Gauge Hierarchy from $AdS_5$ Universe with 3-Branes

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Abstract

If the universe is (or a slice of) $AdS_5$ space with 3-branes, the 5-dimensional GUT scale on each brane can be identified as the 5-dimensional Planck scale, but, the 4-dimensional Planck scale is generated from the low 4-dimensional GUT scale exponentially in our world. The 4-dimensional GUT scales and Planck scale are related to the 5-dimensional GUT scales and Planck scale by exponential factors, respectively. One of such scenarios was suggested by Randall and Sundrum recently. We give another scenario that the 4-dimensional Planck scale is generated from the low five-dimensional Planck scale by an exponential hierarchy, and the mass scale in the Standard Model is not rescaled from the 5-dimensional metric to the 4-dimensional metric. We also argue that the additional constant in the solution might exist, which will rescale the 5-dimensional Planck scale and affect the physical scale picture. Finally, we embed those compactifications to the general compactification on $AdS_5$ space and discuss the origin of the additional constant.

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1 Introduction

Experiments at LEP and Tevatron have given the strong support to the Standard Model of the Strong and electroweak interactions. However, the Standard Model has some unattractive features which may imply the new physics beyond Standard Model. One of these problems is that the gauge forces and the gravitational force are not unified. Another is the gauge hierarchy problem between the weak scale and the 4-dimensional Planck scale. Previously, two solutions to the gauge hierarchy problem have been proposed: one is the idea of the technicolor and compositeness which lack calculability, and the other is the idea of supersymmetry, but we have not found any experimental signal at colliders yet.

More than one year ago, it was suggested that the large compactified extra dimensions may be another solution to the gauge hierarchy problem [1]. If the dimension of the spacetime is $4 + n$ where $n > 1$, the 4-dimensional Planck scale is determined by the fundamental $4 + n$ dimensional Planck scale $M_X$. For the simplest case, we have the relation between these two scales:

$$M_{Pl}^2 = M_X^{n+2} V_n,$$

(1)

where $V_n$ is the physical volume of the extra dimensions. Of course, if we required that $M_X$ is at low energy scale, then we may have the hierarchy between the $M_X$ and $V_n^{-1/n}$ except that one needs to introduce many extra dimensions. If we assume $M_X = 10^5 GeV$, $V_n^{-1/n} = 10^4 GeV$, we obtain:

$$n = 10^{26},$$

(2)

which might not seem possible, but it is not excluded.

Several months ago, Randall and Sundrum [2] proposed another scenario that the extra dimension is an orbifold, and the size of the extra dimension is not large but the 4-dimensional mass scale in the standard model is suppressed by an exponential factor from 5-dimensional mass scale. In addition, they suggested that the fifth dimension might be infinity [3], and there may exist only one brane with positive tension at the origin, but, there exists a gauge hierarchy problem for they thought $k$, which is defined in the following section, is equal to the 5-dimensional Planck scale. The remarkable aspect of the second scenario is that it gives rise to a localized graviton field. Combining those results, Lykken and Randall obtained the following physical picture [4]: the graviton is localized on Planck brane, we live on a brane separated from the Planck brane about 30 Planck lengths along the fifth dimension. On our brane, the mass scale in the Standard Model is suppressed exponentially, which gives the low energy scale. Recently, this kind of compactification or similar idea has attracted much attentions [5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

Before discussing our results, first, we would like to point out that one needs to choose a fundamental metric in the scale discussion. For example, in the weakly coupled string, to be consistent, we use the string metric to discuss the scale. In addition, we assume that all the gauge forces are unified on each 3-brane if there exist
gauge forces. The 5-dimensional GUT scale on each 3-brane and the 5-dimensional Planck scale are defined as the GUT scale and Planck scale in the 5-dimensional fundamental metric (for example, equation (3) or (4)), respectively. The 4-dimensional GUT scale on each brane and the 4-dimensional Planck scale are defined as the GUT scale and Planck scale in the 4-dimensional Minkowski Metric ($g_{\mu\nu}$). In order to avoid the gauge hierarchy problem between the weak scale and the 4-dimensional GUT scale $M_{GUT}$ on the brane which is our world, we assume the low energy unification. The key ansatz is that the 5-dimensional GUT scale on each brane is equal to the 5-dimensional Planck scale, and there is no mass hierarchy between the mass parameters in the 5-dimensional fundamental metric if the fifth dimension is compact.

