Satellite Constellation Reconfiguration Method Using Improved PSO Algorithm for Communication and Navigation

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Abstract. A method of satellite constellation reconfiguration based on an improved PSO algorithm was presented. This method used a multi-satellite launch to reconfigure constellations without affecting the initial constellation configuration. Reconfigurable time and cost metrics were quantified, and the processing of a multi-satellite launch was introduced. Constellation failure cases were divided into multi-satellite failure in a single orbital plane and multi-satellite failure in different orbital planes. Constellation reconfiguration models for different failure cases were established. An improved particle swarm optimization (IPSO) algorithm was presented to solve the models. Simulation results show that the IPSO algorithm has better performance in handling constellation reconfiguration problems.

1. Introduction

When satellites failed due to collision, life-ending, and component damage, constellations' performance was reduced. Constellation reconfiguration is an effective way of restoring the performance of damaged constellations. Especially in the construction of large constellations such as Starlink and OneWeb, the risk of a batch of satellites failure is increasing, posing a severe threat to the constellation's regular operation [1]. The methods to quickly restore the flawed constellations' performance have become a research focus.

A recent study by Fakoor and Soleymani et al. proposed a constellation reconfiguration method based on Lambert's theory [2,3]. This method exploited Lambert transfer manner to maneuver satellites and optimized the velocity increment during the constellation reconfiguration. Ling et al. studied the reconfiguration method of BDS (BeiDou Navigation Satellite System). They proposed a reconfiguration method of constellations by using low, medium, and high orbit satellites and adjacent aircraft [4]. Appel et al. used electric propulsion to maneuver satellites and proposed a combined algorithm to solve the optimization problem of multi-satellite coupling maneuver during the constellation reconfiguration process [5]. Hu et al. studied a re-configuration method of global navigation constellations, maneuvered remaining satellites in the damaged constellation, and optimized the selection of adjusted satellites and the phase of adjusted satellites through genetic algorithms [6].

Xiang et al. proposed reconfiguration strategies of navigation constellation. Reconstruct the damaged constellation using a single satellite maneuver, two adjacent satellite maneuvers, and all satellite maneuvers. Navigation constellation reconfiguration models were established by considering energy consumption, recoverability, and performance improvement [7]. Chen et al. studied on-orbit
reconfiguration methods of reconnaissance constellations. Considering the average revisit time, maximum coverage time, and the total coverage time based on task scheduling, reconfiguration optimization models were established, then a variable dimension multi-objective differential evolution algorithm was proposed, which can effectively solve the coupling optimization problem of maneuvering satellite selection and maneuvering target position [8]. De Weck et al. proposed a reconfiguration method combining multi-satellite launch with on-orbit satellite maneuver. A new orbital plane was formed by the launched satellite. Simultaneously, on-orbit satellites were maneuvered, the final position and maneuver strategies of each satellite were optimized, but the impact of the launch process in the reconfiguration process was not considered [9]. Chen et al. adopted the method of phase reset of the satellites in orbit. The optimization models of communication constellation reconfiguration were established by taking the network transmission rate and delay time as the optimization goals [10].

Most previous constellation reconfiguration methods mentioned above considered on-orbit satellites maneuver to reset constellation configuration. However, there are fewer constellation reconfiguration methods considering the constraints of the launch process of multiple satellites. In actual tasks, launch constraints have an enormous influence on the constellation reconfiguration process. In this paper, we aim to propose a constellation reconfiguration method based improved PSO algorithm. The rest of this paper is organized as follows: the models of reconfigurable cost and time and multi-satellite launch process are described in Section 2. An overview of reconfiguration models is established in Section 3. An improved PSO algorithm is proposed in Section 4. Numerical results of a design example are given in Section 4. Finally, conclusions are presented in Section 5.

2. Problem formulation

2.1. Reconfigurable cost and reconfigurable time

Reconfigurable cost is related to the reconfiguration process. The launch of multiple satellites with one rocket involves two processes. The first process is from issuing the reconfiguration task to entering the target orbit planes. The second process is from entering the target orbit planes to maneuvering to the failed satellite's position. The cost required for the first process can be calculated by the SSCM model, including the cost of payload, satellite platform, final assembly, integration and test, launch, operation support, and ground support equipment.

