The Top Quark Production Asymmetries $A_{FB}^t$ and $A_{FB}^\ell$

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A large forward-backward asymmetry is seen in both the top quark rapidity distribution $A_{FB}^t$ and in the rapidity distribution of charged leptons $A_{FB}^\ell$ from top quarks produced at the Tevatron. We study the kinematic and dynamic aspects of the relationship of the two observables arising from the spin correlation between the charged lepton and the top quark with different polarization states. We emphasize the value of both measurements, and we conclude that a new physics model which produces more right-handed than left-handed top quarks is favored by the present data.

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Introduction. The observed forward-backward asymmetry $A_{FB}^t$ in the rapidity distribution of top quarks [1, 2] at the Tevatron deviates by about two standard deviations (2σ) from standard model (SM) expectations [3]. In addition to $A_{FB}^t$, the D0 group also reports a positive forward-backward asymmetry of charged leptons from top quark decays of $A_{FB}^\ell = (15.2 \pm 4.0)\%$ compared with the small value $2.1 \pm 0.1\%$ from SM [2]. The deviation of the asymmetries may be contrasted with the good agreement of the overall rate for top quark production with SM predictions.

In this Letter, we focus on the kinematic and dynamic relationship between $A_{FB}^t$ and $A_{FB}^\ell$. We investigate how the distribution of leptons in the laboratory frame is related to the polarization state of the top quark parent. We show in a model-independent manner that current data on the ratio of the two asymmetries imply that more right-hand than left-handed top quarks are produced. This is a second and independent indication from asymmetry data of discrepancy from the SM since an equal number of right- and left-handed top quarks is predicted in the SM. We urge confirmation of the D0 result by the CDF collaboration and with the full data set in D0. Measurements of both $A_{FB}^t$ and $A_{FB}^\ell$ are especially valuable because their correlation can be related through top quark polarization to the underlying dynamics of top quark production.

We begin with a discussion of the angular distribution of decay leptons, first in the rest frame of the top quark and then in the laboratory frame. Subsequently, we derive the relationship of $A_{FB}^t$ and $A_{FB}^\ell$ separately for left- and right-handed top quarks. Different models of new physics produce top quarks with different proportions of left- and right-handed polarization. We use a $W'$ model [4] and an axigluon $G'$ model [5] to deduce their different expectations for $A_{FB}^t/A_{FB}^\ell$. The $W'$ model and other models [6] with more right- than left-handed top quarks tend to be preferred by the data provided that the constraint of the overall rate is satisfied.

Kinematics. In the top quark rest frame, the distribution in the polar angle $\theta_{hel}$ of a decay lepton $\ell^+$ is [7]

$$
\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta_{hel}} = \frac{1 + \lambda_t \cos\theta_{hel}}{2}.
$$

where $\lambda_t$ denotes the top quark helicity; $\lambda_t = +$ for a right-handed ($t_R$), and $\lambda_t = -$ for a left-handed top quark ($t_L$). The angle is measured with respect to the direction of motion of the top quark in the laboratory frame. Once the top quark is boosted, the angular distribution of the charged lepton relative to the direction of the top quark is sensitive to the energy of the top quark $E_t$. We derive

$$
\frac{d\Gamma}{\Gamma d\cos\theta_{t\ell}} = \frac{1 - \beta \cos\theta_{t\ell} + \lambda_t (\cos\theta_{t\ell} - \beta)}{2\gamma^2 (1 - \beta \cos\theta_{t\ell})^3},
$$

where $\beta = \sqrt{1 - m_t^2/E_t^2}$, $\gamma = E_t/m_t$, and $\theta_{t\ell}$ is the angle between $\ell^+$ and its parent top quark in the boosted frame. As illustrated in Fig. 1 for $E_t = 200$ GeV, about 60% of $\ell^+$ follow the top quark (i.e., $\cos\theta_{t\ell} > 0$) for a $t_L$, and almost 100% for a $t_R$.

