Directed Acyclic Transformer for Non-Autoregressive Machine Translation

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Abstract
Non-autoregressive Transformers (NATs) significantly reduce the decoding latency by generating all tokens in parallel. However, such independent predictions prevent NATs from capturing the dependencies between the tokens for generating multiple possible translations. In this paper, we propose Directed Acyclic Transformer (DA-Transformer), which represents the hidden states in a Directed Acyclic Graph (DAG), where each path of the DAG corresponds to a specific translation. The whole DAG simultaneously captures multiple translations and facilitates fast predictions in a non-autoregressive fashion. Experiments on the raw training data of WMT benchmark show that DA-Transformer substantially outperforms previous NATs by about 3 BLEU on average, which is the first NAT model that achieves competitive results with autoregressive Transformers without relying on knowledge distillation.

1. Introduction
Transformer has been the most popular architecture for sequence-to-sequence learning, especially for machine translation (Vaswani et al., 2017). Vanilla Transformer adopts the autoregressive approach for generation, which obtains strong results but is inefficient in inference due to its sequential decoding. To tackle the problem, Non-autoregressive Transformers (NATs, Gu et al., 2018; Gu et al., 2019; Ma et al., 2019; Ding et al., 2021a; Gu & Kong, 2021) have been proposed, which significantly reduce the inference latency by predicting all tokens in parallel and achieve reasonably high performances in translation. Notably, an NAT-based system obtain the highest BLEU score in German to English translation of WMT21 (Qian et al., 2021b; Akhbardeh et al., 2021), even better than a line of Autoregressive Transformer (AT) systems.

However, current NATs severely suffer from the multi-modality problem (Gu et al., 2018) in both training and inference. Intuitively, in training, as shown in Fig.1(a), NAT models are trained to predict each token independently, where one position may have several possible tokens as labels from several different translation references. In such a case, an NAT model may learn to generate an implausible output mixing multiple translations. Additionally, in inference, the NAT still cannot sample fluent translations even if it captures multi-modal information in training. Since the NAT model generates all tokens simultaneously, no effective sampling approach can be used on top of it. In contrast, ATs do not have the same problem because of their left-to-right generation, where the multi-modality problem for a later position is not so severe since its prefix has been given.

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Currently, the main solution to address the multi-modality problem is to reduce the data modalities by knowledge distillation (KD, Kim & Rush, 2016; Gu et al., 2018), namely, replacing the original training targets with predicted sentences from an AT teacher. KD is simple yet effective, which always leads to significant BLEU improvements, e.g., about 8 BLEU points on WMT14 En-De for vanilla NATs.

However, we argue that current state-of-art NAT models heavily rely on KD, which has two crucial disadvantages. a) Training NAT models by distilling from AT makes the training process redundant. We need to train an AT model first and then regenerate the whole training data. Such complex pre-processing prevents NATs from being practically used. b) Generally, the student model in KD cannot outperform its teacher model with a large margin. In such a case, KD restricts NAT’s performance by imposing an upper bound (not strict), which seriously hurts the potential of further developing NAT models.

In this paper, we propose Directed Acyclic Transformer (DA-Transformer) for Non-Autoregressive Machine Translation, which directly captures many translation modalities via a Transformer to capture multiple translation modalities simultaneously. Unlike previous studies that construct the word lattices with multiple translations but also enables the generation of diverse translations by sampling different paths. Notice that DA-Transformer predicts all translation words in parallel, and the whole model is trained in an end-to-end fashion, which enjoys all merits of NAT models. We propose an objective that does not require multiple references in training, making it applicable to most translation benchmarks. In inference, we propose several sampling methods to decode a translation from DA-Transformer, which provides flexible quality-latency tradeoff in generation.

Experimental results show that DA-Transformer significantly reduces the gap between NATs and ATs while preserving the inference latency (∼7x speedup over ATs). Especially on WMT17 Zh-En, our best model outperforms autoregressive Transformer by 0.6 BLEU without the help of knowledge distillation. To our best knowledge, it is the first time that a non-iterative NAT model achieves competitive results with AT models without KD. DA-Transformer outperforms existing NATs (including iterative approaches) with a large margin on the raw data of standard En↔DE and En↔Zh benchmarks, which sufficiently shows the effectiveness of our proposed model.

2. Related Work

Non-autoregressive Machine Translation Gu et al. (2018) propose NAT models to reduce the latency in generation or decoding, but there exists a gap in translation quality between NAT and AT models. To bridge the gap, iterative NATs manage to repeatedly refine the generated outputs (Lee et al., 2018; Ghazvininejad et al., 2019; Guo et al., 2020). However, as shown in Kasai et al. (2021), most iterative NATs are not advantageous against ATs in the quality-latency tradeoff. Non-iterative NATs are much faster, whose improvements mainly come from alignment-based objectives (Libovický & Helcl, 2018; Ghazvininejad et al., 2020a; Du et al., 2021), or incorporating extra decoder inputs (Shu et al., 2020; Qian et al., 2021a; Bao et al., 2021). Nevertheless, these NATs heavily rely on knowledge distillation (KD, Gu et al., 2018), which is found very effective in reducing the data modalities (Zhou et al., 2020). A recent study (Huang et al., 2022b) provides a unified perspective showing that most existing methods actually modify targets or inputs to reduce the token dependencies in the data distribution, which eases the NAT training but introduces data distortion.

Unlike existing NATs, our method retains multiple translations instead of dropping the multi-modal information in NAT training. It turns out that our method can effectively tackle the multi-modality problem without modifying the training data and not rely on KD to achieve a good translation performance.

Lattice-based Model in Machine Translation Word lattices have a long history in Statistic Machine Translation. A word lattice is a directed acyclic graph (DAG) with edges labeled with a token and weight, which can represent an exponential number of sentences in the a compact structure. A phrase-based translation system can generate a word lattice during decoding (Ueffing et al., 2002; Och & Ney, 2004). Some models take word lattices as inputs to alleviate input errors brought by word segmentation or speech recognition (Dyer et al., 2008; Koehn et al., 2007; Dong et al., 2014). There are also studies that combine multiple system outputs into a single lattice (Rosti et al., 2007; Feng et al., 2009) and decode a good translation from it (Tromble et al., 2008).

