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Nonlinear adaptive control of COVID-19 with media campaigns and treatment

Boqiang Caoa, b, Ting Kanga, *

a Xinhua College, Ningxia University, Yinchuan, 750021, China
b School of Mathematics, Southeast University, Nanjing, 211189, China

Article info
Article history:
Received 29 December 2020
Accepted 30 December 2020
Available online 9 January 2021

Keywords:
Nonlinear adaptive control
COVID-19
Media campaigns
Treatment
Lyapunov stability

Abstract
Coronavirus disease 2019 (COVID-19) is an infectious disease caused by the infection of severe acute respiratory syndrome coronavirus 2, which is spreading all over the world and causing huge human and economic losses. For these reasons, we study the adaptive control problem of COVID-19 in consideration of media campaigns and treatment in this paper. Firstly, a novel compartment model is constructed by analysing the spread mechanism of COVID-19 and a nonlinear adaptive control problem is established. Then, using the estimation of parameters updated by adaptive laws, the controllers are designed to achieve the control goals. Finally, numerical examples are presented to illustrate the control capability to the outbreak of COVID-19.

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1. Introduction

Since the first COVID-19 case was diagnosed in Wuhan of Hubei Province, China at the end of December 2019, the disease has rapidly spread all over the world and caused heavy human and economic losses, which has been declared as a global pandemic in early March 2020 [1]. Human connectivity, international travel and other factors have brought difficulties to the prevention and control of COVID-19. Currently, the effective way to control the spread of COVID-19 is to combine non-pharmaceutical interventions (quarantines, social distancing policies, curfews, blockades, contact tracing and travel suspensions) with pharmaceutical interventions due to the lack of a fully effective treatment or vaccination [2,3]. Therefore, understanding the effects of these measures on COVID-19 transmission are crucial for curbing the spread of the virus [4–9].

Recently, many scholars have studied the optimal control of COVID-19 by applying the Pontryagin’s maximum principle through introducing different control variables, and given the corresponding control strategies [10–13]. In fact, the optimal control theory can only aim at the systems with known parameters and obtain the system inputs by minimizing the cost function. It is worth noting that there are various uncertainties in the transmission of COVID-19. If there are uncertain parameters in the system, it is impossible for optimal control theory to obtain the desired results, which need to identify the system parameters in advance. Fortunately, adaptive control can update the system parameters by using adaptive laws and guarantee the stability of closed-loop system.

Motivated by the above discussions, we first propose a SEIHR COVID-19 model with media campaigns and treatment by analysing the transmission mechanism. Then, the adaptive controllers are designed and the estimation values of unknown parameters are obtained based on the designed adaptive laws. The rest of this article is organized as follows: In Section 2, the COVID-19 model is proposed and the adaptive control problem is formulated. In Section 3, we analyse the stability of closed-loop system under the designed controllers and adaptive laws. The effectiveness of the designed controllers is illustrated in Section 4. Finally, a brief summary is presented in Section 5.

2. Mathematical formulation of COVID-19 model

In this section, according to the transmission mechanism of COVID-19, we construct the mathematical model and give some explanations to the model parameters. We divide the total human population into five compartments, named the SEIHR model: the number of susceptible, exposed, infected, hospitalized and recovered individuals at time $t$ are denoted by $S(t), E(t), I(t), H(t)$ and...
R(t), respectively, which are abbreviated as S, E, I, H and R in the following for simplicity. Based on the above discussions, the mathematical model is given as follows

\[
\begin{align*}
  \dot{S}(t) &= \Delta - \beta (1 - \theta) S(t) I(t) + \eta E(t) - (\mu + \epsilon_1 u_1(t)) S(t) + \sigma R(t), \\
  \dot{E}(t) &= \beta (1 - \theta) S(t) I(t) + \eta E(t) - (\delta + \mu + \epsilon_1) E(t), \\
  \dot{I}(t) &= \delta E(t) - (\gamma_1 + \gamma_2 + \mu + d_2) I(t), \\
  \dot{H}(t) &= \gamma_2 I(t) - (\gamma_3 + \mu + d_3 + 2\epsilon_2 u_2(t)) H(t), \\
  \dot{R}(t) &= \gamma_1 I(t) + \epsilon_1 u_1(t) (S(t) + (\gamma_3 + 2\epsilon_2 u_2(t)) H(t) - (\mu + \sigma) R(t).
\end{align*}
\]

