Feasibility of Net Energy Gain in Kinematic Nuclear Fusion Devices

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A design principle is suggested to overcome obstacles that prevent positive net energy output in nuclear fusion devices based on electrostatically accelerated ions. Since Coulomb scattering cross-section dwarfs that of nuclear fusion, the focus is on re-capturing energy of elastically scattered ions before the energy is lost to heat. Device configuration to achieve efficient energy re-capturing is proposed and a favorable estimate of net energy gain is obtained.

I. BRIEF PROBLEM OVERVIEW

Nuclear fusion reaction occurs when two ions of certain kind hit each other at energies large enough for them to overcome Coulomb repulsion and approach each other at distances of the order of $10^{-14}$ m, about $10^4$ times smaller than the size of an atom. Consequently, the kinetic energy required is $10^4$ times larger than typical chemical energies: tens or hundreds keV, or, equivalently, hundreds or millions K in the temperature scale.

A majority of fusion experiments attempt to heat plasma to the required temperatures so as to allow random ion collisions to cause fusion reactions. The high temperatures involved require confinement of the plasma, either magnetic or electrostatic, in order to spare the apparatus from the hot plasma inside. Confining and maintaining hot plasma is a formidable task that has yet to result in a controlled sustainable fusion reaction.

An alternative approach is to accelerate the ions by means of electric field, which only requires modest voltages of tens or hundreds kV. However, certain fundamental obstacles reviewed below are believed to preclude net energy gain in such a set up. “Highly nonthermal systems, like the colliding beam reactor proposed by Rostoker et al. would relax to local thermal equilibrium before a significant amount of fusion power could be produced.”[1,2] Here I propose a way to overcome these fundamental obstacles.

The first example of why kinematic approaches do not work is usually to consider an energetic ion beam hitting a solid target. Since the fusion cross-section is so small compared to the squared distances between the atoms in the target, an average ion will have to traverse great many atomic layers before it has a chance of hitting a nucleus. Such an ion will be stopped at a much shorter distance by electrons in the target.

Even in configurations with no electrons present, a fast ion has a much greater chance of hitting another ion close enough to scatter elastically than to hit it dead-on to cause fusion. The ions that scatter elastically redistribute their kinetic energies between them, causing cascading process that quickly leads to thermalization — the loss of the initial kinetic energy to heat. There are only two possible outcomes of thermalization: either the heat leads to the overall temperature sufficient for sustaining fusion, bringing the device to the class of the plasma confinement devices (not discussed here), or (ii) the temperatures are lower than fusion temperatures, in which case the energy lost to heat is unrecoverable fully due to the laws of thermodynamics, thereby precluding net energy gain. In other words, unless a very hot plasma is formed, much more energy has to be spent on accelerating the ions that are unsuccessful at fusion than any gain produced by the very few ones that are successful.

The second example is a fusor device[3] — the simplest device that does achieve fusion reaction by means of electrostatic potentials, albeit no net positive gain has been demonstrated so far.

It is crucial to underscore that the task of achieving fusion is not hard; a fusor is a simple enough device to be built at home or in a garage setting. The difficult task is to make a sustainable fusion reaction that can feed itself through net energy gain. Energy losses in a typical fusor device are five orders of magnitude larger than the fusion power produced[4].

A typical fusor device does not fall into the class of kinetic fusion approaches reviewed here; it is rather described as an Inertial Electrostatic Confinement (IEC) device — the one that still employs hot plasma, shielded from the outside apparatus by electrostatic, rather than magnetic, field (see[5] and bibliography therein). A well-known critique of IEC devices is contained in[6]. The reason for fusors being IECs is a fundamental channel for energy thermalization, similar to that described above. The ions in the fusor device are accelerated by electrostatic bias when they fly from the outer wall towards the center. In the center the average kinetic energy of ions is large enough to undergo fusion. Two cold ions accelerated towards the center from the outer wall have the same energy when they reach the center. They have a chance to hit each other and cause fusion, but they also have a much greater chance to scatter elastically and redistribute their energy between them.

