Verification of bootstrap conditions for amplitudes with quark exchanges in QMRK

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Abstract

The compatibility of the multi–Regge form of QCD amplitudes in the quasi–multi–Regge kinematics (QMRK) and the s–channel unitarity imposes some constraints on the effective jet–production vertices. We demonstrate that these constraints known as bootstrap conditions are satisfied for the amplitudes with the Reggeized quark exchanges.

1 Introduction

This article continues the development of the quark Reggeization theory [1] in QCD. A noticeable progress has been recently achieved here, in particular, the quark Regge trajectory in the next–to–leading approximation (NLA) in D dimensions was found [2,3] and the next–to–leading order (NLO) corrections to the effective vertices appearing in the leading logarithmic approximation (LLA) were calculated [4,5]. All these results were obtained assuming the reggeized form for amplitudes in the multi–Regge kinematics (MRK) in the NLA. It is clear that this assumption, called the quark Reggeization hypothesis, must be proved. However, in the NLO it is tested only in $\alpha_s^2$ order [2,3] so far. Moreover, its complete proof in the LLA for any quark–gluon inelastic process in all orders of $\alpha_s$ was given only recently [6]. This proof is based on the relations required by compatibility of the multi–Regge form of QCD amplitudes with the s–channel unitarity (bootstrap relations). The fulfillment of

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the bootstrap relations is secured by several conditions (bootstrap conditions) on Reggeon vertices and trajectories. An analogous proof can (and has to) be constructed in the NLA as well.

The only kinematics essential in the LLA is MRK, which means that all particles produced in a high–energy process have limited transverse momenta and are well separated in rapidity space. In the NLA, production amplitudes in this kinematics can be obtained by taking one of the effective vertices or Regge trajectory in the NLO. But in the NLA another, quasi–multi–Regge kinematics (QMRK) becomes also important. In this case one of the produced particles is replaced by a jet containing two particles with similar rapidities. At this moment all multi–particle Reggeon vertices required in the NLA are obtained [7]. Therefore, a proof of the quark Reggeization hypothesis concerning the QMRK may be given.

In this paper we prove the quark Reggeization in the QMRK. Our method is the direct continuation of the one used to prove this hypothesis in the LLA. The bootstrap conditions for amplitudes in the QMRK are the same as in the LLA with the substitution of jet production vertices for particle production ones. Hereafter we demonstrate that all these conditions are fulfilled. For convenience we work in the operator formalism, which was introduced in [8] and extended to inelastic amplitudes and quark exchanges in [6].

The paper is organized as follows. The next section contains all necessary denotations and the definition of the QMRK. Section 3 presents the bootstrap conditions in operator formalism. In section 4 we prove these conditions for impact factors and Reggeon–Reggeon–jet (RRJ) effective vertices. Section 5 concludes the paper.

2 Quasi–multi–Regge form of QCD amplitudes

Considering the QMRK we talk about a multiparticle production amplitude as about the amplitude of jet production where one of the jets consists of two particles. Such a jet can be produced either in the fragmentation regions of initial particles, or in the central region, i.e. with the rapidity far away from the rapidities of colliding particles. Let us consider the process $A + B \rightarrow A' + P_1 + \ldots + P_n + B'$ in the QMRK. Using the same denotations as in [6] we introduce light–cone momenta $n_1$ and $n_2$: $n_1^2 = n_2^2 = 0$, $(n_1 n_2) = 1$ and denote $(pn_2) \equiv p^+, (pn_1) \equiv p^-$, so that $pq = p^+ q^- + q^+ p^- + p_1.q_1$. Here the sign $\perp$ means transverse to the $(n_1, n_2)$ plane components. We assume that initial momenta $p_A$ and $p_B$ have predominant components along $n_1$ and $n_2$ respectively. For generality we do not demand that transverse components $p_{A\perp}$ and $p_{B\perp}$ should be zero, but assume $|p_{A\perp}^2| \sim |p_{B\perp}^2| \sim p_A^2 \sim p_B^2 \ll p_A^{\perp} p_B^{\perp}$.
and remain limited (not grow) at $p_\perp^A p_B \to \infty$. For the final jet momenta $p_i$, $i = 0, \ldots, n + 1$, we assume the QMRK conditions:

\[ p_0^- \ll p_1^- \ll \ldots \ll p_n^- \ll p_{n+1}^- , \]
\[ p_{n+1}^+ \ll p_n^+ \ll \ldots \ll p_1^+ \ll p_0^+ , \]

where $p_{i\perp}$ are limited. It ensures that the squared invariant masses $s_{ij} = (p_i + p_j)^2$ are large compared with the squared transverse momenta and invariant masses of the jets. At $i < j$ they have the form

\[ s_{ij} \approx 2p_i^+ p_j^- = \frac{p_i^+}{p_j^+}(p_j^2 - p_{j\perp}^2) = \frac{p_j^-}{p_i^-}(p_i^2 - p_{i\perp}^2) , \]

and at $i < l < j$ submit to relations

\[ s_{il} s_{lj} \approx s_{ij}(p_l^2 - p_{l\perp}^2) . \]

For the momentum transfers $q_i$, $i = 1, \ldots, n + 1$,

\[ q_1 = p_0 - p_A , \quad q_{j+1} = q_j + p_j , \quad (j = 1, \ldots, n) , \]

we have

\[ q_i^2 \approx q_{i\perp}^2 . \]

In the LO the amplitude $A_{2\to n+2}$ of the process $A + B \to A' + P_1 + \ldots + P_n + B'$ has the multi–Regge form

\[ A_{2\to n+2}^R = \tilde{\Gamma}_{A,A'}^R \frac{s_{\omega_1}}{d_1} \gamma_{R_1 R_2} \frac{s_{\omega_2}}{d_2} \cdots \gamma_{R_n R_{n+1}} \frac{s_{\omega_{n+1}}}{d_{n+1}} \Gamma_{B'B}^R , \]

where $\tilde{\Gamma}_{A,A'}^R$ and $\Gamma_{B'B}^R$ are the particle–particle–Reggeon (PPR) effective vertices, describing particle–particle $P \to P'$ transitions due to interaction with Reggeons $R$. For gluon quantum numbers in $q_i$ channel, $\omega_i = \omega_G(q_i)$ is the gluon Regge trajectory and $d_i \equiv d_i(q_i) = q_{i\perp}^2$; for quark numbers, $\omega_i = \omega_Q(q_i)$ is the quark Regge trajectory and $d_i \equiv d_i(q_i) = m - q_{i\perp}$. $\gamma_{R_i R_{i+1}}$ are the Reggeon–Reggeon–particle (RRP) effective vertices describing production of particle $P_i$ at Reggeon transitions $R_{i+1} \to R_i$. In order to be definite we do not consider here antiquark quantum numbers in any of $q_i$ channels. It determines the order of the multipliers in (6). Nonetheless, our consideration does not lose generality because amplitudes with quark and antiquark exchanges are related by charge conjugation.

Since we come to the QMRK replacing one of the particles $P_i$ in the MRK with a pair with fixed invariant mass, QMRK amplitudes have the same form (6) as LO MRK ones, where one of the vertices $\gamma_{P R}^P$ or $\Gamma_{P' P}^R$ is substituted with the jet production vertex $\gamma_{P R}^{P_i P_j}$ or $\Gamma_{P R}^{P_i P_j P_k}$ respectively. Note, that because the QMRK leads to the loss of a large logarithm in the unitarity
relations, energy scales in (6) are unimportant in the NLA. Moreover, we need trajectories and vertices only in the LO there. Assuming similar ordering of longitudinal components one can obtain the more general multi–jet amplitudes $A_{2+n_1\to 2+n_2}^{\mathcal{R}}$ from $A_{2\to n_2}^{\mathcal{R}}$ by usual crossing rules. Note, that as in (6) we can neglect imaginary parts of these amplitudes since in the QMRK they are next–to–next–to–leading. Therefore, as well as for the amplitudes in the LO, crossing rules connecting the QMRK amplitudes do not affect the Regge factors $s_i^{\omega}$. 

Hereafter we work in the physical light–cone gauge

$$(ep) = (e\,n_1) = 0, \quad e = e_\perp - \frac{(ep)_\perp}{p^-}n_1,$$  \hspace{1cm} (7)$$

where $e$ is the polarization vector of a gluon with momentum $p$.

We use the PPR vertices in the LO in this gauge from [6]:

$$\begin{align*}
\Gamma_{G\to G}^G &= -2g\, p^- T^G_{G\to G}(e_{G_1}^*, e_{G_\perp}), \quad \Gamma_{Q\to Q}^G = g\, \bar{u}_Q t^G \gamma^- u_Q, \\
\Gamma_{Q\to Q}^{G'} &= -2g\, p^+ T^G_{G\to G}(e_{G_1}^*, e_{G_\perp}), \quad \Gamma_{Q'\to Q}^G = g\, \bar{u}_Q t^G \gamma^+ u_Q, \\
\Gamma_{Q\to G'}^G &= -g\, \bar{v}_Q t^{G'} \gamma^- v_{Q'}, \quad \Gamma_{Q'\to G}^G = -g\, \bar{v}_Q t^{G'} \gamma^+ v_{Q'}, \\
\Gamma_{Q\to Q}^Q &= -g t^{G'} \bar{e}_{G_\perp} u_Q, \quad \Gamma_{Q'\to Q}^Q = -g t^{G'} \bar{e}_{G_\perp} v_{Q'}, \\
\Gamma_{Q\to G'}^Q &= -g t^{G'} \bar{e}_{G_\perp} u_{Q'}, \quad \Gamma_{Q'\to G}^Q = -g t^{G'} \bar{e}_{G_\perp} v_{Q'}.
\end{align*}$$  \hspace{1cm} (8)$$

Here we denote particles and Reggeons by symbols which accumulate all their quantum numbers. We use the letter $P$ for particles (jets) and the letter $\mathcal{R}$ for Reggeons independently of their nature, letters $G$ and $Q$ for ordinary gluons and quarks and $G'$ and $Q$ for the Reggeized ones. In the gauge (7) the RRP vertices for gluon, quark and antiquark production with momentum $p = q_2 - q_1$ at the transition of Reggeon $\mathcal{R}_2$ (with momentum $q_2$) to Reggeon $\mathcal{R}_1$ (with momentum $q_1$) are as follows [6]:

$$\begin{align*}
\gamma_{G_1G_2}^G &= 2g T^G_{G_1G_2} e^*_{G_1} \left( q_{2\perp} - p_{G_1} \frac{q_{2\perp}}{p_{G_\perp}} \right), \quad \hspace{1cm} (10) \\
\gamma_{Q_1Q_2}^G &= -gt^{G'} e^*_{G_1} \left( \gamma_{G_\perp} - 2(\bar{q}_{2\perp} - m) \frac{p_{G_1}}{p_{G_\perp}} \right), \quad \hspace{1cm} (11) \\
\gamma_{Q_1Q_2}^Q &= g\, \bar{u}_Q \frac{q_{2\perp}}{p_{Q}} t^{G'}_{G_1}, \quad \gamma_{Q_1Q_2}^Q = -gt^{G'} \bar{q}_{2\perp} v_{Q}. \hspace{1cm} (12)
\end{align*}$$

The particle–jet–Reggeon (PJR) and Reggeon–Reggeon–jet (RRJ) effective vertices taken from [9] and [7] can be presented in different forms and our goal is to find the presentation in which subsequent calculations become trivial. Taking this in mind we introduce a set of functions $F_i, K_i, V_i$ (see below
eqs. (20–33) which determine all the PJR and RRJ effective vertices. The selection of these functions is a nontrivial task and a result of the analysis of cancellations during the verification of all QMRK bootstrap conditions.

