Fermion EDMs with Minimal Flavor Violation

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Abstract

We study the electric dipole moments (EDMs) of fermions in the standard model supplemented with right-handed neutrinos and its extension including the neutrino seesaw mechanism under the framework of minimal flavor violation (MFV). In the quark sector, we find that the current experimental bound on the neutron EDM does not yield a significant restriction on the scale of MFV. In addition, we consider how MFV may affect the contribution of the strong theta-term to the neutron EDM. For the leptons, the existing EDM data also do not lead to strict limits if neutrinos are Dirac particles. On the other hand, if neutrinos are Majorana in nature, we find that the constraints become substantially stronger. Moreover, the results of the latest search for the electron EDM by the ACME Collaboration are sensitive to the MFV scale of order a few hundred GeV or higher. We also look at constraints from CP-violating electron-nucleon interactions that have been probed in atomic and molecular EDM searches.
I. INTRODUCTION

Searches for electric dipole moments (EDMs) are a powerful means of probing new sources of the violation of charge parity ($CP$) and time reversal ($T$) symmetries beyond the standard model (SM) of particle physics [1–4]. Recently the ACME experiment [5], which utilized the polar molecule thorium monoxide to look for the EDM of the electron, $d_e$, has produced a new result of $d_e = (-2.1 \pm 3.7_{\text{stat}} \pm 2.5_{\text{syst}}) \times 10^{-29} \text{cm}$, which corresponds to an upper limit of $|d_e| < 8.7 \times 10^{-29} \text{cm}$ at 90% confidence level (CL). This is more stringent than the previous best bound by about an order of magnitude, but still way above the SM expectation for $d_e$, which is at the level of $10^{-44} \text{cm}$ [6]. Hence there is abundant room between the current limit and SM value of $d_e$ where potential new physics may be observed in future measurements. In the quark sector, the EDM of the neutron, $d_n$, plays an analogous role in the quest of new physics. At present its experimental limit is $|d_n| < 2.9 \times 10^{-26} \text{cm}$ at 90% CL [7], while the SM predicts it to be in the range of $10^{-32}$-$10^{-31} \text{cm}$ [8].

Extra ingredients beyond the SM can increase the electron and neutron EDMs tremendously with respect to their SM predictions, even up to their existing measured bounds. Such substantial enlargement may have various causes which could greatly differ from model to model. It is, therefore, of interest to analyze fermion EDMs arising from possible nonstandard origins under a framework that allows one to deal with some general features of the physics without getting into model specifics. This turns out to be feasible under the context of the so-called minimal flavor violation (MFV) which presupposes that the sources of all flavor-changing neutral currents (FCNC) and $CP$ violation reside in renormalizable Yukawa couplings defined at tree level [9–11]. Thus the MFV framework offers a systematic way to explore SM-related new interactions which do not conserve flavor and $CP$ symmetries.

In an earlier paper [12], motivated by the recent ACME data, we have adopted the MFV hypothesis in order to examine $d_e$ in the SM slightly expanded with the inclusion of three right-handed neutrinos and in its extension incorporating the seesaw mechanism for light neutrino mass generation. In the present work, we would like to provide a more extensive treatment of our previous study, covering the EDMs of the other charged leptons as well. For $d_e$ particularly, we demonstrate in greater detail how various factors may affect it within the MFV context, taking into account extra empirical information on neutrino masses. Moreover, we address the possibility that $d_e$ is correlated with the effective Majorana mass that is testable in ongoing and upcoming searches for neutrinoless double-beta decay. We will also perform an MFV analysis on the quark EDMs and estimate the resulting neutron EDM. In addition, we consider the MFV effect on the contribution of the theta term in QCD to the neutron EDM.

The structure of the paper is as follows. In Section II, we describe the MFV framework and its aspects which are relevant to our evaluation of fermion EDMs as probes for the scale of MFV. In Section III we derive the expressions for quark and lepton EDMs from several effective operators satisfying the MFV principle. Section IV contains our numerical analysis. After determining the neutron EDM from the quark contributions and inferring the constraint on the MFV scale from the neutron data, we examine how the contribution of the QCD theta-term is altered in the presence of MFV. In the lepton sector, we devote much of our attention to the electron EDM in light of the ACME data and briefly address its muon and tau counterparts. The acquired constraints
on the MFV scale depend considerably on whether the light neutrinos are Dirac or Majorana in nature, the EDMs in the former case being much smaller than the latter. Subsequently, we look at \(CP\)-violating electron-nucleon interactions, which were also investigated by ACME and other experiments looking for atomic or molecular EDMs. Lastly, we discuss potential constraints from flavor-changing and other flavor-conserving processes. We make our conclusions in Section V. An appendix collects some useful lengthy formulas.

II. MINIMAL FLAVOR VIOLATION FRAMEWORK

In the SM supplemented with three right-handed neutrinos, the renormalizable Lagrangian for the quark and lepton masses can be expressed as

\[
\mathcal{L}_m = \frac{-\bar{Q}_{k,L}(Y_u)_{kl}U_{l,R}\tilde{H} - \bar{Q}_{k,L}(Y_d)_{kl}D_{l,R}H - \bar{L}_{k,L}(Y_e)_{kl}\nu_{l,R}\tilde{H} - \bar{L}_{k,L}(Y_\nu)_{kl}E_{l,R}H}{2} + \text{H.c.} ,
\]

where summation over \(k, l = 1, 2, 3\) is implicit, \(Q_{k,L}\) (\(L_{k,L}\)) represents left-handed quark (lepton) doublets, \(U_{k,R}\) and \(D_{k,R}\) (\(V_{k,R}\) and \(E_{k,R}\)) denote right-handed up- and down-type quarks (neutrinos and charged leptons), respectively, \(Y_{u,d,\nu,e}\) are matrices containing the Yukawa couplings, \(M_\nu\) is the Majorana mass matrix of the right-handed neutrinos, \(H\) is the Higgs doublet, and \(\tilde{H} = i\tau_2H^*\) involving the second Pauli matrix \(\tau_2\). The Higgs' vacuum expectation value \(v \simeq 246\) GeV breaks the electroweak symmetry as usual, which makes the weak gauge bosons and charged leptons massive and also induces Dirac mass terms for the neutrinos. The \(M_\nu\) part in \(\mathcal{L}_m\) plays an essential role in the type-I seesaw mechanism \([13]\).\(^1\) If neutrinos are Dirac particles, however, the \(M_\nu\) terms are absent.

For the quark sector, the MFV hypothesis \([10]\) implies that the Lagrangian in Eq. (1) is formally invariant under the global group \(U(3)_Q \times U(3)_U \times U(3)_D = G_\xi \times U(1)_Q \times U(1)_U \times U(1)_D\), where \(G_\xi = SU(3)_Q \times SU(3)_U \times SU(3)_D\). This entails that the three generations of \(Q_{k,L}, U_{k,R}\), and \(D_{k,R}\) transform as fundamental representations of the \(SU(3)_{Q,U,D}\), respectively, namely

\[
Q_L \rightarrow V_QQ_L , \quad U_R \rightarrow V_QU_R , \quad D_R \rightarrow V_UD_R , \quad V_{Q,U,D} \in SU(3) .
\]

Moreover, the Yukawa couplings are taken to be spurions which transform according to

\[
Y_u \rightarrow V_QY_uV_U^\dagger , \quad Y_d \rightarrow V_QY_dV_D^\dagger .
\]

Consequently, to arrange nontrivial FCNC and \(CP\)-violating interactions satisfying the MFV principle and involving no more than two quarks, one puts together an arbitrary number of the Yukawa coupling matrices \(Y_u \sim (3,\bar{3},1)\) and \(Y_d \sim (3,1,\bar{3})\) as well as their Hermitian conjugates to set up the \(G_\xi\) representations \(\Delta_q \sim (1 \oplus 8, 1,1), \Delta_{u8} \sim (1,1 \oplus 8,1), \Delta_{d8} \sim (1,1,1 \oplus 8), \Delta_u \sim (3,3,1)\), and \(\Delta_d \sim (3,1,3)\), combines them with two quark fields to build the \(G_\xi\)-invariant objects \(Q_L\gamma_\alpha\Delta_qQ_L, \bar{U}_{R}\gamma_\alpha\Delta_{u8}U_R, \bar{D}_{R}\gamma_\alpha\Delta_{d8}D_R, \bar{U}_{R}(1,\sigma_{\alpha\beta})\Delta_uQ_L,\) and \(\bar{D}_R(1,\sigma_{\alpha\beta})\Delta_dQ_L\), includes appropriate numbers of the Higgs and gauge fields to arrive at singlets under the SM gauge group.

\(^1\) An analogous situation occurs in the type-III seesaw model \([14]\).
and contracts all the Lorentz indices. Since $Q_L\gamma_\alpha\Delta_q Q_L$, $U_R\gamma_\alpha\Delta_u U_R$, and $D_R\gamma_\alpha\Delta_d D_R$ in this case must be Hermitian, $\Delta_{q,u,d}$ must be Hermitian as well.

The Lagrangian describing the EDM $d_f$ of a fermion $f$ is $\mathcal{L}_{\text{edm}} = -(id_f/2)\bar{f}\sigma^{\mu\nu}\gamma_5fF_{\mu\nu}$, where $F_{\mu\nu}$ is the photon field strength tensor. Accordingly, among the combinations listed in the preceding paragraph, only $\bar{U}_R\sigma_{\alpha\beta}\Delta_u Q_L$ and $\bar{D}_R\sigma_{\alpha\beta}\Delta_d Q_L$ pertain to our examination of quark EDMs. For $\Delta_{u,d}$, one can take $\Delta_u = Y_u^\dagger\Delta$ and $\Delta_d = Y_d^\dagger\Delta$, where $\Delta$ is built up of terms in powers of $A = Y_u Y_u^\dagger$ and $B = Y_d Y_d^\dagger$, which transform as $(1 \oplus 8, 1, 1)$ under $G_q$.

Formally $\Delta$ comprises an infinite number of terms, namely $\Delta = \sum\xi_{jkl}\cdots A^j B^k A^l\cdots$ with coefficients $\xi_{jkl}\cdots$ expected to be at most of $O(1)$. The MFV hypothesis requires that $\xi_{jkl}\cdots$ be real because complex $\xi_{jkl}\cdots$ would introduce new $CP$-violation sources beyond that in the Yukawa couplings. Using the Cayley-Hamilton identity

$$X^3 = X^2\text{Tr}X + \frac{1}{2}X[\text{Tr}X^2 - (\text{Tr}X)^2] + \mathbb{I}\text{Det}X$$

for an invertible $3 \times 3$ matrix $X$, one can resum the infinite series into a finite number of terms [15, 16]

$$\Delta = \xi_1\mathbb{I} + \xi_2 A + \xi_3 B + \xi_4 A^2 + \xi_5 B^2 + \xi_6 AB + \xi_7 BA + \xi_8 ABA + \xi_9 BA^2 + \xi_{10} BAB + \xi_{11} AB^2 + \xi_{12} ABA^2 + \xi_{13} A^2B^2 + \xi_{14} B^2A^2 + \xi_{15} B^2AB + \xi_{16} AB^2A^2 + \xi_{17} B^2A^2B,$$  

where $\mathbb{I}$ denotes a $3 \times 3$ unit matrix. One can then also utilize this to devise Hermitian combinations such as $\Delta_q = \Delta + \Delta^\dagger$.