With above argument and assumptions, we find out the constant in the solution of 5-dimensional Einstein equation is not a trivial factor, and will affect the 5-dimensional Planck scale and the physical picture if there exists such constant in the fundamental metric, for example, in M-theory on $S^1/Z_2$ [13], only the metric at mid point between two hyperplanes is not changed after one considers the next order correction without five branes [16], and the constant in the metric may have its origin in the compactification on $AdS_5$ space. In short, the general solution to the Einstein equation obtained in five-dimensional model with two boundaries (branes) are:

$$ ds^2 = e^{-2k|y|-2c} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, $$

(3)

$$ ds^2 = e^{2k|y|-2c} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, $$

(4)

where $\mu, \nu$ parametrize the four-dimensional coordinates of Minkowski space, and $y$ is the fifth dimension coordinate.

The first solution is the same as the solution given by Randall and Sundrum except the constant $c$ [3]. The general physical scale picture is the following: the 5-dimensional GUT scale on each brane can be identified as the 5-dimensional Planck scale, but, the 4-dimensional Planck scale is generated from the low 4-dimensional GUT scale exponentially in our world. The 4-dimensional GUT scales and Planck scale are related to the 5-dimensional GUT scales and Planck scale by exponential factors, respectively. In other words, the large mass hierarchy [2] in the Standard Model from the 5-dimensional metric to the 4-dimensional metric is one possible solution of the gauge hierarchy problem in our world, but, not the most general solution. In addition, the fifth dimension may be compact [2], or the fifth dimension may be non-compact. There is a brane with positive tension (call it as “Hidden Brane”) at the origin, the 4-dimensional GUT scale $M_H$ on the hidden brane is larger than or the same as the 4-dimensional GUT scale of our world. We live on

\[\text{We will not explain why } M_{GUT} \text{ can be low energy scale here, but, it is possible if one considers additional particles which change the RGE running. Of course, proton decay might be the problem, but we do not discuss this here.}\]

\[\text{We do not consider the masses of quarks and leptons in this statement.}\]
a brane separated from the hidden brane about a distance \( \ln(M_H/M_{\text{GUT}})/k \) along the fifth dimension where \( k \) is not necessary to be equal to the 5-dimensional Planck scale in this case for the fifth dimension is not compact. Similar to Ref. [3], we also suggest that: we live on a positive tension brane (at origin) which is embedded in a non-compact \( AdS_5 \) space and contains the Standard Model. The five-dimensional Planck scale or the 4-dimensional GUT scale is at intermediate scale \( 10^8 \, \text{GeV} \), then, in this case, \( M_{\text{GUT}}/M_W \simeq M_{\text{top}}/M_{\text{electron}} \).

The second solution tells us that the 5-dimensional Planck scale will be rescaled even though one considers \( c = 0 \). Assuming \( c=0 \), we can obtain the scenario with the following property: the 4-dimensional Planck scale is generated from the low 5-dimensional Planck scale by an exponential hierarchy, and the mass scale in the Standard Model, which is contained in one brane, is not rescaled. In general, for \( c \neq 0 \), the 5-dimensional GUT scale on each brane can be considered as the 5-dimensional Planck scale, but, the 4-dimensional Planck scale is generated from the low 4-dimensional GUT scale exponentially in our world. The 4-dimensional GUT scales and Planck scale are related to the 5-dimensional GUT scales and Planck scale by exponential factors, respectively. The physical scale picture is similar to that in the first solution except that we can not let the fifth dimension non-compact in this case.

We also discuss the compactification on \( AdS_5 \) space, and all of above scenarios can be embedded in this kind of compactification. However, in all of above scenarios, the cosmology constant and brane tensions in the 4-dimensional Minkowski metric can be expressed as simple functions of 4-dimensional GUT scale and Planck scale, therefore, it seems to us that we can not explain the very small cosmology constant \( 10^{-58} \, \text{GeV}^4 \) in this kind of compactification.

Of course, the results in this letter can be generalized to \( AdS_{1+n} \) and can be applied to Type IIB string on \( AdS_5 \times S^5 \) or M-theory on \( AdS_7 \times S^4 \).

### 2 Solutions to the Einstein Equation

The setup is given by Randall and Sundrum [2]. Considering a compact fifth dimension, \(-L \leq y \leq L\) and introducing equivalence relations: \( y \sim y + 2L \) and \( y \sim -y \), we obtain the obifold \( S^1/Z_2 \). The fixed points \( y=0, L \) are taken as the locations of the two 3-branes, which can support 4-dimensional field theories. Let us denote the \( y=0, L \) planes as \( M_0 \) and \( M_1 \) respectively, the 5-dimensional metric in these two branes are:

\[
g_{\mu \nu}^0(x^\mu) \equiv G_{\mu \nu}(x^\mu, y = 0) , \quad g_{\mu \nu}^1(x^\mu) \equiv G_{\mu \nu}(x^\mu, y = L) ,
\]

where \( G_{AB} \) where \( A, B = \mu, y \), is the five-dimensional metric.

