CAN A NUCLEUS BE LARGER THAN AN ATOM
(QUANTUM LAST SUPPER-POSITION)

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Abstract

In this work we consider an extraordinary quantum mechanical effect when, roughly speaking, the nucleus of an atom becomes (linearly) larger than the whole atom. Precisely, we consider Helium ion (in the ground state of the electron) moving translationally with the speed much smaller than speed of the electron rotation. This translation, effectively, changes neither the total momentum, nor the de Broglie wave length of the electron, nor the linear size of the atom corresponding to the diameter of the electron orbit. But, this translation implies a small nucleus momentum and nuclear de Broglie wavelength almost hundred times larger than the electron de Broglie wavelength. In the measurement of the nucleus wavelength using a diffraction apparatus with a characteristic length constant proportional to the proposed nucleus wavelength, according to standard quantum mechanical formalism, the nucleus behaves practically certainly as a wave. Then the unique, irreducible linear characteristic size for such a nucleus is de Broglie wavelength. Such a measurement effectively influences neither the electron dynamics nor linear size of the atom. This implies that, in such measurement, the size of the nucleus is in one dimension larger than the whole atom, i.e. electron orbital. All this corresponds metaphorically to the famous Leonardo fresco "Last Supper" where Jesus' words coming from the nucleus, i.e. center of the composition, cause an expanding "superposition" or dramatic wave-like movement of the apostles.

As it is well-known, according to the usual, classical mechanically simplified, interpretation of the atom structure based on the remarkable Rutherford scattering experiments, an atom is similar to a microscopic solar planetary system. In the center of the atom there is, like to Sun, a small in respect to the whole atom (with linear size proportional to $10^{-15}m$), but massive (few and more thousand times much massive than electron) complex particle, nucleus (consisting of the protons and neutrons). Around the nucleus there is the peripheral shell consisting of the rotating, like to the planets around Sun, simple particles, electrons (with orbit radius proportional to $10^{-10}m$) small and light in respect to the nucleus.
In more accurate, Bohr-Sommerfeld-de Broglie, quasi-classical or naive quantum atomic theory, electrons can be alternatively or complementarily interpreted as the de Broglie waves so that any electron orbit proposed by Bohr quantization postulate has a natural number of corresponding de Broglie wavelengths. Simply speaking, here peripherical electron waves (for small quantum numbers) obtain the linear sizes (wavelengths) proportional to the linear size of the atom. (More accurately, for quantum number $n$ orbit radius is larger $n$-times than wavelength. It implies that, for $n \sim 1$, orbit radius and wavelength, are proportional.) But within the Bohr-Sommerfeld-de Broglie, quasi-classical or naive quantum atomic theory, there are no additional predictions on the (linear size of) atomic nucleus. In respect to the linear size of the whole atom nucleus stands very small.

In this work we shall consider the theory of an extraordinary quantum mechanical effect when, roughly speaking, the nucleus of an atom becomes (linearly) larger than the whole atom. More precisely, we shall consider a once-ionized Helium atom in the ground state of the rotating electron. This ion, as the whole, moves translationally with very small speed, many times smaller than speed of the electron rotation. This translation, effectively, changes neither the total momentum, nor the de Broglie wave length of the electron, nor the linear size of the atom corresponding to the diameter of the electron orbit. Also, this translation implies a small nucleus momentum and nuclear de Broglie wavelength which is close to a hundred times larger than the electron de Broglie wavelength. Further, one can perform measurement of the nucleus wavelength using a diffraction apparatus with a characteristic length constant proportional to the proposed nucleus wavelength. In such measurement, according to standard quantum mechanical formalism [1], [2] and its Copenhagen interpretation [3], [4], the nucleus behaves practically certainly as a wave. Then the unique, irreducible characteristic linear size for such a nucleus is de Broglie wavelength. Such a measurement effectively influences neither the electron dynamics nor linear size of the atom. This implies that, in such measurement, the size of the nucleus is in one dimension larger than the whole atom, i.e. electron orbital. This is not contrary but complementary to measurement of the linear size of the nucleus, as a particle, by means of the remarkable Rutherford experiment. All this corresponds metaphorically and conceptually to the famous Leonardo fresco ”Last Supper” where Jesus’ words (Logos) coming from the nucleus, i.e. center of the composition, cause an expanding ”superposition” or dramatic wave-like movement of the apostles around Jesus [5].

As it is well-known [2] a once-ionized Helium atom behaves similarly to the neutral Hydrogen atom in the sense that it holds only one electron in the electron shell. For this reason the once ionized Helium atom can be exactly quantum dynamically separated in two non-entangled sub-systems, center of mass and relative particle. Center of mass represents a free propagating quantum (sub)system, while relative particle represents the quantum (sub)system in the spherically symmetric Coulomb electrostatic potential.

Since nucleus mass (neglecting mass excess) $M_n$ is here $7348 \text{ or } \sim 10^4$ times larger than electron mass $m_e$, center of mass can be, in an excellent approximation, represented by nucleus only. Linear size of the nucleus determined by remarkable Rutherford scattering experiments equals approximately $10^{-15}m$. Simultaneously, for the same reason, relative particle can be, in an excellent approximation, represented by electron only.

