FULL CONTROL OF A TILTROTOR UAV FOR LOAD TRANSPORTATION

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Abstract— This paper presents the modeling and control of a Unmanned Aerial Vehicle (UAV) to execute tasks of load transportation along a desired trajectory. The system modeling is performed via Euler-Lagrange formulation considering both dynamics of the UAV in Tiltrotor configuration and of the carried load. In order to design linear control laws, the equations of motion of the rotorcraft are linearized around an equilibrium point of the UAV. Moreover, to obtain null steady state tracking error, the integral of the controlled position error is also considered. Two control laws were tuned: one based on D-stability and another obtained through minimization of the $H_\infty$ norm. Simulation results are presented corroborating the effectiveness of the control system even in presence of external disturbances, modeling errors and parametric uncertainties.

Keywords— Load transportation, Tiltrotor UAV, Underactuated mechanical system, $H_\infty$ control.

Resumo— Este trabalho apresenta a modelagem e controle de um Veículo Aéreo Não-Tripulado (VANT) para realizar tarefas de transporte de carga percorrendo uma trajetória desejada. A modelagem do sistema é feita através da formulação de Euler-Lagrange considerando tanto a dinâmica do VANT na configuração Tiltrotor quando a dinâmica da carga transportada. Para projeto de leis de controle lineares as equações de movimento são linearizadas em torno de um ponto de equilíbrio. Ademais, para obter erro nulo no seguimento de trajetória, considerou-se a integral do erro das posições controladas. Duas leis de controle lineares foram ajustadas, uma baseada na D-estabilidade e outra através de minimização da norma-$H_\infty$. Resultados de simulação são apresentados para corroborar a efetividade do sistema de controle em presença de perturbações externas, erros de modelagem e incertezas paramétricas.

Palavras-chave— Transporte de carga, VANT Tiltrotor, Sistema mecânico subatuado, Controle $H_\infty$

1 Introduction

Aerial transport of payloads by towed cables ranges from emergency rescue missions, where individuals are lifted from dangerous situations, to the delivery of heavy equipment to the top of a tall building. Typically, aerial towing is accomplished via a single cable attached to a payload, which limits the controllability of the payload. Multi-rotors UAVs are proper vehicles for autonomous cargo delivery, since it provides high maneuverability, vertical take-off and landing, single-point hover, and ability to carry loads near to their body weight, both onboard as suspended by cables. Control designs for the suspended load transportation using single and multiple micro rotorcrafts were presented in (Maza et al., 2010; Palunko et al., 2012; Sreenath et al., 2013).

One of the main challenges in the design of controllers for this kind of system is due to the underactuation in the task, which requires a stable movement of the load without destabilize the system. Moreover, this is increased when performed by multi-rotors UAVs linked to it, since most of the unmanned aerial vehicles are already underactuated systems due to electromechanical design.

Nevertheless, regarding the UAV configuration, the ability of small and medium aircrafts to access and operate in confined and obstructed environments is a crucial concern for the development of systems with air mobility requirements in order to enable air transportation and execution of complex missions. In order to satisfy that, it is necessary that UAVs are more compact for a given load. Thus, developing a UAV with an alternative concept combining the ability of vertical take-off/landing, cruising flight at high speed with high maneuverability and less energy consumption results in aircrafts with a Tiltrotor configuration.

From control systems perspective, the construction of this kind of miniature UAV is far from simplifying the problem. The torques and forces necessary to control the system are applied not only by aerodynamic effects, but also through the coupling effect that occurs between the dynamics of the rotors and the aircraft’s body. This fact, together with the uncertainties of the model, especially in high frequency bands, makes this system even more complex to be controlled than a standard helicopter, at least when using classic control techniques. Studies found in the literature that implement the concept of Tiltrotor to develop
UAVs are presented in (Sanchez et al., 2008; Amiri et al., 2011; Papachristos et al., 2011).

In this paper it is presented the modeling of a Tiltrotor with suspended load using Euler-Lagrange formulation. To perform load transportation through a desired trajectory, two controllers are synthesized based on the linearized model: the first one is based on pole allocation in desired regions of the complex plane (D-stability pole allocation (Chilali et al., 1999)) and the second one optimizes the first approach by choosing a solution inside the same region that maximizes the disturbance rejection of the aircraft (H\text{\textsubscript{\infty}} control design (Dullerud and Paganini, 2000)).

Section 2 presents the nonlinear dynamic model of the Tiltrotor with suspended load. Section 3 finds the system’s equilibrium point, linearizes the model around this point and presents the design of the aforementioned controllers. Section 4 shows the results obtained when using each one of the controllers. Finally, Section 5 concludes this paper and provides suggestions of future work.

