Cosmic acceleration without dark energy: background tests and thermodynamic analysis

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Received June 26, 2014
Revised September 5, 2014
Accepted September 15, 2014
Published October 17, 2014

Abstract. A cosmic scenario with gravitationally induced particle creation is proposed. In this model the Universe evolves from an early to a late time de Sitter era, with the recent accelerating phase driven only by the negative creation pressure associated with the cold dark matter component. The model can be interpreted as an attempt to reduce the so-called cosmic sector (dark matter plus dark energy) and relate the two cosmic accelerating phases (early and late time de Sitter expansions). A detailed thermodynamic analysis including possible quantum corrections is also carried out. For a very wide range of the free parameters, it is found that the model presents the expected behavior of an ordinary macroscopic system in the sense that it approaches thermodynamic equilibrium in the long run (i.e., as it nears the second de Sitter phase). Moreover, an upper bound is found for the Gibbons-Hawking temperature of the primordial de Sitter phase. Finally, when confronted with the recent observational data, the current ‘quasi’-de Sitter era, as predicted by the model, is seen to pass very comfortably the cosmic background tests.

Keywords: dark matter theory, alternatives to inflation, baryon acoustic oscillations, cosmological parameters from CMBR

ArXiv ePrint: 1406.5538v1
1 Introduction

In the standard cosmological model, the universe is homogeneous and isotropic and the main sources of the gravitational field is a mixture of ideal fluids containing a baryonic plus cold dark matter (CDM) components, and a cosmological constant, $\Lambda$. Due to Lorentz’s invariance, the latter is endowed with a negative pressure that accounts for the present state of accelerated expansion [1–6]. With just an additional free parameter, $\Lambda$, the cosmic concordance lambda cold dark matter model ($\Lambda$CDM) fits rather well the current astronomical data from supernovae type Ia, baryon acoustic oscillations (BAOs), cosmic microwave background (CMB), galaxy clusters evolution, and complementary observations [7–22].

Nevertheless, there are severe and profound drawbacks related to a finite but incredibly small value of $\Lambda$. Firstly, attempts to associate it to the vacuum energy density estimated by quantum field theory leads to a discrepancy of 50 to 120 orders of magnitude with respect to its observed value, about $3 \times 10^{-11} \text{eV}^4$. This implies an extreme fine-tuning problem giving rise to the so-called cosmological constant problem, which also requires an improbable cancellation by some unknown physical mechanism [23–25]. This is why generic proposals replacing $\Lambda$ by some evolving field termed “dark energy” were suggested by many authors, however, the true nature of this field still remains elusive [26–29]. Secondly, there is also the coincidence problem which is related to the question of “why are the energy densities of pressureless matter, $\rho_m$, and vacuum, $\rho_\Lambda = \Lambda/(8\pi G)$, of the same order precisely today in spite of the fact that they evolve so differently with expansion?” [30]. Proposals to alleviate such problems include decaying vacuum models which promote the cosmological constant to a field, $\Lambda(t)$, that varies with time in a suitable manner, and many interacting scalar field descriptions of dark energy [31–62], as well as a single fluid with an antifriction dynamics [63].

On the other hand, the recent accelerating phase of the Universe was probably not the only one. According to the standard cosmological model, the Universe must have experienced a very brief period ($\sim 10^{-30} \text{s}$) of fast accelerated expansion shortly after the big bang, responsible for the observed homogeneity and isotropy of the Universe on large scales, its spatial flatness, and the spatial fluctuations in temperature of the cosmic background (CMB) radiation. Hence, the need arises for introducing a further unknown energy component to account for this. Unfortunately this “solution” comes about with several new problems, like
the initial conditions, the graceful exit and multiverse problems, a combination leading to the existence of new fine tunings and some conceptual problems ([64, 65] and, for a different viewpoint, [66]).

Because of such difficulties, here we suggest another well known proposal for the cosmic acceleration, the gravitational particle production mechanism. The microscopic description of this process was pioneered by Schrodinger’s [67], and developed by Parker and others based on quantum field theory in curved spacetimes [68–72]. There is still an open question concerning the actual range of the particle creation effects in cosmology, since such process involves non-equilibrium quantum field theory in curved space-times, which was not, by now, theoretically developed. We know, however that such a mechanism must in fact take place, and therefore, its possible effects on cosmology should be considered, at least from a phenomenological point of view (for a recent discussion on the dynamic connection between particle production cosmologies and decaying vacuum models see [73]).

