Nuclear matter symmetry energy and the symmetry energy coefficient in the mass formula

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Within the Skyrme-Hartree-Fock (SHF) approach, we show that for a fixed mass number A, both the symmetry energy coefficient \(a_{\text{sym}}(A)\) in the semi-empirical mass formula and the nuclear matter symmetry energy \(E_{\text{sym}}(\rho_A)\) at a subsaturation reference density \(\rho_A\) can be determined essentially by the symmetry energy \(E_{\text{sym}}(\rho_0)\) and its density slope \(L\) at saturation density \(\rho_0\). Meanwhile, we find the dependence of \(a_{\text{sym}}(A)\) on \(E_{\text{sym}}(\rho_0)\) or \(L\) is approximately linear and is very similar to the corresponding linear dependence displayed by \(E_{\text{sym}}(\rho_A)\), providing an explanation for the relation \(E_{\text{sym}}(\rho_A) \approx a_{\text{sym}}(A)\). Our results indicate that a value of \(E_{\text{sym}}(\rho_A)\) leads to a linear correlation between \(E_{\text{sym}}(\rho_0)\) and \(L\) and thus can put important constraints on \(E_{\text{sym}}(\rho_0)\) and \(L\). Particularly, the values of \(E_{\text{sym}}(\rho_0) = 30.5 \pm 3\) MeV and \(L = 52.5 \pm 20\) MeV are simultaneously obtained by combining the constraints from recently extracted \(E_{\text{sym}}(\rho_A = 0.1\) fm\(^{-3}\) with those from recent analyses of neutron skin thickness of Sn isotopes in the same SHF approach.

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I. INTRODUCTION

The study of the nuclear matter symmetry energy \(E_{\text{sym}}(\rho)\), which essentially characterizes the isospin dependent part of the equation of state (EOS) of asymmetric nuclear matter, is currently an exciting topic of research in nuclear physics. Knowledge about the symmetry energy is essential in understanding many aspects of nuclear physics and astrophysics as well as some interesting issues regarding possible new physics beyond the standard model. In recent years, significant progress has been made in determining the density dependence of \(E_{\text{sym}}(\rho)\), especially its value \(E_{\text{sym}}(\rho_0)\) and its density slope \(L\) at saturation density \(\rho_0\). While constraints on \(E_{\text{sym}}(\rho_0)\) and \(L\) from different experimental data or methods become consistently convergent, they are still far from an accuracy required for understanding enough precisely many important properties of neutron stars. To narrow the uncertainty of the constrains on \(E_{\text{sym}}(\rho_0)\) and \(L\) by using more accurate data or new methods is thus of crucial importance.

Recently, based on the calculations within the droplet model and mean field models using a number of different parameter sets, Centelles et al. found that the symmetry energy coefficient \(a_{\text{sym}}(A)\) of finite nuclei with mass number \(A\) in the semi-empirical mass formula can approximately equal to nuclear matter symmetry energy \(E_{\text{sym}}(\rho_A)\) at a reference density \(\rho_A\) in the subnormal density region, i.e., \(E_{\text{sym}}(\rho_A) \approx a_{\text{sym}}(A)\). This relation provides the possibility to directly determine the symmetry energy at subnormal densities from the semi-empirical mass formula and also has many important implications for extracting the symmetry energy from isospin dependent observables of finite nuclei. While this relation has been used to extract information on the symmetry energy around the normal density, its microscopic explanation is still missing.

Within the Skyrme-Hartree-Fock (SHF) energy density functional, we demonstrate in the present work that both \(a_{\text{sym}}(A)\) and \(E_{\text{sym}}(\rho_A)\) can be determined essentially by \(E_{\text{sym}}(\rho_0)\) and \(L\) and meanwhile they display very similar linear dependence on \(E_{\text{sym}}(\rho_0)\) or \(L\), and thus providing an explanation for the relation \(E_{\text{sym}}(\rho_A) \approx a_{\text{sym}}(A)\). Furthermore, we show that a value of \(E_{\text{sym}}(\rho_A)\) can put important constraints on \(E_{\text{sym}}(\rho_0)\) and \(L\). Combining the constraints from recently extracted \(E_{\text{sym}}(\rho_A = 0.1\) fm\(^{-3}\) with those from recent analyses of existing data on neutron skin thickness of Sn isotopes within the same SHF approach leads to stringent constraints simultaneously on \(E_{\text{sym}}(\rho_0)\) and \(L\).

