Investigation of the Effectiveness of Oscillation Control in a Three-Mass System Based on the Additional Feedbacks

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Abstract—Using the example of a three-mass calculating scheme the article presents the results of a study of the effectiveness of using the additional feedbacks synthesized by solving the inverse dynamic problems for a given type of elastic torque to limit the oscillatory motions. In this article, we described a mathematical model of a three-mass mechanical system considering viscous friction forces. The results of numerical modeling of the effectiveness of the proposed additional feedbacks on the velocity and acceleration of the motion drive and on the derivatives of the elastic torque are presented.

Keywords—three-mass system; oscillation control; additional feedbacks; inverse problem

I. INTRODUCTION

The increase speeds and loads of technological and transport machines (metal-cutting machines, industrial robots, rolling mills, lifting and construction machines, shovel and walking excavators, mine hoisting machines, etc.) places high demands on the level of their dynamic calculations. These facts request the inclusion of elastic properties of the construction during the development of methods and means of limiting oscillatory movements of the actuators [1-5]. A distinctive feature of such machines is the presence of elements with pronounced elastic properties (cables, long shafts, span structures of bridge cranes, booms of excavators and cranes), and intermediate concentrated and distributed masses mechanisms for transmitting motion from the drive (gearboxes, drums, pulleys, ball screw gears, etc.). The inertial properties of intermediate mass along with the rotors of the motors, actuators or movable goods need take into account in dynamic calculations [6-11]. Therefore, as being calculation schemes along with dual-mass machines, multi-mass and above all three-mass oscillatory systems are beginning to receive more use as calculation schemes for many modern machines [12–16].

An effective way to limit the oscillatory movements in these machines is to use program motion drive based on the use of elastic deformation feedbacks. To synthesize the structure and parameters of feedback in [17] and [18] it was proposed to use the concept of inverse dynamic problems [4]. In this case, the determination of feedback parameters by elastic deformation and its derivative in [18] was carried out on the basis of dividing a three-mass oscillatory system into two partial subsystems and expressing the connection between the elastic torque and a given torque in one partial subsystem and then establishing the connection of the elastic torque with the driving torque in another partial subsystem. The studies conducted in this work showed the high efficiency of the proposed oscillation control system based on the simplified mathematical model of the three-mass system, which did not take into account viscous friction forces and the selected elastic-inertial parameters corresponded to the weak interconnection between partial subsystems.

This article presents the results of the study of the effectiveness of oscillation control in a three-mass mechanical system based on additional feedbacks synthesized by solving the inverse dynamic problem for a given type of elastic torque based on the more complete mathematical model of this system.

II. OBJECT AND METHOD OF INVESTIGATION

As an object of study, we take a three-mass mechanical system with an elastic connection shown in Fig.1. As mentioned above with the help of this calculation scheme the dynamic properties of many modern technological and transport machines that implement specified technological processes or transfer objects of processing or various goods can be described. In this scheme the following notation was...
adapted: $J_1$ is the reduced moment of inertia of the motor rotor and gearbox; $J_2$ is the reduced moment of inertia of the intermediate masses, which are used for the transmission of motion from the motor to the actuators or movable goods; $J_3$ is the moment of inertia of the actuators (or moving goods); $\phi_1$, $\phi_2$, $\phi_3$ are the generalized coordinates of reduced masses; $c_{12}$ and $b_{12}$ are the stiffness and damping coefficients for mechanical transmission of motion between the motor and intermediate masses; $c_{23}$ and $b_{23}$ are the stiffness and damping coefficients of the second partial subsystem of motion between the intermediate masses and the actuator (or moving good); $M_{dv}$ is the moment of drive; $M_C$ is the moment of load.

Express the connection of the torque

$$M_{12} = (c_{12} + b_{12}s)\Delta \phi_{12},$$

with a given torque value

$$M_{23} = (c_{23} + b_{23}s)\Delta \phi_{23},$$

resolving equations (2) and (3) describing the second partial subsystem with respect to elastic deformation $\Delta \phi_{23}$ provided that $M_C = 0$, we find

$$M_{23}s^2 + b_{23}J_2M_{23}s + \omega_{23}^2M_{23} = (c_{23} + b_{23}s)\frac{M_{12}}{J_2}, \quad (7)$$

where

$$\omega_{23} = \sqrt{\frac{c_{23}(J_2 + J_3)}{J_2J_3}}$$

is the partial oscillation frequency of the second subsystem; $J_{23} = \frac{J_2 + J_3}{J_2J_3}$ is the ratio of the reduced masses in the second partial subsystem.