2 System Modeling

In this section the equations of motion for the Tiltrotor UAV with suspended load are derived using Euler-Lagrange formulation. The whole system is considered as a multibody system composed of four rigid bodies (see Figure 1): the main body (carbon-fiber structure, landing gear, battery and electronic devices); two thrusters groups, one on each side of the aircraft (servomotors with rotors), interconnected to the main body by a revolute joint; and the suspended load attached to the main body via a rigid string with negligible mass.

2.1 Generalized Coordinates

The frames shown on Figure 1 are considered. There are a fixed inertial frame \( I \), a moving frame \( B \) rigidly attached to the geometric center of the main body, a frame \( C_1 \) rigidly attached to the main body’s center of mass, a frame \( C_2 \) rigidly attached to the center of mass of the right servomotor, a frame \( C_3 \) rigidly attached to the center of mass of the left servomotor, and a frame \( C_4 \) rigidly attached to the center of mass of the suspended load.

Moreover, \( \xi = [x_T \ y_T \ z_T ]' \) is the position of the origin of frame \( B \) w.r.t. \( I \), while \( \mathbf{d}_i^B = [d_i^B \ \theta_i^B \ \alpha_i^B]' \) is the position of the origin of frames \( C_i \), for \( i = 1, 2, 3, 4 \), w.r.t. frame \( B \). It should be noted that \( \mathbf{d}_1^B \), \( \mathbf{d}_2^B \) and \( \mathbf{d}_3^B \) are all constants, while \( \mathbf{d}_4^B \) varies due to the degrees of freedom of the suspended load.

In order to calculate the vector \( \mathbf{d}_4^B \), it is used a parametrization that considers the load as a pendulum with a rigid string of length \( l \) and two degrees of freedom, as shown on Figure 2. Thus, the Denavit-Hartenberg method has been used to derive the Forward Kinematic Model (FKM) and \( \mathbf{d}_4^B \) can be obtained from \( \gamma_1 \) and \( \gamma_2 \) (rotations around \( x_5 \) and \( y_5 \), respectively). Table 1 shows the Denavit-Hartenberg parameters for the given system. The FKM of the pendulum subsystem is given by:

\[
\mathbf{d}_4^B = \begin{bmatrix} -C_{\gamma_1}S_{\gamma_2} \\ S_{\gamma_1} \\ -C_{\gamma_1}C_{\gamma_2} \end{bmatrix}
\]

![Figure 1: Tiltrotor UAV frames and variables definition.](image)

![Figure 2: Denavit-Hartenberg parametrization for the two degree-of-freedom pendulum.](image)

The main body attitude in relation with frame \( I \) is described by \( \eta = [\phi \ \theta \ \psi]' \) (Euler angles with the roll-pitch-yaw convention). The attitude of the load is represented with respect to frame \( B \) and its rotation matrix \( \mathbf{R}_5^B \) is obtained using the Denavit-Hartenberg parameters.
The attitude of the rotors \( C_i = [x_{C_i}, y_{C_i}, z_{C_i}] \), for \( i = 2, 3 \) in relation to the main body is also described using Euler angles. However, it is assumed that there is no rotation around axis \( z_{C_i} \), and the rotation around axis \( x_{C_i} \) is constant and defined by \(-\beta\) for \( i = 2 \) and \( \beta \) for \( i = 3 \), where \( \beta \) is a small angle. The angle of rotation around axis \( y_{C_i} \), on the other hand, is variable and is denoted as \( \alpha_R \) for the frame \( C_2 \) and \( \alpha_L \) for the frame \( C_3 \).

Therefore, the generalized coordinates vector \( q \in \mathbb{R}^{10} \) are defined as follows:

\[
q = [\xi \eta \alpha_R \alpha_L \gamma]^T
\]

where \( \gamma = [\gamma_1 \ \gamma_2]^T \).

