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Black Holes in the Real Universe and Their Prospects as Probes of Relativistic Gravity

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1.1 Introduction

Collapsed objects have definitely been observed: some are stellar-mass objects, the endpoint of massive stars; others, millions of times more massive, have been discovered in the cores of most galaxies. Their formation poses some still-unanswered questions. But for relativists the key question is whether observations can probe the metric in the strong-field domain, and test whether it indeed agrees with the Kerr geometry predicted by general relativity.

In a lecture in 1977, Chandrasekhar wrote:

"In my entire scientific life the most shattering experience has been the realisation that exact solutions to Einstein’s theory [the Kerr solutions] provide the absolutely exact representation of untold numbers of massive black holes that populate the Universe."
He was of course referring to the uniqueness theorems, to which Stephen Hawking and several other participants at this conference contributed. At the time that Chandra wrote this, there was some evidence for black holes. Now the evidence is much stronger and my theme in this paper is to summarize this evidence.

I will discuss first the black holes of a few solar masses that are the remnants of supernovae, and then supermassive black holes in the mass range of a million to a billion solar masses, for which there is also good evidence. I shall also mention parenthetically that there may be a population of black holes in the range of hundreds of solar masses, relics of an earlier generation of massive stars. Bernard Carr’s paper discusses primordial black holes, right down to very low masses, for which I think there is no evidence but which would be fascinating if they existed.

By the early 1970s the theoretical properties of black holes – the Kerr metric, the ‘no hair’ theorems, and so forth – were well established. But it took much longer for the observers to discover candidate black holes, and to recognise their nature. A fascinating review of the history has been given by Werner Israel [1]. By now, the evidence points insistently towards the presence of dark objects, with deep potential wells and ‘horizons’ through which matter can pass into invisibility; however, the evidence does not yet allow observers to confirm the form of the metric in the strong-gravity region. I shall briefly summarise the evidence, and then comment on the prospects for testing strong-field gravity by future observations.

1.2 Stellar mass holes

The first black hole candidates to be identified were bodies of a few solar masses, in close orbits around an ordinary companion star, emitting intense and rapidly flickering X-rays. This emission is attributed to inward-spiralling material, captured from the companion star, which swirls inward towards a ‘horizon’. There are two categories of such binaries: those where the companion star is of high mass, of which Cygnus X1 is the prototype, and the low-mass X-ray binaries (LMXBs) where the companion is typically below a solar mass. The LMXBs are sometimes called ‘X-ray novae’ because they flare up to high luminosity: they plainly have a different evolutionary history from systems like Cygnus X1.

The X-ray emission from these objects was distinctively different from the periodic variability of a related class, which were believed to be neutron stars. The masses can be inferred from straightforward Newtonian arguments. It is gratifying that the masses inferred for the periodic sources are in all cases below 2 solar masses (consistent with the theoretically-expected mass range for neutron stars) whereas those that
vary irregularly have higher masses. There are now a dozen or so strong candidates of this type. Of course, the only stellar-mass holes that manifest themselves conspicuously are the tiny and atypical fraction that lie in close binaries where mass transfer is currently going on. There may be only a few dozen such systems in our Galaxy. But there are likely to be at least $10^7$ black holes in our Galaxy. This number is based on the rather conservative estimate that only 1 or 2 percent of supernovae leave black holes rather than neutron stars. There could be a further population of black holes (maybe in the Galactic Halo) as a relic of early galactic history.[2]

Gamma ray bursts are a class of objects, of which about 3000 have been detected, which flicker on time scales of less than a second and last sometimes for only a second; sometimes for a few minutes. They are known to be at great distances: indeed they are so bright that if a gamma-ray burst went off in our galaxy it would be as bright as the Sun for a few seconds. They can be detected by X-ray or gamma-ray telescopes even at very large redshifts. These are probably a rare kind of collapsing star or merger of a compact binary which may signal the formation of a black hole. They are extremely important because they may be objects where we see a non-stationary black hole.

1.3 Supermassive holes

A seemingly quite distinct class of black hole candidates lurk in the centres of most galaxies; they are implicated in the power output from active galactic nuclei (AGNs), and in the production of relativistic jets that energise strong radio sources. The demography of these massive holes has been clarified by studies of relatively nearby galaxies: the centres of most of these galaxies display either no activity or a rather low level, but most seem to harbour dark central masses.

