Chiral Magnetic Effect in QED induced by longitudinal photons

J. L. Acosta Avalo*, and H. Pérez Rojas**,

*Instituto Superior de Tecnologías y Ciencias Aplicadas (INSTEC),
Ave Salvador Allende, No. 1110, Vedado, La Habana, 10400 Cuba, and
**Instituto de Cibernética, Matemática y Física (ICIMAF),
Calle E esq 15, No. 309, Vedado, La Habana, 10400 Cuba.

We demonstrate the existence of the chiral magnetic effect in an electron-positron magnetized gas. A pseudo-vector(conserved)Ohm current is induced by the electric field related to the longitudinal QED mode propagating parallel to the external magnetic field $B$ and separating opposite charges of the same helicity. This current breaks the pre-existing chiral symmetry. From a relation between axial and electromagnetic currents we obtain a non-conserved current leading to an expression close to the usual axial anomaly. The effect is interesting in connection to the QCD chiral magnetic case reported in current literature.

PACS numbers:

I. INTRODUCTION

Nowadays the influence of the magnetic field in relativistic quantum systems, like electron-positron plasma and quark-gluon plasma (QGP) is an important subject for its device applications. In a QGP under the action of an intense external magnetic field, the chiral symmetry is broken even for massless quarks due to the QCD axial anomaly. In we showed that a magnetic field in the presence of imbalanced chirality induces a current along the magnetic field. This is the called Chiral Magnetic Effect in a QGP, which is caused by the topological gluon field configurations that couple to quarks via the axial anomaly. In these papers, the electric current generated along the magnetic field and the associated electric field are a consequence of the breaking of chiral symmetry via QCD axial anomaly. Our approach is different: the nonvanishing electric field, associated to a longitudinal pseudo-vector mode in a medium in presence of an external magnetic field, is responsible of the breaking of chiral symmetry. We recall that in a medium, as different from vacuum, there is a nonvanishing four velocity vector $u_\mu \neq 0$. Also, we must remark that even in absence of the external field in the charged medium there are three electromagnetic modes, two transverse and one longitudinal. These three modes correspond respectively spin projections $\pm 1, 0$ along the photon momentum $k$, their dispersion equations being not on the light cone. The zero spin longitudinal mode dispersion equation in the infrared limit has a solution for $\omega = 0, k \neq 0$ which accounts for the Debye screening, leading to some analogy to the Yukawa force.

One important point is the usual consideration of zero fermion mass, which is essential for the axial anomaly in current literature. The zero mass limit has the advantage that particles have definite helicity, but this means unavoidable difficulties in presence of magnetic fields, since dealing which charged massless particles implies that divergent magnetic moments arise. In what follows the axial character of massive particles in a medium will be understood as given by the average density in momentum space of those having a common helicity.

In a quantum relativistic electron-positron plasma under the action of an external field $B$ generated by a four-potential $A^{\mu}_{\text{ext}}$, the photon self-energy tensor structure determine the modes of propagation in both the $C$ invariant and non-invariant cases. The magnetic field breaks the spatial symmetry, leaving invariances for rotations around $B$, and for space translations along it. This determines different eigenmodes according to the direction of propagation. The dispersion equations for photons propagating in the medium can be solved in any direction. For instance, for propagation parallel to $B$, in which we are interested, for transverse modes it was found the relativistic Hall conductivity and Faraday Effect in . These arise due to the breaking of the electromagnetic wave chiral symmetry induced by the $C$-non-invariance of the transverse modes.

In the present paper we put in evidence that for propagation parallel to $B$ a QED pseudo-vector current is induced by the electric field of the longitudinal mode, leading to an axial anomaly in the magnetized electron-positron medium. The statistical chiral symmetry is broken by the action of the electric field (and this would be valid also for a quark-antiquark gas). The conductivity associated to the pseudo-vector mode is proportional to the corresponding eigenvalue of the photon self-energy tensor in the medium. We discuss the structure of this current in terms of the scattering and pair creation of electrons and positrons resulting from the decay of the longitudinal photons, and show that a current separating charges of opposite sign and the same helicity is produced, which means a chiral magnetic effect in the frame of QED.

