Aberration analysis of AOTF-based stereoscopic spectral imager using optical design software

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Abstract. Stereoscopic spectral imagers using acousto-optic tunable filters (AOTF) provide high-resolution narrow band images acquired from two viewpoints with different polarization in arbitrary spectral intervals, which allows obtaining three-dimensional hyperspectral models of inspected objects for many applications. We discuss modeling of acousto-optic (AO) cell for optical system design and introduce a program module for ray tracing through AO cell compatible with Zemax optical design software. A detailed study of the optical aberrations that limit the image quality in two AOTF-based stereoscopic systems implementing simultaneous AO diffraction of two differently polarized beams in single AO cell is presented. This approach may be used to design various AOTF-based imaging systems, but the limitations of ray tracing analysis should be considered.

1. Introduction

Spectral imaging devices are widely used for visualizing and analyzing the two-dimensional structure of objects in separate spectral intervals \cite{1}. Spectral filtering allows us to work with images exactly in those wavelength intervals where the physical, chemical and other properties of the analyzed objects are most markable. To create such a spectral imager, acousto-optic tunable filters (AOTF) that do not have moving parts and allow fast arbitrary spectral access, provide high spectral resolution (up to 0.1 nm) and allow filtering the light beams that form the image with high spatial resolution (up to 1000×1000), are the most suitable \cite{2}. Stereoscopic spectral imagers simultaneously acquire two images of object from different viewpoints (and possibly with different polarizations) which is necessary for many applications where three-dimensional model of object and distribution of spectral properties should be analyzed jointly \cite{3–5}.

Spectral imaging applications require that Bragg phase matching condition $k_d = k_i + q$ for the wave vectors of ultrasound $q$, incident $k_i$ and diffracted $k_d$ light must be approximately satisfied in a wide range of incident light angles $\theta$. This wide-aperture configuration of acousto-optic (AO) interaction \cite{6} is achievable when the tangents to the wave surfaces for incident and diffracted light are parallel (figure 1(a)). This condition for anisotropic $e\rightarrow o$ and $o\rightarrow e$ AO diffraction in a uniaxial crystal may be written as the relations between the incident angles of light $\theta_e$, $\theta_o$ and the sound angle $\gamma$ (also called a crystal cut angle) in the polar plane $xz$ \cite{7, 8}. The dependencies of $\theta_e$ and $\theta_o$ on $\gamma$ for paratellurite ($\text{TeO}_2$) crystal (the most widely used uniaxial birefringent
Figure 1. Wave diagram (a) and dependence of incident light angle $\theta$ on crystal cut angle $\gamma$ (b) for anisotropic $e\rightarrow o$ (blue) and $o\rightarrow e$ (red) wide-aperture AO diffraction in TeO$_2$. Optical scheme of AO stereoscopic spectral imager using the same diffraction mode for both beams (c) and different modes (d). Ob — inspected object, L1–L3 — lenses, D1, D2 — aperture stops, AOC — acoustic-optic cell, P1–P4 — polarizers, Im1, Im2 — image planes and sensors.

crystal for AOTF) are shown in figure 1(b). For TeO$_2$ $x$ and $z$ axes correspond to [110] and [001] axes. The shape of the curves and the extreme (maximum) value $\gamma_{\text{ext}}$ of the sound angle are defined by the ordinary $n_o$ and extraordinary $n_e (\theta_e)$ refractive indices of the crystal, which depend on the incident light wavelength $\lambda_0$ in vacuum. Thus, for each crystal cut angle $\gamma$, except 0, $\gamma_{\text{ext}}$ and the crossing point of the curves, wide-aperture AO diffraction is possible for four incident light angles ($\theta_{e1}$, $\theta_{o1}$ in near-axis diffraction mode and $\theta_{e2}$, $\theta_{o2}$ in far-off-axis diffraction mode [9]) and the related diffracted light angles ($\psi_{o1}$, $\psi_{e1}$ and $\psi_{o2}$, $\psi_{e2}$).

