The Invisible Axion and Neutrino Masses

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Abstract

We show that in any invisible axion model due to the effects of effective non-renormalizable interactions related to an energy scale near the Peccei-Quinn, grand unification or even the Planck scale, active neutrinos necessarily acquire masses in the sub-eV range. Moreover, if sterile neutrinos are also included and if appropriate cyclic $Z_N$ symmetries are imposed, it is possible that some of these neutrinos are heavy while others are light.

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A natural and elegant way to explain the small value of the active neutrino’s masses is the so-called seesaw mechanism which is implemented when heavy right-handed sterile neutrinos, i.e., transforming as singlet under the standard model $SU(2)_L \otimes U(1)_Y$ gauge symmetry, are added \[1\]. Moreover, depending on future neutrino oscillation data, light sterile neutrinos \[2\] may be a necessary ingredient of the physics beyond the standard model. From neutrino oscillation experiments we already know that active neutrinos have a non-vanishing mass in the sub-eV region \[3, 4\] but, since the $Z^0$ invisible width implies the existence of only three light active neutrinos, any additional light neutrino has to be sterile and there are several possibilities that keep consistency with LEP data \[3, 6\]. The problem is how to implement, in a natural way, light sterile neutrinos. Since they are not protected by the standard model symmetries, they may acquire Majorana masses of the order of the next (if any) energy scale. A solution is the addition of sterile Higgs scalars and a new exact global symmetry \[7, 8\]. The important point is that such a symmetry forbids the Dirac mass term avoiding in this way the seesaw mechanism and the sterile scalar singlet having a vacuum expectation value chosen just to generate light sterile neutrinos. Light sterile neutrino may also appear in supersymmetric models \[9\].

On the other hand, the introduction of a global chiral (Peccei-Quinn) symmetry is an elegant solution to the strong CP problem \[10\] implying in the existence of a pseudo Goldstone boson, the axion \[11\], which besides solving the strong CP problem it is certainly a leading candidate for dark matter. Searches for the axion has been done over the years. Recently, it was obtained an upper limit for the axion-photon coupling $g_{a\gamma\gamma} < 1.16 \times 10^{-10} \text{GeV}^{-1}$ if $m_a \lesssim 0.02 \text{ eV} \ [12, 13]$. In fact, the expected mass for the axion, coming from several experimental or observational constraints, is in the interval $10^{-6} < m_a < 10^{-2} \text{ eV}$ and we see that this interval is near to the neutrino masses required to explain solar and atmospheric neutrino data \[3\]. This fact suggests the existence of a common new energy scale as the responsible for such small masses, which in this case would be that one related to the invisible axion.

Although the existence of a relation between the axion and the seesaw mechanism for generating neutrino masses has already been considered in particular models \[14\], here we will put forward that it is inevitable that neutrino get masses in any invisible axion model. Moreover, if the model have also right-handed neutrinos, some of them may be light but the other ones may be heavy. In fact, we expect that if all sterile neutrinos have the same
physical origin i.e., they are related to the same energy scale, it would be natural that all of
them are either light or heavy. However, if some of them are light and the others heavy this
may be seen as an evidence of energy scales different from the electroweak and Planck scales.
The existence of these energy scales can be masqueraded by imposing discrete symmetries
($Z_N$’s) under which each neutrino can transform in a different way from each other.

The standard model (SM) and many of its extensions can be seen as effective theories below
an energy scale $\Lambda$. The minimal model has doublets of left-handed quarks and leptons, the
respective right-handed singlets for the charged fermions, and one doublet of Higgs scalars,
denoted by $Q_L$, $\psi_L$, $u_R$, $d_R$, $l_R$ and $H$, respectively. Since axion models need more than
one Higgs doublets, we will consider a model that has at least two Higgs doublets. Next,
we introduce $m \geq 1$ right-handed neutrinos, $n_{sR}$, transforming as singlet under the SM
gauge symmetries. An invisible axion model is one in which there is an approximate global
symmetry protecting the $CP$ invariance of the QCD and which is break, beside by non-
perturbative effects, by the VEV of an scalar singlet $\phi$ added to the particle content with
its imaginary part being almost the axion field [15]. Let us suppose also that the invisible
axion is protected against gravitational effects by a local, in the sense of Ref. [16], $Z_N$’s
cyclic symmetries [17, 18]. In general, these symmetries can be anomalous but we will not
address this issue here. On the other hand, since the total lepton number is an accidental
symmetry of the minimal SM without right-handed neutrinos, there are no reasons for this
number to be conserved in extensions of the latter model, so we will also assume that the
total lepton number can be violated explicitly in some non-renormalizable interactions.

