Shedding light on quarkonium formation by producing a photon in association

K. Sridhar

Theory Division, CERN, CH-1211, Geneva 23, Switzerland

Abstract

The process $AB \rightarrow \text{quarkonium} + \gamma + X$, for a heavy quarkonium state such as $J/\psi$ or $\Upsilon$, is considered. The ratio of the cross-sections for this process in $pp$ and $p\bar{p}$ collisions is shown to be a very sensitive probe of models of quarkonium formation.

sridhar@vxcern.cern.ch

CERN-TH.6748/92

December 1992
Over the years, there has been considerable interest in the study of heavy quarkonia because the leptonic decay channels of these resonances provide the cleanest signals of heavy quark formation. The production of quarkonia takes place through subprocesses that are dominated by gluons. These processes have, therefore, been used to study gluon distributions in the hadron.

The cross-section for the production of a $Q\bar{Q}$ bound state factorises into two parts: a short-distance part, which corresponds to the production of a heavy quark pair from a hard collision of the incident particles, while the other part specifies how the $Q\bar{Q}$ pair produced in the collision forms a quarkonium bound state. The short-distance part is, of course, computable in perturbative QCD but the formation of the bound state from the $Q\bar{Q}$ pair can be specified only in the context of some phenomenological model of hadronisation. In particular, it is not known whether the hard scattering subprocess is sensitive to the quantum numbers of the bound state, or whether this information is completely screened by the subsequent hadronisation.

For several years now, two models of quarkonium formation have been used to describe $J/\psi$ and $\Upsilon$ production in high energy collisions: the colour-singlet model [1] and the semi-local duality model [2]. The latter model has also been referred to as the colour-evaporation model in the literature. In the colour-singlet model, one starts with the full $Q\bar{Q}$ production amplitude and then projects out the state with the proper spin, parity and charge-conjugation assignments to describe a $^{2S+1}L_J$ quarkonium state. This state is required to be a colour-singlet, which is achieved by the radiation of a hard parton in the final state. The matrix element so obtained is then convoluted with the modulus-squared of the wave-function at the origin, $|R_0|^2$, (or its derivative, in the case of $P$-wave quarkonia), which finally yields the matrix element for the quarkonium production process. The wave-function and its derivative are fixed from the leptonic or hadronic decay widths of the quarkonia. In this model, because the amplitude for the production of a state with definite $J^{PC}$ are computed, it is possible to predict the production cross section for different resonances in a given family of quarkonia. It is also important to note that the hard scattering vertex is governed by selection rules that involve the quantum numbers of the quarkonium. For example in leptoproduction of charmonia, the $^3S_1$ state (the $J/\psi$) is produced in the subprocess $\gamma g \rightarrow ^3S_1 g$, but the production of the corresponding $P$-states (the $\chi$'s) is disallowed.

In the semi-local duality model, the hard scattering vertex is completely blind to the quantum numbers of the quarkonium resonance. The quantum numbers of the $Q\bar{Q}$ pair produced in the hard scattering are left unspecified, and the $Q\bar{Q}$ pair is not required to be a colour singlet. The colour non-singlet $Q\bar{Q}$ state is assumed to form the physical colour-singlet quarkonium state by multiple soft-gluon emission. In practice, one simply computes the open $Q\bar{Q}$ production cross-section and integrates this cross-section between threshold ($= 2m_Q$) and the open $Q\bar{Q}$ production threshold. Arguments based on semi-local duality are then invoked to relate this integral of the open $Q\bar{Q}$ production cross-section to the sum of the resonance production cross-sections. It is clear that the cross-section for producing a particular resonance with a given $J^{PC}$ cannot be predicted in this
model. The cross-section for the production of a specific bound state is a fraction $f$ of the integral of the open-charm production cross-section over the mass range considered. This fraction $f$ is, however, not a prediction of the model but can be obtained by comparison with experiment.

