Conceptual design of an isochronous ring to generate coherent terahertz synchrotron radiation

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\textbf{Abstract.} A novel coherent light source project in the terahertz wavelength region has been developed at Tohoku University. The project may involve development of high brightness electron guns employing cathode of single crystal LaB$_6$ for production of a very short bunch length less than 100 fs. The light source has been designed based on isochronous ring optics to preserve the short bunch length. Although the ring is not a storage ring, the lattice of isochronous optics has resulted from consideration of path length differences due to the betatron motion. The coherent terahertz photons are emitted from circulating electron bunches injected from the linac. Even though the beam is bent by dipole magnets, the bunch shape does not collapse because of the nearly complete isochronous optics of the ring. Since production of the coherent terahertz radiation requires a bunch length less than 100 fs (stdv, if Gaussian), the maximum path length difference created by passing through the dipoles is controlled to not exceed a couple of tens of femtoseconds. The predicted spectrum of the coherent terahertz radiation and its characteristics are also presented.
1. Introduction

1.1. Advanced linac technologies and next generation light sources

Taking a look at accelerator-based light source developments, it is obvious the most promising source of shorter wavelength radiation below soft x-rays is self-amplified spontaneous emission (SASE) free electron lasers (FELs) [1]. Proof of principle of the SASE-FEL was demonstrated at TESLA-TTF by power saturation of FEL pulses at a wavelength of 80 nm [2], which was soon followed by a visible SASE-FEL at the APS-LEUTL and BNL-VISA [3, 4]. At present, a number of SASE-XFEL projects are being earnestly considered worldwide [5]. The progress of the superconducting accelerating structure is also another motivating power for such extremely high brilliant photon sources.

Another candidate for the next generation light source is high brilliant radiation from an adiabatic damped beam employing isochronous beam transport arcs and high-energy linac connected with low emittance electron guns. This is the so-called energy recovery linacs (ERLs) [6], which have been necessarily grown as high performance photo-injectors have developed. The normalized emittance of extracted beams from the photocathode RF guns presently reaches $\pi \text{ mm mrad}$ with a charge of around $1 \text{nC}$, so that the beam accelerated up to $5 \text{ GeV}$ performs $0.1 \text{ nm rad}$ emittance in both the horizontal and the vertical directions. Target brilliances of the ERLs at the x-ray region are mostly two orders of magnitudes above the third generation light source, even in the time-averaged domain.

In order to reach this higher brightness, a high intensity cw-like beam is of course necessary, which is being developed using the advanced superconducting linacs [7]. In order to recover the huge consumed power for cw beam acceleration in identical accelerating structures, the isochronous arc is necessary to preserve the bunch length. The ERL seems to be a quite challenging accelerator scheme because of many key technologies, i.e., a pretty high brilliant
electron gun in the 6-dimensional phase space, high field gradient superconducting accelerating structures, high efficient energy recovery and power recirculation. In addition, the round beam and few hundred femtoseconds pulse in an ERL offer new applications. The combined complex light source of the ERL with the SASE-FEL has been already proposed [8]. This rapid progress, particularly in linac technologies, will be fruitful competition for the next generation light source, we should notice there are many common technologies and a conceptual approach to be studied and considered.

1.2. Quasi-isochronous ring and coherent synchrotron radiation (CSR)

In contrast to x-ray source targets, we have considered advanced linac beams as also able to give many scientific opportunities with low-energy photons. Particularly intense coherent light via synchrotron radiation (SR) or transition radiation from the extremely short electron bunches at the terahertz frequency region will be a powerful probe for bio-medical science, solid state physics and many related fields.

