Transversal Flow of Pions as a Consequence of Rescattering Process
(in non-central Pb+Pb 158GeV/n collisions)

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Abstract

Aim of this work was to test the idea of J.-Y.Ollitrault about the parallel squeeze-out type of transversal flow in mid-rapidity region [1]. For this purpose we have performed a computer simulation of the expanding pion gas created in non-central Pb-Pb 158GeV/n collisions. A squeeze-out type of asymmetry parallel to impact parameter in azimuthal distribution of pions is found and studied. The asymmetry is explained as a consequence of geometry of non-central collisions and the rescattering process.

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1. Introduction

Azimuthal asymmetries in transverse momentum distributions of particles measured in relativistic heavy ion collisions (HIC) were observed for the first time by plastic-ball detector in Berkeley [2]. Since that pioneering experiment the asymmetries in relativistic HIC were measured also at higher energies [3, 4]. Typical types of asymmetries (bounce-off and squeeze-out) are explained as a consequence of collective behaviour of nuclear matter. This understanding of the origin of azimuthal asymmetries in non-central HIC successfully explains most of the experimental data.

However at AGS experiment with Au+Au 11.4 GeV/n collisions E877 collaboration reported [3] about the unclear origin of a new type squeeze-out effect parallel to impact parameter. It seems that E877 collaboration has found the first experimental indications of the effect predicted by J.-Y.Ollitrault [1].

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This type of squeeze-out effect results from the interaction of produced particles among themselves.

In this work we investigate asymmetry in transverse momentum distribution of pions created in the process of HIC. No collective behaviour of nuclear matter is supposed in the calculation. Asymmetry results from the geometry of non-central HIC and the rescattering process.

Paper is organized as follows: In Section 2 we give an intuitive explanation of the origin of the asymmetry. Description of the computer simulation is given in Section 3. In Section 4 we present results of our simulation - dependence the asymmetry on impact parameter and also other features of the studied effect. We finish this work with short summary and conclusions.

2. Asymmetry as a consequence of rescattering

Usual explanation of the azimuthal asymmetries in momentum distributions of particles in HIC is based on the collective behaviour of nuclear matter. Hydrodynamical models \cite{5, 6} describe the collision process of HIC using the equation of state for nuclear matter. Substantial assumption of these models is a thermalisation process. As we shall see the existence of thermal equilibrium is not required for the existence of our type of asymmetry.

Number of secondary particles created in HIC at present SPS energies (158GeV/n) is substantially larger than the number of nucleons contained in the colliding nuclei. Therefore the situation is different than that at lower energies and the interaction of the secondaries among themselves becomes significant. As a signature of the rescattering process among the produced pions we regard the $\vec{p}_t$ dependence of transverse size parameters $R_t$ \cite{7} extracted by HBT technique \cite{8}. The decrease of $R_t$ with increasing $p_t$ was explained as a consequence of the rescattering of pions in the simulation \cite{9} where the central S-Pb 200 GeV/n collisions were studied.

We think that the rescattering of secondaries can demonstrate itself in the experimental data also as a transverse flow of pions in non-central collisions. For the existence of this effect no collective behaviour of nuclear matter is necessary. Therefore the phenomenon can be observed also in the future HIC experiments (RHIC, LHC) where nuclear transparency region is expected to be reached.

In this section we explain the origin of the predicted transverse flow of pions as a consequence of the geometry of non-central HIC and the rescattering process. On Fig.1a we show the geometry of non-central A+A collision in transverse plane. Secondary particles i.e. also the pions are created mainly in the overlapping region where nucleon-nucleon collisions happen. Shape of the
overlapping region depends on impact parameter $b$ and is azimuthally asymmetric in transverse plane. Since we do not consider any collective behaviour of nuclear matter it is natural to assume that the initial distribution of pions in transverse momenta $\Psi(p_t)$ is azimuthally symmetrical: $\Psi^S(p_t) = \Psi(|p_t|)$. Because of the simplicity of explanation we do not write longitudinal components of $\vec{p}$ and $\vec{x}$ in this section however the simulation described in the next section is performed in 3 dimensions.

