Quintessential inflation with $\alpha$-attractors

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Abstract. A novel approach to quintessential inflation model building is studied, within the framework of $\alpha$-attractors, motivated by supergravity theories. Inflationary observables are in excellent agreement with the latest CMB observations, while quintessence explains the dark energy observations without any fine-tuning. The model is kept intentionally minimal, avoiding the introduction of many degrees of freedom, couplings and mass scales. In stark contrast to $\Lambda$CDM, for natural values of the parameters, the model attains transient accelerated expansion, which avoids the future horizon problem, while it maintains the field displacement mildly sub-Planckian such that the flatness of the quintessential tail is not lifted by radiative corrections and violations of the equivalence principle (fifth force) are under control. In particular, the required value of the cosmological constant is near the electroweak scale. Attention is paid to the reheating of the Universe, which avoids gravitino overproduction and respects nucleosynthesis constraints. Kination is treated in a model independent way. A spike in gravitational waves, due to kination, is found not to disturb nucleosynthesis as well.

Keywords: dark energy theory, inflation, particle physics - cosmology connection

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1 Introduction

The Universe is currently in a phase of accelerated expansion. Since the observational discovery of this from type Ia Supernovae [1, 2], it has been confirmed by several methods, most notably observations of the Cosmic Microwave Background (CMB) [3–6]. The observed dynamics can only be explained via the introduction of some hypothetical substance, called dark energy (for a comprehensive review see ref. [7]). The simplest form of dark energy, which does not require new Physics, is a non-zero cosmological constant corresponding to positive vacuum density. However, since this vacuum density must be comparable to the present density of the Universe (accelerated expansion started in the last billion years only) the value of the vacuum density must be incredibly fine-tuned, down to $\sim 10^{-120} M_P^4$, where $M_P = 1.22 \times 10^{19}$ GeV is the Planck mass, which is the natural scale of gravity in General Relativity [8]. Unless introducing a new scale in Physics and a new hierarchy problem, this explanation of the observed recent accelerated expansion seems unnatural, especially since the dark energy component is thought to make up almost 70% of the current content of the Universe.

Following the success of the inflationary paradigm, a promising alternative to explain the late time acceleration of the Universe is a dynamic scalar field, provided that it avoids the extreme fine-tuning of the cosmological constant. This field has been referred to as quintessence; the fifth element of the current make up of the Universe, after baryonic and cold dark matter,
radiation and neutrinos [9–11]. Quintessence can generate the observed accelerated expansion if it dominates the Universe at present, while rolling down a flat potential, in the same way as the inflaton field drives inflation in the early Universe. However, quintessence suffers from its own tuning problems. Indeed, in fairly general grounds it can be shown that quintessence needs to travel at least over Planckian distances in field space whilst retaining the flatness of its potential against radiative corrections. Also, the effective mass of quintessence is comparable to the Hubble constant $H_0 = 1.43 \times 10^{-33}$ eV, such that its Compton wavelength is comparable to the present horizon. Consequently, if not suppressed, interactions of the quintessence field with the standard model can correspond to a long-range ‘fifth force’, which may result in violations of the equivalence principle [12]. In addition, the introduction of yet another unobserved scalar field (on top of the inflaton field) seems unappealing. Finally, a rolling scalar field introduces another tuning problem, namely that of its initial conditions.

A compelling way to overcome the difficulties of the quintessence scenario is to link it with the rather successful inflationary paradigm. This is quite natural since both inflation and quintessence are based on the same idea; that the Universe undergoes accelerated expansion when dominated by the potential density of a scalar field, which rolls down its almost flat potential. This unified approach has been named quintessential inflation [13]. In quintessential inflation the scalar potential is such that it causes two phases of accelerated expansion, one at early and the other at late times. Apart from using a single theoretical framework to describe both inflation and dark energy, quintessential inflation overcomes the problem of initial conditions of quintessence, because they are determined by the inflationary attractor.

Modelling quintessential inflation is not easy. The two plateaus featured in the potential are bridged by a steep dip over more than a hundred orders of magnitude. Yet, there have been many early attempts [14–25] and since then, the subject has continued to be investigated [26–49]. In quintessential inflation, the scalar field does not decay at the end of inflation because it needs to survive until today, to become quintessence. This is why inflation is non-oscillatory (NO) and instead of the inflaton oscillating around its vacuum expectation value (VEV), it rolls down to the quintessential plateau. Thus, the Universe must be reheated via a mechanism other than the decay of the inflaton field.

A promising such reheating mechanism is instant preheating [50, 51], where, after inflation, the inflaton field crosses an enhanced symmetry point, where it couples to some other field $\chi$. The non-adiabatic change of the $\chi$ effective mass results into copious production of $\chi$ particles, which further decay into the thermal bath of the Hot Big Bang (HBB). Instant preheating can be very efficient, removing a large fraction of the inflaton’s kinetic density. Another reheating mechanism for NO inflationary models is curvaton reheating [52–54], where the Universe is reheated by the decay of some spectator scalar field $\sigma$, that may or may not be responsible for the curvature perturbation (if it is responsible it is called the curvaton). The efficiency of curvaton reheating depends on the density budget of the curvaton at the time of its decay. The above mechanisms, however, introduce an additional field ($\chi$ or $\sigma$), which is to play a crucial role in the Universe history. As such, they are not aligned with the economy principle underlying quintessential inflation.

Fortunately, there is another reheating mechanism, which does not rely on some other scalar field playing a special role. This is the so-called gravitational reheating [55, 56]. Gravitational reheating is due to particle production during inflation of all light fields (i.e. with masses less than the Hubble scale), which are also non-conformally invariant. This is always present in inflation, but the radiation density due to gravitational reheating is negligible in standard oscillatory inflation, so it is ignored. However, in NO inflation, this unavoidable radiation can be the only way to generate the thermal bath of the HBB.
As explained, the potential needs to have a huge drop in energy density between inflation and dark energy times. Soon after the end of inflation, as the potential energy undergoes this massive decrease, the scalar field becomes dominated by its kinetic density. If the latter also dominates over the background density, we enter a period of so-called kination \[14–18\], until the Universe is reheated and the HBB begins. Soon after reheating, the field freezes and remains at a small constant potential density until much later, when it can play the role of quintessence.

Kination typically sends the field down to the quintessential plateau over a super-Planckian displacement in field space. This is a problem because radiative corrections threaten to lift the flatness of the potential. Also, interactions with the standard model, albeit gravitationally suppressed, may become important and challenge the equivalence principle. However, it is desirable that the field moves substantially down the potential, in order for its potential density to massively decrease so that the gap between the energy scales of inflation and dark energy can be bridged. This is a catch-22 problem of quintessential inflation; a super-Planckian displacement threatens the flatness of the quintessential plateau and may generate a fifth-force problem, but a sub-Planckian displacement makes it almost impossible to get from the inflationary to the quintessential plateau in such a way that the potential is not too curved during inflation, so that the generated curvature perturbation remains approximately scale-invariant (this is the \(\eta\)-problem of quintessential inflation \[25\]).

In this paper we attempt to address the above in the context of \(\alpha\)-attractors in inflation model-building. The idea of \(\alpha\)-attractors is that the scalar field has a non-canonical kinetic term, which features poles. Such kinetic terms can be due to specific forms of the Kähler potential in supergravity theories \[57–60\]. The effect of a pole in the kinetic term is that the field cannot travel through it in field space, so it imposes a bound on its value. Switching to a canonical field, transposes the pole to infinity, while “stretching” the scalar potential of the canonical field near the pole, generating thereby a plateau in the potential \[61\]. Because of this, \(\alpha\)-attractors are rather popular for inflation model building \[62–79\], since the latest CMB data favour an inflationary plateau \[4–6\]. For quintessential inflation, we need two flat regions in the scalar potential and we show that this can be naturally generated within the standard \(\alpha\)-attractors framework.\(^1\) The “stretching” effect ensures that the plateaus in the potential are not in danger from radiative corrections, even with a super-Planckian excursion of the canonical field because variation of the non-canonical field can be safely kept sub-Planckian by the bounds due to the poles in the kinetic term. As we explain, this also addresses the danger of the fifth force, so the above catch-22 problem is overcome.\(^2\)

Our paper is structured as follows. In section 2, we introduce our model. In section 3, we discuss inflationary physics and obtain the inflationary observables, such as the spectral index of the scalar curvature perturbations and the ratio between the spectra of tensor to scalar perturbations. We find that our model predictions fall very near the sweet spot of the latest CMB observations, as typical for a plateau inflation model. In section 4, we study in detail the early history of the Universe after inflation. In particular we investigate, in a model independent way, kination and reheating, with emphasis on gravitational reheating.

