The Lightest Neutralino in Supersymmetric Standard Model with Arbitrary Higgs Sectors

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Abstract

We consider the neutralino mass matrix in a supersymmetric theory based on SU(2) × U(1) gauge group with arbitrary number of singlet, doublet and triplet Higgs superfields. We derive an upper bound on the mass of the lightest neutralino, and a lower bound on the mass of the heaviest neutralino, in such a theory. Assuming grand unification of the gauge couplings, the upper bound on the mass of the lightest neutralino can be expressed in terms of the gluino mass. For a gluino mass of 200 GeV, the tree level upper bound on the mass of the lightest neutralino is 92 GeV, which increases to 166.5 GeV for a 1 TeV gluino. We also discuss the effect of dominant one-loop radiative corrections on these bounds.

1Permanent address
In supersymmetric gauge theories, each fermion and boson of the Standard Model is accompanied by its supersymmetric partner, transforming in an identical manner under the gauge group [1]. In supersymmetric theories with R-parity conservation [1], the lightest supersymmetric particle (LSP) is expected to be the lightest neutralino, which is the lightest mixture of the fermionic partners of the neutral Higgs and neutral electroweak gauge bosons. In order to give masses to quarks and leptons, and to cancel triangle gauge anomalies, at least two Higgs doublets $H_1 = (H_1^0, H_1^-)$ and $H_2 = (H_2^+, H_2^0)$, with opposite hypercharge ($Y(H_1) = -1, Y(H_2) = +1$), are required in the minimal version of the Supersymmetric Standard Model (MSSM) [1]. The fermionic partners of these Higgs bosons mix with the fermionic partners of the gauge bosons to produce four neutralino states $\tilde{\chi}_i^0$, $i = 1, 2, 3, 4$, and two chargino states $\tilde{\chi}_i^\pm$, $i = 1, 2$, in the MSSM. In the nonminimal supersymmetric standard model containing a Higgs singlet, besides the two Higgs doublets of the minimal model, the mixing of fermionic partners of neutral Higgs and gauge bosons produces five neutralino states. The neutralino states of the minimal model [2, 3, 4] and those of the nonminimal model [5, 6, 7] have been studied in great detail, because the lightest neutralino, being the LSP, is the end product of any process involving supersymmetric particles in the final state.

Recently we considered the lightest neutralino state in a general supersymmetric theory containing an arbitrary number of singlet, doublet and triplet Higgs superfields under the Standard Model gauge group, and obtained an upper bound on its mass [8]. If we assume the simplest form of grand unification of gauge couplings in such a theory, whereby the triplet and extra doublet Higgs fields are eliminated, then this upper bound is controlled by a soft supersymmetry gaugino mass (which we can take to be the gluino mass), and the vacuum expectation value of the doublet, but not the singlet, Higgs fields. Since the latter are known, the bounds are controlled by the gluino mass alone. For a gluino mass of 200 GeV, the upper bound on the lightest neutralino mass (including the dominant one-loop radiative corrections) was shown to be 62 GeV, which increases to 178 GeV for a 1 TeV gluino.

In this paper we generalize the above result. We consider the neutralino
mass matrix in a general supersymmetric theory with an arbitrary Higgs sector, which includes singlets, doublets and triplets under the Standard Model gauge group. We obtain an upper bound on the mass of the lightest neutralino and a lower bound on the mass of the heaviest neutralino state in such a general supersymmetric theory. These bounds depend on the soft supersymmetry breaking gaugino masses and the vacuum expectation values of the doublet and triplet, but not the singlet, Higgs fields. This is in contrast to the situation that obtains in the Higgs sector of such a general supersymmetric theory \[9\], where the (tree level) upper bound on the lightest Higgs boson mass is controlled by the vacuum expectation value of the doublet Higgs fields and dimensionless parameters only. Nevertheless, if we assume grand unification of such a general supersymmetric theory, then we show that the upper bound is controlled by a soft supersymmetry breaking gaugino mass parameter (which can be taken to be the gluino mass), and \(M_Z\) and \(\theta_W\). Since the latter are known, the former entirely determines the upper bound.

