Are the B decay anomalies related to neutrino oscillations?

Sofiane M. Boucenna,1,* José W.F. Valle,2,† and Avelino Vicente2,3,‡

1INFN, Laboratori Nazionali di Frascati, C.P. 13, 100044 Frascati, Italy.
2Instituto de Física Corpuscular (CSIC-Universitat de València), Apdo. 22085, E-46071 Valencia, Spain.
3IFPA, Dep. AGO, Université Liège, Bat B55, Sart-Tilman B-4000, Liège 1, Belgium

(Dated: April 6, 2015)

Neutrino oscillations are solidly established, with a hint of CP violation just emerging. Similarly, there are hints of lepton universality violation in $b \to s$ transitions at the level of 2.6σ. By assuming that the unitary transformation between weak and mass charged leptons equals the leptonic mixing matrix measured in neutrino oscillation experiments, we predict several lepton flavor violating (LFV) B meson decays. We are led to the tantalizing possibility that some LFV branching ratios for B decays correlate with the leptonic CP phase $\delta$ characterizing neutrino oscillations. Moreover, we also consider implications for $\ell_i \to \ell_j \ell_k \ell_l$ decays.

PACS numbers: 13.25.Hw, 14.60.Pq, 12.60.Cn, 14.60.St, 14.70.Pw

Introduction

The historical discovery of the Higgs boson [1, 2] would have completed our picture of particle physics, were it not for the solid evidence we now have that neutrino flavors interconvert [3]. Apart from neutrino oscillations and cosmology, no other signs of new physics (NP) have been established. However, some indirect signs might have been found by the LHCb collaboration. In 2013, they have published the results of the measurement of a variety of observables in $b \to s$ transitions. In some cases, the experimental result was found to be in clear tension with the Standard Model (SM) prediction. These include angular observables [4–6] in $B \to K^\pm \mu^\mp \mu^-$ [7], as well as a sizable suppression of several branching ratios [8, 9]. Recently, the LHCb announced new results based on the complete LHC Run I dataset [10]. The inclusion of new data has confirmed the anomalies, which are currently at the $\sim 4\sigma$ level. Furthermore, in 2014, the LHCb collaboration found an intriguing indication of lepton universality violation in the ratio [11]

$$R_K = \frac{\textrm{BR}(B \to K \mu^+ \mu^-)}{\textrm{BR}(B \to K e^+ e^-)} = 0.745^{+0.090}_{-0.074} \pm 0.036.$$  (1)

This experimental measurement, obtained in the low dilepton invariant mass regime, is 2.6σ away from the SM result $R_K^{SM} = 1.0003 \pm 0.0001$ [12]. Although the statistical significance of this discrepancy is not enough to claim a discovery, it is highly suggestive that several independent global fits [13–17] have shown that this hint can be explained by the same type of new physics contributions as the previous $b \to s$ anomalies.

The violation of lepton universality usually comes together with the violation of lepton flavor. Based on symmetry arguments, Glashow, Guadagnoli and Lane [18] recently argued that the observation of universality violation in the lepton flavor conserving (LFC) $B \to K \ell_i^+ \ell_i^-$ decays implies the existence of the lepton flavor violating (LFV) processes $B \to K \ell_i^+ \ell_j^-$ (with $i \neq j$). The idea of LFV in $B$ meson decays has been further explored in [19–22].

Here we take a step further in this direction. Since we lack a theory a flavor, we can not make definite predictions for the LFV rates using the LFC ones as input. Hence we make the simplest alternative assumption, namely, that the unitary transformation between weak and mass charged lepton states is given by the leptonic mixing matrix measured in neutrino oscillation experiments. Under this ad hoc assumption we make numerical predictions for several LFV observables in the B system.

General aspects of the $b \to s$ anomalies

The effective hamiltonian describing $b \to s$ transitions can be expressed as:

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V^*_{ts} \frac{e^2}{16\pi^2} \sum_i \left( C_i O_i + C'_i O'_i \right) + \text{h.c.}$$  (2)

