Spin-1 charmonium-like states in QCD sum rule

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Abstract. We study the possible spin-1 charmonium-like states by using QCD sum rule approach. We calculate the two-point correlation functions for all the local form tetraquark interpolating currents with $J^{PC} = 1^{+-}, 1^{-+}$ and $1^{++}$ and extract the masses of the tetraquark charmonium-like states. The mass of the $1^{+-} q\bar{c}q\bar{c}$ state is 4.6 $\sim$ 4.7 GeV, which implies a possible tetraquark interpretation for $Y(4660)$ meson. The masses for both the $1^{+-} q\bar{c}q\bar{c}$ and $s\bar{c}\bar{s}c$ states are 4.0 $\sim$ 4.2 GeV, which is slightly above the mass of $X(3872)$. For the $1^{++}$ and $1^{+-} q\bar{c}q\bar{c}$ states, the extracted masses are 4.5 $\sim$ 4.7 GeV and 4.0 $\sim$ 4.2 GeV respectively.

1 Introduction

The underlying structures of the so-called $X, Y, Z$ states are not understood well \cite{1,2,3}. Some of them do not fit in the conventional quark model easily and are considered as the candidates of the exotic states beyond the quark model, such as the molecular states, tetraquark states, the charmonium hybrid mesons, baryonium states and so on. $X(3872)$ is the best studied charmonium-like state since its discovery by the Belle Collaboration \cite{4}. Although the analysis of angular distributions favors the assignment $J^{PC} = 1^{++}$ \cite{5}, the $2^{++}$ possibility is not ruled out \cite{6}. The mass and decay mode of $X(3872)$ are very different from that of the $2^{3}P_{1} c\bar{c}$ state. Up to now, the possible interpretations of $X(3872)$ include the molecular state \cite{7,8,9,10}, tetraquark state \cite{11,12}, cusp \cite{13} and hybrid charmonium \cite{14}. $Y(4260), Y(4360)$ and $Y(4660)$ are the $Y(J^{PC} = 1^{--})$ family states discovered in the initial state radiation (ISR) process. These new states lie above the open charm threshold. However, the $Y \rightarrow D^{(*)}\bar{D}^{(*)}$ decay modes have not been observed yet \cite{15}, which are predicted to be the dominant decay modes of the charmonium above the open charm threshold in the potential model. In Refs. \cite{16,17}, the authors studied the $1^{--}$ charmonium-like $Y$ mesons using the QCD sum rule approach. Maiani et al. tried to assign $Y(4260)$ as the $s\bar{c}\bar{s}c$ tetraquark in a $P$-wave state \cite{18}. $Y(4260)$ was also interpreted as the interesting charmonium hybrid state \cite{19,20}. $Y(4660)$ was considered as a $\psi(2S)J_{0}(980)$ bound state in Ref. \cite{21}. Recently there have been some efforts on the $1^{+-}$ charmonium-like exotic states. For example, the structure of $X(4350)$ was studied using a $D^{*}_{s}\bar{D}^{*}_{s}$ current with $J^{PC} = 1^{--}$ \cite{22}. Moreover, the newly observed state $Y(4140)$ was argued as a $1^{+-}$ exotic charmonium hybrid state \cite{23}.

In this contribution, we would like to report the systematic study of the tetraquark charmonium-like states with $J^{PC} = 1^{--}, 1^{-+}, 1^{++}$ and $1^{+-}$. After constructing all the tetraquark interpolating currents with definite quantum numbers, we investigate the two-point correlation functions and extract the masses of the charmonium-like states with QCD sum rule. We study both the $qQ\bar{q}\bar{Q}$ and $sQ\bar{s}\bar{Q}$ systems where $Q = c, b$. We also discuss the possible decay modes and experimental search of the charmonium-like states.

