One-loop renormalisation of $\mathcal{N} = \frac{1}{2}$ supersymmetric gauge theory with a superpotential

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We construct a superpotential for the general $\mathcal{N} = \frac{1}{2}$ supersymmetric gauge theory coupled to chiral matter in the fundamental and adjoint representations, and investigate the one-loop renormalisability of the theories.

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1. Introduction

$N = \frac{1}{2}$ supersymmetric theories (i.e. theories defined on non-anticommutative super-space) have recently attracted much attention\cite{1}–\cite{4}. Such theories are non-hermitian and only have half the supersymmetry of the corresponding $N = 1$ theory. These theories are not power-counting renormalisable\cite{5} but it has been argued\cite{7}–\cite{10} that they are in fact nevertheless renormalisable, in other words only a finite number of additional terms need to be added to the Lagrangian to absorb divergences to all orders. Generally speaking, in a non-renormalisable theory, the dimensionality of Green’s functions suffering from logarithmical divergences increases as the number of loops increases, leading inevitably to an infinite number of potential counter-terms; that this does not happen here is due to a set of discrete symmetries whose origin is linked to the non-hermitian nature of the relevant actions. (An elegant analysis of this appearing in Ref.\cite{8} is straightforward to generalise to incorporate Yukawa couplings in the case of adjoint matter that we consider here. We present this analysis in Appendix C, showing in fact that Ref.\cite{8} omitted a few relevant terms). In previous work we have shown that although divergent gauge non-invariant terms are generated at the one-loop level, they can be removed by divergent field redefinitions leading to a renormalisable theory in which $N = \frac{1}{2}$ supersymmetry is preserved at the one-loop level in both the pure gauge case\cite{11} and in the presence of chiral matter in the fundamental representation\cite{12}. In the latter case, the joint requirements of renormalisability and $N = \frac{1}{2}$ supersymmetry impose the choice of gauge group $SU(N) \otimes U(1)$ (rather than $U(N)$ or $SU(N)$). It is interesting to compare our results with those obtained using superfields. The authors of Ref.\cite{13} obtained the one loop effective action for pure $N = \frac{1}{2}$ supersymmetry using a superfield formalism. Although they found divergent contributions which broke supergauge invariance, their final result was gauge-invariant without the need for any redefinition. In subsequent work\cite{14} it was shown that the $N = \frac{1}{2}$ superfield action requires modification to ensure renormalisability, which is consistent with our findings in the component formulation\cite{12}.

It was pointed out in Ref.\cite{1} that an $N = \frac{1}{2}$ supersymmetric theory can also be constructed with matter in the adjoint representation (at the classical level). Our purpose here is to repeat the analysis of Ref.\cite{12} for the adjoint case, then proceed to consider the addition of superpotential terms for both the adjoint and fundamental cases, which will turn out to be a non-trivial task. Our goal is to construct a renormalisable $N = \frac{1}{2}$

\footnote{See Refs.\cite{3} \cite{4} for other discussions of the ultra-violet properties of these theories.}
supersymmetric theory. Renormalisability may require the addition of new terms and associated couplings to the original classical $\mathcal{N} = \frac{1}{2}$ supersymmetric theory in order that all divergences may be removed by adding counterterms to the couplings; but the hope is that this can be done while preserving $\mathcal{N} = \frac{1}{2}$ invariance. We shall find that this can be achieved in the fundamental case, where only mass terms are possible in the superpotential, but not in the adjoint case where trilinear superpotential terms are also allowed. (In the $\mathcal{N} = \frac{1}{2}$ case, these trilinear superpotential terms are accompanied by additional terms with gauge fields; we shall refer to the full set of these terms as the “Yukawa” superpotential.)

2. The classical adjoint action without superpotential

The adjoint action of Ref. [4] was written for the gauge group $U(N)$. As we noted in Refs. [11], [12], at the quantum level the $U(N)$ gauge invariance cannot be retained. As mentioned earlier, in the case of chiral matter in the fundamental representation we were obliged to consider a modified theory with the gauge group $SU(N) \otimes U(1)$. In the adjoint case with a Yukawa superpotential, it will turn out that the matter fields must also be in a representation of $SU(N) \otimes U(1)$. However, for simplicity of exposition we shall start by considering the adjoint case without a superpotential, in other words adapting the calculations of Ref. [12] to the adjoint case. The classical action without a superpotential may be written

$$S_0 = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} A^\mu A^\nu - i \bar{\chi}^A \sigma^\mu (D_\mu \lambda)^A + \frac{1}{2} D^A D^A ight.$$
$$- \frac{1}{2} i C^{\mu\nu\rho} d^{ABC} e^{ABC} F_{\mu\nu} A^B \chi^C$$
$$+ \frac{1}{g^2} |C^2 d^{ab} d^{cd} (\bar{\chi}^A \chi^B)(\bar{\chi}^C \chi^D)| + \frac{1}{4N g_0^4} |C^2 (\bar{\chi}^A \chi^B)(\bar{\chi}^C \chi^D) + \bar{F} F - \bar{\psi} \sigma^\mu D_\mu \psi - D^\mu \bar{\phi} D_\mu \phi$$
$$+ g \bar{\phi} D^F \phi + ig \sqrt{2}(\bar{\phi} F_{\psi} \psi - \bar{\psi} \chi^F \phi)$$
$$+ d^{abc} g C^{\mu\nu} \left( \sqrt{2} D_{\mu} \bar{\phi} \chi^A \sigma_{\nu} \psi^c + i \bar{\phi}^{a\mu} F^{b}_{\mu\nu} F^c \right)$$
$$+ d^{ab} g_0 C^{\mu\nu} \left( \sqrt{2} D_\mu \bar{\phi} \lambda^A \sigma_{\nu} \psi^b + i \phi_{\mu\nu} ^{a\mu} F^b \right)$$
$$+ d^{a00} g_0 C^{\mu\nu} \left( \sqrt{2} \partial_\mu \bar{\phi} \lambda^a \sigma_{\nu} \psi^0 + i \phi_{\mu\nu} ^{a\mu} F^0 \right)$$
$$+ d^{a0} g_0 C^{\mu\nu} \left( \sqrt{2} \partial_\mu \bar{\phi} \lambda^a \sigma_{\nu} \psi^0 + i \phi_{\mu\nu} ^{a\mu} F^0 \right)$$
$$+ d^{ab} g_0 C^{\mu\nu} \left( \sqrt{2} D_\mu \bar{\phi} \lambda^a \sigma_{\nu} \psi^b + i \phi_{\mu\nu} ^{a\mu} F^b \right)$$
$$+ d^{ab} g_0 C^{\mu\nu} \left( \sqrt{2} D_\mu \bar{\phi} \lambda^a \sigma_{\nu} \psi^b + i \phi_{\mu\nu} ^{a\mu} F^b \right)$$
$$- \frac{1}{4} g^2 |C^2 \bar{\chi}^A \chi^A F| \right].$$

$$S_0 = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} A^\mu A^\nu - i \bar{\chi}^A \sigma^\mu (D_\mu \lambda)^A + \frac{1}{2} D^A D^A ight.$$
$$- \frac{1}{2} i C^{\mu\nu\rho} d^{ABC} e^{ABC} F_{\mu\nu} A^B \chi^C$$
$$+ \frac{1}{g^2} |C^2 d^{ab} d^{cd} (\bar{\chi}^A \chi^B)(\bar{\chi}^C \chi^D)| + \frac{1}{4N g_0^4} |C^2 (\bar{\chi}^A \chi^B)(\bar{\chi}^C \chi^D) + \bar{F} F - \bar{\psi} \sigma^\mu D_\mu \psi - D^\mu \bar{\phi} D_\mu \phi$$
$$+ g \bar{\phi} D^F \phi + ig \sqrt{2}(\bar{\phi} F_{\psi} \psi - \bar{\psi} \chi^F \phi)$$
$$+ d^{abc} g C^{\mu\nu} \left( \sqrt{2} D_{\mu} \bar{\phi} \chi^A \sigma_{\nu} \psi^c + i \bar{\phi}^{a\mu} F^{b}_{\mu\nu} F^c \right)$$
$$+ d^{ab} g_0 C^{\mu\nu} \left( \sqrt{2} D_\mu \bar{\phi} \lambda^A \sigma_{\nu} \psi^b + i \phi_{\mu\nu} ^{a\mu} F^b \right)$$
$$+ d^{a00} g_0 C^{\mu\nu} \left( \sqrt{2} \partial_\mu \bar{\phi} \lambda^a \sigma_{\nu} \psi^0 + i \phi_{\mu\nu} ^{a\mu} F^0 \right)$$
$$+ d^{a0} g_0 C^{\mu\nu} \left( \sqrt{2} \partial_\mu \bar{\phi} \lambda^a \sigma_{\nu} \psi^0 + i \phi_{\mu\nu} ^{a\mu} F^0 \right)$$
$$+ d^{ab} g_0 C^{\mu\nu} \left( \sqrt{2} D_\mu \bar{\phi} \lambda^a \sigma_{\nu} \psi^b + i \phi_{\mu\nu} ^{a\mu} F^b \right)$$
$$+ d^{ab} g_0 C^{\mu\nu} \left( \sqrt{2} D_\mu \bar{\phi} \lambda^a \sigma_{\nu} \psi^b + i \phi_{\mu\nu} ^{a\mu} F^b \right)$$
$$- \frac{1}{4} g^2 |C^2 \bar{\chi}^A \chi^A F| \right].$$
Here
\[
\lambda^F = \lambda^a \tilde{F}^a, \quad (\tilde{F}^A)^{BC} = i f^{BAC},
\]
(similarly for \(D^F\)), and we have
\[
D_\mu \phi = \partial_\mu \phi + ig A^F_\mu \phi,
\]
\[
F^{A \mu}_\nu = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - g f^{ABC} A^B_\mu A^C_\nu,
\]
with similar definitions for \(D_\mu \psi, D_\mu \lambda\). If one decomposes \(U(N)\) as \(SU(N) \otimes U(1)\) then our convention is that \(\phi^a\) (for example) are the \(SU(N)\) components and \(\phi^0\) the \(U(1)\) component. (For later convenience we also define \(g_\lambda\) similarly to encompass both \(g_a = g\) and \(g_0\).) Of course then \(f^{ABC} = 0\) unless all indices are \(SU(N)\). We note that \(d^{ab0} = \sqrt{\frac{2}{N}} \delta^{ab}\), \(d^{000} = \sqrt{\frac{2}{N}}\). (Useful identities for \(U(N)\) are listed in Appendix D.) We also have
\[
e^{abc} = g, \quad e^{a0b} = e^{ab0} = e^{000} = g_0, \quad e^{0ab} = \frac{g^2}{g_0}.
\]
We have written the \(\tilde{\phi} \lambda \bar{\lambda} F\) term as it is given starting from the superspace formalism. We note that it has the opposite sign from that given in Ref. [4]. This term is \(\mathcal{N} = \frac{1}{2}\) supersymmetric on its own and so the exact form chosen should not affect the renormalisability of the theory. It is easy to show that Eq. (2.1) is invariant under
\[
\delta A^A_\mu = -i \lambda^A \sigma_\mu \epsilon
\]
\[
\delta \lambda^A_\alpha = i \epsilon_\alpha D^A + (\sigma^{\mu \nu})_\alpha \left[ F^{A \mu}_\nu + \frac{1}{2} i C_{\mu \nu} e^{ABC} d^{ABC} \bar{\lambda}^B \bar{\lambda}^C \right], \quad \delta \bar{\lambda}^A_\alpha = 0,
\]
\[
\delta D^A = -i \sigma^\mu D_\mu \lambda^A,
\]
\[
\delta \phi = \sqrt{2} \epsilon \psi, \quad \delta \bar{\phi} = 0,
\]
\[
\delta \psi^\alpha = \sqrt{2} \epsilon^\alpha F, \quad \delta \bar{\psi}_\dot{\alpha} = -i \sqrt{2} (D_\mu \bar{\phi})(\epsilon \sigma^\mu)_\dot{\alpha},
\]
\[
\delta F^A = 0,
\]
\[
\delta F^0 = -i \sqrt{2} D_\mu \bar{\psi}^\alpha \sigma_\mu \epsilon - 2 ig (\bar{\phi} \epsilon \lambda^F)^a
\]
\[+ 2 g C^{\mu \nu} D_\mu (\bar{\phi} \epsilon \sigma_\nu \bar{\lambda}^c d^{bca} + \bar{\phi} \epsilon \sigma_\nu \bar{\lambda}^0 d^{0ba}) + 2 g C^{\mu \nu} D_\mu (\bar{\phi} \epsilon \sigma_\nu \bar{\lambda}^0 d^{0ba}),
\]
\[
\delta F^0 = -i \sqrt{2} D_\mu \bar{\psi}^0 \sigma_\mu \epsilon
\]
\[+ 2 g C^{\mu \nu} D_\mu (\bar{\phi} \epsilon \sigma_\nu \bar{\lambda}^0 d^{0ba}) + 2 g_0 C^{\mu \nu} D_\mu (\bar{\phi} \epsilon \sigma_\nu \bar{\lambda}^0 d^{000}).
\]
In Eq. (2.1), \(C^{\mu \nu}\) is related to the non-anti-commutativity parameter \(C^{\alpha \beta}\) by
\[
C^{\mu \nu} = C^{\alpha \beta} \epsilon_{\beta \gamma} \sigma^{\mu \nu \gamma},
\]
where
\[
\sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu \sigma^\nu - \sigma^\nu \sigma^\mu),
\]
\[
\bar{\sigma}^{\mu\nu} = \frac{1}{4}(\bar{\sigma}^\mu \sigma^\nu - \sigma^\nu \bar{\sigma}^\mu),
\]
and
\[
|C|^2 = C^{\mu\nu} C_{\mu\nu}.
\] (2.8)

Our conventions are in accord with [3]; in particular,
\[
\sigma^\mu \bar{\sigma}^\nu = -\eta^{\mu\nu} + 2\sigma^{\mu\nu}.
\] (2.9)

Properties of \(C \) which follow from Eq. (2.6) are
\[
C^{\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\gamma} (\sigma^{\mu\nu})^{\gamma\beta} C_{\mu\nu},
\] (2.10a)
\[
C^{\mu\nu} \sigma_{\nu\alpha\beta} = C^{\alpha} \gamma \sigma^{\mu} \eta^{\gamma\beta},
\] (2.10b)
\[
C^{\mu\nu} \bar{\sigma}_{\nu}^{\alpha\beta} = -C^{\beta} \gamma \bar{\sigma}^{\mu\alpha\gamma}.
\] (2.10c)

In Eqs. (2.1), \(C_{1,2}^{\mu\nu} \) will be identical to \(C^{\mu\nu} \) at the classical level; but we have distinguished them to allow for the possibility of different renormalisations (in practice an overall numerical factor) at the quantum level; so that \(C_{1,2}^{\mu\nu} \) will obey properties analogous to Eqs. (2.6), (2.8) and (2.10). It is important to note that this is only compatible with \(\mathcal{N} = \frac{1}{2} \) supersymmetry due to the fact that the \(d^{ab0} \partial_\mu \bar{\phi}^b \lambda^\mu \bar{\sigma}_\nu \psi^b \) term in Eq. (2.1) contains no gauge field; and the variation of the gauge field in \(d^{ab0} D_\mu \bar{\phi}^a \lambda \bar{\sigma}_\nu \psi^0 \) gives zero. This implies that the variations of the terms containing either \(C_1^{\mu\nu} \) or \(C_2^{\mu\nu} \) respectively are self-contained. (By contrast, the variation of the gauge field in the \(d^{abc} D_\mu \bar{\phi}^a \lambda \bar{\sigma}_\nu \psi^c \) term is cancelled by the \(C^{\mu\nu} \) term in the variation of the \(\lambda \) in the \(\bar{\phi} \lambda^\nu \psi^c \) term, which forces the \(C^{\mu\nu} \) in the 6th line of Eq. (2.1) to be equal to that in the pure gauge terms, and similarly for that in the 7th line; the terms in the 8th line do not get renormalised at all.)

We use the standard gauge-fixing term
\[
S_{gf} = \frac{1}{2\alpha} \int d^4x (\partial_\mu A^\mu)^2
\] (2.11)
with its associated ghost terms. The gauge propagators for \(SU(N)\) and \(U(1)\) are both given by
\[
\Delta_{\mu\nu} = -\frac{1}{p^2} \left( \eta_{\mu\nu} + (\alpha - 1) \frac{p_\mu p_\nu}{p^2} \right)
\] (2.12)
(omitting group factors) and the fermion propagator is

\[ \Delta_{\alpha\dot{\alpha}} = \frac{p_\mu \sigma^\mu_{\alpha\dot{\alpha}}}{p^2}, \quad (2.13) \]

where the momentum enters at the end of the propagator with the undotted index. The one-loop graphs contributing to the “standard” terms in the Lagrangian (those without a \( C^{\mu\nu} \)) are the same as in the ordinary \( \mathcal{N} = 1 \) case, so anomalous dimensions and gauge \( \beta \)-functions are as for \( \mathcal{N} = 1 \). Since our gauge-fixing term in Eq. (2.11) does not preserve supersymmetry, the anomalous dimensions for \( A_\mu \) and \( \lambda \) are different (and moreover gauge-parameter dependent), as are those for \( \phi \) and \( \psi \). However, the gauge \( \beta \)-functions are of course gauge-independent. The one-loop one-particle-irreducible (1PI) graphs contributing to the new terms (those containing \( C \)) are depicted in Figs. 1–6. With the exception of Fig. 6 (which gives zero contributions in the case of chiral fields in the fundamental representation) these diagrams are the same as those considered in Ref. [12]. The divergent contributions from these and other diagrams considered later are listed (for the adjoint case) in Appendix A.

3. Renormalisation of the adjoint \( SU(N) \) action

The renormalisation of \( \mathcal{N} = \frac{1}{2} \) supersymmetric gauge theory presents certain subtleties. The bare action is given by

\[
S_B = S_0^B + \frac{1}{N} \kappa_1 g_0^2 |C|^2 \left( \overline{\phi^a} \Phi^a \right) \left( \overline{\psi^b} \right)
- g d^{ab0} \kappa_2 C_2^{\mu\nu} \left( \sqrt{2} D_\mu \phi^a \sigma_\nu \psi^b + \sqrt{2} \overline{\phi^a} \lambda^b \sigma_\nu \psi^b + i \phi^a D_\mu \psi^b \right)
- g_0 d^{ab0} \kappa_3 C^{\mu\nu} \left( \sqrt{2} D_\mu \phi^a \sigma_\nu \psi^b + \sqrt{2} \overline{\phi^a} \lambda^b \sigma_\nu \psi^b + i \phi^a F_\mu \psi^b \right)
\]

where \( S_0^B \) is obtained by replacing all fields and couplings in \( S_0 \) (in Eq. (2.1)) by their bare versions, given below. The terms involving \( \kappa_{1-3} \) are separately invariant under \( \mathcal{N} = \frac{1}{2} \) supersymmetry. Those with \( \kappa_1, \kappa_2 \) must be included at this stage to obtain a renormalisable Lagrangian; those with \( \kappa_3 \) will be required when we introduce a superpotential but could be omitted at present.

