Prescribing net demand for electricity market clearing

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Abstract

We consider a two-stage electricity market comprising a forward and a real-time settlement. The former pre-dispatches the power system following a least-cost merit order and facing an uncertain net demand, while the latter copes with the plausible deviations with respect to the forward schedule by making use of power regulation during the actual operation of the system. Standard industry practice deals with the uncertain net demand in the forward stage by replacing it with a good estimate of its conditional expectation (usually referred to as a point forecast), so as to minimize the need for power regulation in real time. However, it is well known that the cost structure of a power system is highly asymmetric and dependent on its operating point, with the result that minimizing the amount of power imbalances is not necessarily aligned with minimizing operating costs. In this paper, we propose a mixed-integer program to construct, from the available historical data, an alternative estimate of the net demand that accounts for the power system’s cost asymmetry. Furthermore, to accommodate the strong dependence of this cost on the power system’s operating point, we use clustering to tailor the proposed estimate to the foreseen net-demand regime. By way of an

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illustrative example and a more realistic case study based on the European power system, we show that our approach leads to substantial cost savings compared to the customary way of doing.

**Keywords:** OR in Energy, Smart predict, Value-oriented forecasting, Data-driven decision-making under uncertainty, Electricity market

1. Introduction

Many decision-making processes under uncertainty can be modeled by optimization problems where some of the input parameters are not perfectly known. The field of Optimization under Uncertainty focuses on developing tools to tackle these problems depending on the knowledge of those parameters that the decision maker actually has. For example, if these parameters can be modeled reasonably well as random variables following certain probability distributions, then the decision maker should probably resort to stochastic programming techniques [1]. In contrast, if all the decision maker knows about said parameters is their range of variation or support, then she should rather opt for robust optimization methods instead [2].

In the realm of electricity markets and power system operations, there is a vast literature on OR methods, models, and algorithms for market clearing and power dispatch that rely on stochastic programming or robust optimization or hybrids of both. The richness of this literature makes it materially impossible and pointless to embrace it all in this paper. Instead, we refer the reader to monograph [3] and references therein for examples of market-clearing models based on stochastic programming, to the seminal work [4] on the application of robust optimization for unit commitment and power dispatch, and to the recent contribution [5] on a distributionally robust chance-constrained electricity market.
Despite the firm and promising advances in Optimization under Uncertainty, still one of the most widely extended practices in decision making is to replace the unknown parameter with a sensible value or estimate, some sort of “the most likely value” that the parameter can take on. A natural candidate to play that role is the expected value of the parameter. Thus, the decision maker can, in addition, exploit all the powerful tools that the disciplines of Statistics, Forecasting and Machine Learning have developed for decades to estimate that expected value conditional on all the information the decision maker has available at the moment the decision must be made. The adherence of the power sector to this strategy is particularly notorious, essentially because it is argued to be simpler, more transparent, computationally cheaper and easily accepted by the different stakeholders (see, e.g., [6, 7, 8, 9] for further details on this issue). However, OR researchers have repeatedly shown that this strategy results in suboptimal decisions in general, because the conditional expected value of the parameter’s uncertainty on the decision’s value 1.

Against this background, research efforts have been recently placed on finding a compromise solution. Intuitively, the idea is still to replace the uncertain parameter but with a point estimate (generally different from the parameter’s conditional expectation) that is purposely computed to result in the optimal or a nearly optimal decision in view of the parameter’s uncertainty. The term smart predict has been recently coined to refer to this midway solution strategy. In this line, we find a number of research works, e.g., [10, 11, 12, 13]. In particular, the authors in [10] propose a heuristic gradient-based procedure to produce estimates of uncertain parameters in optimization problems based on the objective of the task for which these estimates will be used. In [11], instead, they introduce a bilevel programming
framework to the same end. In [12], they deal with linear programs with an uncertain cost vector, for which they develop a tailored convex loss function to compute an estimate of the cost vector that accounts for the underlying linear optimization problem. Finally, the work in [13] is a continuation of that in [12] where the authors provide bounds on how well a certain method to predict the cost vector from training data generalizes out of sample.

In the field of power systems and electricity markets, it has also been shown that, by smartly tuning the input parameters of current operational and market-clearing procedures, these can mimic the performance of their stochastic-programming-based counterparts to a large extent. For instance, in [6], they propose a bilevel programming model to compute the amount of (uncertain) renewable power generation that must be considered in a forward electricity market to maximize the short-run market efficiency. In the same vein, the authors in [14] show that, by properly setting the (uncertain) reserve requirements in an European-style two-stage electricity market, such a market can be almost as cost-efficient as the ideal two-stage electricity market given by a full stochastic programming approach. All in all, these two works reveal that the power sector can highly benefit from the aforementioned smart-predict strategy. Actually, in [10], they apply it to power generation and grid-scale electricity battery operation, and in [11] to the offering problem of a thermal power producer competing strategically in an electricity market. In [15] and [16], they focus instead on renewable energy producers, for which they propose different smart-predict strategies for energy trading. Lastly, the authors in [17] use a bilevel programming framework similar to that proposed in [11] whereby they train several autoregressive models to estimate the uncertain demand and the size of the energy reserves in a joint reserve allocation and energy dispatch problem.
Within this context, and given a two-stage electricity market, our contributions are the following:

- We propose a mixed-integer program to learn the net-demand value that the forward market must clear so that the power system operating costs are minimized in expectation. This value is, in general, different from the conditional mean of the net demand that is currently employed in standard industry practice.

