Spin transfer torques in the nonlocal lateral spin valve

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Abstract
We report a theoretical study on the spin and electron transport in the nonlocal lateral spin valve with a non-collinear magnetic configuration. The nonlocal magnetoresistance, defined as the voltage difference on the detection lead over the injected current, is derived analytically. The spin transfer torques on the detection lead are calculated. It is found that spin transfer torques are symmetrical for parallel and antiparallel magnetic configurations, in contrast to that in a conventional sandwiched spin valve.

1. Introduction
Due to the increasing interest in nanostructures with a spin degree of freedom incorporated, the local spin valve (LSV), where a layer of normal metal (NM) or insulator is sandwiched by two layers of ferromagnetic metal (FM), has been considered as the prototype of an experimental setup for the demonstration of spin-dependent effects, such as GMR [1], magnetization switching [2–4], etc. However, it is not easy to precisely analyse the spin transport based on an LSV in an experiment. The reason is that, accompanying the electrical current flowing across the detection ferromagnetic contact, spurious effects such as anisotropy magnetoresistance and Hall effect due to the FM contact are also involved [5]. This problem can be partially removed by using a nonlocal lateral spin valve (NLSV), where only spin current flows across the detection ferromagnetic contact, spurious effects such as anisotropy magnetoresistance and Hall effect due to the FM contact are also involved [5]. Recently, several experiments of metallic spin injection and detection had been carried out on NLSV [5–12]. From these experiments, important parameters of spin transport, such as spin diffusion length, are obtained [13].

However, most of those experiments focused on a collinear magnetic configuration, in which the magnetization of the injection source and detection drain are arranged to be parallel or antiparallel. On the other hand, the non-collinear spin transport in an LSV has been studied extensively [14–19] and reveals interesting physics, such as spin transfer torques (STT) and related magnetization switching. Little effort has been put into the non-collinear spin transport in NLSV so far. For NLSV, it is interesting to know whether or not we can also obtain sizeable STT, and how the spin current behaves when carried by the diffusion of spin instead of the electrically assistant drift of spin. Recently, current-induced magnetization switching was realized in NLSV [20], which gave strong evidence for the presence of the STT effect even in NLSV.

In this paper, by combining the diffusion equation and the magnetoelectronic circuit theory [21, 22], we investigate theoretically the spin transport in NLSV with non-collinear magnetic configurations. The angular magnetoresistance (AMR) in NLSV is discussed for systems with metallic FM/NM contacts and tunnelling contacts. It is also shown that, because of the spin accumulation at the normal metal side of the FM/NM contact, STT could be acting on the ferromagnet. When the length of the NM stripe is less than the spin diffusion length in NM, we found that STT in NLSV is comparable with that in an LSV. The angular dependence of the torques is qualitatively different from that in an LSV.

This paper is organized as follows. In section 2, the theoretical framework for dealing with the non-collinear transport in NLSV is presented and analytical expressions for both AMR and STT are derived. In section 3, we calculate the AMR and STT in the NLSV and the properties of torques and voltage difference across the FM/NM junction are discussed. Finally, we summarize our paper in section 4.

2. Theory description and models
Figure 1 is the schematic of the NLSV experimental setup. It consists of one NM lead and two ferromagnetic
leaves FM1 and FM2. These two ferromagnetic leads are separated by a length \( L \) and are aligned parallel to each other. Experimentally, the current \( I_0 \) is injected from FM1 and flows out from the left end of the NM. In this work, the direction of electrical current \( I_0 \) is defined along the direction of the electron (particle) current. After injection, the spin is accumulated in the NM lead. The diffusive spin spreads over the region in the NM lead between the two FM/NM contacts. A voltage difference \( V \) across the FM2/NM contact could be built up \([8, 9]\). For different configurations of magnetization arrangements, the spin-accumulation-induced voltage across the FM2/NM contact is angle-dependent \((V(\theta))\). It can be measured by the nonlocal AMR defined as \( R(\theta) \equiv V(\theta)/I_0 \).

### 2.1. Theoretical framework of transport in NLSV

The transport theory in an NLSV should include three parts: the transport in FM, NM resistors and across the FM/NM contacts. As the dimensions of FM and NM resistors in NLSV typically are much larger than the electron mean-free path, the transport can be described by the diffusion equation in terms of the spatially dependent electrochemical potential \([23]\).