The second process's cost can be calculated based on the mass of propellant consumed by the satellite orbit maneuver. The relationship between the cost of satellite orbit maneuvering and the mass of propellant consumed is shown in Figure 1. There are two sources of propellant required for satellite orbit maneuvering. The first part comes from the conventional maneuvering propellant preset when the satellite is manufactured. The other part comes from the propellant used to maintain the satellite's life. The total amount of propellant in the satellite tank is \( M_n \). The preset propellant threshold for conventional maneuvers is \( M_n \). The propellant consumed by the satellite maneuver in phase 1 is the propellant planned for maneuver. When the propellant consumed by the satellite maneuver is less than \( M_n \), it does not affect the life of the satellite and belongs to normal maneuver. The propellant consumed in phase 2 satellite maneuvering is the quality of the propellant used to maintain the life of the satellite. When the propellant consumed by the satellite maneuvering is greater than \( M_n \), it means that the satellite needs to lose part of its life to provide the maneuver's energy. According to the above analysis, the cost of satellite orbit maneuvering \( C_M \) can be defined as
The cost per kilogram of propellant consumed by satellite maneuver is \( c_s \), which usually sets 1% of one satellite cost. The cost of one satellite is \( C_{sat} \), which can be calculated by the SSCM model. Assuming that the number of launching satellites is \( n \), then, the total cost of reconfiguration can be defined as

\[
C = nC_{sat} + \sum_{i=1}^{n} C_{M_i}
\]

(2)

The reconfiguration time can be divided into the launch time and the phase maneuver time. The launch time refers to the time required for new satellites from launch to enter the target orbit. The phase maneuver is performed immediately after the satellite enters the target orbit. Which refers to the time required for the satellite phase maneuver process. The reconfiguration time can be defined as

\[
T = t_L + t_{phase}
\]

(3)

Where launch time is \( t_L \), and phase maneuver time is \( t_{phase} \).

2.2. Multi-satellite launch process analysis

It is assumed that the mobile launch method is used for the multi-satellite launch process to comply with the actual mission. Before the mission is issued, the preparation and testing of the equipment required for the launch have been completed. The multi-satellite launch process can be divided into the following stages: (1) Launch vehicles maneuvering stage (2) Launch windows waiting stage (3) Launching stage (4) Satellite phase maneuver

Make the following assumptions:

(1) The maneuver launch manner is adopted, an effective launch method to shorten the reconfiguration time.
(2) To save launch cost and orbit cost, adopt the coplanar launch method.
(3) The model and payload of launched satellites are the same as those of failed satellites in orbit.
(4) The longest maneuvering time of launch vehicles \( t_{mavoh} = 6 \) h.
(5) Launch waiting time must be greater than 2h.
Analyze the launch vehicle maneuvering phase and launch waiting phase. Assuming that the maneuver launch site is a circular area with a fixed launch site \((\lambda_0, \phi_0)\) as the center and a radius of \(R_m\), the radius depends on the launch vehicle velocity and maneuvering duration of the maneuvering launch vehicle. The velocity of the maneuvering launch vehicle is \(V_{\text{vehicle}}\). The longest maneuvering time of the launch vehicle is \(t_{\text{maxveh}}\), \(R_m = t_{\text{maxveh}} V_{\text{vehicle}}\). The longitude range \([\lambda_{\text{min}}, \lambda_{\text{max}}]\) and latitude range \([\phi_{\text{min}}, \phi_{\text{max}}]\) of the maneuvering launch area can be calculated from the geometric relationship. The down-orbit launch process is shown in Figure 2. In the constellation reconfiguration method based on the multi-satellite launch, the new satellites' target orbits are the same as the orbit of the failed satellites. Given that the failed satellite orbits' semimajor axis is \(a_f\), the orbital inclination is \(I_f\). The ascension of the ascending node is \(\Omega_f\). The launch waiting time from a point \(L\) in the maneuvering launch area can be calculated by equation (4).

\[
\text{Figure 2. Satellite Down-orbit Launch}
\]

\[
t_w = \frac{1}{w'} \mod(\pi + \Omega_f - \arcsin(\frac{\tan \phi_f}{\tan I_f}) - \vartheta(t) - \lambda_L - 2\pi) - t_{\text{veh}}
\]  
(4)

Where \(\mod\) denoted the remainder of dividing two numbers. \(\vartheta(t)\) denoted Greenwich sidereal time angle at the initial moment \(t\). \(w'\) denoted the rotation angular velocity of the target orbit relative to the earth under the perturbation of the J2. \(t_{\text{veh}}\) denoted launch vehicle maneuver time.

