Table of Running Quark Mass Values: 1994

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Abstract

Running quark mass values $m_q(\mu)$ at some typical energy scales ($\mu = 1$ GeV, $\mu = m_W$, and so on) are reviewed. The values depend considerably on the value of $\Lambda_{MS}$, especially, the value of top quark mass at $\mu = 1$ GeV does so. The relative ratios of light quark masses ($m_u$, $m_d$ and $m_s$) to heavy quark masses ($m_c$, $m_b$ and $m_t$) are still controversial.
§1. Introduction

Recently, there has been considerable interest in phenomenological studies of quark and lepton mass matrices in order to obtain a clue to unified understanding of quarks and leptons. However, for this purpose, we must have the reliable knowledge of running quark mass values $m_q(\mu)$ which are evolved to an identical energy scale $\mu$ (e.g. $\mu = 1$ GeV). Since the earlier work by Gasser and Leutwyler [1], many works [2-5] on estimates of running quark masses have been reported. However, the values of $\Lambda_{\overline{MS}}$ which were adopted in these references [2-5] are not identical. Some of the input data have become older. On the other hand, this year (1994), the first observation [6] of top quark mass value has been reported, and the 1994 version of “Review of Particle Properties” (RPP94) [7] has been published. Therefore, this year is just timely for summarizing these works at present stage, and the review will be useful for physicists who intend to make a model-building of quarks and leptons.

In this review, we will give a summary table of running quark masses $m_q(\mu)$ at $\mu = 1$ GeV, $\mu = m_q$, $\mu = m_W$ and $\mu = \Lambda_W$, where $\mu = \Lambda_W$ is a symmetry breaking energy scale of the electroweak gauge symmetry SU(2)$_L \times$U(1)$_Y$.

$$\Lambda_W \equiv \langle \phi^0 \rangle = (\sqrt{2} G_F)^{-2} / \sqrt{2} = 174 \text{ GeV} . \quad (1.1)$$

In this paper, we use the mass renormalization equation

$$\frac{d}{d\mu} m_q(\mu) = -\gamma(\alpha_s) m_q(\mu) , \quad (1.2)$$

and do not use the renormalization equations for Yukawa couplings. This prescription is applicable only to the energy scale which is below the symmetry breaking energy scale $\Lambda_W$ of the electroweak gauge symmetry SU(2)$_L \times$U(1)$_Y$. If we want to evolve our results $m_q(\mu)$ to an extremely high energy scale far from $\mu = \Lambda_W$ (e.g. $\mu = \Lambda_{GUT}$), we must use the renormalization equations for Yukawa couplings.

In the next section, we review values of light quark masses $m_u(\mu)$, $m_d(\mu)$ and $m_s(\mu)$ at $\mu = 1$ GeV. In §3, we review values of heavy quark masses $m_c(\mu)$, $m_b(\mu)$ and $m_t(\mu)$ at $\mu = m_q$. In order to estimate $m_q(\mu)$ at any $\mu$, we must know the values of the QCD parameters $\Lambda_{\overline{MS}}^{(n)}$ (n=3,4,5,6). In §4, the values of $\Lambda_{\overline{MS}}^{(n)}$ are evaluated. In §5, the values of $m_q(\mu)$ at $\mu = 1$ GeV, $\mu = m_q$, $\mu = m_w$ and $\mu = \Lambda_W$
are estimated. Finally, §6 is devoted to summary and discussion.