Firstly, we introduce \( x_i = k_i^- / k^- \), with \( k_i \) being the momentum of the final particle and \( k = k_1 + k_2 \) the momentum of the jet, so \( x_1 + x_2 \approx 1 \), we also use \( v = x_2 k_{1\perp} - x_1 k_{2\perp} \). The commonly arising denominators may be expressed via

\[
D(p, q) = x_1 p_\perp^2 + x_2 q_\perp^2 \quad \text{and} \quad d(p, q) = (x_1 p_\perp - x_2 q_\perp)^2. \tag{13}
\]

We rewrite all RRP and PJR effective vertices in terms of the functions \( G_i \), \( K_i \), \( V_i \) by means of the following identities

\[
\frac{1}{k_{2\perp} (k_{2\perp}^2 - m^2)} - \frac{x_2}{x_1 (k_{2\perp}^2 - m^2)} \frac{x_2}{D(k_2, k_1) - x_2 m^2} + \frac{x_2}{(k_{1\perp}^2 - m^2)(D(k_2, k_1) - x_2 m^2)}, \tag{14}
\]

\[
\frac{x_1 x_2}{(d(k_2, k_1) - x_2 m^2)(D(k_2, k_1) - x_2 m^2)} = \frac{1}{(k_1 + k_2)^2 - m^2} \left( \frac{1}{d(k_2, k_1) - x_2 m^2} - \frac{1}{D(k_2, k_1) - x_2 m^2} \right), \tag{15}
\]

\[
x_1 x_2 (k_{2\perp}^2 - k_{1\perp}^2) = (x_1 - x_2) d(k_2, k_1) - 2 x_1 x_2 (k_1 + k_2, v) \perp \\
= x_1 x_2 (k_{1\perp}^2 - k_{2\perp}^2) + (x_1 - x_2) D(k_2, k_1), \tag{16}
\]

\[
(e_1, v) \perp (e_2 k_2) \perp + (e_2, v) \perp (e_1 k_1) \perp \\
= x_2 (e_2, k_1 + k_2) \perp (e_1 k_1) \perp - x_1 (e_1, k_1 + k_2) \perp (e_2 k_2) \perp \\
= x_1 (e_2, v) \perp (e_1, k_1 + k_2) \perp + x_2 (e_1, v) \perp (e_2, k_1 + k_2) \perp, \tag{17}
\]

\[
\bar{u}_k, \hat{n}_1 \left( \frac{k_{1\perp} + m}{x_1} \gamma^\mu \frac{k_{2\perp} + m}{x_2} \right) v_k = 2 k^- \bar{u}_k, \gamma^\mu v_k, \tag{18}
\]

\[
\bar{u}_k, \hat{n}_1 \left( \frac{k_{1\perp} + m}{x_1} \gamma^\mu \frac{k_{2\perp} - m}{x_2} \right) u_k = 2 k^- \bar{u}_k, \gamma^\mu u_k, \tag{19}
\]

where \( e_1 \) and \( e_2 \) are the polarization vectors of emitted gluons. Our functions are:

\[
F_1^\mu(k_2, k_1) = \hat{e}_1 \perp \left( 2 x_1 k_{2\perp}^\mu - x_2 (k_{1\perp} + m) \gamma^\mu \right), \\
\hat{F}_1^\mu(k_2, k_1) = F_1^\mu(k_2, k_1)|_{\perp \leftrightarrow 2}, \tag{20}
\]

\[
F_2^\mu(k_2, k_1) = \gamma^\mu (2 x_1 (e_2, v) \perp + x_2 (v + x_2 m)) \perp, \tag{21}
\]
\[ F_3^\mu(k_2, k_1) = \hat{e}_{1\perp} (2v^\mu - x_2 \gamma_\perp \hat{v} (\hat{x}_2 m)), \]
\[ \bar{F}_3^\mu(k_2, k_1) = \hat{e}_{2\perp} (-2v^\mu + x_1 \gamma_\perp \hat{v} + x_1 m)), \]
\[ F_4^\mu(k_2, k_1) = \hat{e}_2 (\gamma^\mu (\hat{v} + m) - 2x_1 v^\mu), \]
\[ \bar{F}_4^\mu(k_2, k_1) = \hat{e}_1 (-\gamma^\mu (\hat{v} - m) + 2x_2 v^\mu), \]
\[ F_5(k_2, k_1) = (\hat{k}_{1\perp} + m)(\hat{k}_{2\perp} + m), \]
\[ K_1^\mu(k_2, k_1) = \frac{2x_1 \hat{e}_{1\perp} (\hat{k}_{1\perp} + m) k_2^\mu}{(D(k_2, k_1) - x_2 m^2)(k_{1\perp}^2 - m^2)}, \]
\[ \bar{K}_1^\mu(k_2, k_1) = K_1^\mu(k_2, k_1)|_{1\leftrightarrow 2} \]
\[ K_2^\mu(k_2, k_1) = \frac{2x_2 \hat{e}_{2\perp} (\hat{k}_{1\perp} + m) k_2^\mu}{(D(k_2, k_1) - x_2 m^2) k_{2\perp}^2}, \]
\[ \bar{K}_2^\mu(k_2, k_1) = K_2^\mu(k_2, k_1)|_{1\leftrightarrow 2}, \]
\[ K_3^\mu(k_2, k_1) = \frac{4x_1 k_{1\perp}^\mu (e_2, k_2)}{D(k_2, k_1) k_{1\perp}^2}, \]
\[ \bar{K}_3^\mu(k_2, k_1) = \frac{4x_2 k_{1\perp}^\mu (e_2, k_2)}{D(k_2, k_1) k_{2\perp}^2}, \]
\[ V_1^\mu(k_2, k_1) = 2x_1 x_2 (e_1 e_2) \perp v^\mu - 2x_1 (e_2, v) \perp e_1^\mu - 2x_2 (e_1, v) \perp e_2^\mu, \]
\[ e_1^\mu V_2^\mu(k_2, k_1) = x_1 x_2 (e_1 e_2) \perp (k_{2\perp}^2 - k_{1\perp}^2) + 2x_1 (e_1, v) \perp (e_2, k_{2\perp}^2) + 2x_2 (e_2, v) \perp (e_1, k_{1\perp}^2), \]
\[ \tilde{V}_{1\perp}^\mu(k_2, k_1) = 2x_2 x_2 \hat{v}_{2\perp} v^\mu = 2x_1 (e_2, v) \perp \gamma_\perp^\mu - 2x_2 \hat{v}_{2\perp} e_2^\mu, \]
where 1 ↔ 2 means \(k_1 \leftrightarrow k_2, x_1 \leftrightarrow x_2, e_1 \leftrightarrow e_2\).

Notice that
\[ e_1^\mu \tilde{F}_{3,4}(k_2, k_1) = e_2^\mu F_{3,4}(k_2, k_1)|_{1\leftrightarrow 2} \]
and
\[ e_1^\mu \tilde{K}_{3,4}(k_2, k_1)|_{1\leftrightarrow 2} = e_2^\mu K_{3,4}(k_2, k_1). \]
The functions \(V_1(k_2, k_1), e_1^\mu V_2^\mu(k_2, k_1)\) are antisymmetric under \((1 \leftrightarrow 2)\) replacement. One can see that \(V_3^\mu(k_2, k_1)\) equals \(V_1^\mu(k_2, k_1)\), where \(e_{1\perp}\) is changed to \(\gamma_\perp\). We also need some of these functions with the substitution \(e_1 \rightarrow n_1\). We mark them with a "\(\sim\)" sign, e.g.
\[ \tilde{F}_{4}^{-\mu}(k_2, k_1) = \tilde{n}_1 (-\gamma^\mu (\hat{v} - m) + 2x_2 v^\mu). \]

Now we are ready to present the PJR effective vertices describing the transition of a particle with momentum \(k + q\) to a jet with momentum \(k = k_1 + k_2\) and a
Reggeon carrying momentum $q$ (everywhere below (1 $\leftrightarrow$ 2) means in addition $G_1 \leftrightarrow G_2$):

$$\Gamma_{\{G_1,G_2\}G} = g^2 \left( t_{G_2 G_1} \gamma_\mu V_\mu(k_2, k_1) \right. \left. \frac{e_2^\mu F_3^\mu(k_2, k_1 + q)}{d(k_2, k_1 + q) - x_2^2 m^2} + (1 \leftrightarrow 2) \right) u_Q, \quad (37)$$

$$\Gamma_{\{G_1,G_2\}Q} = g^2 2k^- e^\mu \left( t_{G_2 G_1} \gamma_\mu \frac{F_3^\mu(k_2, k_1)}{d(k_2, k_1) - x_2^2 m^2} \right. \left. + \left[ t_{G_2 G_1} \right. \right. \left. \gamma_\mu \frac{F_3^\mu(k_2, k_1 + q)}{d(k_2, k_1 + q) - x_2^2 m^2} \right) u_Q, \quad (38)$$

where $e^\mu$ is the polarization vector of the incoming gluon.

$$\Gamma_{\{Q,G\}Q} = -g^2 e_{2,\perp} \bar{u}_Q \left( t_{G_2 G_1} \gamma_\mu \frac{F_3^\mu(k_2, k_1)}{d(k_2, k_1) - x_2^2 m^2} \right. \left. + \left[ t_{G_2 G_1} \right. \right. \left. \gamma_\mu \frac{F_3^\mu(k_2, k_1 + q)}{d(k_2, k_1 + q) - x_2^2 m^2} \right) u_Q, \quad (39)$$

where $k_1$ and $k_2$ are momenta of the emitted quark and gluon and $e_2^\mu$ is the polarization vector of the outgoing gluon.

