Topologically Massive Spin-1 Particles and Spin-Dependent Potentials

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Abstract

We investigate the role played by particular representations of an intermediate massive spin-1 boson in the context of spin-dependent potentials between fermionic sources in the limit of low momentum transfer. A comparison between the well-known Proca case and that of a rank-2 tensor gauge potential coupled to a 4-vector gauge field is investigated in order to extract spin- as well as velocity-dependent profiles of the interparticle potentials. Bounds on some of the coupling parameters are derived and we discuss possible applications.

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I. INTRODUCTION

Most macroscopic phenomena originate either from gravitational or electromagnetic interactions. There has been some experimental effort in the last decades towards the improvement of low energy measurements of the inverse-square law with fairly good agreement between theory and experiment [1][2]. The equivalence principle has also been recently tested to search for a possible spin-gravity coupling [3]. On the other hand, many beyond the Standard Model (BSM) proposals motivated by high energy problems predict very light, weakly interacting sub-eV particles (WISPs) that could generate new long range forces, such as axions [4], SUSY-motivated particles [5] or paraphotons [6][7][8].

The discovery of a new, though feeble, fundamental force would represent a great advance. Besides the Coulomb-like “monopole-monopole” force, it is also possible that spin- and velocity-dependent forces arise from monopole-dipole and dipole-dipole (spin-spin) interactions. Those behaviors are linked to the two main aspects of any interaction: matter and mediator vertices and the mediator’s propagator. This paper deals mainly with the latter and its impact on the potential between two fermionic sources.

The propagator is obtained from the kinetic part of the Lagrangean and is dependent on field characteristics, such as its spin. Most of the literature is concerned with spin-1 bosons in the \( (\frac{1}{2}, \frac{1}{2}) \)-representation of the Lorentz group (e.g., photon). Here we would like to address the following questions: for two fields representing the same spin-1 particle, which role does a particular representation play in the final form of the interaction? Is the form of the mass term (connected to the mass-generation mechanism) determinant for the macroscopic characterization of the interparticle potential?

The amplitude for elastic scattering of two fermions is sensitive to the fundamental, microscopic, properties of the intermediate boson. We set out to study the potential generated by the exchange of two sets of neutral fields: a Proca (vector) boson and a rank-2 anti-symmetric tensor, the Cremer-Scherk-Kalb-Ramond (CSKR) field, coupled to another vector boson [9][10]. Two-form gauge fields are typical of off-shell SUGRA multiplets in four and higher dimensions [11][12][13] and the motivation to take them into consideration is two-fold:

i) They may be the messanger, or the remnant, of some BSM physics. This is why we are interested in understanding whether we may find out the track of a 2-form gauge sector
in the profile of spin-dependent potentials.

ii) In four space-time dimensions a pure on-shell rank-2 gauge potential actually describes a scalar particle. However, off-shell it is not so. This means that the quantum fluctuations of a rank-2 gauge field may induce a new pattern of spin-dependence. Moreover, its mixing with an Abelian gauge potential sets up a different scenario to work out potentials induced by a massive vector particle.

We exploit a variety of couplings to ordinary matter in order to extract possible experimental signatures allowing to distinguish between the two in the realm of low energy interactions. Just as in the usual electromagnetic case where the 4-potential is subjected to gauge-fixing conditions to reduce the number of degrees of freedom (d.o.f.), we shall also impose constraints, especially in the CSKR case, to ensure that only the spin-1 d.o.f. survives. From the physical side, we expect those potentials to exhibit a polynomial correction (in powers of $1/r$) to the well-known $e^{-m_0 r}/r$ Yukawa potential. This implies that a laboratory apparatus with typical dimension of $\sim mm$ could be used to examine the interaction mediated by massive bosons with $m_0 \sim 10^{-3} eV$.

Developments in the measurement of macroscopic interactions between unpolarized and polarized objects \cite{1} \cite{2} \cite{16} were able to constrain many of the couplings between electrons and nucleons (protons and neutrons), so that we can concentrate on a more fundamental questions, such as the impact of the representation of the intermediate boson in the two-fermion potential.

Our paper is outlined according to what follows: in Section II, we introduce the concept of potential and briefly discuss the notation and conventions employed. Next, we calculate the potentials with different couplings for the Proca and CSKR fields in Sections III and IV. We present our Conclusions and Perspectives in Section V. An Appendix follows, where we cast the list of the relevant vertices in the low-energy limit.

II. METHODOLOGY

Let us first establish the kinematics of our problem. We are dealing with two fermions, 1 and 2, which scatter elastically. If we work in the center of mass (CM) we can assign them momenta as indicated in Fig.(1) below, where $\vec{q}$ is the momentum transfer and $\vec{p}$ is the average momentum of fermion 1 before and after the scattering.
Given energy conservation and our choice of reference frame, it is straightforward to show that $\vec{p} \cdot \vec{q} = 0$ and that $q^\mu$ is space-like: $q^2 = -\vec{q}^2$. The amplitude will be expressed in terms of $\vec{q}$ and $\vec{p}$ and we shall keep only terms linear in $|\vec{p}|/m_{1,2}$. It will also include the spin of the involved particles.

