Bose–Einstein condensation at reheating

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We discuss the possibility that a perturbative reheating stage after inflation produces a scalar particle gas in a Bose condensate state, emphasizing the possible cosmological role of this phenomenon for symmetry restoration.

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I. INTRODUCTION

At the end of inflation, the Universe was in a very cold, low entropy phase. In particular, its energy density was still dominated by the vacuum energy of the fields which were previously driving inflation. The process of converting this energy into a hot thermal bath of matter and radiation is known as reheating, and it represents one of the most active areas of research in inflationary cosmology. In the last ten years, following the works \[1,2\], it has been shown that the first stages of reheating can be characterized by interesting nonperturbative phenomena, according to the particular theory one is considering. However, all these different possibilities share the common feature that they are followed by a more standard stage of perturbative particle production leading to the formation of the thermal bath. In the present work we focus on the possibility that after inflation some light boson field \(\chi\) may experience a Bose-Einstein condensation, after production of the corresponding quanta via a perturbative reheating stage. This situation may be naturally realized if these quanta carry a charge which is either exactly conserved or weakly violated by interaction processes with all species produced at reheating. Condensation may then occur if a sufficiently large value for this charge is released during reheating.

The first condition is eventually a statement on the lagrangian density governing the \(\chi\) field dynamics. As an example we consider a simple model of a complex scalar field \(\chi\) whose mass \(m_{\chi}\) is assumed to be dominated by radiative effects. We also assume that the initial distribution has a particle/antiparticle asymmetry,

\[Q \equiv n_{\chi} - n_{\bar{\chi}},\]

and that the charge per comoving volume is conserved (or only very weakly violated) by the self-interactions among these quanta. Without loss of generality, we consider \(Q > 0\). The production of this charge will be discussed in the next section.

II. CONDITIONS FOR A CONDENSATE

In this section we discuss the conditions leading to the formation of a condensate. We consider a distribution of quanta of a light complex scalar field \(\chi\), whose mass \(m_{\chi}\) is assumed to be dominated by radiative effects. We also assume that the initial distribution has a particle/antiparticle asymmetry, \(Q = n_{\chi} - n_{\bar{\chi}}\), and that the charge per comoving volume is conserved (or only very weakly violated) by the self-interactions among these quanta. Without loss of generality, we consider \(Q > 0\). The production of this charge will be discussed in the next section.

*We neglect here interactions with other fields; see the next section for a more general discussion of the system in a cosmological setup.
The evolution of the system is of course strictly dependent on these self-interactions, since they determine the thermalization time-scale as well as the radiative mass. As an example, let us consider the interaction term

$$L_{int} = -\hbar |\chi|^4 .$$

At lowest order in $\hbar$, this term is responsible for $2 \leftrightarrow 2$ scatterings. Combining more vertices, processes which change the total number of particles $n = n_\chi + n_{\bar{\chi}}$ are possible, preserving however the charge $Q$. The efficiency of these processes in establishing a kinetic equilibrium, i.e. the thermalization time-scale, will be discussed in the next section. Here we consider the asymptotic equilibrium state in presence of a fast thermalization.

We start studying the case where a condensate does not form. The equilibrium distribution functions are therefore given by

$$f_{\chi,\bar{\chi}}^{BE}(p) = \left[ e^{\beta(E + \mu)} - 1 \right]^{-1} .$$

In this expression, $E \equiv \sqrt{p^2 + m_\chi^2(T)}$ denotes the energy of the quanta, while $\beta = 1/T$ is the inverse temperature. The presence of the chemical potential $\mu$ is related to a nonvanishing charge $Q$. If $Q$ and $\mu$ are both positive, the minus sign in Eq. (2) refers to particles $\chi$, and the plus sign to antiparticles $\bar{\chi}$. The equality $\mu_\chi = -\mu_{\bar{\chi}} \equiv \mu$ holds in presence of effective number violating interactions such as $\chi \bar{\chi} \chi \bar{\chi} \leftrightarrow \chi \bar{\chi}$.