When two particles of the same energy scatter elastically they may still re-distribute their energies. This can be visualized on a billiard table: it is possible, for instance, for two identical billiard balls with the same speed to collide in such a manner that one of them stops and the other flies away with twice the energy. The process is the exact reverse of a billiard ball hitting a stationary billiard ball. In summary, the energy outcome of an elastic collision depends on timing and mutual trajectories even
for identical particles with identical initial kinetic energies. Energy re-distribution due to Coulomb scattering leads to thermalization.

The same fundamental obstacle applies to other configurations that attempt to achieve sustainable fusion via accelerated ion beams, such as, for instance, in accelerated beam fusion reactors (ABFR). In addition to the usually-quoted problem with beam de-focusing due to internal electrostatic pressure, the same process causes many more ions to be elastically scattered away, carrying their energy with them, than the few that cause fusion, precluding net energy gain.

Here I discuss a device configuration, which circumvents the above fundamental obstacle by allowing efficient recovery of the energy carried away by the elastically scattered ions [7].

II. PREVENTING ENERGY REDISTRIBUTION OF COLLIDING IONS

Energy re-distribution is absent for two particles of the same mass and same but exactly opposite velocities. Elastic scattering in such head-on collisions result in both particles changing the direction of their flight but not their energies. Scattered particles fly away in the opposite directions that can be at any angle to their initial velocities, but their energies remain equal to their initial energies. This property can be used, which is the focus of this work, to help recover the energies of the scattered ions.

Deviation of the collision angle from $180^\circ$ quickly breaks down this property, allowing for energy redistribution, thereby making the task of recovering the energy of the scattered particles much harder, if not impossible. Employing ion species with different masses, such as D–T or p–$^{11}$B, also leads to energy re-distribution. This narrows down the present approach essentially to D–D, T–T or the aneutronic $^3$He–$^3$He fusion reactions.

III. WORKED EXAMPLE

As an illustration of the concept, consider a vacuum-evacuated chamber containing two cold ion injection openings of small angular size $2\alpha$ placed opposite to each other and a coaxial ring-shaped accelerating electrode in the center, negatively-biased with respect to the chamber. If necessary, the paths of the opposite ion beams can be controlled by small deflecting magnetic fields. If the paths are made to collide head-on near the center of the accelerating electrode, the ions will have a chance to undergo a fusion reaction, provided the bias voltage is large enough.

The majority of the ions will not scatter or undergo fusion, but will de-celerate as they reach the opposite side of the chamber, returning their energy back to the cirquit. In the simplest basic design the path of these un-scattered ions can be made to hit the opposite cold ion injector, in which case these ions can be either collected or allowed to return back to the chamber and be accelerated again for another try.

The great majority of the rest of the ions will be scattered elastically in near-head-on collisions. These ions will have a very narrow energy distribution and a certain distribution over scattering angles. These ions too will be de-celerated by the positive electrode and return the bulk of their energy to the circuit. Now, these scattered ions will have to be collected and removed from the chamber by the evacuation system (the positive electrode will have to be made of a mesh).

It is imperative to not allow the ions scattered at angles greater than some value (say, of the order of $\alpha$) to return back to the chamber, as the next scattering event will not be head-on and will cause energy re-distribution.

 Thus, ignoring fusion reactions for the moment, the steady state of the device will have a highly non-thermal spatial and velocity distribution strongly enhancing same-mass same-energy head-on collisions near the center of the chamber.

The concept design resembles an usual fusor device. However, the back energy transfer is made possible by restricting the majority of the collisions to be head-on collisions of same-mass, same-energy ions, whereby the energies of the scattered particles are known exactly to within narrow tolerances. Prior-art fusor designs lack this critical ability to retrieve the post-scattered ion energy fully and to avoid thermalization. Therefore, the operation of the present device differs significantly from that of the prior-art fusor, with high-temperature plasma in the central area replaced with a narrow, highly non-thermal phase space distribution.

