Resolving cosmological singularity problem in logarithmic superfluid theory of physical vacuum

K G Zloshchastiev
Institute of Systems Science, Durban University of Technology, Durban, South Africa
E-mail: k.g.zloschastiev@gmail.com

Abstract. A paradigm of the physical vacuum as a non-trivial quantum object, such as superfluid, opens an entirely new prospective upon origins and interpretations of Lorentz symmetry and spacetime, black holes, cosmological evolution and singularities. Using the logarithmic superfluid model, one can formulate a post-relativistic theory of superfluid vacuum, which is not only essentially quantum but also successfully recovers special and general relativity in the “phononic” (low-momenta) limit. Thus, it represents spacetime as an induced observer-dependent phenomenon. We focus on the cosmological aspects of the logarithmic superfluid vacuum theory and show how can the related singularity problem be resolved in this approach.

1. Introduction

It is a general consensus now that the physical vacuum, or a non-removable background, is a nontrivial object whose properties’ studies are of utmost importance, because it affects the most fundamental notions our physics is based upon, such as space, time, matter, field, and fundamental symmetries. An internal structure of physical vacuum is still a subject of debates based on different views and approaches, which generally agree on the main paradigm, but differ in details; some introduction can be found in monographs by Volovik and Huang [1, 2].

It is probably Dirac who can be regarded as a forerunner of the superfluid vacuum theory (SVT). As early as 50’s, he noticed that if space was filled with a medium of quantum nature then the Michelson-Morley-type experiments would be insensitive to it, unlike a case of the classical aether which was abandoned in 20th century [3]. The reason is that the velocity of the quantum matter would be related to a gradient of its wavefunction’s phase, while such phases are non-observable (at least in a space of trivial topology). In other words, such a quantum “aether”, unlike its classical counterpart, would create no preferred directions in space, therefore an observer would see the mostly isotropic universe around. A simplest analogy of this phenomenon would be an s-wavefunction of a hydrogen atom, which is rotationally invariant despite the atom’s original (classical) two-body Hamiltonian is not.

Dirac’s theoretical views were definitely a step in the right direction, but not a full story as yet. In particular, he did not not explain why this background medium does not slow down celestial bodies moving through it for billions years, as well as how would the Lorentz symmetry emerge in this conventional quantum-mechanical picture. This is where the superfluid vacuum approach takes from.

Superfluid vacuum theory is a post-relativistic approach in high-energy physics and classical and quantum gravity, which advocates that physical vacuum is a superfluid, and all elementary
particles are excitations above its ground state; the latter is assumed to be observed as a Bose-Einstein condensate of some sort. The term ‘post-relativistic’ implies that SVT is generally a non-relativistic theory but contains relativity as a subset, or as a special case or limit with respect to some dynamical value - akin to the Newtonian theory of gravity turned out to be a special limit of the Einstein’s theory of general relativity, at small values of gravitational fields. As for the notion of superfluid itself, then it is usually understood as a non-relativistic quantum liquid with suppressed dissipative fluctuations and absence of macroscopic viscosity [4, 5, 6]. Its laboratory examples include liquid helium-4 below the 2.17 K (at the normal pressure), known as superfluid helium-4 or helium II phase [7, 8], and “bosonized” fermionic fluids of Cooper pairs of electrons in the BCS-type superconductors.

In this paper, we are going to describe why and how the Lorentz symmetry, relativistic gravity and cosmology occur in a superfluid vacuum theory, and discuss the physical implications thereof.

2. Relativity and SVT

An easiest way to see how does relativity arise in superfluid vacuum theory is the energy spectrum of excitations of a typical superfluid. If one plots this energy versus momentum for superfluid helium-4, then one finds out that it has the following distinct shape, predicted by Landau: as the excitation’s momentum increases, energy grows from origin until it reaches a local maximum (called the “maxon” peak), which is crucial for suppressing the dissipative fluctuations. Then it dives down to a local minimum (called the “roton” regime, for historical reasons); then it climbs up again, ad infinitum. In the regime of small momenta, called the phononic regime, the dispersion relation is approximately linear with respect to momentum, which is a behaviour typical for relativistic particles if one replaces speed of light by a speed of sound. As a matter of fact, one can still use some sort of relativistic description (by adding extra fields to account for small deviations from the linear law), until the momentum reaches its value corresponding to the “maxon” peak. From there up, relativistic approximation is no longer robust or natural.

