An angular speed and position FLL-based estimator using linear Hall-effect sensors

LUIS IBARRA¹, RENATO GALLUZZI¹, GERARDO ESCOBAR², and RICARDO A. RAMIREZ-MENDOZA²

¹School of Engineering and Sciences, Tecnologico de Monterrey. Puente 222, Tlalpan 14380, Mexico City, Mexico.
²School of Engineering and Sciences, Tecnologico de Monterrey. Av. Eugenio Garza Sada 2501 Sur, Monterrey 64849, Nuevo León, Mexico.

Corresponding author: Renato Galluzzi (e-mail: renato.galluzzi@tec.mx).

ABSTRACT This paper proposes using a frequency-locked loop-based detector to estimate rotational speed and angle position for an electric machine rotor shaft. The measurement system consists of arrays of permanent magnets fixed to the rotor shaft together with linear Hall-effect sensors attached to a fixed frame. Parametric uncertainties on the sensor assembly lead to significantly noisy signals, exhibiting unbalance and harmonic distortion. To accurately estimate rotational speed and angle, it is proposed to use a frequency-locked loop scheme based on a fourth-order harmonic oscillator (FOHO) to allow the processing of the symmetric components, thus dealing with the unbalance. The scheme also includes an adaptive law to reconstruct the fundamental frequency. Moreover, a harmonic compensation mechanism comprising parallel FOHOs is included; each FOHO is tuned at the spectral component under concern for its cancellation. The proposed algorithm delivers a clean estimate of the positive sequence fundamental component despite the disturbances at the signals provided by the Hall-effect sensor, which is used to reconstruct the rotation angle. The described approach could enhance low-cost sensing solutions in applications where position feedback is mandatory and sensorless control is impossible, not requiring special installation considerations.

INDEX TERMS Angular position, position feedback, frequency-locked loop, phase-locked loop, harmonic oscillator, unbalance, Hall-effect sensor

I. INTRODUCTION

Rotating electrical machines benefit from the use of speed and position signals to attain motion control. With the electrification of numerous applications in the industrial, aerospace, and automotive fields, offered solutions must be precise, robust, and cost-effective. Permanent-magnet synchronous motors (PMSM) are increasingly gaining popularity in the contexts above due to their favorable torque-to-volume ratio compared to other machine technologies. However, position feedback is particularly relevant in synchronous machines to implement the so-called field-oriented control. Furthermore, other machines like induction motors can also exploit motion feedback for improved control performance.

Commonly, electric machines use resolvers or encoders to measure rotor position and speed. Resolvers rely on a magnetic circuit with fixed (stator) and moving (rotor) parts. The rotor of the resolver is mechanically coupled to the motor shaft and has an excitation coil fed by a high-frequency sinusoidal voltage. The stator generally presents two pickup coils in quadrature to measure the back-electromotive force induced by the rotor excitation. The demodulation of pickup-coil signals yields the angular position of the rotor. In contrast, encoders rely on optical readings of slits etched on a rotating disk. Although common and sufficiently precise for most applications, both of these solutions are costly and require cautious installation and tuning before their use.

For this reason, numerous efforts have been devoted to developing sensorless control solutions that estimate the rotor position from other signals, such as machine voltages and currents [1]. In practice, sensorless control eliminates the presence of a transducer from the application. However, motion estimates could face challenges in situations like startup, under heavy transients, and during dynamic load variations. While these drawbacks can be accepted in some systems, many applications still require a more robust sensing solution.

VOLUME 4, 2016

1
Then, robust, low-cost transducers are required. Rotor position can be measured by magnetic means using linear Hall-effect sensors (HES) [2], [3]. From a constructive standpoint, this setup requires the reading of two or more analog signals. Sensors are installed on a fixed frame to measure the magnetic flux density provided by a rotating magnet array. These magnets can be the poles of the rotor’s active part; i.e., measurable, or dedicated elements. Besides cost, magnetic transducers are proper for fluid-embedded applications, such as fluid power actuators [4]. Nevertheless, one of the main drawbacks of this solution is the strong nonlinearity affecting the intensity of the magnetic field. Parametric uncertainties in the geometry and material properties inevitably induce large deviations on the measured field.

Typical alternative solutions for speed/position sensing applications involve discrete Hall-effect sensors (dHES), also called bipolar or switch sensors. The dHESs do not provide a signal matching flux density, but instead turn their state on and off based on the polarity of the detected field. First, the step-like generated signals can be smoothed to approximately achieve a sinusoidal waveform using a low-pass filter (LPF); then, a traditional PLL can be used to estimate speed and position output of this smoothed signal. In [5], a smooth filter was included in the above-described process. The characteristics of this filter are adapted in function of the estimated rotor frequency to eliminate undesired oscillations in the filter response. Likewise, [6] proposed the use of a position observer plus a PLL to suppress noise at the resulting signals. Finally, a synchronous frequency tracking filter was used in [7] to band-pass the estimated frequency only, while eliminating any other spectral component.

However, the main problem with dHES-based solutions is that the sensors are not able to exhibit sufficient triggering at low or zero speeds, thus impeding a proper estimators’ operation. Their performance can be improved by adding HESs, which increase the accuracy. Hence, HESs are able to detect slighter variations. In some applications, HESs are discretized using customized thresholds or comparing their outputs among each other to obtain a more accurate output than dHESs [8]. To attain higher feedback resolution, \( \sin / \cos \) strategies have been adopted using dedicated commercial integrated circuits, which readily estimate an angular position from analog periodic signals using trigonometry [9]. However, the inherent distortion due to placement imperfections requires a calibration process and additional circuitry to correctly operate HESs as dHESs [8], [10].

Therefore, HESs remain an attractive solution, despite the introduction of such strong nonlinearities, which is perhaps the main reason behind the limited use of this magnetic sensing solution. Among the most interesting proposals, the authors in [11] applied an adaptive notch filter and a PLL to obtain a reliable position estimate despite of signals heavily affected by a third-order harmonic. The proposed system exhibited good results, although its implementation requires the sensing of the edge-field between the stator and rotor, which requires a particular installation inside the stator teeth. The authors in [12] proposed the use of a delayed signal cancellation-based (CDSC) PLL over an array of two HESs; however, the HESs were only simulated, and problems involving unbalance, harmonic distortion, and noise were mostly neglected.

