On the uniqueness of $D = 11$ interactions among a graviton, a massless gravitino and a three-form.

III: Graviton and gravitini

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Abstract

Under the hypotheses of smoothness of the interactions in the coupling constant, locality, Poincaré invariance, Lorentz covariance, and the preservation of the number of derivatives on each field in the Lagrangian of the interacting theory (the same number of derivatives like in the free Lagrangian), we prove that in $D = 11$ there are no cross-interactions between the graviton and the massless gravitino and also no self-interactions in the Rarita-Schwinger sector. A comparison with the case $D = 4$ is briefly discussed.

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1 Introduction

Here, we develop the third step of constructing all possible interactions in $D = 11$ among a graviton, a massless Majorana spin-3/2 field, and a three-form gauge field. The previous steps were exposed in [1], where we obtained all the interactions that can be added to a eleven-dimensional free theory describing a massless spin-two field and an Abelian three-form gauge field, and respectively in [2], where the same problem was solved with respect to a massless Rarita-Schwinger field and an Abelian three-form gauge field. Based on the previously mentioned results, in the sequel we analyze the consistent couplings that can be introduced between a massless spin-two field (described in the free limit by the Pauli-Fierz action) and a massless Rarita-Schwinger spinor in eleven spacetime dimensions. Under the hypotheses of smoothness of the interactions in the coupling constant, locality, Poincaré invariance, Lorentz covariance, and the
preservation of the number of derivatives on each field in the Lagrangian of the interacting theory (the same number of derivatives like in the free Lagrangian), we prove that in $D = 11$ there are no cross-interactions between the graviton and the massless gravitini and also no self-interactions among the gravitini. As announced in [2], we comment on the absence of self-interactions among the gravitini in $D = 11$ and argue that this result does not contradict the presence in the Lagrangian of $D = 11$, $N = 1$ SUGRA of a quartic gravitini vertex. We also make the comparison with the case $D = 4$, where gravitini are known to allow self-interactions in the presence of a graviton, such that their ‘mass’ constant becomes related to the cosmological one.

2 Free model: Lagrangian formulation and BRST symmetry

Our starting point is represented by a free model, whose lagrangian action is written as the sum between the action of the linearized version of Einstein-Hilbert gravity (the Pauli-Fierz action) and that of a massless Rarita-Schwinger field in eleven spacetime dimensions

$$S_0^h \left[ h_{\mu \nu}, \psi_\mu \right] = \int d^{11}x \left[ -\frac{1}{2} \left( \partial_{\mu} h_{\nu \rho} \right) \left( \partial^\rho \partial^\nu h_{\mu \rho} \right) + \frac{1}{2} \left( \partial_{\mu} h \right) \left( \partial_{\nu} h \right) - \frac{i}{2} \bar{\psi}_\mu \gamma_{\mu \nu \rho} \partial^\nu \psi^\rho \right]$$

\[ \equiv \int d^{11}x \left( L^h + L_0^\psi \right). \] (1)

We follow closely all the conventions and notations from [1] related to the Pauli-Fierz field and respectively from [2] in relation with the massless Rarita-Schwinger theory. We will need the Fierz identities specific to $D = 11$

$$\gamma_{\mu_1 \cdots \mu_p \nu_1 \cdots \nu_q} = \sum_{p+q-11 \leq k \leq 2M} \delta^{|\mu_1 \cdots \mu_p \nu_1 \cdots \nu_k |} \delta^{|\nu_{k+1} \cdots \nu_q |} \gamma_{\mu_1 \cdots \mu_p \nu_1 \cdots \nu_k \nu_{k+1} \cdots \nu_q},$$

\[ \] (2)

where $M = \min(p, q)$ and also the development of a complex, spinor-like matrix $N$ in terms of the basis $\{1, \gamma_\mu, \gamma_{\mu \nu}, \gamma_{\mu \nu \rho}, \gamma_{\mu \nu \rho \lambda}, \gamma_{\mu \nu \rho \lambda \sigma}\}$

$$N = \frac{1}{32} \sum_{k=0}^{5} \left( -k^{k-1}/2 \right) \frac{1}{k!} \text{Tr} \left( \gamma_{\mu_1 \cdots \mu_k} N \right) \gamma_{\mu_1 \cdots \mu_k}.$$ (3)

We recall that $[\mu_1 \cdots \mu_k]$ signifies complete antisymmetry with respect to the (in this case Lorentz) indices between brackets, with the conventions that the minimum number of terms is always used and the result is never divided by the number of terms. Action (1) possesses an irreducible and Abelian generating set of gauge transformations

$$\delta_\epsilon h_{\mu \nu} = \partial_{(\mu} \epsilon_{\nu)}, \quad \delta_\epsilon \psi_\mu = \partial_\mu \epsilon,$$ (4)
with $\epsilon_\mu$ bosonic and $\varepsilon$ fermionic gauge parameters. In addition $\varepsilon$ is a Majorana spinor.

In order to construct the BRST symmetry for (1) we introduce the fermionic ghosts $\eta_\mu$ corresponding to the gauge parameters $\epsilon_\mu$ and the bosonic, spinor-like ghost $\xi$ corresponding to the gauge parameter $\varepsilon$ and associate antifields with the original fields and ghosts, respectively denoted by $\{h^{\ast\mu\nu}, \psi_\mu^\ast\}$ and $\{\eta^{\ast\mu}, \xi^\ast\}$. The antifields of the Rarita-Schwinger fields are bosonic, purely imaginary spinors.

Like in the previous situations (see [1, 2]), the BRST differential simply decomposes into $s = \delta + \gamma$, where $\delta$ represents the Koszul-Tate differential and $\gamma$ stands for the exterior derivative along the gauge orbits. If we make the notations

$$
\Phi_{A0} = (h_{\mu\nu}, \psi_\mu), \quad \Phi_{A0}^\ast = (h^{\ast\mu\nu}, \psi_\mu^\ast),
$$

(5)

$$
\eta_{A1} = (\eta_\mu, \xi), \quad \eta_{A1}^\ast = (\eta^{\ast\mu}, \xi^\ast),
$$

(6)

then, according to the standard rules of the BRST formalism, the degrees of the BRST generators are valued as: $\text{agh}(\Phi_{A0}) = 0 = \text{agh}(\eta_{A1}^\ast)$, $\text{pgh}(\Phi_{A0}) = 1$, $\text{pgh}(\eta_{A1}) = 0$. The actions of the differentials $\delta$ and $\gamma$ on the generators from the BRST complex are given by

$$
\delta h^{\ast\mu\nu} = 2H^{\mu\nu}, \quad \delta \psi^{\ast\mu} = -i \partial_\mu \bar{\psi}_\lambda \gamma^{\mu\lambda\nu},
$$

(7)

$$
\delta \eta^{\ast\mu} = -2\partial_\mu h^{\ast\mu\nu}, \quad \delta \xi^\ast = \partial_\mu \psi^{\ast\mu},
$$

(8)

$$
\delta \Phi_{A0} = 0 = \delta \eta_{A1}^\ast,
$$

(9)

$$
\gamma \Phi_{A0}^\ast = 0 = \gamma \eta_{A1},
$$

(10)

$$
\gamma h_{\mu\nu} = \partial_\mu \eta_{\nu}, \quad \gamma \psi_\mu = \partial_\mu \xi, \quad \gamma \eta_{A1} = 0,
$$