If one assumes that the actual physical vacuum has a similar energy spectrum of excitations then one can project the above-mentioned picture from the condensed-matter realm to the realm of elementary particle physics and gravity – by replacing phonons with photons, speed of sound with speed of light, etc. [9]. Moreover, one can further explore this analogy to find the analogs of helium-II phenomena in high-energy physics and quantum gravity.

There exists also a more formal way to describe a theory of relativity as a subset of the superfluid vacuum theory. It starts with the mathematical map between the inviscid liquids and manifolds of non-vanishing Riemann curvature, usually referred as the fluid/gravity correspondence [1, 2]. Essentially, it means that the propagation of small acoustic perturbations inside an inviscid irrotational barotropic fluid, described by background values of density $\rho$, pressure $P$ and velocity $u$, is analogous to the propagation of probe particles along the geodesics of a four-dimensional pseudo-Riemannian manifold whose metric is, in Cartesian coordinates:

$$
g_{\mu\nu} \propto \frac{\rho}{c_s} \begin{pmatrix}
-(c_s^2 - u^2) & -u \\
\cdots & \cdots \\
-u & \cdots & I
\end{pmatrix},
$$

where $c_s = \sqrt{\frac{\partial P}{\partial \rho}}$ is a propagation speed of fluid oscillations. The metric tensor is defined up to a constant factor whose value is determined by measurement units and boundary conditions.

Notice that while the background fluid is essentially non-relativistic, the small perturbations themselves couple to the metric which treats space and time as a spacetime. If we regard such fluid as a physical vacuum or a non-removable background then this metric describes the induced spacetime geometry.
This effect should not be confused with the relativistic gravitational effect of the ideal fluid as a source introduced via stress-energy tensor in the Einstein field equations. Instead, for a given metric (1), one can always define the induced matter stress-energy tensor

\[ T^{(\text{ind})}_{\mu\nu} \equiv \kappa^{-1} \left[ R_{\mu\nu}(g) - \frac{1}{2} g_{\mu\nu} R(g) \right], \tag{2} \]

where \( \kappa \) is the Einstein’s gravitational constant, \( R_{\mu\nu}(g) \) and \( R(g) \) are, respectively, Ricci tensor and scalar curvature corresponding to the induced metric \( g_{\mu\nu} \).

Thus, superfluid vacuum theory interprets Einstein field equations not as the differential equations for an unknown metric but rather as a derivation procedure of an induced stress-energy tensor of the matter to which the small fluctuations and probe particles couple. If an observer operates with those only, then this is the matter he is going to observe.

In other words, we reveal two types of observers in the SVT approach. The first type, called the relativistic observer or R-observer, is the one whose measuring apparatus is based on small fluctuations of superfluid vacuum. This observer sees a relativistic picture, the Lorentz symmetry is a fundamental symmetry for him. The second type, called the full observer or F-observer, can measure things with objects which can violate the smallness condition. She measures the Bose liquid or condensate, which technically flows in an empty Euclidean space. Note that a symmetry of the latter is no longer relevant, because this empty space is unobservable to an F-observer as far as the vacuum condensate exists as an integral entity described by its wavefunction.

Obviously, an F-observer is capable of seeing phenomena an R-observer is unable to, therefore her picture of reality must be more consistent and free of any divergences or anomalies. A condensed-matter analogy of such difference would be the so-called sonic black holes in liquids \cite{10}, which “exist” in a picture drawn by phonons (sound waves), but not in a picture drawn by photons. However, our current conventional observer is still of an R-type, therefore the information from F-observer’s models must be translated to an R-observer’s language. The corresponding “dictionary” must be based on a metric (1) and associated Einstein field equations.

3. Spacetime induced by background superfluid

Let us further specify the values of the fluid’s density, velocity and speed of oscillations in eq. (1). Typically, one deals with a quantum liquid described by a condensate wavefunction obeying a nonlinear wave equation. The latter can be chosen to have a minimal \( U(1) \)-symmetric form

\[ i \partial_t \Psi = \left[ -\frac{\hbar}{2m} \nabla^2 + V_{\text{ext}}(x, t) - F(|\Psi|^2) \right] \Psi, \tag{3} \]

where \( m \) is the mass of a constituent particle, \( F(\rho) \) is a differentiable function on a positive semi-axis \( \rho \), \( V_{\text{ext}}(x, t) \) is an external potential representing a trapping potential or container (we shall neglect it in what follows), and \( \Psi \) is a condensate wavefunction which obeys the normalization condition:

\[ \int_V |\Psi|^2 dV = \int_V \rho dV = M > 0, \tag{4} \]

where \( M \) and \( V \) are the total mass and volume of the liquid. Then the condensate wavefunction can be written in the Madelung form \cite{11}:

\[ \Psi = \sqrt{\rho} \exp(iS), \tag{5} \]

where

\[ \rho = |\Psi|^2, \tag{6} \]
and $S = S(x,t)$ is a phase which is related to fluid velocity:

$$u = (\hbar/m)\nabla S.$$  \hspace{1cm} (7)