Although PLLs may represent a straightforward solution to track the angle, the present work proposes to address the negative effects of the HES nonlinearities by using a frequency-locked loop (FLL) scheme. FLLs are intended to reconstruct an estimate of the fundamental voltage component as well as an estimate of the fundamental frequency of a reference signal, usually for grid synchronization purposes. FLLs have demonstrated better performance than PLLs, especially under heavily distorted electric grid signals. In contrast to standard PLLs, FLLs implement a closed loop aiming to reduce the frequency error instead of the phase error. However, as FLLs also estimate the voltage’s positive sequence component, the phase angle can still be calculated out of the loop using, for instance, the arctangent of the signals in quadrature calculated in the process.

We propose to adapt the FLL presented in [13], which involves the knowledge of the nominal frequency. As shown later, the fundamental frequency estimator used here was directly obtained from the Lyapunov stability analysis, avoiding the said manipulation of the frequency estimate. In contrast to [13], the present work includes other perturbations, such as DC-offsets and harmonic distortion with components at frequencies of a fraction of the fundamental, which turn out to be also unbalanced. The effect of the above-mentioned perturbations must be compensated to satisfy the HES application requirements. For this, the harmonic compensation mechanism proposed in [14] was reworked to operate along with the proposed FLL, instead of the PLL originally used in that work.

To the best of the authors’ knowledge, this is the first time an FLL is used in HES-based applications, and in angular position/speed estimation in general. In addition, the proposed setup could be mounted anywhere along the rotating mechanism, avaliable the magnets that are already part of the motor, or using dedicated magnets, i.e., no special installation is required. It was observed from the tests that the resulting estimation exhibits low error despite of the presence of significant input disturbances.

The paper is structured as follows. First, Section II describes the sensor setup and its limitations. Next, Section III provides mathematical means to model the distorted sensor signals. The angular position and speed estimation, together with a harmonic compensation technique to provide clean estimates, is presented in Section IV, where a tuning method for the proposed approach is also suggested. Section V outlines a dedicated testing setup to validate the proposed technique, whereas Section VI discusses the attained results. Finally, Section VII presents some concluding remarks.
II. SENSOR LAYOUT

The present research focuses on a HES-based position sensor installed on a permanent-magnet synchronous motor (PMSM). Figure 1 describes the setup and main parts comprising the sensor and their placement in the PMSM. This setup is used in a rotary regenerative damper [15], where position signals are fundamental to capture the alternating nature of the vehicle suspension stroke.

Button-shaped sensor permanent magnets (PM) are placed on the rear end of the rotor shaft. In particular, it is convenient that the number of sensor PMs matches the number of motor magnets \((N_m)\), where the number of pole pairs is defined as:

\[
p = \frac{N_m}{2},
\]

and the pole pitch angle is defined as

\[
\theta_p = \frac{\pi}{p}.
\]

To measure the axial flux of the sensor PMs array, the motor cover (fixed frame) holds a sensor board with \(N_h\) linear HESs. The location of these sensors obeys the sensor pitch angle. For a two-sensor configuration, transducers are placed in quadrature; hence,

\[
\theta_h(N_h = 2) = \frac{\pi}{2p} \pm k\frac{2\pi}{p}.
\]

Similarly, three-sensor setups use a balanced three-phase distribution, and thus

\[
\theta_h(N_h = 3) = \frac{2\pi}{3p} \pm k\frac{2\pi}{p}.
\]

where index \(k\) can exploit periodicity and fit the sensors in the board without geometrical interference. Industrial applications feature multiple HES configurations according to their needs [8], where the most common involve two or three HESs. In general, field-oriented control requires only two HESs in quadrature. However, other control strategies like the two-phase-on approach for the so-called brushless DC motors commonly use three dHESs. When custom latching thresholds or continuous angular measurements are preferred, dHESs are replaced by linear HESs. From a hardware perspective, this work aims at exploiting those standard configurations using linear sensors. Thus, the problem will be addressed by means of a novel estimation strategy for this type of measurements.

A magnetic finite-element model can help to understand the sensitivity of this setup. The 3D domains in Figure 2 are evaluated using the AC/DC module of COMSOL Multiphysics\textsuperscript{TM}. Periodicity is exploited to reduce the computational overhead; hence, the studied domains belong to a single pole-pair.

After calculations, the axial magnetic flux density \((B_z)\) is obtained at different air gaps, ranging from 1 to 3 mm. The air gap \(g\) represents the axial distance between the sensing element and the surface of the sensor PMs. Furthermore, \(B_z\) is measured along the arc where the HESs are collocated to mimic one electrical rotation. Results for different air gaps along an electrical rotation are illustrated in Figure 3.
The depicted acquired signals show significant variability concerning the air gap \( g \). As shown in Figure 4, if perfect sinusoidal waveforms are assumed, then their amplitude can be adjusted to a decaying exponential function of \( g \), where

\[
B_z(g) = B_0e^{-g/\gamma},
\]

with maximum amplitude \( B_0 = 623.4 \text{ mT} \), and \( \gamma = 0.86 \) mm.

Preliminary results suggest that any variation (or manufacturing tolerances) at the axial assembly of the sensor can significantly affect the measurement output. For example, a slight axial deviation between the transducer and the sensor PM could yield inaccurate voltage readings that require real-time post-elaboration. Figure 5 highlights the inaccuracies in the sensor PM installation on one of the tested prototypes. In addition to sensor PMs placement, other important factors could affect the sensor readings, such as the sensor board fixture, the integrated circuit soldering on the board, and the sensor PMs' magnetization tolerance.

### III. Modeling of the Distorted Measured Signal

Due to the above-described imperfections, the resulting periodic signals from the HESs are not purely sinusoidal, do not have the same amplitude and are not equally phase-shifted among each other. In other words, amplitude unbalance, irregular shifting, DC offset, and harmonic distortion are the significant irregularities that the sensed signal will bear. Therefore, accurate extraction of speed and position from the measured signals becomes quite challenging, as these signals must be processed, and the effects of all imperfections must be minimized. We believe that a successful solution must consider the mathematical modeling of the measured signals. The proposed scheme bases its design in the model description in terms of the fixed-frame (\( \alpha \beta \)) coordinates, which are considered orthogonal, i.e., in quadrature. For the proposed two-sensor array mentioned in Section II, \( v_{\alpha \beta} \) is directly acquired, and is assumed that the coordinates \( v_\alpha \) and \( v_\beta \) are in quadrature (orthogonal), implying that only positive and negative sequences will be present, making the proposed two-phase modeling suitable for the problem at hand.

