Parametrization for the Scale Dependent Growth in Modified Gravity

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We propose a scale dependent analytic approximation to the exact linear growth of density perturbations in Scalar-Tensor (ST) cosmologies. In particular, we show that on large subhorizon scales, in the Newtonian gauge, the usual scale independent subhorizon growth equation does not describe the growth of perturbations accurately, as a result of scale-dependent relativistic corrections to the Poisson equation. A comparison with exact linear numerical analysis indicates that our approximation is a significant improvement over the standard subhorizon scale independent result on large subhorizon scales. A comparison with the corresponding results in the Synchronous gauge demonstrates the validity and consistency of our analysis.

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I. INTRODUCTION

Cosmological data from a wide range of sources including type Ia supernovae [1, 2], the cosmic microwave background [3], baryon acoustic oscillations [4, 5], cluster gas fractions [6, 7], and gamma ray bursts [8, 9] seem to indicate that at least 70% of the energy density in the universe is in the form of an exotic, negative-pressure component, called dark energy. While the standard ΛCDM framework is the minimal model that successfully accounts for observations [4], there remain numerous viable alternatives that also pass current experimental tests. These alternative models can be broadly categorized as quintessence [10, 11] or modified gravity models [12, 13, 14, 15], and both categories can, with sufficient tuning, replicate the expansion history of the universe in consistence with observations. (See [16, 17] for recent reviews). In order to distinguish between these two categories of models, it is therefore important to look beyond the expansion rate.

The growth of structure offers a hope of breaking this degeneracy since different growth histories can arise from models which have similar expansion histories. To examine the growth of structure one examines the evolution of the linear matter density contrast \( \delta \equiv \delta \rho / \rho \) which is given in terms of the background density \( \rho \) and the perturbation \( \delta \rho \). For scales much smaller than the horizon, \( \delta \) satisfies a simple equation called the growth equation, which is scale-independent. Note that in what follows we use the usual definition of 'scale' as either the distance \( \lambda_p \) in physical FRW coordinates or the corresponding wavenumber \( k = \frac{2\pi}{\lambda_p} \). There are many investigations which attempt to characterize the evolution of \( \delta \) through the use of a growth parameterization which assumes a different value depending on the cosmological model used, thereby allowing for models to be distinguished (e.g. [14, 34, 35]). The standard definition of the growth parameter, \( \gamma \), in terms of the growth function \( f \), the matter density \( \Omega_m \) and the scale factor \( a \) is given as

\[
f(a) \equiv \frac{d \ln \delta}{d \ln a} = \Omega_m(a) \gamma.
\] (1)

Once this parameter is determined (for earlier theoretical developments on the parameterization of the growth parameter and experimental constraints on \( \gamma \), see [34, 36–47]) one may then be in a position to determine whether the standard general relativistic (GR) framework of ΛCDM is responsible for the acceleration of the universe, or some other, more exotic process is at work.

However, in [48, 49], it was demonstrated that in the Newtonian gauge (for another interesting look at gauge issues see [50]), the above parameterization can become inaccurate for large subhorizon scales \( \gtrsim 100 h^{-1}\text{Mpc} \). In other words, if the physical growth of structure is correctly described by the Newtonian gauge, then it would show up as inconsistent with scale-independent parameterization in Eq. (1) and (mistakenly) appear to be caused by exotic physics. The reason for the discrepancy was shown to be the scale dependence of the growth of \( \delta \), which becomes important for large subhorizon scales (\( \gtrsim 100 h^{-1}\text{Mpc} \)). An improved version of the growth equation was derived in [48] which incorporates the scale-dependence. In [49], a new scale-dependent parametrization of the growth function \( f \) was proposed, which was shown to account for the evolution of \( f(a) \) on these large scales with considerably greater accuracy than Eq. (1).

