The achievement analysis of Indonesian TIMSS 2011 in mathematics towards didactical situation

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Abstract. This analysis study focuses on Indonesian students’ achievement at TIMSS 2011 in mathematics which was relatively low compared to other ASEAN countries. This condition leads to question about Indonesian mathematical learning profile and it links to TIMSS 2011 achievement. A case study is conducted to analyse and examine three teachers’ mathematical learning profile at secondary level of two province in Indonesia. The learning profile is described from the occurrence of teachers’ behavioural and students’ behavioural based on didactical situation theory (DST) and also teachers’ learning atmosphere to provoke students’ mathematical thinking. This case study used video that was produced at TIMSS Video Study 2011. After analysing the three secondary school videos of “SMP A”, “SMP B”, and “SMP C” and then correlate to DST and students’ mathematical thinking skills framework, it is found out that students at “SMP A” that gained high international benchmark in TIMSS 2011 was taught by teacher who had in line learning profile with DST. Meanwhile, the students at “SMP B” and “SMP C” who got low international benchmark showed that teacher’s learning profile were far out from DST. It is shown that DST leads to high international benchmark achievement of TIMSS in mathematics.

1. Introduction

Education in Indonesia has become one of important issues and significant improvement in curriculum happens during the last couple years. It focuses on children’s learning development and their inquiry in getting knowledge. Hanushek and Woessmann \cite{1} suggest that education must be able to develop cognitive and thinking skills. Bialik and Kabbach \cite{2} found out in their two years research that cognitive skills can be achieved by mathematical learning because one of the goals of mathematics teaching is to improve reasoning proficiency. Moreover, it was also discovered that the brain which is practicing by mathematical tasks has ability to solve the problem and improves higher order thinking skills (HOTS) \cite{2}. The student who is always assessed by higher order task has more critical thinking and problem-solving ability or good in cognitive skills \cite{3}.

There are some international studies to assess student’s ability in HOTS. One of the prestigious studies internationally recognized is Trends in International Mathematics and Science Studies (TIMSS). TIMSS is a four years regular assessment which is applied and assess content domain and cognitive domain. The cognitive domain assesses at TIMSS are knowing, applying, and reasoning.
The reasoning in TIMSS can be categorized as measuring higher order thinking because the area of reasonings’ are analysis, generalization, synthesis, justification, and solving non-routine problem [4].

The burning question rises in relating with 8th grade student of TIMSS in mathematics achievement and a contributing factor of its. This comes since at twice participated at TIMSS in mathematics studies, the 8th grade Indonesian students’ achievement in generally on lower stage if comparing with two others ASEAN countries. The comparison percentage of 8th grade Indonesian achievement at TIMSS in mathematics at years 2007 and 2011 with two others ASEAN countries can be shown in Table 1.

Table 1. TIMSS 2011-8 grade: Trends in percentage of student reaching the international benchmark of mathematics achievement

| Country  | Advance 2007 | Advance 2011 | High 2007 | High 2011 | Intermediate 2007 | Intermediate 2011 | Low 2007 | Low 2011 |
|---------|--------------|--------------|-----------|-----------|-------------------|-------------------|----------|----------|
| Indonesia | 0            | 0            | 4         | 2         | 19                | 15                | 48       | 43       |
| Malaysia | 2            | 2            | 18        | 12        | 50                | 36                | 82       | 65       |
| Thailand | 3            | 2            | 12        | 8         | 34                | 28                | 66       | 62       |

Source: Mullis [5]

Table 1 shows that at TIMSS 2011 and 2007, the Malaysian and Thailand achieved 2% advance level 8th grade students. The Indonesian did not achieve this level meaning that 2% of Malaysian and Thai students had a good reasoning to be applied on their mathematical knowledge in solving nonroutine problem. However, there was a decrease on high level achievement comparing to TIMSS 2007, especially Malaysian students. Table 1 also shows that Malaysian was the most successful country in declining the percentage of students who achieved low international achievement. So, it can be predicted that in the next ten years, when the students are in the working age, they will be more competitive comparing with students of two others ASEAN participated countries. Malaysian has more student in advance, high, and intermediate level achievement. According to Table 1, it can be assumed that Indonesian mathematical learning could not be improving student reasoning.

