THE RESONANCE THEORY OF PROTON AND ALPHA DECAY FROM HOT SOURCES

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Abstract

The consecutive microscopic solution is presented of the problem of tunneling of a particle through a potential barrier. The method is applied to the $\alpha$- and proton decay of compound systems formed in fusion reaction. Appearance of the peaks in the spectrum of emitted particles is predicted. The peaks correspond to quasistationary states inside the potential barrier.
1 Introduction

Prefission emission of alphas and light charged particles provides with a source of information about the time scale in the fusion-fission reaction. On the other side, theoretical description of the processes is usually made in terms of the inverse cross-section. Such an approach assuming time reversibility of the process leaves out of the scope a possibility of its profound experimental check. This is in contrast with a number of indications that violations of the reversibility may arise due to back-transparency of the inner slope of the potential barrier in the ingoing channel [1], or different response of the nuclear surface on the interaction with the emitted and the same incoming particle, or due to temperature effects on the barrier distribution [2], as in the ingoing channel experimental fit of the optical model parameters is only possible for cold nuclei [3].

Moreover, traditional decay theory deals with tunnelling through a barrier of a particle which is in the quasistationary state. This does not involve important cases when a virtual proton or cluster is between the quasistationary states, as in alpha decay from compound systems formed in fusion reaction.

Our approach allows one to calculate the decay width at any energy of the emitted particle. Strong resonance effects are, specifically, predicted in alpha spectra from compound systems produced in heavy-ion collisions.

2 Formalism

2.1 \( \alpha \) decay widths.

Let \( \Psi(r_1, \ldots, r_A) \) be a wave function of the source nucleus with the mass number \( A \). First, it can be expressed in terms of the channel wave-function basis, as products of the wave-function of the daughter nucleus \( \varphi_n(r_1, \ldots, r_{A-4}) \) and the \( \alpha \)-particle w.f. \( \chi_k^{(L)}(r_{A-3}, \ldots, r_A) \):

\[
\Psi = L_0 \sum_{L=0} \sum_{nk} C_{nkL} \varphi_n(r_1, \ldots, r_{A-4}) \chi_k^{(L)}(r_{A-3}, \ldots, r_A) \equiv \\
\equiv L_0 \sum_{L=0} \sum_{nk} C_{nkL} \varphi_n(r_1, \ldots, r_{A-4}) \eta_L^{(f)}(R; r_{A-3}, \ldots, r_A),
\]

(1)

where we selected the angular momentum \( L \) of the relative motion of the \( \alpha \) particle in the nucleus. \( \eta_L^{(f)} \) may be treated as a wave-function of the relative motion of the \( \alpha \) cluster in the mother nucleus which evidently turns out to depend on the relative coordinate \( R \). In simple cases of pure configuration the expansion coefficients \( C_{nkL} \) are reduced to genealogical coefficients.

Let then the nucleus make a transition \( i \rightarrow f \). As a result, in the exit channel we observe the system in a state which is described by a wavefunction as a superposition of the plane wave and ingoing spherical wave \( \Psi \) at large \( \alpha \)-nucleus distances \( R \):

\[
\psi_{f\alpha}(r_1, \ldots, r_A) \sim \varphi_f(r_1, \ldots, r_{A-4}) g_p(r_{A-3}, \ldots, r_A),
\]

(2)

\( R \rightarrow \infty \)
In eq. (3), \( g_p \) is the channel wave-function, which is the eigen function of the \( \alpha \)-nucleus Hamiltonian with an appropriate mean-field single particle potential \( U_\alpha(R) \):

\[
(H - \varepsilon_p)F_p = 0 ,
\]

\[
\varepsilon_p = p^2 / 2 M_\alpha .
\]

Furthermore, taking into account the asymptotics (3), the wave function \( F_p(R) \) can be expressed in terms of the spherical harmonics in a usual way:

\[
F_p(R) = \sum_{\ell=0}^{\infty} i^\ell (2\ell + 1)e^{i\theta_\ell} Y_{\ell m}(\theta, \varphi) .
\]

