Model-independent determination of $|V_{ub}|$

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Abstract
The decay distribution of the kinematic variable $\xi_u$ in inclusive charmless semileptonic decays of $B$ mesons is unique. The novel method for a model-independent determination of $|V_{ub}|$ is described.

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1 Problems and Solutions

The fundamental Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ub}|$ has been determined from inclusive charmless semileptonic decays of $B$ mesons, $B \to X_u \ell \nu$. However, there are experimental and theoretical problems that obstruct a precise determination of $|V_{ub}|$. Experimentally, it is very difficult to separate signals from the rare $B \to X_u \ell \nu$ decay from large $B \to X_c \ell \nu$ background. Theoretically, QCD uncertainties arise in calculations that relate the measured quantity to $|V_{ub}|$. The potential theoretical uncertainties from perturbative and nonperturbative QCD can be comparable.

The solutions for the problems are provided by a novel method. It has been proposed to use the kinematic cut on the variable $\xi_u = (q^0 + |\mathbf{q}|)/M_B$ (q is the momentum transfer to the lepton pair) above the kinematic limit for $B \to X_c \ell \nu$, $\xi_u > 1 - M_D/M_B$, to separate $B \to X_u \ell \nu$ signal from $B \to X_c \ell \nu$ background. Most of $B \to X_u \ell \nu$ events pass the above cut. This kinematic requirement provides a very efficient way for background suppression. $|V_{ub}|$ can then be extracted from the weighted integral of the measured $\xi_u$ spectrum via the sum rule for inclusive charmless semileptonic decays of $B$ mesons with little theoretical uncertainty. The sum rule is derived from the light-cone expansion and beauty quantum number conservation. Thus a model-independent determination of $|V_{ub}|$ can be achieved, minimizing the overall (experimental and theoretical) error.

2 Sum Rule

Because of the large $B$ meson mass, the light-cone expansion is applicable to inclusive $B$ decays that are dominated by light-cone singularities. For inclusive charmless semileptonic decays of $B$ mesons, the light-cone expansion...
and beauty quantum number conservation lead to the sum rule

\[ S \equiv \int_0^1 d\xi_u \left( \frac{1}{\xi_u^5} \frac{d\Gamma}{d\xi_u} (B \to X_u \ell \nu) \right) = |V_{ub}|^2 \frac{G_F^2 M_B^5}{192 \pi^3}. \]  

(1)

This sum rule has the following advantages:

- Independent of phenomenological models
- No perturbative QCD uncertainty
- Dominant hadronic uncertainty avoided

The sum rule (1) establishes a relationship between $|V_{ub}|$ and the observable quantity $S$ in the leading twist approximation of QCD. The only remaining theoretical uncertainty in the relation comes from higher-twist corrections to the sum rule, which are suppressed by a power of $\Lambda_{QCD}^2/M_B^2$.

### 3 The $\xi_u$ Spectrum

Now let me explain why the decay distribution of the kinematic variable $\xi_u$ is unique and why the kinematic cut on $\xi_u$ is very efficient in the discrimination between $B \to X_u \ell \nu$ signal and $B \to X_c \ell \nu$ background.

Without QCD corrections, the tree-level $\xi_u$ spectrum in the free quark decay $b \to u \ell \nu$ in the $b$-quark rest frame is a discrete line at $\xi_u = m_b/M_B$. This is simply a consequence of kinematics that fixes $\xi_u$ to the single value $m_b/M_B$, no other values of $\xi_u$ are kinematically allowed in $b \to u \ell \nu$ decays. This discrete line at $\xi_u = m_b/M_B \approx 0.9$ lies well above the charm threshold, $\xi_u > 1 - M_D/M_B = 0.65$.

The $O(\alpha_s)$ perturbative QCD correction to the $\xi_u$ spectrum has been calculated. The $\xi_u$ spectrum remains a discrete line at $\xi_u = m_b/M_B$, even if virtual gluon emission occurs. Gluon bremsstrahlung generates a small tail below the parton-level endpoint $\xi_u = m_b/M_B$.