\(^1\)See also ref. \[81\], which appeared closely after our work. For pure quintessence with \(\alpha\)-attractors see refs. \[82, 83\].

\(^2\)A super-Planckian excursion of the canonical field may result in the production of sizeable gravitational waves, even though the variation of the non-canonical field is kept sub-Planckian. In this way, one can evade the Lyth bound and obtain a large value of the tensor-to-scalar ratio \(r\) with a sub-Planckian (non-canonical) inflaton \[80\]. In our model, though, we only achieve a modest production of gravitational waves as we find \(r \lesssim 10^{-3}\).
In section 5, we discuss the physics of quintessence and constrain our model parameters such that the dark energy observations are satisfied. In section 6, we discuss the problems of the fifth-force and overproduction of gravitinos and gravitational waves in our model. Finally, we conclude in section 7.

We consider natural units, where \( c = \hbar = 1 \) and Newton’s gravitational constant is \( 8\pi G = m_p^2 \), with \( m_p \equiv M_P/\sqrt{8\pi} = 2.43 \times 10^{18} \text{ GeV} \) being the reduced Planck mass.

2 The model

We consider the following Lagrangian,

\[
\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi \left( 1 - \frac{\phi^2}{6\alpha} \right)^2 m_P^2 - V_0 e^{-\kappa \phi} + \Lambda ,
\]

where the dimensionless scalar field \( \phi \) is measured in units of \( m_P \), \( \alpha \) and \( \kappa \) are dimensionless positive constants, \( V_0 \) is a constant density scale and \( \Lambda \) is the cosmological constant. In the above, the non-canonical kinetic term of the field features poles at \( \phi = \pm \sqrt{6\alpha} \), and has the standard form of \( \alpha \)-attractor models motivated by supergravity [57–60], corresponding to a non-trivial Kähler manifold. In this context, the scalar potential can be due to non-perturbative effects, e.g. gaugino condensation [84–86]. The effect of the scalar potential is to drive \( \phi \) to large values. However, the existence of the poles in the kinetic term has the important consequence that the field cannot traverse through them in field space [57–60].

Quintessence was introduced to explain the dark energy observations [7] without making use of the cosmological constant. The motivation is that the required value of the cosmological constant in \( \Lambda \)CDM is incredibly fine-tuned because the vacuum density today is about \( (10^{-3} \text{ eV})^4 \). In our model we still feature \( \Lambda \) but, as we show, the required value is much more reasonable; at least as large as the electroweak scale. We introduce \( \Lambda \) for the following reason.

As was standard practice until the observation of dark energy, we assume that, due to some unknown symmetry, the vacuum density in the Universe is zero. However, because of the positive pole present in the model, \( \phi \) cannot go to infinity in the vacuum; it is capped at \( \phi = \sqrt{6\alpha} \) because this is the value that corresponds to the smallest possible potential density. This means that zero vacuum energy density requires \( V_0 e^{-\kappa \sqrt{6\alpha}} = \Lambda \). Substituting this back into eq. (2.1), the Lagrangian becomes

\[
\mathcal{L} = \frac{1}{2} (\partial \phi)^2 m_P^2 \left( 1 - \frac{\phi^2}{6\alpha} \right)^2 - V_0 e^{-n} \left[ e^{n(1-\frac{\phi}{\sqrt{6\alpha}})} - 1 \right] ,
\]

where \( n \equiv \kappa \sqrt{6\alpha} \). It is now evident that as \( \phi \to \sqrt{6\alpha} \) the potential density disappears.\(^3\)

Now, the initial value of \( \phi \) needs to be between the poles in the potential. Were initially \( \phi > \sqrt{6\alpha} \) then it would roll down to infinity and the vacuum density would be zero without

\(^3\)One may contemplate adding an increment \( \delta \Lambda \) to the value of the cosmological constant, which would appear in eq. (2.2). This would ensure eternal acceleration a la \( \Lambda \)CDM. In this case, considering the quintessential tail would only provide some dynamics to the effective barotropic parameter of dark energy, which might depart from \(-1\). However, this increment \( \delta \Lambda \) will suffer from the same problem as the cosmological constant in \( \Lambda \)CDM, namely it would have to be incredibly fine-tuned so not to exceed the value of the dark energy density at present \( \sim (10^{-3} \text{ eV})^4 \). In other words, the mechanism that is assumed to eliminate the vacuum density would have to deviate from exactly zero by this tiny amount. We feel that this would negate the need for quintessence and this is why we will not consider this possibility here.
the introduction of Λ. Were initially φ < −√6α then the field would roll down to φ = −√6α only and the required Λ for zero vacuum density would have been Λ = V₀e^(√6α). We do not consider either case. The reasons are practical. In the former case, we have inflation near the pole but the exponential tail is not steep enough to allow for successful quintessence. In the latter case, we have inflation with an exponential potential, which is power-law and contradicts observations (plus, it never ends). Because of the no-hair theorem, the discussion over the initial conditions of the inflaton field is largely academic as it is not testable.

Because of the poles, the potential becomes stretched at φ → ±√6α. Hence, we find two plateaus in the model, with V → V₀e^(-n(e^(2n) - 1)) or V → 0. If the scalar field dominates the Universe, the two plateaus can result in early and late periods of accelerated expansion. Thus, they can play the role of the inflationary and quintessential plateau (also called ‘quintessential tail’).

To assist our intuition and help with studying the model, we make the following field redefinition to obtain a canonical kinetic term

$$\phi = \sqrt{6\alpha} \tanh \frac{\varphi}{\sqrt{6\alpha} m_P},$$

(2.3)

which allows our new canonical field, φ, to take any value whilst our non-canonical degree of freedom, ϕ, remains sub-Planckian at all times, as long as α ≲ \frac{1}{6}. The potential becomes now:

$$V(\varphi) = e^{-2nM^4} \left\{ \exp \left[ n \left( 1 - \tanh \frac{\varphi}{\sqrt{6\alpha} m_P} \right) \right] - 1 \right\}.$$

(2.4)

where we have defined \( M^4 \equiv e^nV_0 \), which stands for the inflation energy scale. Note, also, that Λ = e^{-2n}M. The potential is shown in figure 1.

As \( \tanh(\varphi/\sqrt{6\alpha} m_P) \) approaches a constant value when \( |\varphi| \) is very large, the potential becomes asymptotically constant, featuring plateaus. At the locations of these plateaus the field slow rolls and accelerated expansion occurs. In the following sections we examine these two periods; that of inflation when \( \varphi \rightarrow -\infty \) (φ → −√6α) and that of quintessence when \( \varphi \rightarrow +\infty \) (φ → √6α), as well as the evolution between them.

