Spin–orbit (SO) coupling leads to numerous phenomena in electron systems. Artificial SO coupling in ultracold neutral atoms provides the opportunity to study such phenomena in bosonic systems, which exhibit superfluidity and various symmetry-breaking condensate phases. In general, a richer structure of symmetry breaking results in a nontrivial finite-temperature phase diagram, but the thermodynamics of the SO-coupled Bose gas at finite temperature remains unknown both in theory and experiment. Here we experimentally determine a new finite-temperature phase transition that is consistent with the transition between the stripe ordered phase and the magnetized phase. We also observe that the magnetic phase and the Bose condensate transitions occur simultaneously as temperature decreases. We determine the entire finite-temperature phase diagram of the SO-coupled Bose gas, thus illustrating the power of quantum simulation.

In this work we generate SO coupling in $^{87}$Rb Bose gases using two counter-propagating laser beams, as described in previous works. In this set-up only the motion along the spatial direction of the Raman laser (the $x$-direction) is coupled to spin, and the single-particle Hamiltonian along the $x$-direction is given by ($\hbar = 1$)

$$\hat{H}_x = \frac{(k_z - k_x \sigma_z)^2}{2m} + \frac{\delta}{2} \sigma_x + \frac{\Omega}{2} \sigma_z$$

(1)

where $m$ is the mass of the atoms, $k_z$ is the recoil momentum, $\Omega$ is the Raman coupling strength and $\delta$ is the Raman detuning, $\sigma_z$ and $\sigma_x$ are the Pauli matrices denoting the spin of the particle. We focus on the case $\delta = 0$, where the system has an additional $Z_2$ symmetry ($k_z \rightarrow -k_z$, and $\sigma_x \rightarrow -\sigma_x$ simultaneously). The single-particle dispersion is shown in Fig. 1a. To motivate our study of finite-temperature physics, we shall first summarize what is known at zero temperature.

For $\Omega < \Omega_2 \approx 4E_r$ ($E_r = k_z^2/(2m)$), there are two degenerate single-particle minima, denoted by $\pm k_{\text{min}}$, and their wavefunctions are represented by $\psi_1$ and $\psi_2$, respectively; these two degenerate states have opposite magnetization. As a result of this degeneracy, the wavefunction of Bose condensate should be determined by the interactions in this regime. Theoretical results have shown that for the interaction parameters of $^{87}$Rb atoms, the condensate wavefunction is in a superposition state $|\psi_1 + \psi_2\rangle/\sqrt{2}$ for $\Omega < \Omega_2 \approx 0.2E_r$ and bosons condense either into $\psi_1$ or into $\psi_2$ for $\Omega_2 < \Omega < \Omega_1$. For the former, the condensate exhibits periodic density stripe order and the spatial translational symmetry is spontaneously broken. For the latter, spatial translational symmetry is not broken but the $Z_2$ symmetry is, and the condensate is therefore magnetic. Experimentally, although the stripe order has not been directly imaged so far, a miscible to immiscible transition has been observed at $\Omega \approx \Omega_1$ (ref. 3). Finally, when $\Omega > \Omega_2$, the single-particle dispersion has only one single minimum at zero momentum and its wavefunction, denoted by $\psi_1$, exhibits zero magnetization. A divergent spin susceptibility has been observed at $\Omega = \Omega_2$ (ref. 6).

In short, as shown in Fig. 1a, at zero temperature the system will undergo two successive magnetic phase transitions as $\Omega$ increases, first from the non-magnetic stripe (ST) phase to the magnetized (MG) phase, and then from the MG phase to the non-magnetic zero-momentum (NM) condensate. We remark that the ST phase at $\Omega < \Omega_1$ and the MG phase at $\Omega > \Omega_1$ are two fundamentally different phases, as they exhibit very different behaviours in terms of magnetization, symmetry breaking and low-energy excitation spectra. At zero temperature, the phase boundary between them is determined by the interaction energy only. Although the finite-temperature phase diagram should contain richer physics owing to the interplay between interaction effects and thermal effects, it has

