On the forward-backward charge asymmetry in $e^+e^-$-annihilation into hadrons at high energies

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The forward-backward asymmetry in $e^+e^-$-annihilation into a quark-antiquark pair is considered in the double-logarithmic approximation at energies much higher than the masses of the weak bosons. It is shown that after accounting to all orders for the exchange of virtual photons and $W, Z$-bosons one is lead to the following effect (asymmetry): quarks with positive electric charge (e.g. $u, d$) tend to move in the $e^+$-direction whereas quarks with negative charges (e.g. $d, \bar{u}$) tend to move in the $e^-$-direction. The value of the asymmetry grows with increasing energy when the produced quarks are within a cone with opening angle, in the cmf, $\theta_0 \sim \frac{2M_Z}{\sqrt{s}}$ around the $e^+e^-$-beam. Outside this cone, at $\theta_0 \ll \theta \ll 1$, the asymmetry is inversely proportional to $\theta$.

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I. INTRODUCTION

The standard theoretical description of $e^+e^-$-annihilation into hadrons at high energies starts with the sub-process of $e^+e^-$-annihilation into quarks and gluons, which is then studied with perturbative methods. It is usually considered as mediated by the exchange of all electroweak (EW) bosons: $e^+e^- \rightarrow \gamma^*, Z, W \rightarrow q\bar{q} + $gluons. One of the most well-known and successful predictions of the Standard Model is the forward-backward asymmetry, which has been studied for many years both theoretically and experimentally, particularly around the Z boson \cite{1}. Next linear colliders will explore $e^+e^-$-annihilation at very high energies, probing further the Standard Model and eventually looking for New Physics. As it is well known, pure QED also gives rise to a forward-backward asymmetry even at low energies, albeit small, due to interference of one-photon and two-photon exchange diagrams. This effect persists at asymptotically high energies due to the multiphoton contributions in higher orders in $\alpha$. Such multiphoton contribution in $e^+e^- \rightarrow \mu^+\mu^-$ was studied in Refs. \cite{2} in the double-logarithmic (DL) approximation (DLA). The annihilation process in \cite{2} was considered in the following two kinematical regions:

(i) Forward kinematics, when, in the center of mass frame (cmf), the outgoing $\mu^+$ ( $\mu^-$) goes in the direction of the initial $e^+$ ($e^-$).

(ii) Backward kinematics, when the outgoing $\mu^+$ ($\mu^-$) goes in the $e^-$ ($e^+$)-direction.

These kinematical configurations refer to the case when the initial positive (or negative) electrical charges do not change the direction after the scattering, or they are affected by a major - almost backward - deviation. It was shown in \cite{2} (see also the review \cite{3}) that at high energies the radiative DL corrections to the Born amplitudes are quite different for the forward and the backward kinematics. As a result, the cross section of the forward annihilation dominates over the backward one and therefore there is a charge forward-backward asymmetry: positive muons tend to go in the $e^+$-beam direction and negative muons rather follow the direction of $e^-$. In the present paper we generalize these results, accounting also for DL contributions of multiple W,Z exchanges. We calculate below the forward and backward scattering amplitudes for $e^+e^-$-annihilation into $q\bar{q}$-pair at energies much higher than $M_Z$ and estimate the dependence of the asymmetry on the scattering angle and on the total energy of the process. We account for multiphoton exchanges as well as for multi- W,Z exchanges to all orders in the EW couplings in DLA. To this aim we construct and solve in DLA an infrared evolution equations (IREE) for the backward and forward scattering amplitudes. In doing so we follow the approach of Refs. \cite{4, 5}. Recently this approach was used\cite{6} for studying the double logarithmic asymptotics for lepton-antilepton backward scattering, for

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scattering angles $\theta \sim \pi$ in the EW theory. Furthermore, in Ref. \[3\] it is considered the case where the initial (final) state consists of a left (right) lepton and its antiparticle. However, because there is an essential difference in the EW theory in the description of the processes involving the left and the right fermions, in the present paper we generalize those results and consider the forward and backward annihilation with both left and right initial and final states. We also study these processes in a wider angular region $\theta \ll 1$. Although we use the Feynman gauge through this paper, all our results are gauge invariant. The paper is organized as follows. Instead of calculating amplitudes of $e^+e^- \rightarrow$ quarks directly, we find more convenient to operate with $SU(2)$ - invariant amplitudes of a more general process, the lepton-antilepton annihilation into quark-antiquark pair. Then in Sect. 2 we introduce such invariant amplitudes and show their relation to the forward and backward amplitudes for $e^+e^-$ -annihilation. The IREE for the invariant amplitudes are constructed in Sect. 3 and solved in Sect. 4. The solutions are obtained in terms of the Mellin amplitudes corresponding to collinear kinematics. The IREE for the Mellin amplitudes are obtained and solved in Sect. 5. Then, we define and estimate the forward-backward asymmetry in Sect. 6. Finally, we discuss our results in Sect. 7.

II. INVARIANT AMPLITUDES FOR LEPTON-ANTILEPTON ANNIHILATION INTO $q \bar{q}$.

We are going to account for the DL effects of exchanging the EW bosons to $e^+e^-$ -annihilation into quark-antiquark pairs of different flavours. When multiple $W$ - exchange is taken into account, the flavour of the virtual intermediate fermion state is not fixed, though the initial and final states of the annihilation are well-defined. Because the EW theory organizes all fermions into doublets of the left particles and right singlets, this suggests that is more convenient to calculate first the scattering amplitude of a more general process, the annihilation of a lepton and its antiparticle into a quark - antiquark pair, and only after to specify the flavour of the initial and final states. This turns to be easier because the effects of the violation of the initial $SU(2) U(1)$ symmetry are in many respects neglected within the double-logarithmic accuracy. On the other hand that also means that the DLA can be applied safely only when the energy of the annihilation is much higher than $M_Z$, $M_W$. At such energies the propagators of the $SU(2)$ - gauge bosons $W_a (a = 1, 2, 3)$, $D_{ab}(k) \sim \delta_{ab}/k^2$. The propagator of the $U(1)$ -gauge boson $B$ is $1/k^2$ in the same approximation. The $SU(2)$ vertices of the $W_a$ interaction with the left fermions are $g t^a$, where $t^a$ are the $SU(2)$ generators and $g$ is the coupling, whereas the vertex of the interaction of the field $B$ with the left and the right fermions is $g' Y/2$, $Y$ being the hypercharge and $g'$ being the coupling. As in the most general process both the initial and final particles can be left and/or right, we consider all these cases separately.

