Determination of mass hierarchy with medium baseline reactor neutrino experiments

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I discuss the sensitivity of future medium baseline reactor antineutrino experiments on the neutrino mass hierarchy. By using the standard $\chi^2$ analysis, we find that the sensitivity depends strongly on the baseline length $L$ and the energy resolution $(\delta E/E)^2 = (a/\sqrt{E/\text{MeV}})^2 + b^2$, where $a$ and $b$ parameterize the statistical and systematic uncertainties, respectively. The optimal length is found to be $L \sim 40 - 55$ km, the larger resolution the shorter optimal $L$. For a 5 kton detector (with 12% weight fraction of free proton) placed at $L \sim 50$ km away from a 20 GWth reactor, an experiment would determine the mass hierarchy with $(\Delta \chi^2)_{\text{min}} \sim 9$ on average after 5 (15) or more years of running with the $(a, b) = (2, 0.5\%)$ ($3, 0.5\%)$ energy resolution. This type of experiment can also measure the relevant mixing parameters with the accuracy of $\sim 0.5\%$.

I. INTRODUCTION

Now that a large $\theta_{13}$ has been measured at Daya Bay \cite{1, 2} and RENO \cite{3} experiments accurately, neutrino physics enters a new era. One of the next challenges is determination of the mass hierarchy. Among many ideas proposed, the medium baseline reactor antineutrino experiment \cite{4–10} has stimulated various re-evaluations of its physics potential and sensitivity recently. Some works utilize the Fourier transform technique \cite{11–13}, first discussed in refs. \cite{6–8}, to distinguish the mass hierarchy. The main advantage of this technique is that the mass hierarchy can be determined without precise knowledge of the reactor antineutrino spectrum, the absolute value of the large mass-squared difference $|\Delta m^2_{31}|$, and the energy scale of a detector. Although interesting and attractive, this technique is somewhat subtle to incorporate the uncertainties of the mixing parameters and to estimate its sensitivity to the mass hierarchy.

On the other hand, some works adopt the $\chi^2$ analysis \cite{10, 13–15} and new measure based on Bayesian approach \cite{16}. These methods utilize all available information from experiments, and it is straightforward to incorporate the uncertainties to evaluate the sensitivity, providing robust and complementary results to the Fourier technique.

In this proceedings, we analyze the sensitivity of medium baseline reactor antineutrino experiments to the mass hierarchy for the baseline length of 10–100 km and the energy resolution $(\delta E/E)^2 = (a/\sqrt{E/\text{MeV}})^2 + b^2$ in the range $2% < a < 6\%$ and $b < 1\%$ with the $\chi^2$ analysis. The optimal baseline length and the expected statistical uncertainties of the neutrino parameters, $\sin^2 2\theta_{12}, \sin^2 2\theta_{13}, \Delta m^2_{21}$ and $\Delta m^2_{31}$, are also estimated.

II. REACTOR ANTI NEUTRINO FLUX

In this section, we briefly discuss the evaluation of how many electron antineutrinos, $\bar{\nu}_e$, would be detected at a far detector with a medium baseline length from a reactor.

In a nuclear reactor, antineutrinos are mainly produced via beta decay of the fission products of the four radio-active isotopes, $^{235}\text{U}, ^{238}\text{U}, ^{239}\text{Pu}$ and $^{241}\text{Pu}$, in the fuel. The flux of antineutrinos with energy $E_\nu$ (MeV) at a reactor of $P$ (GWth) thermal power is then expressed as \cite{7}

$$
\frac{dN}{dE_\nu} = \frac{P}{\sum_k f_k \epsilon_k} \phi(E_\nu) \times 6.24 \times 10^{21},
$$

where $f_k$ and $\epsilon_k$ are the relative fission contribution and the released energy per fission of the isotope $k$, respectively \cite{17}. $\phi(E_\nu)$ is the number of antineutrinos produced per fission \cite{18}. 

