Branching Ratio and direct CP Asymmetry for $B \to K^{*}\gamma$ Decay in MSSM

Marina-Aura Dariescu and Ciprian Dariescu*

Institute of Theoretical Science
University of Oregon, Eugene, OR 97403

Abstract

The present paper deals with a next-to-leading order analysis of the radiative $B \to K^{*}\gamma$ decay. Working in the PQCD approach developed by Szczepaniak et al., we compute the correction, coming from a single gluon exchange with the spectator, to the essential form factor. Since the branching ratio gets much above the experimental data, although in agreement with other theoretical models predictions, we take into consideration the effects of complex flavour couplings in the squark sector. Finally, we discuss these SUSY implications on the branching ratio and direct CP asymmetry values and impose bounds on the squark mixing parameter $(\delta_{23}^{d})_{LR}$.

*On leave of absence from Department of Theoretical Physics, Al. I. Cuza University, Bd. Copou no. 11, 6600 Iași, Romania, email address (after May 1): marina@uaic.ro
After the Cabibbo-favoured $b \to s \gamma$ mode was first reported, in 1993, by CLEO II [1] and updated in 1995 [2], the exclusive radiative decays, $B \to K^* \gamma$ and $B \to \rho \gamma$, as well as the inclusive ones, $B \to X_s(d) \gamma$, have become main targets for both experimental and theoretical investigations. The exclusive modes, which are easier to be experimentally investigated [3, 4, 5], but less theoretically clear, have been worked out in different approaches. For example, the spin symmetry for heavy quarks combined with wave function models [6, 7] or the heavy quark effective theory when both $b$ and $s$ are heavy [8] have been used. Also, perturbative QCD (PQCD) formalisms, introduced for exclusive nonleptonic heavy-to-light transitions, have been extended to account for the radiative decays. Recently, detailed analyses of $B \to K^* \gamma$ and $B \to \rho \gamma$, in the next-to-leading order (NLO), with the inclusion of hard spectator and vertex corrections, have been performed [9-11] and a consistent treatment, based on a new factorization formula, has been proposed [12]. Besides an independent determination of the $|V_{td}/V_{ts}|$ ratio, the $b \to s \gamma$ decays are suitable for studying the viability of SUSY extensions of the SM, in view of flavour changing neutral currents (FCNC) and CP tests, and for imposing constraints on the supersymmetric benchmark scenarios [13, 14].

The aim of the present paper is to analyse the $B \to K^* \gamma$ decay, in the minimal supersymmetric SM (MSSM) context. First, at next-to-leading order, we compute the hard-spectator correction to the essential form factor. In this respect, we employ the PQCD approach developed by Szczepaniak et al. [15], for decays dominated by tree diagrams and later extended to “penguin” processes [16]. As the branching ratio gets much above the experimental data, we make use of the mass insertion method to include, in the Wilson coefficients $C_7, C_8$, gluino-mediated FCNC contributions. Finally, the available data on $Br$ and direct CP asymmetry are used to constrain the complex values of the squark mixing parameter $(\delta_{23}^d)^{L,R}$. The effective Hamiltonian which describes the $B \to K^* \gamma$ radiative decay is given by [9, 10]

$$H = \frac{G_F}{\sqrt{2}} \lambda_p [C_7 O_7 + C_1 O_1^p + C_8 O_8],$$

(1)

where $\lambda_p \equiv V_{pb} V_{ps}^*$, with $p$ summed over $u$ and $c$, and $C_1, C_7, C_8$ are the effective Wilson coefficients at $\mu = m_b$. The hadronic matrix elements of the four-fermion operator and of the electromagnetic and chromagnetic penguin
operators
\[
\mathcal{O}_1^p = (\bar{s} \gamma_\mu (1 - \gamma_5) p) (\bar{p} \gamma_\mu (1 - \gamma_5) b)
\]
\[
\mathcal{O}_7 = \frac{e m_b}{8\pi^2} [\bar{s} \sigma^{\mu\nu} (1 + \gamma_5) b] F_{\mu\nu}
\]
\[
\mathcal{O}_8 = \frac{g_s m_b}{8\pi^2} [\bar{s} \sigma^{\mu\nu} (1 + \gamma_5) T^i b] G^i_{\mu\nu}
\]

