Balance of Datum Land Prices Among Cities Based on the City Gravitation Model and Stochastic Diffusion Equation

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Abstract  A balance of urban datum land prices is achieved to harmonize regional land prices and make the prices truly reflect different economic development levels and land prices among cities. The current piecewise linear interpolation balance method widely used has two drawbacks that always lead to an unsatisfactory balance among some cities. When the excess of land price in the central city to the surrounding zone reaches a certain degree, land price in the circumjacent city is not only consistent with the local land grade and land use level, but also influenced by the diffusion of land price in the central city. Thus, a new balanced scheme of datum land prices based on the city gravitation model and stochastic diffusion equation is brought forward. Finally, the new method is examined in the practice of datum land price balance in Hubei Province, China.

Keywords  datum land price balance; city gravitation model; stochastic diffusion equation

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Introduction

Achieving a balance of urban datum land prices started late in China. Even now there is no uniform technical regulation, although work has begun in Shandong, Fujian, Jiangsu and Hubei Province. Under the practice, a mature technical flow forms on the principle of “controlling land price by its grade and examining land grade with its price”. The piecewise linear interpolation balance method, which is widely used for the moment[1], has two drawbacks in balancing prices. First, the method uses crude results of urban land gradation to regulate single datum prices (commerce, housing, and industry), and the mismatch between the two types of data precision will lead to unsatisfactory results in some cities. Second, this method does not consider regulating datum land prices in terms of regional spatial structure of development and the economic capacity of concentration and diffusion in central cities. Thus, it cannot regulate prices properly. To overcome the problems above, the authors propose a new method based on the city gravitation model and stochastic diffusion equation.

The imbalance increase theory from A.O. Hirschman, and diffusion theory from T.Hagerstrand express the point that all forces among cities in the region can be generalized to two types: centralization and diffusion. Lewis Mannford, a famous city-theorist in America, and British scholar Ebenezer Howard, further explained the functionality of centralization and diffusion among cities using the theory of gravitation. The effect of diffusion not only has an impact on capital, goods, labor, technology and information, but
also influences urban land prices. Urban land differential income is influenced by both inner and outer factors because factors such as natural conditions, resources, economic infrastructure and labor take effect among cities and lead to obvious differences in development conditions and land productivity. When the excess of land price in the central city to the surrounding zone achieves a certain degree, land price of the circumjacent city is not only consistent with the local land grade and land use level, but also influenced by the diffusion of land price in the central city. Thus, a new balance scheme of datum land prices will be brought forward in this study. Central cities will be selected according to regional spatial pattern of development and city quality assessment. Various single datum prices will be balanced first, then a land price diffusion model of the central city will be developed. Finally, the confirmed model will be used to balance datum land prices in other cities.

1 Appraisal of city quality

Quality is the basic concept in physics. However, there is no exact “city quality” in the research of cities. City quality has a meaning on two levels on the basis of studies by Junsheng Liu, Pengfei Ni and Jianjun Mao[2]. The core level is the city’s development degree, while the regional level includes the city’s economic capacity of concentration and diffusion. This study uses the urban land gradation method to appraise city quality by establishing general impact indicators of urban socio-economic development.

1.1 Primary appraisal of city quality

The city quality assessment is the bellwether of other works, which proceeds by means of urban land quality gradation in this study. According to the gradation regulation of our country, we first adopt the Karhunen-Loeve method to set up the factor system. Then, a crude gradation result will be acquired by the multifactor comprehensive assessment method[3].

1.2 Examining the primary result based on clustering analysis and central place theory

We introduce the modified K-means clustering method and central place theory to validate and revise the original result. When using multi-dimensional spatial clustering, we should consider both the proximity of locations and the similarity of attributes[4]. \((X_i, Y_i)\) represents a point’s location and \(Z_i = (Z_{i1}, Z_{i2}, ..., Z_{im})\) represents serial attributes of the point. Thus, the generalized Euclidean distance, represented with \(D_{ij}\), has the following definition.

\[
D_{ij} = W_a \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2} + \\
W_p \sqrt{\sum_{k=1}^{m} W_k (Z_{ik} - Z_{jk})^2}
\]

where \(W_a\) and \(W_p\) weigh the distance of attributes and geometry separately.

