Four-Neutrino Oscillations

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Abstract

It is shown that at least four massive neutrinos are needed in order to accommodate the evidences in favor of neutrino oscillations found in solar and atmospheric neutrino experiments and in the LSND experiment. Among all four-neutrino schemes, only two, with a mass spectrum composed of two pairs of neutrinos with close masses separated by the “LSND gap” of the order of 1 eV, are compatible with the results of all neutrino oscillation experiments. In these two schemes the probability of $\nu_e \rightarrow \nu_e$ transitions into other states, the probability of $\nu_\mu \rightarrow \nu_e$ transitions and the size of CP violation effects in $\nu_\mu \leftrightarrow \nu_e$ and $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$ transitions are suppressed in long-baseline experiments.

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I. INTRODUCTION

The existence of neutrino masses and mixing is today one of the hottest topics in high-
energy physics and is considered as one of the best ways to obtain indications on the physics
beyond the Standard Model. If neutrinos are massive and mixed, the left-handed com-
ponents $\nu_{\alpha L}$ ($\alpha = e, \mu, \tau, \ldots$) of the flavor neutrino fields are superpositions of the left-handed
components $\nu_{kL}$ ($k = 1, \ldots, N$) of neutrino fields with definite mass $m_k$, $\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL}$,
where $U$ is a $N \times N$ unitary mixing matrix. In this case neutrino oscillations occur. From the measure-
ment of the invisible decay width of the Z-boson it is known that the number of light active neutrino
flavors is three, corresponding to $\nu_e$, $\nu_\mu$ and $\nu_\tau$. This implies that the number $N$ of massive neutrinos is bigger or equal to three. If $N > 3$, in the flavor basis
there are $N_s = N - 3$ sterile neutrinos, $\nu_{s_1}, \ldots, \nu_{s_{N_s}}$. In this case the flavor index $\alpha$ takes
the values $e, \mu, \tau, s_1, \ldots, s_{N_s}$.

Evidences in favor of neutrino oscillations have been found in solar neutrino experiments
[1], in atmospheric neutrino experiments [2] and in the LSND accelerator experiment [3].
The observed disappearance of atmospheric $\nu_\mu$'s can be explained by $\nu_\mu \rightarrow \nu_\tau$ and/or $\nu_\mu \rightarrow \nu_s$
transitions, the observed disappearance of solar $\nu_e$'s can be explained by $\nu_e \rightarrow \nu_\mu$ and/or
$\nu_e \rightarrow \nu_\tau$ and/or $\nu_e \rightarrow \nu_s$ transitions, and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\nu_\mu \rightarrow \nu_e$ transitions have been observed in the LSND experiment.

II. THE NECESSITY OF AT LEAST THREE INDEPENDENT $\Delta m^2$'S

The three evidences in favor of neutrino oscillations found in solar and atmospheric
neutrino experiments and in the accelerator LSND experiment imply the existence of at
least three independent neutrino mass-squared differences. This can be seen by considering
the general expression for the probability of $\nu_\alpha \rightarrow \nu_\beta$ transitions in vacuum, that can be
written as (see [4])

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \sum_{k=1}^{N} U_{\alpha k}^* U_{\beta k} \exp \left( -i \frac{\Delta m^2_{kj} L}{2 E} \right),$$

(2.1)

where $\Delta m^2_{kj} \equiv m_k^2 - m_j^2$, $j$ is any of the mass-eigenstate indices, $L$ is the distance between the
neutrino source and detector and $E$ is the neutrino energy. The range of $L/E$ characteristic of
each type of experiment is different: $L/E \sim 10^{11} - 10^{12} \text{eV}^{-2}$ for solar neutrino experiments,
$L/E \sim 10^2 - 10^3 \text{eV}^{-2}$ for atmospheric neutrino experiments and $L/E \sim 1 \text{eV}^{-2}$ for the
LSND experiment. From Eq. (2.1) it is clear that neutrino oscillations are observable in an
experiment only if there is at least one mass-squared difference $\Delta m^2_{kj}$ such that $\Delta m^2_{kj} L/2E \gtrsim 0.1$ (the precise lower bound depends on the sensitivity of the experiment) in a significant
part of the energy and source-detector distance intervals of the experiment (if this condition
is not satisfied, $P_{\nu_\alpha \rightarrow \nu_\beta} \simeq |\sum_k U_{\alpha k}^* U_{\beta k}|^2 = \delta_{\alpha \beta}$). Since the range of $L/E$ probed by
the LSND experiment is the smaller one, a large mass-squared difference is needed for LSND
oscillations, $\Delta m^2_{\text{LSND}} \gtrsim 10^{-1} \text{eV}^2$. The 99% CL maximum likelihood analysis of the LSND
data in terms of two-neutrino oscillations gives [3]
0.20 eV² ≤ \( \Delta m^2_{\text{LSND}} \) ≲ 2.0 eV². \( (2.2) \)

