Has cosmological dark matter been observed?

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There are many indications that ordinary matter represents only a tiny fraction of the matter content of the Universe, with the remainder assumed to consist of some different type of matter, which, for various reasons must be nonluminous (dark matter). Among these indications are the inflationary scenarios which predicts that the average energy density of the Universe coincides with the so called critical value (for which the expansion never stops but the rate of expansion approaches zero at very late times). At the same time it is known (from the predictions of Big Bang nucleosynthesis on the abundances of the light elements, other than Helium) that the baryonic energy density (ordinary matter) must represent $(1.5 \pm 0.5)h^{-2} \%$ (where $h$ is the Hubble constant in units of $100 \text{ km s}^{-1}\text{Mpc}^{-1}$) of this critical value [1,2]. We present here evidence supporting the model in which the rest of the energy density corresponds to a scalar field, which can be observed, however indirectly, in the oscillation of the effective gravitational constant, and manifests itself in the known periodicity of the number distribution of galaxies [3,4]. We analyze this model numerically and show that, the requirement that the model satisfy the bounds of light element abundances in the Universe, as predicted by Big Bang nucleosynthesis, yields a specific value for the redshift-galactic-count oscillation amplitude compatible with that required to explain the oscillations described above [3,5], and, furthermore, yields a value for the age of the Universe compatible with standard bounds [2]. The fact that the model has successfully passed these tests lends
support to it and to the conclusion that \( \approx 98\% \) of the energy density of the Universe is stored in this scalar field. This leads to the astonishing conclusion that if the observations of Broadhurst et. al. [3,4] are not a statistical fluke, the cosmological component of dark matter might already have been observed.
It has long been suspected that ordinary matter (photons, ordinary massless neutrinos and atoms) represents only a tiny fraction of the matter content of the Universe. The photons and neutrinos (collectively called radiation, because they are massless particles) represent at the present time a negligible fraction of the energy density corresponding to the atoms. The latter is sometimes called baryonic matter because most of the energy corresponds to the rest mass of protons and neutrons (baryons). The evidence that there is much more matter in the Universe comes from various sources, and it manifests itself through its gravitational influence. One such item of evidence is found in the dynamics of the motion of stars in galaxies, where the use of the standard gravitational laws to explain rotation curves results in the conclusion that the amount of mass contained in the galaxy is about ten times larger than the sum of the masses of the stars, gases and other known luminous constituents of the galaxy \[ F \]. In the behavior of clusters of galaxies there are further indications that there exists much more mass than we observe in form of stars and gases \[ G \]. Finally, the most dramatic indication of the “insignificance” of ordinary matter relative to the total matter content of the Universe comes from the so-called inflationary scenarios \[ H, I, J \]. This indication consists in a concrete prediction that the sum of all contributions to the average energy density of the Universe exactly equals the so-called critical value. These inflationary scenarios that are invoked to solve serious defects of the standard cosmological model indicate that, since the observed ordinary matter is of the order of 1% of the critical value, there must be a large amount of dark matter. Moreover, this dark matter must be mostly exotic matter, since cannot consist of radiation or baryonic matter. This conclusion results from the dependence on the baryonic energy density, of the predicted values of the relative primordial abundances of light elements (Be, Li, D, etc) in Big Bang nucleosynthesis. This restricts the value of the baryonic energy density \( \Omega_{\text{bar}} \) to lie in the range \( (1.5 \pm 0.5)h^{-2} \) % of the critical value \( K, L \), where \( h \) is the Hubble constant in units of 100 km s\(^{-1}\)Mpc\(^{-1}\).

Cosmologists and particle physicists have long been puzzled about what this exotic matter might be \( M, N \). The main models can be divided in two categories. The first one is called hot dark matter which consists of light neutrinos or a similar species, i.e. massive
particles whose number density at freeze out, is determined when they still may be considered as relativistic particles. The cold dark matter model consists of all the remaining weakly interacting massive particles like axions, neutralinos, superheavy monopoles, primordial black holes, etc. These particles were already non relativistic when their number density reached a state of equilibrium in which annihilations freezes out. No independent evidence for the existence of these new types of matter has so far been found.