CSR was first observed using a linac beam at the Laboratory of Nuclear Science, Tohoku University [9]. Recently coherent radiation at the sub-terahertz region was observed on a third generation light source, BESSY-II [10]. In order to realize a very low emittance beam on third generation rings, a large bending radius is generally introduced because the emittance is mostly proportional to the squares of the dispersion function and its slope. Since path length deviation of an off-momentum particle is proportional to the dispersion function, the third generation light sources might be relatively appropriate for approaching isochronous beam optics. On the other hand, recently Vinokurov et al [11] proposed a ring type SASE-FEL based on a complete isochronous transport arc. Experiments of low momentum compaction factor ($\alpha$), namely quasi-isochronous, operations on electron/positron storage rings have been carried out to study fundamental beam dynamics in storage rings and short bunch lengths because the equilibrium bunch length is proportional to the square root of $\alpha$ [12]. The bunch length on storage rings is, however, not only an equilibrium state of energy spread due to SR but is also affected by the ring impedance. Since the longitudinal wakefield strongly depends on the peak current of the bunch, bunch lengthening or instability is likely to occur with decreasing $\alpha$. In fact most low alpha experiments were reported at very low beam currents. It is well known that SR is coherently enhanced when the frequency component of the longitudinal bunch shape (namely the bunch form factor) at the radiation frequency is close to unity. Since the bunch shape in the frequency domain is the Fourier transform of the bunch length in the time domain, a couple of hundred femtoseconds bunch length at least is required for generation of the terahertz coherent radiation. However it seems to be very difficult to realize such very short bunches on storage rings with considerable beam currents. Meanwhile recent achievements whereby an advanced RF gun generates a very low emittance beam and the bunch length can be compressed to sub-picosecond by manipulating the longitudinal phase space in the linacs are very impressive.

The RF gun was invented many years ago, but not with a laser photo-cathode [13]. Since a critical issue of extraordinary cathode heating caused by beam back-bombardment was found early on, the thermionic RF gun has not progressed worldwide. Back-bombardment is a phenomenon where the electrons get into decelerating phase during travelling in the RF gun, then go back and hit the cathode. In particular this effect is serious for long macropulse operation such as infrared FEL drivers [14]. Today, the laser photo-cathode RF gun is regularly employed.
Figure 1. Schematic view of the terahertz CSR source. An electron beam with very short bunch length around 100 fs is produced by a thermionic RF gun and accelerated up to around 200 MeV. The beam is injected into a ring consisting of isochronous transport arcs. The bunch length is preserved because of nearly complete isochronous optics, so that coherent light can be radiated during turns. The beam is kicked out before quality deteriorates and transferred to a beam dump.

for the SASE-FEL projects except for SCSS at SPring-8, in which an optimized DC gun on purpose is employed [15].

Nevertheless the thermionic RF gun is very attractive as a high brilliant electron source because it has not been well studied and still has scope for investigation. We have examined the possible performance of the thermionic RF gun thorough a 3D simulation using an finite difference time domain (FDTD) method as a Maxwell’s equations solver [16]. According to the simulation results, back-streaming electrons are divided into two groups, i.e., a low-energy group and high-energy group, respectively. The high-energy group of the back-streaming electrons can be excluded by applying a weak dipole magnetic field around the cathode. However, a considerable amount of the low-energy electrons hit the cathode, because they turn back immediately and there is not enough drift time to be deflected. We have concluded that the most efficient way to reduce the effect of back-bombardment seems to be reduction of the cathode size.

This paper describes a design study of a novel terahertz CSR source by employing a ring with isochronous beam transport optics in addition to direct use of the extremely short electron bunch from a linac equipped with an RF gun. A schematic layout of the accelerator complex of the light source is shown in figure 1.