§2. Light quark masses

Grasser and Leutwyler [1] have concluded in their review article of 1982 that the light quark masses $m_u(\mu)$, $m_d(\mu)$ and $m_s(\mu)$ at $\mu = 1$ GeV are

\[ m_u(1\text{GeV}) = 5.1 \pm 1.5 \text{ MeV}, \]
\[ m_d(1\text{GeV}) = 8.9 \pm 2.6 \text{ MeV}, \]
\[ m_s(1\text{GeV}) = 175 \pm 55 \text{ MeV}. \] (2.1)

On 1987, Domingues and Rafael [2] have re-estimated those values. They have obtained the same ratios of the light quark masses with those estimated by Grasser and Leutwyler, but they have used a new value of $(m_u + m_d)$ at $\mu = 1$ GeV

\[ (m_u + m_d)_{\mu=1\text{GeV}} = (15.5 \pm 2.0) \text{ MeV}, \] (2.2)

instead of Grasser–Leutwyler’s value $(m_u + m_d)_{\mu=1\text{GeV}} = (14 \pm 4) \text{ MeV}$. Therefore, Dominguez and Rafael have concluded as

\[ m_u(1\text{GeV}) = 5.6 \pm 1.1 \text{ MeV}, \]
\[ m_d(1\text{GeV}) = 9.9 \pm 1.1 \text{ MeV}, \]
\[ m_s(1\text{GeV}) = 199 \pm 33 \text{ MeV}. \] (2.3)

Narison (1989) [3] has obtained

\[ m_u(1\text{GeV}) = 5.2 \pm 0.5 \text{ MeV}, \]
\[ m_d(1\text{GeV}) = 9.2 \pm 0.5 \text{ MeV}, \]
\[ m_s(1\text{GeV}) = 159.5 \pm 8.8 \text{ MeV}, \] (2.4)

by using $(m_u + m_d)_{\mu=1\text{GeV}} = (14.4 \pm 1.0) \text{ MeV}$.

On the other hand, Donoghue and Holstein (1992) [4] have estimated somewhat different quark mass ratios

\[ r_1 = (m_u + m_d)/[m_s + (m_u + m_d)/2] = 0.061, \]
\[ r_2 = (m_d - m_u)/[m_s - (m_u + m_d)/2] = 0.036. \] (2.5)
which lead to

\[ m_d/m_u = 3.49, \quad m_s/m_d = 20.7. \]  

(2.6)

The value of \( m_d/m_u \) is considerably different from the previous values, e.g., Grasser-Leutwyler’s value \( m_d/m_u = 1.75 \). Donoghue and Holstein estimated the values (2.5) from the following four different sources: (1) \( r_1r_2 = 2.11 \times 10^{-3} \) from meson masses \( + (\Delta m_R^2)_{EM} \), (2) \( r_1r_2 = 2.35 \times 10^{-3} \) from \( \eta \to 3\pi \) decay, (3) \( r_1/r_2 = 0.67 \pm 0.16 \) from \( \psi' \to J/\psi + \pi^0(\eta) \), and (4) \( r_1 = 0.067 \pm 0.012 \) from meson masses and \( L_7 \). The values of \( r_1 \) and \( r_2 \) from these sources are still controversial.

Donoghue and Holstein’s value of \( m_s/m_d \) is in good agreement with that estimated by Dominquez and Rafael. Hereafter, we will adopt Dominquez-Rafael’s value (2.3) as light quark mass values at \( \mu = 1 \) GeV.

§3. Heavy quark masses at \( \mu = m_q \)

Pole mass

Sometimes, values of heavy quark masses \( m_c, m_b, \) and \( m_t \) are estimated in terms of the “pole” masses \( M_q^{\text{pole}} \). It is known that the pole mass, \( M_q^{\text{pole}}(p^2 = m_q^2) \), is a gauge-invariant, infrared-finite, renomalization-scheme-independent quantity.

Generally, mass function \( M(p^2) \), which is defined by [1]

\[ S(p) = Z(p^2)/ \left( M(p^2) - \not{p} \right), \]  

(3.1)

\[ Z(p^2) = 1 - \frac{\alpha_s}{3\pi}(a - 3b + \frac{2}{3})\lambda + O(\alpha_s^2), \]  

(3.2)

is related to

\[ M(p^2) = m(\mu) \left[ 1 + \frac{\alpha_s}{\pi}(a + \lambda b) + O(\alpha_s^2) \right], \]  