This rate is then modulated by neutrino oscillation. The $\bar{\nu}_e$ survival probability is expressed as

$$P_{ee} = \left| \sum_{i=1}^{3} U_{ei} \exp \left( -i \frac{m_i^2 L}{2E} \right) U_{ei}^* \right|^2 = 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 (\Delta_{21})$$

$$- \sin^2 2\theta_{13} \sin^2 (|\Delta_{31}|)$$

$$- \sin^2 \theta_{12} \sin^2 2\theta_{13} \sin^2 (\Delta_{21}) \cos (2|\Delta_{31}|)$$

$$\pm \frac{\sin^2 \theta_{12}}{2} \sin^2 2\theta_{13} \sin (2\Delta_{21}) \sin (2|\Delta_{31}|),$$

(2)

where $U_{ei}$ is the neutrino mixing-matrix element relating the electron neutrino to the mass eigenstate $\nu_i$. The variables $m_i$ and $E_i$ are the mass and energy of the corresponding mass eigenstate, while $\theta_{ij}$ represent the neutrino mixing angles. The oscillation phases $\Delta_{ij}$ are defined as

$$\Delta_{ij} \equiv \frac{\Delta m_{ij}^2 L}{4E}, \quad (\Delta m_{ij}^2 \equiv m_i^2 - m_j^2)$$

(3)

with a baseline length $L$. The plus or minus sign in the last term of eq. (2) corresponds to the mass hierarchy; the plus sign is for normal hierarchy (NH), and the minus sign for inverted hierarchy (IH). Note that this last term is the only source of the mass hierarchy difference. We have neglected the matter effect because it is small enough for the energy range and the baseline lengths we concern in this study [19].

Similar as the current reactor experiments, such as Daya Bay [1, 2], RENO [3] and Double Chooz [20], future medium-baseline reactor-antineutrino experiments can also use free protons as targets to detect electron antineutrinos via the inverse neutron-beta-decay (IBD) process, producing a neutron and a positron. The threshold neutrino energy of this process is

$$E_{\text{thr}} \sim m_n - m_p + m_e.$$  

(4)

The produced positron then interacts with scintillator, converting its kinetic energy to photons. Eventually, the positron annihilates with an electron in the detector and emits two $0.5$ MeV photons. The energies of photons are accumulated as the visible energy, $E_{\text{vis}}$, which is the sum of the positron’s total energy and an electron’s rest energy,

$$E_{\text{vis}} \sim E_e + m_e \sim (E_\nu - 0.8) \text{ MeV}.$$  

(5)

The observed antineutrino distribution by a detector with $N_p$ free protons after an exposure time $T$ can then be expressed as

$$\frac{dN}{dE_{\text{vis}}^{\text{obs}}} \propto \frac{N_p T}{4\pi L^2} \int_{E_{\text{thr}}}^{\infty} dE \frac{dN}{dE} \cdot P_{ee}(L, E_\nu) \sigma_{\text{IBD}}(E_\nu) G(E_\nu - 0.8 \text{MeV} - E_{\text{vis}}^{\text{obs}}, \delta E_{\text{vis}}),$$

(6)

where $\sigma_{\text{IBD}}(E_\nu)$ is the cross section of the IBD process [21], $G$ is the detector response function with the energy resolution $\delta E_{\text{vis}}$, and $E_{\text{vis}}^{\text{obs}}$ is the observed visible energy by the detector. In this study, we take the normalized gaussian function as the response function, i.e.,

$$G(E_{\text{vis}} - E_{\text{vis}}^{\text{obs}}, \delta E_{\text{vis}}) = \frac{1}{\sqrt{2\pi}\delta E_{\text{vis}}} \exp \left\{ -\frac{(E_{\text{vis}} - E_{\text{vis}}^{\text{obs}})^2}{2(\delta E_{\text{vis}})^2} \right\}.$$  

(7)

The detector energy resolution, $\delta E_{\text{vis}}$, is parameterized into two parts,

$$\frac{\delta E_{\text{vis}}}{E_{\text{vis}}} = \sqrt{\left( \frac{a}{\sqrt{E_{\text{vis}}/\text{MeV}}} \right)^2 + b^2}.$$  

(8)

The first term in the square root represents the statistical uncertainty, and the second one gives the systematic uncertainty [22].
TABLE I: The input values $Y^{\text{input}}$ and their uncertainties $\delta Y$ taken from refs. \[1, 2, 23\]. The uncertainty of $\sin^2 2\theta_{13}$ can be 5\% or less after 3 years running of Daya Bay experiment \[24\].