To find a transition amplitude, one has to change to the coordinate system of the exit channel \( |f_p\rangle \). The transformation of eq. (1) then conventionally reads as

\[
\Psi = \sum_p \langle \psi_{fp}|\Psi \rangle \psi_{fp} ,
\]

the re-expansion coefficients giving the transition amplitude under consideration. This way is similar to that found by Migdal when solving his classical problem of shake of an atom in \( \beta \) decay [4]. Substituting eqs. (1) together with (2), (3) and (6) into eq. (7), we arrive at the following expression for the transition amplitude:

\[
M_{fp} = \sum_{\ell=0}^{L_0} C_{f\ell} i^\ell e^{i\theta_\ell} Y_{\ell m}(\theta, \varphi) \times
\]

\[
\times \langle R_p(R) Y_{\ell m}(\theta, \varphi) \xi(r_{A-3} - R_1, \ldots, r_A - R) | \eta_{f\ell}(r_{A-3}, \ldots, r_A) \rangle \equiv \sum_{\ell=0}^{L_0} C_{f\ell} \langle p \xi | f \xi \rangle i^\ell e^{i\theta_\ell} Y_{\ell m}(\theta, \varphi) .
\]

Taking into account that the wave functions \( F_p \) are normalized at 1 particle in a unit volume, with the flux \( v \equiv p / M_\alpha \), we obtain from (8) the following expression for the decay probability per a unit time:

\[
\Gamma_p \equiv \frac{d^3W}{d^3p} = |M_{fp}|^2 v .
\]

Inserting \( M_{fp} \) from eq. (8) into eq. (9) and integrating over all the angles of emission within \( 4\pi \), we arrive at the following final expression for the decay width:

\[
\Gamma_\alpha \equiv \frac{dN}{d\varepsilon_\alpha} = 4\pi M_\alpha^2 v \sum_{k=0}^{L_0} \sum_{\ell=0}^{L_0} |C_{f\ell}|^2 |\langle p | f \rangle_\ell|^2 .
\]
3 Method of numerical solution. Eigenvalues

$\alpha$-nucleus potential is characterized by a Coulomb barrier, which is high enough, to form quasibound states inside the barrier (Fig. 1).

These would be usual eigenstates, if the barrier were infinitely broad. The values, however, go over the resonances on the continuum background, whenever the penetrability of the barrier is taken into account. Coupling to the continuum causes the energy shift and broadening of the eigenstates. Affected eigenvalues can be determined as follows.

The Schrödinger equation for an $\alpha$-particle in the field of a nucleus reads as follows:

$$\left\{ -\frac{1}{2m} \left[ \frac{d^2}{dr^2} - \frac{L(L+1)}{r^2} \right] + V(r) \right\} \Psi = E \Psi , \quad (11)$$

with the potential

$$V(r) = V_{SW}(r) + V_C(r), \quad (12)$$

$$V_{SW} = -\frac{V_0}{1 + \exp \left( \frac{r - c}{a} \right) }, \quad (13)$$

and the Coulomb potential was taken into account as due to the sharp-edge charge distribution:

$$V_C(r) = \begin{cases} \frac{\alpha z}{2R_0} \left[ 3 - \left( \frac{r}{R_0} \right)^2 \right] & \text{for } r < R_0, \\ \frac{\alpha z}{r} & \text{for } r \geq R_0, \end{cases} \quad (14)$$

with $R_0$ being the nuclear radius.

