Dirac Neutrino Mass Matrix and its Link to Freeze-in Dark Matter

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Abstract

Using a mechanism which allows naturally small Dirac neutrino masses and its linkage to a dark gauge $U(1)_D$ symmetry, a realistic Dirac neutrino mass matrix is derived from $S_3$. The dark sector naturally contains a fermion singlet having a small seesaw mass. It is thus a good candidate for freeze-in dark matter from the decay of the $U(1)_D$ Higgs boson.
Introduction: It has been shown recently [1] that naturally small Dirac neutrino masses may be linked to a dark $U(1)_D$ gauge symmetry. One specific model is studied here with the inclusion of an $S_3$ family symmetry, so that a realistic Dirac neutrino mass matrix is obtained. The dark sector consists of four singlet Majorana fermions. Its structure allows one to be the lightest from a seesaw mechanism akin to that used in canonical Majorana neutrino mass. It is thus very suitable as freeze-in dark matter which owes its relic abundance from the decay of the $U(1)_D$ Higgs boson.

The simple mechanism in question was first pointed out in 2001 [2]. Consider two Higgs doublets $\Phi = (\phi^+, \phi^0)$ and $\eta = (\eta^+, \eta^0)$, where $\eta$ is distinguished from the standard-model (SM) $\Phi$ by a symmetry to be decided. Whereas $\Phi$ has the usual $\mu^2 < 0$, the corresponding $m^2$ for $\eta$ is positive and large. The aforesaid symmetry is assumed to be broken by the soft term $\mu' \Phi^\dagger \eta + H.c$. The spontaneous breaking of the $SU(2)_L \times U(1)_Y$ gauge symmetry of the SM then results in the usual vacuum expectation $\langle \phi^0 \rangle = v$, but $\langle \eta^0 \rangle = v'$ is now given by $-\mu'^2 v/m^2$, which is suppressed by the small $\mu'$ and large $m^2$.

For neutrino mass, if $\nu_R$ is chosen to transform in the same way as $\eta$, but not the other SM particles, then it pairs up with $\nu_L$ to form a Dirac fermion with mass proportional to the small $v'$. If the symmetry chosen also forbids $\nu_R$ to have a Majorana mass, then the neutrino is a Dirac fermion with a naturally small mass. This idea of achieving a small $v'/v$ ratio is akin to that of the so-called Type II seesaw, as classified in Ref. [3] and explained in Ref. [4]. It is also easily generalized [5] and applicable to light quarks and charged leptons [6].

In this paper, following Ref. [1] which incorporates an anomaly-free $U(1)_D$ gauge symmetry to distinguish $\nu_R$ from the other SM particles, a specific model of two massive Dirac neutrinos is proposed. With the implementation of an $S_3$ discrete family symmetry, a realistic Dirac neutrino mass matrix is obtained. The natural occurrence of light freeze-in dark matter is also discussed.
Outline of Model: The particle content of the proposed model is listed in Table 1.

| fermion/scalar | $SU(2)_L$ | $U(1)_Y$ | $U(1)_D$ | $S_3$ |
|----------------|-----------|-----------|-----------|-------|
| $(\nu, e), [(\nu_\mu, \mu), (\nu_\tau, \tau)]$ | 2         | $-1/2$    | 0         | $1', 2$ |
| $e^c, [\tau^c, \mu^c]$ | 1         | 1         | 0         | $1', 2$ |
| $\nu^c_{2,3}$ | 1         | 0         | $-4$      | 1, 1'  |
| $\psi_{1,2,3}$ | 1         | 0         | 1         | 1, 1, 1|
| $\zeta$ | 1         | 0         | 5         | 1      |
| $\Phi_{1,2} = (\phi_{1,2}^+, \phi_{1,2}^0)$ | 2         | 1/2       | 0         | 1, 1'  |
| $\eta_{1,2,3,4} = (\eta_{1,2,3,4}^+, \eta_{1,2,3,4}^0)$ | 2         | 1/2       | 4         | 1, 1', 2|
| $\chi_1$ | 1         | 0         | 2         | 1      |
| $\chi_2$ | 1         | 0         | 4         | 1      |
| $\chi_3$ | 1         | 0         | 6         | 1      |

Table 1: Fermion and scalar content of Dirac neutrino model with dark $U(1)_D$ and $S_3$ symmetries.

