Unstable vacuum and spectrum of atoms with a superheavy nucleus

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Abstract. We show that the light spectra of atoms with a superheavy nucleus may be important for our understanding of the processes in supercritical fields. The shape is obtained of a spectral line emitted by an atom with a superheavy nucleus. We study the connection of the light part of the spectrum with the parameters characterizing the nucleus decay.

1. Introduction

Optical spectroscopy has played an important part in the development of physics as a whole. Observation of the Balmer series in the Hydrogen emission spectrum inspired the Bohr atomic model and quantum mechanics. The discovery of the Lamb shift led to the creation of quantum electrodynamics (QED). Modern laser spectroscopy allows one to reach a precision of 15 decimal digits and to register radiation of very low intensity, up to one photon, providing a powerful tool for investigation and tests of fundamental physics laws [1]. So, the spectroscopy of muonic hydrogen has led to the well-known proton radius puzzle [2] and made scientists review the fundamentals of QED. Another questionable problem, which provokes interest during last several decades, is the spontaneous real electron-positron \( e^- e^+ \) pair creation from the vacuum and, as a consequence, to the appearance of additional narrow lines with completely defined position and shape in the positron energy spectrum. The existence of these lines has not been unambiguously established experimentally yet [4,5]. The key problem is that there are several mechanisms of positron production during a heavy ion collision, such as induced positron creation and internal pair conversion [6], and it is impossible to distinguish experimentally between the contributions from these mechanisms. Besides, there are still doubts that giant nuclei live long enough for the vacuum decay to occur. Thus, positron spectra provide not enough data and an additional independent source of information is needed to resolve the problem. Optical spectra of heavy ion collisions may serve this source, since their structure is vastly affected by the inner parameter of the giant nucleus. In this paper we derive the form of the spectrum of the optical radiation emitted by an atom with a supercritical nucleus. We show how it is affected by the nuclear characteristics mentioned above. The investigation of optical spectra is universal and may be applied for the study of any other ions, to determine their physical parameters.
2. Methods

In our investigation of the problem we use the generalized dynamical equation (GDE), which in [7] has been derived as a direct consequence of the first principles of quantum physics. Being equivalent to the Schrödinger equation in the case when the interaction in a quantum system is instantaneous, GDE allows one to extend quantum dynamics to the case of nonlocal-in-time interactions. Generalized quantum dynamics (GQD) developed in this way provides a new insight into many problems in quantum physics [8-13]. In GQD a history of a physical process is represented by some version of the time evolution of the system associated with completely specified instants of the beginning and end of the interaction in the system [7]:

\[ U(t,t_0) = 1 + \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \tilde{S}(t_2, t_1), \]

where \( \tilde{S}(t_2, t_1) \) describes a contribution to the evolution operator \( U(t,t_0) \) from the process in which the interaction begins at time \( t_1 \) and ends at time \( t_2 \). GDE allows one to obtain \( S(t_2, t_1) \) for any \( t_1 \) and \( t_2 \), if the operators \( \tilde{S}(t_2', t_1') \) corresponding to infinitesimal duration times \( \tau = t_2' - t_1' \) of interaction are known. Most of the contribution to the evolution operator in the limit \( \tau \to 0 \) comes from processes associated with the fundamental interaction in the system under study, which can be parameterized as an interaction operator \( H_{int}(t_2, t_1) \) [7]. This rise to the following boundary condition for \( \tilde{S}(t_2, t_1) \):

\[ \tilde{S}(t_2, t_1) \xrightarrow{\tau \to 0} H_{int}(t_2, t_1). \]

In the Schrödinger picture the evolution operator can be represented in the form [9]

\[ U_s(t,0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dx \exp(-ix\tau)G(z), \]

with \( G(z) = G_0(z) + G_0(z)T(z)G_0(z) \), where \( G_0(z) = (z - H_0)^{-1} \), \( H_0 \) is the free Hamiltonian, and the operator \( T(z) \) is defined as

\[ T(z) = i \int d\tau \exp(i\tau) \exp(iH_{t_2}t_2) \tilde{S}(t_2, t_1) \exp(iH_0t_1), \]

where \( \tau = t_2 - t_1 \). In terms of the operator \( T(z) \), GDE takes the form

\[ \frac{dT(z)}{dz} = -T(z)(G_0(z))^2T(z), \]

which is used with the boundary condition \( T(z) \xrightarrow{|z| \to \infty} B(z) \), where

\[ B(z) = i \int d\tau \exp(i\tau) \exp(iH_0t_2)H_{int}(t_2, t_1) \exp(-iH_0t_1). \]

