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Ambiguity in a pandemic recession, asset prices, and lockdown policy

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Abstract
Using an asset pricing model of a multisector production economy including pandemic disaster, we explain the average stock price boom and significant cross-sectional variation of stock returns in the United States and Japan during the COVID-19 pandemic recession. We find that two features of the pandemic, namely ambiguity and sector-specific shocks, are critical determinants of the unusual asset price dynamics observed. Extending the model, we analyze the welfare effects of lockdown policy during pandemics for heterogeneous households. We theoretically show that enforcing a lockdown improves the welfare of asset holders and households working in sectors with positive sector-specific shocks. Consequently, a Pareto-optimal lockdown policy controls for the tightness of lockdown to maximize the welfare of households working in sectors with negative sector-specific shocks.

1 | INTRODUCTION

The COVID-19 pandemic has had a huge impact on the global economy. However, despite the declines in GDP and consumption, asset prices, including stock prices, have continued to rise during the resultant economic crisis. This contrasts with the decline in asset prices that typically accompanies a recession. In this paper, using a tractable model including the pandemic disaster, we explain the stock price dynamics and explore the resource allocation among heterogeneous agents and the role of lockdown policy in the economy.

For the economic analysis of the pandemic disaster, we focus on the following three points. First, there has been intense uncertainty about the outlook for the COVID-19 pandemic.
Uncertainty about the infectivity of the virus, uncertainty about the timing of vaccine development, and so forth, provide few clues as to how far into the future this situation will persist. As demonstrated by Altig et al. (2020), the substantial increase in asset price volatility is associated with such uncertainty about the pandemic. Under these circumstances, it is difficult to conduct objective probability assessments, and most individuals can only make decisions based on subjective probability assessments.

Second, the COVID-19 pandemic has had a distinct impact on individual households and firms. There are industries whose activities have been severely curtailed, such as the tourism and food service industries. Other industries, however, such as information technology, are growing rapidly as they serve as essential infrastructure for many economic activities during the pandemic: Figure 1 illustrates the stock price dynamics in the United States and Appendix A provides a detailed explanation. The analysis in this paper captures these heterogeneous impacts of the pandemic on the performance of firms and industries.

Finally, many governments have enforced lockdowns in response to the COVID-19 pandemic. The tightness of these lockdowns varies across countries, but all affect the macroeconomic and distributive dynamics of economies by changing infection rates and cross-sectional demand patterns. The lockdown policies, therefore, should have differing welfare effects across economic agents.

![Figure 1](image_url)  
US stock markets: three high-growth sectors, eight low-growth sectors, the S&P 500 index, and the VIX index. The price index of three high-growth sectors (S&P High sectors), eight low-growth sectors (S&P Low sectors), and the S&P 500 index (S&P) is shown. The S&P 500 index is divided into 11 categories. Among the 11 sectors, stock prices in the following three sectors have risen more than the S&P500 index: Information Technology, Consumer Discretionary, and Communication Services. Stock prices in the remaining eight sectors have not risen as much as the S&P500 index; Health Care, Financials, Industrials, Consumer Staples, Energy, Utilities, Real Estate, and Materials. As the first infection was identified in December 2019 in China, we normalize the values on December 11, 2019, to 100. It also shows the VIX index (VIX), the official name of which is the Chicago Board Options Exchange’s CBOE Volatility Index. The sample period is from November 14, 2018, to February 10, 2021. See Appendix A in more detail.
In this paper, we propose a model that incorporates the role of ambiguity in the pandemic recession and cross-sectional variations in stock returns during the pandemic. Developing the model, we explore the resource allocation dynamics among heterogeneous agents and evaluate the welfare consequences of lockdown policy. More concretely, our model comprises a consumption-based asset pricing model of a multisector production economy including pandemic shocks and ambiguity about the stochastic structure of these shocks. In the model, there are ambiguity-averse capitalists (asset holders) and workers (non-asset holders) belonging to either booming or damaged sectors (hereafter, sectors A and B, respectively).

Using the model without lockdown policy, we show that the consumption/saving choice of capitalists given ambiguity leads to a stock price boom at the time of a very rare pandemic recession. This is because these ambiguity-averse capitalists form a high subjective probability of the continuation of the pandemic and save much to smoothen consumption under the anticipated long recession. This increases asset demand and pushes up asset prices.

Next, following Lucas (1987), we measure the welfare cost over the business cycle from the pandemic for all agents: that is, the capitalists and the workers in sectors A and B. We find that the sizes of the welfare losses differ among them. The most damaged agents are the sector B workers, the capitalists are moderately damaged, and the least damaged are the sector A workers. This is because while capitalists, whose incomes arise from aggregate excess profit, are affected only by the aggregate negative shock of the pandemic, workers receive, in addition, a cross-sectional shock arising from the resultant demand shift, such that sector A workers are the winners and sector B workers are the losers.

Finally, we extend our baseline model to a model incorporating lockdown policy. We show that various patterns of stock price dynamics are realized according to the tightness of lockdown. As discussed in Section 4, our model yields a negative relationship between asset price change and the tightness of lockdown, and this is consistent with the findings of recent empirical studies in the area, including Saito and Sakamoto (2021) and Scherf et al. (2022). We also find that enforcing lockdown improves the welfare of both capitalists and sector A workers, but not necessarily sector B workers. Accordingly, if the effectiveness of a lockdown is low, then introducing a lockdown lowers the welfare of sector B workers. In contrast, if a lockdown is sufficiently effective, then sector B workers are also winners from a mild lockdown (although a tight lockdown harms them). This implies that a Pareto improvement is possible by restricting the economic activities to at least some extent with pandemic recessions.

Using a standard calibration, the numerical analysis demonstrates that the welfare of capitalists and sector A workers is increasing, while that of sector B workers is hump-shaped with respect to the tightness of lockdown. This implies that maximizing the welfare of sector B workers is a Pareto-optimal lockdown policy. However, as demonstrated later, the utilitarian social planning problem selects a stricter lockdown. Thus, we should consider appropriate

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1We assume that agents face uncertainty about the probability that a pandemic continues and employ a subjective decision-making framework incorporating the ambiguity of Gilboa and Schmeidler (1989). In this setting, the ambiguity aversion of agents is represented by a max-min expected utility function, which means that the agents’ subjective assessment of the probability of continuation of the pandemic is as high as possible. Of course, we could employ a more general setting, such as a subjective expectation asset pricing model based on the smooth ambiguity utility representation constructed by Ju and Miao (2012). Following Klibanoff et al. (2005), we obtain the max-min expected utility representation as the limiting case of extremely strong ambiguity aversion. As this is the only tractable case in the smooth ambiguity framework, we adopt the setting of Gilboa and Schmeidler (1989).
redistribution policies if we introduce a tight lockdown to attain both an early end to the pandemic and to avoid a severe economic recession.

### 1.1 Literature review

The work most related to the present paper is that of Herrenbrueck (2021). A characteristic feature of the COVID-19 pandemic recession is that people must curtail their consumption. To reflect this, Herrenbrueck (2021) constructs a representative agent model that imposes an upper bound constraint on consumption. He then shows that stock prices rise in the situation where the consumption constraint binds because it stimulates saving demand. In contrast, our analysis focuses on an alternative mechanism to suppress consumption and increase saving demand, being the ambiguous nature of the pandemic recession. Altig et al. (2020) demonstrated that the COVID-19 pandemic has led to a substantial increase in financial market uncertainty. Following this, we assume that ambiguity-averse individuals will consider the possibility that the pandemic will persist for several years. We show that subjective probability assessments such as this can also raise asset prices, but by employing a mechanism unlike Herrenbrueck’s upper bound constraint on consumption. Elsewhere, Guerrieri et al. (2020) present a mechanism in which supply shocks to specific sectors cause demand shocks in a multisector production economy model. A further novelty of our paper relates to the importance of heterogenous impacts on economic activities. Contrary to Herrenbrueck (2021), we rely on a heterogeneous agent model with multiple production sectors. Therefore, we are able to analyze the impact of lockdown restrictions on cross-sectional variation in stock returns and welfare impacts among heterogeneous individuals.

Many other studies have already analyzed the impact of the COVID-19 pandemic on economic activity and most consider lockdown policy to reduce the mortality rate. For example, Acemoglu et al. (2020), Alvarez et al. (2020), Gollier (2020), Guan et al. (2020), Bosi et al. (2021), and Kubota (2021) incorporate the susceptible, infected, and recovered (SIR) model of epidemiology into a dynamic economic model to derive the optimal extent and timing of lockdown. Adaptively, using an SIR-dead model, Gallic et al. (2021) compare the two most important nonpharmaceutical policy strategies used to combat COVID-19 in Europe: natural herd immunity and avoiding the saturation of hospital capacity. Hritonenko et al. (2021) propose an integral model for a real description of the epidemic dynamics in the COVID-19 infectiousness period and assess how lockdown and reopening policies affect epidemic spread. Pierre and Ponthiere (2022) discuss the welfare implications of lockdown policy in an overlapping generations economy without an SIR structure. Unlike these studies, we do not explicitly consider the relationship between lockdown policy and human lives and instead focus on the role of the former in cross-sectional redistribution.

Atolia et al. (2021) construct a neoclassical growth model with a nondegenerate distribution of initial asset holdings and consider a reopening policy. Once again, we focus on the significance of lockdown in ending the economic disaster and explore not the distribution of asset holding, but the disparity of asset holding between households in different classes. Chan-Lau and Zhao (2020) suggest that the more a country loosens its fiscal stimulus in response to a pandemic, the more negative the impact on the stock market becomes. Likewise, instead of fiscal policy, we focus on lockdown and ambiguity about the persistence of the disaster. Lastly, Dalton et al. (2021) empirically examines the K-shaped recovery from the pandemic observed in
the labor market as a whole, whereas our model captures these dynamics for two contrasting sectors.

Finally, using the framework of the welfare cost of business cycles (Alvarez & Jermann, 2004; Lucas, 1987; Weitzman, 2007), we contribute to the welfare analysis of economic disasters. The seminal work by Barro (2009) suggests that with an asset pricing model of a representative agent economy including economic disaster, the welfare loss from the disaster is substantial. The novelty of our analysis is that we shed light on the heterogeneous impacts on economic agents, such that the divergence in stock prices across sectors closely relates to the underlying disparity in incomes and assets between workers, and this leads to a conflict of interest concerning optimal lockdown policy.

