State space sets with common optimal feedback laws for nonlinear MPC

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Abstract—We extend an approach to nonlinear model predictive control that was recently proposed for linear model predictive control. A solution to an optimal control problem for a point in state space provides an optimal feedback law and a region in state space where this optimal feedback law is valid. Also, an optimal active set results from the solution. We state a criterion for a subset of an active set to already define the optimal feedback law. Any optimal active set containing a subset such that the criterion holds defines the same optimal feedback law as the subset. This enables to identify regions with identical optimal feedback laws if the optimal active sets of the regions are known. We use this result to propose a model predictive control approach that reduces the number of optimal control problems that are solved online. The approach initially unites regions with identical optimal feedback laws offline. This results in large regions with known optimal feedback laws. Online, the calculation of the optimal control problem is avoided whenever the current state is detected to be part of a region with known optimal feedback law. We illustrate the achieved savings with an example.

Index Terms—Nonlinear Model Predictive Control (NMPC), Regional MPC, Constrained Control

I. INTRODUCTION

The ability of model predictive control (MPC) to consider constraints directly in the formulation makes MPC a favorable control scheme for many systems. However, MPC is computationally demanding. An entire field of research focuses on the reduction of the computational effort.

Regional MPC approaches intend to reduce the computational effort by reducing the number of optimal control problems (OCPs) that are solved. This reduces the overall cost of online MPC calculation. Applications of MPC to a battery-powered system, for example, could benefit from this method. Most publications address the linear case and exploit characteristics of the parametric solution of the linear-quadratic OCP, which results in a piecewise-affine feedback law [1], [2]. It is the central idea of regional MPC to reuse the optimal feedback law of the previous state if the current state is located in the same region as the previous state [3]–[6]. In contrast to explicit approaches [1], [7], [8], regional MPC approaches do not require the parametric solution of the OCP. It has been shown in [9] that regional MPC approaches can be extended to the nonlinear case, see also [10], [11].

Recently, a novel regional MPC approach was presented for linear MPC. The approach exploits that in some regions of the piecewise-affine feedback law have the same optimal feedback law in common. The paper presents a simple criterion for subsets of active sets to define the optimal feedback law, and shows that any optimal active set that contains such a subset then defines the same optimal feedback law as the subset [12]. The approach suggested in [12] reuses the optimal feedback law of the previous state if the current state is located in the same region as the previous state or in a region defined by an active set defining the same optimal feedback law as the previous state. The parametric solution is not needed in this approach. This enables reusing the optimal feedback law of the previous state more often and therefore avoids solving OCPs more often. In contrast to other approaches [13]–[15], the calculation of the parametric solution is not necessary.

It is the purpose of the present paper to extend the approach from [12] from the linear to the nonlinear case. We proceed analogously to [12] and, in the first step, establish a simple criterion for subsets to define the optimal feedback in the nonlinear case. In the second step, we show that optimal active sets containing such a subset define the same optimal feedback law as the subset. Due to nonlinearities, the proposed approach is different from the approach from [12] and requires to calculate the regions defined by optimal active sets containing such a subset offline. Finally, the approach and the achieved savings are illustrated with an example.

Section II introduces the class of nonlinear optimal control problems treated here. Section III describes state space sets with common feedback laws. Implementation aspects and an example are discussed in Sects. IV and V, respectively. Brief conclusions are given in Sect. VI.

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Notation
For any $M \in \mathbb{R}^{a \times b}$ and any ordered set $\mathcal{M} \subseteq \{1, \ldots, a\}$ let $M_{\mathcal{M}} \in \mathbb{R}^{|\mathcal{M}| \times b}$ be the submatrix of $M$ containing all rows indicated by $\mathcal{M}$.

II. PROBLEM STATEMENT AND PRELIMINARIES
Consider a discrete-time system

$$x(k+1) = f(x(k), u(k)), \quad k = 0, 1, \ldots \tag{1}$$

that must respect constraints of the form

$$u(k) \in U \subseteq \mathbb{R}^m, \quad x(k) \in X \subseteq \mathbb{R}^n, \quad k = 0, 1, \ldots$$

with input variables $u(k) \in \mathbb{R}^m$, state variables $x(k) \in \mathbb{R}^n$, and a nonlinear function $f: \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$. We assume $f$ is twice continuously differentiable, $f(0, 0) = 0$ holds, and $U$ and $X$ are compact full-dimensional polytopes that contain the origin in their interiors.

