NOVEL FLOWS BETWEEN N=2 LANDAU–GINZBURG THEORIES:
New Directions in Modulii Space via c = 0 Theories

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ABSTRACT
A new method for constructing flows between distinct Landau-Ginzburg theories at fixed central charge is presented. The essential ingredient of the construction is an enlarged moduli space obtained by adding theories with zero central charge. The flows involve only marginal directions hence they can be applied to transitions between string vacua, in particular to the construction of mirror pairs of string ground states described by RG fixed points of N=2 supersymmetric Landau–Ginzburg theories. In contrast to previous methods this new construction of mirror theories does not depend on particular symmetries of the original theory.

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1. Introduction

An important question in quantum field theory is the problem of constructing flows between renormalization group fixed points. A landmark result in this direction is Zamolodchikov’s ‘c–theorem’ [1] which shows that along the RG trajectories the c–function, describing the central charge at the fixed points, decreases monotonically. This result unfortunately seems to imply that RG flows are uninteresting for string theory where the ground states are described by conformal field theories at some fixed central charge. The only possible flows therefore are those generated by marginal operators. But flows along marginal directions, as long as they are regular, only change the Yukawa couplings and do not change the spectrum of the theory. When the flows become (in some sense) singular [2] control of the spectrum of the theory is lost.

It is the purpose of this article to introduce a new type of flow which relates distinct N=2 supersymmetric Landau–Ginzburg theories at some fixed central charge. These flows proceed via marginal operators, hence are of interest for the understanding of string vacua described by RG fixed points of Landau–Ginzburg theories. They lead, in particular, to a new type of mirror map between pairs of string vacua. Most known constructions of mirror theories depend, in general, on orbifolding of the original theory [3–6]. This is unsatisfactory because no general selection rule is known which determines the specific form of the action or even its order. Even though for Fermat type superpotentials both ingredients, the order of the group and its specific form is known [8], for non–Fermat polynomials finding the action is more of an art. Thus it is useful to search for constructions which do not depend on the modding of symmetries.

In Section 2 I will describe the basic idea, keeping the discussion general. Even though the application I will focus on in this paper is mirror symmetry, the basic framework is quite independent of that particular question and may well have other applications. Concrete applications to various types of potentials will be presented in Section 3 and in Section 4 I will discuss mirror pairs of Landau–Ginzburg vacua. Section 5 contains results regarding the explicit construction of mirror vacua via marginal flows.

2. Enlarged Moduli Space via c = 0 Theories: The Basic Idea

To motivate the construction consider a heterotic vacuum described by a GSO–projected Landau–Ginzburg theory defined by a superpotential \( W(\Phi_i) \) that depends on chiral N=2 superfields \( \Phi_i(z, \bar{z}, \theta^\pm, \bar{\theta}^\pm), \ i = 1, \ldots, n \). The mirror question is how to construct the mirror theory, which I assume to be described by the mirror potential \( \tilde{W}(\tilde{\Phi}_i) \) which in turn depends on chiral N=2 superfields \( \tilde{\Phi}_i(z, \bar{z}, \theta^\pm, \bar{\theta}^\pm), \ i = 1, \ldots, \tilde{n} \). As a first step toward an explicit construction of the mirror vacuum \( \tilde{W} \) from \( W \) one may assume a candidate pair of potentials \( (W, \tilde{W}) \) as given and

\[ ^1 \text{An exception is Batyrev’s construction via dual polyhedra [7].} \]
to attempt to somehow ‘move’ from one theory to the other.

In order to achieve this one might imagine adding the two potentials and performing some sort of projection in the path integral which reduces this ‘superposition’ of theories to the individual potentials. This does not work because the central charge is doubled in the process and therefore the theory $W + \tilde{W}$ does not describe a heterotic vacuum of the type necessary. This problem can be circumvented however.