- We introduce a data clustering and partitioning strategy that, on the one hand, increases the prescriptive performance of the net-demand estimate that our approach produces (by making it dependent on the foreseen net-demand regime), and, on the other, substantially decreases the computational effort to solve the associated mixed-integer estimation problem.

- We evaluate the economic benefits that our approach achieves through an out-of-sample test on a stylized version of the European power system that makes use of real data, in particular, of actual and day-ahead predicted net-demand values downloaded from the ENTSO-e Transparency Platform [18].

The rest of this paper is organized as follows. Section 2 describes the two-stage market organization we consider throughout our paper and motivates the ultimate goal of our work. Section 3 formulates the mixed-integer program we use to construct, from the available historical data, estimates of the net-demand intended to minimize the expected system operation costs. The potential of the so-obtained estimates of the net demand is illustrated, discussed and justified using a small power system in Section 4. In Sec-
tion 5 we introduce a procedure based on data clustering and partitioning to enhance the value of our estimate and to speed up the solution of the mixed-integer estimation problem. A case study based on the European power system is used in Section 6 to investigate the benefits of our approach on real data. Finally, Section 7 concludes the paper with some final remarks.

2. Problem description

We consider a two-stage electricity market consisting of a forward and a real-time settlement. The forward market is cleared some time prior to the actual delivery of energy, for instance, from 1 hour to 36 hours in advance. The real-time market processes the energy imbalances with respect to the forward production schedule. Without loss of generality, to keep our model simple and computationally manageable, we make the following assumptions on our two-stage market setup:

- The inter-temporal constraints of power production portfolios, such as ramping limits and minimum-up and -down times, are not explicitly accounted for by the market-clearing algorithm.

- Network constraints are only taken care of in the real-time market using a pipeline representation of the transmission network.

With these two simplifying assumptions, our setup is closer to the European market, in line with the case study we present in Section 6. Nevertheless, any of the two assumptions above could be dropped at the expense of increasing the complexity of the resulting mathematical optimization model.

The forward market thus determines the power dispatch that minimizes
the anticipated electricity production costs as follows:

\[ \min_{p_g, g \in G} \sum_{g \in G} C_g p_g \]

\[ \text{s.t. } \sum_{g \in G} p_g = \hat{L} \]

\[ 0 \leq p_g \leq P_g, \ \forall g \in G \]

where each block \( g \) has associated a production level \( p_g \) and a marginal cost \( C_g \). Equation (1b) enforces the market equilibrium (i.e., production must equal consumption), with parameter \( \hat{L} \) representing a point or single-value estimate of the total net demand in the system \( L \), which is unknown at the moment the forward market is cleared and thus, is to be treated as a random variable. Finally, Equation (1c) sets the size of each production block.

The linear program (1) stands for an economic dispatch problem whereby production blocks are filled up following a cost-merit order, meaning that blocks \( g \) with a lower cost \( C_g \) are dispatched first. To ease the discussion that follows, hereinafter we will consider that the blocks in the set \( G \) are ordered such that \( g < g' \) if and only if \( C_g > C_{g'} \). Hence, if we denote the optimal solution to (1) as \( \{p^*_g\}_{g \in G} \), it holds that \( p^*_g > 0 \) implies that \( p^*_g = P_g \), whenever \( g < g' \).

With abuse of notation, let \( (L_{di})_{d \in D} \) be a certain realization \( i \) of the nodal net loads in the system. The aim of the real-time market is to satisfy those loads in a cost-efficient manner, that is,

\[ \min_{\Xi} \sum_{g=1}^{G} (C^u_{g} - C^d_{g}) \]

\[ \text{s.t. } 0 \leq p^*_g + r^u_g - r^d_g \leq P_g, \ \forall g \in G \]

\[ 0 \leq r^u_g \leq R^u_g, \ \forall g \in G \]
\begin{align}
0 \leq r^d_g & \leq R^d_g, \quad \forall g \in G \tag{2d} \\
\sum_{g \in G(b)} (p^*_g + r^u_g - r^d_g) & = \sum_{d \in D(b)} L_{di} + \sum_{l: o(l) = b} f_l - \sum_{l: e(l) = b} f_l, \quad \forall b \in B \tag{2e} \\
|f_l| & \leq \mathcal{T}_l, \quad \forall l \in \Lambda \tag{2f}
\end{align}

where \( \Xi := \{r^u_g, r^d_g, g \in G; f_l, l \in \Lambda \} \) is the set of decision variables.

The power output of each flexible unit \( g \) may be increased by an amount \( r^u_g \), based on the marginal cost for upward regulation \( C^u_g \), or decreased by an amount \( r^d_g \) of downward regulation, which entails a marginal benefit (linked to fuel-cost savings) of \( C^d_g \). These actions are driven by the nodal power balance equation (2e) and the minimization of the total regulation costs (2a). Naturally, the amount of regulation provided from each production block \( g \), either upward or downward, must be such that the eventual power output from that block (taking into account the forward optimal schedule \( p^*_g \)) is positive and lower than the block size \( P_g \), as stated in Equation (2b). Moreover, constraints (2c) and (2d) limit the amount of up- and down-regulation that can be deployed from each production block to \( R^u_g \) and \( R^d_g \), which are indicative of how flexible the underlying generation asset actually is.

As stated above, Equation (2c) enforces the nodal power balance. To this end, we employ a pipeline model where \( G(b) \) and \( D(b) \) represent the set of power units and loads that are connected to bus \( b \), in that order. In that equation too, \( o(l) \) and \( e(l) \) stand for the origin and ending nodes of line \( l \), respectively. Finally, line capacity limits are imposed by (2d), with \( f_l \) being the power flow through line \( l \).

