Insensitivity of the Yrast Spectra of Even-Even Nuclei to the T=0 two-body interaction matrix elements

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Abstract

Calculations of the spectra of various even-even nuclei in the fp shell ($^{44}$Ti, $^{46}$Ti, $^{48}$Cr, and $^{50}$Cr) are performed with two sets of two-body interaction matrix elements. The first set consists of the matrix elements of the FPD6 interaction. The second set have the same T=1 two-body matrix elements as the FPD6 interaction, but all the T=0 two-body matrix elements are set equal to zero. Despite the drastic differences between the two interactions, the spectra they yield are very similar and indeed it is difficult to say which set gives a better fit to experiment. That the results for the yrast spectra are insensitive to the presence or absence of T=0 two-body matrix elements is surprising because the only bound two nucleon system has T=0, namely the deuteron. Also there is the general folklore that T=0 matrix elements are responsible for nuclear collectivity. Electric quadrupole transition rates are also examined. It is found that the reintroduction of T=0 matrix elements leads to an enhancement of B(E2)'s for lower spin transitions but in some cases for higher spin transitions one gets another surprising result that there is a small suppression.
I. INTRODUCTION

The study of neutron-proton pairing especially in the $T=0$ channel is a particularly prominent topic these days. While the number of journal articles are far too numerous to reference one might begin to make some headway into the varied approaches by starting from the references found in Refs. [1–3]. In so doing one will find a field of study filled with disagreement and occasionally strife.

The approach here is to study the effects of the $T=0$ portion of the neutron-proton interaction by removing it altogether. In this way it is hoped that we can understand where in nuclear structure this portion of the interaction is found to play a large role. This would in turn suggest where $T=0$ pairing might most clearly reveal its presence or absence. In this work we will examine the yrast spectra of the even-even ($fp)^4$ nuclei $^{44}$Ti, $^{46}$Ti, $^{48}$Cr, and $^{50}$Cr with two sets of interaction parameters. First we have the full FPD6 interaction [4], and then we use the same interaction for the two-body $T=1$ matrix elements while setting all the $T=0$ two-body interaction matrix elements to zero. We shall denote this interaction as $T0FPD6$. (This modification of an effective interaction is along the same lines of that used by Satula et al. to examine Wigner energies a few years ago [5].) We have used this modification of FPD6 in the past to study a variety of things and in particular the full fp spectrum of $^{44}$Ti [6–9]. We found to our surprise and to the surprise of many others that one could obtain a fairly decent spectrum for the levels of $^{44}$Ti with the second interaction here labeled $T0FPD6$. This is surprising because the $T=0$ interaction is by no means small. The only bound two nucleon system is the $T=0$ combination of two nucleons, the deuteron. Moreover the largest valued matrix elements in the FPD6 interaction tend to be those in the $T=0$ channel.

With the one nucleus $^{44}$Ti shown in a full fp space calculation in Ref. [6] we have the first clue that the spectra of even-even nuclei are relatively insensitive to the $T=0$ two-body interaction matrix elements. To make this more conclusive however we need to examine more nuclei. We have expanded the examination of the even-even nuclei to those listed above. The sample we have chosen consists of nuclei with the same collective properties - ground state bands which have some rotational properties but are not extremely rotational.

Also we now look at transition rates - perhaps these will prove more sensitive to the presence of $T=0$ two-body matrix elements than the spectra. This will be our interest in a few sections.

It should be noted that in Refs. [6–9] a wide range of topics is addressed beyond the spectra of even-even nuclei. These topics include a partial dynamical symmetry that arises
when one uses the T0FPD6 interaction in a single j shell for $^{43}$Sc and $^{44}$Ti. Also while using the T0FPD6 interaction a subtle relationship between the $T=\frac{1}{2}$ states in $^{43}$Sc and $T=\frac{3}{2}$ states in $^{43}$Ca likewise between the the $T=0$ states in $^{44}$Ti and $T=2$ states in $^{44}$Ca. We also considered even-odd nuclei and addressed the topic of how the $T=0$ two-body matrix elements affect B(M1) transitions - both spin and orbital components, and Gamow-Teller transitions. In many cases the transition rates were very sensitive to the presence or absence of the T=0 matrix elements. This was especially the case for some orbital B(M1)'s and the Gamow-Teller transitions.