The straightforward implementation of AOTF-based stereoscopic spectral imager is to use a two-channel optical system, in each channel of which AOTF is installed [10]. However, this approach requires using a pair of AO cells with identical properties and keep them operating in the same regime. Simultaneous AO diffraction of ordinary and extraordinary polarized incident light waves in a stereoscopic spectral imager with one AO cell and one piezotransducer may be implemented in two ways: using the same diffraction mode for both beams ($\theta_{e1}$, $\theta_{o1}$, see figure 1(c)) or different ones ($\theta_{e1}$, $\theta_{o2}$ or $\theta_{e2}$, $\theta_{o1}$, see figure 1(d)) [8]. Optical design of these systems is rather complicated due to off-axis position of aperture stops D1, D2 for front lens L1 and chromatic shift and distortion introduced by refraction and diffraction in AO cell [11]. Popular optical design software (Zemax/OpticStudio, CODE V, etc.) provide powerful tools for aberration analysis and optimization of optical systems, but do not include modules for AO
devices. This problem may be solved using the extension functionality of Zemax or CODE V and user-provided program libraries (DLL) for ray tracing through AO cell [11–13].

In this paper, we present the improved ray tracing module for AO cell compatible with Zemax and demonstrate its capabilities for stereoscopic spectral imaging system design. The example layouts, details of optical design in Zemax and aberration analysis are provided for both schemes shown in figure 1. Finally, we briefly discuss limitations of our module and Zemax for the design of AOTF-based imaging system.

2. AOTF model for optical design software
The mathematical model of AO cell for ray tracing consists of three surfaces: input facet, intermediate diffraction plane and output facet [11–14]. The ray tracing for e→o case through these surfaces includes the refraction of e-wave (indicated by (1) in figure 2(a)), the e→o diffraction ((2) in figure 2(a)) and the refraction of o-wave ((3) in figure 2(a)). The last refraction is modeled using standard Zemax surface, whereas surfaces (1) and (2) are implemented as user-defined ones using the developed DLLs.

The object coordinate system $x_{ob}$,$y_{ob}$,$z_{ob}$ shown in figure 2 coincides with global coordinate system as it is usually defined in Zemax. The orientation of crystal coordinate system $xyz$ relative to $x_{ob}$,$y_{ob}$,$z_{ob}$ is defined by angle $\theta$. In order to simplify the description in this paper, we assume that the meridional plane $y_{ob}$,$z_{ob}$ and the polar plane $xz$ coincide and that all three surfaces are flat and parallel to $x_{ob}$-axis, however, the developed module does not have these limitations. The tilt angles for the input and output facets are $\alpha$ and $\beta$ correspondingly.

The wave vector $k_{e}$ and the light propagation (group velocity) vector $s_{e}$ after the refraction on surface (1) are calculated for given vector $k_{e}'$ using the equations provided in reference [15]. Zemax traces the ray between surfaces (1) and (2) along $s_{e}$, so the optical path length on surface (2) is corrected by the developed module taking into account the direction of $k_{e}$ and phase change introduced by diffraction. The position of surface (2) is approximately at the center of piezotransducer. Its tilt angle $\zeta$ depends on cut angle $\gamma$ and acoustic walk-off angle in TeO$_2$ [16]

$$\zeta = \arctan \left( \frac{(V_{001}/V_{110})^2 \tan \gamma}{1} \right) + \theta - \pi/2,$$  

where $V_{001}$ and $V_{110}$ are the sound phase velocities along the corresponding axes. The direction of wave vector $k_{e}$ for o-wave after diffraction and the transmission coefficient are calculated according to reference [14]. The ultrasound frequency for these calculations is determined by $\lambda_{0}$ and the specified direction of $k_{e}$ for which the phase matching condition is exactly satisfied, the length of AO interaction is user-defined parameter. Consequently, all rays with other directions of $k_{e}$ have the transmission coefficient less than 1. Usually, the specified direction of $k_{e}$ corresponds to local $z_{ob}$-axis as shown in figure 2.

The ray tracing for o→e case is similar, but the order of surfaces is reversed (see figure 2(b)). The correction of optical path length between surfaces (2) and (1) caused by mismatch of $k_{e}$ and $s_{e}$ is done on surface (1). The optical layout exported from Zemax for four possible cases of e- and o-wave propagation is demonstrated in figure 2(c). We should note that the directions of $s_{e}$ and $k_{e}$ coincide for wide-aperture geometry of AO interaction, so the plotted rays do not change their direction on surface (2).

3. Optical system design and aberration analysis
The optical design of two variants of AOTF-based stereoscopic system shown in figures 1(c) and 1(d) was performed in Zemax in the following order. At the first stage, lenses L1–L3 were modeled by paraxial components and the required geometry of AO cell was defined including all facet angles. Next, lenses L1–L3 were designed and optimized separately taking into account
Figure 2. Three-surface model for ray tracing through AO cell in \(e \rightarrow o\) (a) and \(o \rightarrow e\) (b) cases. The optical layout exported from Zemax for four possible cases of extraordinary and ordinary ray tracing using the developed module (c).

the aperture stop positions defined previously. Finally, the system was assembled, analyzed and optimized.