According to the first Born approximation, the two-fermion potential can be obtained from the Fourier transform of the tree-level momentum-space amplitude with respect to the momentum transfer $\vec{q}$

$$V(r, \nu) = -\int \frac{d^3 \vec{q}}{(2\pi)^3} e^{i\vec{q} \cdot \vec{r}} \mathcal{A}(\vec{q}, m\nu),$$

where $\vec{r}$, $r$ and $\nu = |\vec{p}|/m_{1,2}$ are the relative position vector, its modulus and average velocity of the fermions, respectively. The long-range behaviour is related to the non-analytical pieces of the amplitude in the non-relativistic limit [17]. We evaluate the fermionic currents up to first order in $|\vec{p}|/m_{1,2}$ and $|\vec{q}|/m_{1,2}$, as indicated in the Appendix (an important exception is discussed in Section IV.B in connection with the mixed propagator $\langle A_\mu B_{\nu\kappa} \rangle$ since, in that case, contact terms arise).

![Basic vertex structure and momentum assignments.](image)

We restrict ourselves to tree-level amplitudes since we are dealing with weakly interacting particles, thus carrying tiny coupling constants that suppress higher order diagrams. The typical outcome are Yukawa-like potentials with extra $1/r$ contributions which also depend on the spin of the sources, as well as on their velocity. Contrary to the usual Coulomb case, spin- and velocity-dependent terms are the rule, not exception.
III. THE PURE SPIN-1 CASE: THE PROCA FIELD

In order to set up a comparison we start by quickly reviewing the simplest realization of a neutral massive vector particle, the Proca field \( A_\mu(x) \), described by the Lagrangean

\[
L_{\text{Proca}} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} m_0^2 A_\mu^2
\]

(2)

with the field strength tensor given by \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \).

Since we are concerned with the interaction mediated by such a field, it is necessary to calculate its propagator, \( \langle A_\mu A_\nu \rangle \). The Lagrangean above can be conveniently rewritten as \( \frac{1}{2} A^\mu O_{\mu\nu} A^\nu \), in which the operator \( O_{\mu\nu} \), essentially the inverse of the propagator, is

\[
O_{\mu\nu} = (\Box + m_0^2) \theta_{\mu\nu} + m_0^2 \omega_{\mu\nu},
\]

where we introduced the transverse and longitudinal projection operators defined as

\[
\theta_{\mu\nu} \equiv \eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\Box},
\]

(3)

\[
\omega_{\mu\nu} \equiv \frac{\partial_\mu \partial_\nu}{\Box},
\]

(4)

which satisfy \( \theta^2 = \theta \), \( \omega^2 = \omega \), \( \theta \omega = 0 \) and \( \theta + \omega = 1 \). Due to these simple algebraic properties it is easy to invert \( O_{\mu\nu} \) and, transforming to momentum space, we finally have

\[
\langle A_\mu A_\nu \rangle = -i \frac{k^2 - m_0^2}{k^2 - m_0^2} \left( \eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_0^2} \right),
\]

(5)

allowing us to proceed to the calculation of the potentials.

Let us solve in more detail the case of two fermionic vector currents interacting via the Proca field. Using the parametrization of Fig.(11) and applying the Feynman rules, we obtain

\[
i A^{\text{Proca}_{V-V}}_{V} = \bar{u}(p + q/2) \left\{ i g_1^V \gamma^\mu \right\} u(p - q/2) \langle A_\mu A_\nu \rangle \times
\]

\[
\times \bar{u}(-p - q/2) \left\{ i g_2^V \gamma^\nu \right\} u(-p + q/2)
\]

with \( g_1^V \) and \( g_2^V \) referring to the coupling constants. The equation above can be put in a simpler form as below

\[
A^{\text{Proca}_{V-V}}_{V} = i J_1^\mu \langle A_\mu A_\nu \rangle J_2^\nu.
\]

(6)

If we use that \( q^0 = 0 \) and current conservation, we find that the amplitude is \( A^{\text{Proca}_{V-V}}_{V} = -i \frac{1}{q^2 + m_0^2} J_1^\mu J_2^\mu \) and, according to eq.(A.9), we have \( J_1^\mu J_2^\mu \sim O(v^2/c^2) \). Therefore, only the term \( J_1^0 J_2^0 \approx g_1^V g_2^V \delta_1 \delta_2 \) contributes to the scattering amplitude, thus giving

\[
A^{\text{Proca}_{V-V}}_{V} = -g_1^V g_2^V \frac{\delta_1 \delta_2}{q^2 + m_0^2},
\]

(7)
where \( \delta_i \) \((i = 1, 2 \) labels the particles) is such that \( \delta_i = +1 \) if the \( i \)-th particle experiences no spin flip in the interaction, and \( \delta_i = 0 \) otherwise. In the eq.(7) above, the global term \( \delta_1 \delta_2 \) is present to indicate that the amplitude is non-trivial only if both particles do not flip their respective spins. If one of them changes its spin the potential vanishes. This means that this interaction only occurs with no spin flip. In what follows, we shall come across situations where only a single \( \delta_i \) appears, thus justifying the effort to keep the \( \delta_i \) explicit.

