COEFFICIENTS ASSESSMENT FOR CERTAIN SUBCLASSES OF BI-UNIVALENT FUNCTIONS RELATED WITH QUASI-SUBORDINATION

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Abstract. We investigate specific new subclasses of the function class Σ of bi-univalent function defined in the open unit disc, which is connected with quasi-subordination. We find estimates on the Taylor–Maclaurin coefficients $|a_2|$ and $|a_3|$ for functions in these subclasses. Already pointed out are some documented and new implications of those findings.

1. Introduction

Let $A$ denote the analytic function class in the open unit disc $D = \{z \in \mathbb{C} : |z| < 1\}$ which contains the shape

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \quad (z \in D),$$

then let $S$ be the class of all univalent functions from $A$ in $D$. The Koebe One Quarter Theorem \[7\] states that the image of $D$ beneath every function $f$ from $S$ contains a radius disk of $\frac{1}{4}$. This univalent function, therefore, has an inverse one $f^{-1}$ which satisfies

$$f^{-1}(f(z)) = z, \quad (z \in D) \quad \text{and} \quad f(f^{-1}(w)) = w, \quad (|w| < r_0(f), \quad r_0(f) \geq \frac{1}{4}).$$

In fact the inverse function $f^{-1}$ is given by

$$g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \cdots.$$  

A function $f \in A$ is said to be bi-univalent in $D$ if both $f$ and $f^{-1}$ are univalent in $D$. Let $\Sigma$ denote the class of bi-univalent functions defined in the unit disc $D$. Ma–Minda \[11\] introduce the following classes using subordination:

$$S^*(h) = \left\{ f \in A : \frac{zf'(z)}{f(z)} \prec h(z) \right\},$$

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155
where $h$ is an analytic function with positive real part on $\mathcal{D}$ with $h(0) = 1$, $h'(0) > 0$ which maps the unit disc $\mathcal{D}$ on a starlike area with respect to 1 and which is symmetric consider to the real axis. A function $f \in S^*(h)$ is called Ma–Minda starlike. $C(h)$ is a class of convex function $f \in \mathcal{A}$ for which

$$1 + \frac{zf''(z)}{f'(z)} < h(z).$$

The classes $S^*(h)$ and $C(h)$ contain various well-known subcategories of starlike and convex function as private case. The notion of subordination is propagated in 1970 by Robertson [20] through introducing a new notion of quasi-subordination.

For two analytic functions $f$ and $h$, the function $f$ is quasi subordination to $h$ written as $f(z) \prec h(z) (z \in \mathcal{D})$ in the event of an analytical function $\vartheta$ and $w$, with $|\vartheta(z)| \leq 1$, $w(0) = 0$ and $|w(z)| < 1$ such that $f(z)/\vartheta(z) \prec h(z)$, which is equivalent to $f(z) = \vartheta(z)h(w(z)) (z \in \mathcal{D})$. Note that if $\vartheta(z) = 1$, then $f(z) = h(w(z))$, so that $f(z) \prec h(z)$ in $\mathcal{D}$, also if $w(z) = z$, then $f(z) = \vartheta(z)h(z)$ and it is said that $f(z)$ is majorized by $h(z)$ and written as $f(z) \ll h(z)$ in $\mathcal{D}$. Hence it is perceptible that the quasi-subordination is a popularization of the usual subordination as well as majorization. The labor on quasi-subordination is very extensive and that includes some recent investigations [2, 4, 9, 10, 12, 14, 19, 20].

