Dynamic analysis and active control of hard-magnetic soft materials

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\section*{ABSTRACT}

The hard-magnetic soft materials which can sustain high residual magnetic flux density gradually attract the attention of researchers because of potential applications in soft robotics and biomedical fields. In this work, we focus on the dynamic response of hard-magnetic soft materials. The dynamic motion equations are derived by the Euler-Lagrange equation. The effects of the aspect ratio on the nonlinear vibration of the hard-magnetic soft cuboid under the force and applied magnetic fields in different directions are investigated. The amplitude-frequency curves demonstrate that the aspect ratio also has an influence on the frequency and amplitude of the primary resonance. Moreover, to eliminate undesired vibration responses, the PID controller is applied to the vibration of the hard-magnetic soft materials, and the desired results can be obtained.

\section*{1. Introduction}

Soft active materials can generate large deformation under external stimuli, such as solvents, electric and magnetic fields [1-3]. Among them, the magnetoactive soft materials have attracted the attention of researchers due to their ability such as fast reversible response, large deformation, response to remote stimulus, and safety for most organisms [4-9]. In addition, the magnetoactive soft materials have also been widely used in engineering, such as soft robotics [10,11], biomedical applications [5,12], and soft actuators [13,14].
It is known that the magnetoactive soft materials are fabricated by embedding magnetic particles into soft polymer matrices. The traditional magnetically active materials are fabricated by adding iron or iron oxide to the matrices [7,15]. However, such ferromagnetic particles with low coercivity do not retain strong magnetism when the external magnetic field is removed [16,17]. Recently, some magnetic particles with high coercivity such as NdFeB particles have been used to fabricate the hard-magnetic soft materials which is mechanically soft but magnetically hard [4,10,18]. The hard-magnetic soft material can maintain high residual flux density when the external magnetic field is removed. Hence, the deformations of hard-magnetic soft materials structures can be controllable by designing the magnitude and direction of residual flux density in the sample. Combined with 3D printing technology, the hard-magnetic soft materials structures can achieve more complex deformation and adapt to the complex working environment [4]. It can be seen that the hard-magnetic soft materials will play an important role in the fields of flexible electronics, biomedical devices, and soft robotics [19–22].

Due to the potential applications of magnetoactive soft materials, it is necessary to investigate the mechanical responses of the structures. Some theoretical models for the magnetoactive soft materials about nonlinear magneetoelastic modeling have been proposed [23–27]. However, these works mostly focused on the soft magnetic materials. Recently, Zhao et al. [28] developed a continuum model coupling the magnetic fields and the large deformation to model the ideal hard-magnetic soft materials by using magnetic Cauchy stresses. In addition, the developed model is implemented into the ABAQUS through the user-defined element and validated by a set of experiments. Subsequently, a visco-hyperelastic constitutive model is presented and numerically implemented within an implicit finite element framework [29]. Garcia et al. [30,31] also carried out some modeling works based on the microstructure for magneto-viscoelastic soft materials. Bustamante et al. [32] presented mathematical formulations for elastic magneto-electrically coupled soft materials by using spectral invariants for the energy function. Ye et al. [33] proposed a lattice model for hard-magnetic soft materials and implemented the model into the open-source molecular dynamic package-LAMMPS. Mukherjee et al. [34] proposed an explicit dissipative model for hard-magnetic soft materials which involves the effects of the particle volume fraction and the ferromagnetic hysteresis in the hard-magnetic particles. More recently, a three-dimensional model accounting for stretching, bending and twisting deformations was developed by Chen et al. [35]. The analytical solution based on linear theories for small bending deformation of hard-magnetic soft beams was also obtained [36]. Chen et al. [37,38] theoretically studied the exact solution for large-amplitude responses and the various deformed configurations with designed residual magnetic flux density of hard-magnetic soft beams. In addition, the instability of magnetoactive soft materials such as buckling, wrinkling, and bifurcation has also been studied [39–42].

