Large-eddy simulations of the near field of a turbulent annular jet

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Abstract. Using high-order spectral element method and large eddy simulations we study a turbulent annular jet at the Reynolds number of 8900, based on the bulk velocity in the annular channel and its half-width. The region of high turbulent kinetic energy is located inside the recirculation zone behind the bluff-body where the low-frequency motion is known to occur which is detected with energy spectra. Further analysis includes the azimuthal Fourier and Proper orthogonal decomposition (POD). It is shown that the main eigenmodes with low azimuthal wavenumbers $m$ rotate around the symmetry axis in both directions providing the explanation for the precessing motion of the recirculating zone.

1. Introduction

Annular jets are produced by the flow issuing from the annular channel between two coaxial cylinders. Since the inner cylinder usually appears as a central bluff-body, the flow has a recirculation zone in the near field. This leads to inner and outer mixing layers where the first one is between the jet and recirculation zone and the second one is between the jet and the ambient fluid. A combination of features inherent to jets and separated flows makes an annular jet a complex object. The goal of the present study is to investigate the dynamics of the recirculation zone which is of utmost importance for a wide range of practical applications.

In first experiments Chigier and Beer \cite{1} compared circular and annular jets studying their similarities. Ko and Chan \cite{2, 3, 4} considered different configuration focusing on the near field. Coaxial jets issuing from a double annular channel also attracted a lot of attention due to the combustion applications \cite{5}. While the dynamics of the outer shear layers is similar to that appearing in circular jets, the recirculation zone behind the bluff-body follow its rules. Berger et al. \cite{6} studied the dynamics of the wake of a disk and a sphere at relatively high Reynolds numbers and found very low-frequency oscillations of the recirculating bubble. Similar observations were made for various bluff-body geometries \cite{7, 8, 9, 10}.

Recently Vanierschot and Van den Bulck \cite{11} observed the precessing motion of the recirculating zone in annular jets. While Proper orthogonal decomposition (POD) is found useful to study the low-frequency dynamics \cite{12}, the analysis so far was two-dimensional due to experimental capabilities. In the present work we perform three-dimensional simulations and apply POD to volumetric data focusing on the vortical structures responsible for deformations of the recirculation zone.
2. Computational details and problem

To describe the motion of the incompressible fluid we consider the Navier–Stokes equations:

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \nabla) \mathbf{u} = -\nabla p + \frac{1}{Re} \Delta \mathbf{u} - \nabla \cdot \tau,
\]

\[
\nabla \cdot \mathbf{u} = 0,
\]

where \( \mathbf{u}, p \) are the dimensionless spatially filtered velocity field and pressure, respectively, \( \tau \) represents the subgrid-scale stresses appearing in the large eddy simulations framework. The above described governing equations are implemented in the Nek5000 code [13] with a spatial discretisation based on the spectral-element method using Lagrange polynomials. The Navier–Stokes equations are cast in a weak form and discretised in space by means of Galerkin approximation using \( N \)th-order Lagrange polynomial interpolants on the Gauss-Lobatto-Legendre points for the velocity field and pressure (\( P_N - P_N \) formulation). In the present work the polynomial order \( N \) is set to 8. The semi-implicit time-stepping scheme is of the third-order accuracy. In the LES simulation the two highest modes are filtered with parabolic transfer function, and the filter amplitude was 5\% in the last mode [14].

