Nonlinear finite element analysis for reinforced concrete haunched beams with opening

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Abstract. In the current paper, an analysis was carried out on fourteen simply supported reinforced concrete haunched beams (RCHBs). The members were designed for developing shear failure that presented by Colunga et al. in 2008. The verification process was carried out on three solid RC haunched beams analyzed by nonlinear finite element method by ANSYS to ascertain the accuracy and validity of FE procedure. The results of the validation showed a well matching between the FE and the experimental tests involving the load-deflection curves and load capacities, also parametric study was done by several parameters including an existence of transverse opening in the haunch zone, presence of longitudinal opening and varying the compressive strength. The result showed and confirmed that these beams had a higher deformation and energy dissipation, also the existence of the opening led to decrease of the shear strength of these beams; maximum reduction occurred in the longitudinal opening 125*125 mm by 39%, also the lateral opening 100*100 mm, maximum decreasing occurred when the presence of the opening near the vertex by 30%. In addition, reinforcing the opening and increasing $f'_c$ restored the lost strength and enhanced the ductility.

Keywords: haunched beam, shear capacity, shear failure.

1. Introduction

Since the last century, the new shape of beams had emerged, this type of members called hunched beam; this kind of beams uses often for medium height framed building and in continuous or simply supported or bridges as shown in figure 1. The design of RC hunched beams (RCHBs) had been left to decision and experience of the designer and engineer in professional practice because the available popular codes and precise recommendations such as ACI-318M or BS-5400 do not cover these types of members [1-3]. However, some codes such as the German codes (DIN 1045-1, 2001) [4] cover the plan of RCHBs in some details and rules. Although some handbooks (Park and Paulay 1975[5], MacGregor 1997 [6] and Nielsen 1999 [7]) have some brief recommendations for the shear design of RCHBs, the available experimental data of haunched beams failed in shear is too limited (El-Niema [3], Colunga et al [8], Stefanou [9] and MacLeod and Houmsi [10]). Structural and architectural engineer are often resorting to using such members due to the following benefits when compared with
the prismatic beams: (a) Haunched beams substantially enhance the lateral stiffness of structures, (b) Such these members considered a guidance for the efficiency using of steel rebar and concrete. (c) Decreases the weight of the structure. (d) The employment of these members helps in placement of several types of equipment such as (air conditioning and buildings piping) [8].

Figure 1. Haunched beams in the bridge in Mexico [8].

The main disadvantages of these kinds of members are that their using requires higher costs in construction, also special formwork and requires a highly qualified worker in construction [8].

The previous experimental studies carried out on shear resistance of this type of beams concerning on several parameters such as the presence of stirrups, haunch angle, haunch length, amount of longitudinal reinforcement, the depth of beam at supports and the inclination angle of the beams [9-16]. While the investigations including the effect of the longitudinal and transverse opening in this type of beams were limited [17-18]. So as to ensure the desirable behavior of RCHBs, according to the capacity design rules, it is necessary first to understand how these kinds of members will fail and to examine the haunched beams behavior when it has lateral and longitudinal opening may be the main objective of this research.

2. Description of the tested beams

The dimensions of the specimens were defined according to an experimental research conducted by Colunga et al. [8]. The width (b) for all models is 220 mm, the effective span (L) was 2800 mm and the shear span (a) is 1083 mm. The haunched length (Lh) at both beam ends was one-third of the effective span (Lh=L/3=933 mm). Variable linear tapering dimensions were obtained by keeping constant the overall depth at each beam end (hmax=450 mm) and decreasing the overall depth at the middle to 400, 350 and 300. Therefore, haunched angles from the horizontal (α) were 3.07°, 6.12° and 9.13° respectively. Specimens were simply supported beams and analyzed with two concentrated loads (V) that were applied at distance 100 mm from the vertex. Steel plate dimensions at the loading zone were 25.4*100*220 mm to avoid stress concentration that causing crush in concrete elements. Table 1 and figures (2) to (4) show the details of the analyzed specimens for verification purpose.

Three beams with least steel reinforcement for shear design required by NTCC-04[19] for a prismatic member. Three models of haunched beams had been analyzed. For shear failure, insurance haunch zone, providing the flexural capacity in the principal prismatic length at the middle of a member and keeping continuous the longitudinal reinforcement along the model. In addition, additional shear reinforcement placed in the vertex of the beam with shear reinforcement as illustrated in table 1.
Figure 2. The dimension of the analyzed beam [8].

