Inefficient Cosmic-Ray Diffusion around Vela X: Constraints from H.E.S.S. Observations of Very High-energy Electrons

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Abstract

Vela X is a nearby pulsar wind nebula (PWN) powered by a $\sim 10^4$ year old pulsar. Modeling of the spectral energy distribution of the Vela X PWN has shown that accelerated electrons have largely escaped from the confinement, which is likely due to the disruption of the initially confined PWN by the supernova remnant reverse shock. The escaped electrons propagate to the Earth and contribute to the measured local cosmic-ray (CR) electron spectrum.

We find that the escaped CR electrons from Vela X would hugely exceed the measured flux by High Energy Stereoscopic System (HESS) at $\sim 10$ TeV if the standard diffusion coefficient for the interstellar medium (ISM) is used. We propose that the diffusion may be highly inefficient around Vela X and find that a spatially dependent diffusion can lead to CR flux that is consistent with the HESS measurement. Using a two-zone model for the diffusion around Vela X, we find that the diffusion coefficient in the inner region of a few tens of parsecs should be $\leq 10^{28}$ cm$^2$ s$^{-1}$ for $\sim 10$ TeV CR electrons, which is about two orders of magnitude lower than the standard value for the ISM. Such inefficient diffusion around PWN resembles the case of the Geminga and Monogem PWNe, suggesting that inefficient diffusion may be common in the vicinity of PWNe that span a wide range of ages.

Key words: cosmic rays

1. Introduction

Recent measurements of an increasing cosmic-ray (CR) positron ($e^+$) fraction above 10 GeV by the Payload for Antimatter Exploration and Light-nuclei Astrophysics (PAMELA) and Alpha Magnetic Spectrometer (AMS-02) identify an excess of high-energy positrons relative to the standard predictions for secondary electron–positron production in the interstellar medium (ISM; Adriani et al. 2010; Aguilar et al. 2013), suggesting that these positrons are produced as primary particles. The CR $e^+ + e^-$ spectrum has been measured up to a few TeV by Fermi-Large Area Telescope (-LAT), the Very Large Energetic Radiation Imaging Array System (VERTIAS), the Dark Matter Particle Explorer (DAMPE) (Abdollahi et al. 2017; Staszak & VERITAS Collaboration 2015; DAMPE Collaboration et al. 2017), and to $\sim 20$ TeV by HESS recently (H.E.S.S. Collaboration et al. 2017). As high-energy electrons (hereafter, we do not distinguish positrons from electrons) efficiently cool in the ISM through synchrotron and inverse-Compton radiation, they cannot travel beyond a kiloparsec distance before depleting their energies. Therefore, these high-energy electrons must be produced by nearby sources, such as pulsars (or PWNe), annihilating dark matter particles, and supernova remnants (SNR). While the real sources are still under debate, nearby pulsars such as Geminga, PSR B0656+14, and Vela, which are at distances of only $\sim 200$ pc, are the most attractive candidates (Shen 1970; Atoyan et al. 1995; Hooper et al. 2017).

Vela X, powered by a $\sim 10^4$ year old pulsar (i.e., the Vela pulsar or PSR B0833-45), is one of the most well-studied PWNe. It consists of an extended radio halo of the size of $2^\circ \times 3^\circ$ and a small collimated structure, e.g., the “cocoon.” High-energy gamma-ray emission has been detected from both the halo and the cocoon by Fermi-LAT and HESS. de Jager et al. (2008) propose that there are two distinct populations of electrons: one responsible for the radio and GeV gamma-ray emission and the other for the X-ray and TeV emission. To explain the steep GeV spectrum measured by Fermi-LAT and the dimness of the TeV nebula relative to the spin-down power of the Vela pulsar, Hinton et al. (2011) argue that a significant diffusive escape of electrons from the halo must have occurred. The same conclusion is reached more recently by Tibaldo et al. (2018) with an analysis of $\sim 9.5$ years of data from Fermi-LAT observations. While the confinement of particles in PWNe is thought to be effective in the early stage, the interaction with the SNR reverse shock, which seems to have appeared in Vela X several thousand years ago (Blondin et al. 2001; Gelfand et al. 2009), may have brought an end to the confinement. The asymmetric structure of the PWN with respect to the pulsar supports such an interpretation. The interaction is expected to disrupt the PWN sufficiently enough that diffusion of particles out of the PWN becomes possible.

