Joint Decision Making of Replenishment, Pricing, and Fresh Keeping Input in Fruit and Vegetable Cold Chain: Based on Markov Process

1. Introduction

In recent years, the level of China’s cold chain industry has been further improved in terms of environment and market vitality. According to the analysis report on market prospect and investment strategic planning of China’s cold chain logistics industry, the market scale of China’s cold chain logistics industry increased year by year from 2012 to 2017. In 2018, the total cold chain business revenue of China’s top 100 cold chain logistics enterprises reached 39.824 billion yuan, a year-on-year increase of 53.27%, accounting for 13.79% of the whole cold chain logistics market. There were 25 new enterprises among the top 100 enterprises, It shows that the cold chain industry has full competition and strong market vitality. However, the average profit margin of China’s cold chain industry is about 6.15%–7%, while that of the developed countries such as Europe and the United States can reach 20%–30%. The main reason for the low profit margin of China’s cold chain industry is the economic loss caused by the product consumption. Relevant data show that the annual loss rate of fruits and vegetables produced in China from field to table is as high as 25%–30%, worth hundreds of billions of yuan, while the loss rate of fruits and vegetables in developed countries is less than 5%.

In addition, with the improvement of China’s residents’ income and consumption level, people not only increase the demand for cold chain products, but also increase the requirements for the quality and safety of cold chain products. Consumers are more and more inclined to buy high-quality products. The change of consumption concept puts forward higher requirements for China’s cold chain operation. How to improve the cold chain profit and realize the stable development of the cold chain industry on the premise of meeting consumers’ requirements for product quality is an urgent problem to be solved.

The joint decision-making of cold chain replenishment, pricing, and fresh-keeping investment is an important theoretical basis for cold chain inventory operation management. It is of great significance to reduce the deterioration cost caused by cold chain product corruption,
optimize the related decisions of cold chain upstream and downstream inventory management, and increase the sales profits. However, at this stage, the mathematical description and solution of the relationship between the deterioration characteristics, demand characteristics, and fresh-keeping input of cold chain products, especially fruit and vegetable products, are not deep enough, resulting in the low closeness between the relevant research and the actual cold chain at this stage, which directly affects the in-depth development of theory and practice in this field. Therefore, it is of great theoretical and practical significance to dig deeper into the joint decision-making of replenishment, pricing, and fresh-keeping investment in the cold chain in theory, for a deeper understanding of the corruption characteristics and demand characteristics of cold chain products, for a reasonable and scientific fresh-keeping investment, for improving the level of customer satisfaction, and for enhancing the scientificity of cold chain management.

2. Literature Review

The pricing and inventory optimization of cold chain system has always been a hot issue in the field of cold chain. In recent years, there have been a lot of research on the joint optimization of cold chain inventory and pricing from different angles (Feng et al. [1], Azadi et al. [2]).

Wang and Liu [3]establish a replenishment pricing model with three parameter Weibull distribution characteristics, price-sensitive demand, and randomness. The direct method is used to solve the replenishment pricing strategy of cold chain retailers with profit maximization as the goal and products with non-instantaneous deterioration characteristics. It is pointed out that the nonlinear time-varying consumption rate can more comprehensively describe the consumption characteristics of cold chain products. Neda et al. [4], Mahmoodi [5]establish a replenishment pricing model with piecewise time-varying quantity consumption rate and sensitive demand to price and inventory. The direct method is used to solve the replenishment pricing strategy of cold chain retailers aiming at maximizing the profit. Although the above research reflects the diversity of research in this field in terms of demand assumption and quantity consumption assumption, the assumption of cold chain product consumption rate is limited to quantity consumption, that is, it is assumed that a certain proportion of products completely lose their effectiveness in the sales cycle, while the quality of other products remains intact.

Goyal and Giri [6] first pointed out that the consumption rate of cold chain products can be divided into two types. The first is the quantity consumption rate, which refers to the proportion of commodity consumption quantity in the total quantity of commodities. The second is the quality loss rate, which is the proportion of the decline of commodity quality in the quality of goods in good condition. The deterioration caused by the quantity consumption rate makes the goods completely ineffective, cannot continue to be sold, and needs to be discarded immediately. The deterioration caused by quality loss rate reduces the quality of goods, but it still has a certain commodity utility and can continue to be sold. Obviously, the assumption of only quantity loss is applicable to the links with superior fresh-keeping environment in the circulation of cold chain products, because the quality loss rate of products in good fresh-keeping environment is very low and can be ignored. However, in the links where professional preservation management cannot be carried out for cold chain products, the quality loss is more obvious and cannot be ignored. Especially in the retail link of the cold chain of fruit and vegetable products, retailers need to put fruit and vegetable products on the shelves for sale, which is difficult to create a low-temperature or closed fresh-keeping environment for the products, so that not only the quantity loss but also the obvious quality loss will occur in the sales period after the products are put on the shelves.

Therefore, for such links in the circulation of cold chain products, the assumption of considering quantity loss and quality loss at the same time is obviously more realistic. Now, some studies have further established the internal relationship among quality loss, product quality, and demand in the demand function on the basis of considering the quantity loss. That is, the loss of product quality leads to the decline of product quality, which leads to the reduction of demand. Xu [7]established a cold chain replenishment pricing model considering product quality consumption, and the demand is sensitive to the average freshness and price of products. The direct method is used to solve the optimal pricing strategy under the two-stage pricing model. Qin et al. [8] established a replenishment pricing model considering quantity loss and quality loss at the same time, and the demand is sensitive to price, product quality, and inventory. The direct method is used to solve the retailer replenishment pricing strategy aiming at maximizing profit. The quality loss rate and quantity loss rate are expressed as two-parameter Weibull function, and it is pointed out that the assumption of double loss can more comprehensively describe the characteristics of commodity loss. Rabbani et al. [9] established a replenishment pricing model considering quantity loss and quality loss at the same time, and the product demand is sensitive to price and product quality. The direct method is used to solve the replenishment pricing strategy of cold chain retailers aiming at profit maximization. The quantity loss rate is expressed as a three-parameter Weibull function to reflect the instantaneousity of quality loss, and the quality loss rate is expressed as a two-parameter Weibull function to reflect the noninstantaneity of the quantity loss. The research points out that the assumption of considering quantity loss and quality loss at the same time can more comprehensively describe the quantity loss characteristics of cold chain products and the demand changes caused by quality loss. Although the above documents further describe the quality loss while considering the quantity loss, they do not consider how to properly invest in the preservation technology to slow down the loss of demand caused by product loss.

