DERIVED EQUIVALENCE FOR MUKAI FLOP VIA MUTATION OF
SEMIORTHOGONAL DECOMPOSITION

HAYATO MORIMURA

Abstract. We give a new proof of the derived equivalence of a pair of varieties connected either
by the Abuaf flop, a Mukai flop, or a standard flop along the lines of [Ued], which in turn is based
on [Kuz18].

1. Introduction

Let $G$ be a semisimple Lie group and $B$ a Borel subgroup of $G$. For distinct maximal para-
bolic subgroups $P$ and $Q$ of $G$, three homogeneous spaces $G/P$, $G/Q$, and $G/(P \cap Q)$ form the
following diagram:

\[ F := G/(P \cap Q) \]

\[ P := G/P \]

\[ Q := G/Q \]

We write the hyperplane classes of $P$ and $Q$ as $h$ and $H$ respectively. By abuse of notation, the pull-back to $F$ of the hyperplane classes $h$ and $H$ will be denoted by the same symbol. The morphisms $\varphi_-$ and $\varphi_+$ are projective morphisms whose relative $O(1)$ are $O(H)$ and $O(h)$ respectively. We consider the diagram

\[ \begin{array}{c}
V_- \\
V_0 \\
V_+ \\
\end{array} \]

\[ \begin{array}{c}
\varphi_- \\
\varphi_+ \\
\varphi_- \\
\varphi_+ \\
\end{array} \]

\[ \begin{array}{c}
P \\
V \\
Q \\
\end{array} \]

\[ \begin{array}{c}
F \\
\varphi_- \\
\varphi_+ \\
\end{array} \]

\[ \begin{array}{c}
P \\
V_- \\
V_0 \\
\end{array} \]

\[ \begin{array}{c}
V_+ \\
Q \\
F \\
\end{array} \]

where

- $V_-$ is the total space of $((\varphi_-)_*O(h + H))^\vee$ over $P$,
- $V_+$ is the total space of $((\varphi_+)_*O(h + H))^\vee$ over $Q$,
- $V$ is the total space of $O(-h - H)$ over $F$,
- $\iota$ is the zero section, and
- $\phi_+$ and $\phi_-$ are the affinizations which contract the zero sections.

If $V_-$ and $V_+$ have the trivial canonical bundles, then one expects from [BO02 Conjecture 4.4] or [Kaw02 Conjecture 1.2] that $V_-$ and $V_+$ are derived-equivalent.

When $G$ is the simple Lie group of type $G_2$, Ueda [Ued] used mutation of semiorthogonal
deformations of $D^b(V)$ obtained by applying Orlov’s theorem [Orl92] to diagram (1.2) to prove the derived equivalence of $V_-$ and $V_+$. This mutation in turn follows that of Kuznetsov [Kuz18] closely.
In this paper, by using the same method, we give a new proof to the following theorem, which is originally due to Bondal and Orlov [BO], Kawamata [Kaw02], Namikawa [Nam03], and Segal [Seg16]:

**Theorem 1.1.** Varieties connected either by the Abouaf flop, a Mukai flop, or a standard flop are derived-equivalent.

There are several ways to prove Theorem 1.1. In [Seg16], Segal described a new 5-fold flop and showed that varieties connected by it are derived-equivalent by using tilting vector bundles on them. Since the flop is attributed to Abuaf, we call it the Abuaf flop. Hara [Hara] constructed alternative tilting vector bundles and studied the relation between functors defined by him and Segal.

For a Mukai flop, Kawamata [Kaw02] and Namikawa [Nam03] independently showed the derived equivalence by using the pull-back and push-forward along the fiber product \( V_+ \times V_0 \times V_- \). Addington, Donovan, and Meachan [ADM] introduced a generalization of the functor of Kawamata and Namikawa parametrized by an integer, and discovered that certain compositions of these functors give the \( P \)-twist in the sense of Huybrechts and Thomas [HT06]. They also considered the case of a standard flop, where the derived equivalence is originally proved by Bondal and Orlov [BO]. [ADM] gave two proofs of the derived equivalence for a standard flop, one of which is close, if not identical, to the proof given in this paper. Hara [Hara17] also studied a Mukai flop in terms of non-commutative crepant resolutions.

