A Description of multi Charged Black Holes
in terms of Branes and Antibranes

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ABSTRACT

We describe multicharged black holes in terms of branes and antibranes together with multiple copies of gas of massless excitations. Assuming that energies of these copies of gas are all equal, we find that the entropy of the brane antibrane configuration agrees with that of the multicharged black hole in supergravity approximation, up to a factor $X$. We find that $X = 1$ for a suitable normalisation which admits a simple empirical interpretation that the available gas energy is all taken by one single gas which is, in a sense, a certain superposition of the multiple copies; and that the brane tensions are decreased by a factor of 4. This interpretation renders superfluous the assumption of equal energies, which is unnatural from a physical point of view.

\footnote{Name changed from Sanjay to Sanjay Siwach. Previous papers were written under the name Sanjay.}
1. In a recent paper [1] Danielsson, Guijosa, and Kruczenski (DGK) have given, among other things, a description of certain charged black holes in terms of brane antibrane configurations which is valid in the far extremal and Schwarzschild regime also. This has been generalised to other single brane configurations with or without rotation [2, 3, 4] and to intersecting multi brane configurations [5].

In this letter, we study the description of multicharged black holes in terms of intersecting brane antibrane configurations [6, 7, 8, 9]. Following DGK, we obtain the corresponding multicharged black holes as stacks of intersecting branes and antibranes, together with massless excitations. Such a stack of branes and antibranes can be put together (or “taken apart”) in $2^{K-1}$ different ways, along with two copies of gas of massless excitations for each possibility. ($K$ is the number of charges.) Therefore, it seems necessary to consider stacks of branes and antibranes together with $2^K$ copies of gas of massless excitations not interacting with each other. The dynamics of such a gas can be obtained as in [1] from the near extremal limit of the corresponding supergravity solutions [10].

As we will see in this letter, such a configuration suffices to describe the dynamics and to obtain the resulting entropy of the multicharged black hole in the far extremal regime also. However, in contrast to normal physical situations where one naturally takes temperatures to be equal, here it is necessary to assume that the energies of the copies of the gases are equal. The reason for this assumption, nor a physical mechanism that can enforce it, is not clear. Nevertheless, we assume this to be the case and proceed with the analysis as in [1]. We find that the entropy of the brane antibrane configuration agrees with that of the multicharged black hole in supergravity approximation, upto a deficit factor $X$.

We analyse the deficit factor $X$ by studying how the resulting entropy changes if one normalises the brane tension, gas energy, and the entropy by constant factors. We find that one can indeed have $X = 1$ for a suitable normalisation which admits a simple empirical interpretation as in [5]. It implies that the available gas energy is all taken by one single gas which is, in a sense, a certain superposition of the $2^K$ copies of the gas on the brane antibrane stacks; and that the brane tensions are all to be decreased by a factor of 4. However, the precise nature of this superposition is not clear to us. If this interpretation is correct then the entropy of the brane antibrane configuration is exactly equal to that of the multicharged black
hole in supergravity approximation; and the assumption that energies, not temperatures, of the different copies of gas are all equal - an assumption which is unnatural from a physical point of view - is now rendered superfluous.

This letter is organised as follows. In section 2 we give a brief description of the relevant results of [1]. In section 3, we describe the multicharged black hole in terms of brane antibrane configurations. In section 4, we give an interpretation of the deficit factor. In section 5, we conclude by mentioning a few issues for further study.

2. We give a brief description of the relevant results of [1] for the case of single charged black holes. Consider stacks of $p$-branes which, in supergravity approximation, correspond to single charged black holes in transverse $(d + 3)$-dimensional spacetime. The corresponding $(d + 3)$-dimensional charged black holes are obtained as a system consisting of (i) a stack of branes $N$ in number; (ii) a stack of antibranes $\bar{N}$ in number; and (iii) a gas of massless excitations on each stack of branes, with the gas on different stacks assumed not to interact with each other. Following DGK, we assume that such a system alone suffices to describe the dynamics of charged black holes.

