Seeds of Large Scale Anisotropy in Pre-Big-Bang Cosmology

Proceedings of the Spanish Relativity Meeting
Bilbao, 1999

Mairi Sakellariadou

Institut des Hautes Etudes Scientifiques, 91440 Bures-sur-Yvette, France
DARC, Observatoire de Paris, UPR 176 CNRS, 92195 Meudon Cedex, France
E-mail: mairi@amorgos.unige.ch

Abstract

Within a string cosmology context, the large scale temperature anisotropies may arise from the contribution of seeds to the metric fluctuations. We study the cases of electromagnetic and axion seeds. We find that massless or very light axions can lead to a flat or slightly tilted blue spectrum, that fits current data.

1 Introduction

I will briefly present some results on the seeds of large scale anisotropy in the context of string cosmology. I work on the pre-big-bang scenario (PBB), defined as a particular model of inflation inspired by the duality properties of string theory. The question which I address is whether we can reproduce the observed amplitude and slope of the large scale temperature anisotropy and of large scale density perturbations within the PBB scenario.

First-order scalar and tensor metric perturbations lead to primordial spectra that grow with frequency, with a normalization imposed by the string cut-off at the shortest amplified scales. These blue spectra have too little power at scales relevant for the observed anisotropies in the cosmic microwave background radiation (CMBR). In contrast, the axion energy spectra where found to be logarithmically diverging, leading to red spectra of CMBR anisotropies which are in conflict with observations. These results already ruled out four dimensional PBB cosmology.

However, if one allows for internal contracting dimensions in addition to the three expanding ones, the supersymmetric partner of the dilaton (the universal axion of string theory) can lead to a flat Harrison-Zel’dovich (HZ) spectrum of fluctuations for an appropriate relative evolution of the external and the compactified internal dimensions. Thus, the PBB scenario may contain a natural mechanism for generating large scale anisotropy via the seed mechanism (i.e., fluctuations of one-component of the energy momentum tensor can feed back on the metric through Einstein’s equations).

In what follows, I consider the possibility that vacuum fluctuations of the electromagnetic and of the axion field may act, at second order, as scalar seeds of large scale structure and CMBR anisotropies. The induced perturbations are isocurvature perturbations. More precisely, I examine whether the metric perturbation spectrum triggered by these seeds can be flat enough to match present measurements, consistently with the COBE normalization of the amplitude on large scales, and with the normalization imposed by the string cut-off at the shortest amplified scales.

2 Large scale perturbations in the presence of seeds

I will derive a general formula for large scale CMBR anisotropies in models where perturbations are triggered by seeds. I consider the case of scalar perturbations.
2.1 Cosmological perturbation theory with seeds

We express the Fourier components of the energy momentum tensor of the seeds, \( T_{\mu\nu} \), in terms of four scalar “seed functions” \( f_\rho, f_p, f_v \) and \( f_\pi \):

\[
\begin{align*}
T_{00}(k, \eta) &= M^2 f_\rho(k, \eta), \\
T_{20}(k, \eta) &= -iM^2 k_j f_v(k, \eta), \\
T_{ij}(k, \eta) &= M^2 \left( \left( f_p(k, \eta) + \frac{k^2}{3} f_\pi(k, \eta) \right) \gamma_{ij} - k_i k_j f_\pi(k, \eta) \right). 
\end{align*}
\]  

Here \( M \) denotes an arbitrary mass scale, introduced for dimensional reasons; \( \eta \) denotes conformal time and \( \gamma \) represents a metric of constant curvature.

The perturbed Einstein’s equations read:

\[
\begin{align*}
&k^2 \Phi = 4\pi G \rho a^2 D + \epsilon \left[ f_\rho + 3(\dot{a}/a) f_v \right], \\
&\Phi + \Psi = -8\pi G a^2 k^{-2} \rho \Pi - 2\epsilon f_\pi,
\end{align*}
\]

where \( \epsilon = 4\pi GM^2 \), \( a \) is the scale factor and dot denotes derivative with respect to \( \eta \). \( \Pi \) is the anisotropic stress potential, \( V \) is the peculiar velocity potential, \( D \) (and \( D_g \) which I will use later) is a gauge-invariant density perturbation variable. \( \Phi, \Psi \) are two geometric quantities, called the Bardeen potentials. Since large scale CMBR anisotropies are induced at recombination and later, we set \( \Pi = 0 \).