![Figure 1. Left: annular channel with periodic boundary conditions to generate the inflow in the main simulation. Right: main computational domain of the jet flow and coordinate system.](image)

We consider a turbulent annular jet of incompressible fluid with the Reynolds number \( Re = 8900 \), based on the bulk velocity \( U_b \) and outer diameter \( D \), while \( d = D/2 \) is the inner diameter of the annular channel. Figure 1 shows the annular channel of length \( L = 2.5D \) which is used to simulate with periodic boundary conditions a fully turbulent inflow velocity fields for the main computation of the annular jet. To validate this simulation performed with \( 8 \times 32 \times 20 = 5120 \) spectral elements for \( r, \theta \) and \( x \) directions, the results were compared with DNS by Chung et al. [15] with the same computational domain and flow parameters showing excellent agreement, see Fig. 3. The main region represents a cylinder with a diameter \( D_{\text{main}} = 12D \) and length \( L_{\text{main}} = 17D \), which includes a feeding annular channel with length equal \( 2D \). The thickness of the outer wall of the channel is 0.03\( D \). The velocity field in some \( r-\theta \) plane from a periodic channel simulation was copied each time step to the annular channel of the main computation. The coflow in the main domain was set to 0.04\( U_b \). Zero derivative condition was set on the rest of the boundaries. The main area consisted of about 45 thousand spectral elements (40 \( \times \) 32 \( \times \) 35 elements in \( r, \theta \) and \( x \)). Since we use polynomials of the eighth order, there are \( 9^3 = 729 \) computational nodes inside each spectral element, so that the total number of computational nodes is about \( 32 \times 10^6 \) for the main simulation and about \( 3.7 \times 10^6 \) for the periodic annular channel. The distance between computational nodes at the outer wall inside the periodic channel is \( \Delta r^+ = 0.6, (r\Delta \theta)^+ = 5 \) and \( \Delta x^+ = 50 \) in radial, azimuthal and
longitudinal direction, where the superscript \( '+' \) corresponds to the non-dimensionalization using
the friction velocity and viscosity. The computational grid satisfies the near-wall well-resolved
LES criteria [16]. Inside the recirculation zone we calculate a ratio between spatial grid step and
the Kolmogorov scale, which does not exceed value of two Kolmogorov scales. This fact means
that our calculation corresponds to the DNS accuracy.

3. Results
To get the impression on the main flow features we show the instantaneous and time-averaged
axial velocity fields in \( x - r \) plane, see Fig. 2. The outer and inner shear layers merge after
around \( 3D \) leading to the formation of the jet profile. The recirculation bubble having the length
of \( 0.52D \) has a complex dynamics which is addressed below. Figure 3 shows the radial profiles of
the time-averaged axial velocity and its fluctuations at several axial stations along \( x \) direction.
First profiles correspond to the periodic channel simulations where the results show excellent
agreement with DNS calculation of Chung et al. [15]. This agreement confirms sufficient spatial

Figure 2. Left: instantaneous (top) and time-averaged (bottom) axial velocity fields and
coordinate system. Right: The blow-up of the near-nozzle area and four points used to further
analyze spectral characteristics.

Figure 3. Upper rows: time-averaged radial profiles of axial velocity. Bottom rows: profiles of
axial velocity fluctuations. In the first column the comparison of the periodic channel present
simulation with the DNS [15] (symbols) is shown.
resolution of the computational mesh. Four more axial stations are shown, i.e. \( x/D = 0, 0.25, 0.5 \) and 0.75. The recirculation zone is characterized by the high level of fluctuations especially close to the stagnation point.

Further we investigate the time history and spectral characteristics at four points in the plane \( x-r \) (see Fig. 2) where the velocity field was analyzed during 100\( t U_b/D \) time units. Two points were located in the inner mixing layer and two in the outer mixing layer at a distance of 0.25\( D \) and 0.5\( D \) from the nozzle edge. Figure 4 shows the energy spectra for individual velocity components. It is interesting to note that the Kelvin-Helmholtz instability is most clearly pronounced for the radial and azimuthal velocity components with a non-dimensional frequency \( f_0 = f D/U_b \approx 1 \). This peak is marked with the letter ‘A’ in the figure. A similar effect was observed in the mixing layer for a turbulent circular jet [17]. There is a small peak in the range of lower frequencies (marked as ‘B’) which will be investigated below with a more sophisticated method. Similar results were obtained for the Fourier transform from the autocorrelation functions (not shown here).

