Galactic dynamics and long-range quantum gravity

M. Cadoni$^{ab}$, and M. Tuveri$^{ab}$

$^a$Dipartimento di Fisica, Università di Cagliari
Cittadella Universitaria, 09042 Monserrato, Italy

$^b$I.N.F.N, Sezione di Cagliari, Cittadella Universitaria, 09042 Monserrato, Italy

7th May 2019

Abstract

We explore in a systematic way the possibility that long-range quantum gravity effects could play a role at galactic scales and could be responsible for the phenomenology commonly attributed to dark matter. We argue that the presence of baryonic matter breaks the scale symmetry of the de Sitter (dS) spacetime generating an IR scale $r_0$, corresponding to the scale at which the typical dark matter effects we observe in galaxies arise. It also generates a huge number of bosonic excitations with wavelength larger than the size of the cosmological horizon and in thermal equilibrium with dS spacetime. We show that for $r \gtrsim r_0$ these excitations produce a new component for the radial acceleration of stars in galaxies which leads to the result found by McGaugh et al. by fitting a large amount of observational data and with the MOND theory. We also propose a generalized thermal equivalence principle and use it to give another independent derivation of our result. Finally, we show that our result can be also derived as the weak field limit of Einstein’s general relativity sourced by an anisotropic fluid.

1 Introduction

Quantum mechanics determines the macroscopic behaviour of many physical systems. This was realized since its discovery as it was used to explain macroscopic phenomena for which classical physics failed, like e.g. black body radiation and the heat capacity in solids. By now, we also know very well that quantum macroscopic systems exist as a result of long-range quantum coherent states. We have several examples of this, including Bose-Einstein (BE) condensates, superfluids, neutron stars etc. This is believed to be true for physical systems in which the electromagnetic (or strong) interaction plays the main role, but its extension to gravitational systems is commonly overlooked. Indeed, due to the weakness of the gravitational interaction, quantum gravity effects are usually believed to be relevant only at very small scales of the order of Planck length $l_p = \sqrt{\hbar G}$.

In general, our description of gravitational phenomena is based on two implicit assumptions: 1) quantum gravity corrections are suppressed by inverse powers of the Planck mass, $m_p = \sqrt{\hbar/G}$; 2) effective field theories, give a reliable description of infrared (IR) physics. Thus, quantum gravity is expected to play a role near a spacetime singularity but to be completely irrelevant at larger scales. As a consequence, the gravitational physics at galactic and solar system scales should be fully determined by general relativity (GR) or its eventual classical modifications.

$^*$E-mail: mariano.cadoni@ca.infn.it
$^†$E-mail: matteo.tuveri@ca.infn.it

1 In this paper we use, unless explicitly stated, units $c = k_B = 1$, where $c$ is the speed of light, $k_B$ is the Boltzmann constant and $G$ denotes the Newton constant.
However, it is logically possible that the common wisdom expressed by the two statements above is not always true. Some of the laws governing long times and long distances dynamics in our universe could have, in principle, a quantum mechanical origin. There are several reasons supporting this point of view like, for instance, the fact that long-range quantum effects play an important role in several systems like e.g. Bose-Einstein condensates. Their peculiar features are determined not by the strength of the interaction but, rather, by genuine quantum features like e.g. the Pauli exclusion principle or the existence of quantum collective states. In particular, condensed matter systems offer a nice paradigm for what concerns long-range quantum interactions. When applied to gravity this perspective suggests that macroscopic quantum gravity effects could be originated from the cooperative result of a huge number of quantum modes of extremely large wavelength.

Recently, several proposal of long-range quantum gravity effects have been put forward. The emergent gravity scenario, in its different forms, considers spacetime and gravity as a macroscopic manifestation of microscopic quantum gravity degrees of freedom [11, 2, 3, 5, 6, 7, 8, 9, 10]. For example, in the corpuscular gravity scenario black holes and the de Sitter universe may be explained as quantum coherent states of a large number of gravitons [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23]. Recent investigations on the topic have also suggested that quantum gravity corrections may also be relevant at the horizon of astrophysical black holes [20, 27, 25].

One of the most promising arena for looking for long-range quantum gravity effects is galactic dynamics. Despite its great successes in explaining present experimental data about the accelerated expansion of the universe [28], structure formation, galaxy rotation curves and gravitational lensing, the ΛCDM model of standard cosmology [29, 30] fails to explain the so-called baryonic Tully-Fisher relation [31, 32]. This formula establishes a relation between the asymptotic velocity $v$ of stars in galaxies and the total baryonic mass $m_B$. In particular, the acceleration parameter $a_0$ is of the same order of magnitude of the current value of the Hubble constant $H$, $a_0 \simeq 1.2 \times 10^{-10} \text{ m s}^{-2}$ [33].

At phenomenological level a simple explanation of the Tully-Fisher relation is offered by Milgrom’s MOdified Newtonian Dynamics (MOND) [34, 35]. In the MOND theory, $a_0$ is promoted to a fundamental constant of nature. The modification of Newtonian gravity encoded in MOND can be explained either as a modification of the laws of inertia or as an additional acceleration component, $a_{\text{MOND}} = \sqrt{a_0 m_B}$, where $a_B$ is the Newtonian acceleration due to baryonic matter. From astrophysical observations we know that these new effects arise when $a_B \simeq a_0$, to which corresponds a critical scale $r_0 \simeq \sqrt{Gm_B/a_0}$.

Motivated by these considerations, recently, there have been some alternative proposal to explain the galactic-scale phenomenology commonly attributed to dark matter. For example, in one of these approaches the additional MOND acceleration is associated to a "dark force" (DF) generated by the reaction of dark energy (DE) to the presence of baryonic matter [3, 22, 23, 36, 37, 38]. In another approach, this is the result of an environmental modification of the inertial/gravitational mass ratio [39]. In both approaches the modification of the Newtonian laws of gravity is thought as the macroscopic manifestation of a long-range/late times quantum gravity effect 2.

However, these approaches suffer from a serious drawback: they are able to reproduce the asymptotic MOND formula but their predictions significantly differ from observations in the galactic region [34, 55, 56, 57]. In particular, in the MOND framework, the observational data about rotational curves of galaxies can be reproduced only by introducing a phenomenological function $F$. This function interpolates between the baryonic acceleration due to standard Newtonian gravity near to the galactic core and the (deep) MOND regime in the outer galactic region. An interpolating function $F$ (whose explicit value will be shown in section 3.1) fitting a large amount of observational data coming from galaxies with different shapes (spiral, elliptical, spherical) has been proposed by McGaugh et al. [33, 58] (see however [59, 60]).

The purpose of this paper is to explore in a systematic way the possibility that long-range quantum gravity effects could play a relevant role at galactic scales, derive the macroscopic manifestation of these effects and compare them with observations. We will use simple and general arguments not relying on any specific microscopic theory of gravity, but only on general features of thermodynamics, quantum and

\[^2\text{The idea that MOND can be an expression of quantum gravity has been considered also by by other authors [31, 32, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53].}\]
statistical mechanics and, finally, general relativity.

We will start in Sect. 2 by showing that the presence of baryonic matter breaks the scale isometry of the dS spacetime generating, in this way, a length-scale \( r_0 \) and related long-range quantum gravity effects. The gravitational nature of this long-range interaction implies the existence of a huge number of soft, spin-2, collective bosonic excitations following a thermal Bose-Einstein distribution at the temperature of the dS horizon. In Sect. 2.1 using a simple one-dimensional quantum mechanical model, we will compute the energy of these collective bosonic excitations and the related IR scale \( r_0 \). In Sect. 3, using simple thermodynamical arguments, we will derive the acceleration of a test mass produced at galactic scales by these DE bosonic excitations and compare our result with galactic observations. We will show that our theoretical result reproduces the observational data and the McGaugh interpolating function \([33, 58]\) up to a numerical proportionality factor between the acceleration parameter \( a_0 \) and the cosmological acceleration \( H \). In sect. 4, building up the results of Ref. \([39]\), we will propose a generalized thermal equivalence principle. We will use it to give another independent derivation of our results and to compute the acceleration proportionality factor, i.e. \( 1/2\pi \), in agreement with observational data \([33, 58]\). In sect. 5 we will deal with our collective bosonic excitations in the context of corpuscular gravity. In sect. 6 we will show that our weak field description allows for a general covariant uplifting of the theory in terms of GR sourced by an anisotropic fluid. Finally in Sect. 7 we will present our conclusions.

