Sparse linear array with low mutual coupling ratio for DOA estimation

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Abstract
In order to reduce the mutual coupling ratio of sparse linear array, a new linear array structure with two sparse uniform linear arrays interleaved nested, is proposed in this paper. The new sparse linear array structure is consisted of two subarrays which have \( N \) and \( M \) physical sensors, respectively. By setting appropriate interelement spacing and interleaving, the degrees-of-freedom, uniform degrees-of-freedom and array aperture of the proposed sparse array can reach \( 2NM + 2N, 2(N + 2M) + 1 \) and \( \max\{ (N-1)R_1, (M-1)R_2 + d_1 \} \), respectively. On one hand, the proposed sparse array has closed-form expressions for the sensor locations. On the other hand, through comparative analysis, although the proposed array structure has lower uniform degrees-of-freedom, it can greatly increase the degrees-of-freedom, extend the array aperture, and reduce the mutual coupling ratio between physical sensors, which means better estimation performance of direction-of-arrival estimation can be achieved. Finally, the direction-of-arrival estimation performance of the proposed array structure is verified by numerical simulations.

1 INTRODUCTION

Spatial spectrum estimation, also known as direction-of-arrival (DOA) estimation, is an important research content in the field of array signal processing [1–6], which has been widely applied in wireless communication, radio astronomy, sonar, speech and other fields [7–10]. Most DOA estimation algorithms are designed for uniform linear arrays (ULAs). This mainly originates from the steering vector of ULAs that has the form of Vandermonde matrix, which is convenient for mathematical processing [11–14]. However, ULAs also have distinct disadvantages, such as smaller array aperture, less degrees-of-freedom (DOFs) and higher mutual coupling ratio compared with sparse linear arrays. As we know, more DOFs indicates that more source signals can be recognized, larger array aperture means higher resolution, lower mutual coupling ratio demonstrates less adverse effect on DOA estimation performance [15–18].

A large number of array structures have been proposed to address these shortcomings of ULAs in recent decades, such as minimum redundancy array (MRA) [19], original coprime array (OCA) [20], augmented coprime array (ACA) [21] and nested array (NA) [22]. Although the contributions of array structures to increase the DOFs extend the array aperture and reduce the mutual coupling ratio between the array physical sensors, there are no closed-form expressions of sensor locations for MRAs, NAs have a dense ULA for the first level array, and coprime arrays have less DOFs and smaller array aperture compared with MRAs and NAs. Therefore, many improved linear array structures have been proposed. The uniform DOFs for ACA have been enhanced by the coprime array with compressed inter-sensor spacing (CACIS) [23], shifted coprime array (SCA) [24], and optimized coprime array (OpCA) [25]. Iizuka-nested array [26], Yang-nested array [27] and Huang-nested array [28] have been devised for further enlarging the DOFs and the array aperture of NAs.

In addition, there are several practical methods in the literature to suppress the interactions between the array elements [29–33]. Generally speaking, the larger the array element spacing, the smaller the mutual coupling. Therefore, there are many literatures studying sparse arrays to reduce the influence of mutual coupling. The problem of mutual coupling between sensors has also been considered in sparse arrays. In [34–36], 1-D DOA estimation and 2-D DOA estimation under the condition of mutual coupling is studied. In [37], the influence of mutual coupling...
coupling on DOA estimation is analyzed, and two algorithms for compensating mutual coupling effect are proposed. In [38], in order to reduce the mutual coupling ratio and increase DOFs, a maximum inter-element spacing constraint (MISC) array is proposed. In [41], in order to reduce the mutual coupling ratio of NA, super nested array (SNA) is proposed.

However, there still exists space to be further improved. Based on this, a new sparse linear array structure with two sparse ULA interleaved nested is proposed, which can reduce the mutual coupling ratio, further increase the DOFs and extend the array aperture. The linear array structure proposed in this letter can achieve better DOA estimation performance due to these excellent characteristics. Specifically, the main contributions of this paper can be summarized as follows:

1. We propose a new sparse linear array configuration, that is, interleaved nested array (INA), which has closed form expression for the sensor locations.

