UNCERTAIN KOL SELECTION WITH ADVERTISING VIDEOS CIRCULATION AND KOL SELECTION DIVERSIFICATION IN ADVERTISING PROMOTION

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Abstract. Social media marketing is the mainstream marketing method. Before the launch of a new product, the company will advertise on social platforms. A major problem is how to reasonably arrange KOL (Key Opinion Leader) for advertising promotion in order to achieve the results that the company wants. At this time, KOL selection optimization is an effective method to make the best advertising promotion decision for decision-makers. In addition, the uncertainty in advertising promotion has brought challenges for KOL selection. Therefore, in the absence of historical promotion data, this paper solves the uncertainty of advertising promotion in social media marketing. Taking MCN (Multi-Channel-Network) as the decision maker, maximizing the advertising promotion effect as the goal, considering a variety of realistic constraints and uncertain factors in advertising promotion, the optimal allocation of KOL in advertising promotion is realized by constructing an uncertain chance-constrained programming model. Then, by solving the clear form of the model, the optimal solution of the model is obtained. Finally, the effectiveness of the model is verified by numerical examples, and the effects of KOL selection diversification and advertising video circulation on advertising promotion are discussed, which provides decision support for decision-makers.

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1. Introduction. With the continuous development and improvement of Internet technology and the invasion of COVID-19, the way of product marketing has changed more quickly and widely from the traditional way to the contactless online marketing mode, that is, social media marketing (SMM). There are similarities and differences between SMM and traditional marketing. The similarity is that both of them are oriented to the audience of related products. The difference is that traditional marketing does not have the functions of consumer communication, sharing and evaluation[1].

The goal of SMM is to increase the exposure of the company’s brand and products, broaden the range of customers of the company, and then make more profits by sharing information about a company’s products on social platforms. The existing research of SMM is mainly based on analytic hierarchy process[2], structural equation model [3] -[5], artificial intelligence[6] and other methods, mainly to study its impact on consumer behavior[7] -[10], and carried out from two angles. First, analyze the impact of social media marketing on audience behavior, interests and promotion from the perspective of different industries. Second, analyze the impact of SMM based on different social platforms and predict consumer behavior.

From the perspective of different industries, some scholars have studied the influence of social media marketing on audience behavior, interests and promotion. Liao et al.[11] studied the social media marketing in the film industry, and mentioned that social media has become an important marketing method to attract and retain consumers. Therefore, he proposed an improved model based on refined likelihood model, and explored the influence of different social media channels and events on the box office of movies, in order to understand the factors affecting the box office of movies and promote the promotion of movies. Salem et al.[12] studied the role of social media marketing in value awareness, brand support and brand awareness in fast fashion industry by means of questionnaire survey and statistical analysis, and the results showed that social media had a significant positive impact on the three. Therefore, social media marketing is an effective marketing method at present. Cheung et al.[13] studied the role of social media marketing in consumers’ purchase intention, value creation, continuous purchase and search intention in smart phone industry, collected relevant data from mainland China and Hong Kong, and conducted hypothesis test, which showed the positive influence of SMM on the above four factors. Kafferine et al.[14], taking an island in the Philippines as the research object, used fuzzy cognitive mapping technology and combined with three marketing strategies to study the role of social media marketing policies in all aspects of tourism sustainable development, so as to formulate effective social media marketing policies flexibly. Izakova et al.[15] analyzed the advantages and disadvantages of social media marketing in industrial enterprises, studied the influence of social media marketing on product promotion in industrial enterprises, and studied the influence of social media marketing communication on product sales and search volume in industrial market by using the related methods of statistical analysis and hypothesis testing. Xiong et al.[16] studied the role of key opinion leaders (KOL) in social media marketing in skin care industry, and found that besides KOL, skin care promotion was also influenced by consumers’ skin care awareness. This study shows that KOL is not a single way to promote social media.

From the perspective of different social platforms, some scholars have studied the impact of social media marketing, including Twitter, WeChat, Instagram and
Facebook. The main methods used in this study are questionnaire survey, statistical analysis and hypothesis testing. Jung et al. [17] analyzed the role of social media marketing to startups based on the relevant data of startups on Twitter, and predicted the social media participation of startups by using deep learning method as an index to measure the social media marketing effect of startups. Man et al. [18] analyzed consumer online brand-related activities (COBRAs) on WeChat, studied the effect of using WeChat to drive COBRAs of Chinese luxury cosmetics, and confirmed that SMM is the key factor to drive consumer behavior. Ibrohim et al. [19] used the methods of questionnaire survey and statistical analysis to study the influence of marketing of Instagram on college students’ consumption, and the determination coefficient represents the influence degree of marketing of this social platform on the consumption behavior of respondents. Dolega et al. [20] studied the influence of Facebook marketing on traffic, transaction volume and merchant income based on the data of online retailers for one year. The results show that the spread of social media can increase traffic, but a larger number of social media activities can promote the increase of transaction volume and merchant income. Ibrahim et al. [21] used structural equation modeling method to study the influence of social media marketing activities (SMMA) on undergraduates’ repurchase intention. The results show that consumers’ brand loyalty and trust have a moderating effect on the repurchase intention of social media marketing activities.

To sum up, no matter from the perspective of different industries or different social platforms, it is concluded that social media marketing plays an important role in audience behavior and product promotion. Therefore, it is extremely important to study how to promote through social media and how to achieve optimal promotion.

Through reading and combing the literature, we find that although there are countless ways of promotion, the methods of promotion and optimization are very similar. The main research methods for this problem are to establish a programming model and consider more constraints to improve the programming model. Cohen et al. [22] put forward an improved linear model of promotion optimization based on POP formula in FMCG industry. Cohen et al. [23] expressed the optimization problem of multi-project promotion as a nonlinear integer programming, and took business rules as constraints. Baardman et al. [24] studied the problem of how to promote vehicles to get the maximum profit. The author regarded this problem as a nonlinear dichotomy matching type problem, established a related model, and proposed a polynomial of approximate integer programming. The research results show that the promotion method based on this model can improve the sales profit. Ma et al. [25] studied the problem of maximizing the profits of multiple retailers after promotion, built a nonlinear integer programming model by using the two-stage symbolic constraint regularization method and improving the demand model, and tested several stores with supermarket data. The results showed that the profits were increased by about 17.

