HEAVY QUARK PRODUCTION IN pp COLLISIONS

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Abstract

A systematic study of the inclusive single heavy quark and heavy-quark pair production cross sections in pp collisions is presented for RHIC and LHC energies. We compare with existing data when possible. The dependence of the rates on the renormalization and factorization scales is discussed. Predictions of the cross sections are given for two different sets of parton distribution functions.

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INTRODUCTION

Charm and bottom quark production from initial nucleon-nucleon collisions will be copious at the RHIC and LHC colliders. Heavy quark decay into leptons will represent a significant background to dilepton production in heavy ion collisions. A quantitative knowledge of the production cross section in pp collisions is a prerequisite for the detection of collective effects, such as heavy quark production by rescattering and in the quark-gluon plasma, which appears as a deviation from the simple superposition of hadronic collisions.

The lowest order (Born) calculations of the total cross section predict the correct energy dependence but differ from the experimental measurements by a “K factor” of 2-3. While the single-inclusive distributions as well as the mass and rapidity distributions of QQ pairs are also well described to within a K factor by the Born cross section, the pt and azimuthal double-differential distributions are not calculable at the Born level since the QQ pair is always produced back-to-back in lowest order. For this reason, a next-to-leading order (NLO) calculation is needed. The calculations we present here are done using a Monte Carlo program developed by Nason and collaborators [2, 3, 4]. Similar work on the total cross section and the single inclusive distributions was done by Smith, van Neerven, and collaborators [5].

In this calculation, in addition to the uncertainties in the parton distribution functions, uncertainties arise from the heavy quark mass and the renormalization and factorization scale parameters. At collider energies, the calculations become more uncertain due to the lightness of the heavy quark compared to the center of mass energy, \( m_Q/\sqrt{s} \ll 1 \). We first discuss the Born calculation in some detail and then outline the NLO calculation with its additional uncertainties. We use the available data on \( \sigma_{\text{tot}}(s) \) to fix the charm quark mass and the scale parameters. The resulting parameter set provides a point from which to extrapolate to heavy-ion collider energies. We then compare with single-inclusive and double-differential distributions from charm and bottom data when available. We present estimates of heavy quark production cross sections in proton-proton collisions at RHIC (\( \sqrt{s} = 200 \) and 500 GeV) and LHC (\( \sqrt{s} = 5.5 \) TeV and 14 TeV), according to our present theoretical knowledge. We provide both the Born and NLO results for the total QQ production cross section, single inclusive y and pt distributions, and double differential M, \( \phi \), y and pt distributions.

HEAVY QUARK PRODUCTION IN PERTURBATIVE QCD

The most general expression for the double differential cross section for QQ pair production from the collision of hadrons A and B is

\[
E_Q E_{\bar{Q}} \frac{d\sigma_{AB}}{d^3p_Q d^3p_{\bar{Q}}} = \sum_{ij} \int dx_1 dx_2 F_i(x_1, \mu_F) F_j(x_2, \mu_F) E_Q E_{\bar{Q}} \frac{d\hat{\sigma}_{ij}(x_1P_1, x_2P_2, m_Q, \mu_R)}{d^3p_Q d^3p_{\bar{Q}}}. \tag{1}
\]

Here A and B represent the initial hadrons and i, j are the interacting partons, and the functions \( F_i \) are the number densities of gluons, light quarks and antiquarks.
evaluated at momentum fraction \( x \) and factorization scale \( \mu_F \). The short-distance cross section, \( \hat{\sigma}_{ij} \), is calculable as a perturbation series in \( \alpha_s(\mu_R) \) where the strong coupling constant is evaluated at the renormalization scale \( \mu_R \). Both scales are of the order of the heavy quark mass. At leading order, \( \mu_F = \mu_R = \mu \) where \( \mu = 2m_c \) is commonly used. The scale dependence will be discussed in more detail below.

**Leading Order**

At leading order, \( \mathcal{O}(\alpha_s^2) \), \( Q\overline{Q} \) production proceeds by two basic processes,

\[
q + \bar{q} \rightarrow Q + \overline{Q} \quad (2)
\]
\[
g + g \rightarrow Q + \overline{Q} \quad (3)
\]

The invariant cross section for the process \( A + B \rightarrow H + H \) where the \( Q\overline{Q} \) pair has fragmented into hadrons \( H(Q\overline{Q}) \) and \( H(\overline{Q}q) \) can be written as

\[
E_H E_{\overline{H}} \frac{d\sigma_{AB}}{d^3p_H d^3p_{\overline{H}}} = \int \frac{\hat{s}}{2\pi} dx_1 dx_2 dz_Q d\overline{z}_Q C(x_1, x_2) \frac{E_H E_{\overline{H}}}{E_Q E_{\overline{Q}}} D_{H/Q}(z_Q) D_{\overline{H}/Q}(\overline{z}_Q) \delta^4(p_1 + p_2 - p_Q - p_{\overline{Q}}),
\]

where \( \sqrt{\hat{s}} \), the parton-parton center of mass energy, is related to \( \sqrt{s} \), the hadron-hadron center of mass energy, by \( \sqrt{\hat{s}} = x_1 x_2 s \). The intrinsic transverse momenta of the incoming partons have been neglected. The sum of the leading order subprocess cross sections convoluted with the parton number densities is contained in \( C(x_1, x_2) \) where

\[
C(x_1, x_2) = \sum_q [F^A_q(x_1) F^B_{\overline{q}}(x_2) + F^A_{\overline{q}}(x_1) F^B_q(x_2)] \frac{d\hat{\sigma}_{qq}}{dt} + F^A_g(x_1) F^B_g(x_2) \frac{d\hat{\sigma}_{gg}}{dt}.
\]

Only light quark flavors, those with \( m < m_Q \), are included in the sum over \( q \). The dependence on the scale \( \mu_F \) has been suppressed here.

Fragmentation affects the charmed hadron distributions, not the total \( c\bar{c} \) production cross section. The fragmentation functions, \( D_{H/Q}(z) \), describe the hadronization of the heavy quarks where \( z = |\vec{p}_H|/|\vec{p}_Q| \) is the fraction of the heavy quark momentum carried by the final-state hadron. The \( D \) meson \( x_F \) distribution is harder than the calculated charmed quark distribution in hadron-hadron interactions. Including a fragmentation function that describes \( D \) production in \( e^+e^- \) annihilation softens the distribution due to energy lost to light \( q\bar{q} \) pair production [6]. Event generators such as PYTHIA [7], based on the Lund string fragmentation model, harden the \( D \) distribution. In PYTHIA, the charmed quark is always at the endpoint of a string which pulls the charmed quark in the direction of a beam remnant so that the charmed hadron can be produced at a larger momentum than the charmed quark. Correlations of the produced charmed hadron with the projectile valence quarks, not predicted by perturbative QCD, have been measured. Several possible explanations
have been suggested, see i.e., [3, 4, 5]. This interesting high $x_F$ regime will not be measurable at the RHIC and LHC colliders since the center of mass energy is high and the rapidity coverage is mostly confined to the central region. (The PHENIX muon spectrometer at RHIC will have a larger rapidity coverage, $1.5 \leq y \leq 2.5$ [4], but these effects will probably be out of reach at the maximum collider energy.)

If we ignore fragmentation effects for the moment, after taking four-momentum conservation into account, we are left with

$$\frac{d\sigma}{dp_T^2 dy_Q dy_{\bar{Q}}} = x_1 x_2 C(x_1, x_2) ,$$

where $x_1$ and $x_2$ are

$$x_1 = \frac{\hat{m}_{Q}}{\sqrt{s}} (e^{y_Q} + e^{y_{\bar{Q}}}) ,$$

$$x_2 = \frac{\hat{m}_{Q}}{\sqrt{s}} (e^{-y_Q} + e^{-y_{\bar{Q}}}) ,$$

and $\hat{m}_{Q} = \sqrt{m_Q^2 + p_T^2}$. At $y_Q = y_{\bar{Q}} = 0$, $x_1 = x_2$. The target fractions, $x_2$, decrease with rapidity while the projectile fractions, $x_1$, increase. The subprocess cross sections for $Q\bar{Q}$ production by $q\bar{q}$ annihilation and $gg$ fusion to order $O(\alpha_s^2)$, expressed as a function of $\hat{m}_{Q}$, $y_Q$, and $y_{\bar{Q}}$ are [10]

$$\frac{d\hat{\sigma}_{q\bar{q}}}{dt} = \frac{\pi \alpha_s^2}{9\hat{m}_{Q}^4} \frac{\cosh(y_Q - y_{\bar{Q}})}{(1 + \cosh(y_Q - y_{\bar{Q}}))^3} ,$$

$$\frac{d\hat{\sigma}_{gg}}{dt} = \frac{\pi \alpha_s^2}{96\hat{m}_{Q}^4} \frac{8 \cosh(y_Q - y_{\bar{Q}}) - 1}{(1 + \cosh(y_Q - y_{\bar{Q}}))^3} \left( \cosh(y_Q - y_{\bar{Q}}) + \frac{2m_Q^2}{\hat{m}_{Q}^2} - \frac{2\hat{m}_{Q}^2}{\hat{m}_{Q}^4} \right) .$$

Next-to-Leading Order

We now discuss the NLO, $O(\alpha_s^3)$, corrections to the $Q\bar{Q}$ production cross section. At next-to-leading order, in addition to virtual corrections to these diagrams, production by

$$q + \bar{q} \rightarrow Q + \bar{Q} + g$$

$$g + g \rightarrow Q + \bar{Q} + g$$

$$q(\bar{q}) + g \rightarrow Q + \bar{Q} + (\bar{q})q ,$$

must also be included. The last process, quark-gluon scattering, is not present at leading order. The quark-gluon graphs can be interpreted at the Born level as the scattering of a heavy quark excited from the nucleon sea with a light quark or gluon
and are referred to as flavor excitation [2]. The total short distance cross section \( \tilde{\sigma}_{ij} \) for a given production process can be expressed generally as

\[
\tilde{\sigma}_{ij}(\hat{s}, m_Q, \mu_R) = \frac{\alpha_s^2(\mu_R)}{m_Q^2} f_{ij}(\rho, \mu_R^2/m_Q^2), \tag{13}
\]

where \( \rho = 4m_Q^2/\hat{s} \). The function \( f_{ij} \) can be expanded perturbatively as

\[
f_{ij}(\rho, \mu_R^2/m_Q^2) = f_{ij}^0(\rho) + \frac{\alpha_s(\mu_R)}{4\pi} \left[ f_{ij}^1(\rho) + f_{ij}^1(\rho) \ln(\mu_R^2/m_Q^2) \right] + \mathcal{O}(\alpha_s^2). \tag{14}
\]

