Disorder Parameter of Confinement

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The disorder parameter of confinement-deconfinement phase transition based on the monopole action determined previously in $SU(2)$ QCD are investigated. We construct an operator which corresponds to the order parameter defined in the abelian Higgs model. The operator shows proper behaviors as the disorder parameter in the numerical simulations of finite temperature QCD.

1. Introduction

One of the challenging problems of QCD is to explain the confinement phenomena. The dual superconductor scenario of confinement [1, 2] and the related idea of abelian projection [3] have obtained convincing support in the numerical simulations of the lattice gauge theory [4–6]. In this scenario, the condensation of abelian monopoles is responsible for the confining force. These monopoles in the non-abelian gauge theory are defined after fixing the gauge freedom down to the maximal abelian subgroup [3]. In the compact $U(1)$ gauge theory the condensation of monopoles in the confinement (strong coupling) phase has been proved in various ways [7, 8]. Also in the non-abelian gauge theory, such picture has been shown from the viewpoint of the energy-entropy balance [9, 10]. But the disorder parameter of confinement in the non-abelian case has not been established yet. The promising attempts to prove it were made in [11, 12]. We propose a disorder parameter starting from a monopole-current $U(1)$ action which was derived numerically and so exactly from $SU(2)$ QCD [13].

The theory of monopoles in $SU(2)$ QCD is described by the partition function

$$Z = \sum_{*k \in \mathbb{Z}, \delta^* k = 0} \exp(-S[*k]).$$

where $*k$ is an integer-valued 1-form on a dual lattice, $S[*k]$ is a monopole action defined by the following relations:

$$\exp(-S[*k]) = \int \delta(*k - \delta^* n)e^{-S[U]}\delta(X^\pm)\Delta_{P,U}(U)DU$$

$$= \int DuDc\delta(*k - \delta^* n)e^{-S[u,c]}\delta(X^\pm)\Delta_{U}(U)$$

$$= \int Du\delta(*k - \delta^* n)e^{-S_{eff}[u]}.$$  

Here $\delta(X^\pm)$ is the maximally abelian (MA) gauge fixing condition and $\Delta_{P,U}(U)$ is the Faddeev-Popov ghost term. $\delta(*k - \delta^* n)$ gives a definition of the monopole current. The abelian link variables $u$, $c$ can be separated after fixing MA gauge:

$$U'(s, \mu) = V(s)U(s, \mu)V^\dagger(s + \mu) = c(s, \mu)u(s, \mu),$$

$$u(s, \mu) = \begin{pmatrix} e^{i\theta_\mu(s)} & 0 \\ 0 & e^{-i\theta_\mu(s)} \end{pmatrix},$$

$$\theta_{\mu\nu}(s) = \bar{\theta}_{\mu\nu}(s) + 2\pi n_{\mu\nu}(s), \quad (-\pi < \theta_{\mu\nu}(s) \leq \pi).$$

$n_{\mu\nu}$ is an integer-valued plaquette variable corresponding to Dirac string. By extending Swendsen’s method the monopole action $S[*k]$ has been determined in [13] in the form:

$$S[k_{\mu}'] = \sum_{s, s', \mu} k_{\mu}(s)\mathcal{F}(s, s')k_{\mu}(s').$$

where $\mathcal{F}$ is a functional of $U$ and $k_\mu$ is a monopole action.
This action is our starting point for studying the disorder parameter.

2. Disorder parameter in SU(2) QCD

The monopole currents action can be deformed to the (dual) abelian Higgs model action \[16,17\] under some approximation of \(S[\phi]k\). The order parameter in the abelian Higgs model on the lattice has been defined by Kennedy and King \[18\] in terms of gauge invariant correlation function. Correspondingly, we can define a disorder parameter of confinement in SU(2) QCD.

The monopole action determined in \[12\] has been fitted as follows:

\[ S[\phi]k = (\phi, D\phi) \text{,} \quad \text{(3)} \]

where

\[ D = m_0 + \alpha(b)\hat{\Delta}^{-1} \text{,} \quad \alpha(b) = \frac{1}{2} \left( \frac{4\pi}{g(b)} \right)^2 \text{,} \]

and \(\hat{\Delta}^{-1}\) is a modified Coulomb propagator \[12, 13\], \(b(= na)\) is a renormalized lattice spacing \((a\text{ and }n\text{ imply the lattice spacing and the monopole extendedness, respectively})\) \[12\]. \(g(b)\) is the running coupling constant of SU(2) QCD. Here we assume that \(\hat{\Delta}^{-1}\) can be replaced by the ordinary Coulomb propagator \(\Delta^{-1}\).\[1\] Then \(Z_{\text{mon}}\) can be represented as

\[ Z_{\text{mon}} = \int_{-\infty}^{\infty} D^*C \int_{-\pi}^{\pi} D^*\phi \sum_{l\in Z} \exp \left[ -\frac{1}{4\alpha}\|d^*C\|^2 - \frac{1}{4m_0}\|d^*\phi + 2\pi^*l - D^*C\|^2 \right] . \]