The large scale anisotropies of CMBR are determined by the combination \( \Psi - \Phi \):

\[
\Psi - \Phi \sim \max \left\{ \epsilon f_\pi, \epsilon \eta^2 \left( f_\rho + 3(\dot{a}/a) f_v \right) \right\}.
\]  

2.2 The seed contribution to CMBR anisotropies

I calculate the CMBR anisotropies and their contribution to \( \Delta T/T \) via the Sachs-Wolfe effect. The temperature perturbation reads:

\[
\left< \frac{\delta T}{T} (n) \frac{\delta T}{T} (n') \right> = \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell P_\ell (\cos \vartheta).
\]  

where \( P_\ell \) is the Legendre polynomial of degree \( \ell \). The brackets denote spatial average, or expectation values if perturbations are quantized. To determine the \( C_\ell \) we Fourier-transform Eq. (8), defining

\[
\varphi(k) = \frac{1}{\sqrt{V}} \int_V \varphi(x) e^{ik\cdot x} d^3 x,
\]

For the coefficients \( C_\ell \) of Eq. (8) we obtain:

\[
C_\ell = \frac{2}{\pi} \int \frac{\left< |\Delta_\ell(k)|^2 \right>}{(2\ell + 1)^2} k^2 dk,
\]

where

\[
\frac{\Delta_\ell}{2\ell + 1} = \frac{1}{4} D_g(k, \eta_{\rm dec}) j_\ell(k\eta_0) - j'_\ell(k\eta_0) V(k, \eta_{\rm dec})
+ k \int_{\eta_{\rm dec}}^{\eta_0} (\Psi - \Phi)(k, \eta') j'_\ell(k\eta_0 - k\eta') d\eta'.
\]
and $j'_\ell$ stands for the derivative of $j_\ell$ with respect to its argument. On large angular scales, $k\eta_{\text{dec}} \ll 1$, the SW contribution dominates and we obtain

$$C_{\ell}^{\text{SW}} = \frac{2}{\pi} \int k^4 dk \left\langle \left[ \int_{\eta_{\text{dec}}}^{\eta_{\text{eq}}} (\Psi - \Phi)(k, \eta) j'_\ell (k\eta_0 - k\eta) \, d\eta \right]^2 \right\rangle. \quad (12)$$

We approximate the Bardeen potentials $\Psi, \Phi$ on super-horizon scales by a power-law spectrum: $\langle |\Psi - \Phi|^2 \rangle = C^2(k) (k\eta)^{2\gamma}$. Furthermore, we consider models where the seed contribution does not grow in time on sub-horizon scales. Thus,

$$\Psi - \Phi \approx \begin{cases} C(k)(k\eta)^\gamma, & k \eta \ll 1 \\ C(k), & k \eta \gg 1. \end{cases} \quad (13)$$

We further assume that also $C(k)$ is given by a simple power law. Thus, we have

$$C(k) = \begin{cases} Nk^{-3/2}(k/k_1)^\alpha, & k \leq k_1 \\ 0, & k > k_1, \end{cases} \quad (14)$$

where $N$ is a dimensionless constant, and $k_1$ denotes a comoving cutoff scale. Inserting Eq. (14) in Eq. (12), we obtain

$$C_{\ell}^{\text{SW}} \approx N^2 \frac{2}{\pi} \int_{(k\eta)_{\text{dec}}}^{k_1} \frac{dk}{k} \left( \frac{k}{k_1} \right)^{2\alpha} |I(k)|^2, \quad (15)$$

where

$$I(k) = \int_{(k\eta)_{\text{dec}}}^{1} d(k\eta)(k\eta)^\gamma j'_\ell (k\eta_0 - k\eta) + j_\ell (k\eta_0 - 1). \quad (16)$$

