Significance of Cattaneo-Christov heat flux in Darcy-Forchheimer transport of nanofluid with entropy optimization

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Abstract
Darcy-Forchheimer transport of nanofluid comprising Ethylene glycol in Cattaneo-Christov heat flux across an extended cylinder with various slips is investigated in this paper. The influence of autocatalytic chemical reactions in the governing equations enrich the novelty of the proposed mathematical formula. The current challenge also includes an entropy minimization study of the governing equations. For the conversion of the nonlinear system to ODEs, a relevant transformations technique is used. For the calculation of a nonlinear set of linear equations, the bvp4c (shooting) method is coupled with the MATLAB program. Graphical drawings are used to examine the effects of the leading factors on occupied fields. The results demonstrate that a high magnetic parameter boosts the thermal profile while lowering the velocity. In addition, the velocity is reduced when the slip parameter is estimated. Furthermore, it was determined that as the thermal relaxation parameter was raised as the entropy number increases. The tabular data is included to support the current mathematical model.

Keywords
Cattaneo-Christov heat flux, Darcy-Forchheimer flow, nanofluid, multiple slips, entropy optimization, autocatalytic chemical reaction

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Introduction
The investigation of the dehydration process is very important in the semiconductor sector because excessive heat production can damage or destroy equipment or devices. An increase in heat transfer between such a coolant and a heated surface is comparable to improving convective transfer in cooling systems. All thermo-physical properties enhancement measures will affect these two metrics, convective heat transfer efficiency, and fluid/wall exchange surfaces. The fins primarily enhance the thermal transfer surface, while the heat transfer fluid’s conductance affects the heating and cooling coefficient. To improve the efficiency of a cooling system, many strategies have been used. Large surfaces, including fins, are a dependable, cost-effective, and commonly utilized technique of heat

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dissipation. To maintain a cool, large, the high-powered system needs more heat dissipation. The second approach for eliminating heat is integrating liquid aerodynamic performance with high conductivity of increased resistance. Metal-based water nanofluid has greater thermal conductivity and convective heat transfer than the base liquid. This strategy is undeniably the most appealing for increasing heat transport. In 1993, Masuda et al. were the first to use ultrafine solid particles for distribution in a base fluid. Choi coined the term “nanofluid” in 1995, which traditionally refers to a fluid with nanomaterials suspended in it. Many studies have found that substituting a coolant with a nanofluid enhances a base fluid’s thermal efficiency. The thermal conductivity of most solids is larger than the fluid’s heat transfer coefficients. The dispersion of nanoparticles to the conventional fluids resulted in greater thermophysical properties and improved thermal performance when compared to the base fluid. Muhammad et al. studied the impact of compressing flow on the melting of nanofluids. Goudarzi et al. investigated the effect of nanomaterials with convection and radiation on nanofluid. Sadaf and Abdelsalam studied the effects of nanofluid flow in a curved non-uniform disc with flow separation features. Hussein et al. looked at using a nanofluid to boost the thermal output of photovoltaic panels. Gul et al. studied the impact of MHD dipoles on heat transfer using a nanofluid. Abbas et al. explored the Yamada–Ota and Xue theories of nanofluid on spinning needles. Waini et al. investigated with a forward flow of nanofluid toward such a squeezing cylinder. Non-Newtonian fluids are widely employed in industry and business, which has prompted academics to do research in this area. The chemical industry, which includes paint manufacture, palm increased oil, and conditioner production, and even the food industry, which includes mayonnaise preparation, are all examples of key applications for these fluids. Eyring–Powell fluid and Viscoelastic fluid are the most important non-Newtonian fluids. The Eyring–Powell liquid was investigated in this article. Exercising non-Newtonian fluids is the subject of the following scientific projects. Broadspectrum of products methodologies, including satellite communications vehicles, missile social reintegration, various propulsion equipment for airplanes, an atom bomb power plant, solar energy assimilation, in combustion application fields such as Engine design, furnaces, fires, solar constructions, etc., all require radiative heat transfer inside the flow. The mass and heat transfer scenario happened when the concentration variation of species in a mixture transfers them from a high-concentration area to a low-concentration area. There are various operations in this amazing period, such as absorption, heat resistance, and food preparation, as well as alcohol distillation and water content dispersion over grooves fields, all of which need a mass transfer. Abbas et al. developed an entropy optimized MHD micropolar nanofluid with slip phenomena and osmotic pressure. Kotha et al. investigated the heat generation on MHD flow of nanofluid generated by microorganisms. Shahid et al. used heat radiation to study the MHD flow of nanofluid through a porous layer. Awan et al. investigated nonlinear heat and mass transfer effectiveness in the magnetohydrodynamics flow of nanofluid using the impacts of a solar cell. Mishra and Kumar investigated the effects of velocity and slip flow using a straining cylinder with entropy production and Joule heating. Li et al. explored microbe investigation and Wu’s slip-over flow of nanofluid across a surface. Morteza Mousavi et al. studied the impacts of Joule heating on Magnetohydrodynamics flow nanofluid with heat transmission in microfluidic systems. The MHD generator creates converts electrical energy to kinetic energy directly. The main difference between an MHD generator and a traditional electric alternator is that a magnetohydrodynamics generator seems to utilize ionized fluids as conducting polymers. MHD is the investigation of the magnetic properties of conducting fluids. The connection between fluid metals or magnetized particles inside the present and the electrostatic force is considered via MHD interaction. The electrodynamics Maxwell formulas are coupled with fluid equations that incorporate the Lorentz force in the MHD model. In principle, the Lorentz force and inductive electric current have opposing production mechanisms. An increment in the Joule heating variable causes the temperature to rise, concentration to fall, and velocity to rise. Below are some studies in the fields listed in this paragraph. In a Magnetohydrodynamic generator, an ionized fluid is anticipated to migrate at a specific velocity through such a strong magnetic field, creating an electromagnet that may be used to harvest electric energy by inserting two conductors across the fluid stream. Alshber and Nabwey examined heat and mass transfer phenomena caused by the MHD flow of nanofluid around a rotating frame. The features of MHD flow of nanofluid with heat production were investigated by Oyelakin et al. The effects of dissipative heat energy on MHD flow across a nonlinear sheet were investigated by Baag et al. More work on nanofluid and MHD is carried out. Figure 1 shows a schematic representation of the microbiological, antineoplastic, wound healing, and angiogenic capabilities of zinc oxide nanoparticles in veterinary sciences. ZnO nanoparticles have also been employed in tissue regeneration, as a food additive, and as a feed ingredient.

