Modelling and precision of the localization of the robotic mobile platforms for constructions with laser tracker and SmartTrack sensor

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Abstract. This paper presents a way to expand the field of use of the laser tracker and SmartTrack sensor localization device used in lately for the localization of the end effector of the industrial robots to the localization of the mobile construction robots. The research paper presents the equipment along with its characteristics, determines the relationships for the localization coordinates by comparison to the forward kinematics of the industrial robot’s spherical arm (positioning mechanism in spherical coordinates) and the orientation mechanism with three revolute axes. In the end of the paper the accuracy of the mobile robot’s localization is analysed.

1. Introduction
In this paper, the full localization is defined by the direct or indirect measurement by the instrumentality of sensors for the x, y, z coordinates of the origin and the α, β, γ direction angles of a mobile frame of reference, attached to a mobile object, with respect to fixed frame of reference. The measurements x, y, z, α, β, γ will be further defined localization coordinates.

A mobile construction robot can be considered as being formed by a robotized mobile platform or a carrying module and the robotized work equipment which can be compared to a fixed base industrial robot with respect to the mobile platform, figure 1.
The programming of such a mobile robot is done based on the inverse kinematics models of the mobile platform and of the work equipment respectively. The inverse kinematics model of the mobile platform is generally deducted [1], starting from the relationships for the nonholonomic constraints expressed in velocities and their integrations cannot be determined in coupled form except the specific cases, such as smooth surface movement and specific trajectories in the straight line, circular arc or zero turn radius [2], often used trajectories in the case of mobile construction robots. For the general situation, the integration is done numerically through odometry and practice has proved that the positioning errors are substantial [3]. The inverse kinematics model for the robotized work equipment does not raise any special problems, being determined through relationships for the holonomic constraints which are expressed in relation to the positions.

According to the working technology, in case of mobile construction robots, two cases can be distinguished. In the first case, the robotic mobile platform is fixed and only the working equipment works; in this case a static localization, carried out after the mobile platform is stopped and before starting the work sequence of the equipment, is required. In the second case, performs the technological process during the movement of the carrying module; in this case is necessary to make a dynamic localization.

On a world scale the experts have designed various sensor systems for the localization of the mobile platforms used in various fields of research [4, 5], some of these can also be used in for mobile robots for constructions [6]. One of these is formed by a laser tracker and a “SmartTrack” sensor and is further developed.

2. The localization with laser tracker and smarttrack sensor

2.1. The structure of the measurement system of the localization coordinates

The main components of the localization system are as follows [6]:

a) Laser tracker, figure 2,a. It measures in spherical coordinates (azimuth, elevation, distance) the position of the SmartTrack sensor with respect to its own reference frame and it is capable to track automatically the target if it moves. The distance is optically measured and the azimuth and elevation are mechanically measured by encoders. The specifications of the laser tracker are presented in table 1.

b) SmartTrack sensor, figure 2,b. It is capable of self-automatic orientation with respect to laser tracker and measurement of the direction angles (pitch, yaw, roll) of the laser beam, emitted by the laser tracker, with respect to a reference system attached to the fixed mount. The roll angle is optically measured and the pitch and the yaw angles are mechanically measured by encoders. The specifications of the SmartTrack sensor are presented in table 2.
**Figure 2.** Laser tracker and SmartTrack sensor.

**Table 1.** Laser tracker specifications

| Parameter                  | Specification |
|----------------------------|---------------|
| Range of Measurements      | 100 m         |
| Azimuth                    | ±320°         |
| Elevation                  | +79°/-59°     |
| Angular Resolution         | ±0.018”       |
| Angular Accuracy           | 3.5 μm/m      |
| System Resolution          | 0.1 μm        |
| Maximum Lateral Target Speed| 6 m/s         |
| Maximum Acceleration       | > 2g          |
| Internal Level Accuracy    | ±2”           |
| Static Measurement         | ±10 μm        |

**Table 2.** Specifications of SmartTrack sensor

| Parameter                  | Specification |
|----------------------------|---------------|
| Pitch                      | ±55°          |
| Yaw                        | ±140°         |
| Roll                       | ±30°          |
| Tracking distance          | 40 m          |
| Tracking angular speed     | 50°/s         |
| Optic centering accuracy   | ±25 μm        |
| Angular resolution         | ±3”           |

2.2. *The measurement scheme*

In order to attain the full localization of the robotized mobile platform, the laser tracker is put in the origin of the fixed system and the oriented axes with respect to the system of reference with which the attainment of the localization is intended. The SmartTrack sensor is mounted on the robotized mobile platform at the known coordinates with respect to the reference system attached to the platform.