The leading order part of the cross section is in the function \( f_{ij}^0 \). In this case, \( f_{ij}^0 = f_{ig}^0 = f_{gg}^0 = f_{gq}^0 = 0 \). Only \( f_{gg}^0 \) and \( f_{gq}^0 \) contribute and can be computed from the integration of the cross sections given in (8) and (9). The physical cross section should be independent of the renormalization scale: the dependence in eq. (14) introduces an unphysical parameter in the calculation. If the perturbative expansion is sufficient, \( i.e. \) if further higher-order corrections are small, at some value of \( \mu \) the physical \( \mathcal{O}(\alpha_s^n) \) and \( \mathcal{O}(\alpha_s^{n+1}) \) cross sections should be equal. If the \( \mu \) dependence is strong, the perturbative expansion is untrustworthy and the predictive power of the calculation is weak [10]. The rather large difference between the heavy-quark Born and NLO cross sections suggests that further higher-order corrections are needed, particularly for charm and bottom quarks which are rather “light” when \( \sqrt{\hat{s}} \) is large. Usually the renormalization scale in \( \tilde{\sigma}_{ij} \) and the factorization scale in the parton distribution functions are chosen to be equal. We follow this prescription in our calculations.

We have used two sets of recent parton distribution functions [1], GRV HO [12] and MRS D-′ [13]. The first begins with a low scale, \( Q_{0,GRV}^2 = 0.3 \) GeV\(^2\), and valence-like parton distributions, therefore evolving very quickly with \( Q^2 \). The second, with \( Q_{0,MRS}^2 = 5 \) GeV\(^2\), has sea quark and gluon distributions that grow as \( \sim x^{-1/2} \) when \( x \to 0 \). Both are compatible with the recent deep-inelastic scattering data from HERA [13]. We also include estimates of the total cross section using the MRS D0′ [13] distributions. This set assumes a constant value for the sea and gluon distributions at \( Q_{0,MRS}^2 \) as \( x \to 0 \) and lies below the HERA data. The GRV distributions assume \( \overline{\sigma} = \overline{d}, \) a symmetric light quark sea, and \( x\sigma(x, Q_{0,GRV}^2) = 0 \), increasing to give \( 2\langle x\rangle_{\overline{d}}/(\langle x\rangle_{\overline{d}} + \langle x\rangle_{\overline{u}}) \approx 0.53 \) at \( Q^2 = 10 \) GeV\(^2\) [12]. The MRS D sets allow \( \overline{\sigma} < \overline{d} \) to account for measurements of the Gottfried sum rule and assume \( \overline{\sigma} = (\overline{u} + \overline{d})/4 \) at \( Q_{0,MRS}^2 \) [13]. Thus the MRS distributions, arising from a global fit, provide a somewhat better description of the deep-inelastic scattering data for \( x > 0.01 \) than the GRV distributions [12, 13].

Since we compare two extreme cases for the nucleon parton distributions as \( x \to 0 \), MRS D-′ and GRV HO on one hand and MRS D0′ on the other, our results may be thought of as providing an upper and lower bound to the \( Q\overline{q} \) cross section at heavy-ion collider energies for fixed mass and scale. However, little data exist on the gluon

\[\text{†} \text{The order of the expansion is represented by } n. \text{ For } QQ \text{ production, } n \geq 2. \text{ A calculation to order } \mathcal{O}(\alpha_s^n) \text{ introduces corrections at the order } \mathcal{O}(\alpha_s^{n+1}).\]

\[\text{‡} \text{All available parton distribution functions are contained in the package PDFLIB [13], available in the CERN library routines.}\]
distribution function at low $x$ so that it is poorly known, particularly in the $x$ region accessible at RHIC and LHC, $x \approx 10^{-2}$ and $10^{-4}$ around $y = 0$, respectively. The low $x$ behavior has a significant effect on the shape of the gluon distribution at moderate values of $x$ in the energy range of Fig. 1. Steeply rising gluon distributions at low $x$ are compensated for by a corresponding depletion at moderate $x$.

Heavy quark production by gluon fusion dominates the $pp \to Q\overline{Q}X$ production cross section in the central region. Thus we show the shape of the gluon distributions of the three parton distribution sets are shown in Fig. 1(a) over the $x$ range of the previous $pp$ data, $0.01 < x < 1$. To facilitate comparison, all three are shown at $\mu = 2.4$ GeV. The solid curve is the GRV HO distribution, the dashed, MRS D0, and the dot-dashed, MRS D-'. The GRV distribution at $\mu = 1.2$ GeV is also shown to demonstrate the effect of the $Q^2$ evolution. Since it has a smaller initial scale, the evolution with $\mu$ is quite fast. The D0' distribution can be seen to turn over and begin to flatten as $x$ decreases. However, for much of the range, it is above the D-' distribution, reflected in a larger $\sigma_{\text{tot}}$, as shown in Fig. 3. All three sets, evaluated in the $\overline{\text{MS}}$ scheme, have a similar value of $\Lambda_{\text{QCD}}$. In Fig. 1(b), we show the running of the two loop value of $\alpha_s$,

$$\alpha_s(\mu, f) = \frac{1}{b_f \ln(\mu^2/\Lambda_f^2)} \left[ 1 - \frac{b'_f \ln(\mu^2/\Lambda_f^2)}{b_f \ln(\mu^2/\Lambda_f^2)} \right],$$

where $b_f = (33 - 2f)/12\pi$, $b'_f = (153 - 19f)/(2\pi(33 - 2f))$, $f$ is the number of flavors, and $\Lambda_f$ is the value of $\Lambda_{\text{QCD}}$ appropriate for the number of flavors. In the calculation, the number of flavors depends on the chosen quark mass. For charm, $f = 3$, and for beauty, $f = 4$. At $\mu = m_Q$, $\alpha_s(m_Q, f) = \alpha_s(m_Q, f + 1)$. The running of $\alpha_s$ is visible in the renormalization scale dependence, shown in Fig. 2(e). For the NLO $Q\overline{Q}$ production program, $\Lambda_f$ is chosen by $m_Q$. Note that $\Lambda_3 > \Lambda_4 > \Lambda_5$. Additional uncertainties may arise because the threshold $m_Q$ for a given parton distribution set can differ from our fitted $m_Q$.

While it is often possible to use a general prescription like the principle of minimal sensitivity (PMS) [17] to find values of $\mu_R$ and $\mu_F$ where the scale sensitivity is a minimum, the heavy quark production cross section is very sensitive to changes in $\mu$. In Fig. 2 we show the variation of the $c\overline{c}$ and $b\overline{b}$ production cross sections at RHIC (a), (c) and LHC (b), (d) ion energies. The MRS distributions exhibit an artificial stability for low $\mu$ because for $\mu < 2m_c \approx Q_{0,\text{MRS}}$, the factorization scale is fixed at $Q_{0,\text{MRS}}$ and only $\mu_R$ varies. We use the GRV HO parton distribution functions so that we can show the uncertainty with $\mu = \mu_R = \mu_F$ at lower values of $\mu$ since $\mu_F$ is not fixed until $\mu_F \approx 0.4m_c \approx Q_{0,\text{GRV}}$. When $\mu/m_c \approx 0.2$, the cross section diverges since $(\mu/m_c)/\Lambda_{\text{QCD}} \approx 1$. In any case, such small scales below 1 GeV, are excluded because a perturbative calculation is no longer assumed to be valid. As $\mu/m_c$ increases, the cross section becomes more stable. The behavior we find is similar for RHIC and LHC energies. The $b\overline{b}$ cross section shows a smaller variation with $\mu$, particularly at $\sqrt{s} = 200$ GeV. The variation resembles the running of $\alpha_s$ shown in Fig. 1(b).

Indeed, this running is a major source of instability in the NLO $Q\overline{Q}$ cross sections. However, at $\sqrt{s} = 5.5$ TeV the variation with $\mu$ at the Born level increases since
the cross section becomes more uncertain as \( m_Q/\sqrt{s} \) decreases. The NLO results show less variation at this energy. There is no value of \( \mu \) where the Born and the NLO calculations are equal, suggesting that higher-order corrections are needed for \( m_Q/\sqrt{s} \ll 1 \).

We show the change of the \( c\bar{c} \) cross section at \( \sqrt{s} = 200 \) GeV induced by fixing \( \mu_R = 2m_Q \) and changing \( \mu_F \) in Fig. 2(e) and fixing \( \mu_F = 2m_Q \) and varying \( \mu_R \) in Fig. 2(f). The running of the coupling constant is clearly shown in 2(e). In 2(f), the increase with \( \mu_F \) arises because at values of \( \mu_F \) near \( Q_{0,GRV} \) and low \( x \), the sea quark and gluon distributions show a valence-like behavior, decreasing as \( x \to 0 \), an effect special to the GRV distributions [12]. The results are quite different for the MRS distributions, especially for the equivalent of Fig. 2(f). There is not much change in the cross section with \( \mu_F \), particularly at the Born level, since the parton distribution functions do not change below \( Q_{0,MRS} \).

