The Similarity Renormalization Group for Three-Body Interactions in One Dimension

O. Åkerlund, E. J. Lindgren, J. Bergsten, B. Grevholm, P. Lerner, R. Linscott, C. Forssén, L. Platter
Chalmers University of Technology, Department of Fundamental Physics, SE–412 96 Gothenburg, Sweden

December 10, 2012

Abstract. We report on recent progress of the implementation of the similarity renormalization group (SRG) for three-body interactions in a one-dimensional, bosonic model system using the plane wave basis. We discuss our implementation of the flow equations and show results that confirm that results in the three-body sector remain unchanged by the transformation of the Hamiltonian. We also show how the SRG transformation decouples low- from high-momentum nodes in the three-body sector and therefore simplifies the numerical calculation of observables.

PACS. 21.45.-v – 21.30.-x – 21.10.Dr

1 Introduction

Renormalization group methods have become an important tool in modern physics [1]. In particular, for studies of strongly-interacting many-body systems they frequently facilitate the correct calculation of observables. One of these methods is the similarity renormalization group (SRG), which has been extensively used in condensed matter physics [2] but has recently become of importance also in nuclear physics [3,4]. The SRG essentially constitutes a flow equation generating unitarily equivalent Hamiltonians, which after evolution possess certain features that usually make the calculation of observables easier. The form of the evolved potential will depend on the so-called generator that is an essential ingredient of the flow equation. Frequently used generators drive the two-body potential to the diagonal in momentum space. Off-diagonal elements are thus driven to zero and low momenta effectively decouple from large momenta.

Such capabilities are very important in ab initio nuclear structure physics for which high-performance computing has become of increasing importance and the available computational resources set a hard limit on the number of observables that can be calculated [5]. Recently, the SRG flow equations for three-body interactions were implemented for the truncated harmonic oscillator basis and first nuclear structure calculations with consistently evolved two- and three-body interactions have emerged [6–8]. The SRG therefore seems to provide a way to extend the limits of possible computations. In these calculations the interactions were evolved in the harmonic oscillator basis. However, a calculation of the evolved three-body interaction in the plane wave basis would not only facilitate the projection of the potential on any basis but also the calculation of infinite matter observables such as the energy per particle of nuclear matter. In Refs. [9, 10] it was shown that an evolved two-body interaction in combination with a phenomenological three-body interaction seems to lead to a converging many-body perturbation theory series for infinite matter. However, a missing link in this calculation remains the inclusion of a consistently evolved three-body interaction. Instead, parameters of the leading chiral effective field theory three-nucleon force were refitted and used alongside the evolved two-body potential. Since these positive results for light nuclei and nuclear matter validate also an effort to construct an ab initio density functional based on many-body perturbation theory with evolved interactions [11], the SRG seems therefore to have impact on calculations across the whole chart of nuclides.

In this work we consider the evolution of a one-dimensional Hamiltonian with two- and three-body interaction terms. In Sec. 2 we will introduce the flow equations that determine the evolution of the Hamiltonian. In the following two sections we will then discuss how the flow equations are implemented for two- and three-body interactions in bosonic systems. The presentation of numerical results will focus on illustrating key properties of evolved interactions, i.e. the conservation of observables and (for our choice of generator) the effective decoupling of low and high momenta. In the last section we summarize our results and discuss necessary steps towards an extension of this work to three dimensions and to nuclear systems.

2 The Similarity Renormalization Group

Let us denote with $H_s$ the transformed (and initially unknown) Hamiltonian where $s$ denotes the so-called flow
parameter and provides a measure of how much the Hamiltonian has been transformed. Then, there exists a unitary transformation $U_s$ such that

$$H_s = U_s H U_s^\dagger,$$  

where $H = H_{s=0}$ is the known, original Hamiltonian. We can now calculate the derivative of the above equation with respect to $s$

$$\frac{dH_s}{ds} = \frac{dU_s}{ds} H U_s H U_s^\dagger + U_s H \frac{dU_s^\dagger}{ds}.$$  

Since the transformation $U_s$ is unitary we have $U_s U_s^\dagger = 1$ and therefore

$$\frac{dU_s}{ds} U_s^\dagger = -U_s \frac{dU_s^\dagger}{ds} = \eta_s.$$  

Using this in Eq. (2) gives

$$\frac{dH_s}{ds} = \eta_s H_s - H_s \eta_s = [\eta_s, H_s].$$  

We can then specify the unitary transformation by specifying $\eta_s$, which is subject to the condition

$$\eta_s^\dagger = -\eta_s,$$  

which follows from Eq. (3).

