Spintronic Spin Accumulation and Thermodynamics

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Abstract. The spin degree of freedom can play an essential role in determining the electrical transport properties of spin-polarized electron systems in metals or semiconductors. In this article, I address the dependence of spin-subsystem chemical potentials on accumulated spin-densities. I discuss both approaches which can be used to measure this fundamental thermodynamic quantity and the microscopic physics which determines its value in several different systems.

1 Introduction

The role of the electronic spin degree of freedom in theories of the electrical transport properties of paramagnetic metals is passive; usually it as appears only as an afterthought—a factor of two to account for spin degeneracy. All this changes profoundly in electronic systems with substantial spin-polarization, either spontaneous or induced, particularly so if the system is either electrically or magnetically inhomogeneous. Recently, interest in the role of the electronic spin has increased, in part because of the possibility of fabricating technologically useful magnetoresistive sensors and other devices based on spin-dependent transport effects, particularly giant magnetoresistance [1] and tunnel magnetoresistance [2]. Spintronics [3], the study of spin-dependent electronic transport effects in systems containing metallic ferromagnets, is now a large and active area of basic and applied physics. In this article, I discuss the possibility of using transport experiments not to make devices, but instead to measure a fundamental thermodynamic property of a spin-polarized electron system, the dependence of spin subsystem chemical potentials on accumulated spin-densities. This quantity can be important in modeling some spin-dependent transport effects. I will discuss several examples where it also provides a new and useful test of our understanding of the microscopic physics of a spin-polarized itinerant electron system.

In Section 2, I derive a formally exact expression for the dependence of chemical potentials on subsystem densities by defining a spin-dependent thermodynamic-density-of-states matrix. For bulk three-dimensional charged particle systems, the total density is fixed by electroneutrality requirements. In this case only the dependence of up and down spin chemical potentials on the difference between up and down spin densities is of interest. I show that these two quantities can be expressed in terms of the differential magnetic susceptibility, and a less familiar quantity, the derivative of chemical potential with respect to external field. The
focus of the article is this latter quantity, which I will refer to as the *inverse magnetic compressibility*. In Section 3, I discuss two spintronics experiments which can be exploited to measure its value in particular systems. In Section 4, I discuss the microscopic physics which determines its value in three distinct spin-polarized electron systems. Section 5 contains a brief summary.

2 Thermodynamic Density-of-States Matrix

Non-equilibrium spin accumulation [4, 5] due to electronic transport occurs generically in inhomogeneous spin-polarized electron systems and is a ubiquitous feature of spintronics. Any theory of spin accumulation requires, explicitly or implicitly, a model for the relationship between the up and down spin densities and their chemical potentials. Linearizing around the equilibrium state, we can write

\[ d\mu_{\uparrow} = (D_{\uparrow}^{-1} + F_{\uparrow,\uparrow}) dn_{\uparrow} + F_{\uparrow,\downarrow} dn_{\downarrow}, \]

\[ d\mu_{\downarrow} = F_{\downarrow,\uparrow} dn_{\uparrow} + (D_{\downarrow}^{-1} + F_{\downarrow,\downarrow}) dn_{\downarrow}. \]  

(1)

where \( \mu_\sigma \) is the spin-\( \sigma \) chemical potential and \( n_\sigma \) is the density of spin-\( \sigma \) electrons. In these equations, I treat \( n_\uparrow \) and \( n_\downarrow \) as separate thermodynamic variables, something which is useful in discussing spin-accumulation since the processes which establish equilibrium between spin-\( \uparrow \) and spin-\( \downarrow \) subsystems are often slow. The spin-quantization axis has been chosen to lie along the direction of net spin-polarization [6].

