Quantum ratchet driven by broadband perturbation

D.V. Makarov, L.E. Kon’kov
Laboratory of Nonlinear Dynamical Systems,
Pacific Oceanological Institute of the Russian Academy of Sciences,
43 Baltiiskaya st., 690041 Vladivostok, Russia, URL: http://dynalab.poi.dvo.ru

Abstract

Motion of an atomic ensemble trapped into a deep optical lattice is considered. We propose a novel approach to construct an atomic ratchet by superimposing two additional lattices whose amplitudes are small and subjected to broadband modulation. The broadband modulation is modeled by means of harmonic noise. Directed atomic transport occurs with the properly chosen phase shift between the signals modulating the amplitudes of the additional lattices. It is shown that efficiency of the ratchet depends non-monotonously on the parameter determining the spectral width of modulating signals.

Keywords: quantum transport, ratchet, cold atoms, harmonic noise

1. Introduction

The ratchet effect, i.e., occurrence of directed transport in space-periodic systems under unbiased forcing, has become an object of extensive research in recent years. Quantum ratchets realized with cold atoms in optical lattices represent a promising way for constructing new-age microscopic devices, for example, atomic chips in quantum computers. In the simplest setup for the atomic ratchet, one uses far-detuned deep optical lattices [1, 2, 3, 4, 5, 6]. In this case atomic dynamics can be fairly described in the framework of the semiclassical approximation. Activation of net current requires escaping of atoms from the potential wells. This process is impeded by the classical dynamical barriers in phase space. Therefore, the experiments are usually conducted with relatively strong perturbation of the optical potential, when the barriers don’t survive. Another approach to facilitate escaping is the usage of high-frequency driving leading to the lowering of the effective optical potential [7].

In the present paper we propose a simple way to generate directed atomic transport using a small-amplitude perturbation, even if atoms are initially localized near the minima of the potential. Usage of a small-amplitude perturbation should be favorable because it allows one to avoid overheating of the atomic ensemble. The key idea of our approach is to combine deterministic and stochastic driving. The stochastic part is responsible for the destruction of the dynamical barriers, while the deterministic one provides controllable violation of space-time symmetries, which is necessary for production of atomic current in a desired direction. In addition, a properly constructed deterministic driving allows one to reduce the noise level needed for destruction of the dynamical barriers. The particular scheme we consider is based on the usage of two additional optical lattices whose amplitudes are subjected to broadband modulation.

The paper is organized as follows. In the next section we describe the model considered. Section 3 is devoted to the properties of this model in the classical limit. Quantum atomic dynamics is considered in section 4. In Summary, we give a brief account of the results obtained.

2. The model

In the case of large detuning of the atom-field resonance and in the rotating-wave approximation, motion of a wavepacket corresponding to the two-level atom is described by the Schrödinger equation

\[ i\hbar \frac{\partial \Psi}{\partial T} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial X^2} + \hbar \frac{\Omega^2(X, T)}{\delta} \Psi, \]

where \( X \) is position, \( m \) is the atomic mass, \( \delta \) is the detuning of the atom-field resonance, \( \Omega \) is the Rabi frequency which can be expressed as

\[ \Omega^2(X, T) = \Omega_{\text{max}}^2 U(X, T). \]

After the transformation

\[ t = 2\Omega_{\text{max}} \sqrt{\frac{\omega_r}{\delta}} T, \quad x = 2kX, \]

where \( k \) is the wavenumber of the laser field, \( \omega_r = \hbar k^2/2m \) is the recoil frequency, we rewrite the Schrödinger equation [8] in the following form:

\[ \hbar_{\text{resc}} \frac{\partial \Psi}{\partial t} = -\frac{\hbar_{\text{resc}}^2}{2} \frac{\partial^2 \Psi}{\partial x^2} + U(x, t)\Psi, \]

where the rescaled Planck constant is given by

\[ \hbar_{\text{resc}} = 4 \sqrt{\delta \omega_r / \Omega_{\text{max}}}. \]

