Top Mass Analyses for the Reported
Top-Antitop Production and Decay Events

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Abstract

We have analyzed the available data on $p\bar{p} \rightarrow t \bar{\ell}$ followed by the
decays ($t \rightarrow bW^+$, $\bar{\ell} \rightarrow \bar{b}W^-$) which lead to $e^+\mu^\pm2\text{jets}$ or $l^\pm4\text{jets}$
configurations, using a likelihood method we proposed earlier. The
outcome is compared with the recent CDF analysis. In an appendix,
we discuss the nature of the additional “slow” $\mu^+$ observed in one
CDF dilepton event, concluding that it is most probably a “tertiary
lepton” resulting from the decay sequence $b \rightarrow c + \text{hadrons}$, followed
by $c \rightarrow s\mu^+\nu$. 

1 INTRODUCTION

A major step forward in research on the top quark has recently been achieved by the CDF Collaboration\cite{1} and the D0 Collaboration\cite{2}. Both groups present evidence for the discovery of the heaviest quark. Previously CDF\cite{3, 4} published some of the data on a number of top-antitop production and decay events\cite{3, 4}. These data consist of seven single lepton events with four large $p_T$ jets ($l^\pm 4j$) and two opposite-sign dilepton events with two large $p_T$ jets ($\mu^\pm e^\mp 2j$)\cite{1}. Each of the former has one jet “tagged” as a likely $b$ or $\bar{b}$ jet and passes the many different cuts designed to minimize the contribution from the extensive background\cite{5} expected from “$W + jets$” Standard Model processes. The dilepton events are fewer, since hadronic decays predominate over leptonic decays for the $W$-boson; indeed, for 7 b-tagged $l^\pm 4j$ events\cite{3, 4}, we would expect\cite{6} about $7/(18 \times 0.22) \approx 1.8\mu^\pm e^\mp 2j$ events, compatible with observation. The CDF analysis of the $l^\pm 4j$ events follows established procedures from bubble chamber physics, except that energy measurements by calorimeter have much less percentage error than momentum measurements by magnetic fields. The CDF analysis of the $\mu^\pm e^\mp 2j$ events is only kinematical, based on whether the measured momenta satisfy a set of cuts deemed sufficient to remove all background. The formula for this rate as a function of top mass $m_t$ is then used to deduce a lower limit on $m_t$. This procedure gives no weight to precise detailed features of the event and their relative probabilities, given our knowledge of the quark structure of protons and of the Electroweak decay amplitudes characteristic of the Standard Model, so well confirmed elsewhere today.

It appears worthwhile at present to make independent calculations from these and other data\cite{7, 9, 10} by methods we have already described, as a contribution to the phenomenology of top quarks. This may also help us to assess the relative benefits of the various methods which have been proposed. Further, there are now four $\mu^\pm e^\mp 2j$ events available for analyses, and it will be valuable to compare the top mass estimates from them with those from the $l^\pm 4j$ events. That the two groups of top mass estimates should be in agreement is a powerful test of the methods now being used for top mass

\footnote{These last two events each have three jets, but the third jet has very high momentum with low $p_T$ and must be regarded as due to initial-state inner-bremsstrahlung. In both events the third jet happens to be associated with the antiproton (see Table I).}

\footnote{The overall efficiency of b-tagging\cite{4} is about 0.22\cite{4}.}
determinations.

2 The Data and Analyses of “$\mu^\pm e^\mp 2 jets$” events

Over several years, we \[6, 11, 8\] and K. Kondo, \textit{et.al.} [12, 13], independently, have developed a method for determining whether events of the type reported by the CDF and D0 Collaborations are consistent with the hypothesis

\[
\bar{p} + p \rightarrow \bar{t} + t + \text{other hadrons},
\]

followed by

\[t \rightarrow W^+ b,\]

where

\[W^+ \rightarrow l^+ \nu_l \text{ or } u \bar{d} \text{ or } c \bar{s},\]

and

\[\bar{t} \rightarrow W^- \bar{b}.
\]

where

\[W^- \rightarrow l^- \bar{\nu}_l \text{ or } d \bar{u} \text{ or } s \bar{c}.
\]

The procedure is to take the measured configuration of momenta for the final leptons and jets in a single event and to evaluate the probability $P(m_t)$ that these production and decay processes could produce the observed configuration for an assumed value of the top quark mass. This evaluation takes into account each step in the processes (1) and (2): the parton content of the incoming proton and antiproton; the differential cross section for these partons $q$ and $\bar{q}$ to produce the $\bar{t}t$ pair; the decay probability for the top and antitop quarks to undergo the processes (2) and (4) respectively, with the angles observed in the $t$ and $\bar{t}$ rest frames; and the probabilities for the W bosons to decay into the leptons or quark pairs as observed, retaining the spin and tensor polarization the W bosons have following $t$ or $\bar{t}$ decay.

We applied this procedure to the published dilepton event (which we refer to as CDF-1) from the 1988-89 run by CDF[7, 9]. The primary muon from this event did not go through the muon detector which was in place for that first run, but its muonic character was made quite clear by the evidence from its passage through an adjacent calorimeter. Since one of the
acceptance criteria in that run was that the muon should identify itself by passing through the muon detector, CDF has rejected this event. To avoid such rejection in subsequent runs, CDF has elongated their muon detector, so that an event with the same configuration as CDF-1 would be accepted today. This rejection is undoubtedly sound practice when one is concerned with cross sections and relative rates, but our only concern here is whether or not this event is a satisfactory candidate for interpretation as an example of top-antitop production and decay. Our analysis of this event led to a somewhat low mass value, either \(121^{+18}_{-8} \text{GeV}\) or \(131^{+22}_{-12} \text{GeV}\), depending on the details of the treatment of the transverse momentum distribution of the parton-antiparton pair which generates the final \(t\bar{t}\) state. A similar calculation has been made independently by Kondo et al.\[12, 13\], leading to a mass distribution \(P(m_t)\) almost identical with our distribution with a peak at 121 GeV. The D0 collaboration applied three modified versions of our method to their \(\mu e2\text{jet}\) candidate and obtained a probability distribution which peaks at 141 GeV, with half-maximum values at about 131 GeV and 165 GeV. The input data for this event are not known to us. The D0 detector does not have the capability to measure the charge sign for \(e^\pm\) but the charge sign for the muon can, in principle, be determined by the magnetic field in the muon detector.