**CALCULATIONS OF \( \sigma_{Q\bar{Q}}^{\text{tot}} \)**

Previous comparisons of the total charm production cross sections with calculations [16] at leading order suggested that a constant \( K \) factor of \( \sim 2 \) was needed to reconcile the calculations with data when using \( m_c = 1.5 \) GeV, but not when \( m_c = 1.2 \) GeV was chosen. Initial NLO calculations seemed to suggest that the \( K \) factor was no longer needed with \( m_c = 1.5 \) GeV [17]. However, this result is very dependent upon the chosen scale parameters and the parton distribution functions, particularly the shape of the gluon distribution.

**Comparison With Current Data**

We compare our NLO calculations with the available data [18, 19, 20, 21, 22] on the total \( c\bar{c} \) production cross section from \( pp \) and \( pA \) interactions in Fig. 3. When a nuclear target has been used, the cross section per nucleon is given, assuming an \( A\alpha \) dependence with \( \alpha = 1 \), supported by recent experimental studies of the \( A \) dependence [23]. We assume that we can compare the \( c\bar{c} \) production cross section directly with charmed hadron measurements. Often single charmed mesons, denoted \( D/\bar{D} \) to include all charge states, in the region \( x_F > 0 \) are measured. The \( c\bar{c} \) production cross section is symmetric around \( x_F = 0 \) in \( pp \) interactions so that \( \sigma_{\text{tot}}^{c\bar{c}} = 2\sigma_{c\bar{c}}(x_F > 0) \). While the question of how the \( c\bar{c} \) pair hadronizes into \( D\bar{D}, D\bar{\Lambda}_c, \Lambda_c\bar{D}, \Lambda_c\bar{\Lambda}_c \), etc. remains open, some assumptions must be made about how much of \( \sigma_{\text{tot}}^{c\bar{c}} \) is missing since not all channels are measured. If all single \( D \) mesons are assumed to originate from \( D\bar{D} \) pairs, ignoring associated \( \Lambda_c\bar{D} \) production, then by definition, \( \sigma(D\bar{D}) = \sigma(D/\bar{D})/2 \). Thus the single \( D \) cross section for \( x_F > 0 \) is equal to the \( D\bar{D} \) pair cross section over all \( x_F \). However, the contribution to the \( c\bar{c} \) total cross section from \( D_s \) and \( \Lambda_c \) production has been estimated to be \( \sigma(D_s)/\sigma(D^0 + D^+) \approx 0.2 \) and \( \sigma(\Lambda_c)/\sigma(D^0 + D^+) \approx 0.3 \). Thus to obtain the total \( c\bar{c} \) cross section from \( \sigma(D\bar{D}) \),
\(\sigma(D\bar{D})\) should be multiplied by \(\approx 1.5\) \cite{24}. This is done in our data comparison. The data exist in the range \(19 < \sqrt{s} \leq 63\) GeV, mostly from fixed target experiments. Below the ISR energies, \(\sqrt{s} = 53\)-63 GeV, the total cross section is primarily inferred from single \(D\) or \(D\bar{D}\) measurements. At the ISR, the pair production cross section is obtained from lepton measurements, either \(e\mu\) and electron pair coincidence measurements or a lepton trigger in coincidence with a reconstructed \(D\) or \(\Lambda_c\). Rather large \(\sigma x\) cross sections were inferred from the latter analyses due to the assumed shape of the production cross sections: flat distributions in \(x_F\) for the \(\Lambda_c\) and \((1-x_F)^3\) for the \(D\). The ISR results must thus be taken with some care.

Modern parton distributions with \(\Lambda_{QCD}\) fixed by fits to data cannot explain the energy dependence of the total cross section in the measured energy range when using \(m_c = 1.5\) GeV and \(\mu_F = \mu_R = m_c\). Since \(m_c^2 < Q^2_{0,MRS}\) for the MRS distributions and the scale must be chosen so that \(\mu^2 > Q^2_{0,MRS}\) for the calculations to make sense, we take \(\mu = 2m_c\) and vary \(m_c\) for these distributions. We find reasonable agreement for \(m_c = 1.2\) GeV for the \(D^−\) and \(D^0\)' distributions. The results are shown in the solid and dashed curves in Fig. 3 respectively. Since the GRV HO distributions have a much lower initial scale, \(\mu\) can be fixed to the quark mass. The dot-dashed curve is the GRV HO distribution with \(m_c = 1.3\) GeV and \(\mu = m_c\). All three curves give an equivalent description of the data. Our “fits” to the low energy data are to provide a reasonable point from which to extrapolate to higher energies. It is important to remember that significant uncertainties still exist which could change our estimates considerably when accounted for. These relatively low values of \(m_c\) effectively provide an upper bound on the charm production cross section at high energies. For comparison, we also show the cross section with the GRV distributions and \(\mu = m_c = 1.5\) GeV in the dotted curve. It lies a factor of 2-3 below the other calculations. The smaller value of \(m_c\) is needed for the MRS distributions even with the larger scale because parton distribution functions at lower values of \(Q^2\) would decrease at low \(x\), as demonstrated by the GRV distributions \cite{12}. Note that such small choices of \(m_c\) suggests that the bulk of the total cross section comes from invariant masses less than \(2m_p\). In a recent work \cite{24}, the total cross section data was found to be in agreement with \(m_c = 1.5\) GeV with some essential caveats: the factorization scale was fixed at \(\mu_F \equiv 2m_c\) while \(\mu_R\) was allowed to vary and an older set of parton distribution functions with a range of fits with a different value of \(\Lambda_{QCD}\) for each was used. Decreasing \(\mu_R\) with respect to \(\mu_F\) and increasing \(\Lambda_{QCD}\) both result in a significantly larger cross section for a given \(m_c\). We choose here to use the most up-to-date parton distribution functions and to keep \(\mu_F = \mu_R\), facilitating a more direct extrapolation from the current data to the future collider results.

Since data on \(\sigma x\) and \(b\bar{b}\) production by pion beams are also available at fixed target energies, in Fig. 4 we show this data with the same parton distributions where \(m_c\) and \(\mu\) are fixed by the comparison in Fig. 3. The \(\sigma x\) data \cite{18, 25, 26, 27, 28} is based on the \(x_F > 0\) single \(D\) cross section. However, the \(\pi^- N x_F\) distribution is asymmetric, \(\sigma/\sigma(x_F > 0) \sim 1.6\) so that \(\sigma(D\bar{D})\) is obtained by dividing by 2 to get the pair cross section and then multiplying by 1.6 to account for the partial \(x_F\) coverage. The \(b\bar{b}\) data, taken to be over all \(x_F\), are generally obtained from multi-muon studies \cite{23, 30, 31, 32}. The data, especially for \(b\bar{b}\) production, are not as extensive and have
rather poor statistics. Again, some of the data is from a nuclear target. When a nuclear target has been used, the cross section per nucleon is given, assuming an $A^1$ dependence.

The GRV HO pion distributions are based on their proton set so that the two distributions are compatible. In Fig. 4(a), the charm production cross section is calculated using the GRV proton and pion distributions. The solid curve shows the result with a nucleon target, the averaged distributions for proton and neutron, while the dashed curve is the result for a proton target alone. The results are consistent at $\sqrt{s} = 30$ GeV; at lower energies, the cross section on a proton target is slightly larger than on a nucleon target. The calculations using the MRS distributions do not have the same consistency as those with GRV because their pion distribution functions, SMRS P1 and P2, are based on an older set of proton distributions than the current MRS distributions used here. The SMRS distributions use $\Lambda_4 = 190$ MeV while the MRS distributions have fixed $\Lambda_4 = 230$ MeV. In the calculations, we fix $\Lambda_4$ to the current MRS value. The dot-dashed curve shows the MRS D' distributions with the SMRS P2 pion distributions while the dotted curve is with the P1 set. Both are for a proton target. The P1 set has a steeper gluon distribution than P2. The two calculations begin to diverge as $\sqrt{s}$ increases since the gluon fusion contribution is becoming dominant. At low $\sqrt{s}$, valence quark annihilation is important for $\pi^-p$ interactions. Although the calculations and data are not in exact agreement, they are close enough to assume that the same parameters are reasonable for both pion and proton projectiles. The comparison to the $b\bar{b}$ production cross section is given in Fig. 4(b). The data is very sparse. We use $m_b = 4.75$ GeV and $\mu = m_b$ for both sets of parton distributions. The solid curve is the GRV distribution, the dashed is the MRS D' and SMRS P1 result. The agreement is not unreasonable given the quality of the data on the one hand and the theoretical uncertainties on the other.

Extrapolation To RHIC And LHC Energies

The total $c\bar{c}$ and $b\bar{b}$ cross sections at the top ISR energy, $\sqrt{s} = 63$ GeV, and the proton and ion beam energies at RHIC and LHC are given in Tables 1 and 2 respectively. Both the Born and NLO cross sections are given. The theoretical $K$ factor, $\sigma_{QQ}^{\text{NLO}}/\sigma_{QQ}^{\text{LO}}$, tends to increase with energy and is rather large. There is no a priori reason why it should remain constant, rather the increase at collider energies would suggest that the perturbative expansion is becoming less reliable, as discussed below. Note that even though the MRS D' and GRV HO distributions give an equally valid description of the data at ISR energies and below, they differ at higher energies, partly from the difference in $m_c$ and partly because of our scale difference. The MRS D' distributions evolve faster since $\mu = 2m_c$, rather than $\mu = m_c$ due to their chosen initial scale $Q_{0,\text{MRS}}$, resulting in a larger predicted cross section. Less difference is seen between the GRV and MRS D' distributions for the $b\bar{b}$ cross section since the $m_b$ and $\mu$ are used for both. Note that for $b\bar{b}$ production at 14 TeV, the results differ by 30% while the MRS D' NLO $c\bar{c}$ result is three times larger than the GRV HO result.
at the same energy. The D0’ distributions give smaller cross sections at LHC energies due to the different initial behavior at \( x \to 0 \). We illustrate this effect using the Born contribution to the production cross section at fixed \( M \) and \( y = 0 \), approximated as

\[
\frac{d\sigma}{dMdy}\bigg|_{y=0} \approx \frac{\alpha_s^2}{Ms} \left[F_g\left(M/\sqrt{s}\right)\right]^2
\]

since gluon fusion is the dominant contribution to the Born cross section, \( x = M/\sqrt{s} \) at \( y = 0 \), and at fixed \( M \), \( \sigma_{gg} \) is proportional to \((\alpha_s^2/M^2)F_g^2\). The gluon distribution at low \( x \) and \( \mu = Q_0 \) may be approximated as \( F_g(x) = f(x)/x^{1+\delta} \). For a constant behavior at low \( x \), such as in the MRS D0’ distribution, \( \delta = 0 \) and the cross section is independent of \( \sqrt{s} \). At the other extreme, the MRS D’-’ distribution assumes \( \delta = 0.5 \) at \( Q_0 \) so that the cross section grows as \( s^\delta \sim \sqrt{s} \).