In the situation described above, if the Dirac mass term $\bar{\psi}_L H n_R$ is forbidden by the a
$Z_N$ symmetry, there is always an effective operator,

$$-\mathcal{L}_D = \frac{f_{as}}{\Lambda^{N_\phi}} \bar{\psi}_{iaL} \epsilon_{ik} H_k n_{sR} \phi^{N_\phi} + H.c.,$$

with $N_\phi \geq 1$ and $a$ is generation index, while $i, k$ refer to $SU(2)$ indices, the number of
right-handed neutrinos is $s = 1, \cdots, m$ and we have omitted summation symbols. This
term generates a Dirac mass when both $H_k$ and $\phi$ get a non-zero vacuum expectation value,
$\langle \phi \rangle \approx \Lambda_{PQ}$, $\langle H_k \rangle$ is of the order of the electroweak scale ($\sqrt{\Sigma_k \langle H_k \rangle^2} \sim 246$ GeV). Notice
that this Dirac mass from Eq. (1) is proportional to

$$\mathcal{M}_D = f \left( \frac{\Lambda_{PQ}}{\Lambda} \right)^{N_\phi} \langle H \rangle,$$

(2)
so it may be small, in the sub-eV region even if \( f \sim O(1) \), if \( \Lambda \gg \Lambda_{PQ} \) and depending also on the \( Z_N \) symmetry. However, the dimensionless matrix elements denoted by \( f, f' \) etc, may include loop suppression factors.

In general, we may have also the \( d = 5 \) interaction \([19]\), proportional to \((1/\Lambda)(\psi_{aL})^c \psi_{bL} H H\), that induces a Majorana mass for the left-handed neutrino. However, if this interaction is forbidden by \( Z_N \)'s there are effective interactions like,

\[
- L_L = \frac{f'_{ab}}{\Lambda^{N'_\phi+1}} (\psi_{ai})^c_{L} \epsilon_{ik} \psi_{bpL} \epsilon_{pl} H_k H_1 \phi^{N'_\phi} + H.c.,
\]

with \( i, k, p, l \) are \( SU(2) \) indices, that are allowed with an appropriate \( N'_\phi \geq 1 \). When \( \phi \) (may be \( \phi^* \)) and \( H \) gain a non-zero vacuum expectation value they induces a Majorana mass to the left-handed neutrinos

\[
\mathcal{M}_L = f' \left( \frac{(H)^2}{\Lambda} \right) \left( \frac{\Lambda_{PQ}}{\Lambda} \right)^{N'_\phi},
\]

which is in the sub-eV range, for a given value of \( \Lambda_{PQ} \), depending on the value of \( \Lambda \) and \( N'_\phi \) and without any fine tuning in the dimensionless \( f' \) parameters.

Next, we note that the tree level mass term proportional to \((n_{sR})^c n_{tR} \phi \) with \( s, t = 1, \ldots, m \), induces a large Majorana mass for the right-handed neutrinos, if it is allowed at the tree level \([14]\). However, if this mass term is also forbidden by the \( Z_N \) symmetries, the effective interactions with lower dimension are

\[
- L_R = \frac{f''_{st}}{\Lambda^{N''_\phi}} (n_{sR})^c n_{tR} \phi^{N''_\phi} + \frac{f''_{st}}{\Lambda^{2N_H-1}} (n_{sR})^c n_{tR} (H_1^1 H_2^{N_H}) + H.c.,
\]

will always be possible for \( N''_\phi, N_H \geq 1 \) (here also \( \phi \) may be \( \phi^* \)). The first term in Eq. \([5]\) induce a large Majorana mass if \( \Lambda = \Lambda_{PQ} \), however this is not necessarily the case if \( \Lambda > \Lambda_{PQ} \) and an appropriate \( Z_N \) is introduced (see below). The second term in Eq. \([5]\) arises because axion models need at least two electroweak doublets, say \( H_1 \) and \( H_2 \), and it also implies small Majorana masses for the sterile neutrinos, for instance when \( N_H = 1 \), and \( \Lambda \geq \Lambda_{PQ} \).