2 | MODEL

2.1 | Production

2.1.1 | Final good producers

A representative final good producer faces perfectly competitive markets and produces the final good from intermediate goods $A$ and $B$. The production function has constant elasticity of substitution (CES) as follows.

\[
y_t = x_t \left[ (1 - \alpha_t) \frac{y_{At}}{y_t} \frac{\sigma - 1}{\sigma} + \alpha_t \frac{y_{Bt}}{y_t} \frac{\sigma - 1}{\sigma} \right],
\]

where $y_t$ denotes output, $x_t$ is total factor productivity (TFP), $y_{At}$ and $y_{Bt}$ are intermediate goods from sectors $A$ and $B$, respectively, $\alpha_t$ denotes the demand weight, and $\sigma$ a parameter reflecting the elasticity of substitution between the intermediate goods. The profit of the final good producer is

\[
\Pi_t \equiv y_t - p_{At} y_{At} - p_{Bt} y_{Bt},
\]

where $p_{At}$ and $p_{Bt}$ denote the price of intermediate goods $A$ and $B$, respectively. Here, we normalize the price of final goods to unity.

The profit maximization conditions are

\[
p_{At} = x_t^{\frac{\sigma - 1}{\sigma}} (1 - \alpha_t) \left( \frac{y_{At}}{y_t} \right)^{-\frac{1}{\sigma}},
\]

\[
p_{Bt} = x_t^{\frac{\sigma - 1}{\sigma}} \alpha_t \left( \frac{y_{Bt}}{y_t} \right)^{-\frac{1}{\sigma}}.
\]

These equations represent the inverse demand functions for intermediate goods. As the final good producer faces perfectly competitive markets, profit should be zero: $y_t = p_{At} y_{At} + p_{Bt} y_{Bt}$. 
2.1.2 | Intermediate goods producers

There are two intermediate good producers that monopolistically supply the intermediate good by hiring workers. We identify them by their sectors as $A$ and $B$. Their production functions are assumed linear:

$$y_{At} = l_{At},$$  \hspace{1cm} (5)
$$y_{Bt} = l_{Bt},$$  \hspace{1cm} (6)

where $l_{At}$ and $l_{Bt}$ are the labor inputs of sectors $A$ and $B$, respectively. The profits of these producers are written as follows.

$$\pi_{At} \equiv p_{At} l_{At} - w_{At} l_{At},$$  \hspace{1cm} (7)
$$\pi_{Bt} \equiv p_{Bt} l_{Bt} - w_{Bt} l_{Bt},$$  \hspace{1cm} (8)

where $w_{At}$ and $w_{Bt}$ are the wages of sectors $A$ and $B$.

The profit maximization conditions are

$$w_{At} = x_t^{\frac{\sigma-1}{\sigma}} (1 - \alpha_t) \frac{\sigma - 1}{\sigma} \left( \frac{l_{At}}{y_t} \right)^{\frac{1}{\sigma}},$$ \hspace{1cm} (9)
$$w_{Bt} = x_t^{\frac{\sigma-1}{\sigma}} \alpha_t \frac{\sigma - 1}{\sigma} \left( \frac{l_{Bt}}{y_t} \right)^{\frac{1}{\sigma}}.$$ \hspace{1cm} (10)

These equations serve as demand functions of the factors used for output production.

2.1.3 | Distribution of profits

We assume that the labor supply is fixed to one. Thus, the market clearing conditions for labor are written as $l_{At} = l_{Bt} = 1$. Thus, equilibrium wages are given as follows.

$$w_{At}^* = x_t^{\frac{\sigma-1}{\sigma}} (1 - \alpha_t) \frac{\sigma - 1}{\sigma} y_t^\sigma,$$ \hspace{1cm} (11)
$$w_{Bt}^* = x_t^{\frac{\sigma-1}{\sigma}} \alpha_t \frac{\sigma - 1}{\sigma} y_t^\sigma.$$ \hspace{1cm} (12)

The equilibrium prices of intermediate goods are given as follows.

$$p_{At}^* = x_t^{\frac{\sigma-1}{\sigma}} (1 - \alpha_t) y_t^\sigma,$$ \hspace{1cm} (13)
$$p_{Bt}^* = x_t^{\frac{\sigma-1}{\sigma}} \alpha_t y_t^\sigma.$$ \hspace{1cm} (14)

The equilibrium excess profits of the intermediate goods producers are.
Note that (i) the ratio of aggregate excess profits and aggregate labor income to output are $\frac{1}{\sigma}$ and $1 - \frac{1}{\sigma}$, respectively, and that (ii) the ratio of the excess profits of sector B to aggregate excess profits and the share of labor income of sector B to aggregate labor income are both $\alpha_t$. Therefore, the distributive balance between capital and labor income is constant, but cross-sectional variations arise according to the economic state.

2.2 Time-series properties of pandemic disasters

There is an ambiguous pandemic shock in this economy, under which agents face uncertainty about its stochastic structure. According to realizations of the pandemic shock, there are two economic states: $s_t \in \{n, d\}$, which are normal and pandemic disaster states, respectively. Then, the aggregate demand factor, denoted by $x_t$, depends on the state of the economy: $x_t = x(s_t)$. The stochastic process for the logarithm of the aggregate demand factor is of the form:

$$\ln x(s_{t+1}) - \ln x(s_t) = g + \ln(1 - b) \times \xi(s_{t+1}) + u_{t+1},$$

where $g$ is an exogenous trend and $b$ is the scale parameter of the pandemic shock.\(^2\) If a pandemic occurs in period $t + 1$ (i.e., $s_{t+1} = d$), then $\xi(s_{t+1}) = 1$; otherwise (i.e., $s_{t+1} = n$), $\xi(s_{t+1}) = 0$. The innovation term $u_{t+1}$ is identically and independently distributed as $N\left(0, \sigma_u^2\right)$. We consider these IID process shocks \(\{u_{t+1}\}_{t=0}^{\infty}\) as business cycle shocks.

The pandemic episode, \(\{s_t\}_{t=1}^{\infty}\), follows a Markov process. We denote the probability that a pandemic occurs by $\phi = \Pr(s_{t+1} = d | s_t = n)$, and the probability that a pandemic continues by $\theta = \Pr(s_{t+1} = d | s_t = d)$. Then, the growth rate of the aggregate demand factor is given as follows.

$$E\left[\frac{x_{t+1}}{x_t} | s_{t+1}\right] = \begin{cases} 
\exp\left(g + \frac{\sigma_u^2}{2}\right) & \text{if } s_{t+1} = n, \\
(1 - b)\exp\left(g + \frac{\sigma_u^2}{2}\right) & \text{if } s_{t+1} = d.
\end{cases}$$

The pandemic shock affects not only the aggregate demand factor but also the balance of demand between the two sectors. We assume that the demand parameter for each intermediate good, $\alpha_t$, depends on state $s_t$: $\alpha_t \in \{\alpha_n, \alpha_d\}$.

\(^2\)We may wonder whether the pandemic demand shock to $x_t$ (and $\alpha_t$) is not natural because $x_t$ is TFP (and $\alpha_t$ is the weight on intermediate good $B$) in the production function of the final good, (1). However, Guerrieri et al. (2020) demonstrated that a supply shock caused by coronavirus pandemic results in a demand shock in a multisector production economy. Given this, to keep the model simple, we consider supply side shocks like Pierre and Ponthiere (2022) as the aggregate demand shock from a pandemic.
2.3 | Households

In this economy, there are two classes of households: workers and capitalists. The workers are divided into two types as follows. A continuum of workers supplies one unit of labor services inelastically to sector A, and another continuum of workers does the same for sector B. These workers have no initial asset holdings, consume all their labor income in each period, and do not save at all. The capitalist class consists of a continuum of agents that hold assets, obtain capital income, but do not supply any labor services. The capitalists pursue intertemporal optimization, namely their optimal consumption/saving plan over an infinite horizon.

First, we formulate the capitalists’ behavior. The capitalists face uncertainty about the distribution of \( s_t \), denoted by \( \nu_t \), which is determined by parameters \( \phi \) (the probability that disasters occur) and \( \vartheta \) (the probability that disasters continue). However, we assume that the capitalists know the true value of \( \phi \), because it is widely known from historical experiences that the probability of disasters occurring is close to zero. Thus, the capitalists only face uncertainty about \( \vartheta \) and evaluate the disutility of this ambiguity.

With regard to the utility functions, we adopt Gilboa and Schmeidler’s (1989) max-min utility for ambiguity, where capitalists are extremely ambiguity-averse and seek to maximize their utility under the worst possible distribution. In this framework, we consider utility recursion of the following form:

\[
W_t = u(C_t) + \exp(-\rho)\min_{\vartheta} \mathbb{E}_{\nu_t}(W_{t+1}),
\]

\[
u_t\]

where \( W_t \) is the expected lifetime utility and \( C_t \) is the flow of consumption in period \( t \), \( \rho > 0 \) is the rate of time preference, and \( \gamma > 0 \) is the degree of relative risk aversion or the reciprocal of the intertemporal elasticity of substitution (IES). We denote the worst-case probability by \( \vartheta^* \), which is defined by

\[
\vartheta^* = \arg \min_{\vartheta} \mathbb{E}_{\nu_t}(W_{t+1}),
\]

(18)

and the corresponding subjective distribution by \( \nu_t^* \). Thus, the utility recursion described here reduces to the following expression.

\[
W_t = u(C_t) + \exp(-\rho)\mathbb{E}_{\nu_t^*}[W_{t+1}].
\]

The utility recursion reduces to standard time additive utility, which can be expressed as the following lifetime utility function:

\[
W_t = \mathbb{E}_{\nu_t^*}\left[ \sum_{\tau=0}^{\infty} \exp(-\rho\tau)u(C_{t+\tau}) \right].
\]

Next, let us consider the budget constraint of the representative capitalist. The capitalist trades the equity shares of intermediate goods producers and risk-free bonds. The capitalist’s budget constraint is given as
\[(P_{At} + \pi_{At})e_{At} + (P_{Bt} + \pi_{Bt})e_{Bt} + f_t = C_t + P_{At}e_{At+1} + P_{Bt+1}e_{Bt+1} + Q_t f_{t+1},\]

where \(P_t\) and \(e_t\) are the price and holdings of the equity share of intermediate product sector \(i \in \{A, B\}\) respectively, and \(Q_t\) and \(f_t\) are the price and holdings of the risk-free asset holdings in period \(t\).

Finally, we consider the workers’ choices. Recall that the workers do not participate in financial markets and follow a hand-to-mouth consumption function. As the workers consume their entire labor income in each period, their behavior is simply described as follows.

\[c_{it} = w_l l_{it}, \text{ for all } t \text{ and } i = A, B,\]

where \(c_{it}\) is the consumption level of the workers in sector \(i\).

