Quantum Phase Pumping

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In this Letter we consider the adiabatic charge transport through a normal mesoscopic sample sandwiched by superconductors without modulation of local chemical potentials. The deformation of coherent quasiparticles in the normal metal in the presence of periodically changing phases in the superconductors leads to a charge transport. Both the magnitude and the phase dependence of the charge transferred per period through the sample strongly depend on temperature.

Recently adiabatic charge transport in mesoscopic samples generates much interest both experimentally and theoretically \(^1\). In the presence of slowly and periodically changing external fields, the spatial electron density profile follows periodical modulation of local chemical potentials. In the presence of two a.c. charging gates, \(\{g_1(t), g_2(t)\}\), this leads to a d.c. adiabatic charge transport across the sample in the classical limit. The amount of charge transport equals the amount of "flux" of a topological field \(\pi_{12}(g)\) threading the area enclosed by the trajectory \(C\) swept by \(g = \{g_1(t), g_2(t)\}\) in a 2 dimensional parameter space \(M_2\). At low temperature when the dephasing length is longer than the sample size, electrons remain coherent. \(\pi_{12}(g)\), unlike in the classical limit where it is a constant field, develops very rich structures and is a sample specific, random function of \(g\) in \(M_2\) space. Due to quantum interference effects, the adiabatic charge transport at low temperature therefore is of a random sign and magnitude \(^2\).

It still remains a question how the many body correlation enriches the structure of the topological field \(\pi_{12}(g)\) and affects the adiabatic charge transport. In this letter, we exam an example where a mesoscopic sample is in contact with superconductors. At \(T < \Delta (the superconductor energy gap)\), electron-like excitations in these samples are reflected into hole-like excitations through Andreev reflections at superconductor-metal(SN) interfaces. Quasiparticles inside the normal metal at low temperatures are superpositions of electrons and holes, with zero effective charges \(^3\). For a superconductor-normal superconductor (SNS) junction, the resultant excitation spectrum in the metal has little resemblance to that of a normal Fermi liquid: An energy gap of many body nature is opened up at Fermi energy, which remains that of a normal Fermi liquid: An energy gap of many body correlating spectrum in the metal has little resemblance to zero effective charges \(^4\). For a superconductor-normal superconductor energy gap), electron-like excitations in these samples are reflected into hole-like excitations through Andreev reflections would acquire different phases at SN interfaces. Thus, electron-hole superpositions of quasiparticles depend on phase differences of superconductors. The energy gap in the quasiparticle spectrum decreases from a finite value to zero when the phase difference between two superconductors is changing from zero to \(\pi\). Quasiparticles of energy \(\epsilon\) at time \(t\) can be represented by \((\nu_1(\epsilon,t), \nu_2(\epsilon,t))\), with \(\nu_0 \nu_1(\epsilon,t)\) as the average density of states in region 1, \(\nu_0 \nu_2(\epsilon,t)\) in region 2. (The density of states in the normal metal outside region 1,2 equals its bulk value \(\nu_0\), doesn't depend on \(\chi_{1,2}(t)\).) As time evolves, \(\chi_{1,2}(t)\) changes and \(\nu_{1,2}(\epsilon,t)\) oscillates with a frequency \(2eV/\hbar\). When \(\chi_{1,2}(t)\) is far away from \(\pi\), quasiparticles far below the energy gap extend only outside region 1(2). When \(\chi_{1,2}\) approaches \(\pi\), the gap diminishes; the low energy quasiparticles start to extend into region 1(2). This leads to a continuous deformation of quasiparticles of energy \(\epsilon\), characterized by the motion of \((\nu_1(\epsilon,t), \nu_2(\epsilon,t))\) (see Fig.2). It is worth emphasising that unlike the situations in \(^5\), the local chemical potential in this case remains constant as a function of time and the deformation of many body wave functions is possible only when the phase rigidity is present \(^6\). For this reason, we call the phenomena discussed here as phase pumping.

In the following, we show that deformation of coherent quasiparticle wave packets via phase differences eventually leads to a d.c. current in this case. The adiabatic charge transport in the presence of the changing phase differences of superconductors is determined by the sensitivities of the diffusion constants and the density of states to boundary conditions. At low temperatures, the topological field is localized in the parameter space, leading to a very singular phase dependence of d.c. charge transport. The charge transport approaches zero as a power law function of temperature in both high temperature and low temperature limits.

