STUDYING FREQUENCY RELATIONSHIPS OF KILOHERTZ QUASI-PERIODIC OSCILLATIONS FOR 4U 1636–53 AND Sco X-1: OBSERVATIONS CONFRONT THEORIES

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ABSTRACT

By fitting the frequencies of simultaneous lower and upper kilohertz quasi-periodic oscillations (kHz QPOs) in two prototype neutron star (NS) QPO sources (4U 1636–53 and Sco X-1), we test the predictive power of all currently proposed QPO models. Models predict a linear, power law, or other relationship between the two frequencies. We found that for plausible NS parameters (mass and angular momentum), no model can satisfactorily reproduce the data, leading to very large chi-square values in our fittings. For both 4U 1636–53 and Sco X-1, this is largely due to the fact that the data significantly differ from a linear relationship. Some models perform relatively better but still have their own problems. Such a detailed comparison of data from models enables identification of routes for improving those models further.

Key words: accretion, accretion disks – stars: neutron – X-rays: stars

Online-only material: color figures

1. INTRODUCTION

The launch of the X-ray timing satellite Rossi X-ray Timing Explorer (RXTE) led to the discovery of kilohertz quasi-periodic oscillations (kHz QPOs) of low-mass X-ray binaries (LMXBs) in their X-ray light curves. The frequencies of kHz QPOs range from a few hundred to about 1000 Hz; their timescales correspond to the dynamical time of the innermost regions of the accretion flow. Thus, such signals may carry crucial information about the central neutron star (NS), such as mass, spin frequency, angular momentum, radius, magnetic fields, and so on. Usually, the twin kHz QPOs appear simultaneously, and the lower and upper QPOs are almost directly proportional to each other (see, e.g., Belloni et al. 2005, 2007; van der Klis 2006).

Various theoretical models have been proposed to account for kHz QPO signals. Table 1 shows all the present models we collected. Although each model achieves success to a certain extent, the origin of kHz QPOs is still highly debated. Furthermore, many new models have emerged in recent years, and systematic comparisons of them have not been well studied.

In view of this, we investigate systematically the predictive ability of the present kHz QPO models. We focus on those that predict the frequency relationship of the twin kHz QPOs. The moving hot-spots model is not included. This model performs three-dimensional magnetohydrodynamic (MHD) simulations of the accretion around an NS. The simulations show that the moving hot spots on the surface of an NS can develop oscillations in the light curves. However, this model does not provide any analytic relationship between the twin QPOs (Bachetti et al. 2010).

In this work, we measure the frequency relationships of the twin kHz QPOs for 4U 1636–53 and Sco X-1, and then fit the models with the measured results. We choose these two NS systems for several reasons. First, both of them have strong kHz QPOs over a wide frequency range; both of them have been observed more than 10 years with RXTE. Thus, the bias from the sample selection is minimized. Second, the different properties of these two sources allow us to discuss the predictive ability of the models. They are typical atoll and Z sources, because they display characteristic “atoll” and “Z or i” shapes in their color–color or color–intensity diagrams, respectively. The putative spin frequency for 4U 1636–53 is 581 Hz (Strohmayer 2001; Strohmayer & Markwardt 2002), whereas the spin frequency of Sco X-1 remains unknown.

In the following, we first describe the data reduction procedure. Then, we fit the frequency relationships to all the available models. The predictions of NS properties in each model will be presented. Finally, we discuss and conclude our investigation results.

2. DATA ANALYSIS

We have retrieved all the public archival data of the two sources with the Proportional Counter Array (PCA) onboard RXTE. The observation time is from 1996 February 28 to 2007 September 25 for 4U 1636–53, and from 1996 May 5 to 2006 February 4 for Sco X-1.

For 4U 1636–53, we use the event-mode data for 1156 ObsIDs, with time resolution better than 256 μs and an energy band of 2–40 keV. With a similar analysis procedure to those of Barret et al. (2006) and Boutelier et al. (2009), the Power Density Spectrum (PDS) as well as the QPO parameters (peak frequency, width, and amplitude) in each ObsID are obtained. For the ObsIDs with PDS containing the lower QPO, we track its time evolution every 128 s. Following that in Barret et al. (2005), all instances of the 128 s PDS are aligned in every 30 Hz interval of the lower QPO with the shift-and-add technique (Méndez et al. 1998). Then, we search for the twin QPOs in each interval. For the ObsID with PDS containing only the upper QPOs, we...


187 ObsIDs with time resolution better than 256 $\mu$s. The Astrophysical Journal

For Sco X-1, we analyze the Generic Binned–mode data in Note.

MHD Alfvén-wave oscillation (16)

R-T G. Wave (14, 15)

Rayleigh–Taylor gravity wave (14, 15)


directly align the PDS of each ObsID in every 30 Hz interval of the upper QPO. Again, the twin QPOs in each interval are searched. Finally, the two parts of the results are combined and we obtain the frequency relationship.

For Sco X-1, we analyze the Generic Binned–mode data in 187 ObsIDs with time resolution better than 256 $\mu$s and an energy band of 2–40 keV. Considering the effects of the dead time, we use a model of two Lorentzians plus a power law to fit each PDS. The power-law component denotes the dead-time-modified Poisson noise; the Lorentzians account for the contribution of the twin kHz QPOs. We then apply the shift-and-add technique to the ObsID-averaged PDS as described above on the upper QPOs’ frequency, because the span and significance of the upper QPOs are larger than those of the lower QPOs. The interval of shift-and-add is 50 Hz. Similar to the result in Méndez & van der Klis (2000), our frequency relationship shows some subtle structure when the lower QPO is around 800 Hz.

