A Grey Incidence Based Group Decision-Making Approach and Its Application

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Abstract: We define the group measure matrix of the alternative scheme and the ideal scheme based on the relevant factor sequence and the system characteristic behavior sequence. Furthermore, the information distances of decision makers and decision criteria are defined, respectively. According to the information distance, we obtain each scheme’s grey matrix incidence degree for the scheme ranking. Finally, we use an example to verify the rationality of the model and compare it with other classic methods, such as TOPSIS, VIKOR, MULTI-MOORA. Compared with previous grey incidence analysis model, the proposed model can make full use of information of the decision-maker dimension and the criteria dimension. The proposed model can avoid high-dimensional information loss. The results show that the proposed method has superiority in measuring decision-maker information and decision-making standard information.

Keywords: group decision-making; scheme ranking; grey incidence analysis; scheme matrix

MSC: 20C40

1. Introduction

The grey incidence analysis, underpinning the grey system analysis, grey decision making as well as grey clustering, is essential to the grey system theory. and it is also the cornerstone of grey system analysis, being clearly different from the statistical method which usually require large amounts of samples, the grey incidence analysis caters to the circumstance with small sample size. As a result, it is often used to illustrate the connection or influence degree among system variables. The basic idea is to determine whether the link between different sequences is tight according to the geometry of the sequence curve [1,2]. Due to its unique advantages, the grey incidence analysis model has been becoming a hot issue for domestic and foreign scholars since it was proposed. According to the application scope and data characteristics, grey incidence analysis mainly includes grey absolute incidence degree model, grey similarity analysis model, T incidence degree model, B incidence degree model, C incidence degree model, grey entropy incidence degree model, slope incidence degree model, etc. Scholars have applied the grey incidence analysis as a multi-criteria decision-making (MCDM) method to solve problems in many fields such as international trade comparison [3], project management [4], supplier selection [5] and financial performance assessment [6].

To deal with the MCDM problems more effectively, some scholars combined the grey incidence analysis model with some common MCDM methods such as analytic Network Process (ANP), analytic hierarchy process (AHP), Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) to construct novel MCDM methods based on grey incidence analysis. For example, Pakkar [7] properly selected the attribute weights in grey incidence analysis through using the data envelopment analysis (DEA) and AHP to drive attribute weights reasonably. In addition, Samvedi et al. [8] used fuzzy AHP...
to calculate the priority weights of the criteria, and employed grey incidence analysis to rank the alternatives while Baranitharan et al. [9] compared the differences between grey relational analysis and TOPSIS. To overcome the conflicting nature of the evaluation criteria, Prabhu and Ilangkumaran [10] integrated grey incidence analysis and TOPSIS to construct a novel MCDM method. Yazdani et al. [11] applied quality function deployment (QFD) to consider the external factors of incomplete independent attributes, and proposed a fuzzy multi-attribute decision framework based on grey incidence analysis. Giri et al. [12] extended the grey incidence analysis method for solving SVTNN problem, where the attribute weight information is partially known or completely unknown. With respect to the decision problems with panel data, by considering the attributes with negative relational and space-time, Liu et al. [13,14] established a novel grey incidence analysis model by using the tangent function and grey object matrix incidence clustering model. Considering many noticeable advantages of Grey incidence analysis method, such as clear calculation process, lower data requirement, less workload, and great reduction of the losses caused by information asymmetry, some scholars have also proposed a series of extended grey incidence models based on the classic grey incidence analysis method. Wei [15] proposed three different grey incidence analysis based on real value, interval value and fuzzy value. Sun et al. [16] applied the grey incidence analysis to the Hesitant Fuzzy Sets (HFSs) and proposed the HFSs synthetic grey incidence degree. Based on the definition of traditional grey incidence models, Liu et al. [17] proposed grey similar incidence model and grey close incidence model. Yin et al. [18] constructed a new grey comprehensive relational model on the basis of weighted mean distance and induced intensity. In the aspect of group decision-making with grey incidence analysis, Dey [19] presented a neutrosophic soft multi-attribute group decision making based on grey incidence analysis. Hashemi et al. [20] presented a new group decision model based on IFSs theory, ELECTRE and VIKOR along with grey incidence analysis. Liu et al. [21] used grey incidence analysis to calculate the distance between the ideal bull’s eye and the scheme bull’s eye. Pramanik and Mukhopadhyaya [22] developed an intuitionistic fuzzy multi-criteria group making method with grey incidence analysis for teacher selection. Zare et al. [23] proposed a grey group decision-making approach with VIKOR, TOPSIS and grey incidence analysis. Mousavi et al. [24] introduced grey incidence analysis to solve group decision making problems with conventional fuzzy information. Pamucar et al. [25] didified Normalized weighted geometric Dombi Bonferroni mean operator. Ulutas et al. [26] construct a new integrated grey MCDM model. Badi et al. [27] proposed a combined Grey-MARCOS.Regarding the application of the grey incidence analysis method, existing research mainly involves energy systems evaluation [28], healthcare service [29], performance measurement [30], construction projects [31], qualitative analysis [32], honeycomb core [33], Single Crystal Silicon [34], static tensile properties of structural steel [35], magnetic abrasive finishing [36], etc.

Existing literature on group decision-making based on grey incidence analysis only uses grey incidence analysis method as a supplement to obtain weights or combine with other methods. They did not build a corresponding grey incidence model based on the characteristics of multiple decision makers and multiple evaluation criteria in group decision problems. The dimensionality reduction method of high-dimensional information in the existing research literature is easy to cause information loss, which restricts the application space of grey incidence analysis and makes it difficult to aggregate the information of the decision-maker dimension and the criteria dimension. Considering that the ranking of schemes is mainly affected by decision-maker information and evaluation criteria information in the process of the multi-criteria group decision-making, this paper establishes the grey scheme matrix incidence analysis method to solve the ranking issue.