**SINGLE AND DOUBLE DIFFERENTIAL DISTRIBUTIONS**

We now compare the NLO calculations with data on \( Q \) and \( Q\bar{Q} \) distributions. In the presentation of the single inclusive and double differential distributions, we follow the prescription of Nason and collaborators [3, 4] and take \( \mu_S = \sqrt{m_Q} \) for the single and \( \mu_D = \sqrt{m_Q + (p_T^Q + p_T^{\bar{Q}})^2}/2 \) for the double differential distributions. When using MRS distributions for charm production, \( n = 2 \). For all other cases, \( n = 1 \). A word of caution is necessary when looking at our predictions for \( Q\bar{Q} \) pair distributions. It is difficult to properly regularize the soft and collinear divergences to obtain a finite cross section over all phase space. Soft divergences cancel between real and virtual corrections when properly regularized. The collinear divergences need to be regularized and subtracted. For single inclusive heavy quark production, this is possible because the integration over the partonic recoil variables can be performed analytically and the singularities isolated. In exclusive \( Q\bar{Q} \) pair production, the cancellation is performed within the numerical integration. The price paid for this is often a negative cross section near the phase space boundaries, particularly when \( p_T \to 0 \) for the pair and \( \phi \to \pi \) where \( \phi \) is the difference in the azimuthal angle between the heavy quark and antiquark in the plane transverse to the beam axis. A positive differential cross section for \( p_T \to 0 \) can only be obtained by resumming the full series of leading Sudakov logarithms corresponding to an arbitrary number of soft gluons. This has not been done in the case of heavy quark production [4]. Thus when \( m_Q/\sqrt{s} \ll 1 \), fluctuations in the cross section due to incomplete numerical cancellations can become very large, resulting in negative components in the mass and rapidity distributions. We have minimized the fluctuations by maximizing the event sampling at low \( p_T \) and increasing the number of iterations [35].

**Comparison To Current Data**
First, we compare with the 800 GeV fixed target data of the LEBC-MPS collaboration [19] in Fig. 5. They measured the $x_F$ and $p_T^2$ distributions of single $D$ production. The total cross section, $\sigma(D/\bar{D}) = 48 \pm 11 \mu b$, corresponds to a $D\bar{D}$ production cross section of 24$\pm8$ $\mu b$. The solid curves are the MRS D′ results, the dashed, the GRV HO calculations. Data on correlated $D\bar{D}$ production is also available at 800 GeV, from $p$Emulsion studies [36]. The event sample is rather small, only 35 correlated pairs. We compare the mass and $p_T^2$ of the pair and the azimuthal difference between the pair in Fig. 6 with the calculated NLO distribution s. Again the solid curve is MRS D′, the dashed, GRV HO. The Born invariant mass distribution, given by the dashed curve, is parallel to the NLO results shown in the solid curve.

The $p\bar{p}$ data from UA 1, $\sqrt{s} = 630$ GeV, and CDF, $\sqrt{s} = 1.8$ TeV, include single $b$ quark $p_T$ distributions. The measurements are taken in the central region ($|y| < 1.5$ for UA 1 and $|y| < 1$ for CDF) and are integrated over $p_T$ above each $p_{T,min}$. The comparisons with the NLO calculations are given in Fig. 7(a) for UA 1 [37] and Fig. 7(b) for CDF and D0 [38, 39]. Reasonable agreement is found for both GRV HO and MRS D′ for UA 1 with $\mu_S = \sqrt{m_b^2 + p_T^2}$. However, the results from this same scale choice lie somewhat below the early CDF data where data on $J/\psi$ production was used to determine the $B$ production cross section [40]. As reported in Ref. [40], the scale $\mu = \mu_S/4$ was needed for good agreement with the magnitude of the data when the older MRS D0 distributions were used. More recent data using direct measurement of inclusive $b \to J/\psi$ and $b \to \psi'$ decays has shown that the previous results overestimated the $\psi$ production from $b$ decays [38]. Better agreement with theory is now found for $\mu = \mu_S$, as shown in Fig. 7(b). Again the GRV HO and MRS D′ distributions look similar, differing primarily for $p_{T,min} < 10$ GeV. This difference is increased for the lower scale choice where $\mu_S/4 < Q_0$, GRV, for $p_{T,min} < 7.5$ GeV, cutting off the evolution of the MRS distributions below this $p_{T,min}$. The GRV calculations evolve over all $p_{T,min}$ since $\mu_S/4 > Q_0$, GRV, hence the larger difference.

**Extrapolation To RHIC And LHC Energies**

We now show the predicted heavy quark distributions for RHIC ($\sqrt{s} = 200$ and 500 GeV) and LHC ($\sqrt{s} = 5.5$ and 14 TeV) using the MRS D′ and GRV HO distributions. The results are shown in Figs. 8-23. We use the same scales on the $y$-axes for both sets of parton distributions as much as possible to facilitate comparison. In each figure we show the single quark $p_T$ (a) and $y$ (b) distributions and the $p_T$ (c), rapidity (d), invariant mass (e), and azimuthal angle (f) distributions of the $Q\bar{Q}$ pair. The Born (LO) results are also given in (b), (d), and (e). All the distributions have been divided by the corresponding bin width. The single and pair $p_T$ distributions are also given with the rapidity cuts $y < |1|$ at the LHC and $y < |0.35|$ at RHIC, corresponding to the planned acceptances of ALICE [41] and the PHENIX central detector [4]. These $p_T$ distributions are also divided by the width of the rapidity interval. In

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*The inclusive decay, $B \to J/\psi X$, has a 1% branching ratio (BR) while the channel $B \to J/\psi K$ has an 0.1% branching ratio.*
Tables 3-10 we give the $y$-integrated single $p_T^2$ NLO and Born distributions, the pair $p_T^2$ distributions with the cut on rapidity, and the NLO and Born invariant mass distributions for $c$ and $b$ production at each energy with the MRS D'-partons. Note that all distributions have a 2 GeV bin width and that neither it nor the rapidity bin width has been removed in the tables. The statistical uncertainties are less than 1% at low $p_T^2$ and $M$, increasing to 5-6% in the tails. The uncertainty increases slightly with energy.

The development of a rapidity plateau can be seen in both the single and pair rapidity distributions as the energy increases. This plateau is generally broader for the single quarks than the pair since the pair mass enters into the estimate of the maximum pair rapidity while the smaller quark transverse mass gives the maximum single quark rapidity. The plateau is broader for the MRS D'-parton distributions. In the charm rapidity distributions with the MRS D' partons at 14 TeV, the plateau edge is artificial. The set has a minimum $x$ of $10^{-5}$, reached at $y \sim 2.8$ for a single quark and a somewhat larger $y$ for the pair. The GRV HO distributions have a minimum $x = 10^{-6}$, corresponding to $y \sim 4.5$, off the scale of our graphs. The average quark and pair $p_T$ increases with energy. For charmed quarks, $\langle p_T^2 \rangle$ is larger for the pair than for a single quark. The opposite result is seen for $b$ quarks. The GRV distributions result in larger $\langle p_T^2 \rangle$ than the MRS distributions. Near $p_T \to 0$, the MRS parton distributions show a steeper slope than the GRV distributions. As $p_T$ increases, the slopes become somewhat similar at RHIC energies.

In general, the LO mass and rapidity distributions are nearly equivalent to the NLO results scaled by a theoretical $K$ factor independent of $M$ and $y$. At LHC energies, the expansion parameter becomes $\alpha_s \log(s/m_Q^2)$, of order 1 for $m_Q/\sqrt{s} \ll 1$, spoiling the convergence of the perturbative expansion [24]. This causes our predictions to be less reliable at these energies. Note that using $\mu_S$ for the single inclusive distributions and $\mu_D$ for the double differential distributions leads to somewhat different values of the integrated NLO cross sections than given in Tables 1 and 2, calculated with $\mu = nm_Q$, since the correction terms grow with $\mu$. The effect is relatively small for the Born results since the faster evolution of the parton distribution functions is partly compensated by the decrease of $\alpha_s$ with increasing $\mu$.

