Realization of power law inflation & variants via variation of the strong coupling constant

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Abstract. We present a model of power law inflation generated by variation of the strong coupling constant. We then extend the model to two varying coupling constants which leads to a potential consisting of a linear combination of exponential terms. Some variants of the latter may be self-consistent and can accommodate the experimental data of the Planck 2015 and other recent experiments.

Keywords: inflation, alternatives to inflation, particle physics - cosmology connection

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1 Introduction

The new data from the experiment Planck 2015 [1, 2] combined with the new BICEP/Keck and Planck analysis (BKP) [3] make the contours of the combined Planck 2013+BICEP2+WP+highL [4] completely obsolete, and put quite stringent constraints on any inflationary model. In particular, regarding the tensor-to-scalar ratio $r$ and the spectral index $n_s$ parameters, they place an upper bound on the tensor amplitude which is inconsistent with the combined 2013 contours. Moreover, combining the planck 2015 + BKP with the Baryonic Acoustic Oscillations measurements (BAO) [5] provides much stronger constraints than Planck 2015 alone, and shows the constraints $n_s = 0.968 \pm 0.006$ and $r < 0.11$.

The data of 2013 and 2015 are summarized in figure 1 where the confidence contour levels, sourced from the existing ones in [4] and [5] respectively, are shown for the Planck experiment as well as the combined analyses. Regardless of whether the BICEP2 results point to a signal of type B coming from gravitational waves originating from inflation or the signal is rather due to some dust effect [6], we see that the BICEP results are consistent with Planck in a considerable region of parameter space. Consequently, we shall test our model of inflation in how it accommodates the Planck 2015 and the combined experiments.

Inflation [7] is the commonly accepted theory which solves many of the Big Bang scenario problems, mainly the horizon and flatness problems. All inflationary models introduce a scalar field, the inflaton, responsible for the inflation, whose nature is still unknown. A vast array of models were studied [8], and they differ in the details, but most of which attribute a matter content to the inflaton. Varying speed of light (VSL), [9] was an alternative for solving the Big Bang scenario problems. VSL is however an integral part of “variation of constants” ideas [10, 11]. Experimental data preclude any temporal variation of the electric charge [12], the strong coupling [13] and the Higgs vev [14] going back in time till nucleosynthesis. However, no data exist to preclude variation of constants in the inflationary era. In [15], a link was suggested between variation of constants and inflation, in that it attributed the nature of the inflaton responsible for inflation to a time variation of the strong coupling constant, and the dominant contribution to energy was ascribed to quantum trace anomaly effect.
In this letter, we re-address this link to find an exponential inflaton potential, leading to power law inflation \[16, 17\]. Interest in power law inflation has resurfaced recently in the light of the new experiments results \[18\]. We invoke the trace anomaly just to give a rough estimate of the “thermal” gluon condensate, and contrast the predictions of the model with the experimental data.

Being a power law inflation, the model is clearly inconsistent with the Planck 2015 & joined experiments data because such an inflation overproduces tensor modes. Moreover, the model has a parameter $\ell$ with dimension of length, and in order to be even remotely consistent with data (say, with Planck 2013 as shown in figure 1), this length scale, as we shall see, must be smaller than the Planck length $L_{PL}$.\footnote{In this work, we omit the attribute “reduced” for the Planck length (mass). The condition $\ell > L_{PL} = \sqrt{8\pi G}$, (where $G$ is the gravitational constant) is thus stricter than the version expressed in terms of the “standard” Planck length: $\ell > \sqrt{G}$.}

This is related to the well-known “Lyth bound”, which shows that the field variation in inflation must be large in Planck units for models which predict a tensor/scalar ratio $r$ of order 0.1. Although values for $\ell$ shorter than $L_{PL}$ can be envisaged in some string models \[19\], and despite the fact that the inability of measuring a sub-Planckian length does not exclude the possibility of its existence, however the small ratio of $\ell/L_{PL}$ indicates that the model needs to be supplemented by other mechanisms in order to be self-consistent and to produce inflation with the required properties.

To reconcile the power law inflation with data, one can define a modified model where coupling to gravity is non-minimal \[20\]. Here, we do not follow this path, but would rather assume a second gauge group with, again, a varying coupling constant. The potential becomes now a linear combination of two exponential terms. The second varying coupling constant can be considered independent and representing a free parameter, which leads to a multi-field inflation scenario, or can be assumed related to the variation of the strong coupling constant in such a way to suggest a form for an effective single field potential. In both cases, one can consider variants of the power law model and study whether or not the new forms can accommodate the recent data and allow for above-Planckian length acceptable regions.

## 2 The model

Our starting point is the “time varying” QCD lagrangian stated in \[13\] generalizing the work of Bekenstein \[12\] from QED to QCD:

\[
L = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi^\dagger \phi) + \frac{1}{2f^2} \epsilon_{\mu\nu} \epsilon^{\mu\nu}
\]

where $\epsilon$ is a scalar gauge invariant and dimensionless field introduced to express the time variation of the strong coupling constant $g(x) = g_0 \epsilon(x)$, and whose dynamics is represented by the last term of eq. (2.1) where $\ell$ is the Bekenstein scale length and we are using the signature $(+, -, -, -)$, $a = 1, \ldots, 8$ runs over the gauge group generators ($t^a$) indices, with $[t^b, t^c] = if^{abc}t^a$, the covariant derivative is defined as $D_\mu = \partial_\mu - ig_0 \epsilon(x)A_\mu$, and the gluon tensor field is given by

\[
G^a_{\mu\nu} = \frac{1}{\epsilon} \left[ \partial_\mu (\epsilon A^a_\nu) - \partial_\nu (\epsilon A^a_\mu) + g_0 \epsilon^2 f^{abc} A^b_\mu A^c_\nu \right]
\]

