Research concerning the evaluation of the connection forces in the joints of the sucker rod pumping units mechanism

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Abstract: At present, the sucker rod pumping installations are the most used in the case of the wells in production, when an eruptive exploitation is not possible. The practice has demonstrated that an important role in increasing safety in the operation of the pumping units has the design of the various component bearings because of the extremely high values of the connection forces to which they are loaded. That is why it is necessary to establish as accurately as possible the values of these connecting forces. In this paper is analyzed the dynamics of a conventional pumping unit mechanism. The dynamic study which allows establishing the values of the connecting forces in the joints is performed within the Assur structural groups. The dynamic analysis was implemented into a computer program using Maple programming environment and finally it has been presented some simulation results in the case of a C-320D-256-100 pumping unit.

Keywords: pumping unit, connection forces, dynamics

1. Introduction
At present, since most wells can not provide an eruptive exploitation, this is accomplished by pumping using the sucker rod pumping installations [1]. The functioning analysis of these installations must take into account many factors of which we can mention: the configuration of the pumping unit, the dimensions of the component elements, the pumping speed and the size of the sucker rod and of the tubing which contributes to the greatest extent to the load of the pumping unit. An important role in increasing safety in the operation of the pumping units has the design of the various component bearings because of the extremely high values of the connection forces to which they are loaded. In this regard the values of these connecting forces have to be established as accurately as possible. In this paper it is analyzed the dynamics of the mechanism of the conventional pumping units, thus establishing the variation on the cinematic cycle of the values of the connecting forces. The dynamic study is performed within the Assur structural groups [2-4]. A computer program using Maple programming environment [5] has been developed and some simulation results regarding the variation
on a cinematic cycle of the values of the connection forces in the joints of a C-320D-256-100 pumping unit are presented.

2. Pumping unit mechanism dynamics
A conventional pumping unit which is part of a sucker rod pumping installation is shown in figure 1 and in figure 2 is presented the cinematic scheme of its mechanism.

![Figure 1. Conventional pumping unit.](image)

With $C_1$, $C_2$, and $C_3$ were noted the mass centres of the cranks, connection rods and of the rocker, respectively; $m_{CGR}$ is the total mass of the counterweights; $m_{L1}$ is the mass of the two crank pin bearings; $m_{L2}$ is the mass of the equalizer bearing; $m_v$ is the mass of the equalizer considered to be concentrated in the point $B$; $m_{CH}$ is the mass of the horsehead considered to be concentrated in the point $D'$ on the rocker.

![Figure 2. Cinematic scheme of the mechanism of a conventional pumping unit.](image)

For establishing the position of the connection rods 2 and of the rocker 3, the angles $\varphi_2$ and $\varphi_3$ are calculated by solving the following equations system obtained by projecting the vectorial equation corresponding to the closed contour $O-A-B-C-O$ on the $x$ and $y$ axes:
\[ \begin{aligned}
I_1 \cos \varphi_1 + I_2 \cos \varphi_2 + I_3 \cos \varphi_3 - x_C &= 0 \\
I_1 \sin \varphi_1 + I_2 \sin \varphi_2 + I_3 \sin \varphi_3 - y_C &= 0
\end{aligned} \] (1)

where \( I_1 = OA \); \( I_2 = AB \), \( I_3 = BC \) and \( x_C \) and \( y_C \) are the coordinates of the point \( C \) where is located the centre bearing.

The angular velocities and the angular accelerations of the connection rods and of the rocker may be calculated by deriving in relation to time the expressions of the angles \( \varphi_2(\varphi_1) \) and \( \varphi_3(\varphi_1) \), with the following relations:

\[ \omega_j = \dot{\varphi}_j = \frac{d\varphi_j}{d\varphi_1} \frac{d\varphi_1}{dt} = \omega_1 \frac{d\varphi_j}{d\varphi_1}; \quad j = 2,3 \] (2)

\[ \varepsilon_j = \ddot{\varphi}_j = \varepsilon_1 + \omega_1^2 \frac{d^2\varphi_j}{d\varphi_1^2}; \quad j = 2,3 \] (3)

where \( \omega_1 \) and \( \varepsilon_1 \) are the angular velocity and the angular acceleration, respectively, of the cranks.