![Energy spectra for separate velocity components](image)

**Figure 4.** Energy spectra for separate velocity components (left: \( u \), middle: \( v \), right: \( w \)). Upper rows correspond to the inner mixing layer, bottom rows – to the outer mixing layer. Blue line illustrates the “−5/3” slope.

The round geometry suggests decomposition of the flow field into Fourier modes in the azimuthal direction \( \phi \). The instantaneous velocity field \( \mathbf{u} \) is therefore decomposed into complex Fourier coefficients

\[
\mathbf{u}(r, x, m, t) = \mathbf{u}^m = \frac{1}{2\pi} \int_{0}^{2\pi} \mathbf{u}(r, x, \phi, t)e^{im\phi} d\phi,
\]

where \( m \) is the azimuthal wavenumber. We further apply snapshot Proper Orthogonal Decomposition [18] for each azimuthal wavenumber \( m \) and in a cylindrical subdomain located at a distance of 0.1\( D \) from a nozzle with a length of 1\( D \) and a radius of 1.5\( D \) to an ensemble of \( N = 2660 \) Fourier-decomposed fields \([u_1^m, u_2^m, ..., u_N^m] \) corresponding to subsequent time instants \( t = t_1, ..., t_N \) with \( \Delta t = 10^{-2}D/U_b \):

\[
u_i^m(r, x) = \mathbf{u}(r, x, m, t_i) = \sum_{q=1}^{N} \alpha_q^m(t_i)\lambda_q^m v_q^m(r, x),
\]
Figure 5. Left: energy distribution among most energetic POD modes for low wavenumbers $m = 0−3$. Right: 3D visualization of POD modes. A semitransparent axisymmetric figure is the contour of the time-averaged axial velocity corresponding to the value $u = 0.5$, which represents closely the location of the recirculation zone. The blue and orange isosurfaces correspond to $u = \pm 1.5$, $v = \pm 0.5$ and $w = \pm 1.5$.

where $v_{mq}(r, x)$ and $a_{mq}(t)$ are the non-dimensional complex-valued spatial eigenfunctions and temporal amplitudes satisfying orthonormal conditions

$$2\pi \int_V v_{im}(r, x)v_{jm}(r, x)rdrdx = V\delta_{ij},$$

$$\sum_{n=1}^{N} a_{im}(t_n)a_{jm}(t_n) = N\delta_{ij},$$

where $\lambda_{mq}^n$ are the real eigenvalues, $\delta_{ij}$ the Kronecker delta and $V$ the volume of the considered domain (see [17] for details).

Figure 5 (left) shows the distribution of energy among different POD modes $q$ for various $m$. Note that kinetic energy of a particular mode integrated over a volume of interest is $(\lambda_{mq}^n)^2/2$. The results suggest that the dynamics is governed by energy containing modes with low $m$. The most energetic mode has $m = 1, 2$ and $q = 1, 2$. Figure 5 (right) shows the three-dimensional visualization of these four modes. To investigate the dynamics of these structures the complex-valued temporal coefficient is decomposed as $a(t) = \beta(t)e^{2\pi i\gamma(t)}$. Figure 6 shows that behaviour of real-valued $\beta$ and $\gamma$ for $m = 1$ and $q = 1, 2$. While $\beta(t)$ oscillates, $\gamma(t)$ follows a linear function corresponding to the rotation of the coherent structure around the axis of symmetry with constant angular velocity. Note that the value of $d\gamma/dt$ is the frequency of rotation which is close to the one observed previously with energy spectra ($f_0 \approx 0.2$). The described framework of precessing motion will be further developed for this annular jet flow.

