OPTIMAL AND HEURISTIC ALGORITHMS FOR THE MULTI-OBJECTIVE VEHICLE ROUTING PROBLEM WITH DRONES FOR MILITARY SURVEILLANCE OPERATIONS

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ABSTRACT. During military operations, obtaining information on remote battlefields is essential and recent advances in unmanned aerial vehicle technology have led to the use of drones to view battlefields. However, the use of drones in military operations introduces the new problem of determining travel routes for the drones. This type of problem is similar to the well-known classical vehicle routing problem, but the main difference is its objective function. For maintenance purposes, a minimized difference in travel distances is preferred. In addition, obtaining a shorter route in terms of travel distance is important. In this research, we propose a mathematical formulation and an optimal algorithm for the problem and suggest a simple heuristic to handle the large size instance of the problem. The computational results indicate that this algorithm can solve the real-scale instances of the problem, and the heuristic exhibits good performance even when the instance size of the problem is large.

1. Introduction. In a battlefield with distinctive continuous uncertainty, the armies of many nations have long pursued the goal of obtaining information on remote battlefields in real time. Military commanders who are not located on the battlefield always want to monitor the situations as if they were on site. If commanders can clearly understand the field situation and make reasonable decisions, victory can be pursued more easily.

Some equipment has been used to achieve the visualization of remote battlefields, ranging from classical military communication equipment (VRC, PRC, etc.) to head-mounted displays for individual combatants. In recent years, due to the rapid development of unmanned aerial vehicle technology (commonly known as drones), it has become possible to operate drones in remote battlefields and share the situation in real time (Moreover, drones also can be used for missions that are too dangerous for humans).
In 2017, the Republic of Korea (ROK) army presented so-called, five major game-changers, as key forces of the future army in the days of the chief of staff Yongwoo Kim. The first game-changer is the ultra-precise high-power guided missile. The second is strategic corps with high intelligence and fatal weapons. The third is special forces who eliminate the top enemy commander. The fourth is the drone-bot battle group which combines drones and robots to perform offensive operations. Finally, the last is a warrior platform that is an advanced personal combat system [5]. Among the five game-changers, drones are the fastest replacement for surveillance missions performed by humans.

Meanwhile, the ROK army has faced the problem of population drops due to the low fertility rate. According to the data from the Ministry of National Defense (MND), the enlistable population will reduce by 30% in the 2020’s [10]. Therefore, the application of an unmanned surveillance system is an urgent issue in the ROK army, and surveillance missions using drones are considered successful [6].

![Population](image)

**Figure 1.** 20 years old male population in ROK by year (unit: 1000)

However, the replacement of surveillance personnel with drones has increased the need for drone operating personnel. Because the number of drone operating personnel is limited, this has also led to another shortage of personnel. Currently, the operating personnel are expected to manage about six to eight surveillance targets of interest by operating two drones simultaneously [6]. The synchronous operation of multiple drones has the advantage of widening areas of interest that can be monitored.

Generally, due to maintenance convenience requirements, one type of drone is used by operating personnel, and the drones are required to depart and return to the base station at the same time as much as possible. If drones depart and return to the base station at different times, then the workload of the operating personnel sharply increases. In addition, because the battery in drones must be
replaced regularly, a shorter difference in the returning times of the drones results in a longer drone operation time.

Therefore, this research aims to achieve two objectives while satisfying surveillance requirements (visiting all surveillance targets) using the allocated drones. The first objective function is constructing travel routes for the drones that minimize the difference in the drone arrival times. The second objective function is minimizing the total travel distances of the drones.

To minimize both objective function values, Pareto-based methods, decomposition methods, and the transformation method from multiple-objectives to a single objective are considered. In this research, because advances in battery technology are currently very fast, we assumed that the synchronous operation of multiple drones is a more important issue than pursuing a smaller total travel distances.

In this research, we addressed the multi-objective issue using the Pareto-based approach. We prioritized the first objective function that aims to minimize the value of \( z_{\text{max}} - z_{\text{min}} \). Afterward, the solution with the smallest objective function value in terms of the second objective function was chosen as the best solution.