2. INA is compared with NA, SNA, OCA and ACA for different numbers of physical sensors by the uniform DOF, DOF array aperture, and mutual coupling ratio.

3. It is showed that INA achieves better DOA estimation performance compared with other sparse arrays.

The rest of this paper is organized as follows: The signal model and DOA estimation approach is introduced in Section 2. The proposed array structure is presented in Section 3. Simulation results and conclusion are given in Sections 4 and 5, respectively.

**Notations:** Lower-case and upper-case boldface characters, respectively, denote vectors and matrices throughout this paper. The superscripts \((\cdot)^T\), \((\cdot)^H\), and \((\cdot)^*\), respectively, denote the transpose, conjugate transpose, and complex conjugation. The notation \(E[\cdot]\) represents the statistical expectation, \(\text{rec}(\cdot)\) stands for the vectorization operator that sequentially stacks each column of a matrix, and \(\text{diag}(\cdot)\) stands for a diagonal matrix with the corresponding elements on its diagonal. The \(\bigotimes\) and \(\bigodot\) are the Kronecker product and Khatri–Rao product, respectively. \([\cdot]\) and \([\cdot]_\text{F}\), respectively, denote rounding up and rounding down. \(\| \cdot \|_F\) denotes Frobenius norm. \(\triangleright\) represents a generalized inequality between symmetric matrices. Finally, \(I\) denotes an identity matrix with proper dimension.

## 2 THE SIGNAL MODEL AND DOA ESTIMATION

Assuming that there are \(K\) narrow-band stationary uncorrelated far-field Gaussian electromagnetic signals impinging on a \(W\) physical sensor’s sparse linear array, with their spatial locations \(S_d\), where \(S\) is an integer set, \(d = \Lambda/2\), \(\Lambda\) is the wavelength of the incoming source signals. Therefore, the received signals of the array can be modeled as

\[
y(t) = Ax(t) + n(t),
\]

where, \(x(t) = [x_1(t), x_2(t), \ldots, x_K(t)]^T\) denotes the source signal waveform vector, \(n(t)\) denotes the independent and identically distributed Gaussian white complex noise vector, with \(E[n(t)n(t)^H] = \sigma_n^2I, 1 \leq t \leq T\) representing the time variable, where \(T\) represents the total number of snapshots, and \(A = [a(\vartheta_1), a(\vartheta_2), \ldots, a(\vartheta_K)]\) representing the array steering matrix whose \(k\)th column can be described as

\[
a(\vartheta) = [e^{-j/2\pi n_1 d\sin \vartheta/\Lambda}, e^{-j/2\pi n_2 d\sin \vartheta/\Lambda}, \ldots, e^{-j/2\pi n_p d\sin \vartheta/\Lambda}]^T,
\]

where, \(-90^\circ < \vartheta < 90^\circ\), where \(1 \leq k \leq K\). Here, the position of the \(i\)th physical sensor is represented as \(n_i, n_i \in S\).

The theoretical covariance matrix of \(y(t)\) can be calculated as

\[
R_y = E[y(t)y^H(t)] = AR_xA^H + \sigma_n^2I,
\]

and it converges to \(R_y\) as \(T \to \infty\).

In order to break through the DOFs limitation, the virtual array is obtained by vectorizing \(\hat{R}_y\) as

\[
z = \text{vec}(\hat{R}_y) = BP + \sigma_n^{2H}I_p,
\]

where \(P = [\sigma_1^2, \sigma_2^2, \ldots, \sigma_K^2]^T\), and \(I_p = [e_1^T, e_2^T, \ldots, e_p^T]^T\), and \(e_i\) represents a column vector with all zero elements except for the \(i\)th element equals 1. According to (4), \(z\) can be considered as a single snapshot signal where steering matrix can be defined as

\[
B = A^* \bigotimes A = [b(\vartheta_1), b(\vartheta_2), \ldots, b(\vartheta_K)],
\]

where

\[
b(\vartheta) = a^* (\vartheta) \bigotimes a(\vartheta), k = 1, 2, \ldots, K.
\]

The steering matrix \(B\) corresponds to the difference coarray of the array virtual sensors, and the positions of the virtual sensors can be expressed as

\[
D = \{n_p - n_q | n_p, n_q \in S\}.
\]

