Role of quark-quark correlation in baryon structure and non-leptonic weak transitions of hyperons

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We study the role of quark-quark correlation in the baryon structure and, in particular, the hyperon non-leptonic weak decay, which is sensitive to the correlation between quarks in the spin-0 channel. We rigorously solve non-relativistic three-body problem for SU(3) ground state baryons to take into account the quark-pair correlation explicitly. With the suitable attraction in the spin-0 channel, resulting static baryon properties as well as the parity conserving weak decay amplitudes agree with the experimental values. Special emphasis is placed also on the effect of the SU(6) spin-flavor symmetry breaking on the baryon structure. Although the SU(6) breaking effects on the local behavior of the quark wave functions are considerable due to the spin-0 attraction, the calculated magnetic moments are almost the same as the naive SU(6) expectations.

§1. Introduction

Properties of light baryons have been extensively studied by various models based on the constituent quark picture, in which the constituent quarks are assumed to be identified with quasi-particles of non-perturbative QCD vacuum. Their results for static hadron properties are consistent with experiments including applications for the two nucleon systems, although this model involves several adjustable parameters. Despite the success of this approach, it is not clear whether or not the constituent quark model correctly describes the quark distributions in the baryons, and further provides the non-leptonic weak hyperon decay with \( \Delta I = 1/2 \) rule.

The \( \Delta I = 1/2 \) rule implies dominance of the \( \Delta I = 1/2 \) transitions and strong suppression of \( \Delta I = 3/2 \) process in the non-leptonic hyperon decay\(^1\). This empirical rule does not originate from the theory of the weak interaction itself, and therefore one needs an explanation of some dynamical origin. As we will show in section 2, the non-leptonic weak decay of hyperons is reasonably described by the soft-pion theorem. It leads to the baryon pole approximation which can reproduce

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relative magnitudes of various hyperon decays and $\Delta I = 1/2$ rule. Within this approximation, the weak decay takes place as the two-quark transition process where a $u$-$s$-pair in the initial hyperon with their total spin 0 changes to a spin-0 $u$-$d$-pair in the final state baryon. However, if one calculates its matrix element using the constituent quark model, the absolute value of the amplitude is about a half of the experimental data at most $^2)$. We emphasize here, because of the heavy $W$-boson mass, the weak matrix elements are quite sensitive to the short range quark-quark correlations. Consequently, the failure of the constituent quark model to describe the non-leptonic weak transition may suggest the lack of the quark correlation in the $s = 0$ channel. Several works $^{3,4}$ also suggest the importance of the short-range quark correlation on the $\Delta I = 1/2$ decay. In particular, it is known that the parity-conserving $\Sigma^+ \rightarrow n \pi^+$ decay is free from factorization and penguin contributions, and simply dominated by the two-quark transition process in the baryon pole term. Therefore, this decay mode gives a crucial constraint on the strength of the quark-quark correlation.

On the other hand, it was recently pointed out from both theoretical $^{6,7}$ and phenomenological $^8$ points of view, there exists a strong correlation between quarks in the $s = 0$ channel, corresponding to the correlation between the quark-antiquark spin-0 pair which forms the pion as the highly collective state. The simple constituent quark model has never incorporated such a quark-quark correlation properly.

In this paper we try to clarify the role of the quark-quark correlation in the baryon structure and hyperon non-leptonic weak decays by using the constituent quark model. We assume the short range spin-dependent correlations between quarks together with the confinement force, and calculate the baryon masses and other static properties. In order to deal with the spin-dependent correlation in the correct and systematic way, we must rigorously solve the three-body problem. For this purpose, we adopt the Gaussian expansion method (GEM) for few-body systems, which has been developed by the two of the present authors (E.H. and M.K.) and their collaborators $^{9-14}$ (see Ref. 9 for a review). We assume the isospin symmetry between $u$ and $d$ quarks, and solve the three-body problem without further approximations or assumptions. Our calculations do not rely on the SU(3) flavor symmetry, and hence the strange quark is distinguished from light $u,d$ quarks. To our knowledge, this work is a first attempt to study the non-leptonic hyperon decay and other hadron properties consistently in the framework of the constituent quark model by taking into account the quark-quark correlation.

So far the constituent quark model with the confining force and the perturbative gluon exchange provides reasonable results for masses and radii, but fails in reproducing the hyperon weak decay $^2)$. After introducing the suitable quark-pair correlation, one does not know a priori whether or not the constituent quark model can explain both standard properties and at the same time non-leptonic weak decay matrix elements. One of the main purposes of this paper is to make this issue clear.

$^*$ It may be possible to reproduce the experimental values by artificially taking a smaller baryon radius. Such a procedure cannot be justified, because the baryon radii should be also chosen to be consistent with experiments.
by solving the non-relativistic three-quark problem explicitly.

We also focus on the SU(6) breaking effects on baryon properties. Introduction of the spin-dependent correlation naturally spoils the SU(6) spin-flavor symmetry which is known to work well for e.g. the baryon magnetic moments. Hence, we shall calculate the magnetic moments to estimate the SU(6) breaking effects. We discuss how the SU(6) breaking affects the baryon properties, and point out that this symmetry is still useful for the static baryon properties, although local behavior of the quark wave function considerably departs from the SU(6) symmetric limit.

This paper is organized as follows. In section 2, we introduce the effective weak interaction which includes the renormalization group improved QCD corrections. We show the relevant formulae within the pole approximation, and define a set of matrix elements. In section 3 we construct the non-relativistic potential model which incorporates the confinement force and the quark correlation. Our numerical procedure to solve the three-body problem is described in detail here. In section 4 we calculate the baryon masses, radii, and the non-leptonic weak transition amplitudes. Comparison with experiments will be made there. We also evaluate the magnetic moments of the SU(3) baryons in order to clarify the SU(6) breaking effects in section 5. Final section is devoted to the summary and discussions.

