Impact of the Seventh Harmonic Magnitude for an Optimal Control of Five-phase PMSM

A Dieng
University of Cheikh Anta Diop de Dakar – ESP, Laboratory LER, Senegal
Institut de recherche en Energie electrique de Nantes Atlantique, IREENA, Saint Nazaire, France

abdoulaye.dieng@esp.sn

Abstract. In this paper, the impact of the seventh harmonic magnitude for an optimal torque control of 5-phase PMSM is studied. We showed that the machine evolved in two orthogonal frames and can be decomposed into two fictitious machines called respectively main machine, secondary machine. As the two frames are fully decoupled, the torque control of each fictitious machine is done separately. The goal is to maximize the average torque, to minimize the copper losses and to reduce the torque ripples. The main machine is not impacted but in the secondary machine, it is possible to know the variation of the torque ripples and the copper losses by modifying the imposed current references and taking into account the seventh harmonic of the EMF. An itemized analysis of the impact of the EMF seventh harmonic magnitude is done. Simulation results highlight the performances of the torque control strategy under using a robust controller.

1. Introduction
Polyphase electrical drives offer many advantages. Based on harmonic analysis, it is shown that the five-phase PMSM can be decomposed in three fictitious decoupled machines as proposed in [1][2][3]. Each one is described by one phase electrical circuit. The current quality of the electrical drive depends on the supply mode. Nevertheless, the non-sinusoidal character of its EMF required an optimal control strategy of the torque. Control strategy of five-phase PMSM have already been proposed in [4][5][6][7][8][9][10][11]. The optimization of the torque is linked to the exploitation of the so called secondary machine as well the main machine. The torque control of each fictitious machine is done separately. Thus, a good torque repartition between the different fictitious machines is done. The goal is to maximize the average torque, to minimize the copper losses and to reduce the torque ripples. Here, the main machine is not impacted but the problem in the secondary machine due to the seventh harmonic of the EMF.
In this paper, the impact of the EMF seventh harmonic magnitude is investigated and the torque control of the five-phase PMSM by taking into account the seventh harmonic of the EMF is done. It is possible to know the variation of the torque ripple and the copper losses by taking into account the seventh harmonic magnitude. An itemized analysis is done. Simulation results is presented.
2. Dynamical model in view of control [1]

The dynamical model of this machine in dq frames is given by [1] [2] [3]. For the main machine, the next equations are obtained:

\[
V_{dp} = R I_{dp} + L_p \frac{di_{dp}}{dt} - \omega L_p I_{qp} + E_{dp}
\]

\[
V_{qp} = R I_{qp} + L_p \frac{di_{qp}}{dt} + \omega L_p I_{dp} + E_{qp}
\]

(1)

For the secondary machine, the following equations are obtained:

\[
V_{ds} = R I_{ds} + L_s \frac{di_{ds}}{dt} - 3\omega L_s I_{qs} + E_{ds}
\]

\[
V_{qs} = R I_{qs} + L_s \frac{di_{qs}}{dt} + 3\omega L_s I_{ds} + E_{qs}
\]

(2)

The motor’s torque is the sum of the two torques produced by the two fictitious machines and it is given by:

\[
\Gamma = \Gamma_p + \Gamma_s
\]

Where \(\Gamma_p\): torque produced by the main machine, \(\Gamma_s\): torque produced by the secondary machine.

Generally, they are given by the following expressions:

\[
\Gamma_p = \frac{E_{dp} I_{dp} + E_{qp} I_{qp}}{\Omega}
\]

(4)

\[
\Gamma_s = \frac{E_{ds} I_{ds} + E_{qs} I_{qs}}{\Omega}
\]

(5)

An experimental prototype has been developed in our laboratory. Figure 1 represents the EMF normalized profile of the considered five-phase PMSM.

![EMF profile, 5-phase PMSM](image)

**Figure 1. EMF profile, 5-phase PMSM**

The Fourier analysis of the EMF of this machine is summarized in Table 1. It may be noted that the amplitude of harmonics, particularly the third one, are important and can produce torque.

| (a) main machine | (b) secondary machine |
|------------------|-----------------------|
| EMF Harmonic     | 1  | 9   |
| Magnitude/Fundamental % | 100% | 0.7% |
| EMF Harmonic     | 3  | 7   |
| Magnitude/Third harmonic % | 100% | 20% |

Other Harmonics are equal to zero.