It should be mentioned that the previous works rarely investigated the dynamic response of hard-magnetic soft materials. The nonlinear vibration of soft materials has attracted much attention when the soft materials are subjected to the time-dependent loading [43–48]. It is also an important research topic for the modeling and design of magnetic-response structures such as vibration isolators, vibration absorbers, and dampers [8,49–52]. In addition, due to the wide application of hard-magnetic soft materials in soft robots and medical actuators, precise control over their dynamic responses is important [11,53–56]. In this work, the dynamic response of hard-magnetic soft cuboid subjected to the external force and time dependent magnetic field is investigated. The effects of the direction of residual magnetic flux density and the aspect ratio of hard-magnetic soft material on the dynamic behaviors are discussed. In addition, the closed-loop feedback control is applied on the vibration of hard-magnetic soft materials. The desired results can be achieved by using the PID controller.
2. Dynamic modeling

As illustrated in Figure 1, a hard-magnetic soft cuboid is considered. The hard-magnetic particles can maintain high residual magnetic flux density once they are magnetically saturated compared with the soft-magnetic particles. In addition, the hard-magnetic materials can deform under a wider range of magnetic fields than soft-magnetic materials below the coercive field strength due to the high coercivity of the hard-magnetic particles. For more detailed material preparation process, please refer to the literature [28]. In the reference state, the cuboid has the original size \(2L \times 2L \times 2H\). We assume that the residual magnetic flux density \(\vec{B}^r\) is parallel to the positive z direction. In Figure 1(b), the cuboid is stretched where the direction of the applied magnetic field \(\vec{B}_{\text{applied}}\) is along the positive z direction and tensile force \(P\) is applied. In Figure 1(c), the cuboid is compressed where the applied magnetic field is along the negative z direction and the compression force \(P\) is applied. The coordinate \((x, y, z)\) denotes the material point in the current configuration corresponding to the spatial point \((X, Y, Z)\) in the reference configuration.

Based on the continuum mechanics, the motion of the cuboid is described by

\[
x = x(X, Y, Z, t), \quad y = y(X, Y, Z, t), \quad z = z(X, Y, Z, t)
\]

With the incompressibility condition, we can obtain

\[
x = \frac{X}{\sqrt{\lambda(t)}}, \quad y = \frac{Y}{\sqrt{\lambda(t)}}, \quad z = \lambda(t)Z
\]

and the deformation gradient can be expressed as

\[
F = \begin{pmatrix}
\lambda^{-\frac{1}{2}} & 0 & 0 \\
0 & \lambda^{-\frac{1}{2}} & 0 \\
0 & 0 & \lambda
\end{pmatrix}
\]

Figure 1. (a) The undeformed state of the cuboid with the original size \(2L \times 2L \times 2H\) along the X, Y, and Z directions. (b) The deformed state of the cuboid with the size \(2l_1 \times 2l_1 \times 2h_1\) along the x, y, and z directions under the tensile force and the applied magnetic field. (c) The deformed state of the cuboid with size \(2l_2 \times 2l_2 \times 2h_2\) along the x, y, and z directions under the compression force and the applied magnetic field.
where λ is the principal stretch. Then the governing equation can be derived by the Euler-Lagrange equation [45]

$$\frac{\partial L}{\partial \lambda} - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\lambda}} \right) = 0, \quad L = T - U$$

(4)

where L is the Lagrangian, T represents the kinetic energy of the system, and U is the potential energy.