The escaped electrons will contribute to the CRe spectrum measured at the Earth. Using a diffusion coefficient of $1.07 \times 10^{-27} (E/1 \text{ GeV})^{0.6}$ cm$^2$ s$^{-1}$ and a total electron energy of $6.8 \times 10^{48}$ erg, Hinton et al. (2011) predict a distinct bump in the CR electron spectrum at several TeV. This diffusion coefficient is actually smaller than the standard one for the ISM, which is $D_{\text{ISM}} \sim 3.86 \times 10^{28} (E_{\text{e}}/\text{GeV})^{0.33}$ cm$^2$ s$^{-1}$, as inferred from the boron-to-carbon ratio and other CR secondary-to-primary ratios for electrons with $E \lesssim 10$ TeV (see...
2. Results for Spatially Independent Diffusion

We first study whether a simple one-zone diffusion model, which is usually used for CR propagation in the ISM, can produce a flux consistent with the HESS measurement. This simple model assumes that the diffusion is homogeneous along the path from the PWN to the Earth. The diffusion coefficient is given by the standard one, \(D_{\text{ISM}} \approx 3.86 \times 10^{28} (E_e/\text{GeV})^{0.33} \text{cm}^2 \text{s}^{-1}\).

The transport of CR electrons can be described by the equation:

\[
\frac{\partial}{\partial t} n_e(E_e, \vec{x}, t) = \nabla \cdot \left[ D(E_e) \nabla n_e(E_e, \vec{x}, t) \right] + \frac{\partial}{\partial E_e} \left[ \frac{dE_e}{dt} n_e(E_e, \vec{x}, t) \right] + Q(E_e, \vec{x}, t),
\]

where \(n_e(E)\) is the differential number density of electrons, \(D(E)\) is the diffusion coefficient, and \(Q(E_e, \vec{x}, t)\) is the source term. Energy losses caused by inverse-Compton (IC) and synchrotron processes are described as

\[
\frac{dE_e}{dt} = \frac{4}{5} \sigma_T \rho_e(r) S_i(E_e) \left( \frac{E_e}{m_e} \right)^2 + \frac{4}{5} \sigma_T \rho_{\text{mag}}(r) \left( \frac{E_e}{m_e} \right)^2,
\]

where \(\sigma_T\) is the Thomson cross section, \(\rho_e\) denotes the radiation energy density of background photons, and \(\rho_{\text{mag}}\) denotes the energy density of the magnetic field. Various components of the radiation backgrounds are taken into consideration, including the cosmic microwave background (CMB), starlight (star), ultraviolet emission (UV), and infrared emission (IR). Parameters are adopted as follows for the area surrounding Vela: \(\rho_{\text{CMB}} = 0.260 \text{ eV cm}^{-3}\), \(\rho_{\text{star}} = 0.44 \text{ eV cm}^{-3}\), \(\rho_{\text{UV}} = 0.10 \text{ eV cm}^{-3}\), \(\rho_{\text{IR}} = 0.44 \text{ eV cm}^{-3}\), \(\rho_{\text{mag}} = 0.622 \text{ eV cm}^{-3}\) (corresponding to \(B = 5 \mu \text{G}\)) \(T_{\text{CMB}} = 2.7 \text{ K}\), \(T_{\text{star}} = 7500 \text{ K}\), \(T_{\text{UV}} = 20000 \text{ K}\), and \(T_{\text{IR}} = 25 \text{ K}\) (de Jager et al. 2008; Grondin et al. 2013). We also check the results adopting other values of these parameters (Hooper et al. 2017; Fang et al. 2018b), but negligible differences are found. When \(E_e \gtrsim m_e^2 / 2T\), the suppression of the inverse-Compton scattering by the Klein–Nishina effect cannot be ignored, which can be parameterized by (Longair 2011)

\[
S_i(E_e) \approx \frac{45m_e^2}{64\pi^2 T_e^2} \left( \frac{E_e}{m_c^2} + \frac{E_e}{m_e^2} \right)^3.
\]