A convenient choice of $\eta_s$ is $\eta_s = [G_s, H_s]$ where $G_s$ is a Hermitian operator. It obeys Eq. (5) since $H_s$ is also Hermitian:

$$[G_s, H_s]^\dagger = -[G_s, H_s].$$  

We will store all dependence on the flow parameter $s$ in the potential term of the Hamiltonian, thus writing $H_s = T_{\text{rel}} + V_s$, where $T_{\text{rel}}$ is the relative kinetic energy operator. Expanding the commutators then gives the equation

$$\frac{dH_s}{ds} = \frac{dV_s}{ds} = G_s H_s H_s + H_s H_s G_s - 2H_s G_s H_s.$$  

There is significant freedom in the choice of the generator $G_s$. In this work we have chosen $G_s = T_{\text{rel}}$. This generator is used since it is known to drive the Hamiltonian to the diagonal, which is usually desirable. The reason for this feature is that $T_{\text{rel}}$ is, in itself, diagonal in momentum space.

### 3 Momentum Space Equations

We will define the interactions with respect to the usual three-body Jacobi momenta

$$p = \frac{1}{2}(k_1 - k_2),$$  
$$q = \frac{2}{3}(k_3 - \frac{1}{2}(k_1 + k_2)).$$  

We will be working in a partial-wave projected basis. This needs to be clarified since in one dimension plane waves can only propagate in two directions; backwards and forward. In one dimension, there exist therefore only two partial waves $l = 0$ and $l = 1$, which correspond also to the parity of the state. We will be interested in bosons in this work and therefore assume that the two-body system is symmetric under exchange of the particles. Thus, we will only work with $l = 0$ basis states. In the two-body sector, a complete set of states will therefore be written as

$$1 = \int_0^\infty dp \left| p \right| \left< l = 0 \right| \left< p \right| 0 \right> = \int_0^\infty dp |p| \left< p \right| 0 \right>.$$  

In the three-body sector we will concentrate on states with total angular momentum $L = 0$, which implies that the relative angular momentum associated with the $q$ variable is $\lambda = 0$. The complete set of states in the three-body sector is therefore

$$1 = \int_0^\infty dp \int_0^\infty dq \left| pq \right| \left< 000 \right| \left< pq \right| 000 \right>.$$  

where the parentheses denote that two angular momenta have been coupled to total angular momentum $L$. From now on we will drop all angular momentum information in the bras and kets since we have only one angular momentum state in the two- and three-body system, respectively.

Another quantity useful to define is the hypermomentum $\zeta$, defined by

$$\zeta^2 = p^2 + \frac{3}{4} q^2.$$  

Since this quantity is proportional to the total, relative kinetic energy of the three-body system it also defines a plane in which the three-body interaction will become diagonal through evolution.

#### 3.1 Interactions

**Two-Body Potentials:** We have used different two-body potentials in this work to analyze the features of the SRG evolution. In particular, we have used a separable potential that has the advantage that the binding energy and the two-body t-matrix can be calculated analytically

$$V_{\text{sep}}(p, p') = g \exp(-\frac{p^2}{\Lambda^2}) \exp(-\frac{p'^2}{\Lambda^2}).$$  

As the regulator $\Lambda$ is increased towards infinity the potential takes the form of a delta-function in coordinate space. In this case the binding energy of the $N$-boson state is known analytically and provides an excellent test for our few-body code. For additional benchmarking of numerical results we have also employed a potential that was previously used in Ref. [12]

$$V(p, p') = \sum_{i=1,2} \frac{V_i}{2\pi} \exp\left(-\frac{(p-p')^2}{4\sigma_i^2}\right).$$  

with parameters given in Table 1. It is important to note that we employ the *partial-wave* projected versions of the above potentials. The parameters in Table 1 show that we will use a purely attractive interaction ($V_3$) and an attractive interaction with *short-range* repulsion ($V_4$).
The Three-Body Potential: The SRG evolution will generally induce many-body forces, however, it is expected that the three-body force will dominate over higher many-body forces as long as the flow parameter \( s \) is not too large. To complete our analysis, we also added a three-body force to the unevolved Hamiltonian to mimic general features present in nuclear physics. The three-body potential that we have used is of the same simple form as the one used in Ref. \([12]\)

\[
V_3(p, q, p', q') = \sqrt{3} c_E f_A(p, q) f_A(p', q'),
\]

where \( c_E \) is the strength of the interaction and

\[
f_A(p, q) = \exp \left( -\frac{(2p^2 + \frac{3}{2} q^2)}{\Lambda^2} \right),
\]

where we use \( n = 4 \) and \( \Lambda = 2 \) throughout this work.