Given an arbitrary $N=2$ supersymmetric Landau–Ginzburg theory $W$ with some central charge it is possible to, in a sense, add some new nontrivial theory $\tilde{W}$ without changing the central charge by doing something that I will call ‘trivialization’. The idea is to compensate the central charge of the new theory by introducing additional fields with negative central charge such that the added theory has in fact vanishing central charge. To find such fields and the appropriate action is easy and always possible: for any superpotential $\tilde{W}(\tilde{\Phi}_i)$ the potential

$$\tilde{W}'(\tilde{\Phi}_i, \Psi_i) := \tilde{W}(\tilde{\Phi}_i) + \sum_i g_i \tilde{\Phi}_i \Psi_i$$

leads to a $c = 0$ theory. I will call this theory the ‘trivialized’ theory. Adding this trivialized theory to the original potential $W(\Phi_i)$ therefore does not change the central charge. Furthermore the spectrum remains the same because the chiral ring of the theory $\tilde{W}'$ is trivial owing to the fact that $q_{\Psi_i} = 1 - q_{\tilde{\Phi}_i}$, hence $\mu(\tilde{W}') = 1$.

Adding the potential $\tilde{W}'(\tilde{\Phi}_i, \Psi_i)$ is, however, not merely a trivial rewriting of the original theory because the parameter space of the new theory can contain deformations along ‘mixed’ directions, involving fields of the type $O_{IJ} = \Phi_I \Psi_J$, where $I, J$ are multiindices, i.e. the operators $O_{IJ}$ describe monomials in the variables $\Phi_i$ and $\Psi_j$. The existence of these mixed directions is nontrivial. Assuming they exist the question arises what their general structure is. Since the variables with negative central charge were introduced via terms of the form $\tilde{\Phi}_i \Psi_i$ these new operators are not bilinear but instead will be, in general, of the form $\tilde{\Phi}_i^{\alpha j} \Psi_j$. Denote the potential containing the couplings between the $\Phi_i$’s and the $\Psi_j$’s by $W(\Phi_i, \Psi_j)$. It is then possible to move to a point in the enlarged parameter space where the total potential is of the form

$$W(\Phi_i) + W(\Phi_i, \Psi_j) + \tilde{W}(\tilde{\Phi}_i),$$

i.e. the $\Psi_j$ are completely decoupled from the $\tilde{\Phi}_i$. At this point the theory becomes more singular than the original theory: the singular set is not just an isolated point at the origin but some higher dimensional submanifold. This is precisely what is needed because it allows the spectrum of the theory to change. If it so happens that the first two potentials define a $c = 0$ theory it

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1 It should be emphasized that this does not mean that the dimensions of these additional fields are negative, which would be a disaster.

2 More explicitly this can be seen by considering the ideal generated by the potential $\tilde{W}'$. 

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is possible to split off this new trivialized theory and be left with a potential that describes a different consistent Landau–Ginzburg theory with an isolated singularity, i.e. with a finite chiral ring.

In a nutshell, then, the idea is to add to the moduli space of Landau–Ginzburg theories new directions by adding \( c = 0 \) theories. Moving in this enlarged configuration space allows to connect theories with different spectra by passing through degenerate theories which can be made into regular ones by splitting off different \( c = 0 \) theories.

3. Examples: Coset Landau–Ginzburg Theories and Others

To be concrete consider first a pair of potentials which derives \([9,10]\) from a minimal \( N=2 \) superconformal theory at level \( k \) with central charge \( c = 3k/(k+2) \); the first one

\[
W_k(\Phi) = \Phi^{k+2} \tag{3}
\]

describes the mean field theory of the exactly solvable model endowed with the diagonal affine invariant whereas the second one

\[
\tilde{W}(\tilde{\Phi}_1, \tilde{\Phi}_2) = \tilde{\Phi}_1^k + \tilde{\Phi}_1 \tilde{\Phi}_2^2 \tag{4}
\]

describes the theory with the nondiagonal affine D–invariant.

