Analytical Method of Sunshine Temperature Field of Concrete Silo Considering Stored Material

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Abstract. The solar radiation has a significant influence on large-scale concrete silos. Therefore, this paper investigated temperature distributions of large-scale concrete silos exposed to solar radiation considering the thermal stored material. As a first step, based on heat conduction theory, considering the transient heat conduction among the stored material, the silo wall and the external environment, the mathematical model of the silo was established. Further, the boundary conditions were approximately linearized based on the solar radiation theory. Furthermore, the heat conduction equations were solved by Laplace integral transform and inverse transform, and the analytical calculation results of the temperature field of the silo were simulated with the mathematical software. A comparative analysis was conducted between the results of analytical method and numerical method, it turns out that the results had good agreements. Finally, the analytical method was recommended to provide theoretical basis and references for the design of large-scale silos.

1. Introduction

Concrete cylinder silos play an important role in industrial and agricultural production for the advantages of small floor area and low cost. Since silos are usually built in a natural environment, temperature stresses caused by the temperature difference become one of the major factors which affect the durability of silos [1, 2]. Temperature fields can cause the failure of the silos too [3]. Thus, many scholars have studied the temperature effect of silos.

Priestley [4] studied the temperature difference between the inner and outer of a prestressed storage tank and the thermal stresses under the solar radiation. Numerical analysis results indicated that the temperature difference caused by solar radiation exceeds 30 °C, which should be paid more attention to the influence of the temperature difference on the stress of the tank wall in the design. Based on four years of observation, Sinha et al. [5] found that the temperature of the grain in center was lower than the temperature of the outer surface of the silo wall during summer. Conversely, the outer surface of the wall temperature was lower than the grain temperature at the center of the bulk during winter. Lapko and Prusiel [6] made an on-site measurement of a cement silo in 1998. After that, they conducted a short-term temperature distribution study on the cylindrical silo where grain was stored. The results showed that even if the silo wall thickness was only 20 cm, the temperature was distributed nonlinearly along the wall thickness. The temperature effect of the silo was systematically analyzed by combining the experiment and the finite element method.
All these mentioned above have significance for designers to consider the effect of environmental temperature. However, the temperature effects of the silo are mainly focused on the specific practical project; there is little research on the temperature distribution under solar radiation considering the hot stored material in the silo. The relevant codes for silo design in China specify that the temperature effects should be considered in the design of silos, but how to calculate the temperature effects is not included in most of the codes [7]. Due to the high temperature of the stored material, such as the temperature of cement clinker entering the silo can reach 175 °C [8], heat exchange occurs between the stored material and the silo wall. The thicker the wall thickness is, the larger the temperature difference between the inner and outer surfaces of the wall is. Further, the premise of accurately analyzing the temperature effect is to obtain the temperature distribution of the silo. Therefore, based on the theory of transient heat conduction, this paper theoretically analyzes the temperature distribution of the large-scale concrete silo in case of fully loaded under solar radiation, and obtains the analytical solution of the temperature distribution of the silo, which is verified by the finite element method. It provides a theoretical basis for temperature effect analysis of large-scale silo considering the influence of high temperature stored materials.

2. Mathematical model

The temperature field of the concrete silo under solar radiation should be considered as a three-dimensional transient heat conduction, which is more complicated due to the heat conduction between the wall and the stored material in case of the full silo. To obtain the analytical solution of the wall heat transfer when the silo is fully loaded with high temperature stored material, it is assumed that [9, 10]:

1. the temperature field of the silo wall is simplified into a two-dimensional temperature distribution with a one-dimensional heat conduction problem, which is expressed by \( T_1(r,t) \);

2. a wall thickness of the stored material unit is established to simulate the interaction between the storage and the wall, which is expressed by \( T_2(r,t) \);

3. the stored material in the silo is in good contact with the inner surface of the silo wall, and the temperature and heat flux density on the boundary surface is continuous.

2.1. Heat conduction equations

Based on the assumptions, Figure 1 shows the geometric dimension of silo considering stored material and the following heat conduction formula is obtained.

