Πgβ-connectedness in Intuitionistic Fuzzy Topological Spaces

T. Jenitha Premalatha¹, *, S. Jothimani²

¹Department of Mathematics, Tips Global Institute, Coimbatore, India
²Department of Mathematics, Government Arts College, Coimbatore, India

Email address:
joel.jensi@gmail.com (T. J. Premalatha), jothimanis09@gmail.com (S. Jothimani)
*Corresponding author

To cite this article:
T. Jenitha Premalatha, S. Jothimani. Πgβ-connectedness in Intuitionistic Fuzzy Topological Spaces. Mathematics Letters. Vol. 3, No. 6, 2017, pp. 65-70. doi: 10.11648/j.ml.20170306.12

Received: October 5, 2017; Accepted: October 25, 2017; Published: November 23, 2017

Abstract: The paper aspires to discuss the basic properties of connected spaces. Also the concept of types of intuitionistic fuzzy Πgβ-connected and disconnected in intuitionistic fuzzy topological spaces are introduced and studied. The research paper of topological properties is introduced by making the idea of being connected. It turns out to be easier to think about the property that is the negation of connectedness, namely the property of disconnectedness and separable. Also the concepts of intuitionistic fuzzy ΠgβC₃-connectedness, intuitionistic fuzzy ΠgβC₅-connectedness, intuitionistic fuzzy ΠgβC₆-connectedness, intuitionistic fuzzy Πgβ-strongly connectedness, intuitionistic fuzzy ΠgβSuper Connectedness and intuitionistic Fuzzy Πgβ–strongly Connected

Keywords: Intuitionistic Fuzzy Connected, Intuitionistic Fuzzy Πgβ-connected, Intuitionistic Fuzzy ΠgβC₃-connectedness, Intuitionistic Fuzzy ΠgβC₅-connectedness, Intuitionistic Fuzzy ΠgβC₆-connectedness, Intuitionistic Fuzzy Πgβ-Super Connectedness and Intuitionistic Fuzzy Πgβ–strongly Connected

1. Introduction

A predominant characteristic of a topological space is the concept of connectedness and disconnectedness. The former is one of the topological properties that is used to distinguish topological spaces. Connectedness [3] is a powerful tool in topology. Many researchers have investigated the basic properties of connectedness. The first attempt to give a precise definition of these spaces was made by Weierstrass who in fact instigated the notion of arc wise connectedness. However, the notion of connectedness which is used today was introduced by Cantor (1883) in general topology, Later on Zadeh [12] introduced thenotion of fuzzy sets. Fuzzy topological space was further developed by Chang [5]. Coker [6] introduced the intuitionistic fuzzy topological spaces. Connectedness in intuitionistic fuzzy special topological spaces was introduced by Oscag and Coker [6]. Several types of fuzzy connectedness in intuitionistic fuzzy topological spaces were defined by Turnali and Coker [11] and studies these spaces very extensively and also delved into various generalization too of these spaces. Recently Jenitha Premlalatha and Jothimani [7] proposed herald into a new class of sets called Πgβ-closed sets in intuitionistic fuzzy topological space, and these concepts have been used to define and analyse many topological properties. The aim of this paper is to study Πgβ-connectedness and the notions of Intuitionistic fuzzy Πgβ-separated sets, Intuitionistic fuzzy Πgβ-connectedness and Intuitionistic fuzzy Πgβ-disconnectedness is dealt with in detail. Some of their types and their characterizations in Intuitionistic fuzzy topological spaces is studied. The problem focuses on the results when connectedness is replaced with Πgβ-connectedness in intuitionistic fuzzy topological spaces.

2. Preliminaries

Definition 2.1: [2] An intuitionistic fuzzy (IF) set A in X is an object having the form A={<x, µₐ(x), νₐ(x)>/x∈X} where µₐ(x) and νₐ(x) denote the degree of membership and non-membership respectively, and 0≤µₐ(x)+νₐ(x)≤1

Definition 2.2: [2] Let A and B be IFSs of the form A
be not q-coincident \((A \sim B)\) if and only if \(A \subseteq B\) and \(B \subseteq A\). Then \(A \cap B\) is called an intuitionistic fuzzy \(\pi\)-closed set in \((X, \tau)\) for every IF-\(\pi\)-closed set \(V\) of \((Y, \sigma)\).

