Nonlinear dynamic modeling and analysis for aviation axial piston pump in center spring return mechanism

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Abstract
Aviation axial piston pump is an important energy conversion component in aircraft hydraulic system, and the study on the nonlinearity of key operating mechanism is very necessary for improving the reliable operation of the pump. Addressing the aviation axial piston pump, this article analyzes nonlinear dynamic characteristics of the center spring return mechanism by using theoretical analysis and numerical simulation. Firstly, mathematical models of oil-film forces of slipper/swash plate pair and cylinder/port plate pair are derived. Then, based on the nonlinear dynamic theory, some specific theoretical analysis methods such as Lyapunov index spectrum, bifurcation diagram, time series, phase plane portrait, Poincaré map, and power spectrum are used to carry out the numerical simulation of nonlinear dynamic behaviors. The result indicates that characteristics of the center spring can reveal the reason for the nonlinear vibration of the center spring return mechanism. Meanwhile, the behavior of the oil thickness of slipper/swash plate pair is in the periodic and steady motion, and the variation of the oil thickness of cylinder/port plate pair is in chaotic vibration, which establishes the theoretical foundation of the aviation axial piston pump for the traceability and suppression of vibration. The obtained results are of important guidance significance for optimal design and dynamic control of the center spring return mechanism of aviation axial piston pump.

Keywords
Aviation axial piston pump, center spring return mechanism, nonlinear dynamics, chaotic vibration

Introduction
In order to improve the power density of aviation axial piston pump, it is a great challenge to make the pump at a high speed and high-pressure working condition, when considering the local elastic-plastic stress deformation and thermal deformation of the rotating component. The aviation piston pump is in nature a complex nonlinear, time-variant, and multi-variable system, which contains the correlation among hydraulic system, mechanical system, and the electric control system. The anomalies and failures of the center spring return mechanism (CSRM) are often caused by nonlinear vibration, noise, leakage, impact and so on, which seriously affect the motion stability and control precision of aviation axial piston pump. A specific and accurate mathematical model of CSRM can be beneficial to predict nonlinear dynamic characteristics and calculate practical operation issues of aviation axial piston pump. Therefore, it is significant to study nonlinear dynamic behaviors of CSRM of aviation axial piston pump.

In recent years, nonlinear dynamic behaviors of rotating machinery components of hydraulic system have been investigated by many scholars, mainly including servo hydraulic cylinder and control valve. Jerrelind and Stensson summarized previous works on nonlinear dynamics of engineering products. Wang et al. investigated nonlinear characteristics of hydraulic cylinder. The result shows the creeping motion in hydraulic cylinder is caused by characteristics of soft spring and the self-excited vibration is given to the nonlinear friction force. Zhu et al. summarized previous works on nonlinear dynamics of engineering products.
analyzed the nonlinear dynamics behavior of the electrohydraulic servo system. This research indicates that the variation of excitation force, spring force nonlinear term, and damping can result in complicated nonlinear dynamic behaviors. In addition, Hayashi12 investigated the instability and chaotic phenomenon occurring in a pilot-type poppet valve circuit. Licsko13 established a nonlinear dynamic mathematical model of the single stage relief valve embedded within a simple hydraulic circuit. Meanwhile, chaos suppression and nonlinear controller are widely analyzed in nonlinear system. Lin et al.14 used a sliding mode control to make the chaos suppression in coronary artery chaotic system. Yau and Yan15 proposed a proportional-integral switching surface to assign the performance of the systems in the sliding mode motion easily, and the result shows the obtained controller can achieve high precision and long-range positioning performance and robustness by the proposed control approach.

In this study, by establishing mathematical models of oil-film forces of slipper/swash plate pair and cylinder/port plate pair, the dynamic model of CSRM should be built, which is the precondition of investigating the nonlinear dynamic characteristics of CSRM. Koc and Hooke16 discussed the stress sustained by slipper pair in axial piston pump. The result shows that the compression force of the slipper is supported by the static pressure, and the residual compression force is balanced by the dynamic pressure. Hashemi et al.17 measured the friction force between slipper and swash plate from a newly developed axial piston pump. Xu et al.18 derived a numerical mode of oil-film forces in piston/cylinder pair which was taking influences of roughness, elastic deformation of piston, and pressure–viscosity effect into consideration. Zhang et al.19 investigated a new dynamic seven-stage model for the thickness prediction of the film of cylinder/port plate pair in axial piston pump.