§2. Calculation of weak matrix elements

We write the low energy effective weak interaction Hamiltonian density \( H_W(x) \):
\[
H_W(x) = \frac{G_F \sin \theta \cos \theta}{\sqrt{2}} \sum_i c_i(\mu^2) O_i(x) + \text{h.c.} ,
\]
where
\[
O_1 = [\bar{u}\gamma_\mu(1 - \gamma_5)s][\bar{d}\gamma_\mu(1 - \gamma_5)u]
\]
\[
O_2 = [\bar{d}\gamma_\mu(1 - \gamma_5)s][\bar{u}\gamma_\mu(1 - \gamma_5)u]
\]
\[
O_3 = [\bar{d}\gamma_\mu(1 - \gamma_5)s] \sum_{q=u,s,d} [\bar{q}\gamma_\mu(1 - \gamma_5)q]
\]
\[
O_4 = \sum_{q=u,s,d} [\bar{q}\gamma_\mu(1 - \gamma_5)s][\bar{d}\gamma_\mu(1 - \gamma_5)q]
\]
\[
O_5 = [\bar{d}\gamma_\mu(1 - \gamma_5)s] \sum_{q=u,s,d} [\bar{q}\gamma_\mu(1 + \gamma_5)q]
\]
\[
O_6 = \sum_{q=u,s,d} [\bar{q}\gamma_\mu(1 - \gamma_5)s][\bar{d}\gamma_\mu(1 + \gamma_5)q].
\]

This effective weak Hamiltonian can be obtained by integrating out over \( W \)-boson degrees of freedom in the Standard Model. Current-current operators \( O_1, O_2 \) give dominant contributions in our case, and \( O_3 \sim O_6 \) provide so called penguin contributions. The coefficients \( c_i(\mu^2) \) of the Hamiltonian get QCD radiative corrections calculated by the renormalization group technique, and thus depend on the scale. We take the scale \( \mu^2 = 1\text{GeV}^2 \) as a typical scale of light hadrons, and use values of \( c_i \) given in ref. 15).
Our task here is to evaluate the matrix element \( \langle B_f, \pi^a | H_W(0) | B_i \rangle \) for the strangeness changing process \( B_i \to B_f + \pi^a \). We note that the factorization as well as the penguin contributions are too small to reproduce the \( \Delta I = 1/2 \) amplitudes. QCD radiative corrections to the effective weak Hamiltonian tend to increase the \( \Delta I = 1/2 \) amplitudes, but effects are not enough.

Analysis based on the chiral dynamics of low energy QCD is suitable to deal with the strongly interacting pion-nucleon system. With the help of the soft pion theorem, one can rewrite the transition matrix element as

\[
\langle B_f(p_f) \pi^a(q) | H_W(0) | B_i(p_i) \rangle = \int d^4x e^{-iqx} (-q^2 + m_{\pi}^2) \langle B_f | T \{ \pi^a(x), H_W(0) \} | B_i \rangle = \frac{-1}{M_{\pi}} \left\langle B_f \right| \left[ i \int d^3x A_{\mu}^0(x), H_W(0) \right] \left| B_i \right\rangle + \frac{iq \mu}{M_{\pi}} \int d^4x e^{-iqx} \left\langle B_f \right| T \{ A_{\mu}^a(x), H_W(0) \} \left| B_i \right\rangle. \tag{2.2}
\]

The first term is called the commutator term which gives the parity violating S-wave amplitudes. The second term expresses the baryon pole contribution by inserting the intermediate baryon states between \( A_{\mu}^a(x) \) and \( H_W(0) \), and contributes to both parity conserving and violating amplitudes, as illustrated in Fig. 1. Here, the initial hyperon \( B_i \) changes to an intermediate state baryon \( B_n \) by the weak interaction and then \( B_n \) emits the pion to produce the final state \( B_f \)(and vice versa). In this work, we concentrate on the parity conserving P-wave amplitudes to investigate the quark correlation inside the ground state baryons\(^*\). For example, the parity conserving amplitude for \( A^0 \to n + \pi^0 \) is given by

\[
B(A^0 \to n + \pi^0) = \frac{M_N + M_A}{f_{\pi}} \left[ G_{n0} M_A - M_N \right] \langle n | H_{PC} | A \rangle + \langle n | H_{PC} | \Sigma^0 \rangle \frac{1}{M_n - M_{\Sigma}} G_{\pi0} \Sigma \tag{2.3}
\]

where \( \langle n | H_{PC} | A \rangle \) and \( \langle n | H_{PC} | \Sigma^0 \rangle \) are the matrix elements between appropriate baryon states with \( H_{PC} \) being the parity conserving part of the weak Hamiltonian.

\(^*\) The parity violating amplitudes are not adequate for the study of the quark correlation selectively, since the contributions from the commutator, penguin, factorization, and the baryon pole are all comparable.
Roles of quark-quark correlations in baryon structure

defined in the Appendix. \( G_{B B'}^{\pi a} \) denotes the axial vector coupling constant which gives a probability for the pion emission \( B \to B' + \pi^a \). Here, we consider only the ground state baryon octet as the intermediate states. Formulae for other hyperon decays are found in the Appendix.

The axial vector coupling constants are rather well-known quantities from experiments. We adopt the values of \( G_{B B'}^{\pi a} \) obtained by the SU(3) parameterization for the existing experimental data, since the axial vector coupling seems to be insensitive to the quark correlation due to its one-body operator structure. Therefore, we are now in the position to determine the matrix elements of the weak Hamiltonian, \( \langle n|H_{PC}|\Lambda \rangle \) and \( \langle p|H_{PC}|\Sigma^+ \rangle^* \). We recall that the quark models such as Isgur-Karl Harmonic Oscillator model or MIT bag model\(^{(16)}^{(17)}\) give much smaller values for these matrix elements than the experimental data\(^{(2)}^{(3)}\). It is instructive to rewrite the weak Hamiltonian \( H_{PC} \) in the non-relativistic limit in the coordinate space as

\[
H_{PC} = \frac{G_F \sin \theta \cos \theta}{\sqrt{2}} (c_1 O_1^{NR} + c_2 O_2^{NR}),
\]

(2.4)

where \( a_i, a_i^\dagger \) are annihilation and creation operators of quarks with flavor \( i \). Presence of the spin-projection operator \( \left( 1 - \vec{\sigma}_u \cdot \vec{\sigma}_s \right) \) in \( O_1^{NR}, O_2^{NR} \) tells us that only the \( (us) \) pair with their total spin being 0 can contribute to the weak decay process; namely, the weak transition is generated by the two body process between spin-0 quark pairs; \( (us)^0 \to (ud)^0 \). The isospin of the initial \( (us)^0 \) pair is \( 1/2 \), and the final \( (ud)^0 \) has the isospin-0 due to the antisymmetrization. Thus, this process guarantees the \( \Delta I = 1/2 \) dominance in the non-leptonic hyperon decays as pointed out long ago\(^{(1)}\). Now it is clear that this decay amplitude is very sensitive to the correlation of the spin-0 quark pair in the baryons\(^{(4)}\). The standard constituent quark model never incorporates such a correlation properly. However, in fact, some fundamental studies on non-perturbative QCD\(^{(6)}^{(7)}\) suggest that there exists the strong attractive correlation for the quark-quark pair with \( s = 0 \). These considerations naturally lead us to study the quark structure of baryons by taking into account the attractive correlation which could enhance the weak decay amplitudes.

