Perturbativity and a Fourth Generation in the MSSM

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Abstract

We study an extension of the MSSM with a fourth generation of chiral matter. With this extension no value of $\tan \beta$ allows the theory to stay perturbative up to the GUT scale. We suggest one model with extra vector-like states at the TeV scale that allows perturbativity all the way up to the GUT scale.

1 Introduction

The repetition of the quark-lepton families is one of the great mysteries of particle physics. Despite its great success in describing the nature of strong and electroweak (EW) interactions, the Standard Model (SM) does not predict the number of families. What is the principle limiting the number of chiral families? Why not have a fourth generation or even more? The masses of the three observed families have a strong hierarchical pattern. Only the top quark mass ($m_t \simeq 172.6$ GeV) lies close to the EW symmetry breaking scale. This, within the SM, suggests the Yukawa coupling of the top quark should be $\lambda_t \simeq 1$. All remaining Yukawa couplings are suppressed. Thus, with only three observed families, $\lambda_t$ and the three gauge couplings $g_{1,2,3}$ would play an essential role in dynamics upon performing renormalization group (RG) studies. The situation may be modified within a two Higgs doublet SM and MSSM. In these models, due to the parameter $\tan \beta = v_u/v_d$ (the ratio of the

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VEVs of the up type to the down type Higgses $\lambda_b$ and $\lambda_{\tau}$ can also be large ($\sim 1$ for $\tan\beta \approx 60$). How would the picture change if there were a fourth family?

Current lower limits on the masses of the 4th generation fermions at 95\% C.L. are [1]:

$$m_{t'} \geq 220 \text{ GeV} , \quad m_{b'} \geq 190 \text{ GeV} , \quad m_{\tau'} \geq 100 \text{ GeV} , \quad m_{\nu'} \geq 50 \text{ GeV} .$$

(1)

When these masses are translated to the values of their Yukawa couplings, we find the possibility of couplings larger than $\lambda_t$. Moreover, the bound on $m_{\nu'}$ indicates the existence of at least one massive neutrino with mass near the EW scale.

Due to the possible existence of large new Yukawa couplings, a study should be performed and the validity of the perturbative treatment must be examined. As we will show, within the MSSM with a 4th family, there is no value of $\tan\beta$ that allows the perturbativity of the couplings up to the GUT scale. This fact suggests a lower cutoff scale. If there is such a cutoff scale, it should be related to new physics which take care of the self consistent ultraviolet (UV) completion. Can such a completion be constructed? A positive answer would be encouraging for model building as well as for further investigations with various phenomenological implications.

Even without focusing on UV completion of the theory, any extension of the SM or MSSM should be in accord with low energy observables. Some previous works [2]-[5] have discussed the effect of a 4th generation on the EW precision parameters $S$, $T$. These constrain the masses of $t'$ and $b'$ quarks. In agreement with them, we find the effect on the $U$ parameter is well within the $3\sigma$ limits of PDG.

Assuming that the mixings of the fourth family matter with the observed three generations are minimal, most of the constraints come from the self energy diagrams of $W^\pm$ and $Z^0$ gauge bosons. In Ref. [5] it was found that with $m_{t'} - m_{b'} \approx (1 + \frac{1}{2} \ln \frac{M_h}{115 \text{ GeV}}) \times 50 \text{ GeV}$, the new contributions to the parameters $S$ and $T$ get minimized. In particular, with $M_h = 115 \text{ GeV}$ one obtains $m_{t'} - m_{b'} \simeq 50 \text{ GeV}$. Using analytical expressions given in Ref. [4] and the experimentally allowed ranges of $S$, $T$, and $U$ at $1\sigma$ [1]:

$$S = -0.13 \pm 0.10 ,$$
$$T = -0.13 \pm 0.11 ,$$
$$U = 0.20 \pm 0.12 ,$$

(2)

we can derive further constraints on $m_{t'}$ and $m_{b'}$. In Fig. 1 we show the allowed regions for $m_{t'}$ and $m_{b'}$. For these analysis we have allowed $3\sigma$ deviations in Eq. (2).