We found in Refs. [11], [12] that non-linear renormalisations of \( \lambda \) and \( \overline{F} \) were required; and in a subsequent paper [13] we pointed out that non-linear renormalisations of \( F, \overline{F} \) are required even in ordinary \( \mathcal{N} = 1 \) supersymmetric gauge theory when working in the
uneliminated formalism. Note that in the $\mathcal{N} = \frac{1}{2}$ supersymmetric case, fields and their conjugates may renormalise differently. The renormalisations of the remaining fields and couplings are linear as usual and given by

$$
\begin{align*}
\lambda_B^a &= Z_1^{\frac{1}{2}} \lambda^a, & A_{\mu}^a &= Z_2^{\frac{1}{2}} A_{\mu}^a, & D_B^a &= Z_3^{\frac{1}{2}} D^a, & \phi_B^0 &= Z_4^{\frac{1}{2}} \phi^a, \\
\psi_B^a &= Z_5^{\frac{1}{2}} \psi^a, & \overline{\phi}_B &= Z_6^{\frac{1}{2}} \overline{\phi}, & \overline{\psi}_B &= Z_7^{\frac{1}{2}} \overline{\psi}, & F_B &= Z_8 F, & g_B &= Z_9 g, \\
C_{\mu\nu}^B &= Z_1^{\frac{1}{2}} C_{\mu\nu}^B, & |C|^2_B &= Z_2 |C|^2, & C_{1,2B}^{\mu\nu} &= Z_3 C_{1,2}^{\mu\nu}, & \kappa_{1-3B} &= Z_{1-3}.
\end{align*}
$$

Eq. (3.2)

The corresponding $U(1)$ gauge multiplet fields $\overline{\lambda}_0^a$ etc are unrenormalised (as are the $U(1)$ chiral fields $\phi_0^a$ etc in the case with no superpotential); so is $g_0$. The auxiliary field $F$ is also unrenormalised, i.e. $Z_F = 1$ (though again this will no longer be the case when we later introduce a superpotential). In Eq. (3.2), $Z_{1-3}$ are divergent contributions, in other words we have set the renormalised couplings $\kappa_{1-3}$ to zero for simplicity. The other renormalisation constants start with tree-level values of 1. As we mentioned before, the renormalisation constants for the fields and for the gauge coupling $g$ are the same as in the ordinary $\mathcal{N} = 1$ supersymmetric theory (for a gauge theory coupled to an adjoint chiral field) and are therefore given up to one loop by

$$
\begin{align*}
Z_\lambda &= 1 - g^2 NL (2\alpha + 2), \\
Z_A &= 1 + g^2 NL (1 - \alpha) \\
Z_D &= 1 - 2NL g^2, \\
Z_g &= 1 - 2g^2 NL, \\
Z_\phi &= 1 + 2g^2 (1 - \alpha) LN, \\
Z_\psi &= 1 - 2g^2 (1 + \alpha) LN,
\end{align*}
$$

where (using dimensional regularisation with $d = 4 - \epsilon$) $L = \frac{1}{16\pi^2\epsilon}$. The renormalisation of $\lambda^a_B$ is given by

$$
\begin{align*}
\lambda_B^a &= Z_1^{\frac{1}{2}} \lambda^a - \frac{1}{2} NL g^3 C^{\mu\nu} d^{abc} \sigma_\mu \overline{\lambda} A^{b}_\nu - NL g^2 g_0 C^{\mu\nu} d^{ab0} \sigma_\mu \overline{\lambda}^0 A^b_\nu \\
&\quad + i\sqrt{2} \tau_4 NL g^3 d^{abc} (C_\psi) b^{\overline{c}} b^c + i\sqrt{2} \tau_5 NL g^3 d^{a0b} (C_\psi)^0 b^b, \\
\lambda_B^0 &= \lambda^0 i\sqrt{2} \tau_6 NL g^2 g_0 d^{ab0} (C_\psi)^a b^b,
\end{align*}
$$

Eq. (3.4)
where \((C\psi)^{\alpha} = C^{\alpha \beta} \psi^\beta\). The replacement of \(\lambda\) by \(\lambda_B\) produces a change in the action given (to first order) by

\[
S_0(\lambda_B) - S_0(\lambda) = NL g^2 \int d^4 x \left\{ \tau_4 g \left[ ig d^{abc} f^{cde} \overline{\phi^a} \phi^b \psi^c (C\psi)^d \right] \\
+ \sqrt{2} C^{\mu \nu} d^{abc} \overline{\phi^a} \lambda \overline{\sigma^\nu} D_\mu \psi^c + \sqrt{2} C^{\mu \nu} d^{abc} D_\mu \overline{\phi^a} \lambda \overline{\sigma^\nu} \psi^c \\
+ \tau_5 \sqrt{2} g C^{\mu \nu} d^{abc} (\overline{\phi^a} \lambda \overline{\sigma^\nu} \partial_\mu \psi^0 + D_\mu \overline{\phi^a} \lambda \overline{\sigma^\nu} \psi^0) \\
+ \tau_6 \sqrt{2} g_0 C^{\mu \nu} d^{abc} (\overline{\phi^a} \lambda \overline{\sigma^\nu} \lambda \psi^0) + \ldots \right\},
\]

(3.5)

where the ellipsis indicates the terms not involving \(\tau_{4-6}\) (which were given previously in Ref. [12]). The value of \(\tau_4\) will be chosen so as to cancel the divergent contributions from Fig. 6; \(\tau_{5,6}\) will be specified later when we renormalise the theory with a superpotential.

We now find that to render finite the contributions linear in \(F\) we require

\[
\overline{F}_B^0 = Z_F \overline{F}^0 + i C^{\mu \nu} L g^2 \left\{ gN \left[ (5 + 2\alpha) \partial_\mu A_\nu^a - \frac{1}{4} (11 + 4\alpha) g f^{bde} A_\mu^d A_\nu^e \right] \overline{\phi}^a d^{abc} \\
+ \sqrt{2} N g \left[ 2 ((4 + \alpha) - z_{C1}) \partial_\mu A_\nu^a - (\frac{1}{2} (9 + 2\alpha) - z_{C1}) g f^{abc} A_\mu^a A_\nu^b \right] \overline{\phi}^a \\
+ 2 \sqrt{2} N g_0 (-(1 - \alpha) + z_3) \partial_\mu A_\nu^a \phi^a \right\} \\
+ \frac{1}{8} \overline{L} g^4 |C|^2 \left[ 2 (1 - \alpha) N f^{ace} f^{bde} - 11 N d^{abc} d^{cde} + 4 (\delta^{ab} \delta^{cd} + \delta^{ac} \delta^{bd}) \right] \overline{\phi}^a \lambda \overline{\lambda}^d \\
- L g^3 |C|^2 \left\{ d^{abc} \sqrt{2} N \left[ g \phi^a \lambda \overline{\lambda}^c + 3 g_0 \phi^a \overline{\lambda}^c \lambda \right] \right\} \\
- 2 \sqrt{2} N g_0 |C|^2 \lambda \phi \overline{\lambda}^d + \frac{3}{4} g_0 |C|^2 \overline{\lambda}^a \phi \overline{\lambda}^d \right\},
\]

(3.6)

Writing \(Z^{(n)}_C\) for the \(n\)-loop contribution to \(Z_C\) we set

\[
Z^{(1)}_C = z_C NL g^2
\]

(3.7)

with similar definitions for \(Z^{(n)}_{|C|^2}, Z_{C1,2}, Z_{1-3}\). We now find that with

\[
z_C = z_{|C|^2} = 0, \quad z_{C1} = -z_{C2} = 2, \quad z_1 = -3, \quad \tau_4 = 1, \quad \tau_5 = z_2 - z_{C2}, \quad \tau_6 = z_3,
\]

(3.8)

the one-loop effective action is finite, for arbitrary \(z_2, z_3\). It would appear that we do not have enough information yet to specify the renormalisation constants \(\kappa_2, \kappa_3\) in Eq. (3.4). This apparent arbitrariness is due to the possibility of making non-linear renormalisations of \(\lambda\) as in Eq. (3.4) so that changes in \(z_2\) and \(z_3\) can be compensated by changes in \(\tau_4, \tau_5\). We shall however find ourselves obliged to pick certain values for these constants when we introduce a superpotential. This behaviour is unexpected but we shall find similar features in the case of the fundamental representation later. It would be more satisfying to find some underlying reason for making these choices before introducing the superpotential.
4. The superpotential in the adjoint case

We now consider the problem of adding superpotential terms to the Lagrangian Eq. (2.1). The following potential terms are \( \mathcal{N} = \frac{1}{2} \) invariant at the classical level:

\[
S_{\text{int}} = \int d^4x \text{tr} \left\{ y \left[ \phi^2 F - \psi^2 \phi \right] + \phi \frac{2}{3} F - \psi^2 \phi + \frac{4}{3} i g C^{\mu \nu} \phi \hat{F}_{\mu \nu} + \frac{2}{3} C^{\mu \nu} D_\mu \phi D_\nu \phi \right.
\]
\[
+ m \left[ \phi F - \frac{1}{2} \psi \psi - \frac{1}{2} \psi \psi + i C^{\mu \nu} \phi \hat{F}_{\mu \nu} \phi - \frac{1}{8} g^2 |C| \phi \lambda \phi F^F \lambda \phi \right]\}.
\]

(4.1)

Here in the interests of conciseness we have written the superpotential in index-free form, so that

\[
\phi = \phi^A R^A, \quad \psi = \psi^A R^A, \quad \hat{A}_\mu = g A_\mu^a R^a + g_0 A_\mu^0 R^0;
\]

(4.2)

it then follows that \( \hat{F}_{\mu \nu} = g A F_{\mu \nu}^A R^A \), with \( F_{\mu \nu} \) defined as in Eq. (2.3). We denote the group matrices for the fundamental representation of \( SU(N) \otimes U(1) \) by \( R^A \) where our convention is that \( R^a \) are the \( SU(N) \) generators and \( R^0 \) the \( U(1) \) generator. The matrices are normalised so that \( \text{Tr}[R^A R^B] = \frac{1}{2} \delta^{AB} \). In particular, \( R^0 = \sqrt{\frac{1}{2N}}1 \). It is easy to check that \( S_{\text{int}} \) is \( \mathcal{N} = \frac{1}{2} \) invariant. Except for the last mass term, this superpotential is most readily derived directly from the superspace formalism. Denoting an adjoint chiral superfield as \( \Phi_A \), we have that under a gauge transformation

\[
\Phi_A \rightarrow \Omega * \Phi_A * \Omega^{-1}, \quad \overline{\Phi}_A \rightarrow \overline{\Omega} * \overline{\Phi}_A * \overline{\Omega}^{-1},
\]

so that the gauge interactions are written in superfield form as

\[
\int d^4 \theta \text{tr} [\overline{\Phi}_A * e^V * \Phi_A * e^{-V}].
\]

The following superpotential terms are manifestly also invariant:

\[
\int d^2 \theta \text{tr} \left[ \frac{1}{2} m \Phi_A * \Phi_A + \frac{1}{3} y \Phi_A * \Phi_A * \Phi_A \right]
\]
\[
+ \int d^2 \overline{\theta} \text{tr} \left[ \frac{1}{2} m \overline{\Phi}_A * \overline{\Phi}_A + \frac{1}{3} y \overline{\Phi}_A * \overline{\Phi}_A * \overline{\Phi}_A \right].
\]

(4.3)

Expanded in component fields we have

\[
\Phi_A (y, \theta) = \phi (y) + \sqrt{2} \theta \psi (y) + \theta \theta F (y)
\]
\[
\overline{\Phi}_A (\overline{y}, \overline{\theta}) = \overline{\phi} (\overline{y}) + \sqrt{2} \overline{\theta} \overline{\psi} (\overline{y})
\]
\[
+ \overline{\theta} \left( \overline{F} (\overline{y}) + i g C^{\mu \nu} \partial_\mu \{ \overline{\phi}, A_\nu \} (\overline{y}) - \frac{g^2}{2} C^{\mu \nu} [A_\mu, \{ A_\nu, \overline{\phi} \}] (\overline{y}) \right),
\]

(4.4)
where \( \Phi^\mu = y^\mu - 2i \theta \sigma^\mu \bar{\theta} \). Note the modification of the \( \bar{\theta} \theta \)-term.

If we substitute Eq. (4.4) in Eq. (4.3) we obtain Eq. (4.1) except for the last term. (This can also be expressed in superfields but in a more unwieldy form). The coefficient of this final term is arbitrary since it is separately \( \mathcal{N} = \frac{1}{2} \) invariant; the reason for our particular choice will be explained later (after Eq. (A.18) in Appendix A). A similar set of mass terms is admissible in the case of the fundamental representation, with mass terms coupling the fundamental and anti-fundamental representation fields, as we show later. However, no Yukawa terms are possible in the \( \mathcal{N} = \frac{1}{2} \) case for the fundamental representation. If we have both adjoint and fundamental (anti-fundamental) representations \( \Phi(\bar{\Phi}) \) we can construct \( \mathcal{N} = 2 \)-type invariants, of the form

\[
y \left[ \int d^2 \theta \bar{\Phi} \Phi A \Phi + \int d^2 \bar{\Phi} \Phi A \bar{\Phi} \right]. \tag{4.5}
\]

At the classical level \( \phi \) may be considered as forming a representation of \( U(N) \). However, just as we saw in Ref. [12] for the gauge group, the \( U(N) \) structure is not preserved at the quantum level. The \( \phi^a \) renormalise differently from the \( \phi^0 \) and this means that, for instance, there must be a different mass parameter \( (m, \text{say}) \) for the \( \phi^a F^a \), \( \psi^a \psi^a \) terms than for the \( \phi^0 F^0 \), \( \psi^0 \psi^0 \) terms \( (m_0, \text{say}) \). In the case of the mass terms this does not present serious difficulty since we can separate the mass terms in Eq. (4.1) into separately \( \mathcal{N} = \frac{1}{2} \) invariant sets of terms involving either \( m \) or \( m_0 \). However, in the case of the Yukawa superpotential terms, we need to invoke three separate couplings, one \( (y, \text{say}) \) for \( \phi^a \phi^b F^c \) terms, one \( (y_1, \text{say}) \) for \( \phi^a \phi^0 F^b \), \( \phi^a \phi^0 F^b \) etc and one \( (y_2, \text{say}) \) for \( \phi^0 \phi^0 F^0 \). In the \( \mathcal{N} = 1 \) case the theory would, of course, be renormalisable, with each of \( y, y_1, y_2 \) renormalising differently. By contrast, in the \( \mathcal{N} = \frac{1}{2} \) case many of the \( \bar{\phi}^3 A_\mu \) terms are linked by \( \mathcal{N} = \frac{1}{2} \) transformations to more than one of these groups of terms and so cannot be assigned a unique coupling out of \( y, y_1, y_2 \). So in the presence of Yukawa superpotential terms, the \( \mathcal{N} = \frac{1}{2} \) invariance cannot be maintained at the quantum level. It is this linking of different groups of terms, specifically those corresponding purely to \( SU(N) \) with those containing \( U(1) \) fields, which implies that we cannot have an \( \mathcal{N} = \frac{1}{2} \) theory with a superpotential if the chiral fields belong to \( SU(N) \) alone.
5. The renormalised action with superpotential

As we explained in the previous section, many of the individual terms with couplings $m$ or $y$ in Eq. (4.1) will renormalise differently and hence need to be assigned their own separate couplings. For renormalisability, Eq. (4.1) needs to be replaced by

$$S_{\text{int}} = \int d^4x \left\{ \frac{1}{4} y d^{abc} (\phi^a \phi^b F^c - \psi^a \psi^b \phi^c) + \frac{1}{4} y_1 d^{ab0} (\phi^a \phi^b F^0 + 2 \phi^a \phi^0 F^b - \psi^a \psi^b \phi^0 - 2 \psi^a \psi^0 \phi^b) + \frac{1}{4} y_2 d^{000} (\phi^0 \phi^0 F^0 - \psi^0 \psi^0 \phi^0) + \frac{1}{4} y_3 d^{abc} (\phi^a \phi^b \phi^c - \psi^a \psi^b \phi^c) + \frac{1}{4} y_4 \sqrt{\frac{2}{N}} d^{abcd} (\phi^a \phi^b \phi^c - \psi^a \psi^b \phi^c) \right\}$$

$$+ y_0 C^{\mu
u} F_{\mu
u}^0 \left( \frac{1}{6} y d^{abc} d^{cd} (\phi^a \phi^b \phi^c - \psi^a \psi^b \phi^c) + \frac{1}{3} y_5 \sqrt{\frac{2}{N}} \delta^a \delta^b \delta^c \phi^a \phi^b \phi^c \right)$$

Each of the coefficients $m, y, \text{etc.}$ above will renormalise separately. However, for simplicity when we quote the results for Feynman diagrams, we will use the values of the coefficients as implied by Eq. (4.1), i.e. $y_{1-5} = y, m_0 = \mu_1 = m, \mu_{2-5} = 0$, so that these are effectively the renormalised values of these couplings. Note that the $g_0 C^{\mu
u} F_{\mu
u}^0 \sqrt{\frac{2}{N}} d^{abc} \phi^a \phi^b \phi^c$ and $\frac{1}{N} g_0 C^{\mu
u} F_{\mu
u}^0 \phi^0 \phi^0$ terms only mix with the $d^{abc} \phi^a \phi^b \phi^c$ or $d^{abc} \phi^a \phi^b \phi^c$ terms respectively and hence can be assigned the coupling $y$ or $y_1$ respectively. We emphasise that the mass terms in Eq. (5.1) are $\mathcal{N} = \frac{1}{2}$ invariant although the Yukawa terms are not.
The terms with $\kappa_4$ and that with $\kappa_5$ are separately $\mathcal{N} = \frac{1}{2}$ invariant. As with $\kappa_{1-3}$, we set the renormalised couplings to zero so that $\kappa_4 = Z_4$, $\kappa_5 = Z_5$ with $Z_4$, $Z_5$ divergent. The renormalisation constants $Z_{\phi, \psi}$, $Z_F$ now acquire $y$-dependent contributions, so we have

$$
Z_\phi = 1 + \left[-\frac{1}{4} y^2 + 2g^2(1 - \alpha)\right] LN,
Z_\psi = 1 + \left[-\frac{1}{4} y^2 - 2g^2(1 + \alpha)\right] LN,
Z_{\phi^0} = Z_{\psi^0} = 1 - \frac{1}{4} y^2 LN,
Z_F = 1 - \frac{1}{4} y^2 LN.
$$

(5.2)

Here we write $\phi_B^0 = Z_{\phi^0}^2 \phi^0$, etc, since the $U(1)$ chiral fields are now renormalised. There are now several new one-loop diagrams giving $y^2$-dependent divergent contributions to terms in the action without superpotential, Eq. (2.1) which are cancelled by the $y^2$ terms in Eq. (5.2), but we have not calculated them here; the process of accounting for these divergences would be similar to that elucidated in Ref. [12], and we have preferred to concentrate on the renormalisation of the new terms in Eq. (4.1). Moreover, we have not computed divergent contributions to terms purely involving $\phi$, $\lambda$ and/or $F$, which are individually $\mathcal{N} = \frac{1}{2}$ invariant, nor have we explicitly displayed such terms in the action Eq. (5.1); these contributions would not give any more information about the preservation of $\mathcal{N} = \frac{1}{2}$ supersymmetry.