In the first place, data need dimensionless processing to avoid influences from different units of indicators. Although K-means clustering analysis, which is a widely used partition algorithm, has relatively small time and space complexity, it requires careful initial partition selection. If the initial partition is selected improperly, it will easily cause local convergence. The study uses K-means clustering for urban land gradation based on the generalized Euclidean distance of the second meaning. With reference from clustering analysis results and market-surveying materials, a satisfying result for city quality is achieved by adjusting primary grades.

Walter Kelishidale, a famous German geographer, established the central place theory which covered towns, their functions, sizes and spatial structure composition in a region. He used the hexagon to figuratively generalize the relationship of a city’s grade and scale within a region and put forward the idea that city grade was subject to principles of market, transportation and administration. Urban land grade reflects a city’s general socio-economic strength, and is a proper reflection of the city’s scale. If the spatial distribution of gradation results satisfies the regional development “point-axis” mode and three principles of central place theory, the gradation results then prove reasonable and correct.

2 Balance of datum land price among central cities

2.1 Selecting central cities

The following principles are observed to select
central cities.
   1) Select the city that has high quality, especially that with the higher grade.
   2) Select the city at the regional development point-axis location of the network hub to control the whole region.
   3) The percentage of central cities in the region should not be less than 15%.

2.2 Balance datum land price of central cities

In Shaoxiang Ni and Lingxia Wang’s research, they suggested that urban datum land prices can be divided into two levels\(^5\). One is single datum land prices of commerce, inhabitation and industry; the other is general datum land price generalized from single types. General datum land price is calculated by using the marginal analysis method on the principles of maximal gains and best use. According to the Alonzo rent theory, the rational urban development pattern is that general land price of central area references commercial land price, subsidiary area land price references residential price and other area land price references industrial price. See Reference\(^6\) on how to calculate general datum land price from single ones.

To derive the defect of the piecewise linear interpolation method, the study adopts the concept of general datum land price to agree with city grade. We control general datum land prices by gradation results and then calculate the balanced results of single datum prices based on the relationship of general datum price and single prices.

We adopt the modified piecewise linear interpolation method to calculate the balanced general datum price of the central city, then acquire the balanced results of single ones. Finally, they are adjusted with reference to land price monitoring data and local experts’ opinions.

3 Confirming the affective range of the central city

3.1 City gravitation model

The study uses concepts of physics to refer to the city’s influential area as a “magnetic field” and its impact as “gravitation strength”. According to the Raleigh formula, the city gravitation model is expressed such that within the central city’s affective range, gravitation between two cities is proportional to their quality and is inversely proportional to the square of the distance between them. Therefore, city quality and distance between cities are two important parameters in the model.

\[
F = kmM / r^2
\]

where \(F\) stands for gravitation between two cities; \(m\) stands for quality of the city at a lower level; \(M\) stands for the quality of central city; \(k\) stands for gravitational constant; and \(r\) stands for distance between the two cities.

3.2 Confirming the central city’s affective range

In Kewei Liu’ research, a city’s influential area has three kinds of features. ① Hierarchy and nesting. Cities of different levels have different influential areas. The higher the level is, the bigger the influential area. Influential areas of cities that have fewer levels constitute their superior’s. ② Staggering and transition. A city’s influence is inversely proportional to distance. As a result of the Administrative Division and urban function, there is no balanced boundary between two cities; instead, only a gradual transitional belt exists. ③ Competition and variability. If the scale of one of the two cities increases and its influential area extends, the other’s influential area then shrinks\(^7\).