It is noteworthy that systems comprising more than two phases can also be mapped to the two-phases fixed frame. To this end, we appealed to the three-phase voltage classical vector descriptions involving symmetric components and their equivalences, useful for modeling voltages in power systems [13]. Such description departs from a three-phase electrical signal \( v_{abc} \), which is then transformed into an equivalent two-phase signal in the fixed-frame (\( \alpha \beta \)) using the Clarke’s transformation \( T : \mathbb{R}^3 \rightarrow \mathbb{R}^2 \), yielding \( v_{\alpha \beta} = [v_\alpha \ v_\beta]\) :

\[
v_{\alpha \beta} = T v_{abc}, \quad T = \sqrt{2} \left[ \begin{array}{cc} 1 & -\frac{1}{2} \\ \frac{1}{2} & 1 \end{array} \right].
\]

Notice that vector notation is being used, where vectors and matrices are represented by bold symbols hereinafter.

The resulting signals in \( v_{\alpha \beta} \) are orthogonal, i.e., \( v_\alpha \) and \( v_\beta \) are in quadrature as well. Recall that an unbalanced \( v_{abc} \) comprises positive, negative, and homopolar components. Notice, however, that (6) omits the homopolar component of the complete Clarke transformation. It is true that the homopolar component would yield unmodeled distortion; however, it implies common bias among phases, which can be easily corrected during implementation; hence, it can be neglected without affecting the final results. Then, the distortion brought by HES variations can be modeled as done for the referred power systems, where 3-HESs configurations can be mapped through (6). Then, the HESs unbalancing can be perfectly represented by the proposed modeling approach.

Therefore, in what follows the two-phase modeling is considered as it applies to either the two-sensor or three-sensor arrays, which can be mapped to \( v_{\alpha \beta} \in \mathbb{R}^2 \). To deal with unbalance, the method considers that \( v_{\alpha \beta} \) can be described as the sum of both positive and negative sequences (two counter-rotatory components), i.e., \( v_{\alpha \beta} \triangleq v_{+\alpha \beta} + v_{-\alpha \beta} \), where the superscript sign stands for the rotation direction, and thus it is used to identify the sequence.

First, consider that the signal \( v_{\alpha \beta} \) involves only one operating frequency coming from the operation angular speed,
then a model can be derived for $v_{\alpha\beta}$ considering that

\[ v_{\alpha\beta} = v_{\alpha\beta}^+ + v_{\alpha\beta}^- = e^{\theta\mathcal{J}} v_{\alpha\beta}^+ + e^{-\theta\mathcal{J}} v_{\alpha\beta}^- \]  \hfill (7)

where $v_{\alpha\beta}^+ = [v_{\alpha\beta}^+ v_{\alpha\beta}^-]^T$ and $v_{\alpha\beta}^- = [v_{\alpha\beta}^- v_{\alpha\beta}^+]^T$ are the corresponding coefficient vectors in the synchronous frame (also referred to as phasors), and $\theta$ is the angle. The model is derived by taking the time derivative of $v_{\alpha\beta}$ as follows:

\[ \dot{v}_{\alpha\beta} = \omega \mathcal{J} \left( v_{\alpha\beta}^+ - v_{\alpha\beta}^- \right), \quad \mathcal{J} \equiv \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \]  \hfill (8)

where $\omega = \dot{\theta}$, $\mathcal{J}$ is a skew symmetric matrix, i.e., $\mathcal{J}^T = -\mathcal{J}$ and $\mathcal{J}^T \mathcal{I} = \mathcal{I}$, where $\mathcal{I}$ is the $2 \times 2$ identity matrix.

An auxiliary variable representing the difference $(v_{\alpha\beta}^+ - v_{\alpha\beta}^-)$ must be defined to complete the model. This auxiliary variable is referred, from now on, to as the square phase-portrait:

\[ \varphi_{\alpha\beta} \triangleq (v_{\alpha\beta}^+ - v_{\alpha\beta}^-), \]  \hfill (9)

whose time derivative is given, in its turn, by

\[ \dot{\varphi}_{\alpha\beta} = \omega \mathcal{J} \left( v_{\alpha\beta}^+ + v_{\alpha\beta}^- \right) = \omega \mathcal{J} v_{\alpha\beta}, \]  \hfill (10)

which is an equation that completes the model description.

Summarizing, the dynamic model of the unbalanced signal at one specific angular speed $\omega$ can now be written in its complete form as follows:

\[ \dot{v}_{\alpha\beta} = \omega \mathcal{J} v_{\alpha\beta}, \] \hfill (11)

\[ \dot{\varphi}_{\alpha\beta} = \omega \mathcal{J} \varphi_{\alpha\beta}, \] \hfill (12)

which represents a fourth-order harmonic oscillator (FOHO).

Notice that both state variables $v_{\alpha\beta}$ and $\varphi_{\alpha\beta}$ hold a direct relationship with the positive and negative sequences given by

\[ \begin{bmatrix} v_{\alpha\beta} \\ \varphi_{\alpha\beta} \end{bmatrix} = \begin{bmatrix} \mathcal{I} & \mathcal{I} \\ \mathcal{I} & -\mathcal{I} \end{bmatrix} \begin{bmatrix} v_{\alpha\beta}^+ \\ v_{\alpha\beta}^- \end{bmatrix}. \] \hfill (13)

Moreover, since the matrix in (13) is invertible, then the positive and negative sequences can be reconstructed as follows:

\[ v_{\alpha\beta} = \frac{1}{2} \left( v_{\alpha\beta} + \varphi_{\alpha\beta} \right), \quad v_{\alpha\beta}^- = \frac{1}{2} \left( v_{\alpha\beta} - \varphi_{\alpha\beta} \right). \] \hfill (14)