In addition, there are various uncertainties in the promotion. Li and Yang made optimization research from the perspective of high uncertainty of advertising environment, and put forward a stochastic programming model for grouping advertising keywords, grouping search advertising words to achieve optimal grouping of keywords, and using real data sets to prove the effectiveness of the model, which provided meaningful insights for advertisers’ decision-making [27]. On the other hand, Shin et al. [28] solved the problem of uncertainty in the use of an application launched
by a retail company. Through this application, the company promotes by buying one get one free, which involves the high uncertainty of revisiting date. From the perspective of optimizing the application, the author generalized the optimization, and proposed a robust multi-period inventory model by treating the problem as a multi-stage random variable. The results show that the model provides a robust and stable solution for the worst case. Existing literature studies consider the promotion optimization problem as a programming problem and the optimal solution of the programming problem as an optimal decision, but few studies comprehensively consider the uncertainties in the promotion process, such as environment, promotion effect and promotion results. The uncertainty theory put forward by Liu in 2007[29] and revised in 2010[30] is the theoretical basis for solving subjective uncertainty. At present, to solve the influence of uncertainty, the uncertainty theory has been widely used in the fields of finance[31]-[37], vehicle routing problem[38], intensive production programming[39], advertising promotion[40], etc. Therefore, this paper will innovatively use the uncertainty theory to solve the uncertainty problem in social media advertising promotion.

In view of the advertising behavior of social media, this paper takes the Multi-Channel Network (MCN) as the decision-maker, takes the advertising promotion of new products as the background, and adopts the advertising video released by the Key Opinion Leaders (KOLs) to achieve the goal that increasing the exposure and popularity of new products. With “Tik Tok” as the promotion platform, this paper takes KOLs’ advertising promotion effect, the promotion cost of decision-makers and the forwarding rate of advertising videos as uncertain variables, and takes advertising promotion effect, promotion cost, promotion risk, the advertising videos circulation and the KOL selection diversification as constraints of promotion decision optimization, aiming at maximizing MCN’s own income, and calculates the optimal KOL selection for advertising promotion. The main contributions of this paper are as follows: First, considering that the products to be advertised are generally new products, we lack historical promotion data. Under this condition, this paper innovatively introduces a new method to solve the uncertainty in social media marketing. Second, this paper considers the optimization of advertising promotion from a new angle. In addition, this paper fully considers the practical problems in social media advertising promotion, and reflects them as constraints in the programming model. The rest of this paper is as follows. In section 2, we review some basic concepts of uncertainty theory. In section 3, an opportunity-constrained programming model of uncertain KOL selection considering promotion effect, promotion cost, promotion risk, advertising video circulation and KOL selection diversification is established. In section 4, the uncertain model is transformed into an equivalent clear model. In section 5, the effectiveness and practicability of the model are illustrated by numerical simulation. Finally, some conclusions and prospects are given.

2. Preliminaries. In this section, we will introduce the uncertainty theory. Then, we mainly review some basic definitions and theorems of the uncertainty theory. In order to rationally deal with the likelihood that something will happen, there exist two axiomatic mathematical systems, one is probability theory and the other is uncertainty theory. Probability theory is a branch of mathematics concerned with the analysis of frequency, while uncertainty theory is a branch of mathematics concerned with the analysis of belief degree. In order to use them to handle a
quantity (e.g., stock price, market demand, and product lifetime) in practice, the first action we take is to produce a distribution function representing the likelihood that the quantity falls into the left side of the current point. If you believe the distribution function is to close enough to the real frequency, then you should use probability theory. Otherwise, you have to use uncertainty theory[29].

**Definition 2.1.** [29] Setting $\Gamma$ is a non-empty collection. A collection of $L$ consisting of a subset of $\Gamma$ can be called $\sigma$ algebra. If it meets:

1. $\Gamma \in L$;
2. If $\Lambda \in L$, so there is $\Lambda^c \in L$;
3. If $\Lambda_1, \Lambda_2, \cdots, \Lambda_n \in L$, so there is $\Lambda_1 \cup \Lambda_2 \cup \cdots \cup \Lambda_n \in L$. The elements in collection $L$ are called events. The uncertain measure is a function mapped from $L$ to $[0, 1]$.

In order to ensure the mathematical nature of the uncertain measurement $M \{ \Lambda \}$, Liu[29] puts forward the following four axioms:

Axiom 1 (Normality Axiom): $M \{ \Gamma \} = 1$ for the universal set $\Gamma$.
Axiom 2 (Duality Axiom): $M \{ \Lambda \} + M \{ \Lambda^c \} = 1$ for any $\Lambda \in L$;
Axiom 3 (Subadditivity Axiom): For any countable sequence of events, there is

$$M \left( \bigcup_{i=1}^{\infty} \Lambda_i \right) \leq \sum_{i=1}^{\infty} M \{ \Lambda_i \}.$$  

Axiom 4 (Product Axiom): Let $(\Gamma_k, L_k, M_k)$ be uncertainty spaces for $k = 1, 2, \cdots$. The product uncertain measure $M$ is an uncertain measure satisfying

$$M \left( \prod_{k=1}^{\infty} \Lambda_k \right) = \bigwedge_{k=1}^{\infty} M_k \{ \Lambda_k \},$$

where $\bigwedge_k$ are arbitrarily chosen events from $L_k$ for $k = 1, 2, \cdots$, respectively.

If $M$ satisfies the above four axioms, the set function $M$ is called the uncertainty measure.

**Definition 2.2.** [29] An uncertain variable is a function $\xi$ from an uncertainty space $(\Gamma, L, M)$ to the set of real numbers such that $\{ \xi \in B \}$ is an event for any Borel set $B$ of real numbers. Note that the event $\{ \xi \in B \}$ is a subset of the universal set $\Gamma$, i.e.,

$$\{ \xi \in B \} = \{ \gamma \in \Gamma | \xi(\gamma) \in B \}.$$  

We know that under the system of probability theory, random variables are characterized by probability distribution function or probability density function. By the same token, in the uncertain theoretical system, uncertain variables are described by uncertain distribution functions.

**Definition 2.3.** [29] The uncertain distribution function of the uncertain variable $\xi$ is

$$\Phi(x) = M \{ \xi \leq x \}.$$  

Among them, $x$ is a real number.
Definition 2.4. [29] If \( \Phi(x) \in (0, 1) \) is continuous and monotonically increasing with respect to \( x \) and \( \lim_{x \to -\infty} \Phi(x) = 0, \lim_{x \to +\infty} \Phi(x) = 1 \), then \( \Phi(x) \) is regular.

Definition 2.5. [29] When \( \Phi(x) \) is a regular distribution function of \( \xi \), \( \Phi^{-1}(\alpha) \) denotes the inverse uncertainty distribution of \( \xi \), where \( \alpha \in [0, 1] \).

