STAR FORMATION: STATISTICAL MEASURE OF THE CORRELATION BETWEEN THE PRESTELLAR 
CORE MASS FUNCTION AND THE STELLAR INITIAL MASS FUNCTION

GILLES CHABRIER\textsuperscript{1,2} AND PATRICK HENNEBELLE\textsuperscript{3}
\textsuperscript{1} Ecole Normale Supérieure de Lyon, CRAL (UMR CNRS 5574) 69364 Lyon Cedex 07, France 
\textsuperscript{2} School of Physics, University of Exeter, Exeter, EX4 4QL, UK 
\textsuperscript{3} Laboratoire de radioastronomie, UMR CNRS 8112, Ecole normale supérieure et Observatoire de Paris, 24 rue Lhomond, 75231 Paris cedex 05, France

Received 2010 April 6; accepted 2010 October 29; published 2010 November 22

ABSTRACT

We present a simple statistical analysis of recent numerical simulations exploring the correlation between the core mass function (CMF) obtained from the fragmentation of a molecular cloud and the stellar mass function which forms from these collapsing cores. Our analysis shows that the distributions of bound cores and sink particles obtained in the simulations are consistent with the sinks being formed predominantly from their parent core mass reservoir, with a statistical dispersion of the order of one-third of the core mass. Such a characteristic dispersion suggests that the stellar initial mass function (IMF) is relatively tightly correlated to the parent CMF, leading to two similar distributions, as observed. This in turn argues in favor of the IMF being essentially determined at the early stages of core formation and being only weakly affected by the various environmental factors beyond the initial core mass reservoir, at least in the mass range explored in the present study. Accordingly, the final IMF of a star-forming region should be determined reasonably accurately, statistically speaking, from the initial CMF, provided some uniform efficiency factor. The calculations also show that these statistical fluctuations, due to, e.g., variations among the core properties, broaden the low-mass tail of the IMF compared with the parent CMF, providing an explanation for the fact that the latter appears to underestimate the number of “pre brown dwarf” cores compared with the observationally derived brown dwarf IMF.

Key words: ISM: clouds – stars: formation – stars: luminosity function, mass function

Online-only material: color figures

1. INTRODUCTION

Understanding star formation and the origin of the stellar initial mass function (IMF) remains a major challenge in astrophysics. Various observations have suggested a strong similarity between the IMF and the mass function of gravitationally bound structures in molecular clouds, identified as the prestellar core mass function (CMF), the first one being shifted downward compared to the second one by a nearly mass-independent factor of about 2–3 \cite{Motte+98, Testi&Sargent98, Johnstone+2000, André+2007, André+2009, André+2010, Alvés+2007, Nutter&Ward-Thompson2007, Simpson+2008, Enoch+2008}. These observations suggest that the IMF is essentially determined by the properties of the turbulent self-gravitating gas in the parent molecular cloud, which leads to the core formation. Then, magnetically driven outflows are probably responsible for the subsequent mass-loss between the core mass and the stellar mass, yielding the aforementioned \( \sim 30\%–50\% \) efficiency factor characteristic of the CMF to IMF evolution \cite{Matzner&Mckee00}. An analytical theory for the formation and the mass distribution of unbound overdense “clumps” and gravitationally bound “cores” directly inherited from the global physical properties of the molecular cloud has recently been formalized by Hennebelle & Chabrier \cite{Hennebelle-Chabrier2008, Hennebelle-Chabrier2009}, hereafter HC08 and HC09, respectively and has received some support from numerical simulations of compressible turbulence aimed at exploring this issue \cite{Schmidt+10}.

