Robust Backstepping Control Based on a Lyapunov Redesign for Skid-Steered Wheeled Mobile Robots

Eun-Ju Hwang\textsuperscript{1}, Hyo-Seok Kang\textsuperscript{1}, Chang-Ho Hyun\textsuperscript{2,*} and Mignon Park\textsuperscript{1}

\textsuperscript{1} School of Electrical and Electronic Engineering, Yonsei University, Korea
\textsuperscript{2} School of Electrical Electronic and Control Engineering, Kongju National University, Korea
\* Corresponding author E-mail: hyunch@kongju.ac.kr

Received 12 Oct 2012; Accepted 14 Nov 2012

\textbf{Abstract} This paper represents a robust backstepping tracking control based on a Lyapunov redesign for Skid-Steered Wheeled Mobile Robots (WMRs). We present kinematic and dynamic models that explicitly relate the perturbations to the skidding in order to improve the tracking performance during real running. A robust controller is synthesized in the backstepping approach and the Lyapunov redesign technique, which forces the error dynamics to stabilize to the reference trajectories. We design an additional feedback control - a Lyapunov redesign - such that the overall control stabilizes the actual system in the presence of uncertainty and perturbation with the knowledge of the Lyapunov function. Simulation results are provided to validate and analyse the performance and stability of the proposed controller.

\textbf{Keywords} Skid-Steered Wheeled Mobile Robots, Backstepping, Lyapunov Redesign, Robust Tracking Control

1. Introduction

The control of nonholonomic systems has been an active area of research and development in recent years because nonholonomic constraints arise in many advanced robotic structures, such as mobile robots, space manipulators and multi-fingered robot hands. A mobile robot is one of the well-known systems with nonholonomic constraints and there have been many studies on tracking control [1, 2], such as sliding mode control [3], adaptive control [4-6] and backstepping control [7, 8], as well as intelligent control based on fuzzy systems [9-12]. Most of these previous works assume that nonholonomic constraints are subject to a kind of pure rolling without skidding and slipping. Unfortunately, since wheel skidding is inevitable due to wheel tire deformation, or due to wet or icy roads and rapid cornering, the nonholonomic constraints can be perturbed. Consequently, the stability and performance of these controllers are not guaranteed. Therefore, studies of WMRs with uncertainties and disturbances have provided an attractive issue for tracking control. Most previous studies have only considered kinematic models, but uncertainties and disturbances could not be introduced without the dynamics of the WMRs. Recently, some methods for dynamic models have been proposed [9-13]. In [14], Dixon et al. addressed skidding in order to design robust
tracking and regulation controllers using the kinematic model. Robust control offers uniform boundedness solutions. However, if a high-precision control performance is desired, these control laws would require high control input and fast switching actions. A linear time varying controller was proposed in [15] to achieve uniformly locally exponential stability for the Type (2, 0) unicycle based on the dynamic model. In [16], the model is applied to design a slow manifold controller to solve an output-tracking problem. It should be noted that these dynamic models rely on the accurate measurement of parameters, which is not explicitly expressed in terms of skidding state. Moreover, the parameters are difficult to obtain in practice. Since nonholonomic mobile robot dynamics display highly nonlinear behaviour, it is difficult to estimate the proper norm bound. Based on a consideration of the aforementioned robustness perspective, we have designed a stabilizing control input based on a Lyapunov redesign in order to cancel out the uncertainties and disturbances without the estimation of the unknown skidding and nonlinear term of WMRs. In this paper, the robot dynamics and kinematics have been induced from the perturbed nonholonomic constraints. Consequently, the stability and performance of these controllers are not guaranteed. Since nonholonomic mobile robot dynamics exhibit highly nonlinear behaviour, it is difficult to estimate the proper norm bound. Based on a consideration of the aforementioned robustness perspective, we have designed a stabilizing control input based on backstepping control with a Lyapunov redesign in order to cancel out the uncertainties and disturbances without the estimation of the unknown skidding and nonlinear term of WMRs.

In this paper, we propose a robust backstepping controller based on a Lyapunov redesign for skid-steered WMRs. The proposed backstepping controller applies the kinematics and dynamics to analyse and compensate for skidding effects. The backstepping method is used to analyse the system and to design the controller which is making the error dynamics stable. In the first step, we design a virtual control to stabilize the kinematics. Next, we can choose the input term to make the error dynamics stable. In addition, we do not need to estimate the proper norm bound to design the robust controller because a stabilizing control input is designed on the basis of a Lyapunov redesign in order to cancel out the uncertainties and disturbances [17]. The proposed controller compensates for nonlinear terms such as unknown parameters, skidding, and so on, with a Lyapunov redesign. The simulation results show the greater effectiveness of the proposed controller compared to the feedback controller in maintaining the robot on track in an elliptical reference trajectory.