In most cases the evidence is of two kinds. Stars in the innermost parts of some galaxies are moving anomalously fast, as though ‘feeling’ the gravitational pull of a dark central mass; and in some galaxies with active nuclei, the central mass can be inferred by modelling the properties of the gas which reprocesses central continuum radiation into emission with spectral features. But there are two galaxies where central dark masses are indicated by other kinds of evidence that are far firmer.

The first, in the peculiar spiral NGC 4258, was revealed by amazingly precise mapping of gas motions via the 1.3 cm maser emission line of H$_2$O [3]. The spectral resolution of this microwave line is high enough to pin down the velocities with accuracy of 1 km/sec. The Very Long Baseline Array achieves an angular resolution better than 0.5 milliarc seconds (100 times sharper than the HST, as well as far finer spectral
resolution of velocities!). These observations have revealed, right in NGC 4258’s core, a disc with rotational speeds following an exact Keplerian law around a compact dark mass. The inner edge of the observed disc is orbiting at 1080 km/sec. It would be impossible to circumscribe, within its radius, a stable and long-lived star cluster with the inferred mass of $3.6 \times 10^7 \, M_\odot$.

The second utterly convincing candidate lies in our own Galactic Centre (see [4] for a comprehensive review). Direct evidence used to be ambiguous because intervening gas and dust in the plane of the Milky Way prevents us from getting a clear optical view of the central stars, as we can in, for instance, M31. A great deal was known about gas motions, from radio and infrared measurements, but these were hard to interpret because gas does not move ballistically like stars, but can be influenced by pressure gradients, stellar winds, and other non-gravitational influences. There is now, however, direct evidence from stellar proper motions, observed in the near infrared band, where obscuration by intervening material is less of an obstacle [5,6]. The speeds scale as $r^{-1/2}$ with distance from the centre, consistent with a hole of mass $2.6 \times 10^6 \, M_\odot$. One can actually plot out the orbits of these stars. Some will make a complete circuit in little more than 100 years. Corroboration comes from the compact radio source that has long been known to exist right at the dynamical centre of our Galaxy, which can be interpreted in terms of accretion onto a massive hole [7,8].

There is a remarkably close proportionality [9] between the hole’s mass and the velocity dispersion of the central bulge or spheroid in the stellar distribution (which is of course the dominant part of an elliptical galaxy, but only a subsidiary component of a disc system like M31 or our own Galaxy).

### 1.4 Scenarios for Black Hole Formation

There is no mystery about why high-mass stars may yield gravitationally collapsed remnants of 10 solar masses or more, but the formation route for the supermassive holes is still uncertain. Back in 1978, I presented a ‘flow diagram’ [10] exhibiting several evolutionary tracks within a galaxy, aiming to convey the message that it seemed almost inevitable that large masses would collapse in galactic centres. There was not yet (see [1]) any consensus that active galactic nuclei were powered by black holes (despite earlier arguments by Salpeter [11], Zeldovich and Novikov [12] and, especially, Lynden Bell [13]). We have now got used to the idea that black holes indeed exist within most galaxies, but it is rather depressing that we still cannot decide which formation route is most likely.

The main options are summarised below:
1.4 Scenarios for Black Hole Formation

1.4.1 Monolithic formation of supermassive objects

One possibility is that the gas in a newly-forming galaxy does not all break up into stellar-mass condensations, but that some undergoes monolithic collapse. As the gas evolves (through loss of energy and angular momentum) to a state of higher densities and more violent internal dissipation, radiation pressure would prevent fragmentation, and puff it up into a single superstar. Once a large mass of gas started to behave like a single superstar, it would continue to contract and deflate. Some mass would inevitably be shed, carrying away angular momentum, but the remainder would undergo complete gravitational collapse. The behaviour of supermassive stars was studied by Bardeen, Thorne and others in the 1960s. Because radiation pressure is overwhelmingly dominant, such objects are destabilised, in the post-Newtonian approximation, even when hundreds of times larger than the gravitational radius. Rotation has a stabilising effect, but there has still been little work done on realistic models with differential rotation (see, however [14])

The mass of the hole would depend on that of its host galaxy, though not necessarily via an exact proportionality: the angular momentum of the protogalaxy and the depth of its central potential well are relevant factors too.

1.4.2 Mergers of smaller holes (stellar mass or ‘intermediate mass’) 

Rather than forming ‘in one go’ from a superstar that may already be a million solar masses or more, holes might grow from smaller beginnings. The first-generation stars are thought to have been more massive than those forming in galaxies today – perhaps up to hundreds of solar masses. Such stars live no more than a few million years, and leave black hole remnants if they lie in two distinct mass ranges [15]:

(i) Ordinary massive stars, with helium core masses up to 64 M⊙; and
(ii) ‘Very Massive Objects’ (VMOs) with helium cores above 130 M⊙.