It is normally convenient to measure the local chemical potential of a charged particle system from the local electrostatic potential and this common convention is implicit throughout these notes. Accordingly

\[ \mu_\sigma \equiv \frac{1}{V} \frac{\partial F(T, n_\uparrow, n_\downarrow)}{\partial n_\sigma} \]  

(2)

with the free-energy per volume, \( F/V \), calculated excluding any electrostatic contributions. The matrix of coefficients in Eq. (1) is accordingly given by the matrix of second derivatives of \( F/V \) with respect to \( n_\sigma \), with the electrostatic term (which would diverge because of the long-range of the electron-electron interaction) neglected. (Note that \( F_{\uparrow,\downarrow} = F_{\downarrow,\uparrow} \).) To make contact with familiar descriptions of spin accumulation, I have introduced the band-theory spin-dependent density-of-states per volume, \( D_\sigma \). If contributions due to correlation effects were neglected, the relationship between chemical potentials and densities would be diagonal in spin indices and only the density-of-states terms would appear on the right-hand-side of Eq. (1). The density-of-states contribution to the spin-\( \sigma \) chemical potential change is simply the result of changing the filling of the spin-\( \sigma \) energy band. I will refer to the additional correlation terms in Eq. (1), \( F_{\sigma,\sigma'} \), as local-field corrections. The fact that the local-field corrections are in general off-diagonal in the spin indices can have a qualitative importance.
Eq. (1) is formally exact, however, the values for the local field corrections, are not known in general, and their computation is a challenge to theory. In the following Section, I discuss two experiments which measure a particular combination of these coefficients.

Eq. (1) can be inverted to express spin-dependent density changes in terms of spin-dependent chemical potential changes

\[
dn_\sigma = \sum_{\sigma'} D_{\sigma,\sigma'} d\mu_{\sigma'}
\]

where

\[
D_{\sigma,\sigma} = \frac{D_\sigma (1 + D_\sigma F_{\sigma,\bar{\sigma}})}{1 + D_\sigma F_{\sigma,\sigma} + D_\sigma F_{\bar{\sigma},\sigma} - D_\sigma D_\bar{\sigma} F_{\sigma,\bar{\sigma}}^2},
\]

\[
D_{\sigma,\bar{\sigma}} = - \frac{D_\sigma D_{\bar{\sigma}} F_{\sigma,\bar{\sigma}}}{1 + D_\sigma F_{\sigma,\sigma} + D_\sigma F_{\bar{\sigma},\sigma} - D_\sigma D_{\bar{\sigma}} F_{\sigma,\bar{\sigma}}^2},
\]

where \(\bar{\sigma} = \downarrow\) if \(\sigma = \uparrow\) and vice-versa. \(D \equiv \sum_{\sigma,\sigma'} D_{\sigma,\sigma'}\), the rate of change of total density with chemical potential when the spin-subsystems are in equilibrium, is the total thermodynamic density-of-states of an electron system. If interactions are neglected \(D = D_\uparrow + D_\downarrow\). In spintronics it is useful to generalize this concept by defining a thermodynamic density-of-states matrix as in Eq. (4).

To evaluate the inverse magnetic compressibility, I add to the Hamiltonian a Zeeman coupling [7] term which contributes \(-g^* \mu_B H (n_\uparrow - n_\downarrow) / 2\) to the free energy per unit volume. Here \(g^*\) is the system’s g-factor, \(\mu_B\) is the electron Bohr magneton, and \(H\) is the field strength. It is convenient to use a notation where \(\mu_\sigma\) is defined as the chemical potential without its Zeeman contribution, whereas \(\mu\) is the full field-dependent chemical potential. Then the condition for equilibrium between up and down spins is

\[
\mu = \mu_\uparrow - g^* \mu_B H / 2 = \mu_\downarrow + g^* \mu_B H / 2,
\]

or differentiating with respect to field strength:

\[
\frac{\partial \mu}{\partial g^* \mu_B H} = - \frac{\partial \mu_\uparrow}{\partial g^* \mu_B H} - \frac{1}{2} = \frac{\partial \mu_\downarrow}{\partial g^* \mu_B H} + \frac{1}{2}
\]

At fixed total electron density \((dn_\uparrow + dn_\downarrow = 0)\), differentiating Eq. (3) with respect to \(g^* \mu_B H\) gives

\[
\frac{\partial \mu_\uparrow/\partial H}{\partial \mu_\downarrow/\partial H} = \frac{D_{\uparrow,\downarrow} + D_{\downarrow,\downarrow}}{D_{\uparrow,\uparrow} + D_{\downarrow,\uparrow}} = - \frac{D_{\uparrow}^{-1} + F_{\uparrow,\uparrow} - F_{\downarrow,\uparrow}}{D^{-1}_{\downarrow} + F_{\downarrow,\downarrow} - F_{\uparrow,\downarrow}}.
\]