For a burst-like injection of \(Q(E_e, t) = \delta(t) Q_0 E_e^{-\alpha} \exp(-E_e/E_0)\), the solution to Equation (1) is given by

\[
n_e(E_e, r, t) = \frac{Q_0 E_e^{-\alpha} \exp(-E_e/E_0)}{8\pi^3/2 E_e^2 L_{\text{dif}}^3(E_e, t)} \left[ -\frac{r^2}{4L_{\text{dif}}^2 (E_e, t)} \right],
\]

where \(E_0\) is the initial energy of the electron of energy \(E_e\) at time \(t\) and \(L_{\text{dif}}\) is the diffusion length scale given by

\[
L_{\text{dif}} = \sqrt{\int_{E_0}^{E_e} \frac{D(E')}{dE'/dt} dE'}. \quad (5)
\]

We note that Equation (4) permits a fraction of particles to propagate faster than the speed of light, which is the so-called “superluminal diffusion problem” (Dunkel et al. 2007; Aloisio et al. 2009; Liu et al. 2016). From the mathematical point of view, the superluminal propagation always exists in the solutions of the nonrelativistic diffusion equations, but very frequently the contribution of unphysical regions to the solution is negligibly small. In these cases, one can regard that the diffusion equation gives a correct description of the considered physical phenomenon. However, when \(t \sim r/c\), the problem of superluminal diffusion in Equation (4) becomes severe. Aloisio et al. (2009) find a solution to this problem: if the cooling of particles is unimportant (which is applicable to our case), the probability to find one electron at a distance, \(r\), from the source at a time, \(t\), after its injection can be described by

\[
P(E_e, r, t) = \frac{\theta(ct - r)}{(ct)^3 Z(c^2t/2r)} \left[ 1 - \left( \frac{c^2t}{2r} \right)^2 \right] \exp \left[ -\frac{c^2t}{2r} \sqrt{1 - \left( \frac{c^2t}{2r} \right)^2} \right], \quad (6)
\]

where

\[
Z(y) = 4\pi K_0(y)/y, \quad (7)
\]

with \(\theta(r)\) as the Heaviside function and \(K_0(y)\) as the modified Bessel function. Note that the above formula only works when \(r\) is much smaller than the cooling timescale of electrons, which is true for our following calculation. The electron density at time \(t\) after the injection then can be obtained by

\[
n_e(E_e, r, t) = \int_{-\infty}^{t} P(E_e, t - t', r) \delta(t') Q_0 E_e^{-\alpha} \exp(-E_e/E_0) dt'.
\]

Following Hinton et al. (2011), we take \(\alpha = 1.8\) and \(E_e = 6 \text{ TeV}\) for the electron spectrum. The total energy injected into the initially confined PWN depends on the birth-period, \(P_0\), of the pulsar. The energy in relativistic electrons is about \(\sim 2 \times 10^{49} (P_0/30 \text{ ms})^{-2}\), where \(\epsilon\) is the fraction of spin-down
power converted into relativistic electrons. All previous estimates of the total energy give a value of several $10^{48}$ erg (de Jager et al. 2008; Abdo et al. 2010; Hinton et al. 2011; Grondin et al. 2013). This is consistent with the estimate of the birth-period of $P_0 = 40$ ms that is invoked to account for the ratio between the PWN radius and the SNR radius (van der Swaluw & Wu 2001). In the following calculation, we use a total energy of $6.8 \times 10^{48}$ erg, as obtained from the spectral energy distribution (SED) fit of Vela X by Hinton et al. (2011). We use $r = 287$ pc as the distance between the Earth and Vela. We assume that the electrons are released instantaneously at disruption time $t$ of the initial PWN. The exact disruption time of Vela X is unknown, but the reasonable values should be at least several kyr (e.g., Blondin et al. 2001; Gelfand et al. 2009). Note that the time span between the disruption time of Vela X and the current time is exactly the propagation time of injected electrons $t$.

The results of the CR electron spectrum for various $t$ are shown in Figure 1, where Equation (8) is used to avoid the possible superluminal diffusion problem. Obviously, the CR flux produced by Vela X exceed the measured flux by several order of magnitudes for any reasonable value of $t$. Only for a very small $t$ (i.e., $t < 1100$ years), the CR flux can be consistent with the HESS measurement (see the blue line in Figure 1). Such a small $t$ seems unreasonable since the timescale when the PWN collides with the SNR reverse shock explosion is usually several thousand years (e.g., Blondin et al. 2001; Gelfand et al. 2009). This suggests that the simple one-zone diffusion model does not work for Vela X.