In the present work, we focus on the growth of perturbations in scalar-tensor (ST) theories of gravity. It
is well known that in the sub-Hubble approximation the growth of perturbations in these theories is also described by an equation similar to the growth equation in GR, up to a redefinition of the gravitational constant. Working in the Newtonian gauge, we show that the usual growth equation approximation for $\delta$ becomes unreliable on large scales, for the same reason as in the GR case, i.e. the effects of scale dependence. We derive an improved version of the growth equation relevant for these models and propose a more accurate scale-dependent parameterization for growth.

The layout of our paper is as follows. In Section II we discuss the growth of perturbations in ST theories, demonstrating the failure of the usual growth equation approximation and introducing an improved growth equation and a new parameterization for the growth function in these models. In Section III we compare these approximations to exact solutions to demonstrate their accuracy. Our conclusions can be found in Section IV.

II. GROWTH OF MATTER PERTURBATIONS IN SCALAR-TENSOR GRAVITY BEYOND SUBHORIZON SCALES

ST theories are widely studied as an alternative to GR. These theories are well-motivated from string theory, Randall-Sundrum models, as well as extended and hyperextended inflationary models. (See e.g. and references therein for a review of ST theories.) The deviations from GR predicted by these theories have been investigated (see e.g. ). ST theories have also been used to explain the accelerating Universe, and the cosmological consequences of these models have been widely studied (see e.g. ).

In this work, we focus on the growth of matter perturbations in these theories. As shown in , if one works in the Newtonian gauge and considers scales much smaller than the horizon ($k \gg aH$) then the overdensity $\delta$ obeys an equation very similar to the familiar

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\text{eff}}(t)\rho_m \delta = 0, \quad (2)$$

where dots denote derivatives with respect to cosmic time and $G_{\text{eff}}(t)$ is an effective gravitational constant whose evolution is determined by the scalar field dynamics (see also equation below). We reconsider the growth of perturbations in these models, manifestly retaining the scale-dependent effects, and derive an improved version of the growth equation which models the evolution of $\delta$ with a greater accuracy than the scale-independent equation. Using our improved growth equation, we then propose a new scale-dependent parameterization for growth in these models which is applicable under the assumption of our approximations which involve slow evolution of the scalar field.

A. Scalar-tensor cosmology

We start with the general action of a Universe described by ST gravity (in the Jordan frame) and arbitrary matter fields:

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-g} [F(\Phi) R - Z(\Phi) g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - 2 U(\Phi)] + S_m[\psi_m; g_{\mu\nu}] \quad (3)$$

where $g_{\mu\nu}$ is the metric with determinant $g$ and Ricci scalar $R$. $G_*$ is the bare gravitational coupling constant (henceforth we will set $8\pi G_* = 1$). The scalar field $\Phi$ has a potential $U(\Phi)$ and couples to gravity through the functions $F(\Phi)$ and $Z(\Phi)$. $S_m$ denotes the action of matter fields $\psi_m$. Henceforth we work with the parametrization where $Z(\Phi) = 1$ and $F(\Phi)$ is arbitrary.

We next consider a spatially flat Friedman-Robertson-Walker (FRW) Universe with a background metric (in the Jordan frame):

$$ds^2 = -dt^2 + a^2(t) [dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)] \quad (4)$$

We take the matter content of the Universe to be a perfect fluid with energy-momentum tensor:

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \delta S_m = (p + \rho) u_\mu u_\nu + p g_{\mu\nu}, \quad (5)$$

where $p$, $\rho$ and $u^\mu$ are the pressure, energy density and four-velocity of the matter fluid respectively. For convenience, we will henceforth consider the matter to be pressureless and set $p = 0$.

Finally, we work at linear order in perturbation theory in the Newtonian gauge. The metric is perturbed as follows:

$$ds^2 = -(1 + 2\phi) dt^2 + a^2(t) (1 - 2\psi) dx^2 \quad (6)$$

where $\phi$ and $\psi$ are the spatial and temporal perturbations and $x \equiv (r, \theta, \varphi)$. The matter energy density is perturbed as $\rho \to \rho + \delta \rho$, where $\rho$ is the background density and $\delta \rho$ is the perturbation. For convenience, we define the overdensity $\delta \equiv \delta \rho / \rho$. The perturbations in the velocity field $\delta u_\mu$ are conveniently expressed through the velocity potential $v$ (defined such that $\delta u_\mu = -\partial_\mu v$). The scalar field is perturbed as $\Phi \to \Phi + \delta \Phi$.

