Quantum State Transfer between Distant Cavity-Optomechanics Connected via Fiber

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We propose a scheme for high quantum state transfer efficiency between two distant mechanical oscillators. Through coupling separately to two optical cavities connected by an optical fiber, two distant mechanical oscillators achieve a transfer efficiency closing to unity. In the process of analysis, we employed time-varying optomechanical coupling strength as a Gaussian pulse.

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In recent years, the cavity-optomechanical system has drawn a lot of interest both in theoretical and experimental research. On the one hand, it is because the optomechanical system has high optical detection sensitivity, on the other hand, quantum optomechanical system can use light to detect the mechanical movement in quantum system, and generate nonclassical state light and mechanical movement. A general resonant cavity is formed by an optical cavity coupling with a nano mechanical oscillator. The principle is letting the radiation pressure of the light field intra cavity acting on the mechanical oscillator, so that the mechanical oscillator can vibrate freely near the eigenfrequency, and the vibration will, in turn, modulate the frequency of the light field. This optomechanical system has ultra high sensitivity, ultra light effective mass, ultra high quality factor, and ultra high frequency. It is also based on these characteristics, the optomechanical system has very high application value in precision detection and the measurement of force and displacement [1−3]. With the progress of nanotechnology and the semiconductor industry advances in materials and processes, the size of mechanical vibrators can reach micrometer or even nanometer. With the constantly tending to miniaturization and low dissipation, the model of optomechanical system will provide a feasible scheme for the detection of micromechanical vibrators, both in technical and basic science. It will no longer be used solely for precise measurement, but will also become a new technology for controlling cooling [4−8] and amplifying [9,10]. In addition, the mixing optomechanical system coupling with atoms, electrons or quantum dots is favourable to control the optomechanical system, check the basic quantum laws and study the quantum effects of mesoscopic or macroscopic objects [11−13], such as the preparation of mechanical oscillators’ squeezed state, entanglement state and superposition state [14]. At the same time, the realization of controlling quantum optomechanical system has great application value in the processing of quantum information, and it will greatly promote the development of quantum information [15,16].

In this paper, we consider the transfer of a quantum state between two distant mechanical oscillators which are coupled to two optical cavities separately. High efficient transmission of quantum state is achieved by using experimental parameters [16,17] and controlling Gaussian coupling strength between the mechanical oscillators and cavity.

The theoretical model used in this paper is consisted by two optical cavities and two mechanical oscillators. Each mechanical oscillator is coupled to an optical cavity via radiation pressure. One cavity is connected with another optical cavity which is coupled to the second mechanical oscillator via optical fiber (Fig. 1). At the initial moment, we consider a quantum state encoded onto the first mechanical oscillator and transfer it to the second mechanical oscillator through two middle optical cavities that are connected by fiber. Under the current system, we can realize the energy transfer from mechanical energy to mechanical energy in quantum way.

In the interaction picture, the Hamiltonian of the system can be written as [18−21],

\[
H = \omega_m b_1^\dagger b_1 + \omega_m b_2^\dagger b_2 - \Delta_1 a_1^\dagger a_1 - \Delta_2 a_2^\dagger a_2 + \omega_c c^\dagger c + g \left( a_1^\dagger c + a_2^\dagger c + a_1 c^\dagger + a_2 c^\dagger \right) + G_1 \left( a_1^\dagger b_1^\dagger b_1 + a_2^\dagger b_2^\dagger b_2 \right)
\]

where \( \omega_m \) is the frequency of the mechanical mode \( j \), \( b_j \) is the annihilation operator for the mechanical mode \( j \), \( a_j \) is the annihilation operator for the cavity mode \( j \), \( \Delta_j \) is the detuning of the optical drive applied to the cavity mode \( j \), \( g \) is the coupling strength between the cavity and optical fiber, and \( G_j \) is the coupling strength between the mechanical mode \( j \) and cavity \( j \). Through the unitary transformation, we get the effective Hamiltonian of the system.

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\[
H' = \omega_m b_1^\dagger b_1 + \omega_m b_2^\dagger b_2 - \left( \Delta_1 - \frac{2\epsilon}{\Lambda} \right) a_1^\dagger a_1
- \left( \Delta_2 - \frac{2\epsilon}{\Lambda} \right) a_2^\dagger a_2 + \left( \omega_c - \frac{4\epsilon}{\Lambda} \right) c^\dagger c
+ \frac{2\epsilon}{\Lambda} \left( a_1^\dagger a_2 + a_2^\dagger a_1 \right) + G_1 \left( a_1 b_1^\dagger + a_2^\dagger b_1 \right)
+ G_2 \left( a_2 b_2^\dagger + a_1^\dagger b_2 \right)
\]

Here we choose \( U = e^{\frac{i}{\hbar} \left( a_i \phi^i + a_i^\dagger \phi - a_i \phi^i - a_i^\dagger \phi \right)} \), and \( H' = U H U^\dagger \).

