Valied Asynchronous Byzantine Agreement with Optimal Resilience and Asymptotically Optimal Time and Word Communication

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Abstract

We provide a new protocol for Valied Asynchronous Byzantine Agreement. Valied (multi-valued) Asynchronous Byzantine Agreement is a key building block in constructing Atomic Broadcast and fault-tolerant state machine replication in the asynchronous setting. Our protocol can withstand the optimal number $f < n/3$ of Byzantine failures and reaches agreement in the asymptotically optimal expected $O(1)$ running time. Honest parties in our protocol send only an expected $O(n^2)$ messages where each message contains a value and a constant number of signatures. Hence our total expected communication is $O(n^2)$ words. The best previous result of Cachin et al. from 2001 solves Valied Byzantine Agreement with optimal resilience and $O(1)$ expected time but with $O(n^3)$ expected word communication. Our work addresses an open question of Cachin et al. from 2001 and improves the expected word communication from $O(n^3)$ to the asymptotically optimal $O(n^2)$.

1 Introduction

Byzantine agreement is a fundamental problem in computer science introduced by Pease, Shostak and Lamport [23] in 1980. Bracha [6] shows that even strictly weaker primitives than Asynchronous Byzantine agreement can only be solved when the number of parties $n$ is larger than $3f$ where $f$ is the maximum number of parties the adversary can corrupt. We therefore say that a solution has optimal resilience if it solves Byzantine agreement for $n = 3f + 1$. A theorem of Fischer, Lynch and Paterson [15] states that any protocol solving Asynchronous Agreement must have a non-terminating execution even in the face of a single (benign) failure. Ben-Or [5] shows that randomization can be used to make such non-terminating executions become events with probability 0. Feldman and Micali [13] show that Asynchronous Byzantine Agreement can be solved with optimal resilience $n = 3f + 1$ and with an expected $O(1)$ asynchronous running time (where running time is the maximum duration as defined by Canetti and Rabin [10] and is essentially the number of steps when the protocol is embedded into a lock-step timing model). We therefore say that a solution has asymptotically optimal time if it solves Byzantine agreement using an expected $O(1)$ running time. We show that a recent lower bound of Abraham et al. [1] implies that any protocol solving Asynchronous Byzantine Agreement against an adaptive adversary (and without a constant error probability) must have the honest parties send expected $\Omega(n^2)$ messages (see Appendix B). We therefore say that a solution has asymptotically optimal word communication if it solves Byzantine agreement using an expected $O(n^2)$ messages and each message contains just a single word where we assume a word contains a constant number of signatures and domain values. Renewed interest in Byzantine Agreement follows from the need to implement Atomic Broadcast and fault tolerant state machine replication in the asynchronous setting [9, 20, 12]. In 2001, Cachin, Kursawe, Petzold, and Shoup [7] defined the problems of Atomic Broadcast and Valied Byzantine Agreement to address these types of applications. Valied Byzantine Agreement guarantees a decision on some party’s input satisfying a globally verifiable external validity condition. Cachin et al. [7] show how to obtain Valied Asynchronous Byzantine Agreement (VABA) with optimal resilience, asymptotically optimal time and $O(n^3)$ expected word communication. Improving the expected word communication for VABA from $O(n^3)$ to the $O(n^2)$ is an open problem stated in [7] and has been open for almost 20 years.

This paper presents the first VABA solution with optimal resilience, asymptotically optimal time whose expected word communication is $O(n^2)$, thus closing this gap. More precisely, we prove the following theorem:
**Theorem 1.** There exists a protocol among $n$ parties that solves except with negligible probability Validated Asynchronous Byzantine Agreement (VABA), secure against an adaptive adversary that controls up to $f < n/3$ parties, with expected $O(n^2)$ word communication and expected constant running time.

**Background.** In Byzantine Agreement there are $n$ parties each of which has an input value, at most $f < n$ are corrupted (i.e., controlled by an adversary), and the goal of the honest parties is to decide on a unique value. Many models with various assumptions have been proposed in the literature. Some consider asynchronous communication whereas others rely on synchrony. Some restrict the adversary’s computational power in order to use cryptographic tools whereas others assume an information theoretic model, and some assume authenticated or private communication channels whereas others deal with possible forgeability. In this paper, we assume the practical random oracle model [4, 14, 8]. Namely, an environment with authenticated but asynchronous communication channels and a computationally bounded adversary.

A simplified version of the agreement problem is the binary agreement problem in which the inputs of the parties are restricted to the set $\{0, 1\}$. A fundamental work by Cachin, Kursawe, and Shoup [8] was the first to give an optimal algorithm in terms of resilience and word communication in the random oracle model, which they formalized to fit the distributed settings. In particular, the algorithm withstands up to $f < n/3$ Byzantine failures, runs in constant expected number of asynchronous views (rounds), and the expected communication cost is $O(n^2)$ messages of the size of one or two RSA signatures [25]. A more recent work by Mostéfaoui et al. [21] shows how to achieve the same optimal result without any cryptographic assumptions besides the existence of a common random coin$^1$.

As for the multi-valued Byzantine agreement, the original problem specification due to Lamport et al. [23] was motivated by the following setting: Four computers in control of a space-shuttle cockpit need to reach agreement on a sensor reading, despite one being potentially faulty. The problem was captured via a Weak Validity [11] condition as follows:

**Definition 1** (Weak validity). If all honest parties propose $v$, then every honest party that terminates decides $v$.

Note that while the Weak Validity condition is well defined, it says nothing about a situation in which parties propose different values, allowing them to (1) return some default value ⊥ that indicates that no agreement was reached or (2) agree on a value proposed by a corrupted party. Mostéfaoui et al. [22] consider a slightly stronger property in which only a value proposed by an honest party or ⊥ are allowed to be returned. However, honest parties may still decide ⊥ if they initially disagree. In particular, it is not clear how this slightly stronger validity property can be used to solve Atomic Broadcast [7].

Cachin et al. formulated in [7] a problem specification that captures the practical settings where parties propose updates to a replicated state. Agreement is formed on a sequence of updates, hence a non-default decision is needed in order to make progress. To prevent updates from rogue parties, the model is extended with an External Validity predicate as follows:

**Definition 2** (External validity). If an honest party decides on a value $v$, then $v$ is externally valid.

Mostéfaoui et al. presented in [22] a signature-free deterministic reduction from their binary agreement protocol [21] that solved asynchronous Byzantine Agreement with Weak validity. It has optimal resilience and asymptotically optimal time and word communication. However, the weak validity property seems to prohibit the usefulness as a building block for Atomic Broadcast or any State Machine Replication (SMR) protocol that should maintain liveness in an asynchronous environment.

Cachin et al. gave in [7] a randomized reduction from their binary agreement algorithm [8] to VABA and also showed how to use it in order to implement an atomic broadcast. Their VABA protocol provides external validity, has optimal resilience, asymptotically optimal time, and expected message complexity $O(n^3)$. That paper explicitly mentions the open problem of improving the expected word communication from $O(n^3)$ to $O(n^2)$.

**Our Contribution.** The main contribution of this paper is solving this open question. Just like [7], our protocol solves Asynchronous Byzantine agreement with external validity (VABA), has optimal resilience and asymptotically optimal time. Improving on [7], our expected word communication is also asymptotically optimal. In particular, honest parties send a total expected $O(n^2)$ messages, which is optimal and each message is roughly the size of one or two threshold signatures.

$^1$while the construction of [21] requires only $O(n^2)$ bits given a common coin, the word communication of the resulting binary Byzantine Agreement protocol is dominated by the common random coin protocol that requires threshold signatures and $O(n^2)$ word communication.
Our protocol is secure against an adaptive adversary. This follows from using adaptively secure threshold signatures of Libert et al. [18] and adaptively secure common coin protocol of Loss and Moran [19]. Cachin et al. [7] note that their binary protocol [8] and their Validated protocol [7] also immediately generalize to be secure against adaptive adversaries by using the primitives above.

Techniques and Challenges. Our protocols are based on the Random Oracle model and draw heavily from the framework of Cachin et al. [8, 7]. Unlike previous constructions, our protocol does not go through a randomized binary agreement black-box. Instead, much like Katz and Koo’s Synchronous Byzantine agreement protocol [17], in each view, we run $n$ parallel leader-based threads and then use a random leader election primitive to decide which leader is elected in hindsight. Just like [17], not all honest parties reach agreement in the same view. To guarantee safety between different views we use a view-change protocol that guarantees that new leaders can propose only safe values.

Adopting this approach to get optimal word communication requires to overcome several challenges. On the one hand, to obtain $O(n^2)$ word communication per view, we need each of the $n$ leader-based thread protocols to use just $O(n)$ words and we need the global view change protocol to use $O(n^2)$ words. On the other hand, to guarantee progress we make sure that our view change protocol will allow progress even in asynchronous settings. To balance between frugal communication and liveness we adopt a four step protocol that is inspired by an approach taken in the partial-synchrony model by Yin et al. [26].

To obtain optimal $O(1)$ expected time, the next challenge is to guarantee that when the first honest party enters the leader election phase there is a constant fraction of potential leaders such that if one of them is elected then all honest parties will decide in constant time. Moreover, if the elected leader did not complete its broadcast, we need a mechanism to allow parties to abandon the elected leader’s broadcast before running the global view-change protocol.