We also compare to the leading order charm distributions obtained from HIJING [42] for the ion collider energies, 200 GeV (Figs. 8,9) and 5.5 TeV (Figs. 12,13). HIJING uses the same mass and scale parameters and parton distribution functions as the other calculations. Although only a Born level calculation of $Q\bar{Q}$ production, HIJING includes the effect of multiple parton showers which simulates aspects of higher-order production (NLO includes the effect of only one additional parton). The rapidity distributions, shown for $y > 0$ only, closely resemble the NLO calculations. However, the $p_T^2$ distributions, taken in the rapidity interval $|y| < 2$ for the single $c$ quark and the pair, are softer, especially for the $c\bar{c}$ pair. (Again, the distributions are divided by the rapidity bin width.) The distributions are also not strongly peaked at low $p_T$, as are the NLO calculations, due to initial state radiation. HIJING also includes fragmentation of the $c\bar{c}$ pair into hadrons. The calculated $\phi$ distributions are not as sharply peaked at $\phi = \pi$ as the NLO results. Note also that the $D\bar{D}$ pair $\phi$ distributions from HIJING are more isotropic than the original $c\bar{c}$ pairs.
$Q\overline{Q}$ Decays To Lepton Pairs

Since heavy quark decays are an important contribution to the dilepton continuum, we show $c\overline{c}$ and $b\overline{b}$ decays into dileptons at RHIC and LHC for the MRS D′ sets. Because heavy quark decays are not incorporated into our double-differential calculation, the heavy quark pairs have been created from the final distributions. The heavy quark decays to leptons are thus calculated using a Monte Carlo program based on data from $D$ decays at SLAC [43] and $B$ decays from CLEO [44]. The inclusive branching ratio for $D$ meson decay into a lepton, averaged over charged and neutral $D$’s is $BR(D^0/D^+ \rightarrow l^+X) \sim 12\%$. The corresponding branching ratio for $B$ mesons of unspecified charge is $BR(B \rightarrow l^+X) \sim 10.4\%$ [45]. $B$ decays represent a special challenge since lepton pairs of opposite sign can be produced from the decay of a single $B$ by $B \rightarrow DlX$ followed by $D \rightarrow lX$. Thus the $B$ decays can produce dileptons from the following: a combination of leptons from a single $B$, two leptons from primary $B$ decays, two leptons from secondary decays, and a primary lepton from one $B$ and a secondary lepton from the opposite sign $\overline{B}$. The measurement of Ref. [44] is assumed to be for primary $B$ decays to leptons. The NLO pair distributions $d\sigma/dM$ and $d\sigma/dy$ agree well with a $K$ factor times the Born results. Therefore the correlated distributions, $d\sigma/dMdy$, are calculated at leading order and multiplied by this $K$-factor, while the $p^2_T$ and $\phi$ distributions, unavailable at leading order, are taken from the NLO results. The heavy quark pair is specified according to the correlated distributions from the calculated cross section. The momentum vectors of the individual quarks are computed in the pair rest frame, using the rapidity gap between the quarks. Once the quark four-momenta have been specified, the decays are calculated in the quark rest frame, according to the measured lepton momentum distributions, and then boosted back to the nucleon-nucleon center of mass, the lab frame for RHIC and LHC. Finally, the pair quantities, $M_{ll}$, $y_{ll}$, and $p^{}_{T,\,ll}$, are computed.

The average number of $Q\overline{Q}$ pairs, $N_{Q\overline{Q}}$, produced in a central nuclear collision is estimated by multiplying the cross section from Tables 1 and 2 by the nuclear thickness $T_{AB}(0)$. If $N_{Q\overline{Q}} < 1$, only correlated production is important. The number of correlated lepton pairs can be estimated by multiplying the number of $Q\overline{Q}$ pairs by the square of the meson, $H$, branching ratio to leptons: $N_{Q\overline{Q}}BR^2(H/\overline{P} \rightarrow l^\pm X)$. However, if $N_{Q\overline{Q}} > 1$, dilepton production from uncorrelated $Q\overline{Q}$ pairs should be accounted for as well. Then two $Q\overline{Q}$ pairs are generated from the production cross section and the $Q$ from one pair is decayed with the $\overline{Q}$ from the other. Thus for uncorrelated $Q\overline{Q}$ production, the average number of lepton pairs is approximately $N_{Q\overline{Q}}(N_{Q\overline{Q}} - 1)BR^2(H/\overline{P} \rightarrow l^\pm X)$ when $N_{Q\overline{Q}} \gg 1$. If $N_{Q\overline{Q}} \approx 1$, a distribution in $N_{Q\overline{Q}}$ must be considered to calculate the uncorrelated pairs. In the following figures, we show the correlated dilepton cross section in $pp$ collisions, $\sigma_{ll} = BR^2(H/\overline{P} \rightarrow l^\pm X)\sigma_{Q\overline{Q}}$. In Fig. 27, showing uncorrelated lepton pairs from $D\overline{D}$ decays at the LHC, we give the uncorrelated distributions with the value of the correlated cross section since $N_{Q\overline{Q}} < 1$ in $pp$ collisions. To find the correct scale in central $AB$ collisions, calculate $N_{Q\overline{Q}}$ and then multiply the lepton pair cross section by $T_{AB}(0)(N_{Q\overline{Q}} - 1)$.

In Figs. 24-25, we show the mass (a), rapidity (b), and $p^{}_{T,\,ll}$ (c) distributions for the
lepton pairs from $D\bar{D}$ and $B\bar{B}$ pairs respectively. The average mass of the lepton pairs from $D\bar{D}$ decays at RHIC ion energies is $\langle M_{ll} \rangle = 1.35$ GeV and the average lepton pair $p_T$, $\langle p_{T,ll} \rangle = 0.8$ GeV; from $B\bar{B}$ decays, $\langle M_{ll} \rangle = 3.17$ GeV and $\langle p_{T,ll} \rangle = 1.9$ GeV. A like-sign subtraction should eliminate most of the uncorrelated charm production at RHIC.

At LHC ion energies, the $c\bar{c}$ production cross sections are large enough for uncorrelated charm production to be substantial and difficult to subtract in nuclear collisions. The average mass of the lepton pairs from correlated $D\bar{D}$ decays here is $\langle M_{ll} \rangle = 1.46$ GeV and the $\langle p_{T,ll} \rangle = 0.82$ GeV. When the pairs are assumed to be uncorrelated, then $\langle M_{ll} \rangle = 2.73$ GeV and $\langle p_{T,ll} \rangle = 1$ GeV. The average dilepton mass from uncorrelated $D\bar{D}$ pairs is larger since the rapidity gap between uncorrelated $D$ and $\bar{D}$ mesons is larger on average than between correlated $D\bar{D}$ pairs. The $b\bar{b}$ cross section is still small enough at the LHC for uncorrelated lepton pair production from $B$ meson decays to be small. However, the acceptance for these pairs will be larger than for charm decays since high mass lepton pairs from heavy quark decays have a large rapidity gap. When acceptance cuts are applied, at least one member of a lepton pair will have a large enough rapidity to escape undetected so that high mass pairs from heavy quark decays will have a strongly reduced acceptance. This reduction will occur at larger values of $M_{ll}$ for $B\bar{B}$ than $D\bar{D}$ decays. From all $B\bar{B}$ decays, $\langle M_{ll} \rangle = 3.39$ GeV and $\langle p_{T,ll} \rangle = 2$ GeV. In Figs. 26-28, we show the mass (a), rapidity (b), and $p_T$ (c) distributions for the dilepton pairs from correlated and uncorrelated $D\bar{D}$ and correlated $B\bar{B}$ pairs respectively.

**SUMMARY**

In this overview, we have attempted to use the theoretical state of the art to predict heavy quark production in $pp$ collisions at RHIC and LHC energies. Although much progress has been made in the higher-order calculations of $Q\bar{Q}$ production, this is not meant to be the final word. Fragmentation and decay effects need to be incorporated into our next-to-leading order calculations. More structure function data from HERA, combined with collider data on jets and prompt photons, will produce further refined sets of parton distribution functions. Theoretical progress may allow resummation at low $p_T$ or produce estimates of next-to-next-to-leading order corrections. New scale fixing techniques may result in a reduction of scale uncertainties. Thus, there is still room for improvement in these calculations. Though the agreement with lower energy data allows us to extrapolate these results to RHIC and LHC energies, major uncertainties still exist, particularly at LHC energies. However, given our mass and scale parameters, the GRV HO and MRS D' parton distribution functions provide a rough upper and lower limit on the theoretical predictions. This might be useful in particular for the design of detectors at these facilities.
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| $\sqrt{s}$ (GeV) | MRS D0' $\sigma_{c\bar{c}}^{\text{LO}}$ (μb) | MRS D0' $\sigma_{c\bar{c}}^{\text{NLO}}$ (μb) | GRV HO $\sigma_{c\bar{c}}^{\text{LO}}$ (μb) | GRV HO $\sigma_{c\bar{c}}^{\text{NLO}}$ (μb) | MRS D-' $\sigma_{c\bar{c}}^{\text{LO}}$ (μb) | MRS D-' $\sigma_{c\bar{c}}^{\text{NLO}}$ (μb) |
|-----------------|-------------------------------|-------------------------------|-------------------|-------------------|-------------------------------|-------------------------------|
| 63              | 31.87                         | 75.21                         | 30.41             | 72.09             | 26.88                         | 64.97                         |
| 200             | 105                           | 244.2                         | 122.6             | 350.8             | 139.3                         | 343.7                         |
| 500             | 194.8                         | 494                           | 291.6             | 959               | 449.4                         | 1138                         |
| 5500            | 558.2                         | 1694                          | 1687              | 6742              | 7013                          | 17680                        |
| 14000           | 742.4                         | 2323                          | 2962              | 12440             | 16450                         | 41770                        |

Table 1: Total $c\bar{c}$ production cross sections at collider energies.

| $\sqrt{s}$ (GeV) | MRS D0' $\sigma_{b\bar{b}}^{\text{LO}}$ (μb) | MRS D0' $\sigma_{b\bar{b}}^{\text{NLO}}$ (μb) | GRV HO $\sigma_{b\bar{b}}^{\text{LO}}$ (μb) | GRV HO $\sigma_{b\bar{b}}^{\text{NLO}}$ (μb) | MRS D-' $\sigma_{b\bar{b}}^{\text{LO}}$ (μb) | MRS D-' $\sigma_{b\bar{b}}^{\text{NLO}}$ (μb) |
|-----------------|-------------------------------|-------------------------------|-------------------|-------------------|-------------------------------|-------------------------------|
| 63              | 0.0458                         | 0.0884                        | 0.0366            | 0.0684            | 0.0397                        | 0.0746                        |
| 200             | 0.981                          | 1.82                          | 0.818             | 1.51              | 0.796                         | 1.47                          |
| 500             | 4.075                          | 8.048                         | 4.276             | 8.251             | 3.847                         | 7.597                         |
| 5500            | 40.85                          | 112                           | 88.84             | 202.9             | 98.8                          | 224                           |
| 14000           | 78.46                          | 233.9                         | 222.9             | 538.4             | 296.8                         | 687.5                         |

