Research Article

Properties of Certain Subclasses of Meromorphically $p$-Valent Functions Associated with Certain Integral Operator

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The object of this paper is to derive some interesting properties of certain subclasses of meromorphically $p$-valent functions which are defined by using an integral operator.

1. Introduction

Let $\sum_{p,m}$ denote the class of functions of the form

$$ f(z) = \frac{1}{z^p} + \sum_{k=m}^{\infty} a_k z^k, \quad m > p, \ p \in \mathbb{N} = \{1, 2, 3, \ldots\}, $$

(1)

which are analytic and $p$-valent in the punctured unit disc $U^* = \mathbb{U}/\{0\}$, where $U = \{ z : z \in \mathbb{C}, |z| < 1 \}$. For convenience, we write $\sum_{p^1-p} = \sum_{p}$.

For functions $f(z) \in \sum_{p,m}$ given by (1) and $g(z) \in \sum_{p,m}$ defined by

$$ g(z) = \frac{1}{z^p} + \sum_{k=m}^{\infty} b_k z^k, \quad m > p, \ p \in \mathbb{N}, $$

(2)

the Hadamard product (or convolution) of $f(z)$ and $g(z)$ is given by

$$ (f \ast g)(z) = \frac{1}{z^p} + \sum_{k=m}^{\infty} a_k b_k z^k $$

(3)

$$ = (g \ast f)(z). $$

Let $f$ and $g$ be analytic in $U$. The function $f$ is said to be subordinate to $g$, or $g$ is superordinate to $f$, written $f \prec g$ ($z \in U$), if there exists a Schwarz function $w(z)$ in $U$ with $w(0) = 0$ and $|w(z)| < 1$ ($z \in U$), such that

$$ f(z) \prec g(z), \quad (z \in U). $$

If $g(z)$ is univalent in $U$, then the equivalence (cf., e.g., [1, 2])

$$ f(z) \prec g(z) \Leftrightarrow f(0) = g(0), \ f(U) \subset g(U). $$

(4)

For $0 \leq \mu, \alpha \leq 1, \ m > p, \ p \in \mathbb{N}$, and $f \in \sum_{p,m}$, Saleh et al. [3] introduced the $p$-valent Rafid operator $S_{p,p}^\alpha : \sum_{p,m} \rightarrow \sum_{p,m}$ as follows:

$$ S_{p,p}^\alpha f(z) = \frac{1}{(1-\mu)\alpha+1} \int_0^\infty t^{\alpha+\mu p} e^{-t/(1-\mu)} f(zt)dt $$

$$ = \frac{1}{z^p} + \sum_{k=m}^{\infty} (1-\mu)^{k+p} (\alpha+1)_{k+p} a_k z^k, $$

(5)

where $(\nu)_k$ is the Pochhammer symbol defined, in terms of the Gamma function $\Gamma$, by

$$ (\nu)_k = \frac{\Gamma(\nu+k)}{\Gamma(\nu)}, $$

$$ = \begin{cases} 1, & \text{if } k = 0, \nu \in \mathbb{C}/\{0\}, \\ \nu(\nu+1), \ldots, (\nu+k-1), & \text{if } k \in \mathbb{N}, \nu \in \mathbb{C}. \end{cases} $$

(6)
Note that $S^a_{\mu, p} f(z) = S^b_{\mu, p} f(z)$ (see [4]). It follows from (5) that
\[
z(S^a_{\mu, p} f(z))^\prime = (a + 1)S^a_{\mu, p} f(z) - (p + a + 1)S^a_{\mu, p} f(z),
\]
\[(0 \leq \mu, \alpha, \lambda, \lambda, A, B). \tag{7}\]

By using the integral operator $S^a_{\mu, p} f(z)$, we define a subclass of $\sum_{\mu, p, m}$ as follows.

**Definition 1.** For fixed parameters $A$ and $B$, we say that a function $f(z) \in \sum_{\mu, p, m}$ is in the class $\sum_{\mu, p, m}(\alpha, \mu, \lambda, A, B)$ if it satisfies the following condition:
\[
-\frac{z^{p+1}}{p}\{(1 - \lambda)(S^a_{\mu, p} f(z))^\prime + \lambda(S^a_{\mu, p} f(z))^\prime\} < 1 + A z,\tag{8}\]
\[(z \in U), \quad p \in \mathbb{N}, 0 \leq \mu, \alpha \leq 1, \lambda \geq 0 \text{ and } -1 \leq B < A \leq 1.\]

There are many papers about some subclasses of meromorphic functions associated with several families of linear operators (see, for example, [5–11]). In this paper, we obtain some properties of the class $\sum_{\mu, p, m}(\alpha, \mu, \lambda, A, B).