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1Shanghai Branch, Hefei National Laboratory for Physical Sciences at Microscale and Department of Modern Physics, University of Science and Technology of China, Shanghai 201315, China. 2Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei, Anhui 230026, China. 3Institute for Advanced Study, Tsinghua University, Beijing 100084, China. © 2014 Macmillan Publishers Limited. All rights reserved.
The density of each component is half that of the single-component gas, thus, if one ignores interaction effects, the ratio $T_c/T_c^\infty$ will drop to about $(1/2)^{1/3} \approx 0.79$. When $\Omega > \Omega_2$, the dispersion has only one single minimum, the spin of all atoms will be polarized along the $x$-direction and the system will essentially become a single component with an effective mass $m' = m\Omega/\sqrt{(\Omega - \Omega_2)}$. Thus at large $\Omega_2$, $|T_c/T_c^\infty|$ increases towards unity. The red and blue curves in Fig. 2c represent the theoretical calculations of $T_c/T_c^\infty$ for atom numbers $N=1.0 \times 10^5$ and $1.0 \times 10^6$, respectively, with the interaction effects taken into account.

Furthermore, we can study the interaction shift of $T_c$. Experimentally, for each measurement we can determine $T_c$ and the total number of atoms $N$. With the atom number $N$, trap parameter $\omega$ and SO-coupling parameters $k_0$, $\Omega_0$, the non-interacting critical temperature with trap $T_c^\infty$ can be calculated and then $\Delta T_c = T_c - T_c^\infty$ can be deduced. The experimental results of $\Delta T_c/T_c$ are shown in Fig. 2d. $\Delta T_c$ is always negative because the repulsive interactions decrease the density. When $\Omega < \Omega_2$, we find that $|\Delta T_c/T_c|$ increases from about 0.10 at $\Omega \approx 0$ to about 0.14 at $\Omega \approx \Omega_2$. Moreover, when $\Omega > \Omega_2$, we clearly find $|\Delta T_c/T_c|$ decreases as $\Omega$ increases. The error bar of the data is transferred from the $T_c$ and the calculation of $T_c^\infty$. For comparison, we plot two theoretical curves with different total numbers of atoms and trap frequencies, because in the experiment the total atom number and the trap frequency vary as $\Omega$. These theoretical curves include both the interaction effect using the Hartree–Fock approximation and the trap effect using the local-density approximation (details of the calculation are given in Ref. 18 and in the Supplementary Methods). We find reasonable agreement between theory and experiment. This result shows that the interaction shift of the transition temperature reaches a maximum around $\Omega_2$, where the single-particle dispersion changes from a double minimum to a single minimum and the low-energy density-of-states is maximized.

Next, we turn to a discussion of the magnetic properties of the SO-coupled Bose gas. The magnetization of the gas is defined as

$$M_i = \frac{N_{i+} - N_{i-}}{N_{i+} + N_{i-}}$$

(2)

where $N_{i+}$ and $N_{i-}$ represent the atom numbers in the $|\uparrow\rangle$ ($|m_i = -1\rangle$) and $|\downarrow\rangle$ ($|m_i = 0\rangle$) states, respectively. The index $i=0$ ($i=\text{th}$) represents the condensate atoms (the thermal atoms).
Experimentally, we repeat our measurements thousands of times and record the probability of occurrence of different magnetizations of the atoms to obtain histograms for statistical analysis.

Now we focus on the regime with Raman coupling strength \( \Omega < \Omega_c \), where the ground state at zero temperature is predicted to be a ST condensate. However, so far there is no direct experimental evidence of phase coherence between \( \psi_L \) and \( \psi_R \) or a direct image of stripe order in the ST phase. In this work, we shall distinguish between the ST phase and the MG phase by their magnetic properties. Let us first consider a uniform system without domain walls. When the single-particle spectrum has two minima for \( \Omega < \Omega_c \), \( \psi_L \) and \( \psi_R \) have opposite magnetizations \( M_0 = \pm \sqrt{16 - 2^2/\Omega_0^2} \). Therefore, for the MG condensate, the many-body wavefunction for the condensate is either \( (1/\sqrt{N_0})(\hat{a}_L^\dagger)^{N_0}(0) \) or \( (1/\sqrt{N_0})(\hat{a}_R^\dagger)^{N_0}(0) \), where \( N_0 \) denotes the total number of atoms in the condensate. Thus, in the histogram of condensate magnetization, one expects two peaks at \( M_0 = \pm \sqrt{16 - 2^2/\Omega_0^2} \). However, in the presence of magnetic domains, where both spin up and spin down appear simultaneously while phase separated, the peaks will be smeared and the distribution could be flat. For the ST phase, the many-body wavefunction for the condensate is given by

