Assessment of the Fluidic Control Boundary Nozzle within a Pulse Detonation Rocked Engine by Unstructured Meshes CE/SE Method

Yanyan Wang¹, Xiujuan Wu², Fei Gong³, Bin Wang* and Wenjie Yu¹

¹ School of Electrical Engineering, Nanjing Institute of Industry Technology, Nanjing, Jiangsu, 210023, China
² School of Mechanical Engineering, Nanjing Institute of Industry Technology, Nanjing, Jiangsu, 210023, China
³ College of Automation, Nanjing University of Posts and Telecommunications, Nanjing, Jiangsu, 210023, China

*Corresponding author’s e-mail: wangyyjh@126.com

Abstract. The simulation model of pneumatic Fluidic Control Boundary Nozzle (Fluidic nozzle) is built to study its operation and propulsive augmentation performance in the unsteady PDRE system. The numerical simulation employed an adaptable unstructured-mesh CE/SE method, which is found capable to solve flows with strong gradients in detonation tube and complicated wave interaction patterns in fluidic nozzle. It is found that during the detonation and earlier exhaust stage, the whole section of fluidic nozzle is used to expand and increase the thrust without any over-expansion appearing in the nozzle. Conversely, during the later exhaust stage, purge and refilling stage, the annular control surface by subsonic jet of fluidic nozzle is successfully formed on the outside of main stream to avoid harmful shock waves of over-expansion. Thus, the real expansion ratio and thrust efficient are optimized. It is also found that easier for the bell fluidic nozzle to control the outer interface of main supersonic flow, while it is easier for the conical fluidic nozzle to perfect the real expansion ratio of main flow. The research results enrich the research system of PDRE nozzle.

1. Introduction

Pulse detonation engines (PDEs) have been recently recognised as a novel propulsion system that provides the inherent thermodynamic advantages of a near-constant volume [1]. Detonation combustion systems provide better thermal efficiency than constant-pressure combustion engines due to the lower increase of entropy during combustion. Harris et al. [2] and Ma et al. [3] studied the propulsive performance of PDEs and ramjet engines using system-level numerical analyses and it was demonstrated that PDEs could offer higher fuel-based specific impulses than ramjet engines. Compared with these existing engines, PDREs provide simpler mechanics, lighter weight, higher efficiency and larger thrust-to-weight ratios.

However, a method to utilise the potential capacity of PDEs in providing higher thrust has not yet been determined. One possible approach is to use an exhaust nozzle. Theoretically, an ideal exhaust nozzle shape exists for every combustor-to-ambient pressure ratio [4]. The principle of this design can be implemented in most propulsion systems in which these ratios have small variations, because most
thrust devices are steady. However, for PDREs, this nozzle theory does not apply because PDREs operate over a wide range of conditions produced at various cycle periods including refresh, detonation and blow-down, and each event occurs under various pressure conditions[5]. Detailed histories of the progress of this problem can be found in the literature. Tsuboi et al.[6] and Yan et al.[7] estimated the propulsive performance of PDEs with convergent-divergent exhaust nozzles numerically and experimentally, respectively. They found that the optimum values of $r_{th}/r_{tube}$ for $I_{sp}$ were 0.6 and 0.5 for $M=2.1$ and $M=3.5$, respectively. Wang et al.[8] and Peng et al.[9] experimentally investigated the thrust augmentation of air-breathing PDEs resulting from the addition of end nozzles and ejectors. PDREs with four types of exhaust nozzles were studied both experimentally and numerically by Kimura et al.[10]. For multi-tube PDREs, Mawid[11] developed a Pulse Detonation Engine Multi-tube Cycle Analysis and Performance Prediction code to predict the performances of multi-tube-based PDREs with individual divergent, convergent and convergent-divergent nozzles attached to each tube. Additionally, in an attempt to use the exhaust energy efficiently, J. L. Cambier et al.[12] studied the performance of a Pulse detonation Rocked engine(PDRE) induced by a magneto-hydrodynamics ejector. In addition to simple nozzles, Brophy et al.[5] made a preliminary attempt to study the effects of jet flow field and flight Mach number on the performance of a rectangular cross-section jet nozzle.

Generally, the propulsive performance of PDEs could be increased by using the traditional fixed structure nozzles. But in these situations, for most time the fixed structure nozzles did not work in the best working condition, under which the thermal energy of combustion products was turned into propulsive energy as much as possible by using nozzle. Therefore, a new kind of variable nozzles for unsteady Pulse Detonation Engines is needed to be studied.