This communication analyses the Darcy–Forchheimer permeable medium flow of ZnO w of nanofluid with the effects of homogenous and heterogeneous reactions, MHD, and multiple slips over a cylinder. Here zinc Zn,
zinc oxide ZnO, and base fluid Ethylene glycol C\(_2\)H\(_6\)O\(_2\) are used. The similarity transformations are utilized to solve the collection of nonlinear differential equations arising from the controlling system in curvilinear coordinates. MATLAB’s built-in methodology Bvp4c is used to mathematically and visually assess the flow problem’s outcomes. No investigation on the issue has been performed to our knowledge. As a consequence, this technique will be used to collect essential intake from such flows, which will aid in the resolution of several technical difficulties.

**Physically and mathematically flow modeling**

**Flow description**

Here we considered the incompressible flow of nanofluid including zinc Zn, zinc oxide ZnO, and base fluid Ethylene glycol C\(_2\)H\(_6\)O\(_2\) with the effects of multiple slip conditions through a stretched cylinder. The significance of homogenous and heterogeneous reactions with Darcy–Forchheimer permeable medium is investigated here. The importance of entropy generation and Cattaneo-Christov (C-C) theory of heat profile and with a magnetic field is also examined. The cylinder’s coordinates are chosen so that \(r\) and \(z\) in the horizontal and vertical directions, correspondingly, correspond to the cylinder radius see Figure 2.

Chaudhary and Merkin\(^{31}\) proposed the isothermal homogeneous reaction with cubic autocatalysis in nanofluid flow, which is expressed as follows:

\[
C + 2D \rightarrow 3D. \quad (1)
\]

Hence the heterogeneous reaction can be calculated as follows:

\[
C \rightarrow D. \quad (2)
\]

The homogeneous reaction rate is \((k_1Hb^2) k1Hb2\), while the heterogeneous reaction rate is \((k,H)\). The \(H\) and \(b\) are chemical species with concentrations of \(C\) and \(D\).
**Dimensional non-linear equations**