The branes and antibranes have zero entropy and energies given by $CN$ and $C\bar{N}$ respectively where the constant $C$ includes tension and volume of the branes. The gas on the branes and antibranes have energies $E$ and $\bar{E}$ respectively, and their entropies $S$ and $\bar{S}$ are given by

\[
S = AN^\gamma E^\lambda, \quad \bar{S} = A\bar{N}^\gamma \bar{E}^\lambda,
\]

where $A$ includes brane tension and volume, and $\gamma$ and $\lambda$ are constants. The net charge $q$, the total energy $M$, and the total entropy $S_{tot}$ of the system are then given by

\[
q = N - \bar{N}, \quad M = C(N + \bar{N}) + E + \bar{E}, \quad S_{tot} = S(E) + \bar{S}(\bar{E}).
\]

In canonical formalism, such a system is unstable, for sufficiently small values of $q$, towards creating an infinite number of brane antibrane pairs. Hence, one must work in microcanonical formalism where the net charge $q$ and the total energy $M$ of the system are kept fixed. In normal physical systems, one assumes that the total system has one definite temperature. But, it turns out that one must instead assume that $E = \bar{E}$. However, the physical mechanism which enforces this equality is not well understood. The
equilibrium quantities are then determined by maximising the entropy $S_{\text{tot}}$ of the system with respect to $N$, keeping $q$ and $M$ fixed and setting $E = \bar{E}$.

In supergravity approximation, a stack of $p$–branes describes well the extremal and near extremal limits of single charged black holes which have zero entropy in the extremal limit. The entropy $S(E)$ of the gas on the stack of branes is then the same as that of the charged black holes in near extremal limit.

Following [6], see also [9], consider $p$–branes in $D = d + p + 3$ dimensional spacetime, which will correspond to charged black holes in $D - p = d + 3$ dimensional spacetime. Let $\gamma$ and $\lambda$ be given by

$$\gamma = \frac{2(D - 2)}{2d(p + 1) + a^2(D - 2)} , \quad \lambda = \frac{d + 1}{d} - \gamma$$

where $a$ is related to dilatonic charge. In the following we only consider the case where $\lambda > 0$. (Perhaps $\lambda = 0$ case can also be considered along the lines given in [4].) The mass $M_{sg}$ and the entropy $S_{sg}$ of the corresponding charged black hole in the supergravity approximation can be written as

$$M_{sg} = 2b \left( \lambda \mu + \gamma \sqrt{Q^2 + \mu^2} \right) , \quad S_{sg} = A \left( \sqrt{Q^2 + \mu^2 + \mu} \right)^\gamma (2\lambda b \mu)^\lambda$$

where $Q$ is the black hole charge, $A = \frac{4\pi b}{d} (\lambda b)^{-\lambda}$ and the constant $b$, which includes brane tension and volume, can be obtained from expressions given in [6]. Note that upon defining $Q = \mu \sinh 2\phi$, equations (4) become

$$M_{sg} = 2b \mu (\lambda + \gamma \cosh 2\phi) , \quad S_{sg} = A (\lambda b)^\lambda (2\mu)^{\lambda+\gamma} (\cosh \phi)^{2\gamma}.$$ (5)

The above expressions describe $Dp$–branes for $(D, \gamma, a) = (10, \frac{1}{2}, \frac{p-3}{2})$, $M$–branes for $(D, \gamma, a) = (11, \frac{1}{2}, 0)$, and other branes for other values of $(D, \gamma, a)$: for example, $(6, 1, 0)$ and $(5, \frac{3}{2}, 0)$. See [6].

In the extremal limit where $\mu = 0$, the mass and entropy are given by $M_e = 2b\gamma Q$ and $S_e = 0$ since $\lambda > 0$. Thus, $Q$ can be taken to be the number $N$ of branes in the stack, with $b$ containing the brane tension and volume factors. In the near extremal limit where $\mu$ is small, the brane dynamics can be obtained from the above solutions and can be thought of as arising due to a gas of massless excitations. Defining the energy $E$ of the gas on the branes to be $E = M_{sg} - M_e \simeq 2\lambda b \mu$, one obtains

$$S(E) = AN^\gamma E^\lambda.$$ (6)
We now extremise with respect to $N$ the total entropy $S_{tot}$ in (2), keeping the charge $q = N - \bar{N}$ and the total mass $M$ fixed, and setting $E = \bar{E}$. Also, $C = 2b\gamma$. This then determines $N$ and $E$ to be given by

$$2E = M - 2b\gamma(N + \bar{N}) = 4\lambda b\frac{N^\gamma + \bar{N}^\gamma}{N^{\gamma-1} + \bar{N}^{\gamma-1}}$$

and $\bar{N} = N - q$. For $\gamma = \frac{1}{2}$, the above equations become

$$2E = M - b(N + \bar{N}) = 4\lambda b\sqrt{NN}$$

and can be solved for $N$, $\bar{N}$, and $E$ in terms of $M$ and $q$.\footnote{The above equations can be solved for $\gamma = 1$ also. However, it turns out that the above analysis needs to be generalised when $\gamma = \frac{K}{2}$, with $K$ an integer $> 1$.} The solution can be parametrised as

