The Study of Probability Density Function in The Presence of Self-Absorption in Molecular Clouds

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ABSTRACT

Understanding the formation of stars in molecular clouds is a central topic of modern astrophysics. However, identifying the transition region where the gravity takes over and collapse occurs remains elusive. The column density probability density functions (PDFs) of the mass density is a tool developed to study turbulence and self-gravity in molecular clouds. In this work, we study the effects of self-absorption on the intensity PDFs utilizing three synthetic emission lines of CO isotopologs 12CO, 13CO, and C18O. We find that for supersonic turbulence, the change of PDFs from the log-normal to a distribution that has a power-law part depends on the abundance of molecular species that determine its optical depth. We compute the intensity PDFs for two star-forming regions Vela C and SH2-235, and find the change of PDFs in observation agrees with our numerical results. We identify the gravitational collapsing regions on Serpens and Ophiuchus molecular clouds through the PDFs of H2 column density. We see that the change of CO isotopologs’ PDFs is independent of self-gravity, which makes the PDFs less reliable in identifying gravitational collapsing regions using emission lines.

Keywords: ISM:general—ISM:structure—ISM:magnetohydrodynamics—turbulence—radiative transfer

1. INTRODUCTION

Magnetic and turbulent effects are considered to be the crucial agents affecting the dynamics of the star-forming process in molecular clouds, in combination with gas self-gravity, at all physical scales and throughout different evolutionary stages (Li & Henning 2011; Hull et al. 2013; Andersson et al. 2015; Jokipii 1966; Parker 1965, 1979; Caprioli & Spitkovsky 2014; McKee & Ostriker 2007). To understand this complex interplay, it is essential to explore the properties of turbulence, trace magnetic fields, and identify the transition regions where the gravity takes over and collapse occurs (Shu 1992; Kennicutt 1998a,b; Shu 1977; Shu et al. 1994).

The probability density functions (PDFs) of the column mass density is a statistical tool to get an insight into turbulence and self-gravity in molecular clouds. The PDFs are believed to be log-normal in non-self-gravitating isothermal supersonic turbulence (Klessen 2000; Padoan et al. 2017; Kritsuk et al. 2011; Burkhart 2018; Vazquez-Semadeni et al. 1995; Robertson & Kravtsov 2008; Collins et al. 2012; Burkhart 2018):

\[ P_N(s) = \frac{1}{\sqrt{2\pi}\sigma_s^2} e^{-\frac{(s-s_0)^2}{2\sigma^2}} \]

where \( s = \ln(\rho/\rho_0) \) is the logarithmic density and \( \sigma_s \) is the standard deviation of the log-normal, while \( \rho_0 \) and \( s_0 \) denote the mean density and mean logarithmic density. The log-normal PDFs of column density data are used to predict the sonic Mach number (Burkhart et al. 2010; Price et al. 2011), the star formation rate (Krumholz & McKee 2005; Federrath & Klessen 2012), and the initial mass function (Hennebelle & Chabrier 2008, 2011) in both diffuse and dense ISM medium. In addition, in the presence of gravitational collapsing, the PDFs are believed to be shaped into a log-normal \( (P_N) \) format for low-density gas and a power-law \( (P_L) \) format for high-density gas (Körtgen et al. 2019; Vazquez-Semadeni et al. 1995; Robertson & Kravtsov 2008; Collins et al. 2012; Burkhart 2018):

\[ P_L(s) \propto e^{ks}, s > S_t \]

where \( S_t = \ln(\rho_t/\rho_0) = (-k - \frac{1}{2})\sigma_s^2 \) is the logarithm of the normalized transitional density between the \( P_N \) and \( P_L \). The transition from log-normal to power-law PDFs reveals the density threshold, above which the gas becomes self-gravitating.

However, molecular clouds are not optically thin objects. The effects of self-absorption that vary with both the density and the abundance of molecular species are essential for understanding the emission from molecular clouds (Ostriker et al. 2001; Tafalla et al. 2004). The prior numerical study shows that, in the cases of optically thick media, PDFs are dominated by the velocity dispersion and therefore do not represent the true density distribution in molecular clouds.
(Burkhart et al. 2013). The effects of radiative transfer, therefore, can make a big difference for observations of the measured intensities arising from the gravitational collapsing regions, making the measured intensity PDFs different from the underlying column density PDFs. In this work, we study how observed PDFs of measured intensities are affected by the radiative transfer in both numerical simulations and observations. We generate three synthetic emission lines of CO isotopologs, i.e., $^{12}$CO, $^{13}$CO, $^{18}$CO, utilizing the SPARX radiative transfer code (Hsieh et al. 2019).

In what follows, we theoretically illustrate the determination of CO isotopologs’ column density, in §2. In §3, we give the details of the numerical simulation used in this work. In §4, we present our numerical and observational results. We discuss the physical implication of the PDFs with presence of self-absorbing media in §5 and give our conclusion in §6.