Table 2: Total $b\bar{b}$ production cross sections at collider energies.
### $c\bar{c}$ Production $\sqrt{s} = 200$ GeV

| $p_T^2$ (GeV$^2$) | NLO | LO | $p_T^2$ (GeV$^2$) | NLO | $M$ (GeV) | NLO | LO |
|-------------------|-----|----|-------------------|-----|-----------|-----|----|
| 1                 | 232.5 | 102.2 | 1                 | 30.90 |           |      |    |
| 3                 | 37.93 | 15.14 | 3                 | 3.916 | 3         | 172.8 | 76.41 |
| 5                 | 12.37 | 4.589 | 5                 | 1.548 | 5         | 77.05 | 34.18 |
| 7                 | 5.362 | 1.924 | 7                 | 0.8435 | 7         | 22.60 | 9.611 |
| 9                 | 2.774 | 0.9704 | 9                 | 0.4770 | 9         | 8.548 | 3.429 |
| 11                | 1.589 | 0.5435 | 11                | 0.3287 | 11        | 3.671 | 1.427 |
| 13                | 1.003 | 0.3389 | 13                | 0.2203 | 13        | 1.863 | 0.6871 |
| 15                | 0.6715 | 0.2206 | 15                | 0.1608 | 15        | 0.9122 | 0.3438 |
| 17                | 0.4612 | 0.1542 | 17                | 0.1277 | 17        | 0.5120 | 0.1917 |
| 19                | 0.3291 | 0.1079 | 19                | 0.0925 | 19        | 0.3154 | 0.1095 |
| 21                | 0.2399 | 0.0812 | 21                | 0.0786 | 21        | 0.1883 | 0.0651 |
| 23                | 0.1857 | 0.0602 | 23                | 0.0589 | 23        | 0.1210 | 0.0415 |
| 25                | 0.1369 | 0.0428 | 25                | 0.0478 | 25        | 0.0689 | 0.0245 |
| 27                | 0.1088 | 0.0355 | 27                | 0.0356 | 27        | 0.0520 | 0.0166 |
| 29                | 0.0864 | 0.0280 | 29                | 0.0350 | 29        | 0.0364 | 0.0105 |
| 31                | 0.0697 | 0.0225 | 31                | 0.0282 | 31        | 0.0257 | 0.00785 |
| 33                | 0.0574 | 0.0191 | 33                | 0.0206 | 33        | 0.0151 | 0.00538 |
| 35                | 0.0478 | 0.0160 | 35                | 0.0214 | 35        | 0.0111 | 0.00383 |
| 37                | 0.0400 | 0.0132 | 37                | 0.0160 | 37        | 0.0678 | 0.00222 |
| 39                | 0.0343 | 0.0111 | 39                | 0.0135 | 39        | 0.0480 | 0.00198 |

Table 3:

[The rapidity-integrated $p_T^2$ distribution is given for single charm (NLO and Born) and the $p_T^2$ distribution in the range $|y| < 0.35$ is given for $c\bar{c}$ pair production (NLO only). The tabulated results have not been corrected for the rapidity bin width. The rapidity-integrated pair mass distribution is also given. All distributions are at $\sqrt{s} = 200$ GeV and calculated with MRS D-' parton distributions. Note the 2 GeV bin width for the distributions.]
Table 4:

| $p_T^2$ (GeV$^2$) | NLO | LO | $p_T^2$ (GeV$^2$) | NLO | M (GeV) | NLO | LO |
|-------------------|-----|----|-------------------|-----|---------|-----|----|
| 1                 | 739.7 | 332.0 | 1                 | 68.64 |         |     |    |
| 3                 | 134.8 | 538.7 | 3                 | 12.01 | 5       | 548.1 | 242.7 |
| 5                 | 47.37 | 17.43 | 5                 | 4.874 | 5       | 259.5 | 117.2 |
| 7                 | 22.19 | 7.656 | 7                 | 2.828 | 7       | 82.67 | 35.73 |
| 9                 | 12.08 | 4.054 | 9                 | 1.809 | 9       | 32.71 | 13.72 |
| 11                | 7.336 | 2.400 | 11                | 1.193 | 11      | 15.19 | 6.223 |
| 13                | 4.658 | 1.493 | 13                | 0.8440| 13      | 7.878 | 3.108 |
| 15                | 3.281 | 1.041 | 15                | 0.6417| 15      | 4.623 | 1.734 |
| 17                | 2.343 | 0.7234| 17                | 0.5002| 17      | 2.555 | 1.025 |
| 19                | 1.758 | 0.5370| 19                | 0.3983| 19      | 1.577 | 0.6242|
| 21                | 1.328 | 0.3980| 21                | 0.3345| 21      | 1.143 | 0.4171|
| 23                | 1.034 | 0.3052| 23                | 0.2467| 23      | 0.7373| 0.2623|
| 25                | 0.8118| 0.2512| 25                | 0.2098| 25      | 0.4798| 0.1905|
| 27                | 0.6481| 0.1950| 27                | 0.1596| 27      | 0.3227| 0.1220|
| 29                | 0.5411| 0.1618| 29                | 0.1371| 29      | 0.2817| 0.0886|
| 31                | 0.4544| 0.1284| 31                | 0.1283| 31      | 0.2028| 0.0673|
| 33                | 0.3600| 0.0997| 33                | 0.1137| 33      | 0.1530| 0.0472|
| 35                | 0.3006| 0.0897| 35                | 0.0909| 35      | 0.0997| 0.0379|
| 37                | 0.2701| 0.0754| 37                | 0.0758| 37      | 0.0837| 0.0293|
| 39                | 0.2318| 0.0643| 39                | 0.0750| 39      | 0.0627| 0.0250|

Table 4:

The rapidity-integrated $p_T^2$ distribution is given for single charm (NLO and Born) and the $p_T^2$ distribution in the range $|y| < 0.35$ is given for $c\bar{c}$ pair production (NLO only). The tabulated results have not been corrected for the rapidity bin width. The rapidity-integrated pair mass distribution is also given. All distributions are at $\sqrt{s} = 500$ GeV and calculated with MRS D' parton distributions. Note the 2 GeV bin width for the distributions.
| $p_T^2$ (GeV$^2$) | $d\sigma_{c}/dp_T^2$ (µb/2 GeV$^2$) | $d\sigma_{c}/dp_T^2dy$ (µb/2 GeV$^2$) | $d\sigma_{c}/dM$ (µb/2 GeV) |
|-----------------|---------------------------------|---------------------------------|-----------------------------|
| 1               | 10680.                          | 5146.                           | 1840.                       |
| 3               | 2453.                           | 989.                            | 441.5                       |
| 5               | 974.8                           | 350.1                           | 196.9                       |
| 7               | 502.2                           | 166.9                           | 111.3                       |
| 9               | 289.8                           | 93.10                           | 75.68                       |
| 11              | 186.6                           | 57.12                           | 51.60                       |
| 13              | 126.4                           | 37.65                           | 39.07                       |
| 15              | 90.91                           | 25.96                           | 27.28                       |
| 17              | 68.95                           | 19.99                           | 22.55                       |
| 19              | 51.44                           | 14.43                           | 18.47                       |
| 21              | 41.11                           | 11.17                           | 14.14                       |
| 23              | 33.29                           | 8.965                           | 13.53                       |
| 25              | 27.23                           | 7.328                           | 11.02                       |
| 27              | 22.28                           | 6.031                           | 9.862                       |
| 29              | 18.64                           | 4.836                           | 8.612                       |
| 31              | 16.10                           | 4.203                           | 6.944                       |
| 33              | 13.51                           | 3.417                           | 6.359                       |
| 35              | 11.55                           | 2.961                           | 5.050                       |
| 37              | 9.881                           | 2.548                           | 4.683                       |
| 39              | 9.078                           | 2.212                           | 4.680                       |

Table 5:
[The rapidity-integrated $p_T^2$ distribution is given for single charm (NLO and Born) and the $p_T^2$ distribution in the range $|y| < 1$ is given for $c\bar{c}$ pair production (NLO only). The tabulated results have not been corrected for the rapidity bin width. The rapidity-integrated pair mass distribution is also given. All distributions are at $\sqrt{s} = 5.5$ TeV and calculated with MRS D' parton distributions. Note the 2 GeV bin width for the distributions.]
\[ \sigma^c_\bar{c} \quad \sqrt{s} = 14 \text{ TeV} \]

| \( p_T^2 \) (GeV\(^2\)) | NLO | LO | \( p_T^2 \) (GeV\(^2\)) | NLO | LO | \( M \) (GeV) | NLO | LO |
|----------------|-----|----|----------------|-----|----|------------|-----|-----|
| 1              | 23650. | 11960. | 1              | 4594. |    |            |     |     |
| 3              | 6067.  | 2473.  | 3              | 1129. | 3  | 17250.    | 8046. |     |
| 5              | 2576.  | 918.6  | 5              | 513.6 | 5  | 10240.    | 4960. |     |
| 7              | 1368.  | 452.4  | 7              | 298.9 | 7  | 4119.     | 1840. |     |
| 9              | 838.8  | 256.5  | 9              | 195.3 | 9  | 1875.     | 820.2 |     |
| 11             | 545.2  | 162.7  | 11             | 143.4 | 11 | 986.3     | 413.9 |     |
| 13             | 371.4  | 108.3  | 13             | 103.9 | 13 | 554.6     | 232.4 |     |
| 15             | 273.5  | 78.46  | 15             | 78.28 | 15 | 337.7     | 137.8 |     |
| 17             | 206.6  | 55.28  | 17             | 60.18 | 17 | 226.5     | 88.37 |     |
| 19             | 162.1  | 45.82  | 19             | 51.11 | 19 | 162.      | 57.77 |     |
| 21             | 130.4  | 33.90  | 21             | 40.63 | 21 | 107.4     | 41.12 |     |
| 23             | 102.5  | 26.90  | 23             | 34.76 | 23 | 71.90     | 28.14 |     |
| 25             | 84.26  | 22.64  | 25             | 28.13 | 25 | 59.46     | 21.23 |     |
| 27             | 70.85  | 18.27  | 27             | 24.60 | 27 | 38.62     | 15.25 |     |
| 29             | 60.26  | 15.58  | 29             | 21.12 | 29 | 30.19     | 12.05 |     |
| 31             | 51.43  | 13.08  | 31             | 17.05 | 31 | 25.45     | 8.619 |     |
| 33             | 45.92  | 11.02  | 33             | 17.66 | 33 | 22.84     | 6.839 |     |
| 35             | 40.26  | 9.718  | 35             | 16.21 | 35 | 15.55     | 5.642 |     |
| 37             | 33.92  | 7.860  | 37             | 12.86 | 37 | 13.24     | 4.484 |     |
| 39             | 29.80  | 7.281  | 39             | 10.61 | 39 | 11.64     | 3.454 |     |

