The shapes of light curves of Mira-type variables

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Using a sample of 454 mira light curves from the ASAS survey we study the shape of the light variations in this kind of variable stars. Opposite to earlier studies, we choose a general approach to identify any deviation from a sinusoidal light change. We find that about 30% of the studied light curves show a significant deviation from the sinusoidal reference shape. Among these stars two characteristic light curve shapes of comparable frequency could be identified. Some hint for a connection between atmospheric chemistry and light curve shape was found, but beside that no or only very weak relations between light curve shape and other stellar parameters seem to exist.

1 Introduction

Mira (ο Cet) is the longest known periodic variable. The variability class represented by this object, dubbed miras, is characterized by large amplitude (several magnitudes in \(V\)) and long period (>100 d) variations. From the point of stellar evolution, miras are highly evolved stars of low or intermediate mass that are found on the Asymptotic Giant Branch (AGB) in the Hertzsprung-Russell diagram (HRD). Their high luminosity allows to observe them throughout the Milky Way and the Local Group (e.g. Groenewegen 2004), and even beyond (e.g. Rejkuba 2007). Most of them are radial, fundamental mode pulsators (e.g. Wood & Sebo 1996).

Despite the long history of observations of miras and the large number of light curves obtained – with a key contribution coming from amateur astronomers (AAVSO) – systematic studies on the light curve shape of these stars are rare. This lack is somewhat surprising as several papers pointed out that there seems to be a relation between the shape of the visual light curve and various other parameters, mostly related to the circumstellar material around these stars: Bowers (1975) and Bowers & Kerr (1977) were probably the first to notice that the occurrence of microwave OH emission in miras seems to be linked to the steepness of the rising branch of the visual light curve. Vardy et al. (1986) found that the appearance of silicate emission features at 9.7 and 20 \(\mu\)m is related to the asymmetry of the light curve. This was later confirmed by Onaka et al. (1989). It was again Vardy (1987), who showed that the probability of detecting the \(\text{H}_2\text{O}\) vapor line at 1.35 cm depends on the light curve shape. Le Bertre (1992) and Winters et al. (1994) explored a possible relation between light curve shape and dust formation. Mennessier et al. (1997) discriminated M- and C-classes by using period, amplitude, light asymmetry and IRAS colours. Existing and forthcoming large datasets of light curves of long period variables from groundbased photometric surveys thus motivate a closer look on the possibility to derive basic stellar quantities like evolutionary status or mass loss properties from light curve data. For this, an easily applicable and quantifiable method to describe the light curve shape is needed.

Special features in the light curve shape of these variables have been noted by several authors. Humps on the rising branch of the light curve as well as a clear asymmetry between the rising and descending branch have been noted e.g. by Lockwood & Wing (1971), also summarizing results from earlier studies. But it is also clear that only a fraction of all miras show this behaviour. The humps may be analogs of the humps seen on bump Cepheid light curves and likely result from a 2-to-1 resonance between the fundamental and the first overtone mode (Barthes et al. 1998, Wood et al. 1999, Lebzelter et al. 2005). A few miras have been reported to show even double maxima in their light curve (e.g. Keenan et al. 1974). Comparison with infrared colour-period relations suggest that the true period is actually half the period between deep minima (i.e. only one maximum per period, cf. Feast et al. 1982) in these cases.

Early work on the classification of mira light curves has been summarized by Payne-Gaposchkin & Gaposchkin (1938), pointing out that for correlations of light curve shape with other parameters a numerical value is in advantage over a formal classification system. The most extensive study of the shape of mira light curves up to now is the work of Vardy (1988) who analysed light curve data of 368 miras from the literature. Their paper also outlines the two major approaches used to describe the shape of the light curve, namely on the one hand the qualitative classification scheme given by Ludendorff (1928) and on the other hand the visual light asymmetry factor \(f\), which is defined as the rise time...
over the period (in days). As a result of this study, Vardya found that 80% of the miras lie in the range $0.4 \leq f \leq 0.5$, i.e. only a small fraction of the miras show strong deviations from a symmetric visual light curve. This fraction seems to be dependent on the atmospheric chemistry (M-, S- or C-stars). Vardya also compared the $f$ values with the classes defined by Ludendorff, but no clear correlation could be found. This is also hampered by the fact that the classes of Ludendorff are not very clearly defined and the classification seems to be somewhat subjective. However, there is also some uncertainty in the $f$-parameter. As pointed out by Onaka et al. (1989) this uncertainty may reach a value of 0.1. Therefore, the description of the light curve shape and accordingly the frequency of asymmetries has to be seen as not satisfyingly solved (as was also noted by Vardya 1988).