The colour-singlet model has been applied to the study of quarkonium production in both lepton-hadron and hadron-hadron collisions [1, 3]. In leptoproduction, the heavy-quark pair is produced through the fusion of the incident virtual photon with a gluon from the nucleon. The predictions of this model have been compared with the data on $J/\psi$ production at large $p_T$ and small $z$ in lepton-nucleon collisions [3, 4], where $p_T$ is the transverse momentum of the $J/\psi$ and $z = E_{J/\psi}/E_\gamma$; $E_{J/\psi}$ and $E_\gamma$ are the energies of the $J/\psi$ and the incident virtual photon, respectively. The model gives an adequate description of the $p_T$ and rapidity distributions of the $J/\psi$ for large $p_T$ and for $z \leq 0.7 – 0.8$. The overall normalisation predicted by the model turns out to be lower than the data by a factor of 2.3 [4]. Similarly, data on $J/\psi$ and $\Upsilon$ hadroproduction at large $p_T$ also seem to be consistent with the predictions of this model [3], except for the normalisation. Comparisons of the predictions of the semi-local duality model have also been made with data on leptoproduction and hadroproduction of $J/\psi$ [6]. The remarkable thing is that this simplistic model is also capable of explaining the data at a level comparable to the more elaborate colour-singlet model. A recent analysis of $J/\psi$ production in $\mu - p$ scattering [7] suggests that the colour-singlet model is somewhat favoured in the region $z \leq 0.8$ as compared to the semi-local duality model, but the experimental errors are too large for any definite conclusions to be drawn.

In this letter, we suggest that the production of a photon in association with the quarkonium will help to clearly distinguish between these two models of quarkonium formation. The associated production of $J/\psi$ and photon at large $p_T$ has been studied in the context of collider [8], fixed-target [9] and polarised $pp$ scattering experiments [10], in the framework of the colour-singlet model; it has been shown to be a very sensitive probe of gluon distributions. This is so because, in the colour-singlet model, the only subprocess that contributes to $J/\psi + \gamma$ production, with the $J/\psi$ and $\gamma$ produced back-to-back at large $p_T$, is the following:

$$g + g \rightarrow J/\psi + \gamma.$$  (1)

The colour-singlet model predicts that no $P$-states are produced because the subprocess

$$g + g \rightarrow \chi + \gamma$$  (2)

is disallowed because of $C$-invariance. Moreover, the subprocess

$$g + g \rightarrow \chi \rightarrow J/\psi + \gamma$$  (3)

does not produce a photon at large $p_T$ and does not contribute to $J/\psi + \gamma$ production. Finally, the $q\bar{q}$-initiated subprocess

$$q + \bar{q} \rightarrow J/\psi + \gamma$$  (4)
does not contribute, because the $c\bar{c}$ thus produced will not be in a colour-singlet state.

On the other hand, the $q\bar{q}$-initiated subprocess will contribute in the semi-local duality model. This indicates that these models will strongly differ in their predictions for quarkonium+$\gamma$ production. This difference can be particularly enhanced if we study the ratio of the cross-sections for quarkonium+$\gamma$ production in $pp$ and $p\bar{p}$ collisions. In what follows, we study the ratio

$$R \equiv \frac{d\sigma_{pp}/dp_T}{d\sigma_{p\bar{p}}/dp_T}. \quad (5)$$

In the colour-singlet model, the ratio $R$ is clearly equal to unity, because only the gluon-initiated subprocess contributes. Moreover, this will be true for all $p_T$ and will also be independent of the quarkonium resonance considered. Of course, this is all true at the lowest order in QCD: higher-order diagrams in the colour-singlet model will have contributions from both the gluon- and quark-initiated processes. But the ratio $R$, which we propose to study, will be more resistant to the effect of higher-order corrections than the absolute cross-sections themselves. So we expect that $R$ will not deviate significantly from unity in the colour singlet-model, at least at fixed target energies. However, even at the lowest order in perturbation theory, the semi-local duality model allows for both gluon- and quark-initiated processes. Hence, we expect that $R$ will differ from unity in this model.