We compare $C_{\ell}^{\text{SW}}$ with the inflationary result: $C_{\ell}^{\text{SW}} \propto \Gamma(\ell - 1/2 + n/2)/\Gamma(\ell + 5/2 - n/2)$, where $n$ denotes the spectral index. The scale-invariant spectrum, as it has been found by the DMR experiment, requires $0.9 \leq n \leq 1.5$. Thus, we get

$$-0.05 < \gamma + 1 + \alpha < 0.25, \quad \gamma \leq -1, \quad n \simeq 3 + 2(\alpha + \gamma) \quad (17)$$

and

$$-0.05 < \alpha < 0.25, \quad \gamma > -1, \quad n = 1 + 2\alpha. \quad (18)$$

3 Seeds from string cosmology

In this section we compute the seed functions $f_\rho, f_v, f_\pi$, and we estimate the Bardeen potentials for electromagnetic and axion perturbations.

3.1 Electromagnetic seeds

Consider a stochastic background obtained by amplifying the quantum electromagnetic fluctuations of the vacuum. For purely magnetic seeds (the electric component of the stochastic background is rapidly dissipated, due to the conductivity of the cosmic plasma), on super-horizon scales we obtain $f_v = 0, f_\pi \gg \eta^2 f_\rho$, leading to

$$k^3 |\Psi - \Phi|^2 (k, \eta) \approx N^2(k\eta)^{2\gamma}(k/k_1)^{2\alpha}, \quad (19)$$

with

$$\gamma = \begin{cases} -4, & \mu \leq 3/4 \\ 2\mu - 11/2, & 3/4 \leq \mu \leq 3/2 \end{cases} \quad (20)$$

$$\alpha = \begin{cases} 7/2, & \mu \leq 3/4 \\ 5 - 2\mu, & 3/4 \leq \mu \leq 3/2 \end{cases} \quad (21)$$

$$N = \left( \frac{H_1(M_p)}{4\pi} \right)^2 (k_1\eta_{\text{eq}})^2, \quad \text{in both cases}. \quad (22)$$
(μ < 3/2 to avoid photon overproduction.) \( H_1 \) is the physical cut-off scale at which the universe becomes immediately radiation-dominated, and \( M_p \) is the Planck mass.

Since, in both cases \( \gamma + 1 < 0 \), the seeds decay fast enough outside the horizon. However, in both cases \( \gamma + \alpha = -0.5 \), which implies \( n = 2 \). Such a spectrum grows too fast with frequency to fit the COBE measurements. The quadrupole amplitude

\[ Q_{rms-PS} = \sqrt{\langle 5/4\pi \rangle C_2 T_0} = (18 \pm 2) \mu K \text{ leads to } C_2 = (1.09 \pm 0.23) \times 10^{-10}. \]

Thus, compatibility with the COBE normalization implies

\[ (6 - \alpha) \log_{10}(H_1/M_p) \lesssim 55(\alpha - 2) - 6 + \log_{10}(\gamma + 1)^2. \] (23)

This constraint is easily satisfied by a growing seed spectrum, \( \alpha > 2 \). Even in the limiting case \( \alpha = 2 \), this condition is marginally compatible even with the maximal expected value \( H_1 \sim M_s \sim 5 \times 10^{17} \text{ GeV.} \)

### 3.2 Axionic seeds

We consider pseudo-scalar vacuum fluctuations amplified by the time evolution of a higher dimensional background. We first consider massless axions. If \( \mu < 3/4 \), the situation is like for electromagnetic seeds. The induced CMBR fluctuations have the wrong spectrum, but their amplitude is sufficiently low to avoid conflict with observations. However, if \( 3/4 \leq \mu \leq 3/2 \), we obtain

\[ k^3 |\Psi - \Phi|^2 (k, \eta) \approx \mathcal{N}^2 (k\eta)^2 (k/k_1)^{2\alpha}, \quad \text{with} \quad \gamma = 2\mu - 7/2, \quad \alpha = -2\mu + 3. \] (24)

