Yang-Mills theory, 2 + 1 and 3 + 1 dimensions

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Abstract
I review the analysis of (2+1)-dimensional Yang-Mills ($YM_{2+1}$) theory via the use of gauge-invariant matrix variables. The vacuum wavefunction, string tension, the propagator mass for gluons, its relation to the magnetic mass for $YM_{3+1}$ at nonzero temperature and the extension of our analysis to the Yang-Mills-Chern-Simons theory are discussed. A possible extension to 3 + 1 dimensions is also briefly considered.

1 Introduction
In this talk I shall review a Hamiltonian approach to Yang-Mills theory in two spatial dimensions ($YM_{2+1}$), on which I have been working for the last few years. In this analysis, it is possible to carry out some nonperturbative calculations to the extent that results on mass gap and string tension can be compared with lattice simulations of the theory. I shall also discuss recent attempts to extend this to 3 + 1 dimensions. The work I shall report on was done in collaboration with D. Karabali and Chanju Kim [1, 2, 3] and more recently with A. Yelnikov [4].

Let me begin with a few general comments. The 2 + 1 dimensional case is interesting because it can be a model simple enough to analyze mathematically and yet nontrivial enough to teach us some lessons about (3+1)-dimensional $YM$ theories. Another reason to study $YM_{2+1}$ is its relevance to magnetic screening in $YM_{3+1}$ at high temperature. Gauge theories at finite temperature have worse infrared problems than at zero temperature.
due to the divergent nature of the Bose distribution for low energy modes. A dynamically generated Debye-type screening mass will eliminate some of these, but we need a magnetic screening mass as well to have a perturbative expansion which is well defined in the infrared. The mass gap of the 2 + 1 dimensional theory at zero temperature can be interpreted as the magnetic mass of the 3 + 1 dimensional theory at high temperatures, with a certain relationship between the coupling constants [5]. As for the 3 + 1 dimensional theory at zero temperature, being part of the standard model, its importance needs no emphasis.

For $YM_{2+1}$ the following observations will be useful. The coupling constant $e^2$ has the dimension of mass and it does not run as the 3+1-dimensional coupling does. The dimensionless expansion parameter of the theory is $k/e^2$ or $e^2/k$, where $k$ is a typical momentum. Thus modes of low momenta must be treated nonperturbatively, while modes of high momenta can be treated perturbatively. There is no simple dimensionless expansion parameter. $YM_{2+1}$ is perturbatively super-renormalizable, so the ultraviolet singularities are well under control. The 3 + 1 dimensional theory will require renormalization and the associated dimensional transmutation.

\section{The 2 + 1 dimensional theory}

We shall first discuss $Y M_{2+1}$. Consider a gauge theory with group $G = SU(N)$ in the $A_0 = 0$ gauge. The gauge potential can be written as $A_i = -it^aA^a_i$, $i = 1, 2$, where $t^a$ are hermitian $N \times N$-matrices which form a basis of the Lie algebra of $SU(N)$ with $[t^a, t^b] = if^{abc}t^c$, $Tr(t^a t^b) = \frac{1}{2}\delta^{ab}$. The spatial coordinates $x_1, x_2$ will be combined into the complex combinations $z = x_1 - ix_2$, $\bar{z} = x_1 + ix_2$ with the corresponding components for the potential $A \equiv A_z = \frac{1}{2}(A_1 + iA_2)$, $\bar{A} \equiv A_{\bar{z}} = \frac{1}{2}(A_1 - iA_2) = -(A_z)^\dagger$. The starting point of our analysis is a change of variables given by

$$A_z = -\partial_z MM^{-1}, \quad A_{\bar{z}} = M^{\dagger-1}\partial_{\bar{z}}M^\dagger$$

(1)

Here $M$, $M^\dagger$ are complex matrices in general, not unitary. If they are unitary, the potential is a pure gauge. The parametrization (1) is standard in many discussions of two-dimensional gauge fields. A particular advantage of this parametrization is the way gauge transformations are realized. A gauge transformation $A_i \rightarrow A_i^{(g)} = g^{-1}A_ig + g^{-1}\partial_ig, \; g(x) \in SU(N)$ is obtained
by the transformation $M \rightarrow M^g = gM$. The gauge-invariant degrees of freedom are parametrized by the hermitian matrix $H = M^\dagger M$. Physical state wavefunctions are functions of $H$.