2 Long-range quantum gravity effects

To begin with, we consider our universe as made only by dark energy and baryons, i.e. we do not assume the presence of any exotic form of matter like dark matter. Since we do not know what DE really is, we model it in the simplest way as a positive cosmological constant \( \Lambda \). In absence of baryonic matter, consistently with GR, our universe is described by a de Sitter (dS) spacetime with cosmological horizon size (the dS radius) \( L = \sqrt{3/\Lambda} \). The cosmological acceleration \( H \) is related to \( L \) by \( H = 1/L \). The cosmological horizon has an associated Hawking temperature given by

\[
T_{dS} = \frac{h}{2\pi L}.
\]

(1)

Four dimensional dS spacetime can be defined as an hyperbola embedded in \( \mathbb{R}^{1,4} \). In the static patch the empty, i.e. without baryonic matter, dS universe is described by the following metric

\[
ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2, \quad f(r) = 1 - \frac{r^2}{L^2},
\]

(2)

where \( d\Omega^2 \) is the metric of the 2-sphere. The dS spacetime has an isometry group \( SO(1,4) \) inherited from its embedding in \( \mathbb{R}^{1,4} \) which acts as conformal transformations of the 3-sphere at infinity. In particular, it has an intrinsic scale invariance, which becomes evident by writing the metric in FLRW form and in the flat slicing

\[
ds^2 = -dt^2 + e^{2t/L^2}d\tilde{y}^2,
\]

(3)

where \( d\tilde{y}^2 \) is the flat metric on \( \mathbb{R}^3 \). The metric is invariant under \( d\tilde{y} \rightarrow \lambda d\tilde{y}, t \rightarrow t - L \log \lambda \). The scale invariance means that the dS spacetime cannot be endowed with a length scale. Every comoving length interval can be stretched during the cosmological expansion without changing the form of the metric \( (3) \). In particular, this implies that the dS radius \( L \) does not play any dynamical role rather, from a semi-classical perspective, it has to be thought just as a thermal scale whose inverse gives the (horizon) dS temperature \( [1, 61, 62, 63] \).

Scale invariance is not a peculiarity of the dS background but a general property of the Einstein-Hilbert action with a cosmological constant term. In fact, it has been shown that Einstein general relativity with cosmological constant, but in absence of matter, can be formulated as a scale-free theory \([61]\).

Let us now introduce the baryonic matter in the dS universe. For simplicity, in this paper we consider baryonic matter in the form of a point-like mass \( m_B \), but our computations can be easily generalized to
the case of a spherically symmetric mass distribution $m_B(r)$. Due to the presence of baryonic matter, the solution of Einstein equations of GR will change accordingly and they acquire a new, Schwarzschild-like, term in the metric function $f(r)$ in Eq. (2): $f(r) = 1 - \frac{r^2}{L^2} - \frac{2Gm_B}{r}$. Hence, the presence of the baryonic matter explicitly breaks the scale symmetry of the dS spacetime and a length scale $r_0$ is generated. As discussed in Ref. [63], the breaking of the scale symmetry due to the presence of matter occurs also at the full Einstein-Hilbert action level.

At first sight one could be led to identify $r_0$ with the Schwarzschild radius associated to $m_B$, $r_0 = R_s = 2Gm_B$. However, this is correct only if one assumes that DE does not react to the presence of baryonic matter. The existence of such a DE reaction is quite natural in an emergent gravity [8] or corpuscular gravity [22, 23, 24, 36, 37, 38] scenario and in view of our lack of understanding of DE represents the most plausible assumption. In this paper we will assume that this is the case.

If DE reacts to the presence of baryonic matter the length scale $r_0$ can in principle depend on three scales: the Planck length $l_p$, which sets the microscopic scale of quantum gravity phenomena, $R_s$ and $L$. Using an analogy borrowed from condensed matter physics one can describe DE as a system of size $L$ characterized by a microscopic scale $l_p \ll L$ (i.e. the dimension of its elementary constituents) in which we introduce an impurity of size $R_s$. Generically, one expects the generation of a mesoscopic scale $r_0$ with $l_p \ll r_0 < L$, which does not depend on $l_p$ and is completely determined by $R_s$ and $L$.

We will determine $r_0$ shortly, meanwhile lets us ask the question about the relevance of quantum effects at the mesoscopic scale $r_0$. The question can be answered by comparing the Compton wavelength $\lambda_c$ associated to the impurity with $r_0$. When we assume that DE does not react to baryonic matter we have seen that $r_0$ must be of the order of $R_s$ and $\lambda_c = \hbar/m_B$. As it is well known, in this case the condition $\lambda_c \approx r_0$ determines the typical scale of quantum gravity effects to be of the order of Planck length $l_p \approx 10^{-35}m$. We have reached the usual result that quantum gravity effects are completely negligible at macroscopic scales.

The situation in which DE reacts to the presence of baryonic matter is completely different. In this case, we expect the scale $r_0$ to depend also on $L$ and to become mesoscopic. Moreover, in this picture, the quantum modes of DE can acquire new collective (quantum) gravitational properties, which become relevant at scales of order of $r_0$. The baryonic matter can be seen as an impurity in the dS system whose effects are not localised within its Schwarzschild radius, rather diffuse in a broad region (the galactic region).

To evaluate the Compton wavelength associated to the baryonic mass we must consider a test mass $m$ at distance $r$ from $m_B$ and its classical Newtonian energy,

$$V_N(r) = -\frac{Gmm_B}{r}. \quad (4)$$

The typical scale of quantum gravity effects is determined by

$$\lambda_c \approx \frac{\hbar}{|V_N|} \approx r_0. \quad (5)$$

From Eqs (4) and (5) one finds that quantum effects are negligible for $r \ll r_0R_s m/\hbar$ and become relevant for $r \approx r_0 R_s m/\hbar$. We see that at scales $r = r_0$ quantum effects become relevant when $m = h/R_s$, i.e. when the test mass becomes of the same order of the Compton mass of a black hole with mass $m_B$. In particular we can have quantum gravity effects at galactic scales if $r_0$ is of the order of magnitude of galactic radii. This is a quite interesting result suggesting that we can have long-range quantum gravity effects at large scales when the test particle has a mass of the order of the Compton mass of a black hole built with the corresponding baryonic mass $m_B$. Notice that we can write $m \approx m_p^2/m_B$ so that $m \ll m_p$ if $m_B$ is big enough.

Having established that it is possible to have quantum gravity effects at large (galactic) scales generated by the presence of baryonic matter, we are faced with the problem of identifying the corresponding collective quantum excitation of the DE system. Since we do not have a consistent quantum gravity description of the dS universe (see e.g. [65]), we can only formulate some general guess about the nature of the quantum excitation generated in the DE system by the presence of baryonic matter.

Evidences from GR and various quantum gravity models indicate that the gravitational interaction should be mediated by bosonic spin-2 particles. This is, for instance, quite natural in a corpuscular gravity scenario,
like that considered in Ref. [23] where the bosonic excitations have been identified as "dark gravitons". This suggests that DE excitations can be considered as bosonic quantum states in thermal equilibrium with the dS spacetime described by a Bose-Einstein distribution with zero chemical potential at temperature $T_{dS}$ given by Eq. (1)

$$N(\varepsilon) = \frac{1}{e^{\varepsilon/T_{dS}} - 1},$$

where $\varepsilon$ is the energy of the excitation. In the following we will refer to them as DE bosonic excitations. Notice that we are assuming that the energy spectrum is non-degenerate.

It is well known that bosons can condense to produce a Bose-Einstein condensate (BEC) when the temperature of the system drops below a certain critical temperature $T_c$. In our case we expect this critical temperature to be of the order of $T_{dS}$. The dS universe can be therefore considered as a quantum critical state representing a BEC. The description of the dS spacetime as a BEC also emerges quite naturally in a corpuscular gravity context [18, 22, 23], where it has been considered as a BEC of a large number $N \sim L^2/l_p^2$ of gravitons with energy $\varepsilon \sim \hbar / L$.

It is interesting to notice that a quite similar phenomena happens at scales smaller than the galactic one when one considers black holes. The same relations we have written before for the dS universe also hold for black holes if we replace $L$ with the Schwarzschild radius $R_s$. Black holes can be also described as a BEC constituted by a large number $N_{BH} \sim R_s^2 / l_p^2$ of gravitons with typical energy $\varepsilon_B \sim \hbar / R_s$ [11, 12, 13, 14, 15, 16, 66, 17, 18, 19, 20, 21]. Similarly to the case of the dS universe, also black holes are critical BEC.

We note that our quantum description of the dS universe and of black holes is quite similar to that of typical Bose-Einstein condensates as superfluids (e.g. helium under a critical temperature) or superconductors, where the DE bosonic excitations play the role of phonons. The use of Bose-Einstein condensates in cosmology has been recently proposed also in [67, 68, 69]. We stress the fact that in the case of dS universe, the analogy with condensed matter systems and in particular with superconductors allows us to consider the baryonic matter as an impurity.