Here $G_F$ is the Fermi constant, $e$ the electric charge and $V$ the CKM matrix. The Wilson coefficients $C_i$ and $C'_i$ encode the different (SM and NP) contributions to the effective operators $O_i$ and $O'_i$. The analysis of the available experimental data on $b \to s$ transitions reveals that the effective operators relevant for the resolution of the $b \to s$ anomalies are:

$$O_9 \equiv O_9^{\mu\mu} = (s_{\gamma\alpha} P_L b) (\bar{\mu}_{\gamma\alpha} \mu),$$  (3)

$$O_{10} \equiv O_{10}^{\mu\mu} = (s_{\gamma\alpha} P_L b) (\bar{\mu}_{\gamma\alpha} \gamma_5 \mu),$$  (4)

where $P_L = \frac{1}{2}(1 - \gamma_5)$ is the left-chirality projector. Several independent global fits [14–17] find a significant tension between the SM results for the Wilson coefficients of these operators and the experimental data. This can be clearly alleviated in the presence of NP contributions. According to the global fit [17], the $C_9^{\mu\mu}$ coefficient is the key to improve the fits. More precisely, one finds a reasonable agreement with data when NP provides a negative contribution to $O_9^{\mu\mu}$, with $C_9^{\mu\mu,\text{NP}} \sim -30\% \times C_9^{\mu\mu,\text{SM}}$. 


Similar improvements are found when NP enters in the $SU(2)_L$ invariant direction $C^{\mu\mu,\text{NP}}_9 = -C^{\mu\mu,\text{NP}}_{10}$, with $C^{\mu\mu,\text{NP}}_9 \sim -12\% \times C^{\mu\mu,\text{SM}}_9$.

**Predicting lepton flavor violation in $B$ meson decays**

Here we raise the following question: can the leptonic mixing matrix provide the required lepton flavor structure in $C_{90}$ and $C_{100}$? And if so, what are the predictions for lepton flavor violation in the $B$ sector?

As suggested by global fits, let us assume that the relevant NP operator contains a left-handed leptonic current. In this case, this operator can be generally written in the mass basis as:

$$O^{ij} = \frac{1}{\Lambda^2} J^d_{\alpha} P^a_{\ell_i} J_{\ell_j}^e,$$  \hfill (5)

where

$$J^d_{\alpha} = C^{Q}_{\beta\gamma} \bar{d}_{\beta} \gamma_\alpha P_L s,$$  \hfill (6)

$$J^e_{\ell_i} = C^{L}_{\alpha\beta} \bar{\ell}_{i\beta} \gamma_\alpha P_L \ell_j,$$  \hfill (7)

and $\Lambda$ is the energy scale of the NP inducing this operator. The $i, j$ indices denote the lepton flavor combination characterizing the operator in eq. (6). The $3 \times 3$ matrices $C^Q$ and $C^L$ completely determine the relations among the Wilson coefficients for different flavor choices. On the other hand, in the interaction (gauge) basis, $O$ takes the same form, but the quark and lepton currents are written in terms of gauge eigenstates $d'$ and $\ell'$ as

$$J_{\alpha}^d = \tilde{C}^{Q}_{mn} \bar{d}'_m \gamma_\alpha P_L d'_n,$$  \hfill (8)

$$J_{\ell_i}^e = \tilde{C}^{L}_{ij} \bar{\ell}'_{i\beta} \gamma_\alpha P_L \ell'_{\beta},$$  \hfill (9)

We now focus on the leptons. By combining eqs. (7) and (9) one finds the relation between $C^L$ and $\tilde{C}^L$,

$$C^L = U^L_{\ell} \tilde{C}^L U^L_{\ell},$$  \hfill (10)

where $U_{\ell}$ is the unitary matrix which relates the left-handed charged lepton gauge and mass eigenstates as $\ell' = U_{\ell}\ell$. Similarly, the left-handed neutrino gauge and mass eigenstates are connected by another matrix, $U_{\nu}$, as $\nu' = U_{\nu}\nu$. The product of these two matrices determines the leptonic charged current weak interaction,

$$\mathcal{L}_{ee} = -\frac{g}{2\sqrt{2}} \left( W^e_{\mu} \bar{\nu} \gamma^\mu P_L \nu' + \text{h.c.} \right),$$  \hfill (11)

where $K = U^\dagger_{\ell} U_{\nu}$ is the leptonic mixing matrix measured in neutrino oscillation experiments. If $U_{\nu} = I$, the left-handed neutrino gauge and mass eigenstates are the same and all the mixing is in the left-handed charged leptons. In this case $K = U^L_{\ell}$ and eq. (10) leads to

$$C^L = K \tilde{C}^L K^L,$$  \hfill (12)

We do not attempt to give any model prediction for $\tilde{C}^L$. Instead, we will assume that it is diagonal but with non-universal entries. In that case one can determine the required $\tilde{C}^L$ which, after using eq. (12), leads to a $C^L$ matrix compatible with the observations in $b \rightarrow s$ transitions. In particular, the resulting $C^L$ must have a strong hierarchy between the $ee$ and $\mu\mu$ entries, $C^L_{ee} \ll C^L_{\mu\mu}$, in order to induce a sizable correction to $B \rightarrow K(\ast)\mu^+\mu^-$ and a negligible one to $B \rightarrow K(\ast)\mu^+\nu^-$.  