The classical Lagrangian is given by:

\[
S = S_{\text{gravity}} + S_0 + S_1 ,
\]
\[ S_{\text{gravity}} = \int d^4x \int_{-L}^{L} dy \sqrt{-G} \{ -\Lambda + \frac{1}{2} M_X^3 R \} , \quad (7) \]

\[ S_0 = \int d^4x \sqrt{-g^0} \{ \mathcal{L}_0 - V_0 \} , \quad (8) \]

\[ S_1 = \int d^4x \sqrt{-g^1} \{ \mathcal{L}_1 - V_1 \} , \quad (9) \]

where \( M_X \) is the 5-dimensional Planck scale, \( \Lambda \) is the cosmology constant, and \( V_0, V_1 \) are the brane tensions. The 5-dimensional Einstein equation for the above action is [2]:

\[
\sqrt{-G} \left( R_{AB} - \frac{1}{2} G_{AB} R \right) = -\frac{1}{M_X^3} \left[ \Lambda \sqrt{-G} G_{AB} + V_0 \sqrt{-g^0} g^0_{\mu\nu} \delta^{\mu}_M \delta^{\nu}_N \delta(y) + V_1 \sqrt{-g^1} g^1_{\mu\nu} \delta^{\mu}_M \delta^{\nu}_N \delta(y-L) \right]. \quad (10)
\]

Assuming that there exists a solution that respects 4-dimensional Poincare invariance in the \( x^\mu \)-directions, one obtains the 5-dimensional metric:

\[ ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 . \quad (11) \]

With this metric, the Einstein equation reduces to [2]:

\[ \sigma'' = -\frac{\Lambda}{6M_X^3} , \quad (12) \]

\[ \sigma'' = \frac{V_0}{3M_X^3} \delta(y) + \frac{V_1}{3M_X^3} \delta(y-L) . \quad (13) \]

Before we discuss the phenomenology, we would like to define the generalized scale transformation [1] on the 4-dimensional Lagrangian:

\[ g_{\mu\nu} \rightarrow \lambda^2 g_{\mu\nu} , \quad \phi \rightarrow \lambda^{-1} \phi , \quad (14) \]

\[ \psi \rightarrow \lambda^{-3/2} \psi , \quad A_\mu \rightarrow A_\mu , \quad (15) \]

\[ m \rightarrow \lambda^{-1} m , \quad g_c \rightarrow \lambda^{-\text{dim}[g_c]} , \quad (16) \]

where \( \phi, \psi, A_\mu, m \) and \( g_c \) are the scalar field, spinor field, gauge field, mass and coupling, respectively, and \( \text{dim}[g_c] \) is the mass dimension of the coupling (for example, the gauge coupling and Yukawa coupling have dimension 0). Obviously, if \( \lambda \) is not a function of the space-time index, one can easily check that the generalized scale transformation is invariant even when one considers the gravity, the divergence

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4 Conformal invariance is a consequence of the scale invariance in quantum field theory.
and renormalization. It does not change the relative mass ratios, for example, if we consider the Standard Model and gravity in 4-dimension, $M_W/M_{pl}$ is invariant under this transformation. Although we do not mention it explicitly, this kind of the transformation will be often used in the following discussion because one need to consider the physical mass scale in the 4-dimensional Minkowski metric from the 5-dimensional fundamental metric by the compactification of the fifth dimension. By the way, we can define the generalized scale transformation in any dimension.

Now, we consider the solution.