Substituting the dependence (4) and its derivatives up to the second order into expression (7), we find

$$J_2K_1e^{\lambda_1t} + J_2K_2e^{\lambda_2t} = (c_{23} + b_{23}s)M_{12}, \quad (8)$$

where

$$K_2 = C_1(\lambda_1^2 + \omega_{23}^2 + \lambda_1b_{23}J_{23})$$

and

$$K_1 = C_1(\lambda_1^2 + \omega_{23}^2 + \lambda_2b_{23}J_{23}) .$$

The solution of equation (8) we will find in the form

$$M_{12} = B_1e^{\lambda_1t} + B_2e^{\lambda_2t}, \quad (9)$$

where $B_1$ and $B_2$ are unknown coefficients.

Differentiating by the time the expression (9) and substituting it and its derivative in (8) after a series of transformations we obtain the dependence for finding the unknown coefficients

$$[J_2K_1 - B_1(c_{23} + b_{23}\lambda_1)]e^{\lambda_1t} + [J_2K_2 - B_2(c_{23} + b_{23}\lambda_2)]e^{\lambda_2t} = 0 .$$

Equating the expressions in square brackets in the last equation to zero we will have

$$B_1 = \frac{J_2K_1}{c_{23} + b_{23}\lambda_1}; \quad B_2 = \frac{J_2K_2}{c_{23} + b_{23}\lambda_2} .$$

Expression (9) is the required driving torque $M_{12}$ for the second partial subsystem and the specified law of variation of the elastic torque in the first partial subsystem which must be realized by the drive torque $M_{dv}$.

Let’s find the connection between the driving torque $M_{dv}$ and the torque (9) using the first partial subsystem described by equations (1) and (2). Solving these equations with respect to elastic deformation $\Delta \phi_{12}$ and using expressions (5) and (6) we get

$$M_{dv}$$

Fig. 1 The three-mass calculating scheme of the mechanism

Assuming the reduced masses concentrated and the stiffness and damping coefficients are constants, we obtain the differential equations of motion of the mechanical system in operator form:

$$J_1\phi_1s^2 = M_{dv} - c_{12}\Delta \phi_{12} + b_{12}\Delta \phi_{12}s; \quad (1)$$

$$J_2\phi_2s^2 = c_{12}\Delta \phi_{12} + b_{12}\Delta \phi_{12}s - c_{23}\Delta \phi_{23} - b_{23}\Delta \phi_{23}s; \quad (2)$$

$$J_3\phi_3s^2 = c_{23}\Delta \phi_{23} + b_{23}\Delta \phi_{23}s - M_Cs; \quad (3)$$

where $\Delta \phi_{12} = \phi_1 - \phi_2$ and $\Delta \phi_{23} = \phi_2 - \phi_3$ are deformations of elastic elements; $\frac{d}{dt} = s$ is Laplace operator.

Define the driving torque which provides the desired law of change of the elastic torque $M_{23}$ in the form of the exponential dependence

$$M_{23} = C_1(e^{\lambda_1t} + e^{\lambda_2t}), \quad (4)$$

where $C_1$ is some constant; $\lambda_1$ and $\lambda_2$ are the roots of the equation such that $\text{Re}\lambda_1 < 0$.

To establish the connection between the driving torque $M_{dv}$ and a given value of the elastic torque (4) we use the approach proposed in [18] and divide the three-mass system into two partial subsystems, one of which consists of the first $J_1$ and second $J_2$ reduced masses connected by elastic-damping elements with stiffness $c_{12}$ and damping coefficients $b_{12}$, and the second subsystem consists of $J_2$ and $J_3$ masses connected by elastic-damping elements with stiffness $c_{23}$ and damping coefficients $b_{23}$.
ons of time,

expression (10) the expressions (4 and 9) and their derivatives after some transformations we obtain the dependence for finding the required driving torque $M_{dv}$

$$J_1 E_1 e^{\lambda_1 t} + J_2 E_2 e^{\lambda_2 t} = (b_{12} b_{23} s^2 + (b_{12} c_{23} + c_{12} b_{23}) s + c_{12} c_{23}) M_{dv},$$

where $E_1 = B \left( b_{12} \alpha_1^2 + b_{12} b_{12} \alpha_1^2 + c_{12} \alpha_1^2 + b_{12} c_{12} \alpha_1 + \omega_{12}^2 \right)$

$$+ C \left[ c_{12} c_{23} + c_{12} b_{23} \alpha_1 + b_{12} b_{23} \alpha_1 + c_{12} c_{12} \alpha_1 \right],$$

$E_2 = B \left( b_{12} \alpha_2^2 + b_{12} b_{12} \alpha_2^2 + c_{12} \alpha_2^2 + b_{12} c_{12} \alpha_2 + \omega_{12}^2 \right)$

$$+ C \left[ c_{12} c_{23} + c_{12} b_{23} \alpha_2 + b_{12} b_{23} \alpha_2 + c_{12} c_{12} \alpha_2 \right].$$

The solution of equation (11) we will find in the form

$$M_{dv} = D_1 e^{\lambda_1 t} + D_2 e^{\lambda_2 t},$$

where $D_1$ and $D_2$ are unknown coefficients.

