Application of the mini-batch adaptive method of random search (MAMRS) in problems of optimal in mean control of the trajectory pencils

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Abstract. The article discusses the application of one of the new methods of constrained optimization to solve the problem of finding optimal control of a pencil of trajectories of nonlinear deterministic systems emanating from a given set of initial states. The structure of a feedback system is proposed, which contains a model of a control object, a measurement system model, a state observer that generates an estimate of the state vector from incoming measurements, and a regulator. The quality of control is assessed by the value of the average value of the functional determined on individual trajectories. Unknown control laws for the object model and the state observer are found in the form of expansions in terms of orthonormal systems of basic functions defined on the set of admissible states of the dynamic system. The problem of controlling a pencil of trajectories is reduced to the problem of parametric optimization, which is solved using a mini-batch adaptive random search method. A step-by-step algorithm for solving the problem is proposed, which is demonstrated by solving the problem of tracking various coordinates of a dynamical system according to the measurement results. The influence of the mini-batch size on the achieved tracking accuracy is investigated.

1. Introduction

Applied problems of synthesis of nonlinear deterministic joint estimation and control systems are usually solved by linearization in the vicinity of the reference trajectory and application of the separation principle for linear systems with a quadratic performance criterion, which consists in independently finding the optimal linear regulator and an asymptotic state observer of the full or low order. In [1], a method of approximation by the Bernstein polynomial for nonlinear optimal control problems is presented. However, the task is looking for open-loop control with only one trajectory. Methods and applications of the theory of synthesis of nonlinear observers are described in [2,3]. In contrast to the described works, this paper proposes that the matrix of the gain of a nonlinear observer depends not only on time but also on a given set of estimates of the state vector coordinates. For the general case of nonlinear systems, the problem of finding state observers and object control according to the estimate generated by the observer has not yet been solved. It is proposed to use the structure of a closed loop system, in which the laws of control of the object and the observer are in the class of functions that depend on time and on the generated estimates of the state vector [4]. In contrast to the traditional approach, when the problems of optimal control synthesis and observer synthesis are solved...
independently of each other, it is proposed to implement a unified procedure for joint optimization and modeling.

The behavior of a nonlinear continuous deterministic model of a control plant is considered, the trajectory of which is described by a system of ordinary differential equations. Parallelepiped constraints are imposed on the controls. The initial conditions are given by a compact set of positive measure. The quality of control of a separate trajectory is estimated by the value of the Bolz functional, and the trajectory pencil – by its average value over the set of initial states. The problem is posed to find the controls for the object and the observer that minimize the value of the quality functional of pencil control. The desired control of the plant model with feedback on the estimated coordinates of the state vector is sought in the class of functions with saturation, taking into account the presence of control constraints. Unknown control laws are represented in the form of expansions with unknown coefficients in orthonormal systems of basic functions used in the spectral method of analysis and synthesis of control systems [5,6]. The aim of this work is to solve the formulated parametric problem of controlling a pencil of trajectories, using the developed mini-batch adaptive method of random search, which was previously used in problems of identifying parameters of dynamical systems [7]. It belongs to the metaheuristic algorithms [8,9], which use the ideas applied in the gradient methods of machine learning [10]. A computational algorithm is proposed, the efficiency of which is demonstrated by solving a model example of tracking various coordinates of a chaotic process [11-14].

2. Statement of the problem

The behavior of the nonlinear continuous deterministic model of the control plant is described by the equation

\[ \dot{x}(t) = f(t, x(t), u(t)), \]

where \( t \) – continuous time, \( t \in T = [t_0; t_1] \), moments \( t_0 \) and \( t_1 \) are given; \( x \) – state vector, \( x \in \mathbb{R}^n \); \( u \in U = [a_1, b_1] \times \ldots \times [a_q, b_q] \) – set of possible values of control, \( f(t, x, u) \) – given continuous function.

The initial conditions are given by a compact set \( \Omega \) of positive measure with piecewise smooth boundary:

\[ x(t_0) = x_0 \in \Omega \subset \mathbb{R}^n, \]

where the set \( \Omega \) characterizes the uncertainty in setting the initial conditions.