Denoting the space distribution of the points of creation of pions by $\Phi(\vec{x}_t)$ we have the following initial condition for the pion gas created in HIC:

$$X(\vec{x}_t, \vec{p}_t) = \Phi^A(\vec{x}_t) \cdot \Psi^S(\vec{p}_t)$$ (1)

Because of the rescattering process the original $\vec{x} - \vec{p}$ non-correlated distribution (1) becomes $\vec{x} - \vec{p}$ correlated and (as we shall see) the asymmetry in $\Phi^A(\vec{x}_t)$ leaks into asymmetry in the resulting transverse momentum distribution.

$$\Psi^S(\vec{p}_t) \cdot \Phi^A(\vec{x}_t) \rightarrow X^A(\vec{p}_t, \vec{x})$$ (2)

where $^A$ denotes asymmetry in distribution $X(\vec{p}_t, \vec{x})$ in momentum.

This effect can be understood in the following way: Let us have two groups of pions: A group of parallel pions with the momentum parallel to $\vec{b}$ and the group of orthogonal pions with the momentum orthogonal to $\vec{b}$. Because of the asymmetry in $\Phi^A(\vec{x}_t)$ the probability of collision of the orthogonal pion is (in average) higher than the probability of collision for parallel pion (see Fig.1b).

![Fig.1 Geometry of non-central collision in transversal plane](image)

The reason for this is very simple: Number of pions located in the direction orthogonal to $\vec{b}$ is higher because of the bigger size of initial “cloud” of pions in this direction. Thus during the rescattering process orthogonal pions collide more likely than the parallel pions and consequently collisions of type Fig.1c are more frequent than the collisions of type Fig.1d. Such conditions lead to the excess in the number of parallell pions.
This is undoubtedly a non-equilibrium process. Resulting asymmetry in the momentum distribution of pions freezes in the considered pion gas because of the expansion of the system.

Azimuthal asymmetry in transverse momentum distribution of pions after the rescattering process can be measured by detectors as the excess of pions in the direction parallel to the impact parameter. As we shall see in the next sections predicted asymmetry should demonstrate itself as a second order asymmetry in the fourier analysis of transverse flow [11].

On Fig. 2 we show the result of a toy simulation of the effect described. Initial distribution of pions in momenta was taken from the central S-Pb 200 GeV/n HIC simulation [10, 9] and therefore it was azimuthally symmetric. Initial distribution of pions in $\vec{x}_t$ space is artificially asymmetric as it is shown on Fig. 2b. After 20fm/c of the time evolution the rescattering process leads to the asymmetry in transverse momentum distribution as it is clearly seen from Fig. 2c.

![Fig. 2](image-url)

**Fig.2** Toy simulation of the effect. 

- **a)** is $\vec{p}_t$ distribution before the rescattering. 
- **c)** is $p_t$ distribution after 20fm/c of rescattering process. Cut in transverse momentum $p_t > 300 MeV$ is applied to enhance the visual effect of the asymmetry. 
- **b)** is $\vec{x}_t$ distribution before the rescattering process and **d)** after the 20 fm/c of expansion.

Simulation of the expanding pion gas for "real" Pb-Pb 158 GeV/n non-
central collisions is described in the next section.

3. The Simulation

For the simulation of the expanding pion gas created by "real" Pb-Pb 158 GeV/n non-central collisions two independent programs were used:

1) Cascading generator [12] which generates the initial momenta and positions of pions as a result of independent nucleon-nucleon collisions.

2) Rescattering program [10] which simulates time evolution of the interacting of pions created in HIC.

First we shall describe main interface structure of the simulation shown on Fig.3. Pb-Pb 158 GeV/n collisions were simulated by the cascading generator (CG) for random orientations and selected values of impact parameter $\vec{b}$. Information about initial momenta, time and place of the creation of pions $\pi(\vec{x}, \vec{p}, t)$ produced by CG was used as input for the rescattering program [10]. Final momenta of the interacting pions were selected from the output of the rescattering program and analyzed for transversal asymmetries. Orientation of the impact parameter was not supplied by CG and its determination is described in subsection 3.3.