Definition 2.3: \([4]\) An intuitionistic fuzzy topology (IFT) for short on \(X\) is a family \(\tau\) of IFSs in \(X\) satisfying the following axioms:

(i) \(0, 1 \in \tau\)
(ii) \(G_1 \cap G_2 \in \tau\) for any \(G_1, G_2 \in \tau\)
(iii) \(\bigcup \{G_i / i \in I\} \in \tau\).

Definition 2.4: \([1]\). The intersection of all IF\(\beta\)-closed sets containing \(A\) is called IF\(\beta\)-closure \([2]\) of \(A\), and is denoted by IF\(\beta\)-Cl(\(A\)).

\[\text{Def. 2.4: } \text{IF}^{\beta}\text{-Cl}(A) = \bigcap\{K / K \text{ is IFCS in } X \text{ and } A \subseteq K\}.\]

Definition 2.5: An IF subset \(A\) is said to be IF regular open \([8]\) if \(A = \text{IF}^{\beta}\text{-Cl}(A)\).

The finite union of IF regular open sets is said to be IF\(\beta\)-open \([8]\). The complement of IF\(\beta\)-open set is said to be IF\(\beta\)-closed \([8]\).

Definition 2.6: A is said to be IF\(\beta\)-open \([1]\) if \(A \subseteq \text{IF}^{\beta}\text{-Cl}(\text{IF}(A))\). The family of all IF\(\beta\)-open sets of \(X\) is denoted by IF\(\beta\)O(X).

The complement of a IF\(\beta\)-open set is said to be IF\(\beta\)-closed \([1]\). The intersection of all IF\(\beta\)-closed sets containing \(A\) is called IF\(\beta\)-closure \([2]\) of \(A\), and is denoted by IF\(\beta\)-Cl(\(A\)).

The IF\(\beta\)-Interior \([2]\) of \(A\), denoted by IF\(\beta\)-Int(\(A\)), is defined as union of all IF\(\beta\)-open sets contained in \(A\).

It is well known IF\(\beta\)-Cl(\(A\)) = \(A \cup \text{IF}^{\beta}\text{-Int}(\text{IF}(\text{Int}(A)))\) and IF\(\beta\)-Int(\(A\)) = \(A \cap \text{IF}^{\beta}\text{-Cl}(\text{IF}(\text{Int}(A)))\).

Definition 2.7 \((6)\): A mapping \(f: (X, \tau) \rightarrow (Y, \sigma)\) is called an intuitionistic fuzzy \(\pi\)-\(\beta\)-continuous mapping if \(f^{-1}(V)\) is an IF\(\pi\)-\(\beta\)-closed set in \((X, \tau)\) for every IFCS \(V\) of \((Y, \sigma)\).

Definition 2.8 \((6)\): A mapping \(f: (X, \tau) \rightarrow (Y, \sigma)\) is called an intuitionistic fuzzy \(\pi\)-\(\beta\)-irresolute if \(f^{-1}(V)\) is an IF-\(\pi\)-\(\beta\)-closed set in \((X, \tau)\) for every IF-\(\pi\)-\(\beta\)-closed set \(V\) of \((Y, \sigma)\).

Definition 2.9 \((10)\): Two IF\(\beta\)-A and IF\(\beta\)-B in \(X\) are said to be q-coincident \((A \equiv B)\) if and only if there exists an element \(x\) in \(X\) such that \(\mu_A(x) = \nu_A(x)\) or \(\nu_B(x) = \mu_B(x)\).

Definition 2.10 \((10)\): Two IF\(\beta\)-A and IF\(\beta\)-B in \(X\) are said to be not q-coincident \((A \not\equiv B)\) if and only if \(A \subseteq B^c\).

3. Intuitionistic Fuzzy \(\pi\)-\(\beta\) Connected Spaces and Its Types

Definition 3.1: \([9]\): Two subsets \(A\) and \(B\) in a IF space \((X, \tau)\) are said to be IF\(\pi\)-\(\beta\)-separated if and only if
\[A \cap \text{IF}^{\beta}\text{-Cl}(B) = \emptyset \text{ and } \text{IF}^{\beta}\text{-Cl}(A) \cap B = \emptyset.\]

Remark: 3.1: Each two IF\(\pi\)-\(\beta\)-separated sets are always disjoint, since \(A \cap B \subseteq \text{IF}^{\beta}\text{-Cl}(B) \cap B = \emptyset\).