Similarly, using the ascending orbit launch method, the launch waiting time \(t_a\) is

\[
t_a = \frac{1}{w} \mod(\Omega_f + \arcsin(\frac{\tan \phi_f}{\tan I_f}) - \vartheta(t) - \lambda_L + 2\pi) - t_{\text{veh}}
\]  
(5)

After the launch vehicle maneuvering phase and the launch waiting phase, we analyze the launch phase. When the launch mission satisfies the launch window, the rocket is launched from the maneuver launch point \(L\) and reaches the orbit point \(C\) after flying \(t_c\). Given that the geocentric launch angle is \(\alpha_L\), the rocket launch process is approximately regarded as a two-impulse Kepler orbit. The true anomaly angle of point \(L\) in the launch orbit is \(f_L = \pi - \alpha_L\), and the true anomaly angle of point \(C\) in the target orbit. The orbital mechanic's theories show that point \(L\) and point \(C\) satisfy the formula (6).

\[
\begin{align*}
R_e &= \frac{\rho}{1 + e_L \cos f_L} \\
\alpha_r &= \frac{\rho}{1 + e_C \cos f_C}
\end{align*}
\]  
(6)
Where \( e_L \) denoted launch orbit eccentricity, \( \rho \) denoted launch orbit. Solve the equations (6) to obtain \( \rho \), \( e_L \), then the semimajor axis of launch orbit \( a_L \) is obtained

\[
a_L = \frac{\rho}{1-e_L^2} \tag{7}
\]

According to the relationship between the true anomaly and the mean anomaly, the mean anomaly of the launch point \( L \) \( \mu_L \) can be obtained, and then the flight time of the launch section \( t_L \) can be calculated

\[
t_L = (\pi - \mu_L) \sqrt{\frac{a_L^3}{\mu_e}} \tag{8}
\]

Where \( \mu_e \) denoted earth’s gravitational constant.

Given that the latitude argument \( M_1 \) of the failed satellite at the initial moment is \( t \), and after the launch vehicle maneuvering stage, the launch waiting stage, and the launch stage, the latitude argument \( M_T \) of the failed satellite is shown in equation (9).

\[
M_T = M_1 + n'(t_{ob} + t_w + t_l) \tag{9}
\]

Where \( n' \) denoted the average angular velocity of the satellite under the perturbation of J2.

In the spherical triangle ABD, the latitude argument \( M_L \) of the sub-celestial point B on the target orbit can be obtained. As shown in equation (10), then the latitude argument of the new satellite's orbiting point C is \( M_C = M_L + \alpha_L \), \( \alpha_L \) denoted by the geocentric launch angle.

\[
\sin (\pi - M_C) = \frac{\sin \phi_C}{\sin \alpha_L} \tag{10}
\]

Where \( \phi_C \) denoted latitude value of maneuver launch point.

Similarly, when using the ascending launch manner, the latitude argument \( M_L \) of the sub-celestial point B on the target orbit is shown in equation (11). When launching the ascending orbit, the latitude argument of the new satellites' orbit point C is \( M_C = M_L + \alpha_L \).

\[
\sin M_C = \frac{\sin \phi_C}{\sin \alpha_L} \tag{11}
\]

Be analyzing the phasing stage. The phase maneuver method can be divided into the high-orbit phase maneuver and low-orbit phase maneuver [11]. The high-orbit phase maneuver process is shown in Figure 3. Moving satellite 1 from the current phase to the target phase. The specific implementation is that satellite 1 accelerates into the phase maneuver orbit along the velocity direction. When it reaches the target position after a certain number of revolutions in the phase maneuver orbit, it applies an impulse in the opposite direction of the velocity to enter the target orbit. The low orbit phase maneuver process is the opposite. When the phase maneuver time is fixed, the maneuvers' velocity increment is proportional to the satellite phase adjustment amount. When the phase maneuver amount is \( \Delta m \in (0, 180) \), the velocity increment required for low orbit phase maneuver is smaller. When the phase
maneuver amount is $\Delta M \in (180, 360)$, the high orbit phase maneuver method is used due to the smaller increment of velocity needed.