Even though one starts with all $\xi_{jkl}\cdots$ being real, the resummation process will render the coefficients $\xi_\ast$ in Eq. (5) generally complex due to imaginary parts generated among the traces of the matrix products $A^j B^k A^l\cdots$ with $j+k+l+\cdots \geq 6$ upon the application of the Cayley-Hamilton identity. In Appendix A we show the detailed reduction of one of the lowest-order products which give rise to the imaginary components of $\xi_\ast$. We find that the imaginary contributions are always reducible to factors proportional to $\text{ImTr}(A^2 B A B^2) = (i/2)\text{Det}[A, B]$ which is a Jarlskog invariant and much smaller than one [15].

Taking advantage of the invariance under $G_q$, we will work in the basis where $Y_d$ is diagonal,

$$Y_d = \frac{\sqrt{2}}{v} \text{diag}(m_d, m_s, m_b),$$

and the fields $U_{k,L}$, $U_{k,R}$, $D_{k,L}$, and $D_{k,R}$ belong to the mass eigenstates. Hence we can write $Q_{k,L}$ and $Y_u$ in terms of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix $V_{\text{CKM}}$ as

$$Q_{k,L} = \begin{pmatrix} V_{\text{CKM}} & \text{kl} \end{pmatrix} U_{l,L}, \quad Y_u = \frac{\sqrt{2}}{v} V_{\text{CKM}}^\dagger \text{diag}(m_u, m_c, m_t),$$

where in the standard parametrization [7]

$$V_{\text{CKM}} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix},$$

with $\delta$ being the $CP$ violation phase, $c_{kl} = \cos \theta_{kl}$, and $s_{kl} = \sin \theta_{kl}$. We note that, as a consequence, $\Delta_{u8}$ and $\Delta_{d8}$, whose basic building blocks are $Y_{u8}^\dagger Y_u$ and $Y_{d8}^\dagger Y_d$, respectively, are all diagonal and thus will not bring about new flavor- and $CP$-violating interactions.
For the lepton sector, since it is still unknown whether light neutrinos are Dirac or Majorana particles, we address the two possibilities separately. In the Dirac case, the \( M_\nu \) part is absent from \( \mathcal{L}_m \) in Eq. (1), which is therefore, in the MFV language, formally invariant under the global group \( SU(3)_L \times SU(3)_\nu \times SU(3)_E = G_\ell \times U(1)_L \times U(1)_\nu \times U(1)_E \) with \( G_\ell = SU(3)_L \times SU(3)_\nu \times SU(3)_E \). This means that the three generations of \( L_{k,L}, \nu_{k,R}, \) and \( E_{k,R} \) transform as fundamental representations of \( SU(3)_{L,\nu,E} \) in \( G_\ell \), respectively,

\[
L_L \rightarrow V_L L_L, \quad \nu_R \rightarrow V_\nu \nu_R, \quad E_R \rightarrow V_E E_R, \quad (9)
\]

where \( V_{L,\nu,E} \in SU(3) \), whereas the Yukawa couplings are spurions transforming according to

\[
Y_\nu \rightarrow V_L Y_\nu V_\nu^\dagger, \quad Y_e \rightarrow V_L Y_e V_E^\dagger. \quad (10)
\]

We will work in the basis where \( Y_e \) is already diagonal,

\[
Y_e = \frac{\sqrt{2}}{v} \text{diag}(m_e, m_\mu, m_\tau), \quad (11)
\]

and the fields \( \nu_{k,L}, \nu_{k,R}, E_{k,L}, \) and \( E_{k,R} \) refer to the mass eigenstates. We can then express \( L_{k,L} \) and \( Y_\nu \) in terms of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix \( U_{\text{PMNS}} \) as

\[
L_{k,L} = \left( \frac{(U_{\text{PMNS}})_{kL}}{E_{k,L}} \right), \quad Y_\nu = \frac{\sqrt{2}}{v} U_{\text{PMNS}} \hat{m}_\nu, \quad \hat{m}_\nu = \text{diag}(m_1, m_2, m_3), \quad (12)
\]

where \( m_{1,2,3} \) are the light neutrino eigenmasses and \( U_{\text{PMNS}} \) has the same standard parametrization as in Eq. (8). Thus the discussion for the down-type quarks can be easily applied to the charged leptons by replacing \( V_{\text{CKM}} \) with \( U_{\text{PMNS}}^\dagger \) and employing the building blocks \( A = Y_\nu Y_\nu^\dagger \) and \( B = Y_e Y_e^\dagger \) to construct \( \Delta_\nu \) and \( \Delta_e \), which are the lepton counterparts of \( \Delta_u \) and \( \Delta_d \), respectively.

If neutrinos are of Majorana nature, the \( M_\nu \) part in Eq. (1) is allowed. As a consequence, for \( M_\nu \gg M_D = v Y_\nu/\sqrt{2} \) the seesaw mechanism [13] becomes operational involving the \( 6 \times 6 \) neutrino mass matrix

\[
M = \begin{pmatrix} 0 & M_D \\ M_D^T & M_\nu \end{pmatrix}, \quad (13)
\]

in the \( (U_{\text{PMNS}} \nu_L^c, \nu_R^c)^T \) basis. The resulting matrix of light neutrino masses is

\[
m_\nu = \frac{-v^2}{2} Y_\nu M_\nu^{-1} Y_\nu^T = U_{\text{PMNS}} \hat{m}_\nu U_{\text{PMNS}}^T, \quad (14)
\]

where now \( U_{\text{PMNS}} \) contains the diagonal matrix \( P = \text{diag}(e^{i \alpha_1/2}, e^{i \alpha_2/2}, 1) \) multiplied from the right, \( \alpha_{1,2} \) being the Majorana phases. It follows that \( Y_\nu \) in Eq. (12) is no longer valid, and one can instead take \( Y_\nu \) to be [17]

\[
Y_\nu = \frac{i \sqrt{2}}{v} U_{\text{PMNS}} \hat{m}_\nu^{1/2} O M_\nu^{1/2}, \quad (15)
\]

where \( O \) is a matrix satisfying \( OO^T = \mathbb{1} \) and \( M_\nu = \text{diag}(M_1, M_2, M_3) \). As we will see later, \( O \) can provide a potentially important new source of \( CP \) violation besides \( U_{\text{PMNS}} \). We comment that the presence of \( M_\nu \) breaks the global \( U(3)_\nu \) completely if \( M_{1,2,3} \) are unequal and partially into \( O(3)_\nu \) if \( M_{1,2,3} \) are equal [11].
III. FERMION EDMS IN MFV FRAMEWORK

To explore the MFV contribution to the EDMs of quarks and charged leptons, one needs to construct the relevant operators using $\Delta_{u,d,e}$ in combination with the quark, lepton, Higgs, and gauge fields. At leading order, the operators can be written as [10, 11]

\[
O_{RL}^{(u1)} = g' Y_u^d \Delta_{qu1} \sigma_{\kappa \omega} H^\dagger Q_L B_{\kappa \omega}, \quad O_{RL}^{(u2)} = g' Y_u^d \Delta_{qu2} \sigma_{\kappa \omega} H^\dagger \tau_a Q_L W_{a \kappa \omega},
\]

\[
O_{RL}^{(d1)} = g' Y_d^d \Delta_{qd1} \sigma_{\kappa \omega} H^\dagger Q_L B_{\kappa \omega}, \quad O_{RL}^{(d2)} = g' Y_d^d \Delta_{qd2} \sigma_{\kappa \omega} H^\dagger \tau_a Q_L W_{a \kappa \omega},
\]

\[
O_{RL}^{(e1)} = g' Y_e^d \Delta_{q1} \sigma_{\kappa \omega} H^\dagger L_L B_{\kappa \omega}, \quad O_{RL}^{(e2)} = g' Y_e^d \Delta_{q2} \sigma_{\kappa \omega} H^\dagger \tau_a L_L W_{a \kappa \omega},
\]

where $W$ and $B$ denote the usual SU(2)$_L \times$ U(1)$_Y$ gauge fields with coupling constants $g$ and $g'$, respectively, $\tau_a$ are Pauli matrices, $a = 1, 2, 3$ is summed over, and $\Delta_{qu, qd, \kappa}$ with $\kappa = 1, 2$ have the same form as $\Delta$ in Eq. (5), but generally different $\xi_r$. One can express the effective Lagrangian containing these operators as

\[
\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} \left[ O_{RL}^{(u1)} + O_{RL}^{(u2)} + O_{RL}^{(d1)} + O_{RL}^{(d2)} + O_{RL}^{(e1)} + O_{RL}^{(e2)} \right] + \text{H.c.},
\]

where $\Lambda$ is the MFV scale. In general the operators in $\mathcal{L}_{\text{eff}}$ have their own coefficients which have been absorbed by $\xi_r$ in their respective $\Delta$'s. These coefficients also take into account the possibility that the MFV scale in the quark sector may differ from that in the lepton sector.

Expanding Eq. (18), one can identify the terms relevant to fermion EDMs. In the quark sector the resulting EDMs of up- and down-type quarks are, respectively, proportional to $\text{Im}(Y_u^d \Delta_{qu} V^\dagger_{\text{CKM}})$ and $\text{Im}(Y_d^d \Delta_{qd})$. The contributions of $\Delta_{qu, qd}$ to the EDMs come not only from some of the products of the $A$ and $B$ matrices therein, but also from the imaginary parts of $\xi_r$. As mentioned earlier, $\text{Im} \xi_r$ are always proportional to $J_x \equiv \text{Im} \text{Tr}(A^2 B^2) = (i/2) \text{Det}[A, B]$, or explicitly

\[
J_x = \frac{-64(m_u^2 - m_c^2)(m_c^2 - m_t^2)(m_t^2 - m_b^2)(m_d^2 - m_s^2)(m_s^2 - m_b^2)(m_b^2 - m_t^2)}{v^{12}} J_q,
\]

where $J_q = \text{Im}(V_{us} V_{ub}^* V_{cs}^* V_{cb}) = c_{12} s_{12} c_{23} s_{23} c_{13} s_{13} \sin \delta$ is a Jarlskog parameter for $V_{\text{CKM}}$.

Not all of the products of $A = Y_u Y_u^\dagger$ and $B = Y_d Y_d^\dagger$ in $\Delta_{qu, qd}$ will contribute to quark EDMs. Since $Y_u$ has the form in Eq. (7) and $Y_d$ is diagonal, the Hermiticity of $A$ and $B$ implies that only certain combinations of them are relevant. For example, $(Y_d^d A)_{kk} = \sqrt{2} m_{D_k} A_{kk}/v$ is purely real and hence does not affect $d_{D_k}$. In this case, one needs to have terms in $\Delta_{qd}$, which are not Hermitian in order to have imaginary components in $(Y_u^d \Delta_{qd})_{kk}$. We find that only two terms, proportional to $B^2 AB$ and $B^2 A^2 B$, are pertinent to the up-type quarks’ EDMs and only the $ABA^2$ and $AB^2 A^2$ terms are pertinent to the EDM’s of down-type quarks.

