Comparative analysis of calculation methods for Cable Curve of Landscape Suspension Bridges

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Abstract. The main cable curve control of landscape suspension bridge is not only the premise of achieving the ideal completed state, but also the key problem of suspension bridge construction control. By analysing the characteristics of the main cable of anchored double tower landscape suspension bridge, the analytical equations for calculating the main cable curve of suspension bridge are derived by parabolic method and segmented catenary method. Then combined with practical work, the main cable curve was calculation methods by using the two calculation methods. Results show that both the parabolic method and the catenary method can be used for the preliminary design of the main cable curve of the landscape suspension bridge, but the segmented catenary method with repeated revision iteration has higher accuracy.

1. Introduction

In recent years, suspension bridge has become the first choice for urban landscape bridges due to its characteristics of low price and attractive appearance, which has been favoured by domestic and foreign design builders \cite{1-2}.

Landscape suspension bridges are different from general suspension bridges in that the decorative cable of a landscape suspension bridge does not bear the main structure load\cite{3}. However, accurate analysis of the line-shapes and stress-free length of the main cable is directly related to whether the main cable meets the design’s stress conditions, and it is also the key to construction control. The suspension bridge line shape is easily affected by various factors on the construction site, and it is difficult to adjust the main cable shape after the main cable erection is completed \cite{4-6}. Therefore, reliable data obtained through calculations and analyses is required in order to thoroughly control the main cable shape during construction and to meet the design requirements.

The sag of the main cable of a suspension bridge will change greatly during the installation process, mainly in two equilibrium states—the suspended state and the finished bridge state. Based on the analysis of the two states, the initial pre-bias of the saddle can be determined, and the line shape of the main cable in suspended state can be obtained under the condition that the main cable has no stress length in any state, and according to the line shape in the suspended state, the position of the cable clamp in the main cable can be determined, and the line shape of the main cable in the finished bridge state can be determined.

Based on the analysis of the internal force of the decorative cable of the landscape suspension bridge, the main cable’s parameters when the bridge is completed were deduced via the parabolic method and the segmental catenary method \cite{7}. After being combined with an example, comparisons between the
results of the two methods revealed that the error between the two methods is small, and that the calculation accuracy of the segmental catenary method is higher. This method has specific significance for the calculation of the cable in the actual engineering of an ornamental suspension bridge.

2. Theoretical Analysis of Main Cable Curve Finished Bridge State

The decorative cable of a landscape suspension bridge design is different from that of a general suspension bridge, and, in addition to meeting the requirements to maintain an ideal shape, cable tension in wind and other bad weather conditions must be considered. Therefore, the parameters must be accurately calculated through internal force analysis in order to meet the design requirements.

2.1. Assumptions

The following assumptions are used in the internal force analysis of the main cable: (1) The main cable is made of the ideal flexible materials and cannot withstand pressure and bending. (2) The main cable is a linear elastic material, and the stress-strain relationship conforms to Hooke's law. (3) Under the effect of the external load, the changes in the cross-sectional area and weight of the main cable are ignored [8].

The force of the main cable between the adjacent locking clips is roughly the same in both the balancing period and the initial construction period. In order to facilitate the calculation, the main cable is divided into several micro segments, and one of them is used for stress analysis.

2.2. Parabolic Method

Assuming that the load \( q(x) \) on the main cable is uniformly distributed along the horizontal direction, and that \( q \) is a constant, the main cable is a parabola[9,10].

Figure 1 shows the force analysis of a \( dx \) micro-segment with the horizontal length of the main cable. The main cable is free from the horizontal load. According to the balance in the horizontal direction of the micro-segment, the horizontal component \( H \) is a constant, that is \( H_1=H_2=H \).

\[ \begin{align*}
\text{Figure 1. Force analysis of parabolic cable} \\
\text{Figure 2. Catenary cable force analysis diagram}
\end{align*} \]

According to the balance of forces, the parabolic equation can be expressed as:

\[ y = -\frac{q}{2H}x^2 + \left(\frac{h}{l} + \frac{ql}{2H}\right)x \]  

(1)

where \( H = \frac{ql^2}{8f} \), \( l \) represents the span, \( h \) represents the difference in elevation between two points of the cable segment, \( H \) represents the horizontal component of the main cable tension, and \( q \) represents the uniformly distributed load on the main cable.

Substitute the coordinate points \( A(0, 0) \) and \( B(l, h) \) into Eq. 1. Therefore, the parabolic equation of the middle span main cable is

\[ y = \frac{4fx(l-x)}{l^2} \]  

(2)

The parabolic equation of side span main cable is

\[ y = \frac{4fx_1(l-x)}{l_1^2} + \frac{h}{l_1}x \]  

(3)
where $f$ is the vector height of the middle span, $f_1$ is the vector height of the side span, and $l_1$ is the side span.

