ERMAS: Becoming Robust to Reward Function Sim-to-Real Gaps in Multi-Agent Simulations

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Abstract
Multi-agent simulations provide a scalable environment for learning policies that interact with rational agents. However, such policies may fail to generalize to the real-world where agents may differ from simulated counterparts due to unmodeled irrationality and misspecified reward functions. We introduce \(\epsilon\)-Robust Multi-Agent Simulation (ERMAS), a robust optimization framework for learning AI policies that are robust to such multi-agent sim-to-real gaps. While existing notions of multi-agent robustness concern perturbations in the actions of agents, we address a novel robustness objective concerning perturbations in the reward functions of agents. ERMAS provides this robustness by anticipating suboptimal behaviors from other agents, formalized as the worst-case \(\epsilon\)-equilibrium. We show empirically that ERMAS yields robust policies for repeated bimatrix games and optimal taxation problems in economic simulations. In particular, in the two-level RL problem posed by the AI Economist (Zheng et al., 2020) ERMAS learns tax policies that are robust to changes in agent risk aversion, improving social welfare by up to 15% in complex spatiotemporal simulations.

1. Introduction
Reinforcement learning (RL) can optimize policy decisions in real-world multi-agent systems; for example, to improve traffic flow or to increase economic equality and productivity using AI tax policies (Zheng et al., 2020). Simulations offer a safe and efficient environment to train and evaluate policies for multi-agent systems. The behavior of real-world agents can be emulated using agent-based models with fixed behavioral rules (Holland & Miller, 1991; Bonabeau, 2002), or RL agents that optimize for an adaptive reward function.

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ERMAS (top) trains robust actors with agents that optimize for reward functions in an uncertainty set (purple, dotted) that covers sim-to-real gaps in reward functions. This is effective, since many behavioral sim-to-real gaps (right) are more naturally explained by discrepancies in reward functions (left). In contrast, when perturbing agent behavioral policies (bottom), the uncertainty set (grey, dotted) may exclude behavioral sim-to-real gaps that are rationalized with only a small change in the reward function. Dotted lines denote uncertainty sets, with dashed lines denoting their equivalent regions in the complementary space.

This robustness objective is fundamentally different than those considered by existing literature, which include robustness to changes in transition probabilities, observation spaces, and action probabilities. Although perturbations to environment dynamics, action probabilities, and reward functions may partially overlap, they differ in how they measure the magnitude of perturbations, and as a result, what set of perturbations are considered realistic. For example, a small and realistic change in an agent’s reward function may translate to a large perturbation in their policy and therefore be considered unrealistic according to previous notions of multi-agent robustness (Suh et al., 2019; Nachum et al., 2019). This is shown in Figure 2.

**Contributions.** ERMAS has three key features: (1) It formulates a multi-agent robustness objective equivalent to finding the worst-case $\epsilon$-Nash equilibria. (2) It optimizes a tractable dual robustness objective. (3) It approximates the dual problem using local solution concepts and first-order meta-learning techniques (Nichol et al., 2018; Finn et al., 2017). This approach yields policies that are robust to behavioral deviations up to a regret of $\epsilon$.

We validate ERMAS in repeated bimatrix games, as well as in a complex spatiotemporal economy with agent interactions that features a social planner that learns to set tax rates in order to optimize social welfare. In both settings, we show ERMAS yields robust actors by evaluating them in test environments with agents optimized for reward functions that were unseen during training. This generalization setting emulates the challenge faced in transferring policies to the real world. We show that ERMAS is effective even in settings where baselines fail or become intractable. In particular, ERMAS finds AI Economist tax policies that achieve higher social welfare across a broad range of unseen agent risk aversion objectives.

### 2. Related Work

**The sim-to-real gap problem.** The sim-to-real gap problem considers how sim-to-real gaps, i.e., mismatches between a simulation and reality, can (negatively) impact policies trained in simulation. Robustness across sim-to-real gaps has been studied for deep RL to robotic control with visual inputs (James et al., 2019; Christiano et al., 2016; Rusu et al., 2016; Tzeng et al., 2015). To close sim-to-real gaps, domain adaptation aims to transfer a policy trained in simulation to the real world, mainly in single-agent settings (Higgins et al., 2017; Kim et al., 2019). In comparison, the study of sim-to-real gaps in multi-agent RL (MARL) is relatively nascent (Suh et al., 2019; Nachum et al., 2019).

