Multiresonator Quantum Memory with Single Atoms

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Quantum memory based on a system of resonators each containing one atom and connected to an external waveguide through a common resonator has been proposed. The parameters of the resonators and atoms interacting with them at which the effective transfer of a single-photon signal wave packet from the external waveguide to atomic states for long-term storage of the quantum state of the photon have been determined using reversible properties of the dynamics of the system under study and optimization methods. The advantages of quantum memory under consideration, as well as experimental possibilities of its implementation, have been discussed.

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INTRODUCTION

The development of quantum memory becomes of great interest because of the importance of its application to solve numerous problems of quantum processing and communications, including the creation of a universal quantum computer and a quantum repeater [1—3]. The practically significant long-term quantum memory should satisfy some requirements and there are reasons to believe that this memory will be created with existing quantum information carriers, e.g., rare-earth ions in inorganic crystals, where the record lifetime of spin coherence was reached; color centers in diamond, which can operate at room temperature; and quantum dots and walls well integrated in optical waveguides and resonators [1, 4]. At the same time, the creation of quantum memory requires significant improvement of methods of its implementation, e.g., those based on the use of methods to control slow light and photon/spin echo [5]. One of the most promising approaches to the creation of quantum memory schemes integrated in external devices implies the application of high-Q-factor resonators [6, 7] and systems of resonators [8, 9] to significantly enhance the interaction of atoms with photons. It was recently shown that a system of high-Q-factor resonators that has a given periodic structure of resonance frequencies similar to atomic frequency combs [10—12] opens possibilities for work with broadband signals, as demonstrated in experiments on the system of microwave resonators [8, 13]. Furthermore, it was shown [8, 14, 15] that the use of even several (four and more) resonators in a quantum memory cell can ensure almost 100% efficiency and a high accuracy in the work with broadband signal pulses with an arbitrary time shape. However, to significantly increase the lifetime, long-lived quantum information carriers, e.g., atomic ensembles [16], which have a long-term coherence of electron—nuclear spin transitions that can exist for seconds, minutes, and even hours [4], should be integrated in resonator quantum memory.

In this work, we propose a multiresonator quantum memory scheme, where each resonator contains one resonant atom. A resonator with an atom is considered as a promising elementary cell in the implementation of the coherent control of the quantum state of the atom and is actively used to develop single-resonator quantum memory [6, 17]. At the same time, as will be shown in this work, the use of the system of resonators each containing one atom and matched in the parameters makes it possible to significantly expand the working spectral range of quantum memory, to increase its efficiency, and to pass to storage of quantum states of single-photon wave packets with an arbitrary time shape without exact time synchronization of the external control laser field with the signal photon.
wave packet, which is typical of quantum memory based on the resonator with one atom [17].

THEORETICAL MODEL

Our quantum memory scheme is shown in Fig. 1. Quantum memory includes a common resonator connected to the external waveguide and to a system of eight high-$Q$-factor miniresonators each containing one atom.

Taking into account a long atomic coherence lifetime $\gamma_{\text{a}}^{-1}$ and a high $Q$ factor of the resonators, we neglect damping when describing the dynamics of the studied quantum memory at times $t \ll \gamma_{\text{a}}^{-1}$, $2Q/\omega_0$, $2Q/\omega_b$ ($Q$ is the proper $Q$ factor of resonators; $\omega_0$, $n = 1, \ldots, N$, are the frequencies of miniresonators; and $\omega_b$ is the frequency of the common resonator) and specify the Hamiltonian in frequency units in the form

$$H = \omega_0 a^+ a + \sum_{n=1}^{N} \omega_n b_n^+ b_n + \sum_{n=1}^{N} \omega_n S_n^+ S_n + V.$$  \hspace{1cm} (1)

Here, the interaction is taken in the rotating wave approximation

$$V = \sum_{n=1}^{N} f_n S_n^+ S_n + \sum_{n=1}^{N} g_n S_n^+ S_n + \sum_{n=1}^{N} g_{2n} S_n^+ S_n + \text{h.c.},$$ \hspace{1cm} (2)

where $f_n$ is the coupling constant of the $n$th atom with the mode of the $n$th miniresonator; $g$ and $g_{2n}$ are the coupling constants of the common temperature with the modes of the miniresonator and external waveguide; $a^+$, $a$, and $b_n^+$, $b_n$ are the Bose creation and annihilation operators of the modes of the common resonator and $n$th resonator, respectively; $a_{m,0}^+$ and $a_{m,0}$ are the Bose creation and annihilation operators of the $m$th mode of the waveguide (the subscripts $m = 1$ and 2 stand for the incident and reflected waves, respectively); and $S_n^+$, $S_n$, $S_{2n}^+$, $S_{2n}$ are the spin operators of the two-level atom.