(11)

where $H^{\mu\nu}$ is the linearized Einstein tensor. The full solution to the master equation $(S^{h,\psi}, S^{h,\psi}) = 0$, where $S^{h,\psi}$ is the anticanonical generator of the BRST differential with respect to the antibracket structure, $s = (\cdot, S^{h,\psi})$, reads in our case as

$$
S^{h,\psi} = S_0^L [h_{\mu\nu}, \psi_\mu] + \int d^{11}x (h^{\ast\mu\nu} \partial_\mu \eta_{\nu} + \psi^{\ast\mu} \partial_\mu \xi).
$$

(12)

### 3 Consistent interactions between a graviton and gravitini

In order to investigate the consistent couplings that can be added to the free action (1) we act like in [1, 2] and rely on the reformulation of the interaction problem in the context of the antifield-BRST deformation procedure. Thus, if an interacting gauge theory can be consistently constructed, then the solution $S^{h,\psi}$ to the master equation associated with the free theory, (12), can be deformed into a solution $\bar{S}^{h,\psi}$

$$
S^{h,\psi} \rightarrow \bar{S}^{h,\psi} = S^{h,\psi} + \lambda S_1^{h,\psi} + \lambda^2 S_2^{h,\psi} + \ldots
$$
\[ = S^{h,\psi} + \lambda \int d^D x a^{h,\psi} + \lambda^2 \int d^D x b^{h,\psi} + \cdots \]  

(13)
of the master equation for the deformed theory \((\bar{S}^{h,\psi}, \bar{S}^{h,\psi}) = 0\), such that both the ghost and antifield spectra of the initial theory are preserved. The last equation splits, according to the various orders in the coupling constant \(\lambda\), into the equivalent tower of equations: 

\[
2 \left( S^{h,\psi}_1, S^{h,\psi} \right) = 0, \quad (14) \\
2 \left( S^{h,\psi}_2, S^{h,\psi} \right) + \left( S^{h,\psi}_1, S^{h,\psi}_1 \right) = 0, \quad (15) \\
\vdots
\]

Equation \((S^{h,\psi}, S^{h,\psi}) = 0\) is fulfilled by hypothesis, while the next one requires that the first-order deformation of the solution to the master equation, \(S^{h,\psi}_1\), is a (nontrivial) co-cycle of the “free” BRST differential at ghost number zero, \(S^{h,\psi}_1 \in H^0(s)\). Our main concern is to determine \(S^{h,\psi}_1, S^{h,\psi}_2, \) etc. that comply with all the main hypotheses: smoothness of interactions in the coupling constant, locality, Poincaré invariance, Lorentz covariance, and the preservation of the number of derivatives on each field in the interacting Lagrangian with respect to the free theory.

### 4 First-order deformation

The resolution of equation (14) implies standard cohomological techniques related to the BRST differential of the free model under consideration. The necessary cohomological ingredients have already been discussed in [1, 2], so in the sequel we give the solutions to these equations without going into further details. The nontrivial solution to equation (14) can be shown to expand as 

\[ S^{h,\psi}_1 = \int d^1 x \left( a^{h,\psi}_0 + a^{h,\psi}_1 + a^{h,\psi}_2 \right), \]

where \(a^{h,\psi}_k\) describes the cross-interactions between the two theories (so it effectively mixes both sectors), and \(a^{h,\psi}_k\) involves only the Rarita-Schwinger sector. The components \(a^{h,\psi}_0\) and \(a^{h,\psi}_1\) are given by

\[
a^{h,\psi}_0 = \frac{1}{2} \eta^{\mu\nu} \eta^\rho \partial_{[\mu} \eta_{\nu]}, \quad (17) \\
a^{h,\psi}_1 = h^{\mu\rho} \left( (\partial_{[\mu} \eta^\nu) h_{\rho]} - \eta^\rho \partial_{[\mu} h_{\rho]} \right), \quad (18)
\]

while \(a^{h,\psi}_2\) is the cubic vertex of the Einstein-Hilbert lagrangian plus a cosmological term

\[
a^{h,\psi}_2 = a^{h,\psi}_{0\text{-cubic}} - 2\Lambda h, \quad (19)
\]

\[4\]
with \( \Lambda \) the cosmological constant. Related to the interaction sector, it can be shown that the components \( a_k^{h-\psi} \) read as

\[
a_2^{h-\psi} = \frac{\tilde{k}}{8} \left( -\frac{i}{2} \eta^{*\mu} \xi \gamma_\mu \xi + \xi^{*\gamma\mu\nu} \xi \partial_{[\mu} \eta_{\nu]} \right),
\]

(20)

\[
a_1^{h-\psi} = \frac{\tilde{k}}{4} \left[ i h^{*\mu\nu} \xi \gamma_\mu \psi_\nu + \frac{1}{2} \psi^{*\mu} \gamma^{\alpha\beta} \left( \psi_\mu \partial_{[\alpha} \eta_{\beta]} - \xi \partial_{[\alpha} h_{\beta]\mu] \right)
- 4 \psi^{*\mu} \left( \partial_{[\mu} \psi_{\nu]} \right) \eta^\nu \right],
\]

(21)

\[
a_0^{h-\psi} = \frac{\tilde{k}}{4} \left[ \left( \partial_\alpha \bar{\psi}_\beta \right) \gamma^{\alpha\beta\mu} \psi_\nu (h_{\mu\nu} - \sigma_{\mu\nu} h) + \left( \partial_{[\alpha} \bar{\psi}_{\beta]} \right) \gamma^{\mu\nu\beta} \psi_\nu h_{\alpha\beta}
+ \frac{1}{2} \bar{\psi}^\mu \left( \gamma^{\mu\nu} \gamma^\alpha \psi^\lambda \right) \partial_{[\mu} h_{\nu\lambda]} \right],
\]

(22)

while for the self-interactions of the Rarita-Schwinger field one obtains

\[
a_2^\psi = 0, \quad a_1^\psi = \text{im} \psi_\mu^* \gamma^\mu \xi,
\]

(23)

\[
a_0^\psi = - \frac{9 m}{2} \psi_\mu \gamma^{\mu\nu} \psi_\nu,
\]

(24)

with \( \tilde{k} \) and \( m \) some arbitrary, real constants.

## 5 Second-order deformation

We have seen in the above that the first-order deformation can be written as the sum between the first-order deformation of the solution to the master equation for the Pauli-Fierz theory \( S_1^F = \int d^11 x \left( a_1^{h} + a_1^{\psi} + a_0^{h} \right) \), the ‘interacting’ part \( S_1^{h-\psi} = \int d^11 x \left( a_2^{h-\psi} + a_1^{h-\psi} + a_0^{h-\psi} \right) \), and the Rarita-Schwinger component \( S_1^\psi = \int d^11 x \left( a_1^\psi + a_0^\psi \right) \). Thus, the first-order deformation is parameterized in terms of three real constants: \( \Lambda, \tilde{k}, \) and \( m \).