2. THEORETICAL CONSIDERATION

2.1. Column density determination

The determination of CO isotopologs’ column densities in observation comes from the corresponding measurement of brightness temperature. To estimate the column densities, we assume that the emission a uniform excitation temperature, i.e., $\tau \rightarrow \infty$, and that $T_{\text{mb}}^{\text{CO}}$ is the main beam brightness temperature at the peak of $^{12}$CO’s velocity line, and we can derive the excitation temperature $T_{\text{ex}}$ following Pineda et al. (2008):

$$T_{\text{ex}} = \frac{T_0}{\ln(1 + T_0(T_{\text{mb}}^{\text{CO}} + \frac{T_{\text{bg}}}{e^{\frac{h\nu}{kT_{\text{ex}}}}})^{-1})}$$  \hspace{1cm} (3)

where $T_0 = \frac{h\nu}{k}$, $\nu(^{12}\text{CO}) = 115.3$ GHz is the frequency of $^{12}$CO ($J = 1$-0) transition line, $h$ is the Planck constant, $k_B$ is the Boltzmann constant, and $T_{\text{bg}} = 2.725$K is the CMB radiation temperature. The column densities of $^{12}$CO, $^{13}$CO and $^{18}$CO can be derived using the expression for column density in a single rotational level (Garden et al. 1991):

$$N_{\text{tot}} = \frac{3k_B}{8\pi^2 B\mu^2} T_{\text{ex}} + \frac{hB}{3k_B} \int \tau(v)dv$$  \hspace{1cm} (4)

where $\mu$ is the permanent dipole moment of the molecule and $B$ is the rotational constant. The integration of optical depth is performed along the line of sight (Buckle et al. 2010):

$$\int \tau(v)dv \simeq \frac{1}{J(T_{\text{ex}}) - J(T_{\text{bg}})} \int \tau(v)T_{MB} \frac{1}{1 - e^{-\tau(v)}} dv$$  

$$\simeq \frac{1}{J(T_{\text{ex}}) - J(T_{\text{bg}})} \frac{\tau(v_0)}{1 - e^{-\tau(v_0)}} \int T_{MB} dv$$  \hspace{1cm} (5)

where $v_0$ is the central velocity of the line, $T_{MB}$ is the observed main beam temperature and $J(T)$ is the source function:

$$J(T) = \frac{T_0}{e^{T_0/T} - 1}$$  \hspace{1cm} (6)

The column density of a molecular species is proportional to the integrated intensity along the line of sight:

$$N_{\text{tot}} \propto \int T_{MB} dv$$  \hspace{1cm} (7)

We can, therefore, use the integrated intensity map of spectroscopic data to study the mass density’s PDF of each corresponding molecular specie in this work.

3. SIMULATION DATA

3.1. MHD simulation

We simulate numerical 3D MHD simulations through the ZEUS-MP/3D code (Hayes et al. 2006), assuming a single fluid, operator-split, staggered grid MHD in Eulerian frame. Periodic boundary conditions and solenoidal turbulence injections are applied in our simulations.

The simulated interstellar clouds are isothermal with temperature $T = 10.0$ K and sound speed $c_s = 187$ m/s. To probe the relative importance of gravity and thermal pressure forces, we consider cloud with the size $L = 10$ pc and the initial density of $\rho_0$. We vary the value of $\rho_0$, magnetic field strength $B$ to stipulate different physical environments. The physical conditions are characterized by the Alfvénic Mach numbers $M_A = \sqrt{4\pi \rho_0 v_L / B}$, and the Sonic Mach number $M_S = v_L / c_s$, where $v_L$ is the injection velocity and $v_A$ is the Alfvénic velocity. In the case of $M_A < M_S$, the cloud is highly magnetized while $M_A > M_S$ corresponds to the thermal pressure dominates the cloud. The sound crossing time $t_c = L/c_s$ is $\sim 52.0$ Myr, which is fixed owing to the isothermal equation of state.

We list the simulations in Tab. 1. In the text and figures, we refer to the corresponding simulation by their model name. For simulation $M_A0.4$, we additionally consider the effect of self-gravity. We take the snapshot of this self-gravitating simulation at free-fall time $t_{ff} \approx 0.8$Myr.

In this work, molecular gas density and velocity information is extracted from the MHD data mentioned in Sec. 3.1.

| Model  | $M_S$ | $M_A$ | Resolution | Line of Transitions |
|--------|-------|-------|------------|--------------------|
| $M_A0.2$ | 7.31  | 0.22  | $^{12}$CO, $^{13}$CO, $^{18}$O |
| $M_A0.4$ | 6.10  | 0.42  | $^{12}$CO, $^{13}$CO, $^{18}$O |
| $M_A0.6$ | 6.47  | 0.61  | $^{12}$CO, $^{13}$CO, $^{18}$O |
| $M_A0.8$ | 6.14  | 0.82  | $^{12}$CO, $^{13}$CO, $^{18}$O |
| $M_A1.0$ | 6.03  | 1.01  | $^{12}$CO, $^{13}$CO, $^{18}$O |
| $M_A1.2$ | 6.08  | 1.19  | $^{12}$CO, $^{13}$CO, $^{18}$O |
| $M_A1.4$ | 6.24  | 1.38  | $^{12}$CO, $^{13}$CO, $^{18}$O |
| $M_A1.6$ | 5.94  | 1.55  | $^{12}$CO, $^{13}$CO, $^{18}$O |
| $M_A1.8$ | 5.80  | 1.67  | $^{12}$CO, $^{13}$CO, $^{18}$O |
| $M_A2.0$ | 5.55  | 1.71  | $^{12}$CO, $^{13}$CO, $^{18}$O |
| $M_A0.5$ | 0.47  | 0.15  | $^{12}$CO, $^{13}$CO, $^{18}$O |

