EPR & Klein Paradoxes in Complex Hamiltonian Dynamics and Krein Space Quantization

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Abstract. Negative energy states are applied in Krein space quantization approach to achieve a naturally renormalized theory. For example, this theory by taking the full set of Dirac solutions, could be able to remove the propagator Green function’s divergences and automatically without any normal ordering, to vanish the expected value for vacuum state energy. However, since it is a purely mathematical theory, the results are under debate and some efforts are devoted to include more physics in the concept. Whereas Krein quantization is a pure mathematical approach, complex quantum Hamiltonian dynamics is based on strong foundations of Hamilton-Jacobi (H-J) equations and therefore on classical dynamics. Based on complex quantum Hamilton-Jacobi theory, complex spacetime is a natural consequence of including quantum effects in the relativistic mechanics, and is a bridge connecting the causality in special relativity and the non-locality in quantum mechanics, i.e. extending special relativity to the complex domain leads to relativistic quantum mechanics. So that, considering both relativistic and quantum effects, the Klein-Gordon equation could be derived as a special form of the Hamilton-Jacobi equation. Characterizing the complex time involved in an entangled energy state and writing the general form of energy considering quantum potential, two sets of positive and negative energies will be realized. The new states enable us to study the spacetime in a relativistic entangled “space-time” state leading to 12 extra wave functions than the four solutions of Dirac equation for a free particle. Arguing the entanglement of particle and antiparticle leads to a contradiction with experiments. So, in order to correct the results, along with a previous investigation [1], we realize particles and antiparticles as physical entities with positive energy instead of considering antiparticles with negative energy. As an application of modified descriptions for entangled (space-time) states, the original version of EPR paradox can be discussed and the correct answer can be verified based on the strong rooted complex quantum Hamilton-Jacobi theory [2-27] and as another example we can use the negative energy states, to remove the Klein’s paradox without the need of any further explanations or justifications like backwardly moving electrons. Finally, comparing the two approaches, we can point out to the existence of a connection between quantum Hamiltonian dynamics, standard quantum field theory, and Krein space quantization [28-43].

1. Introduction
Complex spacetime actually stems from complex time, as first proposed by El Naschie [2], according to a special case of $E^\infty$ theory [3-5] and then applied by C. D. Yang in a series of papers [9-18]. El Naschie’s theory provides the probability of constructing a valid relativistic quantum mechanics for particles which are moving faster than light [7] and the two dimensional time introduced in El Naschie’s theory can cause the equipotential effects of gravity [8]. However, the complex spacetime was proposed by C. D. Yang in a series of papers in the form...
Our starting point is the relation (3.6) in ref. [17] by C. D. Yang:

\[ x^\mu = x^{\mu R} + ix^{\mu I}, \quad x^{\mu R}, x^{\mu I} \in \mathbb{R} ; \quad x^\mu = (ct, x, y, z), \]

mentioning that the boundary between classical and quantum mechanics could be broken, since the quantum operators are derived from Hamilton equation of motion [9,10]. These complex Hamilton equations were used to investigate wave-particle duality [11], tunneling [12], atomic shell structure of Hydrogen atom [13], multiple trajectories of particle beams [14], quantum state transition [15] and spin [16]. Therefore, it was shown the relativistic quantum mechanics is described by the complex quantum Hamilton-Jacobi (H-J) equations, i.e. the complex spacetime is a natural consequence of including quantum effects in the relativistic mechanics, and is a bridge connecting the causality in special relativity and the non-locality in quantum mechanics, so that extending special relativity to the complex domain leads to relativistic quantum mechanics and considering both relativistic and quantum effects, the Klein-Gordon equation could be derived as a special form of the Hamilton-Jacobi equation. It was also pointed out that the entangled state, causing the faster-than-light links, is a consequence of an entangled energy, plus a quantum potential, i.e. \( E^2 + 2m_0^2c^2Q \), resulting in a constant quantity. This quantity if used in the relativistic H-J equations, may describe quantum multiple trajectories. In this paper, the basis of our work is on a paper of Yang [17] in which after characterizing the complex time involved in an entangled energy state and writing the general form of energy considering quantum potential, two sets of positive and negative energies could be realized. However, we will realize new states for both positive and negative values of energy and momentum, and we will discuss the complex spacetime in a relativistic entangled “space-time” state leading to 12 extra wave functions than the four solutions of Dirac equation for a free particle. We will see that the entanglement of two particles or two antiparticles could be done only with opposite momenta and the entanglement of particle and antiparticle could be done only with the same momenta, where the latter is in contradiction with experiments. Therefore, in the means of modifying the results, we will realize particles and antiparticles as physical entities with positive energy instead of considering antiparticles with negative energy. Moreover, we will introduce unphysical particle and antiparticle with negative energy, as the complement of the sets of solutions for Dirac equation, in accordance to the concept of Krein quantization. Then, we will rewrite and modify the first introduced 16 wave functions in terms of physical and unphysical particles and antiparticles for different values of positive and negative values of momentum. Finally, we will discuss about the original version of Einstein-Podolsky-Rosen (EPR) paradox and we will verify the correct answer which is a unique solution due to the new results based on quantum Hamiltonian dynamics approach and then, using the unphysical negative energy states, we will remove the Klein’s paradox without the need of any further explanations or justifications like backwardly moving electrons.

