Quantum Level Instability of Transverse Excitation in Electron Flow

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Abstract
In current research, we use the effective Schrödinger-Poisson model to study a new kind of quantum-level instability in an infinite-wall slab electron flow. We use the Madelung fluid representation along with the conventional eigenvalue problem techniques in order to solve the linearized coupled differential equations representing the linear transverse collective excitations in the electron gas of arbitrary degree of degeneracy having a constant perpendicular drift. It is shown that the energy levels of collective electrostatic excitations are doubly quantized due to mutual interactions between single electron oscillations, analogous to the problem of a particle in a box, and collective Langmuir oscillations, which are modulated over single electron quantum state. We also report the transverse excitation instability of plasmon energy level in electron slab flow due to the interplay between the wave-like dispersion and the destabilizing perpendicular electron drift momentum. We further study in detail the parametric dependence of such instability in terms of different aspects of the many-electron system. Such a quantum-level instability may have important applications in characteristic behavior of plasmonic devices and their frequency response. Parametric quantization of drifting electron fluid may also have broad applications in nanoscale quantum device calibration and quantum measurements.

Keyword
Plasmons · Quantum instability · Plasmon level · Collective excitations · Electron gas

Mathematics Subject Classification 52.30.-q · 71.10.Ca · 05.30.-d

Introduction
Plasmon excitations due to collective electrostatic interactions [1, 2] are of primary importance in charged environments and inevitable in many laboratory and astrophysical processes in nature. They manifest ultrafast behavior such as frequency response of multicomponent plasmas to external perturbations leading to many interesting linear and nonlinear waves [3–6]. At the quantum level, the overlap of single-electron fermionic wavefunctions [7] in dense plasmas adds interesting new features to collective electrostatic excitations [8–11] which deserve particular attention and have broad applications in strongly coupled plasmas [12–14], plasmonics [15–17], photonics and optoelectronics [18], semiconductor nanoelectronics [19], and miniaturized quantum electronic devices [8, 20–22]. In solid state, the collective electron excitations affect many of the physical properties such as heat and electric transport, energy band structure, and acoustic and optical properties [23, 24]. The rapidly developing miniaturized low-dimensional semiconductor electronic technology [25] strongly depends on electronic excitations and their terahertz-level frequency response properties [26, 27]. Recent technological advancements have opened a new window to energy extraction and storage of high energy collective excitations from hot electrons in plasmonic devices and advanced solar cells technology [28–35]. In astrophysical plasmas, on the other hand, quantum electron processes play a dominant role in dense objects like planetary cores, warm dense matter, and white dwarf stars [36–41]. The relativistic degeneracy of electrons in astrophysical scale has been shown to have a catastrophic effect on the fate of degenerate stellar objects such as white dwarfs [42].

Study of collective processes in quantum electron system had a long tradition over the past century beginning from pioneering works of scientists like Madelung [43], Fermi [44], Hoyle [45], Chandrasekhar [46, 47], Bohm [48–50], Pines [51], Levine [52], Klomontovich [53], Takabayashi [54–58], Castro [59–62], and Lindhard [63]. Recent advancements in quantum hydrodynamic and magnetohydrodynamic models [64, 65] have made possible