In [6] it is shown that the specific estimate \( \hat{L} \) of the system net demand that is used in (1) to clear the forward market (and thus determine the forward dispatch \( p^*_g \)) may have a major impact on the subsequent regulation
costs (2a) through constraint (2b), which conditions the ability of the generation fleet to deploy down- and upward regulation. For this reason, the authors in [6] formulate a bilevel program to compute the value of \( \hat{L} \) in (1) that minimizes the expectation of the total system costs (i.e., forward production costs plus expected regulation costs). This expectation is estimated using a proxy of the probability distribution of the net demands \( (L_d)_{d \in D} \).

Solving such a bilevel program (every time the forward market (1) is to be cleared) is, however, very difficult and therefore, impedes its practical implementation for power system operations. Current practice opts for a much computationally cheaper solution, which is, nonetheless, potentially suboptimal, namely, \( \hat{L} \) is an estimate of the conditional expectation of \( L \), i.e., \( \hat{L} \approx E[L|\Phi] \), where \( \Phi \) represents all the information on which the distribution of \( L \) is conditioned, usually referred to as the context. This expectation, however, is oblivious to the minimization of the regulation costs that drives the clearing of the real-time market (2).

As a data-driven alternative to the approach in [6], we propose a regression procedure that provides an estimate \( \hat{L} \) based on the context \( \Phi \) that explicitly accounts for the potential impact of \( \hat{L} \) on the subsequent regulation costs. This procedure is described in detail in the following section.

3. Contextual dispatch

Suppose we have a sample of \( N \) i.i.d. data points expressed in the form \( \{(x_i, L_i)\}_{i \in N} := \{(x_1, L_1), \ldots, (x_i, L_i), \ldots, (x_N, L_N)\} \), where \( x \in \mathbb{R}^p \) is a vector of features or covariates making up the context and \( L \in \mathbb{R} \) is the random net system demand.

Our objective is to utilize said sample to infer a functional relation \( \hat{L} = \)
\( h(\mathbf{x}) \), with \( h : \mathbb{R}^p \rightarrow \mathbb{R} \) such that, given the context \( \mathbf{x} \), the provided estimate \( \hat{L} \) is trained to deliver the minimum total systems costs in expectation when inserted into the power balance equation (1b).

For simplicity, and because it proves to perform very satisfactorily in the numerical experiments of Section 6, we restrict \( h \) to the family of affine linear functions, i.e., \( h(\mathbf{x}) = \mathbf{q}^T \mathbf{x} \), with \( \mathbf{q} \in \mathbb{R}^p \) and with one of the features, say \( x_p \), fixed to one. To estimate \( \mathbf{q} \), we solve the following empirical risk minimization problem.

\[
\min_{\mathbf{q}, \Upsilon} \frac{1}{N} \sum_{i \in \mathcal{N}} \sum_{g \in \mathcal{G}} (C_g p_{gi} + C_g^u r_{gi}^u - C_g^d r_{gi}^d) \tag{3a}
\]

\( \text{s.t. } \sum_{g \in \mathcal{G}} p_{gi} = \hat{L}_i, \forall i \in \mathcal{N} \tag{3b} \)

\( 0 \leq p_{gi} + r_{gi}^u - r_{gi}^d \leq \mathcal{P}_{gi}, \forall i \in \mathcal{N}, \forall g \in \mathcal{G} \tag{3c} \)

\( 0 \leq r_{gi}^u \leq \mathcal{R}_{gi}^u, \forall i \in \mathcal{N}, \forall g \in \mathcal{G} \tag{3d} \)

\( 0 \leq r_{gi}^d \leq \mathcal{R}_{gi}^d, \forall i \in \mathcal{N}, \forall g \in \mathcal{G} \tag{3e} \)

\( \sum_{g \in \mathcal{G}(b)} (p_{gi} + r_{gi}^u - r_{gi}^d) = \sum_{d \in \mathcal{D}(b)} L_{di} + \sum_{l : o(l) = b} f_{li} - \sum_{l : e(l) = b} f_{li}, \forall i \in \mathcal{N}, \forall b \in \mathcal{B} \tag{3f} \)

\( |f_{li}| \leq \mathcal{F}_{li}, \forall i \in \mathcal{N}, \forall l \in \mathcal{L} \tag{3g} \)

\( \hat{L}_i = \sum_{j=1}^{p} q_j x_{ji}, \forall i \in \mathcal{N} \tag{3h} \)

\( u_{gi} \mathcal{P}_{gi} \leq p_{gi} \leq u_{(g-1)i} \mathcal{P}_{gi}, \forall i \in \mathcal{N}, \forall g \in \mathcal{G} : g > 1 \tag{3i} \)

\( u_{gi} \mathcal{P}_{gi} \leq p_{gi} \leq \mathcal{P}_{gi}, \forall i \in \mathcal{N}, g = 1 \tag{3j} \)

\( u_{(g-1)i} \leq u_{gi}, \forall i \in \mathcal{N}, \forall g \in \mathcal{G} : g > 1 \tag{3k} \)

\( u_{gi} \in \{0, 1\}, \forall i \in \mathcal{N}, \forall g \in \mathcal{G} \tag{3l} \)

where \( \Upsilon := \{p_{gi}, r_{gi}^u, r_{gi}^d, u_{gi}, f_{li}\}_{i, g, l} \).

Intuitively, the estimation problem (3) replicates the sequential clearing.
of the forward and real-time markets, (1) and (2), in that order, for each sample \((x, L_i), i \in \mathcal{N}\), and computes the coefficient vector \(\mathbf{q} = (q_1, \ldots, q_p)\) such that the total system cost averaged over the sample is minimized. This is the reason why all the decision variables related to the power dispatch and the provision of regulating power, i.e., \(p_{gi}, r_{gi}^u, r_{gi}^d\), appear augmented with the sample index \(i\) in (3).

