Hybrid Model of Singular Spectrum Analysis and ARIMA for Seasonal Time Series Data

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ABSTRACT

Hybrid models between Singular Spectrum Analysis (SSA) and Autoregressive Integrated Moving Average (ARIMA) have been developed by several researchers. In the SSA-ARIMA hybrid model, SSA is used in the decomposition and reconstruction process, while forecasting is done through the ARIMA model. In this paper, hybrid SSA-ARIMA uses two auto grouping models. The purpose of this paper is to analyze seasonal data using the SSA-ARIMA hybrid by auto grouping. The first model namely the Alexandrov method and the second method is alternative auto grouping with long memory approach. The two hybrid models were tested for two types of seasonal pattern, multiplicative and additive seasonal time series data. The analysis results using both methods give accurate result; as seen from the MAPE generated the 12 observations for future, the value is below 5%. For additive seasonal pattern, The hybrid SSA-ARIMA method with Alexandrov auto grouping is more accurate (MAPE = 0.13%) than the hybrid SSA-ARIMA method with Alternative method but for multiplicative seasonal pattern the hybrid SSA-ARIMA with alternative auto grouping is more accurate (MAPE = 3.63%) than the hybrid SSA-ARIMA method with Alexandrov method.

Keywords: ARIMA; Automatic Grouping; Long Memory Effect; Seasonal Pattern, Singular Spectrum Analysis

INTRODUCTION

Singular Spectrum Analysis (SSA) is a relatively new non-parametric method that has proved its capability in various time series types. Solving all these problems correspond to the so-called basic capabilities of SSA. Besides, the method has several extensions. First, the multivariate version of the method permits the simultaneous expansion of several time series data; see, for example, [1]. Second, the SSA ideas lead to several forecasting procedures for time series; see [2]. Third, SSA has been utilized for change-point detection in time series. The SSA technique has been used as a filtering method in [3]. Fifth, a family of the causality test based on the multivariate SSA technique has been introduced in [4]. Sixth, SSA can be applied for missing value imputation [5].

SSA can be applied in various disciplines, from mathematics and physics to economics and financial mathematics, meteorology and oceanography, to social sciences.
For instance, in climatology ([6], [7], [8]) and biomedical data time series analysis [9]. Hybrid modeling of SSA in time series data has been carried out by many researchers. The hybrid model is carried out so that the advantages of two or more models make a positive contribution to the forecasting results.

SSA hybrid model with other time series models includes ARIMA, Neural network, ARIMAX, PAR, VARIMAX, and others. [10], performed hybrid SSA with Neural Network. [11] perform the hybrid SSA-Algorithm Firefly-BP Neural Network process. [12] carried out a hybrid SSA model with ARMAX. [13] Combining the SSA model with PAR(p), this model was applied to wind speed data. [14], built the SSA-VARIMAX hybrid model and used it for climate data. The ARIMA model is often used as a comparison for the SSA model, such as [15], comparing SSA, ARIMA, and other time series models for tourism cases in various countries in Europe. The result has indicated that there is no good time series model for all tourism data. [16] compared SSA and ARIMA for predicting ambulance demand.

The SSA-ARIMA hybrid model studied by [17] was applied to the annual Runoff data. [18], the SSA-ARIMA hybrid model was compared with the basic SSA and ARIMA models. The result showed that the SSA-ARIMA hybrid model was the most accurate. However, many of these papers do not discuss specific data forms (e.g., seasonal patterns), so we consider it necessary to examine this hybrid model for seasonal data. In this study, the SSA and the ARIMA were employed collectively to forecast two types of time series data. Both models run to get fast and accurate computation. In SSA, there are two methods of automatic grouping (Alexandrov and Alternative). The forecasting performance of the hybrid SSA-ARIMA model was compared between the two methods (alternative vs. Alexandrov). This paper contributes to the analysis of the seasonal patterns (additive and multiplicative) by the SSA-ARIMA hybrid. The purpose of this paper was to analyze seasonal data using the SSA-ARIMA hybrid by auto grouping for two types of seasonal patterns.