The present results are based on the general properties of the propagation of electromagnetic waves, and the electromagnetic current vector, in presence of the field $B$, and we use the quantum field theory formalism at finite temperature $T \neq 0$ and density $\mu \neq 0$ (the medium is $C$-non invariant) .
II. CHIRAL CURRENT GENERATION IN QED

We will show that a chiral magnetic effect exists in the magnetized medium, but induced by the electric field associated to the longitudinal QED mode. Let us go in the details. The Schwinger-Dyson equation for the photon in the Fourier space is \[ \{ k^2 a_{\mu\nu} - \Pi_{\mu\nu}(k) A_{\mu}^{\text{ext}}(k) \} a^{\nu}(k) = 0, \]
where \( a_{\mu} \) (radiation field) is a perturbation added to \( A_{\mu}^{\text{ext}} \), so that the total external electromagnetic field is \( A_{\mu}^{\text{ext}} + a_{\mu} \) (the electric field of the wave \( E \ll B \)). The quantum corrections are given by the photon self-energy tensor \( \Pi_{\mu\nu}(k) A_{\mu}^{\text{ext}}(k) \), in terms of the \( e^\pm \) Green function in presence of the external magnetic field \( B \) (which we assume as parallel to the \( z \) axis). The photon self-energy tensor was calculated by Batalin and Shabad in [9–11], the diagonalization of the photon self-energy tensor \( \Pi_{\mu\nu}(k) A_{\mu}^{\text{ext}}(k) \), having three non-vanishing eigenvalues \( \eta_i \) and three eigenvectors \( b_i^{\nu}(k) \) for \( i = 1, 2, 3 \), corresponding to three photon propagation modes, whose electric and magnetic fields were obtained in [12].

The photon four-vector \( k_\nu \) has zero eigenvalue [12]. For each mode it is obtained a dispersion law \( k^2 = \eta_i(k_3, k_1, \omega, B) \), where \( k_3 = k_3^0 + k_1^0 - \omega^2 \), here \( k_3 \) and \( k_1 \) are respectively the components of the photon momentum in directions parallel and perpendicular to \( B \) and \( \omega \) its energy \([9–12]\).

In a charged medium, for propagation along the field \( B \), in addition to the two transverse modes, there is a longitudinal polarization mode, the pseudo-vector \( b_i^{\nu}(k) = R F_{\nu\mu} k^\mu \), where \( R \) is an arbitrary constant and \( F_{\nu\mu} \) is the dual of the electromagnetic field tensor \( F_{\nu\mu} \). Its electric polarization vector being in the direction along \( \mathbf{B} \). The \( b_i^{\nu}(k) \) electric polarization vector can be written as \( E^{(2)} = R(k_3^2 - \omega^2) \mathbf{B} [12] \). As pointed out earlier for QED in a charged \( e^\pm \) medium, the longitudinal mode along \( B \) is not on the light cone, that is \( k_3^2 - \omega^2 \neq 0 [12] \).

 Charged fermions interacting with the longitudinal mode, exchange energy by the transfer of momentum \( k_3 \), while the Landau quantum numbers remain unchanged [14]. We assume low frequencies, so that the electric field \( E^{(2)} \) is approximately constant in some intervals of time. Then it produces an electric current along \( B \) in which opposite charged particles, moving in opposite directions, have the same helicity, as it occurs in the system in equilibrium, see [14] which for \( E^{(2)} > 0 \) is \( R \), and vice-versa.