The remainder of this paper is organized as follows: a grey scheme matrix incidence analysis model is proposed in Section 2. In Section 3, a real case illustrates the validity and rationality of the proposed model. Some conclusions are given in Section 4.
2. A Grey Scheme Matrix Incidence Analysis Model

Because of the variation of attribute values resulting from selecting different decision makers in the multi-criteria group decision-making, the decision state in the decision scheme will be changeable. It can be said that the scheme ranking of group decision making is mainly affected by two factors: decision-maker information and decision-criteria information. In this regard, this study takes decision-making schemes as the research object, and establishes a novel multi-criteria grey scheme matrix incidence analysis model.

For multi-attribute group decision-making problem, its basic elements are the schemes, attributes, decision makers, and the evaluation ranges, so that a multi-criteria group decision making problem could be denoted as a four elements set $S = \{X, C, E, V\}$, $X = \{x_1, \ldots, x_n\}$, $C = \{c_1, \ldots, c_m\}$ and $E = \{e_1, \ldots, e_p\}$ express a set of schemes, criteria and decision makers, respectively, and $V = \cup v^k_j (i = 1, 2, \ldots, n; j = 1, 2, \ldots, m; k = 1, 2, \ldots, p)$ expresses the evaluation range of the schemes for the decision makers over multi-criteria, where $v^k_j$ is the value of $x_i$ under criterion $c_j$ evaluated by decision maker $e_k$. To measure the importance of criteria and decision makers, their weights should be given. Assume that $w_k = (w_{1k}, \ldots, w_{jk}, \ldots, w_{mk})$, $\eta = (\eta_1, \ldots, \eta_{k1}, \ldots, \eta_{kn})$ are the self-cognition criteria weight vector, comprehensive criteria weight vector and decision maker weight vector, respectively. Among them, $0 \leq w_{jk} \leq 1$, $\sum_{j=1}^{m} w_{jk} = 1, 0 \leq \eta_k \leq 1, \sum_{k=1}^{p} \eta_k = 1, w_{jk}$ represents the weight of the criterion $c_j$ given by decision maker $e_k$, $w_{jk}$ represents the weight of the criterion $c_{jk}$, and $\eta_k$ represents the weight of the decision maker $e_k$.

The proposed model mainly includes the following steps.

Step 1 Collect evaluation information given by each decision maker with respect to criteria set, weights of criteria and decision makers, and normalize data.

Due to the complexity and uncertainty of the objective world and the limitations of human understanding, decision makers’ evaluation value of the plan under different standards is given in the form of interval numbers [37]. The upper and lower limit of interval number represents the maximum and minimum evaluation value of each scheme evaluated by the decision maker under the criterion.

**Definition 1.** Let $v_{ij}^k = [v_{ij}^k-, v_{ij}^k+]$ be the initial evaluation interval value of scheme under multiple experts and multiple criteria, $u_{ij}^k = [u_{ij}^k-, u_{ij}^k+]$ be the standardized (dimensionless) interval value of initial evaluation value. Then, the method of converting $v_{ij}^k$ to $u_{ij}^k$ is as follows.

\[
 u_{ij}^k = \begin{cases} 
 v_{ij}^k - \min_{1 \leq p \leq m} v_{ij}^k, & j \in J^+ \\
 \max_{1 \leq p \leq m} v_{ij}^k - v_{ij}^k, & j \in J^- 
\end{cases}
\]

where, $J^+$ represents benefit criteria, $J^-$ represents cost criteria.

**Definition 2.** For $\forall x_i \in X, \forall c_j \in C, \forall e_k \in E$, if $u_{ij}^k (i = 1, 2, \ldots, n; j = 1, 2, \ldots, m; k = 1, 2, \ldots, p)$ represents the dimensionless measure of the evaluation value $v_{ij}^k$, then

\[
 U_i = \begin{bmatrix} 
 u_{i1}^1 & \cdots & u_{i1}^k & \cdots & u_{i1}^m \\
 \vdots & \ddots & \vdots & \ddots & \vdots \\
 u_{ij}^1 & \cdots & u_{ij}^k & \cdots & u_{ij}^m \\
 \vdots & \ddots & \vdots & \ddots & \vdots \\
 u_{im}^1 & \cdots & u_{im}^k & \cdots & u_{im}^m 
\end{bmatrix}
\]

is called as the group measure matrix of the decision-making scheme $x_i$. 
Step 2 Determine group measure matrix of the ideal decision scheme.

In the grey incidence analysis, the system behavior sequence, referred to the system behavior characteristics, must first be determined. However, due to the influence of decision makers’ interests and knowledge, it is difficult to achieve an optimal system behavior scheme, termed as the ideal decision scheme, in group decision-making. Therefore, we have the following Definition 3.

**Definition 3.** For \( \forall x_i \in X, \forall c_j \in C, \forall e_k \in E \), by the transformed formula of interval numbers \([38]\) let \( u_{0ij}^k = \max_i \{(1 - \theta)u_{ij}^{k-} + \theta u_{ij}^{k+}\} \), \( \theta \) reflects the risk attitude of decision maker, generally \( \theta = 0.5 \),

\[
\begin{bmatrix}
u_{01} & \cdots & \nu_{0j} & \cdots & \nu_{0m} \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
\nu_{p1} & \cdots & \nu_{pj} & \cdots & \nu_{pm} 
\end{bmatrix}
\]

is called as the group measure matrix of the ideal decision scheme. In the following, it will be used to calculate the distance to the alternative schemes for the deduction of the grey incidence degree for the ranking of the alternative schemes.