3.2 SRG Equations

Two-Body SRG: With the kinetic energy operator as generator for the SRG equation, Eq. \([7]\) becomes

\[
\frac{dV_2}{ds} = 2TV_2T + V_2V_2T + TV_2V_2 - V_2TT - TTV_2 - 2V_2TV_2.
\]

Using the complete set of states defined in Eq. \([8]\) we can write out the evolution equations in momentum space

\[
\frac{d}{ds} \langle p|V_2|p'\rangle = -(p^2 - p'^2)^2 \langle p|V_2|p'\rangle + (p^2 + p'^2) \int dq \langle p|V_2|q\rangle \langle q|V_2|p'\rangle - 2 \int dq dq' \langle p|V_2|q\rangle \langle q|V_2|p'\rangle.
\]

The first term on the right hand side ensures that the potential is driven to the diagonal.

Three-Body SRG: The equations for the evolution of the three-body potential are significantly more complicated. We write the Hamiltonian in the three-body sector as

\[
H_s = T + V_2^{(1)} + V_2^{(2)} + V_2^{(3)} + V_3,
\]

where \( V_2^{(i)} \) denotes the two-body potential in the three different channels. We have also dropped the subscript \( s \) from the potential and will keep doing this from now on to simplify notation. Since we are working with identical bosons we will assume that the induced three-body interaction is symmetric under the exchange of two particles.\(^1\)

The flow equation in the three-body sector is then written as

\[
\frac{d}{ds} \left( V_2^{(1)} + V_2^{(2)} + V_2^{(3)} + V_3 \right) = \left[ [T, H_s], H_s \right].
\]

The expression above contains the combined evolution of the two- and three-body interaction. It is of general interest to separate the two-body from the three-body evolution but in this case it removes also spectator \( \delta \)-functions that arise from disconnected diagrams and complicate the numerical computation of the evolved potential. Following \([3]\), we circumvent this issue by subtracting from the above expression the evolution of the two-body potentials \( V_2^{(i)} \). This isolates the derivative of the three-body potential and removes aforementioned \( \delta \)-functions

\[
\frac{dV_3}{ds} = \left[ [T, H_s], H_s \right] - \sum_{i=1}^{3} \frac{dV_2^{(i)}}{ds}.
\]

Expanding the commutators and rewriting the two-body differential equations as in previous section gives us

\[
\frac{dV_3}{ds} = O_2 + O_{23} + O_3,
\]

where we have defined

\[
O_2 = \sum_{i, j=1}^{3} (1 - \delta_{ij}) \left( TV_2^{(i)}V_2^{(j)} + V_2^{(i)}V_2^{(j)}T - 2V_2^{(i)}TV_2^{(j)} \right),
\]

\[
O_{23} = \sum_{i=1}^{3} \left( TV_3V_2^{(i)} + V_3V_2^{(i)}T - 2V_3TV_2^{(i)} + TV_2^{(i)}V_3 + V_2^{(i)}V_3T - 2V_2^{(i)}TV_3 \right),
\]

\[
O_3 = 2TV_3T - 2V_3TV_3 + TV_3V_3 + V_3TV_3 - TT_3T - TT_3.
\]

We can express the two-body potentials \( V_2^{(2)} \) and \( V_2^{(3)} \) through the potential \( V_2^{(1)} \) after application of permutation operators

\[
V_2^{(2)} = P_{13}P_{23}V_2^{(1)}P_{12}P_{23},
\]

\[
V_2^{(3)} = P_{13}P_{23}V_2^{(1)}P_{13}P_{23}.
\]

At this point it is useful to define the operator \( P \) given by

\[
P = P_{12}P_{23} + P_{13}P_{23},
\]

where \( P_1 \) denotes the permutation operator that exchanges particles \( i \) and \( j \). It can be shown that the overlap matrix elements for \( P_{12}P_{23} \) and \( P_{13}P_{23} \) in a partial-wave projected basis are identical. We can therefore write

\[
V_2^{(2)} = \frac{1}{4} PV_2^{(3)} P,
\]

\[
V_2^{(3)} = \frac{1}{4} PV_2^{(4)} P.
\]
which simplifies the above equations significantly.

