Compact Lattice QED with Staggered Fermions and Chiral Symmetry Breaking†

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Abstract

Different formulations of the 4d compact lattice QED with staggered fermions (standard Wilson and modified by suppression of lattice artifacts) are investigated by Monte Carlo simulations within the quenched approximation. We show that after suppressing lattice artifacts the system undergoes a phase transition from the Coulomb phase into a presumably weakly chirally broken phase only at (unphysical) negative $\beta$–values.

i) The lattice formulation of any field theory is not unique. On a physical ground one has to decide which version is realized in nature, if different lattice versions do not belong to the same universality class.

At present lattice QED is mostly discussed within a non–compact realization of the gauge part in the lattice action [1, 2]. However, considering QED as arising from a subgroup of a non–abelian (e.g. grand unified) gauge theory we are led to a compact description of the local gauge symmetry.

The aim of this note is to shed some light on the mechanism of spontaneous chiral symmetry breaking (SCSB) in lattice gauge theories. Compact lattice QED

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with staggered fermions serves very well for this purpose. It is comparatively simple, and in the limit of vanishing fermion mass the classical action is chirally symmetric. In case of the standard Wilson gauge action the quantized theory undergoes a phase transition, such that in the zero-mass limit the chiral condensate $\langle \bar{\chi}\chi \rangle$ becomes zero within the Coulomb phase (i.e. at $\beta \equiv 1/e_{\text{bare}}^2 > \beta_0 \approx 1.0$), and non–zero in the confinement phase ($\beta < \beta_0$) [3].

The confinement phase of compact QED does not seem to be realized in nature. It is related to the existence and condensation of De Grand–Toussaint monopoles [4] which we can understand to be artifacts of the realization of the lattice discretization.

In case of the pure gauge theory it has been shown [5, 6, 7] that a complete suppression of these monopoles removes the phase transition at $\beta_0$. One is left with a unique Coulomb phase (at least at positive $\beta$’s) [6]. A further modification of the theory by suppressing additionally negative plaquettes was shown not to change the spectrum of gauge degrees of freedom but to improve the overlap of the plaquette operator with the lowest state, at least, in the case of the photon quantum numbers [6].

A comparative study of the chiral properties of the standard and of the modified theories with compact gauge fields coupled to staggered fermions is the aim of this investigation. Within the quenched approximation we are going to discuss the phase structure of these theories. We will show that the modified theories undergo phase transitions at negative (i.e. unphysical) $\beta$–values only.

ii) We consider compact $U(1)$ gauge fields coupled to staggered fermions defined on a lattice of the size $L^3 \cdot L_t$ (different time and spatial extensions are only introduced for the computation of correlators). The gauge fields are treated with the Wilson action modified by suppressing lattice artifacts, such that the gauge action in general reads as follows

$$S_G(U_l) = \beta \cdot \sum_P (1 - \cos \theta_P^c) + \lambda_K \cdot \sum_c |K_c| + \lambda_P \cdot \sum_P (1 - \text{sign} (\cos \theta_P)) ,$$  \hspace{0.5cm} (1)

where $U_l \equiv U_{x\mu} = \exp(i\theta_{x\mu})$, $\theta_{x\mu} \in (-\pi, \pi]$ are the field variables defined on the links $l \equiv (x, \mu)$. The plaquette angles are $\theta_P \equiv \theta_{x;\mu\nu} = \theta_{x;\mu} + \theta_{x+\hat{\mu};\nu} - \theta_{x+\hat{\nu};\mu} - \theta_{x;\nu}$.