An apparently completely independent problem in cosmology is posed by the recent observations in deep pencil beam surveys \[3,4\] which show that the number distribution of galaxies exhibits a remarkable periodicity. This is a shocking development, since if taken at face value it would imply that we live in the middle of a pattern consisting of concentric two-spheres that mark the maxima of the galaxy number density. This, of course, would lead to catastrophic consequences for our concepts of cosmology. While it is true that such periodicity has been observed only in the few directions that have been explored so far, it would be a remarkable coincidence if it turns out that it is absent in other directions, and we have just happened to have chosen to explore the only directions in which this phenomenon is observed. It seems, therefore, reasonable to assume that the periodicity will be also present in deep pencil beam surveys in other directions, thus forcing us toward the concentric spheres scenario. The seriousness of the situation is such that this type of scenario has indeed been proposed in a model where the formation of these concentric shells is a result of a “spontaneous breakdown of the cosmological principle” via a mechanism that results in the appearance of patches filled with a pattern of concentric spheres, with these patches filling the Universe \[13\]. It would be difficult to explain how we we managed to be living in the center of such a patch (more precisely inside the innermost sphere of one such patch).

The only known escape from this type of scenario is to assume that the spatial periodicity is only an “illusion” and is the result of a true temporal periodicity which affects our observations of distant points in the Universe, and which is mistakenly interpreted as spatial periodicity \[3\]. The models that have been put forward that allow this temporal periodicity involve the oscillation of an effective coupling constant due to a contribution
from the expectation value of some scalar field that actually oscillates coherently in cosmic time at the bottom of its effective potential. It is worth to mention that a completely different scenario consisting of oscillating peculiar velocities of galaxies have also been proposed for explaining the observed periodic redshifts [14].

The specific oscillating-coupling-constant models which have been proposed involve the oscillation of the effective electric charge, electron mass, galactic luminosity or the gravitational constant [5, 16, 17]. Of these, the first two have been shown to conflict with bounds arising from tests of the Equivalence Principle [18]. The model of oscillating galactic luminosity is a vaguely specified scenario which nevertheless seems to require at least two new hypotheses: a new type of star cooling mechanism, and also (as the other alternatives do) an oscillating cosmological scalar field which turns that mechanism on and off periodically in cosmic time. Needless to say, once the scenario is implemented in a specific model, unforeseen new bounds might also have to be overcome.

In this light the *Oscillating G Model* seems to be the most attractive alternative. We also should point out that in the fossil record of marine bivalve shells [16], there seems to be further evidence in support of an effective gravitational constant that oscillates with time. Moreover, cosmological consequences of high-frequency oscillations of Newton’s constant have been also studied [19]. In this work, we consider the model, initially proposed in [5], and further studied in [3, 4, 15, 17], of a massive scalar field non-minimally coupled to gravity leading to an effective gravitational constant which oscillates in cosmic time and in this way produces the illusory spatial periodicity observed in the number distribution of galaxies.

The central feature of this model is a cosmological massive scalar field $\phi$ that is non-minimally coupled to the curvature of spacetime. The corresponding Lagrangian can be written as

$$
\mathcal{L} = \left( \frac{1}{16\pi G_0} + \xi \phi^2 \right) \sqrt{-g} R - \sqrt{-g} \left[ \frac{1}{2} (\nabla \phi)^2 + m^2 \phi^2 \right] + \mathcal{L}_{\text{mat}},
$$

where $G_0$ is the Newtonian gravitational constant, $\xi$ is the non-minimal coupling constant, $m$ is the mass associated with the scalar field, and $\mathcal{L}_{\text{mat}}$ represents a matter Lagrangian.
The non-minimal coupling of the scalar field $\phi$ to curvature results in an effective gravitational “constant” $\tilde{G}_{\text{eff}} = G_0 / (1 + 16\pi G_0 \xi \phi^2)$ which explicitly depends on the cosmic time due to the contribution of the expectation value of the scalar field.

