Supersymmetric Left Right Model, Automatic R-parity Conservation and Constraints on the $W_R$ Mass

R. N. Mohapatra
Department of Physics, University of Maryland, College Park, Md-20742, USA
E-mail: Rmohapat@physics.umd.edu

Supersymmetric left-right models with the see-saw mechanism for the neutrino masses have the attractive property that they conserve baryon and lepton number exactly in the Lagrangian. In this talk, I review the recent results valid for a large class of minimal versions of the model that supersymmetry combined with the requirement that the ground state of the model conserve electric charge constrains the mass of the right handed $W_R$ boson to be in a certain range i.e. $M_{W_R} \leq 10$ TeV or $\geq 10^{10}$ GeV. In the former case (low $M_{W_R}$), the vacuum breaks R-parity spontaneously and the latter case (high $M_{W_R}$) is required if vacuum is to conserve R-parity. In the second case, the effective low energy theory is the MSSM with exact R-parity, nonvanishing neutrino masses and a pair of light doubly charged Higgs fields and their fermionic partners. Exact R-parity conservation via see saw mechanism therefore implies that the neutrino masses must be in the desired range to solve the solar and atmospheric neutrino puzzles.

1 Introduction

Supersymmetry is now widely believed to be the next step beyond the successful standard model. Two primary reasons for this belief are: (i) milder divergence structure of supersymmetry provides a way to maintain perturbative stability of the weak scale (or the Higgs mass) and (ii) it also provides a mechanism to dynamically generate the spontaneous breaking of the gauge symmetry through the use of the renormalization group equations. Thus two of the major unsolved puzzles of the standard model receive a rather satisfactory resolution. The minimal supersymmetric model (MSSM) that leads to the standard model at low energies provides the simplest realization of this idea and has been the subject of extensive investigation. One of the key predictions of the MSSM is the existence of a light neutral Higgs boson with mass less than 130 GeV and can be used to test this model. And also another attractive feature of the MSSM is that the lightest superpartner (LSP) of the standard model fields has all the right property to be the cold dark matter of the universe, if it is stable.

MSSM however comes with its own unpleasant baggage and must necessarily be part of a larger more symmetric model. To get a glimpse of what this larger model looks like, let us recall the problems that beset the MSSM. They are the following:
(i) The MSSM symmetries allow the existence of baryon and lepton number violating terms with arbitrary strength, a feature which not only allows the the LSP to decay in fraction of a few years but more seriously, it also allows the proton to decay in a fraction of a second. This is the so called R-parity problem. For a recent review of some the consequences of R-parity breaking, see Ref. In view of the fact that the standard model led to the conservation of baryon and lepton number automatically (i.e. by virtue of the choice of the gauge symmetry and the field representations), MSSM takes us a step backward.

(ii) A second problem with the MSSM lies in its predictions for the CP violating effects being too large. There are two extra phases in MSSM in its most symmetric version residing in the soft breaking parameter $A$ and the Higgs mixing mass $\mu$. These phases manifest in the electric dipole moment of the neutron, already at the one loop level leading generically to:

$$d_n^e \simeq \frac{e}{16\pi^2 M_\tilde{q}} \text{Arg}(m_\tilde{g}[A - \mu \tan \beta])$$  \hspace{1cm} (1)$$

A simple evaluation of the above down quark electric dipole moment implies that unless either (i) the squark masses are of order $3$ TeV or (ii) $\text{Arg}(m_\tilde{g} A)$ and $\text{Arg}(m_\tilde{g} \mu)$ are less than $10^{-3}$ if squark masses $M_\tilde{g} \simeq 100$ GeV, the edm of the neutron will come out to be three orders of magnitude higher than the present experimental upper bound. In either case, we have a fine tuning problem for the theory, the very problem supersymmetry was supposed to solve. In the first case one has to fine tune to get the Higgs mass of order $m_W$ and in the second case, the new phases of the model (unlike the CP phase of the standard model) has to be tuned down by three orders of magnitude from its natural value.