To calculate the charge transport in this limit we introduce Keldysh technique generalized to Nambu space \(^7\).

\[
\begin{align*}
&i \hat{\tau}_z \frac{\partial}{\partial t} \hat{G} + i \frac{\partial}{\partial t} \hat{G} \hat{\tau}_z + [H, \hat{G}] + I_{col} = 0
\end{align*}
\]

where \(H\) is the Hamiltonian of electrons, including random impurity potentials. \(I_{col}\) is the electron-phonon collision integral. \(\hat{G}(r,r',t,t')\) is Keldysh Green function matrix defined in Nambu space.
\[ \tilde{G} = \left( \tilde{G}^R, \tilde{G}^K \right), \tilde{G}^P = \left( \begin{array}{l} G^P, iF^P \\ -iF^P, -G^P \end{array} \right), \]  
\[ P = R, A, K \text{ and } \tau_z = \left( \tau_z, 0 \pm, 0 \right), \tau_z \text{ is z-component of Pauli matrix in Nambu space. Let } \tilde{G}^P_\mathbf{p}(\epsilon, r, t) \text{ be Keldysh component of the semiclassical Green function defined as} \]
\[ \tilde{G}_\mathbf{p}(\epsilon, r, t) = \frac{i}{\pi} \int d\epsilon' d\mathbf{r}' dt' \exp(i\epsilon't - i\mathbf{p} \cdot \mathbf{r}') \]
\[ \tilde{G}(r + \mathbf{r}'/2, r - \mathbf{r}'/2, t + t'/2, t - t'/2) \]  
where \( \xi_\mathbf{p} = p^2/2m - \epsilon_F, \epsilon_F \) is the fermi energy. In diffusion limit, \( \tilde{G}_\mathbf{p} = \tilde{G}(\epsilon, r, t) + \mathbf{n} \cdot \nabla \tilde{G}(\epsilon, r, t), \mathbf{n} \) is the unit vector along \( \mathbf{p} \). In the adiabatic approximation, the solution of Eq.2 for Keldysh component is written in term of two nonequilibrium functions \( f_1, f_2 \) and \( \tilde{G}^{RA} \),
\[ \tilde{G}^K(\epsilon, r) = (\tilde{G}^R(\epsilon, r) - \tilde{G}^A(\epsilon, r))(1 - 2n_F(\epsilon)) + \tilde{G}^R(\epsilon, r)(f_2 + \tau_z f_1) - (f_2 + \tau_z f_1)\tilde{G}^A(\epsilon, r), \]
with \( n_F \) as Fermi distribution function, \( f_{1(2)}(\epsilon) \) is an even(odd) function of \( \epsilon \).

Following Eqs.2,5, in the first order adiabatic approximation, the equations of \( f_{1,2} \) and \( \tilde{G}^R \) take forms of
\[ D \nabla \cdot D_1(\epsilon, r) \nabla f_1(\epsilon, r) = 0, \]
\[ \nabla \cdot D_1(\epsilon, r) \nabla f_1(\epsilon, r) = 0, \]
\[ i\epsilon [\tau_z, \tilde{G}^R(\epsilon, r)] + D \nabla \cdot (\tilde{G}^R(\epsilon, r) \nabla \tilde{G}^R(\epsilon, r)) = 0, \]
where \( D \) is the diffusion constant in bulk metals; \( \nu(\epsilon, r) = Tr(\tilde{G}^R \tau_z - \tau_z \tilde{G}^A), D_1(\epsilon, r) = Tr(1 - \tilde{G}^R \tau_z \tilde{G}^A \tau_z), D_2(\epsilon, r) = Tr(1 - \tilde{G}^R \tilde{G}^A), Tr \) represents trace in Nambu space. Here we neglect \( f_1 \)'s contribution in first equation in the leading order of \( T/\epsilon_F \). To derive the right hand side of the first equation in Eq.6, we use an identity
\[ \frac{\partial H}{\partial t} \nu(\epsilon, r) = \int_0^\epsilon \frac{\partial}{\partial \epsilon} \nu(\epsilon, r) + \frac{1}{2} \frac{\partial}{\partial \epsilon} \int_{-\infty}^{t+} \nu(\epsilon, r) \]  
which is valid when the external parameters vary little over the characteristic length scale of \( G^{RA} \). The second term in the right hand side of Eq.7 vanishes following the sum rule. Inside the leads, quasiparticles remain in equilibrium and \( f_1(\epsilon, r) = f_2(\epsilon, r) = 0 \) at the lead-metal boundaries. At SN boundaries, \( G^R = 0, \tilde{G}^R = \exp(i\chi) \) at \( \epsilon \ll \Delta \). Electron-phonon collisions are neglected in Eq. 6 in the low temperature limit when the diffusion relaxation dominates.

