Opacity effect on core-shift and the spectral properties of jets

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ABSTRACT

Context. There is theoretical and observational evidence that the jet core position changes with frequency. However, the core position for a given frequency may vary with time in the case of flares or for a precessing jet.

Aims. In this work, we want to explore the changes in core position as a function of frequency, magnetic field alignment, relativistic electron density, and jet inclination angle.

Methods. We use a physical model of a synchrotron-emitting jet. Two cases of the jet are analysed, namely with magnetic field parallel and perpendicular to the jet axis. The evolution of the related spectrum is monitored over the radio band.

Results. We find that a smaller jet inclination angle or a higher electron density causes the jet core position to move downstream of the jet and we demonstrate that this displacement of the core along the jet gives rise to a spectral flattening.

Key words. opacity – astrometry – stars: jets – galaxies: jets – radio continuum: general – black hole physics

1. Introduction

Relativistic jets are produced when the inflowing, accreting plasma around a black hole gets expelled and collimated by magnetic forces (e.g., Meier et al. 2001). When a jet is mapped at high resolution with an interferometer at frequency ν, the surface of the jet where optical depth τν ≈ 1 corresponds to the peak flux in the map, and is referred to as the core (Blandford & Königl 1979). The position of the core is a function of frequency, and this is known as the core-shift effect. This characteristic feature of the jet is supported by very long baseline interferometry (VLBI) observations of X-ray binaries (e.g., Paragi et al. 1999, 2013; Massi et al. 2012; Rushton et al. 2012) and active galactic nuclei (AGNs) (e.g., Lobanov 1998; O’Sullivan & Gabuzda 2009; Hada et al. 2011; Sokolovsky et al. 2011).

The core-shift effect is essential for both astrophysical and astrometric applications. It can help us to understand the physical conditions of the jet, such as the magnetic field and the distance of the core from the base of the jet (Lobanov 1998; Hirotani 2005). Moreover, it affects the precise astrometric measurements performed by VLBI (Riaja et al. 2010; Porcas 2009). The position of the core can depend on the activity state of the source (e.g., Kudryavtseva et al. 2011), with recent studies revealing its variability with time (Rushton et al. 2012; Paragi et al. 2013; Niinuma et al. 2015; Lisakov et al. 2017; Plavin et al. 2019), an effect that, if not taken into account, can further affect the accurate astrometric observations of sources.

From a theoretical point of view, the relationship between core-shift and spectral characteristics was derived by Königl (1981) for synchrotron emission by electrons with a power-law distribution of energy. Using synchrotron emission equations and imposing optical depth equal to unity, the position, l, of the core at various frequencies results in a core-shift of ν−1/k, with k depending on the spectral index, magnetic field, and particle density distribution (Königl 1981).

In the present work, we aim to explore the core position in the jet parameter space, that is, as a function of frequency, magnetic field configuration, relativistic electron density, and jet inclination angle, l ≡ (ν, B, k, η), and to monitor the corresponding evolution of the related spectrum. The importance of adding the inclination angle, η, results from the shown anisotropy of jet emission with the inclination (Zdziarski et al. 2016) and the evidence from observations and simulations that the spectral index α can vary by changing the orientation of the jet (Fine et al. 2011; DiPompeo et al. 2012, see their Fig. 1). We want to test here if the core position shows similar anisotropies to those shown for jet emission and spectral index. In particular, in this paper we analyse jets in microquasars. Our code (Massi & Torricelli-Ciamponi 2014), based on the synchrotron-emitting conical jet by Kaiser (2006), provides us with the relevant information regarding the key parameters at different frequencies, that is, the position of the core (lν)1 along the jet where τ = 1 and also the peak flux. In Sect. 2, we describe the geometry of our synchrotron-emitting jet model. In Sect. 3, we present our results. Sections 3.1 and 3.2 describe the trend of the core position and spectral index, respectively, in response to variation in inclination angle, the electron density of the jet, and different magnetic field configurations. In Sect. 3.3, the relationship between core position and spectral index is analysed. Finally, we present our conclusions in Sect. 4.

2. Jet model

Here, we describe the geometry that we adopt for the self-absorbed stellar radio jet. The jet makes an angle η with respect to the line of sight (LOS) and has a half-opening angle ξ (see Fig. 1). We choose η such that ξ < η ≤ 75° (Table 1). Miller-Jones et al. (2006) calculated the half-opening angles of different X-ray binaries. Using the values from Table 1 provided by these latter authors, that is, for GRS 1915+105, Cygnus X-3, GRO J1655-40, XTE J1550-564, H 1743-322 and IXRS J001442.2+580201, along with SS 433 (Marshall et al. 2013) and an updated value for Cygnus X-1 (Tetarenko et al. 2019), we find the
LOS. The optical depths at $x$ represents the path in a stratified jet as viewed by an observer along the LOS. The optical depths at $A$ and $O$ are $\tau = 0$ and $\tau = \tau_{\text{max}}$, respectively.

average half-opening angle of eight different X-ray binaries to be $\xi \sim 3.5$. We use this value for our model.

