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Inverse identification for the balanced biaxial yield stress of AA5182-O alloy sheet

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Abstract: Advanced yield functions, such as Yld2004, could describe the elastic boundary of materials better than the traditional. However the balanced biaxial yield stress $\sigma_b$ which is essential to determine the parameters of advanced yield functions is hard to measure using frequently used test equipment. This work presented an inverse method to calibrate $\sigma_b$ of AA5182-O alloy sheet based on the Erichsen test. The maximum punch force (MPF) measured from this test was used for the inverse identification. A modification coefficient was used to drop down the simulation MPF from shell element, as the application of shell element result in higher simulation punch force. Then the relationship between $\sigma_b$ and MPF was established based on the plane stress Yld2004. With this relationship and the real measured MPF, $\sigma_b$ could be inversely identified. Additionally, a hydraulic bulge test was performed to verify the accuracy of this inversely obtained $\sigma_b$.

Keywords: Biaxial yield stress  Non-quadratic yield function  Finite element method  Inverse identification  Erichsen test

1. Introduction

Advanced yield functions, such as Yld2000 [1] and Yld2004 [2], can reflect material biaxial behavior better than the traditional yield functions. Finite element (FE) models based on advanced yield functions could give more accurate simulation result [3-7]. Yld2004 has more parameters and could characterize more complicated material yielding behavior than Yld2000 [8]. To determine the parameters of advanced yield functions, other than uniaxial tension yield stress $\sigma_\theta$ and $r_\theta$ value, $\sigma_b$ and $r_b$ are also needed. From balanced biaxial tension test or hydraulic bulge test, $\sigma_b$ and $r_b$ can be measured. $r_b$ can also be obtained by a compression test [9]. All of the experimental data can be measured from conventional tests, but $\sigma_b$. Even though biaxial tensile test or hydraulic bulge test could measure $\sigma_b$, the cruciform specimen in the former test is really hard to design and fabricate [10], and these tests equipment are also costly and far from common
applications. With $\sigma_b$ unavailable, it is impossible to gain the exact Yld2000 or Yld2004, or the accurate simulation results.

Simulation results could give the detailed information of the real forming process. The material model plays almost the most important role for the accuracy of the simulation results. As for the difficulty in parameters determination of the material model, inverse method is more and more popular in this field because there is an inherent relationship between material model parameters, simulation results and real measured results. Yueqi Wang et al. updated the parameters of Hill48 by modifying the FE model and making the simulation result close to the experiment [11]. Kunmin Zhao et al. proposed a new hardening law to describe the post-necking stress–strain curve by using inverse method [12]. As it is almost impossible to measure the high speed flow stress, Hak-Gon Noh et al. applied the inverse method to determine the high speed flow stress of Al 1100-O [13]. Cunsheng Zhang et al. inversely identified the material parameters of Arrhenius-type equation by conducting hot compression tests [14].

Even though the inverse method is effective, the application of this method for identification of $\sigma_b$ is seldom performed up to now. This may be due to the difficulty in finding out a forming test which could well reflect the relationship between $\sigma_b$ and the simulation result. The simulation result should be sensitive to $\sigma_b$. Apart from the inverse method, two other methods were proposed. Firstly, some investigators applied additional plane strain or shear test to calibrate the parameters of Yld2000 due to the unavailable $\sigma_b$, without verification [15,16]. However, as these test results are nearly not sensitive to $\sigma_b$, this method may not be effective. Secondly, Nader Abedrabbo et al. once used traditional yield function Yld91 to calculate $\sigma_b$ at warm temperatures [17]. It should be clarified that traditional yield functions cannot describe the biaxial behavior of materials. This method may be also inaccurate.

In this work the inverse identification of $\sigma_b$ is inspired by the Erichsen test. The stress state of the deformed part in the Erichsen test is mostly biaxial tensioned [11]. Based on advanced yield models, the variation of $\sigma_b$ would change its yield locus in biaxial region, and the corresponding simulation result when the part is biaxial tensioned [18]. As for the Erichsen test, the MPF could be easily measured before blank crack. There may be a relationship between $\sigma_b$ and the MPF calculated by the corresponding Erichsen FE model with advanced yield function included. Then the degree of $\sigma_b$ could be inversely identified with the measured MPF. In this investigation, only plane stress Yld2004 is applied due to the efficient simulation process of shell element. An implicit FE program Abaqus/Standard is selected to carry out all the simulations. A modification coefficient is applied to reduce the simulation MPF calculated from shell element, with the help of Mises and Hill48 yield function. Necessarily, the inverse identified $\sigma_b$ was verified by the measured data from a hydraulic bulge test.
2. Experiments