While the distribution function $f_{\chi}^{BE}$ is always regular, we must impose $\mu < m_\chi$ to ensure the regularity of $f_{\chi}^{BE}$. The values of $\beta$ and $\mu$ can be determined by the two covariantly conserved quantities of the system, that is the total energy $\rho$ and charge $Q$ densities

$$\rho = \int \frac{d^3p}{(2\pi)^3} E [f_{\chi}^{BE}(p) + f_{\bar{\chi}}^{BE}(p)] ,$$

$$Q = \int \frac{d^3p}{(2\pi)^3} [f_{\chi}^{BE}(p) - f_{\bar{\chi}}^{BE}(p)] .$$

Let us consider the ratio

$$R(\mu/T, m_\chi/T) \equiv \frac{Q}{\rho^{3/4}} .$$

Notice that since we deal with radiation this ratio is unaffected by the universe expansion, since both quantities $Q$ and $\rho^{3/4}$ scale as the inverse third power of the scale factor, so that their ratio $R$ is constant. It is easy to see that $R$ monotonically increases with $\mu/T$. In particular, it ranges between 0 and the critical value $R_{cr}(m_\chi/T) \equiv R(m_\chi/T, m_\chi/T)$ as $\mu$ ranges between 0 and $m_\chi$. We have thus a simple criterion to understand whether a condensate forms. From the initial conditions for the system at reheating we calculate the quantity $R = Q/\rho^{3/4}$, which is conserved by the interactions among the quanta $\chi$ and $\bar{\chi}$. If this value is below $R_{cr}(m_\chi/T)$, the equilibrium distributions are just of the form (2), that is the condensate does not form. When instead the initial conditions are such that $R$ is greater than $R_{cr}(m_\chi/T)$, Eqs. (3) cannot represent the final distributions, but an additional singular term is present in $f_{\chi}(p)$

$$f_{\chi}(p) = f_{\chi}^{BE}(p) + (2\pi)^3 Q_\chi \delta_3(p) .$$

This shows the appearance of a condensate at zero momentum. The physical interpretation of this is very simple. Loosely speaking, $R > R_{cr}(m_\chi/T)$ corresponds to a case in which a too large charge $Q$ is present to be stored in a regular distribution function with the given energy density $\rho$. The exceeding part of this charge, denoted by $Q_\chi$, gives rise to the condensate.

The value of $R_{cr}(m_\chi/T)$ can be obtained from expressions (3) and (4) in the limit $\mu \to m_\chi$ for the distribution (2). The calculation can be performed numerically, or analytically when $m_\chi/T$ is much smaller than one. In this regime we find

$$\rho = T^4 \left( \frac{\pi^2}{15} + \mathcal{O} \left( \frac{m_\chi^2}{T^2} \right) \right) ,$$

$$Q = T^3 \left( \frac{1}{3} \frac{m_\chi}{T} + \mathcal{O} \left( \frac{m_\chi^3}{T^3} \right) \right) ,$$

such that the scaling $R_{cr}(m_\chi/T) \approx 0.456 m_\chi/T$ is expected for $m_\chi/T \ll 1$. We plot in Figure 1 both the exact numerical behavior and the analytical approximation for $R_{cr}(m_\chi/T)$. As we see, the analytical approximation is very good even for $m_\chi$ as large as $0.1 \div 0.2 T$.

![Figure 1](image-url)

**FIG. 1.** The value of $R_{cr}(m_\chi/T)$ versus $m_\chi/T$ both numerically (solid line) and with an analytic approximation valid at small arguments (dotted line). Initial conditions such that $R > R_{cr}(m_\chi/T)$ indicate the formation of a condensate.

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1 We use natural units $c = \hbar = k_B = 1$. 
If the field $\chi$ is very light, its mass will be dominated by radiative effects. In particular, from the interaction (1) among particles in the thermal distributions (3) we can perturbatively evaluate the thermal mass for $h \ll 1$. From one loop correction to $\chi$ propagator one easily find

$$m_{\chi}^2 = 4h \int \frac{d^3p}{(2\pi)^32E}\left(f_{\chi}^{BE}(p) + f_{\bar{\chi}}^{BE}(p)\right), \quad (9)$$

and hence

$$m_{\chi}(T) \sim \sqrt{\frac{h}{3}}, \quad (10)$$

Therefore we see that a condensate forms whenever the initial conditions result in

$$R > 0.26 h^{1/2}. \quad (11)$$

In the next sections we will discuss scalar condensation in a cosmological context. It is important to stress again that the expansion of the Universe does not modify the above condition (11), at least until the temperature drops below the value of the (non-thermal) mass of $\chi$.