2 Model

In order to reason about distributed algorithms in cryptographic settings we adopt the model defined in [8, 7]. We consider an asynchronous message passing system consisting of a set $\Pi$ of $n$ parties, an adaptive adversary, and a trusted dealer. The adversary may control up to $f < n/3$ parties during an execution. An adaptive adversary is not restricted to choose which parties to corrupt at the beginning of an execution, but is free to corrupt (up to $f$) parties on the fly. Note that once a party is corrupted, it remains corrupted, and we call it faulty. A party that is never corrupted in an execution is called honest. We denote the set $\Pi_h \subseteq \Pi$ to be the set of all honest parties.

We assume an initial setup before every execution in which the trusted dealer generates the initial states of all parties, and we assume that the adversary cannot obtain the states of honest parties at any time during an execution. In particular, the adversary cannot obtain the initial states of honest parties.

Computation. Following [8, 7], we use standard modern cryptographic assumptions and definitions. We model the computations made by all system components as probabilistic Turing machines, and bound the number of computational basic steps allowed by the adversary by a polynomial in a security parameter $k$. A function $\epsilon(k)$ is negligible in $k$ if for all $c > 0$ there exists a $k_0$ s.t. $\epsilon(k) < 1/k^c$ for all $k > k_0$. A computational problem is called infeasible if any polynomial time probabilistic algorithm solves it only with negligible probability. Note that by the definition of infeasible problems, the probability to solve at least one such problem out of a polynomial in $k$ number of problems is negligible. Intuitively, this means that for any protocol $P$ that uses a polynomial in $k$ number of infeasible problems, if $P$ is correct provided that the adversary does not solve one of its infeasible problems, then the protocol is correct except with negligible probability. We assume that the number of parties $n$ is bounded by a polynomial in $k$.

Communication. We assume asynchronous links controlled by the adversary, that is, the adversary can see all messages and decide when and what messages to deliver. In order to fit the communication model with the computational assumptions, we restrict the adversary to perform no more than a polynomial in $k$ number of computation steps between the time a message $m$ from an honest party $p_i$ is sent to an honest party $p_j$ and the time $m$ is delivered by $p_j$. In addition, for simplicity, we assume that messages are authenticated in a sense that if an honest party $p_i$ receives a message $m$ indicating that $m$ was sent by an honest party $p_j$, then $m$ was indeed generated by $p_j$ and sent to $p_i$ at some prior time. This assumption is

\footnote{Note that although this restriction gives some upper bound on the communication in terms of the adversary local speed, the model is still asynchronous since speeds of different parties are completely unrelated.}
reasonable since it can be easily implemented with standard symmetric-key cryptographic techniques [3] in our model.

**Termination.** Note that the traditional definition of the liveness property in distributed systems, which requires that all correct (honest) parties eventually terminate provided that all messages between correct (honest) parties eventually arrive, does not make sense in this model. This is because the traditional definition allows the following:

- Unbounded delivery time between honest parties, which potentially gives the adversary unbounded time to solve infeasible problems.
- Unbounded runs that potentially may consist of an unbounded number of infeasible problems, and thus the probability that the adversary manages to solve one is not negligible.

Following Cachin et al. [8, 7], we address the first concern by restricting the number of computation steps the adversary makes during message transmission among honest parties. So as long as the total number of messages in the protocol is polynomial in \( k \), the error probability remains negligible. To deal with the second concern, we do not use a standard liveness property in this paper, but instead we reason about the total number of messages required for all honest parties to terminate. We adopt the following definition from [8, 7]:

**Definition 3 (Uniformly Bounded Statistic).** Let \( X \) be a random variable. We say that \( X \) is probabilistically uniformly bounded if there exist a fixed polynomial \( T(k) \) and a fixed negligible functions \( \delta(l) \) and \( \epsilon(k) \) such that for all \( l, k \geq 0 \),

\[
\Pr[X > lT(k)] \leq \delta(l) + \epsilon(k)
\]

With the above definition Cachin et al. [8, 7] define a progress property that makes sense in the cryptographic settings:

- **Efficiency:** The number of messages generated by the honest parties is probabilistically uniformly bounded

The efficiency property implies that the probability of the adversary to solve an infeasible problem is negligible, which makes it possible to reason about the correctness of the primitives’ properties. However, note that this property can be trivially satisfied by a protocol that never terminates but also never sends any messages. Therefore, in order for a primitive to be meaningful in this model, Cachin et al. [8, 7] require another property:

- **Termination\(^3\):** If all messages sent by honest parties have been delivered, then all honest parties terminated.

In this paper we consider both efficiency and termination properties as defined in [8, 7]. However, note that when considering an adaptive adversary, it is also possible to define a slightly weaker termination property:

- **Weak termination:** If all messages sent by parties before they were corrupted have been delivered, then all honest parties terminated.

Note that while any protocol that satisfies termination satisfies weak termination as well, a lower bound for termination does not apply for weak termination. Indeed our lower bound (see Appendix B) is for protocols that obtain the termination property. We leave the study of lower bounds for protocols with weak termination as an open question.

**Complexity.** We use the following standard complexity notions (see for example Cannetti and Rabin [10]). We measure the expected word communication of our protocol as the maximum over all inputs and applicable adversaries of the expected total number of words sent by honest parties where expectation is taken over the random inputs of the players and of the adversary. We assume a finite domain \( V \) of valid values for the Byzantine agreement problem, and say that a word can contain a constant number of signatures (see Section 2.1) and domain values. We measure the expected running time of our protocol as the maximum over all inputs and applicable adversaries of the expected duration where expectation is taken over the random inputs of the players and of the adversary. The duration of an execution is the total

\(^3\)Called liveness in [8], but we find this name confusing since it is not a liveness [2] property.
time until all honest players have terminated divided by the longest delay of a message in this execution. Essentially the duration of an execution is the number of steps taken if this execution is re-run in lock-step model where each message takes exactly one time step.

Following Cachin et al. [7], in order to show that our view-based protocol runs in an expected constant running time and has expected $O(n^2)$ word communication, it is enough to show that:

- every view consists of $R(k) = O(n^2)$ messages that consist of one word, and
- the total number of messages is probabilistically uniformly bounded by $R$.

(Note that the number of parties $n$ is a polynomial in $k$, and thus, so is $R$). This follows from the following Lemma:

**Lemma 1 (CKPS 01 [7]).** Given a probabilistically uniformly bounded by $T$ random variable $X$, there is a constant $c$ s.t. the expected value of $X$ is bounded by $cT(k) + O(k)$, where $c(k)$ is a negligible function.

### 2.1 Cryptographic abstractions

The main focus of this paper is on a novel distributed algorithms. Our protocol uses cryptographic tools as black-boxes. To this end, we present our protocol assuming the existence of the cryptographic abstractions as defined below.

**Threshold signatures scheme.** At the beginning of every execution, every party $p_i$ gets a private function $\text{share-sign}_i(m)$ from the dealer, which gets a message $m$ and returns a signature-share $\sigma_i$. In addition, every party gets the following functions: (1) $\text{share-validate}(m, i, \sigma)$, which gets a message $m$, a party identification $i$, and a signature-share $\sigma_i$, and returns true or false; (2) $\text{threshold-sign}(\Sigma)$, which gets a set of signature-shares $\Sigma$, and returns a threshold signature $\sigma$; and (3) $\text{threshold-validate}(m, \sigma)$, which gets a message $m$ and a threshold signature $\sigma$, and returns true or false. We assume that the above functions satisfy the following properties except with negligible probability:

- **Share validation:** For all $i$, $1 \leq i \leq n$ and for every messages $m_i$, $(1) \text{share-validate}(m_i, i, \sigma) = true$ if and only if $\sigma = \text{share-sign}_i(m_i)$, and (2) if $p_i$ is honest, then it is infeasible for the adversary to compute $\text{share-sign}_i(m_i)$.

- **Threshold validation:** For every message $m$, $\text{threshold-validate}(m, \sigma) = true$ if and only if $\sigma = \text{threshold-sign}(\Sigma)$ s.t. $|\Sigma| \geq 2f + 1$ and for every $\sigma_i \in \Sigma$ there is a party $p_i$ s.t. $\text{share-validate}(m_i, i, \sigma) = true$.

**Threshold coin-tossing.** We assume an unpredictable pseudo random generator (PRG) $G : S \rightarrow \{1, \ldots, n\}$, that is known only to the dealer, which gets a string $s \in S$ and returns a party $p \in \Pi$. Following the standard cryptographic definitions, unpredictability means that if $G$ is not known, then given evaluations of $G$ at all points in some set $Q$, the advantage in evaluating $G$ at a point not in $Q$ is negligible. (See formal definition in [8, 7, 18, 19]). At the beginning of every execution, the dealer gives a private function $\text{coin-share}_i(s)$ to every party $p_i$, which gets a string $s$ and returns a coin share $\sigma_i$. In addition, two public functions are available to all parties: (1) $\text{coin-share-validate}(s, i, \sigma_i)$, which gets a string $s$, a party identification $i$, and a coin share $\sigma_i$, and returns true or false; and (2) $\text{coin-toss}(s, \Sigma)$, which gets a string $s$ and a set of coin shares, and returns a party in $\Pi$.

We assume that the following properties are satisfied except with negligible probability:

- For all $i$, $1 \leq i \leq n$ and for every string $s$, $(1) \text{coin-share-validate}(s, i, \sigma) = true$ if and only if $\sigma = \text{coin-share}_i(s)$, and (2) if $p_i$ is honest, then it is infeasible for the adversary to compute $\text{coin-share}_i(s)$.