We refer to this problem as a multi-objective vehicle routing problem with drones (mVRPD) throughout this paper. Our problem is similar to the famous traveling salesman problem (TSP) or vehicle routing problem (VRP) because each surveillance target is asked to visit exactly once without forming a sub-tour, and the constructed route for any drone begins and ends at the base station. The main difference is that we address the two objective functions; thus, some modifications in the objective function and constraints of the classical VRP formulation are performed.

To the best of the authors’ knowledge, this paper is the first to discuss the mVRPD. We first suggested a mathematical formulation that can be used to generate the optimal solution. We also designed an optimal algorithm to address the mVRPD based on the formulation. However, as the computational results indicate, the optimal algorithm failed to obtain a solution within a reasonable time in large size instances of the mVRPD. Therefore, we designed a simple heuristic based on the mathematical formulation to obtain the solution in a reasonable time.

This paper is organized as follows. A survey and comparison between the previous research and our work are discussed in Section 2. In Section 3, we first define the problem formally and propose a mathematical formulation. Section 4 explains the two algorithms, where one is an optimal algorithm and the second is a heuristic algorithm. Although the optimal algorithm can handle the real-scale of the problem, we propose a heuristic algorithm that can obtain the solution in a reasonable time if the instance size increases. In Section 5, we implement two algorithms and demonstrate the performance of the algorithms. We also compare the generated routes between the optimal algorithm and the drone operating personnel. Finally, Section 6 concludes the paper. Contributions of this research are summarized, and possible future research directions are suggested.

2. Literature review. Recently, as the interest in drones has increased, studies on the VRPD have been conducted. The benefits of using drones were first considered by [18], and they integrated drones into the classical VRP. They defined the VRPD as follows: given a set of vehicles equipped with a set of drones, routes are constructed to serve all customers to minimize completion time. Some recent research includes work by [3, 12, 13, 14, 15, 18, 19, 20].
In addition, recent studies on the TSP with drones (TSP-D) include those by [1, 2, 4, 7, 11, 16, 17, 21]. Recent research on the TSP-D has addressed parcel delivery in which a truck and drone are teamed up to serve a set of customers. (The truck has a large capacity but usually low speed in an urban area due to the possible traffic congestion. The drone is faster and not restricted to road networks, but its carrying capacity is limited to one or two parcels). Many variants of TSP-D exist, and many types of research are being conducted.

Most previous research has addressed the problem using heuristic and meta-heuristic algorithms. In particular, meta-heuristic algorithms cannot guarantee an optimal solution, but have been acknowledged as an efficient method to obtain a solution in a short time for the realistic applications. Recently, studies on improving the performance of these algorithms have been conducted in various fields. For example, [8] studied the flexible task scheduling problem in a cloud computing system and solved the problem by a hybrid discrete artificial bee colony algorithm. Also, [9] confirmed that the proposed improved artificial bee colony algorithm can solve the vehicle routing problem with time windows in prefabricated systems with high efficiency.

The mVRPD has two objectives and two restrictions, as described in Section 1. The first restriction is that each travel route of the drone begins and ends at the base station. The second restriction is that each surveillance target must be visited (incoming and outgoing) exactly once by the drone. The TSP and mVRPD have a common feature in that the salesman (the drone in this paper) starts from the base station and visits all cities (surveillance targets) and returns to the base station. Moreover, the VRP assumes that several vehicles exist, and each has a limited capacity to carry freight. The mVRPD addresses the situation in which several drones exist, and each drone must be used.

The restrictions of the mVRPD are similar to those of the VRP with a time window in the work by [9]. The major differences are that no limits exist for vehicle capacity, no maximum vehicle route time and time window exist, and the total travel distance for each drone should be expressed in the mathematical formulation.

3. Mathematical formulation. In this section, we present a mathematical formulation for the mVRPD, which can be used to obtain the travel routes of the drones. Before introducing our formulation, we review the two objective functions and constraints in more detail. As discussed, we assumed that one type of drone is used; thus, the speeds of the drones are the same.