Finally, in the leading order of \( 1/k_F p \), the current can be expressed as
\[ J = D\nu_0 \int d\epsilon \left[ \frac{\epsilon}{2\epsilon_F} D_2(\epsilon, r) \nabla f_2(\epsilon, r) + D_1(\epsilon, r) \nabla f_1(\epsilon, r) \right], \]
with \( \nu_0 \) the density of states in bulk metals.

For the quasi 1 - D geometry shown in Fig.1 where \( W, L \gg d \gg \sqrt{D/\Delta}, G^{RA}(x, y) \) of junction 1,2 can be approximated with \( G^{RA}(y) \) calculated in the leading order of \( (d/W)^2 \) (L is the length of the sample along x-direction) at given \( \chi_{1,2} \). This allows us to integrate Eq. 6 along y-direction independently. The resultant equations \( f_1, f_2 \) and \( G^{RA} \) have only x-dependences. From Eq.6 and its boundary conditions, we find \( f_1 = 0 \) and
\[ f_2 = \frac{\partial n_F(\epsilon)}{\partial \epsilon} \int_0^x \frac{1}{D_2(\epsilon, x')} \frac{\partial \chi(\epsilon, x')}{\partial t} dt - \int_0^L dx' \frac{1}{D_2(\epsilon, x')} \frac{\partial \chi(\epsilon, x')}{\partial t} \left( \int_0^L dx' \frac{1}{D_2(\epsilon, x')} \right)^{-1}. \]  
Here we introduce \( \chi(\epsilon, x) = \int_0^x dx' \int_0^\epsilon dx'' \nu(\epsilon', x') \), and \( \rho^{-1}(\epsilon, x) = D_2(\epsilon, x) \int_0^1 dx'' D_2^{-1}(\epsilon, x'') \).

We want to emphasis that \( G^{RA} \) depends on time only through boundary conditions \( \chi_{1,2}, \chi_{1,2} \in [-\infty, +\infty] \). Wave functions are periodical in such a 2D space, and \( G^{RA}(\chi_1, \chi_2) = G^{RA}(\chi_1 + 2\pi, \chi_2 + 2\pi) \). Furthermore, time reversal symmetry of the ground state at \( \chi = 0 \) requires that physical quantities like \( D_{1,2} \) are even functions of \( \chi \). Evidently, it is more convenient to introduce a compact 2D space \( M_2 \) of \( g_\alpha, g_\alpha = \cos \chi_\alpha, \alpha = 1, 2, g_\alpha \in [-1, 1] \).

Following Eq. 6, in the adiabatic approximation, the nonequilibrium distribution function depends on time locally through \( \chi_{1,2}(t), \partial_t \chi_{1,2}(t) \) or \( \partial_t g \). Correspondingly, the time dependence of \( \rho(\epsilon, x), \chi(\epsilon, x) \) defined after Eq. 9 can be expressed through their dependence on \( g \). Thus, the time derivative of \( \chi \) is represented by a one-form defined in such a 2D parameter space. Substituting Eq.9 back into Eq. 8 and using Stoke's theory to carry out the integral of the one-form along a close trajectory \( C \) in \( M_2 \), we obtain the charge transport along \( x \) direction per period in term of a two-form defined in \( M_2 \)
\[ Q = \int_S \pi_{\alpha\beta} dg_\alpha \wedge dg_\beta \]
\[ \pi_{\alpha\beta} = \nu_0 \int d\epsilon \frac{\partial n_F}{\partial \epsilon} [\hat{\pi}_{\alpha\beta}(\epsilon, g) - \hat{\pi}_{\beta\alpha}(\epsilon, g)] \]
\[ \hat{\pi}_{\alpha\beta}(\epsilon, g) = \int_0^L dx \frac{\partial \chi(\epsilon, x)}{\partial g_\alpha} \frac{\partial \chi(\epsilon, x)}{\partial g_\beta} \]  
Fig.1. Geometry of the sample, \( W \gg d \). Shaded bars stand for superconductors; the patterned region represents a disordered normal metal.
where \( S \) is the region enclosed by trajectory \( C \) in \( M_2 \).
In the leading order of \((d/W)^2\), \( \pi_{\alpha\beta} \) only depends on the sensitivities of \( D_2, \nu \) with respect to \( g \), which reflects the degree of deformation of coherent wave packets at point \( g \) in \( M_2 \).

