Subcarrier interpolation and DFT channel estimation in ultra-wideband systems

Xing Wan, Min Huang, Wanbo Luo
Leshan Vocational and Technical College, Leshan 614000, China
krantson@163.com

Abstract: Channel estimation algorithm is one of the key research directions in UWB systems. For the ECMA368 standard, there is a wave interference problem in channel estimation. This paper constructs a low-order ZP-OFDM-FAST model for improving channel estimation performance. The interpolation problem of virtual zero subcarriers is studied, and the interpolation DFT channel estimation method is proposed. Finally, a method of frequency domain instantaneous filtering is given.

1. Introduction
An important reason why the OFDM system is used in UWB systems is because the OFDM system can effectively combat multipath fading, but OFDM modulates different speeds by converting high-speed serial data into low-speed parallel data during modulation. For modulating orthogonal subcarriers, it was a need to ensure the orthogonality between subcarriers. Multipath interference will destroy the orthogonality of each subcarrier and interfere with the demodulation of the receiving data. Therefore, when implemented, a guard interval is added before or after each OFDM signal, whose length should be greater than the maximum multipath delay of the wireless channel, so that the signal of the previous OFDM symbol will not interfere with the next symbol. Generally speaking, according to difference in guard interval, system can be divided into frame CP-OFDM[1][2] and frame ZP-OFDM[2][3] systems. There are already multiple channel estimation algorithms for these models. In this paper, interpolation for the virtual zero subcarriers and channel estimation issues will be discussed.

2. Low-order ZP-OFDM-fast model
In the traditional ZP-OFDM-fast model, \( F_p \) matrix is used to process the received signal. Although the channel information can be completely recovered theoretically, the complexity of the equalization is very large. The DFT transform is used for the ZP-OFDM system. In practice, the low-order ZP-OFDM-fast model is used, and following matrix is used processing the received signal:

\[
\begin{bmatrix}
e^{j2\pi 0/M} & e^{j2\pi 1/M} & \ldots & e^{j2\pi (M+D-1)/M} \\
e^{j2\pi 1/M} & e^{j2\pi 1/M} & \ldots & e^{j2\pi (M+D-1)/M} \\
\vdots & \vdots & \ddots & \vdots \\
e^{j2\pi (M-1)/M} & e^{j2\pi (M-1)/M} & \ldots & e^{j2\pi (M+D-1)/M}
\end{bmatrix}
\]

(2.1)
It is actually equal to the following matrix \[ F_{M \times M} \cdot F_{M \times D} \]. \( F_{M \times D} \) is first D column of Fourier matrix \( F_{M \times M} \). In the first D column, the previous formula is transformed as follows:

\[
X_M(i) = F_{M \times M} X_{ZP}(i) \\
= F_{M \times M} \left( H_{(M+D)\times M} F_{M}^H S_M(i) + n_p(i) \right) \\
= C_M(h) S_M(i) + F_{M \times M} \left[ I_{M \times M}, I_{M \times D} \right] \left[ n_m(i), n_p(i) \right]^T \\
= C_M(h) S_M(i) + F_M \left[ I_{D \times M}, 0 \right] n_m(i)
\] (2.2)

So it is exactly the same as the ZP-OFDM-OLD model. Unlike the ZP-OFDM-OLD model which needs to do m-times m-order FFT, the simplified model needs to do m-times p-order FFT at the receiving port. The simplified model will not preprocess the received signal by OLA, so every time when receiving a data point in an OFDM symbol, the model can perform real-time processing without waiting for an OFDM symbol to be received before processing is completed. So simplified model has significant advantages in real-time system.

3. Interpolation problem on virtual zero subcarriers

Since the subcarriers transmitted by the transmission channel estimation sequence on the 1st and 63th to 67th are 0, it is impossible to accurately obtain the channel estimation value by using the FFT channel estimation at these frequencies. Consider three methods to solve this problem: One is to use MMSE frequency domain channel estimation to obtain the estimated value directly. The second is to approximate the frequency response corresponding to zero subcarrier to 0. The third is to use interpolation method to deal with it. The first method utilizes the channel. The second-order statistical property is thus the best. The second method is simple and easy to implement, and the performance loss is small due to the small number of zero subcarriers. The third method of interpolation is to achieve achievability and performance.