Here things will be kept simple and we focus on the spectra and B(E2)'s of the yrast levels of selected even-even nuclei. We will examine the sensitivity of these observables on the T=0 two-body interaction matrix elements by setting them to zero and comparing the results thus obtained with those when the T=0 matrix elements are reintroduced.

II. RESULTS

As mentioned in the introductions we perform calculations of even-even nuclei with and without the T=0 two-body matrix elements of the FPD6 interaction. Thus each of the figures 1 to 6 corresponding to the nuclei $^{44}$Ti, $^{46}$Ti, $^{48}$Cr, and $^{50}$Cr will consist of three columns. The first column is the Yrast spectra calculated with the full FPD6 interaction, the second column with the T0FPD6 interaction, and the third column shows the levels from experiment.

We have previously discussed $^{44}$Ti so we will start with $^{46}$Ti, but before dissecting the details of the spectra, note the exceptional results in column 2 of figure 2. There we see that the results found when all the T=0 two-body interaction matrix elements are set to zero (T0FPD6) in this complete fp calculation that the resulting spectra looks quite reasonable in comparison with both the full FPD6 and the known experimental levels. Indeed it is difficult to choose between T0FPD6 and FPD6 as to which yields a better fit to experiment.

A closer look shows some differences. The odd spin excitation energies come down by about 1 MeV when the T=0 matrix elements are set to zero. Experimentally some odd spin excitation energies are known ($J=1^+$ and $J=11^+$) but not as many as the even spins. The even spin spectrum is slightly more spread out in the full FPD6 calculation that is to say there is more of a tendency towards a rotational spectrum. The low lying spectrum is fit somewhat better with the full FPD6 interaction but there is a better fit to the $J=14^+$ state using the T0FPD6 interaction.

There are similar stories for $^{48}$Cr (Figures 3,4) and $^{50}$Cr (Figures 5 and 6). (These
nuclei are of some interest as they both display backbends [10]- [15].) In $^{44}$Ti there is a bigger difference between FPD6 and T0FPD6 perhaps results from the greater deformation in $^{44}$Ti.

As is the case for $^{46}$Ti and indeed many of the even-even nuclei in the region very few odd spin states are known. If these were to be made available experimentally it might be easier to demonstrate more clearly the preference between the full FPD6 or the modification T0FPD6. The fact that one can even think of offering into competition an interaction in which all the T=0 two-body interaction matrix elements are set to zero is quite remarkable and perhaps a bit disturbing.

III. B(E2) RATES

The B(E2) rates for $^{44}$Ti, $^{46}$Ti, $^{48}$Cr, and $^{50}$Cr are listed in tables I to IV. We allow up to $t$ nucleons to be excited from the $f_{7/2}$ shell to the rest of the fp shell. The values of $t$ used are 4,3,2, and 2 respectively. The effective charges used are the standard 1.5e for the proton and 0.5e for the neutron. The difference in the effective charges from 1 and 0 is intended to take care of the fact that the $\Delta N = 2$ and higher excitations are not present in this model space. The results for FPD6 and T0FPD6 are shown. We also display the ratios of the results for the 2 interactions.

For $^{46}$Ti the reintroduction of the T=0 two-body matrix elements causes an increase (relative to T0FPD6) of a factor of two or more for all the transitions considered. So there is evidence here that the T=0 matrix elements contribute to the collectivity. This is not seen by looking at just the excitation energies in $^{46}$Ti where FPD6 and T0FPD6 give very similar results.