In recent years, among a large number of relevant studies on the joint decision-making of cold chain inventory and pricing, some studies have considered the fresh-keeping input factor in the joint decision-making of cold chain inventory and pricing, and further considered the fresh-
keeping input and established the internal relationship between the four on the basis of establishing the internal relationship among quality consumption, product quality, and demand in the demand function. At present, the fresh-keeping investment involved in such research can be divided into two types. The first is the technical fresh-keeping investment. This kind of fresh-keeping investment can affect the consumption rate of products and slow down the consumption rate of products, such as purchasing low-temperature equipment to create a low-temperature environment for products. Studies involving technical fresh-keeping inputs include Wang and Jiang [10] and Wang et al. [11] which established a replenishment pricing model considering quantity loss and quality loss at the same time, and the demand is sensitive to product quality and price. Genetic algorithm is used to solve the replenishment, pricing, and fresh-keeping investment strategies of secondary and tertiary cold chain aiming at profit maximization. The former constructs the functional relationship among fresh-keeping investment, quality loss rate, and quantity loss rate for the first time and proves its rationality. It is pointed out that for agricultural products with strong time sensitivity, appropriate technical fresh-keeping investment can effectively increase the total profit of the cold chain. In addition, Zou et al. [12] are involved in the research of technical fresh-keeping investment in the cold chain inventory model. His research only considers quality consumption, and the demand is sensitive to product quality and fresh-keeping investment. The second is the appearance fresh-keeping investment, which does not affect the consumption rate of the product, but will affect the final presentation quality of the product and then affect the demand, such as exquisite packaging of the product. The research involving appearance preservation investment includes that Pakhira et al. [13] establish a replenishment pricing model considering quantity consumption and demand, which is sensitive to price and appearance preservation investment and has seasonal variability. The direct method is used to solve the joint decision-making problem of secondary cold chain inventory, pricing, and preservation investment aiming at profit maximization. Feng [14] established a replenishment pricing model considering quantity loss and quality loss at the same time, and the demand is sensitive to price and product quality. The fuzzy analysis method is used to solve the joint decision-making problem of replenishment, pricing, and fresh-keeping investment of cold chain retailers with the goal of profit maximization. The linear relationship between appearance preservation investment and product quality is constructed for the first time, and it is pointed out that appropriate appearance preservation investment can increase the cold chain profit. Soni and Suthar [15] established a replenishment pricing model considering that quantity consumption and demand are sensitive to price and appearance fresh-keeping investment and are affected by random factors. The direct method is used to solve the joint replenishment pricing decision-making problem of cold chain retailers aiming at maximizing profits. Although the above literature considers the impact of fresh-keeping investment on demand, they all express the demand as deterministic demand and do not consider the impact of random factors on demand.

In the actual sales process of fruit and vegetable cold chain retailers, their demand is usually disturbed by random factors, such as weather and traffic conditions. In the research on the joint decision-making of cold chain inventory and pricing, some studies have established the internal relationship between quality consumption, product quality, and demand in the demand function, further considered the random factors, and established the internal relationship between the four. For example, based on Markov process, Zhang and Wang [16] established a replenishment pricing model considering quantity consumption; demand is price sensitive and affected by random factors. The strategy iteration method is used to solve the joint decision-making problem of replenishment, pricing, and fresh-keeping investment of cold chain retailers in the indefinite period with the goal of profit maximization.

To sum up, the existing research on the description of quantity loss has been relatively mature, but there are relatively few studies considering quality loss and quality loss at the same time, and most of them are limited to describing the internal relationship between quality loss, product quality, and demand when expressing the demand function. There are few studies on further considering appearance preservation investment and random factors on the basis of these three. And almost all the existing studies are static models, that is, it is assumed that there is only one replenishment cycle or the replenishment volume of multiple replenishment cycles is the same. Existing studies such as Chen et al. [17], Crama et al. [18] proposed that when the demand is affected by random factors and shows uncertain demand, the solution method of dynamic programming is more scientific.

Different from previous studies, this study considers that a retailer’s inventory is affected by quantity loss and quality loss at the same time, and allows out of stock. Considering Uncertain Demand, this study establishes the internal relationship between appearance fresh-keeping investment, quality consumption, product quality, random factors and demand, with the goal of maximizing the profit. The Markov dynamic programming model is used to solve the replenishment, pricing, and fresh-keeping investment strategies of retailers in multiple replenishment cycles within the unlimited sales period.

3. Problem Description

This study considers a retailer inventory system to make decisions on replenishment and sales strategies for multiple replenishment cycles of a single product in an infinite sales period. At the beginning of each replenishment cycle with equal duration, the retailer makes decisions on replenishment quantity, pricing, and preservation input time according to the current inventory status. After each replenishment, the products are affected by both quantity consumption and demand in a replenishment cycle and begin to decrease. The demand is uncertain, accompanied by random disturbance factors. At the same time, the demand is sensitive to product quality and price, and the product quality is affected by quality consumption and fresh-keeping investment. During this period, retailers invest in one-time appearance preservation of
products at the preservation investment time point, so as to improve the quality of products during the sales. At the end of each replenishment cycle, there is no remaining inventory, but shortage is allowed. The demand for shortage is sensitive to product price and waiting time. The shortage quantity at the end of each replenishment cycle is dynamically modeled as a state variable of dynamic programming.

4. Mathematical Model Construction

4.1. Model Assumptions and Symbol Description

4.1.1. Model Assumptions.

(1) The product demand is a continuous function of time, and the system allows out of stock.

(2) Customers are sensitive to product quality, price, and waiting time at the same time. That is, consumers tend to products with low price, high quality, and short waiting time when out of stock.

(3) Retailers choose a time point in each replenishment interval to make an appearance fresh-keeping investment for the remaining products. Appearance preservation investment will not affect the quality and quantity deterioration process of products, but will affect the quality of products when they are sold.