It is assumed that the control uses the information coming from the model of the measuring system described by the relation:

\[ z(t) = h(t, x(t)), \]

where \( z \in \mathbb{R}^m \) – measurement vector, \( h(t, x) \) – continuous vector-function.

It is assumed that from the information coming from the model of the measuring system, it is possible to obtain an estimate of the state vector using a nonlinear observer of the form:

\[ \dot{x}(t) = f(t, \hat{x}(t), u(t, \hat{x}(t))) + K(t, \hat{x}(t))\left[z(t) - h(t, \hat{x}(t))\right], \]

\[ \hat{x}(t_0) = \hat{x}_0, \]

where \( \hat{x}(t) \) – state vector estimate, \( K(t, \hat{x}) \in \mathbb{R}^{n \times m} \) – unknown continuous \( (n \times m) \) matrix function playing the role of the observation process control.

The set of admissible controls \( U \) is formed by such functions \( (u(t, \hat{x}), K(t, \hat{x})) \) that \( \forall t \in T \) the control of the plant model \( u(t) = u(t, \hat{x}(t)) \in U \) is piecewise continuous, the control of the observer
$K(t) = K(t, \dot{x}(t)) \in \mathbb{R}^{\text{dim}}$ is continuous, and the function $f(t, \dot{x}, u(t, \dot{x}))$ is such that the solution of the system of equations (1), (4) with the initial conditions (2), (5) taking into account (3) exists and unique.

We define the quality functional of control of a separate trajectory:

$$I(x_0, u(t, \dot{x}(t)), K(t, \dot{x}(t))) = \int_{t_0}^{t_1} f^0(t, x(t), u(t, \dot{x}(t)), K(t, \dot{x}(t)))dt + F(x(t_1)), \quad (6)$$

where $f^0(t, x, u, K), F(x)$ — given continuous functions.

To each admissible control $(u(t, \dot{x}), K(t, \dot{x})) \in U$ set $\Omega$ we associate the sheaf of trajectories of the system of equations (1), (4):

$$X(t, u(t, \dot{x}), K(t, \dot{x})) = \bigcup \left\{ x(t, u(t, \dot{x}), K(t, \dot{x}), x_0), \dot{x}(t, u(t, \dot{x}), K(t, \dot{x}), \dot{x}_0) \mid x_0 \in \Omega \right\},$$

that is, the union of solutions to the system of equations (1), (4) for all possible initial states (2) in the presence of measurements (3).

The quality of trajectory pencil control is proposed to be estimated by the value of the functional:

$$J[u(t, \dot{x}), K(t, \dot{x})] = \int_{\Omega} I(x_0, u(t, \dot{x}(t)), K(t, \dot{x}(t))) dx_0 / \text{mes } \Omega, \quad (7)$$

where $\text{mes } \Omega$ — measure of the set $\Omega$.

It is required to find a control $\left(u^*(t, \dot{x}), K^*(t, \dot{x})\right) \in U$ such that

$$J[u^*(t, \dot{x}), K^*(t, \dot{x})] = \min_{(u(t, \dot{x}), K(t, \dot{x})) \in U} J[u(t, \dot{x}), K(t, \dot{x})]. \quad (8)$$

The required control is called optimal on average, since the average value of functional (6) is minimized on the set of initial states $\Omega$.

3. **Solution search strategy**

The proposed approach to the solution consists in the transition from problem (8) to the problem of finite-dimensional optimization, that is, the problem of finding the best values of the undefined elements that define the control structure of the control object model (1) and the state observer (4). This structure takes into account the presence of constraints on the control of the object model.

Further we will use the mathematical apparatus from [4] and the following assumptions.

1. The set of initial states $\Omega$ is a parallelepiped defined by the direct product of segments $[\alpha_i; \beta_i], i = 1, n$, i.e. $\Omega = [\alpha_1; \beta_1] \times \cdots \times [\alpha_n; \beta_n]$. With the help of a step $\Delta x_i$, all line segments are divided into $N_i$ segments, and the parallelepiped $\Omega$ is divided into $N = N_1 \cdot \cdots \cdot N_n$ elementary disjoint subsets $\Omega_k$, $k = 1, N$. In each elementary subset $\Omega_k$, an initial state $x_0^k$ (the center of the parallelepiped $\Omega_k$) is specified.