Combining Eq. (7) and Eq. (6), I obtain that for fixed total density

\[
\frac{\partial \mu}{\partial g^* \mu_B H} = \frac{D_{\uparrow}^{-1} - D_{\downarrow}^{-1} + F_{\uparrow,\uparrow} - F_{\downarrow,\downarrow}}{2[D_{\uparrow}^{-1} + D_{\downarrow}^{-1} + F_{\uparrow,\downarrow} + F_{\downarrow,\downarrow} - 2F_{\uparrow,\downarrow}]}.
\]
Similar considerations lead to the following expression for the differential magnetic susceptibility,

$$\chi_S = \frac{g^* \mu_B}{2} \frac{\partial (n_\uparrow - n_\downarrow)}{\partial H} = \frac{(g^* \mu_B)^2}{D_\uparrow^{-1} + D_\downarrow^{-1} + F_{\uparrow,\uparrow} + F_{\downarrow,\downarrow} - 2F_{\uparrow,\downarrow}}.$$  \(9\)

Both of these expressions are formally exact. I show in the following paragraph that the dependence of the chemical potentials on spin accumulation is specified by the inverse magnetic compressibility and $\chi_S$.

Theories of spin accumulation contain in general three elements: i) a theory of the spin and space dependent transport coefficients which lead to non-equilibrium spin-densities, ii) a theory for the disequilibration $(\mu_\uparrow - \mu_\downarrow)$ produced by these spin densities, and iii) a theory of the relaxation process which attempts to establish equilibrium between the spin subsystems. The thermodynamic property we are discussing is related to the second element. Assuming electroneutrality $(dn_\downarrow = -dn_\uparrow)$ it follows from Eq. (1) that

$$\frac{d\mu_\uparrow}{d(n_\uparrow - n_\downarrow)} = \frac{D_\uparrow^{-1} + F_{\uparrow,\uparrow} - F_{\uparrow,\downarrow}}{2}, \quad \frac{d\mu_\downarrow}{d(n_\uparrow - n_\downarrow)} = -\frac{D_\downarrow^{-1} + F_{\downarrow,\downarrow} - F_{\uparrow,\downarrow}}{2}.$$  \(10\)

Note that $d(\mu_\uparrow - \mu_\downarrow)/d(n_\uparrow - n_\downarrow) = (g^* \mu_B)^2/2\chi_S$. This relationship should not be a surprise since an external magnetic field maintains a chemical potential difference $\mu_\uparrow - \mu_\downarrow = g^* \mu_B H$ and induces a magnetization per volume $m = \chi_S H = g^* \mu_B (n_\uparrow - n_\downarrow)/2$. The linear relationship between $\mu_\uparrow - \mu_\downarrow$ and the non-equilibrium spin accumulations is thus completely characterized by $\chi_S$. It is only if we want to know the chemical potential shifts of up-spin and down-spin subsystems individually that the inverse magnetic compressibility is required. The individual chemical potential shifts driven by a non-equilibrium spin accumulation are:

$$\frac{4\chi_S}{(g^* \mu_B)^2} \frac{d\mu_\uparrow}{d(n_\uparrow - n_\downarrow)} = 1 + 2 \frac{d\mu}{d(g^* \mu_B H)} = \frac{D_\uparrow^{-1} + F_{\uparrow,\uparrow} - F_{\uparrow,\downarrow}}{D_\downarrow^{-1} + D_\uparrow^{-1} + F_{\downarrow,\downarrow} + F_{\uparrow,\downarrow} - 2F_{\uparrow,\downarrow}},$$

$$\frac{4\chi_S}{(g^* \mu_B)^2} \frac{d\mu_\downarrow}{d(n_\uparrow - n_\downarrow)} = -1 + 2 \frac{d\mu}{d(g^* \mu_B H)} = -\frac{D_\downarrow^{-1} + F_{\downarrow,\downarrow} - F_{\uparrow,\downarrow}}{D_\downarrow^{-1} + D_\uparrow^{-1} + F_{\uparrow,\uparrow} + F_{\downarrow,\downarrow} - 2F_{\uparrow,\downarrow}}.$$