Now for the bare action we also need to replace $m_B = Z_m m$, $y_B = Z_y y$ etc in addition to the replacements given earlier. These renormalisation constants are given according to the non-renormalisation theorem by

$$
Z_m = Z_{\Phi}^{-1},
Z_m^0 = Z_{\Phi^0}^{-1},
Z_y = Z_{\Phi}^{-\frac{3}{2}},
Z_{y^1} = Z_{\Phi}^{-1} Z_{\Phi^0}^{-\frac{1}{2}},
Z_{y^2} = Z_{\Phi^0}^{-\frac{3}{2}},
$$

(5.3)

where $Z_{\Phi}$, $Z_{\Phi^0}$ are the renormalisation constants for the chiral superfield $\Phi$ given by

$$
Z_{\Phi} = 1 + \left[-\frac{1}{4} y^2 + 4g^2\right] LN,
Z_{\Phi^0} = 1 - \frac{1}{4} y^2 LN.
$$

(5.4)
The redefinitions of $F$ and $\overline{F}$ found in Ref. [11] need to be modified in the presence of mass terms and the $U(1)$ gauge group. This is easily done following the arguments of Ref. [15]; there are no one-loop diagrams giving divergent contributions to $m_\phi F$ or $m_\phi \overline{F}$ although there are counterterm contributions from $m_B \phi_B F$, $m_B \phi_B \overline{F}$. At one loop we have

\begin{align}
F_{aB}^0 &= F_{aB} + (\alpha + 3) g^2 NL \left( m_\phi^a + \frac{1}{4} y d^{abc} \phi^b \phi^c \right) + \frac{1}{2} (\alpha + 3) y g^2 NL d^{ab0} \phi^b \phi^c \\
F_{0B}^0 &= F_{0B} + \alpha g^2 NL d^{0ab} \phi^a \phi^b \\
F_{aB}^0 &= Z_F F_{aB} + (\alpha + 3) g^2 NL \left( m_\phi^a + \frac{1}{4} y d^{abc} \phi^b \phi^c \right) + \frac{1}{2} (\alpha + 3) y g^2 NL d^{ab0} \phi^b \phi^c \\
F_{0B}^0 &= Z_F F_{0B}.
\end{align}

(5.5)

Here $F_{aB}^0$, etc are as given in Eq. (3.6), though of course using the non-zero $Z_F$ given in Eq. (5.2). We have also included the term with $\tau_7$ which is needed to cancel divergences from Figs. 15 and 16. The new $C$-dependent diagrams in the presence of a superpotential are depicted in Figs. 7–11, and their divergent contributions in the corresponding Tables. Note that we do not show Figs. 11(e)–(v) explicitly; instead they are described in Appendix A. We omit diagrams giving contributions of the form $A_\mu A_\nu \phi^3$ which complete the $F_{\mu\nu}^3$ in $F_{\mu\nu} \phi^3$ contributions; we already have ample evidence that gauge invariance, even when apparently violated, can be restored by making divergent field redefinitions.

We now choose the renormalisation constants at our disposal to ensure finiteness. In order to ensure renormalisability of the action in Eq. (5.1), we find we now need to impose specific values for the hitherto arbitrary coefficients $z_2$, $z_3$, namely

\begin{align}
z_2 &= -4, \quad z_3 = 4.
\end{align}

(5.6)

In other words the effective action is rendered finite by adding counterterms to the fields and parameters in Eq. (5.1), without the need for further couplings. Different choices for $z_2$, $z_3$ would require introducing additional parameters into Eq. (5.1), for instance separate couplings (other than $m$, $m_0$ respectively) for the $C_1^{\mu\nu} d^{0ab} F_{\mu\nu}^0 \phi^a \phi^b$ and $C_2^{\mu\nu} d^{0ab} F_{\mu\nu}^0 \phi^a \phi^b$ terms, spoiling the $\mathcal{N} = \frac{1}{2}$ invariance of the mass terms.
We find moreover

\[ Z_{y_3} = 1 - 6LNg^2, \]
\[ Z_{y_4} = 1 - 4LNg^2, \]
\[ Z_{y_5} = 1 - 2LNg^2, \]
\[ Z_{\mu_1} = 1 + \frac{32}{N}LG^2 \left( 1 - \frac{Q^2}{g_0^2} \right), \]
\[ Z_{\mu_2} = -\frac{4}{g_0^2}L, \]
\[ Z_{\mu_3} = 0, \]
\[ Z_{\mu_4} = 2LNg^2, \]
\[ Z_{\mu_5} = 4LNg^2, \]
\[ Z_4 = \frac{1}{2}LNg^2, \]
\[ Z_5 = 3LNg^2, \]
\[ \tau_7 = NLg^2. \]

(5.7)

We note that \( Z_4 \) and \( Z_5 \) are chosen to cancel the divergences from Figs. 13, 14 and 12. (We have only computed those terms involving \( \kappa_4 \) in Eq. (5.1) which contain a derivative; we assume that the others will be as implied by gauge invariance.)
6. The eliminated formalism in the adjoint case

It is instructive and also provides a useful check to perform the calculation in the eliminated formalism. In the eliminated case Eq. (5.1) is replaced by

$$\tilde{S}_{\text{mass}} = \int d^4x \left\{ -\frac{1}{4}y \delta_{abc} \bar{\psi}^a \psi^b \phi^c - \frac{1}{4}y_1 \delta_{ab0} \left( \bar{\psi}^a \psi^{0} \phi^b + 2 \bar{\psi}^a \psi^{0} \phi^0 \right) - \frac{1}{4}y_2 \delta^{000} \bar{\psi}^0 \psi^0 \phi^0 \\
- \frac{1}{4}y \delta_{abc} \bar{\psi}^a \psi^b \phi^c - \frac{1}{4}y_1 \delta_{ab0} \left( \bar{\psi}^a \psi^{0} \phi^b + 2 \bar{\psi}^a \psi^{0} \phi^0 \right) - \frac{1}{4}y_2 \delta^{000} \bar{\psi}^0 \psi^0 \phi^0 \\
- m^2 \bar{\phi}^0 \phi^0 - \frac{1}{2}m_0 \bar{\phi}^0 \phi^0 - \frac{1}{2}m_0 \bar{\phi}^0 \phi^0 - \frac{1}{2}m_0 \bar{\phi}^0 \phi^0 \\
- (\frac{1}{4}y \delta_{abc} \bar{\phi}^a \phi^c + \frac{1}{4}y_1 \delta_{ab0} \bar{\phi}^a \phi^c + m_0 \phi^c) \left( \frac{1}{4}y \delta_{abc} \bar{\phi}^b \phi^c + \frac{1}{4}y_1 \delta_{ab0} \bar{\phi}^b \phi^c + m_0 \phi^c \right) \\
- \frac{1}{4}y_1 \delta_{ab0} \bar{\phi}^a \phi^b + \frac{1}{4}y_2 \delta^{000} \bar{\phi}^0 \phi^0 + m_0 \phi^0 \right) \left( \frac{1}{4}y_1 \delta_{ab0} \bar{\phi}^a \phi^b + \frac{1}{4}y_2 \delta^{000} \bar{\phi}^0 \phi^0 + m_0 \phi^0 \right) + \frac{1}{4}y \delta_{abc} \bar{\phi}^a \phi^b \phi^c + \frac{1}{4}y_1 \delta_{ab0} \bar{\phi}^a \phi^b \phi^0 \phi^0 + \frac{1}{4}y_2 \delta^{000} \bar{\phi}^0 \phi^0 \phi^0.
$$

while we simply strike out the terms involving $F, \bar{F}$ in Eq. (2.1). Once again note that in quoting diagrammatic results we set $y_{1-5} = y, m_0 = \mu_1 = m, \mu_{2-5} = 0$, so that these are effectively the renormalised values of these couplings. In Table 7, the contributions from Figs. 7(f-k) are now absent while those from Figs. 7(l-r) change sign. Similarly, in Table 8, the contributions from Figs. 8(e-p) are now absent while those from Figs. 8(q-dd) change sign. In Table 9, the contributions from Figs. 9(f-o) are now absent while those from Figs. 9(p-z) change sign. In Table 10, the contribution from Fig. 10(d) is now absent. In
Table 11, the contributions from Figs. 11(j–o) are now absent while those from Figs. 11(p–v) which contain two factors of $d_{abc}$ acquire an additional factor of $(-\frac{1}{2})$. Again, note that we do not show Figs. 11(e)–(v) explicitly, instead describing them in Appendix A. Figs. 12 and 13 are no longer present, of course, while the result of Fig. 14 is still given by Eq. (A.27). In Table 15, the contribution from Fig. 15(b) is absent while that from 15(c) changes sign, and 15(d) must now also be considered. Likewise in Table 16, the contribution from Fig. 16(b) is absent while that from 16(c) changes sign, and 16(e) must now also be considered. The net divergent contribution from Figs. 15 and 16 is therefore unchanged. The results from Figs. 7–11 and Figs. 14–16 now add to

$$\Gamma_{71\text{PIelim}}^{(1)\text{pole}} = i L g^{2} C^{\mu \nu} m \left[ -\frac{1}{2}(7 + 5 \alpha) N g d_{abc} \partial_{\mu} A_{\nu}^{a} \bar{\phi} \phi \right. \left. + 3(1 - \alpha) g \sqrt{2 N} \partial_{\mu} A_{\nu}^{a} \bar{\phi} \phi - 2(5 + \alpha) g \sqrt{2 N} \partial_{\mu} A_{\nu}^{a} \bar{\phi} \phi \right],$$

$$\Gamma_{81\text{PIelim}}^{(1)\text{pole}} = i L g^{4} C^{\mu \nu} m f^{a b e} A_{\mu}^{a} A_{b}^{b} \left[ \frac{1}{2}(5 + 3 \alpha) N d^{c d e} \bar{\phi} \phi + 2 \alpha \sqrt{2 N} \bar{\phi} \phi \right],$$

$$\Gamma_{91\text{PIelim}}^{(1)\text{pole}} = |C|^2 m L \left\{ \left[ 2 g^{2} g \delta^{a b} \delta^{c d} + \frac{1}{2} N \left( 3 + \alpha \right) + \frac{4}{N} \left( 1 - \frac{g^{2}}{g_{0}^{2}} \right) \right] f^{a c e} f^{b d e} \right\} g^{4} \bar{\phi} \phi \lambda \lambda' \lambda' \lambda - 2 g^{4} d^{a b c} \sqrt{2 N} \bar{\phi} \phi \lambda \lambda' - 8 g^{3} g \bar{\phi} \phi \lambda \lambda',$$

$$\Gamma_{101\text{PIelim}}^{(1)\text{pole}} = \Gamma_{101\text{PI}}^{(1)\text{pole}},$$

$$\Gamma_{111\text{PIelim}}^{(1)\text{pole}} = i C^{\mu \nu} g^{2} L \left( -\frac{1}{2} g \left( 3 + 2 \alpha \right) N f^{a b e} f^{c d e} \partial_{\mu} \bar{\phi} \phi \phi A_{\nu}^{d} \right.$$

$$+ \left. \left[ - \left( \frac{3}{4} + \frac{7}{12} \alpha \right) d^{a b e} d^{c d e} + \left( \frac{5}{2} - \frac{7}{6} \alpha \right) \delta^{a b} \delta^{c d} \right] g^{2} \bar{\phi} \phi \phi \phi A_{\nu}^{d} \right.$$

$$- \frac{1}{2} \left( 7 + 5 \alpha \right) g \sqrt{2 N} d^{a b c} \bar{\phi} \phi \phi \phi \partial_{\mu} A_{\nu}^{c} + \frac{3}{2} \left( 1 - \alpha \right) g^{4} \bar{\phi} \phi \phi \phi \partial_{\mu} A_{\nu}^{a}$$

$$- \frac{1}{2} (5 + \alpha) g \sqrt{2 N} d^{a b c} \bar{\phi} \phi \phi \phi \partial_{\mu} A_{\nu}^{c} - 2(5 + \alpha) g \sqrt{2 N} \partial_{\mu} A_{\nu}^{a} \right),$$

$$\Gamma_{141\text{PIelim}}^{(1)\text{pole}} = \Gamma_{141\text{PI}}^{(1)\text{pole}},$$

$$\Gamma_{151\text{PIelim}}^{(1)\text{pole}} = -3 i L N g^{2} m y f^{a b c} \left( C \psi \right)^{b} \psi^{c},$$

$$\Gamma_{161\text{PIelim}}^{(1)\text{pole}} = -3 i L N g^{2} y^{2} f^{a b e} d^{c d e} \left( C \psi \right)^{a} \psi^{b} \psi^{d},$$

$$\Gamma_{171\text{PIelim}}^{(1)\text{pole}} = \frac{1}{2} i m^{2} y g L C^{\mu \nu} F_{\mu \nu}^{a} \bar{\phi} \phi .$$

(6.2)

respectively. The results in Eq. (5.7) are unchanged, which is a very good check on the calculation.

7. $\mathcal{N} = \frac{1}{2}$ supersymmetric theory with chiral matter in the fundamental representation

We now turn to the case of the $\mathcal{N} = \frac{1}{2}$ supersymmetric theory with chiral matter
in the fundamental representation. As we saw in Ref. [12], in this case renormalisability combined with $\mathcal{N} = \frac{1}{2}$ supersymmetry requires us to consider an $SU(N) \otimes U(1)$ gauge theory. The action (with no superpotential) is given by [12]

$$ S_0 = \int d^4 x \left[ -\frac{1}{4} F^{\mu \nu A} F_{\mu \nu}^A - i \lambda^A \sigma^\mu (D_\mu \lambda)^A + \frac{1}{2} D^A D^A ight. 
- \frac{1}{2} i C^{\mu \nu} d^{ABC} e^{ABC} F_{\mu \nu}^A \overline{\lambda}^B \lambda^C 
+ \frac{1}{8} g^2 |C|^2 d^{abc} (\overline{\lambda}^b \lambda^c) (\overline{\lambda}^d \lambda^e) + \frac{1}{4N} g_0^4 |C|^2 (\overline{\lambda}^a \lambda^b) (\overline{\lambda}^d \lambda^e) 
+ \left\{ \overline{F} F - i \psi \sigma^\mu D_\mu \psi - D_\mu \overline{\phi} D_\mu \phi 
+ \overline{\phi} \hat{D} \phi + i \sqrt{2} (\overline{\phi} \dot{\lambda} \psi - \overline{\psi} \dot{\lambda} \phi) 
+ \sqrt{2} C^{\mu \nu} D_\mu \overline{\phi} \lambda \sigma_{\nu} \psi + i C^{\mu \nu} \overline{\phi} \dot{F}_{\mu \nu} F - \frac{1}{4} |C|^2 \overline{\phi} \dot{\lambda} \phi 
+ \frac{1}{N} g_0^2 |C|^2 (\overline{\lambda}^a \lambda^b) (\overline{\lambda}^d \lambda^e) 
- \gamma_2 C^{\mu \nu} g \left( \sqrt{2} D_\mu \overline{\phi} \lambda^a R^a \overline{\sigma}_{\nu} \psi + \sqrt{2} \overline{\phi} \lambda^a R^a \sigma_{\nu} D_\mu \psi + i \overline{F}_{\mu \nu}^a R^a F \right) 
- \gamma_3 C^{\mu \nu} g_0 \left( \sqrt{2} D_\mu \overline{\phi} \lambda^a 0 \overline{\sigma}_{\nu} \psi + \sqrt{2} \overline{\phi} \lambda^a 0 \sigma_{\nu} D_\mu \psi + i \overline{F}_{\mu \nu}^0 0 F \right) 
+ (\phi \rightarrow \tilde{\phi}, \psi \rightarrow \tilde{\psi}, F \rightarrow \tilde{F}, R^A \rightarrow -(R^A)^*, C^{\mu \nu} \rightarrow -C^{\mu \nu} \right\}.$$ (7.1)

where $\gamma_{1-3}$ are constants, and

$$ D_\mu \phi = \partial_\mu \phi + i \hat{A}_\mu \phi, 
(D_\mu \lambda)^A = \partial_\mu \lambda^A - g f^{ABC} A^B_\mu \lambda^C, 
F_{\mu \nu}^A = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - g f^{ABC} A^B_\mu A^C_\nu,$$ (7.2)

(with similar expressions for $D_\mu \tilde{\phi}$, $D_\mu \psi$, $D_\mu \tilde{\psi}$). Here

$$ \hat{A}_\mu = \hat{A}_\mu^A R^A = g A^0_\mu R^a + g_0 A^0_\mu R^0,$$ (7.3)

with similar definitions for $\tilde{\lambda}$, $\tilde{D}$, $\tilde{F}_{\mu \nu}$. We also have

$$ e^{abc} = g, \quad e^{a0b} = e^{a00} = e^{000} = g_0, \quad e^{0ab} = \frac{g^2}{g_0}. \quad (7.4)$$

We include a multiplet $\{\phi, \psi, F\}$ transforming according to the fundamental representation of $SU(N) \otimes U(1)$ and, to ensure anomaly cancellation, a multiplet $\{\tilde{\phi}, \tilde{\psi}, \tilde{F}\}$ transforming according to its conjugate. The change $C^{\mu \nu} \rightarrow -C^{\mu \nu}$ for the conjugate representation is due to the fact that the anticommutation relations for the conjugate fundamental representation differ by a sign from those for the fundamental representation. The group matrices
$R^A$ for the fundamental representation of $SU(N) \otimes U(1)$ are as defined in Section 4. They satisfy

$$[R^A, R^B] = i f^{ABC} R^C, \quad \{R^A, R^B\} = d^{ABC} R^C,$$

(7.5)

where $d^{ABC}$ is totally symmetric. We note that $d^{ab0} = \sqrt{\frac{2}{N}} \delta^{ab}$, $d^{000} = \sqrt{\frac{2}{N}}$.

