Effectiveness of fuzzy sliding mode control boundary layer based on uncertainty and disturbance compensator on suspension active magnetic bearing system

Van-Nam Giap and Shyh-Chour Huang

Abstract
This paper presents the disturbance and uncertainty suppression by using the nonlinear disturbance observer and an extended state observer for a nonlinear active magnetic bearing system. Otherwise, the chattering free is assured by a fuzzy controller, where the fixed sliding mode surface boundary is regulated by fuzzy boundary layer. The stability of the system is guaranteed by Lyapunov condition. First, the nonlinear disturbance observer is presented to estimate the disturbance from outside of the system. Second, the system parameter variations are estimated by an extended state observer with the construction via the estimated disturbance value. Third, the proportional–integral–derivative sliding mode surface has been constructed due to the chattering values that appear from the high-frequency switching control values. Fourth, these chattering values are reduced by using a Mamdani fuzzy logic control. The proposed control methodology was given by the MATLAB simulation. The overshoot value that is equal to zero, narrow settling time, and the average distance tracking error value which is quite small are archived.

Keywords
Nonlinear disturbance observer, extended state observer, fuzzy sliding mode control, boundary layer, chattering free

Date received: 20 September 2019; accepted: 12 January 2020

Introduction
Active magnetic bearing system (AMBS) has been applied in many areas in industrial system. AMBS offer a system with noncontact working between the stator and rotor, which leads the practical system to reduce and maintain costs, device life, and no lubrication. Taking an overview on AMBS, the fuzzy logic control and sliding mode control have been applied to control the suspension AMBS. A robust observer–based optimal linear quadratic tracker for 5-degree-of-freedom sampled data AMBS. The robust nonsingular terminal sliding mode control was applied to suspension AMBS, the proposed method ignored the outside disturbance effects analysis, the method proposed by Su et al. is very good but complicated in real practice, and the method proposed by Tsai et al. is also complicated to understand how much the lumped uncertainties are bounded. This paper reveals new control methodology; it is very easy to understand and apply in practice. Furthermore, the outside disturbance, system uncertainty, and chattering are mostly rejected.

In industrial system, physical process system contains uncertainty and affected by outside disturbance. With an AMBS, the uncertainty occurs from the gravity force, winds, unmolded system dynamic, and the parameter variation. The simple structure of the magnet poles will lead system to very sensitive with outside disturbance. The inability to build dynamic system is the requirement of the disturbance and uncertainty estimation. In recent decades, the disturbance and unknown input estimation is one of the most important topics in process control. The concepts of nonlinear disturbance observer (DOB) were revealed from the past decades. It has been developed to high-order DOB, and implemented to Euler–Lagrange system and underwater
vehicle system. In recent years, nonlinear DOB was designed for robotic manipulators and mobile wheeled inverted pendulum, and combined control of robotic system with exogenous disturbance was presented. The disturbance could be rejected and estimated by equipping the DOB and also the extended state observer. A radial basis function neural network was presented based on an extended DOB for the velocity control loop. The extended state observer has been applied to estimate the disturbance values. The extended state observer–based fuzzy sliding mode control was implemented for 3-degree-of-freedom humanoid arm. This proposed methodology used the fuzzy logic to tune the sliding mode control gain, and disturbance has been calculated by the difference of estimated system state and measured system state. The same way to construct the disturbance value was used. The disturbance and uncertainty estimation has been proposed. The combination of system state feedback and DOB was proposed for the AMBS and for mismatched disturbance system. This paper proposes the disturbance and uncertainty rejection based on extended state observer and via the nonlinear DOB, which means the disturbance needs to be defined first by an observer, and then the system state estimation is constructed with estimated DOB and the error of estimated state and measured state through the extended state formulation. The different value of measured system state and estimated system state is referred to as system parameter variation or is called as uncertainty value. The background controller is used, which is named as sliding mode control.

Sliding mode control was proposed in the middle of 1950s; its property includes the equivalent control and switching control, which forces the system to be in stable state on predefined manifold and converge on the surface, respectively. The main disadvantage problem of the sliding mode control is centered on switching control chattering, which occurred from high hitting control gain and the sliding mode boundary layer value. Many proposed methodologies were implemented to optimize the chattering value, and the boundary layer approximation problems have been revealed in the works of Igor and Chyun. These papers just focused on the effectiveness of the boundary layer thickness. Do et al. applied the nonsingular terminal sliding mode control for a nonlinear robot manipulator. Their proposed method achievements are good, but the sliding mode boundary layer thickness was ignored. This paper proposes method to approximate the sliding mode boundary layer in comparison to previous published papers, that is, a fuzzy logic control to approximate the saturation function can be used to force the system state stay on the bounded area.