The evolution of the background variables is governed by the zero'th order Einstein equations and the equations of conservation of energy-momentum:

$$3FH^2 = \rho_m + \frac{1}{2} \dot{\Phi}^2 - 3H \dot{\Phi} + U \quad (7)$$

$$-2F\dot{H} = \rho_m + \dot{\Phi}^2 + \dot{\Phi} - H \dot{\Phi} \quad (8)$$

$$\dot{\Phi} + 3H \dot{\Phi} = 3F'(\dot{H} + 2H^2) - U' \quad (9)$$

$$\dot{\rho}_m + 3H \rho_m = 0 \quad (10)$$

where primes denote derivatives with respect to the scalar field $\Phi$ and dots denote derivatives with respect to the coordinate time.
The evolution of the perturbations is governed by the first order Einstein and conservation equations. We work in Fourier space assuming a spatial dependence of \( \exp(ik \cdot x) \). The scalar field fluctuation is given by

\[
\delta \Phi + 3H \delta \dot{\Phi} + \left[ \frac{k^2}{a^2} - 3 \left( \dot{H} + 2H^2 \right) F'' + U'' \right] \delta \Phi = \left[ \frac{k^2}{a^2} (\phi - 2\psi) - 3 \left( \ddot{\psi} + 4H\dot{\psi} + H\dot{\phi} \right) \right] F' \\
+ \left( 3\dot{\psi} + \dot{\phi} \right) \dot{\Phi} - 2\phi U'.
\]

The matter density perturbation and velocity potential evolve as

\[
\dot{\delta} = - \frac{k^2}{a^2} \nu + 3\dot{\psi}
\]

\[
\dot{\nu} = \phi
\]

The evolution of the metric perturbations is given by

\[
\psi = \phi \frac{F' \delta \Phi}{F}
\]

\[
2F \left( \dot{\psi} + H\dot{\phi} \right) + \dot{F} \phi = \rho \nu + \phi \delta \Phi + \delta \dot{F} - HF'\delta \Phi
\]

\[
-3\dot{F} \phi = \left( 2\frac{k^2}{a^2} F - \Phi^2 + 3HF' \right) \phi = \rho (\delta + 3H\nu) + U' \delta \Phi + \left( \frac{k^2}{a^2} - 6H^2 - 3\frac{\dot{F}^2}{F^2} \right) \delta F + \phi \delta \Phi
\]

\[
+ 3H \dot{\Phi} \delta \Phi + 3\frac{\dot{F}}{F} \delta \dot{F}
\]

The precise accuracy level however depends on the degree of validity of the above approximation which may vary depending on the details of the field dynamics. Defining

\[
\xi(a, k) \equiv 3a^2 H(a)^2 / k^2 \simeq \frac{3H_0^2 \Omega_{m0}}{a k^2}
\]

(18)

(where the last equation is valid in the matter era) Eq. (11) reduces to:

\[
\frac{\delta \Phi}{\Phi} = \frac{\xi F' F'}{F^2 + 2F'^2 + \xi (FU''/3H^2 - F''U/2H^2 - F'F''/2)}
\]

(19)

Plugging into Eq. (19) we obtain

\[
\frac{k^2}{a^2} \phi = - \frac{1}{2F} \rho (\delta + 3H\nu) g(F, U, \xi),
\]

(20)

where
Using the above, (and eliminating $v$ from Eq. (15) ignoring time derivatives), we obtain the following form for the Poisson equation:

$$
\frac{k^2}{a^2} \phi = -\frac{1}{2F} \rho \frac{g(F,U,\xi)}{1 + \xi g(F,U,\xi) h(F,U,\xi)},
$$

where

$$
h(F,U,\xi) \equiv \frac{2F + 3F'^2 + \xi (2FU''/3H^2 - F'U'/H^2 - F''U/H^2)}{2F + 4F'^2 + \xi (2FU''/3H^2 - 2F'U'/3H^2 - F''U/H^2 - F'F'/H^2)}.\]

