Application of Economic Model Predictive Control in Integrated Energy System

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Abstract. Due to the uncertainty of the energy supply and the power demand, the stability and economic performance of the integrated energy system has become a key problem. In this paper, economic model predictive control with augmented model was directly applied to optimize the performance index while responding power demand. Based on the prices of power and hot water, the economic objective function was designed and two modes of operation of heating have been studied which include providing domestic hot water and space heating. The simulation result shows, compared with traditional model predictive control, economic model predictive control could improve economic performance of system by 20% while providing domestic hot water, and showed similar performance while working on space heating.

1. Introduction

Integrated energy system (IES), because of its high energy efficiency, ability to accommodate renewable energy and satisfying various kinds of loads, has obtained the widespread attention and research, and is also considered as an indispensable part in building new energy system. Compared with traditional energy units, the safe and stable operation of IES could be a more complex problem, due to the high proportion of renewable energy and multiple energy and device combinations. Meanwhile, the development of the IES in China is still at the laboratory stage, and how to run IES economically is the key problem of IES commercialization.

Model predictive control (MPC), which can solve multivariable and constraint problems effectively, is being widely used in process industries [1]. In recent years, a new general methodology called economic model predictive control (EMPC), has emerged based on MPC, which could optimize the economic performance of the process by considering economic factor in a general cost function.

Because IES with renewable energy is still in research stage, EMPC is not widely used in this kind of system. Literature [2] studied sizing and energy management of hybrid renewable energy systems based on EMPC, but lacks research on the dynamic properties of systems as well as process control on shorter time scales. Similarly, literature [3] provided the economic criterion of stand-alone building energy system based on renewable energy sources, but the economic analysis method was used for component sizing problem rather than control problem. At the same time, offset-free operation performance of EMPC also attracted considerable attention. Literature [4] ensured the offset-free asymptotic performance of EMPC by introducing gain compensation of input to correct the output prediction model. Literature [5] used augmented model with states estimated and corrected in operation to achieve offset-free performance of hierarchical structure controller with EMPC.
In this paper, EMPC controller with augmented model was designed to improve the economic performance of an integrated thermoelectric energy system (IES), which consists of a photovoltaic (PV) module, a micro gas turbine (MGT) and an air source heat pump (ASHP) and provides power and heat, under disturbance and power load instruction. The EMPC controller, including fuel cost, heating water revenue, tracking penalty term and constraints associated to the IES, was also compared with tradition MPC controller in simulation, resulting in the main contribution of this paper.

2. System modelling
A schematic diagram of the IES studied in this work is presented in Figure 1, which contains an 80kW MGT unit, a 60kW PV module, a 60Ah lithium battery, an ASHP whose rated input power is 60kW and a Tank rated at 320m³. This system is supposed to supply both electricity and heat for residential usage. The PV module absorbs solar energy and generates environment-friendly electricity. The MGT unit operates in CHP mode, supplying both reliable electricity and heat. The lithium battery plays a vital role to stabilize the grid voltage and store the energy. The Tank is used for storing hot water, and two kinds of hot water mix in Header before into Tank. There are two modes of operation, as shown in Figure 2, one is providing domestic hot water which is consumed by users, and another is for space heating to maintain temperature constant with backwater.

2.1. The PV model
The theoretical model of the PV module is divided into three parts, the PV cell, the MPPT controller and the boost circuit. The perturb-and-observe method for PV maximum power point tracking (MPPT) is adopted in this paper. Figure 3 is the one-diode equivalent circuit of the PV cell and its I-V characteristic equation can be expressed as (1).

\[ I = I_{ph} - I_d - I_{sh} = I_{ph} - I_{ph} \left[ e^{\frac{V + IR}{a R_{sh}}} - 1 \right] - \frac{V + IR}{R_{sh}} \]

\[ a = \frac{n k T}{q} \]
Where $I_{ph}$ is the photo current, $A$; $I_d$ is the current flowing through the diode, $A$; $I_o$ is the cell reverse saturation current, $A$; $q$ is the electron charge, $1.6 \times 10^{-19}$ C; $k$ is the Boltzman constant, $1.38 \times 10^{-23}$ J/K; $n$ is the diode ideal factor; $R_s$ is the series resistance, $\Omega$; $R_sh$ is the shunt resistance, $\Omega$; $T$ is the cell temperature, K; And $a$ is the modified ideal factor.