Figure 3. Flexural and shear reinforcement for the analyzed beams [8].
Table 1. Geometrical and reinforcement details of the verification and parametric beams [8].

| Beam ID | $\alpha$ | Flexural reinforcement Top | Flexural reinforcement Bottom | Stirrups | Vertex stirrups |
|---------|----------|---------------------------|-----------------------------|----------|----------------|
| H1      | 3.07°    | 3#8                       | 4#8                         | 8S@185 mm | 1S#3          |
| H2      | 6.12°    | 3#8                       | 4#8                         | 8S@185 mm | 3S#3@140 mm   |
| H3      | 9.13°    | 3#8                       | 4#8                         | 8S@185 mm | 3S#3@75 mm    |

Figure 4. Cross section of the studied models [8].

3. Material modeling

3.1. Concrete

Development of the model for the concrete behavior is a challenging task. Concrete is a semi-brittle material with different behavior in tension and compression. In compression, the stress-strain curve for concrete is linearly elastic up to about 30% of the maximum compressive strength ($\sigma_{cu}$). Above this point, the stress increases gradually up to the maximum compressive strength. When it reaches to the maximum compressive strength ($\sigma_{cu}$), the curve falls away into the softening zone, and eventually, failure by crushing happens at the ultimate value of the strain ($\varepsilon_{cu}$). In tension, the stress-strain curve for this material is almost linearly elastic up to the maximum value of the tensile strength. After this point, the concrete cracks and the strength losses gradually to zero [20]. Ultimate uniaxial compressive strength ($f'c$), ultimate uniaxial tensile strength (modulus of rupture, $f_t$), elastic modulus ($E$), Poisson’s ratio ($\mu$) and the reduction factor of stiffness for the cracked tensile condition are essential to define concrete behavior in ANSYS. In all cases, a tensile strength of concrete of 10% of the compressive strength was considered [20]. Concrete properties of the RCHBs are described in Table 2.

The concrete constitutive behavior is simulated using plasticity based damage model with a three-dimensional continuum. Inelastic performance of concrete was represented by using the isotropic
hardening plasticity. The formula presented by Kachlakev [20] is applied to compute the stress-strain curve for uniaxial compression response of concrete as follows;

\[
f_c = \varepsilon E_c \quad \text{for} \quad 0 \leq \varepsilon \leq \varepsilon_1
\]

\[
f_c = \frac{\varepsilon E_c}{1 + (\frac{\varepsilon}{\varepsilon_0})} \quad \text{for} \quad 0 \leq \varepsilon \leq \varepsilon_0
\]

\[
f_c = f' c \quad \text{for} \quad \varepsilon_0 \leq \varepsilon \leq \varepsilon_c
\]

where;

\[
\varepsilon_1 = \frac{0.3 f' c}{E_c} \quad , \quad \varepsilon_0 = \frac{2 f' c}{E_c} \quad \text{and} \quad f_c = \text{stress at any strain } \varepsilon
\]

Figure 5. Represent the relationship between stress-strain curve in both tensile and compression as proposed by Kachlakev [20].

| Member | \(\alpha\) (degree) | \(f' c\) (MPa) | \(f_t\) (MPa) | \(E\) (GPa) |
|--------|----------------------|-----------------|---------------|-------------|
| H1     | 3.07                 | 26.38           | 2.64          | 13.11       |
| H2     | 6.12                 | 28.64           | 2.86          | 11.38       |
| H3     | 9.13                 | 28.24           | 2.82          | 11.45       |
3.2. Steel reinforcement, steel loading and bearing plates
All specimens were exposed to four-point bending with a simply supported condition. Steel plates with 250 mm width and 220 mm length, 25 mm thick were placed on the bearing zone, while the plate at loading zone was placed on the hinge-pin supports with dimension 100 mm length, 220 mm width, and the same thickness of bearing plate at supports.

Steel representation is prepared by the bilinear isotropic material according to Von Mises failure principle that has the same behavior in both compression and tension had been used for the modeling of steel reinforcement. steel reinforcement has Poisson’s ratio \( \mu = 0.3 \) was also used in this research. To define tangent modulus of steel and the yield stresses as it can be explained in Table.3. Linear behavior was considered for the steel bearing plates. Bearing plates were represented in ANSYS as they were used with the aim of averting stress concentrations in such zones. Young’s modulus \( E = 210 \) GPa was used for all steel plates [20] as shown in Figure 6.

