Surface Tension at Finite Temperature in the MIT Bag Model

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Abstract

At $T = 0$ the surface tension $\sigma^{1/3}$ in the MIT bag model for a single hadron
is known to be negligible as compared to the bag pressure $B^{1/4}$. We show that at
finite temperature it has a substantial value of 50 - 70 MeV which also differ from
hadron to hadron. We also find that the dynamics of the Quark-Gluon Plasma is
such that the creation of hybrids ($s\bar{s}g$) with massive quarks will predominate over
the creation of ($s\bar{s}$) mesons.

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Recently a possible signature of Quark-Gluon Plasma (QGP) has been proposed by two of us [1]. We showed that the QGP dynamics is such that during hadronisation the creation of hybrids will predominate over the creation of mesons. These hybrids will manifest themselves by significantly modifying the photonic signals of QGP. That calculation [1] was done for 2-flavour massless quarks. One may ask what happens if one takes the massive quarks? In this paper we study this problem and look at the \((s\bar{s}g)\) hybrid with respect to the \((s\bar{s})\) mesons at finite temperature and explore it’s significance for QGP.

Recently it has been pointed out by Swanson [2] that the hybrids may in fact have intrinsic connection with the concept of quark confinement. If this is indeed true then the study of hybrid should be more crucial than was realised earlier. This paper is a continuing venture in that direction.

We also look at the surface tension at finite temperature in the MIT bag model for a single hadron. It is known [3] that at \(T = 0\), the surface tension in the MIT bag model for a single hadron is negligible in comparison to the bag pressure \(B^{1/4}\). (Note that what is called “intrinsic” surface tension in ref.[3] is what we have here and shall call it simply “surface tension”). However for a massive quark (like s-quark) at finite temperature the surface tension \(\sigma^{1/3}\) may be non-zero [4]. Recently some work has been done on the surface tension both at \(T \neq 0\) [5] and at \(T = 0\) [6, 7]. A mass formula for a spherical lumps of three-flavour quark matter at \(T = 0\) was derived within the MIT bag model, taking into account bulk, surface and curvature contributions where an ansatz is provided for the curvature contribution to the density of states for massive quarks [6]. At finite temperature near the confinement phase transition, the free energy of finite droplets of QGP and of finite hadronic bubbles in an bulk plasma has been calculated by Mardor & Svetitsky [5]. They have shown that the curvature term in the
free energy proportional to the radius of the droplet or bubble is more important than the contribution of the surface tension, proportional to the radius squared. But it is not clear what its exact value in the MIT bag should be. Here we find that the (intrinsic) surface tension $\sigma^{1/3}$ at finite temperature has a substantial value and is in the range 50-70 MeV which also depends upon the structure of hadrons. Hence the same (intrinsic) surface tension which is zero at $T = 0$ is shown to have a substantial non-zero value at finite temperature.

1 Formalism:

To study the thermodynamic properties of a hadron (assumed to be in contact with the heat bath mimicking the hot central zone in relativistic heavy ion collision), a bag is treated like a many-body system at a given temperature. The thermodynamic treatment of a hot hadronic bag has been justified by several authors over the years [4, 8, 9, 10, 11]. Which pointed out that at a high temperature a large number of $q\bar{q}$ pairs can be created from the negative energy sea, and so at least in principle it is not a system of only two or three particles. We consider a system of non-interacting quarks, anti-quarks and gluons placed in a heat bath with which it can exchange the energy and the particle numbers, so that one can use the grand canonical ensemble formalism. The quarks and gluons are confined in an MIT bag [12]. We consider the system of single quark flavor (s-quark) with mass $m_s = 150$ MeV where the single particle energies are obtained by solving the equation of motion with linearised boundary condition [13]. The single particle energies for the massive quarks and massless gluons are in units of $\hbar c/R$, where the radius of the bag $R$ is included.
In a statistical approach, the grand canonical partition function is given by

\[ Z_G = Z_{\text{vac}} Z_s Z_{\bar{s}} Z_g \]  

(1)

where \( Z_{\text{vac}} \) takes care of the temperature \( T \to 0 \) limit, and we consider this as

\[ -T \ln Z_{\text{vac}} = BV + C/R \]  

(2)

with \( BV \) being the volume energy of the bag and \( C/R \) as the Casimir energy which is \( T \) independent with the value of \( C \) given in ref.\,[1, 11, 14]. \( Z_s \) and \( Z_{\bar{s}} \) refers to the partition function for the s-quark and s-antiquark, where \( Z_g \) refers to the gluonic part.