In this Sect. we consider the general case of the annihilation of the left lepton $l_k(p_1)$ belonging to the doublet ($\nu$, $e$) and the antilepton $\bar{l}_i(p_2)$ from the charge conjugated doublet into the left quark $q_i(p_1')$ belonging to the doublet ($u$, $d$) and the antiquark $\bar{q}_i(p_2')$ from the charge conjugated doublet. Therefore, the scattering amplitude $M$ of this process is

$$M = \bar{l}_i(-p_2) \bar{q}_i(p_1') \tilde{M}_{ki} ^{i'i} l_k(p_1) q_i'(−p_2')$$

(1)

where the matrix amplitude $\tilde{M}_{ki} ^{i'i}$ has to be calculated perturbatively. We consider the kinematics where, in the cmf, both particles of the produced pair move close to the lepton-antilepton beam. It corresponds to two kinematics:

(i) forward kinematics when

$$-t = -(p_1' - p_1)^2 << s = (p_1 + p_2)^2 \approx -u = -(p_2' - p_1)^2 ,$$

(2)

(ii) backward kinematics when

$$-u = -(p_2' - p_1)^2 << s = (p_1 + p_2)^2 \approx -t = -(p_1' - p_1)^2 .$$

(3)

Then, replacing in (1) the lepton-antilepton pair by $e^-$, $e^+$ and the quark-antiquark pair by $\mu^-$, $\mu^+$ respectively, we see immediately that the electric charge almost does not change its direction in the forward kinematics (2) while it’s reversed in the backward kinematics (3). Obviously that does not apply strictly for the annihilation into quarks because the electric charges of $u$ -quarks and $d$ -quarks are different in sign. Therefore $t$ -kinematics is “forward” for the annihilation into a $d \bar{d}$ -pair and at the same moment it is ”backward” for the annihilation into $u \bar{u}$ quarks. We will come back to this definition of backward and forward kinematics later, when we shall discuss the annihilation into $u \bar{u}$ and $d \bar{d}$ pairs, but until then we refer to (2) as $t$ - kinematics, and (3) as $u$ - kinematics.
In order to simplify the isospin structure, it is convenient to expand the matrix $\tilde{M}_{kk'}^i$ into a sum, each term corresponding to some irreducible representation of $SU(2)$. In the $t$ - kinematics (2), the initial $t$ - channel state is $l^k(p_1)q_\nu(p'_1)$. Obviously,

$$l^k(p_1)q_\nu(p'_1) = \left[ \frac{1}{2} \delta^k_0 \delta^\nu_0 + (\delta^k_0 \delta^\nu_0 - \frac{1}{2} \delta^k_0 \delta^\nu_0) \right] l^0(p_1)q_\nu(p'_1) + \left[ \frac{1}{2} \delta^k_0 \delta^\nu_0 + 2 \sum_c (t_c)^k (t_c)^\nu \right] l^c(p_1)q_\nu(p'_1)$$  \hspace{1cm} (4)

where the first term corresponds to the scalar and the second one – to the triplet representation of $SU(2)$. Eq. (4) suggests the representation

$$\tilde{M}_{kk'}^i = (P_0)^{ii'}_{kk'} \tilde{M}_0 + (P_1)^{ii'}_{kk'} \tilde{M}_1$$  \hspace{1cm} (5)

where $\tilde{M}_{0,1}(s, t)$ are scalar functions and the singlet and triplet projection operators correspondingly are:

$$(P_0)^{ii'}_{kk'} = \frac{1}{2} \delta^k_0 \delta^\nu_0, \quad (P_1)^{ii'}_{kk'} = 2(t_c)^k (t_c)^\nu .$$  \hspace{1cm} (6)

Similarly for the $u$ - kinematics (3), the initial $u$ - channel state is $l^k(p_1)q_\nu(-p'_2)$, irreducible $SU(2)$ representations are obtained by symmetrization and antisymmetrization,

$$\tilde{M}_{kk'}^i = (P_-)^{ii'}_{kk'} \tilde{M}_-(u, s) + (P_+)^{ii'}_{kk'} \tilde{M}_+(u, s) ,$$  \hspace{1cm} (7)

with

$$(P_-)^{ii'}_{kk'} = \frac{1}{2} \left[ \delta^k_0 \delta^\nu_0 - \delta^k_0 \delta^\nu_0 \right], \quad (P_+)^{ii'}_{kk'} = \frac{1}{2} \left[ \delta^k_0 \delta^\nu_0 + \delta^k_0 \delta^\nu_0 \right] .$$  \hspace{1cm} (8)

Using the projectors $P_j , j = 0, 1, -, +$, the invariant amplitudes can be easily obtained:

$$\tilde{M}_j = \frac{(P_j)^{kk'}_{ii'} (\tilde{M})^{ii'}_{kk'}}{(P_j)^{kk'} (\tilde{M})^{ii'}_{kk'}.}$$  \hspace{1cm} (9)

In the Born approximation, the amplitudes $\tilde{M}_j$ defined in Eqs. (6,7) can be written as

$$\tilde{M}_j^{Born} = R A_j^{Born}(s) , \quad A_j^{Born}(s) = \frac{s}{s + \epsilon^+} a_j ,$$

$$R = \frac{\bar{u}(-p_2)[(1-\gamma_5)/2] \gamma_\nu [(1+\gamma_5)/2] u(p_1) \bar{u}(p'_1)[(1-\gamma_5)/2] \gamma_\nu [(1+\gamma_5)/2] u(-p'_2)}{s}$$  \hspace{1cm} (10)

where $R$ denotes the normalized spinor factor and $A_j^{Born}$ are scalar functions of $s$, differing only in constant group factors $a_j$.

As we discuss the particular case of the left fermions we can drop the factors $[(1 \pm \gamma_5)/2]$ and use the following definition:

$$R = \frac{\bar{u}(-p_2) \gamma_\nu u(p_1) \bar{u}(p'_1) \gamma_\nu u(-p'_2)}{s} .$$  \hspace{1cm} (11)

For the left particles, the lepton and the quark hypercharges are $Y_l = -1$ and $Y_q = 1/3$ respectively. The group factors $a_j$ for the Born amplitudes are

$$a_0 = \frac{3g^2 + g'^2 Y_l Y_q}{4}, \quad a_1 = \frac{-g^2 + g'^2 Y_l Y_q}{4}, \quad a_- = \frac{-3g^2 + g'^2 Y_l Y_q}{4}, \quad a_+ = \frac{g^2 + g'^2 Y_l Y_q}{4} .$$  \hspace{1cm} (12)
The contributions proportional to $Y_l Y_q$ in Eq. (12) come from the Born graph where the lepton line is connected to the quark one by the $B$-field, the other contributions come from the Born graphs with propagators of $W_i$-fields. Accounting for all DL corrections transforms the coefficients $a_j$ into invariant amplitudes $M_j$, 

$$\tilde{M}_j = R M_j(s,u,t),$$  
(13)

where, in DLA, $M_j$ depend on $s$, $t$, $u$ through logarithms. When $M_j$ are calculated, Eqs. (13) allow us to immediately express the amplitudes $e^+ e^- \to u\bar{u}$ and $e^+ e^- \to d\bar{d}$ in terms of $M_j$;

$$M(e^+ e^- \to u\bar{u}) = R M_1(s,t),$$
$$M(e^+ e^- \to d\bar{d}) = R [M_0(s,t) + M_1(s,t)]/2,$$  
(14)

for the annihilation in $t$-kinematics (2). Similarly,

$$M(e^+ e^- \to u\bar{u}) = R [M_-(s,t) + M_+(s,t)]/2,$$
$$M(e^+ e^- \to d\bar{d}) = R M_+(s,t),$$  
(15)

for the annihilation in $u$-kinematics (3).