This paper was organized as follows: The current section was an introduction where we briefly outlined the use of SSA and introduced our study. In the next section, the methods section, we described the detailed methodology of SSA and ARIMA, briefly outlined forecasting using a linear recurrent formula, identification of fractional differencing parameter, identification of hidden periodicities based on Periodogram and Automatic grouping on Alexandrov Method ([19], [20]) also alternative Automatic grouping [21]. This section also included a proposed algorithm for automatic hybrid SSA-ARIMA. In the results and discussion section, we demonstrated the abilities of hybrid SSA-ARIMA in real-time series data. In this part, we also investigated three types of time series data: Seasonal with no trend, multiplicative seasonal with the trend, and Additive seasonal with the trend. This section also discussed the comparison result between hybrid SSA-ARIMA with the Alexandrov method and hybrid SSA-ARIMA with an alternative method for real data analysis.

**METHODS**

**Singular Spectrum Analysis**

The (non-parametric) SSA method has received a fair amount of attention in the literature. The first phase of SSA is the decomposition, where the time series are broken down into four components: trend, seasonal, cyclical, and noise. This phase consists of the Embedding and Singular Value Decomposition steps. The second phase, namely the Reconstruction phase, consists of Grouping and Diagonal Average process. The
Forecasting process can be done once the four stages have been completed. For the completeness of presentation of our method, we presented the complete phase of the SSA algorithm in the following section.

**Embedding**

The embedding step will transform one-dimensional time series $X = (x_1, x_2, \ldots, x_T)$ into multi-dimensional series $X_1, X_2, \ldots, X_K$ with vectors $X = (X_i, X_{i+1}, X_{i+2}, \ldots, X_{i+L-1})^T \in \mathbb{R}^L$, where $i = 1, 2, \ldots, K$, $K = T - L + 1$. The parameter window length $L$ defines the embedding process, where $2 \leq L \leq T - 1$ [22]. If we need to emphasize the size (dimension) of the vectors $X_i$, then we shall call them $L$-lagged vectors. The $L$-trajectory matrix (or simply the trajectory matrix) of the series $X$ is defined as

$$ X = \begin{bmatrix} x_1 & x_2 & \cdots & x_K \\ x_2 & x_3 & \cdots & x_{K+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_L & x_{L+1} & \cdots & x_T \end{bmatrix} $$

The lagged vectors $X_i$ are the columns of the trajectory matrix $X$. Both the rows and column of $X$ are sub-series of the original series. The $(i,j)$ element of matrix $X$ is $x_{ij} = x_{i+j-1}$ which yields that $X$ has equal elements on the ‘antidiagonals’ $i+j=\text{const}$. Hence the trajectory matrix is a Hankel matrix.

**Singular Value Decomposition**

The second step, the SVD step, makes the singular value decomposition of the trajectory matrix $X$ and represents it as a sum of rank-one bi-orthogonal elementary matrices. Set $S = XX^T$ and denoted by $\lambda_1, \lambda_2, \ldots, \lambda_L$ the eigenvalues of $S$ taken in the decreasing order of magnitude $(\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_L \geq 0)$ and by $U_1, U_2, \ldots, U_L$ the orthonormal system of the eigenvectors of the matrix $S$ corresponding to these eigenvalues.

$$ d = \text{max}\{i, \text{such that } \lambda_i > 0\} = \text{rank } X $$

If we denote $V_i = \frac{X^T u_i}{\sqrt{\lambda_i}}$, then the SVD of the trajectory matrix can be written as

$$ X = X_1 + X_2 + \cdots + X_d, $$

where eigenvector $U_i$, Eigenvalues $\sqrt{\lambda_i}$ form matrix $V_i^T X$. The three elements of SVD forming are called eigen triple.

**Grouping**

The purpose of this step is to appropriately identify the trend, the oscillatory components with different periods and noise. This step can be skipped if one does not want to extract hidden information by regrouping and filtering components precisely. The grouping procedure partitions the set of indices $1, 2, \ldots, L$ into $m$ disjoint subsets $I = I_1, I_2, \ldots, I_m$, so the elementary matrix in equation (2) is regrouped into $m$ groups. Let $I = \{i_1, i_2, \ldots, i_p\}$. Then the resultant matrix $X_i$ corresponding to the group $i$ is defined as $X_i = X_{i_1} + X_{i_2} + \cdots + X_{i_p}$. The matrices are computed for $I_1, I_2, \ldots, I_m$, and substituted into equation (2) to obtain the new expansion.