A fourth generation of chiral matter would also affect the Higgs sector. This will give more interesting insights [6, 7] within a SUSY framework. As is well known, in MSSM the value $\tan\beta \approx 1$ is disfavored due to the LEP lower bound on a lightest CP even Higgs boson mass $M_h \geq 114.4 \text{ GeV}$. In the MSSM, at tree level $M_h^2 = M_Z^2 \cos^2 2\beta$. Taking $\tan\beta \approx 1$, the tree level mass vanishes. Loop corrections are not sufficient to raise $M_h$. When a 4th generation is added, the situation is even more drastic because in order to preserve perturbativity $\tan\beta$ cannot be much greater than 1. This is an additional motivation for new physics.

This leads us to believe that the MSSM with a 4th family should be extended further. In this paper we suggest one such extension with vector like states having masses at the TeV scale. As
an outcome of the proposed model, we obtain perturbativity of the couplings all the way up to the GUT scale with \( \tan \beta \sim 2 \). This avoids the difficulties discussed above, and is promising for the possibility of embedding the whole scenario in a grand unified theory.

The paper is organized as follows. In section 2 we discuss theoretical bounds: problems arising from perturbativity considerations that limit \( \tan \beta \) and implications on Higgs physics. In section 3 we present our model which allows perturbativity of all couplings up to the GUT scale and extends \( \tan \beta \) up to \( \sim 2 \) such that the LEP bound on \( M_h \) can easily be satisfied. The model has extra vector-like states which can be detected at the LHC. The summary of our work and conclusions are presented in section 4.

2 Theoretical Bounds and Some Implications

In this section we discuss bounds coming from theoretical considerations and discuss some implications of theories with a new heavy chiral fermion family.

2.1 Bounds from Tree Level Unitarity

The upper bound on a heavy chiral fermion’s mass comes from the unitarity of scattering amplitudes. We assume that fermion mass is generated through the Yukawa coupling of the fermion with a fundamental Higgs doublet. In this case, for the heavy quark doublet \( Q \) with mass \( m_Q \) the \( Q\bar{Q} \to Q\bar{Q} \) scattering \( J = 0 \) partial wave amplitude at tree level (at energies \( \sqrt{s} \gg m_Q \)) is given
by \cite{8}:

\[ |a_0| \approx \frac{5}{4\sqrt{2\pi}} G_F m_Q^2, \tag{3} \]

and the unitarity requirement \(|a_0| < 1\) gives the upper bound

\[ m_Q^2 < \frac{4\sqrt{2\pi}}{5G_F} \approx (552 \text{ GeV})^2, \tag{4} \]

as was first obtained in \cite{8}. The analogous bound for the leptonic doublet \(L\)

\[ m_L^2 < \frac{4\sqrt{2\pi}}{G_F} \approx (1.23 \text{ TeV})^2, \tag{5} \]

is higher. As we see, the current experimental direct bounds in Eq. (1) are not in conflict with the theoretical upper bounds of (4) and (5) derived at tree level. As we discuss below, the inclusion of loop corrections and the requirement of perturbativity will imply stringent theoretical bounds on Yukawa couplings.

### 2.2 Bounds from Perturbative RGE

Here we focus on MSSM with a 4th generation. The reason for the SUSY framework is twofold. First of all, low scale SUSY is the most appealing extension of SM in order to solve the gauge hierarchy problem. Second, as it turns out, more stringent bounds are obtained in the SUSY setup and for demonstrative purposes it is most useful. The discussed mechanisms (presented in the next section) for solving various problems could be also applied for SM and on two Higgs doublet SM.