There will be three basic steps in our analysis, namely, the determination of the inner product of the wavefunctions by evaluating the gauge-invariant volume measure, rewriting the Hamiltonian as an operator involving the new variables and finally using these to solve the Schrödinger equation. We will consider these in turn.

The measure of integration over the fields $A, \bar{A}$ is $d\mu(A)/\text{vol}(G_s)$ where $d\mu(A) = \prod_{x,a} dA^a(x) d\bar{A}^a(x)$ is the Euclidean volume element on the space of gauge potentials $A$ and $\text{vol}(G_s)$ is the volume of gauge transformations, viz., volume of $SU(N)$-valued functions on space. From (1) we see that

$$\delta A = -D(\delta MM^{-1}), \delta \bar{A} = \bar{D}(M^\dagger - 1\delta M^\dagger)$$

which gives

$$d\mu(A) = (\det D\bar{D}) d\mu(M, M^\dagger) \quad (2)$$

where $d\mu(M, M^\dagger)$ is the volume for the complex matrices $M, M^\dagger$, which is associated with the metric $ds^2_M = 8 \int \text{Tr}(\delta MM^{-1} M^\dagger - 1\delta M^\dagger)$. This is given by the highest order differential form $dV$ as $d\mu(M, M^\dagger) = \prod_x dV(M, M^\dagger).$

Writing out this form and using $M = U\rho$, $H = \rho^2 = M^\dagger M$,

$$d\mu(M, M^\dagger) = \prod_x dV(M, M^\dagger) \text{vol}(G_s) = \text{vol}(G_s) \prod_x d\mu(H) \quad (3)$$

$$d\mu(H) = \epsilon_{a_1...a_n}(H^{-1}dH)_{a_1}...(H^{-1}dH)_{a_n}$$

$$\text{vol}(G_s) = \prod_x d\mu(U) \quad (4)$$

$d\mu(U)$ is the standard Haar measure for $SU(N)$. The volume element or the integration measure for the gauge-invariant configurations can now be written as

$$\frac{d\mu(A)}{\text{vol}(G_s)} = \frac{[dA_x d\bar{A}_x]}{\text{vol}(G_s)} = (\det D\bar{D}) d\mu(H) \text{vol}(G_s) = (\det D\bar{D}) d\mu(H) \quad (5)$$

The problem is thus reduced to the calculation of the determinant of the two-dimensional operator $D\bar{D}$. This is well known to be given in terms of
the Wess-Zumino-Witten (WZW) action as [6]

$$\begin{align*}
(\det D \bar{D}) &= \left[ \det' \frac{\partial \tilde{D}}{\partial d^2x} \right]^{\dim G} \exp [2c_A S(H)] \tag{6}
\end{align*}$$

where $c_A \delta^{ab} = f^{amn} f^{bmn}$; it is equal to $N$ for $SU(N)$. The WZW action $S(H)$ for the hermitian matrix field $H$ is given by [7]

$$S(H) = \frac{1}{2\pi} \int \operatorname{Tr} (\partial H \bar{D} H^{-1}) + \frac{i}{12\pi} \int \epsilon^{\mu\nu\alpha} \times \operatorname{Tr} (H^{-1} \partial_\mu H H^{-1} \partial_\nu H H^{-1} \partial_\alpha H) \tag{7}$$