Alternatively, some authors \cite{Bate&Bonnell2005, Bate2009} and references therein\) have suggested that the CMF is essentially determined by the various environmental conditions in the star-forming region (nearby massive stars, competitive accretion between prestellar cores, dynamical interactions, \ldots), i.e.,

by the various processes converting gas into stars. Accordingly, these authors argue that there is no correlation between the CMF and the IMF, as any possible link between these two distributions will inevitably be wiped out by these various environmental factors. Another notable difference between these two scenarios of star formation is the reason for the universal behavior of the IMF and the nearly invariance of the location of the peak of the CMF/IMF in various star-forming regions. In the first scenario, this universal property arises from the universal behavior of the turbulent spectrum, which tends to form clouds with similar (Larson-like) properties, whereas the peak invariance arises from the similar but opposite dependence of the Jeans mass and Mach number upon the cloud’s size \cite{HC08, HC09}. In contrast, in the second scenario, the IMF universality stems from competitive accretion and dynamical interactions, which wipe out the initial conditions due to the cloud’s properties.

Finding out which one, if any, of these scenarios is the dominant one, and thus whether or not there is a direct correlation between the CMF obtained from the fragmentation of a molecular cloud and the IMF which forms from these bound cores, represents a major issue to understand the very nature of star formation. Recently, Smith et al. \cite{2009MNRAS.394.2097S}, hereafter S09 have conducted dedicated numerical simulations in order to explore this issue. The original core masses in the simulations are identified from their gravitational potential whereas the final “stellar” masses are identified by sink particles. Because of the dispersion in the final sink mass distribution, these authors argue that there is a poor correlation between the IMF

\footnote{Remember that in the Hennebelle-Chabrier theory, the slope of the IMF and the Larson coefficients are directly related to the turbulence power spectrum index \cite{HC08}.}
and the CMF, and use this argument to invoke environmental conditions, i.e., accretion from the surrounding medium, as a dominant factor in the determination of the final stellar IMF. This conclusion bears important consequences for determining whether or not the final IMF of a star-forming region can be predicted accurately from the observed core mass distribution. Using core mass distributions similar to the ones obtained by S09 and simple statistical calculations, we determine the statistical correlation between the CMF and the sink mass function obtained in these simulations, in terms of a mass variance characteristic of the width of the CMF–IMF dispersion.

2. CALCULATIONS

The simulations of S09 identify two types of structures, as the outcome of the fragmentation of a molecular cloud. The initial overdense structures (denominated “p-cores” by the authors) are identified as peaks in the gravitational potential compared with the surrounding background. Note that some of these “p-cores” can have enough internal energy to prevent collapse and are thus unbound transient structures. Tracing the core binding energy throughout the structure lifetimes, S09 identify the bound cores as the structures with positive binding energy, supposed to represent the gravitationally bound prestellar cores observed in millimeter surveys. About 300 bound cores are found in the simulations, out of 573 initial p-cores. Eventually, these bound cores will collapse or fragment into smaller structures, identified in the simulations as sink particles, which are supposed to represent stars.

We start with a random distribution of about 300 bound cores within a mass range \( [M_{\text{inf}}, M_{\text{sup}}] = [0.2, 2]M_\odot \), similar to the number and mass range of bound cores in the simulations of S09.\(^5\) In order to be consistent with the numerical simulations, the core masses are drawn randomly according to a probability law \( P(m) = \int_{M_{\text{inf}}}^{M_{\text{sup}}} \rho_1(x) \, dx \), where the probability density \( \rho_1(x) \) is given by the Salpeter mass function over the entire mass range, \( \rho_1(x) \propto x^{-2.35} \). Our core population is thus consistent with the one identified in S09 (see their Figure 5).

In order to measure, statistically speaking, the degree of correlation (or lack of) between the initial CMF and the final IMF, we consider the probability to form a sink of a mass \( M_{\text{sink}} \) from a mass reservoir of mean value \( \mu = \varepsilon \times M_{\text{core}} \) to be given by a normal (Gaussian) law, of probability density \( p_2(x) = \exp[-(x - \mu)^2/2\sigma^2] \). Given the large statistical uncertainties entering the core identification and properties (contributions of the different modes of turbulence leading to the velocity dispersion, density distribution within the core, shape of the gravitational potential, etc.), it seems acceptable, in the absence of a more accurate study of systematic effects, to invoke such a normal density probability on the basis of the central limit theorem (e.g., Adams & Fatuzzo 1996). According to the above probability law, the probability for a sink mass \( M_{\text{sink}} \) to be drawn from a reservoir of mean mass \( \varepsilon \times M_{\text{core}} \), with some characteristic mass variance \( \sigma^2 \), is thus given by

\[
M_{\text{sink}} = \varepsilon \cdot \sigma + \varepsilon(t) \times M_{\text{core}},
\]

where the core mass \( M_{\text{core}} \) is sampled from the aforementioned Salpeter distribution and \( \varepsilon \) is a random variable of mean 0 and variance 1. The factor \( \varepsilon(t) \) illustrates the mass fraction accreted from the core mass reservoir onto the sink within a given time \( t \).