possess a general Lorentz decomposition
\[
\langle K^* | \bar{s} \gamma_\mu b | \bar{B} \rangle = \frac{2i V(q^2)}{m_B + m_{K^*}} \varepsilon^{\mu\nu\alpha\beta} \epsilon^\nu P^K_{\mu} P^B_{\beta},
\]
\[
\langle K^* | \bar{s} \gamma_\mu \gamma_5 b | \bar{B} \rangle = 2m_{K^*} A_0(q^2) \langle \epsilon^* q \rangle q^\mu - A_1(q^2)(m_B + m_{K^*}) \left[ \epsilon^*_\mu - \epsilon^* q \frac{q^2}{q^2} q_\mu \right],
\]
\[
\langle K^* | \bar{s} \sigma_{\mu\nu} q^\nu b | \bar{B} \rangle = 2T_1(q^2) \varepsilon^{\mu\nu\alpha\beta} \epsilon^\nu P^K_{\mu} P^B_{\beta},
\]
\[
\langle K^* | \bar{s} \sigma_{\mu\nu} \gamma_5 q^\nu b | \bar{B} \rangle = -iT_2(q^2) \left[ (m^2_B - m^2_{K^*}) \epsilon^*_\mu - (\epsilon^* q)(P_B + P_{K^*})_\mu \right] - iT_3(q^2) \langle \epsilon^* q \rangle \left[ \frac{q^2}{m^2_B - m^2_{K^*}} (P_B + P_{K^*})_\mu \right],
\]

where \( q_\mu \) is the momentum of the photon and \( \epsilon^\nu \) is the \( K^* \) 4-vector polarization. In the heavy quark limit, \( m_b \gg \Lambda_{QCD} \), neglecting the corrections of order \( 1/m_b \) and \( \alpha_s \), one has the following relation among the form factors [9]
\[
\frac{m_B}{m_B + m_{K^*}} V(0) = \frac{m_B + m_{K^*}}{m_B} A_1(0) = T_1(0) = T_2(0) \equiv F_{K^*}(0)
\]

This relation is broken when one includes QCD radiative corrections coming from vertex renormalization and hard gluon exchanges with the spectator. We recommend [9, 10, 12] for detailed analyses of both factorizable and non-factorizable vertex and hard-spectator contributions, involving the operators \( \mathcal{O}_7, \mathcal{O}_8 \) and penguin-type diagrams of \( \mathcal{O}_1 \). However, it has been stated that factorization holds, at large recoil and leading order in \( 1/m_b \) [11] and quantitative tests for proving QCD factorization at the level of power corrections have been provided [17].
For a consistent treatment of radiative decays, at next-to-leading order in QCD, a novel factorization formula have been proposed in [12]. In this approach, the hadronic matrix elements in (1) are written in terms of the essential form factor, which describes the long-distance dynamics and is a nonperturbative object, and of the hard-scattering kernels, $T_I^i$ and $T^{II}_i$, including the perturbative short-distance interactions, as

$$\langle K^*\gamma|O_i|B\rangle = \left[ F_{K^*}(0)T_I^i + \phi_B \otimes T^{II}_i \otimes \phi_{K^*} \right] \cdot \eta,$$  \hspace{1cm} (6)

where $\eta$ is the photon polarization. When the dominant contribution comes from $O_7$, we use (4) to write down the decay amplitude as

$$A^{(0)} = \frac{G_F}{\sqrt{2}} \lambda_p \frac{e m_b(\mu)}{2\pi^2} C_7(\mu) F_{K^*}(0) \times \left[ \epsilon_{\mu\nu\alpha\beta} \eta^\mu \epsilon^{*\nu} P_{K^*}^\alpha P_B^\beta - i (P_{K^*}q)(\eta \epsilon^*) + i(\epsilon^* q)(\eta P_{K^*}) \right],$$  \hspace{1cm} (7)

and consequently the branching ratio reads

$$Br^{LO} = \tau_B \frac{G_F^2 \lambda_p^2}{2\pi^2} \frac{e m_b(\mu)^2}{2\pi^2} \frac{m_B^3}{m_B} \left( 1 - z^2 \right)^3 |C_7(m_b)|^2 |F_{K^*}(0)|^2 ,$$  \hspace{1cm} (8)

with $z = m_{K^*}/m_B$. At next-to-leading order in $\alpha_s$, one has to consider, in (6), the contributions to the hard scattering kernels $T_I^i$ coming from the operators $O_1$ and $O_8$. These have been evaluated in [12] and bring (7) to the expression