The assumed model for the governing flow is given as:

\[(rw)_r + (ru)_r = 0,\]  \hspace{1cm} (3)

\[\rho_nf (ww_z + uw_r) = \mu_nf \left( w_r + \frac{1}{r} w \right) - \sigma_f B_{h\gamma}^2 w - \frac{\mu_nf}{k} w - Fw^2,\]  \hspace{1cm} (4)

\[\rho_nf (wu_z + uu_r) = \mu_nf \left( u_r + \frac{1}{r} u - u \right)^2,\]  \hspace{1cm} (5)

\[(wT_z + uT_r) = \frac{k_{nf}}{(\rho c_p)_w} \left( T_r + \frac{1}{r} T \right) - \lambda_2 \left( w^2 T_z + u^2 T_r + 2 w w T_z + u u T_r + u u T_z + w w T_r \right) - \frac{q^*}{(\rho c_p)_w},\]  \hspace{1cm} (6)

\[uH_r + wH_z = D_A \left( H_r + \frac{1}{r} H \right) - k_1 H b^2,\]  \hspace{1cm} (7)

\[ub_r + wb_z = D_B \left( b_r + \frac{1}{r} b \right) + k_1 H b^2,\]  \hspace{1cm} (8)

The relevant boundary is:

\[
\begin{align*}
&u = 0, \quad w = W_w + L w_r, \\
&T = T_w + I T_r, \\
&D_A H_r = k_1 H, \\
&D_B b_r = - k_1 H a r + a, \\
&u \to 0, \quad T \to T_\infty, \quad H \to a_0, \\
&b \to 0 \text{ as } r \to \infty
\end{align*}
\]  \hspace{1cm} (9)

**Transformation variables**

\[
\begin{align*}
u &= - \frac{c_1}{c_2} f(\xi), \quad w = 2 z c \xi f' (\xi), \\
\Delta T &= T_w - T_\infty, \quad \xi = \left( \frac{T - T_\infty}{T_w - T_\infty} \right), \quad g = \frac{H}{a}, \quad h = \frac{b}{a_0}
\end{align*}
\]  \hspace{1cm} (10)

**Dimension-less equations**

With similarity transformation (10) the equations (4)–(9) obtained the following form:

\[B_1(\xi'''' + f'') + \frac{B_2 \text{Re}}{2} \left( f'''' - f' \right) = 0,\]  \hspace{1cm} (11)

\[B_3(\theta' + \frac{2}{3} \xi''') + A f' + B \theta - 2 \alpha \text{Re Pr} \left( \frac{f^2}{\xi} \theta'' + f f' \theta' \right) - \text{RePr} f \theta' = 0,\]  \hspace{1cm} (12)

\[2 \xi g'' + 2 g' + 2 Sc \text{Re} g' - Sc \text{Re} K g h^2 = 0,\]  \hspace{1cm} (13)

\[2 \xi h'' + 2 h' + 2 \frac{Sc}{\delta} \text{Re} g' - \frac{Sc}{\delta} \text{Re} K g h^2 = 0,\]  \hspace{1cm} (14)

The dimensionless boundary conditions are:

\[
\begin{align*}
f'(1) &= 1 + \lambda f'(1), \quad f(1) = 0, \\
\theta(1) &= 1 + \Omega \theta'(1), \\
g'(1) &= - K_s g(1), \\
\delta h'(1) &= - K_s g(1), \\
f'(\infty) &= 0, \quad \theta(\infty) \to 0, \\
g(\infty) &= 0, \quad h(\infty) \to 0
\end{align*}
\]  \hspace{1cm} (15)

**Reduced parameters**

In the above equations, the Prandtl number is

\[\text{Pr} = \frac{v}{\alpha},\]  \hspace{1cm}

the Reynolds number is denoted by

\[\text{Re} = \frac{\alpha}{\alpha},\]  \hspace{1cm}

the porosity parameter is

\[\gamma = \frac{\alpha}{\alpha},\]  \hspace{1cm}

the Schmidt number is

\[\text{Sc} = \frac{k}{\alpha},\]  \hspace{1cm}

the velocity slip parameter is

\[\lambda = \frac{2 \alpha}{\alpha},\]  \hspace{1cm}

the thermal relaxation parameter is

\[\alpha = \lambda_2 \alpha,\]  \hspace{1cm}

the Strength of homogenous reaction

\[K = \frac{k_{nf}}{T_w},\]  \hspace{1cm}

and the strength of the heterogeneous reaction

\[K_s = \frac{k_{nf}}{T_w}.\]  \hspace{1cm}

The dispersion if (\delta = 1) the \(D_A \text{ and } D_B\) coefficients are assumed to be the same, calculated based on the following relationship:

\[g(\xi) + h(\xi) = 1.\]  \hspace{1cm} (16)