We studied the resulting cosmological model corresponding to an isotropic and homogeneous spacetime; i.e., the one described by the Friedman-Robertson-Walker (FRW) metric. It was also assumed that the scalar field, as well as the matter fields, possess the same symmetries as the spacetime.

We studied the model in detail using numerical methods, and explored its compatibility with standard cosmological tests. The model is known to require a “fortunate phase” in order to satisfy the bounds imposed on the value of $\dot{G} / G$ by the Viking radar echo experiments [20,5], and by the limits on the Brans-Dicke parameter [6]. It has also been argued [3] that it requires fine tuning of the parameters and data in order to satisfy the bounds of Big Bang nucleosynthesis and therefore that the model should not been taken seriously.

We will argue that, if seen in the proper context, this fine-tuning can be taken instead, to be a concrete prediction of the values of parameters and more specifically as relationships among them, a prediction that can be used to rule out or support the model.

Our general philosophy concerning this fine-tuning should be understood in the following context. Scientific models that require a very precise choice of the numerical value of the initial conditions in order to reproduce a given qualitative behavior of the observational data are models that would be considered unnatural, and the choice of the specific initial data is justifiably described as “fine-tuning”. However, models that require a very precise choice of the numerical value of the initial conditions in order to reproduce a specific numerical observational data cannot be considered as unnatural, especially if for every conceivable value of the observational data (at least in some range), there is a corresponding value of the initial data. In this type of models, the particular “preferred” value of the initial data is just the result of a 1 to 1 correspondence between initial conditions and final outcome. The exactness of the predictions so obtained will, of course, depend on
the exactness of the observational data that determine the remaining parameters entering
in the model.

The main point is that in order to reproduce a specific value for the red-shift-galactic-
count oscillation amplitude \( A_0 \) and the corresponding periodicity of 128 Mpc \( h^{-1} \), a very
specific value for the density of the baryonic matter in the Universe emerges. In a previous
work \[17\], for example, we took the quoted observational value of \( A_0 = 0.5 \) \[5\] and \( h = 1 \)
for the Hubble constant. The resulting value, \( \Omega_{\text{bar}} \sim 0.021 \) for which it was possible to
recover the primordial \(^4\text{He} \) abundance from nucleosynthesis and which was also consistent
with the inflationary value \( \Omega = 1 \) lay in a very narrow range that we had assumed to be
\( 0.016 \leq \Omega_{\text{bar}} \leq 0.026 \) \[21\] and thus we took it to be a strong piece of evidence in favor of
the model. Alternatively, if we start from a specific value of the baryonic energy density
(for example one that lies within the range allowed by nucleosynthesis) and the frequency
of the periodicity, the result is a specific value for the amplitude \( A_0 \). Moreover, these
values, together with the value of the Hubble parameter, determine a specific value for
the age of the Universe, and so it is a highly nontrivial question whether these results are
or are not compatible with observations. Our main result is to answer the above in the
affirmative, thus implying that the problems of the nature of cosmological dark matter
and of the periodicity in the galaxy number distribution may be solved simultaneously
within the framework of an oscillating \( G \) model with a massive scalar field.

The field equations following from the Lagrangian \[11\] are similar to Einstein’s equations
with the Newtonian constant replaced by the effective gravitational constant \( G_{\text{eff}} \),
and an energy-momentum tensor which contains contributions from the non-minimal coupling
to curvature, the scalar field, and the matter. The last one consists of two non-
interacting perfect fluids: \( T_{\text{mat}}^{\mu\nu} = T_{\text{bar}}^{\mu\nu} + T_{\gamma}^{\mu\nu} = \sum_{i=1,2} \left( (p_i + e_i)U^\mu U^\nu + p_i g^{\mu\nu} \right) \). The first
one corresponds to pure baryonic matter \( (i = 1) \), and the second one represents a pure
radiation field \( (i = 2) \). Moreover, the scalar field must satisfy the massive Klein-Gordon
equation with an extra term due to the non-minimal coupling to curvature.