The $U(1)_D$ gauge symmetry is anomaly-free because

$$1 + 1 + 1 - 4 - 4 + 5 = 0, \quad 1 + 1 + 1 - 64 - 64 + 125 = 0,$$

which is Solution (C) of Ref. [1]. It is based on the observation [7, 8, 9, 10] that $(-1, -1, -1)$ may be replaced by $(5, -4, -4)$ as $B - L$ charges for gauge $B - L$ symmetry.

The Higgs potential consists of six doublets $\Phi_{1,2}, \eta_{1,2,3,4}$ (which are necessary to enforce the forms of charged-lepton and Dirac neutrino mass matrices to be discussed) and three singlets $\chi_{1,2,3}$ (which are necessary for masses of the dark fermions and the link between the $\Phi$ and $\eta$ doublets). Their quadratic terms are such that $\Phi_{1,2}, \chi_{1,2}$ have negative $\mu_{1,2,3,4}^2$, but $\eta_{1,2,3,4}, \chi_3$ have large positive $m_{1,2,3,4,5}^2$. The terms connecting them are

$$f_1 \chi_1^2 \eta_1^\dagger \Phi_1 + f_2 \chi_2^2 \eta_2^\dagger \Phi_2 + f'_1 \chi_3 \chi_1^\dagger \eta_1^\dagger \Phi_1 + f'_2 \chi_3 \chi_1^\dagger \eta_2^\dagger \Phi_2 + \mu'_1 \chi_2 \eta_1^\dagger \Phi_1 + \mu'_2 \chi_2 \eta_2^\dagger \Phi_2$$
$$+ \mu_{12} \chi_2 \eta_1^\dagger \Phi_2 + \mu_{21} \chi_2 \eta_2^\dagger \Phi_1 + \mu_{31} \chi_3 (\eta_3^\dagger + \eta_4^\dagger) \Phi_1 + \mu_{32} \chi_3 (\eta_3^\dagger + \eta_4^\dagger) \Phi_2$$
$$+ f_3 \chi_3 \chi_1^3 + f_4 \chi_3 \chi_2^2 \chi_1^* + \mu'_3 \chi_3 \chi_2 \chi_1 + \mu'_4 \chi_3 \chi_2 \chi_1^2 + H.c.,$$

(2)
where the $\mu_{12}, \mu_{21}, \mu_{31}, \mu_{32}$ terms break $S_3$ softly. Let $\langle \phi_{1,2}^0 \rangle = v_{1,2}$, $\langle \eta_{1,2,3,4}^0 \rangle = v'_{1,2,3,4}$, $\langle \chi_{1,2,3} \rangle = u_{1,2,3}$, then $v_{1,2}, u_{1,2}$ obtain nonzero vacuum expectation values in the usual way at the breaking scales of $SU(2)_L \times U(1)_Y$ and $U(1)_D$ respectively, whereas $v'_{1,2,3,4}, u_3$ are small because of the large positive $m^2_{1,2,3,4,5}$. Assuming that the soft breaking of $S_3$ preserves the interchange symmetry $\eta_3 \leftrightarrow \eta_4$, so that $m_3 = m_4$ and $v'_3 = v'_4$, the results are

$$v'_{1,2} \simeq \frac{-f_{1,2}u_{1}^2v_{1,2} - \mu_{1,2}u_{1}u_{2}v_{1,2} - \mu_{12,21}u_{2}v_{1,2}}{m^2_{1,2}},$$
$$v'_3 = v'_4 = \simeq \frac{-\mu_{31}u_{2}v_{1} - \mu_{32}u_{2}v_{2}}{m^2_{3}}, \quad u_3 \simeq \frac{-f_{3}u_{1}^2 - f_{4}u_{2}u_{1} - \mu'_3u_{2}u_{1}}{m^2_{5}}. \quad (3)$$