A macroscopic description of the particle production mechanism by the gravitational field was also discussed long ago by Prigogine and collaborators [74]. Later on, Calvão, Lima and Waga proposed a covariant description [75, 76], and the physical difference between particle production and the bulk viscosity mechanism was clarified by Lima and Germano [77]. The particle production process is classically described by a back reaction term in the Einstein field equations whose negative pressure may provide a self-sustained mechanism of cosmic acceleration. Indeed, since the middle of the nineties, many phenomenological accelerating scenarios have been proposed in the literature [78–83].

Some years ago, it was shown that phenomenological particle production can explain not only the present era of cosmic acceleration but also provide a viable alternative to the concordance ΛCDM model [84–87]. Recently, it has been argued that CCDM is observationally degenerate with respect to ΛCDM (dark degeneracy) even at a perturbative level [88]. In principle, one may also think that the mechanism of particle production could account not only for the late time cosmic acceleration but also for the primeval one (i.e., early inflation). In this case, besides evading the problems related to the cosmological constant it may also have some advantages with respect to standard inflation. In fact, as shown by Lima, Basilakos and Costa [91] (LBC hereafter) gravitationally-induced particle production in the course of the expansion can be responsible by the instability of the initial de Sitter state which evolves smoothly to the standard radiation phase when the production of massless particles is suppressed. More interesting, not only the horizon problem is solved, but also the production of relativistic particles during inflation avoids the supercooling and the need for a subsequent reheating phase, thereby solving in a natural way the exit problem. Qualitatively, such a scenario resembles a variant of the so-called “warm inflation” [92–95].

The cosmological scenario proposed here generalizes the LBC model whose thermodynamic behavior was investigated by Mimoso and Pavón [96]. As with the LBC cosmology, this novel scenario is also complete in the sense that it describes the cosmic evolution from an early to a late time de Sitter phase with the accelerating stages powered by the creation of particles by the gravitational field. As we shall see, the Universe starts from a nonsingular unstable de Sitter phase thereby being free of the horizon problem, and, smoothly evolves to the standard radiation-dominated phase. As the Universe keeps on expanding, the radiation component becomes subdominant and pressureless dark matter takes over. It is then when the production of CDM particles gets triggered. Finally, the Universe approaches the second de Sitter era of expansion characterized by thermodynamic equilibrium.
The generalization of the complete particle production scenario intends to improve this model by providing a more embracing and dynamical cosmic evolution, maintaining, however, the ability of recovering the ΛCDM dynamics for certain values of the free parameters. Especially, at the early universe, we expect that the proposed generalization will provide a way toward the conciliation of the model with recent CMB observations by the Planck satellite [11, 64–66].

In order to test the range of the parameter space of the model we also perform a thermodynamic analysis based on the generalized second law (GSL) of thermodynamics. This law, first formulated for black holes and their environment [97, 98] and later extended to cosmic horizons [99–102], establishes that the entropy of the system plus that of the causal horizon enveloping it should never decrease. Further, in the last stages of the evolution the total entropy should also be a concave function. Otherwise, the total entropy (system plus horizon) would increase unbounded without ever reaching equilibrium -the state of maximum entropy compatible with the constraints upon the system [103, 104]. Our aim is to explore which restrictions (if any) the GSL plus the concavity requirement impose on the free parameters of the cosmological model.

The paper is organized as follows. Next section briefly introduces the basics of phenomenological particle production in cosmology. Section 3 considers this effect in the early universe after the initial de Sitter expansion. Section 4 studies the corresponding constraints imposed by the second law of thermodynamics. Section 5 generalizes the scenario of ref. [91]. The thermodynamic analysis based on the GSL is carried out with emphasis on the transition from matter dominated to the second de Sitter phase. Based on recent observations we also determine some constraints on the main parameters of the model. Finally, section 6 summarizes our findings.