In a spin-polarized system, besides the electrochemical potential \( u_0(x) = u_{ch}(x) - e\phi(x) \), where \( u_{ch} \) gives the chemical potential and \( \phi \) gives the electric potential \((-e\phi \equiv \text{denoting the electron charge})\), it is necessary to introduce a quantity \( u_s(x) \) accounting for the spin accumulation in the system \([21, 22]\). The direction of \( u_s(x) \) denotes the direction of spin accumulation in spin space and the magnitude of \( u_s(x) \) gives the energy splitting of the two spins in the local coordinate system. In principle, the direction of \( u_s(x) \) in the normal metal is arbitrary and needs to be determined by boundary conditions. In a ferromagnet, the spin accumulation is \( u_0^F(x) = m(u_0^L(x) - u_0^R(x)) \), where \( m \) is an unit vector along the magnetization in the FM and \( u_0^F(x) \) is the electrochemical potential of the majority (minority) spin in the local coordinate system where the quantized axis is parallel to the magnetization.

**Transport in FM.** As the spin decoherence length is of the order of the lattice constants in a conventional ferromagnet \([24]\), only the components which are parallel or antiparallel to the magnetization direction \( m \) in the FM can survive. Therefore, the electrical and the spin currents in the FM region are \([25, 26]\)

\[
I_F^e(x) = -(S_F/e)(\sigma_F^L \nabla_x u_0^L(x) + \sigma_F^R \nabla_x u_0^R(x)) \quad \text{and} \quad I_F^s(x) = -(S_F/e)\nabla_x (\sigma_F^L u_0^L(x) - \sigma_F^R u_0^R(x)) \mathbf{m}.
\]

Here the transport is assumed to be along the \( x \) axis, \( S_F \) is the area of the cross section in the FM and \( \sigma_F^\parallel \) denotes the conductivity for the majority (minority) spin channel. The bulk parameters, such as conductivity \( \sigma \), are assumed to be spatially uniform in this study.

Correspondingly, with conservation of electrical current \( I_F^e \), the continuity equations of the spin current are \([25]\)

\[
\nabla_x I_F^s(x)/S_F = -e\xi_s(u_0^R(x) - u_0^L(x))/\tau_{z1} + e\xi_s(u_0^F(x) - u_0^F(x))/\tau_{z1} + e\xi_s(u_0^L(x) - u_0^R(x))/\tau_{z1} + e\xi_s(u_0^F(x) - u_0^F(x))/\tau_{z1},
\]

where \( \xi_{z1} \) is the density of states per unit volume at the Fermi level for a single spin, and \( \tau_{z1} \) and \( \tau_{z1} \) are the spin-flip scattering time for majority and minority spins.

Inserting the expression of spin current into the continuity equations and with detailed balance \( \xi_s/\tau_{z1} = \xi_s/\tau_{z1} \), we obtain the conjugated diffusion equations for \( u_0^L(x) \) and \( u_0^R(x) \) in a ferromagnetic metal as

\[
\nabla_x^2 u_0^L(x) = u_0^L(x)/D_{1}\tau_{z1} - u_0^F(x)/D_{1}\tau_{z1},
\]

\[
\nabla_x^2 u_0^R(x) = -u_0^L(x)/D_{1}\tau_{z1} + u_0^F(x)/D_{1}\tau_{z1},
\]

where \( D_{1}\tau_{z1} \) is the diffusion constant and related to \( \sigma_{1}\tau_{z1} \) via the Einstein relation \( \sigma_{1}\tau_{z1} = e^2\xi_{z1}(D_{1}\tau_{z1}) \) \([23]\). Solving the diffusion equations, we obtain the spin-resolved electrochemical potential in the FM \([25]\):

\[
\frac{u_1(x)}{u_1(x)} = (\tilde{A} + \tilde{B}x) \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \tilde{C}e^{i/\tilde{\xi}} \begin{pmatrix} \sigma_{1}^{F-1} \\ -\sigma_{1}^{F-1} \end{pmatrix} + \tilde{D}e^{-i/\tilde{\xi}} \begin{pmatrix} \sigma_{1}^{F-1} \\ -\sigma_{1}^{F-1} \end{pmatrix},
\]