Finally, we take the Fourier transform in order to obtain the potential between two static (vector) currents,

\[
V_{V>V} = \frac{g_V g_2}{4\pi} \delta_1 \delta_2 e^{-m_0 r},
\]

which displays the well-known exponentially suppressed repulsive Yukawa behaviour typical of \( s = 1 \) boson exchange. In our notation the potential is indicated as \( V_{v_1-v_2} \), where \( v_{1,2} \) refer to the vertices related to particles 1 and 2. In the case above the subscripts \( V \) stand for vector currents. The typical decay length is \( 1/m_0 \) and we expect that very light bosons will be measurable in laboratory macroscopic distances, e.g. for masses of \( \sim 10^{-3} \) eV we have ranges of \( d \sim mm \).

Following the same procedure we can calculate other scenarios, namely: vector with pseudo-vector currents and two pseudo-vector currents. The results are

\[
V_{V>PV}^{Proca} = -\frac{g_V g_2}{4\pi} \left\{ \vec{p} \cdot \langle \vec{\sigma} \rangle_2 \delta_1 \left[ \frac{1}{m_1} + \frac{1}{m_2} \right] + \frac{(1+m_0 r)}{2m_1 r^2} \left[ \langle \vec{\sigma} \rangle_1 \times \langle \vec{\sigma} \rangle_2 \cdot \hat{r} \right] \right\} e^{-m_0 r},
\]

and we notice that all kinds of spin-dependent interactions appear whereas the \( r \) factors are limited to \( r^{-2} \). In the next section we will see that richer potentials are generated.

**IV. THE TOPOLOGICALLY MASSIVE SPIN-1 CASE**

The Proca vector field transforms under the \( \left( \frac{1}{2}, \frac{1}{2} \right) \)-representation of the Lorentz group and its Lagrangean is the simplest extension leading to a massive intermediate vector boson, but it is not the only one. A massive spin-1 particle can also be described through a gauge-invariant formulation: a vector and a tensor fields connected by a mixing topological mass term \cite{18, 19}.
Both the vector $A_\mu$ and the tensor $B_{\mu\nu}$ are gauge fields described by the following Lagrangean:

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{6} G_{\mu\nu\kappa}^2 + \frac{m_0}{\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} A_{\mu} \partial_{\nu} B_{\alpha\beta},$$

(11)

where the field strength for the anti-symmetric tensor is $G_{\mu\nu\kappa} = \partial_{\mu} B_{\nu\kappa} + \partial_{\nu} B_{\kappa\mu} + \partial_{\kappa} B_{\mu\nu}$.

The action is invariant under the independent local Abelian gauge transformations given by

$$A'_\mu = A_\mu + \partial_\mu \alpha$$

(12)

$$B'_{\mu\nu} = B_{\mu\nu} + \partial_\mu \beta_\nu - \partial_\nu \beta_\mu,$$

(13)

and it can be shown that together with the equations of motion, the pair $\{A_\mu, B_{\mu\nu}\}$ is described by three degrees of freedom, being, therefore, equivalent to a massive vector field. It is interesting to note that, contrary to the typical Proca case, the topological mass term does not break gauge invariance, therefore characterizing a dynamical - rather than spontaneous (symmetry breaking) - mass generation mechanism.

Even though the Proca field and the mixed $\{A_\mu, B_{\mu\nu}\}$ system describe both an on-shell spin-1 massive particle, these two cases are significantly different when considered off-shell. Our topologically massive spin-1 system displays 6 d.o.f. when taken off-shell (since gauge symmetry allows us to eliminate 4 compensating modes), whereas the Proca field carries 4 off-shell d.o.f. (the subsidiary condition, which is an on-shell statement, eliminates one d.o.f.).

On the other hand, since the potential evaluation is an off-shell procedure, we consider relevant to compare both situations bearing in mind that the potential profiles may indicate - if we are able to set up an experiment - whether a particular mechanism is preferable in the case of a specific physical system. Characteristic aspects of the potentials in these two situations might select one or other mechanism in some possible physical scenario.

Our goal is to obtain the potentials between fermions mediated by the exchange of these fields and compare the spin-, velocity- and distance-dependence against the Proca case. First of all, we need to extract the propagators.

A. The propagators

As with the Proca field, it is important to obtain suitable projection operators in order to obtain the propagators of the model. The projection operators that act on an anti-symmetric
2-form are

\[
(P_b^1)_{\mu\nu,\rho\sigma} = \frac{1}{2} (\theta_{\mu\rho} \theta_{\nu\sigma} - \theta_{\mu\sigma} \theta_{\nu\rho})
\]

which are anti-symmetric generalizations of the projectors \(\theta_{\mu\nu}\) and \(\omega_{\mu\nu}\) \[20\] \[21\] \[22\]. The comma indicates that we have anti-symmetry in changes \(\mu \leftrightarrow \nu\) or \(\rho \leftrightarrow \sigma\). These operators satisfy the following algebra:

