Features of the development of a mathematical model of an electric multipole with memresistive branches for nanoelectronic components of quantum computing systems

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Abstract. The article discusses the features of constructing a mathematical model of a memristor, one of the components of quantum computing systems, in the finite increments of its parameters. A modification of the basic set of elements for the design of nanoelectronic circuits with the addition of a memristor as one of the components is described. As a result of the modification, a mathematical model of an electric multipole with memresistive branches in the finite increments of its parameters is constructed.

1. Introduction
The mathematical model of an electric multipole is formed for a certain basic set of elements. The universal set includes: linear and nonlinear resistor, linear and nonlinear capacitance, linear and nonlinear inductance, independent current and voltage sources, dependent (controlled) sources - a current or voltage controlled current source, and a voltage or current controlled voltage source. Thus, this set contains 12 elements, the parametric equations of which are known in finite increments [1-16].

In 1971, the American researcher Chua L.O. suggested the possibility of creating in the future a fourth basic element of electrical circuits. In fact, at the tip of his pen, he predicted the presence of a physical dependence linking 4 main variables of electrical circuits: current I, voltage U, charge Q and magnetic flux F. Five out of six possible relationships connecting these parameters are well known. For the sixth ratio, Chua suggested that the magnetic flux and electric charge be linked through a new characteristic - memresistivity M. And the "absent" element was called memristor:

$$dF = M \, dQ$$  \hspace{1cm} (1)

Note that flux in this case should be understood as an integral of voltage over time Chua showed that in the general case the memristivity should depend on q [5; 13].
In theory, the memristor model can be represented as follows [13]:

\[ u(t) = (R_{ON} \frac{w(t)}{D} + R_{OFF} \left(1 - \frac{w(t)}{D}\right)) \cdot i(t) \]  \hspace{1cm} (2)

In this case, the border shifts according to the law:

\[ \frac{dw(t)}{dt} = \mu_u \cdot \frac{R_{ON}}{D} \cdot i(t) \]  \hspace{1cm} (3)

where \( \mu_u \) is the average ion mobility. Integration gives the formula for \( w(t) \):

\[ w(t) = \mu_u \cdot \frac{R_{ON}}{D} \cdot q(t) \]  \hspace{1cm} (4)

Considering that \( R_{ON} < R_{OFF} \), we get an expression for memristivity [13]:

\[ M(q) = R_{OFF} \left(1 - \frac{\mu_u \cdot R_{ON}}{D^2}\right) \cdot q(t) \]  \hspace{1cm} (5)

For more than a quarter of a century, the memristor remained a hypothetical circuit element with no material implementation. But in 2008, a team of HP researchers led by Stanley Williams finally created a real memristor. Its properties corresponded to the model proposed by Chua. The memristor, developed by Williams' group, was a thin layer of semiconductor material sandwiched between two metal contacts. On one side of the layer, there is a dopant (positive ions) - in the region of width \( w \) [13; 14].
2. Materials and methods

I-V characteristic of the memristor. Due to the fact that the memristor has moved from the category of hypothetical components to the field of practical implementation and has been used by leading manufacturers of nanoelectronics for a considerable time, it makes sense to take a fresh look at the mathematical modeling of multipoles. Since the characteristics of nanoelectronic components largely depend on the intensity of the magnetic field, for an adequate mathematical description of such systems, it is simply necessary to introduce a linear and nonlinear memristor into the basic set of elements, which is responsible for the interconnection of the electric charge and magnetic field in the structure of nanoelectronic devices.

Obviously, we have to talk about not one, but two new elements of the basic set. This follows from the well-known experimental dependences of the I-V characteristic obtained by Western manufacturers. They showed that if an alternating sinusoidal voltage of a certain frequency is applied to the memristor, its current-voltage characteristic takes a form resembling a Lissajous figure centered at the origin. Therefore, a memristor, unlike a resistor, has a hysteresis. With increasing voltage frequency, the hysteresis curve degenerates into a straight line. This circumstance makes a memristor related to resistance, only in the reverse analogy (at high frequencies, the I–V characteristic of the memristor becomes linear). That is why we are talking about two new elements of the basic set: linear and nonlinear memristors.