Furthermore, from Eq. (2.1) it is clear that a dependence of the oscillation probability from the neutrino energy \( E \) and the source-detector distance \( L \) is observable only if there is at least one mass-squared difference \( \Delta m^2_{kj} \) such that \( \Delta m^2_{kj} L / 2E \sim 1 \). Indeed, the exponentials of all the phases \( \Delta m^2_{kj} L / 2E \ll 1 \) are equal to one and the contributions of all the phases \( \Delta m^2_{kj} L / 2E \gg 1 \) are washed out by the average over the energy and source-detector ranges characteristic of the experiment. Since a variation of the oscillation probability as a function of neutrino energy has been observed both in solar and atmospheric neutrino experiments and the ranges of \( L/E \) characteristic of these two types of experiments are different from each other and different from the LSND range, two more mass-squared differences with different scales are needed:

\[
\Delta m^2_{\text{sun}} \sim 10^{-12} - 10^{-11} \text{eV}² \quad (\text{VO}), \quad \Delta m^2_{\text{atm}} \sim 10^{-3} - 10^{-2} \text{eV}². \quad (2.3)
\]

The condition (2.3) for the solar mass-squared difference \( \Delta m^2_{\text{sun}} \) has been obtained under the assumption of vacuum oscillations (VO). If the disappearance of solar \( \nu_e \)'s is due to the MSW effect (see [4]), the condition

\[
\Delta m^2_{\text{sun}} \lesssim 10^{-4} \text{eV}² \quad (\text{MSW}) \quad (2.4)
\]

must be fulfilled in order to have a resonance in the interior of the sun. Hence, in the MSW case \( \Delta m^2_{\text{sun}} \) must be at least one order of magnitude smaller than \( \Delta m^2_{\text{atm}} \).

It is possible to ask if three different scales of neutrino mass-squared differences are needed even if the results of the Homestake solar neutrino experiment is neglected, allowing an energy-independent suppression of the solar \( \nu_e \) flux. The answer is that still the data cannot be fitted with only two neutrino mass-squared differences because an energy-independent suppression of the solar \( \nu_e \) flux requires large \( \nu_e \to \nu_\mu \) or \( \nu_e \to \nu_\tau \) transitions generated by \( \Delta m^2_{\text{atm}} \) or \( \Delta m^2_{\text{LSND}} \). These transitions are forbidden by the results of the Bugey [5] and CHOOZ [6] reactor \( \bar{\nu}_e \) disappearance experiments and by the non-observation of an up-down asymmetry of \( e^- \)-like events in the Super-Kamiokande atmospheric neutrino experiment [7].

### III. FOUR-NEUTRINO SCHEMES

The existence of three different scales of \( \Delta m^2 \) imply that at least four light massive neutrinos must exist in nature. Here we consider the schemes with four light and mixed neutrinos, which constitute the minimal possibility that allows to accommodate the results of all neutrino oscillation experiments. In this case, in the flavor basis the three active neutrinos \( \nu_e, \nu_\mu, \nu_\tau \) are accompanied by a sterile neutrino \( \nu_s \).

The six types of four-neutrino mass spectra with three different scales of \( \Delta m^2 \) that can accommodate the hierarchy \( \Delta m^2_{\text{sun}} \ll \Delta m^2_{\text{atm}} \ll \Delta m^2_{\text{LSND}} \) are shown qualitatively in Fig. [1]. In all these mass spectra there are two groups of close masses separated by the “LSND gap” of the order of 1 eV. In each scheme the smallest mass-squared difference corresponds to \( \Delta m^2_{\text{sun}} \) (\( \Delta m^2_{21} \) in schemes I and B, \( \Delta m^2_{32} \) in schemes II and IV, \( \Delta m^2_{43} \) in schemes III and A), the intermediate one to \( \Delta m^2_{\text{atm}} \) (\( \Delta m^2_{31} \) in schemes I and II, \( \Delta m^2_{42} \) in schemes III and A), and the largest one to \( \Delta m^2_{\text{LSND}} \).
and IV, $\Delta m^2_{21}$ in scheme A, $\Delta m^2_{43}$ in scheme B) and the largest mass squared difference $\Delta m^2_{41} = \Delta m^2_{41\text{LSND}}$ is relevant for the oscillations observed in the LSND experiment. The six schemes are divided into four schemes of class 1 (I–IV) in which there is a group of three masses separated from an isolated mass by the LSND gap, and two schemes of class 2 (A, B) in which there are two couples of close masses separated by the LSND gap.