(4) The system replenishes goods instantaneously, regardless of the lead time.

(5) Once the product is put into storage, it will deteriorate immediately.

4.1.2. Symbol Description.

(1) inventory model symbol description:

\[ I(t) : \text{inventory during replenishment interval in } v \]
\[ D : \text{demand rate, the demand rate in each replenishment interval is divided into three segments: } D_1, D_2, D_3 \]
\[ Q : \text{single replenishment quantity.} \]
\[ q(t) : \text{quality of product at time } t \]
\[ \theta(t) : \text{quantity loss rate which satisfies the Weibull function distribution, } \theta(t) = a\beta\theta^{\beta-1} \]
\[ y(t) : \text{quality loss rate which satisfies the Weibull function distribution, } y(t) = abt^{b-1} \]
\[ \delta : \text{input of unit product appearance fresh-keeping} \]
\[ \omega : \text{influence degree coefficient of unit appearance preservation investment on product quality.} \]
\[ P : \text{selling price.} \]
\[ K : \text{preservation input time point.} \]
\[ S : \text{out of stock time point.} \]
\[ K_f : \text{unit input cost (/unit)} \]
\[ H_f : \text{unit inventory cost (/unit/hour)} \]
\[ C_f : \text{unit purchase cost (/unit)} \]
\[ P_f : \text{unit deterioration cost (/unit)} \]
\[ S_f : \text{unit shortage cost (/unit)} \]

\( T_f : \text{unit transportation cost (/time/unit)} \)
\( m : \text{retailer replenishment interval.} \)

(2) Symbolic description of Markov model:

State variable:

\[ s^v : \text{Inventory initial status in the replenishment interval } v, \text{ which is equal to the inventory status at the end of the } v-1 \text{ replenishment interval. } s^v = I^{v-1}((v-1)m) \]

Decision variables:

\[ a : \text{behavior vector.} \]
\[ a \equiv [P(s^v), K(s^v), S(s^v)] \]
\[ P(s^v) : \text{selling price.} \]
\[ K(s^v) : \text{preservation input time point.} \]
\[ S(s^v) : \text{out of stock time point.} \]
\[ \pi(s^v) : \text{decision vector, } \pi(s^v) = P(a|s^v) \text{ means the probability of taking action } a \text{ in the initial inventory state } s^v \text{ in the replenishment interval } v. \]

Other symbols:

\[ R_v^a : \text{One-step reward, the profit obtained by the retailer taking action } a \text{ when the initial state of the first replenishment interval is } s. \]
\[ R_v^{a, v-1} : \text{One-step reward: the profit obtained by the retailer taking action } a \text{ when the initial state of the first replenishment interval is } s. \]
\[ P_v^{a, v-1} : \text{One-step reward: the retailer takes action } a \text{ when the initial state of the } v \text{ replenishment interval is } s, \text{ resulting in the profit obtained when the initial state of the } v+1 \text{ replenishment interval is } s. \]
\[ V_v(s^v) : \text{state value function.} \]
\[ V^\ast (s^v) : \text{optimal state value function.} \]
\[ q_v (s^v, a) : \text{action value function.} \]
\[ q^\ast (s^v, a) : \text{optimal action value function.} \]
\[ \varepsilon : \text{random factor of demand function.} \]
\[ A : \text{behavior space.} \]
\[ S^A : \text{state space.} \]

4.2. Retailer Inventory Model. Retailers have similar inventory curves in each replenishment interval, so this chapter will focus on the analysis of retailers’ inventory in the replenishment interval \( v. \)

4.2.1. Change of Retailer’s Inventory during the Replenishment Interval \( v. \)

(1) As shown in Figure 1, when \((v - 1)m < t \leq (v - 1)m + K\), the retailer’s inventory change is affected by demand rate \( D_1 \) and quantity consumption rate, where demand is a function of product quality and price.

(2) As shown in Figure 1, when \((v - 1)m + K < t \leq (v - 1)m + S\), retailers begin to invest in appearance preservation of products, and the change of retailer inventory is affected by demand rate \( D_2 \) and quantity consumption rate, in which demand is a function of product quality, preservation investment, and price.
4.2.2. Construction of Retailer Demand Function in the Replenishment Interval $v$

(1) Demand rate $D_1$ satisfies $D_1(P, q) = A_0 \cdot q \cdot P^{-k} + \epsilon$, in which $q$ is the product quality at time $t$. $q$ satisfies the condition $\frac{dq(t)}{dt} = -\gamma(t) \cdot q(t)$. $\gamma(t)$ is the retailer's inventory quality consumption rate, which satisfies $\gamma(t) = ab^{h(t)}$ and $q((v-1)m) = q_0$ in which $q_0$ is the initial quality of each replenishment point. According to the above conditions, the result is

$$ q(t) = q_0 \cdot e^{-\theta(t)(v-1)m}, $$

According to the different product types and sales investment on product quality, which varies degree coefficient of unit appearance preservation $\theta$. $\theta$ is a random variable.

(2) Demand rate $D_2$ satisfies $D_2(P, q_\delta) = A_0 \cdot q_\delta \cdot P^{-k} + \epsilon$, in which $q_\delta$ is the quality of the product after adding fresh-keeping input, which satisfies $q_\delta(t) = q_0 \cdot e^{-h(t)} + \delta \cdot \delta$. $\delta$ is the input of unit product appearance preservation, which varies according to the different product types and sales periods.

