New gauge bosons and logarithmic corrections in an exact $\text{AdS}_3$

Jin Young Kim

*Department of Physics, Kunsan National University, Kunsan 573-701, Korea*

H.W. Lee and Y. S. Myung

*Department of Physics, Inje University, Kimhae 621-749, Korea*

Abstract

We calculate the absorption cross section by studying the spin-dependent wave equation in three-dimensional anti-de Sitter space ($\text{AdS}_3$). Here the AdS/CFT correspondence is used. It turns out that the new gauge bosons coupled to $(2,0)$ and $(0,2)$ operators on the boundary at infinity receive logarithmic corrections. This shows that the gauge bosons may play the role of singletons in $\text{AdS}_5$. On the other hand, test fields including the intermediate scalars ($\eta, \xi$) and fixed scalar ($\lambda$) do not receive any logarithmic correction in the first-order approximation.
I. INTRODUCTION

Recently there has been great progress in studying the black hole physics using string theory and conformal field theory (CFT) [1–3]. A 5D black hole (M5: D1-D5 brane black hole) contains the BTZ black hole in the near-horizon and thus it is very important to study the BTZ black hole (AdS3) [4,5]. Actually, a 5D black hole has the geometry: AdS3 in the near-horizon (the throat region) but with asymptotically flat space. If one starts with M5×S1×T4 in the ten-dimensional type IIB superstring theory, we have AdS3×S3×T4 in the throat region (r0 ≤ r ≤ R, R = √r1r5) but finds immediately Minkowski space after passing through the remote boundary (r = R) of AdS3. In this sense we call it an AdS3 bubble. Further the AdS/CFT correspondence plays a crucial role in calculating the entropy and greybody factor (absorption cross section). This correspondence implies that the ten-dimensional type IIB bulk theory deep in the throat ((AdS3×S3)R×T4/r1/r5) is related to the two-dimensional gauge theory (dual CFT2) on its remote boundary.

On the other hand, one found the logarithmic corrections to the absorption cross section of minimal scalars from an AdS5 bubble scattering [6] and an AdS3 bubble scattering [3]. Explicitly, these terms can be related to non-conformally invariant operators in their gauge theories. Nowadays it is very important to interpret the CFT results in terms of the bulk AdS results. Here the bulk AdS means an exact AdS geometry which is an infinitely long throat without asymptotically flat space. In the limit of R → ∞, this picture may come out of an AdS bubble.

We propose the AdS/CFT lore that the conformal limit of the gauge theory corresponds to scattering in an exact AdS background. So it appears that the logarithmic term cannot be understood from an exact AdS scattering. However, the scattering analysis in an exact AdS is not an easy matter. This is so because one cannot define ordinary asymptotic states in an exact AdS, due to the timelike boundary and the periodicity of geodesics. To avoid the closed timelike curves, one can use the universal cover by ignoring the periodicity of time. However, the presence of the timelike boundary leads to major differences with physics in
Minkowski space. First one does not have a well-posed initial value problem unless one puts boundary conditions there. Hence we always need boundary conditions at infinity. These are the Dirichlet or Neumann conditions. But instead of these, one can use the non-normalizable modes to obtain the greybody factor [8]. For Minkowski space the boundary conditions can be decomposed into ingoing and outgoing waves, which leads to the usual idea of particle and S-matrix. In AdS background spacetime, solutions to the free wave equation can be classified into normalizable and non-normalizable modes [9]. The two types of modes are distinguished by their asymptotics and so the background can be determined from the boundary condition at infinity uniquely up to a choice of normalizable component [10]. The first(second) correspond to the states of the theory(boundary conditions for the field).

