Knowledge of the constants that describe the current cosmological world model, $H_0$, $t_0$, and the three $\Omega$s is central to physical cosmology. Although there is a vast range of suggested tests and existing constraints much of the recent discussion of cosmological constants involves three time variable photospheres: Cepheids, SNIa, and the CMB last scattering surface. These concluding remarks for the Moriond XXXIII meeting are made at a time when many of the established methods have made careful, interesting, statements about the values of various cosmological constants based on data of small random errors. The flood of new data over the next few years will lead to a satisfying increase in the precision of both direct and model dependent estimates of the main cosmological parameters.

1 Introduction

Commenting on the progress being made in estimating the values of the cosmological parameters has perils not unlike those of critiquing a great artwork, perhaps an opera, while it is being staged for the first time. The problem is that there is no working consensus for the values of the cosmological constants. There are rather wide compromise ranges which accommodate most of the derived values. However the values of the cosmological constants at one end of a compromise range are completely incompatible with those at the other, both in physical meaning and in stated measurement error.

An aspect of cosmological parameter determination that is fascinating for any interested scientist is that it continues to reward an integrated view of the entire subject from trigonometric parallaxes to photometric response functions to radio interferometric mapping to X-ray plasma analysis to the outer limits of particle theory, to name but a few. Martin Rees has described Cosmology as the “Grandest of the Environmental Sciences” which serves to remind us of the difficulty of relating the observations of the universe to the simple cosmological models of
interest. Although the FRW model and an interest in its parameters have been around for a long time, it was not until the 1980's when development of solid state detectors of very low noise and very high quantum efficiency allowed virtually every waveband used by astronomy to greatly increase both the quality and the abundance of data. The objects now examined range from nearby white dwarfs to galaxies at redshifts beyond 5, to precise measurements of the CMB radiation all over the sky.

The following discussion discusses the broad comprise ranges for the basic cosmological parameters. In most cases the data have small random errors. The problems for all methods come in calibration and model uncertainties. For instance Cepheids distances have random errors of about 1%, nevertheless the total error in the Hubble constant is generally quoted to be at least 10 times larger as a result of potential calibration and systematic errors. Even those apparently large error budgets are hard won from vast efforts to control, measure and remove systematic errors. The true error ranges of these are continuing to shrink significantly and steadily, such that sometime in the next decade our understanding will undergo a phase transition, hopefully a crystallization not a meltdown.

2 $t_0$

The age of the Universe is currently most accurately measured from the Globular Clusters in our own galaxy. This is an impressively mature subject which interprets very high precision photometry using immensely detailed physical models of relatively simple stars. There have been two slightly startling developments in the past few years. First, when allowance was made for helium diffusion, the ages of globular clusters came down about 10%. Second, the Hipparcos satellite provided new distances to local subdwarf stars which can be used to calibrate the distance to similar stars in globular clusters. The surprise was that the distances were somewhat larger than anticipated, meaning that the stars were brighter and hence younger. The quoted ages are $11.5 \pm 1.5$ Gyr and a minimum of 13 Gyr. The physical complexities of confidently predicting the model luminosities in the observed filter bands leads to some further uncertainty. Although the results are statistically compatible, within this range one can draw quite different cosmological conclusions. Since one generally adds 1 Gyr to the Globular cluster ages to derive a $t_0$ it appears that the traditional first year astronomy values of 12 to 15 Gyr remain acceptable values. There is great excitement that white dwarfs will soon provide an alternate dating method to some of the nearby globular clusters.

3 $H_0$

In principle, measuring $H_0$ is simple. In practice it is a dimensional constant which effectively requires carrying the standard meter from the earth to cosmological distances where the fractional perturbations to the Hubble flow are small. The classical methods are entirely geometric, relying on parallax and the $1/d^2$ law, but require building a “distance ladder”. A variety of newer methods are single step, but, model dependent. For instance, combining cluster X-ray and Sunyaev-Zeldovich measurements, or, gravitational lenses with measurable time delays between images, gives a quantity that depends on the Hubble constant.