The logarithm of the partition function for the system of s-quark, anti-quark and gluon with the chemical potential \( \mu_{s(\bar{s})} \) for quark(anti-quark) is given by

\[ \ln Z = \sum_i \ln \left( 1 + e^{-\left(\epsilon_i^s - \mu_s\right)/T} \right) + \sum_i \ln \left( 1 + e^{-\left(\epsilon_i^{\bar{s}} - \mu_{\bar{s}}\right)/T} \right) \]

\[ - \sum_i \ln \left( 1 - e^{-\epsilon_i^g/T} \right) \]  

(3)

where \( \epsilon_i^{s(\bar{s})} \) and \( \epsilon_i^g \) are the quark (anti-quark) and gluon single particle energies in the bag respectively.

The number of quark and gluon is given by

\[ N_s = \sum_i 1/\left( e^{\left(\epsilon_i^s - \mu_s\right)/T} + 1 \right) \]  

(4)

and

\[ N_g = \sum_i 1/\left( e^{\epsilon_i^g/T} - 1 \right) \]  

(5)

Whereas the energy and the free energy of the quark gluon system is given by

\[ E(T, R) = T^2 \left( \frac{\partial \ln Z}{\partial T} \right) + \mu_{s(\bar{s})} N_{s(\bar{s})} + BV + C/R \]  

(6)

\[ F(T, R) = -T \ln Z + \mu_{s(\bar{s})} N_{s(\bar{s})} + BV + C/R \]  

(7)
The pressure generated by the participant gas

\[ P = -\left( \frac{\partial F(T, V)}{\partial V} \right)_{T, \mu_s} \]  

is balanced by the bag pressure constant \( B \) leading to the stability condition to the system.

2 Results and discussion:

The physical behaviour of a system at a finite temperature \( T \) is governed by the properties of its free energy. We consider the radius of the bag \( R \) to be a variational parameter in the free energy \( F(T, R) \) to study the stability of the bag. The quarks and gluons single particle energies are in units of \( hc/R \), so for a fixed value of \( T \), we vary \( R \) while adjusting the chemical potential and ensuring that we have one s-quark in the system. For this temperature \( T \) and for the range of \( R \) considered, the gluon number \( N_g \) is calculated from Eq.(5). If this value is less than one (which is true for the low temperatures), we go to higher temperature such that we have one gluon. Now at this temperature \( T \), we calculate the free energy of the (s\(\bar{s}\)g) system with different values of \( R \) and see that there is a radius \( R \) of the bag for which there is one gluon attaches to the (s\(\bar{s}\)) system making it hybrid (s\(\bar{s}\)g) and also at which point the free energy of the hybrid is minimum.

For \( B^{1/4} = 200 \) MeV, we calculate the free energy of the (s\(\bar{s}\)g) system for various \( R \) values at a fixed temperature \( T \) at which \( F \) is minimum at some value of \( R \) where one gluon attaches to the (s\(\bar{s}\)) system making it hybrid (s\(\bar{s}\)g) at that \( T \).

