Probing the isotropy of cosmic acceleration using different supernova samples

Z. Q. Sun$^1$ and F. Y. Wang$^{1,2,*}$

$^1$ School of Astronomy and Space Science, Nanjing University, Nanjing 210093, China
$^2$ Key Laboratory of Modern Astronomy and Astrophysics (Nanjing University), Ministry of Education, Nanjing 210093, China

*fayinwang@nju.edu.cn

ABSTRACT

Recent studies have indicated that an anisotropic cosmic expansion may exist. In this paper, we use three datasets of type Ia supernovae (SNe Ia) to probe the isotropy of cosmic acceleration. For the Union2.1 dataset, the direction and magnitude of the dipole are $(l = 309.3^{\circ}_{-15.7^{\circ}} + 15.5^{\circ}, b = -8.9^{\circ}_{-0.8^{\circ}} + 11.2^{\circ}), A = (1.46 \pm 0.56) \times 10^{-3}$. For the Constitution dataset, the results are $(l = 67.0^{\circ}_{-6.2^{\circ}} + 66.5^{\circ}, b = -0.6^{\circ}_{-26.3^{\circ}} + 25.2^{\circ}), A = (4.4 \pm 5.0) \times 10^{-4}$. For the JLA dataset, no significant dipolar deviation is found. We also explore the effects of anisotropic distributions of coordinates and redshifts on the results using Monte-Carlo simulations. We find that the anisotropic distribution of coordinates can cause dipole directions and make dipole magnitude larger. Anisotropic distribution of redshifts is found to have no significant effect on dipole fitting results.

Subject headings: type Ia supernova - cosmology: observations

1. Introduction

Type Ia supernovae (SNe Ia) are ideal standard candles (Phillips 1993). In 1998, the accelerating expansion of the Universe was discovered using the luminosity-redshift relation of SNe Ia (Riess et al. 1998, Perlmutter et al. 1999). The cosmological principle assumes that the Universe is homogeneous and isotropic at large scales. Based on the cosmological principle and numerous observational facts, the standard ΛCDM model has been established. It can be used to explain various observations.

However, it is worthy to examine the validity of the standard ΛCDM model (Kroupa et al. 2012, Kroupa 2012, Perivolaropoulos 2014, Koyama 2016) and its assumptions, namely
the cosmological principle. Deviation from cosmic isotropy with high statistical confidence level would lead to a major paradigm shift. At present, the standard cosmology confronts some challenges. Observations on the large-scale structure of the Universe, such as “great cold spot” on cosmic microwave background (CMB) sky map (Vielva et al. 2004), alignment of lower multipoles in CMB power spectrum (de Oliveira-Costa et al. 2004; Tegmark et al. 2003), alignment of polarization directions of quasars in large scale (Hutsemékers et al. 2005), handedness of spiral galaxies (Longo 2009), and spatial variation of the fine structure constant (King et al. 2012; Mariano & Perivolaropoulos 2012), show that the Universe may be anisotropic.

The isotropy of the cosmic acceleration has been widely tested using SNe Ia. Generally, there are two different ways to study the possible anisotropy from SNe Ia. The first one is directly fitting the data to a specific anisotropic model (Campanelli, L et al. 2011; Li et al. 2013; Wang & Wang 2018). Many anisotropic cosmological models have been proposed to match the observations, including the Bianchi I type cosmological model (Campanelli, L et al. 2011; Aluri et al. 2013) and the Rinders-Finsler cosmological model (Chang et al. 2014). The extended topological quintessence model with a spherical inhomogeneous distribution for dark energy density is also proposed (Mariano & Perivolaropoulos 2012).

