Optimization Model for System Redundant Reliability Based on Dynamic Programming and its Application in Ball Screw Actuator

Fan Zhang¹, Zhida Zhu², Li Zeng³, Min Dai⁴, Jian Yang⁵, Haijiang Kou⁶
¹ College of Mechanical Engineering, Yangzhou University, Yangzhou, China
*Corresponding Author: zdzhu@yzu.edu.cn

Abstract

In some products or systems, the redundant parts, components or subsystems must be configured for their high reliability, in addition to the high reliability of parts and components. But the systems would pay more “cost” of resources such as cost, weight or volume, than those without redundancy of parts, components and subsystems. Therefore, the study of redundant reliability optimization for the system would be carried out. This paper presents several models for basic component structures of reliability based on the block diagram. According to the block diagram model for series-parallel reliability of components and subsystems, a mathematical model and its optimization method for system redundant reliability optimization are established, which are based on dynamic programming. Then the ball screw actuator system was taken as an example, and the redundant reliability of its parts or subsystems was calculated with the dynamic programming method based on the control sequence method. And the optimized result of the ball screw actuator system was gotten with the system constraints conditions. The redundancy of components at all levels is obtained with the greatest reliability.

Keywords: redundancy reliability optimization; block diagram; dynamic programming method; ball screw actuator.

1. Introduction

Reliability is the ability of a product to perform some specified functions in the prescriptive condition and period [1]. It is one of the quality characteristics of product and system, and also an important performance of product design. In order to improve the reliability of the product, some spare parts with the same performance for the main function or important place can be configured into the product, and the automatic input devices for the spare parts should be designed. The more spare parts are used, and the system would be more reliable. But while more spare parts are used, the cost, quality and volume of the whole system would increase. Therefore, how to select the number of spare parts of each important component with the constraints of cost, quality and volume should be studied, which would maximize the reliability of the whole system. This is the optimization design of reliability.

2. Basic Model of System Reliability

2.1 A. Reliability model of series system

Fig. 1. Block diagram model of series system

Series system refers to a system in which the failure of any unit results in the failure of the whole system, which means that the system can work normally while all units can work normally. Usually, mechanical system is series system, and its reliability model is shown in Fig 1. If the reliability of a unit (subsystem or component) in the system is \( R_i \) (\( i = 1, 2, ..., n \); \( n \) is the number of subsystems or components in the system), the reliability of the series system is

\[
R_s = \prod_{i=1}^{n} R_i
\]  

(1)

It can be seen from equation (1) that the system has more component units and subsystems, its failure rate or probability is higher, and its reliability is lower.

2.2 Reliability model of parallel system

The relationship of subsystems or units in parallel systems is shown in Fig. 2. If one of these subsystems or units does not fail, the whole system would not fail. In other words, while all subsystems and units of the system fail, the whole system would fail. If the reliability of a subsystem or unit in the system is \( R_i \) (\( i = 1, 2, ..., n \); \( n \) is the number of subsystems or components in the system), then the reliability of the parallel system is

\[
R_p = 1 - \prod_{i=1}^{n} (1 - R_i)
\]  

(2)

According to equation (2), the reliability of the parallel system is greater than the reliability of each subsystem or unit in the system. When parallel structure is adopted in the system, the size, weight and price of the system will be increased obviously.

Fig. 2. Block diagram model for parallel system

2.3 Reliability model of series-parallel system

Series-parallel system is the system in which the subsystems or components in the system are firstly parallel connected and then series connected. Its block diagram model is shown in Fig 3. The whole system is finally
composed of some series subsystems or parts, while the subsystems are composed of paralleled units. When calculating the reliability of the system, the paralleled parts and components in subsystem are firstly treated as a unit, and then whole subsystems of the system would be calculated as a series system.

![Block diagram model of series-parallel system](image)

If the system has $m$ series subsystems, and the $i$-th subsystem has $n_i$ parallel units. The reliability of the $j$-th unit in the $i$-th subsystem is $R_{ij}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n_i$, the reliability of the series-parallel system is

$$R_i = \prod_{j=1}^{n_i} \left[ 1 - \prod_{j=1}^{n_i} (1 - R_{ij}) \right]$$  \hspace{1cm} (3)

### 2.4 Reliability model of parallel-series system

Parallel-series system is a parallel structure composed with series subsystems or units, and its model block diagram is shown in Fig 4. When calculating the reliability of the system, the parts or components can be firstly calculated as series subsystem, and then the subsystems would be connected in parallel.

