Probing new physics in $B \to \phi \pi$ decay

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(March 26, 2022)

Abstract

We analyze the rare decay mode $B \to \phi \pi$, which is a pure penguin induced process, receiving dominant contribution from the electroweak penguins. Thus the standard model branching ratio is expected to be very small, which makes it as a sensitive probe of new physics. Using QCD factorization approach, we find the branching ratio in the standard model as $\text{Br}(B^- \to \phi \pi^-) \simeq 5 \times 10^{-9}$. Exploring some of the beyond standard model scenarios the branching ratio is found to be $\sim \mathcal{O}(10^{-8})$ in the minimal supersymmetric standard model (MSSM) with mass insertion approximation and in the extended technicolor model (TC2). The existence of an extra vector-like down quark (VLDQ) model predicts it to be $\sim \mathcal{O}(10^{-7})$. The recent BaBar result on $B^0 \to \phi \pi^0$ might be a strong indication of new physics effect present in the penguin induced process $B \to \phi \pi$.

PACS : 13.25.Hw, 12.60.Jv, 12.60.Nz
1 Introduction

The intensive search for physics beyond the standard model (SM) is performed now a days in various areas of particle physics. In this respect, the $B$ meson system can also be used as a complementary probe to the search for new physics. The main objectives of the ongoing and the future $B$ factory experiments are to explore in detail the origin of CP violation, to test the standard model at an unexpected level of precision and to look for possible existence of new physics effects. In the $B$ experiments new physics beyond the SM may manifest itself in various ways, e.g., (i) decays which are expected to be rare in the SM and are found to have large branching ratios (ii) CP violating rate asymmetries which are expected to vanish or to be very small in the SM are found to be significantly large (iii) the discrepancy between the mixing induced CP asymmetry between various $B$ decay processes which are dominated by a single decay amplitude with same weak phase, i.e., $S_{ψK_S}$ and $S_{φK_S}$ etc.

Thus the rare $B$ meson decays are suggested to give good opportunities for discovering new physics beyond the SM. The discrepancy between the recently measured $S_{ψK_S}$ and $S_{φK_S}$ [1, 2] has already given an indication of the possible existence of new physics in the $B$ decay amplitudes (i.e., in the penguin induced $B \rightarrow φK_S$ decay).

In this paper, we would like to explore the presence of new physics in another pure penguin induced decay mode $B \rightarrow φπ$. It proceeds through the quark level transition $b \rightarrow d\bar{s}s$, which is a flavor changing neutral current (FCNC) process at the one loop level. The interesting feature of this process is that it is dominated by electroweak penguin contribution. The QCD penguins should play a minor role for this transition since $\bar{s}$ and $s$ quarks emerging from the gluons of the usual QCD penguin diagram form a color octet state and consequently cannot build up the $φ$ meson which is a $\bar{s}s$ color singlet state. Therefore the SM prediction for the branching ratio for this process is expected to be quite small. Thus as this mode is highly suppressed in the SM, it may serve as a good hunting ground to look for new physics beyond the SM.

Recently, these decay modes have been searched for by the BaBar collaboration [3, 4]:

$$\text{Br}(B^{±} \rightarrow φπ^{±}) < 0.41 \times 10^{-6},$$
$$\text{Br}(B^{0} \rightarrow φπ^{0}) = (0.2^{+0.4}_{-0.3} ± 0.1) \times 10^{-6},$$
$$< (1.2 ± 0.8) \times 10^{-6}. \quad (1)$$

On the theoretical side the decay mode $B^{-} \rightarrow φπ^{-}$ has been studied recently in Ref. [5]. Using QCD factorization approach the branching ratio has been obtained in the SM and in the constrained minimal supersymmetric standard model. It has also been studied in Ref. [6] in the SM and R-parity violating supersymmetric model and in the Refs. [7, 8] using the standard model approach.

In this paper, we will first reanalyze this process again in the SM for the sake of completeness, using QCD factorization. It should also be noted that there is not good agreement of the SM predictions in the literature. We then consider the minimal supersymmetric model with mass insertion approximation. Basically we will confine ourselves to the case where the
new contributions to the $B \to \phi \pi$ will arise from the gluino mediated $b \to d \bar{s}s$ process induced by the flavor mixing in the down s-quark sector. Next we will calculate the branching ratio in the framework of topcolor assisted technicolor model (TC2) and in the model with an extra vector like down quark (VLDQ).

