Two body decays of the $b$-quark:
Applications to direct CP violation,
searches for electro-weak penguins and new physics.

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Abstract 

A systematic experimental search for two-body hadronic decays of the $b$-quark of the type $b \rightarrow$ quark + meson is proposed. These reactions have a well defined experimental signature and they should be theoretically cleaner compared to exclusive decays. Many modes have appreciable branching ratios and partial rate asymmetries may also be quite large (about $8-50\%$) in several of them. In a few cases electroweak penguins appear to be dominant and may be measurable. CP violating triple correlation asymmetries provide a clean test of the Standard Model.

Hadronic $B$-decays can be divided into two categories: inclusive and exclusive. Inclusive decays involve calculations of quark level processes and theoretical predictions here are relatively firm [1], but well defined experimental signatures are usually rather difficult. On the other hand, the theoretical calculations of exclusive channels [2, 3] entail many ad-hoc assumptions and crude approximations [4]. Consequently, the resulting predictions are highly unreliable but the experimental signatures are clear. In this paper we want to present a systematic study of a class of semi-inclusive decays which lie somewhere in between the above two categories. i.e. they possess improved
prospects for predictive power as well as well defined experimental signatures. Furthermore, we suggest that the class of semi-inclusive processes that we will discuss, 1) should exhibit measurable direct CP asymmetries in many channels, 2) provide clean tests of the Standard Model (SM), 3) are a good probe of electroweak penguins, and 4) should be helpful in clarifying important issues related to hadronization.

We will focus here on two-body decays of the $b$-quark of the type:

$$b \rightarrow M + q_f$$  \hspace{1cm} (1)$$

where $M$ is a spin 0 or spin 1 meson and $q_f$ is a quark in the final state. Note that two decays of the $b$-quark, that have received considerable attention in the past few years, belong to this category; namely $b \rightarrow \gamma s$ (i.e. $B \rightarrow \gamma X_s$) and $b \rightarrow \eta' s$ (i.e. $B \rightarrow \eta' X_s$). Indeed the calculational procedure that we will use for (1) will be a generalization of the method that we used, for $b \rightarrow \eta' s$ leading to $B \rightarrow \eta' X_s$ \cite{5, 6, 7}. Two other cases for which analogous calculations exist are $B \rightarrow \phi + X_s$ \cite{8} and $B \rightarrow K(K^*) + X$ \cite{9}.

We first briefly discuss the experimental signature. There are a few unique kinematic features of this class of events implied by (1) that should prove very helpful in searching for such processes:

1. The energy ($E_M$) of the meson $M$, will be centered around $(m_b^2 + m_M^2 - m_{q_f}^2)/2m_b$ and have a spread of $0(\Lambda_{QCD}) \sim$ a few hundred MeV.

2. The energy of the outgoing quark in (1) is of course also similarly fixed and since it is relatively low ($\sim 2$ GeV), on hadronization, it will lead to fairly low average multiplicity (about 3)/event. Thus the combinatorics problem in discriminating against backgrounds is unlikely to be too difficult. This should be specially helpful in discriminating against the cascade decays of charm hadrons as charmless final states are of great interest for CP violation \cite{1, 2, 10, 11}, for searching for electroweak penguins (EWP) and for clues of new physics.

3. In the rest frame of the $B$, the sum of the momentum transverse to the direction of the meson $M$, over all the other particles in the event, should be severely limited, perhaps to $0(\Lambda_{QCD}) \sim$ a few hundred MeV.

We next want to outline the main reason why we believe that this class of reactions will be theoretically cleaner, compared to exclusive decays (say)
into 2 mesons \[2, 3\]. The starting point for all such calculations is, of course, the short distance Hamiltonian which can be symbolically written as \[1\]

\[ H_{\text{eff}} = \sum_j c_j O_j \]  

(2)

where \(c\)'s are the \(c\)-number coefficients and \(O\)'s the 4-quark operators. Typically \(O\)'s have the form \(\bar{q}_1 \Gamma b \bar{q}_2 \Gamma' q_3\) where \(q\)'s are the appropriate flavors of quarks, \(u, d, s, c\), and \(\Gamma\)'s are Dirac matrices and color indices are suppressed for simplicity. The matrix elements leading from the 4-quark operators to reaction (1) can be symbolically divided into two categories

\[ \langle M_{q_2 q_3} q_f | \bar{q}_1 \Gamma b \bar{q}_2 \Gamma' q_3 | b \rangle \Rightarrow \langle M_{q_1 q_2} | \bar{q}_2 \Gamma' q_3 | 0 \rangle \langle q_f | \bar{q}_1 \Gamma b | b \rangle \]  

\[ \Rightarrow \langle M_{q_1 q_2} | \bar{q}_1 \Gamma'' q_3 | 0 \rangle \langle q_f | \bar{q}_2 \Gamma'' b | b \rangle \]  

(3)

where the subscripts on \(M\) indicates the flavors of quarks that \(M\) is composed of. For an explicit example consider the operator \(\bar{u} \gamma^\mu (1 - \gamma_5) b \bar{d} \gamma^\nu (1 - \gamma_5) u\) leading to the decays \(b \rightarrow \rho^- u\) or \(b \rightarrow \rho^0 (\omega) d\) corresponding to eqns. (3a) and (3b) respectively.

