Mathematical modelling of paddy drying using fluidized bed dryer

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Abstract. Several mathematical models have been developed to illustrate the drying process of crops, and one of them is thin-layer modelling. However, there are still limited information about mathematical modelling in paddy drying. In this paper, thin layer modelling of paddy drying curve on drying using fluidized bed dryer is presented. The drying paddy was done with drying temperature variations of 50, 60, 70, 80, and 90 °C. The initial paddy moisture content is 33 % dry basis. The drying process was done for 1.5 hours. Paddy moisture was measured every 10 minutes using grain moisture meter. The thin layer models used are Newton, Page, Modified Page, Henderson and Pabis, Logarithmic, Two-term, and Wang and Singh models. Model fittings were done using MATLAB 2015 software by sum square error minimization. Experimental results showed that Modified Page model gave the best results, evidenced by the highest average $R^2$ value of 0.9949 and lowest average RMSE (Root Mean Square Error) value of 0.0631, followed by Page model and Henderson and Pabis model. The plotting of MR (Moisture Ratio) and time at five drying temperatures showed that effective diffusivity (Deff) value ranged from $1.0334 \times 10^{-8} \text{ m}^2/\text{s}$ to $2.1983 \times 10^{-8} \text{ m}^2/\text{s}$ and the value increases as drying temperature increases. The plotting results of $\ln D_{eff}$ and $1/T$ (absolute temperature) showed that the value of diffusivity factor ($D_o$) is $8.5293 \times 10^{-6} \text{ m}^2/\text{s}$ and activation energy (Ea) is 18.11 kJ/mol.

1. Introduction
Paddy provides around 20 % of total energy per capita and 13 % protein for global population. In Asia, paddy contributes to 35 % energy and 28 % protein, in South America 12 % energy and 9 % protein. Paddy is the main food in several developing countries, contributing to 4000 kJ energy per capita everyday. Starch is the main component of paddy (±75 %) which located in the endosperm, shaped like granules with the size of 3-10 μm. Protein is the second component in paddy (±8 %), located inside the endosperm, shaped like particles with the size of 1-4 μm. Harvested paddy usually has moisture content of 20 % or more (wet basis), therefore it has to be dried before being kept in storage. In tropical countries, paddy is usually dried by direct sunlight until it reaches moisture content of 14 % (wet basis). At this condition, paddy can be stored for 2-3 months. If further storage time is needed, the paddy’s moisture content must be lowered to 12 % (wet basis) using dryer [1].

Drying is one of the oldest and most commonly used food preserving methods. The most common drying method used until now is by drying directly under the sun (sun drying). However, recently there are lots of improvement and development on mechanical or semi-mechanical drying methods, which are cheap and effective in handling various adversities of sun drying like the need of wide area to dry the material,
fluctuation of solar radiation, weather change, contamination of microorganism, animal, and other chemical reactions. One of the artificial drying methods developed is Fluidized Bed Dryer (FBD) [2]. Fluidized Bed Dryer is a good alternative for paddy drying due to uniform product quality due to complete mixing and its high drying capacity due to high ratio of air mass to mass of product [3]. The drying rate of paddy in fluidized bed dryer was affected by drying air temperature and bed thickness [4]. The maximum drying temperature in fluidized bed was suggested as 115 °C to reduce moisture to 24-25% (d.b.) for ensuring rice quality [5]. The drying temperature in pulsed fluidized bed should be less than 145 °C for initial paddy moisture content of 28% (d.b.) to maintain rice quality [6].

Several mathematical models have been developed to illustrate the drying process. One of the modelling method commonly used on agricultural drying is thin layer modelling. There are several researches about thin layer modelling, such as by Jafari et al. (2017) about paddy drying in semi-industrial continuous band microwave dryer, Behera and Sutar (2018) about parboiling of paddy, and Li et al. (2016) about changes in moisture effective diffusivity and glass transition temperature of paddy during drying [7-9]. However, study about modelling of paddy drying in FBD has yet to be found. Therefore, the aim of this study is to determine the paddy drying kinetics on FBD by thin layer models and determine the value of activation energy, effective diffusivity, and its relation with temperature.