(iii) The MSSM with global R-parity conservation leads to zero mass for the neutrinos. In view of the recent growing experimental evidences for neutrino masses, it is more appropriate to consider extensions of the MSSM that can lead to neutrino masses.

The simplest extension of the MSSM that solves all three of the above problems is the supersymmetric left-right model with the field content chosen to yield naturally small neutrino masses via the seesaw mechanism. The detailed solution to the R-parity and SUSYCP problems in the left-right symmetric models has been discussed in Ref. We will briefly go over these arguments in section 2 of this article, where we also present the field content and the superpotential.

The main focus of this article will be on the constraints on the $W_R$ scale implied by electric charge conservation and R-parity conservation by the ground
The model, which is based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$. In Table I, we give the particle content of the model needed to implement the seesaw mechanism for neutrino masses. We will suppress the $SU(3)_c$ indices in what follows.

| Fields | SU(2)$_L \times$ SU(2)$_R \times$ U(1)$_{B-L}$ | group transformation |
|--------|---------------------------------|----------------------|
| Q      | (2,1, + $\frac{4}{3}$)          | UQ                  |
| $Q^c$  | (1,2, - $\frac{2}{3}$)          | VQ$^c$              |
| L      | (2,1, -1)                       | UL                  |
| $L^c$  | (1,2, + 1)                      | VL$^c$              |
| $\Phi_{1,2}$ | (2,2,0) | $U\Phi V^\dagger$ |
| $\Delta$ | (3,1, + 2) | $U\Delta U^\dagger$ |
| $\Delta^c$ | (3,1, -2) | $U\Delta^c U^\dagger$ |
| $\Delta^c$ | (1,3, + 2) | $V\Delta^c V^\dagger$ |
| $\Delta^c$ | (1,3, -2) | $V\Delta^c V^\dagger$ |
| S      | (1,1,0)                         | S                   |

Table 1: Field content of the SUSY LR model; we assume that $S$ is odd under parity; $U$ and $V$ denote the $SU(2)_L$ transformations respectively.

The superpotential for this theory is given by (we have suppressed the generation index):

$$W = h^{(i)}_q Q^T \tau_2 \Phi_{1,2} \tau_2 Q^c + h^{(i)}_l L^T \tau_2 \Phi_{1,2} L^c$$
$$+ i(f L^T \tau_2 \Delta L + f_c L^T \tau_2 \Delta^c L^c)$$
$$+ M_{\Delta} \left[ Tr(\Delta \Delta^c) + Tr(\Delta^c \Delta^c) \right] + \lambda S(\Delta \Delta^c - \Delta^c \Delta) + \mu SS^2$$
$$+ \mu_{ij} Tr(\tau_2 \Phi^T_i \tau_2 \Phi_j) + W_{NR}$$

where $W_{NR}$ denotes non-renormalizable terms arising from higher scale physics such as grand unified theories or Planck scale effects.

$$W_{NR} = A [Tr(\Delta^c \Delta^c)]^2 / 2 + B Tr(\Delta^c \Delta^c) Tr(\Delta^c \Delta^c) / 2$$

where $A$ and $B$ are of order $1/M_{Planck}$ and we have omitted terms involving left triplet Higgs fields for the reasons stated below.
Our goal is to seek the ground state of this model that conserves electric charge and violates parity. First we note that the presence of the parity odd singlet $S$ enables one to get a parity violating minimum. The effective theory below this scale (i.e. scale $< S > \neq 0$), can be written only in terms of the $\Delta^c$ and $\bar{\Delta}^c$ terms. Therefore, in what follows we drop the $\Delta$ and $\bar{\Delta}$ fields.

