1 Introduction

Current HFAG averages of the mixing and CP violation (CPV) parameters in the $D^0$ meson system is to be reviewed. We present recent Belle measurements of the mixing parameters $x$ and $y$ using $D^0 \to K^0_S \pi^+ \pi^-$, and $y_{CP}$ and $A_{\Gamma}$ from $D^0 \to h^+ h^-$, $D^0 \to h^+ h^-$, and $D^+ \to K^0_S K^+$ decays together with $\Delta A_{CP}^{hh}$ are to be presented.

2 Current HFAG averages [1]

$D^0 - \bar{D}^0$ mixing occurs since the mass eigenstates $D_1$ and $D_2$ are different from the weak eigenstates $D^0$ and $\bar{D}^0$. Assuming CPT is conserved, the mass eigenstates can be written in terms of the weak eigenstates by $|D_1\rangle = p|D^0\rangle - q|\bar{D}^0\rangle$ and $|D_2\rangle = p|D^0\rangle + q|\bar{D}^0\rangle$, where $|p|^2 + |q|^2 = 1$. Thus without CPV $D_1$ and $D_2$ are CP-even and CP-odd, respectively, with the HFAG convention $CP|D^0\rangle = -|\bar{D}^0\rangle$ and $CP|\bar{D}^0\rangle = -|D^0\rangle$. The mixing parameters, $x$ and $y$ can be expressed by the difference of masses and widths between the two mass eigenstates, $x = (m_1 - m_2)/\Gamma$ and $y = (\Gamma_1 - \Gamma_2)/2\Gamma$, where $\Gamma = (\Gamma_1 + \Gamma_2)/2$. CPV parameters are $|q/p|$ and $\phi = \text{Arg}(q/p)$, where the former and the latter are responsible for CPV in mixing and that in interference between the decays with and without mixing, respectively. Without direct CPV, alternative mixing and CPV parameters are $x_{12} = 2|M_{12}|/\Gamma$, $y_{12} = |\Gamma_{12}|/\Gamma$, and $\phi_{12} = \text{Arg}(M_{12}/\Gamma_{12})$, where $M_{12}$ and $\Gamma_{12}$ are off-diagonal elements of the $D^0 - \bar{D}^0$ mass and decay matrices which are responsible for the mixing. Without CPV $x_{12}$, $y_{12}$, and $\phi_{12}$ become $x$, $y$, and zero, respectively. Current HFAG averages of the mixing parameters $x$, $y$, $x_{12}$, and $y_{12}$ rule out the no-mixing hypothesis with more than 10$\sigma$ significance, but they are difficult to be interpreted with the standard model (SM) due to the final state interactions [2]. CPV parameters, $|q/p|$, $\phi$, and $\phi_{12}$ show no indirect CPV at present. Since both $x$ and $y$ favor positive values, the CP-even state in the $D^0$ system is heavier (unlike $K^0$ system) and shorter-lived (like $K^0$ system).
3 CPV and mixing in the charm sector at Belle

3.1 Mixing \((x, y\) from \(D^0 \rightarrow K_S^0 \pi^+ \pi^-\) and \(y_{CP}\), \(A_{\Gamma}\) from \(D^0 \rightarrow h^+ h^-\) and \(D^0 \rightarrow K^- \pi^+\))

Time dependent decay matrix element of \(D^0 \rightarrow K_S^0 \pi^+ \pi^-\) is expressed with
\[
\mathcal{M}(m^2_-, m^2_+, t) = A(m^2_-, m^2_+) \frac{e_1(t) + e_2(t)}{2} - \frac{q}{p} A(m^2_+, m^2_-) \frac{e_1(t) - e_2(t)}{2},
\]
where \(t\) is proper decay time, \(m^2_\pm = m^2(K^0_S \pi^\mp)\), \(e_j(t) = e^{-t(\Gamma_j/2 + i m_j)}\), and \(A(\mathcal{A})\) is the decay amplitude of \(D^0(D^0)\). Thus the mixing parameters, \(x\) and \(y\) can be extracted with time dependent Dalitz analysis of the decay rate, \(\mathcal{M}^2\). The best fit model for \(A(m^2_-, m^2_+)\) is found to be a sum of Breit-Wigner for P-, D-wave resonances, K-matrix [3] and LASS [4] models for \(\pi\pi\) non-resonant decay. Figure 1 shows the 2-D Dalitz plot and fit results as \(m^2_\pm\) and decay time projections. The results from the fit under CP conservation, \(q/p = 1\) and \(A = \mathcal{A}\), are \(x = (0.56 \pm 0.19^{+0.03+0.06}_{-0.09-0.09})\%\), \(y = (0.30 \pm 0.15^{+0.04+0.03}_{-0.05-0.06})\%\), and \(\tau_{D^0} = (410.3 \pm 0.4)\) fs, where \(x\) and \(y\) are the most sensitive to date and \(\tau_{D^0}\) is consistent with world average [5].

