Models of the optimal distribution of fertilizers and vehicles in grain production

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Abstract. The paper considers models of optimal distribution of fertilizers and vehicles in grain production. To ensure the delivery of fertilizer to the destination for the chemical treatment agriculture, the optimal distribution of fertilizers in grain production was solved. On the basis of optimal models, the needs of vehicles for servicing these works are determined. As calculations show, the cost of acquiring machines and operating costs can be reduced by 20-30% due to the correct selection and use of equipment. This made it possible to distribute mineral fertilizers between farms in accordance with the characteristics of their soils. With the help of ground equipment, mineral fertilizers are applied, chemical pest and plant disease control is carried out, weed vegetation is destroyed in grain crops. The optimal plan for the use of machine and tractor fleet can only be determined using optimization methods, since the composition of machines and tractors and their possible use have such a large number of options that it is almost impossible to simply sort them out and select the best. Further, the algorithm and the solution to the problem of optimal distribution of vehicles in grain production are considered.

1 Introduction

One of the most important works in the transport system of the grain processing industry is to ensure the delivery of fertilizers to the destination for the chemicalization of agriculture. In this regard, in determining the transportation of fertilizers, it is necessary to solve the problem of optimal distribution of fertilizers between grain crops, individual farms, as well as districts, oblasts. The need for mineral fertilizers considerably exceeds the possibilities of their production, therefore the problem of developing methods for their most rational use is of extremely great not only sectoral, but also national economic importance. The increase in yield and the amount of net income in agriculture largely depend on the rational distribution of organic and mineral fertilizers between crops and farms.

All currently known methods for solving the problem of fertilizer distribution, taking into account computer-aided implementation programs, can be divided into two groups: 1) based on iterative models; 2) optimization. In the first case, the algorithm of the preferred saturation method is used, in the second – the simplex method. Iterative models make it possible to select the most rational of them through targeted analysis and evaluation of plan variants; the use of optimal methods ensures the selection of the best solution for the given conditions of the problem. Despite the differences in the methods of solution, the formulation of the problem has much in common. In both cases, a plan for distributing a limited fertilizer pool is sought, ensuring maximum efficiency in their use.

The effectiveness of fertilizers is expressed in yield increase, which can be measured in physical and monetary terms. Moreover, the increase in value terms can be commensurate with the costs associated with its receipt, that is, with the costs of purchasing, transporting, storing and applying fertilizers. Based on these indicators, net (conditional) income per 1 ha of sowing from mineral fertilizers is calculated.

The optimization criterion for the distribution of fertilizers is taken as the maximum gross yield of grain crops obtained through their use, or the maximum conditional net income obtained as the difference between the value of the gross yield increase from mineral fertilizers and the costs associated with their use.

In the previously described models for the optimal distribution of mineral fertilizers, practically the same indicators are used as constraints: size of sown areas, limits of mineral fertilizer funds, planned production volumes (increase) of production, doses of mineral fertilizers, increase in yield of grain crops as a result of using mineral fertilizers. However, these models do not always take into account the presence of local fertilizers, soil types, agrochemical characteristics, the effect of fertilizers, as well as the conditions for the preservation and improvement of soil fertility. These factors are taken...

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into account when solving the problem of determining the rational distribution of mineral fertilizers.

Consequently, the statement of the problem can be formulated as follows: determine a plan for the distribution of mineral fertilizers between grain crops and farms, which would provide the maximum conditional net income.

The optimal distribution plan for fertilizers should take into account: the presence of fertilizers on the farm; the area of fields of crop rotations (areas) with different agrochemical characteristics; the priority of fertilizing for the main crops and crops grown on land-reclamation lands. It follows that the constraints of the problem are:

- balances on the use and availability of mineral fertilizers;
- conditions on the areas of grain crops in the context of individual areas that differ in agrochemical characteristics;
- conditions for ensuring the planned production volumes;
- conditions for the preservation and improvement of soil fertility in the farms;
- conditions for ensuring the priority of crops grown on land-reclamation lands.

