Double Transverse-Spin Asymmetries for Drell-Yan Process in $pp$ and $p\bar{p}$ Collisions: Role of Nonleading QCD Corrections at Small Transverse Momentum

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We discuss the double-spin asymmetries in transversely polarized Drell-Yan process, calculating all-order gluon resummation corrections up to the next-to-leading logarithmic accuracy. This resummation is relevant when the transverse-momentum $Q_T$ of the produced lepton pair is small, and reproduces the (fixed-order) next-to-leading QCD corrections upon integrating over $Q_T$. The resummation corrections behave differently between $pp$- and $p\bar{p}$-collision cases and are small for the latter case at the kinematics in the proposed GSI experiments. This fact allows us to predict large value of the double-spin asymmetries at GSI, using the recent empirical information on the transversity.

The double-spin asymmetry in Drell-Yan process with transversely-polarized protons, $p^1\,p^1 \to l^+l^-X$, for azimuthal angle $\phi$ of a lepton measured in the rest frame of the dilepton $l^+l^-$ with invariant mass $Q$ and rapidity $y$, is given by ($d\omega \equiv dQ^2 dy d\phi$, $q = u, \bar{u}, d, \bar{d}, \ldots$)

$$A_{TT} = \frac{d\sigma^{1\dagger}/d\omega - d\sigma^{1\dagger}/d\omega}{d\sigma^{1\dagger}/d\omega + d\sigma^{1\dagger}/d\omega} = \frac{\Delta_T d\sigma/d\omega}{d\sigma/d\omega} = \frac{\cos(2\phi) \sum_\delta e_\delta^2 \delta q(x_1, Q^2) \delta \bar{q}(x_2, Q^2) \ldots}{2 \sum_\delta e_\delta^2 \delta q(x_1, Q^2) \delta \bar{q}(x_2, Q^2) + \ldots}, \quad (1)$$

as the ratio of products of the relevant quark and antiquark distributions, the transversity $\delta q(x, Q^2)$ and the unpolarized $q(x, Q^2)$, and the ellipses stand for the corrections of next-to-leading order (NLO, $O(\alpha_s)$) and higher in QCD perturbation theory. The scaling variables $x_{1,2}$ represent the momentum fractions associated with the partons annihilating via the Drell-Yan mechanism, such that $Q^2 = (x_1 P_1 + x_2 P_2)^2 = x_1 x_2 S$ and $y = (1/2) \ln(x_1/x_2)$, where $S = (P_1 + P_2)^2$ is the CM energy squared of the colliding protons. Thus the transversely polarized Drell-Yan (tDY) data for (1) can provide a direct access to the transversity, and it is important to clarify the role of QCD corrections in the double-spin asymmetries.

It has been shown that the NLO QCD corrections for (1) are not so significant and the resulting $A_{TT}$ is less than a few percent at RHIC, similarly to the LO estimates (see [2]). This reflects that the sea-quark region is probed at RHIC for $Q^2 \gtrsim 10$ GeV$^2$, where the denominator in (1) is enhanced with small $x_{1,2}$. Now, when the transverse momentum $Q_T$ of the final dilepton is also observed in tDY, we obtain the double-spin asymmetry at a measured $Q_T$, as the ratio of the $Q_T$-differential cross sections, $A_{TT}(Q_T) = (\Delta_T d\sigma/dQ_T d\omega)/(d\sigma/dQ_T d\omega)$.

In principle, the relevant parton distributions in this asymmetry may be controlled by the new scale $\sim Q_T$, in contrast to $Q$ in (1). The small-$Q_T$ case is important because the bulk of events is produced for $Q_T \ll Q$. In this case, the cross sections $(\Delta_T) d\sigma/dQ_T d\omega$ receive the large perturbative corrections with logarithms $\ln(Q^2/Q_T^2)$ multiplying $\alpha_s$ at each order by the recoil from gluon radiations, which have to be resummed to all orders [2]. As a result, we get ($b_0 = 2e^{-\gamma_E}$ with $\gamma_E$ the Euler constant)

$$A_{TT}(Q_T) = \frac{\cos(2\phi)}{2} \int d^2 b \ e^{ib\cdot Q_T} e^{S(b, Q)} \sum_\delta e_\delta^2 \delta q(x_1, b_0^2/b^2) \delta \bar{q}(x_2, b_0^2/b^2) + \ldots,$$  