![Figure 2](image)

**Figure 2.** Results of modeling the behavior of the memristor: a - based on the model described by equations (2-5); b - when multiplying the right side of equation (3) by the window function $w(1-w)/D^2$. The axes plotted dimensionless values of voltage, current and time, normalized to $v_0 = 1$ V (Fig. 2, a) and 4V (Fig. 2, b), $i_0 = 10$ mA, $t_0 = 10$ ms, respectively [13].

Topological equations of a multipole. Earlier, a mathematical model of an electric multipole was obtained in a hybrid basis of topological and parametric equations in finite increments of standard components of the base set [12]. With the introduction of a memristor into this set of elements, it becomes necessary to build a new mathematical model that takes into account not only internal connections and processes in current and voltage, but also in magnetic components.
The generalized mathematical model of a multipole network will consist of \( n \) topological and \( n \) parametric equations. Indeed, topological equations in finite increments for the previous base set [12]:

\[
\begin{align*}
D \cdot \Delta I &= 0 \\
B \cdot \Delta U &= 0
\end{align*}
\]  

(6)

where \( D \) is the matrix of principal sections of \( (k-1) \) equations; \( B \) - matrix of main contours of \( (n-k+1) \) equations; \( n \) is the number of branches and \( k \) is the number of nodes; \( \Delta I, \Delta U \) - increments of currents and voltages in the branches of the directed graph of the circuit.

Decomposed form for vectors of currents and voltages increments:

\[
\begin{align*}
\Delta U &= [\Delta U_L^X, \Delta U_H^X, \Delta U_R^X, \Delta U_M^X, \Delta U_C^P, \Delta U_H^P, \Delta U_R^P, \Delta U_M^P] \\
\Delta I &= [\Delta I_L^C, \Delta I_H^C, \Delta I_R^C, \Delta I_M^C, \Delta I_L^H, \Delta I_H^H, \Delta I_R^H, \Delta I_M^H]
\end{align*}
\]  

(7)

where \( \Delta U_L^X, \Delta I_L^C \) - increments of voltages and currents on inductive chords; \( \Delta U_C^P, \Delta I_C^P \) - voltage and current increments on the capacitive edges; \( \Delta U_R^X, \Delta U_R^P, \Delta I_R^C, \Delta I_R^H \) - voltage and current increments on resistive edges and chords; \( \Delta U_H^X, \Delta U_H^P, \Delta I_H^C, \Delta I_H^H \) - voltage and current increments on nonlinear edges and chords; \( \Delta U_M^X, \Delta U_M^P, \Delta I_M^C, \Delta I_M^H \) - voltage and current increments on memresistive edges and chords.

The system of topological equations for currents will take the form:

\[
\begin{bmatrix}
\Delta I_L^C \\
\Delta I_H^C \\
\Delta I_R^C \\
\Delta I_M^C \\
\Delta I_L^H \\
\Delta I_H^H \\
\Delta I_R^H \\
\Delta I_M^H
\end{bmatrix} =
\begin{bmatrix}
D_I & D_{II} & D_{III} & D_{IV} \\
D_V & D_{VI} & D_{VII} & D_{VIII} \\
D_{IX} & D_X & D_{XI} & D_{XII} \\
D_{XIII} & D_{XIV} & D_{XV} & D_{XVI}
\end{bmatrix}
\begin{bmatrix}
\Delta I_L^X \\
\Delta I_H^X \\
\Delta I_R^X \\
\Delta I_M^X
\end{bmatrix}
\]  

(8)

Concerning voltages:

\[
\begin{bmatrix}
\Delta U_L^X \\
\Delta U_H^X \\
\Delta U_R^X \\
\Delta U_M^X \\
\Delta U_L^P \\
\Delta U_H^P \\
\Delta U_R^P \\
\Delta U_M^P
\end{bmatrix} =
\begin{bmatrix}
B_I & B_{II} & B_{III} & B_{IV} \\
B_V & B_{VI} & B_{VII} & B_{VIII} \\
B_{IX} & B_X & B_{XI} & B_{XII} \\
B_{XIII} & B_{XIV} & B_{XV} & B_{XVI}
\end{bmatrix}
\begin{bmatrix}
\Delta U_L^C \\
\Delta U_H^C \\
\Delta U_R^C \\
\Delta U_M^C \\
\Delta U_L^H \\
\Delta U_H^H \\
\Delta U_R^H \\
\Delta U_M^H
\end{bmatrix}
\]  

