Resurrection of Charge Symmetry Violation for $\langle \rho^0 | H_{em} | \omega \rangle \sim -5000 \text{ MeV}^2$

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Abstract

We review the theoretical predictions for $\rho^0 - \omega$ mixing and conclude in many ways that $\langle \rho^0 | H_{em} | \omega \rangle \sim -5000 \text{ MeV}^2$, more in line with earlier analyses of charge symmetry violation and with present data.

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As of the reviews of 1990 and earlier [1], charge symmetry violation (CSV) in nuclear physics was thought to be reasonably well understood. This was in part due to earlier rho-omega ($\rho^0 - \omega$) mixing analyses of Glashow [2], Renard [3] and Coon et al [4], culminating in the Coon and Barrett conclusion [5] in 1987 that the most accurate particle data group measurements lead to the $\Delta I = 1 \ \rho^0 - \omega$ electromagnetic hamiltonian density ($H_{em}$) transition

$$\langle \rho^0 | H_{em} | \omega \rangle = -(4520 \pm 600) \ MeV^2 . \quad (1)$$

Only recently Coon and Scadron [6] have shown on the basis of SU(3) and SU(6) symmetry, that the theoretical versions of both nonstrange (NS) $\omega - \rho^0$ and $\eta_{NS} - \pi^0 \ \Delta I = 1$ transitions have the universal value

$$\langle \rho^0 | H_{em} | \omega \rangle \approx \langle \pi^0 | H_{em} | \eta_{NS} \rangle \sim -5000 \ MeV^2 , \quad (2)$$

for similar mass shells $m_\omega \approx 783 \ MeV$, $m_{\rho^0} \approx 769 \ MeV$, $m_{\eta_{NS}} \sim \frac{1}{2}(m_{\eta'} + m_\eta) \sim 753 \ MeV$.

This universal CSV scale in (2) is also compatible [6] with the Coleman-Glashow [7] and Dashen theorem [8] versions of all 13 electromagnetic (em) ground state pseudoscalar (P), vector (V), octet baryon (B) and decuplet baryon (D) SU(2) mass splittings [9]. Further contact with CSV data follows from constructing [1] class III and class IV $NN$ potentials from the off-shell spacelike $q^2$ CSV $\Delta I = 1$ exchange graph of Fig. 1. These Fourier-transformed potentials are proportional to $e^{-m_V r}$ or $e^{-m_V r}/r$, where $m_V = (m_\rho + m_\omega)/2$. This class III potential is compatible with $NN$ scattering and bound state (Nolen-Schiffer anomaly) data in mirror nuclear systems [1,5,10], with Coulomb displacement energies of isobaric analog states [11], and with isospin-mixing matrix elements relevant to the isospin-forbidden beta decays [12]. The class IV CSV potential [13] is compatible with precise measurements of the elastic scattering of polarized neutrons off of polarized protons [14].

Not withstanding the above rather complete picture of CSV, beginning with ref. [15] in 1992, a new approach to CSV in nuclear physics has begun
to emerge \cite{16–19}. This approach questions \cite{15–19} the validity of both the empirical and theoretical values of the $\rho^\circ–\omega$ mixing transitions in (1) and (2) while claiming instead that there is further off-shell $q^2$ dependence of $\rho^\circ–\omega$ mixing beyond the usual Fourier-transformed $NN$ potential following from Fig. 1.

In this letter we briefly summarize why these new CSV questions since 1992 \cite{15–19} are misdirected and do not negate the (successful) CSV $\rho^\circ–\omega$ mixing picture given in our first two paragraphs.

To begin, we remind the reader that the $\Delta I = 1$ Coleman-Glashow (CG) em hamiltonian density in (1) or (2) has (photon) current-current (JJ) and contact (tadpole) parts

$$H_{\text{em}} = H_{\text{JJ}} + H_{\text{tad}}^3.$$ (3)

While both matrix elements of $H_{\text{JJ}}$ and of $H_{\text{tad}}^3$ are necessary to fit all 13 \cite{6} P, V, B, D em mass differences of (3), only the contact (tadpole) part in (3) is needed for the $\rho^\circ–\omega$ mixing transition, leading to

$$\langle \rho^\circ | H_{\text{em}} | \omega \rangle = \langle \rho^0 | H_{\text{tad}}^3 | \omega \rangle = \Delta m_{K^*}^2 - \Delta m_{\rho}^2 \approx -4666 \text{ MeV}^2.$$ (4)