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2 INDEPENDENT CURRENTS

Firstly, we construct all the local form diquark-antidiquark type of interpolating currents in a systematic way. By considering the Lorentz structures, the charge conjugation properties and the color symmetries of the tetraquark operators, we derive the following tetraquark interpolating currents with $J^{PC} = 1^{+-}, 1^{--}, 1^{++}$ and $1^{+-}$:

- The interpolating currents with $J^{PC} = 1^{+-}$ and $1^{--}$ are:
  \[ J_{1^{+-}} = q_d^c C \gamma_5 s_b (\bar{q}_a \gamma_\mu s_b C \gamma^\nu \bar{q}_b + \bar{q}_b \gamma_\mu s_b C \gamma^\nu \bar{q}_a), \]
  \[ J_{1^{--}} = q_d^c C \gamma_\mu \gamma_\nu s_b (\bar{q}_a \gamma_\mu s_b C \gamma^\nu \bar{q}_b + \bar{q}_b \gamma_\mu s_b C \gamma^\nu \bar{q}_a), \]
  \[ J_{2^{+-}} = q_d^c C \gamma_\mu \gamma_\nu s_b (\bar{q}_a \gamma_\mu s_b C \gamma^\nu \bar{q}_b + \bar{q}_b \gamma_\mu s_b C \gamma^\nu \bar{q}_a), \]
  \[ J_{2^{--}} = q_d^c C \gamma_\mu \gamma_\nu s_b (\bar{q}_a \gamma_\mu s_b C \gamma^\nu \bar{q}_b + \bar{q}_b \gamma_\mu s_b C \gamma^\nu \bar{q}_a), \]
  \[ J_{3^{+-}} = q_d^c C \gamma_\mu \gamma_\nu s_b (\bar{q}_a \gamma_\mu s_b C \gamma^\nu \bar{q}_b + \bar{q}_b \gamma_\mu s_b C \gamma^\nu \bar{q}_a), \]
  \[ J_{3^{--}} = q_d^c C \gamma_\mu \gamma_\nu s_b (\bar{q}_a \gamma_\mu s_b C \gamma^\nu \bar{q}_b + \bar{q}_b \gamma_\mu s_b C \gamma^\nu \bar{q}_a), \]

where $"++"$ corresponds to $J^{PC} = 1^{++}$, "$-\-"$ corresponds to $J^{PC} = 1^{--}$.

- The interpolating currents with $J^{PC} = 1^{++}$ and $1^{-+}$ are:
  \[ J_{1^{++}} = q_d^c C \gamma_\mu \gamma_\nu s_b (\bar{q}_a \gamma_\mu s_b C \gamma^\nu \bar{q}_b + \bar{q}_b \gamma_\mu s_b C \gamma^\nu \bar{q}_a), \]
  \[ J_{1^{-+}} = q_d^c C \gamma_\mu \gamma_\nu s_b (\bar{q}_a \gamma_\mu s_b C \gamma^\nu \bar{q}_b + \bar{q}_b \gamma_\mu s_b C \gamma^\nu \bar{q}_a), \]
  \[ J_{2^{++}} = q_d^c C \gamma_\mu \gamma_\nu s_b (\bar{q}_a \gamma_\mu s_b C \gamma^\nu \bar{q}_b + \bar{q}_b \gamma_\mu s_b C \gamma^\nu \bar{q}_a), \]
  \[ J_{2^{-+}} = q_d^c C \gamma_\mu \gamma_\nu s_b (\bar{q}_a \gamma_\mu s_b C \gamma^\nu \bar{q}_b + \bar{q}_b \gamma_\mu s_b C \gamma^\nu \bar{q}_a), \]
  \[ J_{3^{++}} = q_d^c C \gamma_\mu \gamma_\nu s_b (\bar{q}_a \gamma_\mu s_b C \gamma^\nu \bar{q}_b + \bar{q}_b \gamma_\mu s_b C \gamma^\nu \bar{q}_a), \]
  \[ J_{3^{-+}} = q_d^c C \gamma_\mu \gamma_\nu s_b (\bar{q}_a \gamma_\mu s_b C \gamma^\nu \bar{q}_b + \bar{q}_b \gamma_\mu s_b C \gamma^\nu \bar{q}_a), \]

where $"++"$ corresponds to $J^{PC} = 1^{++}$, "$-\-"$ corresponds to $J^{PC} = 1^{--}$.