The preceding discussions show that the contributions of $\text{Im} \xi_r$ to the EDM of, say, the $u$ ($d$) quark are suppressed by a factor of $m^2/q^2$ ($m^2 m^2/q^4$) compared to the contributions from $B^2 AB$ ($ABA^2$), which has the least number of suppressive factor from $Y_u$ ($Y_d$) among the products in Eq. (5) that can potentially contribute. Hence we can neglect the impact of $\text{Im} \xi_r$ on the quark EDMs. One, however, needs to take $\text{Im} \xi_r$ into account when considering how MFV affects the contribution of the strong theta-term to the neutron EDM, as we demonstrate later.
Simplifying things, we arrive at the leading-order contributions to the $u$- and $d$-quarks’ EDMs

\[
d_u = \frac{\sqrt{2} e v}{\Lambda^2} \text{Im}[Y_u^\dagger (\Delta_{qa1} + \Delta_{qa2}) V_{\text{CKM}}^\dagger]_{11} \\
= \frac{32 e m_u}{\Lambda^2} \left[ \xi_{u12}^d + \frac{2(m_c^2 + m_s^2)}{v^2} \xi_{u16}^d \right] \left( \frac{m_c^2 - m_t^2}{v^2} \right) \left( \frac{m_s^2 - m_b^2}{v^2} \right) \left( \frac{m_c^2 - m_t^2}{v^2} \right) J_q, \\
d_d = \frac{\sqrt{2} e v}{\Lambda^2} \text{Im}[Y_d^\dagger (\Delta_{qd1} - \Delta_{qd2})]_{11} \\
= \frac{32 e m_d}{\Lambda^2} \left[ \xi_{d12}^d + \frac{2(m_s^2 + m_b^2)}{v^2} \xi_{d16}^d \right] \left( \frac{m_s^2 - m_b^2}{v^2} \right) \left( \frac{m_c^2 - m_t^2}{v^2} \right) \left( \frac{m_c^2 - m_t^2}{v^2} \right) J_q,
\]

where $\xi_{u} = \xi_{u1} + \xi_{u2}$ and $\xi_{r} = \xi_{r1} - \xi_{r2}$. The expressions for $d_{c,t}$ and $d_{s,b}$ can be simply derived from Eqs. (20) and (21), respectively, by cyclically changing the quark labels.\(^2\)

In the lepton sector, we get from Eq. (18) the electron EDM

\[
d_e = \frac{\sqrt{2} e v}{\Lambda^2} \text{Im}(Y_e^\dagger \Delta_{\ell 1} - Y_e^\dagger \Delta_{\ell 2})_{11} \\
= \frac{\sqrt{2} e v}{\Lambda^2} \left[ \xi_{e12}^\ell \text{Im}(Y_e^\dagger A B A^2)_{11} + \xi_{e16}^\ell \text{Im}(Y_e^\dagger A B A^2)_{11} \right],
\]

where $\xi_{e} = \xi_{e1} - \xi_{e2}$, we have ignored $\text{Im} \xi_{e}^\ell$, and here $A = Y_\nu Y_\nu^\dagger$ and $B = Y_e Y_e^\dagger$. If neutrinos are Dirac particles, analogously to $d_d$, we obtain

\[
d_e^D = \frac{32 e m_e}{\Lambda^2} \left[ \xi_{e12}^\ell + \frac{2(m_s^2 + m_b^2)}{v^2} \xi_{e16}^\ell \right] \left( \frac{m_s^2 - m_b^2}{v^2} \right) \left( \frac{m_c^2 - m_t^2}{v^2} \right) \left( \frac{m_c^2 - m_t^2}{v^2} \right) J_\ell,
\]

where $J_\ell = \text{Im}(U_{e2} U_{\mu3} U_{\tau3}^* U_{\mu2}^*)$ is a Jarlskog invariant for $U_{\text{PMNS}}$.

In the case of Majorana neutrinos, if $\nu_{k,R}$ are degenerate, $M_\nu = M \mathbb{1}$, and $O$ is a real orthogonal matrix,\(^3\) from Eq. (15) we have

\[
A = \frac{2}{v^2} M U_{\text{PMNS}} \bar{m}_\nu U_{\text{PMNS}}^\dagger
\]

and consequently

\[
d_e^M = \frac{32 e m_e M^3}{\Lambda^2 v^8} \left( m_s^2 - m_b^2 \right) \left( m_1 - m_2 \right) \left( m_3 - m_1 \right) \xi_{e12}^\ell J_\ell,
\]

the $\xi_{e16}^\ell$ term having been neglected. Since $m_s < M$, we can see that $d_e^D$ is highly suppressed relative to $d_e^M$. The formulas for $d_{\mu,\tau}^D$ and $d_{\mu,\tau}^M$ can be readily found from Eqs. (23) and (25), respectively, by cyclically changing the mass subscripts.

\(^2\)It is worth commenting that since $\text{Im} \xi_\ell \propto \text{Det}[A,B]$, due to the reality of the coefficients $\xi_{ijkl}$ in the infinite series expansion of $\Delta$, and since $A$ and $B$ are Hermitian, $d_q$ would be identically zero if there were only one generation of fermions. The same applies to the lepton sector.

\(^3\)Since the lepton Lagrangian with $\nu_{k,R}$ being degenerate is $O(3)_\nu$ symmetric, one could transform this real $O$ into a unit matrix [18].
In the discussion above, $d_e$ arises from the $CP$-violating Dirac phase $\delta$ in $U_{\text{PMNS}}$, and the Majorana phases $\alpha_{1,2}$ therein do not participate. However, if $\nu_{k,R}$ are not degenerate, nonzero $\alpha_{1,2}$ can bring about an additional effect on $d_e$, even with a real $O \neq \mathbb{1}$. With a complex $O$, the phases in it may give rise to an extra contribution to $d_e$, whether or not $\nu_{k,R}$ are degenerate. The formulas for $d_e$ in these scenarios are more complicated than Eq. (25) and are not shown here, but we will explore some of them numerically in the next section.

The various contributions to the fermion EDMs that we have considered have high powers in Yukawa couplings. Since the MFV hypothesis presupposes that all $CP$-violation effects originate from the Yukawa couplings, the high orders in them reflect the fact that nonvanishing EDMs in the SM begin to appear at the three-loop level for quarks and in higher loops for the electron. One may wonder whether these are the only contributions to fermion EDMs under the MFV framework. The answer is no because one can realize fermion EDMs by combining some lower-order Yukawa terms from the MFV operators with SM loop diagrams, such as those contributing to quark EDMs in the SM. Nevertheless, hereafter we will not include such type of possible contributions. The contributions that we have already covered should provide a good idea about how fermion EDMs are generated in the presence of MFV. For definiteness, we will apply numerically the results we have acquired and discuss some of their implications.

IV. NUMERICAL ANALYSIS

We will first treat the neutron EDM, $d_n$, evaluated from the quark contributions and infer from its data a bound on the scale of quark MFV. We will also look at how MFV affects the contribution of the strong $\theta$-term to $d_n$. Proceeding to the lepton sector, we will devote much of the section to the electron EDM, and briefly deal with the muon and tau EDMs, in order to explore limitations on the scale of leptonic MFV. Afterwards, we will examine constraints from $CP$-violating electron-nucleon interactions which were probed by recent searches for atomic and molecular EDMs. Finally, we will address potential restrictions from some $CP$-conserving processes.

A. Neutron EDM

In calculating quark EDMs, as in Eqs. (20) and (21), one needs to take into account the running of the quark masses due to QCD evolution. We adopt the mass ranges $m_u = 0.00139^{+0.00042}_{-0.00041}$, $m_d = 0.00285^{+0.00049}_{-0.00048}$, $m_s = 0.058^{+0.018}_{-0.012}$, $m_c = 0.645^{+0.043}_{-0.085}$, $m_b = 2.90^{+0.16}_{-0.06}$, and $m_t = 174.2 \pm 1.2$, all in GeV, at a renormalization scale $\mu = m_W$ from Ref. [19]. With the central values of these masses and the quark Jarlskog parameter $J_q = (3.02^{+0.16}_{-0.10}) \times 10^{-5}$ from the latest fit by CKMfitter [20], we arrive at

\begin{align}
  d_u &= \frac{1.4 \times 10^{-35} \text{ e cm}}{\Lambda^2/\text{GeV}^2} (\xi^u_{15} + \xi^u_{17}) , \\
  d_d &= \frac{1.3 \times 10^{-29} \text{ e cm}}{\Lambda^2/\text{GeV}^2} (\xi^d_{12} + 0.00028 \xi^d_{16}) , \\
  d_s &= \frac{-2.6 \times 10^{-28} \text{ e cm}}{\Lambda^2/\text{GeV}^2} (\xi^d_{12} + 0.00028 \xi^d_{16}) ,
\end{align}

(26)

where $\xi^u_r = \xi^u_{r1} + \xi^u_{r2}$ and $\xi^d_r = \xi^d_{r1} - \xi^d_{r2}$. Evidently, the $s$-quark effect may be dominant.
To determine the neutron EDM, one needs to connect it to the quark-level quantities. The relation between $d_n$ and $d_{u,d,s}$ can be parameterized as

$$d_n = \eta_n (\rho_n^u d_u + \rho_n^d d_d + \rho_n^s d_s),$$

(27)

where $\eta_n = 0.4$ accounts for corrections due to the QCD evolution from $\mu = m_W$ down to the hadronic scale [21] and the values of the parameters $\rho_n^{u,d,s}$ depend on the model for the matrix elements $\langle n | \bar{q} \sigma^{\kappa \omega} q | n \rangle = \rho_n^\kappa \bar{u}_n \sigma^{\kappa \omega} u_n$. For instance, in the constituent quark model $\rho_n^u = -4/3 = -4 \rho_o^u$ and $\rho_n^d = 0$ [1], whereas in the parton quark model $\rho_n^u = -0.508$, $\rho_n^d = 0.746$, and $\rho_n^s = -0.226$ [3]. From the various models proposed in the literature [1, 3, 22], we may conclude that

$$-0.78 \leq \rho_n^u \leq -0.17, \quad 0.7 \leq \rho_n^d \leq 2.1, \quad -0.35 \leq \rho_n^s \leq 0.$$  

(28)

In view of these numbers and Eq. (26), we can ignore the $d_u$ and $\xi_{16}$ terms. Hence, taking the extreme values $\rho_n^d = 2.1$ and $\rho_n^s = -0.35$, as well as scanning over the quark mass and $J_q$ ranges quoted above to maximize $d_n$, we get

$$d_n = \frac{8.4 \times 10^{-29} \text{ e cm}}{\Lambda^2/\text{GeV}^2} \xi_{12}. $$

(29)

It is then interesting to note that $\Lambda/|\xi_{12}|^{1/2} = 100$ GeV translates into $d_n = 8.4 \times 10^{-33}$ e cm, which is roughly similar to the SM expectation $d_n^{\text{SM}} \sim 10^{-32}$-$10^{-31}$ e cm [8]. Comparing Eq. (29) with the current data $|d_n|_{\text{exp}} < 2.9 \times 10^{-26}$ e cm at 90% CL [7], we extract

$$\frac{\Lambda}{|\xi_{12}|^{1/2}} > 0.054 \text{ GeV}, $$

(30)

which is not strict at all. Less extreme choices of $\rho_n^{d,s}$ would lead to even weaker bounds. We conclude that the present neutron-EDM limit cannot yield a useful restriction on $\Lambda$.