The contribution to the Green operator \( G(z) \), which comes from the processes associated with the self-interaction of particles, has the same structure as the free Green operator \( G_0(z) \). For this reason it is natural to replace \( G_0(z) \) by the propagator \( \hat{G}_0(z) \), which describes the evolution of particles interacting only with the vacuum and hence has the structure

\[ \langle m' | \hat{G}_0(z) | m \rangle = \langle m' | m \rangle (z - E_m - C_m(z))^{-1}, \]
where $|m\rangle$ are the eigenvectors of the free Hamiltonian $\langle H_0|n\rangle = E_n\langle n\rangle$). Correspondingly, the operator $T(z)$ should be replaced by the operator $M(z)$, which describes the evolution of particles interacting not only with the vacuum. These operators are related as follows:

$$G(z) = G_0(z) + G_0(z)T(z)G_0(z) = \tilde{G}_0(z) + \tilde{G}_0(z)M(z)\tilde{G}_0(z).$$

(7)

The self-energy function $C_w(z)$ describes the self-interaction of particles in the state $|m\rangle$, and the condition $z - E_m - C_w(z) = 0$ determines the physical masses of the particles. Taking into account equation (7), one can rewrite (4) in terms of $M(z)$ and $C_w(z)$, with the boundary conditions $M(z) \rightarrow B_r(z)$, $C_w(z) \rightarrow B_d(z)$. Here $B_r(z)$ is the part of the interaction operator describing the proper interaction between particles, and $B_d(z)$ describes their self-interaction: $B(z) = B_r(z) + B_d(z)$. This trick was firstly used in [14].

3. Supercritical atoms

The Dirac equation with the Coulomb potential $V(r) = -Z\alpha r^{-1}$ ($\alpha$ is the fine-structure constant) allows one to find atomic energy levels for any nucleus charge number $Z \leq 137$ in the case of a point nucleus and $Z \leq Z_{cr} \approx 173$ for an extended nucleus [3]. If the charge of the nucleus becomes larger than $Z_{cr}$, the binding energy $E_{1s}$ of the lowest electron becomes smaller than the electron-positron pair rest energy $E_{1s} \leq -2m_e$. The electron K-shell joins the positron energy continuum. If it is empty, an additional vacancy in the lower continuum is created, which may be spontaneously occupied by an electron from this continuum leaving a hole in it, what is equivalent to the positron emission. This process is called the vacuum decay in supercritical fields [3]. It results in the appearance of additional narrow lines in the positron kinetic energy spectrum. The standard approach that uses the Dirac equation predicts the widths of the lines to be several keV, what corresponds to the decay time $\tau_d = h / \Gamma \sim 10^{-19}$ s [3], while the lifetime of giant nuclei formed during the collision is predicted to be $\tau_x \sim 10^{-20}$ s [6], what is obviously much shorter than $\tau_d$. The narrow line structures were not confirmed by the last experiments on heavy ion collisions [4,5]. Thus, the situation remains unclear.

In general case, an unstable state is not an eigenstate of any Hamiltonian and is described by an energy distribution $\omega_0(E)$. The vector of an unstable state may be written in the form $|\psi\rangle = \int a(E)|E, \psi_\nu\rangle dE$, where $|E, \psi_\nu\rangle$ is the eigenvector of the energy operator corresponding to the energy $E$. $\omega_0(E) = \langle a(E) \rangle^2$ determines the decay law of the unstable state unambiguously.