In $^{48}$Cr and $^{50}$Cr a more interesting behavior evolves. In $t=2$ calculations, the B(E2)’s for low spin transitions get an enhancement with the reintroduction of the T=0 two-body matrix elements but for some of the higher spins we get a suppression. The suppression occurs when backbending occurs as in $^{50}$Cr in the $J=8 \rightarrow 10$ transition.

In summary, in studying the problem of the T=0 neutron-proton interaction in a nucleus, it may prove more fruitful to begin by removing this channel altogether as was done here by setting all the T=0 two-body matrix elements to zero and then reintroducing them, rather than adopting the more common approach of investigating the effects of a pairing interaction separated from the rest of the interaction. This may be especially true in the shell model as the suggestion has been made by Satula and Wyss that it may not be appropriate to separate out a pairing interaction from the rest of the Hamiltonian in a shell
model context [16].

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FIG. 1. Full fp calculation for T=0 states in $^{44}$Ti with lowest state at 0 MeV.
| MeV | 46 Ti FPD6 | T0FPD6 | Experiment |
|-----|------------|--------|------------|
| 16  | 16         |        |            |
| 15  | 16         |        |            |
| 14  | 15         |        |            |
| 13  |            |        |            |
| 12  |            |        |            |
| 11  | 13         |        |            |
| 10  | 14         |        | 14         |
| 9   | 13         | 14     |            |
| 8   | 12         |        | 11         |
| 7   | 9          | 12     |            |
| 6   | 10         | 11     | 10         |
| 5   | 8          | 7      | 8          |
| 4   | 5          |        | 1          |
| 3   | 6          | 3      | 6          |
| 2   | 4          | 4      | 4          |
| 1   | 2          | 2      | 2          |
| 0   | 0          | 0      | 0          |

FIG. 2. Full fp calculation for T=1 states in $^{46}$Ti with lowest state at 0 MeV.
| t=2 FPD6 | 48 Cr T0FPD6 | Experimental Levels |
|----------|--------------|---------------------|
| MeV      |              |                     |
| 14       | 15           |                     |
| 13       | 16           |                     |
| 12       | 16           |                     |
| 11       |              |                     |
| 10       | 13           | 14                  |
| 9        | 14           | 13                  |
| 8        | 11           | 12                  |
| 7        | 12           | 12                  |
| 6        | 10           | 9                   |
| 5        | 8            | 7                   |
| 4        | 6            | 5                   |
| 3        | 6            | 6                   |
| 2        | 4            | 4                   |
| 1        | 2            | 2                   |
| 0        | 0            | 0                   |

FIG. 3. t=2 calculation and experimental results for T=0 states in $^{48}$Cr up to J=16
FIG. 4. \( t=2 \) calculation for \( T=0 \) states in \(^{48}\text{Cr} \) \( J=17 \) and greater.
| MeV | \(^{50}\text{Cr}\) t=2 FPD6 | T0FPD6 | Experimental Levels |
|-----|-----------------|--------|-------------------|
| 10  | 14              | 14     | 14                |
| 9   | 13              |        | 13                |
| 8   |                 |        | 12                |
| 7   | 12              | 11     | 10                |
| 6   | 11              | 10     | 9                 |
| 5   | 8               | 7      | 8                 |
| 4   | 3               |        | 1                 |
| 3   | 6               | 3      | 6                 |
| 2   | 4               | 4      | 4                 |
| 1   | 2               | 2      | 2                 |
| 0   | 0               | 0      | 0                 |

**FIG. 5.** t=2 calculation and experimental results for T=1 states in \(^{50}\text{Cr}\) up to J=14
| MeV | \(^{50}\text{Cr}\) t=2 | Experimental Levels |
|-----|---------------------|---------------------|
| 21  | FPD6 t0FPD6         |                     |
| 20  |                     |                     |
| 19  |                     |                     |
| 18  |                     |                     |
| 17  |                     |                     |
| 16  |                     |                     |
| 15  |                     |                     |
| 14  |                     |                     |
| 13  |                     |                     |
| 12  |                     |                     |
| 11  |                     |                     |