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exploration of many interesting features of dense degenerate plasmas with no counterparts in classical charged environments [66–76]. In quantum plasmas, the electron excitations are of dual-tone nature due to both single-electron oscillations and collective electrostatic interactions [77]. An effective quantum model to investigate such excitations is the Schrödinger-Poisson model [78] based on the Madelung quantum fluid formulation and quantum hydrodynamics which simultaneously incorporates both single-electron aspects in addition to collective electrostatic interactions. The model has the advantage of generalization to the electronic system with arbitrary degree of degeneracy. Recent investigation [79–83] of plasmon excitations in arbitrary degenerate electron gas trapped in an infinite wall of one-dimensional box within the frameworks of the Schrödinger-Poisson model reveals energy quantization of collective modes with statefunctions characterized by features present in single-electron wavefunction modulated by fine-structured density oscillations due to electrostatic interaction modeled via Poisson’s relation coupled to the Schrödinger equation. Many other interesting features of collective quantum electrostatic interactions in arbitrary degenerate quantum electron gas, with possible applications in nanoscale plasmonic devices, have been explored recently using the same model. The energy band structure of multistream quantum electron system has been recently calculated using the N-coupled Schrödinger-Poisson model [84]. It is remarked that the electrostatic interactions between single-electron modes can significantly distort the free electron energy bands in long wavelength regime producing variable width large energy gaps. In the current study, we extend our previous investigation of plasmon excitation in electron gas in a one-dimensional infinite box to the case with perpendicular electron drift and show that the transverse dispersion of collective modes and their energy levels can be strongly affected by the perpendicular electron drift momentum. The paper is structured as follows. The basic model is presented in the “Quantum Hydrodynamic Model” section. The dispersion relation of drifting electron gas of arbitrary degeneracy in a one-dimensional infinite well is presented in the “Quantum Dispersion Relation and Instability” section. Energy quantization and the quantum-level instability are investigated in the “Double Quantization of Plasmons” section and conclusions are drawn in the “Conclusion” section.

Quantum Hydrodynamic Model

Let us consider an arbitrary degenerate electron gas freely flowing in y-direction with constant speed \( v_y \) and confined in the slab region \( 0 < x < L \). The closed set of the quantum hydrodynamic equations may be cast into a coupled Schrödinger-Poisson system [77, 78]

\[
\frac{i\hbar}{\partial t} \frac{\partial \mathcal{N}(\mathbf{r}, t)}{\partial t} = -\frac{\hbar^2}{2m} \Delta \mathcal{N}(\mathbf{r}, t) - e\phi(\mathbf{r}, t)\mathcal{N}(\mathbf{r}, t) + \mu \mathcal{N}(\mathbf{r}, t),
\]

\[
\Delta \phi(\mathbf{r}, t) = 4\pi e n(\mathbf{r}, t),
\]

where \( \mu \) is the chemical potential of the electron gas. In the Madelung fluid language [43], the statefunction \( \mathcal{N} = \psi(\mathbf{r}, t) \exp[i\mathbf{s}(\mathbf{r}, t)/\hbar] \) may be used effectively to describe the dynamics of the system, where \( \psi(\mathbf{r}, t)\psi^*(\mathbf{r}, t) = n(\mathbf{r}) \) characterizes the local electron number density distribution and \( \nabla \psi(\mathbf{r}, t) = m\mathbf{u}(\mathbf{r}, t) \). On the other hand, the geometry of current problem imposes conditions in which all hydrodynamic quantities such as the electrostatic potential \( \phi \) and local electron number-density \( n \) vary as \( f(\mathbf{r}, t) \propto g(x) \exp(ik_y y - i\omega t) \) [85] in which the \( z \)-axis is eliminated by symmetry considerations and \( k_y \) denotes the de Broglie’s wavenumber in the flow direction. Therefore, in our particular case, the statefunction is separable in \( x - y \) plane into the form \( \mathcal{N}(x, y, t) = \psi(x) \exp[i(p_y y - \epsilon t)/\hbar] \) in which \( \epsilon = h\omega \) is the energy eigenvalue of collective electron excitations and \( p_y = \hbar k_y \) is the electron de Broglie momentum in flow direction. Note that in the direction of electron flow (y-axis) the plasmon excitations become resonant with the constant flow momentum and the de Broglie wavenumber \( k_y = \hbar v_y / n \) becomes the fundamental wavenumber of propagation in the flow direction [86]. Note also that the equilibrium chemical potential \( \mu_0 \) and electron number density \( n_0 \) are related through an appropriate equation of state (EoS) [87].