2. Complex spacetime in a relativistic entangled “space-time” state

Our starting point is the relation (3.6) in ref. [17] by C. D. Yang:

\[
\psi(t, x, y, z) = (C_0^+ e^{i(k_0/h)t} + C_0^- e^{-i(k_0/h)t})(C_1^+ e^{ik_1x} + C_1^- e^{-ik_1x})
\]

\[
(C_2^+ e^{ik_2y} + C_2^- e^{-ik_2y})(C_3^+ e^{ik_3z} + C_3^- e^{-ik_3z})
\]

the relation (4.1) of ref. [17]:

\[
\psi(t, x, y, z) = (C_0^+ e^{i(k_0/h)t} + C_0^- e^{-i(k_0/h)t})e^{-i(k_1x + k_2y + k_3z)}
\]

which can be rewritten in terms of energy and momentum, respectively as:

\[
\psi(t, \vec{r}) = (C_0^+ e^{i(E/h)t} + C_0^- e^{-i(E/h)t})(C^+ e^{i\vec{p}.\vec{r}/h} + C^- e^{-i\vec{p}.\vec{r}/h})
\]

\[
\psi(t, \vec{r}) = (C_0^+ e^{i(E/h)t} + C_0^- e^{-i(E/h)t})e^{-i\vec{p}.\vec{r}/h}
\]
and finally, the identity (4.20) of ref. [17]: $E^2(t) + 2m_0c^2Q(t) = k_0^2 = \text{Const.}$ in which, both and are time dependent, but their summation appears to be a constant. Now, we can write the Eq. (5) as:

$$E(t) = \pm \sqrt{k_0^2 - 2m_0c^2Q(t)} = \pm \sqrt{(m_0c^2)^2 + c^2p^2 - 2m_0c^2Q(t)} \equiv \pm E_{\pm}$$

(5)

It is clear that for any time $t$, there are two momenta $(p > 0, p < 0)$ and two energies $(E_+ > 0, E_- < 0)$, and we can see that in this general form of energy, the quantum potential $Q(t)$ is nonzero. Also, we should notice that in Eqs. (3.13)-(3.16) of ref. [17]: $\psi_1(t, x, y, z) = Ce^{i(Et-\vec{p}.\vec{r})/h}$, $\psi_2(t, x, y, z) = Ce^{i(Et+\vec{p}.\vec{r})/h}$, $\psi_3(t, x, y, z) = Ce^{i(Et+\vec{p}.\vec{r})/h}$, $\psi_4(t, x, y, z) = Ce^{i(Et+\vec{p}.\vec{r})/h}$, the derived four wave functions were the eigenfunctions solved from the Dirac equation for a free particle and there was considered no entanglement and $Q(t) = 0$; we call it space-time state. In Eq.(2) or (4), the entanglement of time was considered and the wave function was written only for positive values of energy and momentum with $Q(t) \neq 0$; we call it space-entangled time state. In the following, we will realize another states for both:

$$\psi(t, \vec{r}) = (C_0^+ e^{i(E_{\pm}/h)t} + C_0^- e^{-i(E_{\pm}/h)t})(C^+ e^{i(\pm \vec{p}.\vec{r})/h} + C^- e^{-i(\pm \vec{p}.\vec{r})/h})$$

and are time dependent, but their summation appears to be a constant. Now, we can write the $\psi(t, \vec{r})$ as:

- For $E_+$, $E_-$, $p_+ > 0$, $p_+ < 0$:
  - $\psi_1(t, \vec{r}) = Ce^{i(E_{\pm}/h)t} (E > 0, p > 0)$
  - $\psi_2(t, \vec{r}) = Ce^{i(E_{\pm}/h)t} (E > 0, p < 0)$
  - $\psi_3(t, \vec{r}) = Ce^{i(E_{\mp}/h)t} (E < 0, p > 0)$
  - $\psi_4(t, \vec{r}) = Ce^{i(E_{\mp}/h)t} (E < 0, p < 0)$

- For $E_+$, $E_-$, $p_+ > 0$, $p_+ < 0$:
  - $\psi_5(t, \vec{r}) = (C_0^+ e^{i(E_{\mp}/h)t} + C_0^- e^{-i(E_{\mp}/h)t})(C^+ e^{i(\pm \vec{p}.\vec{r})/h} + C^- e^{-i(\pm \vec{p}.\vec{r})/h})$
  - $\psi_6(t, \vec{r}) = (C_0^+ e^{i(E_{\mp}/h)t} + C_0^- e^{-i(E_{\mp}/h)t})(C^+ e^{i(\pm \vec{p}.\vec{r})/h} + C^- e^{-i(\pm \vec{p}.\vec{r})/h})$
  - $\psi_7(t, \vec{r}) = (C_0^+ e^{i(E_{\mp}/h)t} + C_0^- e^{-i(E_{\mp}/h)t})(C^+ e^{i(\pm \vec{p}.\vec{r})/h} + C^- e^{-i(\pm \vec{p}.\vec{r})/h})$
  - $\psi_8(t, \vec{r}) = (C_0^+ e^{i(E_{\mp}/h)t} + C_0^- e^{-i(E_{\mp}/h)t})(C^+ e^{i(\pm \vec{p}.\vec{r})/h} + C^- e^{-i(\pm \vec{p}.\vec{r})/h})$

Now, we can realize four states: (a) Space-time state ($\psi_1, \psi_2, \psi_3, \psi_4$): $Q(t) = 0$ (b) Space-entangled time state ($\psi_5, \psi_6, \psi_7, \psi_8$): $Q(t) \neq 0$ (c) Entangled space-time state ($\psi_9, \psi_{10}, \psi_{11}, \psi_{12}$): $Q(t) \neq 0$ (d) Entangled space-entangled time state ($\psi_{13}, \psi_{14}, \psi_{15}, \psi_{16}$): $Q(t) \neq 0$, leading to the following 16 wave functions:

- For $E_+$, $E_-$, $p_+ > 0$, $p_+ < 0$:
  - $\psi_{11}(t, \vec{r}) = Ce^{i(E_{\mp}/h)t} (E > 0, p > 0)$
  - $\psi_{12}(t, \vec{r}) = Ce^{i(E_{\mp}/h)t} (E > 0, p < 0)$
  - $\psi_{13}(t, \vec{r}) = Ce^{i(E_{\mp}/h)t} (E > 0, p > 0)$
  - $\psi_{14}(t, \vec{r}) = Ce^{i(E_{\mp}/h)t} (E > 0, p < 0)$

The wave functions $\psi_1, \psi_2, ..., \psi_{16}$ after doing the multiplications could be described as:

- $\psi_1$: particle with positive momentum, $\psi_2$: particle with negative momentum, $\psi_3$: antiparticle with positive momentum, $\psi_4$: antiparticle with negative momentum.
- $\psi_5, \psi_7$: entangled particle-antiparticle with positive momenta, $\psi_6, \psi_8$: entangled particle-antiparticle with negative momenta.
- $\psi_9, \psi_{10}$: entangled particle-particle with opposite momenta.
- $\psi_{11}, \psi_{12}$: entangled antiparticle-antiparticle with opposite momenta.
- $\psi_{13}, \psi_{14}, \psi_{15}, \psi_{16}$: entanglement of (entangled particle-antiparticle with positive momenta) to (entangled particle-antiparticle with negative momenta) or entanglement of (entangled particle-particle with opposite momenta) to (entangled antiparticle-antiparticle with opposite momenta).