Figure 3. High-orbit Phase Maneuver

The latitude argument $M_c$ of the new satellite at the moment of entry to orbit is given. Simultaneously, the latitude argument of the failed satellite is $M_f$. The new satellite needs to adjust the phase $\Delta M = M_f - M_c$. When $\Delta M \in (0, 180)$, low-orbit phase maneuver is used. When $\Delta M \in (180, 360)$, a high-orbit phase maneuver is used. It is known that the failed satellites in target orbital planes travel $l$ revolutions. If a low orbit phase maneuver is adopted, velocity $\Delta v_{\text{phase}}$, time $t_{\text{phase}}$, and semimajor axis $a_{\text{phase}}$ of phase maneuver orbit are shown in equation (12).

$$
\begin{align*}
\Delta v_{\text{phase}} &= \frac{2}{a_f} \sqrt{\frac{\mu}{a_f}} \left(1 - \sqrt{2 - \left(1 - \frac{\Delta M}{2\pi l}\right)^2}\right) \\
t_{\text{phase}} &= \frac{a_f^2}{\mu} \\
a_{\text{phase}} &= a_f \left(1 - \frac{\Delta M}{2\pi l}\right)^2
\end{align*}
$$

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$$
\begin{align*}
\Delta v_{\text{phase}} &= \frac{2}{a_f} \sqrt{\frac{\mu}{a_f}} \left(\sqrt{2 - \left(1 + \frac{2\pi - \Delta M}{2\pi l}\right)^2} - 1\right) \\
t_{\text{phase}} &= \frac{a_f^2}{\mu} \\
a_{\text{phase}} &= a_f \left(\frac{2\pi - \Delta M}{2\pi l} - 1\right)^2
\end{align*}
$$

The rocket equation can be used to calculate the mass of propellant required $m$ for each phase maneuver, as shown in equation (14). The cost of constellation reconfiguration can be obtained through equation (1) and equation (2).
Where $m_0$ denoted satellite mass, $e$ denoted universal constant, $g_0$ denoted acceleration of gravity, usually set to 9.98 m/s$^2$, $I_w$ denoted specific propellant impulse, usually set to 300 s.

Assuming that there are $n$ satellites failure in one orbital plane, and $n$ satellites are sorted according to the latitude argument's value. The total reconfiguration time can be calculated by equation (15).

$$T = t_{veh} + I_w + t_i + \max\left(t_{phase}\right)$$  \hspace{1cm} (15)

Where $t_{phase}$ denoted the $i$th new satellite required time for phasing.

3. Reconfiguration Models

3.1. Multi-Satellite Failure in the Same Orbital Plane

When constellation meets multi-satellite failure in the same orbital plane, reconfigure the optimized model can be established. Assuming that the number of failed satellites in the same orbital plane is two or more. Taking the reconfiguration cost as the optimization goal. The reconfiguration cost optimization model is established. There are four optimization variables in the model, which are the longitude value and the latitude value of the launch point, the maneuver time of the launch vehicle, and the launch manner flag.

Optimization goal:

$$\min f = \min C$$  \hspace{1cm} (16)

Where $C$ denoted reconfiguration cost.

Optimization variables:

$$x = (\lambda_L, \varphi_L, t_{veh}, \kappa)$$  \hspace{1cm} (17)

Where $\lambda_L$ denoted the longitude of the Maneuvering launch point, $\varphi_L$ denoted the latitude of the Maneuvering launch point, $t_{veh}$ denoted maneuvering duration of the launch vehicle, $\kappa$ denoted launch mode. When set as 1, the ascending orbit is used for launching, while when set as 0, the descending orbit is used for launching.