The subscripts $a$ and $b$ are the color indices, $q$ denotes $u$ or $d$ quark. It is understood that all the currents in Eqs. (1) - (2) should contain $(u|c \bar{u} + d|c \bar{d})$ in order to have definite isospin and $G$-parity. The details about the current construction could be found in Ref. [24].

3 SPECTRAL DENSITY

In the past several decades, QCD sum rule has been widely used to study the hadron structures and proven to be a very powerful non-perturbative method [25,26]. We consider the two-point correlation function:

\[ \Pi_{\mu\nu}(q^2) = \left. \frac{i \int d^4 x e^{i q x} \langle 0 | J_{\mu}(x) J_{\nu}^\dagger(0) | 0 \rangle}{\langle T | J_{\mu}(0) J_{\nu}^\dagger(0) | 0 \rangle} \right|_{T} = -\Pi_1(q^2) (q_\mu q_\nu - \frac{q_\mu q_\nu}{q^2}) + \Pi_0(q^2) \frac{q_\mu q_\nu}{q^2}, \]

where $J_{\mu}$ is an interpolating current for the tetraquark states. $\Pi_1(q^2)$ is related to the vector meson while $\Pi_0(q^2)$ is the scalar current polarization function. The correlation function $\Pi_{\mu\nu}(q^2)$ can be calculated.
in the operator product expansion (OPE) using perturbative QCD augmented with non-perturbative quark and gluon condensates to describe the large distance physics. At the hadron level, the correlation function is expressed by the dispersion relation with a spectral function:

\[
\Pi(q^2) = \int_{4m_c^2}^{\infty} \frac{\rho(s)}{s - q^2 - i\epsilon} ds,
\]

(4)

In approximation of the infinitely narrow widths of resonances, the spectral function can be expressed as:

\[
\rho(s) \equiv \sum_n \delta(s - m_n^2)(0|\eta n)(n|\eta^*|0) = f_\pi^2\delta(s - m_\pi^2) + \ldots,
\]

(5)

where "..." represents the higher states contribution.

The theoretical basis of the QCD sum rule approach is the assumption of the quark-hadron duality, which ensures the equivalence of the correlation functions obtained at the hadron level and the quark-gluon level. After performing the Borel transformation to the correlation functions, we can extract the mass of the state \(X\):

\[
m_X^2 = \frac{\int_{4m_c^2}^{s_0} dse^{-s/M_B^2} \rho(s)}{\int_{4m_c^2}^{\infty} dse^{-s/M_B^2} \rho(s)},
\]

(6)

where \(s_0\) is the continuum threshold and \(M_B\) is the Borel parameter. We performed the QCD sum rule analysis for all the tetraquark currents in Eqs. (1)-(4). The results of OPE can be found in Ref. [24].

4 QCD Sum Rule Analysis

We use the following parameter values of the quark masses and various condensates [27,28,29] for the numerical analysis: \(m_q(2\text{GeV}) = (4.0 \pm 0.7)\text{MeV}, m_s(2\text{GeV}) = (101^{+29}_{-21})\text{MeV}, m_c(2\text{GeV}) = (1.23 \pm 0.09)\text{GeV}, m_b(m_b) = (4.20 \pm 0.07)\text{GeV}, \langle \bar{q}q \rangle = -0.23 \pm 0.03\text{GeV}^3, \langle \bar{q}q, \sigma Gq \rangle = -M_B^2\langle \bar{q}q \rangle, M_B^2 = (0.8 \pm 0.2)\text{GeV}^2, \langle \bar{s}s \rangle / \langle \bar{q}q \rangle = 0.8 \pm 0.2, \langle q_f^2Gq_f \rangle = 0.88\text{GeV}^2\). There are two important parameters in QCD sum rule analysis: the threshold parameter \(s_0\) and the Borel mass \(M_B\). The stability of QCD sum rule requires a suitable working region of \(s_0\) and \(M_B\). Since the exponential weight function in Eq. (6) the higher state contribution is naturally suppressed for small value of \(M_B\). However, the OPE convergence would become worse if \(M_B\) was too small. In our analysis, we choose the value of \(s_0\) around which the variation of the extracted mass \(m_X\) with \(M_B^2\) is minimum. The working region of the Borel mass is determined by the convergence of the OPE series and the pole contribution. The requirement of the convergence of the OPE series leads to the lower bound \(M_B^2\) of the Borel parameter while the constraint of the pole contribution yields the upper bound of \(M_B^2\).