In the process of group decision-making, the decision makers determine the evaluation values according to the criteria of decision-making scheme. The evaluation values given by the decision makers of different decision scheme under different criteria are different. Therefore, it is necessary to measure the differences of schemes among the decision makers and among the criteria, thereby the grey scheme matrix absolute incidence analysis model is established.

**Step 3 Calculate the distance of schemes among the decision makers and among the criteria.**

**Definition 4.** For \( \forall x_i \in X, \forall c_j \in C, \forall e_k \in E \), if \( u_i^k = \{u_{i1}^k, \cdots, u_{ij}^k, \cdots, u_{im}^k\} \) and \( u_0^k = \{u_{01}^k, \cdots, u_{0j}^k, \cdots, u_{0m}^k\} \) denote the decision scheme \( x_i \) and the ideal decision scheme of decision maker \( e_k \), respectively, then

\[
d_1^{0ij} = \frac{\Delta_1^{0ij}}{\sum_{k=1}^{p} \eta_k u_{0j}^k} = \sqrt{\frac{\sum_{k=1}^{p} \eta_k (\hat{u}_{ij}^k - u_{0j}^k)^2}{\sum_{k=1}^{p} \eta_k u_{0j}^k}} \tag{2}
\]

\[
d_2^{0ij} = \Delta_2^{0ij} = \sum_{k=1}^{p} \eta_k |\hat{u}_{ij}^k - u_{0j}^k| \tag{3}
\]

is called as the decision-maker information distance and decision-criteria information distance over criterion \( c_j \) between decision scheme \( i \) and ideal decision scheme, respectively. In order to simplify the calculation, \( \hat{u}_{ij}^k = (1 - \theta)u_{ij}^{k-} + \theta u_{ij}^{k+} \), is called the the transformed formula of interval number \( u_{ij}^k \).

**Step 4 Calculate grey matrix incidence degree between each scheme and ideal scheme.**

Through the process of Equations (2) and (3), we can aggregate the decision-makers evaluation information and criteria information contained in the scheme evaluation matrix. For convenience, the ideal decision scheme can be denoted as \( x_0 \), and then the grey matrix incidence analysis method is given as follows.
\textbf{Definition 5.} Let $U_i$ and $U_0$ stand for the group measure value matrix of the decision scheme $x_i$ and the ideal decision scheme $x_0$ respectively, for $\forall x_i \in X, \forall c_j \in C, \forall e_k \in E$, the expression

$$\xi_{0ij} = \min_i \min_j \frac{d_{0ij}^3 + \rho \max_i \max_j d_{0ij}^3}{d_{0ij}^3 + \rho \max_i \max_j d_{0ij}^3}$$

(4)

is called as the grey matrix incidence coefficient over the criteria $c_j$ between the decision scheme $x_i$ and ideal decision scheme $x_0$.

Where, $d_{0ij}^3 = ad_{0ij}^1 + (1-a)d_{0ij}^2, 0 \leq a \leq 1, w_j = \sum_{k=1}^{p} \eta_k w_j^k$

$$\xi_i = \sum_{j=1}^{m} w_j \xi_{0ij}$$

(5)

is called as the grey matrix incidence degree over the criteria set between the decision scheme $x_i$ and ideal decision scheme $x_0$.

\textbf{Theorem 1.} The scheme matrix similarity incidence model has the following basic properties:

1. Normalization;
2. Proximity;
3. Comparability;
4. Uniqueness.

\textbf{Proof.} Due to $d_{0ij}^1 \geq 0$, $d_{0ij}^2 \geq 0$, according to the expression (2) and (3), one can obtain $0 \leq d_{0ij}^3 \leq 1, 0 < \xi_{0ij} \leq 1$ such that $0 < \xi_i \leq 1$. $\square$

Proximity, Comparability and Uniqueness is some characteristics of the criteria value and doesn’t to be proved.

\textbf{Theorem 2.} For $0 \leq a \leq 1$, the grey incidence degree of the scheme is an increasing function over parameter $a$.

\textbf{Proof.} For $\forall a_i \in A, \forall c_j \in C, \forall e_k \in E$, it holds true as follows:

$$\xi_{0ij}(a) = \frac{\min_i \min_j \frac{d_{0ij}^3 + \rho \max_i \max_j d_{0ij}^3}{d_{0ij}^3 + \rho \max_i \max_j d_{0ij}^3}}{\min_i \min_j \frac{[ad_{0ij}^1 + (1-a)d_{0ij}^2] + \rho \max_i \max_j [ad_{0ij}^1 + (1-a)d_{0ij}^2]}{[ad_{0ij}^1 + (1-a)d_{0ij}^2] + \rho \max_i \max_j [ad_{0ij}^1 + (1-a)d_{0ij}^2]}}$$

(6)

Let $\min_i \min_j d_{0ij}^3 = A, \rho \max_i \max_j d_{0ij}^3 = B$, such that $\xi_{0ij}(a) = \frac{A + B}{[ad_{0ij}^1 + (1-a)d_{0ij}^2] + B}$.