Classical estimation of $H_0$ has been advanced enormously with CCD photometry and telescopes with greatly improved angular resolution, allowing the brightness of point-like objects to be accurately measured at relatively large distances. Cosmology has always been a matter of dealing with state of the art detection of photons from distant, faint objects which are themselves the focus of research. It is not surprising that it has been slow progress since Hubble’s original discovery (whose 1936 book gave a well considered Hubble constant of about 600 km s$^{-1}$ Mpc$^{-1}$). The Hubble constant search has become a substantial industry with many approaches to the
problem. The HST Key Project to measure Cepheid distances to a representative set of objects in the local supercluster is generally regarded as a piece of bedrock which is a standard rod against which other measurements approaches are calibrated or compared. Although the distances to the various galaxies that have been measured are fairly non-controversial, the implied Hubble constant remains quite controversial. It would be desirable if more SNIa would co-operate and explode in some of these galaxies with carefully measured distances.

It is impressive that in about the last 10 years the scatter in the values of the Hubble constant have decreased by a factor of two or perhaps a little more. Many would accept that the Hubble constant lies someplace in the range of 55 to 75 km s$^{-1}$ Mpc$^{-1}$. No one is really comfortable with such a large range. The good news is that the Key Project has more results to announce, and the rate at which data are being collected in other approaches to this problem is increasing. The next 10 years will see total errors reduced another factor of two, which should lead to consensus of some sort.

Newer methods like the SZ/X-ray technique or the gravitational lens time delay measurements are promising one-step methods with simple physics, however in both cases the results at the moment have a fairly large scatter compared to classical estimators and there remain genuine complications in modelling the potentials involved in the cluster potentials and the the lensing potentials. On the other hand, the interest, effort and data quality of these new methods are ascending very rapidly.

4 The Three $\Omega$s

The three controversial $\Omega$s are the $\Omega_B$, $\Omega_X$, and $\Omega_\Lambda$, the density parameters of the baryons, dark matter and the $\Lambda$ term, respectively. The matter density, $\Omega_M = \Omega_B + \Omega_X$, is the total density of gravitating matter which has no significant pressure. The fourth $\Omega$ is that of radiation, which although by far the smallest at the current epoch, is known to fabulously high precision through the measurement of the spectrum of the CMB radiation.

Over the past 10 years there has been a great deal of new work on the baryonic $\Omega_B$ which probably hasn’t changed the formal error range that much, but, many systematic errors have been reconsidered. The range of quoted values is about a factor of three from the bottom end of the “Helium” value to “low Deuterium” value, $\Omega_B = 0.007 - 0.024 h^{-2}$. Helium is particularly difficult because one needs to derive the primordial abundance from systems that have been partially contaminated with stellar Helium. Deuterium is both very fragile in stars and difficult to observe. Galaxy clusters estimate the baryon fraction, the ratio of the gas plasma baryonic mass to total mass within some radius, $f_B \simeq 0.12 \pm 0.02 h^{-3/2}$, which is consistent with the BBN $\Omega_B$ provided that $\Omega_M$ is less than approximately 0.4. This suggests, but does not yet prove, that clusters contain a representative gas-to-mass ratio.

An extremely significant, and confident result, is that although $\Omega_B$ is about an order of magnitude larger in gas than in stars, it is still about an order of magnitude less than the total gravitating mass. Hence, the unknown dark matter continues to be the dominant mass in the universe. Another puzzle is that the current location of most of the baryons is not known, with the IGM being the usual suspect, although MACHOs are a new possibility.

The total $\Omega_M$ has seen a lot of variation over the past 20 years, coming back to rest someplace near the views of the classic article of the mid-1970’s, which argued for an $\Omega_M \simeq 0.1$. In spite of the apparent lack of change this subject has gone the full circle with tremendous profit to astrophysics. The circle began when Peebles derived the Cosmic Virial Theorem which helped to provide the motivation for large redshift surveys. One of the first redshift surveys was the CfA, which indicated $\Omega \simeq 0.2$ under the assumption that galaxies trace the mass. About the same time there were two theoretical developments which lead to hesitation in accepting this result. First, inflation argues very persuasively that $\Omega_M = 1$. Second, the concept of formation
of objects at peaks of the density fluctuation field immediately demonstrated that galaxies could be clustered much more strongly than the underlying total mass,\[\Omega_M\] hence the derived \[\Omega_M\] could be severe underestimates. Virtually all theorists concluded that in the absence of overwhelming evidence to the contrary, \[\Omega_M = 1\]. A little later there was the largely unanticipated discovery of large scale flows of galaxies\[\Omega_M\] which appeared to be a very strong argument for \[\Omega_M \simeq 1\].