\[
\begin{align*}
\frac{1}{\alpha_1} \frac{\partial T_1}{\partial t} &= \frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r} \frac{\partial T_1}{\partial r} \quad r_1 < r < r_2 \\
\frac{1}{\alpha_2} \frac{\partial T_2}{\partial t} &= \frac{\partial^2 T_2}{\partial r^2} + \frac{1}{r} \frac{\partial T_2}{\partial r} \quad r_2 < r < r_3
\end{align*}
\]

where, \( T_1 \) and \( T_2 \) are the temperature of the silo wall and the stored material unit at time \( t \), respectively (°C); \( r_1, r_2, r_3 \) are the external radius, the internal radius of the silo, and the stored material unit radius, respectively (m); \( t \) is the time (s); \( \alpha_1 = \lambda_1/\rho_1 c_1 \) and \( \alpha_2 = \lambda_2/\rho_2 c_2 \) are the thermal diffusivity of concrete wall and stored material, respectively (m²/s); \( \lambda_1 \) and \( \lambda_2 \) are thermal conductivity of concrete wall and stored material, respectively (W/(m·K)), \( \rho_1 \) and \( \rho_2 \) are the density of concrete and stored material, respectively (kg/m³), \( c_1 \)
and $c_1$ and $c_2$ are the concrete and the stored material specific heat, respectively (J/(kg·K)).

2.2. Initial condition
When solving the transient temperature field, it is considered that temperature at the inner wall, the outer wall and the inner wall of the storage layer in different directions are the same respectively. The temperature gradients between the outer and inner walls, the inner wall and the storage layer are linearly distributed [9]. (Eq. 2).

\[
\begin{align*}
T_1|_{r=0} &= T_{1,c} \\
T_2|_{r=0} &= T_{2,c}
\end{align*}
\]  

where, $T_{1,c}$ is the initial temperature of the concrete wall, while $T_{2,c}$ is the initial temperature of the stored material, (°C).

2.3. Boundary condition

2.3.1. Internal boundary condition. On the boundary surface $\Gamma_2$ where the stored material is in contact with the silo wall, it could be simplified as a complete contact boundary condition according to the assumptions (see Eq. 3). While on the boundary surface $\Gamma_3$, the temperature boundary conditions could be considered as the first boundary condition (Eq. 4).

\[
\begin{align*}
\lambda_1 \frac{dT_1}{dn} |_{r=r_2} &= \lambda_2 \frac{dT_2}{dn} |_{r=r_2} \\
T_2 |_{r=r_3} &= T_3
\end{align*}
\]  

where, $n$ is the normal direction; $T_3$ is the temperature of the stored material at $r = r_3$, (°C).

2.3.2. External boundary condition. The heat exchange between structures and the environment consists of three parts [10]: solar short-wave radiation ($q_s$), thermal convection ($q_c$) and thermal long-wave radiation ($q_r$). The temperature boundary condition of the outer surface of the wall ($\Gamma_1$) could be considered as the second boundary condition (Eq. 5).

\[
\lambda_1 \frac{dT}{dn} |_{\Gamma_1} = q_s + q_c + q_r
\]  

The heat transfer boundary condition is complex and nonlinear; it is very difficult to solve the conduction directly from this boundary condition by an analytical method, especially for the solar radiation intensity $q_s$, which must be reasonably simplified. Therefore, a half-cosine model was used to simplified the solar radiation process curve. On the above simplify, the following Eq. 6–8 [11] have been used:

\[
q_s(t) = q_{ds}(t) + q_{rs}(t)
\]  

\[
q_{ds}(t) = \begin{cases} 
\frac{nH_ds}{2(t_{sr}-t_{rd})} \cos \left( \frac{\pi(0.5(t_{rd}+t_{sr}))}{t_{rd}-t_{sr}} \right) & 0 < t \leq t_{rd} \\
0 & t_{rd} < t \leq t_{sr} \\
\frac{nH_ds}{2(t_{sr})} \cos \left( \frac{\pi(0.5(t_{rd}+t_{sr}))}{t_{rd}-t_{sr}} \right) & t_{sr} < t \leq 24
\end{cases}
\]  

\[
q_{rs}(t) = \begin{cases} 
\frac{nH_rs}{2(t_{sr})} \cos \left( \frac{\pi(0.5(t_{rd}+t_{sr}))}{t_{rd}-t_{sr}} \right) & 0 < t \leq t_{rr} \\
0 & t_{rr} < t \leq 24
\end{cases}
\]  

where: $H_ds$ is the direct radiation intensity of the solar radiation on the vertical wall; $t_{rd}$ and $t_{sr}$ are the time of sunrise and the time of sunset of the vertical wall, respectively. $H_rs$ is the scattering and reflection intensity of the solar radiation; $t_{rr}$ and $t_{sr}$ are the time of sunrise and the time of sunset of the horizontal wall, respectively.
Further, the Eq. 9 is obtained by converting Eq. 5 into the Fourier form:
\[
\left(\lambda_1 \frac{\partial T}{\partial t} + h \frac{T}{t}\right)_{r=r_1} = a_{1,0} + \sum_{n=1}^{\infty} \left(\frac{a_{1,n}}{2} + \frac{b_{1,n}}{2j}\right) e^{in\omega t} + \left(\frac{a_{2,n}}{2} - \frac{b_{1,n}}{2j}\right) e^{-in\omega t}
\]  
(9)
where, \( h \) is the comprehensive heat transfer coefficient between the boundary \( \Gamma_1 \) and the external environment. The detailed parameters \( a_{1,0}, a_{1,n} \) and \( b_{1,n} \) could be refer to the reference [11].