The top quark rapidity is $y_t \equiv \ln (\sqrt{(E_t + p_t^z)/(E_t - p_t^z)})$ where $p_t^z$ is the longitudi-
then (z-component) of the top quark momentum. The forward direction is specified as the direction of the incident proton beam. The probability for finding a positive charged lepton in the forward region when it originates from a top quark with a velocity \( \beta \), rapidity \( y_t \), and polarization \( \lambda_t \) is denoted

\[
R^{\lambda_t}_F (\beta, y_t) = \frac{N_F}{N_F^L + N_B^L}, \tag{3}
\]

where \( N_F (N_B^L) \) is the number of leptons \( \ell^+ \) in the forward (backward) region in the laboratory. After lengthy algebra, we derive

\[
R^{\lambda_t}_F (\beta, y_t) = \frac{1}{2} + \frac{1}{2} \sqrt{1 + 2 \gamma^{-2} \coth^2 y_t} \frac{1}{1 - \beta},
\]

\[
\text{for } y_t \in [0, y^\text{max}_t], \text{ where } y^\text{max}_t = \ln \frac{1 + \beta}{1 - \beta}.
\]

To illustrate the effect of the top quark boost, we plot \( R_F \) as a function of \( y_t \) in Fig. 2(a,b). We choose two characteristic top quark energies, \( E_t = 200 \text{ GeV} \) and 600 GeV. The former energy represents top quarks produced around the threshold region, while the latter pertains for highly boosted top quarks. When a top quark moves perpendicular to the beam line, i.e. \( y_t = 0 \), there is an equal number of leptons in the forward and backward regions, i.e. \( R_F = 0.5 \), independent of \( E_t \) and \( \lambda_t \).

For \( t_R \), \( R_F \) increases rapidly with \( y_t \) because most of the leptons move close to the direction of motion of the top quark after being boosted to the lab frame. We can also see that when \( E_t \) becomes larger, i.e. the top quark is more energetic and the lepton is more boosted, \( R_F \) rapidly reaches its maximum value 1.

On the contrary, in the case of \( t_L \)'s, the ratio \( R_F \) does not vary significantly with \( y_t \) owing to the anti-boost effect on \( \ell^+ \). For \( E_t = 200 \text{ GeV} \), the boost causes \( \ell^+ \) to distribute nearly uniformly, and \( R_F \) is around 0.5. When the energy of \( t_L \)'s is large enough, the large boost forces most of the charged leptons from top quark decays to move along the top quark direction of motion, even if they move against the top quark direction of motion in the top quark rest frame. The boost yields a large value \( R_F \) in the region of large \( y_t \). The competing influences leave the \( t_L \) curve slightly below the \( t_R \) curve.

In Fig. 3 we show how \( R_F \) varies with \( p_T^\ell \) and \( y_t \). The distributions for \( t_R \)'s do not vary greatly with \( p_T^\ell \) because most \( \ell^+ \) follow \( t_R \). However, the shapes of the curves for \( t_L \)'s are very different between the low \( p_T^\ell \) and high \( p_T^\ell \) regions. As the top quark moves forward, i.e. \( y_t > 0 \) for fixed \( p_T^\ell \), the boost becomes more significant as the energy of the top quark is increased. Therefore, more leptons are forced to move along the direction of the top quark. On the other hand, some fraction of the decay leptons which are initially in the forward/backward region \((y_t > 0/y_t < 0)\) will then be in the backward/forward region. In summary, two factors affect \( R_F \): the boost and the rearrangement of the distribution of charged leptons in the forward \((y_t > 0)\) and backward \((y_t < 0)\) regions. The former always increases \( R_F \) while the latter may increase or decrease \( R_F \) depending on \( E_t \) at \( y_t = 0 \).

Generally speaking, when the initial boost is not significant (low \( p_T^\ell \)), \( R_F \) decreases when \( y_t \) increases from \( y_t = 0 \) to \( y_t = 0 \), as we see in Fig. 3(a). For large enough boost \((p_T^\ell > m_t/\sqrt{3})\), \( R_F \) always increases with \( y_t \); the critical value is obtained from \( \frac{\partial R_F}{\partial y_t} |_{y_t=0} = 0 \).