$$\Gamma_{\{Q,G\}G} = -g^2 e^\mu \left( t_{G_2 G_1} \gamma_\mu \frac{F_4^\mu(k_2 + q, k_1)}{d(k_2 + q, k_1) - m^2} \right. \left. + \left[ t_{G_2 G_1} \right. \right. \left. \gamma_\mu \frac{V_1^\mu(k_2, k_1 + q)}{x_1 d(k_2, k_1 + q)} \right) v_Q, \quad (40)$$

where $e^\mu$ is the polarization vector of the incoming gluon and $k_1$ and $k_2$ are momenta of the emitted antiquark and gluon.

$$\Gamma_{\{Q,Q\}Q} = -\frac{g^2}{2k^-} t_{G_2 G_1} \gamma_\mu \bar{u}_Q \gamma_\perp u_Q \left( t_{G_2 G_1} \gamma_\mu \frac{F_4^\mu(k_2, k_1)}{d(k_2, k_1) - m^2} \right. \left. + \frac{g^2}{2k^-} t_{G_2 G_1} \gamma_\mu \bar{u}_Q \gamma_\perp u_Q \left( t_{G_2 G_1} \gamma_\mu \frac{F_4^\mu(k_2 + q, k_1)}{d(k_2 + q, k_1) - x_2^2 m^2} \right) v_Q, \quad (41)$$

where $k_1$ and $k_2$ are momenta of the emitted quark and antiquark.

$$\Gamma_{\{Q,Q\}G} = g^2 e^\mu \bar{u}_Q \left( -t_{G_2 G_1} \gamma_\mu \frac{F_4^\mu(k_2 + q, k_1)}{d(k_2 + q, k_1) - m^2} \right. \left. + t_{G_2 G_1} \gamma_\mu \frac{F_4^\mu(k_2, k_1 + q)}{d(k_2, k_1 + q) - m^2} + \left[ t_{G_2 G_1} \gamma_\mu \frac{F_4^\mu(k_2, k_1)}{d(k_2, k_1) - m^2} \right) v_Q, \quad (42)$$

where $e^\mu$ is the polarization vector of the incoming gluon and $k_1$ and $k_2$ are momenta of the emitted quark and antiquark. Quite analogously, the RRJ
effective vertices describing the production of a jet \( \{P_1 P_2\} \) with momentum \( k = k_1 + k_2 \) at the collision of two Reggeons with momenta \(-q_1\) and \(q_2\) look as follows

\[
\gamma^{\{G_1 G_2\}}_{G_1 \gamma_2} = g^2 T^a_{G_1 G_2} T^a_{\gamma_2 G_1} \left\{ \frac{V_1^\mu(k_2, k_1)}{d(k_2, k_1)} \left( \frac{q_2^2(k_1 + k_2)^2}{(k_1 + k_2)^2} - q_{2\perp}^2 \right) + \frac{e_1^\mu V_2^\mu(k_2, k_1) q_{2\perp}^2}{D(k_2, k_1) (k_1 + k_2)^2} \right\} + g^2 \left( T^{G_1} T^{G_2} \right)_{\gamma_2 G_1} e_{1\perp} \left\{ -\frac{2V_2^\mu(q_2 - k_1, k_1)}{D(q_2 - k_1, k_1)} + q_{2\perp}^2 \left( K_3^\mu(k_2, k_1) - K_3^\mu(q_2 - k_1, k_1) \right) \right\} + (1 \leftrightarrow 2) ;
\]

where \( e_1, e_2 \) are the polarization vectors of the emitted gluons.

\[
\gamma^{\{Q G\}}_{\bar{Q} G} = g^2 \left[ t^{G_1 G_2} \right] \frac{\gamma_1^\mu}{2k_1^\perp} \left( -\frac{2V_2^\mu(k_2, q_2 - k_2)}{D(k_2, q_2 - k_2)} + \frac{q_{2\perp}}{k_2^\perp} K_3^\mu(k_2, q_2 - k_2) \right) - \frac{g^2}{k} \left( t^{\bar{Q} G} \frac{q_{2\perp}^2 F_2^\mu(k_2, k_1)}{d(k_2, k_1) - x_2^2 m^2} - t^{\bar{Q} G} \hat{e}_{2\perp} \left\{ \frac{F_5(k_1, q_2 - k_1)}{D(q_2 - k_1, k_1) - m^2} + 1 \right\} \right) ,
\]

where \( k_1 \) and \( k_2 \) are momenta of the emitted antiquark and gluon and \( e_2 \) is the emitted gluon polarization;

\[
\gamma^{\{Q G\}}_{\bar{Q} G} = g^2 e_{2\perp} \bar{u}_Q \left[ t^{\bar{Q} G} \right] \left\{ \frac{F_1^{-\mu}(q_2 - k_1, k_1)}{D(q_2 - k_1, k_1) - x_2^2 m^2} - K_1^{-\mu}(q_2 - k_1, k_1)(\hat{q}_{2\perp} - m) \right\} - t^{\bar{Q} G} \frac{F_1^{-\mu}(k_2, q_2 - k_2)}{D(k_2, q_2 - k_2) - x_2^2 m^2} - t^{\bar{Q} G} \frac{F_3^{-\mu}(k_2, k_1)}{d(k_2, k_1) - x_2^2 m^2} + t^{\bar{Q} G} \left( K_1^{-\mu}(k_2, k_1) + K_2^{-\mu}(k_2, k_1) - K_3^{-\mu}(k_2, q_2 - k_2) \right)(\hat{q}_{2\perp} - m) + t^{\bar{Q} G} \left( \frac{F_3^{-\mu}(k_2, k_1)}{d(k_2, k_1) - x_2^2 m^2} \right) \frac{1}{k_{1\perp} + k_{2\perp} - m}(\hat{q}_{2\perp} - m)
\]

where \( k_1 \) and \( k_2 \) are momenta of the emitted quark and gluon and \( e_2 \) is the
emitted gluon polarization;

\[
\gamma^{\{QQ\}}_{\bar{q}_2 q_2} = \frac{g^2}{k^-} \bar{u}_Q \left( t_{\bar{q}_2 q_2} \frac{\hat{n}_1 F_5(k_2, q_2 - k_2)}{D(k_2, q_2 - k_2) - m^2} - t_{\bar{q}_2 \bar{q}_2} \frac{\hat{n}_1 F_5(q_2 - k_1, k_1)}{D(q_2 - k_1, k_1) - m^2} \right) u_Q \\
+ \frac{g^2}{k^-} \bar{u}_Q \left[ t_{\bar{q}_2 q_2} \gamma_1 \right] \left( \frac{q_2^+}{(k_1 + k_2)^2} \left( \hat{n}_1 F_5(k_2, k_1) - \frac{(k_1 + k_2)^2}{d(k_2, k_1) - m^2} \hat{F}_{-\mu}^{-\mu}(k_2, k_1) \right) u_Q \right) \\
+ \frac{g^2}{k^-} \bar{u}_Q \left[ t_{\bar{q}_2 \bar{q}_1} \right] \left( -\hat{n}_1 + \frac{q_2^+}{d(k_2, k_1) - m^2} \right) u_Q ;
\]

\[
\gamma^{\{Q \bar{Q}\}}_{Q_1, Q_2} = -\frac{g^2}{2k_2} t^a \gamma_1 \left\{ \bar{u}_Q t^a \left( \frac{F_{-\mu}^{-\mu}(q_2 - k_1, k_1)}{D(q_2 - k_1, k_1) - x_2 m^2} - \frac{K_1^{-\mu}(q_2 - k_1, k_1)(q_2 - m)}{d(k_2, k_1) - m} \right) \\
- \frac{g^2}{k^- (k_1 + k_2)^2} \bar{u}_Q t^a \left( \hat{n}_1 F_5(k_2, k_1) - \frac{(k_1 + k_2)^2}{d(k_2, k_1) - m^2} \hat{F}_{-\mu}^{-\mu}(k_2, k_1) \right) u_Q \right\} \\
- \frac{g^2}{2k^- (d(k_2, q_2 - k_2) - m^2)} \bar{u}_Q t^a \left( \hat{F}_{-\mu}^{-\mu}(k_2, k_1) \right) u_Q ;
\]

where \( k_1 \) and \( k_2 \) are momenta of the emitted quark and antiquark.

### 3 Bootstrap conditions in QMRK

In this article we prove a part of the quark Reggeization hypothesis in the NLA for the QMRK or in other words the multi–Regge form (6) for the QMRK. Here it seems sensible to make two remarks concerning ”Reggeization” and ”signaturization”. These notions were carefully discussed in [6] for the case of the LLA. In the QMRK all conclusions are valid as well with the corresponding substitution of ”jets” for ”particles”, so the reader is referred to the end of Section 2 of [6].

The proof of the form (6) in the QMRK may be done by obvious extension of the corresponding argumentation presented in [6] for the LLA. It is based on the relations required by the compatibility of the quasi–multi–Regge form of the QCD amplitudes with the s–channel unitarity (bootstrap relations). Fulfillment of the bootstrap relations impose some constraints on the Regge vertices and the trajectory (bootstrap conditions). The bootstrap conditions are formulated in this Section and checked in the next one.

It is worth mentioning that besides all, the argumentation in [6] uses the amplitude in Born approximation to calculate loop–by–loop all radiative corrections to Born amplitudes and examine the formula (6). Therefore, the Born
form for the QMRK has to be proved first. It is verified for the corresponding
tree amplitudes from which the vertices $\gamma^{P_1P_2}_\{R\}$ and $\Gamma^{P_1P_2}_R$ were extracted.
For the amplitude with an arbitrary number $n$ of emitted particles the proof
may be performed via the $t$–channel unitarity as it was done in [10,1] for the
MRK.