So far, these present literatures about the nonlinear dynamic characteristics of hydraulic components and system mainly focus on hydraulic servo cylinder, servo valve and servo system; however, the nonlinear dynamic analysis of CSRM of aviation axial piston pump is also significant but the relevant studies are very poor. Moreover, mathematical models of the extrusion force and thermal wedge force of slipper/swash plate pair and the extrusion force of cylinder/port plate pair are rarely analyzed either. In this work, a mathematical model of CSRM of aviation axial piston pump was derived, and equations of oil-film forces of slipper/swash plate pair and cylinder/port plate pair were built. Then, the nonlinear dynamic characteristics of CSRM would be analyzed by using special identification techniques such as bifurcation, Lyapunov index spectrum, time series, phase plane portrait, Poincaré map, and power spectrum.

The rest of this article was organized as follows. In the upcoming section, the nonlinear dynamic model of CSRM of aviation axial piston pump would be derived. Numerical simulations and results were analyzed by using MATLAB software in “The nonlinear dynamic analysis of CSRM” section. Finally, some concluding remarks were presented in the final section.

The CSRM model and mathematical formulation

The diagram of schematic structure and all forces of CSRM system of aviation axial piston pump are shown in Figure 1. The center spring pushes the cylinder block to the port plate, meanwhile, under the action of center spring force, the push-rod can push the spherical hinge to the back-strake plate through the washer, then the back-strake plate can drive the slipper pair returning.

In this work, the CSRM can be equivalently simplified to spring damping system with two mass blocks. The equivalent mass block on the left side of the center spring mainly includes piston, slipper, spherical hinge, push rod, and back strake plate. The equivalent mass block is influenced by the coupling action of multiple forces, mainly including the axial hydraulic pressure, the piston inertia force, the nonlinear spring force, the friction between the piston and cylinder, and the oil-film force of slipper/swash plate pair. The mass block on the right side of the center spring is cylinder. The cylinder is also under the coupling action of multiple forces, including the axial hydraulic pressure, the nonlinear spring force, the friction force and the oil-film force of cylinder/port plate pair. In this study, we can analyze the nonlinear dynamic characteristics of CSRM by researching variations of oil thickness of slipper/swash plate pair and cylinder/port plate pair. The equivalent mass model of CSRM is illustrated in Figure 2.

The notations in following equations are given in the Appendix. The differential equation of CSRM can be obtained by Newton’s second law

$$\begin{align*}
M_1 \frac{d^2x}{dt^2} &= N_n - (F_{np} + F_k + k_{CT}(x+y) + a(x+y) + \lambda(x+y)^3 + F_t - F_g)\cos\beta \\
M_2 \frac{d^2y}{dt^2} &= F_{ic} + F_k + k_{CT}(x+y) + a(x+y) + \lambda(x+y)^3 + F_t - F_{EX}
\end{align*}$$

(1)
where $F_n$ is the axial hydraulic force acting on the bottom of piston, it can be calculated as

$$ F_n = \frac{5}{4} \pi d_p^2 P_d $$

The piston inertia force $F_g$ is

$$ F_g = \sum_{i=1}^{9} m_x a_p = m_x \omega R^2 \tan \beta (\cos \phi - 4 \sqrt{2} \sin \phi) $$

The effect of the nonlinear center spring on the CSRM can be represented by Duffing equation. The center spring force $F_s$ can be calculated as

$$ F_s = F_k + k_{CT}(x + y) + a(\dot{x} + \dot{y}) + \lambda(x + y)^3 $$

where $F_k$ represents the preload force of center spring.

The diagram of schematic structure and all forces of piston are shown in Figure 3. The friction between the piston and cylinder $F_f$ is

$$ F_f = -\frac{\mu \rho P}{h_{cp}} 2 \pi r_p l_{cp} = \frac{2 \pi R \sigma r_p l_{cp}}{h_{cp}} \tan \beta \sin \phi $$

where $h_{cp}$ represents the oil thickness between cylinder block and piston.
The slipper pair not only makes reciprocating rectilinear movement along the cylinder, but also makes a circular motion on the swash plate at a high speed. These oil-film forces of slipper/swash plate pair should be analyzed in detail, which includes the extrusion force of oil film caused by the oil squeeze, and the thermal wedge force caused by the oil thermal expansion. In this study, there are five pistons in the absorption area, and four pistons in the pressure area. The oil-film force of slipper/swash plate pair $N_n$ can be calculated as

