Dynamic Analysis and Model Predictive Control of Excavator’s Working Device

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Abstract—Aiming at the development requirements of automation and intelligence of traditional hydraulic excavator, the robot modeling theory is applied to the modeling process of it, and the general forward and inverse kinematics models of excavator working device including the hydraulic drive structure were given. Based on Lagrange equation, the dynamics model of excavator is established, and a trajectory tracking control method is proposed for this model. The tracking effect of model predictive control method on preset trajectory under different working conditions is studied by computer simulation technology. The research results show that the model predictive control can give a more accurate feedforward control force under complex working conditions, realize accurate tracking of the trajectory, and effectively reduce the control delay caused by large inertia working conditions, and has good robustness.

1. INTRODUCTION
Hydraulic excavator is widely used in bridge construction, mining, tunnel construction and other aspects, is one of the important mechanical equipment of earthwork, plays a crucial role in national economic construction.

In recent years, China's construction machinery industry has made great progress, XCMG, Sany, Zoomlion and other products have gradually opened the domestic and foreign markets, and can compete with caterpillar, Japan komatsu and other traditional strong enterprises. However, the control system of existing hydraulic excavators mostly adopts open-loop control, or adopts simple PID control model \cite{1,2}, which requires personnel operation, low control accuracy and poor stability, and is inconsistent with the development trend of unmanned, automation and intelligence.

In this paper, the robot modeling theory \cite{3} is applied to the excavator, the forward and inverse kinematics equations of the working device are given, and the Lagrange \cite{4} method is used to derive the dynamics equations, and the corresponding model predictive controller is designed \cite{5,6}.

2. KINEMATICS AND DYNAMICS MODELING OF EXCAVATOR

2.1. Forward Kinematics Analysis
Without considering the walking structure of the excavator, the main working parts (moving parts) are rotary mechanism and mechanical arm, and the mechanical arm includes the maneuvering arm, bucket
rod and bucket. Therefore, the working parts have four degrees of freedom in total, and the general structural form is shown in Fig. 1.

![Figure 1. Establishment of frames of excavator working parts](image)

As shown in the Fig. 1, establish the world frame \( \{W\} \), select vertical upward as the positive direction of z axis, horizontal right as the positive direction of x axis, and y axis is determined by the right hand system. The walking mechanism of excavator realizes three degrees of freedom in plane motion through differential speed on both sides, do not discuss here. Therefore, it is assumed that the excavator walking mechanism is static, and is overlapped with the world frame.

It is only assumed that the initial direction of the coordinate axes in the above frames is the same as that in the world frame, and the above frames are all moving frames, then the movement process of the bucket in the world frame is described as follows:

1. The translation distance along the z-axis is \( z_1 \), then the rotation about the z-axis is \( \theta_1 \), and it becomes frame \( \{A\} \).
2. Moves \( z_2 \) along the z-axis, \( x_2 \) along the x-axis, rotates \( \theta_2 \) around the y-axis, and becomes frame \( \{B\} \).
3. Moves \( z_3 \) along the z-axis, \( x_3 \) along the x-axis, rotates \( \theta_3 \) around the y-axis, and becomes frame \( \{C\} \).
4. Moves \( z_4 \) along the z-axis, \( x_4 \) along the x-axis, rotates \( \theta_4 \) around the y-axis, and becomes frame \( \{D\} \).

The above process can be expressed in homogeneous coordinate form as:

\[
T_i = \begin{bmatrix}
 c\theta_i & -s\theta_i & 0 & 0 \\
 s\theta_i & c\theta_i & 0 & 0 \\
 0 & 0 & 1 & z_i \\
 0 & 0 & 0 & 1
\end{bmatrix}
\quad T_i = \begin{bmatrix}
 c\theta_i & 0 & s\theta_i & x_i \\
 -s\theta_i & 0 & c\theta_i & z_i \\
 0 & 0 & 0 & 1
\end{bmatrix}
\]

Therefore, the pose of the frame \( \{D\} \) of the last end bucket is:

\[
T = T_4T_3T_2T_1
\]

The driving part of the excavator is the hydraulic cylinder, and the linear movement of the hydraulic rod is transformed into the rotation of the joint by the restriction of the mechanical structure. As shown in Fig. 1, EF, GH and IJ are hydraulic cylinders in \( \Delta BEF \), \( \Delta CGH \) and \( \Delta DJI \). Assume that the initial lengths of the hydraulic cylinders are \( l_{EF} \), \( l_{GH} \) and \( l_{IJ} \) respectively, the elongations are \( \Delta l_{EF} \), \( \Delta l_{GH} \) and \( \Delta l_{IJ} \) respectively. Select \( \Delta BEF \) as the research object:
\[ \theta_2 = \arccos \left( \frac{l_{aw}^2 + l_{bc}^2 - (l_{aw} + l_{bc})^2}{2l_{aw}l_{bc}} \right) - \arccos \left( \frac{l_{aw}^2 + l_{bc}^2 - l_{cd}^2}{2l_{aw}l_{bc}} \right) \]  

\( \Delta \text{CGH} \) and \( \Delta \text{DIJ} \) are processed in the same way.