We present, in the following, the $p_T$ dependence of the ratio $R$ in the semi-local duality model for $J/\psi + \gamma$ production. In this model, we need to integrate the open charm production cross-section, given by the processes

$$g + g \rightarrow c + \bar{c} + \gamma,$$
$$q + \bar{q} \rightarrow c + \bar{c} + \gamma, \quad (6)$$

with the invariant mass of the $c\bar{c}$ pair restricted to lie between $4m_c^2$ and $4m_D^2$, the latter being the threshold for open charm production. The kinematics is identical to that described in Ref. [11]. The four-momenta of the initial partons are taken to be

$$\frac{\sqrt{s}}{2} (1, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta),$$
$$\frac{\sqrt{s}}{2} (1, -\sin \theta \cos \phi, -\sin \theta \sin \phi, -\cos \theta). \quad (7)$$

The $xy$ plane contains the final particles, and the direction of the photon is taken to define the $+ve$ $x$-axis. The four-momenta of the $c$, $\bar{c}$ and the photon are

$$p_c = \frac{\sqrt{s}}{2} x_3(1, \beta_3 \cos \theta_3, \beta_3 \sin \theta_3, 0),$$
$$p_{\bar{c}} = \frac{\sqrt{s}}{2} x_4(1, \beta_4 \cos \theta_4, \beta_4 \sin \theta_4, 0),$$
$$p_\gamma = \frac{\sqrt{s}}{2} x_5(1, 1, 0, 0), \quad (8)$$
\[ \beta_i = \sqrt{1 - 4m_c^2/x_i^2 s}. \]  

(9)

In the above equations, \( s \) is the subprocess centre-of-mass energy and is related to the hadronic centre-of-mass energy \( S \) by the usual relation, \( s = x_1 x_2 S \), where \( x_1 \) and \( x_2 \) are the momentum fractions of the incident hadrons carried by the partons. From energy conservation we obtain,

\[ x_3 + x_4 + x_5 = 2 \quad (x_i \leq 1, \ i = 3, 4, 5). \]

(10)

The angles \( \theta_{35} \) and \( \theta_{45} \) are given by

\[
\cos \theta_{35} = \frac{(\beta_3 x_3)^2 - (\beta_4 x_4)^2 - x_5^2}{2\beta_3 x_3 x_5}, \\
\cos \theta_{45} = \frac{(\beta_3 x_3)^2 - (\beta_4 x_4)^2 - x_5^2}{2\beta_4 x_4 x_5}. 
\]

(11)

The \( p_T \) of the photon (which is the same as that of the \( J/\psi \)) is given by

\[ p_T = \frac{\sqrt{s}}{2} x_5 \sqrt{\cos^2 \theta + \sin^2 \theta \sin^2 \phi}. \]

(12)

The differential hadronic cross-section is then given by

\[
\frac{d\sigma}{dx_1 dx_2 dx_3 dx_4 d\phi dp_T} = \frac{\alpha^2 \alpha_s^2 p_T}{4\pi s x_5^2 \cos \theta \cos^2 \phi} H(x_1, x_2)|M_{q\bar{q}}|^2 + G(x_1, x_2)|M_{gg}|^2, 
\]

(13)

where \( H(x_1, x_2) \) and \( G(x_1, x_2) \) are parton distribution factors for the \( q\bar{q} \) and the \( gg \) subprocesses, respectively. \( |M_{q\bar{q}}|^2 \) and \( |M_{gg}|^2 \) are the squared matrix elements for the subprocesses given in (6). These matrix elements are obtained from those for heavy quark lepton production by crossing. The matrix elements for the latter process are given in Ref. [12].

Integrating over all variables other than \( p_T \) in Eq. (13), we obtained the cross-sections for \( pp \) and \( p\bar{p} \) collisions as a function of \( p_T \). The integration was done by restricting the invariant mass of the \( c\bar{c} \) pair as described above, and also restricting the rapidities of the \( J/\psi \) and the photon to be both equal to zero. The computations are done for fixed target energies at Fermilab (\( \sqrt{s} \approx 38.75 \text{GeV} \)). For the parton densities Owens’ Set 1.1 distributions [13] were used, and the \( Q^2 \) for the process was taken to be \( 4m_c^2 + p_T^2 \).