## 2 Standard Model contribution

In the SM, the decay process $B \to \phi \pi$ receives contribution from the quark level transition $b \to d \bar{s}s$, which is induced by the pure penguin diagram with dominant contributions coming from electroweak penguins. The effective Hamiltonian describing the decay $b \to d \bar{s}s$ [9] is given as

$$H_{\text{SM}}^{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[V_{qb} V_{qd}^* \sum_{i=3}^{10} C_i O_i \right], \quad (2)$$

where $q = u, c$. $O_3, \ldots, O_6$ and $O_7, \ldots, O_{10}$ are the standard model QCD and electroweak penguin operators, respectively. The values of the Wilson coefficients at the scale $\mu \approx m_b$ in the NDR scheme are given in Ref. [10] as

$$C_3 = 0.014, \quad C_4 = -0.035, \quad C_5 = 0.009, \quad C_6 = -0.041, \quad C_7 = -0.002\alpha, \quad C_8 = 0.054\alpha, \quad C_9 = -1.292\alpha, \quad C_{10} = 0.263\alpha. \quad (3)$$

We use QCD factorization [9] to evaluate the hadronic matrix elements. In this method, the decay amplitude can be represented in the form

$$\langle \phi \pi^- | O_i | B^- \rangle = \langle \phi \pi^- | O_i | B^- \rangle_{\text{fact}} \left[1 + \sum r_n \alpha_s^n + O(\Lambda_{\text{QCD}}/m_b)\right], \quad (4)$$

where $\langle \phi \pi^- | O_i | B^- \rangle_{\text{fact}}$ denotes the naive factorization result and $\Lambda_{\text{QCD}} \sim 225$ MeV, the strong interaction scale. The second and third terms in the square bracket represent higher order $\alpha_s$ and $\Lambda_{\text{QCD}}/m_b$ corrections to hadronic matrix elements.

In the heavy quark limit the decay amplitude for the $B^- \to \phi \pi^-$ process, arising from the penguin diagrams, is given as

$$A^{\text{SM}}(B^- \to \phi \pi^-) = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb} V_{qd}^* \left[a_3^q + a_5^q - \frac{1}{2} (a_7^q + a_9^q)\right] X, \quad (5)$$

where $X$ is the factorized matrix element. The amplitude for $B^0 \to \phi \pi^0$ is related to $B^- \to \phi \pi^-$ by

$$A(B^0 \to \phi \pi^0) = \frac{1}{\sqrt{2}} A(B^- \to \phi \pi^-). \quad (6)$$
Using the form factors and decay constants, defined as [12]

\[
\langle \pi^- (p_\pi) | \bar{d} \gamma^\mu b | B^- (p_B) \rangle = \left[ (p_B + p_\pi)^\mu - \frac{m_B^2 - m_\pi^2}{q^2} \right] F_1(q^2) + \frac{m_B^2 - m_\pi^2}{q^2} q^\mu F_0(q^2),
\]

\[
\langle \phi (q, \epsilon) | \bar{s} \gamma^\mu s | 0 \rangle = f_\phi m_\phi \epsilon^\mu,
\]

we obtain the factorized matrix element \(X\) as

\[
X = \langle \pi^- (p_\pi) | \bar{d} \gamma_\mu (1 - \gamma_5) b | B^- (p_B) \rangle \langle \phi (q, \epsilon) | \bar{s} \gamma^\mu (1 - \gamma_5) s | 0 \rangle = 2 F_1^{B \to \pi} (m_\phi^2) f_\phi m_\phi (\epsilon \cdot p_B).
\]

The coefficients \(a_i^q, s\) which contain next to leading order (NLO) and hard scattering corrections are given as [7, 11]

\[
a_3^u = a_3^c = C_3 + \frac{C_4}{N} + \frac{\alpha_s C_F}{4\pi N} C_4 F_\phi,
\]

\[
a_5^u = a_5^c = C_5 + \frac{C_6}{N} + \frac{\alpha_s C_F}{4\pi N} C_6 (-F_\phi - 12),
\]

\[
a_7^u = a_7^c = C_7 + \frac{C_8}{N} + \frac{\alpha_s C_F}{4\pi N} C_8 (-F_\phi - 12),
\]

\[
a_9^u = a_9^c = C_9 + \frac{C_{10}}{N} + \frac{\alpha_s C_F}{4\pi N} C_{10} F_\phi,
\]