Constraint:

1. The latitude of the maneuvering launch point determines the maximum inclination of the satellite launched from that place, so the latitude $\varphi_L$ of the maneuvering launch point must meet $\varphi_L \leq I_t$.

2. Since the maneuvering launch area is a spherical circle area, the maneuvering launch vehicle's velocity is $V_{vehicle}$, and the maneuver duration of the launch vehicle is $t_{veh}$. In addition to satisfying boundary constraints, the longitude and latitude of the maneuver launch point must be satisfied:

$$\angle \beta \leq \frac{t_{veh} \cdot V_{vehicle}}{R_e}$$  \hspace{1cm} (18)

Where $\angle \beta$ denoted the angle between the maneuvering launch point and the fixed-point launch site.

3. During the satellite phase maneuver process, if $\Delta M \in (0, 180)$, low orbit phasing mode is preferentially selected. Phasing orbits' semimajor axis must not be lower than the minimum allowed...
semimajor axis. Assuming that minimum allowed semimajor axis length is generally 6678km. If not satisfied, high orbit phasing mode is selected. If $\Delta M \in (180, 360)$, high orbit phasing mode is selected.

Similarly, the reconfigurable time model can be obtained by replacing the optimization goal with the reconfiguration time in the case of multi-satellite failure in the same orbital plane.

### 3.2. Multi-Satellite Failure in Different Orbital Planes

The reconfiguration optimization model of multi-satellite failure in different orbital planes is established. The multi-satellite failure in different orbital planes are a combination of the multi-satellite failure in the same orbital plane. Each orbital plane containing the failed satellite can be selected launch reconfiguration method in turn, but it takes a long time. In actual missions, multi-launch areas are generally used to launch multiple rockets to reconfigure the constellation, and the number of launched rockets is the same as the number of orbital planes of the failed satellites. Generally speaking, the number of launch areas is much smaller than the number of rockets required to be launched, so there will be situations where multiple rockets are launched in one launch area. The model can be established as follows: Assuming the number of the failed orbital planes in the constellation is $p$, and the number of maneuvering launch areas is $q$, and $p \geq q$. The number of optimization variables is $pq$.

Optimization goal:

$$\min f = \min \sum_{i=1}^{p} \sum_{j=1}^{q} c_{i,j} x_{i,j}$$  \hspace{1cm} (19)

Where $c_{i,j}$ denoted minimum reconfiguration cost required to reconfigure a failed orbital plane $i$ from the launch area $j$. When a failed orbital plane $i$ is reconfigured from the launch area $j$, $x_{i,j}$ is set as 1. Otherwise, it is set as 0.

Optimization variables:

$$x = (x_{i,j}) (i = 1, 2, \ldots, p) (j = 1, 2, \ldots, q)$$  \hspace{1cm} (20)

Constraint:

(1) The maneuvering launch area's minimum latitude is greater than the orbital plane's inclination to be reconfigured. If it is not satisfied, then $x_{i,j} = 0$

(2) Each failed orbital plane only selects one area from all maneuvering launch areas to reconfigure, avoiding the situation that multiple launching areas launch rockets to reconfigure the same orbital planes.

$$\sum_{j=1}^{q} x_{i,j} = 1 \hspace{1cm} (i = 1, 2, \ldots, p)$$  \hspace{1cm} (21)

(3) To ensure that all launch areas are used, at least one launch mission exists in each maneuvering launch area.

$$\prod_{j=1}^{q} \sum_{i=1}^{p} x_{i,j} \neq 0$$  \hspace{1cm} (22)

Establish a reconfiguration time optimization model in the case of multi-satellite failure in different orbital planes. It is assumed that when multiple missions are launched in the same mobile launch area, they can only be launched in sequence. Therefore, the reconfiguration time is the sum of the time of all reconfiguration tasks. The total reconfiguration time is shown in equation (23).
Where $t_{i,j}$ denoted minimum reconfiguration time required to reconfigure a failed orbital plane $i$ from the launch area $j$.

