Shocks in the low angular momentum accretion flow

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Abstract. We address the variability of low luminous galactic nuclei including the Sgr A* or other transient accreting systems, e.g. the black hole X-ray binaries, such as GX 339-4 or IGR J17091. These sources exhibit bright X-ray flares and quasi-periodical oscillations and are theoretically interpreted as the quasi-spherical accretion flows, formed instead of or around Keplerian accretion disks. In low angular momentum flows the existence of shocks for some range of leading parameters (energy, angular momentum and adiabatic constant of the gas) was studied semi-analytically. The possible hysteresis effect, caused by the fact that the evolution of the flow and the formation of the shock depends on its own history, was discovered. The presence of the shock in the accreted material is important for the observable properties of the out-coming radiation. In the shocked region the gas is dense and hot, thus much more luminous than in the other case. We study the appearance of standing shocks in low angular momentum gas accreting onto a black hole with numerical hydrodynamical simulations, using the ZEUS code with Paczynski-Wiita pseudo-Newtonian potential.

1. Introduction
The possible existence of shocks in low angular momentum flows has been studied from different points of view during the last thirty years. Quasi-spherical distribution of the gas endowed by constant specific angular momentum λ and the arisen bistability was studied already by [1]. More recently, the shock existence was found also for the disc-like structure in hydrostatic equilibrium with low angular momentum both in pseudo-Newtonian potential [2] and in full relativistic approach [3]. Two component advective model was proposed to describe the system with Keplerian disc accompanied by a low angular momentum component – the corona [4]. In this perspective the observed QPOs are also being linked with oscillations of the shock front [5].

In this contribution we follow up with our recent study [6], where more detailed discussion can be found. We proceed with the study of quasi-spherical configuration and perform “1.5-D” numerical simulations of the flow with changing specific angular momentum at the outer boundary in ZEUS code. Our main aim is to address the variability of the outgoing radiation.

2. Quasi-spherical slightly rotating flow
In our study we consider the non-viscous accretion flow with the polytropic equation of state for the gas pressure \( p = K \rho^\gamma \), where \( \gamma \) is the adiabatic index, hence the local sound speed \( a \) is given by the relation \( a^2 = \gamma p / \rho = \gamma K \rho^{\gamma-1} \), where \( \rho \) is the gas density.

The mass accretion rate \( \dot{M} \), which is a constant linked to the chosen density units, and accordingly the entropy accretion rate \( \dot{M}_s \) is in the spherical geometry of the flow given by [4]

\[
\dot{M} = M K^n \gamma^n = u r^2 K^n \gamma^n = u r^2 a^{2n}. \tag{1}
\]
Energy conservation for the steady state is written in the form

\[ \epsilon = \frac{1}{2} u^2 + \frac{a^2}{\gamma - 1} + \frac{\lambda^2}{2 r^2} + \Phi(r), \]  

(2)

where \( \Phi(r) \) is the gravitational potential.

The radial gradient of the flow velocity is obtained from the equations after performing appropriate differentiation (for detailed derivation of all presented equations see [6])

\[ \frac{d u}{d r} = \frac{\lambda^2}{r^3} - \frac{1}{2(r - 1)^2} + \frac{2a^2}{r} \left( \frac{r_c}{4(r_c - 1)^2} - \frac{\lambda^2}{2 r_c^2} \right). \]  

(3)

The last equality holds for the Paczynski-Wiita potential, which in the form \( \Phi(r) = \frac{1}{2(r - 1)} \) mimics the most important features of strong gravitational field near the compact object (comparison of more pseudo-Newtonian potentials could be found in [2]).

