Identification of Misconceptions in Middle School Mathematics Utilizing Certainty of Response Index

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Abstract. Student’s understanding of a material concept differs one to the others. This comprehension will be used to learn the next material. Thus, it is important to know whether or not the student understands a mathematical concept. The aim of the research is to identify students’ misconceptions in mathematics and the possible causes of misconceptions. This qualitative case study approach used a multiple choices test given to 53 middle school mathematics students in grade 9 in Yogyakarta, Indonesia. The identification of students’ misconceptions was based on the Certainty of Response Index (CRI) technique. The result showed that 20.9% of students had misconceptions in working mathematical problems. Meanwhile, the percentage of lucky guess, knowledge of correct concepts and lack of knowledge respectively is 18.8%, 32.7%, and 27.6%. The data indicated that students had misconception in classifying objects based on fulfilling requirements that form concepts, determining the right concepts to solve problems, distinguishing the necessary conditions and/or sufficient conditions for the concept of a circle. The result suggests that in the learning process students should be trained to understand the concept well, not only memorize the material. Furthermore, teachers and students should be given adequate interaction during the learning process.

1. Introduction

The quality of education is not only determined by the plan and development of education but also determined by the quality of its implementation [1]. So that in the teaching and learning process, knowing the level of a student's understanding of a material concept is important. This is in accordance with the statement of Booth, Barbieri, McGinn and Young [2] that mathematics teachers not only focus on mastering procedural skills but also encouraged to foster students' conceptual understanding.

The high level of students' understanding of material usually indicates that the student mastered the concept. Understanding this concept will also be a provision for students to learn the next material. A student is said to understand the concept well if he is able to explain the concept, able to recognize concepts and use them in various situations, and able to develop the consequences of these concepts [3]. This is also supported by [4] which stated that conceptual understanding enables students to solve mathematical problems in various forms, where students with a high level of conceptual understanding are able to solve new problems that have never been encountered before.

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According to [5] students often do not realize that they have problems in mathematics or do not even want to admit it. Sometimes students are also reluctant to ask for help and afraid of embarrassment if they have to ask. So if left unchecked it will cause misconceptions for students. On the other hand, based on the National Assessment of Educational Progress only 42% of grade 4 students and 35% of grade 8 students were at or above the advanced level of mathematics [6]. In other words, it can be said that there is still more than half the number of students whose level of understanding in mathematics is low and even has wrong conceptual ideas (misconceptions).

Misconception is “…misunderstanding and misinterpretations based on incorrect meanings. They are due to ‘naive theories’ that impede rational reasoning of learners” [7]. While according to Hasan, Bagayoko and Kelley [8] misconception is a cognitive structure that is held firmly different from the understanding received in a field and it is considered to interfere with the acceptance of new knowledge. Meanwhile, according to Swan [9] stated that misconception is not a wrong thought, but it is a concept in local generalization made by students. In the stage of natural development, this is very likely to occur. Even though we can and must avoid activities and examples that encourage misconceptions, this misconception cannot be avoided.

This misconception comes up from the fact that a person has their own concept that is formed from reasoning, intuition, culture or other things that explain the phenomenon around them. However, the concept they have understood is different from the actual concept [10]. In line with this, Ojose [11] stated that misconceptions are caused by students' excessive need to understand the instructions they receive. In some cases, misconceptions can last long in students' thinking. This usually happens due to students have memorized the wrong concept for a long time. So when faced with a problem solving, the concept memorized by students is no longer in accordance with the problem. Without guidance, students applying concepts that do not suitable into the problem will produce more mathematical misconceptions.

Misconceptions are caused by factors of students, teachers, textbooks, learning contexts and how the teachers teach. Meanwhile, according to Ojose [11] students have errors that produce misconceptions in two things, namely conceptual errors and execution errors. Conceptual errors are related to students' lack of understanding, while execution errors occur when attempts to perform several procedures are damaged or only partially executed. Examples of misconceptions that are often experienced by middle school students are errors in operating the addition or subtraction of fractions. This student misconception is associated with the transition from the concept of integer operation to operations with fractions. Because of previous knowledge is possessed, students do subtraction or addition operations between numerators and denominators. So students who apply that rules, will get errors in solving problems.