Generally speaking, $A, B$ is generally constant, one can determine that $\xi_i(a) = \sum_{j=1}^{m} w_j \xi_{0ij}(a)$ is a function on $a$, it is

$$\xi_i(a) = \sum_{j=1}^{m} w_j \frac{A + B}{[ad_{0ij}^1 + (1-a)d_{0ij}^2] + B}$$

(7)

Solve the derivative for $\xi_i(a)$ on $a$, one can acquire as follows:

$$\xi_i(a)' = (A + B) \left\{ \sum_{j=1}^{m} \frac{w_j}{[ad_{0ij}^1 + (1-a)d_{0ij}^2] + B} \right\}$$

(8)
where, 
\[
\frac{d}{\partial a} \left( \frac{\left| \omega_1 \right|}{\left| \omega_2 \right|} \right) = \frac{\omega_i \left( d_{im}^2 - d_{im}^0 \right)}{\left| \omega_1 \right|} \geq 0.
\]

Because 
\[
\frac{\omega_i \left( d_{im}^2 - d_{im}^0 \right)}{\left| \omega_1 \right|} \geq 0, i = 1, 2, \ldots, m
\]

one can know that \( \xi_i(a) \) is a monotone increasing function on \( a \). And the grey incidence degree of decision scheme is an increasing function on \( a \).

Therefore, the theorem is determined. □

According to Theorem 2 infers, when \( a \) increases, the grey incidence degree will become larger. In addition, it is indicated that the more information extracted by decision makers, the larger evaluation value of the decision schemes. Moreover, the higher the degree of attention to attribute information, the smaller the evaluation value of the decision-making scheme, which indicates the decision-maker information is more influential than the attribute information. For any two schemes \( a_i, a_s, 0 \leq a \leq 1 \), if \( \xi_i(a) \geq \xi_s(a) \), then the scheme \( x_i \) is not inferior to the scheme \( x_s \), and vice versa.

Step 5 Sort the schemes according to grey matrix incidence degree.

The larger the value of \( \xi_i \), the better the scheme.

To clearly present the decision-making process, the modeling steps of the proposed method are summarized as shown in Figure 1.

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**Step 1** Collect evaluation information given by each decision maker with respect to criteria set, weights of criteria and decision makers, and normalize data by using Equation (1).

**Step 2** Determine group measure matrix of the ideal decision scheme by Definition 3.

**Step 3** Calculate the differences of schemes among the decision makers and among the criteria by Equation (2) and Equation (3).

**Step 4** Calculate grey matrix incidence degree between each scheme and ideal scheme by Equation (4) and Equation (5).

**Step 5** Sort the schemes according to grey matrix incidence degree.

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**Figure 1.** The decision-making process of proposed model.
3. Case Analysis

The pharmaceutical industry is characterized with large investments, high risks and long cycles. For pharmaceutical enterprises, research and development (R&D) on new drug is the basis to obtain core competitiveness and high development. However, due to technology, policy, market demand and other reasons, many new drugs cannot be listed at last. Therefore, R&D should be evaluated for the reduction of risk in new drug projects. The selection process of a new drug R&D project is a complex dynamic process. Thus, a scientific and objective evaluation method must be established to ensure the rational use of R&D investment and reduce, reducing the investment risk, which facilitate the rational selection of new drug R&D projects.

With respect to pharmaceutical enterprises, Investment of a lot of money and man-power is inevitable. New drug research and development projects not only bring new development impetus to enterprises, but also bring huge risks to enterprises. Therefore, factors should be considered in the decision-making process, for example, the company’s comprehensive capabilities, R&D level, management level, corporate financial status, risks and costs of new drug R&D, and future profitability after the drug is launched. With these influencing factors, an attribute system involved in the six aspect of management evaluation, technical evaluation, market evaluation, financial evaluation, risk assessment and cost assessment could be established, as \( c_1, c_2, c_3, c_4, c_5 \), respectively. Based on the principles of science, completeness, validity, operability, comparability and simplicity. For instance, there exists a pharmaceutical enterprise in Wuxi city, and it plans to start a R&D project on new drug. Considering current manufacturing equipment and R&D level, the pharmaceutical enterprise has found 6 candidates R&D projects on new drugs denoted as \( x_1, x_2, x_3, x_4, x_5, x_6 \). To obtain the evaluation values, the pharmaceutical enterprise invites 5 experts in corresponding fields and records them as \( e_1, e_2, e_3, e_4, e_5 \). The five experts are mainly temporarily composed of new drug research and development personnel, new drug market personnel, drug supervision and administration departments, new drug consumer agents and new drug investors. According to the information about management level of enterprise, technical and financial evaluation, enterprise brand effect and market share, project risk, cost assessment, sales of similar drugs in the market, revenue of similar R&D projects, market capacity of drugs, internal rate of return and payback period provided by the pharmaceutical enterprise, the 5 experts will make evaluation information of the 6 R&D projects on new drugs.

Step 1 Collect evaluation information given by each decision maker with respect to criteria set, weights of criteria and decision makers, and normalize data.

Due to the incomplete information provided by the enterprise and the limitation cognition of expert’s, the evaluation values made by 5 experts are given in the form of interval values. The initial evaluation values have been transformed into standard interval evaluation values. The specific evaluation values are shown as follows:

\[
U_1 = \begin{bmatrix}
0.4596, 0.5143 & 0.4982, 0.5226 & 0.4985, 0.5004 & 0.4753, 0.5166 & 0.4462, 0.5064 & 0.4825, 0.5242 \\
0.4902, 0.5054 & 0.4905, 0.5117 & 0.4909, 0.5168 & 0.4988, 0.5069 & 0.4862, 0.5029 & 0.4798, 0.5111 \\
0.4716, 0.5064 & 0.4856, 0.5162 & 0.4988, 0.5178 & 0.4968, 0.5169 & 0.5023, 0.5312 & 0.4689, 0.5001 \\
0.4853, 0.5215 & 0.4762, 0.5059 & 0.4762, 0.5298 & 0.4896, 0.5211 & 0.4962, 0.5139 & 0.4937, 0.5002 \\
0.4925, 0.5187 & 0.5018, 0.5157 & 0.4863, 0.5117 & 0.5024, 0.5241 & 0.4789, 0.4999 & 0.4853, 0.5189 \\
\end{bmatrix}
\]

