The Stückelberg method is powerful in that it clarifies the dynamics of interacting high-spin fields. Here we analyzed a simple high-spin system: a spin-2 field coupled to electromagnetism and found that the theory must have an intrinsic UV cutoff, no higher than $\Lambda = me^{-1/3}$. The pathologies manifest themselves in the scalar Stückelberg sector. However we saw that, for a judicious choice of non-minimal interaction terms, they appear in a regime where we can no longer trust our effective field theory description, at least in the scalar sector.

Generalizations of this example to arbitrary spin $s$ is also being done [13]. Starting from spin-5/2 the story gets complicated by the presence of auxiliary fields that are not (gamma)traces of the high-spin field. The cutoff turns out to be no higher than $\Lambda = me^{-1/(2s-1)}$. This result rules out long-lived high-spin charged particles, and thus may affect directly the strategy for their search in future colliders. Gravitational coupling of high-spin fields is also very important; it should be relatively straightforward in our formalism. Existence of an intrinsic cutoff in this case will signal the ultimate limit of any local effective field theory description of interacting massive high-spin fields.

Does the existence of a UV cutoff necessarily call for new physics? Suppose some nontrivial UV fixed point exists [14]. Then it means that the theory can be resummed, so that the massless limit will be smooth. While this might be true for a spin-2 field with non-minimal electromagnetic interaction, absence of smooth massless limit starting from spin-5/2 [2, 4] rules out such a possibility for higher-spins.

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