Unlike previous studies that construct the word lattices with a search algorithm, our model predicts the whole DAG with multiple translations simultaneously. Moreover, DA-Transformer’s training does not require ground-truth word
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3. Our Proposed Method

In this section, we describe our proposed DA-Transformer in detail. Intuitively, to facilitate the explicitly modeling of multiple modalities, we propose to replace the original Non-autoregressive Transformer decoder with a directed acyclic decoder, whose topological structure is a DAG. Each path of the DAG forms a sequence of hidden states that stores a possible translation, and the whole DAG store multiple translations in different paths. DA-Transformer still generates in a non-autoregressive fashion.

We will first introduce the network structures in Section 3.1, which presents how to construct the DA-Transformer to parameterize the conditional probability. Then in Section 3.2, we will elaborate on the training of DA-Transformer, including how to train it with one reference and the efficient implementation of traversing possible paths. Finally, in Section 3.3, we provide several decoding approaches, aiming to sample fluent sentences efficiently given the well-trained DA-Transformer.

3.1. Architecture of DA-Transformer

DA-Transformer consists of a Transformer encoder and a directed acyclic decoder. The encoder is the same as vanilla Transformer while the decoder organizes its hidden states as a DAG. As shown in Fig. 2, hidden states correspond to vertices of the DAG, which model word distributions in specific positions; and edges of the DAG are transitions between hidden states, which organize generated words into a final sentence.

Intuitively, given a source sentence $X$, the directed acyclic decoder generates a sentence in three steps: (1) receiving the position embeddings as inputs and producing hidden states as vertices; (2) calculating the transition probabilities between the vertices based on the vertex states; (3) sampling a path from the DAG following the transitions, and then predicting target tokens using the vertex states on the path.

Formally, the probability of a target sentence $Y = \{y_1, \ldots, y_M\}$ is formulated as

$$P_{\theta}(Y|X) = \sum_{A \in \Gamma} P_{\theta}(Y, A|X)$$

$$= \sum_{A \in \Gamma} P_{\theta}(A|X)P_{\theta}(Y|A, X),$$

where $A = \{a_1, a_2, \ldots, a_M\}$ is a path represented by a sequence of vertex indexes, and $\Gamma$ contains all paths with the same length of the target sentence $Y$.

Vertex The directed acyclic decoder utilizes the Transformer layers (Vaswani et al., 2017) to predict the vertex states. Unlike the autoregressive decoder that generates tokens from left to right, it generates the vertex states in parallel.

Specifically, we use graph positional embeddings $G = \{g_1, \ldots, g_L\}$ as the decoder inputs, which is identical to the learnable positional embeddings in vanilla Transformer but represents the vertex indexes instead of the token positions. Note that $L$ is the graph size, where we set $L$ to $\lambda$ times the source length $N$ and tune $\lambda$ as a hyper-parameter. The decoder then produces the vertex states $V = [v_1, \ldots, v_L]^T$, which is defined as

$$[v_1, \ldots, v_L] = \text{Transformer-Blocks}(g_1, \ldots, g_L).$$

Transition Each edge of the DAG is assigned the transition probability between the connecting vertices. The transition probabilities are locally normalized, i.e., the probabilities of outgoing edges sum to one. Formally, the probability of path $A$ is defined as

$$P_{\theta}(A|X) = \prod_{i=1}^{M-1} P_{\theta}(a_{i+1}|a_i, X) = \prod_{i=1}^{M-1} E_{a_i, a_{i+1}}.$$
where $E \in \mathbb{R}^{L \times L}$ is the transition matrix normalized by rows. Specifically, the transition matrix is obtained by

$$E = \text{softmax}(\frac{QK^T}{\sqrt{d}}),$$

where $d$ is the hidden size, $W_Q$ and $W_K$ are learnable parameters. To ensure that there is no cycle in the DAG, we apply lower triangular masking on $E$, which only allows transitions from vertices with small indexes to large indexes. Note that the matrix $E$ can be calculated in parallel, thereby facilitating fast sampling of paths.

**Token Prediction** Conditioned on the vertex states in $V$ and the selected path $A$, the decoder predicts the target tokens in parallel. Formally, we have

$$P_\theta(Y|A, X) = \prod_{i=1}^{M} P_\theta(y_i|a_i, X) = \prod_{i=1}^{M} \text{softmax}(W_P v_{a_i}),$$

where $W_P$ are learnable weights, and $v_{a_i}$ is the representation of the $i$-th vertex on the path $A$.

In the implementation, we actually calculate the distributions on all vertices and then skip the vertices not appearing on the chosen path. Specifically, we obtain

$$P = \text{softmax}(VW_P^T),$$

where $P \in \mathbb{R}^{L \times |V|}$ is the matrix containing the token distributions on the $L$ vertices, and $P_\theta(y_i|a_i, X) = P_{a_i, y_i}$. The matrix $P$ facilitates fast calculation for multiple paths since the shared vertices are not calculated twice, which is significant in training and inference introduced later.

### 3.2. Training

To capture multiple translation modalities in training, the proposed directed acyclic decoder arranges words from different modalities in different vertex states of the decoder, which can effectively reduce the inconsistent problem in training. In this section, we will elaborate on training details of DA-Transformer, including training with one reference, efficient implementation of marginalizing all paths in the DAG, and modified glancing training techniques according to the graph structures.