(3.3)

\[ a = \frac{4}{3} - \ln \frac{m^2}{\mu^2} + \frac{m^2 - p^2}{p^2} \ln \frac{m^2 - p^2}{m^2}, \]  

(3.4)

\[ b = -\frac{m^2 - p^2}{3p^2} \left( 1 + \frac{m^2}{p^2} \ln \frac{m^2 - p^2}{m^2} \right), \]  

(3.5)

where \( \lambda \) is given by \( \lambda = 0 \) in the Landau gauge and \( \lambda = 1 \) in the Feynman gauge. For \( p^2 = m^2 \), we obtain \( a = 4/3 \) and \( b = 0 \), so that we obtain the relation

\[ M_q^{\text{pole}}(p^2 = m_q^2) = m_q(m_q) \left( 1 + \frac{4}{3} \frac{\alpha_s}{\pi} + O(\alpha_s^2) \right). \]  

(3.6)
The estimate of the pole mass to two loops has been given by Gray et al [8]:

\[ m_q(m_q) = M_q^{\text{pole}}(p^2 = m_q^2) \left[ 1 - \frac{4}{3} \frac{\alpha_s(M_q^{\text{pole}})}{\pi} - \left(K - \frac{16}{9}\right) \left(\frac{\alpha_s(M_q^{\text{pole}})}{\pi}\right)^2 + O(\alpha_s^3)\right], \]

(3.7)

\[ K = K_0 + \frac{4}{3} \sum_{i=1}^{n-1} \Delta(M_i^{\text{pole}}/M_n^{\text{pole}}) \simeq 17.15 - 1.04n + \frac{4}{3} \times 1.04 \sum_{i=1}^{n-1} \frac{M_i^{\text{pole}}}{M_q^{\text{pole}}}. \]  

(3.8)

Here the sum in (3.8) is taken over \( n - 1 \) light quarks with masses \( M_i^{\text{pole}} (M_i^{\text{pole}} < M_q^{\text{pole}} \equiv M_n^{\text{pole}}) \). The exact expressions of \( K_0 \) and \( \Delta(r) \) are given in Ref. [8]. The numerical values of \( \Delta(M_i^{\text{pole}}/M_n^{\text{pole}}) \) without approximation are tabulated in Surguladze’s paper [9].

Similarly, for the spacelike value of \( p^2, p^2 = -m_q^2 \), we obtain \( a = 4/3 - 2\ln 2 \) and \( b = (2/3)(1 - \ln 2) \), so that we obtain the gauge-dependent “Euclidean” masses

\[ M_q^{\text{pole}}(p^2 = -m_q^2) = m_q(m_q) \left[ 1 + \frac{\alpha_s}{\pi} \left(\frac{4}{3} - 2\ln 2\right) + O(\alpha_s^2)\right]. \]  

(3.9)

**Charm and bottom quark masses**

Gasser and Leutwyler (1982) [1] have estimated charm and bottom quark masses \( m_c \) and \( m_b \) as

\[ m_c(m_c) = 1.27 \pm 0.05 \text{ GeV}, \quad \text{(3.10)} \]
\[ m_b(m_b) = 4.25 \pm 0.10 \text{ GeV}. \quad \text{(3.11)} \]

Narison (1989) [3] has, from \( \psi \)- and \( \Upsilon \)-sum rules, estimated those as

\[ M_c^{\text{pole}}(p^2 = -m_c^2) = 1.26 \pm 0.02 \text{ GeV}, \quad \text{(3.12)} \]
\[ M_b^{\text{pole}}(p^2 = -m_b^2) = 4.23 \pm 0.05 \text{ GeV}, \quad \text{(3.13)} \]

which mean

\[ M_c^{\text{pole}}(p^2 = m_c^2) = 1.45 \pm 0.05 \text{ GeV}, \quad \text{(3.14)} \]
\[ M_b^{\text{pole}}(p^2 = m_b^2) = 4.67 \pm 0.10 \text{ GeV}, \quad \text{(3.15)} \]

with \( \Lambda = 0.15 \pm 0.05 \text{ GeV}. \)

Dominguez and Paver (1992) [5] have estimated the value of \( m_b \) as

\[ M_b^{\text{pole}}(p^2 = m_b^2) = 4.72 \pm 0.05 \text{ GeV}, \quad \text{(3.16)} \]
from the ratio of Laplace transform QCD sum rules in the non-relativistic limit which is not so dependent on the value of \( \Lambda \).