\( A_F^t \) and \( A_F^t B \). Positive \( A_F^t B \) indicates more top quarks are produced in the forward region than in the backward region. Both \( t_R \) and \( t_L \) can generate a positive \( A_F^t B \). However, \( t_L \) would need a large boost along the beam line to overcome the fact that most of \( \ell^+ \) from its decay move against it in its rest frame, while \( t_R \) can yield a positive \( A_F^t B \) even for top quarks near the \( t \bar{t} \) threshold region. Therefore, the observed positive \( A_F^t B \) indicate that the top quark polarization may be playing a non-trivial role. In this section we present a general analysis of the correlation between \( A_F^t B \) and \( A_F^t B \) to prepare for a better understanding of the numerical results derived from new physics (NP) models.

Assuming the large \( A_F^t B \) is generated mainly by NP, \( A_F^t B \) can be divided into the contributions from different polarizations of top quarks:

\[
A_F^t B \approx \rho_{tL} A_F^{tL, NP} + \rho_{tR} A_F^{tR, NP} \times R_N, \tag{5}
\]
where

\[ A_{FB}^{\lambda_1, NP} = \left[ \frac{N_F^{\lambda_1} - N_B^{\lambda_1}}{N_F^{\lambda_1} + N_B^{\lambda_1}} \right]_{NP}, \quad \rho_{\lambda_1} = \frac{N_{\lambda_1, NP}}{N_{\lambda_1, tot}}. \]  

(6)

Here, \( A_{FB}^{\lambda_1, NP} \) denotes the forward-backward asymmetry of the top quark with polarization \( \lambda_1 \) generated only by NP, while \( \rho_{\lambda_1} \) is the fraction of top quark with polarization \( \lambda_1 \) in \( t\bar{t} \) events induced by NP, and \( R^{NP}(= N_{NP}^{\lambda_1, tot}/N_{tot}^{\lambda_1, tot}) \) is the ratio of NP signal events to the total observed \( t\bar{t} \) events. One advantage of decomposing \( A_{FB}^{\lambda_1} \) into different top quark polarizations is to monitor the chirality of the couplings of NP particles to top quarks. Another advantage is to make the connection between \( A_{FB}^{\lambda_1} \) and \( A_{FB}^{\lambda_2} \) more transparent.

As discussed earlier, the ratio \( R^{FB} \) depends on the top quark kinematics (\( \beta, y_t \) and \( \lambda_t \)). To compute the probability for a charged lepton in the forward region, one must convolute the top quark production cross section with \( R^{FB} \) on an event-by-event basis, i.e.

\[ N^{t\bar{t}} \otimes R^{FB, \lambda_t} = \int N^{t\bar{t}}(\beta, y_t, \lambda_t) R^{FB, \lambda_t}(\beta, y_t), \]  

(7)

where \( N^{t\bar{t}} \) labels the \( t\bar{t} \) production rate for a top quark with specific kinematics (\( \beta, y_t, \lambda_t \)). The lepton asymmetry \( A_{FB}^{\lambda_1} \) generated by a top quark with polarization \( \lambda_t \) is, therefore,

\[ A_{FB}^{\ell, \lambda_1} \bigg|_{NP} = \left( \frac{N_F^{\lambda_1} - N_B^{\lambda_1}}{N_F^{\lambda_1} + N_B^{\lambda_1}} \right) \otimes \left( \frac{2R^{FB, \lambda_1} - 1}{N_F^{\lambda_1} + N_B^{\lambda_1}} \right)_{NP}. \]  

(8)

Because \( R^{FB, \lambda_1} \) cannot exceed 1, we have \( A_{FB}^{\ell, \lambda_1} \lesssim A_{FB}^{\ell, \lambda_2} \).

