Systematic Study of Elliptic Flow at RHIC Energy

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We study the elliptic flow systematically from SIS to RHIC energies in a realistic dynamical cascade model. We compile our results with the recent data from STAR and PHOBOS experiments on elliptic flow of charged particles in Au + Au collisions at RHIC energy. From the analysis of elliptic flow as a function of different dynamical variables such as transverse momenta, pseudorapidity and centrality at RHIC energy, we found a good fitting with data at 1.5 times a scaling factor to our simulation model, which characterizes that the model is required to have extra pressure generated from the subsequent parton scatterings. In energy dependence of elliptic flow, we observe a re-hardening nature at RHIC energies, which may probably signal the possible formation of quark-gluon plasma.

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I. INTRODUCTION

The prime aim of ultra-relativistic heavy-ion collisions is to understand the nature of quantum chromo dynamics under extreme conditions of density and temperature. In such extreme conditions, it is expected that nuclear matter undergo a phase transition to quark-gluon plasma (QGP). At present it is of great interest to study the nature of this plasma and understand the phase transition between hadron and QGP phases. For that reason, various high energy heavy-ion collision experiments have been carried out at SIS, AGS, SPS and RHIC energies, and will start at LHC around 2007. Very recently, the new qualitative data have been reported from RHIC energy experiments at BNL. These are glimpse of the wealth of physics to be extracted from four experiments BRAHMS, PHENIX, PHOBOS and STAR at RHIC accelerator.

At RHIC energy, soon after the collisions of heavy nuclei, a huge number of particles are produced and move collectively. The collective motion and the behavior of these particles are called as flow. Recently these flow data have been reported at AGS and SPS besides RHIC energy. At AGS energies, the sideward and elliptic flow are well described by dynamical microscopic simulation models $^1$ in non-central Au+Au collisions. Also, the elliptic flow in non-central and strong radial flow in central Pb+Pb collisions are observed at the SPS $^2$. For non-central collisions, the initial nucleus-nucleus overlap has an almond or elliptic shape. This initial almond shaped overlap region expands and becomes more spherical, quenching the driving force that produces the elliptic flow. The pressure gradient and its anisotropy are much larger in
the initial stage, hence the elliptic flow give more precise information of the initial thermalization and equation of state. At RHIC where deconfined phase is expected to emerge, the elliptic flow would be more sensitive to the parton re-scatterings and thermalization degree in the initial stage than to the later hadronic equation of state. So, the information about the formation of QGP can be drawn from the measure of the final flows, e.g. radial and elliptic flow of the produced particles.

For radial flows, there is a strong evidence that the hadron transverse mass spectra get much stiffer than SPS energy [3]. The stiffness can be realized as follows. According to the large level density of hadrons, the hadronic matter is expected to be softer. However, this phenomena can not persist at such high energies, since hadrons are dissolved into quark and gluons in vacuum. Therefore in that level, pressure grows rapidly and becomes stiffer as a function of energy density. In other words, we say re-hardening of transverse mass spectra at RHIC energies [3,4] is due the probable formation of QGP. This point may be quite premature, because the radial flow is not a direct observable but a quantity extracted through theoretical model analyses, and there may be some other mechanism such as $p_t$ broadning.

For non-central collisions, the overlap geometry between two nuclei is lens or almond shaped. As the initial lens-shape expands, it produces the elliptic flow. The elliptic flow is the anisotropic emission of particles in- and out-of reaction plane defined by the beam and the impact parameter directions. Thus the momentum anisotropy can be translated from the spatial anisotropy in the presence of strong re-scattering and elliptic flow is sensitive to number of interaction. One can measure this by measuring the second Fourier coefficient in the azimuthal distribution of particles with respect to reaction plane and is usually characterized by the particle momenta distribution [5].

$$v_2 = \frac{\langle (p_x^2 - p_y^2)/(p_x^2 + p_y^2) \rangle}{(1)}.$$

Also, the elliptic flow is influenced by the formation of QGP in non-central collisions with function of beam energies, since it depends on the early stages of the system evolution. Then the question arises if the QGP is formed, does it live longer at SPS or at RHIC? Recently, it has been estimated experimentally [3] that the hard QGP phase is expected to live longer at RHIC than at the SPS. If this is true, then the elliptic flow of the produced particles should indicate this difference at the end. Therefore, it is urgently required to estimate the incident energy dependence of the elliptic flow more systematically in order to derive the new physics. Experimentally, it has been found that at higher energies, e.g., at AGS and above, the coefficient $v_2 > 0$, the "in-plan" flow. This fact has been verified and well described by the dynamical transport model with mean field up to AGS energies [4]. In any case, whether the transport model has mean field or not, the elliptic flow $v_2$ is positive at higher energies. Recently, the elliptic flow has been predicted to increase with beam energies by RQMD [5] as well as hydrodynamic models [6].