In the sequel we infer the complete expression of the second-order deformation of the solution to the master equation, \( S_2^{h,\psi} \), which is subject to equation (15). Acting like in the above, we can write the second-order deformation as the sum between the Pauli-Fierz, the Rarita-Schwinger, and the interacting parts, \( S_2^{h,\psi} = S_2^h + S_2^\psi + S_2^{h-\psi} \). The piece \( S_2^h \) describes the second-order deformation in the Pauli-Fierz sector and we will not insist on it since we are merely interested in cross-couplings. The term \( S_2^\psi \) results as solution to the equation

\[
(S_1, S_1)^\psi + 2 S_2^\psi = 0,
\]

(25)

where \( (S_1, S_1)^\psi = \left( S_1^{h,\psi}, S_1^{h,\psi} \right) + \left( S_1^{h-\psi}, S_1^{h-\psi} \right) \psi \). In the last formula the notation \( \left( S_1^{h-\psi}, S_1^{h-\psi} \right) \psi \) signifies the terms from the antibracket \( \left( S_1^{h-\psi}, S_1^{h-\psi} \right) \) that
contain only BRST generators from the Rarita-Schwinger sector. The piece $S_2^{h-\psi}$ is solution to the equation

\[
(S_1, S_1)^{h-\psi} + 2sS_2^{h-\psi} = 0,
\]

(26)

where $(S_1, S_1)^{h-\psi} = 2(S_1^h, S_1^{h-\psi}) + 2(S_1^{h-\psi}, S_1^{h-\psi})$. In the last relation we used the notation $\bar{\Delta}$ where

\[
(S_1^{h-\psi}, S_1^{h-\psi})^{h-\psi} = (S_1^{h-\psi}, S_1^{h-\psi}) - \left(S_1^{h-\psi}, S_1^{h-\psi}\right). \tag{25}
\]

If we denote by $\bar{\Delta}^\psi$ and $\bar{b}^\psi$ the nonintegrated densities of $(S_1, S_1)^\psi$ and respectively of $\bar{S}_2^\psi$, then equation (25) takes the local form

\[
\bar{\Delta}^\psi = -2\bar{b}^\psi + \partial_\mu \bar{n}^\mu,
\]

(27)

with $\text{gh}(\bar{\Delta}^\psi) = 1$, $\text{gh}(\bar{b}^\psi) = 0$, $\text{gh}(\bar{n}^\mu) = 1$, for some local currents $\bar{n}^\mu$. Direct computation shows that $\bar{\Delta}^\psi$ decomposes as $\bar{\Delta}^\psi = \bar{\Delta}_2^\psi + \bar{\Delta}_1^\psi + \bar{\Delta}_0^\psi$, with $\text{agh}(\bar{\Delta}_I^\psi) = I, I = 0, 1, 2$, where

\[
\bar{\Delta}_2^\psi = \partial_\mu \bar{\tau}^\mu_2 + \gamma \left(\frac{i\bar{k}^2}{16}\xi^\gamma \gamma^{\mu\nu} \xi \xi_{\mu\nu}\right),
\]

(28)

\[
\bar{\Delta}_1^\psi = \partial_\mu \bar{\tau}^\mu_1 + \delta \left(\frac{i\bar{k}^2}{16}\xi^\gamma \gamma^{\mu\nu} \xi \xi_{\mu\nu}\right) + \gamma \gamma \left(\frac{i\bar{k}^2}{16}\psi^{\mu\nu} \gamma^{\alpha\beta} \psi_{\mu\nu} \xi_{\alpha\beta}\right)
\]

\[
- \frac{i\bar{k}^2}{16} \psi^{\mu\nu} \gamma^{\alpha\beta} \xi \left(\bar{\psi}_\mu \gamma_{\alpha\psi_{\beta}} + \frac{1}{2} \bar{\psi}_\alpha \gamma_{\mu\psi_{\beta}}\right),
\]

\[
+ \frac{i\bar{k}^2}{8} \psi^{\mu} \left[(\partial_\mu \psi_{\nu}) \xi_{\gamma^\nu}\right] \xi_{\gamma_\rho} \partial_\rho \psi_{\nu} - \frac{1}{2} \bar{\xi}_\mu \partial_\nu \psi_{\rho}\right)\right),
\]

(29)

\[
\bar{\Delta}_0^\psi = \partial_\mu \bar{\tau}^\mu_0 + i \left(180m^2 - \bar{k}A\right) \xi^{\gamma\mu}\psi_{\mu}
\]

\[
- \frac{i\bar{k}^2}{16} \left[(\partial_\alpha \bar{\psi}_\beta) \gamma^{\alpha\beta\gamma\nu} \psi_{\nu} \left(\xi_{(\mu\nu)} - 2\sigma_{\mu\nu}\xi_{\gamma\rho}\psi_{\rho}\right) \right.
\]

\[
+ \bar{\psi}_\mu \gamma^{\mu\nu} \left(\partial_\nu \psi_{\alpha\beta}\right) \xi_{(\alpha\beta)} \xi_{\gamma\rho} \psi_{\mu}\psi_{\nu} + \bar{\psi}_\beta \gamma^\nu \psi_{\mu}\psi_{\nu} \xi^{(\alpha\beta)} (\xi_{\sigma\rho}\psi_{\rho})
\]

\[
+ \bar{\psi}_\beta \gamma^\nu \psi_{\mu}\psi_{\nu} \xi^{(\alpha\beta)} (\xi_{\sigma\rho}\psi_{\rho})
\]

(30)

Since $\bar{\Delta}^\psi$ stops at antighost number two, we can take, without loss of generality, the corresponding second-order deformation to stop at antighost number three, $\tilde{b}^\psi = \sum_{I=0}^3 \tilde{b}_I^\psi$, $\text{agh}(\tilde{b}_I^\psi) = I$, $I = 0, 1, 2, 3$, $\text{agh}(\tilde{n}_I^\mu) = I$, $I = 0, 1, 2, 3$. By projecting (27) on the various (decreasing) values of the antighost number, we obtain the equivalent tower of equations

\[
0 = -2\gamma\bar{b}_3^\psi + \partial_\mu \bar{n}_3^\mu,
\]

(31)
\[ \bar{\Delta}_2^\psi = -2 \left( \delta \bar{b}_2^\psi + \gamma \bar{b}_1^\psi \right) + \partial_\mu \bar{n}_2^\mu, \]  
\[ \Delta_1^\psi = -2 \left( \delta \bar{b}_2^\psi + \gamma \bar{b}_1^\psi \right) + \partial_\mu \bar{n}_1^\mu, \]  
\[ \bar{\Delta}_0^\psi = -2 \left( \delta \bar{b}_1^\psi + \gamma \bar{b}_0^\psi \right) + \partial_\mu \bar{n}_0^\mu. \]  

Equation (31) can always be replaced, by adding trivial terms only, with \( \gamma \bar{b}_3^\psi = 0 \). Thus, \( \bar{b}_3^\psi \) belongs to the Rarita-Schwinger sector of cohomology of \( \gamma, H (\gamma) \). 