Next, according to Lyapunov’s approach, we proposed the following quadratic storage function:

\[ V = \frac{\lambda v_{\alpha\beta}^T v_{\alpha\beta}}{2} + \frac{\lambda \dot{\varphi}_{\alpha\beta}^T \dot{\varphi}_{\alpha\beta}}{2} + \ddot{\omega}^2, \quad \lambda > 0, \] \hfill (20)

whose time derivative is given by

\[ \dot{V} = \lambda \dot{\omega} (v_{\alpha\beta}^T \mathcal{J} \varphi_{\alpha\beta} + \dot{\varphi}_{\alpha\beta}^T \mathcal{J} v_{\alpha\beta}) \] \hfill (21)

where we used the property $\dot{v}_{\alpha\beta}^T \mathcal{J} \dot{\varphi}_{\alpha\beta} = -\dot{\varphi}_{\alpha\beta}^T \mathcal{J} \dot{v}_{\alpha\beta}$. Furthermore, it can be seen that

\[ \dot{v}_{\alpha\beta}^T \mathcal{J} \dot{\varphi}_{\alpha\beta} = \dot{v}_{\alpha\beta}^T \mathcal{J} (\varphi_{\alpha\beta} + \dot{\varphi}_{\alpha\beta}) = -\dot{\varphi}_{\alpha\beta}^T \mathcal{J} v_{\alpha\beta} + \dot{v}_{\alpha\beta}^T \mathcal{J} \dot{\varphi}_{\alpha\beta}, \] \hfill (22)

which substituted into (21) yields

\[ \dot{V} = \lambda \dot{\omega} \dot{v}_{\alpha\beta}^T \dot{\varphi}_{\alpha\beta} - \lambda \gamma_1 \dot{v}_{\alpha\beta}^T \dot{\varphi}_{\alpha\beta} + \ddot{\omega}^2. \] \hfill (23)

Notice that, by proposing $\dot{\omega} = -\lambda v_{\alpha\beta}^T \mathcal{J} \varphi_{\alpha\beta}$ in (23), where $\lambda > 0$ is referred to as the estimation gain, then $\dot{V} \leq 0$ for $\gamma_1 > 0$, $\gamma > 0$. Since $\dot{V}$ is only negative semidefinite, we appeal to the LaSalle’s invariance principle to complete the proof. Out of this, all increments $\dot{v}_{\alpha\beta}^T \varphi_{\alpha\beta}$, $\dot{v}_{\alpha\beta}^T$, and $\ddot{\omega}$ converge to zero, asymptotically. Moreover, as $\dot{V}$ is radially unbounded, then the result is global. Based on the fact that $\dot{\omega} = -\ddot{\omega}$, then the estimation law for $\omega$ can be rewritten as

\[ \dot{\omega} = \lambda v_{\alpha\beta}^T \mathcal{J} \varphi_{\alpha\beta}, \] \hfill (24)

which is referred to as the fundamental frequency estimator (FFE). Notice that the estimation gain $\lambda$ is used to adjust the response speed of the FFE. In brief, the FOHO-AE in (15), (16), together with the FFE (24), ensures $\dot{v}_{\alpha\beta} ightarrow 0$, $\dot{\varphi}_{\alpha\beta} ightarrow 0$, and $\dot{\omega} ightarrow 0$ with global asymptotic stability, as above stated. In other words, this algorithm provides estimation of both the positive and negative sequences $\ddot{v}_{\alpha\beta}$ and $\ddot{\varphi}_{\alpha\beta}$, respectively, as well as the estimation of the fundamental frequency $\ddot{\omega}$ of a voltage measured signal $v_{\alpha\beta}$.
A. ADDING SPECTRAL CONTAMINATION

If the measured signal contains additional components in the frequency spectrum other than the fundamental, then the model can be extended as follows:

\[
\mathbf{v}_{\alpha\beta} = \sum_{k \in \mathbf{H}} \left( \mathbf{v}_{\alpha\beta,k}^+ + \mathbf{v}_{\alpha\beta,k}^- \right) = \sum_{k \in \mathbf{H}} \mathbf{v}_{\alpha\beta,k},
\]

where \( \mathbf{H} \) is a set comprising the indexes of all the harmonic components of the measured signal, e.g., \( \mathbf{H} = \{1, 3, 2/5, 3/5, \ldots\} \), that is, the spectral components of interest are indexed by \( k \), and thus they can be written in terms of the fundamental frequency as \( k \omega \). Normally, \( k \) is either a positive integer \( (k \in \mathbb{N}) \) or a regular fraction in terms of the number of pole pairs \( p \), i.e., \( k = n/p \), \( n \in \mathbb{Z} \).

A model for the \( k \)-th harmonic component can be obtained as an extension of (11)-(12) by including the index \( k \) as follows:

\[
\mathbf{\hat{v}}_{\alpha\beta,k} = k \omega \mathbf{J} \mathbf{\hat{a}}_{\alpha\beta,k},\quad k \in \mathbf{H},
\]

\[
\mathbf{\hat{a}}_{\alpha\beta,k} = k \omega \mathbf{J} \mathbf{\hat{a}}_{\alpha\beta,k},
\]

which is referred to as the FOHO-AE\(_k\). Finally, by extending (14), the positive and negative sequences of the \( k \)-th harmonic component can be derived as follows:

\[
\mathbf{v}_{\alpha\beta,k}^+ = \frac{1}{2} \left( \mathbf{v}_{\alpha\beta,k} + \mathbf{\hat{a}}_{\alpha\beta,k} \right),\quad k \in \mathbf{H},
\]

\[
\mathbf{v}_{\alpha\beta,k}^- = \frac{1}{2} \left( \mathbf{v}_{\alpha\beta,k} - \mathbf{\hat{a}}_{\alpha\beta,k} \right).
\]

Analogously, we propose to extend (15)-(16) to estimate the \( k \omega \) voltage component by including the \( k \)-th index as follows:

\[
\mathbf{\hat{v}}_{\alpha\beta,k} = k \omega \mathbf{J} \mathbf{\hat{a}}_{\alpha\beta,k} + \gamma_k \mathbf{\hat{e}}_{\alpha\beta,k},\quad k \in \mathbf{H},
\]

\[
\mathbf{\hat{a}}_{\alpha\beta,k} = k \omega \mathbf{J} \mathbf{\hat{a}}_{\alpha\beta,k},
\]

which is referred to as FOHO-AE\(_k\).

Notice that the estimated \( \hat{\omega} \), which is common to every FOHO-AE\(_k\), is obtained by the FFE. However, as described in (24), \( \hat{\omega} \) relies on \( \mathbf{v}_{\alpha\beta,1} \) and \( \mathbf{\hat{a}}_{\alpha\beta,1} \), implying that the prevailing spectral contamination mainly reflected in \( \mathbf{v}_{\alpha\beta,1} \) would propagate to the FFE causing a ripple issue on \( \hat{\omega} \).