Currently, insufficient knowledge of soils and agroecosystems as objects of modeling creates the greatest difficulty in developing complex mathematical models of renal fertility. Therefore, the creation of models of soil fertility is still at the stage of modeling the dynamics of its elements.

In the work of Z.O. Zhadlun [1] developed economic and mathematical models of the vital nutrition of plants.

When constructing and studying mathematical models of the optimal distribution of fertilizers, the following objective conditions are taken into account that affect the yield of grain crops:

- soil type and agrochemical characteristics (acidity, provision of mobile phosphorus and potassium compounds);
- weather conditions are average for 5 years;
- precedents based on the data of production experiments, based on the structure of the sown area;
- level of agrotechnology, which took place in the production experiments. It is accepted as advanced and accessible to all households.

At present, a large number of economic and mathematical models in agricultural production have been developed [2-6], in which there are three main areas:

- development and solution of economic and mathematical problems of on-farm analysis;
- development and solution of economic and mathematical problems at the level of agro-industrial associations and individual units of agricultural production;
- development and solution of economic and mathematical problems of industry analysis.

At present, the tasks of the first direction are the most developed and implemented, since the information necessary for them is more accessible and reliable. The objectives of this direction include: optimization of the use of mineral and organic fertilizers; optimization of crop development plan; optimization of the production structure of the grain enterprise, etc.

The second direction, which has arisen in connection with the organization of agro-industrial associations, includes the tasks of optimizing not only the production of grain production, but also its industrial processing within the associations.

The third direction is connected with the development and solution of problems of development of individual links of the grain processing industry at the level of the oblast, krai and republic. The main objective of this direction is the optimal placement and specialization of grain production by regions, as well as the optimization of purchases of grain products by farms, regions, regions and republics [6].

2 Material and methods

Taking into account the above features, a mathematical model of the optimal distribution of fertilizers can be formulated as follows:

Find the maximum net income

$$\sum \sum \sum G_{eqj} X_{eqj} + \sum \sum \sum C_{eqj} Y_{eqj} \rightarrow \max$$

under restrictions

on the distributed fund of mineral and organic fertilizers

$$\sum \sum h_{eqj} X_{eqj} + \sum \sum h_{eqj} Y_{eqj} \leq F_{eqj}, q \in Q, u \in U$$

on conservation and improvement of soil fertility in farms

$$\sum \sum h_{eqj} X_{eqj} + \sum \sum h_{eqj} Y_{eqj} \geq B_{eqj}, q \in Q, u \in U$$

to fulfill the minimum required volume of gross production of grain products by farms

$$a_{eqj} X_{eqj} + Y_{eqj} \geq P_{eqj}$$

to the maximum possible amount of increase in production

$$Y_{eqj} \leq M_{eqj}$$

on fertilized crop areas by crops

$$X_{eqj} \leq S_{eqj}$$

on non-negativity of variables

$$X_{eqj} \geq 0, Y_{eqj} \geq X_{eqj} \geq 0, \ Y_{eqj} \geq 0, e \in E; \ j \in J; \ q \in Q;$$

Where, E - many types of soil, fields;

- e - soil type, field e \( \in E \);
- J - a variety of crops;
- j - grain crop index, j \( \in J \);
- Q - a set of farms - grain production;
- q - farm index, q \( \in Q \);
- U - many types of fertilizers;
- u is the type of fertilizer, u \( \in U \);
- a_{eqj} - average yield (base) j-th culture on th e-th soil of the q-th farm;
recommended according to the zonal agrochemical station. Application rates are determined based on the use of organic fertilizers. For example, the doses of mineral fertilizers can be determined by calculation using the formulas [8]:

- for phosphate and potash fertilizers
  \[D = \frac{(100*B - P*K_r - P'*K_y)}{K_y}\]
  for nitrogen fertilizers
  \[D = \frac{(100*B - P*K_y)}{K_y}\]

Where \(D\) – the dose of fertilizer, kg dv;

\(B\) – removal of nutrients with the planned yield, kg ae. ;

\(P\) – the content of mobile compounds of phosphorus and potassium in the soil, kg dv per 1 ha (determined by multiplying their amount in milligrams per 100 g of soil by 30, since 1 mg of phosphorus and potassium per 100 g of soil corresponds to 30 kg per 1 ha);

\(P'\) is the nutrient content of organic fertilizers applied per 1 ha, kg;

\(K_r\) – the utilization of mobile substances of phosphorus and potassium from the soil, %;

\(K_y\) – the utilization of nutrients from fertilizers in the first year, %;

\(K_r\) – the utilization of nutrients from the manure in the year of application, %.