It can be seen that the entanglement of two particles or two antiparticles could be done only with opposite momenta, but the entanglement of particle and antiparticle could be done only
with the same momenta. Furthermore, we can deduce from (b) and (c) that the entanglement of time causes the entanglement of particle with antiparticle, and the entanglement of space causes the particle-particle or antiparticle-antiparticle entanglement. However, we encounter to a confliction, here. Empirical experiments demonstrate the quantum correlation at a distance of a particle-antiparticle system like kaon and antikaon system which are entwined. Therefore, at this stage, we want to introduce a parallel approach to correct all the being results. It seems as if something is missed here. According to the theory of Dirac, antiparticles are believed to be particles of negative energy. But, due to the fact that antiparticles are detectable, so the physical antiparticles must be of positive energy. Moreover, according to Eq. (5) there are both positive and negative energy states. However, it seems that taking the negative energies as antiparticles, is not covering all the underlying physics [1]. So, we propose that it is rational to accept that positive energy belongs to physical particles and negative energy belongs to unphysical particles. Then, we can say that the solutions of Dirac equation describe both physical particles and antiparticles with positive energy and both unphysical particles and antiparticles with negative energy. Consequently, we should modify the descriptions of wave functions $\psi_1, \psi_2, ..., \psi_{16}$ according to the following:

\[(aa) \psi_1 : \text{physical particle or antiparticle with } E_+ > 0, p > 0. \quad \psi_2 : \text{physical particle or antiparticle with } E_+ > 0, p < 0. \quad \psi_3 : \text{unphysical particle or antiparticle with } E_- < 0, p > 0. \quad \psi_4: \text{unphysical particle or antiparticle with } E_- < 0, p < 0. \quad (bb) \psi_5, \psi_6 : \text{entangled physical (particle-particle or antiparticle-antiparticle or particle-antiparticle or antiparticle-particle with } E_+ > 0 \text{ and } p > 0 \text{, or } p < 0. \quad \psi_7, \psi_8 : \text{entangled unphysical (particle-particle or antiparticle-antiparticle or particle-antiparticle or antiparticle-particle with } E_- < 0 \text{ and } p > 0 \text{, or } p < 0. \quad (cc) \psi_9, \psi_{10} : \text{entangled physical (particle-particle or antiparticle-antiparticle or particle-antiparticle or antiparticle-particle with } E_+ > 0 \text{ and opposite momenta.} \quad \psi_{11}, \psi_{12} : \text{entangled unphysical (particle-particle or antiparticle-antiparticle or particle-antiparticle or antiparticle-particle with } E_- < 0 \text{ and opposite momenta.} \quad \psi_{13}, \psi_{14} : \text{entanglement of two entangled physical (particle-particle or antiparticle-antiparticle or particle-antiparticle or antiparticle-particle with } E_+ > 0 \text{ and the same or opposite momenta.} \quad \psi_{15}, \psi_{16} : \text{entanglement of two entangled unphysical (particle-particle or antiparticle-antiparticle or particle-antiparticle or antiparticle-particle with } E_- < 0 \text{ and the same or opposite momenta.}