The proposed methodologies are implemented to the AMBS. The organization of this paper is as follows. First, a brief introduction about this study and the overview of previous published papers are given. Second, system mathematical model is revealed. Third, proposed control methodology is presented subsequently. Fourth, the results and the data are presented. Finally, the discussion is given.

**Mathematical modeling of AMBS**

The AMBS consists of an eddy-current sensor; the sensor is used to estimate the distance of the thrust disk to the stator. Sensor input is distance value and sensor output are voltage values, where we need to convert these signals to current values by using the 0.5 A/V amplifiers: a computer with 1 multi-channel analog to digital car with 16 bits resolution and 1 multi-channel digital-to-analog car with 16 bits resolution. This research aims to control the position of the embedded thrust disk on the vertical rotor. The details are represented in Figure 1.

![Figure 1. Active magnetic bearing system.](image)

### Mathematical Model of Active Magnetic Bearing System (AMBS)

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\[
F_+ = \frac{B_+^2}{2\mu_0} \quad (1)
\]

\[
F_- = \frac{B_-^2}{2\mu_0} \quad (2)
\]

where \(B_+\) and \(B_-\) are the upper component and the lower component of magnetic density vector. The magnetic force of the Suspension active magnetic bearing system (SAMB) is modeled as

\[
F_x = F_+ - F_-
\]

\[
= K \left( \frac{(i_0 + i_x)^2}{(x_0 - x_s)^2} - \frac{(i_0 - i_x)^2}{(x_0 + x_s)^2} \right) \quad (3)
\]

Using Taylor expansion of equation (3) leads to

\[
F_x = K_{ap} x(t) - K_{ai} i(t) \quad (4)
\]

with \(K_{ap}\) and \(K_{ai}\) are the amplification factors of the rotor position and the magnet coil currents, respectively, where

\[
K_{ap} = \frac{\partial F(x, i)}{\partial x} \bigg|_{x = 0, i = 0} = 2k \left( \frac{(i_0 + i_x)^2}{(x_0 - x_s)^2} - \frac{(i_0 - i_x)^2}{(x_0 + x_s)^2} \right) = \frac{4k^2}{x^3} \quad (5)
\]
The values of the system parameters are shown in Table 1. The system parameters are presented in Table 1.

### Table 1. System parameters.

| Parameters | Description                              | Value | Unit |
|------------|------------------------------------------|-------|------|
| $m$        | The rotor mass value                     | 2.565 | kg   |
| $K_{op}$   | The coefficient of electromagnetic current| 40    | N/A  |
| $K_{ap}$   | The coefficient of electromagnetic position| 25,200| N/m  |
| $x_0$      | Nominal air gap where thrust disk is centered | 1     | mm   |
| $T$        | The thrust disk mass                     | 0.38  | kg   |
| $c$        | Damping constant                         | 0.0001| (N.s)/m|
| $v_0$      | Reference voltage                        | 1.4   | V    |
| $l_0$      | Amplifier range                          | 0.5   | A/V  |

Following the Newton II law

$$m\ddot{x}(t) = F_{xt}(t) - f_{ds}(t)$$  \hspace{1cm} (7)

where $m$ is the mass of the inside rotor and an embedded thrust disk, $F_{xt}(t)$ is the Lorentz force, and $f_{ds}(t)$ is the unexpected output disturbance. Equation (7) can be written as

$$m\ddot{x}(t) = -c\dot{x}(t) + K_{ap}x(t) + K_{ai}(t) + f_{ds}(t)$$  \hspace{1cm} (8)

or

$$\ddot{x}(t) = \frac{1}{m}(-c\dot{x}(t) + K_{ap}x(t) + K_{ai}(t) + f_{ds}(t))$$ \hspace{1cm} (9)

where $A = -c/m$, $B = K_{ap}/m$, $B = K_{ai}/m$, and $d = -f_{ds}(t)/m$. Equation (9) can be rewritten as

$$\ddot{x}(t) = Ax(t) + B\dot{x}(t) + Ci(t) + d$$ \hspace{1cm} (10)

or

$$\ddot{x}(t) = (A_\eta + \Delta A)\dot{x}(t) + (B_\eta + \Delta B)x(t) + (C_\eta + \Delta C)i(t) + d$$ \hspace{1cm} (11)