Therefore, to eliminate the undesired components \( (k \in \mathbf{H} - \{1\}) \), we propose to insert a parallel combination of FOHO-AE\(_k\), which is referred to as the unbalance harmonic compensation mechanism (UHCM). The idea behind the UHCM is to estimate the harmonic components under concern, add them, and then subtract their sum from the measured signal. The effect is equivalent to clean the measured \( \mathbf{v}_{\alpha\beta} \) so as to recuperate its fundamental component only, or in other words, to approximate the error \( \mathbf{v}_{\alpha\beta} \) as \( \mathbf{v}_{\alpha\beta,1} \). Figure 6 shows the UHCM (parallel array of FOHO-AE\(_k\)) which, together with the FOHO-AE\(_1\), allows cleaner estimations of \( \hat{\omega} \) and \( \mathbf{\hat{a}}_{\alpha\beta,1} \). Summarizing, Figure 7 shows the block diagram representation of the overall proposed estimation system, referred to as the FOHO frequency-locked loop (FOHO-FLL), which comprises the FOHO-AE\(_1\), the FFE, and the UHCM.
Based on (28), the estimate of the fundamental component positive sequence $\hat{v}_{\alpha\beta,1}$ can be reconstructed according to
\[
\hat{v}_{\alpha\beta,1}^+ = \frac{1}{2} (\hat{v}_{\alpha\beta,1} + \nu_{\alpha\beta,1}) ,
\]
which represents an approximate of the undisturbed signal $v_{\alpha\beta,1}$ that would have been measured if an ideal sensor had been available under ideal conditions. Finally, we propose to reconstruct an estimate of the motor electrical angle $\hat{\theta}_e$ out of such a clean, balanced, and almost pure sinusoidal vector signal $\hat{v}_{\alpha\beta,1}$ as follows:
\[
\hat{\theta}_e = \tan^{-1}(\hat{v}_{\alpha\beta,1}^+/\hat{v}_{\alpha\beta,1}^+) + \pi, \tag{33}
\]
where an amount of $\pi$ is added to get $\hat{\theta}_e \in [0, 2\pi].$

**Remark 1.** Notice that, $\hat{\omega}_e = \hat{\omega} = \hat{\theta}_e$ must match the estimated frequency $\hat{\omega}$ obtained in the FFE.

### B. FOHO-FLF Tuning

The stability analysis above presented yielded the conditions on the control parameters $\gamma_1, \lambda > 0$ to preserve stability; however, it is still necessary to tune the control parameters to guarantee a preferred dynamic performance of the proposed estimator. Therefore, for a first tuning approach, we propose to consider a linearization of the system model, assuming an ideal operation of the HESSs, which is equivalent to consider a balanced and uncontaminated measured voltage. In other words, $\nu_{\alpha\beta} = v_{\alpha\beta} = v_{\alpha\beta,1}^+,$ where both amplitude and frequency are constant. To simplify the notation, the harmonic order and the sequence superscripts are omitted in what follows. Based on these considerations, the observer can be rewritten as
\[
\dot{\hat{\nu}}_{\alpha\beta} = \hat{\omega} J \hat{v}_{\alpha\beta} + \gamma \hat{v}_{\alpha\beta}, \tag{34}
\]
\[
\dot{\hat{\omega}} = \lambda \hat{v}_{\alpha\beta}^T J \hat{v}_{\alpha\beta}. \tag{35}
\]

The error model, i.e., the model in terms of the increments, is given by
\[
\dot{\nu}_{\alpha\beta} = \omega J v_{\alpha\beta} - (\omega - \hat{\omega}) J \hat{v}_{\alpha\beta} - \gamma \hat{v}_{\alpha\beta}, \tag{36}
\]
\[
\dot{\omega} = -\lambda \hat{v}_{\alpha\beta}^T J \hat{v}_{\alpha\beta}, \tag{37}
\]
where $\nu_{\alpha\beta} \triangleq \nu_{\alpha\beta} - \nu_{\alpha\beta},$ and $\omega \triangleq \omega - \hat{\omega}.$ Notice that $\dot{\omega} = -\dot{\hat{\omega}}$ since $\omega$ is assumed constant.

The error model can now be transformed to yield a more familiar structure by considering the following coordinates transformation:
\[
x_1 \triangleq \hat{v}_{\alpha\beta}^T \nu_{\alpha\beta}, \quad x_2 \triangleq \hat{v}_{\alpha\beta}^T J \hat{v}_{\alpha\beta}, \quad x_3 \triangleq \hat{\omega}. \tag{38}
\]
Out of which, the above error model is transformed to
\[
\dot{x}_1 = -\gamma x_1 - x_2 x_3 + \gamma \|\hat{v}_{\alpha\beta}\|^2, \tag{39}
\]
\[
\dot{x}_2 = -x_1 x_3 - \gamma x_2 + x_3 (\|v_{\alpha\beta}\|^2 - \|\hat{v}_{\alpha\beta}\|^2), \tag{40}
\]
\[
\dot{x}_3 = -\lambda x_2, \tag{41}
\]
where we exploited the properties $J^T J = I,$ $J = -J^T,$ and $z^T J z = 0$ for any $z = [z_1, z_2]^T.$

**Remark 2.** Consider that $\hat{v}_{\alpha\beta} \triangleq v_{\alpha\beta} - \hat{v}_{\alpha\beta},$ then
\[
\hat{v}_{\alpha\beta}^T \hat{v}_{\alpha\beta} = (v_{\alpha\beta} - \hat{v}_{\alpha\beta})^T (v_{\alpha\beta} - \hat{v}_{\alpha\beta}), \tag{42}
\]
\[
= v_{\alpha\beta}^T v_{\alpha\beta} - v_{\alpha\beta}^T \hat{v}_{\alpha\beta} - \hat{v}_{\alpha\beta}^T v_{\alpha\beta} - 2 \hat{v}_{\alpha\beta}^T \hat{v}_{\alpha\beta}, \tag{43}
\]
\[
= \|v_{\alpha\beta}\|^2 - \|\hat{v}_{\alpha\beta}\|^2 - 2 x_1, \tag{44}
\]
where $\| \cdot \|^2$ stands for the squared amplitude.