We have used two different representations of the matrix element of the operator $P$
\[
\langle pq|P|p'q'\rangle = \sum_{x=\pm 1} \delta(p - \pi(q, q', x)) \delta(p' - \pi(q', q, x)),
\]
\[
\langle pq|P|p'q'\rangle = \sum_{x=\pm 1} \delta(p - \tilde{\pi}(p', q', x)) \delta(q - \chi(p', q', x)),
\]
(26)

where
\[
\pi(q, q', x) = \sqrt{\frac{1}{4} q^2 + q'^2 + xqq'},
\]
\[
\tilde{\pi}(p, q, x) = \sqrt{\frac{1}{4} p^2 + \frac{9}{16} q^2 + \frac{3}{4} pqx},
\]
\[
\chi(p, q, x) = \sqrt{p^2 + \frac{1}{4} q^2 - pqx}.
\]
(27)

The obvious consequence of the implementation of the operator $P$ is off-grid momenta in the object it is applied on. This problem can be solved by splining these objects, e.g. a function containing the shifted momentum $\pi(q, q', x)$ will be written as
\[
f(\pi(q, q', x)) = \sum_i N S_i(\pi(q, q', x)) f(q_i).
\]
(28)

We have used the global splines defined in Ref. [16] but also the cubic splines given in Ref. [17]. While the cubic splines provide a speedup in the calculation they also decrease the accuracy of results slightly. The results in this work were therefore all obtained with global splines.

4 Observables

4.1 Two-Body Observables

**Phaseshifts and binding energies**: We have calculated scattering and bound-state properties in the two-body sector. Scattering properties are obtained by solving the Lippmann-Schwinger equation. In operator form it is given by
\[
t = V + V G_0(E) t,
\]
where $G_0(E)$ denotes the free Green’s function
\[
G_0(E) = \frac{1}{E - p^2/m + i\epsilon}.
\]
(30)

The phaseshifts $\delta_l$ are then obtained from the on-shell $t$-matrix using the relation
\[
t_l(p, p) = -\frac{2pe^{i\delta_l}}{m\pi}.
\]
(31)

where we will concern ourselves only with the $l = 0$ phaseshifts.

4.2 Three-Body Observables

In the three-body sector we will focus on binding energies. An effective way to calculate three-body binding energies in momentum space is provided by the Faddeev equation.
\[
|\psi\rangle = G_0 t_2 P |\psi\rangle + G_0 t_2 G_0 t_3 (1 + P) |\psi\rangle,
\]
(32)

where $t_2$ and $t_3$ are the transition operators obtained from the two- and three-body potential terms.

We can then obtain the total wavefunction by
\[
|\Psi\rangle = (1 + G_0 t_3)(1 + P) |\psi\rangle.
\]
(33)
dot-dashed, dotted lines give the result for $\Lambda$ the standard ode solvers available in the corresponding programming language. We did not encounter any stiffness when solving the differential equations.

We have chosen to work in units where $\hbar^2/m = 1$ and all results are given according to this convention. The results for two-body binding energies obtained from unevolved potentials are shown in Table 3 and the first row of Table 2. Three-body binding energies are presented in Table 3. We note that our results for $V_0 + V_4$ differ slightly from the values given in Table II of Ref. 12. However, our results have been reproduced recently 13 and are therefore assumed to be correct.

**Table 3.** Two- and three-body binding energies $B_2$ and $B_3$ for the starting two-body potentials $V_0$ and $V_\beta$ and varying strength of the $s = 0$ three-body interaction.

| $V_0$ | $c_E$ | $B_2$ |
|-------|-------|-------|
| $V_0$ | -0.10 | -0.920 | -3.225 |
| $V_0$ | -0.05 | -0.920 | -2.885 |
| $V_0$ | 0.00  | -0.920 | -2.567 |
| $V_0$ | 0.05  | -0.920 | -2.279 |
| $V_0$ | 0.10  | -0.920 | -2.027 |
| $V_\beta$ | -0.10 | -0.474 | -2.570 |
| $V_\beta$ | -0.05 | -0.474 | -2.132 |
| $V_\beta$ | 0.00  | -0.474 | -1.708 |
| $V_\beta$ | 0.05  | -0.474 | -1.307 |
| $V_\beta$ | 0.10  | -0.474 | -0.952 |