Recently, Tirard and Yuduráin [10] have re-estimated charm and bottom quark masses precisely and rigorously. They have concluded that

\[
M_{c_{\text{pole}}}^2(p^2 = m_{c}^2) = 1.570 \pm 0.019 \mp 0.007 \text{ GeV} ,
\]

(3.17)

\[
M_{b_{\text{pole}}}^2(p^2 = m_{b}^2) = 4.906^{+0.069}_{-0.051} \mp 0.004^{+0.011}_{-0.040} \text{ GeV} ,
\]

(3.18)

\[
m_{c}(m_{c}) = 1.306^{+0.021}_{-0.034} \pm 0.006 \text{ GeV} ,
\]

(3.19)

\[
m_{b}(m_{b}) = 4.397^{+0.007-0.003+0.016}_{-0.002+0.004-0.032} \text{ GeV} ,
\]

(3.20)

where the first- and second-errors come from the use of the QCD parameter \( \Lambda_{\overline{\text{MS}}}^{(4)} = 0.20^{\pm 0.08}_{-0.06} \text{ GeV} \) and the gluon condensate value \( \langle \alpha_s G^2 \rangle = 0.042 \pm 0.020 \text{ GeV}^4 \), and the third error denotes a systematic error. They have used \( K_{c} \simeq 14.0 \) and \( K_{b} \simeq 13.4 \) as the values of \( K_{c} \) and \( K_{b} \) given by (3.8).

Hereafter, we adopt Tirard and Yuduráin’s values (3.19) and (3.20) as \( m_{c}(m_{c}) \) and \( m_{b}(m_{b}) \), although we do not adopt their value \( \Lambda_{\overline{\text{MS}}}^{(4)} = 0.20 \text{ GeV} \) as \( \Lambda_{\overline{\text{MS}}}^{(n)} \). For simplicity, we refer the values (3.19) and (3.20) as

\[
m_{c}(m_{c}) = 1.306^{+0.022}_{-0.035} \text{ GeV} ,
\]

(3.21)

\[
m_{b}(m_{b}) = 4.397^{+0.018}_{-0.033} \text{ GeV} .
\]

(3.22)

**Top quark mass**

Recently, the CDF collaboration (1994) [6] has reported the top quark mass value

\[
m_{t} = 174 \pm 10^{+13}_{-12} \text{ GeV}
\]

(3.23)

from the data of \( p\overline{p} \) collisions at \( \sqrt{s} = 1.8 \text{ TeV} \). The value (3.23) is consistent with the recent standard-model-fitting value [11]

\[
m_{t} = 161^{+15+16}_{-16-22} \text{ GeV} ,
\]

(3.24)

from LEP and \( p\overline{p} \) collider data.

We adopt the value (3.23) as the top quark mass value at \( \mu = m_{t} \). Hereafter, we will simply refer the value (3.23) as

\[
m_{t}(m_{t}) = 174^{+22}_{-27} \text{ GeV} .
\]

(3.25)
Note that usually the so-called standard-model-fitting value of $m_t$ does not correspond to $m_t(m_t)$ but to $M_t^{pole}(p^2 = m_t^2)$. The CDF value of $m_t(m_t)$, (3.25), together with the value of $\Lambda^{(5)}_{MS} = 0.195$ GeV [7] (see the next section), leads to

$$M_t^{pole}(p^2 = m_t^2) = 182^{+23}_{-28} \text{ GeV}. \quad (3.26)$$

§4. Estimates of the values of $\Lambda^{(n)}_{MS}$

Prior to estimates of the running quark masses $m_q(\mu)$, we must estimate the values of $\Lambda^{(n)}_{MS}$. The effective QCD coupling $\alpha_s = g_s^2/4\pi$ is controlled by the $\beta$-function:

$$\mu \frac{\partial \alpha_s}{\partial \mu} = \beta(\alpha_s), \quad (4.1)$$

where

$$\beta(\alpha_s) = -\frac{\beta_0}{2\pi} \alpha_s^2 - \frac{\beta_1}{4\pi^2} \alpha_s^3 + O(\alpha_s^4), \quad (4.3)$$

$$\beta_0 = 11 - \frac{2}{3} n_q, \quad \beta_1 = 51 - \frac{19}{3} n_q, \quad (4.4)$$

and $n_q$ is the effective number of quark flavors, so the $\alpha_s(\mu)$ is given by

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0} \frac{1}{L} \left[ 1 - \frac{2\beta_1}{\beta_0^2} \ln L + O(L^{-2} \ln^2 L) \right]. \quad (4.5)$$

where

$$L = \ln(\mu^2/\Lambda^2). \quad (4.6)$$

At present, we can use only the expression of $\alpha_s(\mu)$ where the higher order term $O$ in (4.5) is dropped. Then, the value of $\alpha_s(\mu)$ is not continuous at $n$th quark threshold $\mu_n$ (at which the $n$th quark flavor channel is opened), because the coefficients $\beta_0$ and $\beta_1$ in (4.2) depend on the effective quark flavor number $n_q$. Therefore, usually,

\footnote{In RPP94 [7] a three-loop expression of $\alpha_s(\mu)$ has been reviewed. However, at the moment, the two-loop expression (4.5) is sufficient for estimating running quark mass values to two-loops.}
we use the expression $\alpha_s^{(n)}(\mu) \ (4.5)$ with a different $\Lambda_{\overline{MS}}^{(n)}$ for each energy scale range $\mu_n \leq \mu < \mu_{n+1}$, where $\Lambda_{\overline{MS}}^{(n)}$ are defined such as $\Lambda_{\overline{MS}}^{(n-1)}$ and $\Lambda_{\overline{MS}}^{(n)}$ satisfy the relation

$$\alpha_s^{(n-1)}(\mu_n) = \alpha_s^{(n)}(\mu_n). \quad (4.7)$$

Therefore, we practically regard $n$th quark mass value $m_{qn}(\mu)$ at $\mu = m_{qn}, m_{qn}(m_{qn})$, as $\mu_n$.

Particle data group (PDG) [7] has concluded that the world average value of $\Lambda_{\overline{MS}}^{(5)}$ is

$$\Lambda_{\overline{MS}}^{(5)} = 195^{+65}_{-50} \text{MeV}. \quad (4.8)$$

On the other hand, in the conventional quark mass estimates since Gasser- Leutwyler [1], the value $\Lambda_{\overline{MS}}^{(3)} = 150 \text{MeV}$ is frequently used, although the value was used in the one-loop expression of $\alpha_s(\mu)$. For reference, we estimate $\Lambda_{\overline{MS}}^{(n)}$ and $m_q(\mu)$ for the case of $\Lambda_{\overline{MS}}^{(3)} = 150 \text{MeV}$ as well as the case of $\Lambda_{\overline{MS}}^{(5)} = 195 \text{MeV}$.

Starting from $\Lambda_{\overline{MS}}^{(5)} \equiv 0.195 \text{GeV}$, by using the continuity condition of $\alpha_s(\mu)$, (4.7), at $\mu_5 = m_b(m_b) = 4.397 \text{GeV}$, $\mu_4 = m_c(m_c) = 1.306 \text{GeV}$, and $\mu_6 = m_t(m_t) = 174 \text{GeV}$, we obtain $\Lambda_{\overline{MS}}^{(4)} = 0.28475 \text{GeV}$. $\Lambda_{\overline{MS}}^{(3)} = 0.33156 \text{GeV}$ and $\Lambda_{\overline{MS}}^{(6)} = 0.07760 \text{GeV}$. These results are summarized in Table IV.