The superpotential couplings involving 4th generation matter superfields are

\[ W_4 = \lambda_{t'} q_d h_u + \lambda_{b'} q_d h_d + \lambda_{\tau'} l_d h_d + \lambda_{\nu_{\tau'}} l_N h_u, \tag{6} \]

where \(N\) is a right handed neutrino (complete singlet of MSSM) responsible for the Dirac mass generation of \(\nu_{\tau'}\). Yukawa couplings defined at corresponding mass scales can be expressed as

\[ \lambda_{\nu'}(m_{\nu'}) = \frac{m_{\nu'}}{|1 + \delta_{\nu'}|v \sin \beta}; \quad \lambda_{\nu}(m_{\nu}) = \frac{m_{\nu}}{|1 + \delta_{\nu}|v \cos \beta}; \]

\[ \lambda_{\tau'}(m_{\tau'}) = \frac{m_{\tau'}}{|1 + \delta_{\tau'}|v \cos \beta}; \quad \lambda_{\nu_{\tau'}}(m_{\nu_{\tau'}}) = \frac{m_{\nu_{\tau'}}}{|1 + \delta_{\nu_{\tau'}}|v \sin \beta}; \tag{7} \]

where \(\delta_{\alpha} (\alpha = t', b', \tau', \nu_{\tau'})\) exhibit the 1-loop finite corrections emerging after SUSY breaking \cite{9}. Since we are dealing with large Yukawa couplings\((\sim 2)\), these corrections can be as large as 25% and should be taken into account. For examining the RG perturbativity, one should take the values for masses satisfying the bounds in Eq. (1) and run each Yukawa coupling from the corresponding mass scales up to higher scales. In Ref. \cite{10} this analysis was done with the fourth generation fermion masses smaller than the top mass. This was in accord with the experimental bounds that
existed at that time. They found that if \( \tan \beta < 3 \) all Yukawa couplings could be perturbative up to the GUT scale. Given the current lower bounds on quark and lepton masses, we find this is no longer the case. When one uses the renormalization group equations for evolving the Yukawa couplings from low scale up to higher energy scales, the couplings rapidly grow and blow up. For example for \( \tan \beta = 2 \), \( \lambda_t' \) becomes non-perturbative at about 1 TeV. As \( \tan \beta \) increases, it is more difficult to tame the Yukawa coupling. This is shown in Fig. 2.

![Figure 2: Plotting \( \tan \beta \) vs. \( \Lambda \), the scale at which one of the Yukawa couplings becomes non-perturbative. Dotted line corresponds case with all Yukawa couplings taken into account. For masses we took lowest allowed values from Eq. (1).](image)

We have assumed the validity of perturbative RG for Yukawa couplings \( < 2.5 \). For these analysis we ignored all \( \delta_\alpha \)'s (i.e. set \( \delta_\alpha = 0 \), keeping in mind that unknown soft breaking terms allow more flexibility), however even the values \( \delta_\alpha \sim 1/4 \) do not change the situation much.

It is clear from this figure that no value of \( \tan \beta \) allows perturbative calculation all the way up to the GUT scale. Perturbativity puts a strict upper bound on the mass of the \( b' \) quark. For \( \tan \beta = 1.5 \) we calculate this limit to be about \( \approx 100 \) GeV. This value is below the experimental lower bound of 190 GeV. If a fourth generation exists, this provides a strong reason to introduce new physics at the TeV scale. In order for this to work, the cutoff scale of the theory should be near the TeV scale. Without any UV completion we have a strongly coupled theory at the TeV scale. What are the solutions to this problem? In section 3 we will introduce a specific model with new physics at the TeV scale that will allow values of \( \tan \beta \) up to \( \sim 2 \) with perturbativity all the way up to the GUT scale \( \approx 2 \cdot 10^{16} \) GeV.
2.3 Implications for Higgs Physics

