A NOVEL GEOMETRICAL ANALYSIS OF THE ARTERIAL PULSE BASED ON THE GOLDEN RATIO $\phi$ (PHI): ASSOCIATION WITH HEART RATE VARIABILITY

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Purpose. This study aimed to quantify the deviation of an arterial pulse (pressure wave) from the „golden” or „divine” pulse, defined according to the golden ratio $\phi$ (phi), and to investigate whether the extent of this deviation is related to the function of autonomic nervous system (ANS) as assessed by 24-hr heart rate variability (HRV).

Methods. Seventy-two healthy subjects underwent 24-hr continuous monitoring of ECG. Applanation tonometry of the radial artery and pulse wave analysis of peripheral and central (mathematically derived) aortic pressure waveforms were performed in all subjects. Time- and frequency domain indices of HRV were computed together with nonlinear dynamic parameters. Two novel indices were computed primarily based on pressure (systolic and diastolic pressure; mmHg) and time (ejection and diastolic durations-intervals; msec) values extracted from the recorded pressure waves.

Results. The new phenotypic, geometrical biomarkers based on the golden ratio ($\phi$) were associated with

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The last few decades scientists began to appreciate the pivotal role of heart rate variability (HRV) in clinical research. HRV describes the variations of both instantaneous heart rate and RR intervals and it has prevailed, as a term, instead of cycle length variability, heart period variability, RR variability, and RR interval tachogram. HRV is increasingly recognized for its ability to indicate or predict various health problems such as autonomic nervous system dysfunction, cardiovascular disorders, fetal distress, anxiety, depression, asthma, diabetes and other diseases. Basically, HRV assesses the balance between the sympathetic and the parasympathetic nervous system and it can provide additional valuable information concerning pathophysiological conditions as well as risk stratification and prediction. A milestone in the clinical significance of HRV was reached in 1980s when it was confirmed that HRV was a strong and independent predictor of mortality after an acute myocardial infarction\textsuperscript{10}. Until now several other studies have demonstrated that HRV can predict mortality in other populations as well\textsuperscript{11}.

Several different methods and techniques are available for (i) the recording of cardiac cycle length (period) and (ii) for the processing and analysis of the acquired time-series (i.e. ECG, pressure, blood flow or other bio signals) for the quantification and assessment of HRV\textsuperscript{8}. Time-domain and spectral (or frequency-domain) analyses are the most commonly used methods for the computation of numerous indices of HRV\textsuperscript{12}. Other geometrical parameters and more recently dynamics have been also applied for HRV assessment often yielding promising diagnostic and prognostic results\textsuperscript{13-14}.

On the other hand, the arterial pulse and more specifically the morphology of the pressure waves has also been extensively analyzed and widely used for the study and diagnosis of several cardiovascular and other pathophysiological conditions. With the advent of technological innovations accurate and non-invasive recording of arterial pressure waves is now possible using high-fidelity sensors such as those implemented in applanation tonometry or photo plethysmography.
In parallel, several computational methods have been proposed for Pulse Wave Analysis (PWA), each one aiming to determine different facets and underlying mechanisms related to pressure wave phenotypes such as pressure and time related characteristics. Beyond the measurement of peak (systolic and diastolic) BP values, pressure waves can be further analyzed for the evaluation of wave reflections, arterial stiffness, arterio-ventricular coupling and cardiac output. The pathophysiological and clinical relevance of wave reflections and arterial stiffness have been extensively examined and quite established the last three decades as predictors of CV risk.

However, the relationship between pulse wave morphology and HRV is still unexplored. Although geometrical methods have been used for HRV analysis, there are limited applications of such geometrical approaches on PWA. One of the most mysterious and absorbing numbers in the history of mathematics, beyond the well known number $\pi$ (3.14), is the "mean and extreme ratio" or else "divine proportions", "golden section", "golden ratio" or "golden number" most often denoted by the Greek symbol-letter $\phi$ (phi). This element represents an ancient concept probably known by Babylonians and Egyptians more than 2500 years BC ago, but it was theoretically described by the mathematician and philosopher Pythagoras (ca 580BC – 500BC) and his School, while it was further established by the mathematician Euclid of Alexandria (Mid-4th century – Mid-3rd century BC) as reported in his work The Elements.