2.2. Micro gas turbine model
The MGT system in the IES consists of a centrifugal compressor, a radial turbine, a combustor and a heat regenerator. Modular modelling method will be used in building MGT model.

The modelling of the compressor is based on the empirical formula of compressor characteristics [6]:

\[
\begin{align*}
\pi_e &= f_e(m_e, n) \\
\eta_c &= f_h(m_h, n) \\
T_{s1} &= T_{s1} \left[1 + \left(\pi_e^{1/n} - 1\right)/\eta_c\right] \\
N_c &= m_c \rho_c c_p c \left(\pi_e^{1/n} - 1\right)/\eta_c \\
\end{align*}
\]

(3)

Where $\pi_e$ is pressure ratio and $\eta_c$ is efficiency; $T_{s1}$ and $T_{s2}$ are the inlet and outlet air temperature respectively, $^\circ C$; $n$ is compressor speed; $N_c$ is the compressor output power, kW; $m_c$ is the air flow rate, kg/s; $c_p c$ is the specific heat capacity of the air, kJ/(kg·K); $k_a$ is the air adiabatic index.

The dynamic model of the rotor is obtained as (4)

\[
J \frac{d\omega}{d\tau} = N_i - N_e - N_p - N_r
\]

(4)

Where $\omega$ is the angular velocity, rad/s; $N_i$ is the mechanical friction loss, kW and $N_e$ is the electricity generation power of the MGT unit, kW, which can be simply defined as (5)

\[
N_e = N_{sd} \left(\frac{n}{n_d}\right)^2
\]

(5)

Where $N_{sd}$ is the rated power of the MGT, kW; $n_d$ is the rated rotational speed of the MGT, m/s.

2.3. Lithium battery model
The modelling of lithium battery mainly focuses on the internal charge-discharge process and characteristics between the current and voltage. The charge and discharge process model can be described as follows [7]

Charging ($i^* > 0$)

\[
f_1(it, i^*, i) = E_0 - K \frac{Q}{Q - it} - K \frac{Q}{Q - it} i^* + \alpha e^{-Bt}
\]

(6)

Discharging ($i^* < 0$)

\[
f_2(it, i^*, i) = E_0 - K \frac{Q}{it + 0.1Q} - K \frac{Q}{it + 0.1Q} i^* + \alpha e^{-Bt}
\]

(7)

Where $K$ is the polarization constant, Ah$^{-1}$; $i^*$ is the low frequency current dynamics, A; $i$ is the battery current, A. $it$ and $Q$ are the extracted capacity and maximum battery capacity respectively, Ah; $\alpha$ with $V$ and $B$ are the exponential voltage and capacity, Ah$^{-1}$.

2.4. Air source heat pump model
The main components of an ASHP system include the compressor, condenser, evaporator and expanding valve. For simplicity, lumped parameter method is adopted for temperature modelling Error! Reference source not found.
Assuming the medium flow rate of the evaporator is the same as the one in compressor and ignoring the pressure and heat loss during evaporation, the thermal balance equation of the cooling water can be established according to the law of energy conservation.

\[
\frac{1}{2} m_e c_{pe} \frac{dt_{ew1} + dt_{ew2}}{d\tau} = G_e c_p (t_{ew1} - t_{ew2}) - Q_e
\]

Where \( m_e \) is the water quantity in the cooling side of evaporator, kg; \( G_e \) is the flow rate, kg/s; \( Q_e \) is the refrigerating capacity, kW; \( t_{ew1} \) and \( t_{ew2} \) are the inlet and outlet water temperature respectively, \(^\circ\)C.

As for the refrigerant side, the state change of the refrigerant approximates to the enthalpy difference between the import and export of the evaporator. According to the law of energy conservation, we can obtain

\[
m_e \frac{dh_e}{d\tau} = Q_e - G_e (h_e - h_t)
\]

Where \( m_e \) is the quantity of the refrigerant in the evaporator, kg; \( h_e, h_t \) are the average, outlet and inlet enthalpy of unit mass medium respectively, \( kJ/kg \), which are the functions of evaporate temperature \( t_e \). Then the heat exchange capacity between the refrigerant and the cooling water can be defined as (10).

\[
Q_e = K_r F_r \left( \frac{t_{ew1} + t_{ew2}}{2} - t_c \right)
\]

Where \( K_r \) is the heat transfer coefficient, kW/(m\(^2\) K); \( F_r \) is the heat transfer area of the evaporate.