(9)

Taking into account the above, the set of topological equations in finite increments in a hybrid basis of currents and voltages, charges and magnetic fluxes will consist of 4 groups:
operating conditions, it is advisable to take this term into account. Thus, we obtain the equivalent circuit increments to be negligible [12, 15, 16]. However, to study the robustness of electronic circuits in real mode.

\[
\begin{align*}
\Delta U_L^X & = \left[ -B_I \quad 0 \right] \cdot \left[ \Delta U_C^P \right] + \left[ -B_{II} \quad 0 \right] \cdot \left[ \Delta U_H^P \right] + \left[ -B_{III} \quad 0 \right] \cdot \left[ \Delta U_R^P \right] + \left[ -B_{IV} \quad 0 \right] \cdot \left[ \Delta U_M^P \right] \\
\Delta I_L^X & = \left[ 0 \quad -D_I \right] \cdot \left[ \Delta U_C^P \right] + \left[ 0 \quad -D_{II} \right] \cdot \left[ \Delta U_H^P \right] + \left[ 0 \quad -D_{III} \right] \cdot \left[ \Delta U_R^P \right] + \left[ 0 \quad -D_{IV} \right] \cdot \left[ \Delta U_M^P \right]
\end{align*}
\]

\[
\begin{align*}
\Delta U_H^X & = \left[ -B_V \quad 0 \right] \cdot \left[ \Delta U_C^P \right] + \left[ -B_{VI} \quad 0 \right] \cdot \left[ \Delta U_H^P \right] + \left[ -B_{VII} \quad 0 \right] \cdot \left[ \Delta U_R^P \right] + \left[ 0 \quad -D_V \right] \cdot \left[ \Delta U_M^P \right] + \left[ 0 \quad -D_{VI} \right] \cdot \left[ \Delta I_H^M \right]
\end{align*}
\]

\[
\begin{align*}
\Delta U_R^X & = \left[ -B_I \quad 0 \right] \cdot \left[ \Delta U_C^P \right] + \left[ -B_{II} \quad 0 \right] \cdot \left[ \Delta U_H^P \right] + \left[ -B_{III} \quad 0 \right] \cdot \left[ \Delta U_R^P \right] + \left[ 0 \quad -D_I \right] \cdot \left[ \Delta I_H^M \right] + \left[ 0 \quad -D_{II} \right] \cdot \left[ \Delta I_R^M \right]
\end{align*}
\]

\[
\begin{align*}
\Delta U_M^X & = \left[ -B_{IV} \quad 0 \right] \cdot \left[ \Delta U_C^P \right] + \left[ -B_{VII} \quad 0 \right] \cdot \left[ \Delta U_H^P \right] + \left[ -B_{VIII} \quad 0 \right] \cdot \left[ \Delta U_R^P \right] + \left[ 0 \quad -D_{IV} \right] \cdot \left[ \Delta I_H^M \right] + \left[ 0 \quad -D_{VII} \right] \cdot \left[ \Delta I_R^M \right]
\end{align*}
\]

3. Results

Parametric memristor equations. We now turn to consideration of parametric equations. Expressions for \( \Delta U_L^X, \Delta U_H^X, \Delta U_R^X, \Delta I_L^X, \Delta I_H^X, \Delta I_R^X, \Delta I_M^X, \Delta I_M^L, \Delta I_M^H, \Delta I_M^R \) were obtained earlier and are known [12]. The unknown are the expressions for \( \Delta U_M^X, \Delta U_M^H, \Delta I_M^X, \Delta I_M^H \).

Consider the parametric equation of a linear memristor. Equation (1) has an analogue according to Ohm's law [17]:

\[
u(t) = M(q) \cdot i(t)
\]

Due to the fact that only electrical parameters voltage and current are considered, then equation (11) in the case of increments can be written in the form:

\[
\Delta u(t) = E_{II}^H + M_E \cdot \Delta i(t)
\]

where \( E_{II}^H = i(t) \cdot \Delta M(q) \) - interval specified independent source of EMF; \( M_E \in [M(q) + \Delta M(q)] \) is the equivalent interval specified value of memristivity for the disturbed mode.