The golden ratio is defined by the proportion of two quantities ($\alpha$ and $\beta$) where the ratio of the sum of the quantities ($\alpha + \beta$) to the larger quantity ($\alpha$) equals the ratio of the larger quantity ($\alpha$) to the smaller one ($\beta$), according to the equation:

$$\frac{\alpha}{\beta} = \frac{\alpha + \beta}{\alpha} = \phi$$

[eq. 1], where $\phi$ is the golden number which approximates 1.618.

A philosophically and historically based concept (rather than clinical), of the "golden" or "divine" arterial pulse, is described in this paper. The main purpose is to quantify the deviation of an arterial pulse from the "golden pulse" and to further explore whether the extent of this deviation is related to the function of autonomic nervous system as assessed by HRV. The possibility to detect or predict specific HRV characteristics via a simple phenotypical, geometrical, analysis of a single arterial pulse wave using the golden ratio $\phi$, either in pressure or time scale (axes), is a challenging and novel hypothesis with substantial clinical relevance.

**Methods**

**Study population**

The study included healthy subjects who underwent a preventive cardiovascular check-up. Exclusion criteria were drug administration for chronic diseases, diabetes, cardiovascular disease, heart failure, renal failure and obesity. A total of 72 subjects were finally examined (mean age 39.6 ± 13.9 years, 28 females and 44 males). Thirty-one subjects were non-smokers, 35 were smokers and 6 ex-smokers. All subjects underwent non-invasive hemodynamic assessment and 24 hours monitoring of heart rate variability as previously described.

**Twenty-four hours recording of ECG**

A continuous electrocardiogram (ECG) was recorded for 24-hours at ambulatory conditions using a Holter device (Synescope, version 3.1, ELA Medical, France). The recording was acceptable if its minimum time duration was 18 hours, necessarily including sleep. All subjects were advised to follow their regular daily schedule at the day of ECG recording. The sampling frequency and time resolution was 500 Hz and 1 ms respectively. The ECG data were digitized by the apparatus and the recorded RR interval series were exported and transferred to a computer for further offline analysis.

**Assessment and analysis of Heart Rate Variability**

Initially the RR-interval time series were analyzed by the software of the Holter device. Manual editing by visual inspection was performed to eliminate noise, premature/ectopic beats and artifacts. Only recordings with >90% normal sinus beats were included in the study and further analyzed.

Time- and frequency-domain analysis was performed by the Synescope software for the calculation of several indices (Table 1). Analysis of heart rate dynamics by methods based on chaos theory and nonlinear system theory was also performed. Four nonlinear dynamic indices were computed: (i) Approximate Entropy (ApEn) and (ii) time scale correlation features ($\alpha_1$, $\alpha_2$) derived from Detrended Fluctuation Analysis (DFA). Nonlinear indices were computed using our custom made nonlinear dynamics tool as previously described. For each nonlinear measure the RR time series was segmented in accordance to previous studies and the average values for each index were computed from segments containing 4000 RR intervals. The RR interval series was extracted from Synescope in a "*.txt" file format and used as an input for our computational tool.
Non-invasive recording of the arterial pressure wave

The arterial pulse was recorded at every subject by applanation tonometry of the radial artery as previously described. Each subject rested for at least 10 min before the measurement which was performed in the supine position. The subjects were examined in a quiet room with constant temperature (~21-23 °C), at least 12 hours from their last meal, coffee or cigarette. Recording and analysis of the peripheral (radial) and aortic pressure waves were performed by using the SphygmoCor apparatus (AtCor Medical Pty. Ltd, Sydney, Australia). In brief, peripheral pressure waves were recorded at the radial artery by applanation tonometry and a single averaged pressure waveform was obtained from approximately 10-20 waveforms. Pressure waves were calibrated using brachial SBP and DBP values. Each recording was assessed by specific quality control criteria according to the manufacturer’s instructions (quality index≥85%). Three measurements with acceptable quality index were averaged to calculate the hemodynamic parameters used in the study. The apparatus software mathematically transformed the radial pressure waveforms by using transfer functions to derive the respective aortic pressure waves and further analyzed them as previously described. The determined pressure and time-related parameters are reported at Table 2.