We start by recalling the neutralino mass matrix in the Minimal Supersymmetric Standard Model \[1\]. In the basis
\[
\psi_j^0 = (-i\lambda', -i\lambda^3, \psi_{H_1^1}, \psi_{H_2^2}), \quad j = 1, 2, 3, 4, \tag{1}
\]
where \(\lambda'\) and \(\lambda^3\) are the two-component gaugino states corresponding to the \(U(1)_Y\) and the third component of \(SU(2)_L\) gauge groups, respectively, and \(\psi_{H_1^1}\) and \(\psi_{H_2^2}\) are the two-component Higgsino states with hypercharge -1 and +1, respectively, the neutralino mass matrix can be written as
\[
M = \begin{bmatrix}
M_1 & 0 & -M_Z \sin \theta_W \cos \beta & M_Z \sin \theta_W \sin \beta \\
0 & M_2 & M_Z \cos \theta_W \cos \beta & -M_Z \cos \theta_W \sin \beta \\
-M_Z \sin \theta_W \cos \beta & M_Z \cos \theta_W \cos \beta & 0 & -\mu \\
M_Z \sin \theta_W \sin \beta & -M_Z \cos \theta_W \sin \beta & -\mu & 0
\end{bmatrix} \tag{2}
\]
where \(M_1\) and \(M_2\) are supersymmetry breaking gaugino masses associated with the \(U(1)_Y\) and \(SU(2)_L\) subgroups of the standard model, respectively, and \(\mu\) is the Higgs(ino) mixing parameter in the superpotential. Here, \(\tan \beta = v_2/v_1\), where \(v_1 = \langle H_1^0 \rangle\) and \(v_2 = \langle H_2^0 \rangle\) are the vacuum expectation values of the neutral components of the two Higgs doublets, and
\( M_Z^2 = (g^2 + g'^2)(v_1^2 + v_2^2)/2 \), with \( g \) and \( g' \) being the gauge couplings associated with the SU(2) and U(1) factors of the standard model gauge group, respectively. Diagonalization of the mass matrix (2) gives the physical neutralino masses and eigenstates \([10]\). We shall denote these neutralino eigenstates by \( \chi_1^0, \chi_2^0, \chi_3^0 \), and \( \chi_4^0 \), labelled in order of increasing mass. Since some of the neutralino masses resulting from diagonalization of the mass matrix (2) can be negative, it is convenient to consider the squared mass matrix \( M^\dagger M \) and the corresponding squared masses of the neutralinos. An upper bound on the squared mass of the lightest neutralino \( \chi_1^0 \) can be obtained by using the fact that the smallest eigenvalue of \( M^\dagger M \) is smaller than the smallest eigenvalue of its upper left 2 \( \times \) 2 submatrix

\[
\begin{bmatrix}
M_1^2 + M_Z^2 \sin^2 \theta_W & -M_Z^2 \sin \theta_W \cos \theta_W \\
-M_Z^2 \sin \theta_W \cos \theta_W & M_Z^2 + M_Z^2 \cos^2 \theta_W 
\end{bmatrix}
\]

thereby resulting in the upper bound \([11]\)

\[
M_{\chi_1^0}^2 \leq \min(M_1^2 + M_Z^2 \sin^2 \theta_W, M_2^2 + M_Z^2 \cos^2 \theta_W).
\]  

(4)

On the other hand, the larger eigenvalue of the upper left 2 \( \times \) 2 submatrix (3) of \( M^\dagger M \) gives a lower bound on the squared mass of the heaviest neutralino:

\[
M_{\chi_4^0}^2 \geq \max(M_1^2 + M_Z^2 \sin^2 \theta_W, M_2^2 + M_Z^2 \cos^2 \theta_W).
\]  

(5)

We note that the bounds (4) and (5) are controlled by, in addition to \( M_Z \) and \( \theta_W \), the soft SUSY breaking gaugino masses, \( M_1 \) and \( M_2 \). This is in contrast to the Higgs sector of MSSM, where the corresponding bounds on the (tree level) masses of the lightest and the heaviest scalar Higgs bosons are controlled by \( M_Z \), and not by supersymmetry breaking masses \([12]\).

