Multiple Shape Coexistence in $^{110,112}$Cd and Beyond
Mean Field Calculations

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Abstract. Detailed spectroscopy employing the $\beta^+$/EC decay of $^{110,112}$In and the $\beta^-$ decay of $^{112}$Ag has been used to study the excited states of $^{110,112}$Cd. Low-energy decay branches from highly excited states have been observed and, combined with level lifetimes from the $(n,n'\gamma)$ reaction, permit $B(E2)$ values to be determined thus revealing rotational-like bands built on excited $0^+$ states and $\gamma$ bands built on the ground and the shape-coexisting intruder states. The excitation energies of the $0^+_4$ states appear incompatible with that expected for a pure $\pi(4\rho6\ell)$ configuration. The experimental results for the $0^+_4$ excited states are compared with beyond-mean-field calculations that suggest they possess different shapes, including prolate, oblate, and triaxial.
1. Introduction

The long-standing view of the structure of the mid-neutron-shell Cd nuclei has been of nearly harmonic, vibrational states built on approximately spherical ground states coexisting with deformed intruder excitations built on $\pi(2p4h)$ configurations. The two-phonon-vibration triplets of states $0^+, 2^+$, and $4^+$ appeared to be well established and there were many studies seeking the higher-lying multiphonon excitations, especially in $^{110,112}$Cd, which were considered as paradigms of harmonic vibrational motion (see, e.g., Refs. [1, 2, 3]) but requiring strong mixing between the spherical phonon states and deformed intruder states [4, 5, 6]. There have been a number of studies [7, 8, 9, 10, 11, 12], however, that have challenged this interpretation and have refuted the strong-mixing scenario. It was argued in Refs. [7, 13] that the mixing was weak, and an analysis of the $E0$ transitions in $^{114}$Cd [12] using a two-state mixing model provided affirmation.

While the data for the Cd isotopes were not in agreement with the picture of nearly harmonic vibrations [11], the nature of the excitations remained somewhat enigmatic. A major step forward resulted from high-precision $g$-factor measurements for states in $^{111,113}$Cd that were compared with detailed calculations using both a particle-vibration model, and a particle-rotor coupling model [14]. That study reached the conclusion that the data were incompatible with the particle-vibration coupling scheme, but were more consistent with a particle-rotor picture that assumes a deformed core. A recent, more in-depth study [15] of the $g$-factors in $^{111}$Cd concluded that, while the particle-rotor model gave an approximate reproduction, there were limitations that likely arose from the stiffness of the deformed core used in the calculations. A shell-model treatment reproduced the data well, and it was suggested that similarity between the two models was due to the Nilsson wave functions being a good approximation to the shell model wave functions at small deformations [15].

Some of the most critical data for the elucidation of structure are those for excited $0^+$ states. In the even Cd isotopes, the energy systematics of the excited $0^+_2$ and $0^+_3$ states display an approximately parabolic trend with a minimum near the neutron mid-shell at $^{114}$Cd. This observation had been used as evidence for an intruder $\pi(2p4h)$ nature for the $0^+_2$ ($0^+_3$ for $N > 66$) levels, together with strong population in the $^{108,110}$Pd($^3$He,$n$)$^{110,112}$Cd reaction [16] (see Fig. 1). It had been noted [13] that the $0^+_2$ level also displays a parabolic trend in its energy which, combined with its strongly favoured decay to the $2^+_1$ level, the $2^+$ rotational band member of the intruder band, led to the suggestion [13] that it may represent the $\pi(4p6h)$ configuration.

