Nature of the Vacuum inside the Color Flux Tube

F. Gliozzi\* and S. Vinti\*  
\*Speaker at the conference

Dipartimento di Fisica Teorica, Università di Torino,  
via P. Giuria 1, 10125 Torino, Italy

The interior of the color flux tube joining a quark pair can be probed by evaluating the correlator of pair of Polyakov loops in a vacuum modified by another Polyakov pair, in order to check the dual superconductivity conjecture, which predicts a deconfined, hot core. We also point out that at the critical point of any 3D gauge theories with a continuous deconfining transition the Svetitsky-Yaffe conjecture provides us with an analytic expression of the Polyakov correlator as a function of the location of the probe inside the flux tube. Both these predictions are compared with numerical results in 3D $Z_2$ gauge model, finding complete agreement.

1. INTRODUCTION

The underlying assumptions of the dual superconductivity of gauge theories, and its appropriateness for describing quark confinement, are not rigorously founded, and it is necessary to perform precise numerical or analytic tests of this conjecture whenever possible.

The internal structure of the color flux tube joining a quark pair provides an important test of these ideas, because it should show, as the dual of an Abrikosov vortex, a very peculiar property: it is expected to have a core of normal, hot vacuum as contrasted with the surrounding medium, which is in the dual superconducting phase. The location of the core would be given by the vanishing of the disorder parameter $\langle \Phi_M(x) \rangle = 0$, where $\Phi_M$ is some effective magnetic Higgs field.

In a pure gauge theory, the formulation of this property from the first principles poses some problems, because no local, gauge invariant, disorder field $\Phi_M(x)$ is known. As a consequence, one cannot define in a meaningful, precise way the notion of core of the dual vortex.

A possible way out is suggested by the fact that in a medium in which $\langle \Phi_M \rangle = 0$ the quarks should be deconfined, then it is expected that the interquark potential inside the flux tube gets modified. As a consequence, one may try to define a gauge-invariant notion of normal core of the flux tube as the region where the interquark interaction mimics a deconfined behavior. Of course one cannot speak of a true deconfinement, as it would require pulling infinitely apart the quarks, while the alleged core has a finite size.

A simple, practical way to study in a lattice gauge theory the influence of the flux tube on the quark interaction is based on the study of the system of four coplanar Polyakov loops $P_1, P_2, P_3$ and $P_4$ following two steps

1. Modify the ordinary vacuum by inserting in the action the pair $P_3, P_4^\dagger$ acting as sources at a fixed distance $R$.

2. Evaluate in this modified vacuum the correlator the other pair $P_1, P_2^\dagger$ of Polyakov loops which are used as probes.

The correlators in the two vacua are related by

$$\langle P_1 P_2^\dagger \rangle_{\bar{q}q} = \frac{\langle P_1 P_2^\dagger P_3 P_4^\dagger \rangle}{\langle P_3 P_4^\dagger \rangle}.$$  \hspace{1cm} (1)

In this note we study some general properties of these correlators for $T \leq T_c$. In particular, we point out that at $T = T_c$ the functional form of these correlators is universal and in some 3D gauge theories can be written explicitly, even in finite volumes.

2. FOUR POLYAKOV LOOPS

Consider the system of four parallel, coplanar Polyakov loops, symmetrically disposed with respect the origin of a cubic lattice with periodic
boundary conditions in the direction of the imaginary time (which coincides with the common direction of the loops). We study their correlator

\[ \langle P_1 P_2^4 P_3 P_4^1 \rangle = \langle P(-\frac{r}{2}) P(\frac{r}{2}) P(-\frac{R}{2}) P(\frac{R}{2}) \rangle \]  \tag{2}

as a function of \( r \leq R \). For large \( R \) and \( r \sim R \) it obeys the asymptotic factorization condition

\[ \langle P_1 P_2^4 P_3 P_4^1 \rangle \sim \langle P_1 P_3^3 \rangle \langle P_2 P_4^3 \rangle . \]  \tag{3}

When \( T < T_c \), assuming the usual area law \( \langle P_1 P_2^3 \rangle \propto \exp(-\sigma r/T) \), where \( \sigma \) is the string tension, yields

\[ \langle P_1 P_2^3 \rangle \sim \exp(\sigma r/T) \sim 1/\langle P_1 P_3^3 \rangle , \]  \tag{4}

which gives an apparent repulsion between the two probes due to the attraction of the two sources.