Stars in between these two ranges leave no compact remnant at all, instead ending their lives by a disruptive explosion induced by the onset of electron-positron pair production.

VMO remnants could have interesting implications in the present universe [2]; such objects, captured by supermassive holes, would yield gravitational radiation signals detectable by LISA out to redshifts of order unity, possibly dominating the event rate. Could there be a link between these ‘intermediate mass’ holes and supermassive holes? There are two possibilities. The most obvious, at first sight, is that a cluster of such objects might merge into one. But it is not easy for a cluster of black holes to merge into a single one. To see this, note that one binary with orbital
speed $10^4$ km/sec (with a separation of 1000 Schwarzschild radii) would have just as much binding energy as a cluster of 10000 holes with velocity dispersion 100 km/sec. Thus, if a cluster accumulated in the centre of a galaxy, the likely outcome would be the expulsion of most objects, as the consequence of straightforward N-body dynamics, leaving only a few.

The prospects of build-up by this route are not quite as bad as this simple argument suggests, because the binding energy of the compact cluster could be enhanced by dynamical friction on lower-mass stars, by gas drag, or by gravitational radiation. This nonetheless seems an inefficient route towards supermassive holes.

Ordinary stars, with large geometrical cross-sections, have a larger chance of sticking together than pairs of black holes. We therefore cannot exclude a ‘scenario’ where a supermassive star builds up within a dense central cluster of ordinary stars. The most detailed calculations were done by Quinlan and Shapiro ([16] and other references cited therein). These authors showed that stellar coalescence, followed by the segregation of the resultant high-mass stars towards the centre, could trigger runaway evolution without (as earlier and cruder work had suggested) requiring clusters whose initial parameters were unrealistic (i.e. already extremely dense, or with implausibly high velocity dispersions). It would be well worthwhile extending these simulations to a wider range of initial conditions, and also to follow the build-up from stellar masses to supermassive objects.

1.4.3 Runaway growth of a favoured stellar-mass hole to supermassive status

Even if a large population of low-mass holes is unlikely to merge together, is it, alternatively, possible for one of them, in a specially favoured high-density environment, to undergo runaway growth via accretion? An often-cited constraint on the growth rate is based on the argument that, however high the external density was, growth could not happen on a timescale less than the classic ‘Salpeter time’[11]

$$t_{\text{Sal}} = 4 \times 10^7 (\varepsilon/0.1) \text{ yrs}$$ (1.1)

For an efficiency $\varepsilon$ of 0.1 this would yield an e-folding timescale of $4 \times 10^7$ years. If these holes started off with stellar masses, or as the remnants of Population III stars, there would seem to be barely enough time for them to have grown fast enough to energise quasars at $z = 6$.

This is not, however, a generic constraint; there are several suggestions in the literature for evading it. [17-20]
1.5 The Galactic Context

1.5.1 The key issues

Physical conditions in the central potential wells of young and gas-rich galaxies should be propitious for black hole formation: such processes, occurring in the early-forming galaxies that develop from high-amplitude peaks in the initial density distribution, are presumably connected with high-\(z\) quasars. It now seems clear that most galaxies that existed at \(z = 3\) would have participated subsequently in a series of mergers; giant present-day elliptical galaxies are the outcome of such mergers. Any black holes already present would tend to spiral inwards, and coalesce [21,22] (unless a third body fell in before the merger was complete, in which case a Newtonian slingshot could eject all three: a binary in one direction; the third, via recoil, in the opposite direction).

The issues for astrophysicists are then:

(a) How much does a black hole grow by gaseous accretion (and how much electromagnetic energy does it radiate) at each stage? Models based on semi-analytic schemes for galaxy evolution have achieved a good fit with the luminosity function and \(z\)-dependence of quasars. Less gas is available at later epochs, and this accounts for the scarcity of high-luminosity AGNs at low \(z\).

and

(b) How far back along the ‘merger tree’ did this process start? A single big galaxy can be traced back to the stage when it was in hundreds of smaller components with individual internal velocity dispersions as low as 20 km/sec. Did central black holes form even in these small and weakly bound systems? This issue has been widely discussed (see for instance [23-26] and references cited therein). It is important because it determines whether there is a population of high-\(z\) miniquasars.

