Higher spin holography and the AdS string sigma model

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Abstract
We analyse the cubic spin-3 interaction in AdS space using the higher spin extension of the string-theoretic sigma-model constructed in our previous work, whose low energy limit is described by the AdS vacuum solution. We find that, in the leading order of the cosmological constant, the spin-3 correlator on the AdS4 string theory side reproduces the structure of the three-point function of composite operators, quadratic in free fields, in the dual d = 3 vector model. The cancellation of holography violating terms in d = 3 is related to the value of the Liouville background charge in d = 4.

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1. Introduction
It is common to think of AdS/CFT holography as a duality between the semiclassical limit of supergravity with a negative cosmological constant and conformal field theory (CFT) living on the boundary of its vacuum solution (AdS space). This, however, is the low energy approximation; in the stronger sense the AdS/CFT conjecture means that the correlation functions of physical vertex operators computed in closed string theory in the anti-de Sitter background must reproduce the correlators of the corresponding conformally invariant observables on the CFT side [1–3]. Regardless of the space-time dimension, higher spin fields in AdS space (with various symmetries) inevitably have to play a critical role in AdS/CFT holography since the overwhelming number of operators on the CFT (gauge theory side), for example, those of the type
\[ \sim \text{Tr}(\phi \nabla_{m_1} \ldots \nabla_{m_s} \phi^I) \] (1)
simply have no choice but to match the higher spin objects propagating in AdS space-time (and possibly polarized along the direction of the boundary). In particular, it has been conjectured [4, 5] that, in the case of AdS4/CFT3, the symmetric fields of spin s in AdS4 described by Vasiliev’s unfolding formalism (e.g. see [6–12]) are dual to the symmetrized objects of the...
type (1) at the conformal points of the $O(N)$ vector model in $d = 3$ for even values of $s$, and the $U(N)$ model for odd spins. This conjecture has been checked explicitly in important papers [13–15] and later analysed in a number of insightful works, e.g. [16–20] whose results suggest the importance of the free field theory limit in the $O(N)/HS$ duality, despite the fact that the Maldacena–Zhiboedov theorem can be circumvented under certain assumptions [18]. Apart from the low-energy limit, the dynamics of these higher spin fields is described by physical vertex operators in open or closed string theories in anti-de Sitter space and their worldsheet correlation functions. In particular, the AdS/CFT duality conjecture strongly suggests the existence of an infinite tower of massless higher spin states in the string spectrum in AdS space-time. In practice, however, little is known about AdS string theory dynamics beyond the semiclassical limit, since straightforward quantization of string theory in AdS space-time is not known (e.g. see [21]). Another important point is that, in the standard description, the string excitations correspond to the space-time fields in the metric [22–24] rather than the unfolded formulation, while it is the unfolded formalism which is the most natural and efficient framework to approach the problem of the higher spin extension of the AdS/CFT duality [4, 5, 13–15, 18, 25]. In a recent work [26] we constructed the string-theoretic sigma model based on hidden space-time symmetry generators in the RNS formalism, realizing the AdS$_d$ isometry group. The model is initially defined in the flat background; however, when perturbed by the vertex operators based on the hidden AdS isometry generators, it flows to the new fixed 2D conformal point, corresponding to the AdS geometry in space-time. This can be shown by analysing the conformal beta-function of the sigma-model, resulting in the low-energy effective equations of motion, describing (in the leading order) the AdS$_d$ vacuum solutions of gravity with negative cosmological constant in the MacDowell–Mansouri–Stelle–West description [27–29]. Remarkably, the closed string vertex operators, constructed in [26], describe the gravitational excitations around the AdS vacuum in the frame, rather than the metric formalism—i.e. in terms of the vielbein and spin connection gauge fields. In this paper we extend our analysis of this sigma-model to include the excitations corresponding to massless higher spin fields in the frame-like Vasiliev approach. The string-theoretic vertex operators for the frame-like higher spin fields have been constructed in our earlier work [30] where we performed their BRST analysis and analysed their correlators in flat space, showing them to lead to Berends–Burgers–Van Dam type [31] cubic spin-3 interactions [32–34, 24, 35–37] in flat space. In the current paper we extend this analysis using the sigma-model approach [26] in order to study the AdS deformations of these cubic interactions and their relevance to higher spin holography problems [18, 13–17, 19, 20, 38–41] in the context of [5]. We find that, in the leading nontrivial order in $\rho^{-1}$, the cubic spin-3 interaction reproduces the correlators of the operators of type (1) in the free field limit of the $U(N)$ model in $d = 3$. In particular, in this limit the cubic spin-3 interaction is dominated by the nine-derivative terms, while the lower derivative terms (posing a potential threat to the holography) are absent as their cancellation is ensured by the ghost number selection rules for the vertex operators and by the value of the Liouville background charge ($q = \sqrt{\frac{5}{2}}$ in $d = 4$). The terms with the lower number of derivatives, however, are generally present in the sigma-model for $d \neq 4$. In addition, in the $d = 4$ case these terms may still appear in the higher order corrections in $\alpha'$ (corresponding to $\frac{1}{q}$ corrections in the dual theory). Next, it is the momentum behaviour and the pole structure of the string-theoretic spin-3 amplitude in the sigma-model that corresponds to the field-theoretic pole structure of the three-point amplitude of the operators (1) of the dual theory in the momentum space. In the following sections, we shall use the AdS string sigma-model to perform the explicit computation of the three-point correlators using the vertex operators for spin-3 fields in the frame-like formalism (in the leading nontrivial orders in $\Lambda$ and $\alpha'$) and discuss physical implications of our results.
2. Sigma-model for AdS strings and vertex operators for frame-like higher spin fields: a brief review

The sigma-model for AdS strings constructed in [26] is based on hidden space-time symmetry generators in the RNS superstring theory. Namely, consider the RNS superstring theory in flat space with the action given by:

\[ S_{RNS} = S_{\text{matter}} + S_{bc} + S_{\beta\gamma} + S_{\text{Liouville}} \]

\[ S_{\text{matter}} = \frac{-1}{4\pi} \int d^2z (\partial X_m \bar{\partial} X^m + \psi_m \bar{\partial} \psi^m + \bar{\psi}_m \partial \bar{\psi}^m) \]

\[ S_{bc} = \frac{1}{2\pi} \int d^2z (b \bar{\partial} c + \bar{b} \partial \bar{c}) \]

\[ S_{\beta\gamma} = \frac{1}{2\pi} \int d^2z (\beta \bar{\partial} \gamma + \bar{\beta} \partial \bar{\gamma}) \]

\[ S_{\text{Liouville}} = \frac{-1}{4\pi} \int d^2z \left( \partial \bar{\phi} \partial \phi + \bar{\partial} \lambda \lambda + \partial \bar{\lambda} \bar{\lambda} + \mu_0 e^{\bar{\phi}} (\lambda \bar{\lambda} + F) \right), \]

where \( \phi, \lambda, F \) are components of the super Liouville field and the Liouville background charge is

\[ q = B + B^{-1} = \sqrt{\frac{9 - d}{2}}. \]

The ghost fields \( b, c, \beta, \gamma \) are bosonized according to

\[ b = e^{-\sigma}, \quad c = e^\sigma \]

\[ \gamma = e^{\phi - \chi} \equiv e^{\phi} \eta \]

\[ \beta = e^{\epsilon - \phi} \partial \chi \equiv \partial \xi e^{-\phi} \]

and the BRST charge is

\[ Q = Q_1 + Q_2 + Q_3 \]

\[ Q_1 = \oint \frac{dz}{2i\pi} (cT - bc\partial c) \]

\[ Q_2 = -\frac{1}{2} \oint \frac{dz}{2i\pi} \left( \gamma \bar{\psi}_m \partial X^m - q \bar{\partial} \lambda \right) \]

\[ Q_3 = -\frac{1}{4} \oint \frac{dz}{2i\pi} b \gamma^2. \]

Then, in the limit \( \mu_0 \to 0 \) the action (2) is symmetric under the global space-time transformations generated by

\[ T^m = \frac{1}{\rho} K \circ \oint dz \partial X^m \]

\[ T^{mn} = K \circ \oint dz \psi_m \psi^n, \]

where the homotopy transform of an operator \( V K \circ V \) is defined according to

\[ K \circ V = T + \frac{(-1)^{N}}{N!} \oint \frac{dz}{2i\pi} (z - w)^{N} : K \partial W(z) : \]

\[ + \frac{1}{N!} \oint \frac{dz}{2i\pi} (z - w)^{N+1} K(z) [Q_{\text{brst}}, U]. \]
where \( w \) is some arbitrary point on the worldsheet, \( U \) and \( W \) are the operators defined according to
\[
[Q_{\text{brst}}, V(z)] = \partial U(z) + W(z),
\]
\[
K = ce^{2X-2\phi}
\]
is the homotopy operator satisfying \([Q_{\text{brst}}, K] = 1\) and \( N \) is the leading order of the operator product
\[
K(z_1)W(z_2) \sim (z_1 - z_2)^NY(z_2) + O((z_1 - z_2)^{N+1}).
\]
The operators \( T_m \) and \( T_{mn} \) then can be shown to satisfy the AdS isometry algebra with the cosmological constant \( \Lambda = -\frac{1}{\rho^2} \) [26]:
\[
[T_{ab}, T_{cd}] = \eta_{ac}T_{bd} - \eta_{ab}T_{cd} - \eta_{cd}T_{ab} + \eta_{bd}T_{ac}
\]
\[
[T_a, T_b] = -\frac{1}{\rho^2} T_{ab}.
\]
The minus sign in the last commutator is actually highly nontrivial and is related to the subtleties of operator product expansion (OPE) and picture equivalence relations analysed in [26]. The AdS isometry algebra (11) also admits another realization in terms of the operators \((S^m, L^m)\) where
\[
L^m = K \circ T^m \equiv K \circ \oint \frac{dz}{2i\pi} \psi^m\psi^m
\]
is the same full rotation operator (6) (where the \( K \circ \) represents the homotopy transformation to ensure the BRST-invariance) while \( S^m \) is the homotopy transformation of the operator \( \oint \frac{dz}{2i\pi} \lambda \psi^m \), representing the rotation in the Liouville-matter plane:
\[
S^m = K \circ \rho^{-1} \oint \frac{dz}{2i\pi} \lambda \psi^m
\]
\[
= \rho^{-1} \oint \frac{dz}{2i\pi} \left[ \lambda \psi^m + 2c e^{X-\phi} \left( \partial \psi^m - \partial X^m \lambda - qP^{(1)} \psi^m \right) - 4\partial c e^{2X-2\phi} \lambda \psi^m \right]
\]
\[
= -4 \left\{ Q, \rho^{-1} \oint \frac{dz}{2i\pi} e^{2X-2\phi} \lambda \psi^m \right\}.
\]
The conformal weight \( n \) polynomials \( P^{(n)}_{\phi^m + \lambda \psi^m + \sigma} \) (where \( a, b, c \) are some constants) are defined according to
\[
P^{(n)}_{\phi^m + \lambda \psi^m + \sigma} = e^{-a\phi(z)}bX(z)-c\sigma(z) \left( \frac{d^n}{dz^n} e^{a\phi(z)+bX(z)+c\sigma(z)} \right)
\]
(with the product taken in the algebraic rather than OPE sense).

