Shahriar Afandizadeh* and Hamid Bigdeli Rad

Developing a model to determine the number of vehicles lane changing on freeways by Brownian motion method

https://doi.org/10.1515/nleng-2021-0036
Received Aug 19, 2021; accepted Oct 17, 2021

Abstract: Lane change maneuvers are essential on car trips. Drivers change lanes to follow the desired route to reach their destination or improve their driving condition or level of service. To change lanes, a driver must consider several factors that affect safety. Due to the lack of appropriate data and consequently the lack of appropriate models to determine the number of lane-changes on the road (as an influential factor in accidents), this study attempts to collect proper data in a new way. Thus, the Qazvin-Karaj freeway was selected as the case study. After installing the imaging cameras and performing the image processing, SPSS and Expert Design statistical software were used to model development. The Brownian motion model was also used to construct the driver change lane model. The results showed that logarithmic model number 2 reported a better coefficient of determination than other models with a value of 0.472. Then models 3 and 9 were ranked with $R^2$ of 0.451 and 0.442, respectively. Also, the Expert Design model with $R^2$ (0.786) could have a better fit. The value of the response variable $(N_{ch} + 0.52)^{0.76}$ was obtained three-dimensionally against the changes of distance from the front vehicle ($D_f$) and distance from the rear vehicle ($D_b$). Variable values of distance from the front car and distance from the rear vehicle have more effective values on the number of lane changes than left and right distance values. The observed and Brownian data had a slight mean difference (0.018), and also, the standard deviation was so small. Also, the correlation in this data pair is 0.912, which is a suitable value and indicates a slight difference between the outputs of the Brownian model and the observations.

Keywords: Lane Change Modeling, Expert Design, Brownian Motion Model, Statistical Analysis

1 Introduction

Pursuing a car and changing lanes are two unavoidable actions while driving. Many car-following models have been used. Classical lane-changing models mainly focus on driver behavior and gap acceptance of lane selection [1, 2]. It can be said that lane changing and vehicle tracking models are the basis of traffic flow theory. With the advent of automatic and semi-automatic vehicles, understanding the accuracy of lane-turning and car-tracking behavior models is critical to the driver’s decisions to ensure the safe operation of these vehicles and the surrounding traffic [3, 4]. Researchers have studied vehicle tracking for over fifty years, while fewer experiments have been performed on lane-changing behavior. This issue may be because of: 1) lane changing involves moving in two dimensions, and 2) relatively more vehicles (about five vehicles) are involved in a lane-changing event. In contrast, a car following usually involves two cars following one another in the same lane [5].

Oliver and Lam (1965) introduced the first nonlinear model to determine the number of lane changes [6]. This model assumed that the number of shift maneuvers from lane i to lane $i + 1$ is proportional to the second power of density in lane $i$ multiplied by the difference between density and critical density in lane $i + 1$. Then there is the Gipps model, which was designed to describe the behavior of cars and trucks entering, passing, and leaving a section of road. To be used in conjunction with a car tracking model [7], which imposes restrictions on a driver’s braking, to calculate the safe speed of a rear-wheel-drive vehicle.

To examine drivers’ intentions to change lanes, many researchers use various methods such as machine learning classification, such as the Hidden Markov Model (HMM) [8–10], Support Vector Machine (SVM) [11, 12], Bayesian network [13–15], artificial neural network [16, 17] and deep
neural network [18, 19]. Zheng and Hansen developed an HMM-based lane-detection model using vehicle dynamic signals. They reported that the model classification could be covered by 80.36 for left-hand lane changing and 83.22 for right-lane changing in a real dataset [10]. Kim and his co-workers have proposed an ANN + SVM model to predict drivers’ lane-changing decisions [20]. Vehicle status and road surface status information are enhanced using ANN models, and added information is transmitted to the SVM to detect drivers’ decisions accurately.

Hu and his co-workers used the Bayesian and decision tree methods to model lane changes. This model predicts driver decisions about lane integration or non-lane connection. The best results were obtained when both the Bayesian tree and decision tree classifiers were combined in a single effect [21].