By substituting the Madelung ansatz into eq. (3), and separating real and imaginary parts, one obtains in the leading-order approximation with respect to the Planck constant,

$$P \approx -\frac{\hbar}{m} \int \rho F'(\rho) \, d\rho, \quad c_s \approx \sqrt{\mp \frac{\hbar}{m} \rho F'(\rho)},$$ \hspace{1cm} (8)

where a prime denotes a derivative with respect to an argument in brackets; the sign $\mp$ differentiates between regimes of stable and unstable flow, and must be chosen in such a way that $c_s$ stays real-valued. The detailed derivation of these formulae can be found in refs. [12, 13].

With these formulae in hand, 4D metric (1) takes the form [12]:

$$g_{\mu\nu} \propto \frac{\lvert \Psi \rvert^2}{c_s} \left[ \frac{\hbar^2}{m^2} \left\{ \nabla \ln \left( \frac{\Psi}{\lvert \Psi \rvert} \right) \right\}^2 \pm \lvert \Psi \rvert^2 F'(\lvert \Psi \rvert^2) \right] : \frac{i\hbar}{m} \nabla \ln \left( \frac{\Psi}{\lvert \Psi \rvert} \right) : I,$$ \hspace{1cm} (9)

where $c_s \approx (\hbar/m)^{1/2} \lvert \Psi \rvert \sqrt{\mp F'(\lvert \Psi \rvert^2)}$. The value $c_s$ thus becomes a maximum attainable propagation velocity of any fluctuation of physical vacuum whose quantum wave amplitude is much smaller than $\lvert \Psi \rvert$. In our case, in the low-momenta (“phononic”) limit

$$c_s \to c_s^{(0)} \approx c,$$ \hspace{1cm} (10)

where $c = 2.9979 \times 10^8 \text{ m s}^{-1}$ is a universal constant, which is historically called the speed of light in vacuum. In the framework of superfluid vacuum theory, $c_s$ is a maximal velocity which can be measured by an R-observer.

4. Why logarithm

The logarithmic fluid is a quantum liquid described by eq. (3), where

$$F(\rho) = b \ln \left( \frac{\rho}{\bar{\rho}} \right),$$ \hspace{1cm} (11)

where $b$ and $\bar{\rho}$ being real-valued parameters; the former is also called the nonlinear coupling. One can show that different signs of $b$ mark two different phases our fluid can be in. According to equations (8), its macroscopic equation of state has an ideal-fluid form, $P \propto \rho$, in the leading-order approximation with respect to Planck constant.

Although a logarithmic nonlinearity itself was studied since the works by Rosen and Bialynicki-Birula and Mycielski [14, 15] (there were also extensive mathematical studies, to mention just a very recent literature [16, 17, 18, 19, 20, 21, 22, 23]), the logarithmic fluid approach itself was proposed relatively recently [12], as a further development and generalization of the non-perturbative theory of quantum gravity with a logarithmically nonlinear wave equation [24, 25]. Currently, there exist at least two independent arguments for why it is this type of fluid, which describes the physical vacuum.

The first argument is of a statistical nature and closely related to a theory of many-body open quantum systems. One can show that the logarithmic nonlinearity universally occurs in leading-approximation models of a large class of condensate-like matter in which the interaction potentials between constituent particles are substantially larger than the kinetic energies thereof.
[26, 27]. According to that approach, the nonlinear coupling must be related to thermodynamic values of the fluid:

\[ b \sim T \sim T_\Psi, \]

(12)

where \( T \) and \( T_\Psi \) are the thermal and quantum-mechanical temperature, respectively; a symbol “\( \sim \)” means “a linear function of”. The quantum-mechanical temperature is defined as a thermodynamical conjugate of the quantum information entropy function

\[ S_\Psi = -\int_V |\Psi|^2 \ln(|\Psi|^2/\bar{\rho}) \, dV, \]

which was proposed and studied by Everett, Hirschman and others, some bibliography can be found in ref. [26]. In particular, this entropy function directly emerges from equations (3) and (11), when averaged using a Hilbert space’s inner product [24, 28, 29].