Theorem 3.1: Let \(A\) and \(B\) be nonempty sets in an IF space \((X, \tau)\). The following statements hold:

(i) If \(A\) and \(B\) are IF\(\pi\)-\(\beta\)-separated and \(A \subseteq A\) and \(B \subseteq B\) then \(A\) and \(B\) are also IF\(\pi\)-\(\beta\)-separated.

(ii) If \(A \cap B = \emptyset\) such that each of \(A\) and \(B\) are both IF\(\pi\)-\(\beta\)-closed (IF\(\pi\)-\(\beta\)-open) then \(A\) and \(B\) are IF\(\pi\)-\(\beta\)-separated.

(iii) If each of \(A\) and \(B\) are both IF\(\pi\)-\(\beta\)-open then \(A\) and \(B\) are IF\(\pi\)-\(\beta\)-separated.

Proof:
(i) Since \(A \subseteq A\) and \(\text{IF}^{\beta}\text{-Cl}(A)\) are IF\(\beta\)-open sets of \(X\), then \(A \cap \text{IF}^{\beta}\text{-Cl}(B) = \emptyset\) and \(\text{IF}^{\beta}\text{-Cl}(A) \cap B = \emptyset\). Hence \(A\) and \(B\) are IF\(\beta\)-separated.

(ii) Since \(A = \text{IF}^{\beta}\text{-Cl}(A)\) and \(B = \text{IF}^{\beta}\text{-Cl}(B)\), then \(A \cap B = \emptyset\). Similarly \(A \cap \text{IF}^{\beta}\text{-Cl}(B) = \emptyset\). Hence \(A\) and \(B\) are IF\(\beta\)-separated.

(iii) If \(A\) and \(B\) are IF\(\pi\)-\(\beta\)-open, then \(A \cap B = \emptyset\). Hence \(A\) and \(B\) are IF\(\pi\)-\(\beta\)-separated.

Theorem 3.2: The sets \(A\) and \(B\) of a IF space \(X\) are IF\(\pi\)-\(\beta\)-separated if and only if there exist \(U\) and \(V\) in IF\(\pi\)-\(\beta\)-O(X) such that \(A \subseteq U\) and \(B \subseteq V\).

Proof:
Let \(A\) and \(B\) be IF\(\pi\)-\(\beta\)-separated sets. Then \(A \cap \text{IF}^{\beta}\text{-Cl}(B) = \emptyset\) and \(\text{IF}^{\beta}\text{-Cl}(A) \cap B = \emptyset\). On the other hand, let \(U, V \subseteq \text{IF}^{\beta}\text{-O}(X)\) such that \(A \subseteq U\) and \(B \subseteq V\). Therefore \(A \cap B = \emptyset\).

Theorem 3.3: Let \(A\) and \(B\) be nonempty disjoint subsets of a IF space \(X\), \(A \neq B\), then \(A\) and \(B\) are IF\(\pi\)-\(\beta\)-separated.

Proof:
Let \(A\) and \(B\) be IF\(\pi\)-\(\beta\)-separated sets. By Definition 3.1, \(A\) contains no IF\(\pi\)-\(\beta\)-lim points of \(B\). Then \(B\) contains all IF\(\pi\)-\(\beta\)-lim points of \(B\) which are in \(A\) and \(B\) is IF\(\pi\)-\(\beta\)-closed in \(A\) and \(B\).

Theorem 3.4: An intuitionistic fuzzy topological space \((X, \tau)\) is said to be intuitionistic fuzzy \(\pi\)-\(\beta\)-connected if there exists an intuitionistic fuzzy \(\pi\)-\(\beta\)-closed set \(A\) in \(X\), \(A \neq \emptyset\), \(B \neq \emptyset\), such that \(\text{AC} \cap B = \emptyset\) and \(\text{AC} \subseteq B\). If \(X\) is not IF\(\pi\)-\(\beta\)-disconnected then it is said to be intuitionistic fuzzy \(\pi\)-\(\beta\)-connected.
Definition 3.7 ([10]):

\( \pi \) Since \( E = G \) \( \pi \)gβ -disconnected. Then there are two nonempty IFπgβ-connected sets in a space \( X \) and \( B, C \) are IFπgβ-connected space if the only IF sets which are disconnected, if there exists an IF set \( A \) \( \sim \) C which contradicts with the hypothesis. Hence one of the conditions (i) and (ii) must be hold. 