where \( \tilde{A}, \tilde{B}, \tilde{C}, \tilde{D} \) are constants to be determined by boundary conditions and \( \tilde{\xi} \) is the spin diffusion length in the FM given by \( (D_{1}\tau_{z1})^{-1} + (D_{1}\tau_{z1})^{-1} \). \( \tilde{\xi} \) is the spin relaxation time in the NM. The electrical and spin currents in the NM are also governed by the diffusion equation as \([21]\)

\[
\nabla_x I_N^e(x) = -(\sigma_N/e)S_N\nabla_x u_0^N(x) \quad \text{and} \quad I_N^s(x) = -(\sigma_N/e)S_N\nabla_x u_0^N(x).
\]

\( S_N \) is the cross section of the NM and \( \sigma_N \) is the conductivity of the NM. Conservation of electrical current requires \( \nabla_x I_N^e(x) = 0 \) which leads to \( \nabla_x^2 u_0^N(x) = 0 \). Experimentally, the sample length of the NM is always comparable or longer than spin diffusion length in the NM. Therefore, the spin-flip scattering cannot be neglected.

The continuity condition of spin current in the NM is \( (1/S_N)\nabla_x I_N^s(x) = -e\xi_F/2u_0^F(x)/\tilde{\xi} \), where \( \tilde{\xi} \) is the total density of states per unit volume at Fermi level in the NM and \( \tilde{\xi} \) is the spin relaxation time in the NM. With the current and continuity equation, the diffusion equation for \( u_0^N(x) \) is

\[
\nabla_x^2 u_0^N(x) = u_0^N(x)/(\tilde{\xi}/\tau_{z1}),
\]

where \( \tilde{\xi}/\tau_{z1} = (D_N\tau_{z1})^{1/2} \) is the spin diffusion length in the NM. \( D_N \) is the diffusion constant and related to \( \sigma_N \) via \( \sigma_N = e^2\xi_F/4D_N \). Solving the diffusion equation, the spin accumulation in the NM can be written in the form

\[
u_0^N(x) = \tilde{E}e^{i/\tilde{\xi}} + \tilde{F}e^{-i/\tilde{\xi}}.
\]
where $\vec{E}$ and $\vec{F}$ are the constant vectors depending on the boundary conditions.

**Transport across FM/NM.** In the absence of the interfacial spin-flip scattering, the electrical current $I_{0}^{\text{NF}}$ and the spin current $I_{x}^{\text{NF}}$ across the FM/NM contact, which are evaluated at the NM side, can be written in terms of electrochemical potential and spin accumulation in the linear response regime as [21]

$$
e I_{0}^{\text{NF}} = (G_{1}^{R} + G_{1}^{I}) (u_{0}^{N} (x_{1}^{-}) - u_{0}^{F} (x_{1}^{+})) + \frac{i}{2} (G_{1}^{R} - G_{1}^{I}) (m \cdot u_{0}^{N} (x_{1}^{-}) - u_{0}^{F} (x_{1}^{+}))$$

and

$$e I_{x}^{\text{NF}} = m [(G_{1}^{R} - G_{1}^{I}) (u_{0}^{N} (x_{1}^{-}) - u_{0}^{F} (x_{1}^{+})) - \frac{i}{2} (G_{1}^{R} + G_{1}^{I}) u_{0}^{N} (x_{1}^{+}) - \frac{i}{2} (2 \text{Re} G_{1}^{R} - G_{1}^{I} - G_{1}^{I}^{*}) m \cdot u_{0}^{N} (x_{1}^{-})] + \text{Re} G_{1}^{I} u_{0}^{N} (x_{1}^{-}) - \text{Im} G_{1}^{I} m \cdot u_{0}^{N} (x_{1}^{+}),$$

where $u_{0}^{F}(x_{1}^{+}) = (u_{0}^{F}(x_{1}^{+}) + u_{0}^{F}(x_{1}^{-}))/2$, the index $I$ refers to the contact and $x_{1}^{\pm}$ denotes the position in the immediate vicinity of the contact at the FM(NM) side. $G_{1}^{(\pm)}$ is the conductance of the FM/NM contact for the majority (minority) spin and the complex quantity $G_{1}^{(\pm)}$ is the mixing conductance describing the non-collinear transport [21]. In a metallic system, the imaginary part of $G_{1}^{(\pm)}$ is usually two orders less than the real part [29] and will be neglected in this work.