Let us next consider the matrix element of the 4-quark operators in the traditional exclusive reactions \([2, 3]\) \(B \rightarrow M_1 M_2\). It may take the generic form:

\[ \langle M_1 M_2 | \bar{q}_1 \Gamma b \bar{q}_2 \Gamma' q_3 | B \rangle \Rightarrow \langle M_1 | \bar{q}_2 \Gamma' q_3 | 0 \rangle \langle M_2 | \bar{q}_1 \Gamma b | B \rangle \]  

(4)

Now recall that the evaluation of the factor \(\langle M_2 | \bar{q}_1 \Gamma b | B \rangle\) in eqn. (4) requires a knowledge of two form factors, (often denoted as \(f_1\) and \(f_0\)) if \(M_2\) is a \(0^-\) meson or of four form factors \((A_{1-3}, V)\) if \(M_2\) is a \(1^-\) meson, at \((p_B - p_{M_2})^2 = m_{M_2}^2\). This represents the one significant theoretical distinction between the semi-inclusive, quasi-two-body reactions, eqn. (4), that we are considering in this paper versus the exclusive decay \(B \rightarrow M_1 M_2\). The latter reaction, even using the factorization approach of eqs. (2-4), entails additional theoretical uncertainties as it requires knowledge of 2–4 form factors for the \(B \rightarrow M_2\) transition that the former reactions manage to evade through summation over an appropriate ensemble of states.

In full generality the final states accessible through eqn. (4) can be subdivided into three categories depending on the type of operators which contribute: 1) tree \(\times\) tree, 2) tree \(\times\) penguin and 3) penguin \(\times\) penguin. There
are interesting physics issues that each type allows us to address. However, for the purpose of this paper we will confine the discussion to only those available via tree × penguin and penguin × penguin.

The results regarding the partial rate asymmetry (PRA) and the branching ratio are presented in Table 1. A figure of merit, often used to get a rough feel for detectability of CP asymmetry, is given by \( N^3_B \) defined by

\[
N^3_B = \frac{9}{\alpha_{\text{PRA}} \cdot \text{Br} \cdot \text{eff}}
\]  

where \( N^3_B \) is the number of \( B-\bar{B} \) pairs needed to establish a PRA to the accuracy of three statistical standard deviations. Here \( \alpha_{\text{PRA}} \) is the PRA, \( \text{Br} \) is the branching ratio and \( \text{eff} \) is the product of all the efficiencies responsible for the signal. A quick calculation shows that with about \( 5 \times 10^6 \) \( B-\bar{B} \) pairs, the asymmetries in the \( \rho^- \), \( K^- \) and \( K^*-u \) channels start to become accessible. With about \( 5 \times 10^7 \) \( B-\bar{B} \), the PRA’s in modes with \( \pi^- \), \( \pi^0 \), \( \rho^0 \), \( \omega^0 \), \( D^- \) and \( D^{*-} \) may also become measurable. Note also that several channels may have 8–50% asymmetries.

It is useful to recall that the existing CLEO analysis is based on about \( 3 \times 10^6 \) \( B-\bar{B} \) pairs and a factor of 2 to 2.5 times more data is expected to be analysed in the next few months. Thus even the existing data sample may well be sufficient to reveal the PRA in some of the channels in the Table. Furthermore, the \( e^+e^- \) based B-factories at Cornell, KEK and SLAC are expected to have about a few times \( 10^7 \) \( B-\bar{B} \) pairs/year starting next year. Thus PRA’s in many of these channels should be observable in the near future.

Using the CKM unitarity and assuming \( SU(3) \) it is easy to see that the difference in partial rates for \( b \to \pi^-u(\rho^-u) \) are equal and opposite to that of \( b \to K^-u(K^*-u) \). Thus, for instance,

\[
\Gamma(b \to \rho^-u) - \Gamma(\bar{b} \to \rho^+\bar{u}) = -\Gamma(b \to K^{*-}u) + \Gamma(\bar{b} \to K^{*+}\bar{u})
\]  

(6)

This implies that to the extent that \( SU(3) \) is a valid symmetry, in the standard model there will be no partial rate asymmetry in \( b \to h^-u \) where \( h^- \) indicates the sum over \( K \) (\( K^* \)) or \( \pi \) (\( \rho \)) final states. Conversely, if this combined asymmetry is very large, it implies that physics beyond the standard model is present. From the experimental point of view, it also illustrates that
in order to see asymmetries which might be present in the standard model, it is important to be able to distinguish $\pi^− (\rho^−)$ from $K^− (K^{*-})$.

