New Dissipation Relaxation Phenomenon in Oscillating Solid $^4$He

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We describe the first observations on the time-dependent dissipation when the drive level of a torsional oscillator containing solid $^4$He is abruptly changed. The relaxation of dissipation in solid $^4$He shows rich dynamical behavior including exponential and logarithmic time-dependent decays, hysteresis, and memory effects.

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The recent discovery [1, 2] and its confirmations [3, 4, 5, 6] of significant low temperature reductions in the rotational moment of inertia of solid $^3$He below 300 mK have generated enormous excitement as possible evidence for the long-sought supersolid behavior [7]. The physical origin of the apparent non-classical rotational inertia (NCRI) extracted from the temperature dependent shift in the torsional oscillator frequency, however, remains unsettled [8]. The majority of the torsional oscillator experiments showing NCRI has been devoted to measuring the oscillator frequency under a variety of conditions in such as crystal quality [3], sample size and geometry, and $^3$He impurity concentration [9]. Relatively little experimental information is available on the dissipation accompanying the torsional oscillation. If the observed reductions in rotational inertia are related, for example, to vortex liquid/solid phenomenon [10, 11], glassy behavior [12, 13], or defect motion [14] in solid $^3$He, then dynamical time-dependent effects might occur in the dissipation in the oscillating solid $^4$He samples. We carried out systematic observations on the time-dependent dissipation accompanying sudden changes in the torsional oscillator displacement amplitude. We have discovered that the dissipation in solid $^4$He shows rich dynamical behavior including exponential and logarithmic time-dependent relaxation, hysteresis and memory effects.

The torsional oscillator utilized was the compound oscillator identical to that described in [6]. Briefly, our oscillator contained an upper “dummy bob” hung by an upper rod and connected to a lower rod attached to the sample chamber containing a cylindrical (10 mm diameter and 8mm height) solid $^4$He. The oscillator resonated at ”in-phase” and ”out-phase” modes at frequencies, $f_1 = 496$ Hz and $f_2 = 1173$ Hz, respectively, whose amplitudes and phases were continuously tracked. The temperature was measured by a calibrated $^3$He melting pressure thermometer mounted onto the vibration isolation copper block to which our oscillator was attached. All of our solid samples were grown from commercially available $^4$He gas with nominal 0.3 ppm $^3$He impurity by the usual blocked capillary method.

The procedure for measuring the time dependent behavior of our torsional oscillator is illustrated in Fig. 1. The $f_2$ mode resonance was continuously tracked by a lock-in amplifier maintaining a constant phase between the drive and the response. The shifts in resonance frequency and the response amplitude were monitored continuously throughout the procedure. The response amplitude was converted to sample rim velocity amplitude, $v_r$. The cell temperature was initially raised to 90 mK, where the NCRI was found to be reversible as a function of drive level [6]. The oscillator was then driven at a given level and allowed to achieve a steady state. The temperature was next lowered within about $10^3$ s to 9.95 mK at the time defined as $t = -t_0 = -5 \times 10^3$ s.

FIG. 1: Time dependence of, ac drive amplitude (a), frequency shift (b), and rim velocity (c) and (d)) at $T = 9.95$ mK. The initial $v_r$ ($\sim 160$ µm/s) corresponds to displacement amplitude of 22 nm. The drive (torque) level is $\propto V_{dc} V_{ac}$, where the dc bias voltage $V_{dc}$ is kept constant. The ring down portion of $v_r$ is not shown in (d). The drive level was increased back to the original level at $t = 5 \times 10^4$ s. The frequency shift does not return to the initial value in accordance with our observation of hysteresis [8]. After the increase in drive, there is no decay similar to that after the first decrease in drive. The ring down time of the empty oscillator itself ranges from 120 to 200 s.
s and subsequently regulated within 10 μK. The drive level was reduced to half of the initial value at \( t = 0 \) (see panel (a)). Following the reduction in drive, the rim velocity (panel (d)) unexpectedly "undershot" and eventually recovered in a complex relaxation process. The usual ring down of the oscillator resonance transpired during the time interval prior to the recovery from the undershooting. When the drive level was restored to the original value at \( t = 4.4 \times 10^4 \) s, \( v_r \) increased more than twice but the relaxation behavior was absent. When the process was repeated to decrease the drive level by half at \( t = 5.4 \times 10^4 \) s, the initial undershoot behavior was now absent but "remembered" its previous state at the same drive level. Thus, the unexpected relaxation phenomenon occurs only in the first decrease from one high drive level to a lower one. Majority of the relaxation phenomena described here was studied with the high drive level to a lower one. Majority of the relaxation phenomena described here was studied with the high drive level to a lower one. The parameters \( v_0 \) and \( B \) are first set by fitting the data in the long time limit \( (t > 1 \times 10^4 \) s), where the exponential term is negligible. The "origin" of time is taken as that when the final temperature is stabilized \( (t = -t_0) \) to account for the decay during the waiting time interval \( t_0 \). Once the logarithmic decay parameters are set, the remainder is fitted with the exponential dependence in Eq. (1). The fits (lines) with parameters shown in Fig. 2 appear adequate. The exponential decay amplitude \( C \) decreases as the temperature is raised and vanishes in the fit at 66.6 mK in Fig. 2.