### 2.2 Kinematics

The relation of a point rigidly attached to the body frame \( p^B \) with respect to the inertial frame \( p^I \) is given by:

\[
p^I = R^B_0 p^B + \xi
\]

where \( R^B_0 \) is the rotation matrix from frame \( B \) to \( I \). This matrix is derived using the roll-pitch-yaw convention and is given by:

\[
R^B_0 = \begin{bmatrix}
C\phi C\theta & C\phi S\theta S\psi - S\phi C\psi & C\phi S\theta C\psi + S\phi S\psi \\
S\phi C\theta & S\phi S\theta S\psi + C\phi C\psi & S\phi S\theta C\psi - C\phi S\psi \\
-S\theta & C\theta S\psi & C\theta C\psi
\end{bmatrix}
\]

Moreover, the relation between a point rigidly attached to frame \( C_i \) in relation to the body frame \( B \) is obtained as follows:

\[
p^B_i = R^B_{C_i} p^B_i + d_i^B, \quad i = 1, 2, 3, 4
\]

Thus, by replacing equation (5) into (3) the rigid motion is computed by:

\[
p^I_i = R^B_{C_i}(R^B_{C_i} p^C_i + d_i^B) + \xi, \quad i = 1, 2, 3, 4
\]

### 2.3 Euler-Lagrange equations

The Euler-Lagrange formulation describes the equations of motion in the following form:

\[
M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = F(q) + F_{ext}
\]

where \( M(q) \in \mathbb{R}^{10 \times 10} \) is the inertia matrix, \( C(q, \dot{q}) \in \mathbb{R}^{10 \times 10} \) is the Coriolis and centrifugal matrix calculated with Christoffel symbols of the first kind, \( G(q) \in \mathbb{R}^{10} \) is the gravitational force vector, \( F(q) \in \mathbb{R}^{10} \) is the generalized input force vector, and \( F_{ext} \in \mathbb{R}^{10} \) represents external disturbances on the system.

The gravitational force vector \( G(q) \) is calculated as follows:

\[
G(q) = \frac{\partial P}{\partial q}
\]

where \( P \) is the potential energy and is given by the sum of the potential energy of the individual bodies \( P_i \) (Shabana, 2013):

\[
P = \sum_{i=1}^{4} P_i
\]

Thus, equation (16) can be rewritten for each body in the form:

\[
p^I_i = \int_{V_i} \rho_i g_i^B p^C_i + d_i^B + \xi
\]

is the volume integral on a body \( i \) with mass density \( \rho_i \), \( g_i^B = [0 0 -g]^T \) is the gravity vector.

By substituting (6) into (10), yields to:

\[
P_i = g^B_i \int_{V_i} \rho_i [R^B_i(R^C_i p^C_i + d_i^B) + \xi] dV_i
\]

Besides, assuming that all bodies are symmetric leads to:

\[
\int_{V_i} \rho_i p^C_i dV_i = 0
\]

Thus, the potential energy of the system is:

\[
P = g^B_i \int_{V_i} \sum_{i=1}^{4} m_i d_i^B + g^B_i m \xi
\]

where \( m_i \) is the mass of body \( i \) and \( m = \sum m_i \). The vector \( G(q) \) can then be found using (8).

The inertia matrix can be found by calculating the system’s kinetic energy and expressing it in the form \( K = \frac{1}{2} q^T M(q) \dot{q} \). The kinetic energy of the whole system is given by the sum of the individual kinetic energy \( K_i \) of the bodies (Shabana, 2013):

\[
K = \sum_{i=1}^{4} K_i
\]

where

\[
K_i = \frac{1}{2} \int_{V_i} \rho_i (v^T_i)'(v^T_i) dV_i
\]
Then, solving equation (15) taking into account (12) and skew-symmetric properties, it is possible to obtain:

\[ K_i = X_i \quad (20) \]

\[ K_i = X_i + Y_i \quad i = 2, 3 \quad (21) \]

\[ K_4 = X_4 + Y_4 + (\omega_{BE}^g)' m_4 S(d_4^g) d_4^g + \frac{1}{2} \mathbf{d}_4^g m_4 d_4^g + \varepsilon_4 m_4 R_{BE}^g \quad (22) \]

where \( X_i \) and \( Y_i \) are given by:

\[ X_i = \frac{1}{2} m_i \dot{\mathbf{q}} \] (23)

\[ Y_i = (\omega_{BE}^g)' R_{BE}^g \mathbf{I} \dot{\omega}_{c_i} + \frac{1}{2} (\omega_{c_i}^g)' R_{c_i}^g \dot{\omega}_{c_i} \] (24)

The inertia tensor of body \( i \) is:

\[ I_i = \int \mathbf{S}(p_{c_i}^g)' \mathbf{S}(p_{c_i}^g) \, dm = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} = \mathbf{W}_{c_i} \] (25)

and \( \mathbf{J}_i \) is the inertia tensor of body \( i \) for a rotation around an axis displaced by a distance \( d_i \) (Steiner’s theorem for parallel axis) given by:

\[ \mathbf{J}_i = \mathbf{R}_{BE}^g \mathbf{I}_i \mathbf{R}_{BE}^g + m_i \mathbf{S}(d_i^g)' \mathbf{S}(d_i^g) \] (26)