The Ministry of Educational and Culture (MOEC) of the Republic of Indonesia with The World Bank Office-Jakarta have conducted the TIMSS Video Study 2011 to complete TIMSS study [6]. The video was made to captured the learning profile of mathematics lessons in schools of participants of TIMSS. Santagata, et al [7] stated that the video of classroom instruction can be described and discussed about classroom practices. Therefore, the video is examined the didactical situation at mathematics teachers’ learning profile and then is linked to 8th grade Indonesian TIMSS 2011 mathematics achievement. D’Amore [8] suggests that didactical situation as the art of forming idea and creating condition that lead to learning of some pieces of mathematical knowledge, and so forth the mathematical thinking ability. The mathematical didactical situation is created in order to improve student’s ability in doing mathematical activities, in discover what is mathematics without any intervene which is organized by teacher.

Thorough study on theory of the didactical situation and student’s mathematical thinking ability, the mathematics learning profile can be predicted and presumed as the result of TIMSS 2011 Mathematics achievement in Indonesian context. Those contribution is expected to be given out from this case study.

2. Method
The main aim of this case study is to find out the learning profile as contributing factor of TIMSS 2011 in Mathematics achievement. Therefore, the three videos from TIMSS Video Study 2011 are chosen to get data in mathematics didactic situation and also mathematical learning atmosphere in provoking student mathematical thinking. Table 2 shows the video which is taken as data in this case study.
Table 2. The video-studied data

| No. | Video | Kind of School | Locality | TIMSS 2011 Achievement |
|-----|-------|----------------|----------|------------------------|
| 1.  | “SMP A” | Public | Not at Java Island | 505 |
| 2.  | “SMP B” | Public | West Java | 391 |
| 3.  | “SMP C” | Private | West Java | 393 |

a The World Bank Office, Jakarta (Unpublished) [9]
b The Centre for Educational Assessment Board, MOEC, Republic of Indonesia (Unpublished) [10]
c SMP is Secondary School

Video of “SMP A” is chosen to represent the mathematical learning that produced high international benchmark achievement of TIMSS 2011 in mathematics. Meanwhile the last two videos, “SMP B” and “SMP C” are chosen to represent the mathematical learning that gained average scale score of Indonesian student or low international benchmark achievement. The videos were observed and the result was transcribed into observational log.

The three videos were analysed to get teachers’-students’ behavioural or learning profile under didactical situation theory (DST). The criteria of teachers’-students’ behaviour that is used as main observable from videos is taken from D’Amore [8] and Radford [11] theoretical framework. The teachers’ behaviour under DST along mathematics lesson that is being analysed are 1) no to tell how to solve the problem; 2) to allow student handle their own problem; 3) to understand students’ mathematical activities, such as not just receiving information, to learn and to send the right and appropriate mathematical messages; 4) to avoid direct teaching; 5) to find out the formal mathematics or contextual problem; and 6) to provoke the expected learning [8,11]. Meanwhile the students’ behaviour under DST that is being analysed are 1) to be independent learning; 2) to have self-concept or rational manners, like as mathematician; 3) to involve in solving mathematical problem, like as an expert; and 4) to have capability in producing, formulating, proving, and constructing model of language/statement, concept, and theory of mathematics [8,11].

As TIMSS 2011 in mathematics assessed cognitive domain, then through the videos also being analysed about teachers’ learning atmosphere to support students’ attitudes in mathematical thinking. To obtain the data then Mason, Burton, and Stacey [12] theoretical review is used. So that, to gain data about teachers’ learning atmosphere to support students’ mathematical thinking, from the videos is analysed the occurrence of such an atmosphere are questioning, challenging, and reflecting that conducted by the teacher. From each atmosphere then being analysed about students’ attitudes in mathematical thinking. When questioning phase, students’ attitudes that being analysed are to identify questions for investigation, to query assumption, and to negotiate meanings of terms. Meanwhile at challenging phase, students’ attitudes that being analysed are to make conjecture, to seek justifying or falsifying arguments, and to check, to modify, to alter. Whilst at reflection phase, students’ attitudes that being analysed are to be self-critical, to expect and to assess’ different approaches, and to shift, to re-negotiate, to change direction. All students’ attitudes are summed up as I CAN [12].

3. Results and discussion

In this case study, learning profile in 3 schools is reviewed based on DST and learning atmosphere to provoke student mathematical thinking skills. Learning profile is obtained from video and its transcript. Learning profile describes mathematics didactical situation and students’ mathematical thinking skills. The mathematics didactical situation in three schools will be reviewed from teacher and student behaviour. Table 3 will discover the occurrence of teachers’ behavioural of three schools.

