Minority game is a model of heterogeneous players who think inductively. In this game, each player chooses one out of two alternatives every turn and those who end up in the minority side wins. It is instructive to extend the minority game by allowing players to choose one out of many alternatives. Nevertheless, such an extension is not straightforward due to the difficulties in finding a set of reasonable, unbiased and computationally feasible strategies. Here, we propose a variation of the minority game where every player has more than two options. Results of numerical simulations agree with the expectation that our multiple choices minority game exhibits similar behavior as the original two-choice minority game.

I. INTRODUCTION

Complex adaptive systems have drawn attention among statistical physicists in recent years. The study of such systems not only provides invaluable insight into the non-trivial global behavior of a population of competing agents, but also has potential application in economics, biology and finance. Moreover, we can study complex adaptive systems from the perspective of statistical physics.

The El Farol bar problem [1], which was proposed by W. B. Arthur in 1994, has greatly influenced and stimulated the study of complex adaptive systems in the last few years. It describes a system of \( N \) agents deciding independently in each week whether to go to a bar or not on a certain night. As space is limited, the bar is enjoyable only if it is not too crowded. Agents are not allowed to communicate directly with each other and their choices are not affected by previous decisions. The only public information available to agents is how many agents came in last week. To enjoy an uncrowded bar, each agent has to employ some hypotheses or mental models to guess whether one should go or not. With bounded rationality, all agents use inductive reasoning rather than perfect, deductive reasoning since the system is ill-defined. In other words, agents act upon their currently most credible hypothesis based on the past performance of their hypotheses. Consequently, agents can interact indirectly with each other through their hypotheses in use. The emerging system is thus both evolutionary and complex adaptive in the presence of those inductive reasoning agents.

Inspired by the El Farol bar problem, Challet and Zhang put forward the Minority Game (MG) [2, 3]. It is a toy model of \( N \) inductive reasoning players who have to choose one out of two alternatives independently at each time step. Those who end up in the minority side (that is, the choice with the least number of players) win. The only public information available to all players in the MG is the winning alternatives of the last \( M \) passes, known as the history. Players have to employ some strategies based on the history to guess the winning choice. In fact, they may employ more than one strategy throughout the game. More precisely, every player picks once and for all \( S \) randomly drawn strategies from a suitably chosen strategy space before playing the game. The performance of each strategy will be recorded during the game. Then players make decision according to their current best performing strategy at every time step.

In spite of its simplicity, MG displays a remarkably rich emergent collective behavior. Numerical simulations showed that there is a second order phase transition between a symmetric phase and an asymmetric phase [4, 5, 6]. There is no predictive information about the next minority group available to agent’s strategies in the symmetric phase, whereas there is predictive information available to agent’s strategies in the asymmetric phase. MG addresses the interaction between agents and public information, that is, how agents react to public information and how the feedback modify the public information itself. Later works revealed that the dynamics of the system in fact minimizes a global function related to market predictability [7, 8]. Therefore, the MG can be described as a disordered spin system [9]. Hart and his coworkers found that the fluctuations arising in the MG is controlled by the interplay between crowds of like-minded agents and their anti-correlated partners [10]. Since MG is a prototype to study detailed pattern of fluctuations, it plays a dominant role in economic activities like the market mechanism. In order to learn more on MG-like systems, much effort was put on the extension of the MG model, such as the introduction of evolution [11] and the modification of MG for modelling the real market [12, 13].

In the real world, however, an agent usually has more than two options. For example, people decide where to dine or which share to buy from a stock market. Consequently, it is worthwhile to investigate the situation where players have more than two choices especially when the number of choices is large. Indeed, D’Hulst and Rodgers [14] has written a paper on the three-choice minority game. In their paper, a symmetric and an asymmetric three-sided models are introduced. Their symmetric model is only a model which mimics the cyclic trading between three players using the same strategy.
formalism as MG and thus cannot be extended to realistic cases with a large number of choices. Furthermore, their asymmetric model is nothing but the original minority game with the possibility allowing players not to participate in a turn. Hence, the player’s choice is not symmetric.