### III. EVOLUTION EQUATIONS FOR THE INVARIANT AMPLITUDES $M_j$.

In this section we calculate $M_j$ in the high energy limit, by constructing and solving an IREE for them, as a generalization of the evolution equations derived earlier in QCD. This approach exploits the evolution of scattering amplitudes with respect to the infrared cut-off $\mu$ in the transverse momentum space. Transverse momenta of all virtual particles are supposed to obey

$$k_{i\perp} > \mu, \quad k_{i\perp} \perp p_1, p_2.$$  
(16)

The value of the cut-off $\mu$ must not be smaller than any of the involved masses, otherwise it is arbitrary. Introducing $\mu$ makes also possible to neglect the masses of all involved quarks and to restrict ourselves to consider the evolution of $M_j$ with respect to $\mu$ only. Then one can take in the final formulas $\mu$ of order of the largest mass involved. In DLA we can also neglect the difference between the masses of EW bosons $M_W$ and $M_Z$, putting in the final expressions

$$\mu = M = M_Z \approx M_W.$$  
(17)

First we consider the annihilation in $t$-kinematics and construct the IREE for $M_j$ with $j = 0, 1$. According to (2), $t$ is small compared to $u, -s$. To bound it from below we assume that

$$s \gg -t \gg \mu^2.$$  
(18)

The main idea of the IREE consists in evolving the invariant amplitudes with respect to the infrared cut-off $\mu$ by applying to them the differential operator

$$-\mu^2 \partial/\partial \mu^2,$$  
(19)

in the form

$$-\mu^2 \frac{\partial M_j}{\partial \mu^2} = \frac{\partial M_j}{\partial \rho} + \frac{\partial M_j}{\partial \eta},$$  
(20)

where we use $u \approx -s$ in this forward kinematics and have introduced the notations

$$\rho = \ln(s/\mu^2), \quad \eta = \ln(-t/\mu^2).$$  
(21)
In order to obtain the rhs of last equation, we have to take into account the factorization of DL contributions of virtual particles with respect to $\mu$, where $\mu$ is the lowest limit of integration over $k_\perp$. In turn, this minimal $k_\perp$ acts as a new cut-off for other virtual momenta (see\ [3] for details). When the virtual particle with the minimal $k_\perp$ is a EW boson, one can factorize its DL contributions as shown in Fig. 1. Applying then the differentiation\ [19] and the projection operators $P_j$ we obtain with the help of Eq. (17) the contributions $G_0, G_1$ to the EW singlet and triplet parts of the IREE respectively.

Before writing the explicit expressions for $G_{0,1}$, we want to discuss the general structure of these contributions. Integration over longitudinal momentum of the factorized boson with momentum $k$ in graphs (a) and (b) in Fig. 1 yields $\ln(-s/k_\perp^2)$ whereas the same integration in graphs (c) and (d) yields $\ln(-u/k_\perp^2)$. Similarly, graphs (e),(f) yield $\ln(-t/k_\perp^2)$. Besides these logarithms, each graph in Fig. 1 contains also an integration of $M_j(s/k_\perp^2,t/k_\perp^2)/k_\perp^2$ over $k_\perp^2$, with the lowest limit $\mu^2$. After differentiation\ [19], we arrive at

$$G_{0,1} = \frac{1}{8\pi^2} \sum_{j=0,1} \left[ \left( b^{(j)}_{s} \right)_{0,1} \ln \left( \frac{-s}{\mu^2} \right) - \left( b^{(j)}_{u} \right)_{0,1} \ln \left( \frac{-u}{\mu^2} \right) \right] M_j \left( \frac{s}{\mu^2}, \frac{t}{\mu^2} \right) - \frac{1}{8\pi^2} h_{0,1} \ln \left( \frac{-t}{\mu^2} \right) M_{0,1} \left( \frac{s}{\mu^2}, \frac{t}{\mu^2} \right),$$

(22)

where the the quantities $\left( b^{(j)}_{s} \right)_{0,1}, ..., h_{0,1}$ will be explicitly given later.

Let us introduce

$$\rho^{(\pm)} = \frac{1}{2} \left[ \ln \left( \frac{-s}{\mu^2} \right) \pm \ln \left( \frac{-u}{\mu^2} \right) \right]$$

(23)

so that $\ln(-s/\mu^2) = \rho^{(+)} + \rho^{(-)}$ and $\ln(-u/\mu^2) = \rho^{(+)} - \rho^{(-)}$. Obviously, $\rho^{(+)}$ and $\rho^{(-)}$ are symmetrical and antisymmetrical functions with respect to replacing $s$ by $u$. It is convenient also to introduce the invariant amplitudes $M_{0,1}^{(\pm)}$ with the same properties :  

$$M_{0,1}^{(\pm)} = \frac{1}{2} \left[ M_{0,1} \left( \frac{s}{\mu^2}, \frac{t}{\mu^2} \right) \pm M_{0,1} \left( \frac{u}{\mu^2}, \frac{t}{\mu^2} \right) \right],$$

(24)

so that $M_{0,1} = M_{0,1}^{(+)} + M_{0,1}^{(-)}$. Then for signature amplitudes $G_{0,1}^{(\pm)}$ defined as

$$G_{0,1}^{(\pm)} = \frac{1}{2} \left[ G_{0,1}(s,t) \pm G_{0,1}(u,t) \right]$$

(25)

from Eq. (22) we obtain the following expressions

$$G_{0,1}^{(+)} = \frac{1}{8\pi^2} \sum_{j=0,1} \left[ \left( b^{(j)} \right)_{0,1}^{(+)} \rho^{(+)} M_j^{(+)} + \left( b^{(j)} \right)_{0,1}^{(-)} \rho^{(-)} M_j^{(-)} \right] + \frac{1}{8\pi^2} h_{0,1} \eta M_{0,1}^{(+)},$$

$$G_{0,1}^{(-)} = \frac{1}{8\pi^2} \sum_{j=0,1} \left[ \left( b^{(j)} \right)_{0,1}^{(+)} \rho^{(+)} M_j^{(-)} + \left( b^{(j)} \right)_{0,1}^{(-)} \rho^{(-)} M_j^{(+)} \right] + \frac{1}{8\pi^2} h_{0,1} \eta M_{0,1}^{(-)}$$

(26)

where

$$\left( b^{(j)} \right)_{0,1}^{(\pm)} = \left( b^{(j)}_{s} \right)_{0,1}^{(\pm)} \mp \left( b^{(j)}_{u} \right)_{0,1}^{(\pm)}.$$

(27)

Besides an EW boson, in kinematics\ [2] a $t$-channel virtual fermion pair, as shown in Fig. 2a, could also attain the minimal transverse momentum. However, DL contributions arising from the integration over this pair momentum could only come from the region $k_\perp^2 > -t \gg \mu^2$. Hence they do not depend on $\mu$ in kinematics\ [18] and must vanish when differentiated with respect to $\mu$. The same is true for the Born amplitudes\ [10].

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1 In the Regge theory, amplitudes $M^{(\pm)}$ are called the positive and negative signature amplitudes, and we use these notation below.
As soon as \( \ln(-s/\mu^2) = \ln(s/\mu^2) - \pi \), in kinematics \([2]\) with \( \rho = \ln(s/\mu^2) \approx \ln(-u/\mu^2) \) we obtain

\[
\rho^{(+)} = \rho - \frac{\pi}{2} \text{sign}(s), \\
\rho^{(-)} = -\frac{\pi}{2} \text{sign}(s). 
\]  

(28)

It is assumed in DLA that \( \ln(s/\mu^2) \gg \pi \). This means that

\[
\rho^{(+) \gg \rho^{(-)}. 
\]  

(29)

Similarly, in DLA in each order of the perturbative expansion the amplitudes \( M_j^{(+)} \) dominate over \( M_j^{(-)} \) for one power of \( \ln(s/\mu^2) \). By the same reason the amplitudes \( M_j^{(+)} \) are mainly real, and we can assume

\[
M_j^{(+)} \approx \Re M_j^{(+)} \gg |M_j^{(-)}|. 
\]  

(30)