Starting from the symmetry generators (6), (13), one can construct the closed string vertex operator describing the dynamics of vielbeins and spin connection gauge fields in space-time [26]
\[
G(p) = e^m(p)F_m - \partial_{\alpha m}(p) \left( F_{m} L_{\alpha} - \frac{1}{2}F_{\alpha m} L_{m} \right) + \text{c.c.}
\]
where
\[
F_m = -2K_{U_1} \oint d\lambda \psi^m e^{i\lambda X}(z)
\]
\[
U_1 = \lambda \psi^m e^{i\lambda X} + \frac{i}{2} Y \lambda \left( \hat{p} \hat{\psi} \psi^m - P_{\alpha m} P^{(1)} \right) e^{i\lambda X}
\]
or manifestly

\[ F_m = -2 \int dz \left\{ \lambda \psi_m (1 - 4 \partial \lambda e^{2\varphi - 2\phi} + 2\kappa e^{2\varphi} (\lambda \partial X_m - \partial \varphi \psi_m + q \psi_m P^{(1)}_{\varphi}) \right. \\
\left. - \frac{i}{2} (\bar{\psi} \bar{\psi} \gamma_5 \psi_m - p_m P^{(1)}_{\varphi - \chi}) \right\} e^{i\pi X}(z), \]  

(17)

where the partial homotopy transform \( T \rightarrow L = K_T \circ T \) of an operator \( T \) based on \( \gamma \) is defined according to

\[ L(u) = K_T \circ T = T + \frac{(-1)^N}{N!} \oint \frac{dz}{2\pi i} (z - w)^N : K \bar{\partial}^N \gamma : (z) + \frac{1}{N!} \oint \frac{dz}{2\pi i} \gamma^{N+1} [(z - w)^N K(z)] K[Q_{\text{rest}}, U] \]  

(18)

where \( N \) is the leading order of the OPE of \( K \) and \( \gamma \). Particularly, if \( [Q, T] = \oint \gamma \), the partial homotopy transform obviously coincides with the usual homotopy transform. Next,

\[ L^a = \int d\bar{z} e^{-3\phi} \left\{ \frac{i}{2} \bar{\psi} \bar{\psi} \gamma^a - 2\bar{\partial} \bar{\partial} X^a + i p^a \left( \frac{1}{2} \bar{\partial} \bar{\lambda} + \frac{1}{q} \bar{\partial} \bar{\varphi} \bar{\lambda} - \frac{1}{2} \bar{\lambda}(\bar{\partial} \bar{\varphi})^2 \right) \right. \\
\left. + (1 + 3q^2) \bar{\lambda} \left( 3 \bar{\partial} \bar{\varphi} \bar{\psi} b - \frac{1}{2q} \bar{\partial} \bar{\varphi} \right) \right\} e^{i\pi X} \]  

(19)

at the minimal negative picture \(-3\) representation and

\[ L^a = K \circ \int d\bar{z} e^{2\varphi} \left\{ \frac{i}{2} \bar{\psi} \bar{\psi} \gamma^a - 2\bar{\partial} \bar{\partial} X^a + i p^a \left( \frac{1}{2} \bar{\partial} \bar{\lambda} + \frac{1}{q} \bar{\partial} \bar{\varphi} \bar{\lambda} - \frac{1}{2} \bar{\lambda}(\bar{\partial} \bar{\varphi})^2 \right) \right. \\
\left. + (1 + 3q^2) \bar{\lambda} \left( 3 \bar{\partial} \bar{\varphi} \bar{\psi} b - \frac{1}{2q} \bar{\partial} \bar{\varphi} \right) \right\} e^{i\pi X} \]  

(20)

at the minimal positive picture \(+1\) representation. (Similar to its holomorphic counterpart \( L^a \).) Then,

\[ F_{ma} = F_{ma}^{(1)} + F_{ma}^{(2)} + F_{ma}^{(3)} \]  

(21)

where

\[ F_{ma}^{(1)} = -4q K_U i \int d\bar{z} c e^{-\varphi} \gamma^a c \psi_m \psi_a \]  

\[ U_2 = [Q = Q_3, c e^{-\varphi} \gamma^a c \psi_m \psi_a e^{i\pi X}] - \frac{i}{2} e^{\lambda} \left( (\bar{\psi} \bar{\psi}) \gamma^a \gamma^a - p_m P^{(1)} c e^{i\pi X} \right) \]  

(22)

\[ F_{ma}^{(2)} = K \circ \int d\bar{z} \psi_m \psi_a e^{i\pi X} = -4 \left\{ Q, \int d\bar{z} c e^{-2\varphi} e^{i\pi X} \psi_m \psi_a (z) \right\} \]  

(23)

and

\[ F_{ma}^{(3)} = K \circ \int d\bar{z} e^{2\varphi} \psi_m \bar{\psi}_m (\bar{\partial} \bar{\partial} X_a) e^{i\pi X}(z). \]  

(24)

In the limit of zero momentum the holomorphic and the antiholomorphic components of the operator correspond to AdS isometry generators in different realizations, as described above. The BRST invariance imposes the following on-shell constraints on vielbein and connection fields:

\[ p^{ab}_{\nu} e_{\mu}^{(a)} (p) - \omega_{\nu}^{b}_{\nu} e_{\mu}^{(a)} (p) = 0 \]

\[ p^{ab}_{\nu} \gamma_{\mu}^{a} (p) = 0 \]

\[ p^{ab}_{\nu} e_{\mu}^{(a)} (p) = 0 \]

\[ p^{ab}_{\nu} \gamma_{\mu}^{a} (p) = 0. \]  

(25)
The first two constraints represent the linearized equations $R^{AB} = 0$ (the first one being the zero torsion constraint $T^a = R^{a0} = 0$ while the second reproduces the vanishing Lorenz curvature $R^0 = 0$). The last two constraints represent the gauge fixing conditions related to the diffeomorphism symmetries. The fact that the BRST invariance leads to space-time equations in a certain gauge is not surprising if we recall that similar constraints on a standard vertex operator of a photon also lead to Maxwell’s equations in the Lorenz gauge. Provided that the constraints (25) are satisfied, the vertex operator $G(p)$ can be written as a BRST commutator in the large Hilbert space plus terms that are manifestly in the small Hilbert space, according to

$$G(p) = \{Q, W(p)\} + \frac{1}{q} K \circ \omega_m^{ab} \int dz e^\theta (\psi^m \partial^2 X_a) - 2 \partial \psi_{\{m} \partial_{\rangle a\rangle}) e^{ipX} (z) \bar{L}_a + \text{c.c.}$$

$$W(p) = 8 \rho_m^a (p) \bar{L}_a \int dz \partial \partial \partial \psi e^{-2\phi} \lambda \psi^m e^{ipX} + \omega_m^{ab} \bar{L}_b \left[ - \frac{4}{q} \int dz \partial \partial \partial \psi e^{-2\phi} \psi \psi^m e^{ipX} \\
+ 4 \int dz (z - w) \partial \partial \partial \partial \partial \partial \psi e^{-3\phi} \lambda \psi^m e^{ipX} \right].$$

(26)

This particularly implies that, modulo gauge transformations, the vertex operator $G(p)$ is the element of the small Hilbert space.

The linearized gauge symmetry transformations for vielbein and connection gauge fields are given by:

$$\delta \alpha^a_m = \partial_m \rho^a + \rho_m^a$$

$$\delta \omega_m^{ab} = \partial_m \rho^{ab} + \rho^{[a \delta^b_m]}$$

(27)

where we write $\rho^{AB} = (\rho^{ab}, \rho^{\lambda 0}) = (\rho^{ab}, \rho^a)$. The variation of $G(p)$ under (27) in the momentum space is

$$\delta G(p) = p^m F_m \rho^a + p^m F_{ma} \rho^{ab}.$$  (28)

The two terms of the variation (28) are BRST exact in the small Hilbert space (and therefore are irrelevant in correlators) since

$$p^m F_m = \{Q : \Gamma : (w) [Q, \xi A]\}$$

$$A = \int dz e^{-3\phi} \partial \lambda ((\bar{p} \partial \bar{X}) \lambda - (\bar{p} \bar{\psi}) \partial \phi + (\bar{p} \bar{\psi}) P^{(1)} \psi_{-(1+q)} X) e^{ipX}$$

(29)

and

$$p^m F_{ma}^{(1)} = 4q \left[ Q, \Gamma (w) \int dz e^{-3\phi} \partial \partial \partial \partial \psi \lambda \psi_a (\bar{p} \bar{\psi}) e^{ipX} \right]$$

$$p^m F_{ma}^{(2)} = \left\{ Q : \Gamma : (w) \int dz \partial \partial \partial \partial \partial \psi (\bar{p} \bar{\psi}) \partial X_a - (\bar{p} \partial \bar{X}) \psi_a e^{ipX} \right\}$$

$$p^m F_{ma}^{(3)} = \left\{ Q, K \circ \int dz \partial \psi_a e^{ipX}, B \right\}$$

$$B = \int dz \partial \partial \psi e^{-3\phi} \partial \lambda ((\bar{p} \bar{\psi} \partial^2 \bar{X}) - 2 \partial \partial \partial \partial \partial \partial \lambda ((\bar{p} \bar{\psi} \partial^2 \bar{X}) - 2 (\partial \bar{\psi} \partial \bar{X}))].$$

(30)

Therefore gauge transformations of $e$ and $\omega$ shift $G(p)$ by terms not contributing to correlators. The $G(p)$ vertex operator, whose construction is explained above, describes the dynamics of the spin $s = 2$ massless field in the closed string spectrum, in terms of vielbein and connection gauge fields. Note that, despite the fact that the unperturbed theory has been originally defined in flat space, in the perturbed theory (which flows to the AdS vacuum) the distinction between the tangent indices $a, b, ...$ and the manifold indices $m, n, ...$ is ensured.
by the corresponding operators of $F_{\nu}$-type and $L^\nu$-type being the elements of different ghost cohomologies and having very different on-shell constraints and gauge symmetries. In the leading order, the vanishing beta-function condition for the sigma-model, given by the RNS action perturbed by the $G(p)$-operator (15) leads to space-time equations for $\omega$ and $\lambda$, given by [26]

$$R^{ab} = d\omega^{ab} + (\omega \wedge \omega)^{ab} - \frac{1}{\rho^2} \epsilon^{ab} \wedge \epsilon^{cd} = 0$$

(31)

and

$$d\epsilon^a + \omega^{ab} \wedge \epsilon^b = 0$$

(32)

with the solution given by the AdS vacuum [27]. From the two-dimensional point of view this means that the RNS theory, initially defined in flat space and perturbed with $G(p)$, flows to the new fixed conformal point corresponding to the theory in the new space-time background, namely, AdS. The next step is to introduce the higher spin excitations. The vertex operators, describing the dynamics of the massless higher spin $s \geq 3$ fields in Vasiliev’s frame-like formalism, can be constructed in the open string sector of the extended RNS superstring theory. Such a construction has been recently performed in [30]. In the frame-like approach, the spin-3 field is described by the dynamical space-time field $\omega^{20}_m \equiv \omega^{20}_m$, as well as by two auxiliary fields $\omega^{21}_m \equiv \omega^{21}_m$ and $\omega^{21}_m \equiv \omega^{21}_m$, related to $\omega^{20}_m$ by generalized zero torsion constraints [30].