The study of Vardya is based on the archive of observations from amateur astronomers (e.g. Campbell 1955). In contrast, nowadays a large number of photometric light curves in the visual range are available from various surveys like MACHO (Wood et al. 1999) or OGLE (Soszynski et al. 2009, survey mainly done in the $I$-band). These data have led to major advances in our understanding of long period variables, especially in terms of the period-luminosity relations (e.g. Wood et al. 1999, Ita et al. 2004, Soszynski et al. 2007) or the study of long secondary periods (e.g. Wood 2000, Soszynski et al. 2007, Wood & Nicholls 2009). However, a systematic study of these data sets in terms of light curve asymmetry has not been done yet.

In this paper the deviation of the shape of the light curves of miras from a sinusoidal variation is investigated for a selected sample of sources in the All-Sky Automated Survey (ASAS 3) variable stars catalogue (Pojmanski 2002). Opposite to other surveys like MACHO or OGLE, ASAS is focusing on the variability of comparably bright field stars. Its detection limit is close to a $V$ magnitude of 14. Photometric accuracy is given to be about 0.05 mag, but may be somewhat worse in some cases. Despite these shortcomings the ASAS catalogue was selected for our study as many of these bright targets have additional information available, e.g. from 2MASS, IRAS or light curves from other sources.

## 2 Sample selection

Starting point for our selection was the ASAS catalogue of variable stars (ACVS). All ASAS data including this catalogue are accessible via the ASAS webpage. The variability type found in that catalogue has been assigned by means of an automated classification scheme using both light curve parameters (period, amplitude and Fourier coefficients) and near infrared colours (taken from 2MASS). See Pojmanski (2000, 2002) and Pojmanski & Maciejewski (2004) for details. In total 2895 objects in the catalogue are classified as miras. We note here that about 850 of the stars classified as miras in the ACVS have a $V$ amplitude of less than 2.5 magnitudes. According to the classical scheme (Khlopov 1985-88, GCVS) such stars would be rather classified as SRa. For the beginning we did not exclude these stars from our sample, but we will later come back to this point.

A quick visual check of all selected light curves was done. During this step, the period determination of several stars was slightly improved and a small number of stars were rejected due to an unclear variability pattern or an obviously low photometric accuracy of the data. In the next step, the time series were transformed into phased light curves ($[0,1]$) using the revised periods from the previous step. Each of these light curves was then split into steps of 0.05 in phase, and the data within each slot were averaged. At this step we kept only those stars that had no gaps in their phased light curves, i.e. none of the slots was empty. Naturally, we lost most stars with periods close to one year in this step, because such objects were always unobservable at the same phase. As a result we had averaged light curves with a step size of 0.05 in phase.

The resulting sample included a remarkable fraction of light curves with extremely broad minima. According to Pojmanski (priv. comm.) such a behaviour may be mimicked by blending with nearby stars. The limited spatial resolution of ASAS does not allow to separate stars closer than 15 arcsec. If the minimum of the mira is at a lower brightness than the blending star, the resulting light curve would show such a broad minimum. To check for this possible disturbance we searched the USNO A2 catalogue for stars within 20 arcsec of the sample miras that show an $R$-magnitude brighter than 15. Removal of these sample stars reduced the number of light curves with such broad minima to very few cases and the total sample to approximately 500 objects, from which 454 stars were then chosen as the final sample (removing some cases of uncertain or possibly variable period or a large scatter of the averaged data points of the mean light curve).