Table 1 summarizes the normalized magnitude of each harmonic of the considered machine. The fundamental and the ninth harmonics project in the main machine, the third and seventh harmonic in the secondary machine. The fifth harmonic is the homopolar and it has no influence because the neutral is not connected. The Fourier analysis of the EMF shows that the ninth harmonic is very low and can be neglected. In this case the main machine can be considered such as a sinusoidal two-phase machine in the \((\alpha_p, \beta_p)\) frame. Thus the same control strategies used for a sinusoidal three-phase machine in the dq frame can be applied. Regarding the secondary machine, the third harmonic contributes to the production of the torque while the seventh harmonic generates a pulsating torque.
and copper losses. This secondary machine behaves as a three-phase machine, non-sinusoidal EMF where the electrical pulse is $3\omega$. Therefore, the control strategy of the torque needed a repartition of the reference torque between the two machines.

Neglecting the ninth harmonic which is low, the profile of the EMF in both dq frames is shown in Figure 2. The $E_{ds}$ component of the EMF is oscillating around zero and $E_{qs}$ is oscillating around constant value.

![Figure 2. EMF waveforms in the dq frames, 5-phase PMSM](image)

As shown in previous Equations (1) and (2) the main and secondary machines are decoupled. As the EMF’s components in the Park frame of the main machine are constants, the torque control of this machine is similar to a classical sinusoidal three-phase machine. The difficulty is the control of the secondary machine.

3. Torque control of the secondary machine

As shown in Figure 2 the EMF’s components in the secondary dq frame are sinusoidal where the frequency is ten times higher than the fundamental frequency.

In order to minimize the copper losses and maximize the torque, the EMF and current vectors of the machine must be collinear:

$$E_{ds}I_{qs} = E_{qs}I_{ds}$$  \hspace{1cm} (6)

$$I_{ds} = \frac{E_{ds}}{E_{qs}}I_{qs}$$  \hspace{1cm} (7)

By introducing references values in Equation (5) and based on Equation (7), the current references can be deduced:

$$I_{dsref} = \frac{E_{ds}}{E_{qs}^2 + E_{qs}^2} I_{sref} \Omega = K_{ds}(\theta)I_{sref}$$  \hspace{1cm} (8)

$$I_{qref} = \frac{E_{qs}}{E_{qs}^2 + E_{ds}^2} I_{sref} \Omega = K_{qs}(\theta)I_{sref}$$  \hspace{1cm} (9)

Where $I_{sref}$ is the reference torque in the secondary machine.

Equations (8) and (9) show that the current references $I_{dsref}$ and $I_{qref}$ are not constants because the EMF components of the secondary machine in the dq frame are not constants where the frequency is ten times higher than the fundamental frequency. By imposing constants currents, the torque is oscillating. Then of constant torque desired the current references ($I_{dsref}$ and $I_{qref}$) are in function of $\theta$.

4. Torque repartition

The global copper losses are given by:

$$\Gamma_j = R\left(I_{qP}^2 + I_{qS}^2 + I_{ds}^2\right)$$  \hspace{1cm} (10)

Taking into account Equation (5), Equation (3) becomes:

$$\Gamma = \Gamma_p + \Gamma_s = \frac{1}{\mu} \left[E_{qp}I_{qp} + \frac{E_{ds}^2 + E_{qs}^2}{E_{ds}} I_{qs}\right]$$  \hspace{1cm} (11)
Based on Equations (10) and (11), for given torque, the minimum of the copper losses is obtained by differentiating Eq. (10) and it is reached when:

\[ x = \frac{I_{qs}}{I_{qp}}, \quad y = \frac{I_{ds}}{I_{qp}} = \frac{E_{ds}}{E_{qp}} \]  

(12)

Thus the reference torques in the main and secondary machines are obtained:

\[ \Gamma_s = \frac{E_{ds} I_{ds} + E_{qs} I_{qs}}{E_{qp} I_{qp} + E_{ds} I_{ds} + E_{qs} I_{qs}} = \frac{x^2 + y^2}{1 + x^2 + y^2} \]  

(13)

\[ \Gamma_p = \frac{E_{qs} I_{qp}}{E_{qp} I_{qp} + E_{ds} I_{ds} + E_{qs} I_{qs}} = \frac{1}{1 + x^2 + y^2} \]  

(14)

\( E_{ds} \) and \( E_{qs} \) are not constants therefore there are two possibilities: a repartition to the average values of the torque or a repartition to the instantaneous values of the torque.