The optimal control problem (OCP) treated in the present paper reads

$$\begin{align*}
\min_{U, X} & \quad x(N)^T P x(N) + \sum_{k=0}^{N-1} (x(k)^T Q x(k) + u(k)^T R u(k)) \\
\text{s.t.} & \quad x(k+1) = f(x(k), u(k)), \quad k = 0, \ldots, N-1 \\
& \quad u(k) \in U, \quad k = 0, \ldots, N-1 \\
& \quad x(k) \in X, \quad k = 0, \ldots, N-1 \\
& \quad x(N) \in T,
\end{align*} \tag{2}$$

where $U = (u^T(0), \ldots, u^T(N-1))^T \in \mathbb{R}^{nN}$ and $X = (x^T(0), \ldots, x^T(N))^T \in \mathbb{R}^n$ collect the inputs and states, respectively, the initial state $x(0)$ is given, $Q, P \in \mathbb{R}^{n \times n}$ and $R \in \mathbb{R}^{m \times m}$, with $Q, P, R > 0$ are the usual weighting matrices, $N \in \mathbb{N}$ is the horizon, and $T \subseteq X$ is the terminal set.

We assume the set $U$ is defined by a finite number of halfspaces. Let $q_U$ denote the number of halfspaces that define $U$. Furthermore, let $q$ and $Q = \{1, \ldots, q\}$ refer to the total number of constraints in (2) and their index set, respectively.

By substituting (1) into (2), the OCP (2) can be stated in the form

$$\begin{align*}
\min_{U} & \quad V(x(0), U) \\
\text{s.t.} & \quad G(x(0), U) \leq 0. \tag{3}
\end{align*}$$

Let $\mathcal{F}$ refer to the set of initial states $x(0)$ such that (3) has a solution. For any $x(0) \in \mathcal{F}$, let $U^*(x(0))$ denote an optimal solution to (3). We call $U^*(x(0))$ an optimal control law for $x(0)$.

In MPC, problem (3) is solved in every time step for the current state $x(0)$ and the first input of an optimal control law $U^*(x(0))$ is applied to the system (1). We call the first input of an optimal control law an optimal feedback law

$$u^*(x(0)) = U^*_1(x(0)).$$

The KKT-conditions for (3)

$$\nabla_U \left( V(x(0), U) + \sum_{i=1}^{q} \lambda_i G_i(x(0), U) \right) = 0,$$

$$\lambda_i G_i(x(0), U) = 0, \quad i = 1, \ldots, q, \tag{4}$$

$$G_i(x(0), U) \leq 0, \quad i = 1, \ldots, q,$$

$$\lambda_i \geq 0, \quad i = 1, \ldots, q,$$

where $\lambda \in \mathbb{R}^q$ denotes the Lagrangian multipliers, are a necessary condition for optimality. Therefore, for every $x(0) \in \mathcal{F}$ and optimal solution $U^*(x(0))$, there exists $\lambda$ such that (4) is satisfied.

We introduce the following sets as a preparation to Prop. 1 below.

For any $x(0) \in \mathcal{F}$ and optimal solution $U^*(x(0))$, let the sets $A(x(0))$ and $\mathcal{I}(x(0))$ refer to the optimal active set $A(x(0)) = \{i \in Q | G_i(x(0), U^*(x(0))) = 0\}$ and the corresponding inactive set $\mathcal{I}(x(0)) = Q \setminus A(x(0))$. The sets $W(x(0))$ and $S(x(0))$ refer to the set of weakly active constraints $W(x(0)) = \{i \in A(x(0)) | \lambda_i^+ G_i^+(x(0), U^*(x(0))) = 0\}$ and the set of strongly active constraints $S(x(0)) = A(x(0)) \setminus W(x(0))$, respectively. We often drop the argument $x(0)$ for brevity.

We say an active set $A$ exists for the problem (3) if $A$ appears as an optimal active set for an $x(0) \in \mathcal{F}$.