Recently, Ein-Dor et al. proposed a multichoice minority game based on neural network [14]. Their model generalizes the El Farol bar problem to the case of multiple choice. Nevertheless, it differs quite significantly from the MG of Challet and Zhang as each player has only one strategy to use and that strategy evolves according to its performance. Besides, no phase transition of any kind is observed in Ein-Dor et al.’s model. Chau and Chow also proposed another multichoice minority game model based on MG [15]. In this model, players choosing their strategies from a reduced strategy space consists of anti-correlated and uncorrelated strategies only.

In this paper, we propose a new model called the multiple choices minority game (MCMG) with a neural network flavor. It is a variation of the MG where all heterogeneous, inductive thinking players have more than two choices. Just like the original MG, strategies in MCMG are not evolving and they are picked in each turn according to their current performance. In Section II, the MCMG model are explained in detail. Results of numerical simulation of our model are presented and discussed in Section III. We also compare our results with those of the original MG as well. In the Section IV, we deliver a brief summary and an outlook of our work.

II. THE MODEL

Let us consider a repeated game of a population of $N$ players. At each time step, every player has to choose one of $N_c$ rooms/choices independently. Here, we assume that $N_c$ is a prime power and we identify the $N_c$ rooms with elements in the finite field $GF(N_c)$. We represent the choice of the $i$th player at time $t$ by $x_i(t)$ which only “takes on” the $N_c$ different rooms. Those players in the room with the least number of players, that is, in the minority side, win. The winning room at time $t$ is the publicly known output of the game $\Omega(t)$. The players of the winning room will gain one unit of wealth while all the others lose one. So the wealth of the $i$th player, $w_i(t)$, is updated by

$$w_i(t+1) = w_i(t) + 2\delta(x_i(t) - \Omega(t)) - 1,$$

where $\delta(x) = 1$ and $\delta(x) = 0$ whenever $x \neq 0$. Note that the output of the last $M$ steps, $\mu(t) = (\Omega(t-M+1), \ldots, \Omega(t-1))$, is the only public information available to all players. Therefore, players can only interact indirectly with each other through the history $\mu(t)$ which can take on $N_c^M$ different values.

Aimed at maximizing one’s own wealth, each player has to employ some strategies to predict the trend of the output of the game. But how to define a strategy? For a minority game with more than two choices, it is not effective to formulate the strategy as in MG. Recall that in MG, a strategy is defined as a number of set of choices corresponding to different histories. In other words, a strategy is a map sending each $\mu(t)$ to the choice $0/1$. Therefore, there are $2^{M}$ different strategies in the full strategy space for MG.

Similar numerical results of the fluctuation arising in the MG are obtained if strategies are drawn from the reduced strategy space instead of the full strategy space [2, 3]. Hence, the reduced strategy space plays a fundamental role in the properties of the fluctuation arising in the MG. The reduced strategy space is formed by strategies which are significantly different from each other. Given a strategy $s$, only its uncorrelated and anti-correlated strategies are significantly different from it. Indeed, the reduced strategy space is composed of two ensembles of mutually uncorrelated strategies where the anti-correlated strategy of any strategy in one ensemble always exists in the other ensemble. For MG, there are $2^{M+1}$ different strategies in the reduced strategy space [2, 3].

For a minority game with $N_c$ rooms using the form of strategy in MG, there are $N_c^{N_c M}$ different strategies in the full strategy space while there are $N_c^{M+1}$ different strategies in the reduced strategy space provided that $N_c$ is a prime power. Because the strategy space size increases rapidly as $N_c$ increases, strategies will quickly get out of control for large $N_c$. Consequently, we would like to define the strategy in a different way.

In our game, we do not restrict players to have only “good strategies” since each player (who thinks inductively) does not know whether a strategy is good or not before commences the game. “Good strategy” is time dependent. In fact, players will adapt with each other in order to use those “good strategies”. Therefore, all strategies must be uniform in the following sense:

1) any input $\mu(t)$ can produce any output $x_i(t)$,

2) there are same probability for any output produced by any input.