(I) Assuming $\Lambda$ is negative, the first solution is:

$$\sigma = |y| \sqrt{-\frac{\Lambda}{6M_X^3}} + c .$$

This solution was obtained by Randall and Sundrum by choosing $c=0$.\footnote{For $kL$ is small, the physical results are similar to those in previous extra dimension proposal\cite{1}, so, we will not discuss it here.} Similar to their result, one can define a new scale $k$ which relates $V_0, V_1, \Lambda$ to $M_X$ as:

$$V_0 = -V_1 = 6M_X^3k , \quad \Lambda = -6M_X^3k^2 .$$

Then, the bulk metric is:

$$ds^2 = e^{-2k|y|-2c}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 .$$

The corresponding 4-dimensional Planck scale is:

$$M^2_{pl} = \frac{M_X^3e^{-2c}}{k}[1 - e^{-2kL}] .$$

We assume that the observable brane is $M_1$, and in the 5-dimensional fundamental metric, the GUT scale $M^{(5)}_{GUT}$ of our world is equal to the Planck scale $M_X$. We obtain our world 4-dimensional GUT scale

$$M^{(5)}_{GUT} = M^{(5)}_{GUT}e^{-kL-c} = M_Xe^{-kL-c} ,$$

and the 4-dimensional Planck scale and GUT scale relation

$$M^2_{pl} = \frac{M^{3}_{GUT}e^{+3kL+c}}{k}[1 - e^{-2kL}] .$$

We can push $M_{GUT}$ to the TeV scale or $10^5$ GeV easily. Assuming that in the fundamental theory we just have one scale, $L^{-1}$ should be close to the $M_X$. Because $kL$ can not be arbitrarily large\footnote{For $kL$ is small, the physical results are similar to those in previous extra dimension proposal\cite{1}, so, we will not discuss it here.} we choose $k = M_X$ for the simplicity and obtain:

$$M^2_{pl} = M^{2}_{GUT}e^{2kL}[1 - e^{-2kL}] .$$
With \( kL = 34.5 \) and 30, we can have \( M_{GUT} \) at the TeV scale and \( 10^5 \) GeV scale respectively. For \( c=0 \) and \( k = M_X \), we obtain \( M_X = M_{pl} \). For \( c = -kL/2 \) and \( k = M_X \), we obtain that for \( M_{GUT} = 10^3 \) GeV and \( 10^5 \) GeV, \( M_X \) is about \( 10^{10} \) GeV and \( 10^{11} \) GeV, respectively. And for \( c = kL \), we obtain \( M_X = M_{GUT} \). In short, for \( c \neq 0 \), the physical scale picture is different from that in Ref. [2,3]. And the result in Ref.[2] is not the most general solution to the gauge hierarchy problem in our world.

One can also consider the cosmology constant and brane tensions in the 4-dimensional Minkowski metric, they are:

\[
A^{(4)} = - \frac{3M^6_{GUT}e^{6kL}}{M^2_{pl}}(1 - e^{-2kL})(1 - e^{-4kL}), \quad (24)
\]

\[
V_0^{(4)} = \frac{6M^6_{GUT}e^{6kL}}{M^2_{pl}}(1 - e^{-2kL}), \quad (25)
\]

\[
V_1^{(4)} = -\frac{6M^6_{GUT}e^{2kL}}{M^2_{pl}}(1 - e^{-2kL}). \quad (26)
\]

Because \( M_{GUT} \) can not be smaller than TeV scale, each of above values is much larger than \( 10^{-58} \text{ GeV}^4 \).

For above case, the \( M_1 \) brane tension is negative (although it is not ruled out), and the space is compact. Now, we consider the case similar to that in the Ref. [3]. There is just one positive tension brane which is located at the origin and the fifth dimension is infinity. The solution of the metric can be written as:

\[
ds^2 = e^{-2k|y|}e^{-2c}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \quad (27)
\]

which is similar to the above case, and \( k \) is defined as before. The scale relations are:

\[
M^2_{pl} = \frac{M^3_X e^{-2c}}{k}, \quad (28)
\]

\[
M_{GUT} = M_X e^{-c}, \quad (29)
\]

\[
M^2_{pl} = \frac{M^3_{GUT} e^c}{k}. \quad (30)
\]

In this case, because the fifth dimension is not compactified, we do not require that \( k \approx M_X \) \footnote{One may think there exists hierarchy between \( k \) and \( M_X \) in 5-dimensional fundamental metric which is not gauge hierarchy, but, \( k \) is related to the cosmology constant and small \( k \) may ameliorate the cosmology constant problem, which can be considered as the motivation of small \( k \).}. We can push \( M_{GUT} \) to TeV or \( 10^5 \) GeV scale by choosing \( e^{-c} k=10^{-27} \) GeV and \( 10^{-21} \) GeV. Taking \( c=0 \), \( M_X=M_{GUT} \), and in general, \( M_X \) might be any number. In order not to be ruled out by experiment, we might require that \( e^{-c} k > \)
Choosing \( c=0 \) and \( k = 10^{-13} \) GeV, we obtain \( M_{GUT} = M_X \approx 10^8 \) GeV, and \( M_{GUT}/M_W \approx M_{top}/M_{electron} \), so, the hierarchy is not too severe. However, we can not obtain small enough cosmology constant in this scenario too, for the cosmology constant and the brane tension in the 4-dimensional Minkowski metric can be expressed as:

\[
\Lambda^{(4)} = -\frac{3M_{GUT}^6}{M_{pl}^2}, \quad V^{(4)}_0 = \frac{6M_{GUT}^6}{M_{pl}^2}.
\] (31)

