Hybrid Online Sensor Error Detection and Functional Redundancy for Artificial Pancreas Control Systems

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Abstract: Artificial pancreas (AP) control systems rely on signals from glucose sensors to collect glucose concentration (GC) information from people with Type 1 diabetes and compute insulin infusion rates to maintain GC within a desired range. Sensor performance is often limited by sensor errors, communication interruptions and noise. A hybrid online sensor error detection and functional redundancy system is developed to detect errors in online signals, and replace erroneous or missing values detected with model-based estimates. The proposed hybrid system relies on two techniques, an outlier-robust Kalman filter (ORKF) and a locally-weighted partial least squares (LW-PLS) regression model. This leverages the advantages of automatic measurement error elimination with ORKF and data-driven prediction with LW-PLS. A novel method called nominal angle analysis is proposed to distinguish between signal faults and large changes in sensor values caused by real dynamic changes in the metabolism. The performance of the system is illustrated with clinical data from continuous glucose monitoring sensors collected from people with Type 1 diabetes.

Keywords: Functional sensor redundancy, sensor error detection, Kalman filter, locally weighted partial least squares regression, glucose concentrations, artificial pancreas.

1. INTRODUCTION

For people with Type 1 diabetes (T1D), frequent glucose concentration (GC) measurements is critical for maintaining their GC in a desired range in spite of disturbances such as meals and exercise by adjusting insulin delivered to their body. Continuous glucose monitoring (CGM) provides real time GC information at high frequency. The Artificial pancreas (AP) system uses GC data to manipulate insulin infusion rates from an insulin pump to the body. The accuracy of GC provided by CGM is critical for GC management with AP systems. Outliers, missing data, signal drift or noise will make the CGM signal unreliable and cause the AP control systems to suggest incorrect insulin infusion flow rates that can drive the AP user to hypoglycemia or hyperglycemia. Hence, a sensor error detection and functional redundancy (SED&FR) module would improve the performance of AP systems.

Methods based on Kalman filters have been proposed to detect CGM failures by setting a threshold of difference between GC prediction from the filter and CGM data (Bequette, 2010). Error detection should be followed by data reconciliation to estimate ‘close enough’ GC values to compute good insulin infusion rates by the AP.

Fault detection and diagnoses (FDD) methods can be grouped into three categories: Quantitative model based, qualitative model based, and process-history-based (data-driven) techniques (Venkatasubramanian et al., 2003). For model-based techniques, prior process knowledge (either quantitative or qualitative) is needed to develop the model for computing residuals between measured and predicted values and compare the residual with a threshold to determine the presence of an error. The models can be first principles, empirical or experiential. For process-history-based techniques, large amounts of historical process data are used to create a database of fault patterns, and compute statistical limits that indicate significant deviations in sensor readings (Raich and Cinar, 1996). Qualitative model-based techniques have also been integrated with data-driven techniques to leverage the power of multivariate statistical approaches and knowledge-based systems (Undey et al., 2003). Error detection methods based on Kalman filter (Kobayashi and Simon, 2007, Wei et al., 2010), artificial neural networks (ANN) (Talebi et al., 2009) and principal components analysis (PCA) (Wang and Cui, 2005) are some of the model-based and data-driven techniques used in many applications.

Proper control decisions require reconciliation of erroneous sensor values. The development of detailed models for data reconciliation can be challenging, and simple adaptive models provide an attractive option. But the use of erroneous data in updating model parameters can yield a model that includes the effects of sensor errors, and affect error detection and data reconciliation. A hybrid online SED&FR is proposed to address this problem, based on two technologies, the outlier-robust Kalman filter (Ting et al., 2007) (ORKF) and the locally-weighted partial least squares (Kim et al., 2013) (LW-PLS). Our results indicate that the hybrid system increases the probability of rapid detection of sensor faults and the accuracy of the estimated values. The final estimate is based on two different methods that use data in different ways to build the models providing modified sensor values.
A new algorithm, nominal angle analysis, is also proposed to discriminate between sensor faults and sensor readings affected by actual changes in process operation.