### 2.4 Market clearing conditions

The market clearing condition for the risk-free bond is \(f_t = 0\), and \(e_{At} = 1\) and \(e_{Bt} = 1\) for the equity share. The market clearing condition for goods and services is \(y_t = C_t + e_{At} + c_{Bt}\). The labor markets are cleared when \(l_{At} = 1\) and \(l_{Bt} = 1\) hold. The following conditions also hold: \(C_t = \frac{y_t}{\sigma}\), \(c_{At} = (1 - \alpha_t) \frac{\sigma - 1}{\sigma} y_t\), and \(c_{Bt} = \alpha_t \frac{\sigma - 1}{\sigma} y_t\).

We note the following. Because \(y_t = x_t\) in equilibrium given (1), the aggregate excess profit is proportional to \(x_t\): \(\pi_{At} + \pi_{Bt} = \frac{1}{\sigma} x_t\). This indicates that the capitalists’ income is affected by the aggregate negative shock of the pandemic, but is independent of the sector-specific shock, \((\alpha_d, \alpha_n)\). Therefore, as shown later, the equilibrium consumption of capitalists does not depend on the size of the sector-specific shock.

### 3 EQUILIBRIUM

The capitalists’ Euler equations, or first-order conditions, for \(e_{it+1}\) and \(f_{t+1}\) are expressed as follows.

\[P_{it} = \mathbb{E}_{\pi_t^*} \left[ \exp(-\rho) \left( \frac{C_{t+1}}{C_t} \right)^{\gamma} (P_{it+1} + \pi_{it+1}) \right], \quad (19)\]

\[Q_t = \mathbb{E}_{\pi_t^*} \left[ \exp(-\rho) \left( \frac{C_{t+1}}{C_t} \right)^{\gamma} \right]. \quad (20)\]

We define the price–dividend ratio of the intermediate product sector \(i \in \{A, B\}\) as \(\omega_{it} \equiv \frac{P_{it}}{\pi_{it}}\). Using (15), (16), and \(\frac{C_{t+1}}{C_t} = \frac{y_{t+1}}{y_t}\), (19) is rewritten as follows.

\[\omega_{At} = \mathbb{E}_{\pi_t^*} \left[ \exp(-\rho) \left( \frac{y_{t+1}}{y_t} \right)^{1-\gamma} \frac{1 - \alpha_{t+1}}{1 - \alpha_t} (\omega_{At+1} + 1) \right], \quad (21)\]
\[ \omega_{Br} = E^{-\rho} \left[ \exp(-\beta) \frac{y_{t+1}}{y_t} \right]^{1-\gamma} \frac{\alpha_{t+1}}{\alpha_t} (\omega_{Br+1} + 1). \] (22)

As pandemic episodes follow a Markov process and the capitalist has a constant relative risk aversion (CRRA) utility function, asset prices are a function of the current state of the economy. We denote the price–dividend ratio of intermediate sector firm \( i \) in state \( s \) by \( \omega_{is} \). Then, (21) can be expressed by the following system of equations:

\[ \omega_{An} = (1 - \phi) \beta (\omega_{An} + 1) + \phi \beta \delta \frac{1 - \alpha_d}{1 - \alpha_n} (\omega_{Ad} + 1), \] (23)

\[ \omega_{Ad} = (1 - \theta^*) \beta \frac{1 - \alpha_n}{1 - \alpha_d} (\omega_{An} + 1) + \theta^* \beta \delta (\omega_{Ad} + 1), \] (24)

where \( \beta \equiv \exp[-\rho + (1 - \gamma) g + \frac{(1 - \gamma)^2}{2} \sigma_u^2] \) and \( \delta \equiv (1 - b)^{1-\gamma} \). Similarly, (22) can be expressed as

\[ \omega_{Bn} = (1 - \phi) \beta (\omega_{Bn} + 1) + \phi \beta \delta \frac{\alpha_d}{\alpha_n} (\omega_{Bd} + 1), \] (25)

\[ \omega_{Bd} = (1 - \theta^*) \beta \frac{\alpha_n}{\alpha_d} (\omega_{Bn} + 1) + \theta^* \beta \delta (\omega_{Bd} + 1). \] (26)

We define \( \eta_i \) as follows.

\[ \eta_A \equiv \frac{1 - \alpha_d}{1 - \alpha_n}, \] (27)

\[ \eta_B \equiv \frac{\alpha_d}{\alpha_n}. \] (28)

Using the above definition, we have the simple expression of intermediate sector \( i \)'s price–dividend ratio and state \( s, \omega_{is} \).

\[ \omega_{in} = (1 - \phi) \beta (\omega_{in} + 1) + \phi \beta \delta \eta_i (\omega_{id} + 1), \] (29)

\[ \omega_{id} = (1 - \theta^*) \beta \eta_i^{-1} (\omega_{in} + 1) + \theta^* \beta \delta (\omega_{id} + 1). \] (30)

\[ W_i \equiv \begin{bmatrix} \omega_{in} \\ \omega_{id} \end{bmatrix} = M \times (W_i + i), \]

where

\[ M \equiv \begin{bmatrix} (1 - \phi) \beta & \phi \beta \delta \eta_i \\ (1 - \theta^*) \beta \eta_i^{-1} & \theta^* \beta \delta \end{bmatrix}. \]

Then, \( W \) is the solution of \( W = (I - M)^{-1} Mi \), where \( I \) is an identity matrix and \( i \) is a column vector whose elements are ones.
3.1 Refinement of subjective beliefs

Here, we can deduce the subjective belief $\theta^*$, which supports the competitive equilibrium. As the representative capitalist is perfectly ambiguity-averse, the subjective belief seems to necessarily correspond with the worst belief possible: $\theta^* = 1$. However, because the subjective belief affects equilibrium behavior and therefore the market clearing condition, we should care about the consistency of subjective beliefs and the existence of competitive equilibria. To ensure the existence of competitive equilibria, the determinant of $I - M$ should be positive, but this depends on $\theta^*$. The determinant is given as $1 - \beta + \beta(1 - \beta \delta) \phi - \beta \delta (1 - \beta) \theta^*$. The following inequality is satisfied for positive equity prices.

$$\theta^* < \frac{1}{\beta \delta} + \frac{1 - \beta \delta}{\delta (1 - \beta)} \phi. \tag{31}$$

To determine $\theta^*$, we adopt the following assumption.

**Assumption 1.** We assume that $\delta \geq 1, \beta < 1, \beta \delta < 1$, and $\kappa < 1$, where $\kappa \equiv (1 - b) \exp \left( g + \frac{\sigma^2}{2} \right)$.

The interpretation of Assumption 1 is as follows. Because $\beta = \exp \left[ -\rho + (1 - \gamma) g + \frac{(1 - \gamma)^2}{2} \sigma^2 \right]$ is the current average value of one unit of the consumption good in the preceding period, $\beta < 1$ implies natural discounting. The condition $\kappa < 1$ is also reasonable, as it implies that the pandemic shock leads to negative growth on average. The condition $\delta = (1 - b)^{1 - \gamma} > 1$ matters. This is equivalent to $\gamma > 1$. Thus, given $\delta > 1$, we assume implicitly that IES, $1/\gamma$, is lower than 1. This is considered to be a quantitatively plausible assumption in the macroeconomics literature. Finally, $\beta \delta < 1$ indicates that the size of the pandemic or the degree of relative risk aversion is sufficiently large, relative to the natural discounting. Although relaxing this condition does not change the results that follow, it makes the exposition of the model very simple without loss of generality and is likely to hold under standard parameter values.

Under Assumption 1, we obtain the subjective distribution of the capitalists.

**Lemma 1.** The subjective distribution of the capitalists is characterized by $\theta^* = 1$.

**Proof.** As capitalists with subjective expectations calculate the expected utility based on the worst-case probability, $\theta^*$ should be the highest possible value for a given $\phi$. The inequality (31) limits the possible highest $\theta^*$ under the existence of the competitive equilibrium. By $\beta \delta < 1$, the right-hand side of (31) is larger than 1. Therefore, $\theta$ can be the maximum, that is, $\theta^* = 1$. □

Although the value of $\theta^*$ is restricted to unity, we use the notation $\theta^*$ in the following equations for comprehensibility.
3.2 | Pandemic and asset prices

Solving the above system of equations, \( W = (I - M)^{-1}Mi \), we obtain the price–dividend ratios in equilibrium using \( \theta^* \).

\[
\omega_{in} = \beta \frac{1 - [1 - \delta (\eta_i + \beta)] \phi - \beta \delta \theta^*}{1 - \beta + \beta (1 - \beta \delta) \phi - \beta \delta (1 - \beta) \theta^*},
\]

(32)

\[
\omega_{id} = \beta \frac{\eta_i^{-1} + \beta \delta \phi - [\eta_i^{-1} - \delta (1 - \beta)] \theta^*}{1 - \beta + \beta (1 - \beta \delta) \phi - \beta \delta (1 - \beta) \theta^*}.
\]

(33)

We investigate the asset price movements when the pandemic shock hits the economy. There are two drivers of asset price fluctuations in this model: aggregate pandemic shock \( \xi \) and sector-specific pandemic shocks. Independently of the sector-specific shocks, the capitalists incorporate \( \xi \) and pursue intertemporal consumption smoothing with strong ambiguity aversion. This affects the stock prices of both sectors in the same manner. In addition, sector-specific shocks skew the cross-sectional balance of demand between the two sectors. This exerts heterogeneous impacts on the two sectors; sector A gains while sector B loses. These distinct disasters, the aggregate and sector-specific shocks, complicate the stock price dynamics in this model. Therefore, we decompose their marginal contributions in the following manner.

**Lemma 2.** Consider a market portfolio that invests in the stocks of both sectors A and B and receives aggregate excess profits \( y_t / \sigma \) as dividends. The price–dividend ratios of this market portfolio equal those of the individual sectors without sector-specific shocks: \( \alpha_n = \alpha_d \).

**Proof.** \( P_t \) denotes the share price of the market portfolio in period \( t \). Because \( P_t \) should be determined by the capitalists’ Euler equations, the following equation holds:

\[
P_t = \mathbb{E}_{\eta^*} \left[ \exp(-\rho) \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( P_{t+1} + \frac{y_{t+1}}{\sigma} \right) \right].
\]

\( \omega_t \) denotes the price–dividend ratio of the market portfolio and is expressed as

\[
\omega_t = \mathbb{E}_{\eta^*} \left[ \exp(-\rho) \left( \frac{y_{t+1}}{y_t} \right)^{1-\gamma} (\omega_{t+1} + 1) \right].
\]

Because the pandemic event follows a Markov process, the price–dividend ratios are state dependent: \( \omega_n \) and \( \omega_d \). In particular, they follow the system of equations (29) and (30) with \( \eta_i = 1 \). Therefore, \( \omega_n \) and \( \omega_d \) can be written as (32) and (33) with \( \eta_i = 1 \).