### 2. Preliminary Lemmas

To establish our main results, in this paper, we shall need the following lemmas.

**Lemma 1** (see [12] and [2]). Suppose that the function $h(z)$ is analytic and convex (univalent) in $U$ with $h(0) = 1$ and $\phi(z)$ given by
\[
\phi(z) = 1 + c_{p+m} z^{p+m} + c_{p+m+1} z^{p+m+1} + \cdots. \tag{9}\]

If
\[
\phi(z) + \frac{zh'(z)}{y} < h(z), \quad (\Re(y) \geq 0, y \neq 0, z \in U), \tag{10}\]
then
\[
\phi(z) < h(z) = \frac{y}{p + m} - \frac{y}{p + m - 1} \int_0^1 \frac{1}{p + m - h(t) < h(z)}, \quad (z \in U), \tag{11}\]
and $\psi(z)$ is the best dominant of (10).

Let $P(y)$ be the class of analytic in $U$ of the form
\[
\varphi(z) = 1 + b_1 z + b_2 z^2 + \cdots, \tag{12}\]
which satisfies the following inequality:
\[
\Re(\varphi(z)) > y, \quad (0 \leq y < 1, z \in U). \tag{13}\]

**Lemma 2** (see [13]). Let the function $\varphi(z)$, given by (12), be in the class $P(y)$. Then,
\[
\Re(\varphi(z)) \geq 2y - 1 + \frac{2(1 - y)}{1 + |z|}, \quad (0 \leq y < 1, z \in U). \tag{14}\]

**Lemma 3** (see [14]). If $\varphi_j \in P(y_j), \quad (0 \leq y_j < 1, j = 1, 2),\]
then
\[
\varphi_1 * \varphi_2 \in P(y_3), \quad y_3 = 1 - 2(1 - y_1)(1 - y_2). \tag{15}\]

The result is the best possible.

Let $a, b, c$ be any real or complex numbers with $c \notin \mathbb{Z}_0 = \{0, -1, -2, \ldots\}$, and consider the function given by
\[
_{2}F_{1}(a, b, c, z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{z^k}{k!} \tag{16}\]
This function, called the Gauss hypergeometric function, is analytic and converges absolutely for $z \in U$ (see [15]).

**Lemma 4** (see [15]). Let $a, b, c$ be any real or complex numbers with $c \notin \mathbb{Z}_0$. Then,
\[
\int_0^1 t^{b-1} (1 - t)^{c-b-1} (1 - zt)^{-a} dt = \frac{\Gamma(b)\Gamma(c - b)}{\Gamma(c)} _2F_1(a, b, c, z), \quad \Re(c) > \Re(b) > 0, \tag{17}\]
\[
_{2}F_{1}(a, b, c, z) = (1 - z)^{-a} _2F_{1}
\]
\[
\left(a - b, c, \frac{z}{z - 1}\right), \tag{18}\]
\[
_{2}F_{1}
\left(a, \frac{a + b + 1}{2}, \frac{1}{2}\right) = \frac{\sqrt{\pi} \Gamma((a + b + 1)/2)}{\Gamma((a + 1)/2)\Gamma((b + 1)/2)}, \tag{19}\]
\[
_{2}F_{1}
\left(1, 1, 2, \frac{z}{z + 1}\right) = \frac{z + 1}{z} - \ln(1 + z), \quad z \neq 0. \tag{20}\]
3. Main Results

Unless otherwise mentioned, we shall assume throughout the sequel that
\[ m > -\alpha, \alpha \in \mathbb{N}, 0 \leq \mu, \alpha \leq 1, \lambda > 0, z \in U \text{ and } -1 \leq B < A \leq 1. \]  

**Theorem 1.** If \( f \in \sum_{p,m} (\alpha, \mu, \lambda, A, B) \), then
\[ -\frac{z^{p+1} (S^a_{p,m} f(z))'}{p} < q_1(z) < \frac{1 + Az}{1 + Bz} \]  
where the function \( q_1(z) \) is given by

\[ q_1(z) = \left[ \frac{A}{B} + \left( 1 - \frac{A}{B} \right) (1 + B) \right] \left( 1, 1, \frac{\alpha + 1}{\lambda (p + m)} + 1, \frac{Bz}{Bz + 1} \right), (B \neq 0), 1 - \frac{\alpha + 1}{\lambda (p + m) + \alpha + 1} A (B = 0). \]  

The result is the best possible.