II. \(E_{\text{sym}}(\rho)\) AND \(a_{\text{sym}}(A)\) IN THE SKYRME-HARTREE-FOCK APPROACH

The EOS of isospin asymmetric nuclear matter, given by its binding energy per nucleon, can be expanded to 2nd-order in isospin asymmetry \(\delta\) as

\[
E(\rho, \delta) = E_0(\rho) + E_{\text{sym}}(\rho) \delta^2 + O(\delta^4),
\]

where \(\rho = \rho_n + \rho_p\) is the baryon density with \(\rho_n\) and \(\rho_p\) denoting the neutron and proton densities, respectively; \(\delta = (\rho_n - \rho_p)/\rho\) is the isospin asymmetry; \(E_0(\rho) = E(\rho, \delta = 0)\) is the binding energy per nucleon in symmetric nuclear matter, and the nuclear symmetry energy is expressed as

\[
E_{\text{sym}}(\rho) = \frac{1}{2!} \left. \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2} \right|_{\delta=0} = E_{\text{sym}}(\rho_0) + L\chi + \frac{K_{\text{sym}}}{2!} \chi^2 + O(\chi^3),
\]

with \(\chi = (\rho_n - \rho_p)/\rho\). The coefficients \(L = 3\rho_0 \left. \frac{dE_{\text{sym}}(\rho)}{d\rho} \right|_{\rho=\rho_0}\) and \(K_{\text{sym}} = 9\rho_0^2 \left. \frac{d^2E_{\text{sym}}(\rho)}{d\rho^2} \right|_{\rho=\rho_0}\) are the slope and cur-
vature parameters of the symmetry energy, respectively. Within the standard SHF approach, the symmetry energy can be written as (see, e.g., Ref. [12])

$$E_{\text{sym}}(\rho) = \frac{\hbar^2}{6m} \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{2/3} - \frac{1}{8} t_0 (2x_0 + 1) \rho - \frac{1}{24} \left( \frac{3\pi^2}{2} \right)^{2/3} \Theta_{\text{sym}} \rho^{2/3} - \frac{1}{48} t_3 (2x_3 + 1) \rho^{2/3}$$

(3)

with \( \Theta_{\text{sym}} = 3t_1 x_1 - t_2 (4 + 5 x_2) \) and \( \sigma, t_0 - t_3, x_0 - x_3 \) being the Skyrme interaction parameters.

As shown in Refs. [18, 20], the 9 Skyrme interaction parameters, i.e., \( \sigma, t_0 - t_3, x_0 - x_3 \) can be expressed analytically in terms of 9 macroscopic quantities \( \rho_0, E_0(\rho_0) \), the incompressibility \( K_0 \), the isoscalar effective mass \( m_{\text{sym}}^* \), the isovector effective mass \( m_{\text{sym}}^* \), \( E_{\text{sym}}(\rho_0) \), \( L \), gradient coefficient \( G_S \), and symmetry-gradient coefficient \( G_V \). In terms of these macroscopic quantities, the symmetry energy can be rewritten as

$$E_{\text{sym}}(\rho) = A_1 E_{\text{sym}}(\rho_0) + B_1 L + C_1,$$

(4)

with

$$A_1 = (\gamma u - u^2)/\gamma - 1 \quad (\gamma - 1)$$

(5)

$$B_1 = (u^2 - u)/[3(\gamma - 1)]$$

(6)

$$C_1 = E_{\text{sym}}^{\text{kin}}(\rho_0) u^{4/3} + Du^{5/3}$$

$$- \frac{(3\gamma - 2) E_{\text{sym}}^{\text{kin}}(\rho_0) + (3\gamma - 5) D}{3(\gamma - 1)} u$$

$$+ \frac{E_{\text{sym}}^{\text{kin}}(\rho_0) - 2D}{3(\gamma - 1)} u^2,$$

(7)

where \( u = \rho/\rho_0 \) is the reduced density; \( E_{\text{sym}}^{\text{kin}}(\rho_0) = \frac{\hbar^2}{6m} \left( \frac{3\pi^2}{2} \rho_0 \right)^{2/3} \) is the kinetic symmetry energy at \( \rho_0 \); and the parameters \( D \) and \( \gamma \) are defined as [21, 22]

$$D = \frac{5}{9} E_0^{\text{kin}} \left( \frac{4m}{m_{\text{sym}}} - \frac{3m}{m_{\text{iso}}} - 1 \right)$$

(8)

$$\gamma = \sigma + 1 = \frac{K_0 + 2E_0^{\text{kin}} - 10C}{3E_0^{\text{kin}} - 9E_0(\rho_0) - 6C},$$

(9)

with \( C = \frac{m - m_{\text{sym}}}{m_{\text{iso}}} E_0^{\text{kin}} \) and \( E_0^{\text{kin}} = \frac{3\hbar^2}{10m} \left( \frac{3\pi^2}{2} \rho_0 \right)^{2/3} \).

