String Theory on \( AdS_3 \) and Symmetric Products

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Abstract: We consider string theory on a background of the form \( AdS_3 \times \mathcal{N} \). Our aim is to give a description of the dual CFT in a general set up. With the requirement that we have \( N = 2 \) supersymmetry in spacetime, we provide evidence that the dual CFT is in the moduli space of a certain symmetric product \( \mathcal{M}^p/S_p \). On the way to show this, we reproduce some recent results on string propagation on \( AdS_3 \) and extend them to the superstring.

1 Introduction

We consider in this contribution string theory on \( AdS_3 \) from the perspective of the \( AdS/CFT \) correspondence. Namely, strings on \( AdS_3 \) are dual to a (spacetime) 2-d CFT which can be thought of as being located on the boundary of the \( AdS_3 \). To be more precise, the background on which the strings propagate is \( AdS_3 \times \mathcal{N} \), where \( \mathcal{N} \) is a CFT with suitable central charge to make the string theory critical. The details of the dual spacetime CFT\(^2\) will depend on the worldsheet CFT \( \mathcal{N} \); for instance (super)symmetries of \( \mathcal{N} \) will be reflected in (super)symmetries in spacetime.

It was known since a long time\(^1\) that the Virasoro algebra of conformal symmetry of a two-dimensional theory can be recovered from diffeomorphisms of a theory of gravity on \( AdS_3 \). This can of course be extended to superconformal symmetry, starting from supergravity on \( AdS_3 \)\(^2\). The central charge of the boundary (super)conformal theory is already fixed classically, and is given by \( c_{st} = \frac{3L}{2G_3} \), where \( L \) is the “radius” of \( AdS_3 \) (i.e. the cosmological constant is given by \( \Lambda = -\frac{1}{L^2} \)) and \( G_3 \) is the 3-dimensional Newton constant.

It was then realized in\(^3\) that the near horizon geometry of a system of \( p \) D1-branes parallel to \( k \) D5-branes wrapped, say, on \( T^4 \), was given by \( AdS_3 \times S^3 \times T^4 \). Rewriting the

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three dimensional constants $L$ and $G_3$ in terms of the stringy ones, the spacetime central charge ends up being:

$$c_{st} = 6kp.$$  

Since the system above can be S-dualized to a system where only NSNS fields are turned on, the possibility opens up to study the above background perturbatively using string theory worldsheet techniques \cite{4, 5}.

To formulate the problem in string perturbation theory, one notices that $AdS_3$ is nothing else than (the universal covering of) the group manifold of $SL(2)$. Thus the worldsheet CFT is taken to be the product of the WZW model on $SL(2)$ at level $k$, times another CFT $\mathcal{N}$. For some specific such $\mathcal{N}$, one gets a specific candidate for the dual CFT $\mathcal{N}_2$. On the other hand, for a general $\mathcal{N}$ one can use the methods of \cite{6, 7} in order to construct the superconformal algebra of the boundary theory. Our aim is to go beyond this result and specify some properties that hold in general for the spacetime CFT. We will indeed find that its chiral spectrum has a specific structure \cite{8}.

2 Review of strings on $AdS_3$

The study of string theory on $AdS_3$ is an “old” issue, whose interest stems from the possibility to handle perturbatively string propagation on backgrounds including a curved time (see \cite{9} for a copious list of early and also more recent references on the subject).

The problems which arose from the beginning were related to the fact that one is considering a non-compact WZW model. More precisely, such a model has a host of negative norm states, and one hopes that in string theory one will get rid of them through imposing the Virasoro constraints (as it is the case with the timelike oscillators in flat spacetime). However it turns out in this case that one of the conditions for the “no-ghost theorem” on $SL(2)_k$ is the requirement that the physical states be built only on the (discrete) $SL(2)$ representations with spin $j$ in the range $-1 < j < \frac{k-2}{2}$. Notwithstanding the long-time open problem of the compatibility of this finite range with modular invariance, the upper bound is also particularly counter-intuitive because of the following. When considering string theory on $AdS_3 \times \mathcal{N}$, the bound $j < \frac{k-2}{2}$ implies, through the on-shell condition, that for physical states there is a bound on the weight $\Delta$ of operators in $\mathcal{N}$. For instance, if there is a cycle $S^3 \subset \mathcal{N}$, then the winding number would be bounded by virtue of the upper bound on $j$.