By means of definitions (10)–(11) we get that \( H (\gamma) \) in the Rarita-Schwinger sector is generated by the objects \( (\psi^{*\mu}, \xi^*, \partial_\mu \psi_{\nu|\rho}) \), by their spacetime derivatives up to a finite order, and also by the undifferentiated ghosts \( \xi \) (the spacetime derivatives of \( \xi \) are \( \gamma \)-exact according to the second relation in (4)). As a consequence, we can write \( \bar{b}_3^\psi = \beta_3^\psi \left( [\partial_\mu \psi_{\nu|\rho}], [\psi^{*\mu}], [\xi^*] \right) e^3 (\xi) \), where \( e^3 (\xi) \) are the elements of pure ghost number three of a basis in the space of polynomials in the ghosts \( \xi \) and the notation \( f ([q]) \) means that \( f \) depends on \( q \) and its spacetime derivatives up to a finite order. Inserting (28) in (32) and using standard cohomological arguments, we reach the conclusion that \( \beta_3^\psi \) are (nontrivial) elements of \( H_3^{inv} (\delta \rho) \), where \( H_3^{inv} (\delta \rho) \) denotes as usually the local cohomology of the Koszul-Tate differential in the space of invariant polynomials in antighost number three for the free theory (1). (By ‘invariant polynomials’ we mean elements of \( H (\gamma) \) at pure ghost number zero.) On the other hand, \( H_3^{inv} (\delta \rho) = 0 \) for the free theory under consideration, such that we can safely take \( \beta_3^\psi = 0 \), which further leads to \( \bar{b}_3^\psi = 0 \).

With this result at hand, from (32) and (28) it follows that

\[ \bar{b}_2^\psi = -\frac{i k^2}{32} \xi^{*\gamma\mu\nu} \xi \gamma_\mu \psi_\nu + \bar{b}_2^\psi, \]  

where \( \bar{b}_2^\psi \) is solution to the equation \( \gamma \bar{b}_2^\psi = 0 \). Looking at \( \bar{\Delta}_1^\psi \) given in (29), it results that it can be written as in (33) if

\[ \bar{\chi} = \frac{i k^2}{8} \psi^{*\mu} \left( \partial_\mu \psi_{\nu|\rho} - \frac{1}{2} (\gamma^{\nu\rho} \xi) \left( \xi \gamma_\rho \partial_\mu \psi_{\nu|\rho} - \frac{1}{2} \xi \gamma_\rho \partial_\mu \psi_{\nu|\rho} \right) \right), \]  

can be expressed like

\[ \bar{\chi} = -2 \delta \bar{b}_2^\psi + \gamma \bar{\rho} + \partial_\mu \bar{l}_\mu, \]  

where

\[ \bar{\rho} = -\frac{i k^2}{16} \psi^{*\mu} \gamma^{\alpha\beta} \left( \psi_\mu \xi \gamma_\alpha \psi_\beta - \xi \left( \psi_\mu \gamma_\alpha \psi_\beta + \frac{1}{2} \psi_\alpha \gamma_\mu \psi_\beta \right) \right) - 2 \bar{b}_1^\psi. \]  

Assume that (37) holds. Then, by taking its left Euler-Lagrange (EL) derivatives with respect to \( \psi^{*\mu} \) and using the commutation between \( \gamma \) and each EL derivative \( \delta^L / \delta \psi^{*\mu} \), we infer the relations

\[ \frac{\delta^L}{\delta \psi^{*\mu}} \left( \bar{\chi} + 2 \delta \bar{b}_2^\psi \right) = \gamma \left( \frac{\delta^L \bar{\rho}}{\delta \psi^{*\mu}} \right). \]
As $\tilde{b}^\psi_2$ is $\gamma$-invariant, then $\delta \tilde{b}^\psi_2$ will also be $\gamma$-invariant. Recalling the previous results on the cohomology of $\gamma$ in the Rarita-Schwinger sector, we find that $\delta \tilde{b}^\psi_2 = e^2(\xi) \psi^{* \mu} v_\mu$, with $v_\mu$ fermionic, $\gamma$-invariant functions of antighost number zero and $e^2(\xi)$ the elements of pure ghost number two of a basis in the space of polynomials in the ghosts $\xi$. By using (36) and the last expression of $\delta \tilde{b}^\psi_2$, direct computation provides the equation

$$\delta L(\bar{\chi} + 2\delta \tilde{b}^\psi_2) / \delta \psi^{* \mu} = \bar{k}^2 / 8 \left[ \frac{1}{2} (\gamma^{\nu \rho} \xi) \left( \bar{\xi} \gamma_\rho \partial_\mu \psi_\nu - \frac{1}{2} \bar{\xi} \gamma_\mu \partial_\nu \psi_\rho \right) \right] + (\partial_\mu \psi_\nu) \bar{\xi} \gamma^\nu \xi + 2e^2(\xi) v_\mu. \tag{40}$$

On the one hand, equation (39) shows that $\delta L(\bar{\chi} + 2\delta \tilde{b}^\psi_2) / \delta \psi^{* \mu}$ is trivial in $H(\gamma)$. On the other hand, relation (40) emphasizes that $\delta L(\bar{\chi} + 2\delta \tilde{b}^\psi_2) / \delta \psi^{* \mu}$ is a nontrivial element from $H(\gamma)$ (because each term in the right-hand side of (40) is nontrivial in $H(\gamma)$). Then, $\delta L(\bar{\chi} + 2\delta \tilde{b}^\psi_2) / \delta \psi^{* \mu}$ must be set zero

$$\delta L(\bar{\chi} + 2\delta \tilde{b}^\psi_2) / \delta \psi^{* \mu} = 0, \tag{41}$$

which yields

$$\bar{\chi} + 2\delta \tilde{b}^\psi_2 = \partial_\mu \bar{p}. \tag{42}$$

By acting with $\delta$ on (42) we deduce

$$\delta \bar{\chi} = \partial_\mu \bar{p}. \tag{43}$$

From (30), by direct computation we find

$$\delta \bar{\chi} = -\frac{k^2}{8} (\partial_\alpha \bar{\psi}_\lambda) \gamma^{\alpha \lambda \mu} \left( (\partial_\mu \psi_\nu) \bar{\xi} \gamma_\nu \xi + \frac{1}{2} (\gamma^{\nu \rho} \xi) \left( \bar{\xi} \gamma_\rho \partial_\mu \psi_\nu - \frac{1}{2} \bar{\xi} \gamma_\mu \partial_\nu \psi_\rho \right) \right). \tag{44}$$

Comparing (43) with (44) and recalling the Noether identities corresponding to the Rarita-Schwinger action, we obtain that the right-hand of (44) reduces to a total derivative iff

$$(\partial_\mu \psi_\nu) \bar{\xi} \gamma_\nu \xi + \frac{1}{2} (\gamma^{\nu \rho} \xi) \left( \bar{\xi} \gamma_\rho \partial_\mu \psi_\nu - \frac{1}{2} \bar{\xi} \gamma_\mu \partial_\nu \psi_\rho \right) = \partial_\mu \bar{p}. \tag{45}$$

Simple computation exhibits that the left-hand side of (45) cannot be written like a total derivative, so neither relation (43) nor equation (37) hold. As a consequence, $\bar{\chi}$ must vanish and hence we must set

$$\bar{k} = 0. \tag{46}$$

In fact, the general solution to equation (42) takes the form $\bar{\chi} + 2\delta \tilde{b}^\psi_2 = u + \partial_\mu \bar{p}$, where $u$ is a function of antighost number one depending on all the BRST generators from the Rarita-Schwinger sector but the antifields $\psi^{* \mu}$. As the antifields $\psi^{* \mu}$ are the only Rarita-Schwinger antifields of antighost number one, the condition $\text{agh}(u) = 1$ automatically produces $u = 0$. 