Hereafter we shall omit the subscript at \( \hbar_{\text{resc}} \).
In the present work, we propose the following configuration of the optical potential:

\[ U = 1 - \cos x + \varepsilon V(x, t), \]  
\[ V(x, t) = 1 + f(t) \sin x - sf(t + \Delta) \cos x, \]

where \( \varepsilon \ll 1 \), \( s \) is a model parameter equal to 1 or −1, and \( f(t) \) is a broadband signal. We model this signal as the so-called harmonic noise \[ \xi(t). \] Harmonic noise is the two-dimensional Ornstein-Uhlenbeck process obeying the following coupled stochastic differential equations

\[ \dot{f} = y, \quad \dot{y} = -\Gamma y - \omega^2_0 f + \sqrt{2\mu} \xi(t), \]

where \( \Gamma \) is a positive constant, and \( \xi(t) \) is Gaussian white noise with \( \langle \xi(t) \rangle = 0, \quad \langle \xi(t) \xi(t') \rangle = \delta(t - t'). \)

It should be emphasized that the terms \( f(t) \) and \( f(t + \Delta) \) in (7) correspond to one and the same realization of harmonic noise, they differ only by the temporal shift \( \Delta. \)

The first two moments of harmonic noise are given by

\[ \langle f \rangle = 0, \quad \langle f^2 \rangle = \frac{\mu}{\omega_0^2}. \]

Power spectrum of the harmonic noise is described by formula

\[ S(\omega) = \frac{\mu \Gamma}{\omega^2 \Gamma^2 + (\omega^2 - \omega_0^2)^2}. \]

In the present work \( \mu \) is taken of 1. Then the perturbation strength is determined by the parameter \( \varepsilon \). In the case of low values of \( \Gamma \), the power spectrum has the peak at the frequency

\[ \omega_p = \sqrt{\frac{\omega_0^2 - \Gamma^2}{2}} \]

with the width

\[ \Delta \omega = \omega_p + \omega_0' - \omega_p - \Gamma \omega_0'. \]

where \( \omega' = \sqrt{\omega_0^2 - \Gamma^2/4}. \) It should be mentioned that harmonic noise had been recently considered in the context of the Landau-Zener tunneling in optical lattices [11][12].

As \( \Gamma \rightarrow 0, f(t) \rightarrow \sin(\omega_0 t + \phi_0), \) where \( \phi_0 \) is determined by initial conditions in (8). Setting \( f(0) = 1, y(0) = 0, \) and

\[ \Delta = \frac{\pi}{2\omega_0}, \]

one obtains \( f = \cos \omega_0 t \) and

\[ V(x, t) \rightarrow \sin(x + \omega_0 t), \]

with \( \Gamma \rightarrow 0. \) Hence, it turns out that \( V(x, t) \) for \( \Gamma > 0 \) behaves like a plane wave with fluctuating phase and amplitude (see Fig. 1). In numerical simulation, we set \( \omega_p = 1 \) and \( \varepsilon = 0.05. \)

Taking into account the periodicity of the potential \( U \), we use the periodic boundary conditions

\[ \Psi(x) = \Psi(x + 2\pi). \]

It means that we consider only the states with zero quasimomentum. This choice is reasonable as long as we consider only the semiclassical regime with fairly small values of the rescaled Planck constant \( \hbar. \) In this case energy bands are flat and tunneling between neighboring potential wells is negligible, that is, a wave function of a single atom is tightly enough localized in space.

3. Classical dynamics

Before considering quantum evolution of the atomic ensemble, it is reasonable to give insight onto atomic dynamics in the classical limit. Trajectories of atoms obey Hamiltonian equations of motion

\[ \frac{dx}{dt} = \frac{\partial H}{\partial p}, \quad \frac{dp}{dt} = -\frac{\partial H}{\partial x}, \]

where the classical Hamiltonian \( H \) is given by

\[ H = \frac{p^2}{2} - \cos x + \varepsilon V(x, t). \]