**FIG. 6.** \(t=2\) calculation for \(T=1\) states in \(^{50}\text{Cr}\) \(J=15\) and greater.
### TABLE I. $^{44}$Ti yrast B(E2) values ($e^2 fm^4$) in FPD6 and T0FPD6

| t=3 | t=3 | ratio |
|-----|-----|-------|
|     |     |       |
| FPD6 | T0FPD6 |       |
| 0 → 2 | 702.2 | 433.2 | 0.617 |
| 2 → 4 | 344.3 | 169.1 | 0.491 |
| 4 → 6 | 233.8 | 70.85 | 0.303 |
| 6 → 8 | 147.8 | 75.64 | 0.512 |
| 8 → 10 | 135.6 | 90.72 | 0.669 |
| 10 → 12 | 75.65 | 55.65 | 0.736 |

### TABLE II. $^{46}$Ti yrast B(E2) values ($e^2 fm^4$) in FPD6 and T0FPD6

| t=3 | t=3 | ratio |
|-----|-----|-------|
|     |     |       |
| FPD6 | T0FPD6 |       |
| 0 → 2 | 672.1 | 472.6 | 0.703 |
| 2 → 4 | N/A | N/A | N/A |
| 4 → 6 | 272.1 | 100.8 | 0.370 |
| 6 → 8 | 221.6 | 93.13 | 0.420 |
| 8 → 10 | 169.4 | 84.84 | 0.501 |
| 10 → 12 | 63.64 | 33.45 | 0.526 |
| 12 → 14 | 46.13 | 22.07 | 0.478 |
| 14 → 16 | 1.407 | 0.5186 | 0.369 |
### TABLE III. \(^{48}\text{Cr}\) yrast B(E2) values \((e^2f m^4)\) in FPD6 and T0FPD6

|        | t=2 FPD6 | t=2 T0FPD6 | ratio |
|--------|----------|------------|-------|
| 0 \(\rightarrow\) 2 | 892      | 691.6      | 0.775 |
| 2 \(\rightarrow\) 4 | 424.7    | 287.8      | 0.678 |
| 4 \(\rightarrow\) 6 | 345      | 169.4      | 0.491 |
| 6 \(\rightarrow\) 8 | 319.5    | 202.1      | 0.633 |
| 8 \(\rightarrow\) 10 | 238.9    | 175.9      | 0.736 |
| 10 \(\rightarrow\) 12 | 168.9    | 133        | 0.787 |
| 12 \(\rightarrow\) 14 | 138      | 120.7      | 0.875 |
| 14 \(\rightarrow\) 16 | 77.60    | 79.22      | 1.021 |
| 16 \(\rightarrow\) 18 | 1.178    | 1.471      | 1.249 |
| 18 \(\rightarrow\) 20 | 2.555    | 0.728      | 0.285 |

### TABLE IV. \(^{50}\text{Cr}\) yrast B(E2) values \((e^2f m^4)\) in FPD6 and T0FPD6

|        | t=2 FPD6 | t=2 T0FPD6 | ratio |
|--------|----------|------------|-------|
| 0 \(\rightarrow\) 2 | 761.8    | 614.6      | 0.807 |
| 2 \(\rightarrow\) 4 | N/A      | N/A        | N/A   |
| 4 \(\rightarrow\) 6 | N/A      | N/A        | N/A   |
| 6 \(\rightarrow\) 8 | 188.3    | 133        | 0.706 |
| 8 \(\rightarrow\) 10 | 46.48    | 69.12      | 1.486 |
| 10 \(\rightarrow\) 12 | 54.83    | 63.79      | 1.163 |
| 12 \(\rightarrow\) 14 | 74.58    | 80.91      | 1.085 |
| 14 \(\rightarrow\) 16 | 6.665    | 3.647      | 0.547 |
| 16 \(\rightarrow\) 18 | 86.84    | 36.68      | 0.422 |
| 18 \(\rightarrow\) 20 | 0.8343   | 1.006      | 1.206 |
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