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Inserting (46) in (28)–(30), we obtain that
\[
\hat{\Delta}^\psi_2 = \partial_\mu \bar{\tau}_2^\mu, \quad \hat{\Delta}^\psi_1 = \partial_\mu \bar{\tau}_1^\mu, \tag{47}
\]
\[
\hat{\Delta}^\psi_0 = \partial_\mu \bar{\tau}_0^\mu + 180i \xi \gamma^\mu \psi_\mu. \tag{48}
\]
From (47) it results that we can safely take \(\bar{b}_2^\psi = 0\) and \(\bar{b}_1^\psi = 0\), which replaced in (34) lead to the necessary condition that \(\hat{\Delta}^\psi_0\) must be a trivial element from the local cohomology of \(\gamma\), i.e. \(\hat{\Delta}^\psi_0 = -2\gamma \hat{b}_0^\psi + \partial_\mu \nabla_0^\mu\). In order to solve this equation with respect to \(\bar{b}_0^\psi\), we will project it on the number of derivatives. Since \(\gamma \hat{b}_0^\psi\) contains at least one spacetime derivative, the above equation projected on the number of derivatives equal to zero reduces to \(\hat{\Delta}^\psi_0 = 180im^2 \xi \gamma^\mu \psi_\mu = 0\), which further implies
\[
m = 0. \tag{49}
\]
Substituting relations (46) and (49) in (20)–(24) we obtain that \(S_h - \psi_1 = 0\) and \(S_\psi_1 = 0\), so equations (25)–(26) possess only the trivial solution \(S_h - \psi_2 = 0\) and \(S_\psi_2 = 0\). The vanishing of \(S_h - \psi_1, S_\psi_1, S_\psi_2\), and \(S_\psi_1\) further leads, via the equations that stipulate the higher-order deformation equations, to the result that we can take
\[
S_h - \psi_i = 0, \quad S_\psi_i = 0, \quad i \geq 1. \tag{50}
\]
In conclusion, under the hypotheses of locality, smoothness of the interactions in the coupling constant, Poincaré invariance, (background) Lorentz invariance, and the preservation of the number of derivatives on each field, there are no cross-interactions between the Pauli-Fierz field and the massless Rarita-Schwinger field and also no self-interactions for the massless Rarita-Schwinger field, both in \(D = 11\).

6 On the quartic SUGRA gravitini vertex

We have seen (here and in [2]) that gravitini allows no self-interactions in \(D = 11\) if separately coupled to a graviton or respectively to a three-form gauge field. Nevertheless, it is known that \(D = 11, N = 1\) SUGRA contains a quartic vertex (at order two in the coupling constant \(\lambda\)) expressing self-interactions among the gravitini. This apparent paradox has a simple explanation. If we start from a free theory with all the three fields (gravitini, graviton, and three-form), then the consistency of the first-order deformation in the Rarita-Schwinger sector becomes
\[
(S_1, S_1)^\psi + 2sS_2^\psi = 0, \tag{51}
\]
where \((S_1, S_1)^\psi\) collects now all the self-interaction terms, coming from \((S_1^\psi, S_1^\psi), (S_1^{h,\psi}, S_1^{h,\psi}), and (S_1^{A,\psi}, S_1^{A,\psi})\) (where \(S_1^{A,\psi}\) represents the first-order deformation):
\[
(S_1, S_1)^\psi = (S_1^\psi, S_1^\psi) + (S_1^{h,\psi}, S_1^{h,\psi}) + (S_1^{A,\psi}, S_1^{A,\psi}). \tag{52}
\]
The notation \((S^{b_{1}}_{\psi}, S^{b_{2}}_{\psi})\) is explained in the above and \((S^{A_{1}}_{\psi}, S^{A_{2}}_{\psi})\) is discussed in \([2]\). Let \(\Delta^{\psi}\) be the nonintegrated density of \((S_{1}, S_{1})\psi\) and \(b^{\psi}\) the nonintegrated density of \(S_{2}^{\psi}\). Then, (51) takes the local form

\[
\Delta^{\psi} = -2s b^{\psi} + \partial^{\mu} n^{\psi}_{\mu},
\]

with \(gh (\Delta^{\psi}) = 1\), \(gh (b^{\psi}) = 0\), \(gh (n^{\psi}_{\mu}) = 1\), for some local currents \(n^{\psi}_{\mu}\). It is easy to see that \(\Delta^{\psi}\) decomposes as \(\Delta^{\psi} = \Delta^{\psi}_{0} + \Delta^{\psi}_{1} + \Delta^{\psi}_{2}\), with \(agh (\Delta^{\psi}_{0}) = I, I = 0, 3\), where

\[
\Delta^{\psi}_{0} = \partial_{\mu} (\bar{x}_{0}^{\mu} + \bar{z}_{0}^{\mu}) + i (180m^{2} - \kappa \Lambda) \xi^{\mu} \psi_{\mu},
\]

\[
- \frac{\kappa}{16} \left[ (\partial_{\lambda} \bar{y}_{\mu}^{\beta}) \gamma^{\alpha \beta \mu} \psi_{\nu} \left( \xi_{(\mu} \psi_{\nu)} - 2\sigma_{\mu \nu} \xi^{\rho} \psi_{\rho} \right) + \bar{y}_{\mu}^{\alpha} \gamma^{\nu \rho} (\partial_{\rho} \psi_{\alpha}) \xi_{(\alpha} \psi_{\beta)} \right] + \bar{y}^{\alpha} \gamma^{\nu \rho} \partial_{\nu} \left( \psi_{(\beta} \psi_{\rho)} - 2\sigma_{\beta \rho} \xi^{\lambda} \psi_{\lambda} \right) + \bar{y}_{\mu}^{\alpha} \gamma^{\nu \rho} \partial_{\nu} \left( \psi_{(\beta} \psi_{\rho)} - 2\sigma_{\beta \rho} \xi^{\lambda} \psi_{\lambda} \right) + i \kappa^{2} \left( \bar{y}^{\alpha \beta} \gamma^{\nu \rho} \psi_{\lambda} \right) + \frac{1}{2} \bar{y}^{\alpha} \gamma^{\nu \rho} \psi_{\lambda} \partial_{\nu} \left( \psi_{(\alpha} \psi_{\beta)} \right) \partial_{\nu} \left( \psi_{(\alpha} \psi_{\beta)} \right) \right].
\]

Related to (54) and (55), we mention that the expression of \(\Delta^{\psi}_{1}\) can be found in \([2]\) (they represent the contributions due to the simultaneous presence of the three-form and gravitini), while \(\Delta^{\psi}_{2}\) and \(\Delta^{\psi}_{3}\) are given in \([28]\) and \([29]\). Since \(\Delta^{\psi}\) has components with the maximum value of the antighost number equal to two, we can take the nonintegrated density of the second-order deformation of the solution to the master equation in the Rarita-Schwinger sector to stop at antighost number three: \(b^{\psi} = \sum_{I=0}^{3} b_{I}^{\psi}\), \(agh (b^{\psi}_{I}) = I, I = 0, 3\), \(n^{\psi \mu} = \sum_{I=0}^{3} n_{I}^{\psi \mu}\), \(agh (n_{I}^{\psi \mu}) = I, I = 0, 3\). By projecting (53) on various, decreasing values of the antighost numbers, we obtain that it becomes equivalent to the equations

\[
0 = -2\gamma b_{3}^{\psi} + \partial_{\mu} n_{3}^{\psi \mu},
\]

\[
\Delta^{\psi}_{2} = -2 (\delta b_{3}^{\psi} + \gamma b_{2}^{\psi}) + \partial_{\mu} n_{2}^{\psi \mu},
\]

\[
\Delta^{\psi}_{1} = -2 (\delta b_{2}^{\psi} + \gamma b_{1}^{\psi}) + \partial_{\mu} n_{1}^{\psi \mu},
\]

\[
\Delta^{\psi}_{0} = -2 (\delta b_{1}^{\psi} + \gamma b_{0}^{\psi}) + \partial_{\mu} n_{0}^{\psi \mu}.
\]