As an illustration, we consider the first term in Eq. \([5]\) which generates the mass term

\[
\mathcal{M}_R = f'' \left[ \frac{\Lambda_{PQ}}{10^{12} \text{GeV}} \left( \frac{\Lambda_{PQ}}{\Lambda} \right)^{N''_\phi} \right] 10^{21} \text{ eV}.
\]

As we said before, in Eqs. \([1]\), \([3]\) and \([5]\), \( \Lambda \) is related to the new physics implying the non-renormalizable interactions. It may be related to the PQ, grand unification (GUT) or even the Planck scale \([9, 20]\). We see from Eq. \([6]\) that, when \( \Lambda = \Lambda_{PQ} \) (or \( N''_\phi = 0 \)), the
right-handed neutrinos are necessarily heavy. We obtain $M_R = f''(10^{-4N''_\phi})10^{21} \text{ eV}$ if the $\Lambda$ is the GUT scale, i.e., $\Lambda = 10^{16} \text{ GeV}$; or $M_R = f''(10^{-7N''_\phi})10^{21} \text{ eV}$ if $\Lambda$ is the Planck scale, with $\Lambda_{PQ} = 10^{12} \text{ GeV}$ in both cases. We see that there exist a $N''_\phi$ which always produce neutrino masses of the order of eV. For instance, if we assume that $f'' \sim O(1)$, we have that $N''_\phi = 5$ (GUT scale) or $N''_\phi = 3$ (Planck scale) given in fact $M_R$ in the eV range. These value for $N''_\phi$ implies an appropriate $Z_N$ symmetry. The important point is that because of this symmetry some sterile neutrinos are light but other are heavy, depending how they transform under $Z_N$.

The general mass matrix for the neutrinos is

$$M^\nu = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}. \quad (7)$$

If there is a hierarchy like $M_L \ll M_D \ll M_R$ the eigenvalues of such a matrix are of the form $M_R$ and $-(M_L + M_D^2/M_R)$. If the Dirac mass is not suppressed with respect to the left- and right-handed Majorana mass terms, i.e., if $N_\phi \ll N'_\phi, N''_\phi$, we have that neutrinos are pseudo-Dirac particles $M_L, M_R \ll M_D$ [21]. Notice that, as we said before, since there are several sterile neutrinos, they may not have all the same $Z_N$ charge, thus some may have the interaction in Eq. (5) with $N''_\phi = 1$ and are heavy, but other get the same interactions with $N''_\phi > 1$ and are light. The exact value depend on the value of the scale $\Lambda$. Hence, some entries of the matrix $M_R$ may be large while other may be small, implementing in this way the seesaw mechanism for the active neutrinos and at the same time allowing light sterile neutrinos.

Let us considered an example of the present mechanism for generating both light and heavy sterile neutrinos in the context of a invisible axion model which is a version of the model of Ref. [22]. The representation content is, in the fermion sector, with three generations, lepton doublets $\psi$, quark doublets $Q$, the respective right-handed singlets $u_R, d_R, l_R$ and $\nu_R$. In the scalar sector, there are four scalar doublets $H_u, H_d, H_l, H_\nu$, a scalar non-hermitian triplet $T$. We avoid the scalar singlet $h^+$ and introduce a fourth right-handed neutrino, $n_{4R}$. The cyclic symmetry is $Z_{13}$ ($\omega_k = e^{2\pi i k/13}; k = 0, \cdots, 6$), but now fields transform as: $\psi \rightarrow \omega_6 \psi, l_R \rightarrow \omega_4 l_R, Q \rightarrow \omega_5 Q, u_R \rightarrow \omega_3 u_R, d_R \rightarrow \omega_5^{-1} d_R$ and $\nu_{aR} \rightarrow \omega_1 \nu_{aR}$ ($a = e, \mu, \tau$), $H_u \rightarrow \omega_{-1} H_u, H_d \rightarrow \omega_5^{-1} H_d, H_l \rightarrow \omega_2 H_l$, and $H_\nu \rightarrow \omega_6^{-1} H_\nu, T \rightarrow \omega_4^{-1} T$, $\phi \rightarrow \omega_1^{-1} \phi, n_{4R} \rightarrow \omega_4^{-1} n_{4R}$, while the gauge fields transforming trivially. Notice that in this
case it is not necessary to introduce a $Z_3$ cyclic symmetry as it was done in Ref. [22]. With these fields we have several effective interactions, the dominant are the following:

$$- \mathcal{L}_Y = \frac{f_{ab}}{\Lambda} \bar{\psi}_{aL} \nu_{bR} \bar{H}_\nu \phi + \frac{f'_{ab}}{\Lambda} \bar{(\psi_{ai})}_L \epsilon_{ik} (\psi_{b} \rho)_L \epsilon_{pl} (H_\nu)_k (H_\nu)_l + \frac{f''_{ab}}{\Lambda} (\nu_{aR})^c \nu_{bR} \phi^2$$

$$+ \frac{f''_{44}}{\Lambda^4} (n_{4R})^c n_{4R} \phi^4 + \frac{h_{4a}}{\Lambda^4} \nu_{aR} \phi^5 + H.c.,$$  