4. Improved PSO Algorithm

After the above analysis and discussion, the reconfiguration optimization models for multi-satellite failure in one orbit, multi-satellite failure in the different orbital planes were established. These models have a single optimization goal and have some or all integral optimization variables. Constraints are mostly nonlinear and cannot be expressed explicitly. If traditional optimization algorithms are used to solve the models, low quality of the optimal reconfiguration scheme and difficulty in handling constraints will arise.

In this section, the PSO algorithm is improved. Firstly, a constraint tolerance parameter is introduced into the feasibility rule. Secondly, an adaptive inertia weight method based on the chaos model and a population update strategy based on the learning mechanism are proposed to enhance the algorithm's fast search in the constraint domain space. Finally, integer variables are transformed into continuous variables, and then the population is updated, which solves the optimization problem of integer variables in the models. The algorithm flow is shown in Figure 4.

\[
T = \max \left( \sum_{j=1}^{N} t_{i,j} x_{ij} \right) \ (i = 1,2,...,p)
\]  

(23)

**4.1. Constraint Handling Method**

Most of the constellation reconfiguration optimization models are nonlinear and non-directive constraints. The standard PSO algorithm cannot solve this kind of constrained optimization problem. It can be used only after adding a constraint processing method to the algorithm. Law of feasibility is a relatively common constraint handling method [12]. This method always rejects infeasible solutions in the evolution process. Although it is conducive to quickly finding the feasible region, its ability to find the feasible region's optimal value is not strong.
As shown in Table 1, the feasibility rule is improved. Firstly, the quality of the infeasible solution is judged through constraint tolerance. When the constraint violation degree of the infeasible solution is lower than the constraint tolerance, it indicates that this infeasible solution is better. Infeasible solutions that can provide information for the optimal value optimization process. After that, a random probability parameter is used to determine the superior infeasible solution's evaluation method. There are two evaluation methods for the superior infeasible solution. One is based on function value and the other is based on a constraint violation value. Based on the function value judgment for the superior infeasible solution, the superior solution's information can be effectively used to guide the evolution process of the feasible solution, especially for optimizing the feasible solution at the boundary of the feasible region.

**Table 1. Improved Feasibility Rule**

| Condition | Rule |
|-----------|------|
| If \(c(X_i) = 0 \) and \(c(X_j) = 0\) | Ranking Based on Function Value |
| Else if \(c(X_i) = 0 \) and \(c(X_j) \neq 0\) | Ranking based on constraint violation method |
| \(c(X_j) \leq \varepsilon\) and \(\text{rand} \leq P_r\) | Ranking Based on Function Value |
| else | Ranking Based on Function Value |
| \(c(X_i) \leq \varepsilon\) and \(c(X_j) \leq \varepsilon\) and \(\text{rand} \leq P_r\) | Ranking based on constraint violation method |

### 4.2. Adaptive Weight Method

Choosing appropriate inertia weights at different stages of evolution can improve the algorithm's ability to optimize in the constrained domain. The proposal of chaos theory provides a new idea for the selection of inertia weight. Logistic is a typical chaos model. Introducing the Logistic chaos model into the method of obtaining inertia weight can enhance the chaos of inertia weight selection, thereby improving the algorithm's handling constraints ability. The Logistic chaos model is shown in equation (24), and the inertia weight can be obtained according to equation (25).

\[
\begin{align*}
l(t + 1) &= 4l(t)(1 - l(t))j \\
w(t) &= l(t)w_{\text{min}} + (w_{\text{max}} - w_{\text{min}})e^{-\frac{t - t_w}{T_{\text{max}}}}
\end{align*}
\]

Where \(l(0)\) is a random number in \([0,1]\), \(t\) is the current iteration; \(T_{\text{max}}\) is the total number of evolution; \(w_{\text{max}}\) is the maximum value of inertia weight; \(w_{\text{min}}\) is the minimum value of inertia weight.

