Weak decay rates for waiting-point nuclei involved in the rp-process

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Abstract.
We study weak interaction rates under the stellar density and temperature conditions encountered in the rp process. Special attention is paid to neutron-deficient waiting-point nuclei. The nuclear structure part of the problem is described within a deformed Skyrme Hartree-Fock + BCS + QRPA approach, which reproduces not only the beta-decay half-lives but also the available Gamow-Teller strength distributions, measured under terrestrial conditions. The various sensitivities of the decay rates to both density and temperature are discussed. In particular, we study the contributions coming from excited states in the parent nucleus, which are thermally populated, and the competition between beta-decays and continuum electron captures.

1. Introduction
A reliable nuclear input is an essential element in most astrophysical simulations of explosive events, which involve in particular, having a knowledge of the nuclear properties of exotic nuclei. Unfortunately, the experimental information on exotic nuclei is still very poor and thus, the astrophysical models of violent stellar phenomena are mainly based on nuclear model predictions. This is the case of type I X-ray bursts, generated by a thermonuclear runaway in the hydrogen-rich environment of a neutron star that is fed from a red giant binary companion. The explosions typically release $10^{38} - 10^{39}$ erg of energy, producing bursts lasting between seconds and a few minutes. These explosions have recurrence periods ranging from hours to days. They reach peak temperatures ($T$) and densities ($\rho$) of about $T = 1-3$ GK and $\rho = 10^6-10^7$ g cm$^{-3}$, respectively.

This scenario allows the development of the nucleosynthesis rapid proton (rp) capture process [1], which is characterized by proton capture reaction rates that are orders of magnitude faster than any other competing process, in particular $\beta$-decay. It produces rapid nucleosynthesis on the proton-rich side of stability toward heavier proton-rich nuclei and explains the energy and luminosity profiles observed in X-ray bursts. The potential impact of X-ray burst nucleosynthesis on the galactic abundances is still an open question. Although ejection of matter is very unlikely, it is not completely discarded. The possible relevance of these events to understand the origin and abundance of some neutron-deficient isotopes is still a matter of debate.

In general, the reaction path follows a series of fast proton-capture reactions until further proton capture is inhibited by photodisintegration. Then, the reaction flow has to wait for a relatively slow $\beta$-decay to proceed further and the respective nucleus is called a waiting point (WP). The half-lives of the WP nuclei finally determine the time scale of the nucleosynthesis process and the produced isotopic abundances.
The nuclear structure problem involved in the calculation of these rates must be treated in a reliable way. This means in particular, that the nuclear models should be able to reproduce at least the experimental information available on the decay properties (Gamow-Teller strength distributions and β-decay half-lives) measured under terrestrial conditions. The model used in this work is the quasiparticle random phase approximation (QRPA) based on a deformed Skyrme Hartree-Fock formalism with pairing correlations in BCS approximation. Measured beta-decay half-lives and Gamow-Teller (GT) strength distributions in neutron-deficient medium-mass nuclei [2, 3] are well reproduced by theoretical calculations based on deformed QRPA [4, 5].

Weak decay rates at high ρ and T are different from the decays observed under terrestrial conditions. First, one has to deal with the decay from thermally populated excited states in the parent nucleus. Secondly, because atoms in these scenarios are completely ionized, the existence of an electron plasma opens the possibility for continuum electron captures (cEC).

2. Theoretical formalism

The general formalism to calculate weak interaction rates in stellar environments as a function of ρ and T was introduced in Ref. [8]. Improvements on the nuclear structure part were subsequently introduced either from the Shell Model [9] or from QRPA [10].

