Statefinder diagnosis of nearly flat and thawing non-minimal quintessence

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Abstract – Non-minimally coupled scalar field models of dark energy are equivalent to an interacting quintessence in Einstein’s frame. Considering two special important choices of the potential of the scalar field, i.e., nearly flat and thawing potential, one has an analytical expression for the equation-of-state parameter as a function of the density parameter of the scalar field for any choice. Here we investigate the non-minimal quintessence model by applying the method of statefinder diagnosis to it and plotting the evolutionary trajectories of the statefinder parameters.

Introduction. – According to the supernova observations [1], our universe has an accelerating expansion in the present epoch [2,3]. Accepting the general relativity theory as the underlying theory for describing the dynamics of the universe, there is a missing energy momentum component of the universe, the dubbed “dark energy” [4], providing the present acceleration. There are several candidates for such a dark energy, the cosmological constant, the Chaplygin gas, quintessence and so on (See ref. [4] and references therein).

Some cosmological models involve an interaction between dark matter and dark energy [5]. This removes the need for a fine-tuned cosmological constant to get the ratio of dark matter to dark energy of order unity nowadays. This is widely known as the coincidence problem [6].

A particular class of these models is the Brans-Dicke (BD) type scalar field model. One can deal with BD scalar field model in two frames, the “Jordan frame” and the “Einstein frame”. In the Jordan frame, the scalar field does not appear in the action of matter fields but couples non-minimally to gravity. It is possible to make a particular conformal transformation, converting the theory to a scalar-tensor model in which the scalar field couples conformally to matter and minimally to gravity. Describing the scalar field as quintessence, hereafter we shall refer to this model as the conformally coupled quintessence model (CCQ). There has been a lot of discussion about the choice of one of these frames as the physical frame [7,8]. Here we pay our attention to the Einstein frame. This frame is a suitable framework to make the model consistent with the solar system constraint and further observational constraints from Big-Bang nucleosynthesis and inflation [9].

Following Caldwell and Linder [10], the scalar field models of quintessence can be divided into two categories, called “freezing” and “thawing” models. In the former, the equation-of-state parameter has a decreasing behavior but in the latter it has a value near \(-1\) initially, and then increases with time to less negative values.

In [11], it is shown that the non-interacting thawing quintessence model with nearly flat potentials (the potentials satisfying the slow-roll conditions: \(\left(\frac{1}{V} \frac{dV}{d\phi}\right)^2 \ll 1, \frac{1}{V} \frac{d^2V}{d\phi^2} \ll 1\)) provides a natural way to produce a value of \(\omega\) near \(-1\) today. Generalizing this to the CCQ model with nearly flat potentials [12] shows that there exists a universal behavior for \(\omega\) which is different from the thawing behavior initially. In a recent paper [13], some conditions on the potential of the scalar field are derived which are different from the slow-roll conditions initially and lead to the thawing behavior for all times.

On the other hand, as demonstrated in ref. [14], it is possible to discriminate different models of dark energy from each other by some parameters, called the statefinder parameters \((r, s)\), firstly proposed in [15] and [16]. The first of these, \(r\), is the jerk parameter and the other is a function of jerk and the decelerating parameters. The statefinder pair depends on the metric of space-time and is constructed using the second and third derivatives of the scale factor as

\[
\begin{align*}
  r &= \frac{\dddot{a}}{aH^3}, \\
  s &= \frac{r - 1}{3(q - 1/2)},
\end{align*}
\]

where \(q = -\dddot{a}/a^2\) is the deceleration parameter.
In this paper we will discuss further the nearly flat and thawing CCQ models of dark energy by statefinder diagnostic. In the next section, we will start with the BD interacting dark energy model. According to [12] and [13], the application of some conditions on the potential, i.e. nearly flat and thawing conditions, leads to an analytical expression for $\omega$ in any case. Then, we apply the statefinder diagnostic to the CCQ model in the third section. In the last section we will give some conclusions. Throughout this work we have chosen the units $8\pi G = c = 1$.