At the same temperature, the free energy of the pure mesonic system (s\(\bar{s}\)) has been calculated as a function of radius \( R \). This variations of free energies for hybrid (s\(\bar{s}\)g) and meson (s\(\bar{s}\)) is displayed at Fig. 1 and the corresponding energy (mass) at the minimum free energy condition is given in Table 1.
From Fig. 1 we see that at the temperature $T = 156$ MeV; the hybrid ($s\bar{s}g$) is more stable (less free energy) than the meson ($s\bar{s}$), whereas from Table 1 we see that the energy (mass) of the hybrid ($s\bar{s}g$) is less than twice the mass of the meson ($s\bar{s}$) hence forbidding the decay of the strange hybrid ($s\bar{s}g$) into a pair of mesons ($s\bar{s}$). So although ($s\bar{s}g$) hybrid decay into a pair of ($s\bar{s}$) mesons at $T = 0$ MeV [15, 16], the strange hybrid ($s\bar{s}g$) does not decay into ($s\bar{s}$) pair at finite T. However unlike the case of non-strange hybrids, as $s\bar{s}g \rightarrow s\bar{u} + \bar{s}u$ is possible and hence strange hybrid may decay through strong interaction. Earlier we had shown [1] that the non-strange hybrids will leave a unique signature in terms of suppression of photonic signals. The same may not be true of strange hybrids.

Note that our picture of the hybrid at finite temperature is akin to what is considered [16] as gluonic excitation of meson. So what may have been a meson at $T = 0$ converts into a hybrid at high temperature due to the gluonic excitation arising therein.

For the case of strange hybrid ($s\bar{s}g$) we see that during the hadronisation of thermalised QGP, the formation of hybrid is more favourable than that of the meson at the same temperature. So the mixed phase in addition to QGP and mesons will contain massive hybrids as well and thereby reduce the life-time of the mixed phase. Hence by studying the non-strange hybrid [1] as well as the strange hybrid we found that the hybrid formation is favoured over the meson formation during the hadronisation of the QGP.

For a system of quarks and gluons confined in a bag of finite size, the finite size corrections are significant. The finite size corrections can be incorporated in two different ways. The first method is to take the sum over the discrete single particle states of quarks and gluons for computing the thermodynamic quantities [11] and also as we have done here. The second method is to replace the sum over discrete states by the integral with
single particle density of states including finite size corrections \([14]\). For reasonably dense system (as we have here) these two methods are equivalent to each other \([3, 11, 14]\). We shall use this equivalence to calculate the (intrinsic) surface tension of the single hadron at finite temperature.

For the interior of the sphere with radius \(R\), the density of state becomes \([3, 14]\)

\[
\rho(k) = \frac{2}{3\pi} R^3 k^2 + C_1 \frac{4\pi}{3} R^2 k + C_2 \frac{8\pi}{3} R
\]  

(9)

Since the surface area term vanishes in \(\rho(k)\) for the massless particles, one gets the mass (energy) of the system of massless quarks, anti-quarks and gluons at the equilibrium condition as \(M = 4BV\). Where \(B\) is the usual bag pressure constant. But for the system of massive quarks the surface term also does contribute. So for the strange quark system we add the surface energy term \(4\pi R^2 \sigma\) in the energy (Eq.(3)) and free energy expression (Eq.(7)). In these expressions we replace the discrete sum over single particle states by the integral with density of states containing the volume and curvature term. Now applying the pressure balance condition as Eq.(8) for stability, the mass (energy) of the system \([4, 14]\) at equilibrium (where the total free energy is minimum) is given by

\[
M = 4BV + 12\pi R^2 \sigma
\]  

(10)

Where we parametrize \(\sigma\) as the coefficient of the surface energy and is known as surface tension.

In our system of strange hybrid \((s\bar{s}g)\) at finite temperature, we get the mass of the hybrid at equilibrium (minimum free energy position) which should be equal to the expression \((10)\) as our system has massive quarks for which surface energy does contribute. Hence we get,
\[ \sigma^{1/3} = \left[ \frac{M - 4BV}{12\pi R^2} \right]^{1/3} \] (11)

With \( B^{1/4} = 200 \text{ MeV} \), for the hybrid of \( R = 0.943 \text{ fm} \), and \( M = 3.086 \text{ GeV} \), we get the value of the surface tension as \( \sigma^{1/3} = 57 \text{ MeV} \) at the temperature \( T = 156 \text{ MeV} \). Whereas at the same temperature \( T = 156 \text{ MeV} \), the surface tension for the meson \((s\bar{s})\) is obtained as \( \sigma^{1/3} = 63 \text{ MeV} \).