Figure 1. Left panel: sink mass distribution obtained from the core mass distribution according to Equation (1) for \( \varepsilon = 0.3 \) and \( \sigma = M_{\text{core}}/3 \) (red triangles). The small blue crosses represent the sinks identified in the simulations of Smith et al. (2009), kindly provided by Rowan Smith, for \( t = t_{\text{dyn}} \). The solid line corresponds to \( M_{\text{sink}} = \varepsilon \times M_{\text{core}} \). Right panel: the two respective mass distributions (solid line: present; dashed line: Smith et al. 2009). Dotted line: Salpeter mass function.

(A color version of this figure is available in the online journal.)

\(^5\) The 300 core calculations allow a direct comparison with the S09 results, but we have verified the robustness of our conclusions by conducting calculations with \( 10^5 \) cores.

The figure illustrates the similarity between the sink mass distributions obtained from Equation (1), with \( \sigma = M_{\text{core}}/3 \), and S09 (their Figure 10) at \( t = t_{\text{dyn}} \) is portrayed in Figure 1. The solid line corresponds to a “perfect” (zero variance) correspondence between the core mass and the sink mass, for a global uniform efficiency factor \( \varepsilon = 0.3 \). The figure illustrates the similarity between the two distributions. This is quantified on the right panel of the figure, which portrays the two sink mass distributions: both distributions agree within less than one Poissonian fluctuation over the mass range presently probed by the simulations.
These results show that the sink masses obtained from the numerical simulations of a collapsing molecular cloud are consistent with these masses being predominantly determined by the initial core mass reservoir, with some inherent statistical dispersion characterized by a standard deviation of the order of one-third of the core mass. The time evolution of this deviation can be inferred from a comparison of Figure 10 of S09, which displays the sink mass distribution for various dynamical times, and the sink mass distribution obtained from Equation (1) for various values of the coefficient $\epsilon$, which kind of mimics a time sequence evolution in the CMF-to-IMF conversion. Such a comparison is portrayed in Figure 2, where the value of $\sigma$, which determines the width of the distribution, is determined by looking for the best agreement between the two distributions. As seen in the figure, only after about three dynamical times in the S09 simulations, does the standard deviation start to increase from the fiducial value $M_{\text{core}}/3$, although even after five dynamical times, the deviation remains of the order of about 0.5–0.6 core mass. However, as discussed in the conclusion, exploring the CMF-to-IMF process after several dynamical times is probably unrealistic in reality as magnetically driven outflows will halt the accretion, and thus the star formation long before.

The sink mass function obtained according to Equation (1) is portrayed in Figure 3, for $\epsilon = 0.3$ and $\epsilon = 1$, as well as the CMF sampled from the Salpeter-like probability law. It is clear that the sink mass function closely resembles the CMF and recovers a Salpeter IMF. This was indeed anticipated from Equation (1). Since, however, this sink mass distribution agrees well with the one obtained in S09 simulations, arising from the star-forming gas cores produced by molecular cloud fragmentation, it is tempting to suggest that the observed resemblance between a prestellar CMF and the resulting stellar IMF indeed arises from the strong correlation, statistically speaking, between the two distributions, characterized by a mass variance $\sigma^2 \simeq (M_{\text{core}}/3)^2$, and a $\sim 30\%$ or so uniform efficiency factor, as suggested by observations and theoretical calculations (see the introduction).