The vertex operators for the dynamical $\omega^{20}_m$-field for the massless spin-3 are given by:

$$V^{(-3)} = H_{\text{bar}}(p)c \epsilon^{-\phi} \partial X^a \partial X^b \psi^m \epsilon^{i0X}$$

at uninverted minimal negative picture and

$$V^{(+1)} = K \circ H_{\text{bar}}(p) \oint d\omega^{20}_m \epsilon^{i0X}$$

at integrated minimal positive picture +1. These operators are the elements of the superconformal ghost cohomology $H_1 \sim H_2$ [30]. The vertex operators for the first auxiliary field $\omega^{21}_m$ are given by

$$V^{(21)}_{-} = 2 \omega^{ab}_m(p) c \epsilon^{-\phi} (\omega^{ab}_m \partial X^a \partial X^b) - 2 \partial X^a \partial X^b \psi^m \epsilon^{i0X}$$

(35)

at negative (uninverted) representation and

$$V^{(21)}_+ = 2 \omega^{ab}_m(p) K \oint d\omega^{20}_m \epsilon^{i0X}$$

(36)

at positive (inverted) representations. The operators (35), (36) are the elements of $H_2 \sim H_4$; the cohomology constraints for $V^{(21)}_{-}$ lead to generalized zero torsion constraints relating $\omega^{21}_m$ and $\omega^{20}_m$,

$$\omega^{abc}_m = 2p^d \omega^{ab}_m - p^a \omega^{bc}_m$$

(37)

modulo BRST exact terms in small Hilbert space. The vertex operators for the second auxiliary field $\omega^{21}_m$ are given by

$$V^{(21)}_{+} = -3 \omega^{abcd}_m(p) c \epsilon^{-\phi} (\omega^{ab}_m \partial X^a \partial X^b - 2 \partial X^a \partial X^b \psi^m \epsilon^{i0X}$$

(38)
at negative (unintegrated) representation and
\[ V_+^{2|2}(p) = -3\omega_m^{abcd}(p) K \circ \oint dz e^{3\phi} \left( \psi^m \partial^2 \psi_r \partial^3 \psi_d \partial X^a \partial X_b - 2\psi^m \partial \psi_r \partial^3 \psi_d \partial X_a \partial^2 X_b \right. \]
\[ + \frac{5}{8} \psi^m \partial \psi_r \partial^2 \psi_d \partial X^3 \partial X_b + \frac{57}{16} \psi^m \partial \psi_r \partial^2 \psi_d \partial^2 X_a \partial^2 X_b \right) e^{i\lambda x} \] (39)
at positive (integrated) representation. The \( V_+^{2|2} \) operators are the elements of \( H_3 \sim H_{-3} \) and the cohomology constraint leads to the second generalized torsion condition relating \( \omega_3^{2i}(p) \) and \( \omega_3^{2i}(p) \) up to BRST exact terms:
\[ \omega_m^{abcd} = 2p^b \omega_0^{bcd} - p^d \omega_0^{adc} + p^d \omega_0^{bad} + p^b \omega_0^{acd}, \quad (40) \]
Combining the AdS sigma-model construction [26] with expressions for vertex operators describing the higher spin excitations in an unfolded formalism, the generating functional for the model describing the higher spin dynamics in AdS space is given by
\[ Z(e_m^{ab}, \omega_m^{ab}, \omega_s^{a-1i}, \rho) = \int D(X, \psi, \bar{\psi}, \text{ ghosts}) e^{-S_{\text{rest}} + \int\, \left(G(p, \rho) + \sum \omega_s^{a-1i}(p) V^{a-1i}(p) \right)}, \quad (41) \]
where \( \{\omega_s^{a-1i}\} \) is the set of dynamical and auxiliary fields for the spin \( s > 2 \) and \( V^{a-1i} \) are the corresponding massless vertex operators in open string theory. In this paper we shall restrict ourselves to the spin-3 case. The correlation functions describing the higher spin interactions in the AdS space are then given by
\[ \langle V_{s-1}^{a_1}(p_1) ... V_{s-1}^{a_n}(p_n) \rangle = \frac{\delta^{(n)} Z(e_m^{ab}, \omega_m^{ab}, \omega_s^{a-1i}, \rho)}{\delta \omega^{a_1-1i_1}(p_1) ... \delta \omega^{a_n-1i_n}(p_n)} \bigg|_{\omega_s^{a-1i} = 0, ... , \omega_s^{a_n-1i_n} = 0}. \quad (42) \]
In the rest of the paper, for the sake of the holographic context, we shall assume that all the operators of the \( d \)-dimensional non-critical superstring theory are polarized along the \( d-1 \)-dimensional subspace and also propagate in this subspace corresponding to the underlying AdS boundary, unless stated otherwise.

Perturbation expansion in the powers of \( \frac{1}{\rho^s} \) then describes the AdS deformations of the higher spin interactions in terms of \( \alpha' \) and the cosmological constant \( \Lambda \) in the frame-like formalism. In the next section we shall use the generating functional (41) in order to compute the AdS deformations of the 3-vertex for the spin-3 fields in the first nontrivial order in \( \Lambda \) and to analyse their relevance to the CFT correlators in the dual model.

3. Holographic spin-3 vertex in AdS background: preliminaries

In the previous work [30] we computed the three-point function of the spin-3 vertex operators (33)–(40) in open string theory in flat space, showing it to reproduce the Berends–Burgers–Van Dam (BBD) type of interaction vertex in space-time [31] for spin-3 in the frame-like formalism. To compute the AdS\(_d\) deformations of this vertex, one has to expand the functional (41) in powers of \( \frac{1}{\rho} \), that is, \( G(p) \). The result significantly depends on the number of space-time dimensions since \( G(p) \) expression (15) depends manifestly on the Liouville background charge. Since the \( G(p) \) operator for the vielbein and spin-2 connection is a closed string excitation and the spin-3 fields vertex operators are in the open string spectrum, the leading order contribution to the AdS\(_d\)-deformation stems from the amplitude on the disc. Furthermore, it is clear that the contribution linear in the spin-2 connection \( \omega_m^{ab} \), which has the order of \( \rho^{-1} \sim \sqrt{\lambda} \), vanishes since the corresponding correlator is linear in the Liouville superpartner \( \lambda \), i.e. is proportional to the vanishing one-point function of \( \lambda \). Similarly, all the contributions proportional to odd powers of \( \rho^{-1} \) or half-integer powers of \( \Lambda \) vanish as well, since they all are given by the correlators containing odd numbers of \( \lambda \) insertions. For this reason, the first
nontrivial leading order contribution to the disc correlator is proportional to the $AdS_d$ vielbein field $e_m(p)$ and is of the order of $p^{-2}$. This is the contribution given by the four-point function on the disc, equivalent to the five-point function on the sphere. The ghost number selection rule therefore requires that the overall left $+$ right $\phi$-ghost number carried by the correlator equals $-2$. This selection rule particularly makes it convenient to take two spin-3 operators at the $\omega^{210}$ representation and at the negative unintegrated ghost picture $-3$ representation (33). It is convenient to locate them at the points $z_{1,2} = \pm i$ on the disc. The third spin-3 operator is convenient to take at the $\omega^{212}$ representation (39) and at the minimal positive integrated ghost picture $+3$ representation. There is no loss of generality here, since the operators for $\omega^{210}$ and $\omega^{211}$ are the elements of different ghost cohomologies and do not contribute due to ghost number selection rules. Similarly, the selection rules exclude the pieces proportional to $|e|^2$ and their derivatives. The manifest form of these operators is irrelevant to us since the pieces proportional to $|e|^2$ are of the order of $e^{-2\frac{c}{2}}$. This is the contribution given by the four-point function $L(p)$ and of the integrated spin-3 operator for $\omega^{212}$, upon applying all the relevant homotopy/partial homotopy transforms:

\[
F_m = -2 \int dz \left\{ \lambda \psi_m (1 - 4 \partial c c e^2 x^2 - 2 \phi^2) + 2 c e^x \phi \left( \lambda \partial X_m - \partial \phi \partial \psi_m + q \phi m \right) \right\} e^{i\phi X}(z).
\]

(43)

and

\[
L^x(p, u) = \frac{1}{2} \int dz (z - u)^2 \left\{ \left( P_{2\phi-2\chi-\sigma}^{(2)} \right) - 24 \partial c c e^2 x^2 - \phi \right) \lambda \partial^2 X^a - 2 \partial \chi \partial X^a d q \partial \sigma \left( \frac{1}{2} \partial^2 \chi + \frac{1}{4} \partial \phi \partial \chi - \frac{1}{2} \partial \chi \partial \sigma \right) + (1 + 3 q^2) \lambda \left( 3 \partial \psi_b \psi^b - \frac{1}{2} \partial \sigma \psi^2 \right) + c e^x G^{(4)}(\phi, \chi, \psi, \lambda, X) \right\} e^{i\phi X}(z).
\]