What is commonly believed is that quantum mechanics do not play any role at intermediate scales, i.e. at galactic and extra-galactic scales, being important only at Planckian scales. However, if the length scale $r_0$ at which these new quantum effects arise is of the order of magnitude of the galaxy size, it is possible that many phenomena we observe at galactic scale could be associated to long-range quantum interaction between dark energy and baryonic matter. In this case, from the BE distribution (6) we see that the hard excitations with $\varepsilon \gg T_{dS}$ are exponentially suppressed. Hence the scale $r_0$ corresponds to the appearance of DE bosonic excitations, i.e. $N = O(1)$, and the energy of the excitation must satisfy

$$\frac{\varepsilon}{T_{dS}} = O(1).$$

As a consequence, we are left with a large number $N \gg 1$ of soft excitations with $\varepsilon \ll T_{dS}$. In this limit, at leading order we have

$$N(\varepsilon) = \frac{T_{dS}}{\varepsilon}.$$
this new "dark" interaction term by introducing a dimensionless coupling constant $\alpha$ characterizing the interaction between the cosmological condensate and baryonic matter. We write

$$\epsilon = \frac{\hbar \alpha}{r},$$

(9)

The coupling constant $\alpha$ will depend on both the baryonic coupling constant $G m_B$ and the properties of the cosmological condensate, i.e. the dS radius $L$. Moreover, being the DE bosonic excitations soft we must require $\alpha < 1$.

In the next subsection we determine the value of $\epsilon$, i.e. of the dimensionless coupling constant $\alpha$, and the value of the scale $r_0$ by considering a simple one-dimensional quantum model of galactic dynamics.

2.1 One-dimensional quantum mechanical model for galactic dynamics

In the region $r > r_0$ there is a new interaction term between the baryonic mass $m_B$ and the test particle of mass $m$ located at distance $r$, which has not classical, but rather quantum mechanical origin. We can think of the test particle as dressed by the presence of dark energy and we want to derive the energy spectrum of the system. Because we do not have any clear understanding of DE microphysics, we use again an analogy with condensed matter physics.

We generically expect the DE excitations introduced in the previous subsection to be collective modes of DE, i.e. quasi particles with effective mass $m^*$ subject to an effective potential $V(x)$. Owing to the spherical symmetry of the problem, we can use a one-dimensional quantum mechanical model described by a single coordinate $x$. The analogy with condensed matter systems becomes more and more stringent when we try to describe, at least at a mesoscopic level, the relation between DE, dS spacetime and the emergent properties of spacetime itself. Indeed, old and new emergent gravity scenarios [11, 2, 13, 15, 16, 8, 9, 10] describe spacetime as a sort of medium where the internal degrees of freedom are located in specific points as they constitute a sort of cosmological lattice. In this way, if spacetime emerges from the dynamics of these microscopic degrees of freedom, it is quite natural to assign to it some elastic properties and, as a consequence, to have the generation of collective modes as quasi-particles, analogous to phonons. For these reasons, assuming that DE has some sort of elastic properties and working in the harmonic approximation, the presence of the baryonic mass $m_B$ will generate an elastic response of the DE medium and, consequently, the harmonic oscillator effective potential,

$$V(x) = \frac{1}{2} K(r, L) x^2.$$  

(10)

The elastic constant $K$ in general will depend both from the distance $r$ from the baryonic mass and from the horizon radius $L$ of the dS spacetime. The simplest choice for $K$ is to take it proportional to the absolute value of the Newtonian potential energy $|V_N(r)|$ of the mass $m^*$ in the potential generated by the baryonic mass $m_B$. The proportionality factor must have dimensions of length $^{-2}$ defining a pivotal distance $\langle x \rangle$ such that $V(x = \langle x \rangle) = |V_N(r)|$. This gives

$$K = \frac{2 G m_B m^*}{r \langle x \rangle^2}.$$  

(11)

The pivotal distance $\langle x \rangle$ must keep information about the two length scales $r$ and $L$ and in view of the difference of many order of magnitude between $r$ and $L$, it is appropriate to take a geometric mean of the two scales, $\langle x \rangle = \sqrt{rL}$. Hence, the elastic constant takes the following form

$$K = \frac{2 G m_B m^*}{r^2 L}.$$  

(12)

The energy spectrum of the DE excitations is now simply given by the energy levels $\epsilon_n$ of the one-dimensional quantum harmonic oscillator of mass $m^*$ with potential given by Eq. (10) and elastic constant (12). We find

$$\epsilon_n = \left( n + \frac{1}{2} \right) \frac{\hbar}{r} \sqrt{\frac{2 G m_B}{L}},$$  

(13)
where the ground state energy is
\[ \varepsilon_0 = \frac{\hbar}{r} \sqrt{\frac{Gm_B}{2L}}. \] (14)

For a given baryonic mass \( m_B \) and distance \( r \) the low-lying excitations dominate. Neglecting a factor of order 1 in Eq. (13) the energy \( \varepsilon \) of DE bosonic excitations is given by
\[ \varepsilon \simeq \sqrt{\frac{Gm_B h}{L r}}. \] (15)

This equation shows that, as expected, the energy of DE bosonic excitations only depends on \( Gm_B, L \) and \( r \). The value of the coupling constant \( \alpha \) in Eq. (9) is \( \alpha \simeq \sqrt{Gm_B/L} \).

It is interesting to note that \( \alpha \) is independent from the microscopic quantum gravity scale \( l_p \), which can enter in \( \alpha \) only through the Newtonian coupling \( Gm_B \). This means that the long-range quantum mechanical properties of dS universe manifest themselves at mesoscopic scales represented by \( r_0 \) and they are different from the quantum gravitational effects one expect at very small scales. As we will see later in this paper, \( r_0 \) represents the scale at which Einstein gravity has to be modified in order to fully describe galactic gravitational dynamics.

We are now able to determine the length scale \( r_0 \). As already mentioned in the previous section, the presence of the baryonic mass will produce in the DE condensate a large number \( N \) of DE bosonic excitations of energy \( \varepsilon \) in thermal equilibrium with dS universe at temperature \( T_{dS} = h/2\pi L \), distributed according to the Bose-Einstein statistics given by Eq. (6). The scale \( r_0 \) corresponds to the appearance of this excitation. By using Eq. (15) into Eq. (7) one finds
\[ r_0 \simeq \sqrt{R_s L} = \sqrt{Gm_B L}, \] (16)
telling us that the mesoscopic scale \( r_0 \) corresponds, again, to the geometric mean between the Schwarzschild radius of the baryonic source and the cosmological scale \( L \). We can now express the coupling constant \( \alpha \) of Eq. (9) in terms of \( r_0 \) and \( L \) only, i.e.
\[ \alpha = \frac{r_0}{L}. \] (17)

The coupling constant \( \alpha \) is suppressed by the inverse of the dS radius \( L \), it is therefore extremely small unless \( r_0 \) becomes of the order of magnitude of galactic scales. This explains why long-range quantum gravity effects become relevant only (at least) at galactic scales. On the other hand the number \( N \) of DE excitations is exponentially suppressed for \( \varepsilon > T_{dS} \), which in view of Eqs. (15) and (16) implies \( r < r_0 \), whereas \( N \) grows according to Eq. (8) for \( \varepsilon < T_{dS} \). This again shows that long-range quantum gravity effects become relevant only at galactic scales where they take the form of a specific new (quantum) "dark interaction" term. Moreover, they manifest themselves in terms of a huge number of extremely soft collective excitations.

Our result for \( r_0 \) confirms previous determination of the scale \( r_0 \) found matching the Newtonian acceleration of baryonic matter with the cosmological acceleration, \( a_B = H [8, 22, 23] \). For a typical spiral galaxy like the Milky way, with \( m_B = 10^{11} M_\odot \), \( r_0 \sim Kpc \) which is of the order of magnitude of the distance at which one observes deviations from Newtons law in galaxies [22].

Let us conclude this section by showing that the ground state energy (14) can be also found by using a very rough, potential well, approximation for the potential. In this case, \( V(x) \) is taken to be zero inside the sphere of radius \( r \) and to jump to the asymptotic value \( |V_N(L)| \) outside the sphere. We have therefore a potential well of height \( V_0 \), where \( V(x) = 0 \) for \( -r < x < r \) and \( V(x) = V_0 \) for \( x < -r \) and \( x > r \), with \( \sqrt{V_0} = V_N(L) = m^*|\phi(L)| = \frac{Gm^*m_B}{Gm^*m_B} \).

The energy spectrum \( \varepsilon \) can be found solving the Schrödinger equation. Considering states of even parity the solution of the Schrödinger equation gives, for \( \varepsilon < V_0 \), the bound states,
\[ \sqrt{2m^*}(V_0 - \varepsilon) = \sqrt{2m^*} \varepsilon \tan \left( \frac{r \sqrt{2m^*} \varepsilon}{\hbar} \right). \] (18)
For very small energy $\varepsilon$ ($\varepsilon \ll V_0, r\sqrt{2m^*\varepsilon} \ll \hbar$) and at leading order Eq. (18) gives for the ground state the energy $\varepsilon = \sqrt{\frac{V_0}{2m^*}}\frac{\hbar}{r}$, which exactly matches with Eq. (14).

