**Article**

**Comparison of Forecasting Ability for Energy Consumption in BRICS: ARIMA (1,1,1) and FGM (1, 1) Models**

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**Abstract:** Brazil, Russia, China, India, and the Republic of South Africa (BRICS) represent developing economies facing different energy and economic development challenges. The current study aims to forecast energy consumption in BRICS at aggregate and disaggregate levels using the annual time series data set from 1992 to 2019 and to compare results obtained from a set of models. The time-series data are from the British Petroleum (BP-2019) Statistical Review of World Energy. The forecasting methodology bases on a novel Fractional-order Grey Model (FGM) with different order parameters. This study contributes to the literature by comparing the forecasting accuracy and the forecasting ability of the FGM(1,1) with traditional ones, like standard GM(1,1) and ARIMA(1,1,1) models. Also, it illustrates the view of BRICS’s nexus of energy consumption at aggregate and disaggregates levels using the latest available data set, which will provide a reliable and broader perspective. The Diebold-Mariano test results confirmed the equal predictive ability of FGM(1,1) for a specific range of order parameters and the ARIMA(1,1,1) model and the usefulness of both approaches for energy consumption efficient forecasting.

**Keywords:** Energy consumption, BRICS, GM (1, 1), Fractional-order, GREY, Forecasting accuracy

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**1. Introduction**

Brazil, Russia, India, China, and South Africa (BRICS countries) belong to the most prominent and fastest developing economies. Although the dynamics of their growth differ across countries, they consume more and more energy. The study aims to forecast energy consumption in Brazil, China, India, Russia, and South Africa at both aggregates and disaggregate levels basing on the time series observed in the years 1992-2019.

Energy plays the most crucial role in the development and achieving the sustainable economic growth of any country. The significance of energy is more critical in countries with less reserve or domestic energy sources (oil, gas, coal, hydro, etc.). BRICS is falling in the list of countries spending many energy resources to fulfill their domestic needs in residential, agricultural, and industrial requirements. The financial spending on the import of crude oil is an extra burden on the economy. Therefore, there is a need for correct forecasting about energy consumption.

In the modern era, due to globalization, the relationship among different countries are more tied up with each other in terms of social, political, and economy-wise. There is fierce competition among the developed as well as developing countries. For fulfilling the economic challenges of the 21st century, every nation is trying to achieve a sustainable level of economic growth, so countries need a sustainable supply of energy to run their economies properly. Ultimately, energy requirements lead the energy consumption in the
country. However, there is vast potential to address this hot issue because massive flaws have been observed due to the traditional techniques.

The global energy consumption in 2019 amounted to 173340 tera-watt hours, while BRICS participated in this consumption in 35.79%. Particularly China is the leading energy consumer globally, consuming up to 22.71% of the global magnitude. It can also be observed that global energy consumption tends to decrease annually by 1-2%. However, in Brazil, China, India, and South Africa, energy consumption exhibits positive growth rates. On the other hand, Russia is reducing its energy consumption, and it follows the global decreasing trend\(^1\).

When looking at the particular energy sources, the global energy consumption consisted of 30.93% oil, 25.30% coal, 22.67% natural gas, 8.00% biofuels and waste, 6.03% hydro, 4.00% nuclear, and 3.07% others in 2019\(^2\). Taking into account global energy consumption structure, in the paper, the focus is put on the aggregate energy consumption and traditional energy sources, which are to be limited over time but still play a crucial role in energy consumption and keeps particular countries far from sustainable development goals. Thus the following disaggregates are included: oil, coal, natural gas, and hydro energy.

The paper's novelty lies in applying the fractional-order \(GM(1,1)\) model (\(FGM(1,1)\), hereafter) proposed by [1] to forecasting energy consumption in BRICS countries at both aggregates and disaggregates levels. This is the first application of this model in the empirical analysis to the authors' best knowledge. That is why the model needs to compare to well-known forecasting techniques based on the time series analysis, such as a standard grey model \(GM(1,1)\) proposed by [2] and Auto Regressive, Integrated, Moving Average (\(ARIMA(1,1,1)\)), which is initially proposed by [3]. The model comparison is two-fold. In the first step, standard measures of forecasting accuracy such as mean square error (MSE) and mean absolute percentage error (MAPE). In contrast, in the second one, the models are compared for equal forecasting ability using the Diebold-Mariano test [4].

The rest of the paper has organized as follows. Subsection 1.1 provides an energy profile of BRICS countries, and subsection 1.2 reports the relevant literature review. Section 2 provides materials and methods. Section 3 presents the empirical results. Section 4 provides the discussion of results. The final section 5 concludes the paper and discusses policy implications.

### 1.1 Energy Profile of BRICS Countries

In this section, we briefly present the energy profile of BRICS. There is enormous potential in the energy sector of BRICS. The facts and figures of the following energy for BRICS have been taken from the BRICS energy report, 2020\(^3\).

Brazil generated 306.8 million tonnes of oil equivalent (mtoe) of primary energy in 2018, with 14 mtoe of unutilized energy and natural gas reinjection (in 2019: 327 and 17 mtoe, respectively). Production of oil surpassed demand by 52.5 percent, accounting for most of the Brazilian surplus (in 2019: 64 percent).

After China and the United States (US), Russia is the world’s third-largest producer and user of energy resources, accounting for 10% of global production and 5% of global consumption. The Russian energy complex, which includes the oil, gas, coal, electricity, and heat supply industries, is a significant source of revenue for the Russian Federation’s budget.

After the US and China, India is the world’s third-largest energy user, producing around 6% of global demand. Between 2010 and 2019, the country’s energy consumption increased by 50%. At the same period, coal accounts for 56% of global primary energy

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\(^1\) [https://ourworldindata.org/energy -production -consumption](https://ourworldindata.org/energy -production -consumption) (accessed 18.03.2021).

\(^2\) [https://ourworldindata.org/energy -production -consumption](https://ourworldindata.org/energy -production -consumption) (accessed 18.03.2021).

\(^3\) [https://eng.brics-russia2020.ru/images/114/89/1148985.pdf](https://eng.brics-russia2020.ru/images/114/89/1148985.pdf) (accessed 18.03.2021).
output. India produces just over half of its oil. The level and structure of energy production have changed significantly between 2010 and 2019: the volume of energy production has increased by 40%. The share of conventional biomass replaced by coal in the energy mix has decreased significantly.

China’s energy output grew steadily in 2018, reaching 3.77 billion tons of coal equivalent, up 5.0 percent year on year and the highest amount in the last six years, accounting for 18.7% of global production. In 2018, fossil fuels accounted for 81.8 percent of China’s energy output, with coal accounting for 69.1% and non-fossil accounting for 18.2%. China has surpassed the US as the world’s largest hydropower, wind power, and solar power installed capacity nation. China’s overall energy consumption in 2018 was 4.64 billion tons of coal equivalent, up 3.3 percent year on year. China’s low rate of energy consumption growth helps to sustain the country’s medium-high-speed economic growth.

The Republic of South African (RAS) is the continent’s second-largest energy user. South Africa’s total primary energy consumption in 2019 was 135 mtoe, down 5.6 percent from 2010. Coal dominates the energy demand structure, accounting for about 75% of total consumption. South Africa is a net energy exporter, exporting more than 45 mtoe of coal to global markets each year, while having minimal domestic oil and natural gas output and relying on imports for most of these fuels. The structure of energy production has remained nearly unchanged since 2010, but overall production has decreased slightly.