Here we have assumed SU(3) symmetry for an $\omega$ as pure nonstrange ($\bar{u}u + \bar{d}d)/\sqrt{2}$ and used the 1996 PDG \cite{9} $K^*$ and $\rho$ masses, $m_{K^{*+}} \approx 891.6$ MeV, $m_{K^{*0}} \approx 896.1$ MeV so that $\Delta m_{K^*}^2 = m_{K^{*+}}^2 - m_{K^{*0}}^2 \approx -8045$ MeV$^2$, along with $m_{\rho^+} \approx 766.9$ MeV, $m_{\rho^0} \approx 769.1$ MeV, so that $\Delta m_{\rho}^2 = m_{\rho^+}^2 - m_{\rho^0}^2 \approx -3379$ MeV$^2$. This results in the $\rho^\circ–\omega$ mixing scale given in eq. (4).

The above CG contact (tadpole) $\rho^\circ–\omega$ mixing prediction (4) must hold by virtue of the Dashen observation \cite{8} $\langle \pi^0 | H_{\text{JJ}} | \eta \rangle = 0$ extended to $\langle \rho^0 | H_{\text{JJ}} | \omega \rangle = 0$ \cite{6}. But it is important to demonstrate that the contact-tadpole graph of Fig. 2 actually recovers the $\rho^\circ–\omega$ mixing scales of eqs. (1), (2), (4). Here the $\Delta I = 1 \delta^0$ pole in Fig. 2 is now called \cite{9} the $a^0_0 (984)$. Reference \cite{6} shows in fact that

$$\langle \rho^0 | H_{\text{tad}}^3 | \omega \rangle \approx -f_\delta 2g_{\delta\omega\rho} \approx -f_\delta m_\delta^2/f_\pi \approx -4400 \text{ MeV}^2.$$ (5)
where the “decay constant” $f_\pi$ is 0.42 MeV as found from the SU(2) to SU(3) symmetry breaking scale of 2% where $f_\pi \approx 93$ MeV. We believe that the close agreement between the $\rho^0$–$\omega$ mixing scales in (1), (2), (4) and (5) are significant—they justify the CG tadpole picture (as do the successful 13 P, V, B, D em mass splittings).

By way of contrast, ref. [15] ignores this CG contact tadpole picture and instead studies the (non-contact) $\rho^0$–$\omega$ GHT [15] quark loop of Fig. 3 (their Fig. 1), where $\delta m = m_u - m_d$ corresponds to the $u$–$d$ current quark mass difference. While it is true [6] that $\delta m$ appears in the CG tadpole hamiltonian $H_3^{\text{tad}} = \delta m(\bar{u}u - \bar{d}d)/2$, the GHT quark loop of Fig. 3 is not a CG contact-tadpole of Figs. 1 or 2. Moreover the GHT quark loop picture does not recover the 13 P, V, B, D em mass differences nor the $\langle \rho^0 | H_{em} | \omega \rangle \Delta I = 1$ scales of eqs. (1), (2), (4), (5) above.

In spite of these (serious) deficiencies, refs. [16–19] continue to study the properties of Fig. 3 and learn that nonstrange quark loops have a large off-shell $q^2$ dependence when coupled to external $\rho^0$ and $\omega$ states. Whether this is true or not is irrelevant because such GHT nonstrange $u$ and $d$ quark loops cannot generate the needed CG contact-tadpole graphs of Figs. 1 or 2, which explain CSV due to the $\Delta I = 1 \rho^0$–$\omega$ contact transition.

As noted in the second reference in [4], the “boosted” form of the contact $\langle \rho^0 | H_{em} | \omega \rangle$ transition is

$$\varepsilon^\mu \langle \rho_\mu, \nu | H_{em} | \omega_\nu \rangle \varepsilon^\nu = \varepsilon^\mu \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{t} \right) \varepsilon^\nu \langle \rho^0 | H_{em} | \omega \rangle,$$

(6)

where $t = q^2$ is the invariant squared momentum of the vector mesons. Because of the conserved isovector $\rho^0$ current, the second term on the right-hand side of (6) vanishes and there is no $q^2$ variation of this contact $\rho^0$–$\omega$ mixing transition (6) since the $\omega(783)$ and $\rho(770)$ are essentially on the same mass shells.