It is thus a not so big surprise that the logarithmic model turns out to be robust for describing microscopic properties of the superfluid component of liquid \(^4\)He: it analytically reproduces with high accuracy three main observable facts: Landau spectrum of excitations, the structure factor, and the speed of sound at normal pressure, whereby using only one non-scale parameter to fit the excitation spectrum’s experimental data [5, 6].

According to an introductory section above, it is natural to expect the physical vacuum being condensate-like matter, composed of a superfluidic condensate and quantum fluctuations thereof. Therefore, the logarithmic fluid should be a robust model here too.

The second argument is not related to any statistics but relies on the correspondence principle. The latter implies that in the low-momenta limit, superfluid vacuum theory has to recover the Einstein’s relativistic postulates. One of them, about a constancy of \( c \), implies that \( c_s \) should not depend on density, at least in the leading order with respect to the Planck constant. Recalling eq. (8), this results in the following differential equation

\[ \rho F'(\rho) \approx \mp mc_s^{(0)2}/\hbar \approx \text{const}, \]

(13)

whose solution is a logarithmic function (11), as one can easily check. Similarly to eq. (8), an approximation sign indicates a leading-order approximation with respect to \( \hbar \).

The solution of the equation above implies that for an R-observer, the physical meaning of the nonlinear coupling is dynamical

\[ b \rightarrow b_s \propto mc_s^{(0)2}/\hbar \propto mc^2/\hbar, \]

(14)

while for an F-observer it is quantum thermodynamical, cf. eq. (12).

In other words, thermal processes inside superfluid vacuum, such as change of temperature, heavily influence dynamical processes therein hence the structure of induced spacetime observed by an R-observer. This should have profound implications in many high-energy and strong-gravity phenomena, including those occurring in cosmology.

It also means that one can use the relativistic approach (with tweaking by adding additional fields) for a large range of energies, all the way up to the “maxon” peak threshold, which can be as high as hundreds TeV and above. These intermediate relativistic models can still provide valuable understanding about various fundamental phenomena, such as the mass generation mechanism and non-zero extent of particles [30, 31]. However, drastically new physics will step in when one manages to reach the “maxon” threshold and thus go into an essentially non-relativistic regime; vacuum Cherenkov radiation and superluminal boom being some examples of phenomena which will occur [9, 25].

5. Superfluid vacuum cosmology

Let us study here a case when background logarithmic superfluid is in a state described by the wavefunction \( \Psi_0(\mathbf{x}, t) \), while quantum fluctuations are disregarded. If we assume the simplest
possible case, when the superfluid’s phase in this state is linear with respect to the radius-vector, we obtain
\[ i \ln (\Psi_0/|\Psi_0|) \sim u^{(0)} \cdot x, \]

hence its gradient is a constant three-vector. Therefore, the mapping (9) gives us the following induced 4D geometry:
\[ ds^2_b \propto \frac{1}{b} |\Psi_0(x,t)|^2 \left[ -c_b^2 dt^2 + (dx - u^{(0)} dt)^2 \right], \]

where \( c_b \) is constant, according to above; we can assume here that \( c_b \approx \sqrt{|b|/\hbar/m} \). From the viewpoint of an R-observer, the value of \( u^{(0)} \) is not observable and can be set to any value by an appropriate coordinate transformation. This confirms our remarks on isotropy made in the introductory section above.

Obviously, for manifolds with the line element (16), the Weyl tensor vanishes. Therefore, they are of the type O, according to the Petrov classification [32]. This is the class where all Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime metrics belong to, including those which describe the worlds expanding with an acceleration. A subtle technical point is that in the SVT approach the induced spacetime metrics come out written in conformally-flat coordinates [33, 34], which thus requires additional coordinate transformations to present metrics in a form which is more popular in relativistic cosmology nowadays.

By using eq. (16) and the conformal rescaling technique, one can derive the induced four-dimensional stress-energy tensor corresponding to this metric, see ref. [12] for details. This stress-energy tensor strongly resembles a theory with the non-minimally coupled scalar field, which can be interpreted as a dilaton or inflaton. One can also demonstrate this process by moving in opposite direction: it was shown that logarithmic nonlinearity appears in field equations when performing an ADM-type reduction of dilatonic gravity [35].

6. Conclusion

In conclusion, let us demonstrate how the cosmological singularity problem gets resolved in the SVT approach. In the previous section, it was shown that if the superfluid vacuum is presented by a logarithmic fluid being in a state with a phase linearly dependent on radius-vector, then our R-observer sees himself embedded into the FLRW-kind Universe.