The momentum data for the two \(\mu^\pm e^{\mp}2\text{jet}\) events observed by CDF\[3, 4\] in their 1992-93 run, mentioned above, are given here in Table 1 in terms of Cartesian momenta. We have analysed these two events by the same procedure as we used\[6, 11\] for the CDF-1 event. In both cases jet 3 is neglected for the reason given in footnote 1. We assigned uncertainties to the jet transverse energies by an algorithm based on the \(\sigma_E\) values presented by CDF for all their different energy jets, in the single lepton \(t\bar{t}\) events\[^3\]. A jet is paired with a lepton to determine a paraboloid of possible top momenta. The second jet is paired with the other lepton to form the paraboloid for the anti-top momenta. Assuming that the \(t\bar{t}\) production occurs with small total transverse momentum, the transverse momentum of the top quark (for a definite mass value \(m_t\)) should nearly cancel the corresponding transverse momentum of the anti-top quark with the same mass \(m_t\). To allow for gluon bremsstrahlung, we took a gaussian distribution of values centered on zero,

\[^3\]A linear interpolation, \(\sigma(E) = (3.4 + 0.1E)\text{GeV}\) was used. This underestimates the uncertainties for some of the low energy jets.
with a representative width of $\rho = 0.1m_t$, to weight the probability of the pair of transverse momenta assigned to the $t$ and $\bar{t}$ jets. The resulting probability distributions are shown on Fig. 1. In both cases, the assignment of jets to the $b$ and $\bar{b}$ quarks is uniquely determined by this mass analysis. It happens that for event 41722/38382, jets 1 and 2 are necessarily $b$ and $\bar{b}$, respectively, and the same holds for event 41540/127085. The latter also has a secondary (slow) $\mu^+$ lepton, which would routinely be assigned to the $\bar{b}$ jet, here jet 2. We discuss, in the Appendix, the possible interpretation for this additional lepton.

The probability distributions $P(m_t)$ from these two new CDF events, shown in Fig. 1, peak at $158^{+7}_{-6}$ GeV for 41540/127085 and at $168^{+24}_{-14}$ GeV for 47122/38382. These independent probability distributions are consistent with the production of top quarks with $158^{+7}_{-6}$ GeV, as seen in Fig. 2, where the two have been multiplied. The mass distributions for the CDF-1 and D0 events are appreciably lower in peak mass, but each has a very gradual fall-off for higher mass values. The joint probability for all four dilepton events is displayed in Fig. 2. It peaks at $156^{+7}_{-6}$ GeV, since event 41540/127085 gives the narrowest distribution. This event also has the lowest likelihood; indeed, if $\rho$, the transverse momentum weighting parameter in eq.(3.21) of ref.[6], is reduced from our adopted value of $0.1m_t$ down to zero, the solution for this event ceases to exist. However, as things are, this joint probability $P(m_t)$ has a convincing shape, although its absolute likelihood is very low\footnote{CDF-1 comes from an earlier run and the data for the D0 event is not published, so we have not been able to normalize the probabilities for all the dilepton events consistently. The two recent CDF events are consistently normalized however, so their likelihoods can be compared.}. It is still quite possible that not all of these events, or even any of them, are due to top-antitop production and decay. More candidate $e^\pm\mu^\pm$ 2j events are needed for study before we can conclude which of these events constitute a group of top-antitop events for a definite mass $m_t$. As already discussed\footnote{CDF-1 comes from an earlier run and the data for the D0 event is not published, so we have not been able to normalize the probabilities for all the dilepton events consistently. The two recent CDF events are consistently normalized however, so their likelihoods can be compared.}, there are non-top sources of background possible which may mimic real top-antitop events.
3 The Data on “$l^\pm 4$ jets” events and Analysis Procedure

We now consider the single lepton events, with at least 4 jets, one of which is tagged as $b$ or $\bar{b}$ quark. The momenta of the leptons and jets in the seven published events are given in Cartesian co-ordinates in Table 2. The jet calorimeter energies are the “corrected values” quoted by CDF\cite{CDF} in their Appendix A, following the calculated scatter plots given in their Fig. 57; they are the CDF estimates for the original parton energies, with well-defined statistical uncertainties. The C.M. energy for the two jets hypothesized to result from W decay is generally rather far from the well-known value\cite{Helm}: $M_W = 80.2(2)$ GeV, and this presents a problem. CDF uses a kinematic fitting procedure, established long ago\cite{kinematic} in bubble chamber work, to manipulate the already corrected transverse energies in order to reproduce the W mass value at the expense of a higher $\chi^2$. We do not use the resulting “best fit values” given by CDF, since we follow a different scheme of analysis\cite{analysis}.

Our analysis of these $l^\pm 4j$ events employs a simple extension of the method used above for dilepton events, which has been laid out in considerable detail in ref. \cite{dilepton}. We sketch it briefly here, for the case of a positively charged lepton $l^+$; the case for $l^-$ follows when every particle is replaced by its antiparticle and vice versa. One jet is chosen tentatively to be the $b_l$ jet associated with $l^+$ and a kinematic paraboloïd is formed, as before, leading to an ellipse in momentum space which includes all momenta consistent with $b_l$ and $l^+$, for an assumed mass $m_t$. The other three jets are assumed to arise from $\bar{t}$ decay where the resulting $W^-$ boson decays hadronically, thus:

$$\bar{t} \to \bar{b} + W^-, \text{ followed by } W^- \to \bar{q}_1 + q_2.$$  \hspace{1cm} (6)

The quark assumed to be $\bar{b}$ will be denoted\footnote{We use the CDF notation.} by $b_j$. The experimental error distributions for these quark energies have been discussed in much detail by CDF\cite{CDF} and we adopt the same algorithm that interpolates their $\sigma_E$ values as stated in footnote 3. A grid of momentum values ($\bar{b}$, $\bar{q}_1$, $q_2$) is laid out and weighted by their probability values at each point, together with a probability weighting $F_W(\bar{q}_1, q_2)$ of Breit-Wigner form to emphasize those grid points at which ($\bar{q}_1 \cdot q_2$) is consistent with the W-boson mass. At each grid point,
there is a definite momentum \((\ell = \bar{b} + \bar{q}_1 + q_2)\) and deduced mass \(m_t\), and this point is then paired with the points on the \(t\)-ellipse for \(m_t\), which also have their weighting factors due to measurement errors. This product of probabilities is finally weighted by a Gaussian factor \(G[(t+\ell)_{\tau}/\rho]\) to represent the effect of limited transverse momentum due to initial state gluon emission, the value 0.1\(m_t\) being adopted for the parameter \(\rho\). Contributions to the net probability for the top quark mass to lie within \((m_t, m_t + \Delta m_t)\) come from all grid points which lie within this band \(\Delta m_t\), and are summed to give the net probability \(P(m_t)\) indicated by this event.