The extended theory

\[
W(\Phi) + \tilde{W}(\tilde{\Phi}_i, \Psi_i) = \Phi^{k+2} + \tilde{\Phi}_1^k + \tilde{\Phi}_1 \tilde{\Phi}_2^2 + g_1 \tilde{\Phi}_1 \Psi_1 + g_2 \tilde{\Phi}_2 \Psi_2. \tag{5}
\]

leads to an enlarged moduli space which contains mixed monomials of charge one, e.g.

\[
\Phi^2 \Psi_1, \quad \Phi^{k-1} \Psi_2 \tag{6}
\]

describing new marginal directions along which the original theory can be deformed. Thus it is possible to move to the point in parameter space defined by

\[
W(\Phi) + g_1 \Phi^2 \Psi_1 + g_2 \Phi^{k-1} \Psi_2 + \tilde{W}(\tilde{\Phi}_i) \tag{7}
\]

where the fields with negative central charge are completely decoupled from \( \tilde{W}(\tilde{\Phi}_i) \). This theory does not have a well–defined chiral ring. However, the first three terms define a \( c = 0 \) theory which can be split off, leaving a nondiagonal theory with a well–defined spectrum which is different from the spectrum of the diagonal model.

It is worth remarking that there exists a framework in which the potentials just described appear in a natural way, namely the level–rank duality \([11]\) which relates at level \( k \) the two
theories

\[
\begin{array}{c|cc}
SU(2) & SU(k+1) \\
\hline
U(1) & SU(k) \times U(1)
\end{array}
\]

\[k \iff 1\]  \hspace{1cm} \text{(8)}

Consider the potentials for these two theories. For any theory of the type

\[
\begin{array}{c|cc}
SU(m+n) & SU(m) \times SU(n) \times U(1) \\
\hline
\end{array}
\]

\[1\]  \hspace{1cm} \text{(9)}

the potential is known \cite{12,13} to be of the form

\[
W = \sum_{n_1+2n_2+\cdots+mn_m=m+n+1} A_{n_1\cdots n_m} \Phi_1^{n_1} \cdots \Phi_m^{n_m}.
\]  \hspace{1cm} \text{(10)}

Thus the potential of the minimal model is

\[
W = \Phi_1^{k+2} + \Phi_2^2
\]  \hspace{1cm} \text{(11)}

whereas the potential of the dual model is in the deformation class \[^3\]

\[
\tilde{W} = \Phi_1^{k+2} + \Phi_2^{(k+2)/2} + \Phi_2 \Phi_2^{k/2} + \Phi_2^{(k+2)/2} + \Phi_2 \Phi_2 + \Phi_2 \Phi_2^{k/2+2} + \Phi_2^{k/2+2} + \cdots
\]  \hspace{1cm} \text{(12)}

It is therefore always possible to split off a trivial \(c = 0\) theory in the level–rank–dual theory to be left with a minimal theory in which the diagonal affine invariant has been replaced by the D–invariant.

With the Landau–Ginzburg theories just described it is possible to analyze a large variety of mirror pairs by tensoring other exactly solvable \(N=2\) models or, more generally, by adding some arbitrary \(N=2\) supersymmetric Landau–Ginzburg potential. For the sake of generality it should be emphasized however that the idea introduced in Section 2 can be applied to Landau–Ginzburg potentials that do not derive from the coset construction. A simple example may illustrate how an iterative application of the technique allows to discuss other types of theories as well. Consider the pair of potentials described by

\[
W(\Phi_i) = \Phi_1^{25} + \Phi_1 \Phi_2^{16}
\]  \hspace{1cm} \text{(13)}

and

\[
\tilde{W}(\tilde{\Phi}_i) = \tilde{\Phi}_1^{25} + \tilde{\Phi}_1 \tilde{\Phi}_2^8 + \tilde{\Phi}_2 \tilde{\Phi}_3^2,
\]  \hspace{1cm} \text{(14)}