The second and the third term in the action have been added in order to suppress monopoles and negative plaquette values, respectively, as lattice artifacts. The monopole currents $K_c$ are defined as follows. The plaquette angle can be splitted

$$\theta_P^c = \bar{\theta}_P + 2\pi n_P, \quad -\pi < \bar{\theta}_P \leq \pi, \quad n_P = 0, \pm 1, \pm 2,$$  \hspace{0.5cm} (2)

where $\bar{\theta}_P$ describes the (gauge-invariant) ‘electromagnetic’ flux through the plaquette and $n_P$ is the number of ‘Dirac’ strings passing through it. The net number of Dirac strings passing through the surface of an elementary 3$d$ cube $c_{x,\mu}$ and taken with the correct mutual orientation determines the monopole charge within
this cube, i.e., a monopole current along the corresponding dual link labeled by \((x, \mu)\)

\[
K_c \equiv K_{x,\mu} = \frac{1}{2\pi} \sum_P \bar{\theta}_P = \frac{1}{2\pi} \sum_P (\theta_P - 2\pi n_P) = -\sum_P n_P .
\]  (3)

In the action \(\Pi\) the parameters \(\lambda_K\) and \(\lambda_P\) play the role of chemical potentials for monopoles and negative plaquettes, respectively. In the present investigations we have chosen the following cases \((\lambda_K, \lambda_P) = (0, 0)\) – standard Wilson action \(WA\), \((\infty, 0)\) – modified action \(MA1\) with total monopole suppression, and \((\infty, \infty)\) – modified action \(MA2\) with simultaneous total monopole and negative plaquette suppression. In practice we realize this total suppression via Kronecker \(\delta\)'s in the measure of the functional integral. They are easily locally taken into account in the heat bath or Metropolis Monte Carlo updating procedure.

The staggered fermion part of the action looks as follows.

\[
S_F(U_l, \bar{\chi}, \chi) = -\bar{\chi}(x)(\mathcal{M} + m)_{xy} \chi(y)
\]  (4)

\[
\mathcal{M}_{xy} = -\frac{1}{2} \sum_\mu \eta_\mu(x)[U_{x\mu} \delta_{y,x+\hat{\mu}} - U_{y\mu}^\dagger \delta_{y,x-\hat{\mu}}]
\]  (5)

where \(\eta_\mu(x) = (-1)^{x_1 + x_2 + \ldots + x_\mu - 1}\), \(\eta_1(x) = 1\), and \(\chi, \bar{\chi}\) represent one-component Grassmanian variables.

Gauge invariance requires the gauge fields to couple to fermions via the phase factors \(U_{x\mu}\), i.e., in a compact way. This holds independently of the compact or non–compact realization of the gauge field action \(S_G\). Therefore, one should keep in mind that lattice QED within the standard compact and the non–compact case agree at strong coupling \(\beta = 0\). On the contrary, the constraints due to the modification terms in \(MA1\) and \(MA2\) change the strong coupling behaviour. Therefore, from the beginning at strong coupling we cannot a priori expect qualitative agreement with non–compact QED.

Here, we treat the fermions within the quenched approximation. We expect that the chiral properties of the full theory will be qualitatively correctly described.

First of all we are going to discuss bulk properties, i.e., we measure the average plaquette action \(\bar{E}_P \equiv \langle 1 - \cos \theta_P \rangle\) as well as the chiral condensate \(\langle \bar{\chi}\chi \rangle = \frac{1}{V} \langle \text{Tr} (\mathcal{M} + m)^{-1} \rangle\). The latter is computed applying the conjugate gradient algorithm combined with the noisy estimator method.

Moreover, in order to distinguish the Coulomb phase for the modified actions even in the negative \(\beta\)-range we calculated the \(J^{PC} = 1^{-+}\) plaquette–plaquette correlator, which for non–vanishing momenta \(p_i = 2\pi k_i/L_s\) with \(k_i = \pm 1, \pm 2, \ldots\) contains the photon–state \((1^{-+})\) as the leading contribution. The photon correlator \(\Gamma(\tau)\) is

\[
\Gamma(\tau) = \langle \Phi^*_p(t+\tau) \cdot \Phi_p(t) \rangle_c ,
\]  (6)
where the operators $\Phi_{\vec{p}}(t)$ are defined as Fourier transforms of $\sin \theta P_{6,8}$. This correlator is expected to behave as

$$\frac{\Gamma(\tau)}{\Gamma(0)} = A \cdot \left[ \exp\left(-\tau \cdot E(\vec{p})\right) + \exp\left(-(L_t - \tau) \cdot E(\vec{p})\right) \right] + \ldots, \quad 0 \leq A \leq 1 \quad (7)$$

with the energy $E(\vec{p})$ in units of the lattice spacing given by the lattice dispersion relation

$$\sinh^2 \frac{E}{2} = \sum_{i=1}^{3} \sin^2 \frac{p_i}{2} \quad (8)$$

for a zero–mass excitation.