An alternative method is directly analysing the SNe Ia data in a model-independent way (Antoniou & Perivolaropoulos 2010; Cai & Tuo 2011; Cai et al. 2013; Mariano & Perivolaropoulos 2012; Zhao et al. 2013; Yang et al. 2013; Wang & Wang 2014; Javanmardi et al. 2015; Jimenez et al. 2015; Lin et al. 2016), which does not depend on the specific cosmological model. The hemisphere comparison (HC) method and dipole fitting (DF) method are usually used in literature. The hemisphere comparison method divides samples into two hemispheres perpendicular to a polar axis, then fits cosmological parameters using samples in each hemisphere independently and compares their differences. The dipole fitting (DF) method assumes a dipolar deviation on redshift-distance modulus relation, then derives the dipole’s direction and magnitude using statistic approaches. Meanwhile, low-redshift SNe Ia are used to estimate the direction and amplitude of the local bulk flow (Bonvin et al. 2006; Schwarz & Weinhorst 2007; Gordon et al. 2008; Colin et al. 2011; Turnbull et al. 2012; Kalus et al. 2013; Appleby & Shafieloo 2014; Huterer et al. 2015). So far, no study has been able to rule out the isotropy at more than 3σ. The gravitational wave as standard siren has also been proposed to probe cosmic anisotropy (Cai et al. 2017).

The directions and magnitudes of anisotropy from previous works are shown in table 1. It’s obvious that different results are derived from different authors. In this paper, we compare the DF fitting results of different SNe Ia samples and try to find the reason for the differences. This paper is organized as follows. In section 2 SNe Ia datasets and DF method
are introduced. The fitting results are shown in section 3. We discuss the possible reasons for the differences in section 4. Finally, we summarize in section 5.

2. Data and Methods

2.1. Datasets

Large-scale systematic sky surveys on SNe Ia have been performed in the past decades. These surveys, which cover a wide range of redshifts from \( z < 0.1 \) to \( z \sim 1 \), include Supernovae Legacy Survey (Astier et al. 2006; Sullivan et al. 2011, SNLS), Sloan Supernova Survey (Holtzman et al. 2008), the Pan-STARRS survey (Rest et al. 2014; Scolnic et al. 2014; Tonry et al. 2012), Harvard-Smithsonian Center for Astrophysics survey (Hicken et al. 2009, CfA), the Carnegie Supernova Project (Stritzinger et al. 2011; Folatelli et al. 2009; Contreras et al. 2009, CSP), the Lick Observatory Supernova Search (Ganeshalingam et al. 2013, LOSS), the Nearby Supernova Factory (Aldering et al. 2002, NSF), etc. Thanks to these sky surveys, a bunch of SNe Ia catalogs has been published, including “SNLS” (Astier et al. 2006), “Union” (Rubin et al. 2009), “Constitution” (Hicken et al. 2009), “SDSS” (Campbell et al. 2013), “SNLS3” (Conley et al. 2011), “Union2.1” (Suzuki et al. 2012), and “Joint Light-curve Analysis(JLA)” (Betoule et al. 2014).

In this paper, we used three SNe Ia catalogs in our analysis: Union2.1, Constitution, and JLA. Union2.1 includes 580 SNe Ia (Suzuki et al. 2012). The catalog covers samples with redshift range \( 0.015 \leq z \leq 1.414 \). Constitution catalog combines samples from Union and CfA3, containing 397 SNe Ia with redshifts in the range \( 0.015 \leq z \leq 1.55 \) (Hicken et al. 2009). The coordinate information of Constitution catalog is adopted from the Open Supernova Catalog in this paper (Guillochon, James et al. 2016). JLA catalog includes several low-redshift samples \( (z < 0.1) \), all three seasons from the SDSS-II \( (0.05 < z < 0.4) \), and three years from SNLS \( (0.2 < z < 1) \). It includes 740 SNe Ia with high-quality light curves (Betoule et al. 2014). It covers redshift range \( 0.01 \leq z \leq 1.30 \).

2.2. Dipole fitting method

Firstly, we briefly introduce the dipole fitting method. For Union2.1 and Constitution datasets, the luminosity distance could be expanded by Hubble series parameters: Hubble parameter \( H \), deceleration parameter \( q \), jerk parameter \( j \) and snap parameter \( s \). These
parameters can be expressed as functions of the scale factor $a$ and its derivatives,

$$\begin{align*}
H &= \frac{\dot{a}}{a}, \quad q = -\frac{1}{H^2} \frac{\ddot{a}}{a}, \\
\dot{j} &= \frac{1}{H^3} \frac{\ddot{a}}{a}, \quad s = \frac{1}{H^4} \frac{\ddot{a}}{a}.
\end{align*}$$

(1)

