Interplay of drag by hot matter and electromagnetic force on the directed flow of heavy quarks

Sandeep Chatterjee and Piotr Bożek

1AGH University of Science and Technology, Faculty of Physics and Applied Computer Science, aleja Mickiewicza 30, 30-059 Krakow, Poland and
2Department of Physical Sciences, Indian Institute of Science Education and Research Berhampur, Transit Campus (Govt ITI), Berhampur-760010, Odisha, India

Abstract

Rapidity-odd directed flow in heavy ion collisions can originate from two very distinct sources in the collision dynamics i. an initial tilt of the fireball in the reaction plane that generates directed flow of the constituents independent of their charges, and ii. the Lorentz force due to the strong primordial electromagnetic field that drives the flow in opposite directions for constituents carrying unlike sign charges. We study the directed flow of open charm mesons $D^0$ and $D^\ast_0$ in the presence of both these sources of directed flow. The drag from the tilted matter dominates over the Lorentz force resulting in same sign flow for both $D^0$ and $D^\ast_0$, albeit of different magnitudes. Their average directed flow is about ten times larger than their difference. This charge splitting in the directed flow is a sensitive probe of the electrical conductivity of the produced medium. We further study their beam energy dependence; while the average directed flow shows a decreasing trend, the charge splitting remains flat from $\sqrt{s_{NN}} = 60$ GeV to 5 TeV.

A strongly interacting medium is expected to be formed in relativistic heavy ion collisions. Transport coefficients of the dense matter are one of the foremost indicators of the nature of the relevant degrees of freedom that constitute this medium. Shear and bulk viscosities which are the transport coefficients corresponding to the energy momentum tensor has been extensively studied and extracted from data leading to considerable understanding of the nature of the strongly interacting quark gluon plasma that is expected to be created in these collisions [1]. The electric conductivity $\sigma$ is the transport coefficient corresponding to the electric charge. An estimate of $\sigma$ in heavy-ion collisions will further add to our understanding of the medium properties of hot and dense QCD matter [2, 3]. Further, in the light of attempts to calibrate the magnitude and temporal dependence of the electromagnetic (EM) field produced in heavy-ion collisions [4] and its phenomenological consequences like the chiral magnetic effect [5], the knowledge of $\sigma$ is of utmost importance.

Heavy quarks (HQs) by virtue of being several times more massive than the highest ambient temperatures achieved in a collision are expected to be produced only in primordial collisions. Thus, they serve as excellent probes that witness the spacetime evolution of the fireball [6]. Charged HQs are formed early and their deflection by the Lorentz force probes the EM fields at the very early stage of the collision. In the following as heavy quarks we study specifically charm and anticharm quarks, observed in the final state in open charm mesons $D^0$ and $D^\ast_0$.

The initial state of a non-central heavy-ion collision is expected to break the forward-backward symmetry by a tilt of the fireball away from the beam axis [7–12]. This is confirmed by the observation of rapidity-odd directed flow $v_1$ of charged particles [13–16]. On the other hand, HQs which are produced according to the profile of the binary collision sources are distributed symmetrically in the forward-backward direction. At nonzero rapidities it results in a shift of the HQ production points from the tilted bulk. Recently, within the framework of Langevin dynamics coupled to a hydrodynamic background, it has been shown that this difference between the bulk matter and the HQ production points can lead to HQ $v_1$ that is of same sign as the bulk but several times larger [17]. Similar trends are also expected from a transport model approach [18]. Such large HQ $v_1$ compared to the charged particle $v_1$ is a clear signature of the tilt of initial source.

A rapidity-odd $v_1$ can also arise due to the presence of EM field [19, 20]. However, unlike the $v_1$ sourced by the expansion of the tilted fireball which is of same sign for both $D^0$ and $D^\ast_0$ [17], the Lorentz force experienced by charm and anti-charm quarks being in opposite direction, the resulting $v_1$ is of opposite sign for $D^0$ and $D^\ast_0$ [20]. In this work, we calculate the directed flow coefficient $v_1$ of $D^0$ and $D^\ast_0$ mesons under the combined influence of the drag from the tilted source and the EM fields.