Pulse wave analysis based on the golden ratio φ (phi)

The golden ratio or divine proportions between two elements α and β that fulfill the equation [1] can be applied for the analysis of the arterial pulse wave at the pressure and time dimension. According to this concept, BP values, namely DBP, pulse pressure (PP), and SBP may correspond to the elements α, α+β and β as illustrated at figure 1. Similarly, at the time scale (axis), the diastolic duration (DD), ejection duration (ED) and cardiac period can be expressed by three lines α, β and α+β, respectively. Theoretically, by this geometrical analysis, the “golden” or “divine”
Table 2. Descriptive, hemodynamic and heart rate variability characteristics of the study population.

| Parameter                  | Value (mean±standard deviation) |
|----------------------------|---------------------------------|
| Age (years)                | 39.6±13.9                       |
| Height (cm)                | 173±9                           |
| Weight (kg)                | 77.9±15.8                       |
| BMI (kg/m^2)               | 25.9±4                          |
| Gender (males, n, %)       | 44 (61.1%)                      |
| Smoking status             |                                 |
| Smoker, %                  | 31 (43.1%)                      |
| Non-smoker, %              | 35 (48.6%)                      |
| Ex-smoker, %               | 6 (8.3%)                        |
| Brachial SBP (mmHg)        | 124.7±21.5                      |
| Brachial DBP (mmHg)        | 80.3±10.6                       |
| Brachial PP (mmHg)         | 44.4±18.7                       |
| Mean pressure (mmHg)       | 95.3±12.6                       |
| Aortic SBP (mmHg)          | 112.2±17                        |
| Aortic DBP (mmHg)          | 81.2±10.7                       |
| Aortic PP (mmHg)           | 30.9±12.3                       |
| Ejection Duration (msec)   | 328±19                          |
| Diastolic Duration (msec)  | 575±129                         |
| Cardiac Period (msec)      | 903±143                         |

Values are expressed as mean±standard deviation for continuous variables or as absolute number and percentage for categorical variables. BMI: body mass index, SBP: systolic pressure, DBP: diastolic blood pressure, PP: pulse pressure.

Table 3. Linear and nonlinear indices of 24 hours heart rate variability.

| Heart Rate Variability indices | Value (mean±standard deviation) |
|--------------------------------|---------------------------------|
| Time domain                    |                                 |
| HR24 (bpm)                     | 76.1±9.6                        |
| SDNN (ms)                      | 154.2±46.2                     |
| SDANN (ms)                     | 137±39.8                       |
| ASDNN (ms)                     | 65.4±31.5                      |
| RMSSD (ms)                     | 39.2±38.8                      |
| PNN50 (%)                      | 11.7±13.8                      |
| Frequency domain               |                                 |
| TP (ms^2)                      | 4985±5471                      |
| TP_log                         | 3.6±0.3                        |
| VLF (ms^2)                     | 2761±2070                      |
| VLF_log                        | 3.3±0.3                        |
| LF (ms^2)                      | 1417±2066                      |
| LF_log                         | 3.0±0.4                        |
| LF (%)                         | 26.4±6.3                       |
| LF_norm (n.u.)                 | 68.8±8.6                       |
| HF (ms^2)                      | 611±1871                       |
| HF_log                         | 2.4±0.5                        |
| HF (%)                         | 8±5.6                          |
| HF_norm (n.u.)                 | 19.6±8.4                       |
| LF/HF                          | 4.3±2.2                        |

Nonlinear dynamics

| ApEn                          | 1.026±0.219                    |
| α1                            | 1.276±0.175                    |
| α2                            | 1.009±0.109                    |

Abbreviations are defined in table 1.

Figure 1. Analysis of the arterial pressure wave using the golden ratio φ (phi).

SBP: systolic blood pressure, DBP: diastolic blood pressure, PP: pulse pressure, ED: ejection duration, DD: diastolic duration, Period: duration of the cardiac cycle.
Table 4: Bivariate linear correlations of the absolute differences of (a) the two time ratios and (b) the two pressure ratios for both brachial and aortic pressure waves with demographic, hemodynamic and heart rate variability parameters.

|                  | Pearson Correlation Coefficients |  |  |
|------------------|----------------------------------|---|---|
|                  | Radial d-ratio(P) | Aortic d-ratio(P) | d-ratio(t) |
| **Age**          | -0.117              | -0.393**            | -0.220     |
| **Height**       | 0.111               | 0.232               | -0.017     |
| **Weight**       | 0.025               | 0.046               | -0.052     |
| **Body mass index** | -0.057              | -0.103              | -0.059     |