It is easy to show that Eq. (7.1) is invariant under

\[
\begin{align*}
\delta A^A_\mu &= - i \bar{\lambda}^A \sigma_\mu \epsilon \\
\delta \lambda^A_\alpha &= i \epsilon_\alpha D^A + (\sigma^{\mu\nu})_\alpha \left[ F^A_{\mu\nu} + \frac{i}{4} C_{\mu\nu} e^{ABC} d^{ABC} \bar{\lambda}^B \lambda^C \right], \quad \delta \bar{\lambda}^A_\dot{\alpha} = 0, \\
\delta D^A &= - \epsilon^{\sigma\mu} D_\mu \bar{\lambda}^A, \\
\delta \phi &= \sqrt{2} \epsilon \psi, \quad \delta \bar{\phi} = 0, \\
\delta \psi^\alpha &= \sqrt{2} \epsilon^\alpha F, \quad \delta \bar{\psi}_{\dot{\alpha}} = - i \sqrt{2} (D_\mu \bar{\phi})(\epsilon^{\sigma\mu})_{\dot{\alpha}}, \\
\delta F &= 0, \quad \delta \bar{F} = - i \sqrt{2} D_\mu \bar{\psi} \sigma^{\mu} \epsilon - 2 i \phi \hat{\lambda} + 2 C^{\mu\nu} D_\mu (\bar{\phi} \epsilon \sigma_{\nu} \lambda). 
\end{align*}
\]

(7.6)

The terms involving $\gamma_1-3$ are separately invariant under $\mathcal{N} = \frac{1}{2}$ supersymmetry and must be included to obtain a renormalisable Lagrangian. In fact only the $\gamma_{1,2}$ terms were required in the case without a superpotential [12]; to ensure renormalisability in the massive case we need to include the $\gamma_3$ terms and also modify $\gamma_2$, with a corresponding change to the bare gaugino $\lambda_B$ (see later).

We now consider the problem of adding superpotential terms to the Lagrangian Eq. (7.1). Again, this problem is most succinctly addressed by returning to the superfield formalism whence the $\mathcal{N} = \frac{1}{2}$ action was originally derived. Denoting fundamental (anti-fundamental) chiral superfield representations as $\Phi$ ($\tilde{\Phi}$) it is simple to see that

$$\int d^2 \theta \tilde{\Phi} \ast \Phi + \int d^2 \bar{\theta} \bar{\Phi} \ast \tilde{\Phi}$$

is gauge invariant, since under a gauge transformation we have

$$\Phi \rightarrow \Omega \ast \Phi, \quad \tilde{\Phi} \rightarrow \bar{\Phi} \ast \Omega^{-1}.$$ 

In the $\mathcal{N} = 1$ case an interaction term is possible for the group $SU(3)$, i.e.

$$\int d^2 \theta \epsilon_{abc} \Phi^a_1 \Phi^b_2 \Phi^c_3 + \text{c.c.}$$

This construction does not, however, generalise to the $\mathcal{N} = \frac{1}{2}$ case, because of the non-anticommutative product.
We may express the superfields in terms of component fields as follows:

\[
\Phi(y, \theta) = \phi(y) + \sqrt{2} \theta \psi(y) + \theta \theta F(y)
\]

\[
\Phi(y, \theta) = \phi(y) + \sqrt{2} \theta \psi(y) + \theta \theta F(y)
\]

\[
\Phi(y, \theta) = \phi(y) + \sqrt{2} \theta \psi(y) + \theta \theta F(y)
\]

\[
\Phi(y, \theta) = \phi(y) + \sqrt{2} \theta \psi(y) + \theta \theta F(y)
\]

where \(\bar{y}^\mu = y^\mu - 2i \theta \sigma^\mu \bar{\theta}\). Note the modification of the \(\theta \theta\)-term [4].

We thus obtain

\[
m \int d^2 \theta \Phi \ast \Phi = m \left[ \phi \bar{F} + F \phi - \psi \bar{\psi} \right]
\]

\[
m \int d^2 \theta \Phi \ast \Phi = m \left[ \phi \bar{F} + F \phi - \psi \bar{\psi} + i C^{\mu \nu} \partial_\mu \phi A_\nu \bar{\phi} - \frac{1}{4} C^{\mu \nu} \phi A_\mu A_\nu \bar{\phi} \right]
\]

In fact, the most general mass term is in components

\[
S_{\text{mass}} = m \int d^4 x \left[ (\phi \bar{F} + \phi \bar{\psi}) + \text{h.c.} + i C^{\mu \nu} \phi A_\mu A_\nu \bar{\phi} \right]
\]

\[
S_{\text{mass}} = m \int d^4 x \left[ (\phi \bar{F} + \phi \bar{\psi}) + \text{h.c.} + i C^{\mu \nu} \phi A_\mu A_\nu \bar{\phi} \right]
\]

As in the adjoint case, the coefficient of the final term in Eq. (7.9) is arbitrary since it is separately \(N = \frac{1}{2}\) invariant; we make a particular choice for similar reasons, as explained after Eq. (B.6) in Appendix B. This final term can also be expressed in superfields but in a more unwieldy form.

The one-loop one-particle-irreducible (1PI) graphs contributing to the new terms (those containing \(C\)) in the absence of a superpotential were given in Ref. [12]; the new diagrams in the presence of the mass terms are depicted in Figs. 7–9. The divergent contributions from these diagrams are listed in Appendix B.

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8. Renormalisation of the action in the fundamental case

The renormalisations of $\lambda, F$ and $\overline{F}$, which are non-linear as in the adjoint case, will be given later. (Note that $F$ is unrenormalised in the absence of Yukawa superpotential terms.) The renormalisations of the remaining fields and couplings are linear as usual and given by

$$
\begin{align*}
\overline{\lambda}_B &= Z_{\overline{\lambda}}^0 \overline{\lambda}, & \overline{\lambda}_B^0 &= Z_{\overline{\lambda}^0}^0 \overline{\lambda}^0, & A_{\mu B}^a &= Z_{A^a}^\mu A^a_{\mu}, & A_{\mu B}^0 &= Z_{A^0}^\mu A^0_{\mu}, \\
D_{\mu B}^a &= Z_{D^a}^\mu D^a, & D_{\mu B}^0 &= Z_{D^0}^\mu D^0, \\
\phi_B &= Z_{\phi}^2 \phi, & \psi_B &= Z_{\psi}^2 \psi, & \overline{\phi}_B &= Z_{\overline{\phi}}^2 \overline{\phi}, & \overline{\psi}_B &= Z_{\overline{\psi}}^2 \overline{\psi}, \\
g_B &= Z_g^2 g, & g_{0 B} &= Z_{g^0}^2 g^0, & m_B &= Z_m^2 m, \\
\gamma_{1-3 B} &= \tilde{Z}_{1-3}, & C_{\mu\nu}^B &= Z_{C}^{\mu\nu} C^{\mu\nu}, & |C|_B^2 &= Z_{|C|^2} |C|^2,
\end{align*}
$$

(8.1)

with similar expressions for $\tilde{\phi}_B, \overline{\psi}_B$ etc. In Eq. (8.1), $\tilde{Z}_{1-3}$ are divergent contributions, in other words we have set the renormalised couplings $\gamma_{1-3}$ to zero for simplicity. The other renormalisation constants start with tree-level values of 1. As we mentioned before, the renormalisation constants for the fields and for the gauge couplings $g, g_0$ are the same as in the ordinary $\mathcal{N} = 1$ supersymmetric theory and are therefore given up to one loop by [17]:

$$
\begin{align*}
Z_\lambda &= 1 - g^2 L(2\alpha N + 2), \\
Z_A &= 1 + g^2 L[(3 - \alpha)N - 2] \\
Z_D &= 1 - 2g^2 L, \\
Z_g &= 1 + g^2 L(1 - 3N), \\
Z_{\phi} &= 1 + 2(1 - \alpha)L\hat{C}_2, \\
Z_{\psi} &= 1 - 2(1 + \alpha)L\hat{C}_2, \\
Z_m &= Z_{\phi}^{-1}, \\
Z_{\Phi} &= 1 + 4L\hat{C}_2,
\end{align*}
$$

(8.2)

where

$$
\hat{C}_2 = g^2 R^a R^a + g^2 R_0^a R_0^a = \frac{1}{2} \left( N g^2 + \frac{1}{N} \Delta \right)
$$

(8.3)

with

$$
\Delta = g_0^2 - g^2.
$$

(8.4)

(For the gauge multiplet, the renormalisation constants given in Eq. (8.2) are those corresponding to the $SU(N)$ sector of the $U(N)$ theory; those for the $U(1)$ sector, namely
$Z_{\Phi}$, $Z_{A^0}$, $Z_{D^0}$ and $Z_{g_0}$, are given by omitting the terms in $N$ and replacing $g$ by $g_0$.) In Eq. (8.2), $Z_\Phi$ is the renormalisation constant for the chiral superfield $\Phi$ so that the result for $m_B$ is the consequence of the non-renormalisation theorem. For later convenience we write (denoting for instance the $n$-loop contribution to $\tilde{Z}_1$ by $\tilde{Z}_1^{(n)}$)

$$\tilde{Z}_1^{(1)} = \tilde{z}_1 L$$

with similar expressions for $\tilde{Z}_{2,3}$. The renormalisation of $\lambda^a$ is given by

$$\lambda_B^a = \frac{2}{\lambda} \lambda^a - \frac{1}{2} NLg^3C^{\mu\nu}d^{abc}\sigma_\mu \bar{X}_a^b A_\nu^b - NLg^2g_0C^{\mu\nu}d^{ab0}\sigma_\mu \bar{X}_a^0 A_\nu^b$$

$$+ i\sqrt{2}Lg\rho_1[\bar{\phi}R^a(C\psi) + (C\bar{\psi})R^a\bar{\phi}],$$

$$\lambda_{B}^{0} = i\frac{Z_{\chi}\lambda^{0}}{2} + i\sqrt{2}Lg\rho_2[\bar{\phi}R^0(C\psi) + (C\bar{\psi})R^0\bar{\phi}],$$

where $(C\psi)^a = C^{a\beta}\bar{\psi}^\beta$. Here $\rho_{1,2}$ are divergent parameters to be defined later. Note that the renormalisation of $\lambda^a$ required in the case of the fundamental representation is different from that required in the adjoint case. The replacement of $\lambda$ by $\lambda_B$ produces a change in the action given (to first order) by

$$S_0(\lambda_B) - S_0(\lambda) = L \int d^4x \left\{ \rho_1 \sqrt{2} g C^{\mu\nu}(\bar{\phi}\lambda^a R^a\sigma_\nu D_\mu \psi + D_\mu \bar{\phi}\lambda^a R^a\sigma_\nu \psi) + \rho_2 \sqrt{2} g_0 C^{\mu\nu}(\bar{\phi}\lambda^0 R^0\sigma_\nu D_\mu \psi + D_\mu \bar{\phi}\lambda^0 R^0\sigma_\nu \psi) + (\phi \rightarrow \tilde{\phi}, \psi \rightarrow \tilde{\psi}, R^a \rightarrow -(R^a)^*, C^{\mu\nu} \rightarrow -C^{\mu\nu} + \ldots) \right\},$$

where the ellipsis indicates the terms not involving $\rho_1, \rho_2$ (which were given previously in Ref. [12]).

The final term in Eq. (7.9) may be decomposed into four terms each of which are separately gauge and $\mathcal{N} = \frac{1}{2}$ invariant and hence can (and do) renormalise separately. Consequently, in order to consider the renormalisation of the theory we need to replace Eq. (7.9) by

$$S_{\text{mass}} = \int d^4x \left\{ m(\phi \tilde{F} + F\tilde{\phi} - \psi\tilde{\psi}) + \text{h.c.} + imC^{\mu\nu}\bar{\phi}\tilde{F}_{\mu\nu}\tilde{\phi} \right.$$

$$- \frac{1}{4} |C|^{2}\tilde{\phi} \left( \frac{1}{2} \mu_1 g^2 d^{abc}R^a \bar{X}^c \bar{X}^b + \frac{1}{2N} \mu_2 g^2 \bar{X} \bar{X} \right)$$

$$+ 2\mu_3 g g_0 R^a \bar{X}^a \bar{X}^{0} + \mu_4 g_0^2 R^0 \bar{X}^0 \bar{X}^{0} \right\},$$

where each of $\mu_{1-4}$ will renormalise separately. However, for simplicity when we quote results for Feynman diagrams, we use the values of the coefficients as implied by Eq. (7.9),...
i.e. $\mu_{1-4} = m$; so that we are setting the renormalised values of $\mu_{1-4}$ to be $m$. In contrast to the adjoint case, where it was impossible to maintain $\mathcal{N} = \frac{1}{2}$ invariance for the Yukawa terms at the quantum level, in the fundamental case where only mass terms are allowed we shall find that $\mathcal{N} = \frac{1}{2}$ invariance can be preserved.

The redefinitions of $F$ and $\overline{F}$ found in Ref. \[15\] need to be modified in the presence of mass terms. As in the adjoint case this is readily done following the arguments of Ref. \[15\]. However, note that due to the afore-mentioned change in sign for the $\overline{\phi \lambda \lambda F}$ term, the result for Fig. 8 in Ref. \[15\] is modified to

$$
\Gamma_{81\text{PI}}^{(1)\text{pole}} = L|C|^2\overline{\phi}\left\{g^2 \left[\frac{1}{8}(13 - 2\alpha)Ng^2 - 2\hat{C}_2\right] \overline{\lambda}^a \lambda^b d^{abc} R^c + g g_0 \left[\frac{1}{4}(13 - \alpha)Ng^2 - 8\hat{C}_2\right] \overline{\lambda}^a \lambda^b R^0 R^a - \left[2\hat{C}_2 + \frac{1}{4}\alpha Ng^2\right] g^2 d^{abc} R^0 \overline{\lambda}^a \lambda^b - 4 g_0^2 \hat{C}_2 \overline{\lambda}^a \lambda^b R^0 R^a\right\} F.
$$

(8.9)

We find

$$
\overline{F}_B = \overline{F} + (\alpha + 3)mL\hat{C}_2 \overline{\phi} + L\left\{\left[\left(7Ng^2 + 2(1 + \alpha)\hat{C}_2 + 2\tilde{z}_2\right) g \partial_\mu A^a_\nu - \left(\frac{16}{8} Ng^2 + (1 + \alpha)\hat{C}_2 + \tilde{z}_2\right) g^2 f^{abc} A^b_\mu A^c_\nu \right] i C^{\mu\nu} \overline{\phi} R^a

- 2 \left((1 + \alpha)\hat{C}_2 + \tilde{z}_3\right) g_0 \partial_\mu A^a_\nu i C^{\mu\nu} \overline{\phi} \tilde{R}^0

+ \frac{1}{8}|C|^2 \left[\left(-19Ng^2 + (17 - \alpha)\hat{C}_2\right) g^2 d^{abc} \overline{\phi} \tilde{R}^c \overline{\lambda}^a \lambda^b - 4 \left(-16Ng^2 + (17 - \alpha)\hat{C}_2\right) g g_0 \overline{\phi} \lambda^a R^0 R^a

+ 2(17 - \alpha)\hat{C}_2 g^2 \overline{\phi} \lambda^a R^0 R^a + \left(-6Ng^2 + (17 - \alpha)\hat{C}_2\right) g^2 d^{abc} \overline{\phi} R^0 R^a \right]\right\},
$$

(8.10)

$$
F_B = F + (\alpha + 3)mL\hat{C}_2 \overline{\phi}.
$$

Again, note that these are different from the corresponding results in the adjoint representation (Eq. (5.3)). We now find that with

$$
Z_C^{(1)} = Z_{|C|^2}^{(1)} = 0, \quad \tilde{z}_1 = -3Ng^2, \quad \tilde{z}_2 = 8(2\hat{C}_2 - Ng^2),
$$

$$
\tilde{z}_3 = 4 \left[4 - 2 \frac{\Delta}{g_0^2}\right] \hat{C}_2 - Ng^2, \quad \rho_1 = 1 + \tilde{z}_2, \quad \rho_2 = \tilde{z}_3,
$$

$$
Z_{\mu_1} = 1 + \left[44 + 64\frac{g^2}{g_0^2}\right] \hat{C}_2 - \left(28 + 32\frac{g^2}{g_0^2}\right) Ng^2\right] L,
$$

$$
Z_{\mu_2} = 1 + \left[44 + 128\frac{g^2}{g_0^2}\right] \hat{C}_2 - \left(28 + 32\frac{g^2}{g_0^2}\right) Ng^2\right] L,
$$

$$
Z_{\mu_3} = 1 + \left(44\hat{C}_2 - 30N g^2\right)L,
$$

$$
Z_{\mu_4} = 1 + 44\hat{C}_2 L,
$$

(8.11)
the one-loop effective action is finite. In fact the massless theory is finite for arbitrary choices of \( \tilde{z}_2, \tilde{z}_3 \); the particular values chosen are necessary to ensure renormalisability of the mass terms in Eq. (8.8), in analogy to the adjoint case. As explained earlier, by renormalisability we mean that the massive theory is rendered finite by adding counterterms to the fields and parameters in Eq. (8.8), without the need for further parameters. The \( \mathcal{N} = \frac{1}{2} \) invariance is thereby retained. Different choices for \( \tilde{z}_2, \tilde{z}_3 \) would require introducing additional parameters into Eq. (8.8), specifically a separate coupling (other than \( m \)) for the \( C^{\mu \nu} \overline{\phi} \tilde{F}_{\mu \nu} \phi \) term, spoiling the \( \mathcal{N} = \frac{1}{2} \) invariance.