3 Inflation

In the limit \( \varphi \rightarrow -\infty \) (φ → −√6α), the potential in eq. (2.4) becomes:

$$V(\varphi) \simeq M^4 \exp \left( -2ne^{\frac{2\varphi}{\sqrt{6\alpha} m_P}} \right).$$

(3.1)

3.1 The scalar spectral index and tensor ratio

In view of the above, the slow roll parameters are

$$\epsilon = \frac{m_P^2}{2} \left( \frac{V'}{V} \right)^2 = \frac{4n^2}{3\alpha} e^{\frac{4\varphi}{\sqrt{6\alpha} m_P}},$$

(3.2)

$$\eta = m_P^2 \frac{V''}{V} = -\frac{4n}{3\alpha} e^{\frac{2\varphi}{\sqrt{6\alpha} m_P}} \left( 1 - 2ne^{\frac{2\varphi}{\sqrt{6\alpha} m_P}} \right),$$

(3.3)

\(^4\)Still a number of authors have considered the likelihood of appropriate initial conditions for inflation (for a recent discussion see ref. [87]). One of the arguments is that the inflationary potential must make contact with the Planck scale, otherwise the Universe cannot exit from the spacetime foam and there is no initial boost for the expansion. The latest CMB observations favour plateau inflationary models, which cannot fulfill this requirement since the potential density is capped at sub-Planckian values, which may pose an initial condition problem for inflation [88]. One way to overcome this problem is by considering a period of power-law proto-inflation, which takes the system from the Planck scale and safely places it at the inflationary plateau [89].
Figure 1. The potential in eq. (2.4). It features two flat regions for $|\varphi| \gg \sqrt{6\alpha} m_P$; the inflationary plateau and the quintessential tail, with a steep dip between them.

where ‘*’ denotes the value at horizon crossing, when cosmological scales exit the horizon and the prime denotes derivative with respect to $\varphi$. From the usual condition of the end of inflation, $\epsilon = 1$, we find:

$$\varphi_{\text{end}} = \frac{\sqrt{6\alpha}}{2} m_P \ln \left( \frac{\sqrt{3\alpha}}{2n} \right).$$  \hfill (3.4)

The slow roll parameters are better expressed as functions of the number of remaining e-folds of inflation at horizon crossing of cosmological scales, defined as:

$$N_* = \frac{1}{m_P^2} \int_{\varphi_{\text{end}}}^{\varphi_*} \frac{V}{V'} d\varphi,$$ \hfill (3.5)

through which we obtain:

$$\varphi_* = \frac{\sqrt{6\alpha}}{2} m_P \ln \left[ \frac{3\alpha}{4n} \left( N_* + \frac{\sqrt{3\alpha}}{2} \right)^{-1} \right],$$ \hfill (3.6)

which can be negative if $\alpha$ is small. Using the above, the slow-roll parameters become

$$\epsilon = \frac{3\alpha}{4} \left( N_* + \frac{\sqrt{3\alpha}}{2} \right)^{-2},$$ \hfill (3.7)

$$\eta = - \left( N_* + \frac{\sqrt{3\alpha}}{2} \right)^{-1} \left[ 1 - \frac{3\alpha}{2} \left( N_* + \frac{\sqrt{3\alpha}}{2} \right)^{-1} \right].$$ \hfill (3.8)
Thus, we obtain the tensor-to-scalar ratio and the spectral index of the scalar curvature perturbation as

\[
r = 16\epsilon = 12\alpha \left( N^* + \frac{\sqrt{3}\alpha}{2} \right)^{-2},
\]

\[
n_s = 1 + 2\eta - 6\epsilon = 1 - \frac{2}{\left( N^* + \frac{\sqrt{3}\alpha}{2} \right)^2} - \frac{3\alpha}{2 \left( N^* + \frac{\sqrt{3}\alpha}{2} \right)} \simeq 1 - \frac{2}{N^*},
\]

where the last equation in eq. (3.10) corresponds to small \( \alpha \). We see that \( n_s \) follows the pattern of the \( \alpha \)-attractors inflationary models \([57–60, 62–79]\). In fact, this is suggested by most plateau inflationary models (e.g. see ref. \([93]\)), like Starobinsky \([94, 95]\) and Higgs \([96]\) inflation, which are favoured by the latest CMB observations \([4–6]\).

From the above, the running of the spectral index is easy to calculate as

\[
n_s' \equiv \frac{\ln n_s}{\ln k} = -\frac{1}{\left( N^* + \frac{\sqrt{3}\alpha}{2} \right)} \left[ \frac{2}{\left( N^* + \frac{\sqrt{3}\alpha}{2} \right)^2} - \frac{3\alpha}{2 \left( N^* + \frac{\sqrt{3}\alpha}{2} \right)} \right] \approx -\frac{2}{N^* - 2N^*},
\]

where again the last equation in the above corresponds to small \( \alpha \).

Here we should briefly consider what it really means when we apply the limits \( \alpha \to 0 \) and \( \alpha \to \infty \). If \( \alpha \to 0 \) then the region between the poles is shrinking, so it becomes increasingly unlikely that \( \phi \) initially finds itself there. Still, as we show below, when \( \alpha \lesssim 0.1 \) or so the value of the spectral index gradually becomes insensitive to \( \alpha \) (see figure 2), which means that there is no point considering \( \alpha \) incredibly small (which would amount to fine-tuning anyway). In the opposite limit, \( \alpha \to \infty \), the poles are transposed to infinity and \( \phi \) becomes canonically normalised. In this case, there are no plateaus to consider and we end up with either power-law inflation that never ends, or with no inflation at all (depending on how big \( \kappa \) is in eq. (2.1)). Barring the extremes \( \alpha = 0, \infty \), the natural value of \( \alpha \) is close to unity.

### 3.2 Constraining \( N^* \) and \( M \)

The number of remaining e-folds of inflation when the cosmological scales exit the horizon, \( N^* \) depends on the expansion history of the Universe. In this model we have a period of kination, where the kinetic energy density of the inflaton is, for a time, the dominant energy density in the Universe and controls its evolution. During this regime \( a \propto \rho^{-1/6} \).

We start from the recognisable equation

\[
e^{N^*} = 2 \frac{H_*}{H_k} \left( \frac{a_{\text{end}}}{a_{\text{reh}}} \right) \left( \frac{a_{\text{reh}}}{a_{\text{eq}}} \right) \left( \frac{a_{\text{eq}}}{a_k} \right),
\]

where subscripts ‘end’, ‘reh’, and ‘eq’ refer to the end of inflation, the onset of radiation domination and the beginning of matter domination respectively, while subscript ‘\( k \)’ corresponds to horizon reentry of the pivot scale \( k = 0.05 \text{Mpc}^{-1} \). From the above, we obtain

\[
N^* \simeq 61.93 + \ln \left( \frac{V_{\text{end}}^{1/4}}{m_P} \right) + \frac{1}{3} \ln \left( \frac{V_{\text{end}}^{1/4}}{T_{\text{reh}}} \right),
\]

where \( T_{\text{reh}} \) is the reheating temperature when the Hot Big Bang (HBB) begins and \( V_{\text{end}} \equiv V(\varphi_{\text{end}}) = M^4 e^{-\sqrt{3}\alpha} \) (cf. eq. (3.4)). This differs slightly from that of a model which
contains no kination [90]. As shown in section 4.2 (cf. eq. (6.3)), for gravitational reheating, $T_{\text{reh}} \propto V_{\text{end}}/m_P^3$. Using this, it is easy to show that $(V_{\text{end}}^{1/4} T_{\text{reh}}^{1/3}) \propto (m_P V_{\text{end}}^{1/4})$. Thus, the dependence on $V_{\text{end}}$ in the second and third terms of eq. (3.13) cancels out and we are left with a constant value for $N_*$, independent of both $\alpha$ and $n$:

$$N_* = 63.49 .$$

(3.14)

Using the above, eqs. (3.10) and (3.11) give $n_s \simeq 0.9685$ and $n'_s \simeq -5.11 \times 10^{-4}$ for negligible $\alpha$, which is in excellent agreement with observations ($n_s = 0.968 \pm 0.006$ and $n'_s = -0.003 \pm 0.007$ [4, 5]).