Example [29]: An uncertain variable \( \xi \) is called normal if it has a normal uncertainty distribution
\[
\Phi(x) = \left(1 + \exp\left(\frac{\pi(\mu - x)}{\sqrt{3}\sigma}\right)\right), \quad x \in \mathbb{R}
\]
denoted by \( N(e, \sigma) \) where \( e \) and \( \sigma \) are numbers with \( \sigma > 0 \). The inverse distribution of normal uncertain variable \( N(e, \sigma) \) is
\[
\Phi^{-1}(\alpha) = \mu + \frac{\sqrt{3}\sigma}{\pi} \ln \frac{\alpha}{1 - \alpha}, \alpha \in [0, 1].
\]

Definition 2.6. [29] The optimistic value and pessimistic value to an uncertain variable \( \xi \) at a given level \( 0 < \alpha < 1 \) are defined as \( \xi_{\text{sup}}(\alpha) = \sup \{\gamma \mid M \{\xi \geq \gamma\} \geq \alpha\} \) and \( \xi_{\text{inf}}(\alpha) = \inf \{\gamma \mid M \{\xi \leq \gamma\} \geq \alpha\} \). If \( \xi \) has a regular uncertainty distribution \( \Phi \), then we have \( \xi_{\text{sup}}(\alpha) = \Phi^{-1}(1 - \alpha) \) and \( \xi_{\text{inf}}(\alpha) = \Phi^{-1}(\alpha) \).

Theorem 2.7. [29] Let \( \xi_i (i = 1,2, \cdots, n) \) be uncertain variables, and let \( f \) be a real-valued measurable function. Then \( f(\xi_1, \xi_2, \cdots, \xi_n) \) is an uncertain variable.

Theorem 2.8. [29] Assume that \( \xi_1, \xi_2, \cdots, \xi_n \) are independent uncertain variables with regular distribution functions \( \Phi_1, \Phi_2, \cdots, \Phi_n \). If \( f(x_1, x_2, \cdots, x_n) \) strongly increases with respect to \( x_1, x_2, \cdots, x_m \) and strongly decreases with respect to \( x_{m+1}, x_{m+2}, \cdots, x_n \), then we get
\[
\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \cdots, \Phi_{m}^{-1}(\alpha), \Phi_{m+1}^{-1}(1 - \alpha), \cdots, \Phi_{n}^{-1}(1 - \alpha)),
\]
which represents the inverse distribution of \( \xi = f(\xi_1, \xi_2, \cdots, \xi_n) \).

Theorem 2.9. [29] Let \( g(x, \xi_1, \xi_2, \cdots, \xi_n) \) be a constrain function, in which \( \xi_1, \xi_2, \cdots, \xi_n \) are independent, with uncertainty distributions \( \Phi_1, \Phi_2, \cdots, \Phi_n \). If \( g(x, \xi) \) strongly increases with respect to \( \xi_1, \xi_2, \cdots, \xi_n \) and strongly decreases with respect to \( \xi_{s+1}, \xi_{s+2}, \cdots, \xi_n \), then \( M \{g(x, \xi_1, \xi_2, \cdots, \xi_n) \leq 0\} \geq \alpha \) is equivalent to
\[
g(x, \Phi_1^{-1}(\alpha), \cdots, \Phi_{s}^{-1}(\alpha), \Phi_{s+1}^{-1}(1 - \alpha), \cdots, \Phi_{n}^{-1}(1 - \alpha)) \leq 0.
\]

Definition 2.10. [29] Assume that \( \xi \) is an uncertain variable, the expected value is
\[
E[\xi] = \int_{0}^{+\infty} M \{\xi \geq x\} dx - \int_{-\infty}^{0} M \{\xi \leq x\} dx.
\]

Among them, at least one of the above two points exists.
Theorem 2.11. [29] Let $\xi$ be an uncertain variable with uncertainty distribution $\Phi$. Then

$$E[\xi] = \int_{0}^{+\infty} (1 - \Phi(x))dx - \int_{-\infty}^{0} \Phi(x)dx.$$ 

Definition 2.12. [29] Let $\xi$ be an uncertain variable with finite expected value $e$. Then the variance of $\xi$ is

$$V[\xi] = E[(\xi - e)^2].$$

This definition tells us that the variance is just the expected value of $(\xi - e)^2$. Since $(\xi - e)^2$ is a nonnegative uncertain variable, we also have

$$V[\xi] = \int_{0}^{+\infty} M \left\{ (\xi - e)^2 \geq x \right\} dx.$$ 

3. Chance-constrained programming models for advertising promotion.

3.1. Problem description. Based on real interpersonal relationship, as an interactive network platform, the advertising marketing value of social media has been paid more and more attention by commercial investors and enterprises. Social media advertising has two main purposes. One is the products promotion in the early stage of marketing, which aims to enhance the popularity and attention of products and pave the way for later marketing. The other is the products promotion during the marketing period, whose main purpose is to sell the products and then obtain profits.

There are two main ways of advertising new products in the early stage of marketing. One is advertising push for information flow. At this time, the advertisement is directly delivered to potential users, and the content and purpose of the advertisement are clear, which is easy to cause users to resent. If you choose this way to promote advertising, the views number of advertising videos will be considerable, but the marketing goal is too clear, and the complete broadcast volume of videos is very small, so you can’t really complete advertising promotion. The other is to choose KOLs reasonably for advertising promotion. In this case, product advertisements are generally added to the video stories designed and interpreted by KOLs. Through the story and interest of the video, users are attracted to watch the video completely, and then get in touch with the product information efficiently.

In the early stage of product marketing, KOLs’ advertising promotion is the most popular way to promote products based on new media platform. The KOLs’ selection determines the promotion effect of product advertisement. Based on the current advertising promotion mode of social media, advertisers either directly connect KOLs for products promotion, or hand over the promotion tasks to corresponding MCN institutions, and then MCN will carry out KOLs’ selection and further promotion tasks. In this paper, advertisers are supposed to entrust MCN with the task of promoting new products. MCN has its own KOL groups and has a mature KOLs classification standard. Generally, MCN will classify KOLs according to the comprehensive evaluation results of KOL’s fan number, audience type, matching degree between the promotion content and KOLs’ style, and KOL’s promotion ability. For
example: If the product to be publicized and promoted is cosmetics, its audiences are mainly women. Therefore, MCN will comprehensively evaluate KOL’s female fans, the matching degree between products and KOLs’ style, and the promotion effect of KOL’s previous products, and further grade them.

This paper will give a simple example to illustrate. In the promotion of game software, the game company will invest a certain cost to publicize the software. Generally, the game company will hand over the promotion to a special organization (MCN). In this case, the game company will pay a certain cost to the MCN, and require MCN to achieve a certain promotion effect. Then, the MCN assigned the publicity task to KOLs and paid KOLs corresponding remuneration. At this time, the profit of MCN itself is the difference between the remuneration paid by the game company to MCN and the total salary paid by MCN to KOLs. Therefore, the goal of MCN is to optimize KOLs’ selection, and then maximize its own revenue on the basis that the final effect of product promotion meets the requirements of advertisers. In this context, this paper will solve the problem of the best choice of KOLs by MCN in advertising promotion by establishing a mathematical model.

