Holographic Particle Image Velocimetry and its Application in Engine Development

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Abstract. This paper reviews Holographic Particle Image Velocimetry (HPIV) as a means to make three-component velocity measurements throughout a three-dimensional flow-field of interest. A simplified treatment of three-dimensional scalar wave propagation is outlined and subsequently used to illustrate the principles of complex correlation analysis. It is shown that this type of analysis provides the three-dimensional correlation of the propagating, monochromatic fields recorded by the hologram. A similar approach is used to analyse the Object Conjugate Reconstruction (OCR) technique to resolve directional ambiguity by introducing an artificial image shift to the reconstructed particle images. An example of how these methods are used together to measure the instantaneous flow fields within a motored Diesel engine is then described.

1. Introduction
Holographic Particle Image Velocimetry (HPIV) can be considered as a derivative of Particle Image Velocimetry (PIV) that uses coherent detection (holography) to record the phase and amplitude of the light scattered by seeding particles suspended within the flow of interest. Since the early 1960s it has been recognised that multiple exposure holographic images contain information concerning the three-dimensional displacement of moving objects. Pioneering work by Trolinger [1] and Thompson [2] demonstrated that, amongst other parameters, velocity could be measured by focusing into a double or multiple pulsed holographic image of a seeded flow using a travelling microscope. However, research into what is now called HPIV began in earnest in the early 1990s when correlation techniques were first applied to double exposure holographic images.

In 1994 Barnhart and Adrian demonstrated the capability of HPIV as a means to make detailed three-component measurements from a three-dimensional flow field [3]. In essence a stereo microscope system was used to interrogate a double exposure hologram producing a pair of two-dimensional projections of the locally reconstructed intensity field, and subsequently correlation of these projections gave the three components of fluid velocity. Following this work many groups have considered holographic recording geometries and data analysis techniques for HPIV and more recently digital holography has been considered. A summary of this work is presented in excellent review articles by Hinsch [4,5].

In 1992 we reported an HPIV method that differs fundamentally from others in that it correlates the complex amplitude rather than the intensity of the reconstructed images [6]. At this time we recognised that optical correlation techniques were capable of producing a three-dimensional optical field that was proportional to the autocorrelation of the complex amplitude of the reconstructed field.
HPIV using complex correlation analysis was further enhanced by Barnhart who proposed Object Conjugate Reconstruction (OCR) as a means to introduce a known image shift, and thereby resolved the directional ambiguity problem[7].

In the past decade we have shown that the complex amplitude method is inherently tolerant to aberrations [8], can be used to measure the velocity of solid surfaces and surrounding fluid simultaneously [9], and can measure sub-wavelength displacements regardless of particle size [10]. With the huge advances in affordable computing, it is now more cost effective to compute the correlation numerically, however, the advantages of complex correlation analysis remain.

In this paper we review the basic theory of complex amplitude correlation and object conjugate reconstruction and present our latest refinements to the technique with a view to measuring fluid velocity within the thick cylinder walls of an optical diesel engine.

2. Complex Amplitude Correlation

We begin our analysis by considering the propagation of optical fields in three-dimensional space. Since the holographic recordings of HPIV are generally of high numerical aperture (NA) we use a wave-vector description of the (scalar) optical field rather than the two-dimensional approach normally used in Fourier Optics [11]. Accordingly, a complex field $U(r)$ at a position vector $r = (r_x, r_y, r_z)$ can be decomposed into it’s spectrum of plane-wave components $S(k)$ defined by the three-dimensional Fourier transform,

$$S(k) = \int_{-\infty}^{\infty} U(r) \exp(-2\pi j k \cdot r) dr$$  \hspace{1cm} (1)

where $k = (k_x, k_y, k_z)$ and $dr$ conventionally denotes the scalar quantity $dr_x dr_y dr_z$. Clearly the field at any point in space can be found by inverse transformation, and is defined as,

$$U(r) = \int_{-\infty}^{\infty} S(k) \exp(2\pi j k \cdot r) dk$$  \hspace{1cm} (2)

We note that for a monochromatic system $|k| = 1/\lambda$ where $\lambda$ is the wavelength.

