NONLOCALITY AS AN AXIOM FOR QUANTUM THEORY*

Daniel Rohrlich and Sandu Popescu

School of Physics and Astronomy, Tel-Aviv University
Ramat-Aviv, Tel-Aviv 69978 Israel

ABSTRACT

Quantum mechanics and relativistic causality together imply nonlocality: nonlocal correlations (that violate the CHSH inequality) and nonlocal equations of motion (the Aharonov-Bohm effect). Can we invert the logical order? We consider a conjecture that nonlocality and relativistic causality together imply quantum mechanics. We show that correlations preserving relativistic causality can violate the CHSH inequality more strongly than quantum correlations. Also, we describe nonlocal equations of motion, preserving relativistic causality, that do not arise in quantum mechanics. In these nonlocal equations of motion, an experimenter “jams” nonlocal correlations between quantum systems.

1. INTRODUCTION

Two aspects of quantum nonlocality are nonlocal correlations and nonlocal equations of motion. Nonlocal correlations arise in settings such as the one discussed by Einstein, Podolsky and Rosen\(^1\). As Bell\(^2\) showed (and Aspect has reviewed in his lecture here) no theory of local variables can reproduce these correlations. The Aharonov-Bohm effect\(^3\) is also nonlocal in that an electromagnetic field influences an electron in a region where the field vanishes. The field induces a relative phase between two sets of paths available to an electron, displacing the interference pattern between the two sets of paths. Thus, the Aharonov-Bohm effect implies nonlocal equations of motion.\(^4\) Both aspects of quantum nonlocality arise within nonrelativistic quantum theory. However, the very definition of a local variable is relativistic: a local variable can be influenced only by events in its backward light cone, and can influence events only in its forward light cone. In this sense, quantum mechanics and relativity together imply nonlocality. They coexist because quantum correlations preserve relativistic causality (i.e. they do not allow us to transmit signals faster than

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light). But quantum mechanics does not allow us to consider isolated systems as separate, as Einstein, Podolsky and Rosen\(^1\) assumed. This violation of not the letter but the spirit of special relativity has left many physicists (including Bell) deeply unsettled. Today, quantum nonlocality seems as fundamental—and as unsettling—as ever. If nonlocality is fundamental, why not make nonlocality an axiom of quantum theory rather than a consequence? Can we then invert the logical order, showing that nonlocality and relativistic causality together imply quantum theory?

2. **NONLOCALITY I: NONLOCAL CORRELATIONS**

Quantum mechanics and relativistic causality together give rise to nonlocal correlations, which many physicists regard as a negative aspect of quantum theory. Here, we regard quantum nonlocality as a positive aspect of quantum theory. What new possibilities does quantum nonlocality offer us? In particular, if we make nonlocality an axiom, what becomes of the logical structure of quantum theory?\(^5\)\(^6\)\(^7\) The special theory of relativity can be deduced in its entirety from two axioms: the equivalence of inertial reference frames, and the constancy of the speed of light. Aharonov\(^7\) has proposed such a logical structure for quantum theory. Let us take, as axioms of quantum theory, relativistic causality and nonlocality. As an initial, immediate result, we deduce that quantum theory is not deterministic, otherwise these two axioms would be incompatible.\(^7\) Two “negative” aspects of quantum mechanics—indeterminacy and limits on measurements—then appear as a consequence of a fundamental “positive” aspect: the possibility of nonlocal action. Moreover, by taking nonlocality as an axiom, we free ourselves of the need to explain it.

We have not yet defined the axiom of nonlocality. Relativistic causality is well defined, but quantum nonlocality arises both in nonlocal correlations and in the Aharonov-Bohm effect. In this section we consider nonlocal correlations. We ask which theories yield nonlocal correlations while preserving causality. Our result is independent of quantum mechanics or any particular model. We find\(^8\) that quantum mechanics is only one of a class of theories consistent with our two axioms, and, in a certain sense, not even the most nonlocal theory.

The Clauser, Horne, Shimony, and Holt\(^9\) (CHSH) form of Bell’s inequality, holds in any classical theory (that is, any theory of local hidden variables). It states that a certain
A combination of correlations lies between -2 and 2:

\[-2 \leq E(A, B) + E(A, B') + E(A', B) - E(A', B') \leq 2. \tag{1}\]

Besides 2, two other numbers, \(2\sqrt{2}\) and 4, are important bounds on the CHSH sum of correlations. If the four correlations in Eq. (1) were independent, the absolute value of the sum could be as much as 4. For quantum correlations, however, the CHSH sum of correlations is bounded\(^\text{10}\) in absolute value by \(2\sqrt{2}\). Where does this bound come from? Rather than asking why quantum correlations violate the CHSH inequality, we might ask why they do not violate it more.