Table 6:

[The rapidity-integrated \( p_T^2 \) distribution is given for single charm (NLO and Born) and the \( p_T^2 \) distribution in the range \(|y| < 1\) is given for \( \sigma^c_\bar{c} \) pair production (NLO only). The tabulated results have not been corrected for the rapidity bin width. The rapidity-integrated pair mass distribution is also given. All distributions are at \( \sqrt{s} = 14 \text{ TeV} \) and calculated with MRS D' parton distributions. Note the 2 GeV bin width for the distributions.]
### $b\bar{b}$ Production $\sqrt{s} = 200$ GeV

| $p_T^b$ (GeV$^2$) | $d\sigma_b/dp_T^b$ (μb/2 GeV$^2$) | $d\sigma_M/dp_T^b dy$ (μb/2 GeV$^2$) | $d\sigma_M/dM$ (μb/2 GeV) |
|------------------|----------------------------------|----------------------------------|-------------------|
|                  | NLO | LO | NLO | M (GeV) | NLO | LO |
| 1                | 0.2201 | 0.1123 | 1 | 0.2073 | |
| 3                | 0.1704 | 0.0883 | 3 | 0.0524 | |
| 5                | 0.1558 | 0.0680 | 5 | 0.0263 | |
| 7                | 0.1064 | 0.0541 | 7 | 0.0170 | |
| 9                | 0.1035 | 0.0577 | 9 | 0.0118 | 9 | 0.0463 | 0.0320 |
| 11               | 0.0863 | 0.0406 | 11 | 0.00814 | 11 | 0.4363 | 0.2100 |
| 13               | 0.0605 | 0.0343 | 13 | 0.00660 | 13 | 0.3184 | 0.1640 |
| 15               | 0.0478 | 0.0255 | 15 | 0.00441 | 15 | 0.1987 | 0.1050 |
| 17               | 0.0458 | 0.0264 | 17 | 0.00341 | 17 | 0.1225 | 0.0637 |
| 19               | 0.0351 | 0.0190 | 19 | 0.00311 | 19 | 0.0753 | 0.0400 |
| 21               | 0.0359 | 0.0186 | 21 | 0.00274 | 21 | 0.0492 | 0.0249 |
| 23               | 0.0300 | 0.0139 | 23 | 0.00237 | 23 | 0.0318 | 0.0160 |
| 25               | 0.0244 | 0.0122 | 25 | 0.00201 | 25 | 0.0214 | 0.0104 |
| 27               | 0.0216 | 0.0116 | 27 | 0.00183 | 27 | 0.0145 | 0.00688 |
| 29               | 0.0202 | 0.0103 | 29 | 0.00156 | 29 | 0.0091 | 0.00466 |
| 31               | 0.0171 | 0.0080 | 31 | 0.00147 | 31 | 0.0069 | 0.00321 |
| 33               | 0.0159 | 0.0083 | 33 | 0.00121 | 33 | 0.0047 | 0.00215 |
| 35               | 0.0125 | 0.0054 | 35 | 0.00111 | 35 | 0.0032 | 0.00154 |
| 37               | 0.0101 | 0.0055 | 37 | 0.00111 | 37 | 0.0022 | 0.00108 |
| 39               | 0.0097 | 0.0049 | 39 | 0.00086 | 39 | 0.0016 | 0.00075 |

Table 7:
[The rapidity-integrated $p_T^b$ distribution is given for single $b$ quarks (NLO and Born) and the $p_T^b$ distribution in the range $|y| < 0.35$ is given for $b\bar{b}$ pair production (NLO only). The tabulated results have not been corrected for the rapidity bin width. The rapidity-integrated pair mass distribution is also given. All distributions are at $\sqrt{s} = 200$ GeV and calculated with MRS D-′ parton distributions. Note the 2 GeV bin width for the distributions.]
### $b\bar{b}$ Production $\sqrt{s} = 500$ GeV

| $p_T^2$ (GeV^2) | NLO | LO | NLO | LO | $p_T^2$ (GeV^2) | NLO | LO | $M$ (GeV) | NLO | LO |
|-----------------|-----|----|-----|----|-----------------|-----|----|-----------|-----|----|
| 1               | 0.9809 | 0.4798 | 1 | 0.3427 | 1 | 0.3427 | 1 | 0.3427 |
| 3               | 0.7911 | 0.4024 | 3 | 0.2503 | 3 | 0.2503 | 3 | 0.2503 |
| 5               | 0.6490 | 0.3362 | 5 | 0.1260 | 5 | 0.1260 | 5 | 0.1260 |
| 7               | 0.5492 | 0.2801 | 7 | 0.0818 | 7 | 0.0818 | 7 | 0.0818 |
| 9               | 0.4528 | 0.2358 | 9 | 0.0558 | 9 | 0.0558 | 9 | 0.0558 |
| 11              | 0.3807 | 0.1987 | 11 | 0.0426 | 11 | 0.0426 | 11 | 0.0426 |
| 13              | 0.3256 | 0.1688 | 13 | 0.0341 | 13 | 0.0341 | 13 | 0.0341 |
| 15              | 0.2781 | 0.1433 | 15 | 0.0285 | 15 | 0.0285 | 15 | 0.0285 |
| 17              | 0.2428 | 0.1248 | 17 | 0.0235 | 17 | 0.0235 | 17 | 0.0235 |
| 19              | 0.2068 | 0.1057 | 19 | 0.0197 | 19 | 0.0197 | 19 | 0.0197 |
| 21              | 0.1824 | 0.0932 | 21 | 0.0169 | 21 | 0.0169 | 21 | 0.0169 |
| 23              | 0.1595 | 0.0811 | 23 | 0.0147 | 23 | 0.0147 | 23 | 0.0147 |
| 25              | 0.1429 | 0.0719 | 25 | 0.0133 | 25 | 0.0133 | 25 | 0.0133 |
| 27              | 0.1240 | 0.0622 | 27 | 0.0122 | 27 | 0.0122 | 27 | 0.0122 |
| 29              | 0.1108 | 0.0557 | 29 | 0.0109 | 29 | 0.0109 | 29 | 0.0109 |
| 31              | 0.0984 | 0.0492 | 31 | 0.0098 | 31 | 0.0098 | 31 | 0.0098 |
| 33              | 0.0898 | 0.0435 | 33 | 0.0085 | 33 | 0.0085 | 33 | 0.0085 |
| 35              | 0.0789 | 0.0387 | 35 | 0.0076 | 35 | 0.0076 | 35 | 0.0076 |
| 37              | 0.0716 | 0.0350 | 37 | 0.0071 | 37 | 0.0071 | 37 | 0.0071 |
| 39              | 0.0646 | 0.0319 | 39 | 0.0074 | 39 | 0.0074 | 39 | 0.0074 |

**Table 8:**  
[The rapidity-integrated $p_T^2$ distribution is given for single $b$ quarks (NLO and Born) and the $p_T^2$ distribution in the range $|y| < 0.35$ is given for $b\bar{b}$ pair production (NLO only). The tabulated results have not been corrected for the rapidity bin width. The rapidity-integrated pair mass distribution is also given. All distributions are at $\sqrt{s} = 500$ GeV and calculated with MRS D' parton distributions. Note the 2 GeV bin width for the distributions.]
Table 9:

| $p_T^2$ (GeV$^2$) | NLO  | LO   | $p_T^2$ (GeV$^2$) | NLO  | LO   | $M$ (GeV) | NLO  | LO   |
|------------------|------|------|------------------|------|------|-----------|------|------|
| 1                | 23.59| 11.22| 1                | -2.366|      |           |      |      |
| 3                | 19.38| 9.650| 3                | 12.80 |      |           |      |      |
| 5                | 16.25| 8.253| 5                | 6.634 |      |           |      |      |
| 7                | 13.84| 7.028| 7                | 4.424 |      |           |      |      |
| 9                | 11.83| 6.065| 9                | 3.303 | 9    | 6.102     | 2.498|      |
| 11               | 10.14| 5.148| 11               | 2.496 | 11   | 42.57     | 19.58|      |
| 13               | 8.916| 4.469| 13               | 1.946 | 13   | 37.41     | 18.51|      |
| 15               | 7.776| 3.890| 15               | 1.726 | 15   | 27.66     | 13.89|      |
| 17               | 6.883| 3.424| 17               | 1.439 | 17   | 20.00     | 9.930|      |
| 19               | 6.132| 3.004| 19               | 1.199 | 19   | 14.41     | 7.187|      |
| 21               | 5.436| 2.650| 21               | 1.073 | 21   | 10.53     | 5.190|      |
| 23               | 4.825| 2.296| 23               | 0.9512| 23   | 8.007     | 3.863|      |
| 25               | 4.357| 2.098| 25               | 0.8151| 25   | 6.028     | 2.911|      |
| 27               | 3.959| 1.875| 27               | 0.7535| 27   | 4.583     | 2.202|      |
| 29               | 3.545| 1.666| 29               | 0.6718| 29   | 3.577     | 1.721|      |
| 31               | 3.208| 1.526| 31               | 0.5796| 31   | 2.879     | 1.342|      |
| 33               | 2.950| 1.367| 33               | 0.5276| 33   | 2.248     | 1.078|      |
| 35               | 2.683| 1.207| 35               | 0.5491| 35   | 1.813     | 0.8730|     |
| 37               | 2.468| 1.131| 37               | 0.4692| 37   | 1.507     | 0.7100|     |
| 39               | 2.255| 1.034| 39               | 0.4334| 39   | 1.261     | 0.5682|     |

