Solving the problem of optimizing the cutting of wood whips with curvature

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Abstract. Stem wood, obtained from the cuttings of care in low-forest areas is of low quality. Such wood has a number of vices, one of which is simple curvature, which has impact on the voluminous and commercial output of round timber. Reducing this influence can be achieved through high-quality bucking. Therefore, the purpose of the work presented is to increase the volume and commercial output of round timber harvested in low-forest areas. The developed technique using computers allows to solve the tasks of modeling the shape of wood whips and logs using multi-critical optimization and bucking whips having curvature. The use of the proposed technique allows for results adequate to the real production conditions, as evidenced by the methods of decision-making used. Scientific research and theoretical developments, taking into account the formed database, allowed to search for the optimal scheme of cutting when bucking round timber, performed according to the algorithm based on the busting of acceptable variants using modern theory of graphs and matrices. As a result, the developed program will reduce the time for data formation, guarantee the accuracy of the results, the program easily adapts to natural and production conditions and will expand the possibilities for Computer-aided design.

1. Introduction

When performing timber harvesting in sparsely wooded areas of the, it is necessary to solve a number of tasks aimed at improving the quality of round timber processing. Wood in low-wooded areas harvested from logging is significantly different in quantitative and qualitative terms from the raw materials harvested during the main use. What does it indicate N Vanzetti and S Sillett in their works [1,2].

Cross-cutting of round timber with a defect in the shape of the trunk, in particular curvature, taking into account the conditions of production and consumption, various optimality criteria should be maximized, reflecting the technical and economic indicators of crosscutting. The percentage of wood raw materials obtained from felling associated with forest maintenance fluctuates in the range of 35...40% and is characterized by an average and low volume of whip. The volumes of processing by enterprises located in low-forested areas are relatively small, and the range of products is large. The existing shortage of high-quality wood raw materials forces to attract low-quality wood into processing thin-dimensional, having various defects in the shape of the trunk [3]. High-quality processing of such raw materials in the conditions of wood-processing workshops depends on the quality of the cross-cutting of tree trunks into assortments. Therefore, the goal is to increase the volumetric and marketable output of round timber when harvesting in low-wooded areas due to mathematical modeling of the shape of round timber with a simple curvature and multi-criteria...
optimization of the cross-cutting process. Thus it is necessary to obtain the maximum number of high quality assortments. To solve such a problem, it is necessary to study the main characteristics of wood whips obtained during harvesting in low-wooded areas, to perform mathematical modeling of round timber, to determine rational cutting schemes. Therefore, the purpose of the presented work is to increase the volume and commodity yield of round timber obtained during logging in low-forest areas from logging by means of more adequate modeling of the shape of whips having a simple curvature and multi-criteria optimization of the cross-sawing process [4].

The description of the stem generator with subsequent modeling is carried out with varying degrees of accuracy, using both simple mathematical expressions, complex polynomials, and modern volumetric scanners [5,6]. The latter give high accuracy, but their use in small enterprises is impossible because they are expensive to operate. In this paper, a new generatrix is proposed that can describe a simple curvature at any point of the trunk. The method of determining the useful volume of sortings received for further processing is justified.

2. Materials and methods

The objects of research are round timber sent for cross-cutting with curvature. Research is carried out in order to determine the optimal cutting patterns for wood whips, taking into account quality indicators.

The optimal scheme for cutting round timber is a scheme that maximally meets the specified requirements in specific natural production conditions of production and consumption. The solution to the problem must be carried out in several stages: at the first stage, develop a model that allows describing the shape of a whip with a simple curvature, as well as taking into account its location along the trunk with the described part of the “strophoid” curve; on the second, determine the area in any section; and finally, determine the optimal cutting pattern based on the theory of graphs and matrices.

2.1 Specifying a centerline as part of a “strophoid” curve

When carrying out research of mathematical functions, it was revealed that the curve of the “Strophoid” on the interval [0;1], quite accurately describes the axis of the whip having a simple curvature (figure 1).

![Figure 1. Place of points $M_1$ and $M_2$ strophoids for which $PM_1 = PM_2 = OP$.](image)
The performed analysis of the “Strophoid” function given by a mathematical equation:

$$y^2 = x^2 \left( \frac{a + x}{a - x} \right)$$

did not give a positive solution allowing to describe the shape of a whip having a simple curvature, as well as to take into account its location along the length of the trunk, since when the length of the trunk changes, it is necessary to re-feed the variables. In order to avoid the influence of different barrel lengths on the accuracy of the description of the axial line of the strophoid function in parametric form, the function reduced to relative coordinates should be considered [5].

As a result, the function, reduced to relative coordinates, adequately represents the behavior of the curvilinear axis of the trunk and allows you to correct the mathematical model to describe a whip with a simple curvature and will allow you to use it when optimizing cross-cutting.

2.2 Determination of the useful overlap area in any section on the interval [0; 1]
Since the trunks and assortments under consideration have a curvature of the trunk shape, the butt and apex ends will receive some displacement [6].