![Block diagram model of parallel-series system](image)

If the system has $m$ parallel subsystems, and the $i$-th subsystem has $n_i$ series units. The reliability of the $j$-th unit in the $i$-th subsystem is $R_{ij}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n_i$, the reliability of the series-parallel system is

$$R_i = 1 - \prod_{j=1}^{n_i} \left[ \prod_{j=1}^{n_i} (1 - R_{ij}) \right]$$  \hspace{1cm} (4)

Series-parallel system and parallel-series system are the simple mixed system reliability models, which are the basis of analyzing the complex system. Series-parallel system and parallel-series system are also called hybrid system.

### 3. Reliability Redundancy Optimization Model Based on Dynamic Programming Method

#### 3.1 Mathematical model for redundancy optimization of system reliability

The mathematical models for reliability redundancy optimization of the system include:

1. Maximizing the reliability of the system when all subsystems are optimal redundancy under the constraint that the system resources are reliability functions, it can be expressed as

$$\max R_i = \prod_{j=1}^{N} R_{ij}, \text{ constraint } \sum_{j=1}^{N} g_j(R_i) \leq b_j, i = 1, 2, \ldots, m$$  \hspace{1cm} (5)

In the equation, $R_i$ is the reliability of the actuator system; $R_{ij}$ is the reliability of the $j$-th component or subsystem; $g_j(R_i)$ is the resource consumed on the $j$-th component or subsystem, and its relationship with the reliability $R_i$ of the component or subsystem can be either linear or nonlinear; $b_j$ is the maximum or minimum total of the $i$-th resource.

2. Maximizing the reliability of the system under functional constraints with redundant resources consumed at each level, it can be expressed as

$$\max R_i = \prod_{j=1}^{N} R_{ij}, \text{ constraint } \sum_{j=1}^{N} g_j(x_j) \leq b_j, i = 1, 2, \ldots, m$$  \hspace{1cm} (6)

In the equation, $R(x_j)$ is the reliability of the system level $j$ (the $j$-th subsystem or component), and is the redundancy of the components or subsystems.

3. When the reliability of subsystems at all levels is a function of redundancy and the constraints of total system reliability are satisfied, the system consumes less resources. It can be expressed as

$$\max R_i = \prod_{j=1}^{N} R_{ij}, \text{ constraint } \prod_{j=1}^{N} R_{ij} \geq R_i$$  \hspace{1cm} (7)

In the equation, $R_i$ is the total resource of the system, which can be the cost of the system, or the volume or weight of the system.

4. For complex systems, redundant reliability can be obtained by conditional probability or network algorithm. The optimization model is

$$\max R_i = f(R_i, R_{i-1}, \ldots), \text{ constraint } \sum_{j=1}^{N} g_j(R_i) \leq b_j, i = 1, 2, \ldots, m$$  \hspace{1cm} (8)

Here, the reliability of the system is a function of the reliability $R_i$ of components or subsystems.

#### 3.2 Redundancy reliability optimization model based on dynamic programming

Dynamic programming is one of the methods of reliability design and optimization. The following three methods of redundant reliability optimization based on dynamic programming are introduced.

1. Basic dynamic programming principle

The reliability optimization equation based on dynamic programming method obtained from document[7-10] is:

$$f_i(b_{i+1}, s_{i+1}) = \max \left\{ R_i'(x_i) g_i(x_i, b_{i+1}, s_{i+1}) \right\}$$  \hspace{1cm} (9)
\[
\begin{align*}
R(x) & \geq s \\
b - g(x) & \geq 0 \\
x & = 0, 1, 2, L
\end{align*}
\]

In the equation \( R = 1 - (1 - R)^{x_k} \), \( x_k \) represents the decision variable of the number of redundant parallel components in the \( k \)-th level subsystem; \( b_k \) represents the state variable of the resources consumed from the first level to the \( k \)-th level system; \( S_k \) is the state variable of the working probability of the first \( k \)-level components or subsystems of the system.

The state transition equation of the system obtained from the constraint condition shown in equation 9 is

\[
b_{k+1} = b_k + g_k(x_k), S_{k+1} = S_k \cdot R_k'(x_k)
\]

Therefore, the reliability optimization model based on dynamic programming is

\[
f_k(b_k, S_k) = \max_{x_k} \left\{ R_k'(x_k) \cdot f_{k+1}(b_k, S_k) \right\}
\]

In the equation, \( x_k \) is the smallest integer of the redundant component of the \( k \) level subsystem obtained by \( R_k(x_k) = S_k \). When the minimum reliability of the system is not unrestricted, \( x_k^* = 1 \); \( x_k^* \) is the largest integer of the redundant components for the \( k \)-th subsystem obtained by \( g_k(x_k) \leq b_k \).

