Model of object non-destructive technology of road surfaces compaction control

A P Prokopev¹, Zh I Nabizhanov, R T Emelyanov and V I Ivanchura
Siberian Federal University, 79, Svobodny ave., Krasnoyarsk, RU-660041, Russia

¹ E-mail: prok1@yandex.ru

Abstract. The results of research on the development of a mathematical model of the object of non-destructive technology for continuous monitoring of compaction of road construction materials are considered. The dynamic model is based on the process of compacting the material with a double drum vibratory road roller, as an element of a cyberphysical road construction system. The main stages of constructing a mathematical model using the state variables method are presented. The state variables are vertical displacement and velocity of the rollers, vertical displacement, velocity and angular displacement of the roller frame. The results of developing a simulation model of a dynamic system implemented in the language of the Simulink program based on a model-oriented approach are presented. Output signals of the dynamic model – displacement, velocity, speeding-up, contact force of the roller drums. A mathematical model of a viscoelastic body is used to describe the dynamics of the compacted material. The results of modelling a numerical example of the dynamical system under study are considered.

1. Introduction
Sealing of asphalt pavement is the completion phase of roadway building. Permanent monitoring of compaction’s quality is required to advance efficiency of tamping machines. This method is relating to non-destructive solutions in roadway building. Technologies of non-destructive compaction monitoring are mainly implemented with point-by-point short-time measurements in the Russian Federation. Such technology keeps out of measurements continuity over constructed road, and it is difficult to provide a safe workplace.

The paper is dedicated to mathematical formulation of nondestructive technology’s object (intelligent compaction, continuous compaction control) and development of simulation object model of nondestructive technology for compaction control of road surfaces. Research object is a process of contact interaction of paving compactor with road construction material. Theoretical and experimental researches of compaction by paving compactors are considering by many Russians and foreign scientists [1-6]. Limitations of predeveloped mathematical, experimental-mathematical models are calculation difficulties in computer process simulation. State space method, which allows accomplish precise formal characterization and automation of computer procedure, is recommended to remove these limitations [7].

2. A model of the compaction process in the state space
Vibration frequency, amplitude, velocity, speeding-up are dynamic parameters of roller’s vibration of paving compactors and particles of compacted material. Vibration amplitude of rollers depends on mechanical-and-physical properties of compacted material and changes during its compaction [8].

Design model of dynamic system of work process of dual-drum paving compactor when interacting with road material is shown in figure 1.

![Design model of dynamic system "vibratory roller - material".](image)

The following keys were used on the figure 1: $m_{d1}$ – weight of roller 1, kg; $m_{d2}$ – weight of roller 2, kg; $m_f$ – weight of frame, kg; $m_a$ – net weight of compacted mixture by roller 1, kg; $m_{a2}$ – net weight of compacted mixture by roller 2, kg; $k_{f1}$, $k_{f2}$ – elasticity coefficient of oscillation absorber of roller 1, roller 2, N/m; $c_{f1}$, $c_{f2}$ – subsidence ratio of oscillation absorber of roller 1, roller 2, N s/m; $k_{a1}$, $k_{a2}$ – elasticity coefficient of compacted mixture by roller 1, roller 2, N/m; $c_{a1}$, $c_{a2}$ – subsidence ratio of compacted mixture by roller 1, roller 2, N s/m; $y_{d1}$, $y_{d2}$, $y_{d1}$, $y_{d2}$, $y_f$ – displacement of elements of dynamic system, respectively, m; $\theta$ – angle of rotation of frame, $J$ – degree; inertia moment of frame, kg/m².

The system of differential equations is obtained, which can describe dynamic of three-mass oscillating system "frame – vibrating rolls – material"