4.1 Vector charmonium-like systems

For the interpolating currents with \(J^{PC} = 1^{-+}\) and \(1^{--}\), we keep the \(m_q\) and \(m_s\) related terms in the spectral densities. These terms give important corrections to the OPE series and are useful to enhance the stability of the sum rule. For the \(q\bar{c}q\bar{c}\) systems, the absolute value of the four quark condensate \(\langle \bar{q}q \rangle^2\) is bigger than other condensates in the region of \(M_B^2 < 3.1\text{GeV}^2\). It is the dominant power contribution to the correlation function in this region. Especially for the currents with \(J^{PC} = 1^{--}\), the quark condensate \(\langle \bar{q}q \rangle\) is proportional to the light quark mass \(m_q\) and vanishes if we take \(m_q = 0\). However, it is proportional to the strange quark mass \(m_s\) and larger than \(\langle \bar{s}s \rangle^2\) for the \(sc\bar{c}\bar{s}\) system since \(m_s \gg m_q\). This is the main difference between the \(q\bar{c}q\bar{c}\) and \(sc\bar{c}\bar{s}\) systems. The similar situation exits in \(1^{-+}\) charmonium-like systems.
After careful study of the OPE convergence and the pole contribution, we find the suitable working region of the Borel parameter for each vector charmonium-like current. The threshold value of $s_0$ is also fixed around which the variation of the extracted mass $m_X$ with $M_B^2$ is minimum. In Fig. 1 and Fig. 2, we show the variation of $m_X$ with the threshold value $s_0$ and Borel parameter $M_B^2$ for the current $J_{1\mu}$ with $J^{PC} = 1^{-+}$ in $qc\bar{q}c$ and $sc\bar{c}c$ systems, respectively. One notes that they are very similar with each other except the chosen $s_0$ mentioned above. The extracted mass of the $qc\bar{q}c$ state is 4.64 GeV, which is consistent with the mass of the meson $Y(4660)$. One may wonder whether $Y(4660)$ could be a tetraquark state. The extracted mass of the $sc\bar{c}c$ state is 4.92 GeV, which is about 0.28 GeV higher than that of the $qc\bar{q}c$ state.

![Fig. 1. The variation of $m_X$ with $s_0(a)$ and $M_B^2(b)$ corresponding to the current $J_{1\mu}$ for the $1^{-+}$ $qc\bar{q}c$ system.](image1)

![Fig. 2. The variation of $m_X$ with $s_0(a)$ and $M_B^2(b)$ corresponding to the current $J_{1\mu}$ for the $1^{-+}$ $sc\bar{c}c$ system.](image2)

