Generalized ordered weighted harmonic averaging operator with trapezoidal neutrosophic numbers for solving MADM problems

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Abstract
Harmonic mean is suitable for algebraic calculation and other mathematical treatments and also suitable for directly aggregated negative indicators. In many different situations, harmonic mean improves the flexibility. In this paper, we develop some new aggregation operators under neutrosophic environment and apply with multi attribute decision making (MADM) problems. First, we provide a Single valued trapezoidal neutrosophic Generalized ordered weighted harmonic averaging (SVTNOWHA) operator which is the extension of single valued trapezoidal neutrosophic ordered weighted harmonic averaging (SVTNOWHA) operator. To fix the operators on the mount, we have tested these methods in few illustrative examples, and the results have been presented.

Keywords Single valued trapezoidal neutrosophic numbers · Harmonic averaging operator · Multi-attribute decision making

1 Introduction
Aggregation operators are mathematical functions used to combine the information. The mathematical and behavioral are properties of aggregation operators. In 1970’s Multi criteria decision making (MCDM) problems active in research area. It is concerned with structure and planning of the problems and to analyzing how to solve these decisions, invoked by multiple criteria. It is the process of selecting the best alternative from the predefined alternatives. The arithmetic mean, geometric mean and harmonic mean are the most well-known aggregation operators. By the comparison of arithmetic and geometric mean operators, the advantage of harmonic mean operator is directly aggregating negative indicators. Zadeh (1965) introduced the concept of fuzzy set, which deals with vagueness and uncertainty of real world situations. Wang and Fan (2003) introduced the concept of fuzzy ordered weighted averaging (FOWA) operator. Xu and Da (2002) developed fuzzy weighted harmonic mean operator, fuzzy OWH operator and fuzzy hybrid harmonic operators, these aggregation operators reduced interval or real numbers. By considering the non-membership degree to the concept of fuzzy set, Atanassov (1986) proposed the concept of an intuitionistic fuzzy set which is characterized by membership degree and non-membership degree. Wang and Zhong (2009) proposed the concept of weighted arithmetic and geometric average operators with intuitionistic environment. Wan and Yi (2016) developed trapezoidal intuitionistic fuzzy numbers with power geometric operators. Das and Guha (2015) proposed new aggregation operators Trapezoidal intuitionistic fuzzy weighted power harmonic mean (TriFWPHM) and discussed with some special case of TriFWPHM operator. Wan and Zhu (2016) proposed triangular intuitionistic fuzzy Bonferroni harmonic aggregation operators. Das and Guha (2017) developed four kinds of aggregation operators which are TriFWHM, TriFOWHM, TriFIOWHM, TrOFhHM based on harmonic mean operators under trapezoidal intuitionistic fuzzy numbers. Many researchers have used neutrosophic sets in decision making.

Smarandache (1998) introduced the concept of neutrosophic set theory which is an extension of fuzzy set and intuitionistic fuzzy set. Wang et al. (2005) developed interval neutrosophic sets, single valued neutrosophic sets and multi-valued neutrosophic sets. Irfan and Yusuf
(2014) introduced the concept of single valued trapezoidal neutrosophic weighted aggregation (SVTNWAO) operator and applied it to the multi criteria decision making problem. Ye (2015a) defined a trapezoidal neutrosophic set and its operational rules such as score and accuracy functions. He proposed trapezoidal neutrosophic number weighted arithmetic averaging (TNNNWA) and trapezoidal neutrosophic number weighted geometric averaging (TNNNWGA) operators to deal with multiple attribute decision making problems. Ye (2016a) developed a multi attribute decision making method based on trapezoidal neutrosophic weighted arithmetic averaging (TNWA) operator and Trapezoidal neutrosophic weighted geometric averaging (TNWGA) operator and investigate their properties. Ye (2015b) presented a simplified neutrosophic harmonic averaging projection measure. Ye (2016) developed expected values of neutrosophic linguistic numbers (NLN), and also he established weighted arithmetic and geometric aggregations operators with NLN. Zhikang and Ye (2017) proposed hybrid weighted arithmetic and geometric aggregation operators, hybrid ordered weighted arithmetic and geometric operator under single valued neutrosophic number information and utilized these operators to solve multiple attribute decision making problem. Deli (2018) introduced geometric and arithmetic aggregation operators including single valued trapezoidal neutrosophic (SVTN) ordered weighted geometric operator, SVTN-hybrid geometric operator, SVTN-ordered weighted arithmetic operator, SVTN-hybrid arithmetic operator and also developed an operator for multi attribute group decision making problem. Deli (2019) proposed novel defuzzification method of SV-trapezoidal neutrosophic numbers and multi-attribute decision making. Deli and Subas (2017) proposed the ratio ranking method which is the extension of the concepts of value and ambiguities ranking function with single valued trapezoidal neutrosophic numbers. Irfan and Yusuf (2017) introduced some weighted geometric operators with SVTrN numbers. Surapati and Rama (2018) extended the TrNWAA operator and Hamming distance which deals with VIKOR strategy to MAGDM problems in neutrosophic environment. Surapati and Rama (2019) developed TODIM strategy under neutrosophic environment.