Together with this older problem, a new one arose in the context of the $AdS$/$CFT$ correspondence, which revived interest in string theory on $AdS_3$. It is the problem of the perturbative formulation of the so-called ‘long strings’ \cite{10, 11, 12}, related to the number $p$. The presence of these long strings was already postulated in the work of \cite{4}, in particular in the way the spacetime central charge \cite{4} was derived. Another crucial remark was the one of \cite{10} that a BPS string in an $AdS_3$ background could nucleate at the origin and expand to the boundary (where it has an infinite spatial extent) at a finite cost in energy. This analysis was refined in \cite{11} where it was shown that the possibility to produce long strings should be seen in the boundary CFT by the presence of a continuum of states (roughly, the radial momentum of the long string) above a gap (the cost in energy to pull a long string to the boundary).

From the above we learn that, if the worldsheet theory gives a good description of the spacetime CFT, we should expect to find there vertex operators corresponding in spacetime to this continuum of states. These vertex operators were not seen in the ‘traditional’ $SL(2)$ WZW approach.
However, Maldacena and Ooguri offered in [13] a new look on the $SL(2)$ WZW model that proposes a solution to both of the above problems. Their main argument is that the Hilbert space of the $SL(2)$ WZW model must include a host of new (affine) representations, obtained from the usual ones by spectral flow. This was first noticed in the work of Hwang and collaborators in Ref. [9]. Most of the new representations feature an $L_0$ which is unbounded from below. This is however not a problem since in a string theory context one has to impose the Virasoro constraints.

Let us sketch very briefly their reasoning, starting by defining the spectral flow transformation:

\begin{equation}
\begin{align*}
J^\pm_n &\to J^\pm_{n+w} \\
J^3_n &\to J^3_n - \frac{k}{2}w\delta_n \\
L_n &\to L_n - wJ^3_n - \frac{k}{4}w^2\delta_n.
\end{align*}
\end{equation}

By applying the above transformations on the various representations of $SL(2)$, one obtains the following. The spectral flow of the discrete representations $\hat{D}_j^\pm$ (which are defined by having an affine primary field with $SL(2)$ spin $j$ real and bounded by $-\frac{1}{2} < j < \frac{k-3}{2}$ [14, 13], and a momentum $m$ given by $m = \pm(J + n)$, with $n$ a positive integer) gives rise to vertex operators whose $SL(2)$ part has an increasingly negative $L_0$, and thus allows the weight $\Delta$ of the part relating to $N$ to grow without bounds. On the other hand, the spectral flow of the continuous representations $\hat{C}_{j,\mu}$ (which are defined by an $SL(2)$ primary with $j = -\frac{1}{2} + is$ and $m = \mu, \mu \pm n$, with $0 \leq \mu < 1$) corresponds to the continuous spectrum of the long strings. Note that the non-spectrally flown ($w = 0$) continuous representation is usually associated with the tachyon, and is actually projected out in the superstring, as we discuss later. Thus here the spectral flow is essential to recover the signature of the long strings in the worldsheet theory. Let us conclude this section by noting that in the work of [13, 15] a modular invariant partition function of the (bosonic) $SL(2)$ WZW model is explicitly worked out.

### 3 Strings on $AdS_3$ and a twist field

Let us now address the same issues as above but in a different formulation that will allow us to extend the results to the superstring and compute the spacetime chiral spectrum in a convenient way [3].

Starting from the bosonic WZW model $SL(2)_k$, the holomorphic currents satisfy the following OPE (the same holds for the anti-holomorphic ones):

\begin{equation}
J^A(z)J^B(0) \sim \frac{k\eta^{AB}}{z^2} + \frac{i\epsilon^{ABC}\eta_{CD}J^D(0)}{z},
\end{equation}

where $A = 1, 2, 3$ and $\eta_{AB} = (+, +, -)$. Rewriting:

\begin{equation}
J^3 = -\sqrt{\frac{k}{2}}\partial X,
\end{equation}

a primary field of the $SL(2)$ WZW model can be written as:

\begin{equation}
\Phi_{jmn} = \Psi_{jmn}e^{\sqrt{k}(mX(z)+\bar{m}\bar{X}(\bar{z}))},
\end{equation}

where $\Psi_{jmn}$ can be thought of as an $SL(2)/U(1)$ parafermion, and the weight of the primary field is:

\begin{equation}
\Delta(\Phi_{jmn}) = -\frac{j(j+1)}{k-2}.
\end{equation}
The primary fields $\Phi_{jm\bar{m}}$ belong to one of the following representations of $SL(2)$: lowest or highest weight discrete representations $D^\pm_j$, or continuous representations $C_{j,\mu}$.