\[
(P_b^1 + P_e^1)_{\mu\nu,\rho\sigma} = \frac{1}{2} (\eta_{\mu\rho} \eta_{\nu\sigma} - \eta_{\mu\sigma} \eta_{\nu\rho}) \equiv 1^{a.s.}_{\mu\nu,\rho\sigma}
\]

\[
(P_b^1)_{\mu\nu,\alpha\beta} (P_b^1)_{\alpha\beta,\rho\sigma} = (P_b^1)_{\mu\nu,\rho\sigma}
\]

\[
(P_e^1)_{\mu\nu,\alpha\beta} (P_e^1)_{\alpha\beta,\rho\sigma} = (P_e^1)_{\mu\nu,\rho\sigma}
\]

\[
(P_b^1)_{\mu\nu,\alpha\beta} (P_e^1)_{\alpha\beta,\rho\sigma} = 0
\]

\[
(P_e^1)_{\mu\nu,\alpha\beta} (P_b^1)_{\alpha\beta,\rho\sigma} = 0.
\]

Before going to the next step, we note that the mixing term between \(A_\mu\) and \(B_{\mu\nu}\) introduces a new operator, \(S_{\mu\nu\kappa} \equiv \epsilon_{\mu\nu\kappa\lambda} \partial^\lambda\), which is not a projector, since

\[
e^{\mu\nu\alpha\beta} A_\mu \partial_\nu B_{\alpha\beta} = \frac{1}{2} \left[ A_\mu S_{\mu\nu\kappa\lambda} B_{\kappa\lambda} - B_{\kappa\lambda} S_{\kappa\lambda\mu} A_\mu \right],
\]

so that we need to study the algebra of \(S_{\mu\nu\kappa}\) with the projectors (14) and (15), giving us

\[
S_{\mu\nu\alpha} S^{\alpha\kappa\lambda} = -2 \Box (P_b^1)_{\mu\nu,\kappa\lambda}
\]

\[
(P_b^1)_{\mu\nu,\alpha\beta} S^{\alpha\beta\kappa} = S_{\mu\nu,\kappa}
\]

\[
S^{\kappa\alpha\beta} (P_b^1)_{\alpha\beta,\mu\nu} = S^{\kappa\mu\nu}
\]
\[(P_\nu^1)_{\alpha\beta, \alpha\beta} S^\alpha\beta\kappa = 0 \]  
\[(25)\]

\[S^\kappa_{\alpha\beta} (P_\nu^1)^{\alpha\beta, \mu\nu} = 0 \]  
\[(26)\]

\[S_{\mu\alpha\beta} S^{\alpha\beta}_{\nu} = -2\Box\theta_{\mu\nu}. \]  
\[(27)\]

Now we have all the operators we need and, according to eq.(27), a closed algebra between \(S_{\mu\alpha\beta}\) and the projectors \(\theta_{\mu\nu}\) and \(\omega_{\mu\nu}\) (as well as their anti-symmetric generalizations). Similarly to the well-known photon case, we have to add gauge fixing terms to eq.(11),

\[L_{g.f.} = \frac{1}{2\alpha} (\partial_\mu A^\mu)^2 + \frac{1}{2\beta} (\partial_\mu B^{\mu\nu})^2 \]  
\[(28)\]

thus defining the full Lagrangean \(L = L_0 + L_{g.f.}\). In terms of the operators discussed above, \(L\) can be cast in a more compact form as follows:

\[L = \frac{1}{2} \begin{pmatrix} A^\mu & B^{\kappa\lambda} \end{pmatrix} \begin{pmatrix} P_{\mu\nu} & Q_{\mu\rho\sigma} \\ R_{\kappa\lambda\nu} & S_{\kappa\lambda, \rho\sigma} \end{pmatrix} \begin{pmatrix} A^\nu \\ B^{\rho\sigma} \end{pmatrix}, \]  
\[(29)\]

where we identify

\[P_{\mu\nu} \equiv \Box\theta_{\mu\nu} - \frac{\Box}{\alpha} \omega_{\mu\nu} \]  
\[(30)\]

\[Q_{\mu\rho\sigma} \equiv m_0 S_{\mu\rho\sigma}/\sqrt{2} \]  
\[(31)\]

\[R_{\kappa\lambda\nu} \equiv -m_0 S_{\kappa\lambda\nu}/\sqrt{2} \]  
\[(32)\]

\[S_{\kappa\lambda, \rho\sigma} \equiv -\Box (P_{\nu}^1)_{\kappa\lambda, \rho\sigma} - \frac{\Box}{2\beta} (P_{\epsilon}^1)_{\kappa\lambda, \rho\sigma}. \]  
\[(33)\]

By following the usual procedure to obtain propagators, we invert the matrix operator in \(29\) and read off the \(\langle A_\mu A_\nu \rangle\), \(\langle A_\mu B_{\kappa\lambda} \rangle\) and \(\langle B_{\mu\nu} B_{\kappa\lambda} \rangle\) momentum-space propagators as given below:

\[\langle A_\mu A_\nu \rangle = -\frac{i}{k^2 - m_0^2} \eta_{\mu\nu} + i \left( \frac{1}{k^2 - m_0^2} + \frac{\alpha}{k^2} \right) \frac{k_\mu k_\nu}{k^2} \]  
\[(34)\]
\[ \langle B_{\mu\nu}B_{\kappa\lambda} \rangle = \frac{i}{k^2 - m_0^2} (P_b^1)_{\mu\nu,\kappa\lambda} + \frac{2i\beta}{k^2} (P_e^1)_{\mu\nu,\kappa\lambda} \]  