Equation (57) possesses only the trivial solution \(b^{\psi}_{0} = 0\) (from precisely the same argument as that used in relation with equation (31)). With this result at hand, from (54) and (58) we get

\[
b^{\psi}_{2} = \delta b^{\psi}_{2},
\]

(61)
where \( \bar{b}_2^\psi \) is expressed by (35). From the expression of \( \Delta_1^\psi \) given in (55) we obtain that (60) holds if

\[
\chi = \frac{i\bar{k}^2}{8} \psi^\mu \left[ (\partial_\mu \psi_\nu) \bar{\xi} \gamma^\nu \xi + \frac{1}{2} (\gamma^\nu \partial_\nu \psi_\rho) \right] \\
- \frac{i\bar{k}^2}{3} \left( \psi_\mu \bar{\gamma}_{\nu\rho\lambda} \xi \right) - \frac{1}{2} \psi_\mu \bar{\gamma}_{\nu\rho\lambda} \xi \bar{\gamma}^{\mu\nu} \partial^{[\nu} \psi_{\rho]} \right] \\
= \bar{\chi} + \tilde{\chi}, \tag{62}
\]

(with \( \bar{\chi} \) as in (30) and \( \tilde{\chi} \) due to the presence of the three-form gauge field [2]) decomposes as

\[
\chi = -2\delta \bar{b}_2^\psi + \gamma \rho + \partial_\mu l^\mu, \tag{63}
\]

where

\[
\rho = -\frac{i\bar{k}^2}{16} \psi^\mu \gamma^{\alpha\beta} \left( \psi_\mu \bar{\xi} \gamma_\alpha \psi_\beta \right) - \xi \left( \psi_\mu \gamma_\alpha \psi_\beta + \frac{1}{2} \psi_\alpha \gamma_\mu \psi_\beta \right) \\
+ \frac{i\bar{k}^2}{3} \left( \psi_\mu \bar{\gamma}_{\nu\rho\lambda} \xi \right) - \frac{1}{2} \psi^{\alpha\beta} \gamma^{\mu\rho\lambda} \xi \bar{\gamma}^{\mu\nu} \partial^{[\nu} \psi_{\rho]} - 2b_1^\psi. \tag{64}
\]

Identities (2) and (3) allow us to rewrite (62) in the form

\[
\chi = \frac{1}{8} \left( \bar{k}^2 + \frac{\bar{k}^2}{32} \right) \left\{ -21 \psi^{\mu\nu} \partial_{[\mu} \psi_{\nu]} \xi \gamma^\mu \xi + \frac{1}{2} \left( -7 \psi^{\alpha\beta} \gamma_{\mu\beta} \partial_{[\alpha} \psi_{\beta]} \right) \\
+ \psi^{\mu\nu} \gamma^{\alpha\beta} \partial_{[\alpha} \psi_{\beta]} \bar{\xi} \gamma^\mu \xi - \frac{1}{24} \left[ \psi_\mu \bar{\gamma}_{\nu\rho\lambda} \xi \partial_{[\alpha} \psi_{\beta]} \right] \bar{\xi} \gamma^{\mu\rho\lambda} \xi \right\} + \delta \Omega, \tag{65}
\]

where we made the notation

\[
\Omega = \left[ \frac{1}{8} \left( 3\bar{k}^2 - \frac{7\bar{k}^2}{9 \cdot 2^7} \right) \psi^\mu \gamma^\nu \bar{\psi}^{\mu\nu} - \frac{1}{16} \left( \bar{k}^2 - \frac{11\bar{k}^2}{9 \cdot 2^6} \right) \psi_\rho \gamma^{\mu\rho\lambda} \bar{\psi}_{\rho} \\
- \frac{1}{6} \left( \bar{k}^2 - \frac{7\bar{k}^2}{3 \cdot 2^8} \right) \psi_\rho \gamma^\nu \bar{\psi}^{\mu\nu} \right] \bar{\xi} \gamma^\mu \xi \\
+ \frac{1}{4} \left( \bar{k}^2 + \frac{11\bar{k}^2}{9 \cdot 2^6} \right) \psi_\rho \gamma_{\mu\rho\lambda} \bar{\psi}^{\mu\lambda} - \frac{4}{3} \left( \bar{k}^2 + \frac{7\bar{k}^2}{3 \cdot 2^8} \right) \psi_\mu \gamma_{\mu\rho} \bar{\psi}^{\rho} \\
- \frac{1}{2} \left( 3\bar{k}^2 + \frac{7\bar{k}^2}{9 \cdot 2^7} \right) \psi_\rho \gamma_{\mu\rho} \bar{\psi}_{\rho} \\
+ \frac{1}{24} \left( 7 \bar{k}^2 + \frac{\bar{k}^2}{2^6} \right) \psi_\mu \gamma_{\mu\rho} \bar{\psi}_{\rho} + \frac{1}{8} \left( \bar{k}^2 + \frac{5\bar{k}^2}{2^7} \right) \psi_\nu \gamma_{\mu\rho} \bar{\psi}_{\rho} \\
- \frac{7}{8} \left( \bar{k}^2 + \frac{\bar{k}^2}{2^6} \right) \psi_\nu \gamma_{\mu\rho\lambda} \bar{\psi}_{\rho\lambda} - \frac{1}{10} \left( \bar{k}^2 + \frac{11\bar{k}^2}{2^9} \right) \psi_\nu \gamma_{\mu\rho\lambda} \bar{\psi}_{\rho\lambda} \times \bar{\xi} \gamma_{\mu\rho\lambda} \xi \right]. \tag{66}
\]
Reprising the procedure employed in the previous section between formulas \((37)\) and \((46)\), we deduce that
\[
\bar{k}^2 + \frac{\bar{k}^2}{32} = 0, \tag{67}
\]
\[
\bar{b}_2^\psi = -\frac{1}{2} \Omega. \tag{68}
\]
Consequently, \((63)\) takes the simple form
\[
\gamma \rho + \partial_\mu b^\mu = 0, \tag{69}
\]
which, since \(\text{agh}(\rho) = 1 > 0\), can be replaced with
\[
\gamma \rho = 0. \tag{70}
\]
Inserting \((68)\) in \((59)\) we obtain the component of antighost number one from the second-order deformation of the solution to the master equation in the Rarita-Schwinger sector as
\[
b_1^\psi = -\frac{i\bar{k}^2}{32} \psi^*_{\rho\gamma} \gamma^{\rho\gamma} \xi \gamma_\mu \psi_\nu - \frac{\bar{k}^2}{9 \cdot 2^{10}} \left( -\frac{11 \cdot 17}{2} \psi^*_{\mu\gamma} \gamma^\mu \bar{\psi}^*_{\nu} \\
+ \frac{29}{2} \psi^*_{\mu\gamma} \gamma^{\mu\nu} \bar{\psi}^*_{\rho} + 31 \psi^*_{\nu\gamma} \bar{\psi}^*_{\gamma} \psi^*_{\mu} \right) \xi \gamma_\mu \xi + \frac{i\bar{k}^2}{6} \left( \psi^*_{[\mu\gamma_{\nu\rho\lambda}] \xi} - \frac{1}{2} \psi^*_{\sigma} \gamma_{\mu\nu\rho\lambda} \xi \right) \bar{\psi}^*_{\mu} \gamma^\nu \psi^*_{\rho} \psi^*_{\lambda} \right). \tag{71}
\]
Expressing \(\bar{k}^2\) in terms of \(\bar{k}^2\) with the help of \((67)\) we find that the pieces of antighost number two and one from the second-order deformation, responsible for the self-interactions among gravitini, become parameterized by a single constant
\[
b_2^\psi = \frac{-i\bar{k}^2}{32} \xi^* \gamma^\mu \xi \bar{\psi}^*_{\rho} \psi^*_{\mu} - \frac{\bar{k}^2}{9 \cdot 2^{10}} \left( -\frac{11 \cdot 17}{2} \psi^*_{\mu\gamma} \gamma^\mu \bar{\psi}^*_{\nu} \\
+ \frac{29}{2} \psi^*_{\mu\gamma} \gamma^{\mu\nu} \bar{\psi}^*_{\rho} + 31 \psi^*_{\nu\gamma} \bar{\psi}^*_{\gamma} \psi^*_{\mu} \right) \xi \gamma_\mu \xi + \frac{29}{4} \psi^*_{\rho} \gamma_{\mu\nu} \bar{\psi}^*_{\rho} \xi \gamma^\mu \xi \tag{71}
\]
\[
b_1^\psi = \frac{-i\bar{k}^2}{32} \psi^*_{\rho\gamma} \gamma^{\rho\gamma} \xi \gamma_\mu \psi_\nu - \frac{\bar{k}^2}{9 \cdot 2^{10}} \left( -\frac{11 \cdot 17}{2} \psi^*_{\mu\gamma} \gamma^\mu \bar{\psi}^*_{\nu} \\
+ \frac{29}{2} \psi^*_{\mu\gamma} \gamma^{\mu\nu} \bar{\psi}^*_{\rho} + 31 \psi^*_{\nu\gamma} \bar{\psi}^*_{\gamma} \psi^*_{\mu} \right) \xi \gamma_\mu \xi + \frac{29}{4} \psi^*_{\rho} \gamma_{\mu\nu} \bar{\psi}^*_{\rho} \xi \gamma^\mu \xi \tag{71}
\]