Hidas introduced a lane-changing model in the Intelligent Transport System Simulator (SITRAS). This model was developed under heavy traffic conditions to change the imposed and participatory lane. Each component of this model is a complex process [22]. Time gaps are a better indicator of driver behavior compared to spatial gaps. Time gaps are also a function of the spatial gaps and speed of the rear vehicle. In this regard, Bham, in 2009, developed a model of forced lane changing with the model of time gap acceptance. Accepted or rejected gaps between the target vehicle and the hypothetical front vehicle and the target vehicle and the hypothetical rear vehicle in the target lane were analyzed when the front-rear vehicle pairs followed each other [23]. In 2016, Balal and his co-workers developed a fuzzy inference system (ANFIS) that models a driver’s binary decision as to whether or not to change lanes on the freeway. This model can be used in the lane-changing recommendation system in smart vehicles [5].

Hetrick also used observers to collect data on vehicles. The lane changing time period was between 3.4 to 13.6 seconds. Younger drivers had shorter time, while older drivers had longer time. The average lane-changing time length was 6 seconds [24]. Lee and co-workers pointed out that the presence of an observer in this research can affect driver behavior and lead to the absence of normal driving behavior [4]. A different approach used by Salvucci and Liu in 2002 uses a driving simulator to evaluate lane-changing behavior. Eleven participants in the experiment were asked to drive on a multi-lane highway equipped with a simulator. Individuals are then asked to report their willingness to lane changing and completing the change. Based on these observations, the average lane-changing period was estimated to be 5.14 with a standard deviation of 0.86 seconds [25].

Brownian motion is the random motion of particles in a liquid due to their collision with atoms or other molecules. A macroscopic (visible) particle can be considered to map Brownian Motion, which is affected by many microscopic accidents. The Brownian motion is named after Robert Brown, a Scottish botanist who observed pollen that moved randomly in water. Brown described the move in 1827 but could not explain it. This phenomenon of particle transport remained unexplained until 1905. That is until Albert Einstein published an article explaining that water molecules move pollens in a liquid. The mathematical description of Brownian motion is a relatively simple calculation that is important not only in physics and chemistry but also in other statistical phenomena [26, 27]. The first person to propose a mathematical model for Brownian motion was Thorvale N. Thiele, who published an article in 1880 that the current model is the Wiener process. Today, mathematical models that describe Brownian motion are used in mathematics, economics, engineering, physics, biology, chemistry, and many other disciplines [28, 29].

It isn’t easy to distinguish between motion due to Brownian motion and motion due to other effects. In biology, for example, observations must tell whether a species of living thing is moving because it can move (it can move on its own) or because it is moving Brownie [30]. Typically, one can distinguish between processes because Brownian motion appears to be irregular and random. In contrast, real motion is often in one direction or in the form of rotating in a particular direction [31].

Another example of the Brownian movement is illustrated: Imagine a person standing in a straight line and wanting to start walking. To select the path, he uses the method of tossing a coin and determining the next step based on it, then stops to take the next step, repeats the same thing, drops the coin again, and moves wherever the coin says. In many natural phenomena, we encounter such a model, and it is essentially this random behavior that determines the direction and dynamics of the system [32].

Lewis Bachlier (1900) first showed that financial markets follow random step processes. Therefore, standard probability accounts can be used to model financial markets. Stochastic step processes are essentially a Brownian motion in which past changes are independent of variable value changes in the future and past [33]. Brownian Motion has well-behaved mathematical properties, in which a pattern can be estimated with high accuracy and probabilities [34]. Thus, analysts often resort to independent trends, such as Brownian Motion, when analyzing a multidimensional process of unknown origin (such as the stock market). Brownian motion theory and random step patterns have been widely used in financial market modeling.

Given that speculation is modeled, one can use Bachler’s extended probabilities, which have continued to be
used to this day [35]. Studies by Osborne have shown that
the natural logarithm of stock price and monetary value can
be influenced by a set of decisions in statistical equilibrium.
And this group of prices logarithms (created over time) is
very similar to the Motion of a large number of molecules of
a substance. The probability distribution function can be
calculated using the probability distribution function and
the randomly selected stock price at a random time in a
steady-state, which is precisely the probability distribution
for a particle (molecule) in Brownian Motion [36].

