Issues on the cosmological constant

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Abstract
Some issues of the cosmological constant or dark energy are briefly reviewed. There are an increasing number of observations that constrain the equation of state of dark energy more stringently and favor the time-independent cosmological constant. Then a plausible model of dark energy would be a theory with degenerate perturbative vacua in which its origin is explained by a nonperturbative effect so that, unlike quintessence, k-essence etc., it is separable from the perturbative problem why its amplitude is smaller than the Planckian density by a factor of $O(10^{-120})$.

1 Introduction

Originally the cosmological constant $\Lambda$ was introduced as an undetermined constant in the left-hand-side of the Einstein equation,

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu},$$

so that this equation had a static cosmological solution, guided by the strong prejudice that our Universe is static \[1\]. As is well known, however, the solution thus obtained is very unstable and is not realistic. As a natural consequence, it turned out that we live in a dynamically expanding universe as theoretically pointed out by Friedmann and observationally discovered by Hubble \[2\].

Nowadays it is more appropriate to put $\Lambda$ in the right-hand-side of the Einstein equation as a part of the energy-momentum tensor. By nature the cosmological constant is equivalent to the vacuum energy density $\rho_v$ with the following relation.

$$\rho_v = \frac{\Lambda}{8\pi G} = \frac{M_{Pl}^2}{8\pi} \Lambda = M_G^2 \Lambda,$$

where $M_{Pl}$ and $M_G \equiv M_{Pl}/\sqrt{8\pi} = 2.4 \times 10^{18}$GeV are the Planck mass and the reduced Planck scale, respectively.

In the field theoretical point of view, if we add the zero-point fluctuation of a quantum field up to a cutoff $k_c$, we find

$$\langle \rho_v \rangle = \frac{k_c^4}{16\pi^2},$$

Taking the cutoff $k_c$ at the Planck scale beyond which classical gravity may not be used, we find $\langle \rho_v \rangle = 4M_G^2$. Thus the natural scale of the vacuum energy is the Planck scale \[3\].

Observationally cosmic energy density today is at most of order of the critical density $\rho_{cr0} \approx 10^{-120}M_G^4$. Confronting such a huge discrepancy of 120 digits between the theoretically natural value and the observationally allowed upper bound, it was considered for a long time that there exist some mechanism to make $\Lambda$ exactly vanish and its quest is the conventional cosmological constant problem to which we refer the Problem I.

Recent advance of observational cosmology, however, has revealed that our Universe is in a stage of accelerated expansion now \[4\]. This means that there exists a tiny magnitude of cosmological constant and/or some unknown form of matter with negative pressure driving accelerated expansion which is generically termed as dark energy. To account for the origin of this small dark energy is the new cosmological constant problem referred to the Problem II.

In this talk I first summarize the currently favored observational values of the cosmological parameters and then move on to Problems I and II and approaches to their solutions. Finally I mention the expected future of our Universe.
2 Cosmological parameters

Current trend of observational cosmology is that the values of cosmological parameters are converging to that of the so-called concordance model. Although the forthcoming data of MAP satellite will certainly provide more precise informations on them, it is still remarkable that combinations of many different means of observational determination of cosmological parameters have already terminated most of the long-standing disputes on them.

2.1 Hubble parameter

The key project of the Hubble Space Telescope has determined the value of the Hubble parameter out to about 400Mpc with various secondary indicators based on the primary cepheid distance which gives $H_0 = 75 \pm 10$ km/s/Mpc. The results of the secondary indicators are summarized as follows.

- Type Ia Supernovae: $H_0 = 71 \pm 2(\text{stat}) \pm 6(\text{syst})$ km/s/Mpc
- Tully-Fisher relation: $H_0 = 71 \pm 3(\text{stat}) \pm 7(\text{syst})$ km/s/Mpc
- Surface brightness fluctuations: $H_0 = 70 \pm 5(\text{stat}) \pm 6(\text{syst})$ km/s/Mpc
- Type II Supernovae: $H_0 = 72 \pm 9(\text{stat}) \pm 7(\text{syst})$ km/s/Mpc
- Fundamental plane of elliptical galaxies: $H_0 = 82 \pm 6(\text{stat}) \pm 9(\text{syst})$ km/s/Mpc

Combining these results Freedman et al. conclude that the final result of HST key project is $H_0 = 72 \pm 8$ km/s/Mpc

Sunyaev-Zel’dovich effect provides other means of determination of $H_0$, although it suffers from fairly large systematic errors. The current result is reported as $H_0 = 60 \pm 4^{+13}_{-18}$ km/s/Mpc, which is consistent with the HST result.

