Quantum transport phenomena in disordered electron systems with spin-orbit coupling in two dimensions and below

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Abstract

Electron transport phenomena in disordered electron systems with spin-orbit coupling in two dimensions and below are studied numerically. The scaling hypothesis is checked by analyzing the scaling of the quasi-1D localization length. A logarithmic increase of the mean conductance is also confirmed. These support the theoretical prediction that the two dimensional metal in systems with spin-orbit coupling has a perfect conductivity. Transport through a Sierpinski carpet is also reported.

Key words: quantum transport, spin-orbit coupling, symplectic class, scaling theory, perfect conductivity

1. Introduction

Research in spintronics and carbon nanotubes sparked renewed interests in the effects of spin-orbit coupling on quantum transport [1,2]. One of the remarkable features of non-interacting disordered electron systems with spin-orbit coupling is the existence of a metallic phase in two dimensions (2D) [3,4]. More exactly the 2D metallic phase appears in systems with symplectic symmetry, i.e., in systems with time reversal symmetry but without spin rotation symmetry. Such systems are an exception to the prediction of Abrahams et al. that there is no metallic phase in 2D disordered electron systems [5].

Recently some 2D metals have been discovered experimentally [6]. Although the mechanism inducing the 2D metal might be different from spin-orbit coupling, we think that detailed understanding of the 2D symplectic systems gives some insights on general transport properties of 2D metals.

A scaling theory for the symplectic systems [3,7,8] can be described by the \( \beta \) function for the conductance \( g \) (in units of \( e^2/h \))

\[
\beta(\ln g) = \frac{d \ln g}{d \ln L},
\]

where \( L \) is the linear size of a system. (The conductance is actually a distributed quantity. See Refs. [9,10,11] for discussions on the scaling hypothesis of the conductance.) Because of the so-called anti-localization effect, the \( \beta \) function shows non-monotonic behavior in the symplectic systems. The asymptotic behavior in the
strongly metallic region in 2D is conjectured to be
\[
\beta(\ln g) = \frac{1}{\pi g} \quad (g \gg 1).
\]
(2)

This indicates that there is a metallic phase in 2D.

In the scaling theory for the symplectic systems in 2D, however, there is a long standing problem for more than two decades [12,13]; the logarithmic divergence of the conductance in the 2D metallic phase in the limit \( L \to \infty \) indicated by Eq. (2). Since the conductivity and the conductance are the same quantity in 2D, this predicts a perfect conductivity, \( \sigma = \infty \), in spite of the system being disordered. This is in contrast to 3D disordered metals, in which the conductivity converges to a finite value.

According to the renormalization group theory of continuous phase transitions, the scaling hypothesis is valid when the correlation length is much larger than all other microscopic lengths. The scaling argument which deduced (2), however, is based on the weak localization theory in which disorder is supposed to be weak. In such a weakly disordered metallic phase this condition on the correlation length may not be satisfied. Therefore, a numerical check of the scaling hypothesis in the 2D metallic region is desirable.

In this paper, we report a numerical check of the scaling hypothesis in the 2D metallic region. We also report numerical calculation of the Landauer conductance in 2D, which is a direct check of the prediction of a perfect conductivity. Last, numerical simulations of transport through a Sierpinski carpet are reported.

2. The SU(2) model

Among the models with symplectic symmetry, we employ the SU(2) model [14,15],
\[
H = \sum_{i,\sigma} \epsilon_i c_i^\sigma c_i^\sigma - \sum_{j, j', \sigma, \sigma'} R(i,j)_{\sigma\sigma'} c_j^{\sigma'} c_j^{\sigma'}.
\]
(3)
The random potential \( \epsilon_i \) is distributed with box distribution in the range \([-W/2, W/2]\). Random spin-orbit coupling is included in the nearest neighbor hopping term. We parameterize the hopping matrix as
\[
R(i,j) = \begin{pmatrix}
  e^{i\alpha_{ij}} \cos \beta_{ij} & e^{i\gamma_{ij}} \sin \beta_{ij} \\
  -e^{-i\gamma_{ij}} \sin \beta_{ij} & e^{-i\alpha_{ij}} \cos \beta_{ij}
\end{pmatrix},
\]
(4)
and we distribute \( \alpha_{ij} \) and \( \gamma_{ij} \) with uniform probability in the range \([0, 2\pi]\), and \( \beta_{ij} \) according to the probability density \( p(\beta) d\beta = \sin(2\beta) d\beta \) in the range \([0, \pi/2]\). In the actual simulations \( R(i,j) \)’s in one direction are set to the unit matrix with the aid of the local SU(2) gauge transformation.

The SU(2) model has an advantage that corrections to scaling arising from irrelevant variables are smaller than other models [14,15], so it is quite a useful model when we study universal aspects of Anderson localization and the Anderson transition.

