Production of Two Non-Conjugate Leptoquarks in $e^- e^-$ Collisions

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Abstract

We study the production of scalar and vector leptoquarks in $e^- e^-$ scattering. We use the most general couplings to the known fermions which are dimensionless, baryon and lepton number conserving, and $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ invariant. Expressions are presented for production cross sections and predictions are given for future linear colliders.
1 Introduction

Leptoquarks are contained in many extensions of the Standard Model which try to unify lepton and quark degrees of freedom. In some of these scenarios the leptoquark states emerge at mass scales below 1 TeV, a range which can be probed with present and future high energy colliders. Since none of the possible extensions of the Standard Model are selected yet by experimental data, the search for leptoquarks should be carried out in a widely model independent framework. Demanding the leptoquark couplings to be dimensionless, $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ invariant, and baryon ($B$) and lepton ($L$) number conserving, there are nine scalar and nine vector states which qualify. Only a subset of these is contained in most of the proposed models.

There are some very stringent low energy limits on the ratios $m_{LQ}^2/\lambda^2$ of the leptoquark masses to their couplings to fermion. However, these bounds vary greatly depending on the structure of the leptoquark-fermion coupling matrix. For some choices there are even no limits at all.

In the above framework, the decay pattern of leptoquarks does not allow a clear distinction of the different species in general. However, the possible production channels are rather selective with respect to the different quantum numbers of the leptoquark states. More specifically, the discrimination between the different types of leptoquarks will be greatly facilitated at a linear collider of the next generation, when it is run in its four different modes: $e^+e^-$, $e^-\gamma$, $\gamma\gamma$ and $e^-e^-$. In this Letter we focus on the $e^-e^-$ collider mode and show that the pair production of leptoquarks in $e^-e^-$ scattering is only possible for a particular subset of combinations complementary to other leptoquark production reaction. After classifying the different combinations of leptoquarks which can be produced in $e^-e^-$ scattering in Section 2, the production cross sections and the discovery potential are derived in Section 3, and the Conclusion is stated in Section 4.

2 Classification of Processes

We consider the effective Lagrangian

\[
\mathcal{L} = \mathcal{L}_{|F|=2} + \mathcal{L}_{F=0},
\]

1 Note that there are models which contain all these states.
which describes the interaction of scalar and vector leptoquarks with the fermions of the Standard Model referring to the two possible classes of leptoquarks distinguished by their fermion number \( F = L + 3B \). Assuming the conservation laws enumerated in the Introduction and non-derivative couplings one has

\[
\mathcal{L}_{|F|=2} = (g_{1L} \bar{q} \gamma \tau_2 l + g_{1R} \bar{u} \gamma e) S_1 \\
+ \tilde{g}_{1R} \bar{d} \gamma e S_1 + g_{3L} \bar{q} \gamma i \tau_2 \bar{l} \tilde{S}_3 \\
+ (g_{2L} \bar{d} \gamma^\mu l + g_{2R} \bar{q} \gamma^\mu e)V_{2\mu} \\
+ \tilde{g}_{2L} \bar{u} \gamma^\mu \bar{l} \tilde{V}_{2\mu} \\
+ h.c.,
\]

\[
\mathcal{L}_{F=0} = (h_{2L} \bar{d} l + h_{2R} \bar{q} i \tau_2 e) R_2 + \tilde{h}_{2L} \bar{l} \tilde{R}_2 \\
+ (h_{1L} \bar{q} \gamma^\mu l + h_{1R} \bar{d} \gamma^\mu e) U_{1\mu} \\
+ \tilde{h}_{1R} \bar{u} \gamma^\mu e \tilde{U}_{1\mu} + h_{3L} \bar{q} \gamma^\mu \bar{l} \tilde{U}_{3\mu} \\
+ h.c.
\]

where \( f^c = C T^T \) and \( \tau_i \) are the Pauli matrices. We use the notation \( l = (e_L, \nu_L) \) and \( q = (d_L, u_L) \) for the left-handed \( SU(2)_L \) lepton and quark doublets, and \( e = e_R, d = d_R \) and \( u = u_R \) for the right-handed singlets. The family indices are suppressed, but it should be borne in mind that in general the constants \( g \) and \( h \) denote \( 3 \times 3 \) coupling matrices. The subscript of the scalar leptoquark fields \( S, \tilde{S}, R, \text{ and } \tilde{R} \), and of the vector fields \( V_{\mu}, \tilde{V}_{\mu}, U_{\mu} \), and \( \tilde{U}_{\mu} \) indicates their \( SU(2)_L \) singlet, doublet or triplet nature. The quantum numbers of the 18 individual states in Eqs (2,3) have been summarized in Refs [2,5].