(3) Demand rate $D_3$ satisfies $D_3(P, t) = A_0 \cdot q_0 \cdot e^{-h(t)} \cdot P^{-k} + \epsilon(v-1)m \leq t \leq (v-1)m + M$,

The above formula meets the following conditions:

Condition 1. $I^z_1((v-1)m + S) = 0$

Condition 2. $I^z_1((v-1)m + S) = 0$

Condition 3. $I^z_1((v-1)m) = P + I^z_1((v-1)m) - I^z_1((v-1)m)$

Condition 4. $I^z_1((v-1)m + K) = I^z_1((v-1)m + K)$

Problem-solving process:

From the solution of condition 3 and (4):

$$ I(t)_1 = e^{-\int_{(v-1)m}^{t} \theta(r)dr} \cdot [Q + I^z_1((v-1)m) + \int_{(v-1)m}^{t} D_1(u) \cdot e^{-\int_{(v-1)m}^{u} \theta(r)dr} du]. $$

Let, $g(t) = \int_{(v-1)m}^{t} \theta(r)dr = \int_{(v-1)m}^{t} a \beta \alpha dr = a \beta \alpha (t - (v-1)m)^{\beta}$

So, $I(t)_1 = e^{-g(t)} \cdot [Q + I^z_1((v-1)m) + \int_{(v-1)m}^{t} D_1(u) \cdot e^{\int_{(v-1)m}^{u} \theta(r)dr} du].$

From the solution of condition 2 and (5):

$$ I(t)_2 = e^{-\int_{(v-1)m}^{t} \theta(r)dr} \cdot \int_{(v-1)m}^{t} D_2(u) \cdot e^{-\int_{(v-1)m}^{u} \theta(r)dr} du = e^{-g(t)} \cdot \int_{(v-1)m}^{t} D_2(u) \cdot e^{\int_{(v-1)m}^{u} \theta(r)dr} du = e^{-g(t)} \cdot \int_{(v-1)m}^{t} D_2(u) \cdot e^{\theta(u)du} du.$$

From the solution of condition 1 and (6):

$$ I(t)_3 = -A_0 \cdot P^{-k} \cdot q_0 \cdot \ln(-\sigma \cdot \nu + \rho \cdot \nu + t \cdot (\sigma - \rho) - 1) \cdot \cdot (\sigma - \rho)^{k} + t \cdot \epsilon + A_0 \cdot P^{-k} \cdot q_0 \cdot \ln(-\sigma \cdot \nu + \rho \cdot \nu + v - \nu - (v - 1)m).$$

4.2.3. Construction of Retailer's Inventory Function in the $v$-th Replenishment Interval. Retailer's inventory change in the first replenishment interval $v$:

$$ \frac{dI(t)}{dt} = -\theta(t)I(t)_1 - D_1(v-1)m \leq t \leq (v-1)m + K, $$

$$ \frac{dI(t)}{dt} = -\theta(t)I(t)_2 - D_2(v-1)m $$

$$ + K \leq t \leq (v-1)m + S, $$

$$ \frac{dI(t)}{dt} = -D_3(v-1)m + S \leq t \leq vm, $$

$(v = 1, 2, 3, \ldots).$
\[ Q = -I_3^{-1}((v-1)m) + \int_{(v-1)m+K}^{(v-1)m+S} D_1(t) \cdot e^{\theta(t)} \, dt \\
- \int_{(v-1)m+K}^{(v-1)m} D_1(t) \cdot e^{\theta(t)} \, dt. \] 

(3)

Therefore, the single replenishment quantity \( Q \) can be obtained when the decision variable \( P, K, S \) is known, so it is not used as the decision variable of this model.

The solution shows that the retailer's inventory function in the replenishment interval \( v \) is

\[ I(t) = e^{-\theta(t)} \left[ Q + I_3^{-1}((v-1)m) + \int_{t}^{(v-1)m} D_1(u) \cdot e^{\theta(u)} \, du \right], \]

\( (v-1)m \leq t \leq (v-1)m+K, \)

\[ I(t) = I(t) + e^{-\theta(t)} \int_{t}^{(v-1)m+S} D_2(u) \cdot e^{\theta(u)} \, du, \]

\( (v-1)m+K \leq t \leq (v-1)m+S, \)

\[ I(t) = -A_0 \cdot P^k \cdot q_0 \cdot \ln(-\sigma \cdot vm + \rho \cdot vm + (v-1)m) \cdot (\sigma - \rho)^{-1} + \rho \cdot \rho \cdot (v-1)m + S \cdot \rho \cdot (v-1)m \]

\[ + (v-1)m + S \cdot e, \]

\( (v-1)m + S \leq t \leq vm, \)

\( v = 1, 2, 3, \ldots. \) 

(4)

The retailer's replenishment volume in the replenishment interval \( v \) is:

\[ Q = -I_3^{-1}((v-1)m) + \int_{(v-1)m+K}^{(v-1)m+S} D_1(t) \cdot e^{\theta(t)} \, dt - \int_{(v-1)m+K}^{(v-1)m} D_1(t) \cdot e^{\theta(t)} \, dt. \] 

(10)

The retailer's weighted inventory during the replenishment interval \( v \) is:

\[ I(t) = \int_{t}^{(v-1)m} I(t)^c \, dt + \int_{(v-1)m+K}^{(v-1)m+S} I(t)^c \, dt. \]

(11)

The demand of retailers in the \( v \) replenishment interval is

\[ Dm = \int_{(v-1)m}^{(v-1)m+K} D_1(t) \, dt + \int_{(v-1)m+K}^{(v-1)m+S} D_2(t) \, dt + \int_{(v-1)m+K}^{vm} D_3(t) \, dt. \] 

(5)

4.2.4. Retailer Profit Calculation. The various costs of the retailer in the replenishment interval \( v \) are:

1. Total inventory cost:

\[ HF = Hf \cdot I(t)m. \] 

(6)

2. Total purchase cost:

\[ CF = Cf \cdot Q. \] 

(7)

3. Total deterioration cost:

\[ PF = Pf \cdot (Q - Dm). \] 

(8)

4. Total fresh-keeping input cost:

\[ KF = Kf \cdot \delta \cdot \int_{(v-1)m+K}^{(v-1)m+S} I_2(t) \, dt. \] 

(9)

5. Out of stock cost:

\[ SF = Sf \cdot \int_{(v-1)m+S}^{vm} D_2(t) \, dt. \] 

(10)

6. Transportation cost:

\[ Tf. \] 

(11)

Then, the total cost of a retailer in the replenishment interval \( v \) is

\[ F = HF + CF + PF + KF + SF + TF. \] 

The total profit of the retailer in the replenishment interval \( v \) is

\[ Rm = Dm \cdot P - F. \] 

(13)

4.3. Construction of Dynamic Programming Model Based on Markov Decision Process. According to the previous section, the next step reward function in the state \( s \) of the replenishment interval \( v \) is

\[ E[\mathbb{R}^n_v(\pi(s'), \epsilon)] = E(Rm) = \sum_{s' \in S} P^n_{s,s'} \cdot \mathbb{R}^n_{s',\pi(s')} \cdot (\pi(s'), \epsilon). \] 

(14)

In the initial state \( s \) of the \( v \) replenishment interval, the one-step reward \( \mathbb{R}^n_{s,v} \) obtained by taking action according to strategy \( \pi \) is a function of strategy \( \pi \) and random variable \( \epsilon \). The one-step reward contains random variables, so there is no exact value, only the expected value, which is equal to the expected value of the retailer's total profit in the replenishment interval \( v \).