The present work studies a Fluidic Control Boundary Nozzle (Fluidic nozzle) for PDRE. In this nozzle, a bit of annular subsonic jet flow is injected around the main supersonic reaction products. The injection flow could make a new fluidic boundary. Thus, the real boundary and real exit area of main supersonic fluid could be changed. In this way, the actual expansion ratio of main flow will be more close to the best one, which will raise PDRE’s thrust coefficient and will avoid pernicious shock wave of over-expansion. Besides, in this paper, the effects of nozzle shape on the real expansion ratio and on thrust coefficient of PDREs are analysed.

2. Theoretical formulation

The formulation is based on the axisymmetric two-dimensional conservation equations of mass, momentum and energy. The vector form of the inviscid part of the conservation laws can be rewritten as follows:

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = R - \alpha \frac{H}{y}$$

where,

$$U = [\phi_{g} \rho_{g}, \phi_{l} \rho_{l}, \phi_{g} \rho_{g} u_{g}, \phi_{l} \rho_{l} u_{l}, \phi_{g} \rho_{g} v_{g}, \phi_{l} \rho_{l} v_{l}, \phi_{g} \rho_{g} E_{g}, \phi_{l} \rho_{l} E_{l}]^T,$$

$$F = [\phi_{g} \rho_{g} u_{g}, \phi_{l} \rho_{l} u_{l}, \phi_{g} (\rho_{g} u_{g}^2 + p), \phi_{l} \rho_{l} u_{l} v_{l}, \phi_{g} (\rho_{g} u_{g}^2 + p), \phi_{l} \rho_{l} u_{l} v_{l}, \phi_{g} \rho_{g} E_{g}, \phi_{l} \rho_{l} E_{l} + p, \phi_{l} \rho_{l} (\rho_{l} E_{l} + p)]^T$$

$$G = [\phi_{g} \rho_{g} v_{g}, \phi_{l} \rho_{l} v_{l}, \phi_{g} \rho_{g} u_{g} v_{g}, \phi_{g} (\rho_{g} v_{g}^2 + p), \phi_{l} \rho_{l} u_{l} v_{l}, \phi_{g} (\rho_{g} v_{g}^2 + p), \phi_{l} \rho_{l} v_{l} \omega_{g} \rho_{g} E_{g} + p, \phi_{l} \rho_{l} v_{l} (\rho_{l} E_{l} + p)]^T$$

$$R = [I_{dx} - I_{dx} u_{i} - F_{dx}, I_{dy} - I_{dy} u_{i} - F_{dy}, -I_{dx} v_{i} + F_{dx}, -I_{dy} v_{i} + F_{dy},$$

$$-Q_{d} + Q_{c} - (F_{dx} u_{i} + F_{dy} v_{i}) + I_{dx}(E_{i} + p / \rho_{i}), Q_{d} + (F_{dx} u_{i} + F_{dy} v_{i}) - I_{dx}(E_{i} + p / \rho_{i})]^T$$

$$H = [\phi_{g} \rho_{g} u_{g}, \phi_{l} \rho_{l} u_{l}, \phi_{g} \rho_{g} u_{g}, \phi_{l} \rho_{l} v_{l}, \phi_{g} \rho_{g} u_{g}, \phi_{l} \rho_{l} v_{l}, \phi_{g} \rho_{g} u_{g}, \phi_{l} \rho_{l} v_{l}, \phi_{g} \rho_{g} E_{g}, \phi_{l} \rho_{l} E_{l} + p, \phi_{l} \rho_{l} v_{l} (\rho_{l} E_{l} + p)]^T.$$

3. Numerical method

3.1. Space-time CE/SE method using unstructured meshes
The new adaptable space-time CE/SE method based on unstructured meshes is used to evaluate the flow field and propulsive performance of PDRE equipped with fluidic nozzle. The CE/SE method was shown to have comparable accuracy with the MOL discretised by the 5th-order weighted non-oscillatory upwind scheme despite having a significantly shorter calculation time [13]. In the CE/SE method, the distributions of variables within the solution elements (such as planes AG'GA, A'E'EA, A'FFA and F'C'G'D'E'B' in Figure 1), are assumed to be continuous and can be approximated using Taylor expansions. While in the conservation elements (such as hexagonal cylinders AG'D'E'AGDE, A'E'B'F'AEBF and AF'C'G'AFCG in Figure 1), the flux integral evaluation requires no interpolation or extrapolation.

