Non-perturbative gauge invariant scalar fluctuations of the metric in Higgs inflation from complex geometrical scalar-tensor theory of gravity.

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In this letter we investigate gauge invariant scalar fluctuations of the metric in a non-perturbative formalism for a Higgs inflationary model recently introduced in the framework of a geometrical scalar-tensor theory of gravity. In this scenario the Higgs inflaton field has its origin in the Weyl scalar field of the background geometry. We found a nearly scale invariance of the power spectrum for linear scalar fluctuations of the metric. For certain parameters of the model we obtain values for the scalar spectral index $n_s$ and the scalar to tensor ratio $r$ that fit well with the Planck 2018 results. Besides we show that in this model the trans-planckian problem can be avoided.

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I. INTRODUCTION

The theory of cosmological perturbations is a very important part in the physical description of the inflationary epoch. It describes the formation and evolution of the seeds of cosmological structure. Some inflationary scenarios are compatible with measurements of cosmic microwave background (CMB) and Planck 2018 Results [1, 2]. In the inflationary models the inflaton field generates the enough vacuum energy to solve the old problems of big bang cosmology. However, the unique scalar particle we have experimental evidence of its existence is the Higgs boson, which has been observed in 2012 with a mass of 125 GeV [3, 4]. This discovery has led several cosmologists to propose that the Higgs scalar field might be the same as the inflaton field [5]. The main problem in regarding this idea is related with the so called “hierarchy problem”. This consists in the observational fact that the Higgs boson mass seems to be sensitive to quantum corrections and the bare Higgs mass then need to be fine-tuned to achieve a physical Higgs boson mass many orders of magnitude smaller than the Planck scale [6]. It basically means that there is a big gap of energy between the electroweak and the Plack scale, and thus the Higgs field in this conditions results to be too small to generate the enough energy to inflate the primordial universe. In particular, in order to have the enough inflation to solve the big bang problems, the inflaton is estimated to have a mass $\sim 10^{13}$ GeV [1, 8]. Among many others, we can find in the literature models with non-minimally coupled inflaton Higgs field trying to alleviate this issue [7, 8]. Some other attempts include models in the Palatini approach [13, 20] and models in non-riemannian geometries [1, 21].

On the other hand, a way to introduce a scalar field in a gravitational framework in a geometrical manner is shown in the recently introduced geometrical scalar-tensor theories of gravity [22, 23]. These theories arose as an attempt to alleviate the Jordan to Einstein frame controversy. In this new approach the scalar-tensor theory is formulated in a non-riemannian geometry which is obtained via the Palatini variational principle.
In this framework the inflaton scalar field is introduced as a part of the affine structure of the geometry and thus it results to be related to Weyl scalar field [21, 22]. Some other topics like (2 + 1) gravity models, inflation and cosmic magnetic fields, quintessence and some cosmological models have been studied in this approach [24–27].

In this letter we use a non-perturbative approach to study linear scalar fluctuations of the metric in a recently introduced Higgs inflationary model [21] developed in the context of a geometrical scalar-tensor theory of gravity. Thus, in section I we give a brief introduction. Section II is devoted to the basic formalism of geometrical scalar-tensor theories. Section III is left for the scalar fluctuations of the metric of arbitrary amplitude. In section IV we study linear fluctuations of the metric and we obtain their power spectrum. Finally, in section V we give some conclusions.

II. BASIC FORMALISM

We start by considering a complex scalar-tensor theory of gravity in vacuum whose action reads

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ \tilde{\Phi} \tilde{\Phi}^\dagger R + \tilde{W}(\tilde{\Phi}) + g^{\mu\nu} \tilde{\Phi}^\dagger_{\mu} \tilde{\Phi}_{\nu} - \tilde{U}(\tilde{\Phi}) \right],$$

(1)

where $R$ denotes the Ricci scalar, $\tilde{W}(\tilde{\Phi})$ is a well-behaved differentiable function of $\tilde{\Phi}^\dagger$, the dagger $\dagger$ denotes transposed complex conjugate and $\tilde{U}(\tilde{\Phi})$ is a scalar potential. By means of the transformation $\tilde{\Phi} = \sqrt{-g} e^{-\varphi}$ the action (1) can be recasted in the more convenient form

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} e^{-(\varphi + \varphi^\dagger)} \left[ \frac{R}{16\pi G} + \hat{\omega}(\varphi + \varphi^\dagger) g^{\mu\nu} \varphi_{\mu} \varphi^\dagger_{\nu} - \hat{V}(\varphi + \varphi^\dagger) \right],$$

(2)

where we have made the identifications $\hat{\omega}(\varphi + \varphi^\dagger) = (1/16\pi) \tilde{W}(\varphi + \varphi^\dagger) e^{\varphi + \varphi^\dagger}$ and $\hat{V}(\varphi + \varphi^\dagger) = (1/16\pi) \tilde{U}(\varphi + \varphi^\dagger) e^{\varphi + \varphi^\dagger}$. Now, to determine the background geometry corresponding to the action (2) we use the Palatini variational principle. Thus we arrive to the compatibility condition

$$\nabla_\mu g_{\alpha\beta} = (\varphi + \varphi^\dagger)_{,\mu} g_{\alpha\beta}.$$  

