Gluon Radiation Off Scalar Stop Particles

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ABSTRACT

We present the distributions for gluon radiation off stop-antistop particles produced in $e^+e^-$ annihilation: $e^+e^- \rightarrow \tilde{t}\bar{\tilde{t}}g$. For high energies the splitting functions of the fragmentation processes $\tilde{t} \rightarrow \tilde{t}g$ and $g \rightarrow \tilde{t}\bar{\tilde{t}}$ are derived; they are universal and apply also to high-energy stop particles produced at hadron colliders.
Introduction. Stop particles are exceptional among the supersymmetric partners of the standard-model fermions. Since the top quarks are heavy, the masses of the two stop particles \( \tilde{t}_1 \) and \( \tilde{t}_2 \), mixtures of the left (L) and right (R) squarks, may split into two levels separated by a large gap \(^1\). The mass of the lightest eigenstate \( \tilde{t}_1 \) could be so low that the particle may eventually be accessible at the existing pp̅ and even \( e^+ e^- \) storage rings. So far the result of search experiments at \( e^+ e^- \) colliders \(^4, 5\) has been negative and a lower limit of 45.1 GeV has been set at LEP \(^6\) for the L/R mixing angle outside the band of \( \cos^2 \theta_t \) between 0.17 and 0.44 and for a mass difference between the \( \tilde{t}_1 \) and the lightest neutralino \( \tilde{\chi}_1^0 \) of more than 5 GeV. The higher energy at LEP2 and dedicated efforts at the Tevatron will open the mass range beyond the current limits soon.

To begin, we briefly summarize the well-known theoretical predictions for the cross section of the production process \([\text{Fig} \, 1\, (a)]\)

\[
e^+ e^- \rightarrow \tilde{t}_1 \tilde{t}_1
\]

For a given value \( \theta_t \) of the L/R mixing angle, the vertices of the \( \tilde{t}_1 \) pair with the photon and the \( Z \) boson may be written as \( i e_0 \tilde{Q}_{[p_{\tilde{t}_1} - p_{\tilde{t}_1}^\mu]} \), where \( p_{\tilde{t}_1} \) and \( p_{\tilde{t}_1}^\mu \) are the 4-momenta of the stop and antistop squarks, and the charges read

\[
\tilde{Q}_\gamma = -e_t \\
\tilde{Q}_Z = (\cos^2 \theta_t - 2 e_t \sin^2 \theta_W) / \sin 2 \theta_W
\]

respectively. \( \theta_W \) is the standard electroweak mixing angle and \( e_0 = \sqrt{4 \pi \alpha} \) is the electromagnetic coupling to be evaluated with \( \alpha^{-1}(M_Z) = 129.1 \) in the improved Born approximation \(^7\). The Z boson coupling vanishes for the L/R mixing angle \( \cos^2 \theta_t \rightarrow 2 e_t \sin^2 \theta_W \approx 0.30 \). Defining the \( \gamma \) and \( Z \) vector/axial-vector charges of the electron, as usual, by \( e_e = -1 \), \( v_e = -1 + 4 \sin^2 \theta_W \) and \( a_e = -1 \), the cross section can be expressed in the compact form \(^8\)

\[
\sigma_B[e^+ e^- \rightarrow \tilde{t}_1 \tilde{t}_1] = \frac{\pi \alpha^2}{s} \left[ \tilde{Q}_\gamma^2 + \left( \frac{v_e^2 + a_e^2}{4 \sin^2 \theta_W} \right) \tilde{Q}_Z^2 \frac{s^2}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} + \frac{v_e \tilde{Q}_\gamma \tilde{Q}_Z}{\sin 2 \theta_W} \frac{s(s - M_Z^2)}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \right] \beta^3
\]

where \( \sqrt{s} \) is the center of mass energy and \( M_Z \), \( \Gamma_Z \) are the mass and the total width of the \( Z \) boson, respectively. The \( P \)-wave excitation near the threshold gives rise to the familiar \( \beta^3 \) suppression, where \( \beta = (1 - 4 m_{\tilde{t}_1}^2/s)^{1/2} \) is the velocity of the stop particles. Angular momentum conservation enforces the \( \sin^2 \theta \) law, \( \sigma_B^{-1} d\sigma_B/d \cos \theta = \frac{3}{4} \sin^2 \theta \), for the angular distribution of the stop particles with respect to the beam axis.