In 1967, Lewin [10] researched the class $\Sigma$ of bi-univalent functions and gained the limit for the second coefficient $a_2$. Brannan and Taha [5] examined specific subclasses of bi-univalent functions similar to the common subclasses of univalent functions consisting of strongly starlike, starlike and convex functions. They introduced the bi-starlike function, bi-convex function classes and acquired non-sharp bounds for the first two Taylor–Maclaurin coefficients $|a_2|$ and $|a_3|$. The study and the investigation of various subclasses of the bi-univalent function class $\Sigma$ was revived in recent years by Srivastava et al. [27] and significantly large number of continuation (see [3, 21, 22, 23, 26, 29, 32]) refer to Srivastava et al. [27]. Recently Ali et al. [1], Deniz [6], Tang et al. [30], Peng et al. [16], Ramachandran et al. [18], Murugusundaramoorthy et al. [13], Srivastava et al. [24, 28], etc., have examined and studied Ma–Minda type subclasses of bi-univalent functions class $\Sigma$. Further generalization of Ma–Minda type subclasses of class $\Sigma$ have been made several authors including [8, 15, 12, 23, 31] using quasi-subordination. Motivated by work in [9, 14] on quasi-subordination, we introduce and investigate here certain new subclasses of class $\Sigma$.

Throughout this sheet, it is assumed that $h(z)$ is analytic and univalent with positive real part in $\mathcal{D}$ and let

$$h(z) = 1 + B_1z + B_2z^2 + \cdots, \quad (B_1 \in \mathbb{R}^+),$$

and $h$ maps $\mathcal{D}$ onto a region starlike with respect to 1 and symmetric with respect to the real axis. Also, let $\vartheta(z)$ be an analytic function in $\mathcal{D}$ and

$$\vartheta(z) = A_0 + A_1z + A_2z^2 + \cdots, \quad (|\vartheta(z)| \leq 1, z \in \mathcal{D}).$$
Definition 1.1. For $0 \leq \delta \leq 1$ and $\lambda \geq 0$, a function $f \in \Sigma$ is said to be in the class $H^\delta_\Sigma(\lambda, \delta, \vartheta)$ if the following quasi-subordination hold

$$
\frac{(1 - \delta)f(z) + \delta zf'(z)}{z} + \lambda zf''(z) - 1 \prec_q (\vartheta(z) - 1)
$$

and

$$
\frac{(1 - \delta)g(w) + \delta wg'(w)}{w} + \lambda wg''(w) - 1 \prec_q (\vartheta(w) - 1)
$$

where the function $g$ is the extension of $f^{-1}$ to $\mathcal{D}$.

Definition 1.2. A function $f \in \Sigma$ is said to be in the class $M^\delta_\Sigma(\beta, \vartheta)$ if the following quasi-subordination hold

$$
\frac{z\Psi'(z)}{\Psi(z)} - 1 \prec_q (\vartheta(z) - 1), \quad \frac{w\Phi'(w)}{\Phi(w)} - 1 \prec_q (\vartheta(w) - 1),
$$

where $\Psi(z)$ and $\Phi(w)$ are as follows:

$$
\frac{1}{\Psi(z)} = \frac{1 - \beta}{f(z)} + \frac{\beta}{zf'(z)} \quad \text{and} \quad \frac{1}{\Phi(w)} = \frac{1 - \beta}{g(w)} + \frac{\beta}{wg'(w)} \quad (\beta \in \mathbb{C}),
$$

and $\Phi$ is the extension of $\Psi^{-1}$ to $\mathcal{D}$.

Lemma 1.1 (See [17]). Let $\mathcal{P}$ be class of all functions $p$ analytic in $U$ for which $\text{Re}(p(z)) > 0$ and have the form $p(z) = 1 + p_1z + p_2z^2 + \cdots$ for $z \in \mathcal{D}$, then $|p_i| \leq 2$ for each $i \in \mathbb{N}$.

2. Main Results

Theorem 2.1. $0 \leq \delta \leq 1$ and $\lambda \geq 0$. If $f \in \Sigma$ of the form [14] belonging to the class $H^\delta_\Sigma(\lambda, \delta, \vartheta)$, then

$$
|a_2| \leq \frac{|A_0|B_1 \sqrt{B_1}}{\sqrt{|(1 + 2\delta + 6\lambda)A_0B_1^2 + (1 + \delta + 2\lambda)^2(B_1 - B_2)|}}
$$

$$
|a_3| \leq \frac{|A_0|^2B_1^2}{(1 + \delta + 2\lambda)^2} + \frac{|A_1|B_1}{1 + 2\delta + 6\lambda} + \frac{|A_0|B_1}{1 + 2\delta + 6\lambda}.
$$