We can determine the inflationary scale by the so-called COBE constraint [91]

$$\sqrt{\mathcal{P}_\zeta} = \frac{1}{2\sqrt{3\pi}} \frac{V^{3/2}}{m_P^2 |V'|} ,$$

(3.15)

where $\mathcal{P}_\zeta = (2.208 \pm 0.075) \times 10^{-9}$, is the spectrum of the scalar curvature perturbation [4, 5]. Using eqs. (3.1) and (3.6) we find:

$$\left( \frac{M}{m_P} \right)^2 = 3\pi \sqrt{2\alpha} \mathcal{P}_\zeta \left( N_* + \frac{\sqrt{3\alpha}}{2} \right) \exp \left[ \frac{3\alpha}{4} \left( N_* + \frac{\sqrt{3\alpha}}{2} \right) \right] .$$

(3.16)

We determine the particular values of $M$ for various $\alpha$ values, shown in table 1. Eq. (3.16) suggests that the inflation energy scale $M$ is also independent of $n$. As expected, $M$ is near the scale of grand unification.

### 3.3 Parameter space from observational constraints

We are able to test the constraints of the model immediately via comparison of the model’s prediction for the tensor-to-scalar ratio to observation. The constraint on the tensor-to-scalar ratio, $r < 0.1$ [92], allows us to constrain the allowed values of $\alpha$:

$$120 \alpha < \left( N_* + \frac{\sqrt{3\alpha}}{2} \right)^2 .$$

(3.17)

Using the value of $N_*$ derived in the previous section, $N_* = 63.49$, results in a bound of $\alpha \leq 39.6$. Currently we have no lower bound on $r$ and hence no lower bound on $\alpha$, but (as shown later) it should not get too small. Requiring the mass scale which suppresses the non-canonical $\phi$ in the kinetic term not to be too small compared to $M$, we have $\sqrt{6\alpha} m_P \gtrsim M$, which results in $\alpha \gtrsim 10^{-7}$.

| $\alpha$ | $M$ (GeV) |
|----------|---------|
| 0.01     | $2.42 \times 10^{15}$ |
| 0.10     | $4.29 \times 10^{15}$ |
| 1        | $7.64 \times 10^{15}$ |
| 10       | $1.41 \times 10^{16}$ |
| 100      | $3.81 \times 10^{16}$ |

Table 1. Values of $M$ calculated from eq. (3.16) for various $\alpha$ values.
Figure 2. The tensor-to-scalar ratio, $r$, versus the spectral index, $n_s$ for our model is displayed in red, overlaid on the Planck 2015 results. $\alpha$ varies from 0 to 39.6, bottom to top. The slope of the line for large values of $\alpha$ is understood as $n_s \to 0$ when $\alpha \gg 1$ (cf. eq. (3.10)). Note that the line corresponding to the values of $n_s$ and $r$ is crooked for small $\alpha$ (values of $\alpha \lesssim 0.1$ or so) so that the spectral index becomes insensitive to $\alpha$ when the latter is small. For a closer look at this region see figure 4.

The corresponding bounds on $n_s$ are

$$0.9585 \lesssim n_s \leq 0.9686,$$

and fall almost entirely within the recent BICEP2 2-$\sigma$ bounds [6]. Figure 2 shows the parameter space for $n_s$ and $r$ in our model, using these values. However, because of the field redefinition in eq. (2.3), to avoid super-Planckian values of $\phi$, we need $\alpha \lesssim \frac{1}{16}$, which places us firmly within the constraints of the tensor-to-scalar ratio at $r \leq 0.00049$. The bounds on $n_s$ with this $\alpha$ constraint are:

$$0.9685 \lesssim n_s \leq 0.9686,$$

which corresponds to the very lowest part of the line in figure 2, well inside the Planck 1-$\sigma$ contour.

4 After inflation

During inflation the inflaton is slow rolling along the inflationary plateau but, after the end of inflation, the inflaton falls down the steep slope of the potential. For a brief period of time, the kinetic energy of the inflaton is the dominant energy density in the Universe, the field is oblivious to the potential and we have a period of so-called kination [14–18]. Kination, however, must end well before Big Bang Nucleosynthesis (BBN). Hence, the Universe should be reheated so that the HBB can begin. Because inflation is non-oscillatory, reheating must occur without the decay of the inflaton field, which is to survive until the present and become quintessence. In the following, we will briefly study kination and reheating.

4.1 Kination

Soon after inflation ends, the inflaton energy density is completely dominated by the kinetic part as the potential density becomes negligible. Being oblivious of the potential, the equation
of motion becomes $\ddot{\varphi} + 3H\dot{\varphi} \simeq 0$, which gives

$$\dot{\varphi} = \sqrt{2} \frac{m_P}{3t}.$$  \hfill (4.1)

Thus, in kination we have $\rho = \rho_{\text{kin}} \equiv \frac{1}{2} \dot{\varphi}^2 \propto a^{-6}$ with $a \propto t^{1/3}$ and barotropic parameter $w = 1$ (stiff fluid). Integrating eq. (4.1) we obtain

$$\varphi = \varphi_{\text{end}} + \sqrt{\frac{2}{3}} m_P \ln \left( \frac{t}{t_{\text{end}}} \right).$$  \hfill (4.2)

Kination has to end and the HBB to begin before BBN takes place. Therefore, the radiation bath of the HBB must be created after inflation. Because radiation density scales as $\rho_\gamma \propto a^{-4}$, once created, radiation eventually takes over, since $\rho_{\text{kin}} \propto a^{-6}$, but this has to happen before BBN. Also, since the inflaton is not oscillating after inflation, reheating of the Universe cannot occur through the decay of the inflaton field.

### 4.2 Reheating

There are various possibilities for reheating the Universe in quintessential inflation scenarios, most promising of which are instant preheating [50, 51] and curvaton reheating [52–54]. If all else fails, the Universe is reheated by so-called gravitational reheating [55, 56]. As mentioned, the latter is due to particle production during inflation of all light fields (i.e. with masses less than the Hubble scale), which are also non-conformally invariant. This is Hawking radiation in de Sitter space, which generates a radiation bath of temperature $T_H = H/2\pi$. Such radiation is always produced at the end of inflation, but its density $\sim T_H^4$ is negligible in standard oscillatory inflation, so it is ignored.

The radiation density produced through gravitational reheating is

$$\left(\rho_{\gamma}^{\text{gr}}\right)_{\text{end}} = \frac{q \pi^2}{30} g^*_{\text{end}} \left( \frac{H_{\text{end}}}{2\pi} \right)^4 \frac{q g^*_{\text{end}}}{480\pi^2} H_{\text{end}}^4,$$  \hfill (4.3)

where $g^*_{\text{end}} = O(100)$ is the number of effective relativistic degrees of freedom at the energy scale of inflation and $q \sim 1$ is some efficiency factor. The above does not imply that radiation produced through gravitational reheating is thermal. Indeed, even though it has been found that $\left(\rho_{\gamma}^{\text{gr}}\right)_{\text{end}} \sim 10^{-2} H_{\text{end}}^4$ [55, 56], as suggested above, thermalisation of the produced radiation may occur much later [56]. This, however, does not make any difference in our considerations, because radiation always scales as $\rho_\gamma \propto a^{-4}$ regardless of whether it is thermalised or not.