**Adversarial Robustness.** Morimoto & Doya (2001) proposed robust RL for reward functions with a worst-case adversarial perturbation, inspired by $H_\infty$ control, and provided analytical solutions in the linear setting. This robust RL problem can be solved using an adversarial player that chooses perturbations. Pinto et al. (2017) optimized for the $\alpha$-quantile of reward perturbations, using a conditional value-at-risk (CVAR) interpretation of adversarial robustness. Other works have studied perturbing transition matrices, observation spaces, and action probabilities, which are relevant to robotics and control applications (Pinto et al., 2017; Tessler et al., 2019; Hou et al., 2020; Li et al., 2019). In contrast, we study a new form of robustness: robustness to perturbations in agent rewards. Therefore, ERMAS is not an alternative to existing techniques, but a novel tool to address an unexplored source of uncertainty. ERMAS can be combined with existing techniques that study robustness to changes in dynamics, observations, or action probabilities, though this beyond the scope of this work.

**Multi-Agent Robustness.** ERMAS studies a more challenging multi-agent setting than previous works. First, agents do not necessarily optimize for the same (robustness) objective. As such, we consider the one-sided robustness problem: only the actor learns to be robust. This setting is significantly more difficult than previous works that assume all agents act robustly (Li et al., 2019). Second, we address a challenging nested RL setting that is key to important real-world applications such as adaptive mechanism design (Zheng et al., 2020). Although we formulate ERMAS as using simultaneous gradient descent, ERMAS can be easily extended to use nested optimization such as competitive
Robust Mechanism Design. Mechanism design aims to design reward functions that have desirable features at equilibrium, e.g., stability (Nash equilibria), truthfulness (auctions), and others (Myerson, 2016). Robust mechanism design (Bergemann & Morris, 2005) has studied how mechanisms can be effective even when the environment is imperfectly known. However, it is often very hard to derive analytical solutions for (robust) mechanisms. Instead, Dütting et al. (2019) used deep learning to learn (close to) optimal auctions, while learning optimal taxes by the AI Economist (Zheng et al., 2020) can be seen as adaptive mechanism design using two-level RL. In such settings, ERMAS implements robust adaptive mechanism design.

Behavioral Economics. A significant source of reward function sim-to-real gaps is human behavior, i.e., bounded rationality. Behavioral economics has studied human reward functions and how human behavior differs from that of idealized agents optimizing simple utility functions (Pesendorfer, 2006). Simon (1976) decomposed bounded rationality into substantive and procedural rationality. Substantively rational agents have incorrect reward function estimates (e.g., humans might be more risk-averse than a simulated agent) or require richer simulations to describe them. Procedurally rational agents are boundedly rational agents which violate rationality assumptions or do not maximize for their reward function (Simon & Schaeffer, 1990).

3. Robustness and Sim-To-Real Gaps in Multi-agent Environments

High-level Motivation. ERMAS trains a robust actor \( \pi_r \) that interacts with an environment featuring \( N \) agents \( \pi \).\(^1\) The robust actor learns to optimize an objective \( J_r(\pi, \pi_r) \) that depends both on its own policy and the behavior of agents in response to that policy. For example, the robust actor might be a central planner optimizing social welfare in a spatiotemporal economy (Zheng et al., 2020), an MARL problem in which the robust actor and agents co-adapt.

In practice, it is often infeasible to optimize the robust actor in the target environment where it will ultimately be used (i.e., the real world). Hence, we train the robust actor in a simulation where agents’ behaviors \( \pi \) emulate those of the agents in the target environment. Importantly, the behaviors \( \pi \) depend on the reward function of the agents, e.g., in an economic simulation, each agent’s reward is its post-tax utility.

As such, we aim to successfully transfer a robust actor \( \pi_r \) from the training to the target environment, which depends on the sim-to-real gap in the agent reward functions and learned behaviors. Our method achieves such robustness by training agents whose behavior is consistent with the worst-case sim-to-real gap (worst-case with respect to the robust actor’s objective \( J_r \)) within a well-defined search space (the uncertainty set).

Figures 1 and 2 illustrate how policies may fail to generalize to test environments due to sim-to-real gap gaps in agent reward functions, and how ERMAS defines a natural uncertainty set to seek robustness to.

Robustness Objective. Formally, we model the MARL problem for \( N \) (non-robust) agents using a partially-observable multi-agent Markov Games (MGs) (Sutton & Barto, 2018), defined by the tuple \( M := (S, A, r, T, \gamma, o, I) \), where \( S \) and \( A \) are the state and action spaces, respectively, and \( I \) are agent indices. We extend this to an MG \( M[\pi_r] \), which includes the dependency on the robust actor’s policy \( \pi_r \). MGs proceed in episodes that last \( H + 1 \) steps (possibly infinite), covering \( H \) transitions. At each time \( t \in [0, H] \), the world state is denoted \( s_t \). Each agent \( i = 1, \ldots, N \) receives an observation \( o_{i,t} \) of the world state, executes an action \( a_{i,t} \) and receives a reward \( r_{i,t} \). The environment transitions to the next state \( s_{t+1} \), according to the transition distribution \( T(s_{t+1}|s_t, a_t) \). Agent policies \( \pi_i \) are parameterized by \( \theta_i \), while the robust actor policy \( \pi_r \) is parameterized by \( \bar{\theta} \).