From equation (13)–(14)

\[2 \xi g'' + 2 g' + 2 Sc \text{Re} g' - Sc \text{Re} K g (1 - g)^2 = 0,\]  \hspace{1cm} (17)

With

\[g'(1) = - K_s g(1), \quad g(\infty) \to 1.\]  \hspace{1cm} (18)

Skin friction coefficient is categorized as

\[C_f = \frac{1}{2} \frac{\tau_w}{W_w^2},\]  \hspace{1cm} (19)

\[\tau_w = \mu_f W_w |r - a|,\]  \hspace{1cm} (20)

Here, appended the dimensionless form of the drag force

\[C_f = f''(1),\]  \hspace{1cm} (21)

**Numerical scheme**

The nonlinear non-dimensional converted problem equations and boundary conditions were addressed
using the nonlinear shooting technique and the MATLAB built-in function bvp4c.\textsuperscript{36-38} For $\zeta_{\text{max}} = 7$ asymptotic convergence is reported to be reached. bvp4c is a boundary value problem solver in the MATLAB package. This problem’s numerical results are all applied to a $10^{-6}$ error tolerance. The following variables are used to transform a system of partial differential equations into a system of first-order ordinary differential equations:

\[
\begin{align*}
&f = s_1, f' = s_2, f'' = s_3, f''' = s_3', \\
&\theta = s_4, \theta' = s_5, \theta'' = s_5', \\
&\phi = s_6, \phi' = s_7, \phi'' = s_7.
\end{align*}
\]

\[
\begin{align*}
s_1 &= \frac{1}{\xi B_1} \left[ -B_1 s_3 - B_2 \text{Re} (s_1 s_3 - s_2^2) + \text{Re} F_s s_2^2 \frac{\lambda \text{Re}}{2B_4} B_1 s_2 + M s_2 \right], \\
s_2 &= \frac{-B_3 s_3 - \alpha s_3 - 2\alpha \text{Re} B_4 \text{Pr} s_3 s_2 s_2 + \text{Re} B_4 \text{Pr} s_3 s_2 s_2}{2\Omega B_3 - 2\alpha \text{Re} B_4 \text{Pr} s_3 s_2 s_2}, \\
s_3 &= \frac{1}{2\xi} \left[ -2s_7 - 2\text{Sc} \text{Re} s_1 s_7 + \text{Sc} \text{Re} K s_6 (1 - s_6) \right], \\
s_4 &= \left( s_4 (1 - 1 - \lambda s_3 (1), s_1 (1), s_4 (1) - 1 - \Omega s_3 (1) \right), \\
s_5 &= \left( s_5 (1) - \frac{\alpha k_f}{2\nu s_5} s_6 (1), s_2 (\infty), s_4 (\infty), s_6 (\infty) - 1 \right).
\end{align*}
\]  

**Entropy profile**

The entropy generation is stated as.\textsuperscript{39,40} The rate of entropy generation is $S_{g}^{m} = \frac{S_{g}^{m}}{S_{0}^{m}}$

\[
S_{0}^{m} = \frac{4k_f (T_w - T_\infty)^2}{L^2 T_w^2}.
\]

By using similarity transformation (10), the entropy generation is transformed into the following dimensionless form

\[
N_S = \frac{S_{g}^{m}}{S_{0}^{m}},
\]

\[
N_S = \left( B_3 \frac{\xi}{\eta} \frac{\theta^2 (\xi) - B_1 \frac{\lambda \text{Re}}{2B_4} \frac{\text{Re} F_s}{\text{Re} B_4} \frac{s_2}{\text{Re} B_4} \frac{s_2}{\text{Re} B_4} \frac{\text{Pr} s_3 s_2 s_2}{\text{Pr} s_3 s_2 s_2}}{\text{Pr} s_3 s_2 s_2} \right),
\]

\[
m = \frac{\mu_f \frac{W_o}{k_f} (z)}{k_f (T_w - T_\infty)}, \quad \Gamma = \frac{T_w - T_\infty}{T_\infty},
\]

The Brinkman number is supposed as

\[
Br = \frac{\mu_f \frac{W_o}{k_f} (z)}{k_f (T_w - T_\infty)}.
\]