(3)

Hence, the background geometry is Weyl-integrable. It must be noted that (3) is invariant under the symmetry group of transformations

$$g_{\mu\nu} \rightarrow e^{f + f^\dagger} g_{\mu\nu},$$

(4)

$$\varphi \rightarrow \varphi + f,$$

(5)

$$\varphi^\dagger \rightarrow \varphi^\dagger + f^\dagger,$$  

(6)

where $f = f(x^\mu)$ is a well defined complex function of the space-time coordinates. Unfortunately, as it was shown in [21, 25, 26] the action (2) does not remain invariant under the symmetry group of the geometry (11–13). Thus it is proposed the action

$$S = \int d^4x \sqrt{-g} e^{-(\varphi + \varphi^\dagger)} \left[ \frac{R}{16\pi G} + \hat{\omega}(\varphi + \varphi^\dagger) g^{\mu\nu} \varphi_{\mu} \varphi^\dagger_{\nu} - \hat{V}(\varphi + \varphi^\dagger) \right],$$

(7)

where $\varphi_{,\mu} = (w) \nabla_{\mu} \varphi + \gamma B_{\mu} \varphi$, is a gauge covariant derivative with $B_{\mu}$ being a gauge vector field, $(w) \nabla_{\mu}$ is the Weyl covariant derivative determined by $\gamma$ and $\gamma$ is a pure imaginary coupling constant introduced to have the correct physical units. Notice that the action (7) correspond to a non-conventional scalar-tensor theory of gravity. The invariance of (7) under (11–13) is guaranteed once the vector field $B_{\mu}$, the function $\omega$ and the scalar potential $V(\varphi)$, obey the transformation rules

$$\varphi B_{\mu} \rightarrow \varphi B_{\mu} - \gamma^{-1} f_{,\mu},$$

(8)

$$\varphi^\dagger B_{\mu} \rightarrow \varphi B_{\mu} + \gamma^{-1} f_{,\mu},$$

(9)

$$\hat{\omega}(\varphi + \varphi^\dagger) \rightarrow \hat{\omega}(\varphi + \varphi^\dagger) - \hat{\omega}(\varphi + \varphi^\dagger),$$

(10)

$$\hat{V}(\varphi + \varphi^\dagger) \rightarrow V(\varphi + \varphi^\dagger).$$

(11)

Notice that (8) and (9) are transformation rules for the product $\varphi B_{\alpha}$. Besides they have the same algebraic form of the elements of the Lie algebra associated to the group $U(1)$ employed in quantum electrodynamics. Thus, we may include a dynamics for $\varphi B_{\alpha}$ extending the action (2) by adding an electromagnetic type term in the form

$$S = \int d^4x \sqrt{-g} e^{-(\varphi + \varphi^\dagger)} \left[ \frac{R}{16\pi G} + \hat{\omega}(\varphi + \varphi^\dagger) g^{\alpha\beta} \varphi_{,\alpha} \varphi_{,\beta} - \hat{V}(\varphi + \varphi^\dagger) - \frac{1}{4} e^{(\varphi + \varphi^\dagger)} H_{\alpha\beta} H^{\alpha\beta} \right],$$

(12)

where $H_{\alpha\beta} = (\varphi B_{\beta})_{,\alpha} - (\varphi B_{\alpha})_{,\beta}$ is the field strength associated to the gauge boson field $B_{\mu}$.

The action (12) is an invariant action compatible with its background geometry and originates a new kind of complex scalar-tensor theory of gravity. This action is written in terms of the metric $g_{\mu\nu}$ which according to Weyl transformations (4) is not a Weyl-invariant. Moreover, the differential line element transforms as

$$d\tilde{s}^2 = e^{f + f^\dagger} ds^2.$$  

(13)

Thus, we introduce the Weyl-invariant metric

$$h_{\mu\nu} = e^{-f - f^\dagger} g_{\mu\nu}.$$  

(14)

In terms of this Weyl-invariant metric the action (12) acquires the form

$$S = \int d^4x \sqrt{-h} \left[ \frac{R}{16\pi G} + \hat{\omega}(\varphi + \varphi^\dagger) h^{\mu\nu} \nabla_{\mu} \varphi \nabla_{\nu} \varphi^\dagger - \hat{V}(\varphi + \varphi^\dagger) - \frac{1}{4} H_{\mu\nu} H^{\mu\nu} \right].$$

(15)
where now the gauge covariant derivative becomes $D_\mu = (\partial \nabla_\mu + \gamma B_\mu)$ and the operator $(\partial \nabla_\mu$ denotes the Riemannian covariant derivative.