The kinetic energy of the system is given as

$$T = \int \frac{1}{2} \rho (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) d\Omega = \frac{2}{3} \rho H L \lambda^3 \lambda^{-3} + \frac{4}{3} \rho H L^2 \lambda^2$$

(5)

where ρ is the density of the hard-magnetic soft cuboid. According to the ideal hard-magnetic soft materials model proposed by Zhao et al. [28], as long as the hard-magnetic particles are magnetically saturated, the residual magnetic flux density of the ideal hard-magnetic soft material keeps unchanged and is not affected by the external magnetic field below the coercivity field. In addition, the direction change of the remnant magnetization and the rotation of particles [57,58] are neglected in this work. Then the Helmholtz free energy density of the system is given as

$$W = W_{\text{elastic}}(F) - \frac{FB' \cdot B^{\text{applied}}}{\mu_0}$$

(6)

where μ₀ is the vacuum permeability. Due to the homogeneity, the potential energy U is obtained by multiplying the free energy density with the volume [28,45]. The potential energy of the system can be written as

$$U = 8H^2 \left[ \frac{G}{2} \left( \lambda^2 + \frac{2}{\lambda} - 3 \right) - \frac{P}{4L^2 \lambda} - \frac{\lambda B' B^{\text{applied}}}{\mu_0} \right]$$

(7)

where the Neo-Hookean model is adopted to describe the strain energy density of the material, and G is shear modulus.

Substituting Equations (5) and (7) into Equation (4), the governing equation is derived as

$$\dot{\lambda} - \frac{1.5\lambda^2}{\lambda + \left( \frac{2H^2}{L^2} \right) \lambda^4} + \frac{6G \lambda^4 - \lambda - \frac{P\lambda^3}{4GL^2} - \frac{B' B^{\text{applied}} \lambda^3}{G\mu_0}}{1 + \left( \frac{2H^2}{L^2} \right) \lambda^2} = 0$$

(8)

As can be seen, the governing Equation (8) involves the first and the second time derivatives of the stretch. As the time dependent terms are removed, the same analytical expression as Zhao et al. [28] can be obtained. This can prove the validity of our model.

### 3. Natural frequency analysis

Then we discuss the natural frequencies of hard-magnetic soft materials. According to Zhu et al. [43], the cuboid may reach an equilibrium state when the force P and the magnetic flux density B^{applied} are both static. Then Equation (8) reduces to

$$g(\lambda, B, P) = \frac{6G \lambda^4 - \lambda - \frac{P\lambda^3}{4GL^2} - \frac{B' B^{\text{applied}} \lambda^3}{G\mu_0}}{1 + \frac{2H^2\lambda^3}{L^2}} = 0$$

(9)
By solving Equation (9), the equilibrium stretch $\lambda_{eq}$ is obtained. As the equilibrium state is perturbed, we have

$$\lambda(t) = \lambda_{eq} + \Delta(t)$$  \hspace{1cm} (10)

where $\Delta(t)$ is the perturbed amplitude. Substituting Equation (10) into the governing Equation (8) satisfies

$$\ddot{\lambda}_{eq} + c\lambda_{eq}^2 + g(\lambda_{eq}, B, P) + \Delta + c\dot{\Delta}_{eq} + \Delta \frac{\partial g}{\partial \lambda} (\lambda_{eq}, B, P) = 0$$  \hspace{1cm} (11)

Since $\lambda_{eq}$ also satisfies the Equation (8), we have

$$\ddot{\Delta} + c\Delta_{eq}^2 + \Delta \frac{\partial g}{\partial \lambda} (\lambda_{eq}, B, P) = 0$$  \hspace{1cm} (12)

The natural frequency of the small-amplitude oscillation around the state of equilibrium is determined by

$$\omega^2 = \frac{\partial g}{\partial \lambda}$$

$$= \frac{6G}{\rho L^2 (1 + \frac{2H}{L^2} \lambda^3)^2} \left(4\lambda^3 - 1 - \frac{3P\lambda^2}{4GL} - 3 \frac{B'B^\text{applied}}{G\mu_0} \lambda^2 \right) \left(1 + \frac{2H^2\lambda^3}{L^2} \right)$$

$$- \frac{6G}{\rho L^2 (1 + \frac{2H}{L^2} \lambda^3)^2} \left(\lambda^4 - \lambda - \frac{P\lambda^3}{4GL} - 2 \frac{B'B^\text{applied}}{G\mu_0} \lambda^3 \right) \left(1 + \frac{6H^2\lambda^2}{L^2} \right)$$ \hspace{1cm} (13)