We now consider a general class of supersymmetric standard models, namely, supersymmetric models based on Standard Model gauge group with an arbitrary Higgs sector. We shall assume \([4]\):
(i) Two Higgs doublets $H_1^{(1)}, H_2^{(1)}$, with hypercharge $Y = \mp 1$, which are coupled to the quarks and leptons in the superpotential

$$W^0 = h_U Q_L U_R^c H_2^{(1)} + h_D Q_L D_R^c H_1^{(1)} + h_E L_L E_R^c H_1^{(1)}.$$  \hspace{0.5cm} (6)

In addition, we assume an arbitrary number of extra pairs of Higgs doublets $H_1^{(j)}, H_2^{(j)}, j = 2, ..., d + 1$, which are decoupled from quarks and leptons so that there are no dangerous flavor changing neutral currents [13].

(ii) Gauge singlets $N^\sigma, \sigma = 1, ..., n_s$.

(iii) SU(2) triplets $\Sigma^a, a = 1, ..., t_0$, with $Y = 0$.

(iv) SU(2) triplets $\Psi_1^m, \Psi_2^m, m = 1, ..., t_1$, with $Y = \pm 2$.

Since the two Higgs doublets $H_1^{(1)}$ and $H_2^{(1)}$ are the minimum number of doublets which are required in MSSM to give masses to all fermions and to cancel triangle gauge anomalies, the above extra Higgs multiplets are the only ones that can provide renormalizable couplings to all possible combinations of $H_1^{(1)}$ and $H_2^{(1)}$, viz. $H_1^{(1)} H_2^{(1)}, H_1^{(1)} H_1^{(1)},$ and $H_2^{(1)} H_2^{(1)}$, in the superpotential. More exotic Higgs representations contribute to the $\beta$-functions of the gauge couplings and give lower values of the upper bound on the lightest Higgs mass in general supersymmetric theories. Since we are interested in absolute upper bounds on the particle masses in these theories, we do not consider such representations. The most general renormalizable superpotential for the above Higgs supermultiplets can be written as

$$W' = W_1 + W_2,$$ \hspace{0.5cm} (7a)

where (repeated indices summed)

$$W_1(H_{1,2}, N, \Sigma, \Psi_{1,2}) = f_{ij}^{\alpha} H_1^{(i)} H_2^{(j)} N^{(\alpha)} + f_{ij}^{\alpha} H_1^{(i)} \Sigma^{(a)} H_2^{(j)} + g_{ij}^{\alpha} H_1^{(i)} \Psi_1^{(m)} H_2^{(j)} + g_{ij}^{\alpha} H_1^{(i)} \Psi_2^{(m)} H_2^{(j)},$$ \hspace{0.5cm} (7b)
\[ W_2(N, \Sigma, \Psi_{1,2}) = h_{alm} \text{tr}(\Sigma^{(a)}\Psi_1^{(l)}\Psi_2^{(m)}) + \frac{X_{abc}}{6} \text{tr}(\Sigma^{(a)}\Sigma^{(b)}\Sigma^{(c)}) \]
\[ + \frac{\lambda_{\mu\nu\sigma}}{6} N^{(\mu)} N^{(\nu)} N^{(\sigma)}, \tag{7c} \]

and where we have represented the triplets by 2 x 2 traceless complex matrices, and the trace in (7c) is over the matrix indices. More explicitly, the triplets are represented as (with multiplicity indices suppressed):

\[ \Sigma = \begin{bmatrix} \xi^0/\sqrt{2} & \xi_2^+ \\ \xi^- & -\xi^0/\sqrt{2} \end{bmatrix}, \tag{8a} \]

\[ \Psi_1 = \begin{bmatrix} \psi_1^+ / \sqrt{2} & -\psi_1^{++} \\ \psi_1^0 & -\psi_1^+ / \sqrt{2} \end{bmatrix}, \quad \Psi_2 = \begin{bmatrix} \psi_2^+ / \sqrt{2} & -\psi_2^{++} \\ \psi_2^0 & -\psi_2^+ / \sqrt{2} \end{bmatrix}. \tag{8b} \]