where \(N = 3\), is the number of colors and \(C_F = (N^2 - 1)/2N\). The parameters present in (9) are given as

\[
F_\phi = -12 \ln \frac{\mu}{m_b} - 18 + f_\phi^I + f_\phi^{II},
\]

\[
f_\phi^I = \int_0^1 dx g(x) \phi_\phi(x),
\]

\[
g(x) = 3 \frac{1 - 2x}{1 - x} \ln x - 3i\pi,
\]

\[
f_\phi^{II} = \frac{4\pi^2}{N} \frac{f_\pi f_B}{F_1^{B \to \pi} (0) m_B^2} \int_0^1 dz \phi_B(z) \int_0^1 dx \phi_\pi(x) \int_0^1 dy \phi_\phi(y).
\]

The light cone distribution amplitudes (LCDA’s) at twist two order are given as

\[
\phi_B(x) = N_B x^2 (1 - x)^2 \exp \left( -\frac{m_B^2 x^2}{2\omega_B^2} \right),
\]

\[
\phi_\pi,\phi(x) = 6x(1 - x),
\]

where \(N_B\) is the normalization factor satisfying \(\int_0^1 dx \phi_B(x) = 1\) and \(\omega_B = 0.4\) GeV. The branching ratio can be obtained using the formula

\[
\text{Br}(B^- \to \phi \pi^-) = \tau_B \frac{|p_{cm}|^3}{8\pi m_\phi^2} |A(B^- \to \phi \pi^-)/(\epsilon \cdot p_B)|^2,
\]

\[
\text{Br}(B^0 \to \phi \pi^0) = \frac{\kappa}{2} \text{Br}(B^- \to \phi \pi^-),
\]

\[\text{(12)}\]
where \( \kappa = \tau_{B^0}/\tau_{B^-} \) and \( p_{cm} \) is the momentum of the outgoing particles in the \( B \) meson rest frame.

For numerical evaluation we have used the following input parameters. The value of the form factor at zero recoil is taken as \( F_{B \to \pi}^{B \to \pi}(0) = 0.33 \), and its value at \( q^2 = m^2_{\phi} \) can be obtained using simple pole dominance ansatz [12] as \( F_{B \to \pi}^{B \to \pi}(m^2_{\phi}) = 0.34 \). The values of the decay constants are as \( f_\phi = 0.233 \text{ GeV} \), \( f_B = 0 \).19 GeV, \( f_\pi = 0.131 \text{ GeV} \), the particle masses and the lifetime of \( B \) mesons \( \tau_{B^-} = 1.674 \text{ ps} \), \( \tau_{B^0} = 1.542 \text{ ps} \) are taken from [13]. For the CKM matrix elements, we have used the Wolfenstein parameterization and have taken the values of the parameters \( A = 0.819 \pm 0.040 \), \( \lambda = 0.2237 \pm 0.0033 \), \( \rho = 0.224 \pm 0.039 \) and \( \eta = 0.324 \pm 0.039 \). With these input parameters, we obtain the branching ratio for \( B^- \to \phi \pi^- \) in the SM as

\[
\text{Br}(B^- \to \phi \pi^-)_{SM} = (5.5 \pm 0.9) \times 10^{-9} ,
\]

\[
\text{Br}(B^0 \to \phi \pi^0)_{SM} = (2.5 \pm 0.4) \times 10^{-9} .
\] (13)

These predicted values are quite below the present experimental upper limit and the central value of \( B^0 \to \phi \pi^0 \) (1). It should be noted that our prediction is in agreement with [5], the slight difference is due to the difference in the used CKM parameters and formfactor.

### 3 Supersymmetric contribution

Now we study the decay process \( B \to \phi \pi \), in the minimal supersymmetric standard (MSSM) model with gluino contributions, because the chargino and charged Higgs contributions are expected to be suppressed by the small electroweak gauge couplings. Thus, the one loop contributions to the above mentioned decay process can be induced by s-quark and gluino penguin and box diagrams. These gluino mediated FCNC contributions are of the order of strong interaction strength, which may exceed the existing limits. It is customary to rotate the effects, so that they occur in s-quark propagator rather than in couplings and to parametrize them in terms of dimensionless parameters. Here we work in the usual mass insertion approximation [14, 15] where the flavor mixing \( i \to j \) in the down-type squarks associated with \( \tilde{q}_B \) and \( \tilde{q}_A \) are parametrized by \( (\delta^d_{ij})_{AB} \), with \( A, B = L, R \) and \( i, j \) as the generation indices. More explicitly \( (\delta^d_{L L})_{ij} = (V^d_{L} M^2_{d} V^d_{L})_{ij}/m^2_{\tilde{q}}, \) where \( M^2_{d} \) is the squared down squark mass matrix and \( m_{\tilde{q}} \) is the average squark mass. \( V_d \) is the matrix which diagonalizes the down quark mass matrix.

