Structural Optimization design of marine hydraulic Tube clamp

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Abstract. The layout design and structure design of rubber elastic tube clamp have a direct impact on the vibration and noise characteristics of pipeline system, as well as the fatigue life of tube clamp itself and related parts in pipeline system. It is a typical multi-objective optimization design problem. How to design an elastic tube clamp that meets the requirements of stiffness, strength and fatigue life at the same time is an important issue in the design of marine pipeline system. In this paper, the structural optimization of rubber elastic tube clamps is studied by using the non-linear multi-objective topology optimization method.

1. Introduction
Outboard tube clamp, as pipeline support device, should have certain stiffness to ensure that the displacement of pipeline is not too large; at the same time, it should be flexible to ensure that the pipeline will not be damaged by excessive stress when it is subjected to certain deformation; at the same time, it should ensure certain vibration isolation capacity, so that the excitation force transmitted to the hull is small and radiated noise is reduced [1]. Therefore, the design of tube clamp is a typical multi-objective optimization problem. In this paper, multi-objective topology optimization method is used to design the tube clamp. Because of the better mechanical properties of rubber material, rubber is used as elastic material in tube clamp, and multi-objective topology optimization technology based on finite element analysis is used to optimize the layout of rubber material in tube clamp. Because the current commercial software doesn’t provide multi-objective non-linear material topology optimization design method, the MATLAB program written by myself is done to complete the tube clamp topology optimization design [2]. Following is a brief introduction of its main technology and process.

2. Constitutive Model of Rubber Material
The constitutive model of rubber material is the basis of finite element analysis of rubber structural parts. Firstly, various constitutive models of rubber materials are studied, and the constitutive parameters of rubber materials used in tube clamp are identified by experiments, which are used for finite element analysis of tube clamp.

Rubber belongs to macromolecule super-elastic material. The so-called super-elasticity means that the material can produce great strain under the action of external force. When unloaded, the strain can be automatically restored [3]. That is to say, the deformation in the process is elastic deformation. At
present, in most commercial non-linear finite element analysis systems, a class of hyper-elastic incompressible material model has been established. The rubber hyper-elasticity model is based on the hypothesis of isotropy and isotherm characterized by a unified physical quantity, which is the strain specific energy function \( U \), which is a scalar function of strain or deformation tensor.

\[
U = U(B)
\]  

Where, \( B \) is Cauchy-Green deformation tensor. The derivative of the strain component is the corresponding stress component:

\[
T = \frac{\partial U}{\partial E}, T_{ij} = \frac{\partial U}{\partial e_{ij}}
\]  

Where, \( T \) is the second-order Piola-Kirchhoff stress tensor and \( E \) is the Green-Lagrange strain tensor are used. Therefore, the Cauchy stress tensor can be expressed as:

\[
\sigma = \frac{1}{\kappa} \text{HTH}^T
\]

\[
H = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}, H_{ij} = \frac{\partial x_i}{\partial X_j}
\]

\[
\kappa = \det(H)
\]

\[
B = HH^T
\]

Where, \( x_i \) represents the current coordinates and \( X_j \) the reference coordinates. For isotropic hyper-elastic materials, the strain specific energy function can be expressed by three invariants \( I_1, I_2 \) and \( I_3 \) of Cauchy-Green strain tensor.

\[
I_1 = \text{trace}(B)
\]

\[
I_2 = \frac{1}{2} \left( I_1^2 - \text{trace}(B^2) \right)
\]

\[
I_3 = \det(B)
\]

So far, the hyper-elastic constitutive model of rubber materials can be generally expressed as follows:

\[
\sigma = -pI + 2 \left( \frac{\partial U}{\partial I_1} + I_1 \frac{\partial U}{\partial I_2} - B \frac{\partial U}{\partial I_2} B^2 \right)
\]