In order to write the kinetic energy in function of the generalized coordinates, the following mappings are applied:

\[ \omega_{BE}^g = \begin{bmatrix} 0 & 0 & -S\theta \\ 1 & 0 & 0 \\ 0 & S\phi & C\phi \end{bmatrix} = \mathbf{W}_g \dot{\mathbf{q}} \] (27)

\[ \omega_{c_i}^g = \begin{bmatrix} \dot{\alpha}_R & 0 & 0 \\ 0 & \dot{\alpha}_L & 0 \\ 0 & 0 & \dot{\gamma}_i \end{bmatrix} = \mathbf{P} \dot{\gamma} \] (28)

\[ \omega_{c_i}^g = \begin{bmatrix} 0 & 0 & \dot{\gamma}_i \\ 0 & \dot{\gamma}_i & 0 \\ \dot{\gamma}_i & 0 & 0 \end{bmatrix} = P \dot{\gamma} \] (29)

Therefore, through equation (14), the inertia matrix is given by:

\[ \mathbf{M}(\mathbf{q}) = \begin{bmatrix} m_{I_{x3x}} & m_{I_{x3y}} & m_{I_{x3z}} & m_{I_{x34}} & m_{I_{x35}} \\ m_{I_{y32}} & W_{y}^g J_{w} & m_{I_{y34}} & m_{I_{y35}} \\ m_{I_{z34}} & m_{I_{z35}} & a' \mathbf{I}_{a} & m_{I_{z34}} & m_{I_{z35}} \\ m_{I_{z34}} & m_{I_{z35}} & a' \mathbf{I}_{a} & m_{I_{z34}} & m_{I_{z35}} \end{bmatrix} = \mathbf{L} \] (30)

\[ \mathbf{M}(\mathbf{q}) = \begin{bmatrix} m_{I_{x3x}} & m_{I_{x3y}} & m_{I_{x3z}} & m_{I_{x34}} & m_{I_{x35}} \\ m_{I_{y32}} & W_{y}^g J_{w} & m_{I_{y34}} & m_{I_{y35}} \\ m_{I_{z34}} & m_{I_{z35}} & a' \mathbf{I}_{a} & m_{I_{z34}} & m_{I_{z35}} \\ m_{I_{z34}} & m_{I_{z35}} & a' \mathbf{I}_{a} & m_{I_{z34}} & m_{I_{z35}} \end{bmatrix} = \mathbf{L} \] (31)

Therefore, through equation (14), the inertia matrix is given by:

\[ \mathbf{M}(\mathbf{q}) = \begin{bmatrix} m_{I_{x3x}} & m_{I_{x3y}} & m_{I_{x3z}} & m_{I_{x34}} & m_{I_{x35}} \\ m_{I_{y32}} & W_{y}^g J_{w} & m_{I_{y34}} & m_{I_{y35}} \\ m_{I_{z34}} & m_{I_{z35}} & a' \mathbf{I}_{a} & m_{I_{z34}} & m_{I_{z35}} \\ m_{I_{z34}} & m_{I_{z35}} & a' \mathbf{I}_{a} & m_{I_{z34}} & m_{I_{z35}} \end{bmatrix} = \mathbf{L} \] (32)

where

\[ m_{I_{x32}} = -R_{BE}^g \mathbf{H} \mathbf{W}_{\eta}, \quad m_{I_{x3z}} = 0, \quad m_{I_{x3z}} = 0, \quad m_{I_{x3z}} = 0, \quad m_{I_{x33}} = 0, \quad m_{I_{x35}} = 0, \quad m_{I_{x35}} = 0, \quad m_{I_{x35}} = 0, \quad m_{I_{x35}} = 0 \] (33)

with \( \mathbf{M} = \sum m_i \mathbf{J}_i, \quad \mathbf{H} = \mathbf{S} (\sum m_i \mathbf{d}_i^g) \).

The Coriolis and centrifugal matrix is obtained from the Inertia Matrix \( \mathbf{M}(\mathbf{q}) \) using Christoffel symbols. Thus, the \((k,j)th\) element of the matrix \( \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \) is defined as:

\[ c_{kj} = \sum_{i=1}^{10} \left( \frac{\partial m_{ij}}{\partial q_k} - \frac{\partial m_{ik}}{\partial q_j} \right) \dot{q}_i \] (34)

where \( m_{ij} \) is the \((i,j)th\) element of \( \mathbf{M}(\mathbf{q}) \).