where $A_\eta$, $B_\eta$, and $C_\eta$ are known system state matrices. The values $\Delta A \subseteq [A_1, A_2]$, $\Delta B \subseteq [B_1, B_2]$, and $\Delta C \subseteq [C_1, C_2]$ are unknown parameters; they represent the uncertainty values of the system and parameter variation. The system model is represented as

$$\ddot{x}(t) = A_\eta \dot{x}(t) + B_\eta x(t) + C_\eta i(t) + L + d$$ \hspace{1cm} (12)

where $L = \Delta A\dot{x}(t) + \Delta Bx(t) + \Delta Ci(t)$ is called the lumped uncertainty value of the system, which occurs from the system parameter variation.

Equation (12) can be written as

$$\dot{X}(t) = \begin{bmatrix} 0 & I \\ B_\eta & A_\eta \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ C_\eta \end{bmatrix} i(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (d + L)$$ \hspace{1cm} (13)

where $L < L, d < \dot{d}$. $X(t) = [x(t), \dot{x}(t)]^T$, and $B = L + \dot{d}$ is a positive-constant upper boundary of the unknown uncertainty and disturbance of the system. This paper presents a generalized proportional integral DOB. The main advantages of the proposed observer are the uncertainty values, system parameter variation, and output disturbances, and all will be rejected approximately; the rejection of perturbation values is aimed to guarantee that the bound of the lump of uncertainty reaches zero in infinite time. The system parameters are shown in Table 1.

### Proposed approach

This section presents the proposed control methodologies. The first one is DOB for AMBS. Second, extended state observer is constructed via the estimated disturbance value, and the disturbance and uncertainty feedback is investigated. Finally, sliding mode control is built with a fuzzy rule for tuning the sliding surface boundary layer thickness.

#### Nonlinear DOB

The system from equation (11) is represented as

$$\dot{X}(t) = \begin{bmatrix} 0 & I \\ B & A \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ C \end{bmatrix} i(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d$$ \hspace{1cm} (14)

or

$$\dot{X}(t) = G_1X(t) + G_2i(t) + G_3d$$ \hspace{1cm} (15)

where the uncertainty is hidden in terms of system state and control matrices, the system state vector. This section presents the nonlinear DOB as

$$\begin{cases} \dot{y} = -l(x)G_3[y + p(x)] - l(x)[G_1x(t) + G_2i(t)] \\ \dot{d} = y + p(x) \end{cases}$$ \hspace{1cm} (16)

where $y$ is an internal state vector of the DOB; $p(x)$ is the auxiliary vector of DOB, which is the nonlinear function; $l(x) = \partial p(x)/\partial x$ is the observer gain; and $\dot{d}$ is the estimated value of disturbance from the system. Nonlinear DOB is used to estimate uncertain unknown disturbance under operating process. There exists a sub-function $y = H(x) \in R^m$, where $H(x)$ is smooth.
function, related to degree from disturbance \(d\) to \(p\) for all system states \(x(t)\). The \(p(x)\) and \(l(x)\) are chosen as

\[
\begin{align*}
\dot{l}(x) &= p_0 \frac{aL_\infty^{-1}p(x)}{\alpha x} \\
p(x) &= p_0 \Delta I \Gamma \dot{l}(x)
\end{align*}
\] (17)

where \(p_0\) is positive constant for tuning the bound of errors. Let \(n_0|I_\Gamma L_G \Delta I \Gamma H(x)|\) is positive scalar. The error of the estimated disturbance and disturbance is

\[
\tilde{d} = d - \hat{d}
\]

Taking the derivative of the error of estimated and real disturbance will lead to

\[
\dot{\tilde{d}} = \dot{d} - \dot{\hat{d}} = \hat{d} - \dot{\hat{d}} - \dot{\hat{d}} = \hat{d} - \dot{\hat{d}} + l(x)G_1(y + p(x) - d) = \hat{d} - \dot{\hat{d}} + l(x)G_2(\hat{d})
\]

(18)

Assumption that \(|\tilde{d}| < \beta\) where \(\beta\) is positive constant given by system lump of uncertainties. The Lyapunov function is

\[
V(\tilde{d}) = \tilde{d}^T \tilde{d}
\]

(19)