2. Experimental methods

2.1. Drying process

This study was performed in the Laboratory of Department of Chemical Engineering, Faculty of Engineering, Diponegoro University. The paddy used in this experiment was taken from Sragen region, Middle Java. 10 grams of paddy was put in a porcelain cup, then placed on 105 °C oven to determine the initial moisture content. The weight of the sample was measured every 10 minutes until it reaches constant value. From this test, paddy initial moisture content of 33 % dry basis was obtained. For drying operation, 200 grams of paddy were used. Paddy was inserted in the fluidization column to be dried for 90 minutes at different temperatures of 50, 60, 70, 80, and 90 °C. Every 10 minutes, a small portion of paddy was taken from the fluidization column to measure its moisture content using grain moisture meter.

2.2. Mathematical modelling

Seven drying models were used in this study. The modelling was done using MATLAB software. The definition of Moisture Ratio (MR) is shown in equation (1)

\[
MR = \frac{M - M_o}{M_e - M_o}
\]  

(1)

Where M is moisture content at certain time, M_o is initial moisture content, and M_e is equilibrium moisture content. To determine the accuracy of the models, there are 2 parameters that can be used, namely Root Mean Square Error (RMSE) and determination coefficient (R^2). The calculations of RMSE and R^2 were done using Microsoft Excel. A model is said to be suitable if it has high R^2 value and low RMSE value. The calculations of RMSE and R^2 are expressed in equation (2) and (3) [10].

\[
RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (MR_{exp,i} - MR_{pre,i})^2}
\]  

(2)

\[
R^2 = \frac{\left[ \sum_{i=1}^{N} (MR_{exp,i} - M_{exp}) (MR_{pre,i} - M_{pre}) \right]^2}{\sum_{i=1}^{N} (MR_{exp,i} - M_{exp})^2 \sum_{i=1}^{N} (MR_{pre,i} - M_{pre})^2}
\]  

(3)

Where Mr_{exp} is experimental Moisture Ratio, Mr_{pre} is Predicted Moisture Ratio, and N is number of observations.
2.3. Effective diffusivity and activation energy

Drying characteristics on falling rate period can be determined using Fick’s diffusion equation. With the assumptions of spherical paddy coordinate, uniform initial moisture content distribution, and long drying time, the relation of Moisture Ratio and Effective Diffusivity can be expressed with equation (4) [8]:

\[ MR = \frac{6}{\pi^2} \exp \left(-\frac{D_{\text{eff}} \pi^2}{r^2} \theta \right) \] (4)

Where \( D_{\text{eff}} \) is effective diffusivity (m\(^2\)/s), \( r \) is the radius of paddy (m), \( \theta \) is drying time (s), and \( \pi \) is a constant. By changing equation (4) into logarithmic form, a new linear equation is obtained, as shown in equation (5).

\[ \ln MR = \ln \left(\frac{6}{\pi^2}\right) - \frac{D_{\text{eff}} \pi^2}{r^2} \theta \] (5)

The value of effective diffusivity (\( D_{\text{eff}} \)) can be determined by plotting the ln MR data versus drying time (\( \theta \)). From the slope value obtained, effective diffusivity can be calculated. Furthermore, the relation of effective diffusivity and drying temperature can be illustrated with Arrhenius equation, as shown in equation (6) [11].

\[ D_{\text{eff}} = D_o \exp \left(-\frac{E_a}{RT}\right) \] (6)

Where \( D_o \) is diffusivity constant at infinite drying temperature (m\(^2\)/s), \( R \) is universal gas constant (8.314 J/mol.K), \( E_a \) is activation energy (kJ/mol), and \( T \) is absolute temperature (K). The value of \( E_a \) and \( D_o \) can be determined by plotting \( \ln D_{\text{eff}} \) and \( 1/T \), after linearization of the equation above, as shown in equation (7).

\[ \ln D_{\text{eff}} = \ln D_o - \frac{E_a}{RT} \] (7)

3. Results and discussion

3.1. Drying curve analysis

The relation of Moisture Ratio and drying time at five different drying temperatures is shown in Figure 1. From Figure 1, it is shown that as drying time increases, the Moisture Ratio decreases, and the higher the drying temperature, the reduction of Moisture Ratio become bigger. As a comparison, on research performed by Kamin and Janaun (2017) about paddy drying in a Laterally Aerated Moving Bed Dryer, at ambient temperature, it took 5 hours to reduce paddy’s moisture ratio from 1 to approximately 0.72 [12]. It can be concluded that the whole process of paddy drying were happened on falling rate period and the most important factor that affects paddy drying is moisture diffusion.
3.2. Modelling results

Seven models of thin layer drying were fitted with experimental Moisture Ratio data at five different temperatures. The models used are shown in table 1. The model with highest $R^2$ value and lowest RMSE value is considered the most suitable model to illustrate paddy drying kinetics on FBD. Modelling results shown that the most suitable model is Modified Page model, followed by Page model and Henderson and Pabis Model. The value of model constants used and the value of $R^2$ and RMSE are shown in table 2 while figure 2 shows the comparison of drying curve between experimental results and modelling results, in this case Modified Page model was used. From figure 2, it can be observed that there is a good agreement between experimental results and Modified Page model.