It has been show that the schemes of class 1 are disfavored by the data if also the negative results of short-baseline $\bar{\nu}_e$ and $\nu_\mu$ disappearance experiments are taken into account \[8\,11\]. This is basically due to the fact that the non-observation of neutrino oscillations due to $\Delta m^2_{41}$ in short-baseline disappearance experiments imply that, in each scheme in Fig. 1, $\nu_e$ and $\nu_\mu$ are mainly superpositions of one of the two groups of mass eigenstates separated by the LSND gap. Hence, in the schemes of class 1 $\nu_e$ and $\nu_\mu$ almost coincide with superpositions of the three grouped mass eigenstates or with the isolated mass eigenstate. Moreover, only the possibility of both $\nu_e$ and $\nu_\mu$ mainly superpositions of the three grouped mass eigenstates allows to explain the results of solar and atmospheric neutrino experiments with neutrino oscillations. This is because disappearance of solar $\nu_e$’s and atmospheric $\nu_\mu$’s is possible only if $\nu_e$ and $\nu_\mu$ have large mixing with the mass eigenstates whose mass-squared differences give $\Delta m^2_{\text{sun}}$ and $\Delta m^2_{\text{atm}}$. In all schemes of class 1 $\Delta m^2_{\text{sun}}$ and $\Delta m^2_{\text{atm}}$ are mass-squared differences between two of the three grouped mass eigenstates neutrinos. However, if both $\nu_e$ and $\nu_\mu$ are mainly superpositions of the three grouped mass eigenstates, short-baseline $\nu_\mu \rightarrow \nu_e$ oscillations due to $\Delta m^2_{41}$ are strongly suppressed and one can calculate that the allowed transition probability is smaller than that observed in the LSND experiment \[8\,10\]. Hence, we conclude that the schemes of class 1 are disfavored by neutrino oscillation data.

The two four-neutrino schemes of class 2 are compatible with the results of all neutrino oscillation experiments if the mixing of $\nu_e$ with the two mass eigenstates responsible for the oscillations of solar neutrinos ($\nu_3$ and $\nu_4$ in scheme A and $\nu_1$ and $\nu_2$ in scheme B) is large and the mixing of $\nu_\mu$ with the two mass eigenstates responsible for the oscillations of atmospheric neutrinos ($\nu_1$ and $\nu_2$ in scheme A and $\nu_3$ and $\nu_4$ in scheme B) is large \[12\,8\,11\]. This is illustrated qualitatively in Figs. 2 and 4, as we are going to explain.

Let us define the quantities $c_\alpha$, with $\alpha = e, \mu, \tau, s$, in the schemes A and B as

![Figure 1](image-url)
Physically \( c_\alpha \) quantify the mixing of the flavor neutrino \( \nu_\alpha \) with the two massive neutrinos whose \( \Delta m^2 \) is relevant for the oscillations of atmospheric neutrinos (\( \nu_1, \nu_2 \) in scheme A and \( \nu_3, \nu_4 \) in scheme B). The negative results of short-baseline disappearance experiments imply that

\[
\begin{align*}
\alpha &\equiv \sum_{k=1,2} |U_{\alpha k}|^2, \\
\alpha &\equiv \sum_{k=3,4} |U_{\alpha k}|^2.
\end{align*}
\]  

(3.1)

The quantities \( a_\alpha^0 \) and \( a_\mu^0 \), that depend on \( \Delta m^2_{41} = \Delta m^2_{\text{SND}} \), are obtained, respectively, from the exclusion plots of short-baseline \( \bar{\nu}_e \) and \( \nu_\mu \) experiments (see [4]). From the exclusion curves of the Bugey reactor \( \bar{\nu}_e \) disappearance experiment [5] and of the CDHS and CCFR accelerator \( \nu_\mu \) disappearance experiments [13] it follows that \( a_\alpha^0 \lesssim 3 \times 10^{-2} \) for \( \Delta m^2_{41} \) in the LSND range (2.2) and \( a_\mu^0 \lesssim 0.2 \) for \( \Delta m^2_{41} \geq 0.4 \text{ eV}^2 \).