This leads to our “improved” growth equation for the evolution of perturbations, accurate for large subhorizon scales:

$$
\dot{\delta} + 2H \delta = \frac{1}{2F} \left[ \frac{g(F,U,\xi)}{1 + \xi g(F,U,\xi) h(F,U,\xi)} \right] \rho \delta = 0. \tag{22}
$$

In the case of general relativity this reduces to the form derived in Ref. [48, 49]

$$
\dot{\delta} + 2H \delta - \frac{4\pi G \rho_m \delta}{1 + \xi(a,k)} = 0 \tag{23}
$$

Note that for scales much smaller than the horizon ($k \gg aH$, or equivalently $\xi \to 0$) equation (22) reduces to the well-known form (see e.g. Eqn. (5.13) of [18]):

$$
\ddot{\delta} + 2H \dot{\delta} - \frac{1}{2F} \left[ \frac{2F + 4F'^2}{2F + 3F'^2} \right] \rho \delta = 0. \tag{24}
$$

Equation (23) may be expressed in terms of the growth factor $f = \frac{d \ln \delta}{d \ln a}$ in the form

$$
f' + f^2 + \left( 2 - \frac{3}{2} \Omega_m(a) \right) f = \frac{3}{2} \frac{\Omega_m(a)}{1 + \xi(a,k)} \tag{25}
$$

where $' \equiv \frac{d}{d \ln a}$, and we have assumed $\Lambda$CDM for $H(a)$.

For sub-Hubble scales $\xi(k,a) \to 0$ and the solution of equation (24) is well approximated by [11] with $\gamma = \frac{6}{17}$.

For larger scales a perturbative approach may be used to derive the scale dependent growth rate $f(a,k)$ of matter perturbations in GR as:

$$
f(a,k) = \Omega_m(a)^\gamma \left( 1 + \frac{3H_0^2\Omega_mK}{ak^2} \right)^{-1}, \tag{26}
$$

with $K = 0$ for the scale-independent growth function and $K = 1$ for the GR scale-dependent approximation.

Following the same reasoning as in Ref. [49], the following parametrization may be derived for the growth function $f(a,k)$ in ST cosmologies:

$$
f(a,k) = \Omega_m(a)^\gamma \frac{1}{F} \left[ \frac{g(F,U,\xi)}{1 + \xi g(F,U,\xi) h(F,U,\xi)} \right]. \tag{27}
$$

Even though this parametrization may be derived as an approximate solution by a perturbative expansion to all orders in $\xi$ as in Ref. [49], its validity for $\gamma \approx 0.55$ (the exact value is obtained by fitting to the exact numerical solution) will also be tested in the next section. The scale dependence of $\gamma$ has also been recently discussed in [83, 84].

### III. COMPARISON TO EXACT RESULTS

We now compare the evolution predicted by Eq. (22) and Eq. (27) to results from the exact evolution.

To solve the system numerically, we choose the following functional forms [85]. We take

$$
F(\Phi) = 1 - \lambda \Phi^2; \quad Z(\Phi) = 1; \quad U(\Phi) = 1 + \exp[-\lambda \Phi] \tag{28}
$$

Then we solve numerically the background equations given by Eq. (17), Eq. (18) and Eq. (17). These are solved in the Newtonian gauge using the background. The initial time is taken to be in the matter dominated epoch, at $z_i = 1000$, and we set $\Phi_i \simeq 0$.