2.1. Geometry of the jet

We follow the model for adiabatic jets provided by Kaiser (2006) and described in Massi & Torricelli-Ciamponi (2014). We consider a conical jet with the jet propagating along the $x$-axis. All quantities are considered to be radially constant and change only along the $x$-axis. Any position along the $x$-axis is defined as $x = x_0 f$ with the jet’s base at position $x_0$, as shown in Fig. 1. The parameter $l$ is a dimensionless coordinate and is equal to unity at the base of the jet. Adopting a distance of 2 kpc and the configuration of its magnetic field. From Table 1 of Kaiser (2006) discovered that the jet maintains a parabolic streamline over a range in size scale equal to $10^{5} R_{\text{sch}}$, while further downstream the jet takes on a conical shape. This result is consistent with the size of the acceleration/collimation region of a jet that is around $10^{4} R_{\text{sch}}$ following Marscher (2006). Thus our jet model starts after the acceleration zone.

We parameterize the evolution of the magnetic field of the jet as $B = B_{0} l^{-\alpha_{B}}$, where $B_{0}$ is the magnetic field strength at the base of the jet and $\alpha_{B}$ can take different values depending on the configuration of its magnetic field. From Table 1 of Kaiser (2006), for a purely parallel magnetic field configuration, $\alpha_{B} = 2$ and for a purely perpendicular magnetic field configuration, $\alpha_{B} = 1$. We consider both of these cases in this work.

The number density of the relativistic electrons producing synchrotron radiation has a power-law distribution in energy,

$$N_{\text{rel}} = \int_{E_{\text{min}}}^{E_{\text{max}}} \kappa E^{-p} \text{d}E,$$

where $E$ is the energy of the electrons. The power-law index $p$ influences the flux density $S \propto \nu^{p}$, with spectral index $\alpha$ related to $p$ as $\alpha = -(p-1)/2$ (Longair 1994). For our model, we use the parameters as given in Table 1. We assume the electron density distribution along the jet to evolve as $\kappa = \kappa_{0} l^{-\alpha_{\kappa}}$, where $\kappa_{0}$ is a parameter related to the relativistic electron density at the base of the jet and $\alpha_{\kappa} = 2(2+p)/3$ as derived by Kaiser (2006) from the mass conservation condition in the case of adiabatic losses. The spectral index range for optically thin emission is $-1 < \alpha < -0.2$ (Fender 2001) which corresponds to $1.4 < p < 3$. For $p = 1.8$, $\kappa_{0}$ is expressed in units of $[\text{erg}^{-0.8} \text{cm}^{-3}]$ throughout the paper. The integral in the above equation is performed from $E_{\text{min}}$ to $E_{\text{max}}$, where $E_{\text{min}}$ and $E_{\text{max}}$ are the minimum and maximum energy of the electron distribution, respectively, and are related to the Lorentz factor, as discussed in Sect. 2.3.

2.2. Synchrotron emission and optical depth

In the framework of the jet model described above and developed by Massi & Torricelli-Ciamponi (2014) for varying inclination angles, here we derive the flux density and opacity of the jet model in terms of inclination angle $\eta$ and length of the jet, $l$. The perpendicular direction to the surface of the approaching jet makes an angle $90^\circ - (\eta + \xi)$ with respect to the LOS, whereas it makes an angle $90^\circ - (\eta + \xi)$ for the receding jet. Thus, the flux emanating from the approaching jet is given by

$$S_{\alpha} = \int_{0}^{l} r_{0} x_{0} I_{\nu}(\eta, l) \sin(\frac{\xi}{\cos \eta}) \frac{\text{d}l}{D^2},$$

where $D$ is the distance of the source, and $I_{\nu}$ is the intensity of the approaching jet. The integral is performed over the length of the jet from $l = 1$ to $L$. We get the same expression for the receding jet flux ($S_{\alpha}$), but the sign of the angle is reversed. In this work, we consider a mildly relativistic jet with Doppler factor $\approx 1$ because we want to study the impact of opacity on jet properties due to the change in $\kappa_{0}$ and $\eta$. On the other hand, a greater Doppler factor increases the opacity (e.g., Eq. (10) in Finke 2019) and only enhances the studied effects, especially at small inclination angles.