Hemodynamic Parameters derived by Pulse Wave Analysis

|                  | Radial d-ratio(P) | Aortic d-ratio(P) | d-ratio(t) |
|------------------|-------------------|-------------------|------------|
| Brachial SBP     | 0.083             | -0.259            | 0.036      |
| Brachial DBP     | 0.061             | 0.224             | -0.027     |
| Mean pressure    | 0.042             | -0.060            | -0.038     |
| Brachial PP      | 0.060             | -0.425**          | 0.057      |
| Aortic SBP       | 0.030             | -0.281            | -0.016     |
| Aortic DBP       | 0.059             | 0.214             | -0.035     |
| Aortic PP        | -0.020            | -0.574**          | 0.010      |
| Ejection duration| 0.028             | -0.144            | 0.239**    |
| Diastolic duration| 0.009             | -0.149            | 0.487**    |
| Cardiac period   | 0.012             | -0.154            | 0.472**    |

Indices of Heart Rate Variability

|                  | Radial d-ratio(P) | Aortic d-ratio(P) | d-ratio(t) |
|------------------|-------------------|-------------------|------------|
| ApEn             | 0.124             | 0.062             | 0.024      |
| α1               | -0.020            | 0.096             | -0.166     |
| α2               | 0.000             | -0.158            | 0.070      |
| HR\_24hr         | -0.062            | 0.245             | -0.366**   |
| SDNN             | 0.076             | 0.049             | 0.439**    |
| SDANN            | 0.055             | 0.053             | 0.456**    |
| ASDNN            | 0.078             | 0.052             | 0.265      |
| RMSSD            | 0.076             | 0.041             | 0.123      |
| PNN50            | 0.098             | 0.070             | 0.326**    |
| TP               | 0.083             | 0.038             | 0.187      |
| TP\_50           | 0.049             | 0.089             | 0.330**    |
| VLF              | 0.108             | 0.029             | 0.425**    |
| VLF\_log         | 0.053             | 0.064             | 0.387**    |
| LF               | 0.040             | 0.038             | 0.062      |
| LF\_log          | 0.043             | 0.128             | 0.220      |
| LF\_normal       | -0.025            | 0.143             | -0.173     |
| LF\_normo        | 0.015             | 0.061             | -0.317**   |
| HF               | 0.064             | 0.034             | -0.009     |
| HF\_log          | 0.020             | 0.091             | 0.323**    |
| HF\_normal       | 0.039             | 0.132             | 0.162      |
| HF\_normo        | 0.027             | 0.070             | 0.339**    |
| LF\_HF           | 0.097             | 0.083             | -0.302**   |

* and ** indicate that correlation is significant at the 0.05 and 0.01 level, respectively

SBP, DBP: systolic and diastolic blood pressure respectively, PP: pulse pressure.
Abbreviations of heart rate variability indices are defined in table 1.
pressure and time ratios of an arterial pressure waveform should follow the golden ratio \( \phi \) according to the equations:

\[
\frac{DD}{ED} = \frac{Period}{DD} \approx \phi \quad \text{and} \quad \frac{DBP}{PP} = \frac{SBP}{DBP} \approx \phi
\]

[eq. 2], where \( \phi = 1.618 \)

Pulse waveforms that deviate from the "golden" pulse will exhibit pressure or time ratios that differ from 1.618. In order to quantify this deviation we determined the absolute difference between the two pressure and time ratios as following:

\[
d-ratio(p) = \text{abs} \left[ \left( \frac{DBP}{PP} \right) - \left( \frac{SBP}{DBP} \right) \right], \quad \text{[eq. 3]}
\]

\[
d-ratio(t) = \text{abs} \left[ \left( \frac{DD}{ED} \right) - \left( \frac{Period}{DD} \right) \right], \quad \text{[eq. 4]}
\]

The net values of the above differences were also determined. Absolute differences close to zero (0) indicate an equality of the two ratios meaning that the pressure or time proportions are equal by definition to the golden number 1.618. As the absolute differences increase then the pressure or time proportions deviate from the golden ratio and thus the pulse differs from the theoretical "golden" pulse.

**RESULTS**

The main demographic and hemodynamic characteristics of the study population are reported in Table 2. The linear (time- and frequency-domain) as well as the nonlinear dynamic indices of 24-hr HRV are reported in Table 3.

The absolute differences between (a) the two time ratios and (b) the two pressure ratios for both radial and aortic pressure waves (figure 1) were analyzed by bivariate correlation coefficients versus demographic, hemodynamic and HRV parameters (Table 3).

It is notable that the absolute difference between the time ratios derived from the radial pressure waves (figure 1) recorded by applanation tonometry were associated significantly with most of the time-domain and frequency domain indices of HRV. More specifically positive correlations were found for SDNN, SDANN, ASDNN, PNN50, HF_{log} and HF_{norm}, whereas negative correlations were observed for HR_{24hr}, LF_{norm}, and HF_{norm} (Table 4). The respective absolute difference between the two pressure ratios had no significant correlation with the computed HRV indices.

