NONLINEAR ELECTRODYNAMICS AND THE SURFACE REDSHIFT OF PULSARS

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ABSTRACT

It is currently argued that the best method of determining neutron star (NS) fundamental properties is by measuring the gravitational redshift \( z \) of spectral lines produced in the stellar photosphere. Measurement of \( z \) at the stellar surface provides a unique insight on the NS mass-to-radius relation and thus on its equation of state, which reflects the physics of the strong interaction between the particles making up the star. Evidence for such a measurement has been provided quite recently by Cottam, Paerels, & Mendez and also by Sanwal and coworkers. Here we argue that although the quoted observations are undisputed for canonical pulsars, they could be misidentified if the NS is endowed with a superstrong magnetic field \( (B) \) as are the so-called magnetars and strange quark magnetars, e.g., the spectral line discovered by Ibrahim and coworkers. The source of this new “confusion” redshift is related to nonlinear electrodynamics (NLED) effects.

Subject headings: line: formation — line: identification — magnetic fields — methods: analytical — pulsars: general — relativity

1. INTRODUCTION

Neutron stars (NSs), the death throes of massive stars, are among the most exotic objects in the universe. They are supposed to be composed of essentially neutrons, although some protons and electrons are also required in order to guarantee stability against Pauli’s exclusion principle for fermions. Among the most exotic objects in the universe. They are supposed to be composed of essentially neutrons, although some protons and electrons are also required in order to guarantee stability against Pauli’s exclusion principle for fermions. Because of its density, a neutron star is also believed to trap in its core even more exotic states of matter. It is almost a consensus that these new states might exist inside the star and may dominate its structural properties. Pion plus kaon Bose-Einstein condensates could appear, as well as “bags” of strange quark matter (Miller 2002). This last is believed to be the most stable state of nuclear matter (Glendenning 1997), which implies an extremely dense medium whose physics is currently under severe scrutiny. The major effect of these exotic constituents is manifested through the NS mass-radius ratio \((M/R)\). Most researchers think that such exotic components not only make the star more compact, i.e., smaller in radius, but also lower the maximum mass it can retain. To get some insight into the neutron star’s most elusive properties, its mass \((M)\) and radius \((R)\), astronomers have several techniques at their disposal, the most promising being the gravitational redshift. Since the redshift depends on the ratio \(M/R\), measuring NS spectral line displacement leads to a direct insight into the dense matter equation of state (EOS).

In recent years strong evidence seems to have gathered around a new and exotic class of hypermagnetized neutron stars, the so-called “magnetars” (Duncan & Thompson 1992). These objects are supposed to be the final stage of newly-born neutron stars on which a classical \( \alpha - \omega \) dynamo mechanism has efficiently acted during the early stages of its evolution, reaching field strengths up to \( B_{\text{sup-crit}} \sim 10^{17} \text{G} \). A peculiar class of gamma-ray sources known as “soft gamma-ray repeaters” (SGRs) (Kouveliotou et al. 1998) and a set of X-ray pulsars known as “anomalous” (AXPs) have been claimed to be associated with these type of stars (Mereghetti 2001).

Although these magnetars are said to be the best model to explain the dynamics of SGRs and AXPs, accretion-driven models (Marsden et al. 2001), strange quark matter stars with normal \( B_s \) (Zhang et al. 2000; Xu & Busse 2001; Hu & Xu 2002; Xu 2002), or even the strange magnetar interpretation (Zhang 2002) have also been proposed as competing scenarios. Note that in a recent paper Pérez Martínez et al. (2003) have provided arguments against the formation of the so-called magnetars in the context of the physics used by their mentors (Duncan & Thompson 1992) and for their occurrence in nature. Pérez Martínez et al. argue that a full description of the physics taking place during the early evolution of NSs should not overlook fundamental issues, such as quantum electrodynamics effects, when discussing the role of superstrong \( B_s \) on the stability of just-born NS pulsars. The positive magnetization of the neutron matter and the appearance of a ferromagnetic configuration in the stellar structure are examples of such effects. Thus the idea of magnetars is still contentious. Despite this lively dispute, in this paper, we present theoretical arguments for the potential effects of NLED on the physics of strongly magnetized NSs.