Combining Eqs. (28,24) and using Eqs. (30,28,29) leads us to the following IREE where negligible in DLA terms \( M^{(-)} \rho^{(-)} \) are dropped and terms \( M^{(-)} \rho^{(+)} \sim M^{(-)} \rho^{(-)} \) are retained:

\[
\begin{align*}
\frac{\partial M_{0,1}^{(+)}(\rho)}{\partial \rho} + \frac{\partial M_{0,1}^{(+)}(\eta)}{\partial \eta} &= \frac{1}{8\pi^2} \rho \left[ \sum_{j=0,1} (b(j))_{0,1}^{(+)} M_{j}^{(+)} \right] + \frac{1}{8\pi^2} \eta M_{0,1}^{(+)}, \\
\frac{\partial M_{0,1}^{(-)}(\rho)}{\partial \rho} + \frac{\partial M_{0,1}^{(-)}(\eta)}{\partial \eta} &= \frac{1}{8\pi^2} \rho \left[ \sum_{j=0,1} (b(j))_{0,1}^{(-)} M_{j}^{(-)} \right] + \frac{1}{8\pi^2} \eta M_{0,1}^{(-)}. 
\end{align*} 
\]  

(31)

Let us proceed now to the \( u \) - kinematics \([3]\). Using projection operators \([8]\) instead of \([2]\) one can consider the annihilation in the \( u \) -kinematics and obtain the IREE for invariant signature amplitudes \( M_{0,1}^{(\pm)} \), \( M_{+}^{(\pm)} \) introduced in a way similar to that one used for \( M_{0,1}^{(\pm)} \). As the amplitudes \( M_{-}^{(\pm)} \), \( M_{+}^{(\pm)} \) correspond to \( SU(2) \) singlet and triplet representations, similarly to \( M_{0,1}^{(\pm)} \), we can easily obtain IREE for \( u \) -kinematics from Eq. (31) with the replacement

\[
t \rightarrow u, \quad "0" \rightarrow "-", \quad "1" \rightarrow "++", \quad Y_q \rightarrow -Y_q. 
\]  

(32)

Indeed, adding the restriction

\[
s \gg -u \gg \mu^2 
\]  

(33)

to Eq. (3), the derivation is quite similar to the previous one done for the \( t \) -kinematics thus leading to the same structure of the IREE for the amplitudes \( M_{0,1}^{(\pm)} \). Therefore one can write down the same IREE for all invariant signature amplitudes \( M_{j}^{(\pm)} \), with \( j = 0, 1, "-", "++", \) generalizing Eq. (31).

The next significant simplification of (32) comes from the explicit calculation of the group factors \((b(j))_{0,1}^{(\pm)}\). It turns out that

\[
(b^{(0)})_{0,1}^{(+)} = (b^{(1)})_{0,1}^{(+)} = (b^{(-)})_{0,1}^{(+)} = (b^{(1)})_{0,1}^{(-)} = 0. 
\]  

(34)

and consequently the IREE for the positive signature amplitudes become linear homogenous partial differential equations. They can be written in a more simple general way:

\[
\frac{\partial M_{j}^{(+)}(\rho)}{\partial \rho} + \frac{\partial M_{j}^{(+)}(\eta)}{\partial \eta} = -\frac{1}{8\pi^2} [b_j \rho + h_j \eta] M_{j}^{(+)}, 
\]  

(35)
where \( b_j, b_j, \) and \( \eta' \) will be specified below, so that the only difference between the equations for different amplitudes comes from the numerical factors \( b_j, h_j: \)

\[
\begin{align*}
b_0 &= \frac{g^2(Y_l - Y_q)^2}{4}, & b_1 &= \frac{8g^2 + g^2(Y_l - Y_q)^2}{4}, \\
b_- &= \frac{g^2(Y_l + Y_q)^2}{4}, & b_+ &= \frac{8g^2 + g^2(Y_l + Y_q)^2}{4},
\end{align*}
\]  

(36)

\[
\begin{align*}
h_0 &= \frac{3g^2 + g^2Y_lY_q}{2}, & h_1 &= \frac{-g^2 + g^2Y_lY_q}{2}, \\
h_- &= \frac{3g^2 - g^2Y_lY_q}{2}, & h_+ &= \frac{-g^2 - g^2Y_lY_q}{2}.
\end{align*}
\]  

(37)

The IREE for the negative signature amplitudes \( M_j(-) \) are also linear partial differential equations, but not being homogenous, they involve positive signature amplitudes through a non-zero matrix \( r_{jj'}: \)

\[
\frac{\partial M_j(-)}{\partial \rho} + \frac{\partial M_j(-)}{\partial \eta'} = -\frac{1}{8\pi^2} \left[ (b_j \rho + h_j \eta') M_j(-) + \left(\frac{-i\pi}{2}\right) \sum_{j'} r_{jj'} M_{j'}(+) \right].
\]  

(38)

In order to write down the IREE for all \( M_j(-) \) and \( M_j(+) \) in the same way, we have used in Eqs. (35,38) the variable \( \eta' \) so that

\[
\eta' \equiv \eta = \ln(-t/\mu^2)
\]  

(39)

for \( t \)-kinematics and

\[
\eta' \equiv \chi = \ln(-u/\mu^2)
\]  

(40)

for \( u \)-kinematics.

The non-zero numerical factors \( r_{jj'} \) in Eq. (38) are:

\[
\begin{align*}
r_{00} &= r_{11} = \frac{g^2(Y_l + Y_q)^2}{4}, & r_{01} &= r_{-+} = 3g^2, & r_{10} &= r_{+-} = g^2, & r_{--} &= r_{++} = \frac{g^2(Y_l - Y_q)^2}{4}.
\end{align*}
\]  

(41)

IV. SOLUTIONS TO IREE FOR THE INVARIANT AMPLITUDES \( M_j(\pm) \).

It is easy to check that a general solution to Eq. (35) is

\[
M_j(+) = \Phi_j(+) (\rho - \eta') \exp[-\phi_j(C, \eta')] ,
\]  

(42)

with

\[
\phi_j = \frac{1}{8\pi^2} \left[ b_j C \eta' + (b_j + h_j) \frac{\eta'^2}{2} \right],
\]  

(43)

\( C = \rho - \eta' = \text{const} \) and \( \Phi_j \) is an arbitrary function. We can specify it by imposing the boundary condition

\[
M_j(+) (\rho, \eta') \bigg|_{\eta' = 0} = A_j(+) (\rho) ,
\]  

(44)
where $A_j^{(\pm)}(\rho)$ are the amplitudes for the annihilation in the “collinear kinematics”, i.e. in the kinematics where quarks are produced close to the direction of the beam of initial leptons, with the value of either $t$ or $u$ much smaller than those fixed by Eqs. (18,33). Of course one must use a separate boundary condition (44) for the $t$ and $u$-kinematics. We define the notation “collinear $t$-kinematics” for

$$-t < \mu^2$$

and the notation “collinear $u$-kinematics” for

$$-u < \mu^2.$$ (46)

It is convenient to use the Mellin transform to represent signature amplitudes $A_j^{(\pm)}$ in the “collinear kinematics” (45,46):

$$A_j^{(\pm)}(\rho) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \left( \frac{s}{\mu^2} \right)^{\omega} \xi^{(\pm)}(\omega) F_j^{(\pm)}(\omega)$$ (47)

where

$$\xi^{(\pm)} = \frac{\exp(-i\pi\omega) \pm 1}{2}$$ (48)

are the well known signature factors. At asymptotically high energy $s$ the region of small $\omega$, $\omega \ll 1$, is dominating in integral (47). This allows one to exploit the following approximations:

$$\xi^{(+)} \approx 1, \quad \xi^{(-)} \approx -\frac{i\pi\omega}{2}$$ (49)