where $S(b, Q)$ is the matrix element for the Drell-Yan process. This gives rise to $Q_T$-dependent corrections, which are important for the small-$Q_T$ data. The cross section in (2) is used to calculate the double-spin asymmetry at GSI. In this limit, the logarithms in (2) receive corrections upon integrating over $Q_T$, and reproduce the (fixed-order) next-to-leading QCD corrections up to the next-to-leading logarithmic accuracy.
where the numerator and denominator are, respectively, reorganized in the impact parameter $b$ space in terms of the Sudakov factor $e^{S(b,Q)}$ resumming soft and flavor-conserving collinear radiation, while the ellipses involve the remaining contributions of the $O(\alpha_s)$ collinear radiation, which can be absorbed into the exhibited terms as $\delta q \to \Delta_T C_{qq} \otimes \delta q$, $q \to C_{qq} \otimes q + C_{qg} \otimes g$ using the corresponding coefficient functions $(\Delta_T)C_{ij}$; there appears no gluon distribution in the numerator of (2), similarly as in (1), because of the chiral-odd nature. Using universal Sudakov exponent $S(b,Q)$ with the first nonleading anomalous dimensions in (2), the first three towers of large logarithmic contributions to the cross sections, $\alpha_s^n \ln^m(Q^2/Q_T^2)/Q_T^2$ ($m = 2n - 1, 2n - 2, 2n - 3$), are resummed to all orders in $\alpha_s$, yielding the next-to-leading logarithmic (NLL) resummation. In addition to these NLL resummed components relevant for small $Q_T$, the ellipses in (2) also involve the other terms of the fixed-order $\alpha_s$, which treats the LO processes in the large $Q_T$ region, so that (2) is the ratio of the NLL+LO polarized and unpolarized cross sections. We include a Gaussian smearing as usually as $S(b,Q) \to S(b,Q) - g_{NP} b^2$, corresponding to intrinsic transverse momentum. The integration of these NLL+LO cross sections $\Delta_T d\sigma/d\omega Q_T$, $d\sigma/d\omega Q_T$ over $Q_T$ coincides (2) with the NLO cross sections $\Delta_T d\sigma/d\omega$, $d\sigma/d\omega$, respectively, associated with $A_{TT}$ of (1); thus the NLO parton distributions have to be substituted into (2) as well as (1).

The resummation indeed makes $1/b \sim Q_T$ the relevant scale. The numerical evaluation of (2) at NLL+LO with RHIC and J-PARC kinematics has revealed (2) that, in small and moderate $Q_T$ region ($Q_T \lesssim Q$), $\mathcal{A}_{TT}(Q_T)$ is governed by the NLL resummed component and is almost constant as a function of $Q_T$, reflecting universality of the large Sudakov effects. The results show $\mathcal{A}_{TT}(Q_T) > A_{TT}$, because the denominator of (2) is not enhanced for $Q_T \ll Q$ compared with that of the corresponding NLO $A_{TT}$ of (1), and also show the tendency that $\mathcal{A}_{TT}(Q_T)$ with resummation at higher level yields the larger value. Using the NLO transversities that saturate the Soffer bound, $2\delta q(x, \mu^2) \leq q(x, \mu^2) + \Delta q(x, \mu^2)$, at a low scale $\mu$ with $\Delta q$ the helicity distribution, the NLO value of (1) at $\phi = 0$ is $\lesssim 4\%$ and $\sim 13\%$ for typical kinematics at RHIC and J-PARC, respectively, and the NLO+LO $\mathcal{A}_{TT}(Q_T)$ for small $Q_T$ using the same transversity are larger than those NLO $A_{TT}$ by about 20-30%. It is also worth noting that, for $Q_T \approx 0$, the $\delta$ integral of (2) is controlled by a saddle point $b = b_{SP}$, which has the same value between the numerator and denominator in (2) at NLL accuracy (2); combined with the almost constant behavior of $\mathcal{A}_{TT}(Q_T)$ mentioned above, the $Q_T$ region, omitting the small corrections from the LO components involved in the ellipses in (2). The saddle-point evaluation does not lose the NLL accuracy of (2); in particular, the $O(\alpha_s)$ contributions from the coefficients $(\Delta_T)C_{ij}$, e.g. those with gluon distribution in the denominator, completely decouple as $Q_T \to 0$. Remarkably (2), $b_0/b_{SP} \simeq 1$ GeV, irrespective of the values of $Q$ and $g_{NP}$. The formula (3) allows quantitative evaluation of (2) to good accuracy, and embodies the above features of $\mathcal{A}_{TT}(Q_T)$ in a compact form.