Similarly, the values of $\Lambda_{\overline{MS}}^{(n)}$ are estimated for the case of $\Lambda_{\overline{MS}}^{(3)} \equiv 0.150 \text{GeV}$. The results are listed in Table IV.

Table IV. The values of $\Lambda_{\overline{MS}}^{(n)}$ in unit of GeV and $\alpha_s(\mu_n)$.

The underlined values are input values. Here, $\mu_4 = m_c(m_c) = 1.306 \text{GeV}$, $\mu_5 = m_b(m_b) = 4.397 \text{GeV}$, $\mu_6 = m_t(m_t) = 174 \text{GeV}$, and $m_Z = 91.187 \text{GeV}$ are used.
§5. Estimates of running quark masses

The scale dependence of a running quark mass $\mu_q(\mu)$ is determined by the equation

$$\mu \frac{d}{d\mu} m_q(\mu) = -\gamma(\alpha_s) m_q(\mu) , \quad (5.1)$$

where

$$\gamma(\alpha_s) = \alpha_s \gamma_0 + \alpha_s^2 \gamma_1 + O(\alpha_s^3) , \quad (5.2)$$

$$\gamma_0 = 2 , \quad \gamma_1 = \frac{101}{12} - \frac{5}{18} n_q , \quad (5.3)$$

so that $m_q(\mu)$ is given by

$$m_q = \tilde{m}_q \left( \frac{1}{2} L \right)^{-2 \gamma_0 / \beta_0} \left[ 1 - \frac{2 \beta_1 \gamma_0 \ln L + 1}{\beta_0 L} + \frac{8 \gamma_1}{\beta_0 L} + O(L^{-2} \ln^2 L) \right] , \quad (5.4)$$

where $\beta_0$ and $\beta_1$ are given in (4.3) and $L = \ln(\mu^2 / \Lambda^2)$. Here, $\tilde{m}_q$ is the renormalization group invariant mass, which is independent of $\ln(\mu^2 / \Lambda^2)$.

Since we interest only in the ratios $m_q(\mu) / \tilde{m}_q$, we define the following quantity

$$R^{(n)} = \left( \frac{1}{2} L \right)^{-2 \gamma_0 / \beta_0} \left( 1 - \frac{2 \beta_1 \gamma_0 \ln L + 1}{\beta_0 L} + \frac{8 \gamma_1}{\beta_0 L} \right) . \quad (5.5)$$

The value of $R^{(n)}$ is not continuous at $\mu = \mu_n$ ($\mu_n$ is the $n$th quark flavor threshold). Therefore, we calculate the evolution of the quark masses $m_q(\mu)$ from $\mu = \mu_A$ ($\mu_m \leq \mu_A < \mu_{m+1}$) to $\mu = \mu_B$ ($\mu_n \leq \mu_B < \mu_{n+1}$) as follows:

$$\frac{m_q(\mu_B)}{m_q(\mu_A)} = \left( \frac{R^{(m)}(\mu_{m+1})}{R^{(m)}(\mu_A)} \right) \left( \frac{R^{(m+1)}(\mu_{m+2})}{R^{(m+1)}(\mu_{m+1})} \right) \cdots \left( \frac{R^{(n-1)}(\mu_n)}{R^{(n-1)}(\mu_{n-1})} \right) \left( \frac{R^{(n)}(\mu_B)}{R^{(n)}(\mu_n)} \right) . \quad (5.6)$$

For example, the ratio $m_t(m_W) / m_t(1\text{ GeV})$ is given by

$$\frac{m_t(m_W)}{m_t(1\text{ GeV})} = \left( \frac{R^{(3)}(m_t)}{R^{(3)}(1\text{ GeV})} \right) \left( \frac{R^{(4)}(m_b)}{R^{(4)}(m_c)} \right) \left( \frac{R^{(5)}(m_W)}{R^{(5)}(m_b)} \right) . \quad (5.7)$$
The values of $R^{(4)}(m_b)/R^{(4)}(m_c)$, $R^{(5)}(m_t)/R^{(5)}(m_b)$, and so on are summarized in Table V.