Table 3. The occurrence of DST based teachers’ behaviour

| Teacher Behavioural | “SMP A” | “SMP B” | “SMP C” |
|---------------------|---------|---------|---------|
| No to tell how      | Teacher gave a problem | Teacher gave a problem and | Teacher gave a |
| Teacher Behavioural | “SMP A” | “SMP B” | “SMP C” |
|---------------------|---------|---------|---------|
| to solve the problem without any explanation about how to solve the problem. | some explanation about how to solve the problem. | problem and some explanation about how to solve the problem. | |
| To allow student handle their own problem Teacher walked around the class after gave the problem. It did to assure him whether all student work on the problem or not. While walked around, he informed to student about the allocated time to do with the problem. Like at minutes 00:19:18, he sounded that time allotted with the problem was 3 minutes. | Teacher walked around the class after gave the problem. Same with teacher at “SMP A”, she did it to assure that all student work on the problem. But, while walked through the class, she always gave some direction to all student about the way to solve the problem. | The classroom climate was like as “SMP B”.

There was a scaffold for some student who faced difficulties in working on problem. But, it done by individually to student who asked him by private.

To understand students’ mathematical activities, such as not just receiving information, to learn and to send the right and appropriate mathematical messages Teacher did not force student to have an answer as he wanted to. This behavioural was taped when he gave an opportunity for student to expose a different answer with others. Such as at minutes to 00:42:27, teacher said, “O... if there is any different answer, go on... it’s ok...” This statement was intended to all students to expose any answer as their already got.

If student made wrong answer, teacher asked them to examine the occurrence on mistake or error at their answer. Like

Teacher forced student to have an answer as rubrics that she has.

Teacher forced student to have an answer as rubrics that she has.
Teacher Behavioural | “SMP A” | “SMP B” | “SMP C”
---|---|---|---
At minutes to 00:16:35, teacher said, “.... who are having the same answer?.... Raise your hand if you have the same answer.... You got the wrong answer?... ya, ok, next time you have to correct your answer.... which part that you’ve got a mistake? What kind is an error?”
To avoid direct teaching | Teacher wait in patiently for student responses along they solved a problem. | Teacher would give the way how to solve or the answer of the problem, if the students could not solve a problem in quickly. | Teacher would give the way how to solve or the answer of the problem, if the students could not solve a problem in quickly.
To find out the problem in formal mathematics or contextual problem | The formal mathematics problem was dominantly along the lesson. But there was some contextual problem that given to student. Like, student asked to observe a thing at environment that use tangent circle principle. The observation result was presented by student, like at minutes to 01:02. | The formal mathematics problem was dominantly along the lesson. | The formal mathematics problem was dominantly along the lesson.
To provoke the expected learning | Teacher rewarded the students to provoke expected learning. It could be formed in giving applause or noting in assessment log to students who be involved along the lesson. | Teacher very focused to her lesson plan, but she forgot about the way to provoke expected learning. It impacts to low of student’s enthusiasm along the learning. | Teacher very focused to her lesson plan, but she forgot about the way to provoke expected learning. It impacts to low of student's enthusiasm along the learning.

Table 3 inform that DST based teacher’s behavioural occurred at “SMP A”. Meanwhile, at “SMP B” and “SMP C”, the teachers’ behavioural did not fully meet with DST. Since learning profile base on DST will be reviewed under teacher’s behavioural and student’s behavioural, the next Table 4 will sound the students’ behavioural along the lesson at the schools.
### Table 4. The occurrence of DST based students’ behaviour

| Student Behavioural                           | “SMP A”                                                                 | “SMP B”                                                                 | “SMP C”                                                                 |
|-----------------------------------------------|-------------------------------------------------------------------------|-------------------------------------------------------------------------|-------------------------------------------------------------------------|
| To be independent learning                    | Teacher assisted students at sometimes. But, in generally, students were given more opportunity to solve the given problem. | Students were directed to be on teacher power control.                  | Students were directed to be on teacher power control.                  |
| To have self-concept or rational manners, like as mathematician | Student did investigation, observed example or non-example about the existence of the common tangent of 2 circles. | Student followed the teacher direction and instruction (in case, teacher presuming this kind of instruction was not the best way). | Student followed the teacher direction and instruction (in case, teacher presuming this kind of instruction was the best way for students’ who attended school at noon). |
| To involve in solving mathematical problem, like as an expert | Student shown enthusiasm when teacher asked them to write their answer on the blackboard. So many students who raised their hands. | Student did not have a “brave heart” to write their answer in front of the class. | Student did not have a “brave heart” to write their answer in front of the class. Even, teacher always reminded them to do an exercise on their book. |
| To have capability in producing, formulating, proving, and constructing model of language/ statement, concept, and theory of mathematics | One student could draw a mathematical conclusion. An example of this occurrence is shown when one student presented her investigation about the thing that use tangent circle principle. | Did not happen.                                                          | Did not happen.                                                          |