The first objective function is minimizing the difference in the travel distances of the drones because drones are controlled by one person. Drones start flying simultaneously; thus, a greater difference in travel distances results in a larger difference in arrival times. A large difference in arrival times makes it difficult to manage the drones. Therefore, the operator aims to minimize the difference in travel distances so that the drones arrive at the base station at the same time as much as possible.

The second objective function is finding travel routes with the smallest total travel distances. Because drones rely on comparatively small batteries for powering flight, it is important to save the remaining capacity of the battery to save energy for a future flight. Therefore, the travel route with the smallest total travel distance is preferred.
In addition, two types of constraints exist for the problem. The first is that all drones are required to start from the base station and return to the station. The second restriction is that every surveillance target must be visited exactly once by the drone. We define the decision variables which are used in the mathematical formulation, as follows.

\[ x_{ij} = \begin{cases} 
1, & \text{if arc } (i,j) \text{ is selected to construct travel route for any drone,} \\
0, & \text{otherwise.} 
\end{cases} \]

\[ y_i = \text{cumulative travel distance from base station to surveillance target } i \]

\[ z_{\text{max}} = \text{maximum among travel distances of drones} \]

\[ z_{\text{min}} = \text{minimum among travel distances of drones} \]

Our first objective function is minimizing the difference in drone travel distances. Therefore, many representations of the difference can be used, such as deviation, variance, maximum − minimum, and so on. In this research, because we use an integer linear programming approach to address the problem, the objective function and constraints are required to take the form of a linear expression. Therefore, the first objective function is represented as \( \text{Minimize } z_{\text{max}} - z_{\text{min}}. \)

Before we represent the restrictions, we perform a small modification on the surveillance area to identify the cumulative travel distance for each drone. We assume that the surveillance area is represented as the graph \( G = (V, A) \) (where \( V \) and \( A \) are the set of surveillance targets including a base station and the set of arcs, respectively). Because drones are not restricted to traffic congestion, each pair of surveillance targets are connected by an arc. For example, if the surveillance area is given with two drones, we find the travel routes of the drones, as shown in Figure 2 (where node 1 represents a base station and nodes 2, 3, 4, and 5 indicate surveillance targets).

**Figure 2.** Travel routes when four surveillance targets and one base station are given with two drones
The travel routes of the drones are $1 \to 2 \to 3 \to 1$ and $1 \to 5 \to 4 \to 1$, respectively. Thus, the selected arcs are $x_{12}$, $x_{23}$, $x_{31}$, $x_{15}$, $x_{54}$, and $x_{41}$, and the values of $y_1$, $y_2$, $y_3$, $y_4$, and $y_5$ are 0, 2, 8, 7, and 5. Therefore, the cumulative travel distances of the two drones are 11(=2+6+3) and 11(=5+2+4), respectively.

The cumulative travel distances of the drones must be stored in the $y_i$ variables to calculate the values of $z_{\text{max}}$ and $z_{\text{min}}$. Therefore, if the travel routes of the drones can be represented as shown in Figure 3, then the values of $z_{\text{max}}$ and $z_{\text{min}}$ can be easily obtained from $y_6$ and $y_7$.

![Figure 3. Example of the graph modification procedure](image)

Therefore, we perform small modifications to the original graph as follows. We first added two (the number of drones) virtual surveillance targets, 6 and 7. These two targets perform the role of duplicated base stations. Moreover, the existing arcs $3 \to 1$ and $4 \to 1$ which indicate the returning route to the base station were replaced by arcs $3 \to 6$ and $4 \to 7$. Then, we added a new virtual surveillance target 8, which performs the role of a sink station, and two dummy arcs $6 \to 8$ and $7 \to 8$ (from the duplicated base stations to the sink station) were added with 0 arc cost. Then, according to the definition of $y_i$, the assigned values of $y_6$ and $y_7$ represent the cumulative travel distances of the two drones, and the travel distance 11 was stored in each.

We formally describe the graph modification procedure below where the surveillance area is given as $G = (V, A)$, where $|V| = m$ and $|A| = n$. Additionally, $k$ drones and nonnegative arc costs $c_{ij}$ (for $(i, j) \in A$) are given. Moreover, $V$ is a set of surveillance targets including a base station, and $A$ is a set of arcs. The base station is designated as node 1.