*Deriving* $C^L$ *from neutrino oscillations*

Barring fine-tunings, we find two generic $\tilde{C}^L$ matrices in the gauge basis that lead to valid $C^L$ matrices in the mass basis. Their forms define our two scenarios:

- **Scenario A**: $\tilde{C}^L = \text{diag}(0, \epsilon, 1)$
- **Scenario B**: $\tilde{C}^L = \text{diag}(\epsilon, 0, 1)$

Here $\epsilon \ll 1$ is a small parameter (note that Ref. [18] considered $\tilde{C}^L = \text{diag}(0,0,1)$, which corresponds to any of our scenarios with $\epsilon = 0$). Using the standard parameterization for the leptonic mixing matrix $K$, one finds that in order to suppress the contributions to the $ee$ Wilson coefficients, $\epsilon$ must be close to

$$\epsilon_A = -\frac{\tan^2 \theta_{13}}{\sin^2 \theta_{12}} \quad \text{in scenario A,} \quad \epsilon_B = -\frac{\tan^2 \theta_{13}}{\cos^2 \theta_{12}} \quad \text{in scenario B}. \quad \text{(13, 14)}$$

Taking 3$\sigma$ ranges for the mixing angles from the latest global fit to neutrino oscillation data [23], one finds the ranges $[-0.10, -0.05]$ for scenario A and $[-0.05, -0.03]$ for scenario B, irrespective of the neutrino mass spectrum; normal and inverted hierarchies giving basically the same results. Interestingly, $\theta_{13} \neq 0$ implies $\epsilon \neq 0$, indicating a suggestive connection between quarks and leptons.

We can now obtain $C^L$ for both scenarios. For simplicity we focus on case A though numerical results for case B are also given below. Assuming $\epsilon = \epsilon_A$ and taking the best-fit values from [23], we find:
where $\delta$ is the Dirac leptonic CP violating phase. In the CP conserving case ($\delta = 0$) this matrix simplifies to

$$C^L = \begin{pmatrix} 0 & 0.094 & 0.128 \\ 0.094 & 0.537 & 0.508 \\ 0.128 & 0.508 & 0.389 \end{pmatrix}. \quad (16)$$

**Lepton flavor violation in the B system**

The matrix $C^L$ can be used to make definite predictions for ratios of branching ratios in $B \to K e^+ e^-$ decays,

$$\text{BR}(B \to K e^+ e^-) = 2 \rho_{NP} \left| \frac{C_{ij}^L}{C_{\mu \mu}^L} \right|^2 \text{BR}(B \to K \mu^+ \mu^-). \quad (17)$$

Here $\text{BR}(B \to K e^+ e^-) = \text{BR}(B \to K e^+ e^-) + \text{BR}(B \to K \mu^+ \mu^-)$ and $\text{BR}(B \to K \mu^+ \mu^-) = (1.19 \pm 0.03 \pm 0.06) \times 10^{-7}$ is the LHCb result [8], measured using the $3 \, \text{fb}^{-1}$ dataset after LHC Run I in the $1 \, \text{GeV}^2 < q^2 < 6 \, \text{GeV}^2$ range, where $q^2 = M^{2}_{\mu \mu}$ is the dimuon invariant mass. This $q^2$ range has been selected in order to avoid pollution from other hadronic resonances. The factor $\rho_{NP}$ is the NP fraction of the $B \to K \mu^+ \mu^-$ amplitude, $\rho_{NP} = M_{NP}/M_{Total}$ [18]. Using the results of the global fit [17], which gives $C_{\mu \mu}^{NP} \sim -12\% \times C_{\mu \mu}^{SM}$, $\rho_{NP}$ is found to be $\rho_{NP} \sim -0.136$.