§3. Constituent quark model for baryons with spin-dependent correlation

Our purpose here is to construct the quark model to deal with the quark pair correlations and thus account for the non-leptonic weak decay. Although there are several efforts from the lattice QCD simulation and phenomenological analysis, our knowledge of the interaction between light quarks is still far from complete understanding. As we have discussed in the introduction, the instanton liquid model of the non-perturbative QCD vacuum provides a spin-dependent correlation between the quark-pair. Due to the finite spatial size of the instantons, typically 0.35fm\(^{(19)}\), such a

\footnote{In this paper, we restrict ourselves to study only \( \Lambda, \Sigma \) hyperon decay. Results including other hyperons will be published in subsequent publication.}
spin-spin interaction should not be point-like but has a finite range. The constituent quarks are also assumed to have their internal structure. Hence, we introduce the Gaussian shape spin-dependent force which acts on only the quark-quark pair with $s = 0$. We neglect possible flavor dependence of the potential. Other spin-dependent pieces like the spin-orbit and tensor interactions are neglected for simplicity, because we shall clarify the effects of the spin-spin interaction on the non-leptonic weak decay amplitudes.

On the other hand, we take the two-body Harmonic Oscillator potential as the confinement force, since the analytical solutions of the three-body system are well known\textsuperscript{16}. In order to check accuracy of our numerical calculations, we can compare analytical results with ours, when we turn off the spin-dependent interaction. Choice of the HO potential is advantageous for us to develop our numerical procedure in this paper, but it can be easily improved in a more realistic way.

Finally, we phenomenologically introduce the effective Hamiltonian which includes the confinement force $V_C$ and the spin-dependent part $V_S$ as

$$H = \sum_i \frac{p_i^2}{2m_i} - T_G + V_C + V_S + V_0,$$

$$V_C = \sum_{i < j} \frac{1}{2} K (x_i - x_j)^2,$$

$$V_S = \sum_{i < j} \frac{C_{SS}}{m_im_j} \exp \left[- \frac{(x_i - x_j)^2}{\beta^2} \right] (\text{spin}=0 \text{ pair})$$

and $V_S = 0$ for spin=1 pair. Here, $m_i$, $x_i$, and $p_i$ are the mass, coordinate and momentum of the $i$-th constituent quark, and $K, C_{SS}, \beta$ are the model parameters which are taken to be common for all the baryons concerned. $T_G$ is the c.m. kinetic energy and $V_0$ is the constant parameter which contributes to the overall shift of the resulting spectrum and is chosen to adjust the energy of the lowest state to the nucleon mass. The quark masses are taken to be $m_u = m_d = 330$ MeV and $m_s = 500$ MeV.

Using this Hamiltonian, we shall solve non-relativistic three-body problem rigorously. We assume only the isospin symmetry between up and down quarks. The quark wave functions are constructed by the antisymmetrization without invoking any further approximations or assumptions. We note that the SU(6) spin-flavor symmetry is broken within our formalism because of the spin-dependent correlation. Namely, the spatial part of the total wave function is dependent on the spins and isospins of the three quarks and is expanded in terms of a number of basis functions so as to describe the spin-dependent short-range correlations.

Since $N, \Delta$ and $\Omega^-$ are composed of three quarks having the same isospin and the other baryons $\Lambda, \Sigma, \Sigma^* \Xi$ and $\Xi^*$ are not, we introduce two different types of total wave functions for the two cases.
3.1. Wave functions of $N, \Delta$ and $\Omega^-$

According to the Gaussian expansion method\(^9\)\(^{14}\), we consider three rearrangement Jacobian coordinates of Fig. 2 and refer them as channels $c = 1$ to $3$; here, $r_k = x_i - x_j$ and $R_k = x_k - (x_i + x_j)/2$ for the cyclic permutations of $(i,j,k)$. We first construct three-body basis functions for the spin, isospin and spatial part of the channel $c = k$ with $J,M$ (total spin and its $z$-component) and $T,T_z$ (total isospin and its $z$-component) as

$$
\Phi^{(c=k)}_{JMT T_z, \xi} = \left[ \chi_{\frac{1}{2}}(i) \chi_{\frac{1}{2}}(j) \chi_{\frac{1}{2}}(k) \right]_S \left[ \phi_{nl}(r_k) \psi_{NL}(R_k) \right]_I \times \left[ \eta_{\tau}(i) \eta_{\tau}(j) \eta_{\tau}(k) \right]_{TT_z}, \quad \xi = \{s, n, l, N, L, I, t\} \quad (3.4)
$$

with the isospin of quarks, $\tau$, to be

$$
\tau = \left\{ \begin{array}{ll}
\frac{1}{2} & \text{for } N, \Delta \\
0 & \text{for } \Omega^- .
\end{array} \right. \quad (3.5)
$$

Here, $\eta_{\tau z}(i)$ is the isospin function of the $i$-th quark, $t$ denoting the isospin of each quark-pair. $\chi_{\frac{1}{2}}(i)$ is the spin function of the $i$-th quark. $s$ and $S$ denote the intrinsic spin of each quark-pair and three quarks, respectively. The numbers $n$ and $l$ ($N$ and $L$) specify respectively the radial and angular-momentum excitations with respect to the Jacobian coordinates $r_c$ ($R_c$), and $l$ and $L$ are coupled to the total orbital angular momentum $I$. Explicit form of the spatial functions $\phi_{nl}(r_k)$ and $\psi_{NL}(R_k)$ will be discussed below. The three-body center-of-mass motion does not appear in our framework.

The totally antisymmetric basis function with the quantum number set $\xi$, $\Phi_{JMT T_z, \xi}$, is obtained by the equal-weight superposition of $\Phi^{(c)}_{JMT T_z, \xi}$ over $c = 1 - 3$, multiplied by the color singlet wave function and posed by the Pauli restriction $1 + s + 2\tau + t + l =$
even:
\[ \Phi_{JMT T_z, \xi} = \Phi(\text{color singlet}) \sum_{c=1}^{3} \Phi^{(c)}_{JMT T_z, \xi}. \] (3.6)

The totally antisymmetric property of the basis functions \( \Phi_{JMT T_z, \xi} \) is explicitly seen by interchanging the particle numbers of any pair.