An important constraint which must be satisfied by the operators $\Phi_{jm\bar{m}}$ for single valuedness of the wave function on $AdS_3$ is that:

$$m - \bar{m} \in \mathbb{Z}.$$  \hspace{1cm} (7)

In [4, 12] the above condition was introduced by hand. Here we wish to impose it more intrinsically, and a way to do that is to introduce “twist fields” which implement (7) by requiring mutual locality of the worldsheet operators with them. Let us stress that we are not twisting an otherwise consistent theory, rather it is consistency of the theory (tree level unitarity and higher loop modular invariance) which requires the inclusion of these twist fields, and of all the twisted operators which arise through OPEs with the twist fields.

We thus introduce the operators:

$$t^w = e^{w\sqrt{2}(X(z)+\bar{X}(\bar{z}))}, \quad w \in \mathbb{Z}.$$  \hspace{1cm} (8)

Then the OPE of $t^w$ with $\Phi_{jm\bar{m}}$ takes the form

$$t^w(z, \bar{z})\Phi_{jm\bar{m}}(z', \bar{z}') \sim (z - z')^{-w(|m| + \bar{m})} - w\Phi_{jm\bar{m}}(z', \bar{z}') = (z - z')^{-m - \bar{m}}|z - z'|^{-2mw}\Phi_{jm\bar{m}},$$  \hspace{1cm} (9)

where

$$\Phi_{jm\bar{m}} = \Psi_{jm\bar{m}}e^{\sqrt{2}[(m+\frac{k}{2}w)X+(\bar{m}+\frac{k}{2}w)\bar{X}]}.$$  \hspace{1cm} (10)

From (8) we see that mutual locality with $t^w$ indeed implies the condition (7). Now, consistency of the theory implies that we must include also the “twisted” operators $\Phi_{jm\bar{m}}^w$.

The scaling dimension of $\Phi_{jm\bar{m}}^w$ is:

$$\Delta(\Phi_{jm\bar{m}}^w) = -\frac{j(j+1)}{k-2} - \frac{k}{4}w^2 - mw.$$  \hspace{1cm} (11)

Then for a vertex operator of $AdS_3 \times \mathcal{N}$ in the $w$ twisted sector:

$$V^w = V_\Delta\Phi_{jm\bar{m}}^w,$$  \hspace{1cm} (12)

where $V_\Delta$ is a primary operator in the CFT on $\mathcal{N}$ with weight $\Delta$, the on-shell condition is:

$$-\frac{j(j+1)}{k-2} - \frac{k}{4}w^2 - mw + \Delta = 1.$$  \hspace{1cm} (13)

In spacetime it creates from the vacuum a state with weight given by:

$$h = |m| + \frac{k}{2}|w|,$$  \hspace{1cm} (14)

where for convenience (see [8] for the details) we choose fields belonging to the $D^-_j$ representation and $w < 0$. (Roughly, recall that the eigenvalue of the spacetime $L_0^t$ is given by minus the eigenvalue of $J^3$ on the worldsheet [4].) When putting together (13) and (14), we get the following expression for the spacetime weight (for $w \neq 0$):

$$h = \frac{k|w|}{4} + \frac{1}{|w|}\left(-\frac{j(j+1)}{k-2} + \Delta - 1\right),$$  \hspace{1cm} (15)

in agreement with [13]. In the untwisted sector $w = 0$ we had of course $h = j + 1$ [4].
The superstrings

Going now to the superstring (we use the NSR formulation), we have to consider the supersymmetric $SL(2)$ WZW model. In addition to the three currents $J^A$ (we will now concentrate for simplicity on holomorphic fields) we have their three fermionic partners $\psi^A$ with OPE:

$$\psi^A(z)\psi^B(0) \sim \frac{\eta^{AB}}{z}. \quad (16)$$

Accordingly, the currents decompose in bosonic and fermionic pieces:

$$J^A = j^A - i\epsilon^{ABC}\psi^B\psi^C, \quad (17)$$

where $j^A$ has a regular OPE with the fermions. Taking the total current to be at level $k$, it follows that the bosonic part has level $k + 2$ since the fermionic piece has level $-2$. Note that this entails a slight change in the value of certain quantities. For instance, the weight of a bosonic primary of the WZW model is now $\Delta(\Phi_{jm}) = -j(j+1)k$, and the range for the $SL(2)$ spin of the discrete representations $\mathcal{D}_j^\pm$ is:

$$-\frac{1}{2} < j < \frac{k-1}{2}. \quad (18)$$

In the following, in order to have some robust information on the spectrum of the space-time theory, we will need to have at least $N = 2$ supersymmetries in spacetime. The conditions to achieve that were studied in [6, 7]. There it was shown that string theory on $AdS_3 \times \mathcal{N}$ gives rise to a boundary $N = 2$ SCFT provided that we can write, at least locally, $\mathcal{N} = U(1)_Y \times \mathcal{N}/U(1)$, and that $\mathcal{N}/U(1)$ is a worldsheet CFT with $N = 2$ supersymmetry (and central charge $c_{\mathcal{N}/U(1)} = 9 - \frac{6}{k}$). For later convenience, we write the current corresponding to the $U(1)_Y$ in terms of the canonically normalized scalar $Y$:

$$J^Y = i\partial Y. \quad (19)$$

With the above conditions, the spacetime supercharges $G_r^\pm$ can be constructed [8, 9], together with all the superconformal generators.

Let us single out the following piece of the algebra generated by the supercharges:

$$\{G_r^+, G_s^+\} = 2L_{r+s} + (r-s)J_0, \quad r, s = \pm \frac{1}{2}, \quad (20)$$

where the zero mode $J_0$ of the spacetime $R$-current is given by the worldsheet operator:

$$J_0 = \sqrt{2k} \int dz J^Y(z). \quad (21)$$

The twist field in the supersymmetric case has to be slightly amended, in order for it to be mutually local with the supercharges, and also convenient for the study of the chiral spectrum. We thus introduce:

$$t^w_{\pm} = e^{-w} \int J^{3\pm w} \int J^Y = e^{w\sqrt{2}(X \pm iY)}. \quad (22)$$

Note that the $J^3$ above is the total current. The twist fields $t^w_\pm$, being mutually local with $G_r^\pm$, survive the GSO projection (thus the twist of a GSO invariant operator is also GSO invariant), have zero weight on the worldsheet, and are “chiral” in spacetime in the sense that they verify $|J^3| = \frac{1}{2}|J^Y|$. 
4 Chiral spectrum, untwisted and twisted

We now give a worldsheet description of operators which are chiral (or anti-chiral) under the spacetime superconformal algebra (20). Let us first of all concentrate on vertex operators in the untwisted sector \( w = 0 \). Moreover, we consider for now only operators whose \( SL(2) \) piece is in discrete representations \( D_{j}^{\pm} \).

Every such vertex operator will include, besides a piece relating to the bosonized superghost, pieces relating to the \( U(1) \), the \( N/U(1) \) and the \( SL(2) \) factors of the worldsheet CFT. The on-shell and the GSO conditions together tie the respective values of the momentum along the \( U(1) \), the weight and \( R \)-charge in \( N/U(1) \) and the \( SL(2) \) spin \( j \).

After imposing all these conditions in both the NS and R sectors, we obtain the following physical vertex operator which are (anti)chiral in spacetime.

In the NS sector, there are two kinds of vertex operators. The first family is given by:

\[
\mathcal{X} = e^{-\varphi} e^{-i\sqrt{2}(j+1)Y} V \Phi_{j=\frac{1}{2}(1-r_{V})-1,m} , \quad \Delta_{V} = \frac{r_{V}}{2} , \quad r_{V} + \frac{1}{k} < 1 , \tag{23}
\]

where the inequality is implied by (18). The spacetime weight and \( R \)-charge of these operators are:

\[
h_{\mathcal{X}} = \frac{k}{2}(1-r_{V}) = -\frac{1}{2}R_{\mathcal{X}}. \tag{24}
\]

The operators \( \mathcal{X} \) are thus antichiral. Note that the corresponding chiral operators (with positive spacetime \( R \)-charge) are simply obtained by charge conjugation (and will involve \( V \) with \( r_{V} < 0 \); here and below we restrict for simplicity to \( r_{V} \geq 0 \)).

