Buckling Analysis of Single and Multi Delamination In Composite Beam Using Finite Element Method

Hans Charles Simanjorang\textsuperscript{1, a}, Hendri Syamsudin\textsuperscript{2, b}, Muhammad Giri Suada\textsuperscript{3, c}

\textsuperscript{1, 2, 3} Lightweight Structure Research Group, Faculty of Mechanical and Aerospace Engineering
Institut Teknologi Bandung, Jl. Ganesha 10 Bandung 40132, Indonesia
\textsuperscript{a}hans.simanjorang@students.itb.ac.id, \textsuperscript{b}hendri.syamsudin@ae.itb.ac.id, \textsuperscript{c}mg@aero.pauir.itb.ac.id

Abstract. Delamination is one type of imperfection in structure which found usually in the composite structure. Delamination may exist due to some factors namely in-service condition where the foreign objects hit the composite structure and creates inner defect and poor manufacturing that causes the initial imperfections. Composite structure is susceptible to the compressive loading. Compressive loading leads the instability phenomenon in the composite structure called buckling. The existence of delamination inside of the structure will cause reduction in buckling strength. This paper will explain the effect of delamination location to the buckling strength. The analysis will use the one-dimensional modelling approach using two-dimensional finite element method.

Keywords: Delamination, buckling load, buckling mode, composite structure, finite element method

1. INTRODUCTION

The development of composite material has been successfully compete with another conventional materials such as metals in many applications. Composite material is material that has high specific strength, high specific stiffness, and low density \cite{1}. Despite it has many advantages compared to metallic materials, composite materials also have weakness in its application. Composite materials especially composite laminates are prone to delamination. This will reduce the strength of composite material due to compressive loading. Sallam and Simitsess (1984) performed one dimensional buckling and post buckling analysis on cross-ply laminate that have delamination to calculate buckling loads. Parlapali, Shu, and Chai (2008) have developed the analytical model with Euler Bernoulli and Classical Lamination Theory basis to analyze buckling on composite beam with two delamination lower and upper bounds. Chen and Qiao (2011) have developed the analytical equation to analyze the buckling delamination on bi-layer column with the effect of transverse shear deformation and local tip deformation. This paper will explain the effect of delamination location, delamination length, and multi delamination to the buckling strength numerically using commercial finite element software.
2. LINEAR BUCKLING ANALYSIS

Numerical modelling using finite element is done with the linear buckling analysis – eigenvalue analysis. The purpose is to calculate the critical load and mode of buckling deformation. To calculate the critical load, it is required the equilibrium position of loads, external load, and internal load acting on the structure as in this following equation.

\[ [K]\{d\} = ([K]_0 + \lambda [K]_\sigma)\{d\} = \{0\} \]  

Where \([K]\) is total stiffness matrices, \([K]_0\) is the stiffness matrices calculated in the span of small displacement, \([K]_\sigma\) is the geometry stiffness matrices, \(\lambda\) is the eigenvalue and \(\{d\}\) is the displacement vector. The equation above is to solve the eigenvalue problem. If the displacement vector is not zero, then the solution of the equation can be obtained using the following characteristic equation.

\[ \det([K]_0 + \lambda [K]_\sigma) = 0 \]  

The equation can be solved for \(\lambda\). The smallest eigenvalue will take into consideration because this value will become the multiplier factor to calculate the critical load on the structure. Eigenvector also can be obtained and it shows the mode of buckling. Using this method, the structure can be assumed as linear until it reaches the instability point [2].

3. LAMINATE BEAM PLATE BUCKLING

In this paper, the thickness of the specimen is very small compared to its length and the force acting on the structure only in x-axis, and the lamina can be assumed as beam [3]. The following is linear differential equation for plate buckling.

\[ \frac{\partial^2 M_{xb}}{\partial x^2} + 2 \frac{\partial^2 M_{xyb}}{\partial x \partial y} + \frac{\partial^2 M_{yb}}{\partial y^2} + \frac{\partial^2 N_{xb}}{\partial x^2} + 2 \frac{\partial^2 N_{xy}}{\partial x \partial y} + \frac{\partial^2 N_{yb}}{\partial y^2} = 0 \]  

\[ \frac{\partial^2 M_{xb}}{\partial x^2} + \frac{\partial^2 M_{xb}}{\partial x \partial y} + N_{x} \frac{\partial^2 w}{\partial x^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_{y} \frac{\partial^2 w}{\partial y^2} = 0 \] ...