The new effective \( \Delta B = 1 \) Hamiltonian relevant for the \( B \to \phi \pi \) process arising from new penguin/box diagrams with gluino-squark in the loops is given as

\[
\mathcal{H}^{SUSY}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \sum_{i=3}^6 \left[ C^{NP}_i O_i + \tilde{C}^{NP}_i \tilde{O}_i \right],
\] (14)

where \( O_i \) are the QCD penguin operators and the \( C^{NP}_i \) are the new Wilson coefficients. The operators \( \tilde{O}_i \) are obtained from \( O_i \) by exchanging \( L \leftrightarrow R \). Thus including the new physics
contribution, one can write the total amplitude for $B \to \phi \pi$ process as

$$A^T = A^{SM} \left[ 1 + \left| \frac{A^{SUSY}}{A^{SM}} \right| e^{i\theta} \right],$$  \hspace{1cm} (15)

where $A^{SUSY}$ is the new physics amplitude arising from minimal supersymmetric model and $\theta$ is the relative phase between the SM and the new physics decay amplitudes. The corresponding branching ratio is given as

$$Br = Br^{SM} \left[ 1 + 2 \cos \theta \left| \frac{A^{SUSY}}{A^{SM}} \right| + \left| \frac{A^{SUSY}}{A^{SM}} \right|^2 \right],$$  \hspace{1cm} (16)

where $Br^{SM}$ is the SM branching ratio. To evaluate the amplitude in the MSSM, we have to first evaluate the Wilson coefficients at the $b$ quark mass scale. At the leading order in mass insertion approximation the new Wilson coefficients corresponding to each of the operator at the scale $\mu \sim \tilde{m} \sim M_W$ are given as \cite{15}

$$C^{NP}_3 \approx -\frac{\sqrt{2} \alpha_s^2}{4 G_F V_{tb} V_{td}^* m_q^2} \left( \frac{\delta_{LL}^d}{13} \right) \left[ -\frac{1}{9} B_1(x) - \frac{5}{9} B_2(x) - \frac{1}{18} P_1(x) - \frac{1}{2} P_2(x) \right],$$

$$C^{NP}_4 \approx -\frac{\sqrt{2} \alpha_s^2}{4 G_F V_{tb} V_{td}^* m_q^2} \left( \frac{\delta_{LL}^d}{13} \right) \left[ -\frac{7}{3} B_1(x) + \frac{1}{3} B_2(x) - \frac{1}{6} P_1(x) + \frac{3}{2} P_2(x) \right],$$

$$C^{NP}_5 \approx -\frac{\sqrt{2} \alpha_s^2}{4 G_F V_{tb} V_{td}^* m_q^2} \left( \frac{\delta_{RR}^d}{13} \right) \left[ \frac{10}{9} B_1(x) + \frac{1}{18} P_1(x) - \frac{1}{2} P_2(x) \right],$$

$$C^{NP}_6 \approx -\frac{\sqrt{2} \alpha_s^2}{4 G_F V_{tb} V_{td}^* m_q^2} \left( \frac{\delta_{LL}^d}{13} \right) \left[ -\frac{2}{3} B_1(x) + \frac{7}{6} B_2(x) + \frac{1}{6} P_1(x) + \frac{3}{2} P_2(x) \right].$$  \hspace{1cm} (17)