- For every string $s$, $\text{coin-toss}(s, \Sigma) = G(s)$ if and only if $|\Sigma| \geq f + 1$ and for every $\sigma \in \Sigma$ there is a party $p_i$ s.t. $\text{coin-share-validate}(s, i, \sigma) = true$.

**Implementations.** Several widely used and established implementations of these abstractions can be found in the literature, e.g., [8, 7, 25], and few more recent ones [18, 19] provide implementations that are also proven to be secure against an adaptive adversary. In all implementations, given an input value of size $B$, the size of the returned shares (from the share-sign and coin-share functions) and the threshold signature (the return value of threshold-sign) is $B + \epsilon(k)$, where $\epsilon(k)$ is a negligible function.
2.2 Validated Asynchronous Byzantine Agreement (VABA)

In this paper we follow Cachin et al. [7] and define a (multi-valued) Byzantine agreement with an external function we call \textit{ex-ba-validation}, that determines whether a value is valid for agreement or not. In addition, we explicitly require a notion of \textit{quality} to capture the “fairness” of the decision value. Note that the protocol of Cachin et al. [8, 7] already obtains this Quality property.

**Definition 4 (Validated Byzantine Agreement).** A protocol solves validated Byzantine agreement with chain quality if it satisfies the following properties except with negligible probability:

- ** Validity: If an honest party decides an a value \(v\), then \textit{ex-ba-validation}(v) = \text{true}.
- ** Agreement: All honest parties that terminates decide on the same value.
- ** Quality: The probability of choosing a value that was proposed by an honest party is at least \(1/2\).
- ** Termination\(^4\): If all honest parties start with externally valid values and all massages sent among honest parties have been delivered, then all honest parties decided.
- ** Efficiency: The number of messages generated by the honest parties is probabilistically uniformly bounded.

3 VABA with Optimal Resilience and Asymptotically Optimal time and Communication

In this section we give a protocol for asynchronous Byzantine agreement, secure against an adaptive adversary that controls up to \(f < n/3\) parties, with expected word communication \(O(n^2)\) and expected running time \(O(1)\). Inspired by Cachin et al. [7, 24, 8] we present a modular implementation of our protocol, which consists of three sub protocols: two broadcast primitives we call \textit{4-Stage-f+1-Provable-Broadcast} and \textit{f+1-Provable-Broadcast}, and a simple and efficient leader election protocol.

3.1 Broadcast primitives

A broadcast primitive is an abstraction for a pre-defined party, which is called a \textit{sender}, to pass a message to all other parties. An instance of a broadcast primitive is identified via an \textit{id}. We present two broadcast abstractions: \textit{f+1-Provable-Broadcast} and \textit{4-Stage-f+1-Provable-Broadcast}, the first is a simple but useful primitive that we use to implement the second, which in turn is used for the Byzantine agreement protocol. In both, a sender broadcasts a message \(m = (v, \sigma)\) consisting of a value \(v\) and a proof \(\sigma\). The proof is provided in order to allow recipients to screen messages. Both abstractions are parametrized with an \textit{external validation} function that parties use for screening messages. These functions implement an important logic that drive the safety properties of our Byzantine agreement protocol.

The API of our broadcasts somewhat differs from other broadcasts in the literature in two ways:

- The sender’s broadcast returns a proof \(\sigma\). Informally, \(\sigma\) proves, via the threshold signature abstraction, that at least \(f + 1\) honest parties delivered \(m.v\).
- Parties can invoke \textit{abandon(id)} to explicitly stop their participation in the broadcast protocol.

3.1.1 f+1-Provable-Broadcast

An f+1-Provable-Broadcast of a message \(m\) with identification \(id\) is denoted \(f+1-PB(id,m)\). The external validation function of a party \(p_i\) is denoted \textit{ex-sbc-validation}, \((id, m)\). \(f+1-PB(id,m)\) satisfies the following properties except with negligible probability:

- ** PB-Integrity: An honest party delivers a message at most once.
- ** PB-Validity: If an honest party \(p_i\) delivers \(m\), then \textit{ex-sbc-validation}(id, m) = \text{true}.
- ** PB-Abandon-ability: An honest party does not deliver any message after it invokes \textit{abandon(id)}.

\(^4\)Called liveness in [8], but we find this name confusing since it is not a liveness [2] property.
• **PB-Provability:** For all \(v, v', \) if the sender can produce strings \(\sigma, \sigma'\) s.t. 
  \(\text{threshold-validate}(\langle id, v, \sigma \rangle) = \text{true} \) and \(\text{threshold-validate}(\langle id, v', \sigma' \rangle) = \text{true}\), then \(1 \leq v = v' \leq f + 1\) honest parties delivered a message \(m \) s.t. \(m.v = v\).

• **PB-Termination:** If the sender is honest, no honest party invokes \(\text{abandon}(id)\), all messages among honest parties arrive, and the message \(m\) that is being broadcast is externally valid, then \((1)\) all honest parties deliver \(m\), and \((2)\) \(f+1\)-PB(id,m) returns (to the sender) \(\sigma\), which satisfies \(\text{threshold-validate}(\langle id, m.v, \sigma \rangle) = \text{true}\).

• **PB-Linear-complexity:** The total number of messages sent by honest players is at most \(2n\).

The pseudocode of \(f+1\)-Provable-Broadcast appears in Algorithms 1 and 2. The sender sends a message \(m = \langle v, \sigma \rangle\) consisting of a value \(v\) and a proof \(\sigma\) to all parties. Once a party gets a message \(m\) from the sender that passes its ex-sbc-validation function for the first time it produces a valid signature share of \(v\) on \(\langle id, m.v \rangle\), delivers the message \(m\), and sends the share \(v\) back to the sender. When the sender receives \(2f + 1\) valid shares it produces a valid threshold signature and returns it. If \(\text{abandon}(id)\) is invoked by a party, it ignores all further messages of this broadcast instance.

**Algorithm 1** \(f+1\)-Provable-Broadcast with identification id: Protocol for the sender

Local variables initialization:
1: \(S \leftarrow \{\}\)

2: upon \(f+1\)-PB(id,\(\langle v, \sigma \rangle\)) invocation do
3: send \(\langle id, send, \langle v, \sigma \rangle \rangle\) to all parties
4: wait until \(|S| = 2f + 1\)
5: return \(\text{threshold-sign}(S)\)

6: upon receiving \(\langle id, ack, \nu_j \rangle\) form party \(p_j\) for the first time do
7: if \(\text{share-validate}(\langle id, v, j, \nu_j \rangle) = \text{true}\) then
8: \(S \leftarrow S \cup \{\nu_j\}\)

**Algorithm 2** \(f+1\)-Provable-Broadcast with identification id: Protocol for a party \(p_i\)

Local variables initialization:
1: \(stop \leftarrow false\)

2: upon receiving \(\langle id, send, \langle v, \sigma \rangle \rangle\) from the sender do
3: if \(stop = false \land \text{ex-sbc-validation}(id, \langle v, \sigma \rangle) = \text{true}\) then
4: \(stop \leftarrow true\)
5: \(\nu_i \leftarrow \text{share-sign}_i(\langle id, v \rangle)\)
6: deliver \(\langle v, \sigma \rangle\)
7: send \(\langle id, ack, \nu_i \rangle\) to the sender

8: upon \(\text{abandon}(id)\) do
9: \(stop \leftarrow true\)

### 3.1.2 4-Stage-\(f+1\)-Provable-Broadcast

A 4-Stage-\(f+1\)-Provable-Broadcast of message \(m\) with identification id is denoted by \(4S-f+1\)-PB-broadcast(id,m). The external validation function of a party \(p_i\) is denoted \(\text{ex-bc-validation}(id, m)\). 4-Stage-\(f+1\)-Provable-Broadcast has three deliveries which we refer to as key, lock, and commit, respectively. These deliveries satisfy similar properties to \(f+1\)-Provable-Broadcast, but where PB-provability additionally guaranteeing that a lock delivery implies that a key delivery has occurred in at least \(f + 1\) honest parties, and a commit delivery implies that a lock delivery has occurred in at least \(f + 1\) honest parties.

4-Stage-\(f+1\)-Provable-Broadcast is implemented on top of \(f+1\)-Provable-Broadcast, and consists of four sequential invocations of \(f+1\)-Provable-Broadcast with identifications \(\langle id, j \rangle\) for \(j \in \{1, \ldots, 4\}\). \(f+1\)-PB(id,j,\(\langle v, (\sigma_{ex}, \sigma_{in}) \rangle\)) is invoked with two proofs: a proof \(\sigma_{ex}\) for external validity condition of ex-bc-validation that implements a logic of the VABA protocol, and a proof \(\sigma_{in}\) for external validity condition of
ex-sbc-validation that implements a logic of the 4-Stage-f+1-Provable-Broadcast. \( f+1-PB(⟨id, 1⟩, ⟨v, ⟨σ_{ex}, σ_{in}⟩⟩) \) takes \( σ_{in} = ⊥ \), while the remaining invocations require \( σ_{in} \) to be a valid output of the previous one.