| $Y$ | $\sin^2 2\theta_{12}$ | $\sin^2 2\theta_{13}$ | $\Delta m^2_{21}$ eV$^2$ | $|\Delta m^2_{31}|$ eV$^2 | f_{\text{sys}}$ |
|-----|-----------------|-----------------|-----------------|-----------------|-----------------|
| $Y^{\text{input}}$ | 0.857 | 0.089 | $7.50 \times 10^{-5}$ | $2.32 \times 10^{-3}$ | 1 |
| $\delta Y$ | 0.024 | 0.005 | $0.20 \times 10^{-5}$ | $0.1 \times 10^{-3}$ | 0.03 |

III. THE SENSITIVITY TO THE MASS HIERARCHY

After obtaining the energy distribution of reactor antineutrinos, we would like to estimate the sensitivity of determining the mass hierarchy using the standard $\chi^2$ analysis \[2, 8, 11, 13–15\].

To set the stage, we introduce the $\chi^2$ function as

$$\chi^2 = \chi^2_{\text{para}} + \chi^2_{\text{stat}}.$$ (9)

The first term summarizes the prior knowledge on fitting parameters. In reactor antineutrino experiments, these are the mixing angles, $\sin^2 2\theta_{12}$ and $\sin^2 2\theta_{13}$, and the two mass-square differences, $\Delta m^2_{21}$ and $|\Delta m^2_{31}|$. In this study we also consider the event-number normalization factor $f_{\text{sys}}$, assuming the 3\% uncertainty. Their contributions look like,

$$\chi^2_{\text{para}} = \left\{ \frac{(\sin^2 2\theta_{12})^{\text{fit}} - (\sin^2 2\theta_{12})^{\text{input}}}{\delta \sin^2 2\theta_{12}} \right\}^2 + \left\{ \frac{(\sin^2 2\theta_{13})^{\text{fit}} - (\sin^2 2\theta_{13})^{\text{input}}}{\delta \sin^2 2\theta_{13}} \right\}^2 + \left\{ \frac{(\Delta m^2_{21})^{\text{fit}} - (\Delta m^2_{21})^{\text{input}}}{\delta \Delta m^2_{21}} \right\}^2 + \left\{ \frac{(|\Delta m^2_{31}|)^{\text{fit}} - (|\Delta m^2_{31}|)^{\text{input}}}{\delta |\Delta m^2_{31}|} \right\}^2 + \left( \frac{f_{\text{sys}}^{\text{fit}} - f_{\text{sys}}^{\text{input}}}{\delta f_{\text{sys}}} \right)^2.$$ (10)

The input values $Y^{\text{input}}$ and their uncertainties $\delta Y$ are listed in Table I.

The second term in \[9\] represents the statistical fluctuation. When we binning w.r.t. $F_{\text{vis}}^{\text{obs}}$, it looks like

$$\chi^2_{\text{stat}} = \sum_i \left( \frac{N_i^{\text{fit}} - N_i^{\text{NH(IH)}}}{\sqrt{N_i^{\text{NH(IH)}}}} \right)^2.$$ (11)

with the summation running over all the bins. Here, $N_i^{\text{NH(IH)}}$ is the event number for the $i_{\text{th}}$ bin when the hierarchy is NH (IH), while $N_i^{\text{fit}}$ is the theoretical prediction of the event number either with right or wrong mass hierarchy, calculated as a function of the four model parameters and the normalization factor $f_{\text{sys}}$, which are all varied under the constraints of \[10\]. In this study we prepare the data $N_i^{\text{NH(IH)}}$ by using eq. \[6\] with the input values of the five parameters for each mass hierarchy.

In the limit of infinitely many events, the bin size can be reduced to zero, and the sum \[11\] can be replaced by an integral. Although a finite bin size is required for actual experiments, we adopt this zero-bin-size limit as a measure of the maximum sensitivity.

