Simulation-Based Speed-Density Relationship Model of Highway Interchange

Yanwei Liu*
School of Highway, Chang’an University, Shaanxi, 710064, China
*Corresponding author’s e-mail: 767460224@qq.com

Abstract. To study the law of traffic wave transmission in highway interchange, this study analysed the traffic characteristics of the interchange section, developed an improved speed-density model of highway interchange based on 5PL logistic model by modifying the turning density and the mixing rate coefficient. The correction coefficients \( \Delta_1, \Delta_2 \) related to the traffic of the ramp were introduced and quantified by the nonlinear fitting and multivariate linear regression analysis of the simulation data under different traffic volumes combinations. The actual operation data of the interchange section was collected on site, and the difference between the calculated value and the actual value of the model was compared to verify the correctness and versatility of the model.

1. Introduction
Interchange is a key node of the highway network, which enhances the connection between the main lines. However, due to the conflicts between the inbound and outbound behaviors of the vehicles on the interchange, this will cause traffic flow disorder and reduce the traffic capacity of the segment, which makes the interchange become a bottleneck of the expressway[1]. By establishing the speed-density model of the interchange section suitable for the actual roads’ operation in China, the traffic operation law of the interchange section and the traffic efficiency will be clear. On this basis, the overall operation state of the road network and the location and range of the congestion can be determined or predicted. It assists the management department in planning effective emergency traffic management measures.

The macroscopic traffic flow theory regards the vehicle running on the road as a compressible continuous fluid medium, studies the overall behavior of the vehicle, and directly describes the internal relations between speed, density and flow, which is more effective to study the change law of traffic flow in the whole road network[2]. Some scholars summarized the traffic flow characteristic parameter relationship model and related application research, and studied the law of congestion spread dissipation based on traffic wave theory[3]. However, the focus of these studies is on the basic sections of expressways, and the impact of diverging and converging traffic near the interchanges is not considered in their theoretical model. The research on traffic flow in interchange sections pays more attention to the micro-behavioral characteristics such as vehicle lane change and follow-up. Based on the microscopic traffic flow theory, Ngoduy et al.[4] proposed a multi-lane continuous traffic flow model under the influence factors such as the traffic of the ramp, the length of the ramp, and the driver’s acceptable minimum clearance of vehicles. Wang T[5] studied the characteristics of crowded traffic flow near the entrance and exit ramps, and established a two-lane lattice hydrodynamic model. Wang L[6] studied the entrance and exit ramp characteristics and the advantages of microscopic traffic simulation to analyze the relationship between vehicle speed and highway capacity. However, the micro-model needs to set much more parameters, which is also greatly affected by individual driver behavior,
application range is relatively limited. It is also difficult to find the connection relationship with the macro-model of the basic road segment.

This paper adopts the macroscopic traffic flow--logistic speed-density relation model as the theoretical basis, considers the influence of the entrance and exit of the highway interchange section, and establishes a relationship model between the speed and density that characterizes the overall state of the traffic flow. When constructing the model, it is necessary to adjust the main line traffic volume, export traffic volume, and entrance traffic volume. If we collected the actual interchange traffic data, it is a huge workload and the range of data is uncontrollable. Therefore, various traffic combinations are simulating under VISSIM traffic simulation application. On this basis, the speed-density relationship model of highway interchange sections is established, which is in accordance with China's traffic conditions. The accuracy and validity of the model are verified by collecting and analyzing the actual road operation data, and the influence of the entrance and exit ramps on the main line traffic flow characteristics is summarized.

2. Logistic Speed-Density Model of Highway Interchange

2.1. Generalized logistic speed-density model

The early speed-density models are single-segment structures, such as the Greendshields model[7], the Greenberg model[8], and the Underwood model[9], which do not fit well with empirical data in free-flow states and crowded flow states. The mathematical expression of the multi-segment structure model that is later established requires a large amount of data to determine the state boundaries. The Logistic model was originally proposed for the simulation of population growth, and was widely used in various fields of research because of its refined and concise mathematical expressions[10-11]. The 5PL logistic model’s performance in terms of relative error and fitting effect on the measured data is much better than the single-segment structure model[12].