$$N = \frac{m}{2}e^{2\theta}, \quad \bar{N} = \frac{m}{2}e^{-2\theta}.$$ 

Then, the required quantities can all be expressed in terms of $m$ and $\theta$. The result is:

$$M = 2bm(\lambda + \frac{1}{2} Cosh 2\theta), \quad q = m Sinh 2\theta$$

$$S_{tot} = 2^{-\lambda} A(\lambda b)^{\lambda}(2m)^{\lambda+\frac{1}{2}} Cosh \theta.$$ 

Comparing with the corresponding quantities in the supergravity approximation after setting $\theta = \phi$ and $m = \mu$ so that $M_{sg} = M$, it can be easily seen that $Q = q$ and

$$S_{tot}(M, q) = 2^{-\lambda} S_{sg}(M, q).$$

This is essentially the description, given in [1], of charged black hole in terms of branes and antibranes.

3. We now consider multicharged black holes. In the extremal and near extremal limit, they can be described as intersecting $p-$branes of String/M theory. Explicit solutions corresponding to such multicharged black holes can be found in [7, 8, 9]. We present here only the expressions for the mass and the entropy of the multicharged black holes in the supergravity approximation; they will suffice for our purposes here. For complete details,
see [7, 8, 9]. Denoting by $K$ the number of charges, the mass $M_{sg}$ and the entropy $S_{sg}$ of the multicharged black holes are given by

$$
M_{sg} = 2b \left( \lambda \mu + \sum_{i=1}^{K} \gamma_i \sqrt{Q_i^2 + \mu^2} \right)
$$

$$
S_{sg} = A \prod_{i=1}^{K} \left( \sqrt{Q_i^2 + \mu^2 + \mu} \right)^{\gamma_i} (2\lambda b \mu)^{\lambda}
$$

(10)

where $Q_i, i = 1, 2, \cdots, K$ are the charges, the constants $A$ and $b$ are given as before in (4) and $\lambda$, assumed to be $> 0$ in the following, is now given by

$$
\lambda = \frac{d+1}{d} - C , \quad C = \sum_{i=1}^{K} \gamma_i .
$$

(11)

Note that upon defining $Q_i = \mu \sinh 2\phi_i$, equations (10) become

$$
M_{sg} = 2b \mu \left( \lambda + \sum_{i=1}^{K} \gamma_i \cosh 2\phi_i \right)
$$

$$
S_{sg} = A(\lambda b)^{\lambda}(2\mu)^{\lambda+C} \prod_{i=1}^{K} (\cosh \phi_i)^{2\gamma_i} .
$$

(12)

In the extremal limit where $\mu = 0$, the mass and entropy are given by $M_e = 2b \sum_i \gamma_i Q_i$ and $S_e = 0$ since $\lambda > 0$. Thus, $Q_i$ can be taken to be the number $\nu_i \equiv N_i$ or $\bar{N}_i$ of branes or antibranes in the stack, with $b$ containing the brane tension and volume factors. In the near extremal limit where $\mu$ is small, the brane dynamics can be obtained from the above solutions and can be thought of as arising due to a gas of massless excitations. Defining the energy $E$ of the gas on the branes to be $E = M_{sg} - M_e \simeq 2\lambda b \mu$, one obtains

$$
S(E) = A \left( \prod_{i=1}^{K} \nu_i^{\gamma_i} \right) E^\lambda .
$$

(13)

We now describe the corresponding multicharged black holes as a system consisting of stacks of intersecting branes and antibranes, with a gas of massless excitation on each stack. In supergravity approximation, a stack of intersecting branes describes well the extremal and near extremal limits of multicharged black holes which have zero entropy in the extremal limit. The
entropy $S(E)$ of the gas on the stack of branes is then the same as that of the charged black holes in near extremal limit, which is given by equation (13).