**Step 1:** We constructed the extended graph by adding $k$ virtual surveillance targets (nodes $m + 1$, $m + 2$, ..., $m + k$), which perform the role of duplicated base stations, and the corresponding $y_i$ variable stores the value of the cumulative travel distance of each drone.

**Step 2:** Arcs $2 \to m + 1$, ..., $2 \to m + k$, ..., $m \to m + 1$, ..., $m \to m + k$ were also added from existing surveillance targets to the newly added virtual surveillance targets, and were assigned the values of $c_{21}$, ..., $c_{21}$, ..., $c_{m1}$, ..., $c_{mk}$ as arc costs.

**Step 3:** To represent the sink station, we added a dummy surveillance target (node $m + k + 1$) to the graph. Then, we connected the added target
Finally, the augmented graph $G' = (V', A')$, where $|V'| = m + k + 1$ and $|A'| = n + (m-1)k + k$, was constructed. If we use the extended graph $G'$, the mathematical formulation of the mVRPD can be represented as follows.

$$
\text{Minimize} \quad z_{\text{max}} - z_{\text{min}}
$$

s.t. \quad \sum_{i=2}^{m} x_{i,j} = k, \quad \forall j = m + 1, m + 2, \ldots, m + k. \quad (1)

$$
\sum_{i=2}^{m} x_{i,j} = 1, \quad \forall j = 2, \ldots, m.
$$

$$
x_{i,m+k+1} = 1, \quad \forall i = m + 1, m + 2, \ldots, m + k. \quad (3)
$$

$$
\sum_{i=1(\neq j)}^{m} x_{i,j} = 1, \quad \forall j = 2, \ldots, m. \quad (4)
$$

$$
\sum_{i=2(\neq j)}^{m+k} x_{j,i} = 1, \quad \forall j = 2, \ldots, m. \quad (5)
$$

$$
y_j + M(1 - x_{i,j}) \geq y_i + c_{i,j}, \quad \forall i, j(\neq i) = 1, \ldots, m + k. \quad (6)
$$

$$
y_j - M(1 - x_{i,j}) \leq y_i + c_{i,j}, \quad \forall i, j(\neq i) = 1, \ldots, m + k. \quad (7)
$$

$$
y_1 = 0. \quad (8)
$$

$$
z_{\text{max}} \geq y_i, \quad \forall i = m + 1, m + 2, \ldots, m + k. \quad (9)
$$

$$
z_{\text{min}} \leq y_i, \quad \forall i = m + 1, m + 2, \ldots, m + k. \quad (10)
$$

$$
x_{i,j} \in \{0, 1\}, \quad \forall i, j(\neq i) = 1, \ldots, m + k + 1. \quad (11)
$$

$$
y_i \geq 0, \quad \forall i = 1, \ldots, m + k + 1. \quad (12)
$$

$$
z_{\text{max}}, z_{\text{min}} \geq 0. \quad (13)
$$

Constraints (2) and (3) indicate that $k$ drones are required to start from the base station (node 1) and return to $k$ duplicated virtual base stations (nodes $m + 1, \ldots, m + k$), respectively. Constraints (4) represent the requirement of returning to the sink station from $k$ duplicated virtual base stations for all drones. Constraints (5) and (6) indicate that surveillance targets must be visited by exactly one drone, incoming and outgoing, respectively. Constraints (7) and (8) define the decision variable $y_i$ as follows. Because the definition of $y_i$ is the cumulative travel distance from the base station to node $i$, if arc $i \rightarrow j$ is used for route construction (if $x_{ij}$ takes the value of 1), then $y_j = y_i + c_{ij}$ (where $y_j \geq y_i + c_{ij}$ is from constraints (7) and $y_j \leq y_i + c_{ij}$ is from constraints (8)). If arc $i \rightarrow j$ is not selected for route construction (if $x_{ij}$ takes the value of 0), then constraints (7) and (8) hold trivially. Constraint (9) indicates that because the cumulative travel distance of the base station itself is 0 by definition, the value of 0 is assigned to the decision variable $y_1$. Constraints (10) and (11) define the decision variables $z_{\text{max}}$ and $z_{\text{min}}$. Lastly, constraints (12), (13) and (14) show the binary and non-negativity requirements of the decision variables.