$$N_n = 4N_{EX} + 9N_{TH} \quad (6)$$

The structure schematic diagram of slipper is shown in Figure 4. When the piston enters the oil pressure zone, there is a certain initial oil thickness of slipper/swash plate pair. Based on Reynolds equation $\frac{\partial}{\partial r} \left( \frac{h_1^3}{12} \frac{\partial p}{\partial r} \right) = r \frac{\partial h_1}{\partial r}$, the oil-film pressure distribution is affected by the oil squeeze, and then the additional bearing force will be formed to withstand the load change. Ignoring the effect of the temperature on the hydraulic oil viscosity, the equation of pressure distribution on the bottom of slipper can be obtained as

$$p = -\frac{3\mu^2}{h_1^3} \frac{\partial h_1}{\partial t} + \frac{\mu C_1}{h_1^3} \ln r + C_2 \quad (7)$$

These constants can be obtained from boundary conditions $P = P_0$ at $r = R_1$, $P = P_d$ at $r = R_2$, and the pressure distribution expression can be calculated as

$$p = P_d - \frac{P_d - P_0}{\ln(R_1/R_2)} \ln(r/R_2) + \frac{3\mu}{h_1^3} \frac{\partial h_1}{\partial t} \left[ (r^2 - R_2^2) - (R_1^2 - R_2^2) \ln(r/R_2) \right] \quad (8)$$

If the lateral of slipper is under the atmospheric pressure, that is $P_0 = 0$. And with the bearing force in oil-in chamber of slipper, the expression of the extrusion force $N_{EX}$ can be given by

$$N_{EX} = \pi R_2^2 P_d - \pi R_1^2 P_0 + \int_0^{2\pi} \int_{R_1}^{R_2} p r dr d\theta = \frac{\pi (R_1^2 - R_2^2)}{2 \ln(R_1/R_2)} P_d + \frac{3\pi\mu}{2h_1^3} \frac{\partial h_1}{\partial t} \left[ (R_1^4 - R_2^4) - (R_1^2 - R_2^2)^2 \ln(R_1/R_2) \right] \quad (9)$$
In equation (9), the former represents the hydrostatic bearing force under the load stabilization, and the latter represents the extrusion bearing force caused by the oil squeeze in case of load variation.

In this work, the slipper pair is parallel to the swash plate, and the loss of leakage and friction power will be converted into energy, then the thermal wedge force will be formed by the oil thermal expansion. The structure of the bottom of slipper is shown in Figure 5.

For simplicity, the bottom of slipper can be divided into two parts, one part represents sector areas from \( u_0 \) to \( \pi - \phi_0 \) in the upper and lower parts, and the other part represents sector areas from \(-\phi_0\) to \(\phi_0\) in the left and right parts. Ignoring the effect of temperature on the hydraulic oil viscosity, the equation of thermal wedge force of the infinitesimal element \( Ldy \) on the bottom of slipper can be written as

\[
dF = \frac{2\beta_z \mu_0 \omega R_1}{C_p \rho h_1^2} L^2 dy
\]

Integrating in two areas separately yields

\[
N_{TH1} = 4 \int_{\phi_0}^{\pi/2} 2\beta_z \mu_0 \omega R_1 \frac{4k R_2 \cos \theta}{C_p \rho h_1^2} (k R_2 \cos \theta)^3 d\theta = \frac{32 \beta_z \mu_0 \omega R_1 R_2^3}{C_p \rho h_1^2} \left( \frac{2}{3} - \sin \phi_0 + \frac{1}{3} \sin^3 \phi_0 \right)
\]

\[
N_{TH2} = 4 \int_{\phi_0}^{\pi/4} 2\beta_z \mu_0 \omega R_1 \frac{4(k - 1)^2 R_2^3 \cos \varphi d\varphi}{C_p \rho h_1^2} = \frac{32 \beta_z \mu_0 \omega R_1}{C_p \rho h_1^2} (k - 1)^2 R_2^3 \sin \phi_0
\]

Then, the thermal wedge force \( N_{TH} \) can be calculated as

\[
N_{TH} = N_{TH1} + N_{TH2} = \frac{32 \beta_z \mu_0 \omega R_1 R_2^3}{C_p \rho h_1^2} \left[ \left( \frac{2}{3} - \sin \phi_0 + \frac{1}{3} \sin^3 \phi_0 \right) + (k - 1)^2 \sin \phi_0 \right]
\]

where \( k = \sin^{-1} \phi_0, \ k = \frac{R_i}{R_o} \).