**Dirac Neutrino Masses and Mixing**: The $S_3$ representation used is that first proposed in Ref. [11] and fully explained in Ref. [12]. Let $(a_1, a_2)$ and $(b_1, b_2)$ be doublets under $S_3$, then

$$a_1b_2 + a_2b_1 \sim 1, \quad a_1b_2 - a_2b_1 \sim 1', \quad (a_2b_2, a_1b_1) \sim 2. \quad (4)$$

The structure of the charged-lepton mass matrix is determined thus by the Yukawa terms $y_1ee^c\tilde{\phi}_1^0, y_2(\mu\mu^c + \tau\tau^c)\tilde{\phi}_1^0$ and $y_3(-\mu\mu^c + \tau\tau^c)\tilde{\phi}_2^0$, so that the $3 \times 3$ mass matrix linking $(e, \mu, \tau)$ to $(e^c, \mu^c, \tau^c)$ is diagonal with $m_e = y_1v_1$, $m_\mu = y_2v_1 - y_3v_2$, $m_\tau = y_2v_1 + y_3v_2$.

There are only two singlet neutrinos $\nu^c_{2,3}$ which couple to $(\nu_e, \nu_\mu, \nu_\tau)$ through $\eta^0_{1,2,3,4}$. One linear combination of the three neutrinos must then be massless. For calculational convenience, $\nu^c_1$ may be added, so that the $3 \times 3$ Dirac mass matrix is of the form

$$M_\nu = \begin{pmatrix} 0 & a & b \\ 0 & c & -d \\ 0 & c & d \end{pmatrix}, \quad (5)$$

where the $(12)$ entry comes from $v'_1$, the $(13)$ entry comes from $v'_2$, the $(22)$ and $(32)$ entries are the same because they come from $(\mu \eta^0_1 + \tau \eta^0_2)\nu^c_2$, whereas the $(23)$ and $(33)$ entries come from $(-\mu \eta^0_1 + \tau \eta^0_2)\nu^c_2$.

The neutrino mixing matrix is then obtained by diagonalizing

$$M_\nu M_\nu^\dagger = \begin{pmatrix} |a|^2 + |b|^2 & ac^* - bd^* & ac^* + bd^* \\ a^*c - b^*d & |c|^2 + |d|^2 & |c|^2 - |d|^2 \\ a^*c + b^*d & |c|^2 - |d|^2 & |c|^2 + |d|^2 \end{pmatrix}. \quad (6)$$
Assuming that $|a|^2|b|^2 < (2|d|^2 + |b|^2)(2|c|^2 + |a|^2)$, the eigenvalues are

$$m_{\nu_1} = 0, \quad m_{\nu_2}^2 = 2|c|^2 + |a|^2, \quad m_{\nu_3}^2 = 2|d|^2 + |b|^2.$$  \tag{7}

Let $b/d = i\sqrt{2}s_{13}/c_{13}$, then

$$\nu_3 = is_{13}\nu_e - \frac{1}{\sqrt{2}}c_{13}\nu_\mu + \frac{1}{\sqrt{2}}c_{13}\nu_\tau.  \tag{8}$$

Let $a/c = \sqrt{2}c_{13}s_{12}/s_{12}$, then

$$\nu_2 = s_{12}c_{13}\nu_e + \frac{1}{\sqrt{2}}(c_{12} - is_{12}s_{13})\nu_\mu + \frac{1}{\sqrt{2}}(c_{12} + is_{12}s_{13})\nu_\tau.  \tag{9}$$

With these choices, the massless eigenstate is automatically

$$\nu_1 = c_{12}c_{13}\nu_e + \frac{1}{\sqrt{2}}(-s_{12} - ic_{12}s_{13})\nu_\mu + \frac{1}{\sqrt{2}}(-s_{12} + ic_{12}s_{13})\nu_\tau.  \tag{10}$$

In other words, a completely realistic neutrino mixing scenario dubbed cobimaximal \cite{13} with $\theta_{23} = \pi/4$ and Dirac CP phase $\delta = -\pi/2$ is possible with Eq. (5). Numerically, using the most recent world averages \cite{14}

$$m_{32}^2 = 2.453 \times 10^{-3} \text{ eV}^2, \quad m_{21}^2 = 7.53 \times 10^{-5} \text{ eV}^2, \quad s_{13}^2 = 0.0218, \quad s_{12}^2 = 0.307,  \tag{11}$$

the values

$$d = 0.035 \text{ eV}, \quad c = 0.0051 \text{ eV}, \quad b/d = 0.21i, \quad a/c = 0.93  \tag{12}$$

are obtained.