![Figure 1: From left, 2-D Dalitz plot, fit projections onto \(m^2_-, m^2_+\), and proper decay time, respectively.](image)

The width asymmetry between neutral \(D\) CP-even and CP-odd eigenstates is referred to as \(y_{CP}\) and the experimental observable for \(y_{CP}\) is the lifetime difference between CP-even \((D^0 \rightarrow h^+ h^-)\) and CP-mixed \((D^0 \rightarrow K^- \pi^+\) states as shown in Eq. (2). The \(y_{CP}\) could be different from \(y\) if CP is violated in charm decays.
\[
y_{CP} = \frac{\Gamma(CP-even) - \Gamma(CP-odd)}{\Gamma(CP-even) + \Gamma(CP-odd)} = \frac{\tau_{D^0 \rightarrow K^- \pi^+}}{\tau_{D^0 \rightarrow h^+ h^-}} - 1 \simeq y.
\]

The width asymmetry between two CP conjugate modes provides the CPV parameter, \(A_{\Gamma}\) which can be measured from lifetime difference between the two CP
conjugate decays. Figure 2 show the results for $y_{CP}$, $A_{\Gamma}$, and $\tau_{D^0\rightarrow K^-\pi^+}$ as a function of the $\cos \theta^*$, where $\theta^*$ is polar angle of $D^0$ at the center-of-mass system (c.m.s.). The averages are $y_{CP} = (1.11 \pm 0.22 \pm 0.11)\%$, $A_{\Gamma} = (-0.03 \pm 0.20 \pm 0.08)\%$, and $\tau_{D^0\rightarrow K^-\pi^+} = (408.56 \pm 0.54) \text{ fs}$, where the last is consistent with world average [5]. Thus we observe $y_{CP}$ with 4.5$\sigma$ significance and find no $CPV$.

3.2 Direct $CPV$ ($A_{CP}$ in $D^+ \rightarrow K_S^{0}\pi^+$, $D^0 \rightarrow h^+h^-$, $D^+ \rightarrow K_S^{0}K^+$, and $\Delta A_{CP}^{hh}$)

The $D^+ \rightarrow K_S^{0}\pi^+$ final state is a coherent sum of Cabibbo-favored and doubly Cabibbo-suppressed decays where no SM $CPV$ in charm decay is expected, while $(−0.332 \pm 0.006)$% of $A_{CP}^{K_S^{0}K^+}$ is expected. Using $\sim 1.74M$ reconstructed $D^+ \rightarrow K_S^{0}\pi^+$

$\text{Figure 2: } y_{CP}$, $A_{\Gamma}$, and $\tau_{D^0\rightarrow K^-\pi^+}$ as a function of the $\cos \theta^*$. Top(bottom) three plots are obtained with 3-layer(4-layer) silicon detector.
decays, the $A_{CP}$ is measured in bins of $|\cos\theta^*_{D^+}|$ and the average of $A_{CP}^{D^+\rightarrow K^0\pi^+}$ is $(-0.363 \pm 0.094 \pm 0.067)\%$ which shows $3.2\sigma$ deviations from zero. This is the first evidence for $CPV$ in charm decays from a single decay mode while the measured asymmetry is consistent with the $A_{CP}^{K^0\pi^+}$.[6], the $CPV$ in charm decay, $A_{CP}^{D^+\rightarrow K^0\pi^+}$[7] is measured to be $(−0.024\pm0.094\pm0.067)\%$.[7]

Belle preliminary using 976/fb

![Graphs showing reconstructed signal distributions and preliminary results of $A_{CP}$ as a function of the polar angle of $D^{*+}$ momentum at the c.m.s.](image)

Figure 3: Top four plots show reconstructed signal distributions described in the text and bottom two plots show preliminary results of $A_{CP}$ as a function of the polar angle of $D^{*+}$ momentum at the c.m.s.

The $D^0 \rightarrow h^+h^−$ final states are singly Cabibbo-suppressed (SCS) decays in which both direct and indirect $CPV$ are expected in the SM [8], while the $CP$ asymmetry

$^2$We neglect doubly Cabibbo-suppressed decay, $D^+ \rightarrow K^0\pi^+$. 
difference between the two decays, \( \Delta A^{hh}_{CP} = A^{KK}_{CP} - A^{\pi\pi}_{CP} \), reveals approximately direct CPV with the universality of indirect CPV in charm decays [9]. Figure 3 shows reconstructed signal distributions showing 14.7M \( D^0 \rightarrow K^-\pi^+ \), 3.1M \( D^{*+} \) tagged \( D^0 \rightarrow K^+\pi^- \), 282k \( D^{*+} \) tagged \( D^0 \rightarrow K^+ K^- \), and 123k \( D^{*+} \) tagged \( D^0 \rightarrow \pi^+\pi^- \), respectively, and the measured \( A_{CP} \) in bins of \( |\cos \theta_{D^{*+}}| \). From the bottom plots in Fig. 3 we obtain \( A^{KK}_{CP} = (-0.32 \pm 0.21 \pm 0.09)\% \) and \( A^{\pi\pi}_{CP} = (+0.55 \pm 0.36 \pm 0.09)\% \) where the former shows the best sensitivity to date. From the two measurements, we obtain \( \Delta A^{hh}_{CP} = (-0.87 \pm 0.41 \pm 0.06)\% \) which shows 2.1\( \sigma \) deviations from zero and supports recent LHCb [10] and CDF [11] measurements. By combining LHCb, CDF, and Belle results, the average of \( \Delta A^{hh}_{CP} \) becomes \((-0.74 \pm 0.15)\% \).