The axial compression force acts on the stage at the bottom of the slipper, which can push the cylinder to the port plate. The equation of axial compression force \( F_{nc} \) can be calculated as

\[
F_{nc} = 5 \left[ \pi \frac{r_2}{p} - \frac{\theta}{2} (r_2^2 - r_1^2) \right] P_d
\]

where \( \theta \) represents the wrap angle of the kidney port of cylinder where piston is located.

In this study, the cylinder block is parallel to the port plate, when the piston enters the oil pressure zone, there is a certain initial oil thickness of cylinder/port plate pair. The structure of port plate is shown in Figure 6.
Based on Reynolds equation \( \frac{d}{dr} \left( \frac{h^3}{12 \mu} \frac{dP}{dr} \right) = r \frac{d}{dr} \left( \frac{\partial h}{\partial r} \right) \) the oil-film pressure distribution is affected by the oil squeeze, and then the additional bearing force will be formed to withstand the load change, the equation of pressure distribution on the external seal belt of port plate can be obtained as

\[
p = \frac{3 \mu r^2}{h^3} \frac{\partial h_2}{\partial t} + \frac{\mu C_1}{h_2^3} \ln r + C_2 \tag{15}
\]

The expression of constants can be obtained from boundary conditions \( P = P_0, \) at \( r = r_1 \), \( P = P_d, \) at \( r = r_2 \) as

\[
\begin{align*}
C_1 &= \frac{(P_0 - P_d) - \frac{3 \mu h_2}{h_2^3} \frac{\partial h}{\partial r}(r_1^2 - r_2^2)}{(\mu/h_2^3) \ln(r_1/r_2)} \\
C_2 &= P_d - \frac{3 \mu r_2^2}{h_3^3} \frac{\partial h_2}{\partial t} \left( \frac{(P_0 - P_d) - \frac{3 \mu h_2}{h_3^3} \frac{\partial h}{\partial t}(r_1^2 - r_2^2)}{\ln(r_1/r_2)} \right) \ln r_2
\end{align*}
\tag{16}
\]

Substituting equation (16) into equation (15), the pressure distribution on the external seal belt of port plate can be calculated as

\[
p = P_d - \frac{(P_d - P_0)}{\ln r_1/r_2} \ln(r/r_2) + \frac{3 \mu h_2}{h_2^3} \frac{\partial h_2}{\partial t} \left( (r^2 - r_2^2) - (r_1^2 - r_2^2) \frac{\ln(r/r_2)}{\ln(r_1/r_2)} \right) \ln r_2
\]

Then, the expression of extrusion force on the external seal belt of port plate can be obtained as

\[
F_{EX1} = \int_0^\Phi \int_{r_1}^{r_2} prdrd\theta = \frac{\Phi}{2} P_0 r_1^2 - \frac{\Phi}{2} P_d r_2^2 + \frac{\Phi (r_1^2 - r_2^2)}{4 \ln(r_1/r_2)} (P_d - P_0) + \frac{3 \Phi \mu h_2}{4h_2^3} \frac{\partial h_2}{\partial t} \left( (r_1^4 - r_2^4) - \frac{(r_1^2 - r_2^2)^2}{\ln(r_1/r_2)} \right) \ln (r_1/r_2) \tag{18}\]

where \( \Phi \) represents the wrap angle in the pressure area of port plate.