On the other hand, the non-metricity associated to the background geometry of the action [11] is $N_{\alpha\beta\gamma} = -[\ln(\Phi\Phi^\dagger)]$. This non-metricity is quadratic in $\Phi$. However, when we implemented the transformation $\Phi = e^{\gamma} e^{-\phi}$ the quadratic dependence in both the non-metricity and the action is lost, as shown in (2) and (3). Thus, in order to restore the quadratic dependence in the scalar field, we introduce the field transformations

$$\zeta = \sqrt{\xi} e^{-\phi},$$
$$A_\mu = B_\mu \ln(\zeta/\sqrt{\xi}),$$

where $\xi$ is a constant introduced in order to the field $\zeta$ has the correct physical units.

Hence, the action (15) rewritten in terms of the fields $\zeta$ and $A_\mu$ becomes

$$S = \int d^4x \frac{\sqrt{-h}}{16\pi G} \left[ \frac{\mathcal{R}}{16\pi G} + \frac{1}{2} \omega(\zeta^\dagger) h^{\mu\nu} D_\mu \zeta(D_\nu \zeta)^\dagger - V(\zeta^\dagger) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right],$$

where $D_\mu \zeta \equiv \xi D_\mu (\ln \zeta) = (\partial \nabla_\mu + \gamma A_\mu) \zeta$ is an effective covariant derivative, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu = -H_{\mu\nu}$ is the Faraday tensor and where we have made the identifications

$$\omega(\zeta^\dagger) = \frac{\omega(\ln \zeta^\dagger)}{\zeta^\dagger},$$
$$V(\zeta^\dagger) = \frac{\hat{V}}{\xi} \left( \ln \frac{\zeta^\dagger}{\xi} \right).$$

The effective background geometry of (18) is riemannian and it results invariant under the gauge transformations

$$\zeta = \zeta e^{\gamma(x)}$$
$$A_\mu = A_\mu - \theta_\mu,$$

where $\theta(x)$ is a well-behaved function. Thus, the last term in (18) together with the transformations [21] and [22], suggest that $A_\mu$ can play the role of an electromagnetic potential. However, it is important to note that the part of (18) that we relate with electromagnetism has its origin in the required Weyl invariance of the action (7).

III. NON-PERTURBATIVE SCALAR FLUCTUATIONS OF THE METRIC IN A HIGGS INFLATION MODEL

In order to study gauge invariant scalar fluctuations of the metric, let us first give the basic formulation of a Higgs inflationary model derived from the formalism explained in the previous sections. In particular we will use the model proposed in [21]. Thus we consider the Higgs potential in the Weyl frame in the form

$$\hat{V}(\Phi \Phi^\dagger) = \frac{\lambda}{4} (\Phi \Phi^\dagger - \sigma^2)^2,$$

where according to the best-fit experimental data $\lambda = 0.129$ and the vacuum expectation value for electroweak interaction $\sigma = 246 GeV$ [28, 29]. Thus, the Higgs potential in terms of the field $\zeta$ in the Riemann frame acquires the form

$$V(\zeta^\dagger) \equiv \frac{\lambda}{4} \left( \frac{\zeta^\dagger}{\xi} - \sigma^2 \right)^2.$$  

The ground state $||\zeta^\dagger|| = \sqrt{\xi} \sigma$ associated with [21] is invariant under [21]. However, the breaking of the symmetry is achieved when we take $\zeta = \zeta^\dagger$ because in this particular case $||\zeta|| = \sqrt{\xi} \sigma$. Thus, excitations around the ground state read

$$\zeta(x^\mu) = \sqrt{\xi} \sigma + Q(x^\mu),$$

where $Q(x)$ denotes the Higgs scalar field. According to [23] the kinetic term in (18) can be written in terms of the Higgs field as

$$\omega(\zeta) = \frac{\omega_{eff}(x)}{2} \left( \partial^\nu Q \partial_\nu Q - \gamma^2 \xi \sigma^2 A^\nu A_\nu \right),$$

where $\omega_{eff}(Q) = \omega(\sqrt{\xi} \sigma + Q)$. Now, to implement the cosmological principle we make the gauge election: $\theta_\mu = A_\mu$ or equivalently $\bar{A}_\mu = 0$. Under this gauge election, the terms in (20) that depend of the electromagnetic field $A_\mu$ become null and thus the action (18) becomes

$$S = \int d^4x \frac{\sqrt{-h}}{16\pi G} \left[ \frac{\mathcal{R}}{16\pi G} + \frac{1}{2} \omega_{eff}(Q) h^{\mu\nu} Q_{\mu\nu} Q - V_{eff}(Q) \right].$$

where $V_{eff}(Q) = V(\sqrt{\xi} \sigma + Q)$. In this manner, in order to have a scalar field with a canonical kinetic term we implement the field transformation

$$\phi(x^\sigma) = \int \sqrt{\omega_{eff}}(Q) dQ.$$  

Thus, the action (27) in terms of $\phi$ results

$$S = \int d^4x \sqrt{-h} \left[ \frac{\mathcal{R}}{16\pi G} + \frac{1}{2} h^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - U(\phi) \right],$$