This probability has been computed for a definite assignment \((b_1; \bar{q}_1, q_2, \bar{b}_j)\) of the four jets. If none of the jets is b-tagged, then for an individual event distribution, we will still have to sum over all permutations of the jet assignments. If one jet is b-tagged by SVX, we shall generally not know whether is is a \(b\)-jet or a \(\bar{b}\)-jet, and we shall then have to sum over the two possible assignments for a jet which may be due to \(b\) or to \(\bar{b}\). It is important to note that each jet is treated in an identical way, so that relative probability between different events and different interpretations can be compared. The probability distributions obtained in this way for each of the seven \(l^{\pm}4j\) events are shown in Fig. 3. In most, but not all, of the events, the combination of jets which has the largest integrated probability is the assignment chosen by CDF’s method\[4\]. However, two exceptions to this observation will be mentioned below. The product of the independent probabilities for these seven \(l^{\pm}4j\) events is plotted in Fig. 2. Their net distribution peaks at \(m_t = 172^{+2}_{-4}\) GeV, in accord with the CDF group’s conclusion for the same seven events.

How should we assess the significance of the integrated probability found for each of these events? They range from a maximum of roughly \(8 \times 10^{-4}\) for event 45880/31838 down to \(5 \times 10^{-7}\) for event 43351/266423.

4 Our Analysis Procedure in the Light of a Monte Carlo Model for \(l^{4}jets\) Events

To address this question of significance, we have carried out the same analysis procedure for a random sample of computer-generated events, a Monte-
Carlo simulation based on tree-level QCD Feynman graphs for top-antitop production, followed by their decay into the final states $l^\pm 4j$. In this way, we generated 100,000 events for mass $m_t = 170$ GeV, of which 38,394 passed all of the appropriate experimental cuts. Each event requires the specification of eleven variables, each arranged to vary over a finite range, and this finite 11-dimensional space was divided into a finite number of cells. The procedure is iterative, based on a program by Ohnemus. Each event had a weight, its fractional contribution to the total cross section. The first 20,000 events, randomly generated, led to a net weight in each cell, and a new set of variables were chosen, leading to a new set of cells, each with about the same net weight. This procedure was iterated five times, for each new set of 20,000 randomly-generated events. The distribution of the final 20,000 events in the 11-dimensional space is then expected to be much closer to the physical reality corresponding to this simple tree-level model, than that of the first 20,000 events. The improvement in each iteration can be tested, for example by comparing the total cross section calculated after each iteration with the directly calculated total cross section for this simple model, as given by Berends et al. The weights are finally used to choose randomly a set of unweighted events to form a representative subsample. The subsample we can analyse is necessary small, of order 100, since our analysis procedure is quite complicated and needs very considerable computer time. In the end, we chose a subsample of 105 events randomly, by the procedure just mentioned above, out of the 1,292 events which passed the experimental cuts, from the first 3,000 events of the last iteration. Our purpose is to compare the observed features of the candidate top-antitop events with the features predicted for these events by this simple QCD model.

Since these are MC events, we know which quarks are which. In analyzing each event, typical measurement errors are assumed for the lepton and the jets. We then use our analysis method to deduce the probability distribution $P(m_t)$ from the event. The integrated probabilities, $IP \equiv \int dm_t P(m_t)$, and the $m_t$ values for the peak in the probability $P(m_t)$ are shown in the “scatter plot” of Fig.4. This shows that the analysis does reproduce the input mass rather well, but that there is a wide range in the IP values. We have plotted the distribution obtained for these IP values in Fig.5, and note that they have a rather wide range. However, this exercise has given us some criterion for recognizing poor fits, when we come to consider the analysis of real events, and this was its purpose.
5 The Analysis of Seven $l^{\pm}4j$ Events

Our analyses of the seven empirical events are summarized in Table 3. Since these are tagged events, the tagged jet is likely to be either a $b$ or $\bar{b}$ jet associated with the $W$ that decayed either leptonically (the $b_l$ jet) or hadronically (the $b_j$ jet). For a given event, each assignment of jets to the $t - \bar{t}$ hypothesis in eqn(6) is analyzed independently, i.e. each particular choice of jets to correspond to the configuration $(b_l; q_1, q_2, b_j)$, is analyzed separately. CDF has done such separate analyses as well. They assign the measured jets and the lepton to a configuration and determine the $\chi^2$ per degree of freedom (there are 18 measurements - momenta, energies and angles, with 20 constraints) that that configuration satisfies the $t - \bar{t}$ hypothesis, based only on the jets’ kinematics, without weighting by the probabilities for such kinematics.

All but two of the seven events give integrated probabilities between $4.3 \times 10^{-4}$ and $1.6 \times 10^{-4}$, significantly lower (by a factor about 1/5) than the majority of our Monte Carlo sample. Two of these events, 45880/31838 and 45879/123158, have two maxima peaking above $10^{-4}$; the former gives $4.3 \times 10^{-4}$ with $m_t = 164 GeV$ for configuration $(b_l; q_1, q_2, b_j) = (1; 2, 4, 3)$ and $3.2 \times 10^{-4}$ with $m_t = 134 GeV$ for $(2; 1, 4, 3)$, and the latter giving $3.8 \times 10^{-4}$ with $m_t = 178 GeV$ for $(2; 3, 4, 1)$ and $2.5 \times 10^{-4}$ with $m_t = 180 GeV$ for $(2; 1, 4, 3)$. CDF’s fit to the latter is for configuration $(1; 3, 4, 2)$ with $\chi^2 = 2.2$ and $m_t = 169(10) GeV$; we do have a fit in this configuration with peak at $168 GeV$, but it has a low integrated probability, $6 \times 10^{-6}$. The events 43090/47223 and 43351/266423 also give much lower integrated probabilities, $3 \times 10^{-6}$ and $5 \times 10^{-7}$, respectively, and we are inclined to reject them as top candidates. The latter is a poor fit also in CDF’s analysis, but CDF found the former to be acceptable. On the other hand, event 45610/139604 gives integrated probability $1.6 \times 10^{-4}$ with $m_t = 180 GeV$ in our analysis; the CDF analysis also found $m_t = 180(9) GeV$ but a poor $\chi^2 = 5.0$.