There is a subtlety now. Description of multicharged black holes involves stacks of intersecting branes and antibranes, with $N_i$ and $\bar{N}_i$, $i = 1, 2, \cdots, K$, being the number of $i^{th}$ type of branes and antibranes. Let

$$\mathcal{N}_I = (\nu_1, \nu_2, \cdots, \nu_K)$$

where $\nu_1 = N_1$, $\nu_i = N_i$ or $\bar{N}_i$ for $i = 2, \cdots, K$, denote the numbers of constituent branes/antibranes in a stack of intersecting brane configuration. We use $\mathcal{N}_I$ to denote also the corresponding stack itself. The subscript $I$, taken to be in the range $I = 1, 2, 3, \cdots, 2^K - 1$, denotes a particular realisation of $\nu_i$, $i = 2, \cdots, K$. Let $\bar{\nu}_i = \bar{N}_i(N_i)$ when $\nu_i = N_i(\bar{N}_i)$ and let

$$\bar{\mathcal{N}}_I = (\bar{\nu}_1, \bar{\nu}_2, \cdots, \bar{\nu}_K).$$

Thus, for example, if $\mathcal{N}_1 = (N_1, N_2, \cdots, N_K)$ then $\bar{\mathcal{N}}_1 = (\bar{N}_1, \bar{N}_2, \cdots, \bar{N}_K)$.

Now, following DGK, multicharged black holes can be described as a system consisting of two stacks, $\mathcal{N}_I$ and $\bar{\mathcal{N}}_I$ for any single $I$, of intersecting branes and antibranes, together with the gas of massless excitations. This pair of stacks will have zero entropy, net charge $q_i = N_i - \bar{N}_i$, $i = 1, 2, \cdots, K$, and mass $E_t$ due to brane/antibrane tension given by

$$E_t = 2b \sum_{i=1}^{K} \gamma_i(N_i + \bar{N}_i).$$

Such a system, consisting of stacks of intersecting branes and antibranes, will have gas(es) of massless excitations living on them. In the single charged case DGK have argued, based on physics involving tachyon condensation, that one copy of gas lives on stack of branes and another on that of antibranes. In the present case, where the system consists of a pair of stacks $\mathcal{N}_I$ and $\bar{\mathcal{N}}_I$ for a single $I$, the corresponding tachyon physics is not clear. Certainly, as in DGK, there should be one copy of gas on each of these stacks. However, once the system is put in place, it can be thought of (or “taken apart”) as a pair of stacks with any value of $I$, along with a copy of gas on it. Hence, we assume that a copy of gas, with energy $E_I$ or $\bar{E}_I$ and entropy $S_I$ or $\bar{S}_I$ corresponding to each stack $\mathcal{N}_I$ or $\bar{\mathcal{N}}_I$, is present in the system for each value
of $I$. Thus, there are $2^K$ copies of gas in total which are further assumed, following [1], not to interact with each other.

The entropies $S_I$ and $\bar{S}_I$ are then obtained from the supergravity description of near extremal dynamics and are given by

$$S_I = A \left( \prod_{i=1}^{K} \nu_i^{\gamma_i} \right)_I E^\lambda_I, \quad \bar{S}_I = A \left( \prod_{i=1}^{K} \bar{\nu}_i^{\gamma_i} \right)_I \bar{E}^\lambda_I.$$ (14)

The subscript $I$ in the above expressions means that $\nu_i$'s and $\bar{\nu}_i$'s are those corresponding to the stacks $N_I$ and $\bar{N}_I$.

Thus, the total energy $M$ and the total entropy $S_{tot}$ of the system of intersecting branes and antibranes and the $2^K$ copies of non interacting gas living on them are now given by

$$M = 2b \sum_{i=1}^{K} \gamma_i (N_i + \bar{N}_i) + \sum_I (E_I + \bar{E}_I)$$ (15)

$$S_{tot} = \sum_I (S_I + \bar{S}_I).$$ (16)

In normal physical systems consisting of multiple components in equilibrium, it is natural to assume that all the components are at the same temperature. This is ensured by interactions between the components, no matter how weak, and the principles of statistical mechanics and ergodicity. It turns out that in the present system, consisting of $2^K$ copies of gas, such an assumption leads to results unconnected to charged black holes. However, if we assume that the energies $E_I$ and $\bar{E}_I$ of the gases are all equal to each other for all $I$, then the resulting dynamics describes that of the charged black hole. We will now assume this to be the case and proceed with the analysis, and comment on it afterwards.