Although the methods used in previous research are
different, all of these methods state that lane changes are
not instantaneous events. Developing a lane-change model
for a multi-lane traffic simulator is a challenging task. There
is no clear and appropriate rule to be used by the majority
of drivers in the decision to change lanes (there is a need
for a random model). Driver behavior in rear and front vehi-
cles usually affects the lane change process. For example,
fast drivers change lanes more than slow drivers. As stated,
the basis for the formation and simulation of stock price
fluctuations with Brownian Motion was experimental. In
this way, the rise or fall of stock prices is considered a ran-
don move (despite economic stimuli and deterrents). In
the present study, due to the random nature of changing
lanes [37] of drivers (and, of course, the existence of obsta-
cles and other vehicles), we have tried to simulate changing
lanes of vehicles with brown movement. Past studies have
generally examined the effect of traffic flow parameters on
lane change, such as: [38–41]. But the lack of previous stud-
ies is that perceptible parameters to drivers can be used in
models. Therefore, in this study, in addition to using the
new method “Brownian Motion” and adapting it to lane-
changing, an attempt has been made to develop the model
with the simplest parameters (i.e., the distance of each vehi-
cle from the other vehicles and the surrounding obstacles)
to fill this gap.

2 Methodology

According to Figure 1, after reviewing literature studies and
realizing the existing shortcomings, statistical models and
the Brownian motion model were selected to determine
the number of lane changes. By selecting the study area,
data collection was done through imaging and processing,
and finally, the models were implemented, and the outputs
were analyzed and compared.

2.1 Statistical analysis and regression model

The model is a symbol of reality, and in situations where
due to economic, technical, and other constraints, it is im-
possible to experience the issues in practice. It is possible
to understand how the system behaves. After reviewing
previous research on models and variables affecting acci-
dents in this study, the study area was determined, and the
required information was collected. Different models were
obtained using regression technique and fitting different
linear and nonlinear functions on the collected data and
their calibration. Finally, by controlling the accuracy and
validity of the fitted models through statistical tests, the
appropriate model is introduced.

Many attempts were made to find the best transfer func-
tion for the dependent variable and combine the indepen-
dent variables to make the most significant possible con-
nection using SPSS and Expert Design software. In this
regard, linear and nonlinear functions were investigated.
Finally, after fitting several models to the data and conduct-
ing preliminary studies regarding statistical tests, the most
appropriate models were selected from dozens of different
models, and the results were discussed. In the literature
review section, the factors affecting the lane-changing were
examined, and in this research, it is decided to examine
the spatial and distance parameters as follows:

- $D_f$: Distance from the vehicle in front
- $D_b$: Distance from the rear vehicle
- $D_r$: Distance from the vehicle (or guard-rail) on the
  right
And for the time-dependent deviation \( \alpha \) and the time-dependent \( \sigma^2 \) are written as Eq. (4).

\[
\begin{align*}
&x_0^i = x_0 + \sum_{i=1}^{t_i} \alpha(u) du + z_i \sqrt{t_i - t_{i-1}} \tag{4} \\
&x_{i+1} = x_i + \int_{t_i}^{t_{i+1}} \alpha(u) du + z_i \sqrt{t_{i+1} - t_i} \sigma^2(u) du
\end{align*}
\]

Next, if \( z_1, z_2, \ldots \), are randomly identical independent variables with distribution \( N(0,1) \), then we have:

\[
\begin{align*}
&x_0 = x_0 \\
&x_i = x_{i-1} + \int_{t_{i-1}}^{t_i} \alpha(u) du + z_i \sqrt{t_i - t_{i-1}} \sigma^2(u) du \quad i = 1, 2, \ldots, k
\end{align*}
\]

In Eq. (5) specifically \( X_i - X_{i-1} \) has a normal distribution \( N \left( \int_{t_{i-1}}^{t_i} \alpha(u) du, \int_{t_{i-1}}^{t_i} \sigma^2(u) du \right) \) which is clearly a random step.

Given the dependence of \( \alpha \) and \( \sigma \) on time, the integral of Eq. (4) may not be efficiently computable. They can be replaced by squaring formulas in the simplest case \( \alpha(t) = 0 \) and \( \sigma(t) = 1 \) on the interval \([t_{i-1}, t_i]\) is approximated. In this case, we replace Eq. (4) as follows.

\[
\begin{align*}
&x_i = x_{i-1} + \alpha(t_{i-1}) (t_i - t_{i-1}) + z_i \sigma(t_{i-1}) \sqrt{t_i - t_{i-1}} - t_i \\
&i = 1, 2, \ldots, k
\end{align*}
\]