(35)

\[ \langle A_\mu B_{\nu\kappa} \rangle = \frac{m_0/\sqrt{2}}{k^2 (k^2 - m_0^2)} \epsilon_{\mu\nu\kappa\lambda} k^\lambda. \]  

(36)

From the propagators above, we clearly understand that the massive pole \( k^2 = m_0^2 \), present in (34)-(36), actually describes the spin-1 massive excitation carried by the set \( \{A_\mu, B_{\mu\nu}\} \).

**B. The potentials**

We have already discussed the procedure to obtain the spin- and velocity-dependent potentials in previous sections. Thus, we will focus on the particular case in which we have the propagator \( \langle B_{\mu\nu}B_{\kappa\lambda} \rangle \) and two tensor currents. In the following we assume the same parametrization of Fig. (1). After applying the Feynman rules, we can rewrite the scattering amplitude for this process as

\[ \mathcal{A}^{(BB)}_{T-T} = i J_1^{\mu\nu} \langle B_{\mu\nu}B_{\kappa\lambda} \rangle J_2^{\kappa\lambda} \]  

(37)

with the tensor currents given by eq. (A.13). Substituting the propagator (35) in eq. (37) and eliminating its longitudinal sector (due to current conservation) we have

\[ \mathcal{A}^{(BB)}_{T-T} = -\frac{1}{q^2 - m_0^2} J_1^{\mu\rho} J_2^{\nu\rho}. \]  

(38)

The current product leads to \( J_1^{\mu\rho} J_2^{\nu\rho} = 2J_1^{0i} J_2^{0i} + J_1^{ij} J_2^{ij} \). However, according to eq. (A.14), we conclude that \( J_1^{0i} J_2^{0i} \sim \mathcal{O}(v^2/c^2) \) does not contribute to the non-relativistic amplitude. The term \( J_1^{ij} J_2^{ij} \) can be simplified using eq. (A.15) (with the appropriate changes to the second current), so that we get

\[ \mathcal{A}^{(BB)}_{T-T} = \frac{1}{2} \frac{g_1^T g_2^T}{q^2 + m_0^2} \langle \vec{\sigma} \rangle_1 \cdot \langle \vec{\sigma} \rangle_2. \]

Performing the well known Fourier integral we obtain the non-relativistic spin-spin potential, namely

\[ V_{T-T}^{(BB)} = -\frac{g_1^T g_2^T}{8\pi} \langle \vec{\sigma} \rangle_1 \cdot \langle \vec{\sigma} \rangle_2 \frac{e^{-m_0 r}}{r}, \]  

(39)

and, similarly, we find the interaction potentials between tensor and pseudo-tensor currents,
\[
V_{T-PT}^{(BB)} = \frac{g_1^T g_2^{PT}}{8\pi r} \left\{ \left( \frac{1}{m_1} + \frac{1}{m_2} \right) \vec{p} \cdot (\langle \vec{\sigma} \rangle_1 \times \langle \vec{\sigma} \rangle_2) + \right. \\
+ \frac{(1 + m_0 r)}{2r} \left( \frac{\delta_2}{m_2} \langle \vec{\sigma} \rangle_1 - \frac{\delta_1}{m_1} \langle \vec{\sigma} \rangle_2 \right) \cdot \hat{r} \left. \right\} e^{-mor} \tag{40}
\]

as well as two pseudo-tensors currents,

\[
V_{PT-PT}^{(BB)} = \frac{g_1^{PT} g_2^{PT}}{8\pi} \langle \vec{\sigma} \rangle_1 \cdot \langle \vec{\sigma} \rangle_2 e^{-mor} \tag{41}
\]

It is worthy comparing the potentials (39) and (41). We observe that they differ by a relative minus sign. This means that they exhibit opposite behaviors for a given spin configuration: one is attractive and the other repulsive. The physical reason is that the \(PT - PT\) and \(T - T\) potentials stem from different sectors of the currents: the \(PT - PT\) amplitude is composed by the \((0i) - (0j)\) terms of the currents; the \(T - T\) amplitude, on the other hand, arises from the \((ij) - (kl)\) components, as it can be seen from eq.(37).

In the light of that, we check the structure of the \(\langle B_{\mu\nu} B_{\kappa\lambda} \rangle\)-propagator and it becomes clear that, in the case of the \(\langle B_{0i} B_{0j} \rangle\)-mediator, an off-shell scalar mode is exchanged. In contrast, in the \(\langle B_{ij} B_{kl} \rangle\)-sector the only exchange is of a pure \(s = 1\) (off-shell) quantum. It is well-known, however, that the exchange of a scalar and a \(s = 1\) boson between sources of equal charges yields attractive and repulsive interactions, respectively, therefore justifying the aforementioned sign difference between eqs.(39) and (41).