QCD corrections. Gluonic corrections modify the cross section \(^9, 10\). The virtual corrections, \( \text{Fig.} \, 1\, (b) \), can be expressed by the form factor

\(^1\)Since we focus on QCD gluon effects for light stop particles in the LEP range, we do not take into account quark-gluino loop effects, assuming the gluino to be heavy; these loop effects have been discussed for squark production at the Tevatron in Ref.\(^11\) and at \( e^+ e^- \) colliders in Ref.\(^12\).
\[ F(s) = \frac{4}{3} \frac{\alpha_s}{\pi} \left\{ \frac{s - 2m_{\tilde{t}_1}^2}{s \beta} \left[ 2 \text{Li}_2(w) + 2 \log(w) \log(1 - w) - \frac{1}{2} \log^2(w) + \frac{2}{3} \pi^2 - 2 \log(w) \right] \right. \\
\left. - \log(w) \log \left( \frac{\lambda^2}{m_{\tilde{t}_1}^2} \right) \right\} - 2 - \log \left( \frac{\lambda^2}{m_{\tilde{t}_1}^2} \right) \right\} \] (2)

where \( \alpha_s \) is the strong coupling constant and the kinematical variable \( w \) is defined as \( w = (1 - \beta)/(1 + \beta) \). The form factor is infrared (IR) divergent. We have regularized this divergence by introducing a small parameter \( \lambda \) for the gluon mass. The IR singularity is eliminated by adding the contribution of the soft gluon radiation [Fig.1(c)], with the scaled gluon energy integrated up to a cut-off value \( \epsilon_g = 2E_{\text{cut}}/\sqrt{s} \ll 1 \). The sum of the virtual correction \( (V) \) and the soft-gluon radiation \( (S) \) depends only on the physical energy cut-off \( \epsilon_g \),

\[ \sigma_{V+S} = \sigma_B \frac{4}{3} \frac{\alpha_s}{\pi} \left\{ \frac{s - 2m_{\tilde{t}_1}^2}{s \beta} \left[ 4 \text{Li}_2(w) - 2 \log(w) \log(1 + w) + 4 \log(w) \log(1 - w) \right] \right. \\
\left. + \frac{1}{3} \pi^2 - 2 \log(w) \log(\epsilon_g) \right\] + \frac{4m_{\tilde{t}_1}^2 - 3s}{s \beta} \log(w) + \log \left( \frac{m_{\tilde{t}_1}^2}{s} \right) - 2 \log(\epsilon_g) - 2 \right\} \]

After including the hard gluon radiation, the dependence on the cut-off \( \epsilon_g \) disappears from the total cross section. The total QCD corrections can finally be summarized in a universal factor \[ (\ref{eq:total}) \]

\[ \sigma[e^+e^- \to \tilde{t}_1 \tilde{t}_1 (g)] = \sigma_B \left[ 1 + \frac{4}{3} \frac{\alpha_s}{\pi} f(\beta) \right] \] (3)

with (Fig.2)

\[ f(\beta) = \frac{1 + \beta^2}{\beta} \left[ 4 \text{Li}_2(w) + 2 \text{Li}_2(-w) + 2 \log(w) \log(1 - w) + \log(w) \log(1 + w) \right] \]

\[ - 4 \log(1 - w) - 2 \log(1 + w) + \left[ 3 + \frac{1}{\beta^3} \left( 2 - \frac{5}{4}(1 + \beta^2)^2 \right) \right] \log(w) + \frac{3}{2} \frac{1 + \beta^2}{\beta^2} \]

Very close to the threshold the Coulombic gluon exchange between the slowly moving stop particles generates the universal Sommerfeld rescattering singularity \[ (\ref{eq:universal}) \] \( f \rightarrow \pi^2/2\beta \), which damps the threshold suppression, yet does not neutralize it entirely. Employing methods based on non-relativistic Green’s functions, an adequate description of stop pair production near threshold has been given in Ref.\[ (\ref{ref:non-relativistic}) \], which also takes into account screening effects due to the finite decay width of the stop particles. In the high-energy limit \[ (\ref{eq:high-energy}) \] the correction factor in eq.\[ (\ref{eq:total}) \] approaches the value \( (1 + 4\alpha_s/\pi) \).

In this note we present a general analysis of hard gluon radiation. We also include stop fragmentation due to collinear gluon emission in the perturbative regime at high energies and we give an account of non-perturbative fragmentation effects.
For unpolarized lepton beams the cross section for gluon radiation off $\tilde{t}_1$ squarks

$$e^+ e^- \rightarrow \tilde{t}_1 \tilde{t}_1 g$$

depends on four variables: the polar angle $\theta$ between the momentum of the $\tilde{t}_1$ squark and the $e^-$ momentum, the azimuthal angle $\chi$ between the $\tilde{t}_1 \tilde{t}_1 g$ plane and the plane spanned by the $e^\pm$ beam axis with the $\tilde{t}_1$ momentum [see Ref.[13]], and two of the scaled energies $x(\tilde{t}_1)$, $\bar{x}(\tilde{t}_1)$, $z(g)$ in units of the beam energy. The energies are related through $x + \bar{x} = 2$ and vary over the intervals $\mu \leq x, \bar{x} \leq 1$ and $0 \leq z \leq 1 - \mu^2$, where $\mu = 2m_{\tilde{t}_1}/\sqrt{s}$ denotes the squark mass in units of the beam energy. For the angles between the squark and gluon momenta we have

