Probabilistic assessment of the buildings and structures construction duration

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Abstract. One of the main conditions for achieving the set value of the buildings and structures construction duration is the construction production planning. Due to the fact that the construction process has an uncertain dynamic nature, it is recommended to use probabilistic analytical models to determine such a calculated parameter as the duration. The article proposes a probabilistic model for estimating the duration of construction of buildings and structures on the basis of the normalized Erlang’s law of the n-th order. This model allows, by changing the number and duration of work lying on the critical path, to assess the construction project completion probability within the prescribed period.

Introduction
The construction production process is a complex dynamic system that combines such elements as control, production and auxiliary subsystems, workers, construction machines and mechanisms, etc. All the subsystems and elements are interconnected. Their interaction in the construction process is aimed at the effective achievement of the main goal: obtaining high-quality construction products in a timely manner and with a given cost [1].

The building construction process is destabilized due to a number of random factors [2, 3]. It is rather difficult to describe their possible impact quantitatively, therefore, when writing this article, the final result of such impact – the probability of deviation of the construction duration from the specified indicator in the project was considered.

In the development of predictive models to estimate the duration of construction regression analysis is used [4], dynamic programming algorithms [5]. The BIM technologies use allows to demonstrate the projects implementation, to deal with downtime and problems associated with the design stage, to ensure resource saving during construction [6,7].

Risk theory, which takes into account unforeseen work and costs, is widely accepted. However, when using the risk theory is not fully investigated and quantified the circumstances and factors that affect the duration of construction.

The purpose of the study is to develop a probabilistic model for assessing the duration of construction of buildings.

It should be noted that the total construction duration depends on the complex of interdependent works, the sequence and duration of which are uniquely defined. Therefore, an increase in the duration of work lying on the critical path for any reason will lead to an increase in the overall duration of the construction of the facility as a whole.
Materials and Methods
Construction is a dynamic process, which is influenced by many factors. Only in rare cases the actual duration and cost of construction and installation works completely coincide with the schedule. The uncertainty that exists in planning leads to a discrepancy between the actual and planned indicators and the need to adjust the calendar plan. In addition, there may be situations of changing the sequence of individual construction works. In this regard, the determination of the design parameter (duration) given the uncertain nature of the construction work is advisable to conduct using probabilistic analytical models [8].

The problems that can be solved with the use of probabilistic models are associated with the determination of the reliability of the design indicators of construction and with the assessment of the probability of completion of works in the construction of buildings and structures.

Results
A probabilistic model for estimating the duration of construction of buildings and structures based on the normalized erlang's law of the n-th order is proposed. This model consists of three stages:

1. Built network diagram that determines the timing of construction of the building, depending on the duration of the work lying on the critical path and time constraints for the timing of their production.
2. Applying the normalized erlang's law of the n-th order, the distribution density and numerical characteristics of the random variable – the length of the critical path of the network graph are calculated.
3. The building timely completion probability is assessed.

Discussion
Let us consider step by step the determination of the calculated parameters of the probabilistic model.

At the first stage, a network schedule is built, in which all the works necessary for the construction of the building are arranged in a technological sequence in strict accordance with the project. The construction works organization methods are taking into account division into the corresponding organizational and structural elements of system of construction production (sites, captures, tiers) shall be observed. When calculating the network schedule, the duration of the critical path is determined and its components are set.

The main design parameter of the model – the value of the critical path \( (T(n)) \) – can be represented as the sum of random values of the duration of all work lying on the critical path (figure 1).

\[
T(n) = T_1 + T_2 + \ldots + T_{n-1} + T_n.
\]  

(1)

\[ \text{Figure 1. The length of the critical path network schedule} \]

At the second stage, the distribution density and numerical characteristics of the random variable \( T(n) \) are calculated.

The distribution law of each of the random variables \( T_1, T_2, \ldots, T_n \) is a composition of the distribution laws. As the distribution law of the random variable \( T(n) \) we use the generalized Erlang's law of n-th order. The disadvantage of applying this law is to increase the value of the expectation and variance.
with increasing number \( n \). Therefore, for calculations we use the normalized Erlang's law of the \( n \)-th order \([9]\), according to which the random variable \( T_{(n)} \) will be distributed as follows:

\[
\tilde{T}_{(n)} = \frac{T_{(n)}}{n}.
\]  

(2)

The density of the distribution of the exponential function is determined by the formula:

\[
\tilde{g}_{(n)}(t) = n\tilde{g}_{(n)}(nt) = \frac{n\lambda(n\lambda t)^{n-1}}{(n-1)!} e^{-n\lambda t}.
\]  

(3)

Distribution function is:

\[
\tilde{G}_{(n)}(t) = 1 - \frac{(n\lambda t)^{n-1}}{(n-1)!} e^{-n\lambda t}.
\]  

(4)

The expectation of a random variable is:

\[
M[\tilde{T}_{(n)}] = M[T_{(n)}]/n = \frac{1}{\lambda}.
\]  

(5)

The variance of the random variable is:

\[
D[\tilde{T}_{(n)}] = \frac{1}{n\lambda^2}.
\]  

(6)

The standard deviation is:

\[
\sigma[\tilde{T}_{(n)}] = \frac{1}{\lambda \sqrt{n}}.
\]  

(7)

Figure 2 shows a family of normalized erlang laws for \( n \) varying from 1 to 7 at \( \lambda=1 \).
The conditions variability and construction technologies can be taken into account by changing the parameter $\lambda$. Figures 3 and 4 show families of normalized Erlang laws at $\lambda=1.25$ and $\lambda=1.5$.

**Figure 2.** A family of normalized Erlang laws at $\lambda=1$

**Figure 3.** A family of normalized Erlang laws at $\lambda=1.25$
At the third stage, the probability of timely completion of construction is estimated. The basis for the calculation is the network schedule (calendar plan), and obtained in the first and second stages of the indicators of the duration of construction.

The probability of completion of construction in the interval $(t_{kp}, T_{kp})$ is found by the formula:

$$
P(t_{kp} < t < T_{kp}) = \tilde{G}(t_{kp}) - \tilde{G}(T_{kp}) = \frac{(n\lambda)^{n-1}}{(n-1)!}(t_{kp}e^{-n\lambda t_{kp}} - T_{kp}e^{-n\lambda T_{kp}}),
$$

where:
- $T_{kp}$ – regulatory, legislative or mounted a production job duration;
- $t_{kp}$ – the duration of the critical path of construction of the building.

The obtained value of the probability of timely completion of the building construction is compared with the established or acceptable level of organizational and technological reliability [10,11].

**Summary**

Thus, the proposed probabilistic model allows, by changing the number and duration of work lying on the critical path, to assess the construction project completion probability in a timely manner. The model can be used in industrial and civil construction.

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