Our analysis consists in evolving the scale factor, the scalar field and the ordinary
matter densities backwards and forwards in cosmic time using the field equations, and
starting from the model parameters $\xi$ and $m$, and data corresponding to today’s values of $H_0$, $\Omega_{\text{bar}}, \Omega_{\text{rad}}$, $\phi_0$ and $\dot{\phi}_0$. With these initial conditions it is then possible to integrate the field equations numerically.

We will work within the standard inflationary scenario, so $\Omega = \Omega_{\text{bar}} + \Omega_{\text{rad}} + \Omega_{\phi} = 1$, where $\Omega_{\text{rad}}$ is the cosmic background radiation (CBR) and its value today is determined from the CBR temperature of 2.735 K resulting in a contribution which is several orders of magnitude smaller than $\Omega_{\text{bar}}$ which is itself known from recent analysis to be in the range $0.01h^{-2} - 0.02h^{-2}$ \[1,2\]. Therefore fixing $\Omega_{\text{bar}}$ is equivalent to fixing $\Omega_{\phi}$.

The initial condition (today) for the time derivative of the scalar field $\dot{\phi}_0 = 0$ was fixed so that it satisfies the Viking radar echo experiments \[20,5\]. The initial condition $\phi_0$ as well as the value of the coupling constant $\xi$ turn out to be expressible in terms of the values of the amplitude $A_0$, the oscillation frequency $\omega$ of the scalar field and the parameter $\Omega_{\phi} = 1 - \Omega_{\text{bar}} - \Omega_{\text{rad}}$. The observations yield a value of $A_0$ of about 0.5 (in fact, it has been argued that $A_0 \geq O(0.5)$ \[3\], see the discussion below), while the observed galactic periodicity of $128 \ h^{-1}$ translates to a value $m \sim 10^{-31} \ eV$ for the scalar-field mass \[4,5\].

As we mentioned, one of the main problems faced by the model is that related to the primordial nucleosynthesis of $^4\text{He}$. The latter is determined by the temperature at which the rate of weak interactions, which convert neutrons to protons, equals the value of the Hubble parameter. According to standard cosmology the value of about 0.7MeV for this temperature corresponds to a $^4\text{He}$ abundance which is the best approximation to observational data (see \[21\] for a review).

The evolution of the scalar field backwards in cosmic time results in it going to $\pm \infty$ depending on the initial data. This suggests that there is a very precise initial data for which it is possible to reach a transition point in which $\phi$ remains steady and close to zero. This “steady state” is represented by a kind of plateau during which $G_{\text{eff}} \rightarrow G_0$ \[17\]. It was possible to correlate the length of this plateau with the recovery of the precise standard freeze-out temperature. The larger the plateau the closer the freeze-out temperature predicted by the oscillating model approached the value 0.7 MeV. The search
for that transition point is what we call “fine-tuning” and it turns out that this can be done by adjusting only the parameters $A_0$ and $\Omega_{\text{bar}}$ (see Fig. 1). In principle it is possible to extend the plateau of $\phi$ to the nucleosynthesis era or even to earlier eras by improving the “fine-tuning” of the values of $\Omega_{\text{bar}}$ or $A_0$.

As explained above, the recovering of the success of standard Big Bang nucleosynthesis requires the implementation of the so-called “fine-tuning” of the parameters and the initial conditions of the model. However, as we previously argued this is not the kind of fine-tuning that implies the dismissal of the model, but it is rather a procedure that becomes necessary in order to recover the observational data extracted from our Universe today. We must also stress that while the fine tuning is completely unnatural when approached, as we have approached to it, from the present to the past, when looked from the opposite, and more natural direction, the situation is quite different. In fact all that seems to be required is for some mechanism to drive the scalar field to an extremely low value before the era of nucleosynthesis. Then, as our calculations show, the field will remain at that value up to and beyond that era so that we will have $G_{\text{eff}} \approx G_0$ and then the success of “Big Bang Nucleosynthesis” will be recovered naturally. The value of the field $\phi$ will later be amplified by the curvature coupling just before the onset of oscillatory behavior.