Even if this GHT quark loop were the origin of $\rho^0$–$\omega$ mixing (which is not physically possible as explained above), a recent analysis [20] using Borel
and finite energy QCD sum rules shows that the inclusion of finite mesonic
widths requires the $\rho^0$--$\omega$ mixing matrix element in the space-like region to
have the same sign and similar magnitude as its on-shell value, eqs. (1), (2),
(4), (5) above. Moreover, ref. [20] demonstrates that the “node theorem” of
ref. [17] for the mixed $\rho^0$ and $\omega$ propagator is spoiled.

This misuse of the “mixed $\rho^0$ and $\omega$ propagator” culminated in the recent
analysis [19] of $e^+ e^- \to \pi^+ \pi^-$ in the $\rho^0$--$\omega$ interference region, claiming that
data from the $e^+ e^- \to \pi^+ \pi^-$ interference region cannot be used to fix the
value of $\rho^0$--$\omega$ mixing in a model-independent way. This conclusion is in con-
trast with eqs. (1), (2), (4), (5) above and follows from a misinterpretation of
the contact-tadpole analysis of Renard [3] calling there $\delta m$ [21], but meaning
the CG contact-tadpole $\langle \rho^0 | H_{em} | \omega \rangle$.

To make our point in more detail, we follow refs. [4] and Fig. 4a to com-
pute $\langle \rho^0 | H_{em} | \omega \rangle$ using the 1996 PDG rates [9] $\Gamma(\rho^0 \to 2\pi) = 150.7 \pm 1.2$ MeV
and $\Gamma(\omega \to 2\pi) = 0.186 \pm 0.025$ MeV:

$$\Gamma(\omega \to 2\pi) \approx \left| \frac{\langle \rho^0 | H_{em} | \omega \rangle}{i m_\rho \Gamma_\rho} \right|^2 \Gamma(\rho^0 \to 2\pi), \quad (7a)$$

$$\langle \rho^0 | H_{em} | \omega \rangle \approx -(4069 \pm 277) \text{ MeV}^2. \quad (7b)$$

The sign of (7b) is uniquely determined from the $\rho^0$ and $\omega$ interference phase
in $e^+ e^- \to \pi^+ \pi^-$ of $\phi \approx 100^\circ$ [4], where this phase was also found by Renard
[3]. A judicious choice of $\omega \to 2\pi$ data increases this scale in (7b) to the value
[5] in (1). This $\omega$--$\rho$ transition in (1) or in (7b) is a Coleman-Glashow [7]
shaded contact (tadpole) interaction of Figs. 1,2,4; it is not a (non-contact)
scaling arising from a $\rho^0$--$\omega$ mixed quark propagator or from the quark bubble
of Fig. 3.

The notion of quantum-mechanical mixing applies to strong interaction
$\phi$--$\omega$ mixing with states

$$|\omega\rangle = \cos \phi_V \, |\omega_{NS}\rangle - \sin \phi_V \, |\omega_s\rangle \quad (8a)$$

$$|\phi\rangle = \sin \phi_V \, |\omega_{NS}\rangle + \cos \phi_V \, |\omega_s\rangle \quad (8b)$$
relative to the nonstrange $|\omega_{NS}\rangle = (|\bar{u}u\rangle + |\bar{d}d\rangle)/\sqrt{2}$ and strange $|\omega_S\rangle = |\bar{s}s\rangle$ quark basis [22]. This mixing angle $\phi_V$ is very small [23] and can be obtained from data using [9] $\Gamma(\phi \to \pi\gamma) = 5.80 \pm 0.58$ keV and $\Gamma(\omega \to \pi\gamma) = 717 \pm 43$ keV:

$$\frac{\Gamma(\phi \to \pi\gamma)}{\Gamma(\omega \to \pi\gamma)} = \frac{p_\phi^3}{p_\omega^3} \tan^2 \phi_V = 0.00809 \pm 0.00094 , \quad (9)$$

requiring $\phi_V \approx 3.4^0$ for $p_\phi = 501$ MeV, $p_\omega = 379$ MeV. There is no point in finding a rediagonalized $\rho^0-\omega$ (or $\pi^0-\eta_{NS}$) mixing angle, but it is small anyway [21].