1.5.2 Tidally-disrupted stars

When the central hole mass is below \(10^8\) \(\text{M}_\odot\), solar-type stars are disrupted before they get close enough to fall within the hole’s horizon. A tidally disrupted star, as it moves away from the hole, develops into an elongated banana-shaped structure, the most tightly bound debris (the first to return to the hole) being at one end. There would not be a conspicuous ‘prompt’ flare signalling the disruption event, because the energy liberated is trapped within the debris. Much more radiation emerges when the bound debris (by then more diffuse and transparent) falls back onto the hole a few months later, after completing an eccentric orbit. The dynamics and radiative transfer are then even more complex and uncertain than in the disruption event itself, being affected by relativistic precession,
as well as by the effects of viscosity and shocks.

The radiation from the inward-swirling debris would be predominantly thermal, with a temperature of order $10^5$ K; however the energy dissipated by the shocks that occur during the circularisation would provide an extension into the X-ray band. High luminosities would be attained – the total photon energy radiated (up to $10^{53}$ ergs) could be several thousand times more than the photon output of a supernova. The flares would, however, not be standardised – what is observed would depend on the hole’s mass and spin, the type of star, the impact parameter, and the orbital orientation relative to the hole’s spin axis and the line of sight; perhaps also on absorption in the galaxy. To compute what happens involves relativistic gas dynamics and radiative transfer, in an unsteady flow with large dynamic range, which possesses no special symmetry and therefore requires full 3-D calculations – still a daunting computational challenge. [27-30]

The rate of tidal disruptions in our Galactic Centre would be no more than once per $10^5$ years. But each such event could generate a luminosity several times $10^{44}$ erg/s for about a year. Were this in the UV, the photon output, spread over $10^5$ years, could exceed the current ionization rate: the mean output might exceed the median output. The radiation emitted from the event might reach us after a delay if it reflected or flouresced off surrounding material. Sunyaev and his collaborators have already used such considerations to set non-trivial constraints on the history of the Galactic Centre’s X-ray output over the last few thousand years.

1.6 Do the Candidate Holes Obey the Kerr Metric?

1.6.1 Probing near the hole

The observed molecular disc in NGC 4258 lies a long way out: at around $10^5$ gravitational radii. We can exclude all conventional alternatives (dense star clusters, etc); however, the measurements tell us nothing about the central region where gravity is strong – certainly not whether the putative hole actually has properties consistent with the Kerr metric. The stars closest to our Galactic Centre likewise lie so far out from the putative hole (their speeds are less than 1 percent that of light) that their orbits are essentially Newtonian.

We can infer from AGNs that ‘gravitational pits’ exist, which must be deep enough to allow several percent of the rest mass of infalling material to be radiated from a region compact enough to vary on timescales as short as an hour. But we still lack quantitative probes of the relativistic region. We believe in general relativity primarily because it has been resoundingly vindicated in the weak field limit (by high-precision observations in the
Solar System, and of the binary pulsar) – not because we have evidence for black holes with the precise Kerr metric.

The emission from most accretion flows is concentrated towards the centre, where the potential well is deepest and the motions fastest. Such basic features of the phenomenon as the overall efficiency, the minimum variability timescale, and the possible extraction of energy from the hole itself all depend on inherently relativistic features of the metric – on whether the hole is spinning or not, how it is aligned, etc. But the data here are imprecise and ‘messy’. We would occasionally expect to observe, even in quiescent nuclei, the tidal disruption of a star. Exactly how this happens would depend on distinctive precession effects around a Kerr metric, but the gas dynamics are so complex that even when a flare is detected it will not serve as a useful diagnostic of the metric in the strong-field domain. There are however several encouraging new possibilities.

1.6.2 X-ray spectroscopy of accretion flows

Optical spectroscopy tells us a great deal about the gas in AGNs. However, the optical spectrum originates quite far from the hole. This is because the innermost regions would be so hot that their thermal emission emerges as more energetic quanta. X-rays are a far more direct probe of the relativistic region. The appearance of the inner disc around a hole, taking doppler and gravitational shifts into account, along with light bending, was first calculated by Bardeen and Cunningham [31] and subsequently by several others (eg [32]). There is of course no short-term hope of actually ‘imaging’ these inner discs (Though an X-ray interferometer called MAXIM, with elements separated by 500 km, is being studied.) However, we need not wait that long for a probe, because the large frequency-shifts predicted in (for instance) the 6.4 keV line from Fe could reveal themselves spectroscopically – substantial gravitational redshifts would be expected, as well as large doppler shifts [33]. Until recently, the energy resolution and sensitivity of X-ray detectors was inadequate to permit spectroscopy of extragalactic objects. The ASCA X-ray satellite was the first with the capability to measure emission line profiles in AGNs. There is already one convincing case [34] of a broad asymmetric emission line indicative of a relativistic disc, and others should soon follow. The value of \((a/m)\) can in principle be constrained too, because the emission is concentrated closer in, and so displays larger shifts, if the hole is rapidly rotating, and there is some evidence that this must be the case in two objects [35, 36].