Based on the above iteration calculation, the optimal value of spare parts redundancy of subsystems at all levels can be obtained by heuristic column search calculation in the order of resource increase such as consumption cost, quality or volume in the range of \( x_k \) value. Class n results: \( (x_{\mu}, x_{\mu-1}, \ldots, x_1) \) is the combination of optimal redundant components at all levels when system reliability is greatest.

(2) Dynamic programming based on Lagrange multiplier

When there are many constraints on the objective function of actuator system reliability optimization, the Lagrange multiplier method can be used to reduce the dimension of the model. If the system constraints are

\[
\begin{align*}
\sum_{j=1}^{n_1} g_{ij}(x_j) & \leq b_1 \\
\sum_{j=1}^{n_l} g_{ij}(x_j) & \leq b_l
\end{align*}
\]

Introducing the Lagrange multiplier lambda as a penalty term, the reliability redundancy optimization model of the system becomes

\[
\max \prod_{j=1}^{n} R_j(x_j) \exp \left[ -\lambda \sum_{j=1}^{n} g_{ij}(x_j) \right], \text{ constraint } \sum_{j=1}^{n} g_{ij}(x_j) \leq b_l
\]

Therefore, the basic equations for dynamic programming optimization of the system are changed into

\[
f_k(b_k, y_k) = \max_{x_k} \left\{ R_k'(x_k) \cdot f_{k+1}(b_k, y_k) \cdot \exp[\lambda g_k(x_k)] \right\}
\]

In the equation, the \( x_k^* \) value is determined by distributing the redundant number on a resource constrained condition. In the assignment process, if the reliability of the last structure \( x \) is \( R(x) \) and no constraint conditions are destroyed, then the set of minimum integers can be obtained by solving \( n \) equations \( (x^k = x_j) \) of \( R(x) \leq 1 - (1 - R_j)^{x_k} \) for \( x_k \).

The value of \( x_k^* \) is determined after the value of \( x_k^* \) is determined according to the system constraints. Determining the value of the upper bound \( x_k^* \) of level \( k \) subsystem can make \( x_k = x_k^* \), \( j = 1, 2, \ldots, n, j \neq k \). \( x_k^* \) is the smallest integer in the set \( \{c_1, c_2, \ldots, c_i\} \).

\[
c_j = \max \left\{ x_j \right\}, \text{ is integer, and } g_k(x_j, L, x_{k+1}, x_l^*),
\]

\[
x_k^* = \{x_j, L, x_{k+1}, x_l^*\}, j = 1, 2, L, r.
\]

The determination of the value of \( \lambda \) requires that the resource constraints should be equal as far as possible, and the optimization results for different values of \( \lambda \) should be different. The value of \( \lambda \) should be chosen to maximize the reliability \( R_k \).

(3) Dynamic programming based on control sequence

The model of reliability redundancy optimization based on control sequence for dynamic programming of the system is

\[
\max R = \prod_{j=1}^{n} \left[ 1 - (1 - R_j)^{x_j} \right], \quad g_j(x_j) \leq b_j, \quad i = 1, 2, \ldots, r
\]

In solving the reliability redundancy optimization of multi-constrained systems, the control conditions of the system structure can be simplified because of the large amount of computational work.

If the constraints that the system satisfies is

\[
\sum_{j=1}^{n} g_j(x_j) \leq b_j, \quad i = 1, 2, \ldots, r, \quad R_j(x_j) \geq R_j(x)
\]

To sum up, we can see that one system structure \( x \) is controlled by the system structure \( x^\ast \). That is, if the control structure of the system is reliable, and the resource consumption is small, and all the resource constraints of equation (14) are satisfied, and the system is not controlled by the distribution sequence of other redundant numbers, it is called a control sequence.

Dynamic programming based on control sequence: the first two levels (i.e. the first and second levels) are combined to produce the control sequence of the structure; then, the third level is combined to produce another control sequence. If a constraint condition is destroyed, its sequence is terminated. From first, second, ... Until the control sequence combination of the N level system produces the optimal system structure. From first, second, ... Until the control sequence combination of the N level system produces the optimal system structure. In order to reduce the length of control sequences, we can use heuristics to determine the upper \( x_j^\ast \) and lower bounds \( x_j \) (j = 1, 2, ... n) of \( x_j \).