\[
U_2 = \begin{bmatrix}
0.4632, 0.5366 & 0.4878, 0.5176 & 0.4932, 0.5118 & 0.4998, 0.5233 & 0.4785, 0.5236 & 0.4826, 0.5089 \\
0.4846, 0.5127 & 0.4977, 0.5063 & 0.4884, 0.5006 & 0.4962, 0.5099 & 0.4816, 0.5069 & 0.4983, 0.5076 \\
0.4875, 0.5309 & 0.4962, 0.5244 & 0.4856, 0.4939 & 0.5047, 0.5066 & 0.4963, 0.5187 & 0.5027, 0.5112 \\
0.4753, 0.5026 & 0.4711, 0.4863 & 0.5013, 0.5234 & 0.4937, 0.5122 & 0.4755, 0.4986 & 0.4735, 0.4981 \\
0.4958, 0.5124 & 0.4811, 0.5012 & 0.4788, 0.5033 & 0.4899, 0.4999 & 0.4935, 0.5088 & 0.5028, 0.5214 \\
\end{bmatrix}
\]
According to the importance and knowledge background of experts and the authority in drug industry, the pharmaceutical enterprise gives the weight of these five experts based on self-cognition. Their weights are: \( \eta = \{ \eta_1, \eta_2, \eta_3, \eta_4, \eta_5 \} = \{0.05, 0.1, 0.3, 0.35, 0.2\} \). In addition, due to differences in background, status, etc., each expert has his own opinion on the weight of attributes. The expert self-cognition weights of attributes are shown below.

\[
\begin{align*}
  w^1 &= (w_{1}^1, w_{2}^1, w_{3}^1, w_{4}^1, w_{5}^1, w_{6}^1) = (0.15, 0.10, 0.15, 0.27, 0.25, 0.08) \\
  w^2 &= (w_{1}^2, w_{2}^2, w_{3}^2, w_{4}^2, w_{5}^2, w_{6}^2) = (0.12, 0.02, 0.30, 0.35, 0.15, 0.06) \\
  w^3 &= (w_{1}^3, w_{2}^3, w_{3}^3, w_{4}^3, w_{5}^3, w_{6}^3) = (0.20, 0.10, 0.32, 0.15, 0.18, 0.05) \\
  w^4 &= (w_{1}^4, w_{2}^4, w_{3}^4, w_{4}^4, w_{5}^4, w_{6}^4) = (0.10, 0.03, 0.28, 0.26, 0.31, 0.02) \\
  w^5 &= (w_{1}^5, w_{2}^5, w_{3}^5, w_{4}^5, w_{5}^5, w_{6}^5) = (0.17, 0.01, 0.09, 0.09, 0.54, 0.10)
\end{align*}
\]

According to the calculation method of \( w_j = \sum_{k=1}^{p} \eta_k w^k_j \), the comprehensive weight of the attribute can be obtained where \( w = (w_1, w_2, w_3, w_4, w_5, w_6) = (0.15, 0.05, 0.25, 0.2, 0.3, 0.05) \).

Step 2 Determine group measure matrix of the ideal decision scheme. Suppose the risk attitude of all decision makers is \( \theta = 0.5 \). By the Definition 3, we can obtain group measure matrix \( U_0 \) of the ideal decision scheme as follows:

\[
U_0 = \begin{bmatrix}
0.5274 & 0.5309 & 0.5106 & 0.5116 & 0.5063 & 0.5065 \\
0.5129 & 0.5026 & 0.5057 & 0.5230 & 0.5022 & 0.5030 \\
0.5140 & 0.5192 & 0.5083 & 0.5135 & 0.5168 & 0.5108 \\
0.5061 & 0.5193 & 0.5124 & 0.5054 & 0.5187 & 0.5057 \\
0.5056 & 0.5088 & 0.5044 & 0.5146 & 0.5012 & 0.5121 \\
0.5061 & 0.5088 & 0.5044 & 0.5146 & 0.5012 & 0.5121 \\
\end{bmatrix}
\]
Step 3 Calculate the differences of schemes among the decision makers and among the criteria. According to Equations (2) and (3), we can calculate \( d_{0ij}^1 \) and \( d_{0ij}^2 \) as follows:

\[
\begin{bmatrix}
\begin{array}{cccccc}
0.0172 & 0.0200 & 0.0065 & 0.0081 & 0.0099 & 0.0161 \\
0.0129 & 0.0265 & 0.0123 & 0.0117 & 0.0196 & 0.0122 \\
0.0441 & 0.0232 & 0.0251 & 0.0173 & 0.0228 & 0.0167 \\
0.0108 & 0.0209 & 0.0231 & 0.0211 & 0.0192 & 0.0225 \\
0.0074 & 0.0217 & 0.0135 & 0.0138 & 0.0280 & 0.0096 \\
0.0136 & 0.0093 & 0.0094 & 0.0097 & 0.0121 & 0.0118 \\
\end{array}
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.0120 & 0.0165 & 0.0051 & 0.0051 & 0.0080 & 0.0138 \\
0.0105 & 0.0219 & 0.0096 & 0.0091 & 0.0149 & 0.0086 \\
0.0432 & 0.0212 & 0.0225 & 0.0152 & 0.0194 & 0.0143 \\
0.0074 & 0.0191 & 0.0223 & 0.0192 & 0.0175 & 0.0194 \\
0.0062 & 0.0199 & 0.0084 & 0.0100 & 0.0206 & 0.0065 \\
0.0103 & 0.0031 & 0.0084 & 0.0070 & 0.0084 & 0.0095 \\
\end{array}
\end{bmatrix}
\]