3. Discussion about EPR and Klein Paradoxes in Complex Hamiltonian Dynamics

In this section, first we discuss about the original version of Einstein-Podolsky-Rosen (EPR) paradox and find out the unique solution, due to the wave functions of $\psi_9, \psi_{10}$ in part (cc). In quantum mechanics, the position of a particle and its momentum have the commutation relation $[\hat{x}, \hat{p}] = i\hbar$. According to EPR, if we consider a two-particle system, the operators $(\hat{x}_1 - \hat{x}_2)$ and $(\hat{p}_1 + \hat{p}_2)$ will commute:

$$[\hat{x}_1 - \hat{x}_2, (\hat{p}_1 + \hat{p}_2)] = 0 \quad (6)$$

meaning that these operators have a common eigenstate with eigenvalues $x_1 - x_2 = a = \text{const.}$ and $p_1 + p_2 = 0 \text{, } p_2 = -p_1$. However, the above mentioned two-system implies a strange state, because the two particles are moving in opposite momentum directions while keeping their distance unchanged, and this is a unimaginable state. In a paper by G-j Ni et al. [19], it was mentioned that by the EPR definition of a two-particle system with the commutation relation (6), it seems that the original version of EPR paradox still remains to be answered. Then, for solving the problem, they propose the commutation relation $[\hat{x}_1 + \hat{x}_2, (\hat{p}_1 - \hat{p}_2)] = 0$ instead of the relation (6), and conclude that the correct answer turns out to be a particle and its antiparticle moving in opposite direction with momentum $p_1$ and $p_2 = -p_1$ and position $x_1$ and $x_2 = -x_1$, which is a unique solution to the original version of EPR paradox. As we can see, the wave functions in part (cc) can describe an entangled particle-antiparticle with $E_+ > 0$
and opposite momenta that can verify the correct answer which is a unique solution due to the new results based on quantum Hamiltonian dynamics approach.

As another example, we can treat Klein’s paradox with full set of Dirac solutions. In 1929, Oskar Klein [20] calculated the reflection and transmission coefficients for an incident beam of electrons of energy $E$, falling on a potential barrier of strength $V_0$. He found out that the unexpected amount of reflected electrons or transmitted electrons with a steady rate causes paradoxical results. Briefly speaking, according to Fig. 1, The momentum of free particles, i.e. the incident current in region (I) would be $p = \frac{1}{c}\sqrt{E^2 - m_0c^4}$, whereas in region (II), it becomes $q = \frac{1}{c}\sqrt{(E_0 - E)^2 - m_0^2c^4}$. Using Dirac’s relativistic theory [21], Klein found that the reflection and transmission coefficients for large potentials are respectively $R = \left(\frac{1+\kappa}{1-\kappa}\right)^2$, $T = \frac{2\kappa}{(1+\kappa)^2}$ and one can see $R + T = 1$. Also, $\kappa$ is a kinematical factor, defined as $\kappa = \frac{q}{p} \frac{E + m_0c^2}{E - ev_0 + m_0c^2}$ and choosing $e = 1$, the common form $\kappa = \frac{q}{p} \frac{E + m_0c^2}{E - ev_0 + m_0c^2}$ is obtained which becomes negative, when $E < V_0 - m_0c^2$. This implies a "paradoxical issue" since in this case, we get $R > 1$ while $T < 0$; the number of reflected particles from the barrier is greater than that of the incident ones on it. This is known as Klein’s Paradox. Another important point here, corresponds to the group velocity. For relation $q^2c^2 = (V_0 - E)^2 - m_0^2c^4$, the group velocity becomes $v_{gr} = \frac{dE}{dq} = \frac{E - v_0}{2c}$. So for left-to-right moving particles in region (II) of Figure 1, $v_{gr} > 0$, and since $E < V_0$, therefore $q$ has to be negative, i.e. $q = -\frac{1}{c}\sqrt{(V_0 - E)^2 - m_0^2c^4}$. This expression for $q$, gives [22,23]

$$\kappa = \left[\frac{(V_0 - E + m_0c^2)(E + m_0c^2)}{(V_0 - E - m_0c^2)(E - m_0c^2)}\right]^\frac{1}{2}. \quad (7)$$