The coordinates measured by the two components of the respective system azimuth, elevation, distance, pitch, yaw and roll, show that there is a correspondence between these and the joint variables of a 6 DOF robot, with positioning mechanism in spherical coordinates and orientation mechanism with three orthogonal revolute axes. Therefore, the localization coordinates are determined the same way as the position and the orientation of the end effector of the spherical robot is determined, through the forward kinematics model [1].

Based on the previous reasons, attaching the reference systems in order to determine the localization coordinates is done by using the Denavit-Hartenberg method, therefore in figure 3:

- the fixed frame is chosen with the origin at the intersection of the measurement axes of the laser tracker, the z₀ axis is oriented in according to the vertical direction and is overlapped on the axis of rotation of the laser tracker around which the azimuth is measured in...
counterclockwise direction; the $x_0$ axis is located in the horizontal plane and is oriented suitable;
- the system attached to the turret of the laser tracker has the same origin as the fixed system, the $z_1$ axis is oriented with respect to the rotation axis of the tracker’s lens around which the elevation is measured and the $x_1$ axis is oriented with respect to an optical axis.
- the mobile system attached to the laser tracker’s lens has the same origin as the fixed system, the $z_2$ is oriented along the optical axis with which the $z_2$ distance is measured and the $x_2$ is oriented down on the vertical direction;
- the mobile system 3 is attached to the laser beam and has the $z_3$ axis oriented along it;
- the mobile system 4 is attached on the lens of the SmartTrack sensor, has the $z_4$ axis oriented with respect to the optical axis of it and the roll angle is measured around it;
- the turret of the SmartTrack sensor has attached to it system 5 with the $z_5$ overlapped with the axis of rotation of the turret and around it is measured the pitch angle of the lens rotation with respect to the turret;
- the last reference system is attached on the mount base of the target, it has the $z_6$ axis oriented on the axis of rotation of the turret with respect to the mount base of the target and around it the yaw angle is measured.

Figure 3. The measurement scheme.

The corresponding Denavit - Hartenberg parameters of the system are centralized in table 3.

| No. | Joint | $\theta_i$ | $d_i$ | $a_i$ | $\alpha_i$ | $q_i$ |
|-----|-------|------------|-------|-------|----------|------|
| 1.  | A     | $\theta_1$| 0     | 0     | 270°     | $\theta_1$|
| 2.  | B     | $\theta_2$| 0     | 0     | 90°      | $\theta_2$|
| 3.  | C     | 0          | $d_3$ | 0     | 0°       | $d_3$ |
| 4.  | D     | $\theta_4$| 0     | 0     | 270°     | $\theta_4$|
| 5.  | E     | $\theta_5$| 0     | 0     | 90°      | $\theta_5$|
| 6.  | F     | $\theta_6$| $d_6$ | 0     | 0°       | $\theta_6$|

Table 3. Denavit - Hartenberg parameters of the system
2.3. Determining the localization coordinates
Considering a coordinate frame that travels through “the kinematic chain” starting from the $Ox_0y_0z_0$ position to the $Ox_6y_6z_6$ position, the coordinate transformations are thereby written consecutively:

$$T_i^{i-1} = \begin{pmatrix}
    c\theta_i & -s\theta_i & 0 & a_i c\theta_i \\
    s\theta_i & c\theta_i & 0 & a_i s\theta_i \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 0
\end{pmatrix} \quad (1)$$

in which $i=1...6$.