The implementation of the model (1) – (7) on a computer technique will be considered using a specific example.

### 3 Results and discussion

Distribute the fund of mineral fertilizers for applying them to grain crops in such a way as to obtain the maximum conditional net income. The total sown area is 2750 hectares. It is planned to cultivate winter wheat, corn for grain, sunflower, corn for green fodder. Funds of mineral fertilizers (in kg of active ingredient) were allocated for these areas: nitrogen – 167400, phosphate – 196200, potash – 136200. Planned to produce 12000 centner grains, sunflower – 10600 centner, green mass of corn – 170000 centner.

The optimal distribution of mineral fertilizer funds by crop is given in Table 1.

In the obtained optimal plan, the fertilized areas with basic yield were: winter wheat – 1000 hectares, corn for grain – 700 hectares, sunflower – 220 hectares, corn for green fodder – 439 hectares. The increase in corn yield was 3,500 c.

| Fertilizers | Dedicated fertilizer and active substance funds | Distribution of mineral fertilizers and active ingredients by crop |
|-------------|-----------------------------------------------|---------------------------------------------------------------|
|             | Winter wheat | Corn for grain | Sunflower | Corn for green fodder | Total |
| Nitrogen    | 1674         | 400            | 588       | 354                  | 332   | 1674 |
| Phosphoric  | 1962         | 450            | 704       | 432                  | 376   | 1962 |
| Potash      | 1362         | 400            | 385       | 279                  | 288   | 1362 |

An economic analysis of the use of fertilizers showed that the mineral fertilizers available on the farm, with their
optimal distribution, make it possible to increase the net income of crop production by 14.6%.

Thus, in solving the optimization problem, factors describing the effect on yield were more fully taken into account than in traditional planning. This made it possible to distribute mineral fertilizers between farms in accordance with the characteristics of their soils.

Receiving sufficiently high and stable yields of grain crops is impossible without the use of chemical agents, the introduction of which can be carried out using the machine-tractor fleet. In this case, there is a need for a coordinated solution of the tasks of the machine-tractor park and the achievement of maximum efficiency from its use in grain farming.

With the help of ground equipment, mineral fertilizers are applied, chemical pest and plant disease control is carried out, weed vegetation is destroyed in grain crops.

Each chemical work (technological operation) can be performed using different types of machine and tractor fleet. To perform the same operation, an unequal number of ground equipment with different performance and operating costs will be required. Therefore, it is necessary to choose the best option for the use of machine and tractor fleet, which will ensure the implementation of a given amount of chemical work in a timely manner at the lowest cost.

The optimal plan for the use of machine and tractor fleet can only be determined using optimization methods, since the composition of machines and tractors and their possible use have such a large number of options that it is almost impossible to simply sort them out and select the best.

Consider the problem of optimal use of machine and tractor fleet.

The mathematical model of the problem has the following form [8].