The superscript (b) shows there are moment and buckling forces and (i) shows the force shortly before buckling and pre-buckling. The complete details to obtain these equations can be seen at reference [4].

In this current paper, the force acts only in axial direction \(N_x\) and all differentiation in \(y\) is eliminated in the linear differential equation of plate buckling and it becomes

\[ \frac{\partial N_{xb}}{\partial x} = 0 \]  

\[ \frac{\partial^2 M_{xb}}{\partial x^2} + N_{x} \frac{\partial^2 w}{\partial x^2} = 0 \]  

For 1-D beam, \(M_{xb} = E_{bol} \frac{\partial^2 w}{\partial x^2}\) and \(N_{x} = b N_{x}\), therefore the equation above becomes

\[ \frac{\partial N_{xb}}{\partial x} = 0 \]
\[ E_x I_y \frac{\partial^4 w}{\partial x^4} + N_x \frac{\partial^2 w}{\partial x^2} = 0 \] ........................................ (9)

\[ \frac{\partial^4 w}{\partial x^4} + \frac{N_x}{E_x I_y} \frac{\partial^2 w}{\partial x^2} = 0 \] ........................................ (10)

Where \( \lambda^2_b = \frac{N_x}{E_x I_y} \)

The general solution of the equation above is

\[ w(x) = c_1 \sin \lambda_b x + c_2 \cos \lambda_b x + c_3 x + c_4 \] .......................................................... (11)

C1, c2, c3, c4, obtained from the boundary conditions. To determine the critical buckling load, for example the length of the beam is L, thickness h, with the fixed supported in both ends, the equation become

\[ w(0) = 0, \quad \frac{\partial w}{\partial x}(0) = 0, \quad w(L) = 0, \quad \frac{\partial w}{\partial x}(L) = 0 \] .... (12)

Therefore, the critical buckling load per unit thickness is

\[ N_{cr} = \frac{\pi^2 E_b}{3 \left( \frac{h^3}{L^2} \right)} \] .......................................................... (13)

From the explanation above, it can be stated that the analysis of 1-D plate buckling is similar with beam/column buckling analysis.

4. MATERIAL PROPERTIES

Material that has been used in this research is fiberglass-epoxy that made from wet-lay-up method. The specimen contains 20 layers with cross-ply orientation. Material made following the standard ASTM 3039. The experiment to obtain the material properties is done by Mulia Minhat (2007) \(^5\). He conducted the tensile test on the specimens. Because of the limitation in the equipment tools, the property obtained from the experiment was only the young modulus. The poison ratio and shear modulus obtained from the literature.

| Table 1. Material property of Fiberglass-Epoxy Isotropic |
|--------------------------------------------------------|
| Fiberglass Property                                    |
| Longitudinal Modulus, \( E_{11} \) 9.25 GPa            |
| Poisson Ratio, \( \nu_{12} \) 0.25                      |
| Shear Modulus, \( G_{12} \) 3.7 GPa                   |

5. FINITE ELEMENT METHOD ANALYSIS

Due to the limitation of the experimental tools, the material will be assumed as isotropic in finite element model. Commercially finite element code MSC Patran Nastran 2011 will be used in the numerical analysis. MSC Patran will be used as pre-processing tool to model the geometry, load, boundary conditions, and input material properties and also to mesh the model. MSC Nastran will be used as processing tool to calculate and obtain eigenvalue and buckling mode. MSC Patran will be used again to show the calculated eigenvalue and mode of buckling. The composite beam will be modelled
as 2-D shell type with CQUAD4 elements. The boundary conditions will be fixed supported at both ends with apply constant displacement in one side.