In this way any monochromatic optical field propagating in an linear isotropic homogeneous medium (LIH) can be decomposed into a plane wave spectrum in wave-vector space that is defined on the surface of a sphere of radius $1/\lambda$. In solid state physics this sphere is usually referred to as the Ewald sphere [12]. By considering propagating monochromatic optical fields as spherical shells in k-space it is possible to apply the usual tools of linear signal processing theory (convolution, correlation, power spectrum analysis etc) to the data stored in three-dimensional holographic images.

Let us consider two optical fields $U_1(r)$ and $U_2(r)$, the first and second exposures of a double exposure hologram respectively, made using a standard recording and reconstruction geometry such as that shown in figure 1.

![Figure 1. Holographic recording and reconstruction.](image-url)
Further let us suppose that the object is subject to a uniform displacement between exposures such that $U_2(r)$ is an identical but shifted version of $U_1(r)$ and can be written,
\[
U_2(r) = \exp(j\phi)U_1(r - \vec{s})
\]
where $\phi$ is a phase constant and the three-dimensional displacement vector is $\vec{s}$. Subject to normal holographic practice, the complex amplitude, $U(r)$, of the double exposure reconstructed field can be written,
\[
U(r) = U_1(r) + \exp(j\phi)U_2(r - \vec{s})
\]
Using the Fourier shift theorem, the k-space representation of this field is given by,
\[
S(k) = \int_{-\infty}^{\infty} U(r) \exp(-2\pi jkr) dr
\]
\[
= S_1(k)(1 + \exp[j(2\pi k \cdot \vec{s} - \phi)])
\]
where $S_1(k)$ is the spectrum of the reconstructed field due to the first exposure. The power spectrum, $P(k)$, is given by,
\[
P(k) = S(k)S^*(k)
\]
\[
= 2P_1(k)(1 + \cos(2\pi k \cdot \vec{s} - \phi))
\]
where $P_1(k) = |S_1(k)|^2$. Finally we define the autocorrelation, $R(r)$ of the reconstructed field,
\[
R(r) = \int_{-\infty}^{\infty} P(k) \exp(2\pi jkr) dk
\]
\[
= R_1(r)*[2\delta(r) + \exp(-j\phi)\delta(r - \vec{s}) + \exp(+j\phi)\delta(r + \vec{s})]
\]
where $*$ denotes convolution and $R_1(r)$ is the autocorrelation of the reconstructed field due to the first exposure.

To illustrate the power spectrum and autocorrelation in three-dimensions, we consider the case of a diffraction limited reconstruction of point object. We define the reconstruction of a unit amplitude point object as a uniform distribution on the Ewald sphere such that,
\[
S_1(k) = \delta(|k| - 1/\lambda)
\]
Performing the necessary integration we find,
\[
U_1(r) = \int S_1(k) \exp(2\pi jkr) dk
\]
\[
= \frac{4\pi^2}{\lambda^2} \sin c\left(\frac{2\pi r}{\lambda}\right)
\]
Similarly the autocorrelation, $R_1(r)$ is,
\[
R_1(r) = \int U_1(n)U_1^*(n-r) dn
\]
\[
= \frac{4\pi^2}{\lambda^2} \sin c\left(\frac{2\pi r}{\lambda}\right)
\]
It is interesting to note that function $\text{sinc}(x)$ is the sinc function $\sin(x)/x$ that is normally associated with the far-field diffraction pattern of a one-dimensional slit. In three-dimensions it is a radially symmetric sinc function, that defines both the reconstructed particle image and its autocorrelation. A slice through the field resulting from a reconstruction of two ideal particles separated by approximately 3 wavelengths is shown in figure 2.
For the case of a double exposure, monochromatic, reconstruction of an ideal point source we note that the power spectrum given by equation 6 is a set of cosinusoidal fringes defined in k-space upon the surface of the Ewald sphere. This is illustrated in figure 3 for the case of particle images separated by approximately three wavelengths. Although the fringes appear circular in perspective, equation 6 shows the fringes are regularly spaced (in this case in the \( k_z \) direction) with a spatial frequency proportional to the particle displacement vector \( \mathbf{s} \). For the case of the autocorrelation defined by equation 7, three peaks with a radially symmetric sinc form are present as illustrated in figure 4. A central peak at the origin and two signal peaks at positions defined by the displacement vector \( \mathbf{s} \). Although the particle displacement could be deduced from either the power spectrum or the autocorrelation, identification and location of the signal peaks in the complex correlation of holographic reconstructions provides a more robust method to measure particle displacements in three-dimensions.