Theorem 3.5: Let \( A \) and \( B \) be subsets in IF space \( (X, \tau) \) such that \( A \subset B \subset \text{IFπgβ-C}(A) \). If \( A \) is IFπgβ-connected then \( B \) is IFπgβ-connected.

Proof: If \( B \) is IFπgβ-connected, then there exists two IFπgβ-connected sets \( A \) and \( V \) such that \( B = A \cup V \). Then either \( A \subset \text{IFπgβ-C}(U) \) or \( V \subset \text{IFπgβ-C}(U) \). Let \( A \subset U \). As \( A \subset U \subset \text{IFπgβ-C}(U) \), which is a contradiction. Therefore \( (X, \tau) \) is an intuitionistic fuzzy \( \pi \)gβ-connected space. This implies \( (X, \tau) \) is a contradiction. Therefore \( (X, \tau) \) is an intuitionistic fuzzy \( \pi \)gβ-connected space.

Theorem 3.6: An IFTS \( (X, \tau) \) is an intuitionistic fuzzy \( \pi \)gβ-connected space if and only if there exist no non-zero intuitionistic fuzzy \( \pi \)gβ-open sets \( A \) and \( B \) in \( (X, \tau) \) such that \( A \neq B \).

Proof: Necessity: Let \( A \) and \( B \) be two intuitionistic fuzzy \( \pi \)gβ-open sets \( A \) and \( B \) in \( (X, \tau) \) such that \( A \neq B \). Therefore \( B \) is an intuitionistic fuzzy \( \pi \)gβ-open set. Since \( A \neq B \), \( A \neq B \). This implies \( B \) is a proper IFS which is both intuitionistic fuzzy \( \pi \)gβ-open and intuitionistic fuzzy \( \pi \)gβ-closed in \( (X, \tau) \). Hence \( (X, \tau) \) is not an intuitionistic fuzzy \( \pi \)gβ-connected space. But this is a contradiction to our hypothesis that there exist no non-zero intuitionistic fuzzy \( \pi \)gβ-open sets \( A \) and \( B \).

Sufficiency: Let \( A \) be both intuitionistic fuzzy \( \pi \)gβ-open and intuitionistic fuzzy \( \pi \)gβ-closed in \( (X, \tau) \) such that \( A \neq B \). Now let \( B = A \). Then \( B \) is an intuitionistic fuzzy \( \pi \)gβ-open set and \( A \neq B \).

This implies \( B = A \neq B \), which is a contradiction to our hypothesis. Therefore, \( (X, \tau) \) is an intuitionistic fuzzy \( \pi \)gβ-connected space.

Theorem 3.10: An IFTS \( (X, \tau) \) is an intuitionistic fuzzy \( \pi \)gβ-connected space if and only if there exist no non-zero intuitionistic fuzzy \( \pi \)gβ-open sets \( A \) and \( B \) in \( (X, \tau) \) such that \( A = B \), \( B = (\beta-\text{Cl}(A)) \) and \( A = (\beta-\text{Cl}(B)) \).

Proof: Necessity: Assume that there exist IF sets \( A \) and \( B \) such that \( A \neq B \) such that \( A = B \), \( B = (\beta-\text{Cl}(A)) \) and \( A = (\beta-\text{Cl}(B)) \). Since \( B = (\beta-\text{Cl}(A)) \) and \( A = (\beta-\text{Cl}(B)) \) are intuitionistic fuzzy \( \pi \)gβ-open sets in \( (X, \tau) \), \( A \) and \( B \) are intuitionistic fuzzy \( \pi \)gβ-open sets in \( (X, \tau) \). This implies \( (X, \tau) \) is not an intuitionistic fuzzy \( \pi \)gβ-connected space, which is a contradiction. Therefore there exist no non-zero intuitionistic fuzzy \( \pi \)gβ-open sets \( A \) and \( B \) in \( (X, \tau) \) such that \( A = B \), \( B = (\beta-\text{Cl}(A)) \) and \( A = (\beta-\text{Cl}(B)) \).

Sufficiency: Let \( A \) be both intuitionistic fuzzy \( \pi \)gβ-open and intuitionistic fuzzy \( \pi \)gβ-closed in \( (X, \tau) \) such that \( A \neq B \). Now by taking \( B = A \), will lead to the contradiction to our hypothesis. Hence \( (X, \tau) \) is an intuitionistic fuzzy \( \pi \)gβ-connected space.

