Chapter 6
The Blue Straggler Population in Dwarf Galaxies

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6.1 Introduction

6.1.1 Blue Stragglers in Globular Clusters

Ever since their first identification in M3 by Sandage [67], the term Blue Stragglers (BSS) refers to the population of stars that are relatively brighter and bluer than the cluster’s main sequence (MS) turn-off point. Several decades after, globular clusters remain the ideal environment where BSS can be identified with certain ease. Indeed, the family of Galactic globular clusters typically shows a turn-off mass of \(~ 0.8 \, M_\odot\), being mostly coeval and \(\gtrsim 10\) Gyr old [39]. Thus, the identification of a \(\sim 1.2 \div 1.5\, M_\odot\) hot stellar population, nowadays, is unexpected since such high mass stars would have evolved already in a \(\gtrsim 10\) Gyr system; thereafter the “straggler” term.

In the context of globular clusters studies the last decade has brought a new wave of interest in these presumably dead systems (see Chap. 5). In particular, thanks to the accumulating evidence of: (i) systematic chemical abundance anomalies [10], coupled with (ii) detection of multiple and discrete red giant branches and/or main sequences [47], the idea that multiple stellar populations coexist within a single cluster is taking the upper hand [62]. This clearly defies the old text-book definition of globular clusters (i.e. ancient, coeval and chemically homogeneous systems).

For what concerns the BSS identification in globular clusters one may wonder if admitting the multiple populations scenario would leave some space for the presence of a “young” population (BSS), the answer is no. Indeed, and based on our current understanding, the complexity of multiple epochs of star formation within globular clusters is always limited to the first \(\lesssim 1\) Gyr since the formation epoch of the cluster.
(some $\sim 13$ Gyr time ago). Thus, and despite these chemical anomalies and multiple generations of stars, there is still no room to accommodate for the presence of a $1.2 \div 1.5\ M_\odot$ hot/blue stellar population (i.e. BSS) within the standard single-star evolution in globular clusters.

The origin of the BSS is usually sought as either the products of (i) dynamical interactions; or (ii) binary evolution (see Chap. 6). The dynamical origin for BSS foresees a continuous production of collisional binaries [27] between single and/or binary MS stars throughout the life of the stellar system. On the other hand, the most likely BSS formation scenario is that involving binary evolution. In this case the origin of the fresh hydrogen that “rejuvenates” the BSS is mass transfer [46]. In particular, mass transfer occurs when the evolved primary (now invisible) fills its Roche lobe and processed material overflows to the secondary MS (which now constitutes the BSS visible component). Within the binary evolution scenario, BSS can also be formed via the coalescence of a binary system made by two “normal” MS stars at the TO level.

### 6.1.2 The Importance of Dwarf Galaxies

Dwarf galaxies represent the dominant population, by number, of the present-day universe and galaxy clusters. These low-mass galaxies are not simply scaled-down versions of giant systems. Indeed, they hold the keys for a deeper understanding of galaxy formation, chemical evolution, star formation processes and dark matter content. In the framework of hierarchical clustering scenarios such as in cold dark matter models [6] dwarf galaxies would have been the first objects to be formed, that would later contribute to the assembling of larger systems [74]. The resultant picture is one in which the dwarfs observed nowadays are those which survived merging events. There are a number of factors that contribute to making the Local Group ($d \lesssim 1.1$ Mpc) a unique laboratory for astronomers; first its dwarf members are close enough to enable determining age, metallicity and their star formation histories from their resolved stellar populations. Second, the Local Group comprises dwarf galaxies of such variety of morphological types, masses, ages, spatial distributions and metallicities that they are statistically representative of other dwarf populations present in other environments, nearby groups or clusters. The two aforementioned properties qualify dwarf galaxies as optimal cases for “near-field cosmology”.

When studying the Blue Stragglers properties in the resolved stellar populations of nearby dwarf galaxies one soon realises a basic limitation: a meaningful analysis is possible only for those galaxies whose imaging surveys reach at least $\sim 1$ magnitude below the old main-sequence turnoff level. Consequently, the current sample of “useful” Local Group dwarf galaxies is limited to systems within $D_\odot \approx 900$ kpc, where the upper limit is set by the Tucana dwarf spheroidal galaxy survey by the Hubble Space Telescope. Nevertheless, one should appreciate that addressing the BSS population in such a distant system implies availing a photometric catalogue.
with a reasonable photometric completeness level at $I \approx 29.0$, which is a major challenge already.

**Fig. 6.1** The effective radius ($R_{\text{eff}}$ [pc]) vs absolute $V$-band magnitude ($M_V$) for the family of stellar systems covering a range of 10 magnitudes in mass. This figure is reproduced from [48] with permission by the RAS.

### 6.1.3 Dwarf Galaxies vs Globular Clusters

It is important to recall that the *fuzzy*, almost featureless dwarf galaxies still represent a different class of objects on their own, which is fundamentally different from that of globular clusters. Misgeld & Hilker [48] searched for *Fundamental Plane* relations for systems ranging from faint galaxies and star clusters of only a few hun-
dred solar masses up to giant ellipticals $10^{12} \, M_\odot$. Their analysis (see Fig.6.1) shows a clear dichotomy between the galaxy and star cluster family. It is straightforward to qualify globular clusters as members of the star cluster family, and that dwarf galaxies occupy the faint tail of the galaxy group. In particular, and over several orders of magnitudes ($-10 \lesssim M_V \lesssim -5$), the typical effective radius of globular clusters ($\sim 3 - 10$ pc) and dwarf galaxies (few hundreds to $\sim 1000$ pc) does not vary significantly with mass.