The results in Ref.[4] can also be applied here. Assuming the brane at the origin as a hidden brane, and our world locates at \( y_0 \), we obtain the scale relations

\[
M_H = M_X e^{-c}, \quad M_{GUT} = M_X e^{-k|y_0| - c},
\] (32)

\[
M_{pl}^2 = \frac{M_H^3}{k}, \quad M_{GUT}^2 = \frac{M_{GUT}^3 e^{3k|y_0| + c}}{k},
\] (33)

where the \( M_H \) is the 4-dimensional GUT scale on the hidden brane, and we have assumed that the 5-dimensional GUT scale \( M_H^{(5)} \) on the hidden brane is equal to the 5-dimensional Planck scale, i. e., \( M_H^{(5)} = M_X \). One can easily solve the gauge hierarchy problem in our world by varying \( y_0 \) or \( c \).

In addition, in 4-dimensional Minkowski metric, the cosmology constant \( \Lambda^{(4)} \) is:

\[
\Lambda^{(4)} = -\frac{3M_{GUT}^6 e^{6k|y_0|}}{M_{pl}^2}.
\] (34)

Therefore, the larger the \( |y_0| \), the larger the magnitude of the \( \Lambda^{(4)} \) for fixed \( M_{GUT} \). We can not explain why the cosmology constant is so small, too.

In short, the general physical scale picture is the following: in 5-dimension, the GUT scale on each brane is the same as the Planck scale, but, in 4-dimension, the Planck scale is generated from the our world low GUT scale exponentially. The 4-dimensional GUT scales and Planck scale are related to the 5-dimensional GUT scales and Planck scale by exponential factors, respectively. In addition, if the fifth dimension is not compact, there is a hidden brane with positive tension at the origin, the 4-dimensional GUT scale \( M_H \) on the hidden brane is larger than or the same as that of our world. we live on a brane separated from the hidden brane a distance about \( \ln(M_H/M_{GUT})/k \) along the fifth dimension where \( k \) is not necessary to be equal to the 5-dimensional Planck scale for the fifth dimension is not compact.

(II) Assuming \( \Lambda \) is negative, the other solution is:

\[
\sigma = -|y|\sqrt{\frac{-\Lambda}{6M_X^2}} + c.
\] (35)

Similar to the previously result, we can define a new scale \( k \) which relates \( V_0, V_1, \Lambda \) to \( M_X \) as:

\[
-V_0 = V_1 = 6M_X^3 k, \quad \Lambda = -6M_X^3 k^2.
\] (36)
And then, the bulk metric is
\[ ds^2 = e^{2k|y|-2c} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \, . \] (37)

The Planck scale in the four-dimensional Minkowski metric is:
\[ M_{pl}^2 = \frac{M_X^3 e^{-2c}}{k} [e^{2kL} - 1] \, . \] (38)

We assume that the brane of our world is \( M_0 \). Using similar ansatz as above, we obtain our world 4-dimensional GUT scale, and the relation between \( M_{pl} \) and \( M_{GUT} \)
\[ M_{GUT} = M_{GUT}^{(5)} e^{-c} = M_X e^{-c} \, , \] (39)
\[ M_{pl}^2 = \frac{M_{GUT}^{3} e^{2kL+c}}{k} [1 - e^{-2kL}] \, . \] (40)

We can push \( M_{GUT} \) to the TeV scale or \( 10^5 \) GeV scale easily. Taking \( k = M_X \) for the fifth dimension is compact in this case, we obtain
\[ M_{pl}^2 = M_{GUT}^2 e^{2kL} [1 - e^{-2kL}] \, . \] (41)

As before, with \( kL = 34.5 \) and 30, we can have the \( M_{GUT} \) at the TeV scale and \( 10^5 \) GeV scale, respectively. But, one should notice that, in this scenario, if we take \( c=0 \), we will not rescale the mass scale in the Standard Model from the 5-dimensional metric to the 4-dimensional metric, but we rescale the Planck scale, the large 4-dimensional Planck scale is generated from the low 5-dimensional Planck scale exponentially.