Taking the derivative of equation (17) will lead to

\[
\dot{V}(\tilde{d}) = 2\tilde{d}^T (\tilde{d} - l(x)G_2 \dot{\hat{d}})
\]

\[
= 2\tilde{d}^T \dot{\hat{d}} - 2\tilde{d}^T l(x)G_2 \dot{\hat{d}}
\]

\[
\leq -2n_0 p_0 \|\tilde{d}\|^2 + 2\|\tilde{d}\|k
\]

\[
\leq -n_0 p_0 \|\tilde{d}\|^2 - \left(\sqrt{p_0 n_0^2}\|\tilde{d}\| - \frac{k}{\sqrt{p_0 n_0}}\right)^2 + \frac{k^2}{p_0 n_0}
\]

(20)

Then, we have

\[
\|\tilde{d}(t)\|^2 \leq \|\tilde{d}(t)\| \exp(-n_0 p_0 t) + \frac{k^2}{p_0 n_0^2}
\]

(21)

The control signal is updated as

\[
I_{\text{control}} = I_c + \alpha \tilde{d}
\]

(22)

where \(I_c\) is the conventional control value. Based on the matching condition, \(\alpha\) is chosen as \(\alpha = -G_2^{-1}/G_3\), in the sense that input-to-state is stable.

**Uncertainty estimation based on extended state observer via estimated disturbance value**

The extended state observer is applied to force the uncertainty of the system to converge to the zero value. The system state is modified as

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= A_n x_2(t) + B_n x_1(t) + C_n \hat{u}(t) + L + d
\end{align*}
\]

(23)

where \(\dot{x}_1(t) = \hat{x}(t)\) and \(\dot{x}_1(t) = \ddot{x}(t)\). The state observer is constructed as

\[
\begin{align*}
\dot{\hat{x}}_1 &= \ddot{x}_2 - \chi_1(x_1(t) - \hat{x}_1(t)) \\
\dot{\hat{x}}_2 &= \ddot{\chi}_2(x_1(t) - \hat{x}_1(t)) + A_n x_2(t) + B_n x_1(t) + C_n \hat{u}(t) \\
\ddot{L} &= \chi_3(x_1(t) - \hat{x}_1(t))
\end{align*}
\]

(24)

The \(\hat{x}_1\) is an estimated value of the \(x_i\), and \(\hat{x}_i\) is an observer gain that needs to be defined. The error value between estimated signals and measured signals is defined as \(\hat{x}_1 = x_1 - \hat{x}_1, \hat{x}_2 = x_2 - \hat{x}_2\), and \(\ddot{d} = d - \ddot{d}\). The error function is

\[
\begin{align*}
\dot{\hat{x}}_1 &= \ddot{x}_2 - \chi_1(x_1(t) - \hat{x}_1(t)) \\
\dot{\hat{x}}_2 &= \ddot{\chi}_2(x_1(t) - \hat{x}_1(t)) + A_n x_2(t) + B_n x_1(t) + C_n \hat{u}(t) \\
\ddot{L} &= L(x(t) - \hat{x}_1(t))
\end{align*}
\]

(25)

The figuration of the uncertainty is considered as \(L + \ddot{d} = l(x(t))\), and the error disturbance and uncertainty are referred to as \(\ddot{d} = L + \ddot{d}\) The disturbance and uncertainty are \(\ddot{d} = L + \ddot{d}\) The characteristic of equation (12) is

\[
s^3 + \chi_1 s^2 + \chi_2 s + \chi_3 = 0
\]

(26)

The parameters of the characteristic equation are chosen as long as equation (26) is stable following Hurwitz’s rule.

**Boundary layer stability of fuzzy sliding mode control**

**Sliding mode control.** The sliding mode controls are included as an equivalent control value and a switching control value. Every system state will slide and converge on a predefined state. The sliding mode control surface needs to be defined such that the term of convergence of the sliding mode surface to zero is as fast as possible. The sliding mode surface is constructed by proportional–integral–derivative (PID) type; the sliding surface input is the different value of the estimated system position and reference signal. The sliding surface is built as

\[
s(t) = k_p \dot{e}(t) + k_i e(t) + k_d \int_0^t e(\tau) d\tau
\]

(27)

Refer \(e(t) = x_m(t) - x_r(t)\), where \(x_m(t)\) is the reference distance value and \(x_r(t)\) is the measured distance. \(k_p, k_i, k_d\) are positive-constant values, and it should be chosen such that the real parts of the roots than sliding surface are satisfied with Lyapunov law as

\[
\dot{V}(t) = \dot{s}(t)s(t) < 0
\]