Similarly, the equation of pressure distribution on the internal seal belt of port plate can be obtained as

\[
p = \frac{3 \mu r^2}{h^3} \frac{\partial h_2}{\partial t} + \frac{\mu C_3}{h_2^3} \ln r + C_4 \tag{19}
\]
The expression of constants can be obtained from boundary conditions \( P = P_d \) at \( r = r_3 \), \( P = P_0 \) at \( r = r_4 \)

\[
\begin{align*}
C_3 &= \frac{(P_d - P_0) - 3\mu \frac{\partial h_2}{\partial t} (r_3^2 - r_4^2)}{(\mu h_2^3) \ln(r_3/r_4)} \\
C_4 &= \frac{P_d - 3\mu r_3^2 \frac{\partial h_2}{\partial t}}{h_2^3 \ln(r_3/r_4)} - \frac{(P_d - P_0) - 3\mu \frac{\partial h_2}{\partial t} (r_3^2 - r_4^2)}{\ln(r_3/r_4)} \ln r_3
\end{align*}
\]

(20)

Substituting equation (20) into equation (19), the pressure distribution on the internal seal belt of port plate can be calculated as

\[
p = P_d + \frac{(P_d - P_0)}{\ln r_3/r_4} \ln(r/r_3) + \frac{3\mu}{h_2^3} \frac{\partial h_2}{\partial t} \left[ (r^2 - r_3^2) - (r_3^2 - r_4^2) \frac{\ln(r/r_3)}{\ln(r_3/r_4)} \right]
\]

(21)

Then, the expression of extrusion force on the internal seal belt of port plate can be obtained as

\[
F_{EX_2} = \int_0^\phi \int_{r_3}^{r} pr dr d\phi = \frac{\Phi}{2} P_d r_3^3 - \frac{\Phi}{2} P_0 r_4^2 + \frac{\Phi}{4} \frac{r_3^2 - r_4^2}{4 \ln(r_3/r_4)} (P_d - P_0) + \frac{3\mu \Phi}{4h_2^3} \frac{\partial h_2}{\partial t} \left[ (r_3^4 - r_4^4) - \frac{(r_3^2 - r_4^2)^2}{\ln(r_3/r_4)} \right]
\]

(22)

With the hydraulic bearing force in the pressure area of port plate, the expression of the extrusion force can be calculated as

\[
F_{EX} = F_{EX_1} + F_{EX_2} + F_{EX} = \int_0^\phi \int_{r_3}^{r_1} p r dr d\phi + \int_0^\phi \int_{r_3}^{r_4} p r dr d\phi + \int_0^\phi \int_{r_3}^{r_2} P_d r dr
\]

(23)

If the lateral of port plate is under the atmospheric pressure, that is \( P_0 = 0 \). After rearranging yields

\[
F_{EX} = \frac{\Phi}{4 \ln(r_1/r_2)} P_d + \frac{\Phi}{4 \ln(r_3/r_4)} P_d + \frac{3\mu \Phi}{4h_2^3} \frac{\partial h_2}{\partial t} \left[ (r_3^4 - r_1^4) - \frac{(r_3^2 - r_1^2)^2}{\ln(r_1/r_2)} + (r_3^4 - r_4^4) - \frac{(r_3^2 - r_4^2)^2}{\ln(r_3/r_4)} \right]
\]

(24)

The nonlinear dynamic analysis of CSRM

In this section, the dynamic characteristics of CSRM can be researched by nonlinear analysis methods, and the numerical analysis of the differential equation of CSRM system can be solved by a variable step continuous solver based on the Runge–Kutta provided by MATLAB software. Moreover, in order to ensure that datum being analyzed can be in the steady state, the first about one million time series datum should be neglected, and the rest of series datum can be retained to carry out the numerical analysis. The major parameters of CSRM of aviation axial piston pump which were used in the numerical study are shown in Table 1.

Based on equation (1), the autonomous equation can be constructed, and the state equation of CSRM of aviation axial piston pump can be obtained as

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{M_1} \left[ N_n - (F_{np} + F_k + k_C T (x + y) + a(x + y) + \lambda(x + y)^3 + F_l - F_g \cos \beta) \right]
\end{align*}
\]

\[
\begin{align*}
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= \frac{1}{M_2} \left[ F_{nc} + F_k + k_C T (x + y) + a(x + y) + \lambda(x + y)^3 + F_l - F_{EX} \right]
\end{align*}
\]

\[
\dot{z} = \omega
\]