For inclination angles $\xi < \eta < 90^\circ$, the observer views a stratified jet with varying plasma conditions along the LOS. For such a case, the total optical depth will have a contribution from each layer along the LOS. Therefore, using the radiative transfer equation, the intensity $I_{\nu}$ emanating from the surface of the jet and in the direction of the observer at an angle $\eta$ is given as

$$I_{\nu}(\eta, l) = \int_{0}^{l} \nu_{0} e^{-\tau_{0}/\cos \eta} \frac{\text{d}l}{D^2},$$

From Longair (1994) and using Kaiser’s notation, $J_{\nu}$ is the emissivity per unit volume, and $\chi_{\nu}$ is the absorption coefficient given by

$$J_{\nu} = J_{0} \nu^{-(p-1)/2} \Gamma^{1-(p+1)/2} \kappa_{0} \nu^{1/2} \, \text{W m}^{-3} \text{Hz}^{-1},$$

and

$$\chi_{\nu} = \chi_{0} \nu^{-(p+4)/2} \Gamma^{-(p+2)/2} \kappa_{0}^{-1} \, \text{m}^{-1},$$

where,

$$J_{0} = 2.3 \times 10^{-25} (1.3 \times 10^{37})^{(p-1)/2} \kappa_{0} B_{0}^{(p+1)/2} \kappa_{0},$$

and

$$\chi_{0} = 3.4 \times 10^{-9} (3.5 \times 10^{18})^{p} b(p) B_{0}^{(p+2)/2} \kappa_{0},$$
with \( a(p) \) and \( b(p) \) being constants as given in Longair (1994). The optical depth is given by (as in Kaiser 2006)

\[
\tau_l = \tau_0 l^C,
\]

where \( C = 1 - a_3 - (p + 2)\alpha_2/2 \). Using Table 1 from Kaiser (2006), all possible values of the exponent in Eq. (8) will be negative, that is, \( C = -5.3 \) and \( C = -3.4 \) for parallel and perpendicular magnetic field, respectively. Therefore, the optical depth is maximum at the base of the jet \( l = 1 \) and is given by

\[
\tau_0 = \tau(l = 1) = \frac{\chi_{0}(0)}{[1 - a_3 - (p + 2)\alpha_2/2]} \gamma^{-(p+4)/2}.
\]

The integral in Eq. (3) is performed over the optical depth along the LOS (along line segment \( AO \) in Fig. 1) from \( \tau = 0 \equiv \tau_1 \) (if it is assumed that no emission or absorption occurs between the jet and the observer) to \( \tau_{\text{end}}(l) \), which represents the maximum optical depth of the jet, that is, until the position at \( O \). For an approaching jet, the optical depth \( \tau_{\text{end}}(l) \) is given as

\[
\tau_{\text{end}}(l) = \tau_0 l^C \left[ \frac{\tan \eta - \tan \xi}{\tan \eta + \tan \xi} \right]^C - 1.
\]

See Eq. (A.5) in the Appendix of Massi & Torricelli-Ciamponi (2014) for a comprehensive description.

To find the position along the jet where the optical depth attains a specific value \( \tau = \tau' \cos \eta \) (see Eq. (3)), Eq. (10) becomes \( \tau_0 l^C f(\eta, \xi) = \tau \cos \eta \). Re-arranging the equation, we have

\[
l(\tau) = \left( \frac{\tau \cos \eta}{\tau_0 \left[ \frac{\tan \eta - \tan \xi}{\tan \eta + \tan \xi} \right]^C - 1} \right)^{1/C}.
\]

As the core position is defined where the optical depth \( \tau = 1 \), we can use Eq. (11) to compute it along the jet \( (l_{\text{core}}) \) for any given frequency \( \nu \), inclination angle \( \eta \), relativistic electron density (dependent on the parameter \( \kappa \); see Sect. 2.3), and magnetic field configuration (dependent on the parameter \( a_3 \)).