Differentiating (12) twice and substituting the obtained expressions in (11) we find

$$J_1 E_1 - D_1 (b_{12} b_{23} \lambda_1^2 + (b_{12} c_{23} + c_{12} b_{23}) \lambda_1 + c_{12} c_{23}) +

J_2 (b_{12} b_{23} \lambda_1^2 + (b_{12} c_{23} + c_{12} b_{23}) \lambda_1 + c_{12} c_{23}) = 0,$$

from which we determine the coefficients:

$$D_1 = \frac{J_1 E_1}{b_{12} b_{23} \alpha_1^2 + (b_{12} c_{23} + c_{12} b_{23}) \lambda_1 + c_{12} c_{23}},$$

$$D_2 = \frac{J_2 E_2}{b_{12} b_{23} \alpha_1^2 + (b_{12} c_{23} + c_{12} b_{23}) \lambda_1 + c_{12} c_{23}}.$$

Expression (12) determines the necessary law of change of the drive torque $M_{dv}$ to obtain a given law (4) of change $M_{23}$.

We will show that the found laws of changes in the driving $M_{dv}$ and elastic $M_{12}$ torques make it possible to determine the required law of change the acceleration coordinate of the drive. Substituting into (1) the expressions (9) and (11) we obtain the differential equation for determining the acceleration of the first mass

$$J_1 \phi s^2 = \left( D_1 - B_1 \right) e^{\lambda_1 t} + \left( D_2 - B_2 \right) e^{\lambda_2 t}.$$

The solution of equation (13) we will find in the form

$$\phi_1 = F_1 e^{\lambda_1 t} + F_2 e^{\lambda_2 t},$$

where $F_1$ and $F_2$ are unknown coefficients.

Differentiating (14) twice in time and substituting in (13) we obtain the dependence for finding the unknown coefficients

$$(F_1 \lambda_1^2 - \frac{D_1 - B_1}{J_1}) e^{\lambda_1 t} + (F_2 \lambda_2^2 - \frac{D_2 - B_2}{J_1}) e^{\lambda_2 t} = 0,$$

from which we get

$$F_1 = \frac{D_1 - B_1}{J_1 \lambda_1^2}; F_2 = \frac{D_2 - B_2}{J_1 \lambda_2^2}.$$

The required laws of change of the driving torque $M_{dv}$ (12), elastic torque $M_{12}$ (9) and coordinate of the displacement of the first mass $\phi_1$ (14) providing the desired character of the change of the elastic torque (4), which are functions of time, make it possible to form various structures of additional feedbacks for controlling the motion in three-mass oscillatory system.

For example, we will form the structure and parameters of the oscillation control system based on additional feedback on the drive coordinate and its derivatives. To this end, using the expression (14) we construct the system of equations to determine the acceleration and speed of the drive

$$\phi_1 s^2 = \lambda_1^2 f_1 e^{\lambda_1 t} + \lambda_2^2 f_2 e^{\lambda_2 t}.$$

Expressing from (15) the time dependencies through the speed and acceleration of the drive:

$$e^{\lambda_1 t} = \frac{\phi_2 \lambda_2 - \phi_1 s_2}{f_1 (\lambda_2 - \lambda_1)},$$

$$e^{\lambda_2 t} = -\frac{\phi_2 \lambda_1 + \phi_1 s_2}{f_2 (\lambda_2 - \lambda_1)},$$

and substituting expressions (16) into the equation for the driving torque (12) we obtain an expression defining the structure and parameters of the oscillation control system based on additional feedback on the speed and acceleration of the drive

$$M_{dv} = \frac{D_2 f_2 \lambda_2 - D_2 f_2 \lambda_1}{f_1 f_2 (\lambda_2 - \lambda_1)} \phi s^2 + \frac{D_2 f_2 \lambda_2 - D_2 f_2 \lambda_1}{f_1 f_2 (\lambda_2 - \lambda_1)} \phi \phi s = K \phi s^2 + K \phi \phi s.$$
where
\[
K_V = \frac{D_2 F_1 \lambda_1^2 - F_1 D_2 \lambda_2^2}{F_1 F_2 \lambda_1 \lambda_2 (\lambda_2 - \lambda_1)} \quad \text{and}
\]
\[
K_C = \frac{D_2 F_1 \lambda_1 - D_1 F_2 \lambda_2}{F_1 F_2 \lambda_1 \lambda_2 (\lambda_2 - \lambda_1)}
\]
are the gains of additional feedbacks on speed and acceleration of the drive.