When a party delivers a message \( ⟨v, ⟨σ_{ex}, σ_{in}⟩⟩ \) in the \( j^{th} \) broadcast instance (the \( f+1 \)-Provable-Broadcast with identification \( ⟨id, j⟩ \)), it delivers key\( ⟨id, ⟨v, σ_{in}⟩⟩ \), lock\( ⟨id, ⟨v, σ_{in}⟩⟩ \), or commit\( ⟨id, ⟨v, σ_{in}⟩⟩ \), respectively. When an abandon\( (id) \) is invoked by a party, it simply invokes abandon\( (⟨id, j⟩) \) for all \( j ∈ \{1, . . . , 4⟩ \). We do not define and prove the properties of the 4-Stage-f+1-Provable-Broadcast. Instead, we use it to describe our validated asynchronous Byzantine agreement protocol, which we prove with the \( f+1 \)-Provable-Broadcast properties directly.

The pseudocode of 4-Stage-f+1-Provable-Broadcast appears in Algorithms 3 and 4.

### Algorithm 3 4-Stage-f+1-Provable-Broadcast with identification \( id \): Protocol for a sender.

1: upon 4S-f+1-PB-broadcast\( (id, ⟨v, σ_{ex}⟩) \) invocation do
2: \( σ_{in} ← ⊥ \)
3: for \( j = 1, . . . , 4 \) do
4: \( σ_{in} ← f+1-PB(⟨id, j⟩, ⟨v, ⟨σ_{ex}, σ_{in}⟩⟩) \)
5: return \( σ \)

### External validation function for the \( f+1 \)-Provable-Broadcast instances:

6: procedure EX-SBC-VALIDATION\( (⟨id, j⟩, ⟨v, ⟨σ_{ex}, σ_{in}⟩⟩) \)
7: if \( j = 1 \) ∧ ex-bc-validation\( (id, ⟨v, σ_{ex}⟩) \) = true then
8: return true
9: if \( j > 1 \) ∧ threshold-validate\( (⟨⟨id, j−1⟩, v⟩, σ_{in}) \) = true then
10: return true
11: return false

### Algorithm 4 4-Stage-f+1-Provable-Broadcast with identification \( id \): Protocol for all parties.

1: upon delivery\( (⟨id, j⟩, ⟨v, ⟨σ_{ex}, σ_{in}⟩⟩) \) do \( \triangleright j ∈ 1, . . . , 4 \)
2: if \( j = 2 \) then
3: deliver key\( (id, ⟨v, σ_{in}⟩) \)
4: if \( j = 3 \) then
5: deliver lock\( (id, ⟨v, σ_{in}⟩) \)
6: if \( j = 4 \) then
7: deliver commit\( (id, ⟨v, σ_{in}⟩) \)
8: upon abandon\( (id) \) do
9: for \( j = 1, . . . , 4 \) do
10: abandon\( (⟨id, j⟩) \)

#### 3.2 Leader election

A leader election abstraction provides one operation to elect a unique party (called a leader) among the parties. An instance of a leader election primitive is identified via an \( id \), and exposes an operation elect\( (id) \) to all parties, which returns a party \( p ∈ Π \). Formal definitions are given below and the pseudocode appears in Algorithm 5.

A protocol for leader election associated with id \( id \) satisfies the following properties except with negligible probability.

- **Termination**: If \( f + 1 \) honest parties invoke elect\( () \), and all messages among honest parties arrive, then all invocations by honest parties return.

- **Agreement**: All invocations of elect\( (id) \) by honest parties return the same party.

- **Validity**: If an invocation of elect\( (id) \) by an honest party returns, it returns a party \( p \) with probability \( 1/|Π| \) for every \( p ∈ Π \).
• **Unpredictability:** The probability of the adversary to predict the returned value of $elect(id)$ invocation by an honest party before it returns is at most $1/|\Pi| + \epsilon(k)$, where $\epsilon(k)$ is a negligible function.

---

**Algorithm 5** Leader election. Protocol for party $p_i$

**Local variables initialization:**
1. $\Sigma \leftarrow \{\}$

2. **upon** $elect(id)$ **do**
3. $\rho_i \leftarrow coin\text{-}share_i(id)$
4. send "SHARE, id, $\rho_i$" to all parties
5. wait until $|\Sigma| = f + 1$
6. **return** $coin\text{-}toss(id, \Sigma)$

7. **upon receiving** "SHARE, id, $\rho_j$" from party $p_j$ for the first time **do**
8. **if** $coin\text{-}share\text{-}validate(id, j, \rho_j) = true$ **then**
9. $\Sigma \leftarrow \Sigma \cup \{\rho_j\}$

---

3.3 **VABA protocol**

3.3.1 **Overview.**

The 4-Stage-f+1-Provable-Broadcast and the leader election abstractions are used as building blocks for our validated asynchronous Byzantine agreement protocol. The protocol works in a view-by-view manner, where each view consists of three phases: Broadcast Phase, Leader-election Phase, and View-change Phase. In the Broadcast Phase, each party invokes 4-Stage-f+1-Provable-Broadcast to broadcast its value. Parties wait to learn that $2f + 1$ instances of 4-Stage-f+1-Provable-Broadcast have completed by returning at their respective senders. Then, in the leader-election Phase, parties choose a leader $p_l$ uniformly at random. Finally, in the View-change Phase, parties learn what happened in the elected leader’s 4-Stage-f+1-Provable-Broadcast. If they learn that some party delivered $commit(v)$ (and has a proof that justifies it), they can decide $v$. Otherwise, they need to carefully adopt a value and continue to the next view. (more details are in the view-change description in Section 3.3.2).

Our protocol guarantees that at least $2f + 1$ 4-Stage-f+1-Provable-Broadcast instances complete in the broadcast phase before a leader is chosen. If the elected leader has completed (implying that at least $f + 1$ honest parties delivered a $commit$) then even an adaptive adversary cannot prevent progress. Otherwise, the view-change phase ensures that agreement is not violated even if a bad leader is chosen. Since the probability to choose a leader that completed its broadcast is constant, the number of views in the protocol is constant in expectation. More concretely, the probability to choose a completed broadcast is greater than $2/3$, and thus the number of views in expectation is less than $3/2$. More details on each phase are given below. The pseudocode appears in Algorithms 7 and 6, and a formal proof is given in Appendix A.

3.3.2 **Protocol for view $j$**

Broadcast phase. Each party broadcasts, using a 4-Stage-f+1-Provable-Broadcast, the value and $KEY$ it has adopted from the previous views, as determined in the View-Change Phase (explained below); at view 1, a party broadcasts its input and an empty key. Parties participate in $n$ concurrent 4-Stage-f+1-Provable-Broadcasts, where their ex-bc-validation function (Algorithm 6 lines 1-9) uses the external validity condition for agreement (i.e., ex-ba-validation function), as well as a condition on the $KEY$ as explained below.

Each party sends a notification about the completion of its own 4-Stage-f+1-Provable-Broadcast carrying its output for proof. When a party receives $2f + 1$ such notifications, it sends a signature share on a “skip” message. Each party waits to obtain (either directly or indirectly) a combined threshold skip signature, forwards it to others, and moves to the Leader-election phase (even if its broadcast has not completed).

Leader election phase. Once a party enters the Leader-election Phase, it abandons all the 4-Stage-f+1-Provable-Broadcast instances. The parties elect a leader via the leader election abstraction and continue to the next phase as if only the the leader’s 4-Stage-f+1-Provable-Broadcast ever occurred.
From now on, we refer to the 4-Stage-f+1-Provable-Broadcast of the leader of a view $j$ as the 4-Stage-f+1-Provable-Broadcast of view $j$, and refers to its delivery events as the deliveries of view $j$.

**View-change phase.** In the View-change Phase of view $j$, parties report their deliveries from the broadcast of this view (the leader’s broadcast) to everyone, including a proof of delivery for each report. Each party waits to collect $2f+1$ reports. Recall that 4-Stage-f+1-Provable-Broadcast provides the following guarantees: If some honest party delivered a commit, then $f+1$ honest parties delivered lock, hence all honest parties collect this lock in the view-change exchange. Similarly, if some honest party delivered a lock, then $f+1$ honest parties delivered key, hence all honest parties collect this key in the view-change exchange.

All parties maintain two cross-view variables, LOCK and KEY:

- The LOCK variable stores the highest view $\mathcal{R}$ for which the party ever received a view-change message that includes a lock.

- The KEY variable stores the highest view $\mathcal{R}$, for which the party ever received a view-change message that includes a key and the key itself.

Once a party has collected $2f+1$ view-change messages, it processes them as follows. If it receives a commit $v$, it decides $v$. Otherwise, if it receives a lock, it increases its LOCK variable to the current view. Last, if it receives a key, it updates its KEY variable to store the current view and the received key. If it did not reach a decision, a party adopts the value $v$ of its (up-to-date) KEY variable and moves to the next view, where it broadcasts $v$ together with $\text{KEY}$ as proof for the external validation function (ex-bc-validation).

As mentioned above, a party participates in a 4-Stage-f+1-Provable-Broadcast only if the message $m = \langle v, (\mathcal{R}, \text{key}) \rangle$ (note that $(\mathcal{R}, \text{key}) = \text{KEY}$) passes its external validation test. The external validation includes a crucial key-locking mechanism (see Algorithm 6, lines 1 to 9). In particular, in view $j > 1$, a party checks that the key is valid for $v$ and $\mathcal{R}$ (meaning that key includes a proof that key($v$) could have been delivered by an honest party in the chosen broadcast of view $\mathcal{R}$), and that the view $\mathcal{R}$ is at least as large as LOCK. We prove in Appendix A that the key-locking mechanism together with the fact that parties abandon all broadcasts before sending the view-change messages guarantee agreement and satisfy progress. Here we give some intuition for the proof:

- Lock Safety: If some party has a proof for commit $v$ in view $\mathcal{R}$, then at least $f + 1$ honest parties previously locked (lock = $\mathcal{R}$) in view $\mathcal{R}$.