**Multiple Linear Regression Analysis**

By multivariate linear regression analysis we further examined whether the observed associations with d-ratio(t) remained significant after adjustment for the potential confounding effect of age, gender, BMI and blood pressure (mean pressure and pulse pressure). It was found that d-ratio(t) was still significantly related with SDNN (b=0.363, p<0.001), SDANN (b=0.402, p<0.001), PNN50 (b=0.233, p=0.03), HR_{24hr} (b=-0.369, p=0.001), TP_{log} (b=0.215, p=0.028), VLF (b=0.322, p=0.001), VLF_{log} (b=0.281, p=0.004), LF_{norm} (b=0.324, p=0.007), HF_{log} (b=0.203, p=0.042), HF_{norm} (b=0.274, p=0.019), and LF/HF (b=-0.256, p=0.031), independently of age, gender, BMI, MAP and brachial PP. The strongest independent association was observed between d-ratio(t) and SDANN with the former index explaining almost the 16% of SDANN variation regardless from age, gender, BMI, MAP and PP.

Still, a critical question is whether the observed associations of HRV indices with d-ratio(t) are superior than the associations of ED, DD, cardiac period with HRV indices, per se.

The absolute difference d-ratio(t) between the ratio(t) and the ratio(_t) (equation 4) which expresses the deviation of the time-intervals' proportions from the golden number 1.618, has no collinearity with ED (r=0.239), DD (r=0.487) and cardiac period (r=0.472).

The HRV indices which are significantly correlated with d-ratio(t) as reported in table 4, were adjusted for ED and DD in a separate multiple regression model (table 5). Surprisingly, it is shown that the d-ratio(t) is associated with SDNN, SDANN, VLF, LF_{norm}, HF_{log}, HF_{norm}, and LF/HF independently of ED and DD (table 5).

**DISCUSSION**

The functional characteristics of the autonomic nervous system and the hemodynamic profile of systemic circulation are both critical factors modulating and reflecting the cardiovascular health. Analysis of HRV and arterial pulse wave morphology provide valuable information concerning ANS function and systemic circulation respectively. For the first time a new geometrical analysis was applied on non-invasively recorded peripheral (radial) and central (aortic) arterial pressure waves revealing some novel pressure wave phenotypes. It was demonstrated that the new phenotypic, geometrical biomarkers based on the golden ratio (\( \phi \)) or divine proportions were significantly and independently associated with HRV indices and especially those computed at the frequency domain in healthy subjects. Specifically the absolute difference of the time ratios DD/ED and Period/ DD (d-ratio(t)) of the arterial pressure wave was a
The arterial pulse and particularly the pressure wave is commonly examined: (a) at the brachial artery with the main purpose to determine its maximum and minimum pressure levels using cuff-based sphygmomanometers and (b) at the radial artery by palpation in order to determine its frequency via the measurement of heart beats per minute and consequently to estimate the pulse period. Gradually, the classic, traditional sphygmomanometric techniques are transforming to more advanced technologies offering the recording and computation of a plethora of hemodynamic biomarkers other than SBP, DBP, MAP and heart rate such as central (aortic) blood pressure, wave reflection indices (augmentation index), pulse transit time and arterial stiffness, cardiac output and others. More interestingly wearable devices have been also developed thus providing the possibility of ambulatory monitoring of the above mentioned parameters.

Nonetheless, there is limited information concerning the association of pressure wave features with HRV indices especially in healthy subjects. Although a relationship between arterial stiffness (assessed by pulse wave velocity) and HRV has been reported in diabetic patients the potential association of HRV indices with pressure wave characteristics is still cloudy. In a previous study of healthy subjects, it was observed that traditional cardiovascular disease factors such as age and heart rate) rather than hemodynamic parameters predominantly influenced long-term, time- and frequency-domain HRV indices. This was also apparent for several pressure markers derived by PWA including central pressure and augmentation index.

In the present study we applied for the first time the golden ratio $\Phi$ on peripheral and central pressure waveforms recorded non-invasively in healthy subjects. Deviation of the arterial pulse from the concept of the “golden” pulse was quantified in both the time and frequency dimensions. Greater differences between these ratios indicate a greater deviation from the golden ratio $\Phi$. The most striking observation of this study is the significant correlation of $d$-ratio(t) with the frequency domain indices of HRV regardless of age, gender, blood pressure levels and BMI. In contrast, BP markers and pulse wave time indices (ED and DD) were not related to most of the spectral characteristics of HRV.