We then define $\Delta \chi^2$ as

$$\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}},$$ (12)

where $\chi^2_{\text{min}}$ is the minimum of $\chi^2$, which is obviously zero in our approximation of neglecting statistical fluctuations in data, $N_i^{\text{NH(IH)}}$. When wrong mass hierarchy is assumed in the fit, the minimum of $\Delta \chi^2$, $(\Delta \chi^2)_{\text{min}}$, will deviate from zero, and the wrong mass hierarchy can be rejected with the significance $\sqrt{(\Delta \chi^2)_{\text{min}}}$.
IV. RESULTS

In this section, we discuss the sensitivity to the mass hierarchy, the optimal length and the statistical uncertainties of the neutrino parameters. All our results are obtained by assuming a reactor of 20 GW$_{th}$ thermal power, a far detector of 5 kton fiducial volume with 12% weight fraction of free proton and 5 years exposure time.

![Graph showing energy distributions for different baseline lengths.](image)

**Fig. 1**: The energy distribution of reactor antineutrinos with 20 GW$_{th}$, 5kt (12% free-proton weight fraction)-5yrs exposure and the baseline length $L = 30$ km (left) and 50 km (right). **Upper**: The case with exact $E_{\nu}$ measurement, where the dashed blue and dashed red curves are for NH and IH, respectively. The solid curve shows the best fit of IH assumption to the NH data. The red arrow points out the energy at which the difference due to mass hierarchy vanishes. **Lower**: $6/\sqrt{E_{\text{vis}}}$ % energy resolution case.

Figures show energy distributions for $L = 30$ km (left) and 50 km (right), in which the exact $E_{\nu}$ measurement is assumed in the upper panel, whereas in the lower panel the energy resolution of $a = 6\%$ with $b = 0$ in eq. (8) is assumed. The dashed blue curve corresponds to the IH case, and the dashed red curve to the IH case, while the solid curve is obtained using the parameter values fitted to the NH data with the “wrong” IH assumption. The red arrow points out the energy at which the difference due to mass hierarchy vanishes (the degeneracy point).

At $L = 30$ km, the solid curve almost coincides with the dashed blue one even with the exact energy measurement, implying that it is almost impossible to distinguish the mass hierarchy by experiments at $L = 30$ km. This is because the small phase shift between the NH and IH predictions can be absorbed by a small shift in $|\Delta m^2_{31}|$ by a fraction of its present uncertainty, $0.1 \times 10^{-3}$eV$^2$.

The situation changes when the second peak of the mass-hierarchy dependent term appears in the energy range. The mass hierarchy difference can no longer be absorbed by a shift in $|\Delta m^2_{31}|$ since the relative phase difference between the NH and IH oscillations changes across the degeneracy point. There is no way to make the differences on the both sides compensated, resulting in the distinct mismatch between the dashed blue curve (the NH data) and the solid curve (the best fit under the IH assumption) as shown in the upper panel of the right plot in Fig. 1 where the antineutrino energy is exactly measured.

Once the finite energy resolution is introduced, the phase difference in the lower energy side of the degeneracy point is significantly smeared out as it oscillates faster w.r.t. $E_{\nu}$ at the low energy. Hence it is easier for one oscillation period to be covered by a sizable Gaussian profile of the detector response function. The remaining difference in the higher energy side can then be absorbed by a small shift in $|\Delta m^2_{31}|$, resulting in an excellent fit (solid curve) to the NH data (blue dashed curve) in the lower panel of the right plot in Fig. 1 shown for $6/\sqrt{E}/\text{MeV}$ energy resolution.

The left plot in Fig. 2 shows the resulted $(\Delta \chi^2)_{\text{min}}$ value as a function of the baseline length $L$, for several energy resolutions, $a = 2, 3, 4, 5$ and 6% (with $b = 0$) in eq. (8), from the top to the bottom. Solid curves are for NH, while dashed curves are for IH. The results clearly show that the mass hierarchy can be determined by those experiments only if the energy resolution of the detector is 3$/\sqrt{E}/\text{MeV}$ or better, and that the optimal baseline length (as shown by the cross symbol) is around 50 km for that resolution. The small $(\Delta \chi^2)_{\text{min}}$ for the baseline length $L < 40$ km and $L > 80$ km is due to a shift in $|\Delta m^2_{31}|$ and low statistics, respectively. For the $a = 5$ and 6% cases $(\Delta \chi^2)_{\text{min}}$ stays almost zero at all $L$. 

