Black Holes Must Die

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Abstract
In light of recent evidence suggesting a nonzero present-day cosmological constant \[1\], Adams, Mbonye, & Laughlin\[2\] have considered the evolution of black holes in the presence of vacuum energy. Using the assumption that \(\Lambda\) remains constant with time and a conjecture based on a paper by Mallett \[3\], they reach the remarkable conclusion that black holes with current mass greater than \(\sim 2 \times 10^{-9} M_\odot\) will not Hawking evaporate in the distant future, but will instead absorb vacuum energy and grow to roughly the de Sitter horizon size. In this letter we reexamine black hole evaporation in the presence of a cosmological constant, and find instead that all known black holes will eventually evaporate.

1 Naive Estimate
It is well known that a black hole of mass \(M\) in otherwise empty spacetime radiates as a blackbody at the Hawking temperature

\[ T_H = 1/(8\pi GM) = 6.15 \times 10^{-8}(M_\odot/M)K. \]

(1)

It is also well known that inertial observers in a de Sitter universe with cosmological constant \(\Lambda\) feel themselves to be immersed in a bath of thermal particles at the Gibbons-Hawking temperature \[4\]

\[ T_{GH} = \frac{1}{2\pi} \left( \frac{\Lambda}{3} \right)^{1/2} \approx 2.3 \times 10^{-30} \left( \frac{\Omega_V}{0.7} \right) \left( \frac{h}{0.7} \right)^2 K. \]

(2)

In the above equation we have used \(\Omega_V = \rho_V/\rho_{crit}\), to relate the vacuum energy density, \(\rho_V = \Lambda/(8\pi G)\), to the critical energy density, \(\rho_{crit} = 3H_0^2/(8\pi G)\), with \(\hbar = c = k_B = 1\). Current measurements imply a Hubble constant \(h = H_0/100\,\text{km/s/Mpc} \approx 0.7\), and \(\Omega_V \approx 0.7\), giving \(\rho_V \approx 3.6\,\text{keV/cm}^3 = 2.78 \times 10^{-11}\,\text{eV}^4\), and \(\Lambda \approx 4.7 \times 10^{-66}\,\text{eV}^2\).

Consider now a black hole in a de Sitter universe, with the black hole horizon at approximately \(r_S = 2GM\) and cosmological de Sitter horizon at approximately \(r_{dS} = \sqrt{3/\Lambda}\). If the two horizons are sufficiently far apart, that is if \(r_S \ll r_{dS}\), then to first approximation one might expect that the Gibbons-Hawking radiation and the black hole’s Hawking radiation would remain essentially unchanged. The black hole should accrete some of the Gibbons-Hawking radiation, and the black hole’s Hawking radiation would remain essentially unchanged. The black hole should accrete some of the Gibbons-Hawking radiation, however, and so the net rate of mass loss should be diminished. Note that for any currently known black hole, accretion of the cosmic microwave background radiation and other interstellar and intergalactic matter is a much larger effect than either of the two terms above, but we are considering the distant future where the de Sitter expansion has rendered such accretion negligible.
An order of magnitude estimate of the black hole’s absorption cross section should be something like $4\pi r_h^2 S$, the black hole’s surface area. Naively, then, the change in the mass of the black hole is $\dot{M} \approx 4\pi \sigma r_h^2 H (T_{\text{3H}} - T_{\text{3H}}^0)$. Thus, there should exist some critical mass above which black holes have a net accretion of energy, and thus never evaporate by the Hawking process. Setting $\dot{M} = 0$ gives $M_{\text{crit}} = (1/4G)\sqrt{3/\Lambda} \approx 2.7 \times 10^{22} M_\odot$ for current estimates of $\Lambda$. Thus, naive considerations lead us to expect that black holes of size comparable to the de Sitter horizon could escape evaporation, whereas all smaller black holes should eventually evaporate. For comparison, all known black holes are in the mass range $1 M_\odot$ to $3 \times 10^9 M_\odot$, comfortably in the evaporation regime.