By removing the duplicate virtual sensors from set \(D\), the positions of the virtual sensors can be expressed as \(D_C \subseteq D\), where \(D_C \subseteq D\). Therefore, the equivalent signals of the virtual array can be expressed as

\[
z_C = \hat{B}P + \sigma_n^{2H}I_C,
\]

where \(\hat{B} \in \mathbb{C}^{|V_C| \times K}\) denotes the steering matrix of the virtual array, \(I_C\) is the corresponding elements in \(I_p\), and \(| \cdot |\) denotes
the set cardinality. However, the virtual array usually includes some missing sensors, namely, holes, resulting in a non-uniform geometry of the virtual array. In order to address the problem of non-uniformity, the general solution of the existing algorithms is to extract the maximum continuous ULA $\mathcal{S}_{UL}$ and ignore the discontinuous virtual sensors $\mathcal{S}_c - \mathcal{S}_{UL}$. However, loss of information is achieved, thus reducing the accuracy of DOA estimation. In order to make the most of the information, the virtual array interpolation is introduced. Therefore, the signals of the holes should be restored as much as possible. To address this problem, an optimization problem to fit the theoretical virtual signal vector is established, and its Toeplitz matrix exhibits the smallest difference with $\mathbf{R}_v$ as

$$
\text{arg min } \text{rank}(T(z))
\text{s.t. } ||M_p(T(z)) - \mathbf{R}_v||_F^2 \leq \varepsilon, \; T(z) > 0',
$$

where $\varepsilon$ represents a predefined fitting error, $T(z) \in \mathbb{Z}^{L_x \times L_z}$ is the Hermite Toeplitz matrix, $z$ denotes the first column of $T(z) \in \mathbb{Z}^{L_x \times L_z}$, $\mathbf{P}$ denotes a binary matrix whose elements corresponding to the virtual sensors in $\mathbf{R}_v$. The elements of $\mathbf{P}$ are 0 corresponding to the virtual sensors which are regarded as broken down; otherwise, the elements will be regarded as 1. $M_p(\cdot)$ is the projection operation of the projection matrix $\mathbf{P}$, and $\mathbf{R}_v$ can be expressed as follows:

$$
\mathbf{R}_v = \begin{bmatrix}
\langle v_1 \rangle_1 & \langle v_1 \rangle_2 & \cdots & \langle v_1 \rangle_{L_z - 1} \\
\langle v_1 \rangle_2 & \langle v_1 \rangle_3 & \cdots & \langle v_1 \rangle_{L_z - 2} \\
\vdots & \vdots & \ddots & \vdots \\
\langle v_1 \rangle_{L_z - 1} & \langle v_1 \rangle_{L_z - 2} & \cdots & \langle v_1 \rangle_{L_z}
\end{bmatrix},
$$

where $\langle v_1 \rangle_q$ represents the $q$th element in $v_1$. Given that the non-convex property of rank function makes Equation (9) difficult to address, it can be relaxed to trace function; therefore, the optimization problem Equation (9) can be transformed as

$$
\text{arg min } ||M_p(T(z)) - \mathbf{R}_v||_F^2 + \eta \text{Tr}(T(z))
\text{s.t. } T(z) > 0,
$$

where, $\eta$ denotes the regularization parameter which can balance the fitting error and $\text{Tr}(T(z))$. Therefore, the optimization problem (12) can be effectively addressed due to the convex characteristics of trace function, and can be solved by SOC programming software packages such as ScDuMi and CVX. Finally, a subspace-based DOA estimation method such as the classical multiple signal classification (MUSIC) can be used to estimate the spatial spectrum as follows:

$$
\frac{1}{\mathbf{a}_l^H(\theta_k) \mathbf{E}_a \mathbf{E}_a^H \mathbf{a}_{l+}(\theta_k)},
$$

where $\mathbf{a}_{l+}(\theta_k)$ represents the virtual steering vector, $\mathbf{E}_a$ is the noise subspace.