The other limit \( r \ll R \) is more interesting, because the kinematics does not force any factorization and different confinement models suggest different behaviors. In particular in the naive string picture one is tempted to assume the factorization even in this limit, because within this assumption the total area of the surfaces connecting the Polyakov loops is minimal. On the contrary, in the dual superconductivity it is expected that the test particles probe the short distance properties of the hot core of the flux tube, thus the correlator in the modified vacuum would approach to a constant \( \langle P_1 P_2^3 \rangle \) from above and

\[ \langle P_1 P_2^3 \rangle \sim \langle P_1 P_2^3 \rangle \quad (r \ll R, T < T_c) . \]  \tag{5}

In the range \( T \geq T_c \), the interior of the flux tube is in the same phase of the surrounding region and the mutual interaction between the two near probes should not depend on the presence of very far sources, then

\[ \langle P_1 P_2^3 \rangle \sim \langle P_1 P_2^3 \rangle \quad (r \ll R, T \geq T_c) . \]  \tag{6}

2.1. Critical Behavior

According to the widely tested Svetitsky-Yaffe conjecture, any gauge theory in \( d + 1 \) dimensions with a continuous deconfining transition belongs to the same universality class of a \( d \)-dimensional \( C(G) \)-symmetric spin model, where \( C(G) \) is the center of the gauge group. It follows that at the critical point all the critical indices describing the two transitions and all the adimensional ratios of correlation functions of corresponding observables in the two theories should coincide. In particular, since the order parameter the gauge theory is obviously mapped in the corresponding one of the spin model, the correlation functions among Polyakov loops should be proportional to the corresponding correlators of spin operators:

\[ \langle P_1 \ldots P_n \rangle_{T=T_c} \propto \langle s_1 \ldots s_{2n} \rangle . \]  \tag{7}

Conformal field theory has been very successful in determining the exact form of these universal functions for \( d = 2 \) even in a finite box, which is a precious information for a correct comparison with numerical simulations.

In particular, using the known results of the 2D critical Ising model in a rectangle \( L_1 \times L_2 \) with periodic boundary conditions we can write explicitly the correlator of any (even) number \( 2 \) of Polyakov loops of any \( 2 + 1 \) gauge theory with \( C(G) = \mathbb{Z}_2 \). Let \( x_j, y_j \) be the spatial coordinates of \( P_j \) and define the complex variables \( z_j = \frac{x_j}{L_1} + i \frac{y_j}{L_2} \) and \( \tau = t L_2 / L_1 \). Then

\[ \langle P_1 \ldots P_{2n} \rangle^2 = c_n \sum_{\nu=1}^4 \sum_{\nu=1}^4 A_{\nu}(z \cdot \bar{z}) \prod_{i<j} B_{ij} \]  \tag{8}

with \( z \cdot \bar{z} = \sum_i \xi_i \bar{z}_i \) and the primed sum is constrained by \( \sum_i \xi_i = 0 \); \( c_n \) is an overall constant that can be expressed by factorization in terms of \( c_1 \). The universal functions \( A_{\nu} \) and \( B_{ij} \) can be written in terms of the four Jacobi theta functions \( \vartheta_\nu(z, \tau) \) as follows

\[
B_{ij} = \frac{\left| \frac{\vartheta_1(z_i - z_j, \tau)}{\vartheta_1(0, \tau)} \right|^{\epsilon_i \epsilon_j / 2}}{\vartheta_1(0, \tau)}, \tag{9}
\]

\[
A_{\nu}(z) = \frac{|\vartheta_\nu(z, \tau)|^2}{|\vartheta_\nu(0, \tau)|}, \tag{10}
\]

In the infinite box limit \( L_1, L_2 \to \infty \), using the Taylor expansion

\[
\vartheta_\nu(z, \tau) = a_\nu(1 - \delta_{1,\nu}) + b_\nu z + O(z^2), \tag{11}
\]

the correlator becomes

\[
\langle P_1 P_2 P_3 P_4 \rangle = \frac{4c_1^2}{(Rr)^{1/2}} \sqrt{\frac{R + r}{R - r}}, \tag{12}
\]

which satisfies both factorizations \( \text{(3)} \), \( \text{(8)} \).
3. CLUSTER ALGORITHM

In order to test the above formulae at criticality it is convenient to perform the numerical simulations in the simplest model belonging to the above-mentioned universality class, which is the the 3D $\mathbb{Z}_2$ gauge model.