Having established the values of the angles \( \varphi_2 \) and \( \varphi_3 \), it were determined the coordinates of the various points on the mechanism, including those of the mass centres of the component elements and of the point \( D \) where acting the force \( F \) (figure 2) at the polished rod. The projections on the \( x \) and \( y \) axes of the speeds and of the accelerations of any point on the mechanism may be calculated by deriving in relation to time the expressions of its coordinates, for example in the case of the point \( D \), by using the following relations:

\[ \begin{aligned}
(v_D)_x &= \dot{x}_D = \frac{dx_D}{d\varphi_1} \frac{d\varphi_1}{dt} = \omega_1 \frac{dx_D}{d\varphi_1} \\
(v_D)_y &= \dot{y}_D = \frac{dy_D}{d\varphi_1} \frac{d\varphi_1}{dt} = \omega_1 \frac{dy_D}{d\varphi_1} \\
(a_D)_x &= (v_D)_x = \varepsilon_1 \frac{dx_D}{d\varphi_1} + \omega_1^2 \frac{d^2x_D}{d\varphi_1^2} \\
(a_D)_y &= (v_D)_y = \varepsilon_1 \frac{dy_D}{d\varphi_1} + \omega_1^2 \frac{d^2y_D}{d\varphi_1^2}
\end{aligned} \] (4)

The mechanism comprises a single Assur structural group, namely a dyad composed of the connection rods and the rocker. Figure 3 shows the loads corresponding to the elements of the dyad, where \( G_j = m_j \cdot g \); \( j = 2,3 \), are the weight forces of the connecting rods and of the rocker and \( m_2 \) and \( m_3 \) are their masses; \( g = 9.81 \text{ m/s}^2 \) is the gravitational acceleration that acts in the opposite direction of the \( y \) axis; \( G_{L1} = m_{L1} \cdot g \) is the weight force of the two crank pin bearings (it has been considered that only half of the weight of these bearings is concentrated on the connecting rods, the rest remaining concentrated on the cranks); \( G_{L2} = m_{L2} \cdot g \) is the weight force of the equalizer bearing; \( G_p = m_p \cdot g \) is the weight force of the equalizer; \( G_{CB} = m_{CB} \cdot g \) is the weight force of the horsehead; \( F_y = -m_j \cdot \vec{a}_C; \quad j = 2,3 \), are the inertia forces corresponding to the connecting rods and to the rocker and \( \vec{a}_C; \quad j = 2,3 \) are the accelerations of their mass centres; \( \vec{M}_y = -J_{Cj} \cdot \vec{e}_j; \quad j = 2,3 \), are the inertia moments, where \( J_{Cj}; \quad j = 2,3 \), are the mass moments of inertia corresponding to the connecting rods and to the rocker; \( \vec{F}_{L1} = -m_{L1} \cdot \vec{a}_A \), \( \vec{F}_{L2} = -m_{L2} \cdot \vec{a}_B \), \( \vec{F}_p = -m_p \cdot \vec{a}_B \) and \( \vec{F}_{CB} = -m_{CB} \cdot \vec{a}_D \) are the inertia forces corresponding to the two crank pin bearings, equalizer bearing, equalizer and to the horsehead, respectively; \( \vec{F} \) is the force acting at the end of the polished rod. Figure 3 shows also the
The projections on $x$ and $y$ axes of the connection forces from the two crank pin bearings ($2\overrightarrow{F}_{12x}$ and $2\overrightarrow{F}_{12y}$) and of the connection force from the centre bearing ($\overrightarrow{F}_{03x}$ and $\overrightarrow{F}_{03y}$).

\begin{equation}
\left( \sum M_B \right)_2 = 0 \Rightarrow 2BA \times (\overrightarrow{F}_{12x} + \overrightarrow{F}_{12y}) + \frac{1}{2} BA \times (\overrightarrow{G}_{L1} + \overrightarrow{F}_{dL1}) + BC \times (\overrightarrow{G}_2 + \overrightarrow{F}_{12}) + \overrightarrow{M}_{12} = 0
\end{equation}

\begin{equation}
\left( \sum M_C \right)_{2-3} = 0 \Rightarrow 2CA \times (\overrightarrow{F}_{12x} + \overrightarrow{F}_{12y}) + \frac{1}{2} CA \times (\overrightarrow{G}_{L1} + \overrightarrow{F}_{dL1}) + CC \times (\overrightarrow{G}_2 + \overrightarrow{F}_{12}) + CB \times (\overrightarrow{G}_{L2} + \overrightarrow{F}_{dL2} + \overrightarrow{G}_{G} + \overrightarrow{F}_{dG}) + CC \times (\overrightarrow{G}_3 + \overrightarrow{F}_{13}) + CD \times (\overrightarrow{G}_{CB} + \overrightarrow{F}_{dCB}) + \overrightarrow{CD} \times \overrightarrow{F} + \overrightarrow{M}_{12} + \overrightarrow{M}_{13} = 0
\end{equation}