We can now write the inner product for states $|1\rangle$ and $|2\rangle$, represented by the wavefunctions $\Psi_1$ and $\Psi_2$, as [8]

$$\langle 1|2 \rangle = \int d\mu(H) e^{2c_A S(H)} \Psi_1^* \Psi_2 \tag{8}$$

The next step is the change of variables in the Hamiltonian. However, there is some further simplification we can do before taking up the Hamiltonian. The wavefunctions, being gauge-invariant, are functionals of the matrix field $H$, but actually they can be taken as functionals of the current of the WZW model (7) given by $J = (c_A/\pi) \partial_\mu H^{-1} H^{-1}$. Equation (8) shows that matrix elements of the theory are correlators of the hermitian WZW model of level number $2c_A$. The properties of the hermitian model of level number $k + 2c_A$ can be obtained by comparison with the $SU(N)$-model defined by $e^{kS(U)}$, $U(x) \in SU(N)$. The hermitian analogue of the renormalized level $\kappa = (k + c_A)$ of the $SU(N)$-model is $-(k + c_A)$. Since the correlators involve only the renormalized level $\kappa$, the correlators of the hermitian model (of level $(k + 2c_A)$) can be obtained from the correlators of the $SU(N)$-model (of level $k$) by the analytic continuation $\kappa \rightarrow -\kappa$. For the $SU(N)_k$-model there are the so-called integrable representations whose highest weights are limited by $k$ (spin $\leq k/2$ for $SU(2)$, for example). Correlators involving the nonintegrable representations vanish. For the hermitian model the corresponding statement is that the correlators involving nonintegrable representations are infinite. In our case, $k = 0$, and we have only one integrable representation corresponding to the identity operator (and its current algebra descendents). Therefore, for states of finite norm, it is sufficient to consider $J [1, 2]$. 
This means that we can transform the Hamiltonian $\mathcal{H} = T + V$ to express it in terms of $J$ and functional derivatives with respect to $J$. For functionals which can be expanded in powers of $J$, the kinetic term may worked out by the chain rule of differentiation, calculating the coefficients of derivatives with respect to the $J$’s using proper regularization. We find

$$
T = \frac{e^2}{2} \int E_i^a E_i^a = m \left[ \int J^a(u) \frac{\delta}{\delta J^a(u)} + \int \Omega_{ab}(u, v) \frac{\delta}{\delta J^a(u)} \frac{\delta}{\delta J^b(v)} \right] \quad (9)
$$

$$
V = \frac{1}{2e^2} \int B^a B^a = \pi \frac{\delta_{ab}}{mc_A} \int \partial J^a(\vec{x}) \partial J^a(\vec{x}) \quad (10)
$$

where $m = e^2 c_A / 2\pi$ and

$$
\Omega_{ab}(u, v) = \frac{c_A}{\pi^2 (u - v)^2} - \frac{f_{abc} J^c(v)}{\pi (u - v)} \quad (11)
$$

The first term in $T$ shows that every power of $J$ in the wavefunction gives a value $m$ to the energy, suggesting the existence of a mass gap.

The next step is to solve the Schrödinger equation. We will consider the vacuum wavefunction, which is presumably the simplest to calculate.

For the vacuum state, we take the ansatz $\Psi_0 = \exp(P)$, where $P$ is taken to be a series in powers of $J$. Substituting this into the Schrödinger equation, with the Hamiltonian given by (9, 10), $P$ can be determined term by term. Upto three powers of the current we find

$$
P = -\frac{1}{2e^2} \int B^a(x_1) K(x_1, x_2) B^a(x_2) + \int f^{abc}(x_1, x_2, x_3) J^a(x_1) J^b(x_2) J^c(x_3) + \cdots
$$

$$
K(x_1, x_2) = \left[ \frac{1}{(m + \sqrt{m^2 - \nabla^2})} \right]_{x_1, x_2} \quad (12)
$$