The total wave function, \( \Psi_{JMT T_z} \), is given as a sum of these basis functions:
\[ \Psi_{JMT T_z} = \sum_{\xi} A_{\xi} \Phi_{JMT T_z, \xi}. \] (3.7)

This form is the most general one of the totally antisymmetric three-quark functions of \( N, \Delta \) and \( \Omega^- \) (and their spatially excited states). The coefficients \( A_{\xi} \) are to be determined by solving the Schrödinger equation
\[ (H - E) \Psi_{JMT T_z} = 0 \] (3.8)
with the Rayleigh-Litz variational principle.

3.2. Wave functions of \( \Lambda, \Sigma, \Sigma^*, \Xi \) and \( \Xi^* \)

In this case, in order to construct totally antisymmetric wavefunctions, we consider according to the Gaussian expansion method\(^{(9)-(14)}\), the nine rearrangement Jacobian coordinates of Fig. 3 (\( \gamma = 1 - 3, c = 1 - 3 \)) in which a particle (illustrated by a double circle) is \( s \) quark and the other two are \( u, d \) quarks for \( \Lambda, \Sigma, \Sigma^* \), and the situation is opposite for \( \Xi, \Xi^* \). The channel name \( \gamma \) indicates the particle number of the sole, different quark (double circle), whereas the channel name \( c \) denotes the particle number which the Jacobian coordinate \( R \) points. In order to make the coupling scheme of the spins and isospins of the three quarks as visual as possible, we always place the sole, different quark as the thirdly (lastly) coupled particle in the spin-isospin space (but not always in this order in the coupling scheme of the coordinate space).

The three-body basis function for the spin, isospin and spatial part of the channel \( \gamma, c \) is given by
\[
\Phi^{(\gamma, c)}_{JMT T_z, \xi} = \left[ \left[ \chi^{(\alpha)}_1(\alpha) \chi^{(\beta)}_2(\beta) \chi^{(c)}_3(\gamma) \right]_S \left[ \phi_{nl}(\text{R}_{\gamma, c}) \psi_{NL}(\text{R}_{\gamma, c}) \right]_I \right]_M \\
\times \left[ \left[ \eta^{(\alpha)}_T(\alpha) \eta^{(\beta)}_T(\beta) \eta^{(\gamma)}_T(\gamma) \right]_T \right]_{TT_z}, \quad \xi \equiv \{s, S, n, l, N, L, I, t\} \quad (3.9)
\]
where \( \alpha, \beta, \gamma \) are given by the cyclic permutations. The isospins \( \tau \) and \( \tau_{\gamma} \) are defined as
\[
\tau = \begin{cases} 
\frac{1}{2} (u, d) & \text{for } \Lambda, \Sigma, \Sigma^* \\
0 (\bar{s}) & \text{for } \Xi, \Xi^* 
\end{cases} \quad \tau_{\gamma} = \begin{cases} 
0 (s) & \text{for } \Lambda, \Sigma, \Sigma^* \\
\frac{1}{2} (u, d) & \text{for } \Xi, \Xi^* 
\end{cases} \quad (3.10)
\]

Using these basis functions, we can construct two types of totally antisymmetric three-quark basis functions:
(i) A-type: \textit{equal-weight} superposition of the three basis functions with $\gamma = c = 1 - 3$ (cf. the left-most column of Fig. 3), multiplied by $\Phi$(color singlet) and posed by the Pauli restriction $1 + s + 2\tau + t + l = \text{even}$ for the quark pair having the same isospin (strangeness):

$$\Phi_{JMT\xi, \xi}^{(A)} = \Phi\text{(color singlet)} \sum_{\gamma=c=1}^{3} \Phi_{JMT\xi, \xi}^{(\gamma,c)} \cdot$$ (3-11)

(ii) B-type: \textit{equal-weight} superposition of the six basis functions with $\gamma \neq c =$
$\Phi^{(B)}_{JMT T, \xi} = \Phi^{(color \ singlet)} \sum_{\gamma=1}^{3} \sum_{c \neq \gamma} \Phi^{(\gamma, c)}_{JMT T, \xi} \cdot (3.12)$

The total wave function is given by a sum of these basis functions:

$\Psi_{JMT T} = \sum_{\xi} A_{\xi} \Phi^{(A)}_{JMT T, \xi} + \sum_{\xi'} B_{\xi'} \Phi^{(B)}_{JMT T, \xi'} \cdot (3.13)$

This is the most general form of the totally antisymmetric three-quark wave functions of $\Lambda, \Sigma, \Sigma^*, \Xi$ and $\Xi^*$ (and their spatially excited states). The A-type basis functions are useful for describing the correlations between the quarks having the same isospin (strangeness) whereas the B-type ones are effective for the correlations between the quarks which have different isospins (cf. Fig. 3). The coefficients $A_{\xi}$ and $B_{\xi'}$ are determined by the variational principle.

It is to be noted that, when calculating the energy $E$ and the coefficients $A_{\xi}$ and $B_{\xi'}$, use of the channel $\gamma = 1$ alone in Eq. (3.12) and (3.13) is sufficient since the strong interaction does not mix the configurations having different $\gamma$. But, use of the full wave function with $\gamma = 1 - 3$ (replacing the so-obtained $A_{\xi}$ by $A_{\xi}/\sqrt{3}$ and $B_{\xi'}$ by $B_{\xi'}/\sqrt{3}$) is necessary in the calculation of the weak decay matrix elements.

3.3. Spatial basis functions

Our wave function allows complicated admixture of spin-isospin states depending on the quark-pair correlations. For the spatial basis functions $\phi_{nlm}(r)$ and $\psi_{NLM}(R)$, we have to employ any functional form which satisfies the following requirements: i) The functions should be very suited for describing both the short-range correlations and the long-range tail behavior. ii) Energy matrix elements should be calculated analytically and easily between the basis functions of the different arrangement set of Jacobian coordinates. iii) Non-linear parameters of the basis functions can be searched quickly.