With the sets just introduced, (4) can be expressed as

$$\nabla_U \left( V(x(0), U) + \sum_{i \in A} \lambda_i G_i(x(0), U) \right) = 0,$$

$$G_A(x(0), U) = 0,$$ 
$$\lambda_{\mathcal{I}, \mathcal{I}^+} = 0,$$ 
$$G_{\mathcal{I}}(x(0), U) < 0,$$ 
$$-\lambda_{\mathcal{I}} < 0. \tag{5}$$

We denote the left-hand side of the equality and inequality constraints in (5) with $F_{eq}^A(U, \lambda, x(0))$ and $F_{ineq}^A(U, \lambda, x(0))$, respectively.

The following proposition follows from Prop. 1 and Lem. 2 in [9].

Proposition 1. Consider an active set $A$ such that a solution to $F_{eq}^A(U, \lambda, x(0)) = 0$ exists. Let the solution be denoted $(U_{sol}^A, \lambda_{sol}^A, x(0)_{sol})$.

If $\frac{\partial F_{eq}^A(U, \lambda, x(0))}{\partial U} |_{sol}$ has full rank, then $F_{eq}^A(U, \lambda, x(0)) = 0$ implicitly defines $U^*(x(0))$ for those $x(0)$ on a region $\mathcal{R}(A)$,

$$\mathcal{R}(A) = \left\{x(0) \in \mathbb{R}^n | F_{ineq}^A(U, \lambda, x(0)) < 0 \right\}.$$ 

It follows from Prop. 1 that an active set that satisfies the conditions in Prop. 1 implicitly defines an optimal control law $U^*$ for all states $x(0) \in \mathcal{R}(A)$.

III. STATE SPACE SETS WITH COMMON OPTIMAL FEEDBACK LAWS

In this section we derive a condition for a subset of an active set to define an optimal feedback law $u^*$. It follows
that every active set with the same subset defines the same optimal feedback law.

We assume the constraints in (2) are ordered such that the input constraints on \( u(0) \) appear first,

\[
\begin{align*}
 u(0) & \in \mathcal{U}, \\
 & \vdots \\
\end{align*}
\]

The order of the remaining constraints is irrelevant. Let the constraint order (6) be preserved in the constraints in (3).

**Lemma 2.** Consider (3) and assume the constraints are ordered as in (6). Recall that according to Sect. II, the set \( \mathcal{U} \) is defined by a finite number of halfspaces. Then, the constraints from (3) can be stated in the form

\[
\begin{pmatrix}
 \tilde{G} u(0) - \tilde{w} \\
 G(q_{r+1} \ldots q_{n}) (x(0),U)
\end{pmatrix} \leq 0,
\]

with \( \tilde{G} \in \mathbb{R}^{n \times m} \) and \( \tilde{w} \in \mathbb{R}^{n_0} \).

**Proof.** The first constraint in (6) is

\[
u(0) \in \mathcal{U}.
\]

By assumption, \( \mathcal{U} \) is a polytope and bounded by \( q,U \) halfspaces. Therefore, the first \( q,U \) rows of \( G(x(0),U) \) are linear and only depend on \( u(0) \). They can be expressed in the form

\[
\tilde{G} u(0) \leq \tilde{w}.
\]

Consider an arbitrary active set \( A \) that exists for (3). Let \( \tilde{A} \) contain the indices that are active in the first rows of (7),

\[
\tilde{A} := A \cap \{1, \ldots, q,U \}. 
\]

With Lem. 2, the constraints that correspond to the active set \( \tilde{A} \) are

\[
\tilde{G} \tilde{A} u(0) - \tilde{w} \tilde{A} = 0. 
\]

If (9) already determines an optimal feedback law \( u(0) \), then this is the optimal feedback law for all active sets \( A' \) that exist for (3) and satisfy

\[
A' \cap \{1, \ldots, q,U \} = \tilde{A}. 
\]

We summarize these active sets in the set

\[
\mathcal{M}(A) = \{ A' \subseteq \mathbb{Q} | A' \cap \{1, \ldots, q,U \} = A \cap \{1, \ldots, q,U \} \}.
\]

It follows that the same optimal feedback law applies to the union of regions

\[
\Gamma(A) = \bigcup_{A' \in \mathcal{M}(A)} \mathcal{R}(A'). 
\]

We point out that this equality only holds for the first entry of the optimal control law \( u(0) \). Of course, the control law \( u(0), \ldots, u(N-1) \) might differ for each region \( \mathcal{R}(A) \) in (10).