Where, \( I \) is the unit matrix. \( p \) is the Hydrostatic pressure introduced by the incompressibility of rubber. A review of constitutive models of rubber Hyper-elastic Materials is provided in reference [1]. The Ogden constitutive model used in this paper is discussed below. The expression of strain specific energy in Ogden model is as follows:

\[
U = \sum_{i=1}^{N} \frac{2 \mu_i}{\alpha_i^2} \left( \sqrt{\lambda_1}^{\alpha_i} + \sqrt{\lambda_2}^{\alpha_i} + \sqrt{\lambda_3}^{\alpha_i} - 3 \right) + \sum_{j=1}^{N} \frac{1}{D_j} (J_{el} - 1)^{2j}
\]

Where, \( N \) is the material parameter; \( \mu_i, \alpha_i, D_i \) is the temperature-related material parameter.

In the Ogden model, the initial shear modulus \( \mu_0 \) and the initial bulk modulus \( K_0 \) satisfy the following relationships:

\[
\mu_0 = \sum_{i=1}^{N} \mu_i
\]

\[
K_0 = \frac{2}{D}
\]
Generally, the elastic modulus and Poisson's ratio of steels with different compositions do not differ greatly. However, rubber is very different from steel. The material properties of rubber with different compositions are quite different. Therefore, although the constitutive model of rubber material is discussed in detail in the previous section, the specific parameters of the model must be obtained through experiments. The common method is to prepare rubber material samples, and carry out uniaxial, biaxial, plane and volume tensile tests on the samples. After collecting the experimental data, the parameters of the constitutive model are obtained by parameter identification method. In this paper, Ogden model ($N = 3$) is used to calculate the model interpolation of the test data, and a good fitting accuracy is obtained (Table 1).

| mu$_i$ | Alpha$_i$ | D$_i$          |
|--------|-----------|----------------|
| 1      | 0.372301454 | 5.4482338     |
| 2      | 6.56215274e-4 | 5.84632117   |
| 3      | 1.703590401e-2 | -1.83456548  |

Table 1. Identification of constitutive parameters of rubber materials

3. **Nonlinear multi-objective structural topology optimization method for flexible rubber tube clamp**

Topology optimization is an important research direction of structural optimization. It aims to help designers find the best material layout to meet certain objectives and constraints in the initial stage of structural design (conceptual design stage). The results of topology design have a crucial impact on the final performance of the structure. The earliest discussion on topology optimization can be traced back to the truss theory proposed by Michell more than a hundred years ago. However, at that time, due to the limitations of structural analysis and optimization methods, the initial development of topology optimization theory was very limited. With the development of computer software and hardware and the maturity of structural finite element analysis and computer-aided optimization methods, topology optimization has developed vigorously since the end of 1980s. The most important milestone of topology optimization is the homogenization method proposed by Bendse and Kikuchi in 1988. This method relaxes the original problem into a continuous variable optimization problem by introducing a composite material model, which can be efficiently solved by Optimal Criteria (OC) and Mathematical Programming (MP). Density method is a variant of homogenization method. There are many specific forms at present, including SIMP method, RAMP method and so on. The common point of density method is that the elastic tensor of material is defined as a monotone function of material density, the original problem is relaxed as a continuous variable optimization problem, and the penalty term is introduced implicitly or explicitly to eliminate the intermediate density.

3.1. **Finite Element Analysis Method of Rubber Flexible Tube Clamp**

Because the material nonlinearity and geometric nonlinearity of flexible rubber tube clamps are related to the history of loading and deformation. In structural analysis, the load is usually divided into several increments. Assuming the displacement $'u_i$ and strain $'e_{ij}$ and the stress $'\sigma_{ij}$ corresponding to the load and displacement conditions at the time $t$ have been found, that the following equations and boundary conditions should be satisfied when time transits to $t + \Delta t$:

$$\nabla \cdot \left( \sigma + \Delta \sigma \right) + 'F + \Delta F = 0 \quad \text{(in } \Omega \text{)}$$

$$\sigma = -p I + 2\left[ \left( \frac{\partial U}{\partial I_1} + I_1 \frac{\partial U}{\partial I_2} \right) B - \frac{\partial U}{\partial I_2} B^2 \right] \quad \text{(in } \Omega \text{)}$$

$$'T + \Delta T = 'T + \Delta T \quad \text{(on } S_\sigma \text{)}$$

$$'u + \Delta u = 'u + \Delta u \quad \text{(on } S_u \text{)}$$

(14)
Where $\Omega$ is the two-dimensional or three-dimensional geometric domain, $S_\sigma$ is the load boundary and $S_u$ is the constraint boundary. The principle of virtual displacement in incremental form is established.