Table V. Values of $R^{(\mu)}(\mu)$ for the case I ($\Lambda^{(5)}_{\overline{MS}} = 0.195$ GeV) and the case II ($\Lambda^{(3)}_{\overline{MS}} = 0.150$ GeV).

|                | $\Lambda^{(5)} = 0.195$ GeV | $\Lambda^{(3)} = 0.150$ GeV |
|----------------|-----------------------------|-----------------------------|
| $R^{(3)}(1\text{GeV})$ | 1.00886 $\equiv$ 1         | 0.738347 $\equiv$ 1         |
| $R^{(3)}(m_c)$    | 0.882993 0.87524            | 0.69043 0.93510            |
| $R^{(4)}(m_c)$    | 0.84169 $\equiv$ 1         | 0.64410 $\equiv$ 1         |
| $R^{(4)}(m_b)$    | 0.60363 0.71716             | 0.52206 0.81052            |
| $R^{(5)}(m_b)$    | 0.55141 $\equiv$ 1         | 0.47152 $\equiv$ 1         |
| $R^{(5)}(m_W)$    | 0.38415 0.69667             | 0.35418 0.75115            |
| $R^{(5)}(m_t)$    | 0.36036 0.65353             | 0.33529 0.71111            |
| $R^{(6)}(m_t)$    | 0.31051 $\equiv$ 1         | 0.28700 $\equiv$ 1         |
| $R^{(6)}(\Lambda_W)$ | 0.31051 1.00000            | 0.28700 1.00000            |

In Table VI, we summarize the running quark mass values at $\mu = m_q, \mu = 1$ GeV, $\mu = m_W (= 80.22$ GeV) and $\mu = \Lambda_W (= 174$ GeV), where $\Lambda_W$ is defined by

$$\Lambda_W \equiv \langle \phi^0 \rangle = (\sqrt{2}G_F)^{-\frac{1}{2}}/\sqrt{2} = 174 \text{ GeV} \ .$$  (5.8)
Table VI. Running quark mass values $m_q(\mu)$ (in unit of GeV) at
$\mu = m_q$, $\mu = 1$ GeV, $\mu = m_W = 80.22$ GeV and $\mu = \Lambda_W = 174$ GeV.
The upper values (lower values) are the running quark mass values in
the case of $\Lambda^{(5)} \equiv 0.195$ GeV (the case of $\Lambda^{(3)} \equiv 0.150$ GeV).

|   | $m_q(m_q)$ | $m_q(1\text{GeV})$ | $m_q(m_W)$ | $m_q(\Lambda_W)$ |
|---|-----------|-------------------|-----------|------------------|
| $m_u$ | $0.3463_{-0.0018}^{+0.0017}$ (0.1631_{-0.0015}^{+0.0015}) | 0.0056 ± 0.0011 (0.0056 ± 0.0011) | 0.00245 ± 0.00048 (0.00319 ± 0.00063) | 0.00230 ± 0.00045 (0.00302 ± 0.00059) |
| $m_d$ | $0.3524_{-0.0015}^{+0.0013}$ (0.169 ± 0.019) | 0.0099 ± 0.0011 (0.0099 ± 0.0011) | 0.00433 ± 0.00048 (0.00564 ± 0.00063) | 0.00406 ± 0.00045 (0.00534 ± 0.00059) |
| $m_s$ | 0.489 ± 0.021 (0.338 ± 0.029) | 0.199 ± 0.033 (0.199 ± 0.033) | 0.087 ± 0.014 (0.113 ± 0.019) | 0.082 ± 0.014 (0.107 ± 0.018) |
| $m_c$ | $1.306_{-0.035}^{+0.022}$ (1.306_{-0.022}^{+0.022}) | 1.492_{-0.037}^{+0.023} (1.397_{-0.024}^{+0.024}) | 0.653_{-0.017}^{+0.009} (0.795_{-0.021}^{+0.013}) | 0.612_{-0.016}^{+0.010} (0.753_{-0.023}^{+0.013}) |
| $m_b$ | $4.397_{-0.033}^{+0.018}$ (4.397_{-0.033}^{+0.018}) | 7.005_{-0.053}^{+0.029} (5.801_{-0.044}^{+0.024}) | 3.063_{-0.023}^{+0.013} (3.303_{-0.025}^{+0.014}) | 2.874_{-0.022}^{+0.012} (3.127_{-0.023}^{+0.013}) |
| $m_t$ | $174_{-27}^{+22}$ (174_{-27}^{+22}) | 424_{-66}^{+54} (323_{-50}^{+41}) | 185_{-29}^{+23} (184_{-29}^{+23}) | 174_{-27}^{+22} (174_{-27}^{+22}) |