3. The Two-zone Diffusion Model

Recent HAWC observations of the PWN regions around Geminga and Monogem provide detailed information about the spatially extended emission of TeV gamma-rays. The spectrum and morphology of the TeV emission can be used to infer the features of underlying high-energy electrons responsible for the IC photons. The angular profiles of the TeV emission observed from Geminga and Monogem indicate that the diffusion is highly inefficient in the regions surrounding these sources (Abeysekara et al. 2017). This is also the first empirical determination of a diffusion coefficient in the region of tens of pc around pulsars in the local Galaxy. The inferred diffusion coefficient is more than two orders of magnitude smaller than the standard diffusion coefficient in the ISM. Furthermore, assuming a spatially dependent diffusion, Fang et al. (2018a) and Profumo et al. (2018) show that nearby pulsars, such as Geminga, could contribute significantly to the CR electron spectrum in 0.1–1 TeV. Motivated by these results, we study whether a spatially dependent diffusion model with inefficient diffusion in the inner region surrounding Vela X could resolve the inconsistency between the predicted CR electron flux and the observed one by HESS. We approximate the diffusion coefficient of the entire space to be a step function of the distance to Vela X, where the diffusion is suppressed with a coefficient of $D_1$ within a few tens of parsecs from Vela X and a standard diffusion coefficient $D_2$ for the outside region, i.e.,

$$D(r) = \begin{cases} D_1, & r \leq r_0 \\ D_2 = D_{\text{ISM}}, & r > r_0 \end{cases}$$

![Figure 2. Schematic picture of the two-zone diffusion model. $r_0$ is the radius of the region with inefficient diffusion, and $r$ is the distance between the Vela pulsar and the Earth.](image)

We develop an analytic approach to solve Equation (1) in the above two-zone diffusion model, as shown in Figure 2. We first use the solution of Equation (4) with a diffusion coefficient, $D_1$, to estimate the number density of electrons at the interface of the two zones (i.e., at the spherical surface with a radius, $r_0$). We note that Equation (4) is strictly correct only when $D_1 = D_2$. So the above obtained number density is a crude estimate when $D_1 \neq D_2$. The flux density at the sphere is $F_D = -\frac{\partial N}{\partial r}$, and the number of particles in unit time passing an surface element on the sphere outwardly is $\Delta N = F_D \Delta S$, with $\Delta S$ as the area of the element surface. We then regard the surface element as a point source with an injection rate, $\Delta N$. Since the flux at the sphere is a function of time, $\Delta N$ is also a function of time. We then regard the sphere consist of many surface elements, each of which is treated as a point source. A convenient choice is to divide the sphere into annular rings perpendicular to the line connecting the Earth and Vela X, so that the distance between the Earth and each surface element on the ring is the same. The radius of the ring is $r_0 \cdot \sin \theta$, and the distance to the Earth is $d = \sqrt{r^2 - 2r \cdot r_0 \cos \theta + r_0^2}$. The area of this ring is $\Delta S_{\text{ring}} = 2\pi r_0^2 \sin \theta d\theta$. The integral can be done over the annular rings on the spherical surface. We further regard the continuous injection from the sphere as the sum of a series of discrete injection so that the electron number density at the radius of the Earth can be calculated with Equation (4). The precision of this analytic method is tested in the Appendix.
We find that the difference of the CR electron flux at the Earth between our analytic results and the numerical approach used by Fang et al. (2018b) is, at most, \(\sim 30\%\). Since the uncertainty of the CR flux measured by HESS at \(\sim 10\ TeV\) is about a factor of two, we consider that the precision of our analytic approach is sufficient for this study.

We note that the burst-like injection is an assumption based on the reverse shock interaction scenario. To be more complete, we also consider the case of continuous injection. Since the injection rate is most likely to be monotonically decreasing, a flat injection profile (i.e., a constant injection rate of electrons) would provide the most conservative estimate of the electron flux arriving at the Earth, given that other parameters are the same. That is, the burst-like profile and the flat-time profile can be regarded as the two extremes for any injection patterns. To deal with the flat injection, we decompose the injection function into many small time bins, treat each time bin as a burst-like injection, and last sum up the contribution from each of them. We compare the results of the burst-like injection and the flat injection in Figure 3 for a different time, \(t\), at which electron injection started. We find that, in the case of the flat-profile injection, a larger diffusion coefficient is obtained. This is due to that a significant part of electrons are injected at a later time in this case, and these electrons have not arrived at the Earth. The peak of the spectrum moves toward lower energy for earlier injection (i.e., larger \(t\)). This is because electrons with lower energy can reach the Earth while electrons with higher energy will be cooled down. We find that, for all the \(t\) considered here, the difference in the obtained \(D_1\) for the two different injection profiles is within a factor of 2. We thus conclude that our result is not sensitive to the injection profile, and in what follows, we only consider the burst-like injection profile for simplicity.