A very interesting example of how this issue could be elusive is provided by the recent discovery by Ibrahim et al. (2002) and subsequent confirmation by Ibrahim et al. (2003) of cyclotron resonance features from the SGR 1806–20. It is well known, in accretion-powered models, that proton \((p)\) and \( \alpha \)-particles (He) produce fundamental resonances of energy

\[
E_{\text{line}} = \begin{cases} 
  \frac{p}{\text{He}} & 6.3 (1+z)^{-1} \left[ \frac{B_{\text{sup-crit}}}{10^{15}\text{G}} \right] \text{keV}, \\
  & (1)
\end{cases}
\]

Ibrahim et al. (2002, 2003) showed that the 5 keV absorption line in the spectrum of SGR 1806–20 is consistent with a proton-cyclotron fundamental resonance in a redshift-dependent supercritical magnetic field \((B)\) of strength...
$B_{\text{sup-crit}} \sim 7.9 \times 10^{14}(1+z)^{-1} \text{ G}$. This translates into a $B_{\text{sup-crit}} \sim 1.0 \times 10^{15} \text{ G}$ for the mass and radius of a canonical NS ($\rho \sim 10^{14} \text{ g cm}^{-3}, R \sim 10 \text{ km}, M \sim 1.4 M_\odot, B \sim 10^{12} \text{ G}$), which agrees with the field strength inferred from the SGR 1806–20 spindown, i.e., from $P$ and $\dot{P}$ (Kouveliotou et al. 1998).

2. GRAVITATIONAL VERSUS NLED REDSHIFT

In particular, we argue that for extremely supercritical magnetic fields, NLED effects force photons to propagate along accelerated curves. When the nonlinear Lagrangian density is a function only of the scalar $F = F_{\mu\nu}F^{\mu\nu}$, e.g., $L(F)$, the force accelerating the photons is given by

$$k_\alpha k^\alpha = \frac{k^2}{2} L_{F|\alpha} - 2(L_{F|F}^{\mu\nu}F^{\mu\nu})_{\alpha}k_{\alpha}k_{\nu},$$

where $k_{\nu}$ is the wavevector and $L_F$ means the partial derivative with respect to $F$ (note that it does not depend on any intrinsic property of the photons). This feature allows for this force, acting along the photons path, to be geometricized (Novello et al. 2000; Novello & Salim 2001) in such a way that in an effective metric,

$$g^{\text{eff}}_{\mu\nu} = g_{\mu\nu} + g_{\mu\nu}^{\text{NLED}},$$

the photons follows geodesic paths, as we show in § 3, in a particular case of the above Lagrangian. The standard geometric procedure used in general relativity (GR) to describe the photons can now be used by substituting the usual metric with the effective metric. In particular, the outgoing redshifts now have a couple of components, one due to the gravitational field and another from $B$.

A direct insight into the GR $z = z(M, R)$ at the surface of a compact star could be attained from the identification of absorption or emission lines from it. NS mass ($M$) can be estimated, in some cases, from the orbital dynamics of binary systems, while attempts to measure the radius ($R$) proceed via high-resolution spectroscopy (Sanwal et al. 2002, for 1E1207.4–5209; Cottam et al. 2002, using type-I X-ray bursts from EXO 0748–676). In these systems, success was achieved in determining these parameters, or the relation between them, by looking at excited ions near the NS surface (arguments favoring a strange star in EXO 0748–676 are given by Xu 2003). Gravity effects cause the observed energies of the spectral lines of excited atoms to be shifted to lower values by a factor

$$\frac{1}{(1+z)} \equiv \left[1 - \frac{2G}{c^2} \left(\frac{M}{R}\right)\right]^{1/2}.$$

Measurements of such line properties, energy, width, and polarization, as here called for, would lead to an indirect, but highly accurate, estimate of the NS mass-to-radius ratio ($M/R$), a tight constraint on its EOS, and strong limits on the $B$ strength (but not on its configuration) at the stellar surface. The above analysis stands whenever effects of NS $B$s are negligible. However, if the NS is pervaded by a superstrong $B$ ($B_{\text{sup-crit}}$), then NLED should be taken into account to describe the overall physics taking place on the pulsar surface. Our major result proves that for extremely high $B$s, the redshift induced by NLED can be as high as that produced by gravity alone, thus making it hard to draw any conclusive claim about NS fundamental properties.