1.2 Literature review

There is a large plethora of literature available on the issue of energy consumption forecasting. Many studies used ARIMA methods for forecasting energy consumption, e.g., [5–15], and some studies were forecasted by comparing the ARIMA approach with some other methods. On the other hand, some studies used the grey methods for energy consumption forecasting. Referring only to the BRICS group of countries, there is numerous literature on energy consumption in China. Besides, Brazil and India are sometimes represented; however, Russia and South Africa are rarely the analysis subjects.

In past research, [10] investigated energy demand in the transport sector using ARIMA, exponential smoothing, and multi regression models. On the other hand, the study of [12] forecasted China’s primary energy consumption by comparing the ARIMA and grey models. In [13], the authors estimated electricity consumption for Brazil by applying the Spatial ARIMA model. There are very few studies that evaluated energy consumption for BRICS by using the ARIMA model. Some studies forecasted energy consumption by using the ARIMA forecasting method like [5–10, 12] for China, [13] for Brazil, and [15] for South Africa. On the other hand, some studies used the grey Markov method with rolling mechanism and singular spectrum analysis for energy consumption forecasting like [16] for India. Similarly countrywide studies are [12, 17–74] for China; [75] for the US; [76] for China and India; [77] for BRICS; [78] for China and US, [79–80] for Brazil, and [81] for Asian countries.

In [14] the authors analyzed several versions of grey models (e.g., grey model including $GM(1,1); GM(1, n); RollingGM(1,1)$ $RollingGM(1,1, X_n)$ and $Rolling NOGMN(1,1)$, and forecasted electricity consumption from 2015 to 2020. Most of the studies used the standard $GM(1,1)$. There is some technical problem in the grey prediction model’s methodology, as most of the studies fail to fulfill the principle of “new information priority” proposed by [2]. The grey forecasting method proposed by [2] has gained popularity among researchers because it is efficient in a small number of observations [17]. To tackle this problem, the study of [23] developed the new grey model based on the initial condition by considering the last data point of the one accumulated generating operation (AGO) sequence as the initial condition. [25] analyzed electricity consumption for China by using the continuous fractional-order grey model and forecasted from 2010 to 2014. [27] analyzed grey $GM(1,1)$, Gross weight grey model, and $GVGM(1,1)$ for China and forecasted from 2010 to 2020. [28] analyzed energy consumption for China by using the improved hybrid grey model (INHGM-Markov) and forecast from 2018 to 2022. Similarly, the grey method is suitable to tackle forecasting in the case of inaccuracy of data.
Some researchers developed the model's extended versions using the standard $GM(1,1)$ grey forecasting model, as [18] proposed an improved version of the seasonal rolling grey prediction model to estimate the accurate forecasting for traffic flow problems. Moreover, [19] proposed fractional-order accumulation techniques and forecasted from 1999 to 2007 for China and from 1999 to 2008 for the US. [20] developed a new time-delayed polynomial grey model, which has shown outstanding results when forecasting China’s natural gas consumption and forecasted from 2005 to 2013 and 2014 to 2020. [21] predicted China’s energy consumption by incorporating genetic programming in the grey prediction approach and forecasted from 2004 to 2007. Similarly, [22] developed a generalized fractional-order grey model using the fractional calculus and forecasted it from 2010 to 2014. [39] forecast coal stockpiles for China using grey spontaneous combustion forecasting models and forecasted from day 11 to 20. [40] analyze the electricity consumption for China by using grey prediction with the nonlinear optimization method and forecasted from 2014 to 2020. [41] examined electricity consumption by using grey $GM(1,1)$ and combined improved grey ($DCOGM(1,1)$) prediction models and forecasted from 2017 to 2021. The results show that $DCOGM(1,1)$ shows better results than the traditional grey $GM(1,1)$ model. [42] analyzed electricity consumption for China by using the grey polynomial prediction model and forecasted from 2011 to 2015. [43] analyzed the energy vehicle industry for China by using grouping approach-based nonlinear grey Bernoulli model (DGA-based $NGBM(1,1)$) and $GM(1,1)$ forecasting from 2018Q1 to 2020Q4 [50] forecast for by using Self-adaptive intelligence grey predictive model and forecasted for 2014. The results indicate that the Self-adaptive grey model shows better results than $GM(1,1)$ and discrete grey ($DGM(1,1)$) models. [81] used hybrid dynamic grey model for forecasted from 1999 to 2007 for China, from 1999 to 2008 for the US.

2. Materials and Methods
This section is divided into three subsections related to data sources description, forecasting models, and forecasting accuracy.

2.1. Data Sources
The present study is based on the secondary data source consisting of annual observations on the BRICS economy for 1992-2019. The starting date is limited by the case of Russia being founded in 1991 after the dissolution of the Soviet Union. The currents study uses the energy consumption in BRICS and for empirical analysis. The data on energy consumption (EC) at aggregate and disaggregate energy consumption components (oil, gas, coal, and hydroelectric) are taken from British Petroleum (BP-2019) Statistical Review of World Energy. All variables are measured in mtoe units, and the description of variables is as: (1) Aggregate energy consumption (agg) (2) Oil consumption (oil) (3) Gas consumption (gas) (4) Coal consumption (coal) (5) Hydroelectric consumption (hydro).
2.2. Methodology

The current study is based on FGM(1,1) grey model. The model was introduced to the literature in 2019. Its application to energy consumption forecasting is still not recognized by the authors of the paper [1] based on simulation results. In the current study, the FGM (1,1) model is compared to a standard GM (1,1) model, as well as the ARIMA \((p,d,q)\) widely recognized in the forecasting literature. We focus on model comparison in terms of forecasting ability.

2.2.1 Unit Root Testing and ARIMA \((p,d,q)\) Model

The most recognized representation for nonstationary time series is the ARIMA model that can be written in the form:

\[
\Phi(L)(1-L)^d x_t = \theta(L) \epsilon_t
\]

(1)

where \(\Phi(L)\) and \(\theta(L)\) are polynomials in the lag operator, \(L\), defined such that \(L^n x_t = x_{t-n}\), \(\mu_t\) is the unconditional mean, \(d\) is the order of integer differencing, and \(\epsilon_t\) is a white noise process (i.i.d. normally distributed) (for further details, see [3, 82]). This model is termed an \(ARIMA(p,d,q)\) to indicate \(p\) lags in the \(AR\) and \(q\) lags in the \(MA\) terms, and \(d\) is an integer differencing. To estimate the parameters of the ARIMA model, the maximum likelihood method is recommended. The model selection procedure, related to lag parameters \(p\) and \(q\), is based on the Akaike information criterion (AIC) and Bayesian information criterion (BIC) [83].

ARIMA model refers to integrated time series, which become stationary after \(d\)-th times differencing. To determine whether a time series is stationary or not, the Augmented Dickey and Fuller test is typically used [84]. It is essential to check the time series stationarity at the beginning of the analysis to fit the correct type of model. The graphical analysis of all series suggests that there would be a need to take a trend in the unit root testing procedure. Hence, the study used the intercept and trend in all cases. The null hypothesis assumes that the time series is nonstationary of order 1, i.e., \(I(1)\), and the alternative hypothesis assumes stationarity (i.e., \(I(0)\)). If the time series is stationary, the \(ARIMA(p,q)\) model is applied.

2.2.2 Fractional-order GM (1,1) Model

The construction of the fractional-order GM(1,1) grey model methodology is explained by [1]. As it is quite a new approach, it is presented in this section.