2.2 Matter density

Nonrelativistic matter consists of baryon and cold dark matter whose identity is still unknown but two primary candidates are neutralinos and axions. Observation of light elements and standard big-bang nucleosynthesis (SBBN) gives $\Omega_{b0} = (0.019 - 0.025)h^{-2}$ with $h = H_0/100$ km/s/Mpc.

There are a number of independent observational constraints on the matter density $\Omega_{m0}$. To name a few, the luminosity density and average mass-to-light ratio of galaxies gives $\Omega_{m0} = 0.19 \pm 0.06$, while cluster baryon fraction from X-ray emissivity supplemented by $\Omega_{b0}$ from SBBN yields $\Omega_{m0} = 0.35 \pm 0.07$. On the other hand, shape parameter of the transfer function of CDM scenario of structure formation, $\Gamma = \Omega_{m0}h$ is fit by observation for $\Gamma \approx 0.25$ which means $\Omega_{m0} \approx 0.35$ for $h \approx 0.7$. Although we cannot pin down the value of $\Omega_{m0}$ yet, currently favored value is around $\Omega_{m0} \approx 0.3$.

2.3 Spatial curvature

The angular power spectrum of cosmic microwave background radiation (CMB) provides a unique probe for the spatial geometry of the Universe. In particular, the location of the first Doppler peak reflects the angular scale subtending the sound horizon at decoupling which is sensitive to spatial curvature $K$. Although the data before WMAP suffer from fairly large errors around the first peak, they favor spatially flat universe. This is a good news because it is in accordance with standard inflation models.

2.4 Vacuum energy density

Observations of the magnitude-redshift relation of high-redshift type Ia supernovae has probed the deceleration parameter $q_0 = (\Omega_{m0} - 2\Omega_{v0})/2$ in addition to $H_0$ and they have found that $q_0$ is negative or our Universe is in a stage of accelerated expansion, which means that significant amount of vacuum-like energy density exists today as $\Omega_{v0} \approx 1.25\Omega_{m0} + 0.5 \pm 0.5$. (4)
2.5 The concordance model

Combining all the above observational data we arrive at a model with \( \Omega_{m0} \approx 0.3, \Omega_{v0} \approx 1 - \Omega_{m0} \approx 0.7. \) Then the plausible range of the cosmic age lies \( t_0 = (0.9 - 1.0)H_0^{-1}. \) The Hubble time corresponding to \( H_0 = 72 \pm 8\text{km/s/Mpc} \) reads \( H_0^{-1} = 12.2 - 13.6 - 16.9\text{Gyr}, \) which in turn means \( t_0 = 11 - 17\text{Gyr} \) with the most likely value around \( t_0 \approx 13\text{Gyr}. \)

Astrophysically, age of globular clusters gives \( t_0 = 11 - 14\text{Gyr} \) while cosmological nuclear chronology yields \( t_0 = 12 - 15\text{Gyr}. \) Ever since Hubble’s determination of the Hubble parameter \([2]\), \( H_0 \sim 500\text{km/s/Mpc} \), we confronted with the cosmic age problem from time to time in the history of cosmology. The above set of cosmological parameters gives consistent cosmic age with astrophysical observations and indeed deserves the name of the concordance model.

Looking back more recent history of cosmology, only one and half decades ago, theorists typically expected that we live in the Einstein-de Sitter universe with \( \Omega_{m0} = 1, \Omega_{v0} = 0, K = 0, \) and \( H_0 \sim 50\text{km/s/Mpc}. \) On the other hand, observational astronomers believed that we live in an open universe with \( \Omega_{m0} \lesssim 0.2, \Omega_{v0} = 0, \) and \( H_0 = 50 \text{ or } 100\text{km/s/Mpc} \) depending on schools \([13, 14]\). Thus both parties can compromise if they admit nonvanishing vacuum energy density taking an intermediate value of the Hubble parameter. A good news with the concordance model is that it supports a theoretical prejudice that the Universe is spatially flat and so is consistent with the prediction of standard inflationary cosmology, whose another prediction, generation of almost scale-invariant adiabatic fluctuations, has also been supported by observations of CMB anisotropy. Thus we can describe evolution of the Universe in the framework of inflationary cosmology. Then the greatest mystery in cosmology is the questions, what is the origin of the vacuum energy? Why is it so small?