The force vector \( \mathbf{F}(\mathbf{q}) \) for the problem of load transportation by the Tiltrotor UAV is composed of:

\[ \mathbf{F}(\mathbf{q}) = [T_x, T_y, T_z, \tau_\alpha, \tau_\beta, \tau_{\alpha_L}, \tau_{\beta_L}, \tau_{\gamma_L}]^T \] (35)

where \( T_i \) represent translational forces along an axis \( i \) and \( \tau_k \) represent rotational torques resulting in the variation of the angle \( k \).

The force of each propeller can be decomposed along frame \( \mathbf{B} \) such as follows:

\[ \mathbf{F}_{R}^B = \begin{bmatrix} f_{Rx}^B \\ f_{Rx}^B \\ f_{Rx}^B \end{bmatrix} = \begin{bmatrix} S_A R \end{bmatrix} \begin{bmatrix} C_A R \end{bmatrix} \] (36)

\[ \mathbf{F}_{L}^B = \begin{bmatrix} f_{Lx}^B \\ f_{Ly}^B \end{bmatrix} = \begin{bmatrix} S_A L \end{bmatrix} \begin{bmatrix} C_A L \end{bmatrix} \] (37)

where \( f_{R} \) and \( f_{L} \) are the right and left propeller thrusts, respectively. The translational forces expressed in the inertial frame are given by:

\[ \mathbf{T}^T = \mathbf{R}_{BE}^g (\mathbf{F}_{R}^B + \mathbf{F}_{L}^B) \] (38)

The rotational torques are obtained by adding the torque generated by the thrust forces of the propellers to the torque caused by the drag of the propellers. The dynamic of the rotor is assumed in steady-state, which leads to the drag torque \( \tau_{\text{drag}} = k_r/b \) (Castillo et al., 2005), where \( k_r \) and \( b \) are known aerodynamic constants. Thus, the main body’s rotational torques are written as follows:

\[ \begin{bmatrix} \tau_0 \\ \tau_\alpha \end{bmatrix} = \begin{bmatrix} f_{Lx} - f_{Rx} \cos(\lambda) r + \frac{k_r}{b} (f_{Rx} - f_{Rx}) \\ \frac{k_r}{b} (f_{Rx} + f_{Rx}) d_i + \frac{k_r}{b} (f_{Ly} + f_{Ly}) \\ \frac{k_r}{b} (f_{Rx} - f_{Rx}) d_i + \frac{k_r}{b} (f_{Ly} + f_{Ly}) \end{bmatrix} \] (39)

where \( r, \lambda, d_i \) and \( d_z \) are function of the geometry of the rotorcraft, which are given by:

\[ d_x = d_2^d = d_{z3}^d, \quad d_y = |d_3^d| = d_{x3}^d, \quad \lambda = \arctan \left( \frac{d_y}{d_x} \right), \quad r = \sqrt{d_x^2 + d_y^2} \]

The torques \( \tau_{\alpha_\ell} \) and \( \tau_{\beta_\ell} \) are direct inputs of the system. Therefore, there is no mapping that should be done from an input to these variables. The torques \( \tau_{\gamma_1} \) and \( \tau_{\gamma_2} \) are always zero, since there is no directly actuation over these variables.

Finally, the input force vector can be expressed in a decoupled form:

\[ \mathbf{F}(\mathbf{q}) = \mathbf{B}(\mathbf{q}) \Gamma \] (40)

where

\[ \Gamma = \begin{bmatrix} f_{R} \\ f_{L} \\ \tau_\alpha \\ \tau_\beta \\ \tau_{\alpha_L} \end{bmatrix} \] (41)
3 Control Design

In this section two linear controllers are designed using Linear Matrix Inequalities (LMI) approach. The control objective of the proposed control strategy is to transport a payload using a Tiltrotor UAV along a predefined trajectory using a single control law, which is able to stabilize the whole system, including the load, even when exogenous disturbances affect the system, and in presence of unmodeled dynamics and parametric uncertainties.

The first controller is based on regional pole allocation (D-Stability): a region on the complex plane is defined and the solver finds a state-feedback control law \( u = Kx \) such that the poles in closed-loop are all inside this region.

This first solution can be satisfactory if the requirement is to stabilize the rotorcraft while following some trajectories. However, it is still possible to optimize the D-Stable solution by finding some pole allocation inside the chosen region that also minimizes some given cost function.

Therefore, a second controller is designed such that it maintains the pole allocation within the region of the first example but also maximizing disturbance rejection. The group of controllers that satisfy the constraint of maximal disturbance rejection is known as \( \mathcal{H}_\infty \) controllers.