2 Cosmic dynamics in models with particle production

Let us consider a flat, homogeneous and isotropic Friedmann-Robertson-Walker (FRW) universe, whose matter content is endowed with the mechanism of particle production. In this case the Friedmann equations can be written as [75–77]:

\[ 8\pi G\rho = 3\frac{\ddot{a}}{a^2} , \]  
\[ 8\pi G(p + p_c) = -2\frac{\ddot{a}}{a} -\frac{\dot{a}^2}{a^2} . \]

where \( \rho \) and \( p \) are the energy density and the equilibrium hydrostatic pressure, \( a \) is the cosmic scale factor (the over-dot means derivative with respect to cosmic time) and \( p_c \) is the creation pressure which is related to the gravitationally induced process of particle production.

As a consequence, the energy conservation law generalizes to

\[ \dot{\rho} + 3H(\rho + p + p_c) = 0 . \]  

Recently, a great deal of attention has been paid to scenarios driven by ‘adiabatic’ particle production. In this case, particles and entropy are generated but the entropy per particle does not vary. Under such ‘adiabatic condition’, the creation pressure can be written as [84, 85, 91, 96]

\[ p_c = -(\rho + p)\frac{\Gamma}{3H} , \]
where the positive-definite quantity $\Gamma$ denotes the rate of particle production. We shall denote the latter by $\Gamma_r$ and $\Gamma_m$ during the early and late phases of expansion, respectively.

By assuming the usual equation of state, $p = w\rho$, it is readily checked from eqs. (2.1)–(2.2) and (2.4) that the Hubble parameter is governed by the differential equation

$$\dot{H} + \frac{3(1+w)}{2}H^2 \left(1 - \frac{\Gamma}{3H}\right) = 0.$$  

(2.5)

Thus, this scenario will be fully determined once $w$ and $\Gamma$ are specified. Note that for $w = \text{constant}$ and $\Gamma \ll 3H$ the standard evolution, $a(t) \propto t^{\frac{2}{3(1+w)}}$, is readily recovered.

3 Production of particles in the early universe

In the previous section we have seen that the ratio $\Gamma/3H$ is the key to determine the dynamics of the model. Due to the absence of a rigorous quantum field theory in curved space-time, including (non-equilibrium) back reaction, from which the expression of $\Gamma$ should be calculated, in [91] a phenomenological early universe model with a particle creation rate, given by

$$\frac{\Gamma_r}{3H} = \frac{H}{H_I},$$  

(3.1)

was introduced. Here, $H_I$ is the constant inflationary expansion rate associated to the initial de Sitter phase ($H \leq H_I$), and $\Gamma_r$ is the creation rate of relativistic particles in the transition from the early de Sitter stage to the radiation dominated phase.

The ratio $H/H_I$ takes into account that the particle production must be strongly suppressed ($\Gamma_r/3H \ll 1$) when the Universe enters the radiation phase. This is so because, according to Parker’s theorem, massless particles cannot be quantum-mechanically produced in that phase [72].

This photon creation model at early times provides an interesting description of the Universe evolution [91]. The latter starts with a de Sitter expansion without initial singularity. This expansion, due to the creation of massless particles, becomes unstable and the universe subsequently enters the conventional radiation-dominated era.

At this point, it is interesting to consider the following generalization of (3.1)

$$\frac{\Gamma_r}{3H} = \left(\frac{H}{H_I}\right)^n,$$  

(3.2)

where $n$ is a non-negative constant parameter to be constrained by observational data. The above expression reduces to the original model [91] in the particular case of $n = 1$.

Replacing this more general expression for the creation rate in eq. (2.5), it follows that the evolution of the Hubble parameter in this case ($w = 1/3$) is governed by

$$\dot{H} + 2H^2 \left[1 - \left(\frac{H}{H_I}\right)^n\right] = 0.$$  

(3.3)

As expected, for $H = H_I$ we obtain an unstable de Sitter solution, $\dot{H} = 0$. The creation of particles, as before, causes a dynamic instability that leads to a transition from a de Sitter regime to the radiation dominated era.