(28)
\[ s(t) \dot{s}(t) = s(t) (k_d \ddot{x}_d(t) - (A_n + \Delta A)x_m(t) - (B_n + \Delta B)x_m(t) - d - (C_n + \Delta C)(I_{sw}(t) + I_{eq}(t)) + k_p \dot{e}(t) + k_i e(t)) \]
\[ = s(t) (k_d \ddot{x}_d(t) - (A_n + \Delta A)x_m(t) - (B_n + \Delta B)x_m(t) - d - (C_n + \Delta C)(I_{sw}(t) + I_{eq}(t)) + k_p \dot{e}(t) + k_i e(t)) + \frac{1}{k_d C_n} \left[ k_d \ddot{x}_d(t) - k_d (A_n \dot{x}(t) + B_n \dot{x}(t) + L + d) + k_p \dot{e}(t) + k_i e(t) + k_s e(t) \right] \]  
\[ + k_p \dot{e}(t) + k_i e(t) \]  
\[ \text{(35)} \]

or
\[ \leq |s(t)| \left[ |k_d| \left( \left| -\Delta A + C_n^{-1} A_n \Delta C \right| |x_m(t)| + \left| \Delta B + C_n^{-1} A_n \Delta B \right| |x_m(t)| \right) - |L + d - (C_n^{-1} \Delta C)(k_d |x_d(t)| + k_i |e(t)|) + k_p |e(t)| \right] - s(t) k_d(C_n + \Delta C)I_{sw}(t) < 0 \]  
\[ \text{(36)} \]

The inequality (equation (36)) is satisfied when the hitting control gain should be defined as
\[ k = (C_n + \Delta C)^{-1} \left( \left| -\Delta A + C_n^{-1} A_n \Delta C \right| |x_m(t)| + \left| \Delta B + C_n^{-1} A_n \Delta B \right| |x_m(t)| \right) \]  
\[ -|L + d - (C_n^{-1} \Delta C)(k_d |x_d(t)| + k_i |e(t)|) + k_p |e(t)| \right) \]  
\[ \text{(37)} \]

Then the inequality (equation (32)) could be written as
\[ s(t) \dot{s}(t) \leq -k |s(t)| \]  
\[ \text{(38)} \]

Chyun\textsuperscript{39} presented a variable thickness boundary layers for the sliding mode control; this paper dealt with the effects of layer boundary to the chattering value. In fact, the chattering also occurs from switching control gain. This paper reveals new ideas to deal with the chattering value. The fuzzy logic controller will be used to force the chattering value to zero.

**Boundary layer regulation–based fuzzy logic control.** Fuzzy logic is another practical mathematical addition to classic Boolean logic.\textsuperscript{31} Fuzzy logic system includes fuzzification, inference engine, and defuzzification. The fuzzification converts the input signal into the fuzzy sets, the inference engine is used to determine the input variable and fuzzy rules, and defuzzification is used to convert the fuzzy crisp of inference engine to output value. In this paper, the fuzzy rules were chosen somehow and the output of the fuzzy is satisfied by the Lyapunov condition. This paper proposes two fuzzy controllers; all fuzzy rules are formed If–Then, and the first fuzzy system is used to approximate the hitting control gain of the sliding mode control, with multi-input and single output form as
If \( x_1 \) is \( A_1 \), \ldots, \( x_n \) is \( A_n \), then \( y \) is \( B \) \hspace{1cm} (39)

where \( x = [x_1 \ldots x_n]^T \in \Sigma \subset \mathbb{R}^n \) and \( y \in \Upsilon \subset \mathbb{R} \) denote the fuzzy input and output variables. \( A_i \) and \( B \) are fuzzy sets in the \( \Sigma \) and \( \Upsilon \), respectively. The defuzzier map is

\[
y = \frac{\int \mu_A(y) \, dy}{\int \mu_B(y) \, dy}
\hspace{1cm} (40)
\]

When chattering occurred, the control value needs to be changed to adapt the chattering variation. In other words, the boundary layer needs to be changed; however, the boundary is bounded. The system state always tend to inside of boundary layer, which caused the chattering is free, to force the control signal more smooth, and vice versa. This paper proposes the control methodology as

\[
I_c = I_{eq}(t) + I_{sw}(t) - k_{sat} \left( \frac{s(t)}{\phi_f} \right)
\hspace{1cm} (41)
\]

In equation (33), the chattering occurs with a high switching frequency bandwidth when the sliding mode control outside the boundary value of the chattering is maximum. If sliding mode surface \( s \) is in \([-\phi, 0]\), the fuzzy output needs to be a small positive value; when \( s \) is in \([0, \phi]\), the fuzzy output needs to be a small negative value. If the sliding mode surface is outside \([-\varepsilon, \varepsilon]\), the fuzzy output needs logic with single input and single output, which is equipped to force the chattering value equivalent at the zero point; the fuzzy rules are as follows:

If \( s(t) \) is negative big, then \( u_f(t) \) is positive big.
If \( s(t) \) is negative medium, then \( u_f(t) \) is positive medium.
If \( s(t) \) is zero, then \( u_f(t) \) is zero.
If \( s(t) \) is positive medium, then \( u_f(t) \) is negative medium.
If \( s(t) \) is positive big, then \( u_f(t) \) is negative big.

where \( u_f(t) \) is the output signal of fuzzy system; the fuzzy membership functions are shown in Figure 2.

In Figure 2, negative big is referred to NB, negative medium is referred to NM, zero is referred to ZO, positive medium is referred to PM, and positive big is referred to PB. This paper used five fuzzy rules to approximate the chattering converge to zero.

Taking the advantage of the polygonal shape, the fuzzy membership functions were selected with a linear characteristic, easy-to-obtain input and output mapping problem, and easy-to-modify parameters of the fuzzy membership function. The empirically chosen fuzzy rules need to be fulfilled in the stability condition. The fuzzy input membership function is in polygonal shape, and fuzzy output membership function is single value function. The combination of the disturbance and uncertainty feedback, the boundary layer regulation, and the system control are constructed, as shown in Figure 3 and Table 2.

![Figure 2. Fuzzy rules: (a) sliding mode surface and (b) fuzzy output signal.](image)

| Parameter | Description | Value |
|-----------|-------------|-------|
| \( k_p \) | Proportional coefficient | 500 |
| \( k_i \) | Integral coefficient | 3000 |
| \( k_d \) | Derivative coefficient | 1 |
| \( k \) | Positive gain | 100 |
| \( \varepsilon \) | Saturation coefficient | 0.8 |
| \( \delta_0 \) | Disturbance coefficient | 30 |
| \( \delta_1 \) | Disturbance coefficient | 3000 |
| \( x_1 \) | Extended state coefficient | 140 |
| \( x_2 \) | Extended state coefficient | 1600 |
| \( x_3 \) | Extended state coefficient | 1 |

![Figure 3. Control system for active magnetic bearing system.](image)

The control methodology is constructed as shown in Figure 3; the sliding mode control input is the different signal between the measured output and desired signal. Disturbance and feedback are estimated by a nonlinear DOB.
The originality of this paper is the lump of uncertainty estimation; there, the extended state observer is built based on the observed disturbance value.

The difference of the estimated and measured states is considered as the uncertainty value; this value is considered as feedback to compensate the system parameter variation.

After all, the control current is

\[
I_c(t) = \frac{1}{k_d C_n} \left[ k_d \dot{x}_d(t) - k_d (A_n \dot{x}(t) + B_n x(t) + \hat{L} + \hat{d} + \hat{u}) + \tilde{u} \right]
\]

The Lyapunov condition is now represented as

\[
s_{\text{new}}(t) \dot{s}_{\text{new}}(t) \leq -k |s_{\text{new}}(t)| + \hat{L} + \hat{d} + \tilde{u}
\]

Figure 4. Output signals: (a) distance response signals in first 20 s, (b) distance response signals in the first 0.05 s, (c) distance response signals at the point disturbance affected on the system, (d) distance tracking error values at first 20 s, (e) sliding mode surfaces, (f) disturbance and uncertainty values, and (g) estimated and measured system states.
or

\[ s_{new}(t) \leq |s_{new}(t)| (k - \overline{L} - \overline{d} - \overline{u}) \quad (44) \]

The Lyapunov is satisfied when \( k > \overline{L} + \overline{d} + \overline{u} \) following the condition given in equations (22) and (25), and when changing small control signal, equation (44) is satisfied with the chosen hitting control gain.

### An illustrative example

This part investigates about the proposed control methodology achievements. All of the results are focused on three cases as follows:

**Case 1.** The sliding mode control for AMBS

The control value is

\[ I_c(t) = \frac{1}{k_d C_n} [k_d \dot{x}_d(t) - k_d (A_n \dot{x}(t) + B_n x(t) + L + d) + k_p \epsilon(t) + k_c (s(t)) + k_{sat} \left( \frac{s(t)}{\phi} \right)] \quad (45) \]

This control methodology cannot guarantee the disturbance and uncertainty, and chattering will be rejected.