There are now two possibilities for the vacuum state: one which conserves R-parity in the process of breaking the gauge symmetries. The Higgs vevs for this case have the following pattern:

$$< \phi > = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}; \quad < \Delta^c > = \begin{pmatrix} 0 & v_R \\ 0 & 0 \end{pmatrix}$$

Similar pattern for $< \bar{\Delta}^c >$ is assumed.

There is however a second possibility where in addition to the above vevs, one could have $< \bar{\nu}^c > \neq 0$. This ground state breaks R-parity spontaneously. As a result, even though this does not allow the LSP to remain stable, baryon number remains a good symmetry and the most disastrous limits on the R-violating couplings are avoided.

Two different bounds on the $W_R$ masses emerge for the two cases: in case (i) where R-parity is exactly conserved, we find that there is a lower bound on the $W_R$ mass i.e. $M_{W_R} \geq 10^{10}$ GeV; in case (ii) on the other hand, where R-parity is spontaneously broken, there is an upper bound on $M_{W_R}$ of less than a few TeV. Below I briefly outline the main arguments leading to these bounds and refer the reader to the original papers for further details.

3 Exact R-parity conservation and lower bound on $M_{W_R}$

Let us first give a group theoretical argument for the existence of the lower bound. Using Eq. 3, we will first show that in the supersymmetric limit, there exist two massless doubly charged superfields if we ignore the higher dimensional terms $A$ and $B$ as well as the leptonic couplings $f$ in the superpotential. It is easy to see that the superpotential in this case has a complexified $U(3)$ symmetry (i.e. a $U(3)$ symmetry whose parameters are taken to be complex) that operates on the $\Delta^c$ and $\bar{\Delta}^c$ fields. This is due to the holomorphy of the superpotential. After one component of each of the above fields acquires vev as in the charge conserving case with $\theta = 0$ (and supersymmetry guarantees that both vev’s are parallel), the resulting symmetry is the complexified $U(2)$.

This leaves 10 massless fields. Once we bring in the D-terms and switch on the gauge fields, six of these fields become massive as a consequence of the Higgs mechanism of supersymmetric theories. That leaves four massless fields in the absence of higher dimensional terms. These are the two complex doubly
charged fields. The existence of these massless particles signals the presence of a flat direction, which has been shown to exist in this case. The existence and the parameterization of the flat direction can be seen by writing down the potential in the supersymmetric limit:

\[ V(\Delta^c, \overline{\Delta}^c) = M^2 (Tr(\Delta^c \overline{\Delta}^c) + Tr(\overline{\Delta}^c \Delta^c)) + D - \text{terms} \]  

(5)

We omit the detailed form of the D-terms except to note that it is only a function of \((Tr(\Delta^c \tau_a \overline{\Delta}^c) - Tr(\overline{\Delta}^c \tau_a \Delta^c))^2\). In the limit of supersymmetry, one must have the absolute values of the vevs of \(\Delta^c\) and \(\overline{\Delta}^c\) equal. It is then easy to see that the flat direction can be parameterized as follows:

\[ <\Delta^c> = v_R \left( \begin{array}{cc} 0 & \sin \theta \\ \cos \theta & 0 \end{array} \right) \]  

(6)

Clearly \(\theta = \pi/2\) corresponds to the charge conserving vacuum.

Let us now add the nonrenormalizable Planck scale induced terms to the superpotential. Of the two possible terms \(A\) and \(B\) given above, only the \(A\)-term has the complexified \(U(3)\) symmetry. Hence the supersymmetric contribution to the doubly charged particles will come only from the \(B\)-term. It is then easy to see that if the nonrenormalizable terms \(A\) and \(B\) are scaled by the Planck mass, \(M_P\), then their contributions to the doubly charged fields is of order \(v_R^2/M_P\). Since the CERN LEP lower bound on the masses of such particles is 45 GeV, this implies that we must have \(v_R \geq 10^{10}\) GeV. Although the leptonic couplings do not respect this symmetry, they are unimportant in determining the vacuum structure and hence do not effect this result.