The shadowed areas in Figs. 2 and 3 illustrate qualitatively the regions in the \( c_e-c_\mu \) plane allowed by the negative results of short-baseline \( \bar{\nu}_e \) and \( \nu_\mu \) disappearance experiments for a fixed value of \( \Delta m^2_{41} \). Figure 2 is valid for \( \Delta m^2_{41} \geq 0.3 \text{ eV}^2 \) and shows that there are four regions allowed by the results of short-baseline disappearance experiments: region SS with small \( c_e \) and \( c_\mu \), region LS with large \( c_e \) and small \( c_\mu \), region SL with small \( c_e \) and large \( c_\mu \), and region LL with large \( c_e \) and \( c_\mu \). The quantities \( c_e \) and \( c_\mu \) can be both large, because the unitarity of the mixing matrix imply that \( c_\alpha + c_\beta \leq 2 \) and \( 0 \leq c_\alpha \leq 1 \) for \( \alpha, \beta = e, \mu, \tau, s \). Figure 3 is valid for \( \Delta m^2_{41} \lesssim 0.3 \text{ eV}^2 \), where there is no constraint on the value of \( c_\mu \) from the results of short-baseline \( \nu_\mu \) disappearance experiments. It shows that there are two regions allowed by the results of short-baseline \( \bar{\nu}_e \) disappearance experiments: region S with small \( c_e \) and region L with large \( c_e \).

Let us take now into account the results of solar neutrino experiments. Large values of \( c_e \) are incompatible with solar neutrino oscillations because in this case \( \nu_e \) has large mixing with the two massive neutrinos responsible for atmospheric neutrino oscillations and, through the
unitarity of the mixing matrix, small mixing with the two massive neutrinos responsible for solar neutrino oscillations. Indeed, in the schemes of class 2 the survival probability \( P_{\nu_e \rightarrow \nu_e}^{\text{sun}} \) of solar \( \nu_e \)'s is bounded by \( P_{\nu_e \rightarrow \nu_e}^{\text{sun}} \geq c_e^2 / 2 \), and its possible variation \( \Delta P_{\nu_e \rightarrow \nu_e}^{\text{sun}}(E) \) with neutrino energy \( E \) is limited by \( \Delta P_{\nu_e \rightarrow \nu_e}^{\text{sun}}(E) \leq (1 - c_e)^2 \). If \( c_e \) is large as in the LS or LL regions of Fig. 2 or in the L region of Fig. 3, we have \( P_{\nu_e \rightarrow \nu_e}^{\text{sun}} \geq (1 - a_e^0)^2 / 2 \simeq 1 / 2 \) and \( \Delta P_{\nu_e \rightarrow \nu_e}^{\text{sun}}(E) \leq (a_e^0)^2 \lesssim 10^{-3} \), for \( \Delta m_{21}^2 = \Delta m_{\text{LSND}}^2 \) in the LSND range (2.2). Therefore, \( P_{\nu_e \rightarrow \nu_e}^{\text{sun}} \) is bigger than 1/2 and practically does not depend on neutrino energy. Since this is incompatible with the results of solar neutrino experiments interpreted in terms of neutrino oscillations, we conclude that the regions LS and LL in Fig. 2 and the region L in Fig. 3 are disfavored by solar neutrino data, as illustrated qualitatively by the vertical exclusion lines in Figs. 2 and 3.

Let us consider now the results of atmospheric neutrino experiments. Small values of \( c_{\mu} \) are incompatible with atmospheric neutrino oscillations because in this case \( \nu_\mu \) has small mixing with the two massive neutrinos responsible for atmospheric neutrino oscillations. Indeed, the survival probability of atmospheric \( \nu_\mu \)'s is bounded by \( P_{\nu_\mu \rightarrow \nu_\mu}^{\text{atm}} \geq (1 - c_{\mu})^2 \), and it can be shown that the Super-Kamiokande up–down asymmetry of high-energy \( \mu \)-like events generated by atmospheric neutrinos, \( A_{\mu} = 0.311 \pm 0.043 \pm 0.01 \) [8], and the exclusion curve of the Bugey \( \bar{\nu}_e \) disappearance experiment imply the upper bound \( c_{\mu} \gtrsim 0.45 = b_{\mu} \). This limit is depicted qualitatively by the horizontal exclusion lines in Figs. 4 and 5. Therefore, we conclude that the regions SS and LS in Fig. 2 and the small-\( c_{\mu} \) parts of the regions S and L in Fig. 3 are disfavored by atmospheric neutrino data.