\[
\begin{align*}
(m_{d1} + m_{a}) \cdot \ddot{y}_{d1} + c_{f1} \cdot (\dot{y}_{d1} - \dot{y}_f) + k_{f1} \cdot (y_{d1} - y_f) + k_{a1} \cdot y_{d1} &= F_{e1} \cdot \sin(\omega_{e1} \cdot t) + (m_{d1} + m_{a}) \cdot g; \\
(m_{d2} + m_{a2}) \cdot \ddot{y}_{d2} + c_{f2} \cdot (\dot{y}_{d2} - \dot{y}_f) + k_{f2} \cdot (y_{d2} - y_f) + k_{a2} \cdot y_{d2} &= F_{e2} \cdot \sin(\omega_{e2} \cdot t) + (m_{d2} + m_{a2}) \cdot g; \\
J \cdot \ddot{\theta} + k_{f1} \cdot (y_{f1} - y_{d1}) + c_{f1} \cdot (\dot{y}_{f1} - \dot{y}_{d1}) + k_{f2} \cdot (y_{f2} - y_{d2}) + c_{f2} \cdot (\dot{y}_{f2} - \dot{y}_{d2}) &= 0; \\
J \cdot \ddot{y}_f + k_{f1} \cdot (y_{f1} - y_{d1}) + c_{f1} \cdot (\dot{y}_{f1} - \dot{y}_{d1}) + k_{f2} \cdot (y_{f2} - y_{d2}) + c_{f2} \cdot (\dot{y}_{f2} - \dot{y}_{d2}) &= 0; \\
\theta &= \frac{L_1}{L_2} \cdot \frac{y_{f1} + L_2 \cdot y_{f2}}{L_1 + L_2}; \\
m_{d1} \cdot \ddot{y}_{d1} + c_{a1} \cdot \dot{y}_{d1} + k_{a1} \cdot y_{d1} - m_{d1} \cdot g - F_{e1} &= 0; \\
m_{d2} \cdot \ddot{y}_{d2} + c_{a2} \cdot \dot{y}_{d2} + k_{a2} \cdot y_{d2} - m_{d2} \cdot g - F_{e2} &= 0; \\
F_{e1} &= m_{a1} \cdot r_{a1} \cdot \omega_{e1}^2 \cdot \dot{y}_{d1} + y_{d1} = y_{d1}, \\
F_{e2} &= m_{a2} \cdot r_{a2} \cdot \omega_{e2}^2 \cdot \dot{y}_{d2} + y_{d2} = y_{d2},
\end{align*}
\]
where $y_d, \dot{y}_d, \ddot{y}_d, y_{d2}, \dot{y}_{d2}, \ddot{y}_{d2}$ – displacement, velocity, speeding-up of vibrating rolls, respectively; $y_f, \dot{y}_f, \ddot{y}_f$ – displacement, velocity, speeding-up of paving compactor frame, respectively; $g$ – speeding-up of gravity, m/s$^2$; $F_{d1}, F_{d2}$ – impact force of vibration generators, N; $t$ – time, s; $m_{d1}, m_{d2}$ – debalance of vibration generators, kg; $r_{d1}, r_{d2}$ – radius of eccentricity of vibration generators debalance, m; $\omega_{d1}, \omega_{d2}$ – angular frequency of revolution of eccentric shaft of vibration generators, rad/s.

State space method allows representation of research object as equations set $[7, 9]$:

$$\dot{x}(t) = A(t) \cdot x(t) + B(t) \cdot u(t);$$

$$y(t) = C(t) \cdot x(t) + D(t) \cdot u(t),$$

where $x(t)$ – vector of state, which components are state variables of $n$-order systems; $y(t)$ – vector of output, which components are response variables of the system; $A(t)$ – system coefficient matrix ($n \times n$); $B(t)$ – input matrix ($r \times n$), $r$ – the number of reactions; $u(t)$ – input vector, which components are input variables of the system; $C(t)$ – output matrix ($n \times p$), $p$ – the number of output variable; $D(t)$ – detour matrix, which define direct dependence of output to input.

Thresholds of state variables were defined in the following way: $\dot{z}_1$ – vertical displacement of roller 1 of paving compactor, $z_1 = y_{d1}$; $\dot{z}_2$ – vertical velocity of roller 1, $z_2 = \dot{y}_{d1}$; $\dot{z}_3$ – vertical displacement of roller 2, $z_3 = y_{d2}$; $\dot{z}_4$ – vertical velocity of roller 2, $z_4 = \dot{y}_{d2}$; $\dot{z}_5$ – vertical displacement of frame, $z_5 = y_f$; $\dot{z}_6$ – vertical velocity of frame, $z_6 = \dot{y}_f$; $\dot{z}_7$ – angular displacement of frame, $z_7 = \theta$; $\dot{z}_8$ – velocity of frame’s angular displacement, $z_8 = \dot{\theta}$.