6 Summary of Conclusions

We may summarize our considerations as follows:

1) Our analysis of the seven $l^{\pm}4j$ events now known is in general accord with the CDF-analysis, especially with their mass estimate of about $175 GeV$. 


Two of the events have very low likelihoods in our analysis, while two of them have relatively large $\chi^2$ in the CDF analysis, one event being rejected by both; four events stand firm in both analyses. The three events rejected may be due to background such as that originating from the processes $W^\pm + 4jets$ with $W^\pm \rightarrow l^\pm$, as discussed by Berends, et.al.\cite{berends}, although those authors show that tagging a single $b(\bar{b})$ quark should significantly reduce that background. Their calculations indicate a suppression by about $10^{-2}$ when both $b$ and $\bar{b}$ are tagged. More estimates from other mechanisms involving $b$-quarks need to be considered quantitatively, within the framework of our analysis procedure.

(ii) The relative rate between $l^\pm 4jet$ and $e^\pm \mu^\mp 2jet$ events must be considered. Accepting that four b-tagged events of the former class have been observed, we need to calculate the expected number of events of the latter class. This is a complicated calculation, which is sensitive to the precise cuts which are imposed and which we do not attempt to carry out here. The efficiencies depend on whether the lepton in question is an electron or a muon. The nature of the identification given by tagging is different for SVX and SLT. SVX does not distinguish $b$ from $\bar{b}$, since it determines only the location of the secondary vertex, while SLT does not give the location of the vertex but does distinguish between $b$ and $\bar{b}$. Since the c-quark decay lifetime is shorter than that for the $b$-quark, there should frequently be seen a tertiary vertex arising from c decay, not far from a secondary $b$- vertex.

(iii) Finally, the peak mass $m_t$ appears to be systematically somewhat lower for $e^\pm \mu^\mp 2jet$ events than for $l^\pm 4jet$ events, the former being $156(8)$ GeV, and the latter being $175(8)$ GeV, a separation of $19(11)$ GeV, although this is still a tolerable difference. We repeat that it is far from sure that all of the latter events are due to $t\bar{t}$ production and decay. However, to strengthen the lower mass value for the former, we can also consider the CDF event 45047/104393, which has a more natural identification with the dilepton variety. Assigning both leptons as “hard”, rather than taking one as “soft”, and combining two jets into a single jet, leads to a good fit as a $t\bar{t}$ event with $m_t = 136^{+18}_{-14}$, consistent with the generally lower masses for the dilepton candidates.
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APPENDIX

The CDF dilepton event 41540/127085 has a unique interpretation when analysed as $\mu^+e^- j(1)j(2)$, $j(1)$ being identified as the $b$-jet and $j(2)$ as the $\bar{b}$-jet (see Table 1). The jet $j(3)$ is close to the initial direction, being most probably due to gluon bremsstrahlung. The event has a third lepton, a "slow" $\mu^+$ of energy 8.9 GeV. The routine choice of associating this lepton with the $\bar{b}$ jet is not convincing since its largest momentum component, $p_x(\mu^+) = 8.7$ GeV, is oppositely directed to the largest component of the jet 2 momentum, $p_x = -50.0$ GeV. It is much more plausible that the slow $\mu^+$ is associated with jet 1, since its momentum is almost parallel with the momentum of jet 1, and in the same direction; its momentum $p_\perp$, transverse to jet 1 is only about 0.6 GeV/c.

The sequence of quark processes which lead to the emission of a tertiary $\mu^+$ lepton, (a) with, and (b) without, a secondary $\mu^-$ lepton, are as follows:

\begin{align}
t &\rightarrow b + W^+ \\
b &\rightarrow c + W^- \\
c &\rightarrow s + W^+ \\
W^+ &\rightarrow \mu^+ + \nu_\mu,
\end{align}

or

\begin{align}
W^- &\rightarrow \mu^- + \bar{\nu}_\mu \\
or \quad W^- &\rightarrow \text{hadrons}(\bar{u}d + \bar{c}s),
\end{align}

the W’s in (8), (9) and (10) being necessarily virtual, of course. The net process for the event 41540/127085 would be the consequence of (7),(9) and (10), thus:

\begin{align}
t &\rightarrow \text{hadrons} + \mu^+(\text{fast}) + \mu^+(\text{slow}) + \text{neutrinos}.
\end{align}
To orient ourselves concerning the final states, we have made some simple model calculations for the momentum distributions for a secondary lepton $l_2$, or for a tertiary lepton $l_3$, appropriate for an initial $b$-jet with momentum about 130 GeV/c. We adopted the fragmentation function of Peterson et.al.\cite{18}, with the parameter $\epsilon_Q = (0.49)/m_Q^2$ GeV$^{-2}$, where $m_Q$ denotes the appropriate heavy quark mass. In the first step, the $b$-quark generally undergoes hadronization to a final ground state meson $B_\pm^u$, $B_d^0$ or $B_s^0$ with spin-parity $0^-$, together with some number of light mesons; we neglect explicit mention of hadronization to $\Lambda_b$ baryons, since such final states contribute much less and do not affect the over-all conclusions. Using the standard model expression for the momentum distribution of the lepton resulting from $b \rightarrow c l_2 \bar{\nu}_l$ in the B-meson rest frame, we obtain the $l_2^-$ momentum distribution in the lab frame by integrating this distribution over the B-momentum distribution given by the fragmentation function. The resulting energy distribution for secondary leptons is given in Fig.6(a). We note that these energies run up to very large values. The mean $l_2^-$ energy is $\approx 33$ GeV/c and 50% of the leptons have energy greater than 29 GeV. The distribution for $p_\perp$, the secondary muon momentum transverse to the B-momentum, is given in Fig.7(a), although we must note that the B-momentum is affected by the gluons and light mesons emitted so that it differs a little from the $b$-jet axis observed. The most probable value for $p_\perp(l_2)$ is 1.4 GeV/c; its median value is 1.35 GeV/c. For 90% of the secondary leptons, $p_\perp(l_2)$ exceeds 0.6 GeV/c; 70% of them have $p_\perp(l_2) \geq 1$ GeV/c. For tertiary leptons, we must first carry out the same calculation for the lab momentum distribution of the c-quarks from the b-jet. Naturally, this distribution is quite different from that for the secondary leptons, because of the large mass value for the c-quark. The lab energy distribution for the $l_3^+$ lepton from $c \rightarrow s l_3^+ \bar{\nu}_l$ decay is then obtained by integrating the latter, as given by the standard model, over the fragmentation function for the ground state $0^-$ ($D_u$, $D_d$ and $D_s$)-mesons from the c-jet distribution just calculated. The resulting energy distribution for the $l_3^+$ lepton is shown in Fig.6(b). We note that these energies are much less than those for secondary leptons but their distribution is very asymmetric; their peak value is $\approx 0.5$ GeV, while their median value is $\approx 5$ GeV. Above 1 GeV, the distribution falls gradually with increasing energy $E(l_3)$, by a factor of 3 from 5 to 15 GeV, and then faster beyond; 25% of the tertiary leptons have $E(l_3) \geq 10$ GeV, but only 9% have energies exceeding 15 GeV. The distribution for the transverse momentum $p_\perp(l_3)$ is shown on Fig.7(b).
It peaks at 0.35 GeV/c and is a little asymmetric; about 30% of the events have \( p_{\perp}(l_3) \geq 0.6 \) GeV/c, about 5% have \( p_{\perp}(l_3) \geq 1.0 \) GeV/c.

We now return to the consideration of event CDF-41540/127085. That the “slow” \( \mu^+ \) lepton is associated with jet 1 is supported by a close examination of the event shown in Fig.10 and Table VII of the CDF paper [4]. There is a displaced vertex shown in the SVX detector, at \( \bar{r} = 0.33 \) cm from the origin of the event. Comparison of the \( \phi \) distribution in their Fig.10(b) with the entry in their Table VII shows us that the secondary vertex shown is associated with the b-jet (jet 1). We are not told where the 8.9 GeV/c \( \mu^+ \) emerged. The two most immediate possibilities are:

(T1) the displaced vertex is a non-leptonic b-decay, the ratio \( \bar{r}/\bar{d}_B \) being 0.26, where \( \bar{d}_B = \gamma_B \tau_B \) is the mean distance of travel by the b-quark before decay, \( \gamma_B \tau_B = 131 \) GeV/c, and \( \tau_B \) being the B-meson lifetime. The resulting c-quark then undergoes decay \( c \rightarrow s \mu^+ \nu_\mu \), leading to a “slow” \( \mu^+ \) which is tertiary. The chance that this c-decay occurs outside the SVX region is about \( e^{-((d-\bar{d}_B)/\bar{d}_D)} \) where \( d = 0.5 \) cm and \( \bar{d}_D = \gamma_D \tau_D \). Taking \( \gamma_D \) to have value about the same as \( \gamma_B \), we then have \( \bar{d}_D \) about 0.37 cm, which gives the chance of the c-quark escaping without detection to be about 40%.

(T2) The vertex observed is a tertiary decay, the slow \( \mu^+ \) being one of the tracks observed (whether or not it is identified) and coming from the transition \( c \rightarrow s \mu^+ \nu_\mu \). The only question is “where is the b-quark decay vertex?” To give rise to what is observed, there should then be a b non-leptonic vertex between the origin and the displaced vertex, but perhaps so close to the tertiary vertex, in view of the rapidity of c-decay relative to b-decay, that it may be difficult to separate the two vertices. Also, in this case, there should necessarily be a \( \mu^+ \) emitted from the displaced vertex, although there is no clear record of this \( \mu^+ \) in the SVX data. It is difficult to estimate the probability for this outcome, without more detailed information. A much closer examination of the SVX data on this event is needed.

Such tertiary leptons will not be rare. The branching fraction (BF) for all leptonic modes is known [4] to be about 21.0(4)% for the b quark and about 23(3)%, on average, for the c quark, assuming that the configurations

\footnote{For the D mesons, the BF’s are 34.4(38)% for \( D^+ \) and 17.7(24)% for \( D^0 \). From their known lifetimes their leptonic decay rates are therefore 3.3(4) \times 10^{11} \) s\(^{-1}\) and 4.3(7) \times 10^{11} \) s\(^{-1}\), respectively, in fair agreement with each other. The well known inequality between their total decay rates (and therefore between their leptonic BF’s) is due to a suppression of the non-leptonic decay modes of \( D^+ \) relative to those for the \( D^0 \).}
(\bar{u}c), (\bar{d}c) and (\bar{s}c) are produced equally often. Neglecting corrections for the efficiencies for detecting SLT’s, generally stated to be about 30% but which may be substantially lower than this for the detection of tertiary leptons, we may estimate that the frequency of tertiary leptons without any secondary lepton is comparable with the frequency of secondary leptons without any tertiary lepton.

However, there is an alternative interpretation possible for the “slow” \( \mu^+ \):

(S\(_d\) or S\(_s\)) The hadronization of the b-jet may lead to a charged \( B^- \) meson or to a neutral meson, \( B^0_d \) or \( B^0_s \). In the latter two cases, the meson may undergo the process of \((B^0_d, \bar{B}^0_d)\) mixing or \((B^0_s, \bar{B}^0_s)\) mixing, and can then emit a \( \mu^+ \) lepton from the secondary process \( \bar{b} \to \bar{c}\mu^+\nu_\mu \), then possible from the \( \bar{b} \)-quark in the \( B^0_d \) or \( B^0_s \) components of the final mixed \((B^0, \bar{B}^0)\)-meson state. From data on b-jet development following the much studied process \( Z^0 \to b\bar{b} \), it is known that the secondary \( \mu^+ \)'s from this source have an intensity 13% of the total from the secondary \((\mu^+ + \mu^-)\) leptons from the initial b-quark. These secondary \( \mu^+ \)'s from mixing will have the same energy spectrum as the \( \mu^- \) secondary leptons from all three kinds of final B-meson, which we have estimated from our model calculation to have the form shown in Fig.6(a), a spectrum much harder than our estimate for the tertiary \( \mu^+ \) spectrum, given in Fig.6(b).