For the mixed propagator \(\langle A_{\mu} B_{\kappa\lambda} \rangle\), eq.(36), we have four possibilities involving the following currents: vector with tensor, vector with pseudo-tensor, pseudo-vector with tensor and pseudo-vector with pseudo-tensor. The results are given below

\[
V_{V-T}^{(AB)} = \frac{g_1^V g_2^T \delta_1}{4\pi \sqrt{2m_0r^2}} \left[ 1 - (1 + m_0 r) e^{-mor} \right] \langle \vec{\sigma} \rangle_2 \cdot \hat{r} \tag{42}
\]

\[
V_{PV-T}^{(AB)} = \frac{g_1^{PV} g_2^{PT}}{4\pi \sqrt{2m_0\mu r^2}} \left[ 1 - (1 + m_0 r) e^{-mor} \right] \left( \langle \vec{\sigma} \rangle_1 \cdot \vec{p} \right) \left( \langle \vec{\sigma} \rangle_2 \cdot \hat{r} \right) \tag{43}
\]

\[
V_{PV-PT}^{(AB)} = \frac{g_1^{PV} g_2^{PT}}{\sqrt{2m_0}} \left\{ \frac{\delta_2}{2m_1 m_2} \left[ 3 \delta^3(\vec{r}) + \frac{m_0^2}{4\pi r} e^{-mor} \right] \langle \vec{\sigma} \rangle_1 \cdot \vec{p} + \right. \\
+ \frac{1}{4\pi r^2} \left[ 1 - (1 + m_0 r) e^{-mor} \right] \left( \langle \vec{\sigma} \rangle_2 \times \langle \vec{\sigma} \rangle_1 \right) \cdot \hat{r} \left. \right\} \tag{44}
\]
The longest and richest potential is between vector and pseudo-tensor sources, given by

\[ V_{V^{-PT}}^{(AB)} = \frac{g_V g_2^{PT}}{\sqrt{2m_0}} \left\{ \frac{\delta_1 \delta_2}{2m_2} \left[ \delta^3(\vec{r}) + \frac{m_0^2}{4\pi r} e^{-m_0 r} \right] + \right. \]
\[ + \frac{\delta_1}{4\pi \mu r^2} \left[ 1 - (1 + m_0 r) e^{-m_0 r} \right] \vec{L} \cdot \langle \vec{\sigma} \rangle_2 + \right. \]
\[ + \frac{1}{2m_1} \left[ \delta^3(\vec{r}) + \frac{m_0^2}{4\pi r} e^{-m_0 r} - \frac{1}{4\pi r^3} \left[ 1 + (1 + m_0 r) e^{-m_0 r} \right] \right] \langle \vec{\sigma} \rangle_1 \cdot \langle \vec{\sigma} \rangle_2 + \right. \]
\[ + \frac{1}{8\pi m_1 r^3} \left[ 3 + (3 + 3m_0 r + m_0^2 r^2) e^{-m_0 r} \right] \left( \langle \vec{\sigma} \rangle_1 \cdot \hat{r} \right) \left( \langle \vec{\sigma} \rangle_2 \cdot \hat{r} \right) \left\} \right. \]

where we introduced the reduced mass of the fermion system \( \mu^{-1} = m_1^{-1} + m_2^{-1} \) and \( \vec{L} = \vec{r} \times \vec{p} \) is the orbital angular momentum.

Naturally, the contact terms do not contribute to a macroscopic interaction. Nevertheless, they are significant in quantum mechanical applications in the case of \( s\)-waves which can overlap the origin. This is a peculiarity of \( \langle A_\mu B_\kappa \lambda \rangle \) sector due to the extra \( q^2 \) factor in the denominator which forces us to keep terms of order \( |\vec{q}|^2 \) in the current products.

Finally, for the propagator \( \langle A_\mu A_\nu \rangle \), eq.(34), we will find the same results of the Proca situation, due to current conservation. This means that, even through the vector field appears now mixed with the \( B_{\mu \nu} \) field with a gauge-preserving mass, for the sake of the potentials, the results are the same as in the Proca case as far as the \( A_\mu \) field exchange is concerned.

V. CONCLUSIONS AND PERSPECTIVES

The model we are investigating describes an extra Abelian gauge boson, a sort of \( Z_\mu^{0'} \), which appears as a neutral massive excitation of a mixed \( \{ A_\mu, B_{\mu \nu} \} \) system of fields. It may be originated from some sector of BSM physics, where the coupling between an Abelian field and the 2-form gauge potential in the SUGRA multiplet may yield the topologically massive spin-1 particle we are considering. To have detectable macroscopic effect, this sort of \( Z_\mu^{0'} \) intermediate particle should have a very small mass, of the order of meV. This would be possible in the class of phenomenological models with SUSY breaking close to the accelerator energies, namely, the TeV scale.