We check the consistency of our expression for \( \tau \) with that of Eq. (1) in Lobanov (1998), which was derived in the limit of a small jet opening angle. This latter equation, with \( N = N_1 (r_1/\gamma)^{\alpha}, B = B_0 (r_1/\gamma)^{\eta}, r_1 = 1 \, \text{pc}, \) and \( \alpha = -0.5 \), i.e., \( s = 1 - 2\alpha = 2 \), in the limit of mildly relativistic flows, reads as

\[
\tau(\kappa)_{\text{Lobanov}} = 6.1 \times 10^{24} N_1 B_0^2 r^{-2(\alpha n - 1)} \nu^{-3} |\phi| / \sin(\theta).
\]

Table 1. Parameters used to study the characteristics of the adiabatic jet model.

| Symbol | Description | Value |
|--------|-------------|-------|
| \( \xi \) | Half-opening angle. | 3.5° |
| \( \eta \) | Jet inclination angle. | 5°−75° |
| \( \kappa_0 \) | Parameter related to the relativistic electron density at the base of the jet (erg\(^{0.8}\) cm\(^{-3}\)). | 6 and 0.6 |
| \( x_0 (B_0) \) | Position of the base of the jet for parallel magnetic field (cm). | 6.0 × 10\(^{12}\) |
| \( x_0 (B_\perp) \) | Position of the base of the jet for perpendicular magnetic field (cm). | 4.7 × 10\(^{12}\) |
| \( r_0 (B_0) \) | Radius of the cone at the base of the jet for parallel magnetic field (cm). | 3.7 × 10\(^{11}\) |
| \( r_0 (B_\perp) \) | Radius of the cone at the base of the jet for perpendicular magnetic field (cm). | 2.9 × 10\(^{11}\) |
| \( B_0 (B_0) \) | Magnetic field strength at the base of the jet for parallel magnetic field (Gauss). | 1.1 |
| \( B_0 (B_\perp) \) | Magnetic field strength at the base of the jet for perpendicular magnetic field (Gauss). | 9.6 |
| \( \rho \) | Power-law index. | 1.8 |
| \( D \) | Distance of the source (kpc). | 2.0 |

From Eq. (11), our expression for \( \tau \) is

\[
\tau = 4.1 \times 10^{24} b(p) \kappa_0 x_0 \nu^{-3} \left( \frac{\tan \eta - \tan \xi}{\tan \eta + \tan \xi} \right)^C - 1.
\]

This equation can be compared with Eq. (1) from Lobanov (1998) (here Eq. (12)), for small \( \xi \) and \( \eta > \xi \) (Konigl 1981).

In this limit, the angular part can be approximated as

\[
\left[ \frac{1 - 2 \frac{\xi}{\tan \eta} \nu^C - 1}{\cos \eta} \right] \approx \left[ \frac{1 - 2 C \frac{\xi}{\tan \eta} \nu^C - 1}{\cos \eta} \right] = -2C \frac{\xi}{\sin \eta} \nu^C.
\]

Setting the normalization distance \( x_0 = 1 \, \text{pc} \) and the exponent \( p = 2 \), as in Lobanov, with \( b(p) = 2.069 \) (Longair 1994),

\[
\tau = 6.8 \times 10^{44} \kappa_0 B_0^2 r^{-2(\alpha n - 1)} \nu^{-3} (\xi/\sin \eta).
\]

This expression is the same as Eq. (12), taking into account the correspondence between our parameters and those of Lobanov (1998), \( a_3 = m, a_3 = n, \xi = \Phi, \) and \( \eta = \theta \) and the different approximations used for synchrotron absorption; in fact we used the formalization of Longair (1994) instead of that of Blumenthal & Gould (1970) and Rybicki & Lightman (1979), as in Lobanov (1998).

2.3. Model parameter \( \kappa \) and relativistic electron density

To understand the physical implications of the parameter \( \kappa \), in this section, we calculate the relativistic electron density for Cygnus X-1, as it is the best example of a system with a flat radio spectrum over the entire radio band until millimetre range (Fender 2001). The range in energy for which these relativistic electrons exist is required to compute the relativistic electron density, and therefore,

\[
E_{\text{min}} = \gamma_{\text{min}} m c^2 \leq E \leq \gamma_{\text{max}} m c^2 = E_{\text{max}},
\]

where \( E_{\text{min}} \) and \( E_{\text{max}} \) is the minimum and maximum energy of the electron distribution, respectively and \( y \) is the Lorentz factor. As for a synchrotron process, \( \gamma_{\text{min}} \gg 1 \) (Dulk 1985), we set \( \gamma_{\text{min}} = 10 \). Integrating Eq. (1) gives

\[
N_{\text{rel}} = \frac{\kappa}{1 - p} \left( E_{\text{max}}^{1-p} - E_{\text{min}}^{1-p} \right). \tag{17}
\]