In order to present the bootstrap conditions in a compact way similar to one in
[6] we use operator formalism as in [6] slightly adopting it for jet production.
So, $\langle G_i |$ and $| G_i \rangle$ are ”bra”– and ”ket”–vectors for $t$–channel states of the
Reggeized gluon with transverse momentum $r_{i\perp}$ and colour index $G_i$. Their
scalar product is

$$\langle G_i | G_j \rangle = r_{i\perp}^2 \delta(r_{i\perp} - r_{j\perp}) \delta_{G_i G_j}. \quad (46)$$

Similarly, we introduce $\langle Q_i |$ and $| Q_i \rangle$ denoting the $t$–channel states of the
Reggeized quark with transverse momentum $r_{i\perp}$, colour index $Q_i$ and spinor
index $\rho_i$ and their scalar product

$$\langle Q_i | Q_j \rangle = (m - \hat{r}_{i\perp})_{\rho_i \rho_j} \delta(r_{i\perp} - r_{j\perp}) \delta_{Q_i Q_j}. \quad (47)$$

Two–Reggeon states are built from the above ones. It is useful to distin-
guish the states $| R_i R_j \rangle$ (the corresponding ”bra”–vector $\langle R_i R_j \rangle$) and $| R_j R_i \rangle$ (”bra”–vector $\langle R_j R_i \rangle$). We associate the first of them with the case when
Reggeon $R_i$ is located in the lower part of Fig. 1, i.e. when it belongs to
$A^{n+2}_{AB \to A'B'}$ in the unitarity relation, and the second with the case when it is
in the upper part of Fig. 1, i.e. in the amplitude $A_{n+2 \to A'B'}^{R}$. We define three
types of states

$$| G_i G_j \rangle = | G_i \rangle | G_j \rangle, \quad | G_i Q_j \rangle = | G_i \rangle | Q_j \rangle, \quad | Q_i G_j \rangle = | Q_i \rangle | G_j \rangle. \quad (48)$$

States of different types are orthogonal one another. All of them create a

![Fig. 1. Schematic representation of the s–channel discontinuity disc\(^S_s A_{AB \to A'B'}\).](image-url)
complete set, i.e.

\[
\langle \Psi | \Phi \rangle = \int \langle \Psi | \mathcal{G}_1 \mathcal{G}_2 \rangle \frac{d^{D-2}r_{1\perp} d^{D-2}r_{2\perp}}{r_{1\perp}^2 r_{2\perp}^2} \langle \mathcal{G}_1 \mathcal{G}_2 | \Phi \rangle \\
+ \int \langle \Psi | \mathcal{Q}_1 \mathcal{G}_2 \rangle \frac{d^{D-2}r_{1\perp} d^{D-2}r_{2\perp}}{(m - \hat{r}_{1\perp}) r_{2\perp}^2} \langle \mathcal{Q}_1 \mathcal{G}_2 | \Phi \rangle \\
+ \int \langle \Psi | \mathcal{G}_1 \mathcal{Q}_2 \rangle \frac{d^{D-2}r_{1\perp} d^{D-2}r_{2\perp}}{(m - \hat{r}_{2\perp}) r_{1\perp}^2} \langle \mathcal{G}_1 \mathcal{Q}_2 | \Phi \rangle,
\]

where summation over colour and spin indices is assumed.

Bootstrap conditions relate jet production operators, impact–factors and jet production effective vertices. We define impact–factors describing jet production in the fragmentation regions of initial particles as the projections of the \(t\)-channel states \(|\{B'_1 B'_2\} B\rangle\) or \(|\{A'_1 A'_2\} \bar{A}\rangle\) on two–Reggeon states (cf. eq.(32-33) in [6]):

\[
\langle \mathcal{R}_1 \mathcal{R}_2 |\{B'_1 B'_2\} B \rangle = \delta(r_{1\perp} + r_{2\perp} - q_{B\perp}) \\
\times \sum_P \left\{ \frac{1}{2p_B} \left( \Gamma^\mathcal{R}_2 \{B'_1 B'_2\} P \Gamma^\mathcal{R}_1 \pm \Gamma^\mathcal{R}_2 \{B'_1 B'_2\} \Gamma^\mathcal{R}_1 \right) \right. \\
+ \frac{1}{2p_{B'_1}} \left( \Gamma^\mathcal{R}_2 \{B'_1 B'\} P \Gamma^\mathcal{R}_1 \{B'_2 B'_2\} P \Gamma^\mathcal{R}_1 \pm \Gamma^\mathcal{R}_2 \{B'_1 B'_2\} \Gamma^\mathcal{R}_1 \{B'_2 B'_2\} P \right) \\
+ \left. \frac{1}{2p_{B'_2}} \left( \Gamma^\mathcal{R}_2 \{B'_2 B'_2\} P \Gamma^\mathcal{R}_1 \{B'_1 B'_2\} P \Gamma^\mathcal{R}_1 \pm \Gamma^\mathcal{R}_2 \{B'_2 B'_2\} \Gamma^\mathcal{R}_1 \{B'_1 B'_2\} P \right) \right\},
\]

(50)

where \(\Gamma^\mathcal{R}_{\{B'_1 B'_2\} B}\) are the particle–jet–Reggeon (PJR) effective vertices describing particle–jet \(P \rightarrow \{P'_1 P'_2\}\) transition due to the interaction with Reggeon \(\mathcal{R}\); the \(+\) (\(-\)) sign stands for the fermion (boson) state in the \(t\)-channel, \(q_B = p_B - p_{B'_1} - p_{B'_2}\), the sum is taken over quantum numbers of particles \(P\) (they can be different in different terms) and the factor \(1/p_{B_i}\) comes from the phase space element in the unitarity relation (see eqs. (27,38) in [6]). The factor 1/2 and the last term in each brackets in (50) stand on account of the "signaturization". The bar over particle symbol means, as usual, antiparticle while \(\Gamma^\mathcal{R}_{\{B'_2 B'_2\} P}\) and \(\Gamma^\mathcal{R}_{\{B'_1 B'_2\} P}\) are obtained from \(\Gamma^\mathcal{R}_{\{B'_2 B'_2\} P}\) and \(\Gamma^\mathcal{R}_{\{B'_1 B'_2\} P}\) correspondingly taking of wave functions (polarization vectors and Dirac spinors) of \(B\) and \(B'_i\) the wave functions of \(B\) and \(B'_i\) from the first term in brackets of (50).

Quite analogously,

\[
\langle \{A'_1 A'_2\} \bar{A} | \mathcal{R}_1 \mathcal{R}_2 \rangle = \delta(r_{1\perp} + r_{2\perp} - q_{A\perp}) \\
\times \sum_P \left\{ \frac{1}{2p_A} \left( \Gamma^\mathcal{R}_2 \{A'_1 A'_2\} P \Gamma^\mathcal{R}_1 \pm \Gamma^\mathcal{R}_2 \{A'_1 A'_2\} \Gamma^\mathcal{R}_1 \right) \right. \\
+ \frac{1}{2p_{A'_1}} \left( \Gamma^\mathcal{R}_2 \{A'_1 A'\} P \Gamma^\mathcal{R}_1 \{A'_2 A'_2\} P \Gamma^\mathcal{R}_1 \pm \Gamma^\mathcal{R}_2 \{A'_1 A'_2\} \Gamma^\mathcal{R}_1 \{A'_2 A'_2\} P \right) \\
+ \left. \frac{1}{2p_{A'_2}} \left( \Gamma^\mathcal{R}_2 \{A'_2 A'_2\} P \Gamma^\mathcal{R}_1 \{A'_1 A'_2\} P \Gamma^\mathcal{R}_1 \pm \Gamma^\mathcal{R}_2 \{A'_2 A'_2\} \Gamma^\mathcal{R}_1 \{A'_1 A'_2\} P \right) \right\},
\]

(51)
\[
+ \frac{1}{2p_{A_1}^+} \left( \Gamma_{A'_2 P_{A_2}^+} R_{A'_1 P_{A_2}^+ A} \pm \Gamma_{A P_{A_2}^+} R_{A_2 P_{A_2}^+} \right)
+ \frac{1}{2p_{A_2}^+} \left( \Gamma_{A'_2 P_{A_2}^+} R_{A'_1 P_{A_2}^+ A} \pm \Gamma_{A P_{A_2}^+} R_{A_2 P_{A_2}^+} \right)
\]
(51)

where \( q_A = p_{A'_1} + p_{A'_2} - p_A \).

The strong bootstrap conditions resulting from the quasi–elastic (one final particle is a two–particle jet) amplitudes impose the following constraints on the impact factors

\[
|\{ \bar{B}_1 B'_1 \} B \rangle = g | R_\omega(q_{B_\perp}) \rangle \Gamma_{\{ A'_1 A'_2 \} B} \},
|\{ A'_1 A'_2 \} \rangle = g \Gamma_{\{ A'_1 A'_2 \} A} \langle R_\omega(q_{A_\perp}) \rangle,
\]
(52)

where \( | R_\omega(q_{\perp}) \rangle \) are universal (process independent) eigenstates of the kernel \( \hat{K} \) (see eq.(43) in [6]) with the eigenvalues \( \omega_R(q) \). From calculations in leading order we know that

\[
\langle G_1 G_2 | R_\omega(q_{\perp}) \rangle = \delta(r_{1\perp} + r_{2\perp} - q_{\perp}) T_{G_1 G_2}^q,
\]

\[
\langle G_1 Q_2 | Q_\omega(q_{\perp}) \rangle = \delta(r_{1\perp} + r_{2\perp} - q_{\perp}) t_{G_1}^q,
\]

\[
\langle Q_1 G_2 | Q_\omega(q_{\perp}) \rangle = -\delta(r_{1\perp} + r_{2\perp} - q_{\perp}) t_{G_2}^q.
\]
(53)

Similarly to the impact factors for scattering jets we define the impact factors for Reggeon–jet transitions (compare with (51)) as

\[
\langle R_1 R_2 | \{ \bar{P}_1 \bar{P}_2 \} R_{j+1} \rangle = \delta(r_{1\perp} + r_{2\perp} - q_{j\perp})
\]
\[
\times \sum_P \left\{ \frac{1}{2k_-} \left( \Gamma_{\{ P_1 P_2 \} P} R_{P R_1 R_{j+1}}^P \pm \Gamma_{P(\bar{P}_1 \bar{P}_2)} R_{P R_2 R_{j+2}}^P \right)
+ \frac{1}{2k_1} \left( \Gamma_{\{ P_1 P_2 \} P} R_{P R_1 R_{j+1}}^P \pm \Gamma_{P(\bar{P}_1 \bar{P}_2)} R_{P R_2 R_{j+2}}^P \right)
+ \frac{1}{2k_2} \left( \Gamma_{\{ P_1 P_2 \} P} R_{P R_1 R_{j+1}}^P \pm \Gamma_{P(\bar{P}_1 \bar{P}_2)} R_{P R_2 R_{j+2}}^P \right) \right\},
\]
(54)

where \( \gamma_{\{ P_1 P_2 \}} \) are the RRJ effective vertices, describing production of jets \{ \( P_1 P_2 \) \} at the Reggeon transition \( R_{i+1} \rightarrow R_i \); \( k = k_1 + k_2 \) is the jet momentum, \( q_{j\perp} = q_{(j+1)\perp} - q_{\perp} \), the \(+\)(−) sign stands for the case when the Reggeon quantum numbers (i.e. quark or gluon) in the \( j \) and \( j + 1 \) channels are equal (different). Analogously

\[
\langle \{ P_1 P_2 \} R_{i+1} R_2 \rangle = \delta(r_{1\perp} + r_{2\perp} - q_{(i+1)\perp})
\]
\[
\times \sum_P \left\{ \frac{1}{2k_+} \left( \Gamma_{\{ P_1 P_2 \} P} R_{P R_1 R_2}^P \pm \Gamma_{P(\bar{P}_1 \bar{P}_2)} R_{P R_2 R_2}^P \right)
+ \frac{1}{2k_1} \left( \Gamma_{\{ P_1 P_2 \} P} R_{P R_1 R_2}^P \pm \Gamma_{P(\bar{P}_1 \bar{P}_2)} R_{P R_2 R_2}^P \right)
+ \frac{1}{2k_2} \left( \Gamma_{\{ P_1 P_2 \} P} R_{P R_1 R_2}^P \pm \Gamma_{P(\bar{P}_1 \bar{P}_2)} R_{P R_2 R_2}^P \right) \right\},
\]
(55)
where \( q_{(i+1)\perp} = q_{i\perp} + k_{\perp} \).