### 2.1.2.1 Multi-dimensional Brownian motion simulation

If the d-dimensional Brownian motion has a covariance matrix \( \Sigma \), then decomposition of \( \Sigma = AA^T \) is necessary. In the simplest case, \( \alpha = 0 \) (no deviation) and \( \Sigma = I \), the random step structure of Eqs (2) and (3) is converted directly to the d-dimensional state. With \( z_i \) and \( w_i \), which are the \( R^d \)-dimensional (Each \( z_i \) is constructed with random digits \( d \) representing the random variables of the iid \( N(0,1) \) distribution). This is equivalent to the independence of simulating each component of a d-dimensional Brownian motion using a one-dimensional Eq. (3). The generalization of Eq. (3) for the d-dimensional model with the initial vector \( x_0 \) follows.

\[
\begin{align*}
&x_0 = x_0 \\
&x_i = x_{i-1} + \alpha(t_{i-1}) + \sqrt{t_i - t_{i-1}} A z_i \\
&i = 1, 2, \ldots, k
\end{align*}
\]

Given \( z_i, x_i \in R^d \), calculating the coefficient matrix at each stage is computationally complex. If \( \alpha \) or especially \( \Sigma \) are dependent on \( t \) (time), the computational costs will increase. Here Eq. (4) is generalized. Given the assumed initial vector \( x_0 \), we have Eq. (8).

\[
\begin{align*}
&x_0 = x_0 \\
&x_i = x_{i-1} + \int_{t_{i-1}}^{t_i} \alpha(u) du + A (t_i - t_{i-1}) Z_i \\
&i = 1, 2, \ldots, k
\end{align*}
\]

In this equation:

\[
A (t_{i-1}, t_i) A (t_{i-1}, t_i)^T = \int_{t_{i-1}}^{t_i} \sigma^2(u) du
\]

In other words, the matrix of coefficients \( A \) in Eq. (8) needs to be calculated as a factorization matrix based on Eq. (9) at each stage. For a standard d-dimensional Brownian motion
(X_t)_{t \geq 0}$ assuming no deviation and $\sum = I_d$ the Brownian bridge structure can easily be applied to each component independently (same Face as described for the random step structure above). Also, for the d-dimensional Brownian motion $(X_t)_{t \geq 0}$ with deviation $\alpha$ and the covariance matrix $\sum$ concerning the Brownian bridge structure still apply the structure $(X_t)_{t \geq 0}$ and $X_t$ is obtained by $X_t = at + AW_t$ assuming $\sum = AA^t$.

3 Case study

The study area is a freeway with a length of 103 km with three lanes in each direction, between Alborz and Qazvin provinces of Iran, and is part of Freeway 2 (Tehran-Tabriz). This freeway is the most accident-prone in the country and is one of the important transportation routes in the country.

4 Results and discussion

4.1 Regression models (by SPSS)

Various models were implemented in SPSS software, and 11 of the models whose significance was confirmed are reported in Table 1. The resulting models failed to provide the proper R-square. However, model number 2, which is logarithmic, reported a better coefficient of determination than other models with a value of 0.472. After that, models 3 and 9 were placed with coefficients of determination of 0.451 and 0.442, respectively. Among these, model number 9 can be a better model due to its simplicity in use and the slight difference in the coefficient of determination.

Figure 2: Recording and Recognition of Distances by the Cameras.
Table 1: Output of models made by SPSS.

| Number | \( N_{ch} \) | \( r \) | R-square Adjusted R2 | Sig. |
|--------|---------------|-------|----------------------|------|
| 1      | 88.12* (-55.45*\( \log D_f \))^{0.6} | 0.496 | 0.246               | 0.239 | <0.0001 |
| 2      | 64.82* (-22.65*\( \log D_f \)) \* (-23.71*\( \log D_b \))^{0.7} | 0.687 | 0.472               | 0.463 | 0.004   |
| 3      | 176.2 - 64.71*\( \log D_f \)^{0.5} - 66.32*\( \log D_b \)^{0.6} | 0.672 | 0.451               | 0.443 | 0.002   |
| 4      | 31.29 - 0.347*\( D_f \) | 0.346 | 0.12                | 0.1   | <0.0001 |
| 5      | 32.36 - 0.303*\( D_f \) - 0.399*\( D_b \) | 0.481 | 0.231               | 0.225 | 0.0012  |
| 6      | 39.25 - 0.301*\( D_f \) - 0.423*\( D_b \) - 0.636*\( D_l \) | 0.490 | 0.24                | 0.221 | <0.0001 |
| 7      | 31.5 - 1.389*\( D_f \)^{0.7} - 5.892*\( D_b \)^{0.5} + 4.673*\( D_l \) + 3.378*\( D_r \) | 0.550 | 0.302               | 0.294 | <0.0001 |
| 8      | 39.25 - 1.823*\( D_f \)^{0.65} | 0.574 | 0.329               | 0.321 | <0.0001 |
| 9      | 40.21 - 1.92*\( D_f \)^{0.4} | 0.665 | 0.442               | 0.436 | 0.0009  |
| 10     | 63.52 - 1.428*\( D_f \)^{0.8} - 5.722*\( D_b \)^{0.6} + 1.268*\( D_l \) | 0.549 | 0.301               | 0.293 | 0.007   |
| 11     | 31.5 - 1.389*\( D_f \)^{0.7} - 5.892*\( D_b \)^{0.5} + 4.673*\( D_l \) + 3.378*\( D_r \) | 0.596 | 0.355               | 0.35  | <0.0001 |