Of course one might argue at this point that once one incorporates supersymmetry breaking terms, the doubly charged particles might pick up masses of order 100 GeV anyway regardless of what the value of \(W_R\) is. However, as was shown in great detail in Ref.11, this does not happen and the bound remains as it is. Let us elaborate on this now. The main point is that in the presence of the supersymmetry breaking terms and in the absence of the nonrenormalizable terms the global minimum of the potential turns out to be at \(\theta = 0\) as shown in Ref.11, which means that electric charge is no more respected by vacuum. This manifests itself in detailed calculation as a negative mass-squared term for the doubly charged Higgs boson \(\Delta^{c++}\). The mass-squared term is of order of the 100 GeV to a TeV. In order to have a charge conserving vacuum, we must seek a positive contribution of the same order to the \(\Delta^{c++}\) term. This is provided by the non-renormalizable terms which contribute an amount \(\frac{v_R^4}{M_P^2}\) to \(M_{\Delta^c}^2\). It is then clear that in order to lead to a charge conserving
vacuum, we must have $\frac{v_R^2}{M_{Pl}} \geq 100 $ GeV leading to the lower limit on the right handed scale of order $10^{10} $ GeV. (We have used $M_{Pl} \simeq 1.2 \times 10^{18}$ GeV.)

In the above discussion, we assumed that the hidden sector supersymmetry breaking scale is in the range of $10^{12} $ GeV or so. There are however the scenarios for supersymmetry breaking where the hidden sector SUSY breaking is transmitted via the known gayuge forces the so called GMSB models where the SUSY breaking scale $\Lambda_S$ could be of order 100 TeV. The $SU(2)_R$ scale in these models could therefore be higher than $\Lambda_S$. It turns out that in these models the effective theory below $v_R$ contains massless doubly charged superfields. It was shown in that once the hidden sector SUSY breaking is turned on, only the scalar component of the doubly charged Higgs superfield picks up mass of order 100 GeV. Therefore one must invoke the higher dimensional terms $A$ and $B$ to generate enough mass for the fermionic component. Again requiring LEP Z-decay bound of 45 GeV for this particle leads to a bound of $10^{10}$ GeV for the $v_R$ scale. Thus even though there is no problem with charge violation by the vacuum, essentially the same bound on $v_R$ emerges.

It is clear that the mass of the doubly charged superfields in both the GMSB as well as the high scale gravity mediated models is given by $\frac{v_R^2}{M_{Pl}}$ and is in the accessible range of accelerator experiments if $v_R \simeq 10^{10} - 10^{11}$ GeV. Implications for collider experiments of such a light doubly charged field has been extensively studied in recent papers.

4 Upper limit on $M_{W_R}$ with spontaneous R-parity violation

There is another way to lower the electric charge conserving vacuum below the one that violates it by giving vev to the $\tilde{\nu}_c$ field as was noted in. So does this mean that in the R-parity violating situation, the $M_{W_R}$ is unrestricted? The answer to this question is ”No” since in order to have $<\tilde{\nu}_c>|\neq 0$, we must have the potential for the field $\tilde{\nu}_c$ must have a ”Mexican” hat form. Let us therefore look at the schematic form of the potential for the $\tilde{\nu}_c$ field.

$$V(\tilde{\nu}_c) = M^2_{\tilde{\nu}_c} \tilde{\nu}_c \tilde{\nu}_c^+ + f M_{SUSY} v_R \tilde{\nu}_c^2 + f^2 v_R^2 |\tilde{\nu}_c|^2 + \text{higher powers of } \tilde{\nu}_c + h.c. \tag{7}$$