4. Optimal and heuristic algorithms. In this section, we propose an optimal algorithm and heuristic algorithm for the mVRPD. As we discussed, two objectives exist in this problem. The first objective is minimizing the value of $z_{\text{max}} - z_{\text{min}}$, and the second objective is minimizing the total travel distance of the drones. In
this research, we prioritized the first objective function, which aims to minimize the value of $z_{max} - z_{min}$. Afterward, the solution with the smallest objective function value in terms of the second objective function was chosen as the best solution.

4.1. Optimal algorithm. The mathematical formulation given in Section 3 minimizes the value of $z_{max} - z_{min}$. Therefore, several multiple optimal solutions can be generated. For example, assume that we have three drones and that the values of the travel distances are 4, 5, and 6. Moreover, assume that, the alternative travel distances, which are 2, 3, and 4, are also available. Two travel routes generate the same $z_{max} - z_{min}$ values (2=6−4 and 2=4−2). However, because the sums of the travel distances are 15 and 9, the second travel route is preferred in terms of the second objective function. Therefore, we propose the following procedure to find the solution, as illustrated in Figure 4.

![Figure 4. The procedure of the optimal algorithm for the mVRPD](image)

First, we solve the formulation optimally and store the assigned values of $y_{m+1}, \ldots, y_{m+k}$. Then, we prepare and assign the variable $Z$ with the sum of the stored values and add the constraint $\sum_{i=m+1}^{m+k} y_i \leq Z - \epsilon$ to the formulation (where $\epsilon$ is a very small positive value). The added constraint represents the sum of the travel
distances of the drones, and the constraint aims to identify a better solution in terms of the second objective function by subtracting \( \epsilon \) from \( Z \). Then, we solve the augmented formulation optimally and determine whether the solution with the same objective function value is achievable. If the same objective function value can be achieved, then we obtain a better solution in terms of the second objective function, and the process continues. Otherwise, if a larger objective function value is obtained, then the travel routes with a smaller travel distance do not exist.

The total number of iterations of the algorithm depends on the value of \( \epsilon \). If the value of \( \epsilon \) is too small, then the consumed CPU time is large. If the value of \( \epsilon \) can be set as a small amount, then our algorithm can be considered an exact optimal algorithm. We set the value of \( \epsilon \) to 1 considering the scale of the surveillance area; thus, we refer to the algorithm as an optimal algorithm.

4.2. **Heuristic algorithm.** In this sub-section, we present a heuristic algorithm that can be used to determine the travel routes efficiently. The optimal algorithm suggested in the previous sub-section requires several optimal solutions of mixed-integer linear programming (MILP) formulation, and the number of iterations can be high if the value of \( \epsilon \) is too small. Therefore, we devised a simple and straightforward heuristic that can obtain the solution in polynomial-time.

The main idea of the heuristic is simple. To solve the formulation, we drop the integrality requirements of the MILP formulation in Section 3 to modify the formulation as linear programming. Because a polynomial-time algorithm exists for linear programming problem, we efficiently obtain the solution of the relaxed formulation. However, because the solution of the linear programming relaxation can be fractional, we suggest the following procedure to obtain an integral solution.

The difference between the optimal algorithm and heuristic algorithm is that the following four steps are used to obtain the integral solution.

**Heuristic to solve the formulation**

**Step 1:** Drop the binary integer requirements of the \( x_{ij} \) variables.

**Step 2:** Solve the linear programming relaxation optimally.

**Step 3:** Find the most fractional \( x_{ij} \) variable that is close to 1, and then fix the value of the \( x_{ij} \) variable to 1(\( x_{ij} = 1 \)) and add to the formulation.

**Step 4:** Solve the augmented linear programming optimally. If no fractional \( x_{ij} \) variable exists, then stop the algorithm and pass the \( y_{m+1}, \ldots, y_{m+k} \) values to the next process. Otherwise, return to **Step 3**.