**Definition1**: The sequence of raw data series \(X^{(0)} = x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\), where \(r \in \mathbb{R}^+\), which is known as \(X^{(r)} = (x^{(r)}(1), x^{(r)}(2), x^{(r)}(1), \ldots, x^{(r)}(n))\) is the \(r\)th order accumulating sequence of \(X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))\) [48] where \(\Gamma(n)\) is denoting the gamma function,

\[
x^{(r)}(k) = \sum_{i=1}^{\Gamma(r+1)} \frac{\Gamma(r+i)}{\Gamma(r) \Gamma(r+i)} x^{(i)}(k), k = 1, 2, \ldots, n
\]

(2)

**Definition 2**: Assume that \(X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))\) is the sequence of raw data, where \(r \in \mathbb{R}^+\), which is known as \(X^{(r)} = (x^{(r)}(1), x^{(r)}(2), \ldots, x^{(r)}(n))\) is the \(r\)th order generating sequence of \(X^{(0)} = x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\). [48] where,

\[
x^{(r)}(k) = \sum_{i=0}^{\Gamma(r+1)} (\Gamma(r+i+1))^{-1} x^{(i)}(k - i)
\]

(3)

**Theorem 1**: Assume that \(X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))\) is the sequence of raw data, \(X^{(p)}\) is the \(p\)th-order generating sequence of \(X^{(0)}\), where \(p \in \mathbb{R}^+\), and \(X^{(q)}\) is known as \(q\)th-order reducing generation sequence of \(X^{(0)}\), where \(q \in \mathbb{R}^+\). It implies that \((X^{(p)})^{(q)}\) is the \(q\)th-order reducing generation sequence of \((X^{(p)})^{(p)}\), and \((X^{(0)})^{(p)}\) is the \(p\)th-order accumulating generation sequence of \((X^{(0)})^{(q)}\). The following conditions will exist if:

- If \(p - q > 0\), \(X^{(p-q)}\) is the \((p - q)\)th-order accumulating generation sequence of \(X^{(0)}\).
- If \(p - q < 0\), \(X^{(p-q)}\) is the \((q - p)\)th-order reducing generation sequence of \(X^{(0)}\).
The fractional-order accumulating generation operator and the reducing generation operator satisfy the commutative and exponential laws.

\[ X^{(p-q)} = (X^{(p)})^{(-q)} = (X^{(-q)})^{(p)} \]  \hspace{1cm} (4)

**Definition 3:** Assume that \( X^{(r)} = (x^{(r)}(1), x^{(r)}(2), \ldots, x^{(r)}(n)) \) is defined as definition 1, and \( X^{(-r)} = (x^{(-r)}(1), x^{(-r)}(2), \ldots, x^{(-r)}(n)) \) follow the definition 2. Thus, \( Z^{(r)} = (z^{(r)}(2), z^{(r)}(3), \ldots, z^{(r)}(n)) \), where

\[ Z^{(r)}(k) = \frac{\sum_{i=1}^{k} x^{(r)}(i) + x^{(r)}(k-1)}{2}, \quad k = 2, 3, 4, \ldots, n. \]  \hspace{1cm} (5)

The model formula

\[ x^{(r-1)}(k) + az^{(r)}(k) = b \]  \hspace{1cm} (6)

is representing the FGM (1,1). The following conditions are distinguished:

- if \( r = 1 \), Eq (6) is representing the GM (1,1)

\[ x^{(0)}(k) + az^{(1)}(k) = b, \]

which is called as standard GM (1,1) model described in [2]

- if \( r = 0 \), Eq (6) is representing the direct GM (1,1) modeling.

\[ x^{(-1)}(k) + az^{(0)}(k) = b. \]

It is expected that the development coefficient \( a \) to be negative and the intension parameter \( b \) to be positive (See, [85] chapter 7, p.149).

**Theorem 2:** We are following definition 3 for FGM (1,1), and the parameter vector is explained as:

\[ x^{(r-1)}(k) + az^{(r)}(k) = b, \quad \hat{a} = [a, b]^T \] is the GM (1,1) parameter, and the least-square estimate of the parameters satisfies the following equations.

\[ \hat{a} = (B^T B)^{-1} B^T Y \]  \hspace{1cm} (7)

The construction of observation matrices is described in [1]. It follows,

\[ Y = \begin{bmatrix} x^{(r-1)}(2) \\ x^{(r-1)}(3) \\ \vdots \\ x^{(r-1)}(n) \end{bmatrix}, \quad B = \begin{bmatrix} -z^{(r)}(2) & 1 \\ -z^{(r)}(3) & 1 \\ \vdots & \vdots \\ -z^{(r)}(n) & 1 \end{bmatrix} \]  \hspace{1cm} (8)

where,

\[ x^{(r-1)}(k) = \left( (x^{(r)})^{-1}(k) - x^{(r)}(k-1) \right) \]

\[ = \sum_{i=1}^{k} \frac{\Gamma(r + k - i)}{\Gamma(k - i + 1) \Gamma(r)} x^{(0)}(i) - \sum_{i=1}^{k-1} \frac{\Gamma(r + k - 1 - i)}{\Gamma(k - i) \Gamma(r)} x^{(0)}(i), \quad k = 2, 3, \ldots, n, k - i \geq 1 \]

\[ x^{(r)}(k) = \frac{x^{(r)}(k) + x^{(r)}(k-1)}{2}, \quad k = 2, 3, \ldots, n \]

Therefore, it follows

\[ Y = \begin{bmatrix} \sum_{i=1}^{2} \frac{\Gamma(r + 2 - i)}{\Gamma(2 - i + 1) \Gamma(r)} x^{(0)}(i) - \sum_{i=1}^{1} \frac{\Gamma(r + 2 - 1 - i)}{\Gamma(2 - i) \Gamma(r)} x^{(0)}(i) \\ \sum_{i=1}^{3} \frac{\Gamma(r + 3 - i)}{\Gamma(3 - i + 1) \Gamma(r)} x^{(0)}(i) - \sum_{i=1}^{2} \frac{\Gamma(r + 3 - 1 - i)}{\Gamma(3 - i) \Gamma(r)} x^{(0)}(i) \\ \vdots \\ \sum_{i=1}^{n} \frac{\Gamma(r + n - i)}{\Gamma(n - i + 1) \Gamma(r)} x^{(0)}(i) - \sum_{i=1}^{n-1} \frac{\Gamma(r + n - 1 - i)}{\Gamma(n - i) \Gamma(r)} x^{(0)}(i) \end{bmatrix} \]

\[ = \frac{r(r-1)}{2} x^{(0)}(1) + (r-1)x^{(0)}(2) + x^{(0)}(3) + \sum_{i=1}^{n} \frac{\Gamma(r + n - i)}{\Gamma(n - i + 1) \Gamma(r)} x^{(0)}(i) - \sum_{i=1}^{n-1} \frac{\Gamma(r + n - 1 - i)}{\Gamma(n - i) \Gamma(r)} x^{(0)}(i) \]
The model properties are defined by Definition 4 and Theorem 3.

**Definition 4:** Assume that \( x^{(r-1)}(k) \) and \( Z^{(r)}(k) \) as defined as in Theorem 1:

\[
\frac{d x^{(r)}}{d t} + a x^{(r)} = b
\]

is known as the whitenization function of the GM(1,1) differential equation.