Harish and Nancy (2018) established novel hybrid aggregation operators based on geometric and arithmetic operators under single valued and interval neutrosophic numbers and applied multi-criteria decision making problem. Pranab et al. (2018) developed distance measure based MADM strategy with interval trapezoidal neutrosophic numbers. Pranab et al. (2018a) developed excepted value of trapezoidal neutrosophic numbers and applied multi attribute group decision making problems. Pranab et al. (2018b) established a Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) Strategy of MADM problems in neutrosophic environment. Jana et al. (2018) introduced the interval trapezoidal neutrosophic number weighted arithmetic averaging operator (ITNNWAA) and the interval trapezoidal neutrosophic number weighted geometric averaging operator (ITNNWG) and developed a multi attribute decision making problem. Harish and Nancy (2019) defined power aggregation operators for the linguistic single valued neutrosophic set (LSVNS) and proposed a group decision making problems. Bharatraj and Anand (2019) introduced a power harmonic weighted aggregation operator with a single valued trapezoidal neutrosophic number and Interval valued neutrosophic set and developed multi criteria decision making problem. Chiranjibe et al. (2020) utilized Hamacher aggregation operators in single valued trapezoidal neutrosophic arithmetic and geometric operator and developed a multi attribute decision making problems. Surapati and Rama (2020) developed multi-objective optimisation by ratio analysis (MOORA) strategy to solve multi-attribute group decision making (MAGDM) in trapezoidal neutrosophic numbers. Tuhin and Nirmal Kumar (2020) presented centroid approach for solving linear programming problems with trapezoidal neutrosophic number environment. Broumi et al. (2020) proposed a new distance measure of trapezoidal fuzzy neutrosophic numbers and applied the measure for software selection process. Shigui et al. (2020) utilize simplified neutrosophic indeterminate elements weighted arithmetic averaging (SIEA) operator and simplified neutrosophic indeterminate elements weighted geometric averaging (SIEWA) operator. Wang et al. (2020) have been developed possibility degree and power weighted aggregation operators of single valued trapezoidal neutrosophic numbers. He utilized power average and geometric operators to single valued trapezoidal neutrosophic numbers to deal with multi criteria decision making problems. Deli and Ozturk (2020) proposed an MCDM method based on the score functions of single valued neutrosophic numbers and reduced single valued trapezoidal neutrosophic numbers to fuzzy numbers. Garai et al. (2020) developed ranking methods for possibility mean with neutrosophic numbers and applied to multi-attribute decision making with single valued neutrosophic numbers.

Literature review reflects that no research has been carried out on weighted harmonic averaging operator with trapezoidal neutrosophic numbers for multi attribute decision making problems. To bridge the gap, we propose harmonic aggregating operators in single valued trapezoidal
Generalized ordered weighted harmonic averaging operator with trapezoidal neutrosophic numbers, such as single valued trapezoidal neutrosophic weighted harmonic averaging (SVTNWHA) operator, single valued trapezoidal neutrosophic ordered weighted harmonic averaging (SVTNOWHA) operator, single valued trapezoidal neutrosophic generalized ordered weighted harmonic averaging (SVTNGOWHA) operator. We can also investigate some of their properties, applying a multi attribution decision making method. The main aim of this proposed operator is to choose the best alternative of the decision making under the preference value of the alternative.

This paper is organized as follows. Section 2 depicts some review of basic concepts. Section 3 presents harmonic operations on single valued trapezoidal neutrosophic numbers. Section 4 discusses method for multi attribute decision making problem. Section 5 conclusion of the paper is given.

2 Preliminaries

In this section, we review some basic concepts about the single valued trapezoidal neutrosophic numbers.

Definition 2.1 (Smarandache 1998) Let X be a non-empty set. Then a neutrosophic set \( \tilde{a} \) of X is defined as

\[
\tilde{a} = \{ x, T_{\tilde{a}}(x), I_{\tilde{a}}(x), F_{\tilde{a}}(x) \mid x \in X \}, T_{\tilde{a}}(x), I_{\tilde{a}}(x), F_{\tilde{a}}(x) \in [0, 1]
\]

where \( T_{\tilde{a}}(x), I_{\tilde{a}}(x), F_{\tilde{a}}(x) \) are truth membership function, indeterminacy membership function and falsity membership function and \( 0 \leq T_{\tilde{a}}(x) + I_{\tilde{a}}(x) + F_{\tilde{a}}(x) \leq 3 \)

Definition 2.2 (Smarandache 1998) A neutrosophic set \( \tilde{a} \) is defined on the universal set of real numbers \( R \) is said to be neutrosophic number if it has the following properties.

1. \( \tilde{a} \) is normal if there exists \( x_0 \in R \), such that \( T_{\tilde{a}}(x_0) = 1, I_{\tilde{a}}(x_0) = F_{\tilde{a}}(x_0) = 0 \)
2. \( \tilde{a} \) is convex set for the truth function \( T_{\tilde{a}}(x) \)
   \[
   T_{\tilde{a}}(x_1 + (1 - \mu)x_2) \geq \min(T_{\tilde{a}}(x_1), T_{\tilde{a}}(x_2)), \forall x_1, x_2 \in R, \mu \in [0, 1]
   \]
3. \( \tilde{a} \) is concave set for the indeterminacy and falsity functions \( I_{\tilde{a}}(x) \) and \( F_{\tilde{a}}(x) \)
   \[
   I_{\tilde{a}}(x_1 + (1 - \mu)x_2) \geq \max(I_{\tilde{a}}(x_1), I_{\tilde{a}}(x_2)), \forall x_1, x_2 \in R, \mu \in [0, 1]
   \]
   \[
   F_{\tilde{a}}(x_1 + (1 - \mu)x_2) \geq \max(F_{\tilde{a}}(x_1), F_{\tilde{a}}(x_2)), \forall x_1, x_2 \in R, \mu \in [0, 1]
   \]

Definition 2.3 (Mohamed Abdel-Basset et al. 2019) Let \( \tilde{a} = (a_1, a_2, a_3, a_4); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}} > 0 \) be a single valued trapezoidal neutrosophic number on the real number set \( R \), whose truth membership, indeterminacy membership and falsity membership functions are given by

\[
T_{\tilde{a}}(x) = \begin{cases} \frac{x-a_2}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, & \text{for } a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3}, & \text{for } a_3 \leq x \leq a_4 \\ 0, & \text{otherwise.} \end{cases}
\]

\[
I_{\tilde{a}}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2 \\ \frac{x-a_4}{a_4-a_3}, & \text{for } a_3 \leq x \leq a_4 \\ 0, & \text{otherwise.} \end{cases}
\]

\[
F_{\tilde{a}}(x) = \begin{cases} \frac{a_3-x}{a_3-a_2}, & \text{for } a_2 \leq x \leq a_3 \\ \frac{x-a_1}{a_1-a_0}, & \text{for } a_1 \leq x \leq a_2 \\ 0, & \text{otherwise.} \end{cases}
\]

respectively, Where \( a_1, a_2, a_3, a_4 \in R \). If \( a_1 \leq 0 \) and at least \( a_4 > 0 \), then the single valued trapezoidal neutrosophic number \( \tilde{a} \) is positive and it is denoted by \( \tilde{a} > 0 \). If \( a_4 \leq 0 \) and at least \( a_1 < 0 \), then the single valued trapezoidal neutrosophic number \( \tilde{a} \) is negative and it is denoted by \( \tilde{a} < 0 \). Without loss of generality, we have considered \( a_2 = a_3 \). Then trapezoidal neutrosophic numbers transform to a triangular neutrosophic numbers. Where \( T_{\tilde{a}}, I_{\tilde{a}} \) and \( F_{\tilde{a}} \) represent the maximum degree of acceptance, an indeterminacy and minimum degree of rejection respectively, such that they satisfy the condition \( 0 \leq T_{\tilde{a}}(x) + I_{\tilde{a}}(x) + F_{\tilde{a}}(x) \leq 3, x \in \tilde{a} \).