Note that if we keep the sign of the first term negative and if $fv_R \leq |M_{\tilde{\nu}_c}|$, then one can have $<\tilde{\nu}_c>|\neq 0$. But once $fv_R \geq |M_{\tilde{\nu}_c}|$, the minimum of the potential under consideration is at $<\tilde{\nu}_c> = 0$. This then means that the charge violating minimum becomes the lower minimum. In other words in the case with spontaneous R-parity violation, there must be an upper limit on the scale $v_R \leq M_{SUSY}/f$. For reasonable value of the parameters, this implies an upper limit on $M_{W_R}$ of at most 10 TeV’s.
5 Testing high scale $SU(2)_R$ theories in neutrinoless double beta decay

The high $SU(2)_R$ breaking scale implied by the minimal SUSYLR models with the seesaw mechanism with R-parity conservation decouples essentially all new particles that are not present in the MSSM except the doubly charged bosons and fermions. They remain light with mass around 100 GeV or so. Thus the low energy effective theory consists of the MSSM spectrum with massive Majorana neutrinos and a pair of doubly charged Higgs superfields. If this theory is embedded into an SO(10) GUT model, then unification of gauge coupling constants demands that the $SU(2)_R$ breaking scale be equal to the GUT scale, $M_U$. The doubly charged fields in this case become superheavy and disappear from the low energy spectrum. Looking for the low energy effects of the doubly charged particles will therefore be a way to test between a grand unified left-right model such as $SO(10)$ and a nonunified SUSYLR model all the way to the Planck or string scale.

One interesting experimental effect of the light doubly charged Higgs bosons is in the neutrinoless double beta decay $^{14}$. Let me explain how this effect arises and how it becomes observable despite the high $v_R$ scale. Note that among the nonrenormalizable operators that can be added to the theory is the operator $\Phi \Phi \Delta^c \Delta^c / M_{Pl}$. This leads to a term in the potential of the form $M_{Pl} \phi \Delta^c \Delta^c$. Since we expect $M \approx v_R$, the strength of this interaction is of order $10^{-8}$. It contributes to neutrinoless double beta decay via the diagram in Fig.1. This leads to a double beta decay amplitude roughly of order

$$M_{\beta\beta} \approx \frac{g^2}{4} \left( \frac{m_d}{m_W} \right)^2 \frac{v_R^2}{M_{Pl}} \frac{f}{M_{Pl}^2 M_{++}^2}$$  \hspace{1cm} (8)

The $M_\phi$ denotes the mass of the bidoublet Higgs field and we have assumed that the Yukawa couplings of the bidoublet Higgs is proportional to the quark masses in analogy with the standard model. In principle, this could be bigger. Therefore our estimate is the most conservative one. For $v_R \approx 10^{10}$ GeV, we find that this leads to $M_{\beta\beta} \approx 10^{-18}$ GeV$^{-5}$, which is roughly where the present Heidelberg-Moscow enriched Germanium limits are. The amplitude depends on the $SU(2)_R$ scale like $v_R^2$ since the mass of the doubly charged Higgs field goes like $M_{++} \sim v_R^2$ and therefore an improvement in the lifetime by a factor of 100 will improve the lower limit on $v_R$ by a factor of three.

6 Comments and Conclusion

In conclusion, we have shown that in the minimal supersymmetric left-right model with the seesaw mechanism, the requirement that vacuum state conserve...
electric charge imposes very stringent limits on the scale of the right handed interactions. First of all there is a whole range of values for the $SU(2)_R$ scale $v_R$ (i.e. $10^3 \leq v_R/GeV \leq 10^{10}$) where the vacuum breaks electric charge and is therefore theoretically ruled out. If we further demand that the vacuum of the theory conserve R-parity automatically, then the entire range below $10^{10}$ GeV is ruled out. On the other hand if we allow the vacuum to break R-parity, then the range above 10 TeV is ruled out. It is interesting to note that the higher mass range seems to be preferred by the conventional neutrino mass schemes being discussed in the literature. It is also important to point out that the lower mass range can be substantially covered by the proposed GENIUS double beta decay experiment\[15] as well as the ATLAS detector in the LHC experiment\[16].