3 Spintronic Thermodynamic Measurements

3.1 Field-dependent Coulomb Blockade Peaks

The first type of experiment I discuss was pioneered by Ono and co-workers [8] and takes advantage of the equally spaced conductance peaks which occur in Coulomb blockade devices. The experimental geometry is that of a single-electron-transistor (SET) in which current flow from source to drain through
a small metallic particle is influenced by a gate voltage. In general source lead, drain lead, and metallic particle can be either paramagnetic or ferromagnetic. For definiteness, I assume that the only the small metallic particle is ferromagnetic and address a situation which is simpler than what has been encountered in experiments by also assuming a single domain. The following paragraph provides a simplified explanation the operation of a SET which is sufficient for present purposes.

The dependence of the ground-state energy of an isolated metallic grain on its net charge is dominated by an electrostatic contribution and has the form

$$E_0(N) = \frac{e^2(N - N_0)^2}{2C} + V\epsilon_0(N/V)$$

(11)

where $e(N - N_0)$ is the net charge on the grain and the effective capacitance of the grain $C \sim R$ where $R$ is the grain diameter. Here $\epsilon_0(n)$ is the energy per unit volume $V$ calculated for a macroscopic grain which is electrically neutral. The conductance for current flow through the grain between lead electrodes is sharply peaked when the addition energy of the island is equal to the chemical potential in the lead electrodes $\mu_L$, i.e. when

$$\mu(N) = E_0(N + 1) - E_0(N) = \mu_L.$$ 

(12)

In a SET the chemical potential for electrons on the metallic grain is manipulated by a gate voltage $U$:

$$E_0(N) \rightarrow \frac{e^2(N - N_0)^2}{2C} + V\epsilon_0(N/V) - NeU.$$ 

(13)

As $U$ is varied, the equilibrium number of particles on the grain changes. The number of particles in the grain’s ground state changes between $N$ and $N + 1$ when Eq. (12) is satisfied:

$$eU^*_N = e^2(N + 1/2)/C + \mu(N/V) - \mu_L.$$ 

(14)

A peak occurs in the source-drain conductance at this value of the gate voltage. In this equation $\mu(n)$ is the bulk chemical potential of an electrically neutral system, the quantity examined in Section 2.

As discussed in Section 2, $\mu(n)$ is field-dependent in general. It follows from Eq. (14) that provided the field-dependence of the lead chemical potentials can be ignored

$$\frac{d\mu(n)}{d(g^*\mu_B H)} = \frac{d(eU^*_N)}{d(g^*\mu_B H)};$$

(15)

the quantity of interest can simply be read off the gate voltage dependence of the Coulomb blockade conductance peak. More realistic models are complicated by geometry dependent cross capacitances between different circuit elements which invalidate the simple relationship between the chemical potential on the ferromagnetic grain and the gate voltage assumed here. However these two quantities are still proportional and the proportionality constant can be sorted out.
experimentally by measuring the spacing between Coulomb blockade conductance peaks at fixed voltage. Estimates of \( d(\mu_0(n)/d(g^*\mu_B H) \) have already been obtained [8] using this approach for one ferromagnetic transition metal.

### 3.2 Field-dependent double-layer compressibility measurements

The second potential experiment I discuss is a variant on one which has been used in the past [9, 10, 11, 12] to measure the compressibility of two-dimensional electron gas layers. It exploits techniques which have been developed [13] to make separate contact to nearby two-dimensional electron layers. The experimental setup can be thought of as a parallel plate capacitor, where one plate is a metal or heavily doped semiconductor and the second plate consists of two separately contacted two-dimensional electron layers, one on top of the other and closer to the metallic gate. A change in the charge density on the surface of the metal induces an opposing charge density distributed between the two two-dimensional electron layers. The equilibrium condition which determines the distribution of charge is:

\[
\mu_T(n_T) = \mu_B(n_B) + 4\pi e^2 d(n - n_T - n_0/2). \tag{16}
\]