where

$$U(\phi) = V_{eff}[Q(\phi)] = \frac{\lambda}{4} \left[ (\sqrt{\xi} \sigma + Q(\phi))^2 - \sigma^2 \right]^2.$$
is the potential associated to the new field $\phi$. The field equations obtained from the action \cite{29} read

\begin{equation}
G_{\alpha\beta} = -8\pi G[\phi,\alpha,\phi,\beta - \frac{1}{2} h_{\alpha\beta} (\phi^\mu \phi,_{\mu} - 2U(\phi))],
\end{equation}

\begin{equation}
\Box \phi + U'(\phi) = 0,
\end{equation}

with $\Box$ denoting the D’Alambertian operator and the prime representing derivative with respect to $\phi$.

Thus, in order to consider the Higgs inflationary model developed in \cite{31}, we will use the anzats

\begin{equation}
\omega_{\text{eff}}(Q) = \frac{1}{\left[1 - \beta^2 (\sqrt{\xi} \sigma + Q)^4\right]^{1/2}},
\end{equation}

where $\beta$ is a constant parameter with units of $M_p^{-2}$. Hence, it follows from \cite{32} that

\begin{equation}
\phi = \frac{\sqrt{\xi} \sigma + Q}{\left[1 - \beta^2 (\sqrt{\xi} \sigma + Q)^4\right]^{1/4}}.
\end{equation}

It can be verified that when $1 - \beta^2 (\sqrt{\xi} \sigma + Q)^4 > 0$ the expression \cite{33} is free of pole singularities. Such condition is fulfilled during inflation. Therefore the potential \cite{30} acquires the form

\begin{equation}
U(\phi) = \frac{\lambda}{4\xi^2} \left(\frac{\phi^4}{1 + \beta^2 \phi^4}\right).
\end{equation}

It is not difficult to see that the choice of the anzats \cite{33} allows the effective Higgs potential \cite{30} to exhibit a plateau for large enough field values, making possible a suitable slow-roll inflation. Something similar is used for example in \cite{31}. After inflation begins the condition $\beta^2 \phi^4 \ll 1$ holds, and the potential \cite{35} can be approximated by

\begin{equation}
U(\phi) \simeq \frac{\lambda}{4\xi^2} \phi^4.
\end{equation}

With the idea in mind to study non-perturbative gauge invariant scalar fluctuations of the metric we will use the non-perturbative formalism introduced in \cite{31}. In this formalism the amplitude of scalar fluctuations is arbitrary. Thus, we consider the perturbed line element

\begin{equation}
ds^2 = e^{2\tilde{\psi}} dt^2 - a^2(t)e^{-2\tilde{\psi}}(dx^2 + dy^2 + dz^2),
\end{equation}

where $\tilde{\psi}(t,x,y,z)$ is a metric function describing gauge invariant scalar fluctuations of the metric in a non-perturbative manner and $a(t)$ is the cosmic scale factor.

Inserting \cite{37} in \cite{31}, the perturbed field equations read

\begin{equation}
e^{-2\psi} \left(3H^2 - 6H\dot{\psi} + 3\dot{\psi}^2\right) + \frac{e^{2\psi}}{a^2} \left[2\nabla^2 \psi - (\nabla \psi)^2\right] = 8\pi G \left[\frac{1}{2} \dot{\phi}^2 - \frac{e^{2\psi}}{2a^2} (\nabla \phi)^2 + U(\phi)\right],
\end{equation}

\begin{equation}
e^{-2\psi} \left(2\dot{\psi}^2 - 5\dot{\psi}^2 + 8H\dot{\psi} - \frac{2\ddot{\psi}}{a} - H^2\right) + \frac{e^{2\psi}}{3a^2} (\nabla \psi)^2 = 8\pi G \left[\frac{1}{2} e^{-2\psi} \phi^2 - \frac{e^{2\psi}}{6a^2} (\nabla \phi)^2 - U(\phi)\right],
\end{equation}

\begin{equation}
\frac{1}{a} \frac{\partial}{\partial x^i} \left(\frac{\partial}{\partial t} (\phi^2)\right) - \frac{\partial \phi}{\partial t} \frac{\partial \phi}{\partial x^i} = 4\pi G \phi \frac{\partial \phi}{\partial x^i},
\end{equation}

where the dot is denoting time derivative. With the help of \cite{32} and \cite{47} the equation that determines the dynamics of $\phi$ is given by

\begin{equation}
\ddot{\phi} + (3H - 4\dot{\psi})\dot{\phi} - \frac{e^{4\psi}}{a^2} \nabla \phi + e^{2\psi} U'(\phi) = 0.
\end{equation}

An algebraic manipulation of \cite{35} and \cite{39} leads to

\begin{equation}
e^{-2\psi} \left(4H^2 + \frac{2\ddot{\psi}}{a} - 2\dot{\psi} - 14H\dot{\psi} + 8\dot{\psi}^2\right) + \frac{e^{2\psi}}{a^2} \left[2\nabla^2 \psi - \frac{4}{3} (\nabla \psi)^2\right] = 8\pi G \left[\frac{2}{3a^2} e^{2\psi} (\nabla \phi)^2 + 2U(\phi)\right],
\end{equation}

which determines the dynamics of the scalar fluctuations of the metric $\psi$.