Definition 3.9: An IFTS \( (X, \tau) \) is said to be an intuitionistic fuzzy \( \pi \)gβ-connected space if every intuitionistic fuzzy \( \pi \)gβ-closed set in \( (X, \tau) \) is an intuitionistic fuzzy \( \pi \)gβ-connected space.
(ii) \((X, \tau)\) is an intuitionistic fuzzy GO-connected space.

(iii) \((X, \tau)\) is an intuitionistic fuzzy \(\pi\beta\)-connected space.

Proof: (i)\(\to\)(ii) is obvious from Theorem 3.8

(ii)\(\to\)(iii) is obvious.

(iii)\(\to\)(i) Let \((X, \tau)\) be an intuitionistic fuzzy \(\pi\beta\)-connected space. Suppose \((X, \tau)\) is not an intuitionistic fuzzy \(\pi\)-connected space, then there exists a proper IFS \(A\) in \((X, \tau)\) which is both intuitionistic fuzzy \(\pi\beta\)-open and intuitionistic fuzzy \(\pi\beta\)-closed in \((X, \tau)\). Since \((X, \tau)\) is an intuitionistic fuzzy \(\pi\beta\)-connected space, it is both intuitionistic fuzzy \(\pi\beta\)-open and intuitionistic fuzzy \(\pi\beta\)-closed in \((X, \tau)\). This implies that \((X, \tau)\) is not an intuitionistic fuzzy \(\pi\beta\)-connected space, which is a contradiction to our hypothesis. Therefore \((X, \tau)\) is intuitionistic fuzzy \(\pi\beta\)-connected space.

Theorem 3.12: If \(f: (X, \tau)\rightarrow(Y, \sigma)\) is an intuitionistic fuzzy \(\pi\beta\)-continuous surjection and \((X, \tau)\) is an intuitionistic fuzzy \(\pi\beta\)-connected space, then \((Y, \sigma)\) is an intuitionistic fuzzy \(\pi\beta\)-connected space.

Proof: Let \((X, \tau)\) be an intuitionistic fuzzy \(\pi\beta\)-connected space. Suppose \((Y, \sigma)\) is not an intuitionistic fuzzy \(\pi\beta\)-connected space, then there exists a proper IFS \(A\) in \((X, \tau)\) which is both intuitionistic fuzzy \(\pi\beta\)-open and intuitionistic fuzzy \(\pi\beta\)-closed in \((Y, \sigma)\). Since \(f\) is an intuitionistic fuzzy \(\pi\beta\)-continuous mapping, \(f^{-1}(A)\) is both intuitionistic fuzzy \(\pi\beta\)-open and intuitionistic fuzzy \(\pi\beta\)-closed in \((X, \tau)\). But this is a contradiction to hypothesis. Hence \((Y, \sigma)\) is an intuitionistic fuzzy \(\pi\beta\)-connected space.

Theorem 3.13: If \(f: (X, \tau)\rightarrow(Y, \sigma)\) is an intuitionistic fuzzy \(\pi\beta\)-irresolute surjection and \((X, \tau)\) is an intuitionistic fuzzy \(\pi\beta\)-connected space, then \((Y, \sigma)\) is also an intuitionistic fuzzy \(\pi\beta\)-connected space.

Proof: Suppose \((Y, \sigma)\) is not an intuitionistic fuzzy \(\pi\beta\)-connected space, then there exists a proper IFS \(A\) in \((X, \tau)\) which is both intuitionistic fuzzy \(\pi\beta\)-open and intuitionistic fuzzy \(\pi\beta\)-closed in \((Y, \sigma)\). Since \(f\) is an intuitionistic fuzzy \(\pi\beta\)-irresolute mapping, \(f^{-1}(A)\) is both intuitionistic fuzzy \(\pi\beta\)-open and intuitionistic fuzzy \(\pi\beta\)-closed in \((X, \tau)\). But this is a contradiction to hypothesis. Hence \((Y, \sigma)\) is an intuitionistic fuzzy \(\pi\beta\)-connected space.

Definition 3.10: An IFTS \((X, \tau)\) is called intuitionistic fuzzy \(\pi\beta\)-connected between two IFSs \(A\) and \(B\) if there is no intuitionistic fuzzy open set \(E\) in \((X, \tau)\) such that \(A \subseteq E\) and \(E \cap B = \emptyset\).