The remainder of the paper is structured as follows. The two techniques (ORKF and LW-PLS), and nominal angle analysis method are described in Section 2. The results of the system performance with data from clinical experiments are given in Section 3. The discussion of results and conclusions are provided in Section 4 and Section 5, respectively.

2. METHODS

2.1 Outlier-robust Kalman Filter

The ORKF method has been used to solve many sensor outlier-related problems in various fields such as GPS data analysis (Agamennoni et al., 2011) and robotic systems (Ting et al., 2007). Sensor measurement data $y_k \in \mathbb{R}^{d_1}$ can be described by a Kalman filter system equations with hidden states $x_k \in \mathbb{R}^{d_2}$ where $d_1$ and $d_2$ are the dimensions of outputs (number of sensors) and state variables, respectively:

$$x_k = Ax_{k-1} + s_k$$

$$y_k = Cx_k + v_k$$

where $C \in \mathbb{R}^{d_1 \times d_2}$ is the observation matrix, $A \in \mathbb{R}^{d_2 \times d_2}$ is the state transition matrix, $v_k \in \mathbb{R}^{d_1 \times 1}$ the observation noise at time step $k$, and $s_k \in \mathbb{R}^{d_2 \times 1}$ the state noise at time step $k$. $v_k$ and $s_k$ are uncorrelated mean-zero Gaussian noise: $v_k \sim \text{Normal}(0,R)$, $s_k \sim \text{Normal}(0,Q)$. $R \in \mathbb{R}^{d_1 \times d_1}$ and $Q \in \mathbb{R}^{d_2 \times d_2}$ are diagonal covariance matrices, with $r \in \mathbb{R}^{d_1 \times d_1}$ and $q \in \mathbb{R}^{d_2 \times d_2}$ on their diagonal, respectively. The coefficient matrices representing the system dynamics ($A$, $C$, $R$ and $Q$) are unknown and need to be adapted to fit dynamic changes of a system with time-varying coefficients. Consequently, a Kalman filter with constant coefficients that considers all samples to be part of the data cloud would not be appropriate and could generate erroneous estimates. To overcome these limitations, a Bayesian algorithm that treats the weights associated with each data sample probabilistically is introduced. A scalar gamma-distributed weight $w_k$ is assigned to each observed data sample $y_k$. This entire problem can be treated as an Expectation-Maximization-like likelihood (EM) learning problem and model parameters are calculated by maximizing the log likelihood log $p$ (Ting et al., 2007). Matrices $C$, $A$, $R$ and $Q$ at time step $k$ ($C_k$, $A_k$, ...) can be solved by incorporating the EM update equations to give the outlier-robust Kalman filter (ORKF):

$$x_k' = A_{k-1}(x_{k-1})$$

$$w_k = \frac{1}{b[ (y_k - C_k x_{k-1} )^T \theta_{k-1}^{-1} (y_k - C_k x_{k-1}) ]}$$

$$C_k = \left[ \sum_{i=1}^{n} \Lambda_i^k w_i y_i x_i^T \right] \left[ \sum_{i=1}^{n} \Lambda_i^k w_i x_i x_i^T \right]^{-1}$$

$$A_k = \left[ \sum_{i=1}^{n} \Lambda_i^k x_i x_i^T \right] \left[ \sum_{i=1}^{n} \Lambda_i^k x_i x_i^T \right]^{-1}$$

$$r_{km} = \frac{1}{n} \sum_{i=1}^{n} \Lambda_i^k w_i [y_{im} - C_k (m,:) x_i]^2$$