To the authors’ opinion, the most suitable are the Gaussian basis functions with range parameters chosen to lie in a geometrical progression$^{9) - 14)}$:

$\phi_{nlm}(r) = N_{nl} r^{l} e^{-(r/r_{n})^2} Y_{lm}(\hat{r}),$

$\psi_{NLM}(R) = N_{NL} R^{L} e^{-(R/R_{N})^2} Y_{LM}(\hat{R}), \hspace{1cm} (3.14) \text{where } N_{nl} \text{ and } N_{NL} \text{ are normalization constants and}$

$r_{n} = r_{\text{min}} a^{n-1} \hspace{0.5cm} (n = 1 \sim n_{\text{max}}),$

$R_{N} = R_{\text{min}} A^{N-1} \hspace{0.5cm} (N = 1 \sim N_{\text{max}}). \hspace{1cm} (3.15)$

Successful applications of the Gaussian expansion method with the use of the above Gaussian basis functions are seen in Refs. 9)-14) for various three- and four-body systems.
3.4. Matrix elements of the Weak Hamiltonian

With the use of the above wave functions, the weak decay matrix elements, $\langle p \mid O^{NR}_{i} \mid \Sigma^{+} \rangle$, $\langle n \mid O^{NR}_{i} \mid \Sigma^{0} \rangle$, $\langle n \mid O^{NR}_{i} \mid \Lambda \rangle$, are calculated in the following manner: Firstly, we consider the operation of the operator Eq. (2.5) on the hyperon wave function. Since $s$ quark changes to $u$ quark and $d$ quark to $u$ quark due to the weak interaction (Fig. 1), the operation on the isospin part of the $\Sigma^{+}$ wave function results in

$$a_{u}^{+} a_{u}^{+} (1 - \sigma_{u} \cdot \sigma_{s}) \delta(x_{u} - x_{s}) a_{u} a_{s} | \eta_{\lambda_{\frac{1}{2}}}(\alpha)\eta_{\lambda_{\frac{1}{2}}}(\beta)\eta_{0}(\gamma) \rangle$$

$$= (1 - \sigma_{\alpha} \cdot \sigma_{\gamma}) \delta(x_{\alpha} - x_{\gamma}) | \eta_{\lambda_{\frac{1}{2}}}(\alpha)\eta_{\lambda_{\frac{1}{2}}}(\beta)\eta_{0}(\gamma) \rangle$$

$$+ (1 - \sigma_{\beta} \cdot \sigma_{\gamma}) \delta(x_{\beta} - x_{\gamma}) | \eta_{\lambda_{\frac{1}{2}}}(\alpha)\eta_{\lambda_{\frac{1}{2}}}(\beta)\eta_{\lambda_{\frac{1}{2}}}(\gamma) \rangle . \tag{3.16}$$

The same operation on the isospin part of the $\Lambda$ wave function gives

$$a_{d}^{+} a_{u}^{+} (1 - \sigma_{u} \cdot \sigma_{s}) \delta(x_{u} - x_{s})$$

$$\times a_{u} a_{s} \left[ \frac{1}{\sqrt{2}} \left[ \eta_{\lambda_{\frac{1}{2}}}(\alpha)\eta_{\lambda_{\frac{1}{2}}}(\beta) - \eta_{\lambda_{\frac{1}{2}}}(\alpha)\eta_{\lambda_{\frac{1}{2}}}(\beta) \right] \eta_{0}(\gamma) \right]$$

$$= \frac{1}{\sqrt{2}} (1 - \sigma_{\alpha} \cdot \sigma_{\gamma}) \delta(x_{\alpha} - x_{\gamma}) | \eta_{\lambda_{\frac{1}{2}}}(\alpha)\eta_{\lambda_{\frac{1}{2}}}(\beta)\eta_{0}(\gamma) \rangle$$

$$+ \frac{1}{\sqrt{2}} (1 - \sigma_{\beta} \cdot \sigma_{\gamma}) \delta(x_{\beta} - x_{\gamma}) | \eta_{\lambda_{\frac{1}{2}}}(\alpha)\eta_{\lambda_{\frac{1}{2}}}(\beta)\eta_{\lambda_{\frac{1}{2}}}(\gamma) \rangle . \tag{3.17}$$

Overlap between the proton wave function and the so-operated $\Sigma^{+}$ wave function, multiplied by $G_{F}\sin\theta\cos\theta(c_{1} - c_{2})/\sqrt{2}$, gives the amplitude $\langle p \mid \mathcal{H}_{pc} \mid \Sigma^{+} \rangle$, and similarly for $\langle n \mid \mathcal{H}_{pc} \mid \Lambda \rangle$. From a simple relation between Clebsch-Gordan coefficients, we have $\langle n \mid \mathcal{H}_{pc} \mid \Sigma^{0} \rangle = \langle p \mid \mathcal{H}_{pc} \mid \Sigma^{+} \rangle / \sqrt{2}$.

§4. Mass spectrum and structure of the baryons

We shall fix the model parameters $K$, $\beta$ and $C_{ss}/m_{u}^{2}$ so as to reproduce the baryon masses and the charge radius. The experimentally measured proton charge radius includes contributions from both valence quark core part and its meson clouds. It is reasonable to subtract the vector meson dominance contribution from the data of the proton electric charge radius $(0.86\text{fm})^{2}$ to obtain the valence quark core radius $\langle r_{p,\text{core}}^{2} \rangle$. From this analysis, we extract $\langle r_{p,\text{core}}^{2} \rangle \sim (0.6\text{fm})^{2}$. By searching the parameters within a reasonable range, we obtain the parameters $K = 0.007\text{GeV}^{3}$, $\beta = 0.55 \text{ fm}$ and $C_{ss}/m_{u}^{2} \simeq 1.10\text{GeV}$ which give $m(\Delta) - m(N) \simeq 293 \text{ MeV}$ and $\langle r_{p,\text{core}}^{2} \rangle = (0.60\text{fm})^{2}$. After this determination, there are no more adjustable parameters in our calculation. With these parameters, we obtain the neutron charge radius square $\langle r_{n}^{2} \rangle = -0.05\text{fm}^{2}$, which is also consistent with the experimental value $\langle r_{n}^{2} \rangle = -0.12\text{fm}^{2}$, after taking into account contributions from the meson cloud.

As for the angular momentum space and the Gaussian range parameters of the basis functions, we examined that the following case is good enough. Contribution of the orbital angular momenta other than $l = L = I = 0$ is found to be negligible.
as long as the calculation is made of the ground states of baryons without non-central forces as in the present study. As an example, spin $s$ and $S$ and Gaussian range parameters employed for the case of nucleon are listed in Table I only for $l = L = I = 0$. The same range parameters are used commonly for any set of spins and isospins of other baryons and are suitable enough to obtain good convergence of the eigenenergies.