Finally, let us consider the results of the LSND experiment. In the schemes of class 2 the amplitude of short-baseline \( (\nu^{-}_{\mu} \rightarrow \nu^{-}_{e}) \) oscillations is given by \( A_{\mu e} = \left| \sum_{k=1,2} U_{ek} U_{\mu k}^* \right|^2 \geq \left| \sum_{k=3,4} U_{ek} U_{\mu k}^* \right|^2 \) (\( A_{\mu e} \) is equivalent to \( \sin^2 2\vartheta \), where \( \vartheta \) is the two-generation mixing angle used in the analysis of the data of short-baseline \( (\nu^{-}_{\mu} \rightarrow \nu^{-}_{e}) \) experiments). The second equality is due to the unitarity of the mixing matrix. Using the Cauchy–Schwarz inequality we obtain

\[
   c_e c_{\mu} \geq A_{\mu e}^{\text{min}} / 4 \quad \text{and} \quad (1 - c_e)(1 - c_{\mu}) \geq A_{\mu e}^{\text{min}} / 4 ,
\]

where \( A_{\mu e}^{\text{min}} \) is the minimum value of the oscillation amplitude \( A_{\mu e} \) observed in the LSND experiment. The bounds (3.3) are illustrated qualitatively in Figs. 2 and 3. One can see that the results of the LSND experiment confirm the exclusion of the regions SS and LL in Fig. 2 and the exclusion of the small-\( c_{\mu} \) part of region S and of the large-\( c_{\mu} \) part of region L in Fig. 3.

Summarizing, if \( \Delta m_{41}^2 \gtrsim 0.3 \text{eV}^2 \) only the region SL in Fig. 2, with

\[
   c_e \leq a_e^0 \quad \text{and} \quad c_{\mu} \geq 1 - a_{\mu}^0 ,
\]

is compatible with the results of all neutrino oscillation experiments. If \( \Delta m_{41}^2 \lesssim 0.3 \text{eV}^2 \) only the large-\( c_{\mu} \) part of region S in Fig. 3, with

\[
   c_e \leq a_e^0 \quad \text{and} \quad c_{\mu} \geq b_{\mu}^{\text{SK}} ,
\]

6
is compatible with the results of all neutrino oscillation experiments. Therefore, in any case $c_e$ is small and $c_\mu$ is large. However, it is important to notice that, as shown clearly in Figs. 2 and 3, the inequalities (3.3) following from the LSND observation of short-billion $\nu_\mu \rightarrow \nu_e$ oscillations imply that $c_e$ and $1 - c_\mu$, albeit small, have the lower bounds

$$c_e \gtrsim A_{\mu e}^{\text{min}}/4 \quad \text{and} \quad 1 - c_\mu \gtrsim A_{\mu e}^{\text{min}}/4.$$  \hfill (3.6)

IV. LONG-BASELINE EXPERIMENTS

The smallness of $c_e$ in the schemes A and B implies that electron neutrinos do not oscillate in atmospheric and long-baseline neutrino oscillation experiments.

The transition probabilities of electron neutrinos and antineutrinos into other states in long-baseline experiments (LBL) are bounded by [15]

$$1 - P_{\nu_e \rightarrow \nu_e}^{(\text{LBL})} \leq a_e^0 \left(2 - a_e^0\right).$$  \hfill (4.1)