| Specimen     | \( f_y \) (MPa) | \( \varepsilon_y \) | Area (mm\(^2\)) |
|--------------|----------------|--------------------|-----------------|
| #8           | 427            | 0.00237            | 507             |
| Tangent modulus | 448          | -                  | -               |
| #3           | 451            | 0.00235            | 71              |
| Tangent modulus | 452          | -                  | -               |

3.3. Element types used for modeling
A Solid65 element was used to represent the concrete elements. This element has eight nodes with three degrees of freedom for each and it is able to plastic distortion, cracking and crushing (Figure 6 a). Using Link180 element for modeling longitudinal rebar and stirrups (Figure 6 b). Link180 is a 3D bar element and it consist of two nodes with three degrees of freedom for each node. (Solid185) was used too in these beams to model the bearing steel plates located at loading and supports points (Figure 6 c). This element has eight nodes [21].

![Figure 6](image_url)

**Figure 6.** Details of the material element; (a) Solid65 element (b) Link180 element (c) Solid185 element [21].
3.4. Element type, mesh and loading

In this numerical study to simulate the behavior of the simply supported RCHBs, the concrete, steel reinforcement and bearing plate elements were represented by using solid elements, so as to be more efficient in describing the boundaries of the elements and modeling the behavior. A fine mesh of three-dimensional eight-node solid elements was used (Figure 7.a). Mesh at vertex had made finer to get high accuracy because this zone is too sensitive. In order to reduce the computational effort, symmetry conditions for the studied RCHB were taken into account, which allowed one-quarter of the beam to be represented. Therefore, the corresponding length for the haunched beam section represented is 1650 mm with a width of 110 mm, as illustrated in Figure 7.

For each beam, the area of flexural steel reinforcement at the top at a line of symmetry is half of the normal area for one bar #8 because one-half of the bar was cut off in the meshing. As it can be seen from Figure 7.b, flexural and shear steel reinforcement have some mutual nodes. For the base reinforcement, shaped by a two-bar bundle, 2#8 bars for each bundle, just a single bundle is spoken to because of the symmetry of the shaft, utilizing an equal zone component. Only half of the stirrup was modeled because of the symmetry of the beam. supports were modeled in such a way that a roller was created. A single line of nodes on the plate was given constraint in the y and z directions. The line of nodes on the plate is located at (50 mm from the vertex) as shown in figure 7.a.

![Figure 7. (a) Mesh of concrete and boundary condition. (b)Mesh of steel reinforcement which shows creating the steel reinforcement at the shared nodes.](image)

4. Comparison between experimental and numerical results

The verification is carried out so as to check the validity and accuracy of the finite element procedure. The results obtained by using FE analysis are compared with the results of the experimental study was presented by Colunga et al. [8] in 2008. The verification process was achieved on three solid RC haunched beams (H1, H2 and H3) analyzed by nonlinear finite element analysis (FEA) by ANSYS. The comparison was done through the load-deflection curves and ultimate load, which showed a good agreement as illustrated in figure 8 and table 4.
Figure 8. Load-deflection curve for verification results; (a) H1 (b) H2 (c) H3.
Table 4. Comparison between the numerical & experimental results in load failure (V) and max. deflection (Δ).

| Beam ID | $V_{\text{ANSYS}}$ (KN) | $V_{\text{exp}}$ (KN) | $V_{\text{ANSYS}}/V_{\text{exp.}}$ | $\Delta_{\text{ANSYS}}$ (mm) | $\Delta_{\text{exp}}$ (mm) | $\Delta_{\text{ANSYS}}/\Delta_{\text{exp.}}$ |
|---------|-------------------------|-----------------------|-------------------------------|-----------------------------|-----------------------------|----------------------------------|
| H1      | 199                     | 210                   | 94.76%                        | 35.5                        | 35                          | 101%                             |
| H2      | 179                     | 170                   | 105%                          | 39.7                        | 40.35                       | 98.38%                           |
| H3      | 125                     | 135                   | 92.6%                         | 41.55                       | 42                          | 99%                              |

5. Parametric Study
The parameters were represented on simply supported RCHBs to study the effect of these parameters on the behavior, shear capacity, crack pattern and the ductility of these beams in comparison with the control beams H1, H2 and H3. These parameters are lateral opening, longitudinal opening and compressive strength. All models properties are shown in table 5 and figures 9-11.

5.1. Transverse opening
Square opening (100 x 100) mm was placed in haunch zone in the mid of the total depth. Location of the opening is far away (230, 415 and 600) mm from the vertex as illustrated in figure 9.