Step 4 Calculate grey matrix incidence degree between each scheme and ideal scheme. According to Equations (4) and (5), let \( \rho = 0.5, a = 0.5 \) the grey matrix incidence degree of each scheme is

\[
\begin{align*}
\xi_1 &= 0.7690, \\
\xi_2 &= 0.7115, \\
\xi_3 &= 0.5711, \\
\xi_4 &= 0.6323, \\
\xi_5 &= 0.6934, \\
\xi_6 &= 0.8030.
\end{align*}
\]

Step 5 Sort the schemes according to grey matrix incidence degree. The ranking result is show as

\[
x_6 \succ x_1 \succ x_2 \succ x_5 \succ x_4 \succ x_3.
\]

According to the scheme ranking, we can know that the project 6 is the optimal project on new drug, so the pharmaceutical enterprise can preferentially support project 6 to industrialize; project 1 and project 2 have certain strengths, and the pharmaceutical enterprises can take them as alternative projects; project 3 and project 4 overall lack competitiveness, and the pharmaceutical enterprise cannot consider supporting.

4. Comparison and Discussion

In the existing research on multi-criteria decision-making methods and group multi-criteria decision-making methods based on grey incidence analysis [39,40], scholars have deeply explored the influence of weights and distinguishing coefficient on decision-making results. The basis of the method in this paper is grey incidence analysis, so this paper focuses on the sensitivity analysis of the parameter \( a \). Let parameter \( a \) verify from 0 to 1 with step 0.1. Based on the proposed model, we calculate grey incidence degree of each scheme with different parameter values and the results are shown in Table 1. And the Figure 2 shows the quadratic fitting curve under different parameter \( a \) according to the grey incidence value from Table 1.

| \( a \) | \( x_1 \) | \( x_2 \) | \( x_3 \) | \( x_4 \) | \( x_5 \) | \( x_6 \) |
|---|---|---|---|---|---|---|
| 0.0 | 0.7517 | 0.6920 | 0.5420 | 0.5981 | 0.6819 | 0.7839 |
| 0.1 | 0.7566 | 0.6975 | 0.5501 | 0.5981 | 0.6851 | 0.7892 |
| 0.2 | 0.7606 | 0.7019 | 0.5567 | 0.6153 | 0.6878 | 0.7936 |
| 0.3 | 0.7638 | 0.7057 | 0.5623 | 0.6218 | 0.6900 | 0.7972 |
| 0.4 | 0.7666 | 0.7088 | 0.5670 | 0.6274 | 0.6918 | 0.8003 |
| 0.5 | 0.7690 | 0.7115 | 0.5711 | 0.6323 | 0.6934 | 0.8030 |
| 0.6 | 0.7710 | 0.7138 | 0.5747 | 0.6365 | 0.6948 | 0.8053 |
| 0.7 | 0.7728 | 0.7159 | 0.5778 | 0.6402 | 0.6960 | 0.8074 |
| 0.8 | 0.7743 | 0.7176 | 0.5805 | 0.6435 | 0.6971 | 0.8092 |
| 0.9 | 0.7757 | 0.7192 | 0.5830 | 0.6446 | 0.6980 | 0.8108 |
| 1.0 | 0.7769 | 0.7207 | 0.5852 | 0.6491 | 0.6989 | 0.8122 |
Table 1 and Figure 1 show that the grey incidence degree of scheme increases when the parameter value \( a \) increases, but their growth rate is different. And no matter how the parameters change, it does not affect the ranking of the schemes, and the results of schemes ranking are shown as follows: \( x_6 \succ x_1 \succ x_2 \succ x_5 \succ x_4 \succ x_3 \). According to the calculation results and Figure 1, we can get some interesting results. When \( a \) is small, the grey incidence degree between project 5 and project 2 is small, and the grey incidence degree between project 5 and project 4 is large, that is, when the information of the decision-maker dimension is reduced, the distinction between project 5 and project 2 is small, and the distinction between project 5 and project 4 is greater. When \( a \) is larger, that is, the information of the decision maker dimension increases while information of the decision attribute decreases. It shows that the change of the parameter \( a \) reflects the emphasis of the dimensional information, resulting in the discrimination of the ranking result. In addition, changes in the parameters can affect the ranking of programs to a certain extent. Due to the particularity of the selected cases in this paper, the influence of varied parameter on the ranking of the schemes is not shown in the final ranking results. However, when a certain decision-making problem requires a higher degree of discrimination between decision-makers and decision-making standards, the method proposed in this paper can show its flexibility and convenience. By choosing appropriate parameters, we can improve the discrimination of ranking results and better assist decision-making.

To further verify the validity and rationality of the proposed model, it should be used to compared with other classic MCDM methods. Considering the modeling conditions of this paper, we choose TOPSIS, VIKOR, MULTIMOORA methods for comparative analysis. It should be noted that each method has its corresponding problems, and they are not completely comparable. The comparison of different methods in this paper is just to express the validity of the model.

The ranking results are shown in Table 2. Based the method proposed in this paper, the ranking is \( x_6 \succ x_1 \succ x_2 \succ x_5 \succ x_4 \succ x_3 \), which is consistent with the TOPSIS method. The proposed method and VIKOR have difference in the ranking of scheme \( x_2 \) and \( x_5 \); the proposed method and MULTI-MOORA have difference in the ranking of scheme \( x_1, x_2, x_3, x_4, \) and \( x_5 \). These differences show that VIKOR and MULTI-MOORA only focus on mining the information of the decision maker or criteria dimension, the proposed model can not only well integrate and aggregate the information of different dimensions but also distinguish the importance of the above two dimensions through the parameter \( a \) according to the actual situation. Although TOPSIS and our method have the same ranking results,
TOPSIS describes absolute differences. The method in this paper describes the overall difference in terms of the geometry of the sequence curve.