\( x^{(r-1)}(k) + a x^{(r)}(k) = b \)

**Theorem 3:** Assume that \( B, Y \) and \( \tilde{a} \) are based on the Theorem 2, as if the following GM(1,1) parameter and least square equation,

\[
\tilde{a} = [a, b]^T = (B^T B)^{-1} B^T Y,
\]

then, the solution (i.e., time response function) of the whitenization function of the FGM(1,1),

\[
\frac{d x^{(r)}}{d t} + a x^{(r)} = b
\]

is given by

\[
\hat{x}^{(r)}(t) = \left( x^{(1)}(1) - \frac{b}{a} \right) e^{-a t} + \frac{b}{a}
\]

The time response sequence of FGM(1,1),

\( x^{(r-1)}(k) + a x^{(r)}(k) = b \), is given in equation (10),

\[
\hat{x}^{(r)}(k+1) = \left( x^{(0)}(1) - \frac{b}{a} \right) e^{a(k+1)} + \frac{b}{a}, k=1,2,\ldots,n
\]

Let \( x^{(1)}(0) = x^{(0)}(1) \), then the restored value of \( x^{(0)}(k) \) is given as in equation (12)

\[
\hat{x}^{(0)}(k) = (\hat{x}^{(r)})^{-r}(k) = \frac{x^{(0)}(1)}{\sum_{i=0}^{k-1} (-1)^i \frac{\Gamma(r+1)}{\Gamma(i+1)\Gamma(r-i+1)} \hat{x}^{(r)}(k-i) \quad k = 2,3,\ldots,n}
\]

2.2.3 Forecasting accuracy measures

The most popular measures of forecast accuracy concentrate on computing forecast errors. There are plenty of such measures, such as MSE and MAPE (See, [86] (p.309). Their usefulness consists of showing the differences in accuracy of computed forecasts but say nothing about the method of forecasting.

In 1995, [4] derived a testing procedure of equal predictive accuracy. The hypothesis to be tested says that the alternative methods are equally accurate on average. The general idea of Diebold-Mariano’s test relies on two-time series, including actual values and forecasts of a predicted variable, say \( y_t \) and \( \hat{y}_t \), as well as on the loss function depending on the forecast and actual values only through the forecast error, defined as: \( g(y_t, \hat{y}_t) = g(\hat{y}_t - y_t) = g(e_t) \). The loss function may take many different forms, which is discussed further in this part. What we compare is a loss differential between the two forecasts, coming from two competing models of the form: \( d(t) = g(e_{1t}) - g(e_{2t}) \). The forecasting methods are equally accurate if \( E(d(t)) = 0 \), which is assumed under the null hypothesis.
This paper applied the Diebold-Mariano test to compare a standard \( GM(1,1) \) model, \( ARIMA(1,1,1) \), and \( FGM(1,1) \) model estimated for different \( r \) values. The loss function based on the mean square error was selected for comparison because the differences between forecast errors were relatively low. It is worth emphasizing that such a comparison is possible only if a sufficient number of observations are necessary to estimate the \( ARIMA \) model.

3. Results

In this section, the data used in the analysis is presented. To begin the analysis, the time series are presented in figure A1 in the appendix. The descriptive statistics of the selected aggregate and disaggregate energy consumption series of BRICS are provided in table A1 in the appendix.

The mean value of “agg” ranges from 111.94 mtoe in South Africa to 1909.42 mtoe in China in BRICS countries. However, the mean value of “coal” ranges from 13.71 mtoe in Brazil to 1298.20 mtoe in China. Similarly, the average “gas” is lowest in South Africa with 2.42 mtoe, while it is highest in Russia with 346.38 mtoe. On the other hand, average “hydro” is lowest in South Africa with 0.30 mtoe and highest in China with 123.78 mtoe. Finally, average “oil” is lowest in South Africa with 24.55 mtoe, while it is highest in China with 360.67 mtoe. The distribution of energy series for each country is skewed in nature (as the mean is different from the median), so the median is a better measure of the center instead of the mean, and interquartile range (IQR) is a better measure of a spread than standard deviation (SD). It can be seen that most of them represent a growing trend, so they are assumed to be nonstationary.

The opposite conclusion can be derived when hydro energy consumption in India, Russia, and South Africa is observed. As far as variation is concerned, South Africa has the least variation (13.51 mtoe), while China has the highest variation (919.14 mtoe) for “agg”. Brazil has the least variation (2.09 mtoe), while China has the highest variation (557.56 mtoe) among the “coal” series. South Africa has the least variation (1.25 mtoe), while China has the highest variation (76.87 mtoe) among the “gas” series. South Africa has the least variation (0.17 mtoe) while China has the highest variation (81.93 mtoe) among the “hydro” series. Finally, South Africa has the least variation (3.05 mtoe) while China has the highest variation (167.76 mtoe) among the “oil” series.

In most cases, the unit root hypothesis was confirmed apart from four series, i.e., Russia’s “agg”, “coal” and “hydro” energy, and South Africa’s “hydro” energy. As concerns Russia, decreasing trends is observed in coal energy consumption. Total (aggregate) energy consumption was decreasing in 1992-1998. Oil and gas consumption decreased in 1992-1996; since that time, a growing tendency has been observed. Testing for normality using Jarque and Bera test [87] indicates that most of the time series satisfied the normal distribution. Only aggregate energy consumption and coal energy consumption in Russia do not satisfy this condition.

The results of forecasting energy consumption using the \( FGM(1,1) \) model are presented in table 1 for aggregate energy consumption and in tables A3-A6 for disaggregates energy consumption. All results of aggregate and disaggregate energy consumption is reported with the values of MAPE, MSE, development coefficient (a), grey input (b) with different orders (r) varying from \( r = 0 \) to \( r = -1.5 \). In table 1, the results of aggregate energy consumption are reported; for Brazil, the order (r) is \( r = 0.9 \) with the minimum MAPE= 3.43 with the appropriate sign of grey parameters, \( a = -0.021 \), and \( b = 125.064 \). For China, the order (r) is \( r = 0.5 \) with the minimum MAPE= 10.030 with the parameters \( a = -0.021 \), and \( b = 125.064 \). For India, the order (r) is \( r = 1 \) with the minimum MAPE= 2.163 with the parameters \( a = -0.051 \), and \( b = 204.588 \). For Russia, the order (r) is \( r = 1 \) with the minimum MAPE= 3.632 with the parameters \( a = -0.003 \), and \( b = 632.45 \). Finally, South Africa has the least MAPE= 21.565 with the parameters \( a = -0.001 \), and \( b = 1.376 \), where the value of \( r = 0 \).
Table 1. Mean absolute percentage error (MAPE) and Mean Square Error (MSE) for Aggregate Energy Consumption of BRICS

|       | r=0     | r=0.01  | r=0.05  | r=0.1   | r=0.5   | r=0.9   | r=1    | r=1.5   | r=0     | r=0.01  | r=0.05  | r=0.1   | r=0.5   | r=0.9   | r=1    | r=1.5   | ARIMA(1,1,1) |
|-------|---------|---------|---------|---------|---------|---------|--------|---------|---------|---------|---------|---------|---------|---------|--------|---------|-------------|
| **MAPE** |         |         |         |         |         |         |        |         |         |         |         |         |         |         |        |         |              |              |
| MSE   | 115.761 | 114.048 | 113.330 | 121.564 | 183.547 | 104.364 | 121.952 | 2365.346 | 118.233 | 137.248 | 194.597 | 120.572 | 153.293 | 71846.171 | 34.952 |
|      | 3.437   | 3.467   | 3.843   | 4.299   | 5.652   | 3.430   | 3.877   | 15.476   | 3.464   | 3.894   | 5.659   | 4.182   | 5.294   | 114.571  | 1.689  |
| a     | 0.016   | 0.018   | 0.024   | 0.028   | 0.009   | -0.021  | -0.027  | -0.054   | 0.014   | 0.004   | -0.012  | 0.771   | 1.475   | 2.876   | ...    |
| b     | 9.516   | 10.424  | 14.011  | 18.400  | 57.183  | 125.064 | 150.880 | 381.851  | 8.604   | 4.943   | 0.586   | 36.399  | 13.890  | -7.802  | ...    |