the latter leading to the \(c = 0\) theory

\[
\tilde{W}'(\tilde{\Phi}_i, \Psi_i) = \tilde{W}(\tilde{\Phi}_i) + \sum \tilde{\Phi}_i \Psi_i.
\]  \hspace{1cm} \text{(15)}

\[^3\text{Note that this potential, as written, appears to describe the sum of two trivial theories with }c = 0.\text{ This is not the case however, since the potential in fact does not lead to a finite chiral ring at all: the theory is singular.}\]
Moving along mixed direction it is possible to arrive at the potential

\[ W(\Phi_i) + \Phi_1 \Psi_1 + \Phi_2^3 \Psi_2 + \Phi_1^{11} \Psi_3 + \tilde{W}(\tilde{\Phi}_i). \tag{16} \]

After splitting off the \( c = 0 \) theory defined by the first four terms one is left with the new theory \( \tilde{W}(\tilde{\Phi}_i) \).

It is easy to see that the example of the last paragraph is just an iteration of the basic construction involving the transition from the diagonal affine invariant of a minimal model at level \( (2b - 2) \), \( W(\Phi_2) = \Phi_2^{2b} \), to the nondiagonal D–invariant \( W(\Phi_i) = \tilde{\Phi}_2^b + \tilde{\Phi}_2 \tilde{\Phi}_2^3 \). The only difference is that the fields which carry the flow are factored into some additional fields. Similarly one can consider further iterations to construct flows between ever more complicated potentials.

There are, however, other potentials that occur in the class of all Landau–Ginzburg vacua [14,15] which are different from the types described so far. Even though it is not, at present, known what precisely the range of applicability of this new construction is, it is clear that it is more general than the infinite series discussed above. An interesting example is the following pair which involves the transposition of a potential, a concept introduced in ref. [6]. The example is also enlightening because it illustrates how the precise vacuum structure comes to the rescue in cases where a naive application of the construction does not work.

Consider the pair of potentials described by

\[ W(\Phi_i) = \Phi_1^5 \Phi_2 + \Phi_2^3 \tag{17} \]

and

\[ \tilde{W}(\tilde{\Phi}_i) = \tilde{\Phi}_1^5 + \tilde{\Phi}_1 \tilde{\Phi}_2^3 \tag{18} \]

neither of which is related to a minimal model. The trivialization of the potential \( \tilde{W} \) introduces two fields \( \Psi_1, \Psi_2 \) with charges 4/5, 11/15. Whereas it is possible to couple \( \Psi_2 \) to the field \( \Phi_1 \), it is not possible to couple \( \Psi_1 \) to either \( \Phi_1 \) or to \( \Phi_2 \). Thus it is impossible to decouple the fields \( \Psi_i \) from the \( \Phi_i \) in order to split off a \( c = 0 \) theory to be left with the second potential describing the new theory.

Since the application I have in mind is to understand string vacua the focus in the following will not be in just any Landau–Ginzburg theory but in special ones, namely those defining consistent ground states. It turns out that the above potential \( \tilde{W} \) indeed appears as part of superpotentials, but it does so together with other fields (in order to bring the central charge up to \( c = 9 \)). An additional field that appears in the context of the two potentials above is a field of charge 1/30 to which the field of weight 11/15 can obviously be coupled. Thus the construction goes through. This example shows that the framework introduced here also throws light on the concept of the transposition of Landau–Ginzburg vacua. Since the applicability of
transpos restricted to particular points in the moduli space it is to be hoped that incorporating it into the present framework allows a generalization away from this restriction.

4. Flows between Mirror Pairs of Landau–Ginzburg Theories

In the following I will discuss applications of the constructions of the previous Sections to a number of examples of string vacua described by different types of Landau–Ginzburg theories in order to illustrate the fact that the range of applicability is actually quite large even though at present it is not known precisely what class of models can be discussed in this way.