Definition 3.11: An IFTS \((X, \tau)\) is called intuitionistic fuzzy \(\pi\beta\)-connected between two IFSs \(A\) and \(B\) if there is no intuitionistic fuzzy \(\pi\beta\)-open set \(E\) in \((X, \tau)\) such that \(A \subseteq E\) and \(E \cap B = \emptyset\).

Theorem 3.14: If an IFTS \((X, \tau)\) is intuitionistic fuzzy \(\pi\beta\)-connected between two IFSs \(A\) and \(B\), then it is intuitionistic fuzzy \(\pi\beta\)-connected between two IF Sets \(A\) and \(B\).

Proof: Suppose \((X, \tau)\) is not intuitionistic fuzzy \(\pi\beta\)-connected between two IFSs \(A\) and \(B\), then there exists an intuitionistic fuzzy \(\pi\beta\)-open set \(E\) in \((X, \tau)\) such that \(A \subseteq E\) and \(E \cap B = \emptyset\). Since every intuitionistic fuzzy open set is intuitionistic fuzzy \(\pi\beta\)-open set, there exists an intuitionistic fuzzy \(\pi\beta\)-open set \(E\) in \((X, \tau)\) such that \(A \subseteq E\) and \(E \cap B = \emptyset\). This implies \((X, \tau)\) is not intuitionistic fuzzy \(\pi\beta\)-connected between \(A\) and \(B\).
**Definition 3.13:** An IFTS $(X, \tau)$ is called an intuitionistic fuzzy \(\pi g\beta\)-super connected space if there exists no intuitionistic fuzzy regular \(\pi g\beta\) open set in $(X, \tau)$.

**Theorem 3.18:** Let $(X, \tau)$ be an IFTS, then the following are equivalent.

(i) $(X, \tau)$ is an intuitionistic fuzzy \(\pi g\beta\)-super connected space.

(ii) For every non-zero intuitionistic fuzzy regular \(\pi g\beta\) open set $A$, \(\pi g\beta\)-Cl($A$) $= 1$.

(iii) For every intuitionistic fuzzy regular \(\pi g\beta\) closed set $A$ with $A = 1$, \(\pi g\beta\)-Int($A$) $= 0$.

(iv) There exists no intuitionistic fuzzy regular \(\pi g\beta\) open sets $A$ and $B$ in $(X, \tau)$ such that $A = 0$ and $B = \pi g\beta$-$Cl(A)$, $A = \pi g\beta$-$Int(B)$.

(v) There exists no intuitionistic fuzzy regular \(\pi g\beta\) closed sets $A$ and $B$ in $(X, \tau)$ such that $A = 1$ and $B = \pi g\beta$-$Int(A)$.

(vi) $(X, \tau)$ is an intuitionistic fuzzy \(\pi g\beta\)-super connected space.

Proof: (i) $\Rightarrow$ (ii) Assume that there exists an intuitionistic fuzzy regular \(\pi g\beta\) open set $A$ in $(X, \tau)$ such that $A = 0$ and \(\pi g\beta\)$-$Cl(A) = 1$. Now let $B = \pi g\beta$-$Int(\pi g\beta$-$Cl(A))$. Then $B$ is a proper intuitionistic fuzzy regular \(\pi g\beta\) open set in $(X, \tau)$. But this is a contradiction to the fact that $(X, \tau)$ is an intuitionistic fuzzy \(\pi g\beta\)-super connected space. Therefore \(\pi g\beta\)$-$Cl(A) = 1$.

(ii) $\Rightarrow$ (iii) Let $A = 1$, be an intuitionistic fuzzy regular \(\pi g\beta\)-closed set in $(X, \tau)$. If $B = A^c$, then $B$ is an intuitionistic fuzzy regular open set in $(X, \tau)$ with $B = 0$. That is \(\pi g\beta\)$-$Int(B) = 0$. Hence \(\pi g\beta\)$-$Int(A) = 0$.

(iii) $\Rightarrow$ (iv) Let $A$ and $B$ be two intuitionistic fuzzy regular \(\pi g\beta\) open sets in $(X, \tau)$ such that $A = 0$ and \(\pi g\beta\)$-$Cl(A) = 1$. Since $B$ is an intuitionistic fuzzy regular \(\pi g\beta\)-closed set in $(X, \tau)$ and $B = 0$, implies $B = 1$, $B = \pi g\beta$-$Cl(\pi g\beta$-$Int(B))$ and we have, \(\pi g\beta\)$-$Int(B) = 0$. But $A \subseteq B^c$.

