CNI Polarimetry and the Hadronic Spin Dependence of pp Scattering

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Abstract

Methods for limiting the size of hadronic spin-flip in the Coulomb-Nuclear Interference region are critically assessed. This work was presented at the High Energy Polarimetry Workshop in Amsterdam, Sept. 9, 1996 and the RHIC Spin Collaboration meeting in Marseille, Sept. 17, 1996.
The interference between the Coulomb spin-flip amplitude $$\phi_5$$ and the hadronic non-flip amplitude $$\phi_1 + \phi_3$$ at small $$t$$ contributes a calculable amount to $$A_N$$, an amount that is small but large enough to be used as a practical polarimeter for colliding proton beams [1]. However, the corresponding interference between the hadronic spin-flip amplitude and the Coulomb non-flip amplitude contributes a piece to $$A_N$$ that has exactly the same shape for small $$t$$ as does the CNI piece. Thus in order for CNI to be useful as a polarimeter it is necessary that a suitably small bound be known for the hadronic spin-flip amplitude. This has been discussed in an earlier note [2] where all of the standard notation is recapitulated. Here we will extend that discussion in some small ways.

The first question is, can the size of $$\phi_5$$ be limited by measurements taken in the same experiment that is used to measure $$A_N$$, in particular the pp2pp experiment at RHIC [3]? To discuss this, we parametrize the hadronic spin-flip amplitude in terms of the non-flip amplitude as

$$\phi_h^5 = \tau \sqrt{-t/m^2} (\phi^h_1 + \phi^h_3)/2$$

(1)

In general $$\tau$$ is a complex function of $$s$$, the total energy squared, but we will assume that it is independent of $$t$$ at least for $$|t| < 0.05\text{GeV}^2$$. It may be constant for a larger region, but it seems unlikely that this will be so for $$|t|$$ much greater than 0.1\text{GeV}^2. In this region for high energy, the analyzing power is given by

$$A_N \frac{d\sigma}{dt} = \frac{\alpha \sigma_{\text{tot}} e^{mt/2}}{2m\sqrt{-t}} \left\{ (\mu - 1) - 2\text{Re}(\tau) - 2\rho \text{Im}(\tau) \right\} + 2\text{Im}(\tau) \frac{\sqrt{-t}}{m} \left( \frac{d\sigma}{dt} \right)_{\text{hadronic}}$$

(2)

From this expression we see two important features. The first is that $$\text{Re}(\tau)$$ just shifts the CNI curve up or down; it does not modify the shape at all. Thus it is impossible to infer a bound on $$\text{Re}(\tau)$$ from the shape of $$A_N$$ in the CNI region. If the best limit we have on $$\text{Re}(\tau)$$ is

$$|\text{Re}(\tau)| \leq \tau^*$$

(3)

then the precision with which the polarization $$P$$ can be measured is limited by

$$\frac{\Delta P}{P} \geq \frac{2\tau^*}{(\mu - 1)}.$$ 

(4)
In particular, a 5% measurement of $P$ requires that $\tau^* < 0.05$.

Second, the shape of the curve is evidently quite sensitive to $\text{Im}(\tau)$ because it leads to purely hadronic spin-flip in $A_N$. However, this contribution is not enhanced at small $|t|$ and so the CNI peak is not sensitive to it. (See Fig.1.) Since a priori the phase of $\tau$ is unknown, this sensitivity is not useful for constraining the hadronic contribution to $A_N$ in the CNI region. Krisch and Troshin [4] have argued that the phase of $\tau$ should not be too small. In that case, one can use data at moderate values of $t$, $0.1 < -t < 0.5$, to bound $\text{Im}(\tau)$ and hence infer a bound on $\text{Re}(\tau)$. On the contrary, if the elastic scattering is dominated by the exchange of $C = +1$ in the $t$-channel, as it would be if dominated by Pomeron and multiple-Pomeron exchange, crossing relations for small $t$ and large $s$ imply that asymptotically the amplitude is pure imaginary. This is true for the spin-flip just as for the non-flip amplitudes [3, 6]. If $|\phi_1|$ and $|\phi_5|$ have exactly the same asymptotic behaviour, then $\text{Im}(\tau) = 0$. One also learns from this argument that $\text{Re}(\phi_5)/\text{Im}(\phi_5) \propto 1/\ln s$. One
might reasonably estimate, by analogy with the non-flip amplitude, that this is of order \( \rho \); i.e. of order 0.1 and slowly falling with energy. If there is significant odderon contribution, having the same asymptotic behaviour but with \( C = -1 \) this restriction on the phase is no longer true. Indeed, the measurement of this phase would be another way of investigating the presence of the odderon in elastic pp scattering.

We have just seen that the shape cannot be used to limit \( \tau^* \). Examination of the standard reference of Buttimore, Gotsman and Leader [7] shows that the spin-flip amplitude \( \phi_5 \) contributes to three other measurable quantities (assuming always that final polarizations are not measured): the differential cross section, the two-spin asymmetry \( A_{NN} \) and the two-spin asymmetry \( A_{SL} \). In none of these is its contribution enhanced by interference with the Coulomb amplitude, and so one expects in advance that it will be difficult to get an adequate constraint from these measurements.

