Production of two electron-positron couples in electroweak $\gamma\gamma$-interaction

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Abstract

We present the calculation of $\gamma\gamma \rightarrow 2e^+e^-$ process cross section. The construction are performed using both helicity amplitude method and method of precision covariant calculation. The magnitude of cross section is obtained by the Monte-Carlo method of numerical integration. Different energies, polarization states and kinematics cuts are considered.

1 Introduction

At future linear collider besides of $e^-e^-$ and $e^+e^-$ interactions $\gamma\gamma$ and $\gamma e$ modes are planned to realize (TESLA, CLIC and others). This possibility will provide a great advantage in study of non-Abelian nature of electroweak interaction, gauge boson coupling as well as couplings of gauge bosons with Higgs particles if it is light enough to be produced. Since $W^\pm$ and Higgs bosons decay within detector they can be investigated via their decay products, for instance four leptons in final state. Because of high accuracy and relatively clean environment provided by a leptonic collider, a precision calculation of backgrounds of $\gamma\gamma \rightarrow \ldots \rightarrow 4l$ processes is necessary.

Total cross sections of processes $\gamma\gamma \rightarrow 2e^-2e^+, \gamma\gamma \rightarrow e^+e^-\mu^+\mu^-, \gamma\gamma \rightarrow 2\mu^-2\mu^+$ have been already calculated [1]-[3] about 30 years ago and were found to be large. However there was used the low energy approximation, and obtained results are not applicable to analyze the results of high energy experiments.

The matrix element of $\gamma\gamma \rightarrow 4l$ process has been constructed also in ref. [4]. However at that paper neither calculation of cross section no numerical analyze are present. So it is impossible to perform any numerical congruence.

This process was also analysed in ref. [5] where some numerical calculations were performed, and the dependence of total cross section from the energy of initial beam was investigated. But authors consider low energy region only (about 1-5 GeV) while high energy experiments require to study $\gamma\gamma$-interaction at beam energy up to 300-500 GeV.

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Beside that the process of four lepton production in $\gamma \gamma$ interaction was considered in ref. [6]. There was applied the algorithm ALPHA for automatic computations of scattering amplitude. However modern high energy experiments require the calculation of cross section at definite polarization states of initial and final particles that ALPHA method doesn’t provide.

2 Construction and calculations

There are six topologically different Feynman diagrams of electroweak interaction describing process $\gamma \gamma \rightarrow 4l$ (see fig.1). Whole set of diagrams can be derived on base of these six ones using C- P- and crossing symmetries.

\[ M_1 = \frac{-ie^4}{(k_1 - p_1 - p_2)^2} \bar{u}(p_1) \tilde{\epsilon}(k_1) \frac{\hat{p}_1 - \hat{k}_1 + m}{(p_1 - k_1)^2 - m^2} \gamma^\mu v(p_2) \bar{u}(p_3) \gamma_\mu \times \]
\[ \frac{\hat{k}_2 - \hat{p}_4 + m}{(k_2 - p_4)^2 - m^2} \tilde{\epsilon}(k_2) v(p_4) - ie^2 \left( \frac{g}{2\cos(\theta_W)} \right) D_{\mu\nu}(k_1 - p_1 - p_2) \bar{u}(p_1) \tilde{\epsilon}(k_1) \times \]
\[ \frac{\hat{p}_1 - \hat{k}_1 + m}{(p_1 - k_1)^2 - m^2} \gamma^\mu (g_V + g_A \gamma_5) v(p_2) \bar{u}(p_3) \gamma^\nu (g_V + g_A \gamma_5) \frac{k_2 - p_4 + m}{(k_2 - p_4)^2 - m^2} \tilde{\epsilon}(k_2) v(p_4), \]

*Fig. 1. Feynman diagrams for process $\gamma \gamma \rightarrow 4l$. 