Figure 1 depicts the physical geometry of our problem. The arbitrary degenerate electron gas is bounded by infinite walls at \( x = 0, L \) flowing with constant speed in

\[
\psi = 0, \quad \psi' = 0, \quad \psi_1 = m_1, \quad \psi_1' = 0, \quad \phi_1 = 0, \quad \phi_1' = 0.
\]

\[ V_y \]

\[ \text{Infinite Well} \]

\[ \text{Electron Slab Current} \]

\[ \text{Infinite Well} \]

\[ \text{Z} \]

\[ \text{L} \]

\[ \text{X} \]
perpendicular \((y)\) direction. The statefunction and its derivative ideally vanish at wall positions. The condition is analogous to the electron gas confined to a hard-wall box in one dimension considered elsewhere \[77\] in addition to having a constant perpendicular momentum. Within the normalization scheme \(\Psi(x) = \psi(x)/\sqrt{n_0}\) with \(n_0\) being the equilibrium electron number density and \(\Phi(x) = e\phi(x)/E_p\) with \(E_p = \hbar\omega_p\) and \(\omega_p = \sqrt{4\pi e^2 n_0/m_e}\) being the plasmon frequency, after separation of variables, we arrive at the following system

\[
\frac{d^2\Psi(x)}{dx^2} + \Phi(x)\Psi(x) + (\epsilon - \mu - k_y^2)\Psi(x) = 0, \tag{3}
\]

\[
\frac{d^2\Phi(x)}{dx^2} - k_y^2\Phi(x) - |\Psi(x)|^2 = 0, \tag{4}
\]

where all wavenumbers including \(k_y\) are normalized to the plasmon wavenumber \(k_p = \sqrt{2m_e E_p/\hbar}\) and the space and time variables are scaled to the plasmon length \(l_p = 1/k_p\) and the inverse of the plasmon frequency, respectively. We would now assume linear perturbations around equilibrium point with equilibrium identities as \(\Psi = 1, \mu = \mu_0\) and \(\Phi = 0\) which lead to the following coupled linear differential equation system

\[
\frac{d^2\Psi(x)}{dx^2} + \Phi(x) + (E - k_y^2)\Psi(x) = 0, \tag{5}
\]

\[
\frac{d^2\Phi(x)}{dx^2} - k_y^2\Phi(x) - \Psi(x) = 0, \tag{6}
\]

where \(E = \mathcal{E} - \sigma\) with \(\sigma = \mu_0/E_p\) is the normalized equilibrium chemical potential and \(\mathcal{E} = \epsilon/E_p\) is the normalized energy eigenvalue.

Figure 2 depicts variations in different parameters of the arbitrary degenerate electron gas. The normalized chemical potential \(\sigma\) at room temperature is seen (Fig. 2(a)) to increase

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Variations in various plasmon parameters, namely, a the normalized chemical potential \(\sigma\) with respect to the equilibrium chemical potential of arbitrary degenerate electron gas at room temperature, b the equilibrium number density versus the equilibrium chemical potential, c the effective plasmon length \(l_p\) in nanometers in terms of the electron number density, and d the plasmon energy \(E_p\) in electronVolts versus the electron concentration.}
\end{figure}
sharply as the chemical potential of electron gas increases in the classical regime ($\mu_0 \ll 1$) and monotonically increases in the quantum regime. Figure 2(b) shows the values of the chemical potential for corresponding electron number densities showing that at room temperature the chemical potential amounts to few electronvolts in typical metallic densities. The variation of plasmon length $l_p$ in terms of the electron number density is shown in Fig. 2(c) in nanometer units. It is remarked that the plasmon length decreases from few tenths of a nanometer in semiconductors to few hundredth of a nanometer in metallic densities. It is also seen from Fig. 2(d) that the plasmon energy varies slowly in semiconductors and increases sharply to few electron volts in the metallic density regime.

