Haemodynamics of giant cerebral aneurysm: A comparison between the rigid-wall, one-way and two-way FSI models

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Abstract. In this paper a computer simulation of a blood flow in cerebral vessels with a giant saccular aneurysm at the bifurcation of the basilar artery is performed. The modelling is based on patient-specific clinical data (both flow domain geometry and boundary conditions for the inlets and outlets). The hydrodynamic and mechanical parameters are calculated in the frameworks of three models: rigid-wall assumption, one-way FSI approach, and full (two-way) hydroelastic model. A comparison of the numerical solutions shows that mutual fluid–solid interaction can result in qualitative changes in the structure of the fluid flow. Other characteristics of the flow (pressure, stress, strain and displacement) qualitatively agree with each other in different approaches. However, the quantitative comparison shows that accounting for the flow–vessel interaction, in general, decreases the absolute values of these parameters. Solving of the hydroelasticity problem gives a more detailed solution at a cost of highly increased computational time.

1. Introduction

A description of the blood flow in a circulatory system is one of the challenging and topical problems in fluid dynamics. Mathematical modelling based on real clinical data allows one to obtain qualitative and quantitative characteristics of the blood flow in healthy and pathological circulation. This brings possibilities to solve such important medical problems as determination of the laws of circulatory system functioning, prognosis for consequences of surgical interventions and natural history of various diseases. One of such dangerous vascular diseases is a cerebral arterial aneurysm (CA), which is an abnormal dilation of a blood vessel wall [1].

Surgical treatment of the CA is carried out mainly with two kinds of interventions: an open surgery and an endovascular operation. In the open surgery, clipping is the most used procedure, in which the aneurysmal sac is removed from the flow domain with a clamp placed over the aneurysmal neck. The endovascular treatment is a minimally invasive procedure, which can be applied to patients with heavy concomitant somatic pathologies, or older patients for which the traditional neurosurgical intervention is contraindicated. Moreover, endovascular operations are used in cases of CA with difficult access or no possibility for the clipping [1]. Medical instruments in the endovascular surgery are delivered through a small cut in the femoral artery or femoral...
Among the tools used for the operation are catheters, guide wires, stents, coils, balloons and other.

When choosing the right technology and technique for the treatment, a neurosurgeon does not have all the information needed for the exact prognosis of the operation outcome. That includes the characteristics of all elements of the system, their properties, the range of admissible external influences on the vessels. The neurosurgeon can follow only his/her previous experience and rely on his/her intuition. The lack of the necessary informational support has led to the absence of a common agreement on the technology and strategy for carrying out the surgical intervention [2]. Thus, it is very important to perform patient-specific preoperative modelling for better treatment strategy planning.

Modelling of the blood flow in vessel is a difficult mathematical problem of fluid–structure interaction (FSI). There are still a number of open questions both in analytical methods and in problems of numerical scheme construction [3]. In the present work based on clinical data for the real patient, a numerical simulation of haemodynamics in cerebral vessels with a giant aneurysm at the bifurcation of the basilar artery is performed. Three different approaches are used to simulate the flow: rigid-wall assumption, one-way FSI approximation and two-way FSI. A comparison between the different solutions is carried out.

2. Mathematical models

One of the most widely-used approach to description of the circulation is a mathematical model in which the blood flow is governed by the Navier–Stokes equations and the vessel walls are considered to be rigid. In such a case, only the hydrodynamic problem for the blood flow is solved. In relatively large cerebral arteries one can consider blood as a viscous incompressible Newtonian fluid [4]. Then the system of equations governing a non-stationary 3D flow reads

$$\nabla \cdot \mathbf{v} = 0, \quad \rho (\mathbf{v}_t + (\mathbf{v} \cdot \nabla) \mathbf{v}) + \nabla p = \mu \Delta \mathbf{v}, \quad \mathbf{x} \in \Omega,$$

where $\mathbf{v}$ is the velocity, $p$ is the pressure, $\mu$ is the dynamic viscosity, $\rho$ is the constant density, and $\Omega$ is the flow domain. The viscosity of blood is taken to be four times larger as that of water: $\mu = 0.004$ Pa s [3].