Proof. Let $f \in H^\delta_\Sigma(\lambda, \delta, \vartheta)$. In view of Definition [14] there exist then Schwarz functions $k(z)$, $s(w)$ and an analytic function $\vartheta(z)$ such that

$$
\frac{(1 - \delta)f(z) + \delta zf'(z)}{z} + \lambda zf''(z) - 1 = \vartheta(z)(h(k(z))) - 1,
$$

$$
\frac{(1 - \delta)g(w) + \delta wg'(w)}{w} + \lambda wg''(w) - 1 = \vartheta(w)(h(s(w))) - 1.
$$

Define the functions $p(z)$ and $q(w)$ by

$$
p(z) = \frac{1 + k(z)}{1 - k(z)} = 1 + c_1z + c_2z^2 + \cdots
$$

$$
q(w) = \frac{1 + s(w)}{1 - s(w)} = 1 + b_1w + b_2w^2 + \cdots,
$$

COEFFICIENTS ASSESSMENT FOR CERTAIN SUBCLASSES OF BI-UNIVALENT FUNCTION 157
or equivalently,
\[(2.7)\]
\[k(z) = \frac{p(z) - 1}{p(z) + 1} = \frac{1}{2} \left( c_1z + \left(c_2 - \frac{1}{2}c_1^2\right)z^2 + \cdots \right),\]
\[(2.8)\]
\[s(w) = \frac{q(w) - 1}{q(w) + 1} = \frac{1}{2} \left( b_1w + \left(b_2 - \frac{1}{2}b_1^2\right)w^2 + \cdots \right).\]

It is clear that \(p(z)\) and \(q(w)\) are analytic and have positive real parts in \(D\). In view of (2.3), (2.4), (2.7) and (2.8) clearly
\[(2.9)\]
\[\frac{(1 - \delta)f(z) + \delta zf'(z)}{z} + \lambda zf''(z) - 1 = \vartheta(z) \left[ h \left( \frac{p(z) - 1}{p(z) + 1} \right) - 1 \right],\]
\[(2.10)\]
\[\frac{(1 - \delta)g(w) + \delta wg'(w)}{w} + \lambda wg''(w) - 1 = \vartheta(w) \left[ h \left( \frac{q(w) - 1}{q(w) + 1} \right) - 1 \right].\]

The series expansions for \(f(z)\) and \(g(w)\) as given in (1.1) and (1.2) respectively, provide us
\[(2.11)\]
\[\frac{(1 - \delta)f(z) + \delta zf'(z)}{z} + \lambda zf''(z) - 1 = \frac{1}{2} A_0 B_1 c_1 z + \frac{1}{2} A_1 B_1 c_1 + \frac{1}{2} A_0 B_1 \left( c_2 - \frac{c_1^2}{2} \right) + \frac{A_0 B_2 c_1^2}{4} z^2 \cdots .\]
\[(2.12)\]
\[\frac{(1 - \delta)g(w) + \delta wg'(w)}{w} + \lambda wg''(w) - 1 = -(1 + \delta) a_2 w + (1 + 2\delta)(2\alpha_2^2 - a_3) w^2 - 2\lambda a_2 w + 6\lambda(2\alpha_2^2 - a_3) w^2 + \cdots .\]

Using (2.5) and (2.6) together with (1.3) and (1.4)
\[(2.13)\]
\[\vartheta(z) \left[ h \left( \frac{p(z) - 1}{p(z) + 1} \right) - 1 \right] = \frac{1}{2} A_0 B_1 c_1 z + \frac{1}{2} A_1 B_1 c_1 + \frac{1}{2} A_0 B_1 \left( c_2 - \frac{c_1^2}{2} \right) + \frac{A_0 B_2 c_1^2}{4} z^2 \cdots \]
\[(2.14)\]
\[\vartheta(z) \left[ h \left( \frac{q(w) - 1}{q(w) + 1} \right) - 1 \right] = \frac{1}{2} A_0 B_1 b_1 w + \frac{1}{2} A_1 B_1 b_1 + \frac{1}{2} A_0 B_1 \left( b_2 - \frac{b_1^2}{2} \right) + \frac{A_0 B_2 b_1^2}{4} w^2 \cdots .\]

