Application of intelligent technologies and neural network modeling methods in the development of a hybrid learning environment

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Abstract. We consider the problems of applying mathematical modeling and intelligent technologies in the development of methods and tools to support educational processes. As a part of creating a hybrid intelligent learning environment, we offer the construction of differential mathematical models with delayed feedback. We characterize a generalized model of the educational process. This model contains a knowledge base module and an interaction module based on artificial intelligence. We study a number of qualitative properties for the dynamic «director–teacher–student» task transfer model with delayed feedback and intelligent components. The described approach to analyzing model properties allows us to consider uncertainties and control actions in hybrid learning environments.

1. Introduction
Research on the digital transformation of mathematical education is an important area of pedagogical science in the modern digital society that uses the achievements of artificial intelligence. Building new models that describe pedagogical processes is a necessary step in creating a hybrid learning environment in the subject area related to teaching mathematics in secondary schools. In the future, digital mathematical education platforms should provide tools for effective student work, improve the means of organizing secondary education, and optimize school time. Nowadays, it is particularly important to design individual educational routes depending on the level of training and individual psychological characteristics of students in a hybrid learning environment [1].

The use of modern digital technologies in the educational process leads, on the one hand, to changes in programs and approaches to training, and on the other hand, to the development of additional instrumental and methodological support. This additional instrumental and methodological support should contribute to the study of processes and phenomena from different points of view, taking into account the subject features of the discipline, psychological characteristics and knowledge of students.

Research and development of tools in the field of digital transformation of mathematical education use the results of interdisciplinary areas related to the analysis and synthesis of complex self-organizing systems. It is important to note that the synergetic approach in education allows us to lay the foundations for designing individual educational environments that consist of educational elements at different levels, taking into account the processes of self-organization of subjects [2, 3]. As it is commonly known,
the synergistic approach is based on mechanisms of interdisciplinary interaction in order to create new, more complex structures with new qualities.

2. Models and methods

We consider a model of multi-stage transfer of educational tasks, given by an m-th order differential equation of the following form:

$$\frac{d^m x}{dt^m} = f(t, x(t), x(t - \delta t), \gamma(x, t)),$$

(1)

where \( x \in \mathbb{R}^n \) is vector of the phase state of the system, \( t \) is time, \( x(t - \delta t) \) is delay vector, \( \gamma(x, t) \) is a function that sets the disturbance vector in the system. One of the important special cases of the model (1) we discuss later in section 3.

A generalized model with control can be represented in the form:

$$\frac{d^n x}{dt^n} = f(t, x(t), x(t - \delta t), \gamma(x, t), u(x, t)),$$

(2)

where part of the input values is explained after the equation (1), \( u(x, t) \) is control vector.

In models (1) and (2), the n-th order of the derivative corresponds to the construction of a chain of \( m \) links for transmitting educational tasks. There is a qualitative dependence of the value of the next link from the previous one in the specified chain. Systems (1), (2) define continuous dynamic models with time delay. The presence of the delay vector is caused by delays in the transfer of tasks. Models (1) and (2) are represented as vector equations as follows:

$$\dot{x} = g(t, x, \delta), \quad x \in \mathbb{R}^n,$$

(3)

$$\dot{x} = g(t, x, \delta, u), \quad x \in \mathbb{R}^n.$$

(4)

Models (3), (4) and their modifications and concreteness can be used in one of the basic blocks of the hybrid learning environment. We call this block the model building block. The methods of studying these dynamic models include, especially, the methods of qualitative theory of dynamic systems and control theory. In particular, the search for equilibrium States and stability analysis in the Lyapunov sense with respect to all and part of the phase variables can be considered important problems [4].

To account for various types of uncertainties, it is possible to make a transition from the model of the form (3) to the corresponding differential inclusions, fuzzy and stochastic differential equations (figure 1).

Based on the principle of reduction of differential inclusions stability problem to the problem of stability of fuzzy and stochastic differential equations, the constructive stability conditions are formulated [5–7]. We suggest using this principle in relation to models of the pedagogical process. In addition, important methods of analysis include numerical methods for finding the trajectories of systems (3) and (4).

In model (2), it is assumed that the disturbances are defined by a function that is a subject to limits. For model (2), you can set an optimal control problem with the specified initial and boundary conditions. The optimal control problem is to find the control function \( u(t, x) \), which corresponds to the trajectory, taking into account the fulfillment of the specified conditions. This problem should consider a fixed time interval \((t_1, t_2)\). In addition, for the model (2), taking into account the imposed conditions, we can also consider the formulation of the speed problem, in which it is necessary to minimize the value of the difference \( t_2 \) and \( t_1 \). Moreover, the corresponding optimal control problem with mixed limits is of our interest.
Methods for analyzing the stability of pedagogical processes models.