3. Solution of Heat Conduction Problem

3.1. Laplace integral transform
The temperature field of the silo can be determined by Laplace integral transform [12]. The Laplace integral transform of the time function \( f(t) \) and the inverse transform of Laplace \( f^{-1} \) are defined as:
\[
\tilde{f}(s) = \int_0^{\infty} f(t) e^{-st} dt
\]  
(10)
\[
f(t) = \int_0^{\infty} \tilde{f}(s) e^{st} ds
\]  
(11)
where, \( s = \sigma + i\omega \), and \( \sigma > 0 \).

Therefore, after the Laplace integral transformation on the boundary conditions, Eq. 3–4 and 9 can be expressed as the following formulas.
\[
\left(\lambda_1 \frac{\partial T}{\partial r} + h \frac{T}{r}\right)_{r=r_1} = a_{1,0} + \sum_{n=1}^{\infty} \left(\frac{a_{1,n}}{2} + \frac{b_{1,n}}{2j}\right) \frac{1}{ysn} + \left(\frac{a_{2,n}}{2} - \frac{b_{1,n}}{2j}\right) \frac{1}{ysn}
\]  
(12)
\[
\begin{align*}
T_1|_{r=r_2} &= T_2|_{r=r_2} \\
\lambda_1 \frac{\partial T_1}{\partial r} &= \lambda_2 \frac{\partial T_2}{\partial r}
\end{align*}
\]  
(13)
\[
T_2|_{r=r_3} = \frac{T_3}{s}
\]  
(14)

3.2. Solution
Frist, the heat conduction Eq. 1 is integrated and transformed, then the general solutions are obtained as follows:
\[
\begin{align*}
T_1 &= m_1 J_0 \left( r \frac{\sqrt{\alpha}}{a_1} \right) + m_2 J_0 \left( r \frac{\sqrt{\alpha}}{a_2} \right) + \frac{T_{1,0}}{s} \\
T_2 &= m_3 J_0 \left( r \frac{\sqrt{\alpha}}{a_3} \right) + m_4 J_0 \left( r \frac{\sqrt{\alpha}}{a_4} \right) + \frac{T_{2,0}}{s}
\end{align*}
\]  
(15)
where, \( J_0(*) \) is the first Bessel function and \( Y_0(*) \) is the second Bessel function. The undetermined coefficients such as \( m_1, m_2, m_3 \) and \( m_4 \) could be obtained by the following calculations.

Further, general solutions are taken into the boundary conditions (Eq. 12–14). Furthermore, the equations are sorted out, and the undetermined coefficients \( m_1 \sim m_4 \) are obtained as Eq. 16.
\[
\begin{align*}
m_1 &= \frac{X_1}{F_2} F_1, \quad m_2 = \frac{X_3}{F_4} F_3, \quad m_3 = \frac{X_4}{F_4} F_3, \quad m_4 = \frac{X_3}{F_4} F_3
\end{align*}
\]  
(16)
where,
\[
\begin{align*}
X_1 &= \frac{T_{1,0} F_2 + T_{2,0} F_1}{s} Q_2 + \frac{T_{2,0} F_2 + T_{3,0} F_1}{s} Q_2 + \frac{T_{3,0} F_2}{s} Q_2 F_3 \frac{P_3}{K_3} \frac{P_2}{K_2}, \\
X_2 &= \frac{i_4}{k_2} \frac{a_2}{a_1} Q_4 F_3 F_2 K_2 \frac{P_3}{K_3} \frac{P_2}{K_2}, \\
X_3 &= \frac{T_{1,0} F_2 + T_{2,0} F_1}{s} Q_4 + \frac{T_{2,0} F_2 + T_{3,0} F_1}{s} Q_4 + \frac{T_{3,0} F_2}{s} Q_4 F_3 \frac{P_3}{K_3} \frac{P_2}{K_2}, \\
X_4 &= \frac{i_4}{k_2} \frac{a_2}{a_1} Q_4 F_3 F_2 K_2 \frac{P_3}{K_3} \frac{P_2}{K_2}, \\
R_1 &= P_1 \frac{K_1}{K_2} P_2, \quad R_2 = P_3 \frac{K_1}{K_2} P_4, \quad R_3 = P_2 \frac{K_1}{K_2} P_1, \quad R_4 = P_4 \frac{K_1}{K_2} P_3,
\end{align*}
\]
\[ F_1 = Q_1 \frac{K_3}{K_4} Q_2, \quad F_2 = \frac{\delta_2}{\delta_1} \sqrt{a_2} \left( Q_3 - \frac{K_3}{K_4} Q_4 \right), \quad F_3 = Q_2 \frac{K_4}{K_3} Q_1, \quad F_4 = \frac{\delta_2}{\delta_1} \sqrt{a_2} \left( Q_4 - \frac{K_4}{K_3} Q_3 \right), \]