**Training DA-Transformer with One Reference** Although DA-Transformer retains multiple translations in the DAG, its training objective only requires one reference per sample, which facilitates efficient training on most translation benchmarks. Specifically, it directly maximizes the log-likelihood $\log P(Y|X)$ by marginalizing all possible paths $A$, which can be formulated as follows,

$$\mathcal{L} = -\log P_\theta(Y|X) = -\log \sum_{A \in \Gamma} P_\theta(Y, A|X),$$

where $\Gamma$ contains all paths with $1 = a_1 < \cdots < a_M = L$.

To understand why a single reference is adequate for the DAG learning, we analyze the training process by inspecting the gradients. Intuitively, we find that the objective assigns a single reference to several paths, where the vertices on the chosen paths are updated to generate the reference tokens, and the other vertices remain unchanged. The sparse assignment is the key to the successful training, which avoids inconsistent labels in token predictions and preserves the unseen translations stored on the unchanged paths. In such a way, the DAG can be learned across different training instances, each of which only provides a single reference, not requiring an instance with multiple references.

Specifically, we inspect the gradient of $\mathcal{L}$ and find that

$$\frac{\partial}{\partial \theta} \mathcal{L} = \sum_{A \in \Gamma} w_A \left( \frac{\partial}{\partial \theta} \mathcal{L}_A \right),$$

where

$$\mathcal{L}_A = -\log P_\theta(Y, A|X),$$

$$w_A = \frac{P_\theta(Y, A|X)}{\sum_{A' \in \Gamma} P_\theta(Y, A'|X)}.$$  \(7\)

$\mathcal{L}_A$ maximizes the likelihood of sampling $Y$ with the path $A$, and $w_A$ is the weight of $\mathcal{L}_A$. Eq(5) indicates that the weights of paths are assigned according to the probability that $Y$ appears on $A$. If a path $A$ is more probable for the target $Y$, then a larger weight will be used in optimizing $\mathcal{L}_A$, which further strengthens its dominance. In contrast, an unlikely path $A$ will get a negligible weight, indicating that the vertices on $A$ are not affected in the update.

A real example is shown in Fig.3. In the early stage, training with one sample will affect all vertices in the DAG. In the late stage, only some vertices are updated, reserving the other vertices for storing unseen translations.

\[\text{For example, the orange line (the second token, need) on the vertex } v_{20} \text{ is the sum of } w_A \text{ for the paths } A = \{a_1, a_2, \cdots, a_5\} \text{ satisfying } a_2 = 20. \text{ } w_A \text{ is defined in Eq(7).}\]
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**Algorithm 1 Greedy / Lookahead Decoding in Pytorch-like Parallel Pseudocode**

**Input:** Graph Size $L$, Transition Matrix $E \in \mathbb{R}^{L \times L}$, Token Distributions $P \in \mathbb{R}^{L \times \mathcal{V}}$

if Using Lookahead then

$E := E \odot [P_{\text{MAX}}(\text{dim}=1), \text{UNSQUEEZE}(\text{dim}=0)]$

# $E$ now jointly considers $P$ and $E$

# $\odot$ is element-wise multiplication

end if

$\text{tokens} := P_{\text{ARGMAX}}(\text{dim}=1)$  # shape: (L)

$\text{edges} := E_{\text{ARGMAX}}(\text{dim}=1)$  # shape: (L)

$i := 1$, output := [tokens[1]]

repeat

$i := \text{edges}[i]$  # jumping along the transition

output.append(tokens[i])

until $i = L$


decides the number of unmasked tokens according to the prediction accuracy.\(^4\) (3) We add $Z$ to the decoder input and train the model by minimizing Eq(8).

3.3. Inference

In inference, DA-Transformer constructs a DAG that stores multiple translations, where we aim to find the most probable one. Compared with existing NATs, DA-Transformer utilizes transitions to distinguish different candidates, which improves fluency and avoids errors like repeated tokens. We propose three decoding strategies to find high-quality translations while keeping low latency.

**Greedy** The simplest strategy is to take the most likely choices for the transitions and tokens. Specifically, we perform parallel argmax operations to obtain the most likely transition and token for each vertex. Then, we generate the translation by collecting the predicted tokens along the chosen path. The greedy decoding is highly efficient that only uses two parallel operations, as shown in Algo.1.

**Lookahead** Lookahead decoding improves the greedy strategy by jointly considering the transitions and the tokens. Specifically, we rearrange $P_\theta(Y, A|X)$ into

$$P_\theta(y_1|a_1, X) \prod_{i=2}^{M} P_\theta(a_i|a_{i-1}, X) P_\theta(y_i|a_i, X),$$

which becomes a sequential decision problem of choosing $a_i$ and $y_i$ in order. We simultaneously obtain

$$y^*_i, a^*_i = \arg \max P_\theta(y_i|a_i, X) P_\theta(a_i|a_{i-1}, X),$$

which can be still implemented in parallel with almost zero overhead, as presented in Algo.1.

**BeamSearch** BeamSearch is a more accurate method for solving the above decoding problem. Following Gu & Kong (2021), we combine an n-gram language model to

\(^4\)The number of unmasked token $t = \tau \sum_{i=1}^{M} [y_i \neq \hat{y}_i]$, where $\hat{y}_i = \arg \max P_\theta(\cdot|a_i, X)$, and $\tau \in [0, 1]$ is a hyper-parameter.

Marginalizing $A$ with Dynamic Programming The objective $\mathcal{L}$ requires marginalizing all paths $A$, which is expensive due to the numerous paths. Similar to Graves et al. (2006), we employ dynamic programming to tackle the issue.

Generally, we recurrently calculate the probability sum of path prefixes that end at the vertex $u$ and generate the target prefix $Y_{\leq i}$, denoted as $f_{i,u}$. Since the path prefixes that end at the vertex $u$ should pass through a vertex $v$ satisfying $v < u$, so $f_{i,u}$ can be obtained from $f_{i-1,v}$. By recurrently calculating $f_{i,u}$, we finally obtain the probability sum of all valid paths and the training objective $\mathcal{L} = -\log f_{M,L}$. Our algorithm reduces the time complexity to $O(M L^2)$ and can be implemented by $O(M)$ PyTorch operations. The detailed formulation is presented in Appendix A.

