Coherent coupling of two semiconducting flux qubits

M Kurpas, J Dajka and E Zipper
Instytut Fizyki, Uniwersytet Śląski, 40-007 Katowice, Poland

Abstract. The entanglement of two distant two-level systems (qubits) built on semiconducting quantum rings is discussed. A mechanism of entangling the qubits by swapping is shown to lead to entanglement of subsystems which have never physically interacted. The numerical calculations of linear entropy being the entanglement measure are presented.

1. Introduction
The interaction of matter and light is one of the fundamental processes occurring in nature. Its most fundamental form is realized when a single atom interacts with a single photon. Recently it has been shown [1, 2] that the solid state two-level systems playing the role of an artificial atoms can be coherently coupled to microwave photons produced in the quantum cavities. The resulting entangled states of qubits with the modes of the electromagnetic field can be exploited in the emerging field of quantum computation and quantum information.

In this paper we show that these states can be used to entangle distant solid state qubits that have never physically interacted.

The scheme of entanglement swapping presented in Fig. 1 is the following [3]. The interaction of qubit $Q_1(Q_2)$ with an electromagnetic field mode $R_1(R_2)$ leads to the entangled state of the partner subsystems $(QR)_1$ and $(QR)_2$ respectively. The state of a pair $(QR)_1$ is factorisable from the state of a pair $(QR)_2$. If one then performs a Bell State Measurement (BSM) on the electromagnetic modes emerging from the resonators $R_1$ and $R_2$, the partner subsystems $Q_1$ and $Q_2$ will collapse to an entangled state although they never physically interacted.
entanglement by swapping has been discussed in recent years [3, 4, 5, 6, 7]. Among the solid state qubits the flux qubits controlled by the magnetic flux are very promising. They can be built on a superconducting mesoscopic ring interrupted by three in-line Josephson junctions [8] or by a thin constriction [9]. Recently a flux qubit based on semiconducting quantum ring with a controllable barrier has been proposed [10]. The basis states carry opposite persistent currents in the presence of static magnetic flux and are coupled by tunneling forming the two superposition qubit states. In the present paper we confine ourselves to semiconducting flux qubits.

Such qubit can be described in the pseudospin notation by the Hamiltonian:

$$H_Q = -\frac{1}{2} B_z \sigma_z - \frac{1}{2} B_x \sigma_x. \quad (1)$$

where

$$B_z = \Delta_0 \left( 1 - 2 \frac{\phi}{\phi_0} \right), \quad (2)$$

$$\Delta_0 = \frac{3\hbar^2}{2m_e}$$

is the quantum size energy gap, $\phi = \phi^{cl}$ is the classical magnetic flux threading the ring, $\phi_0 = \frac{\hbar}{e}$, $B_x$ describes the tunneling amplitude of an electron via a potential barrier.

We then place the qubit in a quantum resonator (cavity) described by the Hamiltonian

$$H_Q = \hbar \omega_R \left( a^\dagger a + \frac{1}{2} \right). \quad (3)$$

The magnetic flux $\phi$ entering the formula for $B_z$ will then have the quantum component $\phi^q$ which leads to the coupling of the QR subsystems

$$\phi = \phi^{cl} + \phi^q \quad (4)$$

$$\phi^q = \sqrt{\frac{\hbar \omega_R L}{2}} \left( a^\dagger + a \right), \quad (5)$$

where $L$ is the inductance of the cavity. The hamiltonian of the composed system after some algebra takes the form

$$H_{QR} = \frac{\hbar \omega_Q}{2} \sigma_z + \hbar \omega_R \left( a^\dagger a + \frac{1}{2} \right) - \hbar g \left( a + a^\dagger \right) [\sigma_x \sin \Theta - \sigma_z \cos \Theta] \quad (6)$$

with

$$g = \frac{\Delta_0}{\phi_0} \sqrt{\frac{\hbar \omega_R L}{2}}, \quad (7)$$

where $\frac{\Delta_0}{\phi_0} = I_0$, $I_0$ is the amplitude of persistent current, $\tan(\Theta) = \frac{B_c}{B_x}$, $\omega_Q$ is the qubit frequency. The coherent evolution of the system takes place at $t \ll T_Q, T_R$ where $T_Q \approx 1\mu s - 10\mu s$ [10] is the qubit decoherence time, $T_R \approx 3\mu s$ (we assume the quality factor of the cavity $\approx 10^6$) is the photon lifetime. In the following we discuss the system in the above time regime and neglect the decoherence effects.
2. Entanglement swapping scheme