Wang H[13] defined the traffic flow meaning of some 5PL logistic model parameters in his research, and proposed the use of Logistic curves to establish the speed-density relationship. Liu[14] further clarified the meaning of the rest parameters by collecting data of China's traffic flow, and established a generalized Logistic speed-density relationship model in line with China's actual traffic conditions, such as equation (1).

\[ V = v_b + \frac{v_f - v_b}{1 + \exp\left(\frac{k - k_i}{b}\right)} \]  

(1)

Where \( V \) is the speed (km/h); \( k \) is density (pcu/km/lane); \( v_f \) is free flow speed; \( v_b \) is the average speed when the vehicle is in the stop and go state; \( k_i \) is the turning density of the traffic flow from free flow to crowded flow; \( b \) is mixing rate coefficient, dimensionless parameter related to the mixing rate of heavy vehicles; \( g \) is a dimensionless parameter related to traffic density in crowded flows.

However, the driving behaviour of vehicles near the interchange section is complicated. Drivers constantly searches for the insertion gap, which causes the traffic on the upstream and downstream of the entrance and exit to be disturbed. Moreover, the loading and unloading effect of the entrance and exit has a significant influence on the background traffic volume, and the original shape of the traffic flow wave is changed. The speed-density model proposed by Liu[14] does not consider the influence of the interchange segment, so the logistic model shown in equation (2) may not be suitable for interchange sections. However, since the logistic model established, many scholars have found that the macroscopic traffic flow is consistent with the logistic relationship through the actual observation of the road traffic state. The research by Xu[15] shows that the logistic model is suitable for various types of data sets and has strong stability. Therefore, it can be considered that the traffic flow of the interchange section still satisfies the logistic curve, and the influence of the entrance and exit is mainly reflected in the change of the model parameter value.
2.2. Speed-Density Model of Highway Interchange

In order to investigate how the traffic in and out of the transition affects the logistic curve, the influence of each parameter of the model on the function curve should be clarified. An intuitive change of the function image under different parameter values is shown in Figure 1. The parameters $v_f, v_b$ respectively control the position of the upper and lower progressive lines of the curve; $b$ controls the rapidity of the curve transition between the asymptotes; the parameter $k_t$ controls the position of the transition region; and the parameter $g$ controls the rate close to the bottom asymptote.

![Figure 1. The effect of different parameters on the logistic curve.](image)

The two traffic extremes are described at both ends of the Logistic curve. The traffic on the road is very small and drivers are completely free; the traffic is extremely large, and the driving environment is extremely limited. In these two states, the influence of the combined traffic of the ramps has been small and can be ignored. Therefore, the logistic speed curve of the interchange section is mainly reflected in the left or right deviation of the middle section of the curve. That is, the change of the mixing ratio parameter $b$ and the transition density $k_t$, and the parameters $v_f, v_b, g$ which characterize the free flow and the crowded flow state don’t change. The correction model is shown in equation (2).

$$V = v_b + \frac{v_f - v_b}{[1 + \exp\left(\frac{k - \Delta k}{\Delta b}\right)]^g}$$

(2)

$\Delta_1, \Delta_2$ are correction coefficients, and other parameters have the same meaning with equation (1). In the study of Liu[14], when the vehicle mixing rate is 0, $v_f = 81.8 km/h$, $v_b = 0 km/h$, $k_t = 33 veh/km/lanes$, $b = 5.402$, $g = 0.2562$. The difference between the highway interchange section and the basic section is the influence of the combined traffic on the ramp. It is reasonable to assume that $\Delta_1, \Delta_2$ are the parameters related to the traffic volume $\sigma_1$ and the traffic volume $\sigma_2$, ie $\Delta_1 = f_1(\sigma_1, \sigma_2)$, $\Delta_2 = f_2(\sigma_1, \sigma_2)$. It should be noted that the logistic speed model of the interchange section established in this paper mainly considers the influence of the vehicle splitting under normal driving conditions. It is considered that all drivers follow the general traffic rules, have good driving habits. The model does not apply to individual drivers who drive improperly near the entrance and exit, such as emergency lane changes, parking, and reverse traffic.
3. Interchange simulation test