According to the previous work [28], the geometrical sizes can affect the mechanical deformation of hard-magnetic soft materials. In this work, we set $H = al$ to investigate the effect of the different geometry sizes on the natural frequencies of hard-magnetic soft cuboid. Figure 2(a) reveals the relationships between the natural frequencies and the applied magnetic field under different aspect ratios for a constant force $P = 100$N. It can be seen that the natural frequency of hard-magnetic soft cuboid decreases with the increase of the aspect ratio under the same applied magnetic field. This may be caused by the change of the inertia effect in the thickness direction. In addition, the natural frequency is almost constant and unaffected by the external magnetic field with $a = 0.5$. As $a < 0.5$ and the applied magnetic field is in the same direction as the residual flux density, the natural frequency increases with the ascending of applied magnetic field. However, when the applied magnetic field is opposite to the residual flux density, the natural frequency decreases with the increase of applied magnetic field. Regarding $a > 0.5$, the natural frequency decreases with the increase of the applied magnetic field which is along the direction of the residual flux density. Nevertheless, the natural frequency shows the incremental trend with the uprising of applied magnetic field as the directions of them are reversed. Figure 2(b) shows the relationships between the natural frequencies and the applied force under different aspect ratios with a constant applied magnetic field $B = 0.2$T. It shows similar trend as that in Figure 2(a). Furthermore, the ranges of frequency that varies with the magnetic field $B$ in the range of $[-1, 1]$ T and force $P$ in the range $[-1200, 1200]$ N are shown in Table 1.

4. Dynamic analysis

In this section, the dynamic response of the hard-magnetic soft cuboid subjected to an external force and a constant magnetic field or a periodic magnetic field is studied.
Figure 2. The relationships between the natural frequencies and (a) the applied magnetic field (b) the applied force with different aspect ratios.
Table 1. The frequency varies with the magnetic field $B$ in the range of $[-1, 1]$ T and force $P$ in the range $[-1200, 1200]$ N.

| Aspect ratio (a) | $a = 0.1$ | $a = 0.3$ | $a = 0.5$ | $a = 0.8$ | $a = 1$ |
|------------------|------------|------------|------------|------------|---------|
| Frequency [rad/s] varies with magnetic field | [895.6, 1018.5] | [848.8, 909.3] | 773 | [650.8, 597.5] | [575.5, 509.2] |
| Frequency [rad/s] varies with force | [897.6, 1208.9] | [849.7, 917.6] | 773 | [648.6, 591.4] | [574.9, 503.1] |

4.1. Dynamic response under a constant magnetic field

In this case, a cuboid structure with $L = H = 50$ mm under a constant magnetic field is considered. In Figure 3, we set constant magnetic field $B = 0.2$ T and tensile forces $P = 100$ N, 500 N, 1000 N. The materials parameters are set as: shear modulus $G = 303$ kPa, residual flux density $B' = 0.143$ T, density $\rho = 2434$ kg/m$^3$, permeability $\mu_0 = 4\pi \times 10^{-7}$ N/A$^2$ [28]. As shown in Figure 3(a), the amplitude of the vibration increases with the force, and the variation of the frequency with force is consistent with Figure 2(b). From Figure 3(b), the phase diagram forms a closed curve which means the cuboid experiences a steady periodic oscillation. Then the dynamic response and phase diagrams of the cuboid with $B = -0.2$ T and compression forces $P = -100$ N, -500 N, -1000 N are shown in Figure 4. The hard-magnetic soft cuboid is compressed and undergoes a steady periodic oscillation. Because the similar vibration behaviors will be obtained as Figures 3(a) and 4(a) except for a smaller amplitude, in order to avoid repetition, the cases under positive magnetic field with compression forces or the negative magnetic field with tensile forces are neglected.