$$q_{kn} = \frac{1}{n} \sum_{i=1}^{n} \Lambda_i^k [x_{in} - A_k (n,:) (x_k)]^2$$

$$\Sigma_k' = Q_k$$

$$S_k' = (C_k x_k' C_k^T + \frac{\Lambda_i^k}{w_k})^{-1}$$

$$K_k' = \Sigma_k' C_k^T S_k'$$

$$x_k = x_k' + K_k' (y_k - C_k x_k')$$

$$\Sigma_k = (I - K_k' C_k) \Sigma_k'$$

where $m=1, \ldots, d_1$; $n=1, \ldots, d_2$; ($x_k$) is the posterior mean vector of the state $x_k$, $\Sigma_k$ is the covariance matrix of $x_k$, and $S_k$ is the covariance matrix of the residual prediction error at step $k$; $r_{km}$ is the $m$th coefficient of the vector $r_k$; $q_{kn}$ is the $n$th coefficient of the vector $q_k$; $C_k (m,:)$ is the $m$th row of the matrix $C_k$; $A_k (n,:)$ is the $n$th row of the matrix $A_k$, and $\lambda$ is the forgetting factor. In the ORKF, weight $w_k$ is assigned in Eq. 4 to determine the parameters in the model, $a=1$ and $b=1$ is set according to reference (Ting et al., 2007). $w_k$ can be assigned values from 0 to 1.5 depending on the magnitude of the estimated measurement error $(y_k - C_{k-1} x_{k-1}^*)$ compared to the expected measurement error ($R_k$) so that if the measurement error is larger than expected, a small $w_k$ is assigned to this measurement and the model will not be updated to follow the wrong tendency.

Forgetting factor $\lambda$ is also introduced to the ORKF. This way, only the data close to current time step are used to generate the model, which enables the model to adapt to variations in a system with time-varying parameters. Smaller $\lambda$ gives more weight on recent data. $\lambda$ should be selected to make the trajectory of ORKF ($x_k$) follow most of the dynamic changes in CGM while robust to abnormal CGM readings and outliers. The initial values of $A$ and $C$ are usually set to identity matrix since the algorithm is relatively insensitive to the initialization of $A$ and $C$ (Ting et al., 2007). Initial values of $R$ and $Q$ are selected based on the initial estimate of the noise in observed data. Each time there is a new measurement update, the ORKF will calculate an estimated value ($\tilde{y}_{0,k}$) that will be used in SED&FR.

2.2 Locally Weighted Partial Least Squares

Locally weighted regression (LWR)(Cleveland, 1979), also called just in time learning or model-on-demand, constructs a local model by prioritizing samples in a database according to the similarity between them and a query sample.

Assume that $E$ experiments have already finished and there are $S_{e,k}$ ($e \in \{1, 2, ..., E\}$) sample steps in each experiment. A database contains measurements in these historical experiments as input $X \in \mathbb{R}^{N \times M}$ and one-step-ahead prediction as output $Y \in \mathbb{R}^{N \times 1}$. $N$ is the number of pairs of $x_n$ and $y_n$ in matrix $X$ and $Y$. $M$ is the number of variables in one sample $x_n$. Raw data from the database are smoothed with a Savitzky-Golay filter (SGF) (Savitzky and Golay, 1964) to reduce noise, and all variables are mean-centered and scaled by their standard deviation. The $n$th row ($n = 1, 2, ..., N$) of $X$ and $Y$ are $x_n^T$ and $y_n^T$, respectively:

$$x_n = [S_{e,k-\alpha} S_{e,k-\alpha+1} ..., S_{e,k-1}, \Delta S_{e,k-\alpha}, \Delta S_{e,k-\alpha+1} ... ]^T$$

$$\Delta S_{e,k-1}, u_{e,k-\alpha}, u_{e,k-\alpha+1}, ..., u_{e,k-1}]^T$$

$$y_n = S_{e,k}$$
where \( S_{e,k} \) is the sensor signal in \( e \)th experiment at step \( k \) (\( k \in \{a+1, a+2, ..., P_e\} \)). \( \Delta S_{e,k} = S_{e,k} - S_{e,k-1} \). \( a \) denotes the sliding past data window (number of past sampling times) used to generate sample \( x_n \). \( u_{e,k} \) is the signal from other related sensors in \( e \)th experiment at step \( k \) (if there exist more than one other sensor whose values are related to target \( S_{e,k} \), these values can also be added at the end of sample \( x_n \)).