- Key Safety: If some party has a proof for commit $v$ in view $\mathcal{R}$, then it is not possible for a party to have a valid key on a value other than $v$ in view higher than or equal to $\mathcal{R}$.

- Key Progress: If some party $p_i$ is locked in view $\mathcal{R}$, then at least $f+1$ honest parties obtained a key in $\mathcal{R}$ before sending the view-change messages of view $\mathcal{R}$, and thus all honest parties will have a KEY with view at least $\mathcal{R}$.

**Communication complexity.** We start by analyzing the word communication of a single view. Recall that a word can contain a constant number of domain values and signatures. We denote by $V$ the domain of valid values for the Byzantine agreement. Note that every message that is broadcasted during the agreement protocol consists of a value $v \in V$ and two proofs - a proof (key) for the ex-bc-validation function of the 4-Stage-f+1-Provable-Broadcast and another proof for the ex-sbc-validation function of every instance of the f+1-Provable-Broadcasts therein. Since a proof is simply a threshold signature on a value and the broadcast identification, we get that the size of the broadcast messages is a single word. The total number of messages sent by honest parties in an f+1-Provable-Broadcast is $2n$. Therefore, since 4-Stage-f+1-Provable-Broadcast uses 4 instances of f+1-Provable-Broadcast, we get that the number of words sent by honest parties in the f+1-Provable-Broadcast protocol is $O(n)$.

Our Byzantine agreement protocol uses $n$ concurrent 4-Stage-f+1-Provable-Broadcasts in phase 1 of every view, which brings the word communication to $O(n^2)$. The word complexity of sending the all to all “skip-share” and “skip” messages as well as the message complexity of the leader election abstraction is $O(n^2)$. Finally, since each party sends $n$ view change messages, of a single word (at most 3 values and 3 proofs), we get that the word communication of the view-change phase is $O(n^2)$. In total, the word communication of a single view in our Byzantine agreement protocol is $O(n^2)$. We prove in Appendix A that our protocol has an expected constant number of views, which implies that the expected word communication of our asynchronous Byzantine agreement protocol is $O(n^2)$. 


Algorithm 6 Validated asynchronous byzantine agreement with identification $id$: protocol for party $p_i$.

Local variables initialization:
\[
\text{LOCK} \leftarrow 0 \\
\text{KEY} \leftarrow \langle 0, v_i, \bot \rangle \text{ with selectors } \text{view}, \text{value}, \text{proof}
\]

for every view $j \geq 1$, initialize:
\[
v_j \leftarrow v_i; \sigma_j \leftarrow \langle 0, \bot \rangle; \text{L}[j] \leftarrow \bot; \text{BCdone}_j \leftarrow 0; \text{BCskip}_j \leftarrow \{\}; \text{skip}_j \leftarrow \text{false}
\]

for every party $p_k \in \Pi$ initialize:
\[
\text{Dkey}[k, j] = \text{Dlock}[k, j] = \text{Dcommit}[k, j] = \langle \bot, \bot \rangle
\]

External validity for the 4-Stage-$f+1$-Provable-Broadcast:

1: procedure EX-BC-VALIDATION($id, \langle v, \langle j, \sigma \rangle \rangle$) ▷ external ABA validity check
2: if ex-ba-validation($v$) = false then ▷ validate the key
3: return false
4: if $j \neq 1 \land \text{threshold-validate}(\langle \langle id, L[j], j \rangle, 1 \rangle, v, \sigma) = \text{false}$ then
5: return false
6: if $j \geq \text{LOCK}$ then ▷ check that key is not smaller than lock
7: return true
8: else
9: return false

Protocol for party $p_i$

10: $j \leftarrow 1$
11: while true do ▷ Broadcast phase
12: for all $k=1, \ldots, n$ do
13: initialize an instance of 4-Stage-$f+1$-Provable-Broadcast with identification $\langle id, k, j \rangle$
14: $\sigma \leftarrow 4S-f+1$-$\text{PB}$-broadcast($\langle id, i, j, \langle v_j, \sigma_j \rangle \rangle$)
15: wait for 4S-$f+1$-$\text{PB}$-broadcast($\langle id, i, j, \langle v_j, \sigma_j \rangle \rangle$) to return or $\text{skip}_j = \text{true}$
16: if $\text{skip}_j = \text{false}$ then
17: send “id, done, j, $\langle \langle v_j, \sigma_j \rangle, \sigma \rangle$” to all parties
18: wait until $\text{skip}_j = \text{true}$ ▷ Leader election phase
19: for all $k=1, \ldots, n$ do
20: abandon($\langle id, k, j \rangle$)
21: $L[j] \leftarrow \text{elect}(\langle id, j \rangle)$ ▷ View-change phase
22: send “View-change, id, j, Dkey[$L[j], j$], Dlock[$L[j], j$], Dcommit[$L[j], j$]” to all parties
23: wait for View-change messages from $2f + 1$ different parties
24: $v_{j+1} \leftarrow \text{KEY.value}$
25: $\sigma_{j+1} \leftarrow \langle \text{KEY.round}, \text{KEY.proof} \rangle$
26: $j \leftarrow j + 1$
Algorithm 7 Validated asynchronous byzantine agreement with identification $id$: messages.

1: upon $key((id, k, j), (v, \sigma))$ do
2: \hspace{1em} $D_{key}[k, j] = (v, \sigma)$

3: upon $lock((id, k, j), (v, \sigma))$ do
4: \hspace{1em} $D_{lock}[k, j] = (v, \sigma)$

5: upon $commit((id, k, j), (v, \sigma))$ do
6: \hspace{1em} $D_{commit}[k, j] = (v, \sigma)$

7: upon receiving “$id, done, j, (v, \sigma)$” from party $p_i$ for the first time do
8: \hspace{1em} if threshold-validate($(⟨⟨⟨id, i, j⟩, 4⟩, v), \sigma)$ then
9: \hspace{2em} $BCdone_j \leftarrow BCdone_j + 1$
10: \hspace{1em} if $BCdone_j = 2f + 1$ and “skip-share” message has not sent yet then
11: \hspace{2em} $\nu \leftarrow$ sigh-share($(id, skip, j)$)
12: \hspace{1em} send “$id, skip-share, j, \nu$” to all parties

13: upon receiving “$id, skip-share, j, \nu$” from party $p_i$ for the first time do
14: \hspace{1em} if share-validate($(id, skip, j), i, \nu)$ then
15: \hspace{2em} $BCskip_j \leftarrow BCskip_j \cup \{\nu\}$
16: \hspace{1em} if $|BCskip_j| = 2f + 1$ then
17: \hspace{2em} $\sigma \leftarrow$ threshold-sign($BCskip_j$)
18: \hspace{1em} send “$id, skip, j, \sigma$” to all parties

19: upon receiving “$id, skip, j, \sigma$” do
20: \hspace{1em} if threshold-validate($(id, skip, j), \sigma)$ = true then
21: \hspace{2em} $skip_j \leftarrow true$
22: \hspace{1em} if “skip” message was not sent yet then
23: \hspace{2em} send “$id, skip, j, \sigma$” to all parties

24: upon receiving “$View-change, j, (v_2, \sigma_2), (v_3, \sigma_3), (v_4, \sigma_4)$” do
25: \hspace{1em} $l \leftarrow L[j]$
26: \hspace{1em} if $v_4 \neq \perp \land$ threshold-validate($(⟨⟨⟨id, l, j⟩, 3⟩, v_4), \sigma_4)$ = true then
27: \hspace{2em} decide $v_4$
28: \hspace{1em} if $v_3 \neq \perp \land j > lock \land$ threshold-validate($(⟨⟨⟨id, l, j⟩, 2⟩, v_3), \sigma_3)$ = true then
29: \hspace{2em} $LOCK \leftarrow j$
30: \hspace{1em} if $v_2 \neq \perp \land j > key.round \land$ threshold-validate($(⟨⟨⟨id, l, j⟩, 1⟩, v_2), \sigma_2)$ = true then
31: \hspace{2em} $KEY \leftarrow (j, v_2, \sigma_2)$
4 Discussion

Our protocol addresses an open problem of Cachin et al. [7] and reduces the expected word communication from $O(n^3)$ to $O(n^2)$ against an asynchronous adaptive adversary. We also show that in the standard definition of an asynchronous adaptive adversary this expected word communication is asymptotically optimal for any protocol that obtains the standard definition of termination (liveness) as defined [8, 7]. An interesting open question is related to protocols that obtain weak termination in the adaptive setting: is there a $\Omega(n^2)$ lower bound against an adaptive adversary that is required to deliver all messages sent by parties before they are corrupted? or does there exist a protocol with near linear expected word communication under this weak termination property?
Appendix A  Correctness proofs

A.1 f+1-Provable-Broadcast

Note that PB-integrity, PB-validity, PB-abandon-ability, and PB-linear-complexity follow immediately from the code. We now prove PB-provability and PB-termination.

Lemma 2. Algorithms 1 and 2 satisfy PB-termination.