The Nash equilibria of \( M[\pi_r] \) are sets of agent policies where any unilateral deviation is suboptimal (Nash, 1950):

\[
\text{NE}(\pi_r) := \{ \pi | \forall i \in [1, N], \forall \pi'_i \in \Pi : J_i(\pi'_i, \pi_{-i}, \pi_r) \leq J_i(\pi_i, \pi_{-i}, \pi_r) \} \tag{1}
\]

where agent \( i \) optimizes \( J_i(\pi, \pi_r) := E_{\pi, \pi_r} \left[ \sum_{t=0}^{H} \gamma^t r_{i,t} \right] \), and \( \Pi \) is the space of all policies. Hence, a rational agent would not unilaterally deviate from \( \pi \in \text{NE}(\pi_r) \).

To evaluate a fixed robust actor policy \( \pi_r \), we could sample outcomes using policies \( \pi \in \text{NE}(\pi_r) \). Also optimizing \( \pi_r \) introduces a form of two-level learning. Under appropriate conditions, this can be solved with simultaneous gradient descent (Zheng et al., 2020; Fieß et al., 2019). We formulate the robust actor’s objective by formalizing reward function sim-to-real gaps as perturbations \( \xi_i \in \Xi \) to agent rewards, where the uncertainty set \( \Xi := (S, A)^H \rightarrow \mathbb{R} \) is the space of possible perturbations and represents uncertainty about agent reward functions. We extend the Nash equilibria in

\(^1\)Bold-faced quantities denote vectors or sets, e.g., \( \pi = (\pi_1, \ldots, \pi_N) \), the set of policies for the \( N \) agents.
where we overloaded the definition of $\NE$:

$$\NE(\pi_r, \xi) := \left\{ \pi \mid \forall i \in [1, N], \pi_i' \in \Pi : J_i^\xi(\pi_i', \pi_{-i}, \pi_r) \leq J_i^\xi(\pi_i, \pi_{-i}, \pi_r) \right\}$$

(2)

$$J_i^\xi(\pi_i', \pi_{-i}, \pi_r) := J_i(\pi_i', \pi_{-i}, \pi_r) + \mathbb{E}_{\tau_i \sim \Pi} \left[ \xi_i(\tau_i) \right]$$

(3)

where $\tau_i$ is a trajectory, i.e., a sequence of state-action pairs.

Following (Morimoto & Doya, 2001), a robust actor optimizes its reward, subject to agents at a perturbed Nash equilibrium $\NE(\pi_r, \xi)$ that maximally penalizes the robust actor:

$$\pi_r^* = \arg \max_{\pi_r} \min_{\xi \in \Xi} \min_{\pi \in \NE(\pi_r, \xi)} J_r(\pi_r, \pi),$$

(4)

Note that agent policies $\pi \in \NE(\pi_r, \xi)$ optimize their own reward function, and an (abstract) adversary chooses $\xi$.

**Bounded Uncertainty Set.** There are two challenges with Equation 4. First, if the adversary can arbitrarily choose $\Xi$, the worst case is uninformative. Second, the uncertainty set $\Xi$ may be high-dimensional and intractable to search. We address these issues by upper-bounding the size of the uncertainty set, or equivalently, the deviations in agent behaviors at a Nash equilibrium:

$$\Xi_\epsilon := \left\{ \xi \mid \sup_{\pi, \pi_r} |\xi_i(\pi, \pi_r)| < \epsilon, \text{ for all } i \in I \right\},$$

(5)

where $\epsilon > 0$ is a tunable hyperparameter. Because the perturbations $\xi_i$ emulate the difference between the reward functions of agents in the training and target environments, $\epsilon$ upper-bounds the size of the sim-to-real gap that ERMAS provides robustness to.