In the case of a repartition to the instantaneous values of the torque, the both torque references, in the main machine and the secondary machine, are oscillating and are in opposite phase. The total torque is constant. The current reference \( I_{qr} \) of the main machine becomes not constant due to the reference torque ripples. Then it will be necessary to have three high dynamical performance controllers to control the currents. The control strategy becomes more and more complex.

A repartition to the average values of the torque is better suited and easy. The control of the main machine is very easier and need to use a simple controller. The problem arises in the secondary machine. Even if its torque reference is constant, the current references are not constants due to the EMF components in the secondary dq frame. Equations (8) and (9) give the optimal current references. Thus a robust controller is needed. It must ensure reference tracking without affecting the signal magnitudes or introduce phase shift. If these conditions are not respected, the torque is oscillating and also for the total torque. Therefore, a good torque control strategy requires a good vectorial control of the secondary machine. Here a fractional order regulator is used as proposed and studied in [12][13][14].

5. Impact of the seventh harmonic magnitude

In this part, we propose to study the impact of the seventh harmonic magnitude on the torque ripples and the copper losses. In the secondary machine, it is possible to know the variation of the torque ripples and the copper losses by modifying the imposed current references. The expressions of \( E_{ds} \) and \( E_{qs} \) can be written in the secondary dq frame (where the number of pole pairs=1):

\[
\begin{bmatrix}
E_{ds} \\
E_{qs}
\end{bmatrix} = \frac{\sqrt{2}}{2} P(-3\theta) [T_s]^1 [E]_s
\]  

(15)

The harmonics that are projected in the secondary machine are the third harmonic and the seventh ones. The positive sequence is indexed on the third harmonic. The seventh harmonic rotates in the opposite direction.

\[
[E]_s = E_3 [T_s] P(3\theta) \begin{bmatrix} 0 & 1 \end{bmatrix} + E_7 [T_s] P(-7\theta) \begin{bmatrix} 0 & -1 \end{bmatrix}, \text{ it becomes:}
\]

\[
\begin{bmatrix}
E_{ds} \\
E_{qs}
\end{bmatrix} = \frac{\sqrt{2}}{2} E_3 P(-3\theta) [T_s]^1 [T_s] P(3\theta) \begin{bmatrix} 0 & 1 \end{bmatrix} + \frac{\sqrt{2}}{2} E_7 P(-3\theta) [T_s] [T_s] P(-7\theta) \begin{bmatrix} 0 & -1 \end{bmatrix}
\]  

(16)

\[
\begin{bmatrix}
E_{ds} \\
E_{qs}
\end{bmatrix} = \frac{\sqrt{2}}{2} E_3 P(0) \begin{bmatrix} 0 & 1 \end{bmatrix} + \frac{\sqrt{2}}{2} E_7 P(-10\theta) \begin{bmatrix} 0 & -1 \end{bmatrix}
\]  

(17)

After developing the following equations are obtained:

\[ E_{ds} = k_7 \Omega \sin(100) \]  

(18)

\[ E_{qs} = k_3 \Omega + k_7 \Omega \cos(100) \]  

(19)

Where \( k_3 \Omega = \frac{\sqrt{2}}{2} E_3 \), \( k_7 \Omega = -\frac{\sqrt{2}}{2} E_7 \).

By substituting (18) and (19) in (8) and (9), Equations (8) and (9) become:
In order to see the impact of a bad estimation of the seventh harmonic magnitude a variable $k \in [0, 1]$ is introduced and the current references in the secondary machine can be modifying:

$$
I_{dsref} = \frac{k_7 \sin(100)}{k_3^2 + k_7^2 + 2k_3 k_7 \cos(100)} \Gamma_{sref}
$$

(20)

$$
I_{qref} = \frac{k_3 + k_7 \cos(100)}{k_3^2 + k_7^2 + 2k_3 k_7 \cos(100)} \Gamma_{sref}
$$

(21)

By substituting (18), (19), (22) and (23) in (5), equation (5) becomes:

$$
\Gamma_s(0) = \frac{k_3^2 + k_7^2 + (1+k)k_3 k_7 \cos(100)}{k_3^2 + k_7^2 + 2k_3 k_7 \cos(100)} \Gamma_{sref}
$$

(24)

The torque ripples can be calculated and it is given by:

$$
\Delta \Gamma_s = \Gamma_{smax} - \Gamma_{smin}
$$

(25)

After differentiating Equation (24), sup and inf values of $\Gamma_s$ are obtained for $\theta = \frac{\pi}{10}$ and $\theta = 0$ respectively. Where $k_7 < 0$.