This way, taking into account that $c\alpha$, are constant and substituting $\sin(\theta) = s\alpha$ and $\cos(\theta) = c\alpha$, the following transformation matrix for the laser tracker results:

$$T_0^3 = T_0^1 * T_1^2 * T_2^3 = \begin{pmatrix}
    c_2 & -s_2 & d_3 c_1 & -d_3 s_1 \\
    s_2 & c_2 & d_3 s_1 & d_3 c_1 \\
    0 & 0 & c_2 & d_2 \\
    0 & 0 & 0 & 1
\end{pmatrix} \quad (2)$$

for the SmartTrack sensor the transformation matrix is obtained:

$$T_3^6 = T_3^4 * T_4^5 * T_5^6 = \begin{pmatrix}
    c_4 c_5 c_6 - s_4 s_6 & -c_4 s_5 s_6 & c_4 c_5 s_6 & c_4 s_5 & d_6 c_4 c_5 s_6 \\
    c_4 s_5 c_6 + c_5 c_6 s_6 & c_4 c_6 - c_5 s_6 & c_5 s_6 & s_5 c_6 & d_6 s_4 s_5 \\
    -c_6 s_5 & c_5 s_6 & d_6 c_5 & 0 & 0 \\
    0 & 0 & 0 & 1 & 0
\end{pmatrix} \quad (3)$$

and for the entire system it is obtained:

$$T_0^6 = T_0^3 * T_3^6 = \begin{pmatrix}
    r_{11} & r_{12} & r_{13} & r_{14} \\
    r_{21} & r_{22} & r_{23} & r_{24} \\
    r_{31} & r_{32} & r_{33} & r_{34} \\
    0 & 0 & 0 & 1
\end{pmatrix} \quad (4)$$

Where:

- $r_{11} = -s_1 (c_4 s_6 + c_2 c_5 s_4) - c_1 c_2 (s_4 s_6 - c_4 c_5 c_6) - c_1 c_6 s_2 s_5$
- $r_{21} = c_1 (c_4 s_6 + c_5 c_6 s_4) - c_2 s_1 (s_4 s_6 - c_4 c_5 c_6) - c_6 s_1 s_2 s_5$
- $r_{31} = s_2 (s_4 + c_4 c_5 c_6) - c_2 c_6 s_5$
- $r_{12} = c_1 s_2 s_5 - c_1 c_2 (c_6 s_4 + c_5 c_6 s_6) - s_1 (c_4 c_6 - c_5 s_4 s_6)$
- $r_{22} = c_1 (c_4 s_6 + c_5 c_6 s_4) - c_2 s_1 (c_6 s_4 + c_4 c_5 s_6) + s_1 s_2 s_5$
- $r_{32} = s_2 (c_6 s_4 + c_4 c_5 s_6) + c_2 s_5$
- $r_{13} = c_1 c_5 s_2 - s_1 s_4 s_5 + c_1 c_2 c_4 s_5$
- $r_{23} = c_5 s_1 s_2 + c_1 s_4 s_5 + c_2 c_4 s_1 s_5$
- $r_{33} = c_2 c_5 - c_4 s_2 s_5$
- $r_{14} = d_3 c_1 s_2 + d_6 c_1 c_5 s_2 - d_6 s_1 s_4 s_5 + d_6 c_1 c_2 c_4 s_5$
- $r_{24} = d_3 s_1 s_2 + d_6 c_1 c_5 s_2 + d_6 c_1 s_4 s_5 + d_6 c_2 c_4 s_1 s_5$
- $r_{34} = d_3 c_2 + d_6 c_2 c_5 - d_6 c_4 s_2 s_5$

From the transformation matrix $T_0^6$ are identified the coordinates of the origin of the mobile system attached to the mount base of the SmartTrack sensor in respect to the fixed system

$$p_x = r_{14} = d_3 c_1 s_2 + d_6 c_1 c_5 s_2 - d_6 s_1 s_4 s_5 + d_6 c_1 c_2 c_4 s_5$$
$$p_y = r_{24} = d_3 s_1 s_2 + d_6 c_1 c_5 s_2 + d_6 c_1 s_4 s_5 + d_6 c_2 c_4 s_1 s_5$$
$$p_z = r_{34} = d_3 c_2 + d_6 c_2 c_5 - d_6 c_4 s_2 s_5 \quad (6)$$