2. An estimate of the set of possible states is known, which is represented by the direct product $Q = [x_1, x_1] \times \cdots \times [x_n, x_n]$, where $x_i, x_i, i = 1, n$ — lower and upper boundaries for each coordinate, respectively, determined by the physical meaning of the problem being solved. Therefore, we can assume that the required estimates of the state vector must satisfy the same conditions $\hat{x}_i \in [x_{\hat{i}}, x_{\hat{i}}], \ldots, \hat{x}_n \in [x_{\hat{n}}, x_{\hat{n}}]$.

3. Components of the control law $u(t, \dot{x}) = \left(u_1(t, \dot{x}), \ldots, u_q(t, \dot{x})\right)^T$ are in the form:

$$u_j(t, \dot{x}(t)) = \text{sat} \left[ g_j(t, \hat{x}_1(t), \ldots, \hat{x}_n(t)) \right], \quad j = 1, q, \quad (9)$$
where

\[
\text{sat} \, v_j(t) = \begin{cases} 
  v_j(t), & a_j < v_j(t) < b_j, \\
  a_j, & v_j(t) \leq a_j, \\
  b_j, & v_j(t) \geq b_j, 
\end{cases}
\]  

saturation function sat guarantees the fulfillment of control constraints of the form

\[ a_j \leq u_j(t) = u_j(t, \hat{x}(t)) \leq b_j \]  

the functions \( g \) is proposed to be sought in the form:

\[ g_j(t, \hat{x}_1, \ldots, \hat{x}_n) = \sum_{i=0}^{L_0} \sum_{j=0}^{L_1} \cdots \sum_{i=0}^{L_0} u_{i_0, i_1, \ldots, i_n}(i, \hat{x}_1) \cdots p_n(i, \hat{x}_n), \]

where \( u_{i_0, i_1, \ldots, i_n} \) – unknown coefficients; \( L_0, L_1, \ldots, L_n \) – scales of truncation in time and coordinates of the state vector estimates used in control; \( q(i, t), \hat{q}(i, t) = 0, L_0 - 1 \) – system of orthonormal time functions (basis functions) defined on the segment \([t_0, t_1]\) and satisfying the condition

\[ \int_{t_0}^{t_1} q(i, t)q(j, t)\, dt = \begin{cases} 1, i = j, \\
 0, i \neq j, \end{cases} \]

(basis functions) defined on an interval \([x_0, x_1]\), \( j = 1, \ldots, n \).

As the basis functions \( q(i, t), p_n(i, \hat{x}_k), k = 1, n \), one can take, for example:

a) Legendre polynomials:

\[ p(n, x) = \sum_{k=0}^{n} \left( \frac{C_n^k}{n} \right)^2 \hat{x}^{n-k} \hat{x}^k, n = 0, L - 1; \]

b) Cosine:

\[ p(n, x) = \cos \left( n \cdot \pi \cdot (2 \cdot \hat{x} - 1) \right), n = 0, L - 1; \]

where \( \hat{x} = (x - x_0) / (x_1 - x_0) \), and other systems of basic functions.

The components \( K_i(t, \hat{x}), i = 1, \ldots, n; j = 1, \ldots, m \) of the observer control law \( K(t, \hat{x}) \) are found by a formula similar to (11).