\[
\frac{1}{\sqrt{N_0!}} \left( \frac{\hat{a}_L^\dagger + \hat{a}_R^\dagger}{\sqrt{2}} \right)^{N_0} |0\rangle = \frac{1}{\sqrt{N_0!}} \sum_m c_m (\hat{a}_L^\dagger)^m (\hat{a}_R^\dagger)^{-m} |0\rangle
\]

with \( c_m = ((N_0/2 - m)!(N_0/2 + m)!)/N_0! \). Each measurement projects the coherent state into a Fock state \( (\hat{a}_L^\dagger)^{N_0/2 - m} (\hat{a}_R^\dagger)^{N_0/2 + m} |0\rangle \) with a probability \( (c_m)^2/N_0! \), with a magnetization \( M_0 = (2m/N_0) \sqrt{16 - 2^2/\Omega_0^2}. \)

Because \( c_m \) is a smooth function peaked at \( m = 0 \), the histogram of the condensate magnetization should also show a distribution centred at \( M_0 = 0 \). Across a transition between the ST phase and the MG phase, one expects that \( \sqrt{\langle M_0^2 \rangle} \) will show a kink at the transition.

In this regime we choose three different Raman coupling strengths \( \Omega = 0.10E_r, 0.15E_r \) and \( 0.18E_r \) and show the histograms of the condensate magnetization for various temperatures below \( T_c \). Just below \( T_c \), when the condensate fraction is very small (\( 0 < f < 0.02 \)), it is very clear from the leftmost plots in Fig. 3a that the histogram shows two sharp peaks around \( M_0 \approx \pm 1 \). This evidence for spontaneous \( Z_2 \)-symmetry breaking of the magnetization strongly supports the condensate not being in the ST phase but rather in the MG phase when the condensate first appears. As the temperature is further reduced, we find that the peaks at \( M_0 \approx \pm 1 \) gradually decrease, and a broad peak centred at \( M_0 = 0 \) gradually develops. Finally, at low temperature, when the condensate fraction \( f > 0.25 \), as shown in the rightmost plots in Fig. 3a, the peaks at \( M_0 \approx \pm 1 \) completely disappear and a Gaussian-like peak develops around \( M_0 = 0 \), which is consistent with the ST state at low temperature, as analysed above. We also vary the evaporation cooling time and the holding time and find the low-temperature behaviour is unchanged. Comparing the three different
Raman coupling strengths, we find that the closer the Raman coupling is to $\Omega_1$, the lower is the transition temperature from the MG phase to the ST phase. Roughly speaking, the transition from the MG phase to the ST phase takes place when the condensate fraction $f \approx 0.05$ for $\Omega = 0.10 \Omega_1$, $f \approx 0.10$ for $\Omega = 0.15 \Omega_1$, and $f \approx 0.20$ for $\Omega = 0.18 \Omega_1$, as indicated by dashed lines in Fig. 3a. This supports the finite-temperature phase diagram being of the type shown in Fig. 1b1—namely, that as the temperature increases the ST phase will first turn into the MG phase before becoming the normal state. This is reminiscent of the transition from the superfluid H-phase to the superfluid A-phase as the temperature increases in Helium-3 (ref. 26). A further analogy is the magnetic transition in undoped iron pnictide$^{22-29}$, in which the low-temperature phase is a spin-density wave that breaks translational symmetry, which as the temperature increases, undergoes a transition to a spin nematic state that restores transitional symmetry but breaks a discrete symmetry, finally becoming a paramagnetic state at higher temperatures.

To determine the phase boundary more quantitatively, we plot $\sqrt{\langle M^2 \rangle}$ as a function of $f$ in Fig. 3b. The plots all exhibit a kink which we use to determine the location of the phase boundary. Thus our measurements give three points in the finite-temperature phase boundary between the ST and MG phases, corresponding to the condensate fraction (measured temperature) of $f = 0.06 \pm 0.02$ ($T = 126 \pm 12$ nK) at $\Omega = 0.10 \Omega_1$, $f = 0.09 \pm 0.02$ ($T = 116 \pm 10$ nK) at $\Omega = 0.15 \Omega_1$, and $f = 0.22 \pm 0.09$ ($T = 62 \pm 27$ nK) at $\Omega = 0.18 \Omega_1$, respectively.