(44)

where $u$ is an arbitrary point whose choice is irrelevant to the correlators since all the $u$-derivatives of $L^x$ are BRST-exact in the small Hilbert space [42]. For our purposes, it shall be particularly convenient to choose $u = -i$ on the unit disc boundary. Finally

\[
V^{(2)}_x(p) = -3 a_m^{(\phi \psi \chi \omega \psi \lambda \chi \lambda \theta)} \left\{ \frac{1}{64} e^3 \phi P_{2\phi-2\chi-\sigma}^{(6)} + 28 \partial c c e^2 x^2 + \phi \right) \lambda \partial^2 X^a - 2 \partial \chi \partial X^a d q \partial \sigma \left( \frac{1}{2} \partial^2 \chi + \frac{1}{4} \partial \phi \partial \chi - \frac{1}{2} \partial \chi \partial \sigma \right) + (1 + 3 q^2) \lambda \left( 3 \partial \psi_b \psi^b - \frac{1}{2} \partial \sigma \psi^2 \right) + c e^x + \lambda G^{(12)}(\phi, \chi, \psi, \lambda, X) \right\} e^{i\phi X}(z).
\]

(45)

where $G^{(4)}(\phi, \chi, \psi, \lambda, X)$ and $G^{(12)}(\phi, \chi, \psi, \lambda, X)$ are certain operators of conformal dimensions 4 and 12 accordingly, depending on derivatives of the matter and Liouville fields $X, \psi$, bosonized ghost fields $\phi$ and $\chi$ and also on $\lambda, \psi$ and their derivatives. The manifest form of these operators is irrelevant to us since the pieces proportional to $\sim c e^x$ in $L^x$ and to $\sim c e^{x+2\theta}$ in $V^{(2)}_x(p)$ do not contribute to the overall correlator due to the ghost number selection rules. Similarly, the selection rules exclude the pieces proportional to $\sim \partial c c e^2 x^2 - \phi \partial \sigma \psi^2$.
and $\sim \varphi e^{i\tau + 2\phi}$ in the expressions (44), (45) for $L^2$ and $V^{12}_m(p)$ accordingly. Finally, the selection rules leave the only relevant term in the expression (17) for $F_m$ proportional to $\sim \oint dz \varphi$, with all others not contributing to the leading order correlator for the same reason. This altogether significantly simplifies the computation of the five-point correlator, making it still cumbersome but no longer insurmountable.

4. Holographic spin-3 vertex in AdS background: the computation

Using the results of the previous section, it is now straightforward to identify the correlator giving the AdS deformation of spin-3 vertex in the leading order:

$$A(p; k_1, k_2, k_3) = e_m^a(p)\omega^{(ab)}_{m_1}(k_1)\omega^{(bc)}_{m_2}(k_2)\omega^{(ca)}_{m_3}(k_3)$$

$$\times \left\{ \int d^2z(z-u)^2 \left[ e^{i\phi(p)}P^{(2)}_{2\phi-2\chi-\sigma} + \left( \alpha \right) \partial \partial \varphi + 3(1 + \frac{1}{3}q^2)\lambda \partial \psi \psi \right] e^{i\phi X} (z) e^{i\phi X} (\bar{z}) + c.c. \right\}$$

$$\times \int dt (\tau - \tau_1)^6 P^{(6)}_{2\phi-2\chi-\sigma} e^{i\phi(\psi \partial \psi, \partial \chi \partial \chi)}$$

$$\times \left\{ \frac{57}{16} e^{i\phi(\psi \partial \psi, \partial \chi \partial \chi)} e^{i\phi X} \psi \partial \psi \partial \chi \partial \chi + \frac{5}{8} e^{i\phi(\psi \partial \psi, \partial \chi \partial \chi)} e^{i\phi X} \right\}$$

where $\tau_1 = -i, \tau_2 = i$, the $\tau$-integration is over the disc boundary and the $z$-integral is over the interior of the disc. In order to simplify the computation, a useful strategy is to first perform the conformal transformation from the disc to the upper half-plane using

$$z \rightarrow i \frac{z + i}{z - i}.$$  \hspace{1cm} (47)

Then the integrand of the correlator (46) can be computed on the half-plane and integrated in $\tau$ (which, upon the conformal transformation, becomes the integral over the real axis). Having done that, we shall conformally map the obtained expression back to the disc, in order to perform the $z$-integration over the disc’s interior. So we start with the first step, that is, computing the integrand of (46) on the half-plane.

The contributions to this correlator are factorized in terms of ghost, $\psi = \lambda$ and $X$-parts. Let us start with the ghost part, given by

$$A_{gh}(\tau, z, \tau_1, \tau_2) = \langle e^{i\phi P^{(6)}(\chi)} e^{i\phi P^{(2)}(\chi)} e^{-i\phi(\psi \partial \psi, \partial \chi \partial \chi)} \rangle.$$  \hspace{1cm} (48)

Note that, upon the conformal transformation (47) we have $\tau_1 = 0, \tau_2 \rightarrow \infty$, so as usual, we only need the leading order of this correlator in $\tau_2$ (all others shall result in expressions with negative powers of $\tau_2$ in the overall correlator, vanishing in the limit $\tau_2 \rightarrow \infty$ and corresponding to the pure gauge contributions). This means that we only should consider the expressions of the ghost polynomials $P^{(2)}_{2\phi-2\chi-\sigma}(z)$ and $P^{(6)}_{2\phi-2\chi-\sigma}(\tau)$ between themselves and with the ghost exponents $e^{i\phi(\psi \partial \psi, \partial \chi \partial \chi)}$ and $e^{i\phi(\chi \partial \chi)}$. First of all, we note (as is straightforward to check) that the contributions between the ghost polynomials are limited to

$$P^{(2)}_{2\phi-2\chi-\sigma}(z)P^{(6)}_{2\phi-2\chi-\sigma}(\tau) = \frac{12}{(z - \tau)^2} P^{(1)}_{2\phi-2\chi-\sigma}(z)P^{(5)}_{2\phi-2\chi-\sigma}(\tau).$$  \hspace{1cm} (49)
Then correlator (48) can be computed using the associate ghost polynomial (AGP) technique, explained in [30]. The table of the relevant AGPs for $P^{(6)}_{2\phi - 2\chi - \sigma}$ is straightforward to compute and is given by (using the same notations as in [30]):

\begin{align*}
P^{(6)}_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= 0 \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= 61P^{(1)}_{2\phi - 2\chi - \sigma} \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= \frac{5}{2} \times 61P^{(2)}_{2\phi - 2\chi - \sigma} \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= \frac{5}{2} \times 61P^{(3)}_{2\phi - 2\chi - \sigma} \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= \frac{5}{2} \times 61P^{(4)}_{2\phi - 2\chi - \sigma} \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= \frac{5}{2} \times 61P^{(5)}_{2\phi - 2\chi - \sigma} \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= \frac{5}{2} \times 61P^{(6)}_{2\phi - 2\chi - \sigma} \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= 7! \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= -6 \times 61P^{(1)}_{2\phi - 2\chi - \sigma} \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= \frac{5}{2} \times 61P^{(2)}_{2\phi - 2\chi - \sigma} \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= -\frac{2}{3} \times 61P^{(3)}_{2\phi - 2\chi - \sigma} \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= \frac{1}{8} \times 61P^{(4)}_{2\phi - 2\chi - \sigma} \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= \frac{5}{2} \times 61P^{(5)}_{2\phi - 2\chi - \sigma} \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= P^{(6)}_{2\phi - 2\chi - \sigma} \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= 5! \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= 5! \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= -6! \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= -\frac{5}{2} \times 61P^{(1)}_{2\phi - 2\chi - \sigma} \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= 3 \times 61P^{(2)}_{2\phi - 2\chi - \sigma} \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= -\frac{19}{12} \times 61P^{(3)}_{2\phi - 2\chi - \sigma} \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= \frac{11}{24} \times 61P^{(4)}_{2\phi - 2\chi - \sigma} \\
P_{2\phi - 2\chi - \sigma \mid \sigma - 3\phi} &= P^{(5)}_{2\phi - 2\chi - \sigma} \\
\end{align*}

Using (49) and the table (50) the ghost correlator (48) is straightforward to compute and is given by:

\[
\frac{1}{2\kappa_0^6} \lim_{t_1 \to \infty} A_{\phi_\chi}(\tau, \phi, \tau_0, \phi_0) = \tau_2 \frac{4}{3} (\tau - \tau_0)^9 (z - \tau_1)^3 (\tau - z)^3 \times \left[ \frac{21}{(\tau - z)^2} + \frac{10}{(\tau_1 - z)^2} + \frac{30}{(\tau - z)(\tau_1 - z)} \right] \times \left[ \frac{7}{(\tau - z)^6} - \frac{30}{(\tau - z)^3 (\tau - \tau_1)} + \frac{50}{(\tau - z)^2 (\tau - \tau_1)^2} - \frac{40}{(\tau - z)^3 (\tau - \tau_1)^3} \right]
\]
This concludes the calculation of the ghost factor of the overall correlator (46). Next, we shall consider the matter factor of the correlator (46). Structurally, the $G(p)$ insertion contributes two different matter pieces: the first resulting from the $L^{(1)}_a$-factor of $G(p)$ containing the matter factor proportional to $L^{(1)}_a \sim \lambda \partial^2 X_0 - 2\partial \lambda \partial X_a + \frac{1}{2} p_{\alpha} \partial^2 \lambda$ with no $\psi$-dependence and the second stemming from the $\psi$-dependent piece of $L_a$ proportional to $L^{(2)}_a \sim (3 + 9\psi^2) p_{\alpha} \lambda \partial \psi \psi^b$. Second, the matter part of the $V^{(2)}_d$ consists of two terms of the type $\partial^{(M)} X_{\alpha_1} \partial^{(M)} X_{\alpha_2} \partial^{(N)} \psi \psi^{\mu} \partial^{(P)} \psi \psi^{\nu} \partial^{(Q)} \psi \psi^{\delta} \partial^{(\delta)} \psi$ with $M_{1,2}$ ranging from 1 to 3, $P_{1,2,3}$ ranging from 0 to 3 and satisfying $M_1 + M_2 + P_1 + P_2 + P_3 = 7$. The structure of the spin-3 interaction is determined by the $\psi$-contractions between themselves and by the $X$-contractions between themselves and with the exponents. The total number of $X$-fields in the correlator (46) is equal to 7, so generically, their contractions with the exponents may bring from one to seven derivatives in the cubic vertex. Since the $\omega^{(2)}$ field already contains two derivatives, the possible types of $X$-contractions result in interaction terms with the number of derivatives ranging from three to nine. The nine-derivative contribution with the maximum number of derivatives (corresponding to the case when all the $X$-derivatives contract to the exponents) is of particular interest to us since, in the case of AdS$_4$, this contribution is related to the holographic correspondence with the $d = 3$ vector model correlator of the type

$$A(\vec{x}_1, \vec{x}_2, \vec{x}_3) = \langle \Phi J \partial_m \partial_n \partial_r \Phi^I(\vec{x}_1) \Phi J \partial_m \partial_n \partial_r \Phi^J(\vec{x}_2) \Phi K \partial_m \partial_n \partial_r \Phi^K(\vec{x}_3) \rangle (52)$$

in the dual vector model; note that

$$\langle \Phi^I \Phi^J(\vec{x}_1) \Phi^J(\vec{x}_2) \Phi^K(\vec{x}_3) \rangle \sim N |\vec{x}_1 - \vec{x}_2|^{-1} |\vec{x}_2 - \vec{x}_3|^{-1} |\vec{x}_1 - \vec{x}_3|^{-1}. (53)$$