The recently-launched Chandra and XMM X-ray satellites are now able to extend and refine these studies; they may offer enough sensitivity, in combination with time-resolution, to study flares, and even to follow a
‘hot spot’ on a plunging orbit.

The swing in the polarization vector of photon trajectories near a hole was long ago suggested \[37\] as another diagnostic; but this is still not feasible because X-ray polarimeters are far from capable of detecting the few percent polarization expected.

1.6.3 The Blandford-Znajek process

Back in 1969 Penrose \[38\] showed how energy could in principle be extracted from a spinning hole. Some years later Blandford and Znajek \[39\] proposed an astrophysically-realistic process whereby this might happen: a magnetic field threading a hole (maintained by external currents in, for instance, a torus) could extract spin energy, converting it into directed Poynting flux and electron-positron pairs.

Can we point to objects where this is definitively happening? The giant radio lobes from radio galaxies sometimes spread across millions of lightyears – \(10^{10}\) times larger than the hole itself. If the Blandford-Znajek process is really going on, these huge structures may be the most direct manifestation of an inherently relativistic effect around a Kerr hole.

Jets in some AGNs definitely have Lorentz factors exceeding 10. Moreover, some are probably Poynting-dominated, and contain electron-positron (rather than electron-ion) plasma. But there is still no compelling reason to believe that these jets are energised by the hole itself, rather than by winds and magnetic flux ‘spun off’ the surrounding torus. The case for the Blandford-Znajek mechanism would be strengthened if baryon-free jets were found with still higher Lorentz factors, or if the spin of the holes could be independently measured, and the properties of jets turned out to depend on \(a/m\).

The process cannot dominate unless either the field threading the hole is comparable with that in the orbiting material, or else the surrounding material radiates with low radiative efficiency. These requirements cannot be ruled out, though there has been recent controversy about how plausible they are. (The Blandford-Znajek effect could be important in the still more extreme context of gamma-ray bursts, where a newly formed hole of a few solar masses could be threaded by a field exceeding \(10^{15}\) G.)

1.6.4 What is the expected spin?

The spin of a hole affects the efficiency of ‘classical’ accretion processes; the value of \(a/m\) also determines how much energy is in principle extractable by the Blandford-Znajek effect. Moreover, the orientation of the spin axis may be important in relation to jet production, etc.
Spin-up is a natural consequence of prolonged disc-mode accretion: any hole that has (for instance) doubled its mass by capturing material that is all spinning the same way would end up with $a/m$ being at least 0.5. A hole that is the outcome of a merger between two of comparable mass would also, generically, have a substantial spin. On the other hand, if it had gained its mass from capturing many low-mass objects (holes, or even stars) in randomly-oriented orbits, $a/m$ would be small.

### 1.6.5 Precession and alignment

Most of the extensive literature on gas dynamics around Kerr holes assumes that the flow is axisymmetric. This assumption is motivated not just by simplicity, but by the expectation that Lense-Thirring precession would impose axisymmetry close in, even if the flow further out were oblique and/or on eccentric orbits. Plausible-seeming arguments, dating back to the pioneering 1975 paper by Bardeen and Petterson [40], suggested that the alignment would occur, and would extend out to a larger radius if the viscosity were low because there would be more time for Lense-Thirring precession to act on inward-spiralling gas. However, later studies, especially by Pringle, Ogilvie, and their associates, have shown that naive intuitions can go badly awry. The behaviour of the ‘tilt’ is much more subtle; the effective viscosity perpendicular to the disc plane can be much larger than in the plane. In a thin disc, the alignment effect is actually weaker when viscosity is low. What happens in a thick torus is still unclear, and will have to await 3-D gas-dynamical simulations.

The orientation of a hole’s spin and the innermost flow patterns could have implications for jet alignment. An important paper by Pringle and Natarajan [41] shows that ‘forced precession’ effects due to torques on a disc can lead to swings in the rotation axis that are surprisingly fast (i.e. on timescales very much shorter than the timescale for changes in the hole’s mass).