Following to the Taylor approximation, the linearization of the system described by (39)-(41) around the equilibrium $[x_1 \ x_2 \ x_3] = [0 \ 0 \ 0]$ (or equivalently $\hat{v}_{\alpha\beta} = 0$ and $\hat{\omega} = 0$) yields
\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
-\gamma & 0 & 0 \\
0 & -\gamma & \|v_{\alpha\beta}\|^2 \\
0 & -\lambda & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}, \tag{45}
\]
which is stable provided $\gamma, \lambda > 0.$ This system comprises two fully decoupled subsystems, a first-order system involving $x_1,$ and second-order system involving states $x_2$ and $x_3.$ In particular, the second-order subsystem can be represented in its transfer function form $G(s) : \Delta \omega \rightarrow x_3/$ as follows:
\[
\frac{\hat{\omega}(s)}{\Delta \omega(s)} = \frac{s^2 + \gamma s}{s^2 + \gamma s + \lambda \|v_{\alpha\beta}\|^2}, \tag{46}
\]
where $\Delta \omega$ is a virtual frequency perturbation (added to the output $x_3.$) This transfer function has a damping factor $\xi = \gamma/(2 \|v_{\alpha\beta}\| \sqrt{\lambda})$ and a natural frequency $\omega_n = \|v_{\alpha\beta}\| \sqrt{\lambda}.$

Out of these, we propose the following tuning rules which consider, in addition, the selection of both a desired settling time $t_s = 4/\xi \omega_n$ and a convenient damping coefficient $\xi:$$
\gamma = 8/t_s, \quad \lambda = 16/(t_s \xi \|v_{\alpha\beta}\|)^2. \tag{47}
\]
These explicit tuning rules, which respond to such a performance criterion, represent, however, a first rough approximation only. Therefore, they require further fine-tuning, which is usually performed by trial-and-error.

### C. Temperature Variation Analysis

In addition to the above discussed disturbances, permanent magnets are also sensitive to environmental variations, among which temperature is paramount. In fact, the magnetic flux density is inversely proportional to temperature. Therefore, if in an application the temperature changes, then it is expected that measurements will vary as well. As linear HESSs are transducers providing an output voltage from a measured flux density, then the acquired voltage’s amplitude is expected to vary accordingly.

Expression (46) shows that modifying $\|v_{\alpha\beta}\|$ would affect the system’s dynamics. In particular, it can be seen that
\[
t_s = 8/\gamma, \quad \xi = \gamma/(2 \|v_{\alpha\beta}\| \sqrt{\lambda}). \tag{48}
\]
Therefore, voltage variations will mainly affect the response’s damping. In applications where wide variations of temperature are expected, magnets with slight dependence on temperature must be selected. Otherwise, it is recommended to tune parameters around a nominal operating point, and
calculate variations with respect to the desired damping against known values of $|v_{\alpha\beta}|$. In more drastic scenarios, real-time temperature monitoring and adaptive tuning could be implemented instead.

### D. UHCM Tuning

The above tuning guidelines focus on the frequency estimation dynamics only. Therefore, to complement the tuning process, the behavior of the FOHO-AE$_k$ has to be further analyzed as described next. Once we consider only the positive sequence component, then the FOHO-AE$_k$ can be rewritten using complex scalar terms as follows:

$$
\hat{v}_{\alpha_k} + j\hat{v}_{\beta_k} = jk\hat{\omega}(\hat{v}_{\alpha_k} + j\hat{v}_{\beta_k}) + \gamma_k(\hat{v}_{\alpha_k} + j\hat{v}_{\beta_k}).
$$

(49)

Since $(\hat{v}_{\alpha_k} + j\hat{v}_{\beta_k}) = (v_{\alpha_k} + jv_{\beta_k}) - (\hat{v}_{\alpha_k} + j\hat{v}_{\beta_k})$, then the system can be rewritten in the Laplace domain as

$$
\frac{\hat{v}_{\alpha_k}(s) + j\hat{v}_{\beta_k}(s)}{v_{\alpha_k}(s) + jv_{\beta_k}(s)} = \frac{\gamma_k}{s + \gamma_k - jk\hat{\omega}},
$$

(50)

which is a band-pass filter (BPF) centered at $\hat{\omega}$, whose only tunable parameter $\gamma_k$ determines its bandwidth (BW) and the cutoff frequency (where the stopband starts). It is essential to notice that $\hat{\omega}$ must be constant to enable the transfer function representation, which entails an additional assumption to the previous analysis.

Now, after substitution of $s \rightarrow j\Omega$, the magnitude of (50) is given by

$$
M = \frac{\gamma_k}{\sqrt{\gamma_k^2 + (\Omega - \hat{\omega})^2}};
$$

(51)

therefore, $\gamma_k$ can be set to satisfy a desired BW. Solving for $\Omega$ in the previous expression yields

$$
\Omega_{1,2} = \hat{\omega} \pm \frac{\gamma_k}{M} \sqrt{1 - M^2},
$$

(52)

which leads to a frequency interval $[\Omega_1, \Omega_2]$ where the magnitude in the passband overpasses $M$. Notice that high values of $\gamma_k$ enlarge the bandwidth of (50), while it becomes more “frequency selective” for smaller $\gamma_k$.

As every FOHO-AE$_k$ is aimed to extract a specific spectral component, then they must be selective enough. Notice that high $\gamma_k$ values would not sufficiently attenuate other frequencies, hindering the overall estimator’s performance.

Therefore, $M$ and $\Omega_{1,2}$ are considered here as auxiliary parameters necessary to design an appropriate $\gamma_k$. Notice that there is a compromise regarding $\gamma_1$ as high values would, on the one hand, accelerate the frequency estimation, but on the other hand, they would widen the BW at the same time.

### V. Experimental Setup and Practical Considerations

An electric machine prototype was equipped with a HES measurement system to test the proposed FLL-based estimator. Then the entire estimation system can be referred to as the FLL-based HES (FLL-HES) measurement system. As depicted in Figure 5, ten button-shaped N45SH-grade NdFeB PMs were installed on the rotor rear end to match the electric machine poles $p = 5$.