We first introduce a transformation which simplifies the formulation. By re-defining the gauge potential and the field tensor as

\[
\tilde{A}^{a\mu} = \epsilon A^{a\mu}, \quad \tilde{G}^{a\mu\nu} = \epsilon G^{a\mu\nu}
\]
we find that the explicit dependence on the field $\epsilon$ disappears in the definition of the new gluon field $\tilde{G}^{\mu \nu} \equiv \tilde{G}^{a \mu \nu} t^a$ in terms of the new gauge potential $\tilde{A}^{\mu} = \tilde{A}^{\mu}_a t^a$, as

$$\tilde{G}^{\mu \nu} = \partial \tilde{A}^\nu - \partial \tilde{A}^\mu - ig_0 [\tilde{A}^{\mu}, \tilde{A}^{\nu}]$$

(2.4)

The QCD-lagrangian can be written as:

$$L = -\frac{1}{4\epsilon^2} \tilde{G}^{\mu \nu} \tilde{G}^{\alpha}_{\mu \nu} + (D_\mu \phi)^\dagger (D^\mu \phi) - V (\phi^\dagger \phi) + \frac{1}{2\ell^2} \epsilon_{\mu \nu} \epsilon^{\mu \nu}$$

(2.5)

in which the covariant derivative is given now by $D_\mu = \partial_\mu - ig_0 \tilde{A}_\mu$. Thus, in addition to the “kinetic” last term of eq. (2.5), the dependence on the $\epsilon$-field appears only in the gauge kinetic term divided by $\epsilon^2$. The equation of motion remains the same in that any zero variation with respect to $[\phi, A_\mu, \epsilon]$ is equivalent to a zero variation with respect to $[\phi, \tilde{A}_\mu, \epsilon]$.

We will neglect the fermionic matter contribution during the inflationary era, whereas a “thermal” average $\langle \tilde{G}^2 \rangle_T$, called henceforth “thermal” gluon condensate, should be taken for the gluon strength field squared in what concerns pure-gauge QCD at temperature $T$ characteristic of inflation. We get

$$L = + \frac{1}{2\ell^2} \epsilon_{\mu \nu} \epsilon^{\mu \nu} - \frac{1}{4\epsilon^2} \langle \tilde{G}^2 \rangle_T$$

(2.6)

We shall assume the condensate $\langle \tilde{G}^2 \rangle_T$ value is approximately constant during the inflationary short period in contrast to [15] where the constancy was assumed to hold approximately for the condensate $\langle G^2 \rangle_T$. 

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**Figure 1.** 1-$\sigma$ and 2-$\sigma$ Contour levels, for $n_s$ versus $r$, of Planck 2013 & 2015 experiments and their Combined analyses with other experiments.
The basic relation between the “thermal” gluon condensate $\langle G^2 \rangle_T$ and the zero-temperature gluon condensate $\langle G^2 \rangle_0$, which corresponds to a vacuum expectation value or a ground state average (estimated by QCD sum rules method \[21\] to give the renormalization-independent quantity $\frac{2\pi}{\alpha_S} \langle G^2 \rangle_0 = 0.012 \pm 0.004 \text{GeV}^4$), can be stated as follows \[22\]:

$$\langle G^2 \rangle_T = \langle G^2 \rangle_0 - \langle \rho_g - 3P_g \rangle_T$$  \hspace{1cm} (2.7)

where the temperature-dependent trace anomaly part $\langle \rho_g - 3P_g \rangle_T$, with $\rho_g$ ($P_g$) denotes the gluon energy density (pressure), is normalized such that it vanishes at zero temperature. For $T \gtrsim T_c$, where $T_c$ is QCD phase transition critical de-condensation temperature (which is of the same order of the chiral symmetry breaking $\sim 200 \text{MeV}$, had we introduced quarks), one can get by perturbation theory for pure gluodynamics the following \[23\]:

$$\frac{\langle \rho_g - 3P_g \rangle_T}{T^4} = 4N'b_0 \frac{N'^2 - 1}{288} g^4(T)$$  \hspace{1cm} (2.8)

where for QCD with vanishing number of quark flavors we have $N' = 3, b_0 = \frac{11N'}{1488\pi^2}$. This perturbative formula fits well with the lattice data available in the range up to $5T_c$, and explains why the “thermal” gluon condensate becomes negative and proportional to $T^4$ as the zero-temperature contribution becomes negligible for $T > T_c$. The ideal non-interacting gas model, which one might suspect to appear at $T > T_c$ because of “asymptotic freedom” in QCD, means zero condensates at high energies, and the negative value of the condensate means that upon raising the temperature through $T_c$, not only do the gluons coming from the interactions in the vacuum condensate de-condense, but the further gluons which are created at the high temperatures also take part in the de-condensation process contributing to the energy momentum trace \[24\].