In view of eq. (4.3), the density parameter of radiation for gravitational reheating at the end of inflation is

$$\left(\Omega_{\gamma}^{\text{end}}\right)^{\text{gr}} \equiv \frac{\rho_{\gamma}^{\text{gr}}}{\rho_{\text{end}}} = \frac{q g^*_{\text{end}}}{1440\pi^2} \left( \frac{H_{\text{end}}}{m_P} \right)^2.$$  \hfill (4.4)

The above is the lowest possible value of the radiation density parameter at the end of inflation $\Omega_{\gamma}^{\text{end}} \equiv (\rho_\gamma/\rho)_{\text{end}}$, which can, in principle approach unity, in the case of instant preheating. Thus, in general we have

$$\left(\Omega_{\gamma}^{\text{end}}\right)^{\text{gr}} \lesssim \Omega_{\gamma}^{\text{end}} \lesssim 1.$$  \hfill (4.5)
The Universe is reheated when the radiation takes over and dominates the kinetic density of the scalar field. This is bound to happen, regardless how small \( \Omega_{\text{end}}^{\gamma} \) is, because \( \rho_{\text{kin}} \propto a^{-6} \), while for radiation we have \( \rho_{\gamma} \propto a^{-4} \). Using that \( a \propto t^{1/3} \) during kination, it is straightforward to show that the time when the HBB begins (i.e. radiation takes over) is

\[
t_{\text{reh}} = (\Omega_{\text{end}}^{\gamma})^{-3/2}t_{\text{end}}.
\]

Then eq. (4.2) gives

\[
\varphi_{\text{reh}} = \varphi_{\text{end}} - \sqrt{\frac{3}{2}} m_{P} \ln \Omega_{\text{end}}^{\gamma}.
\]

Now, considering that radiation is thermalised by the time it comes to dominate the Universe (this is certainly true for gravitational reheating, where \( \Omega_{\gamma} \) is minimal), the reheating temperature is obtained as follows. Since \( \Omega_{\gamma} = \rho_{\gamma}/\rho_{\text{kin}} \propto a^2 \) during kination, it is easy to find that \( \rho_{\text{kin}}^{\text{reh}} = (\Omega_{\gamma}^{\text{end}})^3 \rho_{\phi}^{\text{end}} \), where \( \rho_{\text{kin}}^{\text{reh}} \equiv \rho_{\text{kin}}(t_{\text{reh}}) \). Using that \( \rho_{\gamma}^{\text{reh}} \equiv \rho_{\phi}^{\text{reh}} \), the reheating temperature is

\[
T_{\text{reh}} = \left( \frac{30}{\pi^2} \frac{g_{*}^{\text{reh}}}{g_{*}^{\text{end}}} (\Omega_{\gamma}^{\text{end}})^3 \rho_{\phi}^{\text{end}} \right)^{1/4}.
\]

Combining the above with eqs. (4.4) and (4.5), we find

\[
T_{\text{reh}} \geq \frac{q^{3/4}}{24\pi^2} \left( \frac{g_{*}^{\text{end}}}{g_{*}^{\text{reh}}} \right)^{1/4} \sqrt{\frac{g_{*}^{\text{end}}}{10}} \frac{H_{\text{end}}^2}{m_{P}},
\]

where the equality corresponds to gravitational reheating. For inflation near the grand unified energy scale (cf. table 1) we have \( H_{\text{end}} \sim 10^{12} \text{GeV} \). Taking \( g_{*}^{\text{end}} = \mathcal{O}(100) \) and \( g_{*}^{\text{reh}} = 10.75 \) (assuming late reheating), we find \( T_{\text{reh}} \gtrsim 10^4 \text{GeV} \), which is safely much higher than the temperature at BBN.\(^5\)

### 4.3 Freezing of the scalar field

After kination ends and radiation domination takes over, the field continues to roll for a time until it runs out of kinetic energy and freezes. Indeed, after the onset of the HBB, the field is still kinetically dominated so that the equation of motion is still \( \ddot{\varphi} + 3H \dot{\varphi} = 0 \). However, in radiation domination, this results in

\[
\dot{\varphi} = \sqrt{\frac{2}{3} m_{P} \sqrt{t_{\text{reh}}}}.
\]

Integrating the above we find

\[
\varphi = \varphi_{\text{reh}} + \sqrt{\frac{2}{3} m_{P}} \left( 1 - \sqrt{\frac{t_{\text{reh}}}{t}} \right).
\]

Thus, the field freezes for \( t \gg t_{\text{reh}} \) at the value

\[
\varphi_{F} = \varphi_{\text{end}} + \sqrt{\frac{2}{3}} \left( 1 - \frac{3}{2} \ln \Omega_{\text{end}}^{\gamma} \right) m_{P},
\]

where we considered also eq. (4.7). It should be stressed here that this result is model independent because, while \( \varphi \) is kinetically dominated, it is oblivious to the potential.\(^6\) The evolution of \( \rho_{\varphi} \) is shown in figure 3.

---

\(^5\)Quintessential inflation typically features low values of \( T_{\text{reh}} \). For example, \( T_{\text{reh}} \sim 10^4 \text{GeV} \) is an upper bound in the particular quintessential inflation model in ref. [97].

\(^6\)Of course, just before freezing we have \( \rho_{\text{kin}} \lesssim V \). But the subsequent variation of \( \varphi \) is exponentially suppressed, so \( \varphi \simeq \varphi_{F} = \text{constant} \).
Figure 3. Log-log plot depicting the evolution of the density of the scalar field $\rho_{\phi}$ (solid line) and the radiation density of the HBB $\rho_{\gamma}$, which eventually gives away to matter $\rho_m$ (both depicted by the dashed line). In late times the Universe is dominated by dark energy (DE in the graph).

From eq. (4.12), to maximise the value of $\varphi_F$, in order to achieve a low residual potential density, we see that we have to consider the minimum possible value of $\Omega_{\gamma}$. As suggested by eq. (4.5), this corresponds to gravitational reheating.

Therefore, in this paper we consider gravitational reheating. As explained, gravitational reheating is selected to ensure the field retains enough kinetic density to roll to a low potential density, to align with observations of dark energy today. Additionally, this negates the need to introduce additional scalar fields that play a crucial role in reheating, as in the cases of instant preheating and curvaton reheating. This promotes economy in the model because it ensures our model stays as minimal as possible.

5 Quintessence

In the limit $\varphi \to +\infty$ ($\phi \to \sqrt{6\alpha}$), the potential in eq. (2.4) becomes

$$V \simeq 2ne^{-2nM^4}e^{-\frac{2\varphi}{\sqrt{6\alpha}m_P}}. \quad (5.1)$$

Thus, the potential features a classic exponential quintessential tail, of the form

$$V = V_Q \exp \left(-\frac{\lambda \varphi}{m_P}\right), \quad (5.2)$$

where $V_Q = 2ne^{-2nM^4}$ and $\lambda = 2/\sqrt{6\alpha} = (2/n)\kappa$. The exponential quintessential tail features two attractor solutions, depending on whether the scalar field is dominant or not.
to the background density. Originally, the field is frozen at $\varphi_F$, in eq. (4.12), with potential density $V_F \equiv V(\varphi_F)$. However, when $V_F$ approaches the potential density of the attractor, the field unfreezes and eventually follows the attractor solution. As shown in numerical simulations [98], the system briefly oscillates around the attractor before following it. Below we consider both attractors and the corresponding parameter space. It is easy to check that the attractor solutions in eqs. (5.4) and (5.5) are solutions to the Klein-Gordon equation for the canonical field

$$\ddot{\varphi} + 3H \dot{\varphi} + V' = 0.$$  

(5.3)

### 5.1 Dominant quintessence

If the scalar field is dominant then the attractor is

$$V = \frac{2(6 - \lambda^2)}{\lambda^4} \left( \frac{m_P}{t} \right)^2 \quad \& \quad \rho_{\text{kin}} \equiv \frac{1}{2} \dot{\varphi}^2 = \frac{2}{\lambda^2} \left( \frac{m_P}{t} \right)^2 \quad \Rightarrow \quad \rho_\varphi = \frac{12}{\lambda^4} \left( \frac{m_P}{t} \right)^2,$$

(5.4)

where $\rho_\varphi = \frac{1}{2} \dot{\varphi}^2 + V$. Then, $a \propto t^{2/\lambda^2} = t^{1/\epsilon} \Rightarrow H \propto a^{-\epsilon}$, where $\epsilon \equiv -\dot{H}/H^2 = \lambda^2/2$. Since, $a \propto t^{2/3(1+w)}$ we find that the barotropic parameter of the Universe is $w = -1 + \lambda^2/3$. Thus, in order to have accelerated expansion, we require $w < -\frac{1}{3}$, i.e. $\lambda < \sqrt{2}$. This accelerated expansion is eternal. The Planck constraint on the barotropic parameter of dark energy is $w = -1.006 \pm 0.045$ [4, 5]. This means that $\lambda \leq 0.342$, which results in the bound $\sqrt{6\alpha} = 2/\lambda \geq 5.847$. Such a large $\alpha$ would render $\phi$ super-Planckian, which should be avoided.