To illustrate the method we show in Fig. 1 two examples of mira lightcurves from the ASAS archive. The top row shows the photometry against time directly taken from the archive. It is obvious that the left example (ASAS 154500-2804.4) shows quite strong variations of the amplitude, while in the example on the right a bump can be seen on the rising branch of the light change (ASAS 201445-4659.0). The row below now gives the phased light curve (small open symbols) and on top of it the averaged lightcurve for each of the two objects (filled large symbols). The left example has been rejected from our final sample for two reasons: first, the average of the measured standard deviation of all data points in each bin exceeds a limit of 0.6 magnitude. We did make some experiments with slightly changing this limit, but the effect on the results was minor. We note that we rejected also stars with a very large scatter (exceeding 1 magnitude) in a single bin. Second, the light curve shows a very flat and broad minimum indicating blending with a

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1. http://www.astrouw.edu.pl/asas/

2. It is irrelevant whether the scatter is produced by amplitude or period changes or by short time irregularities in the light change.
more we have only single period objects in our study. Semiregular variables are absent from our sample, further in the classical scheme. Short period or small amplitude mag, i.e. these objects have a too small amplitude for mi-

result of our selection process. It is also clear, that our sam-

tioned above, we have a gap for periods around 1 year as the

period and amplitude plane. The typical period range of

may be affected by this problem and removed unclear cases.

Taking into account the fact that mira pulsation is not

strictly periodic the derived average light curve has to be

seen as a typical light change during the time of the ob-

servations. For stars with a very short observing window at

the site of the ASAS telescope (Las Campanas Observatory,

Chile) and thus a bad phase coverage, asymmetries could

be smeared out or even falsely introduced by cycle-to-cycle

variations in the light amplitude. On the other hand, the

note that this extreme case shows also some cycle to cy-

cles variations in the light amplitude. On the other hand, the

change is smooth. The maximum value found for our sam-

ple stars was \(\chi^2=0.97\), which was measured for the mira

DD Ara (ASAS 171611-6148.0, \(P=284\) d, \(\Delta V=3\) mag). We

found for the star T Col (ASAS 051917-3342.5, \(P=230\) d, \(\Delta V=4.6\) mag). Various examples

light curves are shown in Fig. 4.

\[
\chi^2 = \sum_{i=1}^{20} (X_O(\phi_i) - X_S(\phi_i))^2,
\]

where \(X_O\) and \(X_S\) are the 20 individual data points of

the normalized averaged light curve and the sinusoidal ref-

cence curve, respectively, and \(i\) is an index running from

minimum phase to minimum phase. The light curve of the

target was shifted relative to the sinusoidal reference curve

until a minimum value for the squared difference was reached,

which was used for the further analysis. This was done to

compensate for possible phase shifts.

Figure 3 shows the distribution of the sums of the squared

differences \(\chi^2\) between normalized averaged light curve and reference curve found for our sample stars. Small val-

ues indicate a light change very close to a sinusoidal curve. Inspection of individual cases reveals that up to a value of

\(\chi^2=0.2\) only minor deviations from the reference curve can be seen. The transition towards a clearly non-sinusoidal light change is smooth. The maximum value found for our sample stars was \(\chi^2=0.97\), which was measured for the mira

DD Ara (ASAS 171611-6148.0, \(P=284\) d, \(\Delta V=3\) mag). We

note that this extreme case shows also some cycle to cy-
cles variations in the light amplitude. On the other hand, the

smallest value of \(\chi^2\) was found for the star T Col (ASAS

051917-3342.5, \(P=230\) d, \(\Delta V=4.6\) mag). Various examples

light curves are shown in Fig 4.
For a better characterization we also determined the absolute differences between each point of the normalized averaged light curve and the reference curve and summed them up for the first and the second half of the light cycle separately. We define \( \chi_1 \) as \( \sum_{i=1}^{10} (X_O(\phi_i) - X_S(\phi_i)) \) and \( \chi_2 \) as \( \sum_{i=11}^{20} (X_O(\phi_i) - X_S(\phi_i)) \), respectively. As we set the light curve maximum to 0 and the minimum to 1, a positive difference \( (X_O(\phi_i) - X_S(\phi_i)) \) means that the observed curve is fainter at a given phase than what would be expected from a sinusoidal variation.

Depending on the deviation of the light curve from the sinusoidal variation – resulting either in a more narrow or in a broader peak – this criterion may allow to distinguish various kinds of light curve shapes. Indeed our results seem to prove that. Illustrative examples for various cases are shown in Fig. 4.