In \( e^- e^- \) scattering, the process of leptoquark pair production is characterized by the following quantum numbers defined by the possible initial states:

\[
Q = -2 \\
F = 2 \\
(T, T_3) = \begin{cases} 
(0,0) & RR \\
(\frac{1}{2}, -\frac{1}{2}) & LR \\
(1,-1) & LL 
\end{cases}
\]

with \( Q \) the electric charge, \( F = L + 3B \) the fermion number, \( T \) the weak isospin and \( T_3 \) its third component. Here, \( R \) and \( L \) label the different helicity states of the electrons. Thus, if either only \( |F| = 2 \) or \( F = 0 \) leptoquarks exist they cannot be produced in \( e^- e^- \) scattering by \( 2 \rightarrow 2 \) processes. Pair-production
can only take place in the presence of two *non-conjugate* leptoquarks. The possible leptoquark combinations carrying the quantum numbers \( \{4\} \) of a given \( e^-e^- \) initial state are summarized in Table 1.

| \( Q \) | \( F \) | \( S_{1}^{-1/3} \) | \( S_{3}^{-1/3} \) | \( V_{2}^{-1/3} \) | \( \bar{V}_{2}^{-1/3} \) |
|---|---|---|---|---|---|
| \( T \) | \( T_{3} \) | \( R_{2}^{-5/3} \) | \( U_{1}^{-5/3} \) | \( U_{3}^{-5/3} \) |
| \( Q = -5/3 \) | \( F = 0 \) | \( 1/2 \) | \( -1/2 \) | 1 | 0 |
| \( Q = -1/3 \) | \( F = 2 \) |

Table 1: Leptoquark pairs obeying Eqs (4)

3 Production Cross Sections

According to Table 1, there are in total 18 different reactions which can take place. These can be subdivided into five classes whose typical Feynman diagrams are shown in Fig. 1. They are uniquely characterizable by the weak
isospin \((T, T_3)\) of the leptoquark pair, the leptoquark spins \(s\), and the number \(n\) of scattering channels:

\[
1: \quad (T, T_3) = (0, 0) \quad \text{and} \quad (1, -1) ; \\
2: \quad (T, T_3) = \left( \frac{1}{2}, \frac{1}{2} \right) \quad s = 0 \quad n = 2 ; \\
3: \quad (T, T_3) = \left( \frac{1}{2}, \frac{1}{2} \right) \quad s = 0 \quad n = 1 ; \\
4: \quad (T, T_3) = \left( \frac{1}{2}, \frac{1}{2} \right) \quad s = 1 \quad n = 2 ; \\
5: \quad (T, T_3) = \left( \frac{1}{2}, \frac{1}{2} \right) \quad s = 1 \quad n = 1 .
\]

The corresponding integrated cross sections are given by

\[
\sigma_1 = 4G \left[ S + L \left( D + 2m_S^2 + 2 \frac{m_S^4}{D} \right) \right], \quad (6)
\]

\[
\sigma_2 = 2G \left[ 3S + L \left( D + 2 \frac{m_0^2 m_2^2}{D} \right) \right], \quad (7)
\]

\[
\sigma_3 = G \left[ 2S + LD \right], \quad (8)
\]

\[
\sigma_4 = 8G \left[ 2S + L \left( D + 2(m_0^2 + m_2^2) + 2 \frac{(m_0^2 + m_2^2)^2}{D} \right) \right], \quad (9)
\]

\[
\sigma_5 = G \left[ \frac{S}{6} \left( \frac{D^2}{m_0^2 m_2^2} + 12 \left( \frac{D}{m_0^2} + \frac{D}{m_2^2} \right) + 12 \left( \frac{m_0^2}{m_2^2} + \frac{m_2^2}{m_0^2} \right) - 28 \right) - 4LD \right]. \quad (10)
\]

Here,

\[
G = \frac{3\pi \alpha^2}{s^2} \left( \frac{\lambda}{e} \right)^4 \\
D = s - m_0^2 - m_2^2 \\
S = \sqrt{D^2 - 4m_0^2 m_2^2} \\
L = \ln \frac{D + S}{D - S} , \quad (11)
\]