This paper is organized as follows. Section 2 presents the model of the nonholonomic skid-steered WMRs. The controller design for tracking problems and stability analysis are described in Section 3. Simulation results for the proposed controller with WMRs are given in Section 4, followed by the conclusion in Section 5.

2. Model of Skid-Steered Wheeled Mobile Robots

The dynamics of the nonholonomic mobile robot with two actuated wheels in the absence of wheel skidding is described by the Lagrange equation of motion:

$$M(q)\dddot{q} + V(q, \dot{q})\dot{q} + G(q) + \tau_d = E(q)\tau - A^T(q)\lambda$$

(1)

where $q \in \mathbb{R}^n$ are the generalized coordinates, $\tau_d \in \mathbb{R}^l$ is the disturbance of an input vector, $\tau \in \mathbb{R}^l$ is an input vector, $\lambda \in \mathbb{R}^m$ is a vector of constraints forces, $M(q) \in \mathbb{R}^{n \times n}$ is a symmetric and positive definite inertia matrix, $V(q, \dot{q}) \in \mathbb{R}^{n \times n}$ is the centripetal and Coriolis matrix, $G(q) \in \mathbb{R}^l$ is the gravitational vector, $E(q) \in \mathbb{R}^{m \times l}$ is an input transformation matrix, $A(q) \in \mathbb{R}^{m \times n}$ is a matrix associated with nonholonomic constraints and $l = n - m$. We define $q = [x \ y \ \phi]^T$. The variables $x, y, \phi$ denote the position and orientation of the platform, respectively.

![Figure 1. Model of the wheeled mobile robot](image)

The WMRs have three perturbed nonholonomic constraints in the presence of skidding, represented as follows:

$$\begin{align*}
\dot{y} \cos \phi - \dot{x} \sin \phi &= \mu \\
\dot{x} \cos \phi + \dot{y} \sin \phi + b_\phi &= r_\phi \\
\dot{x} \cos \phi + \dot{y} \sin \phi - b_\phi &= r_\phi 
\end{align*}$$

(2)

where $\mu$ indicates the lateral skidding velocity in mobile robots, which can easily be caused by the centrifugal force generated when mobile robots travel at high speeds on wet or icy roads or around corners. This value is dependent on the wheel and road conditions and may vary with location. Constraints (2) can be represented by:
\[ A(q)q = \mu \] (3)

where:
\[ A(q) = \begin{bmatrix} \sin \theta & -\cos \theta & 0 \end{bmatrix} . \]

\( S(q) \) is in the null space of \( A(q) \). A matrix \( S(q) \) is defined to satisfy \( A(q)S(q) = 0 \).

Then, using the perturbed nonholonomic constraints (2) and the matrix \( S(q) \), the kinematics of skid-steered mobile robots are obtained as:
\[ \dot{q} = S(q)z + \phi(q, \mu) \] (4)

where \( z = \begin{bmatrix} \nu & \alpha \end{bmatrix}^T \); \( \nu = r(\theta_1 - \theta_2) / 2 \) is the forward linear velocity, \( \alpha = r(\theta_1 - \theta_2) / (2b) \) is the angular velocity of the mobile robots and \( \phi(q, \mu) = [-\mu \sin \varphi, \mu \cos \varphi, 0]^T \) denotes the unmatched disturbance vector induced from the perturbed nonholonomic constraints.

**Assumption 1** [15]: The \( \phi(q, \mu) \) is bounded as \( \left\| \phi(q, \mu) \right\| = \left\| \begin{bmatrix} -\sin \varphi & \cos \varphi \end{bmatrix} \right\| = 1 \) where \( \beta \) is an unknown positive constant and the first derivative is also bounded as \( \left\| \dot{\phi}(q, \mu) \right\| = \left\| \begin{bmatrix} -\mu \beta_1 & \beta_2 \end{bmatrix} \right\| \) with \( \beta_1 < \beta \) where \( \beta_1 \) is an unknown positive constant.