Table 1. Description of our MHD simulations. $M_S$ and $M_A$ are the instantaneous values at each the snapshots are taken. The lowest-transition $J=1$-0 is considered for all CO isotopologs.
Figure 1. Examples of how the PDFs change their shape in the presence of self-absorption and in the absence of self-gravity. We consider three kinds of supersonic environments: sub-Alfvenic turbulence (the first row), trans-Alfvenic turbulence (the second row), and super-Alfvenic turbulence (the third row). Each simulation produces four synthetic observation cubes through the radiative transfer calculation, i.e., $^{12}$CO (the first column), $^{13}$CO (the second column), C$^{18}$O (the third column), and H$_2$ (the fourth column). $\tau$ is the optical depth, $k$ is the slope of a power-law fit, and $\rho_0$ is the mean density.

| $M_A$   | 0.22 | 0.42 | 0.61 | 0.82 | 1.01 | 1.19 | 1.38 | 1.55 | 1.67 | 1.71 |
|---------|------|------|------|------|------|------|------|------|------|------|
| $^{12}$CO | 2.56 | 1.37 | 0.74 | 0.48 | 0.33 | 0.21 | 0.17 | 0.13 | 0.10 | 0.08 |
| $^{13}$CO | 2.62 | 2.62 | 2.56 | 2.37 | 2.20 | 2.02 | 1.90 | 1.76 | 1.67 | 1.71 |
| C$^{18}$O | 0.61 | 0.17 | 0.08 | 0.05 | 0.03 | 0.02 | 0.02 | 0.01 | 0.01 | 0.01 |

Table 2. The summary of the optical depth $\tau$ and the shape of corresponding PDF for each model and molecular tracer $^{12}$CO, $^{13}$CO, C$^{18}$O without self-gravity. PDFs are classified as: (i) Left Power-Law (LPL): PDF shows power-law distribution at low-density range; (ii) Right Power-Law (RPL): PDFs appear a power-law distribution at high-density range; and (iii) Log-Normal (LN): PDFs show a single log-normal distribution.

The fractional abundances of the CO isotopologs $^{12}$CO, $^{13}$CO, and C$^{18}$O are set as $1 \times 10^{-4}$, $2 \times 10^{-6}$, and $1.7 \times 10^{-7}$, respectively, following Hsieh et al. (2019). Hence, the $^{12}$CO-to-H$_2$ ratio of $1 \times 10^{-4}$ comes from the cosmic value of C/H = $3 \times 10^{-4}$ and the assumption that 15% of C is in the molecular form. For the abundance of $^{13}$CO, we adopted a $^{13}$CO/$^{12}$CO ratio of 1/69 (Wilson 1999). Hence, the $^{13}$CO-to-H$_2$ ratio is approximately $2 \times 10^{-6}$. With $^{12}$CO/C$^{18}$O = 500 (Wilson et al. 2016), we obtain a C$^{18}$O-to-H$_2$ ratio of $1.7 \times 10^{-7}$. When producing the synthetic molecular channel maps, we focus on the lowest-transition $J$ = 1-0 of the CO isotopologs, in which the LTE condition is satisfied. The required (critical) density for thermally populating the $J$ = 1-0 of CO isotopologs is $\approx 10^{-3} cm^{-3}$, which is comparable to the molecular gas density in the diffuse ISM.

4. RESULTS

4.1. The column density PDFs of CO isotopologs in supersonic and subsonic turbulence

Fig. 1 shows an example of how the column density PDFs change their shape with the presence of self-absorbing media and the absence of self-gravity. We consider the supersonic turbulence $M_S \approx 6.0$ here. In the case of H$_2$, the PDFs exhibit an expected log-normal distribution, and its width is controlled by $M_S$. However, the PDFs become complicated when considering the effect of radiative transfer. For
Figure 2. Examples of how the column density PDFs change their shape in the presence of self-absorbing media. We consider the subsonic ($M_S = 0.47$) and sub-Alfvénic ($M_A = 0.15$) environment in the absence of self-gravity. $\tau$ is the optical depth and $\rho_0$ denotes the mean density $\langle \rho \rangle$.