The ratio \( R \) of the cross-sections computed in this model (curves labelled II) for \( J/\psi \) production is shown in Fig. 1, as a function of \( p_T \). The ratio is significantly smaller than unity and also shows a strong \( p_T \)-dependence. With increasing \( p_T \), larger values of \( x_1 \) and \( x_2 \) are sampled and the \( q\bar{q} \) process becomes more important. Consequently, there is a larger suppression of the ratio \( R \). We have also computed \( R \) for the case of \( \Upsilon \) production, and find that it also depends on the mass of the quarkonium. In the case of \( \Upsilon \), we find that \( R \) is smaller than that obtained for \( J/\psi \). In Fig. 1, we have also shown the curve (labelled...
I) for $R$ expected in the colour-singlet model. The ratio is simply unity, independent of $p_T$ and the mass of the quarkonium.

The experimental feasibility of measuring this process at fixed-target energies was dealt with in detail in Ref. [9]. We would like to simply point out here that one could get a enhancement by a factor of 100 if a nuclear target were used and the cross-sections from $pA$ and $\bar{p}A$ were compared. Nuclear effects would cancel in the ratio.

In conclusion, we find that the production of a photon in association with a quarkonium such as $J/\psi$ or $\Upsilon$, will provide important insights into the nature of quarkonium formation. In particular, by studying the ratio of the $pp$ and $p\bar{p}$ induced cross-sections at fixed-target energies it will be possible to discriminate between the colour-singlet and the semi-local duality models of quarkonium formation. Just as this work was being completed, we received a preprint [14] where leptoproduction of $J/\psi + \gamma$ is discussed as a probe of models of quarkonium formation.
References

[1] M.B. Einhorn and S.D. Ellis, Phys. Rev. D 12 (1975) 2007; C.E. Carlson and R. Suaya, Phys. Rev. D 14 (1976) 3115; E.L. Berger and D. Jones, Phys. Rev. D 23 (1981) 1521; R. Baier and R. Rückl, Z. Phys. C 19 (1983) 251.

[2] H. Fritzsch, Phys. Lett. B 67 (1977) 217; M. Glück and E. Reya, Phys. Lett. B 79 (1978) 453; M. Glück, J.F. Owens and E. Reya, Phys. Rev. D 17 (1978) 2324.

[3] R. Baier and R. Rückl, Nucl. Phys. B 218 (1983) 289.

[4] D. Allasia et al., Phys. Lett. B 258 (1991) 493; P. Amaudruz et al., Nucl. Phys. B 371 (1992) 553.

[5] E.W.N. Glover, A.D. Martin and W.J. Stirling, Z. Phys C 38 (1988) 473.

[6] A.D. Martin, R.G. Roberts and W.J. Stirling, Phys. Rev. D 37 (1988) 1161.

[7] J. Ashman et al., Preprint CERN-PPE/92-102.

[8] M. Drees and C.S. Kim, Z. Phys C 53 (1992) 673.

[9] R.V. Gavai, R.M. Godbole and Sridhar K., Preprint CERN-TH.6679/92 (to appear in Phys. Lett. B).

[10] Sridhar K., Phys. Lett. B 289 (1992) 435.

[11] R.V. Gavai, S. Gupta and Sridhar K., Phys. Lett. 227 (1989) 161.

[12] R.K. Ellis and Z. Kunszt, Nucl. Phys. B 303 (1988) 653.

[13] J.F. Owens, Phys. Lett. B 266 (1991) 126.

[14] C.S. Kim and E. Reya, Dortmund Preprint DO-TH 92/20.
Figure caption

Fig. 1 The ratio $R$ as a function of $p_T$. I and II denote colour-singlet and semi-local duality models, respectively. The quarkonium states to which the curves correspond are shown in parenthesis.