Alternatively, for $\sqrt{2} \leq \lambda < \sqrt{3}$ we still have a negative pressure but it is not negative enough to lead to eternal accelerated expansion. It can lead, however, to transient accelerated expansion [99]. Indeed, as mentioned above, after the field unfreezes, the system briefly oscillates around the attractor. As such, the effective barotropic parameter can temporarily decrease below $-\frac{1}{3}$ so expansion becomes accelerated.

### 5.2 Subdominant quintessence

If the scalar field is subdominant then the attractor is

$$V = \frac{2}{\lambda^2} \left( 1 - w \right) \left( \frac{m_P}{t} \right)^2 \quad \& \quad \rho_{\text{kin}} \equiv \frac{1}{2} \dot{\varphi}^2 = \frac{2}{\lambda^2} \left( \frac{m_P}{t} \right)^2 \quad \Rightarrow \quad \rho_\varphi = \frac{4}{\lambda^4(1+w)} \left( \frac{m_P}{t} \right)^2,$$

(5.5)

where $w$ now is the barotropic parameter of the background matter ($w = 0$ in the case of matter). We know

$$\rho = \frac{4}{3(1+w)^2} \left( \frac{m_P}{t} \right)^2.$$  

(5.6)

So we find

$$\Omega_\varphi \equiv \frac{\rho_\varphi}{\rho} = \frac{3(1+w)}{\lambda^2}.$$  

(5.7)

For subdominant quintessence, $\Omega_\varphi < 1$, which implies $\lambda > \sqrt{3(1+w)}$.

The attractor in the subdominant quintessence case does not lead to accelerated expansion, because the evolution of the density of the scalar field mimics the background density. However, similarly to the previous case, because, after unfreezing, the system briefly oscillates around the attractor, this may result into a bout of transient accelerated expansion, as the scalar field density briefly dominates before settling to its path, just below the background.
density. For this, the quintessence density should not be much smaller than the background density. Numerical studies have shown that this occurs for $\lambda^2 < 24$ or so [25, 100–102].

Thus, for transient accelerated expansion we need

$$\sqrt{2} \leq \lambda \lesssim 2\sqrt{6},$$

while $\lambda < \sqrt{2}$ leads to eternal accelerated expansion. Because $\lambda = 2/\sqrt{6\alpha}$, the corresponding bounds on $\alpha$ for transient accelerated expansion are

$$0.03 \lesssim \alpha \leq 0.33,$$

while for $\alpha > \frac{1}{3}$ we end up with eternal accelerated expansion. However, as explained earlier, to avoid super-Planckian values for the non-canonically normalised $\phi$, we consider $\alpha \lesssim \frac{1}{6}$. Thus, we see that, with $\alpha \sim 0.1$ we attain transient accelerated expansion with mildly sub-Planckian values of $\phi$.

Now, transient accelerated expansion has a clear merit over eternal accelerated expansion, as in $\Lambda$CDM. This is the known future horizon problem of string theory. Eternal accelerated expansion results in a future event horizon. As a result, future asymptotic states are not well defined because space is not causally connected. Thus, the S-matrix in string theory, which defines transition amplitudes, cannot be formulated [103–106]. This may be just a problem of string theory and not a no-go theorem of nature. But still, it is an incentive to avoid eternal acceleration if possible.

### 5.3 The parameter space

We now find the parameter space for $n$ and $\kappa$. To do this, we enforce the requirement that the density of quintessence $V_F$ must be comparable with the density of the Universe at present $\rho_0$ (coincidence requirement). We have

$$\frac{\rho_{\inf}}{\rho_0} \simeq \frac{V_{\inf}}{V_F} \simeq \frac{e^{\lambda\varphi_F/m_P}}{2n e^{-2n}} \sim 10^{108},$$

where we used $V_{\inf} = (1 - e^{-2n}) M^4 \simeq M^4 \sim (10^{15} \text{GeV})^4$ (we will find $n \gg 1$), $\rho_0 \sim (10^{-3} \text{eV})^4$, $V_F = V_Q \exp(-\lambda\varphi_F/m_P)$ and $V_Q = 2n e^{-2n} M^4$. The above leads to

$$2n - \ln(2n) \simeq 108 \ln 10 - \frac{2}{\sqrt{6\alpha} m_P} \varphi_F,$$

where we used $\lambda = 2/\sqrt{6\alpha}$. Ignoring for the moment $\varphi_{\text{end}}$, from eq. (4.12), we have

$$\frac{\varphi_F}{m_P} \simeq \sqrt{\frac{2}{3} \left(1 - \frac{1}{2} \ln \Omega_{\gamma}^{\text{end}}\right)},$$

where, with gravitational reheating, we have $\Omega_{\gamma}^{\text{end}} = (\Omega_{\gamma}^{\text{end}})_{gr}$ given by eq. (4.4), which suggests $(\Omega_{\gamma}^{\text{end}})_{gr} \sim 10^{-3} (M/m_P)^4 \sim 10^{-15}$, where we used that $M \sim \sqrt{H_{\inf} m_P} \sim 10^{15} \text{GeV}$ (cf. table 1). Inserting this number into the above we find $\varphi_F \simeq 43 m_P$. We put this into eq. (5.11) and find the allowed values of $n$. For the range in eq. (5.9), which corresponds to transient accelerated expansion with mildly sub-Planckian $\phi$, we obtain

$$25 \lesssim n \lesssim 92.$$
The corresponding values of $\varphi_{\text{end}}$ can be obtained from eq. (3.4). We find $-3.20 \leq \varphi_{\text{end}}/m_P \lesssim -0.63$ for the range of $\alpha$ considered. This substantiates our assumption to ignore $\varphi_{\text{end}}$ as $|\varphi_{\text{end}}| \ll \varphi_F$. In view of the above range, we also obtain $V_0^{1/4} = e^{-n/4} M = 10^{5-12} \text{GeV}$, which is a quite large range of reasonable intermediate energy scales.

Using the values in eq. (5.13), since $\kappa \equiv n/\sqrt{6} \alpha$, we readily obtain
\begin{align*}
59 \lesssim \kappa \leq 65.
\end{align*}

Thus, because $\kappa \approx 60$, the non-canonical $\phi$ in the exponent of the scalar potential in eq. (2.1) is suppressed by the mass-scale $m_P/\kappa \approx 4 \times 10^{16} \text{GeV} \sim M$. We also find that the scale of the cosmological constant is $\Lambda^{1/4} = e^{-n/2} M \sim 10^{-5} - 10^{10} \text{GeV}$. In particular, for $\sqrt{6} \alpha \lesssim 1$ ($\phi \lesssim m_P$) we have $n = \kappa \sqrt{6} \alpha \lesssim 60$, so that $\Lambda^{1/4} \gtrsim 10^2 \text{GeV}$, which is comparable with the electroweak energy scale.

As both the tensor-to-scalar ratio and the spectral index are independent of $n$, we simply use the constraints on $\alpha$ in eq. (5.9) to update our results. For the spectral index we find a firm prediction
\begin{align*}
n_s = 0.9686,
\end{align*}
which is very close to the value $n_s = 0.9685$ that we obtained for negligible $\alpha$ in section 3.1. For the running of the spectral index, we find the value $n'_s = -5.09 \times 10^{-4}$ which is virtually indistinguishable from the result obtaining in section 3.1 with negligible $\alpha$.

For $N_s = 63.49$, eq. (3.9) gives $r \simeq \frac{\alpha}{3 \kappa^2}$. In view of eq. (5.9) we find the following range of values for the tensor-to-scalar ratio:
\begin{align*}
8.9 \times 10^{-5} \lesssim r \leq 9.7 \times 10^{-4}.
\end{align*}

which can be potentially observable in the near future. These results are plotted in figure 4.

Since refs. [100–102] are somewhat dated, we intend to revise the bound in eq. (5.9) in a future publication, in view of the latest CMB constraints on the equation of state of dark energy. We expect that the parameter space will be somewhat reduced. The predictions of our model, however, are rather robust. For example, this is evident from figure 4 regarding the value of the spectral index. The same is true for the model parameters, since $\kappa \approx 60$ and $V_0$ and $\Lambda$ assume reasonable intermediate density scales in the parameter space in eq. (5.9).