By integrating the last equation we obtain

$$H = \frac{H_I}{(1 + Da^{2n})^{1/n}}.$$  

(3.4)
or, alternatively,

\[
\int_{a_*}^{a} \frac{d\tilde{a}}{\tilde{a}} \left[ 1 + D \tilde{a}^{2n} \right]^{1/n} = H_I t
\]

(3.5)

where \( D \) is a positive definite constant, \( t \) is the time elapsed after the end of the inflationary era, denoted by \( t_* \), hence \( a_* = a(t_*) \). Using the condition \( H(a_*) \equiv H_* \), the constant of integration \( D \) is found to be

\[
D = a_*^{-2n} \left[ \left( \frac{H_I}{H_*} \right)^n - 1 \right].
\]

(3.6)

Integration of eq. (3.5) yields the cosmic time \( t(a) \) in terms of the scale factor\(^1\)

\[
t(a) = \left( \frac{1 + D a^{2n}}{2H_I D a^{2n}} \right)^{\frac{1+n}{2n}} \times F \left[ 1, 1, 1 - \frac{1}{n}; \frac{-1}{D a^{2n}} \right],
\]

(3.7)

where \( F[\alpha_1, \alpha_2, \alpha_3, z] \) is the Gauss hypergeometric function.

It is easy to show that the correct transition from an early de Sitter to the radiation phase is obtained for any positive value of \( n \). Thus, our analysis shows that this generalized model can describe the dynamics of the early universe free of the big bang singularity due to \( D \neq 0 \), and that it overcomes the horizon problem.

Specifically, the Universe starts from an unstable inflationary phase \([H \simeq H_I \text{ with } a(t) \simeq a_* e^{H_I t}]\) powered by the huge value \( H_I \) which may connected to the scale of a grand unified theory or even the Planck scale, then it deflates (with a massive production of relativistic particles), and subsequently evolves towards the radiation-dominated era, \( a \sim t^{1/2} \) (i.e., \( H \simeq a^{-2} \)), for \( Da^{2n} \gg 1 \) in eq. (3.4). Hence, there is “graceful exit” from the inflationary stage which implies that the Universe evolves continuously from inflation towards the standard (FLRW) radiation dominated stage, thereby ensuring that primordial Big Bang Nucleosynthesis (BBN) will not be harmed at all. Indeed the primordial BBN epoch can proceed fully standard, with no interference at all from the particle production which is strongly suppressed \((H/H_I \ll 1)\) in the radiation era and thus the Hubble parameter reduces to its nominal form, namely \( H \simeq a^{-2} \).

Finally, it is useful to point that all the dynamical stages discussed here can be described in a field theoretical language, i.e., in terms of the dynamics of an effective scalar field \((\phi)\). For a similar analysis in the case of bulk viscosity see [117], and for the equivalent decaying \( \Lambda(t) \)-models see refs. [54, 118].

4 Thermodynamic analysis of the early universe

Let us now discuss the thermodynamic behavior of the model in the radiation era. Thermodynamics tells us that the entropy of isolated systems can never diminishes, and it is concave, at least during the last stage of approaching equilibrium (otherwise no entropy maximum could ever be achieved).

Recently it was demonstrated that cosmological apparent horizons are also endowed with thermodynamical properties [105]. One can relate a temperature and entropy to the apparent horizon analogous to the ones associated to the black hole event horizon. Unlike the event horizon, the cosmic apparent horizon always exists and it coincides with the event

\(^1\)In order to obtain (3.7) we have applied the transformation \( u = -1/D a^{2n} \).
horizon in the case of a last de Sitter expansion. So, in accordance with the GSL, the total entropy $S$ must include the entropy of all sources, that is, the fluid inside the apparent horizon and the entropy of the apparent horizon itself. Denoting by $S_\gamma$ the entropy when the Universe is radiation-dominated and $S_h$ the apparent horizon entropy, it thus follows that $S = S_\gamma + S_h$.

The radiation phase is followed by a matter dominated era that eventually will transit to a second de Sitter phase. Accordingly, we expect that in the radiation phase the entropy increases and be a convex function of the scale factor, i.e., $S' > 0$ and $S'' > 0$ (a prime means $d/da$). If it were concave, the Universe would have attained a state of thermodynamic equilibrium (maximum entropy) and would stayed in it for ever unless forced by some “external agent”. However, as is well-known, during the radiation phase the production of particles is suppressed \[72\]; so in this model there would be no external agent to remove the system from thermodynamic equilibrium. This is why we expect the entropy to be convex in this phase.