3.3 Maintaining the Integrity of the Specifications

The template is used to format your paper and style the text. All margins, column widths, line spaces, and text fonts
are prescribed; please do not alter them. You may note peculiarities. For example, the head margin in this template measures proportionately more than is customary. This measurement and others are deliberate, using specifications that anticipate your paper as one part of the entire proceedings, and not as an independent document. Please do not revise any of the current designations.

4. Redundancy Reliability Optimization of Ball Screw Actuators

Before you begin to format your paper, first write and save the content as a separate text file. Keep your text and graphic files separate until after the text has been formatted and styled. Do not use hard tabs, and limit use of hard returns to only one return at the end of a paragraph. Do not add any kind of pagination anywhere in the paper. Do not number text heads-the template will do that for you.

4.1 Electro-hydraulic control system for ball screw actuators

Fig. 5 shows the electro-hydraulic control system of the ball screw actuator, which consists of three main parts: electro-hydraulic control module, hydraulic system valve module and ball screw actuator [11]. Electronic control components include servo controller and amplifier, solenoid valve controller, motor control driver, etc. Pipe valve components include servo controller and amplifier, solenoid valve, relief valve, and hydraulic pipeline, etc. Ball screw actuator is a kind of hydraulic-mechanical compound transmission mechanism which converts linear motion into rotary swing. The structure of the new hydraulic swing motor is shown in Fig 6 [12-13]. It is mainly composed of three parts: ball screw pair, ball spline pair and hydraulic cylinder assembly.

In addition, the ball screw actuator can also be used as the hydraulic oscillating motor of the hydraulic system, which is characterized by high efficiency and large driving torque, and is widely used in the field of construction machinery hydraulic system.

4.2 Composition and reliability requirements of ball screw actuator system

As can be seen from Fig. 5 and the characteristics of the system, the whole system cannot work when any component of the three components fails, which causes the system failure. The system is a three-level series system. In order to improve the reliability of the whole system, the redundant configuration design of the parts of each group (subsystem) is carried out and the parts can be automatically put into the work of the system. Obviously, the more redundant parts, the higher the reliability of the system, but the cost, quality and volume of the whole system resources are increased accordingly, the system mobility and working accuracy will also be reduced. In order to maximize the reliability of the whole system, it is necessary to select the number of redundant parts of each subsystem reasonably.

For the ball screw actuator system, each subsystem can carry out redundant configuration design of components as long as it meets the resource (quality) constraints, but the maximum reliability value should be selected based on the different combination schemes. The reliability and resource "quality" of each subsystem or component of the ball screw actuator system are shown in Table 1. The constraint condition is that the resource (quality) of the whole actuator system should not exceed 40kg. The system reliability is maximized by the redundancy of the components. The solution procedure is as follows.

Table 1. Basic Data

| Unit | Name                        | Reliability (R) | Quality (M) |
|------|-----------------------------|-----------------|-------------|
| I    | Electronic control modules  | 0.9             | 6kg         |
| II   | Value tube modules          | 0.8             | 10kg        |
| III  | Actuator                    | 0.9             | 8kg         |

4.3 Optimization of redundant reliability of ball screw actuator system

The reliability optimization model of actuator system can be obtained by equation (14):

$$\max R = \prod_{j=1}^{n}(1 - (1 - R_j)^{x_j})$$

constraint $\sum m(j)x_j \leq 40$(kg), $x_j$ is integer

To solve the system structure and reliability process of the optimal ball screw actuator, the reliability optimization analysis and calculation of the system are carried out according to the dynamic programming method based on the control sequence.

(1) Determining lower bound $x_{i,k}$ and upper bound $x_{u,k}$

The determination of $x_{i,k}$: if the minimum reliability of the system is not restricted, it is desirable to $x_{i,k}=1$, so $x_{i,k}=1$, ...
\[ x_i' = 1, x_j' = 1; \]

The determination of \( x_i' \): according to the constraint conditions,

\[ x_i' = M_1 + x_i' \leq 1, M_2 + x_j' \leq 40 \]

Bring \( M_1, M_2, M_3, x_i', x_j' \) into equation, then \( x_i' \leq 3.6 \) take \( x_i' = 3, x_j' = 3 \)

\( (2) \) Redundant resources and reliability calculation of subsystems at all levels

First level subsystem, \( x_i' = 4 \)

When \( x_1 = 1, M_1 = 86kg, R_1 = 0.9 \); When \( x_1 = 2, M_1 = 122kg, R_1 = 1 \); When \( x_1 = 3, M_1 = 18kg, R_1 = 1 \); When \( x_1 = 4, M_1 = 24kg, R_1 = 1 \). The reliability is 0.992.