3.2. Parameter representation. Assume that the KOL levels are level 1 to level n. As KOLs’ advertising promotion effect is affected by uncertain factors such as the network environment, the previous promotion data is not enough to explain the future promotion effect. Therefore, this study assumes that KOL’s advertising promotion effect, KOL’s salary level, and the probability of advertising videos being forwarded are all uncertain variables. In order to more clearly describe the advertising promotion problem in an uncertain environment, we use the following symbolic variables to represent:

- $\xi_i$ indicates the advertising promotion effect of KOL at grade $i$, $i = 1, 2, \cdots, n$, $0 < \xi_i < 1$.
- $c_i$ represents the fee paid by the MCN to a single KOL at the level $i$, $i = 1, 2, \cdots, n$, $0 < c_i < \Gamma$.
- $\eta_i$ indicates the forwarding rate of advertising videos published by KOLs, $i = 1, 2, \cdots, n$, $0 < \eta_i < 1$.
- $x_i$ indicates the ratio of quantity for KOLs at the grade $i$ selected by the decision-maker in advertising promotion to the total quantity, $i = 1, 2, \cdots, n$, $0 < x_i < 1$.
- $\omega_{ij} = \{0, 1\}$ indicates whether KOLs’ image is positive. When KOLs’ image is positive, $\omega_{ij} = 1$; When KOL image is negative, $\omega_{ij} = 0$. Among them, $i = 1, 2, \cdots, n$.
- $\upsilon$ indicates the confidence level of KOL selection diversification, $\upsilon > 0$.
- $k$ indicates the confidence level of advertising videos circulation, $k > 0$.
- $X$ indicates the total number of KOLs selected by MCN for advertising promotion.
- $\Gamma$ indicates the funds paid by advertisers to MCN.
- $C$ indicates MCN’s own expected earnings.
- $L$ indicates the maximum fee that MCN can pay KOLs, $0 < L < \Gamma$.
- $H$ indicates the expected effect of advertisers on product promotion.
3.3. **Basic chance-constrained programming model.** Advertising promotion based on social media is usually in the form of short videos. The content of the advertisement is generally embedded in the story interpreted by the short video. If you do not watch the video in its entirety, you will not necessarily pay attention to the advertisement and cannot know the relevant information about the product being promoted, so it cannot be counted as a real advertising promotion. Only when an advertising video is fully broadcast can it be regarded as the completion of an advertising promotion. Therefore, it will be more accurate to use the full number of advertising videos to express the promotion effect of advertising. In addition, social media can check whether the user has played the video completely based on the length of time it takes to browse the video. The completion rate (the ratio of full playback) and the number of views of the advertising video published by KOLs are the data that can be provided by the platform, so you can get the total amount of the video being played in its entirety. Therefore, this paper uses the complete playing times of the advertising video to express the promotion effect of the advertisement. The overall promotion effects of the videos released by KOLs are as follows:

\[
PE_1(x) = X \cdot \sum_{i=1}^{n} \xi_i x_i. \tag{1}
\]

Among them, number of full-play = completion rate * views.

The MCN should meet the promotion goals of the product side. Therefore, the MCN should ensure the promotion effect of the product, and then reduce the risk of breach of appointment, even if the chance that the promotion effect of the advertisement is lower than the threshold is less than a certain probability. Therefore, the average promotion effect of KOLs should meet the following constraints.

\[
M \left\{ \sum_{i=1}^{n} \xi_i x_i \leq H \right\} \leq \alpha. \tag{2}
\]

Among them, \( M \) is the uncertainty measure, \( H \) is the average promotion effect expected by the merchants, and \( \alpha \) is the confidence level.

Decisions are accompanied by risks. It is assumed that the fluctuation of KOLs’ expected promotion effect is regarded as risk. Therefore, the measure of risk can be measured by variance. Therefore, the fluctuation degree of KOLs’ average promotion effect cannot exceed a certain value, which is written as,

\[
R \left\{ \sum_{i=1}^{n} \xi_i x_i \right\} = V \left[ \sum_{i=1}^{n} \xi_i x_i \right] \leq \gamma, \tag{3}
\]

Among them, \( R \) represents the risk measure. \( V \) represents the variance, that is, fluctuation of average promotion effect. And, \( \gamma \) represents the confidence level of decision risk.

As for KOLs’ remuneration, it is also a cost issue for decision makers. At present, there are mainly two ways: one is to charge according to KOL's final promotion effect, and the other is to report KOL’s own fixed price. Obviously, the first charging
method is more reasonable, so this paper adopts the first charging method. Due to various uncertainties in the promotion process, the promotion effect of KOLs is uncertain, so is the cost that decision makers need to pay for KOLs. It is assumed that the cost for decision makers to choose KOLs at the same grade is the same.

It is assumed that the MCN pay remuneration according to the final promotion effect of KOLs. Because advertising promotion is affected by various uncertain factors, the previous promotion data cannot explain the future promotion effect. In addition, the promotion effect of KOLs in the future is uncertain, so is the income of KOLs. On this basis, this paper assumes that the expenses paid by MCN to KOLs at grade \(i\) is uncertain variable \(c_i\). Therefore, the total cost paid by MCN to KOLs, that is, the promotion cost of MCN, is:

\[
PC(x) = X \cdot \sum_{i=1}^{n} c_i x_i. \tag{4}
\]

Among them, \(x_i\) is the decision variable.

As a decision maker, MCN aims to maximize its own interests while meeting the requirements of the product side. In this process, MCN plays an intermediary role between the product side and KOLs. Therefore, the revenue of MCN is the difference between the remuneration paid by the product side to MCN and the total wage paid by MCN to KOLs, which is

\[
\Gamma - PC(x). \tag{5}
\]

From Formula (4), it can be seen that the promotion cost of MCN is uncertain. We cannot guarantee that MCN will reach its expected income level. Therefore, we can only make the revenue of MCN reach the expected measure as large as possible, which is written as follows.

\[
M \left\{ \Gamma - X \cdot \sum_{i=1}^{n} c_i x_i \geq C \right\} \geq \varsigma \tag{6}
\]

Among them, \(M\) is the uncertainty measure. \(C\) is the expected income of MCN. And \(\varsigma\) is the confidence level of decision-making income.

The goal of MCN is to make their own profits more, so they need to make their costs lower. Therefore, the cost constraint is

\[
M \left\{ X \cdot \sum_{i=1}^{n} c_i x_i \leq L \right\} \geq \varsigma. \tag{7}
\]

Among them, \(L\) is the threshold value of promotion cost, there are \(L = \Gamma - C\).