Some cases of misconceptions in previous studies that are often experienced by middle school students in mathematics subjects, included a research by Trivena, Ningsih and Jupri [12] which stated that students still experience difficulties in understanding arithmetic fraction procedures, especially in the addition and subtraction process. Meanwhile in geometry, Özerem [13] found a number of misconceptions and lack of knowledge of grade 7 students related to the subject of geometry. On another occasion, Ningrum, Yuliani, Helingo and Budiarto [10] identify students 'misconceptions on rectangular material, wherein general students' misconceptions are caused by the images given.

Consider this, it is very important to know whether students understand or do not understand and even understand the wrong concept of a material. For this reason, an analysis is needed to find out the misconceptions experienced by students. Basically, there have been many attempts by previous researchers to identify student misconceptions. But there is a weakness in distinguishing between students who experience misconceptions with students who do not know or who lack understanding of a concept.

So to identify students’ misconceptions at once differentiating between students who do not know the concept with students who misconception is use the Certainty of Response Index (CRI) technique that developed by Saleem Hasan et al. According to Hasan, Bagayoko and Kelley [8], CRI is often used in survey research, where respondents are asked to provide a level of certainty they have in their own ability to choose and utilize knowledge, concepts or rule in getting answers. CRI is usually based on a
certain scale. For example, the scale used is a scale of six (0-5). Low confidence scale (CRI 0-2), describes students as having a low level of confidence that shows students’ ignorance of the concept. While the high confidence scale (3-5), describing students having a high level of certainty in giving answers. If (CRI 3-5) and student answers are correct, then this shows a high level of confidence in the truth of knowledge well. Whereas if the students’ answer are wrong, then indicate that a misconception in the knowledge of the student. Whereas to classify the types of errors by students, refers to the six categories of student’ errors in middle school mathematics, namely 1) misused data, 2) misinterpreted language, 3) logically invalid inference, 4) distorted theorem or definition, 5) unverified solution, and 6) technical error [14].

Based on the background and theoretical study, it is important to identify students' misconceptions in mathematics material. Thus, the purpose of this study is to find out students' misconceptions in mathematics material and the possible causes or suspected of misconceptions. The findings of this study can be used as a suggestion for teachers to know misconceptions that occur to students and to predict the possible causes behind the students' misconceptions. Besides that, the findings of this study can also be used by teachers as a basis in design mathematics learning that enhances students' understanding.

2. Research Methodology

2.1. Research Type

This research is a qualitative research using a case study approach that aims to find out students' misconceptions in mathematics. The study was conducted in August 2018. In this study, researchers identified the misconceptions experienced by students when solving circle’ questions by using the Certainty of Response Index (CRI) technique. To estimate Certainty of Response Index (CRI) are by comparing the correct or not answers of students with the high or low CRI index given by students on the problem.

2.2. Research Participants

Participants in this study were 53 students of class IX which consisted of two classes at a private junior high school, Bantul, Special Region of Yogyakarta, Indonesia. Participants consisted of 25 male and 28 female students with heterogeneous academic abilities (low, medium and high). The material used in this study is Circle that is assumed have been studied by participants.

2.3. Instrument dan Procedure

Data were collected using multiple choice test by adding CRI index to fill students' certainty in answering questions. In the multiple choice test, participants were asked to write down the steps from their answers. This test consists of 8 items about circles, where each item is arranged based on indicators of conceptual understanding. Whereas in the CRI index, students are asked to choose the degree of certainty based on the selected answers.