**Quantum Dispersion Relation and Instability**

The linear system (5) has the following straightforward solution for tentative boundary values $\Phi(0) = 1$ and $\Psi(0) = \Psi'(0) = \Phi'(0) = 0$, as

$$\Phi(x) = \frac{(E + a) \cos(k_1 x) - (E - a) \cos(k_2 x)}{2a},$$

$$\Psi(x) = \Lambda \frac{\cos(k_2 x) - \cos(k_1 x)}{2a},$$

**Fig. 3** The figure depicts a the variation in collective quantum electron dispersion curve with changes in the perpendicular electron drift wavenumber, b the plasmon valence band minimum variation with electron drift wavenumber variation, c the stability region (dashed area) for wave-like electron gas excitations, and d the stability region for particle-like single electron excitations. The increase in curve thickness shows the increase in the varied parameter value in plot a and the dashed curve corresponds to the single electron dispersion curve.
where $A_w$ is a normalization constant, and $k_1, k_2$ are respectively the wavenumbers of particle-like and wave-like oscillations given as
\[
k_1 = \sqrt{\frac{1}{2} \left( E - 2k_y^2 + \alpha \right)}, \quad k_2 = \sqrt{\frac{1}{2} \left( E - 2k_y^2 - \alpha \right)}, \quad \alpha = \sqrt{E^2 - 4}.
\]

(9)

Note that the solution (7) admits the simple dispersion relation $E = (k_1^2 + k_2^2) + 1/(k_1^2 + k_2^2)$ which reduces to that in ref. [77] in the limit $k_y = 0$.

Figure 3(a) depicts the variation of plasmon excitation dispersion in the presence of perpendicular electron drift, in plasmon parameters units. It is remarked that in the presence of electron drift the wave-like excitation branch branches at $k = 0$ limit with the intersection energy value decreasing by increase in the value of $k_y$. This effect causes the setup of a new quantum instability in the wave-like oscillations in collective dispersion branch making the high-energy collective excitation above a threshold energy, $E_m$, only of particle-like type. A plasmon energy band $\Delta E_{k_y}$ forms at $2 < E < E_m$ for which collective electron excitations are stable. Moreover, Fig. 3(b) shows the variations in the wavenumber of energy minimum (plasmon band valley wavenumber $k_y$) with changes in $k_y$ in plasmon wavenumber unit. While the energy of valence band stays unaffected, with increase of $k_y$, the valence band minimum wavenumber $k_v = \pm \sqrt{1 - k_y^2}$ decreases from unity to zero as $k_y$ increases over the range $[0, 1]$, in unit of plasmon wavenumber. Figure 3(c), (d) depict the stability (dashed) regions of the wave-like and particle-like plasmon branches in terms of $k_y$, respectively. It is remarked that while the particle-like branch is always stable for $E > 2$ for all values of drift wavenumber, $k_y$, the wave-like excitation branch destabilizes for higher values of $E$ and $k_y$ denoting a maximum stable plasmon energy of $E_s = k_y^2 + 1/k_y^2$ (e.g., wave-like stability range of $2 < E < E_s$). Assuming a general imaginary wavenumber form $k = \beta + i\gamma$
\[
k = \left[ a + (E - 2k_y^2)^2 - 2a^{1/4}(E - 2k_y^2) \cos \left( \frac{1}{2} \arg a \right) \right]^{1/4} \frac{e^{\frac{1}{4} \arg (k - 2ki)}}{\sqrt{2}}.
\]

(10)

one obtains the following imaginary for collective excitations energy $E = E_r + iE_i$.

Fig. 4 The doubly quantized energy level of first eigenvalue ($E_1$) a for box width $L = 100$ in absence of perpendicular electron drift, b for box width $L = 200$ in absence of perpendicular electron drift, c for box width $L = 200$ with perpendicular electron drift $k_y = 0.5$, and d for box width $L = 200$ with perpendicular electron drift $k_y = 0.6$. 
Double Quantization of Plasmons

The normalized wavefunction $\Psi(x)$ may be written in the form

$$\Psi(x) = 2A_x \sin \left( \frac{(k_1 - k_2)x}{2} \right) \sin \left( \frac{(k_1 + k_2)x}{2} \right),$$

being the normalization factor, given by the condition

$$\int_{-\infty}^{\infty} |\Psi(x)|^2 dx = 1,$$

where $A_x$ is the normalization factor.