Now, this gives positive values for $R$ and $T$ since from (7) it is always $\kappa \geq 1$ and also for intermediate potentials ($m_0c^2 < V_0 < E + m_0c^2$), $R$ and $T$ satisfy $R + T = 1$. Nevertheless the incident current is fulfilled in this latest case, the paradox still exists, since for $V_0 \rightarrow \infty$ and a fixed $E$, Eq.(7) results in a finite value for $\kappa$ and as a consequence, we get a non-zero finite limit for $T$. Therefore, it seems that the electrons could have passed the infinitely strong barrier, without any exponential damping. This is, once again, a paradoxical result which is in contrast to the damping, expected from the quantum tunnelling processes. This unexpected tunnelling was named "Klein Tunnelling" [24]. Now, we choose the one-dimensional Dirac equation $\hbar c \gamma_x \frac{d\psi}{dx} + m_0c^2\psi = \gamma_0(E - V_0)\psi$, in which $\gamma_x$ is the $x$-component of Dirac matrices and $\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. The four components of Dirac matrices obey the Clifford algebra $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\eta_{\mu\nu}$, where $\eta_{\mu\nu} = \text{diag}(+1,-1,-1,-1)$ is the Minkowskian flat metric. For

![Figure 1. The incident beam of electrons of energy $E$, in region I, coming down on the potential barrier of strength $V_0$, located in region II.](image-url)
energies in the range $m_0 c^2 < E < V_0 - m_0 c^2$, the incident wave function [22] in region (I), would be
\[ \psi_I(x) = \left( \frac{ip}{E - m_0 c^2} \right) e^{ipx} + A \left( \frac{-ip}{E - m_0 c^2} \right) e^{-ipx}, \]
while this wave function for transmitted electrons in region (II) becomes
\[ \psi_{II}(x) = B \left( V_0 - E - m_0 c^2 \right) e^{-i|q|x}. \]
Corresponding to the particle current $j(x) = \psi_I^\dagger(x) \hat{\alpha}_x \psi(x)$, $\hat{\alpha}(x) = \gamma_x \gamma_0$. The reflected and transmitted currents satisfy
\[ R = \frac{|j_R|}{|j_{inc}|} = \left( \frac{1 + \kappa}{1 - \kappa} \right)^2, \quad T = \frac{|j_T|}{|j_{inc}|} = \frac{4\kappa}{(1 - \kappa)^2}, \tag{8} \]
where $j_{inc}$ is the incident current. Once again, the paradoxical result appears $|j_R| > |j_{inc}|$.

What is important here is that both spin up or down electrons in the incident current, were supposed to be of positive energies. So the negative energies have been ignored in all available approaches. However, if we keep both positive and negative energies as viable values (although the negative energies are related to un-physical states) or in other words, if we take the full set of Dirac solutions or strictly speaking, the negative energies, then we can show that the reflected and transmitted currents are the same for both electrons and positrons and the paradox disappears and this result states that the Klein’s paradox arises because of some lack in initial data, which leads us to consider some reflected electrons and transmitted positrons (Hole theory) [1]. To prove this mathematically, we get back to the kinematical parameter from Eq.(7) for positive energies ($E > 0$). Maintaining the negative mode states, we define another kinematical parameter $\kappa'$ as $\kappa' = \left[ \frac{(V_0 + m_0 c^2) - (E + m_0 c^2)}{(V_0 - m_0 c^2) - (E - m_0 c^2)} \right]^\frac{1}{2}$ for $E < 0$. Note that $\kappa = \kappa' > 1$. This equation implies that for $V_0 \rightarrow \infty$, $\kappa' \rightarrow 1$. According to equation (8), having $\kappa$ and $\kappa'$, one can obtain relations between incident, reflected and transmitted currents for electrons (and positrons) of negative energies. We have
\[ R' = \frac{|j'_R|}{|j'_{inc}|} = \left( \frac{1 + \kappa'}{1 - \kappa'} \right)^2 = \left( \frac{1 + \frac{\kappa}{\kappa'}}{1 - \frac{\kappa}{\kappa'}} \right)^2 = \left( \frac{\kappa + \kappa'}{\kappa - \kappa'} \right)^2, \]
\[ T' = \frac{|j'_T|}{|j'_{inc}|} = \frac{4\kappa'}{(1 - \kappa')^2} = \frac{4\kappa}{(K - \kappa)^2}. \tag{9} \]
Now, we can easily see that maintaining all modes could provide equal values for reflected and transmitted electrons and positrons [42]. According to processes in regions (I) and (II) in Figure 2, and using equation (8) together with (9), one can define