Table 2. The results of the scheme ranking.

| Method     | Ranking Results |
|------------|-----------------|
| TOPSIS     | $x_6 \succ x_1 \succ x_2 \succ x_5 \succ x_4 \succ x_3$ |
| VIKOR      | $x_6 \succ x_1 \succ x_5 \succ x_2 \succ x_4 \succ x_3$ |
| MULTIMOORA | $x_6 \succ x_2 \succ x_1 \succ x_4 \succ x_3 \succ x_5$ |
| This paper | $x_6 \succ x_1 \succ x_2 \succ x_5 \succ x_4 \succ x_3$ |

5. Conclusions

In order to candle the scheme ranking problem in the process of group decision making, this paper constructs a novel grey scheme matrix incidence model from the aspects of the dimensions of decision makers and decision attributes based on grey incidence analysis. Compared with previous grey incidence analysis model, the model can make full use of information of the decision-maker dimension and the criteria dimension. The model can avoid high-dimensional information loss. Considering the dimensions of decision maker and decision attribute, this paper proposed a grey group decision-making approach. The relative distance and absolute distance between group measure matrix of the scheme and the ideal scheme are defined to express decision maker information and decision attribute information. Finally, the proposed model is applied to solve new drug R&D project selection problems. This method can be operated easily, providing a potential decision-making aid for pharmaceutical companies. The future research direction is to study the dynamic group decision-making problem from the perspective of expert change and criteria change. There are also some limitations in this paper, such as decision-maker weights are determined by qualitative analysis and lack of quantitative methods. In addition, how to determine the optimal parameter $a$ is also what we need to solve in the future.