2 Two-dimensional calculation

While the simple estimates above are suggestive, it would be better to perform a more self-consistent calculation. In order to find the black hole’s evaporation rate, we would like to compute the renormalized expectation value of the stress tensor, $\langle T_{\mu\nu} \rangle$, which encodes the flow of energy into and out of the hole. Unfortunately, this is quite difficult to calculate in four spacetime dimensions. It is far easier to calculate the renormalized $\langle T_{\mu\nu} \rangle$ for massless scalar fields in two dimensional spacetimes, by exploiting the conformal triviality of these spacetimes. For spherically symmetric geometries, one may reduce the dimensionality of the problem to two by suppressing the angular coordinates. Ford and Parker [5] have shown that the two dimensional calculation is equivalent to the full four dimensional calculation in the geometric optics limit.

With this in mind, it is possible to compute the two-dimensional $\langle T_{\mu\nu} \rangle$. We may describe the black hole embedded in de Sitter spacetime by the Schwarzschild-de Sitter geometry,

$$ds^2 = -f(r)dt^2 + [f(r)]^{-1}dr^2 + r^2d\Omega^2$$

with black hole mass $M$ and cosmological constant $\Lambda$. Here, $r_h$ and $r_c$ are respectively the radii of the black hole horizon and cosmological horizon [6], found from $f(r_{\text{horizon}}) = 0$. When the two horizons are far from each other $r_h \approx r_S$ and $r_c \approx r_{dS}$ defined above. Suppressing the angular coordinates, and changing to double null coordinates $u = t - r_+, v = t + r_+$, with $r_+$ the usual Regge-Wheeler “tortoise” radial coordinate given by $dr_+ = dr/f(r)$, the line element takes the form $ds^2 = -f(r)du dv$.

With this, we can now compute the components of the stress tensor. For times long after the black hole has formed, the stress tensor becomes

$$T_{uv} = \frac{1}{192\pi} \left[ 2f(r) \frac{d^2f}{dr^2} - \left( \frac{df}{dr} \right)^2 + \left( \frac{df}{dr} \bigg|_{r=r_+} \right)^2 \right]$$

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$$T_{uv} = \frac{1}{96\pi} f(r) \frac{d^2f}{dr^2}$$

where the boundary terms are evaluated at the respective past null infinities, which are the horizon radii $r_c, r_h$. As $u$ is an outgoing coordinate, $T_{uu}$ represents the outwards flow of energy, and similarly $T_{uv}$.
represents the inwards flow of energy. Note that with $\Lambda = 0$, far from the hole $T_{uu}$ is given by the constant term. This is the Hawking term, representing (2D) radiation at the Hawking temperature. This can be seen by noting that the surface gravity at the horizon takes the form

$$\kappa = \frac{1}{2} \left. \frac{df}{dr} \right|_{r=r_h}$$

(8)

and since the Hawking temperature is given by $T_H = \kappa/2\pi$, we have

$$\sigma_{2D} T_H^2 = \frac{1}{192\pi} \left. \left( \frac{df}{dr} \right) \right|_{r=r_h}^2$$

(9)

where $\sigma_{2D} = \pi/12$. In the absence of the cosmological term, this outward flux of positive energy from the black hole would be precisely matched by an inward flux of negative energy given by $T_{vv}(r_h)$ [9]. The cosmological horizon brings an extra term to the ingoing component, representing positive flux of energy into the black hole.

As a side remark, we note that with no black hole ($M = 0$), the stress tensor reduces to $T_{uu} = T_{vv} = 0$, $T_{uv} = f''f/96\pi$. This is just as we expect – the stress tensor is isotropic and proportional to the metric tensor [10]. The conclusions of Adams et al. are based partially upon a calculation by Mallett [11], who left out the boundary term and therefore did not have $T_{\mu\nu} \propto g_{\mu\nu}$ for $\Lambda = 0$.

With the above expressions for $T_{\mu\nu}$, we need only plug in our form for $f$ to compute the stress tensor for a massless scalar. This is straightforward, but for general $r$ the expressions are somewhat lengthy.