4.2. Dynamic response under a tensile force and a periodic magnetic field

In this case, with the tensile force $P = 100$ N and applied magnetic field $B = 0.2$ T, we can obtain the equilibrium stretch by the Equation (9). The initial conditions are set as: $\lambda(0) = 1.03$ and $d\lambda(0)/dt = 0$. The tensile force $P = 100$ N and the periodic magnetic field $B = (0.2 + 0.1 \sin(\omega t))$ T are applied where the excitation frequency is the same as the natural frequency around the equilibrium state. The size is $L = 50$ mm, and the other material parameters are consistent with the previous work [28].

The dynamic response with different aspect ratios is plotted in Figure 5. It reveals that the hard-magnetic soft cuboid shows obvious beating phenomena under the time varying magnetic field. Compared with the case of constant magnetic field shown in Figure 3(a), the sample can exhibit compression, i.e. stretch $\lambda < 1$. In addition, the larger aspect ratio augments the duration of the beating. Here, we define the amplitude as the maximum difference between the peak and valley, and the amplitude reaches a minimum value when the aspect ratio $a = 0.5$. Then the phase paths and Poincare maps are provided to further study the periodicity and stability of hard-magnetic soft cuboid. As depicted in Figure 6, the phase paths do not go to infinity, which means the cuboid with different aspect ratios undergoes a stable vibration, and the Poincare maps forms the closed loops, which means a quasi-periodic vibration.

In what follows, the amplitude-frequency curves by taking the difference (Amp) between the peak stretch and the valley stretch as a function of excitation frequency are shown in Figure 7. As depicted in Figure 7, the amplitude-frequency curves shift to the left with the increase of aspect ratio. The peak of the primary resonance reaches the minimum value at $a = 0.5$, which...
Figure 3. Dynamic response (a) and phase diagrams (b) of the hard-magnetic soft cuboid under magnetic field $B = 0.2T$ and different tensile forces.
Figure 4. Dynamic response (a) and phase diagrams (b) of the hard-magnetic soft cuboid under magnetic field $B = -0.2$T and different compression forces.
Figure 5. Dynamic response of hard-magnetic soft cuboid under a tensile force and a periodic magnetic field in the same direction as the residual flux density with different aspect ratios (a) $a = 0.1, \omega = 944\text{rad/s}$, (b) $a = 0.5, \omega = 770\text{rad/s}$, and (c) $a = 1, \omega = 537\text{rad/s}$. 
Figure 6. Phase diagrams (blue line) and Poincare’ maps (red points) of hard-magnetic soft cuboid under a tensile force and a periodic magnetic field in the same direction as the residual flux density with different aspect ratios (a) $a = 0.1$, $\omega = 944\text{rad/s}$, (b) $a = 0.5$, $\omega = 770\text{rad/s}$, and (c) $a = 1$, $\omega = 537\text{rad/s}$. 
is consistent with trend in Figure 5. In addition, super-harmonic resonance occurs around the half natural frequency and there are also obvious sub-harmonic resonance phenomena at about the twice natural frequency. Moreover, the peak value of sub-harmonic resonance decreases with the increase of aspect ratio.

4.3. Dynamic response under a compression force and a periodic magnetic field

In this case, with the force $P = -100$N and applied magnetic field $B = -0.2$T, we can obtain the equilibrium stretch. The initial conditions are set as: $\lambda(0) = 0.96$ and $d\lambda(0)/dt = 0$. The periodic magnetic field $B = (-0.2 - 0.1 \sin(\omega t))T$ along the opposite direction of the residual flux density and a compression force $P = -100$N are applied.