Eq. (47) implies that the positive signature amplitudes $M_j^{(+)}(\rho, \eta')$ in the kinematic regions (18,33) can be easily expressed through the Mellin amplitudes $F_j^{(\pm)}(\omega)$:

$$M_j^{(+)}(\rho, \eta') = \exp[-\phi_j(C, \eta')] \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} e^{(\rho-\eta')\omega} \xi^{(+)}(\omega) F_j^{(+)}(\omega).$$ (50)

On the other hand, as stated earlier, the IREE (38) for the negative signature amplitudes $M_j^{(-)}$ are not homogeneous, in contrast to Eq. (35). Besides the amplitudes $M_j^{(-)}$, they also involve the positive signature amplitudes $M_j^{(+)}$. In order to solve Eq. (38), we have to use again the boundary condition

$$M_j^{(-)}(\rho, \eta') \bigg|_{\eta'=0} = A_j^{(-)}(\rho).$$ (51)

It is easy to check that the solution to the Eq. (38) satisfying Eq. (51) is

$$M_j^{(-)}(\rho, \eta') = \exp[-\phi_j(C, \eta')] \left[ A_j^{(-)}(\rho - \eta') - \int_0^{\eta'} d\tau \exp[\phi_j(C, \tau)] \left( \frac{-i\pi}{2} \right) \left( \frac{1}{8\pi^2} \right) \sum_j' r_{jj'} M_j^{(+)}(C, \tau) \right].$$ (52)

Applying the Mellin transforms (47) and (50) for the amplitudes in Eq. (52) we can rewrite it through the Mellin amplitudes $F_j^{(\pm)}(\omega)$ as well. In order to simplify this procedure we use the substitution

$$\left( \frac{-i\pi}{2} \right) \xi^{(+)} \approx \frac{1}{\omega} \xi^{(-)},$$ (53)
which follows from the approximation \( \xi \) and is reasonable when the small \( \omega \) region is dominating in the integral. Eventually we arrive at

\[
M_j^{(-)}(\rho, \eta') = \exp[-\phi_j(C, \eta')] \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} e^{\omega(\rho-\eta')} \xi^{(-)}(\omega) \left[ F_j^{(-)}(\omega) - \frac{1}{8\pi^2} \sum_j' r_{jj'} \frac{F_j^{(+)}(\omega)}{\omega} \int_0^{\eta'} d\tau \exp[\phi_j(C, \tau) - \phi_{j'}(C, \tau)] \right].
\]

Eqs. (50,54) show how one can obtain the amplitudes \( M_j^{(+)} \) when the Mellin amplitudes \( F_j^{(+)}(\omega) \) are calculated.

**V. IREE FOR THE MELLIN AMPLITUDES \( F_j^{(\pm)} \).**

In order to calculate \( F_j^{(\pm)} \) we have to construct IREE for “collinear kinematics” amplitudes \( A_j^{(\pm)}(\rho) \). The IREE for them differ from the IREE for amplitudes \( M_j^{(\pm)} \) considered in the previous section, by the following reasons.

(i) The amplitudes \( A_j^{(\pm)} \) in the kinematics \( 43,44 \) depend on \( \rho = \ln(s/\mu^2) \) only, and some graphs in Fig. 1 do not yield DL contributions to IREE for them. In particular, graphs (c) and (f) with factorized \( t \)-channel virtual bosons do not contribute to IREE for \( A_{0,1}^{(\pm)} \) while graphs (c),(d) with factorized \( u \)-channel bosons do not contribute to IREE for \( A_{-,1}^{(\pm)} \). The lhs of the IREE for \( A_j^{(\pm)} \) turns to \(-\mu^2 \partial A_j^{(\pm)}/\partial \mu^2 \) that in terms of the Mellin variable \( \omega \) results in \( \omega F_j^{(\pm)}(\omega) \).

(ii) The DL contributions of the graphs in Fig. 2 also depend on \( \mu^2 \) and therefore do not vanish when differentiated with respect to \( \mu \). These graphs are combinations of two amplitudes, their contributions become simpler after applying the Mellin transform \( 45 \). Then the differentiation \(-\mu^2 \partial/\partial \mu^2 \) of these graphs leads to the following contribution to the IREE for the Mellin transforms \( F_j^{(\pm)} \):

\[
\frac{c_j}{8\pi^2} \left[ F_j^{(\pm)}(\omega) \right]^2,
\]

with \( c_j = 1 \) for \( j = 0,1 \) and \( c_j = -1 \) for \( j = -\), “+, + “.

(iii) Though at the first sight the Born amplitudes \( A_j^{\text{Born}} \) of Eq. 10 do not depend on \( \mu^2 \) it is necessary to replace them by \( a_j s/(s - \mu^2 + \imath \epsilon) \). This form explicitly tells that \( s \) cannot be smaller than \( \mu^2 \) and also it makes the Mellin transform for the Born amplitudes to be correctly defined. The Mellin transforms for Born amplitudes are therefore \( a_j/\omega \). Applying \(-\mu^2 \partial/\partial \mu^2 \) to them results in multiplying by \( \omega \). Hence the contributions of the Born amplitudes to IREE are just the constant terms \( a_j \).

As a result we arrive to the following IREE for Mellin amplitudes \( F_j^{(\pm)}(\omega) \).

\[
\omega F_j^{(+)}(\omega) = a_j + \frac{b_j}{8\pi^2} \frac{dF_j^{(+)}(\omega)}{d\omega} + \frac{c_j}{8\pi^2} \left[ F_j^{(+)}(\omega) \right]^2,
\]

\[
\omega F_j^{(-)}(\omega) = a_j + \frac{b_j}{8\pi^2} \frac{1}{\omega} \left( \omega F_j^{(-)}(\omega) \right) + \frac{c_j}{8\pi^2} \left[ F_j^{(-)}(\omega) \right]^2 - \sum_j' r_{jj'} \frac{F_j^{(+)}(\omega)}{8\pi^2}.
\]

The coefficients \( a_j, b_j, c_j \) and \( r_{jj'} \) are listed in Table 1 for \( t \)-kinematics and in Table 1 for \( u \)-kinematics. Solutions to Eq. (56) can be expressed in terms of the Parabolic cylinder functions \( D_p \).

\[
F_j^{(\pm)}(\omega) = \frac{a_j D_{p_{j-1}}(\omega/\lambda_j)}{\lambda_j D_{p_j}(\omega/\lambda_j)}
\]
\[ \lambda_j = \sqrt{\frac{b_j}{8\pi^2}}, \quad p_j = \frac{a_j c_j}{b_j}. \]

In contrast, solutions to Eq. (57) can be found only numerically. In QED the negative signature amplitudes for the backward \( e^+e^- \rightarrow \mu^+\mu^- \) annihilation were solved in Ref. [1]. It is interesting to note that \( b_0 = 0 \) in the IREE for the forward amplitudes \( M_{ep}^{(\pm)} \) of \( e^+e^- \rightarrow \mu^+\mu^- \)-annihilation, and the differential equations (56) for \( M_{ep}^{(\pm)} \) in kinematics (13) turn into purely algebraic equations. This result was first obtained in Ref. [2]. Later it was proved [3] that the IREE for the (colourless) scalar components of the \( SU(3) \) negative signature amplitudes of quark-antiquark annihilation into another quark-antiquark pair are also algebraic and therefore can be easily solved. The processes mentioned above are the only known examples of solving IREE for negative signature amplitudes. It has been observed [10] in a QCD context, that these amplitudes can be approximated the difference amounts only to a few percents. Equations for the negative signature amplitudes always involve the intercepts of the negative signature amplitudes are greater than those for the positive signature amplitudes, though the difference amounts only to a few percents. Equations for the negative signature amplitudes always involve the positive signature amplitudes. It has been observed [10] in a QCD context, that these amplitudes can be approximated by their Born values with good accuracy. Such an approximation can help in solving Eqs. (57). We do not consider explicit solutions of Eqs. (57) in the present paper. Instead, we consider below only contributions of amplitudes with the positive signature \( M_j^{(+)} \). Combining Eqs. (56, 58) and introducing variable \( x = \omega/\lambda_j \), we arrive at the expression

\[
M_j^{(+)}(\rho, \eta') = a_j \exp[-\phi_j(\rho - \eta', \eta')] \int_{-\infty}^{\infty} \frac{dx}{2\pi i} e^{x(x(\rho-\eta') + D_{p_j}(x))}. \tag{60}
\]