The heuristic works only when the routes from any surveillance target to other surveillance targets are possible. Otherwise, the heuristic may produce an infeasible solution. However, because drones are not restricted to congestion, a complete graph situation is acceptable.

5. **Computational results.** In this section, we report the performance of the optimal and heuristic algorithms, which were proposed in Section 4, to solve the
mVRPD. The two algorithms were implemented using the optimization software Xpress v. 7.9, and all experiments were run on Intel Core™ i5 (1.60GHz) with 8GB RAM. The running time of the algorithm was measured in seconds.

To compare the performance of the optimal algorithm with the performance of the drone operating personnel, we asked the personnel to construct the travel routes when the surveillance area and two drones are given, as in Figure 5. Comparisons of the travel routes and objective function values are given in Table 1, and the results reveal that our algorithm outperformed the route generated by the drone operating personnel. Therefore, if our algorithm can be used in military surveillance operation, the maintenance and operation waste time can be reduced dramatically.

![Figure 5. Surveillance area which is composed of seven surveillance targets](image)

**Table 1. Performance comparison between drone operating personnel and the algorithm**

|                              | Drone operating personnel | Optimal algorithm | Comparison  |
|------------------------------|---------------------------|-------------------|-------------|
| Route for drone 1            | 0→1→2→7→0                | 0→2→3→5→6→0      | -           |
| Route for drone 2            | 0→3→4→5→6→0              | 0→4→7→1→0        | -           |
| Total distance               | 20.4                      | 19.8              | 3% ↓        |
| $z_{\text{max}} - z_{\text{min}}$ | 3.1                       | 0.6               | 81% ↓       |

To simulate the mission profiles of the drones in a military operation, we generated the surveillance area which is placed at random within a 100 × 100 grid
network. The base station is located at the center point (50, 50). In this experiment, the number of surveillance targets (=number of nodes) is a parameter, and we changed the number from 4 to 10 with an increment of one.

Tables 2, 3, and 4 consider the cases in which the number of drones is two, three, and four. Here, $|V|$ denotes the number of surveillance targets, $\text{Max.}$ represents the maximum among the travel distances of the drones, and $\text{Min.}$ means the minimum among the travel distances of the drones. $\text{Total}$ reveals the sum of the travel distances, and finally, $\text{Time}$ demonstrates the consumed CPU time for obtaining the solution. Although the numbers of surveillance targets ($=|V|$) and drones are six and two in a general mission profile, we increased the numbers up to 10 and four, respectively, for performance verification purposes. The results indicate that our optimal algorithm obtained a better solution compared with the solution of the heuristic. However, as the number of surveillance targets increases, the amount of consumed CPU time in the optimal algorithm increases sharply. The CPU time of the heuristic exhibits relatively stable performance; thus, the heuristic can be used even when the instance size of the problem is large (all numerical values are rounded to the second decimal place).

**Table 2.** Computational results when the number of drones is two

| $|V|$  | Optimal algorithm | Heuristic |
|------|-------------------|-----------|
|      | $\text{Max.}$ | $\text{Min.}$ | $\text{Diff.}$ | $\text{Total}$ | $\text{Time}$ | $\text{Max.}$ | $\text{Min.}$ | $\text{Diff.}$ | $\text{Total}$ | $\text{Time}$ |
| 4    | 105.92 | 96.11 | 9.82 | 202.03 | 0.16 | 186.47 | 73.76 | 112.71 | 260.23 | 0.27 |
| 5    | 221.53 | 205.37 | 16.16 | 426.89 | 0.39 | 292.08 | 98.51 | 193.57 | 390.59 | 0.36 |
| 6    | 149.85 | 149.61 | 0.24 | 299.45 | 4.22 | 177.55 | 55.46 | 122.09 | 233.01 | 0.41 |
| 7    | 216.99 | 216.84 | 0.15 | 433.89 | 41.06 | 294.30 | 190.87 | 103.44 | 485.17 | 0.51 |
| 8    | 250.07 | 250.07 | 0.00 | 500.13 | 234.65 | 367.26 | 98.00 | 269.26 | 465.26 | 0.62 |
| 9    | 200.39 | 200.39 | 0.00 | 400.79 | 78.60 | 524.20 | 72.72 | 451.49 | 596.92 | 0.75 |
| 10   | 287.46 | 287.46 | 0.00 | 574.93 | 8.82 | 321.32 | 266.22 | 55.10 | 587.54 | 0.89 |