The forward-backward asymmetry of the initial fireball can originate from an asymmetric deposition of entropy from forward and backward going participants [10, 21, 22]. Such an ansatz in which a participant is postulated to
deposit entropy preferably along its direction of motion, has been successful is describing the observed charged particle directed flow [11].

The initial density $s(\tau_0, x, y, \eta_{||})$ in the Glauber model with asymmetric entropy deposition can be written as [11]

$$s(\tau_0, x, y, \eta_{||}) = s_0 \left[ (1 - \alpha) \left( N^+_{\text{part}} f_+ (\eta_{||}) + N^-_{\text{part}} f_- (\eta_{||}) \right) + \alpha N_{\text{coll}} \right] f (\eta_{||})$$

where $N^+_{\text{part}}$ and $N^-_{\text{part}}$ are the densities of participant sources from the forward and backward going nuclei respectively evaluated at $(x, y)$ and $N_{\text{coll}}$ is the density of binary collisions. $\tau = \sqrt{T^2 - z^2}$ is the proper time and $\eta_{||} = \frac{1}{2} \log \left( \frac{t + z}{t - z} \right)$ is the spacetime rapidity. \(\tau_0\) is the proper time when the HQ starts to interact with the bulk and also the initial proper time to start the hydrodynamic evolution. In principle these time scales could be different and there have been previous studies on the pre-equilibrium dynamics of the HQ [23]. However, in this first study of the combined effect of drag and EM field, we work with the simple ansatz that the HQ interaction with the medium starts at the same time as the hydrodynamic expansion of the bulk. $f (\eta_{||})$ is the rapidity-even profile

$$f (\eta_{||}) = \exp \left( -\theta \left( |\eta_{||} - \eta_0^0| \right) \frac{\left( |\eta_{||} - \eta_0^0| \right)^2}{2\sigma_{\eta}^2} \right)$$

while the tilt is introduced via the factors $f_{+,-}(\eta_{||})$

$$f_+ (\eta_{||}) = \begin{cases} 1, & \eta_{||} > \eta_T \\ \frac{\eta_{||} + \eta_0}{2\eta_T}, & -\eta_T \leq \eta_{||} \leq \eta_T \\ 0, & \eta_{||} < -\eta_T \end{cases}$$

with $f_- (\eta_{||}) = f_+ (-\eta_{||})$. A suitable choice for $s_0, \eta_0^0, \alpha$ and $\sigma_\eta$ are made to reproduce the charged particle distribution in pseudorapidity at different centralities. Finally, $\eta_T$ is adjusted to reproduce the observed rapidity-odd directed flow of charged particles.

All our results are for the 0 – 80% centrality bin. This corresponds to a choice of impact parameter, $b = 8.3$ fm within our optical Glauber model approach to obtain the initial condition. The (3 + 1)-dimensional relativistic hydrodynamic evolution are carried out by the publicly available vHLLE code [24]. The freezeout hypersurface is assumed to be at a constant temperature $T = 150$ MeV, where statistical emission of hadrons happens [25]. Details of the model and parameters of the hydrodynamic model used at the Brookhaven Relativistic Heavy Ion Collider (RHIC) and at the CERN Large Hadron Collider (LHC) energies can be found in [26].

The full spacetime history of the flow velocity and $T$ fields obtained from the hydrodynamic evolution are fed as input to the Langevin dynamics of the HQs

$$\Delta r_i = \frac{p_i}{E} \Delta t$$

$$\Delta p_i = -\gamma p_i \Delta t + \rho_i \sqrt{2D_0 \Delta t} + F_{iEM}$$

where $F_{EM}$ refers to the Lorentz force due to the EM field. The updates of the position and momentum vectors of the HQ in time interval $\Delta t$ are denoted by $\Delta r$ and $\Delta p$ respectively. Here $i = x, y$ and $z$ are the three Cartesian coordinate components. The initial position coordinates are sampled from the binary collision profile while the momenta are generated from p+p events of PYTHIA [27]. The HQ interaction with the medium is encoded in the drag $\gamma$ and diffusion $D$ term [28]. The Lorentz updates are performed followed by the HQ momentum and the EM field are boosted to the local rest frame of the fluid after which the Langevin updates are performed followed by the HQ momentum being reverted back to the lab frame. At the end of the Langevin evolution $D^{0}$ and $D^{\ast}$ mesons are produced following the Peterson fragmentation of HQs [29].