2. Short bunch production using a thermionic RF gun

2.1. Characteristics of thermionic RF guns with a single crystal LaB₆ cathode

A single crystal of LaB₆ has received much attention as a source of a brilliant electron beam [17], and has been widely used in devices for electron microscopy. In addition to infrared FEL drivers, Vanderbilt University and Tokyo University of Science have already employed the LaB₆ cathode [18]. The operating temperature of the LaB₆ cathode is higher (1700–1900 K) than that of a conventional dispenser cathode. This fact is also an advantage for the back-bombardment
issue because the additional heating power is relatively small. The maximum current density that can be extracted is approximately 100 A cm\(^{-2}\) which is sufficiently large to obtain a considerable amount of beam current. We have designed an RF gun employing a LaB\(_6\) cathode with a very small diameter, less than 2 mm, that can still supply a CW electron beam current up to 300 mA. As mentioned previously, the size of the cathode is very significant against the back-bombardment effect in the RF gun. Consequently a smaller cathode is definitely required to reduce the effect.

Since the laser photo-cathode RF gun is operated at proper RF phase with pulse duration of the induced laser, there is no back-bombardment effect in principle. However, a multibunch beam is very difficult to obtain due to the limited repetition rate of the laser pulse. In this terahertz CSR project, a micropulse train is crucial to increase the average power of the radiation, which is another reason why we have chosen the thermionic RF gun.

2.2. Independently-tunable-cell (ITC)-RF gun

A key issue for production of the high brilliant beam using the thermionic RF gun may be proper geometry of the gun and fed powers for cells including the effect of the beam wakefield. Though sacrificing accuracy, the overall beam dynamics in the RF gun is possibly understood from results of the 3D FDTD simulation. In this paper, we have focused on the longitudinal phase space because an extremely short bunch is required for the terahertz CSR.

If we use a multi-cell configuration for the RF gun, the longitudinal phase space strongly depends on the distance between the cells and the strength of the accelerating field in each cell. However once the cavity geometry is determined, the operation parameter is automatically settled to extract the highest performance of the gun. In order to secure freedom of operation, we have decided to employ two independent cells which are not strongly coupled with each other.

Distribution of the accelerating field in the ITC-RF gun at the \(\pi\) mode is shown in figure 2. The unloaded Q value of the each cell at the resonant frequency of 2856 GHz is deduced to be approximately 13000 by the FDTD calculation, which is in good agreement with those calculated by the code SUPERFISH \[19\]. If we accurately correct the resonant frequencies to be the same, coupling between the cells is not negligible even with a very narrow pipe. The calculation of SUPERFISH shows that a very small deviation of the resonant frequency such as 100 Hz to another cell decreases the coupling coefficient by less than \(10^{-4}\). Usually the fabrication error dimension is of the order of 10 \(\mu\)m, which corresponds to a deviation of approximately 100 Hz for the resonant frequency. So we may not care about the large coupling coefficient between the cells.

Since the electrons are continuously extracted during the RF phase from 0 to \(\pi\) in the cathode cell, the head of the beam is followed by the electrons who acquire a higher velocity. Consequently a velocity-bunching effect occurs, so that the electrons concentrate on the head of the beam. On the other hand, a chirped longitudinal phase space of the beam is required in order to compress the bunch length in a magnetic chicane. In particular, linearity of the momentum distribution on the longitudinal position is desirable for designing the bunch compressor. The ITC-RF gun is specially designed to manipulate the longitudinal phase space by varying the phase and the relative strength of the accelerating field.

Figure 3 shows the longitudinal phase space at the gun exit resulting from the FDTD simulation where the electrons are treated as the current density in Maxwell’s equations. Consequently the calculation includes the beam-induced field and the space charge effect as well. Details of the simulation are described elsewhere \[16\].

\textit{New Journal of Physics} 8 (2006) 292 (http://www.njp.org/)
As one can see, a higher accelerating field in the first cell is not always better for energy chirping, which is one of the significant differences in operation of the RF gun as a photo injector. However, the ITC-RF gun may produce a nearly ideal longitudinal phase space for the bunch compression by varying both the RF phase and power. Usually the thermionic RF gun has been operated at a combination with a bunch compressor $\alpha$-magnet [14, 20]. We are, however, going to employ the conventional chicane for the bunch compressor to avoid introducing a strong nonlinear field.