The state transition function is

\[ E[s^{v+1} (s', \pi(s'), \epsilon)] = E(I_3^c (vm)). \] 

(15)

The initial state \( s^{v+1} \) of the replenishment interval \( v + 1 \) is a function of the initial state \( s' \), strategy \( \pi \), and random variable \( \epsilon \) of the replenishment interval \( v \). The state transition function contains random variables, so there is no exact value, only the expected value, which is equal to the expected value of the shortage at the end of the \( v \) replenishment interval.

The objective function is

\[ \max_{\pi \in \Pi} \sum_{v=1}^{n} \gamma^{v-1} E(\mathbb{R}^n_v). \] 

(16)
That is to solve the strategy \( \pi \) to maximize the sum of one-step rewards from the first replenishment interval to the \( n \)th replenishment interval.

This objective function can be solved by strategy iteration or value iteration according to Markov decision process (MDP). In order to facilitate the iterative solution, the relevant variables are handled as follows:

1. **State space**
   - The state space \( S^A \) to which the state variable \( s' \) belongs is divided into equal lengths:
     \[
     S^A = k\Delta, k = 0, 1, 2, \ldots, K_j. \tag{17}
     \]
   - The formula satisfies \( \Delta = s_M/K_s \), \( s_M \) is the lower limit of inventory.

2. **Strategy space**
   - The decision vector \( \pi(s^v) = [P(s^v), K(s^v), S(s^v)] \) is discretized:
     \[
     B = \{\pi(s^v)|\pi(s^v) = (i\Delta_u, j\Delta_o, m\Delta_p)\}, i, j, m \in Z^+ \cup \{0\}. \tag{18}
     \]

   where \( \Delta_p, \Delta_u, \Delta_o \) are the interval length after discretization of the decision variable selling price, fresh-keeping input time point, and out of stock time point.

   Related variable limit:
   \[
   S^M \leq s' \leq 0. \tag{19}
   \]

   The inventory status at the beginning of the guarantee period is greater than the lower limit of inventory and less than zero.
   \[
   (v-1)m \leq K(s^v) \leq S(s^v). \tag{20}
   \]

   Ensure that the input time point of freshness preservation is greater than the time point at the beginning of replenishment cycle and less than the time point of shortage.
   \[
   (v-1)m \leq S(s^v) \leq vm. \tag{21}
   \]

3. **State transition probability**
4. **Discretization of random variable \( \varepsilon \):**
   - \( G = [\varepsilon_k|\varepsilon_k = \varepsilon' + k\Delta, k = 0, 1, \ldots, K]. \tag{22} \)
   - \( \Delta = (\varepsilon'' - \varepsilon')/K. \)
   - \( \phi(\varepsilon_k) \) is used to represent the distribution law of random variable \( \varepsilon_k \) in the range of \([\varepsilon_k, \varepsilon_{k+1}]\):
     \[
     \phi(\varepsilon_k) = \int_{\varepsilon_k}^{\varepsilon_{k+1}} f(\varepsilon)d\varepsilon. \tag{23}
     \]

Then, the state transition probability matrix is
\[
P_{s^v,s^{v+1}}^a = \text{Prob}\{s^{v+1} = j | s^v = i, \pi(s^v)\} = \text{Prob}\{j = S^A (i, s^v, \varepsilon)\}, \tag{24}
\]

When action \( a \) is taken according to strategy \( \pi \) in the initial state \( s^v \) of the replenishment interval \( v \), the probability that the initial state of the \( v + 1 \) replenishment interval is \( s \) is \( P_{s^v,s^{v+1}}^a \).

At this time, the state, behavior strategy, attenuation coefficient, one-step reward, state transition matrix, and objective function of Markov process are all known, and Behrman equation can be used for iterative solution.

### 4.4. Construction of Behrman Equation

Since the one-step reward function contains random variable \( \varepsilon \), according to the state transition matrix, (17) can be transformed into:
\[
\max_{a \in A} \sum_{v=1}^n \sum_{\varepsilon_k \in E} \phi(\varepsilon_k) \gamma^{v-1} \cdot R^a_{sv} = \max_{a \in A} \left[ \sum_{v=1}^n \left( \sum_{s_{v+1} \in S} P_{s^v,s^{v+1}}^a \cdot \gamma^{v-1} R^a_{s_{v+1}} \right) \right]. \tag{25}
\]

That is to solve the strategy \( \pi \) to maximize the sum of the expected value of the retailer’s one-step reward from the first replenishment interval to the \( n \)th replenishment interval.

According to the Behrman optimal equation, the optimal state value function of the replenishment interval \( v \) is defined as \( V^*(s^v) \), then (19) can be transformed into an iterative solution to \( V^*(s^v) \).

Therefore, according to
\[
V^*(s^v) = \max_a q^*(s^v, a) = \max_a \left( \sum_{s_{v+1} \in S} P_{s^v,s^{v+1}}^a \cdot V^*(s_{v+1}) \right) \tag{26}
\]

can be transformed into:
\[
V^*(s^v) = \max_a q^*(s^v, a) = \max_a \left( \sum_{s_{v+1} \in S} P_{s^v,s^{v+1}}^a \cdot \gamma V^*(s_{v+1}) \right). \tag{27}
\]

Solving the optimal state value function \( V^*(s^v) \) is transformed into solving the maximized optimal behavior value function \( \max_a q^*(s^v, a) \). At this time, the problem of solving the optimal strategy \( \pi^* \) is finally transformed into:
\[
\pi^*(a|s^v) = \begin{cases} 
1, & \text{if } a = \arg \max_{a \in A} q^*(s^v, a), \\
0, & \text{otherwise} \end{cases} \tag{28}
\]

4.5. Strategy Evaluation and Greedy Solution

4.5.1. Strategy Evaluation. According to the convergence property of Gauss Seidel iteration, the pseudo code of the strategy evaluation process is as follows:

Input : state $s$, strategy $\pi$, one-step reward $R_s^a$, state transition matrix $P_{s'\mid s,a}$, attenuation coefficient $\gamma$

Initialization status value function $V(s^0) = 0$

Repeat $k = 0, 1, 2 \ldots$

For every $s$ do

$$V(s_{k+1}) = \sum_{a \in A} \pi(a|s) \sum_{s' \in S} P_{s'\mid s,a} (R_{s'}^a + \gamma V(s'))_k.$$  

(29)

End for

until $v_{k+1} = v_k$.