Constraints (3b)–(3g) serve exactly the same purpose as their analogs in (1) and (2). Equation (3h) expresses the estimate \(\hat{L}_i\) of the net system demand \(L\) under context \(x\), as an affine function of the features, whose coefficients are to be estimated by solving (3). Finally, the set of constraints (3i)–(3l) guarantee that the power dispatch \(\{p_{gi}\}_{g \in G}\) coming from (3) for each sample \(i\) is optimal in the forward market (1). To this end, these constraints enforce, for each sample \(i\), the merit-order dispatch of the production blocks \(\{p_{gi}\}_{g \in G}\), forcing that \(p_{g'i} > 0 \implies p_{gi} = P_g\), for all \(g \in G : g < g'\).

Problem (3) is a mixed-integer linear program due to the binary character of variables \(u_{gi}\), which are used to impose the cost-merit order. As such, this problem can be solved using commercially available solvers such as CPLEX [19]. Once we obtain the optimal coefficient vector \(\mathbf{q}^*\), we can produce the net demand estimate \(\hat{L} = (\mathbf{q}^*)^T x\), which is to be fed into (1b) under the context \(x\) to readily obtain the day-ahead dispatch decisions.

As previously mentioned, the point prediction \(\hat{L} \approx \mathbb{E}[L|\Phi]\), which is typically used to clear the forward market (1), is not consistent with the plausible asymmetry in the cost of dealing with the subsequent prediction errors through the real-time market (2). Indeed, it is most often the case that the cost of increasing the electricity production in real time is different from that of diminishing it. In this line, problem (3) offers a handy way to construct a new estimate of \(\hat{L}\) that takes into account the referred cost.
asymmetry. In the next section, we illustrate the advantages of this approach using a small example.

4. Example

Consider the small three-bus system depicted in Figure 1, which is composed of one random demand $L$ at bus 3, two thermal generators, $G_1$ and $G_2$, at buses 1 and 2, respectively; and two lines, Line 1 and Line 2, connecting nodes 1 and 3, and buses 2 and 3, in that order.

![Figure 1. Three-bus power system with one random demand and two thermal generators.](image)

The technical and economic characteristics of generating units $G_1$ and $G_2$ are collated in Table 1. Note that, in comparison, unit $G_1$ is smaller and cheaper than $G_2$. In contrast, the latter is significantly more flexible as it features re-dispatch costs, i.e., $C_u^g$ and $C_d^g$, that are much more competitive. We remark that $C_d^1 = -20\text{€}/\text{MWh}$ implies that this power unit must be paid €20 for each MWh its production is decreased in the real-time market. Unless stated otherwise, line capacities are assumed infinite.

The only demand in the system, namely, $L$, is random. Suppose we have an i.i.d. sample $\{(L_i, x_i)\}_{i=1}^N$, where the feature $x$ here represents an estimate of $\mathbb{E}[L|\Phi]$, that is, a classical point prediction of the demand $L$ built out of whichever available information $\Phi$ the forecaster has at her disposal.
|       | $C_g$ | $C_u^g$ | $C_d^g$ | $P_g$ | $R_u^g$ | $R_d^g$ |
|-------|-------|---------|---------|-------|---------|---------|
| $G_1$ | 5     | 30      | -20     | 60    | 60      | 60      |
| $G_2$ | 15    | 20      | 10      | 150   | 150     | 150     |

Table 1. Illustrative example: Technical and economic specifications of power plants.
Marginal costs are given in €/MWh and capacities in MW.

Consequently, we can write $x_i \approx \mathbb{E}[L|\Phi_i]$. We stress that this setup is very common in reality, where power system operators often count on specialized software to produce good estimates of $\mathbb{E}[L|\Phi]$. Our objective will be to recycle this estimate to fabricate a better value for $\hat{L}$ in Equation (1b).

For this small example, we generate samples in the form $\{(L_i, x_i)\}_{i=1}^N$ as follows. We consider that the per-unit (p.u.) point forecast of the net demand $L$ follows a uniform distribution between $a$ and $b$, that is, $x \sim U(a, b) \text{ p.u.}$ We further assume that the per-unit net demand itself follows a Beta distribution with mean equal to $x$ and standard deviation $\sigma$. Hence, $L \sim \text{Beta}(\alpha, \beta) \text{ p.u.}$, where the scale and shape parameters are related to the mean $x$ and the standard deviation $\sigma$ as follows:

$$x = \frac{\alpha}{\alpha + \beta} \quad (4a)$$

$$\sigma^2 = \frac{\alpha \cdot \beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} \quad (4b)$$

Therefore, given $x$ and $\sigma$, and provided that the system of nonlinear equations (4) has a solution (notice that $\alpha, \beta > 0$), parameters $\alpha$ and $\beta$ can be computed as

$$\alpha = -\frac{1}{\sigma^2}(x^2 - x + \sigma^2)x \quad (5a)$$

$$\beta = \frac{1}{\sigma^2}(x^2 - x + \sigma^2)(x - 1) \quad (5b)$$
To work with MW, we multiply \( x \) and \( L \) by a factor \( \mathcal{L} \) representing the maximum power consumption at bus 3.

