Squeezed spectra of bosons and antibosons with different in-medium masses

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Abstract

We study the influence of the in-medium mass difference between boson and antiboson on their spectra. The in-medium mass difference may lead to a difference between the transverse momentum spectra of boson and antiboson. This effect increases with the increasing in-medium mass difference between boson and antiboson. The difference between the transverse momentum spectra of boson and antiboson increases with the increasing expanding velocity of the source and decreases with the increasing transverse momentum in large transverse mass region ($m_T > 1.6$ GeV). The interactions between the hadron and the medium may increase with the increasing temperature of the medium and the higher freeze-out temperature may lead to a larger mass difference between boson and antiboson, and may give rise to a larger difference between the transverse momentum spectra of boson and antiboson for higher freeze-out temperature.

Keywords: Boson and antiboson; in-medium mass difference; squeezed spectra.

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I. INTRODUCTION

The transverse momentum spectra of particles are important observables in high-energy heavy-ion collisions. They can be utilized to investigate the thermalization and expansion of the systems produced in such collisions [1–4].

The in-medium mass shifts of bosons may cause a squeezing effect, and directly lead to a squeezed back-to-back correlation between boson and antiboson [5–8]. This squeezing effect is related to the in-medium mass of the bosons, through a Bogoliubov transformation between the creation (annihilation) operators of the quasiparticles in the medium and the corresponding free particles [5–8]. The squeezing effect also affects the transverse momentum spectra of bosons [6, 7]. The influence of squeezing effect caused by in-medium mass shift on transverse momentum spectra and elliptic flow of φ meson was studied in previous work [9]. Since the antiparticle of φ meson is itself, the in-medium masses of φ meson and its antiparticle are treated as the same [9]. The interactions of charged particles and their corresponding antiparticles in a medium are different [10–16]. The in-medium mass difference between a boson and an antiboson may lead to different effects on their spectra. It is necessary to study the squeezing effect on spectra of boson and antiboson with different in-medium masses.

This paper is organized as follows. In Sec. II, we present the formulas of the single-particle momentum distributions for boson and antiboson with different in-medium masses. Then, the squeezed spectra of boson and antiboson are shown in Sec. III. Finally, summary and discussion of this paper are given in Sec. IV.

II. FORMULAS

Denote \( a_k (a_k^\dagger) \) as the annihilation (creation) operator of the free boson with momentum \( k \) and mass \( m_a \), and \( b_k (b_k^\dagger) \) as the annihilation (creation) operator of the free antiboson with momentum \( k \) and mass \( m_b \). For a pair of free boson and antiboson, \( m_a = m_b = m \). The invariant single-particle momentum distributions of boson and antiboson can be expressed by

\[
N_a(k) = \omega_{a,k} \frac{d^3 N}{dk} = \omega_{a,k} \langle a_k^\dagger a_k \rangle, \tag{1}
\]

\[
N_b(k) = \omega_{b,k} \frac{d^3 N}{dk} = \omega_{b,k} \langle b_k^\dagger b_k \rangle, \tag{2}
\]
where \( \langle \cdots \rangle \) means the thermal average, and \( \omega_{a,k} = \sqrt{k^2 + m'^2} \) and \( \omega_{b,k} = \sqrt{k^2 + m'^2} \) are the energy of the free boson and antiboson, respectively.

Denote \( a'_k (a'^\dagger_k) \) as the annihilation (creation) operator of the boson with momentum \( k \) in medium, and \( b'_k (b'^\dagger_k) \) as the annihilation (creation) operator of the antiboson in medium. Assuming the energy split between the boson and antiboson in the medium is \( 2\delta \), the operators \( (a_k, a'^\dagger_k, b_k, b'^\dagger_k) \) for the free particles and \( (a'_k, a'^\dagger_k, b'_k, b'^\dagger_k) \) for the quasiparticles were related by the Bogoliubov transformation

\[
a_k = c_k a'_k + s^*_{-k} b'^\dagger_{-k}, \quad b_k = \bar{c}_k b'_k + \bar{s}^*_{-k} a'^\dagger_{-k},
\]

where

\[
c^\pm_k = c_{\pm k} = \bar{c}^\pm_k = \cosh r_k,
\]

\[
s^\pm_k = s_{\pm k} = \bar{s}^\pm_k = \sinh r_k,
\]

\[
r_k = \frac{1}{2} \ln \left( \frac{\omega_k}{\Omega_k} \right),
\]

\[
\Omega_k = \sqrt{k^2 + m'^2 + \delta^2}.
\]

The in-medium masses of the boson and antiboson are:

\[
m'_a = (\Omega_k + \delta)|_{k=0} = \sqrt{m'^2 + \delta^2 + \delta},
\]

\[
m'_b = (\Omega_k - \delta)|_{k=0} = \sqrt{m'^2 + \delta^2 - \delta},
\]

where \( m' \) is the in-medium mass of the boson or antiboson for \( \delta = 0 \). For a pair of boson and antiboson, the subscript \( a \) indicates the particle with larger mass and the subscript \( b \) represents the particle with smaller mass.