4. Conclusion
Using large eddy simulations we studied the near field of the turbulent annular jet. The Reynolds number based on the bulk velocity in the feeding channel and the outer diameter was 8900. The assessment of statistical characteristics showed that the area around the stagnation point in the end of the recirculating region has a high level of turbulent kinetic energy. These fluctuations display a low-frequency coherent motion revealed by energy spectra. To study the spatial shape of these fluctuations we used Fourier and Proper orthogonal decomposition (POD). It is shown that the perturbations with Fourier harmonics $m \neq 0$ rotate around their own axis with low frequency, while the amplitude of the perturbation oscillates.
Figure 6. The time history of $\beta(t)$ and $\gamma(t)$ corresponding to $m = 1$ and $q = 1, 2$.

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References
[1] N. A. Chigier and J. M. Beer. The flow region near the nozzle in double concentric jet. J. Basic Engnrg. 86:797–804, 1964
[2] N. W. M. Ko and W. T. Chan. Similarity in the initial region of annular jets: three configurations. J. Fluid Mech. 84:641–656, 1978.
[3] N. W. M. Ko and W. T. Chan. Coherent structures in the outer mixing region of annular jets. J. Fluid Mech. 89:515–563, 1978.
[4] N. W. M. Ko and W. T. Chan. The inner region of annular jets. J. Fluid Mech. 93:549–584, 1979.
[5] F. H. Champagne and I. J. Wyganski. An experimental investigation of coaxial turbulent jets. Int. J. Heat Mass Transf. 14:1445–1446, 1971.
[6] E. Berger, D. Scholz and M. Schumm. Coherent vortex structures in the wake of a sphere and a circular disk at rest and under forced vibrations. J. Fluid Struct. 4:231–257, 1990.
[7] F. M. Najjar and S. Balachandar. Low-frequency unsteadiness in the wake of a normal flat plate. J. Fluid Mech. 370:101–147, 1998.
[8] J. J. Miau, J. T. Wang, J. H. Chou and C. Y. Wei. Characteristics of the low-frequency variations embedded in vortex shedding process. J. Fluid Struct. 13:339–359, 1999.
[9] O. Lehmkuhl, I. Rodriguez, R. Borrell and A. Oliva. Low-frequency unsteadiness in the vortex formation region of a circular cylinder. Phys. Fluids 25:085109, 2013.
[10] E. Palkin, R. Mullyadzhanov, M. Hadziabdic and K. Hanjalic. Scrutinizing URANS in shedding flows: The case of cylinder in cross-flow in the subcritical regime. Flow Turbul. Combust. 97:1017–1046, 2016.
[11] M. Vanierschot and E. Van den Bulck. Experimental study of low precessing frequencies in the wake of a turbulent annular jets. Exp. Fluids, 50:189–200, 2011.
[12] B. Patte-Rouland, G. Lalizel, J. Moreau and E. Rouland. Flow analysis of an annular jet by particle image velocimetry and proper orthogonal decomposition. Meas. Sci. Tech. 12:1404–1412, 2001.
[13] S. G. Kerkemeier, P. F. Fischer, and J. W. Lottes. Nek5000: Open source spectral element cfd solver. available at http://nek5000.mcs.anl.gov, 2008.
[14] O. Marin and R. Vinuesa and A. V. Obabko and P. Schlatter. Characterization of the secondary flow in hexagonal ducts. Phys. Fluids, 28:1–26, 2016.
[15] S. Y. Chung, G. H. Rheem, and H. J. Sung. Direct numerical simulation of turbulent concentric annular pipe flow. Part 1: Flow field. Int. J. Heat Fluid Flow, 23:426–440, 2002.
[16] U. Piomelli and J. R. Chasnov. Turbulence and transition modeling. Large-Eddy Simulations: Theory and Applications. Springer-Science, 1995.
[17] R. Mullyadzhanov, S. Abdurakipov and K. Hanjalic. Helical structures in the near field of a turbulent pipe jet. Flow Turbul. Combust., 98:367–388, 2017.
[18] L. Sirovich. Turbulence and the dynamics of coherent structures. Part i: Coherent structures. Quart. Appl. Math. 45:561–571, 1987.