The symmetry energy coefficient \( a_{\text{sym}}(A) \) of finite nuclei in the semi-empirical mass formula can be expressed as [16]

$$a_{\text{sym}}(A) = \frac{E_{\text{sym}}(\rho_0)}{1 + x_A}, \quad \text{with} \quad x_A = \frac{9E_{\text{sym}}(\rho_0)}{4Q} A^{-1/3},$$

(10)

where the \( Q \) parameter is the so-called neutron-skin stiffness coefficient in the droplet model [23, 24] and it is related to the nuclear surface symmetry energy [23, 26]. Usually for a given nuclear interaction, the \( Q \) parameter can be obtained from asymmetric semi-infinite nuclear matter calculations [23, 27, 28, 29]. As a good approximation, the \( Q \) parameter can be expressed as [27]

$$Q = \frac{9}{4} \frac{E_{\text{sym}}^{2} (\rho_0)}{\varepsilon_0^\varepsilon}, \quad \text{with} \quad \varepsilon_0^\varepsilon = \frac{2a}{r_{\text{nn}}} \left( L - \frac{K_{\text{sym}}}{12} \right),$$

(11)

where \( r_{\text{nn}} = \left( \frac{4\pi}{3} \rho_0^2 \right)^{-1/3} \) is the radius constant of nuclear matter and \( a \) is the diffuseness parameter in the Fermi-like function from the parametrization of nuclear surface profile of symmetric semi-infinite nuclear matter. Many calculations [25, 27] have indicated \( a \approx 0.55 \) fm and then \( 2a/r_{\text{nn}} \approx 1 \). Therefore, the \( x_A \) parameter can be approximated by

$$x_A = (L - K_{\text{sym}}/12) \frac{A^{-1/3}}{E_{\text{sym}}(\rho_0)}.$$

(12)

It should be noted that Eq. (12) is a good approximation for evaluating the \( a_{\text{sym}}(A) \), and within the standard SHF energy density functional the difference between the value of \( a_{\text{sym}}(A = 208) \) from Eq. (12) and that of using the exact \( Q \) parameter obtained from asymmetric semi-infinite nuclear matter calculations is essentially less than 1 MeV [29]. Furthermore, within the standard SHF approach, \( K_{\text{sym}} \) can be written in terms of the macroscopic quantities as [22]

$$K_{\text{sym}} = 3\gamma L + E_{\text{sym}}^{\text{kin}}(\rho_0) (3\gamma - 2)$$

$$+ 2D(5 - 3\gamma) - 9\gamma E_{\text{sym}}(\rho_0),$$

(13)

and thus we have

$$x_A = \left( \frac{4 - \gamma}{4} L + \frac{3}{4} \gamma E_{\text{sym}}(\rho_0) - (\frac{3\gamma - 2}{12}) E_{\text{sym}}^{\text{kin}}(\rho_0) \right.$$  

$$- (\frac{5 - 3\gamma}{6}) D \bigg) A^{-1/3} E_{\text{sym}}(\rho_0).$$

(14)

For \( |x_A| < 1 \), \( a_{\text{sym}}(A) \) in Eq. (10) can be expanded as

$$a_{\text{sym}}(A) = E_{\text{sym}}(\rho_0) (1 - x_A + x_A^2 - \cdots).$$

(15)

Neglecting the \( x_A^2 \) and higher-order terms in Eq. (15) leads to

$$a_{\text{sym}}(A) = A_2 E_{\text{sym}}(\rho_0) + B_2 L + C_2,$$

(16)

with

$$A_2 = \left( 1 - \frac{3\gamma}{4} A^{-1/3} \right)$$

(17)

$$B_2 = - \frac{4 - \gamma}{4} A^{-1/3}$$

(18)

$$C_2 = \left( \frac{(3\gamma - 2)}{12} E_{\text{sym}}^{\text{kin}}(\rho_0) + \frac{(5 - 3\gamma)}{6} D \right) A^{-1/3}.$$