During the observed decay process, the oscillator frequency also showed time dependence (see Fig. 1(b)). When \( v_r \) was initially decreased, the oscillator frequency (therefore, the NCRI fraction) increased. This is the "critical velocity" effect as seen previously. There was an associated relaxation in frequency shift which appeared with similar dynamical behavior as \( v_r \). Owing to the much smaller effect on the frequency relaxation, we focused on the much larger effects of dissipation dynamics. Observations on the frequency relaxation resulting from temperature changes were reported recently.

To see how the amplitude relaxation phenomena as shown in Fig. 1 depended on temperature, the same protocol was followed except the final temperature. Some of such measurements are shown in Fig. 2. Similar undershoot in rim velocity was seen at temperatures below 50 mK just after the decrease in drive level. Following the relatively rapid recovery, there was a much slower relaxation at \( t \gtrsim 10^4 \) s. The long time tail behavior after the decrease of drive level was complicated. In the long time limit, the rim velocity decreased in the temperature range, 20 mK > \( T > 30 \) mK, but it increased outside of this range.

There have been suggestions that the supersolid state might involve glassy behavior in some way. The decay of magnetization of spin glass systems has been observed under certain conditions to decay with stretched exponential time dependence. Fitting our data to such time dependence was attempted but inadequate. We find that a sum of purely exponential and logarithmic time dependencies can fit our data satisfactorily:

\[
v_r = v_0 + B \ln(t + t_0) - C \exp(-t/\tau)
\]

(1)

where \( v_0 \), \( B \), \( C \) and \( \tau \) are fitting parameters. The short and long time recovery behaviors are described by the exponential and the logarithmic time dependence, respectively. The parameters \( v_0 \) and \( B \) are first set by fitting the data in the long time limit \( (t > 1 \times 10^4 \) s), where the exponential term is negligible. The "origin" of time is taken as that when the final temperature is stabilized \( (t = -t_0) \) to account for the decay during the waiting time interval \( t_0 \). Once the logarithmic decay parameters are set, the remainder is fitted with the exponential dependence in Eq. (1). The fits (lines) with parameters shown in Fig. 2 appear adequate. The exponential decay amplitude \( C \) decreases as the temperature is raised and vanishes in the fit at 66.6 mK in Fig. 2.

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FIG. 2: Time dependence of the sample rim velocity (\( v_r \)) at different temperatures. The same protocol as in Fig. 1 was followed. The temperatures were 9.95, 29.78, 47.85 and 66.60 mK. The lines are fits to Eq. (1) discussed in the text.
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To examine the dependence of the relaxation behavior on the initial drive level, observations were made at 29.45 mK in the same protocol as above keeping the same final \( v_r \) at \( \sim 22 \) μm/s but varying the initial \( v_r \) between 180 and 100 μm/s. As shown in Fig. 3 the undershoot magnitude increased as the difference between the initial and final \( v_r \) was increased. A small dependence on the difference was also present (see below) in the long time tail behavior. When the cell was filled with superfluid \(^4\)He, there was no relaxation other than the oscillator ring down. The inset shows the dissipation in the sample (subsequent to the end of the undershoot) computed from the observed amplitude \( A \) \( (A_0) \) in the presence of solid (superfluid) \(^4\)He sample via \( \delta(1/Q) \equiv (1/Q) - (1/Q_0) \).
The behavior of the parameter $B$ is complicated. It is known that the particular thermal history (13 and our studies) of a solid $^4$He sample affects the oscillator frequency shift and its relaxation. For example, the direction of decay depends on the direction of temperature change. We do not yet have a complete thermal history characterization of the logarithmic decay of the dissipation and it is possible that the observed sign reversal of $B$ is related to some subtle difference in thermal history.

