Hadron-Quark Phase Transition in Quark-Hybrid Stars

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1 Introduction

The recent discovery of the two-solar mass neutron stars $J1614 - 2230 (1.97 \pm 0.04 M_\odot)$ [1] and $J0348 + 0432 (2.01 \pm 0.04 M_\odot)$ [2] allows us to consider the possible existence of deconfined quarks in the cores of neutron stars [3, 4]. Nevertheless if the dense interior of a neutron star is indeed converted to quark matter, it must be three-flavor quark matter since it has lower energy than two-flavor quark matter. And just as for the hyperon content of neutron stars, strangeness is not conserved on macroscopic time scales, which allows neutron stars to convert confined hadronic matter to three-flavor quark matter until equilibrium brings this process to a halt. As first realized by Glendenning [5], the presence of quark matter in neutron stars enables the hadronic regions of the mixed phase to become more isospin symmetric than in the pure phase by transferring electric charge to the quark phase. The symmetry energy can be lowered thereby at only a small cost in rearranging the quark
Fermi surfaces. The stellar implication of this charge rearrangement is that the mixed phase region of a neutron star will have positively charged regions of nuclear matter and negatively charged regions of quark matter \[5\]. This should have important implications for the electric and thermal properties of neutron stars. First studies of the transport properties of quark-hybrid neutron star matter have been reported in \[6, 7\].

It has been shown \[8, 9, 10, 11\] that the appearance of a mixed phase of quarks and hadrons in NSs depends on the surface tension between nuclear and quark matter, which should be around \(5 - 30 \text{ MeV/fm}^3\). Although recent studies about the nucleation process during the phase transition predict the value of the surface tension, their results vary and are strongly model dependent \[12, 13, 14, 15, 16\]. Thus, the discussion concerning the appearance of a mixed phase in NSs remains open.

Our study is based on NSs containing deconfined quark matter, i.e. quark-hybrid stars (QHSs). To describe the quark matter phase, we use a non-local extension of the SU(3) Nambu Jona-Lasinio (NJL) model with vector interactions, whereas to represent the hadronic phase we consider a non-linear Walecka model using parametrization NL3 \[17\]. A phase transition between these two phases can be constructed via the Gibbs conditions, imposing global electric charge neutrality and baryon number conservation. We find that the non-local NJL model predicts the existence of extended regions of mixed quark-hadron (quark-hybrid) matter in high-mass neutron stars with masses of 2.0 to 2.4\(M_\odot\).

2 Modeling of the Mixed Phase

To determine the mixed phase region of quarks and hadrons, we start from the Gibbs condition for pressure equilibrium between confined hadronic \((P_H)\) matter and deconfined quark \((P_q)\) matter. The Gibbs condition is given by \[5\]

\[
P^H(\mu^H_b, \mu^H_e, \{\phi\}) = P^q(\mu^q_b, \mu^q_e, \{\psi\}),
\]

with \(\mu^H_b = \mu^q_b\) for the baryon chemical potentials and \(\mu^H_e = \mu^q_e\) for the electron chemical potentials in the hadronic \((H)\) and quark \((q)\) phase, respectively. By definition, the quark chemical potential is given by \(\mu^q_b = \mu_n/3\), where \(\mu_n\) is the chemical potential of the neutron. The quantities \(\{\phi\}\) and \(\{\psi\}\) in Eq. (1) stand collectively for the field variables and Fermi momenta that characterize the solutions to the equations of confined hadronic matter and deconfined quark matter, respectively. In the mixed phase, the baryon number density, \(n_b\), and the energy density, \(\varepsilon\), are given by \[5\]

\[
n_b = (1 - \chi)n^H_b + \chi n^q_b,
\]

and

\[
\varepsilon = (1 - \chi)\varepsilon^H + \chi\varepsilon^q,
\]
where \( n^H_b(\varepsilon^H) \) and \( n^q_b(\varepsilon^q) \) denote the baryon number (energy) densities of the hadron and quark phase, respectively. The quantity \( \chi \equiv V_q/V \) denotes the volume proportion of quark matter, \( V_q \), in the unknown volume \( V \). By definition \( \chi \) varies between 0 and 1 depending on how much confined hadronic matter has been converted to quark matter \[5\]. In addition to the Gibbs condition \[1\] for pressure, the conditions of global baryon number conservation and global electric charge neutrality need to be imposed on the field equations.

\[
\rho_b = \chi \rho_Q(\mu_n, \mu_e) + (1 - \chi) \rho_H(\mu_n, \mu_e, \{\phi\}) ,
\]

where \( \rho_Q \) and \( \rho_H \) denote the baryon number densities of the quark phase and hadronic phase, respectively. The condition of global electric charge neutrality is given by

\[
(1 - \chi) \sum_{i=B,l} q^H_i n^H_i + \chi \sum_{i=q,l} q^q_i n^q_i = 0 ,
\]

where \( q_i \) is the electric charge of the \( i \)-th specie in units of the electron charge.