A few of recent works shed light on this direction [11–14]. The vacuum correlators \( \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\cdots\mathcal{O}(x_n) \rangle_{\text{CFT}^4} \) of CFT_4 are expressed as truncated n-point functions convolved against the non-normalizable modes. These can be interpreted as an S-matrix in an exact AdS_5 arising from a limit of scattering from an AdS_5 bubble with asymptotically flat space [12]. Furthermore, in the Poincaré coordinates to avoid the closed-time like curves, the transition amplitude between normalizable modes can be constructed to get correlation functions of the dual CFT. Also Giddings showed that a boundry S-matrix is defined as an overlap of “in” and “out” states near the timelike boundary of AdS [13]. This equals the corresponding correlator in the boundary CFT. On the other hand, two authors(Myung and Lee) showed that the S-matrix of an AdS_3 bubble in the dilute gas and low energy limits can be derived from an exact AdS_3 scattering [14]. This was possible by observing two types of potentials and using non-normalizable modes.

In this paper we find the logarithmic terms from an exact AdS_3 scattering of the new gauge bosons with non-zero spin(\( s = 2 \)). It is emphasized that these are states in the CFT_2 but are absent on the supergravity side [15,16]. We introduce the spin-dependent wave equation by switching on the AdS/CFT correspondence. Here the spin is defined as \( s = h_R - h_L \). Although this equation has an exact solution in the global coordinates, it is
very difficult to obtain its solution in the Poincaré coordinates. Hence matching procedure is essential for obtaining the absorption cross sections for the non-zero spin fields in the low temperature limit. In AdS$_3$ bubble scattering, one needs a self-dual point ($r = R$) as a matching point to calculate the greybody factor. Here we don’t need such a self-dual point, as is needed just in the AdS$_3$ bubble approach. Instead, we introduce a spin-dependent matching point ($Y = \sqrt{2s/\omega}, 1 < Y < \infty$) for calculation. Then the normalizable modes become the relevant one.

The organization of our paper is as follows. In Sec. II we derive the spin-dependent wave equation on AdS$_3$ from the AdS/CFT correspondence. And then we calculate the zeroth-order absorption cross section in the low-temperature limit. We compare it with the known results. We compute the first-order correction to the zeroth-order cross section in Sec. III. Finally we discuss our result in Sec. IV.

II. ZEROTH-ORDER MATCHING

We start from the equation of motion of the field with mass $m$ and spin $s$ in the global coordinates ($\tau, \varphi, \rho$) [3,10],

\[
\left( \Box + \frac{s^2}{R^2 \sinh^2 \rho} \right) \psi = m^2 \psi
\]  

(1)

with

\[
R^2 m^2 = 2h_R(h_R - 1) + 2h_L(h_L - 1) - s^2 = (h_R + h_L)(h_R + h_L - 2),
\]  

(2)

and $s = h_R - h_L$. Eq.(1) comes from the correspondence between states in CAdS$_3$(covered AdS$_3$) and in CFT$_2$. In other words, the CAdS/CFT correspondence determines the mass and spin on AdS$_3$ in terms of the conformal weights $(h_R, h_L)$ on the boundary. Its solution ($\psi = e^{-i\omega \tau}e^{im \varphi} \psi(\rho)$) takes the exact form when $m = s, \omega = -(h_R + h_L)$ or $(h_R + h_L = 2)$ as

\[
\psi(\tau, \varphi, \rho) = C_1 \left( \frac{1}{\cosh \rho} \right)^{h_R + h_L} e^{-i\varphi} e^{i(h_R + h_L)\tau} + C_2 (\cosh \rho)^{h_R + h_L - 2} e^{-i\varphi} e^{-i(h_R + h_L - 2)\tau}.
\]  

(3)
The first corresponds to the normalizable mode and the second to the non-normalizable one. Although the CFT is well defined on the cylindrical boundary expressed in terms of the global coordinates, it is important to find a map between the string Hilbert space in PAdS (Poincaré patch of CAdS) and operators of CFT on the plane. This is so because the scattering analysis is not problematic within PAdS. Then the PAdS/CFT duality relates test fields within the Poincaré patch to conformal operators on the planar boundary. For this purpose, we introduce the Poincaré coordinates $(y, t, x)$

\[
\begin{align*}
\frac{1}{y} &= \cosh \rho \cos \tau + \sinh \rho \cos \varphi \\
t &= y \cosh \rho \sin \tau \\
x &= y \sinh \rho \sin \varphi.
\end{align*}
\]