2.1 Uniaxial tension test

The chemical component of the selected AA5182-O alloy sheet with the thickness of 1.2mm is given in Table 1. Uniaxial tension tests with digital image correlation (DIC) technology were carried out, as is shown in Fig. 1. Dimensions of the specimen referred to ASTM E8. To obtain the material uniaxial anisotropy properties, 7 directions tension tests were performed and the angle between the length direction of the specimen and the rolling direction of the sheet were set as 0°, 15°, 30°, 45°, 60°, 75° and 90°. Through the DIC uniaxial tension tests, the directional yield stress $\sigma_\theta$ and anisotropy coefficient $r_\theta$ could be measured ($\theta = 0°, 15°, 30°, 45°, 60°, 75°$ and $90°$). Obtained values of $\sigma_\theta$ and $r_\theta$ are given in Table 2. The plastic work equaling to 20MPa is utilized to calculate the values of $\sigma_\theta$.

![Fig. 1 DIC uniaxial tension test](image)

### Table 1 Chemical component of AA5182-O

|          | Si | Fe | Cu | Mn | Mg | Cr | Zn | Ti | Al |
|----------|----|----|----|----|----|----|----|----|----|
| %Wt.     | 0.2| 0.34| 0.15| 0.39| 4.6| 0.1| 0.25| 0.1| Balanced |
Table 2 Tested direction-dependent yield stress and $r$ value

| $\theta$ | $\sigma_\theta/\sigma_0$ | $r_\theta$ |
|----------|-------------------------|-----------|
| 0        | 1.0000                  | 0.8059    |
| 15       | 0.9987                  | 0.5167    |
| 30       | 0.9838                  | 0.4577    |
| 45       | 0.9840                  | 0.6042    |
| 60       | 0.9945                  | 0.5031    |
| 75       | 1.0089                  | 0.6481    |
| 90       | 1.0114                  | 1.0860    |

2.2 Hydraulic bulge test (measured $\sigma_b$ is used for verification)

A hydraulic bulge test also with DIC method was performed to measure the balanced biaxial yield stress $\sigma_b$ and biaxial value $r_b$ based on the ISO 16808-2014. The obtained part from the hydraulic bulging test is shown in Fig. 2. Based on the measured flow stress, $\sigma_b$ could be determined (the value of the plastic work is also selected as 20MPa). As the strain distribution was measured according to the DIC technology, $r_b$ could also be determined. The obtained balanced biaxial properties of AA5182-O alloy sheet are given in Table 3.
As has been shown by published paper, \( r_b \) could be measured by a simple compression test \([9,1]\). The difficulty is the measurement of \( \sigma_b \) due to the costly equipment. So the key point of this investigation is to inversely identify the value of \( \sigma_b \). Here the directly measured \( \sigma_b \) will be used for the verification of the inversely identified \( \sigma_b \). As the measurement of \( r_b \) is convenient, the \( r_b \) given in Table 3 would be used for the determination of Yld2004 parameters.

### 2.3 Erichsen test

In this investigation the Erichsen test was performed to provide experimental data for the inverse identification of \( \sigma_b \), as shown in Fig. 3. Graphite grease was selected as the lubrication. Before moving the punch, the blank was clamped with 10kN force. Punch speed was set as 5mm/min. The punch stroke after contacting the blank was set as 6.2mm in order to avoid the crack of the deformed blank. The punch force was recorded in the forming process, as shown in Fig. 4. As the MPF will be used for the inverse identification of \( \sigma_b \), the the Erichsen test was repeated twice to reduce experiment error. Just as expected, the punch forces measured from different tests during the forming process were nearly the same. Average of the MPF is 7.0348kN.
3. Material modeling