### 2.2 Inverse Kinematic Analysis

Suppose the pose of the end bucket \{D\} frame is:

\[
T = \begin{bmatrix}
    a_x & b_x & c_x & d_x \\
    a_y & b_y & c_y & d_y \\
    a_z & b_z & c_z & d_z \\
    0   & 0   & 0   & 1
\end{bmatrix}
\]  

(4)

By comparing formulas (2) and (4) it can be found that the position of the \{D\} frame of the end bucket is only related to \( \theta_1 \), \( \theta_2 \) and \( \theta_3 \).

\[
s_1 = \frac{d_y}{c_y} \Rightarrow \theta_1 = \arctan \left( \frac{d_y}{d_x} \right)
\]  

(5)

At the same time, only the \{D\} position of the end bucket is considered, which can be described as:

\[
d_x = c_x (l_{aw} s_y + l_{bc} c_y + l_{cd} c_z) \\
d_y = s_x (l_{aw} c_y + l_{bc} s_y + l_{cd} c_z) \\
d_z = z_x + l_{aw} s_y + l_{bc} c_y + l_{cd} s_z
\]  

(6)

where

\[
l_{aw} s_y = z_x \quad l_{bc} = \sqrt{z_x^2 + z_y^2} \quad l_{cd} = \sqrt{z_x^2 + z_y^2}
\]

Where \( \theta_1 \) is defined as the Angle between AB and the horizontal plane.

Solve the equation (6) and the result is:

\[
\theta_2 = \arccos \left( \left( \frac{d_x}{c_x} - l_{aw} s_x \right)^2 + \left( d_y - z_x - l_{aw} s_y \right)^2 - (l_{aw}^2 + l_{bc}^2) \right) \left( \frac{1}{2l_{aw} l_{bc}} \right)
\]  

(7)

\( \theta_2 \) is determined by \( \theta_3 \).

### 2.3 Dynamics Analysis

Assume that the centroids of the rotary body, maneuvering arm, bucket rod and bucket are \( m_1 \), \( m_2 \), \( m_3 \) and \( m_4 \), the distances to frame \( \{A\} \), \( \{B\} \), \( \{C\} \) and \( \{D\} \) are \( l_{aw} \), \( l_{aw} \), \( l_{aw} \) and \( l_{aw} \) respectively, the coordinates of the center of mass are:

\[
\begin{align*}
\{m_1\}: & \begin{bmatrix} l_{aw} c_{n2}\xi_1 \\ l_{aw} c_{n2}\eta_1 \\ z_x + l_{aw} s_{n2} \end{bmatrix} \\
\{m_2\}: & \begin{bmatrix} c_x (l_{aw} c_{n2} + l_{aw} c_{n2}) \\ s_x (l_{aw} c_{n2} + l_{aw} c_{n2}) \\ z_x + l_{aw} s_{n2} + l_{aw} s_{n2} \end{bmatrix} \\
\{m_3\}: & \begin{bmatrix} s_x (l_{aw} c_{n2} + l_{aw} s_{n2} + l_{aw} c_{n2}) \\ c_x (l_{aw} c_{n2} + l_{aw} c_{n2} + l_{aw} c_{n2}) \\ z_x + l_{aw} s_{n2} + l_{aw} s_{n2} \end{bmatrix} \\
\{m_4\}: & \begin{bmatrix} c_x (l_{aw} c_{n2} + l_{aw} s_{n2} + l_{aw} s_{n2}) \\ s_x (l_{aw} c_{n2} + l_{aw} s_{n2} + l_{aw} c_{n2} + l_{aw} c_{n2}) \\ z_x + l_{aw} s_{n2} + l_{aw} s_{n2} + l_{aw} s_{n2} \end{bmatrix}
\end{align*}
\]  

(8)

Where, \( c_{n2} \) represents the cosine of the Angle between the center of mass and the horizontal plane, \( \theta_{n2} \), rotated by \( \theta_2 \). The centroid velocity can be obtained by taking the time derivative of the joint variables in the above equation.

So the Lagrangian \( L \) of the system is:

\[
L = TV
\]  

(9)
Where $T$ is the system kinetic energy and $V$ is the system potential energy. They are calculated separately from the following equations:

$$T = \frac{1}{2} \sum_{i=1}^{4} m_i \| V_i \|^2$$

$$V = g \sum_{i=1}^{4} m_i z_i$$

(10)

The equation of motion of the system is expressed by Lagrange equation:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \tau$$

(11)

Where, $q_i$ represents the generalized coordinates of $\theta_i \sim \theta_4$ and $\dot{q}_i$ represents the generalized velocities of $\theta_i \sim \dot{\theta}_4$.