Additionally, prior-art fusor devices suffer from energy losses due to the hot ions striking the central accelerating electrode [4], whereas the current design eliminates this second critical energy loss channel, because the 1D path of the ions does not cross the ring-shaped accelerating electrode.

The operation of the present device may have some resemblance with the "star mode" of the usual fusor device, but with the star being only two-pronged.

A. Deviation from Head-on Collisions

Whereas ideally the post-scattered ions all have the same energy, finite injector size and other technological imperfections lead to a (narrow) distribution of energies. The energy recovery electrode surrounding the reaction chamber must be under-biased in order to disallow return of low-energy post-scattered ions to the reaction chamber.

Deviation from the head-on collision by an angle $\delta$ leads to the post-scattered ions having energy slightly above and slightly below the initial energy. The worst-case scenario (the largest energy deviation $dE$) occurs for
the ions scattered at right angle, in which case the velocity component normal to the main axis is either added or subtracted from the post-scattered velocity. In this case the energy excess/deficiency is $dE(90^\circ) = 2E_0 \sin(\delta)$, where $E_0$ is the accelerating electrode bias. Absent additional beam focusing elements, the angle $\delta$ can be estimated as $\delta \approx \alpha$, half the angular size of the ion injection opening, as seen from the center of the device. The energy deficiency $dE$ may not be fully recovered and thus contributes to losses.

Since the great majority of Coulomb scattering events occur to small scattering angles, the areas of the energy recovery electrode near the axis may be biased closer to $E_0$ in order to limit the losses. The energy recovery electrode may be made segmented to achieve this goal.

B. Net Energy Gain Feasibility Estimate

Formally, the long-range nature of Coulomb force in vacuum makes every ion scatter, albeit to a small angle. For the technical purpose of this description I call “un-scattered” the ions that, upon passing the reaction zone, scatter at angles less than $\alpha$.

In the simplest concept design these ions are permitted to be reflected back and to accelerate again towards the center making multiple attempts at the fusion reaction, until they scatter away from the head-on collision trajectory (most likely), leading to some energy loss, or undergo fusion.

I estimate the gain/loss balance assuming a 5 mm ion injection opening diameter in a 30 cm diameter reaction chamber and the bias voltage of 500 kV for D–D reaction. Thus, “un-scattered” ions are those ions that scatter at angles less than $\alpha \approx 1^\circ$.

The ions that scatter at angles greater than $\alpha$ have the Coulomb scattering cross-section

$$\sigma_C = \frac{\pi}{16} \left( \frac{e^2 \cot \alpha / 2}{4\pi\epsilon_0 E_0} \right)^2 = 235 \text{ barns} \quad (1)$$

This large number is to be compared against the DD fusion cross-section $\sigma_F$ of only 0.19 barn — the dramatic mismatch, which exemplifies the hurdles of the kinematic fusion devices, and which the present design is attempting to overcome.

In other words, for every ion pair undergoing fusion reaction, the number $\sigma_C/\sigma_F = 1200$ pairs are scattered elastically away from the head-on trajectory without undergoing fusion. The kinetic energy of these ions needs to be recovered as fully as possible to achieve net energy gain.

On the positive side of the net energy balance is the energy $E_F = 3.61 \text{ MeV}$ released by a successful fusion reaction.

Now I estimate the residual energy loss $dE$ per ion due to deviations from head-on collision and the need to under-bias the energy recovery electrode by this amount, as mentioned. A very crude estimate can use $dE(90^\circ) \approx 2E_0 \sin(\alpha)$, which in our case amounts to about 3.3% of $E_0$.

However, this energy loss can be reduced further by realizing that the greatest fraction of the ions that do scatter are scattered to small angles for which $dE$ is much smaller. The energy of ions scattered at an arbitrary angle $\chi$ lies between $E_0 - dE(\chi)$ and $E_0 + dE(\chi)$ where

$$dE(\chi) = 2 \sin \alpha \sin \chi \ E_0. \quad (2)$$

The highest energy defect, $dE(90^\circ)$, is observed for the ions scattered at the right angle, but the fraction of these ions is very small compared to the fraction of the ions scattered to small angles. For the ions scattered at $\chi = 1^\circ$ the energy defect is only about 0.06% of $E_0$.