To solve this problem, it is necessary to develop a method for taking into account the overlap area in given sections, which correspond to the end in the top cut and in the zone of the maximum deflection (figure 2).

![Diagram](image)

**Figure 2.** Useful area to overlap the top diameter of the assortment $d_1$ and diameter at the point of maximum deflection $d_f$: (a) – possible value, (b) – area for $S_e$ and $S_h$

Let us define $O$ and $O_1$ – the centers of the end sections, respectively, in the zone of the maximum deflection arrow and the end in the top cut of the circumscribed circles. Let’s represent the point $O$ as the origin of the Cartesian coordinate system, the length of the straight line $OO_1$ – by the positive semiaxis $x$, the value of which is equal to the value of the deflection arrow $f$.

Let us set the equations describing the circles:

$$x^2 + y^2 = a^2$$  \hspace{1cm} (1)

$$\left(x - c\right)^2 + y^2 = b^2$$  \hspace{1cm} (2)

In the case of intersection of circles, coordinates, intersection points will satisfy equations (1) and (2). And vice versa, system (1) and (2) has a solution, then there exist $x$ and $y$ that satisfy both equations and represent the coordinates of their intersection points. The number of solutions to the system will be equal to the number of intersection points.

To solve the system of the presented equations (1) and (2), it is necessary to subtract term by term.
We will solve the system of equations (1) and (2). To do this, we first subtract the equations term by term. As a result, we have:

\[ 2cx - c^2 = a^2 - b^2 \]  
(3)

We solve:

\[ x = \frac{a^2 + c^2 + b^2}{2c} \]  
(4)

Replacing the resulting value of x in the first equation, we have:

\[ \left(\frac{a^2 + c^2 + b^2}{2c}\right)^2 + y^2 = a^2 \text{ then } y^2 = a^2 - \left(\frac{a^2 + c^2 + b^2}{2c}\right)^2 \]  
(5)

After transforming the right side of the equality as the difference of squares, we have:

\[ \left(a + \frac{a^2 + c^2 + b^2}{2c}\right) \cdot \left(a - \frac{a^2 + c^2 + b^2}{2c}\right) = \]

\[ = \frac{1}{4c^2} \left(2ac + a^2 + c^2 - b^2\right) \cdot \left(2ac - a^2 - c^2 + b^2\right) = \]

\[ = \frac{1}{4c^2} \left((a + c)^2 - b^2\right) \cdot \left(b^2 - (a - c)^2\right) = \]

\[ = \frac{1}{4c^2} \left(a + b + c\right) \cdot \left(a + c - b\right) \cdot \left(b + a - c\right) \cdot \left(b + a + c\right) \]  
(6)

Eventually,

\[ y^2 = \frac{1}{4c^2} \left(a + b + c\right) \cdot \left(a + c - b\right) \cdot \left(b + a - c\right) \cdot \left(b + a + c\right) \]  
(7)

It can be argued that if \( a + c > b \), \( a + b > c \) and \( b + c > a \) then the right side of the equation is positive, which means that the system has solutions, it should be noted that there will be two given solutions.

In order to determine the overlap area for the sections of the top end and in the zone of the maximum deflection arrow shown in (figure 2a) by hatching, we use expression (7), having previously replaced some of the arguments.

\[ a \rightarrow \frac{d_f}{2}, b \rightarrow \frac{d_y}{2}, c \rightarrow f \]  
(8)

The expression will have:

\[ y = \sqrt{\frac{1}{4f^2} \left(\frac{d_f}{2} + \frac{d_y}{2} + f\right) \cdot \left(\frac{d_f}{2} + f - \frac{d_y}{2}\right) \cdot \left(\frac{d_f}{2} + \frac{d_y}{2} - f\right) \cdot \left(\frac{d_y}{2} + f - \frac{d_f}{2}\right)} \]  
(9)

Hence the value \( x \) using the equation of the circle will be:

\[ x_1 = \frac{\left(\frac{d_f}{2}\right)^2 - \left(\frac{d_y}{2}\right)^2 + f^2}{2f} \]  
(10)
\[ y_1 = \sqrt{\left(\frac{d_f}{2}\right)^2 - x^2} \]  

To determine the area of the segment (a), bounded by points (in figure 2) \((m_1\text{ and } m)\) and a triangle \((om, m)\), will correspond to:

\[
Sc(a) = \frac{\pi}{360} \cdot \varphi_f - S_\Delta (om, m)
\]

\[
\varphi_f = 2 \arccos \left( \frac{x_1}{\frac{d_f}{2}} \right)
\]

\[
S_\Delta (om, m) = \frac{1}{2} \left( \frac{d_f}{2} \right)^2 \cdot \sin(\varphi_f)
\]

The area of the segment (b), bounded by points (in figure 2) \((m_1\text{ and } m)\) and a triangle \((o, m, m)\), will correspond to:

\[
Sc(b) = \frac{\pi}{360} \cdot \varphi_v - S_\Delta (o, m, m)
\]

\[
\varphi_v = 2a \arccos \left( \frac{f - x_1}{\frac{d_v}{2}} \right)
\]

\[
S_\Delta (o, m, m) = \frac{1}{2} \left( \frac{d_v}{2} \right)^2 \cdot \sin(\varphi_v)
\]

The required area in the sections of the apex end and in the zone of the maximum deflection is determined by the sum of the previously calculated segments, and the expression has the form:

\[
S_{ful} = S_{c(a)} + S_{c(b)}
\]

In terms of optimizing the cutting of tree-lengths with curvature, taking into account such criteria as cylindrical volume, as well as the maximum beam output, the value of the maximum usable overlap area should tend to the maximum.