3. Numerical check of the scaling hypothesis in the 2D metallic phase

We analyze the quasi-1D localization length \( \lambda \) on a quasi-1D strip with width \( L \) [16,17]. Periodic boundary conditions are imposed in the transverse direction. The scaling hypothesis implies that the dimensionless quantity \( \Lambda = \lambda/L \) obeys
\[
\Lambda = F_{\pm} \left( \frac{L}{\xi} \right),
\]
(5)
where \( \xi \) is the 2D correlation (localization) length which depends on \( W \) and the Fermi energy \( E \). The subscript \( \pm \) indicates the metallic and insulating phases.

We have calculated \( \Lambda \) for sizes \( L = [16, 128] \) with an accuracy from 0.3% to 1.0%. The ensemble transfer matrix method has been used [18]. In addition to the data at \( E = 1 \) analyzed in Ref. [15], we have also accumulated data at \( E = 2 \) in \( W = [0.0, 4.5] \). From the numerical data the correlation length \( \xi \) at each pair of \( (E, W) \) and the scaling function \( F_{\pm} (L/\xi) \) are estimated using numerical fit described in Ref. [15]. The quality of the fit is assessed by the goodness of fit probability \( Q \). To eliminate the ambiguity of the absolute scale of \( \xi \), we set \( \xi = 10 \) at \( (E, W) = (1, 3) \).

Figure 1 demonstrates the single parameter scaling in the metallic region. All data fall on a common scaling curve when \( \Lambda \) is plotted as a function of \( L/\xi \), indicating the validity of the scaling hypothesis. In a strongly metallic region \( (\Lambda \gg 1) \), the data are well fitted to
\[
\Lambda = b + c \ln(L/\xi) \quad (\Lambda \gg 1),
\]
(6)
as shown in Fig. 2.
3. Single parameter scaling in the metallic region in 2D. The goodness of fit $Q$ is 0.3.

In Ref. [15], the $\beta$ function for $\Lambda$ from the metallic to the insulating limits was estimated. The estimate has not changed significantly when we add data for $E = 2$.

4. Logarithmic increase of the conductance in the 2D metallic phase

In last section and in Ref. [15], the scaling hypothesis has been checked numerically. This supports the scaling theory for the 2D symplectic systems. Furthermore, the logarithmic increase of $\Lambda$ in the strongly metallic region gives the impression that the conductance may also increase logarithmically. In this section, we study the size dependence of the mean conductance directly.

We now attach two perfect leads to the square sample with linear size $L$ and calculate the two terminal Landauer conductance $g$ (in units of $e^2/h$) with the transfer matrix method [19]. A fixed boundary condition is imposed in the direction transverse to the current. The hopping matrices in the transverse direction are set to the unit matrix, and the Fermi energy $E = 1$. We have accumulated 10000 samples for $L = [64, 256]$ and 5000 samples for $L = 384$.

As shown in Fig. 3, the mean conductance $\langle g \rangle$ in the strongly metallic region does show the logarithmic increase

$$\langle g \rangle \approx a + \pi^{-1} \ln L \quad (\langle g \rangle \gg 1), \quad (7)$$

in agreement with the prediction of the scaling theory [3]. The prefactor of the logarithmic term, which is an important parameter for the scaling in the strongly metallic region, is also consistent with it. Note that a perfect conductivity does not mean perfect transmission. The transmission probability per channel rather goes to zero as $L$ increases, so most of incoming electrons are reflected.

It is left for future to understand physics behind this possible perfect conductivity in 2D.
5. Transport through a Sierpinski carpet

Recently we have obtained numerical results which suggest that an Anderson transition occurs even below 2D in the presence of spin-orbit coupling [20]. This is based on numerical simulations of electrons on a Sierpinski carpet SC(5, 1, k), where k is the generation number. In the limit $k \to \infty$, SC(5, 1, k) becomes a true fractal whose spectral dimension is $d_s = 1.940 \pm 0.009$ [21].

Here we study the size dependence of the Landauer conductance through the Sierpinski carpet in a delocalized phase. We have attached two perfect leads to SC(5, 1, k) and have calculated the conductance with the recursive Green's function method [22]. A fixed boundary condition is imposed in the transverse direction. The hopping matrices in the transverse direction are set to the unit matrix. We set the Fermi energy $E = 0.9$ and the disorder $W = 0, 2$ where relatively large mean conductance is obtained. The number of samples is 10000 for $k = 1, 2, 3$ and 500 for $k = 4$.

Figure 4 shows the mean conductance $\langle g \rangle$ as a function of the linear size $L = 5^k$. The mean conductance increases with $L$, possibly indicating a delocalized phase. It also indicates that the increase is slower than that in 2D. It is an open problem whether the conductance in the possible delocalized phase of the Sierpinski carpet diverges.

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