A clustering algorithm (Fig.2) groups all \( x_n \) into five different clusters according to the characteristics of sensor signals. The five different groups are increase period (IP), decrease period (DP), increase followed by decrease period (IDP), decrease followed by increase period (DIP) and steady period (SP) (Fig.1). Assume one period of sensor signal data \( (S_{e,1-a}, S_{e,1-a+1}, ..., S_{e,k-1}) \) as points with coordinates \( XC = [1, 2, ..., a] \) and \( YC = [S_{e,1-a}, S_{e,1-a+1}, ..., S_{e,k-1}] \) in a two dimensional space. \( y_{\text{max}} \) and \( y_{\text{min}} \) are the maximum and minimum values of \( YC \), the tangents of start point \( (1, S_{e,1-a}) \) and the end point \( (a, S_{e,k-1}) \) intersect at \( (X_{\text{int}}, Y_{\text{int}}) \).

The output estimate \( \hat{y}_{LP} \) for one query sample \( x_q \) (having the same format as \( x_n \) using current sensor reading) is calculated by using the following procedure:

Step 1: determine the number of latent variables \( R_{LP} \) and set \( r = 1 \). Step 2: choose the same group of data \( (x_n, y_n) \) in the database clustering algorithm as \( x_q \) using the clustering algorithm in Fig. 2. Assume that there are \( M_o \) samples of \( x_n \) and \( y_n \) in the database that belong to the same group of \( x_q \).

Step 3: calculate the similarity matrix \( \Omega \):

\[
D_n = \sqrt{(x_n - x_q)^T \Omega (x_n - x_q)}
\]

\[
o_n = \exp(- \frac{D_n}{\sigma_d})
\]

\[
\Omega = \text{diag}(\omega_1, \omega_2, ..., \omega_N)
\]

where \( \sigma_d \) is the standard deviation of \( D_n \) (\( n = 1, 2, ..., N \)) and \( \varphi \) is a localization parameter; the similarity decreases steeply when \( \varphi \) is small and gradually when \( \varphi \) is large. \( \theta \in \mathbb{R}^{M_o \times M_o} \) is an input. The LW-PLS reduces to linear PLS when the similarity matrix \( \Omega \) is an identity matrix. Step 4: calculate \( X_r \), \( Y_r \) and \( x_q,r \):

\[
\bar{x}_m = \sum_{n=1}^{N} \omega_n x_{nm} / \sum_{n=1}^{N} \omega_n
\]

\[
\bar{y} = \sum_{n=1}^{N} \omega_n y_n / \sum_{n=1}^{N} \omega_n
\]

\[
X_r = X - 1_N [\bar{x}_1, \bar{x}_2, ..., \bar{x}_M]^T
\]

\[
Y_r = Y - \bar{y}
\]

\[
x_{q,r} = x_q - [\bar{x}_1, \bar{x}_2, ..., \bar{x}_M]^T
\]

where \( 1_N \in \mathbb{R}^N \) denotes the unity vector.

Step 5: derive the \( r \)th latent variable of \( X \):

\[
t_r = X_r V_r
\]

where \( V_r \) is the maximum eigenvector of \( X_r^T \Omega Y_r \) corresponding to the maximum eigenvalue. Step 6: derive the \( r \)th loading vector \( p_r \), and regression coefficient vector \( q_r \):

\[
p_r = \frac{X_r^T \Omega t_r}{t_r^T \Omega t_r}
\]

\[
q_r = \frac{Y_r^T \Omega t_r}{t_r^T \Omega t_r}
\]

Step 7: drive the \( r \)th latent variable of \( x_q \):

\[
t_{q,r} = x_q^* V_r
\]

Step 8: if \( r = R_{LP} \), calculate the output estimation \( \hat{y}_{LP} = \bar{y} + \sum_{r=1}^{R_{LP}} t_{q,r} q_r \) and terminate the estimation. Otherwise, set \( X_{r+1} = X_r - t_r p_r^T \)

\[
Y_{r+1} = Y_r - t_r p_r^T
\]

\[
x_{q,r+1} = x_{q,r} - t_{q,r} p_r
\]

Step 9: set \( r = r + 1 \) and go to step 5.