Then the dynamic response of hard-magnetic soft cuboid with different aspect ratios is shown in Figure 8. It is different from the case where the constant magnetic field is applied. The cuboid shows more elongation than the compression when the time dependent magnetic field is applied. In addition, the hard-magnetic soft cuboid also shows obvious beating phenomena. From the phase paths and the Poincare’ maps shown in Figure 9, the hard-magnetic soft cuboid experiences a stable quasi-periodic vibration.

The amplitude-frequency curves under the compression force and a periodic magnetic field applied in reverse to the residual flux density are shown in Figure 10. It shows the same trend as Figure 7, i.e. the resonant frequency decreases gradually when the value of aspect ratio increases. The peak of the primary resonance declines at $a < 0.5$, and then aggrandizes at $a > 0.5$ which are also consistent with the dynamic response in Figure 8.

5. Active control of the hard-magnetic soft materials vibration

Hard-magnetic soft continuum robots which can be operated and navigated by remote control hold great promise in various fields, particularly in medical applications [11,20,54,55]. However, some nonlinear effects may affect the normal operation of the
robot during the vibration of the materials. Eliminating adverse nonlinear effect and precise control over their dynamic responses are important.

Next, a PID controller is introduced to realize the active control of the hard-magnetic soft materials. The PID controller which combines the proportional, integral, and derivative values of the errors in the system can adjust the outputs to meet the desired results. Figure 11 illustrates the block diagram of the closed loop feedback control system of hard-magnetic soft cuboid with the PID controller, where the error $e(t)$ is the input signal. The error is obtained from the difference between the reference signal and actual output. The part in the dotted box is the structure design of the PID controller. A PID module $f(e, \alpha, \delta)$ is introduced in Equation (14) [59].

$$f(e, \alpha, \delta) = \begin{cases} \frac{|e|^\alpha \text{sign}(e)}{\delta} & |e| \geq \delta \\ e & |e| < \delta \end{cases}$$

(14)

where $\text{sign}(e) = \begin{cases} 1 & e \geq 0 \\ -1 & e < 0 \end{cases}$ and the function $f(e, \alpha, \delta)$ represents the rate of feedback of the errors. The $e_p, e_i, e_d$ are the proportional, integral, and differential of errors. The $\delta$

Figure 8. Dynamic response of hard-magnetic soft cuboid under the compression force and a periodic magnetic field applied in reverse to the residual flux density with different aspect ratios (a) $a = 0.1$, $\omega = 915\text{rad/s}$, (b) $a = 0.5$, $\omega = 770\text{rad/s}$, and (c) $a = 1$, $\omega = 556\text{rad/s}$.
Figure 9. Phase diagrams and Poincare' maps of hard-magnetic soft cuboid under the compression force and a periodic magnetic field applied in reverse to the residual flux density with different aspect ratios (a) $a = 0.1$, $\omega = 915\text{rad/s}$, (b) $a = 0.5$, $\omega = 770\text{rad/s}$, and (c) $a = 1$, $\omega = 556\text{rad/s}$. 
determines the linear range of the function. The output signal of the PID controller $u(t)$ can be expressed as

$$u(t) = K_p f_p(e_p, a_p, \delta_p) + K_i f_i(e_i, a_i, \delta_i) + K_d f_d(e_d, a_d, \delta_d)$$

It is also the input of the controlled object. The detected signal is the deformation of the hard-magnetic soft cuboid. The dynamic governing equation of the system is expressed as follows [60]

$$\ddot{\lambda} + g(\lambda, P, B(\lambda, \dot{\lambda}, K_p, K_i, K_d)) = 0$$

where the $B$ is applied magnetic field. The amplitude of the applied magnetic field can be adjusted by using the appropriate control parameters to obtain the desired results.