The compressor model is based on the performance test data of the specific compressor type. The vital variable representing the characteristic of the compressor is the pressure ratio of the condensing pressure and the evaporating pressure, which is denoted as \( \epsilon = p_c / p_e \), while \( p_c \) and \( p_e \) are determined by the temperature and the characteristic of the refrigerant. Thus, the flow rate of the refrigerant in the compressor is

\[
G_r = \frac{V_r \omega_r \eta_r \lambda_e}{v_d}
\]

Where \( V_r \) is the gas displacement of the compressor, \( m^3 \); \( \omega_r \) is the rotational speed, \( r/s \); \( \eta_r \) is the volume efficiency and \( \lambda_e \) is the volume coefficient of the compressor; \( v_d \) is the inlet refrigerant specific volume, \( m^3/kg \), which also can be determined by the inlet temperature and the characteristic of the medium. Thus, the compressor outlet refrigerant enthalpy is obtained as (12).

\[
h_{t} = h_{t} + \frac{N}{G_r}
\]

Where \( N \) is the actual power of the compressor, kW; which is calculated as

\[
N = \frac{n \eta_r V_r \omega_r p_e}{1 - n} \left[ 1 - \epsilon \frac{n+1}{n} \right]
\]

Where \( N_r \) is the shaft power of the compressor, kW; \( \eta_r \) is the electrical efficiency; \( n \) is the polytropic exponent of the compressor determined by the operating condition.

The energy equivalent equation of the condenser water side is shown as (14).

\[
\frac{1}{2} m_c c_{pc} \frac{dt_{cw1} + dt_{cw2}}{d\tau} = Q_c c_p (t_{cw2} - t_{cw1})
\]

Where the symbols have the same meanings as (8), but the subscript \( c \) representing the condenser.

As for the refrigerant side, the enthalpy difference can be determined as (15).
\[
m_s \frac{dh}{d\tau} = G_s (h_s - h_h) - Q_e
\]

(15)

Where the heat transfer quantity \( Q_e \) of the condenser can be derived as (16).

\[
Q_e = K_i F_i \left( t_c - \frac{t_{in} + t_{out}}{2} \right)
\]

(16)

Assuming the flow rate of the refrigerant remaining unchanged, the enthalpy value before and after flowing through the valve is the same, namely, \( h_3 = h_4 \).

2.5. Header and Tank modeling

The header is considered as a container for mixing two kinds of hot water. The hot water temperature of the mixture is calculated as

\[
\begin{align*}
Q & = c_p m_{MGT} (t_{in} - t_{MGT}) = c_p m_{HP} (t_{HP} - t_{in}) \\
& = \frac{m_{HP} t_{HP} + m_{MGT} t_{MGT}}{m_{HP} + m_{MGT}}
\end{align*}
\]

(17)

Where \( Q \) is the heat exchange capacity during the mixing, \( kW \); \( m \) is the hot water flow rate, \( kg/s \); \( t \) with subscript is the inlet hot water temperature, \( ^oC \); \( t_{in} \) is the temperature of the mixed water, \( ^oC \).

Hot water flows through the pipe to heat space, and the temperature of the water flowing through the Tank and the pipe is described by the following the model

\[
G_{tem} = \frac{1}{s + \tau_1}
\]

(18)

While providing user domestic hot water, excess hot water generated by the IES will be stored in tank for night time and bad weather without PV on. When the IES is supposed to heat space for hotel or school, the flow rate of MGT and ASHP remain constant (\( G_{in,MGT} = 0.45 \) kg/s, \( G_{in,ASHP} = 0.6 \) kg/s).

3. Problem formulation and control system design

3.1. System description

The IES is designed to provide users power, and hot water is also available as by-product. The system works under power demand instruction, and the power demand is set to change every 5 minutes.

Based on the model of the IES, the input-output structure of the IES can be expressed as what shown in Figure 4. The manipulated variables are fuel flow of MGT (\( G_j \), kg/s), heat pump speed (\( n \), r/s) and feed water flow (\( G_c \), kg/s), and the controlled variables are output power of IES (\( P_{net} \), kW), and hot water temperature (\( T_{hot} \), ^oC). The output power of IES is the spare power provided to the users, which including the power PV and MGT generate and the power ASHP consume. The hot water temperature is supposed to be more than 60^oC.