The disruption time \(t\) of the PWN and the radius of the inner zone are two unknown parameters. We use \(r_0 = 30\) pc and \(r_0 = 50\) pc as two reference values for the radius of the inner zone. The CR fluxes for the two-zone model with \(r_0 = 30\) pc and \(r_0 = 50\) pc are, respectively, shown in the left and right panels of Figure 4. For a reasonable range of \(t\) (i.e., \(t > 2.5\) kyr), we find that only when \(D_1(10\ TeV) \lesssim 10^{28}\ cm^2\ s^{-1}\) can the CR electron flux be consistent with the HESS measurement. This value is about two orders of magnitude smaller than the standard diffusion coefficient at \(\sim 10\ TeV\), which is \(D_{\text{ISM}}(10\ TeV) \approx 0.8 \times 10^{30}\ cm^2\ s^{-1}\). The constraint on the diffusion coefficient becomes

![Figure 3](image-url)

**Figure 3.** Comparison between the CR electron flux produced by Vela X in the two-zone model and the measured flux by HESS. \(t\) is the time when the injection started. \(r_0 = 50\) pc is adopted. The red lines show the results for flat injections, while the blue lines show the results for burst-like injections. The diffusion coefficients in both inner and outer zones are assumed to be proportional to \(E^{0.33}\) (specifically, \(D(E) = 3.86 \times 10^{28} (E/GeV)^{0.33}\ cm^2\ s^{-1}\)).
more stringent for a larger $t$. This is because a longer diffusion time requires a smaller diffusion coefficient in the inner zone.

In the above calculation, we have assumed $D(E) \propto E^{0.33}$. As the physics of the diffusion is not well-known, we also consider another case of the energy dependence, i.e., $D(E) \propto E^{0.548}$ for both the ISM and the region surrounding the PWN (Fang et al. 2017). The results are shown in the left and right panels of Figure 5 for $r_0 = 30$ pc and $r_0 = 50$ pc, respectively. We find that, for this kind of diffusion, we also have $D_{10 \text{ TeV}} \approx 10^{-28} \text{ cm}^2 \text{s}^{-1}$, which is almost the same as the $D(E) \propto E^{0.33}$ case.

In the above, we have taken a total injection energy of $6 \times 10^{58}$, as suggested by Hinton et al. (2011). To be more conservative and motivated by HAWC observations of other TeV halos, we also take a total injection energy corresponding to $\sim 10\%$ of the total energy budget, i.e., $1.9 \times 10^{48}$ erg. The results are shown in Figure 6. As shown, the required $D_1$ does not change significantly, as the electron/positron flux is more sensitive to $D_1$ than the normalization of the injection spectrum.

The cutoff energy of the electron spectrum has been taken to be $E_c = 6 \text{ TeV}$ following Hinton et al. (2011). We note that the spectra observed from Geminga and B0656+14 by HAWC favor much larger values of $E_c$, i.e., several tens of TeV. Results with different $E_c$ are shown in Figure 7. For a larger $E_c$, more electrons with higher energy are injected. As these energetic electrons can more easily escape from the slow-diffusion region and reach the Earth, the received flux of the higher-energy electrons are significantly larger. Therefore, the constraint on the diffusion coefficient becomes even more stringent, and a smaller $D_1$ is required to reconcile with the HESS data.

In all of these cases, $D_1(10 \text{ TeV}) \lesssim 10^{28} \text{ cm}^2 \text{s}^{-1}$ can be obtained. Thus, the limit on the diffusion coefficient is robust and independent of uncertainties in the input parameters. We conclude that the diffusion in the immediate vicinity of Vela X must be highly inefficient.