Taylor expansion of luminosity distance could be made in terms of redshift and Hubble series parameters (Visser 2004). However, this expansion diverges at $z > 1$. Thus, another parameter $y = z/(1 + z)$ is introduced to overcome this problem. The luminosity distance can be expanded as a function of $y$ (Cattoën & Visser 2007; Wang, F Y et al. 2009)

$$d_L(y) = \frac{c}{H_0} \left[ y - \frac{1}{2} (q_0 - 3) y^2 + \frac{1}{6} (11 - 5q_0 - j_0) y^3 + \frac{1}{24} (50 - 7j_0 - 26q_0 + 10q_0j_0 + 21q_0^2 - 15q_0^3 + s_0) y^4 + O(y^5) \right].$$

(2)

The distance modulus is defined as

$$\mu_{th} = 5 \log \frac{d_L}{\text{Mpc}} + 25.$$  

(3)

Then the $\chi^2$ can be calculated as

$$\chi^2(H_0, q_0, j_0, s_0) = \sum_i \frac{(\mu_{\text{obs},i} - \mu_{th,i})^2}{\sigma_i^2},$$

(4)

where $\mu_{\text{obs}}$ and $\sigma_i$ are observational values of distance moduli and their errors, respectively. The best-fitting values of parameters could be obtained by minimizing $\chi^2(H_0, q_0, j_0, s_0)$.

For the JLA sample, the observational values of distance moduli are not directly given. Therefore, we use the values obtained in the ΛCDM model to avoid fitting too many free parameters simultaneously. The theoretical luminosity distance in ΛCDM model can be expressed as

$$d_L(z) = \frac{(1 + z)c}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m (1 + z)^3 + (1 - \Omega_m)}}.$$  

(5)

Union2.1 and Constitution datasets already give $\mu_{\text{obs}}$ as a part of the released data. For JLA dataset, $\mu_{\text{obs}}$ can be derived from light curve parameters of SN Ia from (Betoule et al. 2014)

$$\mu_{\text{obs}} = m_B^* - (M_B - \alpha \times X_1 + \beta \times C),$$

(6)

where $m_B^*$ is the observed peak magnitude in the rest-frame of the B band, $X_1$ describes the time stretching of light-curve, $C$ describes the supernova color at maximum brightness and $M_B$ is the absolute B-band magnitude, which depends on the host galaxy properties. $\alpha, \beta$
are nuisance parameters. $M_B$ can be fitted by a simple step function related with $M_{\text{stellar}}$ \citep{2013MNRAS.435.1975J},

$$M_B = \begin{cases} M_B^1 & M_{\text{stellar}} < 10^{10} M_{\odot} \\ M_B^1 + \Delta M & M_{\text{stellar}} \geq 10^{10} M_{\odot}. \end{cases} \quad (7)$$

where $M_B^1$ and $\Delta M$ are nuisance parameters. $C(\alpha, \beta)$ is the total covariance matrix, which can be obtained with JLA data. $\chi^2$ is defined as

$$\chi^2_{JLA}(\alpha, \beta, M_B^1, \Delta M, \Omega_m) = (\mu_{\text{obs}} - \mu_{\text{th}})^\dagger C^{-1} (\mu_{\text{obs}} - \mu_{\text{th}}). \quad (8)$$

By minimizing $\chi^2_{JLA}$, all free parameters mentioned above can be fitted. Our best-fitting results are consistent with those of Betoule et al. (2014).

In order to quantify the anisotropic deviations on luminosity distance, we define the distance moduli with dipole $\textbf{A}$ and monopole $B$ as

$$\tilde{\mu}_{\text{th}} = \mu_{\text{th}}(1 - \textbf{A} \cdot \hat{n} + B), \quad (9)$$

where $\mu_{\text{th}}$ is the theoretical value of distance modulus with dipolar direction dependence, and $\hat{n}$ is the unit vector pointing at the corresponding SN Ia. $\textbf{A} \cdot \hat{n}$ represents the projected dipole magnitude in the direction of the given SNe Ia sample. $\hat{n}$ can be represented in galactic coordinate as

$$\hat{n} = \cos(b) \cos(l) \hat{i} + \cos(b) \sin(l) \hat{j} + \sin(b) \hat{k}. \quad (10)$$

Then the projection is

$$\textbf{A} \cdot \hat{n} = \cos(b) \cos(l) A_x + \cos(b) \sin(l) A_y + \sin(b) A_z. \quad (11)$$

The best-fitting dipole and monopole parameters can be derived with the following steps:

1. Substitute $\mu_{\text{th}}$ with $\tilde{\mu}_{\text{th}}$ in the expression of $\chi^2$ as is shown in equation (4) and equation (8).