Definition 2.4 (Mohamed Abdel-Basset et al. 2019) Let \( \tilde{a} = (a_1, a_2, a_3, a_4); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}} > 0 \) and \( \tilde{b} = (b_1, b_2, b_3, b_4); T_{\tilde{b}}, I_{\tilde{b}}, F_{\tilde{b}} > 0 \) be two single valued trapezoidal neutrosophic numbers. Then

(i) \( \tilde{a} + \tilde{b} = < (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); T_{\tilde{a}} + T_{\tilde{b}}, I_{\tilde{a}} + I_{\tilde{b}}, F_{\tilde{a}} + F_{\tilde{b}} > \)
(ii) \( \tilde{a} - \tilde{b} = < (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4); T_{\tilde{a}} - T_{\tilde{b}}, I_{\tilde{a}} - I_{\tilde{b}}, F_{\tilde{a}} - F_{\tilde{b}} > \)
(iii) \( \tilde{a} \tilde{b} = < (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4); T_{\tilde{a}} T_{\tilde{b}}, I_{\tilde{a}} I_{\tilde{b}}, F_{\tilde{a}} F_{\tilde{b}} > \)
3 Harmonic averaging operators of SVTN numbers

Based on the basis of harmonic operation on single valued trapezoidal neutrosophic numbers, we propose single-valued trapezoidal neutrosophic weighted harmonic averaging (SVTNWHA) operator, single-valued trapezoidal neutrosophic ordered weighted harmonic averaging (SVTNOWHA) operator and single valued trapezoidal neutrosophic generalized ordered weighted harmonic averaging (SVTNOWGA) operator. In this case, the reordering step is developed with order-inducing variables that reflect a more complex reordering process.

Definition 3.1 Let \( \tilde{a}_j = <(a_{j1}, a_{j2}, a_{j3}, a_{j4}); T_{ij}, I_{ij}, F_{ij}> \) be a collection of single valued trapezoidal neutrosophic numbers. Then, SVTNWHA operator is a function SVTNWHA : \( R^n \rightarrow R \) is defined as

\[
SVTNWHA(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \frac{1}{\left( \sum_{j=1}^{n} \frac{a_{j}}{b_{ij}} \right)^{\frac{1}{2}}}
\]

where \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \) is the weighted vector of \( b_{ij} \) and \( \omega_j \in [0, 1], \sum_{j=1}^{n} \omega_j = 1. \) Especially, when \( \omega_j = 0 \) and \( \omega_i = 1, (i \neq j, i = 1, 2, \ldots, n). \) We have SVTNOWHA(\( \tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n \)) = \( \tilde{a}_j; \) when \( \omega = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T, \) we have SVTNWHA(\( \tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n \)) = \( \frac{1}{\left( \sum_{j=1}^{n} \frac{a_{j}}{b_{ij}} \right)^{\frac{1}{2}}} \)

Definition 3.2 Let \( \tilde{a}_j = <(a_{j1}, a_{j2}, a_{j3}, a_{j4}); T_{ij}, I_{ij}, F_{ij}> \) be a collection of single valued trapezoidal neutrosophic numbers. Then, SVTNOWGA operator is a function SVTNOWGA : \( R^n \rightarrow R \) is defined as

\[
SVTNOWGA(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \frac{1}{\left( \sum_{j=1}^{n} \frac{a_{j}}{b_{ij}} \right)^{\frac{1}{2}}}
\]

where \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \) is the weighted vector and \( \omega_j \in [0, 1], \sum_{j=1}^{n} \omega_j = 1. \) Where \( b_{ij} \) is the largest \( j \)th element in the collection of \( \tilde{a}_j, j = (1, 2, \ldots, n). \)

Definition 3.3 Let \( \tilde{a}_j = <(a_{j1}, a_{j2}, a_{j3}, a_{j4}); T_{ij}, I_{ij}, F_{ij}> \) be a collection of single valued trapezoidal neutrosophic numbers. Then, SVTNOWGA operator is a function SVTNOWGA : \( R^n \rightarrow R \) is defined as

\[
SVTNOWGA(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \frac{1}{\left( \sum_{j=1}^{n} \frac{a_{j}}{b_{ij}} \right)^{\frac{1}{2}}}
\]

where \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \) is the weighted vector with \( \omega_j \in [0, 1], \sum_{j=1}^{n} \omega_j = 1. \) Where \( b_{ij} \) is the largest \( j \)th element in the collection of \( \tilde{a}_j, b_{ij} = <(b_{j1}, b_{j2}, b_{j3}, b_{j4}); T_{ij}, I_{ij}, F_{ij}> \) is reordering of the individual collection of \( \tilde{a}_j, \) where \( \lambda \in R \) is a parameter.

By using arithmetic operations on single valued trapezoidal neutrosophic numbers, we get the following theorem.

Theorem 3.4 Let \( \tilde{a}_j = <(a_{j1}, a_{j2}, a_{j3}, a_{j4}); T_{ij}, I_{ij}, F_{ij}> \) be a collection of single valued trapezoidal Neutrosophic number and \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \) be a weighted vector of \( \tilde{a}_j, \omega_j \in [0, 1], \sum_{j=1}^{n} \omega_j = 1 \) and the parameter \( \lambda \in R, \) then the aggregation value by utilizing the operator is defined as
Then the result is true for all \( n \). Similarly the result is considered for \( \lambda < 0 \), theorem can be proved easily. Thus, mathematical induction method, the proof of theorem is completed.

3.1 Analyzing the weighted vector

If the weighted vector \( \omega = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T \), then we have single valued trapezoidal neutrosophic generalized order weighted harmonic averaging (SVTNGOWHA) operator which is reduced to generalized harmonic mean operator with neutrosophic environment.