The Lorentz force $F_{EM}$ is given by,

$$F_{EM} = q \left( \mathbf{E} + \left( \frac{\mathbf{p}}{E} \times \mathbf{B} \right) \right)$$

where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields respectively induced by the protons in the colliding nuclei as well as the backreaction of the fireball with conductivity $\sigma$. The larger the value of $\sigma$ the longer is the lifetime of the EM fields. The symmetry of the problem is such that the only relevant components for the computation of the directed flow are $B_y$ and $E_x$. The calculation of the time dependent EM field follows Refs. [19][30]. We take only the contribution of the spectator protons to the EM fields and use a constant $\sigma$. The important thing to note is that the additional factor of $p/E \sim 0.3$ makes the magnetic force smaller compared to the electric force which makes the slope of the $D^0$ meson $v_1$ larger than $\bar{D}^0$.

To study the dependence on the initial time we vary $\tau_0$ between 0.2 fm to 0.6 fm. Several lattice QCD computations suggest $\sigma \sim 0.023$ fm$^{-1}$ around $2T_c$ [3]. We vary $\sigma$ in the range 0.011 - 0.035 fm$^{-1}$. $D$ could be obtained from the scattering matrix formalism [31]. This also fixes $\gamma$ via Eq. 6. However, we adopt a data driven approach [17][32]. We work with a simple ansatz of $\gamma \propto T^2(T/m)^2$. In Ref. [17], it was shown that one gets a good qualitative description of the $p_T$ dependence of $R_{AA}$ and $v_2$ at mid-rapidity with a choice of $x$ between 0 to 0.5. Hence, we
implement these two extreme choices for $\gamma, \gamma \propto T$ (large drag) and $\gamma \propto T^{1.5}$ (small drag).

In Fig. 1 is shown the $v_1$ of $D^0$ and $\bar{D}^0$ mesons resulting from the dynamics including the combined influence of the drag by matter in the tilted fireball as well as by the Lorentz force. The harmonic flow coefficient $v_1$ is obtained as follows

$$v_1 = \langle \cos (\phi - \Psi) \rangle$$

where $\phi$ is the $D$ azimuthal angle, $\Psi$ is the reaction plane of the event and $\langle \cdot \rangle$ represents an ensemble average over realizations of the Langevin evolution. In our calculation the reaction plane is well defined by the geometry of the event, in experiment it can be reconstructed from the spectators. We use the same definition of the reaction plane as in experimental analyzes [14] [15]; the nucleus flying in the positive $\eta$ direction is located at positive $x$. Note that it is the reverse of the orientation used in Ref. [19].

In terms of the average $v_1^{\text{avg}}$ and difference $v_1^{\text{diff}}$ of $D^0$ and $\bar{D}^0$ flows,

$$v_1^{\text{avg}} = \frac{1}{2} \left( v_1 (D^0) + v_1 (\bar{D}^0) \right)$$

$$v_1^{\text{diff}} = v_1 (D^0) - v_1 (\bar{D}^0)$$

the drag by the tilted fireball alone is expected to give rise to a non-zero $v_1^{\text{avg}}$ and no $v_1^{\text{diff}}$ [17]. On the other hand, the EM force alone can only give rise to $v_1^{\text{diff}}$ and zero $v_1^{\text{avg}}$ [20]. The combined effect due to both gives rise to both $v_1^{\text{avg}}$ as well as $v_1^{\text{diff}}$. Fig. 1 suggests that the drag from the tilted source dominates and one obtains the same sign $v_1$ for both $D^0$ and $\bar{D}^0$. The EM field gives rise to a small charge splitting of the directed flow $v_1^{\text{diff}}$. We now focus on the $v_1$ slope at mid-rapidity to make some quantitative statements on the dependence on various model parameters like $\tau_0, \sigma$ and the collision energy $\sqrt{s_{NN}}$.

The parameter dependence of $D^0$ and $\bar{D}^0$ directed flow

FIG. 1. (Color online) The rapidity dependence of the directed flow coefficient $v_1$ for $D^0$ and $\bar{D}^0$ mesons. The drag by the tilted fireball on $D^0$ and $\bar{D}^0$, being charge independent, creates a rapidity-odd $v_1$ of same sign and strength. The Lorentz force due to the EM field sourced by the protons in the colliding nuclei results in opposite-sign contributions to the rapidity-odd $v_1$ for $D^0$ and $\bar{D}^0$.