![Fig.3 Main interface structure of the simulation](image)

3.2 Rescattering process

Description of internal structure of the program used for the simulation of rescattering process can be found in [10]. This program was build according to the description of the simulation of central S-Pb 200 GeV/n collisions [9]. Results obtained by program [10] for the central S-Pb collisions are in agreement with the results obtained in [9]. We shall sketch here just main features of our program.

Pions are treated as point-like objects in the simulation, position and momentum of each pion is known during the simulation. Pions move in small time steps $\Delta t = 0.1 fm$ as a free particles. Collisions happen if two pions appear to be at a distance smaller than the critical distance $D_k$ which is determined from the isospin averaged total elastic cross-section.
\[ D_k = \sqrt{\sigma(s)/\pi} \] (3)

Momenta of pions after the collision are determined in CMS of the pair according to the isospin averaged differential cross section
\[ d\sigma/d\Omega = a(s) + b(s) \cos^2(\theta) \] (4)
(Numerical values of the functions \(a(s), b(s)\) are calculated from data [13, 14].)

Then new momenta are transformed back to the global frame of the simulation. Test for the relative distance is performed for every pair of pions in each time step. Not all of pions evolve in time at the beginning of simulation. Cascading generator produces also the information about the time of creation for each pion therefore pions are tested for the existence in the global frame of simulation.

**Fig.4** Dynamics of \(\pi\pi\) collisions. Collisions can happen only if both pions are allowed to interact (solid lines).

Moreover existing pions are restricted from the interaction for time \(T_f\) - formation time [15] after their creation and also for time \(T_i\) after each collision (see description in [10]). These two phenomenological parameters allow to influence total number of collisions in the simulation and also some features of the expanding of pion gas (see Tab.1).

### 3.3 Data analysis

Events of the exact values of impact parameter \(b = 3, 5, 7, 9, 11\text{fm}\) were used for the analysis. Approximate orientation of impact parameter was determined from output of the cascading generator. Cascading generator [12] simulates A-A collision with the target nucleus at the center of coordinate system. Therefore the initial positions of pions which lie mainly in the over-
lapping region of nuclei are shifted from the center of coordinate system in the
direction of impact parameter (see Fig.5).

A procedure close to Danielewicz - Odyniec method [10] was used for deter-
mination of the approximate orientation of impact parameter. For each event
vector

\[ \vec{Q} = \sum_{n} \vec{x}_i \]  

(\( \vec{x}_i \) are initial positions of created pions in transversal plane) was cons tructed
and its orientation was used as the orientation of impact parameter \( \vec{b} \). Final
momenta of pions after the rescattering process were rotated to have the same
orientation of the approximate impact parameter.

![Fig.5 Initial positions of pions in transversal plane obtained from the
cascading generator. 10 events for b=3fm and b=7fm are rotated to
have the same orientation of \( \vec{b} \). On figure b=7fm NoR the rotation is
not performed.](image)

Rotated momenta of pions were used for fourier type of analysis of transver-
sal flow [11]. For each pion azimuthal angle of momentum in respect to the
impact parameter was determined and added to histogram of azimuthal dis-
tribution \( R(\phi) \) (see Fig.6). Because of the known character of asymmetry and
the rotation of events into the direction with the same orientation of impact
parameter we have fitted the normalized histograms \( R^N(\phi) \) to the function

\[ R^N(\phi) = 1 + S_2 \cos(2\phi) \]  

Results of the analysis are presented and discussed in the next sections.

5. Results

First the presence of asymmetry was tested on the set of 24 artificial S-
Pb 200 GeV/n asymmetrical events. A strong final asymmetry in azimuthal
distribution of pions in transversal momentum is visible directly from Fig.2. We have tested also dependence of the strength of the effect on the dynamics of rescattering process. Normalized histograms of $R(\phi)$ distribution were fitted to function (6) for different values of parameters $T_f$ and $T_i$. The results obtained are summarized in Table 1.

| $T_f / T_i$ [fm] | 0.2 / 0.2 | 0.2 / 0.5 | 0.2 / 0.8 | 0.5 / 0.2 | 0.5 / 0.5 | 0.8 / 0.0 | 1.0 / 0.0 |
|-------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $S_2$ ± 5%        | 0.43      | 0.36      | 0.42      | 0.42      | 0.41      | 0.36      | 0.29      |
| $N_{coll}/\pi$    | 4.4       | 2.6       | 2.1       | 1.9       | 1.3       | 0.9       | 0.5       |

Table 1. Results of fit for 24 artificial S-Pb events. $N_{coll}/\pi$ is average number of collisions per \pi during the rescattering process. Total number of pions in one event was 700.