3.1 Plane stress Yld2004

As the plane stress yield functions have advantage in decreasing simulation time, plane stress
Yld2004 is considered in this work. The yield condition of Yld2004 is

\[
\phi = |\tilde{S}_1 - \tilde{S}_1''|_m + |\tilde{S}_1 - \tilde{S}_2''|_m + |\tilde{S}_1 - \tilde{S}_2'\prime|_m + |\tilde{S}_2 - \tilde{S}_2''|_m + |\tilde{S}_2 - \tilde{S}_3''|_m + \\
|\tilde{S}_3 - \tilde{S}_1''|_m + |\tilde{S}_3 - \tilde{S}_2''|_m + |\tilde{S}_3 - \tilde{S}_3''|_m - 4\sigma_y^m = 0
\]  

(1)

in which \(\tilde{S}_1', \tilde{S}_2', \tilde{S}_3'\) and \(\tilde{S}_1'', \tilde{S}_2'', \tilde{S}_3''\) are principal values of \(\tilde{s}'\) and \(\tilde{s}''\), and \(\tilde{s}' = C'T\sigma, \tilde{s}'' = C''T\sigma\). \(C', C''\) and \(T\) can be expressed as:

\[
C' = \begin{bmatrix}
0 & -c_{12}' & -c_{13}' & 0 & 0 & 0 \\
-c_{21}' & 0 & -c_{23}' & 0 & 0 & 0 \\
-c_{31}' & -c_{32}' & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & c_{44}' & 0 & 0 \\
0 & 0 & 0 & 0 & c_{55}' & 0 \\
0 & 0 & 0 & 0 & 0 & c_{66}'
\end{bmatrix}
\]

(2)
\[
\mathbf{C}'' = \begin{bmatrix}
0 & -c''_{12} & -c''_{13} & 0 & 0 & 0 \\
-c''_{21} & 0 & -c''_{23} & 0 & 0 & 0 \\
-c''_{31} & -c''_{32} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & c'_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c'_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & c'_{66}
\end{bmatrix}
\]  
(3)

\[
\mathbf{T} = \frac{1}{3} \begin{bmatrix}
2 & -1 & -1 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 & 0 \\
-1 & -1 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 3
\end{bmatrix}
\]  
(4)

When plane stress Yld2004 is applied, all the stress components related to thickness direction should be zero. Then the coefficients \(c'_{55}, c'_{66}, c''_{55}, c''_{66}\) are not needed [19]. As pointed out by van den Boogaard et al., the value of \(c'_{12}, c'_{13}\) could be set as 1 for the plane stress version of Yld2004 [20]. Finally, the number of parameters for plane stress Yld2004 can be reduced from 18 to 12.

To implement the Yld2004 more conveniently, Eq. 1 is modified equivalently as

\[
\phi(\sigma, \bar{\sigma}) = g(\sigma) - \sigma_y = 0
\]  
(5)

where

\[
g(\sigma) = \left[ f(\sigma) / 4 \right]^{1/3}
\]  
(6)

and

\[
f(\sigma) = |\tilde{S}_1 - \tilde{S}_1''|^{1/3} + |\tilde{S}_1' - \tilde{S}_1''|^{1/3} + |\tilde{S}_2' - \tilde{S}_1''|^{1/3} + |\tilde{S}_2' - \tilde{S}_2''|^{1/3} + |\tilde{S}_2' - \tilde{S}_3''|^{1/3} + |\tilde{S}_2' - \tilde{S}_3''|^{1/3} + |\tilde{S}_3' - \tilde{S}_3''|^{1/3} + |\tilde{S}_3' - \tilde{S}_3''|^{1/3}
\]  
(7)

In this work, the associated flow rule is used

\[
\dot{\bar{\varepsilon}} = \dot{\gamma} \mathbf{N}
\]  
(8)

in which \(\bar{\varepsilon}\) is the plastic strain, \(\dot{\gamma}\) is the plastic multiplier, \(\mathbf{N}\) is the plastic flow vector, \(\mathbf{N} = \partial \phi / \partial \sigma\). Detail calculation of \(\mathbf{N}\) have been given by Barlat et al. and will not be introduced here [2].