One may argue that the faintest dwarf galaxies reach low effective radii that are too close to typical values of the clusters group, thereby defying the galaxy definition. Clearly, these are not ordinary dwarf galaxies (e.g. Segue I has $M_V \sim -1.5$ and a $M/L \sim 3000$) rather they probably represent a class of objects losing dynamical equilibrium and close to disruption [23, 55]. Nevertheless, these ultra-faint dwarfs (whose total luminosities can be less than that of individual red giants) show kinematic and metallicity evidence that unambiguously support the dwarf galaxy classification [76] and as such their BSS population (when present) should be included in any review. This is particularly important because the sample of ultra-faint dwarfs is expected to rise thanks to forthcoming Southern hemisphere surveys (e.g. Skymapper [30]).

6.2 BSS Identification in Dwarf Galaxies

Colour-magnitude diagrams of typically old dwarf spheroidal galaxies show the presence of a well-separated blue plume of stars that very much resembles an old BSS population, as that observed in globular and open clusters (see the early studies of Mateo et al. [43, 41]). However, in the context of dwarf galaxies one cannot exclude that blue plume stars may include genuinely young main sequence (MS) stars, i.e. a residual star forming activity. The BSS–young MS ambiguity is simply a hard quest (see discussions in [29, 1, 9]).

Photometric techniques like the $(U - B)$, $(B - V)$ colour-colour diagram are widely used for Galactic Halo BSS and horizontal branch (HB) studies. Respectively, the two colours are proxies for metallicity and temperature and are useful in separating Halo BSS from blue horizontal branch stars. This technique, however, is not necessary for dwarf galaxies studies, because the dwarf galaxies stellar populations are basically projected at the same distance from us. Thus the disentangling of the BSS population in dwarf galaxies relies entirely on the colour-magnitude diagram (as is the case for Galactic globular and open clusters), and consists of a selection region within certain luminosity and colour boundaries.

The luminosity function of BSS in globular clusters has been found to increase from a luminosity cutoff at $M_V \sim 1.9$ down to $M_V \sim 4.0$, at the ancient MS turn-off level [21] while the temperature of BSS are between $\sim 6000 - 7500$K. An examination of Fig.6.2 shows that the colour and luminosity extensions of the LeoII

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1 http://rsaa.anu.edu.au/observatories/siding-spring-observatory/telescopes/skymapper
blue plume population falls within the BSS limits in globular clusters. On the other hand, the right panel of Fig. 6.2 summarises the BSS–young stars ambiguity in dwarf galaxies. The Fornax dwarf galaxy is known to host a recent star formation episode, that occurred some 200 Myr ago [69], and the diagram from the HST survey [28] shows that in such cases one cannot properly reach the ancient MS turn-off level without the inclusion of contaminant young stars. Thus, the selection of a dwarf galaxies sample for BSS studies must filter out all those galaxies that do not allow a clear detection of the ancient MS turn-off level.

In this regards, one should bear in mind that the fainter end of the BSS sequence extends to $\sim 0.6$ magnitude below the ancient MS turn-off level (e.g. the case of M55 by [34]). Given the small number statistics of the BSS stars in dwarf galaxies,
we caution that a conservative BSS selection region (i.e., a bright $M_V \sim 3.0$ cutoff that does not reach the ancient main sequence turn-off level) would heavily underestimate the BSS frequency in dwarf galaxies. For example, Mapelli et al. [36] (using a conservative BSS selection regions) derive a fraction of HB stars over BSS of $F_{BH} = \log(N_{BSS}/N_{HB}) \approx -0.65$ for the Draco and Ursa Minor dwarf galaxies. This is well below the value derived in [50] of $F_{BH} \approx +0.1$, based, however, on deeper and more complete photometric catalogues that allowed reaching the oldest turn-off level. We note that our BSS frequencies [50] for the two galaxies were recently confirmed by Zhao et al. [78] (see also the discussion in [11]). Admittedly however, there exists no standardised selection process ultimately defining the BSS selection region for dwarf galaxies, and much is left to the discretion of the investigators.

6.2.1 The BSS Identification in the Galactic Halo

The low-density environment of dwarf galaxies is very similar to that of the Galactic Halo, hence it is no surprise that a comparison between their BSS populations is made. In the absence of kinematic and chemical studies of BSS in dwarf galaxies, the comparison to the Galactic Halo BSS properties is limited only to the BSS specific frequency. In this regards, it is worth to spend a few words on the identification of the Galactic Halo BSS. The chapter by G. Preston (Chap. 4) reports a detailed presentation of the (i) photometric; (ii) spectro-photometric; and (iii) spectroscopic criteria used to extract and disentangle the BSS from the horizontal branch population. The spectroscopic survey by Preston & Sneden [60] remains a reference point for Milky Way field blue metal-poor (BSS) studies, and they conclude that over 60% of their sample is made up by binaries, and that at least 50% of their blue metal-poor sample are BSS. We all refer to their results for the BSS frequency in the Galactic Halo: $N_{BSS}/N_{BHB} = 4$.

The Halo BSS frequency however has been derived relying on a composite sample of only 62 blue metal-poor stars that are: (i) distributed at different line of sights; (ii) at different distances; and most importantly, (iii) for which no observational BSS-HB star-by-star correspondence can be established (see also [17]). Most importantly, one has to bear in mind that the normalised horizontal branch population is limited to the blue HB only, as these are the only HB component that can be disentangled from the Thin/Thick Disc population. On the other other hand, the majority of BSS frequency studies for globular clusters and dwarf galaxies refer to the entire HB population, i.e., including the blue, the variable and the red components of the HB. Consequently, one has to bear in mind that the adopted $\log(N_{BSS}/N_{BHB}) \approx 0.6$ value for the Galactic Halo is actually a high upper limit, as indeed the inclusion of red horizontal branch stars would lower this value. Indeed, a simple check of the M31 Halo stellar populations, as recently surveyed by the Hubble space telescope, shows a conspicuous red horizontal branch population (see Fig.2 in [8]). Clearly, one has to account for differences in the star formation and chemical enrichment
histories of the two spiral galaxies, nevertheless the Halo stellar populations of the two galaxies cannot be very dissimilar.