**Proposition 3.** Consider an arbitrary active set \( A \) that exists for (3) and assume the constraints are ordered as in (6). Let \( \tilde{A}, \tilde{G} \) and \( \tilde{w} \) be defined as in (8) and (7).

If \( |\tilde{A}| = m \) and if \( \tilde{G}_{\tilde{A}} \) has full rank, then

\[
u^* = \tilde{G}_{\tilde{A}}^{-1} \tilde{w}_{\tilde{A}}
\]

is an optimal feedback law for all \( x \in \Gamma(A) \), where \( \Gamma(A) \) is as in (10).

**Proof.** The constraints can be formulated as in (7), since the assumptions for Lem. 2 hold. All constraints \( i \in A \) hold with equality and \( \tilde{A} \subseteq A \), therefore (9) holds, with \( \tilde{G}_{\tilde{A}} \in \mathbb{R}^{\tilde{A} \times m} \) and \( \tilde{w}_{\tilde{A}} \in \mathbb{R}^{\tilde{A}} \). By assumption, \( \tilde{G}_{\tilde{A}} \) has full rank and is a square matrix, because \( |\tilde{A}| = m \). Hence, the inverse \( \tilde{G}_{\tilde{A}}^{-1} \) exists. Reformulating (9) results in

\[
u(0) = \tilde{G}_{\tilde{A}}^{-1} \tilde{w}_{\tilde{A}}
\]

which provides (11). It follows that under the stated assumptions the active constraints \( \tilde{A} \) define an optimal feedback law \( u(0) \).

For all \( x \in \Gamma(A) \) holds \( \tilde{A} \subseteq A \) (10) which implies (9). Therefore, the optimal feedback law (12) is an optimal feedback law for all \( x \in \Gamma(A) \).}

IV. IMPLEMENTATIONAL ASPECTS

Proposition 3 can be used to identify that regions with common optimal feedback laws also exist for the nonlinear case. The criterion for that is similar to the criterion for the linear case [12, Prop. 1, Lem. 3]. It is therefore an obvious question whether the regional MPC approach for the linear case [12, Prop. 1, Lem. 3] can be transferred to the nonlinear case.

The approach presented in [12, Sect. 4] determines regions in state space with common optimal feedback laws online. In the linear case, the regions are polytopes [1, Sect. 4.1]. The computational effort to calculate the polytope defined by an active set is relatively small and only comprises matrix operations (see e.g. [3, Lem. 2]). In the nonlinear case, the regions are bounded nonlinearly and therefore, the computational effort to calculate the region defined by an active set is in general much higher than in the linear case. Consequently, calculating those regions online is not an option. In order to analyse the benefit of detecting regions with common feedback laws with Prop. 3 in the nonlinear case, we calculate these regions offline. This requires to calculate those regions that are defined by active sets \( A \) such that \( |\tilde{A}| = m \) holds for \( \tilde{A} \) from (8), and \( \tilde{G}_{\tilde{A}} \) has full row rank (see conditions in Prop. 3).

We then form the union of regions defined by active sets with the same subset \( \tilde{A} \) and determine an optimal feedback law \( u^* \) with (11) for each of those unions. Further, each union is underestimated by one or more ellipsoids \( E \) of the form

\[
E = \{ x \in \mathbb{R}^n | (x - x_C)^T E (x - x_C) \leq 1 \},
\]

with \( E \in \mathbb{R}^{n \times n} \), where \( x_C \in \mathbb{R}^n \) denotes the center of the ellipsoid. Finally, all pairs consisting of the ellipsoid and the
corresponding optimal feedback law \((\mathcal{E}, u^*)\) are collected in the set \(S\).

The online part of the approach is described in Alg. 1. It exploits that an optimal feedback law \(u^*\) is known for all states that are an element of an ellipsoid \(\mathcal{E}\). The algorithm tests if the current state is part of one of the ellipsoids (lines 2, 3) and, if so, the optimal feedback law is set to the optimal feedback law that corresponds to that specific ellipsoid (line 4). The OCP is only solved otherwise (line 7).