The simulated beam acceleration in the gun is shown in figure 4, where the peak accelerating fields of the first and second cells are 25 and 50 MeV, respectively, and the phase deviation $\delta \theta$ is $18^\circ$. Snap shots are plotted for the phases of $1/2\pi$, $\pi$, $3/2\pi$ and $2\pi$ at the first cell RF field. The velocity-bunching effect of the extracted beam and the back-streaming electrons can be seen clearly.

If the transverse emittance is much less than the wavelength, it is not very significant for the coherent radiation. However as discussed in following section, the betatron oscillation motion may cause considerable path length difference, which is not negligible in the isochronous transport system for preservation of the bunch length.

The normalized rms emittance is defined as

$$\varepsilon_{n, \text{rms}} = \beta \gamma \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2},$$

Figure 2. Cavity structure of the ITC-RF gun. The strength of the longitudinal electric field on the axis is also shown in the upper part of the figure where the phase difference between two cells is $\pi$ and the peak accelerating fields in both cells are adjusted to 25 MV m$^{-1}$. The spatial distribution of the strength of the longitudinal electric field is indicated by the colour scale.
Figure 3. Simulated results of the longitudinal phase spaces at the exit of the ITC-RF gun. The plotted range of the momentum is 2% from the top. The peak electric field on the surface of the cathode is (a) 25 MV m$^{-1}$, and (b) 30 MV m$^{-1}$. The symbol $E_2$ in the figure indicates the peak accelerating field of the second cell. The phase difference, denoted by $\delta \theta$, indicates the deviation from the $\pi$-mode for the RF phase, fed into the second cell.

where $\beta$ and $\gamma$ are the relative velocity and the relativistic factor, respectively. Transverse displacement and its gradient are denoted by $x$ and $x'$, respectively. Taking a look at the momentum range of $\Delta p/p = 2\%$ from the top energy, a very small normalized emittance less than 1 $\pi$ mm mrad is obtained by the simulation as shown in figure 5.

By rotating the longitudinal phase space, a very short bunch is possibly obtained as one can imagine from figure 3. However, a chicane for the magnetic bunch compression should be carefully designed because of the nonlinear beam dynamics and self-interaction effects of the coherent radiation (in other words, coherent wakefield). Since the wakefield and the coherent radiation have to be consistently treated with the motion of the electrons, the FDTD simulation will also work for designing the chicane, and this study is under way.

3. Isochronous optics

3.1. Concept of a CW terahertz CSR source based on an isochronous transport ring

Coherent radiation from the very short electron bunch can be produced via transition radiation or SR. Coherent transition radiation, which is often used for beam diagnostics, can be a powerful
Figure 4. Snapshots of the beam acceleration in the ITC-RF gun. Particle distributions (dots) and their projections (solid line) onto the longitudinal axis are shown, as well as the RF fields at the time. In the figure at an RF phase of $2\pi (n + 0.25)$, the peak around $z = 0.045$ m is back-streaming electrons that are travelling towards the cathode, while the electrons are continuously extracted from the cathode. A big peak at $z \sim 0.02$ m is velocity-bunched electrons. The peak of back-streaming electrons can be seen at $z \sim 0.03$ m in the figure at an RF phase of $2\pi (n + 0.5)$. We notice that a smaller cathode is effective for avoiding back-bombardment because those are spatially spreading as seen in the figure.

terahertz light source by choosing an appropriate beam energy. However the electron bunch collapses after passing through the radiator, so that the radiation is pulsed and its repetition rate is therefore that of the beam pulse. In addition the number of the light source is only one.