**Deviation from Cobimaximal Mixing** : Using $\nu'_3 = \nu'_4$, the correlation of $\delta = -\pi/2$ to $\theta_{23} = \pi/4$ has been obtained with Eq. (5). Since the most recent data \cite{14} favors $\theta_{23} > \pi/4$, a modification of Eq. (5) is studied to see how $\delta$ changes numerically with $\theta_{23}$. Let

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & a & b \\ 0 & (1 + \epsilon)c & -(1 + \epsilon)d \\ 0 & (1 - \epsilon)c & (1 - \epsilon)d \end{pmatrix},  \tag{13}$$
then
\[
M_\nu^\dagger M_\nu = \begin{pmatrix}
|a|^2 + |b|^2 & (1 + \epsilon)(ac^* - bd^*) & (1 - \epsilon)(ac^* + bd^*) \\
(1 + \epsilon)(a^*c - b^*d) & (1 + \epsilon^2)(|c|^2 + |d|^2) & (1 - \epsilon^2)(|c|^2 - |d|^2) \\
(1 - \epsilon)(a^*c + b^*d) & (1 - \epsilon^2)(|c|^2 - |d|^2) & (1 - \epsilon^2)(|c|^2 + |d|^2)
\end{pmatrix}.
\] (14)

To first order in \(\epsilon\), the mass eigenvalues are the same, with the following changes in the mixing parameters:
\[
s_{23} = -\frac{1}{\sqrt{2}}(1 + \epsilon), \quad c_{23} = \frac{1}{\sqrt{2}}(1 - \epsilon), \quad e^{-i\delta} = ie^{i\bar{\theta}},
\] (15)
\[
b/d = i\sqrt{2}s_{13}e^{i\bar{\theta}}, \quad c/a = \frac{c_{12} - 2ie s_{12}s_{13}}{\sqrt{2}s_{12}c_{13}}, \quad \theta' = \frac{2\epsilon|c|^2 s_{12}c_{13}^2}{|d|^2 c_{12}s_{13}}.
\] (16)

Using the world average [14] \(s_{23}^2 = 0.545\), the deviation from cobimaximal mixing is then
\[
\epsilon = 0.044, \quad \theta' = 8.24 \times 10^{-3}.
\] (17)

**Dark Sector**: The dark sector consists of four fermion singlets \(\psi_{1,2,3} \sim 1\) and \(\zeta \sim 5\). Because of the chosen scalars \(\chi_1 \sim 2, \chi_2 \sim 4\) and \(\chi_3 \sim 6\), there is no connection to the two singlet neutrinos \(\nu_{2,3} \sim -4\). Hence \(\psi_{1,2,3}, \zeta\) may be considered odd under an induced \(Z_2\) symmetry which stabilizes the lightest among them as dark matter. The \(4 \times 4\) Majorana mass matrix spanning \((\zeta, \psi_{1,2,3})\) is of the form
\[
M_{\zeta,\psi} = \begin{pmatrix}
0 & h'_1u_3 & h'_2u_3 & h'_3u_3 \\
h'_1u_3 & h_1u_1 & 0 & 0 \\
h'_2u_3 & 0 & h_2u_1 & 0 \\
h'_3u_3 & 0 & 0 & h_3u_1
\end{pmatrix}.
\] (18)

Recalling that \(u_3 << u_1\) from Eq. (3), it is clear that \(\zeta\) gets a very small mass, i.e.
\[
m_\zeta = -\frac{h'^2_1u_3^2}{h_1u_1} - \frac{h'^2_2u_3^2}{h_2u_1} - \frac{h'^2_3u_3^2}{h_3u_1}.
\] (19)