When \( R^{FB, \lambda_1} \) is close to a constant \( R_C \), e.g. \( R_C \sim 1/2 \) around the \( t\bar{t} \) threshold (\( E_t \sim 200 \text{ GeV} \)) for left-handed top quark or \( R_C \sim 1 \) for a highly boosted top quark, the lepton asymmetry \( A_{FB}^{\ell, \lambda_1} \) can be simplified as

\[ A_{FB}^{\ell, \lambda_1} \bigg|_{NP} = A_{FB}^{\lambda_1, NP} \times (2R_C - 1). \]  

(9)

Equation (9) and Fig. 2 show that:

- \( A_{FB}^{\ell, t\bar{t}} \sim 0 \) when the \( t\bar{t} \) pair is produced around the threshold region;
- \( A_{FB}^{\ell, t\bar{t}} \lesssim A_{FB}^{t\bar{t}} \approx A_{FB}^{\ell, t\bar{t}} \) in the large \( m_{t\bar{t}} \) region.

Although Eq. (9) is approximate, it helps in understanding the NP prediction obtained from a complete numerical calculation.

**New physics models: axigluon and \( W' \).** We examine two models of new physics, an axigluon model \(^4\) and a flavor-changing \( W' \) model \(^4\). In the axigluon (\( G' \)) model we assume for simplicity that the interaction of \( G' \) to the SM quarks is purely pseudo-vector-like

\[ \mathcal{L} = g_s \left( g_t \bar{q} \gamma^\mu \gamma_5 q + g_h \bar{Q} \gamma^\mu \gamma_5 Q \right) G'_\mu, \]  

(10)

where \( q \) denotes the first two generation quarks and \( Q \) the third generation quarks. The coupling \( g_s \) is the strong coupling strength; \( g_t \) and \( g_h \) are the coupling strength of \( G' \) to \( q \) and \( Q \), respectively.

The absence of deviation from the SM expectation in the measured \( m_{t\bar{t}} \) distribution \(^1, 2\) indicates the \( G' \) should be heavy and broad. Its contribution is therefore under interference with the SM channel. The top quarks are generated unpolarized owing to the pseudo-vector coupling of the \( G' \) to the SM fermions, and

\[ \rho_{t\bar{t}} = 1/2, \quad A_{FB}^{t\bar{t}, NP} = A_{FB}^{t\bar{t}, NP} = A_{FB}^{t\bar{t}}/R^{NP} > 0. \]  

(11)

Since the \( t\bar{t} \) cross section is greatest near the threshold region where \( A_{FB}^{t\bar{t}} \approx 0 \) and \( A_{FB}^{t\bar{t}} \approx A_{FB}^{t\bar{t}} \), the expression for \( A_{FB}^{t\bar{t}} \) becomes \( A_{FB}^{t\bar{t}} \approx \frac{1}{2} A_{FB}^{t\bar{t}} \).

We plot our axigluon model predictions for \( A_{FB}^{t\bar{t}} \) and \( A_{FB}^{t\bar{t}} \) in Fig. 4(a). We first scan the theoretical parameter space \((g_t, g_h \text{ and } m_{G'})\) to fit Tevatron data on \( A_{FB}^{t\bar{t}} \) and the \( t\bar{t} \) total production cross section within 1 \( \sigma \). These parameters are then used to calculate \( A_{FB}^{t\bar{t}} \). The figure shows a clear correlation between \( A_{FB}^{t\bar{t}} \) and \( A_{FB}^{t\bar{t}} \). The best fit to the correlation is \( A_{FB}^{t\bar{t}} \approx 0.47 \times A_{FB}^{t\bar{t}} + 0.25 \%. \)
To fit both $A_{FB}^t$ and $A_{FB}'$ within 1σ, the mass of the $G'$ must be greater than 1 TeV. For masses this great, top quarks from $G'$ decays are highly boosted and cause more $\ell\mu$ to move along the direction of the top quarks.