Performing the QCD sum rule analysis, we show the Borel window, the threshold value, the extracted mass and the pole contribution corresponding to the tetraquark currents with $J^{PC} = 1^{-+}$ in Table 1. The results of the $1^{-+}$ system are listed in Table 2. We only present the numerical results for the currents which lead to the stable mass sum rules in the working range of the Borel parameter. For example, only the currents $J_{1\mu}$, $J_{2\mu}$, and $J_{3\mu}$ with $J^{PC} = 1^{-+}$ in $qc\bar{q}c$ systems have the reliable mass sum rules. For $J_{2\mu}$, $J_{3\mu}$, $J_{5\mu}$, and $J_{6\mu}$, the stability is so bad that the extracted mass $m_X$ grows monotonically with the threshold value $s_0$ and the Borel parameter $M_B$. These currents may couple to the $1^{-+}$ states very weakly, leading to the above unstable mass sum rules. We also study the bottomonium-like analogues by replacing $m_c$ with $m_b$ in the correlation functions and repeating the same analysis procedures done above. The numerical results of the $qb\bar{q}b$ and $sb\bar{s}b$ systems are collected in Table 1 and Table 2.
In this channel, the QCD sum rule analysis shows that the quark condensate $\langle \bar{q}q \rangle$ is the dominant correction to the correlation function for all the currents. Although the OPE convergence becomes worse than that in the vector channel, the currents $J_{5\mu}, J_{7\mu}, J_{3\mu}, J_{4\mu}$ have better OPE convergence than that of $J_{1\mu}, J_{2\mu}, J_{3\mu}, J_{4\mu}$. For the interpolating currents $J_{3\mu}$ and $J_{4\mu}$ with $J^{PC} = 1^{-+}$, we obtain

### Table 1. The threshold value, Borel window, mass and pole contribution corresponding to the currents with $J^{PC} = 1^{-+}$ in the $qc\bar{q}c$, $sc\bar{s}c$, $qb\bar{q}b$ and $sb\bar{s}b$ systems.

| Currents | $s_0$(GeV$^2$) | $[M_{\text{min}}, M_{\text{max}}]$(GeV$^2$) | $m_X$(GeV) | PC(%) |
|----------|----------------|---------------------------------|------------|-------|
| $qc\bar{q}c$ system | $J_{1\mu}$ | 5.0$^2$ | $2.9 \sim 3.6$ | $4.64 \pm 0.09$ | 44.1 |
| | $J_{3\mu}$ | 5.0$^2$ | $2.9 \sim 3.6$ | $4.61 \pm 0.10$ | 46.4 |
| | $J_{7\mu}$ | 5.2$^2$ | $2.9 \sim 4.1$ | $4.74 \pm 0.10$ | 47.3 |
| | $J_{1\mu}$ | 5.4$^2$ | $2.8 \sim 4.5$ | $4.92 \pm 0.10$ | 50.3 |
| | $J_{2\mu}$ | 5.0$^2$ | $2.8 \sim 3.5$ | $4.64 \pm 0.09$ | 48.6 |
| $sc\bar{s}c$ system | $J_{3\mu}$ | 4.9$^2$ | $2.8 \sim 3.4$ | $4.52 \pm 0.10$ | 45.6 |
| | $J_{4\mu}$ | 5.4$^2$ | $2.8 \sim 4.5$ | $4.88 \pm 0.10$ | 51.7 |
| | $J_{5\mu}$ | 5.3$^2$ | $2.8 \sim 4.3$ | $4.86 \pm 0.10$ | 46.0 |
| | $J_{6\mu}$ | 4.8$^2$ | $2.8 \sim 3.1$ | $4.48 \pm 0.10$ | 43.2 |
| $qb\bar{q}b$ system | $J_{7\mu}$ | 11.0$^2$ | $7.2 \sim 8.5$ | $10.51 \pm 0.10$ | 45.8 |
| | $J_{1\mu}$ | 11.0$^2$ | $7.2 \sim 8.3$ | $10.60 \pm 0.10$ | 47.0 |
| | $J_{2\mu}$ | 11.0$^2$ | $7.2 \sim 8.4$ | $10.55 \pm 0.11$ | 43.6 |
| $sb\bar{s}b$ system | $J_{3\mu}$ | 11.0$^2$ | $7.2 \sim 8.4$ | $10.56 \pm 0.10$ | 43.7 |
| | $J_{4\mu}$ | 11.0$^2$ | $7.2 \sim 8.4$ | $10.53 \pm 0.11$ | 44.3 |
| | $J_{5\mu}$ | 11.0$^2$ | $7.2 \sim 8.2$ | $10.62 \pm 0.10$ | 42.0 |
| | $J_{6\mu}$ | 11.0$^2$ | $7.2 \sim 8.4$ | $10.53 \pm 0.10$ | 44.1 |