Similarly for \( B^{1/4} = 250 \text{ MeV} \), the \((s\bar{s}g)\) hybrid is formed at the temperature \( T = 195 \text{ MeV} \) with radius \( R = 0.758 \text{ fm} \), and mass \( M = 3.87 \text{ GeV} \), so the value of \( \sigma^{1/3} = 66 \text{ MeV} \) whereas the value of \( \sigma^{1/3} \) for \((s\bar{s})\) meson with radius \( R = 0.647 \text{ fm} \) and mass \( M_{(s\bar{s})} = 2.459 \text{ GeV} \) is 72 MeV. For \( B^{1/4} = 150 \text{ MeV} \), we find \( \sigma^{1/3}(s\bar{s}g) = 48 \text{ MeV} \) and \( \sigma^{1/3}(s\bar{s}) = 53 \text{ MeV} \). Hence at the same temperature and bag pressure, the value of surface tension \( \sigma^{1/3} \) is larger for pure meson than for the hybrid. It is expected that different hadrons should have different surface tensions depending upon their constituent objects [17]. We would like to point out the power of our method is that it allows us to extract surface tension of objects what have gluonic degree as well.

We have looked at the \((s\bar{s})\) system for various temperatures \( T = 5, 10, 50, 100, 150 \text{ MeV} \) for \( B^{1/4} = 200 \text{ MeV} \) and found \( \sigma^{1/3} \) to be almost steady with a slight decrease from 68 MeV to 63 MeV as the temperature increases. However one should be warned that our method is justified for the cases where the continuum approximation is valid, that is at higher temperatures. Also for \( B^{1/4} = 200 \text{ MeV} \), \( \sigma^{1/3}(s\bar{s}g) \) goes from 39 MeV to 75 MeV and \( \sigma^{1/3}(s\bar{s}) \) goes from 43 MeV to 82 MeV as \( m_s \) goes from 50 MeV to 300 MeV. In the same range the temperature changes very slightly from 155 MeV to 157 MeV. In ref. [17] Berger finds that the dynamical surface tension decreases for heavy strange quark masses. This was found to be true as the number density of the strange quarks in their system decreased as the mass was increased. However in our calculation here we
find that the radius of the hybrid and the meson are almost stationary as the strange quark mass increases and also that we fix the number of quark and antiquark. Hence the number density in our calculation does not change as the strange quark mass increases. Therefore the increase in strange quark mass is directly reflected in terms of an increase of the corresponding (intrinsic) surface tension as we find here.

Note that our surface tension here is the intrinsic surface tension \[ \sigma \]. This is different from the surface tension as obtained in the lattice calculations. The latter is that of the hadron phase and the QGP phase as separated by an interface at the phase transition. These values have gone through extreme evolutions. A prior one expects \( \sigma/T^3 \) to be of the order of unity \[ 1 \]. However when calculations were done it was found \[ 1 \] to just have a value of \( \sim 0.24 \). Latter better lattice calculations brought it down to be \( \sim 0.027 \) \[ 1, 20 \]. It is amusing to note that these values are not very different from our results in this paper.

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CAPTIONS

Table 1

The equilibrium radius \((R)\) and the energy (mass) \((E)\) of the hybrid \((s\bar{s}g)\) and pure mesonic system \((s\bar{s})\) is given for bag constant \(B^{1/4} = 200\) MeV.

Figure 1.

The variation of the free energy of the hybrid \((s\bar{s}g)\) and the pure mesonic systems \((s\bar{s})\) with radius \(R\) are displayed at a particular temperature \(T = 156\) MeV and \(B^{1/4} = 200\) MeV.
| System | T(MeV) | R(fm) | E(GeV) |
|--------|--------|-------|--------|
| \((s\bar{s}g)\) | 156    | 0.943 | 3.086  |
| \((s\bar{s})\)  | 156    | 0.808 | 1.996  |
Free Energy (MeV)

\( B^{1/4} = 200 \text{ MeV} \)

Meson

Hybrid

\( R (\text{fm}) \)