Next we discuss the $p\bar{p}$-collision case, $p^+p^- \to t^+t^-X$; here and below, the formal interchange, $\delta q(x_2) \leftrightarrow \delta q(x_2), q(x_2) \leftrightarrow q(x_2)$, for the distributions associated with the variable $x_2$ should be understood in the relevant formulae (1)-(3) for the asymmetries. Thus this case allows us to probe the product of the two quark-transversities, in particular, the valence-quark transversities for the region $0.2 \lesssim x_{1,2} \lesssim 0.7$ in the proposed polarization experiments at GSI (see e.g. (3)). When the transverse-momentum $Q_T$ is unobserved, one obtains $A_{TT}$...
of (1): for this asymmetry at GSI, the NLO \((O(\alpha_s))\) corrections as well as the higher order corrections beyond them in the framework of the threshold resummation are shown to be rather small, so that the LO value of \(A_{TT}\) at LO, which turns out to be large, is rather robust.

We now consider the QCD corrections at a measured \(Q_T\), calculating \(\mathcal{A}_{TT}(Q_T)\) of (2), (3) at GSI kinematics. The numerical evaluation of (2) using the transversity distributions corresponding to the Soffer bound, which are same as in the pp-collision case discussed above, shows [4] that the NLL resummed component dominates \(\mathcal{A}_{TT}(Q_T)\) in small and moderate \(Q_T\) region such that \(\mathcal{A}_{TT}(Q_T)\) is almost constant, with even flatter behavior than for the pp case. It is also demonstrated that \(\mathcal{A}_{TT}(Q_T)\) at NLL+LO has almost the same value as that at LL; i.e., in contrast to the pp case, the resummation at higher level does not enhance the asymmetry. We here note that \(\mathcal{A}_{TT}(Q_T)\) at LL is given by (2) omitting all nonleading corrections, i.e., omitting the ellipses, replacing \(S(b,Q)\) by that at the LL level, and replacing the scale of the parton distributions as \(b_0^2/b_0^2 \to Q^2\), so that the result coincides with \(A_{TT}\) of (1) at LO. Combined with the above-mentioned property of \(A_{TT}\) at GSI, with the large value of the asymmetry which is quite stable when including the QCD (resummation and fixed-order) corrections.

\[
\mathcal{A}_{TT}(Q_T) \simeq A_{TT},
\]

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To clarify the reason behind this remarkable difference between the \(p\bar{p}\)- and pp-collision cases, the saddle-point formula (4) is useful. The simple form of (4) is reminiscent of \(A_{TT}\) of (1) at LO, but is different from the latter, only in the unconventional scale \(b_0^2/b_0^2\). In fact, this scale, \(b_0^2/b_0^2 \simeq 1\) GeV\(^2\) \((\ll Q^2)\) at all GSI kinematics as determined by the saddle point, completely absorbs the nonuniversal effects associated with nonleading (NLL) level resummation, because \(\mathcal{A}_{TT}^{LL}(Q_T) = A_{TT}^{LO}\) as noted above. In the valence region \(0.2 \lesssim x_{1.2} \lesssim 0.7\) relevant for GSI kinematics, the \(u\)-quark contribution dominates in (3) and (2), so that these asymmetries are controlled by the ratio of the quark distributions, \(\delta u(x_{1.2}, \mu^2)/u(x_{1.2}, \mu^2)\), with \(\mu^2 = b_0^2/b_0^2\) and \(Q^2\), respectively. It is straightforward to see that the scale dependence in this ratio almost cancels between the numerator and denominator in the valence region as \(\delta u(x, b_0^2/b_0^2)/u(x, b_0^2/b_0^2) \simeq \delta u(x, Q^2)/u(x, Q^2)\) (see Fig. 3 in [4]), implying (4) at GSI; this is not the case for pp collisions at RHIC and J-PARC, because of very different behavior of the sea-quark components under the evolution between transversity and unpolarized distributions [2]. A similar logic applied to (2) also explains why \(\mathcal{A}_{TT}(Q_T)\) in pp collisions at GSI are flatter than in pp collisions as mentioned above.