Consequently, ring structure is used in this study to demonstrate a central city’s influential area. There exist a “strong field” and a “weak field”. We think cities in a “strong field” are mainly influenced by radiation of the central city that plays a core role. However, cities in a “weak field” may be influenced by radiation of the surrounding multiple centers. Therefore, a city’s “field strength” is a nested result of multiple magnetic fields.

Furthermore, we regard the range of “weak field” as the borderline of two central places of the same scale, which can be calculated by the broken-point formula.

\[
D_r = D_{rp} / (1 + \sqrt[3]{M_p / M_r})
\]
where $D_r$ represents the standard distance between broken point and central city $r$; $D_{rp}$ represents the standard distance between two central cities; and $M_r$ or $M_p$ represents each central city quality.

Theoretically, the “weak field” is an anomaly region with several boundaries of voronoi polygon. In practice, we can use the following formula to calculate its range, which is based on the option that the whole area can be approximately composed of several circle ranges.

$$R = \sqrt{S/N\pi}$$

where $R$ represents city affective range of the current grade; $S$ represents the area of the whole range; and $N$ represents the number of cities with current or higher grade.

The range of “strong field” can also be described by the standard distance from the central city to the surrounding place which lies in its main radiation area and have the farthest distance from the center.

4 Setting up datum price diffusion model of the central city

4.1 Spatial diffusion of central city’s land price in the region

Generally, factors affecting urban land prices are nonhomogeneous, so we should adopt the spatial diffusion equation in the nonhomogeneous field to open out the diffusion law of urban land price. The stochastic movement equation of spatial diffusion in the non-homogeneous field raised by Weidong Shan and Haosheng Bao have a better effect on the study of spatial diffusion of urban datum price\cite{8}. The equation is as follows:

$$D(x,m) = D_{\text{max}} \cdot \text{erf}(x/\sqrt{4Z(m)})m = f_{\text{max}} - f$$

where $D(x,m)$ is the value of diffusion; $D_{\text{max}}$ is the max value of diffusion, namely the max value of a single datum price in the central city; $\text{erf}(x)$ is the Gaussian Surplus function; $Z(m)$ is the diffusion coefficient function; $x$ is the standard distance to the central city; $f_{\text{max}}$ is the quality of the central city; and $f$ is the quality of a circumjacent city.

In this equation, there are two key sub functions. One is Gaussian surplus function $\text{erf}(x)$ and the other is diffusion coefficient function $Z(m)$. $\text{erf}(x)$ and normal school function $\Phi(x)$ has the relation below.

$$\text{erf}(x) = 2 - 2\Phi(\sqrt{2}x)$$

Therefore, the value of $\text{erf}(x)$ can be achieved by $\Phi(x)$ . Of several approximate formula for calculating standard normal school function\cite{5} this study uses the following:

$$\Phi(x) = 1 - \frac{1}{2} \left( 1 + \sum_{i=1}^{4} a_i x^i \right)^{-4}$$

where $a_1 = 0.196854$, $a_2 = 0.115194$, $a_3 = 0.000344$; and $a_4 = 0.019527$.

There is no uniform function for $Z(m)$ until now. We advance its specific form according to the principle of simple to complex. We first suppose a certain form for $Z(m)$. We then examine its rationality by considering the trend among $m$, $x$ and $(x,m)$, and calculating the current formula’s residuals using present data on distance, gradation scores and land prices. Finally, we choose the best form which leads to the least residuals. According to Weidong Shan’s and Haosheng Bao’s researches\cite{8}, the function $Z(m)$ has several ordinary formats, for example:

$$Z(m) = a m^b, \quad Z(m) = a e^{bm}, \quad Z(m) = a + bm$$

In summary, confirming the diffusion model follows several steps: first, confirm $Z(m)$’s function form; then demonstrate Gaussian surplus function $\text{erf}(x)$ approximately with normal school function $\Phi(x)$; finally, confirm the equation’s parameters using the nonlinear regression method.