### 1.6.6 Stars in relativistic orbits?

Gas-dynamical phenomena are complicated because of viscosity, magnetic fields etc. It would be nice to have a ‘cleaner’ and more quantitative probe of the strong-field regime: for instance, a small star orbiting close to a supermassive hole. Such a star would behave like a test particle, and its precession would probe the metric in the ‘strong field’ domain. These interesting relativistic effects, have been computed in detail by Karas and Vokrouhlicky [42,43]. Would we expect to find a star in such an orbit?

An ordinary star certainly cannot get there by the kind of ‘tidal capture’ process that can create close binary star systems. This is because the
binding energy of the final orbit (a circular orbit with the same angular momentum as an initially near-parabolic orbit with pericentre at the tidal-disruption radius) would have to be dissipated within the star, and that cannot happen without destroying it. An orbit can however be ‘ground down’ by successive impacts on a disc (or any other resisting medium) without being destroyed [44]: the orbital energy then goes almost entirely into the material knocked out of the disc, rather than into the star itself. And there are other constraints on the survival of stars in the hostile environment around massive black holes – tidal dissipation when the orbit is eccentric, irradiation by ambient radiation, etc. [45,46]. They can be thought of as close binary star systems with extreme mass ratios.

These stars would not be directly observable, except maybe in our own Galactic Centre. But they might have indirect effects: such a rapidly-orbiting star in an active galactic nucleus could signal its presence by quasiperiodically modulating the AGN emission.

### 1.6.7 Gravitational-wave capture of compact stars

Neutron stars or white dwarfs circling close to supermassive black holes would be impervious to tidal dissipation, and would have such a small geometrical cross section that the ‘grinding down’ process would be ineffective too. On the other hand, because they are small they can get into very tight orbits by straightforward stellar-dynamical processes. For ordinary stars, the ‘point mass’ approximation breaks down for encounter speeds above 1000 km/s – physical collisions are then more probable than large-angle deflections. But there is no reason why a ‘cusp’ of tightly bound compact stars should not extend much closer to the hole. Neutron stars or white dwarfs could exchange orbital energy by close encounters with each other until some got close enough that they either fell directly into the hole, or until gravitational radiation became the dominant energy loss. When stars get very close in, gravitational radiation losses become significant, and tend to circularise an elliptical orbit with small pericentre. Most such stars would be swallowed by the hole before circularisation, because the angular momentum of a highly eccentric orbit ‘diffuses’ faster than the energy does due to encounters with other stars, but some would get into close circular orbits [47,48].

A compact star is less likely than an ordinary star in similar orbit to ‘modulate’ the observed radiation in a detectable way. But the gravitational radiation (almost periodic because the dissipation timescale involves a factor $(M_{\text{hole}}/m^*)$) would be detectable.
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1.6.8 Scaling laws and ‘microquasars’

Two galactic X-ray sources that are believed to involve black holes generate double radio structures that resemble miniature versions of the classical extragalactic strong radio sources. The jets have been found to display apparent superluminal motions across the sky, indicating that, like the extragalactic radio sources, they contain plasma that is moving relativistically. [49]

There is no reason to be surprised by this analogy between phenomena on very different scales. Indeed, the physics is exactly the same, apart from very simple scaling laws. If we define \( l = L/L_{\text{Ed}} \) and \( \dot{m} = \dot{M}/\dot{M}_{\text{crit}} \), where \( \dot{M}_{\text{crit}} = L_{\text{Ed}}/c^2 \), then for a given value of \( \dot{m} \), the flow pattern may be essentially independent of \( M \). Linear scales and timescales, of course, are proportional to \( M \), and densities in the flow scale as \( M^{-1} \). The physics that amplifies and tangles any magnetic field may be scale-independent, and the field strength \( B \) scales as \( M^{-1/2} \). So the bremsstrahlung or synchrotron cooling timescales go as \( M \), implying that \( t_{\text{cool}}/t_{\text{dyn}} \) is insensitive to \( M \) for a given \( \dot{m} \). So also are ratios involving, for instance, coupling of electron and ions in thermal plasma. Therefore, the efficiencies and the value of \( l \) are insensitive to \( M \), and depend only on \( \dot{m} \). Moreover, the form of the spectrum, for given \( \dot{m} \), depends on \( M \) only rather insensitively (and in a manner that is easily calculated).