To derive the skid-steered dynamics model, we substitute (4) into (1). Then, the dynamic model is obtained as:
\[ \tau = H(q)\dot{z} + F_1(q, \dot{q})z + F_2(q)\phi(q, \mu) + F_3(q, \mu) + F_4(q) + \tau_d \] (5)

where:
\[ H(q) = (S^T E)^{-1} S^T M S , \]
\[ F_1(q, \dot{q}) = (S^T E)^{-1} S^T (M S + V S) , \]
\[ F_2(q) = (S^T E)^{-1} S^T M , \]
\[ F_3(q) = (S^T E)^{-1} S^T V , \]
\[ F_4(q) = (S^T E)^{-1} S^T G . \]

\( \tau = \begin{bmatrix} \tau_1 & \tau_2 \end{bmatrix}^T \) is the control input. We can rewrite (5) as follows:
\[ \dot{z} = A(q, \dot{q})z + B(q)\tau - H^{-1}(q)F_2(q)\phi(q, \mu) - H^{-1}(q)F_3(q, \mu) + \Delta A(q, \dot{q})z + \Delta B(q)\tau \] (6)

where \( A(q, \dot{q}) = -H^{-1}(q)F_2(q, \dot{q}) \), \( B(q) = H^{-1}(q) \). We assume that WMRs are operated on level ground without slope, \( F_2(q) = 0 \). In addition, since \( A(q, \dot{q}) \), \( B(q) \) is changed by the value of \( q \), we consider the parameter uncertainties or variation of the dynamics of WMRs. \( \Delta A(q, \dot{q}) \) and \( \Delta B(q) \) are parameter uncertainties and variation due to the skidding of the WMRs, respectively. The control system satisfies the following properties.

**Property 1** [4, 16]: \( H(q) \) is a symmetric and positive-definite matrix, \( H(q) = B^{-1}(q)M(q) \) where \( M(q) = F_3(q)M(q)S(q) \), \( B(q) = F_3(q)E(q) \).

**Property 2** [17]: The matrix \( (T(q) - 2S(q)) \) is skew-symmetric where \( T_m = S^T(q)(M(q)S(q) + V(q)S(q)) \).

**Property 3**: The disturbances are bounded so that \( \| \tau_d \| \leq d_B \).

Parameter uncertainties, parameter variation, disturbance of input and the skidding of the mobile robot are regarded as disturbances, as follows:
\[ D(x) = -H^{-1}(q)F_2(q)\phi(q, \mu) - H^{-1}(q)F_3(q, \mu) + \Delta A(q, \dot{q})z + \Delta B(q)\tau \] (7)

We can simplify (6) according to (7):
\[ \dot{z} = A(q, \dot{q})z + B(q)\tau + D(x) \] (8)

**Definition 1**: A mobile robot is said to be point controllable if there exists a piecewise continuous input to steer the mobile robot’s reference point from an initial point \( (x(t_0), y(t_0)) \) to a final point \( (x(t_f), y(t_f)) \) in a finite-time interval.

**Property 4** [18]: Suppose that the skidding of a mobile robot (4) satisfies Assumption 1; then, the mobile robot (4), considering the wheel’s skidding, is point controllable, but not posture controllable.

**Remark 1**: Assumption 1 is reasonable due to Property 3 in [19]. The assumption means that the proposed approach does not require any knowledge of wheel skidding in order to design the controller.

### 3. Controller design and stability analysis

Given a reference trajectory \( q_r(t) = [x_r(t), y_r(t), \phi_r(t)]^T \) generated by a reference robot whose equations of motion are:
\[ \dot{x}_r = v_r \cos \phi_r \, , \, \dot{y}_r = v_r \sin \phi_r \, , \, \dot{\phi}_r = \omega_r \] (9)

where \( z_r = [v_r, \omega_r]^T \), \( v_r \) is the reference for the forward linear velocity and \( \omega_r \) is the reference for the angular velocity of the mobile robot. A smooth velocity control law \( z_c = f_c(E_p, z_r, K) \) is determined by:
\[ \lim_{t \to \infty} q(t) = q_r(t) \] (10)
where $E_p$, $z_c$, and $K$ are the tracking position error, the reference velocity and the control gain, respectively. Smooth velocity control inputs are designed for the kinematic steering system to make the position error asymptotically stable. Then, we can find the torque input $\tau(t)$ that can be computed without measuring velocities, such that $z(t)$ converges asymptotically to $z_c(t)$ as $t \to \infty$ [20].