It should be noted that the gain $K_V$ should always be negative, while the gain $K_C$ can take both positive and negative values. To implement the control according to the law (17), it is sufficient to use a standard speed sensor with differentiation of its signal to obtain acceleration of the drive.

Now we define the structure and parameters of the oscillation control system based on additional feedbacks on the elastic torque $M_{23}$, differentiating the dependence (4) with respect to the time

\[
\begin{align*}
M_{23} s^2 &= \lambda_1^2 C_1 e^{\lambda_1 t} + \lambda_2^2 C_1 e^{\lambda_2 t}, \\
M_{23} s &= \lambda_1 C_1 e^{\lambda_1 t} + \lambda_2 C_1 e^{\lambda_2 t}.
\end{align*}
\]

Expressing from (18) the functions of the time through speed and acceleration of the elastic torque

\[
\begin{align*}
e^{\lambda_1 t} &= \frac{M_{23} s^2 \lambda_2 - M_{23} s^2}{C_1 \lambda_1 (\lambda_2 - \lambda_1)}; \\
e^{\lambda_2 t} &= \frac{-M_{23} s \lambda_1 + M_{23} s^2}{C_1 \lambda_2 (\lambda_2 - \lambda_1)},
\end{align*}
\]

and substituting (19) into (12) we find the structure and parameters of the control system based on additional feedbacks on speed and acceleration of the change of the elastic torque

\[
M_{\alpha\eta} = \frac{D_2 \lambda_1 - D_1 \lambda_2}{C_1 \lambda_1 \lambda_2 (\lambda_2 - \lambda_1)} M_{23} s^2 + \frac{D_1 \lambda_2^2 - D_2 \lambda_2^2}{C_1 \lambda_1 \lambda_2 (\lambda_2 - \lambda_1)} M_{23} s = (20)
\]

\[
K'C'M_{23} s^2 + K'V'M_{23} s
\]

where $K_V' = \frac{D_1 \lambda_2^2 - D_2 \lambda_2^2}{C_1 \lambda_1 \lambda_2 (\lambda_2 - \lambda_1)}$ and $K_C' = \frac{D_2 \lambda_1^2 - D_1 \lambda_1^2}{C_1 \lambda_1 \lambda_2 (\lambda_2 - \lambda_1)}$

are the gains of additional feedback on the speed and acceleration of the elastic torque $M_{23}$.

The gains of additional feedbacks $K_V$ and $K_C$ in expression (20) should be negative based on the stability conditions. The technical implementation of this type of additional feedback can be carried out on the basis of the use of a strain gauge force-torque sensor with the subsequent differentiation of its signal.

The structure diagram of the control system in a three-mass mechanical system based on additional feedbacks is given in Fig. 2. Number 1 and 2 correspond additional feedback on the derivatives of the elastic torque $M_{23}$ and additional feedback on the speed and acceleration of the drive are presented on this picture.

![Fig. 2 Structure diagram of the three-mass system with additional feedbacks](image)

### III. RESEARCH AND DISCUSSION

In order to test the effectiveness of using the proposed additional feedbacks numerical modeling of the equations of motion of the three-mass mechanical system (1–3) was performed with the driving torques determined by expressions (17) and (20) with the following system parameters:

\[
J_1 = J_2 = J_3 = 4 \, kg \cdot m^2; \quad c_{12} = c_{23} = 3000 \, N \cdot m/ \text{rad};
\]
\[
J_1 = J_2 = J_3 = 4 \, kg \cdot m^2; \quad b_{12} = b_{23} = 15 \, N \cdot m/ \text{s/ rad}.
\]

These parameters are contrasted to the parameters of the three-mass mechanical system used in [18] and provide a strong connection of partial subsystems.

For the accepted values of the parameters, the calculated values of the roots of equation (4) composed $\lambda_1 = -41$ and $\lambda_2 = -40$ for additional feedback on the speed and
acceleration of the motion drive (17), and \( \lambda_1 = -13 \) и \\
\( \lambda_2 = -14 \) for additional feedbacks on the derivatives of the \\
elastic torque (20). Value of roots chosen on the basis of the \\
stability condition and the maximum response time of the \\
system. The value of the constant in expression (4) was \\
taken to be equal \( C_1 = 1 \).