Using the duality transformation it is possible to build up a one-to-one mapping of physical observables of the gauge system into the corresponding spin quantities. A great advantage of this method is that it can be used a non local cluster updating algorithm \[3\], which has been proven very successful in fighting critical slowing down.

In this framework it is easily shown that the vacuum expectation value of any set $\{C_1 \ldots C_n\}$ of Polyakov or Wilson loops of arbitrary shapes is simply encoded in the topology of Fortuin-Kasteleyn (FK) clusters: to each Montecarlo configuration we assign a weight 1 whenever there is no FK cluster topologically linked to any $C_i \in \{C_1 \ldots C_n\}$, otherwise we assign a weight 0. Let $N_0$ and $N_1$ be the number configurations of weight 0 and 1 respectively, then we have simply

$$\langle C_1 \ldots C_n \rangle = \frac{N_1}{N_0 + N_1}. \quad (13)$$

This method provides us with a handy, very powerful tool to estimate the correlator of any set of Wilson or Polyakov loops even at criticality.

4. RESULTS

In order to test the critical behavior of the multiloop correlator one has to know with high precision the location of the critical temperature as a function of the coupling $\beta$. We took advantage of ref.\[3\], where these critical values have been obtained with an extremely high statistical accuracy. We report in Fig.1 some results at $\beta = 0.746035$ corresponding to $1/aT_c = N_{tc} = 6$ and to a string tension $\sigma a^2 = 0.0189(2)$. The open circles are the data for the correlator $\langle P(-\frac{x}{2})P(\frac{x}{2}) \rangle$ in a $N_x \times N_y \times N_y$ lattice with $N_x = 3N_{tc}, N_y = 64$. They are well fitted by the one-parameter formula $c \exp(-\sigma Njr)/\eta(i\frac{N}{2})$, where the Dedekind $\eta$ function takes into account the quantum contribution of the flux tube violations $\[3\]$. The square symbols correspond to the correlator $\langle P(-\frac{x}{2})P(\frac{x}{2}) \rangle$ at the same temperature, in presence of a pair of sources at a distance $R = 24a$. The data in the central region are well fitted by a two-parameter formula $c_{pq} \exp(-\eta Njr)/\eta(i\frac{N}{2}) + b \to c_{pq} \exp(-\eta Njr)/\eta(i\frac{N}{2}) + b$ which simulates a high temperature behavior with a screening mass $m = \sigma N_j$ and an order parameter $\langle P \rangle = \sqrt{b}$. The black circles correspond to $\langle P(-\frac{y}{2})P(\frac{y}{2})P(-\frac{R}{2})P(\frac{R}{2}) \rangle$ evaluated at $T = T_c$ with $R = 16a$. They fit nicely eq.(8) (continuous line). Note that such a curve does not contain any free parameters, being $c_2 = \sqrt{2}c_1^2$ with $c_1 N_x = 0.199(4)$ as estimated by measuring $\langle P_1P_2 \rangle$ on lattices of different sizes at $T = T_c$ and $N_{tc} = 6$.

Figure 1. Correlator of two Polyakov loops inside and outside the flux tube

REFERENCES

1. G.Parisi, Phys. Rev. D 11 (1975) 970; G. ’tHooft, EPS International Conference, Palermo 1975; S.Mandelstam, Phys. Rep. 23 C (1976) 245.
2. P.Di Francesco, H.Saleur and J.B.Zuber, Nucl. Phys. B 290 [FS20] (1986) 527.
3. R.H.Swendsen and J.S.Wang, Phys. Rev. Lett. 58 (1987) 86.
4. M.Caselle and M.Hasenbusch, Nucl. Phys. B 470 (1996) 435.
5. P. Provero, these proceedings.