SAR Image Despeckling Using Quadratic-Linear Approximated $\ell_1$-Norm

Fatih Nar

Speckle noise, inherent in synthetic aperture radar (SAR) images, degrades the performance of the various SAR image analysis tasks. Thus, speckle noise reduction is a critical preprocessing step for smoothing homogeneous regions while preserving details. This letter proposes a variational despeckling approach where $\ell_1$-norm total variation regularization term is approximated in a quadratic and linear manner to increase accuracy while decreasing the computation time. Despeckling performance and computational efficiency of the proposed method are shown using synthetic and real-world SAR images.

Proposed method: Speckle reduction for the SAR image is defined as the minimization of the following variational cost function:

$$J(f) = \frac{1}{2N} \sum_{p=1}^{N} (f_p - g_p)^2 + \ell(\nabla f)_p \right|$$

where $g$ is observed speckled image, $f$ is the desired despeckled image, $N$ is the pixel count, $p$ is the pixel index number, $\ell$ is a positive value determining smoothing level, and $\nabla$ is the gradient operator. In the cost function, the data fidelity term ensures $f$ stays similar to $g$ in $\ell_2$-norm manner and total variation (TV) regularization term implies penalty on the changes in image gradients in $\ell_1$-norm manner.

Although $\ell_1$-norm TV regularization preserves details, its efficient minimization is difficult since it is not differentiable. SDD [2] proposed to approximate the non-differentiable term quadratically as below:

$$\|z\| \approx (|z| + \epsilon)^{-1/2} \approx (|z| + \epsilon)$$

where $\epsilon$ is a small constant positive number. Accuracy of the approximation increases as $\epsilon$ gets closer to 0. In this study, this quadratic approximation is further improved by combining it with a linear approximation as given in equation (3).

$$|z| \approx (1 - \alpha)(|z| + \epsilon)^{-1/2} + \text{sgn}(z)\epsilon$$

where $0 \leq \alpha \leq 1$, $\text{sgn}(.)$ is the sign function, and $\text{sgn}(z)$ is the linear approximation of $|z|$. Equation (3) is convex combination of quadratic and linear approximations and is accurate around $z$ (see Fig. 1). As $z$ goes to 0, linear term vanishes and quadratic-linear (QL) approximation becomes quadratic around 0 which also avoids staircase artifacts.

If we define $|\nabla f|_p$ as $|(\partial_x f)|_p + |(\partial_y f)|_p$ for a 2D SAR image and use the QL approximation given in equation (3) then the cost function in equation (1) becomes as below:

$$J^{(n)}(f) = \frac{1}{2N} \sum_{p=1}^{N} (f_p - g_p)^2 + (f_p - \hat{f}_p)^2$$

$$+ \lambda[(1 - \alpha)(w_{s,p}(\partial_x f)_p^2 + w_{p,y}(\partial_y f)_p^2)]$$

$$+ \alpha(s_{x,p}(\partial_x f) + s_{y,p,y}(\partial_y f)_p))$$

where $n$ is the iteration number, $\hat{f}_p$ is a proxy constant for $f_p$, $(f_p - \hat{f}_p)^2$ is a new regularization term for forcing $f_p$ stays close to $\hat{f}_p$ since QL approximation is only accurate around $\hat{f}_p$. $w_{s,p}$ and $s_{x,p}$ are defined correspondingly. Superscript $n$ in $J^{(n)}(f)$ shows that cost function must be minimized in an iterative manner due to employed QL approximation.

Equation (4) can be represented in matrix-vector form as below:

$$\tilde{J}^{(n)}(f) = \frac{1}{2N} (\vec{v}_f - \vec{v}_g)^\top (\vec{v}_f - \vec{v}_g) + \lambda(1 - \alpha)(\vec{v}_f^\top C_\gamma^2 \vec{w}_{s,f} + \vec{v}_f^\top C_\gamma^2 \vec{w}_{s,g} + \vec{v}_f^\top C_\gamma^2 \vec{w}_{s,f} + \vec{v}_f^\top C_\gamma^2 \vec{w}_{s,g} + \vec{v}_f^\top C_\gamma^2 \vec{v}_f)$$

where $\vec{v}_g$, $\vec{v}_f$, $\vec{v}_f$, $\vec{s}_s$, $\vec{s}_p$ are vector forms of $g_p$, $f_p$, $f_p$, $s_{s,p}$, $s_{p,y}$, and $\vec{w}_{s,f}$, $\vec{w}_{s,g}$ are diagonal matrix form of $w_{s,p}$, $w_{s,g}$, and $C_\gamma^2$ are the Toeplitz matrices as the forward difference gradient operators where derivatives are zero at the right and bottom boundaries respectively.