On the timelike boundary of $y = 0$, this change leads to a transformation: $u/v \equiv x \pm t = \pm \tan \left( \frac{\tau \mp \varphi}{2} \right)$. In terms of these, the three dimensional metric takes a simple form

\[
ds^2 = \frac{R^2}{y^2}(dy^2 + du dv).
\]

However, the wave equation leads to a complicated form as \cite{17}

\[
\left\{ y^2(\partial_y^2 - \frac{1}{y} \partial_y - \partial_t^2 + \partial_x^2) + \frac{4s^2y^2}{4x^2 + (1 - y^2 + t^2 - x^2)^2} \right\} \psi = m^2 R^2 \psi.
\]

This is our key equation for studying the exact AdS$_3$ scattering for a test field $\psi$ with the non-zero spin. It is not easy to find its solution. In the case of $s = 0$, Eq.(6) can be derived from the conventional action on AdS$_3$ as

\[
I(\psi) = \frac{1}{2} \int d^3x \sqrt{g} \left[ (\nabla \psi)^2 + m^2 \psi^2 \right].
\]

In order to get the spin-dependent term, we switch on the CAdS$_3$/CFT$_2$ correspondence as in Eq.(1). As far as we know, there is no way to introduce the spin-dependent potential in Eq.(3) except this method.

Let us study the limiting behavior of the spin-dependent term. In the limit of $y \rightarrow \infty$ (infinity), one can approximate this term to $4s^2/y^2$ regardless of $x$ and $t$. However in the
limit of $y \to 0$ (horizon), we cannot neglect $t$ and $x$ dependence in the denominator, so we will approximate this term $4s^2y^2/R'$, where $R'$ can be considered as the (compactification) scale along the $x$-direction. For a 5D black hole, we have a compactified scale for $x$, but for 6D black string, one has an uncompactified scale \([18]\). Keeping this in mind, we consider a plane wave solution $\psi(y, t, x) = e^{-i\omega t + ipx} \psi(y)$ \([8]\). The equation of motion (4) leads to

$$
y \to \infty : \left\{ y^2 \left( \frac{\partial^2}{\partial y^2} - \frac{1}{y} \partial_y + \omega^2 - p^2 \right) + \frac{4s^2}{y^2} \right\} \psi = m^2 R^2 \psi
$$

(8)

$$
y \to 0 : \left\{ y^2 \left( \frac{\partial^2}{\partial y^2} - \frac{1}{y} \partial_y + \omega^2 - p^2 \right) + \frac{4s^2 y^2}{R'^4} \right\} \psi = m^2 R^2 \psi.
$$

(9)

Now the spacetime is divided into near and far regions defined by $y < Y$ and $y > Y$, where $Y$ is some scale to be determined later. For simplicity we set $p = 0$. We also consider the low energy scattering and so assume $\omega Y < 1$.

In the far region of $y > Y$, we choose a reciprocal variable $z = 2s/y$ in terms of which the wave function $\psi(y) = \phi(z)/z$ satisfies

$$
z^2 \phi'' + z \phi' + (z^2 - \nu^2) \phi = -\frac{4\omega^2 s^2}{z^2} \phi,
$$

(10)

where $\nu^2 = 1 + m^2 R^2 (\nu = h_L + h - 1)$. Here we consider only an integer $\nu > 0$. The $\nu = 0$ case corresponds to the tachyon, which is out of the scope of this paper. The leading order solution of this equation is

$$
\phi(z) = H_{\nu}^{(2)}(z),
$$

(11)

where we choose the solution to be purely infalling at the horizon ($y \to \infty$). We regard the right hand side as a small perturbation in the far region.