However, the way the gluon plasma approaches the ideal non-interacting gluon gas in the limit of very high temperatures, which restores the conformal symmetry and causes the trace anomaly to vanish, is not clear. The lattice data are lacking for a full investigation in this region to which, presumably, inflation belongs.\footnote{The technique of hard thermal loop (HTL) perturbation theory \[25\] allows also to estimate the trace anomaly. It does not agree with lattice data in the region just above $T_c$. However, the study goes up to regions of order $10^3 T_c$ but still far short than inflation temperatures.} We expect (look at eq. (2.7)) the gluon condensate to reacquire a positive value at very high temperatures. The non-perturbative “power corrections”, which are beyond the scope of the radiative corrections accounted for in perturbation theory, may play an important role with effects in the high temperature region. In \[26\], a detailed analysis of the thermal power corrections and the trace anomaly in the deconfined region was carried out. A best fit for the lattice data in the region $1.13T_c < T < 5T_c$ was given as

$$\frac{\langle \rho_g - 3P_g \rangle_T}{T^4} = a_\Delta + b_\Delta \left( \frac{T_c}{T} \right)^2$$  \hspace{1cm} (2.9)

with $a_\Delta = -0.04$ and $b_\Delta = 3.99$. We shall extrapolate this fitting formula up to “inflationary” very high temperatures, and get

$$\langle \tilde{G}^2 \rangle_T \approx -a_\Delta e^2(T)T^4$$  \hspace{1cm} (2.10)

In all, the gluon field thermal averaging leads to an “effective” potential for the $\epsilon$-field, and our concern is to see whether such a potential leads to inflation or not. The positive
value of the “thermal” gluon condensate is crucial in our analysis, otherwise we shall get an instability corresponding to a potential not bounded from below. Moreover, we need to put the kinetic energy term of the $\epsilon$-field in its “canonical” form, and so we define a new field $\chi$ such that:

$$\frac{1}{2} \chi_{\mu} \chi^\mu = \frac{1}{2} \frac{\epsilon_{\mu} \epsilon^\mu}{\epsilon^2} \Rightarrow \epsilon = \epsilon^{\chi}$$

(2.11)

In terms of the new field $\chi$, the lagrangian becomes

$$L = \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{4} \langle \tilde{G}^2 \rangle T e^{-2\ell \chi}$$

(2.12)

We see directly here that our lagrangian can generate a power law inflation with an exponential potential

$$V(\chi) = M_4^4 e^{-\beta \chi} M_{\text{PL}}, \quad M_4^4 = \frac{1}{4} \langle \tilde{G}^2 \rangle_T, \beta = \frac{2\ell}{L_{\text{PL}}}$$

(2.13)

where $M_{\text{PL}}$ is the Planck mass: $M_{\text{PL}} = \frac{1}{L_{\text{PL}}} \sim 10^{18}$GeV.

The physics behind this toy model of a varying QCD coupling leading to inflation is simple, in that we assumed a scalar field $\epsilon$, determining the value of the gauge coupling, which might have assumed a value in the early universe different from its present value, and so the QCD scale is different from today. There is a contribution to the potential of $\epsilon$ going, on dimensional grounds, as $\Lambda_{QCD}^4$, which can generate inflation provided slow roll conditions are satisfied.

3 Analysis of the model

We summarize now the well known results of the power law inflation model (look, say, at [8] and references therein). In fact, for the power law inflation, one can find the exact solutions to the full following equations of motion ($a$ is the scale factor):

$$H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{3M_{\text{PL}}^2} \left[ \chi^2/2 + V(\chi) \right], \quad \ddot{\chi} + 3H \dot{\chi} + \partial_\chi V = 0$$

(3.1)

in the form of:

$$H(t) = H(t_e) \left( \frac{a(t_e)}{a(t)} \right)^{\beta^2/2}, \quad a(t) = a_e \left( \frac{t}{t_e} \right)^{2/\beta^2}$$

(3.2)

where the subscript “e” denotes the end of inflation. We get inflation when $\ddot{a} > 0$ providing $\beta \leq \sqrt{2}$ which means $\frac{t}{L_{\text{PL}}} \leq 1/\sqrt{2}$, so the model does not allow for an above-Planckian length scale $\ell$.

The $\epsilon$-folding number satisfies then $N - N_e = \frac{2}{\beta^2} \log \frac{t}{t_e}$, whereas the time evolution of the “inflaton” field $\chi$ reads:

$$\chi(t) = \chi_e + M_{\text{PL}}^4 \beta \log \frac{t}{t_e}$$

(3.3)

We see that the field rolls down the potential from lower values to its final value $\chi_e$. The inflation duration satisfies

$$t_e^2 = \frac{2 \left( 6/\beta^2 - 1 \right) M_{\text{PL}}^2 \beta^2}{M_4^4 e^{-\beta \chi_e} M_{\text{PL}}}$$

(3.4)

while the equation of state would read

$$\frac{P}{\rho} = -1 + \frac{\beta^2}{3}$$

(3.5)
where \( P(\rho) \) denote the pressure (energy density) of the universe. We see that the limit \( \beta = 0 \) (corresponding to \( \ell \to 0 \)) leads to \( P = -\rho \) where the scale factor \( a \) inflates exponentially.

The slow-roll parameters are defined as

\[
\varepsilon = \frac{M_{\text{pl}}^2}{2} \left( \frac{V_\chi}{V} \right)^2, \quad \eta = M_{\text{pl}}^2 \frac{V_{\chi\chi}}{V}
\]

where \( V_\chi = \frac{dV(\chi)}{d\chi} \) and \( V_{\chi\chi} = \frac{d^2V(\chi)}{d\chi^2} \).

The tensor to scalar ratio and the spectral index are given by

\[
r \approx 16\varepsilon, \quad n_s \approx 1 - 6\varepsilon + 2\eta
\]

In our power law inflation, we get

\[
r \approx 32 \left( \frac{\ell}{L_{\text{pl}}} \right)^2, \quad n_s = 1 - 4 \left( \frac{\ell}{L_{\text{pl}}} \right)^2
\]

Because the slow-roll parameters are constant during inflation, then the predictions of the model do not depend on the energy scale at which the power law inflation ends. We plot in figure 1, the line relating \( n_s \) and \( r \) for the choice \( \beta \in [0, 0.2] \), and, as mentioned before, our power law inflationary model can not accommodate recent data even for a sub-Planckian length scale \( \ell \).