**The model.** – The general action of the BD scalar-tensor theory in the Jordan frame is

$$S_J = \int d^4x \sqrt{-\tilde{g}} \left[ \Phi \tilde{R} - \tilde{\omega} \Phi \Phi^\mu \Phi_{,\mu} - 2U(\Phi) + L_m(\tilde{g}_{\mu\nu}) \right],$$

where $\tilde{R}$ is the Ricci scalar of the metric $\tilde{g}_{\mu\nu}$, $U(\Phi)$ is the potential of the scalar field, $\tilde{\omega}$ is the BD coupling constant and $L_m$ is the matter Lagrangian. Under the conformal transformation $g_{\mu\nu} = e^{2\phi} \tilde{g}_{\mu\nu}$, in which $\ln \Phi = \tilde{\omega} \phi$, one arrives at the action of the BD theory in the Einstein frame:

$$S_E = \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} (\nabla \phi)^2 - V(\phi) + L_m(e^{-\phi} g_{\mu\nu}) \right],$$

where $V(\phi) = e^{-2\phi} U(\Phi(\phi))$.

Now, consider a spatially flat FRW universe occupied by the pressureless matter. The cosmological equations of motion are

$$H^2 = \frac{1}{3}(\rho_\phi + \rho_m), \quad (5)$$

$$\dot{H} = -\frac{1}{2} [\rho_m + \rho_\phi + p_\phi], \quad (6)$$

and the scalar field evolution is governed by the following equation of motion:

$$\ddot{\varphi} + 3H \dot{\varphi} + V_\phi = \frac{\beta}{3} \dot{\varphi} \rho_m, \quad (7)$$

where $\rho_m$, $\rho_\phi = \frac{1}{2} \dot{\varphi}^2 + V(\phi)$ and $p_\phi = \frac{1}{2} \dot{\varphi}^2 - V(\phi)$ denote the energy density of cosmic fluid, the energy density and pressure density of the scalar field in the Einstein frame, respectively, and $\beta = \sqrt{\frac{2}{3}} \tilde{\omega}$. From eq. (7), one can easily see that the energy density of the scalar field satisfies the following conservation law:

$$\dot{\rho}_\phi + 3H (1 + \omega) \rho_\phi = \frac{\beta}{3} \dot{\varphi} \rho_m \quad (8)$$

in which $\omega = p_\phi/\rho_\phi$ is the equation-of-state parameter for the scalar field. Combining the above equations, the continuity equation for the cosmic fluid can be derived as

$$\dot{\rho}_m + 3H \rho_m = -\frac{\beta}{3} \dot{\varphi} \rho_m. \quad (9)$$

By integrating eq. (9), one obtains

$$\rho_m(t) = \rho_{0m} \left( \frac{a}{a_0} \right)^{-3} e^{-\frac{\varphi_0 - \varphi(t)}{\sqrt{2}\varphi}} \quad (10)$$

in which $\rho_{0m}$, $a_0$ and $\varphi_0$ are the current values of the matter density, the scale factor and the scalar field, respectively. Therefore the usual dependence of the non-relativistic matter upon the scale factor is modified by an exponential factor due to the interaction with the scalar field.

Taking the time derivative of $V(\varphi) = (1 - \omega)/2 \rho_\phi$ and using the continuity equation (8), one obtains [17]:

$$\omega' = -3(1 - \omega)^2 \left[ 1 - \frac{\Omega_{\phi} \rho_\phi}{\sqrt{3(1 + \omega)}} \left( -\frac{V_\phi}{V} + \frac{1}{\sqrt{6}} \Omega_{\phi} \right) \right], \quad (11)$$

where $\Omega_{\phi}$ is the density parameter of the scalar field, $(\Omega_{\phi} = \rho_\phi/3H^2)$ and the prime denotes the derivative with respect to $\ln a$. Ignoring the interaction of the scalar field and the cosmic fluid, this equation reduces to the corresponding equation of Steinhardt et al. [18] for a non-interacting scalar field.

Following the same method as refs. [11,12], using the relation $\dot{\varphi}^2 = (1 + \omega)\rho_\phi$, the continuity equation (8) reads as [13]

$$\Omega_{\phi} = 3(1 - \Omega_{\phi}) \left( -\omega \Omega_{\phi} + \frac{1}{6} \sqrt{2(1 + \omega)\Omega_{\phi}} \right), \quad (12)$$

which is a useful relation to derive the redshift dependence of the cosmological quantities in the next section.