Fig. 6.3 HST colour-magnitude diagrams of NGC 7078 in the F336W, F439W and F555W filters, showing a selection of BSS, horizontal and red giant branch populations. The right panel shows the same selection in the \((U - B)\) plane, highlighting how the BSS and horizontal branch stars (sharing almost the same temperature) display the so-called red-incursion [49], which is an instrumental effect.

Lastly, we point to an often neglected effect when surveying the Galactic Halo for BSS populations. Figure 6.3 displays the HST colour-magnitude diagram of NGC 7078. The left panel shows a typical selection of BSS, HB, and RGB populations and these are later searched in the \(V, (U - B)\) plane (right panel). The later diagrams shows the un-expected and un-physical feature (the so-called red-incursion, see [49]) where blue HB stars can appear redder than their red giant equivalents. This feature has been explained as due to a particular dependence of the adopted \(U\) filter (in any given photometric system) and whether it encompasses the Balmer jump. Figure 6.3 shows that the red-incursion affects also the BSS population, which
ends up at redder colours with respect to red giants at the same luminosity level. The occurrence and extent of the red-incursion shows a dependence of the employed $U$ and $B$ filters on the star’s effective temperature, gravity, and metallicity. In particular, the red-incursion is stronger (i.e. shows redder extent) for lower metallicities stars (see Fig.5 of [49]). Thus, photometric techniques employing $U, B$ filters for BSS and HB surveys in the Galactic Halo should take this into account.

6.3 BSS Specific Frequency in Dwarf Galaxies

6.3.1 The Dwarf Galaxies Sample

The large on-sky projection of the closest dwarf galaxies and the faintness of the BSS in the distant ones preclude the availability of a single homogeneous and large-area photometric data-set addressing the BSS population. Although a significant effort has been devoted to such purpose (e.g. the HST/WFPC2 archival survey by [28]) the small field of view of the HST may veil some important spatial distribution gradients of specific stellar populations. For example the Leo1 HST study of [22] did not show the presence of the blue horizontal (HB) branch population, and this led to the conclusion that LeoI might have delayed its first epoch of star formation. However, a wide-area coverage by [26] did reveal a conspicuous, ancient, HB population. Thus, in the context of BSS studies, spatial distribution gradients are important and should be accounted for by wide-area surveys.

For example, the largest available catalogue for the Sagittarius dwarf galaxy is that of Monaco et al. [51], covering $\sim 1^\circ$ square degree. However, the Sagittarius dwarf has a core radius of $\sim 3.7^\circ$, thus the above-mentioned catalogue covers only $\sim 3.5\%$ of the galaxy, or $\sim 6\%$ of its stellar populations. Hence, any estimate of its BSS frequency is to be taken with caution, especially that the inner $\sim 14^\prime \times 14^\prime$ region has to be excluded to account for the coincidence of the galaxy’s centre with the position of its globular cluster M54. Moreover, a delicate aspect of estimating the BSS frequency involves estimating the Galactic foreground/background contribution in the covered area. This is particularly important for galaxies in certain line of sights (e.g. the Sagittarius dwarf suffering severe Galactic Bulge contamination).

To estimate the Galactic contribution in a homogeneous way the TRILEGAL code [24] was used. This online tool provides synthetic stellar photometry of the Milky Way components (Disc, Halo, and Bulge), and star counts were performed on the simulated diagrams (using the same selection boxes) and these subtracted from the observed HB and BSS star counts for the dwarf galaxies.

There are 12 dwarf galaxies for which a reliable BSS frequency could be determined. The basic properties of the selected galaxies (taken from [44] and [45]) are summarised in Table 6.1, and respectively report the absolute visual magnitude, the absolute distance modulus, the reddening, the central $V$ surface brightness, the core and half-light radius and the stellar mass of the galaxy. The BSS specific frequency
— calculated as $F_{BSS}^{HB} = \log(N_{BSS}/N_{HB})$ — is reported in column 3. We emphasise that: (i) photometric incompleteness corrections; (ii) foreground/background subtraction; (iii) possible overlap between old and intermediate age stellar population around the HB level; and (iv) confusion between BSS and normal MS stars, are unavoidable sources of error that affect any analysis addressing the BSS frequency in dwarf galaxies. The reported error bars (reported in column 4) account for the propagation of the Poisson errors on the star counts, but mostly reflect the dependence on the uncertainty in properly defining the HB and BSS selection boxes. In particular, the reported BSS specific frequency have been either taken directly from the literature (as is the case for the Cetus and Tucana dwarf galaxies, [52]) or from estimates based on photometric catalogues made available for the following objects: (i) Sextans [33]; (ii) Ursa Minor [9]; (iii) Sculptor [66]; (iv) Ursa Major by [75]; (v) Bootes [4]; (vi) Sagittarius by [51]; (vii) LeoII [25]; (viii) Draco [1]; and (ix) LeoIV and Hercules by Milone (priv. comm., based on HST diagrams).