The supersonic simulation $M_A = 0.22$, the PDF obtained from $^{12}$CO is not a log-normal distribution, but a power-law distribution. The PDFs of $^{13}$CO further change its shape to a combination of a log-normal distribution at high-density ranges and a power-law distribution at low-density ranges. The PDFs of $^{18}$CO also get different. It is more close to a single log-normal distribution, similar to the case of H$_2$. When physical conditions are changed to trans-Alfvénic and super-Alfvénic, i.e., $M_A \approx 1.0$ and $M_A > 1.0$, we find that the PDF of $^{12}$CO is getting close to the combination of a log-normal distribution at high-density ranges and a power-law distribution at low-density ranges. The PDF of $^{13}$CO tends to be a combination of a log-normal distribution and a power-law distribution, but at low-density ranges and high-density ranges respectively. The PDF of $^{18}$CO is in stable shape, i.e., the combination of a log-normal distribution at low-density ranges and a power-law distribution at high-density ranges. However, this change is expected to be observed in self-gravitating H$_2$ media. We classified the PDFs as (i) Left Power-Law (LPL): PDFs show power-law distribution at low-density range; (ii) Right Power-Law (RPL): PDFs show a power-law distribution at high-density range; and (iii) Log-Normal (LN): PDFs show a single log-normal distribution. In Tab. 2, we calculate the optical depth $\tau$ for each simulation, assuming the background radiation $T_{CMB} \approx 2.725 K$. We find the LPL PDFs only appear in the optically thick case that $\tau > 1.2$. As a result, the optically thick $^{12}$CO synthetic observation data always show LPL PDFs. As for the optically thin media, the PDFs are in LN shape when approximately $0.2 \leq \tau \leq 1.2$, while the PDFs are in LN shape in the case $\tau < 0.2$. Because $^{13}$CO and $^{18}$CO lines tend to have lower optical depths, they can trace molecular gas over
PDF IN THE PRESENCE OF SELF-ABSORPTION

Figure 3. Examples of how the PDFs change their shape in the presence of both self-absorption and self-gravity. Upper left figure depicts the PDF evolution for HI from $t_{ff} = 0$ to $t_{ff} = 0.8$ Myr. For the rest panels we take the snapshot of this self-gravitating simulation $M_\Lambda = 0.4$ at free-fall time $t_{ff} \approx 0.8$ Myr. We consider the J=1-0 transition lines of $^{13}$CO and change the abundance $^{13}$CO/H$_2$ to $2 \times 10^{-6}$ (upper right), $1 \times 10^{-4}$ (bottom left), and $1.7 \times 10^{-7}$ (bottom right) respectively. $\tau$ is the optical depth and $k$ is the slope of the power-law correlation.

a larger range of densities, while optically thick $^{12}$CO traces only the lower-density outer cloud regions. We, therefore, expect the various optical depth would lead to the change of PDFs.

In Fig. 2, we repeat the calculation of PDFs for subsonic turbulence ($M_S = 0.47$). The PDF of $^{12}$CO shows more similarity with the PDF of $^{13}$CO, while the PDF of C$^{18}$O is more close to H$_2$. However, although $^{12}$CO and $^{13}$CO are optically thick ($\tau > 1$), all of these PDFs follow a log-normal correlation. This is due to the effects of self-absorption in the gas. The cloud becomes optically thick before it can be fully sampled, thus masking out density enhancements that could be seen in the cloud.

We further explore the factors that leads to the change of PDFs. In Fig. 3, we additionally take into account the affect from self-gravity and change the abundance of $^{13}$CO, i.e., the ratio $^{13}$CO/H$_2$. For the H$_2$ media, we see that its PDF is a single log-normal distribution in the absence of self-gravity (i.e., $t_{ff} = 0$ Myr), while the PDF of high-density gas evolves into a power-law at $t_{ff} = 0.8$ Myr. As for the PDF of $^{13}$CO with $^{13}$CO/H$_2 = 2 \times 10^{-6}$, it is more close to a single LN distribution with $\tau = 1.23$. In the case of $^{13}$CO/H$_2 = 2 \times 10^{-4}$ ($\tau = 2.63$) which is the typical value of $^{12}$CO abundance (see §3), the PDF becomes a single LPL distribution which is similar to the PDF of $^{12}$CO shown in Fig. 1. We also set $^{13}$CO/H$_2 = 1.7 \times 10^{-7}$ ($\tau = 0.16$) which is the typical value of C$^{18}$O abundance (see §3). The resultant PDF is similar with the RPL PDF of C$^{18}$O shown in Fig. 1 and the PDF of self-gravitating H$_2$. It therefore confirms our earlier conclusion that the change of PDFs is caused by different optical depths of the media. The PDFs of self-absorbing media does not distinguish the self-gravitating regions.

4.2. The quantile-quantile plot

To quantify the change of PDFs, we introduce the quantile-quantile plot (i.e., Q-Q plot). The Q-Q plot is a graphical

\[ k = -3.27 \pm 0.08 \]

\[ M_\Lambda = 0.42 \\
M_S = 6.10 \]

\[ \tau = 2.63 \]

\[ k = 17.58 \pm 0.35 \]

\[ M_\Lambda = 0.42 \\
M_S = 6.10 \\
t_{ff} = 0.8 \text{ Myr} \\
\tau = 2.63 \]

\[ k = -6.80 \pm 0.18 \]

\[ M_\Lambda = 0.42 \\
M_S = 6.10 \\
t_{ff} = 0.8 \text{ Myr} \\
\tau = 0.16 \]
Table 3. Description of four molecular clouds used in our analysis. $v_{\text{max}}$ is the maximum value of LOS velocity and $v_{\text{min}}$ is the maximum value. $v_{\text{max}}$ and $v_{\text{min}}$ give the range of velocity selected for each molecular cloud.