Further, we study the storage and recovery of a single-photon wave packet whose initial quantum state has the form $|\psi(0)\rangle = \int d\omega \phi_0^*(\omega - \omega_0) e^{-i\omega t} |0\rangle$ with the normalization $\int d\omega |\phi_0(\omega - \omega_0)|^2 = 1$, where $|0\rangle$ is the ground state of the modes of the waveguide, resonators, and atoms. This state corresponds to a photon reaching the quantum memory cell at the time $t = 0$.

For the Hamiltonian of the system given by Eq. (1), the wavefunction has the form

$$|\psi(t)\rangle = \left[ a_c(t)a^+ + \sum_{n=1}^{N} x_n(t)b_n^+ + \sum_{n=1}^{N} s_n(t)S_n^+ \right] 0 \rangle + \int d\omega [c_{1,\omega}(t)a_{1,0}^+ + c_{2,\omega}(t)a_{2,0}^+] |0\rangle,$$ \hspace{1cm} (3)

where $a_c(t)$ and $x_n(t)$ are the excitation amplitudes of modes of the common resonator and $n$th miniresonator, respectively; and $s_n(t)$ and $c_{n,\omega}(t)$ are the excitation probability amplitudes of the $n$th atom and the $\omega$th mode of the waveguide.

Formula (3) gives the wavefunction for any number of atoms and describes the distribution of the excitation quantum over the degrees of freedom of the system under study with the conservation of the total probability $\{ |a_c(t)|^2 + \sum_{n=1}^{N} |x_n(t)|^2 + |s_n(t)|^2 \} + \int d\omega [|c_{1,\omega}(t)|^2 + |c_{2,\omega}(t)|^2] = 1$ and is not an approximate eigenfunction of the Hamiltonian specified by Eqs. (1) and (2). Passing to slow variables and using the standard quantum optics approach [18], we obtain the system of linear equations for the amplitudes:

$$[\partial_t + i\Delta_n] s_n(t) + i\gamma_n x_n(t) = 0,$$ \hspace{1cm} (4)

$$[\partial_t + i\Delta_n] x_n(t) + i\gamma_n s_n(t) + i\gamma_a(t) = 0,$$ \hspace{1cm} (5)

$$[\partial_t + k/2] a_c(t) + i\gamma \sum_{n=1}^{N} x_n(t) = \sqrt{k} A_{\text{in}}(t),$$ \hspace{1cm} (6)

where $k = 2\pi g_{2n}^2$ is the damping constant of the mode of the common resonator and $A_{\text{in}}(t) = \frac{-i}{\sqrt{2\pi}} \int d\omega f_0(\omega - \omega_0) e^{-i(\omega - \omega_0)t}$ is the input field in the waveguide, which is related as $A_{\text{in}}(t) + A_{\text{out}}(t) = \sqrt{k} a_c(t)$ to the excitation amplitude of the mode of the common resonator $a_c(t)$ and the output field $A_{\text{out}}(t)$ [14, 18].

As shown in [8, 14], the system of coupled resonators in the absence of atoms allows the efficient transfer of a broadband photon wave packet in the mode of miniresonators under the impedance matching condition $k = 2\pi g_{2n}^2/\Delta$, which corresponds to the practically achievable cooperation parameter $C =$.
$g^2/(k\Delta) = 1/2\pi$. In turn, radiation appearing in miniresonators will excite atoms, and the problem of creation of efficient quantum memory is reduced to the search for the parameters of the system under consideration that allow the complete transfer of the state of the signal photon to atoms. In this case, the condition $a_0(t) = A_{in}(t)/\sqrt{k}$ is valid for the stage of the complete transfer of the photon to atoms; as a result, in terms of the new variable $x_{n}(t) = ix_{n}(t)$, we obtain the equations of motion in the form

$$
[\partial_t + i\Delta_n]y_{n}(t) + f_n x_{n}(t) = 0, \quad (7)
$$

$$
[\partial_t + i\Delta_n]x_{n}(t) - f_n y_{n}(t) - ga_{n}(t) = 0, \quad (8)
$$

$$
\partial_t a_{n}(t) + g \sum_{n=1}^{N} x_{n}(t) = -\frac{1}{2} ka_{n}(t). \quad (9)
$$