The corresponding $\tilde{C}_i$ are obtained from $C^{NP}_i$ by interchanging $L \leftrightarrow R$. The functions appear in these expressions can be found in Ref. \cite{15} and $x = m_q^2/m_b^2$. Because the parity of the vector meson $\phi$ is opposite to that of $B$ and $\pi$ mesons, which are pseudoscalars, the gluino loop effects appear as $\left( \delta_{LL}^d \right)_{13} \left( \delta_{RR}^d \right)_{13}$. The Wilson coefficients at low energy $C^{NP}_i(\mu)$, $\mu \sim \mathcal{O}(m_b)$ can be obtained from $C^{NP}_i(M_W)$ by using the Renormalization Group (RG) equation as discussed in Ref. \cite{10}, as

$$C(\mu) = U_5(\mu, M_W) C(M_W),$$  \hspace{1cm} (18)

where $C$ is the $6 \times 1$ column vector of the Wilson coefficients and $U_5(\mu, M_W)$ is the five-flavor $6 \times 6$ evolution matrix. In the next-to-leading order (NLO), $U_5(\mu, M_W)$ is given by

$$U_5(\mu, M_W) = \left( 1 + \frac{\alpha_s(\mu)}{4\pi} J \right) U_5^{(0)}(\mu, M_W) \left( 1 - \frac{\alpha_s(M_W)}{4\pi} J \right),$$  \hspace{1cm} (19)

where $U_5^{(0)}(\mu, M_W)$ is the leading order (LO) evolution matrix and $J$ denotes the NLO corrections to the evolution. The explicit forms of $U_5(\mu, M_W)$ and $J$ are given in Ref. \cite{10}.
For the numerical analysis, we fix the SUSY parameter as \( m_{\tilde{q}} = m_{\tilde{g}} = 500 \text{ GeV} \), \( \alpha_s(M_W) = 0.118 \), \( \alpha_s(m_b = 4.4 \text{ GeV}) = 0.221 \). Thus, the values of the Wilson coefficients evaluated at the \( b \) quark mass scale are given as (where the common factor \( (\delta_{LL}^d)_{13} \) has been factored out)

\[
C_{NP}^3 = 0.025, \quad C_{NP}^4 = -0.021, \quad C_{NP}^5 = -0.003, \quad C_{NP}^6 = -0.087.
\] (20)

Now evaluating the matrix elements of the operators \( O_{3-6} \) as done in Eqs. (5)-(9) for the SM i.e., replacing the SM Wilson coefficients \( C_{3-6} \) by their corresponding SUSY counterparts, we obtain the total amplitude in the MSSM as

\[
A(B^- \to \phi \pi^-) = A^{SM}(1 + 2 \times 10^{-2})
\] (21)

The constraint on \( (\delta_{13}^d)_{LL} \) is obtained from \( B^0 - \bar{B}^0 \) mixing, however for \( (\delta_{13}^d)_{RR} \), it is not very stringent. We use the conservative limit for \( m_{\tilde{q}}=500 \text{ GeV} \) and \( x = 1 \) as \( (\delta_{13}^d)_{LL} < \sqrt{|\text{Re}(\delta_{13}^d)_{LL}^2|} \approx 9.8 \times 10^{-2} [15] \), \( (\delta_{13}^d)_{LL} \sim (\delta_{13}^d)_{RR} \) and their weak phases to be equal. Thus, we obtain the branching ratio in MSSM as

\[
\begin{align*}
\text{Br}(B^- \to \phi \pi^-)|_{\text{MSSM}} &\leq 1.2 \times 10^{-8}, \\
\text{Br}(B^0 \to \phi \pi^0)|_{\text{MSSM}} &\leq 0.5 \times 10^{-8}.
\end{align*}
\] (22)

Although the predicted branching ratios in MSSM are enhanced by one order from their SM values but they are still well below the present upper limits.

## 4 Contribution from the VLDQ Model

Now we consider the model with an additional vector like down quark [16]. It is a model with an extended quark sector. In addition to the three standard generation of quarks, there is an \( SU(2)_L \) singlet of charge \( -1/3 \). The mixing of these singlet quarks with the three SM down type quarks provides a framework to study the deviation from unitarity constraint of \( 3 \times 3 \) CKM matrix. The important feature of this model is that it allows \( Z \)-mediated FCNC at the tree level. Thus, the presence of an additional singlet down quark implies a \( 4 \times 4 \) matrix \( V_{\alpha i} (i = u, c, t, 4, \alpha = d, s, b, b') \), diagonalizing the down quark mass matrix. For our purpose, the relevant information for the low energy physics is encoded in the extended mixing matrix. The charged currents are unchanged except that the \( V_{CKM} \) is now the \( 3 \times 4 \) upper submatrix of \( V \). However, the distinctive feature of this model is that FCNC enters the neutral current Lagrangian of the left handed downquarks:

\[
\mathcal{L}_Z = \frac{g}{2 \cos \theta_W} \left[ \bar{u}_{Li} \gamma^\mu u_{Li} - \bar{d}_{Li} \gamma^\mu d_{Li} + 2 \sin^2 \theta_W J^\mu_{em} \right] Z^\mu ,
\] (23)

with

\[
U_{\alpha \beta} = \sum_{i=u,c,t} V_{\alpha i}^* V_{4 \beta} = \delta_{\alpha \beta} - V_{4 \alpha}^* V_{4 \beta} ,
\] (24)
where $U$ is the neutral current mixing matrix for the down sector which is given above. As $V$ is not unitary, $U \neq 1$. In particular its non-diagonal elements do not vanish:

$$U_{\alpha\beta} = -V_{4\alpha}^* V_{4\beta} \neq 0 \quad \text{for } \alpha \neq \beta.$$  

(25)

Since the various $U_{\alpha\beta}$ are non vanishing they would signal new physics and the presence of FCNC at the tree level, this can substantially enhance the branching ratio of $B \to \phi \pi$. The observed discrepancy between $S_{\phi K_S}$ and $S_{\phi K_S}$ can be explained in this model [17]. The new element $U_{db}$ which is relevant to our study is given as

$$U_{db} = V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb}.$$  

(26)

Thus the decay modes $B \to \phi \pi$ receive the new contributions from the $Z$-mediated FCNC transitions and the new additional operator is given as

$$O_{VLDQ} = [\bar{d}_\alpha \gamma^\mu (1 - \gamma_5) b_\alpha] (s_\beta \gamma_\mu (C_V^s - C_A^s) s_\beta),$$  

(27)

where $C_V$ and $C_A$ are the vector and axial vector $Zs\bar{s}$ couplings. Using the identity $(C_V^s - C_A^s) = [(C_V^v + C_A^v)(1 - \gamma_5) + (C_V^v - C_A^v)(1 + \gamma_5)]/2$, the effective Hamiltonian for $B \to \phi \pi$ transition is given as

$$H_{eff}^{VLDQ} = \frac{G_F}{\sqrt{2}} U_{db} [(C_V^s + C_A^s)(db)(\bar{s}s)_{V-A} + (C_V^s - C_A^s)(db)(\bar{s}s)_{V+A}],$$  

(28)

where the subscripts in the currents denote the usual $(V - A)$ and $(V + A)$ currents. Now evaluating the matrix elements of the operators the transition amplitude for the process in VLDQ model is given as

$$A_{VLDQ}^{B \to \phi \pi} = \frac{G_F}{\sqrt{2}} U_{db} X (\epsilon \cdot p_B) \left( (C_V^s + C_A^s)(1 + \frac{\alpha_s}{4\pi} F_{\phi}) + (C_V^s - C_A^s) \right),$$  

(29)

where we have also included the leading order nonfactorizable contributions. Now using the value for $C_V^s$ and $C_A^s$ as

$$C_V^s = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W, \quad C_A^s = -\frac{1}{2},$$  

(30)

$\sin^2 \theta_W = 0.23$, along with $|U_{db}| \leq 1.2 \times 10^{-3}$ [18], we obtain the ratio of VLDQ and SM amplitudes as

$$\left| \frac{A_{VLDQ}^{B \to \phi \pi}}{A_{SM}^{B \to \phi \pi}} \right| \leq 8.16.$$  

(31)

The branching ratio in VLDQ model obtained using Eq. (16) as

$$\text{Br}(B^- \to \phi \pi^-)|_{VLDQ} \leq 4.6 \times 10^{-7}$$

$$\text{Br}(B^0 \to \phi \pi^0)|_{VLDQ} \leq 2.1 \times 10^{-7},$$  

(32)

which are two order above the standard model prediction and the branching ratio for $B^0 \to \phi \pi^0$ is in agreement with the present data (1).


5 Contribution from the TC2 Model

Now we calculate the branching ratio for $B \to \phi \pi$ process in the framework of the topcolor assisted technicolor model (TC2) [19, 20]. It is well known that technicolor is one of the important candidates for the electroweak symmetry breaking and the extended technicolor model was proposed to generate the ordinary fermion masses. In order to generate large top quark mass the topcolor model has been constructed recently. Apart from some difference in group structure and/or particle contents, all TC2 models have similar common features. Following the TC2 model of Hill [19], the charmless decays $B \to PP, PV$ are studied in Ref [21], using the generalized factorization approach. In this paper we will also use the same procedure to evaluate the new physics contribution to the $B \to \phi \pi$ mode, but we will use QCD factorization approach to evaluate the hadronic matrix elements.