$$A = \frac{G_F}{\sqrt{2}} \lambda_p \frac{e m_b(\mu)}{2\pi^2} \left[ C_7 + \frac{\alpha_s C_F}{4\pi} \left( C_1 G_1^p + C_8 G_8 \right) \right] F_{K^*}(0) \times \left[ \epsilon_{\mu\nu\alpha\beta} \eta^\mu \epsilon^{*\nu} P_{K^*}^\alpha P_B^\beta - i (P_{K^*}q)(\eta \epsilon^*) + i(\epsilon^* q)(\eta P_{K^*}) \right],$$  \hspace{1cm} (9)
where \( C_F = (N^2 - 1)/(2N), \) \( N = 3, \) and

\[
G_1(s) = -\frac{833}{162} - \frac{20 i\pi}{27} + \frac{8\pi^2}{9} s^{3/2} + \frac{2}{9} \left[ 48 + 30i\pi - 5\pi^2 - 2i\pi^3 - 36\zeta(3) + (36 + 6i\pi - 9\pi^2) \ln s \right. \\
+ (3 + 6i\pi) \ln^2 s + \ln^3 s \big] s + \frac{2}{9} \left[ 18 + 2\pi^2 - 2i\pi^3 + (12 - 6\pi^2) \ln s + 6i\pi \ln^2 s + \ln^3 s \right] s^2 + \frac{1}{27} \left[ -9 + 112i\pi - 14\pi^2 + (182 - 48i\pi) \ln s - 126 \ln^2 s \right] s^3,
\]

\[
G_8 = \frac{11}{3} - \frac{2\pi^2}{9} + \frac{2i\pi}{3},
\]

with \( s_c = m_{cb}^2/m_b^2 \) and \( \mu = m_b. \)

Going further, we add factorizable NLO hard-spectator corrections, to the form factor \( F_K(0). \) For a single gluon exchanged with the spectator (see Figure 1), we extend the PQCD approach, developed by Szczeniak et al. for heavy-to-light transitions dominated by tree diagrams [15], to the so-called penguin processes.

We evaluate the matrix element of the operator \( O_7, \) as the following trace over spin, flavor and color indices, and integration over momentum fractions [16]

\[
T_\mu = \text{Tr} \left[ \overline{\phi}_K \cdot \sigma_{\mu\nu} (1 + \gamma_5) q^\nu \frac{k_b + m_b}{k_b^2 - m_b^2} \gamma_\alpha \phi_B \gamma^\alpha \frac{4g_\alpha}{Q^2} \right] + \text{Tr} \left[ \overline{\phi}_K \cdot \gamma_5 \frac{k_s}{k_s^2} \sigma_{\mu\nu} (1 + \gamma_5) q^\nu \phi_B \gamma^\alpha \frac{4g_\alpha}{Q^2} \right],
\]

where \( Q^2 \approx -(1 - x)(1 - y)m_B^2. \) The \( B \) meson wave function

\[
\phi_B = \frac{f_B}{12} \varphi_B(x)(P_B + m_B)\gamma_5
\]

contains a strongly peaked distribution amplitude, around \( a = \lambda_B/m_B \approx 0.072, \) for \( \lambda_B = 0.38. \) The \( K^* \) is described by the wave function

\[
\phi_{K^*} = \frac{f_{K^*}}{12} \varphi_{K^*}(y) P_{K^*}\gamma_5,
\]

\( \text{(10)} \)
where the light-cone distribution amplitude, \( \varphi_{K^*}(y) \), has the following expansion in Gegenbauer polynomials [18]

\[
\varphi_{K^*}(y) = 6y(1-y)[1 + \alpha_1^{K^*} C_1^{(3/2)}(2y-1) + \alpha_2^{K^*} C_2^{3/2}(2y-1) + ...],
\]

with \( C_1^{3/2}(u) = 3u, C_2^{3/2}(u) = (3/2)(5u^2 - 1) \), \( \alpha_1^{K^*}(m_b) = 0.18 \pm 0.05 \), and \( \alpha_2^{K^*}(m_b) = 0.03 \pm 0.03 \). Performing the calculations in (11) and using the form factors decomposition (4), we identify the spectator contribution to the essential form factor as

\[
F_{sp}(a) = \frac{g_s^2 f_B f_{K^*}}{m_B \lambda_B} \int_0^{1-a} dy \frac{(2-y)}{(1-y)^2} \varphi_{K^*}(y),
\]

where the \( K^* \)-mass is neglected. Since we have introduced a cut-off for \( y \to 1 \), the form factor correction (15) depends on the peaking parameter \( a \) and this is a main uncertainty in our calculations. For the following input values: \( \alpha_s(\mu = Q^2) \approx 0.38 \), \( f_B = 0.180 \text{ GeV} \), \( f_{K^*}^+ = 0.185 \text{ GeV} \) and \( a = 0.072 \), we get

\[
F_{sp}(0.072) = 0.1475
\]