Bounding the magnitude of the reward function perturbations in $\Xi_\epsilon$ is equivalent to using $\epsilon$-Nash equilibria of $M[\pi_r]$:

$$\NE(\pi_r, \epsilon) := \{ \pi \mid \forall i \in [1, N], \forall \pi_i' \in \Pi : J_i(\pi_i', \pi_{-i}, \pi_r) \leq J_i(\pi_i, \pi_{-i}, \pi_r) + \epsilon \},$$

(6)

where we overloaded the definition of $\NE(\pi_r, \epsilon)$ in the second argument for convenience. Using a bounded uncertainty set (Equation 6), the robustness objective in Equation 4 can be simplified to a nested constrained optimization problem:

$$\text{Outer-Loop}(\epsilon) := \arg \max_{\pi_r} J^*_{r, \min}(\pi_r, \epsilon),$$

$$J^*_{r, \min}(\pi_r, \epsilon) := \min_{\pi \in \NE(\pi_r, \epsilon)} J_r(\pi_r, \pi).$$

Agent-Adv-Search(\epsilon)

Using $\NE(\pi_r, \epsilon)$ replaces the problem of intractably searching through $\Xi$ with searching through the bounded space $\NE(\pi_r, \epsilon)$, and thus merges the two nested min operations in Equation 4. Conceptually, this transfers the worst-case search problem to the agents: agents find an adversarial equilibrium in Agent-Adv-Search(\epsilon); Outer-Loop(\epsilon) optimizes the robust actor given adversarial agents. Note that the constraint set in Eq 7 is non-empty for $\epsilon \geq 0$; the constraints (Eq 6) simply upper-bound the regret of agents. By definition, for non-empty bounded $\Pi$, there exists an optimal policy with zero regret.

**4. ERMAS: Robust Policies in Multi-Agent Simulations**

We now introduce ERMAS, an efficient optimization framework to solve the robustness problem in Equation 7. ERMAS proceeds in three steps. First, it dualizes Equation 7 following constrained RL. Second, it defines a trust-region for the uncertainty set $\NE(\pi_r, \epsilon)$, approximating the dual problem. Finally, it uses first-order meta-learning with trust-regions to solve the approximate dual problem. Algorithm 1 and 2 (in the Appendix) describe this procedure. Using this approach, ERMAS yields robustness to $\epsilon$-equilibria and, equivalently, uncertainty in agent reward functions.

**Dualizing Agent-Adv-Search(\epsilon).** The agent search problem in Equation 7 can be formulated similar to a constrained RL problem (Paternain et al., 2019), where the primary objective of the agents is to minimize the robust actor’s reward and the secondary objective is to maximize their own reward, i.e., find a solution in $\NE(\pi_r, \epsilon)$ (Equation 6). While conventional constrained RL enforces a constant lower bound in the constraint, e.g., $J_i(\pi, \pi_r) \geq C$ with $C > 0$, we enforce a dynamic one:

$$\forall i \in 1 \ldots N : J_i(\pi, \pi_r) \geq J_i(\delta_i, \pi_{-i}, \pi_r) - \epsilon,$$

$$\delta_i := \arg \max_{\pi_i' \in \Pi} J_i(\pi_i', \pi_{-i}, \pi_r),$$

(8)

where $\delta_i$ is the optimal unilateral deviation for agent $i$. To enforce these constraints, we use Lagrange multipliers $\lambda$ to
dualize Agent-Adv-Search(ε) (Equation 7) as:
\[
\min_{\pi} J_\pi(\pi, \pi_r) - \sum_{i=1}^{N} \lambda_i [\Delta J_i(\delta_i, \pi, \pi_r) - \epsilon],
\]
where we denoted the policy parameters as
\[
\delta_i > \rho.
\]

Equation 9 is identical to the dualization of constrained RL, whose duality gap is empirically negligible and provably zero under weak assumptions (Paternain et al., 2019). However, it is still challenging to compute the agent regret \(\rho\) efficiently. By instead performing multipliers \(\theta\) for all \(W\) there exists open sets \(\epsilon\) in differentiable games (Ratliff et al., 2014).

\[
\epsilon \text{ can simplify this problem by proposing a refinement of learning. Estimating agent regret (Equation 10) requires local approximations, and the derivatives above using meta-}
\]

\[
\delta_i (\theta, \pi, \pi_r) := J_i(\delta_i, \pi_r, \pi_r) - J_i(\pi, \pi_r).
\]

\[
\Delta J_i(\delta_i, \pi, \pi_r) := J_i(\delta_i, \pi_r, \pi_r) - J_i(\pi, \pi_r).
\]

\[
\nabla_\theta J^+_{\min}(\pi_r, \epsilon) = -\nabla_\theta J_r(\theta) - \lambda_i \nabla_\theta J_i(\theta_i, \theta_{-i}) - J_i(\theta),
\]
where we denoted the policy parameters as \(\theta := \theta_1, \ldots, \theta_N, \tilde{\theta}\) and \(J_r(\theta) := J_r(\pi, \pi_r)\). In addition, \(\theta_i(\theta)\) are the parameters of the optimal unilateral deviation \(\delta_i\) for agent \(i\), i.e., the parameters that minimize local regret, which depends on the current policy parameters \(\theta\). The Lagrange multipliers \(\lambda_i\) are updated as:

\[
\nabla_\lambda_i J^+_{\min}(\pi_r, \epsilon) = \Delta J_i(\delta_i, \pi, \pi_r) - \epsilon.
\]