One can also look at the contributions of quark chromo-EDMs to the neutron EDM [1]. The relevant operators are obtainable from the MFV quark-EDM operators by replacing $W_{a \mu \nu}$ and $\tau_a$ with the gluon field strength tensor $G_{\mu \nu}^c$ and the color SU(3) generators $\lambda_c$, respectively. The extracted constraints on $\Lambda$ are similar.

### B. MFV contribution to strong theta term

Besides the quark (chromo-)EDMs, another contributor to the neutron EDM is the theta term of QCD [23], which in the SM is given by [3]

$$\mathcal{L}_\theta = \frac{-\bar{\theta} g_s^2}{32\pi^2} \epsilon_{\kappa \nu \phi \omega} G^{\kappa \nu} G^{\phi \omega}, $$

(31)

where $\bar{\theta} = \theta + \text{arg Det}(Y_u Y_d)$ involves the bare $\theta$-parameter, $g_s$ is the strong coupling constant, and $\epsilon_{0123} = +1$. The inclusion of MFV causes $\theta$ to be modified to

$$\bar{\theta}_{\text{MFV}} = \theta + \text{arg Det}(\Delta_{qu}^\dagger Y_u \Delta_{qd}^\dagger Y_d) = \bar{\theta} + \text{arg Det} \Delta_{qu}^\dagger + \text{arg Det} \Delta_{qd}^\dagger, $$

(32)
where $\Delta_{qu,qd}$ have the same expression as $\Delta$ in Eq. (5), but generally different coefficients $\xi_r$. Although the addition of these new factors to the Yukawa Lagrangian amounts only to a redefinition of $Y_{u,d}$ and hence has no direct experimental implications after the quark mass matrices are diagonalized, we can expect that $\Delta_{qu,qd}$ are close to the unit matrix. Our interest is in investigating the size of $\arg \text{Det} \Delta_{qu,qd}$ in Eq. (32) and thus whether or not their presence makes the fine tuning between the two terms in $\bar{\theta}$ worse.

To compute $\text{Det} \Delta_{qu}$, we first write the real and imaginary parts of $\xi_r$ in terms of real constants $c_r$ and $i_r$ as

$$\text{Re} \xi_r = c_r, \quad \text{Im} \xi_r = i_r J_\xi,$$

with $J_\xi$ given in Eq. (19). Upon applying the Cayley-Hamilton identity, we then get

$$\text{Det} \Delta_{qu} = \frac{1}{6} (\text{Tr} \Delta_{qu})^3 - \frac{1}{2} \text{Tr} \Delta_{qu} \text{Tr}(\Delta_{qu}^2) + \frac{1}{3} \text{Tr}(\Delta_{qu}^3),$$

which leads us to

$$\text{Re}(\text{Det} \Delta_{qu}) \simeq c_1^3 + c_2^2 (c_2 y_t^2 + c_4 y_t^4),$$

$$J_\xi^{-1} \text{Im}(\text{Det} \Delta_{qu}) \simeq -c_2 (c_3 e_1 + c_3 (e_13 - e_14) + e_3 e_9 + e_7 e_{11}) - e_3 (e_4 e_{12} - e_4 e_{11} - e_6 e_9)
- (c_6 - c_7) (e_2 e_{10} + e_3 e_8 + e_4 e_5 - e_6 e_7) - e_2 (e_4 e_{17} + e_9 e_{13}) y_t^4
+ [-c_2 (e_2 e_{17} + e_4 e_{15} - e_6 e_{14} + e_7 e_{13} + e_9 e_{11}) - e_3 (e_6 e_{12} - e_8 e_9)
- (c_6 - c_7) (e_4 e_{10} - e_6 e_9)] y_t^2
+ c_1 \{ -c_2 e_1 - c_3 e_6 + e_4 e_5 + e_6 e_{13} - e_7 e_{14} + e_8 e_{11} - e_9 e_{10}
+ [2 e_2 \tau_1 - e_6 e_{16} + e_8 (e_{13} - e_{14}) + e_{11} e_{12}] y_t^2
+ (2 e_4 \tau_1 + e_{12} e_{13}) y_t^4 \}$$

where $y_q = \sqrt{2} m_q/v$ and on the right-hand sides we have ignored terms suppressed by powers of $y_{u,c,d,s,b}$. The formulas for $\text{Det} \Delta_{qd}$ are similar.

Since $y_t^2 \sim 1 \gg y_{u,c,d,s,b}^2$, the requirement that $\Delta_{qu,qd} \simeq \mathbb{1}$ implies that

$$c_1 \simeq 1, \quad |c_{2,4}| \ll 1, \quad |c_{3,5,6,17}| \leq \mathcal{O}(1), \quad |i_{1,2,17}| \leq \mathcal{O}(1).$$

Using these conditions and the quark parameter values employed earlier, we have checked numerically that Eqs. (35) and (36) approximate well the exact (but much lengthier) expressions, especially if $|c_{2,4}| \leq \mathcal{O}(0.001)$. Moreover, we get $|\arg \text{Det} \Delta_{qu,qd}| < 10^{-21}$. Obviously, the MFV effect is negligible compared to the present bound $\theta_{\text{exp}} < 10^{-10}$ [3].

C. Electron EDM

To evaluate the EDMs of charged leptons, we need the values of the various pertinent quantities, such as the elements of the neutrino mixing matrix $U_{\text{PMNS}}$ as well as the masses of neutrinos and charged leptons. If neutrinos are Dirac in nature, the parametrization of $U_{\text{PMNS}}$ is the same as $V_{\text{CKM}}$.
in Eq. (8). In Table I, we have listed \(\sin^2 \theta_{kl}\) and \(\delta\) from a recent fit to global neutrino data [24]. Most of these numbers depend on whether neutrino masses fall into a normal hierarchy (NH), where \(m_1 < m_2 < m_3\), or an inverted one (IH), where \(m_3 < m_1 < m_2\). If neutrinos are Majorana particles, \(U_{\text{PMNS}}\) contains an additional matrix \(P = \text{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)\) multiplied from the right, where \(\alpha_{1,2}\) are the Majorana phases which remain unknown.

Also listed in Table I are the differences in neutrinos’ squared masses, which are well determined. If neutrinos are of Dirac nature, we first note that the mass difference definitions in Table I imply that \(\Delta m_{21}^2 = (m_2^2 - m_1^2)\). If neutrinos are Majorana particles, \(d_e\) can be sizable. To see this, we begin with the simplest possibility that \(\nu_{k,R}\) are degenerate, \(M_\nu = M \mathbb{I}\), and the \(O\) matrix in Eq. (15) is real. For

Table I: Results of a recent fit to the global data on neutrino oscillations [24]. The neutrino mass hierarchy may be normal \((m_1 < m_2 < m_3)\) or inverted \((m_3 < m_1 < m_2)\).

| Observable | NH                      | IH                      |
|------------|-------------------------|-------------------------|
| \(\sin^2 \theta_{12}\) | 0.308 ± 0.017            | 0.308 ± 0.017            |
| \(\sin^2 \theta_{23}\) | 0.425^{+0.029}_{-0.027} | 0.437^{+0.059}_{-0.029} |
| \(\sin^2 \theta_{13}\) | 0.0234^{+0.0022}_{-0.0018} | 0.0239 ± 0.0021         |
| \(\delta/\pi\) | 1.39^{+0.33}_{-0.27}     | 1.35^{+0.24}_{-0.39}    |
| \(\Delta m_{21}^2 = m_2^2 - m_1^2\) | \((7.54^{+0.26}_{-0.22}) \times 10^{-5} \text{ eV}^2\) | \((7.54^{+0.26}_{-0.22}) \times 10^{-5} \text{ eV}^2\) |
| \(\Delta m^2 = |m_3^2 - (m_1^2 + m_2^2)/2|\) | \((2.44^{+0.08}_{-0.06}) \times 10^{-3} \text{ eV}^2\) | \((2.40 \pm 0.07) \times 10^{-3} \text{ eV}^2\) |
this scenario, \( d_e \) is already given in Eq. (25), which depends on the choice for one of \( m_{1,2,3} \) after the mass data are included. Scanning again the empirical parameter ranges in Table I to maximize \( d_e^\text{M} \), we obtain for \( m_1 = 0 \) (\( m_3 = 0 \)) in the NH (IH) case

\[
\frac{d_e^\text{M}}{\text{cm}} = 4.7 \times 10^{-23} \left( \frac{\mathcal{M}}{10^{15} \text{GeV}} \right)^3 \left( \frac{\Lambda}{\text{GeV}} \right)^2,
\]

where \( \hat{\Lambda} = \Lambda / |\xi_{12}|^{1/2} \). Then \( |d_e^{\exp}| < 8.7 \times 10^{-29} \text{ cm} \) [5] implies

\[
\hat{\Lambda} > 0.74 (0.24) \text{ TeV} \left( \frac{\mathcal{M}}{10^{15} \text{GeV}} \right)^{3/2}.
\]

Although this might suggest that \( \hat{\Lambda} \) could be extremely high with an excessively large \( \mathcal{M} \), there are limitations on \( \mathcal{M} \). Since the series in Eq. (5), which implicitly incorporates arbitrarily high powers of \( \mathcal{A} \) and \( \mathcal{B} \), has to converge, their eigenvalues need to be capped [12, 16]. Otherwise, the coefficients \( \xi_r \) might not converge to finite numbers after the reduction of \( \Delta \) from its infinite series expansion to Eq. (5). In the lepton sector, we only need to be concerned with \( \mathcal{A} = Y_\nu Y_\nu^\dagger \), as \( \mathcal{B} = Y_e Y_e^\dagger \) already has diminished eigenvalues. Thus one may demand that the eigenvalues of \( \mathcal{A} \) are at most 1. However, since MFV may emerge from calculations of SM loops, the expansion quantities may be more naturally be \( \mathcal{A}/(16\pi^2) \) and \( \mathcal{B}/(16\pi^2) \), in which case the maximum eigenvalue of \( \mathcal{A} \) cannot be more than \( 16\pi^2 \). As another alternative, one may impose the perturbativity condition on the Yukawa couplings, namely \( (Y_\nu)_{jk} < \sqrt{4\pi} \) [30], implying a cap of \( 4\pi \) instead.