\section{IV. Gauge Invariant Scalar Fluctuations of Small Amplitude}

In order to obtain the power spectrum of scalar fluctuations of the metric during inflation it is necessary to consider the impact of quantum amplitudes of $\psi$ on cosmological scales at the end of inflation. Therefore a linear approximation of the equations \cite{40} to \cite{43} will be sufficient to model such small quantum scalar fluctuations $\psi$. Hence, we can use the formula: $e^{\pm n\psi} \simeq 1 \pm n\psi$. In this scenario the gauge invariance of $\psi$ can be assured and the weak field limit for the inflaton holds. Thus, it is valid the semiclassical approximation $\phi(t, x^i) = \phi_0(t) + \delta \phi(t, x^i)$ where $\phi_0(t) = \langle E|\phi|E\rangle$ is the background classical field with $|E\rangle$ denotes a physical quantum state determined by the Bunch-Davies vacuum \cite{32} and $\delta \phi$ describes the quantum fluctuations of the field $\phi$.

In this manner, linearization of the differential line element \cite{44} reads

\begin{equation}
ds^2 = (1 + 2\tilde{\psi}) dt^2 - a^2(t)(1 - 2\tilde{\psi})(dx^2 + dy^2 + dz^2).
\end{equation}
Analogously, a linearization procedure of the field equations (38) and (39) lead to the classical equations
\[ 3H^2 = 8\pi G \left( \frac{1}{2} \frac{\ddot{\phi}_b}{\phi_b} + U(\phi_b) \right), \quad (45) \]
\[ -2\frac{\ddot{a}}{a} - H^2 = 8\pi G \left( \frac{1}{2} \frac{\ddot{\phi}_b}{\phi_b} - U(\phi_b) \right), \quad (46) \]
The quantum part obtained from the linearization of (43) is given by
\[ \ddot{\psi} + 7H\dot{\psi} - \frac{1}{\alpha^2} \nabla^2 \psi + (6H^2 + 2\dot{H})\psi = -8\pi G U'(\phi_b)\delta\phi. \quad (47) \]
With the help of the linearization of (40) and (41) we obtain the relation
\[ \delta\phi = \frac{1}{4\pi G\phi_0} \left( H\psi + \ddot{\psi} \right), \quad (48) \]
Inserting (48) in (47) we arrive to
\[ \ddot{\psi} + \left( 7H + \frac{2U'(\phi_b)}{\phi_b} \right) \dot{\psi} - \frac{1}{\alpha^2} \nabla^2 \psi + \left( 6H^2 + 2\dot{H} + \frac{2U'(\phi_b)}{\phi_b} \right) \psi = 0. \quad (49) \]
The linearization of (42) gives the system
\[ \ddot{\phi}_b + 3H\dot{\phi}_b + U'(\phi_b) = 0, \quad (50) \]
\[ \delta\phi + 3H\delta\phi - 4\phi_b \psi - \frac{1}{\alpha^2} \nabla \delta\phi + U''(\phi_b)\delta\phi + 2U'(\phi_b)\psi = 0. \quad (51) \]
Under the slow-roll condition \(|\dot{\phi}_b^2/2| \ll |U(\phi_b)|\) it follows from (45) and (50) that
\[ \dot{\phi}_b = -\frac{M_p}{\sqrt{3}} \frac{U'(\phi_b)}{\sqrt{U(\phi_b)}}, \quad (52) \]
where we have taken \(M_p = (8\pi G)^{-1/2}\). This equation determines the background inflaton field dynamics.