### Table 6.1  
Basic properties of dwarf galaxies for which the BSS frequency could be reliably derived

| Name       | $M_V$ | $\log(F_{BSS}^{HB})$ | $e_{\log(F_{BSS}^{HB})}$ | $(m-M)_V$ | $E_{(B-V)}$ | $\mu_V [\text{mag}\cdot\text{arcsec}^{-2}]$ | $r_h [\text{arcsec}]$ | $M_* [10^6 M_\odot]$ |
|------------|-------|----------------------|--------------------------|-----------|-------------|----------------------------------|------------------------|---------------------|
| Bootes     | −5.8  | 0.26                 | 0.15                     | 18.9      | 0.000       | 28.05                                           | 12.6                   | 0.029               |
| Ursa Major | −6.8  | 0.20                 | 0.18                     | 17.1      | 0.150       | 27.80                                           | 11.3                   | 0.014               |
| Draco      | −8.6  | 0.09                 | 0.15                     | 19.5      | 0.030       | 25.30                                           | 10.0                   | 0.290               |
| Ursa Minor | −8.9  | 0.13                 | 0.13                     | 19.0      | 0.030       | 25.50                                           | 8.2                    | 0.290               |
| Sextans    | −9.5  | 0.05                 | 0.12                     | 19.7      | 0.040       | 26.20                                           | 27.8                   | 0.440               |
| Sculptor   | −9.8  | 0.01                 | 0.13                     | 19.7      | 0.000       | 23.70                                           | 11.3                   | 0.230               |
| LeoII      | −10.1 | −0.01                | 0.09                     | 21.6      | 0.030       | 24.00                                           | 2.6                    | 0.740               |
| Sagittarius| −13.5 | −0.26                | 0.26                     | 17.1      | 0.150       | 25.40                                           | 342.0                  | 2.600               |
| Cetus      | −10.1 | 0.05                 | 0.05                     | 24.5      | 0.030       | 25.00                                           | 3.2                    | 2.600               |
| Tucana     | −9.5  | 0.02                 | 0.04                     | 24.7      | 0.030       | 25.00                                           | 1.1                    | 0.560               |
| Leo IV     | −5.0  | 0.49                 | 0.16                     | 20.9      | 0.026       | 27.50                                           | 4.6                    | 0.019               |
| Hercules   | −6.4  | 0.21                 | 0.17                     | 20.6      | 0.063       | 27.20                                           | 8.6                    | 0.037               |

#### 6.3.2 The $F_{BSS}^{HB} - M_V$ Anti-correlation

For a wider perspective on the BSS specific frequency in dwarf galaxies, a comparison is made with that derived for Galactic globular and open clusters. Of the original compilation of BSS in Galactic open cluster by de Marchi et al. [14] we filter out clusters for which less than two BSS stars were found. On the other hand, the original compilation of ~3000 BSS in 56 Galactic globular clusters by Piotto et al. [56] was complemented by BSS frequencies for three interesting additional clusters, namely: (i) NGC 1841 [20], which is the Large Magellanic Cloud (LMC) most metal-poor and most distant (10 kpc from the LMC bar) cluster; (ii) NGC 2419,
Fig. 6.4 The BSS frequency \( F_{\text{BSS}}^{\text{HB}} \) vs \( M_V \) diagram for globular clusters [56], open clusters [13], and dwarf galaxies [50]. The horizontal line shows the mean BSS frequency as derived for the Milky Way field stars by [60]. The range of the BSS frequency for the Milky Way Bulge [11] is shown as a bar at \( M_V = -20.4 \). A preliminary upper limit for the Milky Way Halo BSS frequency (as normalised to only the blue horizontal branch stars) is shown at \( M_V = -17.8 \).

which is a massive Milky Way cluster at 90 kpc from the Galactic centre, suspected to be a dwarf galaxy; and (iii) \( \omega \) Cen, which is the most enigmatic Milky Way cluster, also suspected to be an extra-Galactic dwarf galaxy. The three data-points are based on deep HST/ACS, WFPC2 and ACS archival data, respectively. Focusing our attention only on the BSS frequency of Galactic globular cluster, in particular for the three additional clusters, Fig. 6.4 shows that the so-called \( F_{\text{BSS}}^{\text{HB}} - M_V \) anti-correlation [56] [13] is basically universal for all globular clusters, regardless of their specific complexity and origin. The observed anti-correlation implies that more massive globular clusters are surprisingly BSS-deficient, as if their high collision rate had no correlation with the production of collisional binaries. In particular, the \( F_{\text{BSS}}^{\text{HB}} - M_V \) anti-correlation was explained by Davies, Piotto & de Angeli [13].
(see also [37][38]) in the following manner: the number of BSS produced via collisions tends to increase with cluster mass, becoming the dominant formation channel for clusters with $M_V \leq -8.8$. On the other hand, the BSS number originating from primordial binaries should decrease with increasing cluster mass. Accounting for these two opposite trends and binary evolution, the models by Davies et al. [13] reproduce the observed BSS population, whose total number seems independent of the cluster mass.

When plotting the BSS frequency in dwarf galaxies, Fig. 6.4 shows the following general trends: (i) dwarf galaxies with $M_V \leq -8.0$ possess a relatively higher BSS frequency with respect to globular clusters with similar luminosities; and (ii) the lowest luminosity dwarf galaxies with $-8.0 \leq M_V \leq -5.0$ show BSS frequencies that are fully compatible with that observed in open clusters. This compatibility between dwarf galaxies and open clusters may suggest that there exists a “saturation” in the BSS frequency ($F_{BSS, HB}^\approx 0.3 - 0.4$) for the lowest luminosity systems (both for open clusters and dwarf galaxies). In this regards, we note that the globular clusters distribution shows an abrupt cut at $M_V \sim -6.0$. This is only a selection effect, and we remind the reader that there are a dozen of clusters with $M_V \gtrsim -6.0$ that were not included in the Piotto et al. [56] survey. It is of great importance to fill this gap and test the hypothesis that all stellar systems show a saturation of the BSS frequency at $F_{BSS, HB}^\approx 0.3 - 0.4$.

There is a hint of such universal upper limit in the study by Sollima et al. [72] who derive high BSS frequencies for three clusters with $M_V \approx -5.0$. Similarly, Santana et al. [68] derive high BSS frequencies for the faintest tail of the globular clusters at $M_V \approx -5.0$. Unfortunately, these clusters could not be added to Fig. 6.4 because Sollima et al. [72] normalised their BSS star counts with respect to the main sequence stars, while the study of Santana et al. [68] used the red giant branch stars.