\[
\begin{align*}
\text{Figure 2. Ideal particle reconstruction.} & \quad \text{Figure 3. Power spectrum.} \\
\text{Figure 4. Autocorrelation.} & \quad \text{Figure 5. Measuring the Power Spectrum.}
\end{align*}
\]

In a practice the power spectrum, \( P(k) \), can be measured in a straightforward manner (over a finite portion of k-space) since in essence it is a decomposition of the field into its plane wave components. Consequently, it can be deduced from the intensity, \( I(x,y) \), recorded in the back focal plane of a camera where plane waves are brought to focus as illustrated in figure 5. However, we need to consider the geometric mapping of a plane wave defined by a wave vector, \( \mathbf{k} \), onto a point \((x,y)\) in the focal plane. If vignetting and obliquity effects are neglected the k-space power spectrum is given by,

\[
P \left( \frac{x}{\lambda(x^2 + y^2 + f^2)^{1/2}}, \frac{y}{\lambda(x^2 + y^2 + f^2)^{1/2}}, \frac{f}{\lambda(x^2 + y^2 + f^2)^{1/2}} \right) = I(x, y) \quad (11)
\]
where \( f \) is the focal length of the objective. In this way we have an expression for \( P(k) \), and the three-dimensional autocorrelation field can be calculated by direct application of equation 7.

In this outline, the practical case has been simplified considerably. An ideal hologram was assumed, that records the whole of the scattered field and hence forms a complete record of the scattered field in k-space. In addition we have only considered the reconstructed images from a double exposure recording of a single particle. A practical recording would contain many particle images (because velocity measurements are required throughout the flow field) and a hologram of finite aperture would be used. A thorough analysis of HPIV must therefore include the effect of apertures in both the reconstruction space (to isolate the field due to individual particles) and in k-space (resulting in point images that depart from the ideal radial sinc functions). Nevertheless, it remains apparent from the simplified analysis presented here, that the three-dimensional autocorrelation of the complex amplitude describing a holographic reconstruction field can be calculated from a straightforward measurement of the intensity distribution in the back focal plane of a convex lens.

It is noted from this analysis, that a single autocorrelation does not provide the sense (or direction) of the particle displacement. In order to resolve this ambiguity, Barnhart [7] proposed the method of Object Conjugate Reconstruction (OCR), a technique that essentially introduces a constant lateral shift (image shift) to the reconstructed fields. In the following section we outline the OCR method.

3. Object Conjugate Reconstruction (OCR)

Object Conjugate Reconstruction (OCR) was proposed principally as a means to introduce an image shift during reconstruction, however the technique has several other advantages over the basic reconstruction method shown in figure 1. In its original form, the OCR technique was based upon a reflection hologram geometry that was placed in close proximity to the flow to be measured, increasing the numerical aperture (NA) to give high-resolution three-component velocity measurements. However, the reflection geometry required a Holographic Optical Element (HOE) to correct for distortion introduced by the window which provided optical access to the flow. In addition, the aperture of the reconstructed wave front was partially obstructed by the fibre probe, reducing the signal to noise ratio of the process. In fact, neither the HOE nor the reflection geometry are fundamental requirements of OCR, and for this reason, the OCR holographic geometry has been greatly simplified for our engine studies outlined in section 4.