The Table 4 shown that DST base on students’ behavioural, in relatively was occur at “SMP A”. Conversely, as same with teachers’ behavioural, the students’ behavioural also was not happen at “SMP B” and “SMP C”. The impact of this occurrence was student at both schools did not comply with DST yet. So, it can be inferred that DST was not happen along mathematics instruction at “SMP B” and “SMP C”. But,
we could not judge both teachers at “SMP B” and “SMP C” did not have any effort to conduct good instruction. From the video they were looked in trying to do an effective teaching.

The instruction at “SMP B” and “SMP C” was tent to teacher centre. Along the learning session, teachers seemed to change students into their power control area. Teachers always gave explanation about the inside meaning of given problem. Also, the procedure to solve the problem was informed when students faced a stuck. Students did not have much time to think about how to solve the problem. Teachers seemed in a formula-1 rally. It means that students did not have a range to improve their ability in solving problem and so mathematical knowledge as cognitive skills could not be achieved. Moreover, it can be stated that first principle of DST (P1: knowledge results as the “optimal” solution to a certain situation or problem [11,13]) is unfulfilled. Also, it can be mentioned that because of teachers’ instructional design tent to teacher centre therefore it is not appropriate with The Piaget genetic Epistemology Theory [14]. It means that second principle of DST (P2: learning is -in accordance to Piaget’s genetic epistemology- a form of cognitive adaptation [11, 13]) did not happen.

Another interrupting practice at “SMP B” and “SMP C” was the teachers’ intolerance. If the students made an error then teachers would not tolerate it. They would say that the students’ answers were not correct. This occurrence happened at “SMP B” like at the minutes to 72, when one pupil wrote her answer on the blackboard and an error was occur, on directly teacher gave a response. The citation dialogue from teacher (G) is appeared bellow.

G: (stare to a girl who wrote on the blackboard)
G: Please, write the conclusion..... the answer is angle AOB
G: (teacher walked around the class)
G: (Founding one student did an errors) What? This is not a correct answer.... the central angle is equal to two times of circumference angle?.... What? This is not correct.... it must be a half of central angle.... Please, correcting your answer.....

Meanwhile at “SMP C”, this situation occurred at minutes to 38, when teacher walked around the class and found one student did an error, on directly, she gave a feedback. The citation dialogue between teacher (G) and student (S) is shown below.

G: A hundred?... Angle ACB, where is it?... A... C... B... Circumference or central angle?...
Circumference or central?! (Teacher rises her voice)
S: Circumference angle
G: Circumference angle… If it is circumference angle, a half or two times of central angle?.... Why is this a half?.... This is incorrect, your formula is wrong… It must be a half times central angle… Please write, forty times....

The dialogue figured out that teacher hoped a perfect answer from student. But, it made student into situation that they could not present further mathematical ideas [11,13,15]. This happens because of the cognitive conflict was not occurred on students’ mind [16,17]. This intolerance would lead to student with low self-assurance along mathematics learning. It is caused by student were not treated as independent and autonomous learner. An example of this situation was happened at “SMP C” at minutes to 10 to 20. The citation dialogue between teacher (G) and students (S), as follows.

G : what arc were overlooked by angle peripheral CAB? (appointed to figure 1on the board)
S/G: (some student answered with teacher’s voice in dominantly) CB
G : what arc were overlooked the circumference angle EDF? (appointed to figure 1on the board)
S/G: (some student answered with teacher’s voice in dominantly) EF

Those dialogue shown that teacher answered her own question This situation conflicted with the third principle of DST (P3: for every pieces of mathematical knowledge there is a family of situations
to give it an appropriate meaning [11, 13]) and the fourth principle on DST (P4: the student’s autonomy is a necessary condition for the genuine learning of mathematics [11, 13]). When P1, P2, P3, and P4 are not occur, so it can be presumed that learning atmosphere will not fill of the aspect to develop students’ mathematical thinking skills. The proofing analysed is described on Table 5 that drawn teachers’ learning atmosphere to develop students’ mathematical thinking skills.