Then "real" non-central Pb-Pb 158 GeV/n collisions were studied using the output of cascading generator [12]. We have run the program for five sets of 10 events with impact parameter $b = 3, 5, 7, 9, 11 \text{ fm}$ in order to study dependence of the asymmetry on the impact parameter and additional 20 events for $b = 7 \text{ fm}$ were run in order to verify whether the statistics of 10 events per set was sufficient. Pions in rapidity range $(-1, 1)$ were selected for the analysis. Events were rotated to have the same orientation of impact parameter using the procedure described in section 3.3. On Fig.6 we show the histograms of distribution $R(\phi)$ for the ROT - rotated (to the same orientation of $\vec{b}$) and non-rotated (NoR) events for $b=3\text{ fm}$ and $b=9\text{ fm}$. Normalized distributions $R^N(\phi)$ were used for the fit.

\footnote{For $b = 11\text{ fm}$ 20 events were run because of low multiplicity of pions in these events.}
Fig.6 Histograms of the azimuthal distributions of pions for \( b = 3 \text{fm} \) and \( b = 9 \text{fm} \). Numbers of particles in the bins of histogram for \( b = 3 \text{fm} \) are higher than in the case \( b = 9 \text{fm} \) because of the higher total multiplicity of pions in the collisions \( b = 3 \text{fm} \). Normalized histogram \( R^N(\phi) \) for \( b = 3 \text{fm} \) is artificially shifted up.

Results of the fit are summarized in Tab.2. Simulation of 20 additional events for \( b = 7 \text{fm} \) showed that the statistics of 10 events is sufficient for the qualitative analysis we have performed. Main result of our simulation is visible directly from Fig.7: Asymmetry increases with impact parameter in range \( 3 - 9 \text{fm} \) and it falls down for \( b > 9 \text{fm} \). Turnover point of \( S_2 \) lies between 7 and 11 fm. We think that the position of the turnover point depends on \( T_f, T_i \) parameters used in the simulation.

Rapidity dependence of the asymmetry was analyzed for events with impact parameter \( b = 7 \text{fm} \). Our small statistics allowed us to perform only a very rough analysis of rapidity dependence of \( S_2 \) coefficient. Results in the following table are averaged for forward and backward rapidities around central rapidity what is in our simulation \( Y_c = 0.0 \).

\[
\begin{array}{c|c|c|c}
|Y| & \langle 0, 1 \rangle & \langle 1, 2 \rangle & \langle 2, 3 \rangle \\
\hline
S_2 & 0.14 & 0.15 & 0.9 \\
\end{array}
\]

We have tried to find also first order asymmetry (bounce-off) in our data which would demonstrate itself as a non-zero value of \( S_1 \) coefficient in the fourier expansion:

\[
R(\phi) = A_0 \cdot (1 + S_1 \cdot \cos(\phi) + S_2 \cdot \cos(2\phi))
\]

For events \( b = 7 \text{fm} \) we have obtained following results:

\[
\begin{array}{c|c|c}
Y & \langle -3, -1 \rangle & \langle 1, 3 \rangle \\
\hline
S_1 \text{ before} & +0.007 \pm 0.01 & -0.003 \pm 0.01 \\
S_1 \text{ after} & +0.016 \pm 0.01 & -0.02 \pm 0.01 \\
\langle p_x \rangle \text{ in MeV} & 2.22 & -1.96 \\
\langle p_y \rangle \text{ in MeV} & 0.06 & -0.06 \\
\end{array}
\]

\( S_1 \text{ before} \) is calculated from the output of cascading generator and \( S_1 \text{ after} \) was computed for events after the rescattering process (see Fig.3). Slight
signature of the first order asymmetry is present in the output of cascading generator what can be a consequence of the cascading of nucleons in spectator matter included in the generator [12]. However this effect is not above the statistical errors in our set of events $b=7\text{fm}$.