**Proof.** Set
\[ \phi(z) = -\frac{z^{p+1} (S^a_{p,m} f(z))'}{p}. \]  

Then, the function \( \phi(z) \) is of form (9) and is analytic in \( U \). Differentiating (26) and with the aid of identity (7), we obtain
\[ \phi(z) + \frac{\lambda z \phi'(z)}{\alpha + 1} = -\frac{z^{p+1}}{p} \left( 1 - \lambda \right) (S^a_{p,m} f(z))' \]
\[ + \lambda \left( S^a_{p,m} f(z) \right)' < \frac{1 + Az}{1 + Bz}. \]  

\[ q_1(z) = \frac{\alpha + 1}{\lambda (p + m)} z^{-(\alpha + 1)(\lambda (p + m))} \int_0^z t^{-(\alpha + 1)(\lambda (p + m)) - 1} \left( 1 + At \right) \left( 1 + Bt \right) dt \]
\[ = \left[ \frac{A}{B} + \left( 1 - \frac{A}{B} \right) (1 + B) \right] \left( 1, 1, \frac{\alpha + 1}{\lambda (p + m)} + 1, \frac{Bz}{Bz + 1} \right), (B \neq 0), 1 - \frac{\alpha + 1}{\lambda (p + m) + \alpha + 1} A (B = 0). \]  

by change of variables followed by the use of identities (17) and (18) (with \( a = 1, b = (\alpha + 1) / (\lambda (p + m)) \), and \( c = b + 1 \). This proves assertion (22) of Theorem 1. Next, in order to prove assertion (24) of Theorem 1, it suffices to show that
\[ \inf_{|z| < 1} |\Re(q_1(z))| = q_1(-1). \]  

Indeed, for \( |z| \leq r < 1 \),
\[ \Re \left( \frac{1 + Az}{1 + Bz} \right) \geq 1 - Ar. \]  

Setting
\[ G(s, z) = \frac{1 + Asz}{1 + Bsz} \]

\[ d\mu(s) = \frac{\alpha + 1}{\lambda(p + m)} s^\lambda(p + m) - 1 ds, \quad (0 \leq s \leq 1), \]

which is a positive measure on \([0, 1]\), we obtain

\[ q_1(z) = \int_0^1 G(s, z) d\mu(s) \]

so that

\[ \Re(q_1(z)) \geq \int_0^1 \frac{1 - Asr}{1 - Bsr} d\mu(s) = q_1(-r), \quad (|z| \leq r < 1). \]

Applying Theorem 1 with

\[ \rho = \begin{cases} A/B + (1 - A/B)(1 - B)^{-1}I_2 \left( 1, 1, 1 - \frac{\sigma(1 - m)}{\sigma(p + m)}, \frac{Bz}{Bz + 1} \right), & (B \neq 0), \\ 1 - \frac{1 - \sigma(p + 1)}{1 - \sigma(1 - m)} A, & (B = 0). \end{cases} \]

The result is the best possible.

**Remark 1.** For \( m = 0 \) and \( p = 1 \), the result (asserted by Corollary 1) was also obtained by Patel and Sahoo [16] and Lashin [17].

Putting \( \lambda = (\sigma(a + 1))/(1 - \sigma(p + 1)) \) \((0 < \sigma < (1/ (p + 1)))\) in Theorem 1, we obtain the following corollary.

**Corollary 1.** If \( f(z) \in \Sigma_{p, m} \) satisfies

\[ -z^{p+1}\left[ z^\alpha S_{\sc{\alpha}} \left(f(z)\right) + z(\alpha S_{\sc{\alpha}} f(z))'' \right] < \frac{1 + Az}{1 + Bz}, \]

then

\[ -z^{p+1}(S_{\sc{\alpha}} f(z))' < \frac{1 + Az}{1 + Bz} \]

where the function \( q_2(z) \) given by

\[ q_2(z) = \begin{cases} A/B + (1 - A/B)(1 + Bz)^{-1}I_2 \left( 1, 1, 1 - \frac{\sigma(1 - m)}{\sigma(p + m)}, \frac{Bz}{Bz + 1} \right), & (B \neq 0), \\ 1 + \frac{1 - \sigma(p + 1)}{1 - \sigma(1 - m)} A, & (B = 0). \end{cases} \]

The result is the best possible.

**Remark 2.** The result (asserted by Corollary 2) was also obtained by Srivastava and Patel [18].

Taking \( \delta = -(p(\pi - 2))/(4 - \pi) \) in Corollary 2, we have the following corollary.

Letting \( r \rightarrow 1^- \) in the above inequality, we obtain assertion (30). The result in (24) is best possible as the function \( q_1(z) \) is the best dominant of (22).