\[
\tilde{\pi}_{\alpha\beta}(\epsilon, g) = \frac{\partial \Sigma(\epsilon, g_\beta) \partial \Xi(\epsilon, g)}{\partial y_\beta}.
\]

\[
\Sigma(\epsilon, g_\alpha) = \int_0^\epsilon d\epsilon' \nu(\epsilon', g_\alpha),
\]

\[
\Xi(\epsilon, g) = \eta_0 \sum_\gamma \frac{1}{D_2(\epsilon, y_\gamma) \eta_0 + (1 - 2\eta_0)}\left[1 - \eta_\beta + \eta_0 \sum_\gamma \left(\frac{1}{2} \delta_{\beta\gamma} + \frac{1}{\eta_0} \left(\frac{1}{D_2(\epsilon, y_\gamma)} - 1\right)\right)\right] (11)
\]

with \( D_2, \nu \) calculated at given \( g_\alpha \) in the limit \( W \gg d \). \( \eta_0 = W/L, \eta_\beta = (x_\beta + W/2)/L, x_\beta \) is the x-coordinate of the center of \( \beta \) region. \( \theta(x) \) is a step function. In the absence of phase rigidities, \( \Sigma, \Xi \) do not depend on \( g \) and \( \pi_{\alpha\beta} = 0 \). \( Q \) naturally represents a connection over a tangent fiber bundle \( T M \), where the fiber is one-form \( \Sigma(\partial \Xi/\partial y)dg \) embedded in the tangent manifold of \( M_2 \) and the base manifold is \( M_2 \). Such a geometric point of view was first emphasized in \([12,13]\) for isolated quantum systems and generalized to statistical systems in \([14]\), where kinetic processes are important. Generally speaking, \( Q \) is not proportional to the area enclosed by trajectory \( C \) in the base manifold \( M_2 \) because \( \pi_{\alpha\beta} \) depends on \( g \). Absence of the area law, which usually holds in a classical system, signifies strong correlation effects.

Following the normalization condition we rewrite \( G^R = \cos \theta(\epsilon), F^R = \exp(i\chi(\epsilon)) \sin \theta(\epsilon) \), where \( \theta = \theta_1 + i\theta_2 \), which is obtained as a solution to the last equation in Eq. 6. \( D_1(\epsilon) = \cos^2 \theta_1(\epsilon), D_2(\epsilon) = \cos^2 \theta_2(\epsilon) \) and \( \nu(\epsilon) = \cos \theta_1(\epsilon) \cos \theta_2(\epsilon) \). It is important to notice that \( \theta_1(\epsilon) \neq \pi/2 \) or \( \nu(\epsilon) \neq 0 \) only when \( \epsilon > E_g \), where \( E_g \) is a function of \( \chi \)

\[
E_g(\chi \leq \pi) = E_c \left\{ \begin{array}{ll}
C_2(1 - C_1 \chi^2), & \chi \ll \pi; \\
C_3(\pi - \chi), & \pi - \chi \ll \pi.
\end{array} \right.
\]

Here \( E_c = D/d^2, C_1 = 0.91, C_2 = 3.122, C_3 = 2.43 \). Following Eq. 12, the density of states has a gap, which closes only when \( \chi = \pi \). At energies much larger than \( E_c \), the phase dependence of \( \theta_{1,2} \) decays exponentially as a function of energy,

\[
\theta_{1,2} = \exp(-\sqrt{\frac{\epsilon}{E_c}})(1 + g_{1,2}).
\]