The value of the pencil control quality functional (7) is approximately calculated by the formula:

\[ J[u(t, \hat{x}), K(t, \hat{x})] = (1 / N) \sum_{k=1}^{N} I(x_0, u(t, \hat{x}(t)), K(t, \hat{x}(t))). \]

The strategy for solving the problem is to find the best values of the coefficients \( u_{i_0, i_1, \ldots, i_n}^{K_i} \), \( K_{i_0, i_1, \ldots, i_n}^{u_i} \) using a mini-batch adaptive method of random search [4]. The main idea of its application is that, from the trajectories emanating from the set of initial states, for the approximate calculation of functional (12), for each calculation of the criterion, randomly selected non-coinciding trajectories are used that form a mini-batch:

\[ J_d[u(t, \hat{x}), K(t, \hat{x})] = (1 / d) \sum_{k=1}^{d} I(x_0, u(t, \hat{x}(t)), K(t, \hat{x}(t))). \]

The mini-batch size is user-definable: \( 1 \leq d \leq N \) and is usually fixed. Further, for simplicity of presentation, we assume that each component of the control laws \( u(t, \hat{x}) \) and \( K(t, \hat{x}) \) can be associated with a matrix-column of the coefficients of expansions in terms of elements of basis systems. Further, by concatenation, you can represent the entire set of selected parameters in the form of one extended vector. Let us denote it by \( K_d \) and assume that it has dimension \((n \times 1)\). The objective function is denoted by \( J_d(K_d) \).

4. The mini-batch adaptive method of random search (MAMRS)

Denote: \( J_d^s \) – the minimum value of the function after the \( s \)-th run; \( K_d^s \) – best parameter vector column after startup; \( d \) – the size of the mini-batch.
Step 0. Set: $d=1$ - initial mini-batch size (in the general case, one can start with any value of $1 \leq d \leq N$); $\lambda_1, \lambda_2$ - weights; $S_{\text{max}}$ - maximum number of starts; $B_{\text{max}}$ - maximum number of passes; $\alpha = 1.618$ - expansion coefficient; $\beta = 0.618$ - compression coefficient; $M$ - the maximum number of failed tests at the current iteration; $t_0 = 1$ - initial step size (can be set any value $t_0 > R$), $R$ - minimum step size, $L$ - The maximum number of iterations in the startup procedure.

Step 1. Put: $b = 1$ (number of passes counter); $P_d = 0$ (initial value of the sum of the average values of the objective function).

Step 2. Put: $s = 1$ (counter of the number of starts); $J_d^0 = 10^9 + 10^9$; $S_d = 0$ (initial value of the sum of the values of the objective function).

Step 3. Define the initial approximation of the matrix $K$ and the corresponding column vector $K_d^{s,0}$. Put $l = 0, j = 1$.

Step 4. Get a random vector $\xi^j = \left(\xi_{1}^j, \ldots, \xi_{m}^j\right)^T$, where $\xi_{k}^j$ - a random variable uniformly distributed on the interval $[-1, 1]$.

Step 5. Calculate: $y^j = K_d^{s,j} + t\frac{\xi^j}{\|\xi^j\|}$.

Step 6. Generate mini-batch of size $d$. To do this, generate $d$ pair wise mismatched sets of $q_i \in N_1, \ldots, q_n \in N_n$ values defining the initial states $x_{k}^0$ with numbers $k = q_1, \ldots, q_n \in \{1, \ldots, N\}$, or corresponding to the tuple $<q_1, \ldots, q_n>$.

Check the fulfillment of the conditions:

- a) if $J_d(y^j) < J_d(K_d^{s,j})$, a successful step. Put $z^j = K_d^{s,j} + \alpha(y^j - K_d^{s,j})$. Determine if current direction $y^j - K_d^{s,j}$ is successful: if $J_d(z^j) < J_d(K_d^{s,j})$, successful search direction. Put $K_d^{s,j+1} = z^j$, $t_{s+1} = \alpha t_s$, $l = l + 1$ and check the termination condition. If $l < L$, put $j = 1$ and go to step 4. If $l = L$, search complete: $\hat{K} = K_d^{s,j}$, go to step 8; if $J_d(z^j) \geq J_d(K_d^{s,j})$, search direction is unsuccessful, go to step 7;

- b) if $J_d(y^j) \geq J_d(K_d^{s,j})$, unsuccessful step and go to step 7.

Step 7. Estimate the number of failed steps from the current point:

- a) if $j < M$, put $j = j + 1$ and go to step 4;

- b) if $j = M$, check the termination condition: if $t_s \leq R$, process finish: $\hat{K} = K_d^{s,j}$, $J_d = J_d(K_d^{s,j})$, go to step 8; if $t_s > R$, put $t_s = \beta t_s$, $j = 1$ and go to step 4.