There are various approaches to optimal control of systems of the form (2), namely, based on control using PID controllers [8] or by introducing the system into sliding modes [9]. To obtain optimal trajectories of the model (2), we can use global parametric optimization methods (including methods inspired by nature: neural network methods, ant colony methods, etc.) in combination with the use of intelligent technologies.

As a part of the research, the actual problem is to synthesize regulators based on artificial neural networks (ANNs). One of the useful properties of artificial neural networks (ANNs) is their versatility. Problems that can be successfully solved with the help of ANN include the following: formation of models and various systems that are difficult to describe mathematically; decision-making and diagnostics, especially in fields where there are no clear mathematical models. A characteristic feature of ANN is, in particular, that the neural network is a flexible model for nonlinear approximation of multidimensional functions. These features of the ANN can be used quite effectively in the search for optimal trajectories of dynamic models of systems with switching [10].

3. Results

The model for transferring tasks in the "director – teacher – student" chain with delayed feedback, as well as with unknown disturbances, is represented as:

\[
\begin{align*}
\dot{x}_1 &= k_1 x_2 (t - \tilde{t}) + k_3 x_3 (t - 2\tilde{t}), \\
\dot{x}_2 &= k_2 x_1 (t - \tilde{t}) + k_5 x_5 (t - \tilde{t}), \\
\dot{x}_3 &= k_3 + k_4 x_4 (t - 3\tilde{t}) + d_1,
\end{align*}
\]

where \(x_1, x_2, x_3\) are interdependent quantitative indicators of the pedagogical process for the student, teacher, and director, respectively, \(k_1, ..., k_6\) are the coefficients of the model parameters, \(d_1\) is a function of the disturbance, \(\tilde{t}\) is a single delay in the system. Model (5) is a special case of model (1), when the dimension of the model is 3. Computational experiments were performed to find the trajectories of model (5). Some results of the numerical study are shown in figure 2.

![Figure 1. Methods for analyzing the stability of pedagogical processes models.](image)

Figure 2. Dynamics of delay system (5) taking into account the parameter values \(k = [-0.05, 0.01, -0.07, 0.05, 0.2]\).
It should be noted that the construction of the model (5) is performed taking into account some assumptions. In particular, we suppose that the task transfer process is continuous. To get more precise results, we plan to use complex modeling techniques, in particular, models of system dynamics and agent systems.

An important issue in developing the hybrid learning environment is domain analysis. If we consider the educational process as a single system, the complexity of the process lies in the presence of objective and subjective factors. To build a knowledge base (KB) of a hybrid intelligent learning environment (HILE) based on artificial intelligence (AI) technologies, in addition to didactic materials, it is necessary to have source data that represent the main patterns of the educational system functioning. The structure of the learning process model using AI and KB is shown in figure 3.

An aggregation block can be added to the diagram shown in figure 3, taking into account the results of the work [11]. In this work, the authors suggest using a neural network of 2-3 hidden layers with an identical number of neurons (10 each) with output fixation of the state of the neuron as it passes through the hidden layers.

As a part of the creation of HILE for solving current problems of the educational process, we plan to build and analyze differential mathematical models with delayed feedback. When developing such a HILE, the neural network modeling methods proposed in [10]. Questions related to the study of the stability of complex systems, which include hybrid learning systems, can be investigated using the results [12, 13], which contain, in particular, methods for analyzing intelligent control systems. There are various approaches to obtaining source data, in particular: 1) analysis and synthesis; 2) natural experiment; 3) computational experiment.

Analysis and synthesis have limited applicability due to the specifics of the subject field. natural experiment can be considered quite effective, but its organization is the most time-consuming. A computational experiment combines the advantages of the first and second approaches – it can be performed by the researcher independently, and its results can reflect the hidden patterns of the process being studied.

4. Discussions
The results of the analysis of the three-dimensional model of the educational process show that for a given set of initial values and parameter values, the behavior of trajectories has the character of oscillations. A series of computer experiments demonstrates similar behavior of trajectories. A promising direction is the study of a controlled three-dimensional model and the construction of regulators based on global parametric optimization methods.
As a part of the generalization of models (1) and (3), it is interesting to move to the corresponding models with control functions in the right-hand sides of the equations and with the addition of a quality control criterion, which optimizes the quality of sending and completing tasks. To find control functions taking into account the initial values of variables and given sets of parameters, we can use global parametric optimization methods [14] in combination with neural network modeling methods [15] with learning and other methods. We suggest using the developed neural network algorithms and creating modules of the software package for modeling systems that ensure the functioning of a hybrid learning environment using modern approaches to modeling.