With the total form factor \( F_{K^*}(0) + F_{sp}(0.072) = 0.38 + 0.1475 \) in the amplitude (9), the branching ratio gets significantly enhanced to the value \( B_{\gamma NLO} = 6.97 \times 10^{-5} \). This is comparable to the average theoretical prediction \((7.5 \pm 0.3) \times 10^{-5} \) [9, 11, 12], but is much above the experimental data:

\[
\begin{align*}
Br(B^+ \to K^{*+}\gamma) &= \begin{cases} 
(3.83 \pm 0.62 \pm 0.22) \times 10^{-5} \quad \text{(BaBar [3])} \\
(3.76^{+0.89}_{-0.83} \pm 0.28) \times 10^{-5} \quad \text{(CLEO [4])} \\
(3.89 \pm 0.93 \pm 0.41) \times 10^{-5} \quad \text{(Belle [5])}
\end{cases} \\
Br(B^0 \to K^{*0}\gamma) &= \begin{cases} 
(4.23 \pm 0.40 \pm 0.22) \times 10^{-5} \quad \text{(BaBar [3])} \\
(4.55^{+0.72}_{-0.68} \pm 0.34) \times 10^{-5} \quad \text{(CLEO[4])} \\
(4.96 \pm 0.67 \pm 0.45) \times 10^{-5} \quad \text{(Belle[5])}
\end{cases}
\end{align*}
\]

Moreover, the direct CP asymmetry, defined as

\[
a_{CP} = \frac{\Gamma(\bar{B} \to K^*\gamma) - \Gamma(B \to K^*\gamma)}{\Gamma(\bar{B} \to K^*\gamma) + \Gamma(B \to K^*\gamma)},
\]

has been predicted by the SM to be \( a_{CP} < |0.005| \), and this disagrees with the BaBar and CLEO data, \( a_{CP} = -0.044 \pm 0.076 \pm 0.012 \) (BaBar [3]) and
$a_{CP} = 0.08 \pm 0.13 \pm 0.03$ (CLEO [4], for the sum of neutral and charged $B \to K^{*}\gamma$ decays).

So, predictions for measurable values of the direct CP asymmetry, in agreement with more precise measurements, might have a dominantly new physics origin. Following this idea, let us analyse the $B \to K^{*}\gamma$ decay in the MSSM context. Using the mass insertion approximation, [19], we incorporate, in the Wilson coefficients $C_7$ and $C_8$, the FCNC SUSY contributions

$$C_{7}^{SUSY}(M_{SUSY}) = \frac{\sqrt{2} \pi \alpha_s}{G_F (V_{ub} V^*_u + V_{cb} V^*_c) m^2_{\tilde{g}}} \left( \delta^d_{23} \right)_{LR} m_{\tilde{g}} F_0(x);$$

$$C_{8}^{SUSY}(M_{SUSY}) = \frac{\sqrt{2} \pi \alpha_s}{G_F (V_{ub} V^*_u + V_{cb} V^*_c) m^2_{\tilde{g}}} \left( \delta^d_{23} \right)_{LR} m_{\tilde{g}} G_0(x),$$

where

$$F_0(x) = - \frac{4x}{9(1-x)^4} \left[ 1 + 4x - 5x^2 + 4x \ln(x) + 2x^2 \ln(x) \right],$$

$$G_0(x) = \frac{x}{3(1-x)^4} \left[ 22 - 20x - 2x^2 + 16x \ln(x) - x^2 \ln(x) + 9 \ln(x) \right].$$