### Table 2. The threshold value, Borel window, mass and pole contribution corresponding to the currents with $J^{PC} = 1^{-+}$ in the $qc\bar{q}c$, $sc\bar{s}c$, $qb\bar{q}b$ and $sb\bar{s}b$ systems.

| Currents | $s_0$(GeV$^2$) | $[M_{\text{min}}, M_{\text{max}}]$(GeV$^2$) | $m_X$(GeV) | PC(%) |
|----------|----------------|---------------------------------|------------|-------|
| $qc\bar{q}c$ system | $J_{1\mu}$ | 5.1$^2$ | $2.9 \sim 3.9$ | $4.67 \pm 0.10$ | 50.2 |
| | $J_{1\mu}$ | 5.2$^2$ | $2.9 \sim 4.2$ | $4.77 \pm 0.10$ | 47.4 |
| | $J_{7\mu}$ | 4.9$^2$ | $2.9 \sim 3.4$ | $4.53 \pm 0.10$ | 46.3 |
| | $J_{1\mu}$ | 5.0$^2$ | $2.9 \sim 3.4$ | $4.67 \pm 0.10$ | 44.3 |
| | $J_{2\mu}$ | 5.0$^2$ | $2.9 \sim 3.4$ | $4.65 \pm 0.09$ | 45.6 |
| $sc\bar{s}c$ system | $J_{3\mu}$ | 5.1$^2$ | $2.9 \sim 3.7$ | $4.72 \pm 0.09$ | 44.8 |
| | $J_{4\mu}$ | 5.3$^2$ | $2.9 \sim 4.3$ | $4.84 \pm 0.10$ | 47.3 |
| | $J_{5\mu}$ | 5.3$^2$ | $2.9 \sim 4.3$ | $4.87 \pm 0.10$ | 46.2 |
| | $J_{6\mu}$ | 5.2$^2$ | $2.9 \sim 4.1$ | $4.77 \pm 0.10$ | 44.1 |
| $qb\bar{q}b$ system | $J_{7\mu}$ | 11.0$^2$ | $7.2 \sim 8.6$ | $10.53 \pm 0.11$ | 44.2 |
| | $J_{1\mu}$ | 11.0$^2$ | $7.2 \sim 8.6$ | $10.53 \pm 0.10$ | 44.1 |
| | $J_{2\mu}$ | 11.0$^2$ | $7.2 \sim 8.6$ | $10.49 \pm 0.11$ | 44.7 |
| | $J_{3\mu}$ | 11.0$^2$ | $7.2 \sim 8.1$ | $10.62 \pm 0.10$ | 41.2 |
| $sb\bar{s}b$ system | $J_{4\mu}$ | 11.0$^2$ | $7.2 \sim 8.4$ | $10.56 \pm 0.10$ | 43.8 |
| | $J_{5\mu}$ | 11.0$^2$ | $7.2 \sim 8.3$ | $10.63 \pm 0.10$ | 42.4 |
| | $J_{6\mu}$ | 11.0$^2$ | $7.2 \sim 8.3$ | $10.62 \pm 0.09$ | 42.5 |
| | $J_{7\mu}$ | 11.0$^2$ | $7.2 \sim 8.3$ | $10.59 \pm 0.10$ | 43.1 |