(19)
However, the convergence of the expansion in Eq. (15) is usually very slow and thus Eq. (16) is a very bad approximation to $a_{\text{sym}}(A)$ even for heavy nuclei. A much better approximation could be obtained by the two-variable Taylor expansion with respect to $E_{\text{sym}}(\rho_0)$ and $L$ at a point of $E_{\text{sym}}(\rho_0) = S_0$ and $L = L_0$ as

$$a_{\text{sym}}(A) = a_{\text{sym}}(A)|_{E_{\text{sym}}(\rho_0)=S_0, L=L_0} + (E_{\text{sym}}(\rho_0) - S_0) \frac{\partial a_{\text{sym}}(A)}{\partial E_{\text{sym}}(\rho_0)}|_{E_{\text{sym}}(\rho_0)=S_0, L=L_0} + (L - L_0) \frac{\partial a_{\text{sym}}(A)}{\partial L}|_{E_{\text{sym}}(\rho_0)=S_0, L=L_0} + \cdot \cdot \cdot$$

and keeping only the first-order terms leads to

$$a_{\text{sym}}(A) = A_3 E_{\text{sym}}(\rho_0) + B_3 L + C_3,$$  \hspace{1cm} (21)

with

$$A_3 = \frac{1 - \frac{3}{4} \gamma A^{-1/3} + 2 x_0^9}{(1 + x_0^9)^2},$$

$$B_3 = \frac{\gamma^2 A^{-1/3}}{(1 + x_0^9)^2},$$

$$C_3 = \frac{S_0}{1 + x_0^9} - S_0 A_3 - L_0 B_3,$$  \hspace{1cm} (24)

and

$$x_0^9 = \frac{3}{4} \gamma S_0 + \frac{4 - \gamma}{4} L_0 - \frac{(3\gamma - 2)}{12} E_{\text{sym}}(\rho_0) - \frac{(5 - 3\gamma)}{6} A^{1/3} S_0.$$  \hspace{1cm} (25)

### III. NUMERICAL RESULTS AND DISCUSSIONS

One can see from Eq. (14) that $E_{\text{sym}}(\rho)$ is a linear function of $E_{\text{sym}}(\rho_0)$ and $L$ with the coefficients $A_1$ and $B_1$ determined by the density $\rho$ and nuclear matter macroscopic quantities $\rho_0$, $E_{0}(\rho_0)$, $K_0$, $m_s$, and $m_{v,0}$. Meanwhile, from Eqs. (19) and (21), one can see that $a_{\text{sym}}(A)$ is determined by the mass number $A$ and also the nuclear matter macroscopic quantities. In particular, $a_{\text{sym}}(A)$ can also be linear functions of $E_{\text{sym}}(\rho_0)$ and $L$ if the approximation (16) or (21) is valid. As mentioned previously, the relation $a_{\text{sym}}(A) \approx E_{\text{sym}}(\rho_A)$ has been observed within mean field models using a number of different parameter sets for the nuclear effective interactions. Particularly, one finds $a_{\text{sym}}(A = 208) \approx E_{\text{sym}}(\rho_A = 0.1 \text{ fm}^{-3})$, $a_{\text{sym}}(A = 116) \approx E_{\text{sym}}(\rho_A = 0.093 \text{ fm}^{-3})$, and $a_{\text{sym}}(A = 40) \approx E_{\text{sym}}(\rho_A = 0.08 \text{ fm}^{-3})$. This feature implies that $a_{\text{sym}}(A)$ and $E_{\text{sym}}(\rho_A)$ would display similar correlation with each nuclear matter macroscopic quantity among $L$, $G_V$, $G_S$, $E_0(\rho_0)$, $E_{\text{sym}}(\rho_0)$, $K_0$, $m_s$, $m_{v,0}$, and $\rho_0$, which completely determine the 9 Skyrme interaction parameters $\sigma$, $t_0 - t_3$, $x_0 - x_3$. In the following, we show that this is indeed the case by analyzing the correlations of $E_{\text{sym}}(\rho = 0.1 \text{ fm}^{-3})$ and $a_{\text{sym}}(A = 208)$ with the nuclear matter macroscopic quantities. We have also checked the cases of $A = 116$ and 40, and obtained the similar conclusion as in the case of $A = 208$ and confirmed the relations $a_{\text{sym}}(A = 116) \approx E_{\text{sym}}(\rho_A = 0.093 \text{ fm}^{-3})$ and $a_{\text{sym}}(A = 40) \approx E_{\text{sym}}(\rho_A = 0.08 \text{ fm}^{-3})$.