In (19), $x = m_{\tilde{g}}^2 / m_{\tilde{q}}^2$ is expressed in terms of the gluino mass, $m_{\tilde{g}}$, and an average squark mass, $m_{\tilde{q}}$. We underline that, in the expressions of $C_{7,8}^{SUSY}(M_{SUSY})$, we have kept only the left-right squark mixing parameter $\left( \delta^d_{23} \right)_{LR} = (\Delta_{bs}) / m_{\tilde{q}}^2$ since, being proportional to the large factor $m_{\tilde{g}} / m_b$, will have a significant numerical impact on the branching ratio value. The quantities $\Delta_{bs}$ are the off-diagonal terms in the sfermion mass matrices, connecting the flavours $b$ and $s$ along the sfermion propagators [19]. In these assumptions, the total Wilson coefficients, encoding the New Physics, become

$$C_{7,8}^{total}[x, \delta] = C_{7}(m_b) + C_{7}^{SUSY}(m_b),$$

$$C_{8}^{total}[x, \delta] = C_{8}(m_b) + C_{8}^{SUSY}(m_b),$$

where $C_{7,8}^{SUSY}(m_b)$ have been evolved from $M_{SUSY} = m_{\tilde{g}}$ down to the $\mu = m_b$ scale, using the relations [20]

$$C_{8}^{SUSY}(m_b) = \eta C_{8}^{SUSY}(m_{\tilde{g}}),$$

$$C_{7}^{SUSY}(m_b) = \eta^2 C_{7}^{SUSY}(m_{\tilde{g}}) + \frac{8}{3} (\eta - \eta^2) C_{8}^{SUSY}(m_{\tilde{g}}),$$

(21)
with
\[ \eta = \left( \frac{\alpha_s(m_{\tilde{g}})}{\alpha_s(m_t)} \right)^{\frac{2}{21}} \left( \frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right)^{\frac{2}{23}} \]  

Finally, putting everything together, we replace, in (9), the Wilson coefficients \( C_7 \) and \( C_8 \) respectively by \( C_{7 \text{total}}[x, \delta] \) and \( C_{8 \text{total}}[x, \delta] \), the form factor \( F^{K^*}(0) \) by \( F^{K^*}(0) + F^{sp}(a) \) and, consequently, the branching ratio (8) turns into

\[ B_{r \text{total}} = B_{r \text{SM+_SUSY}} = \frac{G_F^2 \alpha m_b^2}{32\pi^4} m_b^3 (1 - z^2)^3 |F_{K^*}(0) + F^{sp}(a)|^2 \]

\[ \times \left| \lambda_p \left[ C_7^\text{total}[x, \delta] + \frac{\alpha_s C_F}{4\pi} \left( C_1 G_1 + C_8^\text{total}[x, \delta] G_8 \right) \right] \right|^2 \]  

One can notice that, for a given \( x \) and \( \delta \equiv \rho e^{i\varphi} \), the total branching ratio is depending on three free parameters: \( a, \rho, \varphi \), while the direct CP asymmetry parameter, (17), is free of the uncertainty \( a \).

In the next coming discussion, we use the following input parameters: \( m_b(m_t) = 4.2 \text{ GeV}, \alpha = 1/137, |V_{tb}V_{ts}^\ast| = 0.0396 \pm 0.002, \tau_{B^0} = (1.546 \pm 0.018) \text{ ps} \), the QCD sum rules analyses result \( F_{K^*}(0) = 0.38 \), and \( m_{\tilde{q}} = 500 \text{ GeV} \). In what it concerns the gluino, as its pair production cross section has large cancellations in the \( e^+e^- \) annihilation, there is hope that the laser-backscattering photons will provide a precise gluino mass determination [21]. For a wide range of squark masses, a gluino mass of 540 GeV may be measured, with a precision of at least \( \pm 2 \ldots 5 \), at the multi-TeV linear collider at CERN.

(Figure 2)

For \( x \) taking the values \( x_l = 0.3, x_0 = (540/500)^2 \) and \( x_g = 3 \) (where \( l(g) \) comes from \( m_{\tilde{g}} \) less (greater) than \( m_{\tilde{q}} \)), and imposing the BaBar constraint [3]

\[ -0.17 < a_{CP} < 0.082 \]  