However, considering the influence of mutual coupling on the received signal vector, Equation (1) can be modeled as

$$
y(t) = \mathbf{C} \mathbf{x}(t) + \mathbf{n}(t).
$$

The mutual coupling matrix $\mathbf{C}$ can be calculated from electromagnetics, and the closed-form expressions for $\mathbf{C}$ are difficult to calculate. When the sensor array is a linear dipole array, $\mathbf{C}$ can be represented as $[42-45]

$$
\mathbf{C} = (Z_d + Z_l)(\mathbf{Z} + Z_l)I^{-1},
$$

where $Z_d$ and $Z_l$ represent the element impedance and load impedance, respectively, and $\langle \mathbf{Z} \rangle_{n_1,n_2}$ can be represented as

$$
\begin{align}
\langle \mathbf{R} \rangle_{n_1,n_2} = & \sin(\beta l)(-\sin(\theta_0) + \sin(\theta_1)) + 2\sin(\theta_d - 2\sin(\theta_1)) \\
& + \sin(\beta l)(\cos(\theta_0) + \cos(\theta_1) - 2\cos(\theta_d + 2\cos(\theta_1)) \\
& + 2\cos(\theta_d + 2\sin(\theta_1) - 2\sin(\beta d_{n_1,n_2})),
\end{align}
$$

where $d_{n_1,n_2} = |n_1 - n_2|\lambda/2$ represents the distance between sensors, and the parameters $\theta_0, \theta_1, \theta_d$ and $\theta_1$ can be written as

$$
\begin{align}
\theta_0 &= \beta \sqrt{d_{n_1,n_2}^2 + l^2 - l}, \\
\theta_1 &= \beta \sqrt{d_{n_1,n_2}^2 + l^2 + l}, \\
\theta_d &= \beta \sqrt{d_{n_1,n_2}^2 + 0.25l^2 - 0.5l}, \\
v_1 &= \beta \sqrt{d_{n_1,n_2}^2 + 0.25l^2 + 0.5l},
\end{align}
$$

$\sin(\theta)$ and $\cos(\theta)$ represent sine and cosine integrals, and can be written as

$$
\begin{align}
\sin(\theta) &= \int_0^\theta \frac{\sin t}{t} \; dt, \\
\cos(\theta) &= \int_0^\infty \frac{\cos t}{t} \; dt.
\end{align}
$$
The parameters of INA

In this section, in order to reduce the mutual coupling ratio, further increasing the DOFs and enlarging the array aperture of sparse linear array, a new sparse linear array structure termed as “interleaved nested array (INA)” with two sparse ULAs inner nested was proposed.

3.1 Sensor locations of INA

As shown in Figures 1 and 2, the array structure proposed in this letter consists of two sparse subarrays. Subarray one contains \( N \) physical sensors; the spacing between the physical sensors is \( R_1 \); Subarray two contains \( M \) physical sensors; the spacing between the physical sensors is \( R_2 \). Therefore, the total number of physical sensors is \( W = N + M \). For a fixed total number of physical sensors, \( W \), make \( N = \lfloor W/2 \rfloor \), \( M = \lceil W/2 \rceil \), \( R_1 = W d, R_2 = R_1 + d \). Let set \( S_1 d \) and set \( S_2 d \), respectively, represent the sensor locations of Subarray one and Subarray two as follows:

\[
S_1 d = \{0, R_1, \ldots, (N - 1)R_1\}, \quad \text{(27)}
\]

\[
S_2 d = \{0, R_2, \ldots, (M - 1)R_2\} + d. \quad \text{(28)}
\]

Therefore, the locations of the physical sensors can be expressed as the union set \( S = S_1 d \cup S_2 d \).