Second level subsystem, \( x_i' = 3 \)

When \( x_2 = 1, M_2 = 10kg, R_2 = 0.8 \); When \( x_2 = 2, M_2 = 22kg, R_2 = 1 \); When \( x_2 = 3, M_2 = 30kg, R_2 = 1 \). The reliability is 0.999.

Third level subsystem, \( x_i' = 2 \)

When \( x_3 = 1, M_3 = 8kg, R_3 = 0.9 \); When \( x_3 = 2, M_3 = 16kg, R_3 = 1 \); When \( x_3 = 3, M_3 = 30kg, R_3 = 1 \). The reliability is 0.999.

(3) Reliability and constraint resource calculation of systems at different levels of redundancy

1) reliability and constrained resource calculation of first and second different redundancy combinations

The control sequence is composed of first levels and second levels, and is obtained by eliminating other control elements. The calculation process is as follows:

When \( x_1 = 1, x_2 = 1 \), the reliability is \( R_{12} = R_1 \cdot R_2 = 0.9 \times 0.8 = 0.72 \). Resources of constraints is \( M_{12} = M_1 + M_2 = 6 \times 10 = 60kg \).

When \( x_1 = 2, x_2 = 1 \), the reliability is \( R_{12} = R_1 \cdot R_2 = 0.9 \times 0.8 = 0.72 \). Resources of constraints is \( M_{12} = M_1 + M_2 = 6 \times 10 = 60kg \).

In this way, all calculations are shown in Table 2.

2) reliability and constrained resource calculation of first, second and third different redundancy combinations

As shown in Table 2, the elements \((x_1, x_2) = \{(4, 2), (2, 3), (3, 3), (4, 3)\}\) have exceeded the quality constraints of the available resources and are therefore not combined with the third level. Combining the remaining elements in Table 2 with different redundancies at Level 3, the process of calculating system reliability and resources is as follows:

When \( x_1 = 1, x_2 = 1 \), the reliability is \( R_{123} = R_1 \cdot R_2 \cdot R_3 = 0.72 \times 0.9 = 0.648 \).

Resources of constraints is \( M_{123} = M_1 + M_2 + M_3 = 16 + 8 = 24kg \).

When \( x_1 = 2, x_2 = 1 \), the reliability is \( R_{123} = R_1 \cdot R_2 \cdot R_3 = 0.72 \times 0.9 = 0.648 \).

Resources of constraints is \( M_{123} = M_1 + M_2 + M_3 = 16 + 8 = 24kg \).

In this way, all calculations are shown in Table 3.

The elements satisfying the quality constraints of resources in Table 3 are the results of the redundancy reliability optimization design of the system based on the dynamic programming method of control sequence. The optimal control sequence obtained from table 3 is:

\[ (x_1, x_2, x_3) = \{(1, 1, 1), (1, 1, 2), (1, 1, 3), (2, 1, 1), (2, 1, 2), (2, 2, 1), (3, 1, 1)\} \]

The reliability and resource (quality) values of each element of the optimal control sequence can be obtained from table 3. Among them, the reliability of the system redundancy combination (2, 2, 1) is the largest. That is, the dynamic programming method based on control sequence is used to solve the optimal reliability of system redundancy.

That is, when the redundancy of the first stage electronic control component is 2, the second stage valve component is 2, and the third stage actuator is 1, the reliability of the system is maximum.

5 Conclusion

The basic block diagram models of series system, parallel system, series-parallel system and parallel-series system are introduced. The mathematical models and optimization methods of redundancy reliability optimization based on dynamic programming method are established respectively.

Based on the dynamic programming method of control sequence, the redundancy reliability of the ball screw actuator system is optimized, and the redundancy of the components with the highest reliability is obtained when the constraints are satisfied in the combination of the electronic control components, valve components and actuators.