In TC2 model, there exist top-pions ($\tilde{\pi}^\pm$ and $\tilde{\pi}^0$), charged and neutral $b$-pions ($\tilde{H}^\pm, \tilde{H}^0$ and $A^0$) and technipions ($\pi^\pm_1$ and $\pi^\pm_8$). The coupling of top-pions to $t$- and $b$-quarks can be written as

$$m^*_t \frac{F_{\tilde{\pi}}}{i\bar{t}t\tilde{\pi}^0 + i\bar{t}_Rb_L\tilde{\pi}^+ + \frac{m^*_b}{m^*_t}\bar{t}_Rb_R\tilde{\pi}^+ + \text{h.c.}},$$

(33)

where $m^*_t = (1 - \epsilon)m_t$ and $m^*_b \sim 1$ GeV denote the masses of top and bottom quarks generated by the topcolor interactions and $F_{\tilde{\pi}}$ is the top-pion decay constant. At low energy, potentially large FCNC arise when the quark fields are rotated from their weak eigenbasis to their mass eigenbasis, realized by the matrices $U_{L,R}$ for the up-type quarks and $D_{L,R}$ for the down type quarks. Thus for example, making the replacement

$$b_L \to D^{bd}_{L}d_L + D^{bs}_{L}s_L + D^{bb}_{L}b_L,$$

$$b_R \to D^{bd}_{R}d_R + D^{bs}_{R}s_R + D^{bb}_{R}b_R,$$

(34)

the FCNC interactions will be induced. In the TC2 model the corresponding flavor changing effective Yukawa couplings are

$$m^*_t \frac{F_{\tilde{\pi}}}{i\bar{t}_L\tilde{\pi}^0 R s_L + D^{bs}_{L}d_L + D^{bb}_{L}b_L + i\bar{t}_R\tilde{\pi}^+ R s_R + D^{bs}_{R}d_R + D^{bb}_{R}b_R + \text{h.c.}},$$

(35)

The constraints on the parameters in the TC2 model are discussed in detail in Ref [21], obtained from various experimental data. For the mixing matrices the “square root ansatz” is considered, i.e. $D^{bd}_{L} = V_{td}/2$ and $D^{bd}_{R} = V_{ts}/2$. The other parameters are given as

$$m_{\pi_1} = 100 \text{ GeV}, \quad m_{\pi_8} = 200 \text{ GeV}, \quad F_{\tilde{\pi}} = 50 \text{ GeV}, \quad F_{\pi} = 120 \text{ GeV} \quad \epsilon = 0.05,$$

(36)

where $F_{\pi}$ is the technipion decay constant.

In this model, the decay process $b \to dss$ is induced by the exchange of the charged top pions $\tilde{\pi}^\pm$ and technipions $\pi^\pm_1$ and $\pi^\pm_8$ through the strong and electroweak penguin diagrams. Combining the new physics contributions with their SM counterparts, the effective Wilson coefficients are evaluated.

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Thus the new strong and electroweak penguin diagrams can be obtained from the corresponding penguin diagrams in the SM by replacing the internal $W^\pm$ lines with the charged top-pions and technipions. The analytic expressions for these diagrams are calculated in the dimensional regularization with $\overline{\text{MS}}$ renormalization scheme in Ref. [21]. The new contributions to the Wilson coefficients arising from these diagrams are given as

$$C_{0}^{TC2} = \frac{1}{\sqrt{2} G_F M_W^2} \left[ \frac{m_\pi^2}{4 F_\pi^2} T_0(y_t) + \frac{m_\pi^2}{3 F_\pi^2} T_0(z_t) + \frac{8 m_\pi^2}{3 F_\pi^2} T_0(\xi_t) \right],$$

$$D_{0}^{TC2} = \frac{1}{\sqrt{2} G_F} \left[ \frac{1}{4 F_\pi^2} F_0(y_t) + \frac{1}{3 F_\pi^2} \left( F_0(z_t) + 8 F_0(\xi_t) \right) \right],$$

$$E_{0}^{TC2} = \frac{1}{\sqrt{2} G_F} \left[ \frac{1}{4 F_\pi^2} I_0(y_t) + \frac{1}{3 F_\pi^2} \left( I_0(z_t) + 8 I_0(\xi_t) + 9 N_0(\xi_t) \right) \right],$$