As claimed here, the shift in energy and width produced by the effective metric “pull” of the star on laboratory-known spectral lines scales directly with the strength of the effective potential associated to the effective metric. Thus, this shift has two contributions: one from gravity and another from NLED. For hypermagnetized stars, e.g., magnetars, and if the near surface multipole field is much stronger than the dipole component (see Duncan 1998 for a possible toroidal configuration in SGR 0526–66 based on global seismic oscillations; § 4 discusses implications for the cyclotron line interpretation), the correction factor from NLED is substantial, with both contributions of about the same order of magnitude. Thus, there is the possibility for a given field strength for gravity effects to be mimicked by electromagnetic (EM) ones and for the phenomenon to entangle the fixing of constraints on the $M/R$ ratio. We suggest this difficulty can be overcome by taking into account that the $B$ contribution, which differs from that of the gravitational field, which is isotropic, depends on the polarization $b^\alpha$ of the emitted photon, being different for the cases $B_\alpha b^\alpha = 0$ and $B_\alpha b^\alpha \neq 0$.

Our warning is, therefore, that the identification and analysis of spectral lines from high $B$ NSs (in outbursts) must take into account the two possible different polarizations of the received photons in order to discriminate between redshifts produced either gravitationally or electromagnetically. Putting this result in perspective, we claim that if the characteristic $z$, or $M/R$ ratio, were to be inferred from this type of source, care should be taken, since for superstrong $B$, such a $z$ becomes of the order of the gravitational redshift expected from a canonical NS. It is, therefore, not clear whether one can categorically assert something about, e.g., the SGR 1806–20 $M/R$ ratio under such dynamical conditions. We prove this claim next.

3. THE MODEL

The propagation of photons in NLED has been examined by several authors (Bialynicka-Birula & Bialynicki-Birula 1970; Garcia & Plebanski 1989; Dittrich & Gies 1998; De Lorenci et al. 2002). In the case of geometric optics, in which the photon propagation can be identified with the propagation of discontinuities of the EM field in a nonlinear regime, a remarkable property appears: the discontinuities propagate along null geodesics of an effective geometry that depends on the EM field of the background (Novello et al. 2000; Novello & Salim 2001). According to quantum electrodynamics, in the Heisenberg & Euler (1936) approximation (see also Schwinger 1951), a vacuum has nonlinear properties, and these novel properties of photon propagation in NLED can show up, in principle, in photons propagating in a vacuum. In this specific case, the equations for the EM field in a vacuum coincide in their form with the equations of continua in which the electric permittivity and magnetic permeability tensors $\varepsilon_{\alpha\beta}$ and $\mu_{\alpha\beta}$ are functions of the electric and magnetic fields determined by some observer represented by its velocity 4-vector $V^\alpha$ (Denisov et al. 2001a, 2001b; Denisov & Svertilov 2003). This first-order approximation is valid for magnetic fields smaller than $B_0$, a parameter that will be defined below. In curved spacetime, these equations are written as

$$D^\alpha_{[\alpha} = 0, \quad B^\alpha_{[\alpha} = 0,$$

and

$$D^\beta \frac{V^\beta}{c} + \eta^{\alpha\beta\sigma} V_\rho H_{\sigma\beta} = 0.$$
\[ B_{[\beta}^{\alpha} \frac{V^\beta}{c} - \eta^{\alpha\beta\mu\nu} V_\mu E_\nu = 0, \] (7)

where the double vertical bars “||” stand for the covariant derivative and \( \eta^{\alpha\beta\mu\nu} \) is the completely antisymmetric Levi-Civita tensor. The 4-vectors representing the EM field are defined as usual in terms of the EM field tensor \( F_{\mu\nu} \) and polarization tensor \( P_{\mu\nu} \):

\[ E_\mu = F_{\mu\nu} \frac{V^\nu}{c}, \quad B_\mu = \epsilon^{\mu\nu\rho\sigma} F_{\nu\rho} \frac{V^\sigma}{c}, \] (8)

\[ D_\mu = P_{\mu\nu} \frac{V^\nu}{c}, \quad H_\mu = \epsilon^{\mu\nu\rho\sigma} P_{\nu\rho} \frac{V^\sigma}{c}, \] (9)

