We report that the hypothesis that the upper bound on the axion decay constant can be moved up beyond $10^{12}$ GeV in models with a stronger QCD in the early universe is not realized. This proof is possible by studying the superpotential in the dual model and obtaining the form of the axion potential respecting the original global symmetries.

I. Introduction

So far it is known that the cosmological upper bound of the axion decay constant

$$F_a \approx 10^{12} \text{ GeV}$$

does not allow the Peccei-Quinn symmetry breaking scale at the grand unification or string scale. Ever since the existence of this upper bound is known, the axion model encounter either it is unattractive because it cannot accomodate the GUT scale or it predicts an intermediate scale. Furthermore in string models with dilaton $S$, the axion scale is expected to be of order Planck scale. With supersymmetry, the QCD coupling at the string scale is given by the vacuum expectation value of the dilaton field $S$, viz.

$$\langle S \rangle$$

where $H$ is the Hubble parameter and $\theta_{\text{eff}}$ is the low energy value of the QCD vacuum angle.

In this talk, we restrict the discussion for the case

$$\Lambda_{\text{QCD}} \gg m_{\text{soft}} \sim 100 \text{ GeV}. \quad (5)$$

The axion potential in the low energy QCD is obtained under the environment of

$$m_q \sim 5 - 10 \text{ MeV}$$,
$$\Lambda_{\text{QCD}} \sim 150 \text{ MeV}$$
$$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle \sim O(300 \text{ MeV}). \quad (6)$$

In supersymmetric QCD (SQCD) at high energy, we have to check whether what of these are modified. Of course, we anticipate an environment of $\Lambda_{\text{SQCD}}$ close to $M_P = 2.44 \times 10^{18}$ GeV. What about $m_q$? The value of $m_q$ is related to vacuum expectation value of Higgs doublet fields, $H_u$ and $H_d$, which can be of Planck scale in chaotic inflationary scenario. Another relevant questions are, “Do quarks and gluinos condense?” and “What is the height of the potential $V$”

II. Strong SQCD

In this section, we obtain the axion potential in SQCD. The supersymmetric standard model has $N_c = 3$ and $N_f = 6$. The superpotential is given by
\[ W = \lambda_u H_u Q u^c + \lambda_d H_d Q d^c. \] (7)

It is known that for vanishing \( W \) the quantum moduli space of degenerate vacua for SQCD (\( N_c = 3, N_f = 6 \)) is the same as the classical one [1]. This vacuum degeneracy is lifted by the superpotential. To study the effect of superpotential, we note the duality of

\[ SU(N_c) \text{ with } N_f \text{ flavors} \]
\[ \text{dual} \]
\[ SU(N_f - N_c) \text{ with } N_f \text{ dual quarks}. \]

The superpotential of the dual model contains the following superpotential

\[ W_D = \Lambda_{\text{QCD}} (\lambda_u H_u T_u + \lambda_d H_d T_d) + T_u Q_D u_D^c + T_d Q_D d_D^c \] (8)

where subscripts \( D \) denote the dual and

\[ T_u = \frac{Q u^c}{\Lambda_{\text{QCD}}}, \quad T_d = \frac{Q d^c}{\Lambda_{\text{QCD}}} \] (9)

denote composite meson fields made of squarks. Then the soft terms in the Lagrangian contains

\[ -\mathcal{L}_{\text{soft}}^{(D)} = A W_D + \sum_i m_i^2 |\phi_i|^2 \] (10)

from which we note that \( H_u, H_d, \{\lambda_u T_u, \lambda_d T_d\} \) have masses of order \( m^2 \) which is of order \( \Lambda_{\text{QCD}}^2 \).

We also note that the original squarks with positive mass squared have positive mass squared in the dual theory. Since mesons are bound states of original squarks, they do not have vacuum expectation values. Dual squarks can be obtained by dissociating the scalar baryons (containing \( N_c \) original squarks) into \( (N_f - N_c) \) pieces; dual squarks do not have vacuum expectation values. Therefore, we conclude

\[ \langle \bar{q} q^c \rangle \propto \langle T \rangle = 0 \]
\[ \langle q q^c \rangle \propto \langle F_T \rangle = 0 \]
\[ \langle \lambda \lambda \rangle \propto \langle T \rangle^{N_f/(N_f - N_c)} = 0. \] (11)

Therefore, a possible complication, if these condensates are present, is absent. Since we are convinced that the condensates are not present which is manifest in the dual theory as we have shown above, we go back to the original theory.