We now consider the case $R > R_{cr}(m_{\chi}/T)$, so that a condensate forms. In general with the distribution function for the scalars $\chi$ given by Eq. (3), both charge $Q$ and energy density $\rho$ are shared by the regular part $f_{\chi,\bar{\chi}}^{BE}$ and the condensate. The analysis greatly simplifies in the limit where most of the charge $Q$ is stored in the condensate. It is worth noticing that this is also the case we are more interested in. When a large charge is initially produced, the formation of the Bose condensate may lead to cosmologically relevant effects. In this case, we have $n_{\chi} \sim Q_{c} \gg n_{\bar{\chi}}$ and $Q_{c} \gg Q_{th}$, where we have defined with $Q_{c}$ and $Q_{th} = Q - Q_{c}$ the charge accommodate in the condensate and in the thermal distribution respectively. Whenever number density is dominated by the condensate, evaluation of the thermal mass using Eq. (3) gives

$$m_{\chi}^2 \sim 4h \int \frac{d^3p}{(2\pi)^32E}\left(f_{\chi}^{BE}(p) + f_{\bar{\chi}}^{BE}(p)\right) + (2\pi)^3 Q_{c} \delta^3(p) \sim 2h \frac{Q_{c}}{m_{\chi}}, \quad (12)$$

and therefore

$$m_{\chi}(Q_{c}) \sim (2h Q_{c})^{1/3}. \quad (13)$$

This result of course holds when $Q_{c}^{1/3} > T$. More generally, for smaller $Q_{c}^{1/3}/T$ it is easy to check that a very good approximation for the thermal mass is given by

$$m_{\chi} \sim \left(2h Q_{c} + \frac{1}{3} h T^3\right)^{1/3}. \quad (14)$$

The total number density can be estimated as follows

$$n \sim Q_{c} + n_{th}(Q_{c}, T) \quad (15)$$

where the thermal contribution $n_{th}(Q_{c}, T)$ can be numerically evaluated using the expression $f_{\chi,\bar{\chi}}^{BE}(p)$ with $\mu_{\chi} = m_{\chi}$, and $m_{\chi}$ given by Eq. (14). In particular when the temperature is larger than the effective mass, though it is still smaller than $Q_{c}^{1/3}$

$$(2h Q_{c})^{1/3} < T < Q_{c}^{1/3}, \quad (16)$$

we have

$$n_{th} \sim \frac{2\chi(3)}{\pi^2} T^3. \quad (17)$$

Similarly the energy density can be written as

$$\rho \sim m_{\chi} Q_{c} + \rho_{th}(Q_{c}, T) \quad (18)$$

where $\rho_{th}(Q_{c}, T)$ and $m_{\chi} Q_{c}$ are the thermal contribution and the energy density stored in the condensate respectively.

![Fig. 2. The ratios $Q_{th}/Q_{c}$ (solid line), $n_{th}/Q_{c}$ (dashed line), $\rho_{th}/m_{\chi} Q_{c}$ (dotted line) and $R$ (dash-dotted line), in logarithmic scale, versus $Q_{c}/T^3$. $Q_{th}$ is the charge stored in the thermal distribution. Results are shown for $h = 0.01$.](image)
on $Q_c$ and $T$ given in Eq. [14]. As a consequence, the energy density stored in the condensate evolves in time as $Q_c^{1/3} \sim a^{-4}$, where $a$ is the scale factor of the Universe.

As we have said, we will be mainly interested in the following in studying the case where $Q_c$ represents the dominant contribution to $n$. This condition is satisfied whenever $Q_c > n_{th}$. Using the behaviour of $n_{th} \sim h$ and $\rho$ with $Q_c/T^3$ this condition can be cast in a lower bound for the ratio $Q/\rho^{3/4}$

$$\frac{Q}{\rho^{3/4}} \lesssim \frac{1}{3} \frac{1}{2} , \quad (19)$$

as $h$ varies in the range $10^{-10} \div 10^{-1}$. Condition (19) is of course more severe than Eq. (13), which merely states the formation of a condensate regardless of its magnitude with respect to $n_{th}$. The order of magnitude of this result, obtained by a numerical study, can be also achieved analytically by noticing that for all values of the coupling we have considered, whenever $n_{th} \sim Q_c$ the energy density receives the dominant contribution by the thermal part of the $\chi - \bar{\chi}$ distribution, so that

$$\rho \sim \rho_{th} = \frac{\pi^2}{15} T^4 . \quad (20)$$

Using the fact that $n_{th}^{3/4} \sim 3n_{th}$ (see Eq. (13)) we obtain that $Q_c$ dominates the number density when $Q_c > n_{th} \sim \rho_{th}^{3/4} / 3$, i.e. whenever $R > 1/3$.