The CP-violating PRA generated in these processes and given in Table 1 originates from the absorptive part of the penguin graph [10]. While traditionally most discussions of CP have centered around the PRA thus obtained, we want to emphasize here that a very clean test of the physics beyond the SM is possible by searching for triple correlation asymmetry (TCA) via reaction (1), specifically $b \to qV$ where $V$ is a spin 1 meson. Notice first that in such decays a CP-odd, TCA can be experimentally searched for by testing if $\langle \sin \phi \rangle \neq 0$, where $\phi$ is the azimuthal angle between the decay plane of the vector $V$ and the plane of the leading two mesons formed by $q$.

We recall that TCA are $T_N$-odd observables which receive contributions from real part of Feynman amplitudes i.e. they do not require the presence of strong phases (unlike PRA). Thus they nicely complement tests of CP violation that use $T_N$-even observables (e.g. PRA). However, for a TCA to be present in $b \to qV$ there must exist a corresponding TCA at the quark level. This can only happen if both left and right helicity quark amplitudes are present with different CP-odd phases. The penguin operators $O_5$ and $O_6$ do in fact contain coupling to quarks of right helicity and since the tree operators (say $O_1$ and $O_2$) couple to left-helicity quarks it would appear that the conditions for TCA $\neq 0$ exist. General considerations of helicity conservation, however, show this not to be the case for these operators if the final meson is a vector. This is because a meson may only be constructed out of a $q\bar{q}$ pair of the same chirality and since both of these operators produce a $q\bar{q}$ pair of right handed chirality and a single $d$ or $s$ quark of left handed chirality, the right handed pair must bind to form the meson and only the left handed $d$ or $s$ is left over in the final state. The overall reaction thus has the same helicity structure as the tree operator or the other penguins which only involve left chirality quarks. Such amplitudes are thus suppressed by the ratios of current quark masses: $m_s/m_b$ or $m_d/m_b$. For the case when $M$ is a spin one meson [17] TCA are, therefore, vanishingly small compared to PRA in the SM [18].

If such TCA’s are detected beyond the level suggested by the the above suppressions, it indicates that some new physics generates effective operators with an odd number of right chirality quarks in the final state. Candidates would include models with right handed $W$ bosons, SUSY and some models with extended higgs sectors.
We have also examined the effect of EWP in this class of reactions. The most interesting cases are those when the EWP contribution is not color-suppressed where it is expected to be the largest (see Table). Notice, in particular that for $b \rightarrow \pi^0 s$ and $\rho^0 s$ EWP dominate over the other contributions. These should be observable with $\sim 5 \times 10^6 B$’s. Again, the existing CLEO data sample may well be enough to reveal the presence of EWP in these channels.

It may be useful to understand the spectrum of the hadrons recoiling against the meson $M$ in the $B$ decay. To do so we need to factor in the Fermi motion of the $b$-quark with respect to the meson. For this purpose we adopt the model of Ali and Greub \cite{19} and use the parameters chosen in the experimental analysis of $B \rightarrow \gamma X_s$ \cite{20}. The resulting spectra are shown in Fig. 1 for a few cases.

Finally it is important to note that in addition to the assumption of factorization, both types of reactions, i.e. $b \rightarrow Mq_f$ and $b \rightarrow M_1 M_2$, require in eqns. (3) and (4) respectively, a numerical value for $n_{eff}$, the effective number of colors. In the calculations presented in Table 1, we have tacitly assumed $n_{eff} = 3$. Hopefully, a more appropriate value for $n_{eff}$ can be extracted from experiment after the branching ratios of a few of the modes in Table 1 are measured \cite{21}. Thus the traditional exclusive reactions ($B \rightarrow M_1 M_2$) require the assumption of factorization, a value for $n_{eff}$ and also the values of the form factors whereas $b \rightarrow qV$ require only the first two. The key point is that the problem of hadronic matrix elements calculations is so vastly complicated that for the class of semi-inclusive reactions of eqn. (4), which require fewer assumptions and approximations, it may be easier to extract information from experiment in an effort to fine tune the calculational procedure for future applications.