(25)
**Nonlinear dynamic simulation**

In this part, some specific nonlinear methods such as bifurcation diagram, the time series, power spectrum, phase plane portrait, and Poincaré map are used to analyze the nonlinear dynamics of CRSM of aviation axial piston pump. By analyzing the oil thickness variation of slipper/swash plate pair and cylinder/port plate pair, the nonlinear behavior of this system can be investigated in this work. The Lyapunov index provides an effective method to analyze the chaotic motion of CRSM. The Lyapunov index decides the average speed of exponential expansion or contraction in the direction of the beginning deviation \( y(0) \) on a trajectory, which is given by 
\[
\hat{\lambda}_i = \lim_{t \to \infty} \frac{1}{t} \ln \left( \frac{\|y(t)\|}{\|y(0)\|} \right),
\]
where the mark \( \| \| \) represents the vector norm, and \( \hat{\lambda}_i \) represents the Lyapunov exponent. If the Lyapunov index is negative or zero, the motion in this direction is in a regular condition, if the Lyapunov index is positive, the chaotic motion will be formed. In order to get a comprehensive analysis of nonlinear characteristics of CSRM, all Lyapunov indexes (Lyapunov exponential spectrum) should be carried out by Graham-Schmitt orthogonalization method. The Lyapunov index of CSRM is shown in Figure 7.

In this work, one of Lyapunov indexes is positive, which indicates the dynamic behavior of CRSM is divergence and instability in this direction and the system is sensitive to initial condition. Meanwhile, other Lyapunov indexes are negative, which represents dynamic behaviors of CRSM are convergent and stability in other

**Table 1.** The major parameters of CSRM of aviation axial piston pump.

| Name                                              | Symbol | Value  |
|---------------------------------------------------|--------|--------|
| The working pressure of axial piston pump (MPa)   | \( P_d \) | 21     |
| The swash plate angle (°)                         | \( \beta \) | 18     |
| The radius of piston distribution circle (mm)     | \( R \) | 16.6   |
| The radius of piston (mm)                         | \( r_p \) | 4.3    |
| The outside radius of piston sealing belt (mm)    | \( R_1 \) | 5.4    |
| The inside radius of piston sealing belt (mm)     | \( R_2 \) | 3.4    |
| The oil original thickness of slipper/swash plate pair (μm) | \( h_{10} \) | 20     |
| The oil original thickness of cylinder/port plate pair (μm) | \( h_{20} \) | 25     |
| The external radius of port plate external sealing (μm) | \( r_1 \) | 22.5   |
| The internal radius of port plate external sealing (μm) | \( r_2 \) | 19.1   |
| The external radius of port plate internal sealing (μm) | \( r_3 \) | 15.0   |
| The internal radius of port plate internal sealing (μm) | \( r_4 \) | 10.9   |
| The density of hydraulic oil (kg/m³)               | \( \rho \) | 880    |
| The hydraulic oil viscosity (m²/s)                 | \( \mu \) | \( 8.7 \times 10^{-3} \) |
| The specific heat capacity of hydraulic oil (J/(kg·°C)) | \( C_p \) | \( 2.3 \times 10^3 \) |

**Figure 7.** The Lyapunov index of the center spring return mechanism.
directions. By analyzing the Lyapunov exponential spectrum, it can be found that dynamic characteristics of CSRM system is in chaotic phenomena at the given speed.

Generally, the bifurcation diagram is used to research the nonlinear dynamics of the system under some controlling parameters. Since the chaotic responses of nonlinear system may be happened in an unstable area of frequency. For the CRSM of aviation axial piston pump, the rotational speed is a significant factor, which can be chosen as the control parameter to research the chaotic response of the system in bifurcation diagram. The bifurcation diagram versus angular velocity is shown in Figure 8.

![Figure 8. Bifurcation diagram: (a) the oil thickness of slipper/swash plate pair and (b) the oil thickness of cylinder and port plate.](image)

![Figure 9. Dynamic behaviors of the oil thickness of slipper/swash plate pair: (a) the time series, (b) power spectrum, (c) phase plane portrait, and (d) Poincaré map.](image)
In this simulation, the speed rate is $300 \text{ rad/s} < \omega < 628 \text{ rad/s}$, and the speed change step size is $1 \text{ rad/s}$. Figure 8(a) shows that bifurcation diagram of oil thickness of slipper/swash plate pair. During the whole working condition, the dynamic behavior of the oil thickness is regular and periodic. As shown in Figure 8(b), when the axial piston pump is running with $300 \text{ rad/s} < \omega < 628 \text{ rad/s}$, the motion of the oil thickness between cylinder block and port plate is in irregular and chaotic movement. The amplitude of CRSM chaotic responses is $0.0018\mu m$ at $300 \text{ rad/s} < \omega < 410 \text{ rad/s}$, after that, although the amplitudes of dynamic responses of system are lower than before, the existence of unstable area indicates that the system of CRSM has susceptibility of chaotic motion. Then, the amplitudes of this system will be heightened chaotic responses at the region $580 \text{ rad/s} < \omega < 650 \text{ rad/s}$.