Equations set (1) is resulted in differential equation system of the first order in Cauchy normal form taking accepted state variables into consideration.

We obtained object’s model in state space as vector-matrix form

$$\dot{x}(t) = [\dot{x}_1(t) \quad \dot{x}_2(t) \quad \dot{x}_3(t) \quad \dot{x}_4(t) \quad \dot{x}_5(t) \quad \dot{x}_6(t) \quad \dot{x}_7(t) \quad \dot{x}_8(t)]^T;$$

$$x(t) = [x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t) \quad x_5(t) \quad x_6(t) \quad x_7(t) \quad x_8(t)]^T;$$

$$A(t)=
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
A & E & 0 & 0 & k_{f1} & c_{f1} & k_{j1} & L_1 \\
A & A & 0 & 0 & k_{f2} & c_{f2} & k_{j2} & L_2 \\
0 & 0 & 0 & 0 & 0 & 0 & -k_{f2} & L_2 \\
0 & 0 & 0 & 0 & 0 & 0 & -k_{f2} & L_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
J & J & J & J & G & K & L & M
\end{bmatrix};$$

$$A = m_{d1} + m_{d1}; B = m_{d2} + m_{d2}; C = -k_{f1} - k_{s1}; D = -k_{f2} - k_{s2}; E = -c_{f1} - c_{s1}; F = -c_{f2} - c_{s2};$$
The model in the state space is template for building simulation model of dynamic system under study.

3. The study of the mathematical model

Object-focused approach was carried out in the design of simulation model of investigated process in terms of MATLAB/Simulink, figure 2.

The following initial data from research paper was used during computational modeling [10]:

\[ k_{11} = 4 \cdot 10^7 \text{ N/m}; \quad k_{12} = (4 \cdot 1.2) \cdot 10^7 \text{ N/m}; \quad c_{11} = 210 \cdot 10^3 \text{ N \cdot s/m}; \quad c_{12} = (210 \cdot 1.2) \cdot 10^3 \text{ N \cdot s/m}; \]

\[ k_{f1} = 140 \cdot 10^3 \text{ N/m}; \quad k_{f2} = 140 \cdot 10^4 \text{ N/m}; \quad f_{c1} = 48 \text{ Hz}; \quad f_{c2} = 48 \text{ Hz}; \quad F_{c1} = 95000 \text{ N}; \quad F_{c2} = 95000 \text{ N}; \]

\[ L_1 = 1.5 \text{ m}; \quad L_2 = 1.5 \text{ m}; \quad c_{f1} = k_{f1} \cdot \eta/\omega_{c1}; \quad \eta = 0.16; \quad c_{f2} = k_{f2} \cdot \eta/\omega_{c2}; \quad \eta = 0.16; \quad \]

\[ m_d = 2750 \text{ kg}; \quad m_i = 0.2 \cdot m_d, \text{ kg}; \quad m_{d2} = 0.2 \cdot m_{d2}, \text{ kg}; \quad m_j = 6000 + 1500 \text{ kg}; \quad J = 12000 \text{ kg \cdot m}^2. \]

Figure 2. Simulation model of the research object in MATLAB/Simulink.

As a result of computational modeling, time dependences of parameters of the vibration process of the rollers and the vibration roller’s frame during the compaction of road building material were obtained: displacement, velocity, speeding-up, contact force.

Time dependences of displacement, velocity and speeding-up of the roller 1 are indicated in figure 3.
Computational modeling results of road material’s vibratory compacting process showed good process repeatability.

4. Conclusions
A mathematical model of an object of non-destructive technologies for compacting a road concrete mix by the state variable method is obtained. A simulation model has been developed in the
MATLAB/Simulink software. It is planned to use the model in design of intelligence (artificial neural network) system of continuous monitoring of road material’s compaction quality (sub-soil, road concrete mix).

Mathematical and simulation models can be used for research and design elements of cab suspension due to adding more elements of dynamic system - shock-absorbers, cabin, operator's work station.

Results of the research are the searching study phase in the field of process automation of compaction by vibratory rollers.

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