The analysis conducted in this study is by looking at the choice of student answers with the chosen CRI degree. The analysis of occurrence of misconception is distinguished by comparing the correct or not answers of students with the high or low CRI index given by students on the problem. This is the CRI scale which refers to the scale developed by Saleem Hasan et al.

| Scale | Category               |
|-------|------------------------|
| 0     | Totally Guess Answer   |
| 1     | Almost Guess           |
| 2     | Not Sure               |
| 3     | Sure                   |
| 4     | Almost Sure            |
| 5     | Certain                |

The combination matrix according to Hasan, Bagayoko and Kelley [8], between right or wrong answers and high or low CRI as follows:
3. Research Result and Discussion

Based on the research that has been done to 53 students in grade IX of junior high school which is divided into two classes, it can be seen that around 20.9% of students had misconceptions in working on mathematical problems. Analyzing students' misconceptions using the Certainty of Response Index (CRI) is by looking at students' test results by checking the answers to multiple choice tests with CRI scales given to students. Then each of them is distinguished by four categories, namely lucky guess, knowledge of correct concepts, lack of knowledge and misconception.

The following are the results of students' answers to each class in the form of tables 2 and 3.

**Table 2.** Percentage of Students Understanding’ Level in Class D

| Item | Lucky guess | Knowledge of correct concepts | Lack of knowledge | Misconceptions |
|------|-------------|-------------------------------|-------------------|----------------|
| 1    | 23.08       | 65.38                         | 7.69              | 3.85           |
| 2    | 15.38       | 30.77                         | 15.38             | 38.46          |
| 3    | 7.69        | 61.54                         | 11.54             | 19.23          |
| 4    | 11.54       | 26.92                         | 23.08             | 38.46          |
| 5    | 23.08       | 46.15                         | 15.38             | 15.38          |
| 6    | 11.54       | 34.62                         | 19.23             | 34.62          |
| 7    | 15.38       | 57.69                         | 23.08             | 3.85           |
| 8    | 15.38       | 3.85                          | 50                | 30.77          |
| Average | 15.38     | 40.87                         | 20.67             | 23.08          |

Based on Table 2, the items that have the highest percentage of “misconceptions” category are 4 questions. These questions are 2, 4, 6, and 8. While based on Table 3 the items that have the highest percentage of “misconceptions” category are 3 questions. These questions are 4, 5, and 8.

**Table 3.** Percentage of Students Understanding’ Level in Class A

| Item | Lucky guess | Knowledge of correct concepts | Lack of knowledge | Misconceptions |
|------|-------------|-------------------------------|-------------------|----------------|
| 1    | 70.37       | 25.93                         | 3.7               | 0              |
| 2    | 18.52       | 11.11                         | 51.85             | 18.52          |
| 3    | 7.41        | 25.93                         | 51.85             | 14.81          |
| 4    | 0           | 0                             | 59.26             | 40.74          |
| 5    | 22.22       | 11.11                         | 33.33             | 33.33          |
| 6    | 14.81       | 59.26                         | 18.52             | 7.4            |
| 7    | 25.93       | 62.96                         | 7.41              | 3.7            |
| 8    | 18.52       | 0                             | 51.85             | 29.63          |
| Average | 22.22     | 24.54                         | 34.72             | 18.52          |
The results of average percentage recapitulation of students' understanding mathematics level are presented in the form of bar charts. This aim to see the overall picture of students' understanding level related to the mathematics questions.

![Bar Chart](image)

**Figure 1.** Percentage recapitulation of the average understanding level IX graders in middle school

Based on Figure 1, it can be seen that the average percentage of students who experienced the greatest misconception in questions 2, 4, 5 and 8.

The data showed that in question 2 students made mistakes when classifying objects based on whether or not the requirements form the concept. As Figure 2, the misunderstanding was made by students when stating the shaded area. They claim that the shaded area is sector. While the correct answer is segment. This showed that students made mistakes in distinguishing circle elements. So that it is generally assumed that students have type of error "distorted theorem or definition" and "logically invalid inference".

**Figure 2.** Question and student’s response of question number 2

Interestingly, students answered this question by giving a high scale of CRI, that was 4 and 5. This means that students did not realize the mistakes made by giving high scale in answering the question.

Next in question number 4 with indicator students can determine the right concept of circle to solve a problem, students made mistakes in solving real problems. As Figure 3, a misconception made by students in stated the fishpond area. Students made mistakes in interpreting real problems by using illogical concepts and errors in applying the circle formula to a real problem. On the other hand, there
were students who are wrong in distinguishing the radius and diameter of the circle, thus affecting the results of their answers.