In the MSSM with large $\tan \beta$ the lightest Higgs boson mass has an upper bound $M_h < \sim 125$ GeV. Even if $\tan \beta$ is large, the mass at tree level can be no larger than $M_Z$. This is an even bigger problem when one introduces a fourth family. The new quarks limit $\tan \beta$ to small values, thus reducing the tree level contribution for the lightest Higgs mass. Luckily at the same time they provide additional loop corrections to the lightest Higgs mass. The one-loop top-stop radiative corrections to the Higgs mass squared can be simplified as:

$$
\Delta(M_h^2) \simeq \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \ln \frac{m_t^2 m_t^2}{m_h^2}.
$$

The new $t'$ and $b'$ quarks and their superpartners will also contribute to the Higgs mass. These corrections can enhance the Higgs mass [12, 13]. When $\tan \beta > 1$, the correction from the $b'$ quark has a similar form, but it is negative. If $m_{t'} > m_{t'}$ then there is a problem, as the overall correction will be negative. When $m_{t'} > m_{t'}$, with constrained mass splitting displayed in Fig. 1, there is still a sizable positive correction of about $(60 \text{ GeV})^2$. With $\tan \beta \sim 2$ this puts an upper bound, $M_h < 130$ GeV, greater than the LEP lower bound of 114 GeV.

The existence of a 4th chiral family in the mass range of $(200 - 300)$ GeV will have a significant impact on the Higgs signals at the LHC [5]. The most dominant production mechanism for the light Higgs boson is its production from gluon-gluon fusion via a top quark loop [11]. With the 4th chiral family, there will be additional contributions from the non-degenerate $t'$ and $b'$ loops. Thus the Higgs productions will be significantly enhanced. Also, for the light Higgs with mass below 130 GeV, the Higgs decaying to two photons is the most clean channel for detection at the LHC. With the additional contributions from the $t'$ and $b'$ quarks in the loops, the two photon branching ratio will also be enhanced. The other possible mode for the light Higgs detection is the $t\bar{t}h$ mode, and the subsequent decay of the Higgs to $b\bar{b}$. This mode has been downgraded by recent studies mainly due to low production rate and large SM background. However, with the 4th family quarks, there will be additional contributions to the Higgs production via the $t\bar{t}h$ and $b\bar{b}h$ modes. Thus the Higgs detection via this channel may become viable.

3 The Model with Perturbative UV Completion

If the LHC discovers a fourth chiral family, it will be a great challenge for theorists to build self consistent models. There are several reasons for this. First of all, from existing experimental bounds it follows that the Yukawa couplings for $t'$ and $b'$ should be large. Let us be more specific. If the theory is one Higgs doublet Standard Model (SM), then the bounds $m_{t'} \geq 220$ GeV and $m_{b'} \geq 190$ GeV imply that near these mass scales we have $\lambda_{t'} \geq 1.26$ and $\lambda_{b'} \geq 1.1$. The situation is more drastic within the MSSM. The above bound for the $m_{t'}$ gives $\lambda_{t'} \geq 1.1 \sqrt{1 + \tan^2 \beta}$ which for $\tan \beta \simeq 3$ gives $\lambda_{t'}(m_{t'}) \geq 3.45$, a non-perturbative value. Therefore, the (tree level) perturbativity suggests the upper bound $\tan \beta \leq 2.5$. However, as we saw in the previous section, after taking into account RGE effects, the requirement of perturbativity up to higher scales prefers even lower
we suggest one simple extension which allows perturbativity up to the GUT scale for $M_{\text{GUT}} \approx 2 \cdot 10^{16}$ GeV. What are the possibilities to overcome these difficulties? The solution is some reasonable extension which modifies RG running above the TeV scale. Here we suggest one simple extension which allows perturbativity up to the $M_{\text{GUT}}$ with less constraint on $\tan \beta$.