With this assumption, namely $E_I = \bar{E}_I = E$ for all $I$, the total mass $M$ and the total entropy $S_{tot}$ of the system now become

$$M = 2b \sum_{i=1}^{K} \gamma_i (N_i + \bar{N}_i) + 2^K E$$

$$S_{tot} = AE^\lambda \prod_{i=1}^{K} (N_i^{\gamma_i} + \bar{N}_i^{\gamma_i})$$ (17)
where we have used the following relation which follows easily:

\[
\sum_I \left( \prod_{i=1}^{K} \nu_i^{\gamma_i} + \prod_{i=1}^{K} \bar{\nu}_i^{\gamma_i} \right) = \prod_{i=1}^{K} (N_i^{\gamma_i} + \bar{N}_i^{\gamma_i}).
\]

As in [1], in canonical formalism, such a system is unstable, for sufficiently small values of \( q_i \), towards creating an infinite number of brane antibranes; whereas, it is stable in microcanonical formalism for any value of \( q_i \). Hence, we work in microcanonical formalism where the total energy \( M \), and the charges \( q_i \equiv N_i - \bar{N}_i, i = 1, 2, \ldots, K \), of the system are kept fixed and the equilibrium quantities are obtained by maximising the entropy \( S_{\text{tot}} \) of the system with respect to \( N_i \). Hence, we maximise with respect to \( N_i \) and \( \bar{N}_i \)

\[
S_{\text{tot}} + \sum_{i=1}^{K} l_i(N_i - \bar{N}_i - q_i)
\]

where \( l_i \)'s are Lagrange multipliers. After a straightforward algebra, we get

\[
2^K E = M - 2b \sum_{j=1}^{K} \gamma_j(N_j + \bar{N}_j) = 4\lambda b \frac{N_i^{\gamma_i} + \bar{N}_i^{\gamma_i}}{N_i^{\gamma_i - 1} + \bar{N}_i^{\gamma_i - 1}} \tag{18}
\]

and \( \bar{N}_i = N_i - q_i \) where \( i = 1, 2, \ldots, K \). For \( \gamma_i = \frac{1}{2} \) for all \( i \), the above equations become

\[
2^K E = M - b \sum_{j=1}^{K} (N_j + \bar{N}_j) = 4\lambda b \sqrt{N_i \bar{N}_i}, \quad i = 1, 2, \ldots, K
\]

and can be solved for \( N_i, \bar{N}_i \), and \( E \) in terms of \( M \) and \( q_i \).

Note that for all intersecting brane configurations in String and M theories, the exponents \( \gamma_i \) are indeed given by \( \frac{1}{2} \). Also, if the exponents are integer multiples of \( \frac{1}{2} \) then they can be obtained by String/M theory intersecting branes by setting suitable number of charges to be equal. Thus, for example, \( \gamma = \frac{3}{2} \) in a single charged case can be obtained from intersecting branes with \( K = 3 \) and setting \( q_i = q, i = 1, 2, 3 \). Indeed, this appears to be the only way of obtaining such values of \( \gamma \). Interestingly, only such values of
γ appear in the known cases \([6, 7, 8, 9]\). Hence, we set \(\gamma_i = \frac{1}{2}\), for all \(i\), in the following.

The corresponding solutions can be parametrised as

\[
N_i = \frac{m}{2} e^{2\theta_i}, \quad \bar{N}_i = \frac{m}{2} e^{-2\theta_i}.
\]

Then, the required quantities can all be expressed in terms of \(m\) and \(\theta_i\). The result is:

\[
M = 2b m (\lambda + \frac{1}{2} \sum_{i=1}^{K} Cosh 2\theta_i), \quad q_i = m Sinh 2\theta_i
\]

\[
S_{tot} = 2^{-\lambda K} A(\lambda b)^{\lambda} (2m)^{\lambda+\frac{K}{2}} \prod_{i=1}^{K} Cosh \theta_i.
\]  \(\text{(19)}\)

Now compare with the corresponding quantities in the supergravity approximation given in equations (12), with \(\gamma_i = \frac{1}{2}\) and thus \(C = \sum_i \gamma_i = \frac{K}{2}\). Setting \(\theta_i = \phi_i\) and \(m = \mu\), we get \(M_{sg} = M\), \(Q_i = q_i\) and

\[
S_{tot}(M, q_i) = 2^{-\lambda K} S_{sg}(M, q_i) \equiv XS_{sg}(M, q_i).
\]  \(\text{(20)}\)

Thus, the two entropies are equal up to a deficit factor \(X\).