### 4.3. Improved Population Update Strategy

The speed update strategy of particles is improved. Firstly, the populations are sorted according to the function value of the particle's historical optimal value. The first \(N_d\) dominant particle is selected as the dominant subpopulation, and the remaining particles learn from the dominant particles. The particle learning strategy is as follows: Choose a dominant particle randomly in the subpopulation, compare the function value of the dominant particle's optimal value and the optimal particle value. If the dominant particle's value is less than the \(j\) optimal particle value, the dominant particle is used instead of the particle's optimal value \(P_{\text{best}}\), which is shown in equation (26). The average value of the dominant subpopulation's optimal value replaces the optimal value of the particle swarm, and the improved speed update term is shown in equation (28).
Where $i$ is the current iteration number; $v$ is the particle velocity; $c_1$ and $c_2$ are learning factors, $w$ is the inertia weight, $P_{\text{better}}$ the optimal value of the randomly selected dominant particle. $G_{\text{better}}$ is the average value of the optimal value of the dominant subpopulation, $r_1$ and $r_2$ are random numbers in the interval $[0,1]$. 

4.4. Conversion of Binary Integer Variables

There are integer variables in the optimization variables of the reconfiguration optimization models. The standard PSO algorithm is only suitable for solving continuous variable optimization problems. If you need to optimize integer variables, the integer variables need to be mapped to the continuous variable space. This paper uses a new type of mapping relationship to convert binary integer variables into continuous variables. The binary integer variable is an optimized variable that only contains two integers of 0 and 1. The particle velocity update strategy remains unchanged, and the position update strategy is modified. The position update relationship associated with the velocity is established through the coefficient of variation probability, as shown in equation (29)

$$v(i + 1) = w \cdot v(i) + c_1 r_1 \left( P_{\text{better}} - X_i \right) + c_2 r_2 \left( G_{\text{better}} - X_i \right)$$

Where $i$ is the current iteration number; $v$ is the particle velocity; $c_1$ and $c_2$ are learning factors, $w$ is the inertia weight, $P_{\text{better}}$ the optimal value of the randomly selected dominant particle. $G_{\text{better}}$ is the average value of the optimal value of the dominant subpopulation, $r_1$ and $r_2$ are random numbers in the interval $[0,1]$. 

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5. Simulation Analysis
Taking the 60/10/1 Walker navigation constellation with a semimajor axis of 7378 km and inclination of 75° as an example, the longitude range of the target area is 90°E ~ 141°E, and the latitude range of the target area is 0° ~ 47°N. The constellation's navigation ability in the full station state to the target area is shown in figure 6, and the average GDOP value of this area is 4.91.

When the constellation faces multi-satellite failure in a single orbital plane, the orbital parameters of the failed satellite at the initial time of simulation are shown in table 2. The average GDOP value of the damaged constellation to the target area is 21.94. Compared with the full station state, the navigation ability is greatly reduced, and the constellation needs to be reconstructed. The simulation parameters are set as follows: the mass of satellite is 120 kg, the mass of tank propellant is 60 kg, the propellant threshold for orbital maneuver is 20 kg, the position of launch field (100.5°E, 41.1°N), the velocity of...
the mobile launch vehicle is 88 km/h, the farthest maneuver time is 6 h, and the geocentric angle of the
launch phase is 10°. Failed satellites are running 5 revolutions during the phasing stage.

Table 2. Orbital Parameters of Failed Satellite for Multi-satellite Failure in A single Orbital Plane

| Serial Numbers | Right Ascension of Ascending Node (°) | Latitude | Argument (°) |
|----------------|--------------------------------------|----------|--------------|
| 1              | 35.99                                | 2.42     |              |
| 2              | 35.99                                | 26.42    |              |
| 3              | 35.99                                | 50.42    |              |
| 4              | 35.99                                | 74.42    |              |
| 5              | 35.99                                | 98.42    |              |

The constellation reconfiguration method based on an improved PSO algorithm is selected to
reconstruct the damaged constellation. The multi-satellite failure model in a single orbital plane is
established. The constellation reconfiguration model is solved by the improved PSO algorithm, the
standard DE algorithm, the standard PSO algorithm, and the solving process shown in figure 7.