The entropy of the apparent horizon is given by $S_h = k_B A / 4 \ell_{pl}^2$ \[106\], where $A = 4 \pi r_h^2$ is the area of the horizon, $k_B$ the Boltzmann’s constant, $\ell_{pl}$ the Planck’s length, and $r_h$ the radius of the horizon. In our case, a spatially-flat Universe, the latter coincides with the Hubble horizon, $H^{-1}$.

On the other hand, the entropy of the radiation fluid can be obtained from Gibbs’s equation,

$$T_\gamma dS_\gamma = d(\rho_\gamma V) + p_\gamma dV,$$  \hspace{1cm} (4.1)

where $V = 4\pi/(3H^3)$ is the spatial volume enclosed by the horizon, $T_\gamma$ the radiation temperature, $p_\gamma = \rho_\gamma/3$ with

$$\rho_\gamma = \frac{\rho_I}{(1 + D a^{2n})^{2/n}}, \quad \rho_I = \frac{3H_I^2}{8\pi G}.$$  \hspace{1cm} (4.2)

In arriving at this expression use of eqs. (2.1) and (3.4) was made.

By deriving $S_h$ and using eq. (3.4) again, we obtain

$$S_h' = \frac{4k_B \pi}{\ell_{pl}^2 H_I^2} D a^{2n-1} \left(1 + D a^{2n}\right)^{\frac{2}{n} - 1}.$$  \hspace{1cm} (4.3)

Clearly, $S_h' > 0$ regardless the value of $n$.

On its part the radiation temperature obeys,

$$T_\gamma = \frac{T_I}{(1 + D a^{2n})^{\frac{1}{2n}}}$$  \hspace{1cm} (4.4)

where $T_I$ is the initial temperature in the de Sitter phase. Note that for $n = 1$ the LBC expression is recovered (see eq. (13) there). Obviously, for $D a^{2n} \gg 1$ (well inside the radiation era) we recover the standard radiation result, i.e., $T_\gamma \propto a^{-1}$.

From Gibbs’s equation (4.1) it follows that

$$T_\gamma S_\gamma' = \frac{16\pi}{3} \rho_I D \frac{H_I}{a^{2n-1}} a^{2n-1} \left(1 + D a^{2n}\right)^{\frac{1}{n} - 1},$$  \hspace{1cm} (4.5)

that is to say, $S_\gamma' > 0$ irrespective of the value of $n$. 

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To discern whether $n$ gets constrained by the convexity of the total entropy we must determine the sign of the second derivatives of both entropies. From (4.3) we readily get

$$S''_h = CD a^{2(n-1)} \left(1 + Da^{2n}\right)^{\frac{2}{n^2}} \left[3D a^{2n} + 2n - 1\right].$$  \hspace{1cm} (4.6)$$

and from (4.4) and (4.5)

$$S''_\gamma = \frac{16\pi}{3} \frac{\rho T^3}{T^3_I H^3_I} a^{2(n-1)} \left(1 + Da^{2n}\right)^{\frac{3}{2n^2}} \left[\frac{2(Da^{2n} + n) - 1}{1 + Da^{2n}}\right].$$  \hspace{1cm} (4.7)$$

Thus the positivity of both second derivatives is ensured whenever $n > 1/2$.

Altogether, while the GSL does not set any constraint on $n$ the convexity of the total entropy during the radiation era does pose a lower bound on this parameter.

4.1 Quantum corrections

It is well-known that quantum effects generalize the Bekenstein-Hawking entropy law for black holes to the expression

$$S_h = k_B \left[\frac{A}{4\ell_{pl}^2} - \frac{1}{2} \ln\left(\frac{A}{\ell_{pl}^2}\right)\right],$$  \hspace{1cm} (4.8)$$

plus higher order terms [107, 108]. As pointed out in [96] the same should apply to causal cosmic horizons. Here we analyze whether our results remain valid when such corrections are not overlooked.