In description of electron motion in the field of a nucleus the Furry picture is used, in which the free Green operator is constructed on the basis of the eigenvectors of Hamiltonian $\tilde{H}_0$ including the Coulomb interaction of the electrons with the nucleus. In our case these are the states of the critical Hamiltonian $\tilde{H}_\nu = i\gamma_\nu(\partial / \partial x_\nu) + m_e - Z_{cr}\alpha r^{-1}$, $\tilde{H}_\nu |\psi_n^{\nu}(z)\rangle = E_n^0 |\psi_n^{\nu}(z)\rangle$. The "free" Green operator (6) in this basis looks as follows:

$$\tilde{G}_0^{\nu}(z) = \sum_n \frac{|\psi_n^{\nu}(z)\rangle\langle \psi_n^{\nu}(z)|}{z - E_n^0 - C_n(z)},$$

(8)

where $C_n(z)$ is the self-energy function of the state $|\psi_n^{\nu}(z)\rangle$, characterizing the interaction of the supercritical atom with the vacuum. The energy distribution $\omega_{1s}(E)$ is defined as follows:

$$\omega_{1s}(E) = \langle a_{1s}(E) \rangle^2 = (2\pi)^2 \left(z - E_{1s} - C_{1s}(z)\right)^2.$$  

(9)
It may be found that in the first order in $\lambda = Z / Z_{cr}$, 
\[
\omega_{i_s}^{(2)} = |a_{i_s}^{(2)}(E)|^2 = \frac{1}{2\pi} \frac{\Gamma}{\left[ z - E_{i_s}^{(0)} - \Delta E_{i_s} - F(z) \right]^2 + \Gamma^2 / 4},
\]
where $\Gamma = 2\pi \left\langle \psi_{i_s}^{(cr)} | V(Z) | \phi_{E}^{(cr)} \right\rangle$, $\Delta E_{i_s} = \langle \psi_{i_s}^{(cr)} | V(Z) | \psi_{i_s}^{(cr)} \rangle$ and $\left| \phi_{E}^{(cr)} \right\rangle$ are the eigenstates of $H_{cr}$, belonging to the negative energy continuum. Expression (10) has the Lorentz form with the width of several keV, what coincides with the results obtained in [3]. But in the next order in $\lambda$ energy distribution $\omega_{i_s}^{(a)}$ is more complicated and can be obtained only within the framework of GQD. In any case, solving the problem by means of the introduction of the overcritical Coulomb potential leads to the width of the energy distribution of several keV. Note, that this quantity characterizes a one-electron state, rather than the vacuum state.

To describe exactly the vacuum decay we need to find the energy of the interaction of a bare superheavy nucleus with the vacuum $C_{cr}(z)$. It can be shown from the dimensional analysis of the GDE for the function $C_{cr}(z)$ that it is of the order of the electron rest energy $C_{cr}(z) \sim m_e$, what corresponds to the decay time magnitude $\tau_d \sim 10^{-21}$ s. This result is in better correspondence with the nuclear lifetime prediction, but leads to a considerable broadening of the spontaneously created positron lines in comparison with the results given in [3].