Table I. Three-body angular-momentum space and Gaussian range parameters (in fm) for the ground state of nucleon ($J = 1/2^+$, $T = 1/2$).

| $l$ | $L$ | $I$ | $s$ | $S$ | $n_{\text{max}}$ | $r_{\text{min}}$ | $r_{\text{max}}$ | $N_{\text{max}}$ | $R_{\text{min}}$ | $R_{\text{max}}$ |
|-----|-----|-----|-----|-----|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0   | 0   | 0   | 0   | 1/2 | 8               | 0.1             | 3.0             | 8               | 0.1             | 3.0             |
| 0   | 0   | 0   | 1   | 1/2 | 8               | 0.1             | 3.0             | 8               | 0.1             | 3.0             |

We show first in Fig. 4 SU(3) baryon mass spectrum. The agreement with the data encourages us to proceed with our approach. It could be possible to adjust the model parameters or to elaborate the form of the potential to get much improved spectrum, but the present results are enough for our purpose here.

![SU(3) baryon mass spectrum](image)

Fig. 4. SU(3) baryon mass spectrum: Calculations are shown by the solid lines, and experiments by the dashed ones.

To clarify the effects of the attractive correlation in the spin-0 quark pair on the nucleon structure, we introduce the quark-pair correlation function.

$$\rho^{(s)}(r) = \langle \Psi_{\frac{1}{2}M_1 \frac{1}{2}T_z} | \delta(x_i - x_j - r) P^{(s)}(ij) | \Psi_{\frac{1}{2}M_1 \frac{1}{2}T_z} \rangle, \tag{4.1}$$

where $r$ stands for the distance between a quark pair and the $P^{(s)}(ij)$ the projection operator of the quark-pair spin $s$; in other words, the quantity $\rho^{(s)}(r)$ is the probability density to find a spin-$s$ quark pair at the distance $r$. The density correlation
function at origin $\rho^{(s)}(0)$ in the $s = 0$ case essentially fixes the value of the weak decay matrix element in Eq. (2.3). $\rho^{(s)}(\mathbf{r})$, which is independent of the angle $\hat{\mathbf{r}}$, is illustrated in Fig. 5. In Fig. 5 we find a large deviation between the $s = 0$ and $s = 1$ density distributions. $\rho^{(s)}(0)$ in the $s = 0$ case is about three times as large as that of the $s = 1$ case. Similar tendency is seen in other baryons. This provides a huge enhancement for the $\Delta I = 1/2$ weak decay amplitude.

![Fig. 5. Quark-pair density distribution $\rho^{(s)}(\mathbf{r})$ with the spin-dependent correlation. Results for $s = t = 1$ and $s = t = 0$ pairs are depicted by solid and dashed curves, respectively.](image)

For the nucleon, we calculate the r.m.s distance $\bar{r}^{(s)}$ and $\bar{R}^{(s)}$ with respect to the Jacobian coordinates $\mathbf{r}$ and $\mathbf{R}$ by $\bar{r}^{(s)} = \left[ \int r^2 \rho^{(s)}(\mathbf{r}) d\mathbf{r} \right]^{1/2}$ and similarly for $\bar{R}^{(s)}$. We get $\bar{r}^{(s=0)} = 0.92$ fm and $\bar{R}^{(s=0)} = 0.97$ fm, while $\bar{r}^{(s=1)} = 1.10$ fm and $\bar{R}^{(s=1)} = 0.81$ fm. Apparently, the $s = 0$ correlation considerably contacts $\bar{r}$ and extends $\bar{R}$ compared with the $s = 1$ case, but the r.m.s matter radius of the total system is almost 0.58 fm both for the $s = 0$ and 1 cases. It seems that the correlation is not so strong to form the so called ‘diquark’-clustering in the nucleon $^4$.

§5. Numerical results for weak decay amplitudes

With the parameters fixed in the previous section we calculate the matrix elements of the weak Hamiltonian shown in Table II. In the left column

| $\langle n | O^{NR}_{1}(|4\rangle$ | $-0.960$ | $-0.399$ |
| $\langle p | O^{NR}_{1}(|\Sigma^+\rangle$ | $2.760$ | $0.977$ |

Table II. Matrix elements of the operator, $O^{NR}_{1}$ (in $10^{-5}$GeV$^3$) with and without the spin-dependent attraction $V_s$. 


we show the matrix elements with the quark correlation and those without the correlations in the right column. In the absence of the correlation $V_S = 0$, a ratio $\langle p | O_{NR1}^+ | \Sigma^+ \rangle / \langle n | O_{NR1}^+ | \Lambda \rangle = -2.45$ shows a perfect agreement with the SU(6) expectation $-\sqrt{6} \simeq -2.4494 \cdots$. This agreement ensures the validity of our numerical calculations (Not only the ratio but also the absolute value have been examined).

In the realistic case with the spin-dependent force, one can observe the substantial enhancement of the matrix elements and the SU(6) breaking effects.

The non-leptonic weak transition parity-conserving amplitudes are tabulated in Table III. We show the pole contributions only in the second column and additional factorization and penguin contributions, taken from Ref. 4, are in the third column. Then, we show the total decay amplitudes in the forth column to be compared with the experiments. It is worth noting that the $\Sigma^+ \to n\pi^+$ parity conserving decay process is completely dominated by the baryon pole diagrams without any factorization or penguin contributions. This fact indicates that the $\Sigma^+ \to n\pi^+$ P-wave amplitude is the most appropriate observable to probe the quark-quark correlation in the baryons. We find a good agreement for $\Sigma \to N\pi$ decays, while the pole contribution of the $\Lambda \to N\pi$ is not enough. In general, our calculations reasonably reproduce the magnitudes of the pionic hyperon decay amplitudes. However, there is a large cancellation between the first and the second terms in Eq. (2.3), which makes the resulting $\Lambda \to N\pi$ amplitude small. Hence, the pole contribution to $\Lambda \to N\pi$ process strongly depends on the values of $G^{\pi A}_{\Lambda \Sigma}$, $G^{\pi n}_n$ as well as the weak matrix elements. Slight variation of $G^{\pi B}_{\Lambda B'} \sim 10\%$ yields about 40\% modification of the $\Lambda \to N\pi$ pole contribution, whereas the decay amplitudes for $\Sigma \to N + \pi^a$ are essentially unchanged. The $\Lambda$ decay amplitudes are also rather sensitive to the detailed shape of the potential.