The general condition is $\pm k_1 \pm k_2 = 2n\pi/L$ which leads to the double energy quantization with the following quantized energy eigenvalues

$$E_{1,2} = \frac{4n^2\pi^2}{L^2} \pm 2\sqrt{1 - \frac{4n^2\pi^2}{L^2} - k_y^2},$$

with the integer value $n$ characterizing the principal quantum number. It is clearly evident that the energy levels are stable for $n < L/(2\pi k_y)$.

The collective electrostatic excitation energy levels of $E_1$ and $E_2$ in the electron gas flow are depicted in Figs. 4 and 5, in plasmon energy unit, for different values of box width $L = 100, 200, 200$ with perpendicular electron drift $k_y = 0.5, 0.6$. The dashed energy band of $0 < E_z < 2$ indicates forbidden collective excitations.

Fig. 5 The doubly quantized energy level of second eigenvalue ($E_2$) a for box width $L = 100$ in absence of perpendicular electron drift, b for box width $L = 200$ in absence of perpendicular electron drift, c for box width $L = 200$ with perpendicular electron drift $k_y = 0.5$, and d for box width $L = 200$ with perpendicular electron drift $k_y = 0.6$. The dashed energy band of $0 < E_z < 2$ indicates forbidden collective excitations.
Fig. 6  The stability region of 
\( a \) first and \( b \) second energy 
eigenvalues as fractional quantum number \((n/L)\) of collective 
electron excitations versus perpendicular electron drift wave-
number. All wavenumbers are in units of plasmon wavenumber.

\[ E_1 > 2, \text{Stability Region} \]

\[ E_2 > 2, \text{Stability Region} \]

Fig. 7  Variation in statefunction 
of arbitrary degenerate electron gas in infinite wall potential in 
the \( a \) absence and \( b \) presence of perpendicular electron flow. The 
plasmon wavefunction profiles for higher energy levels \( c \ n = 2 \) 
and \( d \ n = 3 \), for \( k_y = 0.8 \).
width and \( k_y \). Figure 4(a), (b) show the quantized plasmon excitation energy \( E_1 \) for box length values of \( L = 100 \) and \( L = 200 \), in plasmon length unit. It is remarked that plasmon energy levels resemble that of a single electron trapped in a one-dimensional box of length \( L \) where with increase of the energy the level separation whereas with increase of box length it decreases. It is remarkable that in the presence of perpendicular electron drift energy levels approach an upper limit, i.e., the maximum stable energy level, forming a stable energy band, as shown in Fig. 4(c). Figure 4(d) shows that further increase of \( k_y \) decreases the stable band width. Moreover, Fig. 5 shows the energy band levels for the second energy eigenvalue, \( E_2 \). It is seen that the energy levels start from the lowest value \( E_2 = -2 \). The energy levels in the range \(-2 < E_2 < 2\) are however collectively unstable. It is also remarked that presence of electron drift introduces an upper stable energy level beyond which quantum-level instability sets in. However, for energy eigenvalues \( E_2 \), the level spacing increases, unlike those of \( E_1 \) and the upper stable levels do not approach each together as \( k_y \) is increased. It is further seen that the maximum stable energy level for given values of \( L \) and \( k_y \) for \( E_2 \) is lower as compared to those of \( E_1 \).

Figure 6 shows the stability (dashed) region for \( n/L \) versus \( k_y \) in units of plasmon wavenumber. As previously mentioned, the new quantum instability arises due to the imaginary wavenumber values for wave-like oscillations, whereas, the particle-like oscillations are always stable. In such case, the overcritical perpendicular electron momentum interferes with the transverse collective plasmon excitations leading the wave-like oscillation amplitude in the system to grow exponentially. This kind of instability may have significant physical impact on the electron transport and second quantization of many-electron system in nanoscale electronic devices. It is remarked that, for the first energy eigenvalue \( E_1 \), the stability region extends to \( k_y = 1 \) limit, whereas, for small values of \( k_y \) there is no lower bound on \( n/L \). Furthermore, for the second energy eigenvalue, \( E_2 \), there is an upper
bound stability limit approximately at $k_y \simeq 0.7$ and a lower bound for the box width parameter value of $n/L \simeq 0.4$.