we draw, in Figure 2, the contour plots of constant \( B_{r \text{total}} \) (the dashed lines) and \( a_{CP} \) (the solid lines). When \( \{ \rho, \varphi \} \in [0, 0.03] \times [-\pi/2, \pi/2] \), we get, for the world average branching ratio data, over the \( B^\pm \) and \( B^0 \) decay modes, \( Br_{\text{exp}}(B^\pm \rightarrow K^{\pm}\gamma) = (4.22 \pm 0.28) \times 10^{-5} \), three dashed lines, with increasing thickness, as \( x \) goes from \( x_l \) to \( x_g \). Correspondingly, for \( a_{CP} \), we get three pairs of solid curves: the lower ones, for \( a_{CP} = -0.17 \), and the upper ones, for \( a_{CP} = 0.082 \). These solid contours close inside the values of direct CP asymmetry which do not agree with (24). Now, we are able to put strong
constraints on \( \left( \delta_{23}^{d} \right)_{LR} \), by looking at the segments of the Br-plots outside the solid contours, for each \( x \). We notice that, for \( m_{q} > m_{\bar{q}} \), all the negative phases, with suitable \( \rho \)'s, can accommodate both the relation (24) and the branching ratio data.

(Figure 3)

In Figure 3, we represent, the \( Br_{\text{total}} \) (in units of \( 10^{-5} \)) and \( a_{CP} \) (in units of \( 0.1 \)), with respectively dashed and solid lines, as functions of \( \varphi \), for \( x = 3 \). As \( \rho \) takes the following values: \( \rho \in \{0.005, 0.01, 0.015, 0.02\} \), we get four pairs of curves, with increasing thickness. The horizontal dashed line corresponds to the average \( Br \) data, while the horizontal solid ones stand for the constraint (24).

Finally, let us perform a numerical analyses, for \( x = x_{0} \), and increasing \( \rho \), starting with \( \rho = 0.005 \). As \( \varphi \in [-8\pi/16, -4\pi/16] \cup [3\pi/16, 7\pi/16] \), the \( Br_{\text{total}} \) and the direct CP asymmetry are inside the ranges \( 10^{5} \times Br_{\text{total}} \in [8.3, 3.3] \) and \( [3.1, 8.1] \) and, respectively, \( a_{CP} \in [-0.054, -0.093] \cup [0.096, 0.062] \), accommodating data and other theoretical models predictions. When \( \rho \) goes to bigger values, the two \( \varphi \) ranges, constrained by the allowed branching ratios, get closer and \( a_{CP} \) moves toward much bigger values. For example, for \( \rho = 0.01 \) and \( \varphi \in [-6\pi/16, -4\pi/16] \cup [3\pi/16, 5\pi/16] \), one gets \( 10^{5} \times Br_{\text{total}} \in [8.1, 3.6] \) and \( [3.2, 7.62] \) and, respectively, \( a_{CP} \in [-0.1, -0.16] \cup [0.21, 0.12] \) and we notice that only the negative \( \varphi \)-values lead to \( a_{CP} \) inside the BaBar constraint (24). For \( \rho = 0.015 \) and \( \varphi \in [-4\pi/16, -2\pi/16] \cup [\pi/16, 3\pi/16] \), the values \( 10^{5} \times Br_{\text{total}} \in [7.8, 3.6] \) and \( [3.3, 7.3] \) are compatible with a measurable \( a_{CP} \approx \pm 0.12 \). Starting with \( \rho = 0.02 \), the predictions for branching ratio are above data and other theoretical estimations, the minimum value being \( Br_{\text{total}} = 9 \times 10^{-5} \), for \( \varphi = 0 \), while the corresponding asymmetry is \( a_{CP} = 0.003 \).

In the present paper, we have analysed the radiative \( B \to K^{*}\gamma \) decay, in a combined PQCD and SUSY framework. First, we have used the PQCD approach, developed by Szczepaniak et al. \[15\] and extended to “penguin” processes \[16\], to compute the hard-spectator contribution, \( F_{sp}(a) \), to the essential form factor \( F_{K}^{-}(0) \). For the peaking parameter in the \( B \) wave function \( a = 0.072 \) and \( F_{K}^{-}(0) = 0.038 \), the branching ratio becomes \( Br^{NLO} = 6.97 \times 10^{-5} \), which is above the experimental data, while the direct CP asymmetry predicted by the SM lies much below \[3, 4\]. In order to
find an agreement, we extend our analyses by including, in the Wilson coefficients $C_{7,8}$, the SUSY contributions coming from squark mixing parameter $(\delta_{23})_{LR} = \rho e^{i\varphi}$. Consequently, the total branching ratio depends, besides $a$, on three (SUSY) parameters: $x, \rho, \varphi$, while $a_{CP}$ is free of the uncertainty coming from the form factors. Using the graphs displayed in Figures 2 and 3, one is able to find out allowed ranges for the mass insertion parameter $(\delta_{23})_{LR}$. As an example, for $x = (540/500)^2$, the world average branching ratio, $Br_{exp} = 4.22 \times 10^{-5}$, can be accommodated for $\{\rho, \varphi\} = \{0.005, -\frac{4\pi}{13}\}$ or $\{0.01, -\frac{4\pi}{15}\}$. The corresponding asymmetries, $a_{CP} = -0.085$ and respectively $a_{CP} = -0.147$, are inside the BaBar constraint (24).