Figure 1. Definition of CEs and SEs of CE/SE method using unstructured meshes

When the Gauss’ divergence theorem is applied in the $E_j$-space, Eq. (1) is shown to be the differential form of the integral conservation law. Then it leads to the following result:

$$\left( U - \frac{\Delta t}{2} \frac{Q}{S} \right)^n_{J} = \frac{1}{S} \sum_{J} \left( \Sigma_{1} U + \Sigma_{2} U_{x} + \Sigma_{3} U_{y} \right)^{n-1/2}_{J}$$

In Eq. (2), $U^n_J$ is a function of $U^{n+1/2}_{x}$, $U^{n+1/2}_{y}$ and $(U_{i})^{n+1/2}_{J}$. Therefore, after $U^{n+1/2}_{J}$ is solved and available, $(U_{i})^{n+1/2}_{J}$ and $(U_{i})^{n+1/2}_{J}$ can be obtained using the oscillation-suppressing procedure.

3.2. Treatment of Source

In a PDRE system with fast chemistry, the characteristic chemical reaction time is smaller than the flow residence time by several orders of magnitude. Therefore, the Runge-Kutta algorithm is established to handle the stiff chemical source terms[14]. In other words, the integration of Eq. (1) to a new time step is divided into: (1) the space-time flux of the flow variables in the CE/SE manner to obtain $\left( U_{i} \right)^{n+1/2}_{J}$ and (2) the temporal integration of the source term over a special tailored conservation element using the previous $\left( U_{i} \right)^{n}_{J}$ as the initial value. The time step used in the Runge-Kutta integration is defined as $\Delta t = \frac{N \Delta t_{CESE}}{2N}$, where $N = 5 - 20$.

4. Results and discussion

4.1. PDRE modeling and boundary conditions

An axisymmetric model of a PDRE with fluidic nozzle is used in the simulation (as showing in Figure 2). The diameter and length of PDRE are 0.06 m and 0.7 m, respectively. The Laval nozzle is used as the basic carrier of fluidic nozzle. The length, inlet diameter, throat diameter and exit diameter of fluidic nozzle are 0.1m, 0.06m, 0.038m and 0.086m, respectively. The secondary flow in fluidic nozzle can be extracted from the flight environment. Then it could be accelerated or decelerated through the appropriate processing, such as supersonic inlet pipe. Concrete fluidic nozzle model
structure is shown in Figure 3. The distance of adjacent jets is constant. It is ensured that all jets are in accordance with the same incidence angle ($\alpha$), which is 90 degrees here. The width of all jets on the wall is set as 0.001m.

The simulation model of PDRE with a fluidic nozzle is divided into two stages. The first stage is the first filling stage. This stage begins at the initial time and ends when the gas volume fraction at nozzle exit decreases to the corresponding value under the condition of chemical equivalence ratio. During this stage, the stationary stoichiometric air/gasoline-droplet mixture at 293 K is filled from the head of PDRE, while the subsonic jet with a given parameter is injected from the divergent wall of nozzle. The second stage is the complete cycle of detonation, exhaust, isolation and filling stage when PDRE with fluidic nozzle works normally. In the second stage, the jet on nozzle wall enters incident mode automatically when the pressure in nozzle at the corresponding position is lower than the given total pressure.

Figure 2. Simulation model of PDRE with nozzle

Figure 3. Structure of fluidic nozzle

The initial conditions of model are as follows:

During the first stage, the initial gas pressure, temperature and Mach number in detonation tube and nozzle are 0.3MPa, 298 K and 0.3, respectively, while the initial gas pressure, temperature and Mach number in external flow field are 0.1MPa, 298 K and 0, respectively.

The boundary conditions of model are as follows:

The reflective slip boundary conditions and the axisymmetric boundary conditions are adopted for the wall and the symmetrical axis respectively. The jet on nozzle wall is given total temperature and total pressure boundary condition, which is calculated according to the selected parameters. Here the total pressure of jet 1, jet 2 and jet 3 is 0.167MPa, 0.125MPa, 0.104MPa, respectively.

4.2. Computational meshes

Unstructured triangular grids were used and proved to be convergent in this simulation with a minimum grid area of 1.73mm$^2$, as shown in Table 1. The solution time in the table refers to the time during which the detonation wave forms and propagates to the tube exit. Table 1 shows that the two mesh schemes have approximately the same detonation pressure peaks and propagation speeds, the relative errors of which are both less than 1%. The internal and external flow fields of the PDRE with different grid sizes exhibited the same detailed characteristics. Therefore, mesh Scheme-1 is adaptive for the detonation tube and is able to capture the important detonation parameters and complicated wave interaction patterns.