Applying Theorem 1 with \( A = 1 - (2\delta/p), \quad (0 \leq \delta < p), \quad B = -1, \quad m = 2 - p, \quad \lambda = (\sigma(a + 1)) \) and making use of (19), we obtain the following corollary.

**Corollary 2.** If \( f(z) \in \Sigma_{p, 2-p} \) satisfies the following inequality

\[ \Re\left\{-z^{p+1}\left[ (p + 2)S_{\sc{\alpha}} f(z) + zS_{\sc{\alpha}} f(z)'' \right] \right\} > \delta, \quad (0 \leq \delta < p), \]

then

\[ \Re\left\{-z^{p+1}(S_{\sc{\alpha}} f(z))' \right\} > \delta + (p - \delta)(\pi/2 - 1). \]

The result is the best possible.

**Corollary 3.** If \( f(z) \in \Sigma_{p, 2-p} \) satisfies the following inequality

\[ \Re\left\{-z^{p+1}\left[ (p + 2)S_{\sc{\alpha}} f(z) + zS_{\sc{\alpha}} f(z)'' \right] \right\} > -\frac{p(\pi - 2)}{4 - \pi}, \]

by Replacing \( S_{\sc{\alpha}} f(z) \) by \( f(z) \), then

\[ \Re\left\{-z^{p+1}(S_{\sc{\alpha}} f(z))' \right\} > 0. \]
The result is the best possible.

Remark 3. The result (asserted by Corollary 3) was also obtained by Pap [19]. Applying Theorem 1 with \( A = 1 - (2\delta/p) \), \( 0 \leq \delta < p \), \( B = -1 \), \( m = 1 - p \), and \( \lambda = \alpha + 1 \) and making use of (20), we obtain the following corollary.

Corollary 4. If \( f(z) \in \sum_p \) satisfies the following inequality

\[
\Re \left\{ z^{p+1} \left[ (p+2)(S_{\mu,p}^a f(z))^\prime + z(S_{\mu,p}^a f(z))^{\prime\prime} \right] \right\} > \delta, \quad (0 \leq \delta < p),
\]

then

\[
\Re \left\{ z^{p+1}(S_{\mu,p}^a f(z))' \right\} > p + 2(p - \delta)(\ln 2 - 1).
\]

The result is the best possible.

Replacing \( \phi(z) \) by \( z^p S_{\mu,p}^a f(z) \) in (26) and applying the same method and technique as the proof of Theorem 1, we can prove the following result.

Theorem 2. If \( f \in \sum_{p,m} \) satisfies

\[
z^p \{(1 - \lambda)S_{\mu,p}^a f(z) + \lambda S_{\mu,p}^{a+1} f(z)\} < \frac{1 + Az}{1 + Bz}
\]

then

\[
z^p S_{\mu,p}^a f(z) < q_1(z) < \frac{1 + Az}{1 + Bz},
\]

\[
\Re \left\{ z^p S_{\mu,p}^a f(z) \right\} > \rho,
\]

where \( q_1 \) and \( \rho \) are given as in Theorem 1. The result is the best possible.

Theorem 3. Let \(-1 \leq B_j \leq A_j \leq 1 \quad (j = 1, 2)\). If each of the functions \( f_j(z) \in \sum_p \) satisfies the following subordination condition

\[
z^p \{(1 - \lambda)S_{\mu,p}^a f_j(z) + \lambda S_{\mu,p}^{a+1} f_j(z)\} < \frac{1 + A j z}{1 + B j z}, \quad (j = 1, 2),
\]

then

\[
z^p \{(1 - \lambda)S_{\mu,p}^a H(z) + \lambda S_{\mu,p}^{a+1} H(z)\} < \frac{(1 - (2\gamma/p)z)}{1 - z},
\]

where

\[
H(z) = S_{\mu,p}^a (f_1 \ast f_2)(z),
\]

\[
y = 1 - \frac{4(A_1 - B_1)(A_2 - B_2)}{(1 - B_1)(1 - B_2)} \left[ 1 - \frac{1}{2} \phi_1 \left( 1, 1, \frac{\alpha + 1}{\lambda} + \frac{1}{2} \right) \right].
\]

The result is the best possible when \( B_1 = B_2 = -1 \).