The minimal series:

The simplest situation is of course described by the transition from a minimal diagonal theory to a minimal D–type polynomial. Consider e.g. the vacuum described by the tensor product of four minimal N=2 superconformal theories \((16 \cdot 31 \cdot 86)\) endowed with the diagonal invariant in each factor. The Landau–Ginzburg potential of this theory is described by the Fermat type polynomial

\[
W = \Phi_1^3 + \Phi_2^{33} + \Phi_3^{88} + \Phi_4^8.
\]

The field theoretic limit of this model describes a particular point in the deformation class

\[
\mathfrak{C}_{(88, 3, 3, 66, 99)}[264]^{57}_{48}
\]

the mirror of which can be shown, via the fractional transformations introduced in [4], to be described by the potential

\[
W = \Phi_1^3 + \Phi_2^{33} + \Phi_3^{88} + \Phi_4^4 + \Phi_2\Phi_5^2
\]

This potential leads to the Calabi–Yau configuration

\[
\mathfrak{C}_{(88, 8, 3, 66, 99)}[264]^{81}_{48}
\]

(here the subscripts (superscripts) denote the Euler number (number of (1,1)–forms). The important operation in order to produce the mirror of the ground state thus is to simply replace the diagonal invariant in the level 6 theory by the D–invariant. This operation leaves the first three terms in the potential invariant. The relevant potentials to consider are

\[
W(\Phi_i) = \Phi_i^6 + \Phi_5^2
\]

and

\[
\tilde{W}(\tilde{\Phi}_i) = \tilde{\Phi}_4^4 + \tilde{\Phi}_4\tilde{\Phi}_5^2
\]

\(^4\)The * indicates that the GSO–projection has been implemented.
leaving the other fields as spectators.

These two potentials, however, are just of the type considered in eqs. (3,4) above and it is obvious that the discussion there immediately applies to the mirror pair of Landau–Ginzburg theories. After including the fields \( \tilde{\Psi}_i \) with negative central charge the extended theory becomes

\[
\Phi_3^3 + \Phi_3^{33} + \Phi_3^{88} + \Phi_4^2 + \tilde{\Phi}_4^2 \Phi_8^2 + \Phi_8^4 + \Phi_8^4 \tilde{\Psi}_5.
\]  

and moving to the singular configuration

\[
\Phi_3^3 + \Phi_3^{33} + \Phi_3^{88} + \tilde{\Phi}_4^2 \Phi_8^2 + \Phi_8^2 \Phi_4^2 + \Phi_4^2 \tilde{\Psi}_4 + \Phi_4^2 \tilde{\Psi}_5
\]  

allows to split off the \( c = 0 \) modification of the diagonal configuration, leaving the mirror theory of our starting point. It should be noted that instead of just reinstating the monomials defining the original theory it is also possible to add all marginal operators that can be constructed from the first three scaling fields, thus allowing us to mirror map a submanifold of the moduli space.

It is also possible to consider nondiagonal theories by adding potentials that do not derive from minimal N=2 superconformal theories. An example of such an application is given by the mirror pair

\[
C^{*}_{(3,11,16,10,40)}[80]^{19}_{-16} \ni \{ \Phi_1^{23} \Phi_2 + \Phi_1 \Phi_2^7 + \Phi_3^5 + \Phi_4^8 + \Phi_5^2 = 0 \} 
\]  

and

\[
C^{*}_{(3,11,16,20,30)}[80]^{27}_{16} \ni \{ \Phi_1^{23} \Phi_2 + \Phi_1 \Phi_2^7 + \Phi_3^5 + \Phi_4^4 + \Phi_4 \Phi_5^2 = 0 \}.
\]  