Find a solution that minimizes total costs.

\[
\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{K} C_{ijk} x_{ijk} + \sum_{i=1}^{n} \left( \max_{j} \sum_{k} x_{ijk} \right) \alpha_{i}, \tag{8}
\]

under restrictions

\[
\sum_{i=1}^{n} P_{ijk} x_{ijk} = d_{jk}, \quad j = 1, n; \quad k = 1, K, \tag{9}
\]

\[
x_{ijk} \geq 0, \quad i = 1, m; \quad j = 1, n; \quad k = 1, K, \tag{10}
\]

Where \( i \) is the index of the type of ground equipment, \( i = 1, m \);

\( j \) is the index of the type of chemical works, \( j = 1, n; \)

\( k \) – the index of the type of the calendar period, \( k = 1, K; \)

\( d_{jk} \) – changeable amount of chemical work of the \( j \)-th species, which must be performed in the calendar period \( k,\ ha; \)

\( P_{ijk} \) – replaceable productivity of ground equipment of type \( i \) performing the \( j \)-th type of work in the \( k \)-th calendar period, \( ha/h \);

\( C_{ijk} \) – replaceable costs for performing work of the \( j \)-th type by the \( i \)-th type of ground-based equipment in the \( k \)-th calendar period, independent of the annual load of machines, tenge;
Where \( y^*_{jk} \) – the amount by which the actual volume is more than planned, if we denote by \( h^*_{jk} \) additional costs per unit of work, then the total cost of additional work is
\[
\sum_{j=1}^{n} \sum_{k=1}^{k_i} t_k h^*_{jk} y^*_{jk} \; ;
\]
\( y^-_{jk} \) – the amount by which the actual amount of work was less than planned; set a penalty for unused equipment per unit of work by \( h^-_{jk} \), then the total losses will actually be
\[
\sum_{j=1}^{n} \sum_{k=1}^{k_i} t_k h^-_{jk} y^-_{jk} \; ;
\]
\[
\sum_{j=1}^{n} \sum_{k=1}^{k_i} t_k \left( h^*_{jk} y^*_{jk} + h^-_{jk} y^-_{jk} \right) \; \text{on average were minimal}.
\]

Then we come to the problem of stochastic programming.

Find a plan \( X \) that minimizes total costs,
\[
\min \; F(x) = \sum_{i=1}^{n} \sum_{k=1}^{k_i} C_{jk} x_{jk} t_k
\]
\[
+ \sum_{i=1}^{n} \left( \max_{k=1}^{n} x_{jk} \right) \alpha_i +
\]
\[
+ M \left( \min_{k=1}^{n} \sum_{i=1}^{n} \sum_{k=1}^{k_i} t_k \left( h^*_{jk} y^*_{jk} + h^-_{jk} y^-_{jk} \right) \right)
\]
under restrictions
\[
y^+_{jk} - y^-_{jk} = d_{jk}(W) - \sum_{i=1}^{n} P_{ijk} x_{ijk}, \tag{12}
\]
\[
y^+_{jk} \geq 0; \quad y^-_{jk} \geq 0, \tag{13}
\]
\[
x_{ijk} \geq 0, i = \overline{1, m}; \quad j = \overline{1, n}; k = \overline{1, K}, \tag{14}
\]
where \( d_{jk}(W) \) is a random realization of \( d_{jk} \).

This is a non-linear two-stage stochastic programming problem.

To solve this problem, we use the iterative random search method proposed in [9].

According to this method, at each iteration for fixed \( X_{jk} \) and \( d_{jk}(W) \), we solve the problem dual to the following problem of the second stage:
\[
\Phi(x, d(w)) = \min \sum_{j=1}^{n} \sum_{k=1}^{k_i} t_k (h^*_{jk} y^*_{jk} + h^-_{jk} y^-_{jk}) \tag{15}
\]
under conditions
\[
y^+_{jk} - y^-_{jk} = d_{jk}(W) - \sum_{i=1}^{n} P_{ijk} x_{ijk}, \tag{16}
\]
\[
y^+_{jk} \geq 0, \quad y^-_{jk} \geq 0, \quad j = \overline{1, m}; \quad k = \overline{1, K}. \tag{17}
\]
The dual problem has the form: to find
\[
G(x, d(w)) = \max \sum_{j=1}^{n} \sum_{k=1}^{k_i} \lambda_{jk}(x, d(w)) - \sum_{i=1}^{n} P_{ijk} x_{ijk}, \tag{18}
\]
under conditions
\[
\lambda_{jk}(x, d(w)) \leq h^*_{jk} t_k \tag{19}
\]
\[
-\lambda_{jk}(x, d(w)) \leq h^-_{jk} t_k \tag{20}
\]
Require that the relation be satisfied
\[
h^*_{jk} + h^-_{jk} \geq 0.
\]