Lemma 2 states that the individual stock price without sector-specific shocks serves as the price of the market portfolio. It is useful to understand the following Proposition 1.
**Proposition 1.** Consider the market portfolio or the case where there are no sector-specific shocks: $\alpha_n = \alpha_d$. Suppose that $\phi$ is sufficiently small and $\delta$ is sufficiently greater than one so that the following conditions are satisfied:

$$\phi \leq \tilde{\phi} = \frac{\delta[\beta(1-\kappa) + \kappa] - 1}{\delta[\beta(1-\kappa) + 1] - 1},$$

$$\delta > \frac{1}{\kappa + (1-\kappa)\beta}.$$  

Then, the equity prices of both sectors rise when the aggregate pandemic shock hits the economy: $\frac{P_{id}}{P_{in}} \geq 1$ for $i = A, B$.

**Proof.** As $P_{is} = \omega_{is} \pi_{is}$ and $\alpha_n = \alpha_d$, 

$$\frac{P_{id}}{P_{in}} = \frac{1 + \beta\delta\phi - [1 - (1 - \beta)\delta]\theta^*}{1 + [\delta(1+\beta) - 1]\phi - \beta\delta\theta^*\kappa}.$$  

The condition $\frac{P_{id}}{P_{in}} \geq 1$ is equivalent to the following inequality:

$$dP \equiv -(1 - \kappa) - [\delta - 1 + \beta\delta(1-\kappa)]\phi + [\kappa(\delta - 1) + (1 - \kappa)\beta\delta]\theta^* \geq 0.$$  

(34)

Note that both the numerator and denominator of the coefficient of $\phi$ in the second term on the right-hand side of (34), $\delta - 1 + \beta\delta(1-\kappa)$ and $\beta\delta(1-\kappa) + \kappa(\delta - 1)$, are positive under Assumption 1. Thus, solving (34) with respect to $\phi$, we obtain

$$\phi \leq \frac{[\beta\delta(1-\kappa) + \kappa(\delta - 1)]\theta^* - (1 - \kappa)}{\delta - 1 + \beta\delta(1-\kappa)}.$$  

The right-hand side of this inequality equals $\tilde{\phi} \in (0, 1)$, because $\theta^* = 1$, $\delta > \frac{1}{\kappa + (1-\kappa)\beta}$, and $\kappa < 1$. □

The intuition for Proposition 1 is as follows. The pandemic disaster lowers the sum of expected dividends under the subjective distribution, which has two effects on stock prices. First, the low expected dividend simply lowers the value of shares. Second, and in contrast, this leads to an increase in asset demand because the lower income in the future induces the capitalists to save more for consumption smoothing. When the latter dominates the former, a stock price boom under a pandemic recession occurs.

The ambiguity-averse capitalists believe that the pandemic will be ongoing for a long time. Because of the low expected growth rate of consumption immediately after the pandemic and the low IES $(1/\gamma < 1)$ setting, the income effect dominates the substitution effect.  

3Note that because $\beta > 1$ and $\kappa < 1$, the assumption $\delta > 1/[\kappa + (1-\kappa)\beta]$ ensures $\delta > 1$. This indicates a $1/\gamma$ of less than one given $\delta = (1 - b)^{1-\gamma}$.  

This amplifies the motive of the capitalists for consumption smoothing and fosters the latter effect. Thus, ambiguity drives the stock price boom. In fact, using (34), we find that a stock price boom requires a sufficiently large $\theta^*$. This is because the capitalists’ incentive to save is not as strong if they expect the pandemic to end soon. As shown in the numerical exercises in Section 5, $\theta^*$ needs to be somewhat high to satisfy inequality (34) under parameter values consistent with the economic damage of the COVID-19 pandemic. Therefore, the subjective increase in the value of $\theta^*$ from the ambiguity plays a pivotal role in explaining asset prices.

This mechanism differs from that in Herrenbrueck’s (2021) model, which assumes that stock price appreciation occurs only when the pandemic is short (lasting only a few years). His model relies on logarithmic utility, meaning that the income and substitution effects cancel out. Therefore, if the pandemic persists, stock prices will fall because of lower dividends. Instead, the upper bound constraint on consumption causes people to consume less and save more, which raises asset prices only if the pandemic ends immediately.

What is more, the condition that $\phi$ is sufficiently small is intuitively plausible. When $\phi$ is small, a pandemic is an ex ante rare event. This is in common with Herrenbrueck (2021). He assumes a situation where the pandemic occurs suddenly, which corresponds to our model’s $\phi = 0$ case. Then, the ex post rise in the stock prices is not incorporated well into the ex ante stock prices. Therefore, the difference in stock prices between the normal and pandemic states will be largest when $\phi = 0$. However, this is not necessary for generating stock price booms following the pandemic. The point is that $\phi$ is very small, as we have shown.\footnote{The present paper also contributes to the literature on asset pricing with economic disasters (Barro, 2006; Rietz, 1988). In explaining the observed asset price dynamics, many studies insist that IES must be higher than or equal to one (Bansal et al., 2014; Nakamura et al., 2013; Watcher, 2013), although most empirical studies that directly estimate IES argue that IES should be lower than 0.8 (Havránek, 2015). Saito and Suzuki (2014) demonstrate a stock price boom under a persistent disaster in a model with an IES lower than one, while Suzuki (2012) provides a theoretical model of persistent disasters with an IES lower than one that is quantitatively consistent with the stock market booms observed in European countries during World War II. Note that in European countries during World War II, sustained economic stagnation and rising stock prices coincided (Jorion & Goetzmann, 1999; Oosterlinck, 2010). While these studies use the objective expectations models, we introduce subjective probability assessment into the asset pricing model of disasters, which captures the stark uncertainty of the COVID-19 pandemic.\footnote{\textsuperscript{4}}}

Next, we clarify the role of the sector-specific pandemic shocks.

**Proposition 2.** Suppose that $dP \geq 0$ holds.

(i) *Sector A’s equity price necessarily rises when the pandemic shock hits the economy:*

$$\frac{P_{At'}}{P_{An}} \geq 1.$$ 

(ii) *Sector B’s equity prices fall when the pandemic shock hits the economy,* $\frac{P_{Bt'}}{P_{Bn}} \leq 1$ if the sector-specific shock is sufficiently large, namely $\frac{\alpha_l}{\alpha_a}$ is sufficiently small.

**Proof.**

(i) The rate of change in sector A’s equity prices between the normal and pandemic states is $\frac{P_{At'}}{P_{An}} = \frac{\alpha_l(1 - \alpha_l)x}{\alpha_a(1 - \alpha_a)}$. This reduces to
\[
\frac{P_{Ad'}}{P_{An}} = \frac{[\beta \delta \phi + (1 - \beta) \delta \theta^*](1 - \alpha_d) + (1 - \theta^*)(1 - \alpha_n)}{[1 - (1 - \beta \delta) \phi - \beta \delta \theta^*](1 - \alpha_n) + \delta \phi (1 - \alpha_d)}.
\]

Thus, by (34), \(\frac{P_{Ad'}}{P_{An}} \geq 1\) is equivalent to the following inequality:

\[
dP (1 - \alpha_n) - [(1 - \beta \kappa) \phi - (1 - \beta) \kappa \theta^*] \delta (\alpha_n - \alpha_d) \geq 0.
\] (35)

Note that \(\beta \delta < 1\) implies \(\hat{\phi} < \frac{(1 - \beta) \kappa}{1 - \beta \kappa}\), which ensures \((1 - \beta \kappa) \phi - (1 - \beta) \kappa \theta^* < 0\) for \(\phi < \hat{\phi}\) under \(\theta^* = 1\). Therefore, (35) can be written as follows.

\[
\frac{dP}{[(1 - \beta \kappa) \phi - (1 - \beta) \kappa \theta^*] \delta} \leq \frac{1 - \alpha_d}{1 - \alpha_n} - 1.
\] (36)

This inequality always holds, because the left-hand side is negative, and the right-hand side is positive. Thus, \(dP \geq 0\) is a sufficient condition for \(\frac{P_{Ad'}}{P_{An}} \geq 1\).

(ii) The rate of change in sector B’s equity prices between the normal and pandemic states is \(\frac{P_{Bd'}}{P_{Bn}} = \frac{\omega_{Bd} \sigma_d \kappa}{\omega_{Bn} \sigma_n}\). It is expressed as follows.

\[
\frac{P_{Bd'}}{P_{Bn}} = \frac{[\beta \delta \phi + (1 - \beta) \delta \theta^*] \alpha_d + (1 - \theta^*) \alpha_n \kappa}{[1 - (1 - \beta \delta) \phi - \beta \delta \theta^*] \alpha_n + \delta \phi \alpha_d}.
\]

Thus, by (34), inequality \(\frac{P_{Bd'}}{P_{Bn}} \leq 1\) can be rewritten as

\[
\frac{dP}{(1 - \beta \delta) (\theta^* - \phi)} \leq 1 - \frac{\alpha_d}{\alpha_n}.
\] (37)

Therefore, a sufficiently strong shock to sector B (a sufficiently small \(\frac{\sigma_d}{\sigma_n}\)) causes the equity price to decline. \(\square\)

Part (i) of Proposition 2 suggests that the existence of the positive effect through intertemporal consumption smoothing against aggregate pandemic shocks is a sufficient condition for a rise in sector A’s stock price. This is because a sector-specific shock necessarily encourages production in sector A and boosts its stock prices. In fact, according to (36), we find that a strong sector-specific shock, that is, sufficiently large \(\frac{\sigma_d}{\sigma_n}\), can raise the stock price of sector A even if \(dP < 0\).

In contrast, (ii) of Proposition 2 asserts that \(dP > 0\) is insufficient to ensure a rise in sector B’s stock price. Clearly, this is because sector-specific shocks always discourage sector B and lower its stock price. Thus, even under the positive effect from intertemporal consumption smoothing, the stock price of the damaged sector, sector B, can fall following a strong sector-specific shock.

Immediately from the proof of Proposition 2, we can consider the case where the average stock price declines: \(dP < 0\). In this case, the stock price of sector B necessarily falls. Inverting
the inequality in (36), it can be easily confirmed that the stock price of sector A can also decline if the sector-specific shock is not strong.