$$
\int_{\Omega} \nabla \cdot (\sigma + \Delta \sigma) \delta (\Delta u) d\Omega - \int_{\Omega} (\bar{F} + \Delta \bar{F}) \delta (\Delta u) d\Omega \\
- \int_{S_u} (\bar{T} + \Delta \bar{T}) \delta (\Delta u) dS = 0
$$

Equation (16) is the equivalent integral form of equation (15), which can be solved by finite element method. Firstly, $n$ finite elements are discretized of $\Omega$, and the displacement field $u$ functions are expressed as the following approximate functions:

$$
u = \sum_{e=1}^{n} [N] \{a\}^e = Na
$$

Among them, $\{a\}^e$ is the undetermined coefficients defined on the nodes, $[N]$ are unit interpolation basis functions or shape functions, which are known as complete function sequences and are linearly independent. The so-called complete series of functions means that any function can be represented by this sequence. $n$ represents the number of finite elements in which the domain $\Omega$ is discretized. By using Galerkin method and introducing formula (16) into equation (15), and then arranging and assembling the elements, the following general finite element equations can be obtained:

$$Ka = F
$$

According to the naming method in elasticity, $K$ it is called the global stiffness matrix, which is composed of element stiffness matrix $K_e$. It describes the integral mapping relationship from equation (15) to form shape function $[N]$. $a$ is known as the nodal displacement vector, it is actually a vector composed of undetermined coefficients $\{a\}^e$. It is known by formula (16) that the only definite field function $u$ will be determined by $a$. $F$ is the nodal load vector. Because of the nonlinearity of the equation, where the $K$ is the function of $a$, which can be solved by Newton-Raphson method.

### 3.2. Mathematical Model and Solution Method of Multi-objective Topology Optimization Problem

The mathematical model of multi-objective topology optimization can be generally expressed as follows:

$$\min \Phi = [\Phi_1, \Phi_2,...,\Phi_m]$$

s.t. $g_k(a, \rho) \leq 0$ ($k = 1,2,...p$)

$h_i(a, \rho) = 0$ ($l = 1,2,...q$)

Where, $\Phi$ is the objective function vectors consisting of $m$ objective functions, $g_k$, $h_l$, representing inequality constraints and equality constraints respectively, $a$ are obtained by the finite element equation (17), $\rho$ is the regularized material density. In this paper, SIMP material interpolation method is used to relax the 0-1 programming topology optimization problem into a continuous variable optimization problem. The SIMP method was originally used to solve elasticity problems. The central idea is that the physical parameters $C$ of materials are expressed as the following functions of density $\rho$:

$$C_e = C^{\min} + (C^0 - C^{\min}) \rho_e^{pl}$$

$C$ is a vector composed of many physical parameters. Rubber material uses Mooney-Rivlin model, so it can be recorded as:
\[ C = [C_{10}, C_{01}, \frac{1}{D}]^T \] (20)

The density \( \rho_e \) of the element will be entered into the overall stiffness matrix \( K \) of the finite element equation (17) through equation (19). \( C_e \) is the physical parameters of the unit, when the elasticity problem is discussed. \( \rho_e \) is the regularization density of the corresponding unit. \( C^0 \) is the physical parameter corresponding to the density of material when it is 1. \( C^\text{min} \) is the physical parameter corresponding to the density \( \rho^\text{min} \) of material when it is considered as a hole and no material. In order to avoid solving singularities, usually \( C^\text{min} = 0.001 C^0, \rho^\text{min} = 0.001 \), and the penalty factor introduced by SIMP method is usually taken, and the intermediate density value is eliminated by implicit penalty method.