| **MSE** | 39766.001 | 38854.842 | 35862.192 | 33235.534 | 30342.711 | 38656.280 | 44284.071 | 192619.194 | 40755.833 | 45700.224 | 55315.626 | 934292.685 | 192632.280 | 2535304.295 | 1455.956 |
| MAPE   | 11.649   | 11.426   | 10.681   | 10.145   | 10.030   | 10.567   | 11.432   | 17.974   | 11.882   | 12.919   | 14.596   | 62.912   | 29.165   | 96.097   | 1.889  |
| a      | -0.021   | -0.021   | -0.020   | -0.020   | -0.033   | -0.052   | -0.056   | -0.077   | -0.021   | -0.022   | -0.025   | 0.019    | 0.842    | 2.014    | ...    |
| b      | 57.208   | 60.395   | 73.612   | 91.213   | 288.417  | 665.437  | 808.421  | 2071.191 | 54.069   | 41.990   | 28.009   | 7.239    | 106.628  | -4.462   | ...    |

| **MSE** | 305.050  | 264.233  | 181.024  | 169.925  | 458.723  | 179.881  | 125.988  | 4767.086  | 358.196  | 775.327  | 2398.166 | 2725.071 | 7235.069 | 231705.058 | 75.811  |
| MAPE   | 3.463    | 3.065    | 2.463    | 2.918    | 5.302    | 3.199    | 2.163    | 10.946    | 3.921    | 6.467    | 11.662   | 41.465   | 20.290   | 106.842  | 1.514  |
| a      | -0.042   | -0.041   | -0.038   | -0.035   | -0.034   | -0.047   | -0.051   | -0.070   | -0.043   | -0.048   | -0.054   | 0.131    | 1.543    | 2.960    | ...    |
| b      | 2.785    | 3.621    | 7.118    | 11.814   | 64.392   | 165.641  | 204.588  | 556.371   | 1.965    | 1.147    | 4.601    | 13.565   | 48.380   | -1.380   | ...    |

| **MSE** | 2557.965 | 3397.955 | 10340.732 | 889.763  | 2634.297 | 2009.022 | 1210.561 | 13152.140 | 1928.408 | 758.559  | 309.188  | 594.381  | 16285.304 | 3961075.140 | NA     |
| MAPE   | 6.689    | 7.659    | 15.072   | 3.786    | 9.383    | 4.659    | 3.632    | 11.189    | 5.825    | 3.619    | 2.245    | 3.203    | 18.722    | 293.588  | NA     |
| a      | 0.230    | 0.161    | -0.038   | -0.042   | 0.044    | 0.005    | -0.003   | -0.037    | 0.294    | 0.456    | 0.516    | 0.916    | 1.861     | 2.592    | ...    |
| b      | 149.005  | 107.228  | -27.610  | -29.666  | 237.035  | 525.883  | 632.457  | 1579.718  | 185.251  | 252.662  | 241.844  | 94.585   | 12.318    | -61.080  | ...    |

| **MSE** | 314.658  | 313.896  | 314.996  | 321.056  | 365.344  | 322.015  | 312.152  | 809.537   | 316.329  | 347.810  | 469.286  | 497.888  | 1777.788  | 166344.037 | 11.193  |
| MAPE   | 21.565   | 21.750   | 22.218   | 22.419   | 23.788   | 22.161   | 21.932   | 31.946    | 21.368   | 22.720   | 28.334   | 29.611   | 52.934    | 444.068  | 2.294  |
| a      | -0.001   | 0.003    | 0.018    | 0.033    | 0.027    | -0.008   | -0.015   | -0.046    | -0.005   | -0.013   | -0.002   | 0.336    | 0.587     | 0.604    | ...    |
| b      | 1.376    | 1.962    | 4.654    | 8.259    | 33.693   | 72.221   | 86.774   | 216.973   | 0.845    | -0.527   | -0.133   | 5.297    | -0.143    | -3.801   | ...    |
In table A3, the results of oil consumption are reported. For Brazil, the order \( r \) is \( r=0.9 \) with the minimum MAPE= 5.030 with the appropriate sign of grey parameters, \( a = -0.013 \), and \( b = 60.198 \). For China, the order \( r \) is \( r=0.1 \) with the minimum MAPE= 2.795 with the parameters \( a = -0.029 \), and \( b = 14.555 \). For India, the order \( r \) is \( r=0.9 \) with the minimum MAPE= 2.457 with the parameters \( a = -0.042 \), and \( b = 56.271 \). For Russia, the order \( r \) is \( r=1.5 \) with the minimum MAPE= 5.566 with the parameters \( a = -0.034 \), and \( b = 361.965 \). Finally, South Africa has the least MAPE= 2.151 with the parameters \( a = -0.005 \), and \( b = 17.103 \), where the value of \( r=0.9 \).

In table A4, the results of gas consumption are reported. For Brazil, the order \( r \) is \( r=0.1 \) with the minimum MAPE= 22.262 with the appropriate sign of grey parameters, \( a = -0.001 \), and \( b = 1.441 \). For China, the order \( r \) is \( r=1 \) with the minimum MAPE= 18.884 with the parameters \( a = -0.124 \), and \( b = 9.974 \). For India, the order \( r \) is \( r=0.1 \) with the minimum MAPE= 7.842 with the parameters \( a = -0.001 \), and \( b = 11.814 \). For Russia, the order \( r \) is \( r=1 \) with the minimum MAPE= 3.408 with the parameters \( a = -0.008 \), and \( b = 309.562 \). Finally, South Africa has the least MAPE= 30.042 with the parameters \( a = -0.050 \), and \( b = 0.853 \), where the value of \( r=0.9 \).

In table A5, the results of coal consumption are reported. For Brazil, the order \( r \) is \( r=1 \) with the minimum MAPE= 8.040 with the appropriate sign of grey parameters, \( a = -0.013 \), and \( b = 11.142 \). For China, the order \( r \) is \( r=0.5 \) with the minimum MAPE= 13.971 with the parameters \( a = -0.016 \), and \( b = 246.587 \). For India, the order \( r \) is \( r=1 \) with the minimum MAPE= 4.489 with the parameters \( a = -0.054 \), and \( b = 105.423 \). For Russia, the order \( r \) is \( r=1.5 \) with the minimum MAPE= 12.986 with the parameters \( a = -0.027 \), and \( b = 249.931 \). Finally, South Africa has the least MAPE= 3.335 with the parameters \( a = -0.003 \), and \( b = 57.857 \), where the value of \( r=0.9 \).

In table A6, the results of hydro consumption are reported. For Brazil, the order \( r \) is \( r=0.9 \) with the minimum MAPE= 5.552 with the appropriate sign of grey parameters, \( a = -0.006 \), and \( b = 52.191 \). For China, the order \( r \) is \( r=1 \) with the minimum MAPE= 11.418 with the parameters \( a = -0.081 \), and \( b = 34.593 \). For India, the order \( r \) is \( r=1 \) with the minimum MAPE= 8.642 with the parameters \( a = -0.027 \), and \( b = 15.549 \). For Russia, the order \( r \) is \( r=1.5 \) with the minimum MAPE= 14.648 with the parameters \( a = -0.036 \), and \( b = 96.269 \). Finally, South Africa has the least MAPE= 122.067 with the parameters \( a = -0.040 \), and \( b = .729 \), where the value of \( r=1.5 \).