In this paper we require the eigenvalues of \( \mathcal{A} = Y_\nu Y_\nu^\dagger \) not to exceed unity. Furthermore, in our illustrations we will choose the largest eigenmasses of the right-handed neutrinos subject to this condition. For the example resulting in Eq. (40), this translates into the maximal value \( \mathcal{M} = 6.16 (6.22) \times 10^{14} \text{ GeV} \) in the NH (IH) case and consequently

\[
\hat{\Lambda} > 0.36 (0.12) \text{ TeV}.
\]

This constraint would weaken if \( m_{1,(3)} > 0 \). For comparison with later illustrations, the \( \mathcal{M} \) numbers above translate into \( d_e^\text{M}\hat{\Lambda}^2 = 1.1 (0.13) \times 10^{-23} \text{ cm} \).

Now, with \( \nu_{k,R} \) still degenerate, \( M_\nu = \mathcal{M} \mathbb{1} \), but \( O \) complex, \( \mathcal{A} \) has a less simple expression,

\[
\mathcal{A} = \frac{2}{\nu^2} \mathcal{M} U_{\text{PMNS}} \hat{m}_\nu^{1/2} O O^\dagger \hat{m}_\nu^{1/2} U_{\text{PMNS}}^\dagger,
\]

which is to be applied to \( d_e^\text{M} \) in Eq. (22). From now on, we ignore the \( \xi_{16} \) parts. We can always write \( O O^\dagger = e^{2i\mathbf{R}} \) with a real antisymmetric matrix

\[
\mathbf{R} = \begin{pmatrix}
0 & r_1 & r_2 \\
-r_1 & 0 & r_3 \\
-r_2 & -r_3 & 0
\end{pmatrix}.
\]

Since \( O O^\dagger \) is not diagonal, \( \mathcal{A} \) will in general have dependence on the Majorana phases in \( U_{\text{PMNS}} \) if they are not zero. To concentrate first on demonstrating how \( O \) can give rise to \( CP \) violation beyond that induced by the Dirac phase \( \delta \) in \( U_{\text{PMNS}} \), we switch off the Majorana phases, \( \alpha_{1,2} = 0 \).
Subsequently, for illustrations, we pick two possible sets of $r_{1,2,3}$, namely, (i) $r_1 = -r_2 = r_3 = -\rho$ and (ii) $r_1 = 2r_2 = 3r_3 = \rho$, and employ the central values of the data in Table I, particularly

$$\delta = 1.39\pi \text{ (NH) or } 1.35\pi \text{ (IH)}.$$  \hspace{1cm} (44)

We present in Fig. 1(a)-(d) the resulting $d_c^M \hat{\Lambda}^2$ versus $\rho$ for the NH (IH) of light neutrino masses with $m_{1(3)} = 0$. Since $\delta$ is not yet well-determined, we also depict the variations of $d_c^M$ over the one-sigma ranges of $\delta$ quoted in Table I with the lighter blue and red bands. We remark that the boundaries of the bands do not necessarily correspond to the upper or lower ends of the $\delta$ ranges. Within these bands, the blue and red solid curves belong, respectively, to the NH and IH central values in Eq. (44). We also graph the (dashed) curves for $\delta = 0$ to reveal the $CP$-violating role of $O$ alone. The solid and dashed curves in Fig. 1(a,b) are roughly the mirror images about $\rho = 0$ of the corresponding curves given in Ref. [12] for $r_{1,2,3} = \rho$.

![Diagram](image)

FIG. 1: Dependence of $d_c^M$ times $\hat{\Lambda}^2 = \Lambda^2/|\xi_{12}|$ on the $O$-matrix parameter $\rho$ in the absence of the Majorana phases, $\alpha_{1,2} = 0$, for degenerate $\nu^c_{k,R}$ and complex $O$ with (a,b,e) $r_1 = -r_2 = r_3 = -\rho$ and (c,d,f) $r_1 = 2r_2 = 3r_3 = \rho$, as explained in the text. The lighter blue, red, and green bands reflect the one-sigma ranges of $\delta$, while the solid and dashed curves correspond, respectively, to its central values in Eq. (44) and to $\delta = 0$. In (e,f) and other QD plots below, only the NH scenario is assumed, unless stated otherwise.
In Fig. 1(a)-(d), as well as in Ref. [12], we have only examples where the lightest neutrinos are massless and, consequently, the neutrino masses sum up to \( \Sigma_k m_k = 0.059 \) eV and 0.099 eV in the NH and IH cases, respectively. These numbers satisfy the aforementioned bound from cosmological data, \( \Sigma_k m_k < 0.18 \) eV [27]. In light of the hints of quasidegenerate neutrinos with \( \Sigma_k m_k \sim 0.3 \) eV from other cosmological observations [28, 29], which still need confirmation by future measurements, here we also provide a couple of instances in Fig. 1(e,f) after making the NH choice \( m_1 = 0.1 \) eV \( < m_2 < m_3 \), which translates into \( \Sigma_k m_k = 0.31 \) eV.

All these examples in Fig. 1 clearly indicate that \( O \) can generate potentially significant new effects of CP violation which can exceed those of \( \delta \). The latter point is most noticeable in Fig. 1(b,d) from comparing the IH \( \delta \neq 0 \) regions at \( \rho \sim 0 \) with the extreme values of the corresponding IH \( \delta = 0 \) curves.

With \( \alpha_{1,2} = 0 \), the CP-violating impact of \( O \) can still materialize even if it is real provided that \( \nu_{k,R} \) are not degenerate. In that case

\[
A = \frac{2}{v^2} U_{\text{PMNS}} \hat{m}^{1/2}_\nu O M_\nu O^\dagger \hat{m}^{1/2}_\nu U_{\text{PMNS}}^\dagger
\] (45)

based on Eq. (15). For instance, assuming that \( O \) is real, \( O = e^R \) with \( r_1 = -r_2 = r_3 = -\rho \), and that \( M_\nu = M \, \text{diag}(1, 0.8, 1.2) \), we show the resulting

\[
 d_{\ell A}^{M^2}
\]

versus \( \rho \) in Fig. 2(a), where only

![Graph](image)

**FIG. 2:** Dependence of \( d_{\ell A}^{M^2} \) on \( O \)-matrix parameter \( \rho \) in the absence of Majorana phases, \( \alpha_{1,2} = 0 \), for nondegenerate \( \nu_{k,R} \) with \( M_\nu = M \, \text{diag}(1, 0.8, 1.2) \) and real \( O = e^R \) with (a,c) \( r_1 = -r_2 = r_3 = -\rho \) and (b,d) \( r_1 = 2r_2 = 3r_3 = \rho \), as explained in the text. The lighter blue, red, and green bands reflect the one-sigma ranges of \( \delta \), while the solid curves correspond to its central values in Eq. (44).
the $\delta \neq 0$ curves are nonvanishing and the sinusoidal behavior of $d_e$ is visible. As in the previous figure, we also display the variations of $d_e^M$ over the one-sigma ranges of $\delta$ from Table I. The solid curves in Fig. 2(a) are similar to their $r_{1,2,3} = \rho$ counterparts in Ref. [12]. As another example, we select again $r_1 = 2r_2 = 3r_3 = \rho$, keeping the other input parameters unchanged, and plot Fig. 2(b) which differs somewhat qualitatively from Fig. 2(a). In Fig. 2(c,d) we graph the QD cases with $m_1 = 0.1\,\text{eV} < m_2 < m_3$, which turn out to have much smaller $d_e^M$ ranges. All of these results further demonstrate the importance of $O$ as an extra source of $CP$ violation.

Turning our attention now to the contribution of the Majorana phases, we first illustrate it for $M_\nu = M \mathbb{I}$ and $O = e^{iR}$ with the two sets of $r_{1,2,3}$ chosen in the previous paragraph. Thus, fixing $\alpha_1 = 0$ and $\rho = \frac{1}{2}$, we depict the resulting dependence of $d_e^M$ on $\alpha_2$ in Fig. 3 for nonzero $\delta$ within its one-sigma ranges from Table I and also for $\delta = 0$. For further illustrations, we do the same with $M_\nu = M \text{diag}(1,0.8,1.2)$ and $O = e^{iR}$, displaying the results in Fig. 4. It is noticeable that each of the solid or dashed curves in Figs. 3 and 4 repeats itself after $\alpha_2$ changes by $4\pi$, which is attributable to the $e^{i\alpha_2/2}$ dependence of $d_e^M$ in these cases. Also, one can verify visually that the solid curves in Figs. 1 and 3 (2 and 4) are consistent with each other at $\rho = \frac{1}{2}$ and $\alpha_{1,2} = 0$. It is evident from the instances in Figs. 3 and 4, as well as their counterparts in Ref. [12], that the Majorana phases yield additional important $CP$-violating effects on $d_e$ beyond $\delta$.

It is interesting that some of the $CP$-violating variables which enter $d_e^M$ also affect neutrinoless double-$\beta$ decay due to the Majorana nature of the electron neutrino. This process is of fundamental
importance because it does not conserve lepton number and thus will be evidence for new physics if detected [25]. If there are no other contributions, the rate of neutrinoless double-$\beta$ decay increases with the square of the effective Majorana mass

$$\langle m_{\beta\beta} \rangle = \left| \sum_k U_{e k}^2 \tilde{m}_k \right| = \left| (U_{\text{PMNS}} \tilde{m}_e U_{\text{PMNS}}^T)_{11} \right| = \sqrt{c_{12}^2 c_{13}^2 m_1 e^{i\alpha_1} + s_{12}^2 c_{13}^2 m_2 e^{i\alpha_2} + s_{13}^2 m_3 e^{-2i\delta}}. \quad (46)$$

In Fig. 5 we display several examples of $\langle m_{\beta\beta} \rangle$ versus $\alpha_2$ for $\alpha_1 = 0$, but not those for $\delta = 0$ to avoid crowding the plots. It is obvious that each of the curves repeats itself after $\alpha_2$ changes by $2\pi$, which is due to the presence of $e^{i\alpha_2}$ in $\langle m_{\beta\beta} \rangle$, unlike the $d_e^M$ curves in Figs. 3 and 4. The peak values in the third plot of Fig. 5 are already close to the existing experimental upper limits on $\langle m_{\beta\beta} \rangle$, the best one being 0.12 eV [31]. Thus the QD possibility will be tested by forthcoming searches within the next decade, which are expected to have sensitivities reaching 0.04 eV to 0.01 eV [32].

From Figs. 3-5, one can conclude that $d_e^M$ and $\langle m_{\beta\beta} \rangle$ may be correlated. For the MFV scenario under consideration and the parameter choices we made with the central values from Table I, we show in Fig. 6 some sample relations between the two observables. One can see in particular that the plots in Fig. 6(a,c) [(d,f)] are related to the solid curves in the first and third (green) graphs of Fig. 5, respectively, and the corresponding solid curves in Fig. 3 [Fig. 4] for $r_1 = 2r_2 = 3r_3 = \frac{1}{2}$. In Fig. 6 we have also indicated a projected sensitivity of 0.04 eV in future hunts for neutrinoless double-$\beta$ decay which may be achieved after several years.
FIG. 5: Dependence of effective Majorana mass $\langle m_\beta \rangle$ on $\alpha_2$ for $\alpha_1 = 0$, nonzero $\delta$, and some selections of $m_1$ or $3$. The bands and solid curves have the same meanings as in previous figures.