To calculate Yld2004 coefficients, i.e. components of \(\mathbf{C}'\) and \(\mathbf{C}''\), the yield stresses and anisotropy coefficients along 7 directions, i.e. \(\sigma_0, \sigma_{15}, \sigma_{30}, \sigma_{45}, \sigma_{60}, \sigma_{75}, \sigma_{90}, r_0, r_{15}, r_{30}, r_{45}, r_{60}, r_{75}, r_{90}\) and \(\sigma_b, r_b\) are needed. Calculation of Yld2004 coefficients was given by Barlat et al. and will not be detailed here [2]. With the lack of \(\sigma_b\), the parameters of Yld2004 cannot determined accurately.
3.2 Hardening law

As the Hockett–Sherby law could well describe the flow stress of aluminum, in this investigation this hardening law is selected. The Hockett–Sherby law is suitable for modeling the stress-strain curve of Al-Mg alloy sheet with large strain range [18], and has been applied by many commercial FE programs, such as AutoForm. Based on the measured flow stress of AA5182-O along the rolling direction, the parameters of Hockett–Sherby law could be determined:

\[
\sigma_y = 272.5 - 89.08e^{-20.90\bar{\varepsilon}^{0.9935}}
\]

in which \( \bar{\varepsilon} \) is the plastic strain.

4. Inverse identification of \( \sigma_b \) by using Erichsen FE model

4.1 Erichsen FE model

Fig. 5 shows the built FE model for the Erichsen test. The main dimensions of the FE model refer to Figure 3. To decrease the simulation time, only 1/4 of the blank is modeled as a circle shape and its diameter is slightly less than that of the die. This setting is reasonable because there is nearly no material flow in the flange region. Shell element S4R is used for the blank, and number of the integration point is set as 9 to improve the simulation accuracy. The die and the holder are modeled as discrete rigid bodies. The shown mesh density could well guarantee the accuracy of the simulation punch force based on the authors’ investigation. The punch is modeled as analytical rigid body. 2.5kN force is applied to clamp the blank between the die and the holder. The punch moves 6.2mm once it contacts the blank. The coefficient of friction between the die, holder and the blank is set as 0.7 [11]. As graphite grease was selected as the lubrication, the coefficient of friction between the punch and blank is set as 0.04 [21]. To implement Yld2004 and Hockett–Sherby hardening law, the built UMAT (User Material) subroutine is embedded into the Abaqus/Standard program.

Fig. 5 FE model for the Erichsen test
4.2 Modification coefficient to reduce simulation MPF via shell element

As the shell element neglect the stress components related with the thickness direction of the sheet, the simulation punch force may be inaccuracy. The simulation punch force obtained via solid element could be more realistic. In this investigation, only plane stress Yld2004 is implemented due to its simulation efficiency. The inverse identification result of $\sigma_b$ based on the plane stress Yld2004 may be not accuracy. It is necessary to clarify the relationship of simulation MPF via the shell and solid element. Here the classic 3D yield function could be applied to determine this relationship. It should be noted that the solid element C3D8R should be used instead of S4R when the 3D Mises or Hill48 yield function is applied.

As shown in Fig. 6, the relationship between the MPF via shell element and the MPF via solid element for the simulation of Erichsen test based on Mises yield function could be expressed as:

$$F_{2D\text{Mises}} = \alpha \times F_{3D\text{Mises}}$$  \hspace{1cm} (10)

in which $F_{2D\text{Mises}}$ means the simulation MPF based on the plane stress Mises yield function, $F_{3D\text{Mises}}$ means the simulation MPF based on the 3D Mises yield function, and $\alpha$ is the modification coefficient. The simulation MPF based on plane stress yield functions could be modified to be more realistic with the help of $\alpha$. It is known from Fig. 6 that the modification coefficient $\alpha$ based on Mises yield function is 1.0942. In this investigation the maximum punch stroke 6.2mm is selected to define the value of $\alpha$. This is mainly because the larger value of punch force is convenient for the following inverse identification of $\sigma_b$.

In the following, the simulation MPF via shell element based on plane stress Yld2004 could be modified more realistic with the help of $\alpha$. The accuracy of the given $\alpha$ is of great importance. In the above, value of $\alpha$ is obtained based on Mises yield function. The influence of yield function on $\alpha$ is necessary to be investigated. So the anisotropic yield function Hill48 is selected to calculate $\alpha$ again. As shown in Fig. 7, the obtained value of $\alpha$ based on Hill48 is 1.0922, only
0.18% smaller than the obtained value based on Mises yield function. Therefore, it is believable that the value of the modification coefficient $\alpha$ may not be influenced by different yield functions. An average value of $\alpha$ is selected to be the final value, i.e. $\alpha = 0.5 \times (1.0942 + 1.0922) = 1.0932$. In the following this value of $\alpha$ would be utilized to update the simulation MPF based on plane stress Yld2004.