So let us concentrate on the nine-derivative case first and its relevance to the AdS$_4$/CFT$_3$ duality. Since we are interested in relating the string theory correlator (46) to the $d = 3$ correlator of the type (52) with the generic set of indices $m, a, b$ ($j = 1, 2, 3$), not all of the $\psi$-contractions are actually relevant to us. Some of them would result in the appearance of the scalar products of the momenta in the nine-derivative contribution. Such terms are of no interest to us since, in the duality context, they would correspond to special degenerate correlators in $d = 3$ where the polarizations of $d = 3$ operators at $\vec{x}_{1,2,3}$ are contracted along one or more mutual directions. On the other hand, we are interested in investigating the relevance of the string correlator (46) to the most general form of the $d = 3$ correlator (52), i.e. in the case with no contractions among $m, a, b$. For indices with different $j$. Furthermore, we assume that all the indices are polarized along the $d = 3$ boundary of AdS$_4$. With all these constraints imposed, it is straightforward to check that the only relevant nine-derivative contributions to the correlator (46) stem from the second part of $G(p)$-insertion containing $L^{(2)}_a$-factor, while the first one, with the $L^{(1)}_a$-factor, only gives rise to degenerate terms with
$m_i$, $a_j$, $b_j$-contractions. The reason is that the $\psi$-correlator pattern for all the terms involving $L_{a_b}^{(1)}$, has the form

$$\lim_{\tau \to \infty} \langle \psi^{m_i} g^{(m)}, \psi \psi^{(m)} \rangle \psi^d(\tau) \bar{\psi}^{m_i}(\tau_1) \psi^{m_i}(\tau_2) \rangle$$

$$= \tau_2^{-1} (-1)^{p_1+p_2} p_1! p_2! \times \frac{\eta^{m_i} \eta^{m_d} \eta^{m_i} \eta^{m_d}}{(\tau - \bar{z})^{p_1+1} (\tau - \tau_1)^{p_1+1}} + O(\tau_2^{-2})$$

(54)

leading to unwanted degenerate contractions because of the common $\eta^{m_i}$ factor. Straightforward computation of the relevant matter ($X + \psi$)-part of the integrand of (46) involving $L_{a_b}^{(2)}$ then gives

$$\lim_{\tau \to \infty} A_{\text{matter}}(\tau, \bar{z}, \tau_1, \tau_2) = (3 + 9q^2) \epsilon_0^d (p) \omega_0^{b_1 c_1 d_1} (k_1) \omega_0^{b_1} (k_2) \omega_0^{b_2} (k_3)$$

$$\times \tau_2^{-4} \times 12 \left[ k_{a_1} k_{a_2} \left( \frac{1}{\tau - \tau_1} - \frac{1}{\tau - \bar{z}} \right)^2 + k_{a_2} k_{a_3} \left( \frac{1}{\tau - \tau_1} + \frac{1}{\tau - \bar{z}} \right)^2 \right]$$

$$- (k_{a_1} k_{a_3} + k_{a_2} k_{a_3}) \left( \frac{1}{\tau - \tau_1} - \frac{1}{\tau - \bar{z}} \right) \left( \frac{1}{\tau - \tau_1} + \frac{1}{\tau - \bar{z}} \right)$$

$$\times \left[ \frac{\eta^{m_{a_1}} \eta^{m_{a_2}} \eta^{m_{a_3}} \eta^{m_{b_1}} \eta^{m_{b_2}} \eta^{m_{b_3}}}{(\tau - \tau_1)^3 (\tau - z)^2 (\tau - \tau_1)^3 (\tau - \bar{z})^2 (\tau - \tau_2)^2 (\tau - \bar{z})^2} \right]$$

$$- (\tau - \tau_1)^3 (\tau - z)^2 (\tau - \tau_2)^2$$

$$+ \frac{\eta^{m_{a_1}} \eta^{m_{a_2}} \eta^{m_{a_3}} \eta^{m_{b_1}} \eta^{m_{b_2}} \eta^{m_{b_3}}}{(\tau - \tau_1)^3 (\tau - z)^2 (\tau - \tau_2)^2}$$

$$- 12 \left[ k_{a_1} k_{a_2} \left( \frac{1}{\tau - \tau_1} - \frac{1}{\tau - \bar{z}} \right)^2 - \frac{1}{(\tau - \tau_1)^2} - \frac{1}{(\tau - \bar{z})^2} \right]$$

$$+ k_{a_2} k_{a_3} \left( \frac{1}{\tau - \tau_1} + \frac{1}{\tau - \bar{z}} \right)^2$$

$$+ k_{a_2} k_{a_3} \left( \frac{1}{\tau - \tau_1} - \frac{1}{\tau - \bar{z}} \right)^2$$

$$- k_{a_1} k_{a_3} \left( \frac{1}{\tau - \tau_1} - \frac{1}{\tau - \bar{z}} \right)^2$$

$$\times \left[ \frac{\eta^{m_{a_1}} \eta^{m_{a_2}} \eta^{m_{a_3}} \eta^{m_{b_1}} \eta^{m_{b_2}} \eta^{m_{b_3}}}{(\tau - \tau_1)^3 (\tau - z)^2 (\tau - \tau_2)^2} \right]$$

$$+ \frac{\eta^{m_{a_1}} \eta^{m_{a_2}} \eta^{m_{a_3}} \eta^{m_{b_1}} \eta^{m_{b_2}} \eta^{m_{b_3}}}{(\tau - \tau_1)^3 (\tau - z)^2 (\tau - \tau_2)^2}$$

$$- \frac{5}{2} \left[ k_{a_1} k_{a_2} \left( \frac{1}{\tau - \tau_1} - \frac{1}{\tau - \bar{z}} \right)^2 \left( \frac{1}{(\tau - \tau_1)^3} - \frac{1}{(\tau - \bar{z})^2} \right) \right]$$

$$+ k_{a_1} k_{a_2} \left( \frac{1}{\tau - \tau_1} + \frac{1}{\tau - \bar{z}} \right)^2$$

$$+ k_{a_1} k_{a_2} \left( \frac{1}{\tau - \tau_1} - \frac{1}{\tau - \bar{z}} \right)^2$$

$$- k_{a_1} k_{a_3} \left( \frac{1}{\tau - \tau_1} - \frac{1}{\tau - \bar{z}} \right)^2$$

$$\times \left[ \frac{\eta^{m_{a_1}} \eta^{m_{a_2}} \eta^{m_{a_3}} \eta^{m_{b_1}} \eta^{m_{b_2}} \eta^{m_{b_3}}}{(\tau - \tau_1)^3 (\tau - z)^2 (\tau - \tau_2)^2} \right]$$

$$+ \frac{3 \eta^{m_{a_1}} \eta^{m_{a_2}} \eta^{m_{a_3}} \eta^{m_{b_1}} \eta^{m_{b_2}} \eta^{m_{b_3}}}{(\tau - \tau_1)^3 (\tau - z)^2 (\tau - \tau_2)^2}$$

$$13$$
where we used the momentum conservation along with the on-shell conditions on the space-time fields. The next step is to perform the integrations in $\tau$ and $z$. We start with the integral over $\tau$ which, upon the conformal transformation (47), is along the real line. As was mentioned above, a convenient choice for $\tau_1$ is $\tau_1 = -i$ on the disc corresponding to $\tau_1 = 0$ on the half-plane. The overall integral is given by

$$A(p; k_1, k_2, k_3) = \frac{1}{2i\pi} \int_{-\infty}^{\infty} d\tau \int d^2 z \, z^2 \left\{ A_{\text{matter}}(\tau, z, \bar{z}) A_{\text{gh}}(\tau, \bar{z}) \right\},$$

and we used the fact that the leading order $\sim \tau_1^4$-factor of the ghost part of the correlator is cancelled by the leading order $\sim \tau_1^{-4}$-factor of its matter part. The $\tau$ integral in (46) looks tricky to evaluate. However, for our purposes we only need its asymptotic value in the field theory limit, that is, in the leading order of $\alpha'$. In this limit the $\tau$ integral is dominated by contributions from the region $\tau \sim z$ as the integrand becomes singular when $z$ approaches the real axis. In this case, we shall use the asymptotic formula

$$\lim_{\epsilon \to 0} \int d\tau \int d^2 z f(z, \bar{z}) g(\tau, z) (\tau - z)^{\epsilon - N} \sim \frac{(-1)^{N-1}}{(N-1)!} \left\{ \int d^2 z f(z, \bar{z}) \bar{z}^{N-1} g(z, \bar{z}) + O(\epsilon) \right\}$$

(55)

where $A_{\text{matter}}(\tau, z, \bar{z}) A_{\text{gh}}(\tau, \bar{z}) \equiv A_{\text{matter}}(\tau, z, \bar{z}, \tau_1, \tau_2) A_{\text{gh}}(\tau, \tau_1, \tau_2) |_{\tau_1 = 0, \tau_2 \to \infty}$

(56)

and $A_{\text{matter}}(\tau, z, \bar{z}) A_{\text{gh}}(\tau, \bar{z}) \equiv A_{\text{matter}}(\tau, z, \bar{z}, \tau_1, \tau_2) A_{\text{gh}}(\tau, \tau_1, \tau_2) |_{\tau_1 = 0, \tau_2 \to \infty}$

(57)

(55)

(56)

(57)

(55)

(56)

(57)
where $N_{1,2,3}$ are some integer numbers, different for each of the terms entering (A.2). The integrals of the type (58) are over the upper half-plane and are still tedious to evaluate. It is therefore convenient, by using the overall conformal invariance of the overall correlator (46) to conformally map it back to the unit disc $(z, \bar{z}) \rightarrow (u, \bar{u})$ and introducing $u = re^{i\theta}$ for the disc coordinates. Then, the transformation (47) reduces the integrals of the type (58) to those of the generalized elliptic type:

$$I(k_1, k_2, k_3) \sim 2^{N_1 - N_2} \int_0^1 \frac{dN}{(r^2 - 1)^{N_1 + N_2 - N_3 - 2k_1 k_2 - 2k_1 k_3 - 2k_2 k_3}} \times \int_0^{2\pi} d\theta \left(1 + \frac{2r}{r^2 - 1} \cos \alpha \right)^{-k_1 k_2 - k_3 k_4 + N_2} \left(1 - \frac{2r}{r^2 - 1} \cos \alpha \right)^{-k_1 k_2 - k_3 k_4 + N_1}$$