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References
1. Liu, S.F.; Yang, Y.J.; Lin, Y. Grey Data Analysis: Method, Models and Applications; Springer: Singapore, 2016.
2. Deng, J.L. Spread of grey relational space. *J. Grey Syst.* 1995, 7, 96–100.
3. Ikram, M.; Sroufe, R.; Rehman, E.; Shah SZ, A.; Mahmoudi, A. Do Quality, Environmental, and Social (QES) Certifications Improve International Trade? A Comparative Grey Relation Analysis of Developing vs. Developed Countries. *Phys. A Stat. Mech. Its Appl.* 2020, 545, 1–34. [CrossRef]
4. Javed, S.A.; Liu, S. Bidirectional absolute GRA/GIA model for uncertain systems: Application in project management. *IEEE Access* 2019, 7, 60885–60896. [CrossRef]
5. Rajesh, R.; Ravi, V. Supplier selection in resilient supply chains: A grey relational analysis approach. *J. Clean. Prod.* 2015, 86, 343–359. [CrossRef]
6. Suvvari, A.; Goyari, P. Financial performance assessment using Grey relational analysis (GRA). *Grey Syst. Theory Appl.* 2019, 9, 502–516. [CrossRef]
7. Pakkar, M.S. Multiple attribute grey relational analysis using DEA and AHP. *Complex Intell. Syst.* 2016, 2, 243–250. [CrossRef]
8. Samvedi, A.; Jain, V.; Chan, F.T.S. An integrated approach for machine tool selection using fuzzy analytical hierarchy process and grey relational analysis. *Int. J. Prod. Res.* 2012, 50, 3211–3221. [CrossRef]
9. Baranitharan, P.; Ramesh, K.; Sakthivel, R. Multi-attribute decision-making approach for Aegle marmelos pyrolysis process using TOPSIS and Grey Relational Analysis: Assessment of engine emissions through novel Infrared thermography. *J. Clean. Prod.* 2019, 234, 315–326. [CrossRef]
10. Prabhu, S.R.; Rangakumaran, M. Selection of 3D printer based on FAHP integrated with GRA-TOPSIS. *Int. J. Mater. Prod. Technol.* 2019, 58, 155–177. [CrossRef]
11. Yazdani, M.; Kahraman, C.; Zarate, P.; Onar, S.C. A fuzzy multi attribute decision framework with integration of QFD and grey relational analysis. *Expert Syst. Appl.* 2019, 115, 474–485. [CrossRef]
12. Giri, B.C.; Molla, M.U.; Biswas, P. Grey relational analysis method for SVTRNN based multi-attribute decision making with partially known or completely unknown weight information. *Granul. Comput.* 2020, 5, 561–570. [CrossRef]
13. Liu, Y.; Du, J.L.; Li, J. A novel grey object matrix incidence clustering model for panel data and its application. *Sci. Iran. E* 2021, 28, 371–385. [CrossRef]
14. Liu, Y.; Du, J.L.; Li, J. Analysis and Time Lag Measurement of the Influencing Factors on Universities R&D Based on Grey Incidence Model. *Oper. Res. Manag.* 2020, 29, 150–156.
15. Wei, G. Grey relational analysis model for dynamic hybrid multiple attribute decision making. *Knowl.-Based Syst.* 2011, 24, 672–679. [CrossRef]
16. Sun, G.; Guan, X.; Yi, X.; Zhou, Z. Grey relational analysis between hesitant fuzzy sets with applications to pattern recognition. *Expert Syst. Appl.* 2018, 92, 521–532. [CrossRef]
17. Liu, S.; Xie, N.; Forrest, J. Novel models of grey relational analysis based on visual angle of similarity and nearness. *Grey Syst. Theory Appl.* 2011, 1, 8–18. [CrossRef]
18. Yin, K.; Xu, J.; Li, X. A new grey comprehensive relational model based on weighted mean distance and induced intensity and its application. *Grey Syst. Theory Appl.* 2019, 9, 374–384. [CrossRef]
19. Dey, P.P.; Pramanik, S.; Giri, B.C. Neutrosophic soft multi-attribute group decision making based on grey relational analysis method. *J. New Results Sci.* 2016, 5, 25–37.
20. Hashemi, H.; Mousavi, S.M.; Zavadskas, E.K.; Chalaeke, A.; Turskis, Z. A new group decision model based on grey-intuitionistic fuzzy-ELECTRE and VIKOR for contractor assessment problem. *Sustainability* 2018, 10, 1635. [CrossRef]
21. Liu, Y.; Du, J.; Wang, Y. An improved grey group decision-making approach. *Appl. Soft Comput.* 2019, 76, 78–88. [CrossRef]
22. Pramanik, S.; Mukhopadhyayya, D. Grey relational analysis based intuitionistic fuzzy multi-criteria group decision-making approach for teacher selection in higher education. *Int. J. Comput. Appl.* 2011, 34, 21–29.
23. Zare, A.; Feylimzadeh, M.; Mahmoudi, A.; Liu, S. Suitable computerized maintenance management system selection using grey group TOPSIS and fuzzy group VIKOR: A case study. *Decis. Sci. Lett.* 2018, 7, 341–358. [CrossRef]
24. Mousavi, S.M.; Jalal, F.; Tavakkoli-Moghaddam, R.; Vahdani, B. A fuzzy grey model based on the compromise ranking for multi-criteria group decision making problems in manufacturing systems. *J. Intell. Fuzzy Syst.* 2013, 24, 819–827. [CrossRef]
25. Pamucar, D. Normalized weighted geometric Dombi Bonferroni mean operator with interval grey numbers: Application in multicriteria decision making. *Rep. Mech. Eng.* 2020, 10, 1–44. [CrossRef]
26. Ulutag, A.; Balo, F.; Sua, L.; Demir, E.; Topal, A.; Jakovljević, V. A new integrated grey MCDM model: Case of warehouse location selection. *Facta Univ. Ser. Mech. Eng.* 2021, 19, 515–535. [CrossRef]
27. Badi, I.; Pamucar, D. Supplier selection for steelmaking company by using combined Grey-MARCS methods. *Decis. Mak. Appl. Manag. Eng.* 2020, 3, 37–48. [CrossRef]
28. Arce, M.E.; Saavedra, A.; Miguez, J.L.; Granada, E. The use of grey-based methods in multi-criteria decision analysis for the evaluation of sustainable energy systems: A review. *Renew. Sustain. Energy Rev.* 2015, 47, 924–932. [CrossRef]
29. Aydemin, E.; Sahin, Y. Evaluation of healthcare service quality factors using grey relational analysis in a dialysis center. *Grey Syst. Theory Appl.* 2019, 9, 432–448. [CrossRef]
30. Guru, S.; Mahalik, D.K. A comparative study on performance measurement of Indian public sector banks using AHP-TOPSIS and AHP-grey relational analysis. *Opsresearch* 2019, 56, 1213–1239. [CrossRef]
31. Sheikh AH, A.; Ikram, M.; Ahmad, R.M.; Qadeer, H.; Nawaz, M. Evaluation of key factors influencing process quality during construction projects in Pakistan. *Grey Syst. Theory Appl.* 2019, 9, 321–335. [CrossRef]
32. Delcea, C.; Bradea, I.A. Patients’ perceived risks in hospitals: A grey qualitative analysis. *Kybernetes* 2017, 46, 1408–1424. [CrossRef]
33. Cheng, Y.C.; Yeh, H.C.; Lee, C.K. Multi-objective optimization of the honeycomb core in a honeycomb structure using uniform design and grey relational analysis. *Eng. Optim.* 2022, 54, 286–304. [CrossRef]
34. Sundararaj ON, R.; Sethuramalingam, P. Experimental Investigation of AWJ Slicing of Single Crystal Silicon Using Fuzzy Grey Relational Analysis (FGRA). *Silicon* 2022, 1–22. [CrossRef]
35. Wu, H.; Lei, H.; Chen, Y.F. Grey relational analysis of static tensile properties of structural steel subjected to urban industrial atmospheric corrosion and accelerated corrosion. *Constr. Build. Mater.* 2022, 315, 125706. [CrossRef]
36. Babbar, A.; Sharma, A.; Singh, P. Multi-objective optimization of magnetic abrasive finishing using grey relational analysis. *Mater. Today Proc.* 2022, 50, 570–575. [CrossRef]
37. Liu, Y.; Forrest, J.; Xie, N.M. Ranking grey numbers based on dominance grey degrees. *J. Syst. Eng. Electron.* **2014**, *25*, 618–626. [CrossRef]

38. Liang, D.; Liu, D. Systematic studies on three-way decisions with interval-valued decision-theoretic rough sets. *Inf. Sci.* **2014**, *276*, 186–203. [CrossRef]

39. Quan, J.; Zeng, B.; Wang, L.Y. Maximum entropy methods for weighted grey incidence analysis and applications. *Grey Syst. Theory Appl.* **2018**, *8*, 144–155. [CrossRef]

40. Du, J.; Liu, S.; Liu, Y. A novel grey multi-criteria three-way decisions model and its application. *Comput. Ind. Eng.* **2021**, *158*, 107405. [CrossRef]