To get higher average power and CW coherent radiation, the beam should be non-distractive via the radiation process. In this sense SR is preferred. However, once the electron bunch is deflected by the dipole magnet, the bunch is lengthened due to pass length deviation originating from the energy spread.
Figure 5. Transverse phase space resulting from the FDTD simulation. The peak electric fields of the first cell and the second cell are 25 and 50 MV m$^{-1}$, respectively, and the phase deviation $\delta \theta$ is $18^\circ$. The integrated charge for the momentum range of 2% from the top is approximately 30 pC which corresponds to approximately 85 mA for the S-band (2.856 GHz) RF accelerator.

3.2. General consideration for the isochronous ring

The path length deviation of the off-momentum electrons in a storage ring can be written as

$$\frac{\Delta C}{C} \approx \alpha_0 \frac{\Delta p}{p},$$  \hspace{1cm} (2)

where $\alpha$ is the momentum compaction factor. This is deduced from the equation

$$\alpha = \frac{1}{C} \int \frac{\eta(s)}{\rho(s)} \, ds,$$  \hspace{1cm} (3)

where $\eta$ is the dispersion function and $\rho$ is the bending radius. In general the dispersion function increases as the bending radius increases, so that the third generation light sources have brought low dispersion function because a large bending radius is employed to increase the number of bending magnets and then to reduce the equilibrium emittance. For the direct use of the short bunch from the linacs, there is no significant obstacle to generating the coherent radiation. However if the short bunch length is preserved in a non-beam-destructive radiator, that is SR, a number of radiation points will be secured. In addition, if it is realized as the ring-type light source, a higher average power will be obtained by multiple turns.

In this sense, the isochronous optics has to be carefully investigated. The higher order momentum compaction factor is estimated by a method of iterative analysis of the closed orbit.
in the ring so far [21], so that the path length difference caused by the momentum deviation is precisely predicted. To cancel out the momentum compaction factor, the negative dispersion function introduced in parts of the bending sections may effectively work. The second order momentum compaction factor can be controlled using the sextupole moment, which has already been studied on a couple of storage rings [22].

The path length is calculated from an equation in the curvilinear coordinate,

\[
L = \int \left[ \left( 1 + \frac{x}{\rho} \right)^2 + \left( \frac{dx}{ds} \right)^2 + \left( \frac{dy}{ds} \right)^2 \right]^{1/2} ds. \tag{4}
\]

Thus the first order path length deviation comes from the horizontal displacement in the bending section

\[
\Delta L = \int \frac{x}{\rho} \, ds + O(\Delta^2). \tag{5}
\]

The displacement yields from not only the dispersion function with the momentum deviation but also the betatron function with the finite beam emittance. Assuming a bending radius of 3 m and a relative momentum spread of \(10^{-4}\), the path length deviation after passing through one turn is approximately 60 \(\mu\)m (\(\sim 200\) fs) if the average dispersion function in the bending magnet is 10 cm. On the other hand, the beam emittance after being accelerated up to, for instance, 200 MeV, is expected to be 2.5 nm rad if the normalized emittance of 1 mm mrad is realized, so that the path length deviation reaches 300 \(\mu\)m (\(\sim 1\) ps) assuming an averaged horizontal betatron function of 1 m, and the displacement is simply evaluated as \(x = \sqrt{\varepsilon \beta}\).

These estimations surely depend on the lattice parameters. However it is obvious that the path length deviation due to the betatron motion, namely \(R_{51}\) in the matrix elements in the transport system defined as \(\Delta L = R_{51} \varepsilon\), has to be considered at a stage of the lattice design for the isochronous optics.

### 3.3. Cancellation of the path length deviation caused by the betatron motion

Horizontal displacement of the single particle betatron motion \(x_\beta\) is written as

\[
x_\beta(s) = \sqrt{\varepsilon \beta_\chi(s)} \cos[\psi(s) + \phi_0], \tag{6}
\]

where \(\varepsilon_\chi\) denotes the horizontal emittance, \(\psi(s)\) and \(\phi_0\) are the phase advance and the initial phase, respectively. Inserting (6) into (5)