Output : $v(s)$

$k$ refers to the $k$th iteration, $v(s)$ is the state value of final convergence after iteration under strategy $\pi$.

4.5.2. Greedy Solution.

For every $s$ do

$$\pi(s)_{k+1} \in \arg \max_a q^\pi(s,a).$$  

(30)

Output : $\pi(s)$

$\pi(s)$ is a new strategy, and the optimal strategy is obtained by repeated strategy evaluation and strategy iteration $\pi^*$.

5. Example Analysis

According to the parameters in Table 1, the example analysis results are as follows:

5.1. Conclusion 1. As shown in Figure 2, when the inventory level is $-7$, the optimal state value reaches the maximum, which is defined as "Max optimal state value point." When the inventory level is greater than $-7$ and gradually approaches $0$, the optimal state value shows a decreasing trend, while when the inventory level is less than $-7$ and gradually approaches $-20$, the optimal state value also shows a decreasing trend.

This change trend is similar to the research conclusion of Zhang and Wang [16]. The reason is that although appropriate shortage will produce additional shortage cost, it can save the high inventory cost caused by inventory backlog caused by commodity deterioration. When the inventory is greater than the optimal shortage level, the inventory cost increases, resulting in the reduction of the optimal state value. When the inventory is less than the optimal shortage level, the shortage cost increases, resulting in the reduction of the optimal state value. Max optimal state value point-7 is the optimal inventory point between shortage cost and inventory cost. In addition, it is worth noting that when the inventory gradually moves away from the max optimal state value point-7 from both the sides, the absolute value of the slope of the optimal state value curve gradually decreases, indicating that when the initial inventory of the current period fluctuates greatly under the influence of random factors, the model can control the fluctuation of system profit within a certain range by adjusting the relevant strategies.

The optimal state value under the model algorithm is compared with the state value under the uniform strategy. The uniform strategy is used to represent that the fruit and vegetable cold chain retailers adopt the replenishment, pricing, and preservation investment strategies uniformly and randomly without the model algorithm. The state value solution under the uniform strategy is to replace the bellman optimal (25) with the bellman expectation equation:

$$V(s') = \sum_{a \in A} \pi(a|s') \sum_{s'' \in S} P_{s''\mid s'} (R_{s''}^a + \gamma V(s'')).$$  

(31)

The uniform strategy is: $\pi(a|s') = 1/N^a$. It means that in state $s'$, the probability of taking each behavior $a$ is the ratio of 1 to the number of all behaviors in the behavior space, that is, each strategy is taken uniformly and randomly. The state value of the uniform strategy after strategy evaluation is shown in Figure 3:

It is found that under different opening inventory states, the optimal state value under the optimal strategy is greater than that under the uniform strategy, and the fluctuation range of state value under the uniform strategy is larger. It shows that compared with random decision-making, the model algorithm can improve the cold chain profit and reduce the fluctuation range of profit.

5.2. Conclusion 2. As shown in Figure 4, the fresh-keeping input time point reaches the maximum near the inventory level $-7$, and its two sides gradually decrease. The reason is that according to conclusion 1, when the inventory level is $-7$, the optimal state value reaches the maximum. When the inventory level fluctuates near $-7$, the inventory level is ideal, and there is no need to make a large adjustment to the fresh-keeping input time point. When the inventory level gradually moves away from $-7$ from both sides, the inventory fluctuates obviously, and the input time point of fresh-keeping needs to be greatly adjusted. When the inventory gradually increases from $-7$, it is necessary to reduce the time point of fresh-keeping investment, that is, fresh-keeping investment in advance. The reason is that early fresh-keeping investment can increase the demand, accelerate the sales speed, and reduce the profit cost caused by inventory backlog. It is worth noting that when the inventory gradually decreases from $-7$, the optimal strategy still chooses to increase the profit by investing in preservation in advance. When the shortage is too large, we should appropriately slow down the sales speed and reduce the shortage cost.

5.3. Conclusion 3. As shown in Figure 5, the price increases from 0 to 20. The reason is that the inventory is small. Appropriately increasing the price can improve the profit, slow down the sales speed, reduce the out of stock time, and reduce the out of stock cost.
5.4. Conclusion 4. As shown in Figure 6, the single replenishment quantity is almost unchanged between 0~7, but shows an obvious increasing trend between −7~−20. Combined with conclusion 2 and conclusion 3, it shows that when the stock decreases between 0 and 7, the optimal strategy is mainly to adjust the price and fresh-keeping input time point, and there is no need to significantly adjust the single replenishment quantity. When the inventory decreases between −7 and −20, the optimal strategy significantly increases the replenishment volume on the basis of adjusting and increasing the price and making fresh-keeping investment in advance, which also explains why in conclusion 2, when the inventory gradually decreases from near −7, the optimal strategy still chooses to increase the profit by raising the fresh-keeping investment. The reason is that increasing the single replenishment quantity can reduce the shortage time, and out of stock cost get the optimal combination. This shows that in the decision-making of single replenishment, increasing the price and making fresh-keeping investment in advance, and realize the best trade-off between the single replenishment quantity and the preservation investment time point.