There are many methods to solve multi-objective optimization problems (18), but generally they can be classified into two categories: one is non-priority method, the other is Preference method. Non-priority method means that the priority of each objective function can not be given beforehand, so only a Pareto optimization solution set (surface) can be obtained. Finally, a point on the Pareto surface can be selected as the final optimal solution by knowledge-based method. The principle of priority is to know the priority of each objective function beforehand, and then construct a new objective function to transform the multi-objective problem into a single-objective problem to solve, such as the weighting method. Because the multi-physical field topology optimization problem is a very large-scale optimization problem, it is not suitable to use the first kind of method, and the weighting method is a good choice. At present, the weighted method is widely used in engineering and has been successful. In this paper, the weighted method is used to solve the multi-objective problem. A new optimization problem constructed by weighting method can be expressed as:

\[ \min \Phi = \omega_1 \Phi_1 + \omega_2 \Phi_2 + \ldots + \omega_m \Phi_m \]
\[ \text{s.t. : } g_k(a, \rho) \leq 0 \quad (k = 1, 2, \ldots p) \]
\[ h_l(a, \rho) = 0 \quad (l = 1, 2, \ldots q) \]
\[ \rho^\text{min} \leq \rho \leq 1 \] (21)

Where, the weighted objective function \( \Phi \) is newly constructed, \( \Phi_m \) represents the normalized \( m^{th} \) objective function, and its corresponding weight coefficients \( \omega_m \) are satisfied by \( \sum_{i=1}^{m} \omega_i = 1 \)

### 3.3. Sensitivity analysis method

For optimization problems (21), mathematical programming methods such as sequential linear programming (SLP) and moving asymptote method (MMA) are usually used to solve them. When these gradient-based optimization methods are used, the first-order gradient (sensitivity) of all objective functions and constrained functions with respect to design variables \( \rho \) must be provided. Here, we use adjoint method to derive the first-order analytical sensitivity of the objective function. Consider the \( m^{th} \) objective function

\[ \Phi_m = \int_{\Omega} \varphi(a, \rho) dV \] (22)

Where, functions \( \varphi \) are defined on the domain \( \Omega \). In order to derive the sensitivity of the objective function \( \Phi_m \), we introduce the Lagrange term to construct a new function \( \Phi_m^* \).

\[ \Phi_m^* = \Phi_m + \lambda^T (K \rho - F) \] (23)
Bring equation (17) into (22) knowledge, \( \Phi_m^* = \Phi_m \) and find the first derivative of design variables \( \rho \) for (22)

\[
\frac{d\Phi_m}{d\rho} = \frac{\partial\Phi_m}{\partial \rho} + \frac{\partial\Phi_m}{\partial \mathbf{a}} \frac{d\mathbf{a}}{d\rho} + \lambda^T \mathbf{K} \frac{d\mathbf{a}}{d\rho} + \lambda^T \frac{\partial \mathbf{K}}{\partial \rho} \mathbf{a}
\]

\[
= \frac{\partial\Phi_m}{\partial \rho} + \left( \frac{\partial\Phi_m}{\partial \mathbf{a}} + \lambda^T \mathbf{K} \right) \frac{d\mathbf{a}}{d\rho} + \lambda^T \frac{\partial \mathbf{K}}{\partial \rho} \mathbf{a} \tag{24}
\]

In order to eliminate the implicit term \( \frac{d\mathbf{a}}{d\rho} \), we introduce the adjoint problem:

\[
K \lambda = -\frac{\partial\Phi_m}{\partial \mathbf{a}} \tag{25}
\]

It can be seen from equation (22) that the value of Lagrange multiplier vector \( \lambda \) can be obtained by replacing the load vector \( \mathbf{F} \) in equation (17) with a new vector \( -\frac{\partial\rho}{\partial \mathbf{a}} \). Then the sensitivity of the objective function \( \Phi_m \) is obtained by introducing (21):

\[
\frac{d\Phi_m}{d\rho} = \frac{\partial\Phi_m}{\partial \rho} + \lambda^T \frac{\partial \mathbf{K}}{\partial \rho} \mathbf{a} \tag{26}
\]

Where \( \frac{\partial \mathbf{K}}{\partial \rho} \) can be obtained directly by variational method.