Indeed, it is not completely clear that whether those strategies used in the Ein-Dor et al.’s model are uniform in the above sense. With the above consideration, a strategy $s$ consists of weights $(\omega_1^s, \ldots, \omega_M^s) \in GF(N_c)^M$ and a uniform random variable called bias $\rho_s \in GF(N_c)$ satisfying the following condition:

$$\sum_{j=1}^{M} \omega_j^s = \eta$$

where $\eta$ is a fixed constant in $GF(N_c)$. Note that all the arithmetic used to generate a strategy and the corresponding choice (including Eq. (2) above and Eq. (3) below) are performed in the finite field $GF(N_c)$. The choice is defined to be the sum of the weighted sum of the last $M$ output plus the bias. Namely, the choice of
the $i$th player using the strategy $s$ with history $\mu(t)$ is given by:

$$\chi_{i,s}(t) = \rho_s + \sum_{j=1}^{M} \omega_j^s \Omega(t-j)$$

(3)

Physically, the weight $\omega_j^s$ represents the importance of the output of the game of $j$th pass before on the choice. It is obvious that the strategies of MCMG all fulfil the uniform criteria which was mentioned before.

Since the weights of a strategy are $M$ independent variable in $GF(N_c)$ which satisfy a single constraint, namely Eq. (2), there are $N_c^{M-1}$ different combinations of weights $\omega_j^s$. Moreover, it can be shown that strategies with the same set of weights are anti-correlated with each other while the others are uncorrelated with each other (see Ref. [14]). Hence, it follows that both the full and reduced strategy space size in MCMG is equal to $N_c^M$. Thus, the strategy space size of MCMG is much smaller than that of MG.

In our model, every player picks and sticks to $S$ randomly drawn strategies before commences the game. We denote the strategies of the $i$th player by $(s_i^{(1)}, \ldots, s_i^{(S)})$. But how does a player decide which strategy is the best? Players use the virtual score, which is just the hypothetical profit for using a single strategy in the game, to estimate the performance of a strategy. In each pass, the virtual score of a strategy $s$ of the $i$th player, $U_{i,s}(t)$, is updated by

$$U_{i,s}(t) = U_{i,s}(t-1) + 2\delta(\chi_{i,s}^{\mu(t)} - \Omega(t)) - 1$$

(4)

where $\chi_{i,s}^{\mu(t)}$ is the choice of the $i$th player under history $\mu(t)$ using the strategy $s$. Each player uses one’s own strategy with the highest virtual score. Although our MCMG model is quite similar to the MG model, there are two main differences, namely, in the number of choices of players and in the formalism of strategies.

In our game, the aim of each player is to maximize one’s own wealth which can be in turn achieved by the maximization of the global profit. So the quantity of interest is

$$\sigma_j^2 = \langle (A_j(t))^2 \rangle - \langle A_j(t) \rangle^2$$

$$\equiv \langle (A_j(t) - N/N_c)^2 \rangle,$$

(5)

namely, the variance of the attendance of room $j$, where the attendance of a room is just the number of people chosen that room. Indeed, the maximum global profit will be achieved if and only if the largest possible minority size $[N/N_c]$ is attained. So the expected attendance of all rooms should be equal to $N/N_c$ for players to gain as much as possible. Accordingly, the variance of the attendance of a room represents the loss of players in the game.

In order to investigate the significance of the strategies, we would like to compare the variance with the coin-toss case value in which all the players make their decision simply by tossing an unbiased coin. It is easy to check that the probability for the attendance $A_j(t)$ equal to $x$ in the coin-toss case is given by

$$p(A_j(t) = x) = \frac{N C_x (N_c - 1)^{N-x}}{\sum_{x=1}^{N} N C_x (N_c - 1)^{N-x}},$$

(6)

where $N C_x = N!/[x!(N-x)!]$. So the expectation of $A_j$ and $A_j^2$ in the coin-toss case are given by

$$\langle A_j \rangle = \frac{\sum_{x=1}^{N} x N C_x (N_c - 1)^{N-x}}{\sum_{x=1}^{N} N C_x (N_c - 1)^{N-x}}$$

and

$$\langle A_j^2 \rangle = \frac{\sum_{x=1}^{N} x^2 N C_x (N_c - 1)^{N-x}}{\sum_{x=1}^{N} N C_x (N_c - 1)^{N-x}}.$$

(7)

(8)

### III. RESULTS OF NUMERICAL SIMULATIONS

In all the simulations, each set of data was taken for 1,000 independent runs. In each run, we took the average values on 15,000 steps after running 10,000 steps for equilibrium starting from initialization.