Now, we fix \( \sigma = 0.075 \) p.u. and generate 20 samples \( \{(L_i, x_i)\}_{i=1}^N \) with \( N = 750 \), where each \( x_i \) is randomly drawn from \( U(a, b) \) and each \( L_i \) is randomly taken from \( \text{Beta}(\alpha_i, \beta_i) \) with
\[
\alpha_i = -\frac{1}{\sigma^2}(x_i^2 - x_i + \sigma^2)x_i \quad \text{and} \quad \beta_i = \frac{1}{\sigma^2}(x_i^2 - x_i + \sigma^2)(x_i - 1).
\]
We take the first 500 data points of each sample as the training set and the last 250 as the test set.

We postulate the affine model \( \hat{L} = q_0 + q_1(\mathcal{L}x) \) and solve problem (3) on the training set to estimate coefficients \( q_1 \) and \( q_0 \). Finally, to evaluate the performance of the affine model, for each data point \( (L_i, x_i) \) in the test set, we simulate the sequential clearing of the forward and real-time markets (1) and (2), with \( \hat{L} = q_0 + q_1(\mathcal{L}x) \) in (1b), and \( L_i \) in (2e). We then compute the sum of the forward and real-time production costs averaged over the 250 data points in the test set. This mean sum is further averaged over the 20 samples we generate. Our approach, which uses a prescription of the system net demand for market clearing, is referred to as P-MC. We compare it with the customary practice of directly using the point forecast \( \hat{L} = \mathcal{L}x_i \) in (1b), which is referred to as F-MC. Notice that our approach boils down to the conventional one if \( q_0 = 0 \) and \( q_1 = 1 \). Finally, the relative cost difference between these approaches is denoted as \( \Delta \text{cost} \).

In the results we discuss next, we set a base case with \( a = 0.03, \, b = 0.97, \, \mathcal{L} = 100 \) MW, and the technical and economic parameters of the three-bus system described above. We then define variants of this case by changing one or some of those parameters.

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1 Parameter \( \mathcal{L} \) appears in the affine model to convert per-unit values into MW.
2 We take \( a = 0.03 \) and \( b = 0.97 \) pu to ensure that (1b) has a real solution.
4.1. Impact of power regulation costs

Table 2 provides the cost savings that our approach achieves with respect to the conventional one under different $G_2$’s power regulation costs. For completeness, this table also includes the average cost of these two approaches for the test set and the values of the intercept $q_0$ and the linear coefficient $q_1$ of the affine model for $\hat{L}$ our approach utilizes. These values represent expectations over the test data points of the 20 samples generated as indicated above. Furthermore, the first row in the table corresponds to the base case.

Interestingly, our approach systematically corrects the point forecast of the net demand $L$ downwards, with a linear coefficient $q_1$ which is, on average, lower than or equal to 1, and a negative intercept $q_0$ in expectation. This is so because it is economically advantageous for the system to cope with positive net demand errors (i.e., eventual demand increases) by deploying up-regulation from unit $G_2$. Indeed, the alternative would be to deal with negative demand errors by down-regulating with unit $G_1$, a recourse that is clearly much more expensive.

To further elaborate on this phenomenon, the second row in Table 2 provides results for a variant of the base case in which $C_u^2$ has been decreased from 20 to €15/MWh. Now that up-regulating through unit $G_2$ is even cheaper, the downward correction of our approach to the net demand point forecast is more pronounced and the associated cost savings due to said phenomenon become larger. On the contrary, if it is the provision of downward regulation by $G_2$ what becomes €5/MWh cheaper (see third row of Table 2), the costs savings brought by our approach (with regard to F-MC) become smaller, as the net demand point forecast is barely corrected in this case so as to profit from the free down-regulation. In this situation,
Table 2. Illustrative example: Cost savings in percentage under different values of $G_2$’s power regulation costs $C_2$ and $C_2^d$ (both given in €/MWh).

| $C_2$ | $C_2^n$ | $C_2^d$ | F-MC cost | P-MC cost | ∆cost | $q_0$ | $q_1$ |
|-------|---------|---------|-----------|-----------|-------|-------|-------|
| 15    | 20      | 10      | 415.9€    | 414.5€    | 0.34% | -0.157 | 0.972 |
| 15    | 15      | 10      | 398.3€    | 386.0€    | 3.08% | -0.272 | 0.889 |
| 15    | 20      | 15      | 413.1€    | 412.3€    | 0.18% | -0.276 | 1.000 |

Note that correcting the point forecast upwards, when $C_2^d = €15/MWh$ (and, hence, free) would be counterproductive, as the system may risk having to resort to the high-cost down-regulation of unit $G_1$ in those likely scenarios in which the net demand ends up being lower than the prescribed $\hat{L}$.

4.2. Impact of grid congestion

Here we introduce a variant of the base case in which the capacity of Line 1 has been set to 30 MW. Recall that the capacity of this line in the base case is unlimited, which we denote by symbol “∞” in Table 3. The results collated in this new table are analogous to those in Table 2.

Recall that the estimation problem (3), whereby we determine the affine function $\hat{L} = q_0 + q_1(\hat{x})$, explicitly accounts for network constraints. In contrast, the computation of the net-demand point forecast $x$ is typically based on statistical criteria alone and, consequently, ignores any possible limiting effect of the grid.

When the capacity of Line 1 is limited to 30 MW, our approach strongly corrects the point prediction $x$ downwards, so that $\hat{L}$ is kept in between 16 and 32 MW approximately. Thus, unit $G_1$ is dispatched well below the expected demand. This is clever because, in doing so, no (expensive)
downward regulation from this unit has to be deployed in real time to comply with the limiting capacity of Line 1. In this way, the eventual realized demand at bus 3 can be satisfied, instead, with cheaper up-regulation from unit $G_2$ through Line 2. The ultimate result is that using $\hat{L}$, given by our approach, to clear the forward market is way more profitable than using the raw point forecast $L_x$.

