Determination of Dynamic Elastic Properties of Anah Formation, Near Rawa city / Western Iraq Using Ultrasonic Technique

Salman Z. Khorshid\textsuperscript{1}, Munther D. Al-Awsi\textsuperscript{2}, Emad H. Kadhim\textsuperscript{2}

\textsuperscript{1}Department of Physics, College of Education, University of Tikrit, Tikrit, Iraq
\textsuperscript{2}Department of Petroleum and Mineral Geology, College of Science, University of Diyala, Diyala, Iraq

Received: 15/9/2019 \quad Accepted: 31/10/2019

Abstract

The aim of the current study is to determine the elastic properties of carbonate rocks using ultrasonic method. Forty rock samples of Anah formation were collected at different depths from four wells drilled at the study area. The relationship between wave velocities and elastic properties of rocks was defined. Regression analyses to define these relations were applied. The results indicate that the elastic properties of the rocks show a linear relationship with both P- and S-wave velocities. The best relationship was obtained between both Young's modulus and Shear modulus with Vs in the determination of the coefficient (R\textsuperscript{2}), with values of 0.91 and 0.94, respectively. Bulk modulus and Lame's constant were better correlated with Vp than with Vs in the determination of R\textsuperscript{2}, with values of 0.92 and 0.83, respectively. Poisson's ratio showed a good correlation using the ratio of Vp/Vs in the determination of R\textsuperscript{2}, with a value of 0.81. The main output of this study shows that the ultrasonic method is a useful tool for the prediction of the elastic dynamic properties of sample rocks and that it can be used as an economical, simple and non-destructive method, especially for engineering purposes.

Keywords: Ultrasonic Method, Elastic Properties, Carbonate Rocks, Anah Formation.

تحديد الخواص الديناميكية المرنة لتكوين عانة قرب مدينة راوة / غرب العراق باستخدام تقنية الموجات الصوتية

سلمان زين العابدين خهرشيد\textsuperscript{1}, منذر ظاهر الاوسي*\textsuperscript{2}, عماد حامد كاظم\textsuperscript{2}

\textsuperscript{1}قسم الفيزياء، كلية التربية، جامعة تكريت، تكريت، العراق
\textsuperscript{2}قسم جيولوجيا النفط والمعادن، كلية العلمات، جامعة ديالى، ديالى، العراق

الخلاصة

الهدف من الدراسة الحالية هو تحديد الخواص المرنة للصخور الكربوناتية باستخدام طريقة الموجات فوق الصوتية. تم جمع أربعين عينة صخرية لتكوين عانة من على أسطح مختلفة في أربعة أبواب محفرة في منطقة الدراسة. تم تعريف العلاقة بين سرعة الموجات الزائارية والخصائص المرنة للصخور. تم تصميم اختبار لتحديد هذه العلاقات. بنيت نتائج الدراسة على خصائص المرنة للصخور الموصوفة عبر علاقة خطية مع كل من السرعة العلوية (Vp) والمستعرضة (Vs). أفضل علاقة تم الحصول عليها هي تلك التي تربط بين كل من معامل بيكو ومعامل القص مع سرعة الموجات المستعرضة حيث بلغت قيمة معامل التحديد (R\textsuperscript{2}) 0.91.

*Email: munther.alawsi@yahoo.com
Introduction

Physical and mechanical properties of rocks are of a remarkable interest in many fields, including materials science, petrophysics, geophysics and engineering geology [1]. In rock mechanics, ultrasonic wave velocity tests are becoming very popular due to their non-destructive nature, high precision, easy application, and cost effectiveness [2]. Three types of sonic velocity methods are available, namely the ultrasonic, low frequency sonic wave, and frequency resonant techniques [3, 4]. Among these methods, the ultrasonic technique is the most convenient to be used in rock mechanics. The advantages of this method is its capability to yield both seismic velocities (Vp and Vs) as well as ultrasonic elastic constants that are very useful in characterizing the dynamic properties of rock, such as Young’s modulus, shear modulus, bulk modulus and Poisson’s ratio. This method is simple, fast and can be used in both field and laboratory [5, 6]. Many researchers used ultrasonic methods to characterize different geomaterials, and concluded that these methods are among the best ways to investigate the elastic properties of such materials [7, 8-11]. Several correlations between elastic modulus and physical properties of rocks with ultrasonic velocities were established [2, 3, 12-17]. The results indicate that the seismic velocity is closely related with rock properties. The aim of the current study is to use an ultrasonic method for the prediction of dynamic elastic properties of carbonate rocks of Anah Formation in a cost-effective manner by establishing correlations between elastic wave velocities and rock properties.

Location of the Study Area

The study area is located close to the both banks of the Euphrates River, between latitudes 34° 25' 0"-34° 30' 0" N and longitudes 41° 42’ 0” - 41° 45’ 0” E. The area represents a proposed site for building a new dam near Rawa city northwest of Al-Anbar governorate, western Iraq (Figure-1).