Suppose that electron is in the ground state. Then, in the quasi-classical approximation, i.e. according to Bohr-Sommerfeld atomic theory, electron propagates along first Bohr orbit with radius $R_e = \frac{\hbar^2}{2m_e e^2} \approx 0.2610^{-10}m$ or diameter $D_e = 2R_e \approx 0.5210^{-10}m$. Speed of the electron equals $v_e = \frac{1}{4\pi\epsilon_0 \hbar} \sim 4.410^6m/s$ and corresponding de Broglie wave length, proportional to the first Bohr orbit diameter, $\lambda_e = \frac{\hbar}{m_e v_e} = 2\pi R_e \approx 1.6310^{-10}m$. In this way linear size of the
Once ionized Helium atom, equivalent to the electron first Bohr orbit diameter, is proportional to electron de Broglie wave length on this orbit. Suppose that the once ionized Helium atom as the whole propagates translationally with a speed \( V_n \) much smaller than \( v_e \), i.e.

\[
V_n = 10^{-x} v_e \approx 4.4 \cdot 10^{6-x} \text{m/s}
\]  

(1)

where \( x \) represents a number greater than 1 and, for realistic situations, smaller than 6. Such propagation, in an excellent approximation, does not change the momentum, orbit diameter and de Broglie wave length of the electron.

Propagating nucleus obtains the de Broglie wave length

\[
\lambda_n = \frac{\hbar}{M_n V_n} \approx \frac{\hbar}{10^4 m_e V_n} = \frac{\hbar}{10^{4-x} m_e v_e} = 10^{-4+x} \lambda_e.
\]  

(2)

Obviously for

\[
6 \geq x \geq 5
\]  

(3)

nuclear de Broglie wave length (1) is close to a hundred times larger than electron de Broglie wave length, i.e.

\[
\lambda_n \sim 10^2 \lambda_e \sim 10^{-8} \text{m}.
\]  

(4)

Simultaneously, under the same condition (3), nucleus speed (1) becomes close to 100m/s, i.e.

\[
V_n \sim 100 \text{m/s}
\]  

(5)

representing a very small, atypical, but technically realizable at least in the principle, speed for atomic particles.

Expression (4) holds very interesting implications. Namely, suppose that there is some diffraction apparatus with characteristic length constant proportional to nuclear de Broglie wavelength (4). By interaction with the translationally propagating, once ionized Helium atom such diffraction apparatus will cause diffraction of the nuclear de Broglie wave only without practically any influence at the electron de Broglie wave (propagating through apparatus like geometrical ray).

In connection with the detector mentioned diffraction apparatus can be consequently considered as a typical measuring apparatus for measurement of the wave characteristics of some quantum system.

In our case given diffraction measuring apparatus measures wave characteristic or wavelength of the nucleus and, of course, yields result comparable with (8). Moreover, according to usual, Copenhagen interpretation [3], [4] of the standard quantum mechanical formalism [1], [2], that is in the excellent agreement with subtle experimental facts [6], [7], it must be stated the following. Within given measurement of the wave characteristics nucleus represents a wave exclusively. Then linear size of the nucleus corresponds to characteristic, irreducible linear size of the wave, i.e. to nuclear de Broglie wavelength (4). (This value, of course, according to Heisenberg momentum-coordinate uncertainty relation, corresponds numerically to the particle coordinate uncertainty.)

On the other hand, in our case, such diffraction measuring apparatus practically does not any influence at the electron, precisely electron orbit, momentum and de Broglie wave length on the orbit. It implies that such measurement does not change linear size of the once ionized Helium atom proportional to electron de Broglie wave length.

Finally, it can be consequently concluded the following. Within discussed measurement linear size of the nucleus (representing nuclear de Broglie wave length) becomes approximately hundred
times larger than linear size of the whole atom (proportional to electron de Broglie wave length on the first Bohr orbit). (This is not contrary but complementary to measurement of the linear size of the nucleus, as a particle, by means of the remarkable Rutherford experiment. Namely, Rutherford experiment represents typical measurement of the particle characteristics of the quantum system.) Obtained effect can be metaphorically called "last supper-position" or quantum Leonardo effect since all this corresponds metaphorically and conceptually to famous Leonardo fresco "Last Supper". This painting demonstrates how Jesus’ words (Logos), coming from nucleus, i.e. center of the composition, cause an expanding "superposition” or dramatic wave-like movement of the apostles around Jesus.

In conclusion the following can be shortly repeated and pointed out. In this work we consider the theory of an extraordinary quantum mechanical effect when, roughly speaking, the nucleus of an atom becomes (linearly) larger than the whole atom. More precisely, we consider a once-ionized Helium atom in the ground state of the rotating electron. This ion, as the whole, moves translationally with very small speed, many times smaller than speed of the electron rotation. This translation, effectively, changes neither the total momentum, nor the de Broglie wave length of the electron, nor the linear dimension of the atom corresponding to the diameter of the electron orbit. Also, this translation implies a small nucleus momentum and nuclear de Broglie wavelength which is close to a hundred times larger than the electron de Broglie wavelength. Further, one can perform measurement of the nucleus wavelength using a diffraction apparatus with a characteristic length constant proportional to the proposed nucleus wavelength. In such measurement, according to standard quantum mechanical formalism and its Copenhagen interpretation, the nucleus behaves practically certainly as a wave. Then the unique, irreducible linear characteristic size for such a nucleus is de Broglie wavelength. Such a measurement effectively influences neither the electron dynamics nor linear size of the atom. This implies that, in such measurement, the size of the nucleus is in one dimension larger than the whole atom, i.e. electron orbital. This is not contrary but complementary to measurement of the linear size of the nucleus, as a particle, by means of the remarkable Rutherford experiment. All this corresponds metaphorically and conceptually to the famous Leonardo fresco "Last Supper” where Jesus’ words (Logos) coming from the nucleus, i.e. center of the composition, cause an expanding "superposition” or dramatic wave-like movement of the apostles around Jesus.

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