4.3. Impact of the peak demand

Now we change the peak demand and consider two variants of the base case in which we take $L = 50$ MW and $L = 150$ MW (in the base case, $L = 100$ MW). The results of this new analysis are compiled in Table 4.

Again, as in the analysis of the impact of $G_2$’s power regulation costs in Section 4.1, our approach systematically corrects the net-demand point forecast downwards to reduce the usage of down-regulation from $G_1$ in favor of the up-regulation from $G_2$. However, the cost savings achieved by our approach get diluted as the peak demand is augmented. The reason for this is twofold. First, the probability of events where the net demand takes on a value below the capacity of unit $G_1$ diminishes with growing $L$. For instance, when $L = 50$ MW, the probability that the net demand is smaller than the capacity of $G_1$ is equal to one, which explains why our method delivers the highest cost savings in this variant (from among the three cases considered.

| $\bar{F}_1$ (MW) | F-MC cost | P-MC cost | $\Delta$cost | $q_0$ | $q_1$ |
|------------------|-----------|-----------|--------------|-------|-------|
| $\infty$         | 415.9€    | 414.5€    | 0.34%        | -0.157| 0.972 |
| 30               | 1021.9€   | 714.5€    | 30.08%       | 16.741| 0.162 |

Table 3. Illustrative example: Impact of grid congestion on cost savings.
in this analysis). In contrast, as $\mathcal{L}$ grows, that probability diminishes and the cheaper down-regulation from $G_2$ becomes more available. Second, the regulation costs account for a lower percentage of the total costs as the peak demand $\mathcal{L}$ increases.

4.4. Impact of the net demand regime

We conclude this small example by studying how the net demand regime affects the prescriptive power of the affine function $\hat{L} = q_0 + q_1(\mathcal{L}x)$ that we estimate by way of problem (3). To this end, we modify the support of the uniform distribution from which the net-demand point prediction $x$ is randomly drawn. Thus, we distinguish a low-demand regime, with $x \sim U(0.03, 0.5)$, and a high-demand regime, with $x \sim U(0.5, 0.97)$. We also consider the base case, where $x \sim U(0.03, 0.97)$ and therefore, no demand regime is differentiated. The corresponding results are provided in Table 5.

In line with the observations in the previous analysis of the impact of the peak demand, under a low-demand regime, the expensive, but flexible unit $G_2$ is not dispatched in the forward market. The downward correction to the net-demand point forecast our approach prescribes is then intended to benefit from the up-regulation provided by $G_2$, which is clearly more competitive than the down-regulation offered by $G_1$. The system features,

| $\mathcal{L}$ | F-MC cost | P-MC cost | $\Delta$cost | $q_0$ | $q_1$ |
|---|---|---|---|---|---|
| 50 | 183.1€ | 182.1€ | 0.53% | -0.110 | 0.980 |
| 100 | 415.9€ | 414.5€ | 0.34% | -0.157 | 0.972 |
| 150 | 742.1€ | 741.3€ | 0.10% | -0.230 | 0.986 |

Table 4. Illustrative Example: Impact of peak demand.
Table 5. Illustrative example: Impact of the net-demand regime.

| U(a, b) | F-MC cost | P-MC cost | Δcost | q₀ | q₁ |
|---------|-----------|-----------|-------|----|----|
| U(0.03, 0.97) | 415.9€ | 414.5€ | 0.34% | -0.157 | 0.972 |
| U(0.03, 0.50) | 239.6€ | 234.5€ | 2.11% | -0.102 | 0.917 |
| U(0.50, 0.97) | 587.8€ | 586.4€ | 0.24% | -6.646 | 1.088 |

therefore, a distinct cost asymmetry given by the expensive down-regulation of \( G_1 \) versus the cheap up-regulation of \( G_2 \). Our approach sees this asymmetry and corrects the net-demand point forecast downwards accordingly. In addition, since the beta distribution modeling the point forecast error is right-skewed for low levels of demand, said correction leads to substantial cost savings. In contrast, under a high-demand regime, \( G_2 \) is very likely to participate in the forward dispatch, whereas there is a lower probability that \( G_1 \) be needed to down-regulate, since the distribution of the point forecast error is left-skewed. Consequently, the cost structure of the system looks very different under a high-demand regime, which prompts a quite different affine function and reduces the cost savings obtained from our method.

Most importantly, in the base case, when no net-demand regime is distinguished, most of the benefits our approach can potentially bring for low values of net demand are lost. This motivates us to cluster net-demand observations into different regimes and use optimization problem (3) to compute a possibly different affine model in the form \( \tilde{L} = q_0 + q_1(Tx) \) for each demand regime, similarly to segmented regression in classical statistics. This is formalized in the next section.
5. Data clustering and partitioning

Take \( \mathcal{N} := \{1, \ldots, i, \ldots, N\} \), that is, the index set of the data sample \( \{(x_1, L_1), \ldots, (x_i, L_i), \ldots, (x_N, L_N)\} \). We partition \( \mathcal{N} \) into a collection \( \{\mathcal{N}_k\}_{k=1}^K \) of \( K \) subsets that are pairwise disjoint and whose union is equal to \( \mathcal{N} \). Consider the one-to-one mapping \( \phi : \mathcal{N} \rightarrow \{1, 2, \ldots, K\} \), such that \( \phi(i) = k \) if data point \((x_i, L_i) \in \mathcal{N}_k \). Therefore, \( \mathcal{N}_k := \{i \in \mathcal{N} : \phi(i) = k\} \).