3.1. Interchange analysis unit
In order to establish a simulation model that can accurately simulate the driving state of vehicles on the interchange, this paper designs an interchange analysis unit including the entrance and exit ramps, as shown in Figure 2. The diversion area and the confluence area are respectively affected by the exit and the entrance ramp, and the main line traffic is disturbed; the basic section is the area where the vehicle is not affected by the vehicle lane-change and deceleration. There is no traffic bottleneck effect and traffic conflicts caused by the diversion, and it only assume the function of introducing and exporting vehicles. The expressway can be regarded as a connector of several interchange units and basic road sections. By studying the propagation law of traffic waves on one unit, the propagation law and radiation range of traffic waves in the highway network can be derived. According to the American Capacity Manual, vehicles are affected by the lane change in the 450m range upstream of the exit ramp and downstream of the entrance ramp. Considering the convenience of calculation and model establishment, the section between 500m before the expressway exit and 500m after the entrance ramp is taken as an interchange unit within the error tolerance.

![Figure 2. Interchange analysis unit.](image)

3.2. Vissim traffic simulation design
In order to study the influence of the traffic volume on the traffic flow parameters of the main line, it is necessary to select different combinations of entering and exiting traffic to carry out simulation experiments. Due to the different traffic capacity of the expressway, if the simulation uses the absolute quantity as the input value, and set different volume combinations, the number of experiments is greatly increased. Therefore, the relative value of entering and exiting traffic volume is selected. That is, the ratio of the entering volume and the exit volume to the main traffic volume. The default entering ratio $\sigma_1$ and exiting ratio $\sigma_2$ are 0.1, 0.2, 0.3, 0.4, and 0.5. Select 10 sets of restricted flow state data under the heavy vehicle mixing rate of 0% in Table 1 as the simulation input parameters. For the same main line traffic input, there will be 25 different combinations of entering and exiting traffic. To improve the credibility of the data and eliminate the accidental error, the simulation is carried out after 5 times with different random seeds under the same input parameters.

| NO. | Density | Speed | Volume | Entering | Exiting |
|-----|---------|-------|--------|----------|---------|
| 1   | 33      | 81.8  | 2699   | 0.1      | 0.1     |
| 2   | 37      | 61.2  | 2264   | 0.2      | 0.2     |
| 3   | 41      | 53.2  | 2180   | 0.3      | 0.3     |
| 4   | 45      | 45.2  | 2036   | 0.4      | 0.4     |
| 5   | 49      | 38.0  | 1863   | 0.5      | 0.5     |
| 6   | 53      | 31.7  | 1681   | 0.1      | 0.1     |
| 7   | 58      | 25.2  | 1460   | 0.2      | 0.2     |
| 8   | 62      | 20.9  | 1295   | 0.3      | 0.3     |
| 9   | 66      | 17.3  | 1143   | 0.4      | 0.4     |
| 10  | 70      | 14.3  | 1004   | 0.5      | 0.5     |

Table 1. Simulation experiment input data.