§6. Summary

We have estimated running quark mass values $m_q(\mu)$ at $\mu = m_q$, $\mu = 1$ GeV,
$\mu = m_W = 80.22$ GeV and $\mu = \Lambda_W = 174$ GeV for the two cases, $\Lambda^{(5)} = 0.195$ GeV
($\Lambda^{(3)} = 0.332$ GeV, $\Lambda^{(4)} = 0.285$ GeV, $\Lambda^{(6)} = 0.0776$ GeV) and $\Lambda^{(3)} = 0.150$ GeV
($\Lambda^{(4)} = 0.116$ GeV, $\Lambda^{(5)} = 0.0716$ GeV, $\Lambda^{(6)} = 0.0256$ GeV). Of course, the case of
$\Lambda^{(3)} = 0.150$ GeV has been listed only for reference, it is not our conclusion.

We have adopted the following quark mass values as the input values:
for light quark masses, Domínez-Rafael’s values:

\[
\begin{align*}
m_u(1\text{GeV}) &= 5.6 \pm 1.1 \text{ MeV}, \\
m_d(1\text{GeV}) &= 9.9 \pm 1.1 \text{ MeV}, \\
m_s(1\text{GeV}) &= 199 \pm 33 \text{ MeV},
\end{align*}
\]

for charm and bottom quarks, Tirard and Yuduráin’s values:

\[
\begin{align*}
m_c(m_c) &= 1.306^{+0.022}_{-0.035} \text{ GeV}, \\
m_b(m_b) &= 4.397^{+0.018}_{-0.033} \text{ GeV},
\end{align*}
\]

and, for top quark mass, CDF value:

\[
m_t = 174^{+22}_{-27} \text{ GeV}.
\]

The results are summarized in Table VI. As seen in Table VI, the running quark mass values (especially, those of heavy quarks at \(\mu = 1\) GeV, and those of light quarks at \(\mu = m_W\) and \(\mu = \Lambda_W\)) are highly dependent on the value of \(\Lambda_{\overline{\text{MS}}}\). The value of \(\Lambda_{\overline{\text{MS}}}\) given in (4.8) includes large error values, so that the absolute values of quark masses in Table VI are not conclusive.

Although in Table VI, the values of \(m_q(m_q)\) for light quarks are listed, those values, especially those for \(u\) and \(d\), should not be taken rigidly, because \(\alpha_s(\mu)\) rapidly increases at \(\mu \leq m_s\), so that the perturbative result \(R^{(n)}(\mu)\), (5.5), becomes unreliable in such a region.

The relative ratios among light quark masses at \(\mu = 1\) GeV are fairly reliable, while the absolute values \(m_q(1\text{GeV})\) are still controversial. The relative ratios of light quark masses to heavy quark masses may be somewhat changed in future.

In this paper, we have evaluated \(m_q(\mu)\) only for energy scales \(\mu\) which are below the electroweak symmetry breaking energy scale \(\Lambda_W\). Running quark mass values at such an extremely high energy scale far from \(\Lambda_W\) will be given elsewhere.

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