Combine formula (8)-(11) and sort it out:

$$M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = \tau$$

(12)

Where $M(q)$ is the 4 × 4 mass matrix, $C(q, \dot{q})$ is a 4 × 1 vector of centrifugal and Coriolis terms, $G(q)$ is a 4 × 1 vector of gravity terms, $\tau$ is the joint generalized driving force.

3. MODEL PREDICTIVE CONTROL

3.1. Establishment of Predictive Model

Assuming that during operation, the external force on the excavator is $f$, which is introduced into the dynamic equation of the excavator, it can be obtained:

$$\dot{q} = \left( \tau + J^T f - (C(q, \dot{q}) \dot{q} + G(q)) \right) / M(q)$$

(13)

Where $J^T$ is the transpose of the excavator Jacobian matrix.

Select state vector $Q = [q \quad \dot{q}]^T$, and establish the system state equation:

$$\dot{Q} = AQ + B\tau + C$$

(14)

Where $A = \begin{bmatrix} 0 & 1 \\ C & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \\ \tau \end{bmatrix}$, $C = \begin{bmatrix} 0 \\ J^T f - G(q) \\ M(q) \end{bmatrix}$.

3.2. Discretization of Prediction Models

Assuming the sampling period is $T$, using the forward Euler method to discretize the state equation:

$$\frac{Q(k+1) - Q(k)}{T} = A\bar{Q}(k) + B\bar{\tau}(k) + \bar{C}$$

(15)

Where $A = \begin{bmatrix} 1 & T \bar{C} \bar{Q}(k) \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ T \\ \bar{M}(q(k)) \end{bmatrix}$, $C = \begin{bmatrix} 0 \\ T (J^T f - G(q(k))) \\ M(q(k)) \end{bmatrix}$, $k$ is the k-th sampling period.

3.3. Prediction

The system state predicted in the future $p$ sampling periods is:

$$X_p = \left[ \begin{array}{c} Q(k+1|k) \quad Q(k+2|k) \quad \ldots \quad Q(k+p|k) \end{array} \right]$$

(16)

Where $p$ is the prediction time domain, $(k+1|k)$ indicates the prediction at k-th sampling period of the system state at (k+1)-th sampling period, and so on.

In addition, to predict the future state of a dynamic system, it is necessary to know the control quantity $U_k$ in the prediction time domain:
Through the analysis of the previous chapter, it can be found that the dynamic system is a nonlinear system, and the coefficient of the discrete system is related to the system state at the sampling time. Therefore, we must make necessary simplification, it is assumed that the coefficients \( A, B \) and \( C \) vary linearly over \( p \) prediction time periods (therefore the coefficient \( p \) should be as small as possible), take the value of the intermediate prediction period, and maintain the value within the period \([k, k + p]\):

\[
\begin{align*}
\mathcal{A} &= \begin{bmatrix}
1 \\
TC(q(k+n|k),q(k+n|k)) \\
M(q(k+n|k))
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\mathcal{B} &= \begin{bmatrix}
0 \\
T \\
M(q(k+n|k))
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\mathcal{C} &= \begin{bmatrix}
0 \\
T(J^Tf - G(q(k+n|k))) \\
M(q(k+n|k))
\end{bmatrix}
\end{align*}
\]

(18)

Where \( n = \frac{p}{2} \) if \( p \) is even; When \( p \) is odd, \( n = (p+1)/2 \).

Now, the state of the system for \( p \) control cycles in the future can be predicted successively through the discretized state equation:

\[
Q(k+1|k) = \mathcal{A}Q(k) + \mathcal{B}r(k|k) + \mathcal{C}
\]

\[
Q(k+2|k) = \mathcal{A}^2Q(k) + \mathcal{A}\mathcal{B}r(k|k) + \mathcal{A}\mathcal{C}
\]

\[
Q(k+3|k) = \mathcal{A}^3Q(k) + \mathcal{A}^2\mathcal{B}r(k|k) + \mathcal{A}^2\mathcal{C}
\]

\[
Q(k+p|k) = \mathcal{A}^pQ(k) + \sum_{i=0}^{p-1}\mathcal{A}^{p-i-1}\mathcal{B}r(k+i|k) + \sum_{i=0}^{p-1}\mathcal{A}^{p-i-1}\mathcal{C}
\]

Integrate into matrix form:

\[
X_k = \Psi Q(k) + \Theta U_k + \Xi
\]

(19)

Where \( \Psi = [\mathcal{A}^T \mathcal{A}^T \ldots \mathcal{A}^T] \), \( \Theta = \begin{bmatrix}
0 & 0 & \mathcal{A}^{p-2}\mathcal{B} & \mathcal{A}^{p-2}\mathcal{C} & \ldots & \mathcal{A}^{p-2}\mathcal{C}
\end{bmatrix} \), \( \Xi = \begin{bmatrix}
0 & 0 & \mathcal{A}^{p-2}\mathcal{C} & \ldots & 0 & 0
\end{bmatrix} \).