III. FORMATION OF THE CONDENSATE

In this section we discuss a possible scenario for the formation of a condensate in the early stages of the Universe. \footnote{For related works, see [4–5].} We start with a field $\phi$ which is oscillating around the minimum of its potential $V(\phi)$. \footnote{We assume that the minimum of the potential corresponds to $\phi = 0$ and that anharmonic terms in the Taylor expansion of $V(\phi)$ can be neglected, so that $\phi$ has a constant mass $m_\phi$.} One interesting possibility is that $\phi$ is a modulus field of a supersymmetric theory. SUSY models offer several candidates for moduli fields, since their scalar potential almost unavoidably presents flat directions if supersymmetry is preserved. Most of these directions are expected to acquire a mass of the order of the supersymmetry breaking scale. Actually this scale, in the early Universe, can be very different from the present one, due to the dynamics of the inflaton or of other scalar fields during inflation and the first stages of reheating. See [6] for detailed discussions. Nevertheless, though supersymmetry naturally offers a theoretical framework, we aim to discuss the condensation phenomenon in a more general context. For this reason we leave the mass $m_\phi$ as a free parameter. Also for the initial amplitude $\phi_0$ of the field we have some natural scales, as for example the Planck mass, emerging in the context of supergravity models, or the breaking scale of GUT theories. However, also for $\phi_0$ very different values can be considered in principle, and we will also take it as an input parameter.

At very early times the field $\phi$ is frozen at $\phi_0$, due to the friction provided by the expansion of the Universe. When, at a time $t \equiv t_0$, the expansion rate $H$ falls down $m_\phi$, the field $\phi$ starts oscillating around the minimum $\phi = 0$. The amplitude of these oscillations decreases as $a^{-3/2}$. If this stage is already radiation dominated, we then have

$$\phi(t) = \phi_0 \left( \frac{a_0}{a(t)} \right)^{3/2} , \quad H(t) = m_\phi \left( \frac{a_0}{a(t)} \right)^2 , \quad (21)$$

where $a_0 \equiv a(t_0)$.

If $\phi$ has a sufficiently long lifetime, its coherent oscillations will dominate over the thermal background for $a > a_{dom} \equiv a_0 (3/8\pi) (M_{Pl}/\phi_0)^2$. After this time, the energy density stored in the oscillations redshifts with a matter-like behaviour, so that

$$H(t) = \sqrt{\frac{8\pi}{3}} m_\phi \phi_0 M_{Pl} \left( \frac{a_0}{a(t)} \right)^{3/2} , \quad a > a_{dom} , \quad (22)$$

while the amplitude $\phi(t)$ is still given by Eq. (21).

When eventually the $\phi$ field decays, it produces a reheating stage. In particular, baryon and lepton asymmetries can be generated in a rather natural way through the Affleck-Dine mechanism [3] if $\phi$ evolves along a complex direction and has baryon and lepton violating couplings. The same mechanism can be applied for the production of a generic charge $Q$. Let us indeed suppose that the field $\phi$ mainly decays at the time $t_d$ into the complex scalar $\chi$ considered in the previous section. Through a
mechanism à la Affleck-Dine, these decays produce an asymmetry

\[ Q \equiv n_\chi - n_\chi = fn_\phi(t_d) ~, \]
\[ n_\phi(t_d) \equiv \rho_\phi(t_d)/m_\phi = \phi_d^2 m_\phi \ . \quad (23) \]

Without entering into the details of the production, which is crucially dependent on the specific coupling between \( \phi \) and \( \chi \) and on the dynamics of the scalar field \( \phi \), we have parametrized with \( f \) in Eq. (23) the charge produced per quantum of \( \phi \). We have also parametrized with \( \phi_d \) the amplitude of the oscillations of \( \phi \) at decay.