Therefore $\pi g\beta$-$Int(\pi g\beta$-$Cl(A)) \subseteq \pi g\beta$-$Int(\pi g\beta$-$Cl(B)) = \pi g\beta$-$Int(\pi g\beta$-$Cl(A)) \subseteq \pi g\beta$-$Int(\pi g\beta$-$Cl(\pi g\beta$-$Int(B))) = \pi g\beta$-$Int(B) = 0$. Which is a contradiction. Therefore (iv) is true.

(iv) $\Rightarrow$ (v) Let $A = 1$ be an intuitionistic fuzzy regular \(\pi g\beta\)-open set in $(X, \tau)$.

If we take $B = \pi g\beta$-$Cl(A)^c$, then $B$ is an intuitionistic fuzzy regular \(\pi g\beta\) open set.

(v) $\Rightarrow$ (vi) Let $A = 1$, be an intuitionistic fuzzy regular \(\pi g\beta\) open set in $(X, \tau)$.

If we take $B = \pi g\beta$-$Cl(A)^c$, then $B$ is an intuitionistic fuzzy regular \(\pi g\beta\) open set, since \(\pi g\beta$-$Int(\pi g\beta$-$Cl(B)) = \pi g\beta$-$Int(\pi g\beta$-$Cl(\pi g\beta$-$Cl(A)^c)) = \pi g\beta$-$Int(\pi g\beta$-$Cl(A)^c) = \pi g\beta$-$Cl(A)^c = B$. Hence $A = \pi g\beta$-$Cl(A)^c = 1$.

**Definition 3.19:** An IFTS $(X, \tau)$ is said to be \(\pi g\beta\)-connected if there exists no non-empty IFS's $A$ and $B$ in $X$ such that $A \neq 1$ and $B$ is a contradiction to (v). Therefore $(X, \tau)$ is an intuitionistic fuzzy \(\pi g\beta\)-super connected space.

Proof: Suppose that $Y$ is not \(\pi g\beta\)-connected then there exists IFS's $C$ and $D$ in $Y$ such that $C \neq 0$, $D = 0$. Since $f$ is \(\pi g\beta\)-irresolute, $f$ is \(\pi g\beta\)-connected. Therefore $f(C) \cap f(D) = 0$. Hence $X$ is \(\pi g\beta\)-connected. Thus $(Y, \pi g\beta)$ is \(\pi g\beta\)-connected.

**Definition 3.16:** $A$ and $B$ are non-zero intuitionistic fuzzy sets in $(X, \tau)$. Then $A$ and $B$ are said to be \(\pi g\beta\)-weakly separated if $\pi g\beta$-$Cl(A) \subseteq B$ and $\pi g\beta$-$Cl(B) \subseteq A^c$.

(i) If $B = \pi g\beta$-$Cl(A)$, then $B = 0$. But $A \neq 1$, $B = \pi g\beta$-$Cl(A)^c$. Hence $A = \pi g\beta$-$Int(\pi g\beta$-$Cl(A)) = \pi g\beta$-$Int(1 - 1) = 1 - 1$. This is $A = 1$, which is a contradiction. Therefore $(X, \tau)$ is an intuitionistic fuzzy \(\pi g\beta\)-connected space.

**Definition 3.17:** An IFTS $(X, \tau)$ is said to be \(\pi g\beta\)-$C_\Sigma$-disconnected if there exists non-zero intuitionistic fuzzy sets $A$ and $B$ in $(X, \tau)$ such that $A \cup B = 1$.

**Definition 3.18:** An IFTS $(X, \tau)$ is said to be \(\pi g\beta\)-$C_\Sigma$-connected if there exists non-zero intuitionistic fuzzy sets $A$ and $B$ in $(X, \tau)$ such that $A \cup B = 1$.

**Remark 3.2:** An IFTS $(X, \tau)$ is said to be \(\pi g\beta\)-$C_\Sigma$-connected if and only if $(X, \tau)$ is \(\pi g\beta\)-$C_\Sigma$-connected.
intuitionistic fuzzy πβ extremally disconnected if the πβ closure of every intuitionistic fuzzy πβ open set in (X, τ) is an intuitionistic fuzzy πβ open set.

Theorem 3.20: Let (X, τ) be an intuitionistic fuzzy πβ T_{1/2} space, then the following are equivalent.

(i) (X, τ) is an intuitionistic fuzzy πβ extremally disconnected space.