A simple calculation in the context of our scenario yields

$$S'_h = \frac{k_B \pi}{\ell_{pl}^2 a H^2} \left(1 - \left(\frac{H}{H_I}\right)^n\right) \left[1 - \frac{\ell_{pl}^2 H_I^2}{2\pi(1 + Da^{2n})^{2/n}}\right].$$  \hspace{1cm} (4.9)$$

It is immediately seen that the presence of the factor $\ell_{pl}^2$ in the numerator of the second term in the square parenthesis renders the said term negligible. Thereby our approach is robust against quantum modifications to the horizon entropy in the early universe.

Moreover, in the limit $a \to 0$ the condition $S'_h > 0$ implies the upper bound on the initial expansion rate, $H_I < \sqrt{2\pi}/\ell_{pl}$, independent of $n$. Thus, the generalization of the model does not alter the original sensible result obtained in [96]: the initial Hubble factor cannot be arbitrarily large; his squared value is limited by Planck’s curvature.

It is also interesting that the temperature $T_I$ appearing in the expression (4.4) has also a natural upper limit imposed by the quantum corrections discussed here. In fact, recalling that the initial temperature of the Universe in our scenario can be associated to the Gibbons-Hawking result [99–102], $T_I = H_I/2\pi$ (see LBC), it is easy to check from the above inequality that $T_I < 1/\sqrt{2\pi}\ell_{pl}$. In other words, the quantum corrections to the usual entropy formula imply that the initial temperature of the Universe in our model is slightly smaller than Planck’s temperature, as it should be expected from a classical description.
5 A generalized cosmological scenario

In the context of particle production models it was shown in the LBC paper that the two eras of accelerated expansion can be closely related through a single expression for the particle creation rate. The model was called “a complete cosmological scenario without dark energy” [91]. The resulting cosmology showed consistency with the observational data both at the background and perturbative level. The phenomenological expression proposed in that work for the particle creation rate was

$$\frac{\Gamma}{3H} = \frac{H}{H_I} + \left(\frac{H_f}{H}\right)^2.$$  (5.1)

Notice that the ansatz of the second term in eq. (5.1) and the corresponding dynamics were first presented in [63]. The first and second terms on the right hand side dominate at early and late times, respectively. Since we have already shown that the proposed extension of the first term is compatible with GSL, we now take a step further by generalizing the second term as well.

Let us now introduce a new extended scenario in which the expression for the particle production rate also encompasses the whole cosmic expansion, namely:

$$\frac{\Gamma}{3H} = \left(\frac{H}{H_I}\right)^n + \left(\frac{H_f}{H}\right)^m,$$  (5.2)

where the free parameter $m$ must be non-negative to lead to an acceptable matter-vacuum expansion history. However, the above (5.2) encompasses several dynamical possibilities involving decaying vacuum models $\Lambda(t)$ (i.e., with $\dot{\Lambda}(t) < 0$), which also solves (or, at least, alleviate) the coincidence problem. In fact, as recently shown in [73], particle production by the gravitational field can also mimic the dynamics of $\Lambda(t)$ models by the correspondence

$$\frac{\Lambda(t)}{3H^2} = \frac{\Gamma}{3H},$$  (5.3)

where the evolution of $\Lambda(t)$ is usually described by a phenomenological power law in $H$. Two previous examples considered in the literature are, $\Lambda(t) = \beta H^2$ [35], and $\Lambda(t) = \beta H$ [109, 110]. In both cases $\beta$ is a positive constant. Therefore, by allowing $\Gamma_m/3H$ to be described at late times by a general power law in $H$, we can also include in the description some physically viable decaying vacuum dynamics (see also the discussion in [49–51]). Such models have been proposed in order to alleviate the coincidence and $\Lambda$ problems, and, simultaneously, to describe the cosmic expansion closely to the $\Lambda$CDM at recent times.