We can then consider the fermion interaction with the longitudinal mode as a problem in \((1 + 1)\) dimensions, which is strictly valid if we consider only the LLL. At this point we would like to point out that the two-dimensional Dirac matrices obey the identity [17]:

\[ \gamma^\mu \gamma^\nu = -e^{\nu\mu} \gamma_{\mu\nu}. \]

This implies that the axial \( j^A_{\mu} \) and vector \( j^{\nu} \) currents exchange their (0, 3) components according to the same relation. Thus, in the \((1 + 1)\) case, we can study the properties of the axial vector current by using results already derived for the vector current. The electromagnetic current as a function of \( A_{\mu}^{\text{ext}} + a_{\mu} \) depends on the two relativistic invariants \( \mathcal{F} = \frac{1}{4} F_{\nu\mu} F^{\nu\mu} = \frac{1}{2} (B^2 - E^2) \approx \frac{1}{2} B^2 \), since \( E \ll B \) and \( \mathcal{G} = \frac{1}{4} F_{\nu\mu} F^{\nu\mu} = \mathbf{B} \cdot \mathbf{E} \) (notice that for the case of propagation along \( B \), the pseudo-scalar \( \mathcal{G} \neq 0 \) only for the longitudinal eigenmode \( b_i^{(2)} \), independently of the C-symmetry of the system). An expansion in series gives:

\[ j_\mu (A_{\mu}^{\text{ext}} + a_{\mu}) = j_\mu (A_{\mu}^{\text{ext}}) + (\delta j_\mu / \delta A_{\mu}^{\text{ext}}) a_{\mu} + \ldots, \]

its linear term in \( a_{\mu} \) is the current density [13, 14]:

\[ j_\mu (A_{\mu}^{\text{ext}} + a_{\mu}) = j_\mu (A_{\mu}^{\text{ext}}) + \left( \frac{\partial}{\partial A_{\mu}^{\text{ext}}} \right) a_{\mu} + \ldots, \]
\[ j_i = \Pi_{\mu\nu} a_\nu = Y_{ij} E_j, \quad \nu = 1, 2, 3, 4, \quad i, j = 1, 2, 3, \quad (5) \]

where \( E_j = i(\omega a_j - k_j a_0) \) is the electric field, with \( a_i = i a_0 \) and \( k_i = i \omega \), also \( j_\mu (A^{ext}_\mu) = \rho \delta \mu_4 \), where \( \rho \) is the charge density. The term \( Y_{ij}/i\omega \) is the complex conductivity tensor and admissibility. The third term in \( (5) \) comes from the second one by using the four-dimensional \( b \) eigenmodes \( Y \) of \( j \mu \). \( \Pi \) is the symmetric and antisymmetric parts of the photon self-energy tensor. The symmetric and antisymmetric parts of the photon self-energy tensor. The first term corresponds to the Ohm current and the second is the Hall current \([13]\). The Hall and Faraday effects, which break the photon chiral symmetry, occur for the \( C \)-non-symmetric case, as different of the Chiral Magnetic Effect. The corresponding Ohm current is chiral-symmetric. Here we will only work with the Ohm current associated to the longitudinally polarized mode which was not discussed in the earlier papers \([10]\). Let us consider the specific case of propagation along \( B \), which is chiral non-symmetric. The current density associated to the longitudinal mode can be expressed in the form: \( j_3 = \sigma^{ij}_{33} E_j \), where \( \sigma^{ij}_{33} = Im[\Pi^{ij}_{33}]/\omega = -\omega Im[\sigma]/z_i \) is the chiral conductivity associated to the longitudinal mode, which now will be calculated in a charged \( \pm \) medium. The scalar \( s \) can be written in the one-loop approximation as \([9][12]\):

\[ s = -\frac{e^3 B}{\pi^2} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{dp_3}{2} \frac{\alpha_n \epsilon_{n,0}^2 (2p_3 k_3 + z_1)}{4z_1 p_3^2 + 4p_3 k_3 z_1 + z_1^2 - 4\omega^2 \epsilon_{n,0}^2} \cdot \left[ n^0(\epsilon_q) + n^3(\epsilon_q) - 1 \right], \]