$f^{abc}(x_1, x_2, x_3)$ is given in reference [3]. The first term in (12) has the correct (perturbative) high momentum limit, viz.,

$$
\Psi_0 \approx \exp\left\{-\frac{1}{2e^2} \int_{x,y} B^a(x) \left[ \frac{1}{\sqrt{-\nabla^2}} \right]_{x,y} B^a(y) + \mathcal{O}(3J)\right\} \quad (13)
$$

The terms with higher number of $J$’s can be shown to be small for the low momentum limit and for the high momentum limit.
We now use this result to calculate the expectation value of the Wilson loop operator which is given as

\[ W(C) = \text{Tr} \mathcal{P} e^{-\oint_C (A dz + \bar{A} d\bar{z})} = \text{Tr} \mathcal{P} e^{(\pi/c) \oint_C J} \]

(14)

For the fundamental representation, its expectation value is given by

\[ \langle W_F(C) \rangle = \text{constant} \exp \left[ -\sigma A_C \right] \]

\[ \sqrt{\sigma} = e^2 \sqrt{\frac{N^2 - 1}{8\pi}} \]

(15)

where \( A_C \) is the area of the loop \( C \). \( \sigma \) is the string tension. This is a prediction of our analysis starting from first principles with no adjustable parameters. Notice that the dependence on \( e^2 \) and \( N \) is in agreement with large-\( N \) expectations, with \( \sigma \) depending only on the combination \( e^2N \) as \( N \to \infty \). (The first correction to the large-\( N \) limit is negative, viz., \(-e^2N/2N^2\sqrt{8\pi}\) which may be interesting in the context of large-\( N \) analyses.) Formula (15) gives the values \( \sqrt{\sigma}/e^2 = 0.345, 0.564, 0.772, 0.977 \) for \( N = 2, 3, 4, 5 \). There are estimates for \( \sigma \) based on Monte Carlo simulations of lattice gauge theory. The results for the gauge groups \( SU(2), SU(3), SU(4) \) and \( SU(5) \) are \( \sqrt{\sigma}/e^2 = 0.335, 0.553, 0.758, 0.966 \) [9]. We see that our result agrees with the lattice result to within \( \sim 3\% \).

One might wonder at this stage why the result is so good when we have not included the \( 3J \)- and higher terms in the wavefunction. This is basically because the string tension is determined by large area loops and for these, it is the long distance part of the wavefunction which contributes significantly. In this limit, the \( 3J \)- and higher terms in (12) are small compared to the quadratic term. We expect their contribution to \( \sigma \) to be small as well; this is currently under study.

Another feature of the wavefunction is that \( P \) is nonlocal when expressed in terms of the magnetic field. This is essentially due to gauge invariance combined with our choice of \( A_0 = 0 \); it has recently been shown that a similar result holds for the Schwinger model [10].

Some observations on the magnetic mass of \( YM_{3+1} \) at finite temperature can now be made. The expression (9) shows that for a wavefunction which is just \( J^a \), we have the exact result \( T J^a = m J^a \). When the potential term is added, \( J^a \) is no longer an exact eigenstate; we find
\((T + V)\ J^a = \sqrt{m^2 - \nabla^2} \ J^a + \cdots\), showing that the mass value is corrected to the relativistic dispersion relation. \(J^a\) may be considered as the gauge-invariant definition of the gluon. This result thus suggests a dynamical propagator mass \(m = e^2 c_A / 2\pi\) for the gluon. A different way to see this result is as follows. We can expand the matrix field \(J\) in powers of \(\varphi_a\) which parametrizes \(H\), so that \(J \simeq (c_A/\pi) \partial \varphi_a t_a\). This is like a perturbation expansion, but a resummed or improved version of it, where we expand the WZW action in \(\exp(2c_A S(H))\) but not expand the exponential itself. The Hamiltonian can then be simplified as

\[
\mathcal{H} \simeq \frac{1}{2} \int_x \left[ -\frac{\delta^2}{\delta \phi_a^2(x)} + \phi_a(x)(m^2 - \nabla^2)\phi_a(x) \right] + \cdots \tag{16}
\]

where \(\phi_a(k) = \sqrt{c_A k^2 / (2\pi m)} \varphi_a(k)\), in momentum space. This result is obtained by expanding the currents and also absorbing the WZW-action part of the measure into the wavefunctions, i.e., the operator (16) acts on \(\tilde{\Psi} = e^{c_A S(H)} \Psi\). The above equation shows that the propagating particles in the perturbative regime, where the power series expansion of the current is appropriate, have a mass \(m = e^2 c_A / 2\pi\). This value can therefore be identified as the magnetic mass of the gluons as given by this nonperturbative analysis.