Overall, the dwarf galaxies sample shows a hint of a proper $F_{BSS, HB}^\approx - M_V$ anticorrelation. In [50], and relying on a smaller sample of 8 galaxies, the statistical significance of a $F_{BSS, HB}^\approx - M_V$ anti-correlation was explored, and the probability that a random sample of uncorrelated experimental data points would have yielded a linear-correlation coefficient of 0.984 was found to be extremely low ($\leq 10^{-6}$). With respect to the above [50] study, the present dwarf galaxy sample has four new solid entries, that populate the extreme ends of the dwarf galaxies luminosity distribution. The BSS frequency for the Tucana and Cetus dwarfs at around $M_V \approx -10.0$ are taken directly from Monelli et al. [52], whereas for the LeoIV and Hercules at around $M_V = -5.0$, the frequencies were derived. Repeating the same exercise, the statistical significance of the hinted $F_{BSS, HB}^\approx - M_V$ anti-correlation still holds. Nevertheless, it is populating the dwarf galaxy sample with the ultra-faint dwarfs with $M_V \approx -4.0$ that the anti-correlation can be firmly established. In this regards, we note the extreme difficulty in the process of identification of BSS samples in such galaxies, as this heavily relies on the quality of photometric catalogues. For example, the BSS frequency for the LeoIV and Hercules dwarf galaxies was derived only thanks to yet unpublished Hubble Space Telescope deep diagrams that reveal with confidence the presence of BSS in these systems. On the other hand, ground-based
observations of the same two galaxies did not allow a reliable estimate of the BSS frequency.

Lastly, one should also keep in mind that for the ultra-faint dwarf galaxies even the absolute luminosity of the system is subject to significant variations, i.e. the inclusion or not of few red giant stars (whose census become very sensitive to foreground contamination) has reflected on changes of the order of $\sim 0.6$ magnitude for some galaxies.

**The specific case of the Carina dwarf galaxy:** previously we emphasised the obvious need to exclude the group of gas-rich, star forming dwarf galaxies from our sample. To this constraint, one can also add the group of dwarf galaxies with known, dominant, intermediate-age population. Indeed, the presence of intermediate age population of around $\sim 5$ Gyr would inevitably interfere with the BSS selection process and impose the shifting of the BSS box to brighter magnitudes. A perfect example of such a case is the Carina dwarf galaxy. The colour-magnitude diagrams of Hurley-Keller, Mateo & Nemec [29] and Bono et al. [7] show clearly the presence of multiple subgiant branches separated by $\sim 0.5-0.8$ magnitude. The Carina star formation history reconstructed by Rizzi et al. [65] shows that the bulk of the star formation has taken place in an episode at around $\sim 6$ Gyr. Deriving a BSS frequency for Carina (as done for the other galaxies but with the limitation of not reaching the faintest turn-off level) results in a lower limit of $F_{\text{BSS}}^\text{HB} = 0.4$, which at $M_V = -9.1$ would still reflect a high BSS frequency with respect to other dwarf galaxies of similar luminosity (e.g. the Tucana dwarf at $M_V = -9.5$ has $F_{\text{BSS}}^\text{HB} = 0.0$). This has triggered the use of the $F_{\text{BSS}}^\text{HB} - M_V$ anti-correlation as a diagnostic for the presence of young stellar populations in other galaxies. For example, examining the diagrams of the Canes VenaticiI dwarf ($M_V = -8.6$), Martin et al. [40] estimate the BSS/blue plume frequency to be $F_{\text{BSS}}^\text{HB} = 0.5$, and along with arguments concerning the spatial distribution of this population they conclude that it is best understood as a young stellar population.

**The specific case of the Milky Way Bulge:** thanks to recent and multi-epoch *Hubble Space Telescope* observations of the Galactic Bulge, Clarkson et al. [11] beautifully managed to proper-motion decontaminate the Bulge stellar populations from the foreground disc contribution. Their goal was to investigate the presence (or not) of a genuine young stellar population in the Galactic Bulge. The cleaned colour-magnitude diagram however resembles that of a typical old stellar population with a scarce population of seemingly BSS. They [11] estimate the BSS specific frequency and (as done in this and many studies) use the convention of normalising the BSS numbers as a function of the entire horizontal branch population. Assuming a $M_V = -20.4$ for the Bulge they conclude that the limits of their BSS frequency is consistent (see Fig. 6.4) with the general trend displayed by the $F_{\text{BSS}}^\text{HB} - M_V$ anti-correlation, suggested in our earlier work [50]. The addition of the Bulge extends the $F_{\text{BSS}}^\text{HB} - M_V$ anti-correlation by over 7 magnitudes to the extreme massive systems. As emphasised by Clarkson et al. [11], the interpretation of this agreement remains unclear and awaits further confirmation of the anti-correlation itself.

Lastly, availing recent *SEGUE* survey results for BSS and blue-HB candidates (Santucci R., priv. comm.), a rough and preliminary BSS frequency for the Galactic
Halo of $F_{BSS}^{BH} \sim 0.2$ was derived involving 9,000 and 5,600 stars. To assign an absolute luminosity of the Galactic Halo, we roughly assume that the family of Galactic globular clusters contribute by 1% of the total luminosity, and derive $M_V \sim -17.8$. As argued in Sec. 6.2.1, normalising the BSS star counts to only blue-HB stars would provide a strong upper limit. Nonetheless, we add this point to Fig. 6.4 for comparison purposes and speculate that the Galactic Halo BSS frequency might approach values of $F_{BH}^{BSS} \sim 0.0$ and lower. All together, the addition of 4 new dwarf galaxies, along with that of the Galactic Bulge and evidence from the Anomalous Cepheid frequency (see Sec. 6.6) seem to confirm the $F_{BH}^{BSS} - M_V$ anti-correlation.