3. COMPARISONS BETWEEN MODELS AND DATA

In the following, we restrict the NS parameters $M \in [1.4, 2.4]M_\odot$ and $j \in [0, 0.3]$ ($j \equiv Jc/GM^2$) in our fittings, corresponding to reasonable equations of state (EOS) of NSs (Lattimer & Prakash 2007), where $M$ and $j$ are the mass and dimensionless angular-momentum parameter of an NS, respectively. The fitting results are summarized in Table 2. For some models, the best-fitting values of $M$ and $j$ approach the upper or lower limits. In each of these cases, we made an extended fitting to relax the limits to $M \in [1.0, 4.0]M_\odot$ and $j \in [0, 0.5]$; these results are presented in Table 3 and discussed in Section 4.

3.1. Comparison of the Sonic-point and Spin-resonance Models

The sonic-point and spin-resonance models (Miller et al. 1998; Lamb & Miller 2001, 2003) attribute the formation of the

Table 1

| Models                        | Reference |
|-------------------------------|-----------|
| Sonic point and spin resonance | (1, 2, 3) |
| Orbital resonance (three models) | (4, 5)    |
| Precession (three models)     | (6, 7, 8) |
| Deformed-disk oscillation     | (9)       |
| “−1r, −2v” resonance         | (10, 11)  |
| Higher-order nonlinearity     | (12)      |
| Tidal disruption              | (13)      |
| Rayleigh–Taylor gravity wave | (14, 15)  |
| MHD Alfvén-wave oscillation   | (16)      |
| MHD                           | (17)      |
| Moving hot spots              | (18)      |

References. (1) Miller et al. 1998; (2) Lamb & Miller 2001; (3) Lamb & Miller 2003; (4) Kluźniak & Abramowicz 2001; (5) Abramowicz et al. 2003; (6) Stella & Vietri 1999; (7) Bursa 2005; (8) Suchlňk et al. 2007; (9) Kato 2001; (10) Török et al. 2007; (11) Bakala et al. 2008; (12) Mukhopadhyay 2009; (13) Germanà et al. 2009; (14) Osherovich & Titarchuk 1999; (15) Titarchuk 2003; (16) Zhang 2004; (17) Shi & Li 2009; (18) Bachetti et al. 2010.

Table 2

The Fit Parameters of All Models for 4U 1636–53 and Sco X-1

| Models                        | 4U 1636–53 | Sco X-1 |
|-------------------------------|-----------|---------|
| $M (M_\odot)$                 | j or Spin (Hz) | Reduced $\chi^2$ | dof | $M (M_\odot)$ | j or Spin (Hz) | Reduced $\chi^2$ | dof |
| SP SR Case 1                  |           |         |     |           |         |     |
| Linear relation               | 2.04 ± 0.01 | 0.19 ± 0.01 | 11  | 14 | 2.09 ± 0.01 | 0.07 ± 0.01 | 71  | 7  |
| Forced 1:3                    | 1.815 ± 0.003 | 0.0001 ± 0.0010 | 186  | 16 | 1.97 ± 0.01 | 0.0000 ± 0.0010 | 306  | 9  |
| Forced 1:2                    | 2.095 ± 0.006 | 0.0 ± 0.0 | 28  | 16 | 2.32 ± 0.01 | 0.0 ± 0.0001 | 54  | 9  |
| Rel. Pre.                     | 2.319 ± 0.003 | 0.3 ± 0.0 | 156  | 16 | 2.40 ± 0.00 | 0.250 ± 0.0001 | 244  | 9  |
| Ver. Pre.                     | 2.160 ± 0.003 | 0.3 | 158  | 16 | 2.33 ± 0.001 | 0.299 ± 0.0001 | 230  | 9  |
| Tot. Pre.                     | 1.814 ± 0.003 | 0.0 | 186  | 16 | 1.971 ± 0.001 | 0.0001 ± 0.0 | 306  | 9  |
| Deformed-disk                 | 2.400 ± 0.001 | 0.0 | 31  | 16 | 2.400 ± 0.001 | 0.0 | 447  | 9  |
| “−1r, −2v” Res.              | 2.400 ± 0.001 | 0.238 ± 0.0002 | 154  | 16 | 2.400 ± 0.002 | 0.172 ± 0.0002 | 248  | 9  |
| Hi. nonline. n = 1            | 1.401 ± 0.001 | 0.0107 | 85  | 14 | 1.40 ± 0.10 | 467 ± 1 | 86  | 7  |
| Fix spin                      | 2.10 ± 0.02 | 581 | 211  | 15 | 1.40 ± 0.01 | 294 ± 1 | 70  | 7  |
| Hi. nonline. n = 2            | 1.401 ± 0.001 | 0.0107 | 70  | 14 | 1.40 ± 0.10 | 294 ± 1 | 70  | 7  |
| Tidal disruption              | 2.400 ± 0.003 | 0.166 ± 0.0001 | 18  | 16 | 2.400 ± 0.002 | 0.045 ± 0.0001 | 43  | 9  |
| R-T G. Wave                   | 1.78 ± 0.38 | 371.1 ± 2.7 | 10  | 14 | 1.66 ± 0.34 | 350.4 ± 3.3 | 4.4  | 7  |
| Fix spin                      | 1.86 ± 0.38 | 581 | 38  | 15 |                                    |         |         |   |
| Alfvén Wave Res.              | $A = 0.699 ± 0.002$ | 81  | 17  | 68  | 10 |
| MHD                           | $\epsilon = 0.647 ± 0.013, 0.014$ | 470.3 ± 7.7 | 10  | 16 | $\epsilon = 0.928 ± 0.007, 0.004$ | 635 ± 5.6 | 56  | 9  |
| Fix spin                      | $\epsilon = 0.928 ± 0.003$ | 581 | 21  | 17 |

Note. The errors were computed by setting $\Delta \chi^2 = 1$.  

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where $c_N$ is the characteristic inward radial velocity of gas.