2. Is the $0^+_4$ level the $\pi(4p6h)$ configuration?

There are several possible interpretations of the $0^+_4$ state consistent with its properties. For example, the $0^+_4$ level could have a configuration similar to the intruder $0^+_2$ level, and they could be mixed. However, this type of excitation might be expected to give rise to an observable population in the two-proton-transfer reaction. Figure 1 displays the spectra from Ref. [16] for the $^{108,110}$Pd($^3$He,$n$)$^{110,112}$Cd reactions. The positions of the arrows indicate where the peaks corresponding to the $0^+_4$ states would be located; no excess of counts is observed in either location. While lack of an observation does not constitute proof, the data are consistent with configurations not of the form of those for the intruding $0^+_2$ state. It is also noted that the $0^+_4$ states are observed to be only weakly populated in single-nucleon transfer reactions [17, 18, 19]. Heyde et al. [20] provide an estimate of the excitation energies expected for $\pi(4p4h)$ excitations at closed shells. Starting from the energy required to create a particle-hole pair, $\epsilon_p - \epsilon_h$, there are a number of corrections that arise from correlations. The excitation energy of the intruding state, built on pairs of particles coupled to $0^+$, can be expressed as [20]

$$\Delta E_{int} = \Delta N\pi (\epsilon_p - \epsilon_h) + \Delta E_M + \Delta E_Q - \Delta E_{pair}$$ (1)

where $\Delta N\pi$ is the number of proton particle-hole pairs above the ground state; $\Delta E_M$ is the
monopole energy shift arising from the change in the proton single-particle energies due to the neutron occupancy; $\Delta E_Q$ is the proton-neutron interaction energy approximated by [21]

$$\Delta E_Q = 2\kappa \Delta N_\pi \cdot N_\nu + \frac{3\kappa N_\nu (2N_\nu - 1) \Delta N_\pi}{[2(N_\pi + N_\nu + \Delta N_\pi) - 1][2(N_\pi + N_\nu) - 1]}$$

(2)

with $\kappa$ a strength parameter; $N_\pi$ and $N_\nu$ are the total number of proton and neutron pairs; and finally, $\Delta E_{pair}$ is the pairing energy correction. For $N_{p/h}$, the number of particle ($p$) or hole ($h$) pairs, the pairing energy is given by

$$E_{pair} = -N_{p/h} G \Omega \left(1 - \frac{N_{p/h}}{\Omega} + \frac{1}{\Omega}\right)$$

(3)

with $G$ the pairing strength, and $\Omega$ the shell degeneracy. The pairing correction energy is the difference between $E_{pair}$ for the ground state and the multiparticle-multihole state. The first three terms in Eq. 1 scale such that they are nearly twice the magnitude for a $\pi(4p6h)$ state as that for a $\pi(2p4h)$ configuration. The last term, however, does not scale by a factor of two. Using the parameters $G = -0.3$ MeV [20], and the degeneracies for the holes as $\Omega_h = 6$ (for $g_{9/2}$ and $p_{1/2}$ orbitals), and for the particles of $\Omega_p = 7$ (for $g_{7/2}$ and $d_{5/2}$ orbitals), leads to pairing energies of 5.1 MeV for the $\pi(2p4h)$ configuration, and 7.2 MeV for the $\pi(4p6h)$ configuration. The ground state, assuming a $\pi(2h)$ configuration, has a pairing energy of $-G\Omega = 1.8$ MeV, which yields $\Delta E_{pair}^{2p4h} = 3.3$ MeV for the $\pi(2p4h)$ state and $\Delta E_{pair}^{4p6h} = 5.4$ MeV for the $\pi(4p6h)$ state. Using the observed excitation energies of the $\pi(2p4h)$ $0^+$ state, Eq. 1 can be expressed as

$$E_{obs}^{2p4h} + \Delta E_{pair}^{2p4h} = \Delta N_\pi (\epsilon_p - \epsilon_h) + \Delta E_M + \Delta E_Q$$

(4)

and hence

$$E_{obs}^{4p6h} \approx 2 \left(E_{obs}^{2p4h} + \Delta E_{pair}^{2p4h}\right) - 2\Delta E_{pair}^{2p4h} + \Delta E_{pair}^{4p6h}.$$  

(5)
Figure 2. Selected electromagnetic decay strengths resulting from the BFM calculations (shown in black) for the excited $0^+$ states compared to experimental data (blue) for $^{110}$Cd. The left side reports the $B(E2; 0^+ \rightarrow 2^+)$ values in W.u.; quantities in square brackets are the relative $B(E2)$ values. The right side of the figure displays the $1000 \times \rho^2(E0)$ values for the $0^+ \rightarrow 0^+$ transitions. The energy labels on the levels are given in keV.