Firstly, the frequency domain channel estimation algorithm is analysed. The MMSE frequency domain channel estimation has been obtained as follows:

\[
\hat{H}_{LMMSE} = \frac{1}{\sigma_n^2} \left( R_{HH} + \delta_n^2 \left( \frac{X^H X}{n} \right)^{-1} \right)^{-1} \hat{H}_{LS}
\] (3.1)

\( X \) is a diagonal matrix consisting of a sequence of channel estimates. Due to the presence of zero subcarriers, \( XX^H \) is the singular matrix. Notice that the transmission sequence takes values \([1+j, 1-j, -1+j, -1-j]/\sqrt{2}\) on non-subcarriers, their modulus values are equal, so ensuring that the matrix elements 1-norm are equal, \( XX^H \) can approximate as follow:

\[
XX^H \approx \frac{61}{62} I
\] (3.2)

Another simple approximation is to use \( XX^H \) instead of \( (XX^H)^{-1} \).

Next, the interpolation method is discussed. The interpolation algorithm based on MMSE has the best performance, but because it requires a priori information of channel noise and the computational complexity is too large, it is not actually considered. Further discussion of the OFDM frequency domain periodic interpolation algorithm will be proposed in for the ECMA368 ZP-OFDM-OLD scheme.

The expression of the frequency domain received signal in the ZP-OFDM-OLD model is obtained. For the sake of simplicity, the unnecessary label is omitted, and the data length is \( n \), and the guard interval length is \( l \), which will be expressed. The formula is written as follows:

\[
Y = XH + F(n + n_g)
\] (3.3)
Among them \( n_L = [n_{129}, n_{130}, \ldots, n_{165}, 0, \ldots, 0]^T \), \( n = [n_1, n_2, \ldots, n_{128}]^T \), \( n_i \) Variance \( \sigma_i^2 \)

\[
Y = [Y_1, Y_2, \ldots, Y_{128}]^T,
\]

\[
X = \text{diag}(X_1, X_2, \ldots, X_{128}) \quad \text{and} \quad H = [H_1, H_2, \ldots, H_{128}]^T.
\]

For \( i \in [2, 62] \cup [68, 128] \), FFT is estimated to be:

\[
\widetilde{H}_i = H_i + \frac{N_i}{X_i}
\]

Consider the correlation function between the non-zero subcarrier frequency domain channel estimate and the first subcarrier:

\[
E\left[ H_i \widetilde{H}_1 \right] = E \left[ H_i \left( H_i + \frac{N_i}{X_i} \right) \right]
\]

\[
= E \left[ \sum_{k=0}^{37-1} h_k e^{-\frac{2\pi k}{128}} \left( \sum_{l=0}^{37-1} h_l e^{-\frac{2\pi (l-i)}{128}} \right)^* \right]
\]

\[
= \sum_{k=0}^{37-1} E \left[ h_k \right] e^{-\frac{2\pi k (i-1)}{128}}
\]

When \( \min(i-1, 129-i) \) is smaller, the greater is the correlation, so \( \widetilde{H}_{i,i} = 2, 128, 3, 127 \) with \( H_i \), the correlation is the largest. So you can use the loop characteristics of the FFT spectrum to estimate the frequency domain response. You can select the 128th, 2nd, and 3rd points to estimate the frequency response of the first point. You can also use the 61st, 62nd, and 68th points. Estimating other points.