where the dual tensor \( X^\mu_{\nu} \) is defined as \( X^\mu_{\nu} = \frac{1}{2} \eta_{\alpha\beta\mu\nu} X^\alpha_{\beta} \) for any antisymmetric second-order tensor \( X_{\alpha\beta} \). The meaning of the vectors \( D^\mu \) and \( H^\mu \) comes from the Lagrangian of the EM field, and in the case of a vacuum, they are

\[ H_\mu = \mu_\mu B^\mu, \quad D_\mu = \epsilon_{\mu\nu} E^\nu, \] (10)

where the permeability and permittivity tensors are given as

\[ \mu_{\mu\nu} = \left[1 + \frac{2\alpha_{\mu\nu} B^2}{4\pi B_q c^2} \right] \mu_{\mu\nu} - \frac{7\alpha_{\mu\nu}}{4\pi B_q} E_\mu E_\nu, \] (11)

\[ \epsilon_{\mu\nu} = \left[1 + \frac{2\alpha_{\mu\nu} B^2}{4\pi B_q c^2} \right] \epsilon_{\mu\nu} + \frac{7\alpha_{\mu\nu}}{4\pi B_q} B_\mu B_\nu. \] (12)

In these expressions \( \alpha \) is the EM coupling constant \( (\alpha = e^2 / \hbar c / 137) \), and \( B_q \) is a quantum electrodynamic parameter, \( B_q = m^2 c^3 / \hbar \epsilon c = 4.41 \times 10^{13} \) G, also known as the Schwinger critical B field, i.e., \( B_q \equiv B_{crit} \). The tensor \( \mu_{\mu\nu} \) is the metric induced in the reference frame perpendicular to the observers, determined by the vector field \( V^\mu \). Our main concern in this paper is the behavior of NLED in either a pulsar or a magnetar, so in this particular case, \( E^\nu = 0, \epsilon_{\mu\nu}^0 = \epsilon^0_{\mu\nu} = \epsilon_{\epsilon\mu\nu} = \epsilon_{\rho\mu\nu} = 0 \), and \( \epsilon_{\alpha\beta\mu\nu} = \mu_{\alpha\beta\mu\nu} \). The scalars \( \epsilon \) and \( \mu \) can read directly from equations (11) and (12) as \( \epsilon \equiv \mu = 1 + (2\alpha_{\mu\nu} B^2) B^2 \). We deal with light propagation in NLED in the optical approximation. The EM wave is represented by 3-surfaces of discontinuities of the EM field that propagate in the nonlinear background. As we show below, the EM wave propagation can be described as if the metric of the background were changed from its original form determined by GR into another effective metric that depends on the dynamics of the background EM field. This formalism allows us to use the well-known results from Riemann geometry largely applied in GR.

Following Hadamard (1903), the surface of discontinuity of the EM field is denoted by \( \Sigma \). The field is continuous when crossing \( \Sigma \), while its first derivative presents a finite discontinuity, specified as follows:

\[ [B^\mu]_{\Sigma} = 0, \quad [\partial_\mu B^\alpha]_{\Sigma} = B^\mu_k a_k, \quad [\partial_\mu E^\alpha]_{\Sigma} = \epsilon^\mu_k a_k, \] (13)

where the symbol

\[ [f]_{\Sigma} = \lim (J_{\Sigma+} - J_{\Sigma-}) \] (14)

represents the discontinuity of the arbitrary function \( f \) through the surface \( \Sigma \). The tensor \( f_{\mu\nu} \) is called the discontinuity of the field, and \( k_\mu = \partial_\mu \Sigma \) is the propagation vector. Applying conditions (13) and (14) to the field equations in the particular case of \( E^\alpha = 0 \), we obtain the constraints \( \bar{\epsilon}^\nu e_{\mu\nu} = 0 \) and \( b^\nu_k a_k = 0 \) and the following equations for the discontinuity fields \( e_\alpha \) and \( b_\alpha \):

\[ \bar{\epsilon}^{\gamma\mu} e_{\mu} \alpha \frac{V^\alpha}{c} + \bar{\eta}^\mu_{\nu\rho\sigma} \frac{V^\nu}{c} (\mu\nu _\rho \alpha \beta \gamma \delta \epsilon_\gamma a_k a_\mu) = 0, \] (15)

\[ b^\alpha \alpha = \bar{\eta}^\mu_{\nu\rho\sigma} \frac{V^\nu}{c} (e_\gamma a_\mu) = 0. \] (16)