For \(SU(2)\) our result is \(m \approx 0.32 e^2\). Gauge-invariant resummations of perturbation theory have given the values 0.38\(e^2\) [11] and 0.28\(e^2\) [12]. Lattice estimates of this mass are 0.31\(e^2\) to 0.40\(e^2\) (as a common factor mass for glueballs [13]) and 0.44\(e^2\) to 0.46\(e^2\) in different gauges [14]. An unambiguous lattice evaluation of this would require a gauge-invariant definition a gluon operator and its correlator. Philipsen has recently given a definition of the “gluon” on the lattice. Preliminary numerical estimates then give the mass as 0.37\(e^2\) [15].

Let us now turn to excited states. Even though \(J\) is useful as a description of the gluon, it is not a physical state. This is because of an ambiguity in our parametrization (1). Notice that the matrices \(M\) and \(M\tilde{V}(\tilde{z})\) both give the same \(\tilde{A} = A\), where \(\tilde{V}(\tilde{z})\) only depends on \(\tilde{z}\) and not \(z\). Since we have the same potentials, physical results must be insensitive to this redundancy in the choice of \(M\); in other words, physical wavefunctions must be invariant under \(M \rightarrow M\tilde{V}(\tilde{z})\). \(J\) is not invariant; we need at least two \(J\)’s to form an
invariant combination. An example is

$$\Psi_2 = \int_{x,y} f(x, y) \left[ \bar{\partial} J_a(x) (H(x, \bar{y}) H^{-1}(y, \bar{y}))_{ab} \bar{\partial} J_b(y) \right]$$  \hspace{1cm} (17)$$

This is not quite an eigenstate of the Hamiltonian. Neglecting certain $O(J^3)$-terms, one can show that this is an approximate eigenstate of eigenvalue $E$ if

$$\sqrt{m^2 - \nabla_1^2} + \sqrt{m^2 - \nabla_2^2} + \log(|x - y|^2 / \lambda) \right] f(x, y) = E f(x, y) \hspace{1cm} (18)$$

where $\lambda$ is a scale parameter [2]. An a posteriori justification of this would require that the size of the bound state be not too large on the scale of $1/m$. This does not seem to be the case, so, at least for now, all we can say is that the mass of $\Psi_2 \geq 2m$; see however [2].

### 3 The Yang-Mills-Chern-Simons Theory

We have extended our analysis to the Yang-Mills-Chern-Simons theory by adding a level $k$ Chern-Simons term to the action, which gives a perturbative mass $e^2/4\pi$ to the gluon [16]. The inner product now becomes

$$\langle 1|2 \rangle = \int d\mu(H)e^{(k+2c_A)} S(H) \Psi_1^* \Psi_2 \hspace{1cm} (19)$$

The earlier conformal field theory argument shows that there are now new integrable operators. These lead to screening behaviour for the Wilson loop operator rather than confinement, as expected. The kinetic energy term becomes

$$T = \frac{e^2}{4\pi} (k + 2c_A) \int J^a(u) \frac{\delta}{\delta J^a(u)} + \frac{e^2c_A}{2\pi} \int \Omega_{ab}(u, v) \frac{\delta}{\delta J^a(u)} \frac{\delta}{\delta J^b(v)} \hspace{1cm} (20)$$