We may now use these calculated probabilities curves to assess the relative likelihood of the two hypotheses, T and S, just discussed above.

(S) \( b \to \bar{b} \to \bar{c}\ell^+ \) and \( b \to c\ell^- \).

As noted above, it is known \cite{14} that the rate for \( l^+ \) is \( \epsilon = 0.13 \) times that for \((l^+ + l^-)\) when the sum is over \( B^0_d \) and \( B^0_s \) mesons. We denote the distribution of the final secondary lepton by \( P_2(E_l) \), shown in Fig.6(a), and the distribution of the secondary lepton momentum transverse to the b-jet axis by \( Q_2(p_{l\perp}) \), shown in Fig.7(a). From ref.\cite{14}, we take \( B_{l\ell} = 0.207 \) for the branching fraction \( (b \to all \ l^\pm)/(all \ b \ decays) \). The net rate for \( l^+ \), occurring as secondary leptons, is given by

\[
R_S = \epsilon \cdot B_{l\ell} \cdot P_2(E_l) \cdot Q_2(p_{l\perp}).
\]

(12) per initial b quark.

However, it is leptonic BF’s which are relevant for discussing the possibilities for tertiary leptons. The leptonic BF’s are not known for \( D^+_s \), only the upper limit, < 20%, but its total lifetime is within three standard deviations of that for \( D^0 \).
(T) $b \rightarrow c \rightarrow l^+$, with no secondary lepton.

Here we ignore the SVX detector, i.e. we do not require the second decay to be visible within it. From ref. \[14\], we take $B_{cl} = 0.34$ as the branching fraction $(c \rightarrow \text{all l}^\pm)/\text{(all c decays)}$. The net rate for $l^+$ is now,

$$R_T = (1 - B_{bl}) \cdot B_{cl} \cdot P_3(E_l) \cdot Q_3(p_{l\perp}).$$ \hspace{1cm} (13)

per initial b quark.

For the event of interest, we have $E_\mu = 8.9$ GeV and $p_{\mu\perp} = 0.60$ GeV/c. The interpretation S that the “slow” $\mu^+$ lepton is due to $(\bar{B}^0, B^0)$ mixing gives the rate per initial b quark as

$$R_S = 0.22 \times 0.13 \times 0.018 \times 0.36 = 4.4 \times 10^{-4}. \hspace{1cm} (14)$$

With the tertiary interpretation T, we have the rate

$$R_T = 0.78 \times 0.33 \times 0.035 \times 1.24 = 1.12 \times 10^{-2}. \hspace{1cm} (15)$$

Hence the calculations with our simple model for the decay sequences $b \rightarrow c l^\pm \nu$ and $b \rightarrow c \rightarrow s l^\pm \nu$ indicate that the likelihood that this $\mu^+$ is tertiary relative to the likelihood that it is secondary - but results from $(\bar{B}^0, B^0)$ mixing - is 25:1. The main factor depressing the rate $R_S$ is the low value for $\epsilon_1$; surprisingly, the observed values for $E_l$ and $p_{l\perp}$ do not distinguish clearly between the possibilities S and T.

It is of interest to compare event 41540/127085 with those SLT $4\text{ljets}$ events reported by CDF, which can be assigned uniquely and kinematically to secondary lepton emission. These are the events where the lepton charge has sign in accord with the decay $b \rightarrow cl^- \bar{\nu}$ (or $\bar{b} \rightarrow c l^+ \nu$, for events which stem from $\bar{t}$ production and decay). We note that the primary lepton energies $E_{1l}$ have a reasonable spread of energies, from 24.7 to 117.3 GeV, as shown in Tables 1 and 2. The two energies $E_{1l}$ in the dilepton events lie at energies in the range $\approx 30 - 70$ GeV. The four “slow” leptons available have energies $E_{2l}$ which range from 2.4 to 14.3 GeV, while the $\mu^+$ energy in event 41540/127085 lies in the middle of this range. The same holds for its $p_{l\perp}$ value. Since the b-jet energy in this event has a surprisingly large value, $(E_{bj} + E(\text{SLT}))$ being $\approx 141$ GeV (overlooking the unknown neutrino energy resulting from this b-decay), we might look instead at the weighted energies $E(\text{SLT})/(E(\text{SLT}) + E_{bj})$ and transverse momenta $p_{l\perp}/(E(\text{SLT}) + E_{bj})$, listed in Table 4. Even
then, their values for 41540/127085 still lie within the ranges obtained for these parameters from the four l4jets events. None of these numbers mark out this SLT event as being obviously different from the other SLT events, except for the charge sign for the “slow” $\mu^+$ and the magnitude of the ratio $R_T/R_S$ discussed above.

Finally, we must compare these four SLT l4jets events with the calculated spectra for our simple ($b \to c l^- \bar{\nu}, b \to c l^+ \nu$) model. Figure 6a shows that the median value predicted for secondary lepton energy $E_{2l}$ lies at $\approx 30 \, GeV$ for the dilepton event 41540/127085, with 130 $GeV$ for jet 1 lab energy $E_{bj}$. The four SLT events have lower b-jet energies, ranging from 39 to 97 $GeV$. This does not effect the $p_{l\perp}$ spectrum, but it alters the $E_{2l}$ spectra. For each event the energy spectrum will depend on the boost from the decaying B rest frame to the lab frame (in which the B meson is a fragment of the b-jet). For the event with the lowest associated b-jet energy, event 43351, the corresponding spectrum will have a median of about 9 $GeV$, compared to the measured $E_{2l} = 2.37 \, GeV$. The median energy grows roughly linearly with jet energy, so the secondary lepton in each of these four events have generally lower energy, $E_{2l}$, than the predicted median. Three of these SLT events have $p_{l\perp}$ values that lie below 0.5 $GeV/c$, whereas our model predicts its median to be $\approx 1.35 \, GeV/c$. The accord of these data with our calculations is neither striking nor unfavourable. It may be that the cuts made on the data by the experimenters, aimed at picking out any background events, have a much larger effect on the predicted curves in Figs.6 and 7 than we have anticipated.