(ii) For each intuitionistic fuzzy πβ closed set A, πβ-Int(A) is an intuitionistic fuzzy πβ open set.

(iii) For each intuitionistic fuzzy πβ open set A, πβ-cl(A) = (πβ-cl(πβ-cl(A)))^C.

Proof: (i) → (ii) Let A be any intuitionistic fuzzy πβ closed set. Then A^C is an intuitionistic fuzzy πβ open set. So (i) implies that πβ-Cl(A^C) = (πβ-Int(A))^C is an intuitionistic fuzzy πβ open set. Thus πβ-Cl(A) is an intuitionistic fuzzy πβ closed set in (X, τ).

(ii) → (iii) Let A be an intuitionistic fuzzy πβ open set. Then we have πβ-cl(A) = (πβ-cl(πβ-cl(A))^C. Therefore (πβ-cl(πβ-cl(A))^C)^C = (πβ-cl(πβ-cl(A)))^C. Since A is an intuitionistic fuzzy πβ open set, A^C is an intuitionistic fuzzy πβ closed set. So by (ii) πβ-Int(A^C) is an intuitionistic fuzzy πβ closed set. That is πβ-cl(πβ-Int(A^C)) = πβ-Int(A^C).

Hence (πβ-cl(A)) = (πβ-cl(A))^C.

(iii) → (iv) Let A and B be any two intuitionistic fuzzy πβ-open sets in (X, τ) such that πβ-Cl(A) = B^C. (iii) implies πβ-Cl(A) = (πβ-cl(πβ-cl(A)))^C = (πβ-cl(B))^C = (πβ-cl(B))^C.

(iv) → (i) Let A be any intuitionistic fuzzy πβ open set in (X, τ). Put B = (πβ-cl(A))^C. Then πβ-Cl(A) = B^C. Hence by (iv), πβ-Cl(A) = (πβ-cl(B))^C. Since πβ-cl(B) is an intuitionistic fuzzy πβ-closed set as the space is an intuitionistic fuzzy πβ-T_{1/2} space, it follows that πβ-cl(A) is an intuitionistic fuzzy πβ open set. This implies that (X, τ) is an intuitionistic fuzzy πβ extremally disconnected space.

5. Conclusion

The πβ closed sets are used to introduce the concepts πβ-connected spaces. Also, the characterization and the types of πβ-connected spaces have been framed and analyzed. In general, the entire content will be a successful tool for the researchers for finding the path to obtain the results in the context of connected spaces in bi topology, and can be extended to Group theory. Also it is believed that this approach will prove useful for studying structures in the phase space of dynamical systems.

References

[1] M. E. Abd El-Monsef, S. N. El-Deeb and R. A. Mahmoud, β-open sets and β-continuous mappings, Bull. Fac. Sci. Assiut Univ., 12(1983), 77-90.
[2] M. E. Abd El-Monsef and R. A. Mahmoud, β-irresolute and β-topological invariant, Proc. Pakistan Acad. Sci., 27(1990), 285-296.
[3] A. V. Arhangel’skii, R. Wiegandt. Connectedness and disconnectedness in topology. Top. Appl 1975, 5.
[4] Atanassov, K., Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 1986, 87-96.
[5] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24(1968) 182–190.
[6] S. Ozcag and D. Coker, On connectedness in intuitionistic fuzzy specialtopological spaces, Inter. J. Math. Math. Sci. 21(1998) 33-40.
[7] J. Jenitha Premalatha, S. Jothimani–Intuitionistic fuzzy πgβ closed set– Int. J. Adv. Appl. Math. andMech. 2(2)(2014)92-101.
[8] M. Sarsak, N. Rajesh, π-Generalized Semi-Pre closed Sets, Int. Mathematical Forum 5(2010)573-578.
[9] Sucharita Chakraborti, Hiranmay Dasgupta, International Mathematical Forum, Vol. 8, 2013, no. 38, 1889-1901.
[10] S. Thakur and R. Chaturvedi, Regular generalized closed sets in intuitionistic fuzzy topological spaces, Universitatea Din Bacau, Studii Si CercetariStiintifice, Seria:Mathematica 16(2006) 257–272.
[11] N. Turnali and D. Coker Fuzzy connectedness in intuitionistic fuzzy topological spaces, Fuzzy Sets andSystems 116 (2000)369–375.
[12] L. A. Zadeh, Fuzzy sets, Inform. Control 8 (1965)338–353.