Calculation of average $\langle p_x \rangle$ and $\langle p_y \rangle$ of pions in the resulting events $b=7\text{fm}$ confirms that the first order asymmetry is very weak in our data.

We have computed $R_p$ parameter used in the study of squeeze-out effect at BEVALAC/SIS energies [17]

$$R_p = \frac{\langle p_x^2 \rangle - \langle p_x \rangle^2}{\langle p_y^2 \rangle - \langle p_y \rangle^2}$$

(9)

where $x$ direction is parallel to impact parameter. For $b=7\text{fm}$ our $R_p$ of rotated (ROT) and non-rotated (NoR) events is:

$$R_{p\text{ROT}} = 1.39 \quad R_{p\text{NoR}} = 0.97$$

(10)

An interesting behaviour of asymmetry coefficient $S_2$ was found in our data analysis. As it is shown in Tab.2 coefficient $S_2$ is significantly higher for high $p_t$ pions ($p_t > 300\text{MeV}$).

| $b$        | 3 fm     | 5 fm     | 7 fm /30  | 9 fm     | 11 fm /20   |
|------------|----------|----------|-----------|----------|-------------|
| $(p_t > 0)$| $S_2$    |          |           |          |             |
|           | 0.06 ± 0.01 | 0.09 ± 0.01 | 0.135 ± 0.010 | 0.20 ± 0.02 | 0.13 ± 0.02 |
| $(p_t > 300)$| $S_2$    | 0.13     | 0.15      | 0.21     | 0.30        | 0.21        |
| $N_{\text{coll}}/\pi$ | 4.5      | 4.2      | 3.4       | 2.6      | 1.1         |

Tab.2 Asymmetry coefficients for Pb-Pb 158 GeV/n non-central events. 7 fm/30 means that the fit was performed for 30 events.

We have tested whether this behaviour is present also in $E_t$ sensitive analysis. For this purpose we have filled histograms of $R(\phi)$ distribution by the value $E_t$ for each pion. $P_t$ dependence of $S_2$ coefficient was still present in the results.

5. Summary and Conclusions

We have studied asymmetry in the azimuthal transverse momentum distribution of pions for non-central Pb-Pb 158 GeV/n collisions. For the presence of this type of asymmetry no collective behaviour of nuclear matter is necessary. It is a consequence of the geometry of non-central collisions and the rescattering among the pions.
The simulation showed that the asymmetry increases with the impact parameter in the range up to $9\text{fm}$. Turnover point is located between $7 - 11\text{fm}$ in our data. Decrease of asymmetry coefficient $S_2$ for $b = 11\text{fm}$ is most likely a consequence of low number of pions participating in the rescattering process combined with small total size of the overlapping region in comparison to formation path $L_f \approx c \cdot T_f$ of pions. Non-interacting pions in the ”formation time stage" do not feel the asymmetric shape of the overlapping region of colliding ions and therefore the asymmetry decreases. If this is the main reason of the decrease of asymmetry at peripheral collisions then the position of the turnover point can be sensitive to the value of formation time parameter $T_f$ used in the simulation.

We think the effect studied in this work was already confirmed experimentally by NA49 collaboration [18] using Ring Calorimeter setup [19].

Unexpected $p_t$ dependence of the $S_2$ asymmetry coefficient was found in the analysis of our data. This seems to be an interesting prediction however the origin of this $p_t$ dependence is not clear at present. It can be a consequence of our scenario of rescattering process and therefore it does not need to appear in experimental data. Results obtained from NA49 TPC could answer this question. We hope some data will be presented during QM’96 conference.

![Graph: S2 vs. b(fm) for Pb + Pb at 158 GeV/n](image)
Fig. 7 Dependence of $S_2$ coefficient on impact parameter. For high-$p_t$ pions the $S_2$ coefficient is systematically higher than in the case of full $p_t$ range pions.

At the future HIC experiments the multiplicities of secondaries will be much higher. Because of the nuclear transparency phenomenon the mechanism responsible for the transversal flow of nucleons at lower energies (up to 10GeV/n) can play a little role. In this case the studied effect can be substantial for the transversal flow phenomenon at HIC on RHIC and LHC.

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