To calculate the real physical decay distribution in $B \to X_u \ell \nu$, we must also account for hadronic bound-state effects. In the framework of the light-cone expansion, the leading nonperturbative QCD effect is incorporated in the $b$-quark distribution function

\[ f(\xi) = \frac{1}{4\pi} \int \frac{d(y \cdot P)}{y \cdot P} e^{i y \cdot P} \langle B|\bar{b}(0)\gamma_P \exp[ig_s \int_0^1 dz A_\mu(z)]b(y)|B\rangle|_{y^2=0}, \]  

(2)

where $\mathcal{P}$ denotes path ordering. Although several important properties of it are known in QCD, the form of the distribution function has not been
completely determined. The distribution function $f(\xi)$ has a simple physical interpretation: It is the probability of finding a $b$-quark with momentum $\xi P$ inside the $B$ meson with momentum $P$. The real physical spectrum is then obtained from a convolution of the hard perturbative spectrum with the soft nonperturbative distribution function:

$$\frac{d\Gamma}{d\xi_u}(B \to X_u\ell\nu) = \int_{\xi_u}^{1} d\xi f(\xi) \frac{d\Gamma}{d\xi_u}(b \to u\ell\nu, p_b = \xi P),$$

where the $b$-quark momentum $p_b$ in the perturbative spectrum is replaced by $\xi P$. The interplay between nonperturbative and perturbative QCD effects has been accounted for.

Bound-state effects lead to the extension of phase space from the parton level to the hadron level, also stretch the spectrum downward below $m_b/M_B$, and are solely responsible for populating the spectrum upward in the gap between the parton-level endpoint $\xi_u = m_b/M_B$ and the hadron-level endpoint $\xi_u = 1$. The interplay between nonperturbative and perturbative QCD effects eliminates the singularity at the endpoint of the perturbative spectrum, so that the physical spectrum shows a smooth behaviour over the entire range of $\xi_u$, $0 \leq \xi_u \leq 1$.

Although the monochromatic $\xi_u$ spectrum at tree level is smeared by gluon bremsstrahlung and bound-state effects around $\xi_u = m_b/M_B$, about 80% of $B \to X_u\ell\nu$ events remain above the charm threshold. The uniqueness of the decay distribution of the kinematic variable $\xi_u$ implies that the kinematic cut on $\xi_u$ is very efficient in disentangling $B \to X_u\ell\nu$ signal from $B \to X_c\ell\nu$ background.

4 How Do You Measure $S$?

To measure the observable $S$ defined in Eq. (1), one needs to measure the weighted $\xi_u$ spectrum $\xi_u^{-5} d\Gamma(B \to X_u\ell\nu)/d\xi_u$, using the kinematic cut $\xi_u > 1 - M_D/M_B$ against $B \to X_c\ell\nu$ background. $S$ can then be obtained from an extrapolation of the weighted spectrum measured above the charm threshold to low $\xi_u$.

While the normalization of the weighted spectrum given by the sum rule does not depend on the $b$-quark distribution function $f(\xi)$, thus being model-independent, the shape of the weighted spectrum does. The detailed analysis is presented in Ref. 2. Gluon bremsstrahlung and hadronic bound-state effects strongly affect the shape of the weighted $\xi_u$ spectrum. However, the shape of the weighted $\xi_u$ spectrum is insensitive to the value of the strong coupling $\alpha_s$, varied in a reasonable range. The overall picture appears to be
that the weighted $\xi_u$ spectrum is peaked towards larger values of $\xi_u$ with a narrow width. The contribution below $\xi_u = 0.65$ is small and relatively insensitive to forms of the distribution function. This suggests that extrapolating the weighted $\xi_u$ spectrum down to low $\xi_u$ would not introduce a considerable uncertainty in the value of $S$.

5 Summary

The kinematic cut on $\xi_u$, $\xi_u > 1 - M_D/M_B$, and the semileptonic $B$ decay sum rule, Eq. (1), make an outstanding opportunity for the precise determination of $|V_{ub}|$ from the observable $S$. This method is both exceptionally clean theoretically and very efficient experimentally in background suppression.

There remain two kinds of theoretical error in the model-independent determination of $|V_{ub}|$. First, higher-twist (or power suppressed) corrections to the sum rule cause an error of the order $O(\Lambda_{QCD}^2/M_B^2) \sim 1\%$ in $|V_{ub}|$. Second, the extrapolation of the weighted $\xi_u$ spectrum to low $\xi_u$ gives rise to a systematic error in the measurement of $S$. The size of this error depends on how well the weighted spectrum can be measured, since the measured spectrum would directly determine the form of the distribution function. In addition, the form of the universal distribution function can also be determined directly by the measurement of the $B \to X_s \gamma$ photon energy spectrum. The experimental determination of the distribution function would provide a model-independent way to make the extrapolation, allowing an error reduction.

Eventually, the error in $|V_{ub}|$ determined by this method would mainly depend on how well the observable $S$ can be measured. To measure $S$ experimentally one needs to be able to reconstruct the neutrino. This poses a challenge to experiment. The unique potential of determining $|V_{ub}|$ warrants a feasibility study for the experiment.

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