here \( \epsilon_{n,0} = \sqrt{\epsilon^2 + 2\epsilon B n} \), with \( n = n_l + 1/2 + s_3/2 \), \( \alpha_n = 2 - \delta_{n,0} \), and \( q = (n, p) \). From the expression for the imaginary part of the scalar \( s \) obtained in \([10]\), the chiral conductivity at a finite temperature \( T \), density characterized by a chemical potential \( \mu \), and with \( S_n = \left[ \Delta N + \theta(-\epsilon^2_{n,0} - z_1)\Delta H \right] / \Lambda \) is obtained:

\[ \sigma^{ij}_{33} = \frac{e^3 B \omega}{2\pi z_1} \sum_{n=0}^{\infty} \alpha_n \epsilon_{n,0}^2 S_n, \quad (8) \]

where \( \Delta N = [N(\epsilon_r) - N(\epsilon_r + \omega)] + \Delta H = [H(\epsilon) + H(\omega + \epsilon) - 2] \). The term \( N = n^3(\epsilon_r) + n^0(\epsilon_r) \) while the term \( H = n^3(\epsilon) + n^0(\epsilon, \epsilon) \). Here \( \Lambda = \sqrt{z_1 (z_1 + 4z_1^2) / z_1} \), and \( \epsilon = (\epsilon_3 + |k_3|) / 2z_1 \). The fermion energies in a magnetic field in terms of the longitudinal mode energy \( \omega \) and the momentum \( k_3 \). The term \( \Lambda N\) accounts for the excitation of particles \([\epsilon(p_3, n) \rightarrow \epsilon(p_3, n + k_3)]\) by increasing their momentum along \( B \), while \( \Delta H \) accounts for the creation \( -\omega z_1 + |k_3| / 2z_1 \). Both due to the interaction with the longitudinal mode, the Landau quantum numbers being unchanged \([13]\). Pauli’s principle demands that vacant states must exist both for the occurrence of excitation and pair creation processes, for a fixed \( n \), which is expected at high temperatures. As
we assume $eB > \mu$, the population of Landau states for $n > 0$ is negligible small, and we may take in (8) only the LLL. It is obtained $\sigma_0^{33} = (e^3B/2\pi z_1)n^2S_0$, where $S_0 = \{\Delta N_R + \theta(-4m^2 - z_1)\Delta H_R\}/\Lambda$. Notice that the particles must have right-handed helicity (for $E^{(2)} > 0)$.

The last fact is a consequence of the paramagnetic magnetization of the system, which implies that for the LLL state, the particles have R-helicity, and for higher (degenerate) Landau numbers states with R-helicity are dominant (actually, the expression (8) contain both diamagnetic and paramagnetic terms). The current is a transport phenomenon, since it carries charge, and is a non-equilibrium process. Thus, the action of the external electric field $E_2$ is to break the equilibrium, and its more interesting consequence is to break the chiral symmetry of the system. Taking into account Eq(7) and Eq(8), with $E = E_2\hat{z}$ and $a = -ie^2/2$, it is obtained:

$$k_{\mu}j_A^\mu = a[m\hat{A}(m) + C(m)](e^2/2\pi^2)E \cdot B,$$

where $\hat{A}(m) = (2\pi m/e)\sum_{n=0}^{\infty}\alpha_nS_n$ and $C(m) = 8\pi B\sum_{n=1}^{\infty}nS_n$. Notice that the contribution to the non-conservation of the axial current Eq(2) term may be due to excitation of particles and also to pair creation. The Eq(2) is exactly an anomaly relation in a medium of massive particles, that bears some analogy to the Adler-Bell-Jackiw (ABJ) relation in vacuum [18]. The remarkable difference of Eq(2) with the ABJ relation is that the pseudo-scalar factor $E \cdot B$ appears also multiplying the usual massive term, vanishing only in the $m \to 0$ limit. All this implies that the longitudinal axial photon plays here a similar role than the $\pi^0$-meson at the vertex of Adler-Bell-Jackiw triangle. A closer correspondence with the Adler-Bell-Jackiw triangular anomaly ($\pi^0 \to \gamma\gamma$) would be obtained by taking higher order terms in the expansion Eq(2) by keeping only an odd number of longitudinal mode legs in each term. In our present model the triangle must be understood as the decay of the longitudinal axial photon in one vertex, which is absorbed by an electron or positron which subsequently radiates two transverse photons or either the decay in an electron-positron pair. This pair annihilates in two transverse photons. This means the possibility of longitudinal photon splitting in two transverse ones in a charged $e^\pm$ medium under the action of a very strong field $B$ [19].