**Boundary conditions.** In the steady state, the charge accumulation across the FM/NM contact is invariant, which leads to the conservation of electrical current across the contact as

$$I_{0}^{N} (x_{1}^{-}) = I_{0}^{\text{NF}} (x_{1}^{+}).$$

The transverse spins injected into the FM are suppressed in the scale of spin decoherence length [24] and the component of spin accumulation collinear with the magnetization direction $m$ of the FM should keep invariant in the steady state, which gives the conservation of spin current collinear with magnetization across the contact as

$$m (m \cdot I_{x}^{\text{NF}} (x_{1}^{-})) = m (m \cdot I_{x}^{\text{NF}}) = I_{x}^{F} (x_{1}^{+}).$$

In the adiabatic approximation, the suppression of the non-collinear part of the spin current, in turn, results in the angular momentum to be transferred into the local magnetic moment in the FM. As a consequence, the STT on the ferromagnet generated by the spin current can be expressed as

$$\tau = -\frac{h}{2e} [I_{x}^{\text{NF}} - m (m \cdot I_{x}^{\text{NF}})].$$

STT could raise an additional term in the Landau–Lifshitz–Gilbert equation as $\partial \mathbf{m} / \partial t |_{\text{STT}} = -\kappa_{s} M_{s} \tau$, where $\gamma > 0$ is the gyromagnetic ratio, $M_{s}$ is the magnetization and $V$ is the volume of the ferromagnet.

### 2.2. The nonlocal AMR and STT

To consider the nonlocal AMR defined as $V(\theta)/I_{0}$, with the current $I_{0}$ in FM1 as input condition we need to know the voltage over the FM2/NM contact. By solving the diffusion equations with the boundary conditions, the spatial distribution of electrochemical potentials in the FM and NM resistors can be obtained. For the FM2 lead, the local electrochemical potential far from the FM2/NM contact ($x \to \infty$) gives the experimentally measured voltage across FM2/NM as $V = u^{F}(\infty)/(-e)$, where the zero potential is set at the NM side of FM2/NM. Then, the angular dependence of the nonlocal AMR can be obtained analytically as

$$R(\theta) = \left\{ 2R_{\text{NNe}^{L/Q_{F}}}(\cos \theta \sum_{l=1}^{2} (P_{l}^{1} \eta_{l} + \alpha_{l}^{F} \eta_{l}^{F})) \right. \times \left[ 1 - e^{2L/Q_{F}} \sum_{l=1}^{2} (2\eta_{l}^{I} + 2\eta_{l}^{F} + 1) + \sin^{2} \theta \left[ 1 - e^{2L/Q_{F}} \sum_{l=1}^{2} (2\eta_{l}^{I} + 1) \right] \right]$$

where the subindex $i = 1(2)$ denotes ferromagnetic lead FM1(FM2) and the corresponding contact FM1/NM (FM2/NM). We have introduced three dimensionless quantities, $\eta_{l}^{F} = R_{l}^{F}/[(1 - (P_{l}^{1})^{2})R_{N}]$ and $\rho^{F} = (2 \text{Re} G_{l}^{(1)} + \text{Im} G_{l}^{(1)})/R_{N}$, with interfacial resistance $R_{l}^{F} = (G_{l}^{(1)} + G_{l}^{(1)*})^{1}/R_{N}$, and $\rho^{F} = \rho_{l}^{N} / (\sigma^{N} \sigma^{F})$ and $\rho^{N} = \rho_{l}^{N} / (\sigma^{N} \sigma^{N})$ are the resistances in the FM and NM within the range of non-equilibrium spin-accumulation relaxation length. $P_{l}^{1} = (G_{l}^{(1)} - G_{l}^{(1)*})/(G_{l}^{(1)} + G_{l}^{(1)*})$ is the polarization across the contact. $\sigma_{F}^{F} = \sigma^{F} + \sigma_{F}^{F}$ and $\sigma_{F}^{N} = (\sigma^{N} - \sigma_{F}^{N})/\sigma^{N} (\sigma_{F}^{N} + \sigma_{F}^{F})$ are the conductivity and polarization in the ferromagnet, respectively. For the cases of $\theta = 0$ or $\theta = 180^\circ$, equation (11) reduces to the previous result [26] exactly.