The tracking error is expressed relative to the local coordinate frame fixed on the mobile robot as $E_p = T(q_p - q)$ or:

$$
E_p = \begin{bmatrix} e_1 \\
 e_2 \\
 e_3 
\end{bmatrix} = 
\begin{bmatrix}
\cos \phi & \sin \phi & 0 \\
-\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} x_c - x \\
y_c - y \\
\phi_c - \phi
\end{bmatrix}
$$

(11)

The error rate becomes:

$$
\dot{E}_p = \begin{bmatrix} \omega_1 + v + v_c \cos \epsilon_3 \\
\omega_2 + 2v_1 \epsilon_2 + 2v_3 \epsilon_1 \\
\omega_3 - \omega
\end{bmatrix}
$$

(12)

An auxiliary velocity control input that achieves tracking for the kinematic model (4) with skidding (i.e., with $E_p = 0$, which is uniformly asymptotically stable under the assumption about the reference linear velocity $v_c > 0$) is given by:

$$
z_c = \begin{bmatrix} v_1 \\
 v_2 \\
 v_3
\end{bmatrix} = 
\begin{bmatrix}
v_1 \cos \epsilon_3 + k_1 \epsilon_1 \\
v_2 + k_2 \epsilon_1 v_2 + k_3 \epsilon_1 v_3 \sin \epsilon_3
\end{bmatrix}
$$

(13)

where $k_1, k_2, k_3 > 0$ are the design parameters.

The backstepping control is used to design the controller which is making the error dynamics stable. The control input $\tau$ in (8) converts the dynamic control problem into the kinematic control problem.

**Step 1.** Starting with equation (4), we define a virtual control law $z_{des}$ and let $\epsilon_c$ be an error variable representing the difference between the actual and virtual controls:

$$
e_c = z - z_{des}
$$

(14)

In this step, our objective is to design a virtual control law $z_{des}$ which makes $q \to 0$. We can now select an appropriate virtual control $z_{des}$ which would make the system stabilizable:

$$
z_{des} = -S^{-1} q \triangleq \alpha(q)
$$

(15)

where $S$ is not a square matrix so that a dummy term is added to obtain $S^{-1}$.

Thus, in terms of the new state variable, we can rewrite (4) as:

$$
\dot{q} = -q + \phi(q, \mu)
$$

(16)

Consider a control Lyapunov function of (16):

$$
V_1 = \frac{1}{2}(v_1^2 + v_2^2) + \frac{k}{k_3}(1 - \cos \epsilon_3)
$$

(17)

Differentiating (17) and substituting the values from (12) and (13), we get:

$$
V_1 = -k_3 v_2^2 + k - k_3 v_c \sin^2 \epsilon_3
$$

(18)

This is because the reference linear velocity $v_c \geq 0$, $\dot{V}_1$, is negative definite.

**Step 2.** We rewrite system in (4) and (8) in the $(q, \epsilon_c)$ coordinates in which it takes on a more convenient form. Differentiating (14), the equation can be rewritten as:

$$
\dot{\epsilon}_c = \dot{z} - \dot{z}_{des}
$$

$$
= Az + Br + D - \dot{z}_{des}
$$

(19)

Function $D(x)$ contains all of the WMRs’ unknown parameters, disturbances of input and skidding, etc., and it is regarded as follows:

$$
D(x) = -A(q, \dot{q}) \xi + B(q, \dot{q}) \xi + H^{-1}(q, \dot{q}) F(q, \phi(q, \mu))
$$

(20)

These quantities are unknown and difficult to determine. Therefore, we apply the Lyapunov redesign technique in which the error dynamics of the overall system are stabilized without the complicated calculations of the unknown terms.

To make the error dynamics stable, the input term in (19) is chosen as

$$
\tau = B^{-1}(z_c - \dot{z}_{c}) - K \dot{z}_c + \nu
$$

(21)

where $K$ is a positive constant.

We can design an additional feedback control $\nu$ such that the overall control $\tau$ stabilizes the actual system in the presence of the uncertainty. The design of $\nu$ is called ‘Lyapunov redesign’.

Substituting (21) into (19), the closed-loop error dynamics can be expressed as:

$$
\dot{\epsilon}_c = (A - BK) \epsilon_c + D + \nu
$$

(22)
To implement the control law given by (21), only the values of \( z \) and \( \dot{z} \) are required without measuring the values of \( \phi \) and \( \dot{\phi} \). Thus, wheel skidding information is not required to implement this control law.