The computational domain, $\Omega$, is shown in figure[1] The boundary $\partial \Omega$ of the domain consists of the vessel wall $\Gamma_{wall}$, the inlets $\Gamma_{in}^i$, and the outlets $\Gamma_{out}^j$: $\partial \Omega = \Gamma_{wall} \cup \Gamma_{in}^i \cup \Gamma_{out}^j$, $i = 1, 2$, $j = 1, \ldots, 6$.

For the hydrodynamic problem, a time-dependent normal velocity $v_{\text{clin}}(t)$ is prescribed at the inlets $\Gamma_{in}^i$ and time-dependent pressure $p_{\text{clin}}(t)$ is prescribed at the outlets $\Gamma_{out}^j$:

$$\mathbf{v} = v_{\text{clin}}(t) \mathbf{n} \text{ on } \Gamma_{in}^i, \quad p = p_{\text{clin}}(t) \text{ on } \Gamma_{out}^j.$$ 

In our computations we use clinical data obtained during endovascular surgery with Volcano ComboMap Pressure and Flow System (Volcano Corp., USA) for $v_{\text{clin}}(t)$ and $p_{\text{clin}}(t)$ [6].

On the vessel wall we impose no-slip boundary conditions:

$$\mathbf{v} = 0 \text{ on } \Gamma_{wall}.$$ 

When performing analysis of the blood flow, it is often very useful to consider the shear stress on the vessel wall determined by the formula [7]:

$$\tau = \mu (\nabla \mathbf{v} \cdot \mathbf{n}_{\text{wall}}),$$

where $\mathbf{n}_{\text{wall}}$ is the normal to the vessel wall.

There are two approaches for determination of the deformations and stresses in the vessel wall: the so called one-way fluid–structure interaction (FSI) and two-way FSI. In the former
formulation the counter-reaction of the vessel wall onto the blood flow is neglected, while in the latter one mutual interaction between the wall and the flow is taken into account.

In the one-way FSI approximation the solutions of the hydrodynamic and elasticity problems are split into two consecutive steps. Having found the blood flow in the rigid-wall assumption, the deformation of the vessel wall is sought as a reaction to the flow. In this work the vessel wall is modelled by an isotropic linearly elastic material [8]. The system of equations—the equilibrium equations and the Beltrami–Michell equations—reads as follows:

$$\sum_{i=1}^{3} \sigma_{ij} \frac{\partial}{\partial x_i} = 0, \quad \Delta \sigma_{ij} + \frac{1}{1+\nu} \frac{\partial^2 (\sigma_{11} + \sigma_{22} + \sigma_{33})}{\partial x_i \partial x_j} = 0,$$

where $\sigma = (\sigma_{ij})$ is the stress tensor, $\nu$ is Poisson’s ratio. The stress tensor $\sigma$ and the strain tensor $\varepsilon$ are related through Hooke’s law [9]:

$$\sigma = \lambda \text{tr} \varepsilon \mathbf{I} + 2G\varepsilon, \quad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad G = \frac{E}{2(1+\nu)},$$

where $\lambda$ and $G$ are Lamé parameters, $E$ is Young’s modulus, tr is the trace operator.

For the boundary conditions for the system (2), the solution for the pressure on the flow domain boundary $\Gamma_{wall}$ is used for the inner surface of the vessel wall. On the exterior boundary of the wall, an elastic support, which models the surrounding brain tissue, with coefficient of $10^7$ N/m$^3$ is used. The inlet and outlet sides of the vessel are fixed.

In the full (two-way) statement of the FSI problem one needs to find the fluid flow and the wall’s deformations coupled through the boundary conditions on the fluid–solid interface.