A cable length of $d_s$, a micro segment in the main cable, is analyzed below. The unstressed cable length is the stressed cable length minus the elastic elongation. The stressed cable length can be obtained by integrating the parabolic equation [11]:

$$
\frac{1}{2} \int_0^l \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{1/2} \, dx
$$

(4)

The series is expanded to the first three terms as below:

$$
S = \int_0^l \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{1/2} \, dx = l \left( \frac{h^2}{2l^2} + \frac{8f^2}{3l^2} - \frac{h^4}{8l^4} - \frac{32f^4}{5l^4} - \frac{4f^2h^2}{l^4} \right)
$$

(5)

$$
\Delta S = \int \frac{T}{EA} \, ds = \frac{H}{EA} \int \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{1/2} \, dx = \frac{H}{EA} \left( \frac{h^2}{l} + l + \frac{16f^2}{3l} \right)
$$

(6)

Unstressed cable length can be calculated as follows:

$$
S_0 = S - \Delta S
$$

(7)

2.3. Segmental Catenary Method

Assuming that the load $q(x)$ of the main cable is uniformly distributed along the main cable curve, the main cable line can be regarded as a combination of multi-segment catenary. Figure 2 shows how a force analysis is carried out with a $dx$ micro-segment of a horizontal length of the main cable. The $q$ is transformed into a uniform load, $q_y$, which is equivalent to the horizontal direction, that is $q \cdot ds = q_y \cdot dx$. $T$ represents the tension at both ends of the micro cable segment, and $H$ is the horizontal component of the main cable tension. The differential equation is derived from the equilibrium equation of the force [12]:

$$
H \frac{d^2y}{dx^2} + q \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{1/2} = 0
$$

(8)

Substitute the coordinate points A (0, 0) and B (l, h) into Eq. 8 to obtain the catenary equation:

$$
y = \frac{H}{q} \left[ \cosh \alpha - \cosh \left( \frac{2\beta x}{l} - \alpha \right) \right]
$$

(9)

Where

$$
\alpha = \sinh^{-1} \left( \frac{h\beta}{l\sinh \beta} \right) + \beta, \ \beta = \frac{ql}{2H}
$$

(10)

The main cable with span $L$ and vector height $f$ is divided into $n$ segments. The catenary equation of section $i$ is

$$
y_i = \frac{H}{q} \left[ \cosh \alpha_i - \cosh \left( \frac{2\beta x_i}{l_i} - \alpha_i \right) \right]
$$

(11)

Where
\[ \alpha_i = sh^{-1}\left(\frac{h_i\beta_i}{l_i sh\beta_i}\right) + \beta_i, \beta_i = \frac{ql_i}{2H} \]

The compatibility condition is

\[ H = \frac{dy_{i-1}}{dx_{i-1}|_{x_{i-1}=x_{i-1}}} = H \left. \frac{dy_i}{dx_i}\right|_{x_i=x_0} = P_{i-1} \]  

\[ \sum_{i=1}^{n} h_i = h \]  

Iterative calculation is established according to Eq. 10 [28]:

1) Assuming that the initial iteration value of the horizontal component \( H_0 = \frac{ql^2}{8f} \).

2) Assuming that the vertical force of the first support \( P_0 = \frac{ql}{2} \).

3) By substituting \( H_0 \) and \( P_0 \) into Eq. 10, \( \alpha, \beta, h \) can be obtained.

4) The \( \alpha_2 \) and \( \beta_2 \) of the next paragraph can be obtained from Eq. 11. And then sequentially calculate \( h_2, h_3, h_4, ..., h_n \).

5) Calculating of \( \Delta h \) by, \( \Delta h = \sum_{i=0}^{n} h_i - h \). If the value is too large, it needs to be corrected. Make \( H_0 \) into \( H_0 + \Delta H \) (\( \Delta H \) is the modified value of the horizontal component of cable force determined by the difference value), then cycle steps (1) through (5) until the cable passes the specified point.

Stressed cable length is [12]:

\[ S = \int_0^l \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{1/2} dx = \frac{H}{q} \left[ sh \left( \frac{-ql}{H} + \frac{qt}{H} \right) - sh \frac{qt}{H} \right] \]  

Elastic elongation is:

\[ \Delta S = \int_0^l \frac{H}{EA} \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right] dx = -\frac{H^2}{4EAq} \left[ sh \left( \frac{-2ql}{H} + \frac{2qt}{H} \right) - sh \frac{2qt}{H} \right] + \frac{Hl}{2EA} \]  

Unstressed cable length is:

\[ S_0 = S - \Delta S \]  

3. Main Cable Curve Calculation under Suspended State

3.1. Calculation of Cable Saddle Offset

In the initial construction period, the horizontal force component of the main cable of two adjacent spans is different due to the unstressed length of the main cable of the middle span and the side span, which will make the main tower bear a larger unbalanced force. Therefore, the tension must be adjusted across the main cables by offsetting the saddle to prevent the main tower from maintaining its balance through its own deformation.