2.6 Note added: WMAP results

On February 12, 4AM (JST), the first-year result of the Wilkinson Microwave Anisotropy Probe was disclosed \([15]\). As for the values of the cosmological parameters they basically confirmed those of the concordance model with significantly smaller error bars than before with \( h = 0.71_{-0.03}^{+0.04}, \Omega_{m0}h^2 = 0.0224 \pm 0.0009, \Omega_{m0} = 0.135_{-0.009}^{+0.008}, \Omega_{v0} = \Omega_{tot0} - \Omega_{m0}, \) and \( \Omega_{tot0} = 1.02 \pm 0.02. \) One should note, however, that the WMAP data alone could be fit by other sets of parameters whose values are quite different from those of the concordance model. The cosmic concordance is achieved as a result of combinations of various cosmological observations.

3 Proposed solutions to the Problem I

3.1 Symmetry

Whenever we encounter an unnaturally small quantity, we usually interpret it as realized by virtue of some symmetry reason. For example, in order to stabilize the Higgs’ mass in the standard particle physics model against quantum correction up to grand unification or Planck scales, we usually introduce supersymmetry (SUSY).

As is well known, global SUSY also predicts vanishing vacuum energy density at the supersymmetric ground state. The SUSY generator, \( Q_\alpha, \) satisfies the following anti-commutation relation.

\[
\left\{ Q_\alpha, Q_\beta^\dagger \right\} = (\sigma_\mu)_{\alpha\beta} P^\mu,
\]

with \( P^\mu \) being the four momentum. Since the supersymmetric ground state is annihilated by this operator, we inevitably find vanishing energy density there.

\[
Q_\alpha \left| 0 \right\rangle = Q_\alpha^\dagger \left| 0 \right\rangle = 0 \Rightarrow \left\langle 0 \right| P^\mu \left| 0 \right\rangle = 0.
\]

Unfortunately, this does not help to solve the Problem I at all, because we know that supersymmetry is broken today. On the contrary, it causes a severe disaster since breaking supersymmetry inevitably induces a positive vacuum energy density with SUSY breaking scale, \( \rho_c \sim (1\text{TeV})^4 \sim 10^{29}\text{gcm}^{-3} \) or larger.
This disaster is cured by making SUSY local, namely, introducing supergravity (SUGRA). In SUGRA we do not have a relation like (6) and the vacuum energy density can be either negative or positive. Usually it is negative due to a $-\frac{3}{2}|W|^2$ term in the scalar potential where $W$ is the superpotential. Then we may incorporate a positive contribution from SUSY breaking to fine-tune the resultant vacuum energy density vanishing. To this end, SUGRA does not solve the conceptual Problem I either.

Thus SUSY/SUGRA does not help after all and no other symmetry is known to realize vanishing vacuum energy density even after SUSY breaking.

### 3.2 Adjustment mechanism

This is an attempt to dynamically cancel the cosmological constant toward zero as the passage of time. In 1982 Dolgov proposed a model with a massless scalar field nonminimally coupled to the scalar curvature [16]. He showed that in this model the effective cosmological constant decreases in the manner inversely proportional to the square of cosmic time toward zero no matter how large it may be at the outset. This evolution law is the same as that in scalar-tensor model of Fujii [17] proposed in the same year. The problem with Dolgov’s model is the effective gravitational constant also decreases toward zero at the same time, so it cannot serve as the solution in the real world.

As for the impossibility of finding a successful adjustment mechanism to the Problem I, there is a no-go theorem of Weinberg [3] which states that one cannot find an equilibrium solution of the field equations with vanishing cosmological constant without fine-tuning. As a simple example, consider a model with $N$ scalar degrees of freedom $\phi_i$. Then an equilibrium or stationary configuration is determined from $N$ equations such as

$$\partial_i V[\phi] = 0 \quad \text{for each} \quad i = 1, \ldots, N,$$

where $V$ represents the potential of the system under consideration. In order to realize vanishing vacuum energy at the equilibrium configuration, another equation should be satisfied there to ensure $\Lambda = 0$, which requires a fine-tuning because we have already used $N$ degrees of freedom to specify the equilibrium configuration.