Our proposal is the following. The couplings $\lambda_{\nu'}, \lambda_{\nu}$ and $\lambda_{\tau'}$ are derived quantities in a low energy effective theory. They are generated after decoupling of additional vector like states with mass $\Lambda_4 \sim \text{few} \cdot \text{TeV}$. Above $\Lambda_4$, new interactions appear in the RGE and this makes the theory perturbative all the way up to $M_{\text{GUT}}$. We discuss the realization of this idea within the framework of the MSSM, however, non-SUSY models can be constructed with equal success.

We introduce two additional vector like pairs $(H_u + H_d), (H'_u + H'_d)$ of Higgs superfields, where $H_u, H'_u$ and $H_d, H'_d$ have the same quantum numbers under the MSSM gauge group as the up type $(h_u)$ and the down type $(h_d)$ Higgs superfields. These $H$-states are accompanied by two pairs of vector like quarks $(D^c + \bar{D}^c), (D'^c + \bar{D}'^c)$, where $D^c$ has the quantum numbers of the down type quark $d^c$. Introduction of $D$-states are suggestive: they, together with $H$-states, effectively constitute complete $SU(5)$ multiplets and therefore gauge coupling unification can be maintained at 1-loop approximation.

We will consider the following superpotential couplings

$$ W_4 = \lambda^{(1)}_{\nu'} q_4 u' h_u + \lambda_{\nu} q_4 u' H_u + \lambda^{(1)}_{\nu'} q_4 d'_4 h_d + \lambda_{\nu} q_4 d'_4 H_d + \lambda^{(1)}_{\tau'} l_4 e'_4 h_d + \lambda_{\tau'} l_4 e'_4 H_d + \lambda'_{\nu} l_4 e'_d h_d - M_H H_u H_d - M_H' H'_u H'_d + M H_u h_d + M' H'_u h'_d + M_D D^c D^c - M_D' D'^c D'^c. $$

For simplicity we do not couple $D'^c, \bar{D}'^c$ states with chiral matter and assume that they have mass $M_{D'} \sim M_D$. After integrating out the $H$ and $D$-states one can easily verify that the effective Yukawa interactions are

$$ W_4^{\text{eff}} = \lambda^{(1)}_{\nu'} q_4 u' h_u + \lambda_{\nu} q_4 u' H_u + \lambda^{(1)}_{\tau'} l_4 e'_4 h_d, $$

where:

$$
\begin{align*}
\lambda_{\nu'} &= \lambda^{(1)}_{\nu'} + \lambda_U \cos \gamma' \\
\lambda_{\nu} &= \lambda^{(1)}_{\nu} + \lambda_D \cos \gamma + \lambda_D' \cos \gamma_D \\
\lambda_{\tau'} &= \lambda^{(1)}_{\tau'} + \lambda_E \cos \gamma \\
\tan \gamma' &\approx \frac{M_{H'}}{M'} \\
\tan \gamma &\approx \frac{M_H}{M} \\
\tan \gamma_D &\approx \frac{M_D}{M_D}.
\end{align*}
$$

The relevant diagrams are shown in Fig. 3. With all the mass scales of the same order ($\approx \Lambda_4$) the effective superpotential given above is valid below the scale $\Lambda_4$. With $\cos \gamma \approx \cos \gamma_D \approx \cos \gamma' \approx 1$, we can see that the effective (derived) Yukawas can be non-perturbative ($\approx 3$) while the original Yukawa couplings remain perurbative; for example, $\lambda_{\nu'} \simeq 2.4$ with $\lambda^{(1)}_{\nu'} \simeq \lambda_U \simeq 1.2$. Above the scale
we are dealing with the couplings $\lambda^{(1)}_{\nu', \nu''}$ and $\lambda_{U,D,E}, \lambda'_D$. By making proper choice for the values of these couplings at $\Lambda_4$, we can have a perturbative regime up to the GUT scale. To demonstrate this we take $\Lambda_4 = 1.66$ TeV and set up all RG equations valid above this scale. At 1-loop they are given by