In this paper, we concentrate on the systematic study of elliptic flow,
because of two reasons, (i) we have lots of quality data on elliptic flow from RHIC experiments, and (ii) it is more fundamental to understand observables which are sensitive to the scatterings among produced particles in the initial stage. Therefore, we make an analysis from SIS to RHIC energies, and a detail discussion at RHIC energy as functions of centrality, pseudorapidity and transverse momentum.

II. MODEL

In this work, we analyze the elliptic flow systematically from SIS to RHIC energies using a dynamical hadron-string cascade simulation model, JAM [9]. In this model, the initial primary collisions produce mini-jet partons by using the eikonal approximation as in the HIJING model [10], which later enter into string configurations. Then strings fragment to hadrons using the LUND fragmentation model constructed in the PYTHIA [11] routine. In JAM, partonic interactions between different mini-jets are not included. However, at present very few models (no dynamical models) exist in the literature which hold such a complicated treatments as partonic, string, and hadron multiple interactions at RHIC energy. Thus it is worthwhile to consider the present model, JAM, for the systematics of elliptic flow.

III. RESULTS

Figure 1 displays the calculated results of $v_2$ at mid-rapidity as functions of beam energy in JAM in comparison with experimental data. For completeness, the result of hydrodynamic models [12] are displayed in the figure. It is evident from the figure that the hydrodynamic model for protons is very well on top of the STAR data [13] at RHIC energy, where JAM under-predicts by a factor of 1/2 of data. Whereas the situation is reverse at SPS, where JAM gives a reasonable description of data and hydrodynamic model fails to describe the data, which overestimates the data by a factor of more than 2. Since hydrodynamic models assume complete local thermalization and QGP formation in the initial condition, the agreement of hydrodynamics results with data may be suggesting that the thermalized QGP is already formed in RHIC energy at mid-rapidities. In JAM, although it fails to explain mid-rapidity STAR data at RHIC energy, it gives reasonable values up to SPS energies, where hadronic matter is expected to dominate. This shows that JAM lacks partonic interactions between mini-jets which play essential roles in early thermalization, while hadron and string interactions are well implemented in this model.

In Fig. 2, we display JAM results on elliptic flow of charged particles as functions of transverse momenta for minimum bias events at RHIC energy. In this figure, we notice that our model gives a good qualitative description of data. However, the overall magnitudes are underpredicted by a factor of
1.2-1.5 for pions and charged particles. If we multiply by a factor of 1.5 times the charged particle results, represented as thin dashed line in Fig. 2, the data and model results are in an excellent agreement till $p_t \sim 2$ GeV/c.

Another interesting point to be noticed here is that the calculated elliptic flow is sensitive to the particle masses as a function of $p_t$. The particles which are having smaller masses have higher values of elliptic flow at small $p_t$, such as pions and kaons and these are linear functions of $p_t$. For higher particles masses such as protons, $v_2$ behaves non-linearly with $p_t$. Similar characteristics are observed in the STAR data [14] as well.

In contrast, hydrodynamic models show excellent agreement upto $p_t \sim 1.5$ GeV and in central and semi-central collisions [8], which is not shown in this figure. It fails at high $p_t$, due to saturation and onset of hard processes and fails at peripheral collisions due to incomplete early-time thermalization.

Figure 3 shows the centrality ($N_{ch}/N_{max}$) dependence of the elliptic flow. The data are from STAR [5] and PHOBOS [15] experiments. At central region, our model fits much better (solid line) to data and fall off away at peripheral region. Even at peripheral region, the experimental data are having large error bars, especially those from PHOBOS experiment. Again consistently, if we multiply a factor of 1.5 to our calculation (dashed line), we could describe data well in a wide range of impact parameters within the error bars. In comparison to hydrodynamic model at peripheral region, the hydrodynamic prediction overestimates the elliptic flow data in the most peripheral region, which is not shown in the figure. In over all, a large degree of thermalization are favored in central collisions (in the early stages) and fails at peripheral collisions.