This means that the expansion of Universe is a phenomenon, whose existence and interpretation is a matter of which type of an observer we are talking about: an R-observer sees the expanding 4D spacetime while an F-observer observes a non-relativistic superfluid flow in a 3D space. Interestingly, superfluid vacuum cosmology offers its own explanation for a temperature of the cosmic microwave background (CMB): it is a temperature of photon-type excitations of superfluid vacuum, which are close to being in a thermal equilibrium with the background superfluid itself. Such a conjecture immediately explains, without involving specific models or fine-tuning of the initial conditions, why CMB temperature’s value is so close to a temperature scale of quantum liquids we know of, which is about two Kelvin.

Furthermore, a metric (16) obviously becomes singular in a domain where the factor \( |\Psi_0(x,t)|^2 \) approaches zero. From the viewpoint of an R-observer, this looks like a serious issue: one cannot impose Cauchy-type initial conditions in a singular point, therefore the whole dynamics is ill-defined.

From the viewpoint of an F-observer, however, nothing drastic happens. In a quantum-mechanical theory, be it a theory of point-like particles or Bose liquids and condensates, wavefunctions’ amplitudes can take zero values. This can happen, for example, at a boundary of a system, or even in the origin if a wavefunction is odd. Wavefunctions can also take asymptotically zero values if a system occupies an infinite-size region of space.
As for infinite values of $|\Psi|^2$, then these are usually forbidden by normalization conditions, like the one given by eq. (4), which ensure a probabilistic or condensate interpretation of a wavefunction.

To summarize, cosmological singularities “exist” only in the incomplete picture seen by an $R$-observer whose measuring facilities are restricted to small excitations of vacuum, as discussed in section 2. This illustrates and reaffirms a nearly obvious fact that the Einstein’s theory of relativity, like any other viable physical theory we have dealt with, has a finite applicability domain. Any physical processes in a vicinity of, or resulting from, spacetime singularities must be described by means of post-relativistic theories and notions.

Acknowledgments

Fruitful discussions with M. Znojil and U. Guenther are much appreciated. This work is based on the research supported by the Department of Higher Education and Training of South Africa and in part by the National Research Foundation of South Africa.

References

[1] Volovik G E 2009 The Universe in a Helium Droplet (Oxford: Oxford University Press)
[2] Huang K 2016 A Superfluid Universe (Singapore: World Scientific)
[3] Dirac P A M 1951 Nature 168 906
[4] Tilley D R and Tilley J 1999 Superfluidity and Superconductivity (Bristol: IOP Publishing)
[5] Zloshchastiev K G 2012 Eur. Phys. J. B 85 273
[6] Scott T C and Zloshchastiev K G 2019 Low Temp. Phys. 45 1231
[7] Kapitsa P L 1938 Nature 141 74
[8] Allen J F and Misener A D 1938 Nature 141 75
[9] Zloshchastiev K G 2020 Int. J. Mod. Phys. A 35 2040032
[10] Unruh W G 1981 Phys. Rev. Lett. 46 1351
[11] Rylov Yu A 1999 J. Math. Phys. 40 256
[12] Zloshchastiev K G 2011 Acta Phys. Polon. 42 261
[13] Zloshchastiev K G 2019 J. Theor. Appl. Mech. 57 843
[14] Rosen G 1968 J. Math. Phys. 9 996
[15] Bialynicki-Birula I and Mycielski J 1976 Ann. Phys. (N. Y.) 100 62
[16] Alves C O and de Morais Filho D C 2018 Z. Angew. Math. Phys. 69 144
[17] Alves C O, de Morais Filho D C and Figueiredo G M 2019 Math. Meth. Appl. Sci. 42 4862
[18] Wang Z and Zhang C 2019 Arch. Rational Mech. Anal. 231 45
[19] Zloshchastiev K G 2010 Grav. Cosmol. 16 288
[20] Zloshchastiev K G 2013 Phys. Lett. A 375 2305
[21] Zloshchastiev K G 2018 Z. Naturforsch. A 73 619
[22] Zloshchastiev K G 2018 Europhys. Lett. (EPL) 122 39001
[23] Brasher J D 1991 Int. J. Theor. Phys. 30 979
[24] Avivdenkov A V and Zloshchastiev K G 2011 J. Phys. B: At. Mol. Opt. Phys. 44 195303
[25] Zloshchastiev K G 2013 Central Eur. J. Phys. 11 325
[26] Petrov A Z 1954 Uch. Zapiski Kazan Gos. Univ. 144 55
[27] Infeld L and Schild A 1945 Phys. Rev. 68 250
[28] Tauber G E 1967 J. Math. Phys. 8 118
[29] Scott T C, Zhang X, Mann R B and Fee G J 2016 Phys. Rev. D 93 084017