(1)

All parameters in model (1) are assumed non-negative and described as follows: \(\Delta\) and \(\mu\) are enrolling rate and natural death rate; \(\beta\) is infective rate from S to E; \(d_2(i = 1, 2, 3)\) represent disease-related death rates; \(\delta\) is transmission rate from E to I; \(\sigma\) denotes recovery rate from R to S; \(\gamma(i = 1, 2, 3)\) are the transmission rate from I to R, from I to H, and from H to R, respectively. \(\theta\) is the homestead-isolation rate of the susceptible; \(\eta\) denotes the infective effect of the exposed in incubation period. The control variables \(u_1(t)\) and \(u_2(t)\) in model (1) to reduce the numbers of susceptible and hospitalized individuals, represent the use of media campaigns and treatment, respectively. The control variables are explained as follows:

(i) The control variable \(u_1\) represents the media campaigns for the susceptible. The high-density information dissemination can make people change their behaviour to reduce the contact with others, so as to avoid being infected. In other words, those who change their behaviour are “immune” to the COVID-19 and they move from S to R, i.e., the term \(u_1 S\) in model (1). In addition, considering that different individuals have different sensitivities to media campaigns, we introduce \(\epsilon_1 \in [0, 1]\) to describe the proportion of susceptible individuals who respond to media campaigns.

(ii) The control variable \(u_2\) represents the treatment to hospitalized individuals. Although there is no specific drug for the COVID-19, some existing drugs and treatment measures, such as interferon and Chinese traditional medicine, are effective in treating infected individuals. Meanwhile, taking timely and effective treatment measure can greatly reduce the risk of death. Thus, we choose the treatment to the hospitalized individuals as another control measure (i.e., term \(u_2 H\)). Due to the individual differences of patients, the same treatment measure for different patients will produce different curative effects, so we introduce the effective rate of treatment \(\epsilon_2 \in [0, 1]\) to describe the therapeutic effect.

\[
\begin{align*}
  \Phi_1 &= \begin{bmatrix}
    x_1 S - I, & -E, & R - 1
  \end{bmatrix}^T, \\
  \Phi_2 &= \begin{bmatrix}
    x_2 S - I, & -H, & R - 1
  \end{bmatrix}^T, \\
  \xi_1 &= \begin{bmatrix}
    \alpha \\ x_1 i_1 \\ \beta \eta (1 - \theta) \gamma_1 \\ \mu \\ \gamma_2 \\
  \end{bmatrix}^T, \\
  \xi_2 &= \begin{bmatrix}
    \gamma_3 + \mu + d_3
  \end{bmatrix}^T.
\end{align*}
\]

(7)

3. Development of adaptive control scheme for model (1)

In this section, we give the form of control variables \(u_1\) and \(u_2\) by using the nonlinear adaptive control method. In traditional adaptive control problem, the control variables will be designed such that the state or output of system can track the preset trajectories with small tracking errors, which provide a way to decrease the numbers of susceptible and hospitalized individuals to reach the desired values.

In order to achieve the control goals, \(u_1\) and \(u_2\) should be adjusted dynamically according to state variables in system (1). From the first and forth equations of (1), the control inputs can be formulated as

\[
u_1 = -\frac{\dot{S}}{\epsilon_1 S} - \frac{\Delta}{\epsilon_1 S} \frac{1}{\epsilon_1} \frac{\beta (1 - \theta)}{\epsilon_1} I - \frac{\beta \eta (1 - \theta)}{\epsilon_1} E + \frac{\sigma R}{\epsilon_1 S} \frac{\mu}{\epsilon_1}
\]

(2)

\[
u_2 = -\frac{\dot{H}}{\epsilon_2 H} + \frac{\gamma_2 I}{\epsilon_2 H} \frac{\gamma_3 + \mu + d_3}{\epsilon_2}
\]

(3)