**Optimization.** We first detail the optimization approach for Equation 9. The agents apply gradients:

\[
\nabla_\theta J^+_{\min}(\pi_r, \epsilon) = -\nabla_\theta J_r(\theta) - \lambda_i \nabla_\theta J_i(\theta_i, \theta_{-i}) - J_i(\theta),\n\]

\[
\nabla_\lambda_i J^+_{\min}(\pi_r, \epsilon) = \Delta J_i(\delta_i, \pi, \pi_r) - \epsilon.
\]

**Trust Regions using Local \(\epsilon\)-equilibria.** We now detail how to efficiently compute and approximate regret using local approximations, and the derivatives above using meta-learning. Estimating agent regret (Equation 10) requires knowledge of the optimal unilateral deviations \(\delta_i\). We can simplify this problem by proposing a refinement of \(\epsilon\)-equilibria inspired by the notion of local Nash equilibria in differentiable games (Ratliff et al., 2014).

**Definition 4.1.** A strategy \(\pi\) is a local \(\epsilon\)-Nash equilibrium if there exists open sets \(W_i \subset \Pi^N\) such that \(\pi_i \in W_i\) and for each \(i \in \{1, \ldots, N\}\) we have that \(J_i(\pi'_i, \pi_{-i}) \leq J_i(\pi) + \epsilon\) for all \(\pi'_i \in W_i \setminus \{\pi_i\}\), where \(\epsilon := \epsilon_{\sup_{\pi'_i \in W_i} KL(\pi'_i | \pi_i)}\).

By instead performing Agent-Adv-Search(ε) on the local \(\epsilon\)-Nash equilibria, we can limit the set of unilateral deviations to consider to small trust regions \(\Pi_\rho(\pi_i)\):

\[
\text{NE}(\pi, \eta) := \{\pi | \forall i \in \{1, N\}, \pi'_i \in \Pi_\rho(\pi_i) : \Delta J_i(\delta_i, \pi, \pi_r) \leq \epsilon\},
\]

\[
\Pi_\rho(\pi_i) := \{\pi'_i \in \Pi | KL(\pi_i | \pi'_i) \leq \rho\},
\]

where \(\rho > 0\) defines the size of the trust region. For small \(\rho\), algorithms such as TRPO (Schulman et al., 2017) can be used to efficiently approximate optimal local deviations \(\delta_i\), affording reasonable approximations of the regret \(\Delta J_i(\delta_i, \pi_{-i}, \pi_r)\). Note that our usage of trust region algorithms is not for optimization purposes. ERMAS requires the use of trust region optimization to ensure that the equilibria considered by ERMAS are limited to a local neighborhood of the policy space (Equation 13).

**First-Order Meta Learning Approximation.** The full gradient in Equation 11 is hard to compute exactly. The second term \(-\nabla_\theta J_i(\theta)\) maximizes the performance of the agent’s policy and is simply found with policy gradient. However, the first term \(\nabla_\theta J_i(\theta_i, \theta_{-i})\) is less straightforward: it minimizes the performance of the best agent policy in the current trust region, and corresponds to a meta-learning gradient. To obtain a first-order approximation, we follow REPTILE (Nichol et al., 2018):

\[
\nabla_\theta J_i(\theta_i, \theta_{-i}) = g_i - \frac{\beta}{M} \sum_{i=1}^{M} g_i,
\]

\[
g_i := \nabla_\theta J_i \left( \theta_i + \sum_{j=1}^{i-1} g_j, \theta_{-i} \right),
\]

where \(g_i\) denotes the \(i\)th policy gradient in the direction of \(J_i\), and the hyperparameter \(\beta < 1\) incorporates a inductive bias where maximizing agent reward leads to local maxima.

Using this first-order meta-learning approximation, ERMAS efficiently solves Equation 9.