### 4.2 Axial-vector charmonium-like systems

In this channel, the QCD sum rule analysis shows that the quark condensate $\langle \bar{q}q \rangle$ is the dominant correction to the correlation function for all the currents. Although the OPE convergence becomes worse than that in the vector channel, the currents $J_{5\mu}, J_{7\mu}, J_{3\mu}, J_{4\mu}$ have better OPE convergence than that of $J_{1\mu}, J_{2\mu}, J_{3\mu}, J_{4\mu}$. For the interpolating currents $J_{3\mu}$ and $J_{4\mu}$ with $J^{PC} = 1^{-+}$, we obtain the...
working region of the Borel parameter in $3.0 \leq M_{\text{B}}^2 \leq 3.4 \text{ GeV}^2$ for both the $qc\bar{c}q$ and $sc\bar{s}c$ systems. The extracted mass is about $m_X = 4.0 \sim 4.2 \text{ GeV}$, which is slightly above the mass of $X(3872)$. The $qb\bar{q}b$ and $sb\bar{s}b$ systems can be studied conveniently by replacement of the parameters, including the quark masses and the various condensates. The numerical results are listed in Table 3 for the $1^{++}$ systems and Table 4 for the $1^{--}$ systems.

| Currents     | $s_0(\text{GeV}^2)$ | $[M_{\text{min}}^2, M_{\text{max}}^2](\text{GeV}^2)$ | $m_X(\text{GeV})$ | PC(%) |
|--------------|--------------------|--------------------------------------------------|------------------|-------|
| $qc\bar{c}q$ system | $J_{3\mu}$ | 4.6$^2$ | $3.0 \sim 3.4$ | 4.19 ± 0.10 | 47.3   |
|               | $J_{3\nu}$ | 4.5$^2$ | $3.0 \sim 3.3$ | 4.03 ± 0.11 | 46.8   |
| $sc\bar{s}c$ system | $J_{3\mu}$ | 4.6$^2$ | $3.0 \sim 3.4$ | 4.22 ± 0.10 | 45.7   |
|               | $J_{3\nu}$ | 4.5$^2$ | $3.0 \sim 3.3$ | 4.07 ± 0.10 | 44.4   |
| $qb\bar{q}b$ system | $J_{3\mu}$ | 10.9$^2$ | $8.5 \sim 9.5$ | 10.32 ± 0.09 | 47.0   |
|               | $J_{3\nu}$ | 10.8$^2$ | $8.5 \sim 9.2$ | 10.22 ± 0.11 | 44.6   |
|               | $J_{3\mu}$ | 10.7$^2$ | $7.8 \sim 8.4$ | 10.14 ± 0.10 | 44.8   |
|               | $J_{3\nu}$ | 10.7$^2$ | $7.8 \sim 8.4$ | 10.14 ± 0.09 | 44.8   |
| $sb\bar{s}b$ system | $J_{3\mu}$ | 10.9$^2$ | $8.5 \sim 9.5$ | 10.34 ± 0.09 | 46.1   |
|               | $J_{3\nu}$ | 10.8$^2$ | $8.5 \sim 9.1$ | 10.25 ± 0.10 | 43.3   |
|               | $J_{3\mu}$ | 10.8$^2$ | $7.5 \sim 8.6$ | 10.24 ± 0.11 | 47.1   |
|               | $J_{3\nu}$ | 10.8$^2$ | $7.5 \sim 8.6$ | 10.24 ± 0.10 | 47.1   |

**Table 3.** The threshold value, Borel window, mass and pole contribution corresponding to the currents with $J^{PC} = 1^{++}$ in the $qc\bar{c}q$, $sc\bar{s}c$, $qb\bar{q}b$ and $sb\bar{s}b$ systems.