Equation (25) becomes:

$$
\frac{\Delta \Gamma_s}{\Gamma_s} = \frac{k_3^2 + k_7^2 + (1+k)k_3 k_7 \cos(100)}{k_3^2 + k_7^2 + 2k_3 k_7 \cos(100)} - \frac{k_3^2 + k_7^2 + (1+k)k_3 k_7 \cos(100)}{k_3^2 + k_7^2 + 2k_3 k_7 \cos(100)}
$$

(26)

Where $\Gamma_s = \Gamma_{sref}$ is the reference average torque.

From the Equations (22) and (23) the copper losses in the secondary machine are given by:

$$
\frac{<P_{ls}^s>}{R} = \frac{<I_{dsref}^2 + I_{qref}^2>}{R}
$$

(27)

$$
\frac{<P_{ls}^s>}{R} = \frac{<\frac{1}{k_3^2 + k_7^2 + 2k_3 k_7 \cos(100)>}^{sref}}{R}
$$

(28)

$$
\frac{<P_{ls}^s>}{R} = \frac{1}{2} \left[ \sup \left( \frac{P_{ls}^s}{R} \right) + \inf \left( \frac{P_{ls}^s}{R} \right) \right]
$$

(29)

sup and inf are obtained by differentiating Equation (28) and the result is:

$$
\frac{<P_{ls}^s>}{R} = \frac{k_3^2 + k_7^2}{[k_3^2 + k_7^2]^2 - [2k_3 k_7]^2} \Gamma_{sref}^2
$$

(30)

The following figures show the torque ripples variation and the copper losses variation as a function of $k$ for different values of the seventh harmonic after simulation.

Where $z = \frac{P_{ls}^s}{P_{ls}^s}$, $A = \Delta \Gamma_s$, $B = R \Gamma_{sref}^2$.

![Figure 3. Variation of the torque ripples versus k and z](image-url)
Figures 3 and 4 show that if the ratio z is low, even with a very bad estimation of the variable k (very bad estimation of the seventh harmonic magnitude), the torque ripples and the copper losses are also low. Nevertheless, if the ratio z increases a very bad estimation of the variable k implies a considerable increase in torque ripples and copper losses.

6. Simulation Results

Figure 5 represents the adopted control scheme for this control strategy.

**Figure 4. Variation of the copper losses versus k and z**

**Figure 5. Torque control scheme in the dq frame**
Figure 6 shows $I_{ds\text{meas}}$ and $I_{qs\text{meas}}$ currents compared with reference currents $I_{ds\text{ref}}$ and $I_{qs\text{ref}}$ using respectively classic PI and PI$^d$ controller. Figure 6(b) show a good tracking performance of the PI$^d$ controller used to control the imposed sinusoidal current in dq frame. Figure 7 shows the electromagnetic torque in the secondary machine using respectively classic PI and PI$^d$ controller. We can note that the torque ripples are vanished (see Figure 7(b)). Figure 8 shows the currents and the electromagnetic torque in the main machine using classic PI.

![Figure 6. Currents in the secondary machine](image1)

![Figure 7. Electromagnetic Torque in the secondary machine](image2)

![Figure 8. Currents and Electromagnetic Torque in the main machine](image3)
Besides, as it can be observed from these figures, the waveforms of all the controlled output currents show the robustness of the proposed controller under the non-sinusoidal character of the EMF. One current in abcde frame and its FFT is shown in Figure 9. We can notice the onset of the thirteen harmonic. This harmonic projects into the secondary machine. Figure 10 shows that the current locus in αβ frames.