For \( \gamma_{\text{max}} \gg \gamma_{\text{min}} \), the number of relativistic electrons per cubic centimetre at the base of the jet \( (l = 1) \) is given by

\[
N_{\text{rel}} = \frac{\kappa_0}{p - 1} (\gamma_{\text{min}} m c^2)^{-1} \nu^p. \tag{18}
\]
For Cygnus X-1, the model of Kaiser (2006) with parameters from his Table 2 yield a relativistic electron density $N_{\text{rel}} \approx 7 \times 10^4 \text{ cm}^{-3}$. We therefore chose $k_0 = 6 \text{ erg}^{1/2} \text{ cm}^{-3/2}$ for our jet model, which corresponds to the relativistic electron density $N_{\text{rel}} \approx 7 \times 10^4 \text{ cm}^{-3}$ as for Cygnus X-1. We also study an intermediate case with $k_0 = 0.6 \text{ erg}^{1/2} \text{ cm}^{-3/2}$ with relativistic electron density $N_{\text{rel}} \approx 7 \times 10^5 \text{ cm}^{-3}$. In the latter part of Sect. 3, we generalise our study for $k_0 = 0.06-60 \text{ erg}^{1/2} \text{ cm}^{-3}$.

3. Results

In this section, we use the parameters of Table 1 to analyse the characteristics of the above-described model of a self-absorbed jet as a function of the angle the jet axis makes with respect to the LOS and the injected relativistic electrons. We study the core-shift and spectral properties of the jet for both parallel and perpendicular magnetic field configurations.

3.1. Core-shift

Figures 2 and 3 show the core position ($l_{r=1}$) for different frequencies (2.2, 5, 7, 8.3, and 15 GHz) for parallel and perpendicular magnetic field configurations. The core positions are plotted for different inclination angles, $\eta = 5.2^\circ$ and $7.6^\circ$. Comparing Figs. 2a with 2c, we see that for a jet with the parallel magnetic field, the core is farther away from the base of the jet than that for a perpendicular magnetic field. For instance, the 5 GHz core position is at $l_{r=1} \approx 2.8$ for the parallel magnetic field (Fig. 2a) and at $l_{r=1} \approx 2.0$ for the perpendicular magnetic field (Fig. 2c).

Previous studies of a compact jet reveal that the core position is a function of the observing frequency, $l_{r=1} \propto \nu^{-1/k}$, where $k_0 = (3 - 2a)\xi_0 + 2\alpha - 2)/(5 - 2\alpha)$. The power index $k_0$ depends on the magnetic field, particle density, and electron energy spectrum (Konigl 1981). Therefore, first, we fit a power-law function of the form $l_{r=1} = a\nu^{-1/k_0}$ (blue lines in Figs. 2, 3) where $a$ is a constant depending on the fit. For the performed fits, for the parallel magnetic field, $k_0' = 1.8$ and for the perpendicular magnetic field, $k_0' = 1.2$, irrespective of the angle $\eta$. To calculate the theoretical values of $k_0$, we then use the spectral index values derived from the jet emission calculation for the given inclination angle.

For instance, for $k_0 = 0.6$ and $B_\parallel$, we have $a = -0.13$ for $\eta = 5.2^\circ$ (top panel of Fig. 7). Using $a_2$ and $a_3$ from Sect. 2.1, we get $k_0 = 1.8$. Similarly, for $B_\perp$, we have $a = -0.34$ for $\eta = 5.2^\circ$ (top panel of Fig. 8) and we get $k_0 = 1.2$. Thus, we see that the values of $k_0'$ found by the fit of the derived core positions are equal to the theoretically calculated values of $k_0$ when using the derived spectral index for the same inclination angle.

We also compare our results with Eq. (11) in Lobanov (1998) where we set the bulk Lorentz factor equal to unity and redshift $z = 0$. Assuming equipartition between jet particle and magnetic field energies, the equation predicts the core shift $\Delta r$ at two frequencies in a jet with synchrotron luminosity calculated using the model of Blandford & Königl (1979). To compare our work with Lobanov’s equation we chose the set of parameters of the perpendicular magnetic field (i.e., $a_2 = 1, a_3 = 2.5, p = 1.8$) which are more compatible with Lobanov’s set of parameters (i.e., $m = 1, n = 2, s = 2$). For $\eta = 5.2^\circ, \xi = 3.5^\circ$, using the values from Table 1 for perpendicular magnetic field, namely $x_0 = 4.7 \times 10^{12} \text{ cm}$ and $B_0 = 96 \text{ Gaus}$, and a synchrotron luminosity $L_{\text{sync}} = 1.7 \times 10^{34} \text{ erg s}^{-1}$ (following Eq. (23) in Blandford & Königl 1979), Lobanov’s Eq. (11) results in a shift between 2 GHz and 8 GHz of $\Delta r = 0.015 \text{ mas}$. In addition, one has to consider that our results compare with Lobanov’s results for small $\xi$ and $\eta > \xi$, as shown in Sect. 2.2. Figure 4 shows the ratio between our general equation (Eq. (11)) on which our plots of Figs. 2 and 3 are based, and its approximation for small $\xi$ and $\eta > \xi$. Whereas the ratio tends to 1 for larger angles, at $\eta = 5.2^\circ$ the ratio is equal to 3.3. Taking into account this correction factor of 3.3, the predicted $\Delta r = 0.015 \text{ mas}$ rises to 0.05 mas, well within the range of 0.04 mas–0.07 mas obtained from Figs. 2c and 3c. Namely, the two values correspond to $k_0$.
of 0.6 (Fig. 2c) and 6.0 (Fig. 3c), and from the equipartition the value for \( \kappa_0 \) from \( B_0 = 9.6 \) Gauss is \( \kappa_0 = 3.6 \).