and the direction cosines of the axis of the mobile system with respect to the axis of the fixed reference frame axes

$$n_x = r_{11} = x_0 * x_4 = -s_1 (c_4 s_6 + c_5 c_6 s_4) - c_1 c_2 (s_4 s_6 - c_4 c_5 c_6) - c_1 c_6 s_2 s_5 \quad (7)$$
\[ n_y = r_{21} = y_0 \times x_4 = c_1(c_4s_6 + c_5c_6s_4) - c_2s_1(s_4s_6 - c_4c_5c_6) - c_6s_1s_2s_5 \]
\[ n_x = r_{31} = z_0 \times x_4 = s_2(s_4s_6 - c_4c_5c_6) - c_2c_6s_5 \]
\[ \alpha_x = r_{22} = x_0 \times y_4 = c_1s_2s_6s_5 - c_1c_2(s_6s_4 + c_4c_5s_6) - s_1(c_4c_6 - c_5c_5s_5) \]
\[ \alpha_y = r_{32} = y_0 \times y_4 = c_1c_4c_6 - c_5s_4s_6 - c_2s_1(c_6s_4 + c_4c_5s_6) + s_1s_2s_6s_5 \]
\[ \alpha_x = r_{33} = x_0 \times z_4 = c_1c_5s_2 - s_1s_4s_6 + c_1c_2c_4s_5 \]
\[ \alpha_y = r_{23} = y_0 \times z_4 = c_5s_1s_2 + c_1s_4s_5 + c_2c_4s_1s_5 \]
\[ \alpha_x = r_{33} = z_0 + z_4 = c_2c_5 - c_4s_2s_5 \]

Considering that the SmartTrack sensor is mounted on the mobile platform with the \( O_6 \) origin situated on its reference point and with the \( z_6 \) axis oriented according to the normal of the platform, it results that the localization coordinates have the following expressions:

\[ x(\theta_1, \theta_2, d_3, \theta_4, \theta_5, \theta_6) = p_x = d_3c_1s_2 + d_4c_1c_5s_2 - d_6s_1s_4s_5 + d_6c_1c_2c_4s_5 \]
\[ y(\theta_1, \theta_2, d_3, \theta_4, \theta_5, \theta_6) = p_y = d_1s_1s_2 + d_4c_1s_1s_2d_6c_1s_4s_5 + d_6c_2c_4s_5 \]
\[ z(\theta_1, \theta_2, d_3, \theta_4, \theta_5, \theta_6) = p_z = d_3c_2 + d_4c_2c_5 - d_4c_2c_5s_5 \]
\[ \alpha(\theta_1, \theta_2, d_3, \theta_4, \theta_5, \theta_6) = \acos(n_x) = \acos(-s_1(c_6s_5 + c_6c_6s_4) - c_1c_2(s_4s_6 - c_4c_5c_6) - c_6c_6s_5) \]
\[ \beta(\theta_1, \theta_2, d_3, \theta_4, \theta_5, \theta_6) = \acos(n_y) = \acos(c_1(c_4c_6 - c_5s_4s_5) - c_2s_1(c_6s_4 + c_4c_5s_6) + s_1s_2s_6s_5) \]
\[ \gamma(\theta_1, \theta_2, d_3, \theta_4, \theta_5, \theta_6) = \acos(n_z) = \acos(c_2c_5 - c_4s_2s_5) \]

### 2.4. Localization accuracy

The accuracy of the localization of the robotized mobile platform depends on the accuracy with which the localization coordinates \( x, y, z, \alpha, \beta, \gamma \) are measured. In their turn, the accuracy of localization coordinates depends on the measurement accuracy of the joint variables azimuth, elevation, pitch, yaw and roll, specified by the system manufacturer [6].