Without loss of generality, we can make a unitary transformation in the space of Higgs doublets \( H^{(1)}_1 \) and \( H^{(1)}_2 \) such that only the Higgs doublets \( H^{(1)}_1 \) and \( H^{(1)}_2 \) acquire a non-zero vacuum expectation value (\( \langle H^{(1)0}_1 \rangle = v_1, \langle H^{(1)0}_2 \rangle = v_2 \)). This requires \( \text{I} \) that some of the Yukawa couplings in (7b) vanish:

\[ f_{1j}^{1\sigma} = f_{2j}^{1\a} = g_{1}^{1jm} = g_{2}^{1jm} = 0 \quad (j \neq 1). \tag{9} \]

We will assume this condition in what follows. With this condition, the part of the superpotential (7a) involving only the neutral fields can be written as

\[
W' = - \sum_{\sigma} \left[ \sum_{i=1}^{d+1} f_1^{i\sigma} H_1^{(i)0} H_2^{(j)0} + \sum_{i \neq 1, i \neq j} f_1^{i\sigma} H_1^{(i)0} H_2^{(j)0} \right] N^{\sigma} \\
+ \sum_{a} \left[ \sum_{i=1}^{d+1} f_2^{i\a} H_1^{(i)0} H_2^{(i)0} + \sum_{i \neq 1, i \neq j} f_2^{i\a} H_1^{(i)0} H_2^{(i)0} + \frac{\xi^{(a)0}}{\sqrt{2}} \right] \chi_{abc}^{6} \\
- \sum_{m} \left[ \sum_{i=1}^{d+1} g_1^{i\m} H_1^{(i)0} H_1^{(i)0} + \sum_{i \neq j \neq 1} g_1^{ij\m} H_1^{(i)0} H_2^{(j)0} \right] \psi_{1}^{(m)0}
\]
where

\[ M_{ij} = M_Z \sin \theta_W \cos \beta, \quad M_{14} = M_Z \sin \theta_W \sin \beta, \]

\[ M_{23} = M_Z \cos \theta_W \cos \beta, \quad M_{24} = -M_Z \cos \theta_W \sin \beta, \]
are the matrix elements which enter the neutralino mass matrix (12) of the MSSM, and we have defined the various vacuum expectation values and the couplings as follows:

\[
\begin{align*}
  u^a &= \langle \xi^a \rangle, \quad x^\sigma = \langle N^\sigma \rangle, \quad y_1^i = \langle \psi_1^i \rangle, \quad y_2^i = \langle \psi_2^i \rangle, \\
  f_1^\sigma &= -f_1^{11\sigma}, \quad f_2^a = f_2^{11a}/\sqrt{2}, \quad f_1^{ia} = -f_1^{1ia}, \quad f_2^{ia} = f_2^{1ia}/\sqrt{2}, \\
  f &= (f_1^\sigma x^\sigma + f_2^a u^a), \quad f^i = (f_1^{ia} x^\sigma + f_2^{iua}), \quad f^{ij} = (-f_1^{ij\sigma} x^\sigma + f_2^{ija} u^a)/\sqrt{2}, \\
  g_1^l &= -g_1^{1ll}, \quad g_2^l = -g_2^{1ll}, \quad h'_{alm} = h_{alm}/\sqrt{2}.
\end{align*}
\]  

(13a) 

(13b) 

(13c) 

(13d)