The grouping process is the phase when the $L \times K$ matrix is grouped into several sub-groups, namely trend patterns, seasonal or periodic, and noise patterns. Here, in this paper, the patterns are identified by Fourier series analysis and long-memory analysis. Fourier series analysis is...
used to identify a seasonal pattern, and long memory series analysis is used to identify the differencing parameter of data. We use the GPH method [23] to identify the differencing parameter of time series.

Diagonal Averaging

The next step in Basic SSA transforms each resultant matrix of the grouped decomposition (3) into a new one-dimensional series of length \( N \) and is called (diagonal averaging). Let \( Y \) denote a matrix with orde \((L \times K)\), with the elements \( y_{ij} \), \( 1 \leq i \leq L, 1 \leq j \leq K \), and define \( L^* = \min(L, K), K^* = \max(L, K) \), and \( T = L + K - 1 \). Let \( y_{ij}^* = y_{ij} \) if \( L < K \) and \( y_{ij}^* = y_{ji} \) otherwise. Diagonal averaging transfers matrix \( Y \) into a series \( y_1, y_2, \ldots, y_N \) by the formula.

\[
y_k = \begin{cases} 
  \frac{1}{k} \sum_{m=1}^{k} y_{m,k-m+1}^* & \text{for } 1 \leq k < L \\
  \frac{1}{L^*} \sum_{m=1}^{L^*} y_{m,k-m+1}^* & \text{for } L^* \leq k < K^* \\
  \frac{1}{T-k+1} \sum_{m=k-K^*+1}^{T-k+1} y_{m,k-m+1}^* & \text{for } K^* < k \leq T 
\end{cases}
\]  

(2)

Using equation (3), the resultant matrix \( X_i, i=1,2,\ldots,m \) will form the series \( \{\hat{y}^{(k)}_t\} = \{\hat{y}^{(k)}_1, \hat{y}^{(k)}_2, \ldots, \hat{y}^{(k)}_T\} \); therefore, the original sequence will be decomposed into the sum of the \( m \) series.

Forecasting Using Linear Recurrent Formula

The forecasting method used in this paper is the Linear Recurrent Formula (LRF). In the above notation, the recurrent forecasting algorithm (briefly, R-Forecasting) can be formulated as follows.

1. The time series \( Y_{T+M} = (y_1, y_2, \ldots, y_{T+M}) \) is defined by

\[
y_i = \begin{cases} 
  \hat{x}_i & \text{for } i = 1,2,\ldots,T \\
  \sum_{j=1}^{L-1} a_j y_{i-j} & \text{for } i = T + 1, \ldots, T + M 
\end{cases}
\]  

(3)

2. The numbers \( y_{T+1}, y_{T+2}, \ldots, y_{T+M} \) form the \( M \) terms of the recurrent forecast

Thus, the R-forecasting is formed by the direct use of the LRF with coefficients \( \{a_j, j = 1, \ldots, L - 1\} \).

Autoregressive Integrating Moving Average

A time series \( \{x_t\} \) is said to follow the Autoregressive Integrated Moving Average model if the differenced that is \( W_t = \nabla^d x_t \) then this process is ARMA processes. If \( W_t \) is ARMA\((p,q)\), then \( \{X_t\} \) is ARIMA\((p,d,q)\) process. The mathematic model of ARIMA\((p,d,q)\) is as follow;
\[ \Phi(B)(1 - B)^d(X_t - \mu) = \theta(B)a_t \]  
(4)

Where

- \( t \) = index of observation
- \( d \) = differencing parameter (1,2,3,..)
- \( \mu \) = mean of observation
- \( a_t \sim IID(0,\sigma^2_a) \)

\[ \Phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p \] is a polynomial of AR\( (p) \)

\[ \theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_p B^p \] is polynomial of MA\( (q) \)