### 3.4. Optimization algorithm

Sequential linear programming (SLP) and moving asymptote method (MMA) are commonly used mathematical programming methods in topology optimization. The basic idea of SLP method is to expand the non-linear objective function and constraint function in Taylor series at the initial design point, retain the linear term, form a series of linear sub-optimization problems, and then solve them by linear programming method to obtain new design points. If the convergence requirement is not satisfied, the new design points are used to replace the initial point and re-expand Taylor series to form new linear sub-optimization problems. The problem is solved and iterated repeatedly until the convergence requirement is satisfied. The final design point is the optimal solution of the original problem. By introducing a moving asymptote, MMA transforms the implicit optimization problem into a series of more explicit and strictly convex approximation sub-problems. The approximation function is determined by the derivatives of the left and right asymptotic points, the original objective function and the constraint function at each point. MMA method is currently the most effective mathematical programming method for solving multi-objective topology optimization.

### 3.5. Expressions of stiffness, volume and fatigue objective functions for topological optimization of flexible rubber pipe clamp

In this paper, the multi-objective topology optimization of flexible rubber tube clamps including stiffness, volume and fatigue objectives is discussed. The functional expressions of these objectives are deduced.

1. **Stiffness object**

   The stiffness objective function in the direction \( x, y, z \) can be expressed by the displacement of the load boundary points, namely:
\[ \Phi_{xx} = \frac{1}{\Omega} \int_{\Omega} a_1 \cdot \delta \left( |r - r_{out}| \right) \, d\Omega \]  
(27)

\[ \Phi_{yy} = \frac{1}{\Omega} \int_{\Omega} a_2 \cdot \delta \left( |r - r_{out}| \right) \, d\Omega \]  
(28)

\[ \Phi_{zz} = \frac{1}{\Omega} \int_{\Omega} a_3 \cdot \delta \left( |r - r_{out}| \right) \, d\Omega \]  
(29)

Where, \( a_1, a_2, a_3 \) is the component of node displacement vector \( \mathbf{a} \) in \( x, y, z \) direction respectively, \( r = (x \ y \ z)^T \) is the position vector; \( r_{out} \) is the position vector of displacement output point. \( \delta \) is a Dirac function:

\[ \delta(x) = \begin{cases} 
1 & (x = 0) \\
0 & (x \neq 0) 
\end{cases} \]  
(30)

After introducing the function \( \delta \), the target at the discrete point in the design domain can be represented by the integral function, which forms the same format as the sum formula (22). Thus, the derivation of the sensitivity function above is also valid for the target at the discrete point.

(2) Volume objective function

The volume function can be expressed as

\[ \Phi_v = \frac{1}{|\Omega|} \int_{\Omega} \rho \, d\Omega - f^v \]  
(31)

Where, \(|\Omega|\) represents the total volume of the design domain and \( f^v \) is the percentage of the target volume to the total volume. Since the expression (27) does not contain an explicit \( \mathbf{a} \) substitution (26), the sensitivity of the volume constraint function obtained is as follows:

\[ \frac{d\Phi_v}{d\rho} = \frac{1}{|\Omega|} \]  
(32)

(3) Fatigue objective function

A large number of references reveal that the fatigue failure of rubber is mainly caused by the high strain energy density \( W \) at some points on the boundary of rubber structure. For uniaxial load, the fatigue life of rubber structure is as follows:

\[ N_f = M W^{-F_c} \]  
(33)