For a standard flop, Segal [Seg16] showed the derived equivalence by using the grade restriction rule for variation of geometric invariant theory quotients (VGIT) originally introduced by Hori, Herbst, and Page [HHP]. VGIT method was subsequently developed by Halpern-Leistner [HL15] and Ballard, Favero, and Katzarkov [BFK]. It is an interesting problem to develop this method further to prove the derived equivalence for the Abuaf flop and a Mukai flop.

**Notations and conventions.** We work over an algebraically closed field \( k \) of characteristic 0 throughout this paper. All pull-back and push-forward are derived unless otherwise specified. The complexes underlying \( \text{Ext}^\bullet(−,−) \) and \( H^\bullet(−) \) will be denoted by \( \text{hom}(−,−) \) and \( h(−) \) respectively.

**Acknowledgements.** The author thanks his advisor Kazushi Ueda for guidance and encouragement.

2. Abouaf flop

Let \( P \) and \( Q \) be the parabolic subgroups of the simple Lie group \( G \) of type \( C_2 \) associated with the crossed Dynkin diagrams \( ≠\cong \) and \( \cong=\). The corresponding homogeneous spaces are the projective space \( P = \mathbb{P}(V) \), the Lagrangian Grassmannian \( Q = \text{LGr}(V) \), and the isotropic flag variety \( F = \mathbb{P}(\mathcal{L}_P^\perp / \mathcal{L}_P) = \mathbb{P}(\mathcal{I}_Q) \). Here \( V \) is a 4-dimensional symplectic vector space, \( \mathcal{L}_P^\perp \) is the rank 3 vector bundle given as the symplectic orthogonal to the tautological line bundle \( \mathcal{L}_P \equiv O_P(−h) \) on \( P \), and \( \mathcal{I}_Q \) is the tautological rank 2 bundle on \( Q \). Note that \( Q \) is also a quadric hypersurface in \( \mathbb{P}^4 \). Tautological sequences on \( Q = \text{LGr}(V) \) and \( F \equiv \mathbb{P}(\mathcal{I}_Q^\vee) \) give

\[
0 \to \mathcal{I}_Q \to O_Q \otimes V \to \mathcal{I}_Q^\perp(H) \to 0
\]

and

\[
0 \to O_F(−h + H) \to \mathcal{I}_F^\vee \to O_F(h) \to 0
\]

where \( \mathcal{I}_F := \mathcal{I}_Q \) and \( \mathcal{I}_Q^\perp \) is the symplectic orthogonal to \( \mathcal{I}_Q \). We have

\[
\mathcal{I}_F(\mathcal{I}_Q(H)) = (\mathcal{L}_P^\perp / \mathcal{L}_P) \otimes \mathcal{L}_P^\vee
\]

\[
(\mathcal{I}_F\mathcal{I}_Q(H)) = (\mathcal{L}_P^\perp / \mathcal{L}_P) \otimes \mathcal{L}_P^\vee
\]
and

\[(\varpi_+)_*(O_F(h)) \cong \mathcal{S}_Q^\vee,\]

whose determinants are given by \(O_F(2h)\) and \(O_Q(H)\) respectively. Since \(\omega_F \cong O_F(-4h)\), \(\omega_Q \cong O_Q(-3H)\), and \(\omega_F \cong O_F(-2h - 2H)\), we have \(\omega_V \cong O_V\), \(\omega_V \cong O_V\), and \(\omega_V \cong O_V(-h - H)\).