The conclusion of model comparison using the Diebold-Mariano test given in table 2 is that the \( GM(1,1) \) model and \( ARIMA(1,1,1) \) possess an equal predictive ability for energy consumption, so the model choice must be based on additional measures for forecasts accuracy, as well as on the data availability. Since the observation number is large, one can rely on the stochastic time series model. On the other hand, grey models are beneficial. As concerns \( FGM(1,1) \), the conclusion is quite similar, but for small negative \( r \) values like -1.5, the results are much worse. This conclusion is useful because one can limit the range of possible \( r \) values between (-1.00; +1.00]. The two results (China, hydro \( r=0.00 \) and India coal, \( r=0.05 \), when the null hypothesis of equal predictive ability was rejected, can be considered random.
Table 2. Diebold-Mariano test results (p-values) for comparison of GM(1,1) and FGM(1,1)

| GM(1,1) | Brazil         | China                  | India                  | Russia                  | South Africa            |
|---------|----------------|------------------------|------------------------|-------------------------|-------------------------|
|         | agg | oil  | gas  | coal | hydro | agg | oil  | gas  | coal | hydro | agg | oil  | gas  | coal | hydro | agg | oil  | gas  | coal | hydro | agg | oil  | gas  | coal | hydro |
| r=0     | >0.1 | >0.1  | >0.1  | >0.1  | >0.1  | >0.1 | >0.1  | >0.1  | >0.1  | >0.1  | >0.1 | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  |
| r=0.01  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1 | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  |
| r=0.05  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1 | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  |
| r=0.1   | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1 | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  |
| r=0.5   | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1 | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  |
| r=0.9   | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1 | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  |
| r=1.5   | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1 | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  |
| r=2.0   | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1 | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  |
| r=3.0   | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1 | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  | >0.1  |

Note: Rejecting the null hypothesis is highlighted with the shaded area; agg-aggregate.
4. Discussion

BRICS countries belong to the developing economies group, although China has become a global competitor in some areas [88]. Developing countries desire precise forecasts of both scales of growth and energy consumption. As the economy grows, energy consumption increases as well. However, there are strict limitations to energy consumption. They divide into global and country-specific limitations. Global recommendations origin is in the United Nations' 2030 Agenda for the Sustainable Development Goals (SDGs), which all 193 UN member states accepted. Among 17 SDGs, goal 7 assumes access to affordable, reliable, sustainable, and modern energy for all, and goal 13 covers urgent action to combat climate change and its impacts. Country-specific limitations consist of natural resource exploitation, technology for energy production, and transformation and consciousness of the necessity of rationalization. Considering the above, the energy policy of BRICS countries needs continuous monitoring for forecasting accuracy and its structure according to the SDGs requirements.

The results of the grey and ARIMA models' comparison presented in the current paper revealed as follows:

1. Fractional Grey Model \( FGM(1,1) \) allows a broad spectrum of parameters that adjust to the empirical data. FGM-based approach is more comprehensive than the standard \( GM(1,1) \) model, which is "a special case" of \( FGM(1,1) \) for \( r=1 \).

2. According to the Diebold-Mariano test results, the estimated \( FGM(1,1) \) models - taking parameters' range [-1; 1] confirmed equal predictive ability with \( GM(1,1) \) model as well as \( ARIMA(1,1,1) \) model.

3. Although grey-type models are mostly recommended for short time series, their predictive ability is equal to ARIMA models designed for long time series. However, taking values of MSE and MAPE in empirical study, \( ARIMA(1,1,1) \) model highly outperformed \( FGM(1,1) \) in 19 cases on 25. Only in China's case of oil consumption, the \( FGM(1,1) \) model has the minimum MAPE and MSE values. The remained five series were stationary, so \( ARIMA(1,1,1) \) model was not estimated.

4. For some parameter "\( r \)" values, empirical \( FGM(1,1) \) models do not satisfy the grey model assumption, i.e., \( a < 0 \) and \( b > 0 \). In such circumstances, it is recommended to estimate the model for another "\( r \)" parameter value.

5. Grey-type models are helpful for forecasting in the case when only a few observations are available. Still, for long and nonstationary time series, standard time series models perform better.

In the paper [89], the authors provided a methodological comparison of probability models, fuzzy math, grey systems, and rough sets. It appears that grey models are evidently preferred in the case of small samples and incomplete information sets. They concentrate on the law of reality. On the other hand, stochastic models, such as ARIMA are designed for large samples and follow historical law. The general conclusion that both types of models possess equal predictive ability indicated by the Diebold-Mariano test allows selecting the proper procedure for a given data set and forecasting perspective. The exact values of MAPE and MSE are less informative because they are valid only for a given sample. Therefore, the presented results are in line with both theory and expectations.

5. Conclusions

The BRICS are emerging economies concerning the production and management of resources and require a consistent supply of having energy resources. The BRICS countries should monitor energy consumption, focusing on the supply-demand gap of energy and its components and facilities provided to local and foreign investors. Therefore, forecasting is quite significant for energetic policy projection. Accurate forecasts of energy consumption are vital when demand grows faster. On the other hand, BRICS's energy consumption values can be offered as fluctuating and increasing.

This study aims to compare different decision-making types for energy demand forecasting in BRICS. First, this paper focused on forecasting the annual energy consumption for BRICS. Secondly, it compared ARIMA, and \( FGM(1,1) \) models with actual data in 1992-
2019 using estimated errors (MAPE) and (MSE). Results have revealed that ARIMA and FGM(1,1) models perform close findings. Thirdly, model comparison using the Diebold-Mariano test confirmed the equal predictive ability of ARIMA(1,1,1) and FGM(1,1) unless the FGM parameter ranges [-1, 1].

The empirical findings allow formulating some recommendations. BRICS countries need to follow SDGs concerning energetic policy keeping their economic growth level increasing. It implies a gradual structural change from traditional towards renewable energy sources. A structural change always means a significant limitation of the observations; therefore, the FGM(1,1) model is recommended for predicting energy consumption in aggregate and disaggregate levels.

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### Appendix A

#### Table A1. Summary statistics of energy consumption for BRICS

| Variable | Brazil | Russia | India | China | South Africa |
|----------|--------|--------|-------|-------|--------------|
| Mean     | 224.48 | 97.12  | 17.87 | 13.72 | 75.63        |
| Med      | 214.21 | 91.52  | 17.54 | 13.00 | 78.15        |
| Max      | 296.25 | 129.59 | 36.92 | 17.62 | 95.44        |
| Min      | 139.15 | 64.37  | 3.19  | 10.68 | 53.34        |
| S. D.    | 52.31  | 18.04  | 11.20 | 2.09  | 11.44        |
| Ske      | 0.04   | 0.05   | 0.19  | 0.46  | -0.27        |
| Kurt     | 1.61   | 2.07   | 1.66  | 1.98  | 2.09         |
| J-B      | 2.25   | 1.03   | 2.28  | 2.19  | 1.29         |
| Prob     | 0.32   | 0.60   | 0.32  | 0.52  | 0.01         |
| Obs.     | 28     | 28     | 28    | 28    | 28           |

Note: Med: Median; Max: Maximum; Min: Minimum; S.D.: Standard deviation; Ske: Skewness; Kurt: Kurtosis; J-B: Jarque-Berra; Prob=Probability; Obs=Observations.