Employing (35) and (52) the background field \(\phi\) is given by the equation (33)
\[ t - t_0 + \frac{\beta^2}{6\mu} \left( \phi^4 + \sqrt{1 + \beta^2 \dot{\phi}_b^2} - \phi_0^4 \sqrt{1 + \beta^2 \phi_0^2} \right) + \frac{2}{3\mu} \left( \sqrt{1 + \beta^2 \phi_0^2} - \sqrt{1 + \beta^2 \dot{\phi}_b^2} \right) + \frac{1}{2\mu} tanh^{-1} \left( \frac{1}{\sqrt{1 + \beta^2 \phi_0^2}} \right) - \frac{1}{2\mu} tanh^{-1} \left( \frac{1}{\sqrt{1 + \beta^2 \dot{\phi}_b^2}} \right) = 0, \quad (53) \]
where \(\mu = \sqrt{M_p^3/3\lambda}\) and \(\phi_0 = \phi(t_0)\), with \(t_0\) being the time when inflation begins. Using the potential (36) the expression (53) becomes
\[ \phi_b(t) = \phi_e e^{2M_p \sqrt{\pi}} (t_e - t). \quad (54) \]
where \(\phi_e = \phi(t_e)\) with \(t_e\) denoting the time when inflation ends. Inserting (36) and (51) in (15) we obtain a scale factor of the form
\[ a = a_e \exp \left( \frac{\phi_e^2}{8M_p^2} \left( 1 - \exp \left( 4M_p \sqrt{\frac{\lambda}{3\xi^2}} (t_e - t) \right) \right) \right), \quad (55) \]
When \(t \approx t_e\) the scale factor (55) can be approximated by
\[ a(t) \approx \tilde{a}_e \exp \left( \frac{\phi_e^2}{2M_p^2} \sqrt{\frac{\lambda}{3\xi^2}} (t_e - t) \right), \quad (56) \]
where \(\tilde{a}_e = a_e \exp \left( -\frac{\phi_e^2}{2M_p^2} \sqrt{\frac{\lambda}{3\xi^2}} t_e \right)\). Thus, the Hubble parameter obtained from (55) reads
\[ H(t) = \frac{1}{\sqrt{3}\xi M_p} \sqrt{\frac{\lambda}{\xi^2}} \phi_e^2 \exp \left( 4M_p \sqrt{\frac{\lambda}{3\xi^2}} (t_e - t) \right). \quad (57) \]
Near the end of inflation, according to (58), the Hubble parameter becomes
\[ H_e = H\big|_{t=t_e} \approx \frac{\phi_e^2}{2M_p^2} \sqrt{\frac{\lambda}{3\xi^2}}. \quad (58) \]
On the other hand, Planck data indicate that Higgs inflation requires an energy scale that corresponds to an initial Hubble parameter \(H_0 \approx 10^{11} - 10^{12} GeV\), which is inferred for an average Higgs mass of the order \(M_h \approx 125.7 GeV\). \(34, 35\). Therefore
\[ H_0 \approx \frac{\lambda}{2\sqrt{3} \beta\xi M_p} \approx 10^{11} - 10^{12} GeV. \quad (59) \]
It is not difficult to verify that for \(\lambda = 0.13\) and \(M_p = 1.22 \times 10^{19} GeV\), the parameter \(\xi\) must range in the interval: \([3.7528 \cdot 10^{-14}, 3.7528 \cdot 10^{-13}] (\beta M_p)^{-1}(GeV)^{-1}\).

Now, we are in position to quantize the theory. In order to do that we will follow a canonical quantization procedure. Thus, we impose the commutation relation
\[ [\psi(t, \vec{x}), \Pi_\psi(t, \vec{x}')] = i\delta^{(3)}(\vec{x} - \vec{x}'), \quad (60) \]
where \(\Pi_\psi = \partial L/\partial \dot{\psi}\) is the canonical conjugate momentum to \(\psi\) and \(L\) denotes the lagrangian given in this case by
\[ L = \sqrt{-h} \left[ \frac{R}{16\pi G} + \frac{1}{2} h^{\mu\nu} \phi_\mu \phi_\nu - U(\phi) \right], \quad (61) \]
where \(R\) is the Ricci scalar curvature which has the form
\[ R = \left( 6H^2 + 6\frac{\ddot{a}}{a} - 30H \ddot{\psi} - 9\dot{\psi} + 18\dot{\psi}^2 \right) e^{-2\psi} + \frac{2}{a^2} (\nabla \psi - (\nabla \psi)^2) e^{2\psi}. \quad (62) \]
With the help of equations (61) and (62) the relation (60) becomes

\[ [\psi(t, \bar{x}), \psi(t, \bar{x}')] = i \frac{4\pi G}{\sqrt{-h}} \delta^{(3)}(\bar{x} - \bar{x}'). \]  

(63)

To simplify the structure of (49) we introduce the auxiliary field defined by the formula

\[ \psi(t, \bar{x}) = \exp \left[ -\frac{1}{2} \int (7H + \alpha) dt \right] \zeta(t, \bar{x}), \]  

(64)

where \( \alpha = 2U' \langle \phi_b \rangle / \dot{\phi}_b \). The equation (49) in terms of \( \zeta \) then reads

\[ \ddot{\zeta} - \frac{1}{a^2} \nabla^2 \zeta - \left[ \frac{3}{2} \dot{H} + \frac{1}{2} \dot{\alpha} + \frac{25}{4} H^2 \right] \zeta = 0. \]  

(65)

Expanding the field \( \zeta(t, \bar{x}) \) in Fourier modes we have

\[ \zeta(t, \bar{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left[ a_k e^{i \bar{k} \cdot \bar{x}} \Theta_k(t) + a_k^\dagger e^{-i \bar{k} \cdot \bar{x}} \Theta_k^*(t) \right], \]  

(66)

where \( a_k \) and \( a_k^\dagger \) denote the creation and annihilation operators obeying the commutation algebra

\[ [a_k, a_{k'}^\dagger] = \delta^{(3)}(\bar{k} - \bar{k}'), \]  