The kinds of accretion flow inferred in, for instance, the centre of the giant galaxy M87, giving rise to a compact radio and X-ray source, along with a relativistic jet, could operate just as well if the hole mass was lower by a hundred million, as in the galactic sources. So we can actually study the same processes involved in AGNs in microquasars close at hand within our own galaxy. And these miniature sources may allow us to observe, in our own lifetimes, a simulacrum of the entire evolution of a strong extragalactic radio source, its life-cycle speeded up by a similar factor.

1.6.9 Discoseismology

Discs or tori that are maintained by steady flow into a black hole can support vibrational modes [50-52]. The frequencies of these modes can, as in stars, serve as a probe for the structure of the inner disc or torus. The amplitude depends on the importance of pressure, and hence on disc thickness; how they are excited, and the amplitude they may reach, depends, as in the Sun, on interaction with convective cells and other macroscopic motions superimposed on the mean flow. But the frequencies of the modes can be calculated more reliably. In particular, the lowest g-mode frequency is close to the maximum value of the radial epicyclic frequency \( k \). This epicyclic frequency is, in the Newtonian domain, equal
to the orbital frequency. It drops to zero at the innermost stable orbit. It has a maximum at about $9GM/c^2$ for a Schwarzschild hole; for a Kerr hole, $k$ peaks at a smaller radius (and a higher frequency for a given $M$). The frequency is 3.5 times higher for $(a/m) = 1$ than for the Schwarzschild case.

Novak and Wagoner [52] pointed out that these modes may cause an observable modulation in the X-ray emission from galactic black hole candidates. Just such quasi-periodicities have been seen. The amplitude is a few percent (somewhat larger at harder X-ray energies) suggesting that the oscillations involve primarily the hotter inner part of the disc. In one object, known as GRS 1915+105 (a galactic object which also emits relativistic jets), the fluctuation spectrum showed a peak in Fourier space at around 67 Hz. This frequency does not change even when the X-ray luminosity doubles, suggesting that it relates to a particular radius in the disc. If this is indeed the lowest g-mode, and if the simple disc models are relevant, then the implied mass is $10.2 M_\odot$ for Schwarzschild, and $35 M_\odot$ for a ‘maximal Kerr’ hole [52]. Several other X-ray sources have been found to display quasi-periodicities, and other regularities: for instance, two superimposed frequencies which change, but in such a way that the difference between them is constant. If such regularities can be understood, they offer the exciting prospect of inferring $(a/m)$ for holes whose masses are independently known.

1.7 Gravitational Radiation as a Probe

1.7.1 Gravitational waves from newly-forming massive holes?

The gravitational radiation from black holes offers impressive tests of general relativity, involving no physics other than that of spacetime itself.

At first sight, the formation of a massive hole from a monolithic collapse might seem an obvious source of strong wave pulses. The wave emission would be maximally intense and efficient if the holes formed on a timescale as short as $(r_g/c)$, where $r_g = (GM/c^2)$ – something that might happen if they built up via coalescence of smaller holes (cf ref [21]).

If, on the other hand, supermassive black holes formed from collapse of an unstable supermassive star, then the process may be too gradual to yield efficient gravitational radiation. That is because post-Newtonian instability is triggered at a radius $r_i \gg r_g$. Supermassive stars are fragile because of the dominance of radiation pressure: this renders the adiabatic index only slightly above $4/3$ (by an amount of order $10^{-1/2}$ $M_\odot$). Since $= 4/3$ yields neutral stability in Newtonian theory, even the small post-Newtonian corrections then destabilize such ‘superstars’. The characteristic collapse timescale when instability ensues is longer than $r_g/c$ by
1.7 Gravitational Radiation as a Probe

1.7.1 The post-Newtonian instability is suppressed by rotation. A differentially rotating supermassive star could in principle support itself against post-Newtonian instability until it became very tightly bound. It could then perhaps develop a bar-mode instability and collapse within a few dynamical times. cf ref [14]. To achieve this tightly-bound state without drastic mass loss, the object would need to have deflated over a long timescale, losing energy at no more than the Eddington rate.

The gravitational waves associated with supermassive holes would be concentrated in a frequency range around a millihertz – too low to be accessible to ground-based detectors, which lose sensitivity below 100 Hz, owing to seismic and other background noise. Space-based detectors are needed. There are firm plans, discussed further in the papers by Shutz and Thorne, for the Laser Interferometric Space Array (LISA) – three spacecraft on solar orbit, configured as a triangle, with sides of 5 million km long whose length is monitored by laser interferometry.