The overall amplitude (46) is then given by the lengthy expression (A.2) described in the appendix; the answer, however, simplifies in the field theory limit of $\alpha' \rightarrow 0$. Integrating the amplitude (46) over $k_1, k_2, k_3$ and $p$, using the momentum conservation in the five-point amplitude that eliminates the integral over $p$, and recovering the $\alpha'$ and the cosmological constant factors, the asymptotics of (46) in the field theory limit gives:

$$A(k_1, k_2, k_3) = 691\, 072 \, 283 \, 467 \, 0360 \alpha' \Lambda (k_1 k_2)^{-1} (k_2 k_3)^{-1} (k_2 k_3)^{-3} \alpha_{m_1}^{\alpha_1}(k_1) \alpha_{m_2}^{\alpha_2}(k_2) \alpha_{m_3}^{\alpha_3}(k_3) k_1^3 k_2^3 k_3^3$$

$$+ \left\{ \kappa_1^m(k_2) a_1(k_2) b_1(k_3) \kappa_2^m(k_3) a_2(k_3) b_2(k_3) + \cdots \right\}$$

where we skipped the contact terms (proportional to the delta-functions in the position space), used the zero torsion conditions (37), (40) relating $\omega^{2|2}$ to $\omega^{20}$ along with the on-shell conditions for $\omega$ and neglected the contributions in the subleading order in $\alpha'$. The overall numerical factor in (60) is consistent with the one obtained in the three-point string amplitude of spin-3 particles in flat space, computed in [30]; it can therefore be absorbed by the appropriate rescaling of the vertex operators, making the normalization obtained from string theory, consistent with the one in the BBD vertex [35]. Then the normalization of (60) is consistent with the one in (52), (53) provided that one identifies $(\alpha' \Lambda)^2 = N^{-1}$ and rescales the vector field according to $\Phi' \rightarrow N^2 \Phi'$, which normalizes the current’s two-point correlator by 1. This, up to contact terms (proportional to the delta-functions in the position space), coincides with the correlator (52) transformed to the momentum space.

Next, evaluating the lower derivative terms in the amplitude (56) gives the answer proportional to:

$$A_{\text{lower-der.}}(k_1, k_2, k_3) = (15 - 6q^2) I_{\text{lower-der.}}(k_1, k_2, k_3) + \cdots$$

with the contact terms skipped. The explicit expression for $I_{\text{lower-der.}}(k_1, k_2, k_3)$ is given in the appendix; (61) particularly implies that, apart from the contact terms, the only lower derivative contribution to the overall amplitude (46) is proportional to the factor of $\sim 15 - 6q^2$ which stems from $q$-independent and $q$-dependent $\mathcal{I}_a^{(1)}$ and $\mathcal{I}_a^{(2)}$ pieces of the closed string insertion for the vielbein vertex operator (satisfying the AdS vacuum solution in the leading order of the beta-function). This expression vanishes (up to contact terms and those of the higher order in $\alpha'$) in $d = 4$ where $q = \sqrt{\frac{4d - 6}{2}} = \sqrt{\frac{5}{2}}$. This means that in the special case of $d = 4$ only the nine-derivative contribution survives in the string-theoretic amplitude (46) which gives a precise holographic relation between the AdS string sigma-model (41) in the case of $d = 4$ and the dual free field theory correlator (53) in $d = 3$ for spin-3. Thus, the AdS4/CFT3 holography correspondence for higher spins appears to be surprisingly related to the value of the Liouville background charge value in $d = 4$ which stems from two-dimensional CFT. This fact is by itself quite intriguing and definitely needs further investigation.
5. Conclusion and discussion

In this paper we analysed the AdS$_4$/CFT$_3$ higher spin holography using the string-theoretic sigma-model describing gravity and higher spin perturbations around the AdS background in the low-energy limit. We found that, in the leading order in $\alpha'$ and cosmological constant, the three-point correlator for spin-3 particles in AdS$_4$ reproduces the free-field correlator for the large $N$ vector model in $d = 3$. Surprisingly, we found that this holographic correspondence appears to be related to the value of the Liouville background charge in $d = 4$ which allows for cancellation of the lower-derivative terms (up to contact terms), so the leading order of AdS string theory appears to be in agreement with Maldacena–Zhiboedov’s proposal [16, 17]. One definitely needs to check whether such a cancellation also holds for vertex operators for spins greater than 4 and for higher point correlators in the AdS$_4$ case. On the other hand, the lower derivative terms do persist in the three-point amplitudes for $d \neq 4$. This is the signal that the higher spin/CFT holography has a more complicated character in higher dimensions, where the dual theories are no longer free. Moreover, even in AdS$_4$ the string theory corrections may definitely modify the limit in which Maldacena–Zhiboedov’s theorem holds. We hope to implement these computations in future papers. The results of this paper suggest that a string theoretic approach may provide interesting insights to HS/CFT duality, such as the relevance of the Liouville theory to the $d = 4$ case. It would be interesting to see possible relations of this fact to the AGT conjecture since open string amplitudes for spin 1 in the sigma-model (41) should particularly involve the super Yang–Mills theory in the low energy limit. Another question of immediate interest is to use the sigma-model (41) in order to study the AdS$_4$/CFT$_3$ holography for higher spins. To approach this problem, one has to study the lower derivative terms appearing in the sigma-model correlators, as well as the higher order corrections in the cosmological constant. Finally, in the AdS$_d$/CFT$_d$ case it would be of crucial importance to understand the relation between the string-theory formalism and the twistor space approach used by Vasiliev [18] to study the higher spin holography. This relation may probably, in some form or another, involve modifications of the twistor string theory developed by Witten [43]. Altogether, this gives a list of problems to address in the future, but one which of course is still preliminary and incomplete.

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Appendix

In this section we present explicit expressions for the amplitude (46) leading to the asymptotics (60). To abbreviate the expressions, we adopt the following notations:

\[
a \equiv z
\]
\[
b \equiv \bar{z}
\]
\[
c \equiv k^{a_1} k^{b_1}
\]
\[
d \equiv k^{a_1} k_{a_3} + k^{b_1} k_{b_3}
\]
\[
f \equiv k^{a_2} k^{b_2}
\]
Then the amplitude can be expressed covariantly in a convenient way, suppressing the indices, to lead to the following answer:

\[
L_1 = k_1k_2 + k_1k_3 \\
L_2 = k_1k_2 + k_2k_3 \\
L_3 = k_1k_3 + k_2k_3.
\]

(A.1)

Then the amplitude can be expressed covariantly in a convenient way, suppressing the indices, in terms of the variables \(a, b, c, d, f, g, h, k, q, t, u, L_{1,2,3}\). The evaluation of the \(\tau\) integral then leads to the following answer:

\[
A(k_1, k_2, k_3) = u \int da \, db \, dc \, [\frac{\epsilon^a b^b}{16(a - b)^{1/2} + L_1} - 45(1488k + 122c - 231d)t + 24a^{14}(1488k + 122c - 231d)t + 10b^{14}(k^2 - 203c + 736c - 491d)p - 2414880kq + 89547132bq + 5939800cq + 13527948bcq - 108055dq + 8919888bdq - 83751039 + 44211208bktb - 6170220ct + 40507190bet + 4309920drt - 111451878bdtt)] + a^{14}b^7((b(36984432k + 5216119c - 22448132d) - 30(195240c + 55785c - 179903d)) - 384(b(-31790k + 873c - 13993d) + 105(-16k + 5c + 26d)p + 79359120kq - 51353040alkq + 4399254cq - 7162686bq - 12648735dq + 34255946bdq + 234435600krt + 24441288bkt + 8927160kct - 48353746bect + 26078880drt - 72924756bdtt) + a^{14}b^7(2(b(3716040k + 1992963c - 3283276d) - 15(114032k + 3351c - 10653d) - 192(b(36512k + 782c - 2999d) - 30(80k + c - 4d)p + 1058200kq - 233099616bkq + 302985cq - 6588324bcq - 976095dqc + 20179833bdcq - 102815400krt + 1209791888bkt - 1626570ct + 30153268bcet + 7224210dt - 59663780bdtt) + \frac{a^{14}b^7(2(2b(21928972k + 8677069c - 7234868d) - 15(1299632k + 41244c - 216481d)) - 192(-22800k + 19668hkbk - 23540c + 34411bc + 6495d - 19339bd)p + 31623120kq - 63763176bq + 2049300cq - 28605954bceq - 5653440dq + 22235154bdq - 29344440kt + 37177976dk + 1858710ct + 40276674bect + 6241860dt - 23223764bdtt) - 10a^{12}b^7((b(2054748k + 414913c - 470605d) + 12(-4456k + 17251c - 7576d) - 96(-2688k + 8332b + 338c + 2525bc - 252d - 2369bd)p + 203220kq
\]
\[ -3367 \times 224bkq - 243405cq - 865908bcq + 74475dq + 871407bdq \\
- 695820k + 3149708bkt + 203355c - 12143541bcet + 57.525dt \\
- 1109229bdt - 10a b^{12} (-30(12076k + 3989c - 3504d) + 13b(1044k \\
+ 2573c - 1417d)) + 192(13b(60k + 7c - 11d) + 75(4k + 3c - 2d))p \\
+ 355680kq + 391248bkq + 169920eq + 30342bcq - 129420dq - 64077bdq \\
+ 135120bkt - 1255384bkt - 168120ct - 198978bct + 67170dt + 256412bdt) \\
+ a^{11} b^{2} (b(9576896k + 246342c - 898206d) - 15(15472k + 542c - 1683d)) \\
+ 192(2b(c + 194(-20k + d)) - 15(-16k + d))p + 696240kq \\
- 28709400bkq + 24390cq - 738933bcq - 75735dq + 2693463bdq \\
+ 18384600kt - 181700576bkt + 205590ct - 11289448bct \\
- 556680dt + 6229332bdt) + a^{4} b^{5} (30(438608k + 139259c \\
- 170889d) + b(31195608k + 23625295c + 7077692d))192(30496k \\
- 245c + 130d) + b(-109400k + 42721c + 1074d)p - 34454160kq \\
- 50747856bkq - 8345745cq + 36016704bcq + 10265985dq - 9433944bdq \\
+ 73283520k + 39522728bkt + 15467220ct - 75827612bct - 19635420dt \\
+ 22658110bdt) + a^{4} b^{8} ((-30(-993296k + 65495c + 51431d) \\
+ b(-140574840k + 8076197c + 20592266d)) - 192(-420(-100k + 11c \\
+ 11d) + b(-154056k + 21149c + 3527d))p - 53598240kq \\
+ 278165748bkq + 3934800cq - 4726128bcq + 2016675dq \\
- 42460149bdq + 63153600t - 439526024b - 21984180ct \\
+ 63570616bct + 8993340dt + 52194190bdt) + a^{4} b^{5} (30(632960k \\
+ 25861c - 55751d) + b(-155995128k - 11490326c + 21567689d)) \\
+ 192(b(42216k + 6692c + 19703d) - 30(320k + 23c - 34d))p \\
- 55011960q + 441897552bq - 2069865cq + 30049308bcq + 4492845dq \\
- 53982408bdq + 24705960kq - 1075703328bkt + 12480240ct \\
- 16542330bct - 12267000dt + 64259624bdgt) + a^{4} b^{5} (30(-527920k \\
+ 21473c - 22362d) + b(109033376 - 1998049c + 328756d)) \\
- 192(b(225668k + 1180c - 13133d) - 30(928k + 22c + 13d))p \\
+ 27250560kq - 132110208bkq - 201590eq + 4628397bcq \\
+ 2606355dq - 9431328bdq - 13143120kt + 158121584bkt \\
- 504140ct + 2368354bct - 18233580dt + 75056032bdgt) \\
+ a^{12} b^{2} (120(105076k + 4767c - 12410d)t + b(-192(-16 + d))p \\
+ 46416kq + 1626cq - 5049dq - 347866128kt - 12118888ct \\
+ 34908536dt) + \left\{ \frac{h a^{4} b^{6} c}{8(a - b)^{19} + L_{i}} \right\} \\
\times ab(4a^{4}(8462k + 577c - 1202d) + 2a^{13}(30(8462k \\
+ 577c - 1202d) + b(-12742604k - 632218c + 1559205d))t \\
+ 30b^{13}(4k + c - d)((960 - 416b)kp + 7005kq - 4108bkq \\
- 7260kt + 2106bkt) + a^{4} b^{4}(2(4b(-1601611k + 615629c - 659330d)
+ 15(4848k − 7679c + 6613d)) − 192(−90c + b(−77 608k + 1129c − 646d) 
+ 60(81k + d))p − 348 480kq + 36 705 972bkq + 686 340cq − 14 675 466bcq 
−583 785dq + 15 626 610bdq − 23 306 640kt − 389 991 016bkt − 3838 620ct 
+ 22 950 092bct + 3457 260dt − 73 172 098bdt) + a^{10} b^3 ((b(79 470 528k 
+ 2016 958c − 6521 170d) − 30(123 808 + 3051c − 10 481d)) − 96(b(40 888k 
+ 780c − 2883d) − 30(100k + c − 5d))p + 11 146 320kq − 238 310 688bkq 
+ 274 005cq − 6035 748bcq − 942 075dq + 19 516 749bdq − 77 125 680kt 
+ 841 941 584bkt − 1654 440ct + 26 608 962bct + 5691 120dt − 50 462 452bdt 
+ a^2 b^7 ((b(23 733 672k + 3102 799c − 8854 348d) − 30(889 920k + 38 513c 
− 122 707d)) − 384(b(−46 491k + 10 514c − 1893d) + 105(28k − 5c 
+ 13d))p + 76 062 960kq − 42 926 256bkq + 3536 955cq − 9361 710bcq 
− 10 353 825dq + 17 679 426bdq − 139 153 920kt + 36 775 152bkt 
+ 875 940ct − 7023 550bct + 12 947 220dt − 5302 270bdt) 
+ 30ab^{12}((13b(3084k + 355c − 563d) + 160(44k + 57c − 34d)) 
− 32(45(−4k + c) + 13b(60k + 11c − 13d))p + 44 880kq 
− 154 960bkq − 7500cq − 26 598bcq − 1860dq + 32 669bdq 
− 193 440kt + 246 376bkt − 19 440ct + 49 010bct + 33 900dt − 55 302bdt) 
+ a^4 b^8((30(443 488k + 70 935c − 97 085d) + b(−1151 176k − 7947 881c 
+ 3618 600d)) − 96(b(143 928k + 782c − 28 474d) + 60(−512k + 133c 
− 2d))p − 35 452 800q + 1654 464bq − 5650 695cq + 11 720 508bcq 
+ 7261 695dq − 4865 790bdq + 44 217 240kt − 31 887 760bkt 
+ 4031 520ct + 100 832bct − 5912 910dt + 247 636bdt) 
+ a^{11} b^5((b(9997 696k + 207 342c − 832 814d) − 1155(224k 
+ 6c − 21d)) + 96(−15(−24k + d) + 2b(−544k + c + 193d))p 
+ 776 160kq − 29 992 848bkq + 20 790cq − 622 065bcq − 72 765dq 
+ 2498 523bdq + 11 616 720r − 103 495 104bt + 249 960ct 
− 9790 670bct − 410 820dt + 6061 460bdt) + a^2 b^8((30(314 384k 
− 51 493c + 13 147d) + b(−69 230 024k − 155 149c + 9845 590d)) 
+ 96(210(7c + 40(−5k + d)) + b(85 184k + 39 575c − 45 454d))p 
− 24 069 600kq + 172 712 364bkq + 3804 840cq + 9101 964bcq − 925 515dq 
− 26 732 397bdq − 1771 320kt − 129 295 912bkt − 7515 360ct − 9836 246bct 
+ 6549 780dt + 19 131 876bdt) + a^{10} b^4((30(606 368k + 18 269c − 47 855d) 
+ b(−149 759 256k − 7429 006c + 16 898 695d)) + 96(b(−21 696k + 7168c 
− 32 107d) − 300(20k + 2c − 7d))p − 53 472 600kq + 431 464 896bcq 
− 1644 975cq + 22 336 260bcq + 4232 835dq − 49 353 342bdq + 162 720 240kt 
+ 573 713 128bkt + 7605 180ct + 2790 792bct − 10 584 600dt + 27 614 802bdt) 
+ a^{12} b(30(357 076k + 15 242c − 40 239d) + b(−96(−24k + d))p 
+ 51 744kq + 1386cq − 4851dq − 283 508 960kt − 9804 142ct + 27 924 468d) 
+ 2a^2 b^{11} (−3(16(b(21 044k + 8251c − 6951d)) − 30(572k + 83c
\(-123d))p + (13b(96672k + 48818c - 38053d) + 480(304k + 51c + 4d))q
+ 2(5(-8316k + 27871c - 19191d) + b(84212k - 186857c + 3817d)t))
+ a^8b^{10}(-2(48(b-3582k + 11269c - 571d) - 75(184k + 106c
- 89d)) + 3(b(3524766k + 1583210c - 1123521d) - 60(29492k + 3877c
- 5475d))q + 2(b(7128476k + 410553c - 1514936d) + 45(-57308k
+ 3915c + 6786d))t)) + a^7b^6(3(32(-30(-1192k + 17c + 53d)
+ b(-257540k + 10748c + 41515d)) + p(b(1784512k + 4673403c
- 6328878d) + 15(259520k - 56778c + 80197d))q - 2(b(18432028k
+ 2063821c - 6743278d) + 10(-1406888k + 6877c + 155327d))t))
+ \left[ \frac{g_d^{1-s-3}b^2}{8(a - b)^{4+3t}} \right]((96a^20 + 2a^3(4835 - 8208b)b^{16} + 2a(95 - 48b)b^{18}
- 10b^9 + 114a^2b^{17}(-15 + 16b) - 2a^9(5 + 912b)
+ 2a^{10}b)(3248k - 371c + 575d)t + a^{16}b^2(-30(102632k
+ 6640c - 14211d)t + b(96(-20k + d)p - 37500kq - 1116eq
+ 3735dq + 101842384kt + 4629100ct - 11840166dt))
+ a^{10}b^4(b(96(-124772k + 21901c - 17779d)p + 69337080kq
- 25027488cq + 22411995dq + 47573000kt + 30096024ct - 71921924dt)
- 15(672(-40k + 5c - 52d)p + 4659444kq + 5988cq - 137073dq
- 2235256kt - 5057ct - 607218dt))
+ 2a^5b^{14}(150(4k + c - d)(-9(16p + 79q - 64r))
- 5b(-2496(4k + c - d)p - 26988kq - 6747cq + 6747dq + 7280kt
+ 1820ct - 1820dt)) + a^{12}b^6(-b(1440(684k + 462c - 1727d)p
+ 38564132kq + 4905636cq - 12983877dq + 94219368kt + 9469882ct
- 66033204kt) - 15(-96(520k + 43c - 104d)p - 34077eq + 136692dq
+ 1857640kt - 22998ct + 47306dt)) + a^{15}b^3(-96(-9324k + 2c + 387d)p
+ 22208160q + 500691eq - 1942140dp + 239709600kt + 584382ct
- 16780882dt) + 15(96(-20k + d)p - 1116eq + 3735dq - 809096kt
- 14066ct + 63272dt) + a^{9}b^{10}(96(-1520k + 511c - 134d)p - 229047cq
- 63054dq + 661742ct - 567764dt) + b(-192(-63332k + 8725c + 7387d)p
+ 6143760cq + 14083365dq + 160389568kt + 33349956ct + 1211602dt)
+ a^{4}b^4(-96(180k + 2c - 9d)p - 620448kq - 14901cq + 52512dq
+ 910496kt + 17798cq - 131728dt) + b(96(38700k + 781c - 2491d)p
+ 213241020q + 4928781eq - 16835793dq - 178959408kt + 2472926ct
+ 11129654dt) - a^{7}b^{15}(15(96(1220k + 383c - 439d)p - 2448324kq
- 261879cq + 501360dq + 4165984kt + 202016ct - 70056dt)
+ b(480(-1616k + 4568c - 1991d)p + 38861664kq + 13797912cq
- 13942029d - 86969480kt - 23944502ct + 25663916dt))
+ a^{2}b^6(-672(-400k + 29c + 62d)p - 3995256kq - 19923cq
+ 725346dq + 8950184kt - 237858ct - 907442dt) + b(-96(119620k
- 2079428kq + 4701582kt + 1347536ct + 772200dt))"
to expression (61) that vanishes in four dimensions. For completeness, we finish by presenting

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\[ + b(2(55 \ 580k + 48(1865c - 3(43 \ 676k + 379d))p + 6(16 \ 303 \ 472k + 1230 \ 680c - 2118 \ 925d)q - 407 \ 402 \ 424kt - 14 \ 101 \ 721c + 23 \ 167 \ 123d))) \]

\[ + a^d b^j ((-15(-480(204k + 5c - 33d)p + 633 \ 804kq - 115 \ 959cq + 4059dq - 338 \ 248kt + 265 \ 498ct - 122 \ 678dt) + b(-2(96(15 \ 676k + 5219c - 4699d)p - 3(5168 \ 668k + 279 \ 987c - 754 \ 942d)q + 20(913 \ 672k + 50 \ 071c - 134 \ 207d)\eta)))) + [a \leftrightarrow b]. \]