FIG. 2. (Color online) (a) Charge splitting $\left( \frac{dv_1}{d\eta} \right)_{\eta=0}^{\text{diff}} = \left( \frac{dv_1}{d\eta} \right)_{\eta=0}^{D} - \left( \frac{dv_1}{d\eta} \right)_{\eta=0}^{\bar{D}}$ is plotted as a function of the medium conductivity $\sigma$. Results are shown for two different initial times $\tau_0 = 0.2$ and 0.6 fm. (b) The mean slope of the $D$ meson directed flow , $\left( \frac{dv_1}{d\eta} \right)_{\eta=0}^{\text{avg}} = 0.5 \left( \frac{dv_1}{d\eta} \right)_{\eta=0}^{D} + \left( \frac{dv_1}{d\eta} \right)_{\eta=0}^{\bar{D}}$ is plotted as a function of the initial time $\tau_0$. 

in Au+Au collisions at \( \sqrt{s_{NN}} = 200 \text{ GeV} \) is shown in Fig. 2. Fig. 2 (a) shows the \( \eta \) slope of \( \Delta v_1^{\text{diff}} \) with respect to variation in \( \sigma \). As \( \sigma \) is raised, the charge splitting of the slope at midrapidity \( \Delta v_1^{\text{diff}}/d\eta \) raises by 400\% for \( \tau_0 = 0.6 \text{ fm/c} \) and by 25\% for \( \tau_0 = 0.2 \text{ fm/c} \). Thus, our results show the possibility to extract \( \sigma \) of QCD matter with the observation of the charge splitting at midrapidity \( \Delta v_1^{\text{diff}}/d\eta \). Also, a smaller \( \tau_0 \) consistently yields a larger \( \Delta v_1^{\text{diff}}/d\eta \). This is because, with increasing \( \sigma \) and/or decreasing \( \tau_0 \), the Lorentz force acts for a longer time on the HQs resulting in increasing \( \Delta v_1^{\text{diff}} \). At very early times (\( \tau \lesssim 0.2 \text{ fm} \)), the trend of the electric field is quite different \( [20] \) which might lead to reduction of the charge splitting. This could be relevant within a framework that takes into account preequilibrium dynamics of the HQs.

The dependence of the \( D \) meson directed flow on the collision energy \( \sqrt{s_{NN}} \) is shown for \( \sigma = 0.023 \text{ fm}^{-1} \) and \( \tau_0 = 0.6 \text{ fm} \). The dependence of the charge splitting of \( D \) meson directed flow \( \Delta v_1^{\text{diff}}/d\eta \) on collision energy is mostly flat (Fig. 3 (a)). In Fig. 3 (b) is plotted the slope of the average \( v_1 \) of \( D^0 \) and \( D^0 \), \( \Delta v_1^{\text{avg}}/d\eta \) versus \( \sqrt{s_{NN}} \). The experimental data for the measured charged particle \( v_1 \) slope is also shown for comparison \( [14, 15] \). This data serves to constrain the parameters of hydrodynamic calculation, and is reasonably well described by the model. On the other hand the energy dependence of the directed flow of \( D \) mesons is a prediction. Both the bulk and the open charm \( v_1 \) decrease with increasing \( \sqrt{s_{NN}} \). Results for two choices of \( \gamma \) is shown to gauge the uncertainty in the prediction for different scenarios of the temperature dependence of the drag coefficient. The \( \gamma \propto T \) (stronger drag) results show a larger \( \Delta v_1^{\text{avg}}/d\eta \) and a smaller \( \Delta v_1^{\text{diff}}/d\eta \) while the \( \gamma \propto T^{1.5} \) (weaker drag) results show a smaller \( \Delta v_1^{\text{avg}}/d\eta \) and a larger \( \Delta v_1^{\text{diff}}/d\eta \). This clearly reveals the interplay of drag by the expanding tilted source and the Lorentz force by the EM field; a stronger drag shifts the balance more in favor of the charge independent flow by the tilted bulk resulting in a larger value of \( \Delta v_1^{\text{avg}}/d\eta \) and a smaller value of \( \Delta v_1^{\text{diff}}/d\eta \) and vice versa for a weaker drag.