Fig. 6.5 The BSS frequency as a function of the stellar collision factor for a specific star per year. Globular clusters are plotted as filled squares, while dwarf galaxies are plotted as filled circles. The filled triangle is that of an open cluster.
6.3.3 The Significance of the Anti-correlation

The blue plume of dwarf galaxies (currently not experimenting star formation episodes) does not allow clear cut indications on whether it represents: (i) a genuine BSS population (as that observed in globular clusters); (ii) a young population of recently \( \leq 2 \) Gyr formed stars; or (iii) a combination of both. To tackle this ambiguous problem, and given the faintness of the blue plume stars, one could only address the number frequency of this population and search for general trends as a function of the parent galaxy parameters. To attempt such analysis, one must tacitly assume that the blue plume is made of a genuine BSS population (perform the BSS star counts and normalisation) and then search for a correlation, say, with the parent galaxy luminosity (a proxy of mass). Should the assumption be incorrect — and the blue plume population is instead made of (or contaminated by) young MS stars — then one would expect to find a correlation between the BSS frequency and the galaxy luminosity (i.e. more massive galaxies tend to preserve a low level of residual star formation rate and hence possess a larger fraction of young stars). On the other hand, a flat BSS frequency distribution would need an ad hoc scenario where more massive galaxies conspire and coordinate their star formation rate in a way to mimic a rather flat “BSS” frequency. Instead, and with all necessary caution, we find hints of an anti-correlation. This is unexpected and points to a genuine BSS origin of the blue plume in the studied galaxies. The fact that globular clusters do show a similar anti-correlation makes it easier to suggest that whatever mechanism is at work in globular clusters might be responsible for the milder anti-correlation seen in the dwarf galaxies. Granted the above, does the scenario envisaged by Davies et al. [13] apply also for dwarf galaxies?

The above question implies answering “do dwarf galaxies harbour a significant population of collisional binaries at all?” The answer is no. This lies in the intrinsic properties of dwarf galaxies and their differences from globular clusters. Indeed, it is enough to recall that the central luminosity density of a dwarf galaxy (e.g. Ursa Minor: \( 0.006 L_\odot \text{ pc}^{-3} \) at \( M_V = -8.9 \)) is several orders of magnitudes lower than that found in a typical globular cluster (e.g. NGC 7089: \( \sim 8000 L_\odot \text{ pc}^{-3} \) at \( M_V = -9.0 \)). This implies that the collisional parameter of dwarf galaxies is very low, and unambiguously point to the much slower dynamical evolution of dwarf galaxies. To further emphasise this last point, in Figure 6.5 we show \( F_{\text{HB}}^{\text{BSS}} \) as a function of a calculated quantity: the stellar specific collision parameter \( \log \Gamma_\star \) (the number of collisions per specific star per year). More specifically, following Piotto et al. [56], we estimate \( \log \Gamma_\star \) from the system’s central surface density and core size. To these, we could add the \( \log \Gamma_\star \) corresponding value of an open cluster, thanks to parameters provided by Giovanni Carraro (priv. comm.).

The mean collisional parameter of the 12 studied galaxies is \(-11.5\). The lowest value is that for the Sagittarius dwarf, and this is probably due to its very extended galaxy core. Compared with the mean value of \( \log \Gamma_\star = -7.5 \) for the globular clusters sample, the estimated number of collisions per specific star per year in a dwarf galaxy is \( 10^{-5} \) times lower. This almost precludes the occurrence of collisional binaries in dwarf galaxies, and one may conclude that genuine BSS sequences in dwarf
galaxies are mainly made of primordial binaries. Moreover, the overall log $\Gamma$ distribution of the dwarf galaxies shows no obvious correlation with the BSS frequency. This is in good agreement with the observational fact (see [56]) that the collisional parameter of globular clusters also do not show any correlation with the BSS frequency.

The flatter slope of the potential anti-correlation in dwarf galaxies may be understood in terms of the almost lack of collisional–BSS: i.e. neither created nor destroyed. Moreover, not all primordial binaries, now present in a dwarf galaxy, turn into or are already in the form of BSS. In particular, it is the low exchange encounter probabilities in environments like the Galactic Halo or dwarf galaxies that guarantees a friendly environment and a slower consumption/evolution of primordial binary systems. The BSS production (via evolution off the MS of the primary and the consequent mass transfer to the secondary that may become a BSS) is still taking place in the present epoch and this can explain the high frequency of primordial BSS in dwarf galaxies as well as the Galactic Halo.

6.4 The BSS Radial Distribution and Luminosity Function in Dwarf Galaxies

As argued in the previous section, the collisional rate of dwarf galaxies is $10^{-5}$ times lower than in globular clusters. This practically precludes the BSS collisional formation channel in dwarf galaxies, thereby supporting solely the mass-transfer binaries channel. This conclusion can be confirmed or refuted by examining the radial distribution and luminosity functions of the BSS population in dwarf galaxies. Indeed, models of the globular clusters BSS that are thought to originate via the collisional channel foresee specific radial distribution and luminosity functions signature, and such signatures can be verified in dwarf galaxies.

6.4.1 Radial Distribution

Thanks to their high density cores, globular clusters are the ideal environment where collisional BSS can, and must, form. Interestingly, the BSS radial distribution of the majority of globular clusters is bimodal [21][15][77][31][2]: showing a central peak, followed by a gradual decrease until reaching a minimum at intermediate radii, and then showing a rise at the clusters periphery. This bimodality was explained [37][38] by the joint contribution of: (i) collisional binaries, naturally produced in the highest density regions of the parent cluster, thereby responsible for the central peak; and (ii) mass-transfer binaries which, left peacefully to evolve in the cluster outskirts, avoid sinking towards the cluster centre, and produce the outer peak. A minority of globular clusters [32][3] do not show the external rise and are expected to be poor in
mass-transfer BSS. Overall, all globular clusters show a central peak in their BSS radial distribution and this is attributed to the production of collisional BSS.

The wide-field imaging study of the Draco and Ursa Minor dwarf galaxies by Mapelli et al. [36] shows clearly that the radial distribution of the BSS population is flat (see also [9]), and hardly consistent with a central peak like that observed in globular clusters. If ever, the BSS populations in this study [36] show a hint of a decrease in the central regions. Mapelli et al. [36] conclude that the BSS in the Draco and Ursa Minor dwarf galaxies have radial distributions that are consistent with the expectations of their models of mass-transfer BSS [37, 38]. Similar results were obtained for the case of the Sculptor dwarf by [35], and Cetus and Tucana by [52]. Figure 6.6 shows an excellent example of flat BSS distributions in the Cetus and Tucana dwarf galaxies (from [52]).