At the photon unloading stage, $A_{in}(t) = 0$ and the system of equations describing the emission of the echo signal has the form

$$
[\partial_t + i\Delta_n]y_{n}(t) + f_n x_{n}(t) = 0, \quad (10)
$$

$$
[\partial_t + i\Delta_n]x_{n}(t) - f_n y_{n}(t) - ga_{n}(t) = 0, \quad (11)
$$

$$
\partial_t a_{echo}(t) + g \sum_{n=1}^{N} x_{n}(t) = -\frac{1}{2} ka_{echo}(t), \quad (12)
$$

where it is suggested that photon unloading begins at the time $t = t_0$ when only the atomic amplitudes $s_{n}(t_0)$ are nonzero.

Before the optimization of the parameters of the system, we note that the excitation amplitudes of the atoms and resonator modes satisfy the conditions $s_{n}(t) = s_{n}^*(t)$, $x_{n}(t) = x_{n}^*(t)$ at $\Delta_n = -\Delta_i$ and $\Re[a_{n}(t)] = 0$. After the time reversal $t \rightarrow -t'$ in Eqs. (10)–(12) and change of the variable $x_{n}(t) = -y_{n}(t')$, we arrive at the system of equations for the amplitudes $s_{n}^*(t)$, $y_{n}^*(t')$, and $a_{echo}^*(t')$. In the case $f_n = f_n^*$, this system coincides with Eqs. (7)–(9) for the variables $s_{n}(t)$, $x_{n}(t)$, and $a_{n}(t)$, which describe the dynamics of the interaction of the signal photon wave packet with the system of resonators and atoms. Imposing additional conditions on the atomic amplitudes $s_{n}(t_0) = s_{n}^*(t_0) = s_{n}(t_0)$ in the absence of excitation in resonators $s_{n}^*(t_0) = 0$ at the initial time $t = t_0$ of the photon unloading stage, we conclude that the emission of the echo signal is the time reversed copy of the absorption of the light pulse signal, in much the same way as the time-reversed quantum memory schemes based on the photon echo [19]. Below, we optimize the parameters and frequency characteristics of the studied system of resonators with atoms, which allows the quantum dynamics ensuring the efficient operation of quantum memory.

**OPTIMIZATION OF THE PARAMETERS OF THE QUANTUM MEMORY**

We performed the simulation of the system of eight miniresonators with atoms shown in Fig. 1 under the assumption that resonant frequencies of atoms are equal to those of miniresonators and constitute a symmetric comb $\Delta_n = [-7/2, -5/2, -3/2, -1/2, 1/2, 3/2, 5/2, 7/2]$, with the identical frequency interval between the nearest frequencies $\Delta = 1$. The equality of the frequency of the mode of the miniresonator to the resonant frequency of the atom can be ensured by implementing a Raman transition at the nonresonant interaction of the signal radiation in the three-level atom in the $\Lambda$ scheme of quantum transitions in the presence of the additional control laser field. The coincidence of the frequencies of the atom and resonator is ensured by choosing the frequency $\omega_{c, n}$ of the control laser field [19], which acts on the adjacent transition and satisfies the condition of the Raman resonance with the signal radiation $\omega_n - \omega_{c, n} = \omega_{21}$, which ensures the resonant transition between the two lowest long-lived atomic levels at the frequency $\omega_{21}$.

To optimize the parameters, we use the expression for the transfer function $S(\omega) = A_{out}(\omega)/A_{in}(\omega)$ relating the spectral components of the input and re-emitted fields, which is obtained from the solution of the system of equations (3)–(5):

$$
S(\omega) = \frac{(k - \gamma_0)/2 + i\omega + i \sum_{n=1}^{N} \frac{g^2}{B_n(\omega)}}{(k - \gamma_0)/2 - i\omega - i \sum_{n=1}^{N} \frac{g^2}{B_n(\omega)}}, \quad (13)
$$

where $B_n(\omega) = \Delta_n - \omega - i\gamma_n - \frac{f_n^2}{\Delta_n - \omega - \gamma_n}$ and $\gamma_s = 0$ and $n_0$ are the atomic coherence decay constants for the modes of the common resonator and the $n$th miniresonator, respectively.