In expressions (54),(55) we introduced the effective vertices \( \gamma_{\{P_1 P_2\}}^{R_i R_j} \). One can obtain them from \( \gamma_{\{P_1 P_2\}}^{\{P_i, P_j\}} \) replacing the wave–functions of the emitted particles with the wave–functions of the corresponding incoming antiparticles:

\[
\bar{u}_Q \rightarrow \bar{u}_{\bar{Q}}, \quad v_Q \rightarrow u_Q, \quad e_{\bar{Q}} \rightarrow e_Q
\]  

(56)

and inverting the momenta \( k_{P_i} \rightarrow -k_{\bar{P}_i} \). In fermion case we also have to change the overall sign due to the operator ordering. There is no uniformity in literature on the matter of including this factor \((-1)\) into the definition of the corresponding effective vertices \( \gamma_{\{P_1 P_2\}}^{R_i R_j} \) or into the impact–factor definition. We define all pair–production vertices without it. As for the RRJ effective vertices, we follow the denotations from [6], where \( \gamma_{Q}^{\bar{G}} \) is defined with \((-1)\).

Thus, when this factor arises and it is not included in the RRJ or PPR effective vertex we explicitly write it in the impact–factor.

Finally, we introduce the operator \( \{P_1 P_2\} \) for the production of a jet \( \{P_1 P_2\} \) with the overall momentum \( k = k_{1\perp} + k_{2\perp} \) as having the following matrix elements:

\[
\langle R_1 R_2 \| \{P_1 P_2\} \| R_1' R_2' \rangle = \delta(q_{(i+1)\perp} - k_{\perp} - q_{i\perp}) \\
\gamma_{\{P_1 P_2\}}^{\{P_i, P_j\}} \delta_{R_2 R_2'} \delta(r_{2\perp} - r_{2\perp}') d_{R_2} + \gamma_{\{P_1 P_2\}}^{\{P_i, P_j\}} \delta_{R_1 R_1'} \delta(r_{1\perp} - r_{1\perp}') d_{R_1} \\
\gamma_{\{P_1 P_2\}}^{P_1 P_2} \delta(k_{1\perp} + r_{1\perp} - r_{1\perp}') + \gamma_{\{P_1 P_2\}}^{P_1 P_2} \gamma_{\{P_1 P_2\}}^{P_2 P_1} \delta(k_{2\perp} + r_{1\perp} - r_{1\perp}') \rangle,
\]  

(57)

where \( q_{i\perp} = r_{1\perp} + r_{2\perp} \), \( q_{(i+1)\perp} = r_{1\perp}' + r_{2\perp}' \).

An additional bootstrap condition, which may be obtained from the bootstrap relation for the amplitudes of a process \( A + B \rightarrow A' + \{P_1 P_2\} + B' \), is analogous to the LLA one [6]. For ”ket”–vectors it reads as follows

\[
\langle \{P_1 P_2\} | R_\omega(q_{(i+1)\perp}) \rangle g_{d_{i+1}}(q_{(i+1)\perp}) + \langle \{P_1 P_2\} | R_{i+1} \rangle = |R_\omega(q_{i\perp})\rangle g_{\gamma_{\{P_1 P_2\}}}^{\{P_i, P_j\}},
\]  

(58)

where \( q_{i\perp} = q_{(i+1)\perp} - k_{\perp} \), while for ”bra”–vectors this condition has the form

\[
g_{d_{i}}(q_{i\perp}) \langle R_\omega(q_{i\perp}) \| \{P_1 P_2\} \rangle + \langle \{P_1 P_2\} | R_{i+1} \rangle = g_{\gamma_{\{P_1 P_2\}}}^{\{P_i, P_j\}} \langle R_\omega(q_{(i+1)\perp}) \| \{P_1 P_2\} \rangle,
\]  

(59)

where \( q_{(i+1)\perp} = q_{i\perp} + k_{\perp} \).

Note that the conditions for ”ket”– and ”bra”– vectors in (58–59) and (52) are not independent since these vectors are interrelated. Indeed, the replacement of ”+” and ”−” momenta components turns any of them into the other, so we consider only ”ket”–vectors herein.

Jet production in the central region in the Reggeized gluon collision and in
the fragmentation region with the Reggeized gluon was considered in [11]. We investigate here the bootstrap conditions for Reggeized quarks.

4 Verification of bootstrap conditions on Reggeon vertices

In this Section we explicitly show that bootstrap conditions (52) and (58)–(59) are satisfied by the known expressions for the effective vertices presented in Section 2. For this purpose we have chosen such a spacial parametrization of these Regge vertices that their following insertion into the bootstrap conditions leads to trivial cancellations.

Each bootstrap condition on concrete Regge vertex we check only for \( \langle G_1 Q_2 \rangle \) (or for \( \langle Q_1 G_2 \rangle \)). Looking at the diagrams one can see that in case of \( \langle Q_1 G_2 \rangle \) the calculation is quite analogous being different only in an overall ”-” sign and the replacement \( r_2 \leftrightarrow r_1 \).

4.1 Two–gluon jet production in fragmentation region

We begin with two–gluon jet production in the fragmentation region. In this and the next subsection let us denote the momentum of the incoming quark \( Q_B \) as \( p_B \) and the momenta of the outgoing Reggeized gluon \( G_1 \) and quark \( Q_2 \) as \( r_1 \) and \( r_2 \) respectively, \( r_1 + r_2 = q \). Here the momenta of the emitted gluons are \( k_1 \) and \( k_2 \). We also often meet shifted momenta

\[
k_{1\perp}' = k_{1\perp} + r_{1\perp}, \quad k_{2\perp}' = k_{2\perp} + r_{2\perp}, \quad k_{2\perp}'' = k_{2\perp} + r_{1\perp}, \quad k_{1\perp}'' = k_{1\perp} + r_{2\perp}. \tag{60}
\]

The bootstrap condition for \( \Gamma^Q_{\{G_1 G_2\}Q} \) has the form:

\[
\langle G_1 Q_2 | \{G_1 \bar{G}_2\} Q_B \rangle = \langle G_1 Q_2 | Q_\omega(q_\perp) \rangle g \Gamma^Q_{\{G_1 G_2\}Q_B}, \tag{61}
\]

where

\[
\begin{align*}
\langle G_1 Q_2 | \{G_1 \bar{G}_2\} Q_B \rangle &= \delta(q_\perp + k_\perp - p_{B\perp}) \\
& \times \left[ \frac{1}{4k_{1\perp}} \left( \sum_Q \Gamma^Q_{\{G_1 G_2\}Q} \Gamma^Q_{\bar{G}_1 \bar{G}_2} + \sum_G \Gamma^G_{\{G_1 \bar{G}_2\}G} \Gamma^G_{\bar{G}_1 Q B} \right) \\
& + \frac{1}{2k_{1\perp}} \left( \sum_Q \Gamma^Q_{\{G_1 \bar{G}_2\}Q_B} + \sum_G \Gamma^G_{\{G_1 \bar{G}_2\}G} \Gamma^Q_{\bar{G}_1 Q B(G G_2)} \right) \\
& + 1 \leftrightarrow 2 \right]. \tag{62}
\end{align*}
\]

As in the definition of the effective vertices we use the denotation \( 1 \leftrightarrow 2 \) assuming the replacement \( \{e_{1\perp}, k_{1\perp}, x_1, G_1\} \leftrightarrow \{e_{2\perp}, k_{2\perp}, x_2, G_2\} \). The parts
of (62) yield:

\[
\frac{1}{2k^\perp} \sum_Q \Gamma_{\{Q, G\} Q}^{G_1 G_2} \Gamma_{Q Q B}^{G_1} = g^3 \left( -tG_2 tG_1 tG_1 \frac{e_2 \mu \tilde{F}_3^\mu(k_2, k_1)}{d(k_2, k_1) - x_2^2 m^2} 
+ tG_2 tG_1 tG_1 \frac{\gamma_1^\mu V_1^\mu(k_2, k_1)}{d(k_2, k_1)} + (1 \leftrightarrow 2) \right) u_{Q B},
\]

(63)

\[
\frac{1}{2k^\perp} \sum_G \Gamma_{\{G, G\} Q}^{G_1} \Gamma_{Q Q B}^{G_1} = -g^3 \left( \left[ tG_2 tG_1 \right] V_1^\mu(k_2, k_1) \langle G_{Q B} G \rangle \right) \right) u_{Q B},
\]

(64)

\[
\frac{1}{2k^\perp} \sum_G \Gamma_{\{G, G\} Q}^{G_1 \Gamma_{Q B} G_2} = g^3 \left( \left[ tG_2 tG_1 \right] \frac{e_2 \mu \tilde{F}_3^\mu(k_2, k_1)}{d(k_2, k_1) - x_2^2 m^2} 
+ \left[ tG_2 tG_1 \right] \frac{\gamma_1^\mu V_1^\mu(k_2, k_1)}{d(k_2, k_1)} + (1 \leftrightarrow 2) \right) u_{Q B}.
\]

(65)

\[
\frac{1}{2k^\perp} \sum_G \Gamma_{\{G, G\} Q}^{G_1} \Gamma_{Q B}^{G_2} = g^3 \left( -tG_2 tG_1 \frac{e_2 \mu \tilde{F}_3^\mu(k_2, k_1)}{d(k_2, k_1) - x_2^2 m^2} 
- tG_2 tG_1 \frac{e_2 \mu \tilde{F}_3^\mu(k_2, p_B - k_2)}{d(k_2, p_B - k_2) - x_2^2 m^2} 
+ \left[ tG_2 tG_1 \right] \frac{\gamma_1^\mu V_1^\mu(k_2, k_1)}{d(k_2, k_1)} \right) u_{Q B}.
\]

(66)

Here we use the relation $\hat{e}_{1\perp} \hat{n}_2 F_3^{-\mu} = 2F_3^\mu - \hat{n}_1(\ldots)$ to obtain (65). Now one can clearly see that (61) is an identity.

