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Outer automorphisms of algebraic groups and determining groups by their maximal tori.
(English) [Zbl 1277.20057]
Mich. Math. J. 61, No. 2, 227-237 (2012).

The main goal of this paper is to fill in an important missing case of a structural result about linear algebraic groups which goes back to G. Prasad and A. S. Rapinchuk [Publ. Math., Inst. Hautes Étud. Sci. 109, 113-184 (2009; Zbl 1176.22011)]. Essentially, this result allows one to recognise a simple algebraic group over a number field \( K \) knowing just the \( K \)-isomorphism classes of its maximal \( K \)-tori. The original proof [in loc. cit.] omitted the case of groups of type \( D_{2n} \) for \( 2n \geq 4 \), although the case of \( 2n \geq 6 \) was later filled in by the same authors. In this paper the author presents a new proof of the \( 2n \geq 6 \) case and also settles the case of \( 2n = 4 \).

Reviewer: Michael Bate (York)

MSC:
- 20G07 Structure theory for linear algebraic groups
- 20G25 Linear algebraic groups over local fields and their integers
- 20G30 Linear algebraic groups over global fields and their integers

Keywords:
linear algebraic groups; outer automorphisms; maximal tori; Tits algebra; global fields; local fields

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