The second family of vertex operators is:

\[
\mathcal{W} = e^{-\varphi} e^{i\sqrt{2}Y} V (\psi \Phi_{j=\frac{1}{2}r_{V}})_{j-1,m} , \quad \Delta_{V} = \frac{r_{V}}{2} , \quad r_{V} + \frac{1}{k} < 1. \tag{25}
\]

Their spacetime weight is:

\[
h_{\mathcal{W}} = \frac{k}{2}r_{V} = \frac{1}{2}R_{\mathcal{W}}. \tag{26}
\]

In the Ramond sector we only have one family:

\[
\mathcal{Y} = e^{-\varphi} (Se^{i\sqrt{2}Y} V \Phi_{j=\frac{1}{2}(r_{V}-1)})_{j-\frac{1}{2},m} , \quad \Delta_{V} = \frac{r_{V}}{2} , \quad 1 < r_{V} + \frac{1}{k} < 2 . \tag{27}
\]

Here \( S \) is a spinfield whose precise expression can be found in [8]. The spacetime weight of the \( \mathcal{Y} \) vertex operators is:

\[
h_{\mathcal{Y}} = \frac{1}{2} + \frac{k}{2}(r_{V} - 1) = \frac{1}{2}R_{\mathcal{Y}}. \tag{28}
\]

Note that all the above vertex operators give rise to spacetime (anti-)chiral operators whose weight lies in the range \( 0 \leq h \leq \frac{k}{2} \).

Going now to the twisted sectors, \( w \neq 0 \), the task of finding the new operators which are chiral in spacetime is simplified by our choice of twist fields (22). It turns out that it is sufficient to twist the above physical operators (23), (25) and (27). The physicality condition on the twisted operators will only fix the value of \( m \) (this essentially means that we obtain in spacetime only the Virasoro highest weight). The physical vertex operators
that we get in both the NS and R sector are thus:

\[ \mathcal{X}_w = e^{-i e^{-\sqrt{2}i(j+1-\frac{kw}{2})} Y V \Phi^w_{j=\frac{1}{2}(1-r_v)-1, m=-j-1} , \quad h^w_{\mathcal{X}} = \frac{k}{2}(1-r_v) + \frac{|w|}{2}, \]

\[ \mathcal{W}_w = e^{-i e^{i \sqrt{2}i(j+1-\frac{kw}{2})} Y (\psi \Phi^w_{j=\frac{1}{2}r_v})_{j-1, m=-j} , \quad h^w_{\mathcal{W}} = \frac{k}{2}r_v + \frac{|w|}{2}, \]

\[ \mathcal{Y}_w = e^{-\frac{i}{2} (Se^{i \sqrt{2}i(j-\frac{kw}{2})} Y \Phi^w_{j=\frac{1}{2}(r_v-1)}_{j-\frac{1}{2}, m=-j-\frac{1}{2}} , \quad h^w_{\mathcal{Y}} = \frac{1}{2} + \frac{k}{2}(r_v - 1) + \frac{|w|}{2}. \tag{29} \]

A clear pattern emerges: every operator in the untwisted sector gives rise to a tower of operators, regularly spaced with a step of \( k/2 \), which are all (anti-)chiral in spacetime. We thus have a spacetime theory whose chiral spectrum is ordered according to the following pattern:

\[ h^w = h^0 + \frac{k}{2} |w|. \tag{30} \]

We should now briefly comment on the vertex operators which belong to the continuous representations \( C_{j,\mu} \) of \( SL(2) \). In the untwisted sector, they are tachyonic and do not survive the GSO projection. However, in the twisted sectors such operators can be physical, and give rise to the long string continuous spectrum, as in \[13\]. Moreover, we expect also that some chiral states can be found within the continuous spectrum, but to find and classify them is a more complicated issue since one cannot simply twist already known physical states, as with the discrete spectrum.

5 The spacetime CFT as a symmetric product

We will now provide evidence that the spacetime CFT is a deformation of the following symmetric product CFT:

\[ (\mathcal{M}_{c=6k})^p / S_p, \tag{31} \]

which indeed has a central charge \( c_{st} = 6kp \). We will specify what \( \mathcal{M}_{6k} \) is after showing that the pattern \[30\] is reproduced in the chiral spectrum of a CFT like \[31\]. For earlier studies of symmetric product CFTs, see \[16\].

To be more precise, we focus on the chiral spectrum of “single particle states,” since this is what the string spectrum can be compared to: vertex operators correspond to single string states in spacetime. In a CFT like \[31\], the “single particle states” are identified with the ones coming from each single \( Z_N \) twisted sector of the \( S_p \) orbifold. Note that this includes also the “diagonal” \( \mathcal{M}_{6k} \) as the \( Z_1 \) sector. In order to obtain the spectrum of the \( Z_N \) twisted sector, we consider the \( \mathcal{M}^N / Z_N \) orbifold. There is a one-to-one correspondence between states in \( \mathcal{M} \) with weight \( h \) and \( R \)-charge \( R \) and states in the \( Z_N \) twisted sector of \( \mathcal{M}^N / Z_N \) with \[17\]:

\[ h^N = \frac{h}{N} + \frac{c}{24} \frac{N^2 - 1}{N}, \quad R^N = R. \tag{32} \]