### 4. Periodic interpolation DFT channel estimation algorithm

In the ECMA368 system, combined with the idea of the periodic interpolation method and the conventional time domain filtering proposed in this paper, the interpolation DFT channel estimation method can be used to improve the FFT frequency domain channel estimation:

In the first step, the frequency channel response is estimated using the traditional FFT method:

\[
\hat{H}_{iLS} = X^{-i} Y = \left[ \frac{Y(0)}{X(0)}, \frac{Y(1)}{X(1)}, \ldots, \frac{Y(128-1)}{X(128-1)} \right]^T
\]

In the second step, the channel estimation value at the zero subcarrier is cleared to filter out the noise component:

\[
\hat{H}_{iLS_F} = [0, \frac{Y(1)}{X(1)}, \ldots, \frac{Y(62)}{X(62)}, 0, 0, 0, 0, \frac{Y(68)}{X(68)}, \ldots, \frac{Y(128-1)}{X(128-1)}]^T
\]

The third step is to record the periodic linear interpolation function as \( y = \text{lin}(x) \). The periodic linear interpolation method can be used to estimate the channel response of zero subcarriers:

\[
\hat{H}_{\text{fival}} = \left[ \text{lin}(0), \frac{Y(1)}{X(1)}, \ldots, \text{lin}(63), \frac{Y(67)}{X(68)}, \ldots, \frac{Y(128-1)}{X(128-1)} \right]^T
\]

The 0th point is estimated based on points 127 and 1, and the 63th to 67th points are estimated according to points 62 and 68.

In the fourth step, the interpolated channel estimate is transformed into the time domain:

\[
\hat{h}_{\text{final}} = \mathbf{F}^H \hat{H}_{\text{final}}
\]

In the fifth step, the noise component is filtered out in the time domain (points 38-128). \( I_{37} \) is a representative of first 37 lines of \( I \), the remaining is zero.
In the sixth step, the time domain channel estimate is finally transformed back into the frequency domain:

$$\hat{h}_{en, ls} = \mathbf{I}_N \hat{h}_{en}$$

(4.5)

This idea can also be used to improve the frequency domain MMSE channel estimation, which is not repeated here. This results show a complete interpolated DFT channel estimation. The simulation results in the next page show that the improved channel estimation method is significantly better than the traditional frequency domain FFT channel estimation method. The reason why the method does not use transform domain filtering is because the time domain method is equivalent to the transform domain method.

The following paper will demonstrate the equivalence of transform domain filtering and time domain filtering under additive noise. Assuming that the transmitted sequence has no zero subcarriers, the frequency domain FFT is estimated as $\hat{\mathbf{H}}_{LS}$, then the time domain transform and the transform domain transform are (4.7) and (4.8) respectively, and satisfy the relation (4.9):

$$\hat{\mathbf{h}}_T = \mathbf{F}^H \hat{\mathbf{H}}_{LS}$$

(4.7)

$$\tilde{\mathbf{h}}_c = \mathbf{F} \hat{\mathbf{H}}_{LS}$$

(4.8)

$$\tilde{\mathbf{h}}_c = \mathbf{Q} \hat{\mathbf{h}}_T$$

(4.9)

among them, $\mathbf{Q}$ is the orthogonal variation matrix:

$$\mathbf{Q} = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
0 & 1 & \cdots & 1
\end{bmatrix}$$

(4.10)

In the physical sense, the transform domain signal is a rearrangement of the time domain signal. The assumed time domain signal is $\mathbf{h}_T = [h_0, h_1, h_2, \cdots, h_{N-1}]$, the corresponding transform domain signal is $\tilde{\mathbf{h}}_c = [h_0, h_{N-1}, h_{N-2}, \cdots, h_1]$, so it turns out that the two methods are equivalent.

5. Conclusion

Based on the results and discussions presented above, the conclusions are obtained as below:

(1) Prove the equivalence of classical time domain filtering channel estimation algorithm and transform filtering channel estimation algorithm in additive noise environment.

(2) For the case where there are 6 virtual zero subcarriers for each OFDM symbols in the training sequence of the ECMA368 standard, a solution for interpolating DFT channel estimation algorithm is proposed.

(3) Based on the approximate periodicity of the OFDM spectrum, the frequency interpolation algorithm of OFDM frequency domain is proposed.

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