Consider a control Lyapunov function of (22):

\[
V_2 = \frac{1}{2} e^T \bar{M} e
\]

(23)

Differentiating (23) and substituting the error dynamics from (22), we get:

\[
\dot{V}_2 = \frac{1}{2} e^T (\bar{M} - 2\bar{V}_m) e - e^T \bar{M} e + e^T \bar{M} (\nu + D)
\]

(24)

If \( V_2 < 0 \), the derivative of the Lyapunov function of the system is negative definite. We prove it in theorem 1.

In the new coordinates \((q, e_c)\), the system is expressed as:

\[
\begin{bmatrix}
\dot{q} \\
\dot{e}_c
\end{bmatrix} = 
\begin{bmatrix}
-I & s \\
0 & A + BK
\end{bmatrix}
\begin{bmatrix}
q \\
e_c
\end{bmatrix} + 
\begin{bmatrix}
\phi \\
D + \nu
\end{bmatrix}
\]

(25)

The system matrix of the resulting system has a negative diagonal.

**Theorem 1.** Consider the closed-loop error dynamics given by (25). Let:

\[
V = V_1 + V_2
\]

(26)

where:

\[
V_1 = \frac{1}{2} (e^2_1 + e^2_2) + \frac{1}{k_2} (1 - \cos e_2)
\]

(27)

\[
V_2 = \frac{1}{2} e^T \bar{M} e
\]

(28)

If \( V \) is positive definite and \( \dot{V} \) is negative definite, then the closed-loop error dynamics given by (22) are asymptotically stable with the Lyapunov redesign.

**Proof.** Differentiating (20), we get:

\[
\dot{V} = \dot{V}_1 + \dot{V}_2
\]

(29)

Substituting the values from (12) and (13), we get:

\[
\dot{V}_1 = -k_2 e^2_1 - k_2 e_1 \sin^2 e_3
\]

(30)

Again, this is because the reference linear velocity \( v_r \geq 0 \), \( V_1 \), is negative definite.

Differentiating (22) and substituting the error dynamics from (18), we get:

\[
\dot{V}_2 = \frac{1}{2} e^T (\bar{M} - 2\bar{V}_m) e - e^T \bar{M} e + e^T \bar{M} (\nu + D)
\]

(31)

Owing to the property 2, the matrix \((\bar{M} - 2\bar{V}_m)\) is skew symmetric. We get:

\[
e^T (\bar{M} - 2\bar{V}_m) e = 0
\]

(32)

Substituting (32) into (31):

\[
\dot{V}_2 = -e^T \bar{M} e + e^T \bar{M} (\nu + D)
\]

(33)

Suppose that we know a Lyapunov function for the nominal model without disturbance; then we have the inequality:

\[
\dot{V}_2 \leq -e^T \bar{M} e + w^T \nu + w^T D
\]

(34)

where \( w = e^T \bar{M} \).

Let us suppose that inequality (34) is satisfied with \( \| D \| \leq \rho + k_0 \| \nu \| \)

(35)

where \( \rho : [0, \infty) \times D \rightarrow R \), is a nonnegative continuous function and \( 0 \leq k_0 < 1 \). The estimation in (35) is the only information we need to have as to the uncertain term \( D\).

We have:

\[
w^T \nu + w^T D \leq w^T \nu + \| D \| \leq w^T \nu + \| \rho \| \| \nu \|
\]

Taking \( \nu = -\eta \times \frac{w}{\| \nu \|} \) with a nonnegative function \( \eta \), we obtain:

\[
w^T \nu + w^T D \leq -\eta \| \nu \| + \| \rho \| \| \nu \| + k_0 \eta \| \nu \|
\]

(36)

Choosing \( \eta \geq \rho / (1 - k_0) \) :

\[
w^T \nu + w^T D \leq -\eta \| \nu \| + \| \rho \| \| \nu \| = 0
\]

(37)

Hence, with the control \( \nu = -\eta \times \frac{w}{\| \nu \|} \), the derivative of \( V \) along the trajectories is negative definite. Since \( V_2 < 0 \), then \( V < 0 \). This proves the asymptotic stability of the tracking trajectory of the wheeled mobile robot. It is obvious that \( V(c) \rightarrow \infty \) as \( \| \cdot \| \rightarrow \infty \). Q.E.D.