In the near region of $y < Y$, assuming $4s^2/R'^4 \ll \omega^2$, we introduce a new variable $\sigma = \omega y$ and $\psi(y) = \sigma f(\sigma)$. Then the equation of motion becomes

$$
\sigma^2 f'' + \sigma f' + (\sigma^2 - \nu^2) f = -\frac{4s^2 \sigma^2}{\omega^2 R'^4} f.
$$

(12)

For small $\sigma$, the right hand side is negligible and the leading order solution is given by the Bessel functions.
\[
f_\nu(\sigma) = \alpha J_\nu(\sigma) + \beta Y_\nu(\sigma),
\]

where the first term corresponds to normalizable mode, whereas the second to non-normalizable mode for \(\nu > 1\). The normalizable mode is compatible with Dirichlet boundary condition at infinity(\(\sigma = 0\)), but the non-normalizable mode is not.

As is in the case of AdS bubble [6,7], we can introduce a matching point(\(Y\)) for calculation. \(Y\) is determined by \(\omega Y = 2s/Y\), i.e. \(Y = \sqrt{2s/\omega}\). One finds \(Y > 1, \omega Y < 1\) in the low energy(\(\omega < 1\)). We note that this depends on the spin. Matching the amplitudes of \(\psi\) to leading order at \(y = Y\)

\[
\left. \frac{1}{z}H^{(2)}_\nu(z) \right|_{z = \sqrt{2s/\omega}} = \sigma \{ \alpha J_\nu(\sigma) + \beta Y_\nu(\sigma) \} \left|_{\sigma = \sqrt{2s/\omega}} \right.
\]

(14)

one finds with \(\beta = 0\) that

\[
\begin{align*}
\nu &= 1 : \alpha = \frac{i}{\pi \omega^2 s^2} \\
\nu &= 2 : \alpha = \frac{4i}{\pi \omega^3 s^3} \\
\nu &= 3 : \alpha = \frac{48i}{\pi \omega^4 s^4}.
\end{align*}
\]

(15)

This means that we take only the normalizable modes and turn off the non-normalizable modes. Now we calculate the flux. The asymptotic form of the normalizable mode(\(y \to 0\)) is

\[
\psi(\sigma) = \sigma \alpha J_\nu(\sigma) \simeq \alpha \sqrt{\frac{\sigma}{2\pi}} \exp\{ i(\sigma - \frac{1}{2} \nu \pi - \frac{1}{4} \pi) \}. 
\]

(16)

Then its flux, defined by

\[
F = \frac{1}{2i} \{ \left. \psi^* \frac{1}{y} \partial_y \psi - \psi \frac{1}{y} \partial_y \psi^* \right\}, 
\]

(17)

is given by

\[
F_\infty = \frac{\omega^2 |\alpha|^2}{2\pi}. 
\]

(18)

Since the ingoing part of the wavefunction at horizon (\(y \to \infty\)) is given by
\[ \psi(z) = \frac{1}{z}H^2_\nu(z) \simeq \sqrt{\frac{2}{\pi z^2}} \exp\{-i(z - \frac{1}{2}\nu\pi - \frac{1}{4}\pi)\}, \quad (19) \]

the ingoing flux at horizon is takes the form

\[ F_0 = \frac{1}{2\pi s^2}. \quad (20) \]

Taking the ratio of the flux across the horizon to the flux at infinity, we get the absorption probability

\[ A = \frac{F_0}{F_\infty} = \frac{1}{|\alpha|^2\omega^2 s^2}. \quad (21) \]

The absorption cross section is given by \( \sigma_{\text{abs}} = A/\omega \) in three dimensions. Inserting \( \alpha \) for \( \nu = 1, 2, 3 \), we have

\[
\begin{align*}
\nu = 1 &: \sigma_{\text{abs}} = \pi^2 \omega s^2 \\
\nu = 2 &: \sigma_{\text{abs}} = \frac{\pi^2 \omega^3 s^4}{16} \\
\nu = 3 &: \sigma_{\text{abs}} = \frac{\pi^2 \omega^5 s^6}{64 \times 36}.
\end{align*}
\quad (22)
\]