For $\delta = 0$, we obtain the following predictions for the $B \to K$ LFV transitions in scenario A,

$$\begin{align*}
\text{BR}(B \to K e^+ e^-) &\in [1.2, 1.7] \times 10^{-10}, \quad (18) \\
\text{BR}(B \to K e^+ \mu^-) &\in [1.9, 5.8] \times 10^{-10}, \quad (19) \\
\text{BR}(B \to K \mu^+ e^-) &\in [3.4, 7.2] \times 10^{-9}. \quad (20)
\end{align*}$$

These have been derived using the LHCb central value and taking the leptonic mixing angles in the preferred $1\sigma$ ranges found by the fit [23]. The main generic prediction from our setup is thus

$$\text{BR}(B \to K \mu^+ \tau^-) \gg \text{BR}(B \to K e^+ \mu^-), \text{BR}(B \to K e^+ \tau^-). \quad (21)$$

**Rare B decays and leptonic CP violation**

One can now consider a scenario with a non-zero value of the CP violating phase $\delta$ characterizing neutrino oscillations. In this case, we are led to the fascinating possibility that the LFV branching ratios for B meson decays will depend upon $\delta$. Our results can be found in figs. (1) and (2) corresponding to the decay modes $B \to K \mu^+ e^-$ and $B \to K \tau^+ e^-$ respectively. This would suggest an alternative way of probing $\delta$ by using LFV B meson decays.
\( \ell_i \to \ell_j \ell_k \ell_k \) decays

The same strategy can be extended to other LFV observables if induced mainly by vectorial operators, as in eqs. (3) and (4). Assuming the same leptonic operators, the analogous operators for the purely leptonic LFV processes \( \ell_i \to \ell_j \ell_k \ell_k \) are:

\[
\mathcal{O}_{4\ell} = \frac{1}{N^2} \left( C_{ij}^{\ell_1} \ell_i \gamma_\alpha P_L \ell_j \right) \left( C_{mn}^{\ell_1} \ell_m \gamma_\alpha P_L \ell_n \right).
\]

Here we assume that the scale of the NP responsible for the vectorial LFV operators is the same as the one relevant for \( B \) meson decays, eq. (3), although in full generality these could be unrelated. The flavor structure of \( \mathcal{O}_{4\ell} \equiv \mathcal{O}_{4\ell}^{mn} \) is given by the product \( C_{ij}^L C_{mn}^L \), which, following the same prescription as for the \( B \) meson decays, can be written as

\[
C_{ij}^L C_{mn}^L = \left( K C^L K^\dagger \right)_{ij} \left( K C^L K^\dagger \right)_{mn}.
\]

The \( \mathcal{O}_{4\ell} \) operator induces several \( \ell_i \to \ell_j \ell_k \ell_k \) decay processes: (i) \( \ell_i \to \ell_j \ell_k \ell_k \), and (ii) \( \ell_i \to \ell_j \ell_k \ell_k \) (with \( k \neq j \)). Their branching ratios can be written as [24]

\[
\text{BR}(\ell_i \to \ell_j \ell_k \ell_k) = \kappa \frac{m_i^5}{512 \pi^3 \Gamma_{\ell_i}} \left| M_{ijk} \right|^2 \frac{\Lambda^4}{A^4},
\]

where \( \kappa = 2/3 \) when there are two identical leptons in the final state, and \( \kappa = 1/3 \) otherwise, and \( m_{\ell_i} \) and \( \Gamma_{\ell_i} \) are the mass and decay width of the \( \ell_i \) lepton, respectively. The coefficient \( M_{ijk} \) takes the form \( C_{ij}^L C_{kk}^L \) in case (i), \( C_{ik}^L C_{jk}^L \) in case (ii).

One can now use the experimental limits on these LFV branching ratios to derive bounds on \( \Lambda \). Processes involving \( C_{ij}^L \) are strongly suppressed and thus do not provide meaningful bounds. This is the case of \( \mu^- \to e^- e^- e^+ \), \( \tau^- \to e^- e^- \tau^+ \) and \( \tau^- \to \mu^- e^+ e^- \). In contrast, the Belle limit [25] translates into \( \Lambda \geq 5.8 \) TeV (in both scenarios, A and B). The other \( \tau \) decay modes lead to slightly less stringent bounds. Future \( B \) factories are expected to improve on the search for \( \tau^- \to \mu^- \mu^- \mu^+ \), with sensitivities to branching ratios as low as \( \sim 10^{-9} \) [26], allowing us to probe NP scales up to \( \Lambda \sim 12 \) TeV.