Step 8. Check the improvement in the value of the objective function as a result of the $s$-th run: if $J_d(K_d^{s,j}) < J_d^0$, put $J_d^0 = J_d(K_d^{s,j})$, $\hat{K} = K_d^{s,j}$ and go to step 9; if $J_d(K_d^{s,j}) \geq J_d^0$ go to step 9.

Step 9. Calculate $S_d = S_d + J_d$ and verify that the conditions for the end of the number of starts are met:

- if $s < S_{\text{max}}$, put $s = s + 1$ and go to step 3; if $s = S_{\text{max}}$, put $\hat{K} = \hat{K}^j$ - the best solution during the $b$-th pass for a given $k$; calculate $m_d$, $\sigma_{m_d}$ and go to step 10.

Step 10*. Put $P_d = P_d + m_d$, $m_d^b = m_d$ and check the condition for completing a given number of passes:

- if $b < B_{\text{max}}$, put $b = b + 1$ and go to step 2; if $b = B_{\text{max}}$, calculate: $\bar{m}_d$, $\sigma_{\bar{m}_d}$.

Step 11*. Check the condition for completing studies of the effect of the mini-batch size: if $k < N$, put $k = k + 1, s = 1$ and go to step 1; if $k = N$, go to step 12.
Step 12. As a result, find the best estimate of $\hat{K}_d^*$ after $B_{\text{max}}$ passes and indicators $\bar{m}_d$, $\sigma_b$ for each value of the mini-batch size $d$. To analyze the resulting estimation accuracy, find the value $J(\hat{K}_d^*)$ of the estimation accuracy criterion (4): 
$$J(\hat{K}_d^*) = (1 / N) \sum_{k=1}^{N} \left( \int_{t_0}^{t_f} \left[ \lambda_1 e^2 (t) + \lambda_2 \left\| \hat{K}_d^* \right\| \right] dt \right).$$

Steps 10 and 11 are performed if necessary.

5. Numerical example
The behavior of a nonlinear continuous deterministic model of the control object is described by the system [11-14]
$$\begin{align*}
\dot{x}_1 (t) &= a(x_2 (t) - x_1 (t)), \\
\dot{x}_2 (t) &= c\dot{x}_1 (t) - x_1 (t)x_3 (t) + u(t, \hat{x}(t)), \\
\dot{x}_3 (t) &= -bx_2 (t) + x_1 (t)x_2 (t),
\end{align*}$$

where $a = 10; b = 8 / 3; c = 16$; $t \in T = [0; 4]$; $x \in \mathbb{R}^3$; $u \in U, n = 3, q = 1$. $t_0 = 0, t_i = 4$.

The set of possible initial states $\Omega$: $|x_{10}| \leq 20, |x_{20}| \leq 30, |x_{30}| \leq 7$.

Measuring system equation: 
$$z(t) = x_1 (t) + x_2 (t) + x_3 (t), m = 1.$$

The task is to track the desired value of $w$ by a given coordinate. Therefore a tracking error is introduced $e(t) = x_i (t) - w, i=1,2,3$.

The quality of trajectory pencil control is characterized by the value of the functional (12):
$$J(K_d) = (1 / N) \sum_{k=1}^{N} \left( \int_{t_0}^{t_f} \left[ \lambda_1 e^2 (t) + \lambda_2 \left\| K_d \right\| \right] dt \right),$$

where $\lambda_1, \lambda_2$ – weights.