#### A. Comparison of two-room MCMG with MG

We first want to investigate if the performance of players are different in MG and the two-room MCMG. We applied the two models to study the properties of the mean attendance as a function of the control parameter $\alpha$ as shown in figure 1. The control parameter $\alpha$ measures the ratio of the reduced strategy space size to the number of strategies at play. Specifically, $\alpha = 2^{M+1}/NS$ for the MG and $\alpha = N^{M}/NS$ for the MCMG. We have only studied the properties of the attendance of one of the two rooms as the attendance of the two rooms have the same behavior due to symmetry [1].

In both MG and the two room MCMG, the mean attendance always fluctuates around the expected value $N/2$ no matter how large is the control parameter (see figure 1). Therefore, we believe that players have the same performance in both MG and the two room MCMG if we only consider the mean attendance. To further investigate the difference of the performance of players in MG and the two-room MCMG, we studied the variance of the attendance as a function of the control parameter which was shown in figure 2.

Before making comparison with that in the MCMG, let us first take a brief review on the properties of the variance in MG as shown in figure 2 [10, 11, 12]. In MG, there is a great variance $\sigma^2$ when the control parameter $\alpha$ is small. It is because players use very similar strategies when the reduced strategy space size ($= 2^{M+1}$) is much smaller than the number of strategies at play ($= NS$).
Such overcrowding of strategies will lead to small minority size and also great fluctuation of the attendance. When the reduced strategy space size increases, the variance decreases rapidly since players are able to cooperate more with less overcrowding effect. Subsequently, the order parameter measures the bias of player’s decision to any choice for individual history.

For the MCMG, the variance of the attendance of a room, \( \sigma^2 \), exhibits similar behavior as a function of the control parameter \( \alpha (=N^M_c/NS \text{ for MCMG}) \) to that in MG no matter what the number of strategies \( S \) is. For example, the variance tends to the coin-toss case value for sufficiently large control parameter. Moreover, there is again an indication of a second order phase transition. To check if the phase transition is second order or not, we calculate the order parameter

\[
\theta = \frac{1}{N^M_c} \sum_{\mu} \left\{ \sum_{\Omega} \frac{1}{\Omega^2} \left( \langle p(\Omega | \mu) \rangle - \frac{1}{N^M_c} \right)^2 \right\} \tag{9}
\]

where \( \langle p(\Omega | \mu) \rangle \) denotes the conditional time average of the probability for \( \Omega(t) = \Omega \) given that \( \mu(t) = \mu \). In fact, the order parameter measures the bias of player’s decision to any choice for individual history.

Figure 3 shows that the order parameter vanishes when the control parameter is smaller than its value corresponding to minimum variance. As a result, we confirm that the phase transition is a second order one. Although our two-room MCMG model does not exactly coincide with the MG model, the variance has almost the same properties as a function of the control parameter in both models. However, the behaviour of the variance as a function of the memory size \( M \) are different in MG and MCMG because the reduced strategy space size are different in the two models.
B. The attendance of different rooms in MCMG

Here, we want to study if the properties of the attendance of different rooms are different or not in the MCMG. Figures 4 and 5 show the mean attendance and the variance of the attendance of different rooms versus the control parameter $\alpha$.

We found that the behavior of the attendance of different rooms are almost the same in MCMG with $N_c \geq 2$ because there is same probability for any choice to be the minority side. As we only want to focus on the study of the attendance of a room, not on their high order correlation, so we will stick to one of them from now on.