**5. Experimental Validation in Multi-Agent Simulations**

**5.1. Solving Agent-Adv-Search(ε)**

**Constrained Repeated Bimatrix Game.** We analyze the agent behaviors learned by Agent-Adv-Search(ε) in an extension of the classic repeated bimatrix game (Figure 3), which is well-studied in the game theory literature as its Nash equilibria can be solved efficiently. At each timestep \(t\), a row player (agent 1) and column player (agent 2) choose how to move around a grid, while receiving rewards \(r_1(i, j), r_2(i, j)\), where the current location is the row \(i\) and column \(j\). The row (column) player chooses whether to move up (left) or down (right). We select the payoff matrices \(r_1, r_2\), illustrated in Figure 3, so that only one Nash equilibrium exists and that the equilibrium constitutes a “tragedy-of-the-commons,” where agents selfishly optimizing their own reward leads to less reward overall. We also introduce a passive planner that observes the game and receives a payoff \(r(i, j)\). The planner does not take any actions and its payoff is constructed such that its reward is high when the agents are at the Nash equilibrium. Our experiments treat the planner as ERMAS’s robust actor.

In effect, this toy setting allows us to verify that ERMAS samples realistic worst-case behaviors—i.e., that agents 1 and 2 learn to deviate from their tragedy-of-the-commons
equilibrium in order to reduce the planner’s reward but also without significantly increasing their own regret.

**Discovering $\epsilon$-Equilibria.** Figure 3 (middle) visualizes how the equilibria reached by AI agents balance the reward of the planner (y-axis) and agents (x-axis). Conventional multi-agent RL discovers the Nash equilibrium, which is visualized in the top left. At this equilibrium, the agents do not cooperate and the planner receives high reward. For small values of $\epsilon$, ERMAS also discovers the Nash equilibrium. Because $\epsilon$ acts as a constraint on agent regret, larger values of $\epsilon$ enable ERMAS to deviate farther from the Nash equilibrium, discovering $\epsilon$-equilibria to the bottom-right that result in lower planner rewards.

Deviations from a Nash equilibrium should yield higher regret, meaning that regret should increase with $\epsilon$. Figure 3 (right) clearly demonstrates this trend under ERMAS. As described by the robustness objective in Equation 7, such adversarial agent behavior is key to training a robust agent.

**5.2. All the Components of ERMAS are Necessary**

**Dynamic or Fixed Lagrange multipliers $\lambda$.** The Lagrange multipliers $\lambda$ balance the objectives of seeking stable agent equilibria and minimizing the robust actor’s reward. Recall that smaller values of $\lambda$ yield more antagonistic agent objectives. The $\lambda$ are updated using local estimates of agent regret (Equation 9). We can validate that these updates are necessary by analyzing the equilibria learned by ERMAS when $\lambda$ are not updated. Using a fixed value $\lambda_0$ reduces the dual problem (Equation 9) to learning with shared rewards.

Figure 4 (left) visualizes the equilibria discovered with a fixed $\lambda_0$ in the same format as in Figure 3 (middle). This comparison shows that using a fixed $\lambda_0$ affects the equilibria discovered by ERMAS; the bottom right quadrant which contains the $\epsilon$-equilibria discovered with dynamic $\lambda$ are not reached for any values of $\lambda_0$. This demonstrates that certain $\epsilon$-equilibria are only reachable with dynamic $\lambda$ and hence the updates in 9 are necessary for proper behavior.$^4$

**First-order Approximation.** Figure 4 (right) depicts the learning curve of ERMAS with varying $\beta$ and shows that it gives rapid convergence and good asymptotic behavior for an appropriate range of the meta-learning weight $\beta$. However, very large ($\beta > \frac{1}{6}$) and very small ($\beta \sim 0$) values fail to fully reach and remain in the adversarial region.

We also find that the ERMAS update can significantly speed up convergence. This might be counter-intuitive: the first-order approximation of Equation 15 might slow learning, since we step “away” from the direction that the standard policy gradient suggests. However, we find that larger values of $\beta$ reach improved agent rewards half an order of magnitude faster than lower values of $\beta$.

**5.3. Solving Outer-Loop($\epsilon$)**

We now show that ERMAS yields policies $\pi_r$ which are more robust to uncertainty in agent reward functions. Specifically, we empirically validate that ERMAS can find strong solutions to the nested optimization problems Agent-Adv-Search($\epsilon$) and Outer-Loop($\epsilon$), in two simulations: repeated bimatrix games (Figure 3, left) and economic simulations (Figure 6, left).

$^4$However, we recommend temporarily using a fixed $\lambda_0$ at the top of Algorithm 2 (Appendix) to “warm up” agent policies, akin to curriculum learning.
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Figure 4. **Left:** Using fixed values of $\lambda$ (rather than allowing it to update, as in the full algorithm) distorts performance and prevents agents from reaching the same $\epsilon$-equilibria discovered with learned $\lambda$. **Right:** Rewards for the planner and for agents 1 & 2 in the constrained repeated bimatrix game over training time. Each line represents an average over 10 seeds, with error bars indicating standard error. The lines correspond to runs of ERMAS and are colored according to $\beta$, the weight of the meta-learning term of Equation 15.