3.2 The parameters of INA

Assuming that the sensor locations of Subarray 1 and Subarray 2 are represented by two integer sets as \( S_1 = \{p_1, p_2, \ldots, p_N\} \), \( S_2 = \{q_1, q_2, \ldots, q_M\} \), respectively. The self-difference set and the cross-difference set are defined respectively as follows:

\[
D_{self} = \{p_i - p_j | 1 \leq i, j \leq N\} \cup \{q_i - q_j | 1 \leq i, j \leq M\} \quad \text{(29)}
\]

\[
D_{cross} = \{p_i - q_j | 1 \leq i \leq N, 1 \leq j \leq M\} \cup \{q_i - p_j | 1 \leq i \leq M, 1 \leq j \leq N\} \quad \text{(30)}
\]

Thus, we can get the difference set \( D = D_{self} \cup D_{cross} \). Therefore, the DOFs, array aperture and uniform DOFs can be calculated, respectively [41]. The array aperture denotes \( \max\{|Sd| - \min\{|Sd|\}\}; \) the uniform DOFs stands for the number of the central ULA segment of \( D \); the DOFs represents the total number of distinct elements of \( D \).

Therefore, the array aperture of INA can be expressed as

\[
\text{Array aperture} = \max\{(N - 1)R_1, (M - 1)R_2 + d\}. \quad \text{(31)}
\]

Without loss of generality, set the inter-element spacing \( d = 1 \). Assume that the sensor locations of subarray one and subarray two are represented by two real number sets as \( \{p_1, p_2, \ldots, p_N\} \) and \( \{q_1, q_2, \ldots, q_M\} \), respectively. According to (27), \( p_n \) can be expressed as

\[
p_n = (n - 1)W, \quad \text{(32)}
\]

where \( n = 1, 2, \ldots, N \). According to (28), \( q_m \) can be expressed as

\[
q_m = (m - 1)(W + 1) + 1, \quad \text{(33)}
\]

where \( m = 1, 2, \ldots, M \).
The cross-difference absolute value set between the \( n \)th sensor of Subarray one and the \( n \)th sensor of Subarray two can be expressed as

\[
|p_n - q_m| = |(n - 1)W - (m - 1)(W + 1) - 1|, \quad (34)
\]

where \( m, n = 1, 2, ..., M, N \).

When \( m = n = 1, 2, ..., M \), the cross-difference set can be calculated as

\[
|p_n - q_m| = |(n - 1)W - (m - 1)(W + 1) - 1| = |m| = \{1, 2, ..., M\}. \quad (35)
\]

When \( m = n = 1, 2, ..., M \), the cross-difference set can be calculated as

\[
|p_n - q_m| = |mW - (m - 1)(W + 1) - 1| = |W - m| = \{W - M, W - M + 1, ..., W\}. \quad (36)
\]

Since \( M = \left\lfloor \frac{W}{2} \right\rfloor \), \( (W - M) - (W - 2M) \leq 1 \), according to (35) and (36), we can conclude that \([0, W]\) is continuous.

When \( m = n = 1, 2, ..., M \), the cross-difference set can be calculated as

\[
|p_n - q_m| = |(n - 2)W - (m - 1)(W + 1) - 1| = |W + m| = W + \{1, 2, ..., M\}. \quad (37)
\]

According to (35)–(37), we can conclude that \([0, (W + M)]\) is continuous. That is to say, the continuous lags of the virtual array contain \([-W, W + M]\).

Therefore, the uniform DOFs of INA can be expressed as

\[
\text{Uniform DOFs} = 2(W + M) + 1. \quad (38)
\]

Supposing that there are duplicate values in \( p_n - q_m \), which can be expressed mathematically as:

\[
p_n - q_m = p_{n'} - q_{m'}, \quad (39)
\]

where \( n, n' \in \{1, 2, ..., N\}, m, m' \in \{1, 2, ..., M\} \). Substituting (32) and (33) into (39) yields

\[
(n - 1)W - (m - 1)(W + 1) - 1 = (n' - 1)W - (m' - 1)(W + 1) - 1, \quad (40)
\]

which equals to

\[
\frac{n - n'}{m - m'} = \frac{W + 1}{W}. \quad (41)
\]

Since \( W + 1 \) and \( W' \) are prime numbers, (41) does not hold.