4.2 Balance datum land price of other cities around the central one

After balancing the central city’s datum land price, and confirming its affective range and the land price diffusion model, it is now easy for us to balance datum land prices of other cities within the gravitation field. For those within a “strong field”, each city’s balanced datum land price is the attenuating value from that of the central city. For those within a “weak field”, each city’s balanced results are the sum of values with different weights, which are the “gravitation strength” of surrounding central cities.

In Fig.1, Gong’an is within the “strong field” of Jingzhou, so we suppose its land price to be influenced only by Jingzhou; however, Zhijiang is within
the “weak field” of Yichang, so we suppose its land price to be influenced by both Jingzhou and Yichang. If \( D_{\text{max1}} \) represents balanced datum land price of Jingzhou, \( M_1 \) is its quality, \( r_1 \) is the distance between Jingzhou and Gong’an, \( m_1 \) represents quality of Gong’an, and \( D_1 \) is its balanced datum land price; \( D_{\text{max2}} \) represents balanced datum land price of Yichang, \( M_2 \) is its quality, \( r_2 \) is the distance between Yichang and Zhijiang, \( m_2 \) represents quality of Zhijiang, \( D_2 \) is its balanced datum land price; \( E_1 \) represents Jingzhou’s influential “gravitation strength” to Zhijiang, \( P_1 \) is the diffusion value of its land price; \( E_2 \) represents Yichang’s influential “gravitation strength” to Zhijiang, and \( P_2 \) is the diffusion value of its land price. The formulas for calculating \( D_1 \) and \( D_2 \) are as follows:

\[
D_1 = D_{\text{max1}} \cdot \text{erf}(r / \sqrt{4Z_1(M_1 - m_1)})
\]

\[
Z_1(x) \text{ is the diffusion coefficient function of Jingzhou.}
\]

\[
P_1 = D_{\text{max1}} \cdot \text{erf}(r_1 / \sqrt{4Z_1(M_1 - m_1)})
\]

\[
E_1 = M_1 / r_1^2
\]

\[
E_2 = M_2 / r_2^2
\]

\[
P_2 = D_{\text{max2}} \cdot \text{erf}(r_2 / \sqrt{4Z_2(M_2 - m_2)})
\]

\[
Z_2(x) \text{ is the diffusion coefficient function of Yichang.}
\]

\[
D_2 = (P_1 \cdot E_1 + P_2 \cdot E_2) / (E_1 + E_2)
\]

There are alps in the west and hills in the north. Ji- anghan plain lies in the southern part, while hills and mountainous region are in the east.

The provincial administration is composed of 84 counties and executive organizations, mainly positioned along the Changjiang River, Hanjiang, the main railways and highways. According to the terrain and district developing pattern, the whole province can be divided into four regions: north western mountainous area, south western mountainous area, central and eastern parts. In the north western and south western areas, traffic is inconvenient, economy is inferior and cities with small scale have a very wide distribution. The central part has a complete traffic network, strong agriculture, and moderate city sizes. Moreover, cities along the waterways and traffic lines have a uniformity of distribution. Some big cities, for instance, Jingzhou, Yichang, Jingmen and Xiangfan, belong to the center. In the east, Wuhan City, a megapolis, plays a core part in the area, around which a great deal of cities are compactly distributed. It has advantages of convenient traffic and superior economy in the area, and the scale of cities is also bigger. Besides Wuhan, there are other important industrial cities such as Huangshi and Ezhou in the east.