The diagrams containing charged current exchange are excluded because only processes with four charged leptons in final state are considered. Matrix elements for remaining diagrams (1)-(3) have the following form:
\[ M_2 = \frac{-ie^4}{(p_3 + p_1)^2} \bar{\pi}(p_1) \bar{\varepsilon}(k_1) \left( \frac{\hat{p}_1 - \hat{k}_1 + m}{(p_1 - k_1)^2 - m^2} \gamma^\mu \frac{\hat{k}_2 - \hat{p}_2 + m}{(k_2 - p_2)^2 - m^2} \bar{\varepsilon}(k_2) v(p_2) \right) \]
\[ \times \bar{\pi}(p_3) \gamma_\mu v(p_4) - ie^2 \left( \frac{g}{2 \cos(\theta_W)} \right) D_{\mu\nu}(p_3 + p_4) \bar{\pi}(p_1) \bar{\varepsilon}(k_1) \frac{\hat{p}_1 - \hat{k}_1 + m}{(p_1 - k_1)^2 - m^2} \gamma^\mu \times (2) \]
\[ (g_V + g_A \gamma_5) \frac{k_2 - \hat{p}_2 + m}{(k_2 - p_2)^2 - m^2} \bar{\varepsilon}(k_2) v(p_2) \bar{\pi}(p_3) \gamma^\nu (g_V + g_A \gamma_5) v(p_4), \]
\[ M_3 = \frac{-ie^4}{(p_1 + p_2)^2} \bar{\pi}(p_3) \gamma^\mu \frac{\hat{p}_1 + \hat{p}_2 + \hat{p}_3 + m}{(p_1 + p_2 + p_3)^2 - m^2} \bar{\varepsilon}(k_1) \frac{\hat{k}_2 - \hat{p}_4 + m}{(k_2 - p_4)^2 - m^2} \times \]
\[ \bar{\varepsilon}(k_2) v(p_4) \bar{\pi}(p_1) \gamma_\mu v(p_2) - ie^2 \left( \frac{g}{2 \cos(\theta_W)} \right) D_{\mu\nu}(p_1 + p_2) \bar{\pi}(p_3) \gamma^\mu (g_V + g_A \gamma_5) \times (3) \]
\[ \frac{\hat{p}_1 + \hat{p}_2 + \hat{p}_3 + m}{(p_1 + p_2 + p_3)^2 - m^2} \bar{\varepsilon}(k_1) \frac{\hat{k}_2 - \hat{p}_4 + m}{(k_2 - p_4)^2 - m^2} \bar{\varepsilon}(k_2) v(p_4) \bar{\pi}(p_1) \gamma^\nu (g_V + g_A \gamma_5) v(p_2). \]

Here \( \hat{p}_1 = p_1^\mu \gamma_\mu \), where \( p_1^\mu \) is \( \mu \)-component of four momentum \( p_1 \); \( \bar{\varepsilon}(k_1) = \varepsilon^\mu(k_1) \gamma_\mu \), where \( \varepsilon^\mu(k_1) - \mu \)-component of polarization vector of photon with four momentum \( k_1 \), \( D_{\mu\nu}(q) \) - propagator of \( Z^0 \)-boson with momentum \( q \).

Corresponding cross section has the form:
\[ \sigma = \frac{1}{4(k_1 k_2)} \int |M|^2 d\Gamma, \]

where
\[ d\Gamma = \frac{d^3p_1}{(2\pi)^3 2p_1^0} \frac{d^3p_2}{(2\pi)^3 2p_2^0} \frac{d^3p_3}{(2\pi)^3 2p_3^0} \frac{d^3p_4}{(2\pi)^3 2p_4^0} (2\pi)^4 \delta(k_1 + k_2 - p_1 - p_2 - p_3 - p_4) \]
is phase space element.

In the present paper squared matrix elements are constructed using both helicity amplitude method \[7]-[10] and method of precision covariant calculation (see, for example, refs. [11], [12]). The helicity amplitude method allows to calculate cross section directly for each definite polarization state of initial and final particles. The matrix element constructed by this method consists of invariants without any difficulties in squaring and numerical integration are excluded. The explicit form of all amplitudes obtained in frame of helicity amplitude method one can find in ref. [13]. The method of covariant calculation allows to obtain the matrix element without any approximation and was used for verification of results in each step of our construction and calculation.

For the investigation of total and differential cross section the Monte-Carlo method of numerical integration was applied. If two or more produced particles propagate very closely, the square of matrix element becomes very large. (So-called collinear peak problem is arisen.) To achieve the acceptable precision the method
of Monte-Carlo was adopted. Instead of regular distribution of kinematic variables (such distribution is usually applied in Monte-Carlo generators) we have used irregular one, which is very close to matrix element behavior. This proximity can be obtained by choosing of several free parameters available in the distribution function. So the adopted Monte-Carlo generator gives the results with very small numerical error (about 0.5% − 0.7%).