$$\cos \theta_{\tilde{t}_1 \tilde{t}_1} = \frac{2 - 2(x + \bar{x}) + x\bar{x} + \mu^2}{\sqrt{(x^2 - \mu^2)(\bar{x}^2 - \mu^2)}}$$

$$\cos \theta_{\tilde{t}_1 g} = \frac{2 - 2(x + z) + xz}{z\sqrt{x^2 - \mu^2}}$$

The spin-1 helicity analysis of the cross section results in the following well-known angular decomposition [14]

$$\frac{d\sigma}{dx d\bar{x} d\cos \theta d\chi/2\pi} = \frac{3}{8}(1 + \cos^2 \theta) \frac{d\sigma^U}{dx d\bar{x}} + \frac{3}{4}\sin^2 \theta \frac{d\sigma^L}{dx d\bar{x}}$$

$$-\frac{3}{2\sqrt{2}} \sin 2\theta \cos \chi \frac{d\sigma^I}{dx d\bar{x}} + \frac{3}{4}\sin^2 \theta \cos 2\chi \frac{d\sigma^T}{dx d\bar{x}}$$

(4)

$[U =$ transverse (no flip), $L =$ longitudinal, $I = \text{trv}\ast\text{long}, T = \text{trv}\ast\text{trv}$ (flip)]. If the polar and azimuthal angles are integrated out, the cross section is given by $\sigma = \sigma^U + \sigma^L$.

It is convenient to write the helicity cross sections as

$$\frac{\beta^3}{\sigma_B} \frac{d\sigma^j}{dx d\bar{x}} = \frac{\alpha_s}{4\pi} \frac{S^j + \mu^2 N^j}{(1 - x)(1 - \bar{x})}$$

(5)

The densities $S^j$ and $N^j$ are summarized in Table 1; $p$ is the momentum of the $\tilde{t}_1$ squark, $\bar{p}$ and $k$ are the longitudinal momenta of $\tilde{t}_1$ and $g$ in the $\tilde{t}_1$ direction, and $p_T$ is the modulus of the transverse $\tilde{t}_1$, $g$ momentum with respect to this axis [all momenta in units of the beam energy]. Since $I, T$ correspond to $\gamma, Z$ helicity flips by 1 and 2 units, they are of order $p_T^2$ and $p_T^4$, respectively. Note that the threshold suppression is absent in the $U, I, T$ components and attenuated in the leading longitudinal $L$ term as expected from eq.(3).

**Fragmentation.** In the limit where the gluons are emitted from fast moving squarks with small angles, the gluon radiation

$$\tilde{t}_1 \rightarrow \tilde{t}_1 g$$

can be interpreted as a perturbative fragmentation process. From the form of the differential cross section $d\sigma/dz dp_T^2$ we find in this limit for the splitting functions, in analogy to the
Table 1: Coefficients of the helicity cross sections in eq. (1). The energy and momentum variables are defined in the text.

Weizsäcker-Williams [15] and Altarelli-Parisi splitting functions [16],

\[
P[\tilde{t}_1 \to \tilde{t}_1; x] = \frac{\alpha_s}{2\pi} \frac{8}{3} \frac{x}{1-x} \log\frac{Q^2}{m_{\tilde{t}_1}^2}
\]

(6)

As usual, \(x\) and \(z\) are the fractions of energy transferred from the \(\tilde{t}_1\) beam to the squark \(\tilde{t}_1\) and the gluon \(g\) after fragmentation, respectively; \(Q\) is the evolution scale of the elementary process, normalized by the squark mass rather than the QCD \(\Lambda\) parameter [in contrast to the light quark/gluon sector]. As a consequence of angular-momentum conservation, the gluon cannot pick up the total momentum of the squark beam. [Similar zeros have been found for helicity-flip fragmentation functions in QED/ QCD [16, 17].]

By using the crossing rules \(\{z \to 1, 1 \to x\}\) and \(\{1 - x \leftrightarrow 1 - x\}\), familiar from the analogous splitting functions in QED [18], we derive for the elementary gluon splitting process into a squark-antisquark pair

\[
g \to \tilde{t}_1 \bar{\tilde{t}_1}
\]

the distribution

\[
P[g \to \tilde{t}_1; x] = \frac{\alpha_s}{2\pi} \frac{1}{2} \frac{1}{x} (1-x) \log\frac{Q^2}{m_{\tilde{t}_1}^2}
\]

(7)

after adjusting color and spin coefficients properly. This splitting function is symmetric under the \(\tilde{t}_1 \leftrightarrow \bar{\tilde{t}_1}\) exchange, i.e. \(\{x \leftrightarrow 1 - x\}\). The probability is maximal for the splitting into equal fractions \(x = 1/2\) of the momenta, in contrast to spinor QED/QCD where the splitting into a quark-antiquark pair is proportional to \(x^2 + (1-x)^2\) and hence asymmetric configurations are preferred.