It is important to note that the right-hand side of the multidimensional differential equation (4) can be approximated using fuzzy controlled TS-models [16, 17]. The use of the TS-model apparatus makes it possible to use modern computing systems to solve linear matrix inequalities, describe the properties of the original nonlinear systems, and solve stability and stabilization problems.

5. Conclusion
In this work, we propose the construction of a generalized model of multi-stage transfer of learning tasks. The generalized model is given by m-order differential equations, taking into account the delay and disturbances. The study of the dynamic model of the transfer of tasks "director–teacher–student" with delayed feedback and intelligent components revealed the nature of the trajectories. We describe an approach to analyzing model properties, taking into account the uncertainties and control actions that need to be taken into account when developing hybrid learning environments. The proposed methods for solving problems of stability and optimal management of the pedagogical process in conditions of uncertainty demonstrate a fairly high level of formalizability and applicability of artificial intelligence tools. The obtained results can be used in the development of a hybrid learning environment using artificial intelligence, as well as in solving problems of optimal control for models educational and socio-economic systems.

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References
[1] Basalin P D, Timofeev A E, Kulagina E A, Neymark E A, Fomina I A and Chernyshova N N 2018 Implementation of a hybrid intellectual learning environment of a production type Modern information technologies and IT-education 14(1) 256-67
[2] Dvoryatkina S, Shcherbatykhl S and Shcherbatykhl L 2018 Synergy of Mathematics, Informatics and Innovative Didactics (on the Example of Retraining of Teachers of Mathematics) Proc. of the 12th International Conference on Education and New Learning Technologies (Seville, Spain) 2003-9
[3] Gerasimova E N, Dvoryatkina S N, Korotkikh V I, Masina O N, Puchkov N P, Usachev A V and Shcherbatykhl S V 2017 Pedagogical synergy based on the fractal approach: content selection training control (a case study of mathematical education) Eurasia Journal of Mathematics Science and Technology Education 13(11) 7591-5
[4] Vorotnikov V I and Rumyantsev V V 2001 Stability and Control over the Coordinates of the Phase Vector of Dynamic Systems: Theory Methods and Applications (Moscow: Science world)
[5] Shestakov A A Generalized Direct Lyapunov Method for Systems with Distributed Parameters (Moscow: Nauka, 1990. 2nd ed. Moscow: URSS, 2007)
[6] Merenkov Yu N 2000 Stable-like Properties of Differential Inclusions, Fuzzy and Stochastic Differential Equations (Moscow: RUDN publishing house)
[7] Druzhinina O V and Masina O N 2009 Methods for Studying Stability and Controllability of Fuzzy and Stochastic Dynamical Systems (Moscow: Dorodnicy Computing Centre of Russian Academy of Sciences)
[8] Nikulin E A 2004 *Fundamentals of Automatic Control Theory. Frequency Methods of Analysis and Synthesis of Systems* (SPb.: BHV-Petersburg)

[9] Emelyanov S V, Korovin S K and Levant A 1993 Sliding modes of higher orders in control systems *Differents. equations* **29**(11) 1877-99

[10] Petrov A A 2018 Structure of the software package for modeling technical systems in the conditions of switching operating modes *Electromagnetic waves and electronic systems* **23**(4) 61-4

[11] Smirnov E I and Shcherbatykh S V 2020 Parameters and classifier of neural network databases of educational results *Materials of the International scientific conference "Fundamental problems of teaching Mathematics, Informatics and Informatization of Education", dedicated to the 180th anniversary of pedagogical education in Yelets* (Yelets: Bunin Yelets State University) p 162-3

[12] Druzhinina O V and Masina O N 2016 *Methods of Stability Analysis of Dynamic Systems of Intelligent Control* (Moscow: URSS)

[13] Druzhinina O V, Igonina E V, Masina O N and Petrov A A 2020 Aspects of using prototyping and artificial intelligence technologies in the framework of digital transformation of the educational process *Modern information technologies and it education* **16**(1) 65-74

[14] Price K, Storn R and Lampinen J 2005 *Differential Evolution: A Practical Approach to Global Optimization* (Springer)

[15] Golovko V A 2001 *Neural Networks. Learning, Organization, and Application* **4** (Moscow: IPRZHR)

[16] Takagi T and Sugeno M 1985 Fuzzy identification of systems and its applications to modeling and control *IEEE Trans. Syst., Man and Cybernetics* **15** 116-32

[17] Tanaka K and Wang H O 2001 *Fuzzy Control Systems Design and Analysis: a Linear Matrix Inequality Approach* (N.Y.: Wiley)