Then the solution of the effective problem (18) – (20) is simply:
\[
\lambda_{jk}(x, d(w)) \leq h^*_{jk} t_k \quad \text{if } d_{jk}(w) - \sum_{i=1}^{n} P_{ijk} x_{ijk} \leq 0,
\]
\[
\lambda_{jk}(x, d(w)) = -h^-_{jk} t_k \quad \text{if } d_{jk}(w) - \sum_{i=1}^{n} P_{ijk} x_{ijk} \geq 0,
\]
\[
\lambda_{jk}(x, d(w)) \quad \text{any number on the segment } (h^*_{jk}, h^-_{jk}).
\]

Consider a random vector with components
\[
\xi_{ijk} = C_{ijk} t_k + F(x) - P_{ijk} \lambda_{jk}(X, d(w)) t_k ,
\]
where \( F(x) \) is the gradient of \( \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ijk} \alpha \)
function expression (11).

As shown in [10], the expectation of the vector \( \xi \) coincides with the vector of the generalized gradient of function (11), and the sequence \{ \( x_{(s)} \) \}, whose components
\[
X_{ijk}\{s\} + \max \{ 0, x_{ijk}\} - \rho_s [C_{ijk} t_k + F(x) - P_{ijk} \lambda_{jk}(x, d(w)) t_k] \}
\]
\((\rho_s \) is a step size, \( d(w) \) is an arbitrary realization of the vector \( d(w) \) in the \( s \)-th iteration), converges with probability 1 to the minimum of function (11) with
\[
\rho_s > 0, \sum_{i=0}^{\infty} \rho_s = \infty, \sum_{i=0}^{\infty} \rho_s^2 < \infty.
\]

Thus, we propose the following algorithm for solving problem (11) - (14) by the method of random search. Let the value be obtained at the \( s \)-th step \( X_{ijk}\{s\}\) (the initial plan \( X_{ijk}\{0\}\) is specified).

1. Choose a random implementation \( d_{jk}(w) \) in accordance with the given distribution law.

2. Calculate the value
\[
L(s) = d_{jk}(w(s)) - \sum_{i=1}^{n} P_{ijk} x_{ijk}(s)
\]

3. At \( L(s) \geq 0 \), \( \lambda_{jk}(s) = h^*_{jk} t_k \); at \( L(s) \leq 0 \), \( \lambda_{jk}(s) = h^-_{jk} t_k \); at \( L(s) = 0 \), \( \lambda_{jk}(s) = 0 \).

4. Find
\[
X_{ijk}\{s+1\} = \max \{ 0, x_{ijk}(s) - \rho_s [C_{ijk} t_k + F(x) - P_{ijk} \lambda_{jk}(s) t_k] \}
\]

4 Conclusion

To ensure the delivery of fertilizers to the destination for the chemicalization of agriculture, the problem of optimal distribution of fertilizers in grain production has been solved. Based on optimal models, the needs of vehicles for servicing these works are determined. As the obtained
calculations show, the cost of purchasing machines and operating costs can be reduced by 20-30% due to the correct choice and use of equipment.

We have developed a complex of economic and mathematical models for the use of vehicles (machine and tractor fleet and agricultural aviation) in the chemicalization of grain crops in a deterministic and stochastic setting. The developed models, in contrast to the previously known ones, are built in accordance with the principles of targeting and taking into account the peculiarities of the development of regions.

The method of determining the need for a machine-tractor park of the economy involves the following main steps:

– identification of typical farms in a given zone or area;
– development of technological maps of crop cultivation and the determination of the optimal composition of the machine-tractor park for typical farms of the zone or area;
– development of standards requirements in the technique for groups of farms characterized by selected typical farms;
– determination of the composition of the machine and tractor fleet of any agricultural object, characterized by this typical farm.

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