3 Macroscopic effects of long-range quantum gravity at galactic scales

In the previous sections, we have argued about the presence at galactic scales of a (dark) interaction term between baryonic matter and DE, which is generated by long-range quantum gravity effects in the DE condensate. We have also shown that this implies the presence of a huge number of extremely soft DE bosonic excitations with energy $\varepsilon \ll \hbar/L$ and wavelength $\lambda \gg L$. In this section we discuss the macroscopic effects due to these quantum excitations.

By analogy with other known interactions, the macroscopic, classical, manifestation of this large number of quanta should be a new dark force (DF) term at galactic scales. This possibility is particularly appealing because it could give an explanation of observed galactic dynamics without assuming the presence of dark matter. However, in our case the problem is much more involved than, say, the electromagnetic case, for which we have first formulated a classical theory of forces and then quantized it and found the associated quanta.

We do not have any classical theory of gravitational interaction "dressed" by the presence of DE and we are not able to quantize classical ds spacetime. For this reason, the collective behaviour of these new quanta can only be studied by means of statistical mechanics and thermodynamics. Indeed the analogy with condensed matter systems and the macroscopic properties of DE and baryons discussed in the previous sections allow us to understand the physics behind the macroscopic behaviour of the DE bosonic excitations. This analogy will help us to determine the macroscopic manifestation of long-range quantum gravity effects as a new component in the acceleration felt by stars in galaxies and commonly attributed to dark matter. This acceleration can be thought as a dark acceleration or, equivalently, to a dark force term so we will refer to this new acceleration term as $a_{DF}$. In the next section we will use a generalized thermal equivalence principle to derive $a_{DF}$.

In our point-like approximation baryonic matter in the galactic core is modelled by a point with mass $m_B$ and the test mass $m$ is located at distance $r$ from the baryonic mass $m_B$.

Generically, we expect $a_{DF}$ to be given in terms of the number $N$ and the energy $\varepsilon$ of DE excitations. In a thermodynamical, quantum mechanical picture, the DF acceleration $a_{DF}$ can be thought as generated by the pressure $P$ of the gas of DE bosonic excitations in the sphere of radius $r$. Thus, we can write the acceleration for unit mass as $a_{DF} \sim PV/r \sim PV\varepsilon$, where $V$ is the volume of the sphere and we have used the fact that $\varepsilon$ scales as $1/r$. At galactic scales, $r \ll L$, the thermal contribution of bosonic excitations, $TS$, to the internal energy $U$ is negligible and the variation of $U$ is produced by the work done by the pressure $P$ of the system. Usual extensive thermodynamics then implies $PV \sim N(\varepsilon)\varepsilon$, so that we can write

$$a_{DF}(\varepsilon) = CN(\varepsilon)\varepsilon^2,$$

where $C$ is a constant with dimensions of (energy)$^{-2}$(lenght)$^{-1}$, whose value will be determined shortly. Note that owing to its elastic origin the dark force is attractive, hence the acceleration $a_{DF}$ is negative. In the following, for simplicity, we will only consider the absolute value of forces and accelerations.

The thermal contribution $TS$ to the internal energy becomes comparable to the pressure term for $r_0 \ll r \ll L$ and becomes dominant for $r = L$, i.e. when the thermodynamical behaviour of the system is completely dominated by the quantum properties of the dS horizon. Nevertheless, one can easily realize that the relation (19) holds also in this regime. In fact, we have $TS \sim N/L \sim N\varepsilon$ and the thermal contribution to the acceleration scales as in Eq. (19). It is interesting to notice that, at least in our thermodynamical analogy, the long-range quantum mechanical contribution can be seen as an extensive volume term. Conversely, the short-range thermal contribution associated to the dS horizon is an area term typical of standard black hole (and dS) thermodynamics. This distinction will become clearer in section 5.
The extensive scaling behaviour \( V \sim N, a \sim N \) of our gas of DE bosonic excitations is perfectly consistent with its origin from the constant energy density characterizing the DE. This is the extensive counterpart the of sub-extensive behaviour

\[
a_B \sim \sqrt{N} \epsilon^2, \quad (20)
\]

found for the Newtonian term \( a_B \) in the radial acceleration \( |\alpha| \) \cite{19, 22, 23} (we will give more details about this point in the next Section).

In order to calculate the value of the constant \( C \) in Eq. (19) we consider the limit in which the number of soft DE bosonic excitations becomes very large, \( N \gg 1 \), i.e. when \( r_0 \lesssim r < L \). In this limit, \( 2\pi L \epsilon \to 0 \) and, at leading order in \( 2\pi L \epsilon \), we have \( N(\epsilon_{DF}) = (2\pi L \epsilon)^{-1} \). In the same limit the universe becomes DE dominated, being the contribution of baryonic matter negligible. The behaviour of DE bosonic excitations should match that of the dS universe, i.e. \( \epsilon = h/L \) and \( a_{DF} \) must become the cosmological acceleration \( a_{DF} = H = 1/L \) \cite{22, 23}. This requirement determines the constant \( C \) in Eq. (19) to be

\[
C = \frac{2\pi}{h^2} L. \quad (21)
\]

From a pure quantum mechanical perspective this (singular) limit procedure corresponds to the transition from an excited system to the critical phase of a Bose-Einstein condensate, describing the dS universe, where all the excitations are in the fundamental state. However, since we do not fully understand the quantum nature of this condensate, we are not able to describe this phase transition.

Putting all together, i.e. using Eqs. (11), (13), (15) and (21) into Eq. (19) we find the searched expression for the DF acceleration

\[
a_{DF} = \frac{2\pi a_B}{e^{2\pi \sqrt{2 \epsilon_B}} - 1}, \quad (22)
\]

where \( a_B = G m_B/r^2 \) is the Newtonian acceleration experienced by the test mass. It is important to stress that our result does not explicitly depend on the quantum properties of the dS condensate. This is due to a nice cancellation of \( h \) in Eq. (19). This is an important check of the validity of our result. Indeed, being a macroscopic effect at galactic scales, \( a_{DF} \) must survive in the \( h \to 0 \) limit.

### 3.1 Comparison with observations

Let us now compare our result \( (22) \) with direct observations of galactic dynamics. One of the most striking features of galactic dynamics is the so-called baryonic Tully-Fisher relation \cite{31, 32}. Astrophysical observations indicate that the total radial acceleration experienced by stars in galaxies can be split in two components, the Newtonian contribution \( a_B \) due to purely baryonic matter and an additional (dark force) term \( a_{DF} \): \( a^r = a_B + a_{DF} \). A simple explanation of the Tully-Fisher relation has been given by Milgrom’s MOrified Newtonian Dynamics (MOND) \cite{33, 34, 35} in which no dark matter is present in the model, whereas the additional acceleration component is given by \( a_{DF} = a_{MOND} = \sqrt{a_B a_0} \) and \( a_0 \) is considered as a fundamental constant of nature. Moreover, from astrophysical observations, we know that the MOND component becomes relevant when \( a_B \approx a_0 \) to which corresponds a critical scale \( r_0 \approx \sqrt{G m_B/a_0} \).

The observational data about rotational curves of galaxies can be fully explained by introducing a phenomenological interpolating function \( F(x), x = a_B/a_0 \), such that the total radial acceleration can be written as \( a^r = F(x) a_B \) \cite{34, 70, 71}. For \( x \gg 1 \), near to the galactic core, the function \( F \) must reproduce standard Newtonian gravity. Instead, for \( x \ll 1 \) we have the MOND regime, \( F(x) \simeq \sqrt{1/\sqrt{x}} \) and the radial acceleration is \( a^r = a_{DF} = \sqrt{a_B a_0} = \sqrt{a_0 G m_B/r^2} \).

An interpolating function which satisfies all the conditions above and fits a large amount of observational data coming from galaxies with different shapes (spiral, elliptical, spherical) has been proposed by McGaugh et al. \cite{33, 58}, i.e. \( F(x) = \frac{1}{1-x^{-e}} \). They measure \( F(x) \) in a survey of rotation curves of 153 galaxies in the SPARC database. They measure \( a^r \) at 2693 radii on these rotation curves and, at the same radii, they
estimate the Newtonian gravitational potential from baryons as observed in stars, gas and dust, and so determine \( a_B \).

The McGaugh form for \( F(x) \) leads to the additional acceleration term

\[
a_{DF} = \frac{a_B}{e^{\frac{a_B}{a_0}} - 1}
\]

In Eq. (23), \( a_0 \) is a fitting parameter. The value of \( a_0 \) found by McGaugh et al. corresponds, approximatively, to \( a_0 = H/2\pi \) [33]. As noted in [33, 72] no dark matter is needed to fit the data since, from Eq. (23), it clearly appears that the dynamics of stars in galaxies only depends on the total amount of baryonic matter in the galactic region.