2. Fit the dipole $(A_x, A_y, A_z)$ and the monopole component $B$ by minimizing $\chi^2$.

Finally, we analyze the likelihood of the fitted parameters and the significance of dipole magnitude utilizing Markov Chain Monte Carlo (MCMC) sampling. In order to obtain the significance of dipole anisotropy precisely, we use the Monte Carlo simulation method \citep{2012JCAP...05..041M, 2013JCAP...08..005Y, 2014JCAP...12..019W}. To be specific, we construct a type of synthetic samples based on original datasets by assuming that theoretical values of distance moduli are “real” values. We refer to these synthetic samples as “isotropic” samples. Applying MCMC sampling on these samples, probability distributions of the fitted parameters can be obtained. They are also used to probe the effect of anisotropic factors, which we will discuss in section 4.
3. Results

Fitting results are shown in Table 2. The confidence level is defined as the probability
\[ P(|A_{iso}| < |A_{fit}|) \], where \(|A_{iso}|\) is the dipole magnitude of an arbitrary dataset in “isotropic”
samples, and \(|A_{fit}|\) is the best-fitting dipole magnitude. Best-fitting dipole directions and
1\(\sigma\) errors for Union2.1, Constitutio, and JLA datasets, along with dipole fitting results of
samples are plotted in Figures 1, 2 and 3 respectively.

We generate \(2 \times 10^6\) effective samples for each dataset for MCMC sampling. Probabil-
ity distributions of dipole and monopole parameters for Union2.1, Constitution, and JLA
datasets are shown in Figures 4, 5 and 6 respectively. Note that the best-fitting param-
eters do not coincide with most probable values for some parameters. This is caused by
transformation from rectangular coordinates \((A_x, A_y, A_z)\) used in MCMC sampling to polar
coordinates \((l, b)\) and dipole magnitude \(A\).

For Union2.1 dataset, the direction and magnitude of the dipole are \((l = 309.3^\circ \pm 15.5^\circ, b =
-8.9^\circ \pm 11.2^\circ)\), and \(A = (1.46 \pm 0.56) \times 10^{-3}\). The confidence level of dipolar anisotropy is
98.3%. For Constitution dataset, these parameters are \((l = 67.0^\circ \pm 6.6^\circ, b = -0.6^\circ \pm 2.2^\circ)\), \(A =
(4.4 \pm 5.0) \times 10^{-4}\), and \(B = (-0.2 \pm 2.4) \times 10^{-4}\). The confidence level of dipolar anisotropy
is 19.7%.

It is worth mentioning that, JLA dataset gives null results in dipole fitting. The 1\(\sigma\)
error range of dipole direction covers the whole celestial sphere. The confidence level of
dipole magnitude is merely 0.23%. Furthermore, there is no significant difference between
the likelihood of simulation results in 1\(\sigma\) error range and full results. The same is true for the
likelihood of parameters of “isotropic” samples and original samples. In addition, significant
deviations exist in best-fitting values and most probable values of fitted parameters. Thus,
no significant dipolar anisotropy of redshift-distance modulus relation is found in the JLA
dataset.

For Union2.1 dataset, we get similar results as previous works (Antoniou & Perivolaropou-
los 2010; Cai & Tuo 2011; Cai et al. 2013; Mariano & Perivolaropoulos 2012; Yang et al. 2013;
Wang & Wang 2014; Lin et al. 2016). For Constitution datasets, our results are different
from those of Kalus et al. (2013). Considering different methods, and the weak signal of the
dipole in this dataset, the difference is reasonable.