### 2.1. Contribution from the Core Mass Ranges

Following S09 (see their Figures 11 and 12), we explore the contribution to the final sink mass distribution arising from various core mass ranges. Figure 4 (top) portrays the CMF obtained from the Salpeter-type probability law, within the same characteristic mass range as the simulations, with various symbols and colors denoting different mass domains. Figure 4 (bottom) illustrates the sink mass function obtained with the Gaussian probability law given by Equation (1), for $\epsilon = 1$ and $\sigma = M_{\text{core}}/3$, corresponding to 2 $t_{\text{dyn}}$ in S09 simulations (see Figure 2). The symbols/colors of the sinks are the same as their respective parent cores. The colors in the sink mass function are indeed well mixed, as found in the simulations of S09. Therefore, correlation between the CMF and the IMF does not imply that the IMF reflects the very same distinct color/symbol domains as the CMF. Statistical fluctuations within the core properties, characterized by the mass variance, lead to the color/symbol mixing of the sink IMF. Indeed, a given core mass leads eventually predominantly to a sink mass of similar value (for $\epsilon = 1$) but with some dispersion, producing a “spread” of the initial core mass domain over a larger final sink mass one, as illustrated in the figure.

### 2.2. Affecting the Width of the IMF

It has been suggested in HC08 and HC09 that the theoretical CMF might underestimate the number of (pre)brown dwarfs compared with the observationally derived Chabrier (2003) system IMF. Although this statement should be taken with caution, as the space density of the field or even young cluster...
brown dwarfs remains very uncertain, we show below that the aforementioned statistical fluctuations due to variations among the core properties naturally lead to a broader distribution in the low-mass part of the IMF and thus provide a natural shift of the sink mass function. Short dashed line: Salpeter mass function.

(A color version of this figure is available in the online journal.)

3. CONCLUSION

In this Letter, we have examined the degree of correlation between the prestellar CMF and the (system) stellar IMF, motivated by the remarkable observational resemblance between these two distributions. Note, however, that the bound cores identified in the simulations correspond to a time average, once each core becomes first bound, so that identification with observed cores at a snapshot in time must be done with caution (see Figures 4 and 5 of HC08 or Figures 8 and 9 of HC09). The sink mass distribution is obtained according to Equation (1) for $\epsilon = 1.0$ and $\sigma = 0.5 \times M_{\text{core}}$ (blue dots) or $\sigma = 0.8 \times M_{\text{core}}$ (red crosses). These sink mass spectra are broader than the original CMF and recover reasonably well the Chabrier IMF.

Figure 5. Empty circle: core mass spectrum drawn from a lognormal probability distribution with a mean ($\log 0.22$) (Chabrier 2003) and a standard deviation $\sigma = 0.4$; dots and crosses: sink mass spectrum obtained from this core mass spectrum according to Equation (1) for $\epsilon = 1.0$ and $\sigma = 0.5 \times M_{\text{core}}$ (blue dots) and $\sigma = 0.8 \times M_{\text{core}}$ (red crosses); dash line: Chabrier (2003) system IMF.

(A color version of this figure is available in the online journal.)
decreases significantly after the class-0 phase (about 1–2 \( t_{\text{ff}} \)); furthermore, recent observations suggest that the central protostar builds up essentially all its mass during a few episodes of violent accretion before this latter decreases substantially, with about half the mass of a 0.5 \( M_\odot \) dense prestellar core being accreted during less than 10% of the Class I lifetime, i.e., a fraction of a free-fall time (Evans et al. 2009). A scenario supported by numerical simulations (Vorobyov & Basu 2006). Therefore, it seems unlikely that significant accretion lasts long enough for the strong correlation between the (initial) CMF and the (final) IMF to be completely washed out. This issue needs to be explored with dedicated numerical simulations including large- and small-scale radiative and magnetic feedback processes. In the same vein, the collision timescale between prestellar cores in dense star-forming clumps appears to be significantly longer than the core lifetimes, suggesting that the cores should evolve individually to form a small number of stars rather than competing for gas accretion, leading to a natural mapping of the CMF onto the IMF (André et al. 2007, 2009; Enoch et al. 2008).