If the weighted vector \( \omega = (1, 0, 0, \ldots, 0)^T \), then we get maximum value SVTNGOWHA \((\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_n) = \max \{\bar{a}_j\}\) and if the weighted vector \( \omega = (0, 0, 0, \ldots, 1)^T \), then we get minimum value SVTNGOWHA \((\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_n) = \min \{\bar{a}_j\}\)
3.2 Analyzing the parameter $\lambda$

(i) If the parameter $\lambda = 1$, then the operator $\text{SVTNGOWHA}$ reduces to ordered weighted harmonic averaging (OWHA) operator with trapezoidal neutrosophic number.

\[
\text{SVTNOWHA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \frac{1}{\left(\sum_{j=1}^{n} \omega_j \bar{b}_j\right)} \tag{5}
\]

(ii) If the parameter $\lambda = 2$, then the operator $\text{SVTNGOWQA}$ reduces to generalized ordered weighted quadratic harmonic averaging (GOWQHA) operator with trapezoidal neutrosophic number.

\[
\text{SVTNGOWQA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \frac{1}{\left(\sum_{j=1}^{n} \omega_j^{1/2} \bar{b}_j^{1/2}\right)^2} \tag{6}
\]

(iii) If the parameter $\lambda = -1$, then the operator $\text{SVTNGOWHA}$ reduces to ordered weighted averaging (OWA) operator with trapezoidal neutrosophic number.

\[
\text{SVTNOWA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left(\sum_{j=1}^{n} \omega_j \bar{b}_j\right) \tag{7}
\]

Theorem 3.5 (Monotonicity) Let $\tilde{a}_j = \langle a_{j1}, a_{j2}, a_{j3}, a_{j4}\rangle$; $T_{\tilde{b}_j}, I_{\tilde{b}_j}, F_{\tilde{b}_j}$ and $\tilde{a}'_j = \langle a'_{j1}, a'_{j2}, a'_{j3}, a'_{j4}\rangle; T'_{\tilde{b}'_j}, I'_{\tilde{b}'_j}, F'_{\tilde{b}'_j}$ be two collections of SVTN numbers. If $\tilde{b}_j \leq \tilde{b}'_j$ for $j = 1, 2, \ldots, n$. Then $\text{SVTNGOWHA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq \text{SVTNGOWHA}(\tilde{a}'_1, \tilde{a}'_2, \ldots, \tilde{a}'_n)$

Proof Case 1: For $\lambda > 0$

Since $b_{j1} \leq b'_{j1} \Rightarrow b_{j1}^{\lambda} \leq (b'_{j1})^{\lambda} \Rightarrow \frac{\omega_j}{b_{j1}^{\lambda/2}} \geq \frac{\omega_j}{(b'_{j1})^{\lambda/2}} \Rightarrow (\sum_{j=1}^{n} \frac{\omega_j}{b_{j1}^{\lambda/2}})^{1/\lambda} \leq (\sum_{j=1}^{n} \frac{\omega_j}{(b'_{j1})^{\lambda/2}})^{1/\lambda}$

Similarly, $\frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{b_{j1}^{\lambda/2}})^{1/\lambda}} \leq \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{(b'_{j1})^{\lambda/2}})^{1/\lambda}}$ and $\frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{b_{j1}^{\lambda/2}})^{1/\lambda}} \leq \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{(b'_{j1})^{\lambda/2}})^{1/\lambda}}$

Since $T_{\tilde{b}_j} \leq T'_{\tilde{b}'_j} \Rightarrow \min_j(T_{\tilde{b}_j}) \leq \min_j(T'_{\tilde{b}'_j})$, $I_{\tilde{b}_j} \geq I'_{\tilde{b}'_j} \Rightarrow \max_j(I_{\tilde{b}_j}) \geq \max_j(I'_{\tilde{b}'_j})$, $F_{\tilde{b}_j} \geq F'_{\tilde{b}'_j} \Rightarrow \max_j(F_{\tilde{b}_j}) \geq \max_j(F'_{\tilde{b}'_j})$.
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\[
\left(\frac{1}{\sum_{j=1}^{n} a_j^\lambda}, \frac{1}{\sum_{j=1}^{n} a_j^\lambda}, \frac{1}{\sum_{j=1}^{n} a_j^\lambda}, \frac{1}{\sum_{j=1}^{n} a_j^\lambda}\right) \leq \left(\frac{1}{\sum_{j=1}^{n} a_j^\lambda}, \frac{1}{\sum_{j=1}^{n} a_j^\lambda}, \frac{1}{\sum_{j=1}^{n} a_j^\lambda}, \frac{1}{\sum_{j=1}^{n} a_j^\lambda}\right)
\]

⇒ SVTNGOWHA(\(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n\)) \leq SVTNGOWHA(\(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n\))

**Case II:** For \(\lambda < 0\)

Since \(b_j \leq b_j^+ \Rightarrow b_j^+ \geq (b_j)^+, \forall j\)

\[
\frac{a_j}{b_j} \leq \frac{a_j}{(b_j)^+} \Rightarrow (\sum_{j=1}^{n} \frac{a_j}{b_j})^\lambda \leq (\sum_{j=1}^{n} \frac{a_j}{(b_j)^+})^\lambda
\]

\[
\frac{1}{(\sum_{j=1}^{n} \frac{a_j}{b_j})^\lambda} \leq \frac{1}{(\sum_{j=1}^{n} \frac{a_j}{(b_j)^+})^\lambda}
\]

In the same way as case I it can be proved.

**Theorem 3.6 (Idempotency)** Let \(\tilde{a}_j \equiv (a_1, a_2, a_3, a_4); T_{ij}, I_{ij}, F_{ij}, \forall j \in \{1, 2, \ldots, n\}\) be a collection of single valued trapezoidal neutrosophic number. If all \(\tilde{a}_j\) are equal, \(\tilde{a}_j \equiv \tilde{a}, \forall j \in \{1, 2, \ldots, n\}\), then SVTNGOWHA (\(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n\)) = SVTNGOWHA (\(\tilde{a}, \tilde{a}, \ldots, \tilde{a}\)) = \(\tilde{a}\).

**Theorem 3.7** (Commutativity) If \((\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)\) is any permutation of \((\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)\), then SVTNGOWHA (\(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n\)) = SVTNGOWHA (\(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n\)).