The projections on $x$ and $y$ axes of the connection force $\overrightarrow{F}_{32}$ ($\overrightarrow{F}_{32x}$ and $\overrightarrow{F}_{32y}$) from the equalizer bearing acting on the equalizer may be established from the following dynamic equations in forces:

\begin{equation}
\left( \sum F_x \right)_2 = 0 \Rightarrow \overrightarrow{F}_{32x} + 2\overrightarrow{F}_{12x} + \frac{1}{2} \overrightarrow{F}_{dL1x} + \overrightarrow{F}_{L2x} + \frac{1}{2} \overrightarrow{F}_{dL2x} + \overrightarrow{F}_{inr} = 0
\end{equation}

\begin{equation}
\left( \sum F_y \right)_2 = 0 \Rightarrow \overrightarrow{F}_{32y} + 2\overrightarrow{F}_{12y} + \frac{1}{2} \overrightarrow{F}_{dL1y} + \overrightarrow{F}_{L2y} + \frac{1}{2} \overrightarrow{F}_{dL2y} + \overrightarrow{F}_{inr} - \frac{1}{2} \overrightarrow{G}_{L1} - \overrightarrow{G}_{2} - \frac{1}{2} \overrightarrow{G}_{L2} - \overrightarrow{G}_{G} = 0
\end{equation}

In the previous equations it has been considered that the mass of the equalizer bearing is distributed evenly on the rocker and the equalizer.

The projections on $x$ and $y$ axes of the connection force $\overrightarrow{F}_{03}$ from the centre bearing may be calculated from the following dynamic equilibrium equations in forces:

\begin{equation}
\left( \sum F_x \right)_3 = 0 \Rightarrow \overrightarrow{F}_{03x} + \overrightarrow{F}_{23x} + \frac{1}{2} \overrightarrow{F}_{dL2x} + \overrightarrow{F}_{13x} + \overrightarrow{F}_{CBx} = 0
\end{equation}

\begin{equation}
\left( \sum F_y \right)_3 = 0 \Rightarrow \overrightarrow{F}_{03y} + \overrightarrow{F}_{23y} + \frac{1}{2} \overrightarrow{F}_{dL2y} + \overrightarrow{F}_{13y} + \overrightarrow{F}_{CBy} - \frac{1}{2} \overrightarrow{G}_{L2} - \overrightarrow{G}_{3} - \overrightarrow{G}_{CB} - \overrightarrow{F} = 0
\end{equation}

where $\overrightarrow{F}_{23x} = -\overrightarrow{F}_{12x}$ and $\overrightarrow{F}_{23y} = -\overrightarrow{F}_{12y}$.
Figure 4 shows the loads corresponding to the cranks, where \( G_i = m_i \cdot g \) is the weight force of the cranks \( (m_i) \) is their mass; \( F_{i1} = -m_i \cdot \ddot{\alpha}_C \) is the inertia force corresponding to the cranks and \( \ddot{\alpha}_C \) is the acceleration of their mass centres; \( G_{CG} = m_{CG} \cdot g \) and \( F_{iCG} = -m_{CG} \cdot \ddot{\alpha}_P \) are the weight force and the inertia force, respectively, corresponding to the counterweights; \( \ddot{\alpha}_P \) is the acceleration of point \( A' \) where the total mass of the counterweights is considered to be concentrated. Also, figure 4 shows the projections on \( x \) and \( y \) axes of the connection forces from the two joints connecting the two cranks and the output shaft of the reducer (2\( F_{1lx} \) and 2\( F_{1ly} \)), the connection forces from the two crank pin bearings (2\( F_{2lx} = -2F_{12x} \) and 2\( F_{2ly} = -2F_{12y} \)) and the motor moment \( M_m \) at the cranks shaft.

\[ \sum F_{ix} = 0 \Rightarrow 2F_{0lx} + 2F_{2lx} + \frac{1}{2} F_{d1lx} + F_{i1x} + F_{iCGx} = 0 \]  
\[ \sum F_{iy} = 0 \Rightarrow 2F_{0ly} + 2F_{2ly} + \frac{1}{2} F_{d1ly} + F_{i1y} + F_{iCGy} - \frac{1}{2} G_{L1} - G_i - G_{CG} = 0 \]

Next, the motor moment at the cranks shaft can be established from an equation in moments calculated relative to the point \( O \):

\[ \sum M_{Ox} = 0 \Rightarrow M_m + 2O \cdot A \times \left( F_{2lx} + F_{2ly} \right) + \frac{1}{2} O \cdot A \times (G_{L1} + F_{1lx}) \]

\[ + O \cdot C_1 \times (G_i + F_{1lx}) + O \cdot A \times (G_{CG} + F_{iCG}) = 0 \]