6.4.2 Luminosity Function

Monkman et al. [54] find that bright BSS stars in 47 Tuc tend to be more centrally concentrated. The central distribution of these bright BSS stars strengthen the idea of a collisional origin. Indeed, the collisional formation channel should allow the final product (the BSS star) to retain a large fraction of the original masses involved in the collisions, hence the brighter luminosities. In terms of the luminosity function, this translates into a correlation between the BSS luminosities and their radial distribution. Should the dwarf galaxy lack the presence of collisional binaries (as we have argued) one expects no correlation between an inner and outer BSS luminosity functions.
This hypothesis was tested by Mapelli et al. for the Draco, Ursa Minor dwarf galaxies and for Sculptor \cite{[35],[36]}, and by Monelli et al. \cite{[52]} for the Cetus and Tucana dwarfs. With the only exception of Sextans \cite{[33]}, all these studies proved the absence of a correlation between the BSS luminosity function and the radial distribution. This is best illustrated in Fig.\ref{fig:6.7} taken from \cite{[52]}. For each galaxy, two BSS luminosity functions were derived and compared. The excellent agreement between the inner and outer \(1.5 \times r_c\) selections points to the absence of collisional binaries products, thereby confirming the mass-transfer binaries as the sole origin of BSS in dwarf galaxies.

6.5 Variable BSS in Dwarf Galaxies

SX Phoenicis (SX Phe) are a class of Population II pulsating variables that exhibit short period (\(P < 0.1\) days) variations, having spectral types between A2–F5. SX Phe variables are particularly interesting because the region in the colour-magnitude diagram where they cross the instability strip coincides with the BSS location for globular clusters. Indeed, all SX Phe variables in globular clusters are BSS stars. SX Phe are, however, \(\sim 1 - 2.5\) magnitudes fainter than RR Lyrae and therefore, in the context of distant dwarf galaxies studies, are harder to detect. Mateo, Fischer & Krzeminski \cite{[42]} and Poretti et al. \cite{[58]} were the first to report on SX Phe in the Carina dwarf galaxy.
Poretti et al. [59] presented an intensive survey of the Fornax dwarf (a galaxy known to harbour recent star formation activity and a predominately intermediate age population). Figure 6.8 shows how the group of 85 SX Phe variables is coincident with the expected BSS location and, at the same time, immersed within the blue plume of young stars. Examining the period–luminosity relation for the SX Phe sample, Poretti et al. [59] conclude that the observed scatter exceeds their observational errors and propose a physical rather than an instrumental origin for this scatter. Using the period–luminosity plane, they reported the first identification of a peculiar group of sub-luminous extra-Galactic SX Phe variables. The presence of the sub-luminous SX Phe was confirmed by Cohen & Sarajedini [12] in the Carina dwarf, NGC 2419 and ω Cen. Poretti et al. [59] cautiously speculate that the sub-luminous variables could be the results of merging of a close binary system, and
present arguments in favour for this scenario. Clearly, more observations are needed to confirm this scenario. However, this may open a new field where BSS can be disentangled (via their variability signature) in star forming dwarf galaxies.

6.6 The Progeny of BSS

During their core-helium burning phase, the progeny of the BSS population (i.e. evolved-BSS, hereafter E-BSS) are expected to pile up in a particular location in the colour-magnitude diagram: bluer than the red giant branch and brighter than normal horizontal branch stars. Renzini & Fusi Pecci [63] were the first to suggest the presence of E-BSS. Interestingly, the occurrence of E-BSS is independent of the formation channel of the BSS [71]. Ferraro et al. [16] performed a systematic search for E-BSS, and Fig. 6.9 shows their ultraviolet diagram of M80 where the E-BSS population is clearly present and highlighted by the open box. They derive $N_{\text{BSS}} / N_{\text{E-BSS}} = 16$.

![Fig. 6.9](image)

Fig. 6.9 The left panel displays the HST ultraviolet $U$, $(U-V)$ diagram of M80 where the progeny of BSS are identified within the open box. The right panel displays the optical diagram of the Cetus and Tucana dwarf galaxies, where the boxes also highlight the E-BSS population. The filled stars are the identified Anomalous Cepheid variables, detected by [5]. The left and right figures are reproduced from [52], and [16], respectively, with permission by the AAS.

The right panel of Fig. 6.9 shows the detection of a vertical extension in correspondence of the red HB region in the Cetus and Tucana dwarf galaxies. In the context of dwarf galaxies, this sequence is usually referred to as vertical-clump [22]: helium-burning stars of few hundred Myr to 1 – 2 Gyr old population. Monelli et al. [53] use their best reconstruction of the Cetus star formation history, and derive
a $N_{\text{BSS}}/N_{\text{E-BSS}} = 12$ and 10 upon simulated and observed diagrams, respectively. The agreement between the empirical and synthetic values for the Cetus dwarf and the value derived by [16] all point to a correct identification of the same evolved BSS population in the 2 different systems. We repeated the exercise for the LeoII dwarf galaxy and derive $N_{\text{BSS}}/N_{\text{E-BSS}} = 9$, again indicating a correct identification of E-BSS in dwarf galaxies.