For JLA dataset, we get different likelihood distributions as Lin et al. (2015), which
can be attributed to different fitting parameters used in the MCMC estimation. In this
paper, we fit the dipole by fitting its rectangular components \((A_x, A_y, A_z)\), then convert the
fitting results to spherical coordinate. As shown in Figure 7, non-zero likelihood at \(b = \pm 90^\circ\)
forms an unreasonable ‘spike’ at poles, which depends on the choice of coordinate system.
By comparison, posteriors used in this paper are in accordance with best-fitting values, and joint likelihood contours are smooth oval shape, as shown in Figure 8.

4. Discussions

4.1. Effects of anisotropy in data distribution

As shown in Figures 4, 5 and 6, even if no redshift-distance anisotropy in the input data, fitting results are still distributed an-isotropically (green dotted lines). This indicates other reasons, such as anisotropic coordinate or redshift distribution would affect the results.

In order to determine whether the coordinate distribution of samples would affect the results, we introduce two types of synthetic datasets.

**Type A datasets** substituting the coordinates in the original dataset with random coordinates uniformly distributed in the whole sky. $\mu_{\text{obs}}$ are replaced with synthetic data.

**Type B datasets** substituting the coordinates in the original dataset with random coordinates, but only uniformly distributed in the eastern hemisphere of the celestial sphere. Distance moduli are substituted the same manner as type A datasets.

Using MCMC sampling method introduced in section 2 we find the dipoles in type A datasets are uniformly distributed in the whole sky. However, the dipoles in Type B datasets tend to concentrate in $(l, b) = (90^\circ, 0^\circ)$, $(l, b) = (270^\circ, 0^\circ)$, as shown in Figure 9. Meanwhile, dipole magnitudes are generally larger than it is in Type A datasets, as shown in Figure 10. This indicates that anisotropy of coordinates of samples does affect the results of dipole fitting. For a better understanding of this case, we examine the anisotropy of datasets with the following method:

1. Define the “sample count” of a given coordinate as the number of samples in $90^\circ$ range on the celestial sphere.

2. Calculate the “mean absolute deviation” (MAD) of sample counts. It is defined as $|N_{\text{sc}} - \overline{N}_{\text{sc}}|$, where $N_{\text{sc}}$ is the sample count defined before, $\overline{N}_{\text{sc}}$ is the mean value of sample counts.

3. Draw contours of MAD of sample counts for three datasets, respectively. Then compare them with dipole coordinates in “isotropic” samples.
We find that dipole directions tend to concentrate in places where sample counts deviate from the average most, as shown in Figures 11, 12 and 13. It is also in accordance with the results of previous simulations.

The spatial distribution of redshifts can be anisotropic, i.e., the redshift of samples in one patch of the sky may be generally smaller than another patch of sky. This may also cause influence on fitting results. To extract the effects of anisotropy in redshift distribution from other factors, we introduce another kind of synthetic dataset.

**Type C datasets** shuffle the coordinates in the original dataset, making distance-related data of every sample correspond with a coordinate of another random sample in the dataset. Keep the ‘shuffled coordinates’ order unchanged, then substitute distance moduli as type A datasets.

Using the generating method described above, we can alternate the spatial distribution of redshifts without changing the distribution of coordinates. We find that the fitting results of type C datasets are fairly consistent with “isotropic” samples, as shown in Figure 14. Therefore, the anisotropy of redshift distribution does not cause a significant influence on fitting results.

5. Conclusions

In this paper, we study three different datasets of SNe Ia, namely Union2.1, Constitution, and JLA, to find possible dipolar anisotropy in redshift-distance relation. We fit the dipole and monopole parameters by minimizing $\chi^2$, then run MCMC sampling to determine the error range and confidence level of the fitting parameters.

For Union2.1 dataset, we find the direction and magnitude of the dipole are $(l = 309.3^{\circ}+15.5^{\circ}, b = -8.9^{\circ}+11.2^{\circ})$, $A = (1.46 \pm 0.56) \times 10^{-3}$. The monopole magnitude is $B = (-2.6 \pm 2.1) \times 10^{-4}$. The confidence level of dipolar anisotropy is 98.3%. For Constitution dataset, these parameters are $(l = 67.0^{\circ}+66.5^{\circ}, b = -0.6^{\circ}+25.2^{\circ})$, $A = (4.4 \pm 5.0) \times 10^{-4}$, $B = (-0.2 \pm 2.4) \times 10^{-4}$. The confidence level of dipolar anisotropy is 19.7%. For JLA dataset, fitted parameters are $l = 94.4^{\circ}$, $b = -51.7^{\circ}$, $A = 7.8 \times 10^{-4}$, $B = 1.9 \times 10^{-3}$. The $1\sigma$ error range of dipole direction covers almost the whole sky. The confidence level of dipolar anisotropy is merely 0.23%.