Eq. (23) reproduces the Newtonian regime of gravity for \( a_B/a_0 \to \infty \), implying \( a_{DF} = 0 \) and \( a^r = a_B \), whereas the deep MOND regime, \( a_{DF} = \sqrt{a_B a_0} = \sqrt{a_0 G m_B / r^2} \), can be obtained as the limiting case \( a_B/a_0 \to 0 \).

By comparing Eq. (23) with Eq. (22) we see that our prediction reproduces the phenomenological result in [33] up to a numerical factor, \( 1/(2\pi) \), which appears as a proportionality factor between the MOND parameter \( a_0 \) and the cosmological acceleration \( H \). Indeed, our computation based on scaling arguments and on a simplified quantum one-dimensional model has some intrinsic limitations. In Ref. [73] we have proposed a general principle, namely the equality between the baryonic acceleration and the response of the DE medium, to determine this factor and reproduce in this way the phenomenological acceleration of McGaugh et al. [33, 58]. In the next section we will derive this proportionality factor using a generalized (thermal) equivalence principle.

The Newtonian regime of gravity \( a_B/a_0 \gg 1 \) correspond to hard DE bosonic excitations with \( \varepsilon / T_{dS} \gg 1 \). In this regime \( N \) goes to zero exponentially and the DF acceleration is switched off, i.e. \( a_{DF} = 0 \). Conversely, the MOND regime \( a_B/a_0 \to 0 \) corresponds to an huge number \( N \gg 1 \) of extremely soft DE bosonic excitations with \( \varepsilon / T_{dS} \ll 1 \). The MOND acceleration is therefore the macroscopic manifestation of an huge number of extremely soft long range quantum excitation of the dS universe generated by the presence of baryonic matter.

4 A generalized thermal equivalence principle

In a recent paper Smolin has proposed that the MOND theory corresponds to a quantum gravity, cosmological constant-dominated, regime in which the equivalence principle does not hold in its usual form [59]. This is the same quantum regime of temperatures below the dS temperature and with length scales larger than the cosmological horizon we are considering in this paper. Similarly to our description, the MOND acceleration is seen as a macroscopic long-range quantum gravity effect, which survives in the \( \hbar \to 0 \) limit. Note also that an extension of [59] which points in a direction similar to what we have discussed in the previous sections has been done in Ref. [40], where dark matter effects at galactic scales are described by means of superfluids particles.

Although starting from the same general idea, namely the existence of a long-range, quantum gravity, cosmological constant-dominated regime, our approach differs from that of Ref. [39]. Smolin explains the modification of Newtonian gravity laws for \( a_B < a_0 \) as originated by an environmental-dependent change of the ratio \( m_i/m_g \) between the gravitational and inertial mass, \( m_g \) and \( m_i \), respectively. Conversely, in our approach the modification of Newtonian gravity is a direct effect of long-range quantum DE excitations.

It is quite clear that the crucial question for both approaches concerns the fate of the classical equivalence principle in the region \( a_B \ll a_0 \), when long-range quantum gravity effects become dominant: does a quantum extension of Einstein’s equivalence principle exist?

Actually, as remarked in Ref. [39] there are two formulations of the equivalence principle, which are classically equivalent but which differ when we consider excitations of wavelength of the same order of magnitude of the curvature radius of the spacetime \( R \) (in our case \( L \)). This, in turn, is what we are discussing in this paper. The first formulation (called EP1 in Ref. [39]) asserts that locally, i.e. for observers
with extent \( l \ll R \), a gravitational field can be eliminated by free fall. The second formulation (called EP2 in Ref. [39]) asserts that to zeroth order in \( l/R \) gravity can be mimicked by uniformly accelerated observers.

The quantum version of EP2 proposed in Ref. [39] takes the form of a thermal equivalence principle (TEP) which incorporates universality of free fall by asserting that the temperature \( T \) seen by an observer is related to his acceleration \( a \) by the Deser-Levin (DL) formula

\[
T = \sqrt{T_{\text{dS}}^2 + \left( \frac{\hbar a}{2\pi c} \right)^2},
\]

(24)

where \( T_{\text{dS}} \) is the dS temperature as given in Eq. (1).

An extension of EP1 to the quantum, \( \Lambda \)-dominated, regime is excluded in Ref. [39] with the argument that in this regime the relevant phenomena are not small compared to \( R \) (we are considering excitation modes with wavelength much bigger than \( L \)!) thus making them intrinsically not compatible with the restrictions required by EP1. However, looking for a quantum generalization of the classical EP1, the restriction that \( l \ll R \) seems too strong, particularly in view of the non-local character of quantum effects. Moreover, the TEP as formulated in Ref. [39] shows an intrinsic quantum/classical asymmetry: it predicts the temperature of quantum modes generated by a macroscopic and classical cause (the acceleration) but says nothing about the macroscopic effect of these quantum modes.

We propose to generalize the TEP of Ref. [39] by formulating it in a symmetric way in the sense expressed above. The generalized thermal equivalence principle (GTEP) asserts not only the statement quoted above in the TEP formulation of Smolin but also that whenever we have a thermal ensemble at temperature \( T \) of quantum gravity degrees of freedom, the macroscopic acceleration produced on a test mass is given by the Deser-Levin formula

\[
a = \frac{2\pi c}{\hbar} \sqrt{T^2 - T_{\text{dS}}^2}.
\]

(25)

Let us now show that our GTEP predicts correctly the behaviour of black holes and the dS universe considered as quantum systems. Moreover, we will show that in the case of long-range quantum gravity excitations the GTEP implies our formula (19) and allows us to derive our Eq. (22) with a proportionality factor between \( a_0 \) and \( H \) consistent with observations.

### 4.1 Black holes

Astrophysical black holes correspond to the limiting case \( T \gg T_{\text{dS}} \) of Eq. (25),

\[
a = \frac{2\pi c}{\hbar} T,
\]

(26)

which, as expected, is nothing but the relationship between surface gravity and the Hawking temperature for a Schwarzschild black hole. For a black hole of mass \( M \) and Schwarzschild radius \( R_s = 2GM/c^2 \) this relation can be also written as \( a = \frac{c^2}{2\pi R_s} \). In the most simple-minded quantum model of a black hole, we can consider it as an ensemble of quanta with typical energy

\[
\varepsilon = \frac{\hbar c}{R_s}.
\]

(27)

Using the previous equation into Eq. (26) we find the nice relation between the energy of the quanta and the acceleration

\[
a = \frac{c}{2\hbar} \varepsilon.
\]

(28)

The same result can be obtained in the corpuscular model of black holes of Refs [66, 17, 20, 22, 23]. Staring from the acceleration in Eq. (20), where the number \( N \) of gravitons is related to the black hole mass by \( N \sim M^2/m_p^2 \) and by using Eq. (27), we find Eq. (28) apart from a dimensional proportionality constant to be chosen appropriately.

\[3\]To make clear the nature of the effects we are considering in this section we will use conventional units, i.e. units in which the speed of light \( c \) is not set to 1.
4.2 The de Sitter universe

Similarly to black holes, the GTEP also applies to dS universe. In this case, Eq. (25) takes the form

\[ a = \frac{2\pi c}{\hbar} T_{dS} = \frac{c^2}{L} \]  

(29)

Hence, the dS universe can be thought as an ensemble of quanta with typical energy

\[ \varepsilon = \frac{hc}{L} \]  

(30)

so that we find

\[ a = \frac{c}{\hbar} \varepsilon \]  

(31)

which is similar to Eq. (28) up to a factor of 2.

Also in this case the corpuscular picture of gravity offers a good setup to find Eq. (31) [23, 18]. Here too the acceleration is given by Eq. (20), where the number of gravitons \( N \) is determined by the dS radius as \( N \sim L^2/l_p^2 \). Using this equation together with Eq. (30) into Eq. (20) we find, choosing appropriately the proportionality constant, Eq. (31).

4.3 Long-range quantum gravity regime

Let us now apply the GTEP to the long-range quantum gravity regime proposed in this paper, i.e. to DE bosonic excitations generating the DF. The regime we are considering now is that of a thermal ensemble of DE bosonic excitations with temperature \( T \) very close to \( T_{dS} \). The DL formula (25) holds only for \( T > T_{dS} \) thus we cannot describe the thermal excitations with \( T \leq T_{dS} \). This is consistent with the fact that at \( T \approx T_{dS} \) the system undergoes a phase transition.