4. Summary

There have been suggestions that electrons must have undergone diffusive escape from the the Vela X PWN, indicated by the evidence for a roll-over of the electron spectrum at energies of a few tens of GeV. These escaped electrons may contribute to the CR electron flux at the Earth. In this paper, we have shown that recent HESS data of the CR electron flux at $\sim 10 \text{ TeV}$ place interesting constraints on the diffusion coefficient around Vela X. We find that a highly inefficient diffusion region in the immediate vicinity...
of Vela X must be present, with \( D (10 \text{ TeV}) \lesssim 10^{26} \text{ cm}^2 \text{ s}^{-1} \). The result is consistent with the recent finding that there are inefficient diffusion regions surrounding the Geminga and PSR B0656+14 PWNe, suggesting that such inefficient diffusion regions may be common around PWNe of various ages.

Previous theoretical studies have suggested that CRs can be scattered by the self-generated Alfvén waves induced by streaming instability (Yan & Lazarian 2004; Yan et al. 2012; Malkov et al. 2013; Quenby 2018), whereby the particles are self-confined. If the CR flux is sufficiently high, the growth rate of the streaming instability can dominate the nonlinear damping of background turbulence. We speculate that such a process enhances the CR scattering rate inside \( r_0 \) and hence reduces the diffusion coefficient to the required level. Another possible mechanism is the influences of the Vela SNR surrounding Vela X. The turbulence in the areas swept up by the shock of the SNR can be strong, which may induce a smaller diffusion coefficient (Bell 1978; Li & Chen 2010, 2012).

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Appendix

We first compare the CR flux obtained with our analytic approach for the two-zone model with \( D_1 = D_2 \) and the simple one-zone model with the same diffusion coefficient (i.e., \( D = D_1 \)). The result is shown in Figure 8. One can see that the difference between the two models is negligibly small. This demonstrates that the treatment described by Figure 2 is quite accurate.

However, this test does not account for the influence caused by the difference in the diffusion coefficients of the two zones.

Figure 6. Same as Figure 4, while different total electron energies are adopted. \( n_0 = 50 \text{ pc} \) is adopted. The diffusion coefficients in both the inner and outer zones are assumed to be proportional to \( E^{0.33} \). The red and the blue lines represent the injection spectra with \( E_c = 60 \text{ TeV} \) and 6 TeV, respectively.

Figure 7. Comparison between the CR electron spectrum with different \( E_c \). The total injection energy is \( 6.8 \times 10^{58} \text{ erg} \). The diffusion coefficients in both the inner and outer zones are assumed to be proportional to \( E^{0.33} \). The red and the blue lines represent the injection spectra with \( E_c = 60 \text{ TeV} \) and 6 TeV, respectively.

Figure 8. Comparison between the CR spectrum obtained with our analytic approach for the two-zone model with the spatially independent diffusion coefficient (i.e., \( D_1 = D_2 = 3.86 \times 10^{28} (E_c / \text{GeV})^{0.33} \text{ cm}^2 \text{ s}^{-1} \)) and that obtained with the simple one-zone model (i.e., Equation (4)) with \( D = 3.86 \times 10^{28} (E_c / \text{GeV})^{0.33} \text{ cm}^2 \text{ s}^{-1} \). A burst-like injection is considered. The blue solid line represents the result obtained using Equation (4), while the red solid line and the red dashed line represent the results obtained for the two-zone analytic model with \( r_0 = 30 \text{ pc} \) and 50 pc, respectively. The injection time is taken to be \( t = 2 \text{ kyr} \).
There are two factors that may cause differences between the results in our analytical model and that of the realistic case. The density gradient of electrons at $r_0$ calculated by Equation (4) becomes smaller when $D_2 > D_1$, since a larger diffusion coefficient in the outer zone leads to a faster diffusion outward. Thus, our model underestimates the flux escaping from the spherical surface at $r_0$ in this regard. On the other hand, part of the electrons located at the spherical surface at $r_0$ will go back to the inefficient diffusion zone. These electrons will take a longer time to arrive at the Earth. Our analytic estimate does not take into account this effect, so our result overestimate the CR flux at the Earth in this regard.

To test the precision for the case of $D_1 = D_2$, we compare our analytic result with the numerical result obtained by Fang et al. (2018a), which solves Equation (1) for the case of $D_1 = D_2$ with a numerical method. The results are shown in Figure 9. For the cases of $t = 2500$ years, the difference between our analytic result and that obtained with the numerical method is, at most, 30%. For the cases of $t = 5000$ years, the difference is even smaller.

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