Chiral phase transition of $N_f=2+1$ QCD with the HISQ action

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QCD phase diagram at \( \mu = 0 \)

**columbia plot:**

- **\( N_f=2+1 \) theory:** at \( m=0 \) or \( \infty \) has a first order phase transition
  - Pisarski, Wilczek PRD '84, Alexandrou et al., PRD'99...

- Intermediate quark mass region an analytic cross over is expected

- At physical quark masses, a cross over is confirmed
  - Bernard et al., PRD '05, Cheng et al., PRD '06, Aoki et al., Nature '06...

- Critical lines of second order transition
  - \( N_f=2 \): \( O(4) \) universality class
  - \( N_f=3 \): Ising universality class
  - Ejiri et al., PRD '09, ...
  - Karsch, Laermann, Schmidt PLB '04,...

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🌟 How large is the chiral phase transition \( T_c \) ?

🌟 How large is the influence of scaling regimes to the physical world ?
QCD at low energies can be described effectively by $O(N)$ symmetric spin models

- $SU(2)_L \times SU(2)_R$ is isomorphic to $O(4)$

- $O(4)$ fields: $\sigma = \bar{q}q$, $\pi = \bar{q}\gamma_5 i q$, and $\eta = \bar{q}\gamma_5 q$, $\delta = \bar{q}t^i q$

- external field $H$ corresponds to quark mass $m$

- order parameter “magnetization” $\Sigma = \langle \sigma \rangle$

This description is valid both below and in the vicinity of the chiral phase transition region
chiral phase transition and universal scaling

Behavior of the free energy close to critical lines

\[ f(m,T) = h^{1+1/\delta} f_s(z) + f_{\text{reg}}(m,T), \quad z = t/h^{1/\beta \delta} \]

- \( h \): external field
- \( t \): reduced temperature
- \( \beta, \delta \): universal critical exponents
- \( f_s(z) \): universal scaling function, O(N) etc.

Magnetic Equation of State (MEmoS):

\[ M = -\partial f_s(t,h)/\partial h = h^{1/\delta} f_G(z) \]
\[ f_\chi(z) = h_0^{1/\delta} (m_l/m_s)^{1-1/\delta} \partial M/\partial h \]
Recent $O(N)$ universal scaling studies

$$f(m, T) = h^{1 + 1/\delta} f_s(z) + f_{\text{reg}}(m, T), \quad z = t/h^{1/\beta \delta}$$

$$M_b = m_s \langle \bar{\psi} \psi \rangle / T^4$$

- the scaling window depends on discretization schemes: standard v.s. improved staggered fermions
- scaling violations seen at $m_f/m_s > 1/10$ using p4 action on $N_t=4$ lattices
Recent $O(N)$ universal scaling studies

Reasonably good prediction of chiral susceptibilities using parameters obtained from the scaling fit to chiral condensates

Useful tool to determine the critical temperature, chiral curvature etc.
Nf=2+1 QCD

- Fix the strange quark mass to be physical
- Decrease the light quark mass approaching to the chiral limit
- Simulations on Nt=6 lattices using Highly Improved Staggered Quarks with 5 different quark masses corresponding to $m_\pi$ from 160 MeV down to 80 MeV

- The chiral first order phase transition region shrinks with better improved staggered fermions $m_{\pi}^c \approx 290$ MeV $\rightarrow m_{\pi}^c \approx 45$ MeV

- 2nd order O(4) scaling regime may have more influence on the physical world?

HTD, xQCD 2012, arXiv:1302.5740
volume dependence at physical pion mass

- volume effects are small in 3 largest volume

- $m_\pi L > 4$ is ensured in the following other datasets
  
  48$^3$x6 with $m_\pi=80$ MeV, 40$^3$x6 with $m_\pi=90$ MeV, 
  32$^3$x6 with $m_\pi=110$ MeV, 24$^3$x6 with $m_\pi=160$ MeV
• Mild volume dependence is seen from chiral condensates

• No evidence of linear volume scaling as signatures of first order phase transition

• Volume scaling analysis needs to understand the volume effects
chiral condensates & susceptibilities

- chiral condensates decrease with increasing temperature and decreasing quark mass

- peak heights of chiral susceptibilities increase and peak locations shift to lower temperatures with decreasing quark mass
O(N) scaling behavior

For large negative values of $z$

$$f_G(z) \sim f_G^{-\infty}(z) = (-z)^\beta \left( 1 + c_2 \beta (-z)^{-\beta \delta / 2} \right)$$

$$M = h^{1/\delta} f_G(z) \sim h^{1/\delta} f_G^{-\infty}(z) = (-t)^\beta \left( 1 + c_2 \beta (-t)^{-\beta \delta / 2} \sqrt{h} \right)$$

Engels et al., PLB 514(2001)299

contribution of Goldstone modes to the order parameter $M$

is enclosed in the scaling function in the low temperature

susceptibility of the order parameter $\sim 1/\sqrt{h}$

For large positive values of $z$

$$f_G(z) \sim R_\chi z^{-\beta(\delta-1)}$$

Engels et al., NPB 675(2003)533

$$M = h^{1/\delta} f_G(z) \sim R_\chi t^{-\beta(\delta-1)} h$$

susceptibility of the order parameter is independent of $h$
At very low temperature, the disconnected susceptibilities scale as square root of quark mass.

At $T \sim 170$ MeV, the disconnected susceptibilities seem to be independent on quark mass: a likely indication of $U(1)_A$ symmetry breaking.

Chris Schroeder’s talk on $U(1)_A$ from DWF, 15:30 today
scaling and scaling violation of the chiral condensate
\[ M = -\frac{\partial f_s(t,h)}{\partial h} = h^{1/\delta} f_G(z) \]

- The right plot is generated using the fitting parameters obtained from the fit to the two lightest quark mass shown in the left plot.
- Scaling violation of chiral condensates seen with \( m_{\pi} \geq 110 \text{ MeV} \) \((m_l/m_s \geq 1/40)\) using the HISQ action on \( Nt=6 \) lattices.
scaling and scaling violation of the chiral condensate

\[ M = -\partial f_s(t,h)/\partial h = h^{1/\delta} f_G(z) \]

\[ \frac{M}{h^{1/\delta}} \]

\[ \frac{m_l/m_s}{2/5, 1/5, 1/10, 1/20, 1/40, 1/80} \]

\[ t/h^{1/\beta\delta} \]

\[ z=t/h^{1/\beta\delta} \]

Navigation: \( m_\pi = 160 \text{ MeV} \sim m_l/m_s=1/20, \ m_\pi = 80 \text{ MeV} \sim m_l/m_s=1/80 \)

- scaling violation of chiral condensates seen with \( m_\pi \geq 110 \text{ MeV} \) (\( m_l/m_s \geq 1/40 \)) using the HISQ action on \( N_t=6 \) lattices
- the scaling window shrinks compared to the results obtained using the p4 action on \( N_t=4 \) lattices
After including the regular terms, the chiral condensates can be described by the $O(2)$ scaling function $f_G(z)$.

The susceptibilities can be reasonably reproduced using the fitting parameters obtained from the fit to the chiral condensate.
Summary

• We study the chiral observables on $N_t=6$ lattices using the HISQ action with $m_{\pi}=160, 140, 110, 90$ and $80$ MeV

• No direct evidence of a first order phase transition in current pion mass window is found

• The scaling window shrinks in the HISQ results compared to that in the p4 results

• Regular terms need to be included to extract information on the singular structure