In addition to verifying the known core-shift relationship with frequency and agreement of our findings with theoretical predictions, we obtain a new result. Figure 2 shows that for the same magnetic field configuration, increasing the inclination angle \( \eta \) shifts the core position of a given frequency closer to the base of the jet. For example, for the parallel magnetic field in Figs. 2a,b, the core position for 5 GHz emission is at \( l_{\tau=1} \approx 2.8 \) for \( \eta = 5.2^\circ \) and decreases to \( l_{\tau=1} \approx 1.4 \) for \( \eta = 7.6^\circ \). Similarly, for perpendicular magnetic field (see Figs. 2c,d), let us consider again the core position for 5 GHz emission, which is at \( l_{\tau=1} \approx 2.0 \) for \( \eta = 5.2^\circ \) but for \( \eta \geq 7.6^\circ \) the optical depth becomes \( \tau < 1 \) all along the conical part of the jet. This shows that the core-shift depends not only on the frequency but also on the jet’s inclination angle.

Another important result from our analysis is shown in Fig. 3, which shows the changes in core position for a higher relativistic electron density, that is, for \( \kappa_0 = 6 \). We see that for higher relativistic electrons, the core moves downstream, that is, away from the base of the jet. Let us revisit the core position for 5 GHz. In Fig. 2a, the core position for 5 GHz is at \( l_{\tau=1} \approx 2.8 \) for \( \kappa_0 = 0.6 \), whereas it increases to \( l_{\tau=1} \approx 4.2 \) for \( \kappa_0 = 6 \) as shown in Fig. 3a.

The change in the core position for different inclination angles \( \eta \) is shown in Fig. 5 for 5 GHz and 7 GHz for the parallel magnetic field. The position of the core can be seen to decay with increasing inclination angle. The last angle for 5 GHz where the core is present in the conical jet is \( \eta \approx 9.1^\circ \) and is shown by a blue circle. For larger angles, the emission of 5 GHz becomes optically thin all along the conical jet. This holds true for the particular case of \( \kappa_0 = 0.6 \). As shown in Fig. 6, at larger electron densities, the core persists at much larger angles. In Fig. 5, for 7 GHz, the core is present in the conical jet until \( \eta \approx 7.6^\circ \). We performed a fit with an exponential function of the form, \( l \propto e^{\eta/\eta_0} \), as shown by dashed lines where \( q \) is a constant depending on the fit.

In Fig. 6, we study the presence of the core in the conical jet for each frequency and different relativistic electron densities (ranging from \( \kappa_0 = 0.06 \)–60.0). For a given frequency, each
position marked in the (η, κ₀)–plane refers to the last inclination angle for which the core is present at that frequency, before moving below the base of the jet. This way, the blue circle at (η, κ₀) = (9.1°, 0.6) for 5 GHz represents the same condition as the blue circle in Fig. 5. The dependence of κ₀ on η is fitted with a power-law function for all frequencies. The curves divide the (η, κ₀)–plane into two distinct regions: any position on and above the curve represents the presence of the core in the conical jet for a given frequency; for example, for 2 GHz, it is the region above the blue shaded area. For instance, as shown in the top panel of Fig. 6, for the parallel magnetic field configuration and κ₀ = 0.06, for a frequency of 2 GHz, the core is present until an inclination angle of η ≈ 9.1°. On the other hand, for 15 GHz, the core is present until 5.2° at this relativistic electron density. To observe the core at a greater inclination angle, we would need to increase the number of relativistic electrons in the jet. Therefore, for the core to be present in the conical jet for 2 GHz at ≈27°, we need an increased electron density with κ₀ = 0.6. The same applies to the perpendicular magnetic field configuration (bottom panel of Fig. 6).