According to the present analysis, the final stellar IMF can be determined reasonably accurately, on a statistical basis, i.e., not for individual objects, from the initial core mass distribution in the cloud, i.e., the CMF, with some unavoidable scatter, leading naturally to two similar mass distributions (see Figure 3). Exploring a larger dynamical mass range with simulations, or conducting such a statistical analysis from observed CMF/IMF, a task possibly in reach with the HERSCHEL mission, would certainly help assessing this result. Such a correlation between the CMF and the IMF argues in favor of the IMF being essentially determined by the general properties of the parent cloud (mean temperature and density, large-scale velocity dispersion, with scaling properties following the Larson relations), as recently theorized by Hennebelle & Chabrier (2008, 2009), and being only weakly affected by the various environmental factors beyond the parent core mass reservoir. Accordingly, the final IMF of a star-forming region can be determined reasonably accurately, provided some average uniform efficiency factor, from the initial CMF obtained from millimeter or submillimeter surveys. Interestingly enough, the present calculations also show that the aforementioned statistical variations among the core properties yield a final IMF extending further down in the low-mass domain than the parent CMF, providing a natural explanation for the fact that this latter appears to underestimate the number of “pre brown dwarf” cores compared with the observationally derived brown dwarf IMF. These conclusions will have to be confronted to the wealth of data expected from the HERSCHEL mission, as nailing down this issue bears major consequences to understand the fundamental origin of star formation.

The authors are grateful to Rowan Smith, Ian Bonnell, and Paul Clark for stimulating discussions and for sharing their data. We are also thankful to Philippe André for fruitful conversations and insightful comments and to the anonymous referee for helping us improving the manuscript. We acknowledge funding from the European Research Council under the European Community 7th Framework Programme (FP7/2007-2013 Grant Agreement no. 247060.

REFERENCES

Adams, F., & Fatuzzo, M. 1996, ApJ, 464, 256
Alvés, J., Lombardi, M., & Lada, C. 2007, A&A, 462, L17
André, Ph., Basu, S., & Inutsuka, S.-I. 2009, in Structure Formation in Astrophysics, ed. G. Chabrier (Cambridge: Cambridge Univ. Press), 254
André, Ph., Belloche, A., Motte, F., & Peretto, N. 2007, A&A, 472, 519
André, Ph., et al. 2010, A&A, 518, L102
Bate, M. 2009, MNRAS, 392, 1363
Bate, M., & Bonnell, I. 2005, MNRAS, 316, 1201
Chabrier, G. 2003, PASP, 115, 763
Dib, S., Kim, J., Vazquez-Semadeni, E., Burkert, A., & Shadmehri, M. 2007, ApJ, 661, 262
Enoch, L., Evans, N. J., Sargent, A., Glenn, J., Rosolowsky, E., & Myers, P. 2008, ApJ, 684, 1240
Evans, N. J., et al. 2009, ApJS, 181, 321
Hennebelle, P., & Chabrier, G. 2008, ApJ, 684, 395 (HC08)
Hennebelle, P., & Chabrier, G. 2009, ApJ, 702, 1428 (HC09)
Johnstone, D., Fich, M., Mitchell, G., & Moriarty-Schieven, G. 2001, ApJ, 559, 307
Johnstone, D., Wilson, C., Moriarty-Schieven, G., Joncas, G., Smith, G., Gregersen, E., & Fich, M. 2000, ApJ, 545, 327
Matzner, C. D., & McKee, C. 2000, ApJ, 545, 364
Motte, F., André, P., & Neri, R. 1998, A&A, 336, 150
Nutter, D., & Ward-Thompson, D. 2007, MNRAS, 374, 1413
Offner, S., Klein, R., McKee, C., & Krumholz, M. 2009, ApJ, 703, 131
Schmidt, W., Kern, A., Federrath, C., & Klessen, R. 2010, A&A, 516, 25
Simpson, R., Nutter, D., & Ward-Thompson, D. 2008, MNRAS, 391, 205
Smith, R., Clark, P., & Bonnell, I. 2009, MNRAS, 396, 830 (S09)
Testi, L., & Sargent, A. 1998, ApJ, 508, L91
Tilley, D., & Pudritz, R. 2004, MNRAS, 353, 769
Vorobyov, E., & Basu, S. 2006, ApJ, 650, 956