### 3.3 Welfare of capitalists and workers

We derive the analytical formulations of the welfare of capitalists and workers. It is well known that the lifetime utility of capitalists is closely related to the price–dividend ratio of the market portfolio that pays the aggregate excess profits or the consumption of the capitalists. Note that \( \omega_c \) is the price–dividend ratio of the market portfolio. This is equal to the price–dividend ratio of a stock. We can also derive the welfare of workers.

As the growth rates of the aggregate demand factor follow a Markov process, the lifetime utility functions are state dependent. Defining the welfare–utility ratio of capitalists \( V_t \equiv \frac{W_t}{U(C_t)} \), the utility recursion \( W_t \) can be written as \( V_t = 1 + \exp(-\rho) \mathbb{E}_{\nu_t} \left[ \frac{u'(C_{t+1})}{u'(C_t)} V_{t+1} \right] \). The Markov properties allow us to express this in state dependent form \( V_s = 1 + \exp(-\rho) \mathbb{E}_{\nu_s} \left[ \frac{u'(C_t)}{u'(C_s)} V_{s'} \right] \).

This is equivalent to \( V_s - 1 = \exp(-\rho) \mathbb{E}_{\nu_s} \left[ \frac{u'(C_t)}{u'(C_s)} + \frac{u'(C_t)}{u'(C_s)} (V_{s'} - 1) \right] \) or

\[
V_n - 1 = (1 - \phi)\beta + \phi\beta\delta + (1 - \phi)\beta (V_n - 1) + \phi\beta\delta (V_d - 1), \\
V_d - 1 = (1 - \theta^s)\beta + \theta^s\beta\delta + (1 - \theta^s)\beta (V_n - 1) + \theta^s\beta\delta (V_d - 1).
\]

Note that the equations above are like equations (29) and (30). The price–dividend ratio without sector-specific shocks is obtained by subtracting one from the welfare–utility ratio \( \omega_c = V_s - 1 \). Because the conditional welfare \( W_t \) is also state dependent \( W_s \), it can be written as \( W_s = u(C_s) (\omega_c + 1) \).

Let us evaluate welfare in terms of certainty-equivalent consumption \( \check{C}_s \) conditional on state \( s \). Suppose that lifetime utility from a sequence of certain consumption \( \check{C}_s \) equals welfare \( \check{W}_s \): \( \sum_{\tau=1}^{\infty} \exp(-\rho \tau) \frac{\check{C}_s^{1-\gamma}}{1-\gamma} = \frac{\check{C}_s^{1-\gamma}}{[1 - \exp(-\rho)](1 - \gamma)} = \check{W}_s \). This is equivalent to \( \check{C}_s = \{1 - \exp(-\rho)\} (1 - \gamma) W_s \). Therefore, the ratio of certainty-equivalent consumption in state \( d \) to state \( n \) serves as a welfare measure:

\[
\frac{\check{C}_{d'}}{\check{C}_n} = \frac{1 - \beta (\theta^s - \phi)}{1 - \beta \delta (\theta^s - \phi)} \frac{1}{1-\gamma}, \tag{38}
\]

and \( 1 - \frac{\check{C}_{d'}}{\check{C}_n} \) is welfare loss from the pandemic for capitalists.

Note that because the utility level takes a negative value when the utility function is CRRA and IES is less than one \( (\gamma > 1) \), lifetime utility also takes a negative value. Thus, a higher value of the lifetime utility to current utility ratio means that lifetime utility takes lower values when IES is less than one. The opposite holds when IES is greater than one.

We can derive the welfare measure of workers by applying a similar procedure. Suppose that \( \check{c}_i \) denotes the certainty-equivalent consumption of workers in sector \( i \) conditional on state

---

5We replace the ratio of current consumption \( \frac{C_t}{c_n} \) with \( \kappa = \exp \left( g + \frac{2d}{x} \right) (1 - b) \), which is the ex post consumption growth rate from state \( n \) to \( d \).
which provides the same lifetime utility as welfare from an uncertain consumption stream. The welfare measure of workers in sector \(i\) is as follows:

\[
\frac{\tilde{e}_{id'}}{\tilde{c}_{in}} = \kappa \left[ \frac{1 - \beta (\theta^* - \phi) - (1 - \beta + \beta \phi) \left( 1 - \eta^*_i \right)}{1 - \beta \delta (\theta^* - \phi) - \beta \delta \phi \left( 1 - \eta^*_i \right)} \right]^{\frac{1}{\gamma'}}. \tag{39}
\]

Thus, \(1 - \frac{\tilde{e}_{id'}}{\tilde{c}_{in}}\) is welfare loss from the pandemic for sector \(i\) workers.

We obtain the following proposition.

**Proposition 3.** \(\frac{\tilde{c}_{ar}}{\tilde{c}_{an}} \geq \frac{\tilde{c}_a}{\tilde{c}_n} \geq \frac{\tilde{c}_{ar}}{\tilde{c}_{ln}}\) holds.

**Proof.** \(\frac{\tilde{c}_{ar}}{\tilde{c}_{an}} \geq \frac{\tilde{c}_a}{\tilde{c}_n}\) holds if the following inequality holds:

\[
1 - \beta (\Theta^* - \phi) - (1 - \beta + \beta \phi) \left( 1 - \eta^*_i \right) < \frac{1 - \beta (\Theta^* - \phi)}{1 - \beta \delta (\Theta^* - \phi)}.
\]

Because \(\alpha_a > \alpha_d\) and \(1 - \eta^*_i > 0\), the above inequality holds if \(\Theta^* \leq \frac{1}{\beta \delta} + \frac{1 - \beta \delta \phi}{(1 - \beta) \delta \phi}\). This inequality is the same as the condition for the existence of competitive equilibrium with positive equity prices (31). Therefore, \(\frac{\tilde{c}_{ar}}{\tilde{c}_{an}} \geq \frac{\tilde{c}_a}{\tilde{c}_n}\) always holds. A similar argument is applicable for the condition where \(\frac{\tilde{c}_{ar}}{\tilde{c}_{ln}} \leq \frac{\tilde{c}_a}{\tilde{c}_n}\).

The implications of Proposition 3 are as follows. Pandemics permanently depress macroeconomic growth rates. For capitalists, this macroeconomic shock is the only source of welfare losses. On one hand, for sector A workers, the pandemic will boost demand in their sector while having adverse macroeconomic effects. Therefore, their welfare loss will be less than that for capitalists. On the other hand, in addition to the adverse macroeconomic impact, sector B workers will also experience the cross-sectional negative effect of reduced demand in their sector. As a result, their welfare will deteriorate significantly more than that of capitalists.

### 4 | A MODEL WITH LOCKDOWN

#### 4.1 | Modeling lockdown

In this section, we extend the model to analyze lockdown policies. Here, lockdown policies are those that suppress economic activity in a particular sector but mitigate the spread of the pandemic. So far,
we have considered $\alpha_d$, which is the demand for the intermediate goods of sector B, to be an exogenous variable. However, in the actual economy, this is determined by the government, which aims to reduce the spread of infectious diseases. In modeling lockdown policy, we focus on the relationship between lockdown and economic disasters. Guan et al. (2020) and Zhang et al. (2022) report that shortening the pandemic state through stricter lockdown policies also mitigates economic losses and helps avoid lasting economic disasters. Therefore, we introduce a setting in which suppressing demand for sector B reduces people’s subjective (and potentially also the actual) probability of a continuing economic disaster due to a pandemic.

We introduce a parameter that represents the strictness of lockdown $L \in [0, 1]$. This affects the economy in the following ways. First, a strict lockdown decreases the demand for the goods and services supplied by sector B. Thus, we consider the following relationship:

$$\alpha_d = (1 - L)/2.$$

That is, the more the government implements a strict lockdown, the lower the demand for goods and services supplied by sector B becomes. Second, we assume that the more the lockdown is stringent, the more people expect the pandemic to end early:

$$\theta^* = \exp(-\lambda L).$$

$\lambda$ is a parameter that indicates the perceived effectiveness of the lockdown. If no restrictions on economic activities are assigned, $L = 0$, people form pessimistic expectations and then $\theta^* = 1$. However, if strict restrictions are assigned, $L = 1$, people expect $\theta^* = \exp(-\lambda)$. The expected average duration of the pandemic is $1/[1 - \exp(-\lambda)]$. As the government conducts stricter lockdowns, people have more optimistic expectations $\frac{\partial \theta^*}{\partial L} < 0$.

### 4.2 Asset prices

We confirm that Propositions 1 and 2 hold under certain conditions for $\lambda$ and $L$. The inequality (34) in Proposition 1 is expressed as the following inequality.

$$\theta^* \geq \frac{1 - \kappa + [\delta - 1 + \beta \delta (1 - \kappa)] \phi}{\beta \delta (1 - \kappa) + \kappa (\delta - 1)}.$$  \hspace{1cm} (41)

As already explained, strong ambiguity aversion amplifies the saving demand and drives the stock price boom. Therefore, (34) indicates that a stock price boom requires a sufficiently large $\theta^*$. This inequality (41) is reduced to the following inequality because $\theta^* = \exp(-\lambda L)$:

$$\lambda \leq \hat{\lambda}(L) \equiv -\frac{1}{L} \ln \left\{ \frac{1 - \kappa + \beta \delta (1 - \kappa)}{\beta \delta (1 - \kappa) + \kappa (\delta - 1)} \right\}.$$  \hspace{1cm} (42)

Note that $\hat{\lambda}$ takes a positive value because the term in $\{\}$ is positive and less than one as Proposition 1 asserts. When $L$ is zero, $\hat{\lambda}(0) = \infty$, and when $L$ is one, $\hat{\lambda}(1) = -\ln \left\{ \frac{1 - \kappa + \beta \delta (1 - \kappa)}{\beta \delta (1 - \kappa) + \kappa (\delta - 1)} \right\} > 0$. For a given strictness of lockdown $L$, stock price booms occur only if the lockdown $\lambda$ is less effective.

The inequality (36) in Proposition 2 can be written as follows.

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Barro (2015) introduces this type of formulation for the probability of disaster and policy interventions in the context of the risk of economic damage from climate change.
The inequality (43) is equivalent to the condition for \( \frac{P_{Ad}}{P_{An}} \geq 1 \) and always holds when \( dP \geq 0 \). In the case of \( dP < 0, \frac{P_{Ad}}{P_{An}} \geq 1 \) holds only if \( \lambda \leq \hat{\lambda}_A(L) \). We can easily confirm that \( \hat{\lambda}_A(L) \geq \hat{\lambda}(L) \). In particular, note that \( \hat{\lambda}_A(0) = \hat{\lambda}(0) = \infty \) and \( \hat{\lambda}_A(L) > \hat{\lambda}(L) \).