5.2. Longitudinal opening
Making a longitudinal square opening with different sizes at the center of the cross-section, length of its rib (50, 100 and 125) mm. The size of the opening is constant along the beam as illustrated in figure 10.

5.3. Reinforcing the opening.
To avoid the expected reduction in beam strength due to the existence of the opening by steel reinforcing has been placed around the opening by placing the reinforcing rebar in the perpendicular direction to the expected cracks direction as illustrated in figure 11. According to the ACI requirement of the needed amount of diagonal reinforcement bars $A_d$ according to the equation (4):

$$A_d = V_u (\phi f_{yd} \sin \alpha)^{-1}$$  \hspace{1cm} (4)

When $\alpha = 45^\circ$ is the inclination angle of the bent bar.

5.4. Compressive stress
Changing the compressive strength ($f_c'$) for the models that having an opening with the same geometry and details, to restore the lost strength due to the existence of the opening.
### Table 5. Properties of the analyzed beams

| Models   | Angle ($\alpha$) (degree) | $f'_{c}$ (MPa) | Area of the opening |
|----------|---------------------------|----------------|---------------------|
| H1       | 3.07                      | 26.38          | -                   |
| H2       | 6.12                      | 28.64          | -                   |
| H3       | 9.13                      | 28.24          | -                   |
| S1-LT    | 6.12                      | 28.64          | 100*100             |
| S1-RT    | 6.12                      | 28.64          | 100*100             |
| S1-CR    | 6.12                      | 28.64          | 100*100             |
| S1-RC    | 6.12                      | 28.64          | 100*100             |
| S1-P1    | 6.12                      | 28.64          | 50*50               |
| S1-P2    | 6.12                      | 28.64          | 100*100             |
| S1-P3    | 6.12                      | 28.64          | 125*125             |
| S1-M1    | 6.12                      | 40             | 100*100             |
| S1-M2    | 6.12                      | 50             | 100*100             |
| S1-N1    | 6.12                      | 40             | 125*125             |
| S1-N2    | 6.12                      | 50             | 125*125             |

**Figure 9.** Geometrical and reinforcement description of F1 models.
6. Result and Discussion
The numerical studies are performed to predict the capability of a constitutive modeling of simulating the behavior of RCHBs under static shear loading and with variables compared with the reference models (H1, H2, and H3), it should be noted that the researcher used E less than the real popular value [8]. When the other beams in this investigation are with a real value of $E = 4700\sqrt{f'c}$.

6.1 General behavior (load-deflection curve)
In this analysis, the load-deflection curve for all beams is linearly elastic up to about 12% of the maximum failure load ($V_u$). Overhead this point, the load increases gradually up and reaches the nonlinearity zone to the maximum load capacity.

The influence of making a transverse square opening (100*100) mm at the center of the depth(h) with different a location in the haunch zone, which far away (230, 415 and 600) mm from the vertex. This parameter is carried out on a model with haunch angle (6.12°). The existence of an opening which showed a decreasing in the shear strength and stiffness of RCHB. The reduction of ultimate capacity is
(15, 22 and 30%) compared with the solid beam. Hence, the reduction magnitude of strength depends on the opening location. The position of the opening has a large influence on the shear capacity, this effect becomes large when the opening position is near the vertex due to the decreasing in depth. In RCHBs with an opening near the vertex caused high decreasing in the stiffness and the maximum reduction in ultimate load was about (30%) as shown in figure 12 a. More shear cracks will develop at the corner of the opening, all opening models have reduced crack propagation and concentrated it at the opening, in comparison with the control beam H2 which showed the maximum crack propagation occurred at the vertex as shown in figure 13.

The effect of creating a longitudinal square opening at the center of the cross-section along the beam with different size. Rib of the square opening is (50, 100 and 125) mm respectively. The effect of the opening seems more effecting whenever we move away from the support due to the decreasing in the depth. The presence of the opening caused a reduction in the beam strength by (13, 30 and 38.5%). Max reduction occurred in the beam S1-P3 about 38.5% as illustrated in figure 12 b. Crack pattern in these beams showed max propagation of cracks near the support and slightly less than amount at the vertex in comparison with the control beam H2 which showed the maximum crack propagation occurred at the vertex as illustrated in figure 13 h.

Reinforcing the opening with ø10 mm according to ACI requirement; which shows restoring most of the beam strength that reduced due to the existence of the opening. The presence of opening with size (100 *100) mm on solid beam produced a reduction in ultimate strength with the ratio 22%. When steel reinforcement used around the opening with an arrangement, led to increasing the strength of beams by 14%. This treatment showed 8% reduction of strength RCHB with opening compared with the solid beam as shown in figure 12 c. Also, the presence of reinforcement around the opening make a better crack distribution, little amount of cracks was developed at the corner of the opening as illustrated in figure 13 g.