Here \( n_T \) is the areal density in the top two-dimensional layer, \( n_B \) is the density in the bottom two-dimensional layer, \( d \) is the separation between the two layers, and \( n = n_T + n_B \) is the total density in the two-layers. The second term on the right hand side of Eq. (16) is the electrostatic potential drop due to the electric field which exists between top and bottom two-dimensional layers. Note that the electric field between the gate and the top layer has magnitude \( 4\pi e(n - n_0/2) \), i.e. \( e n \) equals the surface charge density on the metallic gate up to a constant. If the top two-dimensional layer is held at ground, the gate voltage \( V_G \) is therefore proportional to \( n \). The magnitude of the electric field below the bottom two-dimensional layer, \( 4\pi e n_0/2 \), does not change during the experiment.

To determine the compressibility [9], it is necessary only to measure the current which flows to the bottom layer when the gate voltage \( V_G \) (or equivalently \( n \)) is changed. Differentiating Eq. (16) with respect to \( n \) with \( n_B + n_T = n \) leads to [9] a relationship between a fundamental thermodynamic quantities and a conveniently measurable dimensionless experimental quantity exploited in previous experimental work:

\[
\frac{dn_B}{dn} = \frac{d\mu_T/dn_T}{4\pi e^2d + d\mu_T/dn_T + d\mu_B/dn_B} \tag{17}
\]

Since the first term in the denominator of Eq. (17) is normally dominant the experiment provides a direct measurement of the dependence of chemical potential on density, inversely proportional to the compressibility, for the top layer.

I propose using the same experimental setup to measure the dependence of the charge in the bottom layer on magnetic field at fixed gate voltage. Such a
measurement can be related to the inverse magnetic compressibility; differentiating Eq. (16) with respect to field at fixed \( n \) I find that

\[
\frac{dn_B}{d(g^*\mu_B H)} = \frac{d\mu_T/d(g^*\mu_B H) - d\mu_B/d(g^*\mu_B H)}{4\pi e^2d + d\mu_T/dn_T + d\mu_B/dn_B}
\]  

(18)

Since the denominator is dominated by the first term, the experiment provides a direct measurement of the difference between the rate of change chemical potential with field in top and bottom 2D layers.

4 Microscopic Theory of the Thermodynamic Density-of-States Matrix

4.1 Band Ferromagnets

In order to contextualize the issues raised by these relatively new measurement possibilities, I first discuss the estimate for \( d\mu/d(g^*\mu_B H) \) which follows from the simplest possible mean-field theory of a ferromagnetic metal, Stoner-Wohlfarth [14] theory. In modern work this approach is wrapped in the cloak of spin-density-functional theory [15]. In the Stoner-Wohlfarth theory, quasiparticle spin-up and spin-down energies for each band and for each wavevector in the crystal’s Brillouin-zone are split by an amount

\[
\Delta = 2\mu_B H_{\text{eff}} = 2(I m + \mu_B H).
\]  

(19)

The effective magnetic field includes an exchange contribution which is proportional to the magnetic moment per Bohr magneton per volume of the system,

\[
m = \frac{M}{\mu_B} = n_\uparrow - n_\downarrow.
\]  

(20)

where \( M = \mu_B(N_\uparrow - N_\downarrow) \) is the total magnetic moment. The exchange integral \( I \) is a phenomenological material property which is characteristic of a given system. In a given field, \( m \) and the chemical potential \( \mu \) are determined by self-consistently occupying spin-split bands. At \( T = 0 \) the self-consistent mean-field equations are

\[
m = \int_{\mu - \Delta/2}^{\mu + \Delta/2} d\epsilon \frac{d\epsilon}{e}\frac{d\epsilon}{D(\epsilon)}
\]

\[
n = 2 \int_{-\infty}^{\mu - \Delta/2} d\epsilon D(\epsilon) + \int_{\mu - \Delta/2}^{\mu + \Delta/2} d\epsilon D(\epsilon)
\]  