The connection between the two sets of accuracies is attained by Taylor series expansion of the localization coordinates expressions (8) and the maintaining of the expansion terms all the way to degree one. Under these terms, after the derivations execution it results that:

\[ \Delta x = \frac{\partial x}{\partial \theta_1} \Delta \theta_1 + \frac{\partial x}{\partial \theta_2} \Delta \theta_2 + \frac{\partial x}{\partial \theta_3} \Delta \theta_3 + \frac{\partial x}{\partial \theta_4} \Delta \theta_4 + \frac{\partial x}{\partial \theta_5} \Delta \theta_5 + \frac{\partial x}{\partial \theta_6} \Delta \theta_6 \]
\[ \Delta y = \frac{\partial y}{\partial \theta_1} \Delta \theta_1 + \frac{\partial y}{\partial \theta_2} \Delta \theta_2 + \frac{\partial y}{\partial \theta_3} \Delta \theta_3 + \frac{\partial y}{\partial \theta_4} \Delta \theta_4 + \frac{\partial y}{\partial \theta_5} \Delta \theta_5 + \frac{\partial y}{\partial \theta_6} \Delta \theta_6 \]
\[ \Delta z = \frac{\partial z}{\partial \theta_1} \Delta \theta_1 + \frac{\partial z}{\partial \theta_2} \Delta \theta_2 + \frac{\partial z}{\partial \theta_3} \Delta \theta_3 + \frac{\partial z}{\partial \theta_4} \Delta \theta_4 + \frac{\partial z}{\partial \theta_5} \Delta \theta_5 + \frac{\partial z}{\partial \theta_6} \Delta \theta_6 \]
\[ \Delta \alpha = \frac{\partial \alpha}{\partial \theta_1} \Delta \theta_1 + \frac{\partial \alpha}{\partial \theta_2} \Delta \theta_2 + \frac{\partial \alpha}{\partial \theta_3} \Delta \theta_3 + \frac{\partial \alpha}{\partial \theta_4} \Delta \theta_4 + \frac{\partial \alpha}{\partial \theta_5} \Delta \theta_5 + \frac{\partial \alpha}{\partial \theta_6} \Delta \theta_6 \]
\[ \Delta \beta = \frac{\partial \beta}{\partial \theta_1} \Delta \theta_1 + \frac{\partial \beta}{\partial \theta_2} \Delta \theta_2 + \frac{\partial \beta}{\partial \theta_3} \Delta \theta_3 + \frac{\partial \beta}{\partial \theta_4} \Delta \theta_4 + \frac{\partial \beta}{\partial \theta_5} \Delta \theta_5 + \frac{\partial \beta}{\partial \theta_6} \Delta \theta_6 \]
\[ \Delta \gamma = \frac{\partial \gamma}{\partial \theta_1} \Delta \theta_1 + \frac{\partial \gamma}{\partial \theta_2} \Delta \theta_2 + \frac{\partial \gamma}{\partial \theta_3} \Delta \theta_3 + \frac{\partial \gamma}{\partial \theta_4} \Delta \theta_4 + \frac{\partial \gamma}{\partial \theta_5} \Delta \theta_5 + \frac{\partial \gamma}{\partial \theta_6} \Delta \theta_6 \]

\[ \Delta x = \{-d_3s_1s_2 - d_6[c_5s_1s_2 + s_5(c_1s_4 + c_2c_4s_1)]\} \Delta \theta_1 + \{d_3c_1c_2 + d_6c_1(c_2c_5 - c_4s_2s_5)\} \Delta \theta_2 + [c_1c_2s_3d_3 - d_6s_5(c_4s_1 + c_1c_2s_3)] \Delta \theta_4 + \{d_6[c_1(c_2c_4c_5 - s_2s_5) - c_5s_1s_4]\} \Delta \theta_5 \]
\[ \Delta y = \{d_3c_1s_2 + d_6c_1(c_2s_5 + s_2c_4s_1) - s_1s_4s_5\} \Delta \theta_1 + \{d_3c_2s_1 + d_6s_1(c_2c_5 - c_4s_2s_5)\} \Delta \theta_2 + [c_1s_1s_2s_3d_3 + d_6s_5(c_4s_1 - c_2c_2s_3)] \Delta \theta_4 + \{d_6[c_1c_5s_4 + s_1(s_2s_5 + c_2c_2c_5)]\} \Delta \theta_5 \]
\[ \Delta z = [-d_5s_2 - d_6(c_5c_2 + c_4c_5s_2)] \Delta \theta_2 + c_2d_3 + d_6s_5s_4s_3 \Delta \theta_4 - d_6(c_2s_5 + c_4c_5s_2) \Delta \theta_5 \]
\[ \Delta \alpha = \frac{\sqrt{1 - [c_1c_5s_2 - (s_1s_4 - c_1c_2c_4)]^2}}{\sqrt{1 - [-s_1s_4s_5 + c_1(c_5s_2 + c_2c_4s_5)]^2}} \Delta \theta_2 \]
The following facts are deducted from the analysis of the relationships (10):
- the accuracies of the localization coordinates are not constant in the measurement range. They are variable according to the $\theta_1, \theta_2, d_3, \theta_4, \theta_5, \theta_6$ joint variables and their $\Delta \theta_1, \Delta \theta_2, \Delta d_3, \Delta \theta_4, \Delta \theta_5, \Delta \theta_6$ accuracies;
- the accuracies of the $\Delta x, \Delta y, \Delta z, \Delta \alpha, \Delta \beta, \Delta \gamma$ localization coordinates are not dependent, each on its own of the variables $\theta_1, \theta_2, d_3, \theta_4, \theta_5, \theta_6$ of the measurement system.