Furthermore, we also carried out magnetization histogram analysis at the lowest temperature of $T < 20$ nK with a condensate fraction $f > 0.9$. Zero-temperature theoretical calculations have determined the ST-MG transition at $\Omega \approx \Omega_0 = 0.2E_r$ (refs 3,12) and experimental evidence of this transition has also been obtained at the lowest temperature$^7$. As shown in Fig. 4, the magnetization distribution for $\Omega < \Omega_1$ shows a single peak around $M_0 = 0$, whereas the central peak gradually becomes flat and two peaks at $M_0 \approx \pm 1$ start to emerge when $\Omega > \Omega_1$. The plot of $\sqrt{\langle M^2 \rangle}$ versus $\Omega$ in Fig. 4b exhibits a kink at $\Omega = 0.20 \pm 0.02 \Omega_1$, fully consistent with the known zero-temperature transition. This provides a benchmark for our finite-temperature phase boundary.

Thus, our measurements support the scenario that the phase boundary bends towards the ST phase side, although a more complicated scenario can not be completely ruled out. Here we present a simple and quite general symmetry argument to explain why the MG phase is more favourable than the ST phase as the temperature increases. Because the ST phase breaks both phase symmetry and spatial translational symmetry along the $\sigma$-direction, there will be two linear Goldstone modes located at $\pm k_{\text{min}}$ (ref. 30), as schematically shown in the insets of Fig. 5. In contrast, the MG phase breaks only one continuous phase symmetry, and there is only one linear Goldstone mode. For instance, if atoms condense in $\psi_\pi$ located at $k_{\text{min}}$, the spectrum behaves linearly around $k_{\text{min}}$, while remaining quadratic around $-k_{\text{min}}$ with a vanishingly small roton gap in the critical regime$^{21}$, as shown schematically in the insets of Fig. 5. Thus the MG phase has a lower low-energy density-of-states than the ST phase, meaning the MG phase can gain more entropy from thermal fluctuations and become more favourable.

We next move to the regime with $\Omega_1 < \Omega < \Omega_2$, where the ground state is the MG phase. Here the low-temperature phase exhibits both Bose condensation which breaks $U(1)$ phase symmetry and spontaneous magnetization which breaks $Z_2$ symmetry. The question is whether these two different symmetry breaking mechanisms occur at one single phase transition or two separated phase transitions. This issue can be addressed in our experiment because the condensate fraction and magnetization can be measured simultaneously.
Figure 4 | Phase transition between the ST condensate and the MG condensate at very low temperatures. (a) Magnetization histograms with varying $\Omega$ for a nearly pure condensate ($T < 20\text{ nK}$). (b) $\sqrt{(\langle M^2 \rangle)}$ as a function of $\Omega$ at $T < 20\text{ nK}$, the error bars are the standard statistical error transferred from the measurement. The fitting curve is a guide to the eye.

Figure 5 | Finite-temperature phase diagram of spin–orbit coupled Bose gas. Finite-temperature phase diagram of spin–orbit coupled bosons, the error bars are transferred from the measured $f - T$ relation from Fig. 2b. Insets: Schematic low-energy spectrum for the stripe and magnetized phases.

Here we choose $\Omega = 0.6E_r$ and $\Omega = 3.6E_r$. As shown in Fig. 6a, we find that when the condensate just starts to appear and the condensate fraction $f < 0.05$, the histogram graph clearly shows two peaks located at the condensate magnetizations $M$ of approximately $\pm 0.95$ for $\Omega = 0.6E_r$ and $\pm 0.55$ for $\Omega = 3.6E_r$. Thus, although we cannot unambiguously conclude that there is only one single phase transition, at least it shows Bose condensation and magnetization occur in a very narrow temperature window. Unlike for $\Omega < \Omega_1$, in this regime the double-peak structure persists to a lower temperature, which is consistent with the MG ground states. At the lowest temperature, the double peak structure is not pronounced for $\Omega = 0.6E_r$ (as shown in Fig. 4a). However, no signature of a phase transition on lowering the temperature has been found (for details, see Supplementary Methods). The suppression of the double-peak structure may be due to the formation of ‘magnetic domains’ in the system, meaning that both spin up and spin down atoms could stay in the trap while they are phase separated. This is because, at very low temperatures, it requires a longer time to reach global thermal equilibrium than at higher temperatures. Therefore more domains will be formed at very low temperatures than at higher temperatures. This is also consistent with experiments1 at very low temperatures.