#### Table A2. Unit Root (ADF) Testing for BRICS

| Level     | Variable | Brazil | Russia | India | China | South Africa |
|-----------|----------|--------|--------|-------|-------|--------------|
| Statistic | -0.992   | -0.718 | -1.575 | -1.682 | -3.761 | -2.499       |
| Prob.     | 0.741    | 0.958  | 0.826  | 0.481 | 0.335 | 0.009*       |
| First difference | -4.052 | -4.186 | -3.516 | -5.597 | -4.586 | -3.286 |
| Statistic | -1.456   | -3.713 | -3.653 | -3.737 | -3.749 | -2.202 |
| Prob.     | 0.005*   | 0.004* | 0.018* | 0.000* | 0.001* | 0.000*       |

Note: * indicate the rejection of the null hypothesis of a unit root at the 1% significant levels, respectively; agg=aggregate.
Figure A1. Graphical trends of energy consumption for BRICS
### Table A3. Mean absolute percentage error (MAPE) and Mean Square Error (MSE) for Oil Consumption of BRICS

| r | MSE | MAPE | a | b |
|---|-----|------|---|---|
| MAPE | | | | |
| r=0 | 188.868 | 5.180 | 0.053 | 6.992 |
| r=0.01 | 115.046 | 5.386 | 0.232 | 8.143 |
| r=0.05 | 73.023 | 4.954 | 0.062 | 9.248 |
| r=0.1 | 61.483 | 4.356 | 0.062 | 11.640 |
| r=0.5 | 52.035 | 4.356 | 0.062 | 29.519 |
| r=0.9 | 42.264 | 4.356 | 0.062 | 60.981 |
| r=1 | 47.209 | 4.356 | 0.062 | 73.023 |
| r=1.5 | 554.869 | 4.356 | 0.062 | 181.164 |
| r=0 | 173.090 | 4.356 | 0.062 | 145.729 |
| r=0.01 | 163.078 | 4.356 | 0.062 | 135.180 |
| r=0.05 | 118.839 | 4.356 | 0.062 | 91.542 |
| r=0.1 | 145.729 | 4.356 | 0.062 | 68.716 |
| r=0.5 | 316.868 | 4.356 | 0.062 | 84.703 |
| r=0.9 | 348.328 | 4.356 | 0.062 | 91.542 |
| r=1 | 433.358 | 4.356 | 0.062 | 112.864 |
| r=1.5 | 470.281 | 4.356 | 0.062 | 209.504 |
| MAPE | | | | |
| r=0 | 3.866 | 5.180 | 0.053 | 6.992 |
| r=0.01 | 3.716 | 5.180 | 0.053 | 8.143 |
| r=0.05 | 3.198 | 5.180 | 0.053 | 9.248 |
| r=0.1 | 2.795 | 5.180 | 0.053 | 11.640 |
| r=0.5 | 2.356 | 5.180 | 0.053 | 29.519 |
| r=0.9 | 1.954 | 5.180 | 0.053 | 60.981 |
| r=1 | 1.536 | 5.180 | 0.053 | 73.023 |
| r=1.5 | 1.015 | 5.180 | 0.053 | 181.164 |
| r=0 | 3.248 | 5.386 | 0.232 | 8.143 |
| r=0.01 | 3.343 | 5.386 | 0.232 | 8.143 |
| r=0.05 | 3.651 | 5.386 | 0.232 | 11.243 |
| r=0.1 | 4.148 | 5.386 | 0.232 | 14.555 |
| r=0.5 | 5.602 | 5.386 | 0.232 | 51.763 |
| r=0.9 | 17.790 | 5.386 | 0.232 | 124.539 |
| r=1 | 21.358 | 5.386 | 0.232 | 152.515 |
| r=1.5 | 26.741 | 5.386 | 0.232 | 403.135 |
| MSE | | | | |
| r=0 | 483.268 | 4.954 | 0.062 | 9.248 |
| r=0.01 | 438.328 | 4.954 | 0.062 | 84.703 |
| r=0.05 | 346.868 | 4.954 | 0.062 | 84.703 |
| r=0.1 | 231.141 | 4.954 | 0.062 | 84.703 |
| r=0.5 | 145.729 | 4.954 | 0.062 | 84.703 |
| r=0.9 | 348.328 | 4.954 | 0.062 | 84.703 |
| r=1 | 433.358 | 4.954 | 0.062 | 84.703 |
| r=1.5 | 470.281 | 4.954 | 0.062 | 84.703 |

**Note:** MSE=Mean Standard Error; MAPE=Mean absolute percentage Error.
## Table A4. Mean absolute percentage error (MAPE) and Mean Square Error (MSE) for Gas Consumption of BRICS

|          | FGM (1,1) for Brazil | ARIMA(1,1,1) | FGM (1,1) for China | ARIMA(1,1,1) | FGM (1,1) for India | ARIMA(1,1,1) | FGM (1,1) for Russia | ARIMA(1,1,1) | FGM (1,1) for South Africa | ARIMA(1,1,1) |
|----------|----------------------|--------------|---------------------|--------------|---------------------|--------------|----------------------|--------------|---------------------------|--------------|
|          | r=0                  | r=0.01       | r=0.05              | r=0.1        | r=0.5               | r=0.9        | r=0.01               | r=0.05       | r=0.1                     | r=0.5        |
| MSE      | 13.971               | 13.862       | 13.461              | 13.031       | 11.422              | 15.033       | 17.749               | 60.883       | 14.085                    | 14.589       |
|          | 22.107               | 22.113       | 22.170              | 22.262       | 24.350              | 29.581       | 31.343               | 43.135       | 22.101                    | 22.173       |
|          | 0.008                | 0.007        | 0.004               | -0.001       | -0.035              | -0.062       | -0.092               | 0.009        | 0.012                     | 0.016        |
|          | 1.166                | 1.192        | 1.300               | 1.441        | 2.972               | 5.820        | 6.883                | 16.021       | 1.140                     | 1.038        |
|          | 1597.445             | 60.179       | 13.971              | 2.972        | 0.049               | 0.551        | 8.607                | 0.008        | 0.012                     | 0.108        |
|          | 1070.673             | 58.943       | 13.862              | 5.820        | 0.070               | 0.496        | 8.498                | 0.787        | 0.108                     | 0.012        |
|          | 2386.569             | 2286.868     | 1933.065            | 1576.655     | 422.742             | 236.068      | 233.796              | 455.330      | 2491.238                   | 2964.885     |
|          | 60.179               | 58.943       | 54.298              | 49.114       | 26.881              | 19.728       | 18.884               | 19.937       | 61.446                    | 66.841       |
|          | 0.108                | -0.109       | -0.109              | -0.115       | -0.122              | -0.124       | -0.133               | -0.108       | -0.107                    | -0.107       |
|          | 0.756                | 0.787        | 0.920               | 1.101        | 3.377               | 8.018        | 9.794                | 0.725        | 0.608                     | 0.477        |
|          | 211.700              | 338.965      | 17.565              | 17.909       | 22.225              | 24.800       | 24.043               | 24.014       | 0.229                     | 0.140        |
|          | 8.607                | 8.498        | 8.143               | 7.842        | 8.005               | 10.882       | 12.105               | 22.167       | 8.727                     | 9.325        |
|          | -0.0023              | -0.0021      | -0.0011             | -0.0018      | -0.040              | -0.064       | -0.070               | -0.003       | -0.005                    | -0.008       |
|          | 1.364                | 1.432        | 1.709               | 2.072        | 5.985               | 13.333       | 16.108               | 40.412       | 0.928                     | 0.477        |
|          | 668162.502           | 670.465      | 353.199             | 353.199      | 304.432             | 304.432      | 304.432              | 304.432      | 304.432                    | 304.432      |
|          | 11.005               | 8.753        | 3.498               | 2.980        | 5.601               | 4.213        | 3.408                | 13.490       | 11.384                    | 7.178        |
|          | -0.0423              | -0.0460      | -0.0190             | -0.0210      | -0.039              | -0.001       | -0.008              | -0.041       | -0.028                    | 0.152        |
|          | -13.414              | -14.730      | -3.854              | 16.874       | 120.034             | 257.855      | 309.562              | 770.353      | 8.978                     | 43.940       |
|          | 0.051                | 0.539        | 0.496               | 0.454        | 0.315               | 0.330        | 0.350                | 0.687        | 0.563                     | 0.622        |
|          | 48.536               | 47.852       | 45.358              | 42.625       | 31.557              | 30.042       | 30.360              | 34.505       | 49.252                    | 52.327       |
|          | -0.012               | -0.012       | -0.013              | -0.028       | -0.050              | -0.078       | -0.072              | -0.001       | -0.011                    | -0.010       |
|          | 0.049                | 0.053        | 0.070               | 0.092        | 0.360               | 0.853        | 1.035                | 2.592        | 0.046                     | 0.032        |
Table A 5. Mean absolute percentage error (MAPE) and Mean Square Error (MSE) for Coal Consumption of BRICS