The inequality (37) in Proposition 2 can be written as follows.

\[
\lambda \geq \hat{\lambda}_B(L) \equiv -\frac{1}{L} \ln \left( \frac{1 - \kappa + \beta \delta (1 - \kappa)\phi + (1 - \beta \kappa)\phi L}{\beta \delta (1 - \kappa) + \kappa (\delta - 1) + (1 - \beta)\kappa L} \right), \quad (44)
\]

where \( L < \frac{\kappa (\delta - 1) + (1 - \kappa)\beta \delta}{(1 - \beta)\delta} \) holds. The inequality (44) is equivalent to the condition for \( \frac{P_{Ad}}{P_{An}} \leq 1 \). Note that \( \hat{\lambda}_2(0) = \hat{\lambda}_4(0) = \infty \) and the term in \{\} increases as \( L \) increases. Then, if \( L = \bar{L} = \frac{\kappa (\delta - 1) + (1 - \kappa)\beta \delta - 1 + \beta \delta (1 - \kappa)\phi}{(1 - \beta)\delta (1 - \phi)} \), the term in \{\} equals zero and \( \bar{\lambda}_5(\bar{L}) = 0 \) holds.

Therefore, on a plane where the horizontal axis is \( L \) and the vertical axis is \( \lambda \), these relationships define downward-sloping curves. Figure 2 plots the relationship between \( L \) and \( \lambda \) indicated by \( \lambda = \hat{\lambda}(L), \lambda = \hat{\lambda}_A(L), \) and \( \lambda = \hat{\lambda}_B(L) \) on the plane where the horizontal axis is \( L \) and the vertical axis is \( \lambda \). Then, we demonstrate the following Proposition 4.

**Proposition 4.** Stock prices can change in one of the following four ways.
(1) both sectors A and B (and thus, the average) increase: \( \frac{P_{aA}}{P_{aB}} > 1, \frac{P_{bA}}{P_{bB}} > 1, \) and \( dP > 0. \)

(2) sector A and the average increase, but only sector B decreases: \( \frac{P_{aA}}{P_{aB}} > 1, \frac{P_{bA}}{P_{bB}} < 1, \) and \( dP > 0. \)

(3) only sector A increases, and the average and sector B decrease: \( \frac{P_{aA}}{P_{aB}} > 1, \frac{P_{bA}}{P_{bB}} < 1, \) and \( dP < 0. \)

(4) both sectors A and B (and thus, the average) decrease: \( \frac{P_{aA}}{P_{aB}} < 1, \frac{P_{bA}}{P_{bB}} < 1, \) and \( dP < 0. \)

As the lockdown becomes stricter \((L)\) increases, the case moves from (1) to (4). If the lockdown is very effective, \((\lambda)\) is higher than \(\hat{\lambda}_A(1)\), and (1) to (4) can all occur depending on the level of lockdown severity \(L\). When the effectiveness \((\lambda)\) is intermediate \((\hat{\lambda}_A(1) > \lambda > \hat{\lambda}(1))\), only (1) to (3) can occur. If the lockdown is less effective, \((\lambda)\) is lower than \(\hat{\lambda}(1)\), only (1) to (2) can occur.

Proof. As shown in Figure 2, \(\hat{\lambda}(L), \hat{\lambda}_A(L)\), and \(\hat{\lambda}_B(L)\) defined in (42), (43), and (43) are downward-sloping curves. Because

\[
\begin{align*}
1 - \kappa + [\delta - 1 + \beta \delta (1 - \kappa)] \phi - (1 - \beta) \delta L \\
\frac{1 - \kappa + [\delta - 1 + \beta \delta (1 - \kappa)] \phi}{\beta \delta (1 - \kappa) + \kappa (\delta - 1) - (1 - \beta) \delta L} \\
\leq \\
\frac{1 - \kappa + [\delta - 1 + \beta \delta (1 - \kappa)] \phi}{\beta \delta (1 - \kappa) + \kappa (\delta - 1) + (1 - \beta) \kappa L}
\end{align*}
\]

holds for \(0 < L < 1\), \(\hat{\lambda}_A(L)\) \((\hat{\lambda}_B(L))\) will be the curve that is higher (lower) than \(\hat{\lambda}(L)\). If \(\lambda\) is higher than \(\hat{\lambda}_A(1)\), depending on the size of \(L\), cases (1) through (4) could occur. If \(\lambda\) is lower than \(\hat{\lambda}_A(1)\) but higher than \(\hat{\lambda}(1)\), depending on the size of \(L\), cases (1) through (3) could occur. If \(\lambda\) is lower than \(\hat{\lambda}(1)\), depending on the size of \(L\), cases (1) through (2) could occur.

Proposition 4 suggests that regardless of the effectiveness of the lockdown \(\lambda\), stock prices on average can boom, whereas stock prices in sector B remain stagnant if lockdowns are not strictly enforced. In addition, it also indicates that various stock price patterns arise because of the strictness of lockdown policy \(L\).

Here, we refer to an empirical implication of Proposition 4. Using data from advanced countries, Saito and Sakamoto (2021) and Scherf et al. (2022) report a negative relationship between the strictness of lockdown policy and asset price changes: the tighter the lockdown policy is, the lower the (average) stock prices become. Using the result for the average stock price change in Proposition 4, our model explains this relation in light of the ambiguity aversion of asset holders.

4.3 Welfare

The welfare measure of capitalists and workers, (38) and (39), is written as follows.
Proposition 3 continues to hold. Then, we obtain the following Proposition 5 for the optimal lockdown level.

**Proposition 5.** Optimal degrees of lockdown for capitalists and workers are characterized as follows.

i) Capitalists’ welfare improves with lockdown for any value of $\lambda$. That is, the optimal lockdown rate is $L = 1$.

ii) The welfare of workers in sectors A improves at $L = 0$ for any value of $\lambda$. That is, the optimal lockdown rate is $L > 0$.

iii) The welfare of workers in sector B improves at $L = 0$ only if the efficiency of the lockdown $\lambda$ is sufficiently high but is reduced at $L = 1$ for any value of $\lambda$. That is, if $\lambda$ is high enough, the optimal lockdown severity is greater than zero but less than one. However, if $\lambda$ is sufficiently low, $L = 0$ is optimal.

**Proof.** See Appendix D.

Lockdown has two different effects on economic welfare: the first is a dynamic effect, in which the lockdown leads to an early end to the pandemic; the second is a cross-sectional effect, where the lockdown exerts different effects across the sectors. Note that the dynamic effect benefits both capitalists and workers, whereas the cross-sectional effect has heterogeneous impacts on their welfare. Proposition 5 shows that lockdown has different welfare effects for capitalists and workers because capitalists’ income and consumption are not directly affected by the cross-sectional effect. Thus, a strict lockdown benefits them because of the dynamic effect. However, workers’ income and consumption are affected by the cross-sectional effect. Sector B workers suffer from declines in labor income and consumption directly because of the cross-sectional effect, which reduces their welfare. Only if the dynamic effect is effective in terminating the pandemic does it increase income and consumption. Sector B workers then only appear one-sided victims of the lockdown policy, but if the dynamic effect is sufficiently strong, they will benefit from the policy. Thus, there is the possibility of a Pareto improvement.

5 | **NUMERICAL EXERCISE**

This section presents a numerical example and confirms the properties of the theoretical model. We show that under certain canonically reasonable parameters, the rate of change of stock prices is quantitatively close to that observed in the United States.
As calibration parameters, we set the time preference rate \( \rho \) to 0.03, the trend growth rate \( g \) to 0.025, and the volatility of iid shocks \( \sigma_u \) to 0.03. These are standard values in the field of asset pricing. The elasticity of substitution parameter for production \( \sigma \) is assumed to be two. Labor supply \( l_{An}, l_{Ad}, l_{Bn}, \) and \( l_{Bd} \) are fixed equal to one. \( \alpha_n \) is normalized to 0.5. The impact of the pandemic on TFP growth \( b \) is set to 0.06, which brings the overall GDP growth rate \( \kappa \) to 0.964. That is, GDP declines by \(-0.035\) when the pandemic occurs. This is consistent with US experience.

In an exercise to confirm the theoretical properties of the model, we try various values of \( \gamma \). We specify \( \gamma = 1.5 \) (IES is about 0.67) as a benchmark value as in other studies. Havránek (2015) conducts a meta-analysis on the estimation of the IES. He argues that the average estimate of IES is about 0.5 and that using a value higher than 0.8 is inconsistent with the empirical evidence. However, if we set the IES to 0.5 or smaller in our model, the assumption of \( \beta^\delta < 1 \) is not satisfied. Thus, a value of 0.66 will satisfy this assumption and be empirically reasonable. In Figure 3, we see asset price dynamics in the case of \( \phi \) ranging from 0.001 to 0.1. In Figures 4–7, we fix \( \phi \) to 0.01 as a benchmark value.\(^ \text{10} \)

Although \( \lambda \) is a critical parameter in the lockdown model, it is not easy to specify. Now, \( \lambda \) governs the probability of the continuation of the pandemic by \( \exp(-\lambda L) \). In the strictest lockdown, \( L = 1 \), the probability of continuation is \( \exp(-\lambda) \). We denote that the expected duration of the pandemic with \( L = 1 \) is \( T \) years. Then, \( T = 1/(1 - \exp(-\lambda)) \). In other words, the relationship is \( \lambda = -\ln(1 - 1/T) \). Therefore, \( \lambda \) is chosen based on the expected duration of the pandemic under \( L = 1 \). We examine the following two cases: \( T = 1.5, \lambda = 1.099 \) and \( T = 5, \lambda = 0.223 \). Note that \( \hat{\lambda}(1) \) is 0.690, \( \hat{\lambda}(1) \) is 0.589, and \( \lambda = 0.010 \).