(21)

where \( D(\epsilon) \) is the density-of-states per volume per spin at \( \Delta = 0 \). Let \( \Delta_0 \) and \( I \) denote the self-consistent exchange splitting of the band and the self-consistent
value of $m$ in the absence of an external field respectively. Expanding $\mu$ and $\Delta$ to first order in $H$, I find that

$$\frac{d\mu}{d(g^*\mu B H)} = \frac{D_{\downarrow}^{-1} - D_{\uparrow}^{-1}}{2[D_{\downarrow}^{-1} + D_{\uparrow}^{-1} - 4I]} \tag{22}$$

Here $D_{\uparrow} = D(\mu_0 + \Delta_0/2)$ and $D_{\downarrow} = D(\mu_0 - \Delta_0/2)$ are the majority and minority spin densities of states in the absence of a field.

This expression can be recognized as a special case of the general result derived in Section 2 which is obtained by holding the bands rigid and taking $F_{\uparrow,\downarrow} = -F_{\uparrow,\uparrow} = F_{\downarrow,\downarrow} = I$. In the formally exact theory, the three local-field-factors are independent. Density-functional theory [15] is a practical approach to many-particle physics which has been applied successfully to evaluate the thermodynamic properties of metals and underpins the modern theory [16] of ferromagnetic transition metals. In density-functional theory, the local field factors arise from an interplay between band-structure details and exchange-correlation single-particle potentials. Expectations based on the electron gas case, discussed for two-dimensions below, suggest that the diagonal local-field-factors should be negative and larger in magnitude than the positive off-diagonal local-field-factor. It is not at all obvious that these expectations apply to transition metals, especially because the majority spins at the Fermi energy tend to have predominantly itinerant $s$-electron character while the minority spins tend to have predominantly localized $d$-electron character. Comparison of theoretical and experimental values for both the differential magnetic susceptibility, and the magnetic compressibility, presents a serious and interesting challenge to the density-functional theory of metallic magnetism.

### 4.2 Zero Field Two-Dimensional Electron Gas

The ground state of the two-dimensional electron gas is not ferromagnetic except at extremely low densities [18, 19]. Spin-polarization can, however, be induced by application of an external magnetic field [20]. I now discuss a simple theory for the value of $d\mu/d(g^*\mu B H)$ in a spin-polarized two-dimensional electron gas which is based on the Hartree-Fock approximation [21] for its energy:

$$\frac{E_{n_\uparrow, n_\downarrow}}{A} = \frac{n_\uparrow^2 + n_\downarrow^2}{2D_2D} - \frac{8e^2[n_\uparrow^{3/2} + n_\downarrow^{3/2}]}{3\pi^{1/2}} \tag{23}$$

Here $D_0 = m^*/(2\pi \hbar^2)$ is the density-of-states per area $A$ in a 2D system. The first term on the right hand side of Eq. (23) is the band energy and the second term is the exchange energy. Differentiating twice with respect to density we obtain for the diagonal local-field-correction

$$F_{\sigma,\sigma} = -\frac{2e^2n_\sigma^{-1/2}}{\pi^{1/2}}. \tag{24}$$
The off diagonal local-field-corrections vanishes in this approximation because of the neglect, in the Hartree-Fock approximation, of correlations between electrons of opposite spin. In the presence of an external magnetic field the partitioning of density between spin-subsystems is determined by setting 
\[ \mu_\uparrow - \mu_\downarrow = g^* \mu_B H \]
where \( \mu_\sigma = n_\sigma / D_{2D} - 4e^2 n_\sigma^{1/2} / \pi^{1/2} \). For electron-gas density parameter \( r_s = 2 \), the Hartree-Fock ground state is paramagnetic and the spin-polarization is initially linear in field. At stronger fields, the exchange term causes the minority-spin chemical potential to increase with decreasing density. The polarization must then increases more rapidly with field in order to establish the required chemical potential difference. The polarization eventually jumps from a partial value to full polarization when \( g^* \mu_B H \sim 0.034 \text{Ry} \). (The atomic energy unit 
\[ 1 \text{Ry} = e^4 m^*/2 \bar{\hbar}^2 \approx 5.5 \text{meV in GaAs}. \]