However, in the above forms of \(u_1\) and \(u_2\), since \(\dot{S}\) and \(\dot{H}\) are usually difficult to obtain, we replace them by “equivalent inputs” \(x_1\) and \(x_2\) [14]:

\[
u_1 = -\frac{x_1}{\epsilon_1 S} \frac{\Delta}{\epsilon_1 S} \frac{1}{\epsilon_1} \frac{\beta (1 - \theta)}{\epsilon_1} I - \frac{\beta \eta (1 - \theta)}{\epsilon_1} E + \frac{\sigma R}{\epsilon_1 S} \frac{\mu}{\epsilon_1}
\]

(4)

\[
u_2 = -\frac{x_2}{\epsilon_2 H} + \frac{\gamma_2 I}{\epsilon_2 H} \frac{\gamma_3 + \mu + d_3}{\epsilon_2}
\]

(5)

where the variables \(x_1\) and \(x_2\) are chosen as

\[
x_1 = \hat{S}_d - \alpha_1 \hat{S}, \quad x_2 = \hat{H}_d - \alpha_2 \hat{H}
\]

(6)

Here \(\hat{S}_d\) and \(\hat{H}_d\) denote the desired values, \(\hat{S} = S - \hat{S}_d\) and \(\hat{H} = H - \hat{H}_d\) are the tracking errors to \(\hat{S}_d\) and \(\hat{H}_d\), respectively, and \(\alpha_1, \alpha_2 > 0\) are designed parameters. It follows from (6) that \(\hat{S} = \hat{S}_d - \alpha_1 \hat{S}\) and \(\hat{H} = \hat{H}_d - \alpha_2 \hat{H}\), i.e., \(\hat{S}(t) = \hat{S}_d(t) e^{-\alpha_1 t}\) and \(\hat{H}(t) = \hat{H}_d(t) e^{-\alpha_2 t}\), which imply that \(\hat{S}(t) \to 0\) and \(\hat{H}(t) \to 0\) as \(t \to +\infty\). The above forms of \(u_1\) and \(u_2\) are very complicated since the known state variables S, E, I, H, R and unknown parameters \(\epsilon_1, \epsilon_2, \Delta, \beta, \theta, \eta, \sigma, \mu, \gamma_3, \gamma_3, d_3\) are mixed. By rearranging (4) and (5) in the linearly parameterized forms, we get

\[
u_1 = \Phi_1^T (x_1, S, E, I, R) \xi_1, \quad u_2 = \Phi_2^T (x_2, I, H) \xi_2,
\]

where \(\Phi_1\) and \(\Phi_2\) are vectors containing the known and measurable variables, vectors \(\xi_1\) and \(\xi_2\) include the unknown COVID-19 model parameters that can be estimated by the update rules, which have the following forms

\[
\begin{align*}
  \Phi_1 &= \begin{bmatrix}
    x_1 S - I, & -E, & R - 1
  \end{bmatrix}^T, \\
  \Phi_2 &= \begin{bmatrix}
    x_2 S - I, & -H, & R - 1
  \end{bmatrix}^T, \\
  \xi_1 &= \begin{bmatrix}
    \alpha \\ x_1 i_1 \\ \beta \eta (1 - \theta) \\ \mu \\ \gamma_2 \\
  \end{bmatrix}^T, \\
  \xi_2 &= \begin{bmatrix}
    \gamma_3 + \mu + d_3
  \end{bmatrix}^T.
\end{align*}
\]

(7)

Considering uncertainties of parameters in the COVID-19 model, the adaptive controllers based on estimated parameters can be designed as follows

\[
\begin{align*}
  \tilde{u}_1 &= -\frac{\hat{S}_d - \alpha_1 \hat{S}}{\epsilon_1 S} - \frac{\Delta}{\epsilon_1 S} \frac{1}{\epsilon_1} \frac{\beta (1 - \theta)}{\epsilon_1} I - \frac{\beta \eta (1 - \theta)}{\epsilon_1} E + \frac{\sigma R}{\epsilon_1 S} \frac{\mu}{\epsilon_1}
\end{align*}
\]

(8)

\[
\begin{align*}
  \tilde{u}_2 &= -\frac{\hat{H}}{\epsilon_2 H} + \frac{\gamma_2 I}{\epsilon_2 H} \frac{\gamma_3 + \mu + d_3}{\epsilon_2}
\end{align*}
\]
\[ \ddot{\xi}_1 = 5S\Gamma_1 \Phi_1, \quad \ddot{\xi}_2 = HH\Gamma_2 \Phi_2. \]  

(12)

where \( \Gamma_1 \) and \( \Gamma_2 \) are defined positive definite matrices. Thus, we can choose suitable values of matrices \( \Gamma_1 \) and \( \Gamma_2 \) to obtain the estimated values of unknown parameters, and then the values of control variables \( \ddot{\xi}_1 \) and \( \ddot{\xi}_2 \) in (11) can be calculated to adjust the values of state variables in system (1).