The bootstrap condition for antiquark–gluon production reads as

\[
\langle G_1 Q_2 |(\bar{Q} G)Q B \rangle = \langle G_1 Q_2 |Q_\omega(q_\perp) \rangle g \Gamma_{\{Q G\} Q B}^G,
\]

(67)

but we need not check it because $\Gamma_{\{Q G\} Q B}^G$ is connected with the two–gluon production effective vertex $\Gamma_{\{\bar{G}, G\} Q B}^Q$ by crossing rules.

### 4.2 Quark–antiquark jet production in fragmentation region

We denote the momenta of the emitted quark $Q_1$ and antiquark $\bar{Q}_2$ as $k_1$ and $k_2$. The bootstrap condition for $\Gamma_{\{Q_1, \bar{Q}_2\} Q}$ has the form:
\begin{equation} 
\langle G_1 Q_2 | \{ \bar{Q}_1 Q_2 \} Q_B \rangle = \langle G_1 Q_2 | Q_\omega (q_\perp) \rangle g \Gamma_{(Q_1 Q_2)Q} \tag{68} 
\end{equation}

where

\begin{equation} 
\langle G_1 Q_2 | \{ \bar{Q}_1 Q_2 \} Q_B \rangle = \delta(q_\perp + k_\perp - p_{B\perp}) 
\times \left[ \frac{1}{2k^2} \left( \sum_Q \Gamma^{Q_2}_{Q_1 Q_2} \Gamma^{\bar{Q}_1}_{Q Q_B} + \sum_G \Gamma^{\bar{Q}_1}_{Q_1 Q_2} \Gamma^{Q_2}_{Q G} \right) + \frac{1}{2k^2} \sum_Q \Gamma^{\bar{Q}_1}_{Q Q_B} \Gamma^{Q_2}_{Q B(\bar{Q} Q_2)} \right] + \frac{1}{2k^2} \left( \sum_G \Gamma^{Q_2}_{Q_1 G(\bar{Q} Q_1) Q_B} + \sum_Q \Gamma^{\bar{Q}_1}_{Q_1 Q_2} \Gamma^{Q_2}_{Q B(\bar{Q} Q_2)} \right). \tag{69} 
\end{equation}

The parts of (69) yield:

\begin{equation} 
\frac{1}{2k^2} \sum_Q \Gamma^{Q_2}_{Q_1 Q_2} \Gamma^{\bar{Q}_1}_{Q Q_B} = \frac{g^3}{2k^2} \left[ -t^a t^{\bar{G}_1 \gamma_\perp \mu} u_{Q_B} \otimes \bar{u}_Q, \frac{t^a F^{-\mu}_{4}(k_2, k_1)}{d(k_2, k_1) - m^2} v_{Q_2} \right] + \frac{t^a \gamma^\mu_{x_2}}{x_2} v_{Q_2} \otimes \bar{u}_Q, \frac{t^a t^{\bar{G}_1} F_{3}^{-\mu}(k_2', k_1)}{d(k_2', k_1) - x^2 m^2} u_{Q_B}. \tag{70} \end{equation}

\begin{equation} 
\frac{1}{2k^2} \sum_G \Gamma^{\bar{Q}_1}_{Q_1 Q_2} \Gamma^{Q_2}_{Q B(\bar{Q} Q_2)} = -\frac{g^3}{2k^2} \left[ -t^a \gamma^\mu_{x_2} u_{Q_B} \otimes \bar{u}_Q, \frac{t^a t^{\bar{G}_1} F_{4}^{-\mu}(k_2', k_1)}{d(k_2', k_1) - m^2} v_{Q_2} \right] + \frac{t^a \gamma^\mu_{x_2}}{x_2} v_{Q_2} \otimes \bar{u}_Q, \frac{t^a t^{\bar{G}_1} F_{3}^{-\mu}(k_2', k_1)}{d(k_2', k_1) - x^2 m^2} u_{Q_B}. \tag{71} \end{equation}

\begin{equation} 
\frac{1}{2k^2} \sum_Q \Gamma^{Q_2}_{Q_1 Q_2} \Gamma^{\bar{Q}_1}_{Q Q_B} = -\frac{g^3}{2x^2 k} \left[ -t^a \gamma^\mu_{x_2} u_{Q_B} \otimes \bar{u}_Q, \frac{t^a t^{\bar{G}_1} F_{3}^{-\mu}(k_2', k_1)}{d(k_2', k_1) - x^2 m^2} v_{Q_2} \right] + \left[ t^a t^{\bar{G}_1} \right] \frac{F_{3}^{-\mu}(p_B - k_1, k_1)}{d(p_B - k_1, k_1) - x^2 m^2} - \left[ t^a t^{\bar{G}_1} \right] \frac{F_{3}^{-\mu}(k_2', k_1)}{d(k_2', k_1) - x^2 m^2} u_Q, \tag{73} \end{equation}

\begin{equation} 
\frac{1}{2k^2} \sum_G \Gamma^{\bar{Q}_1}_{Q_1 Q_2} \Gamma^{Q_2}_{Q B(\bar{Q} Q_2)} = \frac{g^3}{2k} \left[ -\bar{u}_Q, \frac{t^a t^{\bar{G}_1} F_{4}^{-\mu}(k_2', k_1)}{d(k_2', k_1) - m^2} v_{Q_2} \otimes t^a \gamma^\mu_{x_2} u_{Q_B} \right] + \frac{t^a t^{\bar{G}_1} \gamma^\mu_{x_2}}{x_2} v_{Q_2} \otimes \bar{u}_Q, \frac{t^a F_{3}^{-\mu}(p_B - k_1, k_1)}{d(p_B - k_1, k_1) - x^2 m^2} u_{Q_B}. \tag{74} \end{equation}

Now one can clearly see that (68) is an identity.
4.3 Two–gluon jet production in central region

Here and in the following subsections we denote the momentum of the incoming Reggeon as $q_1$ and the momenta of the outgoing Reggeons $R_1$ and $R_2$ (the corresponding "bra"–vector $\langle R_1 R_2 \rangle$) as $r_1$ and $r_2$ correspondingly, $r_1 + r_2 = q_1$. The momenta of the emitted particles (here they are gluons $G_1$ and $G_2$) are $k_1$ and $k_2$ ($k_1 + k_2 = k$). The bootstrap condition for $\gamma_{Q_1, Q_2}^{G_1, G_2}$ reads as follows

$$
\langle Q_1 G_2 \rangle \{ G_1 G_2 \} | Q_\omega (q_{2\perp}) \rangle g (m - \tilde{q}_{2\perp}) + \langle Q_1 G_2 \rangle \{ G_1 G_2 \} Q_2
= -\delta (q_{1\perp} + k_{\perp} - q_{2\perp}) t^{G_2} g \gamma_{Q_1, Q_2}^{G_1, G_2},
$$

where $\{ G_1 G_2 \}$ is the operator of two gluon production with the matrix element

$$
\langle Q_1 G_2 \rangle \{ G_1 G_2 \} | Q'_1 G'_2 \rangle = \delta (q_{1\perp} + k_{\perp} - q_{2\perp})
\times \left( \gamma_{Q_1, Q'_1}^{G_1, G_1} \delta_{G_2 G'} \delta (r_{2\perp} - r'_{2\perp}) \bar{r}_{2\perp} \right)
\gamma_{Q_1, Q_2}^{G_1, G_2}
\gamma_{Q'_1, Q'_2}^{G_1, G_2}
\delta_{r_1 - r'_1} (m - \tilde{r}_{1\perp})
\gamma_{G_1, G_2}^{G_1, G_2}
\delta (r_{2\perp} + k_{2\perp} - r'_{2\perp})
+ \gamma_{G_1, G_2}^{G_1, G_2}
\delta (r_{2\perp} + k_{1\perp} - r'_{2\perp})
$$

and

$$
\langle Q_1 G_2 \rangle \{ G_1 G_2 \} Q_2
= \delta (q_{1\perp} + k_{\perp} - q_{2\perp})
\times \left[ \frac{1}{4k_{\perp}} \left( \sum_G \Gamma^2_{G_1 G_2} \gamma_{Q_1, Q_2}^G + \sum_Q \Gamma^{Q_1}_{Q_1} \gamma_{Q_1 Q_2}^{G_2} \right) \right]
+ \frac{1}{2k_{\perp}} \left( \sum_G \Gamma^2_{G_1 G_2} \gamma_{Q_1 Q_2}^{G G_2} + (-1) \sum_Q \Gamma^{Q_1}_{Q} \gamma_{Q Q_2}^{G_2} \right) + (1 \leftrightarrow 2)
$$

The term $\gamma_{G_1, G_2}^{G_2} \gamma_{Q_1, Q'_1} \delta (r_{2\perp} + k_{2\perp} - r'_{2\perp})$ in bootstrap condition (76) is transformed via (14) as

$$
\gamma_{G_1, G_2}^{G_2} \gamma_{Q_1, Q'_1} \delta (r_{2\perp} + k_{2\perp} - r'_{2\perp}) = \delta (q_{1\perp} + k_{\perp} - q_{2\perp}) g^{3t_{G_1}} [ t_2^{G_2} t_{G_2}^{G_2} ]
\times \left\{ e^{G_2}_{2\perp} \left( K^t_2 (k_2', k_1') - K^t_2 (k_2, k_1) \right) + K^t_1 (k_2, k_1) - K^t_1 (k_2', k_1') \right\}
+ e^{G_2}_{1\perp} \left( K^t_3 (k_2, k_1) - K^t_3 (k_2', k_1) - K^t_3 (k_2', k_1') \right) \right\} (\tilde{q}_{2\perp} - m),
$$