In the past few years, the \[\Omega_M\] tide has reversed to generally favouring lower \[\Omega_M\] values again. That is, evidence for a low \[\Omega_M\] that strikes many as overwhelming is gradually being accumulated. A prediction of high \[\Omega_M\] is that clusters should have an increasing M/L with radius, as the Milky Way galaxy does beyond the disk. However, X-ray, redshift survey and weak lensing data all find that cluster galaxies are distributed like the dark matter to the virial radius. Furthermore, as newer catalogues of peculiar velocities and nearly-all sky surveys become available it appears that the flow fields are actually consistent with \[\Omega_M\] as low as 0.2 or so.\[\Omega_M\] Galaxies cannot be completely unbiased with respect to the total matter content of the universe, since various galaxy types and luminosities have different clustering properties. However, it is gradually coming clear that the bias cannot be very large. Much of the community would agree that \[\Omega_M\] is someplace in the range 0.1 \(-\) 0.4. Direct dynamical data favour \[\Omega \simeq 0.2\], but CMB fitting suggests the higher values. Now that much of the community has adopted a low density universe as the most likely model, there is a growing tension between very low \[\Omega_M\], those less than 0.2, and moderate \[\Omega_M\], those greater than 0.3, which point to some unresolved systematic error in the various data involved.

The most uncertain, and puzzling, of the \[\Omega\]s is \[\Omega_\Lambda\]. Here the measurement situation is far from mature. An upper limit on \[\Omega_\Lambda\] comes from the number of split images of distant sources produced by gravitational lensing from intervening galaxies. The very few splittings seen argues that \[\Omega_\Lambda\] is less than about 0.7. This argument hasn’t changed a lot since the 1980’s but it has become a lot more secure as worries about optical selection and evolution in the lensing galaxies have been addressed. The preliminary evidence from the CMB fluctuation measurements (see various contributions to this meeting) is that if \[\Omega_M\] is low there must be a nonzero \[\Omega_\Lambda\] to produce a peak in the angular power spectrum at a sufficiently large angle.

A scientific banquet of ground based, balloon borne and satellite CMB experiments is beginning, such that this meeting stands at a moment when exponential growth of effort is happening in the kitchen, with the expectation that a feast of high quality results begin in about a year. Many of these experiments have made impressive claims for the precision of their measurements, which are certainly true if the instruments perform to specification and the universe and the foregrounds correspond to the various model assumptions.

A completely classical approach to \[\Omega_\Lambda\] is the magnitude-redshift relation, which is underway with the SNIa measurements. These objects can be detected and reliably studied up to redshift of one, which provides considerable leverage on the \[\Omega_M - \Omega_\Lambda\] pair. The obvious concern is that some unknown evolution or systematic measurement error could creep in. The two teams are well aware of these possibilities and appear to be recalibrating, worrying about the astrophysics and the astronomy as much as anyone else. As much as joint analyses have the power to produce better parameter estimates, one looks forward to independent methods giving completely independent results with small errors, that do agree.

It is impossible to avoid a philosophical note when contemplating the \[\Omega\] values. That is surely the point. On the other hand, the universe can do what it wants and it is our job to measure and try to understand the results, which is a lesson from the history of particle physics. However, one cannot help but note the conundrums of a \[\Lambda\] dominated universe. If \[\Omega_\Lambda\] was much larger than unity, the universe would have started to exponentiate before galaxies (and presumably most stars) ever formed. Similarly even if \[\Omega_\Lambda + \Omega_M = 1\], then we are currently well into a round of inflation. We might ask about the prospects for our scientific descendents. Taking a rather modest extrapolation, by astrophysical standards, of 20 times the current age
of the universe, then at that future time there will have been about 20 e-folds of expansion, so
the CMB temperature will be nearly zero, the local group will merge with a few satellites left
orbiting, and, all other galaxies will have receded to unobservable redshifts. The result is that
future astronomers are left in a universe that has one significant galaxy: the merger product of
ours and M31. Most peculiar, but, evidently possible within this model framework.

5 Conclusion

The current situation is precisely why the measurement of the cosmological parameters is the
primary activity of many astronomers and astrophysicists. The exciting likelihood is that most
of the major cosmological constants will be known to a satisfying degree of on the time scale of
a decade.

The Moriond meetings provide an ideal format for frank discussion of extremely controversial
cosmological issues. I thank the organizers for the splendid job they did in managing to bring
us all together and providing a never ending flow of food and stimulus for mind and body.

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