**CONCLUSIONS**

This finding implies that the specific geometrical analysis of the peripheral arterial pulse reflects or characterizes specific HRV features. The potential pathophysiological background and the underlying mechanisms of this association can be hardly

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**Table 5.** Correlation coefficients of specific time measures used for the calculation of $d$-ratio(t) which indicates the deviation from the golden ratio ($\Phi$) versus heart rate variability indices.

| Parameter | ED | DD | Period | $ratio(t)$ | $Net\ d$-ratio(t) | $d$-ratio(t) |
|-----------|----|----|--------|------------|------------------|-------------|
| SDNN      | 0.309** | 0.483** | 0.478** | 0.463** | -0.362** | 0.441** | 0.439** *§† |
| SDANN     | 0.330** | 0.485** | 0.483** | 0.458** | -0.358** | 0.436** | 0.456** *§† |
| ASDNN     | 0.173 | 0.329** | 0.320** | 0.333** | -0.269** | 0.319** | 0.265** |
| PNN50     | 0.131 | 0.315** | 0.302** | 0.323** | -0.232** | 0.302** | 0.326** *§† |
| HR_Ed     | -0.423** | -0.605** | -0.604** | -0.587** | 0.534** | -0.579** | -0.366** *§† |
| TP_log    | 0.229 | 0.383** | 0.377** | 0.379** | -0.303** | 0.363** | 0.330** *§† |
| VLF       | 0.226 | 0.468** | 0.454** | 0.475** | -0.374** | 0.453** | 0.425** *§† |
| VLF_freq  | 0.276 | 0.458** | 0.451** | 0.452** | -0.369** | 0.434** | 0.387** *§† |
| LF_norm   | -0.021 | -0.123 | -0.114 | -0.118 | 0.011 | -0.090 | -0.317** *§† |
| HF_log    | 0.152 | 0.251** | 0.248 | 0.242 | -0.150 | 0.220 | 0.323** *§† |
| HF_freq   | 0.006 | 0.066 | 0.061 | 0.058 | 0.061 | 0.027 | 0.339** *§† |
| LF/HF     | -0.100 | -0.114 | -0.116 | -0.085 | -0.014 | -0.059 | -0.302** *§† |

ED: ejection duration, DD: diastolic duration, $ratio(t)$=Period/DD, $d$-ratio(t): the absolute difference between $ratio(t)$ and $ratio(t)$; Abbreviations of heart rate variability indices are defined in table1.

§ remained significant ($p<0.05$) in a multivariate regression model adjusting for age, gender, mean arterial pressure, brachial pulse pressure and body mass index.

† remained significant ($p<0.05$) in a multivariate regression model adjusting for ejection (ED) and diastolic (DD) duration.

ED: ejection duration, DD: diastolic duration, $ratio(t)$=Period/DD, $d$-ratio(t): the absolute difference between $ratio(t)$ and $ratio(t)$; Abbreviations of heart rate variability indices are defined in table1.

§ remained significant ($p<0.05$) in a multivariate regression model adjusting for age, gender, mean arterial pressure, brachial pulse pressure and body mass index.

† remained significant ($p<0.05$) in a multivariate regression model adjusting for ejection (ED) and diastolic (DD) duration.
explained by the current study design. Also, it is not possible yet to discriminate hemodynamic, autonomic nervous function or mathematical/statistical influences that might contribute to this finding. Undoubtedly, the concept of the „golden“ or „divine“ pulse and its potential research or clinical utility and relevance is very challenging. The application of the golden ratio $\phi$ for the analysis of the arterial pulse merits further investigation and may open a new window to the evaluation of the cardiovascular and autonomic nervous systems.

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Compliance with Ethics Requirements:
„The authors declare no conflict of interest regarding this article“
„The authors declare that all the procedures and experiments of this study respect the ethical standards in the Helsinki Declaration of 1975, as revised in 2008(5), as well as the national law. Informed consent was obtained from all the patients included in the study“
„All institutional and national guidelines for the care and use of laboratory animals were followed“