Another useful property of \( N = 2 \) SCFTs is the existence of a spectral flow between the R and the NS sectors \[15\]:

\[ h_R = h_{NS} - \frac{1}{2} R_{NS} + \frac{c}{24}, \quad R_R = R_{NS} - \frac{c}{6}. \tag{33} \]

We can now proceed to show that every chiral operator of \( \mathcal{M} \) gives rise to a chiral operator in the \( Z_N \) twisted sector of \( \mathcal{M}^N / Z_N \), and thus of each \( Z_N \) twisted sector of \( \mathcal{M}^p / S_p \).
Starting from a chiral state of $\mathcal{M}$ with $R$-charge $R$ and weight $h = R/2$, the spectral flow (33) to the R sector of $\mathcal{M}$ gives a state with $h_R = \frac{c}{2}$ and $R_R = R - \frac{c}{6}$, which is a Ramond ground state. Going then to the R sector of $\mathcal{M}^R/Z_N$ using (32), we get a state with $h^R_N = \frac{c}{24}$ and $R^R_N = R - \frac{c}{6}$, which is still a Ramond ground state, but of the $Z_N$ orbifold CFT. Eventually, we go to the NS sector of $\mathcal{M}^N/Z_N$ using (33) backwards, and get a state with $h^N_{NS} = R^2 + \frac{c}{2}(N-1)$ and $R^N_{NS} = R + \frac{c}{6}(N-1)$, which is thus chiral. For $c = 6k$, we get the following pattern:

$$h^N = h^1 + \frac{k}{2}(N-1).$$

It is exactly the same pattern as (30) if we identify $h^w=0$ and $|w|$ there with $h^{N=1}$ and $N-1$ in (34). Namely, we take $\mathcal{M}_{c=6k}$ such that its chiral spectrum is the one computed from string vertex operators coming from the untwisted sector $w = 0$.

Note that here we have a precise bound on $N$, namely the largest cycle is $Z_p$ and thus we have $N \leq p$. On the other hand, a bound on $w$ is not seen in the worldsheet analysis because it should be non-perturbative in nature, $p$ being related to the string coupling constant by $g_s^2 \sim 1/p$.

6 Conclusions and Discussion

In this contribution, we have provided evidence that the single particle chiral spectrum of superstring theory on $AdS_3 \times \mathcal{N}$ and of the symmetric product CFT $(\mathcal{M}_{6k})^p/S_p$ agree, where we take $\mathcal{M}_{6k}$ to have the chiral spectrum provided by the $w = 0$ sector of the worldsheet theory. More precisely, the $Z_N$ twisted sectors of $(\mathcal{M}_{6k})^p/S_p$ contains (single string) states associated with the worldsheet theory sector related to the presence of $N$ long strings.

It is likely that the same pattern exists (albeit deformed by some amount) for the non-chiral spectrum, and for less or non supersymmetric string theories.

Let us end with a particular example in which we can be more specific, and which reveals more structure. Consider string theory on $AdS_3 \times S^3 \times T^4$ which has (small) $N = 4$ superconformal symmetry. Here the dual CFT is supposed to be a deformation of $(T^4)^{k/p}/S_{kp}$ (plus something that is not seen on the worldsheet, see [19]). Our conjecture being (31), we would like to postulate that $(T^4)^{k/p}/S_{kp}$ is related in some way to the “double” symmetric product $((T^4)^{k}/S_k)^p/S_p$, thus identifying $\mathcal{M}_{6k}$ with a theory in the moduli space of $(T^4)^k/S_k$. Given the conjecture (31), the above argument is compatible with U-duality of the brane configuration leading to that particular background. Note however that (as it was remarked in [11]) there are missing chiral states in the worldsheet description. This was argued [14] to be traced to the fact that the CFT dual to the stringy description is actually sitting on a singular point of the moduli space (roughly because all the RR fields are vanishing in this background). A manifestation of the singularity is the presence of the continuum of the long strings).

This phenomenon where the theory $\mathcal{M}_{6k}$ could itself be in the moduli space of a symmetric product, is however not general. Counter examples can be found in less supersymmetric cases, for instance the pure $N = 2$ case of [20] or the $N = 3$ case of [21], where it is shown that $\mathcal{M}_{6k}$ cannot be a symmetric product of a smaller CFT.

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