The angular dependence of $R(\theta)$ is introduced by the cosine function on the numerator and the term containing $\sin^{2} \theta$ on the denominator. As will be illustrated in the next part, the cosine function gives the configuration symmetry between the two leads while the $\sin^{2} \theta$-related term describes the non-collinear transport across the FM/NM contact. If the FM/NM contact does not dominate the transport of the circuit, the $\sin^{2} \theta$-related term will not give an obvious effect on the angular dependence and $R(\theta)$ takes the form of the cosine function.

According to equation (11), the increase of interfacial polarization $P_{l}$ and ferromagnetic polarization $\sigma_{F}$ could increase AMR, as in this case the injected spin accumulation in the NM resistor could be enhanced. In the limit of heavy spin-flip scattering in the NM resistor, namely $I_{0}^{N} \to 0$, equation (11) gives the vanishing AMR, which is expected as the spin accumulation is completely consumed in the NM resistor.

The analytical result obtained in equation (11) is universal for the diffusive metallic systems without spin-flip scattering at contacts. For a special case with tunnelling contacts (e.g. with several oxidant metallic layers located at contact [6]), the transport properties of the system are dominated by the contact as $R_{l}^{1} \gg R_{N}^{2}(R_{F}^{2})$, which means $\eta_{l}^{F} \gg \eta_{l}^{F}$ in our formalism. Then, for AMR for tunnelling contact is found to be

$$R(\theta) = -\frac{1}{2} \frac{P_{l}^{1} P_{l}^{2} N e^{-L/Q_{F}} \cos \theta}{1 - \sin^{2} \theta [1 - e^{2L/Q_{F}} \sum_{l=1}^{2} (2\rho_{l}^{I} + 1)]^{-1}}.$$
For any type of contact, following equation (10), the STT exerted on FM2 is obtained formally as
\[
\tau = -\frac{\hbar}{2e^2} \text{Re} \, G_{1,1}^{G} \, m_2 \times u_l^{N}(x_l) \times m_2, \tag{13}
\]
where \( m_2 \) denotes the direction of magnetization in FM2 and \( u_l^{N}(x_l) \) is the spin accumulation at the NM side of FM2/NM. According to equation (13), the STT on FM2 is proportional to the spin accumulation, which restores the form of STT in the LSV [22]. The magnitude and direction of spin accumulation in the NM should be solved with the help of the boundary conditions both at the FM1/NM and FM2/NM contacts.

STT \( \tau \) given in equation (13) could be formally rewritten as [3]
\[
\tau = -\delta(\theta) I_0 (m_2 \times m_1 \times m_2), \tag{14}
\]
where \( I_0 \) is the electron current and \( \delta(\theta) \) yields an effective spin torque, which directly scales the critical current of magnetization switching and switching time in dynamics [27]. The analytic expression for \( \delta(\theta) \) is
\[
\delta(\theta) = \frac{2}{e^2} \text{Re} \, G_{1,1}^{G} \, \frac{R(\theta)}{\cos(\theta)} \tag{15}
\]
where the angular-independent coefficient \( T = 2\rho_1^2 \Phi/\Omega \), where
\[
\Phi = 1 + e^{2L/\alpha_l} \sum_{i=1}^{2} (2\rho_i^l + 1)(2n_i^F + 1) - e^{2L/\alpha_l} \sum_{i \neq j}^{2} (2\rho_i^l + 1)(2n_j^F + 1) \tag{16}
\]
and
\[
\Omega = -\left[ 1 - e^{2L/\alpha_l} \sum_{i=1}^{2} (2\rho_i^l + 1) \right] \times \left( \Phi_1^{N1} + \alpha_1^{N1} \right) \times \left[ 1 -(2\rho_1^l + 1)(2n_1^F + 1)e^{2L/\alpha_l} \right]. \tag{17}
\]
As we can see, for \( \delta(\theta) \), the angular dependence comes only from the term \( R(\theta)/\cos(\theta) \). As the numerator of \( R(\theta) \) also has a term in \( \cos(\theta) \), the angular dependence of \( \delta(\theta) \) is only determined by \( \sin^2 \theta \). Obviously, \( \delta(0^\circ) \) exactly equals \( \delta(180^\circ) \), which is quite different from that in the conventional FM spin valve. Such symmetry comes from the fact that no electric current flows in the FM2 lead. Detailed analysis will be given in the next section.

