**0ν2β decay and neutrino magnetic moment**

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We propose a new channel of the neutrinoless double beta decay, based on the fact that in the presence of an external nonuniform magnetic field, the transition between neutrino and antineutrino can take place through the induced neutrino magnetic moment. We calculate the analog of the effective neutrino mass for this channel and show that, for certain values, the resonant enhancement can be obtained.

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I. INTRODUCTION

The neutrinoless double beta decay ($0\nu2\beta$) is a hypothetical process in which a nucleus undergoes two beta transitions, while neutrinos are not emitted, $(A, Z) \rightarrow (A, Z + 2) + 2e$, $(A, Z) \rightarrow (A, Z - 2) + 2\bar{e}$. This decay violates the electron lepton number by two units and we know from neutrino oscillations that the flavor lepton numbers are not conserved, in general. The $0\nu2\beta$ decay also violates the total lepton number $\Delta L = 2$; therefore, it requires some mechanism which allows for the total lepton number to be broken.

The observation of the $0\nu2\beta$ decay will also prove that neutrinos are Majorana particles [1], regardless of the underlying nonstandard mechanism driving this process. It is therefore of great importance to study this problem.

It is well known that neutrinos may undergo resonant flavor transitions when propagating through matter [2]. It has also been shown [3, 4] that the additional presence of a nonuniform magnetic field may induce transitions between neutrinos of different helicities. Thus, to obtain the Pontecorvo oscillations $\nu_\alpha \rightarrow \bar{\nu}_\alpha$, where $\alpha = e, \mu, \tau$, a two-step process is necessary. This process will be driven by the interplay between the neutrino mixing matrix $U$ and the neutrino magnetic moment $\mu_{\alpha\beta}$.

The only magnetic moment for neutral particles can be induced in the second-order processes, and for neutrinos in the standard model (SM) this means a one-loop process defined by the interaction vertices of the form $\nu \rightarrow W + e$, which conserve the electrical charge and lepton number [5]. The induced neutrino magnetic moment in the SM has been estimated to be $3.2 \times 10^{-19}(m_\nu)\mu_B$ [6], where $m_\nu$ is the neutrino mass and $\mu_B = e/(2m_e)$ is the Bohr magneton. For the neutrino mass $m_\nu = 0.05$ eV this yields $1.6 \times 10^{-20}\mu_B$ [7]. Going beyond the standard model, e.g., introducing supersymmetry, it is possible to enhance its value [8, 9]; however, experimental bounds point towards values not greater than $\approx 10^{-11}\mu_B$ [10].

In the following we discuss the process of neutrino-antineutrino oscillations in nuclear matter with a nonuniform external magnetic field. We show that this process may be resonantly enhanced, boosting the half-life of the neutrinoless double beta decay.

II. CORRECTIONS TO THE NEUTRINO MASS

The presence of matter changes the energy levels of neutrinos through neutral and charged current reactions. The neutral current potential is given, in general, by (see, e.g., Ref. [12])

$$V_{nc} = \sqrt{2}G_F \sum_{f=e,n,p} n_f \left( I_3^{(f)} - 2q^{(f)} \sin^2 \theta_W \right),$$

where $G_F$ is the Fermi constant, $\theta_W$ the Weinberg angle, $I_3$ the third component of the weak isospin, $q$ the electric charge, and $n_f$ denotes the number densities of the fermions. In the specific case of nuclear matter, the electrons are absent, $n_e = 0$, and the contributions come from the nucleons only. As for the charged current interactions, they come from interactions between electron (mu, tau) neutrinos and electrons (mu, tau leptons), so in our case they will be absent as well. We conclude therefore that nuclear matter shifts the overall scale of the neutrino energy levels but does not distinguish between flavors or chiralities.

The presence of an external magnetic field, however, distinguishes neutrinos from antineutrinos. Let us denote by $B = B(t)$ the component of the magnetic field projected on the plane perpendicular to the neutrino momentum. The direction of $B(t)$ is described by the angle $\phi(t)$, and by $\dot{\phi} = d\phi(t)/dt$ we denote its time derivative. In what follows we summarize the main points that lead to neutrino-antineutrino transition, as they were discussed in Refs. [4, 5]. First, we switch to the coordinate system which rotates together with $B$. In such a situation the energy correction to the spin $-1/2$ neutrinos will be $+ \dot{\phi}/2$, while for the spin $+1/2$ antineutrinos, it will be $- \dot{\phi}/2$.