The proposed observer equation is:
$$\begin{align*}
\dot{x}_1 (t) &= a(\hat{x}_2 (t) - \hat{x}_1 (t)) + [x_2 (t) + x_3 (t) + x_4 (t) - \hat{x}_2 (t) - \hat{x}_3 (t) - \hat{x}_4 (t)]K_1 (t, \hat{x}(t)), \\
\dot{x}_2 (t) &= c\hat{x}_1 (t) - \hat{x}_1 (t)\hat{x}_3 (t) + [x_1 (t) + x_3 (t) + x_4 (t) - \hat{x}_1 (t) - \hat{x}_3 (t) - \hat{x}_4 (t)]K_2 (t, \hat{x}(t)) + u(t, \hat{x}(t)), \\
\dot{x}_3 (t) &= b\hat{x}_2 (t) + \hat{x}_1 (t)\hat{x}_2 (t) + [x_1 (t) + x_2 (t) + x_3 (t) - \hat{x}_1 (t) - \hat{x}_2 (t) - \hat{x}_3 (t)]K_3 (t, \hat{x}(t)),
\end{align*}$$

Legendre polynomials are used as basis functions:
$$p(n, x) = \sum_{k=0}^{n} \left( \frac{C_n^k x^n}{k!} \right)^2 \hat{x}^{n-k} \hat{x}, n = 0, \ldots, L - 1; \hat{x} = (x - \bar{x}) / (\bar{x} - \bar{x}).$$

We put the truncation scales in (11) $L_1 = 2, L_2 = 2, L_j = 2$, the coefficients $K_i (t, \hat{x}(t)) = K_i = \text{const}$, vectors $\bar{x} = (-18; -10; -12), \bar{x} = (18; 10; 12), M = 3$ – the maximum number of unsuccessful tests at the current iteration, $R = 8 \cdot 10^{-6}$ – the minimum step size.

We consider three cases of tracking the coordinates of the state vector.

Case (A): Tracking the desired value of the coordinate $x_1$.

It is required to find controls for the object and the observer that ensure the fulfillment of the condition $x_i (t) = w = 2$, i.e. tracking a given value of the coordinate $x_i$. Therefore, the error in (15) has the form $e(t) = x_i (t) - 2$.

Table 1 presents the solution depending on mini-batch size. In all tests $\Omega = N = 175$ – number of elementary subsets, $N_1 = 7$, $N_2 = 5$, $N_3 = 5$, $\Lambda_1 = 1, \Lambda_2 = 0$, initial state estimate vector $\hat{x}_0^* = \{18, 10, 12\}$. 

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**Table 1**

| Mini-batch Size | Estimation Accuracy |
|-----------------|---------------------|
| $d = 4$         | $\bar{m}_d = 4$     |
| $d = 8$         | $\bar{m}_d = 8$     |
| $d = 16$        | $\bar{m}_d = 16$    |
Table 1. Result of the solution using the mini-batch adaptive method of random search.

| d   | J[K_d]   |
|-----|----------|
| 1   | ∞        |
| 5   | 11.50127 |
| 10  | 10.09180 |
| 25  | 8.62730  |
| 50  | 8.08472  |
| 100 | 7.10427  |

Figure 1 shows a graph of tracking a given value of the \( x_i = 2 \) coordinate under the initial conditions \( x_1(0) = -2, x_2(0) = 10, x_3(0) = 12 \), as well as a graph of the change in the estimate \( \hat{x}_i(t) \), showing the convergence of \( \hat{x}_i(t) \) to \( x_i(t) \). Figure 2 shows a graph of pencil tracking for a given value of the \( x_i = 2 \) coordinate under various initial conditions. In equations (14), (16), the control law of the form (11), the matrices \( K = [21.8418; 117.0079; 10.9109] \) and \( u = [17.10, 182.33, 226.17, -262.65, 171.35, -66.69, -154.42, -47.43] \) are used, found by the MAMRS with a mini-batch size \( d = 100 \). The figure shows that throughout the entire time interval and at the final moment, the trajectories pass close to the desired value.

**Figure 1.** Numerical results in case A.  
**Figure 2.** Pencil of trajectories in case A.

**Case (B):** Tracking the desired value of the coordinate \( x_2 \).

It is required to find a control that ensures the fulfillment of the condition \( x_2(t) = w = 2 \), i.e. tracking a given value of the \( x_2 \) coordinate. Therefore, the error in (15) has the form \( e(t) = x_2(t) - 2 \).