3. Numerical results and discussion

The AMR of NLSV could be directly measured in experiments and be used to test our theoretical prediction. For the non-collinear NLSV we considered, the permalloy is taken as the ferromagnetic leads while the copper is the normal lead. The material parameters entering our formalism adapt the values extracted from the experiments [28]. The parameter \( G_{1,1}^{G} \) for the Py/Cu contact follows that in [19]. For tunnelling contact, according to \textit{ab initio} calculation [29] the contact resistance could be taken 11 times that of the metallic contact for a thick barrier and the mixing conductance \( G_{1,1}^{G} \) is almost unchanged. The contact area of FM2/NM is assumed to be constant with variation of \( \theta \). The two ferromagnetic leads are also assumed to be identical.

\textbf{AMR in NLSV.} The AMR with different distance \( L \) between FM1 and FM2 is shown in figure 2(a) for metallic contact. It is found that the absolute value of \( R(\theta) \) decreases with increase of \( L \). This is due to the fact that the spin-flip scattering could kill the spin memory in normal metal. With increasing \( L \), the spin accumulation at the NM side of the FM2/NM contact decreases. For a metallic system, the dimensionless parameters \( \eta^l, \eta^F \) and \( \rho^l \) are always less than unity. Therefore, the third terms in the denominator in equation (11) can be neglected compared with the other two terms. As a consequence, AMR is governed by the numerator of equation (11), and gives a cosine lineshape of \( R(\theta) \), which was discussed by Kimura \textit{et al} recently [30].

Figure 2(b) presents the AMR with tunnelling contact, where \( L = 200 \) nm. Because the spin transport is dominated by contact, the lineshape shows very different from that with a metallic contact. Such variation in the lineshape of AMR implies that \( R(\theta) \) decreases more quickly in tunnelling contact when FM1 and FM2 are in a non-collinear configuration. The reason is that the non-collinear spin accumulation in the NM resistor could leak out more efficiently with tunnelling contact comparing with metallic contact. It is known that the drift of the non-collinear spin accumulation across the FM/NM contact decreases. For a metallic system, the dimensionless parameters \( \eta^l, \eta^F \) and \( \rho^l \) are always less than unity. Therefore, the third terms in the denominator in equation (11) can be neglected compared with the other two terms. As a consequence, AMR is governed by the numerator of equation (11), and gives a cosine lineshape of \( R(\theta) \), which was discussed by Kimura \textit{et al} recently [30].

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The magnitude reaches its minima at θ = 90°, which is the magnetization of FM2 is reversed. The induced voltage across FM2/NM will not change when the contact is perpendicular to the magnetization of FM2. The spin of electrons arriving at the NM side of the FM2/NM will be polarized along the direction of FM2/NM is vanishing. After the injection from FM1, the electrons will be polarized along m↑ at first. For θ = 90°, the spin of electrons arriving at the NM side of the FM2/NM contact is perpendicular to the magnetization of FM2. The induced voltage across FM2/NM will not change when the magnetization of FM2 is reversed.

**Spin Accumulation in NLSV.** As the Im G↑↓-related term in equation (7) is disregarded in the metallic system, the spin accumulation u↑N(x↑1) at the NM side of FM2/NM is in the plane spanned by m↑ and m↓. Figure 3(a) presents the dependence of the relative angle α between the direction of u↑N(x↑1) and m↑. As we discussed above, when θ = 90°, FM2/NM is equivalent to an unpolarized contact and α = 90° is expected. Figure 3(b) gives the dependence of the magnitude of u↑N(x↑1) normalized by the injected current I0. The magnitude reaches its minima at θ = 90° while gives an identical value for parallel (θ = 0°) and antiparallel (θ = 180°) configurations, which is quite different from that in the LSV. Such a discrepancy can be identified through the equivalent circuit of LSV and NLSV as shown in figure 4, where the circuits follow the collinear magnetic configuration of LSV and NLSV with r↑F1 = l↑F / (σ↑F(x↑1) S↑F) + 1 / G↑↓(x↑1) and r↑N = 2G↑↑(x↑1) / (σ↑↑N S↑↑).