![Figure 9. Currents in abcde frame and its FFT](image)

**Figure 9. Currents in abcde frame and its FFT**

![Figure 10. Current locus in αβ frames](image)

**Figure 10. Current locus in αβ frames**

7. Conclusion

In this paper, an impact of the seventh harmonic magnitude for an optimal control of 5-phase PMSM is studied. For a good torque control a repartition of the reference torque between the two machines has been done. Simulation results prove the effectiveness of the torque control strategy. The torque ripples are vanished and we have a good current regulation. In conclusion, the analysis has shown that the minimization of the torque ripples and the copper losses depends on a good estimation of the seventh harmonic magnitude of the EMF. The obtained results highlight the performances of the Fractional controller used to control the imposed optimal current and the robustness of the proposed controller under the non-sinusoidal character of the EMF. In the case of a fault operation mode, it will be interesting to apply our approach with the proposed control strategy.
8. References

[1] Dieng A, Le Claire J C, Mboup A B, Benkhoris M F and Ait-Ahmed M, *Analysis of five-phase permanent magnet synchronous motor*, Revue Roumaine Sciences Techniques – Electrotechnique et Énergétique. Vol. 61, 2, pp. 116–120, Bucarest, 2016

[2] Kestelyn X, Semail E and Hautier J P, *Vectorial Multi-machine modeling for a five-phase machine*, International Congress on Electrical Machines, ICEM, Belgium, 2002, CD-ROM

[3] Robert-Dehault E, Benkhoris M F and Zaim M E, *Control strategy of a five-phase synchronous machine*, Second International Conference on Power Electronics, Machines and drives, University of Edinburgh, UK, 2004, (CD-ROM)

[4] Toliyat H. A, *Analysis and simulation of five-phase variable-speed induction motor drives under asymmetrical connections*, IEEE Transactions on Power Electronics, Vol. 13, No. 4, pp. 748-756, 1998

[5] Zong Z L, Wang K, Zhang J. Y, *Control strategy of five-phase PMSM utilizing third harmonic current to improve output torque*, 2017 Chinese Automation Congress (CAC), 20-22 Oct. 2017, Jinan, China

[6] Baudart F, Dehez B, Matagne E, Telteu-Nedelcu D, Alexandre P and Labrique F, *Torque control strategy of polyphase permanent-magnet synchronous machines with minimal controller reconfiguration under open-circuit fault of one phase*, IEEE Transactions on Industrial Electronics, Vol. 59, Iss.6, pp. 2632-2644, 2012

[7] Baudart F, Matagne E, Dehez B and Labrique F, *Optimal current waveforms for permanent magnet synchronous machines with any number of phases in open circuit*, Mathematics and Computers in Simulation, Elsevier, Vol. 90, pp.1-14, 2013

[8] Kestelyn X and Semail E, *A vectorial approach for generation of optimal current references for multiphase permanent-magnet synchronous machines in real time*, IEEE Transactions on Industrial Electronics, Vol. 58, No. 11, pp. 5057–5065, 2011

[9] Dieng A, Le Claire J C, Mboup A B, Benkhoris M F and Ait-Ahmed M, *An improved torque control strategy of five-phase PMSG-PWM rectifier set for marine current turbine applications*, 2019 IEEE 13th International Conference on Compatibility, Power Electronics and Power Engineering, CPE-POWERENG 2019, Sonderborg, Denmark

[10] Guibin L, Jiefeng H, Yongdong Li and Jianguo Zhu, *An Improved Model Predictive Direct Torque Control Strategy for Reducing Harmonic Currents and Torque Ripples of Five-Phase Permanent Magnet Synchronous Motors*, IEEE Transactions on Industrial Electronics Vol. 66, Issue 8, pp. 5820 – 5829, 2019

[11] Dieng A, Benkhoris M F, Ait-Ahmed M and Le Claire J C, *Fault-Tolerant Control of 5-Phase PMSG for Marine Current Turbine Applications Based on Fractional Controller*, 19th IFAC World Congress 2014, Vol 19, pp. 11950-11955, August 2014.

[12] Cao J Y, Liang J and Cao B G, *Optimization of fractional order PID controllers based on genetic algorithms*, Proceedings of the Fourth International Conference on Machine Learning and Cybernetics, Guangzhou, China, pp. 5686-5689, 2005

[13] Dieng A, Benkhoris M F and Ait-Ahmed M, *Torque Ripples Reduction of five phases PMSM using Fractional Order Regulator*, XXth International Conference on Electrical Machines, ICEM, Marseille, France, pp. 1114-1120, 2012

[14] Charef A, *Analogue realization of fractional-order integrator, differentiator and fractional – controller*, The Institution of Engineering and Technology, IEE Proceedings Control Theory and Application, Vol. 153, Iss. 6, pp. 714-720, 2006.