Now, the overall amplitude of the CMB anisotropies leads to an estimation of the inflation duration in that we require [8]

\[
\frac{V_e^{1/4}}{M_{\text{pl}}} \sim 10^{-4} \Rightarrow M_{\text{pl}}^4 e^{-\beta \chi_e/M_{\text{pl}}} \sim 10^{-16} M_{\text{pl}}^4
\]

which via eq. (3.4) and for \( \beta \sim 0.15 \) gives an acceptable \( t_e \) of order \( 10^{-33} \)s well earlier than the EW breaking time of \( 10^{-12} \)s.

Although the observational predictions of the model cannot depend on \( \chi_e \) which would be an irrelevant parameter called in just to assume an exit scenario where a tachyonic instability is triggered, as the inflation cannot stop by slow-roll violation, however one can take by convention \( \epsilon_e = 1 \) (corresponding to \( \chi_e = 0 \)), corresponding to strong coupling constant taking its present energy dependent value, that was reached during the inflation starting from far smaller values, and so we have \( e^{-\beta \chi_e/M_{\text{pl}}} \sim 1 \). This helps to give a rough estimate for the reheating temperature at the end of inflation as \( 10^{-4} M_{\text{pl}} \sim M \sim (\langle \tilde{G}^2 \rangle/4)^{1/4} \) leading to (where we assume the “constant” gluon condensate still satisfies eq. (2.10) after reheating, with \( \epsilon(T_r) \approx \epsilon_e = 1 \)):

\[
3.1 \times 10^{-4} M_{\text{pl}} \sim T_r
\]

so that \( T_r \sim 10^{14} (GeV) \).

If we increase the value of the only free parameter \( \ell \), then we see that the predictions become worse concerning the data. It was shown in [27] that the possibility of having several fields can support inflation even if \( \beta \geq \sqrt{2} \) in eq. (3.2), which would mean in our case varying various gauge couplings. This motivates us to investigate in the next section variants of the model with two varying coupling constants.
4 Variants of the model: two varying coupling constants

We assume that we have two groups $G_1 = SU(3)_c$ and $G_2$ with varying coupling constants $g_1$ and $g_2$ expressed through two length scales $\ell_1$ and $\ell_2$ and two fields $\epsilon_1$ and $\epsilon_2$. Restricting to pure gauge, we have a generalization of eq. (2.6):

$$L = \sum_{k=1}^{k=2} \left( \frac{1}{2\ell_k^2} \epsilon_k \epsilon_k^\mu \epsilon_k^\nu + \frac{1}{4\epsilon_k^2} (\tilde{G}_k^2) T \right)$$

(4.1)

Again defining:

$$\chi_k = \frac{\ln \epsilon_k}{\ell_k} \Rightarrow \epsilon_k = e^{\ell_k \chi_k}$$

(4.2)

we get a generalization of eq. (2.12):

$$L = \sum_{k=1}^{k=2} \left( \frac{1}{2} \partial_\mu \chi_k \partial^\mu \chi_k - \frac{1}{4} (\tilde{G}_k^2) T e^{-2\ell_k \chi_k} \right)$$

(4.3)

and so the potential is expressed as:

$$V(\chi_1, \chi_2) = M^4 \left( e^{-2\ell_1 \chi_1} + \mu e^{-2\ell_2 \chi_2} \right)$$

(4.4)

where $\mu = \frac{\langle \tilde{G}_2^2 \rangle_T}{\langle \tilde{G}_1^2 \rangle_T}$ and $M^4 = \frac{1}{4} \langle \tilde{G}_1^2 \rangle_T$.

4.1 Multi-field inflation

In order to study inflation in this double field setting, we need to introduce the corresponding slow roll parameters as follows [29].

$$\varepsilon_k = \frac{1}{2} \left( \frac{V_{\chi_k}}{V} \right)^2, \quad \eta_{kj} = \frac{V_{\chi_k \chi_j}}{V}$$

$$\varepsilon = \varepsilon_1 + \varepsilon_2, \quad \tan \theta = \frac{\varepsilon_2}{\varepsilon_1}$$

$$\eta_{\sigma \sigma} = \eta_{11} \cos^2 \theta + 2 \eta_{12} \sin \theta \cos \theta + \eta_{22} \sin^2 \theta$$

$$\eta_{\alpha \sigma} = (\eta_{22} - \eta_{11}) \sin \theta \cos \theta + \eta_{12} \left( \cos^2 \theta - \sin^2 \theta \right)$$

$$\eta_{ss} = \eta_{11} \sin^2 \theta - 2 \eta_{12} \sin \theta \cos \theta + \eta_{22} \cos^2 \theta$$

(4.5)

Then, for the slow roll regime, where all the $\varepsilon_k$ and $\eta_{kj}$'s are small, we have approximately the following for the tensor to scalar ratio and the spectral index:

$$r \approx 16\varepsilon, \quad n_s \approx 1 - 6\varepsilon + 2\eta_{\sigma \sigma}$$

(4.6)

We shall also consider the possibility of a term representing the coupling between the two fields, and so we get the potential in the form

$$W(\chi_1, \chi_2) = V(\chi_1, \chi_2) + \lambda \chi_1 \chi_2$$

(4.7)

where $\lambda$ is a positive coupling when $\mu > 0$ in order to keep the potential bounded from below.
4.2 Single field inflation

Alternatively, and provided one knows the actual trajectory of the compound field $\chi = (\chi_1, \chi_2)$, one can classically use it to get an approximative effective single field inflation. We illustrate this in two cases where we expect the path to pass through the origin $\chi_1 = \chi_2 = 0$ corresponding to $\epsilon_1 = \epsilon_2 = 1$ which, conventionally, means an end to the time variation of the coupling constants (i.e. the coupling constant settles to its current “energy-dependent” law).