Table 1. Thin layer drying models

| No | Model              | Formula                                      | References            |
|----|--------------------|----------------------------------------------|-----------------------|
| 1  | Newton             | $\text{MR} = \exp(-kt)$                      | Ayensu [13]           |
| 2  | Page               | $\text{MR} = \exp(-kt^n)$                    | Menges and Ertekin [14]|
| 3  | Modified Page      | $\text{MR} = \exp(-kt) \times k$            | White et al. [15]     |
| 4  | Henderson and Pabis| $\text{MR} = a \exp(-kt)$                    | Kashaninejad et al. [16]|
| 5  | Logarithmic        | $\text{MR} = a \exp(-kt) + c$               | Togrul dan Pehlivan [17]|
| 6  | Two-term           | $\text{MR} = a \exp(-kt) + b \exp(-kt^2)$   | Wang et al. [18]      |
| 7  | Wang and Singh     | $\text{MR} = 1 + at + bt^2$                  | Wang et al. [18]      |

Figure 2. Drying curve comparison between experimental results and prediction results of Modified Page model
Table 2. Thin layer modelling results and value of constants used on paddy drying using FBD

| Model               | T (°C) | k    | n    | a    | b    | c    | k₁   | k₂   | R²     | RMSE  |
|---------------------|--------|------|------|------|------|------|------|------|--------|-------|
| Newton              | 50     | 0.0083 |      |      |      |      |      |      | 0.9898 | 0.0750 |
|                     | 60     | 0.0109 |      |      |      |      |      |      | 0.9784 | 0.1227 |
|                     | 70     | 0.0144 |      |      |      |      |      |      | 0.9700 | 0.1630 |
|                     | 80     | 0.0183 |      |      |      |      |      |      | 0.9687 | 0.1803 |
|                     | 90     | 0.0235 |      |      |      |      |      |      | 0.9759 | 0.1712 |
| Page                | 50     | 0.0272 | 0.6985 |      |      |      |      |      | 0.9960 | 0.0408 |
|                     | 60     | 0.0269 | 0.7709 |      |      |      |      |      | 0.9980 | 0.0300 |
|                     | 70     | 0.0286 | 0.8241 |      |      |      |      |      | 0.9908 | 0.0818 |
|                     | 80     | 0.0293 | 0.8785 |      |      |      |      |      | 0.9843 | 0.1208 |
|                     | 90     | 0.0308 | 0.9277 |      |      |      |      |      | 0.9840 | 0.1345 |
| Modified Page       | 50     | 0.0062 | 0.7499 |      |      |      |      |      | 0.9984 | 0.1119 |
|                     | 60     | 0.0086 | 0.711  |      |      |      |      |      | 0.9992 | 0.0156 |
|                     | 70     | 0.0112 | 0.6694 |      |      |      |      |      | 0.9990 | 0.0198 |
|                     | 80     | 0.0163 | 0.5978 |      |      |      |      |      | 0.9970 | 0.0454 |
|                     | 90     | 0.0237 | 0.5007 |      |      |      |      |      | 0.9811 | 0.1227 |
| Henderson and Pabis | 50     | 0.0073 | 0.9633 |      |      |      |      |      | 0.9873 | 0.0393 |
|                     | 60     | 0.0095 | 0.9489 |      |      |      |      |      | 0.9735 | 0.0734 |
|                     | 70     | 0.0116 | 0.9082 |      |      |      |      |      | 0.9855 | 0.0867 |
|                     | 80     | 0.0132 | 0.8451 |      |      |      |      |      | 0.9471 | 0.0859 |
|                     | 90     | 0.0155 | 0.7878 |      |      |      |      |      | 0.9459 | 0.0972 |
| Logarithmic         | 50     | 0.0089 | 0.8598 | 0.1137 |      |      |      |      | 0.9911 | 0.0363 |
|                     | 60     | 0.0107 | 0.8154 | 0.1139 |      |      |      |      | 0.9778 | 0.0483 |
|                     | 70     | 0.0134 | 0.7772 | 0.1114 |      |      |      |      | 0.9662 | 0.0624 |
|                     | 80     | 0.0161 | 0.7258 | 0.1105 |      |      |      |      | 0.9603 | 0.0755 |
|                     | 90     | 0.0201 | 0.6901 | 0.1045 |      |      |      |      | 0.9651 | 0.0942 |
| Two-term            | 50     | 0.9125 | 0.1152 | 0.0079 | 0.0221 |      |      |      | 0.9939 | 0.0960 |
|                     | 60     | 0.8766 | 0.1155 | 0.0097 | 0.022 | 0.0221 |      |      | 0.9810 | 0.1066 |
|                     | 70     | 0.8331 | 0.1151 | 0.012 | 0.0211 | 0.1151 |      |      | 0.9660 | 0.1110 |
|                     | 80     | 0.7839 | 0.1112 | 0.0141 | 0.0214 | 0.1112 |      |      | 0.9562 | 0.0997 |
|                     | 90     | 0.7341 | 0.1099 | 0.0172 | 0.0207 | 0.1099 |      |      | 0.9557 | 0.0870 |
| Wang and Singh      | 50     | 0.0062 | -0.0002 |      |      |      |      |      | 0.6783 | 0.8202 |
|                     | 60     | 0.0061 | -0.0002 |      |      |      |      |      | 0.5413 | 0.8455 |
|                     | 70     | 0.0062 | -0.0003 |      |      |      |      |      | 0.3922 | 1.6431 |
|                     | 80     | 0.0063 | -0.0003 |      |      |      |      |      | 0.5116 | 1.5674 |
|                     | 90     | 0.0061 | -0.0003 |      |      |      |      |      | 0.5490 | 1.5393 |