**4. Simulation Results**

Simulations for the tracking control of wheeled mobile robots are performed to demonstrate the validity of the proposed control law (15) using MATLAB. In most previous studies, simulation works used a straight line or circle path as a reference trajectory. However, a reference
path that is an ellipse is more suitable than a straight line
or circle path because the reference velocities are usually
not constant in practical situations. Therefore, an elliptical
reference trajectory is used in our simulations, as given
by $x_r = 10 \cos t, \ y_r = 6 \sin t$ and $\varphi_r = \tan^{-1}(\dot{y}_r/\dot{x}_r)$.
The reference linear and angular velocity is given by:

$$v_r(t) = \sqrt{\dot{x}_r^2(t) + \dot{y}_r^2(t)}, \ \omega_r(t) = \frac{\ddot{y}_r(t)\dot{x}_r(t) - \ddot{x}_r(t)\dot{y}_r(t)}{\dot{x}_r^2(t) + \dot{y}_r^2(t)} \quad (38)$$

The trajectory starts from $q_r(0) = \begin{bmatrix} 1 & 0 & \pi/2 \end{bmatrix}^T$ and the
robot’s initial Position is taken as $q(0) = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}^T$. The
parameters of the controller are chosen as $k_1 = 1, k_2 = 100$
, $k_3 = 10$ and $K = \text{diag}(10 \ 10)$. The parameter values of the
WMRs are taken as $m_c = 30 \text{kg}$, $m_w = 1 \text{kg}$, $r = 0.15 \text{m}$,
$b = 0.15 \text{m}$, $d = 0.3 \text{m}$, $I_r = 15.625 \text{kg} \cdot \text{m}^2$, $I_w = 0.005 \text{kg} \cdot \text{m}^2$
and $I_m = 0.0025 \text{kg} \cdot \text{m}^2$.

The skidding of the WMRs is unknown, but it is bounded
by assumption 1. In this paper, we assume that the wheel
skidding is a kind of white noise. When a path following
control problem is solved by a GPS-based control scheme
for WMRs with wheel skidding in [21], the perturbations
computed from GPS measurements have a zero mean,
like white noise.

We compare the performance of the proposed controller
with that of the feedback controller.

![Figure 2. Trajectory tracking of the wheeled mobile robot](image)

In Fig. 2, the dotted line is a reference trajectory and
the red line is the trajectory of the proposed control.
The blue line is the trajectory of the feedback
linearization control. Fig. 3 shows the position error $x$
and $y$ of the WMRs. The tracking errors are bounded
and converge to zero.

The figures show that the proposed controller has
better tracking performance than the feedback
linearization controller in elliptical path tracking
control. However, because the skidding effects are not
impressive in this case, the difference in performance
between the proposed controller and the feedback
linearization controller is not remarkable. The
following simulation results show the robustness
of the proposed controller in the situation in relation to
effective skidding. The proposed controller shows
better tracking performance than the feedback
linearization controller.

![Figure 3. Tracking position error comparison with the proposed controller and the feedback controller](image)

![Figure 4. Trajectory tracking of the wheeled mobile robot](image)

![Figure 5. Tracking error of x](image)
The proposed control method effectively overcomes the skidding and shows robustness in the presence of unknown parameters and disturbances for tracking problems.

5. Conclusions

In this paper, we present a robust controller based on a Lyapunov redesign for a skid-steered wheeled mobile robot. The backstepping method is used to analyse the system and to design the controller thereby making the error dynamics stable. The proposed controller compensates for nonlinear terms such as unknown parameters and skidding, etc. We set up the control input with a Lyapunov redesign to make the system uniformly bounded when $v_r \geq 0$. The proposed robust controller using the Lyapunov redesign technique forces the error dynamics to stabilize to the elliptical reference trajectory. We do not need to estimate the proper norm bound to design the robust controller because a stabilizing control input is designed on the basis of a Lyapunov redesign to cancel out the uncertainties and disturbances. It is shown that the designed controller is robust to uncertainty from skidding. Finally, the simulation results show the effectiveness of the proposed controller against unknown disturbances.

6. Acknowledgments

This work was supported by the research grant of the Kongju National University in 2012.