ACKNOWLEDGMENTS

The authors gratefully acknowledge the kind hospitality and fertile environment of the University of Oregon where this work has been carried out. Professor N.G. Deshpande’s clarifying discussions and constant support are highly regarded. M.A.D.’s gratitude goes to the U.S. Department of State, the Council for International Exchange of Scholars (C.I.E.S.) and the Romanian-U.S. Fulbright Commission, for sponsoring her participation in the Exchange Visitor Program no. G-1-0005.

References

[1] R. Ammar et al. (CLEO Coll.) Phys. Rev. Lett. 71 (1993) 674.
[2] K.W. Edwards et al. (CLEO Coll.) Preprint CLEO CONF95-6 (July 1995).
[3] B. Aubert et al. (BaBar Coll.) Phys. Rev. Lett. 88 (2002) 101805.
[4] T.E. Coan et al. (CLEO Coll.) Phys. Rev. Lett. 84 (2000) 5283.
[5] Y. Ushiroda et al. (Belle Coll.) hep-ex/0104045.
[6] A. Ali and C. Greub, Phys. Lett. B 259 (1991) 182; 361 (1995) 146.
[7] A. Ali, T. Ohl and T. Mannel, Phys. Lett. B 298 (1993) 195.
[8] S. Veseli and M.G. Olsson, Phys. Lett. B 367 (1996) 309.
[9] A. Ali and A.Y. Parkhomenko, Eur. Phys. J. C 23 (2002) 89.
[10] A. Parkhomenko, hep-ph/0209346.
[11] M. Beneke, T. Feldmann and D. Seidel, Nucl. Phys. B 612 (2001) 25.
[12] S.W. Bosch and G. Buchalla, Nucl. Phys. B 621 (2002) 459.
[13] A. Ali and E. Lunghi, Eur. Phys. J. C 26 (2002) 195.
[14] M. Battaglia et al., Eur. Phys. J. C 22 (2001) 535.
[15] A. Szczepaniak, E.M. Henley and S.J. Brodsky, Phys. Lett. B 243 (1990) 287.
[16] M.A. Dariescu and C. Dariescu, Phys. Lett B 367 (1996) 349; 398 (1997) 361.
[17] A.L. Kagan and M. Neubert, Phys. Lett. B 539 (2002) 227.
[18] P. Ball, JHEP 09, 005 (1998); P. Ball and V.M. Braun, hep-ph/9808229.
[19] F. Gabbiani et al., Nucl. Phys. B 477, 321 (1996).
[20] X.G. He, J.Y. Leou and J.Q. Shi, Phys. Rev. D 64, 094018 (2001).
[21] S. Berge and M. Klasen, hep-ph/0303032.
FIGURE CAPTIONS

Fig.1. The Feynman contributing diagrams in the hard scattering amplitude $T_\mu$. The gluon and photon are respectively represented by dotted and dashed lines.

Fig.2. Contour plots of total branching ratio fitting the world average data, in units of $10^{-5}$, (the dashed lines) and the BaBar constraint (24) on direct CP asymmetry (the solid lines), as functions of $\rho$ and $\varphi$. The thickness of contours is increasing as $x$ takes the values: $x = \{0.3, 1.16, 3\}$. The solid curves close inside the values of $a_{CP}$ which disagree with the constraint (24).

Fig.3. The total branching ratio, in units of $10^{-5}$, (the dashed lines) and $10 \times a_{CP}$ (the solid lines), as functions of $\varphi$, for $x = 3$. The thickness of plots increases as $\rho$ is respectively: $\rho = \{0.005, 0.01, 0.015, 0.02\}$. The horizontal dashed line corresponds to the world average branching ratio and the horizontal solid lines are for the constraint (24).