The critical points position is given as the position \( r_c \), where the numerator and denominator of equation (3) are both equal to zero, and is found as a solution of the equation

\[ \epsilon - \frac{\lambda^2}{2 r_c^2} + \frac{1}{2(r_c - 1)^2} = \frac{\gamma + 1}{2(\gamma - 1)} \left( \frac{r_c}{4(r_c - 1)^2} - \frac{\lambda^2}{2 r_c^2} \right) = 0. \]  

(4)

For a certain subset of the parameters \( (\epsilon, \lambda, \gamma) \) equation (4) yields the existence of three different critical points, two of which \( r_{in} \) and \( r_{out} \) are of the saddle type, hence locally the accretion flow can pass through them (inner and outer branch of solution, respectively). However, because the inner critical point is homoclinic, the appropriate inner solution orbit returns to it after making a loop in the \( r - M \) diagram. Hence, the accretion flow which is coming from infinity down onto the compact object cannot follow this branch only. Nevertheless, if Rankine-Hugoniot conditions, which are in our case expressed by the relation for the preshock \((-\)) and postshock \((+)\) Mach number \( M_{\pm} \) as

\[ \frac{(\frac{1}{M_{\pm}^2} + \gamma M_{\pm}^2)}{2 - (\frac{1}{M_{\pm}^2} + \gamma M_{\pm}^2)} = \frac{(\frac{1}{M_{\pm}^2} + \gamma M_{\pm}^2)}{2 - (\frac{1}{M_{\pm}^2} + \gamma M_{\pm}^2)}, \]  

(5)

are fulfilled at certain radius \( r_s \), then the shock can form at that position. This causes the flow jump from the outer branch with a lower entropy accretion rate \( \dot{M}_{out} \) to the inner branch with a higher entropy accretion rate \( \dot{M}_{in} \), while entropy is being produced at the shock front.

3. Numerical simulations

Using the above semi-analytical solution describing the accretion flow with shock as a initial condition we follow the dynamical evolution of the flow with Paczynski-Wiita pseudo-Newtonian potential included in the simulations with the hydrodynamical code ZEUS. The detailed description of the routine used for the initialization of the grid flow variables and the boundary conditions is given in [6].

In the multicritical region two different accretion solutions are possible, the outer branch without shock, and the solution with shock, connecting the outer and inner branch. These two types of accretion flows are depicted in Fig. 1, where the radial profiles of the most important variables \( (M, u, \rho) \) are plotted for both. The shock location \( r_s \) is the point, where the supersonic flow \( (M > 1) \) becomes suddenly subsonic \( (M < 1) \), the velocity of the flow decreases and the density rises significantly. As can be seen in this Figure, the difference between the two possible
densities is about one order of magnitude, hence the total mass of the gas enclosing the compact object up to a certain radius is very different.

The increase of density in the inner region of the flow with a constant mass accretion rate $\dot{M}$ depends on the specific angular momentum $\lambda$. In the right panel of Fig. 1 the density at a given radius ($r = 10M$) is plotted against $\lambda$ for outer branch and the shock solution. For low value of $\lambda$ no shock solution can be found and only the outer accretion flow exists. For $\lambda$ lying in the shock existence interval the density of the shocked flow rises much faster with $\lambda$ than the density of the outer branch. For a high $\lambda$ their values differ by almost three orders of magnitude.

Hence for particular conditions at infinity given by the value of energy $\epsilon$ and mass accretion rate $\dot{M}$, which is related to the density at infinity $\rho_\infty$ (see [6]), the accretion flow profile differs significantly with changing $\lambda$. Therefore we study the behaviour of the flow when the specific angular momentum of matter incoming through the outer boundary changes periodically with time. The boundary value of the angular velocity is given by the relation

$$v_3(t) = \lambda(t)/r, \quad \lambda(t) = \lambda_0 + \Delta\lambda \sin \left(\frac{2\pi t}{T_v}\right),$$

where $\Delta\lambda$ is the amplitude of angular momentum variation and $T_v$ is the period of the variation.

Due to the fact, that for parameters in the multicritical region two different steady solutions exist, the “hysteresis loop”-like behaviour can occur, as was suggested by [3]. In that case different initial conditions lead to different steady solution. Hence, in dynamical evolution the choice between these two states depends on the history of the flow. Namely when the flow is going through the outer type of critical point only (as is the only possibility for low $\lambda$), then the shock will not create even when the angular momentum rises into the multi-critical region. On the other hand, when the flow already had passed through the inner critical point (as the only solution for a very high $\lambda$ does), then for decreasing $\lambda$ the shock appears.