It can be seen from the figure 7 that the optimal cost reconfiguration scheme is to launch five satellites
from [100.49°E, 41.54°N] with one rocket and multiple satellites in descending orbit, and the mobile
time of the launch vehicle is 1.8171×10^4 s. Similarly, the optimal time reconfiguration scheme is to
launch five satellites from [100°E, 41.54°N] with one rocket and multiple satellites in descending orbit,
and the mobile time of the launch vehicle is 3.3406×10^3 s. By comparing the performance of the three
algorithms, it can be seen that the IPSO algorithm can find the optimal solution within 10 generations,
indicating that the IPSO algorithm has more robust convergence in solving the reconfiguration
optimization model.

When the constellation face multi-satellite failure in different orbital planes, some orbital parameters
of the failed satellite at the initial time of simulation are shown in table 3. The average GDOP value of
the damaged constellation to the target area is 26.19. The three launch regions positions are L1 (100.5°E,
41.1°N), L2 (102.03°E, 28.25°N), and L3 (111.01°E, 19.68°) N). The remaining parameters are
consistent with a single-orbit multi-satellite failure reconstruction case.
Table 3. Orbital Parameters of Failed Satellites for Multi-satellite Failure in Different Orbital Planes

| Orbital Number | Right Ascension of Ascending Node/° | Latitude Argument/° |
|----------------|------------------------------------|---------------------|
| 1              | 0                                  | 0, 24, 48, 72, 96   |
| 2              | 35.99                              | 2.42, 26.42, 50.42, 74.42, 98.42 |
| 3              | 143.94                             | 9.82, 33.82, 57.82, 81.82, 105.82 |
| 4              | 215.94                             | 14.62, 38.62, 62.2, 86.62, 110.62 |

The optimal reconfigurable costs of five orbital planes from launching rockets in three maneuvering launch regions are calculated. All costs can be represented by matrix $C$. $c_{ij}$ represents the minimum cost to reconfigure orbit $i$ from the launch region $j$, unit K$.

$$
C = \begin{bmatrix}
1.51\times10^4 & 1.11\times10^4 & 9.93\times10^4 \\
1.29\times10^5 & 8.12\times10^4 & 1.32\times10^4 \\
8.39\times10^4 & 9.45\times10^4 & 9.23\times10^4 \\
1.22\times10^5 & 1.51\times10^5 & 1.47\times10^5 \\
9.44\times10^4 & 1.45\times10^4 & 1.14\times10^4
\end{bmatrix}
$$

(32)

Establish an optimization model about multi-satellite failure in different orbital planes, select IPSO, PSO, DE algorithm to solve this model. The process is shown in figure 8.

Figure 8. Reconfiguration Model of Multi-Satellite Failure in Different Orbital Planes

It can be seen from figure 8 that the optimal cost scheme is to reconfigure the failure orbit numbers of 2, 4, 5 by launching five satellites from the L1 region. It is reconfiguring the failure orbit numbers of 3 by launching five satellites from the L2 region. It is reconfiguring the failure orbit numbers of 1 by launching five satellites from the L3 region, and the optimal cost is $4.23\times10^5$ K$. Similarly, the optimal time reconstruction scheme is to reconfigure the failure orbit numbers of 1 and 3 by launching five satellites from the L1 region. It is reconfiguring the failure orbit numbers of 2 by launching five satellites from the L2 region. It is reconfiguring the failure orbit numbers of 4 and 5 by launching five satellites from the L3 region, and the optimal reconstruction time is $1.098\times10^5$ s.
6. Conclusion
In this paper, the multi-satellite launch is applied to the reconstruction process of large-scale LEO constellations, and the standard PSO algorithm is improved. A simulation scenario is designed for verification. When a Walker constellation faces multi-satellite failure in a single orbital plane, the method mentioned above is selected to reconfigure the constellation. The results are that the optimal reconfigurable cost and reconfigurable time are $1.29 \times 10^5 \text{ K$}$ and $4.2 \times 10^3 \text{ s}$. When a Walker constellation faces multi-satellite failure in different orbital planes, the simulation results show that the optimal reconfigurable cost and time are $4.23 \times 10^5 \text{ K$}$ and $1.098 \times 10^5 \text{ s}$. Simulation results show that the proposed constellation design method can quickly restore the constellations' performance without changing its configuration. Compared with PSO and DE algorithms, the improved PSO algorithm effectively solves the constellation reconstruction models.

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