We compute \( K \) affine models of the form \( \hat{L} = q_k^T x \), \( k \leq K \), by solving the estimation problem (3) for each subset sample \( \mathcal{N}_k \). In practice, this means replacing \( N \) and \( \mathcal{N} \) in (3) with \( |\mathcal{N}_k| \) and \( \mathcal{N}_k \), respectively.

To construct a meaningful mapping \( \phi \), we employ the \( K \)-means algorithm that is implemented in the Python package scikit-learn [20], using the Euclidean distance. We note that, to construct \( \phi \), this algorithm receives the feature sample \( \{x\}_{i \in \mathcal{N}} \) as input. In addition, the algorithm allows extrapolating the mapping \( \phi \) to new outcomes of the feature vector \( x \). That is, given a new observation of \( x \), say \( x_{N+1} \), \( \phi(x_{N+1}) = k \) means that \( x_{N+1} \) is predicted to belong to partition \( \mathcal{N}_k \), and therefore, \( \hat{L} = q_k^T x_{N+1} \) is to be used in the clearing of the forward market (1).

On a different issue, the estimation problem (3) is a MIP program and, as such, computationally expensive in general. Actually, the size of (3) grows linearly with the sample size. To keep the time to solve (3) reasonably low, we reduce the cardinality of subsets \( \{\mathcal{N}_k\}_{k=1}^K \) by means of the PAM K-medoids algorithm [21] through the Python package implementation scikit-learn-extra. This algorithm selects the most representative data points within each subset \( \mathcal{N}_k \), the so-called medoids, by minimizing the sum of distances between each point in \( \mathcal{N}_k \) and said medoids. We remark that this reduction process results in data points (the medoids) with unequal
probability masses, so extra care should be taken when formulating objective function (3a) for each subset $\mathcal{N}_k$ considering the medoids only. More specifically, the uniform weight $\frac{1}{N}$ appearing in the objective function (3a) should be replaced with a medoid-dependent weight representing the probability mass assigned to each medoid as a result of the reduction process.

6. Case Study

In this section we assess the performance of our approach in a realistic case study that is based on the stylized model for the European power system that is described in [22]. Accordingly, we consider a pipeline network model with 28 nodes, each representing an European country. The capacities of the lines are also obtained from [22], in particular, we take the values from “Table 14. Transmission capacities between model regions (GW)” that correspond to the year 2020. We assume that each node in the network (i.e., each European country) includes two types of power plants technologies, which we denote as base and peak, respectively. Again, the available capacity of both technologies has been assigned based on the data in [22] corresponding to 2020 for each country. More specifically, the base power-plant capacities have been obtained by adding up the installed capacities of the technologies “Nuclear”, “Hard coal”, “Oil” and “Lignite” and the peak power-plant capacities from the technologies “Natural Gas”, “Waste” and “Other gases”. The nodes of the system and the resulting generation capacities of each type are listed in Table 6.

To build a data sample of the form $\{(L_i, x_i)\}_{i \in \mathcal{N}}$, we have collected the actual aggregate hourly demand, wind, solar and hydro energy production for each country (node of the system) in 2020 from the ENTSO-e Trans-
Table 6. Base and peak generation capacity (GW) installed per node of the European network.

| Country | AT | BE | BG | CH | CZ | DE | DK | EE | ES | FI | FR | GB | GR | HR |
|---------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| base    | 0.4 | 6.1 | 6.7 | 3.4 | 14.4 | 46 | 2.4 | 2  | 16.3 | 6.5 | 68.3 | 20.6 | 3.9 | 1.3 |
| peak    | 5.9 | 6.8 | 1   | 0.6 | 1.3 | 27.9 | 1.7 | 0.2 | 29.6 | 3.6 | 11.9 | 31.2 | 4.9 | 0.7 |

| Country | HU | IE | IT | LT | LU | LV | NL | NO | PL | PT | RO | SE | SI | SK |
|---------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| base    | 3.3 | 1.8 | 8.7 | 0  | 0  | 0  | 4.5 | 0  | 27.8 | 1.8 | 5.4 | 11.1 | 1.8 | 2.7 |
| peak    | 4.1 | 4.2 | 46.2 | 1.8 | 0.1 | 1.2 | 19.3 | 0  | 3.5  | 4.6 | 2.9 | 1.1  | 0.7 | 1.5 |

parency Platform [18]. We have also retrieved the day-ahead forecast of the hourly demand and the produced wind and solar energy from this platform. To get series of net demand values (both forecast and actual), we have subtracted the respective wind, solar and hydro power data series from the aggregate day-ahead forecast/actual demand series. We clarify that no day-ahead forecast for the hydro power production is available in [18], so the series of real hydro power production has been used (instead of the missing day-ahead hydro forecast) for the computation of day-ahead forecasts of the nodal net demands. Some minor gaps in the data extracted from [18] have been filled through linear interpolation.

The marginal costs of energy generation and up- and down-regulation of each unit are randomly sampled from the uniform distributions specified in Table 7. The so-obtained values for these costs have remained fixed throughout the experiments performed in this section. We point out that in the uniform distributions of Table 7, we have considered that base power units are cheap but inflexible, and thus, with costly regulation. In contrast, peak power plants are expensive, but flexible, and hence, with more competitive regulation costs.
We conduct a rolling simulation on the data of 2020, in which we gradually select non-overlapping windows of 150 points each. From each window, we randomly sub-sample (without replacement) the indexes corresponding to the training and test sets, which are eventually made up of 100 and 50 samples, respectively. We take ten windows over which we average the results that follow.

As in the example of Section 4, we consider a feature vector $\mathbf{x}$ made up of the day-ahead forecast of the system net demand (measured in MWh), enlarged with an additional feature fixed to one to accommodate the intercept of the affine models $\hat{L} = q^T_k \mathbf{x} = q_{1k} x + q_{0k}, k \leq K$.