Eventually, by the above discussions, we can establish the following theorem.

Theorem 3.2. For the closed-loop system (1), the designed control variables (11) and the adaptive laws (12) can guarantee that: (i) All the variables involved in the closed-loop system are bounded; (ii) The state variables \( S \) and \( H \) converge to the desired values \( S_d \) and \( H_d \), respectively.

Proof. Consider the following Lyapunov function

\[ V = \frac{1}{2} \left[ S^2 + H^2 + \epsilon_1 \xi_1^T \Gamma_1^{-1} \xi_1 + \epsilon_2 \xi_2^T \Gamma_2^{-1} \xi_2 \right]. \]  

(13)

where \( \xi_1 = \ddot{\xi}_1 - \dot{\xi}_1 \) and \( \xi_2 = \ddot{\xi}_2 - \dot{\xi}_2 \) are the error estimates of \( \ddot{\xi}_1 \) and \( \ddot{\xi}_2 \), respectively. From \( \ddot{S} = S - S_d, \ddot{H} = H - H_d \), and (1), the dynamics of \( S \) and \( H \) can be expressed as

\[ \dot{S} = \Lambda - \beta (1 - \theta) S + \beta \eta (1 - \theta) S - \mu S + \sigma \epsilon_1 \epsilon_1^T S - S_d \hat{H}, \quad \hat{H} = \gamma_2 \left( (\gamma_3 + \mu + d_3) H - \epsilon_2 \epsilon_2^T H - H_d \right) \]  

(14)

Thus, the derivative of Lyapunov function (13) is

\[ \dot{V} = \ddot{S}(S - S_d) + \ddot{H}(H - H_d) + \epsilon_1 \xi_1^T \Gamma_1^{-1} \xi_1 + \epsilon_2 \xi_2^T \Gamma_2^{-1} \xi_2 \]  

\[ = S(\Lambda - \beta (1 - \theta) S - \beta \eta (1 - \theta) S - \mu S + \sigma \epsilon_1 \epsilon_1^T S) - S_d \ddot{S} \]  

\[ + H(\gamma_2 \left( (\gamma_3 + \mu + d_3) H - \epsilon_2 \epsilon_2^T H - H_d \right)) \]  

\[ + \epsilon_2 \xi_2^T \Gamma_2^{-1} \xi_2 \]  

(15)

Substituting (8) and (9) into (15) and rearranging the terms on right hand, one has

\[ \dot{V} = - \alpha_1 S^2 - \epsilon_1 \xi_1 \left( \frac{\Lambda}{\epsilon_1} - \frac{\lambda}{\epsilon_1} \right) + \epsilon_1 \xi_1 \left( \frac{\beta (1 - \theta)}{\epsilon_1} - \frac{\beta (1 - \theta)}{\epsilon_1} \right) + \epsilon_1 \xi_1 \left( \frac{\beta \eta (1 - \theta)}{\epsilon_1} \right) \]  

\[ + \epsilon_1 \xi_1 \left( \frac{\sigma}{\epsilon_1} \right) \]  

\[ + \epsilon_2 \xi_2 \left( \frac{1}{\epsilon_2} \right) \]  

(16)

\[ \dot{V} = - \alpha_1 S^2 - \alpha_2 H^2 - \epsilon_1 \xi_1 \Phi_1^T \gamma_2^T + \epsilon_2 \Phi_2 \Gamma_2^{-1} \xi_2. \]  