The \( D^+ \) decaying to the final state \( K^0_s K^+ \) proceeds from \( D^+ \rightarrow \bar{K}^0 K^+ \) decay which is SCS, where direct CPV is predicted to occur [8]. The decay \( D^+ \rightarrow \bar{K}^0 K^+ \) shares the same decay diagrams with \( D^0 \rightarrow K^+ K^- \) by exchanging the spectator quarks, \( d \leftrightarrow u \). Therefore, neglecting the helicity and color suppressed contributions in \( D^+ \rightarrow \bar{K}^0 K^+ \) and \( D^0 \rightarrow K^+ K^- \) decays, the direct CPV in the two decays is expected to be effectively the same. Thus, as a complementary test of the current \( \Delta A^{hh}_{CP} \) measurement, the precise measurement of \( A_{CP} \) in \( D^+ \rightarrow \bar{K}^0 K^+ \) helps to pin down the origin of \( \Delta A^{hh}_{CP} \) [12]. Figure 4 shows invariant masses of \( D^+ \rightarrow K^0_s K^\pm \) together with the fits that result in \( \sim 277k \) reconstructed decays and the measured \( A_{CP} \) in bins of \( |\cos \theta_{D^{*+}}| \). From the right plot in Fig. 4 we obtain \( A^{D^+\rightarrow K^0_s K^+}_{CP} = (-0.246 \pm 0.275 \pm 0.135)\% \). After subtracting experiment dependent \( A^{\bar{K}^0}_{CP} \) [9], the CPV in charm decay, \( A^{D^+\rightarrow K^0 K^+}_{CP} \), is measured to be \( (+0.082 \pm 0.275 \pm 0.135)\% \). The current average of \( \Delta A^{hh}_{CP} \) measurements as well as the Belle preliminary result of \( A^{KK}_{CP} \) favor a negative value. Our result, on the other hand, does not show this tendency for \( D^+ \rightarrow \bar{K}^0 K^+ \) decays, albeit with a significant statistical uncertainty.

![Figure 4: Left two plots show \( M(K^0_s K^+) \) and \( M(K^0_s K^-) \) distributions, respectively, and right plot shows preliminary result of \( A_{CP} \) as a function of the polar angle of \( D^+ \) momentum at the c.m.s.](image-url)
4 Summary

In summary, we review the current HFAG averages of CPV and mixing parameters in the charm sector and recent relevant measurements from Belle. Belle measurements of mixing and CPV parameters are consistent with current HFAG averages. No direct CPV in charm decays has been observed from Belle to date.

References

[1] Y. Amhis et al. (Heavy Flavor Averaging Group), arXiv:1207.1158v1[hep-ex] and online update at http://www.slac.stanford.edu/xorg/hfag/.

[2] Z.-Z. Xing, Phys. Rev. D 55, 196 (1997); S Bianco, F. L. Fabbri, D. Benson, and I. Bigi, Riv. Nuovo Cim. 26N7, 1 (2003); G. Burdman and I. Shipsey, Ann. Rev. Nucl. Part. Sci. 53, 431 (2003).

[3] V. V. Anisovich and A. V. Sarantsev, Eur. Phys. J. A 16, 229 (2003).

[4] D. Aston et al. (LASS), Nucl. Phys. B 296, 493 (1988).

[5] J. Beringer et al. (Particle Data Group), Phys. Rev. D 86, 010001 (2012).

[6] Y. Grossman and Y. Nir, JHEP 2012, 04 (2012) 002.

[7] B. R. Ko et al. (Belle Collab.), Phys. Rev. Lett. 109, 021601 (2012), Erratum-ibid. 109, 119903 (2012).

[8] F. Buccella et al., Phys. Rev. D 51, 3478 (1995).

[9] Y. Grossman, A. L. Kagan, and Y. Nir, Phys. Rev. D 75, 036008 (2007).

[10] R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 108, 111602 (2012);

[11] T. Aaltonen et al. (CDF Collaboration), Phys. Rev. Lett. 109, 111801 (2012).

[12] B. Bhattacharya, M. Gronau, and J. L. Rosner, Phys. Rev. D 85, 054104 (2012).