**Algorithm 1:** NMPC using predefined regions with corresponding optimal feedback laws

1. **Input:** current state \(x(0)\), \(S\) (determined offline)
2. **for every** \((\mathcal{E}_i, u_i^*) \in S\) **do**
3.   **if** \(x(0) \in \mathcal{E}_i\) **then**
4.     set \(u^*(x(0)) = u_i^*\)
5.     break
6. **if** \(u^*(x(0))\) is undefined **then**
7.     solve (3) for \(u^*(x(0))\)
8. **Output:** \(u^*(x(0))\)

The fact that the united regions were underestimated by ellipsoids reduces the performance of the proposed algorithm because fewer states are identified to be part of a region with a known optimal feedback law \(u^*\). The underestimation is necessary to keep the computational effort for the membership test in line 2 low, where solving simple inequalities is sufficient to test whether a state is part of an ellipsoid (13). This is essential because, in case none of the ellipsoids applies, we still solve the OCP (line 7). Note that the underestimation does not affect the optimal feedback law which is the output of Alg. 1.

**V. Example**

We illustrate Prop. 3 and the approach presented in Sect. IV with the following example.

**Example 4.** Consider the system [16, Sect. VI A, C 2]

\[
x(k + 1) = \left( \begin{array}{c} x_1(k) + u(k) \\ bx_2(k) + u(k)^2 \\ \end{array} \right),
\]

with \(|u(k)| \leq 1\), \(\mathcal{T} = \{x(k) \in \mathbb{R}^2 | x(k)^T P x(k) \leq \alpha\}\), \(Q = I^2, R = 1\) and \(N = 3\), where \(I\) denotes the identity matrix, \(b = 0.9\), \(P = \begin{pmatrix} 4 & 0 \\ 0 & 10.53 \end{pmatrix}\) and \(\alpha = 1.1\).

The set \(\mathcal{F}\) for Example 4 is shown in Fig. 1. It was generated by determining a global optimal solution to (3) for all states \(x(0)\) on a cartesian grid in the range \(x_{1,2} = [-6, 6]\) with a step size of 0.05. For all states such that an optimal solution exists, an optimal active set was calculated. In total there are 26 different optimal active sets. Fig. 1 illustrates all states such that a solution to (3) exists, states with the same optimal active set are illustrated in the same color.

For Example 4 the sets \(\mathcal{F}\) such that the conditions in Prop. 3 hold result \(\mathcal{F} = \{1\}\) and \(\mathcal{F} = \{2\}\). The corresponding optimal feedback laws with (11) are \(u^* = -1\) and \(u^* = 1\), respectively. In this example, seven different optimal active sets in the solution contain the subset \(\{1\}\) and seven contain the subset \(\{2\}\). Fig. 2 shows all states such that the optimal active set contains the subsets \(\{1\}\) and \(\{2\}\) by blue and red color, respectively, states such that the optimal active set does not contain any of these subsets are shown in black. With Prop. 3, all optimal active sets that contain the subsets \(\{1\}\) and \(\{2\}\) define the optimal feedback laws \(u^* = -1\) and \(u* = 1\), respectively.

The red and blue regions in Fig. 2 are underestimated by hand with two ellipsoids each. Fig. 3 illustrates a sample closed-loop trajectory. For all states that are included in an ellipsoid (white circles), an optimal feedback law is known and no OCP is solved (lines 3, 4 in Alg. 1). For all states that are not part of an ellipsoid (light red triangles), an OCP is solved to determine an optimal feedback law (lines 6, 7 in
Alg. 1). Note that the third state of the trajectory is part of the blue region where an optimal feedback law is known, but it is not part of an ellipsoid. Therefore, an OCP is solved for this state.

Fig. 3. Detail of Fig. 2 with closed-loop trajectory that results for $x(0) = (3, 4)^T$ for Example 4. Trajectory states that are part of an ellipsoid are marked with white circles, all other states are marked with light red triangles.

To analyze the computational effort of the approach proposed in this paper, we calculate the optimal feedback law $u^*$ for 5000 randomly chosen states $x(0) \in F \setminus T$ for Example 4 with two different methods: first, by solving an OCP as is done in classical MPC for the current state, and second, by using the approach proposed in this paper (Alg. 1). Note that 46.3% of the 5000 states are included in an ellipsoid. Both methods are implemented on a 2,7 GHz Dual-Core Intel Core i5 using MATLAB. All OCPs are solved to local optimality as usual.