Figure 5. Robust learning in the repeated bimatrix game. Average planner (the robust actor, when using ERMAS) performance (at convergence) as a function of the reward perturbation coefficient $Q$ in the test environment. Each point represents an average over 5 seeds, with error bars indicating standard error. The vertical dashed line denotes the training environment $Q$. Note: $Q$ is set to 0.5 during training for the line labeled “MARL ($Q=0.5$”).

**Evaluation Workflow.** To measure robustness, we evaluate trained planners in test environments with sim-to-real gaps due to deviations in agent reward functions. The test environments contain agents that are optimized for test reward functions that are unseen while training the planner. We proceed as follows: (1) Train the agents and planner. (2) Fix the planner and transfer it to a test environment, then train new agents from scratch in the test environment with reward functions unseen during Step 1. (3) Report the value of the planner objective after the new agents have converged.

**Augmented Repeated Bimatrix Games.** Our previous analyses studied the constrained repeated bimatrix game, where the planner was passive; we now extend the bimatrix game to a full, nested RL problem by allowing the planner to select an integer $a \geq 0$ that modifies its own reward function: $\tilde{r}(i,j) = a \cdot r(i,j) - a^2$. This function is chosen for its quadratic form: $a$ scales the reward but adds a cost $a^2$ which disincentives large values of $a$. Agents are modified to receive a new reward function $r'_i = r_i - Q\tilde{r}$, where $r_i$ is its original reward function and $Q \leq 1$ is an unknown scalar which may differ between training and testing environments (representing a sim-to-real gap). If $Q = 0$, agents act identical to the experiments in Figure 3 (middle). When $Q > 0$, agents have an incentive to be adversarial.

Figure 5 shows the test performance of ERMAS against vanilla MARL policies learned with $Q_{\text{train}} = 0.5$ or with $Q_{\text{train}} = 0$. Naturally, MARL trained with $Q_{\text{train}} = 0.5$ underperforms MARL trained with $Q_{\text{train}} = 0$ when evaluated in environments where $Q_{\text{test}} < 0.3$. However, MARL trained with $Q_{\text{train}} = 0.5$ outperforms when $Q_{\text{test}} > 0.3$.

Agent-Adv-Search($\epsilon$) finds agents consistent with the the worst-case perturbations, and yields a planner policy at least as good as MARL trained with $Q_{\text{train}} = 0.5$. This shows that ERMAS can produce policies which are robust to uncertainty in other agents’ objectives and is a tractable adversarial robustness algorithm.

**Baselines.** Existing single-agent and multi-agent robust RL algorithms optimize for different robustness objectives than ERMAS. However, some general techniques to address sim-to-real gap can be extended to provide an alternative to ERMAS’s adversarial approach.

Our first baseline extends domain randomization (DR) by applying random perturbations to agent rewards. For DR, agents receive $r'_i = r_i + \sigma$, where $\sigma \sim U[-1, 1]$ (recall $r_i \in [0, 1]$). These perturbations are randomized periodically between episodes. However, DR does not scale well to perturbing reward functions. Even in a simple 4-by-4 bimatrix game, there are $2304$ (16 states $\times$ 16 states $\times$ 9 actions) possible reward values to perturb; a $2304$-dimensional space is too high-dimensional to cover thoroughly using uniformly random noise. Furthermore, in contrast to DR of visual inputs or dynamics, there is a natural latency to the effect of DR of agent rewards: randomization only has an effect if agents learn to adapt to their perturbed rewards.
Our second baseline, risk robustness (RR), applies a concavity to planner rewards, e.g., log $r$. This encourages robustness to environment stochasticity, including the randomness of agent policies. Figure 5 shows neither DR nor RR yield meaningful robustness, even in this simple setting.

ERMAS for Two-Level RL. We now demonstrate ERMAS is effective in complex spatiotemporal environments, using the use-case of optimal tax policy design. We train a robust AI Economist: a robust actor for optimal taxation that acts in a spatiotemporal economic simulation. See Zheng et al. (2020) for a detailed description, model hyperparameters which we replicated exactly, and a link to the simulation environment source code. This setting is very challenging given the scale of the simulation and agent affordances. In particular, adversarial and randomization baselines do not scale to this setting.