9. The eliminated formalism in the fundamental case

Once again it is a useful check to perform the calculation in the eliminated formalism. In the eliminated case Eq. (8.8) is replaced by

\[
\tilde{S}_{\text{mass}} = \int d^4x \left\{ -m^2 (\phi \overline{\phi} + \phi \overline{\phi}) - m (\psi \overline{\psi} + \overline{\psi} \psi) - i m C^{\mu \nu} \overline{\phi} \left[ (1 - 2 \gamma_2) g F_{\mu \nu}^a R^a + (1 - 2 \gamma_3) g_0 F_{\mu \nu}^0 R^0 \right] \overline{\phi} - \frac{1}{4} |C|^2 \overline{\phi} \left( \frac{1}{2} (\mu_1 - 2m) g^2 d^{abc} R^c \overline{\lambda} \lambda + \frac{1}{2N} (\mu_2 - 2m) g^2 \overline{\lambda} \lambda \right) + 2 (\mu_3 - 2m) g g_0 R^a R^0 \overline{\lambda} \lambda + (\mu_4 - 2m) g_0^2 R^0 R^0 \overline{\lambda} \lambda) \right\}. \tag{9.1}
\]

while we simply strike out the terms involving \( F, \overline{F} \) in Eq. (7.1). In Table 17, the contributions from Figs. 7(f-k) are now absent while those from Figs. 7(l-r) change sign. Similarly, in Table 18, the contributions from Figs. 8(e-p) are now absent while those from Figs. 8(q-dd) change sign. In Table 19, the contributions from Figs. 9(f-o) are now absent while those from Figs. 9(p-z) change sign. The results from Figs. 7, 8 and 9 now add to

\[
\begin{align*}
\Gamma^{(1)\text{pole}}_{B71\text{Pelim}} &= i L g C^{\mu \nu} \partial_\mu A_\nu^a \overline{\phi} R^A \left\{ -(68 + 4 \alpha) + 32 \frac{A_0}{g_0} \delta A^0 \right\} \hat{C}_2 \\
&+ \left\{ (29 - \alpha) c^A + 16 \delta A^0 \right\} N g^2 \overline{\phi}, \\
\Gamma^{(1)\text{pole}}_{B81\text{Pelim}} &= i L N g^2 C^{\mu \nu} f^{abc} A_\mu^a A_\nu^b \overline{\phi} \left[ 2 (17 + \alpha) \hat{C}_2 - (13 - \alpha) N g^2 \right] R^c \overline{\phi}, \\
\Gamma^{(1)\text{pole}}_{B91\text{Pelim}} &= -\overline{\phi} \left\{ \left[ \frac{1}{4} (25 + \alpha) + 8 \frac{g^2}{g_0} \right] \hat{C}_2 - \left[ \frac{1}{4} (11 - \alpha) + 4 \frac{g^2}{g_0} \right] N g^2 \right\} g^2 d^{abc} R^c \overline{\lambda} \lambda \\
&+ \left\{ \left[ \frac{1}{4} (25 + \alpha) + 16 \frac{g^2}{g_0} \right] \hat{C}_2 - \left[ \frac{1}{4} (11 - \alpha) + 4 \frac{g^2}{g_0} \right] N g^2 \right\} \frac{1}{N} g^2 \overline{\lambda} \lambda \lambda \\
&+ \left\{ (25 + \alpha) \hat{C}_2 - \frac{1}{2} (27 - \alpha) g^2 N \right\} g g_0 R^a R^0 \overline{\lambda} \lambda \\
&+ \frac{1}{2} (25 + \alpha) g_0^2 \hat{C}_2 R^0 R^0 \overline{\lambda} \lambda \right\} \overline{\phi}, \tag{9.2}
\end{align*}
\]

where \( c^A = 1 - \delta A^0 \). The results in Eq. (8.11) are again unchanged, giving a convincing check on the calculation.
10. Conclusions

We have repeated our earlier one-loop analysis of $\mathcal{N} = \frac{1}{2}$ supersymmetry for the case of chiral matter in the adjoint representation. We have constructed an $\mathcal{N} = \frac{1}{2}$ invariant set of mass terms and an $\mathcal{N} = \frac{1}{2}$ invariant set of Yukawa terms for this case. The $\mathcal{N} = \frac{1}{2}$ invariance of the Yukawa terms requires that the chiral matter be in the adjoint representation of $U(N)$ rather than $SU(N)$ at the classical level. However, once we consider quantum corrections, the $U(1)$ chiral fields will renormalise differently from the $SU(N)$ fields and so at the quantum level we are obliged to consider $SU(N) \otimes U(1)$ rather than $U(N)$. On the other hand, the $\mathcal{N} = \frac{1}{2}$ transformations mix superpotential terms with different kinds of field ($SU(N)$ or $U(1)$) and so it is clear that the $\mathcal{N} = \frac{1}{2}$ invariance of the Yukawa terms cannot be preserved at the quantum level. This is because separate couplings must be introduced for most of the $C$-dependent superpotential terms. The only remaining vestige of $\mathcal{N} = \frac{1}{2}$ supersymmetry is that since the $g_0 C_{\mu \nu} F_{\mu \nu}^0 \sqrt{\frac{2}{N}} d^{abc} \phi_0 \phi_0 \phi_0$ and $\frac{1}{N} g_0 C_{\mu \nu}^0 \phi_0 \phi_0 \phi_0$ terms only mix with the $d^{abc} \phi_0 \phi_0 \phi_0$ or $d^{abc} \phi_0 \phi_0 \phi_0$ fields respectively, they can be assigned the coupling $y$ or $y_1$ respectively which are already in the $\mathcal{N} = 1$ part of the theory. In contrast, we have shown that the $\mathcal{N} = \frac{1}{2}$ invariance of the mass terms is preserved at the one-loop level. However the invariance is assured by making a particular choice of the parameters $\kappa_2, \kappa_3$ (in Eq. (3.1)), as determined by Eq. (5.6). This also implies (through Eq. (3.8)) a particular choice of renormalisation for the gaugino $\lambda$, parametrised by $\tau_5$ (in Eq. (3.4)).

We have also constructed a set of mass terms for the $\mathcal{N} = \frac{1}{2}$ supersymmetric theory with chiral matter in the fundamental representation, and we have shown that the one-loop renormalisation presents similar features to the adjoint case. The $\mathcal{N} = \frac{1}{2}$ invariance is preserved at one loop since the Yukawa terms which presented difficulties in the adjoint case are absent, leaving only the mass terms. Once again the invariance is assured by making a particular parameter choice, in this case of the parameters $\gamma_2, \gamma_3$ (in Eq. (7.1)) combined with a particular choice of renormalisations for the gaugino $\lambda$, parametrised by $\rho_1, \rho_2$ (in Eq. (8.1)). These choices were listed in Eq. (8.11).

The necessity for the above choices in both the fundamental and adjoint cases seems somewhat counterintuitive as these renormalisations are all present in the theory without superpotential and yet there appeared to be nothing in the theory without superpotential to enforce these choices. The fact that the same feature appears in both cases is at least an indication that this really is a generic property of the theory. However it would
be reassuring if some independent confirmation could be found for these particular values. Presumably the necessity for the non-linear renormalisations we are compelled to make lies in our use of a non-supersymmetric gauge (the obvious choice when working in components, of course). So the answer to this puzzle might lie in a close scrutiny of the gauge-invariance Ward identities. Of course a calculation in superspace would also be illuminating. It is always tempting to investigate whether the behaviour at one loop persists to higher orders but the proliferation of diagrams in this case would almost certainly be prohibitive.

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Appendix A. Results for one-loop diagrams

In this Appendix we list the divergent contributions from the one-loop diagrams.

The contributions from the graphs shown in Fig. 1 are of the form

$$\sqrt{2} N g^2 g_B L C^{\mu \nu} d^{ABC} \left( \partial_\mu \bar{\phi}^A X_1^{ABC} \mathcal{X}^B \sigma_\nu \psi^C + \bar{\phi}^A Y_1^{ABC} \mathcal{X}^B \sigma_\nu \partial_\mu \psi^C \right)$$  \hspace{1cm} (A.1)

where \(X_1^{ABC}\) and \(Y_1^{ABC}\) consist of a number \(X_1, Y_1\) multiplying a tensor structure formed of a product of terms like \(c^A\) or \(d^A\), where \(c^A = 1 - \delta^{A0}\), \(d^A = 1 + \delta^{A0}\). The \(X_1, Y_1\) and the tensor structures are given separately in Table 1. (The contributions from Figs. 2–4, 7, 8 also involve tensors \(X_i^{ABC}\), \(Y_i^{ABC}\) etc (for Fig. \(i\)) which can be decomposed similarly and will be similarly presented.)

| Fig. | \(X_1\) | \(Y_1\) | Tensor |
|------|--------|--------|--------|
| 1a   | \(\frac{3}{2}\) | \(-\alpha\) | \(c^A c^B d^C\) |
| 1b   | \(\alpha\) | \(\alpha\) | \(c^A c^B d^C\) |
| 1c   | \(\alpha\) | 0       | \(d^A c^B c^C\) |
| 1d   | 1       | \(-1\)  | \(c^A c^B d^C\) |
| 1e   | 1       | 0       | \(d^A c^B c^C\) |
| 1f   | \(-\frac{1}{2}(1 - 2\alpha)\) | 0       | \(c^A d^B c^C\) |
| 1g   | 1       | 0       | \(c^A d^B c^C\) |
| 1h   | 1       | \(-1\)  | \(c^A d^B c^C\) |
| 1i   | 0       | 1       | \(c^A d^B c^C\) |
| 1j   | \(-3\)  | 0       | \(c^A\) |

*Table 1: Contributions from Fig. 1*
The sum of the contributions from Table 1 can be written in the form

$$\Gamma_{11\text{PI}}^{(1)\text{pole}} = Ng^2\sqrt{2}LC_{\mu\nu}^{\lambda\sigma} \left[ (2 + 3\alpha)gd^{abc}\partial_\mu \overline{\phi^\lambda} \bar{\sigma}_\nu \psi^c - gd^{abc}\overline{\phi^\lambda} \bar{\sigma}_\nu \partial_\mu \psi^c \right. \right.$$

$$+ 2(1 + \alpha)gd^{ab0}\partial_\mu \overline{\phi^\lambda} \bar{\sigma}_\nu \psi^0 - 2gd^{ab0}\overline{\phi^\lambda} \bar{\sigma}_\nu \partial_\mu \psi^0$$

$$+ 2\alpha g_0d^{ab0}\partial_\mu \overline{\phi^\lambda} \bar{\sigma}_\nu \psi^b$$

$$\left. + 2(1 + \alpha)gd^{ab0}\partial_\mu \overline{\phi^\lambda} \bar{\sigma}_\nu \psi^b \right]$$

(A.2)

The contributions from the graphs shown in Fig. 2 are of the form

$$\sqrt{2}g^3g_C NLC_{\mu\nu} A_\mu^A B_\nu^C \bar{\sigma}_\nu \psi^D \left( X_{2ABCD} f_{BAE} d^{CDE} \right.$$  

$$+ Y_{2ABCD} f_{DAE} d^{CBE} + Z_{2ABCD} f_{BDE} d^{CAE} \right)$$

(A.3)

where $g_c \equiv g$. The $X_2$, $Y_2$, $Z_2$ and tensor products in the decomposition of $X_{2ABCD}$, $Y_{2ABCD}$ and $Z_{2ABCD}$ (as described earlier) are shown in Table 2:
| Fig. | $X_2$   | $Y_2$   | $Z_2$   | Tensor                        |
|------|---------|---------|---------|-------------------------------|
| 2a   | $\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $c^A c^B c^C d^D$          |
| 2b   | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $c^A c^B c^C d^D$          |
| 2c   | 1       | -1      | 1       | $c^A c^B c^C d^D$          |
| 2d   | -1      | -1      | -1      | $c^A c^B c^C d^D$          |
| 2e   | 1       | 0       | 0       | $c^A c^B c^C d^D$          |
| 2f   | $-\frac{1}{4}(1 - \alpha)$ | $\frac{1}{4}(1 - \alpha)$ | $-\frac{1}{4}(1 - \alpha)$ | $c^A c^B c^C d^D$          |
| 2g   | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $c^A c^B c^C d^D$          |
| 2h   | $\frac{1}{2} \alpha$ | 0       | 0       | $c^A c^B$                  |
| 2i   | $\frac{3}{4} \alpha$ | 0       | 0       | $c^A c^B$                  |
| 2j   | $-\frac{3}{4}(3 + \alpha)$ | 0       | 0       | $c^A c^B$                  |
| 2k   | $\frac{1}{8} \alpha$ | $-\frac{1}{8} \alpha$ | $\frac{1}{8} \alpha$ | $c^A c^B c^C d^D$          |
| 2l   | $-\frac{3}{8}(1 - \alpha)$ | $-\frac{1}{8}(1 - \alpha)$ | $\frac{1}{8}(1 - \alpha)$ | $c^A c^B c^C d^D$          |
| 2m   | $\frac{1}{2} \alpha$ | $\frac{1}{2} \alpha$ | $-\frac{1}{2} \alpha$ | $c^A c^B c^C d^D$          |
| 2n   | $\frac{1}{2} \alpha$ | $-\frac{1}{2} \alpha$ | $\frac{1}{2} \alpha$ | $c^A c^B c^C d^D$          |
| 2o   | $-\frac{1}{4} \alpha$ | $-\frac{1}{4} \alpha$ | $\frac{1}{4} \alpha$ | $c^A c^B c^C d^D$          |
| 2p   | $\frac{3}{8}(3 + \alpha)$ | $-\frac{1}{8}(3 + \alpha)$ | $\frac{1}{8}(3 + \alpha)$ | $c^A c^B c^C d^D$          |
| 2q   | $\alpha$ | 0       | 0       | $c^A c^B c^C d^D$          |
| 2r   | $-\frac{1}{4} \alpha$ | $\frac{1}{4} \alpha$ | $-\frac{1}{4} \alpha$ | $c^A c^B c^C d^D$          |
| 2s   | $\frac{3}{4}(1 + \alpha)$ | $\frac{3}{4}(1 + \alpha)$ | $-\frac{3}{4}(1 + \alpha)$ | $c^A c^B c^C d^D$          |
| 2t   | $-\frac{1}{2} \alpha$ | $-\frac{1}{2} \alpha$ | $-\frac{1}{2} \alpha$ | $c^A c^B c^C d^D$          |
| 2u   | $\frac{1}{2} \alpha$ | $\frac{1}{2} \alpha$ | $\frac{1}{2} \alpha$ | $c^A c^B c^C d^D$          |
| 2v   | $-\frac{3}{8} \alpha$ | $-\frac{3}{8} \alpha$ | $\frac{3}{8} \alpha$ | $c^A c^B c^C d^D$          |
| 2w   | $-\frac{1}{4}(3 + \alpha)$ | $-\frac{1}{4}(3 + \alpha)$ | $-\frac{1}{4}(3 + \alpha)$ | $c^A c^B c^C d^D$          |
| 2x   | $\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $c^A c^B c^C d^D$          |
| 2y   | 1       | -1      | 1       | $c^A c^B c^C d^D$          |

Table 2: Contributions from Fig. 2
The sum of the contributions from Table 2 can be written in the form

$$\Gamma_{21\text{PI}}^{(1)\text{pole}} = \sqrt{2}g^3LC_{\mu\nu}A_{\mu} \left[ \frac{7}{2}(1 + \alpha)f^{bae}d^{cde} - f^{dae}d^{cbe} + \frac{1}{2}f^{bde}d^{cae} \right] Ng\phi^{\lambda} \overline{\sigma}_{\nu}\psi^{d}$$

$$- \frac{1}{2}(1 + 5\alpha)\sqrt{2}Ng f^{abc}\phi^{b} \overline{\phi}^{c} \overline{\sigma}_{\nu}\psi^{0} - \frac{1}{2}(7 + 5\alpha)\sqrt{2}Ng f^{abc}\phi^{b} \overline{\phi}^{c} \overline{\sigma}_{\nu}\psi^{0} \right]$$

(A.4)

The contributions from Fig. 3 are of the form

$$ig^3NL_{c\nu}(\partial_{\mu}A_{\nu}^{A}B X_{3}^{ABC} F^{C} + A_{\nu}^{A} \partial_{\mu}B Y_{3}^{ABC} F^{C})d^{ABC}$$

(A.5)

where the $X_3$, $Y_3$ and tensor products in the decomposition of $X_3^{ABC}$ and $Y_3^{ABC}$ are given in Table 3:

| Fig. | $X_3$ | $Y_3$ | Tensor |
|------|-------|-------|--------|
| 3a   | 0     | 3     | $c^{A}d^{B}d^{C}$ |
| 3b   | 0     | -2    | $c^{A}d^{B}d^{C}$ |
| 3c   | 1     | 1     | $c^{A}d^{B}d^{C}$ |
| 3d   | -(5 + \alpha) | 0 | $c^{A}$ |
| 3e   | 2\alpha | -2 | $c^{A}d^{B}d^{C}$ |

Table 3: Contributions from Fig. 3

The contributions from Table 3 add to

$$\Gamma_{31\text{PI}}^{(1)\text{pole}} = iNg^3LC_{\mu\nu}A_{\mu} \left[ -(4 - \alpha)d^{abc}\phi^{b} \partial_{\mu}A_{\nu}^{a}F^{c} \right.$$  

$$- 3(1 - \alpha)d^{abc}\phi^{b} \partial_{\mu}A_{\nu}^{a}F^{b} - (5 + \alpha)d^{abc}\phi^{b} \partial_{\mu}A_{\nu}^{a}F^{b} \right].$$

(A.6)

The contributions from Fig. 4 are of the form

$$ig^4NL_{c\nu}A_{\mu}^{A}A_{\nu}^{B}(X_{4}^{ABCD} f^{ABE}d^{CDE} + Y_{4}^{ABCD} f^{ACE}d^{BDE})\phi^{C} F^{D}$$

(A.7)
where the $X_4$ and $Y_4$ and tensor products in the usual decomposition are given in Table 4:

| Fig. | $X_4$ | $Y_4$ | Tensor |
|------|-------|-------|--------|
| 4a   | $-\frac{3}{4}\alpha$ | 0     | $e^A e^B e^C d^D$ |
| 4b   | $\frac{1}{2}\alpha$  | $\alpha$ | $e^A e^B e^C d^D$ |
| 4c   | $-\frac{1}{4}\alpha$ | $-\alpha$ | $e^A e^B e^C d^D$ |
| 4d   | 0     | 0     | $e^A e^B e^C d^D$ |
| 4e   | $\frac{1}{4}(2 + \alpha)$ | 2 + $\alpha$ | $e^A e^B e^C d^D$ |
| 4f   | $-\frac{1}{2}$ | 1     | $e^A e^B e^C d^D$ |
| 4g   | $-\frac{5}{2}\alpha$ | 0     | $e^A e^B$ |
| 4h   | $\frac{3}{4}(1 + \alpha)$ | 0     | $e^A e^B$ |
| 4i   | $-\frac{1}{4}(3 + \alpha)$ | $-(3 + \alpha)$ | $e^A e^B e^C d^D$ |
| 4j   | $\frac{1}{2}\alpha$ | 0     | $e^A e^B e^C d^D$ |
| 4k   | $-\frac{3}{4}\alpha$ | 0     | $e^A e^B e^C d^D$ |
| 4l   | 0     | 0     | $e^A e^B e^C d^D$ |

Table 4: Contributions from Fig. 4

The contributions from Table 4 add to

$$
\Gamma_{411\Pi}^{(1)\text{pole}} = i g^4 L C^{\mu \nu} A^a_\mu A^b_\nu \left( \frac{1}{7}(3 - 4\alpha)N f^{abc} d^{cde} \phi^a \right) F^d \\
- 2\alpha \sqrt{2N} f^{abc} \phi^a \phi^b F^d + \frac{3}{2} \sqrt{2N} f^{abc} \phi^a \phi^b F^c \right). 
$$

(A.8)