The calculation of an OCP is avoided for more than 40% of the initial states, and the average computational time to compute the optimal feedback law is reduced by more than 80%. Drawbacks of the approach presented in this paper are its required offline effort and offline memory demand. The approach requires to calculate, unite, and underestimate regions with common optimal feedback laws offline. This limits the approach to small problems.

VI. CONCLUSION

We established a simple criterion for a subset of an active set to define an optimal feedback law. Optimal active sets that contain such a subset define the same optimal feedback law. We used the criterion and proposed an algorithm to reduce the number of OCPs that are solved. The reduction of the computational online effort was illustrated with an example: The calculation of an OCP is avoided for more than 40% of the initial states, and the average computational time to compute the optimal feedback law is reduced by more than 80%.

REFERENCES

[1] A. Bemporad, M. Morari, V. Dua, and E. N. Pistikopoulos, “The explicit linear quadratic regulator for constrained systems,” Automatica, vol. 38, pp. 3–20, 2002.
[2] M. M. Seron, G. C. Goodwin, and J. A. De Doná, “Characterisation of receding horizon control for constrained linear systems,” Asian Journal of Control, vol. 5, pp. 271–286, 2003.
[3] M. Jost, M. Schulze Darup, and M. Mönigmann, “Optimal and sub-optimal event-triggering in linear model predictive control,” in Proc. of the European Control Conference (ECC), 2015, pp. 1153–1158.
[4] K. Königs and M. Mönigmann, “Regional MPC with active set updates,” IFAC-PapersOnLine, vol. 50, pp. 11859–11864, 2017.
[5] K. Königs and M. Mönigmann, “Regional MPC with nonlinearly bounded regions of validity,” in Proc. of the European Control Conference (ECC), 2018, pp. 294–299.
[6] P. S. Berner and M. Mönigmann, “Event-based networked model predictive control with overlocked local nodes,” in Proc. of the European Control Conference (ECC), 2018, pp. 306–311.
[7] P. Tøndel, T. A. Johansen, and A. Bemporad, “An algorithm for multi-parametric quadratic programming and explicit MPC solutions,” Automatica, vol. 39, pp. 489–497, 2003.
[8] T. A. Johansen, “On multi-parametric nonlinear programming and explicit nonlinear model predictive control,” in Proc. of the 41st IEEE Conference on Decision and Control (CDC), 2002, pp. 2768–2773.
[9] M. Mönigmann, J. Otten, and M. Jost, “Nonlinear MPC defines implicit regional optimal control laws,” IFAC-PapersOnLine, vol. 48, no. 23, pp. 142–147, 2015.
[10] L. F. Domínguez and E. N. Pistikopoulos, “A novel mp-NLP algorithm for explicit/multi-parametric NMPC,” IFAC Proceedings Volumes, vol. 43, no. 14, pp. 539–544, 2010.
[11] L. F. Domínguez and E. N. Pistikopoulos, “Recent advances in explicit multi-parametric nonlinear model predictive control,” Industrial & Engineering Chemistry Research, vol. 50, no. 2, pp. 609–619, 2011.
[12] K. Königs and M. Mönigmann, “Accelerating MPC by online detection of state space sets with common optimal feedback laws,” arXiv preprint arXiv:2009.08764, 2020.
[13] T. Geyer, F. D. Torrisi, and M. Morari, “Optimal complexity reduction of polyhedral piecewise affine systems,” Automatica, vol. 44, pp. 1728–1740, 2008.
[14] M. Kvasnica and M. Fikar, “Clipping-based complexity reduction in explicit MPC,” IEEE Transactions on Automatic Control, vol. 57, pp. 1878–1883, 2012.
[15] M. Kvasnica, J. Hedlík, I. Rauová, and M. Fikar, “Complexity reduction of explicit model predictive control via separation,” Automatica, vol. 49, pp. 1776–1781, 2013.
[16] G. Pannocchia, J. B. Rawlings, and S. J. Wright, “Inherently robust suboptimal nonlinear MPC: theory and application,” in 5th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC), 2011, pp. 3398–3403.