(67)

\[ [a_k, a_{k'}] = [a_k^\dagger, a_{k'}^\dagger] = 0. \]  

(68)

The asterisk mark (*) is denoting complex conjugate and the dagger (†) transpose complex conjugate. Thus, it follows from (65) that the modes \( \Theta_k \) obey

\[ \ddot{\Theta}_k + \left[ \frac{k^2}{a^2} - \frac{3}{2} \dot{H} - \frac{1}{2} \dot{\alpha} - \frac{25}{4} H^2 - \frac{5}{2} \alpha H - \frac{1}{4} \alpha^2 \right] \Theta_k = 0. \]  

(69)

Using (66), (68) and (63) we obtain

\[ \Theta_k + \left[ \frac{k^2}{a^2} - \frac{3}{2} \dot{H} - \frac{1}{2} \dot{\alpha} - \frac{25}{4} H^2 - \frac{5}{2} \alpha H - \frac{1}{4} \alpha^2 \right] \Theta_k = 0. \]  

(70)

Thus, at the end of inflation (70) can be approximated by

\[ \alpha \bigg|_{t = t_e} \simeq -\frac{6 \gamma_e H_e}{\phi^3_e}, \]  

(71)

where

\[ \gamma_e = \frac{\phi^3_e}{1 + \beta^2 \phi^4_e} - \frac{\beta \phi^7_e}{(1 + \beta^2 \phi^4_e)^2}. \]  

(72)

It follows from (63), (69), (71) and (68) that the normalization condition is given by

\[ \Theta_k^\dagger \Theta_k - \Theta_k^\dagger \Theta_k = i \frac{4\pi G}{9a^3_e} \Theta_k = 0. \]  

(74)

Thus considering a Bunch-Davies vacuum condition the normalized solution of (73) results to be

\[ \Theta_k(t) = \frac{1}{6M_P} \sqrt{\frac{\pi}{2a^2_e H_e}} \mathcal{H}_\nu(\nu) |z(t)|, \]  

(75)

where \( \mathcal{H}_\nu(\nu) \) is the second kind Hankel function, the index \( \nu = (-6\gamma_e + 5\phi^2_e)/(2\phi^3_e) \) and

\[ z(t) = \frac{k}{a_e H_e} e^{-H_e t}. \]  

(76)

The squared quantum fluctuations of \( \psi \) in the IR-sector (on cosmological scales) are given by

\[ \langle \psi^2 \rangle = \frac{1}{2\pi^2} \int \psi^2 \left( \frac{6\gamma_e}{\phi^3_e} \right) d^3k \Theta_k \Theta_k^\dagger |_{IR}, \]  

(77)

where \( \epsilon = k_{IR}/k_p \ll 1 \) is a dimensionless parameter, \( k_{IR} = k_e(t_e) \) is the wave number related to the Hubble radius at the time when the modes re-enter the horizon \( t_r \) and \( k_p \) is the Planckian wave number. For a number of e-foldings \( N = 63 \) the parameter \( \epsilon \) ranges between \( 10^{-5} \) and \( 10^8 \), which corresponds to a Hubble parameter at the end of inflation of order \( H_e = 0.5 \cdot 10^{-9} M_p \).

At the end of inflation, on cosmological scales, we can use \( \mathcal{H}_\nu(\nu) |z(t)| \simeq (\pi/\nu!) \Gamma(\nu) |z(t)|^{-\nu} \). Thus according to (77) and (75) we obtain

\[ \langle \psi^2 \rangle = \frac{2^{2\nu-4} \Gamma^2(\nu)}{9\pi^2} \left( \frac{H_e^2}{M_p^2} \left( \alpha_e H_e \right)^{3-2\nu} \right) \int_0^{k_h} \frac{dk}{k} k^{3-2\nu}, \]  

(78)

where \( k_h = \tilde{a}_e \sqrt{(25/4)H_e^2 + (5/2)\alpha_e H_e + (1/4)\alpha^2_e} \). Hence, the corresponding power spectrum reads

\[ P_s(k) = \frac{2^{2\nu-2} \Gamma^2(\nu)}{9\pi^2} \left( \frac{H_e}{2\pi} \right)^2 \left( \frac{H_e}{M_p^2} \left( \alpha_e H_e \right)^{3-2\nu} \right) \left( \frac{k}{\tilde{a}_e H_e} \right)^{3-2\nu}. \]  

(79)

Notice that for nearly scale invariant: \( \nu \simeq 3/2 \) it follows from (79) that \( P_s(k) \big|_{\nu=3/2} \simeq H_e^2/4\pi^2 \). The scale invariance is achieved when \( \gamma_e \simeq (1/3)\phi^3_e \). This condition leaves to

\[ \frac{1}{1 + \beta^2 \phi^4_e} - \frac{\beta^2 \phi^7_e}{(1 + \beta^2 \phi^4_e)^2} \simeq \frac{1}{3}. \]  