To make use of the entanglement swapping one needs to have two pairs of entangled particles. In our consideration each pair is composed of a qubit and a photon mode in a quantum cavity. The qubit-resonator \((QR)\) \((i = 1, 2)\) system is described by a state vector \(|\psi_{(QR)}(t)\rangle_i\) which at \(t = 0\) is the factorisable state

\[
|\psi_{QR}(0)\rangle_i s = |\sigma\rangle_i \otimes |n\rangle_i,
\]

where \(\sigma\) represents the qubit pseudospin states \(\{e, g\}\), \(|n\rangle\) are the photon number eigenstates, forming the so called Fock basis, \(n = 0, 1, 2, \ldots 9\). The unitary evolution of the system generated by the Hamiltonian \((6)\) leads to the entangled states

\[
|\psi_{QR}(t)\rangle_1 = \sum_n [a_n(t)|gn\rangle_1 + b_n(t)|en\rangle_1]\]

\[
|\psi_{QR}(t)\rangle_2 = \sum_n [\tilde{a}_n(t)|gn\rangle_2 + \tilde{b}_n(t)|en\rangle_2]
\]

The two \(QR\) systems do not interact with each other and their state remains separable

\[
|\psi_{QR}(t)\rangle = |\psi_{QR}(t)\rangle_1 \otimes |\psi_{QR}(t)\rangle_2
\]

\[
\rho_{QR} = |\psi_{QR}(t)\rangle \langle \psi_{QR}(t)|.
\]

The BSM is performed on electromagnetic modes in Fock basis \([4]\) and projects the formerly independent qubits onto an entangled state described by \(\rho_{QQ}(t)\). As an example in this paper we discuss the projection onto the \(|\phi^-\rangle\) state,

\[
|\phi^-\rangle = \frac{1}{\sqrt{2}} (|0\rangle_1 |0\rangle_2 - |1\rangle_1 |1\rangle_2).
\]

\[
\rho_{QQ}(t) = Tr_R (|\phi^-\rangle \langle \phi^-| \rho_{QR}(t))
\]

The trace is taken with respect to photonic degrees of freedom. One of the entanglement measures which can be used in the discussed case is the linear entropy \(S_{L}(\rho)\) \([11]\).

\[
S_{L} := 1 - Tr (\rho_{red})^2 \in [0, \frac{1}{2}]
\]

where

\[
\rho_{red} = Tr_Q (\rho_{QQ}(t))
\]

is the reduced \(QQ\) density matrix. \(S_{L} = 0\) for disentangled state and increases to \(1/2\) with increasing the degree of entanglement.

3. Results

In this section we present numerical results for different values of parameters and for different initial states. The top parts of the figures show the dynamics of the \((QR)\) systems reflected in a time evolution of the linear entropies \(S_{L(QR)}\). The dashed line denotes the qubit-qubit linear entropy \(S_{LQQ}\). The bottom parts show the probabilities \(P_k\) of finding the \(QQ\) system in the \(k-th\) state after the BSM, \(k \in \{|ee\}, |eg\}, |ge\}, |gg\}\). Each point in the \(S_{LQQ}\) and \(P_k\) curves corresponds to the output of the BSM performed at the moment \(\omega_R t\). The resulting qubit-qubit state is the final state in our experiment, as we do not consider here the influence of the environment (decoherence processes) on the \(QQ\) state.

All values of \(\omega_Q\) and \(g_i\) are in the units of \(\omega_{R1} = \omega_{R2} = 50GHz\). All plots are made for the angle \(\theta = \frac{\pi}{4}\) for which the interaction terms with \(\sigma_x\) and \(\sigma_z\) have the same weights.
In Figure 2 we present the results for the resonant case when the systems are identical i.e. $g_1 = g_2 = 0.1$, $\omega_{Q_1} = \omega_{Q_2} = 1$ and the initial state is $|\psi_{QR} (0)\rangle = |e0\rangle_1 \otimes |e0\rangle_2$. We see that the curves are regular what is characteristic for the resonance. The $S_{LQQ}$ has the same periodicity as $S_{L(QR)_1}$, and the maximum of $S_{LQQ}$ almost coincides with the maximum of $S_{L(QR)_1}$. In the bottom part we see that only the probabilities $P_{ee}$ and $P_{gg}$ are substantial and oscillate between 0 and 1 whereas $P_{eg}$ and $P_{ge}$ are $\approx 0$ (not visible). One could read the plot of $S_{LQQ} (t)$ as an instruction when the BSM should be done to obtain a strongly entangled $QQ$ state.

![Figure 2. Entropies and probabilities at resonance, $g_1 = g_2 = 0.1$, $\omega_{Q_1} = \omega_{Q_2} = 1$. The initial state is $|\psi_{QR} (0)\rangle = |e0\rangle_1 \otimes |e0\rangle_2$.](image)

One might conjecture (taking as an example the entanglement swapping for entangled photons [3] or Figure 2) that when both $(QR)_i$ are maximally entangled the BSM results in a maximally entangled $QQ$ states. However it is not always true. Figure 3 shows the results out of resonance for $g_1 = g_2 = 0.2$, $\omega_{Q_1} = 0.8$, $\omega_{Q_2} = 1$. Comparing two points $\omega_R t \approx 16$ and $\omega_R t \approx 31$ we see that in both cases the $QR$ entanglement is strong but the $QQ$ system is almost maximally entangled only in the first case. The reason of such behaviour is the following. The $QR$ entropies are determined by the whole $QR$ Hilbert space (we take the dimension of the photon space $N=10$) and each of the states can influence the $S_{LQR}$. The BSM process cuts off most of the states and only those with 0 and 1 photon excitation states ($|e0\rangle, |e1\rangle, |g0\rangle, |g1\rangle$) determine the resulting $QQ$ density matrix and the $S_{LQQ}$ entropy. That is why the maximum of $S_{LQQ}$ does not overlap, in general, with the maximum of $S_{L(QR)_1}$.