Let us say that of the two axioms proposed above, the axiom of nonlocality implies that quantum correlations violate the CHSH inequality at least sometimes. We may then guess that the other axiom, relativistic causality, might imply that quantum correlations do not violate it maximally. Could it be that relativistic causality restricts the violation to \(2\sqrt{2}\) instead of 4? If so, then the two axioms determine the quantum violation of the CHSH inequality. To answer this question, we ask what restrictions relativistic causality imposes on joint probabilities. Relativistic causality forbids sending messages faster than light. Thus, if one observer measures the observable \(A\), the probabilities for the outcomes \(A = 1\) and \(A = -1\) must be independent of whether the other observer chooses to measure \(B\) or \(B'\). However, it can be shown\(^\text{8,11}\) that this constraint does not limit the CHSH sum of quantum correlations to \(2\sqrt{2}\). For example, imagine a “superquantum” correlation function \(E\) for spin measurements along given axes. Assume \(E\) depends only on the relative angle \(\theta\) between axes. For any pair of axes, the outcomes \(|\uparrow\uparrow\rangle\) and \(|\downarrow\downarrow\rangle\) are equally likely, and similarly for \(|\uparrow\downarrow\rangle\) and \(|\downarrow\uparrow\rangle\). These four probabilities sum to 1, so the probabilities for \(|\uparrow\downarrow\rangle\) and \(|\downarrow\uparrow\rangle\) sum to 1/2. In any direction, the probability of \(|\uparrow\rangle\) or \(|\downarrow\rangle\) is 1/2 irrespective of a measurement on the other particle. Measurements on one particle yield no information about measurements on the other, so relativistic causality holds. The correlation function then satisfies \(E(\pi - \theta) = -E(\theta)\). Now let \(E(\theta)\) have the form

(i) \(E(\theta) = 1\) for \(0 \leq \theta \leq \pi/4\);

(ii) \(E(\theta)\) decreases monotonically and smoothly from 1 to -1 as \(\theta\) increases from \(\pi/4\) to \(3\pi/4\);
(iii) \( E(\theta) = -1 \) for \( 3\pi/4 \leq \theta \leq \pi \).

Consider four measurements along axes defined by unit vectors \( \hat{a}', \hat{b}, \hat{a}, \) and \( \hat{b}' \) separated by successive angles of \( \pi/4 \) and lying in a plane. If we now apply the CHSH inequality Eq. (1) to these directions, we find that the sum of correlations

\[
E(\hat{a}, \hat{b}) + E(\hat{a}', \hat{b}) + E(\hat{a}, \hat{b}') - E(\hat{a}', \hat{b}') = 3E(\pi/4) - E(3\pi/4) = 4
\]
violates the CHSH inequality with the maximal value 4. Thus, a correlation function could satisfy relativistic causality and still violate the CHSH inequality with the maximal value 4.

3. NONLOCALITY II: NONLOCAL EQUATIONS OF MOTION

In one version of the Aharonov-Bohm effect, an isolated magnetic flux, inserted between two slits, shifts the interference pattern of electrons passing through the slits. It thereby affects the electron’s momentum, since the electron arrives at a different point than it would without the electromagnetic field. Thus, the Aharonov-Bohm effect implies non-local equations of motion. Aharonov has shown that a physical quantity, the modular momentum of the flux, is uncertain exactly as required to keep us from seeing a violation of causality. In general, modular momentum is measurable and obeys a nonlocal equation of motion. But when the flux is located between the slits, its modular momentum is completely uncertain.

Is quantum mechanics the only relativistically causal theory with nonlocal equations of motion? As in the last section, we may approach this question by looking for a theory not equivalent to quantum mechanics that obeys relativistic causality and nonlocality. We have considered a model in which action by an experimenter affects (“jams”) nonlocal correlations between systems measured at spacelike separations from the action. For example, Shimony considers the effect of a laser beam crossing the path of one of the photons in a singlet pair, after the photon has already passed. We find that while nonlocal “jamming” is not possible in quantum mechanics, it could be consistent with relativistic causality. If jamming is realized in nature, then perhaps, as suggested by Grunhaus, it is possible to jam nonlocal quantum correlations.

We briefly summarize the model. Two experimenters, call them Alice and Bob, make measurements on systems that have locally interacted in the past. Alice’s measurements are
spacelike separate from Bob’s. A third experimenter, Jim (the jammer), presses a button on a black box. This event is spacelike separate from Alice’s measurements and from Bob’s. The black box acts at a distance on the correlations between the two sets of systems. We find no conflict with relativistic causality if jamming satisfies two conditions. The \textit{unary} condition requires that neither Alice, from her results alone, nor Bob, from his, can tell whether Jim has pressed the button. Then jamming cannot carry a signal to either Alice or Bob. The unary condition implies indeterminism. The \textit{binary} condition restricts the range of jamming. If $A$ and $B$ denote Alice’s and Bob’s measurements, and $J$ Jim’s pressing of the button, the overlap of the forward light cones of $A$ and $B$ must lie entirely within the forward light cone of $J$.

4. \textbf{SUMMARY}

We have seen that quantum mechanics is not the only theory combining relativistic causality with nonlocality, nor even, in a sense, the most nonlocal one. We found that both “superquantum” correlations and a model for nonlocal “jamming”—a stronger form of nonlocality than arises in quantum mechanics—can be consistent with relativistic causality. The question remains, from what minimal set of physical principles can we derive quantum mechanics?

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