**Table 3.** Computational results when the number of drones is three

| $|V|$  | Optimal algorithm | Heuristic |
|------|-------------------|-----------|
|      | $\text{Max.}$ | $\text{Min.}$ | $\text{Diff.}$ | $\text{Total}$ | $\text{Time}$ | $\text{Max.}$ | $\text{Min.}$ | $\text{Diff.}$ | $\text{Total}$ | $\text{Time}$ |
| 4    | 80.55 | 63.81 | 16.71 | 212.45 | 0.16 | 80.56 | 37.58 | 42.99 | 181.95 | 0.33 |
| 5    | 110.15 | 81.65 | 28.49 | 277.04 | 0.27 | 160.84 | 18.87 | 141.97 | 264.95 | 0.43 |
| 6    | 119.65 | 116.00 | 3.65 | 355.18 | 1.44 | 178.16 | 46.69 | 131.47 | 351.18 | 0.53 |
| 7    | 133.07 | 128.67 | 4.40 | 385.43 | 9.35 | 257.85 | 91.93 | 165.91 | 459.31 | 0.61 |
| 8    | 157.83 | 157.10 | 0.72 | 472.54 | 599.82 | 325.39 | 48.37 | 277.01 | 461.24 | 0.75 |
| 9    | Fail to obtain solution within 10 minutes | 236.46 | 159.86 | 76.58 | 573.02 | 1.33 |
| 10   | Fail to obtain solution within 10 minutes | 218.13 | 134.78 | 83.35 | 504.72 | 1.50 |

6. Conclusions and future works. In this research, we considered the multi-objective vehicle routing problem with drones. When the surveillance area $G = (V, A)$ is given with $k$ drones, the first purpose of the problem is minimizing the difference in the travel distances of the drones. The second objective is minimizing the total travel distances. These objectives are due to the requirements of maintenance convenience and battery saving. We proposed a mathematical formulation to address the problem and suggested an optimal algorithm and heuristic algorithm. The suggested mathematical formulation can be used to obtain an optimal solution.
in a small instance size of the problem. The computational results indicate that our formulation outperformed the route generated by the drone operating personnel. Moreover, the computational results indicate that the optimal algorithm can obtain the solution within the real-scale problem (the numbers of surveillance targets ($V$) and drones are six and two in a general mission profile), and the heuristic can generate the solution in a reasonable time.

### Table 4. Computational results when the number of drones is four

| $|V|$ | Optimal algorithm | Heuristic |
|-----|-------------------|-----------|
|     | Max. | Min. | Diff. | Total | Time | Max. | Min. | Diff. | Total | Time |
| 5   | 103.71 | 34.99 | 68.73 | 263.60 | 0.09 | 103.71 | 34.99 | 68.73 | 263.60 | 0.22 |
| 6   | 83.45 | 66.03 | 17.42 | 308.09 | 0.19 | 99.44 | 25.06 | 74.38 | 291.21 | 0.48 |
| 7   | 90.83 | 68.00 | 22.83 | 309.91 | 0.61 | 86.70 | 44.72 | 41.98 | 272.22 | 0.59 |
| 8   | 97.02 | 93.38 | 3.63 | 381.29 | 2.72 | 190.32 | 51.23 | 139.10 | 484.29 | 0.68 |
| 9   | Fail to obtain solution within 10 minutes | 229.64 | 95.52 | 134.12 | 676.03 | 1.35 |
| 10  | Fail to obtain solution within 10 minutes | 243.52 | 59.23 | 184.29 | 635.55 | 1.55 |

However, the optimal algorithm still has room for improvement in terms of the computational time, and the heuristic requires improvement in terms of the quality of the solution. Some other good meta-heuristics, such as the artificial bee colony algorithm or ant colony optimization, are required to be applied to the problem. Furthermore, although only one type of drone is used in this paper, several types of drones will be used soon; thus, the formulation and algorithms must be enhanced.