iii) First of all we have studied the phase structure of the two modified theories (MA1 and MA2) in comparison with the standard theory (WA). The simplest way is to consider the average plaquette $E_P$ as a function of $\beta$. We extend the range to be investigated to negative (i.e., a priori unphysical) $\beta$ values.

For the standard theory the plaquette action shows two transition points at $\beta \simeq \pm 1.0$, related to the symmetry $\beta \rightarrow -\beta$ and $U_{x\mu} \rightarrow \eta_{\mu}(x) \cdot U_{x\mu}$ (full circles in Fig.1a). In the same figure we have plotted (open squares) the result for the modified action MA1. Obviously the transition at positive $\beta$ is removed leaving us only with a rather strong discontinuity at $\beta \simeq -0.7$. For the further modification by suppressing additionally negative plaquettes (MA2) this discontinuity is shifted even further down to a $\beta_c \simeq -1.68$ (full squares in Fig.1a). There we have seen a clear bi–stable behaviour indicating the existence of a first order phase transition (Fig.1b).

The interesting lesson from these calculations is that in the strong coupling region (as well as at negative $\beta$’s) there is a very strong influence of the lattice artifacts considered.

We can ask, whether the negative $\beta$–range $\beta_c < \beta \leq 0$ can be interpreted as an (analytical) continuation of the Coulomb phase at positive $\beta$. In order to answer this question we computed the photon propagator as explained before. The results are shown in Fig.2. For the case MA2 we have plotted $\Gamma(\tau)$ for $\beta = 0.9$ and $\beta = -1.1$, for comparison, at lowest non–vanishing momentum. The theoretically expected zero–mass curve is also shown (broken line). All the data points are in perfect agreement with a zero–mass photon.

In Fig.3 we have plotted the chiral condensate $\langle \bar{\chi} \chi \rangle$ for fixed fermion mass $m = 0.04$ (in units of the lattice spacing) as a function of $\beta$ for the standard action WA and for the modified one MA2. Whereas for WA $\langle \bar{\chi} \chi \rangle$ shows clearly chiral symmetry breaking at $\beta \simeq 1.0$ (as well as a sign of the symmetric transition at $\beta \simeq -1.0$) it stays small and approximately constant for the modified action down to $\beta_c \simeq -1.68$. Thus, it does not differ from the behaviour at $\beta > 1.0$, i.e., in the Coulomb phase. However, we observed fluctuations of the chiral order
parameter becoming stronger the nearer we came to the phase transition point \( \beta_c \). At \( \beta < \beta_c \) the fluctuations are very strong making it hard to measure \( \langle \bar{\chi}\chi \rangle \) in the zero–mass limit. The rise of the chiral condensate in this range, nevertheless, could indicate the possibility of a weak spontaneous chiral symmetry breaking.

At several \( \beta \) values we have studied the mass dependence of the chiral condensate in the range \( m = 0.01 \ldots 0.05 \) for different lattice sizes up to \( 16^4 \) (see Table). As an example in Fig.4 we have plotted the mass dependence for \( \beta = 0 \). The finite size effects turn out to be small. The extrapolation down to \( m = 0 \) provides a chiral condensate value compatible with zero. An analogous behaviour has been seen at \( \beta = 0.5, 1.1 \) and \(-1.1\).

iv) Our main conclusion is the following. At real bare coupling, i.e., for positive \( \beta = 1/e_{bare}^2 \) the modified theory MA2 (at least in quenched approximation) is fully in the Coulomb phase, and there is no indication for a phase transition and for spontaneous chiral symmetry breaking. Therefore, we conclude that SCRB in the standard theory is due to lattice artifacts. This is similar to the situation of compact QED with Wilson fermions \([3, 11]\).