For simulating this effect we need the change of $\lambda$ to be high enough to cross both lower and upper boundary of the multi-critical region. At the same time, we need to be aware of the numerical limitations given by the size of the grid and the chosen boundary conditions, which are not consistent with the inner type of solution. Therefore the angular momentum must not be too high. Otherwise the solution in the outer part would change and would interfere with the outer boundary, causing the production of some spurious reflecting waves.
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Figure 2. Accretion rate $\dot{M}_n(t)$ (1st row left) and total mass of gas enclosed inside the radius $r = 100M$ (1st row right). Position of shock $r_s$ with respect to $\lambda(t, r = 100M)$ (2nd row left). $\lambda_t(t)$ (dashed line) and $\lambda(r = 100M)$ (solid line, 2nd row right). For no shock we set $r_s = 0$. Snapshots of the accretion flow profiles at $t_1 = 2.01$ (3rd row left), $t_2 = 2.57$ (3rd row right), $t_3 = 2.575$ (4th row left), $t_4 = 2.695$ (4th row right) $10^{-7}M$ – left $y$-axis stands for dimensionless $\mathfrak{M}$, $u[c]$ and $\rho[\rho_\infty]$, right $y$-axis stands for $\lambda[M]$. $\epsilon = 0.0001$, $\gamma = 4/3$, $\lambda_0 = 3.74M$, $\Delta \lambda = 0.18M$, $T_v = 10^7M$. 

Legend:

- $\mathfrak{M}$ – black
- $u$ – dashed
- $\rho$ – dotted
- $a$ – dot-dashed
- $\lambda$ – double-dashed

- Position of shock $r_s$.
- Total mass of gas enclosed inside the radius $r = 100M$.
- Accretion rate $\dot{M}_n(t)$.
- Dimensionless $\mathfrak{M}$.
- $u[c]$.
- $\rho[\rho_\infty]$.
- $\lambda[M]$. 

- $t_1 = 2.01$ (3rd row left).
- $t_2 = 2.57$ (3rd row right).
- $t_3 = 2.575$ (4th row left).
- $t_4 = 2.695$ (4th row right). 

- $10^{-7}M$. 

- $\epsilon = 0.0001$, $\gamma = 4/3$, $\lambda_0 = 3.74M$, $\Delta \lambda = 0.18M$, $T_v = 10^7M$. 

- $10^{-4}$.
- $10^{-3}$.
- $10^{-2}$.
- $10^{-1}$.
- $10^{0}$.
- $10^{1}$.
- $10^{2}$.
- $10^{3}$.
- $10^{4}$.
- $10^{5}$.
- $10^{6}$.
- $10^{7}$.
- $10^{8}$.
- $10^{9}$.
- $10^{10}$.
- $10^{11}$.
- $10^{12}$. 

- Data points and lines for different times.
In Fig. 2 one example of such solution for $\lambda_0 = 3.74 M, \Delta \lambda = 0.18 M$ and $T_v = 10^7 M$ is shown. In the top left panel the time dependence of the normalized accretion rate through the inner boundary of the grid, defined as $\dot{M}_n(t) = \dot{M}(t)/\dot{M}(t=0)$ is plotted. Broad and sharp peak could be seen, for more details see [6].