Recall from [Be˘ı78] that

\[D^h(P) = \langle O_F(-2h), O_F(-h), O_F, O_F(h) \rangle,\]

and from [Kuz08] (cf. also [Kap88]) that

\[D^h(Q) = \langle (O_Q(-H), \mathcal{S}_Q^\vee(-H), O_Q, O_Q(H) \rangle.\]

Since \(\varphi_{\pm}\) are blow-ups along the zero-sections, it follows from [Orl92] that

\[D^h(V) = \langle (\imath, \varpi_-^* D^h(P), \Phi_-(D^h(V_-))) \rangle\]

and

\[D^h(V) = \langle (\imath, \varpi_+^* D^h(Q), \Phi_+(D^h(V_+))) \rangle,\]

where

\[\Phi_- := ((-) \otimes O_V(H)) \circ \varphi_-^* : D^h(V_-) \to D^h(V)\]

and

\[\Phi_+ := ((-) \otimes O_V(h)) \circ \varphi_+^* : D^h(V_+) \to D^h(V).\]

By abuse of notation, we use the same symbol for an object of \(D^h(F)\) and its image in \(D^h(V)\) by the push-forward \(\imath_*\). (2.5) and (2.7) give

\[D^h(V) = \langle O_F(-2h), O_F(-h), O_F(h), O_F(h), \Phi_-(D^h(V_-)) \rangle.\]

By mutating the first term to the far right, we obtain

\[D^h(V) = \langle O_F(-h), O_F, O_F(h), \Phi_-(D^h(V_-)), O_F(-h + H) \rangle\]

since \(\omega_V \cong O_V(-h - H)\). By mutating \(\Phi_-(D^h(V_-))\) one step to the right, we obtain

\[D^h(V) = \langle O_F(-h), O_F, O_F(h), O_F(-h + H), \Phi_1(D^h(V_-)) \rangle\]

where

\[\Phi_1 := R_{O_F(-h + H)} \circ \Phi_-\]

Note that the canonical extension of \(O_F(h)\) by \(O_F(-h + H)\) associated with

\[\text{hom}_{O_V}(O_F(h), O_F(-h + H)) \cong \text{hom}_{O_V}([O_V(2h + H) \to O_V(h)], O_F(-h + H))\]

\[\cong O_F(2h + H) \cong h((O_F(-2h + H) \to O_F(-3h)))\]

\[\cong h(O_F(-2h + H))\]

\[\cong h((\varpi_+)_* O_F(-2h) \otimes O_Q(H))\]

\[\cong h(O_Q(-1))\]

\[\cong h(k[-1])\]

is given by the short exact sequence (2.2). By mutating \(O_F(-h + H)\) to the left, we obtain

\[D^h(V) = \langle O_F(-h), O_F, \mathcal{S}_F^\vee, O_F(h), \Phi_1(D^h(V_-)) \rangle.\]

By mutating \(O_F(-h)\) to the far right, we obtain

\[D^h(V) = \langle O_F, \mathcal{S}_F^\vee, O_F(h), \Phi_1(D^h(V_-)), O_F(H) \rangle.\]
By mutating $\Phi_1(D^b(V_-))$ one step to the right, we obtain
\begin{equation}
D^b(V) = \langle O_F, \mathcal{F}, O_F(H), \Phi_2(D^b(V_-)) \rangle
\end{equation}
where
\begin{equation}
\Phi_2 \coloneqq R_{(O_F(H))} \circ \Phi_1.
\end{equation}

One can easily see that $O_F(h)$ and $O_F(H)$ are orthogonal, so that
\begin{equation}
D^b(V) = \langle O_F, \mathcal{F}, O_F(H), O_F(h), \Phi_2(D^b(V_-)) \rangle.
\end{equation}

By mutating $\Phi_2(D^b(V_-))$ one step to the left, we obtain
\begin{equation}
D^b(V) = \langle O_F, \mathcal{F}, O_F(H), \Phi_3(D^b(V_-)), O_F(h) \rangle
\end{equation}
where
\begin{equation}
\Phi_3 \coloneqq L_{(O_F(h))} \circ \Phi_2.
\end{equation}

By mutating $O_F(h)$ to the far left, we obtain
\begin{equation}
D^b(V) = \langle O_F(-H), O_F, \mathcal{F}, O_F(H), \Phi_3(D^b(V_-)) \rangle.
\end{equation}