| out | in   | $\Delta_1$ | $\Delta_2$ | $R^2$ | $\Delta_1$ | $\Delta_2$ | $R^2$ | $\Delta_1$ | $\Delta_2$ | $R^2$ | $\Delta_1$ | $\Delta_2$ | $R^2$ | $\Delta_1$ | $\Delta_2$ | $R^2$ |
|-----|------|------------|------------|-------|------------|------------|-------|------------|------------|-------|------------|------------|-------|------------|------------|-------|
| 0.1 | 1.06 | 0.96       | 0.96       | 1.14  | 1.05       | 0.95       | 1.24  | 1.13       | 0.96       | 1.32  | 1.26       | 0.95       | 1.43  | 1.35       | 0.95       |
| 0.2 | 0.96 | 0.84       | 0.96       | 1.03  | 0.96       | 0.95       | 1.14  | 1.03       | 0.95       | 1.23  | 1.13       | 0.95       | 1.33  | 1.24       | 0.95       |
| 0.3 | 0.84 | 0.74       | 0.95       | 0.92  | 0.86       | 0.94       | 1.03  | 0.92       | 0.95       | 1.13  | 1.01       | 0.95       | 1.22  | 1.14       | 0.95       |
| 0.4 | 0.73 | 0.64       | 0.96       | 0.81  | 0.74       | 0.95       | 0.91  | 0.82       | 0.95       | 1.02  | 0.90       | 0.95       | 1.11  | 1.02       | 0.95       |
| 0.5 | 0.63 | 0.53       | 0.95       | 0.72  | 0.63       | 0.95       | 0.82  | 0.71       | 0.95       | 0.93  | 0.79       | 0.95       | 1.01  | 0.91       | 0.95       |

Table 2. Fit values under different entering and exiting traffic.
4. Model parameter quantification

The logistic nonlinear fitting was performed on 10 sets of data of different main line background traffic under different combinations of import and exit ratios. The best values for 25 groups of $\Delta_1$ and $\Delta_2$ were obtained with 95% confidence is in Table 2.

In order to study the variation law of the modified parameters under the influence of a single independent variable, the entering and exiting ratios are controlled separately. The relationship between the modified parameters and another variable is plotted, as shown in Figure 3 and 4. It can be observed that it is an obvious linear function that satisfies the relationship $\Delta_1 = a_1\sigma_1 + b_1\sigma_2 + c_1$, $\Delta_2 = a_2\sigma_1 + b_2\sigma_2 + c_2$. The functional equations were determined using linear regression and the results are shown in Table 3.

| variable | $\Delta_1 - \sigma_1, \sigma_2$ | $\Delta_2 - \sigma_1, \sigma_2$ |
|----------|-------------------------------|-------------------------------|
| function | $-1.05\sigma_1 + 0.95\sigma_2 + 1.06$ | $-1.08\sigma_1 + 0.95\sigma_2 + 0.97$ |
| SSE      | 0.0015                        | 0.0033                        |
| $R^2$    | 0.9985                        | 0.9969                        |
| Adj-$R^2$| 0.9984                        | 0.9966                        |
| RMSE     | 0.0083                        | 0.0122                        |

Figure 3. $\Delta_1, \Delta_2$ changes with the entering ratio. Figure 4. $\Delta_1, \Delta_2$ changes with the exiting ratio.

According to the above analysis results, the logistic speed-density model of the interchange segment of equation (3) can be obtained, and the parameters have the same meaning as before. To verify the accuracy and applicability of the model, this paper selects the Xi’an Taiyi Palace interchange as an experimental point and multiple sampling periods were selected for data collection. Substituting multiple sets of actual traffic data into the model, the function output values are shown in Table 4, and the comparison chart is shown in Figure 5.

\[
V = v_k + \frac{v_f - v_k}{[1 + \exp\left(\frac{k - \Delta_1}{\Delta_2 b}\right)]^{\frac{1}{e}}} \begin{cases} 
\Delta_1 = -1.05\sigma_1 + 0.95\sigma_2 + 1.06 \\
\Delta_2 = -1.08\sigma_1 + 0.95\sigma_2 + 0.97
\end{cases}
\]  

(3)
Table 4. Measured real data & model calculation data

| Exiting Ratio | Entering Ratio | Real Density | Real Speed | Cal. Speed | Dif. |
|---------------|----------------|--------------|------------|------------|------|
| 0.14          | 0.42           | 65.0         | 40.55      | 35.31      | 13%  |
| 0.11          | 0.35           | 65.9         | 35.24      | 31.90      | -9%  |
| 0.31          | 0.10           | 33.7         | 45.36      | 51.77      | 14%  |
| 0.3           | 0.24           | 15.8         | 93.10      | 81.09      | 13%  |
| 0.06          | 0.17           | 18           | 71.8       | 81.13      | 13%  |
| 0.23          | 0.15           | 25.2         | 86.78      | 77.08      | 11%  |

Figure 5. Comparison of Real & Model value.