4. To understand further the deficit factor \(X\), we study how the resulting total entropy \(S_{tot}\) changes if one normalises the brane tension, the gas energy, and entropy by constant factors, as in \([5]\). For this, we consider the total energy and entropy of the configuration to be given, under the same assumptions as before, for example \(E_I = \bar{E}_I = E\) for all \(I\), by

\[
M = 2ab \sum_{i=1}^{K} \gamma_i (N_i + \bar{N}_i) + 2^K E
\]

\[
S_{tot} = \sigma A(\epsilon E)^{\lambda} \prod_{i=1}^{K} (N_i^{\gamma_i} + \bar{N}_i^{\gamma_i}).
\]  \(\text{(21)}\)

The factor \(\alpha\) normalises brane tensions\(^3\), \(\sigma\) the gas entropy, and \(\epsilon\) the energy available to each copy of the \(2^K\) copies of the gas. Maximising the total

\(^3\text{In equation (21), } \alpha\text{ normalises only the total brane energy which includes tensions, volumes, and number of branes. However, it is natural to take } \alpha\text{ as normalising brane tensions, and thereby the total brane energy.}\)
entropy $S_{\text{tot}}$ with respect to $N_i$, setting $\gamma_i = \frac{1}{2}$, solving for $N_i$, etcetera as before, one obtains the result

$$M = 2\alpha b m (\lambda + \frac{1}{2} \sum_{i=1}^{K} \text{Cosh} \ 2\theta_i), \quad q_i = m \text{Sinh} \ 2\theta_i$$

$$S_{\text{tot}} = \sigma \left( \frac{\epsilon}{2K} \right)^{\lambda} A(\lambda \alpha b)^\lambda (2m)^{\lambda+\frac{K}{2}} \prod_{i=1}^{K} \text{Cosh} \ \theta_i.$$  \hspace{1cm} (22)

We define the supergravity charges $Q_i$ to be $Q_i = \alpha q_i = \alpha (N_i - \bar{N}_i)$, or equivalently set $\mu = \alpha m$. This should be so since $\alpha$ normalises brane tension and, hence, the charges. Setting $\theta_i = \phi_i$ and comparing with the corresponding supergravity quantities, we get

$$S_{\text{tot}}(M, Q_i) = XS_{sg}(M, Q_i), \quad X \equiv \sigma \alpha^{-\frac{K}{2}} \left( \frac{\epsilon}{2K} \right)^{\lambda}.$$  \hspace{1cm} (23)

That this is the correct expression for the deficit factor, with scalings included, can be checked explicitly for simple cases like Schwarzschild black hole ($N_i - \bar{N}_i = 0$) or for a single charged black hole ($K = 1$), by expressing the entropies explicitly as a function of $M$ and $Q$. For example, for the later case, one gets after some algebra

$$S_{sg}(M, Q) = A\lambda^{\lambda} b^{-\gamma} \left( \frac{\lambda - \gamma \sqrt{Z}}{\lambda - \gamma} \right)^{\lambda} \left( \frac{1 + \sqrt{Z}}{2} \right)^{\gamma} \left( \frac{M}{\lambda + \gamma} \right)^{\lambda+\gamma}$$

$$S_{\text{tot}}(M, q) = X A\lambda^{\lambda} b^{-\frac{1}{2}} \left( \frac{\lambda - \gamma \sqrt{z}}{\lambda - \gamma} \right)^{\lambda} \left( \frac{1 + \sqrt{z}}{2} \right)^{\gamma} \left( \frac{M}{\lambda + \gamma} \right)^{\lambda+\gamma}$$

$$X \equiv \sigma \alpha^{-\frac{1}{2}} \left( \frac{\epsilon}{2} \right)^{\lambda}.$$  \hspace{1cm} (24)

where $Z = 1 + \frac{4b^2 Q^2 (\lambda^2 - \gamma^2)}{M^2}$, $z = 1 + \frac{4\alpha^2 b^2 \gamma^2 (\lambda^2 - \gamma^2)}{M^2}$, $K = 1$, and $q = N - \bar{N}$. $\gamma$ is arbitrary in the expression for $S_{sg}$ and $= \frac{1}{2}$ in that for $S_{\text{tot}}$. With $Q = \alpha q$ and taking $\gamma = \frac{1}{2}$, we get $Z = z$ and

$$S_{\text{tot}}(M, Q) = XS_{sg}(M, Q)$$  \hspace{1cm} (25)

which agrees with equation (23).

Clearly, the total entropy $S_{\text{tot}}$ of the intersecting brane antibrane configurations will be exactly equal to the entropy $S_{sg}$ of the corresponding
multicharged black hole in supergravity approximation if the deficit factor
\[ X = \sigma \alpha - \frac{3}{2} \epsilon^{2 - \lambda K} = 1 \]
for any value of \( \lambda \) and \( K \).