Under the condition of setting the pre-deflection of the cable saddle, the lengths of the unstressed cables in each span of the initial construction state and the finished bridge state are equal.
Assuming pre-deflection and horizontal component $H'$, substitute them in Eq. 13, 14, and 15. Iteration is carried out according to the conditions that the length of the unstressed cable is equal in two states, and that $d$ and $H'$ are obtained.

3.2. Line-shapes in the Initial Construction Period
Based on pre-deflection and horizontal component $H'$ of the cable saddle obtained above, the coordinates of each lifting point in the initial construction state can be obtained via the principle that the unstressed cable length is the same in two states, thereby obtaining the line-shapes of the main cable in the initial construction state.

4. The Engineering Application
The Biyang bridge is a ground anchored concrete suspension bridge with decorative main cables. The length of the bridge is 240 m, and The span of the bridge is arranged as 60+120+60 m. The ratio of vector to span is 1: 9.756, the height of the vector is $f = 12.3$ m, and the spacing of the boom is 5 m. The main beam is simply supported by concrete with a height of 0.95 m. The cable tower is composed of a reinforced concrete portal tower with a height of 15 m, a double tower, and a cable plane structure. Each cable consists of 22 $\phi 15.2$ stranded wires. The cross-sectional area of a single main cable is $A = 0.1$ m$^2$, $E = 1.95 \times 10^5$ MPa. The parabolic method and the segmented catenary method were used to calculate the bridge line shape and the unstressed length of the main cable, and comparative analysis was also carried out, as shown in Tables 1 and Table 2. By using the parabolic method and the segmented suspension line method to calculate the stress-free cable length of the main cable, conclusions can be drawn from the analysis and comparison data as follows:

1) The error of the unstressed cable length-calculated by the two methods is very small. The middle span is 0.008 m and the side span is 0.048 m.

2) In the completed bridge, the maximum difference of the main cable coordinates of the middle span is 0.003 m, and the error values are all positive. Compared with the segmental method, the vertical coordinates calculated by the parabolic method are higher from the bridge deck.

### Table 1. Internal forces and cable lengths in the completed bridge

|                | Stress cable length /m | Elastic elongation /m | Unstressed cable length /m |
|----------------|------------------------|-----------------------|----------------------------|
|                | Middle span | Side span | Middle span | Side span | Middle span | Side span |
| Parabolic method | 123.282      | 2×62.513 | 0.252      | 2×0.126   | 122.973     | 2×62.362 |
| Sub-catenary method | 123.241 | 2×62.455 | 0.247      | 2×0.121   | 122.963     | 2×62.331 |
| Difference     | 0.041       | 2×0.058 | 0.005      | 2×0.005   | 0.01        | 2×0.031 |
| Error%         | 0.033       | 0.093 | /           | /         | 0.008       | 0.048 |

### Table 2. Main cable coordinates of middle span in the completed bridge

| x/m | 0  | 10 | 20 | 30 | 40 | 60 | 60 |
|-----|----|----|----|----|----|----|----|
| Parabolic method | 3.895 | 7.087 | 9.572 | 11.368 | 12.454 | 12.841 |
| Sub-catenary method | 3.893 | 7.086 | 9.571 | 11.365 | 12.451 | 12.839 |

| y/m | 0  | 0.002 | 0.001 | 0.001 | 0.003 | 0.003 | 0.002 |
|-----|----|------|------|------|------|------|------|
| Parabolic method | 3.895 | 7.087 | 9.572 | 11.368 | 12.454 | 12.841 |
| Sub-catenary method | 3.893 | 7.086 | 9.571 | 11.365 | 12.451 | 12.839 |

Comparisons and analysis reveal that the unstressed cable length and coordinate difference of the main cable in the completed bridge calculated by the parabolic method and the segmental catenary method are small, both being less than 0.4%. The segmental method is highly accurate by way of continuous iteration, which can be used to obtain accurate solutions for a long-span suspension bridge. Although
the parabolic calculation is convenient, it relies on many assumptions and the margin of error is large, which is not suitable for generating accurate solutions of the stress-free cable length of long-span suspension bridges. Therefore, the segmental method is generally used to calculate the main cable shape of landscape decorative suspension bridges.

5. Summary
Based on the force analysis of the urban landscape decorative suspension bridge cable, the parabolic method and the segmental catenary method were summarized in order to study the decorative cable. Combining the hypothesis of the theory with the comparison of the Biyang suspension bridge, it can be found that both the parabolic method and the segmental suspension line method can be used to determine the preliminary calculation of the main cable of the decorative cable of the landscape suspension bridge. The calculation of the parabolic method is relatively simple, but its precision is not high. The segmental catenary method has higher precision due to its continuous iteration. Therefore, the segmental catenary method is more suitable for the calculation of the main cable alignment of long-span suspension bridges, which can be used as a reference for the decorative cable design of landscape suspension bridges in the future.

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