**Glancing Training on Graph** Previous work shows that glancing training (Qian et al., 2021a) can significantly improve the translation quality of non-iterative NATs. Here we present a modified glancing training technique on DAG, which still requires only one reference per sample.

Specifically, to improve the training of DA-Transformer via glancing training, we add a masked target to the decoder input and train the model by reconstruction, which promotes the learning of dependency between vertices. Formally, the objective of glancing is defined as

$$\mathcal{L}_{\text{Glancing}} = -\log P_\theta(Y|X, Z),$$

where $Z = [z_1, \ldots, z_L]$ is a randomly masked target provided as an extra decoder input, and $P_\theta(Y|X, Z)$ is similarly defined as Eq(4).

The glancing training follows three steps, as shown in Fig.4. (1) We assign the target tokens to appropriate vertices since the decoder input is longer than the target sentence. The assignment follows the most probable path $A = \arg \max_{A \in \mathcal{G}} P_\theta(Y, A|X)$, which requires a forward pass of the decoder and dynamic programming. (2) We obtain $Z$ by masking some tokens. We utilize the masking strategy proposed by GLAT (Qian et al., 2021a), which
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Table 1. Results on WMT14 En↔De and WMT17 Zh↔En. We present DA-Transformer’ results with mean and standard deviation of three runs with different random seeds. Best performance of non-iterative NATs (iter=1) are **bolded. Average Gap** is the gap of BLEU against the best AT model, excluding the missing values. * indicates results of our re-implementation. Our autoregressive transformer is better than previously reported results because we use the same training setting as NATs (300k steps, 64k tokens/batch; previous results use 100k steps, 32k tokens/batch). † uses reranking methods in NAT decoding (LPD, Wei et al., 2019).

| Model                  | Iter # | WMT14 En-De Raw KD | WMT14 En-De Raw KD | WMT17 En-Zh Raw KD | WMT17 Zh-En Raw KD | Average Gap | Speedup |
|------------------------|--------|--------------------|--------------------|--------------------|--------------------|-------------|---------|
| Transformer            | M      | 27.6               | 27.8               | 31.4               | 31.3               | 34.3        | 34.4    | 23.7        | 24.0        | 0.45          | 0.49        | 1.0x       |
| Transformer (Ours)     | M      | 28.075*            | 28.54*             | 31.94*             | 31.54*             | 34.89*      | 34.69*  | 23.89*      | 24.68*       | 0             | 0           | 0.0x       |
| CMLM (Ghazvininejad et al., 2019) | 10     | 24.61              | 27.03              | 29.40              | 30.53              | 33.19       | -       | 23.21       | 30.00         | 0             | 1.37        | 2.2x       |
| SMART (Ghazvininejad et al., 2020b) | 10     | 25.10              | 27.65              | 29.58              | 31.27              | 34.06       | -       | 23.78       | 2.67          | 0.67         | 2.2x       |
| DhCo (Kasai et al., 2020) | ≈4     | 25.64              | 27.34              | -                  | 31.31              | -           | 34.63   | -           | 23.83         | 2.43          | 0.59        | 3.5x       |
| Imputer (Saharia et al., 2020) | 8      | 25.0               | 28.2               | -                  | 31.8               | -           | -       | -           | 3.07          | 0.04         | 2.7x       |
| CMLMC (Huang et al., 2022a) | 10     | 26.40              | 28.37              | 30.92              | 31.41              | -           | -       | -           | 1.35          | 0.15         | 1.7x       |
| Vanilla NAT (Gu et al., 2018) | 1      | 11.79*             | 19.99*             | 16.27*             | 25.77*             | 18.92*      | 25.84*  | -           | 15.78         | 8.26         | 15.3x       |
| CTC (Libovický & Helcl, 2018) | 1      | 18.42*             | 25.52              | 23.65*             | 28.73              | 26.84*      | 31.39*  | 12.23*      | 19.93*        | 9.41          | 3.47        | 14.6x      |
| AXE1† (Ghazvininejad et al., 2020a) | 1      | 20.40              | 24.63              | 24.90              | 27.90              | -           | 30.88   | -           | 19.79         | 7.36          | 4.34        | 14.2x      |
| GLAT (Qian et al., 2021a) | 1      | 19.42*             | 25.21              | 26.51*             | 29.84              | 29.79*      | 32.22*  | 18.88*      | 21.84*        | 6.05          | 2.59        | 15.3x      |
| OuXe1† (Du et al., 2021) | 1      | 22.4               | 26.11              | 26.8               | 30.2               | -           | 32.9    | -           | 22.1          | 5.4           | 2.0         | 14.2x      |
| CTC + GLAT (Qian et al., 2021a) | 1      | 25.02*             | 26.39              | 29.14*             | 29.54              | 30.65*      | 32.51*  | 19.92*      | 23.11*        | 3.52          | 1.98        | 14.6x      |
| CTC + DSLP (Huang et al., 2022a) | 10     | 24.81              | 27.02              | 28.33              | 31.61              | -           | -       | -           | 3.44          | 0.73         | 14.0x       |
| DA-Transformer + Greedy (Ours) | 1      | 26.08±2        | 27.31±2           | 30.48±4          | 31.30±2            | 33.17±5      | 33.80±10 | 22.66±2      | 24.04±0.05    | 1.58          | 0.75        | 14.0x      |
| + Lookahead            | 1      | 26.57±2        | 27.49±2           | 30.68±2          | 31.37±2            | 33.83±15     | 34.08±4 | 22.82±2      | 24.23±1.15    | 1.22          | 0.57        | 13.9x      |
| + BeamSearch           | 1      | 27.02±2        | 27.78±2           | 31.24±10         | 31.80±8            | 34.21±12     | 34.35±12 | 24.22±2      | 24.90±0.16    | 0.53          | 0.16        | 7.1x       |
| + BeamSearch + 5-gram LM | 1      | 27.25±1.12    | 27.91±1.17        | 31.54±0.20       | 31.95±0.6          | 34.23±12     | 34.27±0.7 | 24.49±0.6    | 25.01±0.14    | 0.32          | 0.08        | 7.0x       |

It should be noticed that BeamSearch requires sequential operations and does not preserve the non-autoregressive nature. However, such sequential operations do not involve deep network computations and can still be very efficient, with about 7 times speedups compared with AT models.⁵

4. Experiments

**Dataset** We conduct experiments on two benchmarks, WMT14 En↔De (4.5M) and WMT17 Zh↔En (20M), where we follow Zhou et al. (2020); Kasai et al. (2020) for pre-processing. For knowledge distillation, we follow Du et al. (2021) to use Transformer-big as our teacher model and generate the distilled data with a beam size of 5.