However, let us expand the DL formula (25) near a temperature \( T_1 = \sigma T_{dS} \) where \( \sigma > 1 \) and is of order 1. We find, at leading order in \( T^2 \),

\[ a = \rho H + \eta L^2 T^2 \]  

(32)

where \( H \) is the cosmological acceleration and \( \rho, \eta \) are dimensionless constants, which, as expected, blow up for \( \sigma = 1 \). The first term describes the background cosmological dS acceleration, whereas the second one describes the thermal bath of DE bosonic excitations. Taking into account that the typical energy of an excitation of the thermal bath is \( \varepsilon \sim T \) the second term in Eq. (32) can be written in the following form,

\[ a \sim \eta \frac{L}{\hbar^2} \varepsilon^2 \]  

(33)

This formula is very similar to Eq. (19), which gives the acceleration produced by a number \( N \) of bosonic DE excitations of energy \( \varepsilon \). In order to have a complete matching between Eq. (33) and Eq. (19), we have to consider in Eq. (19) the contribution of a single quantum mode \( (N(\varepsilon) \sim 1) \) and to fix the proportionality constant \( \eta \) in Eq. (33) to the same value \( 2\pi/\hbar^2 \) given by Eq. (21). We get the significant result

\[ a = \frac{2\pi L}{\hbar^2} \varepsilon^2 \]  

(34)

Let us stress again that this results only holds in the regime in which the temperature \( T \) is close to \( T_{dS} \).

A comparison of this result with those derived for black holes in Eq. (28) and for the dS universe in Eq. (31) can be very instructive. All the three equations link the gravitational acceleration to the energy of
the quanta and are a direct consequence of the GTEP applied in different quantum gravity regimes. The black hole and the dS cases in Eq. (25) and Eq. (31), respectively, although corresponding to completely different length scales regimes, share the same result: the gravitational acceleration scales linearly with the energy of the quanta. This is a quantum relativistic effect because it goes to zero when the speed of light $c$ goes to zero. Conversely, in the case of DE bosonic excitations (34), the acceleration scales quadratically with the energy of the quanta. Moreover, being independent of the speed of light, the effect survives in the $c \to 0$ limit. This is perfectly consistent with the non-relativistic nature of the MOND theory.

In section 3 we have derived the McGaugh et al. phenomenological formula (23) up to the proportionality factor between the MOND and the cosmological acceleration, $a_0$ and $H$, respectively. This uncertainty was inherited from the approximations we have used quantum model used in sect. 2.1 to determine the energy of DE bosonic excitations, $\varepsilon$.

Let us now show how to recover the exact proportionality factor $(1/2\pi)$ in the context of the GTEP. For a single quantum state, the gravitational acceleration in Eq. (34) becomes equal to the gravitational acceleration generated by the baryonic mass $m_B$, i.e. $a = a_{DF} = a_B$. Using this equation in Eq. (34) we get

$$\varepsilon = \sqrt{\frac{Gm_B h}{2\pi L r}}.$$ (35)

This allows us to compute the DF acceleration felt by stars in galaxies. Using Eqs. (35), (6) and (21) into Eq. (19) we get

$$a_{DF} = \frac{a_B}{e^{\sqrt{2\pi a_B H}} - 1},$$ (36)

which is exactly the McGaugh et al. expression (23), with $a_0 = H/2\pi$ and improves Eq. (22) by determining the proportionality factor between $a_0$ and $H$. Note that the equation $a_{DF} = a_B$ has been used in Ref. [73] as a general principle to determine the energy of the DE bosonic excitations.

5 Galactic dynamics in a corpuscular gravity picture

In the previous sections, we have derived the phenomenological expression (23) using general features of quantum and statistical mechanics, thermodynamics, general relativity and a (thermal-)quantum generalization of the equivalence principle.

A drawback of our approach is that we do not have a detailed description of the microphysics involved in the gravitational interactions between baryonic matter and the DE medium which, in turns produces the long-range bosonic excitations discussed in the previous sections.

In order to have a more definite picture of the microphysics involved, we will work out Eq. (15) in the corpuscular gravity scenario of Refs. [8, 22, 23]. The same approach has been used in [22, 23] to determine the asymptotic behaviour of $a_{DF}$ in the deep MOND regime.

An important feature of the corpuscular gravity picture of Refs. [22, 23] is the use of Verlinde’s idea [8] about the transition between the Newtonian and the DE-dominated regimes of gravity as a competition between area and volume contributions to the number of degrees of freedom (entropy in Verlinde’s paper) of the two systems. This is a crucial point to determine the "dark matter effects" as a reaction of DE to the presence of baryonic matter. Moreover, the dS universe is described as a BEC of gravitons and, similarly, the interaction between baryonic matter and DE is mediated by gravitons called "dark gravitons". Differently from what we have discussed in the previous sections, in the corpuscular gravity approach both the graviton number as well as the gravitons energy depend on the size of the condensate [11, 12, 13, 14, 15, 16, 17, 18, 66]. We recall here the typical scaling of the number of gravitons for DE, $N_{DE}$, and for baryonic matter $N_B$ in terms of the size $r$ of the system and of the baryonic mass:

$$N_{DE}(r) \sim \frac{r^3}{L^2}, \quad N_B(r) \sim \frac{r^2}{L^2}, \quad N_B(m_B) \sim \frac{m_B^3}{L^2}.$$ (37)
The proportionality factor for $N_{DE}(r)$ is chosen in such a way that it matches with the total number $N_{dS} = L^2/l_p^2$ of DE gravitons in the dS spacetime at $r = L$. The number of gravitons associated to baryonic matter and dS universe scales holographically. There is a nice correspondence between the scaling of the graviton number on the one side and the acceleration formulae on the other side. Different scalings for the graviton numbers and different acceleration formulae characterize different regimes of the gravitational interaction. Eq. (37) tells us that the graviton number $N_{DE}$ associated with DE has a volume contribution which becomes relevant at intermediate scales. Translated in the language of the previous sections, this means that the long-range quantum mechanical effects of DE generated by the presence of baryonic matter are responsible for the extensive scaling $N_{DE}$ at intermediate scales shown in Eq. (37). In the corpuscular picture, critical gravitational systems as black holes and dS universe can be described in terms of the scaling of the gravitons number $N$, which behaves holographically.

This has a counterpart in the non-extensive behaviour of Eq. (20) for the gravitational acceleration and in its linear form as a function of the energy of the gravitons, $\varepsilon$ given in Eqs. (28), (31). Conversely, the extensive behaviour for the DE gravitons’ number is associated to non-critical gravitational systems (the long-range quantum gravity regime). It is in correspondence with the extensive behaviour expressed in Eq. (19) for the acceleration and with its quadratic expression in terms of $\varepsilon$ given in Eq. (34).

In the Newtonian regime, the universe is dominated by baryonic matter, $N_B > N_{DE}$. The Newtonian potential can be described by means of a quantum coherent state of $N_B$ gravitons with Compton length $r$, i.e. of energy

$$\varepsilon_N \simeq \frac{\hbar}{r}.$$ (38)

The transition from the Newtonian to the MOND regime of gravity occurs when $N_B(r) = N_{DE}(r)$. This determines the scale $r_0$ at which the dark force effects become relevant [8, 23] and it is exactly given by Eq. (16). Following Ref. 23 (see also 8) the effect of baryonic matter on the DE condensate is the subtraction of DE gravitons according to the formula

$$\delta N_{DE} = -N_B.$$ (39)

The dark force is then interpreted as the response of the DE condensate to the presence of baryonic matter so that the energy of the gravitons mediating the interaction will be modified as follows

$$\varepsilon_N + \delta \varepsilon_N = \varepsilon_N + \varepsilon_{DF} \simeq \frac{\hbar}{r} - \frac{\hbar}{r} \frac{\delta r}{r}.$$ (40)

The first terms describes the Newtonian interaction whereas the second term represents the energy of the gravitons producing the dark force. Because it is relevant only at scales of order $r_0$, the latter can be written as

$$\varepsilon_{DF} = -\frac{\hbar \alpha}{r}, \quad \alpha = \frac{\delta r}{r} \bigg|_{r=r_0}.$$ (41)

The term $\delta r|_{r=r_0}$ can be computed using Eqs. (37) into Eq. (39), giving $\delta r|_{r=r_0} \simeq -Gm_B$. It is interesting to notice that $\delta r$ evaluated at the scale $r_0$ has the same sign and the same order of magnitude of $\delta r$ evaluated at the scale of the cosmological horizon: $\delta L = -2Gm_B [23, 8]$. Physically this means that baryonic matter subtracts a certain amount of gravitons (entropy in [8]) from the DE condensate. Using this result and Eq. (16) one easily obtains

$$\alpha \simeq \sqrt{\frac{Gm_B}{L}}.$$ (42)

which as expected reproduces Eq. (15).