Figure 3. Question and student’ response of question number 4

So that, it was generally assumed that students have type of error "misinterpreted language" and "distorted theorem or definition". Interestingly, not a few of students answered this question with a high scale of CRI, such as 4 and 5. This shows students claim that they understand the material well. But in facts they made mistake.

Next, the student’ test result number 5 with indicator students can give example and not example of a circle concept that is learned. Question number 5 can be seen in Figure 4.

Figure 4. Question number 5

In this problem, students made mistake in distinguishing the example of inner tangent between two circles and outer tangent between two circles. In answering this question, students give a high level of CRI, such as 4 and 5. The interesting facts were there are students who conclude illogically, such as stating the line segment PT as tangent to the circle, whereas clearly drawn PT is the radius of the circle. And the scale of CRI of student in answering this question is high, namely 4.

So, it can be said that students claim themselves understand the material well. Even though the fact is understood the wrong concept. So that it is generally assumed that students have type of error "unverified solution", "logically invalid inference", and "misinterpreted language".

The nest question that students had misconceptions is the question number 8 with indicator students can develop the necessary requirements and/or sufficient terms of a circle concept. In this problem, students made mistakes in solving problems that require necessary conditions and enough of a circle concept, such as errors in determining the radius of the circle and then associating it with the area formula of the circle. In addition, there are students who have not understood the question well, even used solutions that do not make sense.

Figure 5. Question and student’ response of question number 8
Interestingly, students answered this question with a high scale of CRI, such as 4 and 5. This showed that students understand the wrong concept but do not realize it. So that it is generally assumed that students have a type of error "misused data" and "logically invalid inference".

Analyzing the result of student tests on several questions regarding circle material above, students have errors in understanding or known as misconceptions. The most common misconception for students is in classifying objects based on the fulfillment of the requirements that form the concept, where students do the types of errors in "logically invalid inference" and "distorted theorem or definition". Furthermore, it also occurs in determining the right concept to solve a problem, where students do the type of error in "misinterpreted language" and "distorted theorem or definition". Another misconception experienced by students is to differentiate examples and not examples of circle concept, where students do types of errors in "misinterpreted language", "logically invalid inference" and "unverified solution". The next factor that causes misconception is in terms of developing necessary conditions and/or sufficient conditions for a circle concept, where students do the type of error in "misused data" and "logically invalid inference". This is reinforced by the opinion of [15] which stated that students' misconceptions on geometry are caused by students not understanding the questions and not knowing what to do to find the results. In addition, students do not have conceptual knowledge so there are students who do not respond to the questions in the exam at all. Even according to [16] conceptual errors that occur on students can be caused by a lack of knowledge of mathematics teachers, in this case pre-service teachers, about students’ mathematical understanding.

The misconceptions experienced by students that have been described previously are often caused by many factors. First, students before entering the material taught by the teacher do already have errors in their cognitive structure, so that they continue the understanding that has been understood without looking at the new knowledge given by the teacher. Second, because students often apply the concept of memorization, so that when faced with a new problem they use the wrong concept. This is supported by the opinion of [17] which stated that the majority of student errors are due to lack of student attention, basic mathematical errors and misunderstandings on the material previously studied. Thus it needs to pay attention to the substantive aspects of mathematics learning to find students' learning difficulties and minimize misconceptions in students [18,19].

4. Conclusions

This study revealed that the misconceptions that often occur to students in circle material are in classifying objects based on the fulfilling requirements that form the concept, determining the right concept to solve problems, distinguishing examples and not examples of circle concept, and developing the necessary conditions and/or sufficient conditions for the concept of a circle. Factors that are suspected to influence this are due to students’ prior knowledge of errors and because students often did not understand the concept well because of they often apply memorizing the concept. Consider the findings in this study, in the learning process students should be trained to understand the concept well, not only memorize the material. Furthermore, teachers and students should be given adequate interaction during the learning process. So that the teacher can monitor the development of students’ understanding. This study explained students’ misconceptions in circle material but only from the students’ view. Therefore, further research needs to include how teachers view the misconceptions of students and include more mathematical material.

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