For \( \mu = 3/2 \) we obtain a Harrison-Zel’dovich spectrum with amplitude \( \mathcal{N} \sim (H_1/M_p)^2 \). The non-conformal coupling of the axions to the metric leads to an additional amplification of perturbations after the matter-radiation equality. The normalization of the axion spectrum to the COBE amplitude imposes the constraint

\[ \log_{10} \frac{H_1}{M_p} \approx \frac{164 - 116\mu}{1 + 2\mu}, \quad \text{with} \quad 1.4 < \mu < 1.5, \] (25)

implying

\[ 3 \times 10^{-3} \lesssim (H_1/M_p) \lesssim 2.6. \] (26)

This condition is perfectly compatible with \( H_1 \sim M_s \sim 5 \times 10^{17} \text{ GeV.} \)

Let us now turn to the case of massive axions. In this case, the \( f_\pi \) contribution to \( \Phi, \Psi \) is negligible when the super-horizon modes are already non-relativistic at the time of decoupling, and we obtain constant Bardeen potentials with

\[ \gamma = 0, \quad \alpha = 3 - 2\mu, \quad \mathcal{N} = (H_1/M_p)^2 (m/H_{eq})^{1/2}, \] (27)

where \( m \) denotes the axion mass. For \( \mu = 3/2 \) we obtain a flat Harrison-Zel’dovich spectrum. The amplitude of perturbations is enhanced by the factor \( (m/H_{eq})^{1/2} \). Thus, the axion mass, \( m \), has to be bounded to avoid conflicting with the COBE normalization \( C_2 \approx 10^{-10} \). In addition, we impose \( 1.4 < \mu < 1.5 \), \( m > H_{dec} \sim H_{eq} \), and we require that the present axion energy density is constrained by the critical energy density. We find that for a typical scale \( H_1 \sim M_s \sim (10^{-1} - 10^{-2}) M_p \), the maximal allowed axion mass window is

\[ 10^{-27} \text{ eV} \lesssim m \lesssim 10^{-17} \text{ eV}. \] (28)

### 4 Conclusions

In this talk, I briefly discussed, in the context of the PBB scenario, the possibility that the large scale temperature anisotropies may arise from the contribution of seeds to the metric fluctuations. In particular, I considered the cases in which the seed inhomogeneity spectrum is due to vacuum fluctuations of the electromagnetic field and of the (Kalb-Ramond) axion field.
In the first case, I showed that electromagnetic fluctuations lead to a spectrum that grows too fast with frequency to be compatible with COBE observations. Since the contribution of electromagnetic seeds to the large scale anisotropy is negligible, there are no constraints from the COBE normalization to the production of seeds for generating the galactic magnetic fields, via the amplification of electromagnetic vacuum fluctuations due to a dynamical dilaton background

In the second case, I discussed how a stochastic background of massless axions, produced within the context of the PBB scenario, is a possible candidate for an explanation of the large scale anisotropy measured by COBE satellite. Regarding massive axions, I showed that if the axion mass is such that all modes outside the horizon at decoupling are already non-relativistic, then a slightly tilted blue spectrum is still compatible with the amplitude and slope measured by COBE satellite, provided the axion mass is inside an appropriate window, in the ultra-light mass region.

As a next step, one has to study the predictions of this model regarding the acoustic peaks in the CMBR anisotropy power spectrum and the linear dark matter power spectrum and compare them with currently available experimental and observational data. Some preliminary results are discussed in Ref. [11]. The authors found a strong dependence of their predictions on the overall evolution of extra dimensions during the PBB phase. In other words, further experimental and observational data coming from the CMBR anisotropies and the galaxy distribution, may provide some information about the evolution of string theory’s extra dimensions.

Acknowledgments

It is a pleasure to thank the organizers of the 1999 Spanish Relativity Meeting for inviting me to present this work. Many thanks also to Ruth Durrer, Maurizio Gasperini and Gabriele Veneziano, with whom I collaborated on the papers I presented here at Bilbao.

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