The contributions from Fig. 5 are of the form

$$
X_5^{ABCD} |C|^2 g^2 g C D L \phi^A \lambda^C \lambda^D F^B
$$

(A.9)

where $X_5^{ABCD}$ is given in Table 5. In Table 5 we have introduced the notation $(\tilde{D}^A)^{BC} = d^{ABC}$. Using results from Appendix D, the contributions from Table 5 add to

$$
\Gamma_{511\Pi}^{(1)\text{pole}} = g^4 L |C|^2 \left[ -\frac{1}{2}(3 + \alpha)N f^{ace} f^{bde} + \frac{11}{8} N d^{abe} d^{cde} \\
- \frac{1}{2} \delta^{ab} \delta^{cd} - \frac{1}{2} \delta^{ac} \delta^{bd} \right] \phi^a \phi^b \phi^c \phi^d F^b \\
+ d^{abc} g^3 L |C|^2 \sqrt{2N} \left[ g_0 \phi^a \lambda^b \lambda^c F^c + 3 g_0 \phi^b \lambda^a \lambda^c F^c + 2 g_0 \phi^c \lambda^a \lambda^b F^c \right] \\
+ 4 g^3 g_0 L |C|^2 \left( \phi^a \lambda^b F^a + 2 \phi^b \lambda^a F^a \right) .
$$

(A.10)
The divergent contributions to the effective action from the graphs in Fig. 6 are of the form

$$i L N g^3 X^6 \alpha \beta \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi \phi \psi \psi \phi
The divergent contributions to the effective action from the graphs in Fig. 7 are of the form

\[ \text{im} L Ng^{2} g_{A} X_{7}^{ABC} C^{\mu\nu} d^{ABC} \partial_{\mu} A_{\nu}^{A} \phi^{B} \phi^{C} \]  

(A.13)

where the contributions from the individual graphs to \( X_{7} \) and the associated tensors in the usual decomposition are given in Table 7:

| Fig. | \( X_{7} \) | Tensor |
|------|-------------|--------|
| 7a   | 2           | \( c^{A} c^{B} d^{C} \) |
| 7b   | -1          | \( c^{A} c^{B} d^{C} \) |
| 7c   | -1          | \( c^{A} c^{B} d^{C} \) |
| 7d   | 0           |        |
| 7e   | -4          | \( d^{A} c^{B} c^{C} \) |
| 7f   | -2\( \alpha \) | \( d^{A} c^{B} c^{C} \) |
| 7g   | -2          | \( d^{A} c^{B} c^{C} \) |
| 7h   | -\( \frac{1}{2} \) | \( c^{A} c^{B} d^{C} \) |
| 7i   | -1          | \( c^{A} c^{B} d^{C} \) |
| 7j   | -\( (1 + 2\alpha) \) | \( c^{A} c^{B} d^{C} \) |
| 7k   | \( \frac{3}{2} \) | \( c^{A} c^{B} d^{C} \) |
| 7l   | -\( \frac{1}{2} (5 + \alpha) \) | \( c^{A} \) |
| 7m   | \( \alpha \) | \( d^{A} c^{B} c^{C} \) |
| 7n   | 1           | \( d^{A} c^{B} c^{C} \) |
| 7o   | \( \frac{1}{2} \) | \( c^{A} c^{B} d^{C} \) |
| 7p   | 1           | \( c^{A} c^{B} d^{C} \) |
| 7q   | 1 + 2\( \alpha \) | \( c^{A} c^{B} d^{C} \) |
| 7r   | -\( \frac{3}{2} \) | \( c^{A} c^{B} d^{C} \) |

Table 7: Contributions from Fig. 7

These results add to

\[ \Gamma_{\text{71P1}}^{(1)\text{pole}} = -\frac{1}{2} (5 + \alpha) i L g^{2} C^{\mu\nu} m \left[ 3 N g d^{abc} \partial_{\mu} A_{\nu}^{a} \phi^{b} \phi^{c} \right. \]
\[ \left. + 2 g \sqrt{2N} \partial_{\mu} A_{\nu}^{a} \phi^{b} \phi^{c} + 4 g_{0} \sqrt{2N} \partial_{\mu} A_{\nu}^{0} \phi^{a} \phi^{b} \right] \]  

(A.14)
(Note that the contributions from Figs. 7(h-k) cancel those from Figs. 7(o-r) respectively.)

The divergent contributions to the effective action from the graphs in Fig. 8 are of the form

\[ imL \ln g^A X^A_{\mu
u} f^{ABE} d^{CDE} A^A_{\mu} A^B_{\nu} \phi^C \phi^D \]

(A.15)

where the contributions from the individual graphs to \( X_8 \) and the associated tensors in the usual decomposition are given in Table 8:

| Fig. | \( X_8 \) | Tensor |
|------|----------------|--------|
| 8a   | -2             | \( c^A c^B c^C c^D \) |
| 8b   | 1              | \( c^A c^B c^C c^D \) |
| 8c   | 1              | \( c^A c^B c^C c^D \) |
| 8d   | 2              | \( c^A c^B c^C c^D \) |
| 8e   | \( \alpha \)   | \( c^A c^B c^C c^D \) |
| 8f   | 1              | \( c^A c^B c^C c^D \) |
| 8g   | \( -\frac{1}{4}(3 + \alpha) \) | \( c^A c^B c^C c^d^D \) |
| 8h   | 0              |        |
| 8i   | 0              |        |
| 8j   | -2             | \( c^A c^B c^C c^d^D \) |
| 8k   | \( \frac{3}{4}\alpha \) | \( c^A c^B c^C c^d^D \) |
| 8l   | \( -\frac{1}{2}\alpha \) | \( c^A c^B c^C c^d^D \) |
| 8m   | \( \frac{3}{4}\alpha \) | \( c^A c^B c^C c^d^D \) |
| 8n   | \( \frac{1}{4}(2 + \alpha) \) | \( c^A c^B c^C c^d^D \) |
| 8o   | 0              |        |
| 8p   | 0              |        |
| 8q   | \( -\frac{3}{4}\alpha \) | \( c^A c^B \) |
| 8r   | \( \frac{3}{4}(1 + \alpha) \) | \( c^A c^B \) |
| 8s   | \( -\frac{1}{2}\alpha \) | \( c^A c^B c^C c^d^D \) |
| 8t   | \( -\frac{1}{2} \) | \( c^A c^B c^C c^d^D \) |

*Table 8: Contributions from Fig. 8*
These results add to

\[
\Gamma_{81PI}^{(1)\text{pole}} = i L g^4 C^{\mu
u} m f^{abe} A^a_{\mu} A^b_{\nu} \left[ \frac{1}{4} (13 + 2\alpha) N d^{cde} d^{ed} + \frac{3}{2} \sqrt{2} N \phi \phi \phi \right] \quad (A.16)
\]

(Note that the contributions from Figs. 8(g-p) cancel those from Figs. 8(u-dd) respectively.)

The contributions from Fig. 9 are of the form

\[
X_{9}^{ABCD} g^2 g_{C} g_{D} m L |C|^{-2} \phi \phi \phi \lambda \lambda \lambda \lambda . \quad (A.17)
\]

The contributions from the individual graphs to \(X_{9}^{ABCD}\) are given in Table 9. The results in Table 9 add to

\[
\Gamma_{91PI}^{(1)\text{pole}} = |C|^2 m L \left\{ \left[ \frac{11}{8} N d^{abe} d^{cde} - \frac{1}{2} \left( 1 - 4 \frac{g^2}{9_0} \right) \delta^{ab} \delta^{cd} - \frac{1}{2} \delta^{ad} \delta^{bc} \right. \right.
\]

\[
+ \frac{N}{4} \left( 1 - \frac{g^2}{9_0} \right) f^{ace} f^{bde} g_{\phi} g_{\phi} \lambda \lambda \lambda \lambda \right\} \quad (A.18)
\]

(Note that the contributions from Figs. 9(h–m) cancel those from Figs. 9(u–z); this is analogous to the situation with Figs. 7 and 8, and is a consequence of our choice of coefficient for the last term in Eq. (1.1).)
| Fig. | Equation |
|------|----------|
| 9a   | $\frac{1}{2} \alpha \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{D}^C \tilde{D}^D]$ |
| 9b   | $\frac{1}{2} \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{D}^C \tilde{D}^D]$ |
| 9c   | $\frac{1}{2} (3 + \alpha) \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{D}^C \tilde{D}^D]$ |
| 9d   | $-\alpha \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{D}^C \tilde{D}^D]$ |
| 9e   | $N d^{ABE} d^{CDE} c^A c^B c^C c^D c^E - 2c^C c^D \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{D}^C \tilde{D}^D] + \frac{4}{N} f^{ACE} f^{BDE}$ |
|      | $+ \frac{2a^2}{g^2} (c^A c^B c^C c^D \delta^{AB} \delta^{CD} - \frac{2}{N} f^{ACE} f^{BDE})$ |
| 9f   | $\frac{1}{2} \alpha \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{F}^C \tilde{F}^D]$ |
| 9g   | $\frac{1}{2} \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{F}^C \tilde{F}^D]$ |
| 9h   | $-\text{tr}[\tilde{F}^A \tilde{F}^B \tilde{F}^C \tilde{F}^D] - \frac{1}{2} N f^{ACE} f^{BDE}$ |
| 9i   | $-\frac{1}{2} \alpha \left( \text{tr}[\tilde{F}^A \tilde{F}^C \tilde{F}^B \tilde{F}^D] - \frac{1}{2} N f^{ACE} f^{BDE} \right)$ |
| 9j   | $-2 \alpha \text{tr}[\tilde{F}^C \tilde{F}^A \tilde{D}^B \tilde{D}^D]$ |
| 9k   | $-4 \text{tr}[\tilde{F}^C \tilde{F}^A \tilde{D}^D \tilde{D}^B]$ |
| 9l   | $\frac{1}{2} \alpha N c^A d^B c^E d^{ABE} d^{CDE}$ |
| 9m   | $2 \text{tr}[\tilde{F}^A \tilde{D}^C \tilde{F}^D \tilde{D}^B]$ |
| 9n   | 0 |
| 9o   | 0 |
| 9p   | $-\frac{1}{2} \alpha N d^{ABE} d^{CDE} d^C c^D c^E$ |
| 9q   | $\frac{1}{2} (1 + \alpha) N d^{ABE} d^{CDE} c^E$ |
| 9r   | $-\frac{1}{2} (3 + \alpha) \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{F}^C \tilde{F}^D]$ |
| 9s   | $-\frac{1}{2} \alpha \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{F}^C \tilde{F}^D]$ |
| 9t   | $-\frac{1}{2} \text{tr}[\tilde{F}^A \tilde{F}^B \tilde{F}^C \tilde{F}^D]$ |
| 9u   | $\text{tr}[\tilde{F}^A \tilde{F}^B \tilde{F}^C \tilde{F}^D] + \frac{1}{2} N f^{ACE} f^{BDE}$ |
| 9v   | $\frac{1}{2} \alpha \left( \text{tr}[\tilde{F}^A \tilde{F}^C \tilde{F}^B \tilde{F}^D] - \frac{1}{2} N f^{ACE} f^{BDE} \right)$ |
| 9w   | $2 \alpha \text{tr}[\tilde{F}^C \tilde{F}^A \tilde{D}^B \tilde{D}^D]$ |
| 9x   | $4 \text{tr}[\tilde{F}^C \tilde{F}^A \tilde{D}^D \tilde{D}^B]$ |

*Table 9: Contributions from Fig. 9*
The results from Fig. 10 are of the form

\[ Nyg^2LX_{10}C^{\mu\nu} f^{abc} \partial_\mu \phi^a \partial_\nu \phi^b \phi^c \]  

(A.19)

and the contributions from the individual graphs to \( X_{10} \) are given in Table 10.

The results in Table 10 add to

\[ \Gamma^{(1)\text{pole}}_{10\text{PI}} = \frac{1}{2} Nyg^2L C^{\mu\nu} (1 + \alpha) f^{abc} \partial_\mu \phi^a \partial_\nu \phi^b \phi^c \]  

(A.20)

We have not explicitly drawn most of the diagrams (labelled Fig. (11a,b...)) giving contributions of the form

\[ iC^{\mu\nu}yg^2g_D L \left( X_{11}^{ABCD} \partial_\mu \phi^A \phi^B \phi^C A^D_\nu + Y_{11}^{ABCD} \phi^A \phi^B \phi^C \partial_\mu A^D_\nu \right), \]  

(A.21)

since they can be obtained by adding external scalar lines to the diagrams of Fig. 7. Thus Figs. 11(e-o) are obtained from Figs. 7(a-k) by adding an external scalar (\( \phi \)) line at the position of the cross. Figs. 11(p-v) are obtained from Figs. 7(l-r) by adding an external
The scalar ($\tilde{\phi}$) line at the position of the crossed circle. The remaining Figs. 11(a–d) are depicted in Fig. 11. The individual contributions to $X_{11}^{ABCD}$ and $Y_{11}^{ABCD}$ in Eq. (A.21) are given in Table 11.

| Fig. | $X_{11}^{ABCD}$ | $Y_{11}^{ABCD}$ |
|------|----------------|-----------------|
| 11a  | $-\alpha r[\hat{F}^A \hat{F}^B \hat{F}^C \hat{F}^D]$ | 0 |
| 11b  | $-\text{tr}[\hat{F}^A \hat{F}^B \hat{F}^C \hat{F}^D]$ | 0 |
| 11c  | $\frac{1}{2} \alpha (\text{tr}[\hat{F}^A \hat{F}^B \hat{F}^C \hat{F}^D] - \frac{1}{2} N f^{ABE} f^{CDE})$ | 0 |
| 11d  | 0 | 0 |
| 11e  | $-4\text{tr}[\hat{D}^B \hat{D}^A \hat{F}^C \hat{F}^D]$ | 0 |
| 11f  | $2\text{tr}[\hat{D}^A \hat{D}^B \hat{F}^C \hat{F}^D]$ | 0 |
| 11g  | $2\text{tr}[\hat{D}^A \hat{F}^B \hat{D}^C \hat{F}^D]$ | 0 |
| 11h  | $-2N f^{ABE} f^{CDE}$ | 0 |
| 11i  | 0 | $-4\text{tr}[\hat{D}^B \hat{F}^C \hat{D}^D \hat{F}^A]$ |
| 11j  | 0 | $-2\alpha r[\hat{F}^B \hat{F}^C \hat{D}^D \hat{D}^A]$ |
| 11k  | 0 | $-2\text{tr}[\hat{F}^B \hat{F}^C \hat{D}^D \hat{D}^A]$ |
| 11l  | $\text{tr}[\hat{D}^A \hat{D}^B \hat{F}^C \hat{F}^D]$ | 0 |
| 11m  | $2\text{tr}[\hat{F}^D \hat{F}^A \hat{D}^B \hat{D}^C]$ | 0 |
| 11n  | $2\text{tr}[\hat{D}^B \hat{D}^C \hat{F}^D \hat{F}^A]$ | $-2\alpha r[\hat{D}^B \hat{D}^C \hat{F}^D \hat{F}^A]$ |
| 11o  | $-3\text{tr}[\hat{D}^B \hat{D}^C \hat{F}^D \hat{F}^A]$ | 0 |
| 11p  | 0 | $\frac{1}{6}(5 + \alpha) \left( \text{tr}[\hat{F}^A \hat{F}^B \hat{F}^C \hat{F}^D] - N c^D d^{ABE} d^{CDE} \right)$ |
| 11q  | $\frac{2}{3} \alpha \left( \text{tr}[\hat{F}^A \hat{F}^B \hat{F}^C \hat{F}^D] - \frac{1}{2} N f^{ABE} f^{CDE} \right)$ | $\frac{1}{3} \alpha \left( 4\text{tr}[\hat{F}^A \hat{F}^B \hat{D}^C \hat{D}^D] + N c^B c^C d^E d^{ADE} d^{BCE} \right)$ |
| 11r  | $\text{tr}[\hat{F}^A \hat{F}^B \hat{F}^C \hat{F}^D]$ | $\frac{1}{3} \text{tr}[\hat{F}^A \hat{F}^B \hat{F}^C \hat{F}^D]$ |
|      | $\frac{1}{3} \text{tr}[\hat{F}^A \hat{F}^B \hat{F}^C \hat{F}^D]$ | $\frac{1}{3} \text{tr}[\hat{F}^A \hat{F}^B \hat{F}^C \hat{F}^D]$ |
| 11s  | $\frac{1}{3} \left( (3 + \alpha) N f^{ABE} f^{CDE} - \text{tr}[\hat{F}^A \hat{F}^B \hat{F}^C \hat{F}^D] \right) + \frac{1}{6} \left( 4\text{tr}[\hat{F}^A \hat{F}^D \hat{D}^B \hat{D}^C] + c^A c^D d^E N d^{ADE} d^{BCE} \right)$ | $\frac{1}{6}(1 + \alpha) \text{tr}[\hat{F}^A \hat{F}^B \hat{F}^C \hat{F}^D]$ |
|      | $\frac{1}{6} \left( 4\text{tr}[\hat{F}^A \hat{F}^D \hat{D}^B \hat{D}^C] + c^A c^D d^E N d^{ADE} d^{BCE} \right)$ | $\frac{1}{6} \left( 4\text{tr}[\hat{F}^A \hat{F}^D \hat{D}^B \hat{D}^C] + c^A c^D d^E N d^{ADE} d^{BCE} \right)$ |
| 11t  | $\frac{1}{6} \left( 4\text{tr}[\hat{F}^A \hat{F}^D \hat{F}^B \hat{F}^C \hat{F}^D] - \alpha N f^{ABE} f^{CDE} \right)$ | $-\frac{1}{3} \alpha r[\hat{F}^A \hat{F}^B \hat{F}^C \hat{F}^D]$ |
|      | $-\frac{1}{3} \left( 4\text{tr}[\hat{F}^D \hat{F}^A \hat{D}^B \hat{D}^C] + N c^A c^D d^E d^{ADE} d^{BCE} \right)$ | $-\frac{1}{3} \alpha r[\hat{F}^A \hat{F}^B \hat{F}^C \hat{F}^D]$ |

**Table 11:** Contributions from Fig. 11
Table 11: Contributions from Fig. 11 (continued)

The results sum to

\[
\Gamma_{11\Pi}^{(1)\text{pole}} = iC^{\mu\nu}g^2L \left( -\frac{1}{2}g \left( 3 + \frac{7}{3}\alpha \right) Nf^{abc}f_{cde}\partial_\mu \phi \phi \partial_\nu A^d + \left[ -\left( \frac{5}{4} - \frac{1}{6}\alpha \right) Nd^{abc}d^{cde} + \left( 3 + \frac{7}{3}\alpha \right) \delta^{ab}\delta^{cd} \right] g\phi \phi \partial_\mu A^d \right) 
\]

(A.22)

The divergent contributions to the effective action from the graphs in Fig. 12 are of the form

\[
iLN g^2 yX_{12}f^{abc}F^a(C\psi)^b\psi^c
\]

(A.23)

where the contributions from the individual graphs to \(X_{12}\) are given in Table 12:

| Fig. | \(X_{12}\) |
|------|-------------|
| 12a  | 1           |
| 12b  | 1           |

Table 12: Contributions from Fig. 12

The contributions from Table 12 add to

\[
\Gamma_{12\Pi}^{(1)\text{pole}} = 2iLN g^2 yf^{abc}(C\psi)^b\psi^c.
\]

(A.24)

The divergent contributions to the effective action from the graphs in Fig. 13 are of the form

\[
iLN g^2 myX_{13}C^{\mu\nu}F_{\mu\nu}^a F^a
\]

(A.25)
where the contributions from the individual graphs to $X_{13}$ are given in Table 13:

| Fig. | $X_{13}$ |
|------|----------|
| 13a  | $-1$     |
| 13b  | $\frac{1}{2}$ |

*Table 13: Contributions from Fig. 13*

The contributions from Table 13 add to

$$\Gamma_{13\text{PI}}^{(1)\text{pole}} = -\frac{1}{2} i L N g^2 m y C^\mu_{\nu} F^a_{\mu\nu} F^a. \quad (A.26)$$

The divergent contribution from Fig. 14 is

$$-\frac{1}{\sqrt{2}} L N g^2 m y C^\mu_{\nu} \bar{\phi} \sigma_\nu \partial_\mu \psi^a. \quad (A.27)$$

The divergent contributions to the effective action from the graphs in Fig. 15 are of the form

$$i L N g^2 m y X_{15} f^{abc} \bar{\phi} (C\psi)^b \psi^c \quad (A.28)$$

where the contributions from the individual graphs to $X_{15}$ are given in Table 15:

| Fig. | $X_{15}$ |
|------|----------|
| 15a  | $-1$     |
| 15b  | $-1$     |
| 15c  | $1$      |
| 15d  | $-1$     |

*Table 15: Contributions from Fig. 15*

The contributions from Table 15(a)–(c) add to

$$\Gamma_{15\text{PI}}^{(1)\text{pole}} = -i L N g^2 m y f^{abc} \bar{\phi} (C\psi)^b \psi^c. \quad (A.29)$$

Fig. 15(d) is only present in the eliminated case and is discussed separately in Section 5.