**The case of Anomalous Cepheids:** dwarf galaxies (whether purely ancient systems like Ursa Minor or intermediate-age ones like LeoI) are known to host a peculiar class of Cepheid variables known as Anomalous Cepheids (ACs). RR Lyrae and ACs are core helium-burning pulsational variables passing through the instability strip and co-exist in low-metallicity systems. However, ACs are 2 – 3 times more massive than RR Lyrae [64, 18]. The origin of ACs is sought as either due to: (i) relatively young ($\sim 1 – 6 \text{ Gyr}$) single stars; or (ii) the progeny of BSS, formed through mass transfer in primordial (10 Gyr) binary systems. The early study of Renzini, Mengel & Sweigart [64] shows how the chances of survival of such primordial binaries are strictly correlated with the stellar density of the parent system: ACs are easily destroyed via dynamical encounters in high density environments like globular clusters. On the other hand, the low stellar densities of dwarf galaxies offer a peaceful environment and allow the system to evolve. This 1977 prediction is proven correct by the continuous and successful surveys of ACs in dwarf galaxies — e.g. Bernard et al. [5] for the detection of 8 and 6 ACs in Cetus and Tucana, and Fiorentino et al. [20] for the detection of 51 ACs in LeoI. Most interestingly, the prediction of the almost lack of ACs in globular clusters still holds as well. The exception being the detection of a single AC detected in NGC 5466 [79], which is a low-density cluster.

![Fig. 6.10 The Anomalous Cepheid frequency (logS, per $10^5 L_\odot$) as a function of the $M_V$ of the parent dwarf galaxy (i.e. the ACs frequency–$M_V$ anti-correlation). Open circles highlight purely ancient galaxies while starred symbols indicate galaxies with large intermediate-age populations. The solid line shows an excellent fit to only the ancient systems (included in this review). This figure is reproduced from [19] with permission by Astronomy & Astrophysics, © ESO.](image-url)
Fiorentino & Monelli [19] complemented the earlier studies by Mateo et al. [41] and Pritzl et al. [61] and reinvestigated the correlation between the ACs frequency and the absolute visual luminosity of the parent galaxy. Figure 6.10 displays the Anomalous Cepheid frequency (log $S$, per $10^5 L_\odot$) as a function of the $M_V$ of the parent dwarf galaxy. Open circles show purely ancient galaxies while starred symbols indicate galaxies with large intermediate-age populations. The sample of ancient galaxies includes: Ursa Minor, Draco, Sculptor, LeoII, Sextans, Tucana and Cetus (all of which are included in the present BSS study) confirm that these possess a genuine BSS population. Figure 6.10 shows that for these galaxies there exists a clear anti-correlation between the ACs frequency and the luminosity of the parent galaxy. The solid line shows the fit to only the ancient systems while the dashed line is the fit for the intermediate-age galaxies. The interpretation of the apparent ACs frequency–$M_V$ anti-correlations for the two groups is not straightforward. However, intermediate-age galaxies systematically show higher ACs frequencies, and this goes along with the fact that these systems can host ACs originating from 2 different channels (i.e. recently formed young stars as well as evolving primordial binaries). Fiorentino & Monelli [19] note, however, that for their sample of ancient systems one expects to find ACs originating only through the binary channel, and therefore imply that binary systems have a higher chance of survival in low-mass galaxies.

6.7 Conclusion

During the workshop, George Preston proposed us a nice definition of blue straggler stars:

BSS are a subset of the interacting-binary field stars whose end products are hotter than the main sequence turn-off of the parent stellar population.

In his 1993 review L. Stryker [73] concludes:

This makes forming conclusion about the origin of BS in dwarf galaxies difficult, if not impossible, at the present time.

Twenty years after, I believe that the recent imaging surveys of these distant systems have allowed us to get a deeper insight on the general properties and origin of their BSS population. Indeed, and thanks to the new generations of ground and space-based telescopes, the star formation and chemical enrichment history for the majority of Local Group galaxies have been derived. This has allowed us to scrutinise and filter-out those galaxies with hints of recent star formation activity. Consequently, clean samples of “ancient” galaxies are available to address the BSS-young stars ambiguity in a reasonable manner.

Our current understanding of the possible BSS formation channels are mass-transfer binaries, collisions and mergers, and all three channels must manifest themselves in globular clusters. The central peak of the BSS radial distribution is still
best explained as a signature of collisional BSS. Similarly, the segregation of the brighter BSS in the cluster centres is also attributed to collisional BSS. Assuming that the previous two interpretations are true, one can confidently affirm that both these collisional BSS “signatures” are lacking in dwarf galaxies. This proves the long-suspected prediction that BSS in dwarf galaxies form preferentially through mass-transfer binaries, as is the case for the Galactic Halo.

Figure 6.4 shows that the lowest luminosity dwarf galaxies with \(-8.0 \leq M_V \leq -5.0\) have BSS frequencies that are in perfect agreement with that observed in Galactic open clusters. Evidence that the lowest-luminosity globular clusters share the same BSS frequency as in open clusters and dwarf galaxies allow us to suggest that there exists an empirical upper limit to the BSS frequency in any given stellar system. Figure 6.4 also shows that the BSS frequency in dwarf galaxies with \(M_V \approx -9.0\) is higher than that of globular clusters at the same luminosity. This is in perfect agreement with the notion that higher density environments, with their higher collisional rates, perturb the evolution of mass transfer in primordial binaries and end up being BSS deficient. Dwarf galaxies on the other hand offer a more friendly environment for mass-transfer binaries and tend to preserve their initial binary population. The recent derivation of the BSS frequency in the Galactic Bulge, and arguments presented for the BSS frequency in the Galactic Halo, seem to provide more evidence for the dwarf galaxies BSS frequency–\(M_V\) anti-correlation. The mechanism responsible for the decreasing BSS frequency with increasing mass remains unclear, and awaits further confirmation of the anti-correlation. Interestingly, a tight and strong anti-correlation (Fig 6.10) has been observed between the frequency of Anomalous Cepheids (a progeny of BSS) and \(M_V\) for ancient dwarf galaxies with \(-11.0 \leq M_V \leq -9.0\).

All together, I believe that the past few years have been very fruitful for the study of BSS in dwarf galaxies, and the best (deeper imaging and kinematic and chemical surveys) are yet to come.

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