**Theorem 3.8** (Boundedness) Let \(\tilde{a}_j \equiv (a_1, a_2, a_3, a_4); T_{ij}, I_{ij}, F_{ij}, \forall j \in \{1, 2, \ldots, n\}\) be a collection of SVTN numbers and Let \(\tilde{a}_j^\pm = \{(\min b_1, \min b_2, \min b_3, \min b_4); \min (T_{ij}), \max (I_{ij}), \max (F_{ij})\}\)

\(\tilde{a}_j^- = \{(\max b_1, \max b_2, \max b_3, \max b_4); \min (T_{ij}), \max (I_{ij}), \max (F_{ij})\}\)

Then \(\tilde{a}^- \leq SVTNGOWHA(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq \tilde{a}^+\).

**Proof Case I:** For \(\lambda > 0\)

Since \(\min b_j \leq b_j \leq \max b_j\)

\[
\Rightarrow \left(\frac{\min b_j}{b_j}\right)^\lambda \geq \left(\frac{\max b_j}{b_j}\right)^\lambda \Rightarrow (\sum_{j=1}^{n} \frac{\min b_j}{b_j})^\lambda \geq (\sum_{j=1}^{n} \frac{\max b_j}{b_j})^\lambda
\]

\[
\frac{1}{(\sum_{j=1}^{n} \frac{\min b_j}{b_j})^\lambda} \leq \frac{1}{(\sum_{j=1}^{n} \frac{\max b_j}{b_j})^\lambda}
\]

Similarly,

\[
\frac{1}{(\sum_{j=1}^{n} \frac{\max b_j}{b_j})^\lambda} \leq \frac{1}{(\sum_{j=1}^{n} \frac{\min b_j}{b_j})^\lambda}
\]

Also \(\min T_{ij} \leq T_{ij} \leq \max T_{ij}, \forall j; \min I_{ij} \leq I_{ij} \leq \max I_{ij}, \forall j; \min F_{ij} \leq F_{ij} \leq \max F_{ij}, \forall j\). By using the properties of monotonicity and idempotency, we get

\(SVTNGOWHA(\tilde{a}^-) \leq SVTNGOWHA(\tilde{a}) \leq SVTNGOWHA(\tilde{a}^+)\).

**Case II:** For \(\lambda < 0\)

| Table 1 | Decision matrix provided by expert \(d_1\) |
|---------|-----------------------------------------------|
| Alternatives | \(\tilde{e}_1\) | \(\tilde{e}_2\) | \(\tilde{e}_3\) | \(\tilde{e}_4\) |
| \(\tilde{a}_1\) | \(< (2, 4, 6, 8)\) | \(< (2, 4, 6, 7)\) | \(< (17, 18, 19, 20)\) | \(< (3, 4, 6, 7)\) |
| | \(0.5, 0.4, 0.8 >\) | \(0.7, 0.2, 0.5 >\) | \(0.6, 0.3, 0.4 >\) | \(0.7, 0.1, 0.4 >\) |
| \(\tilde{a}_2\) | \(< (3, 5, 6, 7)\) | \(< (15, 17, 19, 20)\) | \(< (3, 4, 5, 6)\) | \(< (4, 5, 6, 7)\) |
| | \(0.6, 0.3, 0.4 >\) | \(0.7, 0.2, 0.4 >\) | \(0.7, 0.2, 0.6 >\) | \(0.6, 0.4, 0.3 >\) |
| \(\tilde{a}_3\) | \(< (1, 2, 3, 4)\) | \(< (2, 3, 4, 5)\) | \(< (2, 4, 5)\) | \(< (15, 16, 18, 20)\) |
| | \(0.7, 0.2, 0.5 >\) | \(0.5, 0.4, 0.3 >\) | \(0.6, 0.4, 0.2 >\) | \(0.8, 0.1, 0.2 >\) |
Table 2 Decision matrix provided by expert $d_2$

| Alternatives | $\tilde{c}_1$ | $\tilde{c}_2$ | $\tilde{c}_3$ | $\tilde{c}_4$ |
|--------------|---------------|---------------|---------------|---------------|
| $\tilde{a}_1$ | $(15, 16, 17, 20); (2, 4, 5, 7); (2, 5, 6, 8); (3, 5, 6, 7);$ | $(0.9, 0.1, 0.4);$ | $(0.5, 0.3, 0.6);$ | $(0.7, 0.2, 0.5);$ | $(0.8, 0.1, 0.3);$ |
| $\tilde{a}_2$ | $(4, 5, 6, 7);$ | $(16, 17, 19, 20);$ | $(3, 4, 5, 6);$ | $(4, 5, 6, 9);$ |
| $\tilde{a}_3$ | $(1, 3, 5, 6);$ | $(2, 3, 4, 6);$ | $(2, 3, 4, 5);$ | $(3, 4, 18, 20);$ |

Table 3 Decision matrix provided by expert $d_3$

| Alternatives | $\tilde{c}_1$ | $\tilde{c}_2$ | $\tilde{c}_3$ | $\tilde{c}_4$ |
|--------------|---------------|---------------|---------------|---------------|
| $\tilde{a}_1$ | $(4, 5, 6, 7);$ | $(1, 2, 3, 4);$ | $(17, 18, 19, 20);$ | $(3, 4, 5, 6);$ |
| $\tilde{a}_2$ | $(3, 5, 6, 7);$ | $(2, 3, 4, 6);$ | $(3, 4, 5, 6);$ | $(16, 17, 19, 20);$ |
| $\tilde{a}_3$ | $(16, 17, 18, 20);$ | $(4, 5, 6, 7);$ | $(2, 4, 5, 6);$ | $(3, 4, 6, 7);$ |

Table 4 Normalized decision matrix provided by expert $d_1$

| Alternatives | $\tilde{c}_1$ | $\tilde{c}_2$ | $\tilde{c}_3$ | $\tilde{c}_4$ |
|--------------|---------------|---------------|---------------|---------------|
| $\tilde{a}_1$ | $(0.05, 0.108);$ | $(0.05, 0.108);$ | $(0.405, 0.49);$ | $(0.07, 0.108);$ |
| $\tilde{a}_2$ | $(0.2, 0.33);$ | $(0.2, 0.29);$ | $(0.63, 0.83);$ | $(0.2, 0.29);$ |
| $\tilde{a}_3$ | $(0.5, 0.4, 0.8);$ | $(0.5, 0.4, 0.8);$ | $(0.5, 0.4, 0.8);$ | $(0.5, 0.4, 0.8);$ |