We remark here that if the $G'$ is found as a resonance in the $t\bar{t}$ mass distribution, the chirality structure of its coupling to $t\bar{t}$ can possibly be determined at the LHC [8].

A different class of NP models to explain the $A_{FB}^t$ is based on $t$-channel kinematics. A model with a non-universal massive neutral vector boson $Z'$ [9] is disfavored because it implies an excessive rate for same-sign top quark production at the 7 TeV LHC [10].

We consider in this paper a flavor-changing $W'$ which couples an incident $d$-quark to the produced $t$-quark [4],

$$\mathcal{L} = g_2 g_R d_\nu^\mu P_R t W'_\mu + h.c.,$$

where $g_2$ is the weak coupling. In the $W'$ model, in addition to the SM process the $t\bar{t}$ pair can also be produced via a $t$-channel process with a $W'$ mediator. In the region $\beta \approx 1$, the nonzero helicity amplitudes $\mathcal{M}_{W'}^t(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ are

$$\mathcal{M}_{W'}^t(+-+-) \sim 2r_W^2(1 - \cos \theta),$$

$$\mathcal{M}_{W'}^t(+--+) \sim 4(1 + \cos \theta)$$

where $r_W = m_t/m_{W'}$. In order to produce top quarks in the forward region, one needs $2r_W^2 < 4$, which is always true for the region of $W'$ masses (heavier than the top quark) considered in this paper. At the Tevatron the $\beta$ distribution of the top quark in $t\bar{t}$ production peaks around 0.6, and therefore most of the top quarks are not significantly boosted. We can also easily see that $\rho_{t\bar{t}} > \rho_{t\bar{t}}$ in the $W'$ model. Since the $t$-channel propagator contributes a minus sign, $A_{FB}^t$ arises from a competition between the square of the purely NP term and the interference term of NP with the SM. The strong correlation is fit well by $A_{FB}^t \simeq 0.75 \times A_{FB}' - 2.1\%$. Moreover, for a relatively light $W' (\lesssim 600)$ GeV, both $A_{FB}^t$ and $A_{FB}'$ can be consistent with the D0 data within 1σ.

The ratio of the predicted $A_{FB}^t$ to $A_{FB}'$ peaks near 50% in the axigluon model and near 62% in the $W'$ model. The data from D0 shows about 78 ± 33%. The ratio in the SM is close to 40%. The $W'$ model generates a larger $A_{FB}'$ than the axigluon $G'$ model because it produces more right-handed top quarks. The comparison to the D0 point shown in Figs. 3a,b indicates that top quark events with a large proportion of right-handed top quarks are favored. Constraints on flavor-changing currents in the $W'$ model allow only right-handed couplings to the top quark, consistent with the D0 $A_{FB}^t$ results. There is no direct evidence of the handedness of the coupling in the massive gluon models. The D0 result could be interpreted as an indirect cue for the chiral couplings of the massive gluon.

Summary. We study the kinematic and dynamic aspects of the relationship between the asymmetries $A_{FB}^t$ and $A_{FB}'$ based on the spin correlation between charged leptons and the top quark with different polarization states. Owing to the spin correlation in top quark decay, $A_{FB}^t$ and $A_{FB}'$ are strongly positively correlated for right-handed top quarks. However, for left-handed top quarks, the nature of the correlation depends on how boosted the top quark is. For large enough $E_t$, $t_L$ will also generate a large $A_{FB}^t$, similar to that for $t_R$. However, if $t_L$ is not boosted, $A_{FB}^t$ from it will be less than $A_{FB}'/2$ for a positive $A_{FB}^t$. Since most of the $t\bar{t}$ events are produced in the threshold region, one may use the large positive values of $A_{FB}^t$ and $A_{FB}'$ measured at D0 to conclude that production of left-handed top quarks is disfavored. Confirmation of the D0 result and greater statistics are essential. There is great value in making measurements of both $A_{FB}^t$ and $A_{FB}'$ because their correlation can be related through top quark polarization to the underlying dynamics of top quark production.

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