In our numeric solution the initial value $\Phi_i$ is chosen so that at early times the deviation in the expansion rate from the $\Lambda$CDM one is not larger than around 5%. This deviation is roughly given by $\frac{H_{\text{ST}} - H_{\text{CDM}}}{H_{\text{CDM}}} \sim F(\Phi_i)^{-1/2} - 1$, which then sets an upper bound $\Phi_{i,\text{max}}$ for
FIG. 1: For three choices of scale, plot of the exact \( \delta_m(k, a) \) (blue solid line), along with two approximations: (i) the solution of the scale-dependent ST growth equation (solid line) and (ii) the solution of the scale-independent ST sub-Hubble growth equation (dot-dashed line). The apparent scale dependence of the present time value of \( \delta_m \) in the subhorizon approximation is due to the scale dependent initial conditions considered needed to secure a scale independent value of the initial metric perturbations \( \Phi \). Although \( \Phi_i/\Phi_{i,\text{max}} \sim 1 \) results in an expansion rate consistent with the \( \Lambda \)CDM one, we find that such a choice gives rise to deviations between the analytic approach in Eq. (22) and the numeric solution for \( \delta_m \). This is because larger \( \Phi \) implies larger effective potential energy of the field, and hence larger \( \dot{\Phi} \) when the field oscillates due to its coupling to curvature and therefore to matter. Above a certain threshold, \( \dot{\Phi} \) grows large enough so that one cannot keep neglecting it in order to arrive to Eq. (22). In what follows we consider \( \Phi_i/\Phi_{i,\text{max}} \lesssim 0.30 \). For this initial condition we secure relatively small deviation from GR and from the \( \Lambda \)CDM expansion rate (consistent with observations) and small field time derivative (consistent with our approximation). We thus find that the solution to Eq. (22) deviates from the numeric solution for \( \delta_m \) by less than 1% on subhorizon scales.

We first compare the exact numerical solution of the ST perturbation system [13], [14] (potentials given by

FIG. 2: Plots of the % difference between the exact numerical solution for \( \delta_m \) (blue line) and the scale-dependent ST approximation (solid line). The % difference corresponding to the subhorizon ST approximation is not included since this is much bigger than the corresponding to the scale-dependent approximations. The values used for the plot are \( \lambda_f = 5 \) and \( \lambda = 10 \).
Assuming matter domination (\(p = 0\)) and dropping the index \((N)\) for the Newtonian gauge we find
\[
\delta_m = \frac{\delta \rho}{\rho + p} + 3Hv \tag{29}
\]
where \(v\) is the velocity potential. This quantity may be evaluated in the synchronous and in the Newtonian gauges leading to
\[
\delta_m = (\frac{\delta \rho}{\rho + p})^{(S)} = (\frac{\delta \rho}{\rho + p})^{(N)} + (3Hv)^{(N)} \tag{30}
\]
where the superscripts \((S)\) and \((N)\) imply evaluation in the Synchronous and in the Newtonian gauge respectively. Assuming matter domination (\(p = 0\)) and dropping the index \((N)\) for the Newtonian gauge we find
\[
\delta = \delta_m + 3Hv \tag{31}
\]
The above calculations have been made in the conformal Newtonian gauge. In this section we briefly discuss the gauge dependence of the density perturbation \(\delta\). Let us consider the gauge invariant quantity

\[
\delta_m = \frac{\delta \rho}{\rho + p} + 3Hv \tag{29}
\]
which is scale independent. The corresponding equation in the Newtonian gauge is eq. (32). When properly related initial conditions are used in the solutions of eqs. (26) and (32), their solutions should satisfy eq. (31). This proves the validity of our analysis and demonstrates that the difference of $3H\psi$ between the matter density perturbations in the two gauges can be significant. It is demonstrated in Fig. 5 where we consider a large scale $(k = 0.001 h Mpc^{-1})$ and show the evolution of $\delta(a)$ (Newtonian gauge, blue continuous line), $\delta^{(S)}(a)$ (black dot-dashed line) and $\delta(a) + 3H\psi$ (red dashed line which coincides with the line of $\delta^{(S)}(a)$ as anticipated).