4.2 Regression models (by Expert Design)

Due to the low R-square values of the models in the previous section, with the help of Expert Design software, it was tried to examine many more models according to the appropriate capabilities of this software. By entering the data into the hypothetical software, a large number of different models were extracted and compared, and finally, the most appropriate model in Eq. (10) was obtained:

\[
(N_{ch} + 0.52)^{0.74} = -0.426D_f - 0.753D_b + 11.03D_l + 3.35D_r - 0.895D_Df + 0.0039D_f^2 \\
+ 0.008D_b^2 - 0.838D_l^2 \\
+ 0.0006D_r^2 D_f
\]  

The model presented in Eq. (10) has a higher coefficient of determination than the models in Section 4.1; as seen in Table 2, the significance of the model is confirmed (Sig. <0.005). According to the coefficient of determination of 0.786, the quality of the model increased significantly. Also,
according to Figure 3, a good report of the normality and model fit was obtained. But it is still possible to increase the coefficient of determination by thinking of measures. The value of the Lambda parameter (\( \lambda \)), according to Figure 4, was determined to be 0.74 by its many tests in its optimal state. The constant coefficient (k) was also set at 0.52 to obtain the superior model among the available options.

As shown in Figure 5, the response variable value \((N_{ch} + 0.52)^{0.74}\) is depicted three-dimensionally against the combined effect of changes in distance from the front \((D_f)\) and distance from the back \((D_b)\). The distance from the front and the distance from the back have more effective values than the values of the distance from the left and right on the number of lane changes. This diagram is also shown as a contour in Figure 6, expressing the effects more clearly. But then, we tried to provide a simpler model in terms of implementation and application and obtain a higher coefficient of determination. By increasing the value of the parameter \(D_f\) and \(D_b\), the response variable decreases to its minimum value. In the other three corners of the graph, where the variable values of the response have large values, there are points where one of the parameters \(D_f\) or \(D_b\) has a low value, i.e., the distance from the front or back is much reduced.

**4.3 Brownian motion model**

Due to the low values of R-square, different regression models tried to study how to change the driving lane and check the graph of the recorded data more carefully. Figure 7 shows the distribution of lane-changing data in a part of the path. A great similarity was revealed between this distribution and Brownian motion. For this reason, according to Section 3.3, the Brownian motion model was simulated, and the following outputs were obtained.

The data collected through imaging and distance recording were entered into the Brownian model, and its output data were compared with the values observed through the paired statistical test. According to Table 3, the observed and Brownian data pairs had a small mean difference (0.018). Also, the standard deviation was so slight. According to Table 4, the correlation is 0.912. It is a good value and indicates a small difference between the outputs of the Brownian model and the observations (this correlation is also shown in Figure 8). This correlation is also due to the meager value of Sig.

The results of the developing model by the Brownian motion method are investigated in Figures 9A–9D. Thus, as shown in Figure 9A (as a single factor), the number of lanes

---

**Figure 5:** Three-dimensional diagram of the output of the Expert Design model.

**Figure 6:** Contour diagram of the Expert Design model output.

**Figure 7:** Distribution of lane changing data in a part of Qazvin-Karaj freeway.

**Figure 8:** Correlation of observed data and Brownian model.
Developing a model to determine the number of vehicles lane changing on freeways

Table 3: Paired test results of observed data and Brownian model.

| Pair 1 | Observed – Brownian | Mean | Std. Deviation | Std. Error Mean | t | Sig. (2-tailed) |
|--------|---------------------|------|----------------|-----------------|---|----------------|
|        |                     | 0.01800 | 0.42219        | 0.05971         | 0.301 | 0.764         |