4. Light spectrum of supercritical atoms

The emission of photons by a superheavy atom is caused by the transition of electrons between the energy levels with high principal quantum numbers $n$ and $m$. Let us consider the following evolution process of an atom with a supercritical double-uranium nucleus $Z = 188$. An atomic electron transits from an excited state $|N_i\rangle$ to any lower state $|N_f, \bar{k}, \epsilon_\lambda \rangle$ and emits an optical photon with the energy $\omega$. Here $N_i$ and $N_f$ stand for the entire set of discrete and continuous variables that characterize respectively the initial and the final states of the system in full (for example, $N_i = np_{1/2}$ and $N_f = m_{3/2}$), $\bar{k}$ and $\epsilon_\lambda$ are momentum and polarization vector of the photon. After the transition the atomic nucleus splits into two Uranium nuclei, each having the energy $E_U$. The natural broadening of spectral line profiles is determined by the formula [15]:
\[
P(\omega) = \frac{dW(\omega)}{d\omega} = A' \omega \sum \Omega \left| \left\langle N_j; \bar{k}, \epsilon_\lambda | U(t, 0) | N_i \right\rangle \right|^2,
\]
where $A'$ is a normalization factor. The full Green operator corresponding to the above process is:
\[
\left\langle N_j; \bar{k}, \epsilon_\lambda | G(z) | N_i \right\rangle = \sum_n \sum_{\nu'} \left\langle N_j; \bar{k}, \epsilon_\lambda | \hat{G}_0(z) | n'; \bar{k}, \epsilon_\lambda \right\rangle \left\langle n'; \bar{k}, \epsilon_\lambda | M(z) | n \right\rangle \left\langle n | \hat{G}_0 | N_i \right\rangle \frac{1}{z - 2E_U - \omega},
\]
where $\left\langle N_j | \hat{G}_0 | N_i \right\rangle = \left[ z - E_N - C(z) \right]^{-1}$ and $\left\langle N_j; \bar{k}, \epsilon_\lambda | \hat{G}_0 | N_j; \bar{k}, \epsilon_\lambda \right\rangle = \left[ z - E_N - \omega - C(z - \omega) \right]^{-1}$ are the diagonal elements of the free Green operators before and after the transition in the Furry picture and $E_N$ is the energy of the giant nucleus ($E_N = 2E_U + \Delta M$, $\Delta M$ is the mass excess). $C(z)$ is the self-energy function of the giant double-uranium nucleus. Substituting (12) into (11), we obtain the following form of the evolution operator:
\[
\left\langle N_j; \bar{k}, \epsilon_\lambda | U(t, 0) | N_i \right\rangle = \frac{1}{2\pi} \int \frac{dz}{(z - 2E_u - \omega)(z - E_N - \omega - C(z - \omega))(z - E_N - C(z))} \left\langle N_j; \bar{k}, \epsilon_\lambda | M(z) | N_i \right\rangle e^{-\omega z} dz.
\]
Here we have taken into account that the most contribution to the self-energy function of the supercritical atom comes from processes of the interaction of its nucleons with the vacuum and as a consequence it is approximately equal to the self-energy function of the nucleus. At the first stage we can take $M(z)$ to be equal to the electromagnetic interaction Hamiltonian $M(z) = H_i = e \int j_\mu A^\mu d^3 x$, which is of the following form for dipole transitions: 

\[
\langle N_f; \vec{k}, \epsilon \mid H_i \mid N_i \rangle = 2\pi \epsilon_\beta \langle N_f; \vec{k}, \epsilon \mid \vec{\pi} \times \hat{d} \mid N_i \rangle,
\]

$\hat{d}$ being the dipole moment operator, $\vec{\pi} = \vec{k} / \omega_\beta$ and $\omega_\beta$ is the atomic transition frequency. Integrating (11) over $z$ and $\Omega$, summing over $\lambda$, we obtain the following expression:

\[
P(\omega) = \frac{A}{\Delta M + C(2E_\mu)} \left( \frac{\omega}{\omega - \Delta M - C(2E_\mu + \omega)} \right) \omega,
\]

where $A$ is a new normalization factor: $A = 4\pi A|V_\beta|^2$, with $|V_\beta|^2 = 2\pi \epsilon_\beta \langle N_f; \vec{k}, \epsilon \mid \hat{d} \mid N_i \rangle^2$. The behavior of the self-energy function $C(z)$ near the point $z = E_\mu$ determines the decay law of the unstable nucleus. In general this law is not exponential one and is characterized not only by a decay width but also by other parameters determining the behavior of $C(z)$ near the point $z = E_\mu$:

\[
\Re C(z) = (z - E_\mu)B_1, \quad \Im C(z) = \Gamma_0 + (z - E_\mu)B_2,
\]

where $\Gamma_0$ is the decay width of the giant double-uranium nucleus, $B_1 = d \Re C(z) / dz|_{z=E_\mu}$ and $B_2 = d \Im C(z) / dz|_{z=E_\mu}$. Here we have taken into account that $\Re C(z = E_\mu) = 0$ because of the fact that we deal with the renormalized nucleus mass. The dimensional analysis of the GDE applied to the problem shows that $B_1 = \frac{\mu}{\Delta M} \cdot I_1$ and $B_2 = \frac{\mu}{\Delta M} \cdot I_2$, where $I_2$ is of order $O(1)$. The dimensionless parameter $I_1$ should be much smaller and will be neglected in further discussion. Taking into account that $\Delta M$ is much smaller than the reduced mass $\mu$ of two Uranium nuclei, we finally get