### 3.3 Quantum Cosmological Approach

This approach was quite fashionable more than a decade ago. It is based on the wave function of the universe. In Hartle-Hawking’s approach [18], the wave function with three geometry $h_{ij}$ and matter configuration $\phi$ is expressed by an Euclidean path integral over all the compact manifolds $C'$ as

$$\Psi_{HH} [h_{ij}, \phi] = \int_{C'} [dg][d\phi]e^{-I[g,\phi]}.$$  

(7)

In this prescription, the expectation value of an operator $Y$ is given by a path integral over all the compact Euclidean manifolds $C$ as

$$\langle Y \rangle_{HH} = \frac{\langle \Psi_{HH}, Y \Psi_{HH} \rangle}{\langle \Psi_{HH}, \Psi_{HH} \rangle} = \frac{\int_{C'} [dg][d\phi]Y e^{-I[g,\phi]} \Psi_{HH}}{\int_{C'} [dg][d\phi]e^{-I[g,\phi]} \Psi_{HH}}.$$  

(8)

If this path integral is dominated by the de Sitter instanton with the action

$$I = -\frac{1}{16\pi G} \int (R - 2\Lambda)\sqrt{g}d^4x = -\frac{3\pi}{G\Lambda},$$  

(9)

the wave function is peaked at $\Lambda = 0$.

$$\Psi_{HH} [h_{ij}, \phi] = \int_{C'} [dg][d\phi]e^{-I[g,\phi]} \propto \exp\left(\frac{3\pi}{G\Lambda}\right),$$  

(10)

suggesting the vanishing cosmological constant [19].

Later on Coleman proposed to incorporate sum over topology with wormhole configurations connected to mother and baby universes. Then all the physical constants become a variable dependent on the state of baby universes $\alpha$ and the expectation value reads,

$$\langle Y \rangle = \frac{\int [d\alpha] e^{-\frac{3\pi}{2G\Lambda}Z(\{\alpha\})} \langle Y \rangle_{\alpha} Z(\{\alpha\})}{\int [d\alpha] e^{-\frac{3\pi}{2G\Lambda}Z(\{\alpha\})}} \propto \exp\left[\exp\left(\frac{3\pi}{G\Lambda(\{\alpha\})}\right)\right].$$  

(11)
Thus we find an even sharper peak at $\Lambda = 0$. This mechanism may also be used to fix other physical parameters. For example, strong CP problem may be solved without introducing axions.

In these Euclidean approaches there is no time dependence in the expressions of expectation values and they should rather be interpreted as an average expectation value throughout the possible cosmic history which would coincide with that in the most stable ground state.

There are a number of serious problems in these approaches. First, the Euclidean path integral is not well-defined, because, once gravity is included, the action is not positive definite even after Wick rotation. Second, no conserved and positive-definite probability current is known in this approach, which makes its interpretation difficult. Third, in the original calculation of wormhole and baby universe system, phase of sum over spheres was miscounted and the sharper peak observed in (11) diminishes in the correct calculation.

For these reasons, although this approach is attractive in that it is a quantum theory, this solution is not taken seriously now.

### 3.4 Higher dimensional models

Higher dimensional theories may open up a new perspective to the cosmological constant problem, because lower dimensional subspace may have flat geometry even if energy-momentum tensor is nonvanishing. Hence if we could succeed in removing the gravitational effect of vacuum energy on cosmic expansion of our three-brane, this could serve as a solution to the problem. For example, if we found a modified Friedmann equation of our three-brane like

$$H^2 = f(\rho, p)(\rho + p) + \ldots,$$

after an appropriate dimensional reduction, such an equation would allow for the Minkowski solution $H = 0$ in the presence of arbitrary vacuum energy density with $\rho_v = -p_v$ on the three-brane. While many ideas have been proposed these days I do not go in detail of them because there will be many brane world talks in this workshop and I hope this issue will be covered there. It seems, however, many of them require fine-tuning, some of which are as severe as the original one, to realize such an effective equation on the brane so they could hardly evade the no-go theorem.