\[
\Delta L_\beta = \int \frac{\sqrt{\varepsilon \beta_\chi(s)} \cos[\psi(s) + \phi_0]}{\rho(s)} \, ds \tag{7}
\]

is obtained. After passing through one bending section, the path length deviation is approximately expressed as

\[
\Delta L_\beta \sim F[\sin(\psi_0 + \phi_0) - \sin \phi_0] = F[\sin \phi_0 \cdot (\cos \psi_0 - 1) + \cos \phi_0 \cdot \sin \psi_0], \tag{8}
\]
where $F \equiv \sqrt{\varepsilon \bar{\beta}_x^{3/2}/\rho}$ and $\bar{\beta}_x$ and $\psi_b$ are the average horizontal beta function and the phase advance in the bending section, respectively. Averaging over the initial phase, we obtain

$$
\langle \Delta L_{\beta} \rangle = F \sqrt{\frac{(\cos \psi_b - 1)^2 + \sin^2 \psi_b}{2}} = F \sqrt{1 - \cos \psi_b},
$$

(9)

which means the path length deviation at the end of a bending magnet is cancelled by choosing a phase advance of $2\pi$ in the bending magnet, and the maximum deviation is $\sqrt{2}F$ at the centre of the magnet. Though the $2\pi$ phase advance is hard to realize in one dipole magnet, the equation implies that $\langle \Delta L_{\beta} \rangle$ can be cancelled out by choosing proper phase advances in the bending sections and straight sections even employing a multi-bend system. For instance, in the case of two identical bending magnets separated by a straight section, (8) becomes

$$
\Delta L_{\beta} \sim F \{[\sin(\psi_b + \phi_0) - \sin \phi_0] + [\sin(2\psi_b + \psi_s + \phi_0) - \sin(\psi_b + \psi_s + \phi_0)]\},
$$

(10)

where $\psi_s$ is the phase advance in the straight section. Choosing the phase advance in a bending magnet as $\psi_b = (2n + 1)\pi$, the path length deviation (or spread) is then $\langle \Delta L_{\beta} \rangle = 2F \sqrt{1 - \cos \psi_s}$. By choosing $\psi_s = 2n\pi$, $\langle \Delta L_{\beta} \rangle$ is therefore cancelled out.

A tentative lattice design is shown in figure 6. In order to suppress both the betatron function and the dispersion function, we have introduced combined-function dipole magnets for the

**Figure 6.** An example of the lattice functions of a half of the ring. Dipoles containing the focusing quadrupole moment are used in the normal cell (#2, 3, 4, 6 and 7), while those containing the defocusing one are used in the dispersion suppressors (#1 and 8).
In figure 7, simulated bunch shapes while passing through the ring arc of figure 6 are shown, which are obtained by a tracking simulation of particles with an initial distribution of no bunch length. Since complete adjustment of the phase advance is difficult and the higher-order effect remains, the longitudinal particle distribution does not come back to the $\delta$-function completely. However, the final width of the bunch is very narrow as shown in figure 7(d); consequently a nearly isochronous system against the betatron motion may be possible. In the bending arc, the pass length deviation is mostly less than 70 fs and around 30 fs at the centre of the bending magnets as shown in figure 8, so that the bunch length would not increase so much if the rms bunch length of 100 fs were realized.

Major parameters of the isochronous ring, of which the lattice is already shown in figure 5, are listed in table 1. Note the horizontal damping partition number and damping time are negative,

\[ \langle \Delta L_\beta \rangle \]

normal cell, which contain the focusing quadrupole magnetic field. The phase advance of the betatron oscillation has been chosen to minimize the path length deviation in the centre of the bending magnets in the normal cell, and to nearly cancel it out after passing through half of the ring. To verify suppression of the path length deviation originating from the betatron motion, a tracking simulation is performed. Since the momentum compaction has not been compensated, only the contribution of the beam emittance for $\langle \Delta L_\beta \rangle$ is evaluated.