In the following, the linearized state-space model and synthesis of the controllers are presented.

3.1 Linearized state-space model

Since the controllers designed in this paper are based on linear systems, in order to perform the mentioned control laws the equations of motion of the system have to be linearized around an operating point.

Initially, consider the nonlinear continuous state-space representation:

\[
\dot{x} = F(x, u, d) = f(x) + g_1(x)u + g_2(x)d
\]

with

\[
x(t) = \begin{bmatrix} x_1 \\ \vdots \\ x_{20} \end{bmatrix}, \quad u = \Gamma, \quad d = F_{ext}.
\]

The dynamic equations (7) can be rewritten in a state-form by using:

\[
\dot{q} = M^{-1}[B(q)\Gamma + F_{ext} - C(q, q)q - G(q)]
\]

from where the maps \( f(x), g_1(x) \) and \( g_2(x) \) are directly obtained.

To linearize the system, first the equilibrium point is computed when it fulfills \( \dot{x}(t) = [q \dot{q}] = 0 \) assuming a scenario with no external disturbances \( (F_{ext} = 0) \). Thus, the equilibrium values of the inputs, \( \Gamma \), and generalized coordinates, \( q \), which ensure hovering of the tiltrotor UAV, are obtained by:

\[
B(q)\Gamma - G(q) = 0 \tag{44}
\]

Since this system has ten equations and fourteen variables, to avoid infinite solutions, four variables are freely chosen and others ten are computed from (44). The state variables \( x, y \) and \( z \) do not appear on the above equations, what would allow infinite equilibrium points. The fourth chosen variable is \( \psi \). Therefore, these four state variables are fixed as \( x_R, y_R, z_R \) and \( \psi_R \). Therefore, a system defined around an equilibrium point \( x_R = [q_R \ 0] \) and reference forces \( u_R = \Gamma_R \) is obtained.

Equation (43) can then be rewritten on the equilibrium point as:

\[
\dot{q} = M(q_R)^{-1}[B(q_R)\Gamma_R + F_{ext} - G(q_R)]
\]

It is then possible to obtain a linearized error model given by:

\[
\Delta \dot{x} = A\Delta x + B_\psi \Delta u + B_u d \tag{46}
\]

where \( \Delta x = x - x_R \) and \( \Delta u = u - u_R \) with

\[
A = \frac{\partial F(x, u, d)}{\partial x} \bigg|_{x=x_R, u=u_R, d=d_R}, \quad B_\psi = \frac{\partial F(x, u, d)}{\partial u} \bigg|_{x=x_R, u=u_R}, \quad B_u = \frac{\partial F(x, u, d)}{\partial d} \bigg|_{x=x_R, u=u_R}
\]

In order to avoid steady-state errors in closed loop in presence of persistent disturbances, an integral action is added for each element of the trajectory. Thus, the new state vector \( x \in \mathbb{R}^{24} \) is defined as follows:

\[
\dot{x} = [x_1 \ldots x_{24}]' = [q' \dot{q} \int x \int y \int z \int \psi]'
\]

3.2 Control via D-Stability

As stated before, the control design via D-Stability is based on finding a controller that allocates closed-loop poles inside a desired region in the complex plane (Chilali et al., 1999). Three different type of regions are used when designing the proposed controller: 1) Left half-plane where \( Re(s) < \sigma \). This guarantees that the closed-loop poles possess time constants \( \tau_s < 1/\sigma \); 2) Region inside a left-oriented cone with \( 2\varphi \) as internal angle, where \( \varphi \) is the angle between the Real axis and the cone’s side. This guarantees that the closed-loop poles possess damping ratio \( \zeta < \cos(\varphi) \); 3) Region inside a circle with center in \( (c, c, c) \) and radius \( R \).

It is possible to find a solution that encompasses all of these three constraints simultaneously. Since the system has \( n \) states and \( n_u \) inputs, it is necessary to find a matrix \( Q \in \mathbb{R}^{n \times n} \) with \( Q > 0 \) and \( Q' = Q \) and a matrix \( Y \in \mathbb{R}^{n_u \times n} \) that satisfy the following inequalities:

\[
2\sigma Q + J + J' < 0 \tag{48}
\]

\[
\begin{bmatrix}
\sin(\varphi)(J + J') & \cos(\varphi)(J + J') \\
\cos(\varphi)(-J + J') & \sin(\varphi)(J + J')
\end{bmatrix} < 0 \tag{49}
\]

\[
\begin{bmatrix}
-JQ & -\epsilon Q + J' \\
-\epsilon Q + J' & -RQ
\end{bmatrix} < 0 \tag{50}
\]
where \( J = AQ + B_e Y \) and the controller \( K = YQ^{-1} \). Equation (48) is the solution for the Region 1, equation (49) is the solution for the Region 2, and equation (50) is the solution for the Region 3.