I would like to briefly mention, however, an exception which makes use of infinite-volume extra-dimensions with large-distance modification of gravity on the brane proposed by Dvali, Gabadadze, and Shifman. They start with an $4+N$ dimensional action,

$$S = M_s^{2+N} \int d^4x d^N y \sqrt{G} R_{4+N} + \int d^4x \sqrt{g} \left( M_G^2 R_4 + L_{SM} + \Lambda_4 \right),$$

where $M_s$ is the fundamental Planck scale and $M_G$ is the four dimensional counterpart induced by quantum correction of matter in four dimension with the Lagrangian $L_{SM}$. In this model the four dimensional graviton $h_{\mu\nu}$ satisfies a modified equation,

$$\left[ 1 + \frac{F_N}{r_c^2 \Box} \right] h_{\mu\nu} = 0,$$

where $F_N$ is a constant and $r_c \equiv M_G/M_s^2$ is the crossover distance beyond which gravity becomes weak. It should satisfy $r_0 \gtrsim H_0^{-1} \approx 10^{28}$ cm to ensure standard cosmology. Hence we should take $M_s \lesssim 10^{-3}$ eV. Since the cosmological constant problem is a far-infrared (large-scale) problem, four dimensional curvature is insensitive to it in such a modified theory of gravity. Thus we may naturally have an almost flat four dimensional spacetime no matter how large $\Lambda_4$ may be. Although this model relies on a huge hierarchy $M_s/M_G \lesssim 10^{-30}$, it is stable under the assumption of unbroken supersymmetry in the bulk.

### 3.5 Anthropic principle

Since the cosmological constant problem is such a difficult problem to solve, it is natural that many people are tempted to resort its solution to the anthropic consideration in which one attempts to interpret natural phenomena based on our existence.
Consider a proposition: We, human being, cannot live in a universe with a large cosmological constant, positive or negative. We can easily convince ourselves that this proposition is correct at least qualitatively, and more quantitative proofs can be found in the literature [25]. Now consider its contrapositive: The cosmological constant in our Universe is small, because human being exists. Since a proposition and its contrapositive always have the same truth value, once we convince ourselves that the above proposition is correct, it can serve as a solution to the cosmological constant problem at least logically.

But this fact alone does not guarantee that the anthropic consideration is useful in theoretical physics. Let us consider a perhaps more illustrative question: Why does our Universe have three spatial dimensions? A possible answer based on the anthropic principle is that if the spatial dimension is not equal to three, an orbit of a planet is not closed, so that its climate would be too unstable to accommodate human life. Again this solution is logically correct answer to the original question, but it is clear that such an answer provides no insights for the compactification mechanism of extra dimensions which is one of the central issue in higher-dimensional theories. Thus in this sense this approach does not contribute to progress in theoretical physics.

I do not claim that we should not use the anthropic principle at all. On the contrary, I admit that this could be useful at some stage of investigation. As for the cosmological constant, we might use it once we have reached an ultimate theory which clarifies what physics controls it but does not predict of its value itself.

The anthropic principle resembles sake: if you use an appropriate amount at night, you can be happy, but if you drink it too much in the morning, you will lose your job. I think it is clear that the situation of investigation of cosmological constant problem is yet in the morning or in the premature stage and use of the anthropic principle now could be dangerous because we might well miss a chance to reach an essence of theoretical physics.

Nevertheless there is an exception. We may or even should resort the solution of what is called the coincidence problem or why-now problem, namely the question why vacuum energy is dominating only recently, to the anthropic principle, because “now” is defined by our very existence.

4 Proposed solutions to the Problem II

I now move on to various proposals to explain the origin of the tiny dark energy density observed. They are based on a common assumption that there exists some ground state with a vanishing vacuum energy density where the Problem I is solved. I would also like to note that two of the suggested solutions to the Problem I, namely the anthropic consideration [25] and Dvali et al’s higher dimensional model [23], may also naturally accommodate a tiny nonvanishing dark energy density.