In what follows, two cases are considered as examples to illustrate the validity of PID controller to tune the vibration of hard-magnetic soft material structure, where the size $L = 50$mm and aspect ratio $a = 0.5$. In the first case, the PID controller is used to eliminate the beating phenomenon in the vibration process of the hard-magnetic soft cuboid. Figure 12(a) shows the beating phenomenon without control under a tensile force $P = 100$N and a magnetic field along the direction of the residual flux density. The dynamic response with PID controller is plotted in Figure 12(b), and the target output is
\[ y(t) = 1.03 + 0.1 \sin(770t) \]. The control parameters are \( K_p = 90, K_i = 8, K_d = 1.2, \alpha_p = 0.5, \alpha_i = -0.1, \alpha_d = 1.1, \) and \( \delta = 4h \), where \( h \) is sampling time. As shown in Figure 12(b), the controlled dynamic response is a sinusoidal response, and the beating phenomenon is eliminated. The errors between the expected results and the controlled outputs are small in Figure 12(c) except in the initial stages. Then the dynamic response of the cuboid under a compression force \( P = -100N \) is considered, where the magnetic field is along the opposite direction of the residual flux density. The target output is \( y(t) = 0.96 + 0.1 \sin(770t) \). The control parameters are \( K_p = 115, K_i = 37, K_d = 0.6, \alpha_p = 0.1, \alpha_i = -0.03, \alpha_d = 1.5, \) and \( \delta = 3h \). It can be seen from Figure 13, the beating phenomenon is also eliminated well which indicates that the nonlinear PID controller is effective.

In the second case, the beating phenomenon is eliminated, and the phase shift is also achieved by the PID controller. Based on Figure 12(a), the solid line in Figure 14 is the

![Figure 12](image-url)

**Figure 12.** Dynamic response of hard-magnetic soft cuboid (a) without control and (b) with control under a tensile force and a magnetic field along the direction of the residual flux density. (c) Absolute errors between the expected results and the controlled outputs.
Figure 13. Dynamic response of hard-magnetic soft cuboid (a) without control and (b) with control under a compression force and a magnetic field along the opposite direction of the residual flux density. (c) Absolute errors between the expected results and the controlled outputs.
result of eliminating beating phenomena and achieving phase shift under a tensile force $P = 100\text{N}$ and a magnetic field in the same direction as the residual flux density. It can be seen that the desired results are obtained. The dash line is the reference signal for phase shift. In Figure 14, the expected result is $y(t) = 1.03 + 0.1 \sin(770t + 600)$, and the control parameters are $K_p = 500, K_i = 95, K_d = 7, a_p = 0.1, a_i = -0.1, a_d = 1.1$ and $\delta = 2h$. When the external magnetic field is opposite to the residual flux density, and the compression force $P = -100\text{N}$ is applied, the controlled results are shown in Figure 15. It can also be found that the beating phenomenon is eliminated, and the phase shift is achieved. In this situation, the expected result is $y(t) = 0.96 + 0.1 \sin(770t + 600)$, and the control
parameters are $K_p = 200$, $K_i = 100$, $K_d = 19$, $a_p = 0.1$, $a_i = -0.1$, $a_d = 1.1$ and $\delta = 2h$. According to the above discussion, the outputs of the hard-magnetic soft cuboid can be controlled to meet the desired results by the PID controller.

6. Conclusions

In this paper, the Euler-Lagrange equation is used to derive the dynamic governing equations of the hard-magnetic soft cuboid. Under a constant magnetic field, the hard-magnetic soft cuboid shows a stable periodic vibration. Under a periodic magnetic field and a constant force, the hard-magnetic soft cuboid shows obvious beating phenomena and stable quasi-periodic vibration. The influence of the aspect ratio on the dynamic response of the hard-magnetic soft cuboid under different loading conditions is also analyzed. The aspect ratio can affect the duration and the amplitude of the beating phenomena, as well as the frequency and amplitude of the primary resonance. Furthermore, a PID active control method is employed on the vibration of the hard-magnetic soft cuboid and desired control effect is achieved. The method can be applied to the active control of soft robot applications.

Disclosure statement

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