Note that the Poincaré coordinates \((y, t, x)\) are dimensionless. By switching on \( R \) we can recover the correct scale for cross section as

\[
\begin{align*}
\nu = 1 &: \sigma_{\text{abs}} = \pi^2 \omega R^2 s^2 \\
\nu = 2 &: \sigma_{\text{abs}} = \frac{\pi^2 \omega^3 s^4 R^4}{16} \\
\nu = 3 &: \sigma_{\text{abs}} = \frac{\pi^2 \omega^5 R^6 s^6}{64 \times 36}.
\end{align*}
\quad (23)
\]

The power of \( R \) is in parallel with \( s \).

It is very important to compare our results (23) with those in the literature. The general formula for the greybody factor is given by up to the constant factor \( C \)

\[
\sigma_{\text{abs}}^{h_R,h_L} = C \frac{(2\pi RT_R)^{2h_R-1}(2\pi RT_L)^{2h_L-1}}{\omega \Gamma(2h_R)\Gamma(2h_L)} \sinh\left(\frac{\omega}{2T_H}\right) \\
\times \left|\Gamma(h_R - \frac{\omega}{4\pi T_R})\Gamma(h_L - \frac{\omega}{4\pi T_L})\right|^2, \quad (24)
\]
where
\[
\frac{2}{T_H} = \frac{1}{T_L} + \frac{1}{T_R}
\]
and \(\Gamma\) is the gamma function. Here \(C\) is \(2(h_R + h_L - 1)^2\) in the effective string method by Gubser [19] and in the boundary CFT approach by Teo [20]. Further \(C = 2(h_R + h_L - 1)\) in the AdS3-bulk calculation by Lee and Myung [17] and \(C = (h_R + h_L)(h_R + h_L - 1)\) in the bulk-boundary calculation by Müller-Kirstern, et al. [21]. The last three calculations are valid only for \(h_L = h_R\). Gubser’s calculation can accommodate the case of non-zero spin \((h_L \neq h_R)\), where \(h_L + h_R\) is an integer. But his result is still problematic in the case of \(h_R = 0\)(or \(h_L = 0\)). Since our calculation is valid for the test fields with non-zero spin, we will compare the two results. For \(\nu = 1\), we put \((h_R, h_L) = (0, 2)\) in \(\sigma_{abs}^{h_R, h_L}\) and compare the result with (23). Note that \((2, 0)\) and \((0, 2)\) operators are coupled to gauge bosons in AdS3. The result is
\[
\sigma_{abs}^{0, 2} = \frac{2\pi^2 R^2 \omega}{\Gamma(4)\Gamma(0)} \left\{1 + \left(\frac{2\pi T_L}{\omega}\right)^2\right\} \frac{\sinh\left(\frac{\omega}{T_H}\right)}{\sinh\left(\frac{\omega}{T_L}\right) \sinh\left(\frac{\omega}{T_R}\right)}.
\]
Before we proceed, we note that \(\sigma_{abs}^{h_R, h_L}\) is derived from the BTZ coordinates \((r, t, x^5)\), while (23) is calculated with the Poincaré coordinates \((y, t, x)\). Hence, to compare our result of (23) with (26), one has to take the low temperature limit. Taking this limit \((T_L, T_R \ll \omega < 1)\), then one finds from (26)
\[
\sigma_{abs}^{0, 2} \rightarrow \frac{2\pi^2 R^2}{\Gamma(4)\Gamma(0)} \omega,
\]
which agrees with the leading term of (23) for \(s = 2, \nu = 1\) up to some factor. Although \(\sigma_{abs}^{0, 2}\) depends on \(\omega\), it includes a singular term of \(1/\Gamma(0)\). Hence the effective string approach does not give us the precise result for calculations of \(\sigma_{abs}^{0, 2}, \sigma_{abs}^{0, 3}\), and \(\sigma_{abs}^{0, 4}\). We note here that our analysis for \(s = 2, \nu = 1\) is unique because the gauge bosons are absent on the supergravity side. Our method seems to be very useful for calculations with \(h_R = 0\)(or \(h_L = 0\)). For \(\nu = 2\), we can compare the result for \((h_R, h_L) = (1, 2)\) and \((2, 1)\) which are coupled to two intermediate scalars \((\eta, \xi)\) in AdS3. For \((2, 1)\), we have
\[ \sigma_{abs}^{2,1} = \frac{8\pi^2 R^4 \omega^3}{\Gamma(4)\Gamma(2)} \left\{ 1 + \left( \frac{2\pi T_L}{\omega} \right)^2 \right\} \frac{\sinh \left( \frac{\omega}{2T_H} \right)}{\sinh \left( \frac{\omega}{2T_L} \right) \sinh \left( \frac{\omega}{2T_R} \right)}. \]  