However, there is another possibility: magnetization occurs above the condensation temperature—that is, in thermal gases. To examine this possibility, the magnetization histograms of thermal atoms are shown in Fig. 6b. For each of the coupling strengths ($\Omega = 0.1E_r, 0.6E_r, 3.6E_r$) and the temperature range ($T > T_c, T \approx T_c$), a single narrow peak at $M_\theta = 0$ is shown, indicating that the thermal atoms are non-magnetic over a large temperature range across $T_c$ and such a possibility is ruled out. Theoretically, spontaneous magnetization in a thermal gas is due to unequal interaction strengths between different spin species. For $^{87}\text{Rb}$ atoms, differences in the $s$-wave scattering lengths are tiny and the density of a thermal gas is very low, which is consistent with the theoretical expectations that a thermal gas for $T > T_c$ will not exhibit magnetization.

In summary, we experimentally mapped out the finite-temperature phase diagram of an SO-coupled Bose gas of $^{87}\text{Rb}$ atoms realized by a Raman coupling scheme and determined several key features at finite temperature. These results demonstrate the
true power of quantum simulation to guide our understanding and reveal more interesting critical phenomena for the phase transition between two different types of superfluid. Our method can also be applied to similar systems with other atoms, such as dysprosium, where the phase diagram may be qualitatively different. Moreover, both the equilibrium and the dynamic behaviour of superfluidity in the critical regime are intriguing subjects for future studies. These studies will greatly enrich our knowledge of superfluidity with its internal structures.

Methods
Preparation and measurement. The set-up of this experiment is the same as in our previous work. The $^{87}$Rb atoms are trapped and cooled in an optical dipole trap. A pair of Raman lasers with a wavelength of $\lambda=803.2$ nm and an incidence angle of $105^\circ$ in the $x$-$y$ plane are applied to couple the internal states of the $F=1$ manifold. A bias magnetic field $B=8.4$ gauss along the $z$-direction is applied to generate the Zeeman splitting. Here the quadratic Zeeman shift $\epsilon = 10.14$ KHz, which is 4.6 times the recoil energy $E_r = 2\pi \times 2.21$ KHz. It effectively suppresses the $|m_j=1\rangle$ state and SO coupling is generated between the $|m_j=-1\rangle$ state, as $\parallel$↑ and the $|m_j=0\rangle$ state, as $\parallel$↓.

The SO-coupled Bose gas at any finite temperature is prepared as follows in our experiment. A single-component Bose gas in the $|m_j=-1\rangle$ state is first prepared at approximately $330 \text{nK}$ with an atom number of $1 \sim 2 \times 10^6$ in an optical dipole trap. Then the Raman lasers are adiabatically turned on, normally from $100 \text{ms}$ to $500 \text{ms}$, which ensures that atoms are loaded into the lower-energy dressed state with SO coupling. Contrary to the previous experiment, where Raman lasers are turned on when the condensate has already formed, in this case the atoms are still thermal. At the same time, further evaporative cooling is performed to lower the temperature for another $2 \text{s}$, followed by holding the trap depth for an additional $500 \text{ms}$ to reach equilibrium.

For detection, the dipole trap and Raman lasers are switched off suddenly in $1 \mu$s, and the atoms are projected back to the bare states $|m_j=-1\rangle$ and $|m_j=0\rangle$. A time-of-flight absorption image is taken after $24 \text{ms}$, with a gradient magnetic field along the $z$-axis to separate the $|m_j=-1\rangle$ and $|m_j=0\rangle$ states in the vertical direction. The image resolves the spin and the momentum distribution of the atoms simultaneously.

Heating rate of the dipole trap and the Raman lasers. The limitation in Raman induced spin–orbit coupling is the heating effect. This prevents us from reaching very low temperatures and obtaining a high condensate fraction for large $\Omega$. The heating rate of the dipole trap is measured to be $18 \text{nK} \text{s}^{-1}$, mainly as a result of photon scattering and the intensity noise of the dipole trap. The heating rate of
the Raman lasers with $\delta = 0$ is measured to be 180 nK s$^{-1}$ for $\Omega = 1.0E_r$, which comes from a two-photon process with momentum transfer. This is about an order of magnitude higher than that induced by photon scattering.

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Author contributions

S.C. and J-W.P. planned and supervised the project. S-C.J., J-Y.Z., Z-D.D. and S.C. performed the experiments, L.Z., W.Z., Y-J.D. and H.Z. provided theoretical support, and all the authors contributed to analysis of the data and writing the manuscript.

Additional information

Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to S.C. and J-W.P.

Competing financial interests

The authors declare no competing financial interests.