Table 5 Normalized decision matrix provided by expert $d_2$

| Alternatives | $\tilde{c}_1$ | $\tilde{c}_2$ | $\tilde{c}_3$ | $\tilde{c}_4$ |
|--------------|---------------|---------------|---------------|---------------|
| $\tilde{a}_1$ | $(0.36, 0.47);$ | $(0.048, 0.12);$ | $(0.048, 0.147);$ | $(0.07, 0.15);$ |
| $\tilde{a}_2$ | $(0.57, 0.91);$ | $(0.17, 0.32);$ | $(0.2, 0.364);$ | $(0.2, 0.32);$ |
| $\tilde{a}_3$ | $(0.5, 0.3, 0.6);$ | $(0.5, 0.3, 0.6);$ | $(0.5, 0.3, 0.6);$ | $(0.5, 0.3, 0.6);$ |

Since $\min \{b_j\} \leq b_{ji} \leq \max \{b_j\}$,$(\forall j > 0) \Rightarrow (\sum_{j=1}^{n} \frac{a_{ji}}{\min \{b_j\}})^{\frac{1}{2}} \leq \frac{a_{ji}}{b_{ji}} \leq \frac{a_{ji}}{b_{ji}} \Rightarrow (\sum_{j=1}^{n} \frac{a_{ji}}{\max \{b_j\}})^{\frac{1}{2}} \leq \frac{1}{(\sum_{j=1}^{n} \frac{a_{ji}}{b_{ji}})^{\frac{1}{2}}} \leq \frac{1}{(\sum_{j=1}^{n} \frac{a_{ji}}{b_{ji}})^{\frac{1}{2}}}$ In the same way as case I it can be proved.
4 Decision making method with trapezoidal neutrosophic number

In this section, we present an approach to multi attribute decision making based on the SVTNGOWHA operator with the help of score function for trapezoidal neutrosophic numbers.

Let \( \tilde{a} = (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_m) \) be a set of m attributes and \( \tilde{c} = (\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_n) \) be the set of n attributes related to alternatives weighted vector \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \) being its weighting vector, which is used to represent the importance weights of different attributes,

where \( 0 \leq \omega_j \leq 1 (j = 1, 2, \ldots, n) \) and \( \sum_{j=1}^{n} \omega_j = 1 \). Let \( D = (d_1, d_2, \ldots, d_i) \) be the set of decision makers (Expert) with weighting vector \( \eta = (\eta_1, \eta_2, \ldots, \eta_i)^T \).

where \( 0 \leq \eta_k \leq 1 (k = 1, 2, \ldots, i) \) and \( \sum_{k=1}^{i} \eta_k = 1 \). In this problem, decision makers evaluate each alternative with trapezoidal neutrosophic numbers according to each criterion. Thus, we obtain trapezoidal neutrosophic decision matrix as follows:

\[
\tilde{a}_k = (\tilde{a}_k^1, \tilde{a}_k^2, \tilde{a}_k^3, \tilde{a}_k^4) = \left( a_{k1,q1}, a_{k2,q2}, a_{k3,q3}, a_{k4,q4}, T_{q1}, I_{q2}, F_{q3} \right)_{(m \times n)}
\]

provided by an expert decision maker \( D \), where \( T_{q1}, I_{q2}, F_{q3} \) are defined on truth, an indeterminacy and falsity membership function and \( T_{q1}, I_{q2}, F_{q3} \in [0, 1] \) with \( 0 \leq T_{q1} + I_{q2} + F_{q3} \leq 3, a_{q1}, a_{q2}, a_{q3}, a_{q4} \in R \). Multi-attribute decision making problem contains benefit and cost attribute.

Table 6 Normalized decision matrix provided by expert \( d_j \)

| Alternatives | \( \tilde{c}_1 \) | \( \tilde{c}_2 \) | \( \tilde{c}_3 \) | \( \tilde{c}_4 \) |
|--------------|-----------------|-----------------|-----------------|-----------------|
| \( \tilde{a}_1 \) | (0.11, 0.15, 0.21, 0.32, 0.5, 0.4, 0.5, 0.6, 0.3, 0.8) | (0.03, 0.06, 0.1, 0.16, 0.5, 0.4, 0.5, 0.6, 0.8) | (0.45, 0.55, 0.66, 0.8) | (0.08, 0.12, 0.17, 0.24) |
| \( \tilde{a}_2 \) | (0.08, 0.15, 0.21, 0.29, 0.6, 0.3, 0.8) | (0.05, 0.09, 0.14, 0.25, 0.5, 0.4, 0.5) | (0.05, 0.12, 0.17, 0.25) | (0.4, 0.59, 0.66, 0.83) |
| \( \tilde{a}_3 \) | (0.4, 0.49, 0.6, 0.8) | (0.1, 0.14, 0.19, 0.28, 0.5, 0.4, 0.5) | (0.05, 0.11, 0.167, 0.24) | (0.08, 0.11, 0.2, 0.28) |

Table 7 Individual overall attributes values with SVTNGOWHA operator

| Alternatives | \( \tilde{c}_1 \) | \( \tilde{c}_2 \) | \( \tilde{c}_3 \) | \( \tilde{c}_4 \) |
|--------------|-----------------|-----------------|-----------------|-----------------|
| \( \tilde{a}_1 \) | (0.08, 0.143, 0.216, 0.342, 0.5, 0.4, 0.8) | (0.041, 0.089, 0.144, 0.24, 0.5, 0.4, 0.8) | (0.136, 0.3, 0.4, 0.607) | (0.071, 0.118, 0.19, 0.276) |
| \( \tilde{a}_2 \) | (0.081, 0.140, 0.195, 0.275, 0.6, 0.4, 0.8) | (0.138, 0.22, 0.323, 0.475, 0.5, 0.4, 0.8) | (0.08, 0.11, 0.161, 0.235) | (0.105, 0.147, 0.203, 0.328) |
| \( \tilde{a}_3 \) | (0.026, 0.088, 0.168, 0.259, 0.5, 0.4, 0.7) | (0.048, 0.099, 0.159, 0.267, 0.5, 0.4, 0.7) | (0.047, 0.105, 0.163, 0.240) | (0.182, 0.263, 0.421, 0.538) |