It is immediate to see how large \( f \) should be in order to produce a condensate in the \( \chi \) quanta distribution. Since \( Q/\rho^{3/4} = f \rho_\phi(t_d)^{1/4}/m_\phi \), we see that the necessary condition for a condensate, Eq. (11), is met for

\[ f^2 \gtrsim 0.07 \frac{m_\phi}{\phi_d} \ . \quad (24) \]

The more stringent requirement that the number density of \( \chi \) is dominated by the condensate, Eq. (19), rewrites instead

\[ f^2 \gtrsim (0.1 \div 0.25) \frac{m_\phi}{\phi_d} \ . \quad (25) \]

The interaction terms responsible for the decay of \( \phi \) into \( \chi \) will also give a mass to \( \chi - \chi \) quanta through radiative corrections. This mass vanishes once \( \phi \) has decayed, so that the considerations of the previous section (for example, Eq. (14)) remain unchanged. However, it could prevent \( \phi \) from decaying, whenever \( m_\chi(\phi_d) > m_\phi/2 \) (we do not consider here nonperturbative decay process, where this bound may not apply). This gives an upper bound on \( \phi_d \), which depends on the specific interaction terms between \( \phi \) and \( \chi \). For interactions of the form \( \sigma \phi \chi^2 + h.c. \), one finds for the decay rate

\[ \Gamma(\phi \rightarrow \chi \chi) \sim \frac{\sigma^2}{16\pi m_\phi^2} \sqrt{m_\phi^2 - 4m_\chi^2(\phi)} \ , \]
\[ m_\chi^2(\phi) = \sigma \phi \quad (26) \]

which, equated to the expansion rate of the Universe, gives

\[ \frac{\phi_d}{m_\phi} \sim \frac{m_\phi}{\sigma} \left(-2r^2 + r \sqrt{1 + 4r^2}\right) \ , \quad (27) \]

where we have defined

\[ r \equiv \left[\frac{3}{8\pi} \frac{\sigma^3 M_{Pl}}{16\pi m_\phi^3} \right]^{1/3} \sim 0.007 \frac{\sigma^3 M_{Pl}}{m_\phi^3} \ . \quad (28) \]

We are assuming that the scalar \( \phi \) decays after it dominates. Using Eq. (23), this translates in the bound

\[ \phi_d < \phi_0 \frac{(\phi_0/M_{Pl})^3}{\phi_0} \ . \quad (29) \]

We show in Fig. 4 how the ratio \( \phi_d/m_\phi \) changes for different values of \( m_\phi \) and \( \sigma \). As it is clear from the above discussion, see for example Eq. (24), this is a crucial quantity, since an increase of \( \phi_d \) over \( m_\phi \) corresponds to an increase of the number of produced quanta over their energy. We first note that, for any fixed \( m_\phi \), there exists only a finite interval for \( \sigma \) where \( \phi_d \) is enhanced. This is easily understood, since both a very low and a very high \( \sigma \) reduces the \( \phi \) decay rate (24), so that when the decay eventually takes place the \( \phi \) amplitude has been strongly damped by the expansion of the Universe. The decrease of \( \Gamma(\phi \rightarrow \chi \chi) \) with decreasing \( \sigma \) is rather obvious, since \( \sigma \) measures the strength of the \( \chi-\phi \) coupling. In the limit of high \( \sigma \), Eq. (27) reduces instead to \( \phi_d/m_\phi \simeq m_\phi/(4 \sigma) \). This shows that, in this limit, the \( \phi \) particles are prevented from decaying by the high effective mass \( m_\chi(\phi) \), and only once \( \phi_d \) is sufficiently reduced the decay can occur.

![FIG. 4. Contours for the ratio \( \phi_d/m_\phi \) in the \( \log(m_\phi/M_{Pl}) - \log(\sigma/m_\phi) \) plane. Continuous lines correspond, from right to left, to \( \phi_d/m_\phi = 1, 10, 10^2, 10^3 \).](image)

For a given \( m_\phi \), the maximum of \( \phi_d \) corresponds to \( r = 1/\sqrt{3} \), that is for \( \sigma_* \) such that \( \sigma_*/m_\phi \simeq 4 (m_\phi/M_{Pl})^{1/3} \). From Fig. 4 it is possible to understand the range for the coupling between the two species such that a sizeable condensate would appear. If \( \sigma \) is not too far from this value, we have from Eq. (27)

\[ \frac{\phi_d}{m_\phi} \simeq 0.05 \left(\frac{M_{Pl}}{m_\phi}\right)^{1/3} \ , \quad (30) \]

that is \( \phi_d/m_\phi \) increases as \( m_\phi \) decreases, as also clear from Fig. 4. Therefore, formation of a condensate is favoured at small \( m_\phi \). For \( \sigma \) close to the above value, the necessary condition Eq. (24) and the more stringent condition of Eq. (24) can indeed be cast in the form