Table 4: Correlation results of observed data and Brownian model.

| Pair 1 | Observed – Brownian | Correlation | Sig. |
|--------|---------------------|-------------|------|
|        |                     | 0.912       | 0.000|

changes in terms of the distance of the vehicle understudy from the front vehicles. When this distance is short, the number of lane changes is at its peak, and as this distance increases, the number of lane changes also decreases. But the interesting point in the results is when the distance exceeds a specific value (65 meters), the number of lanes changes again increases slightly. Figure 9B shows the distance from the rear car, which has caused more oscillations. It is almost similar to Figure 9A, but with the difference that the return point has reached near 50 meters and the increasing slope of lane-changing after this point has become steeper. Figures 9C and 9D evaluate the distance parameters from left and right. They have almost similar diagrams, which increase the number of lane changes by increasing the distance to 5 or 6 meters and then decreasing. It can be seen that the effect of these two variables is less than the two variables of distance from front and rear.

Finally, an attempt was made to find the relationship between traffic density on the desired lane and the number of lane changes made by the vehicle in the same lane per hour. Thus, in Figure 10A, lane 3 (or speed lane) is evaluated. As the density increased, the number of lane changes also increased to a density of about 18 (Veh/km), after which the number of lane changes took a downward trend in this lane. This indicates that the speed lane is very sensitive to density, and if it is too high, the number of lane changes will decrease. Figure 10B shows that with increasing density in lane 2, the number of lane changes increases with a smooth slope and remains almost constant in the range of 20 (Veh/km) onwards and has no downward trend. This can be due to the greater possibility of changing the lane in lane 2 (if the density increases) to go both left and right.

Figure 9: Number of lane changes according to the distance of the vehicle from the front (A), rear (B), left (C), and right (D).
Figure 10: Relationship between traffic density on lanes (3 (a), 2 (b), and 3 (c)) and the number of lane changes made by the vehicle in question on the same lane per hour.

right. In Figure 10C, the number of lanes changes with density increases similar to lane 2 but with a greater slope. Of course, the density in this lane is less than the previous two lanes, which can be a reason to increase the number of lane changes at the end of the chart.

5 Conclusion

Due to the need to develop appropriate models for determining the number of lanes changing on the road (as an influential factor in accidents), this study tried to collect pertinent data in a new way and provide the best models. Also, for the first time, the Brownian motion model was used and adapted to lane-changing data. Various models were implemented in SPSS software, and 11 models which significance was confirmed. The resulting models failed to provide the proper R-square. However, model number 2, which is logarithmic, reported a better coefficient of determination than other models with a value of 0.472. Then models 3 and 9 were ranked with coefficients of determination of 0.451 and 0.442, respectively. The expert Design software was used to improve the responses, and after reviewing a large number of models, model (10) with a value of R-square (0.786) was able to have a better fit. The response variable value \(N_{ch} + 0.52^{0.74}\) was plotted three-dimensionally against changes in distance from the front \(D_f\) and distance from the back \(D_b\). The values of distance from the front and distance from the back have more effective values than the values of distance from left and right on the number of lane changes. Due to the low values of R-square, different regression models tried to study how to change the driving lane and check the graph of the recorded data more carefully.

Figure 7 shows the distribution of lane change data in a part of the path; a great similarity was revealed between observed data distribution and Brownian Motion. The data collected through imaging and distance recording were entered into the Brownian model, and its output data were compared with the values observed through the paired statistical test. The observed and Brownian data pairs had a small mean difference (0.018), and also the standard deviation was very small. Also, the correlation in this data pair is 0.912, which is a good value and indicates a slight difference between the outputs of the Brownian model and the observations. This correlation is also due to the very low value of Sig. According to the obtained results, considering the similarity of the nature of Brownian motion and lane change data and the practical confirmation of the accuracy of this claim, further details of the Brownian model can be studied to gain a better understanding of the driver’s lane changing. Distance-based models can give users a better view to gain a better understanding of lane changing. Naturally, the driver must change lanes for a reason (in rare
Author contributions: All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

Conflict of interest: The authors state no conflict of interest.

References

[1] Zhou B, Wang Y, Yu G, Wu X. A lane-change trajectory model from drivers' vision view. Transp Res, Part C Emerg Technol. 2017;85:609–27.