\(^{10}\phi = 0.01 \) is chosen because the 1918 Spanish flu pandemic, which delivered the starkest economic disaster in recent history, occurred a century ago. However, globally spreading infectious diseases such as the Black Death in the 14th century and the 2009 influenza pandemic that took place only 10 years ago have also been observed in history. For this reason, we consider various values of \( \phi \) in Figure 3.
5.2 Asset prices

Proposition 1 shows that $\delta$ and $\phi$ play an important role in determining asset prices. Figure 3 illustrates the rate of stock price change $\frac{P_d}{P_{in}}$ with $\alpha_n = \alpha_d$ when the state moves from the normal state to the pandemic state. The figure shows the rate of stock price change for values of $\gamma$ from 0.5 to 1.5, and $\theta^*$ equal to one and thus clarifies the relationship between $\delta$, $\gamma$, and $\frac{P_d}{P_{in}}$. When $\gamma = 1$ or $\delta = 1$, the price–dividend ratio is constant. The unit value of the IES indicates that the capitalists consume a constant rate of their wealth in each period. Therefore, even if a pandemic occurs and income and consumption continue to decline in the long run, asset prices will be unaffected. If $\gamma < 1$ or $\delta < 1$, the long-lasting pandemic implies that the expected dividend will decline for a long time, which will lower stock prices. If $\gamma > 1$ or $\delta > 1$, however, the expectation of a pandemic continuing for a long time will increase saving demand because of the consumption-smoothing motive and will then increase stock prices.
At the same time, Figure 3 also clarifies the relationship between $\phi$ and average stock prices. The smaller the value of $\phi$ is, the larger the change in stock prices immediately after a pandemic will be. Now that $\theta = 1$, the price–dividend ratio in the pandemic state $d$ depends only on $\beta$ and $\delta$. The price–dividend ratio in state $n$ then depends on that in state $d$. Thus, the closer $\phi$ is to one, the smaller the difference between $\omega_n$ and $\omega_d$ becomes. Therefore, the lower the value of $\phi$ is, the more extreme the stock price change between states becomes.

We now consider whether our model can explain the stock price dynamics observed in the United States. The annual average stock index for the high-growth sector was 91 in 2019 and 112 in 2020, growing by 22.3%. The annual average stock price index for the low-growth sector was 94 in 2019 and 88 in 2020, representing growth of $-6.3\%$. The annual average stock price index for the entire market was 92 in 2019 and 103 in 2020, growing by 11.3%.

As already explained, strong ambiguity aversion amplifies saving demand and drives the stock price boom. Therefore, (34) indicates that a stock price boom requires a sufficiently large

**FIGURE 6** $P_{id}/P_{in}$ for various $L$ with $\lambda = 1.099$ and $T = 1.5$.

**FIGURE 7** $P_{id}/P_{in}$ for various $L$ with $\lambda = 0.223$ and $T = 5$. 
\( \theta^* \). Under the current parameter setting, the inequality (41) that ensures \( dP \geq 0 \) is \( \theta^* \geq 0.555 \). Therefore, in order for \( \frac{dP}{dL} = 0.113 \), as observed in the United States, \( \theta \) must be much higher than 0.555; \( \theta^* = 0.86 \) in the following numerical examples. Alternatively, empirical estimates of past disasters, such as the Great Depression or World War II, suggest that \( \theta = 0.5 \).\(^{11}\) This is lower than the minimum value of \( \theta^* 0.555 \). Therefore, the assumption of ambiguously risk-averse capitalists is critical for generating the stock price booms observed during the recent COVID-19 pandemic recession.

We turn to the impact of lockdown policy on stock prices. In the model described in Section 5, the strictness of lockdown \( L \) affects \( \alpha_d \) and \( \theta^* \) and causes a divergence between the stock prices of sectors A and B. Figure 4 displays the case of \( T = 5 \) or \( \lambda = 0.223 \) and Figure 5 the case of \( T = 1.5 \) or \( \lambda = 1.099 \). In Figure 4, when \( L \) is less than 0.60, the stock price of sector B also rises; when \( L \) is greater than 0.61, only the stock price of sector B falls. Because \( A(1) = 0.589 \) and the corresponding \( T \) is 2.25, the average stock price does not fall, even under the strictest lockdown scenario of \( L = 1 \). The case of \( L = 0.85 \) is worth considering; the stock price of sector A rises by about 23%, and the stock price of sector B falls by about 6%, resulting in an average rise of 9.4%. These values are quite close to those observed in the United States. Note that when \( L \) is 0.85, \( \theta^* \) is 0.83. The expected duration of the pandemic, \( T \), is 5.85 years. However, in Figure 5, when \( L \) is less than 0.33, the stock price of sector B also rises; when \( L \) is between 0.34 and 0.53, the stock price of only sector B falls; when \( L \) is between 0.54 and 0.94, the average stock price also falls; when \( L \) is greater than 0.95, the stock price of sector A also falls. Because \( A(1) = 1.099 > A(1) = 0.690 \) \( \alpha_d \) in both figures is higher. Therefore, the welfare of sector B workers is at a maximum at values of \( L \) higher than zero and less than one. Indeed, it is maximized at \( L = 0.58 \) in Figure 6 and at \( L = 0.54 \) in Figure 7. Sector A workers can see that \( L = 1 \) increases welfare the most in both figures, although the proposition clearly does not.\(^{12}\)

These numerical examples have the following normative implications for lockdown policy. First, if the lockdown is sufficiently effective, the introduction of lockdown improves the welfare loss.

We show the impact of lockdown on the welfare loss from a pandemic for capitalists and sector A and B workers, \( 1 - \tilde{C}_d/d \) and \( 1 - \tilde{c}_{id}/c_{in} \). Figure 6 plots the welfare loss when \( L \) changes from zero to one under \( T = 5 \) or \( \lambda = 0.223 \). Figure 7 depicts the results under \( T = 1.5 \) or \( \lambda = 1.099 \). These correspond to Figures 4 and 5, respectively. As Proposition 3 shows, under any \( L \), the welfare loss of sector A workers is lowest, followed by that of capitalists, and that of sector B workers is highest. Furthermore, as Proposition 5 shows, capitalist welfare loss is minimized at \( L = 1 \). Now, \( A(1) \) is about 0.010, and \( \lambda \) in both figures is higher. Therefore, the welfare of sector B workers is at a maximum at values of \( L \) higher than zero and less than one. Indeed, it is maximized at \( L = 0.58 \) in Figure 6 and at \( L = 0.54 \) in Figure 7. Sector A workers can see that \( L = 1 \) increases welfare the most in both figures, although the proposition clearly does not.\(^{12}\)

\(^{11}\)Cecchetti et al. (1990) and other studies estimate a Markov switching model and conclude that \( \theta \) is about 0.5. That is, past disasters have typically caused economic damage over 2 years.

\(^{12}\)In no other numerical example could we find a case where the welfare of sector A workers is maximized at the interior point of \( L \).
welfare loss of capitalists, sector A workers, and sector B workers. That is, lockdown policy is desirable in a Pareto sense. However, a lockdown that is too severe will improve the welfare of capitalists and sector A workers, but seriously damage the welfare of the sector B workers directly affected by the lockdown. Therefore, the lockdown strictness that sector B workers find most desirable forms a Pareto-efficient lockdown policy.

5.4 Evaluating lockdown policy

We also consider a social planning problem to derive another criterion for a desirable level of lockdown. Given that the welfare of the workers in each sector \( i \) is also state dependent, we denote it by \( W_{si} \) for state \( s \).\(^{13}\) Because we assume that the population of capitalists and workers of sectors \( A \) and \( B \) are all equal, we define \( SW_s \equiv W_s + W_{As} + W_{Bs} \) as the social welfare in state \( s \). We also define the rate of change of social welfare between the pandemic and normal state \( SW_{dn} \) as a measure of social welfare.

It may also be possible to improve overall welfare in the economy by strengthening lockdown policy and conducting an income redistribution policy. The lockdown level that minimizes the welfare loss of sector B workers is \( L = 0.58 \) in Figure 6. Hereafter, we refer to this as the egalitarian level of lockdown. The lockdown level that maximizes the social welfare measure \( SW_{dn}/SW_n \) is \( L = 0.72 \) in Figure 6. Hereafter, we refer to this as the utilitarian level of lockdown. The utilitarian level is 0.14 stricter than the egalitarian level. In Figure 6, the capitalist’s welfare loss is 0.27 under the utilitarian level but 0.31 under the egalitarian level. Similarly, that of sector A workers is 0.19 under the utilitarian level but 0.24 under the egalitarian level. Compared to the egalitarian level, the welfare loss of capitalists (sector A workers) is about 4 (5)% smaller under the utilitarian level. Conversely, the welfare loss for sector B workers is 0.47 under the utilitarian level, but 0.46 under the egalitarian level. Compared to the egalitarian level, the utilitarian level reduces the welfare loss by 1%. Figure 7 illustrates a similar result. This numerical example suggests that a strict lockdown accompanied by an appropriate redistribution policy is socially beneficial.

This result is consistent with recent empirical studies of the economic impact of lockdown. For example, by considering the supply chain propagation mechanism, Guan et al. (2020) and Zhang et al. (2022) demonstrate that a strict lockdown to end the pandemic quickly would also be desirable in terms of value-added losses. Nevertheless, they do not consider the impact of lockdown on the distribution of income or wealth. Inoue and Todo (2020) also analyze the impact of lockdown by considering a spatial supply chain structure. They find that even if the lockdown is limited to specific regions, it can spread to other industries and regions through the supply chain. On this basis, they argue that strict lockdowns should only be a measure of last resort because the overall effect of a lockdown on the entire economy is huge. In this regard, our results are most consistent with Inoue and Todo (2020) because we highlight the severe negative economic impact of lockdown on certain individuals.

Finally, we note the positive implications of these numerical examples. At about \( L = 0.85 \) in Figure 4, the observed pattern in United States stock prices is reproduced. Correspondingly, at \( L = 0.85 \) in Figure 6, the welfare loss is 0.53%. This is higher than not only the egalitarian but also the utilitarian level. Therefore, the observed level of lockdown might be too severe and

\(^{13}\)The details are in Appendix B.
excessive in terms of the optimal level for sector B workers. In any event, the implementation of redistributive policies alongside that of lockdown policy is extremely important for welfare.

6 | CONCLUDING REMARKS

This paper presents a theoretical model to explain the K-shaped asset price fluctuations exhibited during the 2020 coronavirus pandemic recession. We hypothesize that strong uncertainty about the pandemic recession led to extreme saving demand, pushing up overall stock prices. At the same time, lockdowns may have caused a significant divergence in stock prices between sectors. We also analyze the optimal level of lockdown. We find that a lockdown that maximizes the welfare of households most adversely affected could be optimal in a Pareto sense. However, stricter lockdown policies chosen under the utilitarian objective damage these households and benefit both other workers and capitalists. This indicates that a stricter lockdown policy accompanied by an appropriate income redistribution policy among heterogeneous households is one of several good policy options available.

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DATA AVAILABILITY STATEMENT

Stock price data for the United States are based on the 11-sector S&P and available from the Wall Street Journal website (https://www.wsj.com/market-data). The VIX index data are from the CBOE Volatility Index: VIX available at the FRED (https://fred.stlouisfed.org/series/VIXCLS). The data for Japan are based on the Japanese stock index (sector) tracking ETFs (TOPIX-17) sold by Nomura Asset Management Co. Ltd. They are available at its website (https://global.nomura-am.co.jp/nextfunds/product/). The data are indices standardized to 100 for December 11, 2019. Weekly data use values from Wednesdays. The data for the Nikkei VI index are available at Yahoo Finance (https://stocks.finance.yahoo.co.jp/stocks/history/?code=2035.T).