1.7.2 Gravitational waves from coalescing supermassive holes.

The strongest signals are expected when already-formed holes coalesce, as the aftermath of mergers of their host galaxies. Many galaxies have experienced a merger since the epoch $z > 2$ when, according to ‘quasar demography’ arguments they acquired central holes. When two massive holes spiral together, energy is carried away by dynamical friction (leading, when the binary is ‘hard’, to expulsion of stars from the galaxy) and also by drag on gas. Eventually the members of the binary get close enough for gravitational radiation (with a timescale proportional to the inverse 4th power of separation) to take over and drive them towards coalescence. In their final coalescence, up to $\sim 10$ per cent of their rest mass as a burst of gravitational radiation in a timescale of only a few times $r_g/c$. These pulses would be so strong that LISA could detect them with high signal-to-noise even from large redshifts. Whether such events happen often enough to be interesting can to some extent be inferred from observations (we see many galaxies in the process of coalescing), and from simulations of the hierarchical clustering process whereby galaxies and other cosmic structures form. The merger rate of the large galaxies believed to harbour supermassive holes: it is only about one event per century, even out to redshifts $z = 4$. However, big galaxies are probably the outcome of many successive mergers. We still have no direct evidence – nor firm theoretical clues – on whether these small galaxies harbour black holes (nor, if they do, of what the hole masses typically are). However it is certainly possible that enough holes of (say) $10^5 M_\odot$ lurk in small early-forming galaxies to yield, via subsequent mergers, more than...
one event per year detectable by LISA [53].

LISA is potentially so sensitive that it could detect the nearly-periodic waves from stellar-mass objects orbiting a $10^5 - 10^6 \, M_\odot$ hole, even at a range of a hundred Mpc, despite the $m^*/M_{\text{hole}}$ factor whereby the amplitude is reduced compared with the coalescence of two objects of comparable mass $M_{\text{hole}}$. The stars in the observed 'cusps' around massive central holes in nearby galaxies are of course (unless almost exactly radial) on orbits that are far too large to display relativistic effects. Occasional captures into relativistic orbits can come about by dissipative processes – for instance, interaction with a massive disc [44]. But unless the hole mass were above $10^8 \, M_\odot$ (in which case the waves would be at too low a frequency for LISA to detect), solar-type stars would be tidally disrupted before getting into relativistic orbits. Interest therefore focuses on compact stars, for which dissipation due to tidal effects or drag is less effective. As already described [47,48], compact stars may get captured as a result of gravitational radiation, which can gradually 'grind down' an eccentric orbit with close pericenter passage into a nearly-circular relativistic orbit. The long quasi-periodic wave trains from such objects, modulated by orbital precession (cf refs [22,23]) in principle carries detailed information about the metric.

The attraction of LISA as an ‘observatory’ is that even conservative assumptions lead to the prediction that a variety of phenomena will be detected. If there were many massive holes not associated with galactic centres (not to mention other speculative options such as cosmic strings), the event rate would be much enhanced. Even without factoring on an ‘optimism factor’ we can be confident that LISA will harvest a rich stream of data.

1.7.3 Gravitational-wave recoil

Is there any way of learning, before that date, something about gravitational radiation? The dynamics (and gravitational radiation) when two holes merge has so far been computed only for cases of special symmetry. The more general problem – coalescence of two Kerr holes with general orientations of their spin axes relative to the orbital angular momentum – is a ‘grand challenge’ computational project being tackled at the Einstein Institute in Potsdam, and at other centres. When this challenge has been met (and one hopes it will not take all the time until LISA flies) we shall find out not only the characteristic wave form of the radiation, but the recoil that arises because there is a net emission of linear momentum.

There would be a recoil due to the non-zero net linear momentum carried away by gravitational waves in the coalescence. If the holes have unequal masses, a preferred longitude in the orbital plane is determined
by the orbital phase at which the final plunge occurs. For spinning holes there may, additionally, be a rocket effect perpendicular to the orbital plane, since the spins break the mirror symmetry with respect to the orbital plane. [54]

The recoil is a strong-field gravitational effect which depends essentially on the lack of symmetry in the system. It can therefore only be properly calculated when fully 3-dimensional general relativistic calculations are feasible. The velocities arising from these processes would be astrophysically interesting if they were enough to dislodge the resultant hole from the centre of the merged galaxy. The recoil might even be so violent that the merged hole breaks loose from its galaxy and goes hurtling through intergalactic space. This disconcerting thought should at least impress us with the reality and ‘concreteness’ of the extraordinary entities to whose understanding Stephen Hawking has contributed so much.

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