As a reference for the correlation analyses based on the standard SHF energy density functional, we use in the present work the MSL0 parameter set [18], which is obtained by using the following empirical values for the macroscopic quantities: $\rho_0 = 0.16 \text{ fm}^{-3}$, $E_0(\rho_0) = -16 \text{ MeV}$, $K_0 = 230 \text{ MeV}$, $m_s = 0.8m$, $m_{v,0} = 0.7m$, $E_{\text{sym}}(\rho_0) = 30 \text{ MeV}$, and $L = 60 \text{ MeV}$. to and $G_S = 5 \text{ MeV} \cdot \text{fm}^3$, and $G_S = 132 \text{ MeV} \cdot \text{fm}^3$. And the spin-orbit coupling constant $W_0 = 133.3 \text{ MeV} \cdot \text{fm}^3$ is used to fit the neutron $p_{1/2} - p_{3/2}$ splitting in $^{16}$O. It has been shown [18] that the MSL0 interaction can describe reasonably the binding energies and charge rms radii for a number of closed-shell or semi-closed-shell nuclei. It should be pointed out that the MSL0 is only used here as a reference for the correlation analyses. Using other Skyrme interactions obtained from fitting measured binding energies and charge rms radii of finite nuclei as in usual Skyrme parametrization will not change our conclusion.

Shown in Fig. 1 are $E_{\text{sym}}(\rho_A)$ with $\rho_A = 0.09, 0.10, 0.11 \text{ fm}^{-3}$ as well as $a_{\text{sym}}(A = 208)$ (from Eq. (14) with approximation in Eq. (16)) and its approximation with Taylor expansion in Eq. (21) (with $S_0 = 30 \text{ MeV}$ and $L_0 = 60 \text{ MeV}$) obtained from SHF with MSL0 by varying individually $L$ (a), $G_V$ (b), $G_S$ (c), $E_0(\rho_0)$ (d), $E_{\text{sym}}(\rho_0)$ (e), $K_0$ (f), $m_s$ (g), $m_{v,0}$ (h), $\rho_0$ (i), and $W_0$ (j).
our conclusions. It is interesting to see that, within the uncertain ranges considered here for the macroscopic quantities, $a_{\text{sym}}(A = 208)$ displays strong correlations with both $E_{\text{sym}}(\rho_0)$ and $L$ while it is almost no dependence on other macroscopic quantities (From Eqs. 10 and 13, $a_{\text{sym}}(A)$ is independent of $G_V$, $G_S$, and $W_0$). This is understandable since $a_{\text{sym}}(A)$ is determined uniquely by the three lowest-order characteristic parameters of the symmetry energy, i.e., $E_{\text{sym}}(\rho_0)$, $L$, and $K_{\text{sym}}$ as seen in Eqs. 10 and 13 while $K_{\text{sym}}$ has been found to strongly correlate with $E_{\text{sym}}(\rho_0)$ and $L$ but exhibit very weak dependence on other macroscopic quantities within the standard SHF energy density functional as shown in Ref. 20. Furthermore, $a_{\text{sym}}(A = 208)$ displays approximately linear correlations with both $E_{\text{sym}}(\rho_0)$ and $L$ which is demonstrated by the good approximation of Eq. 21 to $a_{\text{sym}}(A = 208)$ observed in Fig. 1.