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**APPENDIX A: ASSET PRICES DURING COVID-19**

We characterize stock price behaviors during the COVID-19 pandemic using data from the US and Japanese markets. We focus not only in the market index, but also on cross-sectional variation across industries. For the United States, the S&P 500 index is divided into 11 categories. We compare the performance of stock prices in each sector to the S&P 500 index. As the first infection was identified in December 2019 in China, we compared values on December 11, 2019, to values on February 10, 2021. Among the 11 sectors, stock prices in the following three sectors have risen more than the S&P500 index; Information Technology, Consumer Discretionary, and Communication Services. Stock prices in the remaining eight sectors have not risen as much as the S&P500 index; Health Care, Financials, Industrials, Consumer Staples, Energy, Utilities, Real Estate, and Materials.
Figure 1 shows the price returns and volatility of three high-growth sectors, eight low-growth sectors, and the S&P 500 index. It also shows the VIX index, the official name of which is the Chicago Board Options Exchange’s CBOE Volatility Index. It measures the stock market’s expectation of volatility based on S&P 500 index options. The sample period is from November 14, 2018, to February 10, 2021. The data are weekly using data for the Wednesday of each week. Figure 1 shows little variation in returns (stock price changes) across sectors during 2019. However, after the crash in February–April 2020, there was substantial cross-sectional variation in stock returns across sectors. The three high-growth sectors experienced rapid growth; compared with December 2019, the share prices were 44% higher. In contrast, the eight low-growth sectors have only barely recovered their December 2019 levels. Interestingly, the overall market average has recovered by 24% compared with December 2019. In addition to the stock prices behavior, the VIX index also shows important movements. The VIX index averaged about 16 during 2019, before rising to 80 during the crash in February–April 2020, after which it declined. However, it has remained high since April 2020, averaging around 28.

This stock market behavior is not unique to the United States. A similar pattern has been observed in the Japanese market, where the number of infected people is much smaller than in the United States, but the economic impact is comparable. With regard to the Japanese stock markets, TOPIX-17 Series Indices are created by dividing the TOPIX index, which is a comprehensive market index tracking all domestic companies on the Tokyo Stock Exchange’s First Section, into 17 sectors. Among the 17 sectors, stock prices of the following four sectors have risen more than the TOPIX index: Machinery, Electric Appliances & Precision Instruments, Electric Appliances & Precision Instruments, and Retail Trade. Stock prices of the remaining 13 sectors have not risen as much as the TOPIX index: Foods, Energy Resources, Construction & Materials, Raw Materials & Chemicals, Pharmaceutical, Automobiles &
Transportation Equipment, Steel & Nonferrous Metals, Electric Power & Gas, Transportation & Logistics, Commercial & Wholesale Trade, Banks, Financials (Ex Banks), and Real Estate.

Figure A1 shows the volatility data for the four high-growth sectors, 13 low-growth sectors, and the TOPIX index. It also shows the movement of the Nikkei VI index, which measures the stock market’s expectation of volatility based on Nikkei 225 index options. The Nikkei 225 index is another representative market index for the Japanese stock market. As the stocks included in the Nikkei 225 index are different from those in the TOPIX, it is not possible to directly measure the volatility of the TOPIX. However, it could be an indicator of the uncertainty of the outlook for the Japanese market. From Figure A1, we can see that the movements of the Japanese stock market are very similar to those of the United States.

In summary, the characteristics of the stock market in the COVID-19 pandemic are as follows. Despite the restricted level of macroeconomic activity, stock prices have experienced high growth rates on average. However, there are large cross-sectional disparities in asset returns. As illustrated by the VIX index, people are facing great uncertainty about the future of markets and the economy.

APPENDIX B: DERIVATION OF $\frac{c_{it}}{c_{in}}$

We assume that workers have the same utility functions as the capitalists’. But they do not make inter-temporal decision making and move their workplace. Because $c_{it} = (1 - \alpha_s)\frac{\varepsilon - 1}{\varepsilon}y_t$ and $c_{in} = \alpha_s\frac{\varepsilon - 1}{\varepsilon}y_n$. The welfare from uncertain stream of consumption $\{c_{it+\tau}\}_{\tau > 0}$ is denoted as $W_t$. Defining the welfare-utility ratio $V_{it} \equiv \frac{W_{it}}{u(c_{it})}$. Similar to the case for capitalists, the welfare-utility ratio can be written as a state dependent form $V_{is}$. Then one subtract from $V_{is}$ is $V_{is} - 1 = \exp(-\rho) E_v v_{is} = \left[\frac{u'(c_{it})}{u'(c_{na})} + \frac{u'(c_{it})}{u'(c_{na})}(V_{is} - 1)\right]$. Let us define $v_{is} \equiv V_{is} - 1$. $v_{is}$ must satisfy the following equations:

$$v_{ln} = (1 - \phi)\beta (v_{ln} + 1) + \phi \beta \delta \eta_{i}^{1-\gamma} (v_{id} + 1), \quad (B1)$$

$$v_{id} = (1 - \phi^*)\beta \eta_{i}^{1-\gamma} (v_{ln} + 1) + \phi^* \beta \delta (v_{id} + 1), \quad (B2)$$

Note that the similarity between (29) and (B1), and between (30) and (B2).

In these equations, $\omega_{is}$ are replaced by $v_{is}$ and $\eta_{i}$ is replaced by $\eta_{i}^{1-\gamma}$, respectively. Therefore, replacing $\eta_{i}$ with $\eta_{i}^{1-\gamma}$ on the right side of (32) and (33) gives the solutions for $v_{is}$. Then, we obtain (39) by the same procedure as the derivation of (38).

APPENDIX C: PROOF OF $L < 1$

$L < 1$ is easily demonstrated. This inequality is written as follows: $-(1 - \beta \delta) - \beta \delta (1 - \kappa) - \delta (1 - \kappa) < [-(1 - \beta \delta) - \beta \delta (1 - \kappa)] \phi \iff \frac{-(1 - \beta \delta) - \beta \delta (1 - \kappa) - \delta (1 - \kappa)}{-(1 - \beta \delta) - \beta \delta (1 - \kappa)} > \phi$. This is because the coefficient of $\phi$ is negative and the left-hand side of the inequality is higher than 1.

APPENDIX D: PROOF OF PROPOSITION 5

i) An increase in $L$ raise $\frac{c_{it}}{c_{in}}$ because of
\[
\frac{\partial}{\partial L} \left( \frac{\tilde{C}_{d'}}{\tilde{C}_n} \right) = \frac{\kappa}{1 - \gamma} \left[ \frac{1 - \beta (\exp(-\lambda L) - \phi^*)}{1 - \beta \delta (\exp(-\lambda L) - \phi^*)} \right] \frac{1 - \delta}{(1 - \beta \delta (\exp(-\lambda L) - \phi^*))^2} > 0.
\]

ii) The sign of partial derivative of \( \frac{\delta c_{Ad}}{\delta c_{An}} \) with respect to \( L \) is

\[
\text{sign} \left[ \frac{\partial}{\partial L} \left( \frac{\tilde{c}_{Ad'}}{\tilde{c}_{An}} \right) \right] = (\gamma - 1)(1 + L)^{-\gamma}[1 - \beta + \beta \phi(1 - \beta \delta)]
\]

\[
- \exp(-\lambda L)[(1 - \beta \delta) \beta \lambda + (1 - \beta) \beta \delta (1 + L)^{-\gamma}[\gamma - 1 - (1 + L)\lambda]].
\]

The condition for \( \frac{\delta c_{Ad}}{\delta c_{An}} > 0 \) with \( L = 0 \) is expressed as

\[
\lambda > \frac{(\gamma - 1)(1 - \beta \delta)(1 - \beta + \beta \phi)}{\beta (1 - \delta)}.
\]

Because the right-hand side is negative, \( \frac{\partial}{\partial L} \left( \frac{\tilde{c}_{Ad'}}{\tilde{c}_{An}} \right) > 0 \) with \( L = 0 \) holds.

iii) The sign of partial derivative of \( \frac{\delta c_{Bd}}{\delta c_{Bn}} \) with respect to \( L \) is

\[
\text{sign} \left[ \frac{\partial}{\partial L} \left( \frac{\tilde{c}_{Bd'}}{\tilde{c}_{Bn}} \right) \right] = (1 - \gamma)(1 - L)^{-\gamma}[1 - \beta + \beta \phi(1 - \beta \delta)]
\]

\[
+ \exp(-\lambda L)[(1 - \beta \delta) \beta \lambda + (1 - \beta) \beta \delta (1 - L)^{-\gamma}[\gamma - 1 - (1 - L)\lambda]].
\]

Then \( \frac{\partial}{\partial L} \left( \frac{\tilde{c}_{Bd'}}{\tilde{c}_{Bn}} \right) \geq 0 \) with \( L = 0 \) holds if and only if

\[
\lambda \geq \hat{\lambda} \equiv \frac{(\gamma - 1)(1 - \beta \delta)[1 - \beta(1 - \phi)]}{\beta(\delta - 1)}.
\]

On the other hand \( \frac{\partial}{\partial L} \left( \frac{\tilde{c}_{Bd'}}{\tilde{c}_{Bn}} \right) \) with \( L = 1 \) is examined by the following procedure. \( \frac{\partial}{\partial L} \left( \frac{\tilde{c}_{Bd'}}{\tilde{c}_{Bn}} \right) \) can be expressed as

\[
\text{sign} \left[ \frac{\partial}{\partial L} \left( \frac{\tilde{c}_{Bd'}}{\tilde{c}_{Bn}} \right) \right] = \exp(-\lambda L)(\beta \delta - 1) \beta \lambda + (1 - L)^{-\gamma}\exp(-\lambda L)(1 - \beta) \beta \delta \lambda (1 - L)
\]

\[
+ (1 - L)^{-\gamma}(\gamma - 1)[\exp(-\lambda L)(1 - \beta) \beta \delta - 1 + \beta - \beta \phi(1 - \beta \delta)]
\]

Because \( \exp(-\lambda L)(1 - \beta) \beta \delta - 1 + \beta - \beta \phi(1 - \beta \delta) < 0 \), \( \frac{\partial}{\partial L} \left( \frac{\tilde{c}_{Bd'}}{\tilde{c}_{Bn}} \right) = -\infty \) with \( L = 1 \) without any requirements for the value of \( \lambda \).