The Bejan number is defined as

\[
Be = \left( \frac{\frac{k_f}{T_w^2} \frac{T_{\infty}}{T_w} (H_{T_\infty}) + \frac{\frac{\text{Re} \text{Pr} \text{Sc} \text{Re} s_6 (1 - s_6)}{2\xi} \frac{T_{\infty}}{T_w} (H_{T_\infty})}{2\xi}}{\frac{\frac{k_f}{T_w^2} \frac{T_{\infty}}{T_w} (H_{T_\infty}) + \frac{\frac{\text{Re} \text{Pr} \text{Sc} \text{Re} s_6 (1 - s_6)}{2\xi} \frac{T_{\infty}}{T_w} (H_{T_\infty})}{2\xi}}{\frac{\text{Re} \text{Pr} \text{Sc} \text{Re} s_6 (1 - s_6)}{2\xi} \frac{T_{\infty}}{T_w} (H_{T_\infty})}} \right)
\]

\[
S_{g}^{m} = \left( \frac{\frac{k_f}{T_w^2} \frac{T_{\infty}}{T_w} (H_{T_\infty}) + \frac{\frac{\text{Re} \text{Pr} \text{Sc} \text{Re} s_6 (1 - s_6)}{2\xi} \frac{T_{\infty}}{T_w} (H_{T_\infty})}{2\xi}}{\frac{\text{Re} \text{Pr} \text{Sc} \text{Re} s_6 (1 - s_6)}{2\xi} \frac{T_{\infty}}{T_w} (H_{T_\infty})} \right)
\]
The non-dimensional form of the Bejan Number
\[ \text{Be} = \left( \frac{\partial \theta^2}{\partial \zeta^2} + \frac{\partial \theta g'}{\partial \zeta} + \frac{\partial \theta g''}{\partial \zeta^2} \right) \left( \frac{\partial \theta^2}{\partial \zeta^2} + \frac{\partial \theta g'}{\partial \zeta} + \frac{\partial \theta g''}{\partial \zeta^2} \right)^{-1} \]  
(35)

### Results and discussion

The governing prominent parameters such as magnetic parameter, thermal relaxation parameter, Brinkman number, the velocity slip parameter, Reynolds number, the nanoparticles volume fraction, the thermal slip parameter, and temperature difference parameter against the temperature, concentration distribution, velocity, and entropy generation are deliberated through graphs by MATLAB software (see Table 1 and Table 2). The results of \(f'\) the positive values \(M\) are deliberated in Figure 3. The velocity concentration \(f'\) has negative nature for the positive magnitude of the \(M\). The outcomes of a velocity profile \(f'\) for the distinct values of the velocity slip parameter are plotted in Figure 4. From the curves, it is observed that the larger valuation of the velocity slip parameter \(l\) reduces the velocity distribution \(f'\). Figure 5 displays the consequences of the volume fraction of nanoparticles \(\phi\) on the temperature concentration \(\theta\). It is noted that the thermal profile \(\theta\) upsurge for the larger values of volume fraction of nanoparticles \(\phi\). The thickness of the associated boundary layer is enhanced with the augmentation of the volume fraction of nanoparticles \(\phi\). Noticeably, the basic element is that the increment in nanoparticles expands the heat conveyance and cohesive forces among the fluid particles; as a result, the fluid temperature is improved. Figure 6 is illustrated to

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Table 1. The properties of nanofluid.

| Properties          | Notations                          | Nanofluid                                      |
|---------------------|------------------------------------|-----------------------------------------------|
| Dynamic viscosity   | \(B_1 = \frac{\mu_n}{\mu_m} = \mu_{nf}\) | \(\mu_{nf} = \frac{\mu_n}{\left(1 - \phi\right)\tau}\) |
| Heat capacity       | \(B_4 = \frac{(\rho c)_n}{\rho c_m} = (\rho c)_{nf}\) | \((\rho c)_{nf} = (1 - \phi\)(\rho)\_f + (\rho)\_f\phi,\) |
| Density             | \(B_2 = \frac{\rho_n}{\rho_m} = \rho_{nf}\) | \((\rho)\_f(1 - \phi) + (\rho)\_f\phi,\) |
| Thermal conductivity| \(B_3 = \frac{k_n}{k_m} = k_{nf}\) | \(k_{nf} = \frac{(1 - \phi) + 2\phi\alpha n \ln \left(\frac{\alpha_n}{\alpha_m}\right)}{(1 - \phi) + 2\phi\alpha_m \ln \left(\frac{\alpha_n}{\alpha_m}\right)},\) |