Acknowledgment

The authors gratefully acknowledge the reviewers for their comments and suggestions. This research was supported by the Natural Science Foundation of Jiangsu Province [grant number BK20190912], the Natural Science Foundation of China [grant number 61701430], the Project of ‘Lyu Yang Jin Feng’ Science and Technology Program.

References

(1) Xiaochun Zhang, Yuqi Han, “Optimization of cost and reliability,” Journal of Nanjing University of Science and Technology, Vol. 25, No. 5, 2001, pp. 550-553.

(2) Li-Yang Xie, Xue-Hong He, Jia Li, “Reliability and safety design of electromechanical system,” Harbin Institute of Technology Press, 2006.

(3) Xingchun Gen, Guoxiang Lin, Jinhao Ye, “Dynamic optimization design of mechanical system reliability,” Mechanical Design and Manufacturing, No. 9, 2009, pp. 109-110.

(4) Jing Chen, Xiaoxiang Hao, Feng-Xiang Shao, “Application of redundant reliability optimization design,” China Coal, Vol. 36, No. 10, 2011, pp. 76-77.

(5) Ning Huang, Weizhi Chen, Shuai Shi, “Software reliability allocation method based on dynamic programming,” Computer Application and Software, Vol. 28, No. 3, 2011, pp. 119-120.

(6) De Li, Songdi Qian, “Operations research,” Beijing: Tsinghua University Press, 1985, pp. 498-514.

(7) Ri-Shuang Wang, Bing Xu, “Application of dynamic programming,” Beijing: National Defense Industry Press, 1987.
Table 2. Reliability and Constrained Resource Calculation of First and Second Redundancy Combinations (x₁, x₂)

| Calculation of Reliability R₁₂ and Resources M₁₂ | x₁=1 | x₁=2 | x₁=3 | x₁=4 |
|-----------------------------------------------|------|------|------|------|
|                                              | R₁  | M₁  | R₁  | M₁  | R₁  | M₁  | R₁  | M₁  |
|                                              | 0.9 | 6kg | 0.99| 12kg| 0.99| 18kg| 0.999| 24kg|
| x₂=1                                          | R₂  | 0.8 | 0.72| 0.792|0.7992|0.7999|
|                                              | M₂  | 10kg| 16kg| 22kg| 28kg| 34kg|
| x₂=2                                          | R₂  | 0.96| 0.864|0.9504|0.9590|0.9599|
|                                              | M₂  | 20kg| 26kg| 32kg| 38kg| 44kg|
| x₂=3                                          | R₂  | 0.992|0.8928|0.9821|0.9910|0.9919|
|                                              | M₂  | 30kg| 36kg| 42kg| 48kg| 54kg|

Table 3. Reliability and Resource Calculation Of First and Second and 3 Level Redundancy Combinations (x₁, x₂, x₃)

| Calculation of Reliability R₁₂₃ and Resources M₁₂₃ | x₁, x₂ | x₁, x₃ | x₂, x₃ |
|--------------------------------------------------|--------|--------|--------|
|                                                  | R₁     | M₁     | R₁     | M₁     | R₁     | M₁     | R₁     | M₁     |
| (1, 1)                                           | 0.72   | 0.648  | 0.7128 | 0.7193 |
|                                                  | 16kg   | 24kg   | 32kg   | 40kg   |
| (2, 1)                                           | 0.792  | 0.7128 | 0.7841 | 0.7912 |
|                                                  | 22kg   | 30kg   | 38kg   | 46kg   |
| (1, 2)                                           | 0.864  | 0.7776 | 0.8554 | 0.8631 |
|                                                  | 26kg   | 34kg   | 42kg   | 50kg   |
| (2, 2)                                           | 0.9504 | 0.8554 | 0.9410 | 0.9495 |
|                                                  | 32kg   | 40kg   | 48kg   | 58kg   |
| (3, 1)                                           | 0.7992 | 0.7193 | 0.7912 | 0.7984 |
|                                                  | 28kg   | 36kg   | 44kg   | 52kg   |
| (1, 3)                                           | 0.8928 | 0.8035 | 0.8839 | 0.8919 |
|                                                  | 36kg   | 44kg   | 50kg   | 60kg   |
| (3, 2)                                           | 0.9590 | 0.8631 | 0.9494 | 0.9580 |
|                                                  | 38kg   | 46kg   | 54kg   | 62kg   |
| (4, 1)                                           | 0.7999 | 0.7199 | 0.7919 | 0.7991 |
|                                                  | 34kg   | 42kg   | 50kg   | 58kg   |