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**REFERENCES**

1. N. Agatz, P. Bouman and M. Schmidt, Optimization approaches for the traveling salesman problem with drone, *Transportation Science*, 52 (2018), 965–981.
2. M. Dell’Amico, R. Montemanni and S. Novellani, Matheuristic algorithms for the parallel drone scheduling traveling salesman problem, *Annals of Operations Research*, 289 (2020), 211–226.
3. K. Dorling, J. Heinrichs, G. G. Messier and S. Magierowski, Vehicle routing problems for drone delivery, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 47 (2017), 70–85.
4. Q. M. Ha, Y. Deville, Q. D. Pham and M. H. Ha, On the min-cost traveling salesman problem with drone, *Transportation Research Part C: Emerging Technologies*, 86 (2018), 597–621.
5. G. Kim, What happened to the five major game-changer?, *E-daily*, (2019). www.edaily.co.kr.
6. J. Kim, N. Ahn, S. Kim, M. Kim and N. Cho, A study on the construction method of surveillance system using drone in transport command, *Korea Military Academy Technical report*, 18 (2018).
7. P. Kitjacharoenchai, M. Ventresca, M. Moshref-Javadi, S. Lee, J. M. Tanchoco and P. A. Brunese, Multiple traveling salesman problem with drones: Mathematical model and heuristic approach, *Computers & Industrial Engineering*, 129 (2019), 14–30.
8. J. Li and Y. Han, A hybrid multi-objective artificial bee colony algorithm for flexible task scheduling problems in cloud computing system, *Cluster Computing*, 23 (2020), 2483–2499.
9. J. Li, Y. Han, P. Duan, Y. Han, B. Niu, C. Li, Z. Zheng and Y. Liu, Meta-heuristic algorithm for solving vehicle routing problems with time windows and synchronized visit constraints in prefabricated systems, *Journal of Cleaner Production*, 250 (2020).
10. Ministry of National Defense (MND), *Military Reform Plan 2014~2030*, Seoul, 2014.
11. C. C. Murray and A. G. Chu, The flying sidekick traveling salesman problem: Optimization of drone-assisted parcel delivery, *Transportation Research Part C: Emerging Technologies*, 54 (2015), 86–109.
[12] S. Poikonen, X. Wang and B. Golden, The vehicle routing problem with drones: Extended models and connections, *Networks*, 70 (2017), 34–43.
[13] D. Sacramento, D. Pisinger and S. Ropke, An adaptive large neighborhood search metaheuristic for the vehicle routing problem with drones, *Transportation Research Part C: Emerging Technologies*, 102 (2019), 289–315.
[14] D. Schermer, M. Moeini and O. Wendt, Algorithms for solving the vehicle routing problem with drones, *Asian Conference on Intelligent Information and Database Systems*, 10751 (2018), 352–361.
[15] D. Schermer, M. Moeini and O. Wendt, A matheuristic for the vehicle routing problem with drones and its variants, *Transportation Research Part C: Emerging Technologies*, 106 (2019), 166–204.
[16] S. A. Vásquez, G. Angulo and M. A. Klapp, An exact solution method for the TSP with drone based on decomposition, *Comput. Oper. Res.*, 127 (2021), 105127.
[17] K. Wang, B. Yuan, M. Zhao and Y. Lu, Cooperative route planning for the drone and truck in delivery services: A bi-objective optimisation approach, *Journal of the Operational Research Society*, 71 (2020), 1657–1674.
[18] X. Wang, S. Poikonen and B. Golden, The vehicle routing problem with drones: Several worst-case results, *Optimization Letters*, 11 (2017), 679–697.
[19] Z. Wang and J. B. Sheu, Vehicle routing problem with drones, *Transportation Research Part B: Methodological*, 122 (2019), 350–364.
[20] E. Yakici, Solving location and routing problem for UAVs, *Computers & Industrial Engineering*, 102 (2016), 294–301.
[21] E. E. Yurek and H. C. Ozmutlu, A decomposition-based iterative optimization algorithm for traveling salesman problem with drone, *Transportation Research Part C: Emerging Technologies*, 91 (2018), 249–262.

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