4.2.1 $\mu > 0$

Locally, one can approximate the trajectory of $\chi$ around the origin by a straightline, and during the slow-regime we have the equations of motion

$$H^2 \propto V, \quad 3H \dot{\chi} + \nabla V \approx 0 \quad (4.8)$$

which leads to

$$\chi_2 = \alpha \chi_1, \quad \alpha = \frac{\ell_2 \mu}{\ell_1} \quad (4.9)$$

Replacing this in the Lagrangian, and defining a canonical field $\psi$ such that

$$\frac{1}{2} \partial_1^2 + \frac{1}{2} \partial_2^2 = \frac{1}{2} \partial \psi^2 \quad (4.10)$$

we find

$$\psi = \sqrt{1 + \alpha^2 \chi_1} \quad (4.11)$$

Thus, the potential is given as:

$$W(\psi) = M^4 \left( e^{-2\ell_1 \psi/\sqrt{1+\alpha^2}} + \mu e^{-2\ell_2 \psi/\sqrt{1+\alpha^2}} \right) \quad (4.12)$$

Again, as the vanishing minimum is attained at infinite values of the field, an unknown parameter $\psi_v$ should interfere to mark the appearance of new physics ending the inflation (around 0).

4.2.2 $\mu < 0$

We do not have experimental information about the condensate of $G_2$, so we can treat $\mu$ as a free real parameter. We see that $\mu$ needs to be negative in order to get a trajectory shape with a local minimum. However, the potential with $\mu < 0$ is unstable as it is unbounded from below. We seek, in order to simplify the study and get a single effective field inflation, a simple path which could mimic the slow roll regime of the inflation as long as possible before the advent of the tachyonic instability, which would call for a new mechanism for an exit scenario. Were we to assume $\epsilon_1 = \epsilon_2$, then we would reobtain the power law with a new length scale $\frac{1}{\ell_1} = \frac{1}{\ell_1} + \frac{1}{\ell_2}$ and a new condensate $\langle \tilde{G}^2 \rangle_T = \langle \tilde{G}_1^2 \rangle_T + \langle \tilde{G}_2^2 \rangle_T$. Rather, if we assume that the fields $\epsilon_1, \epsilon_2$ vary in a way that gives rise to a single canonical field $\chi_1 = \chi_2 = \chi$, which means:

$$\epsilon_1^{L_{\text{pl}}/\ell_1} = \epsilon_2^{L_{\text{pl}}/\ell_2} \Rightarrow \chi_1 = \chi_2 = \chi, \quad (4.13)$$

then we see in the right side of figure 2 that this path ($\chi_1 = \chi_2$) has a plateau followed by a minimum, resembling standard single field inflation potentials. Defining now a new canonical field

$$\psi = \sqrt{2} \chi \quad (4.14)$$
we end up
\[ L = \frac{1}{2} \partial_{\mu} \psi \partial^{\mu} \psi - M^4 e^{-\sqrt{2} \ell_1 \psi} - M^4 \mu e^{-\sqrt{2} \ell_2 \psi} \]  
(4.15)

and so the potential (multiplied by \(-1\)) is the sum of the second and third terms above. We impose now an end to the reheating period, with the field \(\psi\) at the minimum, corresponding to \(\epsilon_1 = \epsilon_2 = 1\), and so the minimum should happen at \(\psi = 0\). This leads to \(\mu = -\frac{\ell_1}{\ell_2}\) and \(V(\psi = 0) = 1 - \frac{\ell_1}{\ell_2} < 0\) with \(\ell_2 < \ell_1\) in order to have a minimum. Moreover, it is true that the potential can not be defined up to an additive constan t, as everything is coupled to gravity through Einstein’s equations, however the negative minimal value of the potential corresponds to an anti-de-Sitter universe. Within the mult i field “spirit” we imagine a static third field uplifting the universe to become of Minkowski type which agrees with observations. Hence, we get finally the potential in the form:
\[ V(\psi) = M^4 \left[ e^{-\sqrt{2} \ell_1 \psi} \frac{\ell_1}{\ell_2} e^{-\sqrt{2} \ell_2 \psi} + \left( \frac{\ell_1}{\ell_2} - 1 \right) \right] \]  
(4.16)

This form puts the following constraint on the condensates:
\[ \frac{\langle \tilde{G}_2^2 \rangle_T}{\langle \tilde{G}_1^2 \rangle_T} = -\frac{\ell_1}{\ell_2} \]  
(4.17)

We do not justify here the assumptions of eqs. (4.13) and (4.17), but rather take them at face value for the sake of providing a modification of the power law model making it acceptable in a way not meant to be unique. In fact, the path \(\chi_1 = \chi_2\) is not stable for \(V(\chi_1, \chi_2)\) as the right side of figure 2 shows. However, we have simulated two trajectories (blue curves in the 3D right side of figure 2), starting from a point satisfying \(\chi_1 = \chi_2\) at the plateau,\(^4\) one without an initial velocity showing clearly that the path can not be an actual trajectory, whereas the second, with a suitably chosen initial velocity, shows that the trajectory follows

\(^4\)This can be arranged, with a suitable matter content, via running of \(g_1\) and \(g_2\) from a common value satisfying \(\epsilon_1 = \epsilon_2\) at unification scale, which is higher than the inflation starting scale, to the desired initial value.
the flat path long enough before falling to the instability. One needs, however, for such trajectories to verify not only the “potential” slow roll conditions, but also the conditions on the “dynamic” Hubble slow roll parameters, which might require fine tuning of the initial velocity in order to assure that the path along the plateau, representing the slow roll regime, is followed long enough before the end of inflation or before the field $\chi$ departing from it. In all, one can thus argue that the potential $V(\psi)$ of eq. (4.16) may reflect to a large extent the essence of the slow roll regime, and we shall consider it our starting point with no further justification.