Similarly, one can see from Fig. 1 that $E_{\text{sym}}(\rho_0)$ with $\rho_A = 0.09, 0.10$, and $0.11$ fm$^{-3}$ display strong linear correlations with both $E_{\text{sym}}(\rho_0)$ and $L$ while they are almost independent of other macroscopic quantities except with a small dependence on $\rho_0$ (Note $E_{\text{sym}}(\rho_0)$ is independent of $G_V$, $G_S$, and $W_0$). These results indicate that $a_{\text{sym}}(A = 208)$ and $E_{\text{sym}}(\rho_0)$ at subsaturation reference densities $\rho_A = 0.09, 0.10$, and $0.11$ fm$^{-3}$ are essentially determined by $E_{\text{sym}}(\rho_0)$ and $L$. Furthermore, Fig. 1 shows that both $a_{\text{sym}}(A = 208)$ and $E_{\text{sym}}(\rho_0)$ display very similar linear dependence on $E_{\text{sym}}(\rho_0)$ or $L$. Especially, it is seen from Fig. 1 that $E_{\text{sym}}(\rho_0)$ with $\rho_A = 0.10$ fm$^{-3}$ gives the best fit to $a_{\text{sym}}(A = 208)$. In other words, for any Skyrme force determined by the 9 parameters $\sigma$, $t_0 - t_3$, $x_0 - x_3$ or equivalently the 9 macroscopic quantities $L$, $G_V$, $G_S$, $E_0(\rho_0)$, $E_{\text{sym}}(\rho_0)$, $K_0$, $m^*_s$, $m^*_c$, $\rho_0$, the value of $E_{\text{sym}}(\rho_A = 0.10$ fm$^{-3}$) will approximately equal to that of $a_{\text{sym}}(A = 208)$, and this explains the relation $E_{\text{sym}}(\rho_A = 0.10$ fm$^{-3}$) $\approx a_{\text{sym}}(A = 208)$ observed within mean field models using a number of different parameter sets. Furthermore, one can see that the possible small deviations between $a_{\text{sym}}(A = 0.10$ fm$^{-3})$ and $a_{\text{sym}}(A = 208)$ observed for some different parameter sets 10 may be mainly due to the different values of $L$, $E_{\text{sym}}(\rho_0)$, and/or $\rho_0$.

Since $E_{\text{sym}}(\rho_A = 0.10$ fm$^{-3}$) is essentially determined by $E_{\text{sym}}(\rho_0)$ and $L$ and displays linear correlations with the latter, a determination of $E_{\text{sym}}(\rho_A = 0.10$ fm$^{-3}$) will then put important constraints on $E_{\text{sym}}(\rho_0)$ and $L$. As a matter of fact, the value of the symmetry energy around 0.1 fm$^{-3}$ has been heavily investigated in recent years in the literature 31–41. For example, an analysis of the giant dipole resonance (GDR) of $^{208}$Pb with Skyrme forces suggests a constraint $E_{\text{sym}}(\rho_A = 0.10$ fm$^{-3}$) $= 23.3 - 24.9$ MeV 42 while a relativistic mean-field model analysis of the GDR of $^{132}$Sn leads to $E_{\text{sym}}(\rho_A = 0.10$ fm$^{-3}$) $= 21.2 - 22.5$ MeV 43. In a recent work 17, Liu et al. extracted the symmetry energy coefficients $a_{\text{sym}}(A)$ for nuclei with mass number $A = 20 - 250$ from more than 2000 measured nuclear masses and they obtained a value of 20.22 – 24.74 MeV for $a_{\text{sym}}(A = 208)$ within a 2$\sigma$ uncertainty and thus we have $E_{\text{sym}}(\rho_A = 0.10$ fm$^{-3}$) $= 20.22 - 24.74$ MeV according to $E_{\text{sym}}(\rho_A = 0.10$ fm$^{-3}$) $\approx a_{\text{sym}}(A = 208)$. In the following, as a conservative estimate, we will use $E_{\text{sym}}(\rho_A = 0.10$ fm$^{-3}$) $= 20.22 - 24.74$ MeV since it is essentially consistent with the other two constraints from the GDR of $^{208}$Pb and $^{132}$Sn.

Shown in Fig. 2 are the contour curves in the $E_{\text{sym}}(\rho_0)$-$L$ plane for $E_{\text{sym}}(\rho = 0.1$ fm$^{-3}$). The region between the two thick solid lines represents the constraint obtained in the present work with $20.22$ MeV $\leq E_{\text{sym}}(\rho_A = 0.10$ fm$^{-3}$) $\leq 24.74$ MeV while the region between the two thick dashed lines is the constraint from the SHF analysis of neutron skin data of Sn isotopes within a 2$\sigma$ uncertainty 18. The shaded region represents the overlap of the two constraints.