**5 Results**

We have implemented the evolution of the two- and three-body potentials in Python and MATLAB. Equations (17) and (21) were discretized and written as a number of matrix multiplications. They were then solved using one of the standard ode solvers available in the corresponding

**Two-Body Evolution/Decoupling:** We have evolved all previously defined two-body interactions. The diagonalization as a result of the evolution in the plane that is spanned by incoming and outgoing relative momenta is common to all starting interactions. In Fig. 1 we show the SRG transformation of interactions $V_0$, $V_\beta$ and $V_{sep}$, respectively. With increasing flow-parameter $s$ the potential becomes increasingly more diagonal. The area in the low-momentum region of these figures that does not get diagonal with the evolution indicates the presence of a low-momentum two-body bound state. This is a typical feature of the $T_{rel}$ generator and is not present for all other possible generators such as the Wegner generator used in Ref. 13.

The decoupling of large and small momenta in the potential through the SRG evolution is one of the most important features that result from employing the diagonal $T_{rel}$ generator. Low-energy observables can therefore be calculated correctly with a decreased momentum-space cutoff after sufficient evolution. We illustrate this important feature of the SRG in Figs. 2 and 3 where we plot the phaseshifts and two-body binding energy (obtained with $V_0$) as a function of a sharp cutoff $A_{cut}$ in the momentum-space Schrödinger equation for different values of $s$. With increasing flow parameter $s$, observables become less sensitive to such a truncation indicating thereby the decoupling of small from large momenta in the potential.

**Three-Body Evolution/Decoupling:** We have evolved all two- and three-body potentials and obtained not only the corresponding two-
body potential but also the induced three-body interaction term. Our generator diagonalizes the Hamiltonian in terms of the total kinetic energy. In Figs. 4 and 5, we have therefore chosen to plot the three-body interaction in the plane of incoming and outgoing hypermomentum $\zeta$ and $\zeta'$ for three different hyperangles $\theta \equiv \arctan (\sqrt{3}p/(2q))$. The induced three-body potential obtained in the absence of an initial three-body force is shown in Fig. 4. Note that, in this case, the three-body potential is identical to zero for $s = 0$. It can be seen clearly how the evolution induces a three-body potential and that the strength of off-diagonal elements in hypermomentum space depends strongly on the flow parameter $s$. The three-body potential is weak for small $s$ parameter but also couples strongly off-diagonal elements in the hypermomentum plane. Then, when $s$ increases the three-body potential becomes stronger but also more diagonal in the hypermomentum plane. In Fig. 5, we show the same evolution of the three-body interaction where the Hamiltonian contains the initial three-body force as defined in Eq. (14) with $c_E = -0.05$.

Furthermore, we have tested decoupling in the three-body sector numerically. This was achieved by multiplying the two- and three-body potentials with regulators that are functions of the two- and three-body kinetic energy, respectively. In particular, we modify the evolved potential

$$V_s(p, q, p', q') \rightarrow \exp(- (\zeta^2 + \zeta'^2)/\Lambda_{\text{cut}}^2) V_s(p, q, p', q').$$

Equation (34)

In Fig. 6, we show the three-body binding energy as a function of $\Lambda_{\text{cut}}$ for different values of the flow parameter $s$. The longer evolution, measured by $s$, the smaller is the minimal truncation cutoff $\Lambda_{\text{cut}}$ for which the three-body binding energy remains unchanged. The decoupling of small and large hypermomenta due to the SRG is therefore clearly visible.

6 Summary and Outlook

In this work, we have presented results for the SRG evolution of three-body interactions for one-dimensional, bosonic model systems performed in a plane-wave basis. We showed that the evolution in the plane-wave basis is easily implemented and that observables in the three-body sector remain unchanged when evaluated with the evolved potentials. We showed explicitly that the SRG induces a three-body potential that becomes increasingly diagonal in the hypermomentum plane when evolved to larger flow parameter $s$. We tested decoupling explicitly by calculating observables with truncated two- and three-body interactions. Decoupling works well as illustrated for binding energies and phaseshifts in the two-body sector and for binding energies in the three-body sector.