In this study, the time series, power spectrum, phase plane portrait, and Poincaré map will be used together to analyze nonlinear dynamic behaviors of CSRM of aviation axial piston pump comprehensively. The dynamic behaviors of the oil thickness of slipper/swash plate pair are shown in Figure 9. The dynamic behaviors of the oil thickness of cylinder/port plate pair are shown in Figure 10.

**Nonlinear dynamic analysis**

Figure 9 shows the numerical results of dynamic behaviors of oil thickness between slipper/swash plate pair based on time series, power spectrum, phase plane portrait, and Poincaré map, respectively. The time series curve changes periodically during the whole simulation process. The phase plane portrait diagram consists of one curve, which can repeat in a certain region. Meanwhile, peaks appear at the fundamental and doubling frequencies on the power spectrum, and the Poincaré map has one single point. For a periodic motion, there are a few of isolated points in the Poincaré map. For a chaotic behavior, the return points on the Poincaré map will form the geometrical fractal structure. In conclusion, the behavior of the oil thickness of slipper/swash plate pair is in the periodic and steady motion.

![Figure 10](image-url). Dynamic behaviors of the oil thickness of cylinder/port plate pair: (a) the time series, (b) power spectrum, (c) phase plane portrait, and (d) Poincaré map.
In Figure 10, dynamic behaviors of oil thickness of cylinder/port plate pair can be researched too. There is no obvious regularity in the time series diagram. The phase plane portrait diagram consists of multiple irregular closed curves, and these curves are filled in the whole region. The power spectrum diagram seems continuous or broadband spectrum. At last, the strange attractor in the Poincaré map has the fractal structure. In conclusion, the result indicates the variation of oil thickness of cylinder/port plate pair is in chaotic motion, which establishes the theoretical foundation of the aviation axial piston pump for the traceability and suppression of vibration.

Summary and conclusions
Nonlinear dynamic characteristics of CSRM of aviation axial piston pump are discussed in this work. Mathematical equations of oil-film forces of slipper/swash plate pair and cylinder/port plate pair are derived, including the extrusion force based on Reynolds equation and the thermal wedge force. Through the theoretical analysis and simulation experiment, some conclusions about nonlinear dynamic behaviors of CRSM of aviation axial piston pump can be obtained.

The effect of nonlinear center spring force on the CSRM can be described by Duffing equation. Based on specific nonlinear theories about Lyapunov index spectrum, bifurcation diagram, time series, phase plane portrait, Poincaré map, and power spectrum, nonlinear dynamic characteristics of CSRM can be analyzed. In this work, the variation of oil thickness of slipper/swash plate pair is in the periodic and steady motion. Meanwhile, the motion of oil thickness of cylinder/port plate pair is in chaotic vibration, which leads to the mechanical nonlinear vibration of CSRM. This article can help to predict the nonlinear dynamics of the aviation axial piston pump, and to provide the theoretical basis for the structure design of axial piston pump in the future.

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**Appendix**

**Notation**

- $a$: center spring damper
- $f$: friction coefficient
- $h_1$: hydraulic oil thickness of slipper/swash plate pair
- $h_2$: hydraulic oil thickness of cylinder/port plate pair
- $k_{CT}$: center spring stiffness
- $m_s$: mass of slipper pair
- $M_1$: equivalent mass of block on the left side of the center spring
- $M_2$: mass of cylinder
- $x$: oil thickness variation of slipper/swash plate pair
- $x_1$: oil thickness variation of slipper/swash plate pair
- $x_2$: oil thickness change rate of slipper/swash plate pair
- $y$: oil thickness variation between port plate pair and cylinder
- $y_1$: oil thickness variation between port plate pair and cylinder
- $y_2$: oil thickness change rate between port plate pair and cylinder
- $z$: coefficient of autonomous equation
- $a_o$: coefficient of thermal expansion of hydraulic oil
- $\beta_e$: bulk modulus of hydraulic oil
- $\lambda$: nonlinear coefficient of center spring
- $\phi$: any rotation angle of the piston
- $\omega$: angular velocity of aviation axial piston pump