(28)

and similar one for (1,2) by exchanging \( T_L \) and \( T_R \). In the low temperature limit, one finds \( \sigma_{abs}^{2,1} \propto \omega^3 \). We note that in this limit, the effective string calculation agrees with that of the AdS_3 bubble scattering for \((\eta,\xi)\) [22]. Also this agrees with (23) for \( \nu = 2 \). It is straightforward to show that the fixed scalar \( \lambda \) of \( \nu = 3 \) case in (23) agrees in the low temperature limit of \( \sigma_{abs}^{3,1} \) and \( \sigma_{abs}^{1,3} \), and with the AdS_3 bubble calculation [23]. The above confirms clearly that our result (23) is correct in the low temperature limit.

### III. FIRST-ORDER MATCHING

As one would expect, the dominant corrections to the absorption cross section may arise from the matching at the dual point \( y = Y = \sqrt{2s/\omega} \). We will follow the approach of [8-10] to look for the field solution as power series in \( \omega \) and \( s \). In the region of \( y < Y \) the right hand side of (12) is considered as a small correction to the zeroth-order solution. However as we approach the dual point \( y = Y \), this term acts as a correction to the zeroth-order solution of the same order. In the near region of \( y < Y \), we look for a perturbative solution 

\[ f_\nu(\sigma) = f_\nu^0(\sigma) + f_\nu^1(\sigma) \]

where the zeroth-order solution is

\[ f_\nu^0(\sigma) = \alpha J_\nu(\sigma) \]  

(29)

and \( f_\nu^1 \) satisfies the inhomogeneous equation

\[ \sigma^2 f_\nu'' + \sigma f_\nu' + (\sigma^2 - \nu^2) f_\nu^1 = -\frac{4s^2 \sigma^2}{\omega^2 R^4} f_\nu^0. \]  

(30)

Solving this second-order equation for \( f_\nu^1 \), we have

\[ f_\nu^1(\sigma) = \frac{\pi}{2} \int_\sigma d\sigma' \frac{4s^2 \sigma'}{\omega^2 R^4} f_\nu^0(\sigma') \{ J_\nu(\sigma) Y_\nu(\sigma') - J_\nu(\sigma) Y_\nu(\sigma') \}. \]  

(31)

There is ambiguity in \( f_\nu^1(\sigma) \) in the sense that one can add to it any solution of the homogeneous equation. We can fix this ambiguity by demanding that the solutions of the near
and far regions match to order of \( \omega s \) (or \( \omega s \ln(\omega s) \)) for \( \nu = 1 \) case in the transition region. Substituting the form of \( f_\nu^0(\sigma) \) and retaining the leading terms in \( \omega s \) (\( \omega s \ln(\omega s) \)), we find that the first-order solution in the far region \( \psi = \sigma f_\nu(\sigma) = \sigma(f_\nu^0(\sigma) + f_\nu^1(\sigma)) \) is given by

\[
\begin{align*}
\nu = 1 : \psi(\sigma) &= \frac{\alpha}{2} \sigma^2 \left(1 - \frac{1}{8} \sigma^2 + \frac{s^2}{2\omega^2 R^4 \sigma^2}\right) \\
\nu = 2 : \psi(\sigma) &= \frac{\alpha}{8} \sigma^3 \left(1 - \frac{1}{12} \sigma^2 + \frac{1}{3} \frac{s^2}{\omega^2 R^4 \sigma^2}\right) \\
\nu = 3 : \psi(\sigma) &= \frac{\alpha}{48} \sigma^4 \left(1 - \frac{1}{16} \sigma^2 + \frac{1}{4} \frac{s^2}{\omega^2 R^4 \sigma^2}\right)
\end{align*}
\] (32)