The decay rate of the parent nucleus is given by [8]

$$\lambda = \sum_i \lambda_i \frac{2J_i + 1}{G} e^{-E_i/(kT)}; \quad G = \sum_i (2J_i + 1) e^{-E_i/(kT)}; \quad \lambda_i = \sum_f \ln \frac{2}{6146} B_{if} \Phi_{if}(\rho, T),$$

where thermal equilibrium is assumed. In principle, the sum over initial states extends over all populated states in the parent nucleus below the proton separation energy. However, since the range of T for the rp process peaks at $T = 1.5 \text{ GK}$ ($kT \sim 300 \text{ keV}$), only a few low-lying excited states are expected to contribute significantly in the decay. Specifically, we consider in this work all the (collective) low-lying excited states below 1 MeV.

The rates are decomposed into a nuclear structure part $B_{if}$ and a phase space factor $\Phi_{if}$. In this work, $B_{if}$ is given by the transition probabilities of allowed GT transitions, which are dominant,

$$B_{if}(GT) = \frac{1}{2J_i + 1} \left( \frac{g_A}{g_V} \right)^2 \langle f | | \sum_k \sigma^k t^k_+ | i \rangle^2,$$

where $(g_A/g_V)_{\text{eff}} = 0.74 (g_A/g_V)_{\text{bare}}$ is an effective quenched value.

The theoretical formalism used to calculate the GT strengths is based on the QRPA. The quasiparticle basis corresponds to a deformed selfconsistent Skyrme Hartree-Fock calculation with the density-dependent Skyrme SLy4 force and pairing correlations treated in BCS approximation. The residual interactions are spin-isospin forces in both particle-hole and particle-particle channels. The initial and final states in the laboratory frame are expressed in terms of the intrinsic states using the Bohr-Mottelson factorization, which is a very good approximation for well deformed nuclei. The effect of these ingredients (deformation, pairing, residual forces) on the decay rates has been studied elsewhere [11, 12]. More details of the formalism can be found in Ref. [13, 14]. In particular, the parameters for residual forces used here are the same as those used in Ref. [4], where good agreement was obtained with the experimental GT strength distributions and β-decay half-lives. The success of this theoretical formalism in reproducing terrestrial decay rates supports its application to the calculation of the weak interaction rates in stellar matter.
Figure 1. Weak decay rates $\lambda \, (s^{-1})$ for $^{74}\text{Sr}$ as a function of temperature for various densities.

Figure 2. Same as in Fig. 1, but for $^{76}\text{Sr}$.

Figure 3. Same as in Fig. 1, but for $^{78}\text{Sr}$.

The phase space factors in Eq. (1) are decomposed into their $\beta^+$ and $cEC$ contributions,

$$\Phi_{ij} = \Phi_{ij}^{cEC} + \Phi_{ij}^{\beta^+},$$

where

$$\Phi_{ij}^{cEC} = \int_{\omega_i}^{\infty} \omega \sqrt{\omega^2 - 1} (Q_{ij} + \omega)^2 F(Z, \omega) \times S_\nu(\omega) \, [1 - S_\nu(Q_{ij} + \omega)] \, d\omega,$$

and

$$\Phi_{ij}^{\beta^+} = \int_{\omega_i}^{\infty} \omega \sqrt{\omega^2 - 1} (Q_{ij} + \omega)^2 F(Z, \omega) \times S_\nu(\omega) \, [1 - S_\nu(Q_{ij} + \omega)] \, d\omega.$$
for continuum electron capture, and
\[
\Phi^{\beta^+}_{ij} = \int_1^{Q_{ij}} \omega \sqrt{\omega^2 - 1}(Q_{ij} - \omega)^2 F(-Z + 1, \omega) \times \left[ 1 - S_p(\omega) \right] \left[ 1 - S_\nu(Q_{ij} - \omega) \right] d\omega , \tag{5}
\]
for positron emission. In these expressions \( \omega \) is the total energy of the electron (positron) in units of \( m_e c^2 \) and \( F(Z, \omega) \) is the Fermi function \([15]\) that takes into account the distortion of the \( \beta^- \) particle wave function due to the Coulomb interaction.
\[
Q_{ij} = \frac{1}{m_e c^2} (M_p - M_d + E_i - E_f) \tag{6}
\]
is the total energy available in the decay in units of \( m_e c^2 \). It is written in terms of the nuclear masses of parent \( (M_p) \) and daughter \( (M_d) \) nuclei and their excitation energies \( E_i \) and \( E_f \), respectively. In Eq. (4), the lower integration limit is given by \( \omega_l = 1 \) if \( Q_{ij} > -1 \), or \( \omega_l = \left[ Q_{ij} \right] \) if \( Q_{ij} < -1 \). \( S_e, S_p, \) and \( S_\nu \), are the electron, positron, and neutrino distribution functions, respectively. Its presence inhibits or enhances the phase space available. In \( rp \) scenarios the commonly accepted assumptions state that \( S_\nu = S_p = 0 \).