| r   | FGM (1,1) for Brazil | ARIMA(1,1,1) |
|-----|-----------------------|--------------|
|     | MSE                   | MSE          |
|     | MAPE                  | MAPE         |
| a   |                       |              |
| b   |                       |              |

FGM (1,1) for China

| r   | FGM (1,1) for China | ARIMA(1,1,1) |
|-----|---------------------|--------------|
|     | MSE                 | MSE          |
|     | MAPE                | MAPE         |
| a   |                      |              |
| b   |                      |              |

FGM (1,1) for India

| r   | FGM (1,1) for India | ARIMA(1,1,1) |
|-----|---------------------|--------------|
|     | MSE                 | MSE          |
|     | MAPE                | MAPE         |
| a   |                      |              |
| b   |                      |              |

FGM (1,1) for Russia

| r   | FGM (1,1) for Russia | ARIMA(1,1,1) |
|-----|----------------------|--------------|
|     | MSE                  | MSE          |
|     | MAPE                 | MAPE         |
| a   |                      |              |
| b   |                      |              |

FGM (1,1) for South Africa

| r   | FGM (1,1) for South Africa | ARIMA(1,1,1) |
|-----|---------------------------|--------------|
|     | MSE                       | MSE          |
|     | MAPE                      | MAPE         |
| a   |                           |              |
| b   |                           |              |
|        | r=0          | r=0.01        | r=0.05        | r=0.1        | r=0.5        | r=0.9        | r=1          | r=1.5        |
|--------|--------------|---------------|---------------|--------------|--------------|--------------|--------------|--------------|
| MSE    | 26.193       | 25.638        | 24.188        | 23.534       | 26.283       | 29.985       | 39.439       | 422.779      |
| MAPE   | 4.903        | 4.871         | 4.786         | 4.766        | 5.250        | 5.552        | 6.888        | 20.785       |
| a      | 0.067        | 0.071         | 0.079         | 0.081        | 0.035        | -0.006       | -0.014       | -0.047       |
| b      | 6.286        | 6.809         | 8.695         | 10.716       | 25.763       | 52.191       | 62.277       | 152.041      |

**FGM (1,1) for Brazil**

|        | r=0.01       | r=0.05       | r=0.1        | r=0.5        | r=0.9        | r=1          | r=1.5        |
|--------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| MSE    | 215.014      | 240.196      | 832.985      | 360.018      | 19.201       | 20.629       | 22.744       |
| MAPE   | 94.467       | 81.96        | 164.57       | 19.201       | 66.711       | 113.149      | 193.771      |
| a      | 1.851        | -0.041       | -0.044       | -0.007       | -0.097       | -0.039       | -0.033       |
| b      | -19.890      | 3.993        | 4.499        | 5.175        | 23.589       | 34.593       | 88.877       |

**ARIMA(1,1,1)**

|        | r=0.01       | r=0.05       | r=0.1        | r=0.5        | r=0.9        | r=1          | r=1.5        |
|--------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| MSE    | 215.014      | 240.196      | 832.985      | 360.018      | 19.201       | 20.629       | 22.744       |
| MAPE   | 94.467       | 81.96        | 164.57       | 19.201       | 66.711       | 113.149      | 193.771      |
| a      | 1.851        | -0.041       | -0.044       | -0.007       | -0.097       | -0.039       | -0.033       |
| b      | -19.890      | 3.993        | 4.499        | 5.175        | 23.589       | 34.593       | 88.877       |

**FGM (1,1) for China**

|        | r=0.01       | r=0.05       | r=0.1        | r=0.5        | r=0.9        | r=1          | r=1.5        |
|--------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| MSE    | 1102950.215  | 344.024      | 316.631      | 289.252      | 199.725      | 215.014      | 240.196      |
| MAPE   | 94.467       | 81.96        | 164.57       | 19.201       | 66.711       | 113.149      | 193.771      |
| a      | 1.851        | -0.041       | -0.044       | -0.007       | -0.097       | -0.039       | -0.033       |
| b      | -19.890      | 3.993        | 4.499        | 5.175        | 23.589       | 34.593       | 88.877       |

**FGM (1,1) for India**

|        | r=0.01       | r=0.05       | r=0.1        | r=0.5        | r=0.9        | r=1          | r=1.5        |
|--------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| MSE    | 5.671        | 5.531        | 5.410        | 5.421        | 6.653        | 5.422        | 5.055        |
| MAPE   | 8.768        | 8.602        | 8.617        | 8.788        | 10.103       | 9.148        | 8.642        |
| a      | -0.062       | -0.057       | -0.039       | -0.022       | 0.002        | -0.021       | -0.027       |
| b      | -0.786       | -0.666       | -0.173       | 0.449        | 5.315        | 12.772       | 15.549       |

**FGM (1,1) for Russia**

|        | r=0.01       | r=0.05       | r=0.1        | r=0.5        | r=0.9        | r=1          | r=1.5        |
|--------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| MSE    | 9.288        | 22.673       | 3.967        | 4.429        | 11.002       | 5.421        | 3.322        |
| MAPE   | 6.744        | 10.793       | 4.074        | 4.359        | 7.240        | 5.056        | 4.248        |
| a      | -0.006       | -0.058       | -0.018       | 0.103        | 0.062        | 0.010        | 0.001        |
| b      | -0.221       | -2.277       | -0.493       | 6.097        | 17.020       | 33.298       | 39.591       |

**FGM (1,1) for South Africa**

|        | r=0.01       | r=0.05       | r=0.1        | r=0.5        | r=0.9        | r=1          | r=1.5        |
|--------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| MSE    | 0.049        | 0.047        | 0.041        | 0.036        | 0.024        | 0.026        | 0.028        |
| MAPE   | 54.051       | 51.839       | 50.271       | 51.327       | 58.643       | 67.637       | 72.447       |
| a      | -0.005       | -0.002       | 0.011        | 0.030        | 0.067        | 0.014        | 0.003        |
| b      | -0.002       | -0.001       | 0.005        | 0.016        | 0.136        | 0.269        | 0.317        |
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