We can compute now the slow-roll parameters $\varepsilon, \eta$ using eq. (3.6) with $V$ given by eq. (4.16) and replacing $\chi$ by $\psi$. The canonical field $\psi$ starts at the instant $t_i$ in a large value $\psi_i$ and slowly rolls down the potential till it reaches the end of inflation at the instant $t_e$ where the value $\psi_e$ makes $\varepsilon \simeq 1$ or $\eta \simeq 1$ (whichever earlier):

$$\psi_e = \max \left\{ \psi : M_{\text{Pl}}^2 \frac{V'}{V} = \sqrt{2} \quad \text{or} \quad M_{\text{Pl}}^2 \frac{|V''|}{|V|} = 1 \right\} \quad (4.18)$$

During the slow-roll regime, we have the approximate equations of motion (look at eq. (3.1)):

$$H^2 \approx \frac{V(\psi)}{3M_{\text{Pl}}^2}, \quad 3H \dot{\psi} + V' \approx 0 \quad (4.19)$$

and we have also $\rho \approx V$. The e-folding number is defined as

$$N_e = \int_{\psi_i}^{\psi_e} \left| \frac{V}{V'} \right| d\psi \quad (4.20)$$

and we find numerically that it differs little from $N_0 = \int_{\psi_i}^{\psi_i} \left| \frac{V}{V'} \right| d\psi$.

For the “formation of structure” problem, let us sketch now how to estimate the fractional density fluctuations using the relativistic theory of cosmological perturbations [30, 31]. The CMB anisotropies give:

$$10^{-5} \sim \frac{\delta M}{M} \bigg|_{t_e} = \frac{\delta M}{M} \bigg|_{t_i} \frac{1}{1 + \frac{\rho}{\dot{\rho}}} \bigg|_{t_i} \quad (4.21)$$

where $\delta M$ represents the mass perturbations. The initial fluctuations are generated quantum mechanically and estimated by:

$$\frac{\delta M}{M} \bigg|_{t_i} \sim \frac{V' \dot{H}}{\dot{\rho}} \quad (4.22)$$

whence, using the continuity equation $\dot{\rho} + 3(\rho + p)H = 0$ and the slow roll approximation (eq. (4.19))

$$10^{-5} \sim \frac{1}{M_{\text{Pl}}^2} \frac{V' V}{\dot{\rho}} \bigg|_{t_i} \quad (4.23)$$

In order to evaluate $\dot{\rho}$, we use $\dot{\rho} \approx \dot{V} = V' \dot{\psi}$ which, using again eq. (4.19), gives us the final result:

$$10^{-5} \sim \frac{\sqrt{3}}{M_{\text{Pl}}^3} \frac{V \sqrt{V}}{V'} \bigg|_{t_i} \quad (4.24)$$

$^5$Other more refined analyses ([32]) lead to the same estimate multiplied by a factor of order 1.
This helps to get an estimate of the assumingly constant condensate:

\[ \langle \tilde{G}_1^2 \rangle_T |_{t_e} \approx \langle \tilde{G}_1^2 \rangle_T |_{t_i} \approx 4 \times 10^{-10} M_{\text{Pl}}^6 \left( \frac{F'}{F \sqrt{F}} \right)^2_{\psi = \psi_i} \text{ where } F(\psi) = \frac{V(\psi)}{M^4} \]  

(4.25)

We shall not treat the reheating regime in this scenario, but just estimate \( T_r \). The inflation starts with the condensate \( \langle \tilde{G}_1^2 \rangle_T \) getting, via conformal anomaly, a non vanishing value at \( t_i \) and remains constant henceforth (no explicit time dependence, but with an implicit one through energy dependence), and the inflation ends at \( t_e \) starting the reheating oscillatory period ending with \( T_r \).

If we assume now eq. (2.10) to remain valid at the end of reheating, then we can estimate the reheating temperature, where again \( \epsilon(T_r) \approx 1 \), as satisfying:

\[ T_r = \frac{10^{-2} M_{\text{Pl}}^{3/2} \sqrt{F'}}{3^{1/4} \sqrt{F \sqrt{F}}} |_{\psi = \psi_i} \]  

(4.26)

We can evaluate the duration of the inflation using eq. (3.6) and get

\[ \Delta t \approx \int_{\psi_i}^{\psi_e} \frac{\sqrt{3}}{M_{\text{Pl}}} \frac{\sqrt{V}}{V'} d\psi \]  

(4.27)

We need also to check the observational bound [1, 2] that during inflation and in Planckian units we have \( H < 10^{-5} \), by evaluating \( H \) at the end of inflation which gives the constraint:

\[ \frac{\sqrt{F(\psi_e)}}{4 \sqrt{3}} \sqrt{\frac{\langle \tilde{G}_1^2 \rangle_T}{M_{\text{Pl}}^2}} < 10^{-5} \]  

(4.28)

5 Discussion and conclusion

Figure 3 represents the \( n_s - r \) analysis for the three (\( \mu > 0 \))-variants \((W(\psi), V(\chi_1, \chi_2))\) and \((W(\chi_1, \chi_2))\) of eqs. (4.12), (4.4) and (4.7) respectively.