![Graph showing modification coefficient for MPF based on Hill48](image)

**Fig. 7 Modification coefficient for MPF based on Hill48**

### 4.3 Relationship between $\sigma_b$ and updated MPF

It has been published that the level of $\sigma_b$ influence the simulation result of bulging test a lot [18]. In the bulging test, the blank is almost biaxial tensioned. The variation of $\sigma_b$ would change the yield locus (advanced yield function) in the biaxial tension region a lot. Accordingly, this would change the simulation result of the bulging test. The blank is also mostly biaxial tensioned in the Erichsen test. So it could be deduced that the simulation MPF may be influenced by $\sigma_b$ based on the plane stress Yld2004. Further, there may be a relationship between $\sigma_b$ and the updated MPF, with the help of the modification coefficient $\alpha$. To clarify this point of view, several levels of $\sigma_b$ are assumed to determine the parameters of Yld2004. The input data to determine the parameters of plane stress Yld2004 is given in Table 1. In Table 4, 5 levels of $\sigma_b$ are assumed. Therefore, the parameters of plane stress Yld2004 under different levels of $\sigma_b$ could be determined and they are given in Table 5. The corresponding yield locus is shown in Fig. 8. The curvature of the Yld2004 yield locus in balanced biaxial tension region improves as $\sigma_b$ increases.

| $\theta$ | $\sigma_b/\sigma_0$      | $\tau_0$      |
|---------|-----------------|------------|
| 0       | 1.0000 (measured)| 0.8059 (measured) |
| $\sigma_b / \sigma_0$ | $c'_{12}$ | $c'_{13}$ | $c'_{21}$ | $c'_{23}$ | $c'_{31}$ | $c'_{32}$ | $c'_{44}$ | $c''_{12}$ | $c''_{13}$ | $c''_{21}$ | $c''_{23}$ | $c''_{31}$ | $c''_{32}$ | $c''_{44}$ |
|---------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 0.98                | 1.0000    | 1.0000    | 0.8337    | 5.4197E-2 | 0.1882    | 1.0060    | 0.9039    | 0.4026    | 1.3442    | 0.4770    | 0.9399    | -0.2865   | 0.9734    | 0.8455    |
| 0.99                | 1.0000    | 1.0000    | 0.8334    | 9.5041E-2 | 0.1288    | 0.9635    | 0.8932    | 0.4112    | 1.3787    | 0.4632    | 0.9805    | -0.3271   | 0.9307    | 0.8495    |
| 1.00                | 1.0000    | 1.0000    | 0.8344    | 0.1368    | 6.9538E-2 | 0.9202    | 0.8832    | 0.4185    | 1.4131    | 0.4497    | 1.0215    | -0.3669   | 0.8875    | 0.8525    |
| 1.01                | 1.0000    | 1.0000    | 0.8378    | 0.1776    | 1.3943E-2 | 0.8778    | 0.8741    | 0.4233    | 1.4440    | 0.4377    | 1.0620    | -0.4024   | 0.8462    | 0.8546    |
| 1.02                | 1.0000    | 1.0000    | 0.8521    | 0.1932    | 9.5901E-4 | 0.8540    | 0.8644    | 0.4191    | 1.4376    | 0.4391    | 1.0935    | -0.3980   | 0.8338    | 0.8596    |

Table 5 Parameters of Yld2004 under different levels of biaxial yield stress
Fig. 8 Yield locus of Yld2004 under different levels of biaxial yield stress

The determined Yld2004 under different levels of $\sigma_b$ are implemented for the simulation of the Erichsen test. All of the simulations are successfully carried out. Fig. 9 shows the plastic strain distribution of the deformed blank when $\sigma_b/\sigma_0 = 1.00$. The obtained MPF under different levels of $\sigma_b$ are given in Table 6. With the help of the modification coefficient $\alpha$, the updated MPF are also calculated. Then the relationship between the updated MPF and $\sigma_b$ could be established, as is shown in Fig. 10. A linear function is applied to describe their relationship:

$$F_{\text{update}} = 5.911(\sigma_b/\sigma_0) + 1.054 \quad (11)$$

in which $F_{\text{update}}$ means the updated MPF. The correlation coefficient $R$ and the value of RMSE (root mean square error) are also given to evaluate the effectiveness of the fitting function. As the value of $R$ is (almost) equal to 1 and RMSE is very low, it is believed that the relationship between the updated MPF and $\sigma_b$ could be described by Eq. 11.
Fig. 9 Simulation result of plastic strain distribution when $\frac{\sigma_b}{\sigma_0} = 1.00$