The era with $\phi \approx 0$ ends just before the Hubble parameter approaches to the value of $m$, a situation that is followed by an amplification and then the onset of oscillations of $\phi$.

But the point is that this would actually represent no “fine-tuning” at all (if looked in the right perspective), because all that it will mean is that (given the physical constant precise values) the standard inflationary mechanism will ensure that the energy densities of the various components, i.e., scalar field, baryon matter and radiation, add up to $\Omega = 1$, which will then correspond then to a Universe at our time with precise values of the densities, expansion rate etc. In particular the precise current value of the scalar field amplitude and phase arise from a particular precise value of the parameters at early times, among them the baryon content of the Universe.

Thus it is possible that starting from an arbitrary value of $\phi$ near the Big Bang, a mechanism related to inflation would drive the scalar field to a value near zero, where
it will remain until just before $H \approx m$ when amplification and then oscillations would occur. Now, since the behavior of $G_{\text{eff}}$ is determined by the expectation value of $\phi$, oscillations of $\phi$ induce oscillations in $G_{\text{eff}}$ [17], and so we are led to the scenario in which the temporal oscillations of the effective gravitational constant manifest themselves in an apparent spatial periodicity of the number distribution of galaxies [3][5][17].

As we have said, our initial research was carried out by taking very specific values of the observed quantities [17]. The present analysis is motivated by our previous results, and by the need to study the compatibility of the model given the uncertainties in the most up-dated observational data. In particular we consider the constraints arising from the lower bound on the age of the Universe for which we take the most conservative lower bound of 11.5 billion years obtained from a giant-branch fitting [3]. In addition, observations of the galaxy number distribution do not fix the amplitude, but instead indicate the lower bound $A_0 \geq O(0.5)$. Finally, for the Hubble parameter we will use the range of values $0.65 \leq h \leq 0.75$ which is the intersection of the ranges allowed by recent astronomical observations (data from Hubble Space Telescope, and studies of type I supernovae [22,23,2]). It is to be emphasized that the largest uncertainty in the observed galactic periodicity of $128h^{-1}$ is that contained in the value of $h$. Therefore, we will investigate the sensitivity of our results with respect to the values of $A_0$, and $h$.

For each value of $h$ in the above range we obtain a corresponding allowed range of $\Omega_{\text{bar}}$, and for each value of $\Omega_{\text{bar}}$ we can obtain by means of the “fine-tuning” mechanism a value of $A_0$ and a resulting value for the age of the Universe. We have performed this analysis with $h = 0.65$ and $h = 0.75$ the extreme values of the interval. In the first case, the range allowed by light-element nucleosynthesis corresponds to $0.023 \leq \Omega_{\text{bar}} \leq 0.047$, the corresponding range in $A_0$ was found to be $[0.490, 0.499]$ and the age of the Universe lies in the range $[11.77, 12.06]$ Gy which satisfies the 11.5 Gy bound. In the second case the range allowed by light-element nucleosynthesis corresponds to $0.023 \leq \Omega_{\text{bar}} \leq 0.046$, the corresponding range in $A_0$ is $[0.4950, 0.5015]$ and the age of the Universe lies in the range $[10.32, 10.57]$ Gy which fails to satisfy the 11.5 Gy bound.

Similar results can be obtained for other values of $h$ contained in the range $0.65 \leq$
$h \leq 0.75$, from which we find that the upper value of $h$ that satisfies the age bound is $h \sim 0.68$.

These results show that there is a range of the parameters of the model that satisfies all known cosmological constraints while at the same time explaining both the nature of the cosmological dark matter and the observed periodicity in the galaxy distribution with red-shift. Figure 2 shows the red-shift periodicity of the Hubble parameter as calculated by the oscillating $G$ model.