![Figure 6a](OCR recording)
![Figure 6b](OCR reconstruction)

The essential requirement of OCR is a holographic recording of the scattered field in k-space (ie a Fourier or far field recording). It is convenient (but not essential) to make these recordings with two plane reference beams with a small relative tilt as shown in figure 6a). The convex lens in this configuration ensures that individual plane wave components in the scattered fields are mapped onto distinct points in the plane of the hologram as defined by equation 11. Rather than reconstruct using a reference beam, OCR uses a diverging wave from a fibre optic probe that is placed relative to the holographic plate, at a place of interest as shown in figure 6b). The effect of the developed hologram is to perform a multiplication in k-space and is defined as follows.
In a similar manner to the previous analysis let the scattered fields at the time of each exposure be $U_1(r)$ and $U_2(r)$ and the corresponding k-space spectra be $S_1(k)$ and $S_2(k)$. The reference waves are tilted plane waves in the plane of the hologram in this case generated by a diverging wave in the front focal plane of the convex lens. In k-space, the reference waves, $P_1(k)$ and $P_2(k)$, can therefore be written as unit amplitude linear phase ramps,

\[ P_1(k) = \exp(j2\pi k \cdot r_1') \]  
\[ P_2(k) = \exp(j2\pi k \cdot r_2') \]

where $r_1'$ and $r_2'$ are the origins of the first and second exposure reference waves respectively. According to normal holographic practice, the developed hologram will have a transmission function that will contain the terms,

\[ \tau(k) = S_1^T(k)\exp(j2\pi k \cdot r_1') + S_2^T(k)\exp(j2\pi k \cdot r_2') \]

The diverging wave from the fibre optic probe can also be treated as an ideal source and in k-space the probe beam will have a similar form to each reference beam. In this way the probe beam can be written,

\[ P(k) = \exp(j2\pi k \cdot p) \]

where $p$ is the position of the probe. As mentioned previously, the probe beam is multiplied by the transmission function of the hologram, giving the reconstructed field represented by $S(k)$ in k-space,

\[ S(k) = P(k)\tau(k) = S_1^T(k)\exp(j2\pi k \cdot r_1' + p) + S_2^T(k)\exp(j2\pi k \cdot r_2' + p) \]

Finally we transform back to Cartesian space. Physically this is done using the second convex lens. In fact a second forward Fourier transform is performed which results in an inverted co-ordinate system specified by position vector $r' = (r_x', r_y', r_z')$ as shown in figure 6b). Accordingly we find,

\[ U'(r') = \int S(k)\exp(-2\pi j k \cdot r') dk \]

\[ = U_1^T(r' - p - r_1') + U_2^T(r' - p - r_2') \]

Equation 17 shows that the double exposure reconstruction consists of two fields that are the complex conjugates of the first and second exposure scattered fields. The positions of these fields becomes apparent if the co-ordinate system is redefined once again such that $r'' = r'' - r_1'$, to give,

\[ U''(r'') = U_1^T(r'' - p) + U_2^T(r'' - p - (r_2' - r_1')) \]

The first term in equation 18 is identified as a real image of the particles at the probe position, at the time of the first exposure. Similarly, the second term is a real image of the particles at the probe position at the time of the second exposure, with an additional shift equal to $(r_2' - r_1')$. This is image shift is defined by the relative tilt of the reference waves and can be chosen arbitrarily. If it is greater than the particle displacement then the latter can be determined without ambiguity.