Figure 1- Location map showing the wells from which samples were collected.
Geology of the Study Area

According to the four wells located at the study area, Lithostratigraphy can be described from the oldest to the youngest as follows [18, 19]; Baba Formation (U.Oligocene) comprised of hard dolomitic limestone of white color with a thickness varying between 99 and 149 m. The formation that overlies Baba is Anah Formation which has a thickness in the typical section ranging between 7 and 99 m. and composed of chalky limestone, coral reef and breccias. The Euphrates Formation (L. Miocene) is mainly composed of basal conglomerate, dolomitic limestone chalky limestone, and marly limestone, with a thickness ranging between 28 and 75 m. The Fatha Formation (M. Miocene) is composed of marl, limestone, gypsum, bituminous gypsum, and Claystone. Quaternary deposits comprised of river terrace(sandy gravel) sediments of the Pleistocene are also present. Pleistocene–Holocene deposits are represented by slope sediments, floodplain sediments, valley and depression fill sediments, and Sabkha's sediments, including salt crust mixed with silt and clay.

Samples collection and preparation

Forty rock samples of Anah formation were collected from four wells drilled at the study area. The vertical interval distances between the collected samples were irregular. The thickness of Anah formation and the number of the collected samples in each well are illustrated in table 1. Particular attention was dedicated to the collection procedure to avoid sampling in altered layers. All samples were calibrated to be in a cylindrical shape with an approximate length of around 10-16cm and approximate diameter of 5 cm. End surfaces of the samples were polished to a sufficiently smooth plane to provide good coupling between the transducer face and the rock surface.

| Table 1-Number of Collected Samples in the Study Area |
| Well No. | Thickness of Anah Formation (m.) | Number of samples | Vertical interval distance between samples (m.) |
|----------|----------------------------------|------------------|-----------------------------------------------|
| 1        | 22                               | 8                | 2                                             |
| 2        | 7                                | 2                | 5                                             |
| 3        | 13                               | 2                | 6                                             |
| 4        | 32                               | 28               | 1                                             |

Density measurements

The density of each core sample was measured after removing moisture. The density of dry samples was obtained from the following formula:

$$\rho \ (\text{kg/m}^3) = \frac{\text{mass of sample}}{\text{volume of sample}}$$ (1)

The density of samples at the study area shows relatively wide range of dry density due to the effect of compaction and micro fractures. The value of dry density varied from 2029 kg/m$^3$ to 2839 kg/cm$^3$ with an average value of 2506 kg/m$^3$.

Velocity measurements

The velocities of compressional and shear waves were measured on core samples using a New Sonic Viewer (model-5217A) available in Baghdad University. The ultrasonic impulses are transmitted to the core sample and then the transit time (T) is recorded. The core sample was positioned between the transmitter and receiver and the transducers were pressed to the ends of the sample until a stable transit time was recorded. In order to ensure good contact between the surface of sample and transducers, special grease was used in this study. Compressional and shear wave velocity values ($V_p$ and $V_s$) were calculated as follows:

$$V_p = \frac{L}{T_p}$$ (2)

$$V_s = \frac{L}{T_s}$$ (3)

where:

L = length of the sample
Tp and Ts = transit time (P and S wave)

The compressional velocity ($V_p$) values of samples at the study area varied from 3197 m/s to 6944 m/s with an average value of 5559 m/s, while the shear velocity ($V_s$) values varied from 1636 m/s to 3476 m/s with an average value of 2725 m/s.
Dynamic elastic moduli

The elastic properties of rocks can be described by many types of elastic moduli termed Young’s modulus, bulk modulus, Poisson’s ratio, shear modulus, and Lamé’s constant [20]. The knowledge of Vp and Vs and densities (ρ) of the substance allow calculating values of elastic moduli, according to the formulas below:

\[ E = \frac{\rho v_p^2 (v_p^2 - 4v_s^2)}{v_p^2 - v_s^2} \]  \hspace{1cm} (4)

\[ K = \rho \left( \frac{v_p^2 - 4v_s^2}{3v_s^2} \right) \]  \hspace{1cm} (5)

\[ \sigma = \frac{v_p^2 - 2v_s^2}{2(v_p^2 - v_s^2)} \]  \hspace{1cm} (6)

\[ G = \rho v_s^2 \]  \hspace{1cm} (7)

\[ \lambda = \rho (v_p^2 - 2v_s^2) \]  \hspace{1cm} (8)

where E is Young’s modulus, K is the bulk modulus, \(\sigma\) is the Poisson’s ratio, G is the shear modulus, \(\lambda\) is the Lamé’s constant, \(\rho\) is the density, \(v_p\) is P-wave velocity, and \(v_s\) is the S-wave velocity.