![Contour curves](image-url)
The simultaneously extracted values of \( E_{\text{sym}}(\rho_0) \) are 30.5 ± 3 MeV and \( L = 52.5 ± 20 \) MeV from the same SHF approach within a 2\( \sigma \) uncertainty are essentially overlapped with other constraints extracted from different experimental data or methods in the literature \cite{11, 12, 14, 16, 22, 23, 17, 21, 17, 19} (see Ref. \cite{14} for a recent summary) but with higher precision. In particular, these extracted values are in remarkably good agreement with the \( E_{\text{sym}}(\rho_0) = 31.3 ± 4.5 \) MeV and \( L = 52.7 ± 22.5 \) MeV extracted most recently from global nucleon optical potentials constrained by world data on nucleon-nucleus and \((p,n)\) charge-exchange reactions \cite{14}. The extracted value of \( L = 52.5 ± 20 \) MeV also agrees well with the value of \( L = 58 ± 18 \) MeV obtained from combining the constraint from the neutron skin data of Sn isotopes \cite{18} with those from recent analyses of isospin diffusion and the double neutron/proton ratio in heavy-ion collisions at intermediate energies \cite{11}. Furthermore, the extracted value of \( L = 52.5 ± 20 \) MeV is consistent with the value of \( L = 66.5 \) MeV obtained from a recent systematic analysis of the density dependence of nuclear symmetry energy within the microscopic Brueckner-Hartree-Fock approach using the realistic Argonne V18 nucleon-nucleon potential plus a phenomenological three-body force of Urbana type \cite{5d}. It should be stressed that the simultaneously extracted values of \( E_{\text{sym}}(\rho_0) \) and \( L \) in the present work are obtained from the same Skyrme-Hartree-Fock energy density functional.

### IV. SUMMARY

We have analyzed the correlations of the nuclear matter symmetry energy \( E_{\text{sym}}(\rho) \) at a sub-saturation reference density \( \rho_A \) and the symmetry energy coefficient \( a_{\text{sym}}(A) \) of finite nuclei in the semi-empirical mass formula with nuclear matter macroscopic quantities within the Skyrme-Hartree-Fock energy density functional. We have shown that \( E_{\text{sym}}(\rho_A) \) displays explicitly linear correlations with \( E_{\text{sym}}(\rho_0) \) and \( L \) and it is essentially determined by the latters but almost no dependence on other macroscopic quantities except a small dependence on the saturation density \( \rho_0 \). These features imply that a fixed value of \( E_{\text{sym}}(\rho_0) \) will lead to strong linear correlation between \( E_{\text{sym}}(\rho_0) \) and \( L \). Furthermore, we have found that the two macroscopic quantities \( E_{\text{sym}}(\rho_0) \) and \( L \) essentially determine the value of \( a_{\text{sym}}(A) \) and the latter displays approximately linear correlations with both \( E_{\text{sym}}(\rho_0) \) and \( L \). In particular, the correlation between \( a_{\text{sym}}(A) \) and \( E_{\text{sym}}(\rho_0) \) (\( L \)) is found to be very similar to that between \( E_{\text{sym}}(\rho_A) \) and \( a_{\text{sym}}(A) \) obtained in mean field models using a number of different parameter sets.

Using the relation \( E_{\text{sym}}(\rho_A) \approx a_{\text{sym}}(A) \), we have demonstrated that within the Skyrme-Hartree-Fock energy density functional, the value of \( E_{\text{sym}}(\rho = 0.10 \text{ fm}^{-3}) = 20.22 \)–\( 24.74 \) MeV extracted recently from nuclear masses within a 2\( \sigma \) uncertainty \cite{17} can put important constraints on \( E_{\text{sym}}(\rho_0) \) and \( L \). Combining these constraints with those from recent analyses of existing data on neutron skin thickness of Sn isotopes based on the same Skyrme-Hartree-Fock approach within a 2\( \sigma \) uncertainty \cite{18} allows us to extract simultaneously the values of both \( E_{\text{sym}}(\rho_0) \) and \( L \), i.e., \( E_{\text{sym}}(\rho_0) \approx 30.5 ± 3 \) MeV and \( L = 52.5 ± 20 \) MeV. These extracted values are essentially consistent with other constraints extracted from different experimental data in the literature but with higher precision.

In the present work, all analyses are based on the standard Skyrme-Hartree-Fock energy density functional. It will be interesting to see how our results change if different energy-density functionals are used. On the other hand, it will be also interesting to see how our results, especially the new constraints on \( E_{\text{sym}}(\rho_0) \) and \( L \) obtained in the present work, can give implications for the neutron-skin thickness of heavy nuclei, the isovector giant dipole resonance of finite nuclei, and properties of neutron stars. These studies are in progress.

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