The MCN needs to minimize the pessimistic value of cost constraints in order to maximize their own profits. Therefore, the objective function is

\[
\max_x \min_L L. \tag{8}
\]
Among them, min $L$ is $\varsigma$-pessimistic value of objective function.

Therefore, under the uncertain environment, the KOL selection programming model in advertising promotion is:

$$
\begin{array}{l}
\max_{x} \min_{L} \frac{L}{L} \\
\text{s.t.} \\
M \left\{ X \cdot \sum_{i=1}^{n} c_{i} x_{i} \leq L \right\} \geq \varsigma \\
M \left\{ \sum_{i=1}^{n} \xi_{i} x_{i} \leq H \right\} \leq \alpha \\
V \left\{ \sum_{i=1}^{n} \xi_{i} x_{i} \right\} \leq \gamma \\
0 < x_{i}, \xi_{i} < 1 \\
0 < \sum_{i=1}^{n} c_{i} < \Gamma \\
\sum_{i=1}^{n} x_{i} = 1, i = 1, 2, \cdots, n \\
\end{array}
$$

(9)

3.4. *Chance-constrained programming model considering realistic constraints.* In order to better achieve the expected promotion effect of advertisers, the MCN often consider more realistic constraints when making decisions, such as the mobility of advertising videos and the diversification of spokesperson selection.

The purpose of advertising promotion by the product side is to increase the exposure and popularity of its own products, and then pave the way for the marketing after the products go on the market. Therefore, MCN should ensure the promotion effect of product advertisement when making decisions, and then reduce the risk of breaking the contract. The average promotion effect of advertisements should meet the following constraints, which are recorded as:

$$
M \left\{ \sum_{i=1}^{n} \xi_{i} x_{i} \leq H \right\} \leq \beta.
$$

(10)

Among them, $M$ is the uncertainty measure, $H$ is the average promotion effect expected by the decision maker, $\beta$ is the confidence level.

The forwarding of advertising video can cause fission in the number of video transmission, which in turn causes fission in the promotion effect. Therefore, this paper takes the mobility of advertising video into account. Liquidity represents the fission ability of advertising videos. The mobility of advertising video can be reflected by the forwarding rate. In addition, a user can browse the same video many times, but can only like it once. And, usually, the number of likes is more than the number of forwards. Therefore, this paper defines the forwarding rate as the proportion of the forwarding number of an advertisement video on social media to the number of likes in a certain period of time. The numerical change of forwarding rate is uncertain. The previous data cannot represent the future forwarding situation. Assuming that the forwarding rate is an uncertain variable $\eta_{i}$, the average forwarding rate of advertisements should satisfy the following constraints:
Among them, $\eta_i$ represents the forwarding rate of KOLs at grade $i$. And, $k$ is the lower limit of the average value of the forwarding rate.

The factors of KOL grading are the number of fans, the matching degree between the promotion content and KOLs’ style, and the promotion ability of KOLs. Moreover, the thermal entropy in thermodynamics indicates the degree of molecular state confusion. Shannon’s information entropy solves the problem of information quantification. In addition, Li[36] has successfully used entropy to measure the diversification of securities selection. Therefore, entropy is used to describe the diversity of KOL selection. It is not difficult to understand that the more diversified KOL is, the wider the audience will be, and the better the effect of advertising promotion will be. Therefore, advertisers will ask to introduce the KOL selection diversification constraint. MCN should meet the following constraints when selecting KOLs for advertising promotion.

$$E_n = -\sum_{i=1}^{n} x_i \ln(x_i + \varepsilon) \geq v$$

(12)

Among them, in order to avoid taking 0 for $x_i$, a small enough positive number is introduced. $v$ represents the confidence level of KOL selection diversification.

Atigan[41] pointed out that the positive image of goods and shops has a positive effect on consumers’ buying behavior. Similarly, KOL’s personal image also determines the promotion effect of advertising. If KOL maintains a positive image, it will not have a negative impact on promotion. However, if KOL has some negative events that damage its image, it will have a negative impact on the promotion effect of advertising. Example: Simba’s account was blocked because of the “selling fake bird’s nest” incident. Although the account was unsealed later, the negative event also hindered the realization of Simba’s traffic. Therefore, the KOL selection by decision-makers must have a positive image. Therefore, the number of KOLs with positive images must be greater than the total number of KOLs we want to advertise. Let’s assume that the KOL under MCN is divided into $n$ grades, and there are $m$ KOLs under each grade. Therefore, KOL selection programming should meet the following constraints:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \omega_{ij} \geq X.$$

(13)

Among them, $\omega_{ij} = \{0, 1\}$ is a binary variable, which indicates whether the image of $j$-th KOL at grade $i$ is positive or not. If $\omega_{ij} = 1$, the KOL’s image is positive, it will not have a negative impact on the promotion effect; If $\omega_{ij} = 0$, the image of KOL is negative, the KOL will not be selected. $X$ indicates the total number of KOLs selected by decision makers for advertising promotion.

As a decision-maker, MCN aims to maximize its own profits on the basis of meeting the expectations of advertisers. Therefore, considering realistic constraints such as the promotion effect, the promotion risk, advertising video circulation and
KOL selection diversification, and KOL’s image, the programming model of KOL selection in advertising promotion under uncertain environment is as follows:

$$\begin{align*}
\max_{x} \min_{L} & \quad L \\
\text{s.t.} & \quad M \left\{ X \cdot \sum_{i=1}^{n} c_{i}x_{i} \leq L \right\} \geq \zeta \\
& \quad M \left\{ \sum_{i=1}^{n} \xi_{i}x_{i} \leq H \right\} \leq \beta \\
& \quad V[\sum_{i=1}^{n} \xi_{i}x_{i}] \leq \gamma \\
& \quad E[\sum_{i=1}^{n} \eta_{i}x_{i}] \geq k \\
& \quad - \sum_{i=1}^{n} x_{i} \ln(x_{i} + \varepsilon) \geq v \\
& \quad \sum_{i=1}^{n} \sum_{j=1}^{m} \omega_{ij} \geq X \\
& \quad \omega_{ij} \in \{0, 1\} \\
& \quad \sum_{i=1}^{n} x_{i} = 1 \\
& \quad 0 < x_{i}; \xi_{i}, \eta_{i} < 1, 0 < \sum_{i=1}^{n} c_{i} < \Gamma \\
& \quad i = 1, 2, \ldots, n, j = 1, 2, \ldots, m.
\end{align*}$$

(14)

4. **Model transformation.** In order to facilitate the solution of the model, the models (8) and (14) can be transformed into clear equivalent models through the algorithms of uncertain measures and uncertain variables. The details are shown in Theorem 4.1 and Theorem 4.2.