![Figure 1. Single delamination model](image1)

![Figure 2. Single delamination model in FEM](image2)

![Figure 3. Multi delamination model](image3)

![Figure 4. Multi delamination model in FEM](image4)

### Table 2. Model Dimensions

| Model dimensions   |       |
|--------------------|-------|
| Length, L          | 240 mm|
| Width, t           | 2.5 mm|
| Thickness, w       | 40 mm |
| Delamination length, a | Varying variable |
| Delamination location, h1 | Varying variable |
| Delamination location, h2 | Varying variable |
Table 3. Variation of a/L to h/t in single delamination

| Variation of a/L to h/t | h/t | a/L                      |
|------------------------|-----|--------------------------|
|                        | 0.1 | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 |
|                        | 0.2 | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 |
|                        | 0.3 | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 |
|                        | 0.4 | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 |
|                        | 0.5 | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 |

Table 4. Variation of a/L to h/t in multi delamination

| Variation of a/L to h/t | h1/t | h2/t | a1/L                      | a2/L                      |
|------------------------|-----|-----|--------------------------|--------------------------|
|                        | 0.1 | 0.9 | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 |
|                        | 0.2 | 0.8 | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 |
|                        | 0.3 | 0.7 | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 |
|                        | 0.4 | 0.6 | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 |
|                        | 0.5 | 0.5 | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 |

6. RESULTS AND ANALYSIS

Table 5. Critical load of a perfect structure

| Equation             | Critical Load (Newton) | % difference with theoretical value |
|----------------------|------------------------|------------------------------------|
| Theoretical: column model | 330.2                  | 0.00 %                             |
| Theoretical: 1-D plate           | 330.2                  | 0.00 %                             |
| Numerical FEM          | 330.3                  | 0.03 %                             |

It can be seen that the analysis using finite element model in perfect structure is valid with small difference with the theoretical values. This calculation is done to see the accuracy and also to know the critical load of a perfect structure that later on will be compared with the delaminated structure.
In the single delamination, it can be seen that the results are consistent with the results that obtained by previous researchers, Simitsess [6] and Mulia Minhat [5]. From this results, the longer the delamination, the buckling strength of the composite laminate structure will decrease quadratically. This results is match with the equation that derived by Euler that the longer the delamination, it will reduce the buckling strength quadratically.

Besides that, the delamination occur near to the upper or lower surface will reduce the buckling strength significantly and creates the local buckling phenomenon. This is caused by the small moment inertia in the delaminated area. It can be stated that for the delamination near to the structure surface, the structure will fail first by delamination and then by buckling when the compression force applied on it.
Figure 6 (left). \( P_{cr}/P \) to delamination length in single delamination \( h/t=0.1 \) and multi delamination \( h/t=0.1 \) and \( h/t=0.9 \)

Figure 6 (right). \( P_{cr}/P \) to delamination length in single delamination \( h/t=0.2 \) and multi delamination \( h/t=0.2 \) and \( h/t=0.8 \)

Figure 7 (left). \( P_{cr}/P \) to delamination length in single delamination \( h/t=0.3 \) and multi delamination \( h/t=0.3 \) and \( h/t=0.7 \)

Figure 7 (right). \( P_{cr}/P \) to delamination length in single delamination \( h/t=0.4 \) and multi delamination \( h/t=0.4 \) and \( h/t=0.6 \)
Table 7. Buckling mode in multi delamination

| a/L , h/t | 0.1 & 0.9   | 0.2 & 0.8   | 0.3 & 0.4   | 0.4 & 0.6   | 0.1 & 0.5   |
|-----------|-------------|-------------|-------------|-------------|-------------|
| 0.1       | Mixed       | Global      | Global      | Global      | Mixed       |
| 0.2       | Local       | Mixed       | S-shape     | Mixed       | Local       |
| 0.3       | Local       | Mixed       | S-shape     | Mixed       | Local       |
| 0.4       | Local       | Mixed       | Mixed       | Mixed       | Local       |
| 0.5       | Local       | Mixed       | Mixed       | Mixed       | Local       |
| 0.6       | Local       | Local       | Mixed       | Mixed       | Local       |
| 0.7       | Local       | Local       | Mixed       | Local       | Local       |
| 0.8       | Local       | Local       | Local       | Local       | Local       |
| 0.9       | Local       | Local       | Local       | Local       | Local       |

From the results of multi delamination, it can be seen that the critical buckling strength will decrease with the presence of multi delamination. It can be seen at from figure 6 to 7. This parametric study shows if there are multi delamination in the composite laminate structure, it will make the buckling strength decreases more than with the single delamination.