Fig. 1 Ranking values of alternatives with respect to parameter \( \lambda \) in SVTNGOWHA operator

In this procedure, we consider linear scale transformation (sum) which divides the performance ratings of each attribute by the sum of performance ratings for that attribute. Herein, the following algorithm is proposed to obtain the solution of the multi-attribute decision-making problem with the trapezoidal neutrosophic numbers information by using SVTNGOWHA operator with score function.
Table 8  Decision-making results of different aggregation operators

| Method                      | Operator       | Ranking order | Best alternative |
|-----------------------------|----------------|---------------|------------------|
| Ye (2016a)                  | TNWAA          | \( \tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3 \) | \( \tilde{a}_2 \) |
|                            | TNWGA          | \( \tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3 \) | \( \tilde{a}_2 \) |
| Bharatraj and Anand (2019) | PHWAOSVTrNN    | \( \tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3 \) | \( \tilde{a}_2 \) |
| Chiranjibe et al. (2020)   | SVTNHWA        | \( \tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3 \) | \( \tilde{a}_2 \) |
|                            | SVTNHWGA       | \( \tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3 \) | \( \tilde{a}_2 \) |
| Wang et al. (2020)         | SVTNP          | \( \tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3 \) | \( \tilde{a}_2 \) |
|                            | SVTNPG         | \( \tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3 \) | \( \tilde{a}_2 \) |
| Proposed Method             | SVTNOWHA       | \( \tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3 \) | \( \tilde{a}_2 \) |
|                            | SVTNGOWHA      | \( \tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3 \) | \( \tilde{a}_2 \) |

Table 9  Linguistic values of trapezoidal neutrosophic numbers for the linguistic term set

| Linguistic term                  | Linguistic value                |
|---------------------------------|---------------------------------|
| Extremely low priority (ELP)    | \(<0,0,0.1,0.2;0.6,0.2,0.4>\)   |
| Low priority (LP)               | \(<0.1,0.1,0.2;0.3;0.5,0.1,0.3>\)|
| Simple priority (SP)            | \(<0.2,0.3,0.4,0.5;0.8,0.2,0.2>\)|
| Medium priority (HP)            | \(<0.4,0.5,0.6,0.7;0.9,0.2,0.1>\)|
| High priority (HP)              | \(<0.6,0.7,0.8,0.9;0.9,0.1,0.1>\)|

Table 10  Evaluation of criteria by three experts using linguistic variables

| Expert 1 | \( \tilde{c}_1 \) | \( \tilde{c}_2 \) | \( \tilde{c}_3 \) | \( \tilde{c}_4 \) |
|----------|-------------------|-------------------|-------------------|-------------------|
| \( \tilde{a}_1 \) | ELP               | 1/SP              | MP                | 1/LP              |
| \( \tilde{a}_2 \) | SP                | ELP               | MP                | 1/LP              |
| \( \tilde{a}_3 \) | 1/MP              | 1/MP              | ELP               | 1/MP              |
| \( \tilde{a}_4 \) | LP                | LP                | MP                | ELP               |
| Expert 2 | \( \tilde{c}_1 \) | \( \tilde{c}_2 \) | \( \tilde{c}_3 \) | \( \tilde{c}_4 \) |
| \( \tilde{a}_1 \) | ELP               | 1/LP              | SP                | 1/LP              |
| \( \tilde{a}_2 \) | LP                | ELP               | SP                | ELP               |
| \( \tilde{a}_3 \) | 1/SP              | 1/SP              | ELP               | 1/SP              |
| \( \tilde{a}_4 \) | LP                | ELP               | LP                | ELP               |
| Expert 3 | \( \tilde{c}_1 \) | \( \tilde{c}_2 \) | \( \tilde{c}_3 \) | \( \tilde{c}_4 \) |
| \( \tilde{a}_1 \) | ELP               | 1/LP              | LP                | 1/SP              |
| \( \tilde{a}_2 \) | LP                | ELP               | MP                | LP                |
| \( \tilde{a}_3 \) | 1/MP              | 1/MP              | ELP               | 1/SP              |
| \( \tilde{a}_4 \) | SP                | 1/LP              | SP                | ELP               |
Step 1: Since the given attributes are benefit criteria. So, we need to normalize the decision matrix. Then the computations of normalization matrices are given by Tables 4, 5 and 6.

Step 2: Utilize SVTNOWHA aggregation operator. Assume the parameter $\lambda = 1$ and the associated weighted vector $W = (0.067, 0.666, 0.267)^T$ which can be obtained by the fuzzy linguistic quantifier “most” with the pair of $(\alpha, \beta) = (0.3, 0.8)$. Then final aggregated values are given Table 7.

Step 3: Utilize the SVTNGOWHA aggregation operator.

Table 11 Individual overall attributes values with SVTNGOWHA operator

| Alternatives | $\tilde{c}_1$ | $\tilde{c}_2$ | $\tilde{c}_3$ | $\tilde{c}_4$ |
|--------------|--------------|--------------|--------------|--------------|
| $\tilde{a}_1$ | $\langle (0, 0, 0.162, 0.209)\rangle$ | $\langle (2.81, 3.95, 4.054)\rangle$ | $\langle (0.16, 0.209, 0.320)\rangle$ | $\langle (2.81, 3.95, 4.054)\rangle$ |
| $\tilde{a}_2$ | $\langle (0.104, 0.115, 0.167, 0.281)\rangle$ | $\langle (0.104, 0.115, 0.167, 0.281)\rangle$ | $\langle (0.104, 0.115, 0.167, 0.281)\rangle$ | $\langle (0.104, 0.115, 0.167, 0.281)\rangle$ |
| $\tilde{a}_3$ | $\langle (1.852, 2.274, 2.918, 4.054)\rangle$ | $\langle (1.458, 1.708, 2.054, 2.587)\rangle$ | $\langle (0.1, 0.2)\rangle$ | $\langle (1.852, 2.274, 2.918, 4.054)\rangle$ |
| $\tilde{a}_4$ | $\langle (0.12, 0.1395, 0.167, 0.281)\rangle$ | $\langle (0.12, 0.1395, 0.167, 0.281)\rangle$ | $\langle (0.12, 0.1395, 0.167, 0.281)\rangle$ | $\langle (0.12, 0.1395, 0.167, 0.281)\rangle$ |

Step 4: Finally, the aggregation results are obtained by method of score function of single valued trapezoidal neutrosophic number.