Proof. Consider a f+1-Provable-Broadcast instance with identification id in which an honest sender \( p_i \) broadcasts an externally valid message \( m \), no honest party invokes abandon(id), and all messages among honest parties arrived. Since the sender is honest, it sent a message to all honest parties, and since all messages among honest parties arrived, we get that all honest parties received a message from the sender and thus all honest parties delivered a message and (1) is satisfied. For (2), note that the sender accept an ack message only if it contains a valid share. Since there are at least \( 2f + 1 \) honest parties, the sender gets at least \( 2f + 1 \) valid shares, and thus by the threshold validation property, it produces a valid threshold signature except with negligible probability.

□

Lemma 3. Algorithms 1 and 2 satisfy PB-provability.

Consider an f+1-Provable-Broadcast instance with identification id and assume that a sender can produce two strings \( \sigma_1, \sigma_2 \) s.t. \( \text{threshold-validate}((id, v_1), \sigma_1) = \text{threshold-validate}((id, v_2), \sigma_2) = true \) for some values \( v_1, v_2 \). By the threshold validation property, except with negligible probability, for \( j \in \{1, 2\} \), \( \sigma_j = \text{threshold-sign}(\Sigma_j) \) s.t. \( |\Sigma_j| \geq 2f + 1 \) and for every \( \sigma_j' \in \Sigma_j \) there is a party \( p_i \) s.t. \( \text{share-validate}((id, v_j), i, \sigma_j') = true \). For \( j \in \{1, 2\} \), let \( P_j = \{ p_i | \exists \sigma_j' \in \Sigma_j \text{ s.t. share-validate}((id, v_j), i, \sigma_j') = true \} \), and note that \( |P_j| \geq 2f + 1 \). Therefore, there are at least \( f + 1 \) honest parties in \( P_j \), \( j \in \{1, 2\} \), and thus there is an honest party \( p_i \) that is in both \( P_1 \) and \( P_2 \). By the code, \( p_i \) computes a share-sign(v) at most once. Therefore, by the share validation property we get that \( v_1 = v_2 = v \) except with negligible probability, which proves (1). For (2), note that an honest party deliver a message \( m \) before computing a share on \( m.v \).

A.2 Validated asynchronous byzantine agreement

Notations. Consider a byzantine agreement instance with identification id. All parties start at view 1, and we say that a party completes view \( j \geq 1 \) and moves to view \( j + 1 \) when it executes line 26 in Algorithm 6 for the \( j \)th time. We say that view \( j \) completes when at least \( f + 1 \) honest parties move to view \( j + 1 \). The leader of view \( j \) is the party returned by the elect((id, j)) invocations by honest parties at view \( j \), and we say that it is elected when the first such invocation returns. We say that a key = (j, \( \sigma \)) is valid for a value \( v \) if \( \text{threshold-validate}((\langle id, l, j \rangle, 1), v), \sigma ) = true \), where \( l \) is the index of the leader of view \( j \).

Consider a 4-Stage-f+1-Provable-Broadcast with identification id. For simplicity of exposition, we denote the delivery key = (id, m) by deliver4(id, m), lock = (id, m) by deliver2(id, m), and commit = (id, m) by deliver4(id, m). In addition, the variables \( D_{\text{key}}, D_{\text{lock}}, D_{\text{commit}} \) from Algorithm 6 are called here \( D_2, D_3, D_4 \), respectively. We say that a 4-Stage-f+1-Provable-Broadcast with identification id completes when \( f + 1 \) honest parties deliver4(id, \( \langle id, v, \sigma \rangle \)) for some \( \sigma \) and \( v \), and its completion-proof is a string \( \sigma' \), which satisfies threshold-validate(\( \langle id, 4, v \rangle, \sigma' ) = true \).

Properties of the 4-Stage-f+1-Provable-Broadcast. The next four lemmas prove important properties of the 4-Stage-f+1-Provable-Broadcast.

Lemma 4. Consider a 4-Stage-f+1-Provable-Broadcast with identification id. For every value \( v \) and \( j \in \{2, 3, 4\} \), if some honest party gets \( \sigma \) s.t. \( \text{threshold-validate}((\langle id, j \rangle, v), \sigma ) = true \), then at least \( f + 1 \) honest parties previously deliver4(id, \( \langle v, \sigma' \rangle \)) for some \( \sigma' \).

Proof. By the second part of the PB-provability property of the f+1-Provable-Broadcast, at least \( f + 1 \) honest parties deliver4(id, \( \langle v, \sigma_1 \rangle \)) with some \( \sigma_1 \) in the \( j \)th instance of the f+1-Provable-Broadcast. Therefore, by the code, at least \( f + 1 \) honest parties previously deliversj(id, \( \langle v, \sigma_1 \rangle \)).

□

Lemma 5. Consider a 4-Stage-f+1-Provable-Broadcast with identification id. For every value \( v \) and \( j \in \{2, 3, 4\} \), if some honest party deliversj(id, \( \langle v, \sigma \rangle \)), then \( \text{threshold-validate}((\langle id, j - 1 \rangle, v), \sigma ) = true \).
Proof. Let \( p_i \) be an honest party that delivers \((i, (v, \sigma_{in})) \) for some \( v \), and \( j \in \{2, \ldots, 4\} \). By the code, \( p_i \) delivers \((\langle (i, j), (v, (\sigma_{ex}, \sigma_{in})), \rangle) \) in an \( f + 1 \)-Provable-Broadcast with identification \((i, j)\) and some \( \sigma_{ex}\), and by its validity property, \( \text{ex-sbc-validation}, (\langle (i, j), (v, (\sigma_{ex}, \sigma_{in})), \rangle) = \text{true} \). Therefore, since \( j > 1 \), we get by the code of \( \text{ex-sbc-validation} \) that \( \text{threshold-validate}, (\langle (i, j - 1), v \rangle, \sigma_{in}) = \text{true} \).

\[ \square \]

**Lemma 6.** Consider a \( 4\)-Stage-\( f + 1 \)-Provable-Broadcast with identification \( id, p_1, p_2 \), and two values \( v_1, v_2 \). For every \( j_1, j_2 \in \{2, 3, 4\} \), if \( p_1 \) gets \( \sigma \) s.t. \( \text{threshold-validate}, (\langle (id, j_1), v_1 \rangle, \sigma) = \text{true} \) and \( p_2 \) gets \( \sigma \) s.t. \( \text{threshold-validate}, (\langle (id, j_2), v_2 \rangle, \sigma) = \text{true} \), then \( v_1 = v_2 \).

**Proof.** Consider three cases:

- \( j_1 = j_2 \). The lemma follows from the first part of the provability property of the \( f + 1 \)-Provable-Broadcast.
- \( j_1 = j_2 + 1 \). Since \( p_1 \) gets \( \sigma \) s.t. \( \text{threshold-validate}, (\langle (id, j_1), v_1 \rangle, \sigma) = \text{true} \), by the second part of the provability property of the \( f + 1 \)-Provable-Broadcast, we get that at least one honest party delivers \( \langle v_1, (\sigma_{ex}, \sigma_{in}) \rangle \) for some \( (\sigma_{ex}, \sigma_{in}) \) in the \( j_1 \)-th instance of the \( f + 1 \)-Provable-Broadcast. By the external validity property of \( f + 1 \)-Provable-Broadcast, we get that \( \text{threshold-validate}, (\langle (id, j_2), v_1 \rangle, \sigma_{in}) = \text{true} \). Therefore, by the first part of the provability property of \( f + 1 \)-Provable-Broadcast, we get that \( v_1 = v_2 \).
- \( j_1 = j_2 + 2 \). Follows by transitivity.

\[ \square \]

**Lemma 7.** Consider a \( 4\)-Stage-\( f + 1 \)-Provable-Broadcast instance with identification \( id \). If the sender is honest, no honest party invokes \( \text{abandon}(id) \), all messages among honest parties arrived, and the message \( m \) that is being broadcast is externally valid, then the broadcast completes and returns a completion-proof to the sender.

**Proof.** Consider a 4S-f+1-PB-broadcast\((id, v)\) invocation by party \( p_i \). By the code of \( \text{ex-sbc-validation} \), any message is externally valid for the the first instance of an \( f + 1 \)-Provable-Broadcast consisting it. Therefore, by the PB-termination property of \( f + 1 \)-Provable-Broadcast, \( p_i \) gets \( \sigma \) that satisfies \( \text{threshold-validate}, (\langle (id, 1), v \rangle, \sigma) = \text{true} \), and thus the conditions of the PB-termination property is satisfied in the second instance of \( f + 1 \)-Provable-Broadcast as well \( (\langle v, \sigma \rangle) \) is externally valid). So by the PB-termination property of \( f + 1 \)-Provable-Broadcast again, (1) all honest party delivers \( \langle id, (v, \sigma) \rangle \); and (2) \( p_i \) gets \( \sigma \) that satisfies \( \text{threshold-validate}, (\langle (id, 2), v \rangle, \sigma) = \text{true} \). The lemma follows by applying the same arguments for the remain two \( f + 1 \)-Provable-Broadcast instances.

\[ \square \]

**Key protocol property.** The next Lemma use two of the 4-Stage-f+1-Provable-Broadcast properties proven above to prove a key property for both safety and progress of the protocol.

**Lemma 8.** Consider a view \( j \) and let \( p_i \) be the chosen leader of view \( j \). For every \( i \in \{2, 3\} \), if an honest party gets a “view-change” message that includes \( (v, \sigma) \) s.t. \( \text{threshold-validate}, (\langle (id, l, j), i, v \rangle, \sigma) = \text{true} \), then all honest parties get a “view-change” message that includes \( (v, \sigma') \) s.t. \( \text{threshold-validate}, (\langle (id, l, j), i - 1, v \rangle, \sigma') = \text{true} \).