Equation (5) is strictly convex and differentiable; thus, one can take its derivative with respect to $\vec{v}_f$ and equalize it to zero to obtain its minimum which leads to a linear system as given below:

$$A^{(n+1)} f = b$$

where $A = 2I + (1 - \alpha)(C_\gamma^2 w_{s,f} + C_\gamma^2 w_{s,g})$, $I$ is identity matrix, and $b = -\vec{v}_g + \vec{v}_f - \lambda(1/2)(\vec{s}_s + C_\gamma^2 \vec{s}_p)$. Iteration number $n$ for the $A$, $\vec{w}_{s,f}$, $\vec{w}_{s,g}$, $\vec{v}_f$, $\vec{v}_f$, $\vec{s}_s$, and $\vec{s}_p$ is only explicitly stated. Pseudo-code of the proposed method is given in algorithm 1 where implementation of all the steps are easy and computationally cheap, except for solving the linear system in line 11. To obtain computational efficiency in line 11, preconditioned conjugate gradient (PCG) with incomplete Cholesky preconditioner (ICP) is used where maximum PCG iteration is set to $10^5$ and convergence tolerance is set to $10^{-4}$. Note that, all the matrices $(C_\gamma^2$, $C_\gamma^2$, $w_{s,f}$, $w_{s,g}$, $A$) in algorithm 1 are sparse.

Algorithm 1: Quadratic-Linear Approximated $\ell_1$-norm Despeckling

1: procedure SDD-QL($g$, $\ell$, $\alpha$, nmax) $\triangleright$ SDD with QL (SDD-QL)
2: \hspace{1cm} $\vec{v}_f \leftarrow \vec{v}_g$ $\triangleright$ $g$ is observed speckled image
3: \hspace{1cm} for $n=1$ to nmax $\triangleright$ nmax is the maximum iteration
4: \hspace{1.5cm} $\vec{v}_f \leftarrow \tilde{J}^{(n)}(f)$
5: \hspace{1.5cm} $\vec{w}_{s,f} \leftarrow -\text{diag}([C_\gamma f]_f + |\epsilon|^{-1})^{-1}$ $\triangleright$ $C_\gamma f$ is $x$-derivative of $\vec{v}_f$
6: \hspace{1.5cm} $\vec{w}_{s,g} \leftarrow -\text{diag}([C_\gamma f]_g + |\epsilon|^{-1})^{-1}$ $\triangleright$ $C_\gamma f$ is $y$-derivative of $\vec{v}_f$
7: \hspace{1.5cm} $\vec{s}_s \leftarrow \text{sgn}(C_\gamma f)$ $\triangleright$ sign of $x$-derivative of $\vec{v}_f$
8: \hspace{1.5cm} $\vec{s}_p \leftarrow \text{sgn}(C_\gamma f)$ $\triangleright$ sign of $y$-derivative of $\vec{v}_f$
9: \hspace{1.5cm} $A \leftarrow 2I + (1 - \alpha)(C_\gamma^2 w_{s,f} + C_\gamma^2 w_{s,g})$
10: \hspace{1.5cm} $b \leftarrow -\vec{v}_g - \lambda(\alpha/2)(C_\gamma^2 s_{s,f} + C_\gamma^2 s_{p,y})$
11: \hspace{1.5cm} solve $A^{(n+1)} f = b$ $\triangleright$ solve equation 6 to find $\vec{v}_f$ for the next iteration
12: end for
13: $f \leftarrow \vec{v}_f$
14: return $f$
15: end procedure
16: return despeckled image

As $\alpha$ gets closer to 1, A becomes more diagonally dominant and efficiency for solving the linear system increases. However, in that case diffusion process is calculated in a local manner which leads to tiny dithering artifacts in the result. For $\alpha < 1$, A becomes diagonal and solution of the linear system in equation (6) becomes very efficient but more outer iterations (nmax) are required. For $\alpha < 1$, A becomes a positive definite and sparse 5-point Laplacian matrix which can be solved with an efficient iterative solver such as PCG. As $\alpha$ gets closer to 0, A becomes less diagonally dominant; therefore, efficiency for solving the linear system decreases while diffusion becomes more global and only few outer iterations (nmax = 5) are required. In SDD-QL, best accuracy and computational efficiency is achieved when $\alpha$ is around 0.5.