| Nozzle type | Mesh scheme | Grids number | Minimum grid area (mm$^2$) | Solution time (second) | Detonation pressure peak (MPa) | Detonation propagation speed (m/s) |
|-------------|-------------|--------------|----------------------------|-------------------------|-----------------------------|----------------------------------|
| Non-nozzle  | Scheme-1    | 55,161       | 1.73                       | 9,388                   | 2.02                        | 1,452                             |
|             | Scheme-2    | 85,899       | 1.31                       | 6,377                   | 2.04                        | 1,455                             |

4.3. Comparison of experimental and numerical results

To test the ability of the unstructured-mesh CE/SE method to capture the detonation wave, the PDRE which was of the same size and the same operation conditions with the numerical PDRE was experimentally studied and the results were compared (Figure 4). The simulated variation of pressure with time at the end of the PDRE tube agreed well with that obtained experimentally. With respect to
the detonation fracture surface, the strong gradient shown in the experimental results is well captured by the simulations using the unstructured-mesh CE/SE method.

Figure 4. The evolution of pressure with time at the end of the PDE detonation tube

4.4. Flow field analysis in PDRE with fluidic nozzle

4.4.1. Flow field in the first stage process. In order to analyze the optimization effect of fluidic nozzle, the corresponding Laval nozzle is simulated here for comparison. Figure 5 shows that the Mach number contours and streamline diagram inside and outside PDRE with Laval nozzle in the first filling stage. The arrow line in diagram is streamline. In the whole cycle of PDRE, the highest pressure in detonation tube appears in the detonation stage and early exhaust stage. In order to make full use of the internal energy of exhaust gas, the expansion ratio of the nozzle needs to take a larger value. So that, the high temperature gas can be fully expanded in these two stages. However, the nozzle which meets these requirements will have an over-expansion phenomenon in the following stages, as shown in Figure 5. The oblique shock wave by over-expansion can be captured in the divergent part of nozzle when the pressure at nozzle exit reduces. The supersonic flow in nozzle is interrupted by the emergence of this shock wave, and then decays to subsonic flow. This shock wave in nozzle obviously destroys the original intention of using nozzle to increase the thrust of engine, and is one of the worst cases of overexpansion.

Figure 5. Instantaneous Mach number contours of the PDRE with Laval nozzle in filling stage.

Figure 6. Instantaneous Mach number contours of PDRE with fluidic nozzle in the first stage.

Figure 6 shows the Mach number contours of the gasoline/air PDRE with fluidic nozzle at in the first stage, when the working state of the fluidic nozzle is basically stable. This fluidic nozzle in model is composed of above Laval nozzle and several jets on divergent wall. The flow phenomena in the diagram represent the working state of the fluidic nozzle at the later exhaust stage and filling stage. In these stages, the pressure in detonation tube decreases, which causes the pressure in nozzle inlet decreases. Without any other measures, it will lead to over-expansion in nozzle, which will reduce the thrust coefficient. The fluidic nozzle interferes with the flow by introducing a jet, forming a subsonic flow area near the nozzle wall, thus successfully reducing the actual effective exit area of nozzle. Therefore, the expansion ratio is reduced, and the thrust coefficient of the engine is increased. From
Figure 6, it can be seen that the streamline is no longer divergent, and the oblique shock disappears. The supersonic mainstream is close to the central axis because of the control of jet flow.

4.4.2. Flow field in the second stage process of PDRE with fluidic nozzle. Figure 7 shows the Mach number contours in the detonation, exhaust, isolation and filling stage (the second process of computational model) of a gasoline/air PDRE with a fluidic nozzle. The letter \( t_i \) stands for the moment of ignition. Figure 7(a) shows that PDRE enters the detonation stage and the detonation wave propagates to \( x=0.56 \)m. At this time, the supersonic mainstream near the central axis is still surrounded by the subsonic flow area. At \( t=t_i+0.66\)ms (Figure (b)), the detonation wave propagates to the divergent section of nozzle, and a curved shock surface appears outside the supersonic mainstream in nozzle. It means that the detonation wave has crossed the position of the wall jet, and the pressure in nozzle increases with the arrival of detonation wave. Therefore, the total pressure of jet is less than that in the nozzle. So, the jet can no longer be incident anymore. At \( t=t_i+0.73\)ms (Figure 7(c)), the detonation wave travels out of the nozzle and the engine enters exhaust stage. The pressure inside nozzle is greater than the total pressure of jet, and then the jet does not work in this process.