Results and discussion

The main properties obtained from the velocity measurement for the core samples collected from wells 1, 2, 3, and 4 are presented in Table 2. The elastic properties of core samples showed differences in values. The young modules (E) values ranged from 1.44E+10 to 8.94E+10, the bulk modulus (K) ranged from 1.35E+10 to 8.86E+10, the poisons ratio (\(\sigma\)) ranged from 0.109 to 0.42, the shear modulus (G) ranged from 0.54E+10 to 3.43E+10, and the Lamé’s constant (\(\lambda\)) ranged from 0.53E+10 to 7.73E+10 Pascal. The high difference between these two values reflects structural and textural variations that occurred in rocks of Anah formation in the investigated area. In order to describe the relation between ultrasonic wave velocity and elastic modulus of the tested core samples, a regression analysis was achieved. The equations of the best fit line and the regression coefficients (\(R^2\)) were established for each relation. The present section aims to present and discuss correlations that enable the estimation of rocks properties from ultrasonic velocities.

Table 2- The main properties obtained from the velocity measurement for the core samples at the study area.

| No. of Sample | Vp m/s | Vs m/s | \(\rho\) (dry density) kg/m³ | \(\sigma\) | K (E+10) Pascal | E (E+10) Pascal | G (E+10) Pascal | \(\lambda\) (E+10) Pascal |
|---------------|--------|--------|--------------------------|--------|----------------|----------------|----------------|----------------|
| 1             | 4852   | 2578   | 2439                     | 0.303  | 3.58           | 4.23           | 1.62           | 2.5            |
| 2             | 4312   | 2653   | 2372                     | 0.195  | 2.18           | 3.99           | 1.6            | 1.07           |
| 3             | 6172   | 3062   | 2735                     | 0.336  | 7              | 6.86           | 2.56           | 5.29           |
| 4             | 5624   | 2941   | 2525                     | 0.311  | 5.07           | 5.73           | 2.18           | 3.62           |
| 5             | 4069   | 2022   | 2301                     | 0.336  | 2.56           | 2.51           | 0.94           | 1.93           |
| 6             | 5054   | 2450   | 2516                     | 0.346  | 4.41           | 4.07           | 1.51           | 3.41           |
| 7             | 4296   | 1790   | 2317                     | 0.394  | 3.29           | 2.07           | 0.74           | 2.79           |
| 8             | 6358   | 2739   | 2437                     | 0.386  | 7.41           | 5.07           | 1.83           | 6.19           |
| 9             | 4494   | 2975   | 2162                     | 0.109  | 1.82           | 4.25           | 1.91           | 0.53           |
| 10            | 4374   | 2147   | 2275                     | 0.341  | 2.95           | 2.81           | 1.05           | 2.26           |
| 11            | 4999   | 2407   | 2464                     | 0.349  | 4.25           | 3.85           | 1.43           | 3.3            |
| 12            | 5681   | 2870   | 2430                     | 0.328  | 5.17           | 5.32           | 2              | 3.84           |
| 13            | 3301   | 1694   | 2401                     | 0.321  | 1.7            | 1.82           | 0.68           | 1.24           |
Correlation between $V_p$ and $V_s$

The correlation between $V_p$ and $V_s$ were established. A positive linear relationship with a determination coefficient of $R^2 = 0.66$ is shown in Figure-2. The equation for these relations is given below:

$$V_s = 0.3888V_p + 563.08$$

where both $V_p$ and $V_s$ are in m/s.

![Figure 2](image-url)
Correlation between $\rho$ and $V_p$ and $V_s$

The correlation between ($\rho$) and ($V_p$- and $V_s$) for rock samples is plotted in Figures-(3,
4), respectively. According to the plots, $V_p$ and $V_s$ increased linearly with increasing ($\rho$). The empirical relations between ($\rho$) and $V_p$ and $V_s$ are given below:

$$
\rho = 0.1504V_p + 1670.7 \\
\rho = 0.2656V_s + 1783.2
$$

![Figure 3](image3.png)

**Figure 3**-The relation between $V_p$ and $\rho$ in the study area.

![Figure 4](image4.png)

**Figure 4**-The relation between $V_s$ and $\rho$ in the study area.

Correlation between ($E$) and ($V_p$ and $V_s$)

Young’s modulus measures the resistance of a material to elastic deformation under load. Compacted cemented rocks have a higher Young’s modulus than the porous ones. This means that the greater value of Young’s modulus, the larger stress that is required to accomplish the deformation. Figures-5 and 6 show a linear relationship with high regression coefficient values ($R^2=0.76$ and 0.91) between $E$ and ($V_p$- and $V_s$) wave velocities, respectively. The empirical relations between $E$ and $V_p$ and $V_s$ are given below:

-
### Figure 5

The relation between $V_p$ and $(E)$ in the study area.