Fig. 1. Quadratic and QL ($\alpha = 0.5$) $\ell_1$-norm approximations at 0.5 and 4.
Results and analysis: In this section, SDD with QL (SDD-QL), is analyzed qualitatively and quantitatively to show its despeckling accuracy and computational efficiency. SDD and SDD-QL are both developed in C++ using the coding optimizations given in [2] and compiled as 64 bit executables. In all the experiments, (a) Intel i7-6700K 4 GHz CPU is used as hardware, (b) TerraSAR-X sample SAR image of India Visakhapatnam port (spot-mode, 16 bit, VV polarization, resolution ≈ 1 meter, number of looks ≈ 1) is used as test image, and (c) λ = 100, ε = 10^{-4}, α = 0.5, and n_{max} = 5 are default parameters.

As seen in Figure 2, SDD and SDD-QL produce similar despeckling results since SDD-QL is a variant of SDD. However, SDD-QL preserves reflectivity levels in each region better due to the applied improvements on SDD while homogeneous regions are smoothed equivalently. Better reflectivity levels in each region better due to the applied improvements since SDD-QL is a variant of SDD. However, SDD-QL preserves has SNR = 12 dB with SSIM = 0.969. As seen in Figure 2, SDD and SDD-QL produce similar despeckling result are shown. For this synthetic data, Figure 3 shows signal to noise ratio (SNR) and structural similarity (SSIM) index values for different λ parameters. In this experiment, speckled image has SNR = 12.960 dB with SSIM = 0.725, best result of SDD-QL has SNR = 21.395 dB with SSIM = 0.956, and best result of the SDD has SNR = 21.133 dB with SSIM = 0.907. As seen in Figure 4, both methods achieve similar level of SNR while SDD-QL achieves higher value of SSIM which shows that SDD-QL preserves edges better than SDD.

Conclusion: In this letter, approximation of the TV regularization term in SDD method is improved by fusion of a quadratic and linear approximators. Presented quadratic-linear approximator is derived for ℓ_1-norm, but it can be easily extended to other norms that provides sparsity. Experiments show that, proposed method leads to more accurate despeckling with up to 3 times faster execution times comparing to SDD even though SDD already uses satisfactory ℓ_1-norm approximation and an efficient numerical schema.

Acknowledgment: Author would like to thank Atilla Ozgur, Osman Erman Okman, and Mujdat Cetin for their useful suggestions. Fatih Nar (Konya Food and Agriculture University (KFAU), Turkey) E-mail: fatih.nar@gidatarim.edu.tr

References
1. Argenti, F. and Lapini, A. and Bianchi, T. and Alparone, L.: ‘A tutorial on speckle reduction in synthetic aperture radar images’, IEEE Geosci. Remote Sens. Mag., 2013, 1 (3), p. 6-35
2. Ozcan, C., Sen, B., and Nar, F.: ‘Sparsity-driven despeckling for SAR images’, IEEE Geosci. Remote Sens. Lett., 2015, 13 (1), p. 115-119
3. Perona, P. and Malik, J.: ‘Scale space and edge detection using anisotropic diffusion’, Phys. D, 1992, 60, p. 259-268
4. Rudin, L., Osher, S., and Fatemi, E.: ‘Nonlinear total variation based noise removal algorithms’, Phys. D, 1992, 60, p. 259-268
5. Yu, Y. and Acton, S.T.: ‘Speckle reducing anisotropic diffusion’, IEEE Trans. Image Process., 2002, 11 (11), p. 1260-1270
6. Fabbri, L. and Greco, M. and Messina, M. and Pinelli, G.: ‘Improved anisotropic diffusion filtering for SAR image despeckling’, Electron. Lett., 2013, 49 (10), p. 672-674