It is quite interesting to notice that the expression for the DF coupling constant $\alpha$ we have found can be rewritten in terms of a simple ratio between the baryonic mass $m_B$ and the total DE mass, $m_{DE}$ or,
equivalently, between the baryonic gravitons’ number \( N_B \) and the total number of DE gravitons in the dS spacetime, \( N_{dS} \). Indeed, exploiting the quadratic scalings of the masses and graviton number given in Eq. (38), we can rewrite Eq. (42) as follows

\[
\alpha \simeq \sqrt{m_B/m_{dS}} \simeq \left( \frac{N_B}{N_{dS}} \right)^{1/4},
\]

where \( m_{dS} \) is the total mass of the dS universe. Because dark force gravitons are pulled out from the DE condensate by the baryonic matter, Eq. (43) tells us that the DF coupling constant is determined completely by the fraction of gravitons of the DE condensate subtracted by baryonic matter.

6 General relativity uplifting and effective fluid description

One common problem of MOND and others approaches where infrared modifications of the laws of gravity are used to describe the "dark matter phenomenology" is the difficulty to perform a “metric-covariant uplifting” of the theory [74]. They are usually formulated in the weak-field regime, whereas gravity must allow for the metric-covariant description given by GR. In our approach the gravitational interaction at galactic scales has an infrared contribution coming from long-range quantum effects. Therefore, fluid space-time models are the natural candidates for providing us with a simple way to perform a metric-covariant uplifting of our theory. We look for IR modifications of Einstein gravity in which the Einstein-Hilbert action remains unchanged and long-range quantum effects are modelled as a fluid sourcing a classical gravitational field.

In Refs. [22, 23] it has been shown that the emergent laws of gravity based on a BEC of gravitons can be described as GR sourced by an anisotropic fluid. In this description the radial pressure of the fluid describes the dark force. Since the latter is originated from the quantum long-range properties of DE, if we neglect them, the DE is in general described as an isotropic fluid with constant energy density and equation of state \( p = -\rho \). The simplest way to take into account at the GR level the reaction of DE to the presence of baryonic matter responsible for the dark force effects is, therefore, to consider an anisotropic fluid as source in Einstein’s field equations.

This effective fluid description can be easily generalised to the case under consideration in this paper, considering a GR completion of the weak-field description given by Eq. (23). Following Refs. [22, 23], the total radial acceleration has a Newtonian and a pressure term

\[
a^r = a_B + 4\pi G r p_\parallel.
\]

The pressure profile \( p_\parallel \) has to be chosen in such way that Eq. (44) matches Eqs. (23):

\[
p_\parallel(r) = \frac{1}{4\pi} \frac{m_B}{r^3} \frac{1}{e^{1/2\sqrt{Gm_B/a_0}} - 1},
\]

and the radial acceleration is

\[
a^r = \frac{G m_B}{r^2} \frac{1}{1 - e^{1/2\sqrt{Gm_B/a_0}}}.
\]

The full metric solution can be obtained solving Einstein field equations sourced by an anisotropic fluid with a pressure profile given by Eq. (45). The spacetime metric is taken of the form \( ds^2 = -f(r)e^{\Gamma(r)}dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 \) and it is given by

\[
f = 1 - \frac{2G m_B}{r}, \quad \Gamma' = f^{-1} \left( \frac{2G m_B}{r^2} \frac{1}{e^{1/2\sqrt{Gm_B/a_0}} - 1} \right).
\]

In the weak field limit of the metric one finds the potential \( \phi = \frac{1}{2}(f e^{\Gamma}) \), from which one can easily derive the form of the DF acceleration and check that it exactly reproduces Eq. (23). In the MOND regime of Eq. (23), i.e. \( a_B/a_0 \rightarrow 0 \), one obtains the same expansion of the gravitational potential generated by a point-like particle given in Ref. [22] with the characteristic logarithmic behaviour of MOND and an extremely tiny Machian contribution to the Newtonian potential (see Ref. [22]).
7 Conclusions

In this paper we have argued that long-range quantum gravity effects could play a relevant role at galactic scales. The non-Newtonian behaviour of the radial acceleration at these scales is explained, without assuming the presence of dark matter, as the macroscopic effect of a huge number of extremely soft bosonic DE excitations with wavelength much bigger than the size of the cosmological horizon, in thermal equilibrium with de Sitter spacetime.

We have used simple and general arguments not relying on any specific assumption of the would-be microscopic theory of gravity but, rather, only on general features of thermodynamics, quantum and statistical mechanics and general relativity.

Our formula for the additional "dark" component of the acceleration agrees with the phenomenological relation obtained by McGaugh et al. by fitting a large amount of observational data and with the MOND theory. The functional form of the dark acceleration is a direct consequence of the Bose-Einstein thermal distribution for the number of DE excitations.

The BE distribution, in particular the exponential suppression of the number of hard modes, together with the expression for the energy $\varepsilon$ of these modes, explain why the number of DE modes becomes significant only at galactic scales, i.e. at scales larger than the IR scale $r_0$ at which the dark matter effects arise in galaxies. For this reason, they only affect the gravitational interaction at galactic scales, leaving unaltered the usual Newtonian contribution at smaller ones. Moreover, the BE distribution and the explicit expression of $\varepsilon$ lead to the correct value of $r_0$.

Remarkably enough, we have derived the same results using a generalized thermal equivalence principle (GTEP), which promotes the Deser-Levin formula in Eq. (24) to an universal dynamical feature of thermal quantum gravity systems. Moreover, the GTEP allows us to determine the value $1/2\pi$ for the proportionality factor between the MOND acceleration parameter $a_0$ and the cosmological acceleration $H$. This value is in accordance with the results found in [33, 58] obtained by fitting a large amount of observational data.

Last but not least we have shown that our formula appears as the weak field limit of Einstein’s general relativity sourced by an anisotropic fluid.

A weakness of our paper is the lack of a quantum theory describing the DE bosonic excitations, which play a crucial role in our derivation. Presently, we do not have a consistent theory of quantum gravity of the dS spacetime, which, in some way, should provide us the consistent description of these DE bosonic excitations. In particular, we expect that this theory should reproduce our result (15). Similar considerations hold for the GTEP we have proposed in this paper. Owing to its intrinsic quantum origin we expect this principle to find an explanation in the thermal description of the quantum theory of the dS spacetime.

Our explanation of galactic phenomenology as dominated, at large distances from the galactic core, by long-range quantum gravity effects presently covers only the case of galactic rotational curves. Gravitational lensing effects are also very important for validating or confuting our approach. These effects could be described by using the covariant uplifting of our weak field description, namely Einstein’s general relativity sourced by an anisotropic fluid presented in sect. (6). We have not addressed this important point in this paper.

It must be also clearly stated that at present stage of development of the subject, the long-range quantum gravity approach we have presented here cannot represent a full alternative to dark matter and the $\Lambda$CDM model. All the cosmological implications of our approach, in particular those concerning structure formation and the dynamical process of the bullet cluster have to be worked out if long-range quantum gravity effects have to represent a viable alternative to dark matter.

References

[1] A. D. Sakharov, “Vacuum quantum fluctuations in curved space and the theory of gravitation,” Sov. Phys. Dokl. 12 (1968) 1040–1041. [Sov. Phys. Dokl. 12 (1967)].

[2] T. Jacobson, “Thermodynamics of space-time: The Einstein equation of state,” Phys. Rev. Lett. 75 (1995) 1260–1263 arXiv:gr-qc/9504004 [gr-qc]
[3] T. Padmanabhan, “Thermodynamical Aspects of Gravity: New insights,” Rept. Prog. Phys. 73 (2010) 046901, arXiv:0911.5004 [gr-qc].

[4] T. Padmanabhan, “Gravity and Spacetime: An Emergent Perspective,” in Springer Handbook of Spacetime, A. Ashtekar and V. Petkov, eds., pp. 213–242. 2014.

[5] T. Jacobson, “Entanglement Equilibrium and the Einstein Equation,” Phys. Rev. Lett. 116 no. 20, (2016) 201101, arXiv:1505.04753 [gr-qc].

[6] D. Oriti, “The universe as a quantum gravity condensate,” Comptes Rendus Physique 18 (2017) 235–245, arXiv:1612.09521 [gr-qc].

[7] T. Padmanabhan, “The Atoms Of Space, Gravity and the Cosmological Constant,” Int. J. Mod. Phys. D25 no. 07, (2016) 1630020, arXiv:1505.08653 [gr-qc].

[8] E. P. Verlinde, “Emergent Gravity and the Dark Universe,” SciPost Phys. 2 no. 3, (2017) 016, arXiv:1611.02269 [hep-th].

[9] N. S. Linnemann and M. R. Visser, “Hints towards the emergent nature of gravity,” Stud. Hist. Philos. Mod. Phys. B64 (2018) 1–13, arXiv:1711.10503 [physics.hist-ph].

[10] S. De, T. P. Singh, and A. Varma, “Quantum gravity as an emergent phenomenon,” arXiv:1903.11068 [gr-qc].

[11] G. Dvali and C. Gomez, “Self-Completeness of Einstein Gravity,” arXiv:1005.3497 [hep-th].

