2- and 3-Point Gluon Correlation Functions on the Lattice

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I present some preliminary results, obtained in collaboration with C. Bernard and A. Soni, for the lattice evaluation of 2- and 3-point gluon correlation functions in momentum space, with emphasis on the amputated 3-gluon vertex function. The final goal of this approach is the study of the running QCD coupling constant as defined from the amputated 3-gluon vertex.

1. INTRODUCTION

The task of measuring the QCD coupling constant at energies of the order of a few GeVs is a major challenge for the lattice community, because of the deep phenomenological and fundamental implications of such a measurement. Many different methods have been proposed to do this, based on the study of the interquark static potential, charmonium spectra and other quantities [1]. In spite of the success of some of these approaches [2], yet it is still poorly understood. Secondly, from \( G^{(2)} \) and \( G^{(3)} \) one can define non-perturbatively the lattice version of the amputated, 1PI 3-gluon vertex function:

\[
\Gamma_{lat}(p_1, p_2, p_3) \equiv G^{(3)}_{lat}(p_1, p_2, p_3) \times \prod_{i=1}^{3} \left[ G^{(2)}_{lat}(p_i^2) \right]^{-1}
\]  

Evaluating the above quantity is the crucial step in order to define the QCD coupling constant from the lattice 3-gluon vertex, as we explain in the following.

In order to determine a convenient kinematical setup, we consider the general form of the continuum, off-shell 3-gluon vertex [3]. Such an expression contains 6 independent scalar functions, but for the purpose of computing the coupling constant renormalization one only needs to determine the function which multiplies the tree-level vertex. Of course this is the only one which is divergent when the UV cutoff is removed.

If one evaluates the continuum vertex function \( \Gamma^{(3)}_{cont \ \alpha \beta \gamma}(p_1, p_2, p_3) \) at the "asymmetric" point defined by

\[
\alpha = \gamma \neq \beta, \quad p_1 = p_\beta, \quad p_2 = 0, \quad p_3 = -p_1
\]

then it can be written as

\[
\Gamma^{(3)}_{cont \ \alpha \beta \gamma}(p_\beta, 0, -p_\beta) = -2 \ F(p^2) \ p_\beta
\]

The above expression is proportional to the continuum tree-level vertex evaluated at \([4]\), and the
proportionality factor $F(p^2)$ diverges when removing the UV cutoff. One can show that the leading term of the 1-loop lattice calculation of the 3-gluon vertex for the same kinematics is indeed consistent with (6). Thus we decide to calculate the lattice vertex function $\Gamma_{\text{lat}}^{(3)}$ at (4) and we set (neglecting terms of higher order in the external momentum):

$$
\Gamma_{\text{lat}}^{(3)}(p_\alpha,0,-p_\beta) = -2 F_{\text{lat}}(p^2,a) p_\beta
$$

(6)

where $a$ is the lattice spacing. Then, when $a \rightarrow 0$, one can set

$$
F_{\text{lat}}(p^2,a)|_{p^2=q^2} = Z_g^{-1}(a^2 q^2) g_0(a)
$$

(7)

for a generic momentum $q^2$. Finally, following [4], we define the renormalized, ”running” coupling as

$$
g_R(q^2) = Z_A^{3/2}(a^2 q^2) Z_g^{-1}(a^2 q^2) g_0(a)
$$

(8)

where $Z_A$ is obtained from the relation

$$
G_{\text{lat}}^{(2)}(p^2)|_{p^2=q^2} = T_{\mu\nu}(q) Z_A(a^2 q^2) \frac{1}{q^2}
$$

(9)

with $T_{\mu\nu}(q)$ being the projector on transverse fields (we will work in the Landau gauge).

2. THE GLUON PROPAGATOR

The first step in the above program is the evaluation of $G_{\text{lat}}^{(2)}(p^2)$. The lattice gluon field can be defined as (3):

$$
A_{\text{lat}} \mu = \frac{U_\mu - U_\mu^\dagger}{2ia} - \frac{1}{3} \text{Tr} \left( \frac{U_\mu - U_\mu^\dagger}{2ia} \right)
$$

(10)

Earlier lattice studies of $G_{\text{lat}}^{(2)}(p^2 = 0,t) \equiv \sum_{i=1}^{3} G_{\text{lat}}^{(2)}(p^2 = 0,t)$ reported evidence of an effective gluon mass which increases with the time separation, for short time intervals. Moreover, the evaluation of $G_{\text{lat}}^{(2)}(p^2) \equiv \sum_{\mu=1}^{4} G_{\text{lat}}^{(2)}(p^2)$ has allowed a more detailed investigation of the mechanism of dynamical gluon mass generation and other non-perturbative phenomena [9,10]. Our group [9] evaluated $G_{\text{lat}}^{(2)}(p^2 = 0,t)$ and $G_{\text{lat}}^{(2)}(p^2)$ on different sets of (quenched) configurations with $\beta$ ranging between 5.7 and 6.3.

For the purpose of defining the amputated vertex function, the most convenient set of configurations among the ones considered is the set of 25 configurations on a $16^3 \times 40$ lattice at $\beta = 6.0$. This because it meets requirements of good statistics, low infrared cutoff and stable data for the propagator (see Fig.1). All the numerical results shown here refer to such set of configurations.

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Figure 1. $G_{\text{lat}}^{(2)}(p^2)$ vs. $p$ in GeV on the $16^3 \times 40$ lattice at $\beta = 6.0$. We assume $a^{-1} = 2.1$ GeV.

All the calculations are performed after gauge-fixing to the so-called minimal Landau gauge [10] (see [9] for a short review), implemented through the iterative minimization of

$$
H_U[g] \equiv -\frac{1}{V} \sum_{n,\mu} \text{Re} \text{Tr} \left( U_{\mu}^\dagger(n) + U_{\mu}^\dagger(n-\hat{\mu}) \right)
$$

(11)

where $V$ is the lattice volume and $U_{\mu}^\dagger(n)$ indicates the gauge-transformed link $U_{\mu}(n) \equiv g(n) U_{\mu}(n) g^\dagger(n+\hat{\mu})$.

3. THE 3-GLUON VERTEX

After calculating the propagator, we proceed to the evaluation of the complete 3-point function $G_{\text{lat}}^{(3)}(p_\alpha,0,-p_\beta)$. We set $\alpha = 1$, $\beta = 4$, to be able to inject momentum in the longer (time) lattice direction, and we call $p_i$ the injected momentum. As expected, the numerical results are
consistent with an odd function of $p_t$ (see Fig.2). For large values of $p_t$ it gets damped, as a result of the propagators in momentum space corresponding to the external legs. At this point, in order to define the amputated vertex function, we multiply the complete 3-point function shown above by the inverse propagators, according to \( \Gamma_{1\,4\,1}(p_t, 0, -p_t) \). The resulting function \( \Gamma_{1\,4\,1}(p_t, 0, -p_t) \)

is shown in Fig.3, where the error is a jackknife one. The amputated vertex function looks roughly proportional to the external momentum, as expected from 1-loop calculations.

4. CONCLUSIONS

We have discussed a method to measure on the lattice the 3-gluon vertex function and to define from it the running coupling constant. Preliminary numerical results suggest that the vertex function can indeed be non-perturbatively defined and measured. A careful analysis of the role of IR and UV lattice artifacts is needed. In particular, having defined the coupling at a point in momentum space where one of the external momenta is zero, it will be crucial to investigate finite volume effects.

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