Where, \( M \) is a constant related to the material; \( F_c \) is a value related to the cyclic load. Therefore, the objective function of fatigue life can be expressed as:

\[ \Phi_{dua} = \int_{\Omega} \max(W) \, d\Omega = \int_{\Omega} \max(\{a\}^T K_e \{a\}^T) \, d\Omega \]  
(34)

Because the max function is a non-continuous function, it will make the optimization process difficult to converge. Therefore, the new objective function of fatigue life can be expressed by relaxing formula (34) to improve the differentiability of the function.

\[ \Phi_{dua} = \int_{\Omega} \delta \left[ \frac{1}{4} (\nabla \cdot \mathbf{a})(\nabla \cdot \mathbf{a})^T - \psi \right] \mathbf{a} K_e^T \, d\Omega \]  
(35)
The $\delta$ is Dirac function, $\psi$ is used to represent the relaxation factor of fatigue life target. So far, $\psi = 0.03$ the objective function of fatigue life and the general objective function (22) have the same format, so the previous derivation of the sensitivity function is also valid for the fatigue life target.

4. Design Example of DN32 Flexible tube clamp

Because the pipe clamp is axisymmetric, in order to solve the problem faster, the topology optimization problem of the tube clamp structure is simplified to a 2D axisymmetric problem as shown in Figure 1. Among them, the red area is the design area, and its material is rubber; the green area represents the non-design area, and its material is steel. Compared with the rubber material, it has great stiffness. Therefore, it is assumed that it is a rigid body, and the action point of force P is connected with it through the rigid element. The force condition and restraint condition of the flexible pipe clamp are shown in the figure. The flexible pipe clamp is made of rubber with the same batch of material experiments. The parameter identification shows that the Ogden model is used to describe the isotropic hyper-elastic rubber material, and its constitutive parameters are shown in Table 1.

\[ \text{Figure 1. Design Domain and Boundary Conditions Diagram} \]

F1 = 1000N and F2 = 1000N represent the axial and radial forces acting on the flexible tube clamp under two working conditions respectively. According to the overall dynamic index of outboard pipeline system, the rigidity requirements of the flexible pipe clamp are: under the action of F1 and F2, the target displacement vectors are $r_{\text{out1}} = [2 \ 0]^T \text{ mm}$, $r_{\text{out2}} = [0 \ 2]^T \text{ mm}$ respectively,. Therefore, the multi-objective topology optimization problem taking volume fraction $f^V = 0.5$ and considering the x-direction, Y-direction stiffness and fatigue life of flexible pipe clamps can be described as:
\[
\min \Phi = [\phi_{xx}, \phi_{sy}, \phi_{duraF1}, \phi_{duraF2}]
\]
s.t.: \(\phi_k \leq 0 \quad (k = 1, 2, ..., p)\)

\[
\phi_{xx} = \frac{1}{\Omega} \int_{\Omega} x \cdot \delta(|r - r_{out1}|) d\Omega
\]

\[
\phi_{sy} = \frac{1}{\Omega} \int_{\Omega} y \cdot \delta(|r - r_{out2}|) d\Omega
\]

\[
\phi_v = \frac{1}{\Omega} \int_{\Omega} \rho d\Omega - f^v
\]

\[
\phi_{dura} = \int_{\Omega} \delta \left[ \frac{1}{4} (\nabla \cdot a)(\nabla \cdot a)^T - \gamma \right] aK \mathbf{a}^T d\Omega
\]

The weighted method is used to solve this multi-objective topology optimization problem. The optimization results are shown in Figure 2. The red area represents the area with material, the blue area represents the area without material, and the transition color between blue and red represents the intermediate density unit, which is removed in the detailed design stage.

![Figure 2. Topology optimization results of flexible tube clamp](image)

### 5. Conclusion

In this paper, non-linear multi-objective topological optimization method is used to study the multi-objective structural topological optimization of rubber elastic tube clamps. The elastic tube clamp which satisfies the requirements of stiffness, strength and fatigue life is designed, and good application results are obtained.

### References

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