The frame of the model, the X-ray source is an NS with a surface magnetic field about $10^7$–$10^{10}$ G and a spin of a few hundred Hz that accretes gas via a Keplerian disk. At the sonic point $r_{sp}$, some of the accreting gas is channeled by the magnetic field and then impacts the NS surface to produce the lower QPO. Some remain in clumps with the Keplerian disk flow, producing the upper QPO. Therefore, the upper frequency $\nu_2$ is the Keplerian frequency $\nu_K$ at $r_{sp}$; the lower frequency $\nu_1$ is the beat frequency between $\nu_K$ and the NS spin $\nu_s$, i.e., $\nu_1 \approx \nu_B = \nu_K - \nu_s$. The first version of the model (Miller et al. 1998) leads to a constant peak separation $\Delta \nu$, close to $\nu_s$. As pointed out in Psaltis et al. (1998), the varying $\nu_s$ observed in both Z and atoll sources suggests that the above assumptions of $\nu_2$ and $\nu_1$ in the model should be relaxed. The second version (Lamb & Miller 2001) introduced inward drifts of gas to make $\Delta \nu$ dependent on $\nu_1$ (or $\nu_2$). The inward drifts make $\nu_1$ greater than $\nu_B$ and $\nu_2$ less than $\nu_K$ for a prograde gas flow:

$$\nu_1 \approx \nu_B/(1 - \nu_{cl}/\nu_B),$$

$$\nu_2 \approx \nu_K \left(1 - \frac{1}{2}\nu_{cl}/\nu_B\right),$$

where $\nu_{cl}$ is the inward radial velocity of clumps near $r_{sp}$ and $\nu_B$ is the characteristic inward radial velocity of gas. $\nu_{cl}$ and $\nu_B$ are supposed to be approximately constant during the lifetime of a clump, and $\nu_{cl} \ll \nu_B$.

Lamb & Miller (2003) proposed the third version to explain that the frequency separation is close to $\nu_{spin}$ in some stars but close to $\nu_{spin}/2$ in others. The upper QPO is likewise close to the Keplerian frequency $\nu_K$ at $r_{sp}$. They showed that magnetic and radiation fields rotating with the star will preferentially excite vertical motion in the disk at the “spin-resonance” radius $r_{sr}$, where $\nu_K - \nu_s$ is equal to the vertical epicyclic frequency. There are two cases in this model. Case 1 supposes that the flow at $r_{sr}$ is relatively smooth, and the vertical motion excited at $r_{sr}$ modulates the X-ray flux at $\nu_1 \approx \nu_2 - \nu_s$. Case 1 is fully compatible with the second version (Lamb & Miller 2001). Case 2 assumes that the flow at $r_{sr}$ is highly clumped. In this case, the vertical motion excited at $r_{sr}$ modulates the X-ray flux at $\nu_1 \approx \nu_2 - \nu_s/2$.

Figure 1 (top panel) displays the fitting result for 4U 1636–53. We set $\nu_1 \approx \nu_2 - \nu_s/2$ because 4U 1636–53 belongs to the second case in Lamb & Miller (2003). The ratio $\nu_{cl}/\nu_B$ is represented by a free parameter, the torque coefficient $c_N$ (see Lamb & Miller 2001, for detail). Our fitting result is $M = 1.545 M_\odot$, $\nu_s = 652 \pm 5$ Hz, and $c_N = 0.00274$. The fitting does not give a spin frequency close to 581 Hz. Moreover,
our measured $\Delta \nu$ ranges from 220 to 340 Hz, which could be smaller or larger than $\nu_\star/2$ (290.5 Hz). In fact, such behavior of $\Delta \nu$ is already shown in Jonker et al. (2002). Though successful in explaining a changing $\Delta \nu$ close to half of the spin frequency, the latest version predicts that $\Delta \nu$ is always smaller (larger) than $\nu_\star/2$ for a prograde (retrograde) flow. When $\nu_\star$ is below about 800 Hz, $\Delta \nu$ is larger than 290.5 Hz; otherwise, it is smaller than 290.5 Hz. Finally, one can note that the fitting result by fixing $\nu_\star = 581$ Hz shows larger deviations from the data points.

The fitting result for Sco X-1 is shown in Figure 1 (bottom panel), giving $M = 1.80 M_\odot$, $\nu_\star = 329.5$ Hz (or $\nu_\nu = 659$ Hz), and $\epsilon_N = 0.00216$, depending on which case we choose. Comparing this to the fitting result in Lamb & Miller (2001) and Lamb & Miller (2003), we get a larger $M$ and a slightly smaller $\nu_\star$. The reduced chi-square is slightly larger, partly due to our more precise results with smaller error bars.

3.2. Comparison of Orbital Resonance Models

Kluźniak & Abramowicz (2001) and Abramowicz et al. (2003) introduced several kHz QPO models based on the idea of the resonances between the radial and vertical frequencies in orbital motion.

3.2.1. The 2 : 3 Parametric-resonance Model

A parametric-resonance instability occurs near $\omega_\nu = 2\omega_\nu/n$ for $n = 1, 2, 3, \ldots$ in an oscillator that obeys a Mathieu-type equation of motion,

$$\delta \dot{\theta} + \omega_\nu^2 [1 + h \cos(\omega_\nu t)] \delta \theta = 0,$$

where $\delta \theta$ is the small deviation of elevation $\theta$, the dot denotes the time derivative, and $h$ is a known constant. $\omega_\nu = 2\pi \nu_\nu$ and $\omega_\nu = 2\pi \nu_\nu$, where $\nu_\nu$ and $\nu_\nu$ are the radial and vertical epicyclic frequencies, respectively (Abramowicz et al. 2003).

The model predicts $\nu_\nu : \nu_\nu = 2 : n$. When $n$ has the smallest possible value, the strongest resonance is excited. Because $\nu_\nu < \nu_\nu$, the smallest possible value for resonance is $n = 3$, meaning that $\nu_\nu : \nu_\nu = 2 : 3$. Simply supposing $\nu_1 = \nu_\nu$ and $\nu_2 = \nu_\nu$ (Kluźniak & Abramowicz 2002), one can infer the 2 : 3 ratio of the twin kHz QPO peak frequencies. The excitation of the resonance has been studied with numerical simulations (Abramowicz et al. 2003) and an analytic method (Rebusco 2004).