![Figure 4. Input-output structure](image)

![Figure 5. Controller structure diagram](image)

3.2. Control system design

The basic control method used in this paper is MPC. And the control objective is to track load setting value while keeping the system working safely and steadily. The main problem of control system design is dynamic differences due to multiple device combinations. To cover the dynamic
characteristics, long prediction horizon is more suitable.

Another problem brought about by PV module is the disturbance caused by solar radiation intensity and environmental temperature, which would influence the output power of the IES. Several offset-free methods could help to solve this problem, such as disturbance observer and disturbance model. In this paper, integral embedded augmented model was used, which can be directly taken as prediction model while eliminating disturbance. The control structure diagram is as Figure 5 shown and schematic diagram of IES system control structure is as Figure 6 shown.

The state model of the IES is shown as (19), and the integral embedded augmented model is shown in (20).

\[
\begin{align*}
\begin{cases}
x_j(k+1) = A_jx_j(k) + B_ju(k) \\
y(k) = C_jx_j(k)
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
\Delta x_j(k+1)
\end{cases}
\end{align*}
\begin{align*}
y(k+1) = \begin{bmatrix}
A_j & O \\
C_jA_j & I
\end{bmatrix}
\begin{cases}
\Delta x_j(k) + B_ju(k)
\end{cases}
\begin{bmatrix}
y(k)
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
y(k) = \begin{bmatrix}
O & I
\end{bmatrix}
\begin{cases}
\Delta x_j(k)
\end{cases}
\begin{bmatrix}
y(k)
\end{bmatrix}
\end{align*}
\]

3.3. EMPC controller design

In the design of EMPC, the economic stage cost or performance index \( l_e(x,u) \) is chosen as the objective function. Without loss of generality, the optimization problem is:

\[
\begin{align*}
\min_{u} & \sum_{t} l_e(x,u) \\
\text{s.t.} & \quad x(k+1) = Ax(k) + Bu(k) \\
& \quad y(k) = Cx(k) \\
& \quad u(k) \in U \\
& \quad y(k) \in Y
\end{align*}
\]

Where, \( U \) and \( Y \) are constraints of manipulated variables and controlled variables. Due to the change of objective function, it is no longer necessarily true that object function satisfies stability condition of conventional MPC [1]. Terminal constraint was introduced to make sure stability, which is set as power reference value to meet the power demand. And the optimization problem would be:

\[
\begin{align*}
\min_{u} & \sum_{t} l_e(x,u) \\
\text{s.t.} & \quad x(k+1) = Ax(k) + Bu(k) \\
& \quad y(k) = Cx(k) \\
& \quad u(k) \in U \\
& \quad y(k) \in Y
\end{align*}
\]
The stage economic cost function \( l_e(x,u) \) consists of two parts. One is the cost of running, which in the IES is the fuel cost; and the other is the final output, including power and hot water. And it can be defined as follows:

\[
l_e(x,u) = p_{fuel}G_f - p_{water}G_{water}
\]

Where \( p_{fuel} \) is the price of fuel (4.3 CNY/kg); \( p_{water} \) is the difference between the prices of hot water and supply water (0.02618 CNY/kg).

4. Simulation results

Assuming the system is operating at the initial state of Table 1, the simulation, shown in Figure 7, contains two work periods (600 seconds). And the change of radiation intensity will occur in the first work period. As the Fig.4 shows, the control system was put into operation at the operating point, and after 150s first power demand instruction would arrive. The next power demand instruction would be introduced at 450s, and radiation intensity would change at 250s and at 850s, an external disturbance of temperature occurs, which perhaps caused by the pipe cooling change or the change of building heat dissipation. The simulation work is based on the Simulink software of MATLAB R2018b.

4.1. Simulation1: providing domestic hot water

In this simulation, the temperature of hot water is set over 60°C. The basic hot water demand is 1.05 kg/s, and excess hot water will be stored.

Three different controllers had been applied in this simulation.