We repeat the same procedure in the far region \( y > Y \) to find that the first-order solution in the transition region \( \psi = \phi(z)/z = (\phi_0(z) + \phi_1(z))/z \) is given by

\[
\begin{align*}
\nu = 1 : \psi(z) &= \frac{2i}{\pi z^2} \left\{1 - \frac{1}{2} z^2 \ln z + \frac{\omega^2 s^2}{2 z^2} (1 + \frac{3}{2} z^2 \ln z)\right\} \\
\nu = 2 : \psi(z) &= \frac{4i}{\pi z^3} \left(1 + \frac{1}{4} z^2 + \frac{\omega^2 s^2}{3 z^2}\right) \\
\nu = 3 : \psi(z) &= \frac{16i}{\pi z^4} \left(1 + \frac{1}{8} z^2 + \frac{\omega^2 s^2}{4 z^2}\right)
\end{align*}
\] (33)

Now we compare these two solutions at the matching point \( y = Y \). Since \( \omega s \) is much smaller than one, both \( \sigma \) and \( z \) are also small in the matching region \( \sigma = z = \sqrt{2\omega s} \), and our perturbative expansion is valid. The mismatch between these two solutions requires that one take

\[
\begin{align*}
\nu = 1 : \alpha &= \frac{i}{\pi \omega^2 s^2} \left\{1 - \frac{1}{2} \omega s \ln(2\omega s)\right\} \\
\nu = 2 : \alpha &= \frac{4i}{\pi \omega^3 s^3} \left\{1 + \left(\frac{5}{6} - \frac{2}{3} \frac{s^2}{\omega^2 R^4}\right)\omega s\right\} \\
\nu = 3 : \alpha &= \frac{48i}{\pi \omega^4 s^4} \left\{1 + \left(\frac{1}{2} - \frac{1}{2} \frac{s^2}{\omega^2 R^4}\right)\omega s\right\}
\end{align*}
\] (34)

This implies that the absorption cross sections behave as

\[
\begin{align*}
\nu = 1 : \sigma_{abs} &= \frac{\pi^2 \omega R^2 s^2}{16} \left\{1 + \omega s \ln(2\omega s)\right\}, \\
\nu = 2 : \sigma_{abs} &= \frac{\pi^2 \omega^3 R^4 s^4}{64} \left\{1 - \left(\frac{5}{3} - \frac{4}{3} \frac{s^2}{\omega^2 R^4}\right)\omega s\right\}, \\
\nu = 3 : \sigma_{abs} &= \frac{\pi^2 \omega^5 R^6 s^6}{64 \times 36} \left\{1 - \left(1 - \frac{s^2}{\omega^2 R^4}\right)\omega s\right\}
\end{align*}
\] (35)