For easier calculation, we can choose \( \Delta_1 = \Delta_2 = -\Delta - \omega_c \), at the same time it’s under the case of large detuning conditions, \( \Delta \gg g \). In the Heisenberg picture, using input-output theory, we obtain the quantum Langevin equations for the cavity and mechanical mode operators

\[
\dot{a}_1 = i \left( \Delta_1 - \frac{4\epsilon}{\Lambda} \right) a_1 - iG_1 b_1 - \frac{\gamma_1}{2} a_1 + \sqrt{\kappa} a_{in,1}
\]

\[
\dot{a}_2 = i \left( \Delta_2 - \frac{4\epsilon}{\Lambda} \right) a_2 - iG_2 b_2 - \frac{\gamma_2}{2} a_2 + \sqrt{\kappa} a_{in,2}
\]

\[
\dot{b}_1 = -i\omega_m b_1 - iG_1 a_1 - \frac{\gamma_1}{2} b_1 + \sqrt{\gamma_1} f_1
\]

\[
\dot{b}_2 = -i\omega_m b_2 - iG_2 a_2 - \frac{\gamma_2}{2} b_2 + \sqrt{\gamma_2} f_2
\]

where \( \kappa_j \) is the damping rate of the cavity mode \( j \), \( \gamma_j \) is the damping rate of the mechanical oscillator mode \( j \), \( f_j \) is the noise operator for the mechanical oscillator bath \( j \), while \( a_{in,j} \) describes the quantum noise of the vacuum field incident on the cavity mode \( j \). Under ideal conditions, we can choose \( \kappa_1 = \kappa_2 = \gamma_1 = \gamma_2 = 0 \). And only considering the interaction terms, we get the Langevin equations without dissipative terms.

\[
\dot{a}_1 = -iG_1 b_1 - i\frac{2\epsilon}{\Lambda} a_2
\]

\[
\dot{a}_2 = -iG_2 b_2 - i\frac{2\epsilon}{\Lambda} a_1
\]

\[
b_1 = -iG_1 a_1
\]

\[
b_2 = -iG_2 a_2
\]

To analyze the quantum state transfer from mechanical oscillator mode 1 to mechanical oscillator mode 2, we use the corresponding classical equations for Eqs. (7)-(10):

\[
\alpha_1 = -iG_1 \beta_1 - i\frac{2\epsilon}{\Lambda} \alpha_2
\]

\[
\alpha_2 = -iG_2 \beta_2 - i\frac{2\epsilon}{\Lambda} \alpha_1
\]

\[
\beta_1 = -iG_1 \alpha_1
\]

\[
\beta_2 = -iG_2 \alpha_2
\]

We assume that only the mechanical oscillator mode 1 has excitation phonon at the initial moment. To solve the above equations numerically, we assume \( \beta_1(0) = 1, \alpha_1(0) = 0, \alpha_2(0) = 0, \beta_2(0) = 0 \), and the transfer efficiency of quantum state as \( \eta = |\beta_2(t)|^2 / |\beta_1(0)|^2 \).

Before simulating evolution, we choose a Gaussian pulsed optomechanical coupling strength of the form \( G_1(t) = G_0 e^{-(t-t_0)^2/s^2} \), \( G_2(t) = G_0 e^{-(t-t_0)^2/s^2} \), where \( G_0 \) is the maximum optomechanical coupling strength, \( s \) is the width of the Gaussian pulses, and \( t_0 \) are the time at which the optomechanical coupling strength is maximum and the quantum state transfer from one mode to the other occurs. In addition, the form

\[
G(t) = G_0(t) G_0(t)/G_0
\]

\[
G_0(t) = G_0 e^{-(t-t_0)^2/s^2}
\]

\[
G_0(t)/G_0
\]
of the coupling strength between cavity and optical fiber is $g = g_0$ when $t < 9g_0$, and if $t \geq 9g_0$, then $g = 0$.

From Fig. 2-4, we obtain the procedure values of the maximum optomechanical coupling strength $G_0$ and detuning $\Delta$ which maximize the transfer efficiency $\eta$. The values are $G_0 = 2.5g_0$, $\Delta = 10.5g_0$. In such a case, we can get a high transfer efficiency, just as shown in Fig. 5. When considering the dissipation $\kappa$ and $\gamma$, we get Fig. 6. Here the value of dissipations are $\kappa_1 = \kappa_2 = \gamma_1 = \gamma_2 = 0.01g_0$.

Through analyzing the transfer of a quantum state between two distant mechanical oscillators which are separately coupled to two optical cavities, we get the optimal parameter solution for maximizing the transfer efficiency. In the process of analysis, we employed time-varying optomechanical coupling strength as a Gaussian pulsed. Choosing appropriate parameters, we got a high transfer efficiency, which is very closing to unity.

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