Most researchers discard the last term of equations similar to Eq. (12), considering the product of two increments to be negligible [12, 15, 16]. However, to study the robustness of electronic circuits in real operating conditions, it is advisable to take this term into account. Thus, we obtain the equivalent circuit of a linear memristor, shown in figure 3.

\[\text{Figure 3. Equivalent circuit of a linear memristor in finite increments.}\]
Next, consider a nonlinear memristor with an $I$-$V$ characteristic similar to the Lissajous figure (see figure 2, a). In the nominal mode, the nonlinear memristor will be described by the following system of equations:

$$\begin{align*}
\frac{du(t)}{dt} &= \alpha \cdot \sin \omega t \\
i(t) &= \beta \cdot \sin(\omega t + \varphi)
\end{align*}$$

(13)

where $\alpha$ and $\beta$ are constant coefficients of the loop width of the $I$-$V$ characteristic; $\omega$ - frequency of current and voltage; $\varphi$ is the angle of inclination of the $I$-$V$ characteristic figure.

Using the standard expansion of the sine of the sum, substituting the first equation of system (13) into the second and expressing the cosine of the function in terms of the sine, we obtain an expression for the nonlinear memristor in the nominal mode:

$$i(t) = u(t) \cdot \frac{\beta}{\alpha} \cdot \cos \varphi - u(t) \cdot \frac{\beta}{\alpha} \cdot \sin \varphi \cdot \sqrt{1 - \left(\frac{u^2(t)}{\alpha^2}\right)}$$

(14)

In perturbed mode:

$$\begin{align*}
i(t) + \Delta i(t) &= u(t) \cdot \frac{\beta}{\alpha} \cdot \cos \varphi + \Delta u(t) \cdot \frac{\beta}{\alpha} \cdot \cos \varphi - u(t) \cdot \frac{\beta}{\alpha} \cdot \sin \varphi \cdot \sqrt{1 - \left(\frac{(u + \Delta u)^2(t)}{\alpha^2}\right)} - \\
&- \Delta u(t) \cdot \frac{\beta}{\alpha} \cdot \sin \varphi \cdot \sqrt{1 - \left(\frac{(u + \Delta u)^2(t)}{\alpha^2}\right)}
\end{align*}$$

(15)

The equation in finite increments for a nonlinear memristor takes the form:

$$\Delta i(t) = \Delta M^{-1}(q) \cdot \Delta u(t) - J_H^Z - J_N^N$$

(16)

where $\Delta M^{-1}(q) = \frac{\beta}{\alpha} \cdot \cos \varphi$ is the increment of inverse memresistivity (by analogy with conductivity);

$J_H^Z = \Delta u(t) \cdot \frac{\beta}{\alpha} \cdot \sin \varphi \cdot \sqrt{1 - \left(\frac{(u + \Delta u)^2(t)}{\alpha^2}\right)}$ - dependent current source; $J_N^N = u(t) \cdot \frac{\beta}{\alpha} \cdot \sin \varphi \cdot \sqrt{1 - \left(\frac{(u + \Delta u)^2(t)}{\alpha^2}\right)}$ - independent current source.

In this case, the equivalent circuit of the nonlinear memristor will take the form shown in figure 4.
\[
\begin{bmatrix}
\Delta U^X_{MH}
\\Delta I^P_{MH}
\end{bmatrix}
= \begin{bmatrix}
0 & \mathcal{M}^{-1} \\
\mathcal{M} & 0
\end{bmatrix}
\begin{bmatrix}
\Delta U^P_{MH}
\\Delta I^X_{MH}
\end{bmatrix}
- \begin{bmatrix}
E^M_N
\\bar{J}^M_N
\end{bmatrix}
- \begin{bmatrix}
E^M_Z
\\bar{J}^M_Z
\end{bmatrix}
\]
(18)

4. Discussion
Model synthesis. To construct a mathematical model of an electric multipole in a hybrid basis of topological and parametric equations, we will use the previously obtained expressions [12]. The introduction of linear and nonlinear memristors into the basic set of elements will not affect the properties of the reactive and resistive branches of the multipole. Therefore, the expressions for them will remain unchanged.