As in [14], the process of optimization included two stages and was based on the search for the smoothed time delay $\tau(\omega) = -i\arg[S(\omega)]/\omega$ in the emission of the echo signal. At the first stage, the system of miniresonators without atoms ($f_n = 0$) operating in the fixed time delay mode [8] was optimized. With the determined parameters $g_{op}$ and $k_{op} = 2\pi g_{op}^2/\Delta$, the optimal value $f_{op}$ same for all miniresonators $f_n = f_{op}$ was determined at the second stage and all parameters were obtained:

$$
g_{op} = 1.4266, \quad k_{op} = 12.7781, \quad f_{op} = 0.2499. \quad (14)
$$

A small deviation of the relative time delay $\tau(\omega)/\tau(0)$ for the transfer function $S(\omega) = A_{out}(\omega)/A_{in}(\omega)$ within 3% in the working range of frequencies of quantum memory $\omega = [-2, 2]$ corresponds to a high accuracy and efficiency of quantum memory.
RECONSTRUCTION OF A PHOTON IN AN ECHO SIGNAL AND THE UNLOADING STAGE

Figure 2 shows the numerically calculated behavior of the signal photon wave packet having the Gaussian time mode \( f(\omega) = \exp(-\omega^2/(4\sigma^2)) \) with the half-width \( \sigma = 1 \) and the re-emitted photon wave packet, i.e., the echo signal in the output waveguide. As seen in Fig. 2, the echo signal has the form of a symmetric pulse with the maximum at the time \( t = 4\pi/\Delta \) with approximately the same Gaussian shape. Almost all energy stored in atoms is emitted in this pulse (the efficiency of emission was about 98%). We note that the introduction of atoms almost doubled the delay time \( \tau_{ra} \), more precisely, \( \tau_{ra} = 1.9992 \times 2\pi/\Delta \), whereas the delay time in the absence of atoms was \( \tau_r = 1.00015 \times 2\pi/\Delta \); i.e., \( \tau_{ra} \approx 2\tau_r \). This also means that the introduction of atoms into resonators almost halved the frequency interval between the neighboring frequencies of the comb of resonance lines of the total system including resonators and atoms. We recall that the used equations are valid at a sufficiently weak effect of decoherence; consequently, the decay constants should satisfy the condition \( \gamma_{eff} \ll 1/\tau_r \approx 0.01 \).

Since the two systems of Eqs. (4)–(6) and Eqs. (10)–(12) are reversible, it can be expected that the photon wave packet can be completely transferred to the system of atoms near the intermediate time \( t = \tau_{ra}(0)/2 \) in the presence of atoms. Below, we analyze this problem taking the initial conditions with the presence of single-atom excitation in the system of atoms and studying the possibility that atoms effectively emit a photon in the form of a wave packet in the same time mode into the waveguide. The effective emission of such a photon wave packet will mean a high probability of the transfer of the quantum state of the signal photon to the atomic state at the time \( t = \tau_{ra}(0)/2 \approx \tau_{r}(0) \).

The calculations of the unloading of the photon wave packet were performed with the atomic amplitudes at the initial time \( t_0 = 0 \) specified in the form of the distribution \( s_n(t) = [1, 3, 6, 8, 8, 6, 3, 1]/N_m \) (where \( N_m \) is the normalization factor) symmetric in the spectral detuning in the absence of excitation of modes of miniresonators and the common resonator. Figure 3 shows the calculated time behavior of the excitation probability of atoms, modes of resonators, and the waveguide and the probability density of finding the photon at the output of the common resonator \( J(t) = |A_{out}(t)|^2/\sum_{n=1,...,N} |s_n(0)|^2 \). The calculations confirmed a high emission probability of the symmetric photon wave packet corresponding to the echo signal in Fig. 2 calculated beginning with the photon loading stage to atoms. As expected, the photon emission period corresponds to half the period of the total cycle of the operation of quantum memory from the time of arrival of the photon wave packet to its complete re-emission to the waveguide. Thus, the independent calculations confirm the presence of the periodic structure of lines with the period \( \Delta/2 \). It is noteworthy that this structure of lines appears in the system of interacting resonators and atoms and, correspondingly, has a hybrid character in contrast to existing periodic frequency combs implemented in the quantum memory schemes based on the system of atoms [12], electron spins [19], and resonators [8]. Moreover, the atomic and resonator modes are differently involved in the operation of quantum memory in the process of cascade transfer of the photon through...
the resonator modes to atoms at a certain time; at the same time, the miniresonator modes are in the vacuum state.