The gluon mass is now $(k + 2c_A)e^2/4\pi$ reflecting dynamical mass generation as well by nonperturbative effects. The vacuum state and a number of excited states have also been constructed in this case [16].
4 Towards the 3 + 1 dimensional theory

The success of our analysis for $YM_{2+1}$ depended largely on the use of the parametrization (1). This facilitated the calculation of the inner product and the change of variables in the Hamiltonian. So, as a first step in attempting a generalization to $YM_{3+1}$, we can try to get a suitable parametrization for a gauge potential $A^a_i$, $i = 1, 2, 3$, on $\mathbb{R}^3$. ($A_0$ can be taken zero as before.) Recently, A. Yelnikov and I have found a parametrization which may be appropriate [4]. For an $U(N)$ gauge field, the matrix $M$ in terms of which the potentials can be parametrized must take values in $U(2N, \mathbb{C})$; this leads to more redundancy than in the $2 + 1$ dimensional case and will require further constraints. Specifically, let $t^a$ be a basis of the Lie algebra of $U(N)$ (considered as $N \times N$ matrices), and let $\sigma_i$ denote the Pauli matrices. Then a basis for $U(2N, \mathbb{C})$ is given by $\{1 \otimes t^a, 1 \otimes it^a, \sigma_i \otimes t^a, \sigma_i \otimes it^a\}$. One can then show that for every gauge potential $A^a_i$, one can construct a $U(2N, \mathbb{C})$-valued matrix $M$ by solving the equation

$$\sigma \cdot A = -\sigma \cdot \partial M \ M^{-1} \quad (21)$$

Conversely, if an arbitrary $M \in U(2N, \mathbb{C})$ is given, one can obtain a gauge potential $A^a_i$ from the above equation, provided $M$ obeys the further constraints

$$\text{Tr}(t^a \sigma \cdot \partial M \ M^{-1} - \text{h.c.}) = 0$$

$$\partial_i (M^\dagger \sigma_i M) = 0 \quad (22)$$

These results hold for a neighbourhood of the flat potential in the space of potentials. If we expand $M$ as $M \approx 1 + it^a \varphi^a + i\sigma_k t^a \Theta^a_k + \cdots$, it is easy to see that these reproduce known Abelian gauge field parametrizations. In an $N \times N$ splitting, we may write

$$M = \begin{pmatrix} U & 0 \\ 0 & U \end{pmatrix} \quad (23)$$

where $N$ can be obtained as an algebraic function of the gauge-invariant quantities $H = M^\dagger M$ and $W_i = M^\dagger \sigma_i M$. $U$ can be removed by a gauge choice. The invariant measure of integration is then given by

$$\frac{d\mu(A)}{vol(G)} = d\mu(H, W) \exp(\Gamma + \bar{\Gamma})$$

$$\delta[\sigma \cdot \partial NN^{-1} + \text{h.c.}] \delta[\text{Tr} t^a \sigma \cdot \partial NN^{-1} - \text{h.c.}] \quad (24)$$
$d\mu(H,W)$ is the product over the spatial points of the volume on $U(2N, C)/U(N)$ and can be explicitly given \cite{4}. Unlike the lower dimensional case, $\Gamma$ can only be calculated in powers of $F_{ij}$; to the lowest order it is given by

$$\Gamma + \bar{\Gamma} = -\frac{c_A}{128} \int F_{ij}^a \left[ \frac{1}{\sqrt{-(\partial + A)^2}} \right]^{ab} F_{ij}^b + \cdots \quad (25)$$

We are now trying to work out the Hamiltonian in these variables. The hope is that the use of $\Gamma + \bar{\Gamma}$, up to the order calculated, will suffice to show that the wavefunctions should contain a term like $\exp(-\int F^2/8\mu)$ with a dimensional parameter $\mu$ (related to the $\Lambda$ parameter). In this case, by comparison with $YM_{2+1}$, one can conclude that the string tension should be $\sqrt{\sigma} = \mu \sqrt{(N^2 - 1)/8\pi}$. This is under investigation.

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