The expected excitation energies for the $\pi(4p6h)$ configurations, using the observed energies of 1.5, 1.2, and 1.1 MeV for the $\pi(2p4h)$ states in $^{110,112,114}$Cd, are 4.2, 3.5, and 3.4 MeV, respectively. These values are much higher than the observed energies of 2.1, 1.9, and 1.9 MeV for the $0^+_4$ states in $^{110,112,114}$Cd, suggesting that there are additional correlations not taken into account in Eq. 1 and that they are not pure $\pi(4p6h)$ excitations.

3. Nature of the excited $0^+$ states in $^{110,112}$Cd

In order to explore the nature of the excited $0^+$ states in $^{110,112}$Cd, the $\beta^+$/EC decays of $^{110,112}$In and the $\beta$-decay of $^{112}$Ag were studied using the $8\pi \gamma$-ray spectrometer [22] at the TRIUMF-ISAC facility. The goals of the measurements were the observation of weak, low-energy $\gamma$-ray decay branches from levels at high excitation energy, similar to the sensitivity achieved in earlier published studies [13, 23]. Combined with level lifetimes determined by using the $(n, n'\gamma)$ reaction [13, 24], $B(E2)$ values are available for a large number of transitions. We have observed bands in $^{110,112}$Cd built on excited $0^+$ states, and have identified a sequence of states that resemble $\gamma$ bands built on both the ground states and the $0^+_2$ intruder configurations. The results are presented in detail in Refs. [25, 26]. In this contribution, we focus on the results for the excited $0^+$ states.

Shown in Figs. 2 and 3 are portions of the low-lying level schemes for the $0^+$ states in $^{110,112}$Cd predicted with beyond-mean-field (BMF) calculations [25, 26] employing the Gogny D1S energy-density functional and the self-consistent configuration mixing (SCCM) method [27]. The calculated energies and decay transition rates of the first three excited $0^+$ states are compared with experimental quantities that are given using blue-coloured labels. The observed $E2$ decays of the $0^+$ band heads compare favourably to the results of the calculations.
Figure 3. Comparison of the results of the BFM calculations for the excited 0\(^+\) states to the experimental data for \(^{112}\)Cd. See caption to Fig. 2.

Figure 4. Results of the BMF calculations for \(^{110}\)Cd employing the SCCM method. The contours in the \(\beta\)-\(\gamma\) plane represent the wave function probability distributions for the 0\(^+\) states indicated. Plotted for each 0\(^+\) state is the shape of the ellipsoid for the mean values of \(\beta\) and \(\gamma\) extracted from the wave functions.

Unfortunately, the lifetimes of many of the excited 0\(^+\) states remain unknown, and thus only relative \(B(E2)\) values can be presented. The calculations predict enhanced \(E2\) decays from the 0\(^+_3\) states to the \(\gamma\)-band heads – predicted at 2185 and 1798 keV and observed at 1476 and 1312 keV in \(^{110,112}\)Cd, respectively – and 0\(^+_4\) states to the 2\(^+\) members (predicted at 2109 and 1587 keV, respectively) of the 0\(^+_2\) intruder bands; experimentally these decays are enhanced...
(where measured) or strongly favoured over other possible decays. The strengths of the $E0$ transitions, where known, also compare favourably. The calculations suggest that all $0^+$ states possess significant deformation, and have different mean shapes consisting of prolate, oblate, and triaxial shapes. These are plotted in Fig. 4 for $^{110}\text{Cd}$ (the results for $^{112}\text{Cd}$ are very similar).

4. Summary

The mid-shell Cd isotopes, $^{110,112}\text{Cd}$, used as textbook examples of spherical vibrational motion, are suggested to possess significant deformation and exhibit multiple shape coexistence. This alternative interpretation requires confirmation, but fortunately, being stable, these nuclei are amenable to study using, for example, Coulomb excitation and multi-nucleon transfer reactions. Studies in this direction have been initiated, with the first experiments employing the Coulomb excitation of $^{110}\text{Cd}$ with a $^{32}\text{S}$ beam performed at the Heavy Ion Laboratory in Warsaw [28].

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