[12] G. Dvali, S. Folkerts, and C. Germani, “Physics of Trans-Planckian Gravity,” Phys. Rev. D84 (2011) 024039, arXiv:1009.6084 [hep-th].

[13] G. Dvali, G. F. Giudice, C. Gomez, and A. Kehagias, “UV-Completion by Classicalization,” JHEP 08 (2011) 108, arXiv:1010.1415 [hep-ph].

[14] G. Dvali, C. Gomez, and A. Kehagias, “Classicalization of Gravitons and Goldstones,” JHEP 11 (2011) 070, arXiv:1103.5963 [hep-th].

[15] G. Dvali and C. Gomez, “Black Hole’s Quantum N-Portrait,” Fortsch. Phys. 61 (2013) 742–767, arXiv:1210.3359 [hep-th].

[16] G. Dvali and C. Gomez, “Black Holes as Critical Point of Quantum Phase Transition,” Eur. Phys. J. C74 (2014) 2752, arXiv:1207.4059 [hep-th].

[17] G. Dvali and C. Gomez, “Black Hole’s 1/N Hair,” Phys. Lett. B719 (2013) 419–423, arXiv:1203.6575 [hep-th].

[18] P. Binetruy, “Vacuum energy, holography and a quantum portrait of the visible Universe,” arXiv:1208.4645 [gr-qc].

[19] W. Mueck, “On the number of soft quanta in classical field configurations,” Can. J. Phys. 92 no. 9, (2014) 973–975, arXiv:1306.6245 [hep-th].

[20] R. Casadio, A. Giugno, and A. Giusti, “Matter and gravitons in the gravitational collapse,” Phys. Lett. B763 (2016) 337–340, arXiv:1606.04744 [hep-th].

[21] R. Casadio, A. Giugno, A. Giusti, and M. Lenzi, “Quantum corpuscular corrections to the Newtonian potential,” arXiv:1702.05918 [gr-qc].

[22] M. Cadoni, R. Casadio, A. Giusti, W. Mueck, and M. Tuveri, “Effective Fluid Description of the Dark Universe,” Phys. Lett. B776 (2018) 242–248, arXiv:1707.09945 [gr-qc].
[23] M. Cadoni, R. Casadio, A. Giusti, and M. Tuveri, “Emergence of a Dark Force in Corpuscular Gravity,” Phys. Rev. D97 no. 4, (2018) 044047, arXiv:1801.10374 [gr-qc].

[24] A. Giusti, “On the corpuscular theory of gravity,” Int. J. Geom. Meth. Mod. Phys. 16, no. 03, 1930001 (2019). doi:10.1142/S0219887819300010

[25] A. Blommaert, T. G. Mertens and H. Verschelde, “Clocks and Rods in Jackiw-Teitelboim Quantum Gravity,” arXiv:1902.11194 [hep-th].

[26] G. Compère, “Are quantum corrections on horizon scale physically motivated?,” arXiv:1902.04504 [gr-qc].

[27] G. t. Hooft, “The quantum black hole as a theoretical lab, a pedagogical treatment of a new approach,” in 56th International School of Subnuclear Physics: From gravitational waves to QED, QFD and QCD (ISSP 2018) Erice, Italy, June 14-23, 2018. 2019. arXiv:1902.10469 [gr-qc].

[28] Supernova Search Team Collaboration, A. G. Riess et al., “Observational evidence from supernovae for an accelerating universe and a cosmological constant,” Astron. J. 116 (1998) 1009–1038, arXiv:astro-ph/9805201 [astro-ph].

[29] A. A. Penzias and R. W. Wilson, “A Measurement of excess antenna temperature at 4080-Mc/s,” Astrophys. J. 142 (1965) 419–421.

[30] Planck Collaboration, P. A. R. Ade et al., “Planck 2013 results. XVI. Cosmological parameters,” Astron. Astrophys. 571 (2014) A16, arXiv:1303.5076 [astro-ph.CO].

[31] R. B. Tully and J. R. Fisher, “A New method of determining distances to galaxies,” Astron. Astrophys. 54 (1977) 661–673.

[32] S. S. McGaugh, J. M. Schombert, G. D. Bothun, and W. J. G. de Blok, “The Baryonic Tully-Fisher relation,” Astrophys. J. 533 (2000) L99–L102, arXiv:astro-ph/0003001 [astro-ph].

[33] S. McGaugh, F. Lelli, and J. Schombert, “Radial Acceleration Relation in Rotationally Supported Galaxies,” Phys. Rev. Lett. 117 no. 20, (2016) 201101, arXiv:1609.05917 [astro-ph.GA].

[34] M. Milgrom, “A Modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis,” Astrophys. J. 270 (1983) 365–370.

[35] M. Milgrom, “MOND theory,” Can. J. Phys. 93 no. 2, (2015) 107–118, arXiv:1404.7661 [astro-ph.CO].

[36] S. Hossenfelder, “Covariant version of Verlinde’s emergent gravity,” Phys. Rev. D95 no. 12, (2017) 124018, arXiv:1703.01415 [gr-qc].

[37] D.-C. Dai and D. Stojkovic, “Comment on 'Covariant version of Verlinde’s emergent gravity',” Phys. Rev. D96 no. 10, (2017) 108501, arXiv:1706.07854 [gr-qc].

[38] R.-G. Cai, S. Sun, and Y.-L. Zhang, “Emergent Dark Matter in Late Universe on Holographic Screen,” arXiv:1712.09326 [hep-th].

[39] L. Smolin, “MOND as a regime of quantum gravity,” Phys. Rev. D96 no. 8, (2017) 083523, arXiv:1704.00780 [gr-qc].

[40] S. Alexander and L. Smolin, “The Equivalence Principle and the Emergence of Flat Rotation Curves,” arXiv:1804.09573 [gr-qc].

[41] M. Milgrom, “The modified dynamics as a vacuum effect,” Phys. Lett. A253 (1999) 273–279, arXiv:astro-ph/9805346 [astro-ph].
[60] C. Di Paolo, P. Salucci, and J. P. Fontaine, “The Radial Acceleration Relation (RAR): Crucial Cases of Dwarf Disks and Low-surface-brightness Galaxies,” Astrophys. J. 873 no. 2, (2019) 106, arXiv:1810.08472 [astro-ph.GA].

[61] H. Narnhofer, I. Peter, and W. E. Thirring, “How hot is the de Sitter space?,” Int. J. Mod. Phys. B10 (1996) 1507–1520.

[62] S. Deser and O. Levin, “Accelerated detectors and temperature in (anti)-de Sitter spaces,” Class. Quant. Grav. 14 (1997) L163–L168 arXiv:gr-qc/9706018 [gr-qc].

[63] T. Jacobson, “Comment on ‘Accelerated detectors and temperature in anti-de Sitter spaces’,” Class. Quant. Grav. 15 (1998) 251–253 arXiv:gr-qc/9709048 [gr-qc].

[64] M. Cadoni, “Conformal symmetry of gravity and the cosmological constant problem,” Phys. Lett. B642 (2006) 525–529, arXiv:hep-th/0606274 [hep-th].

[65] E. Witten, “Quantum gravity in de Sitter space,” in Strings 2001: International Conference Mumbai, India, January 5-10, 2001. 2001. arXiv:hep-th/0106109 [hep-th].

[66] G. Dvali and C. Gomez, “Quantum Compositeness of Gravity: Black Holes, AdS and Inflation,” JCAP 1401 (2014) 023 arXiv:1312.4795 [hep-th].

[67] S. Das and R. K. Bhaduri, “Dark matter and dark energy from a Bose-Einstein condensate,” Class. Quant. Grav. 32 no. 10, (2015) 105003, arXiv:1411.0753 [gr-qc].

[68] S. Das and R. K. Bhaduri, “Bose-Einstein condensate in cosmology,” arXiv:1808.10505 [gr-qc].

[69] S. Das and R. K. Bhaduri, “On the quantum origin of a small positive cosmological constant,” arXiv:1812.07647 [gr-qc].

[70] S. McGaugh, “Milky Way Mass Models and MOND,” Astrophys. J. 683 (2008) 137–148, arXiv:0804.1314 [astro-ph].

[71] R. H. Sanders, “A historical perspective on modified Newtonian dynamics,” Can. J. Phys. 93 no. 2, (2015) 126–138 arXiv:1404.0531 [physics.hist-ph].

[72] P. D. Mannheim, “Is dark matter fact or fantasy? – clues from the data,” arXiv:1903.11217 [astro-ph.GA].

[73] M. Tuveri and M. Cadoni, “A new perspective on galactic dynamics,” arXiv:1904.08209 [gr-qc].

[74] B. Famaey and S. McGaugh, “Modified Newtonian Dynamics (MOND): Observational Phenomenology and Relativistic Extensions,” Living Rev. Rel. 15 (2012) 10 arXiv:1112.3960 [astro-ph.CO].