The solid line in Fig. 4 shows the corresponding limit obtained from the 90\% CL exclusion plot of the Bugey experiment. The shadowed region in Fig. 4 is allowed if $\Delta m_{41}^2$ lies in the LSND range (2.2). The dash-dotted line in Fig. 4 shows the upper bound for the transition probability of $\bar{\nu}_e$'s into other states obtained from the final 90\% exclusion plot of the CHOOZ [6] experiment for $\Delta m_{41}^2 \gtrsim 3 \times 10^{-3}$ eV$^2$ (the final 95\% exclusion plot of the CHOOZ experiment gives $P_{\nu_e \rightarrow \nu_e}^{(\text{LBL})} \lesssim 0.6$). One can see that the results of the CHOOZ experiment agree with the upper bound (4.1), that is more stringent than the CHOOZ bound for $\Delta m_{41}^2$ in the LSND range.
The probability of $\nu_\mu \rightarrow \nu_e$ transitions in vacuum in LBL experiments is limited by [15]

$$\frac{1}{4} A_{\mu e}^{\text{min}} \leq P_{\nu_\mu \rightarrow \nu_e}^{(\text{LBL})} \leq \min \left[ a_\mu^0 \left( 2 - a_\mu^0 \right), a_e^0 + \frac{1}{4} A_{\mu e}^0 \right],$$

where $A_{\mu e}^0$ is the upper bound for the amplitude $A_{\mu e}$ of short-baseline $\nu_\mu \rightarrow \nu_e$ transitions measured in accelerator neutrino experiments and $A_{\mu e}^{\text{min}}$ is the minimum value of $A_{\mu e}$ observed in the LSND experiment. The bound obtained with Eq. (4.2) from the 90\% CL exclusion plots of the Bugey experiment and of the BNL E776 [16] and KARMEN [17] experiments is depicted by the dashed line in Fig. [8]. The dark shadowed region is allowed by the results of the LSND experiment, taking into account the lower bound in Eq. (4.2). The solid line in Fig. [8] shows the upper bound on $P_{\nu_\mu \rightarrow \nu_e}^{(\text{LBL})}$ in the K2K experiment [18] taking into account matter effects [15]. In this case there is no lower bound and the dark plus light shadowed regions are allowed by the results of the LSND experiment. The expected 90\% CL sensitivity of the K2K long-baseline accelerator neutrino experiment for $\Delta m_{41}^2 \gtrsim 3 \times 10^{-3} \text{eV}^2$ is indicated in Fig. [8] by the dash-dotted line. It can be seen that the results of short-baseline experiments indicate an upper bound for $P_{\nu_\mu \rightarrow \nu_e}^{(\text{LBL})}$ smaller than the expected sensitivity of the K2K experiment, unless $\Delta m_{41}^2 \simeq 0.2 - 0.3 \text{eV}^2$.

Let us emphasize that the upper bounds for the oscillation probabilities in long-baseline experiments presented in Figs. [4] and [5] depend on $\Delta m_{41}^2$, that is the mass-squared difference relevant for oscillations in short-baseline experiment. The transition probabilities measured in each long-baseline experiment can be much smaller that the maximal one, that lies below the upper bounds in Figs. [4] and [5], if $\Delta m_{atm}^2$ is much smaller than the mass-squared difference to which the experiment is most sensitive.
A further consequence of the smallness of $c_e$ and $1 - c_\mu$ in the schemes A and B is the existence of a stringent upper bound for the size of CP or T violation that could be measured in long-baseline experiments in the $\nu_\mu \leftrightarrow \nu_e$ and $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$ channels \[19\]. On the other hand, the effects of CP violation in long-baseline $\nu_\mu \leftrightarrow \nu_\tau$ and $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_\tau$ transitions can be as large as allowed by the unitarity of the mixing matrix \[19\].

V. CONCLUSIONS

We have seen that only the two four-neutrino schemes A and B of class 2 in Fig. 1 are compatible with the results of all neutrino oscillation experiments. These two schemes are equivalent for the phenomenology of neutrino oscillations. We have shown that the quantities $c_e$ and $1 - c_\mu$ in the schemes A and B are small. Physically $c_\alpha$, defined in Eq. (3.1), quantify the mixing of the flavor neutrino $\nu_\alpha$ with the two massive neutrinos whose $\Delta m^2$ is relevant for the oscillations of atmospheric neutrinos ($\nu_1$, $\nu_2$ in scheme A and $\nu_3$, $\nu_4$ in scheme B). Considering long-baseline neutrino oscillation experiments, the smallness of $c_e$ implies stringent upper bounds for the probability of $\nu_\tau$ transitions into other states, for the probability of $\bar{\nu}_\mu \rightarrow \nu_\tau$ transitions and for the size of CP or T violation effects in $\nu_\mu \leftrightarrow \nu_e$ and $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$ transitions.
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