The accuracy of approach based on the helicity amplitude method was estimated by comparing with the precision covariant one. Since the results of both method for cross section of two electron-positron pair production have excellent agreement at each kinematics point, the mass contribution is practically negligible at least if TESLA energy and cuts are used.

3 Conclusion

In this paper the squared matrix elements of process $\gamma \gamma \rightarrow 2e^- 2e^+$ have been constructed using of the helicity amplitude method as well as the method of precision covariant calculations. Numerical integration of obtained cross sections were performed using adopted Monte-Carlo generator. The value of differential and total cross section both at averaged and fixed polarization states were calculated at different energy and kinematics cuts on polar angles.

![Fig.2. Spin average differential cross section of $\gamma \gamma \rightarrow 2e^- 2e^+$ process at c.m. energy of $\gamma \gamma-$ beam 0.5 TeV. $\theta_1(2)$ is the angle between the directions of the first(second) photon and the electron. The values of polar angle cut and cut of angle between any final particles are 11° and 3° respectively.](image-url)
Table 1. The dependence of total cross section on energies and kinematics cuts. Here the notation $(\alpha, \beta)$ for describing of kinematics cuts is used, where $\alpha$ is the cut of angle between directions of any final particles, $\beta$ – the cut of polar angle.

| energy (TeV) | cut      | $\sigma$ (fb) |
|-------------|----------|---------------|
| 0.3         | $(3^\circ, 7^\circ)$ | $76.41 \pm 0.47$ |
| 0.3         | $(3^\circ, 11^\circ)$ | $35.38 \pm 0.23$ |
| 0.5         | $(3^\circ, 7^\circ)$ | $31.96 \pm 0.19$ |
| 0.5         | $(3^\circ, 11^\circ)$ | $15.32 \pm 0.11$ |
| 1           | $(3^\circ, 7^\circ)$ | $9.90 \pm 0.07$ |
| 1           | $(3^\circ, 11^\circ)$ | $4.81 \pm 0.03$ |

Following notation is used in fig. 3 and fig. 4 for describing spin configuration: $(+, -,-, +, -, +)$ means $(\lambda_1 = +1, \lambda_2 = -1, \lambda_3 = -1, \lambda_4 = +1, \lambda_5 = -1, \lambda_6 = +1)$; $(+, +, +, +, -, -)$ means $(\lambda_1 = +1, \lambda_2 = +1, \lambda_3 = +1, \lambda_4 = +1, \lambda_5 = -1, \lambda_6 = -1)$, where $\lambda_{1,2}$ corresponds to polarization of photon with four momentum $k_{1,2}$; $\lambda_{3,4,5,6}$ – helicity of lepton with four momentum $p_{1,2,3,4}$ respectively.

Fig. 3. The differential cross section of $\gamma \gamma \rightarrow 2e^- 2e^+$ process at c.m. energy of $\gamma \gamma$-beam 0.5 TeV at fixed polarization states of interacting particles. $\theta_{1(2)}$ is the angle between the directions of the first(second) photon and the electron. The values of polar angle cut and cut of angle between any final particles are $11^\circ$ and $3^\circ$ respectively.
Fig.4. The differential cross section of $\gamma \gamma \rightarrow 2e^-2e^+$ process at c.m. energy of $\gamma \gamma -$ beam 0.5 TeV at fixed polarization states of interacting particles. $\theta_1(2)$ is the angle between the directions of the first(second) photon and the electron. The values of polar angle cut and cut of angle between any final particles are 11° and 3° respectively.

It is discovered the total and differential cross sections have strong dependence on kinematics cuts and energy of initial beam that table 1 clearly demonstrated. The cross sections increase with decreasing of energy of interacting particles because of they have reverse dependence on scalar production ($k_1k_2$) (see eq. (4)). Magnitude of differential cross section strongly increases if polar angles get close to 0 or $\pi$ and is on decrease at middle region of kinematic field (fig. 2). Differential cross sections at fixed polarization states have symmetric(asymmetric) form in case of similar(opposite) polarization states of initial particles (figs. 3 and 4).

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