We can roughly define two main groups of light curves significantly deviating from a sinusoidal variation:

- group 1: \( \chi_1 \) and \( \chi_2 \) are both positive (upper row of Fig. 4). This is the classical case of an asymmetric mira light curve, and it is characterized by a broad and flat minimum followed by a rather steep rise to a narrow maximum. The way back to the minimum is less steep than the rising branch. Humps may occur on the lower part of the rising branch, in rare cases also on the declining branch.

- group 2: \( \chi_1 \) and \( \chi_2 \) are both negative (middle row of Fig. 4). Here the minimum phase is rather narrow. The rising branch may again be quite steep, but the curve continues into a broad maximum. Special cases in this group are light curves with two maxima, which may be related to stars showing a hump on the upper part of the rising branch and which are also found in this group.

Most stars with \( \chi^2 \) values above 0.2 can unambiguously be attributed to either of these groups. The bottom row of

Fig. 4 shows some examples of light curves close to a sinusoidal variation.

In Fig. 5 we plot \( \chi_1 \) against \( \chi_2 \). Stars in the upper right corner belong to group 1, stars in the lower left to group 2. It can be seen that both groups include a similar number of objects. If we count only the stars with \( \chi^2 > 0.2 \) (filled symbols in Fig. 5), we end up with 74 stars (or 16 % of the total sample) in group 1 and 54 objects (12 %) in group 2. About 20 stars with \( \chi^2 > 0.2 \) fall outside these two groups. None of them shows extreme \( \chi_1 \) or \( \chi_2 \) values. Checking these stars in detail reveals that they are a mixture of stars that do indeed show a light curve shape different from group 1 and group 2, and stars that are close to our scatter limit around the mean light curve (i.e. typically stars with cycle-to-cycle variations). For the first case, the number of stars is too small for a more detailed study.

To allow for an at least approximate comparison to older studies we compare in Fig. 6 the asymmetry parameter \( f \) taken from Vardya (1988) with the value of \( \chi^2 \) derived in this study. We have 89 stars in common with the Vardya study. A nice relation of a decreasing value of \( f \) with an increasing value of \( \chi^2 \) can be clearly seen below \( \chi^2 = 0.2 \). Above that value the relation is somehow lost in a large scatter and in the low number of available objects, but a fraction of the data points seem to still follow the relation suggested from the objects with \( \chi^2 < 0.2 \). Stars are separated here into group 1 (open triangles) and group 2 (open boxes). We note that there is no star in the sample of Vardya (1988) with a \( f \)-value below 0.3. On the other hand, for the most extreme objects in our sample like DD Ara it is very difficult to determine the exact time of the minimum due to its broadness.

We see also a branch of stars around \( f = 0.5 \). These are stars,
Fig. 4  Examples for normalized averaged light curves of miras from the ASAS variable star catalogue. A value of 0 on the y-axis corresponds to light maximum, while a value of 1 marks the minimum. The observed data are indicated by filled symbols, while the sinusoidal comparison curve is marked by open symbols. For illustrative purpose the first and last data point of each light curve has been repeated and the phase of the light maximum ($\Phi=0$) has been shifted to the centre of each panel. The numbers in the upper right corner of each panel gives the corresponding value of $\chi^2$, $\chi_1$, and $\chi_2$, respectively. Top row (group 1): ASAS 171611-6148.0 (DD Ara), ASAS 201445-4659.0 (R Tel), ASAS 202916-4025.1 (U Mic). Second row (group 2): ASAS 162522-5827.8 (EQ Nor), ASAS 173706+1813.1 (FR Her), ASAS 205300+2322.3 (RX Vul). Bottom row (sinusoidal): ASAS 051917-3342.5 (T Col), ASAS 233227-4559.3 (V Phe), ASAS 134821-3651.7 (RT Cen).

which are symmetric but not sinusoidal. Also the star with the largest value of $\chi^2$ in this comparison belongs to this group. These findings reveal an advantage of our method in comparison with the older approach, namely the ability to identify and characterize not only asymmetric but any non-sinusoidal light variation.