The above splitting functions provide the kernels for the shower expansions in perturbative QCD Monte Carlos for \(e^+e^-\) annihilation such as Pythia [19] and Herwig [20]. They serve
the same purpose in the hadron-hadron versions of these generators as well as Isajet [21]. Of course, the interpretation of the radiation processes as universal fragmentation processes becomes increasingly adequate with rising energy of the fragmenting squarks/gluons.

If the $\tilde{t}_1$ squark is lighter than the top quark, the lifetime will be long, $\tau \geq 10^{-20}$ sec, since the dominant decay channel $\tilde{t}_1 \rightarrow t + \chi^0$ is shut off [$\chi^0 = LSP$]. The decay widths corresponding to the 2-body decay $\tilde{t}_1 \rightarrow c + \chi^0$ and 3-body slepton decays involve the electroweak coupling twice and hence will be very small [2]. As a result, the lifetime is much longer than the typical non-perturbative fragmentation time of order $1 \text{ fm}$ [i.e. $O(10^{-23}$ sec)] so that the squark has got enough time to form ($\tilde{t}_1 \bar{q}$) and ($\tilde{t}_1 qq$) fermionic and bosonic hadrons. However, the energy transfer due to the non-perturbative fragmentation, evolving after the early perturbative fragmentation, is very small as a result of Galilei’s law of inertia.

Describing this last step in the hadronization process of a $\tilde{t}_1$ jet by the non-perturbative fragmentation function à la Peterson et al. [22] (which accounts very well for the heavy-quark analogue), we find

$$D(x)_{NP} \approx \frac{4\sqrt{\epsilon}}{\pi} \frac{1}{x[1 - 1/x - \epsilon/(1 - x)]^2}$$

with the parameter $\epsilon \sim 0.5 \text{ GeV}^2/\text{m}^2_{\tilde{t}_1}$. Here, $x = E[(\tilde{t}_1 \bar{q})]/E[\tilde{t}_1]$ is the energy fraction transferred from the $\tilde{t}_1$ parton to the ($\tilde{t}_1 \bar{q}$) hadron etc. The resulting average non-perturbative energy loss

$$<1 - x>_{NP} \sim \frac{2\sqrt{\epsilon}}{\pi} \left[ \log \left( \frac{1}{\epsilon} \right) - 3 \right]$$

is numerically at the level of a few percent.

Monte Carlo programs for the hadronization of $\tilde{t}_1$ squarks link the early perturbative fragmentation with the subsequent non-perturbative hadronization. The relative weight of perturbative and non-perturbative fragmentation can be characterized by the average energy loss in the two consecutive steps. The overall retained average energy of the $\tilde{t}_1$ squarks factorizes into the two components,

$$<x> = <x>_{NP} <x>_{PT}$$

Summing up the energy loss due to multiple gluon radiation at high energies, we find in analogy to heavy-quark fragmentation [23]

$$<x>_{PT} = \left( \frac{\alpha_s(m^2_{\tilde{t}_1})}{\alpha_s(E^2)} \right)^{-8/3b}$$

with $b = (11-2n_f/3)+(-2-n_f/3)$ being the LO QCD $\beta$ function including the colored supersymmetric particle spectrum. At high energies, the perturbative multi-gluon radiation has a bigger impact than the final non-perturbative hadronization mechanism, e.g. $<x>_{PT} \approx 0.93$ for a $\tilde{t}_1$ beam energy $E = 1 \text{ TeV}$ and $m_{\tilde{t}_1} = 200 \text{ GeV}$ as compared to $<x>_{NP} \approx 0.98$. At low energies the two fragmentation effects are of comparable size.

After finalizing the manuscript, we received a copy of Ref.[10] in which the total cross sections for squark pair production in $e^+e^-$ annihilation have been discussed including squark-gluon and quark-gluino loops, yet not the gluon-jet distributions analysed in the present note.
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Figure 1: Generic diagrams for $\tilde{t}_1 \tilde{t}_1$ production in $e^+e^-$ collisions. (a) Born level; (b) virtual QCD corrections; (c) gluon radiation.
\[ e^+ e^- \rightarrow \tau_1 \bar{\tau}_1 (g) \]

QCD correction:
\[ \sigma = \sigma_B \left[ 1 + \frac{4 \alpha_s}{3 \pi} f(\beta) \right] \]

Figure 2: Coefficient of the QCD correction to the total cross section; shown is \( \beta f(\beta) \), cf. eq. (3), with \( \beta = (1 - 4m_{\tau_1}^2/s)^{1/2} \).