| Molecular cloud | Vela C | SH2-235 | Ophiuchus | Serpens |
|-----------------|--------|---------|-----------|---------|
| Line of transitions | $^{12}\text{CO} (J = 1-0)$ | $^{12}\text{CO} (J = 2-1)$ | $^{12}\text{CO} (J = 1-0)$ | $^{12}\text{CO} (J = 2-1)$ |
|                  | $^{13}\text{CO} (J = 1-0)$ | $^{13}\text{CO} (J = 2-1)$ | $^{13}\text{CO} (J = 1-0)$ | $^{13}\text{CO} (J = 2-1)$ |
|                  | C$^{18}\text{O} (J = 1-0)$ | $^{12}\text{CO} (J = 3-2)$ | H$_2$ | H$_2$ |
| $v_{\text{max}}$ [km/s] | 14.89 | 0.70 | 6.57 | 12.78 |
| $v_{\text{min}}$ [km/s] | -2.64 | -29.00 | -1.05 | 0.78 |
| Data source      | Fissel et al. (2016) | Bieging et al. (2016) | Ridge et al. (2003) | André et al. (2010) |
|                  |        |        | André et al. (2010) |        |

Figure 4. The quantile-quantile plot (i.e., Q-Q plot) of the PDFs calculated from self-gravitating simulation $M_0.4$ at free-fall time $t_{ff} \approx 0.8\text{Myr}$ which has been used in Fig. 3. We use $R_0$ denotes the case that $^{13}\text{CO}/H_2 = 2.0 \times 10^{-6}$, $R_1$ denotes $^{13}\text{CO}/H_2 = 1.0 \times 10^{-4}$, and $R_2$ is $^{13}\text{CO}/H_2 = 1.7 \times 10^{-7}$. The dashed line with slope $\alpha = 1$ indicates the expected correlation if two distributions are identical. If the general trend of the QQ plot is flatter (i.e., $\alpha < 1$), the distribution plotted on the horizontal axis is more dispersed than the distribution plotted on the vertical axis. Conversely, if the general trend of the QQ plot is steeper (i.e., $\alpha > 1$) the distribution plotted on the vertical axis is more dispersed than the distribution plotted on the horizontal axis.

Figure 4 shows the change of density PDFs in the presence of self-absorption in observation, we select four molecular clouds: Vela C, SH2-235, Ophiuchus, and Serpens. Vela C is a massive ($\sim 10^5 M_\odot$), relatively nearby GMC(distance ~900 pc), which appears to be relatively young and un-evolved (Hill et al. 2011). The $^{12}\text{CO} (J = 1-0)$, $^{13}\text{CO} (J = 1-0)$, C$^{18}\text{O} (J = 1-0)$ data of Vela C are mapped from a large-scale molecular line survey made with the 22-m Mopra Telescope (Fissel et al. 2016). The SH2-235 molecular cloud complex is one of the most optically prominent H II regions with mass $\sim 5.2 \times 10^4 M_\odot$. SH2-235 is situated at a distance between 1.6 kpc and 2.5 kpc in the Perseus Spiral Arm. The Serpens cloud ($\sim 1450 M_\odot$) is a low-mass star-forming cloud in the Gould Belt at a distance ~415 pc. The $^{12}\text{CO} (J = 2-1)$, $^{13}\text{CO} (J = 2-1)$ data of SH2-235 and Serpens, including the
Figure 5. Observational examples of how the PDFs change their shape for molecular clouds (a) Vela C, and (b) SH2-235. Panel (a): the first row shows the intensity maps of each molecular tracer (left: $^{12}CO$ J = 1-0, middle: $^{13}CO$ J = 1-0, and right: $C^{18}O$ J = 1-0) for Vela C. The second row gives the intensity PDFs. The dashed black line is the best fitting for the power-law part of PDFs within 95\% confidential level and $k$ denotes its slope. $S_t$ indicated by dashed red line is the transition density at which the log-normal PDF changes to a power-law correlation or vice versa. We also plot the corresponding contours of $S_t$ in intensity maps. Panel (b): the notation follows Panel (a), but using $^{12}CO$ (J = 2-1, left), $^{13}CO$ (J = 2-1, middle), and $^{12}CO$ (J = 3-2, right) data for SH2-235.
\(^{12}\text{CO} (J = 3-2)\) data of SH2-235, are observed by the Arizona Radio Observatory Heinrich Hertz Submillimeter Telescope (Burleigh et al. 2013; Bieging et al. 2016). As for Ophiuchus, it is also a low-mass star-forming cloud located at a distance of \(\sim 145\) pc. The \(^{12}\text{CO} (J = 1-0)\) and \(^{13}\text{CO} (J = 1-0)\) data of Ophiuchus are obtained from the COMPLETE Survey of Star-Forming Regions (Ridge et al. 2003), while the H\(_2\) column density data for both Serpens and Ophiuchus are obtained from Herschel Gould Belt Survey (André et al. 2010). See Tab. 3 for more details about each data set. The velocity range is selected to cover the main structures of the cloud, and we mask the pixels which show negative value for plotting PDFs.