This is the partition function for the abelian Higgs model with non-compact dual gauge field \(*C\) and fixed length (dual) scalar field \(*\Phi = \exp(i*\phi)\). In \[13\] the order parameter for the abelian Higgs model has been introduced. In our notations the order parameter for the abelian Higgs model \[13\] reads: \(G^\infty \equiv \lim_{|x-y|\to\infty} G_{x,y}\), \(G_{x,y} \equiv \langle e^{i(\phi, \delta_x - \delta_y)} \cdot e^{-i(*C,h)} \rangle\).

\[ G_{x,y} = \frac{1}{Z_{\text{mon}}} \sum_{k(\epsilon_\gamma)\in Z, \delta, k=0} \exp \left[ -m_0\|*h\|^2 \right] \times \exp \left[ -(*k + *h - *\omega, D(*k + *h - *\omega)) \right] , \]

where \(*\omega\) is a string(\(\delta \omega = \delta_x - \delta_y\)) connecting \(x\) and \(y\), and \(*h\) is a smeared string satisfying the relation:

\[ \delta *h = \delta_x - \delta_y \text{.} \]

\(*h\) is not unique. Following \[13\], we choose

\[ *h = d\Delta^{-1}(\delta_x - \delta_y) \text{.} \]

Then \(G_{x,y}\) takes the form

\[ G_{x,y} = \frac{1}{Z_{\text{mon}}} \sum_{k(\epsilon_\gamma)\in Z, \delta, k=0} \exp \left[ -(*k - *\omega, D(*k - *\omega)) + \alpha(*h, \Delta^{-1}*h) \right] . \]

Note that the monopole currents, \(*k\), do not couple to the smeared string, \(*h\) In this representation it seems that \(*h\) is not important for the disorder parameter behavior. But it guarantees the invariance of \(G_{x,y}\) with respect to the dual \(U(1)\) gauge transformation \[13\].

3. Numerical simulations

The simulations have been done in finite temperature SU(2) QCD on \(16^3 \times 4\) and \(12^3 \times 4\) lattices. We have adopted \(2^3\) extended monopoles and the anti-periodic boundary conditions for space directions. Figure \[1\] shows our results for \(G_{x,y}\) as a function of \(\beta\). The distance \(|x-y|\) is taken to be 6 and 8 for \(12^3 \times 4\) and \(16^3 \times 4\) lattices respectively. One can see clearly that \(G_{x,y}\) is finite for both lattice sizes in the confinement phase(monopole currents condensation). While it is small in the deconfinement phase and it seems to go to zero in the \(|x-y|\to\infty\) limit.

4. Conclusions

We have proposed a new approach to evaluate disorder parameter in SU(2) QCD. Our definition shows numerically the typical behaviors as the disorder parameter of confinement. This
suggests abelian monopoles are condensed in the confinement phase, while they are not condensed in the deconfinement phase. It is worth noting that our definition of the disorder parameter corresponds to definitions used in 2d Ising model [20] and in 4d compact U(1) gauge model [10].

One can take other choices for $h$. For example we have taken also as $h$ a magnetic field, $B = d_3 \Delta_3^{-1}(\delta_x - \delta_y)$ where $d_3$ and $\Delta_3$ are 3d exterior differential and 3d laplacian which are defined on a given dual time slice. Then we obtained similar results.

In order to make more precise measurements of $G$ larger distances $|x - y|$ and consequently larger lattices are necessary. It is also important that the monopole currents action $S[k]$ is determined more precisely. This enables us to consider other effective theories derived from the current action, e.g. $Z$-gauge theory or abelian Higgs model are interesting. It is expected that in these models, the behaviors of disorder parameter are clearer.

This work is financially supported by JSPS Grant-in Aid for Scientific Research (B) (No.6452028).

One of us (V.B.) thanks the Local Organizing Committee and Russian Ministry of Science for partial support.

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