12
\[
\Delta^\psi_0 = \partial_\mu (\bar{\tau}^\psi_0 + \tilde{\tau}_0^\psi) + i (180m^2 - \bar{k}^2) \bar{\xi}\gamma^\mu \psi_\mu - 2\delta b_1^\psi
\]

Relation (67) and (56) lead, by direct computation, to

\[
\Delta^\psi_0 = \partial_\mu (\bar{\tau}^\psi_0 + \tilde{\tau}_0^\psi) + i (180m^2 - \bar{k}^2) \bar{\xi}\gamma^\mu \psi_\mu - 2\delta b_1^\psi
\]

\[
\begin{align*}
&+ \frac{1}{2} \bar{\psi}\gamma^\mu \gamma^\nu \gamma^\rho \xi \\
&+ \frac{1}{2} \bar{\psi}\gamma^\mu \gamma^\nu \gamma^\rho \xi \\
&+ \frac{1}{2} \bar{\psi}\gamma^\mu \gamma^\nu \gamma^\rho \xi \\
&+ \frac{1}{2} \bar{\psi}\gamma^\mu \gamma^\nu \gamma^\rho \xi
\end{align*}
\]

(72)

Relation (67) and (56) lead, by direct computation, to

\[
\Delta^\psi_0 = \partial_\mu (\bar{\tau}^\psi_0 + \tilde{\tau}_0^\psi) + i (180m^2 - \bar{k}^2) \bar{\xi}\gamma^\mu \psi_\mu - 2\delta b_1^\psi
\]

\[
\begin{align*}
&+ \frac{1}{2} \bar{\psi}\gamma^\mu \gamma^\nu \gamma^\rho \xi \\
&+ \frac{1}{2} \bar{\psi}\gamma^\mu \gamma^\nu \gamma^\rho \xi \\
&+ \frac{1}{2} \bar{\psi}\gamma^\mu \gamma^\nu \gamma^\rho \xi \\
&+ \frac{1}{2} \bar{\psi}\gamma^\mu \gamma^\nu \gamma^\rho \xi
\end{align*}
\]

Relation (67) and (56) lead, by direct computation, to

\[
\Delta^\psi_0 = \partial_\mu (\bar{\tau}^\psi_0 + \tilde{\tau}_0^\psi) + i (180m^2 - \bar{k}^2) \bar{\xi}\gamma^\mu \psi_\mu - 2\delta b_1^\psi
\]

\[
\begin{align*}
&+ \frac{1}{2} \bar{\psi}\gamma^\mu \gamma^\nu \gamma^\rho \xi \\
&+ \frac{1}{2} \bar{\psi}\gamma^\mu \gamma^\nu \gamma^\rho \xi \\
&+ \frac{1}{2} \bar{\psi}\gamma^\mu \gamma^\nu \gamma^\rho \xi \\
&+ \frac{1}{2} \bar{\psi}\gamma^\mu \gamma^\nu \gamma^\rho \xi
\end{align*}
\]

(73)

where \(b_1^\psi\) reads as in (72). Comparing the right-hand sides of (60) and (73) we finally identify the second-order Lagrangian in the Rarita-Schwinger sector, which indeed includes quartic gravitini vertices

\[
\Delta^\psi_0 = \partial_\mu (\bar{\tau}^\psi_0 + \tilde{\tau}_0^\psi) + i (180m^2 - \bar{k}^2) \bar{\xi}\gamma^\mu \psi_\mu - 2\delta b_1^\psi
\]

\[
\begin{align*}
&+ \frac{1}{2} \bar{\psi}\gamma^\mu \gamma^\nu \gamma^\rho \xi \\
&+ \frac{1}{2} \bar{\psi}\gamma^\mu \gamma^\nu \gamma^\rho \xi \\
&+ \frac{1}{2} \bar{\psi}\gamma^\mu \gamma^\nu \gamma^\rho \xi \\
&+ \frac{1}{2} \bar{\psi}\gamma^\mu \gamma^\nu \gamma^\rho \xi
\end{align*}
\]

(74)

and also deduce the identity

\[
180m^2 - \bar{k}^2 = 0.
\]

(75)

Thus, we have shown that it is precisely the presence of all three types of fields which induces the appearance of quartic self-interactions among gravitini of the type (74). The main argument is quite clear now: if one considers the spin-3/2 field coupled to either a spin-two field or a three-form, then (67) reduces to either \(\bar{k}^2 = 0\) or respectively to \(\tilde{k}^2 = 0\), which gives \(b_1^\psi = 0\) or respectively \(\tilde{b}_1^\psi = 0\). Another interesting remark is that equation (75) apparently allows the presence of cosmological and gravitini ‘mass’ constants, which are known to be forbidden in \(D = 11, N = 1\) SUGRA. We will see in [3] that equation (75) actually sets zero both constants, \(\Lambda = 0 = m\).
7 Comparison with the case $D = 4$