![Fig.1. Characteristics of sensor signals in different clusters.](image)

![Fig.2. Flow diagram of clustering algorithm for LW-PLS.](image)

### 2.3 Nominal Angle Analysis

For model-based SED&FRs, false positive detections due to model inaccuracy and erroneous signal reconciliation caused by false detections are important problems. This occurs more frequently when the system has large and/or rapid dynamic changes, and the model used in SED&FRs does not have enough information to adapt to such dynamic changes. Since the criterion for error detection is based on the residual between the measured and predicted values, large number of false positives can occur. A simple threshold for signal change \( \Delta S_k \) (\( S_k \), \( \Delta S_k \) and \( u_k \) without subscript \( e \) denote online measurements) is not reliable to distinguish between sensor errors and correct sensor readings caused by rapid changes in sensor readings, since large real dynamic changes in sensor readings can also cause large values of \( \Delta S_k \). From the points of signal smoothness, a continuous process signal with no bias will be smoother than bias signals in general.
Traditionally, a specific threshold is designed for sensor signal change, varying as $\Delta S_k - \Delta S_{k-1}$ instead of $\Delta S_k$, and the sensor reading is considered to be correct if the difference is less than the threshold. But a constant threshold is not suitable for all situations. If a large dynamic change occurs, the sensor signal and its slope will change rapidly. In this case, a large threshold is needed in order not to classify these real sensor readings as faulty. Conversely, sensor readings will not change much during a period where the system is not subjected to large variations, and a small threshold is needed to detect small magnitude sensor bias or early stages of a drift.

The nominal angle analysis (NAA) concept is proposed to distinguish sensor bias from real dynamic changes in sensor readings, and integrated with the SED&FR.

![Fig. 3. Examples for nominal angle $\Phi$ construction](image)

Consider a triangle ($\Delta abc$) illustrated with three examples in Fig. 3. The length ($l_{oa}$) which is the height of the triangle indicates the nominal sensor signal change ($\Delta S^N$), usually, $|\Delta S|_{ave} + |\Delta S|_{std}$ is used as $\Delta S^N$ ($|\Delta S|_{ave}$ is the average deviation of $|\Delta S|_{k}$, $|\Delta S|_{std}$ is the standard deviation of $|\Delta S|_{k}$).

The lengths of segments $l_{ob}$ and $l_{oc}$ are equal to the absolute value of sensor signal change at step $k$ ($\Delta S_k$) and step $k-1$ ($\Delta S_{k-1}$) respectively, and displayed as the basis of the triangle. The location of $l_{oa}$ on the right side or left side depends on the sign of the sensor signal change (positive or negative). The nominal angle $\Phi$ is generated by having $a$ as the vertex and by connecting the point $a$ with $b$ and $c$.

The signal value at time $k$ is assumed to be without a fault if $\Phi < \Phi^{max}$. By determining the values of two tuning parameters $\Delta S^N$ and $\Phi^{max}$, a dynamically varying threshold for $\Delta S_k - \Delta S_{k-1}$ is provided for sensor readings. The up and down thresholds can be determined by $\Phi^{max}$ and $\Delta S^N$:

$$T_{up} = \Delta S^N \left( \tan^{-1} \left( \frac{\Delta S_{k-1}}{\Delta S^N} \right) + \Phi^{max} \right) - \Delta S_{k-1}$$

$$T_{down} = -\Delta S^N \left( \frac{\Delta S_{k-1}}{\Delta S^N} - \tan^{-1} \left( \frac{\Delta S_{k-1}}{\Delta S^N} - \Phi^{max} \right) \right)$$