It is useful here to split \( \phi_j \) defined by Eq. (43) and to combine its part depending on \( \rho - \eta' \) with the exponent of the integrand in Eq. (60). Then changing the integration variable \( x \) to \( \eta \), where \( x = \eta' + \lambda_j \eta' \) we finally obtain

\[
M_j^{(+)}(\rho, \eta') = a_j \exp \left[ -\frac{b_j + h_j}{8\pi^2} \eta'^2 \frac{2}{2} \right] \int_{-\infty}^{\infty} \frac{d\eta'}{2\pi} e^{\lambda_j x(\rho-\eta') + D_{p_j}(x)} \frac{D_{p_j}(l + \lambda_j \eta')}{D_{p_j}(l + \lambda_j \eta')} \tag{61}
\]

where, for the case of \( t \)-kinematics with \( j = 0, 1, \eta' = \eta = \ln(-t/\mu^2) \) and for the case of \( u \)-kinematics with \( \eta' = \chi = \ln(-u/\mu^2) \), \( j = \sigma - \sigma', \sigma = \sigma' \). The exponential factor in front of the integral in Eq. (61) is of Sudakov type. Actually it is a product of the Sudakov form factors of the left lepton and of the left quark. As follows from Eqs. (56, 57), it does not depend on \( j \), i.e. is same for all invariant amplitudes,

\[
S = \exp \left[ -\frac{1}{8\pi^2} \left( \frac{3}{2} g^2 + \frac{Y_t^2 + Y_q^2}{4} g^2 \right) \frac{\eta'^2}{2} \right]. \tag{62}
\]

It corresponds to DL contributions of soft virtual EW bosons and vanishes in the final expressions for the cross sections when bremsstrahlung of soft EW bosons are taken into account. Assuming this to be done we can omit such Sudakov factors.

Until now we have discussed the annihilation \( \bar{t}t \rightarrow q\bar{q} \) for the case when the both initial and final particles were left, i.e. the spinors in Eq. (11) were actually \( [(1 + \gamma_5)/2]u \). It is clear that applying the same reasoning it is easy to construct IREE for amplitudes in \( t \) and \( u \)-kinematics with right fermions. Solutions to such IREE can be presented in the same form of Eq. (61) with \( j = RR \) for both right leptons and quarks, and \( j = LR \) when the initial leptons are left whereas the final quarks are right, or \( j = RL \) vice versa, with:

\[
a_{RR} = \frac{g^2 Y_t^2}{4}, \quad \lambda_{RR} = \sqrt{\frac{2a_{RR}}{8\pi^2}}, \quad b_{RR} = \frac{g^2 (Y_t^2 + Y_q^2)}{4}, \quad h_{RR} = \pm \frac{g^2 Y_q}{2}, \tag{63}
\]
\[
a_{LR} = \frac{g^2 Y_t^2}{4}, \quad \lambda_{LR} = \sqrt{\frac{2a_{LR}}{8\pi^2}}, \quad b_{LR} = \frac{3g^2 (Y_t^2 + Y_q^2)}{4}, \quad h_{LR} = \pm \frac{g^2 Y_q}{2}, \tag{63}
\]

\[2\] In the context of the EW theory, the IREE for the backward scattering amplitude with the negative signature was solved in [4] for the unrealistic case of a complex value of the Weinberg angle.
where $\mp$ signs in $b$ and $\pm$ signs in $h$ correspond to $t$ and $u$ - kinematics respectively. It is worthwhile to remind here that in the numerical estimations when using Eq. (63) one has to substitute for $Y_L, Y_R$ for right and left fermions the correct EW hypercharges: $Y = 2Q$ for right fermions and $Y = 2(Q - T_3)$ for left fermions. The same formulae for the invariant amplitude $M_{L,R}^{(+)}$ in the collinear $u$ - kinematics, with $\chi = 0$, can be also obtained from results of Ref. [3].

\[ \text{VI. FORWARD – BACKWARD ASYMMETRY} \]

In this section we consider in detail the $e^+e^- -$ annihilation into quark-antiquark pairs of different flavours in the configuration of forward and backward kinematics. According to the terminology introduced in Ref. [2], also discussed earlier in section II, the forward kinematics for the annihilation of $e^+e^- \to \mu^+\mu^-$ - kinematics defined by Eq. (2) because the electric charges of the electron and the $u$ - quark are opposite. On the contrary, the forward kinematics for the annihilation of $e^+e^- \to d\bar{d}$ corresponds to the $t$ - kinematics, as we have defined in Eq. (3). Similarly, the backward kinematics for $e^+(p_1)e^-(p_2) \to u(p'_1)\bar{u}(p'_2)$ -annihilation is a $t$ - kinematics and that for $e^+e^- \to d\bar{d}$ is the $u$ - channel one. From the results we have obtained in the section II it follows that the amplitude for forward $e^+e^- \to uu$ -annihilation, $M_u^F$ is expressed in terms of amplitudes $M_-, M_+$ of Eq. (61):

\[ M_u^F = \frac{M_- + M_+}{2} \approx \frac{M_-^{(+)} + M_+^{(+)}}{2}, \]

whereas the backward amplitude $M_u^B$ for the same quarks is expressed through the amplitude $M_1$ of Eq. (61):

\[ M_u^B = M_1 \approx M_1^{(+)}. \]

Similarly, the forward amplitude $M_d^F$ for $e^+e^- \to d\bar{d}$ is

\[ M_d^F = \frac{M_0 + M_1}{2} \approx \frac{M_0^{(+)} + M_1^{(+)}}{2}, \]

while the backward amplitude for this process is

\[ M_d^B = M_0 \approx M_0^{(+)}. \]

By forward kinematics for $e^+e^- \to q\bar{q}$ -annihilation we mean that quarks with positive electric charges, $u$ and $\bar{d}$, are produced around the initial $e^+$ - beam, in the cmf, within a cone with a small opening angle $\theta$,

\[ 1 \gg \theta \geq \theta_0 = \frac{2M}{\sqrt{s}} \]

By backward kinematics we means just the opposite – the electric charge scatters backwards in a cone with the same opening angles.