Let \( \sigma = \alpha = 1 \) as in [1]. Then the deficit factor \( X = 1 \) for any value of \( \lambda \) if
\( \epsilon = 2^K \). This is as if each copy of the gas carries \( 2^K \) times the energy assumed to be available to it [1]. If true, this would violate energy conservation.

However with \( \sigma \) and \( \alpha \) present and \( \neq 1 \), the deficit factor \( X = 1 \) for any value of \( K \) and \( \lambda \), if we set
\[ \epsilon = 2^K, \quad \sigma = \frac{1}{2^K}, \quad \alpha = \frac{1}{4}. \]

These values for \( \epsilon, \sigma, \) and \( \alpha \) admit a simple empirical interpretation [5]. The value of \( \epsilon \) means that the gas energy is to be increased by a factor of \( 2^K \) and the value of \( \sigma \) means that the total gas entropy is to be decreased by a factor of \( 2^K \). Empirically, they can be taken together to mean simply that the available gas energy is not shared equally by the \( 2^K \) copies of the gas but, instead, is all taken by one single gas with its entropy given by the average entropy of the \( 2^K \) copies of the gases. \( \alpha = \frac{1}{4} \) means that the brane tension is to be decreased by a factor of 4, which can perhaps be thought of as a net effect of non trivial dynamics of intersecting branes and antibranes.

The precise nature of the single gas mentioned above is not clear. In the case of the corresponding Schwarzschild black hole [5], \( N_i = \bar{N}_i \) and, hence, the \( 2^K \) copies of gas are identical to each other\(^4\), and also to the single gas. Thus, this single gas can be taken to be one copy - or, more generally, to be one linear combination - of the \( 2^K \) copies, which has all the available energy \( = 2^K E \) in it.

In the case of charged black hole, the \( 2^K \) copies of the gas are in general different from each other\(^5\), and also from the single gas mentioned above. Then, this single gas can perhaps be thought of as a gas which has all the available energy \( = 2^K E \) in it. Furthermore, it must perhaps be thought of as sloshing back and forth as a whole between the \( 2^K \) stacks of branes/antibranes, spending equal time on each of the stack and, thereby, having an entropy equal to the average entropy of the \( 2^K \) copies of the gases. In this sense, this single gas can be thought of as a certain superposition of all the \( 2^K \) copies. However, the precise nature and dynamics of the superposition is not clear to us.

\(^4\) in the sense of having identical entropy vs energy relation
\(^5\) in the sense of having different entropy vs energy relation
If this interpretation is correct then the entropy of the brane antibrane configuration is exactly equal to that of the multicharged black hole in supergravity approximation. Another attractive feature of this interpretation is the following. Note that energy is now conserved since all the available energy is taken by one single gas. Moreover, the assumption that energies, not temperatures, of the different copies of the gas are identical becomes superfluous. Such an assumption is unnatural from a physical point of view, and is hard to realise physically. But, in the present interpretation, it is rendered superfluous.

It is conceivable that the above interpretation somehow captures the essence of brane antibrane dynamics relevant for the description of charged black holes. Then, understanding the detailed properties of the single gas mentioned above will lead to the description of the charged black hole. On the other hand, the deficit factor $X$ may simply be due to the ‘binding energy’ of branes and antibranes, an interpretation advocated recently in [3]. To understand these issues fully one needs a rigorous study of brane antibrane dynamics at finite temperature which, however, is likely to require the full arsenal of string field theory techniques [11].

5. We described multicharged black holes using brane antibrane configurations, generalising those in [1]. The agreement between the entropy of the intersecting brane antibrane configurations and that of the corresponding multicharged black hole in supergravity approximation is impressive. But these two entropies differ by a deficit factor. We provided an empirical interpretation of it.

We conclude by mentioning a few issues that can be studied further. It will be interesting to understand the (near) extremal dynamics of the multicharged black holes in terms of the brane antibrane configurations, along with the gas of massless excitations living on them. This is particularly so since in the present interpretation, there is only one single gas living on brane/antibranes (as opposed to two copies in [1] which play an important role in the (near) extremal description).

More detailed description of multicharged black holes, such as the emission and absorption cross section, requires a better understanding of finite temperature brane antibrane dynamics. String field theory techniques [11] are essential towards a study of such issues. For some ideas in this context, see [1, 3] and references therein.

Note: While this paper was being written, there appeared a paper [12]
which has some overlap with the present one.

References

[1] U. H. Danielsson, A. Guijosa, and M. Kruczenski, JHEP 09 (2001) 011, hep-th/0106201; Rev. Mex. Fis. 49S2 (2003) 61, gr-qc/0204010.