The results for the Zeeman coupling dependence of subsystem chemical potentials and spin-polarization, shown in Fig.( 1) and Fig.( 2) respectively, can lead to substantial values of \( d\mu/d(g^* \mu_B H) \) which can even exceeds one over a wide range of Zeeman coupling strength. This is despite the fact that the minority and majority spin density-of-states are identical in two-dimensions so that the inverse magnetic compressibility would be identically zero for a non-interacting electron system. Numerical results for the inverse magnetic compressibility for this illustrative example are shown in Fig.( 3). Of course, we do not expect the Hartree-Fock approximation theory to be reliable in an electron gas for \( r_s > 1 \).

The large spin-polarizations at relatively weak Zeeman couplings in Fig. (2), occur because the Hartree-Fock approximation ground state is ferromagnetic for \( r_s > \pi/2^{1/2} \). We know from reliable quantum Monte Carlo calculations\[18, 19\] that, in reality, this instability does not occur until much lower densities are reached. Nevertheless illustrative calculation does suggest what should be expected in low-density two-dimensional electron gas layers. For modern samples,
Fig. 2. Hartree-Fock approximation $m/n \equiv (n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow)$ for a two-dimensional electron gas with density parameter $r_s = 2$ vs. Zeeman coupling strength $g^* \mu_B H$ in atomic units ($e^4 m^*/2\hbar^2$). For GaAs the Zeeman coupling strength in atomic (Ry) units is $0.0052 H[\text{Tesla}]$. The system jumps to full polarization at $g^* \mu_B H \sim 0.034 \text{Ry}$.

Fig. 3. Hartree-Fock approximation $d\mu/d(g^* \mu_B H)$ vs. Zeeman coupling strength, for a two-dimensional electron gas with density parameter $r_s = 2$. For $g^* \mu_B H > 0.034$, only the majority spins are occupied and $d\mu/d(g^* \mu_B H) \equiv -0.5$.

which have high mobility at densities a few \times 10^{10} \text{cm}^{-2}, it is clear that a substantial degree of spin-polarization (and possibly complete spin polarization) can be achieved in laboratory superconducting magnets. Measurement of the inverse magnetic compressibility, coupled with simultaneous measurements of the spin polarization, will provide a strict test of our understanding of magnetism in strongly correlated electron gas systems.

4.3 Quantum Hall Ferromagnets

In the quantum Hall regime, the 2D electron density is measured in units of the density which can be accommodated by a full Landau level; the Landau level filling factor $\nu \equiv (2\pi \ell^2) n$ where $\ell \equiv (\hbar c/eB)^{1/2}$ is the magnetic length. At Landau level filling factor $\nu = 1$, the 2D electron system has a strong ferromagnet ground state, i.e. all spins can be aligned with the field by arbitrarily weak Zeeman cou-
aling. Quantum Hall ferromagnets have unusual charged excitations known as Skyrmions [22, 23]. Skyrmions are topologically non-trivial configurations of the magnetization orientation distribution which occur in any 2D magnet, but carry an electrical charge $e$ only in the case of quantum Hall ferromagnets. Skyrmions have an internal integer quantum number $K$ which specifies the number of reversed spins in their interior. When the Landau level filling factor is close to $\nu = 1$, the low energy states of the system can be described in terms of Skyrmion degrees of freedom.

A simple model of a 2DES, valid at low temperature in this regime because Skyrmions are dilute, is obtained by ignoring Skyrmion-Skyrmion interactions. The following grand-canonical ensemble expressions specify the occupation probabilities of the $N_\phi = A/(2\pi\ell^2)$ Skyrmion quasielectron and quasihole states with $K$ excess reversed spins:
\begin{align}
  n_{Ke} &= f(\epsilon_K + K\mu_\uparrow - (K+1)\mu_\downarrow) \\
  n_{Kh} &= f(\epsilon_K + (K+1)\mu_\uparrow - K\mu_\downarrow)
\end{align}
(25)