To describe the interaction of the blood flow with the vessel wall, the Navier–Stokes equations (1) and the elasticity equations (2) are solved simultaneously. In the latter system,
the equations of motion instead of the equilibrium equations are used:

\[ \rho_w \frac{\partial^2 w_i}{\partial t^2} = \sum_{i=1}^{3} \sigma_{ij} \frac{\partial}{\partial x_i} + \frac{1}{1+\nu} \frac{\partial^2 (\sigma_{11} + \sigma_{22} + \sigma_{33})}{\partial x_i \partial x_j} = 0, \]

where \( w = (w_i) \) is the displacement vector, \( \rho_w \) is the density of the vessel wall.

On the fluid–solid interface the kinematic and dynamic boundary conditions

\[ \nu = \dot{w}, \quad \sigma \cdot n_w + \tau \cdot n_f = 0, \]

with stress tensor \( \sigma \), normal vector \( n_w \) for the wall and \( \tau, n_f \) for the fluid, are prescribed [3].

In our computations we consider the vessel wall having the following parameters: the density \( \rho_w = 997 \text{ kg/m}^3 \), Young’s modulus \( E = 10 \text{ MPa} \), Poisson’s ratio \( \nu = 0.49 \), and the wall thickness is 0.4 mm [4].

3. Numerical computations
The geometry of the cerebral vessels with the giant aneurysm is reconstructed from the magnetic resonance scans of a real patient. The segmentation is performed with the specialised ITK-SNAP software [10]. The intraoperative endovascular measurements of the velocity and pressure are carried out in this patient during the surgery [6]. The measured clinical data are denoised and rescaled to be a 1 s periodic function of time (figure 2). The calculations are performed on a 3-second interval with a 0.01 s time step. The periodic solution is formed during the first two seconds of the numerical simulation and the solution within the last period is taken to be the sought periodic solution.

The calculations are performed using the finite volume method with the ANSYS CFX solver. The computational mesh has the following parameters: number of nodes is 437,458, number of elements is 2,263,675, maximum size of the element is 0.2 mm, minimum edge length is 0.13 mm. The mesh is unstructured, tetrahedral, and adaptive (the number of elements is increased in the high curvature regions).

The deformation of the vessel wall is found using the finite element analysis with the ANSYS Mechanical solver. The computational mesh consists of 104,295 nodes and 104,878 elements with maximum size of 0.2 mm and minimum edge length of 0.13 mm.

The two-way FSI problem is solved with the CFX and ANSYS solvers with the CFX coordinating the interaction between the solvers. We note that the full hydroelasticity problem is very computationally expensive. Characteristic durations of the simulations on a 6-core station are

- hydrodynamics: 1 hour 51 min 30 s,
- 1-way FSI: 3 hours 36 min 52 s,
- 2-way FSI: 6 days 19 hours 2 min 50 s.
Figure 3. Hydrodynamic parameters at systole \((t = 2.22 \text{ s})\): (a) streamlines, (b) pressure, (c) wall shear stress

Figure 4. Solution of the one-way FSI at systole \((t = 2.22 \text{ s})\): (a) strain, (b) displacement, (c) von Mises stress

4. Results

The following hydrodynamic parameters are computed for the hydrodynamic problem \([1]\): velocity and streamlines, pressure, and vessel wall shear stress (figure 3).

The flow in the aneurysmal sac is vortical with a high-speed jet flow with the velocity greater than 1 m/s along the anterior wall of the aneurysm. In the centre of the aneurysmal sac, the flow is slower with the velocity less than 0.3 m/s. The pressure reaches 117 mm Hg on the aneurysm wall.

The region of a high wall shear stress (19.53 Pa) is located along the jet flow in the aneurysm. The wall shear stress is also large (13.02 Pa) at the aneurysmal neck and the upper wall of the aneurysm. It is known that high shear stresses on the vessel wall can result in an emaciation and degradation of the vessel wall, which can contribute to the further aneurysm’s growth and rupture \([1]\).