Moreover, by using eq. (10), one can express eqs. (5) and (6) in the following form, convenient for the construction method proposed by [19]:

$$\frac{1}{9H_0^2} V(x) = \frac{H^2}{H_0^2} - \frac{x}{6H_0^2} \frac{dH^2}{dx} = \frac{1}{2} \Omega_{0m} e^{-\frac{\varphi - \varphi_0}{\sqrt{2}\varphi}} x^3, \quad (13)$$

where $x = 1 + z$ in which $z = a_0/a - 1$ is the redshift parameter and $a_0$ is the present scale factor of the universe. These are two coupled equations allowing one to reconstruct the potential $V(z)$ and the scalar field $\varphi(z)$ knowing $\Omega_{0m}$, $\varphi_0$ and also using the observed $H(z)$ from the luminosity distance.

Since the left-hand side of eq. (14) is non-negative, the dynamical expansion of the universe is restricted by the following inequality:

$$\frac{dH^2}{dz} \geq 3H_0^2 \Omega_{0m} (1 + z)^2 e^{-\frac{\varphi - \varphi_0}{\sqrt{2}\varphi}}. \quad (15)$$

This is the weak energy condition for the BD cosmology in Einstein’s frame. We see that although it is not possible to express the Hubble parameter as an explicit function of
the cosmological redshift in this model, there is a necessary condition on the $H(z)$ in this case. Ignoring the interaction and thus the exponential factor, inequality (15) reduces to what is derived by Sahni and Starobinsky in [2] for the non-interacting case.

**Statefinder diagnostic of the CCQ model of dark energy.** According to the previous section, using the BD theory of gravity in Einstein’s frame leads to a scalar-tensor theory in which there is a particular interaction between the non-relativistic matter and the scalar field. It is a well-known fact that the coupled scalar field model with $-1 \leq \omega \leq -1/3$ (called usually interacting quintessence) has the capability of explaining the current cosmic acceleration [20]. However, to make these models acceptable, some limitations on the form of $V$ has to be set, depending on the form of the interaction term [18,21,22].

In [13], the necessary conditions for the existence of the thawing behavior for the CCQ model is found. Assuming the thawing behavior, the following relation, called thawing condition, should be satisfied [13]:

$$\lambda = -\frac{V_{,\varphi}}{V} \simeq \lambda_0 - \frac{1}{\sqrt{6}} \frac{\Omega_m}{\Omega_\varphi},$$

(16)

which shows that it is necessary that $\lambda$ increases with time when $\varphi$ and $\Omega_\varphi$ are increasing functions. Dividing eq. (11) by eq. (12), one arrives at a differential equation for $\omega$ as a function of $\Omega_\varphi$ [13]. Replacing $\lambda$ with expression (16) and retaining terms to the lowest order in $1 + \omega$ ($\omega$ is near $-1$), the differential equation of $\omega$ is exactly solvable. The resulting analytical expression for the state parameter is as follows [13]:

$$1 + \omega = \left(\frac{1 - \Omega_\varphi}{\Omega_\varphi}\right)^{2A} \left[\lambda_0 + \frac{2\lambda_0 \Omega_\varphi^{1/2 + A}}{\sqrt{3} (1 + 2A)^2} F_1 \right] \times \left(\frac{1}{2} + A, 1 + A, \frac{3}{2} + A, \Omega_\varphi\right)^2,$$

(17)

where $\lambda_0$ is a positive constant, $\chi_0$ is an integration constant, $A = 1 + \beta \sqrt{\frac{2}{3}} \lambda_0$ and $F_1$ is the Gauss hypergeometric function.