Figure 3 shows the bounds imposed on $\Omega_{\text{bar}}$ with $h = 0.65$ and $h = 0.75$, and on the age of the Universe from a giant-branch fitting [2]. A triangle indicates a configuration calculated with our model with $\Omega \sim 0.0336$ and $\mathcal{A} = 0.495$. Figure 4 shows the freeze-out temperature predicted by the same configuration ($\sim 0.703$ MeV). The age of the Universe extracted from data of Fig. 5 corresponds to $0.792H_0^{-1}$ which translates into $11.917$ Gy with $h = 0.65$.

The cross in Fig. 3 represents a configuration with $h = 0.75$ and $\Omega \sim 0.0210$ and $\mathcal{A} = 0.5$. Although the results are compatible with almost all bounds, the resulting age of the Universe $10.505$ Gy turns to be too small and thus this configuration must be ruled out.

To conclude, the oscillating $G$ model is certainly the most attractive model for explaining the observed periodicity in the galactic distribution, and it should also be considered as a missing mass model with the scalar field playing the role of cosmological dark matter which is, however, indirectly observable in the oscillation of the galactic distribution. The requirement that the scalar field go through a plateau phase during which $G_{\text{eff}} \approx G_0$ then yields a relationship between the value of the baryonic energy density and the value of the redshift-galaxy-count oscillation amplitude. The fact that these can be accommodated within the allowed ranges inferred from observations, together with the fact that they lead to an acceptable value for the age of the Universe is taken as support for the model, and thus for the idea that the rest of the energy density of the Universe is stored in the oscillating scalar field. This leads to the astonishing conclusion that, if the observations of Szalay et. al. are not a statistical fluke, the cosmological component of the dark matter
might have been observed.

One remaining problem to be faced is that of the phases which according to the latest observations are not the same in all the explored directions. One possible way out of this difficulty has been suggested in [6] where it is argued that a modification of the form of the scalar potential helps to ameliorate the problem. A careful analysis of such a scenario should probably be done within the context of the analysis of local inhomogeneities. Moreover the model should of course be extensively tested, and in particular further and more precise observations will be needed before one could claim with any degree of certainty that indeed cosmological dark matter has been observed.
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FIGURES

FIG. 1. The fine-tuned scalar field amplitude as a function of \( \ln[a/a_0] \) for a flat Universe \((\Omega = 1)\) with \( \Omega_{\text{bar}} \sim 0.033\,689 \) and \( A_0 = 0.495 \) at the onset of oscillations. At present time \((\alpha = 0)\) the initial amplitude is \( \phi_0 \sim 3.26 \times 10^{-3} \) and \( \xi \sim 6.2859 \). Computations were stopped at \( \alpha \sim 1.5 \).

FIG. 2. Periodic distribution of the Hubble parameter with respect the red-shift. The distribution extends from \( z \sim -0.5 \) to \( z \sim 0.5 \).

FIG. 3. Bounds imposed by observations on the age of the Universe (horizontal dashed line), on the values of \( h \) and therefore on the resulting allowed ranges of the baryonic component of matter (vertical lines) obtained from \( 0.01 \leq \Omega_{\text{bar}} h^2 \leq 0.02 \). The triangle depicts the configuration corresponding to Figs. 1, 2, 4, 5. The cross represents a configuration with \( h = 0.75 \) (see text). The square indicates a configuration barely allowed by the age-of-the-Universe bound. This corresponds to the values \( h = 0.68, \Omega_{\text{bar}} \sim 0.0222 \) and \( A_0 = 0.4995 \).

FIG. 4. Expansion (solid line) and neutron-to-proton-weak-interaction transition (dashed line) rates in terms of the blackbody temperature. The asterisk depicts the freezeout temperature \( \sim 0.7 \text{ MeV} \) at which nucleosynthesis takes place as predicted by the standard cosmological models. The crossing point of the curves indicates the corresponding freezeout temperature \( \sim 0.7 \text{ MeV} \) for the oscillating model of previous figures.

FIG. 5. The scale factor of the oscillating model of the previous figures in units of its value today as a function of cosmic time (in units of \( H_0^{-1} \)). The present time \( t_0 \) has been taken to be \( t \sim 1H_0^{-1} \).