Assuming that we have only two flavors of neutrinos and working in the basis

$$(\nu_e, \nu_\mu, \bar{\nu}_e, \bar{\nu}_\mu)^T,$$

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the mixing matrix depends on one vacuum mixing angle only:

\[ U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \tag{3} \]

Furthermore one can write the Hamiltonian in the general form

\[ H = \begin{pmatrix} H_\nu & \Phi \\ -\Phi^T & H_\bar{\nu} \end{pmatrix}, \tag{4} \]

where the neutrino and antineutrino blocks are given by

\[ H_\nu = E + V_{\text{nc}} + \frac{1}{2} \phi + \frac{1}{2} \mathcal{M}, \tag{5} \]

\[ H_\bar{\nu} = E + V_{\text{nc}} - \frac{1}{2} \phi + \frac{1}{2} \mathcal{M}, \tag{6} \]

with

\[ \mathcal{M} = \begin{pmatrix} m_1^2 \cos^2 \theta + m_2^2 \sin^2 \theta & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & m_1^2 \sin^2 \theta + m_2^2 \cos^2 \theta \end{pmatrix}. \tag{7} \]

and \( \Delta m^2 = m_2^2 - m_1^2 \). To obtain Eqs. (4)-(7) we have assumed that the energies of the mass eigenstate neutrinos \( E_i \) are \( E_i = p + m_i^2/(2p) \) in the relativistic case, for which the average neutrino energy \( E \) is approximately equal to its momentum \( p \).

The off-diagonal blocks are

\[ [\Phi] = \begin{pmatrix} 0 & B_{\mu\nu} \\ -B_{\mu\nu} & 0 \end{pmatrix}, \tag{8} \]

where \( B_{\mu\nu} \) is the Majorana neutrino transition magnetic moment. For Majorana neutrinos the magnetic moment is antisymmetric; therefore, transitions between same flavors are forbidden.

After the diagonalization of the Hamiltonian (4) one finds its eigenvalues in the form

\[ E + V_{\text{nc}} + \frac{m^2}{2E}, \tag{9} \]

where the corrected neutrino masses are

\[ m^2 = \frac{1}{2} \left( m_1^2 + m_2^2 \pm \sqrt{(4EB_{\mu\nu})^2 + (E\phi + \Delta m^2)^2} \right). \tag{10} \]

In this process the neutrino is a virtual particle; as such it is not on its mass shell, so it is not bound by the relation \( E^2 = p^2 + m^2 \). We identify the small splitting induced by the \( E\phi \) term with the neutrino-antineutrino masses, while the big splitting defines the first and second mass eigenstates. One sees clearly that in the absence of the \( \phi \) term, the neutrinos and antineutrinos will have degenerate masses.

![FIG. 1: Internal line of the Feynman diagram describing the 0ν2β decay driven by the magnetic moment of the neutrino.](image-url)

**III. NEUTRINOLESS DOUBLE BETA DECAY RATE**

We are interested in the process in which the electron neutrino emitted in one beta vertex is being absorbed as an electron antineutrino in the second beta vertex. This will describe, apart from the nuclear matrix element, the 0ν2β process. The internal line of a Feynman diagram in question is shown in Fig. 1.

One has to notice first that neutrinos are produced and absorbed as flavor eigenstates (electron neutrinos) in each beta vertex, but they propagate through matter as mass eigenstates \( \nu_i \). Therefore, the magnetic moment \( \mu_{ij} \) connects neutrino mass eigenstate \( i \) with an antineutrino mass eigenstate \( j \) and is given by

\[ \mu_{ij} = \sum_{\alpha,\beta} U_{\alpha i}^* \mu_{\alpha\beta} U_{\beta j}. \tag{11} \]