The lower triangle form in the above formula can directly reflect the causal relationship of the system in time, that is, the future time has no influence on the current output, and the output of the system at the future time is completely determined by the current system state and system input.

3.4. Optimization

The goal of the control system is to make the system state of excavator track the desired trajectory, define the desired trajectory as the reference value, and select the reference value sequence in the prediction time domain \( p \):

\[
R_k = \begin{bmatrix}
r(k+1)^T & r(k+2)^T & \ldots & r(k+p)^T
\end{bmatrix}^T
\]

(20)
We hope to find the best control quantity $U_k$, so that the state output in the time domain of control prediction is as close as possible to the reference value. This is an open-loop optimal control problem, which can provide more accurate feedforward input value of the system after optimization. Therefore, we define the cumulative error between the predicted state vector and the reference value as the optimization objective function of the system:

$$J(U_k) = (X_k - R_k)^T V (X_k - R_k)$$  \hspace{1cm} (21)

In general, we don’t expect the control instruction to be too large, so we can add constraints on the control quantity in the optimization objective function:

$$J(U_k) = (X_k - R_k)^T V (X_k - R_k) + U_k^TWU_k$$  \hspace{1cm} (22)

Therefore, the optimization problem can be described as:

$$\min_{U_k} J(U_k)$$

s.t. $|\tau(k+i|k)| \leq \tau_{max}, i = 0, 1, 2, \ldots, p-1$  \hspace{1cm} (23)

Expand formula (22) and unite like terms:

$$J(U_k) = (X_k - R_k)^T V (X_k - R_k) + U_k^TWU_k$$

$$= (E + \Theta U_i)^T V (E + \Theta U_i) + U_k^TWU_k$$

$$= E^TVE + (\Theta U_i)^T V (\Theta U_i) + 2E^T V(\Theta U_i) + U_k^TWU_k$$

$$= U_k^T \left( \Theta^T Y + W \right) U_k + (2E^T Y) U_i + E^TVE$$

$$= U_k^T H U_k + f^T U_i + E^TVE$$  \hspace{1cm} (24)

In the above formula, $E^TVE$ is constant and has no effect on the minimization, $H = 2(\Theta^T Y + W)$, $f^T = 2E^T Y$, the above equation is a standard quadratic optimization objective function. The optimization sequence $U_i$ can be solved and the first element of $U_i$ can be applied to the control at time $k$.

![Model predictive control block diagram of excavator.](image)

**Figure 2.** Model predictive control block diagram of excavator.

4. **Simulation establishment and result analysis**

In order to quickly verify the effectiveness of the control algorithm, a control program is built in Matlab Simulink.

Mining loading action is adopted in simulation. Mining loads a cyclic simulation action model referring to industry standards, and the action process is shown in Fig. 4. In order to better simulate the effectiveness of the algorithm, no-load and load excavation experiments are carried out, in which the load excavation experiment assumes that the earthwork weight is known in the excavation process.
4.1. Mining simulation experiment

The difference between the experimental process in this section and the mining process in section 3.1:

In stage b), increasing horizontal resistance and vertical force are added to the bucket to simulate the accumulation process of earthwork excavation;

In stage c), the decreasing horizontal resistance is increased to 0 and the increasing vertical force is added to the bucket;

Maintain maximum vertical force in stage d);

The vertical force is reduced to 0 in the stage e).

Assume that the above processes are uniformly applied, as shown in the figure:

4.2. Simulation results and analysis

In the working condition of section 3.1, the track tracking effect of each joint of excavator is good. The comparison of two experiments shows that the trajectory control algorithm has universality and can adapt to different working conditions. The change of force curve is also stable, and the difference of control force between the two simulations is large, indicating that the change of terminal working condition will be transmitted to the joint successively through the series arm, which is consistent with the force derivation of Newton-Euler recursive method in the mechanical arm, and the control effect of this control algorithm is good.
8

5. CONCLUSIONS
The following conclusions are obtained: (1) Ignoring the influence of hydraulic rod, the dynamic modeling of excavator can be adopted in the same way as that of mechanical arm; (2) The control effect of model predictive control is good, but it needs a specific dynamic model and requires a large amount of calculation; (3) Model predictive control can effectively eliminate the jitter caused by the traditional control method, and the control is more rapid and accurate; (4) The algorithm can adapt to different working conditions, but it needs to detect the working conditions, and can be obtained by parameter identification.

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