Fig. 3 shows the collision energy dependence of the ratio of \( \Delta v_1^{\text{avg}}/d\eta \) for \( D \) mesons to the value for charged particles. While as seen in Fig. 3 (b), both the numerator and denominator show a decreasing trend with \( \sqrt{s_{NN}} \) due to a reduction of the tilt angle, we find that the decrease of the heavy flavor \( v_1 \) slope is smaller compared to that of the bulk resulting in the increasing trend for their ratio in Fig. 4. This increasing ratio stems from the fact that with increasing \( \sqrt{s_{NN}} \), the fireball is denser that calls for stronger drag by the bulk on the HQ and hence a relatively larger \( v_1 \) of the final \( D \) mesons at LHC energies.

HQs serve as good probes of the initial condition in heavy-ion collisions by virtue of being produced only in the initial state. We study the combined effects of two initial phenomena on the flow pattern of HQs, the drag...
The energy dependence of the observed phenomena stems from three main effects. A decrease of the fireball tilt with energy, resulting in a decrease of the directed flow for both charged particles and $D$ mesons. An increase of the fireball temperature, which makes the fireball more opaque to HQ. This effect counterbalances to some extent the first one for $D$ mesons. When going from RHIC to LHC the average directed flow of $D$ mesons is reduced less than that of charged particles. The third effect, taken into account in our calculation, is the energy dependence of the dynamics of the EM fields in the collision. The resulting directed flow splitting for $D^0$ and $\overline{D}^0$ is found to have an almost a flat dependence on $\sqrt{s_{NN}}$.

This work is the first study of the combined effect of the initial tilt of the fireball and the large initial EM fields on the directed flow of $D^0$ and $\overline{D}^0$ mesons. There are several systematic effects which could be investigated to quantify their influence on the numerical estimates that are presented here. Apart from fragmentation, hadronization could also take place via quark recombination or coalescence \[33\]. Since, the light flavor $v_1$ is much smaller than heavy quarks, we expect small influence on the final state heavy flavor $v_1$. The effect of the hadronic rescattering phase post chemical freezeout on the $D^0$ and $\overline{D}^0$ $v_1$ could also be studied in the future \[34\] [35].

Note added: After the completion of this work preliminary experimental data on the directed flow of charm mesons appeared. The results from the STAR Collaboration for Au-Au collisions at $\sqrt{s_{NN}} = 200$ MeV \[36\] is in qualitative agreement with our predictions. The directed flow of charm mesons is large and the charge splitting of the charm meson flow is small (if any). Our calculation predicts a weak energy dependency of the charge splitting of the charm meson directed flow. Preliminary results of the ALICE Collaboration for Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV \[37\] indicate an order of magnitude larger charge splitting of the directed flow. In our calculation such a strong change with energy cannot be explained using a mechanism based on the diffusion of heavy quarks in electromagnetic fields using similar model parameters at RHIC and LHC energies.

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\[1\] J.-Y. Ollitrault, J. Phys. Conf. Ser. \textbf{312}, 012002 (2011) U. Heinz and R. Snellings, Ann. Rev. Nucl. Part. Sci. \textbf{63}, 123 (2013) C. Gale, S. Jeon, and B. Schenke, Int. J. Mod. Phys. A\textbf{28}, 1340011 (2013)

\[2\] S. Gupta, Phys. Lett. B\textbf{597}, 57 (2004) Y. Hirota, M. Hongo, and T. Hirano, Phys. Rev. C\textbf{90}, 021903 (2014) W. Cassing, O. Linnyk, T. Steinert, and V. Ozenchuk, Phys. Rev. Lett. \textbf{110}, 182301 (2013) Y. Yin, Phys. Rev. C\textbf{90}, 044903 (2014) S. I. Finazzo and J. Noronha, \textit{ibid}, D\textbf{89}, 106008 (2014) A. Puglisi, S. Plumari, and V. Greco, \textit{ibid}, D\textbf{90}, 114009 (2014) M. Greif, I. Bouras, C. Greiner, and Z. Xu, \textit{ibid}, D\textbf{90}, 094014 (2014) P. K. Srivastava, L. Thakur, and B. K. Patra, \textit{ibid}, C\textbf{91}, 044903