\(\lambda = \sqrt{hg}\) being the geometric mean of the leptoquark-lepton-quark couplings, and \(m_0, m_2, m_S\) denoting the mass of the \(F = 0, F = 2\) and the scalar leptoquark, respectively. Furthermore, \(\sqrt{s}\) is the center of mass energy.
In general, the dependence of the cross sections on the Yukawa couplings is slightly more involved than what is shown above, where we have dropped the chirality labels $L$ and $R$. They can be easily restored from the Feynman diagrams of Fig. 1 using Eqs 2 and 3, for the processes of class 1 ($h_{RG}h_{LR}$ or $h_{LR}h_{RG}$), and class 3 and 5 ($h_{LR}h_{RG}$ or $h_{RG}h_{LR}$). Class 2 and 4, however, simultaneously involve two diagrams with different couplings, $h_{LR}h_{RG}$ and $h_{RG}h_{LR}$. The results for $\sigma_2$ and $\sigma_4$ given in Eqs 7 and 9, respectively, apply only to the case where $h_{LR}h_{RG} = h_{RG}h_{LR}$. Obviously, an even clearer discriminating production pattern arises if the $L$- and $R$-couplings cannot be sizable simultaneously, as suggested by low-energy bounds [4]. In the limit where either $h_{LR}h_{RG} = 0$ or $h_{RG}h_{LR} = 0$ the cross sections for class 2 and 4 become those of class 3 and 5 respectively: $\sigma_2 \rightarrow \sigma_3$ and $\sigma_4 \rightarrow \sigma_5$.

In Figs 2 and 3, we have plotted the energy and mass dependence of these cross sections assuming 100% polarized beams, equal masses $m_0 = m_2 = m_S = m_{LQ}$ for both leptoquarks, and $\lambda = e$. The electron and quark masses are set to zero. The high energy ($\sqrt{s} \gg m_0, m_2$) behaviour of these cross sections is as follows:

$$\sigma_{1-4} \propto \frac{1}{s} \ln \frac{s}{m_0 m_2} \quad \sigma_5 \propto s. \quad (12)$$

At threshold ($\sqrt{s} \approx m_0 + m_2$), on the other hand, one has

$$\sigma_{1-4} \propto \sqrt{s - (m_0 + m_2)^2} \quad \sigma_5 \propto s - (m_0 + m_2)^2 \quad (13)$$

The pathological breaking of perturbative unitarity in the processes of class 5 is due to its purely $t$-channel nature. For non-gauge leptoquarks, this behaviour is not worrisome, since in this case new physics effects are anyway expected to set in at some higher energy scale.

For gauge leptoquarks, though, a gauge bilepton [3] may be exchanged in the $s$-channel. If it couples with strength $\lambda$ to the leptoquarks and electrons, the complete cross section becomes:

$$\sigma_5' = G \left\{ \frac{S}{6} \left[ 62 + \frac{2m_B^2 - m_0^2}{m_2^2} + \frac{2m_B^2 - m_2^2}{m_0^2} - \frac{m_0^4}{m_0^2 m_2^2} \right] \right.$$ 

$$+ \frac{2}{s - m_B^2} \left( 5m_0^2 + 5m_2^2 + 22m_B^2 - \frac{m_B^6}{m_0^2 m_2^2} \right)$$ 

$$+ \frac{9m_0^2 m_B^2 - 5m_0^4 - 3m_B^4}{m_2^2} + \frac{9m_2^2 m_B^2 - 5m_2^4 - 3m_B^4}{m_2^2} \right)$$ 

$$- \frac{1}{(s - m_B^2)^2} \left( 8m_0^4 + 8m_2^4 - 32m_B^4 - 32m_0^2 m_B^2 - 32m_2^2 m_B^2 \right.$$

$$- \frac{m_0^2 m_B^2}{m_2^2} + \frac{m_2^2 m_B^2}{m_0^2} \right). \quad (14)$$
This cross section has the same high energy and threshold behaviors as the class 1 - 4 reactions. It is also shown on the plots as the dotted curve, for the bilepton mass $m_B = m_0 = m_2$.

The leptoquarks which can be produced according to the diagrams of Fig. 1 all decay into a charged lepton and a jet with a substantial branching ratio. If they couple only to the first generation of leptons, the decay lepton is an electron. Close to threshold, the signature is thus a pair of high transverse momentum electrons, a pair of high transverse momentum jets and no missing energy. If we cut out the (very few) jet pairs with an invariant mass around the $Z^0$, the lowest order remaining background originates from the $e^{-}e^{-} \rightarrow e^{-}e^{-} q \bar{q}$ continuum. It should be tiny. If one allows for a leptoquark-fermion coupling matrix which mixes different families, like a diagonal matrix for instance, the reaction does not necessarily conserve lepton flavour. In this case there is no Standard Model background at all.

To estimate the discovery potential, we have plotted in Fig. 4 the boundary in the $(m_{LQ}, \lambda/e)$ plane, above which more than 10 events are produced in the class 4 reaction. For this we consider 5 different collider energies and assume 10 fb$^{-1}$ of accumulated luminosity. Again the leptoquarks have the same common mass. In general, an osculating parabola can be fitted to this family of curves. Its equation is approximately given by

$$
\frac{\lambda}{e} = 0.35 \sqrt{m_{LQ}/\text{TeV}} \left( \frac{N}{A \ L/\text{fb}^{-1}} \right)^{1/4} ,
$$

where $N$ is the average number of events, $L$ is the integrated luminosity and $A = 6, 3, 1, 24, 12$ for reactions belonging to class 1, 2, 3, 4, 5', respectively. These numbers originate from the omitted factors in Eqs (12).