### Identification of Fraction differencing parameter

The fractional differencing operator is described as a limitless binomial series growth in powers of the backward-shift operator [24]. One of the methods to estimate the value of the fractional differencing parameter is the Geweke and Porter-Hudak method (GPH). According to the GPH method, the differencing parameter of time series data can be estimated by the least square method of the ARFIMA (autoregressive fractionally integrated moving average) model. The GPH procedure can be described as follows. For the ARFIMA model given in (4), let \( W_t = (1 - B)^\delta Z_t \) and let \( f_W(\omega) \) and \( f_Z(\omega) \) be the spectral density function of \( \{W_t\} \) and \( \{Z_t\} \), respectively. Then,

\[ f_Z(\omega_j) = f_W(\omega_j) \left( \frac{2sin\left(\frac{\omega_j}{2}\right)}{\omega_j}\right)^{-2\delta}, \omega_j \in (-\pi,\pi) \]  
(5)

where;

\[ f_Z(\omega_j) = \frac{\sigma^2}{2\pi} \left| \phi_p(e^{-i\omega_j}) \right|^2, \omega_j \in (-\pi,\pi) \] is the spectral density of a regular ARMA\( (p,q) \) model.

\[ \ln[f_Z(\omega_j)] = \delta \ln\left\{ \frac{2sin\left(\frac{\omega_j}{2}\right)}{\omega_j}\right\}^{-2} + \ln[f_W(\omega_j)], \]

\[ \ln[f_Z(\omega_j)] = \ln[f_W(0)] + \delta \ln\left\{ \frac{2sin\left(\frac{\omega_j}{2}\right)}{\omega_j}\right\}^{-2} + \ln\left[ \frac{f_W(\omega_j)}{f_W(0)} \right]. \]

Replacing \( \omega_j \) by the Fourier frequencies \( \omega_j = \frac{2\pi j}{T} \) and adding \( \ln[I_Z(\omega_j)] \) to both sides, the periodogram\( \{Z_t\} \) gives

\[ \ln[I_Z(\omega_j)] = \ln[f_W(0)] + \delta \ln\left\{ \frac{2sin\left(\frac{\omega_j}{2}\right)}{\omega_j}\right\}^{-2} + \ln\left[ \frac{I_Z(\omega_j)}{f_Z(\omega_j)} \right] + \ln\left[ \frac{f_W(\omega_j)}{f_W(0)} \right]. \]

For \( \omega_j \) closes to zero, i.e. for \( j = 1, 2, \ldots, T/2 \) such that \( m/T \to \infty \) we have \( \ln\left[ \frac{f_W(\omega_j)}{f_W(0)} \right] \approx 0 \), thus,

\[ Y_j = \beta_0 + \beta_1 X_j + e_j, \quad j = 1, 2, \ldots, m \]

\[ Y_j = \ln[I_Z(\omega_j)], \quad X_j = \ln\left( \frac{1}{4sin^2\left(\frac{\omega_j}{2}\right)} \right) \]

\[ \beta_0, \beta_1 \] are estimated by the least square method.
The least-square estimator of long memory coefficient is given by
\[
\hat{\beta}_1 = \frac{\Sigma_{j=1}^m (X_j - \bar{X})(Y_j - \bar{Y})}{\Sigma_{j=1}^m (X_j - \bar{X})^2}; \quad m = g(T) = T^\alpha, 0 < \alpha < 1
\] (7)

Identification of hidden Periodicities based on Periodogram

Identification of hidden periodicities based on Periodogram analysis can be described as follows;
1. Fit the data to equation (7):
   \[X_t = \sum_{k=0}^{[T/2]} (a_k^* \cos \omega_k t + b_k^* \sin \omega_k t),\]...

2. Compute \(a_k^*\) and \(b_k^*\) by (8) and (9):
   \[a_k^* = \begin{cases} \frac{1}{T} \sum_{t=1}^{T} x_t \cos \omega_k t & \text{for } k = 0 \text{ and } k = \frac{T}{2} \text{ if } T \text{ even} \\ \frac{2}{T} \sum_{t=1}^{T} x_t \cos \omega_k t & \text{for } k = 1, 2, ..., \frac{T-1}{2} \end{cases}\]
   \[b_k^* = \frac{2}{T} \sum_{t=1}^{T} x_t \sin \omega_k t, \quad k = 1, 2, ..., \frac{T-1}{2}\] (10)