Numerical simulation was carried out in Matlab Simulink \\
environment with a step change in the driving and load \\
torques. The values of the elastic torques \( M_{12} \) and \( M_{23} \), \\
and the drive torque \( M_{dv} \) were recorded.

Studies have shown the high efficiency of using the \\
proposed additional feedbacks to reduce oscillatory \\
movements in a three-mass mechanical system. These \\
feedbacks allowed to reduce the amplitudes of elastic \\
ocillations and the time of transient processes in the system.

Oscillograms of changes elastic torques \( M_{12} \) and \( M_{23} \), \\
drive torque \( M_{dv} \) for the original mechanical system (1), \\
systems with additional feedback on the speed and the \\
acceleration of motion drive (2), and with additional \\
feedbacks on derivatives of the elastic torque \( M_{23} \) (3) are \\
shown in Fig. 3 and Fig. 4.

Analysis of oscillograms leads to the following 

conclusions:

1. In the start-up mode in the initial system (curve 1) there are 

significant oscillations of the elastic torques \( M_{12} \) and \( M_{23} \) 

(see Fig. 3). At the same time, the overshoot value for the 

torque \( M_{12} \) is 51% with the decrement of oscillations 

\( \delta = 0.0912 \), for \( M_{23} \) overshoot is 114% and decrement is 

\( \delta = 0.283 \). Duration of transient processes is \( t_{fp} = 1.5 \) sec.

The use of additional feedback on the speed and acceleration 

of the drive (curve 2) allows to reducing the overshoot of the 

torque \( M_{12} \) in the start-up mode to 42% and almost 

completely eliminating repeated oscillations. The amplitude of 

the oscillations of the torque \( M_{23} \) is reduced to 14% and the 

decrement of the oscillations increases to \( \delta = 0.4 \) compared 

with the original system. The duration of the transition process 

is reduced to \( t_{fp} = 0.7 \) sec.

2. The change in the load in the initial system (curve 1) leads 

to significant oscillation in the elastic torques: the overshoot 

for the torque \( M_{12} \) is 35% with the decrement of oscillations 

\( \delta = 0.0912 \), and for the torque \( M_{23} \) overshoot is 36% and 

decrement is \( \delta = 0.09 \) (see Fig. 3). The duration of transient 

processes is \( t_{fp} = 1.5 \) sec. The use of additional feedback on 

the speed and acceleration of the drive (curve 2) makes it 

possible to significantly reduce torque oscillations \( M_{12} \) and 

\( M_{23} \), with almost no overshoot and reducing the time of 

transient process to \( t_{fp} = 0.7 \) sec. The use of additional 

feedbacks on derivative of elastic torque \( M_{23} \) (curve 3) provides 

a monotonous transient process for the torque \( M_{23} \), 

however, it leads to significant oscillations of the elastic 

torque \( M_{12} \) with the duration of the transient process 

\( t_{fp} = 0.7 \) sec. It should also be noted that the use of additional 

feedback on the speed and acceleration of the drive allows to 

getting the maximum response time for control response to the 

load without overshooting with the minimum oscillatory 

component of \( M_{23} \) compared to additional feedbacks on the 

derivatives of the elastic torque \( M_{23} \).
3. The analysis of the oscillograms of the driving torque $M_{dv}$ (see Fig. 4) shows that additional feedbacks lead to high-frequency oscillations of the driving torque at the start mode, and when additional feedbacks on the derivatives of the elastic torque $M_{23}$ (curve 3) were used the greatest oscillation was observed in the load-change mode.

Fig. 4 Oscillograms of drive torque $M_{dv}$

It should be noted that when using additional feedbacks on speed and drive acceleration (curve 2) the greatest oscillation of the driving torque was observed at the start mode, and when additional feedbacks on the derivatives of the elastic torque $M_{23}$ (curve 3) were used the greatest oscillation was observed in the load-change mode.

IV. CONCLUSION

Studies have shown the efficiency of the used method of synthesis of motion control systems of technological and transport machines based on solving inverse dynamic problems for a given type of elastic oscillation. The use of procedure of transferring a given type of motion to elastic oscillation, movement of masses and driving torque allows solving the problem effectively of choosing the structure and parameters of additional feedbacks implement the found control actions. The application of the proposed method allowed to reduce the amplitudes of elastic oscillations and the time of transient processes in a three-mass mechanical system with a strong connection between the partial subsystems.

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