For reactive branches:
\[
\begin{bmatrix}
\Delta U^X_L \\
\Delta I^P_L
\end{bmatrix}
= \begin{bmatrix}
\mathcal{C} & 0 \\
0 & \mathcal{C}
\end{bmatrix}
\frac{d}{dt}
\begin{bmatrix}
\Delta U^P_L \\
\Delta I^X_L
\end{bmatrix}
+ \begin{bmatrix}
0 & Z_E \\
Z_E & 0
\end{bmatrix}
\begin{bmatrix}
\Delta U^P_L \\
\Delta I^X_L
\end{bmatrix}
+ \begin{bmatrix}
E^L_E \\
\bar{J}^L_E
\end{bmatrix}
\]
where \(\mathcal{L} = \frac{d}{dt}\begin{bmatrix}
\text{diag}\{L_k + \Delta L_k\} \\
0
\end{bmatrix}, \mathcal{C} = \frac{d}{dt}\begin{bmatrix}
\text{diag}\{C_k + \Delta C_k\} \\
0
\end{bmatrix}\) is the matrix of inductances \((k = 1, n^X_R, l = 1, n^X_C), n^X_R + n^X_C = \text{dim}\Delta U^P_L\); \(\mathcal{C} = \frac{d}{dt}\begin{bmatrix}
\text{diag}\{C_k + \Delta C_k\} \\
0
\end{bmatrix}\) is the capacity matrix \((k = 1, n^X_C, l = 1, n^X_C), n^X_C + n^X_C = \text{dim}\Delta U^P_L\); \(Z_E = \begin{bmatrix}\text{diag}\{Z_E + \Delta E(\Delta I)\}\end{bmatrix}\) - matrix of equivalent resistances and \(V_E = \begin{bmatrix}\text{diag}\{G_E + \Delta G(\Delta U)\}\end{bmatrix}\) - equivalent conductivities of nonlinear inductances and capacities; \(E^L_E = \begin{bmatrix}\text{diag}\{G_E + \Delta G(\Delta U)\}\end{bmatrix}\), \(\bar{J}^L_E = \begin{bmatrix}\text{diag}\{G_E + \Delta G(\Delta U)\}\end{bmatrix}\) - equivalent vectors of current and voltage sources [12].

For resistive branches:
\[
\begin{bmatrix}
\Delta U^X_R \\
\Delta I^P_R
\end{bmatrix}
= \begin{bmatrix}
\bar{R} & 0 \\
0 & \bar{R}
\end{bmatrix}
\begin{bmatrix}
\Delta U^P_R \\
\Delta I^X_R
\end{bmatrix}
+ \begin{bmatrix}
E^R_E \\
\bar{J}^R_E
\end{bmatrix}
\]
(20)

where \(\bar{R} = R + \Delta R\) is the resistance matrix of the branch, \(\bar{Y} = Y + \Delta Y\) is the matrix of the branch conductivity; \(E^R_E, \bar{J}^R_E\) - equivalent vectors of independent voltage and current sources on resistive branches [12].

And for nonlinear branches, taking into account nonlinear memristivities (18), the system of hybrid equations takes the form:
\[
\begin{bmatrix}
\Delta U^X_H \\
\Delta I^P_H
\end{bmatrix}
= \begin{bmatrix}
\Delta M^{-1} \\
G_p(\Delta U^P_H)
\end{bmatrix}
\begin{bmatrix}
\Delta U^P_H \\
\Delta I^X_H
\end{bmatrix}
+ F^t
\]
where
\[
F^t = \begin{bmatrix}
E & E & 0 & 0 & -E & -E & 0 & 0 \\
0 & 0 & E & E & 0 & 0 & -E & -E
\end{bmatrix}
\]
and \(Z_X(\Delta I^X_H)\) and \(G_p(\Delta U^P_H)\) are diagonal matrices of equivalent resistances and conductivities; \(\Delta M^{-1}\) and \(\Delta M\) are diagonal matrices of equivalent inverse and forward memristivities; \(E\) is the identity matrix; \(E^H_E, E^H_F, J^H_E, J^H_F, E^M_E, J^M_E, E^M_F, J^M_F\) and \(\bar{J}^M_E\) are equivalent sources of EMF and current on nonlinear chords and edges.
Since the left sides of equation (21) and the second equation of system (10) are equal, equating their right sides, we get:

$$
\frac{\Delta U^P_H}{\Delta I^H_R} = A_1 \frac{\Delta U^P_C}{\Delta I^C_L} + A_2 \frac{\Delta U^P_V}{\Delta I^V_R} + A_3 \frac{\Delta U^P_M}{\Delta I^M_K} - H \cdot F^I
$$