### III. Axion Potential in MSSM

In this talk, we concentrate on the minimal supersymmetric standard model (MSSM). To obtain the axion potential we include the \( \mu \) term also,

\[ W \rightarrow W + \mu H_u H_d \] (12)

from which we obtain the soft terms

\[ -\mathcal{L}_{\text{soft}} = \frac{1}{2} m_1 \lambda \lambda + A(\lambda_u H_u \bar{Q} u^c + \lambda_d H_d \bar{Q} d^c) + B \mu H_u H_d + \frac{1}{2} \sum_i m_i^2 |\phi_i|^2 + \text{h.c.} \] (13)

The theory has the following global symmetry

\[ G_{\text{MSSM}} = SU(3)_Q \times SU(3)_{u^c} \times SU(3)_{d^c} \times U(1)_A \times U(1)_X \times U(1)_R \] (14)

if we assign appropriate transformation properties for the coupling parameters. For the global non-abelian transformation under \( SU(3)_Q \times SU(3)_{u^c} \times SU(3)_{d^c} \), \( \lambda \)'s transform as

\[ \lambda_u \sim (\bar{3}, 3, 1), \quad \lambda_d \sim (\bar{3}, 1, 3). \] (15)

To introduce soft supersymmetry breaking systematically, let us introduce spurion superfields

\[ \eta = (1 + m_t^2 \theta^2) \]
\[ Y = (1 + 16 \pi^2 m_{1/2}^2 \theta^2) \tau \]
\[ Z_{u,d} = (1 + A \theta^2) \lambda_{u,d} \]
\[ Z_\mu = (1 + B \theta^2) \mu \]

where \( \tau = (8 \pi^2 / g^2) + i \theta_{\text{QCD}} \). From the spurion superfields, both supersymmetric couplings and soft terms are given systematically. To have the \( G_{\text{MSSM}} \) symmetry the \( U(1) \) global charges of the coupling parameters are assigned as given in Table 1.

| \( U(1)_A \) | \( U(1)_X \) | \( U(1)_R \) |
|------------|------------|------------|
| \( Q \)    | 1          | 0          | 1          |
| \( u^c, d^c\) | 1          | -1         | 1          |
| \( H_u, H_d \) | 0          | 1          | 0          |
| \( e^-, \nu \) | 12         | -6         | 6          |
| \( Z_{u,d} \) | -2         | 0          | 0          |
| \( Z_\mu \) | 0          | -2         | 2          |
| \( d^\theta \) | 0          | 0          | -2         |

**TABLE I:** Quantum numbers of superfields and spurions in MSSM.
Let us introduce a superfield $A \equiv \frac{1}{F_a}(s + ia + a\theta + f_a\theta^2)$ (17)

which is interpreted as a fluctuation of $Y$, i.e. under axion shift

$$Y \rightarrow Y + A.$$  (18)

The effective Lagrangian of spurions is obtained from

$$\int d^2\theta d^2\bar{\theta} K_{\text{eff}}(Y, Y^*, Z, Z^*, \eta)$$  
$$+ \int d^2\theta W_{\text{eff}}(Y, Z) + h.c.  \quad (19)$$

For $\langle\lambda \eta\rangle \sim e^{-\theta_0/2}/N$, then $e^{-Y/N}$ is present in the effective Lagrangian. But we have shown that the unique ground state which preserves the chiral symmetries has no branch cut. Thus instantons would induce a term of the form

$$e^{-nY}\omega(Z) \quad (20)$$

in $W_{\text{eff}}$. But selection rules of $G_{\text{MSSM}}$ does not allow any holomorphic $\omega$ which is finite at $\mu \rightarrow 0$. Therefore, we conclude that the axion potential arises from the first term of Eq. (19), i.e. from the effective Kähler function $K_{\text{eff}}$.

For $n = 1$,

$$K_{\text{eff}} \propto e^{-Y}\text{Det}(Z_uZ_d)Z_u^*F(\eta) + h.c. \quad (21)$$

where $F(\eta)$ is an arbitrary function of $\eta$. Note that $K_{\text{eff}}$ is invariant under $U(1)_A \times U(1)_X \times U(1)_B$. To obtain the axion potential, one must insert $D_\eta$, or $F_YZ$ and $F_Y^*Z$ insertions.

From Eq. (21), we represent the order of soft parameters as

$$[m_{\text{soft}}]^2 = \{m_1^2, AB^*, 16\pi^2m_{1/2}B^*\}. \quad (22)$$

Since instantons give dominant contributions for $\rho \sim \Lambda_{\text{QCD}}^{-1}$, the axion potential is estimated as

$$V_a \approx e^{ia/F_a} \left(\frac{1}{16\pi^2}\right)^6 \mu^3\text{Det}(\lambda_u\lambda_d)$$  
$$[m_{\text{soft}}]^2\Lambda_{\text{QCD}}^{-1} + h.c. \quad (23)$$

A careful study of this axion potential is given in Ref. [1]. Actually, one instanton diagram giving Eq. (23) can be found as shown in Fig. 1. Twelve quark lines and six gaugino lines (corresponding to twice the index of adjoint representation of $SU(3)_{\text{color}}$) are coming out from the instanton vertex. Quark lines have Yukawa couplings to $H_{u,d}$ and gaugino and quark lines have gauge couplings to squarks. Two gauginos have soft gaugino mass coupling, two Higgs doublets have the $B\mu$ term coupling, and two squarks and a Higgs doublet have the $A$ term coupling: thus leading to Eq. (23). In Eq. (23), only $[m_{\text{soft}}]^2$ is given, but from Fig. 1 we can see the explicit dependence. In Fig. 1, the $\mu^*\lambda_{u,d}$ vertices arise from $|\partial W/\partial H_{u,d}|^2$ term. Thus from Fig. 1, we obtain

$$(\theta_{\text{eff}})_{\text{MSSM}} = \theta_{\text{QCD}} + \text{Arg(Det}\lambda_u\lambda_d)$$
$$+ 3\text{Arg}m_{1/2} - 3\text{Arg}(\mu B) \quad (24)$$

where $\theta_{\text{QCD}} = \langle a \rangle/F_a$.