For both types of spin valve, the spin accumulation in the normal metal equals the potential difference between node 1 and node 2 (see figure 4). In the LSV, as the particle current flows from FM1 to FM2, the switching of magnetization of FM2 will interchange the resistors r↑F2 and r↑F1, which could change the potential on node 1 and node 2. However, in NLSV, the electrical current I0 flows from electrode A to electrode B and no net electrical current flows to the detection lead FM2, namely, electrode C. Only spin current, which is denoted as I↑F, flows to F2. It is obvious that the interchange of r↑F1 and r↑F2 in figure 4(b) does not affect the current I↑F. As a result, the potential difference between nodes 1 and 2 will not be changed. In the non-collinear magnetic configuration of NLSV, the equivalent circuit in figure 4(b) is not valid anymore. Due to non-collinear transport, more channels will be opened [21] and new resistors could directly connect node 1 and node 2. The potential difference will be changed with variation of the direction of FM2.

**STT in NLSV.** The spin accumulation near the FM2/NM contact could induce an STT on the detection lead FM2. For the metallic and tunnelling contacts, we have calculated δ(θ) and presented the results in figure 5. Even though τ is always zero when two magnetizations are aligned collinearly, δ(0°) and δ(180°) show nonzero values as in the LSV. For a typical space of L = 200 nm, the STT obtained is smaller than that in the LSV [16], but still of the same order of magnitude. The spin-flip scattering in the NM could suppress STT as the space L increases. The space L dependence of δ(0°) is shown in the inset of figure 5(a). As we can see, sizeable STT could be expected even in NLSV with L comparable to or less than l↑N, which is 700 nm in this study. The δ in the NLSV with tunnelling contacts could be even larger because, in this case, the contact dominates the spin transport and with higher spin injection efficiency [6] the spin accumulation in the normal electrode is essentially enhanced per unit current.

Interestingly, the spin torques in NLSV still change their signs when the injected current is reversed. As the electrical (electron) current I0 injected from electrodes A to B as shown in figure 4(b), for the materials we discussed (r↑F < r↑F),
spin accumulation parallel to \( \mathbf{m}_1 \) could be built up in the NM, which will exert STT on FM2. When we reverse the current \( I_0 \), electrons come from electrode B to A and the spin-dependent reflection at the FM1/NM contact will build up a spin accumulation antiparallel to \( \mathbf{m}_1 \) in the NM. So the STT changes its sign.

Contrasting to the STT in the LSV [31], \( \delta(\theta) \) is symmetrical for the parallel and antiparallel magnetic configuration of FM1 and FM2. See equation (13); the symmetry comes from the symmetrical angular dependence of \( \mathbf{u}_0^s(x_L^s) \) shown in figure 3(b). This implies that the critical current should be identical for parallel to antiparallel and antiparallel to parallel.

Switching behaviour in the NLSV has been observed by Kimura et al [20], even though they only observed antiparallel to parallel switching, where the NM lead in figure 1 is replaced by an NM cross and the FM leads are placed on two opposite arms of the cross. The spin accumulation could leak from those arms not in contact with ferromagnetic leads. Therefore, the magnitude of STT could be 2–3 times weaker than that in NLSV discussed here.

4. Summary

Based on the diffusion equation and magnetoelectronic circuit theory, the non-collinear spin transport in NLSV is treated analytically and numerically in the diffusive regime. The analytical expression of AMR defined in NLSV is derived. For the system with metallic contacts, the AMR gives a cosine function like angular dependence. For the system with tunnelling contacts, the AMR shows complicated angular dependence and could be used to extract mixing conductance from experiments. The STT in NLSV has the same order of magnitude as that in the LSV but shows a qualitative difference in the angular dependence. The STT in NLSV is found to be symmetrical when the two FM leads are parallel and antiparallel to each other. The symmetry comes from the fact only spin current flows across the detection lead. Our study implies that the critical current of magnetization switching in NLSV could be identical for parallel configuration and antiparallel configuration.

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Figure 5. The angular dependence of spin torques on FM2 of NLSV with \( L = 200 \) nm, (a) for the metallic contact and (b) for the tunnelling contact. The inset of (a) gives the space \( L \) dependence of \( \delta(0^\circ) \).
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