The behavior of the frequency relationship in the parametric-resonance model is shown in Figure 2, in comparison to our measured data. At first, one can note that the linear 2 : 3 frequency relationship (dashed) is not in agreement with the observations. Then, we also note that the model with $\nu_2 = \nu_\nu$ and $\nu_2 = \nu_\nu$ deviates much more from the data (dotted); it is even unable to follow the basic downward-bending track of the data points. Actually, the assumption $\nu_1 = \nu_\nu$ is inappropriate for an NS. Under the condition of $M \in [1.4, 2.4] M_\odot$ and $j \in [0, 0.3]$, theoretical $\nu_\nu$ has a maximum value of about 635 Hz when $M = 1.4 M_\odot$ and $j = 0.3$ (see, e.g., Stella & Vietri 1999, the equations of orbital frequencies). However, the measured $\nu_\nu$ reaches to as large as 800–1000 Hz. As shown in the figure, the rightmost dotted curve represents the predicted frequency relationship with $M = 1.4 M_\odot$, $j = 0.3$; and the leftmost one shows the same with $M = 2.4 M_\odot$, $j = 0$. The predicted curves with other values of $M$ and $j$ lie between them. Finally, a linear frequency relationship, i.e., $\nu_2 = \nu_\nu$ and $\nu_2 = k \nu_\nu + b$, is proposed in Abramowicz et al. (2005). We use it to fit the data and get $k = 0.840, b = 395$ Hz for 4U 1636–53 and $k = 0.805, b = 422$ Hz for Sco X-1. Then, we obtain the ratio of $\nu_1$ to $\nu_2$ for these two sources: 0.725 and 0.683, respectively, close to but higher than 2 : 3. We will discuss the ratios in Section 4. Here, we just show that the linear fit is naturally disfavored by the data points with apparent nonlinearity.

3.2.2. The Forced 1 : 2 and 1 : 3 Resonance Models

In the numerical simulations of oscillations of a perfect fluid torus (Abramowicz et al. 2003), there is an evident resonant forcing of vertical oscillations. The forcing is caused by radial oscillations through a pressure coupling. This result supports another possible resonance model (Abramowicz & Kluźniak 2001; Abramowicz et al. 2004). In the model, the resonances...
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occur in a forced nonlinear oscillator,
\[ \delta \ddot{\theta} + \omega_0^2 \delta \theta + [\text{nonlinear terms in } \delta \theta] = h(r) \cos(\omega_0 t), \quad \omega_0 = n \omega_r, \] (4)
again where \( \omega_r = 2\pi v_r \) and \( \omega_0 = 2\pi v_0 \).

This model predicts that one of the combination frequencies, i.e., \( v_- = v_\theta - v_r \) and \( v_+ = v_\theta + v_r \), has a \( 2 : 3 \) ratio to the vertical frequency. For \( n = 2 \), the forced epicyclic resonance \( v_r : v_\theta = 1 : 2 \),
\[ v_1 = v_\theta, \quad v_2 = v_+, \] (5)
and for \( n = 3 \), the forced epicyclic resonance \( v_r : v_\theta = 1 : 3 \),
\[ v_1 = v_- , \quad v_2 = v_\theta. \] (6)

We fit the observed QPO frequency relationships to the forced-resonance models in Figure 3. The forced \( 1 : 2 \) model predicts that \( v_2 \) climbs up to a maximum value at \( v_1 \approx 950 \) Hz, and then decreases rapidly. However, our analysis results for the two LMXBs do not show the trend that\( v_2 \) should decrease as \( v_1 \) increases. Regarding the forced \( 1 : 3 \) model, it cannot adequately describe the data points, especially at the high and low frequencies. The \( v_2 \) predicted is higher than that observed at low frequencies, but lower than that observed at high frequencies. This means that the observed \( \Delta v \) does not decrease as sharply as the model predicts. In addition, these two forced models give very large reduced \( \chi^2 \).

### 3.3. Comparison of Precession Models

This section investigates the predictive ability of three precession models, namely, the relativistic precession model, the vertical precession model, and the total precession model. In these precession models, QPOs can be excited by various resonances with the precession frequencies and orbital frequencies under certain conditions, such as inhomogeneities orbiting the inner-disk boundary (Stella 2001).

For the well-known relativistic precession model (Stella & Vietri 1999), the upper QPO \( v_2 \) is assumed to be the azimuthal frequency \( v_\phi \), and the lower QPO \( v_1 \) is expressed as the relativistic periastron precession frequency:
\[ v_1 = v_\phi - v_r, \] (7)
\[ v_2 = v_\phi. \] (8)

In the vertical precession model (Bursa 2005), \( v_1 \) is the same as that in Equation (7); \( v_2 \) is hypothesized as \( v_\phi \).

In the total precession model (Stuchl´ık et al. 2007), \( v_1 \) is the total precession frequency and \( v_2 \) is ascribed to the Keplerian frequency \( v_K \) (or the vertical frequency \( v_\theta \)):
\[ v_1 = v_\theta - v_r, \] (9)
\[ v_2 = v_K \quad \text{or} \quad v_2 = v_\theta. \] (10)

Figure 4 illustrates the comparison of the precession models to the observed data. Note that the models almost overlap and have the same deviation as the forced \( 1 : 3 \) resonance model—\( v_2 \) is predicted too low at high frequencies and too high at low frequencies. Hence, the deviation from the observations increases significantly at high and low frequencies. In addition, the relativistic and vertical precession models give large \( M \) and \( j \). Though the total precession model gives smaller \( M \) and \( j \), the fit has the largest \( \chi^2 \) among the precession models.