To summarize, semi-inclusive decays of $B$ mesons emanating from two-body decays of the $b$-quark have distinctive experimental signature and they are theoretically cleaner compared to exclusive decays. Moreover, as a rule, this class of reactions have larger branching ratios \cite{22} compared to exclusive channels and are expected to exhibit observable CP-asymmetries with data samples $\gtrsim 5 \times 10^6 B\bar{B}$ pairs. These reactions also provide good testing ground for the SM and clues for new physics and are a good probe for electroweak penguins. It is also important to note that since the underlying decays are that of a $b$-quark, the corresponding reactions can be studied using all types of $B$ mesons ($B_u$, $B_d$, $B_s$, $B_c$) and at all kinds of $B$-facilities. Detailed
studies of this class of $B$ decays could prove very helpful in clarifying many issues in QCD dynamics governing weak decays which could in turn yield another route to extraction of the CP violating CKM phases and tests of CKM unitarity. Systematic and dedicated experimental searches for this class of modes are strongly encouraged.

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[16] Note that triple correlation asymmetries are not necessarily CP-odd. To establish the CP-odd nature may require comparison of the asymmetry in $b$ and $\bar{b}$ decays.

[17] When $M$ is spinless, TCA do not vanish in the SM even for massless $d$ or $s$ quarks; however, they are expected to be small.

[18] We emphasize that this argument holds even in the presence of electroweak penguins.

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[21] It is quite possible that $n_{eff}$ is not the same for all the modes in the Table. In particular, final states with $c\bar{c}$ in them could have $n_{eff}$ different from other entries in the Table.
Interesting exception to the rule is the case when the tree contribution to $b \to q + M$ is color suppressed (e.g. $b \to \omega s$), and the corresponding exclusive two meson case is color allowed (e.g. $B \to \omega K$). In such cases the presence of the spectator can make a large correction.

Table 1: Some modes of interest; $Br$, PRA and $N_B^{\Delta\sigma}$ along with EWP contributions (to color allowed channels only) are shown [12]. Note $\gamma \equiv \arg(-V_{ub}^*V_{ud}/V_{cb}^*V_{cd})$.

| Mode     | $Br$         | $\mathbf{[PRA]/\sin\gamma(\%)}$ | $N_B^{\Delta\sigma} \sin^2\gamma\epsilon_{\text{eff}}/10^6$ | $Br$ due to EWP |
|----------|--------------|----------------------------------|------------------------------------------------|-----------------|
| $\pi^-u$ | $1.3 \times 10^{-4}$ | 8 | 12 | |
| $\rho^-u$ | $3.5 \times 10^{-4}$ | 8 | 4 | |
| $\pi^0d$ | $2.4 \times 10^{-6}$ | 36 | 28 | |
| $\rho^0d$ | $5.9 \times 10^{-6}$ | 38 | 10 | $4.7 \times 10^{-8}$ |
| $\omega d$ | $5.8 \times 10^{-6}$ | 39 | 10 | $1.3 \times 10^{-7}$ |
| $\phi d$ | $2.3 \times 10^{-7}$ | 0 | 7.0 $\times 10^{-9}$ |
| $K^0s$ | $2.5 \times 10^{-6}$ | 5 | 1200 | |
| $K^{*0}s$ | $2.9 \times 10^{-6}$ | 16 | 120 | |
| $D^-c$ | $1.7 \times 10^{-3}$ | 2 | 17 | |
| $D^{*-}c$ | $2.2 \times 10^{-3}$ | 2 | 13 | |
| $K^-u$ | $2.9 \times 10^{-5}$ | 33 | 3 | |
| $K^{*-}u$ | $5.1 \times 10^{-5}$ | 51 | 1 | |
| $K^0d$ | $2.0 \times 10^{-5}$ | 1 | 3000 | |
| $K^{*0}d$ | $2.6 \times 10^{-5}$ | 3 | 540 | |
| $\pi^0s$ | $9.8 \times 10^{-8}$ | 0 | 1.6 $\times 10^{-6}$ | |
| $\rho^0s$ | $2.5 \times 10^{-7}$ | 0 | 4.3 $\times 10^{-6}$ | |
| $\omega s$ | $1.3 \times 10^{-6}$ | 0 | 4.7 $\times 10^{-7}$ | |
| $\phi s$ | $6.3 \times 10^{-5}$ | 0 | 4.7 $\times 10^{-7}$ | |
| $D^-c$ | $4.2 \times 10^{-2}$ | 0.1 | 300 | |
| $D^{*-}c$ | $5.3 \times 10^{-2}$ | 0 | 300 | |
Figure 1: The normalized recoil spectra for the quasi-two-body decays, $b \rightarrow \pi^- u$ (solid), $K^*^- u$ (dashed) and $D^- c$ (dotted) are shown.