Investigation of viscous dissipation on free convection in a vertical annular cylinder embedded with porous medium

N Ameer Ahmad1 and Abdulgaphur Athani2

1Mathematics Department, Faculty of Science, University of Tabuk, P.O.BOX. 741, Zip.71491, Saudi Arabia.
2Anjuman Institute of Technology and Management, Bhatkal, 581320 Karnataka India

Corresponding authors: n.ameer1234@gmail.com, abbu.bec@gmail.com

Abstract: In this paper, investigation of viscous dissipation on heat transfer by supplying heat at upper portion of the vertical annular cylinder is carried out. The governing partial differential equations are non-dimensionalised and solved using finite element method. The inner surface of the cylinder is maintained at isothermal temperature and the outer surface at ambient temperature. The top, bottom and horizontal surfaces of the vertical annular cylinder are adiabatic. The effect of viscous dissipation for different values of Rayleigh number is discussed.

Keywords: Porous medium, vertical annular cylinder, viscous dissipation, FEM

1. Introduction

Heat transfer in porous medium can be found in plenty of applications that has led to huge interest in scientific community over the years. Some of those applications can be listed as geothermal energy, underground coal gasification and gas drainage, petroleum reservoirs, nuclear reactors, drying, environmental pollution, fuel cells, nano-manufacturing and nano-material processing etc. Some of the popular books such as Nield and Bejan [1], Ingham and Pop [2], Vafai [3], Pop and Ingham [4] and Bejan and Kraus [5], in the area of porous medium gives deep insight of topic and various issues related with the subject. Free convection in porous medium bounded by various geometries has been studied extensively by researchers is well documented [6-23]. The heat transfer in porous medium is studied by employing two different strategies i.e. either considering a thermal equilibrium among the fluid and solid phase [10, 11, 17, 22-29] or also by considering the thermal discrepancy among those two phases [13, 16, 19, 30-35]. In case of thermal equilibrium, only one energy equation is used to represent fluid as well as solid temperature whereas two energy equations corresponding to fluid and solid phases are used in thermal non-equilibrium model. The viscous dissipation is another important phenomenon that affects the heat transfer behavior in porous medium due to friction between the fluid and solid matrix of medium which can be simulated by taking into account the fluid viscosity and velocity [36-39]. This can be assumed as internal heat generation when fluid and solid phases of porous medium interacts with each other leading to additional thermal energy inside the porous medium. This phenomenon is not dealt in detail as compared to only natural convection or some other phenomenon. The present study is undertaken to investigate the viscous dissipation on free convection in a vertical annular porous medium by using the Darcy model especially for the case of segmental heating of annulus. The emphasis is given to find out the heat and fluid flow behavior due to viscous dissipation.
2. Mathematical formulation
A vertical annular cylinder of inner radius $r_i$ and outer radius $r_o$, as depicted by the schematic diagram in Figure 1, is considered to investigate the heat transfer behavior subjected to viscous dissipation. The coordinate system is chosen such that the $r$-axis points towards the width and $z$-axis towards the height of the cylinder respectively. Because of the annular nature, two important parameters emerge which are aspect ratio $(A_r)$ and radius ratio $(R_r)$ of the annulus. They are defined as 

$$A_r = \frac{H}{r_o}, \quad R_r = \frac{r_o}{r_i}$$

Where $H$ is the height of the cylinder.

![Fig 1: Porous Annulus](image)

The simulation of heat and fluid flow in the porous medium embedded with a cylindrical geometry requires the governing equations to be solved in the cylindrical co-ordinates. It is worth mentioning that the flow in a cylindrical geometry can be described by taking only 2-dimensions due to axisymmetric nature of the cylinder. The following assumptions are considered:

- Porous medium is saturated with fluid.
- The convective fluid and the porous medium are everywhere in local thermodynamic equilibrium.
- The properties of the fluid and of the porous medium are homogeneous and isotropic.
- Fluid properties are constant except the variation of density.

The applicable governing equations in non-dimensional form are

$$\frac{\partial^2 \theta}{\partial z^2} + P \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \theta}{\partial r} \right) \right) = \overline{Pr} \frac{\partial \overline{T}}{\partial \overline{r}}$$

(1)
The corresponding boundary conditions are:

at \( r = r_l \) and \( \frac{H}{2} \leq Z \leq H \), \( T_w = 1 \) \hspace{1cm} (3)

at \( r = r_o \), \( T = T_o \) \hspace{1cm} (4)

Equation 1-2 subjected to boundary conditions 3-4 are converted into finite element equations using triangular elements. The resulting equations are solved with the help of an in-house developed computer code.

3. Results and discussion

The results are presented in terms of streamlines and isotherms. Figure 2 shows the streamlines and isothermal lines distribution inside the porous medium of the vertical annular cylinder for various values of viscous dissipation parameter \( (\varepsilon) \). As the viscous dissipation parameter \( (\varepsilon) \) increases, the streamlines and isothermal lines move away from the hot wall and reaches nearer to the cold wall of the vertical annular cylinder. When viscous dissipation is absent then the circulation of the fluid covers the whole domain of the vertical annular cylinder, i.e., by seeing figure 2a. The circulation of the fluid increases with the increase in viscous dissipation parameter \( (\varepsilon) \). This is due to the reason that the viscous dissipation parameter \( (\varepsilon) \) is basically production of heat due to local friction between moving fluid and the solid matrix of the porous medium. The generation of heat due to Viscous dissipation parameter \( (\varepsilon) \) effect increases the temperature inside the medium, which is reflected in terms of greater area of porous medium occupied by increased temperature lines at the upper portion of the vertical annular cylinder.

The figure 3 illustrates the streamlines and isothermal lines distribution inside the porous medium for various values of Rayleigh number. It is observed that in figure 3a, a weak cell is formed at the lower left corner of the vertical annular cylinder. However this weak cell disappears with the increase in Rayleigh number. The thermal boundary layer thickness decreases as Rayleigh number increases. The magnitude of the streamlines increases as the Rayleigh number increases. This is due to the reason that the increased Rayleigh number promotes the fluid movement due to higher buoyancy force, which in turn allows the convection heat transfer to take dominant position. The increased Rayleigh number particularly enhances the heat transfer rate at upper portion of hot and cold walls of vertical annular cylinder respectively. The fluid circulation moves towards the upper portion of cold wall when Rayleigh number is increased.
Fig 2: Streamlines (left) and Isotherms (Right) for $R_r = 1$, $A_r = 0.5$, $Ra=100$

a) $\varepsilon = 0$

b) $\varepsilon = 0.005$

c) $\varepsilon = 0.01$
Fig 3: Streamlines (Left) and Isotherms (Right) for $A_r=0.5$, $R_r=1$, $\varepsilon=0.01$

a) $Ra=25$ b) $Ra=50$ c) $Ra=100$
4. Conclusion
In the present study, heat transfer analysis in a vertical annular cylinder embedded with porous medium is carried out considering the viscous dissipation and free convection. Finite element method is applied to solve the governing equations. The following conclusions are drawn for the present study:

- It is found that in absence of viscous dissipation, the fluid covers the whole domain of the porous medium.
- As the viscous dissipation is increased, the isothermal lines are moving towards the cold surface from the hot surface, which increases the generation of heat in the fluid.
- The increase in Rayleigh number, increases the heat generation in the porous medium and enhances the fluid to move towards the cold surface of the vertical annular cylinder.

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