The differential cross section $d\sigma_F$ for the forward annihilation is

\[ d\sigma_F = d\sigma^{(0)} \left[ |M_u^F|^2 + |M_d^F|^2 \right] \equiv d\sigma^{(0)} \left[ F_u + F_d \right], \]

and similarly, the differential cross section $d\sigma_B$ for the backward annihilation is

\[ d\sigma_B = d\sigma^{(0)} \left[ |M_u^B|^2 + |M_d^B|^2 \right] \equiv d\sigma^{(0)} \left[ B_u + B_d \right], \]

where $d\sigma^{(0)}$ stands for the Born cross section, though without couplings. It absorbs the factor $|R|^2$ defined by Eq. (11) which cancels in the expression for the forward-backward asymmetry which we define as

\[ A = \frac{d\sigma_F - d\sigma_B}{d\sigma_F + d\sigma_B} = \frac{F - B}{F + B}. \]
where

\[ F = F_u + F_d, \quad B = B_u + B_d. \] (72)

Before presenting numerical results we would like to discuss the asymptotic behaviour of the asymmetry \( A \). Let’s discuss the angular region defined by (68). The Regge factors \((s/t)^\omega, (s/u)^\omega\) are equal to \(\theta^{-2\omega}\) in this region. Eq. (61) states that the asymptotics of all scattering amplitudes \( M^{(+)} \) is determined by the position of the rightmost zero of the Parabolic cylinder function \( D_{p_j}(z) \). The location of the zeros of \( D_{p_j}(z) \) depends on the value of \( p_j \). In vicinity of the rightmost zero \( z_0(p_j) \), \( D_{p_j}(z) \) can be represented as

\[ D_{p_j}(z) = D'_{p_j}(z_0(p_j))(z - z_0(p_j)) \] (73)

which, after substitution in Eq. (61) leads to the following expression for the high energy asymptotics of \( M^{(+)}_j \):

\[ M^{(+)}_j \propto \theta^{-2\lambda_j z_0(p_j)}. \] (74)

When \( p_j < 0 \), all zeros of \( D_{p_j}(z) \) are in the left half of the complex \( z \)-plane and their real parts are negative, so that \( M^{(+)}_j(\theta) \) oscillates and decreases when \( \theta \to \theta_0 \). We remind that all \( \lambda_j \) are positive.

When \( 0 < p_j < 1 \), the rightmost zero becomes real but still negative, so in this case \( M^{(+)}_j(\theta) \) again decreases when \( \theta \to \theta_0 \), though without oscillations.

Finally, when \( p_j > 1 \), the rightmost zero is real and positive, so that \( M^{(+)}_j(\theta) \) increases when \( \theta \to \theta_0 \).

On the other hand, in the angular region of small \( \theta \)

\[ \theta_0 \geq \theta \] (75)

in Eq. (74) \( 1/\theta^2 \) must be replaced by \( s/\mu^2 \). Therefore, \( M^{(+)} \) grows with \( s \) in the angular region (73) when \( p_j > 1 \) and decreases when \( p_j < 1 \).

Basically, finding zeros of \( D_p(z) \) functions is a rather tedious procedure. In order to simplify it, one can use the fact that, at integer \( p \),

\[ D_{p-1}(z)/D_{p}(z) = \sqrt{2}H_{p-1}(z/\sqrt{2})/H_{p}(z/\sqrt{2}) \] (76)

where \( H_p \) are the Hermite polynomials. Now, finding the zeros of a polynomial is easier that for \( D_p(z) \). As \( p_j \) are expressed through \( a_j, b_j, c_j \) (see Eq. (59)), with \( a_j, b_j \) and \( c_j \) given in Table 1, it is easy to see that \( M^{u,d}_{p,0} \) grow while \( M^{u,d}_{p,1} \) fall when \( \theta \to 0 \). Indeed, the small - \( \theta \) asymptotics of \( M^{u}_p \) is controlled by the rightmost zeros of \( D_{p-} \) and \( D_{p+} \).

Eq. (59) and Table 1 give \( p_+ < p_- \) and

\[ p^u_+ \equiv \max \{ p_-, p_+ \} = p_- = \frac{3g^2 - g'^2Y_iY_q}{g'^2(Y_i + Y_q)^2} \approx 25. \] (77)

The small - \( \theta \) asymptotics of \( M^{d}_p \) is controlled by the rightmost zeros of \( D_{p_0} \) and \( D_{p_1} \). Eq. (59) and Table 1 give \( p_1 < p_0 \) and

\[ p^d_+ \equiv \max \{ p_0, p_1 \} = p_0 = \frac{3g^2 - g'^2Y_iY_q}{g'^2(Y_i + Y_q)^2} \approx 6. \] (78)

Both \( u \)- quarks in Eq. (77) and \( d \)- quarks in Eq. (78) are left ones. Amplitudes \( M^{B}_{u}, M^{B}_{d} \) are controlled by \( D_p \) with \( p^B_u \equiv p_1 \) and \( p^B_d \equiv p_+ \) respectively, and both \( p^B_u \) and \( p^B_d \) are negative. Therefore, the asymmetry \( A \), which is small at relatively large \( \theta \), grows up when \( \theta \) decreases down to \( \theta_0 \). When the produced quarks are within the “collinear” angular region (73), the asymmetry \( A \) does not depend on \( \theta \) but turns to depend on \( s \). Besides, the forward amplitudes \( M^{F}_{u,d} \) increase with \( s \) in this angular region whereas the backward amplitudes \( M^{F}_{u,d} \) decrease. As
a result, the asymmetry $A$ is $s$-dependent in the “collinear” angular region \( \theta \geq \theta_0 \) and increases up to 1 when $s \to \infty$. Results of numerical calculations of $A$ in “collinear kinematics” for different quark flavours are presented in Fig. 3.

The curves below 100 GeV were calculated in pure QED with the infrared cut-off $\mu = 300$ MeV for the case of light $u$, $d$ and $s$- quarks and with the cut-off $\mu$ equal to quark mass for heavy quark flavours. Obviously, the value of the asymmetry is the same for the produced quarks and antiquarks (though they go in opposite directions). The energy region in vicinity of 100 GeV is close to the threshold of EW- boson production and therefore the DL radiative corrections at such energies come from virtual photon exchanges whereas radiative corrections involving the weak bosons are non-logarithmic.

Experimentally, quarks manifest themselves through hadron jets. Then to observe the effect of the asymmetry one could measure asymmetry in production of a leading charged hadron $H^\pm$ (a meson $M^\pm$ or a barion $B^\pm$) inside a narrow cone along initial $e^-$ (or $e^+$) beam:

$$A(H) \equiv \frac{d\sigma(H^-) - d\sigma(H^+)}{d\sigma(H^-) + d\sigma(H^+)}.$$  \hspace{1cm} (79)

As $\mathcal{M}^- = \{UD\}$ and $\mathcal{B}^- = \{U\bar{U}D\}$ where $U$ denotes any of $u$, $c$, $t$- quarks and $D$ stands for any of $d$, $s$, $b$- quarks, one should expect a more rapid rise with the annihilation energy $\sqrt{s}$ for $A(\mathcal{M})$ than for $A(\mathcal{B})$. The reason is that at very high energies only $\bar{U}$ and $D$ are produced in $e^-$- beam direction whereas production of $\bar{U}$ and $\bar{D}$- quarks is suppressed as negative. Both $\bar{U}$ and $\bar{D}$- quarks can become constituent quarks of a leading $\mathcal{M}^-$ and cannot be constituent quarks of a leading $\mathcal{M}^+$. Non-constituent quark contributions due to fragmentation $\bar{U} \to \bar{M}^+$ and $D \to M^+$ are subtracted in the numerator of Eq. (79) and can be suppressed in the denominator by choosing the minimal momentum fraction $x_{min}$ in the definition of the leading hadron to be large enough. For the case of barion production a leading $\bar{U}$ can become a constituent quark of $\mathcal{B}^-$, but now a leading $D$- quark can become a constituent of $\mathcal{B}^+$ as well. So, a leading proton can also be observed in the direction of $e^-$- beam. At asymptotically high energies, the contribution of $\bar{U}$- quark will dominate over contribution of $D$- quark and $A(\mathcal{B})$ will tend to unity, though slower than $A(\mathcal{M})$. However the numerical estimations for high but finite energies shown in Fig. 4 demonstrate a sizable difference in the energy dependence between $A(\mathcal{B})$ and $A(\mathcal{M})$. It is worthwhile to note here that the phenomenological hadronization factors for mesons, $W(\bar{U} \to \mathcal{M}^-) = W(D \to \mathcal{M}^-) = W(U \to \mathcal{M}^+) = W(\bar{D} \to \mathcal{M}^+)$, are cancelled in Eq. (79) for $A(\mathcal{M})$. But the barion asymmetry $A(\mathcal{B})$ involves the ratio

$$R = \frac{W(U \to B^+)}{W(D \to B^+)}. \hspace{1cm} (80)$$

The barion curve in Fig. 4 corresponds to $R = 2.5$ which is in a good agreement with $u$ and $d$- quarks fragmentation simulated with the JETSET for $x_p > 0.3$ and $E_{\text{quark}} = 200 \div 2000$ GeV. A more detailed study of hadron asymmetry with thorough account of nonleading and non-constituent contributions will be done elsewhere.