Considering \( \ddot{\xi}_i = \dddot{\xi}_i - \gamma_i, (i = 1, 2) \), and using (7) and (10), (16) can be simplified to the following form

\[ \dot{V} = - \alpha_1 S^2 - \alpha_2 H^2 - \epsilon_1 \xi_1 \Phi_1^T \gamma_2^T + \epsilon_2 \Phi_2 \Gamma_2^{-1} \xi_2. \]  

(17)

The inequality (17) implies that \( V \) is negative semi-definite, which means variables \( H, S \), \( \ddot{\xi}_1 \), and \( \ddot{\xi}_2 \) are bounded from (13). According to the boundedness of \( S_d \) and \( H_d \), we easily get the boundedness of \( S \) and \( H \). Meanwhile, from \( \dddot{\xi}_1 = \dddot{\xi}_1 - \dot{\xi}_1 \) and \( \dddot{\xi}_2 = \dddot{\xi}_2 - \dot{\xi}_2 \), one obtains that \( \dddot{\xi}_1 \) and \( \dddot{\xi}_2 \) are bounded. Based on the above discussions, we conclude the proposed controllers can ensure the stability of system (1). In the following, we show the asymptotic stability of system (1). In order to do this, the derivative of \( V \) can be calculated as
\[ \dot{V} = -2\alpha_1 S \dot{S} - 2\alpha_2 H \dot{H}. \]

It is easily to obtain that the positively invariant set of model (1) is

\[ \Omega = \left\{ (S, E, I, H, R) \in \mathbb{R}^5 : S \geq 0, E \geq 0, I \geq 0, H \geq 0, R \geq 0, S + E + I + H + R \leq \frac{\lambda}{2} \right\}. \]

Thus, we know that \( \dot{S} \) and \( \dot{H} \) are bounded from (1), which, together with the relationships \( \dot{S} = S - S_d, \) \( \dot{H} = H - H_d \) imply that \( \dot{S} \) and \( \dot{H} \) are also bounded. This means \( \dot{V} \) is bounded, and hence \( V \) is uniformly continuous. Applying Barbalat’s lemma [14, Lemma 4.2], the tracking errors \( \dot{S} \) and \( \dot{H} \) converge to zero as \( t \to \infty \), i.e., \( S \to S_d, H \to H_d \) \((t \to \infty)\). This completes the Proof.

4. Numerical simulation

This section is devoted to illustrating the effects of the proposed controllers and studying the influences of control measures to the spread of COVID-19 by three numerical examples.

In the early stage of COVID-19, we want to reduce the number of susceptible individuals \( S(t) \) as soon as possible to avoid large numbers of individuals being infected, so the control goal of \( S(t) \) sets as \( S_d(t) = (S(0) - a_1) e^{-\alpha_1 t} + a_1 \), where \( a_1 > 0 \) is the desired steady state value (final value) of \( S(t) \) and \( a_2 > 0 \) is the reduction rate of \( S(t) \). Since the hospitalized population \( H(t) \) is closely related to the infected population \( I(t) \), the control goal of \( H(t) \) sets as \( H_d(t) = t_1 \int_{t_0}^{t} I(v)dv \), where \( t_1 > 0 \) is the considered time span and \( t_2 > 0 \) is a coefficient. The parameters of model (1) are given as: \( \lambda = 1 \) day\(^{-1} \), \( \beta = 0.00014 \) day\(^{-1} \) [15], \( \mu = 5.48 \times 10^{-5} \) day\(^{-1} \) [16], \( \delta = 1/ \) 5.2 day\(^{-1} \) [7], \( \sigma = 0.01 \) day\(^{-1} \), \( d_1 = 0.046 \) day\(^{-1} \) (Larger than \( d_2 \)), \( d_2 = 0.0037 \) day\(^{-1} \) [17], \( \gamma_1 = 0.1 \) day\(^{-1} \), \( \gamma_2 = 1 \) day\(^{-1} \) [6], \( \gamma_3 = 0.1 \) day\(^{-1} \) [6], \( \theta = 0.7 \) [5], \( \eta = 0.1 \) day\(^{-1} \) [5], and the initial value is \( (S(0), E(0), I(0), H(0), R(0)) = (10000, 10, 1, 1, 0) \). The designed parameters are chosen as: \( e_1 = 3 \times 10^{-9} \), \( e_2 = 3 \times 10^{-6} \), \( \epsilon_5 = \epsilon_H = 1 \), \( a_1 = 10 \), \( a_2 = 200 \), and \( \alpha_1 = \alpha_2 = \alpha \).