C. The attendance in MCMG with different $N_c$

Now, we investigate the properties of the attendance for MCMG with different number of rooms $N_c$. Figure 6 depicts the dependence of the mean attendance on the control parameter $\alpha$ for MCMG with $N_c = 3$ to 7. In MCMG with different $N_c$, the mean attendance fluctuates around $N/N_c$ irrespective of the control parameter $\alpha$. It is reasonable as the maximum global profit will be achieved if and only if the largest possible minority size $\lfloor N/N_c \rfloor$ is attained.

We also studied the dependence of the variance of the attendance on the control parameter $\alpha$ for MCMG with $N_c = 3$ to 7 as shown in figure 7. In this figure, we have divided the variance by $N/N_c$ ($\approx$ largest possible minority size) in order to have an objective comparison of the variance for different $N_c$.

No matter what is the value of $N_c$, there is always a cusp of the variance which strongly suggests the occurrence of a second order phase transition (see figure 7). We calculate the order parameter $\theta$ (introduced in Eq.(10)) to identify the order of the phase transition as shown in figure 8. In MCMG with different $N_c$, the order parameter vanishes when the control parameter is smaller than its value at the cusp. Therefore, we conclude that there is a second order phase transition in MCMG with different $N_c$.

Moreover, the variance tends to a constant value when the control parameter $\alpha \to \infty$. But is such a constant...
FIG. 7: The variance $\sigma^2$ versus the control parameter $\alpha$ in MCMG with different $N_c$ where $S = 2$.

FIG. 8: The order parameter $\theta$ versus the control parameter $\alpha$ in MCMG with different $N_c$ where $S = 2$.

value consistent with the coin-toss case value as in MG? Table 1 shows the value of $\sigma^2$ at $\alpha \approx 100$ in contrast with the coin-toss case value for MCMG with different $N_c$.

![Graph](image)

**TABLE I:** The value of the variance $\sigma^2$ at $\alpha \approx 100$ and the coin-toss limit value in MCMG with different $N_c$ where $S = 2$.

| $N_c$ | $\sigma^2$ at $\alpha \approx 100$ | Coin-toss limit |
|-------|----------------------------------|-----------------|
| 3     | $0.217 \pm 0.010$                | 0.2222          |
| 4     | $0.184 \pm 0.008$                | 0.1875          |
| 5     | $0.156 \pm 0.007$                | 0.1600          |
| 7     | $0.120 \pm 0.006$                | 0.1224          |

It was shown clearly that the two values agree with each other. In other words, the variance does tend to the coin-toss case value as $\alpha \to \infty$ in MCMG with different $N_c$. As a result, we conclude that the variance in MCMG, no matter what $N_c$ is, has very similar properties with respect to the control parameter as in MG.

Since the variance shows a second order phase transition, it is instructive to investigate if $\alpha_o$, the value of $\alpha$ corresponding to minimum $\sigma^2$, depends on $N_c$ or not.

The relationship between the minimum variance $\sigma^2_o$ and $N_c$ is also worth to study. We estimated the value of $\alpha_o$ and $\sigma^2_o$ by polynomial interpolation around the point of minimum variance. The error of $\alpha_o$ and $\sigma^2_o$ was estimated to be the difference of the value of $\alpha_o$ and $\sigma^2_o$ found for different degrees of interpolation.

![Graph](image)

**TABLE II:** The estimated minimum variance $\sigma^2_o$ and corresponding $\alpha_o$ in MCMG with different $N_c$ where $S = 2$.

| $N_c$ | $\alpha_o$ | $\sigma^2_o/(N/N_c)$ |
|-------|------------|----------------------|
| 3     | 0.574 ± 0.007 | 0.274 ± 0.001       |
| 4     | 0.415 ± 0.009 | 0.272 ± 0.002       |
| 5     | 0.331 ± 0.001 | 0.283 ± 0.002       |
| 7     | 0.308 ± 0.013 | 0.318 ± 0.002       |

Table 2 summarises the estimated values of $\alpha_o$ and $\sigma^2_o$ for different $N_c$. The estimated values of $\alpha_o$ and $\sigma^2_o$ against $N_c$ with fixed memory size $M$ are also shown in figure 9. We again divided the variance by $N/N_c$ in order to have an objective comparison of the variance for different $N_c$.