3. Simulation results

The cinematic and dynamic analysis of the conventional pumping units mechanism has been transposed into a computer program using Maple programming environment. Further, are presented some simulation results in the case of a C-320D-256-100 pumping unit. The dimensions of the component elements of a C-320D-256-100 pumping unit (figure 2) produced by \textit{Lufkin} \[6\] are: \( OA = 42 \) in. (1.0668 m); \( AB = 132 \) in. (3.3528 m); \( BC = 110.07 \) in. (2.8211 m); \( CD = 129 \) in. (3.2766 m). The coordinates of the point \( C \) are \([6]\): \( x_C = 111 \) in. (2.8194 m) and \( y_C = 136 \) in. (3.4544 m). The total length of the cranks is: \( OA' = 95 \) in. (2.413 m). The functioning angular speed of the cranks expressed in rotations per minute is equal to 5 rot/min. The simulations have been accomplished by considering the following values of the other parameters involved in the calculation of the connection forces and of the motor moment: \( OA' = 65 \) in. (1.651 m); \( CD' = 110 \) in. (2.794 m); \( m_{L1} = 55 \) kg; \( m_{L2} = 125 \) kg; \( m_{p} = 400 \) kg; \( m_{CB} = 443 \) kg; \( q_1 = 566 \) kg/m; \( q_2 = 25.7 \) kg/m; \( q_3 = 217.5 \) kg/m \((q_1, q_2 \) and \( q_3 \) are the linear masses of the cranks, connecting rods and of the rocker, respectively). In figure 5 is presented the variation on a cinematic cycle of the force \( F \) acting at the end of the polished rod.
In figure 6 it is presented the variation on a cinematic cycle of the motor moment $M_m$ and in figures 7÷10 are presented the variations on a cinematic cycle of the connection forces $F_{01}, F_{12}, F_{32}$ and $F_{03}$, respectively. The curves 1 correspond to the case when the angular speed of the cranks is equal to 5 rot/min and the curves 2 and 3 correspond to the cases when the angular speed of the cranks is equal to 7 rot/min and 10 rot/min, respectively. In these two last cases we have considered an increase of the force $F$ values compared to the initial situation when the cranks speed is 5 rot/min, with 8% and 19.5%, respectively.

Figure 6. The variation on a cinematic cycle of the motor moment $M_m$ when the angular speed of the cranks is equal to 5 rot/min (curve 1), 7 rot/min (curve 2) and 10 rot/min (curve 3).

Figure 7. The variation on a cinematic cycle of the connection force $F_{01}$ when the angular speed of the cranks is equal to 5 rot/min (curve 1), 7 rot/min (curve 2) and 10 rot/min (curve 3).
Figure 8. The variation on a cinematic cycle of the connection force $F_{12}$ when the angular speed of the cranks is equal to 5 rot/min (curve 1), 7 rot/min (curve 2) and 10 rot/min (curve 3).

Figure 9. The variation on a cinematic cycle of the connection force $F_{13}$ when the angular speed of the cranks is equal to 5 rot/min (curve 1), 7 rot/min (curve 2) and 10 rot/min (curve 3).

Figure 10. The variation on a cinematic cycle of the connection force $F_{03}$ when the angular speed of the cranks is equal to 5 rot/min (curve 1), 7 rot/min (curve 2) and 10 rot/min (curve 3).

Figure 6 shows that the motor moment variation curve retains the same allure in all three analyzed cases and its extreme values are relatively close. Figures 7–10 show that the connecting forces in the bearings of the analyzed pumping unit mechanism have extremely high values, in this regard
highlighting the $F_{03}$ connecting force in the central bearing. On the other hand it can be noticed that with the increase of the values of the force $F$ there is an important increase of the maximum values of all the connecting forces in the bearings of the pumping unit mechanism.

4. Conclusions
In this paper it was analyzed the dynamics of the mechanism of the conventional sucker rod pumping units. The dynamic study was first performed within the component dyad composed of the connecting rods and the rocker and then it was continued on its leader cranks. A computer program using Maple programming environment has been developed and so it was possible to establish the variation of the values of the connecting forces in the mechanism bearings and of the motor moment on the cinematic cycle, simulation results being presented in the case of a C-320D-256-100 pumping unit. So, it have been highlighted the extremely high values of these connecting forces, in particular the values of the connection force acting in the centre bearing. Also it have been highlighted the increases in the maximum values of these connection forces with the increase of the values of the force at the polished rod.

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