We have obtained the agreement when we take the Gaussian size parameter $\beta = 0.5 \sim 0.6$ fm for the spin-dependent potential. If we choose smaller values of the size parameter $\beta$, e.g. 0.2 fm, the weak transition matrix element is enhanced and overestimates the experimental data by order of magnitudes. In this work, we have retained the non-relativistic kinematics so far. Since the mass of the constituent quark is comparable with the kinetic energy, we should consider relativistic corrections to both the wave function and the calculation of the weak decay matrix elements. Inclusion of such relativistic corrections also changes the value of size parameter $\beta$. The study of such a dependence is in progress.

In this paper, we concentrate on the calculation of the parity-conserving $P$-wave amplitudes, since they are particularly sensitive to the correlation strength.

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On the other hand, for the parity violating S-wave amplitudes, contributions from commutator, factorization, penguins and the negative parity baryon pole terms are comparable. This fact suggests that the S-wave amplitudes are not suitable to quantify the quark-quark correlation. Nevertheless, it is possible to calculate the S-wave amplitude with our wave functions. Results for the commutator contributions amplitudes are also consistent with the data after including the penguin, factorization and negative baryon pole contributions. Such a study will be discussed in forthcoming paper.

§6. SU(6) symmetry breaking effects on the magnetic moment

In the previous sections we have evaluated the pionic weak transition amplitudes as well as the mass spectrum. Due to the strong spin-dependent attraction our wave function enhances the weak decay amplitudes and violates the naive SU(6) spin-flavor symmetry. As we have shown in Table II, the ratio of the matrix elements of the weak Hamiltonian $\frac{\langle p|O_1^{NR}|\Sigma^+\rangle}{\langle n|O_1^{NR}|\Lambda\rangle}$ shows a clear evidence for the SU(6) breaking. This ratio becomes $-2.45$ without the correlation in agreement with the SU(6) result $-\sqrt{6}$, while $-3.02$ with the quark correlation. Therefore, we estimate the size of the SU(6) breaking effects to be about 20%, which is significant.

On the other hand, it is historically known that the light baryon magnetic moments are well reproduced in the naive quark model by virtue of the SU(6) spin-flavor symmetry. Hence, it is also interesting and important to estimate effects of the SU(6) breaking by calculating the baryon magnetic moments.

Neglecting contributions from the quark orbital angular momentum, the operator for the baryon magnetic moment is simply given by

$$\vec{\mu}_{\text{mag}} = \sum_i \mu_i \sigma_i^z,$$

$$= \sum_i \frac{e}{2m_i} Q_i \sigma_i^z,$$  \hspace{1cm} (6.1)

where $Q_i$ is the electric charge operator of the $i$-flavor quark, and $i$ runs over $i = 1 \sim 3$. The magnetic moment of the baryon $B$ is obtained by directly calculating the matrix element

$$\mu_B = \langle B|\vec{\mu}_{\text{mag}}|B\rangle$$  \hspace{1cm} (6.2)

in terms of our wave functions. This quantity certainly depends on the internal spin-flavor structure of the baryon through $m_i, Q_i$ and $\sigma_i^z$.

Our results are shown in Table IV. In the first column, we show the results with the spin-dependent force, and those without the correlations in the second column which are the same as the naive SU(6) results. It is manifest that the results are almost unchanged even after introducing the spin-dependent correlations. The differences are of order of a few % in any cases.

These results tell us that the global spin-flavor structure of quark wave functions (integrated over volume) is quite insensitive to the existence of the quark-quark correlations. As a result, the success of the SU(6) symmetry for bulk baryon properties
Table IV. Baryon magnetic moments

|       | with $V_s$ | without $V_s$ | Exp.    |
|-------|------------|---------------|---------|
| $\mu_p$ | 2.78       | 2.84          | 2.792847|
| $\mu_n$ | -1.83      | -1.90         | -1.913042|
| $\mu_A$ | -0.602     | -0.613        | -0.613 ± 0.004|
| $\mu_{\Sigma^+}$ | 2.69       | 2.73          | 2.458 ± 0.010|
| $\mu_{\Sigma^-}$ | -1.05      | -1.06         | -1.160 ± 0.025|
| $\mu_{\Xi^0}$ | 0.817      | 0.836         | -       |
| $\mu_{\Xi^+}$ | -1.410     | -1.449        | -1.250 ± 0.014|
| $\mu_{\Xi^-}$ | -0.507     | -0.502        | -0.6507 ± 0.0025|
| $\mu_{\omega}$ | -1.84      | -1.84         | -2.02 ± 0.05|

seems to be maintained. On the other hand, the introduction of the quark correlation modifies the local structure of the quark wave function substantially shown in Fig. 5, and thus enhances the weak decay matrix elements.

The violation of the SU(6) spin-flavor symmetry in the local quark distributions can be found in other experiments, namely, the deep inelastic lepton scattering off the nucleon. Measured ratio of the momentum distribution functions $d(x)/u(x)$ at $x \sim 1$ tends to 0 in contrast to the SU(6) value $1/2$. This is one of the examples that demonstrates the breakdown of the local SU(6) spin-flavor symmetry. We also note that such a flavor dependence of the quark distribution functions is also explained by considering the quark-quark correlation in the $s = 0$ channel.

§7. Summary

In conclusion, we have studied the role of the spin-dependent quark-quark correlation in the baryon structure. We have pointed out that the non-leptonic weak transition of the hyperon is an unique quantity to investigate the quark correlation in the spin-0 channel. In particular, $\Sigma^+ \rightarrow n\pi^+\ P$-wave decay is free from the factorization or penguin contributions as shown in Table III, and thus serves a severe constraint on the quantitative understanding of the spin-0 correlation. In order to demonstrate its importance for the weak transition, we have introduced the non-relativistic constituent quark model with the spin-dependent attraction. We have solved three-body problem rigorously using Gaussian expansion method. Such a spin-dependent interaction may originate from the non-perturbative QCD dynamics. Results for static baryon properties reasonably agree with the empirical values.