**FIG. 1.** Instanton graph for the axion potential Eq. (22) of the MSSM. The solid lines with and without waves around the instanton denote the gluino and the quark modes, respectively, while dotted lines are Higgs and squark fields. The dark blobs represent the insertions of complex couplings.

### IV. In Early Universe

The early universe values of couplings

$$\lambda_{u \in}, \lambda_{d \in}, \mu_{\in}, A_{\in}, B_{\in}, m_{1/2 \in}$$  (25)

and present values of couplings

$$\lambda_{u \text{ eff}}, \lambda_{d \text{ eff}}, \mu_{\text{eff}}, A_{\text{eff}}, B_{\text{eff}}, m_{1/2 \text{ eff}}$$  (26)

should be almost the same for the conditions (3) and (4) to be satisfied. Then we expect

$$\theta_{\text{eff}} \sim \theta_{\text{in}}.$$  (27)

Referring Eq. (24), we define $\delta \theta$ as the value of $\theta$ except $\langle a \rangle/F_a$. If Eqs. (3) and (4) are satisfied, the strong QCD in the early universe determines the
present value $\theta_{\text{eff}}$ at almost 0 and then the axion energy crisis does not occur even if $F_a \sim M_P$.

Certainly $\theta_{\text{in}}$ is the value where $V_a$ is the minimum. If $\theta_{\text{in}}$ is also the value where $V_a$ is the minimum, then there is a possibility that Eq. (27) is satisfied. However, for this to happen the axion potential must be steep enough so that the minimum of the axion potential is quickly achieved in the early universe, i.e. $(\langle a \rangle / F_a)_{\text{in}} = 0$ and $\delta \theta \sim 0$. However, the axion potential is sufficiently suppressed by $m_{\text{soft}}$ and hence $\theta_{\text{in}}$ does not reach $\theta_{\text{eff}}$ in the early universe.

Our question is, if $\Lambda_{\text{QCD}} \gg m_{\text{soft}}$, then can $F_a \gg 4 \times 10^{12}$ GeV? For $a$ to roll down the hill in the early universe,

$$m_a \geq H$$

where $H$ is the Hubble parameter in the early universe. As we have seen above, the axion potential is sufficiently suppressed in MSSM

$$\mu^{3/2} [m_{\text{soft}}]^2.$$  

To raise $F_a$, we need

$$\delta \theta_{\text{in}} = \frac{\langle a \rangle}{F_a} - \theta_{\text{eff}} \leq 10^{-2} - 10^{-3}.$$  

We estimate

$$\left( \frac{m_a}{M_m} \right)_{\text{MSSM}} \approx 10^{-11} C^{-1/2} \left( \frac{4 \times 10^{12}}{F_a} \right) \left( \frac{M_m}{10^5} \right)^{3/2} \left( \frac{10^3}{\Lambda_{\text{QCD}}} \right)^{1/2}$$

where $M_m$ is the messenger scale for the supersymmetry breaking and every mass is in GeV units. Therefore, we conclude $m_a \ll H$. An extension of the MSSM to the next minimal supersymmetric standard model (NMSSM) does not improve the result.

V. Conclusion

We have shown that the strong QCD in the early universe cannot raise the cosmological upper bound on the axion decay constant $F_a$ in the MSSM and NMSSM. We also have shown the method to calculate the form of the axion potential in supersymmetric models from the symmetry argument.

Acknowledgments

This work is supported in part by KOSEF through CTP of Seoul National University, Ministry of Education BSRI-96-2418, and SNU-Nagoya Collaboration Program of Korea Research Foundation.

References

1. K. Choi, H. B. Kim and J. E. Kim, preprint hep-ph/9606372.
2. J. Preskill, M. B. Wise and F. Wilczek, Phys. Lett. B 120, 127 (1983); M. Dine and W. Fischler, Phys. Lett. B 120, 133 (1983); L. F. Abbott and P. Sikivie, Phys. Lett. B 120, 137 (1983).
3. G. Dvali, preprint IFUP-TH 21/95.
4. For reviews, see, J. E. Kim, Phys. Rep. 150, 1 (1987); H.-Y. Cheng, ibid 158, 1 (1987); R. D. Peccei, in CP Violation, ed. C. Jarlskog (World Scientific, Singapore, 1989). For recent pedagogical reviews, see, J. E. Kim, J. Kor. Phys. Soc. 29, S167 (1996); R. D. Peccei, ibid 29, S199 (1996).
5. K. Intriligator and N. Seiberg, preprint hep-ph/9509066.