References
1. Heart rate variability: standards of measurement, physiological interpretation and clinical use. Task Force of the European Society of Cardiology and the North American Society of Pacing and Electrophysiology. Circulation 1996; 93(5):1043-65.
2. Bigger JT Jr., Fleiss JL, Steinman RC, Rolnitzky LM, Kleiger RE, Rottman JN. Frequency domain measures of heart period variability and mortality after myocardial infarction. Circulation 1992; 85(1):164-71.
3. Kleiger RE, Miller JP, Bigger JT Jr., Moss AJ. Decreased heart rate variability and its association with increased mortality after acute myocardial infarction. Am J Cardiol 1987; 59(4):256-62.
4. Lee CH, Lee JH, Son JW, et al. Normative values of short-term heart rate variability parameters in Koreans and their clinical value for the prediction of mortality. Heart Lung Circ 2017; doi: 10.1016/j.hlc.2017.04.009.
5. Nakane T, Nakamae M, Koh H, et al. Autonomic nervous system pretransplant malfunction is a powerful predictor of survival after allogeneic hematopoietic cell transplantation. Transplantation 2017; 101(11):2801-2809.
6. de Castilho FM, Ribeiro ALP, da Silva JLP, Nobre V, de Sousa MR. Heart rate variability as predictor of mortality in sepsis: A prospective cohort study. PLoS One 2017; 12(6):e0180060.
7. Zhao S, Chen K, Su Y, et al. The role of variability in nighttime mean heart rate on the prediction of ventricular arrhythmias and all-cause mortality in implantable cardioverter defibrillator patients. Europace 2015; 17 Suppl 2:a176-82.
8. Heart rate variability. Standards of measurement, physiological interpretation, and clinical use. Task Force of the European Society of Cardiology and the North American Society of Pacing and Electrophysiology. Eur Heart J 1996; 17(3):354-81.
9. Suzuki M, Hiroshi T, Aoyama T, et al. Nonlinear measures of heart rate variability and mortality risk in hemodialysis patients. Clin J Am Soc Nephrol 2012; 7(9):1454-60.
10. Huiukri HV, Perkiomaki JS, Maestri R, Pinna GD. Clinical impact of evaluation of cardiovascular control by novel methods of heart rate dynamics. Philos Trans A Math Phys Eng Sci 2009; 367(1892):1223-38.
11. Lombardi F. Chaos theory, heart rate variability, and arrhythmic mortality. Circulation 2000; 101(1):8-10.
12. Wang KL, Cheng HM, Sung SH, et al. Wave reflection and arterial stiffness in the prediction of 15-year all-cause and cardiovascular mortalities: a community-based study. Hypertension 2010; 55(3):799-805.
13. Cheng HM, Chuang SY, Wang JJ, et al. Prognostic significance of mechanical biomarkers derived from pulse wave analysis for predicting long-term cardiovascular mortality in two population-based cohorts. Int J Cardiol 2016; 215:388-95.
14. Ikonomidou I, Tsartzis S, Papaioannou T, et al. Incremental value of arterial wave reflections in the determination of left ventricular diastolic dysfunction in untreated patients with essential hypertension. J Hum Hypertens 2008; 22(10):678-98.
15. Lekakis JP, Ikonomidou I, Proterogou AD, et al. Arterial wave reflection is associated with severity of extraocular atherosclerosis in patients with coronary artery disease. Eur J Cardiovasc Prev Rehabil 2006; 13(2):236-42.
16. Papaioannou TG, Karatzis EN, Papamichael CM, et al. Circadian variation of arterial pressure wave reflections. Am J Hypertens 2006; 19(3):259-63.
17. Stamatelopoulos KS, Kalpakos D, Proterogou AD, et al. The combined effect of augmentation index and carotid intima-media thickness on cardiovascular risk in young and middle-aged men without cardiovascular disease. J Hum Hypertens 2006; 20(4):273-9.
18. Proterogou AD, Safar ME, Papaioannou TG, et al. The combined effect of aortic stiffness and pressure wave reflections on mortality in the very old with cardiovascular disease: the PROTEGER Study. Hypertens Res 2011; 34(7):803-8.
19. Proterogou AD, Papaioannou TG, Vlachopoulos C. Arterial stiffness mapping: a better navigation to Ithaca. J Am Coll Cardiol 2014; 63(17):1748-50.
20. Vlachopoulos C, Xaplanteris P, Aboyans V, et al. The role of vascular biomarkers for primary and secondary prevention. A position paper from the European Society of Cardiology Working Group on peripheral circulation: Endorsed by the Association for Research into Arterial Structure and Physiology (ARTERY) Society. Atherosclerosis 2015; 241(2):507-32.