We have developed the procedure to calculate the matrix elements of the weak Hamiltonian with a number of the rearrangement channels as discussed in section 3. Same technique can be applied to the strangeness $-2$-system, $\Xi$ hyperons, and such a study is in progress. Calculated transition amplitudes without the spin-dependent quark correlation agree with the SU(6) predictions perfectly. This result guarantees the accuracy of our numerical calculation. Resulting parity-conserving weak transition amplitudes are consistent with the experiments, when the quark correlation is turned on. At the present, quantitative description of the non-leptonic weak
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hyperon decay is still difficult in any models of the baryons\textsuperscript{2), 20, 21). Naive chiral
perturbation theory cannot explain the parity-conserving and violating amplitudes
simultaneously, and convergence of the chiral expansion seems to be worse\textsuperscript{22). Here,
we present a possible improvement of this long standing problem in the framework of
the constituent quark model, but still have several things to work out. For example,
the $\Lambda \to N\pi$ transition amplitudes are underestimated due to the large cancella-
tion in the pole formula, although they are strongly dependent on the choice of the
parameters.

Introduction of the quark correlation, which is strong enough to explain the non-
leptonic weak decay, modifies the valence quark structure of the baryons considerably.
The distance between quarks in the spin-0 pair becomes shorter by 20%. Although
the modification is not so large to induce the diquark-clustering of the nucleon, such
a tendency is consistent with the several phenomenological studies\textsuperscript{8), 23).

Although the spin-dependent correlation certainly violates the SU(6) spin-flavor
symmetry for the wave functions, we have found that the baryon magnetic moments
are almost unchanged compared with the prediction of the naive SU(6) model, as
demonstrated in Table IV. This is because the magnetic moments are quite insen-
sitive to the variation of the wave function at short distances $r, R \sim 0$. There is
a historical argument that the SU(6) symmetry assumption in the quark model is
indispensable to keep the impressive agreement of the magnetic moment with the
data. However, it is now evident that, even after including the SU(6) spin-flavor
breaking effects on the wave function, one can obtain quite reasonable values for the
baryon magnetic moments.

Recently, strong evidence for a new five-quark baryon state $\Theta^+(1540)$ (known as
a pentaquark) has been reported by several groups\textsuperscript{24)- 26). In Ref. 27), it is suggested
that the baryon is a bound state of four quarks and an antiquark, containing two
highly correlated $ud$ pair. The quark-quark correlation potential obtained in the
present work to explain the weak decay of hyperons as well as the baryon mass
spectrum would be useful in the study of the structure of the five-quark baryon.

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Appendix

We give some formulae to calculate the weak decay amplitudes. First we sum-
mearize the expressions for the baryon pole contributions to the parity conserving $\Lambda$
and $\Sigma$ weak decay.

$$B(\Lambda \to n\pi^0) = \frac{M_N + M_\Lambda}{f_\pi} \left[ G_{\pi n} \frac{1}{M_\Lambda - M_N} \langle n | H_{PC} | \Lambda \rangle + \langle n | H_{PC} | \Sigma^0 \rangle \frac{1}{M_N - M_\Sigma} G_{\Lambda \Sigma} \right] (A\cdot 1)$$
\[ B(\Sigma^+ \to p\pi^0) = \frac{M_N + M_\Sigma}{f_\pi} \left[ G^\pi p_{pp} \frac{1}{M_\Sigma - M_N} \langle p|H_{PC}|\Sigma^+\rangle + \langle p|H_{PC}|\Sigma^+\rangle \frac{1}{M_N - M_\Sigma} G^\pi_{p\Sigma^+} \right] \]

\[ B(\Sigma^+ \to n\pi^+) = \frac{M_N + M_\Sigma}{f_\pi} \left[ G^\pi_{pn} \frac{1}{M_\Sigma - M_N} \langle n|H_{PC}|\Sigma^+\rangle + \langle n|H_{PC}|\Sigma^+\rangle \frac{1}{M_N - M_A} G^\pi_{\Sigma^+ A} \right] \]

where \( H_{PC} \) is the parity conserving part of the weak Hamiltonian;

\[ H_{PC} = G_F \sin \theta \cos \theta \sqrt{2} \left( c_1 [(\bar{d}\gamma^\mu u)(\bar{u}\gamma^\mu s) + (\bar{d}\gamma^5\gamma^\mu u)(\bar{u}\gamma^5\gamma^\mu s)] 
+ c_2 [(\bar{u}\gamma^\mu u)(\bar{d}\gamma^\mu s) + (\bar{u}\gamma_\mu\gamma^\mu u)(\bar{d}\gamma_\mu\gamma^\mu s)] \right), \] (A-4)

where we use only the current-current operators \( O_1, O_2 \) which give dominant contributions here. We omit the penguin contributions, since we are interested in the absolute magnitudes of the weak transition amplitudes. We use the following parameters of the weak Hamiltonian, \( G_F = 1.16639 \times 10^{-5}\text{GeV}^{-5} \), \( \sin \theta = 0.219 \), \( \cos \theta = 0.975 \), and \( f_\pi = 92\text{MeV} \).

As for the axial vector coupling constant, we adopt the SU(3) Goldberger-Treiman relation following to the standard approach.

\[ G^\pi_{nn} = G^\pi_{pp} = \frac{f_\pi}{2M_N} (f + d) g \] (A-5)

\[ G^\pi_{A\Sigma^0} = G^\pi_{\Sigma^+ A} = \frac{2f_\pi}{\sqrt{3}(M_N + M_A)} d g \] (A-6)

\[ G^\pi_{\Sigma^+ \Sigma^+} = -G^\pi_{\Sigma^+ \Sigma^0} = \frac{2f_\pi}{2M_\Sigma} f g \] (A-7)

\[ G^\pi_{p\Sigma^0} = \sqrt{2} \frac{f_\pi}{2M_N} (f + d) g \] (A-8)

where \( g \) is the strong \( \pi NN \) coupling constant, and \( f, d \) the SU(3) coupling with a condition \( f + d = 1 \). We use \( f = 0.38 \) and \( d = 0.62 \) to obtain the numerical results presented in this paper. As pointed out in the text, the pole contribution \( \Lambda \to N\pi \) is very sensitive to the choice of \( f, d \).

The matrix elements of the weak Hamiltonian must be evaluated with calculated quark wave functions. The isospin symmetry for \( u \) and \( d \) quarks implies a relation

\[ \langle n|H_{PC}|\Sigma^0\rangle = \frac{1}{\sqrt{2}} \langle p|H_{PC}|\Sigma^+\rangle \] (A-9)

which exactly holds with our wave functions.

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