In [13] it is shown that in the case of the CCQ model, $\lambda$ neither is a small value nor is a constant. Therefore the nearly flat potentials do not lead to the thawing behavior. However, recently, the study of the behavior of the equation-of-state parameter for nearly flat potentials has attracted a lot of attention [11,12]. The authors of [12] have shown that the equation-of-state parameter firstly increases with time and then approaches asymptotically to a value near to $-1$. As mentioned in [12], assuming the slow-roll conditions for the potential, one can show that $|\lambda| \ll 1$ which ensures that $\lambda$ is approximately constant up to now, i.e.

$$\lambda \simeq \lambda_0 = -\frac{V_{,\varphi}}{V} \bigg|_{\varphi_0},$$

(18)

where $\lambda_0$ is a small constant evaluated at the initial value of $\varphi_0$. Combining eqs. (11) and (12) and making two assumptions, the first one is that $\omega$ is near $-1$ and the second is that the condition (18) is satisfied, these yield again to a differential equation for $\omega$ as a function of $\Omega_\varphi$ [12] which gives the following analytical expression for the equation-of-state parameter:

$$1 + \omega = \left[\frac{\lambda_0}{\sqrt{3 \Omega_\varphi}} - \frac{1}{\Omega_\varphi} \right] \left(\frac{\lambda_0}{2\sqrt{3}} - \frac{\beta}{3}\right) \times \ln \left(\frac{1 + \sqrt{\Omega_\varphi}}{1 - \sqrt{\Omega_\varphi}}\right) - \alpha \right]^2,$$

(19)

where $\alpha = -\lambda_0 \frac{2\sqrt{3} \Omega_\varphi}{1 - \Omega_\varphi} - \frac{2\sqrt{3} \beta}{3} \Omega_1$ in which $\Omega_1$ is some small initial value of $\Omega_\varphi$ such that $\omega_i = -1$. From eqs. (5) and (6), it is straightforward to show that the deceleration parameter takes the following form as:

$$q = \frac{1}{2} (1 + 3\omega \Omega_\varphi).$$

(20)

After differentiating eq. (6), using relations (8) and (9), one finds:

$$r = 1 + \frac{9}{2} \omega (1 + \omega) \Omega_\varphi - \frac{3}{2} \omega' \Omega_\varphi,$$

$$- \frac{3}{2\sqrt{2}} \omega \sqrt{1 + \omega} \Omega_\varphi (1 - \Omega_\varphi).$$

(21)

Inserting (20) and (21) in (2) gives

$$s = 1 + \omega - \frac{\omega'}{3\omega} - \frac{1}{3\sqrt{2} \Omega_\varphi} \sqrt{1 + \omega},$$

(22)

which explicitly depends on $\Omega_\varphi$ in contrast to the non-interacting quintessence.

Let us now have a detailed look at how the statefinder parameters behave for the thawing and nearly flat CCQ model of dark energy. As we have seen before, in these cases, it is possible to derive an analytical expression for the statefinder pairs as a function of the density parameter of the scalar field, $\Omega_\varphi$, without considering a special form for the potential. To do this, one can use the relation (11), the thawing condition (16) and the analytical expression of $\omega$, eq. (17), for the case of the thawing CCQ model and the corresponding relations (11), (18) and (19) in the case of the nearly flat CCQ model.

The time evolution of the statefinder pairs $(r, s)$ for the thawing CCQ model has been shown in fig. 1 with $\lambda_0 = 0.9$. This value of $\lambda_0$ is chosen such that $\omega$ has a value near $-1$ today. Also the constant $\chi_0$ is chosen such that the initial condition $\omega = -1$ holds for $\Omega_\varphi = 0.001$ and $\beta$ has been set equal to 0.5. The $\Lambda$CDM ($\Lambda$-cold dark matter) universe corresponds to the fixed point $(1, 0)$. We see that the evolution of trajectories of statefinders pairs passes from the point $(r, s) \simeq (1, -0.07)$ in the past when $z \simeq 4.62$ and $\Omega_\varphi \simeq 0.01$ and then after passing the $\Lambda$CDM
fixed point, \( r \) decreases whereas \( s \) increases to the point \((r, s) \approx (0.04, 0.27)\) at \( z \approx -0.74 \) (\( \Omega_\phi \approx 0.99 \)) in the future. The location of today’s point is \((r, s) \approx (0.57, 0.15)\) when \( \Omega_\phi \approx 0.7 \). This shows the present “distance” of the thawing CCQ model of dark energy from the \( \Lambda \)CDM model. The time evolution of the pairs \((r, q)\) is indicated by fig. 2. The dashed, thick and thin curves have \( \lambda_0 = 0.8, 0.9, 1 \) respectively. We see that both \( \Lambda \)CDM and thawing CCQ models start evolving from the same point, \((r, q) = (1, 0.5)\) which corresponds to the SCDM (standard cold dark matter) universe. In the \( \Lambda \)CDM scenario, the evolution is along a horizontal line which ends at SS (steady-state) fixed point, \((r, q) = (1, -1)\), corresponding to the de Sitter expansion. However, in our model, \( q \) has a decreasing behavior whereas the value of \( r \) first increases and then monotonically decreases. The trajectory goes to \((r, q) \approx (0.04, -0.68)\) at \( z \approx -0.7 \) in the future.