\[
\Gamma_1 = \begin{bmatrix}
1 & -0.2 & 0.1 & -0.1 & 0.1 \\
0 & 2 & -0.2 & 0 & 0 \\
0 & 0.2 & 4 & 0 & 0 \\
0.2 & -0.1 & -0.4 & 3 & -0.2 \\
0.1 & 0 & -0.1 & 0.2 & 2 & -0.1 \\
0 & 0 & -0.2 & 0 & 2 & 1
\end{bmatrix},
\Gamma_2 = \begin{bmatrix}
1 & -0.3 & -0.3 \\
0.3 & 1 & -0.3 \\
0.3 & -0.3 & 1
\end{bmatrix}.
\]

We can easily verify that \( \Gamma_1 \) and \( \Gamma_2 \) are positive definite matrices.

Example 4.1. The effects of \( u_1 \) and \( u_2 \) under different control goals.

Firstly, we study the spread of COVID-19 under control \( u_1 \). Choosing \( a_1 = 2000 \) (Low final value), 4000 (High final value) and \( a_2 = 6 \times 10^{-4} \) (Low reduction rate), \( 9 \times 10^{-4} \) (High reduction rate), we consider the following four cases of \( S_d(t) \): Case A1: Low final value and reduction rate; Case A2: Low final value but high reduction rate; Case A3: High final value but low reduction rate; Case A4: High final value and reduction rate. The simulation results are shown in Fig. 1(a)–(f). From Fig. 1(a) and (d), we see that the trajectories of \( S(t) \) can track \( S_d(t) \) with small tracking errors, which indicate the effectiveness of the proposed controller. Following from Fig. 1(a)–(f) that if both \( u_1 \) and \( u_2 \) are in absence, COVID-19 will breakout on a large scale; the numbers of individuals in compartments \( E \) and \( I \) have reached about 700 and 115, respectively. Under different \( S_d \), we can see from Fig. 1(a)–(f) that the best prevention and control effect is achieved by adopting the goal in Case A2, and the control effect is the worst when high final value and low reduction rate (Case A3) are used. By comparing Cases A1 and A2 (or Cases A3 and A4), we found that although they have same final values, the number of infected individuals has increased significantly in Case A1 (or A3) due to the slow implementation of control measure \( u_1 \) (see Fig. 1(b) and (c)). After implementing \( u_1 \), the number of hospitalized individuals \( H(t) \) has decreased significantly, and the number of individuals with “immunity” has increased dramatically (see Fig. 1(d) and (e)). Therefore, high-density media campaigns \( u_1 \) is of great significance for the prevention and control of COVID-19.

Secondly, we study the spread of COVID-19 under control \( u_2 \). Consider the following four cases: Case B1: \( \tau_1 = 5, \tau_2 = 0.6 \); Case B2: \( \tau_1 = 8, \tau_2 = 0.6 \); Case B3: \( \tau_1 = 5, \tau_2 = 0.9 \); Case B4: \( \tau_1 = 8, \tau_2 = 0.9 \). The simulation results are presented in Fig. 1(g)–(i). As can be seen from Fig. 1(g)–(i), under the control measure \( u_2 \), although the peak values of \( H(t) \) decrease to a certain extent (see Fig. 1(i)), it is almost impossible to effectively control the spread of COVID-19 (see Fig. 1(g) and (h)), and even second and third outbreaks occur near the 300th and 600th days, respectively. Therefore, it is very dangerous to only take treatment measure, which will lead to people being trapped in COVID-19 outbreaks.

Thirdly, we consider the spread of COVID-19 under both controls \( u_1 \) and \( u_2 \). The values of \( a_1, \tau_1, \tau_2 = (1.2) \) in \( S_d \) and \( H_d \) are chosen from Cases A1–A4 and B1–B4, which are specified as follows: Case C1: A2+B1; Case C2: A1+B2; Case C3: A4+B3; Case C4: A3+B4. It can be seen from Fig. 2 that COVID-19 can be more effectively controlled under media campaigns and treatment. Compared with the cases of no control or only one control, two controls can give full play to their advantages: \( u_1 \) can quickly reduce the number of susceptible individuals, thereby preventing too many people from infecting COVID-19, and \( u_2 \) can reduce the number of hospitalized individuals and relieve pressure on hospitals. At the same time, the joint control measures have also effectively avoided the secondary or even multiple outbreaks of COVID-19.