**Proof.** By Lemma 4, there exists a set \( P \in P_h \) of at least \( f + 1 \) honest parties that delivers \( \langle (id, l, j), (v, \sigma') \rangle \) for some \( \sigma' \), and by Lemma 5, \( \text{threshold-validate}, (\langle (id, l, j), i - 1, v \rangle, \sigma') = \text{true} \). By the code, the parties in \( P \) set their \( D_{[l, j]} = \langle v, \sigma' \rangle \), and by the PB-abandon-ability property of the \( f + 1 \)-Provable-Broadcast, it happened before they invoke \( \text{abandon}(id, l, j) \), and thus before they send their “view-change” messages. Since every honest party waits to receive view-change messages from \( 2f + 1 = n - f \) different parties, we get that every honest party gets a view-change message that includes \( (v, \sigma') \) s.t. \( \text{threshold-validate}, (\langle (id, l, j), i - 1, v \rangle, \sigma') = \text{true} \).

\[ \square \]
Agreement proof.

Lemma 9. If a party $p$ decides on a value $v$ in a view $j$, then the LOCK variables of all honest parties are at least $j$ when they move to view $j + 1$.

Proof. Let $p_j$ be the leader of view $j$. By the code, since $p$ decides $v$ in view $j$, we know that $p$ received a “view-change” message in view $j$ with $(v, \sigma)$ s.t. threshold-validate($\langle\langle\langle(id, l, j), 3\rangle, v\rangle, \sigma\rangle = true$. By Lemma 8, we get that all honest parties received a “view-change” message that includes $(v, \sigma')$ s.t. threshold-validate($\langle\langle(id, l, j), 2\rangle, v\rangle, \sigma'\rangle = true$. Therefore, by the code, all honest parties set their LOCK variables to $j$ (if it was smaller than $j$).

The next corollary follows from Lemma 9 and the fact that the LOCK variables are never decreased.

Corollary 10. If a party $p$ decides on a value $v$ in a view $j$, then the LOCK variables of all honest parties are at least $j$ when they start any view $j' > j$.

Lemma 11. If an honest party $p$ decides on $v$ in view $j$, then all honest parties that decide in view $j$, decide $v$ as well.

Proof. Let $p_j$ be the leader of view $j$. By the code, an honest party decides $v$ in view $j$ if and only if it gets a “view-change” message with $(v, \sigma)$ s.t. threshold-validate($\langle\langle(id, l, j), 3\rangle, v\rangle, \sigma\rangle = true$. The lemma follows from the first part of the PB-provability property of the third instance of the $f+1$-Provable-Broadcast.

The next lemma shows that all keys for views higher than or equal to a view in which a decision on a value $v$ was made are valid only for $v$.

Lemma 12. Assume an honest party $p$ decides on $v$ in a view $j$. Than for every view $j' \geq j$, and every honest party that gets $(v', \sigma')$ s.t. threshold-validate($\langle\langle(id, l', j'), 1\rangle, v', \sigma'\rangle = true$, where $p_{j'}$ is the leader of view $j'$, we get that $v' = v$.

Proof. We prove by induction on the view number $j'$.

Base: $j' = j$. By the code, $p$ gets a “view-change” message with $(v, \sigma)$ s.t. threshold-validate($\langle\langle(id, l', j'), 3\rangle, v\rangle, \sigma\rangle = true$. Therefore, by Lemma 6, we get that if an honest party gets $(v', \sigma')$ s.t. threshold-validate($\langle\langle(id, l', j'), 1\rangle, v', \sigma'\rangle = true$, then $v' = v$.

step: Assume the lemma holds for all $j'', j \leq j'' \leq j'$, we now show that it holds for $j' + 1$ as well. Assume by a way of contradiction that some honest party gets $(v', \sigma')$ s.t. threshold-validate($\langle\langle(id, l' + 1, j' + 1), 1\rangle, v', \sigma'\rangle = true$ and $v' \neq v$. By the second part of the PB-provability property, at least one honest party $p_1$ delivers $(v', \langle\sigmaex, \sigmain\rangle)$ for some $(\sigmaex, \sigmain)$ in the $f+1$-Provable-Broadcast instance with identification $\langle\langle(id, l' + 1, j' + 1), 1\rangle$. By Corollary 10, the LOCK variable of $p_1$ at the beginning of view $j' + 1$ is at least $j$. Therefore, by the PB-validity property of the $f+1$-Provable-Broadcast and the definition of its external validation for $p_1$, we get that there is a view $j''$, $j \leq j'' \leq j'$ s.t. threshold-validate($\langle\langle(id, l', j''), 1\rangle, v', \sigmain\rangle = true$. A contradiction to the inductive assumption.

Lemma 13. Algorithms 6 and 7 satisfy agreement.

Proof. Let party $p$ be the first to decide, let $j$ be the view in which $p$ decides, and let $v$ be the value $p$ decides on. First, by Lemma 11, all honest parties that decide in view $j$, decides $v$. Now consider view $j' > j$. By Lemma 12, if an honest party gets $(v', \sigma')$ s.t. threshold-validate($\langle\langle(id, l', j'), 1\rangle, v', \sigma'\rangle = true$, where $p_{j'}$ is the leader of view $j'$, then $v' = v$. Thus, by Lemma 6, if an honest party gets $(v', \sigma')$ s.t. threshold-validate($\langle\langle(id, l', j'), 3\rangle, v', \sigma'\rangle = true$, where $p_{j'}$ is the leader of view $j'$, then $v' = v$. Therefore, since an honest party decides on $v'$ in view $j'$ only if it gets a “view-change” message with $(v', \sigma')$ s.t. threshold-validate($\langle\langle(id, l, j), 3\rangle, v\rangle, \sigma'\rangle = true$, we get that if an honest party decide $v'$ in view $j'$, then $v' = v$. 

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Termination and validity proofs.

Lemma 14. Consider a view $j$ and assume that some honest party $p$ sets its LOCK variable to $j$ in view $j$, then all honest parties set their KEY variable to $(j, v, \sigma)$ s.t. $\text{key} = (j, \sigma)$ is valid for $v$ before moving to view $j + 1$.

Proof. Let $p$ be the leader of view $j$. By the code, since $p$ sets its LOCK variable to $j$ in view $j$, we know that $p$ received a “view-change” message in view $j$ with $(v, \sigma)$ s.t. $\text{threshold-validate}((\langle id, l, j \rangle, 2), v, \sigma) = \text{true}$. By Lemma 8, we get that all honest parties received a “view-change” message that includes $(v, \sigma')$ s.t. $\text{threshold-validate}((\langle id, l, j \rangle, 1), v, \sigma') = \text{true}$. Therefore, by the code, all honest parties set their KEY variables to $(j, v, \sigma)$ s.t. $\text{key} = (j, \sigma)$ is valid for $v$.

Lemma 15. If all honest parties start with externally valid values for byzantine agreement, then for every view $j \geq 1$, all messages broadcast by honest parties are externally valid for the 4-Stage-$f+1$-Provable-Broadcast.

Proof. First consider view $j = 1$. Since all honest parties start with externally valid values for byzantine agreement, and since by the code of ex-bc-validation (lines 1–9 in Algorithm 6), a valid key is not required in view $1$, we get that all messages broadcast by honest parties in view $1$ are externally valid for the 4-Stage-$f+1$-Provable-Broadcast.

Now consider a view $j > 1$. Let $i = \text{max}(\{k \mid \text{there is an honest party } p \text{'s lock} = k \text{ when it begins view } j\})$.

By the code of the external validity for 4-Stage-$f+1$-Provable-Broadcast, we need to show that when a party begins view $j$, its KEY variable is equal to $(k, v, \sigma)$ such that:

1. $(k, \sigma)$ is a valid key for $v$,
2. $k \geq i$, and
3. $v$ is an external valid value for byzantine agreement.

By Lemma 14, all honest parties set their KEY variable to $(i, v, \sigma)$ when they end view $i$ s.t. $\text{key} = (j, \sigma)$ is valid for $v$. By the code, after view $i$, the KEY variable is updated to $(k, v', \sigma')$ by an honest party $p$ only if $p$ receives a “view-change” message with $(v', (k, \sigma'))$ s.t. $k > i$ and $(k, \sigma')$ is a valid key for $v'$. Therefore, we get that (1) and (2) are satisfied. Now by the PB-provability property of $f+1$-Provable-Broadcast, we get that at least one honest party delivers $(v, \sigma'')$ for some $\sigma''$ at some instance of $f+1$-Provable-Broadcast, and thus by the code of ex-sbc-validation and ex-bc-validation functions, $v$ is an external valid value for byzantine agreement.

The following corollary follows immediately from Lemma 15.

Corollary 16. Algorithms 6 and 7 satisfy validity.

The following observation follows from the fact that honest parties echo “skip” messages before setting $\text{skip} = \text{true}$. 

Observation 1. Consider a view $j$, which all honest parties start. If all messages sent by honest parties in a view $j$ arrived and some honest party sets $\text{skip} = \text{true}$ in view $j$, then all honest parties set $\text{skip} = \text{true}$ in view $j$.