In Fig. 5 (a), we plot three intensity maps and PDFs of each molecular tracer \(^{12}\text{CO} (J = 1-0)\), \(^{13}\text{CO} (J = 1-0)\), and \(^{13}\text{CO} (J = 1-0)\) for Vela C respectively. For the intensity PDF of \(^{12}\text{CO} (J = 1-0)\), we see it appears as a power-law at low-intensity regions with the slope \(k = 2.32 \pm 0.10\). The transition density \(S_t\), at which the log-normal PDF changes to a power-law correlation or vice versa, is approximately -0.78. The intensity PDF of \(^{13}\text{CO} (J = 1-0)\) still exhibits as a power-law correlation at low-intensity regions, while the slope gets steeper \(k = 4.40 \pm 0.11\) and \(S_t \sim -1.04\). As for \(^{13}\text{CO} (J = 1-0)\) the PDF becomes completely different. It is a combination of log-normal at low-intensity region and power-law at high-intensity region. The slope of the negative power-law correlation is \(k = -4.13 \pm 0.46\) and \(S_t \sim 0.76\). The corresponding optical depth has been calculated by Fissel et al. (2019). They showed that the optical depth \(\tau_{18}\) for \(^{12}\text{CO}\) ranges from 0.015 to 0.18, with a median value of 0.026. Assuming a \([^{13}\text{CO}/^{12}\text{CO}]\) ratio of 10 and a \([^{12}\text{CO}/^{13}\text{CO}]\) ratio of 400, this implies a typical \(\tau_{12} \approx [^{13}\text{CO}/^{12}\text{CO}]\) in the range of 6 to 72, and \(\tau_3\) in the range of 0.15 1.8. The results agree with our numerical studies in §4.1.

We present the study of SH2-235 in Fig. 5 (b) using higher level transition lines. We plot three intensity maps and PDFs of molecular tracers \(^{13}\text{CO} (J = 2-1)\), \(^{13}\text{CO} (J = 2-1)\), and \(^{13}\text{CO} (J = 3-2)\). The intensity PDF of \(^{12}\text{CO} (J = 2-1)\) still appears as a power-law in low-intensity region, with \(k = 2.59 \pm 2.59\) and \(S_t \sim -0.79\). As for the high-intensity region, these three log-normal peaks indicating the existence of three independent components in the intensity map. In the case of \(^{13}\text{CO} (J = 2-1)\), its PDF gets changed significantly. The low-intensity part is log-normal, while the high-intensity part is power-law. The slope \(k = -1.65 \pm 0.08\) and \(S_t \sim 0.29\). The PDF of \(^{12}\text{CO} (J = 3-2)\) shows only log-normal features. Here, we follow the calculation of \(\tau_3\) used in Pineda et al. (2008), assuming that the excitation temperature of the \(^{13}\text{CO}\) line is the same as for the \(^{12}\text{CO}\) line:

\[
T_{ex} = \frac{5.5K}{\ln(1 + \frac{5.5K}{T_{max}^{5.5K} + 0.82K})}
\]

\[
\tau_3 = -\ln(1 - \frac{T_{13}^{13\text{CO}}}{T_{max}^{13\text{CO}}} / 1/(e^{5.5K/T_{ex}} - 1) - 0.16)
\]

where \(T_{12\text{CO}}^{max}\) and \(T_{13\text{CO}}^{max}\) are the main beam brightness temperatures at the peak of \(^{12}\text{CO}, ^{13}\text{CO}\)’s velocity lines respectively. From our calculation, the mean optical \(\tau_{13}\) for \(^{13}\text{CO}\) is 0.17. Assuming the \([^{12}\text{CO}/^{13}\text{CO}]\) ratio of 80 for SH2-235 (Bieging et al. 2016), we have \(\tau_{12} \sim |^{12}\text{CO} (J = 2-1)\)\(^{13}\text{CO} (J = 2-1)\)\] \(\tau_{13} = 13.60\). Since the \(J = 3-2\) line is excited at relatively higher temperatures than the \(J = 2-1\) line, it offers a useful probe of denser gas associated with regions of current star formation. The abundance of \(^{12}\text{CO} (J = 3-2)\) should be less than \(^{13}\text{CO} (J = 2-1)\). The \(^{13}\text{CO} (J = 3-2)\) mean brightness temperature of is 3.88K for \(^{12}\text{CO} (J = 2-1)\), 1.71K for \(^{13}\text{CO} (J = 3-2)\), and 0.63K for \(^{13}\text{CO} (J = 3-2)\) respectively (Bieging et al. 2016). We can therefore expect the mean optical depth \(\tau\) of \(^{12}\text{CO} (J = 3-2)\) is larger than \(\tau_{13}\) but less than \(\tau_{12}\). According to our numerical analysis, the value should in the range \(0.2 \leq \tau \leq 1.2\).