\[ f^2 \gtrsim 1.4 h \left(\frac{m_\phi}{M_{Pl}}\right)^{1/3} \quad \text{and} \quad f^2 \gtrsim (2 \div 5) \left(\frac{m_\phi}{M_{Pl}}\right)^{1/3} \ . \quad (31) \]
However, both the absolute value of $\phi_d$ and of $Q \sim f m_\phi \phi^2$ decrease as $m_\phi$ becomes smaller. Hence, depending on which cosmologically scales one is interested in, the mass of the field $\phi$ cannot be taken arbitrarily small, if the condensate is supposed to play any relevant role. We will discuss this point in more details in the following section.

Until now we have simply assumed that, when all necessary conditions are fulfilled, the condensate forms due to a fast thermalization of the $\chi$ quanta. The actual generation of the condensate can be discussed by studying the typical timescales of self-interactions of the $\chi$ particles.

Kinetic equilibrium among the quanta of $\chi$ is largely governed by $2 \leftrightarrow 2$ scatterings, allowed at tree level by the interaction term $\chi^2 \phi$. At the initial stages, when the $\chi$ are produced by the $\phi$ field the corresponding rate can be estimated to be

$$\Gamma_{2 \leftrightarrow 2} \sim h^2 n / \langle E \rangle^2 \sim h^2 \phi_d^2 / m_\phi . \quad (32)$$

where $n \geq Q$ is the number density of the particles in the initial state (initially $n \sim n_\phi$ given in Eq. (23) and $\langle E \rangle$ their average energy, initially of the order of $m_\phi$. However, a complete thermalization requires that also processes which violate the particle numbers are in equilibrium. Among these, the ones which appear at the lowest order in the coupling $h$ are $4 \leftrightarrow 2$ processes, whose rates can be estimated as follows

$$\Gamma_{4 \rightarrow 2} \sim h^4 \langle E \rangle^4 / \langle E \rangle^8 , \quad \Gamma_{2 \rightarrow 4} \sim h^4 n / \langle E \rangle^2 . \quad (33)$$

For a thermal distribution without a condensate, one has the simple relations $n \sim \langle E \rangle^3 \sim T^3$. However, we will always consider systems for which $n > \langle E \rangle^3$. In the initial configuration, this occurs for

$$\phi_d > m_\phi . \quad (34)$$

Actually, we typically require $\phi_d \gg m_\phi$, i.e., as we have seen, small couplings $\phi < m_\phi$, and $m_\phi \ll M_{Pl}$, see Eq. (27). The initial distribution of $\chi$, $\bar{\chi}$ is thus characterized by high particle density and small energy of the individual quanta, so that the first stages of thermalization proceed via particle fusion. The decrease of $n$ and increase of $\langle E \rangle$ have the general effect of decreasing the interaction rates. However, the condition $n > \langle E \rangle^3$ can be never violated if the system has a sufficiently high charge $Q > \rho^{3/4}$. This is equivalent to require that most of the particle in the final distribution will be stored in the condensate, Eq. (22). When this condition is met, all the interaction rates are thus always greater than the ones the $\chi$, $\bar{\chi}$ would have if they were in a thermal distribution with the same energy $\rho_N \simeq m_\phi^2 \phi_d^2$, and without any condensate. If we therefore evaluate all relevant rates in the latter case, this will be a sufficient condition to ensure that the system we are considering rapidly approaches its equilibrium form.

Among the interactions we have considered, $2 \rightarrow 4$ processes have the lowest interaction rate. For a thermal distribution with temperature $T \sim \left( m_\phi^2 \phi_d^4 \right)^{1/4}$, we can estimate $\Gamma_{4 \rightarrow 2} \sim h^4 T$, so that thermalization is ensured, a fortiori, for

$$h \gtrsim h_\ast \equiv \left( \frac{\sqrt{m_\phi \phi_d}}{M_{Pl}} \right)^{1/4} . \quad (35)$$

If the coupling $h$ satisfies this bound, the $\chi$, $\bar{\chi}$ system is expected to quickly thermalize right after it is formed at the decay time $t_d$, leading to the formation of the condensate, mainly populated by $\chi$ quanta, and a thermal distribution of both $\chi$ and $\bar{\chi}$.