Table 12 Decision-making results of different aggregation operators

| Method | Operator | Ranking order | Best alternative |
|--------|----------|---------------|------------------|
| Ye (2016a) | TNWAA | $\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$ | $\tilde{a}_1$ |
| | TNWGA | $\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$ | $\tilde{a}_1$ |
| Bharatraj and Anand (2019) | PHWAOSVTNN | $\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$ | $\tilde{a}_1$ |
| Wang et al. (2020) | SVTNHPA | $\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$ | $\tilde{a}_1$ |
| | SVTNPG | $\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$ | $\tilde{a}_1$ |
| Proposed Method | SVTNWHA | $\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$ | $\tilde{a}_1$ |
| | SVTNOWHA | $\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$ | $\tilde{a}_1$ |
| | SVTNGOWHA | $\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$ | $\tilde{a}_1$ |
will be first choice, \( \tilde{a}_3 \) and \( \tilde{a}_2 \) are second and third choice. Hence, the best enterprise is \( \tilde{a}_1 \).

Furthermore, we analyze the different parameter \( \lambda \) that deals with the aggregation results provided by best decision maker. We can consider different values of \( \lambda : -15, -5, -1, -0.8, 1, 6.6, 10, \ldots 12, 15 \) which are provided by the best decision maker. The variation of the aggregation results with parameter \( \lambda \) is shown in Fig. 1. We observed that the aggregation results, if \( \lambda \) decreases, \( (\lambda < 0) \) the values will increase and if \( \lambda \) increases, \( (\lambda > 0) \) the values will decrease. If we compared with different type of parameter \( \lambda \) the aggregation results of decision maker chosen the best alternative is \( \tilde{a}_2 \). Compared with other operators, we find that the main advantage of using the SVTNNGOWHA operator we can consider a range. In this paper, the different values of parameter \( \lambda \) are considered sufficiently while Bharatraj and Anand (2019), Chiranjibe et al. (2020), Wang et al. Wang et al. (2020) did not consider the decision makers preference.

Table 8 will show that existing works and the proposed method which develop decision making approach using single valued trapezoidal neutrosophic numbers.

**Example 2** To identify the effective allocation of the COVID-19 vaccine for priority groups, decision-makers must involve experts from multiple fields to get benefit from their experiences in setting priorities and principle guidelines. After pre-evaluation four alternatives \( \tilde{a}_i, (i = 1, 2, 3, 4) \) are selected for further evaluations. The expert select the best priority group of the basis of the following four attributes:

- \( \tilde{c}_1 \): Old, Adult and kids peoples with health problems,
- \( \tilde{c}_2 \): People with high risk health problems,
- \( \tilde{c}_3 \): Breastfeeding problem,
- \( \tilde{c}_4 \): Healthcare personnel and Essential workers.

\( \tilde{a}_1 \): Age index (AC)
\( \tilde{a}_2 \): Health state index (HS)
\( \tilde{a}_3 \): Women state index (WS)
\( \tilde{a}_4 \): Job kind index (JK)

Let \( \omega = (0.15, 0.35, 0.3, 0.2) \) be the weight vector of these four attributes. We obtain the decision matrices are listed in Tables 9 and 10.

**Step 1:** Since the given attributes are normalized benefit criteria. So, we need not normalize the decision matrix.

**Step 2:** Utilize SVTNNGOWHA aggregation operator. Assume the parameter \( \lambda = 1 \) and the associated weighted vector \( W = (0.067, 0.666, 0.267) \) which can be obtained by the fuzzy linguistic quantifier “most” with the pair \( \omega (\alpha, \beta) = (0.3, 0.8) \). Then final aggregated values are given Table 11.

**Step 3:** Utilize the SVTNGOWHA aggregation operator.

\[
S_1(\omega) = < (0, 0, 0.3965, 0.6601); 0.5, 0.2, 0.4 > \\
S_2(\omega) = < (0, 0, 0.1564, 0.2812); 0.5, 0.2, 0.4 > \\
S_3(\omega) = < (0, 0, 0.3039, 0.5808); 0.5, 0.2, 0.4 > \\
S_4(\omega) = < (0, 0, 0.1767, 0.2963); 0.5, 0.2, 0.4 > 
\]

**Step 4:** Finally, the aggregation results are obtained by method of score function of single valued trapezoidal neutrosophic number.

\[
S_1(\omega) = 0.1673, S_2(\omega) = 0.0693, S_3(\omega) = 0.1401, S_4(\omega) = 0.0749 
\]

Then, we can rank the alternatives \( \tilde{a}_i \) according to \( S_i(\omega), i = 1, 2, 3, 4 \). \( \tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2 \). Hence, the best enterprise is \( \tilde{a}_1 \).

Furthermore, we analyze the different parameter \( \lambda \) that deals with the aggregation results provided by best decision maker. The variation of the aggregation results with parameter \( \lambda \) is shown in Fig. 2.

Table 12 will show that existing works and the proposed method which develop decision making approach using single valued trapezoidal neutrosophic numbers.

**5 Conclusion**

This paper introduced the single valued trapezoidal neutrosophic numbers with generalized ordered weighted harmonic averaging (SVTNOWHA) operator, which provides general formulation that includes a wide range of aggregation operators and it combines with the generalized mean and the weighted harmonic averaging operator under single valued trapezoidal neutrosophic numbers. It can be applied in the selection of financial products, engineering, soft computing a decision theory under neutrosophic environment. The main advantages of the decision making approach based on the SVTNOWHA operator is that the decision maker can obtain a complete view of decision making problem. The application of enterprise selection problem shows the feasibility and effectiveness for multi attribute decision making problems. In the future research, we can establish approaches of aggregation operators with single valued neutrosophic number and apply them in the fields of medical diagnosis, forecasting and project investment.

**Declarations**

**Conflict of interest** The authors declare that they have no conflict of interest.

**Ethical approval** This article does not contain any studies with human participants or animals performed by any of the authors.

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