In this analysis we have demonstrated the key features of the OCR method as a method to introduce an image shift into the holographic reconstruction process. It has the additional advantage that the reconstructed images are found at a fixed position in space and the hologram can be interrogated using a light and easily positioned fibre optic probe. In practice the “4F” configurations of figures 6a) and 6b) are complex and a geometry with without lenses (or HOEs) is possible. In simplified geometries, however, care must be taken to ensure that the power spectrum of the reconstructed images is correctly measured without probe dependent shift or distortion. In the following section a practical OCR geometry is discussed with reference to these issues.

4. HPIV in Engine Development

Much of our recent work has concerned the application of the HPIV techniques described above to the measurement of transient flows within concentric glass cylinders in optical Diesel engines. Figure 7.
shows an optical Diesel engine with cylinder head removed. As optical windows, these cylinders introduce significant aberration since they are both thick and have large curvatures. In this instance HPIV has considerable advantage over planar imaging techniques in that the gross aberrations are greatly reduced in the reconstruction process. Furthermore, by correlating the complex amplitude, rather than the intensity of the reconstructed images, residual aberrations have no effect [8].

**Figure 7.** This photograph shows an optical Diesel engine with the cylinder head removed. Optical access is through two concentric cylinders. The outer cylinder must be thick to withstand the high pressures incurred within a Diesel engine. The inner cylinder reciprocates with the piston and defines the extent of the combustion chamber.

An enhanced OCR geometry applied to the problem of measuring fluid flows within a diesel engine is shown in figure 8a). A double exposure transmission hologram is orientated so that two collimated reference beams enter from the same side as the scattered light from particles suspended in the flow. Although this is not strictly a Fourier hologram, the area of interest in the flow is small (approximately 100μm) and the holographic plate is effectively records the far-field diffraction from this region.

**Figure 8a).** OCR recording in practice

Reconstruction of the developed hologram takes place using a probe consisting of a single mode optical fibre with a lens arrangement to increase the divergence (NA). The probe is positioned at the desired measurement location with respect to the holographic plate as shown in figure 8b). The small relative tilt between the collimated reference waves causes an image shift of 15-20μm in the reconstructed images that are relayed to the sampling aperture. In this way, the fibre probe determines where (relative to the hologram) measurements are made and the size of the aperture and numerical aperture of the hologram weight the effect of light scattered by particles in the flow field. Finally, a second convex lens and CCD is used to measure the power spectrum of the field transmitted by the sampling aperture.

In terms of ray optics the power spectral density can be considered to be the power radiated in a finite bundle of rays propagating in the direction of the wave vector \( \mathbf{k} \). In the absence of any windows the plane wave components of the scattered field propagate to infinity with no deviation. However, it is clear that thick optical windows cause significant deviation of these components as illustrated in
figure 9. It can be seen that rays propagating in the same direction (with identical wave vectors), but from particles in two different locations, are deflected by different amounts by the thick cylindrical window shown, and, arrive at different position on the CCD array. In order to determine particle displacement it is therefore necessary to calculate the mapping between each scattered wave vector in the engine cylinder and its image at the CCD array, as a function of probe position. We have achieved this k-space mapping using ray tracing as described elsewhere[10].

![Figure 9. The k-space mapping problem](image1)

![Figure 10. Measured velocity field](image2)

A typical result showing the flow at the end of the compression stroke is shown in figure 10. Further work will provide a statistical analysis of this and other similar measurements to find the dominant flow patterns.

5. Conclusion and Discussion
In this paper we have used a three-dimensional formulation of scalar diffraction theory to describe the principles of HPIV using complex amplitude correlation analysis and object conjugate reconstruction. In essence a hologram records the three-dimensional field scattered from particles seeding the fluid flow of interest. If the particles move the field scattered from each particle will be displaced similarly in three-dimensional space. It follows that the displacement of a given particle or group of particles can be determined by measuring the local shift in the scattered field and spatial correlation analysis is the most suitable (linear) tool for this purpose. The theory here shows that the method we refer to as complex correlation analysis is exactly this operation.

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