3.3. The effective diffusivity and activation energy in paddy drying using FBD
The effective diffusivity of paddy on drying using FBD at five different temperatures can be determined by plotting ln MR vs time, as shown in figure 3. Table 3 shows paddy’s effective diffusivity at different temperatures. From Table 3, it can be observed that the value of effective diffusivity increases as drying temperature increases. Therefore, it can be implied that high temperature will cause water inside the paddy to diffuse to drying air because of the difference in temperature and pressure between drying air and paddy [19]. As a comparison, effective diffusion determination with the same Fick’s law on the drying of macroalgae (*Oedogonium* sp.) using solar drying performed by Hammond et al. (2018) shows that, at the drying temperature of 25-60 °C, the optimum effective diffusivity of macroalgae obtained was $5.67 \times 10^{-9} \text{m}^2/\text{s}$ [20].
Figure 3. Plotting of ln MR versus drying time

Table 3. Effective diffusivity of paddy dried at different temperatures

| No | Temperature (°C) | Effective Diffusivity (m²/s) |
|----|------------------|------------------------------|
| 1  | 50               | 1.033 x 10⁻⁸                 |
| 2  | 60               | 1.214 x 10⁻⁸                 |
| 3  | 70               | 1.439 x 10⁻⁸                 |
| 4  | 80               | 1.732 x 10⁻⁸                 |
| 5  | 90               | 2.198 x 10⁻⁸                 |

Figure 4 shows the relation of effective diffusivity (ln \(D_{eff}\)) and absolute temperature. From Figure 4 it can be implied that the relation of effective diffusivity and absolute temperature is linear, which relates to Arrhenius relation between diffusivity coefficient and temperature [19]. According to Figure 4, the \(R^2\) value (determination coefficient) is 0.9872. From the plotting results, the value of diffusivity constant \((D_o)\) obtained is 8.529 x 10⁻⁶ m²/s and the value of activation energy (Ea) obtained is 18.11 kJ/mol.

Figure 4. Plotting of ln \(D_{eff}\) versus 1/T
4. Conclusions
Modified Page model is the most suitable model to illustrate the kinetics of paddy drying using Fluidized Bed Dryer (FBD). The value of effective diffusivity ranges from $1.033 \times 10^{-8}$ to $2.198 \times 10^{-8}$ m$^2$/s and will increase as drying temperature increases. The moisture diffusivity of paddy greatly depends on temperature, which follows the Arrhenius relation. From the plotting results of ln $D_{eff}$ and 1/T, the value of diffusivity constant ($D_c$) obtained is $8.529 \times 10^{-8}$ m$^2$/s and the value of activation energy (Ea) obtained is 18.11 kJ/mol.

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