### 3.4. Comparison to Disk Oscillation Models

The resonances between specific modes in an accretion disk are also studied for exciting the observed kHz QPOs. In this section, we investigate two models of this kind.

#### 3.4.1. The Deformed-disk Oscillation Model

Kato (2001) brought forward the deformed-disk resonance model. kHz QPOs are excited by a horizontal resonance in a deformed (warped or eccentric) disk under inviscid and adiabatic perturbations. The perturbations vary as \( \exp\left[ i (\omega t - m \phi) \right] \), where \( \omega \) is the frequency of the perturbations and \( m = 0, 1, 2, \ldots \) denotes the number of arms in the azimuthal direction. Various modes of perturbations are considered subsequently (Kato 2003, 2005, 2009). In the model, QPOs are inertial-acoustic
Figure 4. Fitting results to the precession models for 4U 1636–53 (top) and Sco X-1 (bottom). The solid, dotted, and dashed curves represent the predictions of the relativistic, vertical, and total precession models, respectively. The three curves almost overlap, especially the relativistic and vertical models. (A color version of this figure is available in the online journal.)

Figure 5. Fit results to the deformed-disk resonance model for 4U 1636–53 (top) and Sco X-1 (bottom). The solid curve represents the best-fit result, with $M \approx 2.4 M_\odot$, $j = 0$. The predicted $M$ and $j$ are almost identical in the two LMXBs. (A color version of this figure is available in the online journal.)

oscillations ($p$-mode) and gravity oscillations ($g$-mode), or a combination. The twin kHz QPOs are

$$v_1 = 2(\nu_K - \nu_r),$$

$$v_2 = 2\nu_K - \nu_r.$$  \hspace{1cm} (11, 12)

Figure 5 exhibits the best fits for 4U 1636–53 and Sco X-1. The fit results ($M \approx 2.4 M_\odot$, $j = 0$) are consistent with those in Kato (2007). The model gives a large NS mass. At the same time, the fits to the two different NS systems give the same set of $M$ and $j$. However, the model describes the measured data points relatively better than most of the other models, though it predicts $v_2$ slightly larger than that observed at high frequencies.

3.4.2. The “$-1r$, $-2v$” Resonance Model

Unlike the deformed-disk oscillation model, the perturbations in the “$-1r$, $-2v$” resonance model (Török et al. 2007; Bakala et al. 2008) are not stressed in the azimuthal direction. In this model, the kHz QPOs are excited by the resonance between the radial $m = 1$ and vertical $m = 2$ modes. The excited QPOs are supposed to be

$$v_1 = \nu_K - \nu_r,$$

$$v_2 = 2\nu_K - \nu_r.$$  \hspace{1cm} (13, 14)

The fit results to the model are displayed in Figure 6. As with the precession and forced 1 : 3 resonance models, the model predictions cannot reproduce the observations, especially at low and high frequencies. The fits have very large $\chi^2$ and give $M = 2.4 M_\odot$, reaching the upper limit in the fit.

3.5. Comparison of the Higher-order Nonlinearity Model

Mukhopadhyay (2009) treated accreting systems as damped harmonic oscillators. These oscillators exhibit epicyclic
oscillations with higher-order nonlinear resonance. The resonance is expected to be driven by the coupling between the strong disturbance from an NS and the weaker one from the flow. In the model, the lower and upper kHz QPOs are proposed to be

\[ \nu_1 = \nu_0 - \frac{\nu_s}{2}, \quad \nu_2 = \nu_s + \frac{n}{2} \nu_s. \]  

(15)

(16)

In the disk around an NS, \( n = 1 \) corresponds to a nonlinear coupling, resulting in \( \Delta \nu = \nu_2 - \nu_1 \sim \nu_s/2 \), whereas \( n = 2 \) corresponds to a linear coupling, resulting in \( \Delta \nu \sim \nu_s \). However, \( n = 3 \) may correspond to a higher-order coupling, which is expected to be too weak to produce any observable effects. For \( n = 1 \) and \( n = 2 \), the model divides NSs into fast and slow rotators.

To compute the QPO frequencies, the spin parameter \( j \) should be determined in the following way. If an NS is considered to be spherical in shape with equatorial radius \( R \), spin \( \nu_s \), mass \( M \), and radius of gyration \( R_G \), then the moment of inertia and the spin parameter are computed by

\[ I = MR_G^2, \quad j = \frac{I\Omega_s}{GM^2/c^2}. \]  

(17)

where \( \Omega_s = 2\pi \nu_s \). It is known that, for a solid sphere, \( R_G^2 = 2R^2/5 \) and, for a hollow sphere, \( R_G^2 = 2R^2/3 \). However, a very fast-rotating NS is expected to be ellipsoidal and not completely solid. Therefore, \( 0.35 \leq (R_G/R)^2 \leq 0.5 \) is chosen in the model.

The model parameters, i.e., \( M, \nu_s, R \), and \((R_G/R)^2\), can be obtained by the fit. For 4U 1636–53, Mukhopadhyay (2009) treated it as a fast rotator with \( \nu_s = 581 \) Hz and fit the frequency relationship to \( n = 1 \). In his work, only six data points were collected in the diagram of \( \Delta \nu \) versus \( \nu_1 \). Then, he excluded the data points at low frequencies, corresponding to the ones with large deviations from the model. Finally, he gave a low NS mass of about 1.2–1.4 \( M_\odot \). For Sco X-1, the fit was done both to \( n = 1 \) and \( n = 2 \), and he argued that Sco X-1 is a slow rotator with \( n = 2 \) and \( \nu_s \) about 300 Hz.