Buttimore [8] has worked out a constraint coming from the differential cross section which we can write in the form

\[
|\tau| < \sqrt{\frac{bm^2}{2} \left( \frac{16\pi b\sigma_{el}}{(1 + \rho^2)\sigma_{tot}^2} - 1 \right)}.
\]  

(5)

Because \( bm^2 > 10 \) even a 10% measurement of \( \Delta P/P \) would require that the combination in the parenthesis be known to better than three parts in \( 10^3 \). A 1% measurement of that combination would lead to a bound of \( |\tau| < 0.24 \) and so limit the precision of \( \Delta P/P \) to about 27%. The use of this bound requires a further analysis of how the quantities \( b, \sigma_{el}, \) and \( \sigma_{tot} \) are determined. For example, if \( \sigma_{tot} \) is determined by extrapolation of \( d\sigma/dt \) accompanied by the usual assumption of spin-independence, viz. \( \phi_1 = \phi_3 \) and \( \phi_2 = 0 \) additional uncertainties are introduced. See the talk by André Martin at the Marseille conference [9].

Rather than pursue this, let us consider the differential cross section directly. Continuing with the assumptions of Eq.(1), we have

\[
\frac{d\sigma}{dt} = \frac{1}{16\pi}(1 + \rho^2)\sigma_{tot}^2 (1 - 2|\tau|^2 \frac{t}{m^2}) e^{bt},
\]

(6)

where \( b \) is the slope of the diffraction peak in the small \( t \) region, say \( 4 \times 10^{-4} \leq |t| \leq 0.2 \text{GeV}^2 \)
as planned in the pp2pp experiment. This form yields an effective slope $b'$, where

$$
b' = b - 2\frac{|\tau|^2}{m^2} + 2\frac{|\tau|^4t}{m^4} + O(t^2).
$$

(7)

It is essential to limit the third term because the second term just shifts $b$. For statistical errors of the magnitude which are the goal of pp2pp, about $10^{-3}$, this yields at best a useless bound of $\tau^* < 0.6$, even disregarding other possible sources of $t$ dependence of $b$. Because $\tau$ enters to the fourth power, it seems hopeless to pursue this line.

The situation with $A_{NN}$ is more problematic because, in addition to $\phi_5$ it also depends on the unknown combination

$$
\text{Re}(\phi_1^*\phi_2 - \phi_3^*\phi_4).
$$

(8)

This piece is very likely to prevent a useful bound from being obtained here. If we disregard the purely hadronic contribution to this piece we find that

$$
A_{NN} \frac{d\sigma}{dt} = \sigma_{tot} \exp^{b't/2} \frac{\alpha}{4m^2} \frac{\alpha - 1}{\alpha - 1} \left\{ \left( \mu - 1 \right) \rho - 4(\rho \text{Re}(\tau) - \text{Im}(\tau)) \right\} - 2\frac{t}{m^2} |\tau|^2 \left( \frac{d\sigma}{dt} \right)_{\text{hadronic}}
$$

(9)

If the errors on the double spin asymmetry are of order $10^{-3}$ for $t$ between 0.002 and 0.05 the best bound this gives for $\tau^*$ is about 0.15. Unfortunately, rather small double flip amplitudes $\phi_2$ and $\phi_4$ can cancel this small asymmetry and destroy even this weak bound. This is illustrated in Fig.2. Here we have assumed that

$$
\phi_1 = \phi_3
$$

(10)

$$
\phi_2 = -\phi_4,
$$

(11)

and introduced the natural parametrization

$$
\phi_2 = -\delta \frac{t}{m^2} \phi_1,
$$

(12)

The amplitudes $\phi_2$ and $\phi_4$ are nearly two orders of magnitude smaller than $\phi_5$ in this region, but essentially cancel its effect, because these enter $A_{NN}$ by interfering with the principal amplitudes $\phi_1$ and $\phi_3$. 

4
It is interesting to explore the possibility of using the fact that the single spin asymmetry is proportional to $P$ while the double spin asymmetry is proportional to $P^2$. Perhaps if one could get the errors down this could be used to extract $P$? We continue using the parametrization just introduced. Then $A_{NN}$ is the only two-spin asymmetry which depends on $\phi_5$; in particular, $A_{SL} = 0$, independent of $\phi_5$. Again, it is very easy to find values of $\tau$ and $\delta$ such that $A_{NN}$ has the same shape as it would have for these parameters set to zero, and furthermore it is shifted in magnitude by the square of the factor that $A_N$ is; i.e. if we write

$$P_{\text{apparent}}(\tau) = P_{\text{true}} \frac{A_N(\tau \neq 0)}{A_N(\tau = 0)}$$  \hspace{1cm} (13)$$

then we find over the CNI region that

$$A_{NN}(\tau = 0.1, \delta = -0.01) \approx A_{NN}(0, 0) \left( \frac{P_{\text{apparent}}(0.1)}{P_{\text{true}}} \right)^2$$  \hspace{1cm} (14)$$

and so if in fact $\tau = 0.1$ and $\delta = -0.01$ while we assume that $\tau = \delta = 0$ we would infer consistent but erroneous values of the polarization from the two different measurements. (See Fig.3.)