The manner in which the localization coordinates accuracies is variable in function of each joint variable, for the equipment described in §2.1 is illustrated in figure 4. In order to plot these graphics the nature of the variables has been respected, such as distance or angles. Thereby, in the first six diagrams the variations of localization coordinates accuracies $\Delta x, \Delta y, \Delta z, \Delta \alpha, \Delta \beta, \Delta \gamma$ are presented in function to the $\theta_1, \theta_2, \theta_4, \theta_5, \theta_6$ angular variables of the measurement systems, in the seventh diagram it is shown the variation of the $\Delta \alpha, \Delta \beta, \Delta \gamma$ angular coordinates accuracies as a function of the $d_3$ distance, while in the last diagram it is shown the variation of the linear coordinate accuracies of the localization as a function of the $d_3$ distance. The measurement units used in the diagrams are expressed in [m] for the linear coordinates and for linear accuracies and [rad] for the angular coordinates and angular accuracies.

Considering the attained results and the graphic presentations it can be observed that:
- the $\theta_1, \theta_2, d_3$ joint variables of the laser tracker have a substantially bigger influence on the accuracy of the localization than the $\theta_4, \theta_5, \theta_6$ joint variables of the SmartTrack sensor.
- the maximum localization error is of approximately 150 $\mu$m for the linear coordinates and of approximately $8 \times 10^{-6}$ rad or 5 arc seconds for the angular coordinates.

\[
\begin{align*}
\Delta \beta &= \frac{s_5(c_1c_4s_4 + c_2s_4c_5)}{\sqrt{1 - [c_1c_5s_2 - s_5(s_1s_4 - c_1c_2c_4)]^2}} \Delta \theta_4 + \frac{c_1(s_2s_5 - c_2c_4c_5) + c_5s_1s_4}{s_1(c_2c_5 - c_4s_5)} \Delta \theta_5 \\
\Delta \gamma &= \frac{c_5s_2 + c_2c_4s_5}{\sqrt{1 - (c_2c_5 - c_4s_2s_5)^2}} \Delta \theta_2 - \frac{s_2s_4s_5}{\sqrt{1 - (c_2c_5 - c_4s_2s_5)^2}} \Delta \theta_4 + \frac{c_2c_5 + c_4c_5s_2}{\sqrt{1 - (c_2c_5 - c_4s_2s_5)^2}} \Delta \theta_5
\end{align*}
\]
Figure 4. Accuracy variation of the localization.
3. Conclusions
The field use of the localization equipment for the industrial robots end-effector is extended to localization of mobile construction robots. By using equipment made of a laser tracker and a SmartTrack sensor, a full localization, static or dynamic, can be made for a mobile construction robot in an area of approximately 40 m. The calculus relationships for the localization coordinates are deducted by analogy with the forward kinematics model of the industrial robot with positioning mechanism in spherical coordinates and orientation mechanism with three orthogonal revolute axes. The accuracy with which the localization is performed is sufficient for the construction field, even for the finishing.

4. References
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