We have
\begin{align}
\hom_{O_Y}(O_F, \mathcal{F}) &\simeq \hom_{O_Y}(\{O_Y(h + H) \to O_Y\}, \mathcal{F}) \\
&\simeq h(\{\mathcal{F} \to \mathcal{F}(-h - H)\}) \\
&\simeq h(\mathcal{F}) \\
&\simeq V,
\end{align}

and the dual of (2.1) shows that the kernel of the evaluation map $O_F \otimes V \to \mathcal{F}$ is $(\mathcal{F}^\vee)^*(-H)$.
By mutating $\mathcal{F}$ to the left, we obtain
\begin{equation}
D^b(V) = \langle O_F(-H), (\mathcal{F}^\vee)^*(-H), O_F, O_F(H), \Phi_3(D^b(V_-)) \rangle.
\end{equation}

By comparing (2.33) with (2.8), we obtain a derived equivalence
\begin{equation}
\Phi \coloneqq \Phi_1 \circ \Phi_3 : D^b(V_-) \to D^b(V_+),
\end{equation}
where
\begin{equation}
\Phi_1(-) \coloneqq (\varphi_+)_* \circ ((-) \otimes O_Y(-h)) : D^b(V) \to D^b(V_+)
\end{equation}
is the left adjoint functor of $\Phi_+$.

3. **MUKAI FLOP**

For $n \geq 2$, let $P$ and $Q$ be the maximal parabolic subgroups of the simple Lie group of type $A_n$ associated with the crossed Dynkin diagrams $\ldots \longrightarrow$ and $\longleftrightarrow$. The corresponding homogeneous spaces are the projective spaces $P = \mathbb{P} V, Q = \mathbb{P} V'$, and the partial flag variety $F = F(1, n; V)$, where $V$ is an $(n + 1)$-dimensional vector space. Since $\omega_P \equiv O(-(n + 1)h)$, $\omega_Q \equiv O(-(n + 1)H)$, and $\omega_F \equiv O(-nh - nH)$, we have $\omega_{V_-} \equiv O_{V_-}, \omega_{V_+} \equiv O_{V_+}$, and $\omega_V \equiv O(-(n - 1)h - (n - 1)H).

**Lemma 3.1.** $O_F(-ih + jH)$ and $O_F(-(i + 1)h + (j - 1)H)$ are acyclic for $1 \leq j \leq n - 1$ and $1 \leq i \leq n - j$.

**Proof.** Since $j - n \leq -i \leq -1$ and $j - n - 1 \leq -i - 1 \leq -2$, the derived push-foward of $O_F(-ih + jH)$ and $O_F(-(i + 1)h + (j - 1)H)$ vanish by [Har77] Exercise III.8.4 unless $i = n - 1$ and $j = 1$, in which case the acyclicity of $O_F(-nh)$ is obvious.

**Lemma 3.2.** $\hom_{O_Y}(O_F(ih - jH), O_F) \simeq 0$ for $1 \leq j \leq n - 1$ and $1 \leq i \leq n - j$.
\textbf{Proof.} We have

\begin{equation}
\text{hom}_{O_V} (O_F(ih - jH), O_F) \approx \text{hom}_{O_V} ([O_V((i + 1)h - (j - 1)H) \to O_V(ih - jH)], O_F)
\end{equation}

\begin{equation}
\approx \mathfrak{h} ([O_F(-ih + jH) \to O_F(-(i + 1)h + (j - 1)H))
\end{equation}

which vanishes by Lemma 3.1.