Time-dependent perturbation \( V(x, t) \) leads to the nonintegrability of equations (16) and onset of chaos. For \( \Gamma = 0 \) and \( \varepsilon \ll 1, \) chaotic motion occurs only inside a relatively small near-separatrix layer acting as a “bridge” between the domains of finite and ballistic motion. In this case, the perturbation is a plane wave of the form (13), and the equations of motion read

\[ \frac{dx}{dt} = p, \quad \frac{dp}{dt} = -\sin x - \varepsilon \cos(x + s\omega_0 t). \]
One can easily ensure that both the shift symmetry
\[ x \rightarrow -x, \quad t \rightarrow t + \frac{\pi}{\omega_0}, \tag{19} \]
and the time-reversal symmetry
\[ x \rightarrow -x, \quad t \rightarrow -t, \tag{20} \]
are broken in this case, making the onset of directed transport possible. The plane wave \( V(x, t) \) tends to carry atoms in the direction \( x \rightarrow -\infty \) (\( x \rightarrow \infty \)) for \( s = 1 \) (\( s = -1 \)), i.e., along the lines of the of the constant phase in Fig. 1. It results in the asymmetry of the chaotic layer in the momentum space, as it is demonstrated in Fig. 2. This asymmetry expects emergence of the constant phase lines for \( V(x, t) \) are biased in the same direction for all values of \( \Gamma \), despite of the gradual pattern randomization with increasing \( \Gamma \).

It should be noted that the stable equilibrium point in the Poincaré plots presented in Fig. 2 is displaced from the origin \( x = 0, p = 0 \). This occurs due to nonlinear resonance of multiplicity 1:1 between oscillations of the perturbation with the frequency \( \omega_0 = 1 \), and the unperturbed oscillations in the vicinity of the original equilibrium point. As the resonance 1:1 has relatively large width in phase space, the atoms it traps can undergo large-amplitude oscillations in the energy space, remaining inside the regular domain though. If \( \Gamma > 0 \), the presence of such oscillations facilitates activation of atoms initially located near the center of phase space, because the resonance acts as a “lift” carrying atoms from the center upwards in energy, thus reducing their activation energy. In this way, \( \omega_0 = 1 \) seems to be close to the optimal choice of the driving frequency [17, 18]. Indeed, it is well-known that the lowest-order resonances typically have the largest widths in the energy space, thereby providing the most efficient excitation.

\section*{4. Quantum transport}

Let’s proceed with considering quantum evolution by solving numerically the Schrödinger equation (4). Given a realization of the modulation \( f(t) \), one can calculate the asymptotic current defined as [19]
\[ J(t) = \frac{1}{i} \int_0^t p_q(t') \, dt', \tag{21} \]
where \( p_q \) is quantum-mechanical momentum expectation value
\[ p_q(t) = \int dx \hat{\Psi}^* \hat{\rho} \hat{\Psi}, \quad \hat{\rho} = -i\hbar \frac{\partial}{\partial x}. \tag{22} \]
Averaging over realizations of \( f(t) \), we obtain the statistical mean of the asymptotic current
\[ \langle J \rangle = \frac{1}{N} \sum_{n=1}^N f^{(n)}. \tag{23} \]
where integer \( n \) labels the realizations. Our computations were performed with \( N = 100 \). As long as we consider the case of a deep optical lattice, the rescaled Planck constant \( \hbar \) was taken of 0.1. Initial state used is a random superposition of coherent states:
\[ \Psi(t = 0) = A \sum_{n=1}^{N_0} \phi(x_0, p_n), \tag{24} \]
\[ \phi(x_0, p_n) = (2\pi)^{-1/4} \Delta_x^{-1/2} e^{-\frac{(x-x_0)^2}{4\Delta_x^2} - \frac{|p-p_n|}{2\Delta_p}}, \]
where \( x_0 = 0, \Delta_x = 0.25, N_{ts} = 10^4 \), and \( p_n \) is random quantities obeying the Gaussian statistics with zero mean and variance 0.25. This initial condition corresponds to a wavepacket tightly localized near the stable equilibrium point of the unperturbed system \( x = 0, p = 0 \).