**Table 5.** The occurrence of teachers’ learning atmosphere in developing students’ mathematical thinking

| Teacher learning atmosphere | “SMP A”                                                                 | “SMP B”                                                                 | “SMP C”                                                                 |
|-----------------------------|-------------------------------------------------------------------------|-------------------------------------------------------------------------|-------------------------------------------------------------------------|
| Questioning                | In generally the cognitive level of the given item problem to student are C1 (knowing), C2 (understanding), and C3 (applying) [18]. Here is an example of teacher given problem. | In generally the cognitive level of the given item problem to student are C1 (knowing), C2 (understanding), and C3 (applying) [18]. Here is an example of teacher given problem. | In generally the cognitive level of the given item problem to student are C1 (knowing), C2 (understanding), and C3 (applying) [18]. Here is an example of teacher given problem. |
| Challenging                | If $OA=20 \text{ cm}$ $OB=12 \text{ cm}$ Find out the length of tangent line AB This item problem requires only applying Pythagorean Theorem into tangent line circle. | If $\angle BAC=30^\circ$ Find $\angle ABC=x^\circ$? This item problem only requires to students’ knowing about the measure of each angle of the isosceles-triangle and the understanding of relationship between central and circumference angle. | If $\angle KML=48^\circ$ Find $\angle KOL?$ This item problem only requires to students’ understanding about the relationship between central and circumference angle. |

If $\angle BAC=30^\circ$ Find $\angle ABC=x^\circ$?

This item problem requires only applying Pythagorean Theorem into tangent line circle.

Challenging On minutes to 50-80, teacher asked student to examine and discuss the tangent line between two circles. The figure of two circles would be lead to

Did not happen. Did not happen.
Teacher learning atmosphere

| Understanding about common tangent line of 2 circles. Teacher constructed this instruction through example and non example of common tangent line. | “SMP A” | “SMP B” | “SMP C” |
|---|---|---|---|
| (I Can: to seek justifying or falsifying arguments, to check, to modify, and to alter) | | | |

Reflecting

| There was the process of drawing the conclusion about common tangent line. But the real reflection was happened at minutes to 16 when teacher asked them to examine an error while solving the problem. | Teacher asked students about their feeling along the learning. And of course, they answered would be the good thing about their feeling. This kind of reflection is not related to mathematical proficiency | Did not happen |
| (I can: To be self-critical, to expect, and to assess different approaches) | | |

Table 5 indicates that questioning at “SMP B” and “SMP C” was constructed in low order thinking cognitive level or at C1, C2, and C3 as Bloom revised cognitive stages. This situation could not push student to face extremely challenging for forcing their thinking skills. The reflecting aspect was not generated in optimally too at “SMP B” and “SMP C”. Teachers at both school recognized reflecting as drawing conclusion at the end of the lesson and asking students feeling about the learning. It showed that questioning, challenging, and reflecting aspects were not fully understood as important part of the learning by teachers at those schools, so that student metacognition, thinking about thinking, could not be developed [19-22]. The inexistence of learning atmosphere will direct to student with no autonomy. It means P4 did not happen. The inexistence of P4 will lead to no appearance of situation to provoke student mathematical thinking skills. So, we can conclude that student at “SMP B” and “SMP C” did not achieve I CAN level. Thus, the mathematical concepts would not be mastered in well [23-25].

From the earlier premises, it was predicted that learning situation at “SMP B” and “SMP C”, which was not fully meet with DST orientated to low international benchmark achievement at TIMSS 2011 in mathematics. The different situation occurred at “SMP A”. At this school, P1, P2, P3, and P4 were almost completely done. It means that DST was conducted with almost properly. Because of DST was implemented, it produced a good learning atmosphere which was endorsing I CAN level. So, it can be predicted that the learning atmosphere at “SMP A” directed to high international benchmark achievement.

4. Conclusion

In this case study there are two kind of examining schools. The first school (“SMP A”) is a kind of school with high international benchmark achievement in TIMSS 2011 in mathematics. Meanwhile, the
second and third school (“SMP B” and “SMP C”) is a kind of school with low international benchmark achievement. The learning profile at school with high international benchmark was looked appropriate with all DST principles. And, in the schools with low international benchmark achievement, the learning profile tent to be far out of DST principles. It can be concluded that the instruction under DST principles is going to be able to provoke high international benchmark achievement.

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