I conclude from this analysis that it will not be possible to constrain the size of the
hadronic spin-flip amplitude using only data from the pp2pp experiment. We then ask if we can get useful constraints from other experiments.

The natural thing to do is to look at the size of $A_N$ as determined from lower energy experiments at Fermilab and at CERN, in the 1970’s. One can then try to extrapolate these measurements to higher energy by fitting their energy dependence.

A glance at the data indicates that $A_N$ is falling very fast with energy and so it is tempting to believe that the hadronic spin-flip amplitude will fall off to negligible levels by the time RHIC energy is reached. In order to test this quantitatively we have taken a collection of data from various experiments at different energy and all for $t = -0.15 \text{GeV}^2$ (or interpolated from nearby values), the smallest $|t|$ for which there is sufficient data to do this $^{[10]}$. We have tried a fit suggested by Regge poles, namely $a + b/\sqrt{p_L} + c/p_L$, where $p_L$ denotes the lab momentum for these fixed target experiments. This is shown in Fig.4. The $\chi^2$ is quite good. The relevant result is that

$$a = 0.023 \pm 0.012$$

(15)

which will be the extrapolated value of $A_N$ to very high energy. This is not very well determined; it is consistent with pure CNI which is approximately equal to 0.01 at this $t$-value. It is also consistent with $\text{Re}(\tau) = -1$! Hence it can certainly not be used to limit the
Figure 4: Energy dependence of $A_N$ at $t = -0.15\text{GeV}^2$. The best fit gives the asymptotic value $2.3 \pm 1.2$. The CNI value at 300 GeV is 1.1

magnitude of Re($\tau$) to any useful value, even if we are willing to use the CNI parametrization to values of $|t|$ this large. (I have also done a similar fit at $t = -0.3\text{GeV}^2$ but, while there is more data available there, the scatter from experiment to experiment is quite large and the $\chi^2$ is not good. The fit obtained is consistent with pure CNI.) A fit with a pure $1/p_L$ behaviour, the type suggested by the kinematic factor $\sin \theta/2$ in $\phi_5$, has a much worse $\chi^2$.

Experiment 704 at Fermilab [11] is potentially the most relevant experiment for our question. It measures $A_N$ by elastically scattering protons of known polarization in the CNI region. The errors are unfortunately rather large, but even so this experiment probably gives us the best bound we have on Re($\tau$). I have fit the data up to $|t| < .05\text{GeV}^2$ with the simple form that I have used throughout, assuming Im($\tau$) = 0 and find the best fit is within 1% of the pure CNI curve. The error is about 15%. See Fig.5. If, however, we allow a non-zero Im($\tau$) the best fit gives Re($\tau$) = $0.2 \pm 0.3$ and Im($\tau$) = $0.03 \pm 0.03$. The $\chi^2$ are about the same for the two fits. This is also consistent with pure CNI, but the errors are even larger. These are compared in Fig.6. (Akchurin, Buttimore and Penzo [12] have fit this data and
some other data over a wider $t$-range with a somewhat more complex form. They also find a large error in the determination of $\tau$.

There is an ancient “theoretical prejudice” [9] that scattering amplitudes will become spin-independent at high energy. I don’t know where that comes from; there were extensive theoretical and phenomenological studies in the 1970’s, mainly in the context of Regge theory; for some examples, see [13, 14]. These indicate no deep reason for this simplicity to occur. If there were to be one it would have to lie within the details of the strong interactions, of QCD applied at small $t$. After all, the QED spin dependence does not vanish asymptotically. There are many indications from these and more recent theoretical work that quantitatively the spin-flip amplitude becomes small; the fit to the energy dependence I showed demonstrates this, too. But how small? Small enough that we can safely conclude that $\tau$ is small enough to allow a 5% CNI measurement of the polarization? I believe this remains an open question. It is, I think, an interesting question, too. Thus the two most recent works which address the spin-flip amplitude, by Anselmino and Forte [15] and by Goloskokov and Selyugin [16] use quite different non-perturbative approaches to calculate this. The former uses an instanton generated vertex; it behaves rather like a mass insertion and leads to a small value of $\tau$ which falls with energy as $1/\sqrt{s}$. The latter uses a semi-phenomenological picture, rather...
like Pumplin and Kane [13] used years ago – effectively Regge cuts. In this case $\tau$ is also small but it does not fall with energy. Rather than use these predictions as support for CNI, it seems to me important to study them experimentally, since they represent very different physics.

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