This work presents the first step towards a consistent SRG evolution of two- and three-nucleon interactions in the plane-wave basis. The extension to three dimensions and to nuclear systems will require small modifications of the spatial part of the equations plus the addition of angular momentum, spin and isospin recoupling to the problem. The latter is a usual part of standard few-body equations and we do therefore expect these changes to be straightforward. The immediate benefit of this implementation is the increased consistency in a calculation of nuclear matter observables. A plane-wave formulation will also allow to test more reliably the importance of four-body forces in a description of nuclear systems. It was claimed recently that these become of increasing importance (after SRG evolution) in certain nuclei [8]. However, the results in Ref. [8] were obtained with a SRG evolution in the harmonic-oscillator basis that could in principle be associated with truncation errors. Furthermore, a formulation in a plane-wave basis opens up the possibility of identifying relevant scattering observables in the four-body sector where an emerging four-body interaction should also be visible.

Another possible avenue for the SRG in three dimensions is the Efimov effect in three-body systems of identical bosons with large scattering length. It is an open question how the renormalization group limit cycle, that was found to occur in this problem [18], manifests itself as the SRG parameter is varied and the induced three-body interaction changes. Decoupling in the three-body sector might also provide another path to extract universal properties of the four-body sector. The SRG might be used to remove three-body bound states and to calculate the universal properties of highly-excited four-body states.
Fig. 5. The evolved three-body potential as a function of the initial- and final-state hypermomentum $\zeta$ and $\zeta'$ for the hyperangles $\theta = \pi/12$ (upper panel), $\theta = \pi/4$ (middle panel), $\theta = \pi/2$ (lower panel). The starting two-body potential is $V_\alpha$ as defined in Eq. (13) and Table 1, the starting three-body interaction is given in Eq. (14), where $c_E = -0.05$.

without the need of a scattering calculation as done for example in Ref. [19].

We thank K. Wendt, D. Lee, E. Jurgenson, R. J. Furnstahl for useful discussions. This work was supported by the Chalmers eScience Center, the Swedish Research Council (CF,LP), and the European Research Council under the FP7 (CF).

References

1. Y. Meurice, R. Perry and S.-W. Tsai, Phil. Trans. R. Soc. 369, 2602 (2011).
2. S. Kehrlein, (2006) “The Flow Equation Approach to Many-Body Problems,” Springer-Verlag Berlin Heidelberg.
3. S. K. Bogner, R. J. Furnstahl, R. J. Perry, Phys. Rev. C75, 061001 (2007).
4. S. K. Bogner, R. J. Furnstahl, A. Schwenk, Prog. Part. Nucl. Phys. 65, 94-147 (2010).
5. J. P. Vary, P. Maris, E. Ng, C. Yang, M. Sosonkina, J. Phys. Conf. Ser. 180, 012083 (2009).
6. E. D. Jurgenson, P. Navratil, R. J. Furnstahl, Phys. Rev. Lett. 103, 082501 (2009).
7. E. D. Jurgenson, P. Navratil, R. J. Furnstahl, Phys. Rev. C83, 034301 (2011).
8. R. Roth, J. Langhammer, A. Calci, S. Binder, P. Navratil, arXiv:1105.3173 [nucl-th].
9. S. K. Bogner, A. Schwenk, R. J. Furnstahl, A. Nogga, Nucl. Phys. A763, 59-79 (2005).
10. K. Hebeler, S. K. Bogner, R. J. Furnstahl, A. Nogga, A. Schwenk, Phys. Rev. C83, 031301 (2011).
11. J. E. Drut, R. J. Furnstahl, L. Platter, Prog. Part. Nucl. Phys. 64, 120-168 (2010).
12. E. D. Jurgenson, R. J. Furnstahl, Nucl. Phys. A818, 152-173 (2009).
13. K. A. Wendt, R. J. Furnstahl, R. J. Perry, Phys. Rev. C83, 034005 (2011).
14. E. Anderson, S. K. Bogner, R. J. Furnstahl, E. D. Jurgenson, R. J. Perry, A. Schwenk, Phys. Rev. C77, 037001 (2008).
15. K. Wendt, private communication.
16. W. Glöckle, G. Hasberg, A. R. Neghabian, Zeitschrift für Physik A Hadrons and Nuclei, 3, 217 (1982).
17. D. Huber, H. Vitala, A. Nogga, W. Glocckle, H. Kamada, Few Body Syst. 22, 107-134 (1997).
18. P. F. Bedaque, H. W. Hammer, U. van Kolck, Nucl. Phys. A646, 444-466 (1999).
19. A. Deltuva, Phys. Rev. A82, 040701 (2010).