In addition to the two massive star forming regions Vela C and SH2-235, we verify our discovery in two low-mass star forming regions Ophiuchus and Serpens. For Ophiuchus shown in Fig. 6 (a), we use the \(^{12}\text{CO} (J = 1-0)\), \(^{13}\text{CO} (J = 1-0)\), and \(H_2\) column density data. The change of PDFs on Ophiuchus is more significant. The PDF of \(^{12}\text{CO} (J = 1-0)\) does not show the features of power-law in both low-intensity and high-intensity region, but more close to a log-normal format. As for \(^{13}\text{CO} (J = 1-0)\), the PDF becomes almost RPL in high-intensity region with the slope \(k = -0.83 \pm 0.05\) and \(S_t \sim -0.94\) which highlights a high-intensity region indicated by the red contour in Fig. 6 (a). We compare this high-intensity region with the one outlined by the PDF of \(H_2\). The PDF of \(H_2\) column density also shows power-law tail at high-intensity region, which is expected to be gravitational collapsing. We see there exit two power-laws with different slope, i.e., \(k_1 = -1.03 \pm 0.07\) and \(k_2 = -3.00 \pm 0.20\). The characteristic slope of that power law changes in roughly the cloud mean free fall time from steep \(k \sim -3\), to shallow \(-1.5 < k < -1\) (Burkhart et al. 2017; Girichidis et al. 2014). \(S_{t1}\) and \(S_{t2}\) therefore give two high-density regions under different collapsing stage (see Fig. 6 (a)). The PDF of \(^{12}\text{CO} (J = 1-0)\) provides no information about gravitational collapsing regions, while the high-intensity region identified from \(^{13}\text{CO} (J = 1-0)\)’s PDF is much wider than the one given by \(H_2\)’s PDF.

The situation gets changed for Serpens, see Fig. 6 (b). The PDF of \(^{12}\text{CO} (J = 2-1)\) is a single log-normal distribution. However, the PDF of \(^{13}\text{CO} (J = 2-1)\) is not a RPL distribution, but LPL with slope \(k \sim 3.28 \pm 0.14\). As a result, the transition density \(S_t \sim -0.93\) outlines a low-intensity region in the \(^{13}\text{CO} (J = 2-1)\) map, but not high-intensity region. According to our above studies, the PDF of \(^{13}\text{CO} (J = 2-1)\) is expected to change to a RPL distribution, since the optical depth \(\tau_{3}\) is thinner than \(\tau_{12}\). We thus expected the physical conditions on Serpens are different from Vela C, SH2-235, and Ophiuchus. There exist yet another factor that leads to the change of PDFs, which is not included in our numerical simulations. For instance, the effects of line-of-sight contamination on the column density structure also change the PDFs (Schneider et al. 2015a,b; Law et al. 2019) This issue will be addressed in our further studies. The \(H_2\)’s PDF still exhibits two power-laws with \(k_1 = -2.77 \pm 0.16\) and
Figure 6. Observational examples of how the PDFs change their shape for molecular clouds (a) Ophiuchus, and (b) Serpens. Panel (a): the first row shows the intensity maps of each molecular tracer (left: $^{12}\text{CO} J = 1-0$, middle: $^{13}\text{CO} J = 1-0$, and right: $\text{H}_2$) for Ophiuchus. The second row gives the intensity PDFs. The dashed black line is the best fitting for the power-law part of PDFs within 95% confidential level and $k$ denotes its slope. $S_1$ indicated by dashed red line is the transition density at which the log-normal PDF changes to a power-law correlation or vice versa. We also plot the corresponding red/green contours of $S_2/S_1$ in intensity maps. Panel (b): the notation follows Panel (a), but using $^{12}\text{CO} (J = 2-1$, left), $^{13}\text{CO} (J = 2-1$, middle), and $\text{H}_2$ (right) data for Serpens.
$k_2 = -1.79 \pm 0.14$. The slope at high-density region becomes shallower, which indicates that all the high-density gas having collapsed into isothermal cores (Burkhart, & Mocz 2019). The high-density regions determined by the PDF of $^{13}CO$ ($J = 2-1$) does not coincide with the regions identified from $S_1 \sim 0.22$ and $S_2 \sim 1.02$. The PDFs of CO isotopologs, therefore, are less efficient in identifying the gravitational collapsing regions in molecular clouds.

In Fig. 7, we employ the Q-Q plot to quantify our observational results. We choose the PDF of $^{13}CO$ as reference distribution. For Vela C, we see that the slope $\alpha$ of $^{12}CO$ is initially steeper than one but gets close to 1 at the end, i.e., high-intensity range. The PDF of $^{12}CO$ is, therefore, more dispersed than the one of $^{13}CO$ at low-intensity range. As for $^{13}CO$, the slope is getting steeper from low-intensity range to high-intensity range. The negative skew shape of $^{12}CO$ and positive skew shape of $^{13}CO$ indicates their PDFs are LPL and RPL, respectively. In terms of SH2-235, the slope $\alpha$ of $^{12}CO$ ($J = 2-1$) goes from steeper to shallower, while the $\alpha$ of $^{12}CO$ ($J = 3-2$) is close to a constant but shallower than $\alpha = 1$. Therefore, the PDF of $^{13}CO$ ($J = 2-1$) is more dispersed and the PDF of $^{13}CO$ ($J = 3-2$) is similar to a log-normal distribution. The $\alpha$ of H$_2$ shows a similar trend for both Serpens and Ophiuchus, i.e., goes from steeper to shallower. The PDFs of H$_2$ are, therefore, more dispersed than $^{13}CO$ in low-intensity range but less dispersed in high-intensity range. Also, the PDF of $^{12}CO$ ($J = 2-1$) on Serpens are less dispersed than $^{12}CO$ ($J = 2-1$). While the PDF of $^{12}CO$ ($J = 1-0$) goes from significant dispersed at low-intensity range to less dispersed at the high-intensity range.