In the later exhaust stage (Figure 7(d)), the pressure in the detonation tube decreases. Then the pressure difference between the nozzle inlet and outlet decreases, and the over-expansion occurs slowly. Until \( t=t_i+2.96\)ms, the oblique shock wave caused by the over-expansion develops to the nozzle divergent part, and the thrust loss occurs greatly. At this time, the jet in the nozzle begins to work. By \( t=t_i+3.10\)ms (Fig. 6.5 (e)), the jet interference effect is basically realized. Under the condition of low total pressure at the nozzle inlet, the jets enter into nozzle and interfere with the overexpansion process. The annular control surface by subsonic jet is successfully formed. The fluidic nozzle again realizes the jet boundary working state as shown in Fig. 6.

(a) in the detonation stage, \( t=t_i+0.42\)ms,

(b) in the later detonation stage, \( t=t_i+0.66\)ms,

(c) in the earlier exhaust stage, \( t=t_i+0.73\)ms,
(d). in the later exhaust stage, \( t = t_i + 2.96 \text{ms} \),

(e). in the later exhaust stage, \( t = t_i + 3.10 \text{ms} \),

Figure 7. Instantaneous Mach number contours of PDRE with fluidic nozzle in the second stage.

4.5. The influence of nozzle type on fluidic nozzle

Figure 8 shows the Mach number contours of fluidic nozzle with different profiles. Figure 8 (a) is the fluidic nozzle with conical divergent part, while Figure 8(b) is the fluidic nozzle with bell-shaped one. The throat diameter, outlet diameter and jet aerodynamic parameters of the two nozzles are the same.

It is found that both of the two types of nozzles can successfully control the main flow. However, the supersonic mainstream obtained from the conical fluidic nozzle is closer to the central axis. The outer boundary of the mainstream is more flat, and the expansion angle is smaller. While the outer boundary of the mainstream is farther from the axis in bell-shaped fluidic nozzle, due to the jets on nozzle wall are located slightly away from the axis.

(a). fluidic nozzle with conical divergent part.  
(b). fluidic nozzle with bell-shaped divergent part.

Figure 8. Instantaneous Mach number contours of fluidic nozzle with different profiles.

For system analysis, the actual expansion ratio of each nozzle is calculated in Table 2. For comparison, the optimum expansion ratio under corresponding pressure and Mach number conditions is calculated by using the classical theory of steady flow nozzle, which is also list in Table 2.

\[
\frac{A_s}{A_t} = \frac{1}{M_e} \left[ \left( 1 + \frac{\gamma - 1}{2} M_e^2 \right) \left( \frac{2}{\gamma + 1} \right) \right]^{\frac{\gamma + 1}{2(\gamma - 1)}}
\]  

(3)

Comparing the data in Table 2, it is found that the expansion ratio of fluidic nozzle is closer to the optimum one at both the initial filling stage and the later filling stage. Conversely, the expansion ratio of a Laval nozzle with a fixed wall is far from the optimum one, which will cause a serious overexpansion in the nozzle. Comparing the conical fluidic nozzle with the bell fluidic nozzle, it is found that the expansion ratio of the conical one is closer to the optimum value than that of the bell one at both working conditions. Although the conical fluidic nozzle is not as good as the bell-shaped one in controlling the external interface of the main flow, its actual effect of controlling the exit cross-section area of nozzle is better.

Table 2. Comparing of actual expansion ratio of different nozzles

| Nozzle type       | Laval nozzle | Conical fluidic nozzle | Bell fluidic |
|-------------------|--------------|------------------------|--------------|
|                   |              |                        |              |

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### 5. Conclusion

In this study, the inner and outer flow field characteristics and propulsive performances of unsteady PDRE systems with fluidic nozzle are numerically evaluated. It is found that:

- The unstructured-mesh CE/SE used here is capable to solve flows with strong gradients in detonation tube and complicated wave interaction patterns in fluidic nozzle.

- In the later exhaust stage, when the Laval nozzle is used, overexpansion phenomenon such as oblique shock wave appears in nozzle, which seriously reduces the propulsion performance of PDRE. Conversely, the use of fluidic nozzle at these stages improves the situation. The jet introduced from nozzle wall interferes with the supersonic mainstream and by forming a new subsonic flow wall.

- In the whole cycle of PDRE, the fluidic nozzle can work in full flow at the initial stage of detonation and exhaust. The whole section of nozzle expansion is used for expansion and thrust augmentation. Correspondingly, in the later exhaust, isolation and filling stage, the fluidic nozzle can be automatically switched to the variable cross-section working mode, which improves the thrust coefficient of the engine obviously.

- The bell fluidic nozzle is good at controlling the outer interface of main supersonic flow, while the conical fluidic nozzle is good at perfecting the real expansion ratio of main flow.

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