Recall from [Bel78] that

\begin{equation}
\text{D}^b(P) = \langle O_P, O_P(h), \cdots, O_P(nh) \rangle
\end{equation}

and

\begin{equation}
\text{D}^b(Q) = \langle O_Q, O_Q(H), \cdots, O_Q(nH) \rangle.
\end{equation}

Since \( \varphi \) are blow-ups along the zero-sections, it follows from [Orl92] that

\begin{equation}
\text{D}^b(V) = \langle l, \varpi^* \text{D}^b(P), \cdots, l, \varpi^* \text{D}^b(P) \otimes O_V((n-2)H), \Phi_-(D^b(V_-)) \rangle
\end{equation}

and

\begin{equation}
\text{D}^b(V) = \langle l, \varpi^* \text{D}^b(Q), \cdots, l, \varpi^* \text{D}^b(Q) \otimes O_V((n-2)H), \Phi_+(D^b(V_+)) \rangle,
\end{equation}

where

\begin{equation}
\Phi_- := ((-) \otimes O_V((n-1)H)) \circ \varphi^- : D^b(V_-) \to D^b(V)
\end{equation}

and

\begin{equation}
\Phi_+ := ((-) \otimes O_V((n-1)H)) \circ \varphi^+ : D^b(V_+) \to D^b(V).
\end{equation}

We write \( O_{i,j} := O_F(ih + jH) \). (3.3) and (3.5) give a semiorthogonal decomposition of the form

\begin{equation}
\text{D}^b(V) = \langle A_0, \Phi_-(D^b(V_-)) \rangle
\end{equation}

where \( A_0 \) is given by

\begin{equation}
\begin{array}{cccccccc}
O_{0,0} & O_{1,0} & \cdots & O_{n-2,0} & O_{n-1,0} & O_{n,0} & O_{n,1} & O_{n+1,1} \\
O_{1,1} & \cdots & \cdots & O_{n-2,1} & O_{n-1,1} & O_{n,1} & \cdots & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
O_{n-2,n-2} & O_{n-1,n-2} & O_{n,n-2} & O_{n+1,n-2} & \cdots & O_{2n-2,n-2}.
\end{array}
\end{equation}

Note from Lemma 3.2 that there are no morphisms from right to left in (3.10). Since \( \omega_V \cong O_{-(n-1), -(n-1)} \), by mutating first

\begin{equation}
\begin{array}{cccccccc}
O_{0,0} & O_{1,0} & \cdots & O_{n-2,0} & O_{n-2,1} \\
O_{1,1} & \cdots & \cdots & O_{n-2,1} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
O_{n-2,n-2}
\end{array}
\end{equation}

and then \( \Phi_-(D^b(V_-)) \) to the far right, we obtain

\begin{equation}
\text{D}^b(V) = \langle A_1, \Phi_1(D^b(V_-)) \rangle
\end{equation}

where

\begin{equation}
\Phi_1(D^b(V_-)) := R(O_{n-1,n-1}, \cdots, O_{2n-2,n-2}) \circ \Phi_-
\end{equation}
and $\mathcal{A}_1$ is given by

\[
\begin{array}{cccc}
O_{n-1,0} & O_{n,0} & \cdots & O_{n,n} \\
O_{n-1,1} & O_{n,1} & \cdots & O_{n,n+1} \\
\vdots & \vdots & \ddots & \vdots \\
O_{n-1,n-2} & O_{n,n-2} & \cdots & O_{n-1,n-1} \\
O_{n-1,n-1} & O_{n,n-1} & \cdots & O_{n,n+1} \\
O_{n,n} & O_{n+1,n} & \cdots & O_{n+1,n+1} \\
\vdots & \vdots & \ddots & \vdots \\
O_{n+1,n+1} & \cdots & O_{n+1,n+1} & \cdots \\
\cdots & \ddots & \cdots & \ddots \\
\end{array}
\]

(3.14)

By mutating $\Phi_1(D^b(V_-))$ one step to the left, and then $O_{2n-2,n-2}$ to the far left, we obtain

\[
D^b(V) = \langle \mathcal{A}_2, \Phi_2(D^b(V_-)) \rangle
\]

where

\[
\Phi_2(D^b(V_-)) := L_{O_{2n-2,n-2}} \circ \Phi_1
\]

and $\mathcal{A}_2$ is given by

\[
\begin{array}{cccc}
O_{n-1,-1} & O_{n,-1} & \cdots & O_{n,n} \\
O_{n-1,0} & O_{n,0} & \cdots & O_{n,n+1} \\
\vdots & \vdots & \ddots & \vdots \\
O_{n-1,n-2} & O_{n,n-2} & \cdots & O_{n-1,n-1} \\
O_{n-1,n-1} & O_{n,n-1} & \cdots & O_{n,n+1} \\
O_{n,n} & O_{n+1,n} & \cdots & O_{n+1,n+1} \\
\vdots & \vdots & \ddots & \vdots \\
O_{n+1,n+1} & \cdots & O_{n+1,n+1} & \cdots \\
\cdots & \ddots & \cdots & \ddots \\
\end{array}
\]