Lemma 17. Assume that all honest parties start with externally valid values for byzantine agreement. Consider a view $j$, which all honest parties start. If all messages sent by honest parties in view $j$ arrived and no honest party sets $\text{skip} = \text{true}$ in view $j$, then all 4-Stage-$f+1$-Provable-Broadcasts issued by honest parties have completed and returned completion-proofs.

Proof. Since no honest party sets $\text{skip} = \text{true}$ in view $j$, by the code, no honest party invokes abandon in a the 4-Stage-$f+1$-Provable-Broadcasts of view $j$. By Lemma 15, all messages broadcast by honest parties are externally valid for the 4-Stage-$f+1$-Provable-Broadcast. Therefore, by Lemma 7, all broadcasts issued by honest parties have completed and returned completion-proofs.

Lemma 18. Consider a view $j$, which all honest parties start. If all messages sent among honest parties in a view $j$ arrived and all 4-Stage-$f+1$-Provable-Broadcasts issued by honest parties in view $j$ have been completed and returned completion-proofs, then all honest parties set $\text{skip} = \text{true}$ in view $j$.
Proof. By the code, all honest parties sent a “done” message with a completion-proof (of their broadcast) to all other parties. Therefore, since all messages sent among honest parties in a view \(j\) arrived, we get that all honest parties sent a “share-skip” message to all other parties. Since there are at least \(2f + 1\) honest parties, all honest parties computed a valid threshold signature on “skip” and sent it to all parties. Thus, all honest parties gets a “skip” message with a valid threshold signature and set \(skip = true\) in view \(j\).

\[ \square \]

**Lemma 19.** Assume all honest parties start with externally valid values for byzantine agreement, and consider a view \(j\), which all honest parties start. If all messages sent among honest parties in a view \(j\) arrived, then view \(j\) completed.

Proof. We first show that all honest parties set \(skip = true\) in view \(j\). By Observation 1, if some honest party sets \(skip = true\) in view \(j\), then all honest parties set \(skip = true\) in view \(j\). Now assume by a way of contradiction that no honest party sets \(skip = true\) in view \(j\). Therefore, by Lemma 18, all 4-Stage-f+1-Provable-Broadcasts issued by honest parties have completed and returned completion-proofs, and thus, by Lemma 19, all honest party set \(skip = true\) in view \(j\). A contradiction.

It remains to show that no honest party waits forever for \(elect(j)\) to return or to a “view-change” message from \(2f + 1\) different parties. Since, all honest parties set \(skip = true\) in view \(j\), we get by the code that they all invoke \(elect(j)\), and thus by the termination property of the leader election abstraction we get that all invocations of \(elect(j)\) by honest parties return. Therefore, all honest parties send a “view-change” messages to all other parties, and again, since all messages sent among honest parties in a view \(j\) arrived, we get that all honest parties get “view-change” messages from at least \(2f + 1\) different parties. Hence, all parties move to view \(j + 1\).

\[ \square \]

**Lemma 20.** Algorithms 6 and 7 satisfy termination.

Proof. By applying inductively Lemma 19, we get that the number of views in the protocol is unbounded, and thus honest parties never stop sending new messages. Therefore, termination trivially follows.

\[ \square \]

The secret sauce and the efficiency proof.

**Lemma 21.** Consider a view \(j\) of the protocol with identification \(id\). If an honest party invokes \(elect(⟨id, j⟩)\), then at least \(2f + 1\) 4-Stage-f+1-Provable-Broadcasts have completed before.

Proof. By the code, if an honest party invokes \(elect(⟨id, j⟩)\), then it sets its \(skip_i = true\) before. Thus it got a “skip” message with a valid threshold signature. Thus, some party got \(2f + 1\) “skip-share” messages, and thus at least one honest party \(p\) sent a “skip-share” message. Therefore, by the code again, there is a set \(P\) of \(2f + 1\) parties s.t. for every \(p_i \in P\), \(p\) receives \((v, σ)\) s.t. threshold-validate\((⟨⟨⟨id, l, j⟩, 4⟩, v⟩, σ)\). Therefore, by the PB-provability property, for every \(p_i \in P\), at least \(f + 1\) honest parties deliver a message \(m = ⟨v, ⟨(σ_{ex}, σ_{in})⟩⟩\) for some \((σ_{ex}, σ_{in})\) in the \(4^{th}\) \(1-f+1\)-Provabe-Broadcast instance of the 4-Stage-f+1-Provable-Broadcast issued by \(p_i\). Hence, for every \(p_i \in P\), at least \(f + 1\) honest parties \(deliver_4(⟨id, l, j⟩, ⟨v, σ_{in}⟩)\), and thus we get that at least \(2f + 1\) 4-Stage-f+1-Provable-Broadcasts complete in view \(j\).

\[ \square \]

**Lemma 22.** Consider a completed view \(j\) of the protocol with identification \(id\). If the broadcast issued by the leader of view \(j\) completed before the leader was elected, then all honest parties decide in view \(j\).

Proof. Let \(p_l\) be the leader of view \(j\). Since the broadcast issued by \(p_l\) completed before it was elected, we get that at least \(f + 1\) honest parties \(deliver_4\) a message \((v, σ)\) in \(p_l\)’s broadcast before sending their “view-change” massages. Thus, since honest parties wait for \(2f + 1\) “view-change”, every honest party gets a “view-change” with \((v, σ)\) s.t. threshold-validate\((⟨⟨⟨id, l, j⟩, 3⟩, v⟩, σ)\) = true. Therefore, by the code, all honest parties decide in view \(j\).

\[ \square \]

**Lemma 23.** Assume all honest parties start with externally valid values for byzantine agreement, and consider a completed view \(j\). Then there is a probability of at least \(\frac{2}{3}\) that all honest parties decided in view \(j\).

\[ \square \]
Proof. Since view $j$ is completed at least $f + 1$ honest parties invoked $\text{elect}(j)$, and let $p_l$ be the leader of view $j$. By Lemma 21, at least $2f + 1$ 4-Stage-f+1-Provable-Broadcasts have completed before the first honest party invokes $\text{elect}(j)$. Therefore, by the validity and unpredictability of the leader election abstraction, we get that the 4-Stage-f+1-Provable-Broadcast issued by the leader $p_l$ completed before it was elected with a probability of $\frac{2f+1}{4f+1} > \frac{2}{3}$. Therefore, by Lemma 22, all honest parties decide in view $j$ with probability of at least $\frac{1}{3}$.

Observation 2. An honest party cannot move to view $j + 2$ before view $j$ was completed.

Lemma 24. Algorithms 6 and 7 satisfy efficiency.

Proof. First note that since the number of messages sent by honest parties in an $f+1$-Provable-Broadcasts at most $2n$, we get that every view has $O(n^2)$ messages sent by honest parties. Denote this number by $T$, and let $X$ be the total number of messages sent by honest parties in the protocol before all honest parties decide. By Observation 2, $P_r[X > lT]$ is equal to the probability that some honest parties do not decide in the first $l$ views. By Lemma 23, the probability of this is less than $\frac{1}{3}$, and the lemma follows.

Appendix B Lower Bound on the number of messages for Asynchronous Byzantine Agreement With Adaptive Adversary

A recent theorem of Abraham et al. [1] provides a lower bound for Synchronous Byzantine Agreement against a strongly rushing adaptive adversary:

Theorem 2 (ADDNR [1]). If a protocol solves Synchronous Byzantine broadcast with $(1/2) + \epsilon$ probability against a strongly rushing adaptive adversary, then in expectation, honest parties collectively need to send at least $(\epsilon f / 2)^2$ messages.

The strongly rushing adaptive adversary assumed in Abraham et al. [1] can adaptively decide which $f$ parties to corrupt and when to corrupt them. In particular, the adversary is allowed to decide to corrupt a party $p$ after observing the messages sent by $p$ in round $r$. In addition, after corrupting $p$, the adversary can remove $p$'s round-$r$ messages from the network before they reach other honest parties.

Note that while Theorem 2 is proven for Byzantine broadcast, it immediately induces a lower bound for binary Byzantine agreement. We now use it to prove the following lower bound

Theorem 3. If a protocol solves binary Byzantine agreement with $(1/2) + \epsilon$ probability against an asynchronous adaptive adversary, then in expectation, honest parties collectively need to send at least $(\epsilon f / 2)^2$ messages.

Proof. If a protocol sends at most $(\epsilon f / 2)^2$ messages against any asynchronous adaptive adversary then it will clearly send at most $(\epsilon f / 2)^2$ messages against any restricted adversary that works in the synchronous model of communication (where any message sent in round $r$ by an honest party must arrive by the end of round $r$).

The core observation is that the asynchronous adaptive adversary has the ability to not deliver messages of parties it corrupts. This follows from the definition of Termination - the protocol must terminate even if some messages sent by parties that are corrupted will never be delivered. This is true even if some of these messages have been sent before the adversary decided to corrupt this party.

Hence even when we restrict the asynchronous adaptive adversary to a synchronous message passing model the adaptive adversary can still essentially remove messages of a party $p$ sent in round $r$ where $r$ is the round that the adversary decided to corrupt party $p$. Therefore the adaptive asynchronous adversary can fully simulate the synchronous strongly rushing adaptive adversary. In particular, the adaptive asynchronous adversary can simulate the behaviour as in Theorem 2 and cause the protocol to have an $1/2 - \epsilon$ error probability.

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5In comparison, a standard adaptive adversary (in the synchronous model) cannot “take back” or remove $p$’s round-$r$ messages to other honest parties.
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