This allows more variation in sensor dynamics when $\Delta S_k - \Delta S_{k-1}$ is in the same direction as $\Delta S_{k-1}$. Larger $\Phi^{max}$ and $\Delta S^N$ will both relax the threshold, and larger $\Delta S^N$ makes the threshold more sensitive to $\Delta S_{k-1}$. According to the dynamics of sensor data, $\Delta S^N$ and $\Phi^{max}$ should be tuned to minimize the ratio between false positives and true positives.

Besides providing the initial check for sensor signals to reduce the false positives (declaring error when the sensor is working correctly) and the false negatives (missing sensor errors when they exist), the NAA can also be used for data reconciliation. When sensor bias is detected with a hybrid method such as proposed in this work, each method provides a reconciled value. These reconciled values may be different for each method. The reconciled sensor reading can be based on ORKF, LW-PLS, or a combination of the two estimates (in this paper, average of ORKF and LW-PLS estimations is used). If sensor bias is detected, the angles of retuning candidates are compared (Fig. 4). With estimated sensor values $\Delta S^R_{ORKF}$, $\Delta S^R_{LW}$ and $\Delta S^R_{AVE}$ (Eqs. 34-36), the angles $\Phi_{ORKF}$, $\Phi_{LW}$ and $\Phi_{AVE}$ are generated similarly to the method described earlier. In this way, $\Phi_{ORKF}$, $\Phi_{LW}$ and $\Phi_{AVE}$ are compared with $\Phi_{original}$ generated by $\Delta S_k$. If the $\Phi_{original}$ is the smallest angle, then no error is declared. Otherwise, the estimated value with smallest angle is selected to replace the erroneous sensor signal.

![Fig.4. Data reconciliation candidate selection with NAA](image)

$$\Delta S^R_{ORKF} = \hat{y}_{0,k} - S_{k-1}$$

$$\Delta S^R_{LW} = \hat{y}_{LW,k} - S_{k-1}$$

$$\Delta S^R_{AVE} = (\hat{y}_{0,k} + \hat{y}_{LW,k}) / 2 - S_{k-1}$$

2.4 Sensor error detection and retuning

Three types of sensor errors are considered in this paper: missing signal, signal stuck to a constant value (sensor keeps displaying same value) and biased signal (step change, drifts, white noise, and continuous outliers). Missing signals and stuck signals are easy to detect: if there is no signal at step $k$, then there is a missing signal and if the sensor is repeating the same value, then it is signal stuck. For biased signals, two model prediction values from ORKF and LW-PLS are compared with the real sensor signal to find the signal bias.

Figure 5 shows the process for sensor error detection. If the sensor signal is missing or stuck at a constant value, data reconciliation by functional redundancy is initiated. Otherwise, the signal reading is sent to the NAA algorithm to determine if the change in signal value can be considered as erroneous. Then, ORKF and LW-PLS provide estimates of the current signal value ($\tilde{y}_{0,k}$ and $\tilde{y}_{LW,k}$). If the differences between these estimates and $S_k$ for both methods are beyond...
their threshold (T) and any of the estimates of signal value have angles smaller than $\Phi_{\text{original}}$ in NAA, sensor bias will be reported and the signal value is replaced with the estimate with the smallest angle in NAA. The two methods are looking at different aspects of the signal, ORKF is more focused on the smoothness of current signal (if there is any abnormal rapid change), and LW-PLS is based on historical trends (if the current signal trend is similar to the historical reference). Since rapid changes may exist in sensor signal due to large dynamic change in the process, ORKF may indicate a false positive. If there is new dynamic variation that is not included in the database, LW-PLS may report a false positive. By requiring that both methods must report that the sensor signal is beyond threshold makes error detection more reliable and reduces the false alarms.