| Currents     | $s_0(\text{GeV}^2)$ | $[M_{\text{min}}^2, M_{\text{max}}^2](\text{GeV}^2)$ | $m_X(\text{GeV})$ | PC(%) |
|--------------|--------------------|--------------------------------------------------|------------------|-------|
| $qc\bar{c}q$ system | $J_{3\mu}$ | 4.6$^2$ | $3.0 \sim 3.4$ | 4.16 ± 0.10 | 46.2   |
|               | $J_{3\nu}$ | 4.5$^2$ | $3.0 \sim 3.3$ | 4.02 ± 0.09 | 44.6   |
| $sc\bar{s}c$ system | $J_{3\mu}$ | 4.6$^2$ | $3.0 \sim 3.4$ | 4.00 ± 0.11 | 46.0   |
|               | $J_{3\nu}$ | 4.5$^2$ | $3.0 \sim 3.4$ | 4.14 ± 0.09 | 47.0   |
| $qb\bar{q}b$ system | $J_{3\mu}$ | 4.7$^2$ | $3.0 \sim 3.6$ | 4.24 ± 0.10 | 49.6   |
|               | $J_{3\nu}$ | 4.6$^2$ | $3.0 \sim 3.5$ | 4.12 ± 0.11 | 47.3   |
|               | $J_{3\mu}$ | 4.5$^2$ | $3.0 \sim 3.3$ | 4.03 ± 0.11 | 44.2   |
|               | $J_{3\nu}$ | 4.6$^2$ | $3.0 \sim 3.4$ | 4.16 ± 0.11 | 46.0   |
| $sb\bar{s}b$ system | $J_{3\mu}$ | 10.6$^2$ | $7.5 \sim 8.5$ | 10.08 ± 0.10 | 45.9   |
|               | $J_{3\nu}$ | 10.6$^2$ | $7.5 \sim 8.5$ | 10.07 ± 0.10 | 46.2   |
|               | $J_{3\mu}$ | 10.6$^2$ | $7.5 \sim 8.4$ | 10.05 ± 0.10 | 45.3   |
|               | $J_{3\nu}$ | 10.7$^2$ | $7.5 \sim 8.7$ | 10.15 ± 0.10 | 47.6   |

**Table 4.** The threshold value, Borel window, mass and pole contribution corresponding to the currents with $J^{PC} = 1^{--}$ in the $qc\bar{c}q$, $sc\bar{s}c$, $qb\bar{q}b$ and $sb\bar{s}b$ systems.

## 5 Conclusion

We have performed the QCD sum rule analysis with tetraquark charmonium-like currents in vector and axial-vector channels. The two-point correlation functions and the spectral densities for all the interpolating currents have been evaluated at the quark-hadron level. The numerical analysis shows that the four quark condensate $(\bar{q}q)^2$ is the dominant power contribution to the OPE series for all the vector channel currents. In the situation of the axial-vector channel currents, however, the most
important corrections are the quark condensate $\langle \bar{q}q \rangle$. These properties of the spectral densities lead to a better OPE convergence for the currents in the vector channel than that in the axial-vector channel. The $m_t$ related terms in the OPE series of $sc\bar{c}e$ systems lead to more stable mass sum rules than that of the $q\bar{c}q\bar{c}$ systems. In the working range of the Borel parameter, only the currents $J_{1\mu}$, $J_{3\mu}$ and $J_{5\mu}$ with $J^{PC} = 1^{+}$ display stable QCD sum rules in the $q\bar{c}q\bar{c}$ system. The extracted mass is around $4.6 \sim 4.7$ GeV from these currents, which is consistent with the mass of the meson $Y(4660)$. This result implies a possible tetraquark interpretation for $Y(4660)$. In the $sc\bar{c}e$ system, all currents except $J_{5\mu}$, $J_{6\mu}$ have stable QCD sum rules and the extracted mass is about $4.6 \sim 4.9$ GeV. The Borel window for the currents in the axial-vector channel is very small because of the bad OPE convergence. For the currents with $J^{PC} = 1^{+}$ in the $q\bar{c}q\bar{c}$ system, only $J_{3\mu}$ and $J_{5\mu}$ have reliable QCD sum rules. The same situation occurs in the $sc\bar{c}e$ system. The extracted masses are about $4.0 \sim 4.2$ GeV, which is $0.1 \sim 0.3$ GeV higher than the mass of $X(3872)$.

The possible decay modes of these charmonium-like states are also studied by considering the conservation of the angular momentum, P-parity, C-parity, isospin and G-parity [24].

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