In figure 7, simulated bunch shapes while passing through the ring arc of figure 6 are shown, which are obtained by a tracking simulation of particles with an initial distribution of no bunch length. Since complete adjustment of the phase advance is difficult and the higher-order effect remains, the longitudinal particle distribution does not come back to the $\delta$-function completely. However, the final width of the bunch is very narrow as shown in figure 7(d); consequently a nearly isochronous system against the betatron motion may be possible. In the bending arc, the pass length deviation is mostly less than 70 fs and around 30 fs at the centre of the bending magnets as shown in figure 8, so that the bunch length would not increase so much if the rms bunch length of 100 fs were realized.

Major parameters of the isochronous ring, of which the lattice is already shown in figure 5, are listed in table 1. Note the horizontal damping partition number and damping time are negative,
which means this lattice does not work as a storage ring. However, it can be anticipated that the beam is able to circulate many turns, so that the average power of the coherent light would be greatly enhanced. Taking the energy loss per turn into account, the tracking simulation shows the beam can turn more than 100 times. At the moment an accurate number of possible turns cannot be estimated because the intrabeam scattering has been ignored. Since the beam energy is relatively lower and the emittance is smaller, the multiple Coulomb scattering may be the most significant effect for the beam lifetime. This issue has been investigated by developing the tracking code.

If a large bending radius is employed and then the damping partition becomes positive, this lattice may have very low equilibrium emittance because of the very small dispersion function. Although a considerable amount of the momentum compaction factor still remains, it is possible to reduce it adequately by introducing inverted bending magnets between a normal cell and the dispersion suppressor. A tentative proposal of the isochronous lattice in this paper is presented for which the path length deviation due to the betatron motion can be reduced to almost zero by choosing the appropriate phase advances.

4. Terahertz CSR

4.1. CSR

It is well known that SR is coherently enhanced when the bunch form factor at the frequency is close to unity. The double-differential intensity of the SR at the frequency $\omega$ is written as

$$\frac{d^2 I}{d\omega d\Omega} = [N[1 - f(\omega)] + N^2 f(\omega)] \times \frac{e^2}{4\pi^2 c} \left| \int_{-\infty}^{+\infty} e^{i\omega(t + \frac{\vec{R}}{c})} \vec{n} \times [(\vec{n} - \vec{\beta}) \times \vec{\beta}] \frac{dt}{(1 - \vec{\beta} \cdot \vec{n})^2} \right|^2,$$

(11)
where \( N \) is the number of electrons in a bunch, \( \vec{R}, \vec{\beta}, \vec{\dot{\beta}} \) and \( \vec{n} \) are the observation point, the relative velocity of the electron, its acceleration and the unit vector of \( \vec{R} \), respectively. The latter term of the right-hand side of (11) is the so-called Lienard–Wiechert potential. The form factor of the electron bunch \( f(\omega) \) is calculated using the spatial particle distribution \( S(\vec{r}) \)

\[
f(\omega) = \left| \int_{-\infty}^{+\infty} S(\vec{r}) e^{i\omega \vec{n} \cdot \vec{r}/c} d\vec{r} \right|.
\]  

(12)

The form factor is then the frequency spectrum of the bunch shape, so that the coherent part in the radiation strongly depends on the longitudinal shape of the bunch.

Usually the intensity of CSR is estimated by independently calculating the longitudinal form factor and the incoherent SR. However, the form factor has to be evaluated on an axis to the observation point as (12) indicates. Particularly CSR from the lower energy electrons has to be carefully evaluated because SR is widely spread.

We performed a multi-particle numerical simulation by integrating the equation of the Lienard–Wiechert potential in (11) [23], and investigated the characteristics of the CSR depending on the bunch length.