The similar procedure helps us to rewrite the contribution of $\gamma_{G_1, G_2}^{G_1} \gamma_{Q_1, Q'_1}^G$. Performing cancellations (which are trivial due to our effective vertex presenta-
(80)

One can easily check that the sum of (79) and (80) gives the r.h.s. of (75) fulfilling bootstrap condition (58) for this case.
4.4 Quark–antiquark jet production in central region

We denote the momenta of the emitted quark and antiquark as \(k_1\) and \(k_2\) respectively. The bootstrap condition for \(\gamma_{Q_1\bar{Q}_2}\) has the form:

\[
\langle Q_1 G_2 | \{\bar{Q}_1 Q_2\} | Q_\omega (q_{2\perp}) \rangle g (m - \bar{q}_{2\perp}) + \langle Q_1 G_2 | \{Q_1 Q_2\} Q_2 \rangle = \langle Q_1 G_2 | Q_\omega (q_{1\perp}) \rangle g \gamma_{Q_1\bar{Q}_2}, \tag{81}
\]

where \(\{\bar{Q}_1 Q_2\}\) is the operator of quark-antiquark production with the matrix elements

\[
\langle Q_1 G_2 | \{\bar{Q}_1 Q_2\} | Q'_1 G'_2 \rangle = \delta(q_{1\perp} + k_{\perp} - q_{2\perp}) - \left[ \gamma_{Q_1\bar{Q}_1'} \delta_{Q_2 G'_2} \delta(r_{2\perp} - r'_{2\perp}) + \gamma_{Q_1\bar{Q}_2} \delta_{Q_1 G'_1} \delta(r_{1\perp} - r'_{1\perp}) (m - \bar{r}_{1\perp}) \right], \tag{82}
\]

and

\[
\langle Q_1 G_2 | \{\bar{Q}_1 Q_2\} Q_2 \rangle = \delta(q_{1\perp} + k_{\perp} - q_{2\perp}) \gamma_{Q_1\bar{Q}_2} \gamma_{Q_1 Q_2} \delta(r_{2\perp} + k_{1\perp} - r'_{2\perp}), \tag{83}
\]

\[
\langle Q_1 G_2 | \{\bar{Q}_1 Q_2\} | Q_\omega (q_{2\perp}) \rangle g (m - \bar{q}_{2\perp}) \] yields

\[
\delta(q_{1\perp} + k_{\perp} - q_{2\perp}) g \gamma_{Q_1\bar{Q}_2} \gamma_{Q_1 Q_2} \frac{\gamma_\perp}{2k_2} u_{Q_2} \times u_{Q_1} \gamma_{Q_1 G_2} \left( K_{1\perp -} (k''_2, k_1) + K_{2\perp -} (k''_2, k_1) - K_{1\perp -} (k''_2, k''_1) - K_{2\perp -} (k''_2, k''_1) \right) (\bar{q}_{2\perp} - m). \tag{85}
\]
We can present the result for \( \langle Q_1 G_2 | \{ \bar{Q}_1 Q_2 \} Q_2 \rangle \) in the following form:

\[
\langle Q_1 G_2 | \{ \bar{Q}_1 Q_2 \} Q_2 \rangle = \delta(q_{1\perp} + k_{\perp} - q_{2\perp}) \frac{g^3}{k^-} \left[ t^a \frac{\gamma^\mu}{2 x_2} u_{Q_2} \right.
\]

\( \otimes \bar{u}_{Q_1} \left\{ t^{\bar{g}_2 t^a} \left( \frac{F^{-\mu}_1 (q_2 - k_1, k_1)}{D (q_2 - k_1, k_1) - x_2 m^2} - K^{-\mu}_1 (q_2 - k_1, k_1) (\hat{q}_{2\perp} - \hat{m}) \right)
\right.
\]

\( - t^{\bar{g}_2 t^a} \left( K^{\mu}_1 (k_2^\prime, k_1) + K^{\mu}_2 (k_2^\prime, k_1) - K^{\mu}_2 (k_2^\prime, k_1) - K^{\mu}_1 (k_2^\prime, k_1) \right)
\)

\( (\hat{q}_{2\perp} - \hat{m}) 
\]

\( \left. - t^{\bar{g}_2 t^a} \left( \frac{F^{-\mu}_1 (k_2^\prime, k_1)}{D (k_2^\prime, k_1) - x_2 m^2} \frac{1}{\hat{q}_{2\perp} - \hat{r}_{2\perp} - \hat{m}} - K^{-\mu}_1 (k_2^\prime, k_1) (\hat{q}_{2\perp} - \hat{m}) \right) \right\}
\]

\( + (\hat{q}_{2\perp} - \hat{m}) \left\{ \gamma^\mu \frac{1}{2} \left[ t^{\bar{g}_2 t^a} - \frac{t^{\bar{g}_2 t^a}}{q_{2\perp} - \hat{r}_{2\perp} - \hat{m}} \right] - \frac{t^{\bar{g}_2 t^a}}{(q_{2\perp} - \hat{r}_{2\perp} - \hat{m})} \right\}
\]

\( \otimes \bar{u}_{Q_1} \left\{ t^{\bar{g}_2 t^a} \left( \frac{F^{-\mu}_1 (k_2^\prime, k_1)}{d (k_2^\prime, k_1) - m^2} \right) u_{Q_2} \right\} \right] . \quad (86)
\]

The contribution of \( \langle Q_1 G_2 | \{ \bar{Q}_1 Q_2 \} | Q_\omega (q_{2\perp}) \rangle g (m - \hat{q}_{2\perp}) \) into the bootstrap relation reads as follows:

\[
\langle Q_1 G_2 | \{ \bar{Q}_1 Q_2 \} | Q_\omega (q_{2\perp}) \rangle g (m - \hat{q}_{2\perp})
\]

\[
= \delta(q_{1\perp} + k_{\perp} - q_{2\perp}) \frac{g^3}{k^-} \left[ t^a \frac{\gamma^\mu}{2 x_2} u_{Q_2} \right.
\]

\( \otimes \bar{u}_{Q_1} \left\{ t^a \bar{g}_2 \left( \frac{F^{-\mu}_1 (k_2^\prime, k_1)}{D (k_2^\prime, k_1) - x_2 m^2} \frac{1}{\hat{q}_{2\perp} - \hat{r}_{2\perp} - \hat{m}} - K^{-\mu}_1 (k_2^\prime, k_1) (\hat{q}_{2\perp} - \hat{m}) \right)
\right.
\]

\( + t^{\bar{g}_2 t^a} \left( K^{\mu}_1 (k_2^\prime, k_1) + K^{\mu}_2 (k_2^\prime, k_1) - K^{\mu}_2 (k_2^\prime, k_1) - K^{\mu}_1 (k_2^\prime, k_1) \right)
\)

\( (\hat{q}_{2\perp} - \hat{m}) 
\]

\( \left. - (\hat{q}_{2\perp} - \hat{m}) t^a \right\}
\]

\( \otimes \bar{u}_{Q_1} \hat{n}_{\mu} \left[ \left[ t^a \bar{g}_2 \right] + \left[ t^a \bar{g}_2 F_5 (k_2^\prime, k_1) \right] \frac{1}{D (k_2^\prime, k_1) - m^2} - \frac{t^{\bar{g}_2 t^a} F_5 (k_2^\prime, k_1)}{D (k_2^\prime, k_1) - m^2} \right] u_{Q_2}
\]

\( + \left\{ \gamma^\mu \frac{1}{2} \left[ \frac{t^{\bar{g}_2 t^a}}{\hat{q}_{2\perp} - \hat{r}_{2\perp} - \hat{m}} - \frac{t^{\bar{g}_2 t^a}}{(q_{2\perp} - \hat{r}_{2\perp} - \hat{m})} \right] \hat{q}_{2\perp} - \hat{m} \right\}
\]

\( \otimes \bar{u}_{Q_1} \left\{ \frac{t^{\bar{g}_2 t^a}}{d (k_2^\prime, k_1) - m^2} u_{Q_2} + (\hat{q}_{2\perp} - \hat{m}) t^{\bar{g}_2 t^a} \right\}
\]

\( \otimes \bar{u}_{Q_1} \left\{ \frac{t^{\bar{g}_2 t^a}}{d (k_2^\prime, k_1) - m^2} \frac{1}{\hat{n}_1} + \frac{1}{\hat{n}_1} F_5 (k_2^\prime, k_1) \right\}
\]

\( \left. - \frac{\hat{n}_1 F_5 (k_2^\prime, k_1)}{d (k_2^\prime, k_1) - m^2} - \frac{(k_1 + k_2)^\mu}{d (k_2, k_1) - m^2} \right\} u_{Q_2} \right] . \quad (87)
\]

The sum of the two last expressions gives the r.h.s of (81) fulfilling bootstrap condition for \( \gamma^{\{Q_1 Q_2\}} \).
4.5 *Quark–gluon jet production in central region*

We denote the momenta of the emitted quark and gluon as \( k_1 \) and \( k_2 \). The bootstrap condition for \( \gamma_{G_1Q_2} \) has the form:

\[
\langle G_1Q_2 \mid \{ Q_1G_2 \} \mid Q_\omega(q_{2\perp}) \rangle \, g \, (m - \hat{q}_{2\perp}) + \langle G_1Q_2 \mid \{ \bar{Q}_1\bar{G}_2 \} \mid Q_2 \rangle = \langle G_1Q_2 \mid Q_\omega(q_{1\perp}) \rangle \, g \, \gamma_{G_1Q_2},
\]

(88)

where \( \{ Q_1G_2 \} \) is quark–gluon production operator with the matrix elements

\[
\langle G_1Q_2 \mid \{ Q_1G_2 \} \mid Q'_1Q'_2 \rangle = \delta(q_{1\perp} + k_\perp - q_{2\perp}) \times \left[ \gamma_{G_1Q'_1} \delta_{G_2Q'_2} \delta(r_{2\perp} - r'_{2\perp})r_{2\perp}^2 + \gamma_{G_2Q'_2} \gamma_{G_1Q'_1} \delta(r_{2\perp} + k_\perp - r'_{2\perp}) \right],
\]

(89)

and

\[
\langle G_1Q_2 \mid \{ \bar{Q}_1\bar{G}_2 \} \mid Q_2 \rangle = \delta(q_{1\perp} + k_\perp - q_{2\perp}) \times \left[ \gamma_{\bar{G}_2Q}_2 \delta_{G_1Q'_1} \delta(r_{1\perp} - r'_{1\perp})r_{1\perp}^2 + \gamma_{\bar{G}_1Q'_1} \gamma_{\bar{G}_2Q_2} \delta(r_{2\perp} + k_\perp - r'_{2\perp}) \right].
\]