Proof. If we let

\[
\phi_j(z) = z^p \{(1 - \lambda)S_{\mu,p}^a (f_j(z)) + \lambda S_{\mu,p}^{a+1} (f_j(z))\}, \quad (j = 1, 2),
\]

then, by the hypothesis of Theorem 3, we have

\[
\phi_j(z) \in P(\gamma_j), \quad \left( \gamma_j = \frac{1 - A_j}{1 - B_j}, \quad j = 1, 2 \right).
\]

Using identity (7), (53) can be written as

\[
S_{\mu,p}^a (f_j(z)) = \frac{\alpha + 1}{\lambda} z - p \int_0^z t \frac{\alpha + 1}{\lambda} - 1 \varphi_j(t) dt,
\]

\[
(j = 1, 2).
\]

From (51) and (55), we obtain

\[
S_{\mu,p}^a H(z) = \frac{\alpha + 1}{\lambda} z - p \int_0^z t \frac{\alpha + 1}{\lambda} - 1 \varphi_0(t) dt,
\]

where

\[
\varphi_0(z) = z^p \{(1 - \lambda)S_{\mu,p}^a H(z) + \lambda S_{\mu,p}^{a+1} H(z)\}
\]

\[
= \frac{\alpha + 1}{\lambda} z - \frac{\alpha + 1}{\lambda} \int_0^z t \frac{\alpha + 1}{\lambda} - 1 (\varphi_j \ast \varphi_2)(t) dt.
\]

Since \( \varphi_1(z) \in P(\gamma_1) \) and \( \varphi_2(z) \in P(\gamma_2) \), it follows from Lemma 3 that

\[
(\varphi_1 \ast \varphi_2)(z) \in P(\gamma_3), \quad \left( \gamma_3 = 1 - 2(1 - \gamma_1)(1 - \gamma_2) \right).
\]

According to Lemma 2, we have

\[
\Re \left\{ (\varphi_1 \ast \varphi_2)(z) \right\} \geq 2\gamma_3 - 1 + \frac{2(1 - \gamma_3)}{1 + |z|}.
\]

Now, by using (59) in (57) and then appealing to Lemma 4, we obtain
\[
\Re \{ \varphi_o(z) \} = \frac{\alpha + 1}{\lambda} \int_0^1 u^{(\alpha+1)/\lambda - 1} \Re \{ (\varphi_1 \ast \varphi_2) (uz) \} du \geq \frac{\alpha + 1}{\lambda} \int_0^1 u^{(\alpha+1)/\lambda - 1} \left( 2y_3 - 1 + \frac{2(1-y_3)}{1 + u|z|} \right) du
\]

\[
> \frac{\alpha + 1}{\lambda} \int_0^1 u^{(\alpha+1)/\lambda - 1} \left( 2y_3 - 1 + \frac{2(1-y_3)}{1 + u} \right) du
\]

\[
= 1 - \frac{4(A_1 - B_1)(A_2 - B_2)}{(1-B_1)(1-B_2)} \left[ 1 - \frac{\alpha + 1}{\lambda} \int_0^1 u^{(\alpha+1)/\lambda - 1} (1 + u)^{-1} du \right]
\]

\[
= 1 - \frac{4(A_1 - B_1)(A_2 - B_2)}{(1-B_1)(1-B_2)} \left[ 1 - \frac{1}{2} F_1 \left( 1, 1, \frac{\alpha + 1}{\lambda} + 1, \frac{1}{2} \right) \right]
\]

\[
= \gamma,
\]

which completes the proof of assertion (49).

When \( B_1 = B_2 = -1 \), we consider the functions \( f_j(z) \in \sum_{p,m} \) \( (j = 1, 2) \) defined by

\[
S_{\mu,p}^a(f_j(z)) = \frac{\alpha + 1}{\lambda} z - \frac{\alpha + 1}{\lambda} - p \int_0^z \frac{\alpha + 1}{\lambda} - 1 \left( \frac{1 + A_1}{1-t} \right) dt, \quad (j = 1, 2).
\]

Now, by using Lemma 4 and (57), we have

\[
\varphi_o(z) = \frac{\alpha + 1}{\lambda} \int_0^1 u^{(\alpha+1)/\lambda - 1} \left( 1 - (1 + A_1)(1 + A_2) + \frac{(1 + A_1)(1 + A_2)}{1 - uz} \right) du
\]

\[
= 1 - (1 + A_1)(1 + A_2) + (1 + A_1)(1 + A_2)(1 - z)^{-1} 2F_1 \left( 1, 1, \frac{\alpha + 1}{\lambda} + 1, \frac{z}{z+1} \right) \to 1
\]

\[
- (1 + A_1)(1 + A_2) + \frac{1}{2} (1 + A_1)(1 + A_2) 2F_1 \left( 1, 1, \frac{\alpha + 1}{\lambda} + 1, \frac{1}{2} \right).
\]

Remark 4. For \( p = 1 \), the result (asserted by Corollary 5) was also obtained by Yang [20].

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares no conflicts of interest.

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