**Metrics** For fair comparisons with previous work, we use tokenized BLEU (Papineni et al., 2002) for all benchmarks except WMT17 En-Zh, where we use sacreBLEU (Post, 2018). The latency speedup is evaluated on WMT17 En-De test set with a batch size of 1.

**Hyper-parameters** Our models generally use the hyper-parameters of transformer-base (Vaswani et al., 2017). For regularization, we set dropout to 0.1, weight decay to 0.01, and label smoothing to 0.1. All models, including ATs, are trained for 300k updates with a batch of 64k tokens. The learning rate warms up to 5 · 10⁻⁴ within 10k steps and then decays with the inverse square-root schedule. We evaluate the BLEU scores on the validation set every epoch and average the best 5 checkpoints for the final model. For DA-Transformer, we use λ = 8 and Lookahead Decoding unless otherwise specified. We linearly anneal τ from 0.5 to 0.1 for glancing training. For BeamSearch, we set beam size to 200, γ to 0.1, and tune α from [1, 1.4] on the validation set. The training lasts approximately 32 hours on 16 Nvidia V100-32G GPUs.

⁵We will release an efficient C++ implementation at https://github.com/thu-coai/DA-Transformer.
Figure 5. Quality-latency tradeoff on WMT14 En-De and WMT17 Zh-En with varying graph sizes and beam sizes. The graph size is λ times the source length. We use Beamsearch + 5-gram LM with the beam size (bs) of 200, 100, 50. bs = 1 indicates Lookahead Decoding.

Figure 6. Effects of λ on WMT14 En-De. The graph size (DA-Transformer) or the output length (CTC) is λ times the source length.

Figure 7. Ablation study of training objectives on WMT14 En-De. Max/Sum represent using the max operation or the sum operation in marginalizing all paths in Eq.(4). We compare three masking strategies in obtaining the decoder input: All Masked, Uniform (Ghazvininejad et al., 2019), and Adaptive (Qian et al., 2021a).

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2) Lower inference latency compared with ATs and iterative NATs. DA-Transformer achieves 7x~14x speedups over ATs, where the remaining BLEU gaps are about 0.32 on average. Especially on WMT17 Zh-En, our best model with BeamSearch outperforms ATs by 0.6 BLEU. Moreover, DA-Transformer dominates all iterative NATs on both BLEU and latency for all benchmarks except WMT14 En-DE with KD, which shows the great potential of our model.

3) Flexible quality-latency tradeoff. Comparing the decoding strategies of our method, we find that Lookahead Decoding consistently outperforms Greedy Decoding, and the n-gram LM usually benefits BeamSearch, with almost zero overheads. To better show the quality-latency tradeoff, we tune the graph size and beam size with our decoding strategies. As shown in Fig.5, our method significantly outperforms existing NATs and provides flexible quality-latency tradeoff for non-autoregressive translation.

4.2. Ablation Study

In this section, we investigate the effects of the graph size and training methods on the raw data of WMT14 En-De.

Graph Size DA-Transformer utilizes a DAG with L vertices, which is empirically set to λ times the source length. A large DAG can model more translations. However, it also makes the transition predictions difficult. We manually tune λ from 2 to 16, as shown in Fig.6.

The results show that larger graphs improve the translation quality until λ exceeds 12, where λ is not sensitive around its best value. We compare our methods against CTC, which also utilizes a similar hyper-parameter to determine the output length (Libovický & Helcl, 2018). Although CTC+GLAT has a similar performance with DA-

Transformer when λ = 2, the BLEU score does not increase for a larger λ. We attribute the problem to the inconsistent label problem: a longer output sequence does not help CTC to reduce the inconsistent labels in training, where DA-Transformer benefits from larger graphs by assigning different tokens to distinct vertices. Considering the performance and computation cost, we choose λ = 8 and apply it to all other datasets.

Training Objectives DA-Transformer is trained with a glancing objective that only requires one reference for each sample, where we investigate two important designs: First, we marginalize all possible paths to obtain \( L \), which is equivalent to optimizing the paths with different weights as discussed in Sec.3.2. We compare it with the objective only optimizing the most probable path, i.e., replacing the sum operation by the max operation in Eq.(4). Second, we use glancing training with a masked target as inputs, where the masked tokens are adaptively chosen according to the prediction accuracy (Qian et al., 2021a). We compare it with two other strategies: masking all inputs (i.e., do not use glancing training), uniform random masking (Ghazvininejad et al., 2019).

The results are shown in Fig.7. First, marginalizing all paths (Sum) outperforms choosing the most probable path (Max). One possible reason is that the max operation makes sharp weight assignments in the early training, leading to a premature convergence in which only several paths are
Table 2. Token accuracy under the best assignment. An assignment matches each reference token with a predicted token, which is called an alignment in CTC or a path in DA-Transformer.

| Model       | WMT14 En-De |          | WMT17 Zh-En |          |
|-------------|-------------|----------|-------------|----------|
|             | Train       | Valid    | Train       | Valid    |
| Vanilla NAT | 29.7        | 29.6     | 39.8        | 22.4     |
| CTC + GLAT  | 50.2        | 51.7     | 47.1        | 32.3     |
| DA-Transformer | 69.3        | 69.9     | 80.1        | 67.0     |

used. Second, the glancing training (Uniform or Adaptive) is better than the vanilla training (All Masked), which improves the translation quality by promoting representation learning. Moreover, the adaptive strategy can further boost performance by choosing the masking ratio dynamically.