21. Kyrilagkitis S, Papaioannou TG, Gialafos E, et al. Relationships between heart rate variability and aortic hemodynamic variables in healthy subjects. Hellenic J Cardiol 2016; 57(5):359-362.
22. Kyrilagkitis S, Papaioannou TG, Gialafos E, et al. Assessment of non-linear dynamics of RR-intervals using different computational tools: a comparative study. Archives of the Balkan Medical Union 2014; 49(4):401-410.
23. Pikkujamasi SM, Makikalio TH, Sourander LB, et al. Cardiac interbeat interval dynamics from childhood to senescence: comparison of conventional and new
measures based on fractals and chaos theory. Circulation 1999; 100(4):393-9.
24. Makikalio TH, Seppanen T, Niemela M, Airaksinen KE, Tulppo M, Huikuri HV. Abnormalities in beat to beat complexity of heart rate dynamics in patients with a previous myocardial infarction. J Am Coll Cardiol 1996; 28(4):1005-11.
25. Yamada A, Hayano J, Sakata S, et al. Reduced ventricular response irregularity is associated with increased mortality in patients with chronic atrial fibrillation. Circulation 2000; 102(3):300-6.
26. Vrachatis D, Papaioannou TG, Konstantopoulou A, et al. Effect of supine versus sitting position on noninvasive assessment of aortic pressure waveform: a randomized cross-over study. J Hum Hypertens 2014; 28(4):236-41.
27. O’Rourke MF, Pauca A, Jiang XJ. Pulse wave analysis. Br J Clin Pharmacol 2001; 51(6):507-22.
28. Karamanou M, Papaioannou TG, Tsoucalas G, Tousoulis D, Stefanadis C, Androutsos G. Blood pressure measurement: lessons learned from our ancestors. Curr Pharm Des 2015; 21(6):700-4.
29. Papaioannou TG, Protogerou AD, Stamatoopoulos KS, Vavuranakis M, Stefanadis C. Non-invasive methods and techniques for central blood pressure estimation: procedures, validation, reproducibility and limitations. Curr Pharm Des 2009; 15(3):245-53.
30. Vardoulis O, Papaioannou TG, Stergiopulos N. Validation of a novel and existing algorithms for the estimation of pulse transit time: advancing the accuracy in pulse wave velocity measurement. Am J Physiol Heart Circ Physiol 2013; 304(11):H1558-67.
31. Townsend RR, Wilkinson IB, Schiffrin EL, et al, American Heart Association Council on H. Recommendations for improving and standardizing vascular research on arterial stiffness: a scientific statement from the American Heart Association. Hypertension 2015; 66(3):698-722.
32. Papaioannou TG, Soulis D, Vardoulis O, et al. First in vivo application and evaluation of a novel method for noninvasive estimation of cardiac output. Med Eng Phys 2014; 36(10):1352-7.
33. Papaioannou TG, Vardoulis O, Stergiopulos N. The “systolic volume balance” method for the noninvasive estimation of cardiac output based on pressure wave analysis. Am J Physiol Heart Circ Physiol 2012; 302(10):H2064-73.
34. van Drumpt A, van Bommel J, Hoeks S, et al. The value of arterial pressure waveform cardiac output measurements in the radial and femoral artery in major cardiac surgery patients. BMC Anesthesiol 2017; 17(1):42.
35. Khir AW, Parker KH. Wave intensity in the ascending aorta: effects of arterial occlusion. J Biomech 2005; 38(4):647-55.
36. Vardoulis O, Papaioannou TG, Stergiopulos N. On the estimation of total arterial compliance from aortic pulse wave velocity. Ann Biomed Eng 2012; 40(12):2619-26.
37. Wassertheurer S, Kropf J, Weber T, et al. A new oscillometric method for pulse wave analysis: comparison with a common tonometric method. J Hum Hypertens 2010; 24(8):498-504.
38. Protogerou AD, Argyris A, Nasothimiou E, et al. Feasibility and reproducibility of noninvasive 24-h ambulatory aortic blood pressure monitoring with a brachial cuff-based oscillometric device. Am J Hypertens 2012; 25(8):876-82.
39. Jensen-Urstad K, Reichard P, Jensen-Urstad M. Decreased heart rate variability in patients with type 1 diabetes mellitus is related to arterial wall stiffness. J Intern Med 1999; 245(1):57-61.
40. Cardoso CR, Moraes RA, Leite NC, Salles GF. Relationships between reduced heart rate variability and pre-clinical cardiovascular disease in patients with type 2 diabetes. Diabetes Res Clin Pract 2014; 106(1):110-7.