Possible association has been suggested between the observed NCRI phenomena and the dislocation line network depending on the number, $n_3$, of condensed $^3$He impurities on the lines (14). We consider the following effect of dislocation on our observed dissipation relaxation phenomena. Increasing the drive level might partially breakaway $^3$He from the dislocation lines. Then, the distance ($L_i$) between adjacent pinning sites at $^3$He locations increases (20). Just after the drive level is decreased at $t = 0$, $n_3$ is initially too small for the new lower drive level and there is an excess dissipation resulting in the undershooting of the measured amplitude. As more $^3$He recondenses onto the dislocation lines, the dissipation might decrease as seen by the subsequent increase in amplitude. A difficulty with this scenario is that assuming a uniform dislocation line density $\Lambda \sim 10^5$ cm$^{-2}$ (average separation between lines $\sim 17\mu$m) and taking the diffusion coefficient $D$ of $^3$He impurities (of concentration $x_3$) in solid $^4$He as $Dx_3 \approx 3 \times 10^{-11}$ cm$^2$/s at low temperatures (21) give a diffusion time less than 10 s, short compared to $\tau$ in Fig. 4. The observed $\tau$ might instead be related to the time for $^3$He to migrate along dislocation lines towards an equilibrium distribution.

Another possible mechanism for the observed relaxation dynamics is the effect on the number of dislocations induced by the change in oscillation amplitude. Hiki and Tsuruoka (22) showed that suddenly imposed changes in temperature $\Delta T$ could exert internal shear stresses and thereby change the number of dislocations. The measured relaxation in ultrasonic attenuation was interpreted as evidence for quantum mechanical tunneling of dislocations (22). The tunneling rate increased as the imposed change in temperature was increased. If the change in oscillator amplitude in our experiment were equivalent to (but likely to be much less than) $\Delta T = (1/10)/T$, the resulting tunneling time constant would be $10^3$ s, much longer than the observed value. The quantum mechanical tunneling mechanism also leads to a temperature-independent relaxation time contrary to our finding. Thus, it is not likely that the relaxation time we observe is related to the dislocation tunneling. The observed long-term memory effects and logarithmic time dependence are also difficult to explain by the dislocation-related mechanisms.

We discuss next our observations in terms of a vortex liquid model (10) of supersolid state. In the model, NCRI is interpreted as the rotational susceptibility of the quantized vortex tangle and the dissipation arises from the interaction between the torsional oscillation and the...
thermally fluctuating vorticies. Prior to the first change in the drive level, our procedure of initiating the oscillation at a high temperature is likely to have brought the sample solid $^4$He to a quasi-equilibrium array of vortices for the drive level at the temperature of measurement. The observed logarithmic time dependence probably arises from the thermally activated motion of vortices from one metastable state to another. This process continues throughout the measurement while the sample is maintained at the constant temperature. Just after the drive level is decreased, the existing vortices are out of equilibrium with the new drive level. The observed undershoot may be a transient excess dissipation owing to the "wrong" number of vortices. The vortices now must adjust to the new drive level by, say, moving out of the sample. The characteristic time involved in the macroscopic motion of the vortices is then the observed time constant $\tau$. The motion presumably involves processes occurring both within the sample and at the surface boundaries. The absence of overshoot and exponential decay in both the oscillator response and NCRI fraction implies that the drive reversal does not replenish vortices into the sample. This indicates the presence of surface barrier which inhibits creation of vortices in the system. When the drive level is decreased again, the number of vortices is already near that appropriate for the velocity and there is no undershoot nor accompanying exponential relaxation. The time constant becomes shorter as the vortices begin to move more freely. This vortex liquid state appears to occur above about 60 mK. When the vortex liquid "freezes," the time constant would diverge. Our sample at the lowest temperature apparently did not attain a totally frozen vortex state.

In conclusion, we studied the dynamics of dissipation imparted by cylindrical solid $^4$He samples onto a torsional oscillator by suddenly decreasing the oscillator drive. Unexpectedly, the rim velocity initially undershoots, recovers from the undershoot exponentially with a characteristic time constant and eventually decays logarithmically in the long time limit. The time constant tends to diverge as temperature is decreased below 15 mK.

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