(80)
Solving this equation we obtain that it is satisfied for the value: \( \phi_c = \left[ (\sqrt{3} - 1) \beta^2 \right]^{1/4}/ \beta \). The spectral index is then \( n_s = 4 - 2\nu = (6\gamma_c/\phi_c^2) - 1 \). Hence it is not difficult to show that

\[
n_s = 6 \left[ \frac{1}{1 + \beta^2 \phi_c^4} - \frac{\beta^2 \phi_c^4}{(1 + \beta^2 \phi_c^4)^2} \right] - 1. \tag{81}
\]

The Planck 2018 observational results indicate that the spectral index ranges in the interval \( n = 0.968006 \pm 0.002 \). It follows from \([31]\) that the inflation field at the end of inflation in terms of \( n_s \) is given by the formula

\[
\phi_c = \frac{1}{\sqrt{\beta}} \left[ \frac{6}{\sqrt{6(1 + n_s)}} - 1 \right]^{1/4}. \tag{82}
\]

In this manner, we obtain \( \phi_c < M_p \) when the condition \( \beta > \left[ (6 \sqrt{6(1 + n_s)} - 1) \right]^{1/2} M_p^{-2} \). For example when \( n_s = 0.9735 \) the previous condition reduces to \( \beta > 0.8623 M_p^{-2} \).

On the other hand, the scalar to tensor ratio is given by \([33]\)

\[
r \simeq \frac{128}{\beta^{2/3} M_p^{2/3} (24N)^{5/3}}, \tag{83}
\]

where \( N \) is the number of e-foldings at the end of inflation. For \( N = 63 \) we obtain that \( r < 0.10 \), as indicated by Planck observations \([36]\), when \( \beta > 5.15 \cdot 10^{-4} M_p^{-2} \). Thus for the aforementioned limit \( \beta > 0.8623 M_p^{-2} \) the condition \( r < 0.10 \) can be perfectly satisfied. For example, for \( \beta = 0.9 M_p^{-2} \) we obtain \( r = 6.8 \cdot 10^{-4} \). For this particular case we obtain from \([32]\) that \( \phi_c = 0.97 \). For \( \beta = 16.29053 M_p^{-2} \) we obtain a scalar to tensor ratio \( r = 1 \cdot 10^{-4} \). This value corresponds to \( \phi_c = 0.23 M_p \). Thus the transplanckian problem is avoided in this model.

V. FINAL REMARKS

In this letter we have studied gauge invariant fluctuations of the metric in the framework of a recently proposed Higgs inflationary model were the Higgs field has a geometrical origin. The model has been developed in the theoretical context of a geometrical complex scalar-tensor theory of gravity in which the scalar field form part of the affine structure of the space-time manifold and the gravitational field has a scalar and tensor components. The background geometry was determined via a Palatini variational principle. The description is made from two equivalent frames related by the Weyl transformations in such manner that the Ricci tensor remains unaltered avoiding in this way the unitarity problem \([21, 37]\). The Higgs scalar field plays the role of the inflaton field and it has its origin from the geometrical Weyl scalar field by means of a particular Weyl transformation. In the model the original Higgs potential is rescaled by the non-canonical kinetic function \( \omega(Q) \) associated to the Weyl-scalar field, physically making possible to have the enough energy to inflate the universe. We have considered an ansatz for the \( \omega(Q) \) function in order to create the enough plateau for the inflationary potential to achieve an energy scale for Higgs inflation corresponding to an initial Hubble parameter \( H_0 \approx 10^{11} - 10^{12} \text{GeV} \), which is in agreement with the requirements of PLANCK data for this kind of inflation \([34, 35]\).

In order to study gauge invariant scalar fluctuations of the metric we started obtaining the dynamical field equations \([38, 41]\) for the metric function \( \psi \) that describes the aforementioned fluctuations in a non-perturbative approach, as the one described in \([31]\). As a particular case we have focused in the linear fluctuations and we obtain a nearly scale invariant spectrum at the end of inflation when \( \gamma_c \approx (1/3) \phi_c^3 \). We get a scalar index \( n_s = 0.9735 \) and a scalar to tensor ratio \( r \approx 6.8 \cdot 10^{-4} \) when \( \beta = 0.9 M_p^{-2} \). Moreover, for \( \beta = 16.29053 M_p^{-2} \) we obtain a scalar to tensor ratio \( r = 1 \cdot 10^{-4} \). This value corresponds to \( \phi_c = 0.23 M_p \). Thus, we can say that our model fits well with the Planck 2018 observational data related with the inflationary epoch. In addition, these values for \( \phi_c \) indicate that this model is free of the transplanckian problem. Finally, we would like to mention that there are several inflation models with very small value of the tensor to scalar ratio \( r \), as for example Kähler-moduli and some D-brane, inflationary models. In fact, it may be expected a next generation of cosmological observational data with a best defined uncertainty for \( r \), with upper limits shorter than the have nowadays, \( r < 0.002 \) (95% C.L.) \([38]\).

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