Table 2 presents the solution depending on mini-batch size. In all tests \( \Omega : N = 105 \) – number of elementary subsets, \( N_1 = 5, N_2 = 7, N_3 = 3, \lambda_1 = 1, \lambda_2 = 0 \), initial state estimate vector \( \hat{x}_0 = \{10, -3, 5\} \).
Table 2. Result of the solution using the mini-batch adaptive method of random search.

| d    | J[\text{K}_d] |
|------|--------------|
| 1    | 88.66673     |
| 5    | 19.33560     |
| 10   | 15.72685     |
| 25   | 14.99191     |
| 50   | 14.75473     |
| 100  | 14.69290     |

Figure 3 shows a graph of tracking a given value of the \( x_2 = 2 \) coordinate under the initial conditions \( x_i(0) = 1, x_i(0) = 6, x_i(0) = 2 \), as well as a graph of the change in the estimate \( \hat{x}_i(t) \), showing the convergence of \( \hat{x}_i(t) \rightarrow x_i(t) \). Figure 4 shows a graph of pencil tracking for a given value of the \( x_2 = 2 \) coordinate under various initial conditions. In equations (14), (16), the control law of the form (11), the matrices \( K = [109.0897; 51.1093; 4.1479] \) and \( u = [192.02, 296.66, -68.6, -123.42, 19.21, -152.96, 20.55, -123.3] \) are used, found by the MAMRS with a mini-batch size \( d = 100 \).

**Case (C):** Tracking the given value of the coordinate \( x_3 \).

It is required to find a control that ensures the fulfillment of the condition \( x_i(t) = w = 2 \), i.e. tracking a given value of the \( x_3 \) coordinate. Therefore, the error in (15) has the form \( e(t) = x_i(t) - 2 \).

Table 3 presents the solution depending on mini-batch size. In all tests \( \Omega_i : N = 125 \) — number of elementary subsets, \( N_1 = 5, N_2 = 5, N_3 = 5, \lambda_1 = 1, \lambda_2 = 0 \), initial state estimate vector \( \hat{x}_0^k = \{5,1,1\} \).
Table 3. Result of the solution using the mini-batch adaptive method of random search.

| d  | $J[K_d]$       |
|----|----------------|
| 1  | $\infty$      |
| 5  | 6084.75957    |
| 10 | 27.89230      |
| 25 | 27.80911      |
| 50 | 27.79167      |
| 100| 27.76462      |

Figure 5 shows a graph of tracking a given value of the $x_j = 2$ coordinate under the initial conditions $x_1(0) = -1, x_2(0) = 3, x_3(0) = 4$, as well as a graph of the change in the estimate $\hat{x}_1(t)$, showing the convergence of $\hat{x}_1(t) \rightarrow x_1(t)$. Figure 6 shows a graph of pencil tracking for a given value of the $x_j = 2$ coordinate under various initial conditions. In equations (14), (16), the control law of the form (11), the matrices $K = \begin{bmatrix} 181.4409; & 5.7833; & -35.0755 \end{bmatrix}$ and $u = \begin{bmatrix} 227.93, & 148.88, & 27.49, & -90.32, & 146.14, & -247.28, & -182.53, & 64.96 \end{bmatrix}$ are used, found by the MAMRS with a mini-batch size $d = 100$.

Figure 5. Numerical results in case C.  
Figure 6. Pencil of trajectories in case C.

6. Conclusion
In this work, a strategy, a step-by-step algorithm and the corresponding software for an approximate solution of the problem of finding the optimal control of pencils of trajectories of continuous deterministic dynamic systems under conditions of uncertainty in setting the initial conditions are developed. The control of the object and the control of the observer depend on time and on a different set of estimates of the state vector coordinates. Due to joint estimation and control, it is possible to reduce estimation errors and improve the quality indicators of a closed-loop control system. The presented algorithm and software are tested on a model example of solving the problem of tracking the selected coordinate of a chaotic dynamic system. The influence of the mini-batch size on the quality of the result is investigated. Recommendations on the algorithm parameters choice are given. The mini-batch adaptive method of random search can be used in aerospace problems, such as the problem of
stabilizing a satellite, the problem of controlling aircraft, helicopters, the problem of approaching stationary and moving targets, and others.

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