Double-tagging, the combination of the secondary vertex detector (SVX) and the observation of secondary leptons (SLT) together should provide a powerful means for interpretation of the nature of individual events, without a full dynamical analysis (which would at best be possible only rarely). The above analysis of event 41540/127085 illustrates this point quite strongly.

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Table 1. Dilepton event data reported by CDF[4]. Comments on the right are from CDF.

| Run 41540 Event 127085 |   |   |   |
|-------------------------|---|---|---|
|                         |  $p_x$ |  $p_y$ |  $p_z$ | $E$(GeV) |
| $e^-$                   |  18.657 |  11.658 |  20.731 |  30.229 |
| $\mu^+$                 |   46.089 |  11.491 |   8.114 |   48.188 |
| $\mu^+$                 |    8.714 | -1.225 |   1.593 |   8.943 |
| jet 1                   | 129.725 | -18.232 |  14.439 | 131.793 |
| jet 2                   | -49.968 | -34.988 | -34.564 |  70.112 |
| jet 3                   |  -9.740 |  24.107 |-245.219 | 246.593 |
|                         |        |        | backward anti-proton jet | |
| Run 47122 Event 38382  |   |   |   |
|                         |  $p_x$ |  $p_y$ |  $p_z$ | $E$(GeV) |
| $e^+$                   |  45.859 |  21.384 |  54.141 |  74.105 |
| $\mu^-$                 |  37.209 |   2.602 | -30.191 |  47.987 |
| jet 1                   | -66.981 | -52.331 |  58.191 | 103.010 |
| jet 2                   |  24.993 |  -7.167 |  46.244 |  53.052 |
| jet 3                   |  17.303 | -4.962 |-246.138 | 246.795 |
|                         |        |        | backward anti-proton jet | |
Table 2. Single lepton data with one jet tagged as a $b$-jet via the Silicon Vertex Detector (SVX) or the emission of a Soft Lepton (SLT). The $j_1, ...$ labels correspond to CDF’s jet numbers.

| Run | Event | Lepton | $p_x$ | $p_y$ | $p_z$ | $E$(GeV) | Tag |
|-----|-------|--------|-------|-------|-------|---------|-----|
| 40758 | 44414 | $e^+$ | -94.313 | -50.113 | 48.523 | 117.306 | SVX |
|     |       | jet j1 | 86.267 | 26.685 | -21.881 | 92.913 |
|     |       | j2     | -26.220 | 74.310 | 23.996 | 82.373 |
|     |       | j3     | 46.052 | 47.417 | 43.659 | 79.217 |
|     |       | j4     | 30.613 | -22.003 | 76.790 | 85.545 |
| 43096 | 47223 | $e^-$ | 21.753 | 21.093 | -27.316 | 40.795 | SVX |
|     |       | j1     | 78.068 | 100.425 | 2.544 | 127.225 |
|     |       | j2     | -70.137 | 29.785 | 137.091 | 156.845 |
|     |       | j3     | 10.642 | -66.960 | 81.787 | 106.235 |
|     |       | j4     | -34.707 | -14.202 | 138.856 | 143.831 |
| 43351 | 266429 | $\mu^-$ | 24.577 | -1.062 | -1.723 | 24.660 |
|     |       | j1     | 109.365 | -75.333 | 195.687 | 236.494 |
|     |       | j2     | -85.879 | 6.159 | -15.582 | 87.499 |
|     |       | j3     | 24.815 | -21.905 | 7.680 | 33.979 |
|     |       | j4     | -3.595 | 36.122 | 14.128 | 38.953 | SLT |
|     |       | $\mu^-$ | -0.605 | 2.032 | 1.057 | 2.369 | $p_{t\perp} = 0.46$ |
| 45610 | 139604 | $\mu^+$ | 52.325 | 11.153 | -9.682 | 54.369 | SVX |
|     |       | j1     | 11.612 | 76.423 | -58.639 | 97.025 |
|     |       | j2     | -13.843 | -71.064 | -74.320 | 103.755 |
|     |       | j3     | 3.167 | -36.061 | -77.935 | 85.932 |
|     |       | j4     | -19.286 | -9.042 | 1.492 | 21.352 |
| Run 45705 Event 54765 |
|-----------------------|
| \( e^- \) | 12.221 | 54.445 | 42.329 | 70.038 |
| \( j_1 \) | -74.864 | -49.953 | 81.137 | 121.174 |
| \( j_2 \) | -51.229 | 4.189 | -11.399 | 52.649 |
| \( j_3 \) | 15.072 | -54.971 | 41.817 | 70.694 SLT |
| \( j_4 \) | 31.974 | -9.305 | 113.471 | 118.256 |
| \( e^+ \) | 1.523 | -10.995 | 8.984 | 14.280 \( p_{\perp} = 1.62 \) |

| Run 45879 Event 123158 |
|-----------------------|
| \( \mu^+ \) | 52.586 | 4.746 | -11.170 | 53.969 |
| \( j_1 \) | -75.575 | 27.724 | -197.545 | 213.317 |
| \( j_2 \) | 45.871 | -84.446 | -10.592 | 96.682 SVX & SLT |
| \( j_3 \) | 33.259 | 25.812 | 5.488 | 42.456 |
| \( j_4 \) | -36.642 | -4.359 | -16.765 | 40.530 |
| \( \mu^- \) | 6.680 | -11.732 | -1.488 | 13.582 \( p_{\perp} = 0.27 \) |

| Run 45880 Event 31838 |
|-----------------------|
| \( e^- \) | -5.942 | -25.106 | 4.146 | 26.131 |
| \( j_1 \) | 98.037 | -7.188 | -19.791 | 100.273 |
| \( j_2 \) | -26.071 | -55.037 | 80.329 | 100.804 |
| \( j_3 \) | 18.082 | 39.344 | 16.853 | 46.464 SLT |
| \( j_4 \) | -22.051 | -13.776 | -16.246 | 30.658 |
| \( e^+ \) | 0.838 | 2.440 | 1.088 | 2.800 \( p_{\perp} = 0.27 \) |
Table 3. Output from our analysis of 7 single lepton events. CDF’s fitted values\cite{3} using a kinematical program\cite{5} are listed below their preferred jet assignment.