For $h < h_\ast$ thermalization can occur only when the scale factor of the Universe becomes $a \sim a(t_d) (h_\ast / h)^4$, since the ratio $\Gamma / H$ increases linearly with $a$. Therefore in this case the final effects of the condensate is expected to be strongly damped by the dilution of $Q_c$, due to the expansion of the Universe. The case $h \sim h_\ast$ requires more care, since (as we remarked) self-interactions between particles which are forming the condensate are much more efficient than in the thermal case ($n \gg \langle E \rangle^3$) and thus the formation of the condensate may be expected to hold in this case as well. However, we believe that a more precise answer in this regime cannot be obtained without a numerical study of the relevant Boltzmann equations. For this reason, hereafter the relation $h \gtrsim h_\ast$ will be always assumed.

Before concluding this section, it may be convenient to summarize the above results. We have considered some light scalars $\chi$, $\bar{\chi}$ with an initial asymmetry $Q$. Eq. (24) is the necessary condition for a condensate to appear in the distribution of $\chi$. The distributions for $\chi$ and $\bar{\chi}$ also keep a regular part (2), which contributes to the energy density of $\chi$ quanta. The total energy density redshifts as the one of radiation, as it is clear from Eqs. (18) and (19). The condition (25) corresponds to the case where most of the particle in the final distribution are stored in the condensate, so that the final number density approximately coincides with $Q_c$. Of course this condition is a stronger requirement than just the formation of the condensate of Eq. (24). In the specific example we have discussed, the quanta of $\chi$ are produced by the decay of a massive scalar $\phi$. We parametrized the energy density and the decay time of $\phi$ by its classical amplitude $\phi_d$ when it decays, given by Eq. (27). We have assumed that $\phi$ decays after it dominates, which is ensured by Eq. (29). Finally, (35) is a sufficient condition to guarantee fast self-interactions among the quanta of $\chi$, $\bar{\chi}$, so that the condensate actually forms right after the decay of $\phi$.

**For recent studies in thermalization, see [10–12].
IV. COSMOLOGICAL IMPLICATIONS

The aim of this section is to discuss the possible implications of the condensate of \( \chi \) particles. In particular, we show that it may lead to symmetry restoration. As it is well known, this issue has crucial consequences for cosmology, since when symmetry eventually breaks again, once the expansion of the Universe dilutes the condensate, topological defects can be formed, reintroducing a monopole problem. The possibility of symmetry restoration after nonperturbative creation at preheating has been emphasized in several works [13-19]. Here we show that this may happen also in the ordinary, perturbative reheating stage, which is generally expected to occur in all models of inflation.

We suppose that the field \( \chi \) is coupled to some other scalar field \( \psi \). For definiteness we will consider \( \psi \) to be complex, though our result applies to a real scalar field as well. Besides this interaction, the field \( \psi \) is assumed to have the potential

\[
V_\psi = \kappa \left( |\psi|^2 - \psi_0^2 \right)^2 ,
\]

and to be already settled down in a state with \( |\langle \psi \rangle| = \psi_0 \).

Due to the coupling to the \( \psi \) field, the scalar \( \chi \) acquires in general an effective mass \( m_\chi(\psi) \) and a modified dispersion relation, \( \omega^2 = k^2 + m_\chi^2(Q_c, T) + m_\chi^2(\psi) \), due to forward-scatterings, which however do not modify the distribution function of the \( \chi \) quanta as long as \( m_\chi(Q_c) \gg m_\chi(\psi) \). On the other hand this coupling also results in a contribution to the \( \psi \) one-loop effective potential [20-21],

\[
\frac{\partial}{\partial m_\chi^2(\psi)} \frac{\partial \Delta V_\psi}{\partial m_\chi^2(\psi)} = \frac{1}{(2\pi)^3} \int d^4 p \delta \left( p^2 - m_\chi^2(Q_c, T) - m_\chi^2(\psi) \right) \cdot (f_\chi(p) - f_\chi(p)) .
\]

(37)

In case the \( \chi \) number density is dominated by the condensate, \( f_\chi(p) \sim (2\pi)^3 Q_c \delta^3(p) \), this relation gives

\[
\Delta V_\psi = \sqrt{m_\chi^2(Q_c, T) + m_\chi^2(\psi)} Q_c ,
\]

where as discussed before we are considering \( Q_c \sim n_\chi \gg n_\chi \).

This term may shift the ground state of \( \psi \) to zero, thus restoring any symmetry which may be spontaneously broken when instead \( |\langle \psi \rangle| = \psi_0 \).