*Iterating the minimal series:*

There exist, of course, also potentials which do not contain any minimal factor at all and an obvious question is whether the idea I introduced here also applies to such theories. In fact it does, as the pair of models

\[
C^{*}_{(4,6,5,15,25)}[100]^{33}_{-8} \ni \{ \Phi_1^{25} + \Phi_1 \Phi_2^{16} + \Phi_3^2 + \Phi_4^5 \Phi_5 + \Phi_5^4 = 0 \}
\]  

and

\[
C^{*}_{(4,12,44,15,25)}[100]^{37}_{8} \ni \{ \tilde{\Phi}_1^{25} + \tilde{\Phi}_1 \tilde{\Phi}_2^6 + \tilde{\Phi}_2 \tilde{\Phi}_3^2 + \tilde{\Phi}_4 \tilde{\Phi}_5 + \tilde{\Phi}_5^4 = 0 \}
\]

shows which uses the potentials (13) and (14) discussed above.

*Transposed Mirror Pairs:*

An example where the transition from the original theory to the mirror is accomplished by a simple transposition the potential is furnished by the following pair:

\[
C^{*}_{(2,15,15,8,20)}[60]^{30}_{-12} \ni \{ \Phi_1^{30} + \Phi_2^4 + \Phi_3^4 + \Phi_4 \Phi_5 + \Phi_5^3 = 0 \}
\]
and
\[
\mathbf{C}^*_{(2,15,15,12,16)}[60]_{12}^{36} \ni \{ \Phi_1^{30} + \Phi_4^2 + \Phi_3^4 + \Phi_4^5 + \Phi_4 \Phi_5^2 = 0 \}
\] (32)

None of the above:

An instructive example which involves neither the infinite series nor the transposition procedure is provided by the pair
\[
\mathbf{C}^*_{(3,5,16,16,40)}[80]_{-8}^{33} \ni \{ \Phi_1^{25} \Phi_2 + \Phi_2^{16} + \Phi_3^5 + \Phi_4^2 + \Phi_5^2 = 0 \}
\] (33)
and its mirror
\[
\mathbf{C}^*_{(3,5,16,24,32)}[80]_{8}^{37} \ni \{ \tilde{\Phi}_1^{25} \tilde{\Phi}_2 + \tilde{\Phi}_2^{16} + \tilde{\Phi}_3^5 + \tilde{\Phi}_3 \tilde{\Phi}_5^2 + \tilde{\Phi}_5 \tilde{\Phi}_4^2 = 0 \}.
\] (34)

In this case no truncation of the potential will do in order to be able to decouple the \(\Psi_i\)'s from the original theory. Naively it seems that since the first part of the two potentials agrees we only need to consider the last three terms. Suppose we try to trivialize only the latter part of the second theory, i.e. we consider
\[
\tilde{\Phi}_3^5 + \tilde{\Phi}_3 \tilde{\Phi}_5^2 + \tilde{\Phi}_5 \tilde{\Phi}_4^2 + \tilde{\Phi}_3 \Psi_3 + \tilde{\Phi}_4 \Psi_4 + \tilde{\Phi}_5 \Psi_5,
\] (35)
thereby introducing fields \(\Psi_i\) of weights \(\frac{4}{5}, \frac{3}{5}, \frac{7}{10}\). If only the fields \(\Phi_{3,4,5}\) are available for couplings to the \(\Psi_i\) it is clear that the field \(\Psi_3\) cannot be decoupled from the \(\tilde{\Phi}_i\)'s. The way out of this is by considering the remaining fields as well.