Conclusions and discussion

In summary, we have suggested that the universality and flavor violating \( b \to s \) anomalies may be related to the pattern of neutrino oscillations. By assuming that the unitary transformation between weak and mass charged lepton eigenstates is given by the leptonic mixing matrix measured in neutrino oscillations we predict several lepton flavor violating \( B \) meson decay rates. Moreover we are led to the thrilling possibility that some of the rare LFV \( B \) decay branching ratios correlate with the leptonic CP phase \( \delta \) that characterizes neutrino oscillations. Other lepton flavor violating processes processes such as \( \ell_i \to \ell_j \ell_k \ell_k \) have been considered in a similar manner. Improved measurements at Belle should probe new physics scale at the level \( \Lambda \sim 12 \) TeV. Relevant scenarios involve additional neutral currents, such as schemes containing an extra \( Z' \) boson with lepton universality violation in \( B \) decays [27–30], or possibly some realizations of the electroweak symmetry \( SU(3)_C \otimes SU(3)_L \otimes U(1)_X \) [31–36]. Such schemes should be taken seriously should the observed hints in the \( B \) sector persist.

Note added

A few days ago, an update of [17] was presented in [37]. While this would change slightly the value of \( \rho_{NP} \) used in our analysis, our results are essentially left unchanged.

Acknowledgments

We are grateful to Javier Virto for enlightening discussions. Work supported by the Spanish grants FPA2014-58183-P and Multidark CSD2009-00064 (MINECO), and the grant PROMETEOII/2014/084 from Generalitat Valenciana. SMB acknowledges financial support from the research grant “Theoretical Astroparticle Physics” number 2012CPPYP7 under the program PRIN 2012 funded by the Italian “Ministero dell’Istruzione, Università e della Ricerca” (MIUR) and from the INFN “Iniziativa Specifica” Theoretical Astroparticle Physics (TAsP-LNF). AV acknowledges partial support from the EXPL/FIS-NUC/0460/2013 project financed by the Portuguese FCT.

* Electronic address: boucenna@lnf.infn.it
† Electronic address: valle@ific.uv.es
‡ Electronic address: avelino.vicente@ulg.ac.be

[1] ATLAS Collaboration, G. Aad et al., “Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC,” Phys.Lett. B716 (2012) 1–29, arXiv:1207.7214 [hep-ex].
[2] CMS Collaboration, S. Chatrchyan et al., “Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC,” Phys.Lett. B716 (2012) 30–61, arXiv:1207.7235 [hep-ex].
[3] M. Maltoni, T. Schwetz, M. Tortola, and J. Valle, “Status of global fits to neutrino oscillations,” New J.Phys. 6 (2004) 122, arXiv:hep-ph/0405172 [hep-ph].
[4] J. Matias, F. Mescia, M. Ramon, and J. Virto, “Complete Anatomy of \( B_d \to K^{+} (K \pi) l^+ l^- \) and its angular distribution,” JHEP 1204 (2012) 104, arXiv:1202.4266 [hep-ph].
S. Descotes-Genon, J. Matias, M. Ramon, and J. Virto, “Implications from clean observables for the binned to analysis of $BK \to \mu^+\mu^-$ at large recoil,” *JHEP* **1301** (2013) 048, [arXiv:1207.2753](https://arxiv.org/abs/1207.2753) [hep-ph].

S. Descotes-Genon, T. Hurth, J. Matias, and J. Virto, “Optimizing the basis of $B \to K^{(*)}\ell^+\ell^-$ observables in the full kinematic range,” *JHEP* **1305** (2013) 137, [arXiv:1303.5794](https://arxiv.org/abs/1303.5794) [hep-ph].

C. Langenbruch, “Latest results on rare decays from LHCb,” [arXiv:1412.7164](https://arxiv.org/abs/1412.7164) [hep-ph].

S. Sahoo and R. Mohanta, “Scalar leptoquarks and the rare B meson decays,” [arXiv:1501.05193](https://arxiv.org/abs/1501.05193) [hep-ph].

I. d. M. Varzielas and G. Hiller, “Clues for flavor from rare lepton and quark decays,” [arXiv:1503.01084](https://arxiv.org/abs/1503.01084) [hep-ph].

D. Forero, M. Tortola, and J. Valle, “Neutrino oscillations refitted,” *Phys.Rev.* **D90** no. 9, (2014) 093006, [arXiv:1405.7540](https://arxiv.org/abs/1405.7540) [hep-ph].

B. Bhattacharya, A. Datta, D. London, and B. Gripaios, M. Nardecchia, and S. Renner, “Composite W, Z: electroweak couplings unification,” [arXiv:1411.0565](https://arxiv.org/abs/1411.0565) [hep-ph].