**Theorem 4.1.** Assume that $\xi_{i}$ and $c_{i}$ are independent uncertain variables with continuous uncertain distribution functions $\Phi(x)$ and $\Psi(x)$ for any $i = 1, 2, \ldots, n$. Then, the uncertain chance-constrained programming model (8) with uncertain variables is equivalent to the clear programming model (15).

$$\begin{align*}
\min & \quad L' \\
\text{s.t.} & \quad L' = X \cdot \sum_{i=1}^{n} \Psi^{-1}(\zeta)x_{i} \\
& \quad \sum_{i=1}^{n} \Phi^{-1}(\xi_{i})x_{i} \geq H \\
& \quad \int_{0}^{1} (\Phi^{-1}(x))^{2} dx - (\int_{0}^{1} \Phi^{-1}(x)dx)^{2} \leq \gamma \\
& \quad \sum_{i=1}^{n} x_{i} = 1 \\
& \quad 0 < x_{i} < 1 \\
& \quad i = 1, 2, \ldots, n
\end{align*}$$

(15)
Proof of Theorem 4.2. First, the determined equivalent form of the objective functions is proved. (1) For the objective function

\[ g_1 = X \cdot \sum_{i=1}^{n} c_i x_i \]

is an uncertain variable.

Since \( X \cdot x_i \geq 0 \), we know that \( g_1 \) strongly increases with respect to \( c_i \). By Theorem 2.8, \( g_1 \) has following inverse uncertainty distribution

\[ \Theta^{-1}(\theta) = L(\Psi^{-1}(\theta)) = X \cdot \sum_{i=1}^{n} \Psi^{-1}(\theta)x_i. \]

And from Definition 2.6, the \( \varsigma - pessimistic \) value of \( g_1 \) is

\[ L' = \Theta^{-1}(\varsigma) = X \cdot \sum_{i=1}^{n} \Psi^{-1}(\varsigma)x_i. \]

(2) For the promotion effect constraint of product advertising

\[ \sum_{i=1}^{n} \xi_i x_i \] strongly increases with respect to \( \xi_1, \xi_2, \cdots, \xi_n \). By Theorem 2.9, we have

\[ M \left\{ \sum_{i=1}^{n} \xi_i x_i \leq H \right\} \leq \alpha \iff \sum_{i=1}^{n} \Phi^{-1}(\alpha)x_i \geq H. \]

(3) For the promotion risk constraint of product advertising

\[ V[\sum_{i=1}^{n} \xi_i x_i] = \int_{0}^{1} (\Phi^{-1}(x))^{2} dx - \int_{0}^{1} \Phi^{-1}(x) dx \]

\[ = \int_{0}^{1} (\Phi^{-1}(x))^{2} dx - \int_{0}^{1} 2\Phi^{-1}(x) dx \int_{0}^{1} \Phi^{-1}(s) ds + \left( \int_{0}^{1} \Phi^{-1}(s) ds \right)^2 \]

\[ = \int_{0}^{1} (\Phi^{-1}(x))^{2} dx - 2 \left( \int_{0}^{1} \Phi^{-1}(x) dx \right)^2 + \left( \int_{0}^{1} \Phi^{-1}(s) ds \right)^2 \]

\[ = \int_{0}^{1} (\Phi^{-1}(x))^{2} dx - \left( \int_{0}^{1} \Phi^{-1}(x) dx \right)^2. \]

As mentioned above, model (8) is transformed into model (15). \qed
Theorem 4.2. Assume that $\xi_i$, $c_i$, and $\eta_i$ are independent uncertain variables with continuous uncertain distribution functions $\Phi(x)$, $\Psi(x)$ and $\Lambda(x)$ for any $i = 1, 2, \cdots, n$. Then, the uncertain chance-constrained programming model (14) with uncertain variables is equivalent to the clear programming model (16).

$$\begin{align*}
\min & \quad L' \\
\text{s.t.} & \quad L' = X \cdot \sum_{i=1}^{n} \Psi(\varsigma)x_i \\
& \quad \sum_{i=1}^{n} \Phi(\beta)x_i \geq H \\
& \quad \int_{0}^{1} (\Phi^{-1}(x))^2 dx - (\int_{0}^{1} \Phi^{-1}(x)dx)^2 \leq \gamma \\
& \quad \int_{0}^{1} \Lambda^{-1}(x)dx \geq k \\
& \quad - \sum_{i=1}^{n} x_i \ln(x_i + \varepsilon) \geq v \\
& \quad \sum_{i=1}^{n} \sum_{j=1}^{m} \omega_{ij} \geq X \\
& \quad \omega_{ij} \in \{0, 1\} \\
& \quad \sum_{i=1}^{n} x_i = 1 \\
& \quad 0 < x_i < 1 \\
& \quad i = 1, 2, \cdots, n, j = 1, 2, \cdots, m.
\end{align*}$$

(16)

Proof of Theorem 4.2. First, the determined equivalent form of the objective functions is proved. (1) For the objective function

The process of proof is the same as Theorem 4.1. (2) For the promotion effect constraint of product advertising

$$\sum_{i=1}^{n} \xi_i x_i (1 + o_i) f(\delta)(1 + r)$$

strongly increases with respect to $\xi_1, \xi_2, \cdots, \xi_n$. By Theorem 2.9, we have

$$M \left\{ \sum_{i=1}^{n} \xi_i x_i (1 + o_i) f(\delta)(1 + r) \leq H \right\} \leq \beta \Rightarrow \sum_{i=1}^{n} \Phi(\beta) x_i (1 + o_i) f(\delta)(1 + r) \geq H.$$  

(3) For the promotion risk constraint of product advertising

The process of proof is the same as Theorem 4.1. (4) For the liquidity constraint of product advertising By Theorem 2.11,

$$E(\eta) = \int_{0}^{+\infty} (1 - \Lambda(x))dx - \int_{-\infty}^{0} \Lambda(x)dx.$$  

Because $\eta$ is an uncertain regular variable, we have

$$E(\eta) = \int_{0}^{1} \Lambda^{-1}(x)dx.$$
So, \( E[\sum_{i=1}^{n} \eta_i x_i] \geq k \Rightarrow \int_0^1 \Lambda^{-1}(x)dx \geq k. \)

As mentioned above, model (14) is transformed into model (16).