FIG. 4. (Color online) The beam energy dependence of the ratio of the average heavy quark $v_1$ slope to that of the charged particle $v_1$ is shown. Calculations are performed for $\sigma = 0.023$ fm$^{-1}$ and $\tau_0 = 0.6$ fm and two choices of the temperature dependence of the HQ drag coefficient.

from the expanding tilted fireball and the large EM field in the early stage of the collision. Both of these give rise to HQ directed flow. While the charge independent drag by the matter in the fireball gives rise to the same $v_1$ for $D^0$ and $\overline{D}^0$, the charge dependent Lorentz force by the EM fields gives rise to unlike sign contribution to $v_1$ for $D^0$ and $\overline{D}^0$. We find that the HQ drag contribution dominates resulting in same sign $v_1$ for both $D^0$ and $\overline{D}^0$ with their average $v_1$ being 10 times larger than their difference. The sensitivity of the charge splitting and average $v_1$ of $D^0$ and $\overline{D}^0$ on the model parameters like $\sigma$, $\tau_0$ and $\gamma$ is studied. A smaller $\tau_0$ and/or larger $\sigma$ lengths the time over which the Lorentz force acts on the HQs. This results in larger charge splitting $dv^{\text{diff}}/d\eta$. Also a smaller $\gamma$ reduces the opacity of the tilted source and hence again raises $dv^{\text{diff}}/d\eta$. A smaller initial time $\tau_0$ for the formation of the fireball means that HQ feel the drag of the opaque, dense matter in fireball for a longer time.
(2015) S. Ghosh, ibid. D95, 036018 (2017) K. Hattori and D. Satow, ibid. D94, 114032 (2016) B. Feng, ibid. D96, 036009 (2017) L. Thakur, P. K. Srivastava, G. P. Kadam, M. George, and H. Mishra, ibid. D95, 096009 (2017) S. Mitra and V. Chandra, ibid. D96, 094003 (2017) S. Ghosh, S. Mitra, and S. Sarkar, Nucl. Phys. A969, 237 (2018)

[3] H. T. Ding, A. Francis, O. Kaczmarek, F. Karsch, E. Laermann, and W. Soeldner, Phys. Rev. D83, 034504 (2011) A. Amato, G. Aarts, C. Allton, P. Giudice, S. Hands, and J.-I. Skullerud, Phys. Rev. Lett. 111, 172001 (2013) B. B. Brandt, A. Francis, H. B. Meyer, and H. Wittig, JHEP 03, 100 (2013)

[4] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, Nucl. Phys. A803, 227 (2008) K. Tuchin, Phys. Rev. C82, 034904 (2010), [Erratum: Phys. Rev.C83,039903(2011)] A. Bzdak and V. Skokov, Phys. Lett. B710, 171 (2012) V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya, V. P. Konchakovski, and S. A. Voloshin, Phys. Rev. C83, 054911 (2011) W.-T. Deng and X.-G. Huang, ibid. C85, 044907 (2012)

[5] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D78, 074033 (2008)

[6] R. Rapp and H. van Hees (2010) pp. 111–206, arXiv:0903.1096 [hep-ph] A. Andronic et al., Eur. Phys. J. C76, 107 (2016) G. Aarts et al., ibid. A53, 93 (2017)

[7] L. P. Csernai and D. Rohrich, Phys. Lett. B458, 454 (1999)

[8] R. J. M. Snellings, H. Sorge, S. A. Voloshin, F. Q. Wang, and N. Xu, Phys. Rev. Lett. 84, 2803 (2000)

[9] M. A. Lisa, U. W. Heinz, and U. A. Wiedemann, Phys. Lett. B489, 287 (2000)

[10] A. Adil and M. Gyulassy, Phys. Rev. C72, 034907 (2005)

[11] P. Bozek and I. Wyskiel, Phys. Rev. C81, 054902 (2010)

[12] J. Steinhauer, J. Auvinen, H. Petersen, M. Bleicher, and H. Stöcker, Phys. Rev. C80, 054913 (2014)

[13] B. B. Back et al. (PHOBOS Collaboration), Phys. Rev. Lett. 97, 012301 (2006)