The size of the inspected object \(O_b\) was 12×16 mm\(^2\), it was set up in the front focal plane of \(L_1\). The focal distances of \(L_1\), \(L_2\) and \(L_3\) were 100 mm, 30 mm and 30 mm correspondingly. Thus, the sizes of images \(I_m1\) and \(I_m2\) corresponded to \(1/3``\) sensor (3.6×4.8 mm\(^2\)). The diameter of aperture stops was 8 mm, the distance between their centers was 14 mm. Optimization of the optical systems was performed for spectral range \(\lambda_{\text{min}} = 0.45 - 0.9\ \mu m\), the primary wavelength was chosen as \(\lambda_{\text{prim}} = 0.6328\ \mu m\).

Variant 1 implements the scheme shown in figure 1(c) where both beams are in near-axis diffraction mode and the cut angle \(\gamma\) is 7\(^\circ\). The input facet of AO cell for o-wave is perpendicular to the axial ray (along \(z_{ob}\)-axis), so \(\alpha_{oe} = 0\). The tilt angle \(\alpha_{eo}\) of this facet for e-wave is optimized using Zemax to make the direction of \(s_e\) parallel to \(z_{ob}\)-axis and \(k_o\). This condition provides wide-aperture diffraction geometry for both beams. The tilt angle \(V\) of the output facet can be chosen either to make the output rays in air parallel to the input ones (i.e. \(k'_e \parallel k''_o\) and \(k'_o \parallel k''_e\)) or to minimize chromatic shift introduced by the AO cell. In the last case, \(\beta\) is optimized using Zemax to force parallel directions of the output rays in air for \(\lambda_{\text{min}}\) and \(\lambda_{\text{max}}\) (i.e. \(k''_o(\lambda_{\text{min}}) \parallel k'_e\) and \(k''_o(\lambda_{\text{max}}) \parallel k'_o\)). The optical layout exported from Zemax for achromatic choice of \(\beta\) is shown in figure 3(a). The values of angles for parallel and achromatic variants are given in table 1. In order to provide enough space necessary for polarizers, lenses and sensors, the mirror prism was placed after the AO cell and its angles \(\xi_{oe}\) and \(\xi_{eo}\) were calculated to make the axial rays parallel to \(y_{ob}\)-axis for both channels. The lenses \(L_2\) and \(L_3\) are identical. The rear facets of polarizers \(P_1\) and \(P_2\) act as aperture stops, so actual entrance pupils are located approximately 135 mm from the object and the distance between their centers (the base distance of the stereoscopic system) is about 19 mm.

Variant 2 corresponds to the scheme shown in figure 1(d) with near-axis diffraction mode for e-wave and far-off-axis diffraction mode for o-wave, the cut angle \(\gamma\) is equal to 12\(^\circ\). At smaller angles \(\gamma\), the diffraction efficiencies in the stereoscopic channels differ significantly. In this case,
Figure 3. The optical layout of AOTF-based stereoscopic imager using the same diffraction mode for both beams (a, Variant 1) and different ones (b, Variant 2). The colors of rays indicate the object point position, the rays for $e \rightarrow o$ and $o \rightarrow e$ channels are plotted with solid and dashed lines correspondingly. The axial rays for both channels are bold blue dash-and-dot lines. The incident light angles $\theta$ are shown separately on $xz$-diagrams.

Using the refraction on the input facet to provide the desired AO interaction geometry is not optimal, so we added TeO$_2$ prisms to the AO cell as shown in figure 3(b). The value of $\eta_{oe}$ angle is 23.599°. Again, the output facet angles $\beta$ may be chosen for parallel and achromatic variant. In the last case, the optical components after the AO cell and the image planes are tilted relative to the parallel variant, the corresponding angles are $\xi_{eo}$ and $\xi_{oe}$.

The image quality of the designed systems was compared using the square root of the mean square (RMS) of monochromatic transverse ray aberrations from the centroid (RMS spot radius).
The RMS values calculated in Zemax for the achromatic choice of $\beta$ are presented in figure 4 for several object points along $x_{ob}$ and $y_{ob}$ axes and several wavelengths. The changes of spot size are mainly characterized by field curvature and longitudinal chromatic aberration introduced by the lenses. Thus, the best quality is achieved for 0.5—0.65 $\mu$m in the center of the image and for 0.75—0.9 $\mu$m at the edge of the image. The impact of AO cell is negligible since it works in almost parallel beams, so the values for the parallel choice of $\beta$ are almost identical to the presented ones.