To demonstrate that this $\phi-\omega$ mixing angle analysis of (8) and (9) can also be used to compute the $\rho^0-\omega$ transition of interest (here we prefer not to call this transition $\rho^0-\omega$ “mixing”), we see from eqs. (8a) and (7) that

$$\langle \rho^0|H_{em}|\omega_{NS}\rangle = \frac{1}{\cos \phi_V} \langle \rho^0|H_{em}|\omega\rangle \approx -4076 \text{ MeV}^2 , \quad (10)$$

near (7b) because $\omega$ is 97% nonstrange. Likewise the $\phi \to 2\pi$ rate [9] found from the total $\phi$ rate 4.43 $\pm$ 0.05 MeV and branching ratio $(8 \pm 4) \cdot 10^{-5}$ gives the $\rho^0-\phi$ contact em transition of Fig. 4b as

$$\Gamma(\phi \to 2\pi) \approx \left| \frac{\langle \rho^0|H_{em}|\phi\rangle}{im_\rho \Gamma_\rho} \right|^2 \Gamma(\rho \to 2\pi) \quad (11a)$$

or $\langle \rho^0|H_{em}|\phi\rangle \approx -178$ MeV$^2$. Although this latter $\Delta I = 1$ em transition is $\sim$ 30 times smaller than (1) or (10), the small observed mixing angle $\phi_V \approx 3.4^0$ enhances this scale to

$$\langle \rho^0|H_{em}|\omega_{NS}\rangle = \frac{1}{\sin \phi_V} \langle \rho^0|H_{em}|\phi\rangle \approx -3020 \pm 710 \text{ MeV}^2 . \quad (11b)$$

The latter large error suggests (11b) is compatible with the $\rho^0-\omega$ $\Delta I = 1$ scale in (10) or in (1), (2), (4), (5). If we select the Vasserman et al $\phi \to 2\pi$ data [9] then (11b) increases to

$$\langle \rho^0|H_{em}|\omega_{NS}\rangle \approx -4700 \text{ MeV}^2 , \quad (11c)$$

more in line with (1), (2), (4), (5).

This $\phi-\omega$ mixing angle digression in eqs. (8)–(11) consistently using a $\Delta I = 1$ contact (tadpole) along with $\rho^0-\omega$ and $\rho^0-\phi$ transitions is in direct
conflict with ref. [19] (and also [15]–[18]). They deal with (unphysical) non-contact quark loop graphs and a mixed \( \rho^0 - \omega \) off-diagonal propagator while assuming (with no support from data) that the pure isoscalar omega has an isospin violating coupling to two pions and indirectly suggest that the result (11) could not possibly hold.

In this letter we have briefly reviewed four determinations of the charge symmetry violating (CSV) scale \( \langle \rho^0 | H_{em} | \omega \rangle \sim -5000 \text{ MeV}^2 \) in equs. (1), (2), (5), (7) while giving two new derivations of this CSV scale in equs. (4) and (11).

These six above determinations of \( \langle \rho^0 | H_{em} | \omega \rangle \) all assume that nonphotonic \( \Delta I = 1 \) transitions are always of the contact-tadpole \( u_3 = \bar{q} \lambda_3 q \) form (proportional to \( H_{tad}^3 \) in this letter) as originally proposed by Coleman and Glashow [CG] and supported by Weinberg [7]. This \( u_3 \) contact term is represented by the shaded \( \Delta I = 1 \) interactions in our Figs.1,2,4. Support for this CG scheme is given by the universal electromagnetic SU(2) mass splittings of all (13) ground state P,V,B,D hadrons as summarized in the appendix of ref. [6.] However, this successful analysis is not related to the quark-loop mixed \( \rho^0 - \omega \) propagator formalism recently developed in [15–19].

In conclusion, we believe that charge symmetry violation in nuclear physics is well understood [1–6]. It is based on the particle physics \( \Delta I = 1 \rho^0 - \omega \) contact-tadpole transition [6–8], which is also well understood.

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Figure Captions

Fig. 1 Nucleon-nucleon CSV vector meson exchange graph.

Fig. 2 Delta meson tadpole dominance of the CSV \( \Delta I = 1 \langle \rho^0 | H_{em} | \omega \rangle \) contact transition.
Fig. 3 Nonstrange quark loops for $\rho^0 - \omega$ mixing.

Fig. 4 $\omega - \rho^0$ $\Delta I = 1$ contact graph dominating $\omega \rightarrow 2\pi$ (a); $\phi - \rho^0$ $\Delta I = 1$ contact graph dominating $\phi \rightarrow 2\pi$ (b).
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