For the quark sector, within the non-local NJL model, the mean-field thermodynamic potential at zero temperature is \[4\]

\[
\Omega^{NL}(M_f, 0, \mu_f) = -\frac{N_c}{\pi^2} \sum_{f=u,d,s} \int_0^\infty dp_0 \int_0^\infty dp \ln \left\{ \frac{\hat{\omega}_f^2 + M_f^2(\omega_f)}{\omega_f^2 + m_f^2} \right\} - \frac{N_c}{\pi^2} \sum_{f=u,d,s} \int_0^\sqrt{\mu_f^2 - m_f^2} dp \left[ (\mu_f - E_f)\theta(\mu_f - m_f) \right] - \frac{1}{2} \sum_{f=u,d,s} \left( \bar{S}_f \sigma_f \bar{S}_f + \frac{G_s}{2} \bar{S}_f^2 \right) + \frac{H}{2} \bar{S}_u \bar{S}_d \bar{S}_s - \frac{\varpi_f^2}{4G_V} ,
\]

where \( N_c = 3 \), \( E_f = \sqrt{p^2 + m_f^2} \), and \( \omega_f^2 = (p_0 + i\mu_f)^2 + p^2 \). The constituent quark masses \( M_f \) are treated as momentum-dependent quantities and are given by

\[
M_f(\omega_f^2) = m_f + \sigma_f g(\omega_f^2) ,
\]

where \( g(\omega_f^2) \) is the form factor, which we take to be Gaussian \( g(\omega_f^2) = \exp\left(-\omega_f^2/\Lambda^2\right) \).

The inclusion of vector interactions shifts the quark chemical potential as

\[
\mu_f \rightarrow \tilde{\mu}_f = \mu_f - g(\omega_f^2)\varpi_f ,
\]

where \( \varpi_f \) represents the vector mean fields related to the vector current interaction. The inclusion of the form factor in Eq. \[8\] is a particular feature of the non-local
model, which renders the shifted chemical potential momentum dependent. Accordingly, the four momenta \( \omega_f \) in the dressed part of the thermodynamic potential are modified as

\[
\omega_f^2 \rightarrow \tilde{\omega}_f^2 = (p_0 + i \tilde{\mu}_f)^2 + p^2.
\]

(9)

Note that the quark chemical potential shift does not affect the non-local form factor \( g(\omega_f^2) \), as discussed in \[18, 19, 20\], avoiding a recursive problem. In this work we use for the NJL model the parameters listed in Refs. \[3, 4\].

Within the stationary phase approximation, the mean-field values of the auxiliary fields \( \bar{S}_f \) turn out to be related to the mean-field values of the scalar fields \( \bar{\sigma}_f \) \[21\]. They are given by

\[
\bar{S}_f = -16 N_c \int_0^{\infty} dp_0 \int_0^{\infty} \frac{dp}{(2\pi)^3} g(\omega_f^2) \frac{M_f(\omega_f^2)}{\tilde{\omega}_f^2 + M_f^2(\omega_f^2)}.
\]

(10)

Due to the charge neutrality constraint, for the quark phase we consider the three mean-field flavors \( \bar{\sigma}_u, \bar{\sigma}_d, \text{ and } \bar{\sigma}_s \), which can be obtained by solving the “gap” equations given by \[21\]

\[
\begin{align*}
\bar{\sigma}_u + G_S \bar{S}_u + \frac{H}{2} \bar{S}_d \bar{S}_s &= 0, \\
\bar{\sigma}_d + G_S \bar{S}_d + \frac{H}{2} \bar{S}_u \bar{S}_s &= 0, \\
\bar{\sigma}_s + G_S \bar{S}_s + \frac{H}{2} \bar{S}_u \bar{S}_d &= 0,
\end{align*}
\]

(11)

and \( \varpi_f \) are obtained via minimizing the thermodynamic potential, \( \frac{\partial \Omega_{NL}}{\partial \varpi_f} = 0 \).

For the hadronic phase we have used the same model and parameters detailed in Refs. \[3, 4\], also considering for this work universal coupling constants.

### 3 Results and Conclusions

Depending on the strength of quark vector repulsion, we find that an extended region made of a mixed phase of quarks and hadrons may exist in high-mass neutron stars with masses up to \( 2.0 - 2.4 M_\odot \) as can be seen in Fig 1. The radii of these objects are between 12 and 13 km, as expected for neutron stars. Table 1 lists the strength of the vector interaction, the maximum mass stars, the corresponding fraction of quark matter, and the range of the mixed phase.
Figure 1: Mass versus central density of QHSs computed for the EoS constructed through the non-linear Walecka and the non-local NJL models. Vertical bars indicate the beginning and the end of the mixed phase. Dots refer to the maximum mass star for different values of the vector interaction.

Table 1: Different strengths of the vector interaction ($G_V$) for the quark phase, maximum mass star ($M_{\text{Max}}$), the corresponding fraction of quark matter at $M_{\text{Max}}$ ($\chi$), and the range of the mixed phase.

| $G_V/G_S$ | $M_{\text{Max}}$ ($M_\odot$) | $\chi$ at $M_{\text{Max}}$ | Mixed phase ($\rho_0$) |
|-----------|-------------------------------|-----------------------------|------------------------|
| 0.00      | 2.04                          | 0.72                        | 2.71 − 6.87            |
| 0.05      | 2.21                          | 0.85                        | 3.10 − 6.77            |
| 0.09      | 2.35                          | 0.92                        | 3.25 − 6.53            |

For the non-local NJL model and the NL3 parametrization for the hadronic phase, we find that pure quark matter could not exist in stable neutron stars. Only neutron stars that lie on the left of the mass peak are dense enough to contain quark matter. However, these stars are unstable against radial oscillations and thus could not exist stably in the universe. According to what is shown on Fig. 2, with increasing stellar mass, all the stellar cores are composed of either nucleons, nucleons and hyperons, or a mixed phase of nucleons, hyperons, and quark matter.
Figure 2: Mass-Radius relationship of QHSs computed for the EoS constructed for the non-linear Walecka and the non-local NJL models.

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