(22)

where

$$
A_1 = \begin{bmatrix} \Delta M^{-1} & \bar{Z}_X(\Delta I^H_R) \end{bmatrix} \cdot \begin{bmatrix} -B_V & 0 & 0 & -D_V \\ 0 & -D_V & 0 & -D_V \\ 0 & 0 & -D_V & 0 \end{bmatrix},
A_2 = \begin{bmatrix} \Delta M^{-1} & \bar{Z}_X(\Delta I^H_R) \end{bmatrix} \cdot \begin{bmatrix} -B_V & 0 & 0 & -D_V \\ 0 & -D_V & 0 & -D_V \\ 0 & 0 & -D_V & 0 \end{bmatrix},
A_3 = \begin{bmatrix} -B_{VII} & 0 & 0 & -D_{VII} \\ 0 & -D_{VII} & 0 & -D_{VII} \end{bmatrix} \cdot \begin{bmatrix} \Delta M^{-1} & \bar{Z}_X(\Delta I^H_R) \end{bmatrix}.
$$

Because the right-hand sides of the third equation of system (10) and expression (20) are equal, giving similar terms we get the following expression:

$$
\frac{\Delta U^P_M}{\Delta I^M_K} = G_1 \frac{\Delta U^P_C}{\Delta I^C_L} + G_2 \frac{\Delta U^P_V}{\Delta I^V_R} + G_3 \frac{\Delta U^P_H}{\Delta I^H_R} - W \left[ \begin{array}{c} E^H_N \\ J^R_N \end{array} \right]
$$

(23)

where

$$
G_1 = \begin{bmatrix} B_{XI} & \bar{R} & 0 \\ \bar{Y} & D_{XI} & 0 \\ 0 & 0 & 0 \end{bmatrix}^{-1},
G_2 = \begin{bmatrix} B_{XI} & \bar{R} & 0 \\ \bar{Y} & D_{XI} & 0 \\ 0 & 0 & 0 \end{bmatrix}^{-1},
G_3 = \begin{bmatrix} B_{XI} & \bar{R} & 0 \\ \bar{Y} & D_{XI} & 0 \\ 0 & 0 & 0 \end{bmatrix}^{-1}.
$$

Let us compare the fourth equation of system (10) and expression (17) and, applying similar reasoning as for resistive branches, we obtain:

$$
\frac{\Delta U^P_M}{\Delta I^M_K} = S_1 \frac{\Delta U^P_C}{\Delta I^C_L} + S_2 \frac{\Delta U^P_V}{\Delta I^V_R} + S_3 \frac{\Delta U^P_H}{\Delta I^H_R} - K \left[ \begin{array}{c} E^M_N \\ J^M_N \end{array} \right]
$$

(24)

where

$$
S_1 = \begin{bmatrix} -B_{XVI} & 0 & -D_{XVI} \\ \bar{M}^{-1} & 0 & -D_{XVI} \end{bmatrix}^{-1},
S_2 = \begin{bmatrix} -B_{XVI} & 0 & -D_{XVI} \\ \bar{M}^{-1} & 0 & -D_{XVI} \end{bmatrix}^{-1},
S_3 = \begin{bmatrix} -B_{XVI} & 0 & -D_{XVI} \\ \bar{M}^{-1} & 0 & -D_{XVI} \end{bmatrix}^{-1}.
$$

Comparing the right-hand sides of the first equation of system (10) and (19), we obtain an expression in a hybrid basis for reactive branches:

$$
\begin{bmatrix} 0 & \bar{L} \\ \bar{C} & 0 \end{bmatrix} \frac{d}{dt} \frac{\Delta U^P_C}{\Delta I^C_L} = Q_1 \frac{\Delta U^P_C}{\Delta I^C_L} + Q_2 \frac{\Delta U^P_H}{\Delta I^H_R} + Q_3 \left[ \begin{array}{c} E^H_N \\ J^R_N \end{array} \right] + Q_4 \frac{\Delta U^P_M}{\Delta I^M_K} - E^E_I
$$