(8)

where where $\tilde{H}_\nu = \epsilon H_\nu^*$. These interactions imply

$$m_D \sim f_{ab} \frac{\Lambda_{PQ}}{\Lambda}, m_L \sim f'_{ab} \frac{\Lambda_{PQ}^2}{\Lambda}, m_R \sim f''_{ab} \frac{\Lambda_{PQ}^5}{\Lambda^4}, m_{44} \sim f''_{44} \frac{\Lambda_{PQ}^5}{\Lambda^4}, m_{4a} \sim h_{4a} \frac{\Lambda_{PQ}^5}{\Lambda^4}. $$

(9)

The values for these entries of the neutrino mass matrix depend on the actual value of $\langle H_\nu \rangle$, $\Lambda_{PQ}$ and $\Lambda$. Let us suppose, just for illustration that $\langle H_\nu \rangle = 100$ GeV, $\Lambda_{PQ} = 10^{12}$ GeV and $\Lambda = 10^{19}$ GeV. In this case we have $m_D \sim 10^4 f_{ab}$ eV, $m_L \sim 10^{-6} f'_{ab}$ eV, $m_R \sim f''_{ab} 10^5$ GeV, $m_{44} \sim 10^{-7} f''_{44}$ eV, and $m_{4a} \sim 10^{-7} h_{4a}$ eV. We see that the general matrix as in Eq. (7), in this particular case, allows a light sterile neutrino which is most $n_{4R}$ while $\nu_{aR}$ are heavy ones. If $\Lambda_{PQ} = 10^9$ GeV and $\Lambda = \Lambda_{GUT} = 10^{16}$ GeV we have: $m_D \sim 10^4 f_{ab}$ eV, $m_L \sim 10^{-5} f'_{ab}$ eV, $m_R \sim 100 f''_{ab}$ GeV, $m_{44} \sim 10^{-10} f''_{44}$ eV and $m_{4a} \sim 10^{-10} h_{4a}$ eV. The scalar Higgs sector in this model is the same (up to the singlet $h^+$) with that of Ref. [22], and there it was shown that the scalar potential with this particle content is consistent with the discrete $Z_{13}$ symmetry.

The above mechanism is also implemented in models in which the main part of the axion field is in a non-trivial representation. For instance the $SU(5)$ invisible axion model [23] in which the axion is primarily the antisymmetric part of the singlet component of a complex 24, $\Sigma$. If right-handed neutrino singlets are introduced the effective interaction

$$\mathcal{O}_{st}^{IR} \propto \frac{1}{\Lambda} (n_{sR})^c n_{tR} \Sigma^* \Sigma,$$

(10)

implies heavy right-handed sterile neutrinos. Since $\langle \Sigma \rangle$ breaks down also $SU(5) \rightarrow SU(3) \otimes SU(2) \otimes U(1)$, the unification scale is of the order of the PQ scale. It is interesting that in the axion model considered above [22], the PQ symmetry is automatic, the axion is protected from gravitational effects and unification occurs near the PQ scale having still an stable proton [24]. This is just an illustration of how the issues of grand unification, the axion and the generation of neutrino masses can be related to each other. It is interesting to note that if it does not exist any energy scale between the electroweak and the Planck scale, with only active neutrinos, it would be difficult to generate neutrino masses of the order of eV
since in this case the only possible effective operator is $(1/\Lambda_{\text{Planck}})^c \bar{\psi}_{aL} \psi_{bL} HH$ and the suppression factor is too large.

Finally, a remark is in order. Since we are considering generic invisible axion models the couplings of the axion with all fermions, but the heavy sterile neutrinos, are strongly suppressed. It means that there is no potential conflict of these mechanisms for generating neutrino masses neither with big-bang nucleosynthesis nor with the cosmic microwave background [8, 25]. However these issues deserve a more carefully analysis because such constraints are highly model dependent.

Summarizing, we have shown that an invisible axion implies light massive active neutrinos. Furthermore, if the model has also several right-handed sterile neutrinos, some of them get large and others small Majorana masses, depending on how they transform under discrete $Z_N$ symmetries. The seesaw mechanism is implemented by the heavy neutrinos and, since there may be light sterile neutrinos, some neutrinos may be pseudo-Dirac particles.

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