5.5. Conclusion 5. As shown in Figure 7, although in actual operation, it is only necessary to make decisions on fresh-keeping input time point, price and single replenishment quantity, and the out of stock time point is only to simplify the calculation as the decision variable of the model, it can be seen from the relationship diagram between out of stock time and inventory that when the stock in the inventory increases between −7~0, the out of stock time fluctuation is very small. Combined with conclusion 2, conclusion 3, and conclusion 4, it shows that the shortage time that should have been reduced tends to be stable due to the preservation investment in advance and reducing the price. The reason is that the preservation investment in advance and reducing the price can increase the sales speed, so as to increase the shortage time and reduce the inventory cost. When the inventory decreases between −7 and −20, the out of stock time shows a weak increasing trend. Combined with conclusion 2, conclusion 3, and conclusion 4, it shows that the optimal strategy will eventually lead to the increase of out of stock time when it makes the trade-off between increasing the single replenishment quantity, increasing the price, and making the fresh-keeping investment in advance. This strategy makes the inventory cost, fresh-keeping investment cost, and out of stock cost get the optimal combination. This shows that in the decision-making of single replenishment quantity, fresh-keeping investment and price, the optimal decision cannot be judged intuitively. The calculation of
model algorithm is needed to make an effective trade-off between the three according to different inventory conditions.

To sum up, compared with the uniform strategy, the optimal strategy can improve the cold chain profit and reduce the fluctuation range of profit. When the initial stock of the current period increases from the value point of Max optimal state, the retailer should invest in appearance preservation in advance and reduce the price. When the initial stock of the current period decreases from the value point of Max optimal state, the retailer should increase the single replenishment quantity, increase the price and invest in external preservation in advance, so as to control the fluctuation of system profit within a certain range.

6. Sensitivity Analysis

(1) The sensitivity analysis of shape factor $b$ of product quality loss rate $y(t)$ is carried out, and the results are as follows:

Conclusion 1. As shown in Figure 8, with the increase of $b$ value, the optimal state value gradually decreases. The reason is that the increase of $b$ factor accelerates the deterioration speed of goods, generates additional deterioration costs, and reduces the profits. As shown in Figure 9, as the value of $b$ increases, the single replenishment volume decreases gradually. The reason is that the increase in the value of factor $b$ accelerates the deterioration of goods and reduces the demand rate, resulting in inventory backlog and additional inventory costs. Reducing the single replenishment can reduce the inventory cost caused by the inventory backlog. As shown in Figure 10, with the increase of $b$ value, the value of preservation input time point gradually decreases. The reason is that the increase of factor $B$ makes the deterioration speed of goods faster and the demand rate lower. The investment in preservation in advance can improve the demand rate. As shown in Figure 11, as the value of $b$ increases, the pricing level gradually increases. It is well recognized that when the deterioration rate of products accelerates, the price should be reduced to avoid the increase of inventory cost caused by inventory backlog. The reason is that when the deterioration speed of products is accelerated, reducing the single replenishment volume and fresh-keeping investment in advance will significantly accelerate the sales speed of products and significantly increase the out of stock time. At this time, it is necessary to appropriately increase the price to slow down the sales speed, shorten the out of stock time, and avoid high out of stock costs. It shows that when making decisions on single replenishment quantity, fresh-keeping investment, and price, the optimal decision cannot be judged intuitively, and the calculation of model algorithm is needed to make an effective trade-off between the three according to different inventory conditions. As shown in Figure 12, with the increase of $b$ value, the out of stock time gradually increases, but the change is small. The reason is that when the deterioration speed of products is accelerated, reducing the single replenishment volume and fresh-keeping investment in advance will significantly accelerate the sales speed of products and increase the out of stock time, but the increase of commodity price will shorten the out of stock time.

Conclusion 2. As shown in Table 2, as the value of $b$ increases, the changes of the optimal state value, the optimal single replenishment quantity, and the optimal preservation input time point increase. When
the value of factor $b$ fluctuates below 1, the changes of the optimal state value, the optimal single replenishment quantity, and the optimal preservation input time point are small, while when the value of factor $b$ fluctuates above 1, the changes of the three are more obvious. The reason is that factor $b$ is the shape factor of quality loss rate $c(t)$. According to the study of two-parameter Weibull function by fariborz (2006), when the value of shape factor $b$ is between 0 and 1, $c(t)$ is the decreasing function. When the value of shape factor $b$ is between 1 and 2, $c(t)$ is an increasing function. At this time, the quality loss rate of goods is significantly accelerated with time, the demand rate is significantly reduced, and it is easy to produce serious inventory backlog. At this time, it is necessary to make a large adjustment to the single replenishment quantity and the time point of fresh-keeping investment. At the same time, the optimal state value is also significantly reduced due to the accelerated quality loss rate of goods. This conclusion shows that cold chain retailers should pay attention to the quality loss characteristics of products in different sales environments, and the optimal decision under different loss speeds is also different.

To sum up, with the increase of the shape factor $b$ of the quality loss rate $c(t)$, the optimal state value, optimal single replenishment quantity, and optimal fresh-keeping input time point all decrease, and the optimal price and optimal out of stock time increase. When the value of $b$ fluctuates above 1, the changes of the optimal state value, the optimal single replenishment quantity, and the optimal fresh-keeping input time point are more obvious.
The sensitivity analysis of shape factor $\beta$ of product quantity consumption rate $\theta(t)$ is carried out, and the results are as follows:

With the change of $\beta$ value, the changes of optimal state value and optimal single replenishment quantity are shown in Figures 13 and 14, while the optimal price, optimal fresh-keeping input time point, and optimal out of stock time point do not change after retaining two decimal places, and their values are the same as the results of example analysis.

Conclusion 1. As shown in Figure 13, the value of optimal state decreases with the increase of $\beta$ value. The reason is that the increase of $\beta$ value increases the quantity consumption rate, increases the deterioration cost in the sales period, and reduces the value of the optimal state. As shown in Figure 14, as the value of $\beta$ increases, the optimal single replenishment quantity decreases. The reason is that when the value of $\beta$ increases, the quantity loss rate becomes larger. In this model, the quantity loss is expressed as the proportion of the commodity loss quantity in the total quantity of commodities. Reducing the single replenishment quantity can reduce the quantity of deteriorated commodities and reduce the cost.

Conclusion 2. As shown in Table 3, as the value of $\beta$ increases, the fluctuation range of the optimal state value becomes larger. The reason is that according to the definition of quantity loss rate in the cold chain model by Goyal et al. [6], the quantity loss rate is the loss in the process of product handling, which is related to the handling technology and conditions of workers. The deterioration caused by the quantity loss rate makes the goods completely useless.