Very long baseline array (VLBA) astrometry of the microquasar LS I +61°3303 (Wu et al. 2018) can be better understood in light of our results. A sequence of ten observations was spaced 3–4 days apart and the core of the precessing jet was expected to follow a regular path as well, that is, with regularly spaced astrometric points. Instead, VLBA astrometry (Wu et al. 2018) shows that the positions of the cores are displaced from the jet base, with the largest offsets for those observations corresponding to the smallest inclination angles. A future quantitative comparison of our determined angular distances projected on the sky with observed astrometry will make it possible to constrain important parameters such as κ₀ and B₀ for LS I +61°3303 and other microquasars.

### 3.2. Spectral index analysis

Figures 7 and 8 show the spectra of the jet for different inclination angles for parallel and perpendicular magnetic field cases, respectively. We show the power-law fit required to get the spectral index α only for higher frequencies, i.e., excluding 2 GHz. This is because the 2 GHz core survives the largest inclination angle, and including it would bias the fit, resulting in large uncertainties for the spectral index. For instance, as shown by the blue dashed line, the spectra between 2 GHz and 5 GHz are inverted, contrary to the spectra between other frequencies (top panel of Fig. 7).

Considering the spectra from 5 to 15 GHz in the top panel of Fig. 7 for κ₀ = 0.6, the spectral index indicates optically thin emission (α < 0) at all inclination angles while becoming more negative for increasing angles. For the same electron density, but for the perpendicular magnetic field (top panel of Fig. 8), the emission is again optically thin. However, the spectral index
is more negative than for the corresponding angles of the parallel magnetic field. When we increase the relativistic electron density to \( k_0 = 6 \) (bottom panel of Fig. 7), the spectra become inverted \((\alpha > 0)\) and only become flat around 73°. For the perpendicular case (bottom panel of Fig. 8), the spectra are initially flat at 5.2° and later become optically thin. Our analysis shows that if we increase the relativistic electron density, the spectra become flat or even inverted. Thus we see that the spectral index changes significantly depending on the inclination angle, magnetic field configuration, and relativistic electron density, similar to core position changes as shown in the previous section.

We compare our results with 6 yr of radio observations of LS 1 +61°303. Spectral index data for LS 1 +61°303 are shown in Fig. 5 of Massi & Kaufman Bernadó (2009). The spectral index versus orbital phase presents a recurrent pattern: \( \alpha \geq 0 \) twice along the orbit, during the maximum of the long-term flux density modulation where \( \Theta = 0.0 - 0.1 \), and \( \Theta = 0.9 - 1.0 \), i.e., for ejections at small inclination angles (Massi & Torricelli-Ciamponi 2014). The trend changes and evolves to \( \alpha < 0 \) along the orbit during the minimum of the long-term flux density modulation where \( \Theta = 0.4 - 0.5 \), i.e., for ejections at the largest inclination angles (Massi & Torricelli-Ciamponi 2014).

### 3.3. Relationship between core position and spectral index

We analysed the core positions and spectral indices in previous sections by changing the inclination angles and relativistic electron density. In this section, we study their mutual relationship.

In Fig. 9a, we show the change in the spectral index between 5 GHz and 7 GHz for different inclination angles. For \( k_0 = 0.6 \), we see that the spectrum is inverted for angles below \( \eta = 19° \) and becomes optically thin from \( \eta = 19.1° \) (marked by the green square). As noted above, the core position at the jet base, \( l_{\tau=1} = 1 \), is present until \( \eta = 9.1° \) for 5 GHz (blue circle in Figs. 5 and 9a) and until \( \eta = 7.6° \) for 7 GHz (red circle in Fig. 5). This is different from the expected angle of \( \eta = 19.1° \). This means that even though the core is not present, \( \alpha \) still remains positive near the base of the jet. For the spectrum to evolve from inverted (i.e., \( \alpha > 0 \)) to optically thin (i.e., \( \alpha < 0 \)), the optical depth must fulfil the condition \( \tau \ll 1 \) all along the jet, including at the base of the jet. We quantified this value \( \tau \ll 1 \) with our code. For that purpose, we performed several iterations of the code assuming different values for \( \tau = \tau' / \cos \eta \) (see Eq. (3)). At each iteration, we checked for the spectral index transition from \( \alpha > 0 \) to \( \alpha < 0 \). We find that for our jet model, the transition occurs when \( \tau' / \cos \eta = 0.08 \) at the base of the jet \((l = 1)\), confirming that the condition for an optically thin jet is \( \tau = \tau' / \cos \eta \ll 1 \).