For the controller design were chosen \( \sigma = -2.4, \varphi = \pi/10, c = 0 \) and \( R = 80 \). The parameters chosen for the circle guarantees that the allocated poles will not be too far away from the origin of the plane, avoiding too large gains in the controller. Figure 3 shows the poles in open loop along with its allocation after closed loop.

![Figure 3: Allocation of poles via D-Stability.](image)

\[ 3.3 \ H_\infty \ controller \ design \]

For this section, the following extended linear system is considered:

\[
\begin{align*}
\dot{x} &= Ax + B_e u + B_\omega d \\
z &= C_z x + D_{wz} u + D_{wz} d
\end{align*}
\]

where \( z \) is the error signal to be minimized, \( C_z, D_{wz} \) and \( D_{wz} \) are constant matrices that are determined while designing the controller.

The \( H_\infty \) controller belongs to the class of optimal controllers, which exponentially stabilizes the system while also minimizing the \( H_\infty \) norm, which is given by \( ||H(s)||_\infty = \sup_{w} (\max_{\omega} |H_{wz}(j\omega)|) \), with \( \sigma \) as the singular value of a transfer function \( H_{wz}(j\omega) \). Since the system dealt in this paper is MIMO, there are multiple transfer functions \( H_{wz}(j\omega) \). Thus, the \( H_\infty \) norm is given by the system’s highest frequency response gain of the transfer function with the highest singular value. The \( H_\infty \) norm can be interpreted as the system’s highest gain due to a disturbance input.

Therefore, by minimizing the \( H_\infty \) norm, the effect of external disturbances on the system are also minimized. This can be solved using the LMI given by (52) (Dullerud and Paganini, 2000). This approach finds a level attenuation \( \tilde{\gamma} \) and guarantees that \( ||H(s)||_\infty < \sqrt{\tilde{\gamma}} \). In order to combine this solution with the previous controller, equation (52) is solved altogether with equations (48)-(50).

\[
\begin{bmatrix}
J + J' & B_e \\
B_e' & -\tilde{\gamma} I_{n_w} & Y' D_{wz} \\
C_z Q + D_{wz} Y & D_{wz} & -\tilde{\gamma} I_{n_x}
\end{bmatrix} < 0
\]

The following controller matrices were ad-

justed heuristically:

\[
D_{wz} = \begin{bmatrix}
0_{10,3} & 0_{10,2} & 0_{10,2} \\
0_{13,3} & 0_{13,3} & 0_{13,2} \\
0_{3,3} & 0_{3,3} & 0_{3,2} \\
0_{2,3} & 0_{2,3} & 0_{2,2} \\
0_{2,3} & 0_{2,3} & 0_{2,2} \\
0_{1,3} & 0_{1,2} & 0_{1,2}
\end{bmatrix}
\]  \hspace{1cm} (53)

\[
C_z = \text{diag}(3,4,5,2,2,0,2,0,2,3,3,3,2,2,3,1/3,1/3,3,5,1/30,1/30,0.8,0.8,10.5,10.5,10,10)
\]  \hspace{1cm} (54)

\[
D_{wz} = \begin{bmatrix}
0_{10,2} & 0_{10,2} \\
0.5 + 1_{3 \times 2} & 0_{3,2} \\
0_{2,2} & 1_{2 \times 2} \\
0_{1,2} & 5 + 1_{1 \times 2} \\
0_{2,2} & 5 + 1_{2 \times 2} \\
0_{2,2} & 4 + 1_{2 \times 2}
\end{bmatrix}
\]  \hspace{1cm} (55)

with \( b = [0 \ 0 \ 1] \) and \( 1_{n \times m} \) is a matrix that has \( n \) lines and \( m \) columns filled with 1’s.

As a result, a solution with \( ||H(s)||_\infty < 34.11 \) was found. The pole allocation can be seen on Figure 4.