4.1 Decaying \( \Lambda \) or quintessence

This is an attempt to identify the vacuum-like energy density with a dynamical, slowly-changing and possibly spatially inhomogeneous component with negative pressure [16, 17, 20, 27]. Typical example is a scalar field \( Q \), now called quintessence, slowly rolling down its extremely flat potential [26, 28]. By virtue of its time dependence, the Problem II, or why the observed value of the dark energy today is unnaturally small compared with, say, the Planck scale, may be relaxed. In fact, by choosing an appropriate shape of the potential \( V[Q] \) it has been advocated that one can find a natural solution to the coincidence problem in which the energy density of the quintessence field decreases during radiation domination tracking evolution of the total energy density and becomes dominant some time after the equality time. Such a tracker solution is possible with a potential like \( V[Q] = M^4(M_G/Q)^n \) with \( n > 0 \) or \( V[Q] = M^4 [\exp (M_G/Q) - 1] \), both with a canonical kinetic term. By nature of the tracker solution, the scalar field is still evolving today in the time scale of cosmic expansion, which means that the equation of state satisfies

\[
\frac{\rho_Q}{\rho} = \frac{\dot{Q}^2}{2} - \frac{V[Q]}{2 + V[Q]} \geq -0.75,
\]

and that its mass is comparable to the Hubble parameter today, \( m_Q = \sqrt{V''[Q]} \approx H_0 = 10^{-33}\text{eV}. \)
As will be seen below the above characteristic of the equation of state is now confronting with observational tests. Furthermore from theoretical viewpoint it is difficult to build a sensible particle physics model to realize such a small effective mass with a huge hierarchy \( m_Q/M_w = 10^{-44} = 10^{-26}M_w/M_G \), where \( M_w \) is the weak scale. Nevertheless such a small mass might be stably realized if protected by some symmetry. One example is the model proposed by Nomura, Watari, and Yanagida [29] in which the quintessence field is identified with a pseudo Nambu-Goldstone boson. This model, unfortunately, does not lead to a tracker behavior.

4.2 Kinetically driven quintessence

It was observed by Armendáriz-Picón, Damour, and Mukhanov that a scalar field can drive inflation even if it has no potential provided that it has an appropriate non-canonical kinetic term [30]. This idea was first applied to explain the accelerated expansion observed today by Chiba, Okabe, and Yamaguchi under the name of kinetically driven quintessence [31]. Now it is abbreviated as k-essence [32, 33]. The evolution of the equation of state, \( w \), is highly nontrivial in this class of models and may be observationally tested. An example of the scalar Lagrangian giving rise to a desired recent cosmic acceleration is given by [32]

\[
L = \frac{1}{\varphi^2} \left( 2\sqrt{1 + X} - 2.01 + 0.03 \left(10^{-5}X \right)^{3} - (10^{-6}X) \right). X \equiv -\frac{1}{2} \left( \partial \varphi \right)^2 = \frac{1}{2} \dot{\varphi}^2. \tag{15}
\]

4.3 QCD trace anomaly

The QCD trace anomaly contributes to the vacuum energy density with the amount

\[
\langle F_{a\mu}F_{a}^{\mu} \rangle = \mathcal{O} \left( \Lambda_{QCD}^4 \right) \approx \mathcal{O} \left( (100\text{MeV})^4 \right). \tag{16}
\]

Suppose that this contribution is canceled by the intrinsic cosmological constant in Minkowski vacuum state which is a part of the usual assumption that the Problem I is solved in the ground state. Then Schützhold [34] calculates the same quantity in the Friedmann vacuum under this assumption and finds a weakly time-dependent dark energy density

\[
\rho_v = \mathcal{O} \left( \Lambda_{QCD}^3 H \right). \tag{17}
\]

Although this proposal is attractive in that it is a result of serious calculation of quantum field theory in the curved spacetime, the resultant value might be too large, \( \Lambda_{QCD}^3 H_0 \gtrsim 30\rho_{cr0} \) for \( \Lambda_{QCD} \gtrsim 100\text{MeV} \).

4.4 False vacuum energy

All the proposals to the problem II discussed so far predict time-dependent dark energy. But we should also consider the possibility that we live in a false vacuum state with a finite vacuum energy density. Most people believe that such a possibility is unnatural due to the smallness of the observed vacuum energy, \( \rho_v = 10^{-120}M_G^4 = (\text{meV})^4 \). Several attempts do exist, however, to explain the existence of a false vacuum state with the desired magnitude of vacuum energy density naturally by introducing some discrete symmetry [35] or global symmetry [36] and then breaking it with some appropriate nonrenormalizable terms.