The weighted average $\overline{dE}$ over all scattering angles $\chi > \alpha$ is about 0.13% of $E_0$, assuming the energy recovery electrode is made segmented, each segment at angle $\chi$ being biased with the bias defect of $dE(\chi)$.

Assuming efficiency $\eta$ of the fusion energy recovery, the energy balance has, on the gain side, $\eta \sigma_f (E_F + 2E_0)$ per ion pair vs. 0.0013$E_0\sigma_C$ per ion on the loss side. The Gain/Loss ratio is, therefore,

$$G/L = \frac{\eta \sigma_f (E_F + 2E_0)}{2\sigma_c \overline{dE}} \approx 2.8\eta, \quad (3)$$

attesting to technical feasibility of the overall scheme.

Granted, it is still not a trivial task to achieve $G/L > 1$ in a practical device; however, the proposed approach replaces the fundamental obstacle with an engineering challenge.

Higher practical $\eta$ values are facilitated for D–D reaction by the fact that 63% of the fusion yield is carried away by charged particles (vs. only 20% for D–T), allowing direct energy conversion.

$G/L$ grows with $E_0$ due to decreasing $\sigma_C$. The 500 kV bias seems to be within the bounds imposed by electrical vacuum breakdown $\mathcal{S}$, as is the electric field $\sim 2 \cdot 10^7 \text{ V/m}$ at the central electrode (6 cm diameter assumed). If necessary, the field parameters can be relaxed by scaling up the linear dimensions of the device.

C. Beam Defocusing Estimate

1. Defocusing due to the initial ion temperature.

Assuming the cold ion injector at temperature $T$, the normal component of the thermal motion of ions is of the order $v_n \sim \sqrt{k_B T / m}$, where $m$ is the ion mass. In the simplest concept design under consideration, absent additional beam focusing devices, this velocity component contributes to beam defocusing and consequent deviation from head-on collision via the time-of-flight for the ions. Depending on the device parameters and dimensions, the
cold ion injector may need to be kept at cryogenic temperatures to limit thermal defocusing.

The time-of-flight to the center is

\[ t \approx \frac{R_2}{v_0} \times \frac{\pi}{2} \sqrt{\frac{R_2}{R_1}}, \]  

(4)

where \( R_2 \) is the radius of the reaction chamber, \( R_1 \) is the radius of the accelerating electrode, and \( v_0 = \sqrt{2E_0/m} \) is the hot ion velocity. For \( R_1 = 3 \text{cm} \) this leads to thermal defocusing of less than 0.1 mm for injector at room temperature (0.2 mm contributed to the beam diameter).

2. Internal Coulomb defocusing

The usual critique of accelerated beam fusion reactors conclude that the beam densities necessary to achieve certain energy output lead to beam self-defocusing because of internal Coulomb repulsion [9, 10]. Here I do not make any claim of large energy output, which may be limited by this and other considerations. The goal here is only to design a device with net-positive energy output, albeit possibly small.

On the other hand, Coulomb defocusing limitations can be alleviated or relaxed by including beam focusing elements (conveniently made easier by the 1-dimensional spatial distribution) or employing more involved designs, including e.g. separating ion injectors and ion collectors via deflecting magnetic fields, replacing fusor design with beam storage rings as in Ruggiero [11], employing a single 8-shaped self-crossing ring etc.

Any such design needs to implement the basic design principle to (i) strongly enhance same-energy head-on collisions in the reaction zone, (ii) efficiently evacuate the ions scattered to angles inconsistent with the 1D phase space distribution maintained and (iii) to efficiently collect the kinetic energy of such post-scattered ions and return it back to the electric circuit before thermalization occurs.

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