(3.17)

By comparing (3.15) with (3.4) and (3.6), we obtain a derived equivalence

\[
\Phi := (\varphi_+)_* \circ ((-) \otimes O_{(2n-2),0}) \circ \Phi_2 : D^b(V_-) \sim D^b(V_+).
\]

(3.18)

4. Standard flop

For $n \geq 1$, let $P$ and $Q$ be the maximal parabolic subgroups of the semisimple Lie group $G = \text{SL}(V) \times \text{SL}(V^\vee)$ associated with the crossed Dynkin diagram $\rightarrow \otimes \rightarrow \otimes \rightarrow$. The corresponding homogeneous spaces are the projective spaces $\mathbb{P} = \mathbb{P}V$, $\mathbb{Q} = \mathbb{P}V^\vee$, and their product $\mathbb{F} = \mathbb{P}V \times \mathbb{P}V^\vee$. Since $\omega_P \cong O(-(n+1)h)$, $\omega_Q \cong O(-(n+1)H)$, and $\omega_F \cong O(-(n+1)h - (n+1)H)$, we have $\omega_{V_-} \cong O_{V_-}$, $\omega_{V_+} \cong O_{V_+}$, and $\omega_{V} \cong O(-nh - nH)$.

Lemma 4.1. $\text{hom}_{O_V}(O_F(ih - jH), O_F) \cong 0$ for $1 \leq j \leq n-1$ and $1 \leq i \leq n - j$.

Proof. We have

\[
(4.1) \quad \text{hom}_{O_V}(O_F(ih - jH), O_F) \cong \text{hom}_{O_V}(\langle O_V((i+1)h - (j-1)H) \rightarrow O_V(ih - jH) \rangle, O_F)
\]

(4.2) \hspace{1cm} \cong h(\langle O_F(-ih + jH) \rightarrow O_F(-(i+1)h + (j-1)H) \rangle)

which vanishes for $1 \leq i \leq n - j \leq n - 1$. \hfill \square
It follows from [Orl92] that
\begin{align}
(4.3) & \quad D^b(V) = \langle t, \sigma^* D^b(P), \ldots, t, \sigma^* D^b(P) \otimes O((n-1)(h+H)), \Phi_-(D^b(V_-)) \rangle \\
(4.4) & \quad D^b(V) = \langle t, \sigma^* D^b(Q), \ldots, t, \sigma^* D^b(Q) \otimes O((n-1)(h+H)), \Phi_+(D^b(V_+)) \rangle,
\end{align}
where
\begin{align}
(4.5) & \quad \Phi_- := (-) \otimes O_V(n(h+H)) \circ \varphi^*_+ : D^b(V_-) \to D^b(V) \\
(4.6) & \quad \Phi_+ := (-) \otimes O_V(n(h+H)) \circ \varphi^*_+ : D^b(V_+) \to D^b(V).
\end{align}
We write \( O_{i,j} := O_\mathfrak{F}(ih + jH) \). (3.3) and (4.3) give a semiorthogonal decomposition of the form
\begin{align}
(4.7) & \quad D^b(V) = \langle \mathcal{A}_0, \Phi_-(D^b(V_-)) \rangle \\
where \quad \mathcal{A}_0 \text{ is given by}
\begin{array}{cccccccc}
O_{0,0} & O_{1,0} & \cdots & O_{n-2,0} & O_{n-1,0} & O_{n,0} \\
O_{1,1} & O_{1,0} & \cdots & O_{n-2,1} & O_{n-1,1} & O_{n,1} & O_{n+1,1} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
O_{n-2,n-2} & O_{n-1,n-2} & O_{n,n-2} & O_{n+1,n-2} & \cdots & O_{2n-2,n-2} \\
O_{n-1,n-1} & O_{n,n-1} & O_{n+1,n-1} & \cdots & O_{2n-2,n-1} & O_{2n-1,n-1} \\
O_{n-2,n-2} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\end{array}
\end{align}