### Figure 6

The relation between $V_s$ and $(E)$ in the study area.

**Correlation between (K) and (Vp and Vs)**

Bulk modulus is another elastic constant that has been investigated in this study. This property is to examine the resistance of rocks to compression. A high value of $K$ can be obviously noted in the compacted cemented rocks than porous rocks. The relations between $K$ and (P- and S-wave) velocities are plotted in Figures-7 and 8, respectively. The high regression coefficient ($R^2 = 0.92$) reveals a strong linear positive correlation between the $K$ and $V_p$, while the low regression coefficient ($R^2=0.43$) reveals weak linear positive correlation between the $K$ and $V_s$. The following equation defines these relationships:

$$E = 2 \times 10^7 V_p - 4 \times 10^{10}$$

$$E = 4 \times 10^7 V_s - 5 \times 10^{10}$$
Correlation between $(\sigma)$ and velocity Ratio $(Vp/Vs)$

Velocity Ratio $(Vp/Vs)$ and Poisson’s ratio$(\sigma)$ are paramount in engineering studies for the assessment of lithology, quality and strength of the rocks[21], based on the fact that $Vp/Vs$ is a better indicator of lithology than individual velocity values $(Vp$ and $Vs)$[22]. The relationship between $Vp/Vs$ and $(\sigma)$ is plotted in Figure-9. It shows a linear relationship with high a regression coefficient $(R^2=0.81)$, which indicates that the Poisson’s ratio increases linearly with increasing $(Vp/Vs)$. The following equation defines this relationship:

$$\sigma=0.2232(Vp/Vs) - 0.1293$$

**Figure 7**-The relation between $Vp$ and $(K)$ in the study area.

**Figure 8**-The relation between $Vs$ and $(K)$ in the study area.

**Figure 9**-The relation between $(Vp/Vs)$ and $(\sigma)$ in the study area.
Correlation between \((G)\) and \((V_p \text{ and } V_s)\)

The shear modulus or modulus of rigidity is defined as the ratio of shear stress and shear strain. This property is to examine how stiff a rock is to shearing strain with no change in the volume [20]. Rigid compacted rocks have higher shear modulus than soft rocks. This implies that the greater value of shear modulus, the larger stress is required to produce deformation. Figure-10 shows a linear relationship between \(V_p\) and \(G\), with a regression coefficient of 0.69. There is a similar relationship between \(V_s\) and \(G\) (Figure 11) but with a larger determination coefficient \((R^2=0.94)\). The following equation defines this relationship:

\[
G = 6E+06V_p - 1E+10 \\
G = 1E+07V_s - 2E+10
\]

**Figure 10**-The relation between \(V_p\) and \(G\) in the study area.

**Figure 11**-The relation between \(V_s\) and \(G\) in the study area.

Correlation between \((\lambda)\) and \((V_p \text{ and } V_s)\)

The correlation between \((\lambda)\) and \((V_p \text{ and } V_s)\) for rock samples are plotted. Figure-12 shows a linear relationship with high regression coefficient \((R^2=0.83)\) between \(V_p\) and \(\lambda\), which implies that the \(\lambda\)
increases linearly with increasing $V_p$. The empirical relations between $\lambda$ and $V_p$ are given below:

$$\lambda = 2E+07V_p - 6E+10$$

![Figure 12](image12.png)

Figure 12- The relation between $V_p$ and $\lambda$ in the study area.

The dispersion of data is illustrated in Figure-13. It shows a weak relationship with a low regression coefficient ($R^2=0.27$) between $V_s$ and $\lambda$, which indicates that the value of $V_s$ cannot be used to estimate $\lambda$ with an acceptable accuracy. The empirical relations between ($\lambda$) and $V_p$ is given below:

$$\lambda = 2E+07V_s - 2E+10$$

![Figure 13](image13.png)

Figure 13- The relation between $V_s$ and Lame’s constants ($\lambda$) in the study area.

CONCLUSIONS

The current study aimed to determine some of the elastic properties of carbonate rocks by establishing correlations between rock properties and ultrasonic wave velocities. The results indicate that the dynamic elastic properties of carbonate rock showed linear relationships with both P- and S-wave velocities, but with different values of determination coefficient ($R^2$). Some parameters
such us density, bulk modulus and lame’s constant are strongly related with the P-wave velocity compared with the S-wave velocity, where the determination coefficient between P-wave velocity and these parameters is higher than 0.67. Other parameters such us Young’s modulus and Shear modulus are strongly related with S-wave velocity compared with P-wave velocity, where the determination coefficient between S-wave velocity and these parameters higher than 0.91, while Poisson’s ratio is strongly related with velocity ratio in the determination of $R^2$, with a value of 0.81. Empirical equations obtained in the current study revealed that the elastic properties of carbonat rocks at the study area can be estimated by determining ultrasonic wave velocities.

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