(90)

The contribution of \( \gamma_{G_1G_2}^G \gamma_{G_2Q_2}^Q \) into \( \langle G_1Q_2 \mid \{ Q_1G_2 \} \mid Q_\omega(q_{2\perp}) \rangle \, g \, (m - \hat{q}_{2\perp}) \) yields

\[
\delta(q_{1\perp} + k_\perp - q_{2\perp}) \, g^3 t_{G_2} [ t^G_1 t^{G_2} ] \\
\times e_{2\perp}^\mu \bar{u}_{Q_1} \left( K_2^\mu (k_2', k''_2) + K_1^\mu (k''_2, k''_1) - K_2^\mu (k_2', k_1) - K_1^\mu (k''_2, k_1) \\
- K_2^\mu (k_2', k_1') - K_1^\mu (k_2', k_1') + K_2^\mu (k_2, k_1) + K_1^\mu (k_2, k_1) \right) (\hat{q}_{2\perp} - m)
\]

(92)

and the contribution of \( \gamma_{G_2Q_2}^G \gamma_{G_1Q_1}^Q \) can be obtained from (92) by the substitution \( r_1 \leftrightarrow r_2, \ G_1 \leftrightarrow G_2 \).
We can present the result for \( \langle G_1 G_2 \mid Q_1 G_2 \rangle Q_2 \) in the following form:

\[
\langle G_1 G_2 \mid Q_1 G_2 \rangle Q_2 = \delta(q_{1\perp} + k_{\perp} - q_{2\perp}) g^3 e_{2\perp}^\mu \bar{u}_{Q_1} \left( -t_{G_2} [t_{G_1} G_2] - \frac{F_3^\mu (k_2, k_1)}{d(k_2, k_1) - x_2^2 m^2} \right)
+
\left[ t_{G_2} [t_{G_1} G_2] \right] \left( \frac{F_1^\mu (q_2 - k_1, k_1)}{D(k_2, k_1) - x_2^2 m^2} - K_1^\mu (q_2 - k_1, k_1) (q_{2\perp} - m) \right)
-
\left[ t_{G_2} [t_{G_1} G_2] \right] \left( \frac{F_3^\mu (q_2 - k_2, k_2)}{D(k_2, k_1) - x_2^2 m^2} + K_2^\mu (q_2 - k_2, k_2) (q_{2\perp} - m) \right)
+
\left\{ \frac{F_3^\mu (k_2, k_1)}{d(k_2, k_1) - x_2^2 m^2} \frac{t_{G_1} t_{G_2} G_2}{t_{G_2} G_1} \left( K_1^\mu (k_2, k_1) - K_1^\mu (k_2, k_1') \right) \right\} (q_{2\perp} - m) \right)
\]

The contribution of \( \langle G_1 G_2 \mid Q_1 G_2 \mid Q_\omega (q_{2\perp}) \rangle g (m - \hat{q}_{2\perp}) \) into the bootstrap relation reads as follows:

\[
\langle G_1 G_2 \mid Q_1 G_2 \mid Q_\omega (q_{2\perp}) \rangle g (m - \hat{q}_{2\perp}) = \delta(q_{1\perp} + k_{\perp} - q_{2\perp}) g^3 e_{2\perp}^\mu \bar{u}_{Q_1} \times \left( \left[ t_{G_2} [t_{G_1} G_2] \right] \left( K_1^\mu (k_2, k_1) + K_2^\mu (k_2, k_1) \right) \right)
-
\left[ t_{G_2} [t_{G_1} G_2] \right] \left( K_1^\mu (k_2, k_1') - K_1^\mu (k_2, k_1) + K_2^\mu (k_2, k_1') - K_2^\mu (k_2, k_1) \right)
+
\left[ t_{G_2} [t_{G_1} G_2] \right] \left( K_1^\mu (k_2, k_1') - K_1^\mu (k_2, k_1') + K_2^\mu (k_2, k_1') - K_2^\mu (k_2, k_1') \right)
+
\left[ t_{G_2} [t_{G_1} G_2] \right] \left( \frac{F_3^\mu (k_2, k_1)}{d(k_2, k_1) - x_2^2 m^2} + \frac{F_3^\mu (k_2, k_1)}{D(k_2, k_1) - x_2^2 m^2} \right) \frac{1}{k_{\perp} - m} - K_1^\mu (k_2, k_1) \right)
We denote the momenta of the emitted antiquark and gluon as \( q_{2\perp} \) and \( k_{2\perp} \) respectively. The bootstrap condition has the form:

\[
\begin{align*}
&- F_3^\mu (k_2, k_1) \frac{d (k_2, k_1) - x_2^2 m^2 q_{2\perp} - \hat{r}_{1\perp} - m}{F_1^\mu (k_2, k_1)} + F_3^\mu (k_2, k_1) \frac{d (k_2, k_1) - x_2^2 m^2 \hat{q}_{2\perp} - \hat{r}_{2\perp} - m}{F_1^\mu (k_2, k_1)} \\
&- \left[ t^G_2 t^G_2 \right] t^G_1 \left( \frac{1}{D (k_2', k_1)} - x_2^2 m^2 \frac{q_{2\perp}}{q_{2\perp} - \hat{r}_{2\perp} - m} - K_1^\mu (k_2', k_1) \right) \\
&+ \left[ t^G_2 t^G_1 \right] t^G_2 \left( \frac{1}{D (k_2', k_1)} - x_2^2 m^2 \frac{q_{2\perp}}{q_{2\perp} - \hat{r}_{2\perp} - m} - K_1^\mu (k_2', k_1) \right) \\
&\left( q_{2\perp} - m \right).
\end{align*}
\]

The sum of the two last expressions precisely gives the r.h.s. of (88).

### 4.6 Antiquark–gluon jet production in central region

We denote the momenta of the emitted antiquark and gluon as \( k_1 \) and \( k_2 \) respectively. The bootstrap condition has the form:

\[
\begin{align*}
&\langle G_1 Q_2 | \{ \hat{Q}_1 G_2 \} | G_2 (q_{2\perp}) \rangle g q_{2\perp}^2 + \langle G_1 Q_2 | \{ Q_1 \hat{G}_2 \} G_2 \rangle \\
&= \langle G_1 Q_2 | Q_2 (q_{1\perp}) \rangle g \gamma_{Q_2 g_2},
\end{align*}
\]

(94)

where \( \{ Q_1 \hat{G}_2 \} \) is the operator of antiquark–gluon production with the matrix element

\[
\begin{align*}
&\langle G_1 Q_2 | \{ \hat{Q}_1 G_2 \} | G'_2 G'_2 \rangle = \delta(q_{1\perp} + k_{\perp} - q_{2\perp}) \\
&\times \left( \gamma_{Q_2 g'_2} \delta_{g'_{1} g'_{2}} \delta(r_{1\perp} - r'_{1\perp}) r_{1\perp}^2 + \gamma_{Q_2 \hat{g}_2} \gamma_{Q_2 g_2} \delta(r_{2\perp} + k_{1\perp} - r'_{2\perp}) \right)
\end{align*}
\]

(95)

and

\[
\begin{align*}
&\langle G_1 Q_2 | \{ Q_1 \hat{G}_2 \} G_2 \rangle = \delta(q_{1\perp} + k_{\perp} - q_{2\perp}) \\
&\times \left[ \frac{1}{2 k_{\perp}} \left( \sum_G \Gamma_{Q_2}^{Q_1 G_2} \gamma_{\hat{g}_1 g_2} - \sum_Q \gamma_{Q_2}^{Q_2 \hat{g}_2} \sum_{Q_1} \Gamma_{Q_2}^{Q_1 G_2} \right) \\
&+ \frac{1}{2 k_{\perp}} \left( \sum_G \Gamma_{Q_2}^{Q_1 G_2} \gamma_{\hat{g}_1 g_2} - \sum_Q \gamma_{Q_2}^{Q_2 \hat{g}_2} \sum_{Q_1} \Gamma_{Q_2}^{Q_1 G_2} \right) \right].
\end{align*}
\]

(96)
The contribution of $\gamma^G_{Q_1} \gamma^G_{Q_2} \gamma^Q_{Q_1} \gamma^Q_{Q_2}$ into $\langle G_1 Q_2 | \{ \hat{Q}_1 G_2 \} | G_2 | \{ Q_1 G_2 \} \rangle$ yields

$$\delta(q_{1\perp} + k_{\perp} - q_{2\perp}) g^3 \left[ \left[ t G_1 G_2 \right] t G_2 \right] \frac{q_{2\perp}}{2k_1}$$

$$\times \left( K^\mu_3 (k_2, k_1') + K^\mu_3 (k_2', k_1) - K^\mu_3 (k_2, k_1') - K^\mu_3 (k_2', k_1) \right) \gamma^\mu_1 \gamma_{Q_1}.$$  (97)

We can present the result for $\langle G_1 Q_2 | \{ Q_1 G_2 \} G_2 \rangle$ in the following form:

$$\langle G_1 Q_2 | \{ Q_1 G_2 \} G_2 \rangle$$

$$= \delta(q_{1\perp} + k_{\perp} - q_{2\perp}) g^3 \left[ \left[ t G_1 G_2 \right] t G_2 q_{2\perp} \frac{(q_2 - r_1)^\mu q_{2\perp}}{D(k_2, k_1) - x_{2\perp}^2 m^2} \right]$$

$$\left\{ \begin{array}{l}
\hline
\end{array} \right.$$
5 Summary

The further development of the quark Reggeization theory in QCD demands a proof of the quark Reggeization hypothesis in the NLO. Besides the MRK the NLA also includes another, quasi–multi–Regge kinematics, in which one of the produced particles is replaced by a jet containing two particles with similar rapidities. In this paper we have proved the quark Reggeization in the QMRK by means of the method analogous to the proof performed in the LO. It is based on explicit verification of the so–called bootstrap conditions — the constraints on the effective Reggeon vertices. These conditions are imposed by the bootstrap relations which are required by the compatibility of the s–channel unitarity with the QMRK form of amplitude (6). We formulate these conditions in the operator formalism in the transverse momentum, colour and spin space. This formalism was firstly introduced in [8], then extended to inelastic amplitudes in [6] and adopted here for the QMRK. The direct insertion of the effective vertices into the bootstrap conditions leads to extremely cumbersome and tedious calculations, which are almost completely cancellations. We make these cancellations transparent for verification introducing nontrivial parametrization of the Reggeon vertices through the set of functions $F_i, K_i, V_i$ (20-33).

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