Another consequence of the similar logic is that \(\delta u(x, 1\text{ GeV}^2)/u(x, 1\text{ GeV}^2)\) as a function of \(x\) directly determines the \(Q\) as well as \(S\)-dependence of the value of (4) at GSI, with \(x_{1.2} = (Q/\sqrt{S})e^{-\gamma}\). In Fig. 1, using the NLO transversity directions corresponding to the Soffer bound, the symbols “Δ” plot \(\mathcal{A}_{TT}(Q_T)\) of (2) at NLL+LO as a function of \(Q\) with \(y = \phi = 0\) and \(Q_T \simeq 1\) GeV, in the fixed-target \((S = 30\text{ GeV}^2)\) and collider \((S = 210\text{ GeV}^2)\) modes at GSI [4]. The dashed curve draws the result using (3); this simple formula indeed works well. Also plotted by the two-dot-dashed curve is \(A_{TT}\) of (1) at LO with the transversities corresponding to the Soffer bound at LO level, to demonstrate (4). The \(Q\)- and \(S\)-dependence of these results reflects that the ratio \(\delta u(x, 1\text{ GeV}^2)/u(x, 1\text{ GeV}^2)\) is an increasing function of \(x\) for the present choice. These results using the Soffer bound show the “maximally possible” asymmetry, i.e., optimistic estimate. A more realistic estimate of (2) and (3) is shown in Fig. 1 by the symbols “○” and the dot-dashed curve, respectively, with the NLO transversity distributions assuming \(\delta q(x, \mu^2) = \Delta q(x, \mu^2)\) at a low scale \(\mu\), as suggested by various nucleon models and favored by the results of empirical fit for

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transversity $\delta$. The new estimate gives smaller asymmetries compared with the Soffer bound results because the $u$-quark transversity is considerably smaller, but still yields rather large asymmetries $A_{TT}$. Based on $A_{TT}$, these results also give estimate of $A_{TT}$ of $\delta_q(x)$.

At present, empirical information of transversity is based on the LO global fit, using the semi-inclusive deep inelastic scattering data and assuming that the antiquark transversities in the proton vanish, $\delta\bar{q}(x) = 0$, so that the corresponding LO parameterization is available only for $u$ and $d$ quarks $\delta_q(x)$. Fortunately, however, the dominance of the $u$-quark contribution in the GSI kinematics allows quantitative evaluation of $A_{TT}$ at LO using only this empirical information $\delta_q(x)$: the upper limit of the one-sigma error bounds for the $u$- and $d$-quark transversities obtained by the global fit $\delta_q(x)$ yields the “upper bound” of $A_{TT}$ shown by the dotted curve in Fig. 1. Using $\delta_q(x)$, this result would also represent estimate of $A_{TT}(Q_T)$. In the small $Q$ region, our full NLL+LO result of $A_{TT}(Q_T)$, shown by “▽”, can be consistent with estimate using the empirical LO transversity, but these results have rather different behavior for increasing $Q$, because the $u$-quark transversity for the former lies, for $x \gtrsim 0.3$, slightly outside the one-sigma error bounds of the global fit $\delta_q(x)$. Thus, the asymmetries to be observed at GSI, in particular, the behavior of $A_{TT}(Q_T)$ as well as $A_{TT}$ as functions of $Q$, will allow us to determine the detailed shape of transversity distributions. Other interesting DY spin-asymmetries at GSI are the longitudinal-transverse asymmetry $A_{LT}$ and the single transverse-spin asymmetry $A_{TT}$, which are sensitive to twist-3 effects inside proton.

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References

[1] Slides: http://indico.cern.ch/contributionDisplay.py?contribId=269&sessionId=22&confId=24657

[2] H. Kawamura, J. Kodaira, H. Shimizu and K. Tanaka, Prog. Theor. Phys. 115 667 (2006); H. Kawamura, J. Kodaira and K. Tanaka, Nucl. Phys. B777 203 (2007); H. Kawamura, J. Kodaira and K. Tanaka, Prog. Theor. Phys. 118 581 (2007), and references therein.

[3] H. Shimizu, G. Sterman, W. Vogelsang and H. Yokoya, Phys. Rev. D71 114007 (2005); V. Barone, A. Caffarelli, C. Coriano, M. Guzzi and P. G. Ratcliffe, Phys. Lett. B639 483 (2006).

[4] H. Kawamura, J. Kodaira and K. Tanaka, Phys. Lett. B662 139 (2008).

[5] M. Anselmino et al., Phys. Rev. D75 054032 (2007), and in these proceedings, arXiv:0807.0173 [hep-ph].

[6] Y. Koike, K. Tanaka and S. Yoshida, arXiv:0805.2289 [hep-ph] (2008).

[7] X. Ji, J. W. Qiu, W. Vogelsang and F. Yuan, Phys. Rev. Lett. 97 082002 (2006); Y. Koike and K. Tanaka, Phys. Lett. B646 232 (2007).