Next, we perform the conformal transformation from the half-plane coordinates \( z \equiv a, \bar{z} \equiv b \) to the \( r, \alpha \) coordinates in the disc \( 0 \leq r \leq 1, 0 \leq \alpha \leq 2\pi \) using the prescription (58)-(59) and replacing each of the half-plane integrals of the type \( \int da db (a - b)^\nu \bar{a}^\gamma b^\rho \) with corresponding integrals on the disc. Straightforward evaluation of the asymptotics of the integrals in the field theory limit \( \alpha' \to 0 \) then leads to (60), skipping the contact terms. Finally, the lower-derivative contributions to the correlator (46) are given by the overall expression (61) that vanishes in four dimensions. For completeness, we finish by presenting explicit expressions for \( I_{\text{lower-\text{dec}}} (k_1, k_2, k_3) \) entering (61). It is convenient to cast it according to

\[ I_{\text{lower-\text{dec}}} (k_1, k_2, k_3) = I_1 + I_2 + I_3 \quad (A.3) \]

with \( I_p \) being the \( p \)-derivative pieces. The integration procedure is then identical to the one explained for the \( n \)-derivative contribution. Introducing further convenient abbreviations:

\[ u_1 = \omega_{m_1}^{b_1} (k_2) \omega_{m_2}^{b_2} (k_3) \omega_{m_3}^{b_3} (k_1) \eta^{a_1} (k_1^a + k_2^a + k_3^a) \left( k_1^b + k_2^b + k_3^b \right)^3 \]

\[ u_2 = \omega_{m_1}^{b_1} (k_2) \omega_{m_2}^{b_2} (k_3) \omega_{m_3}^{b_3} (k_1) \eta^{a_2} (k_1^a + k_2^a + k_3^a) \left( k_1^b + k_2^b + k_3^b \right)^3 \]

\[ v_1 = \omega_{m_1}^{b_1} (k_2) \omega_{m_2}^{b_2} (k_3) \omega_{m_3}^{b_3} (k_1) \eta^{a_1} (k_1^a + k_2^a + k_3^a) \left( k_1^b + k_2^b + k_3^b \right)^3 \]

\[ v_2 = \omega_{m_1}^{b_1} (k_2) \omega_{m_2}^{b_2} (k_3) \omega_{m_3}^{b_3} (k_1) \eta^{a_2} (k_1^a + k_2^a + k_3^a) \left( k_1^b + k_2^b + k_3^b \right)^3 \]

\[ \lambda_1 = \omega_{m_1}^{b_1} (k_2) \omega_{m_2}^{b_2} (k_3) \omega_{m_3}^{b_3} (k_1) \eta^{a_1} \left( k_1^a + k_2^a + k_3^a \right) \times \left( k_1^b + k_2^b + k_1^b + k_2^b + k_3^b \right)^3 \]

\[ \lambda_2 = \omega_{m_1}^{b_1} (k_2) \omega_{m_2}^{b_2} (k_3) \omega_{m_3}^{b_3} (k_1) \eta^{a_2} \left( k_1^a + k_2^a + k_3^a \right) \times \left( k_1^b + k_2^b + k_1^b + k_2^b + k_3^b \right)^3 \]
\[ \lambda_3 \equiv \omega_{m_1}^{b_1} (k_2) a_{m_2}^{b_2} (k_2) a_{m_3}^{a_3c} (k_1) \eta^{a_3a_4} (k_1^a + k_2^a + k_3^a) \times (k_1^b k_2^b + k_2^b k_3^b + k_3^b k_1^b), \]
\[ \lambda_4 \equiv \omega_{m_1}^{b_1} (k_2) a_{m_2}^{b_2} (k_3) a_{m_3}^{a_3c} (k_1) \eta^{a_3a_4} (k_1^a + k_2^a + k_3^a) \times (k_1^b k_2^b + k_2^b k_3^b + k_3^b k_1^b), \]
\[ \rho_1 \equiv \omega_{m_1}^{b_1} (k_2) a_{m_2}^{b_2} (k_3) a_{m_3}^{a_3c} (k_1) \eta^{a_3a_4} (k_1^a + k_2^a + k_3^a) \]
\[ \rho_2 \equiv \omega_{m_1}^{b_1} (k_2) a_{m_2}^{b_2} (k_3) a_{m_3}^{a_3c} (k_1) \eta^{a_3a_4} (k_1^a + k_2^a + k_3^a) \]
\[ \times (k_1^b k_2^b + k_2^b k_3^b + k_3^b k_1^b), \]
\[ \text{the asymptotics of the seven-derivative contribution is computed to give} \]
\[ \lambda_1 \sim (k_1 k_2)^{-1} (k_1 k_3)^{-1} (k_2 k_3)^{-1} \{|\omega_1 (1493 472p + 1195 776q + 505 040r) + g(2051 748p} \]
\[ + 2154 326q + 908 764r) + h(2214 564p + 4140 804q + 764 500r) \]
\[ - 122 p + 36 605 620r + h(-44 287 732p + 10 608 920q - 764 824r) \]
\[ + v_1(f(132 128p + 898 622p \]
\[ + 8267 294q + 30 168 844r + h(-44 287 732p + 10 608 920q - 764 824r) \]
\[ + v_2(f(8467 662p + 5883 454q + 936 720r) + g(12 845 628p \]
\[ + 9832 562q + 964 355r) + h(20 605 796p + 1557 672q - 2062 380r) \]
\[ + v_3(f(7884 240p - 3167 390q - 1010 042r) + g(8690 112p + 40 306q \]
\[ + 2087 600r) + h(-3860 368p - 894 766q + 705r)) - \lambda_1 (9085 \]
\[ - 30 492 495q + 735 944r) + \lambda_2(2235 256p - 41 879 080q - 538 906r) \]
\[ + \lambda_3(1264 326p + 24 246 476q + 426r) + \lambda_4(130p + 284 472q + 3280 845r) \]
\[ + \rho_1(c(20047 925p + 2823 510q + 724 612r) + d(2240 244p + 3171 313q \]
\[ + 462 884r) + k(1021 245p + 2436 124q + 6523 480r) \]
\[ + \rho_2(c(-4086 001p + 2049 050q + 206 800r) + d(2006 114p \]
\[ - 929 683q + 96 492r) + k(-24p + 907 200q + 1920 160r)) + \cdots. \]
\[ \text{Next, to describe the five-derivative piece, we shall adopt the notations:} \]
\[ \gamma_1 \equiv \omega_{m_1}^{b_1} (k_2) a_{m_2}^{b_2} (k_2) a_{m_3}^{a_3c} (k_1) \eta^{a_3a_4} (k_1^a + k_2^a + k_3^a)(k_1^b k_2^b) \]
\[ \gamma_2 \equiv \omega_{m_1}^{b_1} (k_2) a_{m_2}^{b_2} (k_3) a_{m_3}^{a_3c} (k_1) \eta^{a_3a_4} (k_1^a + k_2^a + k_3^a)(k_1^b k_2^b) \]
\[ \gamma_3 \equiv \omega_{m_1}^{b_1} (k_2) a_{m_2}^{b_2} (k_3) a_{m_3}^{a_3c} (k_1) \eta^{a_3a_4} (k_1^a + k_2^a + k_3^a)(k_1^b k_2^b) \]
\[ \gamma_4 \equiv \omega_{m_1}^{b_1} (k_2) a_{m_2}^{b_2} (k_3) a_{m_3}^{a_3c} (k_1) \eta^{a_3a_4} (k_1^a + k_2^a + k_3^a)(k_1^b k_2^b) \]
\[ \delta_1 \equiv \omega_{m_1}^{b_1} (k_2) a_{m_2}^{b_2} (k_3) a_{m_3}^{a_3c} (k_1) \eta^{b_1b_2} \eta^{a_3a_4} (k_1^a k_1^b + k_2^a k_2^b + k_3^a k_3^b) \]
\[ \delta_2 \equiv \omega_{m_1}^{b_1} (k_2) a_{m_2}^{b_2} (k_3) a_{m_3}^{a_3c} (k_1) \eta^{b_1b_2} \eta^{a_3a_4} (k_1^a k_1^b + k_2^a k_2^b + k_3^a k_3^b) \)
\[ \times (k_1^b k_2^b + k_2^b k_3^b + k_3^b k_1^b), \]
\[ \text{Then the asymptotics of the five-derivative contribution is computed to give} \]
\[ \lambda_5 \sim (k_1 k_2)^{-1} (k_1 k_3)^{-1} (k_2 k_3)^{-1} \{|\gamma_1 (1864 828p + 2866 523q - 303r) \]
\[ + \gamma_2(20078 254p + 652q - 1887 348r) + \gamma_3(1542 620p - 1288 760q - 1052r) \]
\[ + \gamma_4(1264 326p + 24 246 476q + 426r) + \gamma_5(3280 845r) \]
\[ + \gamma_6(2006 114p - 929 683q + 96 492r) + k(-24p + 907 200q + 1920 160r)) + \cdots. \]
+ γ_4 (30 884 240p + 6268 906q + 9208r) + δ_1 (2305 266p + 764 080q
− 2006 875r) + δ_2 (2640 884p + 306 708q + 9861 808r) + ϵ_1 (3077 454p
− 708 616q − 979 072r) + ....  \tag{A.7}

Finally, to describe the three-derivative contributions (up to contact terms) we denote

\begin{align}
σ_1 & \equiv \alpha^{a_1b_1}_m (k_3) \alpha^{a_2b_2}_m (k_3) \alpha^{a_3b_3}_m (k_3) \eta^{a_1a_2} \eta^{a_2a_3} \eta^{b_1b_2} k_3 \\
σ_2 & \equiv \alpha^{a_1b_1}_m (k_3) \alpha^{a_2b_2}_m (k_3) \alpha^{a_3b_3}_m (k_3) \eta^{a_1a_2} \eta^{a_2a_3} \eta^{b_1b_2} k_3. \tag{A.8}
\end{align}

Then the asymptotics of the three-derivative contribution is computed to give

\begin{align}
I_3 \sim (k_1k_3)^{-1} (k_2k_3)^{-1} [\sigma_1 (−10 887 660p + 30 458 284q + 41 010 562r)
+ \sigma_2 (10 865 492p − 30 460 944q − 41 010 934r)] + ....  \tag{A.9}
\end{align}

This concludes the evaluation of the I(k_1, k_2, k_3) factor describing the lower derivative contributions to the correlator (46), modulo contact terms.

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