\[ M = R_{E>0} + T'_{E<0} = \left( \frac{1+\kappa}{K-\kappa} \right)^2 + \frac{4\kappa\kappa}{(K-\kappa)^2} \]

which is the summation of reflected electrons of positive energies and transmitted ones of negative energies, and

\[ N = R'_{E<0} + T_{E>0} = \left( \frac{K+\kappa}{K-\kappa} \right)^2 + \frac{4\kappa\kappa}{(1-\kappa)^2} \]

which is the summation of the reflected positrons of negative energies and transmitted ones of positive energies, as it is in Figure 2. Dealing with numerators, it is simple to see that quantities \( M \) and \( N \) coincide, showing that the reflected and transmitted currents are the same for both electrons and positrons.

As a consequence, what we argue here is that negative energies could be as important as positive ones, since they contribute in variety of processes and some outstanding physicists have pointed out to their importance. According to Dirac himself: “Negative energies and probabilities should not be considered as nonsense. They are well-defined concepts mathematically, like a negative sum of money, since the equations which express the important properties of energies and probabilities, can still be used when they are negative. Thus, negative energies and probabilities should be considered simply as things which do not appear in experimental results. The physical interpretation of relativistic theory involves these things and is thus in contradiction with experiment. We therefore, have to consider ways of modifying or supplementing this interpretation” [25,26]. Also, Feynman has discussed the negative probabilities as viable concepts in quantum physics [27,28]. Negative energy states play the main role in the concept of Krein quantization, too, which is a naturally re-normalized theory. This theory by taking the full set of Dirac solutions, is able to remove the propagator Green’s function’s divergences and automatically without any normal ordering, cancel the expected value for vacuum state energy. Krein quantization was also used to re-interpret physical phenomena like Casimir effect. However, since it is a purely mathematical theory, the results are under debate and some efforts are devoted to include more physics in the concept [25-38]. Whereas Krein quantization is a pure mathematical approach, complex quantum Hamiltonian dynamics is based on the strong foundations of Hamilton-Jacobi equations and therefore on classical dynamics. Hence, it seems as if quantum Hamiltonian dynamics may construct a connecting bridge between standard quantum field theory and Krein space quantization in order to explain the reason for the practical ugly mathematics of renormalization and provide an answer when asked for what he had won the Nobel prize, Feynman replied, “For sweeping the infinities under the rug”?

4. Conclusion
Due to complex quantum Hamilton-Jacobi theory, complex spacetime is a natural consequence of including quantum effects in the relativistic mechanics. Characterizing the complex time involved in an entangled energy state and writing the general form of energy considering quantum potential, two sets of positive and negative energies could be realized that are in accordance with Krein space Quantization. Here, we realized new states for both positive and negative values of energy and momentum and then, we discussed the complex space-time in a relativistic entangled “space-time” state leading to 12 extra wave functions than the four solutions of Dirac equation for a free particle. We found that the entanglement of two particles or two antiparticles could be done only with opposite momenta and the entanglement of particle and antiparticle could be done only with the same momenta, where the latter was in contradiction with experiments, since experiments show the quantum correlation at a distance of a particle-antiparticle system like kaon and antikaon system which are entwined. Therefore, we came about to correct the obtained results. It seemed us as if something was missed there. Due to the theory of Dirac, antiparticles are believed to be particles of negative energy. But, according to the fact that antiparticles are detectable, so the physical antiparticles must be of positive energies i.e. taking the negative energies as antiparticles, is not covering all the underlying physics [1]. So, in order to correct
the results, along with a previous investigation [1], we realized particles and antiparticles as physical entities with positive energy instead of considering antiparticles with negative energy. Then, as an application of modified descriptions for entangled (space-time) states, we discussed the original version of EPR paradox and verified the correct answer based on the strong rooted complex quantum Hamilton-Jacobi theory [2-27] and as another example we mentioned that applying negative energy states removes the Klein’s paradox without the need of any further explanations or justifications like backwardly moving electrons [1,42]. Finally, comparing the two approaches, we pointed out the existence of a connection between quantum Hamiltonian dynamics, standard quantum field theory, and Krein space quantization [28-43].

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