4.3. Analysis

This section verifies that DA-Transformer benefits from assigning tokens to vertices in training and explicitly considers the transitions in inference. It also shows some cases of learned DAGs. We present more analyses in the appendix, including the translation performance on different lengths (Appendix C.1), performance with controlled training time (Appendix C.2), and some statistics of DAGs (Appendix E).

DA-Transformer improves token accuracy. In training, we assign tokens of different translations to different vertices, which avoids the inconsistent labels in training and thus improves the token accuracy in inference. We compare our model against two baselines, Vanilla NAT and CTC+GLAT. Note that CTC utilizes an alignment-based objective, which also assigns the reference tokens to different positions of Transformer. We calculate the accuracy under the best assignment following two steps: We first obtain the most probable assignment that matches each reference token to a prediction. Then, we calculate the accuracy by comparing the predicted tokens on the best assignment (i.e., the best path in DA-Transformer) against the reference. In CTC, a reference token may be matched with several predictions, so we average the accuracies for the reference token. The special empty tokens are not counted in the accuracy.

DA-Transformer facilitates diverse generation. In inference, DA-Transformer utilizes the transition matrix to avoid incorrect outputs caused by mixing multiple translations. We evaluate the ability to distinguish different translations by sampling diverse translations from the DAG. Specifically, we begin at the start vertex and repeatedly use Nucleus Sampling (top-p sampling, Holtzman et al., 2020) to choose the next vertex and token according to Eq. (9). We use $p = 0.8$ and vary the temperature from 0.4 to 1.0.

We follow Shen et al. (2019) to evaluate the quality and diversity by multi-reference BLEU and pairwise BLEU. We compare our model against AT and GLAT+CTC, the best non-iterative NAT baseline. We obtain the hypotheses from GLAT+CTC by replacing the argmax operations in decoding with Nucleus Sampling with the same $p$ and temperature.

The results are shown in Fig. 8. Compared with GLAT+CTC, DA-Transformer achieves a better tradeoff between quality and diversity. With the same temperature, the generated samples (without KD) by our model are far more diverse than GLAT+CTC. It shows that our model can learn multiple diverse translations and further decode them in inference. Compared with Transformer, DA-Transformer (without KD) is slightly less diverse but achieves a close tradeoff on WMT17 Zh-En, which shows the great potential of our model. Moreover, we find that applying KD to DA-Transformer improves the quality but sacrifices the diversity because KD reduces the data modalities.

Case Study We choose a test sample of WMT17 Zh-En and illustrate the DAG predicted by our model. For a clear presentation, we use $\lambda = 4$ for a small graph and further remove some useless vertices and edges. Specifically, we remove all vertices with passing probabilities smaller than 0.1, where the passing probabilities represent how likely the vertex will appear on a randomly sampled path. We only show the transitions in the top 90% of probabilities.

As shown in Fig. 9, the predicted DAG is highly reasonable. Following the transitions, we can clearly distinguish translation expressions, which avoids the errors like repeated tokens shown in the vanilla NAT’s output. We present the
Figure 9. A test sample of WMT17 Zh-En with the DAG and BeamSearch results. We present the top candidates on each vertex and the transition probabilities between vertices. We also present the passing probabilities representing how likely a vertex will appear on a sampled path. We remove vertices/edges with small passing/transitoin probabilities for a clear presentation. The vanilla NAT mixes the tokens from different translations, which can be avoided in DA-Transformer inference. We use the BPE tokenzer (Sennrich et al., 2016), where a subword prefix is marked by `-`. See more examples in Appendix D.

top-5 hypotheses produced by BeamSearch, which are fluent and diverse.

However, we can still find errors in the predicted DAG, e.g., a possible incorrect translation “Does that sounds ...”.

Although the error does not easily occur in Lookahead or BeamSearch decoding, it shows that there is still space for improving the consistency between the tokens in our model.

5. Conclusion

In this paper, we propose DA-Transformer for non-autoregressive machine translation. Unlike previous NAT models relying on knowledge distillation, DA-Transformer tackles the multi-modality problem by capturing multiple translations with a directed acyclic decoder. Experimental results show that DA-Transformer outperforms all NAT baselines on raw training data and achieves competitive results with AT models. The best model of DA-Transformer even outperforms the autoregressive Transformer by 0.6 BLEU on Zh-En, which demonstrates the potential of the proposed approach.

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A. Dynamic Programming for Training

Figure 10. An overview of dynamic programming for DA-Transformer training. Each valid path in the DAG corresponds to a path in the grid that starts from (1, 1) to (M, L). M is the target length, L is the graph size. \( P(Y, A|X) \) can be calculated by multiplying the token probabilities and the transition probabilities. In Dynamic Programming, we recurrently calculate \( f_{i,u} \), which is the probability sum of all paths that start from (1, 1) and end at (i, u). E.g., \( f_{3,4} \) is the sum of two paths’ probabilities, \( A_1 \) and \( A_2 \). Our training objective \( \mathcal{L} \) is equal to \(-\log f_{M,L}\).

The training objective of DA-Transformer is formulated in Eq(4), which requires marginalizing all possible paths \( A \). To avoid the expensive cost of enumerating the paths, we employ dynamic programming that reduces the time complexity to \( \mathcal{O}(ML^2) \), where \( M \) is the target length, \( L \) is the graph size.