At late times, pressureless matter takes over radiation as the dominant energy component (we denote the former by a subscript $m$), and the Hubble factor satisfies $H_f \leq H \ll H_I$. Accordingly, at the epoch when the second term in (5.2) dominates the evolution of $H$ is dictated by

$$\dot{H} = \frac{3}{2}H^2 \left[1 - \left(\frac{H_f}{H}\right)^m\right].$$  (5.4)

Its solution in terms of the scale factor can be written as

$$H(a) = \left(C a^{-3m/2} + H_f^m\right)^{1/m},$$  (5.5)

where $C = H_0^m - H_f^m$. 

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Figure 1. Likelihood contours for $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$ equal to 2.32, 6.18 and 11.83, corresponding to 1$\sigma$, 2$\sigma$ and 3$\sigma$ confidence levels, in the $(\Omega_{m0}, m)$ plane using the SNIa/BAO/CMB$_{\text{shift}}$ overall statistical analysis (see text). The cross signals the best fit model $(\Omega_{m0}, m) = (0.283, 1.934)$. The inset shows the solution space for the concordance $\Lambda$CDM model.

By evaluating the above expression at the present time $[a = a_0 = 1, H(a = 1) = H_0]$ we obtain

$$H(a) = H_0 \left( \Omega_{m0} a^{-3m/2} + \tilde{\Omega}_{\Lambda0} \right)^{1/m},$$

(5.6)

where $\Omega_{m0} = 1 - (H_f/H_0)^m$ and $\tilde{\Omega}_{\Lambda0} = 1 - \Omega_{m0}$. Obviously, for $m = 2$, we recover, in an effective way, the evolution of the $\Lambda$CDM model. By using the above Hubble parameter, we have performed a joint statistical analysis involving the latest observational data, namely: SNIa-Union2.1 [9], BAO [111, 112] and the Planck CMB shift parameter [11, 113]. The corresponding covariances can be found in Basilakos et al. [114] for the SNIa/BAO data and in [113] for the Planck CMB shift parameter, respectively. Specifically, the joint $\chi^2$ function is given by $\chi^2(p) = \chi^2_{\text{SNIa}} + \chi^2_{\text{BAO}} + \chi^2_{\text{CMB}}$, where $p$ is the statistical vector that contains the free parameters of the model, namely $p = (\Omega_{m0}, m)$. The expressions of the individual chi-square functions: $\chi^2_{\text{SNIa}}, \chi^2_{\text{BAO}}$ and $\chi^2_{\text{CMB}}$ can be found in [114].

As it turns out, the overall likelihood function peaks at $\Omega_{m0} = 0.283 \pm 0.012$, $m = 1.934 \pm 0.06$ with $\chi^2_{\text{min}}(\Omega_{m0}, m) \simeq 563.6$, resulting in a reduced value of $\chi^2_{\text{min}}/\text{dof} \sim 0.96$. Figure 1 shows the 1$\sigma$, 2$\sigma$ and 3$\sigma$ confidence contours in the $(\Omega_{m0}, m)$ plane of the joint analysis. Alternatively, considering the $\Lambda$CDM theoretical value of $m = 2$ and minimizing with respect to $\Omega_{m0}$ we find $\Omega_{m0} = 0.292 \pm 0.008$ (see the inset in figure 1) with $\chi^2_{\text{min}}(\Omega_{m0})/\text{dof} \simeq 567.5/585$.

We also made use of, the relevant to our case, corrected Akaike information criterion (AIC) [115, 116], defined, for the case of Gaussian errors, by

$$\text{AIC} = \chi^2_{\text{min}} + 2k,$$

(5.7)
where $k$ denotes the number of free parameters. A smaller value of AIC indicates a better model-data fit. However, small differences in AIC are not necessarily significant and therefore, in order to assess, the effectiveness of the different models in reproducing the data, one has to investigate the model pair difference $\Delta \text{AIC} = \text{AIC}_y - \text{AIC}_x$. The higher the value of $|\Delta \text{AIC}|$, the higher the evidence against the model with higher value of AIC; a difference $|\Delta \text{AIC}| \geq 2$ indicates a positive such evidence while if $|\Delta \text{AIC}| \geq 6$ the evidence is strong. In its turn, if $|\Delta \text{AIC}| \leq 2$ consistency among the two models under comparison should be assumed. In our case, the value of the particle creation model AIC$_\Gamma$ ($\sim 567.6$) is smaller than the corresponding one AIC$_\Lambda$ ($\sim 569.5$), which suggests that the particle creation cosmological model provides a better fit to the current data than the concordance $\Lambda$CDM cosmology. On the other hand, the $|\Delta \text{AIC}|=|\text{AIC}_\Lambda - \text{AIC}_\Gamma| \simeq 2$ value points out that the said data can hardly tell apart the $\Lambda$CDM from the particle creation model.