Figure 6 shows that the results for single and multi-delamination with delamination near to the surface is close. This is because the similarity in the buckling mode which is local buckling that decreases the buckling strength drastically and also the location of delamination where near to the surface of the structure. It is different with figure 7, where the multi delamination decrease the structure buckling strength more than single delamination.

From this results we can see that the location of the delamination plays major role in reducing the buckling strength. If the delamination occur near to the surface whether it’s single or multi delamination, serious concern should be taken on it. But if the delamination occur far from the surface structure, it’s also need to be checked whether the delamination is single or multi. If its multi delamination, serious concern should be taken because it will reduce the buckling strength more than single delamination.

The delamination near to the structure surface will make the buckling strength decreases significantly. This is happen because of the small moment inertia in the delaminated area. It can be seen that for the delamination near to the surface, the structure will fail first because of the delamination then by buckling.

Table 6 and 7 show the buckling mode of the structure in singe and multi delamination cases. There are three types of buckling mode namely local buckling, global buckling, and mixed mode buckling (local buckling and global buckling) [7].

Majority delamination near to the surface will lead to the local buckling, but when the delamination far from the surface the buckling mode will be in global or mixed buckling. From the results, we can conclude that the local buckling decreases the buckling strength of the structure drastically than global and mixed buckling mode.
7. CONCLUSIONS

The conclusions of this research are numerical modelling using finite element method with 2D Shell CQUAD4 element produce very good results that can show the decrease of buckling load phenomenon in the structure with delamination.

The longer the delamination in every location in the structure will lead to the decreasing of structure buckling load. The location of delamination will affect the buckling load of the structure, where the delamination near to the surface will make the buckling strength of structure decrease significantly.

The buckling strength of structure with multi delamination is lower than with single delamination. Local buckling makes the buckling load of the structure decreases significantly comparing to the other modes of buckling. The buckling phenomenon in the structure dominantly affected by geometrical factor, moment inertia and also the length of the structure.

There are three types of buckling namely local buckling, global buckling, and mixed mode buckling. In this research local buckling occur mainly when the delamination is located near to the surface. The global and mixed mode occur mainly when the delamination far from the surface of the structure. The local buckling mode play major role in decreasing the buckling strength significantly compared to the global and mixed buckling mode.

8. REFERENCES

[1] R.M. Jones, “Mechanics of Composite Materials”, 2nd edition, Taylor & Francis, Inc, 1999.

[2] S. Sallam and G.J Simitses, “Delamination Buckling and Growth of Flat, Cross-Ply Laminates”, Composite Structures, Vol.4, pg 361-381, 1984.
[3] J.N. Reddy, “Mechanics of Laminated Composite Plates and Shells – Theory and Analysis”, CRC Press, pg 165 – 182, 2004.

[4] M.J Sewell, “The Static Perturbation Techniques in Buckling Problems”, Mech. Phys. Solids, 1965, Vol.13, pg. 247-265.

[5] Mulia Minhat, “Buckling Analysis of Delaminated Composite Beams Using Finite Element Method”, Master Degree Thesis Insitut Teknologi Bandung, 2007.

[6] G.J Simitsess, S. Sallam, “Effect of Delamination of Axially Loaded Homogeneous Laminated Plates”, AIAA Journal, pg 1437-1444, 1985.

[7] I. Sheinmann, M. Bass and O. Ishai, “Effect of Delamination on Stability of Laminated Composite Strip”, Composite Structures, Vol. 11, pg 227-242, 1989.