The illustrations in Fig. 6 suggest that, if searches in coming years still yield null results, the acquired limits on $d_e$ and $\langle m_\beta \rangle$ will impose significant restrictions on various scenarios based on lepton MFV. On the other hand, unambiguous observations of $d_e^M$ and/or neutrinoless double-beta decay will help pin down the favored underlying model and parameter space, under the assumption that the latter process is mediated by a light Majorana neutrino [25]. The information to be gained from the direct neutrino-mass determination in planned tritium $\beta$-decay experiments, with

FIG. 6: Sample correlations between $d_e^M \Delta^2$ and $\langle m_\beta \rangle$ over $0 \leq \alpha_2 \leq 4\pi$ for $\alpha_1 = 0$ and the central values of $\delta$ in the cases of (a,b,c) degenerate $\nu_{k,R}$ and $O = e^R$ and (d,e,f) nondegenerate $\nu_{k,R}$ and $O = e^R$, all with $r_1 = 2r_2 = 3r_3 = \frac{1}{2}$, as described in the text. The vertical dashed lines mark a possible sensitivity in future searches for neutrinoless double-$\beta$ within decay the next decade.
expected sensitivities as low as 0.2 eV [25], and the total neutrino mass to be inferred from upcoming cosmological data with improved precision will supply complementary constraints and cross checks.

Before moving on, we would like to make some remarks on the situation in which only two right-handed neutrinos are added into the theory. In that case, \( Y_\nu \) and \( M_\nu \) as defined in Eq. (1) are 3×2 and 2×2 matrices, respectively. As a natural consequence [33], it is straightforward to realize from Eq. (14) that \(|\text{Det} m_\nu| = m_1 m_2 m_3 = 0\), indicating that one of \( m_{1,2,3} \) has to vanish. Another difference is that the \( O \) matrix in Eq. (12) is now 3×2. Accordingly, with \( m_1 = 0 \) or \( m_3 = 0 \) we can write respectively [34]

\[
O = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} O_2 \quad \text{or} \quad O = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} O_2
\]

where \( O_2 \) is a complex 2×2 matrix satisfying \( O_2 O_2^T = 1_2 \), where \( 1_2 \) is 2×2 unit matrix. Thus \( O_2 \) has 2 free real parameters, whereas \( O \) in the presence of 3 right-handed neutrinos has six. All this implies that the specific examples we have provided so far with \( m_1 \) or \( m_3 \) set to zero are applicable to the situation with only 2 right-handed neutrinos, as the 2 parameters of \( O_2 \) are functions of the 6 parameters of \( O \) in the case of 3 right-handed neutrinos with \( m_{1,3} = 0 \). We conclude that for \( d_e \) the situations with 2 and 3 right-handed neutrinos are similar.

D. Muon and tau EDMs

If neutrinos are of Dirac nature, the muon and tau EDMs will be tiny, like \( d_{e}^{D} \). Therefore here, and in the rest of the section, we suppose that neutrinos are Majorana fermions. Furthermore, for definiteness and simplicity, we consider only the scenario in which the right-handed neutrinos are degenerate, \( M_\nu = M \mathbb{1} \), and the orthogonal matrix \( O \) is real. For the neutrino parameters, we will adopt the specific values which yielded Eq. (39) and \( M = 6.16 \times 6.22 \times 10^{14} \text{GeV} \) in the NH (IH) case with \( m_{1(3)} = 0 \).

Accordingly, from Eq. (25) we easily infer the muon and tau EDMs, respectively, to be

\[
\begin{align*}
    d_\mu^M &= d_e^M \frac{m_\mu (m_\tau^2 - m_e^2)}{m_e (m_\mu^2 - m_\tau^2)}, \\
    d_\tau^M &= d_e^M \frac{m_\tau (m_e^2 - m_\mu^2)}{m_e (m_\mu^2 - m_\tau^2)},
\end{align*}
\]

with \( d_e^M \) in Eq. (39). Since \( d_e^M \sim -0.06 d_\mu^M \) and the experimental information on \( d_\tau \) is still imprecise [7], we will not deal with it further.

Hence we get \( d_\mu^M = -2.3 \times 0.26 \times 10^{-21} \text{GeV}^2/\hat{A}^2 \). Currently the best measured limit on the muon EDM is \( |d_\mu|_{\text{exp}} < 1.8 \times 10^{-19} \text{e cm} \) at 95% CL, set by the Muon \((g - 2)\) Collaboration [35]. This implies

\[
\hat{A} > 0.11 (0.038) \text{ GeV} \,
\]

which are not competitive to the bounds in Eq. (41) from \( |d_e|_{\text{exp}} \).
E. \(\text{CP}\)-violating electron-neutron interactions

Searches for atomic and molecular EDMs may be sensitive to other mechanisms possibly responsible for them besides the electron EDM, such as the EDMs of nuclei and \(\text{CP}\)-violating electron-nucleon interactions. In this section we are interested in the third possibility, particularly that described by [3, 4]

\[
\mathcal{L}_{eN} = \frac{-iC_S G_F}{\sqrt{2}} \bar{e} \gamma_5 e \bar{N} N - \frac{iC_P G_F}{\sqrt{2}} \bar{e} e \bar{N} \gamma_5 N - \frac{iC_T G_F}{\sqrt{2}} \bar{e} \sigma^{\mu\nu} \gamma_5 e \bar{N} \sigma_{\mu\nu} N .
\]

(50)

The recent ACME experiment has set the best limit on the first coupling, \(|C_S|_{\text{exp}} < 5.9 \times 10^{-9}\) at 90% CL [5]. The strictest limits on the other two, \(|C_P|_{\text{exp}} < 5.1 \times 10^{-7}\) and \(|C_T|_{\text{exp}} < 1.5 \times 10^{-9}\) at 95% CL, were based on the latest search for the EDM of the \(^{199}\text{Hg}\) atom [36].

These interactions may originate from MFV in the lepton sector as well as the quark sector, which has to be included for a consistent analysis. The Lagrangian for the relevant lowest-order operators is

\[
\mathcal{L}_{\ell q} = \frac{1}{\Lambda^2} \left( \bar{U}_R Y_a^\dagger \bar{\Delta}_{qu1} i \tau_2 Q_L \bar{E}_R Y^\dagger L_L + \bar{Q}_L \bar{\Delta}_{qd1} Y_d D_R \bar{E}_R Y^\dagger L_L \\
+ \bar{U}_R \sigma^{\mu\nu} Y_a^\dagger \bar{\Delta}_{qu2} i \tau_2 Q_L \bar{E}_R \sigma_{\mu\nu} Y^\dagger L_L \\
+ \bar{Q}_L \sigma^{\mu\nu} \bar{\Delta}_{qd2} Y_d D_R \bar{E}_R \sigma_{\mu\nu} Y^\dagger L_L \right) + \text{H.c. ,}
\]

(51)

where \(\bar{\Delta}_{qu,qd} (\bar{\Delta}_{\ell1,\ell2,\ell3,\ell4})\) are the same in form as \(\Delta\) in Eq. (5) and contain the quark (lepton) Yukawa couplings. The leptonic contributions to \(C_{S,P,T}\) turn out to be dominant.

To determine \(C_S\), we need the matrix elements \(\langle N | m_{\ell q} | N \rangle = g_{\ell q}^N \bar{u}_N u_N v\). Thus, we derive

\[
C_S = \frac{16 \sqrt{2} m_e \Lambda^3}{\Lambda^2 G_F v^9} (m_1^2 - m_2^2) (m_1 - m_2) (m_2 - m_3) (m_3 - m_1) \\
\times \left[ (g_u^N + g_c^N + \kappa_{u1} g_\ell^N) \tilde{\xi}_{12}^{\ell1} - (g_d^N + g_s^N + \kappa_{d1} g_\ell^N) \tilde{\xi}_{12}^{\ell2} \right] J_\ell ,
\]

(52)

where \(\tilde{\xi}_{12}^{\ell1,\ell2}\) belong to \(\bar{\Delta}_{\ell1,\ell2}\) and have absorbed the first coefficients \(\xi_{1a}^{u1,du1}\) of \(\bar{\Delta}_{qu1,qd1}\), respectively, and \(\kappa_x \simeq 1 + (\xi_2^x + \xi_\ell^x)/\xi_1^x\) are numbers expected to be at most \(\mathcal{O}(1)\). Numerically, we adopt the chiral Lagrangian estimate [37]

\[
g_u^N = 0.04 (0.12) \times 10^{-3}, \quad g_d^N = 0.08 (0.21) \times 10^{-3},
\]

(53)

\[
g_s^N = 0.25 (2.88) \times 10^{-3}, \quad g_{c,h,t}^N = 0.26 (0.05) \times 10^{-3},
\]

(54)

corresponding to the so-called pion-nucleon sigma term \(\sigma_{\pi N} = 30 (80)\) MeV, which is not yet well-determined [38, 39]. Then, using the maxima of \(g_q^N\) and assuming \(\kappa_x = 1\), we can neglect the \(\tilde{\xi}_{12}^{\ell1}\) part in Eq. (52) to obtain from \(|C_S|_{\text{exp}} < 5.9 \times 10^{-9}\)

\[
\frac{\Lambda}{|\tilde{\xi}_{12}^{\ell2}|^{1/2}} > 0.27 (0.091) \text{ GeV}
\]

(55)

4 Lattice QCD computations [38] tend to produce results smaller than those of chiral Lagrangian calculations and some other methods [39]. As a consequence, employing the lattice values of \(g_q^N\) in Eq. (52) would yield even looser limits than in Eq. (55).
in the NH (IH) neutrino parameter values specified in the preceding subsection. These restraints are far weaker than those from \(|d_e|_{\text{exp}}\).

For \(C_F\), the expression is the same as that for \(C_\xi\) in Eq. (52), except \(g^N_q\) is replaced by \(\zeta_q h^N_q m_N/v\) with \(\zeta_q = +1 (-1)\) if \(q = u, c, t (d, s, b)\) and \(h^N_q\) defined by \(\langle N| m_q \bar{q} \gamma_5 \bar{q} |N\rangle = h^N_q m_N \bar{u}_N \gamma_5 u_N\). Since for mercury \(C_F\) is estimated to be mostly from the neutron contribution [4], we focus on it. Ignoring the effects of \(h_{c,b,t}\), we can relate \(h^u_{u,d,s}\) to the axial-vector charges \(g_A^{(0,3,8)}\) by \(6h_u^u = 2g_A^{(0)} - 3g_A^{(3)} + g_A^{(8)}\), \(6h_d^d = 2g_A^{(0)} + 3g_A^{(3)} + g_A^{(8)}\), and \(3h_s^s = g_A^{(0)} - g_A^{(8)}\), where \(g_A^{(0)} = 0.33 \pm 0.06\), \(g_A^{(3)} = 1.270 \pm 0.003\), and \(g_A^{(8)} = 0.58 \pm 0.03\) were measured in baryon \(\beta\)-decay and deep inelastic scattering experiments [40]. Taking \(\xi_{12}^\ell = \tilde{\xi}_{12}^\ell\) and maximizing \(C_F\), we obtain from \(|C_F|_{\text{exp}} < 5.1 \times 10^{-7}\)

\[
\frac{\Lambda}{|\xi_{12}^{1/2}|} > 0.020 (0.0068) \text{ GeV},
\]

less restrictive than Eq. (55) by more than an order of magnitude.