It can be seen that the difference between the output value and the measured value of the logistic model of the interchange segment established in this paper is small, which proves the versatility and accuracy of the logistic fast-seam model based on the VISSIM simulation data.

5. Conclusion

Starting from the macroscopic traffic flow theory, this paper establishes the simulation environment through VISSIM. Through adjusting the entering and exiting traffic volume combination of the interchange, we obtain the simulation data under different traffic conditions. In this paper, multiple linear regression is used to analyse and process the data, and the speed-density model of the interchange is established. However, the logistic speed-density model still has certain limitations. First, the logistic macroscopic traffic flow model cannot effectively analyse the highly discrete speed and density data in the free-flow state. Secondly, the model ignores the time variable and cannot deal with real-time traffic problems. Therefore, future research will focus on two aspects: 1) the relationship between the model and microscopic traffic behaviour; 2) consider the characteristics of the road network, to promote the scope of application of the model.

References

[1] Song, H., Zhang, W., Yang, X. (2012) Capacity Research on Cloverleaf Interchange by Using Simulation Loading Method. In: 12th International Conference of Transportation Professionals. Beijing. pp. 707-17.
[2] Wu, Q., Jiang, R., Li, X., et al. (2005) Combing Macroscopic Method with Microscopic Method and Promoting New Development of Traffic Flow Theory. Communication and Transportation Systems Engineering and Information, 3: 108-115+126.
[3] Zhang, X. (2014) FCD-based Identification Method for Urban Recurrent Traffic Congestions. Journal of Transport Information and Safety, 32(01): 5-9+15.
[4] Ngoduy, D., Hoogendoorn, S.P., Van Zuylen, H.J. (2006) Continuum Traffic Model for Freeway with On-And Off-Ramp to Explain Different Traffic-Congested States. Transportation research record, 1965(1): 90-102.
[5] Wang, T., Zhang, J., Li, S., et al. (2018) Dynamic Congested Traffic States of Lattice Hydrodynamic Model with An On-Ramp and Its Immediate Upstream Off-Ramp. International Journal of Modern Physics B, 32(29): 1850325.
[6] Wang, L., Wang, Y., Wang, L.Y. (2013) Simulation Research of Flow and Speed on Urban Expressway Interchange Entrance. Materials Transportation and Environmental Engineering, 779: 796-799.
[7] Greenshields, B.D. (1935) in Proceedings of the Highway Research Board. Highway Research Board, 14: 448.
[8] Greenberg, H. (1959) An Analysis of Traffic Flow. Operations research, 7(1): 79-85.
[9] Underwood, R.T. (1961) Speed, Volume, and Density Relationship: Quality and Theory of Traffic
Flow. Yale bureau of highway traffic, 141–188.

[10] Verhulst, P.F. (1847) Deuxième mémoire sur la loi d'accroissement de la population. MEMOIRES de l'académie Royale des sciences, des lettres et des beaux-arts de Belgique, 20:1-32.

[11] Gottschalk, P.G., Dunn J.R. (2005) The Five-Parameter Logistic: A Characterization and Comparison with The Four-Parameter Logistic. Analytical Biochemistry, 343(1): 54–65.

[12] Lee H.Y., Lee H.W., Kim, D. (1998) Origin of Synchronized Traffic Flow on Highways and its Dynamic Phase Transitions. Physical Review Letters, 81(5): 1130-3.

[13] Wang, H.Z., et al. (2011) Logistic Modeling of The Equilibrium Speed–Density Relationship. Transportation research part A: policy and practice, 45(6): 554-566.

[14] Liu, X., Xu, J., Li, M., et al. (2019) General-Logistic-Based Speed-Density Relationship Model Incorporating the Effect of Heavy Vehicles. Mathematical Problems in Engineering.

[15] Xu, C., Chen, X. (2014) Characteristic Analysis of Traffic Flow Velocity-Density Model. Journal of Highway and Transportation Research and Development, 2: 114-120.