Therefore, the loss caused by the increase of quantity consumption cannot be reduced through the strategic adjustment of pricing and fresh-keeping investment (which also explains the reason why the optimal price, optimal fresh-keeping investment time point, and optimal out of stock time point have not changed). The variable quality cost can only be reduced by reducing the single replenishment quantity. After the single replenishment quantity is reduced, the adjustment space of profit is reduced. The fluctuation range of the optimal state value becomes larger. The conclusion shows that the greater the quantity consumption rate, the more difficult it is for the cold chain system to maintain the stability of profit fluctuation. Fruit and vegetable cold chain retailers should pay attention to improving the handling technology of workers and the quality of handling machines, so as to improve the stability of cold chain profit.

To sum up, with the increase of the shape factor $\beta$ of the quantity consumption rate $\theta(t)$, the optimal state value decreases, the fluctuation range of the optimal state value increases, and the optimal single replenishment volume decreases, while the optimal price, the optimal fresh-keeping

| Change of $b$ | Change of $V^*$ | Change of $V^*$ (%) | Change of $Q$ | Change of $Q$ (%) | Change of $K$ | Change of $K$ (%) |
|---------------|----------------|---------------------|---------------|-------------------|--------------|-------------------|
| $b=0.7 \rightarrow b=0.9$ | $-17.4$ | $-3.8\%$ | $-1.2$ | $-1.24\%$ | $-0.01$ | $-1.3\%$ |
| $b=0.9 \rightarrow b=1.1$ | $-28.3$ | $-6.4\%$ | $-1.3$ | $-2.19\%$ | $-0.02$ | $-2.7\%$ |
| $b=1.1 \rightarrow b=1.3$ | $-56.8$ | $-16.3\%$ | $-2.7$ | $-14.72\%$ | $-0.12$ | $-31.6\%$ |
| $b=1.3 \rightarrow b=1.5$ | $-61.4$ | $-22.5\%$ | $-3.1$ | $-14.93\%$ | $-0.13$ | $-53.8\%$ |
| $b=1.5 \rightarrow b=1.7$ | $-88.0$ | $-46.3\%$ | $-9.1$ | $-17.3\%$ | $-0.14$ | $-75.0\%$ |

Table 2: Relationship between $V^*$, $K$, $Q$ and $b$ value change.

| $\beta$ | Fluctuation range of $V^*$ | Fluctuation difference of $V^*$ |
|---------|----------------------------|-------------------------------|
| 1.7     | 300.9~3246.8               | 54.1                          |
| 1.5     | 369.5~317.6                | 51.9                          |
| 1.3     | 422.4~378.8                | 43.6                          |
| 1.1     | 452.5~411.8                | 40.7                          |
| 0.9     | 500.9~462.2                | 38.7                          |
| 0.7     | 525.8~487.7                | 38.1                          |

Table 3: $V^*$ with $\beta$ Fluctuation of value change.
input time point, and the optimal out of stock time point do not change.

7. In Summary

Taking the fruit and vegetable cold chain retailer as the research object, considering the simultaneous quantity and quality consumption of products, the stock is allowed to be out of stock, the demand is uncertain, and the appearance preservation investment of products can be carried out. Aiming at maximizing the retailer’s profit in an indefinite period, this study uses the Markov dynamic programming method to analyze the replenishment at the beginning of each replenishment cycle. The pricing and fresh-keeping investment decision-making problems are mathematically modeled and solved by examples, and the sensitivity of the shape factors of quality loss rate and quantity loss rate is analyzed.

The results show that, first, when the initial stock of the current period fluctuates under the influence of random factors of demand, compared with the uniform strategy, the optimal strategy under the model can improve the cold chain profit and reduce the fluctuation range of profit. The model can maximize profits and control the fluctuation of cold chain profits within a certain range by adjusting the relevant strategies. When the initial stock of the current period increases from the value point of Max optimal state, retailers should invest in appearance preservation of products in advance and appropriately reduce the price. When the initial stock of the current period is reduced from the value point of Max optimal state, the retailer should increase the single replenishment, invest in the preservation of appearance products in advance, and appropriately increase the price. Second, the sensitivity analysis shows that: 1. when the shape factor of quality loss rate increases, the cold chain profit decreases. Retailers should reduce the single replenishment volume, invest in the appearance preservation of products in advance, and increase the price to maximize the profit. The adjustment degree of single replenishment quantity and fresh-keeping input time point is closely related to the value range of quality loss rate shape factor. Retailers should pay attention to the quality loss characteristics of products in order to make scientific decisions. 2. When the shape factor of quantity consumption rate increases, the fluctuation range of cold chain profit becomes larger. Retailers should reduce the single replenishment volume to maximize profits. When the quantity consumption is large, retailers should pay attention to reducing relevant technical losses to improve profit stability. In the future, we can further study the non-homogeneous characteristics of the state transition matrix of the opening stock according to the actual sales situation.

The internal relationship between fresh-keeping input, consumption rate, and random demand are mathematically described. The research results supplement the relevant theories in the field of joint decision-making of cold chain inventory and pricing. The model construction improves the closeness between the cold chain mathematical model and the actual operation of the cold chain. In the actual operation of cold chain, as fresh products are easy to deteriorate, how to reasonably invest in fresh-keeping technology and make replenishment and pricing decisions according to the deterioration characteristics and demand characteristics of fresh products to maximize profits is a problem to be solved in the actual operation of this field at this stage. According to the actual characteristics of fresh products, the current situation of fresh market and the development of fresh-keeping technology, this paper accurately describes the input, consumption rate, and demand of cold chain fresh-keeping and uses appropriate algorithms to solve them. On this basis, the best decision-making scheme of replenishment, pricing, and fresh-keeping input is obtained. The model construction of this paper improves the closeness between the cold chain mathematical model and the actual operation of the cold chain. The research conclusion of this paper is conducive to the cold chain operators to reduce the deterioration cost of fresh products and improve the operating profit.

Data Availability

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Conflicts of Interest

The author states that this article has no conflicts of interest.

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