As an example, we consider an interaction term of the form \( g^2 |\psi|^2 |\chi|^2 \). In order to use the analysis contained in the previous sections, we first have to constrain our study to the case where the \( \chi \) quanta mass is still dominated by the radiative term of Eq. (38), i.e.

\[
g \psi_0 < (2h Q_c)^{1/3} .
\]

(39)

From Eq. (38) we therefore get

\[
\Delta V_\psi \sim \left( m_\chi(Q_c, T) + \frac{g^2 \psi^2}{2m_\chi(Q_c, T)} \right) Q_c ,
\]

(40)

which shifts the minimum of the total potential \( V_\psi + \Delta V_\psi \) to \( \psi = 0 \) for

\[
g^2 Q_c \geq 4 \kappa m_\chi(Q_c) \psi_0^2 .
\]

(41)

The charge per unit volume \( Q_c \) takes its largest value at the decay of the \( \phi \) field introduced in the previous section, and is then diluted by the expansion of the Universe. To see whether the symmetry broken by \( \psi \) is restored we evaluate the conditions (39) and (11) at the decay time \( t_d \). Using the expression (23) for \( Q_c \), we have

\[
\frac{\psi_0}{(m_\phi \phi_d^2)^{1/3}} \leq \min \left[ \frac{(2h f)^{1/3}}{g} , \frac{f^{1/3}}{2^{7/6} \kappa^{1/2} h^{1/6}} \right] .
\]

(42)

For definiteness, and to get the largest scale which can be affected by the formation of the condensate, we consider the case where the ratio \( \phi_d/m_\phi \) is the largest compatible with Eq. (27), for each \( m_\phi \). This is equivalent to the choice \( \sigma_c/m_\phi \), so that from Eq. (40) we have

\[
\frac{\psi_0}{m_\phi} \leq \left( \frac{M_{Pl}}{m_\phi} \right)^{2/9} \min \left[ \frac{0.2 (h f)^{1/3}}{g} , \frac{0.06 f^{1/3}}{\kappa^{1/2} h^{1/6}} \right] .
\]

(43)

Once some specific values for the parameters are chosen, Eq. (40) indicates which is the maximal scale \( \psi_0 \) which can be restored by the condensate. It is not inconceivable that scales as high as the GUT one can be restored, provided \( m_\phi \) is taken sufficiently high and the coupling \( \sigma \) is fixed to give the maximal \( \phi_d \) as in Eq. (39). On the contrary, if we are interested in restoring a symmetry at a smaller scale, more freedom is allowed for \( m_\phi \), as well as for the value of \( \phi_d \), as it is clear from Eq. (42). We remind that these conditions must be supplemented by the ones found in the previous section, essentially Eq. (24), necessary condition for the condensate, and Eq. (15), implying a rapid thermalization of the system. It is worth noticing that these constraints are all mutually compatible.

V. CONCLUSIONS

In this paper we have considered the possibility that a light scalar field, whose quanta are excited at a perturbative reheating stage, may produce a large Bose-Einstein condensate. This phenomenon takes place if these quanta carry a conserved charge and the reheating stage produces large value for this charge. The motivation for this study relies on the fact that in scenarios as the Affleck-Dine models for baryon and lepton number generation, the decay of a modulus field may typically result in a quite large charge per comoving volume in the final relativistic degrees of freedom, and this charge is conserved or only weakly violated by the self-interaction processes after the reheating stage.
We have studied in details the conditions which should be fulfilled for the formation of the condensate for a complex scalar field with a simple quartic self-interaction term, and how the condensation process depends on the time scale for thermalization via scattering and number changing processes. For a wide range for the mass of the field whose decays lead to reheating and for the relevant coupling constants, the condensation phenomenon may easily occur and, moreover, the condensed particles can represent the main contribution to the total number per comoving volume.

We have finally considered the role that the condensate may have in restoring a symmetry which was broken at a larger energy scale. It is interesting to notice that also a grandunified symmetry, with scale $M_{\text{GUT}} \sim 10^{15} \div 10^{16} \text{ GeV}$, can in principle be restored, although this requires some fine tuning in the parameters of the model. Restoration can occur more naturally for smaller scales, until the expansion of the Universe dilutes the charge per comoving volume which is stored in the condensate. This possibility would represent a new intriguing scenario in the evolution of the primordial Universe.

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