The IREE (56,57) also describe $l\bar{l}$ -annihilation into another lepton-antilepton pair, with the lepton being left, providing $Y_q$ is replaced by the lepton hypercharge. Therefore, Eq. (79) can be used also for studying the forward-backward asymmetry for $l\bar{l} \to l\bar{l}'$ -annihilation. Let us note that for such purely leptonic processes, the forward (backward) kinematics is identical to $t \ (u)$ - kinematics.

VII. DISCUSSION

Forward-backward asymmetry in $e^+e^-$ annihilation into a quark-antiquark pair has been considered in the double-logarithmic approximation at energies much higher than the masses of the weak bosons, taking into account to all orders the exchange of virtual photons and $W, Z$ -bosons. In deriving our results we have not considered other channels than the annihilation into a quark-antiquark pair. Of course, other channels could be present, which could change the cross section of the reaction, but will not contribute in DLA to the forward-backward asymmetry. Therefore we expect, at very high energies, that accounting for other channels will not affect the asymmetry. For example, besides EW effects, there are perturbative QCD corrections to both the forward and backward annihilation. In DLA, these corrections result into multiplying $M_F$ and $M_B$ by the same Sudakov form factor

$$S_{\text{QCD}} = \exp \left[ -\frac{\alpha_s}{3\pi} \ln^2 \left( \frac{s}{\mu^2} \right) \right]\hspace{1cm} (81)$$

which suppresses the soft gluon emission. However, these Sudakov form factor effects cancel when the gluon bremsstrahlung is taken into account. The same is true for the soft photon emission.
In addition to the soft emission, there are also important QED and QCD corrections accounting for hard photon and gluon bremsstrahlung. Indeed, the impact of this “harder” emission should be considered in more detail. Here we just notice that according to the results of Ref. the forward-backward asymmetry is due to the DL contributions from the multiphoton exchanges only. As stated earlier, the process obtained in that analysis coincides with our corresponding case of Eqs. (61), (63). Also, for annihilation at low cm energies, when the asymmetry $A$ in QED is due to the DL contributions from the multiphoton exchanges only, we can separate the arguments of Eqs. (61) but not change the position of $z_0(p)$ and therefore it should not lead to big changes in our formulae for the forward-backward asymmetry. As stated earlier, the $l^+ \rightarrow l^+$-annihilation in the kinematics $s \gg -u \sim M^2$ and with chiralities of the fermions corresponding to the particular case $j = LR$ of Eqs. (61) (corresponding to the EW singlet amplitude) was considered in Ref. We are grateful to V.A. Khoze and M.G. Ryskin for discussion of effects of hadronization to the barion asymmetry.

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\[ F_0 = \frac{3g^2+g^2(3g^2)}{4}, \quad F_1 = \frac{-2g^2+g^2(3g^2)}{4}, \quad F_{RR} = \frac{g^2}{4}, \quad F_{LR} = \frac{g^2}{4} \]

| $F_i$ | $a_j$ | $b_j$ | $c_j$ | $r_{ij}$ | $r_{ij}$ | $p_i$ | $q_i$ |
|-------|-------|-------|-------|----------|----------|-------|-------|
| $F_0$ | $3g^2$ | $g^2/4$ | $3g^2$ | $g^2/4$ | $3g^2/4$ | $q_i$ |
| $F_1$ | $-2g^2$ | $g^2/4$ | $-2g^2$ | $g^2/4$ | $-2g^2/4$ | $q_i$ |
| $F_{RR}$ | $g^2/4$ | $g^2/4$ | $g^2/4$ | $g^2/4$ | $g^2/4$ | $q_i$ |
| $F_{LR}$ | $g^2/4$ | $g^2/4$ | $g^2/4$ | $g^2/4$ | $g^2/4$ | $q_i$ |

TABLE I: The coefficients of IREE Eqs. (60, 61) for $t$-kinematics. The angle $\theta$ here is the Weinberg angle.

\[ F_0 = \frac{-3g^2+3g^2(Y_1-Y_2)^2}{4}, \quad F_1 = \frac{-3g^2+3g^2(Y_1-Y_2)^2}{4}, \quad F_{RR} = \frac{3g^2}{4}, \quad F_{LR} = \frac{3g^2}{4} \]

| $F_i$ | $a_j$ | $b_j$ | $c_j$ | $r_{ij}$ | $r_{ij}$ | $p_i$ | $q_i$ |
|-------|-------|-------|-------|----------|----------|-------|-------|
| $F_0$ | $-3g^2$ | $g^2/4$ | $-3g^2$ | $g^2/4$ | $-3g^2/4$ | $q_i$ |
| $F_1$ | $-3g^2$ | $g^2/4$ | $-3g^2$ | $g^2/4$ | $-3g^2/4$ | $q_i$ |
| $F_{RR}$ | $g^2/4$ | $g^2/4$ | $g^2/4$ | $g^2/4$ | $g^2/4$ | $q_i$ |
| $F_{LR}$ | $g^2/4$ | $g^2/4$ | $g^2/4$ | $g^2/4$ | $g^2/4$ | $q_i$ |

TABLE II: The coefficients of IREE Eqs. (60, 61) for $u$-kinematics. The angle $\theta$ here is the Weinberg angle.
FIG. 1: Contribution to IREE from soft EW boson factorization in different channels: s-channel – a and b, u-channel – c and d, t-channel – e and f.

FIG. 2: Contribution to IREE from soft fermion intermediate state in t-channel – a and in u-channel – b.
FIG. 3: Charge asymmetry $A$ for $e^+e^- \rightarrow \bar{q}q$ for different quark flavors in the “collinear angular region”. Curves 1–4 were obtained in pure QED (valid below threshold of EW bosons production):

1 - for $u$- quark with the infrared cut-off $\mu = 300$ MeV;
2 - for $d$ and $s$ -quarks with the same cut-off;
3 - for $c$- quark with the infrared cut-off $\mu = 1.4$ GeV;
4 - for $b$- quark with the infrared cut-off $\mu = 4.5$ GeV.

Curves $U$ and $D$ correspond to any of $u, c, t$- quarks and $d, s, b$- quarks respectively and were calculated in DLA for EW theory with the infrared cut-off $\mu = 100$ GeV.

FIG. 4: Estimation of charge asymmetry $A$ of leading charged hadrons in $e^+e^-$ annihilation: The curve $\mathcal{M}$ is for the meson asymmetry and the curve $\mathcal{B}$ is for the asymmetry of barions.