Since \(\chi_3\) has large and positive \(m_3^2\) so that \(u_3\) is very small, the breaking of \(U(1)_D\) is mainly through \(\chi_{1,2}\). The relevant part of the Higgs potential is then
\[
V = -\mu_3^2\chi_1^*\chi_1 - \mu_4^2\chi_2^*\chi_2 + [\mu_4'\chi_2^*\chi_1^2 + H.c.] \\
+ \frac{1}{2}\lambda_1(\chi_1^*\chi_1)^2 + \frac{1}{2}\lambda_2(\chi_2^*\chi_2)^2 + \lambda_3(\chi_1^*\chi_1)(\chi_2^*\chi_2).
\] (20)
Let $H_{1,2} = \sqrt{2}Re(\chi_{1,2})$, then the $2 \times 2$ mass-squared matrix spanning $H_{1,2}$ is
\[
\mathcal{M}_H^2 = \begin{pmatrix}
2\lambda_1 u_1^2 & 2\lambda_3 u_1 u_2 + 2\mu'_4 u_1 \\
2\lambda_3 u_1 u_2 + 2\mu'_4 u_1 & 2\lambda_2 u_2^2 - \mu'_4 u_1^2 / u_2
\end{pmatrix}.
\] (21)

Let $H_2$ be the lighter, with mixing $\theta$ to $H_1$. Now $H_2$ does not couple to $\psi\psi$, but $H_1$ does and through $\psi - \zeta$ mixing to $\zeta\zeta$ with Yukawa coupling $y_H = m_\zeta / 2\sqrt{2}u_1$. From Eqs. (3) and (19), it is clear that $m_\zeta << u_1$, hence $y_H$ is very much suppressed.

Consider now the very light Majorana fermion $\zeta$ as dark matter. It has gauge interactions, but if the reheat temperature of the Universe is much below the mass of the $U(1)_D$ gauge boson as well as $m_{H_1}$, then it interacts only very feebly through $H_2$. Its production mechanism in the early Universe is freeze-in [15] by $H_2$ decay before the latter decouples from the thermal bath. The decay rate of $H_2 \rightarrow \zeta\zeta$ is
\[
\Gamma_{H_2} = \frac{y_H^2 \theta^2 m_{H_2}}{8\pi} \sqrt{1 - 4r^2} (1 - 2r^2),
\] (22)
where $r = m_\zeta / m_{H_2}$. For $r << 1$, the correct relic abundance is obtained for [16]
\[
y_H \theta \sim 10^{-12} r^{-1/2}.
\] (23)

This translates to
\[
\frac{m_\zeta \theta}{(u_1^2 m_{H_2})^{1/3}} \sim 2 \times 10^{-8},
\] (24)
which may be satisfied for example with $u_1 = 10^7$ GeV, $m_{H_2} = 600$ GeV, $\theta = 0.1$, and $m_\zeta = 80$ MeV. The decoupling temperature for $\zeta$ is roughly $T \sim 1 \text{ MeV}(m_{Z_D}/m_Z)^{4/3} = 5.2$ TeV. This analysis follows that of Ref. [17], where the SM Higgs decay to a light seesaw dark fermion from the decomposition of $SU(10) \rightarrow SU(5) \times U(1)_\chi$.

**Concluding Remarks**: The right-handed neutrino $\nu_R$ has been proposed as the link [1] to dark matter by having it transform under a new $U(1)_D$ gauge symmetry. The small Dirac neutrino masses are enforced by a seesaw mechanism proposed in Ref. [2] using new Higgs doublets also transforming under $U(1)_D$. With the help of the non-Abelian discrete
symmetry $S_3$, it is shown how a realistic neutrino mixing matrix may be obtained which is approximately cobimaximal \cite{13}.

After the spontaneous breaking of $U(1)_D$, a dark parity remains for four Majorana fermions, the lightest of which has a seesaw mass. It is suitable as freeze-in dark matter with its relic abundance coming from the decay of the $U(1)_D$ Higgs boson.

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