Our fit results to different \( n \) are shown in Figure 7. Here, we fit all the data points in the two NS systems without any exclusions. For 4U 1636–53, the best-fit value is \( M = 1.40 M_\odot \), \( \nu_s = 489 \) Hz, \( R = 23.1 \) km, and \((R_G/R)^2 = 0.38\) to \( n = 1 \), and \( M = 1.40 \), \( \nu_s = 307 \) Hz, \( R = 26.5 \) km, and \((R_G/R)^2 = 0.46\) to \( n = 2 \). We find that under \( n = 1 \) and \( n = 2 \), the model gives the curves almost superimposed (solid and dashed in the figure). Moreover, \( \nu_2 \) predicted by the model is too high at low frequencies. By setting \( \nu_s = 581 \) Hz and \( n = 1 \), we obtain results with much larger deviations (dotted curve). For Sco X-1, the model curves under \( n = 1 \) and \( n = 2 \) almost overlap, resulting in 467-Hz and 294-Hz spin frequencies. The best-fit value of other parameters is \( M = 1.40 M_\odot \), \( R = 23.0 \) km, and \((R_G/R)^2 = 0.40\) to \( n = 1 \) and \( M = 1.40 \), \( R = 27.3 \) km, and \((R_G/R)^2 = 0.45\) to \( n = 2 \). For \( n = 2 \), our results are consistent with those in Mukhopadhyay (2009).

3.6. Comparison of the Tidal Disruption Model

Tidal disruption of the orbits of low-mass satellites around a Schwarzschild black hole has recently been studied by Cadez et al. (2008). In the clumps of material orbiting such a black hole, a spherical blob can be squeezed and stretched by tidal forces into a ring-like shape along the orbit, thus producing radial oscillations (Germanà et al. 2009). With simulations of such accretion processes, they generated simulated light curves and fit the power spectra of the light curves. Both twin kHz QPOs are found, and the peak frequencies are supposed to be

\[ \nu_1 = \nu_K, \quad \nu_2 = \nu_K + \nu_r. \]  

(18)

(19)

Figure 8 shows the best fits to this model in the two NS systems. As we see, the model describes well the main parts of the frequency relationships, particularly at low frequencies (\( \nu_1 \approx 800 \) Hz). At high frequencies, however, the model predicts the maximum value of \( \nu_2 \) and then a sharp decrease. It is not supported by observations. Another incompatibility is the high NS mass predicted, which is up to 2.4 \( M_\odot \) in this model.
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3.7. The Rayleigh–Taylor Gravity Wave Model

Osherovich & Titarchuk (1999) and Titarchuk (2003) described QPOs by the Rayleigh–Taylor instability associated with Rossby waves and rotational splitting. Twin kHz QPOs are explained as oscillations of large-scale inhomogeneities (hot blobs) thrown into the NS’s magnetosphere. Participating in the radial oscillations with the Keplerian frequency $v_K$, such blobs are also simultaneously under the influence of the Coriolis force. For such oscillations, $v_2$ and $v_K$ hold an upper hybrid frequency relationship: $v^2_2 - v^2_K = 4v^4_m$, where $v_m$ is the rotational frequency of the magnetosphere near the equatorial plane. If the magnetosphere corotates with the NS (solid-body rotation), then the spin rotation of the NS would be determined. For the first-order approximation, $v_m = v_s = \text{const}$. Within the second-order approximation, the slow variation of $v_m$ as a function of $v_K$ reveals the structure of the magnetospheric differential rotation. Hence, in the model

$$v_1 = v_K,$$  \hspace{1cm} (20)

$$v_2 = (v^2_K + 4v^4_m)^{1/2},$$  \hspace{1cm} (21)

and within the dipole–quadrupole–octupole approximation of the magnetic field, the rotational frequency of the magnetosphere is

$$v_m(v_K) = C_0 + C_1v^{4/3}_K + C_2v^{8/3}_K + C_3v^4_K,$$  \hspace{1cm} (22)

where $C_0 = v_s$, $C_2 = 2\sqrt{C_1C_3}$.

Our fit treats $M$, $C_0$, $C_1$, and $C_3$ as free parameters. The fit results are shown in Figure 9. For 4U 1636–53, the best fit (solid curve) returns $M = 1.78M_\odot$, $C_0 = 371$, $C_1 = -0.050$, $C_3 = 0.060$. 

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**Figure 7.** Fit results to the higher-order nonlinearity model for 4U 1636–53 (top) and Sco X-1 (bottom). Top panel: the solid curve represents the best-fit result to $n = 1$. The dashed curve, which almost overlaps the solid one, is the fit to $n = 2$. The dotted and dot-dashed curves denote the fits by setting $v_s = 581 \text{ Hz}$ to $n = 1$ and $n = 2$, respectively. Bottom panel: the solid and dashed curves indicate the best-fit results to $n = 1$ and $n = 2$. (A color version of this figure is available in the online journal.)

**Figure 8.** Fit results to the tidal disruption model for 4U 1636–53 (top) and Sco X-1 (bottom). The curves exhibit the disagreement with the data points when $v_1 > 800 \text{ Hz}$. (A color version of this figure is available in the online journal.)
flows along the field lines to the polar cap of the NS. Some resonant modes may be excited by the perturbations at the magnetospheric radius (Zhang 2004; Shi & Li 2009).

3.8.1. The MHD Alfvén-wave Oscillation Model

Zhang (2004) explained the twin kHz QPOs with the MHD Alfvén-wave oscillations excited by the distortion of the NS magnetosphere. The model assumes that the infalling MHD material of the Keplerian accretion flow distorts the magnetosphere in the regions with enhanced mass-density gradients, leading to resonant-shear Alfvén waves. In this model, the upper frequency QPO is the Keplerian orbital frequency,

\[ v_2 = v_K = 1850AX^{3/2} \text{ Hz}, \]  

with the parameters \( X = R/r \) and \( A = (m/R_0^2)^{1/2} \), where \( R_0 = R/10^6 \) (cm) and \( m = M/M_\odot \) are the NS radius \( R \) and mass \( M \) in units of \( 10^6 \) cm and solar masses, respectively. The quantity \( A^2 \) is proportional to the average mass density of the NS, expressed as \( \rho = 3M/(4\pi R^2) \approx 2.4 \times 10^{14} \text{ (g cm}^{-3}\text{)} /A(0.7)^2 \).