3. DETECTION OF ERRORS IN CONTINUOUS GLUCOSE SENSOR READINGS

Commercially available glucose sensors have a sampling time of 5 minutes (sampling time used in this study), and the computation time for SED&FR module is 0.2 seconds (CPU: Intel i5-3470, RAM: 8GB, Matlab 2015a). Insulin infusion rate data are also used for building the models for LW-PLS analysis.

For ORKF, the optimal value of $\lambda$ is found to be 0.7 by testing different values in the range [0.5 - 0.95] in increments of 0.05. We set one-step-ahead CGM prediction as the output of the model ($d_1 = 1$). We set 4 hidden states ($d_2 = 4$) in the ORKF to balance maximizing the ratio between number of outliers eliminated and real BG dynamic changes neglected. It is assumed that regular sensor noise is smaller than the dynamic changes in BG, consequently, $R$ and $Q$ are set as $R = 10I$ and $Q = 10^2I$, where $I$ is the identity matrix.

For LW-PLS, a database contains independent clinical data from five different patients with CGM, CGM change and insulin infusion rate as input $X$ and one-step-ahead prediction CGM as output $Y$. Eqs. 14 and 15 can be rewritten as:

$$x_n = \begin{bmatrix} G_{e,k-\alpha} & G_{e,k-\alpha+1} & \ldots & G_{e,k-1} & \Delta G_{e,k-\alpha} & \Delta G_{e,k-\alpha+1} \\ \vdots & \Delta G_{e,k-1} & I_{e,k-\alpha} & \ldots & I_{e,k-1} & \end{bmatrix}^T$$

$$y_n = G_{e,k}$$

where $G_{e,k}$ is the CGM value in $e$th historical experiment at step $k$, $\Delta G_{e,k} = G_{e,k} - G_{e,k-1}$, and $I_{e,k}$ is the insulin infusion rate in $e$th historical experiment at step $k$. The long-term insulin effect on BGC horizon (Hovorka, 2006), $a$, is selected to be 12. Clinical data for 5 patients provide about 10,000 data samples for building the database for LW-PLS. $ST_1$, $ST_2$ are set to 0.05 and -0.05 to make each group have about the same number of samples in the database. The statistic information of the clinical data is given in Table 1.

| $G_{\text{ave}}$ (mg/dl) | $G_{\text{std}}$ (mg/dl) | $|\Delta G|_{\text{ave}}$ (mg/dl/sample) | $|\Delta G|_{\text{std}}$ (mg/dl/sample) | $l_{\text{ave}}$ U/hour | $l_{\text{std}}$ U/hour |
|------------------------|-------------------|---------------------------|---------------------------|-----------------|-----------------|
| 157                    | 58                | 5.23                      | 5.28                      | 2.14            | 4.58            |

Threshold for NAA and model residual $\Delta^S$, $\Phi^{\text{max}}$ and $T$ are set to 10, 20 degrees and 10, respectively to minimize the ratio between false positives and true positives.

Fig.6. Examples of online CGM signal reconciliation (Errors are introduced to CGM data between vertical dashed lines)

Clinical CGM and insulin infusion rate data from 10 subjects with T1D are used for testing the proposed SED&FR module. Since the raw CGM data may have unknown noise, raw data are filtered by SGF to reduce noise. Each patient has 56 hours of data that include CGM and insulin infusion rates. A sensor error generator introduces additional errors to the noise free CGM data. Sensor errors include missing signal, stuck signal value, white noise (add a random number between $\pm \Phi^{\text{max}}$), step change noise (add a constant noise for one period of CGM signal), drift change (a drift increase or decrease), and polynomial outliers (a symmetric second order polynomial shaped noise). Various types of sensor errors are added to data randomly at the frequency of every 2 hours. Certain error amplitudes are used if the error type is sensor bias. Small amplitude white noise (average magnitude of 2 mg/dl) is also added to CGM data to conduct more realistic tests of the system to detect real sensor errors with normal signal noise. Different maximum sensor bias $\Phi^{\text{max}}$ (20, 30, 40 mg/dl) are tested and durations of 2 to 4 samples are assigned to each error. Examples of online CGM signal reconciliation by functional redundancy in all kinds of fault introduced are illustrated in Fig.6. The sensor fault detection results in different test conditions are displayed in Table 2.