Isolating the discontinuity field from (15), substituting in equation (16), and expressing the products of the completely antisymmetric tensors \( \eta_{\mu\nu\rho\sigma} \alpha \beta \gamma \delta \epsilon \) in terms of delta functions, we obtain

\[ b^\gamma \gamma = \left( \frac{\mu^\gamma}{\mu} - \frac{\beta B^\gamma B^\gamma}{\mu - \beta B^2} \right) k^\gamma + \left( \frac{\beta B^\gamma B^\gamma}{\mu - \beta B^2} \right) - \frac{\beta B^\gamma B^\gamma}{\mu - \beta B^2} \right) + \left( \frac{\beta B^\gamma B^\gamma}{\mu - \beta B^2} \right) \right) \] (17)

This expression is already squared in \( k_\mu \) but still has an unknown \( b_\alpha \) term. To get rid of it, one multiplies by \( B_\gamma \) to take advantage of the electromagnetic wave polarization dependence. By noting that if \( B_\gamma b_\gamma = 0 \), one obtains the dispersion relation by separating out the \( k^\gamma k^\gamma \) term; what remains is the \((-\) effective metric. Similarly, if \( B_\gamma b_\gamma = 0 \), one simply divides by \( B_\gamma b_\gamma \) so that by factoring \( k^\gamma k^\gamma \), what results is the \((+\) effective metric. For the case \( B_\gamma b_\gamma = 0 \), one obtains

\[ g^{\gamma\beta}_{\alpha \beta} k_\gamma k_\beta = 0, \] (18)

whereas for the case \( B_\gamma b_\gamma = 0 \), the result is

\[ \left( \frac{\mu^\gamma}{\mu} + \frac{\beta B^\gamma B^\gamma}{\mu - \beta B^2} \right) - \frac{\beta B^\gamma B^\gamma}{\mu - \beta B^2} \right) \right) \] (19)

where \( \mu \) is indicated \( d/dB \); we define \( \beta = 7\alpha_{\mu\nu} / 4\pi B_q^2 \) and \( B^\gamma \equiv B_\gamma / |B_\gamma B_\gamma|^{1/2} \).

From the above expressions we can read the effective metric \( g_{\gamma\beta}^{\alpha \beta} \) and \( g_{\alpha \beta}^{\alpha \beta} \), where the labels “+” and “−” refer to extraordinary and ordinary polarized rays, respectively. To determine the redshift, we need the covariant form of the metric tensor obtained from the expression \( g_{\mu\nu}g^{\alpha\beta} = \delta^{\alpha\beta}_{\mu\nu} \), which reads

\[ g_{\mu\nu} = g_{\mu\nu}, \] (20)

and

\[ g_{\mu\nu}^{\gamma\beta} = \left( 1 + \frac{\mu^\gamma B}{\mu} + \frac{\beta B^\gamma B^\gamma}{\mu - \beta B^2} \right)^{-1} \] (21)
The function $\mu' B / \mu$ can be expressed in terms of the magnetic permeability of the vacuum and is given as

$$\frac{\mu' B}{\mu} = 2 \left( 1 - \frac{1}{\mu} \right).$$  \hspace{1cm} (22)

In the particular case that we are focusing on, both the emitter and observer are in inertial frames, that is, $V^\mu = \delta_0^\mu / (g_{00})^{1/2}$; therefore, the “+” mode component of the effective metric above becomes

$$g^{\text{eff}}_{00} = \frac{g_{00}}{1 + B B^2 / (\mu - B^2)}. \hspace{1cm} (23)$$