For \( W(\psi) \), the results (green colored) are lying adjacent to the power law straightline with some points further from the data. Moreover, the parameter \( \ell_1 \) should be sub-Planckian in order to be nearest to the data.

For the potential \( V(\chi_1, \chi_2) \), the (brown colored) points agree with data regarding \( n_s - r \). However, they are not physically acceptable, as the corresponding initial fields are positive with very high values, and it is not plausible at all that they can be “attracted” to the origin, which lies at a far higher value for the potential.

For this, we introduced the potential \( W(\chi_1, \chi_2) \), and although length scales are still sub-Planckian, however some points corresponding to negative-value initial fields are acceptable physically (look at the blue colored points).

In table 1, we put for each studied pattern of \( \mu > 0 \) the free parameters and their scanned intervals, followed by the constraints taken on the acceptable points, then the extreme values for the acceptable points, and finally their number out of \( 10^7 \) points randomly scattered in the scanned intervals. We opted to scan any length parameter \( \ell_{1,2} \) over \([0, 1] L_{\text{Pl}} \) when no acceptable points exist for above-Planckian length values, whereas we chose to scan over the interval \([1, 11] L_{\text{Pl}} \) in the opposite case.

\footnote{Through eq. (2.5), one can see the possibility of the decay \( \chi \to \tilde{A} \tilde{A} \tilde{A} \to \Phi \Phi \Phi \Phi \) via the terms \( e^{-2f \chi} \tilde{G} \tilde{G} \supset \chi \tilde{A}^4 \) and the term \( D\Phi D\Phi \supset \tilde{A}^2 \Phi^2 \). However, this represents an unrenormalizable term.}
Figure 3. Predictions of the two-Exponentials potential, with $\mu > 0$, compared to 2015 experimental data.

| Potential                        | free parameters | constraints         | Accepted boundaries                  | accepted points |
|----------------------------------|-----------------|---------------------|--------------------------------------|-----------------|
| $W(\psi)$                        | $\ell_1 \in [0, 1]$ | $0.962 < n_s < 0.98$ | $0 < r < 0.21$                      | 186             |
|                                  | $\ell_2 \in [1, 11]$ |                      | [0.0708, 0.0809]                     |                 |
|                                  | $\mu \in [0, 1]$   |                      | [1.0005, 10.936]                     |                 |
|                                  | $\psi \in [-10, 0]$ |                      | [0, 0.0152]                          |                 |
|                                  |                  |                      | [-9.9326, -0.00026]                  |                 |
| $V(\chi_1, \chi_2)$             | $\ell_1 \in [0, 1]$ | $0.962 < n_s < 0.974$ | $0 < r < 0.14$                      | 958             |
|                                  | $\ell_2 \in [1, 11]$ |                      | [0.186, 0.416]                       |                 |
|                                  | $\mu \in [1, 10^4]$ |                      | [1.0005, 10.994]                     |                 |
|                                  | $\chi_1 \in [0, 10^4]$ |                      | [15.792, 9.995 $\times$ 10^3]       |                 |
|                                  | $\chi_2 \in [0, 10^6]$ |                      | [4.466, 9.995 $\times$ 10^3]        |                 |
|                                  |                  |                      | [2.684, 9.959 $\times$ 10^2] $\times$ 10^4 |                 |
| $W(\chi_1, \chi_2)$             | $\ell_1 \in [0, 1]$ | $0.962 < n_s < 0.974$ | $0 < r < 0.15$                      | 63211           |
|                                  | $\ell_2 \in [0, 1]$ |                      | [0, 0.926]                           |                 |
|                                  | $\mu \in [1, 10]$  |                      | [0, 0.903]                           |                 |
|                                  | $\chi_1 \in [-10, 0]$ |                      | [2 $\times$ 10^{-4}, 10]            |                 |
|                                  | $\chi_2 \in [-10, 0]$ |                      | [-10, -0.0006]                       |                 |
|                                  | $\lambda \in [0, 1]$ |                      | [-10, -0.0003]                       |                 |
|                                  |                  |                      | [0.0065, 0.999]                      |                 |

Table 1. Accepted points of the three variants with $\mu > 0$ (out of $10^7$ scanned points). Dimensional quantities are measured in Planckian units ($L_{PL}, m_{PL}$).
Figure 4. Predictions of the two-Exponentials potential, with $\mu < 0$, compared to 2015 experimental data.

As for the pattern $V(\psi)$ of eq. (4.16) with $\mu < 0$, we carry out now a complete analysis beyond that of $(n_s - r)$. The parameter space is 3-dimensional with the free parameters $\ell_1, \ell_2$ and $\psi_i$. We take also randomly $10^7$ points in this space verifying:

$$\ell_1, \ell_2 \in [1, 11] L_{PL}, \psi_i \in [0, 10] m_{PL}$$  \hspace{1cm} (5.1)

corresponding to above-Planckian-length points, and we forced the experimental constraints:

$$0.962 \leq n_s \leq 0.974 \hspace{1cm} 0.001 \leq r_i \leq 0.11$$

$$\ell_1 \geq \ell_2 \hspace{1cm} N_e \geq 50$$  \hspace{1cm} (5.2)

The number of acceptable points was 1134, and are represented by the dots colored in green in figure 4. We got an upper bound $r < 0.002$ for the points which passed the experimental constraints, and this is in line with Planck 2015 findings that for models satisfying the Lyth bound we have typically $r \leq 2 \times 10^{-5}$ for $n_s \sim 0.96$. In order to use eqs. (3.6), (3.7), (4.18), (4.20), (4.25), (4.26), (4.27), (4.28), we choose three benchmark points P1, P2 and P3 for this above-Planckian-length model, and summarize the findings in table 2. We point out in figure 5 the contrast between P1 and P3 in computing $\psi_e$, in that it is $|\eta| (\varepsilon)$ which reaches first the value 1 for the point P3 (P1).