3. Compute the values of ordinate \(I(\omega_k)\) by formula (10);
   \[I(\omega_k) = \begin{cases} Ta_0^2 & k = 0 \\ \frac{T}{2} (a_k^2 + b_k^2) & k = 1, 2, ..., \frac{T-1}{2} \\ Ta_k^2 & k = \left[\frac{T}{2}\right]\end{cases}\] (11)

4. Test for significance of every Fourier frequency as follow;
   \(H_0: \alpha = \beta = 0\) (data do not have a seasonal pattern)
   \(H_1: \alpha \neq 0 \quad \text{or} \quad \beta \neq 0\) (data have a seasonal pattern)
   Statistic test:
   \[T = \frac{I^{(1)}(\omega_{(1)})}{\Sigma_{k=1}^{[T/2]} I(\omega_k)}\] (12)
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where,

\[ I^{(1)}(\omega_i) \] : maximum ordinate of periodogram of Fourier frequency

\[ I(\omega_k) \] : Value of periodogram ordinate at k-th' Fourier frequency.

Test criteria:
Reject \( H_0 \) if \( T > g_\alpha \) with \( \alpha = \) significant level. The value of \( g_\alpha \) can be seen in table Fisher [25].

Automatic Grouping in SSA

In this section, we explain two methods SSA automatic grouping based on trend extraction based namely Alexandrov, and SSA automatic grouping based on long memory approach, namely Alternative method. First, we introduce the periodogram of a time series. Let us consider the Fourier representation of the elements of a time series \( X = (x_1, x_2, ..., x_T) \) of length \( T \).

\[
X_t = C_0 + \sum_{1 \leq k \leq \frac{N-1}{2}} \left( c_k \cos \left( \frac{2\pi tk}{T} \right) \right) + \sum_{1 \leq k \leq \frac{N}{2}} \left( s_k \cos \left( \frac{2\pi tk}{T} \right) \right) + (-1)^t C_{T/2},
\]

Where \( k \in \mathbb{Z}, 0 \leq t \leq T - 1 \) and \( c_{T/2} = 0 \) if \( T \) is an odd number. The periodogram of \( X \) at the frequencies \( \{ k/N \}_{k=0}^{\lfloor T/2 \rfloor} \) is defined as

\[
I_X(k/T) = \frac{T}{2} \begin{cases} 
2c_0^2 & k = 0 \\
(c_k^2 + s_k^2) & 0 < k < T/2 \\
2c_{T/2}^2 & k = T/2, T = \text{even}
\end{cases}
\]

Define the \( k \)th element of the discrete Fourier transform of \( X \) as

\[ F_k(X) = \sum_{t=0}^{T-1} e^{-i2\pi tk/T} x_t. \]

The periodogram \( I_X(k/T) \) at the frequencies \( \omega \in \{ k/T \}_{k=0}^{\lfloor T/2 \rfloor} \) can be simplified as

\[
I_X(k/T) = \frac{1}{T} \left( \frac{|F_k(X)|}{|F_k(X)|} \right)^2 \quad \text{for } 0 < k < T/2
\]

\[
I_X(k/T) = 1 \quad \text{for } k = 0 \text{ or } T \text{ is even and } k = T/2
\]

All frequency in the interval \( (0,0.5) \) is multiplied by two. This is done to ensure the following property;

\[
\|X\|^2 = \sum_{t=0}^{T-1} x_t^2 = \sum_{k=0}^{\lfloor T/2 \rfloor} I_X(k/T)
\]

Where \( \pi_X(\omega) = \sum_{k=0}^{\lfloor k \leq \omega \} I_X(k/T), \omega \in [0,0.5] \) is the cumulative contribution of the frequencies \([0,\omega]\)? Then, for given \( \omega_0 \in (0,0.5) \), [19] define the contribution of low frequencies from the interval \([0,\omega_0]\) to \( X \in \mathbb{R}^T \) as

\[
C(X, \omega_0) = \frac{\pi_X(\omega_0)}{\pi_X(0.5)}
\]

Then, given parameters \( \omega_0 \in (0,0.5) \) and \( \omega_0 \in [0,1], [19] \) proposed to select those SVD components whose eigenvectors satisfy the following criteria:

\[
C(U_j, \omega_0) \leq C_0
\]
where $U_j$ is a corresponding $j$th eigenvector.