We are interested specifically in $T_{vv}(r_h)$, since this tells us the ingoing flux of energy into the horizon. Looking at eqn. [10], we see that

$$T_{vv}(r_h) = \frac{1}{192\pi} \left[ \left. \left( \frac{df}{dr} \right) \right|_{r=r_c}^2 \right] - \left. \left( \frac{df}{dr} \right) \right|_{r=r_h}^2 = \sigma_{2D} (T_{GH}^2 - T_H^2)$$

(10)

The surface gravity takes the form $\kappa = GM/r_H^2 - 2\Lambda r_H/3$; inserting the expressions for $r_c, r_h$ and dividing by $2\pi$ gives $T_{GH}, T_H$. The exact expressions are lengthy, but to leading order in $GM\sqrt{\Lambda}$ the horizon temperatures become

$$T_H = \frac{1}{8\pi GM} \left[ 1 - \frac{16G^2M^2\Lambda}{3} + \mathcal{O}(G^4M^4\Lambda^2) \right]$$

(11)

$$T_{GH} = \frac{1}{2\pi} \sqrt{\frac{\Lambda}{3}} \left[ 1 - 2GM \sqrt{\frac{\Lambda}{3}} + \mathcal{O}(M^2\Lambda) \right]$$

(12)

Since $GM\sqrt{\Lambda} \approx 1.6 \times 10^{-23}(M/M_\odot) \ll 1$ for all known black holes, the correction to the ordinary temperatures are negligible. Clearly, all known black holes will evaporate.

With equation [11] we may also find the critical mass above which the black hole will accrete energy overall, and therefore never evaporate. This occurs when $T_{vv}(r_h) = 0$, implying that the two horizon temperatures are equal. Evaluating the two horizon surface gravities and equating them gives $r_c = r_h$; that is, the critical mass is that for which the black hole radius equals the cosmological radius. This occurs at $M_{\text{crit}} = 1/(3G\sqrt{\Lambda}) \approx 2.1 \times 10^{-22}M_\odot$, close to the result from the naive estimate above. However, for this value of $M$, $f = 0$ and the time coordinate $t$ is nowhere timelike, so our coordinate system is not
valid and our analysis breaks down. See Bousso and Hawking\[12\] for a detailed discussion of black holes with masses near and at \(M_{\text{crit}}\). In any case, for all \(M < M_{\text{crit}}\) our analysis should be valid, showing that the Hawking radiation exceeds the Gibbons-Hawking accretion. We may safely conclude that black holes in the observed mass range evaporate.

3 Adams, Mbonye, and Laughlin calculation

Adams, Mbonye, & Laughlin\[2\] discuss the Gibbons-Hawking correction to the Hawking radiation and state that it is orders of magnitude too small to affect normal black hole evolution, just as we concluded above. However, to take account of the cosmological constant, they rely on a conjecture that

\[\dot{M}_H \approx 4\pi \sigma r_H^2 \left(T_{\text{Vac}}^4 - T_H^4\right),\]  

(13)

where the effective temperature \(T_{\text{Vac}}\) is found from setting \(\rho_V = aT_{\text{Vac}}^4\), with \(a = 4\sigma = \pi^2/15\). Thus \(T_{\text{Vac}} = 30(\Omega_V/0.7)\)K, and the corresponding critical mass black hole from \(\dot{M} = 0\) is \(M_{\text{crit}} = 2.1\times10^{-9}M_\odot\).

This critical black hole size implies that all known black holes are accreting vacuum energy at a rate larger than they are emitting Hawking radiating, and so will never evaporate.

The conjecture of Adams, Mbonye, and Laughlin is based upon a paper by Mallett\[3\] which considered black hole evaporation during GUT scale inflation, and in particular his equation 4.8, (which is same as equation \[13\] with \(T_{\text{Vac}}\) replaced by \(T_{\text{GUT}}\)). Clearly, since \(T_{\text{Vac}} \neq T_{\text{GH}}\), this equation conflicts with our formulae above, and we argue that Mallett’s equation 4.8 is in error. Our argument is strengthened by the fact that Mallett\[11\] derives an expression for \(T_{\text{vv}}\) which, besides the error mentioned in the previous section, has the same form as ours in the relevant limit, and so gives nearly the same critical mass as we find above. Thus Mallett’s equation 4.8 seems to be in conflict with his calculation of \(T_{\text{vv}}\).