Let the uncertain variables \( \xi_i, c_i \) and \( \eta_i \) satisfy respectively \( \xi_i \sim N(\mu, \sigma) \), \( c_i \sim N(e, \delta) \) and \( \eta_i \sim L(a, b) \). Without considering the constraints of reality, we can build the model (17).

\[
\begin{cases}
\min L \\
\text{s.t.} \\
M \left\{ X \cdot \sum_{i=1}^{8} c_i x_i \leq L \right\} \geq \varsigma \\
M \left\{ \sum_{i=1}^{8} \xi_i x_i \leq H \right\} \leq \alpha \\
\left( \sum_{i=1}^{8} \sigma_i x_i \right)^2 \leq \gamma \\
0 \leq x_i, \xi_i < 1 \\
0 < \sum_{i=1}^{8} c_i < \Gamma \\
\sum_{i=1}^{8} x_i = 1
\end{cases} \tag{17}
\]

By Theorem 4.1, model (17) is converted to an equivalent crisp model (18).

\[
\begin{cases}
\min L' \\
\text{s.t.} \\
L' = X \cdot \sum_{i=1}^{8} \Psi^{-1}(\varsigma) x_i \\
\sum_{i=1}^{8} \Phi^{-1}(\alpha) x_i \geq H \\
\left( \sum_{i=1}^{8} \sigma_i x_i \right)^2 \leq \gamma \\
\sum_{i=1}^{8} x_i = 1 \\
0 \leq x_i < 1 \\
i = 1, 2, \cdots, 8
\end{cases} \tag{18}
\]

Then, considering the constraints of reality, we can build the model (19).
By Theorem 4.2, model (19) is converted to an equivalent crisp model (20).

\[
\begin{align*}
\text{min } & \quad L' \\
\text{s.t. } & \quad L' = X \cdot \sum_{i=1}^{8} \Psi^{-1}(\zeta)x_i \\
& \quad \sum_{i=1}^{n} \Phi^{-1}(\beta)x_i \geq H \\
& \quad \left(\sum_{i=1}^{8} \sigma_i x_i \right)^2 \leq \gamma \\
& \quad \sum_{i=1}^{8} x_i \frac{a_i + b_i}{2} \geq k \\
& \quad -\sum_{i=1}^{8} x_i \ln(x_i + \varepsilon) \geq v \\
& \quad \sum_{i=1}^{8} \sum_{j=1}^{5} \omega_{ij} \geq X \\
& \quad \omega_{ij} \in \{0, 1\} \\
& \quad \sum_{i=1}^{8} x_i = 1 \\
& \quad 0 \leq x_i, \xi_i < 1, 0 < \sum_{i=1}^{8} c_i < \Gamma \\
& \quad i = 1, 2, \ldots, 8, j = 1, 2, \ldots, 5. \\
\end{align*}
\]

5. Results and analysis. In order to make the MCN organization better apply the uncertain selection programming model of KOLs in advertising promotion, it is illustrated by a numerical example. The advertising promotion effect of KOLs is
affected by many uncertain factors, which makes the future promotion effect different from the historical situation. Therefore, it is not complete to predict the future promotion effect based on the past advertising promotion data. For simplicity, the MCN organization use “Tik Tok” as a publicity platform, and according to the number of fans for KOLs, the type of audience, the matching degree between the required promotion content and KOL’s own style, as well as the comprehensive evaluation results of the promotion ability of KOLs, KOLs are divided into 8 levels, and each level contains 5 KOLs. For example: if the product to be promoted is a game app, its audience is mainly the young men. Therefore, the decision makers should make a comprehensive evaluation and further grading according to the number of young male fans for KOLs, the matching degree between the product and the style of KOLs, and the promotion effect of such products in the past.

In this paper, the expert experience method of uncertain programming theory is adopted to estimate the promotion effect, promotion cost and video transmission rate, and the least square method[30] is used to predict the uncertain distribution of the extension effect. The values of related uncertain parameters are shown in Table 1.

| Level | \( \mu_i \) | \( \sigma_i \) | \( e_i \) | \( \delta_i \) | \( a_i \) | \( b_i \) |
|-------|---------|---------|---------|---------|---------|---------|
| 1     | 26000   | 50      | 3000    | 33      | 0.011   | 0.019   |
| 2     | 51000   | 100     | 4500    | 87      | 0.009   | 0.024   |
| 3     | 100000  | 150     | 9900    | 57      | 0.007   | 0.016   |
| 4     | 340000  | 300     | 19000   | 75      | 0.013   | 0.032   |
| 5     | 1100000 | 350     | 120000  | 89      | 0.008   | 0.015   |
| 6     | 8700000 | 500     | 780000  | 96      | 0.008   | 0.025   |
| 7     | 15000000| 800     | 1400000 | 125     | 0.006   | 0.031   |
| 8     | 37000000| 1000    | 2900000 | 200     | 0.005   | 0.026   |

The strong fans effect can double the advertising promotion effect of the KOLs. Therefore, suppose the decision maker decides to select 40 KOLs for advertising promotion, the overall expected promotion effect of KOLs is 1200 million with the confidence level 0.36. The maximum tolerance of average risk is 3000000. In addition, the confidence level of cost is 0.8. Then, according to the data in table 1 and the model (18), we can run the Matlab R2020a and use the fmincon algorithm to get the results shown in Table 2. At this time, the minimum cost in KOL selection programming without considering realistic constraints is 9.4003e+07. At this time, the optimal KOL selection is \((0,0,0,8,0,0,0,32)\).

When the parameter value is set to Table 1, the iterative convergence graph of the objective function value is Fig 1. As shown in Fig 1, the convergence speed of the fmincon algorithm is very fast. The algorithm stops running when the number of iterations is 15, which shows that the fmincon algorithm is an effective algorithm for solving uncertain programming models. Then, this paper will add the two realistic constraints that KOL selection diversification and advertising video circulation, and explore the impact of these two realistic constraints on advertising promotion.
In the actual advertising promotion, advertisers may require MCN to be as diversified as possible when making KOL choices. Advertisers believe that choosing different types and levels of KOL for advertising promotion will make the promotion scope wider, and then promote the promotion effect. Therefore, as shown in formulas (21) and (22), we consider KOL selection diversification as the constraint of advertising promotion programming model, and take promotion effect and promotion risk as objective functions respectively, and further discuss the influence of KOL selection diversification constraints on the value of objective function.

\[
\begin{align*}
\min & \sum_{i=1}^{8} \Phi^{-1}(\beta)x_i \\
\text{s.t.} & \\
& -\sum_{i=1}^{8} x_i \ln(x_i + \varepsilon) \geq v \\
& \sum_{i=1}^{8} x_i = 1, 0 < x_i < 1
\end{align*}
\] (21)

| Level i | Allocation ratio |
|---------|------------------|
| 1       | 0.0000           |
| 2       | 0.0000           |
| 3       | 0.0000           |
| 4       | 0.1909           |
| 5       | 0.0000           |
| 6       | 0.0000           |
| 7       | 0.0000           |
| 8       | 0.8091           |
\[
\begin{align*}
\min & \left( \sum_{i=1}^{8} \sigma_i x_i \right)^2 \\
\text{s.t.} & -\sum_{i=1}^{8} x_i \ln(x_i + \varepsilon) \geq v \\
& \sum_{i=1}^{8} x_i = 1, 0 < x_i < 1 
\end{align*}
\] (22)