The second procedure (alternative method) of the automatic Grouping Algorithm can be described using the following steps:

1. Denote the original series as $X = (x_1, x_2, ..., x_T)$, the univariate time series data with length $T$.
2. Define Window Length $(L) 2 \leq L \leq T/2$.
3. Determine the shape of the trajectory matrix based on $L$ as in equation (3).
4. Find the Eigen Triple $\left( \sqrt{\lambda_i}, V_i, U_i \right)$, $i = 1, 2, ..., d$ ($d$ is the number of positive eigenvalues) of the $XX^T$ matrix, where $X$ is the Trajectory matrix.
5. Identify the $\hat{\delta}$ (Order difference) of each series $U_i$ by equation (6) in section 2.3.
6. Make a differencing for all $U_i$ based on the value of $\hat{\delta}$ (step 5) to obtain the $UD_i$ series. Using this step, then we obtain data with the stationary pattern.
7. Specify the maximum value of seasonal order of $P_i$ of $UD_i$. Here, the $P_i$ values are between 0 and 24.
8. Group $UD_i$ series based on $P_i$ values obtained from the previous step 7, a grouping of $UD_i$ is based on periods, 3, 6, 12, and 24, $UD_i$ with $P_i=0$ is grouped as non-seasonal data, where when $P_i>24$ is considered as cycle pattern.
9. Create diagonal averaging for grouping components from step 8; see equation (3).
10. Forecasting (Linear Recurrent Method) with equation (4).
11. Determine MAPE values based on the number of our sample data. The MAPE of the $h$-step ahead forecast is defined as

$$ MAPE = \left( \frac{1}{h} \sum_{i=1}^{h} \left| \frac{y_{n+h} - \hat{y}_{n+h}}{\hat{y}_{n+h}} \right| \right) \times 100\% $$

(17)

where $n$ denotes the last data used in the in-sample data and $\hat{y}_{n+h}$ denotes the forecast values.
12. Go to step 2 until all of the $L$ choices have been calculated. The best model will give the smallest MAPE value.

**Algorithm for hybrid SSA-ARIMA**

In this section, we proposed our new automatic algorithm on Singular Spectrum Analysis. The procedure of our automatic Grouping Algorithm can be described using the following steps:

1. Denote the original series as $X = (x_1, x_2, ..., x_T)$, the univariate time series data with length $T$.
2. Define Window Length $(L) 2 \leq L \leq T/2$.
3. Determine the shape of the trajectory matrix based on $L$ as in equation (3).
4. Find the Eigen Triple $\left( \sqrt{\lambda_i}, V_i, U_i \right)$, $i = 1, 2, ..., d$ ($d$ is the number of positive eigenvalues) of the $XX^T$ matrix, where $X$ is the Trajectory matrix.
5. Use one method of automatic grouping procedures (section 2.4). This procedure takes the best parameter (window length ($L$)).
6. Build Model ARIMA of all grouping (RC's) by Auto-ARIMA procedures.
7. Forecast a new series of all groups by model ARIMA.
8. Sum all results of forecast to get predicted of new time series.
9. Determine MAPE values based on the number of our sample data (17). This procedure is described in figure 1.

![Flowchart of Hybrid SSA-ARIMA](image)

**Figure 1.** Flowchart of Hybrid SSA-ARIMA

**RESULTS AND DISCUSSION**

For our empirical study, we analyzed *AirPassengers*, and CO$_2$ data, available in R package *datasets*. Here, we consider two seasonal data, namely *AirPassengers*, which has a trend and multiplicative seasonal pattern; and CO$_2$, which contains an additive seasonal pattern with the trend.
Figure 2 reveals that there has been a gradual increase in several passengers with a multiplicative seasonal pattern from 1949 to 1960. Figure 3 shows that there has been a sharp increase in Mauna Loa Atmospheric CO₂ Concentration with an additive seasonal pattern.