\[ P_1 = J_0(r_2 \sqrt{a_1}), \quad P_2 = Y_0(r_2 \sqrt{a_1}), \quad P_3 = J_1(r_2 \sqrt{a_1}), \quad P_4 = Y_1(r_2 \sqrt{a_1}), \]

\[ Q_1 = J_0(r_2 \sqrt{a_2}), \quad Q_2 = Y_0(r_2 \sqrt{a_2}), \quad Q_3 = J_1(r_2 \sqrt{a_2}), \quad Q_4 = Y_1(r_2 \sqrt{a_2}), \]

\[ K_1 = h_1 J_0(r_2 \sqrt{a_1}) \lambda_1 J_1(r_2 \sqrt{a_1}), \quad K_2 = h_1 Y_0(r_2 \sqrt{a_1}) \lambda_1 Y_1(r_2 \sqrt{a_1}), \]

\[ K_3 = J_0(r_3 \sqrt{a_2}), \quad K_4 = Y_0(r_3 \sqrt{a_2}), \]

\[ F_s = \frac{a_1 e^{h_1 T_k}}{s} + \sum_{n=1}^{\infty} \left[ \frac{a_{2n} + b_{1n}}{2j} \right] \frac{1}{s^{jn}} + \left[ \frac{a_{2n} - b_{1n}}{2j} \right] \frac{1}{s^{jn}} \]

### 3.3. Numerical calculation of Laplace inverse transform

Since it is hard to obtain analytical solutions in Laplace domain, the Stehfest method is adopted to numerically calculate the inverse transform of Laplace [13] (Eq. 17 and 18).

\[
\begin{align*}
T_1(r, \tau) &= \frac{\ln 2}{\tau} \sum_{i=1}^{N} \nu_i T_1 \left( \frac{\ln 2}{\tau} \right) \\
T_2(r, \tau) &= \frac{\ln 2}{\tau} \sum_{i=1}^{N} \nu_i T_2 \left( \frac{\ln 2}{\tau} \right)
\end{align*}
\]

\[ \nu_i = (-1)^{i+1} \sum_{k=0}^{N} \frac{N!}{k!(k-1)!(i-k)!(2k-i)!} \]  

where \( N, k, i \) are positive integers; \( \tau \) represents an independent variable; \( \nu_i \) is an intermediate function.

### 4. Calculation and analysis

Take a large-scale concrete clinker silo as an example; the silo belongs to a shallow bin with a diameter of 50 m, a wall thickness of 0.8 m and a height of 20 m. After the silo was fully loaded, the calculated was chosen on 9\(^{th}\) March. The temperature range was 4 \( ^\circ \)C ~ 15 \( ^\circ \)C.

The Laplace inverse transform was inverted using mathematical software Maple to obtain the temperature distribution of the silo on the calculated day. Besides, the temperatures obtained from the analytical solution and from the finite element method (FEM) was compared to verify the analytical method, which are shown in Figure 2.

It can be seen from Figure 2 that there is a good agreement between the analytical calculated results and the FE results. Although the initial temperature is the same, it is assumed that the points on the silo wall have the same temperature in the analytical calculation, which is different from the initial calculation of the finite element method. Therefore, the finite element results are slightly larger than the analytical calculation results before sunrise. Since the solar radiation intensity is slightly different from the theoretical solar radiation intensity during the linearization of the boundary conditions. Besides the boundary conditions of the inner surface of the storage layer are different, the analytical calculation results of the outer surface and inner surface of the walls are slightly different from the FE results. However, the error between these two methods is within 10\%. The comparison verifies that the analytical method described in section 3 could be successfully used in the analysis of the temperature distribution of the large-scale silos.
5. Conclusion
(1) In order to obtain a complete analytical solution, the assumption of one-dimensional heat conduction with two-dimensional temperature distribution is adopted, and the boundary conditions of solar radiation are simplified, which is different from the actual situation. Therefore, there is slight difference between the analytical calculation results and the finite element results.
(2) The temperature along the wall thickness at any time can be easily obtained with the analytical calculation method, and the obtained results are recommended as references for the design of temperature effects of silos, especially for large-scale silos.

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