Note from Lemma 4.1 that there are no morphisms from right to left in (4.8). Since \( \omega_V \cong O_V(-nh - nH) \), by mutating first
\begin{align}
(4.9) & \quad O_{0,0} & O_{1,0} & \cdots & O_{n-2,0} \\
& O_{1,1} & O_{1,0} & \cdots & O_{n-2,1} \\
& \vdots & \vdots & \ddots & \vdots \\
& O_{n-2,n-2} \\
\end{align}
to the far right, and then \( \Phi_-(D^b(V_-)) \) to the far right, we obtain
\begin{align}
(4.10) & \quad D^b(V) = \langle \mathcal{A}_1, \Phi_1(D^b(V_-)) \rangle \\
where \quad \Phi_1(D^b(V_-)) := R_{O_{n,n}, \ldots, O_{2n-2,2n-2}} \circ \Phi_-
\end{align}
and \( \mathcal{A}_1 \) is given by
\begin{align}
(4.11) & \quad \begin{array}{cccccccc}
O_{n-1,0} & O_{n,0} \\
O_{n-1,1} & O_{n,1} & O_{n+1,1} \\
\vdots & \vdots & \ddots & \vdots \\
O_{n-1,n-1} & O_{n,n-1} & O_{n+1,n-1} & \cdots & O_{2n-2,n-1} & O_{2n-1,n-1} \\
O_{n,n} & O_{n+1,n} & O_{2n-2,n} & O_{2n-1,n} \\
O_{n+1,n+1} & \cdots & O_{2n-2,n+1} \\
\vdots & \vdots & \ddots & \vdots \\
O_{2n-2,2n-2}.
\end{array}
\end{align}
By mutating \( \Phi_1(D^b(V_-)) \) one step to the left, and then \( O_{2n-1,n-1} \) to the far left, we obtain
\begin{align}
(4.12) & \quad D^b(V) = \langle \mathcal{A}_2, \Phi_2(D^b(V_-)) \rangle \\
where \quad \Phi_2(D^b(V_-)) := L_{O_{2n-1,n-1}} \circ \Phi_1
\end{align}
and $\mathcal{A}_2$ is given by

$$
\begin{align*}
O_{n-1,-1} & \quad O_{n,0} & \quad O_{n+1,1} \\
O_{n-1,0} & \quad O_{n,1} & \\
\vdots & \quad \vdots & \quad \vdots \\
O_{n-1,n-1} & \quad O_{n,n-1} & \quad O_{n+1,n-1} & \quad \cdots & \quad O_{2n-2,n-1} \\
O_{n,n} & \quad O_{n+1,n} & \quad \cdots & \quad O_{2n-2,n} \\
O_{n+1,n+1} & \quad \cdots & \quad O_{2n-2,n+1} \\
\vdots & \quad \vdots & \quad \ddots & \quad \vdots \\
O_{2n-2,2n-2} & \quad \vdots & \quad \cdots & \quad \vdots \\
O_{2n-2,2n-2} & \quad O_{2n,2n} & \\
\end{align*}
$$

\tag{4.15}

By comparing (4.13) with (3.4) and (4.4), we obtain a derived equivalence

$$
\Phi := (\varphi_+)_* \circ ((-) \otimes O_{-(2n-1),0}) \circ \Phi_2 : D^b(V_-) \xrightarrow{\sim} D^b(V_+).
$$

\tag{4.16}

#### References

[ADM] N. Addington, W. Donovan, and C. Meachan, \textit{Mukai flops and $\mathbb{P}$-twists}, Journal für die Reine und Angewandte Mathematik. https://doi.org/10.1515/crelle-2016-0024

[Be˘ı78] A. Be˘ılinson, \textit{coherent sheaves on $\mathbb{P}^n$ and problems in linear algebra}, Funktsional. Rossiiiskaya Akademiya Nauk. Funktsional’nyı Analiz i ego Prilozheniya. 12(3), 68-69 (1978).