It is clear that the larger number of off-shell d.o.f. of the CSKR model accounts for the variety of potentials presented above. In order to distinguish between the two models the
experimenter may choose a set-up consisting of a neutral and a polarized sources (1 and 2, respectively). In this case we must collect the terms proportional to $\langle \vec{\sigma} \rangle_2 \equiv \langle \vec{\sigma} \rangle$, namely

$$V_{\text{Proca\ mon-dip}} = -\frac{g^2 e^{-m_0 r}}{\mu} \vec{p} \cdot \langle \vec{\sigma} \rangle$$

$$V_{\text{CSKR\ mon-dip}} = -\frac{g^2 e^{-m_0 r}}{\mu} \vec{p} \cdot \langle \vec{\sigma} \rangle +$$

$$-\frac{g^2 (1 + m_0 r) e^{-m_0 r}}{m_1 r^2} \hat{r} \cdot \langle \vec{\sigma} \rangle +$$

$$+\frac{g^2 [1 - (1 + m_0 r) e^{-m_0 r}]}{m_0 r^2} \hat{r} \cdot \langle \vec{\sigma} \rangle +$$

$$-\frac{g^2 m_0 e^{-m_0 r}}{m_1 m_2 r} \vec{p} \cdot \langle \vec{\sigma} \rangle +$$

$$+\frac{g^2 [1 - (1 + m_0 r) e^{-m_0 r}]}{\mu m_0 r^3} (\vec{r} \times \vec{p}) \cdot \langle \vec{\sigma} \rangle,$$

where, for simplicity, we have omitted the labels in the coupling constants. If we consider the case in which the source 1 carries momentum so that $\vec{p} // \langle \vec{\sigma} \rangle$, the last term above vanishes. Similarly, it is easy to see that the third term is essentially constant while the fourth one is negligible, since $m_0 |\vec{p}| / m_1 m_2 \ll 1$ by definition. In Fig. (2), we plot the two potentials.

![](image.png)

**FIG. 2.** Monopole-dipole potentials with $m_1 = m_e = 10^5 eV$, $m_0 = 10^{-3} eV$ and source 1 velocity of order $v/c \simeq 10^{-6}$. The scale is irrelevant and coupling constants were not included for simplicity.

It is then possible, in principle, to determine which field representation, Proca or CSKR, is describing the interaction at hand. It is worth mentioning that this difference is regulated by the $1/m_1$ factor in the second term of eq. (47), so that only the lightest fermions (i.e., electrons and not the protons or neutrons, provided that, in a macroscopic source, we can safely neglect the internal quark-gluon structure of the nucleons) would be able to contribute sufficiently.
The calculation we performed is based on the quantum field theoretical scattering amplitude in the non-relativistic limit and the potential obtained is also suitable to be introduced in the Schrödinger equation as a perturbation to the full Hamiltonian. If we take the second line of eq. (45), for example, we notice a coupling of the angular momentum of the first fermion with the spin of the second.

Such a spin-orbit coupling is also found in the hydrogen atom, contributing to its fine structure. Supposing that the proton and electron are charged under the gauge symmetries leading to the CSKR fields, we can calculate a correction to the energy levels of their bound state due to $\langle A_\mu B_{\kappa\lambda} \rangle$ exchange.

If we expand $1 - (1 + m_0 r) e^{-m_0 r}$ we obtain $m_0^2 r^2/2$ as the first non-zero term (to second order), so that the spin-orbit term simplifies to

$$V_{LS}^{VPT} = \frac{\sqrt{2} g_Y g_{PT} m_0}{8\pi \mu} \frac{1}{r} L \cdot S$$

with $S = \langle \vec{\sigma} \rangle_2/2$. Applying first-order perturbation theory to this potential gives a correction to the energy of $\Delta E^{LS} = \frac{9}{8\pi \sqrt{2} \mu} \frac{g_Y g_{PT} m_0}{n^2 a_0}$. Given that the reduced mass and the Bohr radius are $\mu \simeq m_e = 5.11 \times 10^5$ eV and $a_0 = 2.69 \times 10^{-4}$ eV$^{-1}$, respectively, we can constrain $\Delta E^{LS}$ to be smaller than the current spectroscopic uncertainties of one part in $10^{14}$ [23]. We have, thus

$$|g_Y g_{PT} m_0| < 10^{-10} \text{eV},$$

which poses a rather loose, but consistent [8], upper bound on the couplings.

At last but not least, we indicate that it is possible to assign certain $CP$ transformation properties to the fields $A_\mu$ and $B_{\mu\nu}$ so that the topological mass term in eq. (11) violates $CP$. This would induce an electric dipole moment (EDM) if we couple our model to fermionic fields. Following the procedure employed by Mantry et al [24] in the context of axions, we could also use information from the EDM to find further bounds on the coupling constants and the mass of the intermediate spin-1 boson.

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Appendix: Currents in the non-relativistic approximation

In the following we present a brief summary of the conventions and main decompositions employed in the calculations performed in the previous sections.

1. Basic conventions

The basic spinors used to compose the scattering amplitude are the positive energy solutions to the Dirac equation in momentum space \[25\], namely

\[
u(p) = \begin{pmatrix} \xi \\ \frac{\vec{\sigma} \cdot \vec{p}}{2m} \xi \end{pmatrix}
\]  

(A.1)

where \(\xi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\) or \(\xi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\) for spin-up and -down, respectively. Above we have assumed the non-relativistic limit \(E + m \approx 2m\). The orthonormality relation \(\xi_r^\dagger \xi_s = \delta_{rs}\) is admitted to hold and we will usually suppress spinor indices.