Here $f(\epsilon) = (\exp(\epsilon/k_B T) + 1)^{-1}$ is a Fermi factor [25], $\epsilon_K$ is the energy of a Skyrmion quasiparticle, $(2\pi\ell^2)^{-1}$ is the density of a full Landau level, and we have chosen the zero of energy so that quasielectron and quasihole skyrmion states have the same energy[26]. When the spin subsystems are in equilibrium, we can use Eqs. (25) to calculate the chemical potential, given the Landau level filling factor. The Landau level filling factor is increased by quasielectron excitations and decreased by quasihole excitations:
\begin{equation}
  \nu = 1 + \sum_K (n_{Ke} - n_{Kh}).
\end{equation}
(26)

Eq. (25) follows from the property that formation of the $K$-th quasielectron Skyrmion requires the addition of $K + 1$ spin-down electrons and the removal of $K$ spin-up electrons from the $\nu = 1$ ground state, while formation of the $K$-th quasihole Skyrmion requires the addition of $K$ spin-down electrons and the removal of $K + 1$ spin-up electrons. For non-interacting electrons only the $K = 0$ quasiparticles occur; for typical 2DES’s, on the other hand, the lowest energy quasiparticles have $K = 3$; these quasiparticles dominate the low-temperature properties of the system for $\nu$ close to 1. From Eqs. (25) I obtain the following thermodynamic density-of-states matrix:
\begin{align}
  D_{\uparrow,\uparrow} &= (2\pi\ell^2)^{-1} \sum_K [(K+1)^2 \Delta(\epsilon_K + \mu) + K^2 \Delta(\epsilon_K - \mu)] \\
  D_{\uparrow,\downarrow} &= -(2\pi\ell^2)^{-1} \sum_K K(K+1) [\Delta(\epsilon_K + \mu) + \epsilon_K - \mu)]
\end{align}
(27)
(28)

where $\Delta(x) = \text{sech}^2(x/2)/4k_B T$.

These expressions have been used previously to analyze [28] spin bottlenecks which have been observed [29] in the quantum Hall regime. Here I evaluate the...
dependence of the equilibrium chemical potential on Zeeman coupling strength. The possibility of measuring this quantity in two-dimensional electron systems was discussed in the previous section. Zeeman coupling adds \[ |22, 26|\] to the Skyrmion quasiparticle energies, \( \epsilon_K \rightarrow \epsilon_K + g^* \mu_B H (K + 1/2) \). In order for the filling factor to be held fixed as the Zeeman coupling strength varies, the chemical potential must change:

\[
\frac{\partial \mu}{\partial (g^* \mu_B H)} = - \frac{\partial n / \partial (g^* \mu_B H)|_\mu}{\partial n / \partial \mu|_H} = \frac{\sum_K (K + 1/2)[\Delta(\epsilon_K - \mu) - \Delta(\epsilon_K + \mu)]}{\sum_K [\Delta(\epsilon_K - \mu) + \Delta(\epsilon_K + \mu)]}
\]

We see in Fig. (4) that for an ideal disorder-free two-dimensional electron system, \( d\mu/d(g^* \mu_B H) \) is typically larger than one and changes sign when the filling factor crosses \( \nu = 1 \). These results were calculated using Skyrmion state energies evaluated by Palacios et. al. [27]. The value of this quantity gives information about the Skyrmion system complementary to that available from other experiments[23]. Its value depends both on the average \( K \) value of the Skyrmions present in the system, the quantity revealed by Knight shift measurements, but also on the way in which the distribution of \( K \) values changes with field. Unlike transport experiments, this thermodynamics measurement would be sensitive to Skyrmions localized by disorder in the system.

Fig. 4. \( d\mu/d(g^* \mu_B H) \) vs. Landau level filling factor for a quantum Hall system near \( \nu = 1 \) at temperature \( k_B T = 0.02 e^2/\ell \).
5 Summary

In this article, I have discussed the dependence of chemical potential on external magnetic field in a spin-polarized electron system. This quantity appears directly in theories of electron systems disturbed by non-equilibrium spin accumulations generated by spin and space-dependent electronic transport. I have discussed techniques which can be used to measure this fundamental thermodynamic quantity, both in two-dimensional electron systems and in ferromagnetic metals. I believe that its measurement can provide an important and interesting test of theory in several quite different spin-polarized electron systems.

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