To utilize dynamic programming, we first represent the valid paths in a \( M \times L \) grid, where each valid path of the DAG corresponds to a path in the grid that starts from the left-upper corner (1, 1) and ends in the right-bottom corner (\( M, L \)), as shown in Fig.10. Formally, the path \( A = \{a_1, \ldots, a_M\} \) satisfying \( 1 = a_1 < \cdots < a_M = L \) corresponds to a path in the grid that passes through \((1, a_1), (2, a_2), \ldots, (M, a_M)\).

Then we find that the probability on the path \( A \), i.e., \( P_\theta(Y, A|X) \), can be decomposed and calculated by multiplying the probabilities of the token predictions and transitions. Specifically, we have

\[
P_\theta(Y, A|X) = \prod_{j=1}^M P_\theta(y_j|a_j, X) \prod_{j=2}^M P_\theta(a_j|a_{j-1}, X)
\]

\[
= \prod_{j=1}^M P_{a_j,y_j} \prod_{j=2}^M E_{a_{j-1},a_j}.
\]

where \( P_{a_j,y_j} \) can be regarded as the token probability on the point \((j, a_j)\), and the \( E_{a_{j-1},a_j} \) can be regarded as the transition probability on the edge connecting \((j-1, a_{j-1})\) with \((j, a_j)\).

Recall that our objective requires the sum of the probabilities of all valid paths. We can recurrently calculate \( f_{i,u} \), which is defined as the probability sum of the paths that start from (1, 1) but end at (i, u). Since each valid path that ends at (i, u) must pass through a point \((i-1, v)\) where \( v \in [1, u) \), we reach a recursion formula that obtains \( f_{i,u} \) from \( f_{i-1,v} \):

\[
f_{i,u} = \mathbf{P}_{u,y_i} \sum_{v=1}^{u-1} f_{i-1,v} \mathbf{E}_{v,u} \quad (2 \leq i \leq M, 1 \leq u \leq L),
\]

where the boundary conditions are:

\[
f_{1,1} = \mathbf{P}_{1,y_1}; \quad f_{1,u} = 0 \quad (2 \leq u \leq L).
\]

**Algorithm 2** Dynamic Programming Algorithm in Pytorch-like Parallel Pseudocode

**Input:** Target Length \( M \), Graph Size \( L \), Target Sentence \( Y \), Transition Matrix \( \mathbf{E} \in \mathbb{R}^{L \times L} \), Token Distributions \( \mathbf{P} \in \mathbb{R}^{L \times |V|} \)

1. Initialize a zero matrix \( f \in \mathbb{R}^{M \times L} \)
2. \( f[1,1] := 1 \)
3. for \( i = 2, 3, \ldots, M \) do
   4. \( f[i,: := \mathbf{P}[i,: \odot (f[i-1,: \odot \mathbf{E}) \quad \# \odot \) is the element-wise multiplication, \( \odot \) is the vector-matrix multiplication] end for
5. Update the model by minimizing \( \mathcal{L} = -\log f[M,L] \).
Finally, the loss can be obtained by $\mathcal{L} = - \log f_{M,L}$.

Since the product-sum operations can be calculated by matrix multiplications, the above recurrent process can be implemented with $\mathcal{O}(M)$ parallel operations, as shown in Algorithm 2.

### B. Implementation of Beam Search

![Beam Search Diagram](image)

**Step 0:** Insert $b_{init} = [s>]$ into $g_1$. Set $s_1(b_{init}) = 1$

**Step 1:**

| Rank | Beams in $g_1$ | Beam Score |
|------|----------------|------------|
| 1    | $b_{init} = [s>]$ | $\log s_1(b_{init}) / \text{len}(b_{init}) = 0$ |

Candidate 1: $v = 2, t = \text{Yes}$

$B_1 = [s>, \text{Yes}]$, add $B_1$ to $g_2$

$s_2(B_1) = s_1(b_{init}) \times E_{1,2} \times P_{2,\text{Yes}} = 0.4$

Candidate 2: $v = 3, t = \text{Yes}$

$s_3(B_1) = s_1(b_{init}) \times E_{1,1} \times P_{3,\text{Yes}} = 0.4$

*One beam may appear at different vertices.*

**Step 2:**

| Rank | Beams in $g_2$ | Beam Score |
|------|----------------|------------|
| 1    | $B_1 = [s>, \text{Yes}]$ | $\log s_2(B_1) / \text{len}(B_1) = -0.11$ |

Candidate 1: $v = 3, t = \text{Yes}$

$B_2 = [s>, \text{Yes}, \text{Yes}]$, add $B_2$ to $g_3$

$s_2(B_2) = s_1(B_1) \times E_{2,3} \times P_{3,\text{Yes}} = 0.032$

**Step 3:**

| Rank | Beams in $g_3$ | Beam Score |
|------|----------------|------------|
| 1    | $B_1 = [s>, \text{Yes}]$ | $\log s_2(B_1) / \text{len}(B_1) = -0.11$ |
| 2    | $B_2 = [s>, \text{Yes}, \text{Yes}]$ | $\log s_2(B_2) / \text{len}(B_2) = -1.15$ |

Candidate 2: $v = 4, t = <$

$s_4(B_3) = s_3(B_2) \times E_{3,4} \times P_{4,\text{Yes}} = 0.36$

**Step 4:**

| Rank | Beams in $g_4$ | Beam Score |
|------|----------------|------------|
| 1    | $B_1 = [s>, \text{Yes}, <$ | $\log s_4(B_3) / \text{len}(B_3) = -0.09$ |

The final result is $B_3 = [s>, \text{Yes}, <$

*Updated by merging probabilities of multiple paths.*

Figure 11. An step-by-step example of the beam search algorithm presented in Algorithm 3. One beam represents a translation prefix, which may appear on multiple paths. For demonstration, we only preserve the top-1 beam in each step, limit the candidate number when expanding the beams, and set $\alpha = 1$ for the length penalty and $\gamma = 0$ to disable the n-gram language model.