**Case 2.** The disturbance and uncertainty rejection–based observers for AMBS

The control signal is

\[ I_c(t) = \frac{1}{k_d C_n} [k_d \dot{x}_d(t) - k_d (A_n \dot{x}(t) + B_n x(t) + L + d + \overline{d}) + k_p \epsilon(t) + k_c (s(t)) + k_{sat} \left( \frac{s(t)}{\phi} \right)] \quad (46) \]

The disturbance and uncertainty are now free.

**Case 3.** Boundary layer regulation–based fuzzy and sliding mode and disturbance and uncertainty rejection for AMBS

The control signal is presented in equation (42). The proposed method achievements are shown in Figure 4.

In comparison with each methodology for AMBS, the proposed control methodology with a fuzzy logic control to regulate the sliding mode boundary layer thickness, disturbance, and uncertainty-based observers is performing good that the settling time is small, overshoot is approximated to zero, and distance tracking error values are quite small. The comparison data are described in Table 3.

### Conclusion

A designed methodology is named effectiveness of fuzzy sliding mode control boundary layer based on disturbance, and uncertainty observers were constructed to control suspension AMBS successfully. A PID sliding surface is utilized to construct the sliding mode control surface, and the fuzzy logic control is used to regulate the layer thickness of the sliding mode controller. All models are built with an effort to keep the highly nonlinear bearing system stable. In some way, the Lyapunov is guaranteed for system stability.

A nonlinear DOB is equipped to estimate the unknown outside disturbance, and unknown parameter variation is estimated by an extended state observer. The given outside testing disturbance is almost rejected completely. The archived results are figured out that proposed methodology is good at tracking the dynamic input signal, the proposed controller can reject the output disturbance and the variation of the parameters, and chattering is almost free. The main advantages are as follows: the settling time of the proposed controller is very less, overshoot value is small, and the average distance tracking error value is quite small.

### Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

### Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

### ORCID iDs

Van-Nam Giap [https://orcid.org/0000-0002-6230-278X](https://orcid.org/0000-0002-6230-278X)
Shyh-Chour Huang [https://orcid.org/0000-0002-7158-3720](https://orcid.org/0000-0002-7158-3720)

### Table 3. Proposed controllers output value.

| Values                     | Case 1            | Case 2            | Case 3            |
|----------------------------|-------------------|-------------------|-------------------|
| Overshoot                  | 5.1475 μm        | 4.6805 μm        | 2.2246 μm        |
| Distance tracking error    | 5.7062 μm        | 5.7062 μm        | 0.4418 μm        |
| Settling time              | 0.0419 s         | 0.0423 s         | 0.0126 s         |
| Maximum tracking error     | 11.4125 μm       | 11.4125 μm       | 2.2245 μm        |
| DUE rejection              | No                | Yes               | Yes               |
| Chattering rejection       | No                | No                | Yes               |

| Overlap |
|---------|
| 2023-10-01 |

In comparison with each methodology for AMBS, the proposed control methodology with a fuzzy logic control to regulate the sliding mode boundary layer thickness, disturbance, and uncertainty-based observers is performing good that the settling time is small, overshoot is approximated to zero, and distance tracking error values are quite small. The comparison data are described in Table 3.
References

1. Schweitzer G and Maslen EH. Magnetic bearings: theory, design, and application to rotating machinery. Dordrecht: Springer, 2009.

2. Su TJ, Li TY, Tsou TY, et al. Proportional–integral–derivative/fuzzy sliding mode control for suspension of active magnetic bearing system. Adv Mech Eng. Epub ahead of print 14 December 2017. DOI: 10.1177/168781401736654.

3. Tsai JSH, Su TJ, Cheng JC, et al. Robust observer-based optimal linear quadratic tracker for five-degree-of-freedom sampled-data active magnetic bearing system. Inter Jour Sys Sci 2018; 49(6): 1273–1299.

4. Chen Y and Lin FJ. Robust non-singular terminal sliding mode control for nonlinear magnetic bearing system. IEEE T Contr Syst Tech 2011; 19(3): 636–643.

5. Ohishi K, Ohishi K and Miyachi K. Torque-speed regulation of dc motor based on toad torque estimation. Pro IEEE IPEC Conf Jap 1983; 2: 1209–1218.

6. Chen HW. Disturbance observer based control for nonlinear systems. IEEE ASME T Mech 2004; 9(4): 706–710.

7. Kim KS, Rew KH and Kim S. Disturbance observer for estimating higher order disturbances in time series expansion. IEEE T Automat Contr 2010; 55(8): 1905–1911.