In this simulation, agents earn income $z_{i,t}$ from labor $l_{i,t}$, and optimize their expected utility $\mathbb{E}_{\pi,\pi'} \left[ \sum_{t=0}^{T} r_{i,t} \right]$. The utility $r_{i,t}$ has a standard isoelastic form (Arrow, 1971):

$$\tilde{z}_{i,t} = z_{i,t} - T(z_{i,t}), \quad \tilde{x}_{i,t} = \sum_{t' \leq t} \tilde{z}_{i,t}, \quad (17)$$

$$r_{i,t}(\tilde{x}_{i,t}, l_{i,t}) = \frac{\tilde{x}_{i,t}^{1-\eta} - 1}{1 - \eta} - l_{i,t}, \quad \eta > 0, \quad (18)$$

where $\tilde{x}_{i,t}$ is the post-tax endowment of agent $i$, and $\eta$ sets the degree of risk aversion (higher $\eta$ means higher risk aversion). Agents pay taxes $T(z_{i,t})$ over their income $z_{i,t}$; taxes are set by a social planner.

The planner optimizes social welfare $swf$:

$$swf := eq(x) \cdot prod(x), \quad (19)$$

Here $eq(x) := 1 - \frac{N}{N-1} \text{gini}(x)$, and $prod(x) := \sum_{i=1}^{N} x_i$. This shows that ERMAS improves $swf$ even if agent risk aversion significantly increases or decreases (e.g., due to economic cycles or exogenous shocks).

In fact, ERMAS outperforms the non-robust AI Economist for the original setting of $\eta = 0.23$. This suggests that robustness and performance do not necessarily pose a zero-sum game: ERMAS can find equilibria with high performance and strong generalization. It has been observed empirically in various studies that single-agent robust RL similarly yields performance improvements even in the absence of environment perturbations (Pinto et al., 2017).

6. Discussion

Future work could investigate integrating ERMAS with existing techniques that study robustness to different sim-to-real gaps, such as changes in dynamics or observations. Furthermore, it would be interesting to apply ERMAS to problems that feature more general solution concepts, such as Bayesian Nash equilibria or correlated equilibria.
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Algorithm 1 ERMAS outer loop: Outer-Loop($\epsilon$).

**Output:** Robust robust actor policy $\pi_r$.

**Input:** Initial robust actor policy $\pi_r$ and agent policies $\pi$.

for $i = 1, \ldots, n$

  Update parameters of agent policies $\pi$: $\theta \leftarrow \text{Agent-Adv-Search}(\epsilon)$, see Algorithm 2.

  Update robust actor policy $\pi_r \leftarrow \pi_r + \nabla_{\pi_r} J_r(\pi_r, \pi)$.

end for

Algorithm 2 ERMAS inner loop: Agent-Adv-Search($\epsilon$).

**Output:** Parameters $\theta$ of agent policies.

**Input:** Robust robust actor’s objective $J_r$, agent objectives $J_1, \ldots, J_N$.

**Input:** Robust robust actor policy $\pi_r$, reward slack $\epsilon$, trust region radius $\rho$, Lagrange multiplier $\lambda$, learning rate $\alpha$.

**Input:** Number of warm-up steps $k$, number of overall training steps $n_{\text{train}}$, and number of inner learning steps $n_{\text{inner}}$.

Warm up agent policies $\theta$ with $k$ steps of vanilla policy-gradients on $-J_r(\theta) + \sum_{i=1}^{N} \lambda_i J_i(\theta)$.

Copy agent parameters into placeholder parameters $\theta' \leftarrow \theta$.

for $j = 1, \ldots, n_{\text{train}}$ do

  Execute $\pi_{\theta}$ to accumulate batch of experiences $B_0$ with mean rewards $\mu_0$.

  Apply standard policy gradient: $\theta' \leftarrow \theta' + \nabla_{\theta} \left(-J_r(\theta) + \sum_{i=1}^{N} \lambda_i J_i(\theta)\right)$.

  for $i = 1, \ldots, N$ do

    Initialize parameters: $\theta^*_i \leftarrow \theta_i$. // agent $i$ will unilaterally deviate from $\pi_{\theta}$

    for $k = 1, \ldots, n_{\text{inner}}$ do

      Update $\theta^*_i \leftarrow \theta^*_i + \text{TRPO} \left(\theta^*_i, \theta_{-i}, \pi_r, \frac{\rho}{n_{\text{inner}}}, B_{k-1}\right)$ to increase $J_i(\theta^*_i, \theta_{-i})$.

      Execute $\pi_{\theta^*_i}, \pi_{\theta_{-i}}$ to accumulate batch of experiences $B_k$ with mean rewards $\mu_k$.

    end for

    Approximate meta-learning correction to parameters: $\theta'_i \leftarrow \theta'_i - \lambda_i \theta^*_i$.

    Update multiplier $\lambda_i \leftarrow \lambda_i - \alpha \lambda_i (\mu_0, i - \epsilon - \mu_0, i)$.

  end for

  Copy $\theta \leftarrow \theta'$.

end for