Taking into account these results, we find

\[
\tilde{\pi}_{\alpha\beta}(\epsilon, g) = \left\{ \begin{array}{ll}
\eta_0^2 e^{-\sqrt{\epsilon/E_c}}[\theta(\eta_\alpha - \eta_\beta) + \eta_\beta - 1], & \epsilon \gg E_c; \\
F(g_\alpha + 1, g_\beta + 1, \epsilon/E_c), & \epsilon \ll E_c.
\end{array} \right.
\]

Fig. 2. a). Distribution of \( \pi_{\alpha\beta} \) in \( M_2 \) at \( kT \ll E_c \). \( \pi_{\alpha\beta} \) is nonzero only in the shaded region of size \( kT/E_c \). Trajectory \( \alpha \) represents the case \( \delta = \pi/2 \) and trajectory \( \beta \) for \( \delta \sim kT/E_c \). b). \( Q(\epsilon) \) as a function of \( \delta \) at \( kT \gg E_c \) (dashed line) and \( kT \ll E_c \) (solid line). c), d), e), f) are plots for \( (\nu_1(T, t), \nu_2(T, t)) \) at different \( \delta \) and \( kT \ll E_c \). c). \( \delta \gg kT/E_c \); d) \( \delta \) approaches \( kT/E_c \); e) \( \delta \sim kT/E_c \); f). \( \delta \sim 0 \).

\( F \) is zero when \( |g_{\alpha,\beta} + 1| \gg (\epsilon/E_c)^2 \) and approaches unity when \( |g_{\alpha,\beta} + 1| \sim (\epsilon/E_c)^2 \). Eqs. 11, 14 indicate that \( \tilde{\pi}_{\alpha\beta}(\epsilon, g) \) is exponentially small and is a simple sinusoidal function of \( \chi \) at \( \epsilon \gg E_c \). In the opposite limit, the phase dependence of \( \tilde{\pi}_{\alpha\beta} \) depends strongly upon the high harmonics.

At high temperatures, the main contribution to \( \pi_{\alpha\beta} \) is from electrons of energies of order \( E_c \ll kT \) since contributions of electrons with energies of order \( kT \) are exponentially small. Substituting Eq.14 into Eq. 10, we obtain the charge transport at high temperature due to deformation of quasiparticles,

\[
Q(T) = N \left(\frac{W}{L}\right)^2 \left(\frac{E_c}{e^2}\right)^2 \frac{E_c}{kT} F_1(\delta).
\]

\( N \) is the number of electrons inside the sample. \( F_1(\delta) \) is a smooth function of \( \delta \), as shown in Fig. 2b.(the dashed line); \( F_1(\delta = 0, \pi) = 0 \). The small factor \( E_c/\epsilon_F \) is from the asymmetry of the electron-hole spectrums at Fermi energy; \( E_c/kT \) originates from the fact that only wave packets of this small fraction of quasi-particles can be deformed. When \( kT \) is larger than the Josephson coupling energy, the thermal phase slippage takes place and \( Q(T) = 0 \).

At low temperature, \( \pi_{\alpha\beta} \) is determined by \( \epsilon \sim kT \ll E_c \). Following Eq. 14, it is nonzero only in a small region where \( |g_{\alpha,\beta} + 1| \) is smaller than \( kT/E_c(\ll 1) \), as shown in Fig 2a. At zero temperature, \( \pi_{\alpha\beta} \) becomes completely localized at \( g_\alpha = g_\beta = -1 \) and equals zero in the rest of \( M_2 \) plane. The localization of \( \pi_{\alpha\beta} \) in \( M_2 \) indicates a very different phase dependences of charge transport in
the low temperature and high temperature limits. Consider the case $\delta = \pi/2$. The corresponding trajectory $\alpha$ in Fig. 2a does not contain the shaded region where $\pi_{\alpha\beta} \neq 0$. Following Eq. 10, $Q$ equals zero in this case. To have nonzero charge transport, trajectory $C$ is required to enclose a small portion in $M_2$ of radius $kT/E_c$, where $\pi_{\alpha\beta} \neq 0$, and $\delta$ needs to be close to $kT/E_c$. Physically, charge can be transferred only when the density of states at energy $kT$ in regions 1,2 (see Fig. 1) changes simultaneously. It can be demonstrated more explicitly by directly looking at deformation of wave packets, which can be characterized by the trajectory of the vector $(\nu_1(T, t), \nu_2(T, t))$ in a 2-D space, as shown in Fig. 2c). d). e). f). Here $\nu_0 \nu_{1,2}(T, t)$ are the density of states of energy $kT$ in region 1,2 at time $t$. According to Eq. 12, the energy gap $E_g$ in the quasiparticle spectrum closes only at $\chi = \pi$ and the density of states at low energy remains zero for most of the period unless $\chi$ is in the vicinity of $\pi$. When $\delta \gg kT/E_c$, $\nu_1$ changes while $\nu_2$ remains zero and vice versa. The trajectory therefore repeats itself when the time evolves and the area enclosed is zero. When $\delta$ approaches $kT/E_c$, $\nu_2$ becomes nonzero while $\nu_1$ is changing. The trajectory starts to enclose a finite area. The area reaches a maximum when $\delta \sim kT/E_c$. When $\delta$ decreases further towards zero, $\nu_1$ and $\nu_2$ start to change in phase; the trajectory becomes a narrow strip with its width proportional to $\delta$. As $\delta \rightarrow 0$, the trajectory again repeats itself in the 2D space and the area enclosed is zero. The trajectory of $\delta < 0$ and that of $\delta > 0$ obeys mirror symmetry, i.e. the trajectory of $\delta < 0$ can be obtained by reversing the direction of the trajectory of $-\delta$. As a consequence,