4 Discussion and Conclusions

We find that the energy absorbed by black holes in a de Sitter Universe is characterized by the very low Gibbons-Hawking temperature and not the equivalent temperature of a radiation field with a density of the vacuum energy as suggested by Mallett\[3\]. Adams, Mbonye, and Laughlin conjecture that the cosmological constant is caused by a sea of virtual particles, and that black holes may swallow those virtual particles and thereby accrete energy. An argument against this idea comes from the difference between the vacuum expectation value (VEV) of a field, \(\langle \phi \rangle\), and its potential \(V(\langle \phi \rangle)\). Adams et al. note that the sea of virtual particles “must contribute a net positive energy density”, but this fact is actually irrelevant. What matters for \(\Lambda\) is not just the vacuum energy, but the renormalized vacuum energy, which need not be positive despite the nonzero VEV. Indeed, even if \(\Lambda = 0\) there still exist fields with nonzero VEV, e.g. the Higgs condensate. Since Adams et al. accept that their conjectured process would not occur for \(\Lambda = 0\), and since shifting the zero point of the potential will shift \(\Lambda\) without changing the VEV, we see no reason that an “extra” accretion of particles should exist for nonzero \(\Lambda\).

Besides the above, there is another argument against this conjecture. Adams et al. attribute the unexpected increase in black hole mass in their model to swallowing of virtual particles. Accepting for argument’s sake this heuristic description, we note that in order to have any effect on the black hole mass, these virtual particles must go on mass shell. Within the context of this heuristic picture, this “on-shell” requirement is why black holes can Hawking radiate. One member of a virtual pair can go on shell inside the horizon, and carry with it negative energy as viewed from infinity, while its partner
can go on shell with positive energy and escape to infinity. In the ordinary (no $\Lambda$) Hawking process, the black hole cannot gain mass by such processes; the negative energy particle can’t propagate outside the horizon, and if the positive energy particle joins its partner and is swallowed, nothing has changed. Adams et al. conjecture that a cosmological constant alters this process in such a way that a thermal bath of particles at temperature (viewed from infinity) of $\sim 30$K can go down the hole. Note that there is no restriction upon positive energy particles; they can exist equally well far from the hole as near the horizon. So if there were 30K particles going down the hole, a 30 K thermal bath would also be seen elsewhere (e.g. near the Earth), in contradiction with experiment.

In summary, there are several arguments against the heuristic picture that gave rise to the conjecture that black holes accrete vacuum energy, and our calculations above show that the only accretion is of the Gibbons-Hawking radiation. Finally, several previous authors such as Bousso and Hawking[12] and Davies, Ford, and Page[7] reach conclusions about black hole evaporation in concordance with ours. We conclude that in a Universe dominated by vacuum energy, all isolated black holes will eventually evaporate.

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References
[1] S. Perlmutter et al. (The Supernova Cosmology Project), Astrophys. J. 517 565 (1999).
[2] F.C. Adams, M. Mbonye, & G. Laughlin, Phys. Lett. B 450 339 (1999).
[3] R. L. Mallett, Phys. Rev. D 33 2201 (1986).
[4] R. M. Wald, Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics, University of Chicago Press, Chicago (1994).
[5] L. H. Ford & L. Parker, Phys. Rev. D 17 1485 (1978).
[6] S. Tadaki & S. Takagi, Prog. Theo. Phys. 83 941 (1990).
[7] P. C. W. Davies, L. H. Ford & D. N. Page, Phys. Rev. D 34 1700 (1986).
[8] P. C. W. Davies, S. A. Fulling, and W. G. Unruh, Phys. Rev. D 13 2720 (1976).
[9] W. A. Hiscock, Phys. Rev. D 16 2673 (1977).
[10] S. A. Fulling, J. Phys. A 10 917 (1977).
[11] R. L. Mallett, Phys. Rev. D 34 1916 (1986).
[12] R. Bousso and S.W.Hawking, Phys. Rev. D 57 2436 (1998).