$$E_{0}^{TC2} = \frac{1}{2 \sqrt{2} G_F} \left[ \frac{1}{4 F_\pi^2} K_0(y_t) + \frac{1}{3 F_\pi^2} \left( K_0(z_t) + 8 K_0(\xi_t) + 9 L_0(\xi_t) \right) \right],$$

where $y_t = m_t^2/m_\pi^2$ with $m_t^* = (1 - \epsilon)m_t$, $z_t = (\epsilon m_t)^2/m_\pi^2$, and $\xi_t = (\epsilon m_t)^2/m_\pi^2$. The loop functions $T_0(x)$, $F_0(x)$, $I_0(x)$, $K_0(x)$, $L_0(x)$, $N_0(x)$ are found from Ref [21]. Using the top pion and technipion masses to be 200 GeV, the values of the $C_0$, $D_0$, $E_0$ functions at the $M_W$ mass scale are obtained as [21]

$$\{C_0, D_0, E_0, E_0^{TC2}\}_{\mu=M_W} = \{1.27, 0.27, 0.66, -1.58\}.$$  (38)

Now combining these values with the corresponding SM values at the $W$-boson mass scale ($\{C_0, D_0, E_0, E_0^{SM}\}_{\mu=M_W} = \{0.81, -0.48, 0.27, 0.19\}$), and running the resulting contributions to the $b$ quark mass scale ($\mu = 2.5$ GeV) the effective Wilson coefficients are obtained as [21]

$$C_3 = 0.0195, \quad C_4 = 0.0441, \quad C_5 = 0.0111, \quad C_6 = -0.0535, \quad C_7 = 0.0026, \quad C_8 = 0.0018, \quad C_9 = -0.0175, \quad C_{10} = 0.0049.$$  (39)

Now substituting these values in (5) the branching ratios in the TC2 model are found to be

$$\text{Br}(B^- \to \phi \pi^-)|_{TC2} = 1.2 \times 10^{-8},$$

$$\text{Br}(B^0 \to \phi \pi^0)|_{TC2} = 0.5 \times 10^{-8}.$$  (40)

Although the branching ratios are one order higher than the corresponding SM value but they are well below the present experimental limits.

### 6 Conclusion

In this paper, we have analyzed the decay mode $B \to \phi \pi$ both in the standard model and some beyond standard model scenarios. This decay mode proceeds through the quark level...
FCNC transition $b \rightarrow d\bar{s}s$, receiving contributions only from one-loop penguin diagrams. However, because of OZI suppression, the QCD penguins play only a minor role in this case and the dominant contributions coming from the electroweak penguins. Therefore, in the standard model these decays are highly suppressed, which makes them a very sensitive probe for new physics.

Using QCD factorization approach, we found the branching ratios in the SM for these modes as $\sim \mathcal{O}(10^{-9})$, which are quite below the present experimental upper limits $\mathcal{O}(10^{-6})$. We have also calculated the branching ratios in the MSSM with mass insertion approximation, in the VLDQ model and in the topcolor assisted technicolor model. The branching ratios obtained in the MSSM and TC2 models are of the order of $\mathcal{O}(10^{-8})$, whereas they are found to be $\mathcal{O}(10^{-7})$ for VLDQ model.

Recently, BaBar Collaboration [4] has reported the first measurement of the branching ratio for $B^0 \rightarrow \phi \pi^0$ process as $(0.2^{+0.4}_{-0.3} \pm 0.1) \times 10^{-6}$. Although the error bars are quite large, but this preliminary experimental value is almost 2 orders larger than the standard model prediction. If in future the data will remain in this order, i.e., $\mathcal{O}(10^{-7})$, then it will give a clear signal of new physics effect present in the penguin induced process $B \rightarrow \phi \pi$. As shown, only the VLDQ model can predict such a large branching ratio. Therefore, the future experimental data on $B \rightarrow \phi \pi$ will serve as a very good hunting ground for the existence of new physics beyond the SM and also support/rule out some of the existing new physics models.

7 Acknowledgement

We thank Y. Grossman for useful discussions and valuable comments. R.M. would like to thank the HEP theory group at the Technion for the warm hospitality. The work of AKG was supported by Lady Davis fellowship and the work of RM was partly supported by Department of Science and Technology, Government of India, through Grant No. SR/FTP/PS-50/2001.

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