[BFK] M. Ballard, D. Favero, and L. Katzarkov, \textit{Variation of geometric invariant theory quotients and derived categories}, arXiv:1203.6643.

[BO] A. Bondal and D. Orlov, \textit{Semiorthogonal decomposition for algebraic varieties}, arXiv:alg-geom/9506012.

[BO02] A. Bondal and D. Orlov, \textit{Derived categories of coherent sheaves}, Proceedings of the International Congress of Mathematicians, Vol. II (Beijing, 2002), Higher Ed. Press, Beijing, 2002, pp. 47–56. MR 1957019

[Har77] R. Hartshorne, \textit{Algebraic geometry}, Graduate Texts in Mathematics, vol. 52, Springer-Verlag, 1977, xvi+496 pp. ISBN: 0-387-90244-9.

[Hara17] W. Hara, \textit{Non-commutative crepant resolution of minimal nilpotent orbit closure of type A and Mukai flops}, Advances in mathematics. 318, 355-410 (2017).

[Hara] W. Hara, \textit{On derived equivalence for Abuaf flop: mutation of non-commutative crepant resolutions and spherical twists}, arXiv:1706.04417.

[HHP] M. Herbst, K. Hori, and D. Page, \textit{Phases of N=2 theories in 1+1 dimensions with boundary}, arXiv:0803.2045.

[HL15] D. Halpern-Leistner, \textit{The derived category of a GIT quotient.}, Journal of the American Mathematical Society. 28(3), 871-12 (2015).

[HT06] D. Huybrechts and R. Thomas, \textit{P-objects and autoequivalences of derived categories}, Mathematical Research Letters. 13(1), 87-98 (2006).

[Kap88] M. kapranov, \textit{On the derived categories of coherent sheaves on some homogenous spaces}, Inventiones Mathematicae. 92(3), 479-508 (1988).

[Kaw02] Y. Kawamata, \textit{D-equivalence and K-equivalence}, Journal of Differential Geometry. 61(1), 147-171 (2002).

[Kuz08] A. Kuznetsov, \textit{Exceptional collections for Grassmannians of isotropic lines}, Proceedings of the London Mathematical Society. Third Series. 97, 155-182 (2008).

[Kuz18] A. Kuznetsov, \textit{Derived equivalence of Ito–Miura–Okawa–Ueda Calabi–Yau 3-folds}, Journal of the Mathematical Society of Japan. 70(3), 1007-1013 (2018).

[Orl92] D. O. Orlov, \textit{Projective bundles, monoidal transformations, and derived categories of coherent sheaves}, Rossiiskaya Akademiya Nauk. Izvestiya. Seriya Matematicheskaya. 56, 852-862 (1992).

[Nam03] Y. Namikawa, \textit{Mukai flops and derived categories}, Journal für die Reine und Angewandte Mathematik. 560, 65-76 (2003).

[Seg11] E. Segal, \textit{Equivalence between GIT quotients of Landau-Ginzburg B-models}, Communications in Mathematical Physics. 304(2), 411-432 (2011).

[Seg16] E. Segal, *A new 5-fold flop and derived equivalence*, Bulletin of the London Mathematical Society. 48, 533-538 (2016).

[Ued] K. Ueda, *G2-Grassmannians and derived equivalences*, Manuscripta Mathematica. https://doi.org/10.1007/s00229-018-1090-4.

Graduate School of Mathematical Sciences, The University of Tokyo, 3-8-1 Komaba, Meguro-ku, Tokyo, 153-8914, Japan.

E-mail address: morimura@ms.u-tokyo.ac.jp