Eq. (15) conveniently summarizes the leptoquark discovery potential of $e^{-}e^{-}$ collisions, provided the common leptoquark mass lies safely below the pair production threshold of the largest attainable center of mass energy $m_{LQ} < 0.43 \sqrt{s}$. This limit approximately corresponds to the intersection of the oscu-
ating parabola with each curve of a certain center of mass energy $\sqrt{s}$.

Assuming five events should suffice to establish a discovery, an average number of $N = 9.15$ Poisson distributed events is needed, such that at least 5 events are observed with 95% probability. A typical $e^+e^-$ design luminosity scaling relation is

$$L_{e^+e^-} = 200s \ [\text{TeV}^2].$$

Accounting for the luminosity loss due to anti-pinching at the interaction point, we have for the $e^-e^-$ luminosity

$$L_{e^-e^-} = \frac{L_{e^+e^-}}{2}.$$ 

Taking into account the energy scan capability of a linear collider whose maximum center of mass energy is $\sqrt{s}$, non-conjugate leptoquarks can thus be discovered in $e^-e^-$ scattering with 95% confidence up to the mass

$$m_{LQ} \lesssim \sqrt{s} \min \left( 27 \left( \frac{\lambda}{e} \right)^2 \sqrt{A}, 0.43 \right).$$

The first upper bound corresponds to the osculating parabola, whereas the second bound corresponds to the point in the $(m_{LQ}, \lambda/e)$-plane where the parabola terminates for the given nominal collider energy.

4 Conclusion

Leptoquark production provides another good example for the complementarity of the $e^+e^-$ and $e^-e^-$ modes of linear colliders. Through lowest-order pair-production, $e^+e^-$ collisions mainly probe the interactions of leptoquarks with the electroweak gauge bosons of the Standard Model, while $e^-e^-$ collisions allow for powerful tests of the couplings of leptoquarks to electrons and quarks.

Assuming the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ symmetry of the Standard Model to be preserved by the leptoquark interactions, the strength of the couplings to the gauge bosons is fixed by the gauge principle, whereas the couplings to fermion fields remain essentially undetermined. It is therefore important to have independent tests of the latter. Such tests are possible by analyzing leptoquark decays and, more directly, by studying the production in the $e^-e^-$ and $eq$ channels. These channels are particularly selective with respect
to the fermion number and the family structure of the leptoquark-fermion interactions. Moreover, if polarized beams are available they also map out the chiral properties in an unambiguous way.

In this Letter, this is shown for the $e^- e^-$ mode of future linear colliders in the TeV-energy range. Unlike in $e^+ e^-$ reactions where leptoquarks are produced in conjugate pairs, two non-conjugate leptoquarks emerge from $e^- e^-$ scattering. In addition to the Standard Model gauge symmetry, we have assumed, for this study, lepton and baryon number conservation and non-derivative Yukawa-type leptoquark-fermion couplings.

If the couplings are of electromagnetic strength one can expect cross sections which are sufficiently large to be observed up to the kinematical pair production threshold. In this case one also predicts sizable effects in $e^+ e^-$ collision on top of the production cross sections due to the exchange of gauge bosons. Comparison of measurements in the two modes will then provide useful cross-checks. However, if the Yukawa couplings are considerably weaker than the electromagnetic coupling, they are not visible in $e^+ e^-$ production, but can still be tested in $e^- e^-$ scattering down to $\lambda/e \approx 0.1$. Furthermore, if the leptoquarks couple to only one lepton and quark helicity state, only a fraction of the class 1 – 5 reactions considered here can proceed revealing clearly the chiral structure of the leptoquark-fermion interactions.

In summary, although $e^+ e^-$ is the better discovery mode, a high-energy $e^- e^-$ collider is superior in finding out the strength and structure of the leptoquark couplings to normal matter. The latter is important in understanding the elementarity or compositeness of these hypothetical particles.

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Fig. 1. Typical Feynman diagrams for leptoquark pair production in $e^-e^-$ collisions. The quark generation indices are suppressed.
Fig. 2. Cross section as a function of the collider energy. The common mass of the produced leptoquarks is 200 GeV.
Fig. 3. Cross section as a function of the common mass of the produced leptoquarks. The collider energy is 1 TeV.
Fig. 4. Loci of $\sigma_4 = 1$ fb, as a function of the common leptoquark mass and coupling to fermions. The collider energies are .5, 1, 2, 5 and 10 TeV. The thinner osculating parabola is given by Eq. (15).