Next, we separately analyzed the chromatic shift for chief rays calculating transverse ray aberrations in the image planes as follows: $\Delta x_{im}(\lambda, x_{ob}) = x_{im}(\lambda, x_{ob}) - x_{im}(\lambda_{prim}, x_{ob})$, $\Delta y_{im}(\lambda, y_{ob}) = y_{im}(\lambda, y_{ob}) - y_{im}(\lambda_{prim}, y_{ob})$. The results for the achromatic choice of $\beta$ are presented in figure 5. The lateral chromatic aberrations introduced by the lenses are clearly identified by the deviations $\Delta x_{im}$ in the sagittal plane, whereas the impact of AO cell may be assessed from the values of $\Delta y_{im}(\lambda, y_{ob} = 0)$ for the axial ray. Thus, the value of chromatic image shift is comparable to the RMS spot size and does not lead to significant relative shift of images on different wavelengths and information loss near the edges of the images. The chromatic shift for the parallel choice of $\beta$ is considerably higher (see figure 6). It weakly depends on the object height and reaches 100 $\mu$m and 150 $\mu$m at $\lambda = 0.45$ $\mu$m for Variants 1 and 2 correspondingly. For Variant 1, the magnitudes of the chromatic shift for e→o and o→e channels are almost

**Figure 4.** Dependence of RMS spot radius on object height along $x_{ob}$ (dashed) and $y_{ob}$ (solid) axes for several wavelengths (a). Dependence of RMS spot radius on wavelength for e→o (solid) and o→e (dashed) channels for several object points (b).
Table 1. Optical system parameters.

| Variant | Direction | Achromatic | Parallel |
|---------|-----------|------------|----------|
| 1       | e→o       | 77.850     | 73.850   |
|         |           | 3.350      | 3.350    |
|         |           | -1.619     | 0        |
|         |           | 40.637     | 40.637   |
|         |           | 43.954     | 43.956   |
|         | o→e       | 75.746     | 75.746   |
|         |           | 0          | 0        |
|         |           | 4.980      | 3.350    |
|         |           | 40.637     | 40.637   |
|         |           | 43.956     | 43.954   |
| 2       | e→o       | 61.477     | 14.278   |
|         |           | 0          | 0        |
|         |           | -8.010     | 4.015    |
|         |           | 39.374     | 39.374   |
|         |           | 3.274      | -1.627   |
|         | o→e       | 14.278     | 14.278   |
|         |           | 0          | 0        |
|         |           | 2.861      | 2.861    |
|         |           | 39.374     | 39.374   |

Figure 5. Dependence of chromatic image shift $\Delta x_{im}$ on object height along $x_{ob}$-axis (dashed) and $\Delta y_{im}$ on object height along $y_{ob}$-axis (solid) for several wavelengths (a). Dependence of chromatic shift ($\Delta y_{im}$ for the first three object points and $\Delta x_{im}$ for the last one) on wavelength for e→o (solid) and o→e (dashed) channels (b). The results are presented for the achromatic choice of $\beta$.

identical, because both channels use the same near-axis diffraction mode, but the directions of the chromatic shift are opposite. In contrast, the magnitudes of the chromatic shift for e→o and o→e channels are different for Variant 2 which uses different diffraction modes. The reverse sign
of chromatic shift for o→e channel is caused by the reflection before AO diffraction.

Finally, we analyzed monochromatic image distortion using chief rays at primary wavelength $\lambda_{prim} = 0.6328 \, \mu m$. The ideal image positions for regular grid of object points with 1 mm step are plotted in figure 7 by gray lines. For clarity, we magnified differences between real and ideal image positions by 5 times and plotted the distorted grid by red lines. The results are presented for the achromatic choice of $\beta$, the pattern for the parallel choice is visually similar. The values of image shift $\Delta x_{im}$ and $\Delta y_{im}$ in sagittal and meridional planes are presented in figure 8 for achromatic as well as for parallel choice of $\beta$.

For Variant 1, the distortion for o→e channel is similar to the distortion for e→o channel reflected about $x_{ob}z_{ob}$ (or $x_{im}z_{im}$) plane. For Variant 2, the distortion in both channels is almost identical because the rays in o→e channel are reflected before AO diffraction. As we can see, increasing the tilt angle $\beta$ of the output facet for the reduction of chromatic image shift does not lead to significant image distortion.