In (13a) we have represented the Higgs components of the superfields by the same symbol as the superfields themselves. The upper bound on the squared mass of the lightest neutralino can be obtained by examining the upper left 2 x 2 submatrix of \( M^\dagger M \) corresponding to \( M \) of (12). This submatrix can be written as

\[
\begin{bmatrix}
  M_1^2 + M_Z^2 \sin^2 \theta_W + 6g^2y^2 & -M_Z^2 \sin \theta_W \cos \theta_W - 2gg'y^2 \\
  -M_Z^2 \sin \theta_W \cos \theta_W - 2gg'y^2 & M_1^2 + M_Z^2 \cos^2 \theta_W + 2g^2u^2
\end{bmatrix},
\]

(14a)

where

\[
y^2 = \sum_i [(y_1^i)^2 + (y_2^i)^2], \quad u^2 = \sum_a (u^a)^2,
\]

(14b)

and where we have used the expressions for \( W \) and \( Z \) masses appropriate for the general theory that we are considering:

\[
M_Z^2 = \frac{1}{2}(g^2 + g'^2)[v_1^2 + v_2^2 + 4y^2],
\]

(15a)
\[ M_{W}^2 = \frac{1}{2} g^2 [v_1^2 + v_2^2 + 4 u^2 + 2 y^2]. \]  

(15b)

The vacuum expectation values that enter into the \( W \) and \( Z \) masses (15a) and (15b) are experimentally constrained by the \( \rho \) parameter. From a recent global fit, which includes the CDF data, we have [16]

\[ \rho = \frac{M_{W}^2}{M_{Z}^2 \cos^2 \theta_W} = 1.0002 \pm 0.0013 \pm 0.0018, \]  

(16)

where the second error is due to the Higgs mass. This result is remarkably close to the expected Standard Model value of \( \rho = 1 \). Taking a value of \( \rho = 1 \) implies, through (15a) and (15b), the following relation between the triplet vacuum expectation values:

\[ 4 u^2 = 2 y^2. \]  

(17)

We shall henceforth assume (17) to be true. With constraint (17), the \( W \) and \( Z \) masses can be written as [17]

\[ M_{W}^2 = M_{Z}^2 \cos^2 \theta_W = \frac{1}{2} g^2 (v_1^2 + v_2^2 + 4 y^2). \]

Thus, the combination \((v_1^2 + v_2^2 + 4 y^2)\) is determined to be \( \simeq (174 \text{ GeV})^2 \), but the ratio of the triplet to the doublet vacuum expectation values \( y/(v_1^2 + v_2^2)^{1/2} \) is not fixed. Using (17), the 2 x 2 submatrix (14a) can be written as

\[
\begin{bmatrix}
M_1^2 + M_Z^2 \sin^2 \theta_W + 6 g^2 y^2 & -M_Z^2 \cos \theta_W \sin \theta_W - 2 g g' y^2 \\
-M_Z^2 \cos \theta_W \sin \theta_W - 2 g g' y^2 & M_Z^2 + M_Z^2 \cos^2 \theta_W + g^2 y^2
\end{bmatrix}.
\]  

(18)

The smallest eigenvalue of the 2 x 2 matrix (18) serves as the upper bound on the squared mass of the lightest neutralino in the general supersymmetric Standard Model:

\[ M_{\chi^0_1}^2 \leq \min(M_1^2 + M_Z^2 \sin^2 \theta_W + 6 g^2 y^2, M_2^2 + M_Z^2 \cos^2 \theta_W + g^2 y^2). \]  

(19)

On the other hand, the heaviest neutralino \((\chi^0_n)\) mass is bounded from below by

\[ M_{\chi^0_n}^2 \geq \max(M_1^2 + M_Z^2 \sin^2 \theta_W + 6 g^2 y^2, M_2^2 + M_Z^2 \cos^2 \theta_W + g^2 y^2). \]  

(20)
These bounds depend on a priori unknown vacuum expectation values of the triplet Higgs fields \((\psi_1^{(i)}, \psi_2^{(i)})\), and the supersymmetry breaking gaugino mass parameters. Nevertheless, as we shall see these bounds can become meaningful in theories with gauge coupling unification [18]. We note that the singlet vacuum expectation values decouple from these bounds.