The propagation between two beta vertices, as depicted in Fig. 1, is then described by

\[ \chi = \sum_{\alpha,\beta} \mu_{\alpha\beta} \times \left( \sum_{i=1,2} U_{\alpha i} \hat{p} + m_i^\prime U_{\alpha i}^* \right) \left( \sum_{j=1,2} U_{\beta j}^* \hat{p} + m_j^\prime U_{\beta j} \right), \]

where \( p \) is the momentum exchanged between the beta vertices. In the two-neutrino case we have explicitly

\[ \chi = \mu_{\mu\mu} \sum_{i,j} \left( U_{\mu i} U_{\mu i}^* \frac{\hat{p} + m_i^\prime}{\hat{p} - m_i^\prime} U_{\alpha i}^* \right) \left( U_{\beta j}^* \frac{\hat{p} + m_j^\prime}{\hat{p} - m_j^\prime} U_{\beta j} \right), \tag{13} \]

where the antisymmetric property of the magnetic moment has been used. We point out that a nonzero \( \chi \) can have two sources. The difference in effective masses of neutrinos and antineutrinos, \( m_i^\prime \neq m_i \), is the first and most obvious one. However, even for degenerate masses, a nonzero \( \chi \) appears if the \( U \) matrix is not unitary. The general form of the neutrino mixing matrix contains complex phases which describe possible CP violation in neutrino oscillations. These phases are unknown, although recent best-fit values [13] suggest a small CP violation present in \( U \).

The factor \( \chi \) is related to the half-life of the 0ν2β decay through

\[ \left( T_{1/2}^{0\nu} \right)^{-1} = G^{0\nu} |M^{0\nu}|^2 |B\chi|^2, \tag{14} \]
where \( C^{0\nu} \) is the phase space factor and \( M^{0\nu} \) is the nuclear matrix element calculated within a certain (approximate) nuclear model.

It follows from Eq. (13) that the factor \( \chi \) is resonantly enhanced around the poles in the denominators. The equality \( m = p \approx E \) yields, from Eq. (10),

\[
E \dot{\phi} = \pm \sqrt{(2E^2 - (m_1^2 + m_2^2))^2 - (4EB\mu_p)^2} \pm \Delta m^2.
\]

(15)

As the magnetic field corrections are energy dependent, and the typical energy exceeds the mass eigenvalues by many orders of magnitude, the neutrino masses in Eq. (11) are dominated by the factors proportional to \( E \). The average momentum transfer, which for relativistic neutrinos is comparable to the total energy \( E \), in a neutrinoless double beta decay is assessed from the nuclear radius and usually taken to be \( \approx 100 \) MeV. The magnetic field strength \( B \) can be realistically estimated to be of the order of \( O(1) \) T. The magnetic moment, on the other hand, does not exceed \( 10^{-11} \mu_B \) [11]. Neutrino masses squared \( m_1^2 \) and \( m_2^2 \) are of the order of \( 10^{-5} \) eV [13]. Taking all of this into account, one finally arrives at the relation

\[
\dot{\phi} \approx 2E,
\]

(16)

which defines, to a good approximation, the resonance condition. In the typical case of \( E = 100 \) MeV, this yields \( \dot{\phi} \approx 10^{23} \) Hz.

IV. SUMMARY

Neutrino propagating in a nonuniform magnetic field may undergo Pontecorvo transition to a \( CP \)-conjugated state of the same flavor, as it was shown in [4, 5]. In this communication we have pointed out that this process may mediate a neutrinoless double beta decay. The difference between this and previously discussed channels, like the mass mechanism, pion mechanism, sparticle mediation and others, is that a resonancelike dependence on the frequency of the external magnetic field exists. Thus, in principle, it may be possible to enhance the rate of the \( 0\nu\beta\beta \) decay by controlling the field.

One should notice that when the resonance condition is met, the propagator of a certain neutrino mass eigenstate is boosted. This means that also the usual mass mechanism of the \( 0\nu\beta\beta \) decay should also be enhanced. Being a higher order process, the magnetic moment channel seems to be subdominant to the mass channel. However, one can also tune the magnetic field to resonantly enhance the propagation of the antineutrino mass eigenstate. In this case the magnetic moment channel should dominate all others. A more detailed discussion of this problem will appear in an upcoming paper. At present, we are not aware of any experimental possibilities to discriminate between these two cases.

We have discussed a simplified two-neutrino case in which the third mass eigenstate is decoupled from the electron neutrino. A full discussion of the realistic three-neutrino case together with a more thorough numerical analysis will be presented in a subsequent paper.

Acknowledgments

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