5.2 City quality calculation

5.2.1 Evaluate city grades preliminarily

We adopt the system of 3-level-factors in urban land gradation of Hubei Province. Referring to the national regulation of urban land grading and correlative results, we choose 9 Level-1 factors for urban land grading as shown in Table 1. The location condition factor contains economical location condition and traffic condition. The former represents the intensity of economic communication between a city and surrounding ones, while the latter reflects the convenience status of a city’s external communications. Based on factors that include acreage of build-up, civil population scale, civil population density plus proportions of the second- and third-leading industries in terms of GDP contribution, the agglomeration scale factor represents the agglomeration situation of a city. The regional economic development level factor covers several aspects such as the economic level,
financial status, investment of permanent assets and foreign trading situation. The ecological environment factor includes two parts: individual green land area and waste disposal. The infrastructure factor is a reflection of status of civil traffic, gas supply, water supply, drainage condition and heat supply. The utilities factor reflects the management of civil schools, sanitation, research and amusement establishments. Lastly, integrative service capabilities such as finance, information, and technology might be mirrored by the regional service capacity factor. We then adopt a traditional method to get factor powers and calculate the general score for each city by the multifactor comprehensive assessment method. Finally, we evaluate city grades preliminarily using a histogram method.

### Table 1 Level-1 gradation factors of Hubei Province

| Level-1 factor’s name | Weight | Level-1 factor’s name | Weight |
|-----------------------|--------|-----------------------|--------|
| Location condition    | 0.1947 | Infrastructure        | 0.1023 |
| Agglomeration scale   | 0.1447 | Input-output level of urban land | 0.1047 |
| Utilities             | 0.1033 | Regional economic development level | 0.1208 |
| Ecological Environment| 0.1127 | Regional land-provision potential | 0.0459 |
| Regional service capacity | 0.0710 | |

#### 5.2.2 Clustering analysis of gradation results

To increase convergence speed of the algorithm and to avoid local convergence, we make the following settings referencing primary gradation results. The city of Wuhan with obvious advantage is set as Grade-1. Referencing primary gradation excluding Wuhan, we set the number of clusters to 8. According to primary gradation results, we use the geometric center of each grade as the initial center of the cluster. After repeated attempts, we set generalized parameters of Euclidean distance $W_p$ and $W_a$ with 0.2 and 0.8 and then set the largest offset threshold 0.02. Fig.2 shows that results of clustering method agree with primary grades in general.

#### 5.2.3 Central plot theory to examine gradation results

Using clustering analysis results and market-surveying materials, a satisfying result of city quality is achieved by adjusting primary grades, and the final urban land grade map is shown in Table 2.

### Table 2 Urban land gradation results of Hubei Province

| Grade | Number | City name |
|-------|--------|-----------|
| 1     | 1      | Wuhan     |
| 2     | 7      | Huangshi, Yichang, Xiangfan, Jingzhou, Shiyian, E’zhou, Jingmen |
| 3     | 5      | Suizhou, Xiaogan, Xiantao, Qianjiang, Tianmen |
|       |        | Xiangyang, Zaoyang, Macheng, Zhongxiang, Yiling, Huangpi, Yunneng, Dangyang, Yingcheng, |
|       |        | Xinzhou, Hanchuan, Dongxiu, Caidian, Enshi, Huangzhou, Yido, Jiangxia, Daye, Gong’an, |
|       |        | Wuxue, Xianning, Chibi, Shishou, Tongcheng |
| 4     | 24     | Danjiangkou, Laohekou, Guangshui, Nanzhang, Yicheng, Anlu, Jingshan, Shayang, Xishui, |
| 5     | 19     | Qichun, Zhijiang, Hannon, Songzhi, Huangmei, Jiayu, Jiangling, Honghu, Jianli, Yangxin |
| 6     | 11     | Yuxian, Yuan’an, Gucheng, Baokang, Xiaochang, Dawu, Tuanfeng, Hong’an, Zhengui, Tongshan, |
|       |        | Chongyang |
| 7     | 6      | Fangxian, Xingshan, Changyang, Luotian, Yingshan, Lichuan |
| 8     | 9      | Yuansi, Zhushan, Zhuxi, Jiashi, Badong, Yi’en, Xianfeng, Laifeng, Hefeng |
| 9     | 2      | Wufeng, Shengnongjia |
be Level-1 central plot, cities of Grades 2 and 3 are supposed to be Level-2 central plots, cities of Grades 4 and 5 are supposed to be Level-3, cities of Grades 6 and 7 are supposed to be Level-4 and the others are Level-5 central plots. According to regional development and terrain features in Hubei Province, we examine the gradation results through 4 regions separately: the northwest mountain area, southwest mountain area, middle “point to axis” main frame district (north area and Jianghan Plain) and eastern industrial clusters (Wuhan and its surroundings). We come to the following conclusions from figures below.