| Table 2. Summary of sensor fault detection and reconciliation results |
|---------------------------------------------------------------|
| Duration of the error | 2 samples | 3 samples | 4 samples |
|------------------------|-----------|-----------|-----------|
| $\Phi^{\text{max}}$ (mg/dl) | 20 | 30 | 40 | 20 | 30 | 40 |
| EDRS | 276 | 315 | 330 | 271 | 305 | 337 | 227 | 263 | 296 |
| EDRF | 18 | 12 | 11 | 27 | 10 | 7 | 34 | 29 | 16 |
| FN | 90 | 57 | 43 | 86 | 69 | 40 | 107 | 76 | 56 |
| FP | 112 | 101 | 91 | 135 | 148 | 130 | 137 | 174 | 150 |
| Errors detected (%) | 76.6 | 85.2 | 88.8 | 77.6 | 82.0 | 89.6 | 70.9 | 79.4 | 84.8 |
| Successful reconciliation (%) | 93.9 | 96.3 | 96.8 | 90.9 | 96.8 | 98.0 | 87.0 | 90.1 | 94.9 |

The formula for computing percentages in Tables 2 is given in Eqs. 39-41 (EDRS (Error detected and reconciled...
successfully), EDRF (Error detected but reconciliation failed), 
FN (False positives), FP (False negative).

Percentage of errors detected = \frac{EDRS + EDRF}{EDRS + EDRF + FP} \quad (39)

Percentage of successful reconciliation = \frac{EDRS}{EDRS + EDRF} \quad (40)

R_{FE} = \frac{FP}{EDRS + EDRF + FP} \quad (41)

3408 errors were introduced to the data. EDRS indicates that 
during one continuous sensor error episode, the error is 
detected and the reconciled (estimated) value is close to the 
noise free data for sensor bias or stuck signal, and the 
absolute difference between the estimated value and the noise 
free CGM is smaller than 10 mg/dl for missing signal errors. 
If the estimated value does not satisfy these criteria, the error 
is classified as detected but reconciliation failed (EDRF). If 
no error is reported when there is a sensor error, it is 
classified as missed error, and if a sensor error is reported 
when no error was added to data, it is classified as false 
positive (FP). Overall, 81.66% of sensor errors are detected, 
among which 93.86% of reconciled values successfully 
replace the erroneous sensor signal with a better value. The 
average ratio between the false positives reported and the 
errors detected \(R_{FE}\) is 0.347. As expected, the number 
of false alarms decreases and error detection improves with 
increasing error magnitude. (Table 2). But it may not be 
easier to detect sensor errors that last for a longer period. This 
is because the error is easier to detect at the first sample of a 
continuous error, since the angle for CGM in NAA is larger 
especially for step change and drift change.

4. DISCUSSION

A hybrid SED&FR system that can detect the errors 
accurately is reported. The errors are flagged when both 
methods that depend on different approaches indicate 
the existence of errors. For algorithms that rely only on ORKF or 
LW-PLS, the ratio between the wrong errors reported and the 
errors detected are 1.62 and 2.53, respectively.

This paper focuses on CGM sensor errors. Other AP system 
errors like insulin pump leakage, controller performance 
failures may affect CGM data as well. In these cases, 
additional fault classification should be made to diagnose the 
faült and the smearing effects after an error is detected.

5. CONCLUSIONS

The hybrid error detection and analytical redundancy module 
proposed combines two technologies, ORKF and LW-PLS, 
that together successfully detected most of the sensor errors 
and reconciled sensor readings for almost all of the detected 
errors with values close to the real reported value while the 
number of erroneous and missed error detections is kept low.

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