The electron distribution is described by a Fermi-Dirac distribution
\[
S_e = \frac{1}{\exp \left[ (\omega - \mu_e)/(kT) \right] + 1} , \tag{7}
\]
assuming that nuclei at these temperatures are fully ionized and the electrons are not bound to nuclei. The chemical potentials \( \mu_e \) are determined from \( \rho \) and \( T \).

3. Results and discussion

Figs. 1, 2, and 3 show the decay rates versus \( T \) for \(^{74}\text{Sr}, \(^{76}\text{Sr}, \) and \(^{78}\text{Sr}, \) respectively. In panel (a) one can see the decomposition of the total rate into their contributions from the decay of the ground state \( 0^+_g \) and from the decay of the excited \( 2^+ \) states. Panel (b) contains the decomposition of the rates into their \( \beta^+ \) and \( cEC \) components evaluated at different densities. Panel (c) shows the total rates at various densities.

The results decomposed into their contributions from various parent states (a) show that the contributions of the decays from excited states increase with \( T \), as they become more and more thermally populated. In general they do not represent significant contributions to the total rates and can be neglected in most cases \([6,7]\). However, in those cases where the excitation energy of the \( 2^+ \) excited state is very low, as in the cases discussed here for Sr isotopes, the contributions of the low-lying excited states compete with those of the ground state already at temperatures in the range of \( rp \) process.

Concerning the competition between \( \beta^+ \) and \( cEC \) rates (b) one can clearly distinguish between the three isotopes. The most exotic isotope (Fig. 1) shows a clear dominance of the \( \beta^+ \) rates over the \( cEC \) ones that can be neglected except at very high densities beyond \( rp \)-process conditions. On the other hand, the opposite is true with respect to the more stable isotopes (Fig. 3), where the \( \beta^+ \) rates are completely negligible. The origin of these features can be understood from the behavior of the phase space factors as a function of the available energy \( Q_{ij} \). In the case of the \( N = Z \) WP nuclei (Fig. 2), there is a competition between \( \beta^+ \) and \( cEC \) rates that depends on the nucleus, on the temperature, and on the density and that must be analyzed case by case.

Finally, the total rates in (c) are a consequence of the competition between \( \beta^+ \) and \( cEC \) rates mentioned above. Since the \( \beta^+ \) decay rate is independent of the density and depends on \( T \) only through the contributions from excited parent states, the total rates are practically constant for
the most exotic isotope in Fig. 1, only modulated by the small contribution from cEC. But in general this is not true for WP nuclei. Fig. 4 contains the half-lives ($T_{1/2} = \ln 2/\lambda$), as a function of $T$ at a fixed $\rho Y_e = 10^6$ mol cm$^{-3}$ for different isotopes. The decrease of $T_{1/2}$ as $T$ increases is apparent.

Summarizing, we have analyzed the relevant ingredients to describe the decay rates at the peak rp-process conditions of $\rho$ and $T$. We have studied the contributions to the decay rates coming from excited states in the parent nucleus, which are populated as $T$ raises. It is found that they start to play a role above $T = 1\text{--}2$ GK. The effect of the cEC rates is enhanced as $T$ and $\rho$ increase and they compete with the $\beta^+$ decay rates in the WP nuclei at rp peak conditions.

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