Modeling of the random texture surface based on self-similar structures

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Abstract. The construction realized of random texture surfaces based on self-similar ones with the addition of certain rules of the accidental element in the construction process. An example of a random fractal widespread in nature is obtained in the process of so-called diffusion-precise aggregation. I investigated spatial spectrum of the formed surfaces on the basis of the multidimensional Fourier transform. I used software package ParaView to visualize 3D textures.

1. Introduction

In recent years, interdisciplinary research in modern Earth Sciences is becoming increasingly important. Research is increasingly conducted at the junction of different scientific areas [1]. For a comprehensive assessment of the current state of various natural structures, it is necessary to conduct research using not only classical scientific methods developed and tested in the system of Earth Sciences, but also the latest physical, mathematical, computer knowledge and technologies. One of these tools is fractal analysis. This method allows to estimate the structure of self-similarity of a natural object, to reveal its fractal properties [2].

Complex assessment of the state of various natural structures by their images requires not only classical methods of modeling and forecasting trends in the properties of natural objects [2-3]. The fractal approach can be applied to natural objects (in particular, to describe landscape images), demonstrating the properties of self-similarity in a relatively wide range of characteristic scales. Such methods use fractional topological dimension of signals, as well as properties of self-similarity or scaling [3]. It should be noted, that such properties of optical fields as self-similarity [2-3], self-reproduction [4-6], periodicity [7-8] and invariance [9-11] are close and interrelated [12-13].

Note that certain properties of statistical fractals, such as aerosols, smoke, moiré [14-17] coincide, what is very important for the transmission of optical signal through a non-uniform or random medium [18-21].

One of the most important characteristics of fractals is the spatial spectrum [22-26].

In this paper, the construction of random texture surfaces on the basis of self-similar structures with the introduction of a probabilistic element in the construction process. An example of a random fractal common in nature, which is obtained in the process of so-called diffusion-limited aggregation, is considered. The spatial spectra of the formed surfaces are calculated on the basis of the multidimensional Fourier transform.
2. Research result
Consider one of the simplest fractal structures called the Harter – Heituey dragon [27].

![Fractal Structure](image1)

**Figure 1.** The view of the fractal structure dragon Harter – Hatuey (a) and its spatial spectrum (b) for $N=23$ level of the fractal.

In figure 1 it is clearly seen that the spatial spectrum of this structure contains vortex twist associated with the construction of a fractal.

The process of building a dragon-like structure with the introduction of a random angle into the construction process was used when forming a random field. The result is shown in figure 2 and 3.

![Random Field](image2)

**Figure 2.** (a) Regular draconian construction; (b) Spatial spectrum of the regular draconian construction.

![Random Field](image3)

**Figure 3.** (a) The random dragon-like field; (b) Spatial spectrum of the random dragon-like field.

Thus, the introduction of a random element in the process of fractals building allowed to form a random field.

Let us consider the examples of formation of random two-dimensional and three-dimensional fractals.

The fractal is modeled by generating a random Cantor set. Each random Cantor set is assigned the same random process, that is, the removal of a part of the segment, and the length of the segment and the parts separated by it are set randomly within the acceptable value.
At the same time, a certain degree of regularity, retains since at each step of the fractal construction process, the removal is repeated randomly on both sides of the remote segment.

You can build a two-dimensional fractal simply as a product of one-dimensional ones, and you can enter a scale transformation along different axes. Moreover, in this case it is possible to use fractals of different levels. Then the spatial spectrum for the two-dimensional triad Cantor fractal can be estimated by the equation [23, 24, 28, 29]:

\[
F_{S,P}(u,v) = 2^{-S-P} \left[ \prod_{s=0}^{S-1} \cos \left( 2\pi \cdot 3^s \alpha u \right) \right] \left[ \prod_{p=0}^{P-1} \cos \left( 2\pi \cdot 3^p \beta v \right) \right] \text{sinc} (\alpha u) \text{sinc} (\beta v),
\]

(1)

where \(S\) – the level of the fractal.

Figure 4 shows examples of random fractals obtained on the basis of the Serpinsky carpet.

\[\text{Figure 4. Random fractal structure view (left) and its spatial spectrum (right) for different fractal levels: a) } S=3; \text{ b) } S=4.\]

To obtain a three-dimensional random fractal, let’s use a single cube at the first step \(E_0 = [0,1] \times [0,1] \times [0,1]\). At the next step (level) the fractal is set as \(E_1 = ([0,a_1] \cup [b_1,1]) \times ([0,a_2] \cup [b_2,1]) \times ([0,a_3] \cup [b_3,1])\), where \(a_1, a_2, a_3, b_1, b_2, b_3\) – the parameters of the fractal set in the range of \((0,1)\), moreover \(a_1 < b_1, a_2 < b_2\) and \(a_3 < b_3\). The generation of a three-dimensional fractal, as well as for the one-dimensional and two-dimensional case, occurs on the principle of the formation of cracks with a growing frequency as the fractal level increases. But some corrections are introduced so that the length at each step is set randomly.

\[\text{Figure 5. Type of three-dimensional random fractal structure (a) and its spatial spectrum (b) for } S=5 \text{ level of the fractal.}\]
Figure 6 shows the two-dimensional fractal obtained from the Serpinsky carpet and its spatial spectrum.

Figure 6. The two-dimensional fractal structure (Sierpinsky carpet) (a) and its spatial spectrum (b) for $S=5$.

Figure 7. The cut-out part of the spectral characteristic taken a) in the middle, b) from the first quarter (left), the obtained two-dimensional fractal (center) and its spatial spectrum (right).

Figure 8. The cut out part of the spectral characteristic taken with the upper line (left), the resulting two-dimensional fractal (center) and its spatial spectrum (right).
The spectral pattern was studied. A part of the spectral pattern was taken for this purpose. The inverse Fourier transform was obtained from it and the direct transformation was applied for this purpose.

Figures 7-9 show the spectral response from part of the initial spatial spectrum shown in figure 6.

![Image of Figures 7-9]

Figure 9. The cut-out part of the spectral characteristic taken with the diagonal line (left), the resulting two-dimensional fractal (center) and its spatial spectrum (right).

3. Conclusion
In this paper are described a two-dimensional and three-dimensional case of obtaining a fractal structure of a random form. The introduction of a random element in the process of fractals building allowed to form a random field. The developed approach can serve as a supplement to the known methods of a surface roughness modeling [30-31] which used in the analysis of manufacturing errors [32-36]. The spatial spectrum of the formed fractals is obtained. The preservation of the fractal structure of the field constructed only on a part of the spatial spectrum including information on the diagonal directions is shown.

4. References
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