In the general supersymmetric theory that we are considering, the renormalization group equations (RGEs) [15] for the standard SU(3) x SU(2) x U(1) gauge couplings can be written as \((g_1^2 = (5/3)g'^{2}, \ g_2^2 = g^2, \ \tan \theta_W = g'/g, \ g_3\) is the SU(3) gauge coupling constant)

\[
16\pi^2 \frac{dg_1}{dt} = \left[ \frac{33}{5} + \frac{3}{5}(6t_1 + d) \right] g_1^3,
\]

\[
16\pi^2 \frac{dg_2}{dt} = \left[ 1 + 2t_0 + 4t_1 + d \right] g_2^3,
\]

\[
16\pi^2 \frac{dg_3}{dt} = -3g_3^2.
\]

(21)

Obviously, these RGEs depend on the number and the type of the Higgs representations \((d,t_0,t_1)\). We note that the additional doublets increase the \(\beta\) functions of the gauge couplings, even though the VEVs of the additional doublets have been rotated away. If we assume that the gauge couplings \(g_1, g_2, \) and \(g_3\) unify at some grand unification scale, i.e. \(g_1(M_U) = g_2(M_U) = g_3(M_U)\), with \(M_U\) the unification scale, then the simplest choice is [19]

\[
d = t_0 = t_1 = 0,
\]

(22)

with \(n_s\) arbitrary, i.e. MSSM with an arbitrary number of singlet superfields. In other words, only Higgs singlets, besides the two Higgs doublets of MSSM, are consistent with the simplest form of unification [19]. The bounds (19) and (20) now reduce to

\[
M_{\chi_1^0}^2 \leq \min(M_1^2 + M_2^2 \sin^2 \theta_W, \ M_2^2 + M_Z^2 \cos^2 \theta_W),
\]

(23)

\[
M_{\chi_n^0}^2 \geq \max(M_1^2 + M_2^2 \sin^2 \theta_W, \ M_2^2 + M_Z^2 \cos^2 \theta_W),
\]

(24)

which are identical to the corresponding bounds (11) and (13) in the minimal supersymmetric standard model [8].
However, gauge coupling unification can be achieved even when \(d, t, 0 < t_1 > 0\) by adding colored multiplets or introducing an intermediate scale in the theory \([20]\). Therefore, it is important to ask what are the bounds on the mass of the lightest neutralino in the most general case without assuming any particular form of grand unification with a specific particle content. To evaluate the upper bound \([19]\) in the most general case, we recall that the gaugino mass parameters satisfy the renormalization group equations (RGEs) \([21]\):

\[
16\pi^2 \frac{d M_i}{dt} = b_i M_i g_i^2,
\]

where the coefficients \(b_i\) are the same that occur in the evolution \([21]\) of the corresponding gauge couplings. Equations \([21]\) and \([25]\) imply \((\alpha_i = g_i^2/4\pi, \alpha_U = g_U^2/4\pi)\),

\[
M_1(M_Z)/\alpha_1(M_Z) = M_2(M_Z)/\alpha_2(M_Z) = M_3(M_Z)/\alpha_3(M_Z) = m_{1/2}/\alpha_U,
\]

where \(M_{1/2}\) is the common gaugino mass at the grand unification scale, and \(\alpha_U\) is the unified coupling constant. It is important to note that \([20]\), which is a consequence of one-loop renormalization group equations, is valid in any grand unified theory irrespective of the particle content. Two-loop effects to \([26]\) are expected to be numerically small \([22]\). We also note that the GUT relation can be violated only at order \(\alpha_U/\pi\) \([23]\). The relation \([24]\) is the same which occurs in the MSSM with grand unification. It reduces the three gaugino mass parameters to one, and since the the gluino mass \(m_{\tilde{g}}\) is equal to \(|M_3|\), it is, therefore, convenient to let the gluino mass be the free parameter. The other two gaugino mass parameters are then determined through