\[
C(2E_\mu + \omega) = i\Gamma_0 + i\frac{\mu}{\Delta M} \cdot I \cdot \omega
\]

with $I = I_2$. Substituting expression (17) into (14) we obtain:

\[
P(\omega) = \frac{A}{\Delta M^2 + \Gamma_0^2} \left( \frac{\omega}{\omega - \Delta M} \right)^2 \left( \frac{\omega}{\omega - \Delta M} \right)^2\left( \Gamma_0 + i\frac{\mu}{\Delta M} \cdot I \cdot \omega \right)^2.
\]

The slope of curve (18) is as follows:

\[
\frac{dP(\omega)}{d\omega} = \frac{A}{\Delta M^2 + \Gamma_0^2} \left[-\omega^2 \left[1 + \left( \frac{\mu}{\Delta M} \cdot I \right)^2 \right] + \Delta M^2 + \Gamma_0^2 \left( \omega - \Delta M \right)^2 \right] \left( \Gamma_0 + i\frac{\mu}{\Delta M} \cdot I \cdot \omega \right)^2.
\]

Taking into account that for the double uranium nucleus the theoretically predicted mass excess is

$\Delta M \approx 9 \cdot 10^8 eV$ [3],  

$\mu_{238} \approx 1.1 \cdot 10^{11} eV$,  

$\frac{\mu}{\Delta M} \cdot I \approx 1.2 \cdot 10^2$,  

and the optical photon energy is $\omega \sim 2 eV$, 

\[
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\]
we can neglect 1 in comparison with \( \frac{\mu}{\Delta M} \cdot I \) in the numerator and \( \omega \) in comparison with \( \Delta M \) in the denominator. Equation (19) then reads:

\[
\frac{dP(\omega)}{d\omega} = A \frac{-\omega^2 \left( \frac{\mu}{\Delta M} \cdot I \right)^2 + \Delta M^2 + \Gamma_0^2}{\Delta M^2 + \left( \Gamma_0 + \frac{\mu}{\Delta M} \cdot I \cdot \omega \right)^2}.
\]

(20)

The greatest quantity here is the mass excess \( \Delta M \). The spectrum will be determined substantially by this parameter and hence may serve an experimental test for its theoretical value. The contribution from the nuclear decay width \( \Gamma_0 \) is smaller, but still very important. Let us consider two cases. If the decay time is \( \tau_N \sim 10^{-21} \) s, as it is predicted by nuclear physics, then the decay width is \( \Gamma_0 \approx 5 \cdot 10^5 \) eV, and its contribution to the spectrum will be essential. In particular, we can neglect the term \( \frac{\mu}{\Delta M} \cdot I \cdot \omega \) in comparison with the decay width in (20). Otherwise, if \( \tau_N \sim 10^{-19} \) s, as it is desirable for the spontaneous vacuum decay to occur, then the decay width is \( \Gamma_0 \approx 5 \cdot 10^4 \) eV. In this case \( \Gamma_0 \) and \( \frac{\mu}{\Delta M} \cdot I \cdot \omega \) are comparable, and initial expression (20) should be used.

5. Conclusion

We have obtained the shape of a spectrum line emitted by an atom with a superheavy nucleus. It is mainly determined by the mass excess \( \Delta M \) of the giant nucleus, and hence is an important tool for experimental test of the theoretical value of this parameter. The nuclear decay parameters \( \Gamma_0 \) and \( I \) are determined by the form of the light part spectrum that looks as continuous frequency spectrum. Thus, optical methods provide an additional possibility of independent estimation of the superheavy nucleus characteristics, such as the mass excess and the decay parameters, the ratio of which is crucial for the experimental observation of the spontaneous vacuum decay. Although the energy of the photons with which we deal in the case is very small, here it is multiplied by the factor 100, and hence the magnitude of the response will be large enough to be observable on the background. Moreover the differential spectrum is considered, since it is known to provide greater accuracy. The method presented in this paper is universal and may be applied for the study of other unstable nuclei.

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