[14] B. I. Abelev et al. (STAR Collaboration), Phys. Rev. Lett. 101, 252301 (2008)

[15] B. Abelev et al. (ALICE), Phys. Rev. Lett. 111, 232302 (2013)

[16] S. Singha, P. Shanmuganathan, and D. Keane, Adv. High Energy Phys. 2016, 2836989 (2016)

[17] S. Chatterjee and P. Bozek, Phys. Rev. Lett. 120, 192301 (2018)

[18] M. Nasim and S. Singh, Phys. Rev. C97, 064917 (2018)

[19] U. Gaursoy, D. Kharzeev, and K. Rajagopal, Phys. Rev. C89, 054905 (2014)

[20] S. K. Das, S. Plumari, S. Chatterjee, J. Alam, F. Scardina, and V. Greco, Phys. Lett. B768, 260 (2017)

[21] S. J. Brodsky, J. F. Gunion, and J. H. Kuhn, Phys. Rev. Lett. 39, 1120 (1977)

[22] A. Bialas and W. Czyz, Acta Phys. Polon. B36, 905 (2005)

[23] P. M. Chesler, M. Lekaveckas, and K. Rajagopal, JHEP 10, 013 (2013) S. K. Das, M. Ruggieri, S. Mazumder, V. Greco, and J.-e. Alam, J. Phys. G42, 095108 (2015)

[24] I. Karpenko, P. Huovinen, and M. Bleicher, Comput. Phys. Commun. 185, 3016 (2014)

[25] M. Chojnacki, A. Kisiel, W. Florkowski, and W. Broniowski, Comput. Phys. Commun. 183, 746 (2012)

[26] P. Bozek, Phys. Rev. C85, 034901 (2012) P. Bozek and I. Wyskiel-Piekarska, Phys. Rev. C85, 064915 (2012)

[27] T. Sjostrand, S. Mrenna, and P. Z. Skands, JHEP 05, 026 (2006) Comput. Phys. Commun. 178, 852 (2008)

[28] M. He, H. van Hees, P. B. Gossiaux, R. J. Fries, and R. Rapp, Phys. Rev. E88, 032138 (2013)

[29] C. Peterson, D. Schlatter, I. Schmitt, and P. M. Zerwas, Phys. Rev. D27, 105 (1983)

[30] K. Tuchin, Phys. Rev. C88, 024911 (2013)

[31] H. van Hees and R. Rapp, Phys. Rev. C71, 034907 (2005) G. D. Moore and D. Teaney, C71, 064904 (2005) S. S. Gubser, D76, 126003 (2007) W. M. Alberico, A. Beraudo, A. De Pace, A. Molinari, M. Monteno, M. Nardi, and F. Prino, Eur. Phys. J. C71, 1666 (2011) H. Berrehrah, P. B. Gossiaux, J. Aichelin, W. Cassing, J. M. Torres-Rincon, and E. Bratkovskaya, Phys. Rev. C90, 051901 (2014) F. Scardina, S. K. Das, V. Minissale, S. Plumari, and V. Greco, C96, 044905 (2017)

[32] Y. Xu, J. E. Bernhard, S. A. Bass, M. Nahrang, and S. Cao, Phys. Rev. C97, 044907 (2018)

[33] V. Greco, C. M. Ko, and R. Rapp, Phys. Lett. B595, 202 (2004) H. van Hees, V. Greco, and R. Rapp, Phys. Rev. C73, 034913 (2006) S. Cao, G.-Y. Qin, and S. A. Bass, C88, 044907 (2013) T. Song, H. Berrehrah, D. Cabrera, W. Cassing, and E. Bratkovskaya, C93, 034906 (2016) S. Plumari, V. Minissale, S. K. Das, G. Coci, and V. Greco, Eur. Phys. J. C78, 948 (2018)

[34] S. K. Das, F. Scardina, S. Plumari, and V. Greco, Phys. Lett. B747, 260 (2015)

[35] S. Cao, G.-Y. Qin, and S. A. Bass, Phys. Rev. C92, 024907 (2015)

[36] S. Singha (STAR Collaboration), Nucl. Phys. A982, 671 (2019)

[37] F. Grosa (ALICE Collaboration), PoS HardProbes2018, 138 (2018)