### Table 1: DOFs of different array structures

| \( W \) | NA | SNA | OCA (M, N) | ACA (M, N) | INA |
|---|---|---|---|---|---|
| 20 | 219 | 219 | 129 (10, 11) | 159 (5, 11) | 237 |
| 25 | 335 | 335 | 189 (11, 15) | 247 (7, 12) | 359 |
| 30 | 479 | 479 | 269 (15, 16) | 353 (8, 15) | 507 |
| 35 | 645 | 645 | 357 (17, 19) | 459 (11, 14) | 679 |
| 40 | 839 | 839 | 459 (20, 21) | 619 (10, 21) | 877 |
| 45 | 1055 | 1055 | 699 (21, 25) | 963 (11, 24) | 1099 |

### Table 2: Uniform DOFs of different array structures

| \( W \) | NA | SNA | OCA (M, N) | ACA (M, N) | INA |
|---|---|---|---|---|---|
| 20 | 219 | 219 | 41 (10, 11) | 119 (5, 11) | 61 |
| 25 | 335 | 335 | 51 (11, 15) | 181 (7, 12) | 75 |
| 30 | 479 | 479 | 61 (15, 16) | 255 (8, 15) | 91 |
| 35 | 645 | 645 | 71 (17, 19) | 329 (11, 14) | 105 |
| 40 | 839 | 839 | 81 (20, 21) | 439 (10, 21) | 121 |
| 45 | 1055 | 1055 | 91 (21, 25) | 549 (11, 24) | 135 |

Supposing there are duplicate elements in \( p_n - q_m \) and \( q_{m'} - p_{n'} \), it can be expressed mathematically as:

\[
p_n - q_m = q_{m'} - p_{n'}. \quad (42)
\]

Substituting (32) and (33) into (42) yields

\[
\frac{n + n'}{m + m'} = \frac{W + 1}{W}. \quad (43)
\]

(43) holds only in case \( W = 2M + 1 \) or \( W = 2N - 1 \). That is to say, when \( W = 2M + 1 \) or \( W = 2N - 1 \), there are two duplicate elements. However, there are \( 2N + 1 \) elements in the self-difference of Subarray one, different from the elements in the cross-difference of Subarray one and Subarray two. Therefore, there are at least \( 2MN - 1 + 2N + 1 = 2MN + 2N \) different elements.

\[
\text{DOFs} > 2NM + 2N. \quad (44)
\]

### 3.3 Compared with other array structures

In order to quantitatively analyze the performance parameters of INA, we obtain the DOFs, uniform DOFs, array aperture, and mutual coupling ratio of the NA [22], SNA [41], OCA [20], ACA [21], and INA for different total numbers of physical sensors, \( W \).

The DOFs, uniform DOFs, array aperture and mutual coupling ratio of different array structures are summarized in Tables 1–4.
TABLE 3 Array aperture of different array structures

| W  | NA | SNA | OCA (M, N) | ACA (M, N) | INA |
|----|----|-----|-----------|-----------|-----|
| 20 | 109| 109 | 100 (10, 11) | 99 (5, 11) | 190 |
| 25 | 167| 167 | 154 (11, 15) | 156 (7, 12) | 300 |
| 30 | 239| 239 | 225 (15, 16) | 225 (8, 15) | 435 |
| 35 | 322| 322 | 306 (17, 19) | 294 (11, 14) | 595 |
| 40 | 419| 419 | 400 (20, 21) | 399 (10, 21) | 780 |
| 45 | 527| 527 | 504 (21, 25) | 504 (11, 24) | 990 |

TABLE 4 Mutual coupling ratio of different array structures

| W  | NA | SNA | OCA (M, N) | ACA (M, N) | INA |
|----|----|-----|-----------|-----------|-----|
| 20 | 0.3376 | 0.1962 | 0.1732 (10, 11) | 0.1824 (5, 11) | 0.1222 |
| 25 | 0.3465 | 0.1942 | 0.1557 (11, 15) | 0.1601 (7, 12) | 0.1094 |
| 30 | 0.3422 | 0.1918 | 0.1418 (15, 16) | 0.1462 (8, 15) | 0.0999 |
| 35 | 0.3483 | 0.1907 | 0.1313 (17, 19) | 0.1340 (11, 14) | 0.0924 |
| 40 | 0.350 | 0.3350 | 0.1227 (20, 21) | 0.1265 (10, 21) | 0.0913 |
| 45 | 0.3423 | 0.3250 | 0.1174 (21, 25) | 0.1218 (11, 24) | 0.0873 |

It can be observed from Table 1 that: the INA will always produce more DOFs with the same total number of physical sensors compared with the OCA, ACA, SNA and NA, which indicates that more source signals can be recognized.