Figure 7 shows the wavefunction $\Psi_n(x)$ for given box length $L$ and $k_y$ values. The dual lengthscale variation corresponds to coupled wave- and particle-like excitations in the electron gas. By comparing Fig. 7(a), (b), it is clearly remarked that the existence of electron flow strongly affects the long wavelength excitations. This is also confirmed by the dispersion curve of excitations in Fig. 3(a) that the long wavelength collective excitations in the electron gas are most affected due to presence of electron drift. It is remarked that the small wavelength collective perturbations are perfectly fitted within the similar wavefunction profile, as obtained by Schrödinger equation for a single electron. Figure 7(c), (d) depict the higher-order plasmon excitation modes ($n = 2, 3$) manifesting the mutual interactions between single-particle and collective oscillations in the electron gas. The important interplay and energy exchange between wave-particle oscillations in electron gas has been reported to lead to the resonant electron-plasmon interactions in the direction of electron flow \[86\].

Figure 8 depicts the electron density distribution for drifting electron gas in a one-dimensional infinite-walls box. The electron density fringes are due to the single-particle and collective electrostatic interactions in the electron gas vanishing at the boundaries. Figure 8(a) shows the transverse electron density distribution in the absence of perpendicular electron drift. In Fig. 8(b), introducing the electron flow in $y$-direction significantly affects the transverse electron density distribution, altering the fringe pattern. It is remarked that the fringe separation and their effective width is significantly altered which is due to the interaction between the flow momentum and the collective electrostatic interactions. Further increase in perpendicular electron gas momentum, as shown in Fig. 8(c), leads to more separation and thickening of density fringes. Finally, for over-critical drift
momentum value, the transverse plasmon excitation instability sets in, leading to complete elimination of density fringe pattern.

Figure 9(a), (b) show the maximum energy levels of quantum collective excitations corresponding to the first and the second energy eigenvalues as altered by the changes in the different parameters. It is remarked from Fig. 9 that these maximum values are quantized within the considered length scales. Figure 9(a) denotes that, for given value of $k_y$, $E_{1m}$ ($E_{2m}$) increases (decreases) by increase of the quantum box-length until a critical box length is reached and then collapses into the same level and this process occurs periodically. The similar feature is observed when one varies the drift momentum $k_y$ for a fixed box length, as shown in Fig. 9(b). Figure 9(c), (d) show the maximum wavenumbers of quantum collective excitations corresponding to the particle-like and the wave-like oscillations as altered by the changes in different parameters. Figure 9(c) again indicates the collective quantization feature in wavenumber values due to the variation in box length $L$ for a fixed value of drift momentum $k_y = 0.5$ and Fig. 9(d) depicts quantization of excitation wavenumber where $k_y$ is varied for the fixed box length $L = 100$. These quantum features may be of high importance in nanoscale device calibration or various quantum measurements.

**Conclusion**

We have used the quantum electron fluid model in order to investigate the transverse plasmonic excitations in a slab current with infinite wall potential. The linearized coupled Schrödinger-Poisson system is solved and the statefunctions representing the transverse collective electron excitations in the presence of perpendicular electron momentum are obtained. We show that the transverse collective electrostatic excitations are doubly quantized with the upper energy levels becoming unstable for overcritical electron drift wavenumbers. It is also shown that the quantization of collective electronic excitation statefunction profiles is reminiscent of single-electron wavefunction trapped in an infinite potential modulated with small wavelength perturbations caused by many-electron electrostatic interactions. We further report a new kind of instability on wave-like excitations on electron gas due to interaction with perpendicular electron drift. The new instability may play a fundamental role in collective excitations in nano-fabricated plasmonic devices.

**Data Availability** The data that support the findings of this study are available from the corresponding author upon reasonable request.

**Declarations**

**Competing Interests** The author declares no competing interests.

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