Table 6 Simulation maximum punch force under different levels of biaxial yield stress

| $\frac{\sigma_b}{\sigma_0}$ | MPF (kN) | Updated MPF (MPF/$\alpha$, kN) |
|---------------------------|---------|-------------------------------|
| 0.98                      | 7.4846  | 6.8465                        |
| 0.99                      | 7.5495  | 6.9059                        |
| 1.00                      | 7.6143  | 6.9651                        |
| 1.01                      | 7.6789  | 7.0242                        |
| 1.02                      | 7.7430  | 7.0829                        |
4.4 Inverse identification of $\sigma_b$ and verification

In the previous sections the tested MPF, the relationship between the updated MPF and $\sigma_b$ through FE method have been obtained. Then the value of the $\sigma_b$ could be inversely determined. As shown in Fig. 11, with the help of the tested MPF and the given relationship between the updated MPF and $\sigma_b$, the value of $\sigma_b$ is inversely identified. The inversely identified value of $\sigma_b/\sigma_0$ is 1.0118. Its value cannot be larger or smaller with the constraint of the tested MPF. With the help of the inversely obtained $\sigma_b$ and the measured value given in Table 4, the parameters of plane stress Yld2004 can be determined. The inversely identified $\sigma_b$ and the corresponding parameters of plane stress Yld2004 are given in Table 7.
Table 7 Obtained biaxial yield stress and corresponding Yld2004 parameters from different methods

| $\sigma_b/\sigma_0$       | $c'_{12}$ | $c'_{13}$ | $c'_{21}$ | $c'_{23}$ | $c'_{31}$ | $c'_{32}$ | $c'_{44}$ | $c''_{12}$ | $c''_{13}$ | $c''_{21}$ | $c''_{23}$ | $c''_{31}$ | $c''_{32}$ | $c''_{44}$ |
|---------------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 1.0160 (measured)         | 1.0000    | 1.0000    | 1.0351    | 2.1564E-6 | 0.3426    | 1.0136    | 0.8772    | 0.2230    | 1.0638    | 0.5551    | 1.0000    | 1.0657    | 0.5552    | 2.5448E-6 |
| 1.0118 (inversely identified) | 1.0000    | 1.0000    | 1.0372    | 2.5448E-6 | 0.3477    | 1.0190    | 0.8831    | 0.2160    | 1.0657    | 0.5552    | 0.9698    | 2.4674E-6 | 1.0497    | 0.8581    |

To illustrate the effectiveness of the inverse method, the measured $\sigma_b$ and the corresponding parameters of Yld2004 are also given in Table 7. Clearly, the determined $\sigma_b/\sigma_0$ from different methods are nearly the same. The inversely identified $\sigma_b/\sigma_0$ is only 0.41% lower than that of the measured. Values of the parameters of plane stress Yld2004 determined with the help of measured or inversely identified $\sigma_b$ are also closely. Then the accuracy of the inversely identified $\sigma_b$ can be verified.

5. Conclusions

The value of $\sigma_b$ is inversely identified in this investigation, with the help of Erichsen test and plane stress Yld2004. Only the punch force of the Erichsen test is needed for this inverse identification. The parameters of plane stress Yld2004 under different levels of $\sigma_b$ can be determined with the tested $\sigma_\theta$, $r_\theta$, $r_b$ and supposed $\sigma_b$. Then the relationship between updated MPF and $\sigma_b/\sigma_0$ could be obtained with the help of the modification coefficient $\alpha$. With the tested MPF, the value of $\sigma_b/\sigma_0$ can be inversely determined. The directly measured $\sigma_b$ from hydraulic bulge test verifies the accuracy of the inversely identified $\sigma_b$. The proposed inverse method for the identification of $\sigma_b$ is convenient, and may have its applications for the determination of $\sigma_b$. 
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Declaration

1. Availability of data and material

The datasets used or analysed during the current study are available from the submitted supplementary materials.

2. Competing interests

The authors declare that we have no competing interests.

3. Funding

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4. Authors' contributions

Dateng Zheng and Yong Zhang are responsible for the overall design. Qing Zhang mainly finished material modeling. Yuantao Sun performed the experimental operation. all authors discussed the results and revised the manuscript.

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