For a homogeneous source with volume \( V \) and temperature \( T \), the single particle momentum distributions of boson and antiboson can be expressed by

\[
N_a(k) = \frac{V}{(2\pi)^3} \frac{1}{\omega_k} \left\{ |c_k|^2 n_{a,k} + |s_{-k}|^2 (n_{b,-k} + 1) \right\},
\]

\[
N_b(k) = \frac{V}{(2\pi)^3} \frac{1}{\omega_k} \left\{ |\bar{c}_k|^2 n_{b,k} + |\bar{s}_{-k}|^2 (n_{a,-k} + 1) \right\},
\]

\[
n_{a,k} = \frac{1}{\exp[(\Omega_k + \delta)/T] - 1},
\]

\[
n_{b,k} = \frac{1}{\exp[(\Omega_k - \delta)/T] - 1}.
\]
For hydrodynamic sources, the single particle momentum distributions of boson and antiboson become \[6, 8, 18, 21\]

\[
N_a(k) = \int \frac{g_i}{(2\pi)^3} d^4\sigma_\mu(r) k^\mu \left\{ |c'_{k'}|^2 n'_{a,k'} + |s'_{-k'}|^2 \left[ n'_{b,-k'} + 1 \right] \right\}, \tag{14}
\]

\[
N_b(k) = \int \frac{g_i}{(2\pi)^3} d^4\sigma_\mu(r) k^\mu \left\{ |c'_{k'}|^2 n'_{b,k'} + |s'_{-k'}|^2 \left[ n'_{a,-k'} + 1 \right] \right\}, \tag{15}
\]

where \(g_i\) is the degeneracy factor for hadron species \(i\). \(d^4\sigma_\mu(r)\) is the four-dimension element of freeze-out hypersurface, and \(u_\mu(r), T(r)\) are the source four-velocity and the freeze-out temperature, respectively. \(k^\mu = (\omega_k, k)\) is the four-momentum of the particle, and \(k'\) is the local-frame momentum corresponding to \(k\). If the energy split between the boson and antiboson in the medium \(\delta\) is 0, the \(N_a(k)\) will be equal to \(N_b(k)\).

The source distribution in our calculations is taken as

\[
\rho(r) = Ce^{-r^2/(2R^2)} \theta(r - 2R), \tag{21}
\]

where \(C\) and \(R\) are the normalization constant and the source radius, respectively. The expanding velocity of the source is taken as

\[
v(x) = \frac{u}{2R} r, \tag{22}\]

where \(u\) is a parameter. For the considered source and with the sudden freeze-out assumption, the single particle momentum distributions of boson and antiboson become \[6, 8, 18\]

\[
N_a(k) = \int d^3r \frac{g_i}{(2\pi)^3} \omega_\kappa \rho(r) \left\{ |c'_{k'}|^2 n'_{a,k'} + |s'_{-k'}|^2 \left[ n'_{b,-k'} + 1 \right] \right\}, \tag{23}
\]

\[
N_b(k) = \int d^3r \frac{g_i}{(2\pi)^3} \omega_\kappa \rho(r) \left\{ |c'_{k'}|^2 n'_{b,k'} + |s'_{-k'}|^2 \left[ n'_{a,-k'} + 1 \right] \right\}. \tag{24}
\]
III. RESULTS

The transverse momentum spectra for Kaon with various mass-shift $\delta m (\delta m = m' - m)$ are shown in Fig. 1(a) and (b). Here, the freeze-out temperature $T_f$ of Kaon is taken as 150 MeV \[22\] and the energy split between the boson and antiboson in the medium $\delta$ is taken as 0. The mass-shift of Kaon in a hadronic medium is about -10 MeV at 150 MeV \[23\]. In the calculation, the mass-shift of Kaon is taken as -5 MeV and -10 MeV. The source radius $R$ is taken as 5 fm in this paper. The mass-shift leads to an increase in the yield of Kaon in large transverse momentum regions. This effect increases with the increasing mass-shift and decreases with the increasing expanding velocity. This phenomenon is similar to the effect of mass-shift on the transverse momentum spectra of $\phi$ meson \[9\].

FIG. 1: (Color online) The transverse momentum spectra for Kaon with $\delta m = 0, -5$ MeV and -10 MeV.

FIG. 2: (Color online) The transverse momentum spectra for Kaon and anti-Kaon with $\delta = 10$ and 20 MeV for $T_f = 150$ MeV. The experimental data of the central Au + Au collision at $\sqrt{s_{NN}} = 200$ GeV are measured by the PHENIX Collaboration \[24\].