The divergent contributions to the effective action from the graphs in Fig. 16 are of the form

$$L N g^2 y^2 X_{16} f^{abc} d^{cde} (C\psi)^a \psi^b \psi^c \bar{\phi} \phi \quad (A.30)$$
where the contributions from the individual graphs to $X_{16}$ are given in Table 16:

| Fig. | $X_{16}$ |
|------|---------|
| 16a  | $-\frac{1}{4}$ |
| 16b  | $-\frac{1}{4}$ |
| 16c  | $\frac{1}{4}$ |
| 16d  | 0 |
| 16e  | $-\frac{1}{4}$ |

*Table 16: Contributions from Fig. 16*

The contributions from Table 16(a)–(d) add to

$$\Gamma^{(1)\text{pole}}_{16\text{PI}} = -\frac{1}{4} LN g^2 y^2 f^{a c e} d^{c d e} (C\psi)^{a}_{\psi} b^c c^d. \quad (A.31)$$

Fig. 16(e) is only present in the eliminated case and again is discussed separately in Section 5.

Finally the contribution from Fig. 17 is

$$\Gamma^{(1)\text{pole}}_{17\text{PI}} = \frac{1}{2} im^2 yg LC^{\mu\nu} F^a_{\mu\nu} \phi^d. \quad (A.32)$$

This is only present in the eliminated case.

**Appendix B. Results for one-loop diagrams in the fundamental case**

In this Appendix we list the divergent contributions from the one-loop diagrams for the fundamental case.

The divergent contributions to the effective action from the graphs in Fig. 7 are of the form

$$im L g^A C^{\mu\nu} \partial_\mu A^A_\phi \overline{R^A X^A_{B7}}$$

where the contributions to $X^A_{B7}$ from the individual graphs are given in Table 17:
These results add to

\[
\Gamma_{B71P1}^{(1)\text{pole}} = \text{im} L g C^{\mu\nu} \partial_\mu A^A_\nu \bar{\phi} R^A \left[ \left\{ - (76 + 4\alpha) + 32 \frac{\Delta}{g_5} \delta A^0 \right\} \hat{C}_2 
\right.
\]
\[
+ \left\{ (21 + \alpha) c^A + 16 \delta A^0 \right\} N g^2 \bar{\phi}. \tag{B.2}
\]

(Note that the contributions from Figs. 7(h-k) cancel those from Figs. 7(o-r) respectively.)

The divergent contributions to the effective action from the graphs in Fig. 8 are of the form

\[
\text{im} L g C^{\mu\nu} X_{B8} f^{abc} A^a_\mu A^b_\nu \bar{\phi} R^c \phi \tag{B.3}
\]
where the contributions to $X_{B8}$ from the individual graphs are given in Table 18:

| Graph | $X_{B8}$ |
|-------|----------|
| 8a    | $4(2\hat{C}_2 - Ng^2)$ |
| 8b    | $2Ng^2$ |
| 8c    | $-2(2\hat{C}_2 - Ng^2)$ |
| 8d    | $32\hat{C}_2 - 12Ng^2$ |
| 8e    | $2\alpha(2\hat{C}_2 - Ng^2)$ |
| 8f    | $2(2\hat{C}_2 - Ng^2)$ |
| 8g    | $-\frac{1}{2}(3 + \alpha)Ng^2$ |
| 8h    | 0 |
| 8i    | 0 |
| 8j    | $-2(2\hat{C}_2 - Ng^2)$ |
| 8k    | $\frac{3}{2}\alpha Ng^2$ |
| 8l    | $-\alpha Ng^2$ |
| 8m    | $\frac{3}{2}\alpha Ng^2$ |
| 8n    | $\frac{1}{7}(2 + \alpha)Ng^2$ |
| 8o    | 0 |
| 8p    | 0 |
| 8q    | $-\frac{3}{2}\alpha Ng^2$ |
| 8r    | $\frac{3}{2}(1 + \alpha)Ng^2$ |
| 8s    | $-\alpha(2\hat{C}_2 - Ng^2)$ |
| 8t    | $-(2\hat{C}_2 - Ng^2)$ |
| 8u    | $\frac{1}{7}(3 + \alpha)Ng^2$ |
| 8v    | 0 |
| 8w    | 0 |
| 8x    | $2(2\hat{C}_2 - Ng^2)$ |
| 8y    | $-\frac{3}{2}\alpha Ng^2$ |

*Table 18: Contributions from Fig. 8 (fundamental case)*
These results add to

$$\Gamma_{BS1PI}^{(1)\text{pole}} = \text{im}LN g^2 C^{\mu\nu} f^{abc} A^a_{\mu} A^b_{\nu} \phi \left[ 2(19 + \alpha) \hat{C}_2 - \left( \frac{23}{2} + \alpha \right) N g^2 \right] R^c \phi$$  \hspace{1cm} (B.4) 

(Note that the contributions from Figs. 8(g–p) cancel those from Figs. 8(u–dd) respectively.)

The divergent contributions to the effective action from the graphs in Fig. 9 are of the form

$$mL |C|^2 g^A g^B X^{AB} \tilde{\phi} X^{AB} \tilde{\phi}$$  \hspace{1cm} (B.5) 

where the contributions to $X^{AB}_{B9}$ from the individual graphs are given in Table 19. The results from Table 19 add to

$$\Gamma_{BS91PI}^{(1)\text{pole}} = mL |C|^2 \phi \left( \left[ \frac{1}{4} (2 - \alpha) - \frac{4g^2}{g_6} \right] N g^2 + \left[ \frac{1}{4} (9 + \alpha) + \frac{8g^2}{g_6} \right] \hat{C}_2 \right) g^2 d^{abc} R^c \tilde{\phi} \tilde{\phi}$$

$$+ \left\{ - \left[ \frac{1}{4} (11 + \alpha) + 4g^2 \right] N g^2 + \left[ \frac{1}{4} (9 + \alpha) + \frac{16g^2}{g_6} \right] \hat{C}_2 \right\} \frac{1}{N} g^2 \bar{\lambda}^{a} \lambda^{a}$$

$$+ \left\{ (9 + \alpha) \hat{C}_2 - \frac{1}{2} (1 + \alpha) N g^2 \right\} g g_0 R^a R^0 \bar{\lambda}^{a} \lambda^{a}$$

$$+ \frac{1}{2} (9 + \alpha) g_0^2 \hat{C}_2 R^0 R^0 \bar{\lambda}^{a} \lambda^{a} \right) \tilde{\phi}.$$  \hspace{1cm} (B.6) 

(Note that the contributions from Figs. 9(h–m) cancel those from Figs. 9(u–z), in analogy to the situation with Figs. 7 and 8; this is a consequence of our choice of coefficient for the last term in Eq. (7.9).)
| Graph | $X_{B9}^{ab}$ | $X_{B9}^{a0}$ | $X_{B9}^{00}$ |
|-------|----------------|----------------|----------------|
| 9a    | $\alpha \left( \frac{1}{2N} \Delta R^a R^b + \frac{1}{4} g^2 \delta^{ab} \right)$ | $\alpha \frac{1}{2N} \Delta R^a R^0$ | $\alpha \hat{C}_2 R^0 R^0$ |
| 9b    | $\left( \frac{1}{2N} \Delta R^a R^b + \frac{1}{4} g^2 \delta^{ab} \right)$ | $\frac{1}{2N} \Delta R^a R^0$ | $\hat{C}_2 R^0 R^0$ |
| 9c    | $(3 + \alpha) \left( \frac{1}{2N} \Delta R^a R^b + \frac{1}{4} g^2 \delta^{ab} \right)$ | $(3 + \alpha) \frac{1}{2N} \Delta R^a R^0$ | $(3 + \alpha) \hat{C}_2 R^0 R^0$ |
| 9d    | $-2\alpha \left( \frac{1}{2N} \Delta R^a R^b + \frac{1}{4} g^2 \delta^{ab} \right)$ | $-2\alpha \frac{1}{2N} \Delta R^a R^0$ | $-2\alpha \hat{C}_2 R^0 R^0$ |
| 9e    | $\left( Ng^2 + 4 \frac{g^2 \Delta}{g_0^2} \right) d^{abc} R^c + 2 \frac{g^2}{g_0^2} (2g^2 - g_0^2 + 4 \frac{\Delta}{N^2}) \delta^{ab}$ | $0$ | $0$ |
| 9f    | $\alpha \left( \frac{1}{2N} \Delta R^a R^b + \frac{1}{4} g^2 \delta^{ab} \right)$ | $\alpha \frac{1}{2N} \Delta R^a R^0$ | $\alpha \hat{C}_2 R^0 R^0$ |
| 9g    | $\left( \frac{1}{2N} \Delta R^a R^b + \frac{1}{4} g^2 \delta^{ab} \right)$ | $\frac{1}{2N} \Delta R^a R^0$ | $\hat{C}_2 R^0 R^0$ |
| 9h    | $-2 \left( \frac{1}{N} \Delta R^a R^b + \frac{1}{4} g^2 \delta^{ab} \right)$ | $-\frac{1}{2} \left( 2\hat{C}_2 + \frac{3}{N} \Delta \right) R^a R^0$ | $-4\hat{C}_2 R^0 R^0$ |
| 9i    | $\frac{1}{4} \alpha g^2 N d^{abc} R^c$ | $\frac{1}{2} \alpha N R^a R^0$ | $0$ |
| 9j    | $-\alpha g^2 N d^{abc} R^c$ | $-2\alpha N R^a R^0$ | $0$ |
| 9k    | $8 \left( \frac{1}{N} \Delta R^a R^b + \frac{1}{4} g^2 \delta^{ab} \right)$ | $2 \left( 2\hat{C}_2 + \frac{3}{N} \Delta \right) R^a R^0$ | $16\hat{C}_2 R_0 R_0$ |
| 9l    | $\alpha g^2 N d^{abc} R^c$ | $2\alpha N R^a R^0$ | $0$ |
| 9m    | $-4 \left( \frac{1}{2N} \Delta R^a R^b + \frac{1}{4} g^2 \delta^{ab} \right)$ | $-\frac{2}{N} \Delta R^a R^0$ | $-4\hat{C}_2 R^0 R^0$ |
| 9n    | $0$ | $0$ | $0$ |
| 9o    | $0$ | $0$ | $0$ |
| 9p    | $-\alpha g^2 N d^{abc} R^c$ | $-2\alpha N R^a R^0$ | $0$ |
| 9q    | $(1 + \alpha) g^2 N d^{abc} R^c$ | $2(1 + \alpha) N R^a R^0$ | $0$ |
| 9r    | $-\frac{1}{8} (3 + \alpha) g^2 (N d^{abc} R^c + 2g^2 \delta^{ab})$ | $0$ | $0$ |
| 9s    | $-\frac{1}{2} \alpha \left( \frac{1}{2N} \Delta R^a R^b + \frac{1}{4} g^2 \delta^{ab} \right)$ | $-\frac{1}{2} \alpha \frac{1}{2N} \Delta R^a R^0$ | $-\frac{1}{2} \alpha \hat{C}_2 R^0 R^0$ |
| 9t    | $-\frac{1}{2} \left( \frac{1}{2N} \Delta R^a R^b + \frac{1}{4} g^2 \delta^{ab} \right)$ | $-\frac{1}{2} \frac{1}{2N} \Delta R^a R^0$ | $-\frac{1}{2} \hat{C}_2 R^0 R^0$ |
| 9u    | $2 \left( \frac{1}{N} \Delta R^a R^b + \frac{1}{4} g^2 \delta^{ab} \right)$ | $\frac{1}{2} \left( 2\hat{C}_2 + \frac{3}{N} \Delta \right) R^a R^0$ | $4\hat{C}_2 R^0 R^0$ |
| 9v    | $-\frac{1}{4} \alpha g^2 N d^{abc} R^c$ | $-\frac{1}{4} \alpha N R^a R^0$ | $0$ |
| 9w    | $\alpha g^2 N d^{abc} R^c$ | $2\alpha N R^a R^0$ | $0$ |
| 9x    | $-8 \left( \frac{1}{N} \Delta R^a R^b + \frac{1}{4} g^2 \delta^{ab} \right)$ | $-2 \left( 2\hat{C}_2 + \frac{3}{N} \Delta \right) R^a R^0$ | $-16\hat{C}_2 R_0 R_0$ |

*Table 19:* Contributions from Fig. 9 (fundamental case)
Appendix C. Analysis of terms required for renormalisability

In this Appendix we perform a systematic analysis of the terms which can be generated by radiative corrections, focussing on the adjoint case. We follow the broad outlines of the analysis of Lunin and Rey\cite{8}, with modifications to accommodate the presence of Yukawa couplings.

We therefore start by assuming that the couplings for the antichiral fields, $\overline{y}$ and $\overline{m}$ are distinct from those for the chiral fields, $y$ and $m$. The most general operator which can appear through radiative corrections may be written schematically as

$$O = \Lambda^{\beta} y^{\delta} \overline{y} m^{\mu} \overline{m} C_{\mu \nu} \partial^{\alpha_0} \phi^{\alpha_1} \overline{\phi}^{\alpha_2} F^{\alpha_3} \overline{F}^{\alpha_4} \psi^{\alpha_5} \overline{\psi}^{\alpha_6} A^{\alpha_7} \lambda^{\alpha_8} X^{\alpha_9} D^{\alpha_{10}}, \quad (C.1)$$

where $\Lambda$ is an ultraviolet cutoff scale and $\alpha, \beta, \delta, \mu, \alpha_i, \overline{\alpha}_i$ are non-negative integers. For a dimension four operator

$$\beta - \alpha + \alpha_0 + \alpha_1 + \overline{\alpha}_1 + \alpha_4 + 2(\alpha_2 + \overline{\alpha}_2 + \alpha_6) + \frac{3}{2}(\alpha_3 + \overline{\alpha}_3 + \alpha_5 + \overline{\alpha}_5) + \mu + \overline{\mu} = 4. \quad (C.2)$$

There is also a pseudo-R-parity which acts as

$$\phi \rightarrow e^{-i\rho} \phi, \quad F \rightarrow e^{i\rho} F, \quad \lambda \rightarrow e^{-i\rho} \lambda, \quad (C.3)$$

$$C^{\mu \nu} \rightarrow e^{-2i\rho} C^{\mu \nu}, \quad y \rightarrow e^{i\rho} y,$$

$\overline{\phi}, \overline{F}, \overline{\lambda}$ and $\overline{y}$ transforming with opposite charge and all other fields being neutral. This entails

$$-2\alpha + \alpha_2 + \overline{\alpha}_1 + \overline{\alpha}_5 - \overline{\alpha}_2 - \alpha_1 - \alpha_5 + \delta - \overline{\delta} = 0. \quad (C.4)$$

Finally there is the pseudo $U_A(1)$ chiral symmetry which acts as

$$\phi \rightarrow e^{i\gamma} \phi, \quad m \rightarrow e^{-2i\gamma} m, \quad y \rightarrow e^{-3i\gamma} y, \quad (C.5)$$
and acts in the same fashion on $\psi$ and $F$ as on $\phi$; the barred fields transform with opposite charge. This leads to

$$\alpha_1 + \alpha_2 + \alpha_3 - \bar{\alpha}_1 - \bar{\alpha}_2 - \bar{\alpha}_3 + 3(\bar{\delta} - \delta) + 2(\bar{\mu} - \mu) = 0. \quad (C.6)$$

Combining Eqs. (C.4), (C.6) we have

$$\bar{\alpha}_1 = 3\alpha + \alpha_1 - 2(\alpha_2 - \bar{\alpha}_2) - \frac{1}{2}(\alpha_3 - \bar{\alpha}_3) + \frac{3}{2}(\alpha_5 - \bar{\alpha}_5) + \mu - \bar{\mu} \quad (C.7)$$

and substituting in Eq. (C.2) we find

$$\beta + 2\alpha + \alpha_0 + 2\alpha_1 + 4\bar{\alpha}_2 + \alpha_3 + 2\bar{\alpha}_3 + \alpha_4 + 3\alpha_5 + 2\alpha_6 + 2\mu = 4. \quad (C.8)$$

For simplicity we shall start by analysing the massless case, i.e. $\mu = \bar{\mu} = 0$. For $\alpha = 2$ we find from Eq. (C.8)

$$\beta = \alpha_0 = \alpha_1 = \bar{\alpha}_2 = \alpha_3 = \bar{\alpha}_3 = \alpha_4 = \alpha_5 = \alpha_6 = 0 \quad (C.9)$$

and hence Eqs. (C.6) and (C.7) become

$$\bar{\alpha}_1 = 6 - 2\alpha_2 - \frac{3}{2}\bar{\alpha}_5,$$

$$\bar{\alpha}_1 - \alpha_2 - 3\bar{\delta} + 3\delta = 0. \quad (C.10)$$

The only solutions to Eq. (C.10) are

$$\begin{align*}
\bar{\alpha}_5 &= 4, \quad \alpha_2 = 0, \quad \bar{\alpha}_1 = 0, \quad \bar{\delta} - \delta = 0, \\
\bar{\alpha}_5 &= 2, \quad \alpha_2 = 0, \quad \bar{\alpha}_1 = 3, \quad \bar{\delta} - \delta = 1, \\
\bar{\alpha}_5 &= 2, \quad \alpha_2 = 1, \quad \bar{\alpha}_1 = 1, \quad \bar{\delta} - \delta = 0, \\
\bar{\alpha}_5 &= 0, \quad \alpha_2 = 0, \quad \bar{\alpha}_1 = 6, \quad \bar{\delta} - \delta = 2, \\
\bar{\alpha}_5 &= 0, \quad \alpha_2 = 1, \quad \bar{\alpha}_1 = 4, \quad \bar{\delta} - \delta = 1, \\
\bar{\alpha}_5 &= 0, \quad \alpha_2 = 2, \quad \bar{\alpha}_1 = 2, \quad \bar{\delta} - \delta = 0, \\
\bar{\alpha}_5 &= 0, \quad \alpha_2 = 3, \quad \bar{\alpha}_1 = 0, \quad \bar{\delta} - \delta = -1.
\end{align*} \quad (C.11)$$

These solutions correspond to terms

$$|C|^2(\lambda \lambda)^2, \quad \bar{\gamma} |C|^2\lambda \lambda \phi^3, \quad \bar{\gamma}^2 |C|^2 \bar{\phi} \rho^6, \quad |C|^2 \bar{\phi} \lambda \lambda F, \quad |C|^2 \bar{\phi} \lambda \lambda F^2, \quad \bar{\gamma} |C|^2 \bar{\phi} \bar{\phi}^4 F, \quad \bar{\gamma} |C|^2 \bar{\phi} \rho F^3. \quad (C.12)$$
Here arbitrary powers of $y\overline{y}$ and of course $g$ are suppressed. The terms in Eq. \((C.12)\) without a $y$ or $\overline{y}$ are already in the Lagrangian, with the exception of the $\overline{\phi}^2 F^2$ term. As stated earlier, in the case with superpotential we have not computed terms purely involving $\phi$, $\overline{\lambda}$ and/or $F$, which are individually $\mathcal{N} = \frac{1}{2}$ invariant.