Two arrays of Hall-effect sensors (Allegro\textsuperscript{TM} A1326) were tested, one with $N_h = 2$ sensors (providing $v_{\alpha\beta}$) and another one with $N_h = 3$ sensors (providing $v_{abc}$). Moreover, an additional motor phase-to-phase back-EMF measurement system was also included, which provides the signals vector $v_{abc}$ that serves as a reference for the validation of the HES measurement system with the proposed post-processing scheme. An schematic of the experimental setup is depicted in Figure 8. Notice that the machine’s rotor was coupled to a driving machine to spin the prototype at a mechanical speed $\omega_{em} = p\omega$ for testing purposes. HES signals together with the additional motor phase-to-phase back-EMF waveforms were synchronously sampled at 32 kHz using a Siemens\textsuperscript{TM} LMS SCADAS unit through its signature acquisition time-domain data logging module. These signals were subsequently loaded into a Simulink\textsuperscript{TM} model for post-processing.

Notice that the back-EMF signals do not bear the discussed magnetic disturbances, and thus they can be used to obtain cleaner and more accurate motor’s angular speed and position. Figure 9 depicts the calculations involved to obtain these signals, which are described next. According to Faraday’s law, the induced voltage’s amplitude is proportional to the motor’s angular speed $\omega_{em}$, i.e.,

$$
\omega_{em} = \rho \sqrt{v_{\alpha\beta}^\top v_{\alpha\beta}},
$$

(53)

where $v_{\alpha\beta} = \begin{bmatrix} v_{\alpha} & v_{\beta} \end{bmatrix}^\top$ is the Clarke-transformed induced voltage at the three phases of the motor, $v_{abc}$, and $\rho \in \mathbb{R} > 0$ is a proportionality constant. On the other hand, the rotor electrical angle $\theta_{em}$ can be directly computed as

$$
\theta_{em} = \tan^{-1}(v_{\beta}/v_{\alpha}).
$$

(54)

Finally, $\omega_{em}$ and $\theta_{em}$ were post-processed offline with a zero-phase digital low-pass filter to attenuate implementation noise.

Besides, the HES signals $v_{\alpha\beta}$ were processed through the FOHO-FLL as depicted in Figure 7. As discussed in Section II, two-sensor PMs were placed per motor’s pole, implying that each HES would face five cycles per rotor mechanical turn. Then, the fundamental frequency component acquired by each HES corresponds with the electrical speed.
of the motor. Similarly, the non-idealities distortion (mainly air gap variations) is present among every sensor PM; therefore, $n/5$-th subharmonic components will contaminate the HES acquired signals. Out of this, the UHCM was prepared with four FOHO-AE$_{n/5}$, $n = [1, 4]$, as shown in Figure 6.

Two tests were run to verify the FLL-HES performance. Firstly, the driving motor was held at quasi-constant speed, and two HESs in quadrature were used to extract $v_{\alpha\beta}$ directly. This process serves as an initial functional verification of the setup and the estimation strategy. Then, a profile with high dynamic content was imposed to emulate realistic load variations. In this case, two HESs were used to extract $v_{\alpha\beta}$.

The HES signals were normalized such that $|v_{\alpha\beta}| = 1$, and the FOHO-FLL was fine-tuned to $\gamma_1 = 80$ and $\gamma = 10^4$. Note that $\lambda$ was increased to shorten the response time of the frequency estimator, whereas $\gamma_1$ is relatively high, trying to cope with the response speed but attaining an inconvenient passband width of 280 rad/s with $M = 0.5$. Then, the $n/5$-th subharmonics would not be sufficiently attenuated by the FOHO-AE$_1$. As each FOHO-AE$_n$ must address $v_{\alpha\beta,n}$ individually, it is important for the FOHO-AE$_{4/5}$ not to overlap with the FOHO-AE$_1$, requiring a small $\gamma_{4/5}$. The FOHO-AE blocks corresponding to lower subharmonics can relax such consideration progressively, aiming for faster operation. Then, the following tuning was considered: $\gamma_{4/5} = 1$, $\gamma_{3/5} = 2$, $\gamma_{2/5} = 3$, and $\gamma_{1/5} = 4$. Finally, a DC offset compensation mechanism was added to eliminate the DC component, comprising an integrator with gain $\lambda_1$, as depicted in Figure 6. In this work, $\lambda_1 = 1$ was considered.

VI. EXPERIMENTAL RESULTS

The experimental setup in Section V was used to verify the validity of the described approach. Two types of test, which include constant and variable speed conditions for 2-HESs and 3-HESs configurations, respectively, are presented in the following.

A. CONSTANT SPEED TEST, 2-HES CONFIGURATION

The first test considered a speed step variation to analyze the speed estimation capabilities during constant speed intervals. Figure 10 shows that the speed is correctly approximated throughout the test; however, the response exhibits a slight delay during startup. It is important to consider that having an initial estimated frequency $\hat{\omega}_0 = 0$ hinders the performance of the FOHO-AE$_k$ blocks at the beginning of the test. As discussed in Section IV-B, $\gamma_k$ would not provide sufficiently selective passbands because they operate over proportional $\omega_k^2$ components. Then, the system only relies on the FOHO-AE$_1$ to estimate the frequency, whose $\gamma_1$ is more significant. As $\gamma_1$ imposes a response speed/selectivity trade-off, it is necessary to tune it according to the intended application.

Notice that the estimation of $\omega_{rem}$ resulted from the back-EMF signals $v_{\alpha\beta}$ at the motor’s stator, which were barely disturbed as shown in Figure 11. Besides, $v_{\alpha\beta}$ was measured from the 2-HESs configuration, exhibiting large unbalancing and subharmonic distortion. Figure 12 shows that, despite of such disturbances, the rotor speed and position were effectively estimated during the constant speed interval, where both errors achieve a relatively small value. Moreover, both errors exhibited normal distributions with standard deviations of $\sigma_\omega = 7.36$ rad/s and $\sigma_\theta = 0.07$ rad, respectively, which implies that 95% of the time the errors were below 14.7 rad/s and 0.14 rad, respectively. If the speed error is computed relative to the reference speed, then 95% of the time the speed error was found below 2.19%.