\[
\langle J \rangle (t) \text{ or } \langle E \rangle (t) \text{ is readily controlled by the sign of } s.
\]

\[
\text{Figure 3: Time dependence of the ensemble-averaged asymptotic current for } s = -1 \text{ (solid) and } s = 1 \text{ (dashes). In both cases } \Gamma = 0.4.
\]

\[
\text{Figure 4: Dependence of the ensemble-averaged asymptotic current at } t = 200\pi \text{ on } \Gamma.
\]

\[
\text{It is reasonable to examine how current varies with increasing of the noise contribution which can be quantified by } \Gamma. \text{ Indeed, small noise contribution anticipates weak diffusion and slow crossing of the dynamic barriers in phase space. On the other hand, large amount of noise should destroy the plane-wave form of the perturbation due to the loss of coherence between the modulating signals } f(t) \text{ and } f(t + \Delta). \text{ As long as the plane-wave form is responsible for violation of space-time symmetries, resulting transport should become undirected. Consequently, there should be some intermediate range of } \Gamma \text{ values corresponding to the most efficient activation of directed current. To find it, we consider finite time interval } [0 : 200\pi] \text{ and calculate the dependence of the mean asymptotic current at the endpoint of the interval on } \Gamma. \text{ The result computed with } s = 1 \text{ is presented in Fig. 4.}
\]

\[
\text{According to above expectations, the dependence of } J_{\text{end}} \text{ on } \Gamma \text{ is not monotonic, and the most efficient generation of ballistic current corresponds to } 0.3 < \Gamma < 0.6. \text{ It should be mentioned that directed transport is almost absent in the purely deterministic case } \Gamma = 0. \text{ It unambiguously indicates on the importance of noise in the activation of ballistic atomic current.}
\]

\[
\text{To underline the link between the onset of directed current and heating of atoms, we calculate the mean energy determined as}
\]

\[
\langle E \rangle (t) = \frac{1}{N} \sum_{n=1}^{N} dx \Psi^*(t) \hat{H}^\text{eff} \Psi(t), \tag{25}
\]

\[
\text{where}
\]

\[
\hat{H}^\text{eff} = -\frac{\hbar^2}{2} \frac{\partial^2}{\partial x^2} + 1 - \cos x + \epsilon V^\text{eff}(x, t)
\]

\[
\text{is the Hamiltonian operator corresponding to the } n\text{-th realization of perturbation } V(x, t). \text{ Dependence of } E_{\text{end}} \equiv \langle E \rangle (t = 200\pi) \text{ on } \Gamma, \text{ demonstrated in Fig. 5 reveals that the range of the } \Gamma \text{ values corresponding to the most efficient current generation, } 0.3 \leq \Gamma \leq 0.6, \text{ simultaneously corresponds to the most efficient heating. Non-monotonous dependence on the noise level indicates on the significance of factors of heating, which are not concerned with noise. In our case, the role of such factor is played by classical resonance 1:1 whose impact decreases as } \Gamma \text{ grows. Thus, the optimal activation occurs when resonance-assisted and diffusive mechanisms of atom heating accompany each other constructively. It should be noted that decreasing of } E_{\text{end}} \text{ for } \Gamma > 0.6 \text{ is significantly slower than the corresponding decreasing of } |J_{\text{end}}| \text{ (see Fig. 4). This can be understood as noise-induced recovery of space-time symmetries due to loss of correlations between harmonic noise values } f(t) \text{ and } f(t + \Delta).
5. Summary

We presented a novel approach to produce directed atomic current in an optical lattice by means of weak unbiased perturbation consisted of two lattices with broadband amplitude modulation. In this work we model the broadband modulation as harmonic noise. The approach presented allows for the current generation even if atoms are tightly confined by the lattice potential at the initial moment. It is shown that the efficiency of generation depends non-monotonously on the parameter describing the spectral width of the perturbation. We suppose that the approach reported can be used for production of directed current in a more complicated configurations of the optical potential, for example, in potentials with random disorder.

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