I wish to thank Z. Chacko, B. Dutta and G. Senjanović for many discussions on the topics covered in this talk and Pran Nath for hospitality during the PASCOS98 workshop. This work is supported by the National Science
foundation grant no. PHY-9802551.

1. R. Arnowitt, A. Chamseddine and P. Nath, *Applied N=1 Supergravity*, World Scientific (1984); R. Barbieri, S. Ferrara and C. Savoy, Phys. Lett. **119B**, 343 (1982); J. Polonyi, Budapest preprint, KFK-1977-93 (1977); H. P. Nilles, Phys. Rep. **110**, 1 (1984); H. Haber and G. Kane, Phys. Rep. **117**, 76 (1984); for a recent, see S. Martin, hep-ph/9709350.

2. For a recent review of the situation, see G. Bhattacharyya, *Supersymmetry’96: Theoretical Perspectives and Experimental Outlook*, ed. R. N. Mohapatra and A. Rasin, (North Holland, 96); p. 83.

3. R.N. Mohapatra, Phys. Rev. **D34**, 3457 (1986); A. Font, L.E. Ibanez and F. Quevedo, Phys. Lett. **B228**, 79 (1989); S.P. Martin, Phys. Rev. **D46**, 2769 (1992).

4. See for instance R. Garisto, Phys. Rev. **D49**, 4820 (1994).

5. R.N. Mohapatra and A. Rašin, Phys. Rev. Lett. **76**, 3490 (1996); Phys. Rev. **D54**, 5835 (1996); R. Kuchimanchi, Phys. Rev. Lett. **76**, 3486 (1996); R. N. Mohapatra, A. Rašin and G. Senjanović, Phys. Rev. Lett. **79**, 4744 (1997).

6. J.C. Pati and A. Salam, Phys. Rev. **D10**, 275 (1974); R.N. Mohapatra and J.C. Pati, *ibid* **D11**, 566, 2558 (1975); G. Senjanović and R.N. Mohapatra, *ibid* **D12**, 1502 (1975).

7. M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, edited by D. Freedman and P. Van Nieuwenhuizen, North Holland (1979); T. Yanagida, KEK lectures (1979) and R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).

8. Z. Chacko and R. N. Mohapatra, Phys. Rev. **D** (to appear).

9. C. S. Aulakh, A. Melfo and G. Senjanović, Phys. Rev. **D57** (1998).

10. R. Kuchimanchi and R. N. Mohapatra, Phys. Rev. Lett. **75**, 3989 (1995).

11. R. Kuchimanchi and R. N. Mohapatra, Phys. Rev. **D 48**, 4352 (1993).

12. For a recent review and references, see G. Giudice and R. Rattazzi, hep-ph/9801277.

13. K. Huitu, J. Maalampi and M. Raidal, Nucl. Phys. **B420**, 449 (1994); Phys. Lett. **B328**, 60 (1994); F. Cuypers and S. Davidson, hep-ph/9609487; F. Cuypers and M. Raidal, hep-ph/9704224; K. Huitu, J. Maalampi, A. Pietila and M. Raidal, hep-ph/9606311; J. Gunion, C. Loomis and K. T. Pitts, hep-ph/9610237; B. Dutta and R. N. Mohapatra, hep-ph/9804277. For earlier applications of the light doubly charged Higgs bosons of left-right symmetric models, see P. Herczeg and R. N. Mohapatra, Phys. Rev. lett. **69**, 2475 (1992); A. Halprin, Phys. Rev. lett. **48**, 1313 (1982). J. Gunion, J. Grifols, B. Kayser, A. Mendez and
14. R. N. Mohapatra and J. Vergados, Phys. Rev. Lett. \textbf{47}, 1713 (1981).
15. H. Klapdor-Kleingrothaus, Invited talk at the Neutrino98 conference, Takayama, Japan, June 4-10, 1998; hep-ex/9802007 and references therein.
16. R. Collot and R. Ferrari, (ATLAS Collaboration), WEIN98 poster (1998).