However, the changes of the accretion rate at the inner boundary are not the only important quantity for the out-going radiation. Namely for higher $\lambda$ the matter is gathered up in the innermost part of the accretion flow (within the inner several tens of Schwarzchild radii), where the density increases significantly (see Fig. 1), even though the mass accretion rate remains constant. Thus for higher $\lambda$ there is much more matter very close to the compact object, presumably being hot and emitting a lot of radiation. In the top right panel of Fig. 2 the total mass $M_g^{100 M}$ enclosed inside the radius $r = 100 M$ is depicted. Its very sharp increase corresponds to the abrupt switch between the outer branch and shock solution, when the angular momentum in the inner region approaches its maximal value. (See the snapshots at time $t_2 = 2.57 \cdot 10^7 M$ (3rd row right) and $t_3 = 2.575 \cdot 10^7 M$ (bottom left)). This is connected with a temporal switch off of the accretion rate, because the matter is stopped at several Schwarzchild radii. Then slower decline for decreasing angular momentum is seen accompanied by a peak in $M_n$ owing to the gas overcoming the decreasing barrier and plunging into the black hole. Similar behaviour shows also the shock position $r_s$, which time dependence is plotted in the 2nd row right panel of Fig. 2. The second row left panel shows the loop behaviour of the shock, when the shock position decreases with angular momentum (measured at some chosen radius in the inner region, $r = 100 M$), but is not present (then we set $r_s = 0$) when the angular momentum is increasing. The points $r_s(t)$ versus $\lambda(t, r = 100 M)$ are plotted for $t > 1.025 \cdot 10^7 M$, which covers four consecutive loops, during which repeating creation and disappearance of the shock can be seen.

In case when $\Delta \lambda$ is smaller so that the angular momentum does not leave the multicrotical region, the evolution is a bit different, mainly the shock does not disappear or create, depending on the initial conditions. Such example is given in Figs. 3 and 4 with $\gamma = 4/3, \epsilon = 0.0001, \lambda_0 = 3.77 M, \Delta \lambda = 0.2 M, T_v = 10^6 M$ (eventhough the amplitude is quite high, the period is so short, that the maximal value cannot pass through the last cells of the grid). The period was chosen with regard to the microquasar IGR J17091-3624 which in a “heartbeat” mode exhibits quasi-periodic flares with a period of a few tens of second. For the estimate of its mass $M \sim 6 M_\odot$ [7], the period $T_v = 10^6 M \sim 29.55s$. For such parameters we obtain very high and sharp peaks in $\dot{M}_n$ (Fig. 3 left). But the quantity $M_g^{\infty}$ shows periodical peaks with a shape very similar to the light curve of IGR J17091-3624, namely peaks with slow rise and sharp decline. The choice

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**Figure 3.** Normalized accretion rate through the inner boundary $\dot{M}_n(t)$ (left) and the total mass of gas $M_g(t)$ enclosed inside the radius $r_1 = 100 M, r_2 = 50 M, r_3 = 200 M$ (right).
of the threshold radius $r_1$ affects only slightly the height and the shape of the peak, and in Fig. 3 we show the comparison for $r_1 = 100M$, $r_2 = 50M$ and $r_3 = 200M$. Of course, to obtain a light curve predicted by our model more precisely, radiation characteristic at each radius and the possible contribution of another component of the accretion flow has to be taken into account.

We conclude that our simplified model offers a reasonable qualitative explanation of quasi-periodic X-ray flares originating in the low angular momentum flows. The periodic change of the angular momentum at some specified threshold radius may be responsible for the flaring activity behind this radius, with timescales and profiles’ shape similar to the observed ones. In particular, for the microquasar IGR J17091-3624, the thermal instability of the Keplerian, geometrically slim accretion disk was proposed to account for the variability observed in soft X-rays [8]. The hard X-ray emitting corona, which forms above the disk in its innermost radial zone, may possess a sub-Keplerian quasi-spherical configuration. Since $T_v$ at the radius where the coronal flow develops is of the same order as the flaring period, the hard X-ray flares will be connected with the conditions in the disk/corona boundary and the evolution of angular momentum in the coronal flow. The point that this phenomenon cannot be connected with e.g., the orbital motion of the companion star, which is by orders of magnitude longer, is further supported by the fact that in another microquasar showing this kind of activity, GRS 1915+105, the companion star and binary parameters are very different than in IGR J17091-3624 [9].

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