Abstract: The first example of a quantum group was introduced by P.Kulish and N.Reshetikhin. In their paper ”Quantum linear problem for the sine-Gordon equation and higher representations" published in Zap. Nauchn. Sem. LOMI, 1981, Volume 101 (English version: Journal of Soviet Mathematics, 1983, 23:4), they found a new algebra which was later called $U_q(sl(2))$. Their example was developed independently by V.Drinfeld and M.Jimbo, which resulted in the general notion of quantum group. Recently, the so-called Belavin-Drinfeld cohomologies (twisted and untwisted) have been introduced in the literature to study and classify certain families of quantum groups and Lie bialgebras. Later, the last two authors interpreted non-twisted Belavin-Drinfeld cohomologies in terms of non-abelian Galois cohomology $H^1(F, H)$ for a suitable algebraic $F$-group $H$. Here $F$ is an arbitrary field of zero characteristic. The untwisted case is thus fully understood in terms of Galois cohomology. The twisted case has only been studied using Galois cohomology for the so-called (”standard”) Drinfeld-Jimbo structure. The aim of the present paper is to extend these results to all twisted Belavin-Drinfeld cohomologies and thus, to present classification of quantum groups in terms of Galois cohomologies and orders. Our results show that there exist yet unknown quantum groups for Lie algebras of the types $A_n, D_{2n+1}, E_6$. 