The bottom part of Figure 3 shows the probabilities of the resulting $QQ$ states. We see that the maximum of the $S_{LQQ}$ is reached when the probabilities $P_{ee}$ and $P_{gg}$ are equal what means that those states are strong entangled. One can also see that the most probably states are $|ee\rangle$ and $|gg\rangle$ whereas $|eg\rangle$ and $|ge\rangle$ have probabilities almost zero. It is the characteristic feature of the projection onto $|\phi^-\rangle\langle\phi^-|$.

Figure 4 and 5 show $S_{LQQ}$ entropies and probabilities for the initial state $|\psi_{QR} (0)\rangle = |e0\rangle_1 \otimes |g1\rangle_2$. The top part of Figure 4 presents results for the systems at resonance for the same values of parameters as in Figure 2.

Comparing Figure 4 with Figure 2 we see that $QQ$ entanglement depends on the initial state. $S_{LQQ}$ looses its periodicity and one cannot again directly infer the $QQ$ entanglement from the $QR$ correlations. The qubit-qubit system is strongly entangled for most of the time what is reflected in the probabilities shown in the bottom part.

In Figure 5 we show the results for out of resonance case. We can deduce that detunning the
Figure 3. The top part shows $QR$ (solid line, crosses) and $QQ$ (dashed line) linear entropy. The systems parameters are $g_1 = g_2 = 0.2$, $\omega_{Q_1} = 0.8$, $\omega_{Q_2} = 1$, the initial state $|\psi_{QR}(0)\rangle = |e0\rangle_1 \otimes |e0\rangle_2$. The bottom part show the probabilities $P_k$ of finding the $QQ$ system at the $k$ state after the BSM.

two $QR$ subsystems reduces the time the $QQ$ system remains strongly entangled. It is also seen in the probabilities plot where the system exists mostly in the well defined state.

Figure 4. $QR$ (solid line, crosses) and $QQ$ (dashed line) linear entropy. The systems parameters are $g_1 = g_2 = 0.1$, $\omega_{Q_1} = \omega_{Q_2} = 1$, the initial state $\psi_{QR}(0) = |e0\rangle_1 \otimes |g1\rangle_2$.

4. Conclusions
In this paper we presented the scheme for entangling two flux qubits using entanglement swapping protocol. We have discussed the transfer of quantum information between systems having different physical nature and defined in Hilbert spaces of different dimensions. We have shown how the degree of entanglement depends on the initial states and on the system parameters. The qubit-resonator unitary evolution does not take into account decoherence processes and therefore is valid at time shorter than decoherence times which are of the order...
Figure 5. Results for the same initial state as in Figure 4. The system is out of resonance ($g_1 = g_2 = 0.1$, $\omega_{Q_1} = 0.9$, $\omega_{Q_2} = 1$).

of $\mu$s. The quantum nature of microwave electromagnetic field interacting with the mesoscopic solid state qubits allows to use them in the quantum information processing.

Acknowledgments
Work supported by the Polish Ministry of Science and Higher Education under the grant N 202 131 32/3786 and by the EU project RITA-CT-2003-0506095

References
[1] Blais A, Huang R-S, Wallraff A, Girvin S M and Schelkopf R J 2004 Phys. Rev. A 69 062320
[2] Wallraff A, Schuster D I, Blais A, Frunzio L, Huang R-S, Majer J, Kumar S, Girvin S M and Schoelkopf R J 2004 Nature 431 9 162
[3] Zukowski M, Zeilinger A, Horne M A and Ekert A 1993 Phys. Rev. Lett 71 4287
[4] de Riedmatten H, Marcikic I, van Houwelingen W, Tittel W, Zbinden H and Gisin N 2005 Phys. Rev. A. 71 050302
[5] Pan J-W, Bouwmeester D, Weinfurter H and Zeilinger A 1998 Phys. Rev. Lett 80 3891
[6] Moehring D L, Manuz P, Olmschenk S, Younge K C, Matsukevitch D N, Duan L-M and Monroe C 2007 Nature 449 6 68.
[7] Barrett S D and Kok P 2005 Phys. Rev. A. 71 060310
[8] Mooij J E, Orlando T P, Levitov L, Tian L, van der Wal C H and Lloyd S 1999 Science 285 1036
[9] Mooij J E and Harmans C J P M 2005 New J. Phys. 7 219
[10] Zipper E, Kurpas M, Szelag M, Dajka J and Szopa M 2006 Phys. Rev. B 74 125426
[11] Mintert F, Carvalho A R R, Kuś M and Buchleitner A 2005 Phys. Rep. 415 207