$$16\pi^2 \frac{d}{dt} \lambda_{\nu'}^{(1)} = \lambda_{\nu'}^{(1)} \left( S_q + S_{u'} + S_{h_u} - c_i^u g_i^2 \right) \quad (13)$$

$$16\pi^2 \frac{d}{dt} \lambda_{\nu''}^{(1)} = \lambda_{\nu''}^{(1)} \left( S_q + S_{d''} + S_{h_d} - c_i^d g_i^2 \right) \quad (14)$$

$$16\pi^2 \frac{d}{dt} \lambda_{\nu'''}^{(1)} = \lambda_{\nu'''}^{(1)} \left( S_I + S_{h_d} - c_i^e g_i^2 \right) \quad (15)$$

$$16\pi^2 \frac{d}{dt} \lambda_t = \lambda_t \left( 6\lambda_t^2 + 3 \left( \lambda_{\nu'}^{(1)} \right)^2 - c_i^u g_i^2 \right) \quad (16)$$

$$16\pi^2 \frac{d}{dt} \lambda_U = \lambda_U \left( S_q + S_{u''} + 3\lambda_U^2 - c_i^u g_i^2 \right) \quad (17)$$

$$16\pi^2 \frac{d}{dt} \lambda_D = \lambda_D \left( S_q + S_{d''} + 3\lambda_D^2 + \lambda_E^2 - c_i^d g_i^2 \right) \quad (18)$$

$$16\pi^2 \frac{d}{dt} \lambda'_D = \lambda'_D \left( S_q + S_{h_d} + 2\lambda_D^2 - c_i^d g_i^2 \right) \quad (19)$$

$$16\pi^2 \frac{d}{dt} \lambda_E = \lambda_E \left( S_I + 3\lambda_D^2 + \lambda_E^2 - c_i^e g_i^2 \right) \quad (20)$$

where

$$S_q = \left( \lambda_{\nu'}^{(1)} \right)^2 + \left( \lambda_{\nu''}^{(1)} \right)^2 + \lambda_U^2 + \lambda_D^2 + \lambda_E^2 \quad (21)$$

$$S_{u''} = 2 \left( \lambda_{\nu''}^{(1)} \right)^2 + 2\lambda_U^2 \quad (22)$$

$$S_{d''} = 2 \left( \lambda_{\nu'''}^{(1)} \right)^2 + 2\lambda_D^2 \quad (23)$$

$$S_I = 3 \left( \lambda_{\nu'''}^{(1)} \right)^2 + 3\lambda_E^2 \quad (24)$$

$$S_{h_u} = 3 \left( \lambda_{\nu'}^{(1)} \right)^2 + 3\lambda_I^2 \quad (25)$$

$$S_{h_d} = 3 \left( \lambda_{\nu''}^{(1)} \right)^2 + \left( \lambda_{\nu'''}^{(1)} \right)^2 + 3\lambda_D^2 \quad (26)$$

$$c_i^u = \left( \frac{13}{15}, 3, \frac{16}{3} \right), \quad c_i^d = \left( \frac{7}{15}, 3, \frac{16}{3} \right), \quad c_i^e = \left( \frac{9}{5}, 3, 0 \right), \quad (27)$$

and $t = \ln \mu$. We have ignored bottom and tau Yukawa couplings because we still work in a low tan $\beta$ regime. Also the Dirac Yukawa coupling of the fourth left handed neutrino with the ‘right handed’ singlet $N$ is neglected, because assuming $m_{\nu'} \simeq 50$ GeV we get $\lambda_{\nu'} \simeq 0.25$ which is small.