The lower-frequency QPO is identified as the Alfvén-oscillation frequency, given as

\[ v_1 = v_2 X^{3/4} \sqrt{1 - \sqrt{1 - X}}. \]  

Because \( X \) is eliminated in the fit process, we only have one parameter, \( A \). The comparisons between the model predictions and observations are shown in Figure 10. Similar to the precession models, this model also predicts \( v_2 \) to be too high at low frequencies and too low at high frequencies, leading to increased deviations from the observations at low and high frequencies. The result also indicates that \( \Delta\nu \) predicted by this model decreases too sharply compared to the observations. The fit for Sco X-1 is somewhat better than that for 4U 1636–53. Our result of \( A \approx 0.7 \) agrees with that obtained by Zhang et al. (2008), in which the relationship of \( \Delta\nu \) versus \( v_2 \) was fit and the result showed a discrepancy with the observations.

3.8.2. The MHD Model

Shi & Li (2009) presented another explanation for kHz QPO signals in LMXBs based on MHD oscillation modes in an NS magnetosphere. Several MHD wave modes are derived by solving the dispersion equations. They proposed that the coupling of the two resonant MHD modes may lead to the twin kHz QPOs. Finally, they presented the following linear frequency relationships:

\[ v_2 = \sqrt{1 + \delta^2} (v_1 + v_a) \]  

(LSCS)  

\[ v_2 = \frac{1}{\sqrt{1 + \varepsilon^2}} (v_1 + v_a) \]  

(SSCS)  

where \( \delta^2 = (\lambda^2 - \eta^2)/(1 + \eta^2) \) and \( \varepsilon^2 = (\eta^2 - \lambda^2)/(1 + \lambda^2) \). Here, \( \lambda \) and \( \eta \) are two constants linking the Alfvén velocity, acoustic velocity, and Keplerian velocity of MHD waves in the model. The model divides the twin kHz QPOs into two groups: the slope of \( v_2/v_a \) and \( v_1/v_a \), which are either larger or smaller than 1.0, i.e., the large-slope coefficient sources (LSCS) and the small-slope coefficient sources (SSCS), respectively. With our fit, we find that both 4U 1636–53 and Sco X-1 belong to the latter group, because Equation (25) gives bad fit results and reduced \( \chi^2 > 10^5 \).
The fit results are plotted in Figure 11. First, both for 4U 1636–53 and Sco X-1, the observed relationships of $\nu_2$ to $\nu_1$ are not linear; the data points have a track bending downward. Second, for 4U 1636–53, the best fit predicts $\epsilon = 0.65$ and $v_s = 470$ Hz. The spin frequency is not close to 581 Hz. Also, Shi & Li (2009) cannot fit 4U 1636–53 well by holding $v_s = 581$ Hz. Their result gives $\epsilon = 0.77$ and a large reduced $\chi^2 = 23$. In the case of Sco X-1, our fit gives $v_s = 525$ Hz, considerably larger than that from other models.

4. DISCUSSION AND CONCLUSIONS

We have presented newly obtained and more accurate results regarding the frequency relationships of twin kHz QPOs for 4U 1636–53 and Sco X-1. The peak frequencies of lower and upper QPOs are almost directly proportional to each other. The data points tend to bend downward between 500 and 1250 Hz in the diagram of $\nu_1$ versus $\nu_2$. Both frequency relationships show some subtle structure around $\nu_1 \approx 800$ Hz.

Based on the frequency relationships, we have systematically investigated the predictive ability of all currently available models with which the frequency relationship can be calculated. Our conclusions are as follows.

1. The sonic-point and spin-resonance models seem to be suitable only for Z sources. The models describe well the frequency relationship for Sco X-1. However, in the case of 4U 1636–53, the models predict that $\Delta \nu$ is always less than half of the spin frequency for a prograde flow, while the observed $\Delta \nu$ can be larger and smaller than $v_s/2$. This indicates that a single rotational direction of accretion flow cannot explain the behavior of $\Delta \nu$. According to the model, such behavior of $\Delta \nu$ may be produced by a flow that retrogrades when it is far away from the NS, but then switches to a prograde orbit at some special radius when it is closer to the NS. However, no evidence is found to support such a sudden switch of orbital motion.
2. The $2 : 3$ parametric-resonance model predicts a frequency relationship bending upward. Within the limit of NS parameters for reasonable NS EOS, it cannot give $v_1$ as high as in the observations. Furthermore, as claimed in previous papers (Belloni et al. 2005; Zhang et al. 2006), this model leads to the predicted $\Delta v$ increase with the QPO frequency. However, the observed downward-bending track in the diagram of $v_1$ versus $v_2$ indicates a rough inverse proportion between $\Delta v$ and $v_1$ (or $v_2$). Regarding the forced-resonance model, the $1 : 2$ model predicts a decrease of $v_2$ at high frequencies, while the $1 : 3$ model predicts $v_2$ higher (lower) than the observed values at low (high) frequencies. In fact, all the orbital-resonance models are introduced to explain the observed clustering of $2 : 3$ ratios between $v_1$ and $v_2$. However, Belloni et al. (2005) have demonstrated that a simple random walk of the QPO frequencies can reproduce qualitatively the observed distributions of frequency and frequency ratio. Later, Boutelier et al. (2010) have pointed out that the clustering originates naturally from the sensitivity-limited observations and does not support preferred frequency ratios in NS systems. Our results therefore suggest that the orbital-resonance models should be further investigated in order to improve their predictive power for the frequency relationship.