V. CONCLUSIONS

We have derived and tested numerically a simplified ordinary differential equation whose solution describes fairly accurately the growth of linear cosmological perturbations in ST cosmologies up to Hubble scales (beyond $10^{4}h^{-1}Mpc$). A corresponding analytic form of the scale dependent growth rate $f(a,k)$ was also presented and tested numerically in comparison with the exact linear
result. This is a significant improvement over the previously known sub-Hubble which breaks down on scales larger than about $10^2 h^{-1} \text{Mpc}$.

We have thus demonstrated that in the context of the Newtonian gauge, the comparison of cosmological large scale structure data with corresponding theoretical predictions in ST based cosmologies on scales larger than a few hundred $\text{Mpc}$ should not be based on the sub-Hubble scale independent approximation. Instead it requires either use of the full linear numerical solution of the cosmological perturbation equations in each theory or the imposed scale dependent approximation presented in the present study. Thus, previous work using the scale independent approximation to make predictions for the growth of perturbations in ST theories is reliable on scales up about $200 h^{-1} \text{Mpc}$. However, such results should be used with care on larger scales. For example, on scales of a few hundred $\text{Mpc}$ the error induced in the growth rate by not using the scale dependent effects is about $10\%$ on redshifts $z > 3$ while on scales larger than $1 G\text{pc}$ the corresponding error exceeds $50\%$ even at low redshifts $z \approx 1$.

We have also shown that our results are consistent with a corresponding calculation in the synchronous gauge and verified that the quantity $\delta\dot{\psi}$ is gauge dependent and there can be a significant difference between its forms in different gauges on large scales.

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Appendix A: Growth in synchronous and conformal Newtonian gauges

In this appendix we discuss the appropriate (scale-dependent) initial conditions that must be used in the Newtonian and synchronous gauges in order to verify Eq. (31), i.e., equations (34)-(37).

Starting with Eq. (10), and using the initial conditions

\[
\delta \Phi_i = \delta \phi_i = \dot{\psi}_i = 0, \quad \psi_i \simeq -10^{-5} \neq 0 \quad (A1)
\]

(\text{a subscript } i \text{ will always refer to the value of a quantity at the initial time}) we obtain the following relation for $\delta_i$

\[
\delta_i = \frac{1}{\rho_{mi}} \left( \frac{2}{a^2} F_i - \dot{\Phi}_i^2 + 3 H_i \dot{\Phi}_i \right) \psi_i + 3 H_i \psi_i \rho_{mi} \quad (A2)
\]

Using Eq. (A3) with the above initial conditions (A1) we arrive at the relation

\[
3 H_i \psi_i = \frac{1}{\rho_{mi}} \left( 6 H_i^2 F_i + 3 H_i \dot{F}_i \dot{\phi}_i \right) \psi_i \quad (A3)
\]

Next combining Eqs. (A2) and (A3), and ignoring time derivatives, we can then recover the initial condition for $\delta_i$

\[
\delta_i \simeq - \left( \frac{2}{a^2} + 6 H_i^2 \right) \frac{F_i \psi_i}{\rho_{mi}} \quad (A4)
\]

From Eq. (A3), using Eqs. (12), (13), and (14), as well as the initial conditions (A1), we find

\[
\dot{\delta}_i = \frac{k^2}{a^2} \frac{1}{\rho_{mi}} \left( 2 H_i F_i + F_i \dot{\phi}_i \right) \psi_i \quad (A5)
\]

Also from Eqs. (7) and (8), ignoring time derivatives, we have the approximate relationship

\[
\dot{H}_i \simeq \frac{1}{2 F_i} \rho_{mi} \simeq - \frac{3}{2} H_i^2 \quad (A6)
\]

Lastly, ignoring time derivatives in Eqs. (A4) and (A5), and using Eq. (A6), we obtain equations (34) and (35).

For the synchronous gauge, we start with (Eq. (31)), use equations (A2) and (A6), and ignore time derivatives to obtain Eq. (35). Then from Eqs. (12), using equations (A1), (A3), (A5) and (A6), and ignoring time derivatives, we obtain Eq. (37).

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