To evaluate \(C_T\), we need the matrix elements \(\langle N| \bar{q} \sigma^{\alpha\omega} q |N\rangle = \rho^\alpha_N \bar{u}_N \sigma^{\alpha\omega} u_N\), where \(\rho^\alpha_N\) have the values in Eq. (28) for light quarks, assuming isospin symmetry, and vanish for heavier quarks. This leads us to

\[
C_T = \frac{32\sqrt{2} m_e M^3}{\Lambda^2 G_F v^{10}} (m_e^2 - m_\mu^2) (m_1 - m_2) (m_2 - m_3) (m_3 - m_4) \rho^u_u m_u \tilde{e}_{12}^3 J_\ell,
\]

where \(\tilde{e}_{12}^3\) belongs to \(\bar{\Delta}_{ee}\) and has absorbed \(\tilde{e}_{12}^{u2}\) from \(\bar{\Delta}_{qu2}\). The contributions of the down-type quarks cancel due to the relation \(\bar{q} \sigma^{\alpha\omega} \gamma_5 q \bar{e} \sigma^{\alpha\omega} e = \bar{q} \sigma^{\alpha\omega} q \bar{e} \sigma^{\alpha\omega} e\). Hence, with the largest \(m_u\) from Section IV A and \(\rho^u_u = -0.78\), we get from \(|C_T|_{\text{exp}} < 1.5 \times 10^{-9}\)

\[
\frac{\Lambda}{|\xi_{12}^{1/2}|} > 0.033 (0.011) \text{ GeV},
\]

comparable to Eq. (56).

**F. Muon \(g - 2\), \(\mu \to e\gamma\), nuclear \(\mu \to e\) conversion, \(\bar{B} \to X_s\gamma\)**

The MFV coefficient \(\xi_{12}\) that determines the electron EDM also enters the anomalous magnetic moment of the muon \((g_\mu - 2)\) and the rates of the radiative decay \(\mu \to e\gamma\) and nuclear \(\mu \to e\) conversion, the latter two being still unobserved. Since \(g_\mu - 2\) has been very precisely measured and the experimental limits of the flavor-changing transitions are stringent, it is important to check if these processes can yield stronger bounds on \(\bar{\Lambda} = \Lambda / |\xi_{12}^{1/2}|\) than those evaluated in the preceding subsections. Although the other \(\xi_{\tau \neq 12,16}\) terms may contribute to these processes as well and therefore may reduce the impact of the \(\xi_{12}\) term, one also cannot rule out the possibility of a scenario in which the latter dominates the other contributions.

The anomalous magnetic moment \(a_\ell\) of lepton \(\ell\) is described by \(L_{a_\ell} = [e a_\ell / (4m_\ell)] \bar{\ell} \sigma^{\alpha\omega} l F^{\alpha\omega}\).

From Eq. (18) we have

\[
L_{E_i \to E_k \gamma} = \frac{e}{2\Lambda^2} \bar{E}_k \sigma^{\alpha\omega} \left\{ m_{E_k}(\Delta_\ell)_{ki} + m_{E_i}(\Delta_\ell)^*_{ik} - \left[m_{E_k}(\Delta_\ell)_{ki} - m_{E_i}(\Delta_\ell)^*_{ik}\right] \gamma_5 \right\} E_i F^{\alpha\omega},
\]
where \((E_1, E_2, E_3) = (e, \mu, \tau)\) and \(\Delta_\ell = \Delta_{\ell 1} - \Delta_{\ell 2}\). It follows that

\[
a_{E_k} = \frac{4m_{E_k}^2}{\Lambda^2} \text{Re}(\Delta_\ell)_{kk} .
\]

Thus, with the NH neutrino parameter values specified in Section IV D, we have

\[
a_\mu = \frac{4m_\mu^2}{\Lambda^2} \text{Re}(\Delta_\ell)_{22} = \left(45 \xi_1^\ell + 23 \xi_2^\ell + 20 \xi_4^\ell + 0.00085 \xi_8^\ell + 0.00094 \xi_{12}^\ell \right) \frac{\text{GeV}^2}{10^4 \Lambda^2} \tag{61}
\]

where terms with numerical factors much smaller than that of \(\xi_{12}^\ell\) have been dropped. The corresponding numbers in the IH case are roughly similar. Currently the experimental and SM values differ by \(a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (249 \pm 87) \times 10^{-11}\) [41], which suggests that we can require \(|a_\mu| < 3.4 \times 10^{-9}\). For the \(\xi_{12}^\ell\) term alone, this translates into the rather loose limit \(\hat{\Lambda} > 17\ \text{GeV}\), which may be weakened in the presence of the other terms in Eq. (61).

From Eq. (59), one can also calculate the branching ratio \(B(\mu \rightarrow e\gamma)\) of \(\mu \rightarrow e\gamma\). In the \(m_e = 0\) limit

\[
B(\mu \rightarrow e\gamma) = \frac{\tau_\mu e^2 m_\mu^5}{4\pi \Lambda^4} |(\Delta_\ell)_{21}|^2 ,
\]

where \(\tau_\mu\) is the muon lifetime. In the NH case

\[
(\Delta_\ell)_{21} = (0.061 - 0.1i)\xi_2^\ell + (0.011 - 0.11i)\xi_4^\ell - \left[ (26 + 48i)\xi_8^\ell + (6 + 51i)\xi_{12}^\ell \right] \times 10^{-7} ,
\]

where again terms with numerical factors less than that of \(\xi_{12}^\ell\) have been ignored. The \((\Delta_\ell)_{21}\) numbers in the IH case are comparable in size. If only \(\xi_{12}^\ell\) is nonvanishing in \((\Delta_\ell)_{21}\), then the experimental bound \(B(\mu \rightarrow e\gamma)_{\text{exp}} < 5.7 \times 10^{-13}\) [42] implies

\[
\hat{\Lambda} > 2.0\ \text{TeV} .
\]

This is stronger by up to \(\sim 20\) times than those in Eq. (41) from the electron EDM data. However, the other terms in \((\Delta_\ell)_{21}\), some of which are potentially much bigger than the \(\xi_{12}^\ell\) contribution, can in principle decrease the impact of the latter, thereby lessening the restriction on \(\hat{\Lambda}\). Consequently, \(d_e\) provides a less ambiguous probe for \(\hat{\Lambda}\).

Measurements on \(\mu \rightarrow e\) conversion in nuclei can provide constraints on new physics competitive to those from \(\mu \rightarrow e\gamma\) searches [43]. The relation between the rates of \(\mu \rightarrow e\) conversion and \(\mu \rightarrow e\gamma\) produced by possible new physics is available from Ref. [44]. Assuming that the MFV dipole interactions described by Eq. (17) saturate \(\mu \rightarrow e\) conversion in nucleus \(N\), we can express its rate divided by the rate \(\omega_{\text{capt}}^N\) of \(\mu\) capture in \(N\) as

\[
B(\mu N \rightarrow e N) = \frac{e^2 m_\mu^5 |(\Delta_\ell)_{21}|^2 D_N}{4\Lambda^4 \omega_{\text{capt}}^N} ,
\]

where \(D_N\) represents the dimensionless overlap integral for \(N\) and for the NH parameter choices \((\Delta_\ell)_{21}\) is given in Eq. (63). Based on the existing experimental limits on \(\mu \rightarrow e\) transition in various nuclei [7] and the corresponding \(D_N\) and \(\omega_{\text{capt}}^N\) values [44], significant restrictions can be expected from \(B(\mu \text{Ti} \rightarrow e \text{Ti})_{\text{exp}} < 6.1 \times 10^{-13}\) [45] and \(B(\mu \text{Au} \rightarrow e \text{Au})_{\text{exp}} < 7 \times 10^{-13}\) [7]. From
these data, if only the $\xi_{12}^\ell$ term in $(\Delta \epsilon)_{21}$ is nonvanishing, employing $D_{Ti} = 0.087$, $D_{Au} = 0.189$, $\omega_{\text{capt}}^{Ti} = 2.59 \times 10^6/s$, and $\omega_{\text{capt}}^{Au} = 13.07 \times 10^6/s$ [44], we extract

$$\hat{\Lambda}_{Ti} > 0.49 \text{ TeV} , \quad \hat{\Lambda}_{Au} > 0.47 \text{ TeV} ,$$

which are stricter than the results in Eq. (55) by up to a few times, but weaker than Eq. (64). Upcoming searches for $\mu \to e$ in the next several years will, if it still eludes detection, lower the limits to the $10^{-16}$ level or better [43], which will push $\hat{\Lambda}$ higher. Nevertheless, since again the other $\xi_{r}^\ell$ terms are generally present in $(\Delta \epsilon)_{21}$, these bounds on $\hat{\Lambda}$ are not unambiguous. Thus $d_e$ provides the best probe for $\hat{\Lambda}$ in connection with $CP$ violation.

Since there is a possibility that the MFV scales in the lepton and quark sectors are equal or related to each other, it is of interest to check if there are any quark processes that can also offer bounds stronger than those on $\hat{\Lambda}$ from $d_e$. Since, as we saw in Section IV A, the neutron EDM could not provide a competitive constraint, we need to look at other processes. The most stringent restriction on the quark MFV scale turns out to be from the rare decay $\bar{B} \to X_s \gamma$ [10]. Its experimental and SM branching ratios are $B(\bar{B} \to X_s \gamma)_{\text{exp}} = (3.43 \pm 0.22) \times 10^{-4}$ [46] and $B(\bar{B} \to X_s \gamma)_{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4}$ [47] both for the photon energy $E_\gamma > 1.6 \text{ GeV}$. To isolate the MFV contribution, we adopt from Ref. [10] the relation

$$B(\bar{B} \to X_s \gamma)_{\text{exp}} \simeq (1 - 2.4 C_{\gamma \gamma}^{\text{MFV}}) B(\bar{B} \to X_s \gamma)_{\text{SM}} ,$$

where $C_{\gamma \gamma}^{\text{MFV}}$ is evaluated at $\mu = m_W$ and enters the effective Lagrangian

$$\mathcal{L}_{b \to s \gamma} = \frac{e G_F m_b}{8\sqrt{2} \pi^2} V_{ts}^* V_{tb} (C_{\gamma \gamma}^{\text{SM}} + C_{\gamma \gamma}^{\text{MFV}}) \bar{s} \sigma^{\mu \nu} (1 + \gamma_5) b F_{\kappa \omega} ,$$

implying that

$$C_{\gamma \gamma}^{\text{MFV}} = \frac{4\sqrt{2} \pi^2 (\Delta_{qd})_{32}^2}{\Lambda^2 G_F V_{ts}^* V_{tb}} .$$