Table 3. The Influence of KOL Selection Diversification on Advertising Promotion.

| $v$ | 0.51 | 0.61 | 0.71 | 0.81 | 0.91 |
|-----|------|------|------|------|------|
| $OFV_e$ | 3.0946e+04 | 3.2739e+04 | 3.4947e+04 | 3.7714e+04 | 4.1310e+04 |
| $OFV_r$ | 3.5142e+03 | 3.8804e+03 | 4.33e+03 | 4.8971e+03 | 5.6126e+3 |
| Relative Risk | 0.4178 | 0.4360 | 0.4562 | 0.4777 | 0.4998 |

Table 3 shows the change of objective function value of promotion effect ($OFV_e$) and promotion risk ($OFV_r$) with KOL selection diversification. As shown in Table 3, with the enhancement of KOL’s diversification, the better the advertising promotion effect. At the same time, however, the promotion risk also increases with the strengthening of KOL’s diversified choices. Moreover, through calculation, it is found that the relative risk ($\text{Relative Risk} = \frac{OFV_r}{OFV_e}$) also increases with KOL’s diversification.

Figure 2. The graph of OFV changing with $v$.

5.2. Influence of advertising video circulation on advertising promotion. In advertising promotion, the act of forwarding advertising video means that the forwarder approves the video information on the one hand, and promotes the video transmission on the other hand. Then, in advertising promotion, if we consider the advertising video circulation as a constraint condition, will it promote the effect of advertising promotion? In addition, what is the impact of the advertising videos circulation on promotion risk?

Formulae (23) and (24) are advertising promotion programming models that consider the advertising videos circulation constraint and take advertising promotion effect and promotion cost as objective functions respectively.
Figure 3. The graph of Relative Risk changing with $v$.

\begin{align}
\begin{aligned}
\min & \sum_{i=1}^{8} \Phi^{-1}(\beta) x_i \\
\text{s.t.} & \sum_{i=1}^{8} x_i \frac{a_i + b_i}{2} \geq k \\
& \sum_{i=1}^{8} x_i = 1, 0 < x_i < 1
\end{aligned}
\end{align}

\begin{align}
\begin{aligned}
\min & \left( \sum_{i=1}^{8} \sigma_i x_i \right)^2 \\
\text{s.t.} & \sum_{i=1}^{8} x_i \frac{a_i + b_i}{2} \geq k \\
& \sum_{i=1}^{8} x_i = 1, 0 < x_i < 1
\end{aligned}
\end{align}

Table 4. The Influence of Video Circulation on Advertising Promotion.

| $k$  | 0.13 | 0.15 | 0.17 | 0.19 | 0.21 |
|------|------|------|------|------|------|
| $OFV_e$ | 2.59618e+04 | 2.59620e+04 | 5.09250e+04 | 5.09265e+04 | 5.09270e+04 |
| $OFV_r$ | 2.5001e+03 | 2.5001e+03 | 1.3611e+04 | 3.3611e+04 | 6.2500e+04 |
| Relative Risk | 0.354265 | 0.354262 | 0.267275 | 0.659990 | 1.227247 |

It can be seen from Table 4 that with the enhancement of advertising video circulation, the effect and risk of advertising promotion increase. However, when the video circulation degree is less than a certain degree ($k \leq 0.17$), the relative risk is reduced. In addition, the relative risk is increasing, and the growth rate is very large. From the above results, we can find that although the advertising videos circulation can improve the promotion effect, compared with the enhancement of promotion effect, the promotion risk is too high. Therefore, due to the excessive increase of promotion risk, blindly considering advertising video circulation constraints can have the opposite effect.
5.3. Influence of KOL selection diversification and advertising video circulation on advertising promotion. In addition, when considering the realistic factors of KOL selection in advertising promotion, the final objective function value is 9.4106e+07. Among them, under the condition that the parameters are consistent with those in Table1, the values of other parameters are shown in Table 5. Therefore, Table 6 shows the decision values of model (20). At this time, the optimal KOL selection is (2,2,2,2,1,0,0,31).

| $k$  | $\varepsilon$ | $\nu$ |
|------|---------------|-------|
| 0.015 | 0.0001        | 0.81  |

As shown in Fig. 6, the algorithm ends when the number of iterations is 18, so the fmincon is an effective algorithm of model (20). Comparing the OFV of Fig. 1 and Fig. 2, it is not difficult to find that the promotion cost considering KOL selection diversification constraint and advertising video circulation constraint is higher than that of the basic model. Furthermore, from 5.1 and 5.2, it can be seen that the addition of these two constraints will enhance the advertising promotion risk. However, it can also improve the promotion effect to a certain extent. Therefore,
decision-makers could choose different promotion schemes according to the different needs and psychological tendencies of advertisers.

6. Conclusions. Uncertainty is a major challenge in allocating KOLs’ grades and quantities in the process of advertising promotion. Based on the uncertainty theory, the basic KOL selection programming model and the KOL selection programming model with realistic constraints are established in this study, which are used to optimize KOL selection in advertising promotion. In the model, uncertain variables are introduced to deal with the uncertainty of factors in promotion, and uncertain measures are used to evaluate the possibility of uncertain events. At the same time, MCN is regarded as the decision maker, and the maximization of self-profit is considered as the objective function, and then the objective function is transformed into the minimization of promotion cost. On the premise of meeting the opportunity constraints, the optimal KOL selection and allocating ratio in advertising promotion is solved, and a clear equivalent programming model is established. Finally, a numerical example is used to verify the effectiveness of the model. The results of this study provide a strong theoretical support for decision makers in advertising promotion.

KOL selection programming model can be used as an effective tool to solve the problem of optimal allocation of KOL in advertising promotion. If the decision maker is MCN, it is the decision goal to maximize its own profit under the constraint of meeting the requirements of advertisers; if the decision maker is advertiser, it is the decision goal to obtain the maximum promotion effect on the basis of minimum

| Level | Allocation ratio |
|-------|------------------|
| 1     | 0.0395           |
| 2     | 0.0407           |
| 3     | 0.0387           |
| 4     | 0.0488           |
| 5     | 0.0172           |
| 6     | 0.0067           |
| 7     | 0.0005           |
| 8     | 0.8079           |
cost. In the actual advertising promotion based on social media, a lot of advertising promotion is based on multi-platform and multi-period promotion. Therefore, in the follow-up research, we can further diversify the choice of social media in advertising promotion and multi-cycle promotion.

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