6 Additional considerations

6.1 The fifth force problem and radiative corrections

Our scalar field may be coupled to the standard model fields. As is typical for a quintessence field, the mass of our field at present is of the order of the Hubble constant $H_0 = 1.43 \times 10^{-33} \text{eV}$. This means that its Compton wavelength is of the order of the size of the horizon and the quintessence field can result in a long-range interaction (fifth force), which is in danger of violating the equivalence principle. To quantify this, any interaction terms in the Lagrangian density must be taken into account. Such terms are of the form [107]
\begin{align*}
\beta_i \frac{\phi}{m_P} \mathcal{L}_i
\end{align*}
Figure 4. The tensor-to-scalar ratio, $r$, versus the spectral index, $n_s$ for $\alpha = 0.03(0.33)$ left to right. All of these results are well within the Planck 1-\(\sigma\) range. Values of $r \sim 10^{-3}$ are potentially observable in the near future.

where $i$ runs over the various different interactions, so that the dimensionless couplings $\beta_i$ may be different for each gauge-invariant dimension-four operator $L_i$ of the Lagrangian. For example, for the electromagnetic field, the Lagrangian density can be of the form

$$L_{\text{em}} = -\frac{1}{4} e^{-4\beta_{\text{em}}(\phi/m_P)} F_{\mu\nu} F^{\mu\nu} \simeq -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \beta_{\text{em}} \frac{\phi}{m_P} F_{\mu\nu} F^{\mu\nu},$$ (6.2)

where $F_{\mu\nu}$ is the field strength tensor [108]. The above interaction can be responsible for variation of the fine-structure constant [109–111].

Therefore, if the field is sub-Planckian, these interactions are suppressed. As we have seen, kination sends the canonical scalar field $\phi$ to strongly super-Planckian values, which means that substantial fine-tuning of the $\beta_i$s is needed to avoid the so-called fifth force problem of traditional quintessence [12]. This is because, the above gravitationally suppressed interactions can result in potentially observable violations of the equivalence principle.

In our model, however, it is the non-canonical field $\phi$ which is expected to feature in the interaction terms of the form in eq. (6.1). Because, in our model, $\phi$ avoids being super-Planckian as long as $\sqrt{6\alpha} \leq 1$, the fifth force problem can be avoided with only a mild tuning of the $\beta_i$ coefficients. At late times, in our model, we find the couplings to be $\beta_i \sqrt{6\alpha} \mathcal{L}_i \sim \beta_i \mathcal{L}_i$, where we considered $\alpha \sim 0.1$, as suggested by eq. (5.9). Therefore, for mildly suppressed values of $\beta_i$ transient accelerated expansion is possible without sizeable violations of the equivalence principle.

A related issue is the lifting of the flatness of the quintessential tail by radiative corrections. Indeed, as we have seen, kination propels the field to a distance more that $40 m_P$
in field space. A perturbative potential is not valid over such super-Planckian field displacements, as one expects non-renormalisable corrections of the form $\sim \varphi^{2i+1}/m_P^i$ (with $i \geq 1$) to become important. Of course, ours is a non-perturbative potential (possibly originating from gaugino condensation) but still we expect that the flatness of the quintessential tail would be threatened by non-renormalisable terms. However, in the context of $\alpha$-attractors, it is the non-canonical field $\phi$ (and not $\varphi$) we should be concerned about, because this is the fundamental degree of freedom that appears in our original Lagrangian in eq. (2.1), while the canonically normalised $\varphi$, introduced in eq. (2.3) is merely a mathematical tool to help study the model. As we saw, the variation of $\phi$ is kept mildly sub-Planckian ($\alpha \sim 0.1$), which means that radiative corrections are kept under control.

6.2 Overproduction of gravitinos

Our model is rooted in supergravity, since scalar fields with non-canonical kinetic terms naturally arise in this framework. Therefore, one limitation which should be taken into account is the overproduction of gravitinos.

The gravitino is the super-partner of the graviton and it is expected to have a mass of the order of TeV. This presents two options: either the gravitino is stable (e.g. being the lightest supersymmetric particle) and its mass contributes to dark matter, in which case overproduction of gravitinos overcloses the Universe. Or, the gravitino is unstable and decays to other particles. Gravitinos can only decay via gravitationally suppressed interactions, which progress very slowly, giving them a long life-time and meaning they exist past the time of BBN. However, the channels of decay a gravitino can use, necessarily produce particles energetic enough to destroy the nuclei created during BBN. Hence, we need to avoid gravitino overproduction entirely.

The gravitinos can be produced by either a thermal or non-thermal mechanism. Because the inflaton does not oscillate around its VEV in our model, we are only concerned with the thermal production - their creation via scatterings in the thermal bath produced via gravitational reheating. The relative abundance of produced gravitinos depends strongly on the reheating temperature $T_{\text{reh}}$, i.e. the relevant temperature in the transition to radiation domination. To avoid overproduction, $T_{\text{reh}}$ is constrained to be below $10^8 - 10^9$ GeV [112–114] (but sometimes it can be much lower than that; of order $10^6$ GeV or so [115].)

From eq. (4.9), for gravitational reheating we have

$$T_{\text{reh}} = \frac{g^3}{36\pi^2} \left( \frac{g_{\text{end}}}{g_{\text{reh}}^*} \right)^{1/4} \sqrt{\frac{g_{\text{end}}^*}{10}} \frac{V_{\text{end}}}{m_P^3},$$

(6.3)

where $V_{\text{end}} \equiv V(\varphi_{\text{end}})$ and we have taken $V_{\text{end}} \simeq \frac{3}{2} H_{\text{end}}^2 m_P^2$. Using eqs.(3.1) and (3.4) we find that $V_{\text{end}} = e^{-\sqrt{3} H_{\text{end}}^2} M_4^4$. Then, taking $g_{\text{end}}^* = O(100)$ and $g_{\text{reh}}^* = 10.75$, and considering also eq. (5.9) and table 1 we find $T_{\text{reh}} \sim 10^4$ GeV (cf. also section 4.2). Thus, $T_{\text{reh}}$ is well below the upper-bound to avoid gravitino overproduction. Moreover, because $T_{\text{reh}} \gg T_{\text{BBN}}$, where $T_{\text{BBN}} \simeq 0.5$ MeV is the temperature at BBN, we find that latter is safely not affected.

6.3 Overproduction of gravitational waves

The non-decaying mode for a gravitational wave with superhorizon wavelength has a constant amplitude until it re-crosses the horizon and then it undergoes damped oscillations, decreasing as $1/a$. This is a general result valid for any equation of state, however because the damping is affected by the scale factor the energy density spectrum of the gravitational waves scales
differently in different epochs of Universe expansion. During the radiation era, the barotropic parameter of the Universe is \( w = \frac{1}{3} \) and the gravitational wave spectrum is flat. However, during kination we have \( w = 1 \) which produces a spike in the spectrum of gravitational waves at high frequencies. In order to ensure the generated gravitational waves do not destabilise BBN, an upper bound is imposed on their density fraction [118]:

\[
I \equiv h^2 \int_{k_{BBN}}^{k_{end}} \Omega_{GW}(k) \, dk \leq 1 \times 10^{-5},
\]

(6.4)

where \( h = 0.678 \). We focus on the modes which re-enter the horizon during the kination, i.e. the spike in the spectrum, as this is the dominant contribution to \( \Omega_{GW} \). In this regime, we have [116, 117]

\[
\Omega_{GW}(k) = \varepsilon \Omega_\gamma(k_0) h_{GW}^2 \left( \frac{k}{k_{reh}} \right)^2 \left[ \ln \left( \frac{k_{end}}{k_{reh}} \right) \right]^2 \quad \text{for} \quad k_{reh} < k \leq k_{end},
\]