One can try an analytical continuation to negative \( \beta \) values. We have seen the Coulomb phase to be extended down to a certain value \( \beta_c < 0 \) depending on the concrete realization of the modified gauge action (called MA1 or MA2). There, we observed a phase transition presumably of first order. The phase beyond \( \beta_c \) seems to exhibit weak chiral symmetry breaking. But, more thorough investigations with very high statistics are required before drawing any conclusions in this respect.

The question arises if the inclusion of dynamical fermions could change the picture. However, presumably one should not expect qualitative changes as we know from studies of both compact and non–compact QED cases.

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Table

The chiral condensate $\langle \bar{\chi} \chi \rangle$ for MA2 at different values of $\beta$, $m$ and for different lattice sizes. The given errors are purely statistical.

| $\beta$ | $L$ | $m = 0.01$ | $m = 0.02$ | $m = 0.03$ | $m = 0.04$ | $m = 0.05$ |
|---------|-----|----------|----------|----------|----------|----------|
| 0.0     | 8   | .0099(6) | .0198(8) | .0299(7) | .0378(6) | .04601(8) |
| 0.0     | 12  | .00956(5)| .0213(5) | .0286(2) | .0380(4) | .0469(2)  |
| 0.0     | 16  | .00934(2)| .01923(9)| .0288(1) | .03748(7)| .04644(6)|
| 0.5     | 8   | .00854(2)| .01705(4)| .02554(3)| .03432(8)| .04264(6) |
| 0.5     | 12  | .0090(1) | .01739(9)| .0277(4) | .0360(3) | .0430(2)  |
| 0.5     | 16  | .00871(4)| .0181(2) | .0262(1) | .0345(1) | .0450(3)  |
| 1.1     | 8   | .00784(4)| .0159(1) | .0239(2) | .0328(3) | .0404(6)  |
| 1.1     | 12  | .008124(6)| .0169(3) | .0257(4) | .0316(3) | .0394(2)  |
| 1.1     | 16  | .0085(2) | .0171(2) | .0245(2) | .0331(2) | .0417(2)  |
| -1.1    | 12  | .01260(4)| .02376(3)| .03595(4)| .04651(4)| .05967(6) |
| -1.1    | 16  | .01202(1)| .02384(2)| .03556(4)| .04729(4)| .05879(4) |
| -2.0    | 12  | .0343(2) | –        | .0852(3) | –        | .1252(2)  |
| -2.0    | 16  | .02822(4)| .0585(1) | .0807(1) | –        | .1236(1)  |

Figure Captions

Fig. 1a: Plaquette energy $E_P$ as a function of $\beta$ for standard Wilson action WA and modified actions MA1, MA2. Lattice sizes are $6^4$ for WA and MA2 and $4^4$ for MA1.

Fig.1b: Time history for the plaquette energy $E_P$ at the transition point $\beta_c = -1.68$ for the modified action MA2. The lattice size is $6^4$.

Fig.2: The photon correlator $\Gamma(\tau)$ measured at $\beta = 0.9$ and $-1.1$ with lowest non-vanishing momentum on a $12\cdot 6^3$ - lattice for MA2.

Fig.3: Chiral order parameter $\langle \bar{\chi} \chi \rangle$ as a function of $\beta$ for fixed fermion mass $m = 0.04$ for WA and MA2, respectively. The lattice size is $6^4$.

Fig.4: Mass dependence of $\langle \bar{\chi} \chi \rangle$ for the modified action MA2 at $\beta = 0.$ and different lattice sizes $(8^4, 12^4, 16^4)$. 
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FIG. 4

\[ \langle \chi \chi \rangle \]
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