8. Mohammadi A, Marquez HJ and Tavakoli M. Nonlinear disturbance observers design and applications to Euler-Lagrange systems. IEEE Cont Syst Magazine 2017; 37(4): 50–72.

9. Liu S, Liu Y and Wang N. Nonlinear disturbance observer-based backstepping finite-time sliding mode tracking control of underwater vehicles with system uncertainties and external disturbances. Nonlinear Dyn 2016; 88(1): 465–476.

10. Mohammadi A, Tavakoli M, Marquez HJ, et al. Nonlinear disturbance observer design for robotic manipulators. Control Eng Pract 2013; 21(3): 253–267.

11. Huang J, Ri S, Liu L, et al. Nonlinear disturbance observer-based dynamic surface control of mobile wheeled inverted pendulum. IEEE T Control Syst Tech 2015; 23(6): 2400–2407.

12. Liu X and Liu DD. Composite control of nonlinear robotic system with exogenous disturbance. IEEE Access 2019; 7: 19564–19571.

13. Li MG, Shi WZ, Wei JH, et al. Parallel velocity control of an electro-hydraulic actuator with dual disturbance observers. IEEE Access 2019; 7: 56631–56641.

14. Wang YL, Shi RJ and Wang HB. ESO-based fuzzy sliding-mode control for a 3-DOF serial-parallel hybrid humanoid arm. Conr Sci and Eng 2014; 2014: 15.

15. Wang JX, Li SH, Yang J, et al. Extended state observer-based sliding mode control for PWM-based DC-DC buck power converter systems with mismatched disturbances. IET Contr Theo Appl 2014; 9(4): 579–586.

16. Shi SL, Li JX and Fang YM. Extended-state-observer-based chattering free sliding mode control for nonlinear systems with mismatched disturbance. IEEE Access 2018; 6: 22952–22957.

17. Ren C, Li XH, Yang XB, et al. Extended state observer-based sliding mode control of an omnidirectional mobile robot with friction compensation. IEEE T Ind Electron 2019; 66(12): 9480–9489.

18. You SS, Gil J and Kim W. Extended state observer based robust position tracking control for DC motor with external disturbance and system uncertainties. Electric Eng Tech 2019; 14(4): 1637–1646.

19. Shang W, Tang SJ, Guo J, et al. Robust sliding mode control with ESO for dual-control missile. Syst Eng Electron 2016; 27(5): 1073–1082.

20. Stobart RK, Kuperman A and Zhong QC. Uncertainty and disturbance estimator based control for uncertain LTI SISO systems with state delays. J Dyn Syst Meas Contr 2011; 133(2): 024502.

21. Chang JL and Wu TC. Disturbance observer based output feedback controller design for systems with mismatched disturbance. Inter J Cont Automat Syst 2018; 16(4): 1775–1782.

22. Utkin V. Variable structure systems with sliding modes. IEEE T Aut Contr 1997; 22(2): 212–222.

23. Ablay G. Variable structure controllers for unstable processes. J Pro Contr 2015; 32: 10–15.

24. Yoo BK and Ham WC. Adaptive fuzzy sliding mode control of nonlinear system. IEEE T Fuz Syst 1998; 6(2): 315–321.

25. Wai RJ. Fuzzy sliding-mode control using adaptive tuning technique. IEEE T Ind Electron 2007; 54(1): 586–594.

26. Fang JW, Zhang LF, Long ZL, et al. Fuzzy adaptive sliding mode control for the precision position of piezo-actuated nano positioning stage. Inter J Pre Engi Manu 2018; 19(10): 1447–1456.

27. Saghatfania A, Ping HW and Uddin MN. Fuzzy sliding mode control based on boundary layer theory for chattering-free and robust induction motor drive. Inter J Adv Manu Tech 2014; 41(1–4): 57–68.

28. Igor MB. Chattering in sliding mode control systems with boundary layer approximation of discontinuous control. Inter J Syst Sci 2013; 44(6): 1126–1133.

29. Chyun CF. Variable thickness boundary layers for sliding mode control. J Mar Sci Tech 2008; 16(4): 288–294.

30. Duc TM, Hoa NV and Dao TP. Adaptive fuzzy fractional-order nonsingular terminal sliding mode control for a class of second-order nonlinear systems. J Compt Nonlinear Dyn 2018; 13(3): 031004.

31. Ross TJ. Fuzzy logic with engineering applications. 3rd ed. Chichester: John Wiley & Sons, 2010.