The influence of increasing the compressive strength of the beam S1-RT to 40 and 50 MPa to restore the lost strength due to the existence of the opening near the vertex zone, which revealed that using 40 MPa regained about 83% from the loss in strength and decreased the deflection about 4.5%. Increasing in the compressive strength to 50 MPa of the model S1-RT led to increasing in the failure load about 64% in comparison with model S1-RT and the behavior of beams seems a significant improvement of ductility and high energy absorption as shown in figure 12 d. The effect of increasing the compressive strength of the model S1-P3 to 40 MPa showed a reduction on the percentage of loss in the strength to 17% when it was 38.5%. Using \( f_c' \) equals to 50 restored all the loss in strength and enhanced it by 4% additional strength with increase in the ductility as shown in figure 12 e. Changing the \( f_c' \) did not affected the crack pattern propagation and shear stress distribution.

Shear stress distribution in all parameters models showed different distribution. Level of intensity ranged from colors (blue-green- yellow-brown-red) respectively in concentration, the minimum intensity is blue while maximum intensity referred in red color. In models of series F1, maximum shear stress distribution of the models S1-LT, S1-CT and S1-RT occurred near the support and at the corner of the opening as shown in figure 14 d, e and f. Beam S1-RC displayed shear stress under the opening, near the support and towards the loading point as shown in figure 14 g, while the maximum shear stress in models S1-P1 at the edges of the longitudinal opening near the support and at the vertex as illustrated in figure 14 h.

Results of the parameters and its effect on the shear behavior of RC haunched beams are showed in the table below when the result involved crack pattern, damage, and shear stresses distribution (Table 9).
Table 6. The failure load for shear and max. displacement.

| Model   | Vu(KN) | ∆u(mm) |
|---------|--------|--------|
| H1      | 186    | 22.54  |
| H2      | 164    | 25.43  |
| H3      | 117    | 25.92  |
| S1-LT   | 140    | 22.35  |
| S1-RT   | 115    | 28.94  |
| S1-CR   | 128    | 21.7   |
| S1-RC   | 152    | 23.3   |
| S1-P1   | 143    | 23.2   |
| S1-P2   | 115    | 18.56  |
| S1-P3   | 101    | 15.87  |
| S1-M1   | 156    | 27.75  |
| S1-M2   | 189    | 29.74  |
| S1-N1   | 136    | 20.3   |
| S1-N2   | 170    | 28.24  |

(a) [Graph](image1)

(b) [Graph](image2)
Figure 12. Load-displacement curve for all models, (a) F1 (b) S1-Pi (c) S1-RC (d) S1Mi (e) S1-N1.
Figure 13. Crack pattern for all models.
7. Conclusion
The presented study focused on the numerical modeling using nonlinear finite elements of simply supported reinforced concrete haunched beams with an opening designed to develop a shear failure under static loading. The purpose of this study was to assess the ability and limitations of simple and complex analytical models to predict the structural behavior of RCHBs with opening failing in shear.

Based on the results of this numerical investigation, it could be concluded that:

1. In slender RCHBs when the tensile bar is bent, major cracks start from the changing portion of the cross-section near the support and toward the loading point.
2. Increasing in $f'c$ leading to increasing shear strength capacity for RCHBs with and increasing in ductility.
3. Presence of a lateral opening (100*100) mm reduces the shear strength of beam according to the opening location. More shear cracks will develop at the corner of the opening. The position of the opening has a large influence; this effect is large when the opening position is near the vertex due to the decrement in depth. In RCHBs with an opening at the shear zone, the maximum reduction in ultimate load was about (30%).
4. Presence of a longitudinal opening reduces the shear strength of beam according to the opening size, maximum reduction occurred in the opening (125*125) mm by 39% approximately. Cracks started from the support and towards the loading point with an intensity around the opening.
5. Reinforcement around the opening restored about 63% of the reduction, the value of the reduction became 8% only, and the Cracks developed at the corner of the opening.
6. Increasing the compressive strength of the beams having an opening to 40 and 50 MPa restored the lost strength with improving in ductility.
7. In RCHBs, the crack pattern doesn’t affect by changing $f'c$.
8. The bent tensile rebar has a negative contribution to the shear capacity due to stress concentration.

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