The number of variants of possible schemes for cross-cutting of the whip can range from several tens to several thousand. Determining the optimal one among such an array of data is a rather serious and difficult task, which can be solved only with the use of computers, and modern mathematical methods, taking into account many factors [7-10].

The initial values for solving the problem of optimal transverse cutting of trunks with simple curvature include the following ordered sets.

To describe the main characteristics of the whips, we will use:

1. Length of round timber with simple curvature.

\[
\hat{L}_{il} = \{L_{il}^1, L_{il}^2, \ldots, L_{il}^{m_l}\}
\]
2. The value of the median diameter of round timber.

\[ \hat{d}_{0.5}^m = \{d_{0.5}^1, d_{0.5}^2, \ldots, d_{0.5}^m \} \]  

(20)

3. The value of the maximum deflection of round timber with a simple curvature.

\[ \hat{f}_{0.5}^m = \{f^1, f^2, \ldots, f^m \} \]  

(21)

4. On the basis of statistical observations, an interval along the length of the trunk was obtained on which the determination of the deflection deflection values is recorded.

\[ 0.1 L_{1g}^m < l^f_{1g} < 0.8 L_{1g}^m \]  

(22)

To describe the main characteristics of assortments, we will use:

1. The length of the consonant GOST obtained assortments.

\[ \hat{l}_a = \{l^1_a, l^2_a, \ldots, l^n_a \} \]  

(23)

2. The value of the top diameters of the resulting assortments:

\[ \hat{d}_a = \{d_a^1, d_a^2, \ldots, d_a^n \} \]  

(24)

3. Values of the curvature of the resulting assortments:

\[ \hat{f}_a = \{f_a^1, f_a^2, \ldots, f_a^n \} \]  

(25)

The ranges of the indicated sets are adjusted for the species composition, the place of harvesting, and the assortment plan of the enterprise in accordance with GOST.

The process of crosscutting round timber is carried out in a specific sequence using modern mathematical procedures for each optimization stage.

3. Results and discussion

The conducted scientific research and theoretical studies were aimed at creating an information database and developing an algorithm that made it possible to search for the best cross-cutting scheme for round timber. The developed algorithm is based on a sequential enumeration of all admissible schemes for cross-cutting wood using the modern theory of graphs and matrices.

The use of the proposed software is able to reduce to zero the total time for data generation, to guarantee their reliability. The versatility of the software is the ability to adapt to the specified characteristics of the stand of the developed cutting areas.

In particular, the regularities of the change in the cylindrical volume, the volume of the bar and the volume of the bar cut according to the traditional position were established, depending on the size of the deflection arrow and its position along the length. These patterns are reflected in the graphs of dependence at fixed ratios \( \frac{f}{d_v} \). Of greatest interest are graphs with critical values (figure 3), in which a clear predominance of one volume over another is revealed (1 – cylindrical volume, 2 – optimal volume of a bar, 3 – actual volume of a bar).

The point of intersection of the curves of the volumes indicates the advisability of using as an estimated indicator of the volume, whose curve tends to further increase with the movement of the maximum deflection arrow along the length of the whip. It was noted that the timber cut from the usable area has a larger volume than the timber sawn using the traditional method.
Figure 3. The graph of the dependence of the volume of the log at the size of the deflection boom: $\frac{f}{d_v} = 0.6$. 1 – cylindrical volume; 2 – volume of the bar; 3 – volume of the bar are cut out according to the standard method. (a) butt log; (b) median log; (c) top log.

4. Conclusion

The analysis of methods and methods for describing the forming whips revealed that the most suitable for describing the curved axis of the whip is the equation “strophoids” on the section from [0,1], given in parametric form.

When studying the behavior of the curvature, it was found that the possible position of the maximum deflection boom is in the range from 0.2 to 0.8 along the length of the whip. In addition, a significant influence of curvature on the bucking process was revealed. It determines the length of the timber and reduces the yield of the cylindrical volume, as a result-the loss of valuable wood up to 25% to waste.

On the basis of the new generatrix, models of pine whips have been developed and a method for determining the useful volume of sortings received for further processing has been substantiated. At the same time, the points of critical curvature are established. A selection was made and a system of criteria and restrictions was justified. The developed software uses the minimum number of input values, while characterizing the timber entering the processing with sufficient accuracy, which corresponds to real production conditions.

The results obtained have been tested at a number of enterprises located in sparsely wooded areas. They can be used at any logging enterprise engaged in the primary processing of roundwood, both in Russia and abroad.

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