in the low temperature limit.
IV. DISCUSSION

From the AdS$_3$/CFT$_2$ correspondence($\nu = h_R + h_L - 1$), the $\nu = 1$ case contains a minimally coupled scalar $\Phi$ which couples to (1,1) operator, and the new gauge bosons to (2,0) and (0,2) operators [3]. The $\nu = 2$ case involves a new field to (3/2, 3/2) operator, and two intermediate scalars ($\eta, \xi$) to (2,1), (1,2) operators [22]. The $\nu = 3$ accommodates the dilaton (fixed scalar $\nu$) to (2,2), and the fixed scalar $\lambda$ to (2,2), (3,1), and (1,3) operators [23]. Taylor-Robinson showed that a minimally coupled scalar with $s = 0, \nu = 1$ receives logarithmic corrections in the cross section by semi-classical calculation, effective string approach, and AdS$_3$/CFT$_2$ correspondence [7]. Her geometry corresponds to an AdS$_3$ bubble (the near-horizon AdS$_3 \times$S$^3$ but with asymptotically flat space). Hence she needs a self-dual point which is the effective radius($R$) of the AdS$_3$ bubble for matching procedure. Here the self-dual point plays a role of the transition point from AdS$_3$ (near-horizon) to Minkowski space (asymptotically flat space). We remind the reader that our geometry is an exact AdS$_3$. Thus we don’t need to introduce such a self-dual point that exactly plays the same role of $R$. Instead, we introduce a spin-dependent matching point ($Y = \sqrt{\frac{2\omega}{s}}$) for computation.

In this work we consider only the test fields with non-zero spin. Unfortunately we do not thus obtain any information for a minimally coupled scalar with zero spin. Improved matching in an exact AdS$_3$ for gauge bosons leads to a logarithmic correction to the absorption cross section. These gauge bosons appear in the resolution to the puzzle of the missing states between CFT$_2$ and supergravity [15,16]. Actually one cannot find these on the supergravity side. In this sense they do not belong to physical fields. But these are chiral primaries which correspond to the descendent of the identity operator in the CFT$_2$ [3,16]. Here we can include these gauge bosons to study the exact AdS$_3$ scattering by considering the spin-dependent wave equation. On the other hand, the $\nu = 2, 3$ cases including three physical fields($\eta, \xi, \lambda$) do not contain any logarithmic correction in the first-order approximation.

How do we interpret this result? First let us introduce the exact AdS/CFT lore which
states that the conformal limit of gauge theory corresponds precisely with scattering from an exact AdS\(_3\) background. So there is no chance that the logarithmic term appears in an exact AdS\(_3\) scattering. For example, this lore was proven partly for a minimally coupled scalar(Φ) and the dilaton(ν) in an exact AdS\(_3\) scattering [14,8]. Here we have proven this lore for three fields(η, ξ, λ). If we follow this lore, it is very hard to understand our logarithmic correction to the cross section for the gauge bosons. Now we assume that the AdS/CFT lore is valid only for the physical test fields. Then there is a chance with logarithmic term in the exact AdS\(_3\) scattering of the new gauge bosons, since these belong to unphysical fields.

In this direction we can relate the new gauge bosons to singletons in AdS\(_5\)×S\(_5\). These are AdS\(_5\) degrees of freedom that are pure gauge in the bulk and can be gauged away completely except at the boundary [26,27]. Here the relevant AdS\(_5\) supergroup is SU(2,2|4). For AdS\(_3\)×S\(_3\), the supergroup is given by SU(1,1|2) and thus the gauge bosons do not have an exact singleton representation. But something similar happens in our case. Let us consider SU\(_R\)(1,1|2)×SU\(_L\)(1,1|2) Chern-Simons theory. It does not have any propagating degree of freedom in an exact AdS\(_3\) spacetime because it belongs to a topological field theory. However the gauge field is subject to a boundary condition that contains fields living at the boundary. These fields are generators of the right and left-moving chiral algebras. Although the gauge bosons do not belong precisely to the singleton representation, these take similar properties as singletons [28]. More recently, Kogan proposed a relationship between singletons in AdS and logarithmic conformal field theories on its boundary [29]. He showed explicitly that the bulk AdS Lagrangian for a singleton dipole pair induces the two-point correlation function for logarithmic pair on the boundary. This means that although we do not study the corresponding CFT\(_2\) on the boundary, our logarithmic correction (35) through (33) to the new gauge bosons may be understood in relation to singletons. Consequently, our logarithmic correction represents that the new gauge bosons play the role of singletons. This is also compatible with the AdS/CFT lore if this lore is suitable for the physical fields such as a minimally coupled scalar(Φ), dilaton(ν), intermediate scalars(η, ξ), and the fixed scalar(λ).
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