(25)

where

$$
Q_1 = \left[ \begin{array}{c} -B_{II} & 0 & -D_{II} \\ V_E & -D_{IV} \end{array} \right],
Q_2 = \left[ \begin{array}{c} -B_{II} & 0 & -D_{II} \\ 0 & -D_{IV} \end{array} \right],
Q_3 = \left[ \begin{array}{c} -B_{II} & 0 & -D_{II} \\ 0 & -D_{IV} \end{array} \right],
Q_4 = \left[ \begin{array}{c} -B_{II} & 0 & -D_{II} \\ 0 & -D_{IV} \end{array} \right]
$$

Then we compare the second equation of system (10) and expression (21), we get:

$$
P_1 (\Delta I^H_R, \Delta U^H_H) \cdot \frac{\Delta U^P_C}{\Delta I^C_L} + P_2 \frac{\Delta U^P_H}{\Delta I^H_R} = F^{II}
$$

(26)

where

$$
P_1 = \left[ \begin{array}{c} B_V \bar{Z}_X(\Delta I^H_R) \\ G_P(\Delta U^P_H) \end{array} \right],
P_2 = \left[ \begin{array}{c} -B_{IV} & 0 & -D_{IV} \\ 0 & -D_{IV} \end{array} \right],
F^{II} = \left[ \begin{array}{c} B_{VI} \\ 0 \end{array} \right] \cdot \begin{array}{c} E^H_N \\ J^R_N \end{array} - F^{II}_N,
G_1 = \left[ \begin{array}{c} \Delta U^P_C \\ \Delta U^P_H \end{array} \right] + G_2 \left[ \begin{array}{c} \Delta U^P_C \\ \Delta U^P_H \end{array} \right] - M \left[ \begin{array}{c} E^H_N \\ J^R_N \end{array} \right].
$$
Combining expressions (22-25), we obtain a full-size mathematical model of an electric multipole with memristive branches in a full hybrid basis using the branch decomposition method, which will look like this:

\[
\begin{bmatrix}
0 & L \\
\dot{L} & 0
\end{bmatrix}
\frac{d}{dt}
\begin{bmatrix}
\Delta U_C^E \\
\Delta I_C^E
\end{bmatrix} = Q_1
\begin{bmatrix}
\Delta U_C^E \\
\Delta I_C^E
\end{bmatrix} + Q_2
\begin{bmatrix}
\Delta U_R^H \\
\Delta I_R^H
\end{bmatrix} + Q_3
\begin{bmatrix}
E_R^R \\
E_R^H
\end{bmatrix} + Q_4
\begin{bmatrix}
\Delta U_M^R \\
\Delta I_M^R
\end{bmatrix} - \left[ E_N^R \\
J_N^R
\right]
\]

\[
\begin{bmatrix}
\Delta U_C^E \\
\Delta I_C^E
\end{bmatrix} = A_1
\begin{bmatrix}
\Delta U_C^E \\
\Delta I_C^E
\end{bmatrix} + A_2
\begin{bmatrix}
\Delta U_R^H \\
\Delta I_R^H
\end{bmatrix} + A_3
\begin{bmatrix}
\Delta U_M^R \\
\Delta I_M^R
\end{bmatrix} - H \cdot F^l
\]

\[
\begin{bmatrix}
\Delta U_R^H \\
\Delta I_R^H
\end{bmatrix} = G_1
\begin{bmatrix}
\Delta U_C^E \\
\Delta I_C^E
\end{bmatrix} + G_2
\begin{bmatrix}
\Delta U_R^H \\
\Delta I_R^H
\end{bmatrix} + G_3
\begin{bmatrix}
\Delta U_M^R \\
\Delta I_M^R
\end{bmatrix} - W
\begin{bmatrix}
E_N^R \\
J_N^R
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Delta U_M^R \\
\Delta I_M^R
\end{bmatrix} = S_1
\begin{bmatrix}
\Delta U_C^E \\
\Delta I_C^E
\end{bmatrix} + S_2
\begin{bmatrix}
\Delta U_R^H \\
\Delta I_R^H
\end{bmatrix} + S_3
\begin{bmatrix}
\Delta U_M^R \\
\Delta I_M^R
\end{bmatrix} - K
\begin{bmatrix}
E_N^R \\
J_N^R
\end{bmatrix}
\]

\[
(27)
\]

5. Conclusion

Expression (27) can be applied in circuit design and analysis of circuits of nanoelectronic devices, and the element base of quantum computers. The methods for solving the obtained mathematical model are subject to further research using the apparatus of interval arithmetic.

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