B. VARIABLE SPEED TEST, 3-HES CONFIGURATION

A 3-HESs configuration ($N_h = 3$), originally intended for discrete operation, was also tested here. In this case, the three analog signals $v_{\alpha\beta}$ were considered in the proposed post-
Ibarra et al.: An angular speed and position FLL-based estimator using linear Hall-effect sensors

processing method, while the calibration process mentioned in Section II was avoided. The resulting signals were largely disturbed, as shown in Figure 13, where it is possible to directly compare \( v_{abc} \) and \( v_{abc} \) as three HESs were used. Signals \( v_{abc} \) were first mapped to \( v_{\alpha\beta} \) using (6) as shown in Figure 13. This process not only highlights the ability of the FLL-HES to manage heavily distorted input signals, but also outlines it as an enabler of sensors re-purposing.

Figure 15 shows that, despite exhibiting larger disturbances than those present in the two-sensor array signals (compare \( v_{\alpha\beta} \) from Figure 11 and Figure 13), the proposed post-processing scheme correctly estimated the rotor speed as shown in Figure 14, exhibiting low errors comparable to those from the constant speed test. Speed and angle errors exhibited Normal distributions with standard deviations of \( \sigma_\omega = 6.57 \text{ rad/s} \) and \( \sigma_\theta = 0.096 \text{ rad} \), respectively, implying that 95% of the time the errors were below 13.14 rad/s and 0.192 rad, respectively. If the speed error is computed relative to the reference speed, it remained below 1.37% for 95% of the time.

VII. CONCLUSION

An FLL-based estimator of angular speed and position of rotating machinery was introduced and tested, aiming for dealing with severely disturbed input signals coming from linear Hall-effect sensors. The estimator was proven successful in the implemented tests employing two different sensors’ configurations. The results showed a favorable dynamic performance despite DC offset, high subharmonic contamination, and varying speed profiles. Moreover, the proposed setup did not imply specific installation requirements and was able to amend errors coming from misalignment, manufacturing tolerances, and an uneven air gap. It was shown that the proposed estimator yielded speed and angle errors below 2.2% and 0.14 rad, respectively, for the two-sensor configuration. Likewise, for the three-sensor configuration, it led to speed and angle errors below 1.37% and 0.192 rad, respectively, even when facing variable speed profiles and disturbances. Further steps aim at improving the proposed tuning process, providing robustness against temperature variations, and using the estimator in a closed-loop application.

ACKNOWLEDGMENT

The authors give special thanks to the Research Group of Energy and Climate Change of Tecnologico de Monterrey. The authors would also like to thank the Mechatronics Laboratory staff at Politecnico di Torino for their valuable
help in the acquisition of the data presented in this paper.

REFERENCES

[1] R. Bojoi, M. Pastorelli, J. Bottomley, P. Giangrande, and C. Gerada, “Sensorless control of PM motor drives - A technology status review,” in 2013 IEEE Workshop on Electrical Machines Design, Control and Diagnosis (WEMDCD). IEEE, 2013, pp. 168–182.

[2] D. Reigosa, D. Fernandez, C. Gonzalez, S. B. Lee, and F. Briz, “Permanent Magnet Synchronous Machine Drive Control Using Analog Hall-Effect Sensors,” IEEE Transactions on Industry Applications, vol. 54, no. 3, pp. 2358–2369, 2018.

[3] J. Kim, S. Choi, K. Cho, and K. Nam, “Position Estimation Using Linear Hall Sensors for Permanent Magnet Linear Motor Systems,” IEEE Transactions on Industrial Electronics, vol. 63, no. 12, pp. 7644–7652, 2016.

[4] R. Galluzzi, Y. Xu, N. Amati, and A. Tonoli, “Optimized design and characterization of motor-pump unit for energy-regenerative shock absorbers,” Applied Energy, vol. 210, pp. 16–27, 2018.

[5] Y. Zhao, W. Huang, J. Yang, F. Bu, and S. Liu, “A PMSM rotor position estimation with low-cost Hall-effect sensors using improved PLL,” in 2016 IEEE Transportation Electrification Conference and Expo, Asia-Pacific (ITEC Asia-Pacific), Jun. 2016, pp. 804–807.

[6] Z. Wang, K. Wang, J. Zhang, C. Liu, and R. Cao, “Improved rotor position estimation for permanent magnet synchronous machines based on hall-effect sensors,” in 2016 IEEE International Conference on Aircraft Utility Systems (AUS), Oct. 2016, pp. 911–916.

[7] G. Liu, B. Chen, and X. Song, “High-Precision Speed and Position Estimation Based on Hall Vector Frequency Tracking for PMSM With Bipolar Hall-Effect Sensors,” IEEE Sensors Journal, vol. 19, no. 6, pp. 2347–2355, Mar. 2019.

[8] M. Morse, “Linear Hall Effect Sensor Angle Measurement Theory, Implementation and Calibration,” Texas Instruments, Tech. Rep., 2018.

[9] Allegro, “Hall-Effect IC Applications Guide,” Oct. 2019.

[10] Allegro, “Hall-Effect IC Applications Guide,” Oct. 2019.

[11] S.-Y. Jung and K. Nam, “PMSM Control Based on Edge-Field Hall Sensor Signals Through ANF-PLL Processing,” IEEE Transactions on Industrial Electronics, vol. 58, no. 11, pp. 5121–5129, 2011.

[12] M. Wang, Y. Niu, Y. Tang, R. Yang, C. Zhong, and L. Li, “A Hall Sensor-Based Position Estimation Method for PMLSM with an Improved Phase Locked Loop,” in 2019 22nd International Conference on Electrical Machines and Systems (ICEMS), Aug. 2019, pp. 1–5.

[13] G. Escobar, M. F. Martinez-Montejano, A. A. Valdez, P. R. Martinez, and M. Hernandez-Gomez, “Fixed-Reference-Frame Phase-Locked Loop for Grid Synchronization Under Unbalanced Operation,” IEEE Transactions on Industrial Electronics, vol. 58, no. 5, pp. 1943–1951, May 2011.

[14] G. Escobar, S. Pettersson, and C. Ho, “Phase-locked loop for grid synchronization under unbalanced operation and harmonic distortion,” in IECON 2011 - 37th Annual Conference of the IEEE Industrial Electronics Society, Melbourne, Vic, Australia: IEEE, Nov. 2011, pp. 675–680.

[15] R. Galluzzi, S. Circosta, N. Amati, and A. Tonoli, “Rotary regenerative shock absorbers for automotive suspensions,” Mechatronics, vol. 77, p. 102580, 2021.