It is useful to make a short comparison between the cases $D = 11$ and $D = 4$ since it is known that the same free theory (describing a graviton and a massless 3/2-field) allows for self-interactions among gravitini in $D = 4$ and, meanwhile, forces an algebraic relation between the gravitini ‘mass’ constant and the cosmological one. In four dimensions the first-order deformation of the solution to the master equation takes a form similar to $D = 11$, excepting the ‘mass’ terms, which are written as $a_{D=4}^\psi = m (i \psi_\mu^* \gamma^\mu \xi - \bar{\psi}_\mu \gamma^\mu \psi_\mu)$. Similarly to the case $D = 11$, we will denote by $\Delta_{D=4}^\psi$ and $b_{D=4}^\psi$ the nonintegrated densities of $(S_1, S_1)^\psi$ and of $S_2^\psi$ respectively. The consistency of the first-order deformation in the Rarita-Schwinger sector is equivalent to the equation $\Delta_{D=4}^\psi = -2sb_{D=4}^\psi + \partial_\mu n_{D=4}^\mu$, where $\Delta_{D=4}^\psi = \Delta_{D=4=2}^\psi + \Delta_{D=4=1}^\psi + \Delta_{D=4=0}^\psi$, with $\Delta_{D=4=2}$ and $\Delta_{D=4=1}$ having exactly the expressions (28) and (29), but in $D = 4$, and $\Delta_{D=4=0}$ being given by

$$\Delta_{D=4=0}^\psi = i (12m^2 - \bar{k}A) \bar{\xi} \gamma^\mu \psi_\mu$$

$$-\frac{i k^2}{16} \left[ (\partial_\alpha \bar{\psi}_\beta) \gamma_\alpha^\beta \mu \nu \xi (\bar{\xi}(\gamma^\mu \psi_\nu) - 2\sigma_\mu \nu \bar{\xi} \gamma^\mu \psi_\nu \rho) + \bar{\psi}_\mu \gamma^\mu \rho \rho \sigma_\alpha \bar{\psi}_\alpha \xi - \bar{\psi}_\alpha \gamma^\rho \rho \xi \psi_\mu \sigma_\beta + \bar{\psi}_\mu \gamma^\rho \rho \xi \psi_\mu \sigma_\beta + \bar{\psi}_\alpha \gamma^\rho \rho \xi \psi_\mu \sigma_\beta \right].$$

(76)

The key point in $D = 4$ is that the analogue of $\bar{\chi}$ expressed by (80) can be rewritten as

$$\chi_{D=4} = \partial_\mu \theta^\mu + \delta \left[ -\frac{1}{27} \bar{\xi} \gamma_\mu \xi \left( \psi^\star_\alpha \gamma^\rho \psi_\nu + \frac{3}{2} \psi^\star_\nu \gamma^\mu \psi_\rho \right) \right] + \frac{1}{27} \bar{\xi} \gamma_\mu \xi \left( \psi^\star_\alpha \gamma^\mu \psi_\nu - \frac{1}{2} \psi^\star_\nu \gamma^\mu \psi_\rho - 2 \psi^\star_\mu \psi^\star_\nu \right).$$

(77)

With these result at hand we find in $D = 4$ that $b_{D=4=3}^\psi = 0$ and

$$b_{D=4=2}^\psi = -\frac{i k^2}{32} \xi^{\star \gamma^\mu \rho_\lambda \psi_\alpha \psi_\beta} + \frac{1}{28} \bar{\xi} \gamma_\mu \xi \left( \psi^\star_\alpha \gamma^\rho \psi_\nu + \frac{3}{2} \psi^\star_\nu \gamma^\mu \psi_\rho \right)$$

$$-\frac{1}{28} \bar{\xi} \gamma_\mu \xi \left( \psi^\star_\alpha \gamma^\rho \psi_\nu - \frac{1}{2} \psi^\star_\nu \gamma^\mu \psi_\rho - 2 \psi^\star_\mu \psi^\star_\nu \right),$$

(78)

$$b_{D=4=1}^\psi = \frac{i k^2}{32} \left[ \psi^\star_\alpha \gamma^\rho \psi_\alpha \psi_\beta - \psi^\star_\mu \gamma^\alpha \psi_\beta \right] + \frac{1}{2} \bar{\psi}_\nu \gamma_\mu \psi_\nu \right],$$

(79)

while (78) can be put in the form

$$\Delta_{D=4=0}^\psi = i (12m^2 - \bar{k}A) \bar{\xi} \gamma^\mu \psi_\mu$$

$$+ \delta \left[ \frac{i k^2}{16} \left( \psi^\star_\alpha \gamma^\rho \psi_\alpha \psi_\beta - \psi^\star_\mu \gamma^\alpha \psi_\beta \right) \right].$$

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\[
\gamma \left[ \frac{\bar{k}^2}{16} \left( \bar{\psi}^\alpha \gamma^\mu \psi_\mu \bar{\psi}_\alpha \gamma^\nu \psi_\nu - \frac{1}{4} \bar{\psi}^\alpha \gamma^\mu \psi_\mu \left( \bar{\psi}_{\alpha} \gamma_\mu \psi_\beta + 2 \bar{\psi}_{\alpha} \gamma_\beta \psi_\mu \right) \right) \right]. \quad (80)
\]

Relation (80) provides the piece of antighost number zero from the second-order deformation of the solution to the master equation in the Rarita-Schwinger field

\[
b_{\mu=4, \nu=0}^\psi = - \frac{\bar{k}^2}{32} \left( \bar{\psi}^\alpha \gamma^\mu \psi_\mu \bar{\psi}_\alpha \gamma^\nu \psi_\nu - \frac{1}{4} \bar{\psi}^\alpha \gamma^\mu \psi_\mu \left( \bar{\psi}_{\alpha} \gamma_\mu \psi_\beta + 2 \bar{\psi}_{\alpha} \gamma_\beta \psi_\mu \right) \right) \quad (81)
\]

and also enforces the condition \(12m^2 - \bar{k}\Lambda = 0\), which expresses the well-known relation between the coupling constants \(m\) and \(\bar{k}\) and the cosmological constant \(\Lambda\). Finally, we remark that it is the same object, namely (30), which in \(D = 4\) does satisfy an equation of the type (42) (see (77)) and thus ensures the existence of interactions, but in \(D = 11\) cannot satisfy such an equation and consequently forbids the presence of interactions.

### 8 Conclusion

To conclude with, in this paper we have investigated the eleven-dimensional couplings between a massless spin-two field (described in the free limit by a Pauli-Fierz action) and a massless Rarita-Schwinger spinor using the powerful setting based on local BRST cohomology. Under the hypotheses of locality, smoothness of the interactions in the coupling constant, Poincaré invariance, Lorentz covariance, and the preservation of the number of derivatives on each field, we have shown that in \(D = 11\) there are no consistent cross-interactions among the graviton and the massless gravitino and also no self-interactions in the Rarita-Schwinger sector, unlike the case \(D = 4\), where such cross-interactions exist.

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