1) In the northwest and southwest of Hubei, the spatial distribution of cities’ grades meets the administrative principle. The region, with few development axes, is in contact with outsiders mainly by highways and railways. Around the higher center, there are 6 or 7 sub centers that meet the quantity relation of $K=6$ and are in good line with the administrative principle.

2) In the middle “point to axis” main frame district, principles of traffic and market both play a dominant role. The region has three big scale cities (Yichang, Jingzhou and Xiangfan), five middle scale cities and several good axes such as the Changjiang River, Hanjiang River, Jingguang Railway and Jiaoliu Railway. Spatial distribution of city grades meets traffic principle mostly near axes and market principle mostly in the macro region.

3) In eastern industrial clusters, market principle plays a dominant role. The region, which accounts for 40% of the Hubei economy, has its center in Wuhan, with its high density of river networks, roads and railways. The spatial distribution of city grades is similar to a formal hexagon.

5.3 Balance datum land price of central cities

In practice we select 10 central cities according to regional development pattern and urban land gradation results as showed in Fig.3 and Table 2. Using a modified piecewise linear interpolation method, we balance their general datum land prices, and the function curve is displayed in Fig.4. Then we balance their single datum land prices, and the result is shown in Table 3.

5.4 Confirming the central city’s affective range

We use the approximate formula to calculate the “strong field” and “weak field” of each central city, and the result is shown in Table 4.

5.5 Confirming the spatial diffusion equation of each central city

The land price diffusion model differs within the gravitational field of each central city because of diverse distances ($x$) and city quality margins ($m$). Take the strong gravitational field of Xiangfan for example. We calculate the function form of $Z(m)$ using the method described in Reference [8] on the basis of data on city quality, distances and standardized datum land prices. We first confirm $Z(m) = ae^{bm}$ (commerce) and $Z(m) = a + bm$ (housing or industry), then analyze pre-balancing data (Table 5 and Table 6) using Marquardt regression to set parameters.
Table 5  Data of city quality and distances in the “strong field” of Xiangfan/ km

| City         | Xiangfan | Zaoyang | Laohekou | Nanzhang | Yicheng | Gucheng |
|--------------|----------|---------|----------|----------|---------|---------|
| Integrated value | 89.25    | 59.51   | 58.54    | 53.45    | 57.08   | 45.52   |
| Distance     | 0        | 36.4    | 35.3     | 28.7     | 28.4    | 36.5    |

Table 6  Data of city quality and pre-balanced single datum price in the “strong field” of Xiangfan /Yuan • m⁻²

| City         | Xiangfan | Zaoyang | Laohekou | Nanzhang | Yicheng | Gucheng |
|--------------|----------|---------|----------|----------|---------|---------|
| Integrated value | 89.25    | 59.51   | 58.54    | 53.45    | 57.08   | 45.52   |
| Max of pre-balanced single datum prices | Commerce | 1320    | 872      | 515      | 763     | 600     |
|              | Housing  | 977     | 350      | 346      | 349     | 350     |
|              | Industry | 571     | 327      | 273      | 216     | 234     |

6  Conclusions

The traditional balancing method of datum land price focuses mainly on the sequence relationship between integrated gradation value and standardized datum land prices, excluding the regional economic development pattern and central city’s capacity of concentration and diffusion. In this study, the balancing method we proposed is shown to have a good effect on reflecting the essence of why regional datum land prices differ in terms of economic geography and the central city land price diffusion model. The method has been examined in the balance of datum land prices of Hubei Province. We will put emphasis on how to confirm the land price diffusion coefficient function more effectively in the future.

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