We also study the effects of anisotropy of coordinate and redshift distribution in dipole fitting method, and find that anisotropy distribution of coordinates can cause fitted dipole direction to concentrate in places where the mean absolute deviation is larger, cause dipole...
magnitude to become larger. However, anisotropy distribution of redshifts does not have a significant influence on fitting results.

In future, the next generation cosmological surveys, such as LSST \cite{Ivezic2008}, Euclid \cite{Amendola2016}, and WFIRST \cite{Hounsell2017} will observe much larger SNe Ia datasets with enhanced light-curve calibration, which may shed light on the anisotropy in redshift-distance relation.

Acknowledgments

This work is supported by the National Basic Research Program of China (973 Program, grant No. 2014CB845800), the Excellent Youth Foundation of Jiangsu Province (BK20140016) and the National Natural Science Foundation of China (grants 11422325 and 11373022).

REFERENCES

Aldering, G., Adam, G., Antilogus, P., et al. 2002, Proc.SPIE, 4836, 61
Aluri, P. K., Panda, S., Sharma, M., & Thakur, S. 2013, J. Cosmology Astropart. Phys., 12, 003
Amendola, L., Appleby, S., Avgoustidis, A., et al. 2016, arXiv:1606.00180
Antoniou, I., & Perivolaropoulos, L. 2010, J. Cosmology Astropart. Phys., 12, 12
Appleby, S., & Shafieloo, A. 2014, J. Cosmology Astropart. Phys., 10, 070
Astier, P., Guy, J., Regnault, N., et al. 2006, A&A, 447, 31
Betoule, M., Kessler, R., Guy, J., et al. 2014, A&A, 568, A22
Bonvin, C., Durrer, R., & Kunz, M. 2006, Phys. Rev. Lett., 96, 191302
Cai, R., & Tuo, Z. 2011, J. Cosmology Astropart. Phys., 2012, 6
Cai, R.-G., Liu, T.-B., Liu, X.-W., Wang, S.-J., & Yang, T. 2017, arXiv:1712.00952
Cai, R. G., Ma, Y. Z., Tang, B., & Tuo, Z. L. 2013, Phys. Rev. D, 87, 123522
Campanelli, L, Cea, P, Fogli, G L, & Marrone, A. 2011, Phys. Rev. D, 83
Campbell, H., D’Andrea, C. B., Nichol, R. C., et al. 2013, ApJ, 763, 88
Cattoëen, C., & Visser, M. 2007, Classical and Quantum Gravity, 24, 5985
Chang, Z., Li, X., Lin, H.-N., & Wang, S. 2014, EPJC, 74, 2821
Chang, Z., Lin, H. N., Sang, Y., & Wang, S. 2017, arXiv:1711.11321
Colin, J., Mohayaee, R., Sarkar, S., & Shafieloo, A. 2011, MNRAS, 414, 264
Conley, A., Guy, J., Sullivan, M., et al. 2011, ApJS, 192, 1
Contreras, C., Hamuy, M., Phillips, M. M., et al. 2009, AJ, 139, 519
de Oliveira-Costa, A., Tegmark, M., Zaldarriaga, M., & Hamilton, A. 2004, Phys. Rev. D, 69, 063516
Folatelli, G., Phillips, M. M., Burns, C. R., et al. 2009, AJ, 139, 120