| jets          | IP     | $m_t$  | Best Fit       | CDF $m_t$ | CDF $\chi^2$ |
|---------------|--------|--------|----------------|-----------|--------------|
| $(b_t,q_1,q_2,b_j)$ | $(x,\bar{x})$ | $m(tt)$ |                |           |              |
| Event 40758   |        |        |                |           |              |
| $(j_4,j_2,j_3,j_1)$ | $4.1 \times 10^{-4}$ | $170^{+13}_{-9}$ | $(0.395,0.201)$ | 507        | 172 ± 11     | 0.1 |
| $(j_2,j_3,j_4,j_1)$ | $3.7 \times 10^{-6}$ | 184     | $(0.481,0.177)$ | 526        | 498          |     |
| Event 43096   |        |        |                |           |              |
| $(j_1,j_3,j_4,j_2)$ | $3.0 \times 10^{-6}$ | $162^{+8}_{-4}$ | $(0.521,0.139)$ | 484        | 166 ± 11     | 2.0 |
| $(j_4,j_2,j_3,j_1)$ | $5.3 \times 10^{-8}$ | 224     | $(0.529,0.152)$ | 511        | 481          |     |
| Event 43351   |        |        |                |           |              |
| $(j_2,j_1,j_3,j_4)$ | $5.0 \times 10^{-7}$ | $160^{+12}_{-6}$ | $(0.447,0.168)$ | 493        | 158 ± 18     | 6.1 |
| Event 40758   |        |        |                |           |              |
| $(j_2,j_3,j_4,j_1)$ | $1.6 \times 10^{-4}$ | $180^{+7}_{-13}$ | $(0.110,0.379)$ | 367        | 180 ± 9      | 5.0 |
| $(j_1,j_3,j_4,j_2)$ | $8.2 \times 10^{-6}$ | 112     | $(0.093,0.446)$ | 365        | 369          |     |
| Event 45705   |        |        |                |           |              |
| $(j_3,j_1,j_2,j_4)$ | $4.2 \times 10^{-4}$ | $190^{+14}_{-14}$ | $(0.409,0.148)$ | 443        | 188 ± 19     | 0.4 |
| $(j_4,j_1,j_2,j_3)$ | $1.1 \times 10^{-5}$ | 156     | $(0.593,0.097)$ | 430        | 411          |     |
| Event 45879   |        |        |                |           |              |
| $(j_2,j_3,j_4,j_1)$ | $3.8 \times 10^{-4}$ | $179^{+12}_{-10}$ | $(0.140,0.406)$ | 423        | 438          |     |
| $(j_2,j_1,j_4,j_3)$ | $2.5 \times 10^{-4}$ | 180     | $(0.131,0.452)$ | 438        | 396          |     |
| $(j_1,j_3,j_4,j_2)$ | $5.4 \times 10^{-6}$ | 168     | $(0.130,0.273)$ | 396        | 419          | 2.2 |
| Event 45880   |        |        |                |           |              |
| $(j_1,j_2,j_4,j_3)$ | $4.3 \times 10^{-4}$ | $164^{+15}_{-10}$ | $(0.225,0.176)$ | 358        | 358          |     |
| $(j_2,j_1,j_4,j_3)$ | $3.2 \times 10^{-4}$ | $134^{+6}_{-8}$ | $(0.295,0.358)$ | 358        | 365          | 1.7 |
Table 4. Events available having a secondary or tertiary lepton. Energies in GeV, momenta in GeV/c. Bracket denotes possible tertiary lepton.

| Event | 43351 | 45705 | 45879 | 45880 | 41540 |
|-------|-------|-------|-------|-------|-------|
| $E_{1l}$ | 24.7 | 70.0 | 54.0 | 26.1 | 48.2 |
| $E_{bj}$ | 39.0 | 70.7 | 96.7 | 46.5 | 131.8 |
| $E_{2l}$ | 2.37 | 14.28 | 13.58 | 2.80 | (8.9) |
| $E_{2l}/(E_{2l} + E_{bj})$ | 5.7% | 16.0% | 12.6% | 5.7% | (6.8%) |
| $p_{l\perp}$ | 0.46 | 1.615 | 0.27 | 0.27 | 0.60 |
| $p_{l\perp}/(E_{2l} + E_{bj})$ | 1.1% | 1.45% | 0.25% | 0.55% | (0.43%) |
FIGURE CAPTIONS

1. P(m_{t}) plots (not normalized) for the published dilepton events.
2. P(m_{t}) plots for the published events of type “l\pm + 4jets” interpreted as examples of t\bar{t} production and decay.
3. Combined P(m_{t}) plots for (i) the dilepton events, (ii) the “l\pm + 4jets” events, and (iii) all of these events, taken together.
4. Scatter plot for “m_{t} vs. Log.(integrated probability)” for events generated by Monte Carlo calculations, using QCD tree graphs for m_{t} = 170 GeV, using only the right final configurations in analysing 100 randomly chosen MC events.
5. The projection of the scatter plot of Fig.4 onto the Log.(Int.Prob.) axis.
6(a). The energy distribution in the Lab. frame, for the secondary leptons resulting from the decay b \rightarrow c + l^- + \nu_{l}, for a b-quark jet of initial energy 130 GeV.
6(b). The energy distribution in the Lab. frame, for tertiary leptons resulting from the decay c \rightarrow s + l^+ + \nu_{l}, for a b-quark jet of initial energy 130 GeV.
7(a). The distribution of the momentum transverse to the b-jet axis, for secondary leptons resulting from the decay b \rightarrow c + l^- + \nu_{l}, for a b-quark jet of initial energy 130 GeV.
7(b). The distribution of the momentum transverse to the b-jet axis, for tertiary leptons resulting from the decay c \rightarrow s + l^+ + \nu_{l}, for a b-quark jet of initial energy 130 GeV.
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