[2] A. Guijosa, H. H. Hernandez Hernandez, and H. A. Morales Tecotl, hep-th/0402158 which describes rotating black holes.

[3] O. Saremi and A. W. Peet, hep-th/0403170;

[4] O. Bergman and G. Lifschytz, JHEP 04 (2004) 060, hep-th/0403189.

[5] S. Kalyana Rama, Phys. Lett. B 593 (2004) 227, hep-th/0404026.

[6] I. R. Klebanov and A. A. Tseytlin, Nucl. Phys. B 475 (1996) 164, hep-th/9604089. See also [9] below.

[7] M. Cvetic and A. A. Tseytlin, Nucl. Phys. B 478 (1996) 181, hep-th/9606033.

[8] I. R. Klebanov and A. A. Tseytlin, Nucl. Phys. B 479 (1996) 319, hep-th/9607107.

[9] A variety of (intersecting) brane configurations have been obtained in the literature by many methods, for example by applying $S, T, U$ dualities to string and M-theory branes. We mention below a few articles on this subject where further references can also be found. M. J. Duff and K. S. Stelle, Phys. Lett. B 253 (1991) 113; R. Güven, Phys. Lett. B 276 (1992) 49; P. K. Townsend, Phys. Lett. B 350 (1995) 184, hep-th/9501068; M. J. Duff, R. R. Khuri, and J. X. Lu, Phys. Rep. 259 (1995) 213, hep-th/9412184; G. Papadopoulos and P. K. Townsend, Phys. Lett. B 380 (1996) 273, hep-th/9603087; A. A. Tseytlin, Nucl. Phys. B 475 (1996) 149, hep-th/9604035; Nucl. Phys. B 487 (1997) 141, hep-th/9609212; Phys. Lett. B 395 (1997) 24, hep-th/9611111; Class. Quant. Grav. 14 (1997) 2085; hep-th/9702063; J. G. Russo and A. A. Tseytlin, Nucl. Phys. B 490 (1997) 121, hep-th/9611047; J. P. Gauntlett, D. A. Kastor, and J. Traschen, Nucl. Phys. B 478
(1996) 544, hep-th/9604179; J. P. Gauntlett et al, Nucl. Phys. B 500 (1997) 133, hep-th/9702202; R. Argurio, F. Englert, and L. Houart, Phys. Lett. B 398 (1997) 61, hep-th/9701042; N. Ohta, Phys. Lett. B 403 (1997) 218, hep-th/9702164; N. Ohta and J. -G. Zhou, Int. Jl. Mod. Phys. A 13 (1998) 2013, hep-th/9706153; E. Bergshoeff et al, Nucl. Phys. B 494 (1997) 119, hep-th/9612095; I. Ya. Aref’eva, M. G. Ivanov, and I. V. Volovich, Phys. Lett. B 406 (1997) 44, hep-th/9702079; I. Ya. Aref’eva et al, Nucl. Phys. Proc. Supp. B 56 (1997) 52, hep-th/9701092; M. Cvetic and C. M. Hull, Nucl. Phys. B 480 (1996) 296, hep-th/9606193; M. S. Costa, Nucl. Phys. B 490 (1997) 202, hep-th/9609181; Nucl. Phys. B 495 (1997) 195, hep-th/9610138.

[10] There is an enormous amount of literature. We mention below a few review articles where further references can also be found. J. M. Maldacena, Ph. D. Thesis, hep-th/9607235; M. Cvetic, Nucl. Phys. Proc. Suppl. 56B (1997) 1, hep-th/9701152; G. T. Horowitz, gr-qc/9704072; D. Youm, Phys. Rep. 316 (1999) 1, hep-th/9710046; A. W. Peet, Class. Quant. Grav. 15 (1998) 3291, hep-th/9712253; A. W. Peet, hep-th/0008241; S. R. Das and S. D. Mathur, Ann. Rev. Nucl. Part. Sc i. 50 (2000) 153, gr-qc/0105063.

[11] We mention below a few review articles where further references can also be found. A. Sen, hep-th/9904207; A. Lerda and R. Russo, Int. Jl. Mod. Phys. A 15 (2000) 771, hep-th/9905006; J. H. Schwarz, hep-th/9908144; M. R. Gaberdiel, Class. Quant. Grav. 17 (2000) 3483, hep-th/0005029; K. Ohmori, hep-th/0102085.

[12] G. Lifschytz, hep-th/0405042.