For $\alpha = 1$ we need either a $\sigma_{\mu\nu}$, a $\sigma_{\mu\nu}$, a $\sigma_{\mu\nu}$, a $\sigma_{\mu\nu}$, or a $D_{\mu}D_{\nu}$ to contract with the $C^{\mu\nu}$. However, $C^{\mu\nu} \sigma_{\mu\nu} = 0$ due to the self-duality of $C^{\mu\nu}$, and we see from Eq. \((C.8)\) that $\alpha_2 = \alpha_5 = 0$ so no $\lambda$ may appear. So $\sigma_{\mu\nu}$ could only appear in the form $\psi \sigma_{\mu\nu} \psi$ and we need $\alpha_3 = 2$. Eqs. \((C.6)\) and \((C.7)\) then give

$$\alpha_1 = 2 - 2\alpha_2 - \frac{3}{2}\alpha_5,$$

(C.13)

whose solutions are

$$\alpha_5 = 0, \quad \alpha_2 = 1, \quad \alpha_1 = 0, \quad \delta - \overline{\delta} = -1,$$

$$\alpha_5 = 0, \quad \alpha_2 = 0, \quad \alpha_1 = 2, \quad \delta - \overline{\delta} = 0.$$  

(C.14)

These solutions may be written in the form

$$yF\psi C\psi, \quad \overline{\phi}^2 \psi C\psi$$

(C.15)

where $(C\psi)^\alpha = C^\alpha_\beta \psi^\beta$. These terms are not in the original Lagrangian but can be generated at one loop (see Figs. 6 and 12); in fact the $F\psi C\psi$ term is individually $\mathcal{N} = \frac{1}{2}$ invariant. They are both discussed in detail in the main text.

If we have one derivative then $\alpha_0 + \alpha_4 = 1$ which implies from Eq. \((C.8)\) that $\alpha_1 = \alpha_3 = \alpha_6 = 0$ and $\alpha_3 = 1$. So $\overline{\lambda} \sigma_{\mu}\psi$ is the only possibility for the fermion fields, i.e. $\alpha_5 = 1$. Eqs. \((C.6)\) and \((C.7)\) then give

$$\alpha_1 = 1 - 2\alpha_2,$$

(C.16)

whose only solution is $\alpha_1 = 1$, $\alpha_2 = \delta - \overline{\delta} = 0$ which corresponds to $C^{\mu\nu} D_{\mu} \overline{\phi} \lambda \sigma_{\nu} \psi$ which is already in the Lagrangian.

Finally $D_{\mu}D_{\nu}$ corresponds to $\alpha_0 + \alpha_4 = 2$ which from Eq. \((C.8)\) gives $\alpha_1 = \alpha_3 = \alpha_3 = \alpha_6 = 0$. Eqs. \((C.6)\) and \((C.7)\) then give

$$\alpha_1 = 3 - \frac{3}{2}\alpha_5 - 2\alpha_2,$$

$$\alpha_1 = 0,$$

(C.17)
whose solutions are
\[ \begin{align*}
\alpha_5 &= 0, \quad \alpha_2 = 1, \quad \alpha_1 = 1, \quad \delta - \delta = 0, \\
\alpha_5 &= 0, \quad \alpha_2 = 0, \quad \alpha_1 = 3, \quad \delta - \delta = 1, \\
\alpha_5 &= 2, \quad \alpha_2 = 0, \quad \alpha_1 = 0, \quad \delta - \delta = 0.
\end{align*} \] (C.18)

These solutions correspond to
\[ C_{\mu \nu} \phi F_{\mu \nu} F, \quad C_{\mu \nu} F_{\mu \nu} \lambda \lambda, \quad \gamma C_{\mu \nu} \phi D_\mu \phi D_\nu \phi, \quad \gamma C_{\mu \nu} \phi^3 F_{\mu \nu}, \] (C.19)

all of which are already in the Lagrangian.

The inclusion of mass terms is straightforward but laborious and we shall confine ourselves to stating the results. Firstly we invariably find \( \mu = 0 \). For \( \alpha = 2 \) we find
\[ \begin{align*}
\mu &= 1, \quad \alpha_5 = 0, \quad \alpha_2 = 0, \quad \alpha_1 = 5, \quad \delta - \delta = 1, \\
\mu &= 1, \quad \alpha_5 = 0, \quad \alpha_2 = 1, \quad \alpha_1 = 3, \quad \delta - \delta = 0, \\
\mu &= 1, \quad \alpha_5 = 0, \quad \alpha_2 = 2, \quad \alpha_1 = 1, \quad \delta - \delta = -1, \\
\mu &= 1, \quad \alpha_5 = 2, \quad \alpha_2 = 0, \quad \alpha_1 = 2, \quad \delta - \delta = 0, \\
\mu &= 1, \quad \alpha_5 = 2, \quad \alpha_2 = 1, \quad \alpha_1 = 0, \quad \delta - \delta = -1, \\
\mu &= 2, \quad \alpha_5 = 0, \quad \alpha_2 = 0, \quad \alpha_1 = 4, \quad \delta - \delta = 0, \\
\mu &= 2, \quad \alpha_5 = 0, \quad \alpha_2 = 1, \quad \alpha_1 = 2, \quad \delta - \delta = -1, \\
\mu &= 2, \quad \alpha_5 = 0, \quad \alpha_2 = 2, \quad \alpha_1 = 0, \quad \delta - \delta = -2, \\
\mu &= 2, \quad \alpha_5 = 2, \quad \alpha_2 = 0, \quad \alpha_1 = 1, \quad \delta - \delta = -1, \\
\mu &= 3, \quad \alpha_5 = 0, \quad \alpha_2 = 0, \quad \alpha_1 = 3, \quad \delta - \delta = -1, \\
\mu &= 3, \quad \alpha_5 = 0, \quad \alpha_2 = 1, \quad \alpha_1 = 1, \quad \delta - \delta = -2, \\
\mu &= 3, \quad \alpha_5 = 2, \quad \alpha_2 = 0, \quad \alpha_1 = 0, \quad \delta - \delta = -2, \\
\mu &= 4, \quad \alpha_5 = 0, \quad \alpha_2 = 0, \quad \alpha_1 = 2, \quad \delta - \delta = -2, \\
\mu &= 4, \quad \alpha_5 = 0, \quad \alpha_2 = 1, \quad \alpha_1 = 0, \quad \delta - \delta = -3, \\
\mu &= 5, \quad \alpha_5 = 0, \quad \alpha_2 = 0, \quad \alpha_1 = 1, \quad \delta - \delta = -3.
\end{align*} \] (C.20)

These solutions correspond to terms
\[ \begin{align*}
\gamma m |C|^2 \phi^5, \quad m |C|^2 \phi^2 F, \quad \gamma m |C|^2 \phi^2, \quad m |C|^2 \phi^2 \lambda \lambda, \quad \gamma m |C|^2 F \lambda \lambda, \quad m^2 |C|^2 \phi^4, \\
\gamma m^2 |C|^2 \phi^2 F, \quad \gamma m^2 |C|^2 \phi^2, \quad m^2 |C|^2 \phi \lambda \lambda, \quad \gamma m^4 |C|^2 \phi^3, \quad m^2 |C|^2 \phi F, \\
y^2 m^3 |C|^2 \lambda \lambda, \quad y^2 m^4 |C|^2 \phi^2, \quad y^3 m^4 |C|^2 F, \quad y^3 m^5 |C|^2 \phi.
\end{align*} \] (C.21)
The possible terms without gauge fields in Eq. (C.21) have already been given in Ref. [7]. In any case all these terms are individually $\mathcal{N} = \frac{1}{2}$ invariant and we have not calculated their counterterms (with the exception of $m|C|^2 \bar{\phi} \bar{\lambda} \lambda$ which was in the classical lagrangian).

For the case $\alpha = 1$ we find solutions with $\mu = \alpha = 1$, $\alpha_3 = 2$, $\delta - \delta = -1$ and $\mu = 2$, $\alpha_3 = 2$, $\delta - \delta = -2$, all the other parameters zero, corresponding to $ym\bar{\phi}\psi C\psi$ and $y^2 m^2 \bar{\phi} C\psi$ respectively. The $ym\bar{\phi}\psi C\psi$ term has one loop contributions shown in Fig. 15, which are cancelled by our choice of $\rho_7$; we could find no one-loop diagrams contributing to the $y^2 m^2 \bar{\phi} C\psi$ term. There is also a solution with $\alpha_3 = \alpha_0 + \alpha_4 = \mu_0 = \delta - \delta = 1$, all the other parameters zero, corresponding to $y \bar{m} C^{\mu
u} \bar{\lambda} \sigma_{\mu} D_{\nu} \psi$. This has a one-loop contribution shown in Fig. 14, which is cancelled by $\gamma_4$ in Eq. (4.1). We also find solutions with $\alpha_0 + \alpha_4 = 2$:

$$
\begin{align*}
\mu = 1, & \quad \alpha_5 = 0, \quad \alpha_2 = 0, \quad \alpha_1 = 2, \quad \delta - \delta = 0, \\
\mu = 1, & \quad \alpha_5 = 0, \quad \alpha_2 = 1, \quad \alpha_1 = 0, \quad \delta - \delta = -1, \\
\mu = 2, & \quad \alpha_5 = 0, \quad \alpha_2 = 0, \quad \alpha_1 = 1, \quad \delta - \delta = -1, \\
\mu = 3, & \quad \alpha_5 = 0, \quad \alpha_2 = 0, \quad \alpha_1 = 0, \quad \delta - \delta = -2.
\end{align*}
$$

(C.22)

These correspond to terms

$$
C^{\mu\nu} m^2 \phi \mu\nu, \quad C^{\mu\nu} y m F \mu\nu, \quad C^{\mu\nu} y m^2 \bar{\phi} F \mu\nu.
$$

(C.23)

The first term is already in the classical lagrangian; the second gets contributions from Fig. 13 which again are cancelled by the $\gamma_4$ term in Eq. (4.1); and we have not been able to find any one-loop diagrams contributing to the third term.

This exhausts all the solutions.

We now turn to the case of the fundamental/anti-fundamental representation. We can derive the possible terms by selecting terms above with no $y$ or $\bar{y}$ (since there are now no Yukawa couplings) and taking care to replace $\phi$s (for example) by $\bar{\phi}$s and $\bar{\phi}$s in gauge-invariant combinations. This analysis was of course already performed by Lunin and Rey[8] and we reproduce their results; however with the addition of a $m|C|^2 (\bar{\phi} F)(\bar{\phi} \bar{\phi})$ term, a $m|C|^2 (\bar{\phi} \bar{\phi})(\bar{\phi} \bar{\phi})$ term, a $m^2|C|^2(\bar{\phi} \bar{\phi})^2$ term and a $C^{\mu\nu} m \bar{\phi} F_{\mu\nu} \bar{\phi}$ term. It is straightforward to construct one-loop diagrams which give divergent contributions to these terms and so we see no reason to omit them.
Appendix D. Group identities for $U(N)$

The basic commutation relations for $U(N)$ are (for the fundamental representation):

$$[R^a, R^b] = if^{abc} R^c, \quad \{ R^A, R^B \} = d^{ABC} R^C,$$

where $d^{ABC}$ is totally symmetric. Defining matrices $\tilde{F}^A$, $\tilde{D}^A$ by $(\tilde{F}^A)^{BC} = if^{BAC}$, $(\tilde{D}^A)^{BC} = d^{ABC}$, useful identities for $U(N)$ are

$$\Tr[\tilde{F}^A \tilde{F}^B] = N\delta^{AB}, \quad \Tr[\tilde{D}^A \tilde{D}^B] = N\delta^{AB},$$

$$\Tr[\tilde{F}^A \tilde{F}^B \tilde{D}^C] = \frac{N}{2} d^{ABC} c^A c^B d^C, \quad \Tr[\tilde{F}^A \tilde{D}^B \tilde{D}^C] = i \frac{N}{2} f^{ABC},$$

$$f^{ABE} d^{CDE} + f^{ACE} d^{DBE} + f^{ADE} d^{BCE} = 0,$$

where $c^A = 1 - \delta^A_0$ and $d^A = 1 + \delta^A_0$, and also

$$\Tr[\tilde{F}^A \tilde{F}^B \tilde{F}^C \tilde{F}^D] = c^A c^B c^C c^D \left[ \frac{1}{2} \delta^{(AB}\delta^{CD)}
\right.$$}

$$+ \frac{N}{4} \left( d^{ABE} d^{CDE} + d^{ADE} d^{BCE} - d^{ACE} d^{BDE} \right),$$

$$\Tr[\tilde{F}^A \tilde{F}^B \tilde{F}^C \tilde{D}^D] = - \frac{N}{4} i \left( d^{ABE} f^{CDE} + f^{ABC} d^{CDE} \right) c^A c^B c^C d^D,$n

$$\Tr[\tilde{F}^A \tilde{F}^B \tilde{D}^C \tilde{D}^D] = \left[ \frac{1}{2} c^A c^B c^C c^D \left( \delta^{AB} \delta^{CD} - \delta^{AC} \delta^{BD} - \delta^{AD} \delta^{BC} \right)
\right.$$}

$$+ \frac{N}{4} c^A c^B c^C d^D \left( d^{ABE} d^{CDE} + d^{ADE} d^{BCE} - d^{ACE} d^{BDE} \right),$$

$$\Tr[\tilde{F}^A \tilde{D}^B \tilde{F}^C \tilde{D}^D] = \left[ c^A c^B c^C d^D \left( \delta^{AB} \delta^{BD} - \delta^{AC} \delta^{CD} - \delta^{AD} \delta^{BC} \right)
\right.$$}

$$+ \frac{N}{4} c^A d^B c^C d^D \left( d^{ABE} d^{CDE} + d^{ADE} d^{BCE} - d^{ACE} d^{BDE} \right).$$

(D.3)
Fig. 1: Diagrams with one gaugino, one scalar and one chiral fermion line; the dot represents the position of a $C$. 
Fig. 2: Diagrams with one gaugino, one scalar, one chiral fermion and one gauge line; the dot represents the position of a $C$. 
Fig. 2 (continued).
Fig. 3: Diagrams with one gauge, one scalar and one auxiliary line; the dot represents the position of a $C$. 
Fig. 4: Diagrams with two gauge, one scalar and one auxiliary line; the dot represents the position of a C.
Fig. 5: Diagrams with two gaugino, one scalar and one auxiliary line; the dot represents the position of a $C$ or $|C|^2$. 
Fig. 6: Diagrams with two scalar and two chiral fermion lines; the dot represents the position of a $C$. 

(a) 

(b) 

(c) 

(d)
Fig. 7: Diagrams with two scalar, one gauge line; a dot denotes a $C$, a cross a mass and a crossed circle a vertex with both a mass and a $C$. 
Fig. 7 (continued)
Fig. 8: Diagrams with two scalar, two gauge lines; a dot denotes a $C$, a cross a mass and a crossed circle a vertex with both a mass and a $C$. 
Fig. 9: Diagrams with two scalar, two gaugino lines; a dot denotes a $C$, a cross a mass and a crossed circle a vertex with both a mass and a $C$. 
Fig. 9 (continued)
Fig. 10: Diagrams with three scalar lines; a dot represents the position of a C, a cross a Yukawa vertex and a crossed circle a Yukawa vertex with a C.

Fig. 11: Diagrams with three scalar lines and one gauge line; a crossed circle represents a Yukawa vertex with a C. Figs. 11(e-v) are not depicted explicitly. Figs. 11(e-o) are obtained from Figs. 7(a-k) by adding an external scalar (\(\phi\)) line at the position of the cross. Figs. 11(p-v) are obtained from Figs. 7(l-r) by adding an external scalar (\(\phi\)) line at the position of the crossed circle.
Fig. 12: Diagrams with two chiral fermion lines and one auxiliary line; the dot represents the position of a $C$ and the cross a Yukawa vertex.

Fig. 13: Diagrams with one gauge line and one auxiliary line; a cross represents a mass insertion or Yukawa vertex and a crossed circle a vertex with both a mass and a $C$.

Fig. 14: A diagram with one chiral fermion lines and one gaugino line; a cross represents a mass insertion or Yukawa vertex.
Fig. 15: Diagrams with two chiral fermion lines and one scalar; a dot represents the position of a $C$, a cross represents a mass insertion or Yukawa vertex, a crossed circle a vertex with both a mass and a $C$, and a box a mass-Yukawa vertex.

Fig. 16: Diagrams with two chiral fermion lines and two scalars; a dot represents the position of a $C$, a cross a Yukawa vertex and a crossed circle a Yukawa vertex with a $C$. 
Fig. 17: *A diagram with one scalar and one gauge line; a crossed circle represents a vertex with both a mass and a C, and a box a mass-Yukawa vertex.*
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