The dots colored in red in figure 4 represent acceptable points which correspond to sub-Planckian lengths. For this, we scanned $10^5$ points with $\ell_1, \ell_2 \in [0, 1] L_{PL}$, while the other constraints are kept the same, except that for visualization purposes we limited ourselves
Figure 5. $\epsilon$ (grey) and $|\eta|$ (brown)–parameters for the parameter space points P1 and P3.

|                  | P1     | P2     | P3     | P4     |
|------------------|--------|--------|--------|--------|
| $l_1/L_{PL}$     | 3.8280 | 8.5857 | 1.0091 | 0.7249 |
| $l_2/L_{PL}$     | 1.0954 | 2.2939 | 1.0045 | 0.0489 |
| $\psi_e/M_{PL}$  | 3.3725 | 2.1375 | 4.4985 | 9.9499 |
| $\varepsilon_i$  | $6.9 \times 10^{-5}$ | $9.3 \times 10^{-6}$ | $1.1 \times 10^{-4}$ | 0.0033 |
| $|\eta|i$        | 0.018  | 0.014  | 0.018  | 0.0056 |
| $n_{si}$         | 0.9631 | 0.9719 | 0.9626 | 0.9692 |
| $r_i$            | 0.0011 | $1.5 \times 10^{-4}$ | 0.0019 | 0.0522 |
| $\psi_e$         | 0.967  | 0.848  | 0.888  | 1.15   |
| $N_e$            | 52     | 70     | 54     | 50     |
| $\langle \tilde{G}^1_{12} \rangle_{T}/M_{PL}^4$ | $7.4 \times 10^{-15}$ | $9 \times 10^{-16}$ | $6.8 \times 10^{-12}$ | $1.3 \times 10^{-13}$ |
| $T_r/M_{PL}$     | $6.5 \times 10^{-4}$ | $3.8 \times 10^{-4}$ | $36 \times 10^{-4}$ | $13 \times 10^{-4}$ |
| $M_{PL}\Delta t$ | $1.6 \times 10^{16}$ | $1.6 \times 10^{17}$ | $4 \times 10^{14}$ | $6 \times 10^{14}$ |
| $\Delta t$ (s) ≈ | $10^{-27}$ | $10^{-26}$ | $2 \times 10^{-29}$ | $3 \times 10^{-29}$ |
| $10^3H_e/M_{PL}$ | 0.11   | 0.05   | 0.23   | 0.24   |

Table 2. P1, P2, P3 (P4) are above(sub)-Planckian-length benchmark points, for $V(\psi)$ with $\mu < 0$.

to points satisfying $0.02 \leq r_i \leq 0.11$. We got 559 acceptable points, of which we take a representative benchmark point P4 whose results are also shown in table 2.

For all the benchmark points, we see that the reheating temperature would be similar to that of the power law ($\sim 10^{14}$GeV), whereas the duration of the inflation would be larger, but well shorter than the EW breaking constraint ($\sim 10^{-12}$s). Also, the observational bounds on $H_e$ are well respected in all benchmark points.

It may seem that the model under study is very specific, and that any future data imposing a lower bound on $r > 0.002$ will invalidate its above-Planckian-length version, casting doubts on it. However, one can argue that such a model is an element of an entire
class of scenarios where gauge couplings at early universe might depend both on energy and explicitly on time as well. Actually, the sub-Planckian-length benchmark point P4 can be transformed into an above-Planckian-length point by generalizing the procedure, of going from \( V(\chi_1, \chi_2) \) to \( V(\psi) \), to \( n \) groups with varying coupling constants under the “strong” assumption \( \epsilon_k^{1/l_k} = \epsilon_1^{1/l_1} \). This leads to a canonical kinematic term of a field \( \psi \) satisfying \( \psi = \sqrt{n} \chi = \sqrt{n} \log \frac{1}{\ell_k} \), and the pure gauge potential will be given by

\[
\sum_{k=1}^{k=n} \frac{1}{4} \langle \tilde{G}_k^2 \rangle_T e^{-2t_k \chi} = \frac{1}{4} \sum_{k=1}^{k=n} \eta_k e^{-2t_k \sqrt{n} \psi}
\]

with \( \eta_k = \langle \tilde{G}_k^2 \rangle_T / \langle \tilde{G}_1^2 \rangle_T \). It is clear now that if the condensates satisfy: \( \eta_2 = -\ell_1 / \ell_2, \eta_k = 0, k = 3, \ldots, n \) then we get the same potential as in eq. (4.16), but now the length scales at the point P4 will be multiplied by the factor \( \sqrt{n/2} \), and for large enough \( n \) one can make P4 above-Planckian-length point.

To summarize, we suggested a seemingly novel mechanism to reinterpret inflation as a time varying coupling constant à la Bekenstein. With only one varying coupling constant, one can realize the power law inflation which is excluded now by experiment. However, assuming more than one varying coupling constant the model opens up to several variants which can accommodate the experimental data, and may be self-consistent at the same time.

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