4 Discussion

Approximately 30% of the mira light curves deviate significantly from a sinusoidal variation (see Fig. 3). This value is rather independent from the inclusion of lower amplitude stars. If we exclude all stars from our sample with a visual amplitude of less than 2.5 mag, we find 28% stars to
show a non-sinusoidal light change (compared to 33 % in the other case). This result is very similar to the fraction of stars showing an asymmetry value \( f \) outside the range \( 0.45 < f < 0.50 \) according to Vardya (1988). However, by using the whole light curve instead of only two points (maximum and minimum) we think that our method leads to a more robust tool for studying the light curve shape circumventing the problem of determining the minimum point in a rather flat and broad minimum. Furthermore, the group of stars with \( \chi^2 > 0.2 \) includes also several stars with symmetric but non-sinusoidal light changes (group 2). In combination with the two values \( \chi_1 \) and \( \chi_2 \), defined as the absolute deviation of the normalized averaged light curve from the sinusoidal reference curve for the first and second half of the light change, two interesting classes of non-sinusoidal light curves could be detected and well separated.

In the following we want to briefly investigate a possible connection between non-sinusoidal light change and other stellar parameters. Two parameters directly result from the light curves, namely the stellar period and the visual amplitude. None of the two parameters shows any clear correlation with the calculated values \( \chi^2 \), \( \chi_1 \) or \( \chi_2 \). We note, that Mennessier et al. (1997) come to the same result by using the \( f \) value (see their Fig. 2).

The ASAS catalogue further gives various colours for each object from the 2MASS and the IRAS database. The colour of a sample star will depend both on the surface temperature and the circumstellar reddening (plus some contribution from interstellar reddening, which was not taken into account here). Both aspects are likely related to the mass loss, and from previous studies described in the introductory section we would suspect to see a relation here. However, only a weak trend of \( \chi_1 \) with \((J - K)\) could be found (Fig. 7: stars with high (positive) values of \( \chi_1 \) (group 1) are not found among the reddest objects in our sample. Other colours tested, namely \((V - K)\), \((V - [12])\) and \(([12] - [25])\), all show no relation between any of our three parameters and colour. We note that the variability of the objects will naturally introduce some scatter in the colour, which might affect this conclusion.

236 stars of our sample have a spectral type in the GCVS (linked via the variable name given in the ASAS catalogue). Of these, 23 (9 %) are C-stars (including 1 star classified as CS), 9 (3 %) are S-stars, two objects are K-type stars, and the remaining stars are all of spectral type M. Due to the small fraction of C- and S-stars it is difficult to reach reliable conclusions on the relation between light curve shape and the atmospheric chemistry. Discriminating between sinusoidal and non-sinusoidal light variations based on a \( \chi^2 \) limit of 0.2, we find that about 34 % of the S and C stars are showing non-sinusoidal variations, while only 25 % of the oxygen-rich stars are found in this group. All S-stars with non-sinusoidal variations belong to group 2, i.e. they show narrow minima and broad or double-peaked maxima. For the C-stars there is no clear trend visible.

5 Conclusions

Revisiting the question of the shape of mira light curves we find that about 30 % differ significantly from a sinusoidal light change. A deviation in both directions is observed resulting in two main groups of light curve shapes. While the study of Vardya (1988) focused only on asymmetric light curves (our group 1) we present here for the first time statistics on the second group showing broad or even double peak maxima (group 2). This kind of light change is observed almost as frequent as the well known asymmetric light curve.

We found no or only very weak correlations with colour or light curve parameters like period or visual amplitude. The strongest correlation seems to exist with atmospheric chemistry as S-type stars seem to deviate only into one direction (group 2). In general, S and C-stars show a higher fraction of non-sinusoidal variation than the M-type stars. Based on our sample from the ASAS database we conclude that there is no simple relation between the light curve shape and various observables tested here.
Ongoing and future multiple epoch all-sky surveys will provide a wide potential area for applying the method described in this paper. These datasets will allow to re-investigate the question on which stellar properties finally determine the shape of the light curve. Interest on the typical light curve shape of miras comes from various sides, e.g. to constrain dynamical model atmospheres for AGB stars (Nowotny, Höfner & Aringer 2010), or to simulate mira lightcurves in the preparation of data analysis software. This study was done in preparation of the variability analysis of data from the Gaia satellite.

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