For the benefit of the reader we would like to mention that the CCDM perturbation analysis has been presented in Lima et al. [91] (see also [89]). Briefly, the latter authors have found that the CCDM model provides growth rates that match sufficiently well with the observed growth rate of structure, making CCDM a viable model for cosmology (in this connection see also [88]).

Finally, in order to see which values of the parameter $m$ are thermodynamically allowed we write the first derivative of the entropies of the horizon and matter. The first one is,

$$S'_h = \frac{2k_B \pi}{H^2 a} \left[ \frac{3}{2} \left( 1 - \left( \frac{H_f}{H} \right)^m \right) \right]. \quad (5.8)$$

As for the entropy of the matter fluid inside the horizon, it suffices to realize that every single dust particle contributes a given bit, say, $k_B$. So, $S_m = k_B 4 \pi r^3_h n_p / 3$, where the number density of particles, $n_p$, obeys the conservation equation $n'_p = (n_p / a) \left[ (\Gamma_m / H) - 3 \right]$ with $\Gamma_m = H (H_f / H)^m$. Hence,

$$S'_m = \frac{4k_B \pi n_p}{3a H^2} \left[ \frac{3}{2} \left( 1 - \left( \frac{H_f}{H} \right)^m \right) \right]. \quad (5.9)$$

As in the radiation case, the GSL, $S' = S'_m + S'_h \geq 0$, only constraints $m$ to be positive (something previously demanded to satisfy cosmic dynamics). Now, the condition that the total entropy approaches a maximum in the long run does not impose any further condition on $m$. Indeed, from eq. (5.5) we see that $H \to H_f$ when $a \to \infty$, therefore both $S'_h$ and $S'_m$ tend to zero in that limit. On the other hand, since both first derivatives are positive for finite scale factor, we conclude that $S'$ tends to zero from below; hence, $S''(a \to \infty) \leq 0$ which can be realized for positive values of $m$ only. Altogether, the generalized complete model [91] is consistent with thermodynamics also at late times for any positive value of the free parameter $m$.

6 Conclusions

In this work a generalized cosmological scenario of particle production, evolving from de Sitter to de Sitter was presented. Its thermodynamic viability, according to the GSL and the thermodynamic requirement that the entropy of the total system (fluid plus horizon) tends to a maximum in the long run, was investigated. As it turns out, the parameter $n$ [see eq. (5.2)] must be larger than $1/2$ while any positive value of $m$ shows compatibility with
thermodynamics. Further, the inclusion of quantum corrections [107, 108] in the limit $a \to 0$ sets a very reasonable upper bound on the initial Hubble rate, $H_I$, and on the Gibbons-Hawking temperature, $T_I$, which cannot be obtained by purely classical methods.

The statistical analysis of the model shows that, when confronted with current observational data, it performs not less well than the concordance $\Lambda$CDM model. We believe that the cosmological model proposed here provides a viable scenario that closely relates the two accelerated phases of the Universe through a single and simple ansatz, eq. (5.2). Moreover, in simplifying the dark sector it evades the coincidence and the cosmological constant problems of the $\Lambda$CDM model, and the need to introduce unknown components, like dark energy, or unobserved extra dimensions.

Acknowledgments

J.A.S.L. is partially supported by CNPq and FAPESP under grants 304792/2003-9 and 04/13668-0, respectively, and L.L.G. by a PhD fellowship from FAPESP (grant 09380-8). D.P. acknowledges support from the “Ministerio de Economía y Competitividad, Dirección General de Investigación Científica y Técnica”, Grant No. FIS2012-32099. SB acknowledges support by the Research Center for Astronomy of the Academy of Athens in the context of the program “Tracing the Cosmic Acceleration”.

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