### Table 1. Grouping and Contribution of Data CO₂

| RC   | Hybrid-SSA-ARIMA (Alexandrov Grouping) | Contribution | Hybrid-SSA-ARIMA (Alternative Grouping) | Contribution |
|------|----------------------------------------|--------------|----------------------------------------|--------------|
|      | Principle Component (PC)               |              | Principle Component (PC)               |              |
|      | PC1,PC2,PC3,PC6                        | 99.49%       | PC1,PC8,PC9,PC16,PC17                  | 98.40%       |
|      | PC7,PC8                                |              | PC21,PC23,PC25                          |              |
| RC1  | PC4,PC5,PC12,PC15,PC16                 | 0.28%        | PC2,PC3,PC10,PC12,PC13                 | 0.93%        |
|      | PC17                                   |              | PC14,PC15,PC18,PC19,PC20               |              |
| RC2  | PC10,PC11,PC18,PC19                    | 0.07%        | PC24                                   | 0.01%        |
|      | PC22,PC28,PC29                         |              |                                        |              |
| RC3  | PC13,PC14,PC26,PC30,PC31               | 0.06%        | PC4,PC5,PC6,PC22,PC28                  | 0.40%        |
|      | PC31                                   |              |                                        |              |
| RC4  | PC20,PC21,PC23,PC24,PC25               | 0.06%        | PC11,PC26,PC27,PC37,PC39               | 0.06%        |
|      | PC27                                   |              |                                        |              |
| RC5  |                                        |              |                                        |              |

CO₂ data had been successfully decomposed into five groups through SSA, each group consisting of 5 or 6 Principle Components. PC1 to PC31 were PCs that had successfully identified the pattern, while PC32 and so on were PCs with random patterns. RC1 was the largest contribution of 99.49%; this showed that the trend pattern was very dominant in this data.

On the right side of this table, CO₂ data had been successfully decomposed into five groups through SSA; the number of PCs for each RC is uneven. RC3 only consists of 1 PC. PC1 to PC41 were PCs that had successfully identified the pattern, while PC42 and so on were PCs with random patterns. RC1 had the largest contribution of 98.49%; this showed that the trend pattern was very dominant in this data.

### Table 2. Separability of CO₂

| RC   | Hybrid-SSA-ARIMA (Alexandrov Grouping) | Hybrid-SSA-ARIMA (Alternative Grouping) |
|------|----------------------------------------|----------------------------------------|
|      | RC1 RC2 RC3 RC4 RC5                   | RC1 RC2 RC3 RC4 RC5                   |
| RC1  | 1 0.021 0.023 0.006                    | 1 0.010 0.006 0.351* 0.000             |
| RC2  | 0.021 1 0.000 0.001                    | 0.010 1 0.000 0.000 0.000              |
| RC3  | 0.023 0.000 1 0.001                    | 0.006 1 0.000 0.000 0.002              |
| RC4  | 0.000 0.001 0.001 1                    | 0.351* 0.000 0.000 1 -0.002            |
| RC5  | 0.000 0.001 0.001 1 0.001              | 0.000 0.001 0.002 -0.002 1             |
This table shows the correlation values between groups from RC1 to RC5. This best model turned out to not have a very good separability which can be seen from the correlation between RC1 and RC4, which was quite large. However, the correlation values for the others were very small.

The right side of this table shows the correlation values between groups from RC1 to RC5. This best model turned out to not have a very good separability. It can be seen from the correlation between RC1 and RC4, which is quite large. However, the correlation values for the others were very small.

| RC | Principle Component (PC) | Contribution | Principle Component (PC) | Contribution |
|----|--------------------------|--------------|--------------------------|--------------|
| RC1 | PC1,PC2,PC3,PC6,PC11     | 83.58%       | PC15,PC19                | 0.80%        |
|    | PC14,PC15                |              |                          |              |
| RC2 | PC4,PC5,PC16,PC17,PC23   | 7.20%        | PC1,PC2,PC3,PC16,PC17,PC18 | 81.51%       |
|    | PC24,PC29                |              | PC21,PC22,PC23,PC28,PC29 |              |
| RC3 | PC9,PC10,PC19,PC20      | 3.26%        | PC4,PC5,PC6,PC20,PC24,PC26,PC30 | 8.54%         |
|    | PC33,PC35,PC36          |              |                          |              |
| RC4 | PC7,PC8,PC25,PC26,PC30  | 3.13%        | PC7,PC8,PC9,PC10,PC11   | 5.98%        |
|    | PC31                     |              | PC33,PC34,PC36,PC37     |              |
| RC5 | PC12,PC13,PC18,PC21,PC22| 2.91%        | PC31,PC32                | 0.31%        |
|    | PC27,PC28,PC34          |              |                          |              |