At scale $\Lambda_4$, for boundary conditions we take

$$\text{at } \mu = \Lambda_4 = 1.66 \text{ TeV : } \lambda_{\nu'}^{(1)} = \lambda_{\nu''} = 0.697,$$
\[ \lambda_{\nu}^{(1)} = 0.828 , \quad \lambda_D = \lambda_D' = 0.852 , \quad \lambda_{\tau'}^{(1)} = \lambda_E = 0.616 , \]

and run the couplings up to \( \mu = M_{\text{GUT}} \). The numerical solutions are displayed in Fig. 4. For completeness we have also included 2-loop contributions. As we see from Fig. 4, all couplings remain perturbative. Note that the boundary values in (28) with \( \cos \gamma \approx \cos \gamma' \approx 1 \) for \( \tan \beta \approx 2 \) give values for \( m_{\nu'}, m_{\nu'}, m_{\nu'} \) (evaluated at their own mass scales) satisfying current experimental bounds. Thus, our solution is fully consistent.

With this extension and values of the couplings given in (28), the gauge coupling unification occurs at relatively high scale \( M_{\text{GUT}} \approx 7.8 \cdot 10^{16} \text{ GeV} \) with perturbative unified gauge coupling \( \alpha_{\text{GUT}}^{\pi} \approx 0.046 \). The corresponding picture is shown in Fig. 5. For this case, for the mass of \( D'^c, \bar{D}^c \) states we took \( M_{D'} \approx 2.09 \text{ TeV} \). In our analysis we have included 2-loop corrections and also weak scale threshold effects due to \( m_t, m_{\nu'}, m_{\nu'} \) and \( m_{\nu'} \). Note that the gauge coupling unification scale is somewhat higher compared to that of the usual SUSY GUT \( (\approx 2 \cdot 10^{16} \text{ GeV}) \). This will help to alleviate the proton decay problem in SUSY GUT.

We have demonstrated that with a simple extension one can make the MSSM with four chiral generations perturbative all the way up to the GUT scale. This gives firm ground for embedding the whole scenario in a Grand Unified Theory. Other variations of the construction of the effective Yukawa sector are possible, however, we have limited ourselves here with one example because it solves the problems in a simple and efficient way. We hope that our studies will motivate others in further investigations.

## 4 Conclusions

We have investigated the implications of the presence of a 4th chiral family of fermions in the MSSM as well as the SM. The precision EW parameters \( S \) and \( T \) set constraints on the masses of the 4th family and the splitting between the up and down type quarks \( (t' \text{ and } b') \). We have plotted the allowed regime of 4th generation quark masses in Fig. (1).

We also investigated the constraint on the 4th family from the perturbativity condition on the corresponding Yukawa couplings, and found that in MSSM, there is no allowed value of \( \tan \beta \) for which the couplings remain perturbative all the way up to the GUT scale. As a result, if a 4th family is discovered at the LHC, then for the theory to make sense perturbatively, there must be additional new physics with a suitable ultraviolet completion. We have presented such a model with additional vector-like states, at the TeV scale. In our model, only the very narrow range of \( \tan \beta < 2 \) is allowed.

In addition to observing the 4th chiral family of fermions at the LHC, the model has several predictions, such as the existence of vector-like down type quarks at the TeV scale which can be pair produced by gluon-gluon fusion, enhanced decay of the lightest Higgs boson to two photons, and enhanced Higgs production from gluon-gluon fusion due to the \( t' \) and \( b' \) quarks. These predictions of the model can be tested at the LHC.
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Figure 3: Diagrams generating Yukawa couplings $\lambda'_t$, $\lambda'_y$ and $\lambda'_{\tau'}$. 
Figure 4: Plots at left hand side: running of Yukawa couplings $\lambda_t$, $\lambda_t^{(1)}$, $\lambda_b^{(1)}$ and $\lambda_{\tau'}^{(1)}$. Right hand side: running of couplings $\lambda_U$, $\lambda_D$, $\lambda_{\bar{D}}$, $\lambda_E$. 
Figure 5: Gauge coupling unification. $M_{\text{GUT}} = 7.77 \cdot 10^{16}$ GeV, $\frac{\alpha_{\text{GUT}}}{4\pi} = 0.046$