Figure 1: $\alpha$-nucleus potential for the system of $^{131}$La + $\alpha$ with the zero angular momentum.
On the radius segment between the origin \( r = 0 \) and the first turning point \( R_{c1} \): 
\[ 0 < r \leq R_{c1}, \]  
eq. (1) was integrated numerically with the initial condition
\[ \Psi(r) \sim r^L, \quad r \to 0. \]  
(15)

General solution of the Schrödinger equation under the barrier is a linear combination of the two linearly independent solutions. One of them exponentially vanishes, and the other exponentially increases with increasing \( R \). The coefficients can be obtained by sewing the functions at the internal turning point. In principle, the eigenvalues may be obtained from a condition that the coefficient for the exponentially increasing solution vanishes. In the first approximation, this can be achieved by sewing to the Airi function \([4]\). Actually, the eigen solutions were obtained by numerical integration from the external turning point \( R_{c2} \) towards the internal one, with somewhat an arbitrary initial condition
\[ y(R_{c2}) = 1, \quad y'(R_{c2}) = -0.25. \]  
(16)

The derivative in eq. (6) is negative, as the solution is assumed to exponentially decrease under the barrier. In the course of integration, only the right solution survives, which exponentially increases with decreasing \( R \) under the barrier, the other exponentially vanishes, in so far that the eigenvalue obtained practically very weakly depends on the concrete numbers in eq. (16).

In general case, the fundamental set was obtained by numerical integration from \( R_{c2} \) to \( R_{c1} \) with two different initial conditions:
\[ y(R_{c2}) = 1, \quad y'(R_{c2}) = \pm 1. \]  
(17)

After sewing at \( r = R_{c1} \), the resulting solution increases under the barrier (see below Figs. 3 and 4) if not an eigenstate, in contrast with the behavior of each of the fundamental solutions. This demonstrates mathematical correctness of the method. For the numerical integration, the Runge-Kutta-Nyström method was used. The Shtermer method was also tried, with essentially the same results. Behind the barrier, the both solutions oscillate.

4 Results and discussion

Calculations were performed with the Saxon-Woods potential \([13]\), with the parameters \( V_0 = 100 \text{ MeV}, \quad s = 2.3 \text{ Fm}, \quad c = 1.2A^{1/3} \text{ Fm} \). Representative wavefunctions for various energies are presented in Figs. 2–4 for the system \( \alpha + {^{131}\text{La}}, \quad L = 0. \)

Fig. 2 answers the eigenvalue of \( E_\alpha = 10.79 \text{ MeV} \). Corresponding wavefunction has a large amplitude inside the barrier. Therefore, the overlapping integral is also expected to be large in this case.

In Figs. 3 and 4 we present the wavefunctions aside the resonance, for the energies of 11 and 14 MeV, respectively. The wavefunctions are normalised at \( \delta(p - p') \). These figures are in drastic contrast with the resonance one, presented in Fig. 2. The amplitude of the wavefunction within the barrier is much smaller than outside. As a result, the overlapping integral is expected to be small in the nonresonance case, depressing nonresonance \( \alpha \) decay.
Finally, in Fig. 5 we present the calculated line $\Gamma(E)$. That has a typical resonance shape with a half-width of around 200 eV. Therefore, spectrum of the subthreshold $\alpha$ particles turns out to be modulated, directly indicating the resonance states inside the barrier. Confirmation of this effect in experiment would really mean discovering new physics.

In heavy-ion collisions, this effect may be smoothed by mixed multipolarities. The effect must also manifest itself in usual $\alpha$- or proton decay, specifically, of nuclei far from the drip line. In this case, set of the allowed $L$ values is usually not large. Moreover, a partial wave with a certain $L$ may make predominant contribution, which can be
Figure 4: $\alpha$ wavefunction for the system of $^{131}$La + $\alpha$, with the $\alpha$ energy of 14 MeV.

exploited for direct check of the theory presented herein. This study is planned to be made separately, in due course.

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Figure 5: Profile of the $\alpha$ decay line in an anticipated sub-barrier spectrum, half-width being around 200 eV. (The system is $^{131}$La + $\alpha$, for the alphas emitted with L = 0.)
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