Finally, Fig. 4 displays the calculated minimum bias elliptic flow of charged particles as a function of pseudorapidity in comparison with data from PHOBOS collaboration [15] at RHIC energy ($\sqrt{s_{nn}} = 130$ GeV). In this figure we observe that the calculated results agree with the data very well in the fragmentation region ($|\eta| > 2$) labeled as "Cascade" in the figure. If we multiply a factor of 1.5 to our results (dashed line) as in previous figures, we can describe the PHOBOS data at mid-rapidities. On the other hand, a full 3D hydrodynamical model explains the strong elliptic flow at mid-rapidities, while it gives very flat elliptic flows and consequently overestimates the data at large rapidities for all reasonable initial conditions [16]. These findings implies that well thermalized matter is produced at mid-rapidities, where hydrodynamical evolution from QGP initial condition would be justified, and that hadron-string gas still dominates at large pseudorapidity region, where a jet-implemented hadron-string cascade model works well.

**IV. SUMMARY AND DISCUSSION**

In this work, we have made systematic analyses of the elliptic flow at beam energies ranging from SIS to RHIC, and we have also discussed its dependence on transverse momentum, pseudorapidity and centrality at RHIC energy.
From the systematic analysis of elliptic flow at mid-rapidities with beam energy, which is more fundamental about the collision dynamics with dynamical simulation models, we have learned that the elliptic flow shows re-hardening behavior at RHIC energy in non-central collisions. The similar feature was observed from the analysis of radial flow in the central collisions. A jet-implemented hadron-string cascade model, JAM, reasonably describes the elliptic flows up to SPS energies, but it underestimates at RHIC by a factor around two at the centrality where $v_2$ becomes maximum. Transverse momentum dependence also shows that hadron-string cascade does not give strong enough elliptic flows and underestimates the magnitudes around 1.2-1.5 in minimum bias events, although it describes the overall trend very well. Since the elliptic flow is sensitive to the thermalization in the early stage, the above observations indicate that it is necessary to include additional processes which are effective in early thermalization than hadron-string cascade processes.

In the analysis of centrality dependence, we find that the underestimates of $v_2$ in JAM mainly comes from semi-central collisions ($0.1 < N_{ch}/N_{max} < 0.5$). It may be possible to interpret this underestimates as the lack of cooperative processes in JAM. In semi-central collisions, number density of produced particles are smaller than in central collisions, then it is generally more difficult to achieve thermalization of the system. But in the case of cooperative processes such as the phase transition from superheated liquid to gas, the transition proceeds catastrophically from a small seed region.

Finally, in the analysis of pseudorapidity dependence, it is found that the elliptic flows in fragmentation region ($|\eta| > 2$) are well described in a hadron-string cascade, while hydrodynamical description works at mid-pseudorapidities ($|\eta| < 2$). It suggests that the early thermalization is achieved at mid-rapidities where there are a huge number of particles produced by mini-jets.

In summary, we find that a hadron-string cascade gives reasonable descriptions of elliptic flows up to SPS energies and fragmentation region at RHIC energy, although it underestimates the elliptic flow at mid-rapidities especially in semi-central collisions. The most natural explanation of this underestimates is to assume that partons produced in mini-jets interact frequently in the early stage at mid-rapidities and thermalized QGP is formed through these re-scatterings, which is not included in the present model. Actually, this is supported by the full 3D hydrodynamical model calculation, which assumes complete local thermalization and reproduces the data at mid-rapidities. In order to confirm QGP formation theoretically, it is desirable to incorporate partonic interactions in a dynamical model such as JAM. In this regard, some steps has been taken in the literature, but still it is not complete.

On the other hand, it would be also necessary to make more sophisticated analyses in extracting elliptic flows from experimental data. The elliptic flow data were measured using conventional method of correlating particles with an event plane. In this method the observed elliptic coefficient is not
correct, if it is not corrected by dividing by the resolution of the event plane, since the observed event plane is not the true reaction plane. The flow coefficient can be determined without referring to an event plane by the multiparticle correlation method [19] using cumulants. The four-particle correlation method is more advantageous than two-particle method, due to elimination of two-particle non-flow effects. However in four-particle correlation method, the natural statistical errors are larger than the two-particle analysis because of fourth root of the result. We would like to mention here that in Figs. 2 and 3, the data may come further down due to elimination of non-flow effects, measurements of elliptic flow by using four-particle correlations method [19]. In that case, our prediction of 1.5 times the original result of our model is more encouraging and a positive forward step.

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FIG. 1. Elliptic flow as a function of energy for non-central collisions

FIG. 2. Elliptic flow as a function of transverse momenta for minimum bias events for charged particles
FIG. 3. Elliptic flow as a function of centrality for minimum bias events for charged particles

FIG. 4. Elliptic flow as a function of pseudorapidity for minimum bias events for charged particles