AirPassengers data had been successfully decomposed into five groups through SSA with each group consisted of 6 to 8 Principle Components. PC1 to PC36 were PCs that had successfully identified the pattern, while PC37 and so on were PCs with random patterns. RC1 had the largest contribution of 83.589% which indicated that the trend pattern dominated in this data.

AirPassengers data had been successfully decomposed into five groups via SSA, RC1 and RC5 consisted of 2 PCs, RC2 consists of 11 PCs, RC3 consisted of 7 PCs, and RC4 9PCs. PC1 to PC36 were PCs whose patterns had been identified, while PC38 and so on were PCs with random patterns. RC2 had the most considerable contribution of 81.51%, which indicated that seasonal patterns dominated in this data.

This table showed the correlation values between groups from RC1 to RC5. This best model displayed good separability, which could be seen from RC's very small correlation value. The Hybrid-SSA-ARIMA (Alternative Grouping) model showed the correlation value between groups from RC1 to RC5. This best model showed good separability which could be seen from the very small correlation value between RC.
Table 5. MAPE of Two Methods from AirPassengers dan CO₂

| Data     | Hybrid-SSA-ARIMA (Alternative Grouping) | Hybrid-SSA-ARIMA (Alexandrov-Grouping) |
|----------|----------------------------------------|----------------------------------------|
| AirPassengers | 3.63%                                  | 4.64%                                  |
| CO₂      | 0.48%                                  | 0.13%                                  |

This table showed that the AirPassengers data using the Hybrid-SSA-ARIMA (Alternative Grouping) method was more accurate than the hybrid-SSA-ARIMA (Alexandrov grouping) method. The MAPE value of the Hybrid-SSA-ARIMA (Alternative Grouping) method was 3.63%, while the the value of Hybrid-SSA-ARIMA (Alexandrov grouping) method was 4.64%. On the CO₂ data, on the other hand, the Hybrid SSA-ARIMA (Alexandrov grouping) method was more accurate than the Hybrid SSA-ARIMA (Alternative Grouping) method because the MAPE value for CO₂ data using the hybrid-SSA-ARIMA (Alexandrov grouping) method was 0.13%, while for the hybrid-SSA-ARIMA (Alternative Grouping) method, the value was 0.48%.

CONCLUSIONS

Based on the analysis results, it appeared that the SSA-ARIMA method with Alexandrov auto grouping was better for data with additive seasonal patterns such as CO₂ data. The MAPE value for this method was 0.13% for CO₂ data while the SSA-ARIMA method with alternative auto grouping was 0.48% for CO₂ data. On the other hand, for data with multiplicative seasonal patterns such as AirPassengers data, the SSA-ARIMA hybrid method with alternative auto grouping produced more accurate forecasts with the MAPE value of 3.63%. In comparison, the SSA-ARIMA hybrid method with Alexandrov auto grouping containing the same data gave a MAPE value of 4.64%.

In general, the separability value had good separability as the correlation value was close to zero - only one significant correlation value is 0.351. The correlation value between RC1 and RC4 for CO₂ data generated from the SSA-ARIMA hybrid method with alternative auto grouping. As for the contribution, in general, it is dominated by RC1, except for AirPassengers data which was analyzed through the SSA-ARIMA auto hybrid method with alternative auto grouping; the contribution is dominated by RC2, amounting to 81.51%. In general, both the SSA-ARIMA with Alexandrov auto grouping and the SSA-ARIMA with alternative auto grouping provide satisfactory forecast results. The MAPE value of the two methods was below 5% for the two different data patterns.

ACKNOWLEDGMENTS

The author thanks the Rector of Universitas Padjadjaran who has provided financial assistance for dissemination lecturer and student research through the Academic Leadership Grant through contract number:1959/UN6.3.1/PT.00/2021.

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