5. DISCUSSION

5.1. Self-similarity of self-absorbing media

Burkhart et al. (2013) firstly studies the effects of the radiative transfer on the PDFs of molecular MHD turbulence. They numerically concluded that the integrated intensity maps of $^{13}CO$ ($J = 21$) transition line generally follow a
log-normal distribution, with the cases that have $\tau \approx 1$ best matching the PDF of the column density. Our work extends the study to three different CO isotopologs $^{12}CO$, $^{13}CO$, $C^{18}O$. Our results also show that the PDFs are in log-normal shape when $0.2 \leq \tau \leq 1.2$, which agrees with Burkhart et al. (2013). Later, Schneider et al. (2015a,b); Law et al. (2019) also reported the deviation of PDFs using observational dust extinction and CO isotopologs data. They concluded that this power-law correlation at low-density structures is due to the effects of line-of-sight contamination on the column density structure. However, Bialy et al. (2017) found that the shapes of PDFs do not follow a log-normal law at low-density range in photodissociation regions. They explained that the deviation is caused by the interaction of the propagating radiation with the nonuniform gas and the effect on the H$_2$ self-shielding. Here we see this deviation can also come from insufficient optical depth. We, therefore, expect there exist several factors leading to the LPL PDFs, which can explain the unexpected behavior of PDFs of Serpens.

The change of PDFs is only seen in the supersonic turbulence. We find this change is independent of molecular species, but the optical depth or molecular abundance. We expect that for optically thick cases, the cloud cannot be adequately sampled, thus masking out any shock enhancements. The power-law feature that appears in the PDFs is believed to be caused by the self-similarity in the medium. In the case of gravitational collapsing, all materials are accelerating into the center of gravity, and then the collapsing process leads to a power-law in the high-density range of corresponding PDFs. However, considering the self-absorption, there also exists a power-law feature in the low-density range of corresponding PDFs, even without self-gravity. It indicates the self-similarity appears in the low-density regions of the optically thick medium, for instance, $^{12}CO$. This self-similarity possibly comes from the effects of radiative transfer, although it requires further studies.

5.2. Identifying gravitational collapsing regions through velocity gradients

The power-law feature that appears in the PDFs is widely used to identify the gravitational collapsing regions in molecular clouds. However, our studies show the PDFs are valid only for H$_2$ column density data. It is still challenging to apply it to the intensity maps of molecular emissions. The Velocity Gradients Technique (VGT) provides an alternative way to solve this problem.

VGT is developed as a new technique to study the magnetic fields in ISM (González-Casanova & Lazarian 2017; Yuen & Lazarian 2017a; Lazarian & Yuen 2018a), based on the advanced understanding of MHD turbulence. The measurements from VGT is shown to reveal the POS and LOS magnetic field morphology (Lazarian & Yuen 2018b; Zhang et al. 2019), the magnetization level (Lazarian & Yuen 2018a), and turbulent properties of ISM gas (Hu et al. 2019c). It has been numerically and observationally tested in a wide range of column densities from diffuse neutral hydrogen media (Hu et al. 2018, 2019d,c; González-Casanova & Lazarian 2019; Yuen et al. 2019) to molecular self-absorbing dense gas (Hsieh et al. 2019; Hu et al. 2019a,b; González-Casanova et al. 2019). One of the most critical properties of velocity gradients is that the gradients flip their orientation by, 90°, i.e., get changed from perpendicular to magnetic fields to parallel with magnetic fields, in the cases of gravitational collapsing (Yuen & Lazarian 2017b; Hu et al. 2019a). The reaction of gradients with respect to self-gravity can, therefore, be used to identify gravitational collapsing regions.

Hu et al. (2019b) firstly showed the possibility of constructing the 3D magnetic fields model in molecular clouds utilizing different emission lines through VGT. For instance, the velocity gradients tell us about the POS component of the magnetic field over three different density ranges using $^{12}CO$, $^{13}CO$, and $C^{18}O$. We can, therefore, expect that VGT can reveal the density range in which the collapsing occurs. The corresponding work will be carried out by Hu et al. (2020, in prep.)

6. CONCLUSION

The density PDF has been wildly used to study the turbulent properties of ISM. It is also a useful tool in identifying the self-gravitating gas, calculating the star-forming rate in molecular clouds using H$_2$ column density data. In both numerical and observational, we extend the study of PDFs, taking into account the presence of self-absorption. To summarize:

1. For subsonic turbulence without self-gravity, the PDFs appear as a log-normal distribution for all three CO isotopologs $^{12}CO$, $^{13}CO$, $C^{18}O$.

2. For supersonic turbulence, the PDFs partially change its shape to a power-law distribution, which effectively depends on the abundance of molecule species or optical depth. In particular:

(a) The Left Power Law tail of PDFs appears in the optically thick case that $\tau > 1.2$;

(b) PDFs are in log-normal shape when $\tau$ is approximately in the range $0.2 \leq \tau \leq 1.2$;

(c) The Right Power Law tail of PDFs appear in the case of $\tau < 0.2$.

3. The observed shape of the PDFs is strongly affected by self-absorption. This can strongly interfere with the attempts to use the PDFs for identifying gravitational collapsing regions in molecular clouds.

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