Thus adding the trivialization of the full second theory leads to
\[
\Phi_1^{25} \Phi_2 + \Phi_2^{16} + \Phi_3^5 + \Phi_3^2 + \Phi_4^5 + \Phi_4^2 + \Phi_5^2 + \Phi_5 \Phi_4^2 + \Phi_5 \Phi_3^2 + \Phi_3 \Phi_5^2 + \Phi_3 \Phi_4^2 + \sum_{i=1}^{5} \tilde{\Phi}_i \Psi_i.
\] (36)

Moving to the singular configuration
\[
\Phi_1^{25} \Phi_2 + \Phi_2^{16} + \Phi_3^5 + \Phi_4^2 + \Phi_5^2 + \Phi_1 \Psi_1 + \Phi_2 \Psi_2 + \Phi_3 \Psi_3 + \Phi_3^2 \Psi_4 + \Phi_4 \Psi_5 + \tilde{\Phi}_1^{25} \tilde{\Phi}_2 + \tilde{\Phi}_2^{16} + \tilde{\Phi}_3^5 + \tilde{\Phi}_3 \tilde{\Phi}_5^2 + \tilde{\Phi}_5 \tilde{\Phi}_4^2
\] (37)
and splitting off the trivialization of the first one leaves the mirror.

5. Constructing Mirrors via Marginal Singular Flows

So far I have always assumed as given a pair of theories and then attempted to construct a flow from one to the other. The question arises whether it is possible to let the construction determine to which theory the deformed model wants to flow. This is a somewhat involved problem which I will discuss only briefly in the context of the simplest theory encountered above.
Consider again the potential \( \tilde{W} = \tilde{\Phi}_1^4 + \tilde{\Phi}_1 \tilde{\Phi}_2^2 \) with central charge \( c = 9/4 \) as starting point. The fields \( \Psi_i \) that appear in the trivialized theory have charges \( 3/4, 5/8 \) respectively. We are interested in all potentials \( W(\Phi_1, \Phi_2) \) which satisfy the following constraints:

- \( W(\Phi_1, \Phi_2) \) describes a theory of central charge \( c = 9/4 \)
- Marginal operators of the form \( \Phi_i^{a_i}\Psi_j \) exist
- \( W(\Phi_1, \Phi_2) \) has an isolated singularity at the origin.

In the case at hand there are only a few possible operators that can appear. (I) The operators can be of the form \( \Phi_1^{a_1}\Psi_1, \Phi_2^{a_2}\Psi_2 \). The central charge condition then dictates that the only possible solution is \( a_1 = 1 = a_2 \) which are the operators used to trivialize the original theory. (II) The operators are of the form \( \Phi_1^{a_1}\Psi_1, \Phi_2^{a_2}\Psi_2 \), in which case \( 2a_2 = 3a_1 \). Thus \( a_1 \) must be even \( a_1 = 2n \) and \( q_1 = 1/8n \) and \( q_2 = \frac{1}{8}(5 - \frac{1}{n}) \). The requirement that the potential \( W \) has an isolated singularity only at the origin finally determines \( n = 1 \) and hence we have derived with the trivialized theory (24) the potential (23) dressed up with a trivial factor.

6. Conclusion

RG flows between different vacua described by conformal field theories at some fixed central charge (e.g. \( c = 9 \) theories in the case of left–right symmetric compactifications of the heterotic string) can proceed only via marginal operators. Marginal deformations do not, generically, change the spectrum of the theory but instead change the Yukawa couplings and the symmetries of the theory under consideration.

It has been shown in this article that it is possible to flow between Landau–Ginzburg vacua with different spectra via marginal deformations. The essential new ingredient are composite marginal operators that lead to a singular theory, in the sense that the chiral ring becomes ill-defined. By splitting off singular subtheories a regular theory is obtained. The existence of such composite marginal is nontrivial, hence the flows do not connect arbitrary pairs of Landau–Ginzburg theories. The results presented here show that the class of potentials which is amenable to this new construction contains polynomials that occur in mirror pairs of string vacua described via Landau–Ginzburg theories. An immediate consequence of these flows therefore is a novel type of mirror map which proceeds via singular marginal flows.

The general picture that emerges is one of a space of trivialized Landau–Ginzburg theories of vanishing central charge in which particular subspaces describe regular Landau–Ginzburg at some fixed central charge \( c \neq 0 \). These regular theories are obtained by splitting off large parts of the moduli space which describe theories with vanishing central charge.
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