4.5 Non-perturbative explanation with degenerate vacua

Another possibility that predicts time-independent dark energy without introducing any tiny numbers is a theory with degenerate perturbative vacua. It starts with the observation that the cosmological constant Problem I is a perturbative problem, that is, why we observe vanishingly small vacuum energy even after perturbative quantum correction, [37], and we assume that this problem is solved by a yet unknown mechanism as assumed in all the other proposals above. If there exist two or more degenerate perturbative vacua, say, \( |+\rangle \) and \( |−\rangle \) whose vacuum energy is vanishing under the assumption that the perturbative Problem I is solved there, and if quantum tunneling between perturbative vacua is possible, these states are no longer the real ground state. Including the effect of quantum tunneling, which we
assume is described by an instanton with the Euclidean action $S_0$, the energy eigen states are given by superposition of perturbative vacua,

$$|S\rangle = \frac{1}{\sqrt{2}} (|+\rangle + |\rangle), \quad |A\rangle = \frac{1}{\sqrt{2}} (|-\rangle + |\rangle),$$  \hspace{1cm} (18)

where $|S\rangle$ is the real ground state with $\rho_v = -m^4 e^{-S_0}$ and the first excited state is $|A\rangle$ with $\rho_v = m^4 e^{-S_0}$, with $m$ being the energy scale associated with the quantum transition. Thus each perturbative vacua is expressed by the superposition of energy eigenstates with exponentially small energy gap, and if we still live in one of the perturbative vacua we observe a finite magnitude of dark energy density $\rho_v = m^4 e^{-S_0}$ with the probability of 50% in this case. In order to match the predicted value with the observed value, we should take $S_0 = 120 \ln 10 + 4 \ln(m/M_G)$, and the longevity of the perturbative vacua requires $m \gtrsim M_G$ [37].

This solution is attractive in two aspects: one is that the Problems I and II are properly separated and the other is that, as mentioned above, it involves no small numbers and the smallness of the observed dark energy is the largeness of the instanton action. The former feature may particularly be important because in other proposals no explanations have been given why only the contribution of quintessence or k-essence fields to the vacuum energy density remains finite after solving the Problem I.

5 Observational constraints on dark energy

In order to examine which, if any, of the above proposals is the right one, it is important to observationally constrain the equation of state, $w = p/\rho$, of dark energy. An increasing number of observational analysis have been done these days.

For example, Corasaniti and Copeland used SNIa data and the location of the three peaks of CMB anisotropy and obtained an ambitious constraint $-1 \leq w \leq -0.93$ [38].

Melchiorri, on the other hand, performed a joint analysis of CMB, SNIa, HST, and the large-scale structures and concluded $w \leq -0.85$ at $1\sigma$ and $w \leq -0.72$ at $2\sigma$ [39]. His result is perfectly consistent with the cosmological constant $w = -1$.

Percival et al. finds from 2dF data combined with CMB anisotropy $w \leq -0.51$ at $2\sigma$ and the probability distribution of $w$ is peaked around $w = -1$, namely, they favor the cosmological constant [40].

From REFLEX sample of the redshift distribution of X-ray clusters combined with SNIa data, Schuecker et al. find $w = -0.95^{+0.30}_{-0.35}$ under the assumption of spatially flat Universe [41] and claims that the most natural interpretation of the data is the cosmological constant.

Thus we may conclude that the tracker solution of the quintessence scenario is already in a difficult situation and that currently available data seem to favor the cosmological constant. Note that CMB data used above are pre-WMAP ones. The new constraint obtained with WMAP is $w < -0.78$ at $2\sigma$ [15].

6 The future of the Universe dominated by $\Lambda$

If our Universe is dominated by time-independent vacuum energy density as suggested by numerous observations and if it is stable against phase transitions, we can draw some interesting conclusions about the future of our Universe [42, 43].

The existence of a positive cosmological constant means that our Universe is in a stage of asymptotically de Sitter expansion. So there will be an event horizon at the distance $\approx 5.1$ Gpc and we cannot go to any galaxies with $z > 1.8$ not only in practice but also in principle. We can observe a galaxy at $z = 5$ only for 6.4 Gyr from now.

Future evolution of nearby large-scale structure in our Universe dominated by a cosmological constant has been studied by Nagamine and Loeb using an N-body simulation [44]. They find:

- Evolution of large-scale structure continues for 28 Gyrs.
- Our local group will not be bound to the Virgo cluster.
- Our galaxy is likely to merge with Andromeda galaxy within the Hubble time.
• This will be the only galaxy within the horizon 100 Gyr later.
This means that extragalactic astronomy and cosmology must be solved within 100 Gyr. So we should hurry up!

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