A quantum arrow of time

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It is shown that position-momentum correlation is never decreasing and therefore it is a good candidate as a quantum arrow of time devoid of shortcomings of other proposals.

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I. INTRODUCTION

Since most fundamental laws of physics are time reversal invariant we need to refer to some basic irreversible processes in order to fix a direction of time. So, several arrows of time have been identified. The most widely used is the thermodynamic arrow based on the never decreasing entropy of isolated systems. Other arrows of time related with physical precesses are the cosmological arrow, the radiation arrow, the quantum arrow, and the weak arrow. [1] Interesting enough, the most obvious arrow of time, the sense of becoming, is perhaps the less well understood. Whether all these arrows are equivalent and lead to a unique concept of time is an open question.

The proposed quantum mechanical arrow of time is based on the irreversible state collapse occurring in a measurement. However this is not very convenient because the measurement in a quantum system is related with an observer and is one of the most controversial aspects in the foundations of quantum mechanics. In a better proposal, the quantum arrow of time can be related with the decoherence of an initially coherent entangled state. This has the inconvenience that it requires a particular class of states. In this short note an alternative quantum arrow of time well understood, observer independent and state independent is proposed, based on the never decreasing position-momentum correlation of a free particle system.

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II. POSITION-MOMENTUM CORRELATION OBSERVABLE

Given the position observable of a free particle $X$, the momentum observable $P$ is defined by the operator that generates space translations. From this definition the commutation relation $[X, P] = i \hbar$ can be derived. The position-momentum correlation is defined as

$$C = \frac{1}{2} (XP + PX) \tag{1}$$

with commutation relations

$$[X, C] = i \hbar X \quad \text{and} \quad [P, C] = -i \hbar P. \tag{2}$$

Position-momentum correlations have a simple explanation in an interpretation of quantum mechanics suggested by quantum field theory. In this interpretation we can view the “probability cloud” as a permanent creation, propagation and annihilation of virtual particles in an indefinite number making up the quantum field associated to some particle type. We can think that the virtual particles are the components of the field that have objective but ephemeral existence with position and momentum. In this view, Feynman graphs are not only mathematical terms of a perturbation expansion but represent real excitations of the quantum field.

Let us imagine then virtual components of the field created at a location at “the right” of the one dimensional distribution $\rho(x)$, that is, with a positive value for the observable $X - \langle X \rangle$. If these components are moving with momentum smaller than the mean value, that is, with negative value for $P - \langle P \rangle$ the relative motion will be towards the center and the distribution will shrink. Similarly, the components created at the left and moving to the right have the two offsets $X - \langle X \rangle$ and $P - \langle P \rangle$ with different sign, that is, their (symmetrized) product is negative.

For simplicity, let us assume that in this state we have $\langle X \rangle = \langle P \rangle = 0$ (the general state is obtained with the translation and impulsion operator). Therefore the product of the two offsets in position and momentum is precisely the correlation observable and the previous argument means that if the correlation is negative the space distribution shrinks. We can prove this with rigour: let us calculate the time derivative of the width of the distribution $\Delta^2 x = \langle X^2 \rangle$. In the Heisenberg picture, assuming a nonrelativistic hamiltonian $H = P^2/2m$, we have

$$\frac{dX^2}{dt} = \frac{-i}{\hbar} [X^2, H] = \frac{-i}{2\hbar m} [X^2, P^2] = \frac{1}{m} (XP + PX) = \frac{2}{m} C. \tag{3}$$
Taking expectation values we conclude that states with negative correlation shrink and states with positive correlation expand, as expected from the heuristic argument given above.

The momentum distribution for a free particle is time independent and if the state is shrinking, that is, with negative correlation, we are approaching the limit imposed by Heisenberg indeterminacy principle. This principle will not be violated because the correlation will not remain always negative: at some time it will become positive and the state will begin to expand. In fact, we can prove that the correlation is never decreasing in time:

\[
\frac{dC}{dt} = -\frac{i}{\hbar} [C, H] = -\frac{i}{4\hbar m} [XP + PX, P^2] = \frac{1}{m} P^2 = 2H, \tag{4}
\]

and this is a nonnegative operator. If a state is shrinking, at some later time it will be spreading. Gaussian states of this sort have been reported \[2\] in a very comprehensive paper.

Position-momentum correlation, like entropy in thermodynamics, is never decreasing and can be used to define a quantum mechanical arrow of time without recourse any particular class of states or to the controversial state collapse.

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[1] There are many books where these arrows are presented: S.F. Savitt (Ed.), *Time’s arrows today* Cambridge Univ. Press (1997); Huw Price, *Time’s arrow and Archimedes point* Oxford Univ. Press (1996); P. C. W. Davies, *The Physics of Time Asymmetry* University of California Press (1977); H. D. Zeh, *The Physical Basis of the Direction of Time* Springer Verlag (2007); etc.

[2] R. W. Robinett, M. A. Doncheski, L. C. Bassett, “Simple examples of position-momentum correlated Gaussian free-particle wavepackets in one-dimension with the general form of the time-dependent spread in position” Found. of Phys 18, 455-475, (2005).