3. All the precession models nearly overlap with each other. Their predicted $v_2$ is higher (lower) than that observed at low (high) frequencies. The deviations from observations increase significantly at high and low frequencies. In more detail, the relativistic and vertical precession models predict an NS mass higher than $\sim 2.2 M_\odot$. As seen in Table 3, when we relax the fit limits of $M \in [1.4, 2.4]$ and $j \in [0, 0.3]$, the extended analysis shows that these two models could describe the data points better with relatively higher $M$ and $j$. Essentially, the inferred high NS mass may arise from the periastron precession term which is only valid for an NS in a vacuum environment (Zhang et al. 2009). It should be mentioned that, considering a small eccentricity ($\lesssim 0.1$) that decreases with increasing $v_3$, the relativistic precession model would explain the frequency relationship better (Stella & Vietri 1999). In this paper, we do not consider the effect of eccentricity on the frequency relationship because we focus on the NS properties. The total precession model gives lower values of $M$ and $j$ but larger reduced $\chi^2$.

4. The deformed-disk resonance model describes the observations relatively better than most of the models. The fits give the same $M$ and $j$ for the two LMXBs. This suggests that the model may reveal some common properties of atoll and Z sources. Nevertheless, the high mass predicted and the deviations at high frequencies show that the model could be modified. For example, the effect of a magnetic field could be taken into account. The “$-1r, -2v$” resonance model behaves like the precession models. Considering that the best-fit results of these two models approach the upper limit of $M$, we also performed extended fitting. Both for 4U 1636–53 and Sco X-1, the fit results do not improve significantly.

5. The higher-order nonlinearity model classifies NSs based on the values of $n$. After the investigation, one can note that the model predicts the nearly identical frequency relationships under all values of $n$ taken here. Thereby, given that we do not know the spin frequency of Sco X-1, the classification that Sco X-1 is a slow rotator with $n = 2$ is not well founded. The superimposed fit curves under $n = 1$ and $n = 2$ for 4U 1636–53 also indicate some kinds of ambiguity of the classification. Apart from that, when $n = 1$ is chosen, like that in Mukhopadhyay (2009), the spin frequency predicted is not close to 581 Hz. Besides, the model predicts a very low NS mass, reaching the lower limit in our fit. Our extended analysis shows that the NS masses for the two sources are predicted down to $1.0 M_\odot$, quite low for most known NS EOS.

6. The tidal disruption model can describe the main part of the observed frequency relationships, though it predicts a high NS mass. Perhaps this is due to the essential difference between NS and black hole systems. This implies that the kHz QPOs should be greatly affected by the surroundings close to the central object. After all, the agreement with the observations at low frequencies is remarkable, corresponding to the place relatively distant to the center. At that place, the clumps of material (or particles) are not being exposed to the compact object so much. As can be found from Table 3, the fit results of the model can be improved greatly in our extended fitting, in favor of very large NS masses. The model should be modified to better describe the data points at high frequencies and to get a more reasonable mass for an NS.

7. The Rayleigh–Taylor gravity-wave model can follow the frequency relationship in Sco X-1, but for 4U 1636–53 it cannot predict the 581-Hz spin frequency. This may be because the dipole–quadrupole–octupole approximation is not sufficiently accurate for the magnetic field. Therefore, this model seems promising for explaining the origin of kHz QPOs if its description of NS magnetic fields can become more accurate. For the sake of comprehensiveness, it should be noted that this model predicts not only the high-frequency QPOs, but also the low-frequency ones. When the low QPOs are also considered, the fits do not always work, unless, for one particular frequency range, one of the low-frequency QPOs is assumed to be a harmonic of an unseen one, whereas in the other intervals it is the fundamental frequency.

8. The MHD Alfvén-wave oscillation model has the same problem as the precession models with increased deviations from observations at high and low frequencies. It should be noted that the model was put forward based on the analogy of the solar coronal atmosphere of an NS system. Though the solar coronal atmosphere has been studied a lot, the mechanism of Alfvén-wave oscillations in an NS system remains unclear. The model’s performance in our fits indicates that such a mechanism should be investigated further.

9. The MHD model predicts a linear frequency relationship, which is inconsistent with the measured frequency relationships. At the same time, the model cannot predict reasonable spin frequencies for the two NSs. Generally, we also find that these models diverge strongly in their predictions of NS properties. Different models predict spin frequency from less than 300 Hz to more than 600 Hz. The angular momentum is predicted to be from 0 to 0.3, covering entirely the range of the limit in the fit. The predicted NS mass from different models also covers the whole range: $[1.4, 2.4] M_\odot$.

10. The problem of increased deviations at high and low frequencies exists in six models: the forced $1 : 3$ model, the three precession models, the “$-1r, -2v$” resonance
model, and the MHD Alfvén-wave oscillation model. The first five models almost overlap in the plot of their fit results. Because \( v_K \approx v_0 \) (exactly equal if \( v_s = 0 \)), these five models have nearly identical expressions of \( v_1 \) and \( v_2 \). Those five models form the group of the largest \( \chi^2 \) in Table 2. The MHD Alfvén-wave oscillation model performs slightly better than those five models, despite the fact that its \( \chi^2 \) remains much larger than that of the other remaining models. All six models propose that the upper-frequency QPO is Keplerian, i.e., \( v_2 = v_K \). One can infer that for these models, the interpretation is not favored by the data.

12. Those models that include the effects of magnetic fields obtain the best-fit results, such as the sonic-point beat frequency and the Rayleigh–Taylor gravity-wave models. They can at least depict the frequency relationship for Sco X-1.

Finally, one should note the fact that no model gives a statistically acceptable \( \chi^2 \) in the fittings. We argue that all models predicting a linear, power law, or any other frequency relationship are not fully supported by the observations, at least for these two sources.

After the investigation, because three of the models we investigated here (the deformed-disk resonance model, the tidal disruption model, and the Rayleigh–Taylor gravity-wave model) have performed relatively better than other models, we speculate that a model that combines these three models could reveal the physical origin of the observed kHz QPO signals. It is worth noting that each of them still has its own problems.

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