4.2. Result of numerical simulation

The numerical simulation was performed under conditions of the normalized rms emittance of 1 \( \pi \) mm mrad and a beam energy of 200 MeV. In addition, a Gaussian longitudinal distribution is assumed. Figure 9 shows a 2D spectrum of the SR from the bending magnet #4 indicated in figure 6, in which the observation point is located on the tangent of the ideal central orbit and 5 m from the source point. Although the number of particles is only 1000, CSR is clearly seen at the terahertz region (wavelength around 300 \( \mu \)m) in the figure for an initial rms bunch length of 100 fs. In the incoherent region the intensity of the radiation is just 1000 times larger than that of the radiation from the single electron. Obviously the coherent part arises at the longer wavelength region and its intensity is almost proportional to the square of the particle number. It should be noted that the coherent radiation is emitted from every bending angle because of the almost isochronous optics. As one can see, to cover the terahertz frequency region, the rms bunch length is at least required to be less than 100 fs if the bunch shape is Gaussian.

Note that the very intense radiation observed from both the entrance and the exit of the bending magnet is the so-called edge radiation, which is a kind of transition radiation having longer wavelength [24].

5. Summary

We have proposed an intense coherent terahertz radiation source based on both an isochronous optics ring and very short electron bunches generated from a thermionic RF gun.

As a study of the beam dynamics, preservation of the micro-bunching is a very fascinating objective in general. This paper addressed a bunch length of the order of hundreds of femtoseconds, which is the crucial bunch length for coherent terahertz radiation. Although the path length deviation due to the momentum spread has been discussed so far, the betatron motion is found to dominate the path length (\( R_{51} \) in the transport system); even the momentum
Figure 9. Results of numerical simulation for SR using the retarded Lienard–Wiechert potential. The simulation was performed with the isochronous lattice shown in figure 5. Spectra of radiation from the bending magnet #4 ($\rho = 3\, \text{m}$) are presented. The observation point is 5 m far from the source points, which is indicated as bending angle. The beam energy is 200 MeV and the initial Gaussian shape bunch lengths are (a) 300 and (b) 100 fs. Since the particle number is only 1000, random interferences can be seen in the incoherent spectra. Coherent parts of the radiation are, however, clearly seen since there are no statistical fluctuations.

Table 2. Radiation powers for different terahertz light sources.

| Source                     | Peak power (Micropulse length) | Average power (Macropulse length) | Average power |
|----------------------------|--------------------------------|-----------------------------------|---------------|
| p-Ge Laser                 | $1\, \text{W (10}\, \mu\text{s})$ | $-$                               | $100\, \mu\text{W}$ |
| YAG + NOE$^a$              | $300\, \text{mW (4}\, \text{ns})$ | $-$                               | $60\, \text{nW}$   |
| FEL                        | $10\, \text{kw (1}\, \text{ps})$ | $100\, \text{W (10}\, \mu\text{s})$ | $10\, \text{mW}$ |
| Isochronous terahertz ring$^b$ | $100\, \text{kw (250}\, \text{fs})$ | $70\, \text{W (10}\, \mu\text{s})$ | $350\, \text{mW}$ |

$^a$ Nonlinear optical element.

$^b$ Assuming a total charge of 30 pC in a micropulse and an rms bunch length of 100 fs. Powers are counted for a bandwidth of 0.01% of the wavelength.

$^c$ Assuming the beam survives for 100 turns.

Compaction factor ($R_{56}$ in the transport system) is very much reduced. However, by choosing appropriate phase advances of the betatron motion, the path length deviation can be mostly cancelled.

Recently a laser-based terahertz source has been rapidly developed. However the accelerator-based terahertz source is not a rival because the characteristics of the radiation are completely different from those of the laser-based source. Typical values of peak and averaged powers for
different terahertz sources are shown in table 2. Though wideband radiation, the extremely high power of the terahertz light from the relativistic electron beam is a particular feature. Accordingly in vivo terahertz imaging of large samples is a promising application in the fields of bioscience and medical science. To extract the highest performance of the coherent terahertz radiation, handling of the intense light will be a key issue for the applications.

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