For the central values of the quark masses quoted in Section IV A

$$\frac{(\Delta_{qd})_{32}}{V_{ts}^* V_{tb}} = \xi_d y_t^2 + \xi_d^d y_t^4 + y_b^2 (\xi_{7d} y_t^2 + \xi_{8d} y_t^4 + \xi_{9d} y_t^4 + \xi_{12d} y_t^6) ,$$

where the imaginary parts and other $\xi_d^d$ terms are negligible, $y_b^2 \simeq 1$, and $y_t^2 \simeq 0.0003$. Combining the errors in quadrature for the ratio of branching ratios in Eq. (67) and assuming that $\xi_{r \neq 12}^d = 0$, we obtain at 90% CL

$$\frac{\Lambda}{|\xi_{12}^d|^{1/2}} > 0.19 (0.11) \text{ TeV}$$

if $C_{\gamma \gamma}^{\text{MFV}}$ has destructive (constructive) interference with the SM term. These numbers are somewhat lower than those in Eq. (41) and, as in the lepton cases, may go down in the presence of the other $\xi_d^d$ terms in Eq. (70).
V. CONCLUSIONS

We have explored $CP$ violation beyond the SM via fermion EDMs under the framework of minimal flavor violation. The new physics scenarios covered are the standard model slightly expanded with the addition of three right-handed neutrinos and its extension including the seesaw mechanism for endowing neutrinos with light mass. Addressing the quark sector first, we find that the present empirical limit on the neutron EDM implies only a loose constraint on the scale of quark MFV. Moreover, we show that the impact of MFV on the contribution of the strong theta-term to the neutron EDM is insignificant. Turning to the lepton sector, we demonstrate that the current EDM data also yield unimportant restraints on the leptonic MFV scale if neutrinos are of Dirac nature. In contrast, if neutrinos are Majorana particles, the constraints become tremendously more stringent and, in light of the latest search for $d_e$ by ACME, restrict the MFV scale to above a few hundred GeV or more. Furthermore, $d_e$ can be connected in a complementary way to neutrinoless double-$\beta$ decay if it is induced mainly or solely by the exchange of a light Majorana neutrino. We find in addition that constraints on the MFV scale inferred from the $CP$-violating electron-nucleon couplings probed by ACME and the most recent search for the EDM of mercury are relatively weak as well. Finally, we take into account potential restrictions from the measurements on the muon $g - 2$, radiative decays $\mu \rightarrow e\gamma$ and $\bar{B} \rightarrow X_s\gamma$, and $\mu \rightarrow e$ conversion in nuclei, which are not sensitive to $CP$ violation.

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Appendix A: Evaluation of some products of $A$ and $B$ matrices

From the Cayley-Hamilton identity in Eq. (4) with $X = aA + bB$, where and $a$ and $b$ are free parameters, one can extract [15]

\[
A^2B + ABA + BA^2 = A^2\langle B \rangle + (AB + BA)\langle A \rangle + A(\langle AB \rangle - \langle A \rangle \langle B \rangle) + \frac{1}{2}(\langle A^2 \rangle - \langle A \rangle^2)B \\
+ \mathbb{I}\left[\frac{1}{2}(\langle A \rangle^2 - \langle A^2 \rangle)\langle B \rangle + \langle A^2B \rangle - \langle A \rangle \langle AB \rangle\right]
\]  
(A1)

and an analogous expression for $ABB + BAB + BBA$, where $\langle \cdots \rangle = \text{Tr}(\cdots)$. These relations can be used to derive other combinations of $A$ and $B$. For instance, by replacing $B$ with $B^2 ([B, AB])$ in Eq. (A1), we can write $A^2B^2 + AB^2A + B^2A^2$ in terms of lower-ordered products of these matrices. After further algebra, we arrive at

\[
A^2BAB^2 = \zeta_1 \mathbb{I} + \zeta_2 A + \zeta_3 B + \zeta_4 A^2 + \zeta_5 B^2 + \zeta_6 AB + \zeta_7 BA + \zeta_8 ABA + \zeta_9 BA^2 \\
+ \zeta_{10} BAB + \zeta_{11} AB^2 + \zeta_{12} ABA^2 + \zeta_{13} A^2B^2 + \zeta_{14} B^2A^2 + \zeta_{15} B^2AB \\
+ \zeta_{16} AB^2A^2 + \zeta_{17} B^2A^2B
\]
(A2)
where

\[
\zeta_1 = \frac{\langle A^2 B A B^2 \rangle + \langle A B \rangle \langle A^2 B^2 \rangle + \langle A^2 B \rangle \langle A B^2 \rangle}{3} + \langle A \rangle \langle B \rangle \frac{4 \langle A^2 \rangle \langle B^2 \rangle - 6 \langle A^2 B^2 \rangle - 3 \langle A \rangle^2 \langle B \rangle^2}{6} \\
+ \frac{(\langle B \rangle^3 - \langle B \rangle \langle B^2 \rangle) \text{Det} A + (\langle A \rangle^3 - \langle A \rangle \langle A^2 \rangle) \text{Det} B}{6} + \langle A B \rangle \frac{13 \langle A \rangle^2 \langle B \rangle^2 - 3 \langle A^2 \rangle \langle B^2 \rangle}{12} + \langle A^2 B \rangle \langle B \rangle \frac{\langle A \rangle^2 - \langle A^2 \rangle}{6} \\
- \langle A \rangle \langle B^2 \rangle \frac{\langle A^2 \rangle \langle B \rangle + 2 \langle A^2 B \rangle}{6}
\]

(A3)

\[
\zeta_2 = -\frac{\langle A^2 \rangle \text{Det} B}{3} + \langle A B \rangle \frac{4 \langle A^2 B \rangle - 5 \langle A \rangle^2 \langle B \rangle - 3 \langle A \rangle \langle B \rangle}{6} + \langle B \rangle \frac{7 \langle A \rangle^2 + \langle A^2 \rangle}{12} \\
- \langle B \rangle \frac{2 \langle A \rangle \langle A B^2 \rangle + \langle A^2 B \rangle}{3} + \langle B \rangle^2 \frac{2 \langle A^2 B \rangle}{3} + \langle A \rangle \langle B \rangle \frac{9 \langle A \rangle^2 - \langle A^2 \rangle}{12}
\]

(A4)

\[
\zeta_3 = -\frac{\langle B \rangle^2 \text{Det} A}{3} + \langle A B \rangle \frac{2 \langle A^2 B \rangle - 5 \langle A \rangle^2 \langle B \rangle - \langle A^2 \rangle \langle B \rangle}{6} + \langle A \rangle \langle B \rangle \frac{3 \langle B \rangle^2 - \langle B^2 \rangle}{12} \\
- \langle A \rangle \frac{\langle A^2 B \rangle \langle B \rangle + \langle A^2 B \rangle}{3} + \langle A \rangle \langle B \rangle \frac{\langle A \rangle^2 + \langle A^2 \rangle}{6} + \langle A \rangle \frac{9 \langle B \rangle^2 + \langle B^2 \rangle}{12}
\]

(A5)

\[
\zeta_4 = \frac{\langle A \rangle \text{Det} B}{3} + \langle B^2 \rangle \frac{2 \langle A B \rangle - 7 \langle A \rangle \langle B \rangle}{6} - \langle A \rangle \langle B \rangle \frac{\langle A \rangle^3}{3} + \langle B \rangle \frac{\langle A^2 B \rangle}{3}
\]

(A6)

\[
\zeta_5 = \frac{\langle B \rangle \text{Det} A}{3} + \langle A \rangle^2 \frac{2 \langle A B \rangle - 5 \langle A \rangle \langle B \rangle}{6} - \langle A \rangle \langle B \rangle \frac{\langle A \rangle^3}{3}
\]

(A7)

\[
\zeta_6 = \langle A^2 \rangle \frac{\langle B^2 \rangle + \langle B \rangle^2}{6} - \langle A \rangle^2 \frac{\langle A^2 \rangle + 7 \langle B \rangle^2}{6} + \frac{2 \langle A \rangle \langle A^2 B \rangle + \langle B \rangle \langle A^2 B \rangle - 2 \langle A^2 B \rangle}{3}
\]

(A8)

\[
\zeta_7 = \langle A^2 \rangle \frac{\langle B^2 \rangle^2 - \langle B \rangle^2}{12} - \langle A \rangle^2 \frac{5 \langle B^2 \rangle + 11 \langle B \rangle^2}{12} + \frac{2 \langle A \rangle \langle A^2 B \rangle + \langle B \rangle \langle A^2 B \rangle - 2 \langle A^2 B \rangle}{3}
\]

(A9)

\[
\zeta_8 = \langle A \rangle \frac{5 \langle B \rangle^2 + 3 \langle B^2 \rangle}{6} - \frac{2 \langle A^2 B \rangle}{12}
\]

\[
\zeta_9 = \frac{\langle A \rangle \langle B^2 \rangle - \langle A^2 B \rangle}{3}
\]

(A10)

\[
\zeta_10 = \langle B \rangle \frac{5 \langle A \rangle^2 + \langle A^2 \rangle}{6} - \frac{\langle A^2 B \rangle}{3}
\]

\[
\zeta_{11} = \frac{\langle A^2 B \rangle - \langle A^2 \rangle \langle B \rangle}{3}
\]

(A11)

\[
\zeta_{12} = -\frac{\langle B \rangle^2}{2} - \frac{\langle B^2 \rangle}{6}
\]

\[
\zeta_{13} = \frac{4 \langle A \rangle \langle B \rangle + \langle A B \rangle}{3}
\]

(A12)

\[
\zeta_{14} = \langle A \rangle \langle B \rangle - \frac{\langle A \rangle \langle B \rangle}{3}
\]

\[
\zeta_{15} = \frac{-\langle A \rangle^2}{2} - \frac{\langle A^2 \rangle}{6}
\]

(A13)

\[
\zeta_{16} = \frac{2 \langle B \rangle}{3}
\]

\[
\zeta_{17} = \frac{2 \langle A \rangle}{3}
\]

(A14)
The Hermiticity of $A$ and $B$ implies that all the traces and determinants in $\zeta_{1,2,\ldots,17}$ are purely real, except $\langle A^2 BAB^2 \rangle$ in $\zeta_1$ which has an imaginary component

$$J_\xi = \text{Im} \langle A^2 BAB^2 \rangle = \frac{i}{2} \text{Det}[A, B]$$

(A15)

obtainable from the Cayley-Hamilton identity

$$(A, B)^3 = 1 \text{ Det}[A, B] + \frac{1}{2} [A, B] (\langle [A, B]^2 \rangle - \langle [A, B] \rangle^2) + [A, B]^2 \langle [A, B] \rangle.$$ \hspace{1cm} (A16)

Clearly the reduction of $A^2 BAB^2$ into a sum of matrix products with lower orders causes the coefficient $\zeta_1$ to gain an imaginary component equal to $J_\xi$. It follows that higher-order matrix products containing $A^2 BAB^2$ will lead to contributions to the coefficients $\xi_r$ with imaginary parts which are always proportional to $J_\xi$. 

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