This implies that the transition of \( \alpha \) to negative values on the jet surface does not happen at \( l_{\tau=1} = 1 \), that is, at the core at the base of the jet, but at \( l_{\tau=0.08} = l(\tau' / \cos \eta = 0.08) = 1 \), that is, when the jet becomes completely optically thin until the base. Applying the condition \( \tau = 0.08 \) in Eq. (11), for \( k_0 = 0.6 \), the optically thin emission begins at \( \eta = 19.1° \) (marked by the green square) as shown in Figs. 9b,c.

### 4. Conclusions

We studied the synchrotron-emitting conical, self-absorbed jet initially proposed by Blandford & Königl (1979), later developed by Kaiser (2006), and finally modified to vary the inclination angle by Massi & Torricelli-Ciamponi (2014). The aim of this study is to explore the core position, \( l \), as a function of the long-term flux density modulation where \( \Theta = 0 \) to optically thin jet is \( \tau = \tau' / \cos \eta \ll 1 \). The green square represents the angle where the emission becomes optically thin. Red and blue circles are the same as in Figs. 5 and 6. (b),(c) Positions of the jet surface where optical depth \( \tau = 0.08 \) for 5 GHz and 7 GHz, respectively, with \( k_0 = 0.6 \). \( l_{\tau=0.08} = 1 \) represents the position on the jet after which the emission becomes optically thin all along the jet, i.e., when \( \tau \ll 1 \).

Fig. 9. (a) Spectral index between 5 GHz and 7 GHz as a function of the inclination angle for \( k_0 = 0.6 \). The green square represents the angle where the emission becomes optically thin. Red and blue circles are the same as in Figs. 5 and 6. (b),(c) Positions of the jet surface where optical depth \( \tau = 0.08 \) for 5 GHz and 7 GHz, respectively, with \( k_0 = 0.6 \). \( l_{\tau=0.08} = 1 \) represents the position on the jet after which the emission becomes optically thin all along the jet, i.e., when \( \tau \ll 1 \).

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frequency, magnetic field alignment, relativistic electron density, and jet inclination angle.

First, we checked that our derived expression for the optical depth, that is, the fundamental equation in our analysis, is equal to the commonly used Eq. (1) in Lobanov (1998). From our jet model results, we find that without changing the inclination angle of the jet or without changing the density of the relativistic electrons, the position of the core for a higher frequency is closer to the base of the jet than for lower frequencies, as expected in light of the synchrotron self-absorption effects (Konigl 1981). We then checked the consistency of our results, specifically paying attention to the obtained position of the cores at various frequencies with the derived core-frequency relationship of the jets, $I \propto \nu^{-1/k_r}$ by Konigl (1981) and also with the derived shift of the core position between two frequencies calculated by Eq. (11) in Lobanov (1998).

Finally, we proceeded to study the core position in the jet at different inclination angles and electron densities. Our results in Figs. 2 and 3 show that for a given frequency, the greater the inclination angle, the closer the core is to the base of the jet. This implies that the core position can vary in an irregular way if the jet is precessing. Therefore, the deviations from a regular path are expected, as indeed traced by astrometric observations by Wu et al. (2018) which are discussed here in Sect. 3.1. As shown in Fig. 3, the core is more displaced from the jet base for higher electron densities. This is consistent with the observations where increased relativistic electron density due to flares shows a displacement of the core position downstream from the jet (Niinuma et al. 2015; Lisakov et al. 2017; Plavin et al. 2019). Figure 6 summarise the results of variations in inclination angle, electron density, and magnetic field configuration. The curves in the $(\eta, \kappa_0)$-plane show where for a given frequency the conical jet becomes entirely optically thin. The corresponding spectra, functions of the same parameters, are shown in Figs. 7 and 8. Comparing the spectral index for different relativistic electron densities, we see that an increase in electron density causes the spectra to become less steep or even inverted. Such results have been confirmed by observations of radio flares (e.g., Lisakov et al. 2017). Moreover, we see that the spectral index varies by changing the jet orientation, with a steep spectrum at large inclination angles getting flatter as the angle decreases (top panel of Fig. 7 and bottom panel of Fig. 8), or a flat spectrum at large inclination angles becoming inverted as the angle decreases (bottom panel of Fig. 7). These results for the relationship between spectral index and inclination angle are consistent with observations, as discussed in Sect. 3.2. Moreover, simulations (Fig. 1 in DiPompeo et al. 2012) show the same trend between $\alpha$ and $\eta$ as in Fig. 9. In Fig. 9a, the spectral index at low inclination begins with a high positive value (inverted spectrum). Increasing the value of $\eta$ decreases $\alpha$ until it crosses the zero level, becomes negative (steep spectrum) and steeper at larger inclination angles. It is worth noting that in our work, we can follow the change of the spectral index with corresponding displacements of the cores along the jet (Figs. 9b,c).

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