III. CONCLUSIONS

We conclude that as a consequence of general properties of propagation of an electromagnetic wave parallel to a constant strong magnetic field in a dense medium, a chiral magnetic effect exists in QED, manifested in the induction of an electric pseudo-vector current, separating charges of the same helicity along the external magnetic field in analogy to the QCD chiral magnetic case. We emphasize here that photon chirality is Faraday effect exists if the pseudovector $S \neq 0$, whereas fermion chirality exists because the pseudoscalar $E \cdot B \neq 0$.

The chiral magnetic effect we have discussed in QED in a medium can be extended to quark-antiquark electromagnetic interactions and be combined with QCD effects. A separate study of this topic will be addressed in a future work.

Acknowledgments

The authors thanks the Abdus Salam ICTP OEA Office for support under Net-35 and to A. Cabo, A. Pérez Martínez and E. Rodríguez Querts for fruitful discussions.

[1] T. D. Lee and G. C. Wick, Phys. Rev. D9, 2291 (1974).
[2] B. L. Ioffe, arXiv:0809.0212v2 [hep-ph] (2008).
[3] K. Fukushima, D. E. Kharzeev and H. J. Warringa, arXiv:0808.3382v1[hep-ph] (2008).
[4] H. J. Warringa, arXiv:0805.1394 [hep-ph] (2008).
[5] G. ’t Hooft, Phys. Rev. Lett. 37, 8 (1976); G. ’t Hooft, Phys. Rev. D. 14, 3432 (1976).
[6] A. A. Belavin, A. M. Polyakov, A. S. Schwarz and Yu Tyupkin, Phys. Lett. 59B, 85 (1975).
[7] N. S. Manton, Phys. Rev. D 28, 2019 (1983).
[8] E. S. Fradkin, Quantum Field Theory and Hydrodynamics, Proc. of the F.N. Lebedev Inst. No. 29, Consultants Bureau (1967).
[9] H. Pérez Rojas and A. E. Shabad, Ann. Phys. 121, 432-55 (1979).
[10] H. Pérez Rojas and A. E. Shabad, Ann. Phys. 138, 1-35 (1982).
[11] I. A. Batalin and A. E. Shabad, Zh. Eksp. Teor. Fiz. 60, 894 (1971).
[12] A. E. Shabad, Ann. Phys. 90, 166-195 (1975).
[13] L. Cruz Rodríguez, A. Pérez Martínez, H. Pérez Rojas and E. Rodríguez Querts, Phys. Rev A 88, 052126 (2013).
[14] R. G. Felipe, A. Pérez Martines, H. Pérez Rojas, Modern Physics Letters B, Vol. 4, No. 17, 1103-1109 (1990).
[15] M. H. Johnson and B. A. Lippmann, Phys. Rev. 76, 828-32 (1949).
[16] M. Chaichian, S. S. Masood, C. Montonen, A. Prez Martinez, and H. Prez Rojas, Quantum Magnetic Collapse, Phys. Rev. Lett. 84 (2000).
[17] An Introduction to Quantum Field Theory, M. E. Peskin and D. V. Schroeder, Perseus Books Publishing, L.L.C. (1995).
[18] S. L. Adler, Phys. Rev. 177, 2426 (1969). J. S. Bell and R. Jackiw, Nuovo Cim. 60A, 47 (1967).
[19] V. N. Baier, A. I. Milstein and R. Zh. Shaisultanov, arXiv.org/pdf/hep-th/9604028(1996).