Example 4.2. The spread of COVID-19 when the control measures are cancelled after implementing for a period of time. We consider the spread of COVID-19 for cancelling one or two measures after \( T_1 = 50 \) or \( T_2 = 400 \) and obtain the following seven cases: Case D0: implementing two measures at all time; Case D1: cancelling \( u_1 \) after \( T_1 \); Case D2: cancelling \( u_2 \) after \( T_2 \); Case D3: cancelling both \( u_1 \) and \( u_2 \) after \( T_1 \); Case D4: cancelling \( u_1 \) after \( T_2 \); Case D5: cancelling \( u_2 \) after \( T_2 \); Case D6: cancelling both \( u_1 \) and \( u_2 \) after \( T_2 \).

Under the above cases, we obtain the simulation results presented in Fig. 3: In Cases 1 and 3, the outbreak of COVID-19 cannot be controlled, and there are second and third outbreaks, while the measures in other cases could effectively control the spread of COVID-19. By comparing Cases 1, 2 and 3, media campaigns are extremely important for controlling COVID-19 in the early stage of outbreak. At the same time, if \( u_1 \) is cancelled prematurely, the effect of previous containment will be destroyed by more serious secondary or even multiple outbreaks. As can be seen from Cases 4, 5 and 6, COVID-19 will not outbreak again if high-density media campaigns and treatment are implemented before \( T_2 = 400 \), regardless of whether partial or full measures are cancelled after \( T_2 \). Based on the above discussions, we must take timely and strict control measures when COVID-19 first outbreaks. Once the COVID-19 is under control, the governments can gradually relax or even cancel the control measures to reduce economic cost.

Example 4.3. The influences of uncertainties for parameters \( \hat{a}_1 \) and \( \hat{a}_2 \) on tracking performance. In Examples 4.1 and 4.2, we assume that the parameters of model (1) are completely known and
Fig. 1. The spread of COVID-19 under only control (a) or by (b)–(o).

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Fig. 2. The spread of COVID-19 under both controls $u_1$ and $u_2$. 

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Fig. 3. The spread of COVID-19 when the control measures are cancelled after implementing for a period of time.

Fig. 4. The effects of uncertainties for parameters $b_x(0)$ and $b_x(0)$. 

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used to determine the initial value of the adaptive laws (12), but this may unreasonable in practice. Thus, we study the tracking performance when the initial value of the adaptive laws deviates from the real value.

For $\xi_1$, we set $\hat{\xi}_1(0) = (1 + r_1)\xi_1$ with $r_1 = -30\%, 0\%, 30\%$ and 60%. From Fig. 4 (a) and (b), we see that the tracking errors of $S(t)$ increase with the increasing of deviation $r_1$, while the change of $r_1$ has little effect on the tracking errors of $H(t)$. For $\hat{\xi}_2$, we simulate the tracking performance under $\hat{\xi}_2(0) = (1 + r_2)\xi_2$ with $r_2 = -30\%, 0\%, 30\%$ and 60% and the results are presented in Fig. 4 (c) and (d). As can be seen from them, the change of $r_2$ has almost no influence on the tracking errors of $S(t)$, while it has some influences on the tracking errors of $H(t)$. However, the range of tracking errors is small and acceptable. Based on the above discussions, we may conclude that the proposed adaptive controllers have good robustness.

5. Concluding remarks

In this paper, considering media campaigns and treatment, the adaptive control problem has been investigated. Two controllers have been designed to achieve the preset control goals and the proposed controllers can ensure the stability of closed-loop system. Finally, three numerical examples are provided to verify the effectiveness and robustness of controllers, from which we have found that the media campaigns are the most important control measure to curb the spread of COVID-19.

Acknowledgements

This work was supported by Ningxia Natural Science Foundation Project (2019AAC03069), and 2020 Ningxia Hui Autonomous Region Top Young Talents Training Project.

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