![Figure 4: Allocation of poles via \( H_\infty \).](image)

\[ 4 \ \text{Simulations and analysis} \]

The proposed control system has been simulated in Matlab Simulink 2012a with the model parameters shown on Table 2. It is assumed that all states are measured. In order to show robustness of the designed controllers, the UAV tracked a predefined trajectory while some disturbance forces \( F_{xext}, F_{yext} \) and \( F_{zext} \) affected the vehicle on the geometric center (Figure 5). Moreover, the simulation considered that the model’s masses \( m_i \) and inertia tensors \( I_i \), for \( i = 1, 2, 3, 4 \) had all uncertainties randomly ranging from -30% to 30% of their nominal values. In addition, a linear feed-forward term has been added to the control action \( u = K \hat{x} + u_R \), where:

\[
\begin{align*}
u_R &= B^+(q_R)[M(q_R)\hat{x}_{Ref} + C(q_R, q_R)\hat{x}_{Ref} + G(q_R)] \\
B^+ &= (B^TB)^{-1}B^T
\end{align*}
\]  \hspace{1cm} (56)

Figure 6 shows a 3D view of the trajectory that the aircraft followed. The set point \( \psi_{Ref} \) was kept constantly equal to zero. Figure 7 shows the tracking error of the generalized coordinates \( x, y, z \) and \( \psi \) defined as: \( \varepsilon_q = q_{Ref} - q \). It can be
Table 2: System Parameters

| Parameter | Value |
|-----------|-------|
| $m_1$     | 1.243 Kg |
| $m_2, m_3$| 0.150 Kg |
| $m_4$     | 0.050 Kg |
| $d_1$     | $(-0.00065, -0.0072, -0.046)$ m |
| $d_2$     | $(1.7e - 2, -0.27, 0.43)$ m |
| $d_3$     | $(1.7e - 2, 0.27, 0.43)$ m |
| $I_{1xx}$ | $0.018891956$ m |
| $I_{1yy}$ | $0.005237518$ m |
| $I_{1zz}$ | $0.018027985$ m |
| $I_{2xx}, I_{3xx}$ | $(1.7e - 2, 0.27, 0.43)$ m |
| $I_{2yy}, I_{3yy}$ | $0.000077509$ m |
| $I_{2zz}, I_{3zz}$ | $0.000069700$ m |
| $I_{4xx}, I_{4yy}, I_{4zz}$ | $0.000002645$ m |
| $g_z$     | 9.81 |
| $k_r$     | $1.7e - 7$ N.m.s$^2$ |
| $b$       | $9.5e - 6$ N.s$^2$ |
| $\beta$  | $5^\circ$ |
| $l$       | 0.5 m |

Table 3: Mean-square-error of the simulations

|          | $D$ - stable | $H_\infty$ |
|----------|--------------|-------------|
| MSE$x$   | $1.61.10^{-4}$ | $1.04.10^{-4}$ |
| MSE$y$   | $6.76.10^{-4}$ | $5.48.10^{-4}$ |
| MSE$z$   | $4.70.10^{-7}$ | $6.05.10^{-7}$ |
| MSE$\psi$| $2.70.10^{-2}$ | $1.62.10^{-3}$ |

seen from this image that the $H_\infty$ controller presented better results on tracking reference and disturbance rejection when compared to the D-stable controller. The time evolution of the remaining generalized coordinates are shown in Figures 8 - 9. It is possible to see that the designed control laws maintain them all stabilized. Figure 10 shows the system's control inputs along with time. It is possible to verify that the inputs for the $H_\infty$ showed to be more aggressive than the pure D-stable controller.

Table 3 shows the mean square error of the trajectory of the aircraft in relation with its set point. It can be seen that except for the trajectory in the direction $z$ the $H_\infty$ controller achieved better results.

Figure 5: System disturbances in function of time

5 Conclusion

In this work the modeling of a Tiltrotor UAV for load transportation has been derived using the
Euler-Lagrange formulation. To design the controller, the system has been linearized around its equilibrium point. Two controllers were synthesized: one based on D-Stability and another one improving the first one by choosing closed-loop poles that optimizes disturbance rejection using the $H_\infty$ norm. Both controllers presented robustness against external disturbances, unmodeled dynamics and parametric uncertainties when evoked to stabilize the aircraft and perform path tracking. In general the $H_\infty$ controller achieved better results than the D-Stable one, specially when disturbances were applied.

However, there are some drawbacks on using linear controllers such as the designed ones. Given that the system should always be nearby the operating point, a large deviation from this neighborhood may destabilize the UAV. This problem is being solved by the authors of this paper, who are presently studying nonlinear control techniques for the Tiltrotor UAV.

As future work, the designed controllers should be implemented in a real Tiltrotor UAV to perform load transportation. This is part of the ProVANT project, which is being developed in a partnership between UFMG and UFSC.

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