Our concept of beam is similar to the prefix beam search (Hannun et al., 2014), where a beam represents a translation prefix but may appear on multiple paths. E.g., in Fig.11, $[s>, \text{Yes}]$ is a beam that appears on two paths, $\{v_1, v_2\}$ and $\{v_1, v_3\}$. Our beam search aims to calculate the probability sum of all paths that produce the same translation, which approximates $P(Y | X)$ and works better than finding a single path that maximizes $P(Y | A, X)$.

To achieve an effective calculation of the scores, we maintain the probability sum for a beam $B$ during the beam search. Specifically, we define $s_i(B)$ as the probability sum of the paths ends at vertex $i$. When sorting the beams, we use the beam score defined in Eq(11), where $P(Y | X)$ is equal to the probability sum of all paths, i.e., $\sum_{i=1}^{L} s_i(B)$.

Our algorithm is presented in Algorithm 3 with an example shown in Fig.11. We further apply some tricks to reduce the computation costs:

- Unlike vanilla beam search that all beams have the same length in each step, our algorithm may compare beams with different lengths. To avoid a length bias in the selected beams, we only preserve the top-10 for each length. If the total number of beams is still too large, we choose the top-200 beams.

- When expanding beams, we only use the top-5 candidates. A candidate is a $(v, t)$ pair, indicating the next vertex and token, where we jointly consider their probabilities as Eq(10).

### C. More Analyses

#### C.1. Translation Performance on Different Lengths

To investigate the translation performance on different lengths, we split the test set into 6 buckets according to reference lengths and evaluate the BLEU score in each bucket as shown in Fig.12. Compared with NAT baselines, DA-Transformer has...
Algorithm 3 BeamSearch for DA-Transformer

**Input:** Graph Size $L$, Transition Matrix $E \in \mathbb{R}^{L \times L}$, Token Distributions $P \in \mathbb{R}^{L \times |V|}$

For $i \in [1, L]$ and any possible beam $B$, initialize $s_i(B) := 0$ that stores the probability sum of all $B$’s paths that end at vertex $i$

Initialize $\mathcal{G}_1, \cdots, \mathcal{G}_L$ as $L$ empty sets that store the beams at the vertex $i$

Insert a beam $B_{\text{init}}$ with the start token in $\mathcal{G}_1$. Set $s_1(B_{\text{init}}) := 1$

for $i = 1, 2, \cdots, L - 1$ do

# Filter the beams
Sort the beams in $\mathcal{G}_i$ by the score defined in Eq(11)
For each beam length in $\mathcal{G}_i$, preserve the top-10 beams and remove the other beams
Considering all beams in $\mathcal{G}_i$, preserve the top-200 beams and remove the other beams

# Expand the beams
for each beam $B$ in $\mathcal{G}_i$ do

for $(v, t)$ in $B$’s top-5 next candidates do

# $v$ is the next vertex; $t$ is the next token
Insert $B' = B + \{t\}$ as a new beam into $\mathcal{G}_v$.
Update $s_v(B') := s_v(B') + s_i(B) \times P_{v,t} \times E_{i,v}$

end for
end for

end for

Output the best beam in $\mathcal{G}_L$

a substantial improvement for sentences longer than 20. These long sentences usually have more modalities in translation, which are challenging in previous NATs but can be better handled in DA-Transformer.

![Figure 12. The BLEU score on WMT14 En-De and WMT17 Zh-En bucketed by the reference length.](image)

**C.2. Performance with Controlled Training Time**

One update step of DA-Transformer’s training is slower than many previous NATs because our Directed Acyclic Decoder has to process a longer sequence whose length is about 8 times of the original target. In Fig. 13, we show that DA-Transformer still substantially outperforms strong NAT baselines when the training time is controlled. Moreover, we observe that our performance is more stable than the baselines during the training process.

![Figure 13. The valid BLEU on WMT14 En-De. We do not apply the checkpoint average trick. We evaluate the model approximately every 8 GPU-hours. The training costs 500 GPU-hours, which has about 300k, 490k, 970k updates for DA-Transformer, GLAT+CTC, GLAT, respectively.](image)
D. More Cases

Two more test cases from WMT17 Zh-En are presented in Fig.14.

**Source** 这太令人难以置信了。  **Reference** It was just incredible.

**DA-Transformer**

Source 这名女性当场死亡。  **Reference** She died at the scene.

**DA-Transformer**

**E. Statistics of DAGs**

For a better understanding of DA-Transformer, we collect some statistics of predicted DAGs on WMT17 Zh-En. We use a DA-Transformer with $\lambda = 4$.

**Figure 15.** Statistics of DAGs. (a) The distribution of vertices’ passing probabilities and max token probabilities. The passing probability represents how likely a vertex would appear on a randomly sampled path. The max token probability represents the probability of the most probable token on the vertex. (b) The distribution of numbers of vertices’ outgoing edges. We only consider the most likely edges accounting for 80% of the transition probabilities and ignore the vertices with passing probabilities smaller than 0.2. We further merge the edges that are linked to vertices predicting the same token.
In Fig. 15 (a), we present the distribution of vertices with passing probability and max token probability. We generally divide the vertices into three categories:

- Vertices with Passing Prob > 0.5 (accounting for 19.6%): They are very likely to appear in the generated translation. Since the average target length is about $\frac{1}{3} = 25\%$ of the graph size, these vertices generate most of the tokens in the outputs.
- Vertices with Passing Prob < 0.5 and Max Token Prob > 0.5 (accounting for 40.2%): They have high confidence in predicting tokens but do not usually appear in the translation. They may contain some rare expressions.
- Vertices with Passing Prob < 0.2 and Max Token Prob < 0.2 (accounting for 15.4%): These vertices do not have specific meanings. We think the vertices are not well learned. It may be helpful if we encourage them to be more confident in generating some specific tokens.

In Fig. 15 (b), we present the number of outgoing edges of vertices. We find that half of the vertices have only one outgoing edge, and the other half have multiple edges. The result shows that the predicted DAGs have complicated structures, which do not degenerate into chains.