$$Q(T) = N \left( \frac{W kT}{L \epsilon_F} \right)^2 F_2(\delta, kT/E_c), \quad (16)$$

where

$$F_2(\delta, \delta_0) = \text{sign} \delta \begin{cases} \frac{\delta}{\delta_0}, |\delta| < \delta_0; \\ \frac{\delta_0}{\delta}, |\delta| \sim \delta_0; \\ \exp(-|\delta|/\delta_0), |\delta| \gg \delta_0. \end{cases} \quad (17)$$

The contribution to the charge transport in the limit $|\delta| \gg \delta_0$ is from the quasiparticles of energy of order $\delta \times E_c$, population of which is much smaller than that of typical excitations of energy $kT$. Though there is an energy gap in the spectrum, charge transport is not quantized in this case because $kT$ can be higher than $E_g(\chi)$ when $\chi \rightarrow 0$. From Eqs. 15, 16, we find the charge transport vanishes as the temperature goes to zero because of the electron-hole symmetry and decreases as $1/T$ at high temperature limit due to the suppression of electron-hole coherence. $Q$ as a function of temperature develops a maximum at $kT \sim E_c$. At asymptotically low temperature, one can no longer neglect the finite dwell time of the electrons in region 1,2. $Q(T)$ saturates at the value $N(d/W)^2(E_c/\epsilon_F)^2$ when $kT \sim E_c(d/W)^2$. Another type of quantum interference effect, i.e. mesoscopic fluctuations is negligible in the present case because the dimension along $z$ direction in Fig. 1. is assumed to be infinite.

In conclusion, we find that electrons can be transferred out of dots with the help of the phase rigidity of coherent wave packets. In the presence of small amount magnetic impurities, the gap of quasi particle spectrums closes as far as phase differences are in the vicinity of $\pi$, the size of which is determined by the pair breaking rate. In this case, $\pi_{\alpha\beta}$ would be delocalized in $M_2$ space and the pronounced peak of charge transport as a function of $\delta$ as shown in Fig. 2b will be smeared out. When the depairing rate is further increased, coherence effects become exponentially small and $Q(T) \approx 0$.

Finally, in the metallic limit studied in this letter, quantum phase slippage effects are negligible.

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References:

[1] M. Switkes, et.al, Science 283, 1905(1999).
[2] B. Spivak, F. Zhou, M. T. Beal Monod, Phys. Rev. B 51, 13226(1995).
[3] P. Brouwer, Phys. Rev. B 58, 10135(1998).
[4] F. Zhou, et.al., Phys. Rev. Lett. 82, 608(1999).
[5] D. J. Thouless, Phys. Rev. 27, 110(1980).
[6] A. I. Larkin, Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 47, 6083 (1983).
[7] A. I. Larkin, Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 73, 299(1977).
[8] L. Keldysh, Zh. Eksp. Teor. Fiz. 47, 1515(1964). [Sov. Phys. JETP 20, 1018(1965)].
[9] J. E. Avron, et.al., Rev. Mod. Phys. 60, 873(1988).