From Table 2, it is known that SNA and NA produces more continuous lag of virtual array because their virtual arrays have hole-free property. However, the proposed array configuration produces more continuous lags compared with OCA. Although holes in the array may cause ambiguity in DOA estimation, it does not affect when the number of signals is small.

From Table 3, the INA will always obtain larger array aperture with the same total number of physical sensors compared with the OCA, ACA, SNA and NA, which indicates higher resolution.

From Table 4, the INA will always obtain lower mutual coupling ratio with the same total number of physical sensors compared with the OCA, ACA, SNA and NA, which indicates that INA has sparser spaced physical sensors which eventually induce relatively lower mutual coupling effect.

4 SIMULATION RESULTS

In order to prove the superiority of INA through the performance of DOA estimation in the case of mutual coupling and without mutual coupling, OCA, ACA and NA are selected for comparison. Root mean square error (RMSE) is used to quantitatively describe the resolution of DOA estimation as follows:

\[
RMSE = \sqrt{\frac{1}{KC} \sum_{k=1}^{K} \sum_{c=1}^{C} (\hat{\theta}_{k,c} - \theta_{k})^2}. \tag{45}
\]

where \(K\) and \(C\) represent the number of the source signals and the number of the Monte Carlo trials, respectively, and \(\hat{\theta}_{k,c}\) is the estimator of \(\theta_{k}\) for the \(c\)th Monte Carlo trial.

4.1 DOA estimation without mutual coupling

Figures 3 and 4 illustrate the RMSE of DOA estimation versus SNR and snapshots for uncorrelated Gaussian white noise without mutual coupling respectively, where \(W = 20\), \(K = 15\), \(C = 1000\), \(\theta_{k}\) is uniformly distributed in \([-80^\circ, 80^\circ]\). In Figure 3, the SNRs are varied uniformly from \(-15\) to 15 dB, \(T = 500\). In Figure 4, the \(T\) varies from 100 to 1000, \(SNR = 0\) dB, other parameters are the same as before. As shown in Figures 3 and 4, it can be noted that all examined arrays can obtain reliable DOA estimations. What is more, INA can obtain lower RMSE than OCA, ACA and NA due to the more DOFs and the larger array aperture, which means higher resolution and better estimation performance of DOA estimation.
4.2 DOA estimation with mutual coupling

Figures 5 and 6 demonstrate the RMSE of DOA estimation versus respective SNR and snapshots for uncorrelated Gaussian white noise with mutual coupling. In Equation (10), we select $B = 50$, $c_0 = 1$, $c_1 = 0.4e^{\pi/3}$ and $c_\ell = e^{-\ell(\ell-1)\pi/8}$ for $2 \leq \ell \leq B$; other parameters are the same as before. As shown in Figures 5 and 6, DOA estimation performance of all arrays examined is degraded to varying degrees. However, the RMSE DOA estimation performances of INA deteriorates less than OCA, ACA and NA when mutual coupling is considered due to the smallest mutual coupling ratio, which means that the DOA estimation performance of INA is less affected by mutual coupling.

5 CONCLUSION

This paper introduces a new sparse linear array structure called “INA”. A fixed number of sensors, although the INA has lower uniform degrees-of-freedom, have larger array aperture, higher DOFs, and lower mutual coupling ratio compared with NA, SNA, OCA and ACA. Meanwhile, it is shown that the continuous lags in the virtual array of the proposed array configuration contain $[-(W + M), (W + M)]$. The numerical simulation results in the case of mutual coupling and without mutual coupling have confirmed the superiority and effectiveness of the proposed array structure. In future work, 2D and 3D DOA estimation problems will be studied based on the proposed sparse linear array structure.

ACKNOWLEDGEMENTS

This work is supported by the National Natural Science Foundation of China (No. 61903375).

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FIGURE 5 RMSE of DOA estimation versus SNR

FIGURE 6 RMSE of DOA estimation versus snapshots
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