\[
M_2(M_Z) = (\alpha/\alpha_3 \sin^2 \theta_W) m_{\tilde{g}} \simeq 0.27 m_{\tilde{g}},
M_1(M_Z) = (5\alpha/3\alpha_3 \cos^2 \theta_W) m_{\tilde{g}} \simeq 0.14 m_{\tilde{g}},
\]

where \(\alpha\) is the fine structure constant, and where we have used \([24, 25, 26, 27]\) the values of couplings at the \(Z^0\) energy as

\[
\alpha^{-1}(M_Z) = 127.9, \ \sin^2 \theta_W = 0.2315, \ \alpha_3 = 0.123.
\]

Using \([27]\) and the most conservative estimate of the upper bound on the triplet vacuum expectation value, \(y^2 \leq (87\text{GeV})^2\), following from \((v_1^2 + v_2^2 + 4y^2) \simeq (174\text{GeV})^2\), the upper bound \([19]\) can be written as
\[ M_{\chi_0^2}^2 \leq M_1^2 + M_2^2 \sin^2 \theta_W + 6g'^2y^2 \simeq (0.02m_\tilde{g}^2 + 7733.5) \text{GeV}^2. \]  

(29)

Note that the lower bound (20) on the mass of the heaviest neutralino cannot be evaluated in the most general case that we are considering. For a gluino mass of 200 GeV, the upper bound (29) on the mass of the lightest neutralino is 92 GeV. Similarly, for a gluino mass of 1 TeV, the upper bound becomes 166.5 GeV. This should be compared with the upper bound of 52 GeV and 148 GeV, respectively, for gluino mass of 200 GeV and 1 TeV, which one obtains when one assumes a simple form of gauge coupling unification (22).

The bounds (19), (20) and (29) are our main results. While the former are valid in a supersymmetric theory based on a standard model gauge group and an arbitrary Higgs sector, the latter is valid only when we impose the constraint of grand unification on gauge couplings in such a supersymmetric theory. Thus, although a bound on the lightest neutralino mass exists in an arbitrary supersymmetric theory, it is calculable in terms of gluino mass only under the assumption of gauge coupling unification. We further note that since the soft supersymmetry breaking gaugino masses satisfy the GUT relation (26) in any grand unified theory based on a simple group independent of the breaking pattern to the Standard Model gauge group [28], our result (29) is valid in any such supersymmetric grand unified theory.

Finally, we discuss the effect of radiative corrections on the tree level upper bound on the lightest neutralino mass derived above. The dominant radiative corrections arise from top-stop loops. If the neutralino is predominantly Higgsino-like, then a simple estimate of the radiative corrections to its mass arising from a top-stop loop is given by

\[ \frac{\Delta M_{\chi_0^2}}{M_{\chi_0^2}} \simeq 3 \frac{h_t^2}{16\pi^2} \ln \left( \frac{m_t^2}{m_{\tilde{t}}^2} \right) \simeq 5\%, \]  

(30)

where the factor 3 comes from color, \( h_t^2 \) comes from the top Yukawa couplings at the two vertices in the loop \( (h_t \equiv h_{U33}, \text{see (8)}, \text{and } 1/16\pi^2 \text{ is the usual loop factor. In making the above estimate, we have taken the squark mass to be equal to 1 TeV. A similar estimate holds for a gaugino-like neutralino, and
also yields an estimate for the correction of order 5%. These estimates are very close to the generic results which emerge from detailed calculations carried out in the context of the minimal supersymmetric standard model, although, for some extreme values of parameters, the radiative corrections to the lightest neutralino mass can be as large as 20%. Taking these results as indicative of the radiative corrections in the general model that we are considering, we estimate a conservative radiatively corrected upper bound on the mass of the lightest neutralino to be about 110 GeV for a 200 GeV gluino, which increases to 200 GeV for a gluino mass of 1 TeV.

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