[2] Knoop VL, Hoogendoorn SP, Shiomi Y, Buisson C. Quantifying the Number of Lane Changes in Traffic empirical analysis. Transp Res Rec. 2016;2278(1):31–41.

[3] Ngoduy D, Lee S, Treiber M, Keyvan-Ekbatani M, Vu HL. Lanechanging method for a continuous stochastic car-following model and its stability conditions. Transp Res, Part C Emerg Technol. 2019;105:599–610.

[4] Lee S, Ngoduy D, Keyvan-Ekbatani M. Integrated deep learning and stochastic car-following model for traffic dynamics on multi-lane freeways. Transp Res, Part C Emerg Technol. 2019;106:360–77.

[5] Balal E, Cheu RL, Sarkodie-Gyan T. A binary decision model for discretionary lane changing move based on fuzzy inference system. Transp Res, Part C Emerg Technol. 2016;67:47–61.

[6] Oliver RM, Lam T. Statistical experiments with a two-lane flow model. University Berkeley Operations Research Center. 1965. https://doi.org/10.21326/AD0622500.

[7] Gipps PG. A model for the structure of lane-changing decisions. Transp Res, Part B: Methodol. 1986;20(5):403–14.

[8] Rad VB, Najafpour H, Ngah I, Shieh E, Rashvand P, Rad HB. What Are The Safety Factors affecting Road Safety Using Fuzzy Hierarchical Analysis. J Transpers Res. 2020;17(3):33–44.

[9] Abdal A, Mosadeq Z, Bigdeli Rad H. Prioritizing Factors Affecting Road Safety Using Fuzzy Hierarchical Analysis. J Transpers Res. 2020;17(3):33–44.

[10] Zheng Y, Hansen JH. Lane-change detection from steering signal using spectral segmentation and learning-based classification. IEEE Trans Intell Veh. 2017;2(1):14–24.

[11] Ramyar S, Homaiifar A, Karimoddini A, Tunstel E. Identification of anomalies in lane change behavior using one-class SVM. IEEE International Conference on Systems, Man, and Cybernetics (SMC); 2016 Oct 9-12; Budapest, Hungary. IEEE; 2016. p. 004405-004410.
[30] Rudzis P. Brownian Motion [dissertation]. Seattle: University of Washington; 2017.
[31] Joyce P. Brownian Motion. In: Practical Numerical C Programming. Berkeley (CA): Apress; 2020. p. 179–84.
[32] Sung W. Brownian Motions. In: Statistical Physics for Biological Matter. Dordrecht: Springer; 2018. p. 241–68.
[33] Sinha AK. The reliability of geometric Brownian motion forecasts of S&P500 index values. J Forecast. 2021 Apr;for.2775.
[34] Imperial F. Modelling Stock Prices and Stock Market Behaviour using the Irrational Fractional Brownian Motion: An Application to the S&P500 in Eight Different Periods [dissertation]. Madrid: IE Business School; 2018.
[35] Lin Q, Dejian T, Weidong T. A generalized stochastic differential utility driven by G-Brownian motion. Math Financ Econ. 2020;14(3):547–76.
[36] Ramic A, Paulshus O. An empirical application of Black and Scholes option pricing with fractional Brownian motion [dissertation]. Handelshøyskolen BI: Norwegian Business School; 2018.
[37] Gu X, Yu J, Han Y, Han M, Wei L. Vehicle lane change decision model based on random forest. IEEE International Conference on Power, Intelligent Computing and Systems (ICPICS); 2019 Jul 12-14; Shenyang, China. IEEE; 2019. p. 115-120.
[38] Wang G, Hu J, Li Z, Li L. Cooperative lane changing via deep reinforcement learning. arXiv:1906.08662 [Preprint]. 2019 Jun 20 [cited: 19 Aug 2021]. Available from: https://arxiv.org/abs/1906.08662.
[39] Wang T, Cheng R, Ge H. Analysis of a novel two-lane lattice hydrodynamic model considering the empirical lane changing rate and the self-stabilization effect. IEEE Access. 2019;7:174725–33.
[40] Tang TQ, Wang YP, Yang XB, Huang HJ. A multilane traffic flow model accounting for lane width, lane-changing and the number of lanes. Netw Spat Econ. 2014;14(3):465–83.
[41] He J, He Z, Fan B, Chen Y. Optimal location of lane-changing warning point in a two-lane road considering different traffic flows. Physica A. 2020;540:123000.