Table 2. Thermophysical properties of nanoparticles ZnO&Zn and base fluid C\(_2\)H\(_6\)O\(_2\). \(^{41,42}\)

| Physical properties | C\(_2\)H\(_6\)O\(_2\) | ZnO | Zn |
|---------------------|-----------------------|-----|----|
| \(\rho_n/\rho_m\)   | 1115                  | 5600| 7140|
| \(C_p\) \(kJ/kgK\) | 2430                  | 495.2| 390|
| \(K_n/k_m\) \(W/mK\) | 0.253                 | 13  | 116 |
| \(\sigma/\mu_m\)   | \(1.1 \times 10^{-4}\) | 0.01| 1.69 \times 10^7 |

Figure 3. Aspects of velocity profile for \(M\).

Figure 4. Aspects of velocity profile for \(\lambda\).
reveal the effects of magnetic parameters $M$ on the thermal profile $\theta$. The mounting valuation $M$ enhanced the thermal profile $\theta$. Figure 7 depicts the characteristics of the temperature slip $\Omega$ on the $\theta$. From the figure, results are concluded that the swelling values of the temperature slip parameter $\Omega$ fall the temperature concentration $\theta$. The salient features of the thermal relaxation parameter $\alpha$ on temperature concentration $\theta$ are premeditated in Figure 8. It is apprehended that the escalating variations of the thermal relaxation parameter $\alpha$ reduce the $\theta$. Figure 9 designates the features of $\beta$ on $\theta$. It is regarded that the temperature distribution $\theta$ declined for fluctuating values of the heat source-sink parameter $\beta$. Figure 10 is grabbed to picture the influence of the $\Sigma e$ on the $\phi$. From the arcs, it is reflected that the higher variations of the Schmidt number have negative trends for concentration. The upshots of the strength of heterogeneous reaction $K_\phi$ on the concentration distribution are
pondered in Figure 11. The increasing value strength of heterogeneous reaction \( K_s \) diminishes the in the fluid. The influence of growing values of Reynolds number \( Re \) on the entropy generation is delighted in Figure 12. The entropy generation was boosted with the evaluated variation of the Reynolds number. Figure 13 exposed the impacts of the Brinkman number \( Br \) on the \( NG \). The amended variation in the Brinkman number \( Br \) augmented the entropy generation of the nanofluid. Figures 14 and 15 present the results of streamlining for nanofluid when the velocity slip parameter is \( l_0 = 0 \) and \( l_0 = 0.8 \). Figures 16 and 17 show the significance of streamlining for nanofluid when the magnetic parameter is \( M = 0.0 \) and \( M = 1.0 \). Figures 18 and 19 analyze the effects of the Contour line for Prandtl number, thermal radiations parameter, velocity slip parameter, and heat source-sink parameter via Nusselt number. Figures 20 and 21 analyze the effects of 3D plotting for...
Figure 15. Aspects of streamlining for $\lambda = 0.8$.

Figure 16. Aspects of streamlining for $M = 0.0$.

Figure 17. Aspects of streamlining for $M = 1.0$.

Figure 18. Aspects of Contour line for Pr&Rd.

Figure 19. Aspects of Contour line for $\lambda$ & $\beta$.

Figure 20. Aspects of the 3D plot for $\lambda$ & $\beta$. 
Prandtl number, thermal radiations parameter, velocity slip parameter, and heat source-sink parameter via Nusselt number.

Conclusions

Here the Darcy–Forchheimer flow of ZnO/C2H6O2&Zn/C2H6O2 based nanofluid with influences of multiple slips effects, and HOM–HET chemical reaction across a cylinder was studied. The proposed mathematical formulation is also subjected to an entropy generation and Cattaneo-Christov heat flux assessment. The following are the model’s significant assumptions:

- The velocity distributions profile decreased for the rising values of porosity parameter and magnetic parameter ZnO/C2H6O2&Zn/C2H6O2 based nanofluid
- The temperature distributions increased for the growing variations of magnetic parameter and volume fraction of nanoparticles
- The heat profile diminished for the higher values of thermal relaxation parameter and slip parameter for ZnO/C2H6O2&Zn/C2H6O2 based nanofluid
- The concentration distributions profile declined for the rising estimation of Schmidt number and heterogeneous reaction
- The entropy generations profile is enhanced for the higher values of Reynolds number and Brinkman number for ZnO/C2H6O2&Zn/C2H6O2 based nanofluid

Declaration of conflicting interests

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