![Graph](image)

**FIG. 9:** Estimated values of $\alpha_o$ and $\sigma^2_o$ in MCMG with different $N_c$ where $S = 2$.

We found that in MCMG, $\alpha_o$ decreases as $N_c$ increases. We may explain this phenomenon as follows: For MCMG with more number of rooms, more strategies at play are required such that players can use all the strategies in the
reduced strategy space. Since maximum cooperation is attained when all the strategies in the reduced strategy space are used by players, so the value of $\alpha_0$ is larger for MCMG with more number of rooms. We also found that the scaled variance $\sigma^2/(N/N_c)$ increases as $N_c$ increases. It is due to the increase of difficulty for players to cooperate with each other when there are more choices for them.

On the other hand, we also studied the behavior of the attendance as a function of the control parameter $\alpha$ in MCMG with different number of strategies $S$. Figures 10 and 11 display the results for $N_c = 3$. The two figures indicate that the behavior of the attendance as a function of the control parameter $\alpha$ for different number of strategies $S$ are similar, just like the case of MG and MCMG with $N_c = 2$. In conclusion, the attendance in MCMG has very similar properties with respect to the control parameter as in MG no matter how many choices and strategies players have.

### D. Comparison of player’s wealth for different $N_c$

We proceed to study the properties of player’s wealth in the MCMG. The mean and maximum player’s wealth as a function of the control parameter $\alpha$ for $N_c = 3$ to 7 were shown in figures 12 and 13.

![FIG. 12](image1.png)  
**FIG. 12:** Comparison of the mean player’s wealth versus the control parameter $\alpha$ in MCMG with different $N_c$ where $S = 2$.

![FIG. 13](image2.png)  
**FIG. 13:** Comparison of the maximum player’s wealth versus the control parameter $\alpha$ in MCMG with different $N_c$ where $S = 2$.

From figures 12 and 13, we notice that player’s wealth displays similar behavior for MCMG with different number of rooms $N_c$. When the reduced strategy space size is much smaller than the number of strategies at play, the mean player’s wealth remains almost constant as the control parameter $\alpha$ increases. When $\alpha$ is small, the system is in the overcrowding phase where all the rooms
have the same chance to “win”. Therefore, all the players, on average, always win the same number of times and make the same amount of profit for small $\alpha$. Then the mean player’s wealth attains a maximum value near the point of minimum variance when the reduced strategy space size is approximately equal to the number of strategies at play. If the control parameter $\alpha$ increases further, the number of players $N$ is comparable to the number of rooms $N_c$. Thus, finite size effect is important. In this case, some of the rooms will have a higher chance to “win”. Then some of the players will always lose while some of them will always win in the game. As a result, the mean player’s wealth decreases rapidly when the control parameter $\alpha$ increases further.

The properties of the maximum player’s wealth is similar to the mean player’s wealth except there is a significant peak corresponding to maximum cooperation of players. Although players on average always perform the same for small $\alpha$, but the smart players can perform much better when the small control parameter $\alpha$ increases. Therefore, those smart players is wealthier when the control parameter $\alpha \to 1^+$. 

IV. CONCLUSIONS

Although the MCMG and MG models are not exactly the same, our work shows that the attendance of a room in MCMG has similar behavior to that in MG as a function of the control parameter. Besides, we found that the attendance of different rooms displays almost the same behavior in MCMG with number of choices $N_c \geq 2$. Moreover, we observed that both the attendance and player’s wealth displays similar properties as a function of the control parameter in MCMG with different $N_c$. As all the above mentioned features can be explained reasonably, so we concluded that we have successfully built a computationally feasible model of multi-choice minority game — MCMG.

Various extensions of the MCMG model could be studied. For example, the multi-choice game with zero-sum and the multi-room game with agents who can invest different amount. We hope the study of the extensions of the MCMG model can give us more insight on more realistic complex adaptive system.

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