7. References

[1] Kanayama Y, Kimura Y, Miyazaki F and Noguchi T (1991) A Stable Tracking Control Method for a Nonholonomic mobile robot, IEEE/RSJ Int. Workshop Intelligent Robots and Systems. 1236–1241.
[2] Samson C and Ait-Abderrahim K (1991) Feedback Control of a Nonholonomic Wheeled Cart in Cartesian Space, IEEE Int Conf Robot Autom. 1136–1141.
[3] Chwa D (2004) Sliding-mode Tracking Control of Nonholonomic Wheeled Mobile Robots in Polar Coordinates, IEEE Trans. Control Syst. Technol. 12: 637–644.
[4] Fukao T, Nakagawa H and Adachi N (2000) Adaptive Tracking Control of a Nonholonomic Mobile Robot, IEEE Trans. Robot. Autom. 16: 609–615.
[5] Dixon WE, De Queiroz M, Dawson DM and Flynn T (2004) Adaptive Tracking and Regulation of a Wheeled Mobile Robot with Controller/Update Law Modularity, IEEE Trans. Control Syst. Technol. 12: 138-147.
[6] Dong W and Kuhnert KD (2005) Robust Adaptive Control of Nonholonomic Mobile Robot with Parameter and Nonparameter Uncertainties, IEEE Trans. Rob. 21: 261-266.
[7] Cheng L, Cao L, Wu H, Zhu Q, Xu W and Liu L (2011) Trajectory Tracking Control of Nonholonomic Mobile Robots by Backstepping, Proc. Int. Conf. Modeling, Identification and Control. 134-139.
[8] Filipescu A, Minzu V and Filipescu A (2011) Backstepping Control of Wheeled Mobile Robots, Proc. Int. Conf. System Theory, Control, and Computing. 1-6.
[9] Martinez R, Castillo O and Aguilar LT (2008) Optimization with Genetic Algorithms of Interval Type-2 Fuzzy Logic Controllers for an Autonomous Wheeled Mobile Robot: A Comparison under Different Kinds of Perturbations, FUZZ-IEEE 2008: 901-908.
[10] Astudillo L, Castillo O, Aguilar LT and Martinez R (2007) Hybrid Control for an Autonomous Wheeled Mobile Robot Under Perturbed Torques, IFSA (1) 2007: 594-6.
[11] Martinez R, Castillo O and Aguilar LT (2009) Optimization of Interval Type-2 Fuzzy Logic Controllers for a Perturbed Autonomous Wheeled Mobile Robot Using Genetic Algorithms, Inf. Sci. 13: 2158-2174.
[12] Castillo O, Martinez-Marroquin R, Melin P, Valdez F and Soria J (2012) Comparative Study of Bio-inspired Algorithms Applied to the Optimization of Type-1 and Type-2 Fuzzy Controllers for an Autonomous Mobile Robot, Inf. Sci. 192: 19-38.
[13] Fierro R and Lewis F (1995) Control of a nonholonomic mobile robot: backstepping kinematics into dynamics, Proc. 34th IEEE Conf. Decision and Control. 3805-3810.
[14] Dixon W, Dawson D, and Zergeroglu E (2000) Robust control of a mobile robot system with kinematic disturbances, in Control Applications Proc. IEEE International Conference on, 437-442.
[15] Lerouquis W and d’Andrea-Novel B (1996) Modeling and control of wheeled mobile robots not satisfying ideal velocity constraints: the unicycle case, in Decision and Control, Proceedings of the 35th IEEE, pp. 1437-1442.
[16] Motte I and Campion G (2000) A slow manifold approach for the control of mobile robots not satisfying the kinematic constraints, Robotics and Automation, IEEE Trans. Rob. Autom, 16: 875-880.
[17] Khalil H. (2001) Nonlinear Systems 3rd ed., Prentice-Hall, New Jersey.
[18] Yoo S (2010) Adaptive tracking control for a class of wheeled mobile robots with unknown skidding and slipping, IET Control Theory Appl. 4: 2109–2119.
[19] Kim M, Shin J and Lee J (2000) Design of a robust adaptive controller for a mobile robot, Proc. IEEE/RSJ Int. Conf. Intelligent Robots and Systems. 1816–1826.
[20] Slotine J and Li W (1991) Applied Nonlinear Control, Prentice-Hall, Englewood Cliffs, NJ.
[21] Low C and Wang D (2008) GPS-based path following control for a car-like wheeled mobile robot with skidding and slipping, IEEE Trans. Control Syst. Technol. 16: 340–347.
[22] Tamoghna D and Intra N (2006) Design and Implementation of an Adaptive fuzzy logic-based controller for wheeled mobile robots, IEEE Trans. Control Syst. Technol. 14: 501–510.