(6.5)

where \( k \) is the mode’s physical momentum and \( \Omega_\gamma(k_0) \) is the present density fraction of radiation, on horizon scales. We also have \( h_{GW}^2 = \frac{1}{32\pi} \left( \frac{H_{end}}{m_p} \right)^2 \) and \( \varepsilon = 2 R_i \left( \frac{g_{end}}{g_h} \right)^{1/3} \), where \( R_i = \frac{81}{32\pi} \) is the contribution of each massless scalar degree of freedom to the energy density of the amplified fluctuations [116, 117]. Using eq. (6.5) in (6.4) we calculate

\[
I = 2 \varepsilon h^2 \Omega_\gamma(k_0) h_{GW}^2 \left( \frac{k_{end}}{k_{reh}} \right) \left\{ 2 \left( \frac{k_{end}}{k_{reh}} \right) - \left[ \ln \left( \frac{k_{end}}{k_{reh}} \right) + 1 \right]^2 \right\}.
\]

(6.6)

Because \( k_{end} \gg k_{reh} \) the first term in the brackets dominates and we simply obtain

\[
I \approx 2 \varepsilon h^2 \Omega_\gamma(k_0) h_{GW}^2 \left( \frac{k_{end}}{k_{reh}} \right).
\]

(6.7)

Since \( k = aH \) and during kination \( a \propto \rho^{-1/6} \) we have:

\[
I = 2 \varepsilon h^2 \Omega_\gamma(k_0) h_{GW}^2 \left( \frac{\rho_{reh}}{\rho_{end}} \right)^{1/6} \left( \frac{H_{end}}{H_{reh}} \right),
\]

(6.8)

Employing that \( \frac{1}{2} \rho_{reh} \approx \rho_{\gamma}^{reh} = \frac{\pi^2}{30} g_s^{reh} T_{reh}^4 \) and \( \rho_{end} \approx 2 V_{end} \) we obtain

\[
I = \frac{2 \varepsilon h^2 \Omega_\gamma(k_0)}{\pi^{2/3}} \left( \frac{30}{g_s^{reh}} \right)^{1/3} h_{GW}^2 V_{end}^{1/3} T_{reh}^{4/3}.
\]

(6.9)

Inserting eq. (6.3) we find

\[
I = \frac{360 \pi \varepsilon h^2 \Omega_\gamma(k_0)}{q g_s^{end}} \left( \frac{21/3}{2} \right),
\]

(6.10)

where we have substituted in the value of \( h_{GW}^2 \). Using \( \Omega_\gamma(k_0) = 2.6 \times 10^{-5} h^{-2} \) and introducing the values for the remaining constants \( \varepsilon \) and \( g_s^{end} = 106.75 C \), the bound of \( I \leq 1 \times 10^{-5} \) in eq. (6.4) gives

\[
q \geq 3.54/C,
\]

(6.11)

where \( C = 1 \) for the standard model but \( C \gtrsim 2 \) for supersymmetric theories. Hence, a reheating efficiency \( q = O(1) \) satisfies the bound in eq. (6.4), which means that BBN remains undisturbed from gravitational reheating, as we have assumed.
7 Discussion and conclusions

We have presented and analysed a new model of quintessential inflation within the context of \(\alpha\)-attractors, motivated by supergravity theories. We have assumed a simple exponential potential for the non-canonical inflaton field, which may originate from gaugino condensation. We also considered a vanishing vacuum density, due to some unknown symmetry, as was standard practice before the observation of dark energy. However, this does not imply a zero cosmological constant, because the value of our scalar field cannot go to infinity (where its exponential potential density would tend to zero) due to a bound imposed by a pole in the non-canonical kinetic term. The energy scale found for the cosmological constant (\(\gtrsim 10^2\) GeV) is comparable to the electroweak energy scale, in stark contrast with the required value in \(\Lambda\)CDM (\(\sim 10^{-3}\) eV).

Our model performs very well both as an inflation model and as quintessence. The inflationary observables are near the sweet spot of the latest CMB observations, with the spectral index found as \(n_s \approx 0.9686\) or so and the tensor-to-scalar ratio as \(r \sim 10^{-4} - 10^{-3}\) that is potentially observable in the near future. These values lie deep inside the 1-\(\sigma\) contour of the Planck and BICEP2 results. This is not surprising for a plateau inflationary model. For quintessence, we have shown that the model leads to transient accelerated expansion at present for rather natural values of the model parameters. Indeed, we found that \(\alpha \sim 0.1\) and \(\kappa \sim m_P/M\) in eq. (2.1), with \(M\) being the inflation energy scale, which is close to the energy of grand unification. We also found a large but reasonable range of intermediate density scales for \(V_0\) (ranging as \(V_0^{1/4} \sim 10^5-12\) GeV).

Transient accelerated expansion improves over the usual eternal accelerated expansion of \(\Lambda\)CDM in that it does not lead to a future horizon problem, which otherwise undermines the formulation of the S-matrix in string theories. It also results in an ultimate future for our Universe different from the \(\Lambda\)CDM scenario. In \(\Lambda\)CDM, eternal accelerated expansion results to all unbound or loosely bound systems (like galactic clusters) eventually dissolving and pulled beyond the constant horizon, while our galaxy, merged with all other objects in the local group, remains the only object surviving within our observable Universe, which will be filled with Hawking radiation at temperature \(T \sim H_0 \sim 10^{-33}\) eV. In contrast, with transient accelerated expansion, the horizon will continue to expand with the speed of light, while ever more objects will enter the observable Universe. All mass concentrations most probably will eventually become black holes that will evaporate in diminishing radiation with temperature that will asymptote to zero.

Ours is a non-oscillatory inflation model, so that the inflaton field does not decay at the end of inflation but survives until today to become quintessence. Therefore, we need to reheat the Universe by means other than the decay of the inflaton field. In an effort to keep our model minimal, we have not utilised the help of other degrees of freedom, which would play a crucial role in reheating the Universe. Instead, we considered gravitational reheating, which is based on particle production of all light (and not-conformally invariant) fields during inflation. Such production always occurs but it is negligible in the usual oscillatory models of inflation. Hence, gravitational reheating is a neat mechanism to reheat the Universe, since it is unavoidable. We find the reheating temperature \(T_{\text{reh}} \sim 10^4\) GeV, which means that reheating occurs safely before big bang nucleosynthesis. Also, our \(T_{\text{reh}}\) satisfies even the most stringent constraints due to gravitino overproduction.

Between inflation and reheating, there is a period of kination, when the density of the Universe is dominated by the kinetic density of our scalar field. We have studied kination
in a model independent way. We showed that, soon after kination ends, the scalar field freezes at a constant value, where it remains dormant until late times, when it can become quintessence. With gravitational reheating, we found that the displacement, during kination, of the canonical field is strongly super-Planckian $\sim 45 m_P$. This would have meant that the flatness of our quintessential plateau could be lifted by radiative corrections, while the long-range fifth force mediated by quintessence could lead to violations of the equivalence principle.

In the context of $\alpha$-attractors, though, since the non-canonical field may avoid being super-Planckian, the above dangers can be avoided with only a mild tuning of the gravitationally suppressed couplings between quintessence and the standard model fields. The period of kination results in a spike in the spectrum of gravitational waves generated during inflation. We have calculated this spectrum and found that the energy of the gravitational waves is not threatening big bang nucleosynthesis.

In summary, we have studied a new quintessential inflation model, in the context of $\alpha$-attractors. We have found excellent agreement of the inflationary predictions with CMB observations. We have shown that successful quintessence is achieved with natural values of the parameters, including a cosmological constant near the electroweak energy scale. Our model gives rise to transient accelerated expansion, dispensing with the future horizon problem of $\Lambda$CDM. Our setup is purposefully minimal, without many degrees of freedom, mass scales and couplings. We considered gravitational reheating, which does not overproduce gravitinos nor does it affect nucleosynthesis. Finally, a period of kination in our model produces a spike in the spectrum of gravitational waves generated during inflation, which does not disturb big bang nucleosynthesis.

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