In figs. 3 and 4, we have shown the deceleration parameter and the equation-of-state parameter as functions of the redshift parameter. These figures have been plotted numerically using the relations (12), (17) and (20).

In fig. 5 we have shown the time evolution of the statefinder pairs for the nearly flat CCQ model of dark energy with \( \lambda_0 = 0.4 \). This small value of \( \lambda_0 \) ensures that the variation of the potential during the evolution of the universe is very small. Assuming that for an initial value of \( \Omega_\phi \), say \( \Omega_i \), \( \Omega_i = 0.001 \), we have \( \omega = -1 \). This determines the constant \( \alpha \) in the relation (19) setting \( \beta = 0.5 \) here. The \((r, s)\) trajectory has two branches. We find that the first branch comes asymptotically from \( r \approx 1 \), \( s \to -\infty \), goes to \( r \approx 1.33 \), \( s \to +\infty \) for a change of \( \Omega_\phi \) in the interval \([0.001, 0.17]\). Another branch comes along \( r \approx 1.33 \) asymptote, passes from the \( \Lambda \)CDM point and then goes to \((r, s) \approx (0.78, 0.05)\) at \( z \approx -0.7 \) (\( \Omega_\phi \approx 0.99 \)) in the future. The divergent behavior and discontinuity of \( s \) occurs at \( \Omega_\phi \approx 0.17 \) and is due to vanishing \( \omega \) at this point. Along the first branch, both \( r \) and \( s \) are increasing, however, for the other branch \( r \) monotonically decreases, whereas \( s \) increases. The location of today’s
point is \((r, s) \simeq (0.94, 0.02)\) when \(\Omega_\phi \simeq 0.7\). This shows that this model has less distance from the \(\Lambda\)CDM model in comparison to the thawing model. This result is satisfied in both models independently of the allowed chosen value of \(\lambda_0\).

The evolutionary track in the \((r, q)\)-plane is shown in fig. 6. The thick and thin curves correspond to \(\lambda_0 = 0.4\) and 0.1, respectively. The evolutionary track starts from the SCDM point, the same as the \(\Lambda\)CDM evolutionary path, but first \(r\) decreases and \(q\) increases slightly, then, after passing a period in which \(r\) and \(q\) have increasing behavior, they decrease monotonically. This period corresponds to a very high redshift. The trajectory goes to \((r, q) \simeq (0.78, -0.9)\) at \(z \simeq -0.71\) (\(\Omega_\phi \simeq 0.99\)) in the future. At the end, we present the plot of \(q\) and \(\omega\) with respect to \(z\).

\textbf{Conclusion.} – In this paper, we have applied the statefinder diagnostic to the CCQ model of dark energy and we have plotted the trajectories in the \((r, s), (r, q), (q, z)\) and \((\omega, z)\) planes for nearly flat and thawing potentials. As is apparent from figs. 3 and 7, the deceleration parameter decreases monotonically with redshift for both the nearly flat and thawing potentials, however, it goes to more negative values at future in the case of the nearly flat model. Moreover, nearly flat potentials force the equation-of-state parameter to change in a wide range with redshift, whereas it remains near \(-1\) for the thawing model, as one expected. This is exactly what one finds out from figs. 4 and 8. Also the value of \(r\) decreases with redshift steadily for both kinds of potentials as well as in the non-interacting quintessence model [15].

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