![Graph showing $(\Delta \chi^2)_{\text{min}}$ as a function of baseline length for different energy resolutions.](image)
FIG. 2: $(\Delta \chi^2)_{\text{min}}$ for mass hierarchy discrimination shown as a function of the baseline length $L$ for $20 \text{GW} \times 5 \text{kt}$ (12% free-proton weight fraction)·5yrs exposure. The solid curves are for the NH cases and dashed curves for the IH cases. The cross symbols mark the optimal baseline lengths. **Left:** The energy resolution in eq. (8) is varied with $a = 2$ to 6% and $b = 0$, from the top to the bottom. **Right:** The energy resolution is varied with $a = 2\%$ and $b = 0\%, 0.5\%, 0.75\%, 1\%$, from the top to the bottom.

The right plot in Fig. 2 shows the $(\Delta \chi^2)_{\text{min}}$ value as a function of the baseline length $L$ for different $b$ values with $a = 2\%$. The curves from the top to the bottom are obtained for $b = 0\%, 0.5\%, 0.75\%$ and 1%, respectively. The effect of the systematic uncertainty is significant as discussed in ref. [13], reducing the peak value of $(\Delta \chi^2)_{\text{min}}$ from 11.0 ($b = 0\%$) to 9.7 ($b = 0.5\%$), 8.4 ($b = 0.75\%$) and 6.9 ($b = 1\%$) for NH. The optimal $L$ shortens from 51 km for $(a, b) = (2, 0)%$ to 47 km for $(a, b) = (2, 1)%$.

In addition, the neutrino parameters, $\sin^2 2\theta_{12}$, $\Delta m^2_{21}$ and $|\Delta m^2_{31}|$, can be measured accurately with statistical uncertainties shown in Fig. 3. We find

$$\delta \sin^2 2\theta_{12} \sim 4 \times 10^{-3} (0.5\%),$$

$$\delta \Delta m^2_{21} \sim 3 \times 10^{-7} \text{eV}^2 (0.4\%),$$

$$\delta |\Delta m^2_{31}| \sim 7 \times 10^{-6} \text{eV}^2 (0.3\%),$$

with the energy resolution of $(a, b) = (3, 0.5)\%$ at $L = 50$ km; the percentage values in the parentheses denote...
the relative accuracy of the measurement. Those uncertainties are almost independent of the mass hierarchy and of the energy resolution, with the only exception of the $|\Delta m^2_{31}|$ uncertainty for which the larger resolution results in the larger uncertainty. $\sin^2 2\theta_{13}$ and $|\Delta m^2_{31}|$ are measured most accurately around $L \sim 1$ km.

V. DISCUSSIONS AND CONCLUSION

In this proceedings we have investigated the sensitivity of medium-baseline reactor-electron-antineutrino oscillation experiments for determining the neutrino mass hierarchy by performing the standard $\chi^2$ analysis.

We have carefully studied the impacts of the energy resolution $(\delta E/E)^2 = (a/\sqrt{E/\text{MeV}})^2 + b^2$ and find that the sensitivity strongly depends on it. The optimal baseline length is found to depend slightly on the energy resolution, preferring the length slightly shorter than 50 km for the energy resolution of $(a, b) = (2, 0.75)\%$ and $(2, 1)\%$. At the optimal baseline length, the energy resolution better than the $3%/\sqrt{E/\text{MeV}}$ level is needed to determine the neutrino mass hierarchy pattern. For a 5 kt detector (with 12% weight fraction of free proton) placed at $L \sim 50$ km away from a 20 GWth reactor, an experiment would determine the mass hierarchy pattern with $\Delta \chi^2 \sim 9$ on average after five or more years of running if the energy resolution of $(a, b) = (2, 0.5)\%$ is achieved, while a factor of three larger or longer experiment is needed to achieve the same goal for the energy resolution of $(a, b) = (3, 0.5)\%$.

It is also found that this experiment can measure the neutrino parameters, $\sin^2 2\theta_{12}$, $\Delta m^2_{21}$, and $|\Delta m^2_{31}|$, very accurately as shown in [13] for an experiment of 20 GWth·5kt (12% free-proton weight fraction)·5yrs at $L \sim 50$ km.

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