The gamma matrices are chosen as

\[
\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix},
\]  

(A.2)

and the metric and Levi-Civita symbol are defined so that \(\eta^{\mu\nu} = \text{diag}(+,-,-,-)\) and \(\epsilon^{0123} = +1\), respectively. We adopt natural units \(\hbar = c = 1\) throughout, except when checking orders of magnitude.

2. Current decompositions

In order to calculate the spin-dependent potentials it is useful to have the non-relativistic limit of the source currents, in which we assume

1) \(|\vec{p}|^2/m^2 \sim O(v^2/c^2) \to 0\)

2) small momentum transfer: \(|\vec{q}|^2/m^2 \to 0\)
3) The cross product tends to zero if $|\vec{p}|/m$ and $|\vec{q}|/m$ are small. Energy-momentum conservation implies $\vec{q} \cdot \vec{p} = 0$

Here we show the results of the main fermionic currents. We adopt the parametrization for the first current (i.e., first vertex), following Fig. (1). We denote the generators of the boosts and rotations by

$$\Sigma^\mu_\nu \equiv -\frac{i}{4} [\gamma^\mu, \gamma^\nu], \quad (A.3)$$

and $\langle \sigma_i \rangle \equiv \xi^t \sigma_i \xi$. In the Dirac representation, $\gamma_5$ is given by

$$\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (A.4)$$

Thus, making use of the Dirac spinor conjugate, $\bar{u} \equiv u^\dagger \gamma^0$, we have the following set of identities, omitting the coupling constants:

1) Scalar current ($S$):

$$\bar{u}(p + q/2) u(p - q/2) \approx \delta. \quad (A.5)$$

2) Pseudo-scalar current ($PS$):

$$\bar{u}(p + q/2) i\gamma_5 u(p - q/2) = -\frac{i}{2m} \vec{q} \cdot \langle \vec{\sigma} \rangle \quad (A.6)$$

3) Vector current ($V$):

$$\bar{u}(p + q/2) \gamma^\mu u(p - q/2) \quad (A.7)$$

3i) For $\mu = 0$,

$$\bar{u}(p + q/2) \gamma^0 u(p - q/2) \approx \delta \quad (A.8)$$

3ii) For $\mu = i$,

$$\bar{u}(p + q/2) \gamma^i u(p - q/2) = \frac{\vec{p}_i}{m} \delta - \frac{i}{2m} \epsilon_{ijk} \vec{q}_j \langle \sigma_k \rangle \quad (A.9)$$

4) Pseudo-vector current ($PV$):

$$\bar{u}(p + q/2) \gamma^\mu \gamma_5 u(p - q/2) \quad (A.10)$$
4i) For \( \mu = 0 \),

\[
\bar{u}(p + q/2) \gamma^0 \gamma_5 u(p - q/2) = \frac{1}{m} \langle \vec{\sigma} \rangle \cdot \vec{p}
\]  

(A.11)

4ii) For \( \mu = i \),

\[
\bar{u}(p + q/2) \gamma^i \gamma_5 u(p - q/2) \approx \langle \sigma_i \rangle
\]  

(A.12)

5) Tensor current (\( T \)):

\[
\bar{u}(p + q/2) \Sigma^{\mu\nu} u(p - q/2)
\]  

(A.13)

5i) For \( \mu = 0 \) and \( \nu = i \),

\[
\bar{u}(p + q/2) \Sigma^{0i} u(p - q/2) = \frac{1}{2m} \epsilon_{ijk} p_j \langle \sigma_k \rangle + \frac{i}{4m} \delta q_i
\]  

(A.14)

5ii) For \( \mu = i \) and \( \nu = j \),

\[
\bar{u}(p + q/2) \Sigma^{ij} u(p - q/2) \approx -\frac{1}{2} \epsilon_{ijk} \langle \sigma_k \rangle
\]  

(A.15)

6) Pseudo-tensor current (\( PT \)):

\[
\bar{u}(p + q/2) i \Sigma^{\mu\nu} \gamma_5 u(p - q/2)
\]  

(A.16)

6i) For \( \mu = 0 \) and \( \nu = i \),

\[
\bar{u}(p + q/2) i \Sigma^{0i} \gamma_5 u(p - q/2) \approx \frac{1}{2} \langle \sigma_i \rangle
\]  

(A.17)

6ii) For \( \mu = i \) and \( \nu = j \)

\[
\bar{u}(p + q/2) i \Sigma^{ij} \gamma_5 u(p - q/2) = \frac{1}{2m} (\vec{p}_i \langle \sigma_j \rangle - \vec{p}_j \langle \sigma_i \rangle) + \frac{i}{4m} \delta \epsilon_{ijk} \vec{q}_k
\]  

(A.18)

In the above we kept the \( rs \) indices implicit in the \( \delta_{rs} \), as done in the main text, pointing out only the particle label. Due to momentum conservation and our choice of reference
frame (CM), the second current (or second vertex) can be obtained by making the changes
\( q \rightarrow -q \) and \( p \rightarrow -p \) on the first one.

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