The rapidity-integrated $p_T^2$ distribution is given for single $b$ quarks (NLO and Born) and the $p_T^2$ distribution in the range $|y| < 1$ is given for $b\bar{b}$ pair production (NLO only). The tabulated results have not been corrected for the rapidity bin width. The rapidity-integrated pair mass distribution is also given. All distributions are at $\sqrt{s} = 5.5$ TeV and calculated with MRS D' parton distributions. Note the 2 GeV bin width for the distributions.
**Table 10:**

The rapidity-integrated $p_T^2$ distribution is given for single $b$ quarks (NLO and Born) and the $p_T^2$ distribution in the range $|y| < 1$ is given for $b\bar{b}$ pair production (NLO only). The tabulated results have not been corrected for the rapidity bin width. The rapidity-integrated pair mass distribution is also given. All distributions are at $\sqrt{s} = 14$ TeV and calculated with MRS D′ parton distributions. Note the 2 GeV bin width for the distributions.

| $p_T^2$ (GeV$^2$) | NLO | LO | $p_T^2$ (GeV$^2$) | NLO | $M$ (GeV) | NLO | LO |
|-----------------|-----|----|-----------------|-----|----------|-----|----|
| 1               | 68.43 | 32.54 | 1               | -13.36 | 9 | 17.57 | 6.876 |
| 3               | 56.73 | 28.24 | 3               | 34.99 | 11 | 124.0 | 55.90 |
| 5               | 47.74 | 24.25 | 5               | 17.94 | 13 | 112.4 | 54.74 |
| 7               | 41.32 | 20.92 | 7               | 11.83 | 15 | 85.11 | 42.17 |
| 9               | 35.45 | 18.10 | 9               | 8.519 | 17 | 62.92 | 30.97 |
| 11              | 30.61 | 15.55 | 11              | 6.833 | 19 | 46.41 | 22.58 |
| 13              | 27.07 | 13.60 | 13              | 5.537 | 21 | 34.27 | 16.62 |
| 15              | 23.97 | 11.93 | 15              | 4.665 | 23 | 26.12 | 12.44 |
| 17              | 21.22 | 10.41 | 17              | 3.813 | 25 | 19.89 | 9.457 |
| 19              | 18.86 | 9.192 | 19              | 3.392 | 27 | 15.51 | 7.304 |
| 21              | 16.84 | 8.225 | 21              | 3.125 | 29 | 11.93 | 5.673 |
| 23              | 15.20 | 7.227 | 23              | 2.618 | 31 | 9.610 | 4.538 |
| 25              | 13.71 | 6.477 | 25              | 2.328 | 33 | 7.908 | 3.587 |
| 27              | 12.61 | 5.878 | 27              | 2.112 | 35 | 6.267 | 2.966 |
| 29              | 11.20 | 5.215 | 29              | 1.772 | 37 | 5.132 | 2.402 |
| 31              | 10.43 | 4.710 | 31              | 1.811 | 39 | 4.323 | 2.017 |
| 33              | 9.520 | 4.368 | 33              | 1.588 |       |       |     |
| 35              | 8.651 | 3.962 | 35              | 1.409 |       |       |     |
| 37              | 7.795 | 3.492 | 37              | 1.349 |       |       |     |
| 39              | 7.272 | 3.245 | 39              | 1.279 |       |       |     |

**b\bar{b} Production $\sqrt{s} = 14$ TeV**
Figure Captions

1. (a) Gluon distributions from GRV HO (solid), MRS D0’ (dashed), MRS D’ (dot-dashed) at $Q = 2.4$ GeV and GRV HO (dotted) at $Q = 1.2$ GeV. (b) The running of the coupling constant with scale.

2. Investigation of uncertainties in the total cross section as a function of scale. Variation of the $c\bar{c}$ production cross sections with scale at (a) RHIC and (b) LHC. Variation of the $b\bar{b}$ production cross sections with scale at (c) RHIC and (d) LHC. Variation of the $c\bar{c}$ production cross sections at $\sqrt{s}$ at 200 GeV with $\mu_R$ at fixed $\mu_F$ (e) and with $\mu_F$ at fixed $\mu_R$ (b). In each case, the circles represent the NLO calculation, the crosses, the Born calculation.

3. Total charm production cross sections from $pp$ and $pA$ measurements \cite{18, 19, 20, 21, 22} compared to calculations. The curves are: MRS D’ $m_c = 1.2$ GeV, $\mu = 2m_c$ (solid); MRS D0’ $m_c = 1.2$ GeV, $\mu = 2m_c$ (dashed); GRV HO $m_c = 1.3$ GeV, $\mu = m_c$ (dot-dashed); GRV HO $m_c = 1.5$ GeV, $\mu = m_c$ (dotted).

4. (a) Total charm production cross sections from $\pi^- p$ measurements \cite{18, 23, 24, 25, 26, 27, 28} compared to calculations. The curves are: GRV HO $m_c = 1.3$ GeV, $\mu = m_c$ on a nucleon (solid) and proton target (dashed); MRS D’ $m_c = 1.2$ GeV, $\mu = 2m_c$ with SMRS P2 (dot-dashed) and SMRS P1 (dotted) on a proton target. (b) The $b\bar{b}$ production cross section from $\pi^- p$ interactions \cite{29, 30, 31, 32}. The calculations use $m_b = 4.75$ GeV and $\mu = m_b$. The curves use GRV HO (solid) and MRS D’ with SMRS P1 (dashed).

5. Comparison with $D$ meson (a) $p_T^2$ and (b) $x_F$ distributions at 800 GeV \cite{19}. The NLO calculations are with MRS D’ (solid) and GRV HO (dashed) parton distributions.

6. Comparison with $D\bar{D}$ production for (a) $p_T^2$ and (b) $M$ and (c) $\phi$ at 800 GeV \cite{30}. The NLO calculations are with MRS D’ (solid) and GRV HO (dashed) parton distributions.

7. Comparison with $b$ quark production cross sections at (a) UA1 [37] and (b) CDF [38]. The NLO calculations are with MRS D’ (solid) and GRV HO (dashed) parton distributions.

8. Predictions for $c$ and $c\bar{c}$ production at $\sqrt{s} = 200$ GeV with MRS D’ distributions. The $c$ quark $p_T$ distributions at NLO (solid) are shown in (a) and the rapidity distributions at LO (dashed) and NLO (solid) are shown in (b). The $c\bar{c}$ pair distributions are shown in (c)-(f). The LO (dashed) distributions are shown only for mass and rapidity. Additionally, the $p_T$ and $p_T^p$ distributions are shown with a central cut in rapidity. (The rapidity bin widths are removed.) The corresponding distributions
from HIJING are also shown, again with the rapidity bin width divided out.

9. Predictions for $c$ and $\bar{c}c$ production at $\sqrt{s} = 200$ GeV with GRV HO distributions. The corresponding distributions from HIJING are also shown.

10. Predictions for $c$ and $\bar{c}c$ production at $\sqrt{s} = 500$ GeV with MRS D-′ distributions.

11. Predictions for $c$ and $\bar{c}c$ production at $\sqrt{s} = 500$ GeV with GRV HO distributions.

12. Predictions for $c$ and $\bar{c}c$ production at $\sqrt{s} = 5.5$ TeV with MRS D-′ distributions. The corresponding distributions from HIJING are also shown.

13. Predictions for $c$ and $\bar{c}c$ production at $\sqrt{s} = 5.5$ TeV with GRV HO distributions. The corresponding distributions from HIJING are also shown.

14. Predictions for $c$ and $\bar{c}c$ production at $\sqrt{s} = 14$ TeV with MRS D-′ distributions.

15. Predictions for $c$ and $\bar{c}c$ production at $\sqrt{s} = 14$ TeV with GRV HO distributions.

16. Predictions for $b$ and $\bar{b}b$ production at $\sqrt{s} = 200$ GeV with MRS D-′ distributions.

17. Predictions for $b$ and $\bar{b}b$ production at $\sqrt{s} = 200$ GeV with GRV HO distributions.

18. Predictions for $b$ and $\bar{b}b$ production at $\sqrt{s} = 500$ GeV with MRS D-′ distributions.

19. Predictions for $b$ and $\bar{b}b$ production at $\sqrt{s} = 500$ GeV with GRV HO distributions.

20. Predictions for $b$ and $\bar{b}b$ production at $\sqrt{s} = 5.5$ TeV with MRS D-′ distributions.

21. Predictions for $b$ and $\bar{b}b$ production at $\sqrt{s} = 5.5$ TeV with GRV HO distributions.

22. Predictions for $b$ and $\bar{b}b$ production at $\sqrt{s} = 14$ TeV with MRS D-′ distributions.

23. Predictions for $b$ and $\bar{b}b$ production at $\sqrt{s} = 14$ TeV with GRV HO distributions.
24. Dilepton (a) mass, (b) rapidity, and (c) $p_T$ distributions at $\sqrt{s} = 200$ GeV from $c\bar{c}$ decays calculated using MRS D′ distributions are shown.

25. Dilepton distributions at $\sqrt{s} = 200$ GeV from $b\bar{b}$ decays.

26. Dilepton distributions at $\sqrt{s} = 5.5$ TeV from correlated $c\bar{c}$ decays.

27. Dilepton distributions at $\sqrt{s} = 5.5$ TeV from uncorrelated $c\bar{c}$ decays.

28. Dilepton distributions at $\sqrt{s} = 5.5$ TeV from $b\bar{b}$ decays.
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