In the analysis we conduct next, we consider various values for $K$ (number of partitions and hence of affine models) and several percentage reductions of the number of data in each partition $\mathcal{N}_k, k \leq K$. The results of this analysis are summarized in Table 8 where “$r\%$” in the first column means that only the $\left\lceil \frac{r}{100} |\mathcal{N}_k| \right\rceil$ medoids in the partition $\mathcal{N}_k$ have been used to estimate the affine function $\hat{L} = q^T_k \mathbf{x}$ through (3). This table shows, on the one hand, the cost savings achieved by our approach in percentage with respect to the cost of the conventional one and, on the other, the average time the solution to the K estimation problems (3) takes. The reported cost savings have been computed out of sample, that is, on the test sets. Beyond the

|   | $C$      | $C^u$    | $C^d$    |
|---|----------|----------|----------|
| base | U(8, 12) | U(60, 70) | U(-40, -50) |
| peak | U(36, 44) | U(45, 50) | U(30, 35) |

Table 7. Uniform distributions from which the marginal production, up- and down-regulation costs of the units in the European system have been sampled.
| $K$ | 1   | 2   | 5   | 7   | 1   | 2   | 5   | 7   |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|   100% | 2.83% | 4.29% | 4.74% | 4.75% | 2127.7 | 283.7 | 75.9 | 28.0 |
|   50%  | 2.67% | 4.23% | 4.39% | 4.06% | 180.0 | 27.2 | 7.4  | 5.5  |
|   20%  | 2.38% | 4.12% | 4.12% | 3.97% | 8.3  | 3.2  | 1.1  | 1.4  |

(a) Cost savings (out-of-sample results).

(b) Computational time (s).

Table 8. Average cost savings and average time to solve the estimation problem (3) for a number $K$ of partitions and various levels $r$ of reduction in the size of the original training sets (in percentage).

fact that these savings are significant in general, it is clear that our prescriptive approach benefits from exploiting different affine models under different net-demand regimes, which confirms the preliminary conclusion we draw in this regard through the small example of Section 4. Nevertheless, it is also true that the added benefit rapidly plateaus as $K$ grows. Actually, the bulk of the economic gains we get through the partitioning of the data sample is already reaped with $K = 2$. On the other hand, increasing $K$ has a positive side effect: It remarkably reduces the time to solve the MIP problem (3). In addition, this time can be shortened even further, with a tolerable reduction in cost savings, by using only the medoids of the partitions $N_k, k \leq K$, when estimating the affine models through (3).

To comprehend where those cost savings our approach yields come from, in Figure 2 we plot the predicted aggregate net demand $x$ against the one prescribed by our method, i.e., $\hat{L}$. The plot corresponds to one window of 150 data points taken at random out of the ten we have considered in the rolling-window simulation. Furthermore, the figure depicts results from the case with five partitions ($K = 5$). It can be seen that, when the system net demand is predicted low, our method prescribes to overestimate it. This
prescription is motivated by two facts. On the one hand, the overestimation of the net demand in the forward market is covered by cheap power plants, whereas it reduces the need for upward regulation. On the other, even though it slightly increases the demand for downward regulation, the group of units that down-regulate remains the same in any case, i.e., with and without the overestimation, due to the limitations of the network. As a result, the cost savings linked to the reduction in up-regulation outweigh the extra costs incurred by the increase in down-regulation. It is interesting to note that system operators, based on their accumulated experience, often introduce an upward bias into the net demand forecast [23].

As the level of net demand grows, the overestimation of the system net load that our method prescribes diminishes to a point where the prescribed amount flattens (see partitions $N_4$ and $N_5$). Again, this phenomenon is caused by the network and the limitations it imposes. Indeed, our method avoids dispatching power plants in the forward market that, despite being their turn in the cost-merit order, they will have to be irretrievably down-regulated in real time because of network bottlenecks. For instance, in partition $N_4$, F-MC consistently dispatches the DE base generator, with its massive 46 GW, to maximum capacity. However, due to grid constraints, this unit is subsequently down-regulated to around 30 GW. On the contrary, P-MC takes into consideration that this power plant is one of the latest to be scheduled in this partition and foresees the limitation that the grid will impose on the power flow, thus constraining the aggregated energy production and systematically dispatching such a unit to the previously mentioned 30 GW.
7. Conclusions

In this paper, we have proposed a data-driven method to prescribe the value of net demand that the forward settlement in a two-stage electricity market should clear in order to minimize the expected total cost of operating the underlying power system. For this purpose, we have formulated a mixed-integer linear program that trains an affine function to map the predicted net demand into the prescribed one.

Numerical experiments conducted out of sample on a stylized model of the European electricity market reveal that the cost savings implied by the estimated affine mappings are substantial, well above 2%. Furthermore, on the grounds that the cost structure of a power system is highly dependent on its operating point, and hence, on the level of net demand, we have devised a $K$-means-based partition strategy of the data sample to train different affine mappings for different net-demand regimes. The utilization of this
strategy is shown to have a positive twofold effect in the form of substantially increased costs savings and a remarkable drop in the computational burden of the proposed MIP training model. Finally, we have further complemented the partitioning of the data sample with a medoid-based reduction in the size of the partitions, achieving additional speedups in solution times. All this together opens up the possibility to leverage our prescriptive approach in larger instances.

Future work will include attempts to optimize the partitioning of the data sample by embedding it into the MIP training model. We also intend to exploit our prescriptive approach in other applications inside and outside the power systems field.

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