(1) MPC1. Integral embedded augmented model was used as the predictive model and a Kalman filter was designed for this MPC. The objective function was set as \( \sum_{i=1}^{N} \| y_{i+1} - y_{set} \|_{2} + \| Au \|_{2} \), and prediction horizon \( N = 300 \), control horizon \( M = 1 \).

(2) EMPC1. Integral embedded augmented model (as formula (22) shown) and a Kalman filter was used. And the objective function was set as \( \sum_{i=1}^{N} l_e(x,u) + \| y(2)_{i+1} - T_{set} \|_{2} \), and prediction horizon \( N = 300 \), control horizon \( M = 1 \).
EMPC2. Integral embedded augmented model (as formula (22) shown) and a Kalman filter was used. And the objective function was set as 
\[
\sum_{i=1}^{N} u_{i}(x,u) + 0.1\left(\|y(2)_{i} - \Phi - T_{i} + 0.2\left(\|y(1)_{i} - \Phi - P_{i}\right)\right).
\]
And prediction horizon \(N = 300\), control horizon \(M = 1\).

The final simulation results are as Figure 8-10 shown. Economic profits over the entire simulation period were shown in Error! Reference source not found.2.

All controllers could track setting value under disturbance of PV output. Compared with traditional MPC, EMPC could find a new economic equilibrium point which produced more hot water and have better economic performance than MPC, but are little weak in control performance. Compared with EMPC1, the output power of EMPC2 is more stable due to the penalty term of output power in the objective function, but more volatile in output temperature which indicate the penalty terms of output in the objective function would affect each other. The simulation results in this part shows the design of EMPC controller is trade-off between control performance and economic performance.

![Figure 8. Simulation1 result of output power](image)

![Figure 9. Simulation1 result of hot water temperature](image)

![Figure 10. Simulation1 result of input (from top to bottom: fuel flow of MGT, heat pump speed and feed water flow)](image)

Table 2. Profits of different controllers in simulation1 （the cost of fuel and hot water profit）

| Controller | Profits (CNY) |
|------------|--------------|
| MPC1       | 203.8010     |
| EMPC1      | 257.5368     |
| EMPC2      | 221.5570     |

4.2. Simulation2: space heating

In this simulation, the flow rate of hot water is supposed to be constant (\(G_{lw,MGT} = 0.45kg/s\), \(G_{hw,ASHP} = 0.6kg/s\)), which means feed water flow of ASHP (\(G_{c}\), kg/s) can’t be regard as manipulated variable. The control objective is to keep the hot water temperature constant.

Three different controllers had been applied in this simulation

(1) MPC2. Integral embedded augmented model was used as the predictive model and a Kalman filter was designed for this MPC. The objective function was set as 
\[
\sum_{i=1}^{N} u_{i}(x,u) + 0.1\left(\|y_{i} - \Phi - T_{i} + 0.2\left(\|y_{i} - \Phi - P_{i}\right)\right),
\]
and prediction horizon \(N = 300\), control horizon \(M = 1\).
(2) EMPC2.1. Integral embedded augmented model (as formula (22) shown) and a Kalman filter was used. And the objective function was set as \( \sum_{k=0}^{N} (p_{net}[G_f + y(2)_{k+1}^T - T_{ref}]) \), and prediction horizon \( N = 300 \), control horizon \( M = 1 \).

The simulation result is shown in Figure 11 and Figure 12. The economic profits shown in Table 3, only contains the cost of fuel, excluding the hot water profits (which is a constant).

As the pictures shown, compared with the result of simulation 1, the difference of the IES output and economic performance difference of the two controllers above are tiny, which indicated there is little room for improvement in economic performance under such objective function.

| Controller   | Profits (CNY) |
|--------------|---------------|
| MPC2         | -16.8208      |
| EMPC2.1      | -16.8196      |

5. Conclusions
The Integrated energy system as the important part of future energy system development, is bothered with the disturbance of renewable energy. The purpose of this paper is to study the application of economic model predictive control in the system, trying to improve the economic performance while meeting the power demand stably.

Two modes of operation of the integrated energy system have been studied. Compared with conventional model predictive control, economic model predictive control controllers were designed to obtain more profits under the disturbance of PV modules while satisfying power demand. And according to the simulation results, providing domestic hot water seems to be more economical with more operational degrees of freedom and energy storage device could help to improve the control performance of system with renewable sources.

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