Ganeshalingam, M., Li, W., & Filippenko, A. V. 2009, AJ, 139, 120
Gordon, C., Land, K., & Slosar, A. 2008, MNRAS, 387, 371
Guillochon, James, Parrent, Jerod, Kelley, Luke Zoltan, & Margutti, Raffaella. 2016, ApJ, 835
Hicken, M., Wood-Vasey, W. M., Blondin, S., et al. 2009, ApJ, 700, 1097
Holtzman, J. A., Marriner, J., Kessler, R., et al. 2008, AJ, 136, 2306
Hounsell, R., Scolnic, D., Foley, R. J., et al. 2017, arXiv:1702.01747
Huterer, D., Shafer, D. L., & Schmidt, F. 2015, J. Cosmology Astropart. Phys., 12, 033
Hutsemékers, D., Cabanac, R., Lamy, H., & Sluse, D. 2005, A&A, 441, 915
Ivezic, Z., Tyson, J. A., Abel, B., et al. 2008, arXiv:0805.2366
Javanmardi, B., Porciani, C., Kroupa, P., & Pflamm-Altenburg, J. 2015, ApJ, 810, 47
Jimenez, J. B., Salzano, V., & Lazkoz, R. 2015, Phys. Lett. B, 741, 168
Johansson, J., Thomas, D., Pforr, J., et al. 2013, MNRAS, 435, 1680
Kalus, B., Schwarz, D. J., Seikel, M., & Wiegand, A. 2013, A&A, 553, 56
King, J. A., Webb, J. K., Murphy, M. T., et al. 2012, MNRAS, 422, 3370
Koyama, K., 2016, Rept. Prog. Phys., 4, 046902
Kroupa, P., 2012, PASA, 29, 395
Kroupa, P., Pawlowski, M., & Milgrom, M. 2012, IJMPD, 21, 1230003
Li, X., Lin, H.-N., Wang, S., & Chang, Z. 2013, EPJC, 73, 2653
Lin, H. N., Li, X., & Chang, Z. 2016, MNRAS, 460, 617
Lin, H. N., Wang, S., Chang, Z., & Li, X. 2015, MNRAS, 456, 1881
Longo, M. J. 2009, arXiv:0904.2529
Mariano, A., & Perivolaropoulos, L. 2012, Phys. Rev. D, 86
Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, ApJ, 517, 565
Perivolaropoulos, L. 2014, Galaxies, 2, 22
Phillips, M. M. 1993, AJ, 413, L105
Rest, A., Scolnic, D., Foley, R. J., et al. 2014, ApJ, 795, 44
Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, AJ, 116, 1009
Rubin, D., Linder, E. V., Kowalski, M., et al. 2009, ApJ, 695, 391
Schwarz, D. J., & Weinhorst, B. 2007, A&A, 474, 717
Scolnic, D. M., Riess, A. G., Foley, R. J., et al. 2014, ApJ, 780, 37
Stritzinger, M., Phillips, M. M., S, L. B., et al. 2011, AJ, 142
Sullivan, M., Guy, J., Conley, A., et al. 2011, ApJ, 737, 102
Suzuki, N., Rubin, D., Lidman, C., et al. 2012, ApJ, 746, 85
Tegmark, M., De Oliveira-Costa, A., & Hamilton, A. J. S. 2003, Phys. Rev. D, 68
Tonry, J. L., Stubbs, C. W., Lykke, K. R., et al. 2012, ApJ, 750, 99
Turnbull, S. J., Hudson, M. J., Feldman, H. A., et al. 2012, MNRAS, 420, 447
Vielva, P., Martínez-Gonzalez, E., Barreiro, R. B., et al. 2004, ApJ, 609, 22
Visser, M. 2004, Classical and Quantum Gravity, 21, 2603
Table 1: Incomplete list of previous works on cosmological preferred directions.