The general expression for the redshift is then given as

$$\frac{\nu_B}{\nu_A} = \frac{\lambda_B}{\lambda_A} = \left( \frac{g_{00}(\nu)}{g_{00}(\nu_0)} \right)^{1/2}, \hspace{1cm} (24)$$

or

$$z = \frac{\lambda_B - \lambda_A}{\lambda_A} = -1,$$  \hspace{1cm} (25)

where $g_{00}(\nu)$ and $g_{00}(\nu_0)$ stand for the time-time effective metric components at emission and observation, respectively. Hence, for observations very far from the star, the redshift can be approximated as

$$z + 1 = \left( \frac{g_{00}(\nu_0)}{g_{00}(\nu)} \right)^{1/2} = \left( \frac{1 - 2GM / c^2 R}{1 + \beta B^2 / (\mu - B^2)} \right)^{1/2}, \hspace{1cm} (26)$$

$$z + 1 \approx \left( \frac{1 - 2GM / c^2 R}{1 + \beta B^2} \right)^{1/2} = \left( \frac{1 - 0.3M / R}{1 + 0.19B^2_{15}} \right)^{1/2}, \hspace{1cm} (27)$$

where $M$ is the star mass in units of $M_\odot$, $R$ is its radius in units of 10 km, and $B_{15}$ is the $B$ field in units of $10^{15}$ G.

Note, however, that in the present case the correction on the gravitational redshift $z$ by the nonlinear contribution of the magnetic field does depend on the polarization $b^h$ of the emitted photons. Therefore, it is straightforward to verify that because of the appearance of the two different effective metrics in equations (21) and (20), which exhibit the phenomenon of birefringence, one may in principle disentangle the two components of the total pulsar surface redshift by a direct observation.

4. DISCUSSION AND CONCLUSION

The 5.0 keV feature discovered with the Rossi X-Ray Timing Explorer is strong, with an equivalent width of $\sim 500$ eV and a narrow width of less than 0.4 eV (Ibrahim et al. 2002, 2003). When these features are viewed in the context of accretion models, $M/R > 0.3 M_\odot$ km$^{-1}$, which is inconsistent with NSs, or a low $B \sim (5-7) \times 10^{11}$ G is required, which is said not to correspond to any SGR (Ibrahim et al. 2003). In the magnetar scenario, meanwhile, the features are plausibly explained as being ion-cyclotron resonances in an ultralong $B$ field, $B_{\text{sup-crit}} \sim 10^{15}$ G, whose energy and width are close to model predictions (Ibrahim et al. 2003). According to Ibrahim et al. (2003), the confirmation of this finding would allow estimation of the gravitational redshift, mass, and radius of the supposed magnetar SGR 1806–20.

Here we point out that this feature could also be due to NLED in the same strongest $B$ field of SGR 1806–20, as suggested by equation (26). To obtain our conclusion, we used $B \sim 5 \times 10^{15}$ G, which is within the uncertainty of the $B$ field strength estimate from $P$ and $\dot{P}$ and the likely $B$ field near-surface multipole structure, as suggested by various authors in the field. In particular, Duncan (1998) interpreted the 23 ms global oscillations observed in the "magnetar-like" object SGR 0526–66 as being a fundamental toroidal mode, assuming a field $B \sim 4 \times 10^{15}$ G lies underneath the star crust. Other authors hint at the coexistence of poloidal configurations as well. For such fields, the cyclotron viewpoint could be sustained only whenever the dipole component is the dominant emission mechanism. If this were the case, no conclusive assertion about the $M/R$ ratio of the compact star glowing in SGR 1806–20 could be consistently made, since the NLED redshift might well be mimicking the standard gravitational redshift associated with the pulsar surface. More fundamentally yet, if new spectral lines were measured with high precision (as in Cottam et al. 2002) from heavy elements in a compact object with fields $B \gtrsim 10^{15}$ G, then the $\Delta \gamma \geq 10$ $z$-correction brought by NLED would prove critical regarding both its $M/R$ ratio and its EOS.

As a worthy remark, the attentive reader must realize that there exists a hidden divergence in the effective metric here derived. It appears when the magnetic field strength achieves values around $B \longrightarrow B_s \sim 10^{3.5}$ G. We stress that such a divergence is inherent to the sort of approximation we are using, that is, the Heisenberg & Euler (1936) Lagrangian, which is not an exact one, of which we take into account only the first term in its expansion. We advance, meanwhile, that such divergence can be removed by taking advantage of a very different sort of nonlinear electrodynamics Lagrangian, like the exact one introduced by Born & Infeld (1934). This new approach is the subject of a forthcoming paper.

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