Now equating (2.11) and (2.13) in view of (2.9) and comparing the coefficients of \(z\) and \(z^2\), we have
\[(2.15)\]
\[(1 + \delta + 2\lambda) a_2 = \frac{1}{2} A_0 B_1 c_1,\]
\[(2.16)\]
\[(1 + 2\delta + 6\lambda) a_3 = \frac{1}{2} A_1 B_1 c_1 + \frac{1}{2} A_0 B_1 \left( c_2 - \frac{c_1^2}{2} \right) + \frac{A_0 B_2 c_1^2}{4} .\]
Similarly, (2.10) given us
\[-(1 + \delta + 2\lambda)a_2 = \frac{1}{2}A_0B_1b_1,\]
\[(2.17) (1 + 2\delta + 6\lambda)(2a_2^2 - a_3) = \frac{1}{2}A_1B_1b_1 + \frac{1}{2}A_0B_1 \left(b_2 - \frac{b_1^2}{2}\right) + \frac{A_0B_2b_1^2}{4}.\]

From (2.15) and (2.17) we find that
\[(2.19) c_1 = -b_1\]
and (2.16), (2.17) and (2.19) yields
\[a_2^2 = \frac{A_0^2B_1^2(b_2 + c_2)}{4(1 + 2\delta + 6\lambda)A_0B_1^2 - 4(1 + \delta + 2\lambda)^2(B_2 - B_1)}.\]

Now further computations (2.16) to (2.18) lead to
\[a_3 = \frac{A_1B_1(c_1 - b_1) + A_0B_1(c_2 - b_2)}{4(1 + 2\delta + 6\lambda)} + \frac{A_0^2B_1^2(c_2^2 + b_2^2)}{8(1 + 2\delta + 2\lambda)^2}.\]

Using the above results and in view of the inequalities \(|c_i| \leq 2\) and \(|b_i| \leq 2\) \((i = 1, 2)\) for functions with positive real part yield the requested estimate in (2.1) and (2.2).

**Remark 2.1.** For \(\delta = 0\), a function \(f \in \Sigma\) defined in (1.1) is said to be in the class \(H_q\Sigma(\lambda, \vartheta)\) if the following conditions are satisfied
\[\frac{f(z)}{z} + \lambda zf''(z) - 1 \prec q\left(\vartheta(z) - 1\right)\]
and \(g(w) = g(w) - 1 \prec q\left(\vartheta(w) - 1\right)\).

For \(\delta = 0\), we have the class \(H^q\Sigma(\lambda, 0, \vartheta) = H^q\Sigma(\lambda, \vartheta)\).

**Corollary 2.1.** Let \(f(z)\) given by (1.1) belong to the class \(H^q\Sigma(\lambda, \vartheta)\) then
\[|a_2| \leq \frac{|A_0|B_1\sqrt{B_1}}{\sqrt{|(1 + 6\lambda)A_0B_1^2 + (1 + 2\lambda)^2(B_2 - B_1)|}},\]
\[|a_3| \leq \frac{|A_0|^2B_1^2}{(1 + 2\lambda)^2} + \frac{|A_1|B_1}{1 + 6\lambda} + \frac{|A_0|B_1}{1 + 6\lambda}.\]

By putting \(\delta = 1\) in Theorem 2.1, we have the following corollary.

**Corollary 2.2.** Let \(f(z)\) given by (1.1) to the class \(H^q\Sigma(\lambda, 1, \vartheta)\) then
\[|a_2| \leq \frac{|A_0|B_1\sqrt{B_1}}{\sqrt{|3(1 + 2\lambda)A_0B_1^2 + 4(1 + \lambda)^2(B_2 - B_1)|}},\]
\[|a_3| \leq \frac{|A_0|^2B_1^2}{4(1 + \lambda)^2} + \frac{|A_1|B_1}{3(1 + 2\lambda)} + \frac{|A_0|B_1}{3(1 + 2\lambda)}.\]
Theorem 2.2. If \( f \in M^3_\Omega(\beta, \vartheta) \), then

\[
|a_2| \leq \frac{|A_0|B_1\sqrt{B_1}}{\sqrt{|(1 + \beta^2)A_0B_2^2 + (1 + \beta)^2(B_1 - B_2)|}}
\]

\[
|a_3| \leq \frac{|A_0|^2B_2^2}{1 + \beta^2} + \frac{|A_1|B_1}{2|1 + 2\beta|} + \frac{|A_0|B_1}{2|1 + 2\beta|}.
\]