4. Discussion

The developed program module (DLLs) allows to perform ray tracing through AO cell, so it is well suited for geometric aberration analysis of AOTF-based optical system in sequential mode of Zemax. We have demonstrated its usage for the estimation of transverse aberrations in image plane, RMS spot size, lateral chromatic image shift and distortion. However, the ray

Figure 6. Dependence of chromatic image shift ($\Delta y_{im}$ for the first three object points and $\Delta x_{im}$ for the last one) on wavelength for e→o (solid) and o→e (dashed) channels. The results are presented for the parallel choice of $\beta$.

Figure 7. The ideal image positions (gray) and the distorted ones (red, the deviations are magnified by 5 times) for regular grid of object points with 1 mm step. The results are presented for the achromatic choice of $\beta$ and $\lambda_{prim} = 0.6328 \, \mu m$. 

Figure 8. The dependence of image shift $\Delta x_{\text{im}}$ on object height along $x_{\text{ob}}$-axis (dashed) and $\Delta y_{\text{im}}$ on object height along $y_{\text{ob}}$-axis (solid) for the achromatic (a) and parallel (b) choice of $\beta$ at $\lambda_{\text{prim}} = 0.6328 \, \mu$m.

tracing nature of these calculations does not allow to take into account some physical factors affecting the propagation of light, primarily the divergence of parallel beam after diffraction on ultrasonic wave. Hence, the image quality in AOTF-based system tends to be worse than predicted by this analysis. This limitation may be overcome either by using non-sequential mode of Zemax and implementing ray scattering in user-provided DLL or by numerical simulation of wave propagation. Both solutions require large amounts of computation time and are beyond the scope of usual aberration analysis.

Since AOTF-based spectral imager sequentially captures narrow-band images on different wavelengths, the optimization of its optical system may be done to achieve minimum RMS spot size separately for several wavelengths across the working spectral range. The chromatic image shift is more acceptable because it does not significantly affect quality of narrow-band images. The relative shift of images on different wavelengths may be compensated by calibration, but it leads to information loss near the edge of the image if this shift reaches significant values. This optimization was demonstrated in this paper using the developed program module for AO cell. The analysis for monochromatic input ray bundles allows to estimate the dependence of transmission coefficient of AO cell on incident angle, but it does not consider the dependence of wavelength with maximum transmission on incident angle [14]. Thus, it does not take into account the deviation of central wavelength of spectral transmission window at the edges of the image which occurs in AOTF-based spectral imager. Hence, it is not clear if we should use the monochromatic transmission coefficient of AO cell for the optimization of optical system for AOTF-based spectral imager. We did not consider it in the example presented in this paper. The analysis for broadband light source may be done using non-sequential mode of Zemax, but
it is computationally intensive and is not suitable for optimization and aberration analysis of imaging systems.

The analysis of two optical systems for AOTF-based stereoscopic spectral imager implementing simultaneous AO diffraction of o- and e-waves in one AO cell demonstrates that they have similar image quality which is mainly limited by the lenses. The chromatic image shift for both schemes may be reduced by the proper choice of tilt angle for the output facet of the AO cell without significant increase of other aberrations. Variant 1 provides the same diffraction mode for both beams, so it allows to use single ultrasonic frequency. Its practical usage is limited by the large size of crystal required for the AO cell. Variant 2 allows to reduce the size of AO cell by means of additional prisms, but it complicates the design and manufacture. Another issue is the significant difference in diffraction efficiencies and ultrasonic frequencies for near-axis and far-off-axis diffraction modes [9] which leads to the choice of large cut angles $\gamma$ and makes this variant impractical.

5. Conclusion
We have discussed the modeling of AO cell for the optical system design using specialized software and developed the program module for the ray tracing through AO cell. The module allows to perform the geometrical aberration analysis and the optimization of optical systems in the sequential mode of Zemax optical design software. We have used it for the design of two optical systems for AOTF-based stereoscopic spectral imager implementing two combinations of diffraction modes and compared the image quality provided by these systems. The considered approach may be used for the design of various AOTF-based imaging systems, but the limitations of the analysis based on ray tracing should be taken into account. We analyzed the image quality in two of the most promising schemes of single-volume dual-channel polarization sensitive 3D spectral imager. We have shown the dependence of the aberrations in these schemes on the AO cell geometry. Based on a combination of factors, we recommend using Variant 1 with the same diffraction mode for both stereoscopic channels.

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