For \( c=0 \) and \( k = M_X \), we obtain \( M_X = M_{GUT} \). For \( c = kL/2 \) and \( k = M_X \), we obtain that for \( M_{GUT} = 10^3 \) GeV and \( 10^5 \) GeV, \( M_X \) is about \( 10^{10} \) GeV and \( 10^{11} \) GeV, respectively.

One can also consider the cosmology constant and brane tensions in the 4-dimensional Minkowski metric, they are:
\[ \Lambda^{(4)} = -\frac{3M_{GUT}^6 e^{6kL}}{M_{pl}^2} (1 - e^{-2kL})(1 - e^{-4kL}) \, , \] (42)
\[ V_0^{(4)} = -\frac{6M_{GUT}^6 e^{2kL}}{M_{pl}^2} (1 - e^{-2kL}) \, , \] (43)
\[ V_1^{(4)} = \frac{6M_{GUT}^6 e^{6kL}}{M_{pl}^2} (1 - e^{-2kL}) \, . \] (44)

Obviously, because \( M_{GUT} \) can not be smaller than TeV scale, each of those values is much larger than \( 10^{-58} \) GeV.

Furthermore, this solution is related to the first solution by the following transformation: \( c \rightarrow c + kL \) and \( V_0 \leftrightarrow V_1 \).
3 Compactification on $AdS_5$ Space

In all the solutions of above section, the space is a slice of $AdS_5$ space, therefore, we can consider the compactification on the $AdS_5$ space. The $AdS_5$ metric is:

$$ ds^2 = \frac{r^2}{R_{ads}^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R_{ads}^2}{r^2} dr^2, \quad (45) $$

where $R_{ads}$ is the “radius” of $AdS_5$ space, and $R_{ads}^2 = \alpha' \sqrt{4\pi g_5 N}$ for Type IIB string on $AdS_5 \times S^5$ [17] (we do not write down the metric on $S^5$). Obviously, there exists singularity when $r$ is 0 or infinity.

In order to be easy to compare with results of the previous section, we make the following transformation

$$ r = R_{ads} e^{y/R_{ads}}. \quad (46) $$

Then, the $AdS_5$ metric becomes:

$$ ds^2 = e^{2y/R_{ads}} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2. \quad (47) $$

This metric is invariant under the following transformation:

$$ x^\mu \rightarrow e^{-\lambda/R_{ads}} x^\mu, \quad y \rightarrow y + \lambda. \quad (48) $$

Assuming that the minimum and maximum of $y$ are $a$ and $b$ which can be considered as cut-offs, and the brane of our world is located at $y_o$ where $a \leq y_o \leq b$ [1], we obtain the following scale relations:

$$ M_{pl}^2 = M_X^3 \frac{R_{ads}}{2} e^{2b/R_{ads}} (1 - e^{-2(b-a)/R_{ads}}), \quad (49) $$

$$ M_{GUT} = M_{GUT}^{(5)} e^{y_o/R_{ads}} = M_X e^{y_o/R_{ads}}, \quad (50) $$

$$ M_{pl}^2 = M_{GUT}^3 \frac{R_{ads}}{2} e^{(2b-3y_o)/R_{ads}} (1 - e^{-2(b-a)/R_{ads}}). \quad (51) $$

If $a$ and $b$ are finite, we can assume that $R_{ads} = M_X^{-1}$, and obtain

$$ M_{pl}^2 = M_{GUT}^2 \frac{R_{ads}}{2} e^{2(b-y_o)/R_{ads}} (1 - e^{-2(b-a)/R_{ads}}). \quad (52) $$

With $2(b - y_o)/R_{ads} = 34.5$ and 30, we can push $M_{GUT}$ to the TeV scale and $10^5$ GeV scale, respectively. But, one should notice that, if $y_o = 0$, we will not rescale the mass scale in the Standard Model, which is different from previous result [3]. If $a$ is negative infinity, the physical scale picture will be similar to that in the first solution with non-compact fifth dimension in above section, with $c = -b/R_{ads}$.

In above section, the first solution with compact fifth dimension can be considered as $a = y_o = b - L$ and $c = -b/R_{ads}$, and the second solution can be considered as $a = y_o = b - L$ and $c = -a/R_{ads}$ by noticing that $k = R_{ads}^{-1}$. Of course, the value of $a$ might be negative infinity, the result in the first solution with non-compact fifth dimension can be considered as $c = -b/R_{ads}$.

We do not address the issue of determining the locations of the branes here.
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