The solution of the one-way FSI problem is depicted in figure 4. The results show that the strains in the aneurysmal wall are larger by 0.11 than those in the vessel wall. Maximum strain of 0.16 is located at the aneurysmal neck. Maximal displacement of 0.92 mm is reached on the aneurysmal wall. In structural analysis, von Mises stress is often used for characterisation of the stress state of the material. The von Mises stress is computed from the stress tensor by formula \([11]\):

\[
\sigma_{VM} = \sqrt{\frac{(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2}{2}}.
\]

The distribution of the von Mises stress is similar to the strain. Maximum stress of 1.6 MPa is located at the aneurysmal neck.
In figure 5, the solution to the full hydroelasticity problem is shown. The pressure on the aneurysmal wall is distributed uniformly with maximal value of 136 mm Hg. Maximum values of the stress are reached in the aneurysmal sac and neck (0.4 MPa). The displacements reach maxima on the aneurysmal sac and at the junctions with left superior cerebellar artery and left posterior cerebral artery. The solution qualitatively agrees with the two previous ones.

5. Comparison of the solutions

Rigid wall vs two-way FSI In figure 6 the quantitative comparison of the fluid velocity is shown. A difference between the velocity fields clearly shows that the second high speed jet flow in the aneurysmal sac exists when the fluid–solid interaction is taken into account. The difference in velocity magnitude is approximately 0.75 m/s near the aneurysm neck. The existence of the

Figure 5. Solution of the two-way FSI at systole ($t = 2.22$ s): (a) pressure, (b) vessel wall displacement, (c) stress in vessel wall

Figure 6. Comparison of the velocity ($t = 2.22$ s): (a) rigid walls, (b) two-way FSI, (c) the difference of the solutions

Figure 7. Comparison of the wall shear stress ($t = 2.22$ s): (a) rigid walls, (b) two-way FSI, (c) difference of the solutions
Figure 8. Comparison of the pressure \((t = 2.22 \text{ s})\): (a) rigid walls, (b) two-way FSI, (c) difference of the solutions.

Figure 9. Comparison of the displacement \((t = 2.22 \text{ s})\): (a) one-way FSI, (b) two-way FSI, (c) difference of the solutions.

Figure 10. Comparison of the stress \((t = 2.22 \text{ s})\): (a) one-way FSI, (b) two-way FSI, (c) difference of the solutions.

The second jet flow can be also seen in the difference of the wall shear stress shown in figure 7. The difference is approximately 10 Pa.

The pressure in the rigid-wall assumption is higher than that in the two-way hydroelasticity (figure 8). The difference is about 30 mm Hg.

One-way FSI vs two-way FSI. The comparison between the displacements in one-way FSI and two-way FSI solutions is shown in figure 9. In case of the one-way interaction, the displacements are higher by approximately 30%. The difference in stress reaches 0.72 MPa (figure 10) with higher values in the one-way FSI approximation.

Thus, the one-way FSI approximation results in higher values of stress, strain and displacement. This is due to the usage of the static equations 2 in which the inertia of the vessel walls is neglected.
6. Conclusion
In this paper a computer simulation of a blood flow in cerebral vessels with a giant saccular aneurysm at the bifurcation of the basilar artery is performed. The modelling is based on patient-specific clinical data (both flow domain geometry and boundary conditions for the inlets and outlets). The hydrodynamic and mechanical parameters are calculated in the frameworks of three models: rigid-wall assumption, one-way FSI approach, and full (two-way) hydroelastic model.

A comparison of the numerical solutions shows that mutual fluid–solid interaction can result in qualitative changes in the structure of the fluid flow. Other characteristics of the flow (pressure, stress, strain and displacement, except the wall shear stress, which depends of the flow pattern) qualitatively agree with each other in different approaches. However, the quantitative comparison shows that accounting for the flow–vessel interaction, in general, decreases the absolute values of these parameters. Thus, solving of the hydroelasticity problem gives a more detailed solution at a cost of highly increased computational time.

Acknowledgments
The theoretical research of this paper was supported by the Russian Foundation for Basic Research (Project No.16-31-00223). The endovascular measurements of the blood flow were supported by the Russian Foundation for Basic Research (Project No.14-01-00036). The blood vessel segmentation, 3D reconstruction from MR scans, and numerical simulations were supported by the Russian Science Foundation (Project No.14-35-00020).

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