In Fig. 2(a) and (b), we show the transverse momentum spectra for Kaon and anti-Kaon with $\delta = 10$ and 20 MeV. The experimental data of the central Au + Au collision are
measured by the PHENIX Collaboration [24]. The velocity parameter $u$ is taken as 0.6 and it may be suitable for the central Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV. It can be seen that $\delta$ leads to a difference between the transverse momentum spectra of Kaon and anti-Kaon. To analyze the difference quantitatively, we define a new quantity as

$$F = \frac{|N_a(k) - N_b(k)|}{N_a(k) + N_b(k)}.$$  

(25)

![Figure 3](image1.png)

**FIG. 3:** (Color online) The quantity $F$ in $k_T - \delta$ plane for Kaon, where $T_f$ is taken as 150 MeV.

![Figure 4](image2.png)

**FIG. 4:** (Color online) The quantity $F$ for Kaon with different expanding velocity, where $T_f$ is taken as 150 MeV.

In Fig. 3 the quantity $F$ is shown in $k_T - \delta$ plane for Kaon. The difference between the transverse momentum spectra of Kaon and anti-Kaon increases with the increasing $\delta$. The quantity $F$ for Kaon with different expanding velocity is shown in Fig. 4. For fixed $\delta m$
and $\delta$, the difference between the transverse momentum spectra of Kaon and anti-Kaon is almost the same in small momentum region ($k_T < 1.5$ GeV) but decreases with the increasing momentum in large momentum region ($k_T > 1.5$ GeV). In small momentum region ($k_T < 1.5$ GeV), the difference is not dependent on the expanding velocity. The difference increases with the increasing expanding velocity in large momentum region ($k_T > 1.5$ GeV).

![Graph](image)

**FIG. 5:** (Color online) The quantity $F$ for Kaon with different freeze-out temperature.

Hadron mass shifts are caused by interactions in a dense medium and therefore vanish on the freeze-out surface [6]. The interactions may increase with the increasing temperature of the medium and the higher freeze-out temperature may lead to a greater mass difference (or energy difference) between boson and antiboson.

The quantity $F$ for Kaon with different freeze-out temperature $T_f$ is shown in Fig. 5. For fixed $\delta$, the difference between the transverse momentum spectra of Kaon and anti-Kaon decrease slightly with the increasing freeze-out temperature. The energy difference between Kaon and anti-Kaon may increase with the increasing temperature of the medium. The difference between the transverse momentum spectra of Kaon and anti-Kaon increase obviously with the increasing energy difference $\delta$. Thus, the difference between the transverse momentum spectra of Kaon and anti-Kaon may increase with the increasing freeze-out temperature.

In Fig. 6, the quantity $F$ is shown in $k_T - \delta$ plane for $D$ meson. The freeze-out temperature of $D$ meson is taken as 150 MeV [17] and the $\delta m$ is taken as -3 MeV and -5 MeV [17, 25]. The difference between the transverse momentum spectra of $D^+$ and $D^-$ increases with the increasing $\delta$ and decreases with the increasing momentum. The quantity $F$ for $D$ meson with different expanding velocity is shown in Fig. 7. The difference increases with the increasing expanding velocity.

The quantity $F$ for $D$ meson with different freeze-out temperature $T_f$ is shown in Fig. 8.
FIG. 6: (Color online) The quantity $F$ in $k_T - \delta$ plane for $D$ meson.

FIG. 7: (Color online) The quantity $F$ for $D$ meson with different expanding velocity for $T_f = 150$ MeV.

FIG. 8: (Color online) The quantity $F$ for $D$ meson with different freeze-out temperature.
The difference between the transverse momentum spectra of $D^+$ and $D^-$ for $\delta m = -3$ MeV is little affected by the freeze-out temperature. For $\delta m = -5$ MeV, the difference between the transverse momentum spectra of $D^+$ and $D^-$ increase slightly with the increasing freeze-out temperature for fixed $\delta$. Thus, with the increasing freeze-out temperature, the energy difference increases and the difference between the transverse momentum spectra of $D^+$ and $D^-$ increases.

Comparing the above results of $D$ meson and Kaon, the difference between the transverse momentum spectra of boson and antiboson increases with the increasing expanding velocity of the source and decreases with the increasing transverse momentum in large transverse mass region ($m_T = \sqrt{k_T^2 + m^2} > 1.6$ GeV). The difference between the transverse momentum spectra of boson and antiboson may increase with the increasing freeze-out temperature.

**IV. SUMMARY AND DISCUSSION**

In high-energy heavy-ion collisions, the interactions of charged particles and their corresponding antiparticles in a medium are different [10–16]. Thus, the in-medium masses of charged particles and their corresponding antiparticles are different. In this paper, we study the effect of in-medium mass difference between a boson and an antiboson on their spectra. The in-medium mass difference between a boson and an antiboson leads to a difference between the transverse momentum spectra of boson and antiboson. This effect increases with the increasing in-medium mass difference between boson and antiboson. The difference between the transverse momentum spectra of boson and antiboson increases with the increasing expanding velocity of the source and decreases with the increasing transverse momentum in large transverse mass region ($m_T > 1.6$ GeV). The interactions between the hadron and the medium may increase with the increasing temperature of the medium and the higher freeze-out temperature may lead to a larger mass difference between boson and antiboson, and may give rise to a larger difference between the transverse momentum spectra of boson and antiboson for higher freeze-out temperature.

The transverse momentum spectra of particles are important observables in high-energy heavy-ion collisions. The net-charge multiplicity distributions are applied to probe the signatures of the QCD phase transition and critical point [26, 27]. Based from above results, it is necessary to take the effect of in-medium mass difference between a boson and an
antiboson on their spectra in the calculations.

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