| Authors                  | Sample Used | Method   | \(l, b\)                  | Anisotropy Level | C.L. |
|--------------------------|-------------|----------|---------------------------|------------------|------|
| Antoniou & Perivolaropoulos (2010) | Union2 HC | (306°, 15°) | \(\Delta \Omega_m/\Omega_m = 0.42\) | 70%              |
| Cai & Tuo (2011)          | Union2 HC  | (314°±2°, 28°±11°) | \(\Delta q_0/q_0 = 0.79^{+0.27}_{-0.28}\) |                 |
| Cai et al (2013)          | Union2+67GRB MC | (305°, -13°) | \(g_0 = 0.030_{-0.030}^{+0.010}\) | \(2\sigma\) |
| Mariano & Perivolaropoulos (2012) | Union2 DF | (309° ± 18.0°, -15.1° ± 11.5°) | \(A = (1.3 ± 0.6) \times 10^{-3}\) | \(2\sigma\) |
| Kalus et al. (2013)       | Keck+VLT HC | (320.5° ± 11.8°, -11.7° ± 7.5°) | \(A = (1.02 ± 0.25) \times 10^{-5}\) | \(3.9\sigma\) |
| Yang et al. (2013)        | Constitution HC | (-35°, -19°) | \(\Delta H/H \sim 0.026\) | 95%              |
| Wang & Wang (2014)        | Union2.1 DF | No Significance | No Significance | \(A = (1.2 ± 0.5) \times 10^{-3}\) | 95.45% |
| Lin et al. (2015)         | Union2.1+116GRB DF | (307.2° ± 16.2°, -14.3° ± 10.1°) | \(A = (1.37 ± 0.57) \times 10^{-3}\) | 97.29% |
| Lin et al. (2016)         | JLA DF | No Significance | No Significance | \(\Delta \Omega_m/\Omega_m = 0.306\) | 37%  |
| Zhao, W., Wu, P. X. & Zhang, Y., 2013, Int. J. Mod. Phys. D, 22, 1350060 | Union2 HC | (241°, -19°) | \(D = (1.2 ± 0.5) \times 10^{-3}\) | 95.9% |
Table 2: Best-fitting results of dipole and monopole for three datasets

| Datasets | Union 2.1       | Constitution     | JLA   |
|----------|-----------------|------------------|-------|
| $l$      | $309.3^\circ \pm 15.3^\circ$ | $67.0^\circ \pm 66.5^\circ$ | $94.4^\circ$ |
| $b$      | $-8.9^\circ \pm 11.2^\circ$ | $-0.6^\circ \pm 25.2^\circ$ | $-51.7^\circ$ |
| $A$      | $(1.46 \pm 0.56) \times 10^{-3}$ | $(4.4 \pm 5.0) \times 10^{-4}$ | $7.8 \times 10^{-4}$ |
| $B$      | $(-2.6 \pm 2.1) \times 10^{-4}$ | $(-0.2 \pm 2.4) \times 10^{-4}$ | $1.9 \times 10^{-3}$ |
| C.L.     | $98.3\%$        | $19.7\%$         | $0.23\%$ |

Fig. 1.— Best-fitting dipole direction (star) and 1σ error range of Union2.1 dataset. Scatter points represents dipole fitting results of simulating samples.
Fig. 2.— Similar as Figure 1 for Constitution dataset.

Fig. 3.— Best-fitting dipole direction (star) of JLA dataset. Scatter points represents dipole directions and magnitudes generated by MCMC sampling of original samples.
Fig. 4.— Blue lines show marginalized likelihoods of dipole $A$, monopole $B$ and $(l,b)$ for Union2.1 dataset, and black vertical lines represent best-fitting values. Green dotted lines represent results of “isotropic” samples.
Fig. 5.— Similar as Figure 4 for Constitution dataset.
Fig. 6.— Similar as Figure 4 for JLA dataset.
Fig. 7.— Probability distribution functions of rectangular components of dipole $A$ and monopole $B$ for JLA dataset directly using $(l, b, A)$ as fitted parameters.
Fig. 8.— Probability distribution functions of rectangular components of dipole $A$ and monopole $B$ for JLA dataset directly using $(A_x, A_y, A_z)$ as fitted parameters.
Fig. 9.— Distribution of dipole directions of type B samples. Dipole directions concentrate near crossed positions.
Fig. 10.— Likelihood of dipole magnitude $A$ of different samples. Blue solid line indicates result of synthetic samples with isotropically distributed coordinates, which tend to reduce the dipole magnitude. Green dotted line indicates result of synthetic samples with extremely an-isotropically distributed coordinates, which increase the dipole magnitude.
Fig. 11.— Sample count and dipole distribution of Union2.1 dataset. The contour shows mean absolute deviations, which represent how far the sample density near a specific point differs from the average density.

Fig. 12.— Similar as figure 11 but for Constitution dataset.
Fig. 13.— Similar as figure 11 but for JLA dataset.
Fig. 14.— Likelihood of parameters for type C samples and isotropic samples. No significant deviation caused by different spatial distribution of redshifts is found.