As seen in Fig. 3, radiation is unloaded from atoms quite smoothly and there is a certain time interval $\delta t$ (about 1/20 of the total period) at the beginning of unloading (near $t = 0$ in Fig. 3) where almost all excitation in atoms can be transferred to their long-lived levels using the action of control laser pulses. The long-term storage of the photon would be impossible without such an interval. We note that the existence of the available time interval $\delta t$ is unique for the operation with Gaussian signal pulses, which is due to the frequency comb in the system of atoms and resonators, which exists not for all configurations of the interaction of atoms with resonators and not for all parameters of the system under study.

**DISCUSSION AND PROSPECTS**

To summarize, we have proposed a multiresonator quantum memory scheme with single atoms that makes it possible to transfer the quantum states of broadband photon wave packets to the atomic state for the long-term storage of quantum information. The optimization of the parameters has shown that the efficiency of such quantum memory can be increased to almost 100% by changing the initial frequencies of miniresonators even with a few miniresonators [14]. A significant advantage of the proposed quantum memory is the possibility of operation with an arbitrary time form of the photon wave packet, which simplifies operation compared to quantum memory based on the atom in a single resonator, which requires a strong time synchronization of the time modes of the signal and control fields. It is noteworthy that the operation of quantum memory has a cascade fractal character: the photon moving toward atoms and entering the memory cell first arrives at the common resonator, then its spectral components are separated through trajectories to different resonators, and then resonator modes transfer excitation to their atoms. The quantum state of the signal photon distributed over several atoms can be stored for a long time at a long lifetime of spin quantum coherence of atoms and can then be recovered on request following time-reversed dynamics. We note that the number of resonators can differ from eight but it should be no less than four in order to store the signal pulse with the Gaussian time mode profile [14]. In this case, eight resonators make it possible to store already two Gaussian time modes in each of two quartets of resonators. In addition, our calculations show that, in contrast to four resonators [14], the initial frequencies of the resonators in the scheme with eight resonators are not significantly shifted, which facilitates calculations and creation of this scheme.

It seems possible to apply the described quantum memory scheme to atoms and color centers in diamond, using the Raman scheme of atomic quantum transitions to spin levels having a long coherence time at room temperature [20] under a significant enhancement of the Purcell effect and the interaction of the photon with phononless optical transitions of centers in high-$Q$-factor miniresonators. The development of photon nanoresonators recently made it possible to significantly increase the coupling constant of resonant atoms with the mode of such resonators [21], which provides great possibilities in the implementation with single atoms.
Three-level quantum dots can be used as resonant atoms [22], where a large interaction constant of quantum dots with the resonator mode can be achieved [23] in high-$Q$-factor photon micro- and nanoresonators fabricated on semiconductor columns by means of the formation of Bragg gratings made of AlGa and GaAs layers. Such microresonators can be effectively integrated in general schemes and external devices [24]. An ultrahigh coupling constant of quantum dots with the miniresonator mode makes it possible to reach a large cooperation parameter $C = g^2/(\gamma k)$, which is a measure of the coherent interaction of a dot with the photon in a resonator, e.g., $C = 150$ in [25], where $\gamma/(2\pi) = 0.28$ GHz is the decay constant of optical coherence, $g/\gamma = 14$, and $g/k = 5.3$. Such parameters of the quantum dot and resonator satisfy requirements for the implementation of the proposed quantum memory based on the nonresonant Raman interaction of the photon with the quantum dot. Namely, the cooperation parameter for the atom in the resonator in our case is $C = f^2/(g\gamma_{\text{eff}}) = 4.5$ (at $\gamma_{\text{eff}} = 0.01$), which is much smaller than the achievable value $C = 150$ for quantum dots [25]. Correspondingly, the quantum dot allows one to ensure the necessary strength of the coherent interaction by choosing the optimal amplitude of control laser radiation in the nonresonant interaction [19]. Furthermore, it is noteworthy that quantum dots can have long-lived electron spin coherence with the decay constant $\gamma_e \ll \gamma$, which will significantly facilitate the implementation of quantum memory involving nonresonant Raman transitions. The authors of [26] mentioned that the lifetime of spin coherence in quantum dots can further be significantly increased by choosing optimal geometrical dimensions of the quantum dots, taking into account the features of spin interactions and the effect of the external magnetic field.

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