Proof. Since \( f \in M^3_\Omega(\beta, \vartheta) \) and \( \Phi = \Psi^{-1} \) then there exist analytic functions \( k, s : \mathcal{D} \to \mathcal{D} \) with \( k(0) = s(0) = 0 \) satisfying

\[
z\frac{\Psi'(z)}{\Psi(z)} - 1 = \vartheta(z) (h(k(z)) - 1),
\]

\[
\frac{w\Phi'(w)}{\Phi(w)} - 1 = \vartheta(w) (h(s(w)) - 1).
\]

For \( p(z) \) and \( q(w) \) as given in (2.5) and (2.6), respectively, in view of (2.20), (2.21), clearly

\[
z\frac{\Psi'(z)}{\Psi(z)} - 1 = (1 + \beta)a_2z + [2(1 + 2\beta)a_3 + (\beta^2 - 4\beta - 1)a_2^2] z^2 + \cdots,
\]

\[
\frac{w\Phi'(w)}{\Phi(w)} - 1 = -(1 + \beta)a_2w + [-2(1 + 2\beta)a_3 + (\beta^2 + 4\beta + 3)a_2^2] w^2 + \cdots.
\]

The right-hand sides of (2.22) and (2.23) are given by (2.13) and (2.14) respectively. Now using (2.13) and (2.14) in (2.24) and comparing the coefficients of \( z \) and \( z^2 \), we get

\[
(1 + \beta)a_2 = \frac{1}{2}A_0B_1c_1
\]

\[
2(1 + 2\beta)a_3 + (\beta^2 - 4\beta - 1)a_2^2 = \frac{1}{2}A_1B_1c_1 + \frac{1}{2}A_0B_1 \left( c_2 - \frac{c_1^2}{2} \right) + \frac{A_0B_2c_1^2}{4}.
\]

Similarly, it follows from (2.14), (2.25) and (2.24) that

\[
(1 + \beta)a_2 = \frac{1}{2}A_0B_1b_1,
\]

\[
-2(1 + 2\beta)a_3 + (\beta^2 + 4\beta + 3)a_2^2 = \frac{1}{2}A_1B_1b_1 + \frac{1}{2}A_0B_1 \left( b_2 - \frac{b_1^2}{2} \right) + \frac{A_0B_2b_1^2}{4}.
\]

From (2.20) and (2.28) it follows that

\[
c_1 = -b_1
\]

and (2.21), (2.24) and (2.25), yield

\[
a_2^2 = \frac{A_0^2B_1^4(b_2 + c_2)}{4[(1 + \beta^2)A_0B_2^2 - (1 + \beta)^2(B_2 - B_1)]}.
\]
Now further computation (2.27) to (2.29) leads to
\[ a_3 = \frac{A_1 B_1 (c_2 - b_2)}{8(1 + 2\beta)} + \frac{A_0 B_1 (c_2 - b_2)}{8(1 + 2\beta)}. \]
□

**Remark 2.2.** Putting \( \vartheta(z) = 1 \) in Theorem 2.2, we get the corresponding result given by Deniz [6], for \( \beta = 0 \) the above Theorem 2.2 reduces to the following corollary.

**Corollary 2.3.** If \( f \in M_2^q(0, \vartheta) \), then
\[ |a_2| \leq \frac{|A_0|B_1 \sqrt{B_1}}{\sqrt{|A_0|B_2^2 + (B_1 - B_2)|}}, \quad \text{and} \quad |a_3| \leq \frac{|A_0|^2 B_1^2 + \frac{|A_1|B_1}{2} + \frac{|A_0|B_1}{2}}{2}. \]

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