Three-point non-associative supersymmetry generalization and weak version of non-commutative coordinates

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We consider a non-associative generalization of supersymmetry based on three-point associators \([Q_x, Q_y, Q_z]\) for \(Q_{a, \dot{a}}\) supersymmetric generators. Such associators are connected with the product \(Q_{a, \dot{a}} P_{\mu}\). We show that such associators lead to a weak version of non-commutative spacetime: in front of the coordinate commutator there is a prefactor.

PACS numbers: 11.30.Pb; 02.40.Gh
Keywords: supersymmetry, nonassociativity

I. INTRODUCTION

Whether non-associativity could play a fundamental role in the formulation of physics is a question that has been raised from time to time. In Ref. [1–3] the authors develop and investigate the quantization techniques for describing the nonassociative geometry probed by closed strings. In Ref. [4] non-associative structures have appeared in the study of D-branes in curved backgrounds are investigated.

In Ref.[5] we have discussed a possible generalization of supersymmetry with associator having four factors. Here we want to discuss a possible generalization of supersymmetry with associators having three factors (three-point associator). We will assume that such associators are connected with coordinates.

II. ASSOCIATIVE SUPERSYMMETRIC PRELIMINARIES

In this section we would like to remember basic properties of the simplest supersymmetry algebra (see, for example, textbook [6]). The most important for us is the anticommutator for \(Q_a, Q_{\dot{a}}\) supersymmetry generators

\[
\{Q_a, Q_{\dot{a}}\} = Q_a Q_{\dot{a}} + Q_{\dot{a}} Q_a = 2 \sigma_{a\dot{a}}^\mu P_\mu,
\]

and all other commutators and anticommutators are zero

\[
\{Q_a, Q_b\} = \{Q_{\dot{a}}, Q_{\dot{b}}\} = 0, \quad [Q_a, P_\mu] = [Q_{\dot{a}}, P_\mu] = 0,
\]

\[
[P_\mu, P_\nu] = 0,
\]

here \(P_\mu = -i \partial_\mu\) is the momentum operator; \(\mu = 0, 1, 2, 3; \ a = 1, 2; \ \dot{a} = \dot{1}, \dot{2}\). Pauli matrices \(\sigma^\mu_{a\dot{a}}, \sigma^{a\dot{a}}_\mu\) are defined in the standard way

\[
\sigma^\mu_{a\dot{a}} = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right\}, \quad \sigma^{a\dot{a}}_\mu = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\}
\]

with orthogonality relations for Pauli matrices

\[
\sigma^\mu_{a\dot{a}} \sigma^{\nu\dot{a}}_{a\dot{a}} = 2 \delta^\nu_\mu, \quad \sigma^{a\dot{a}}_\mu \sigma^{a\dot{a}}_\mu = 2 \delta^a_{\dot{a}} \delta^\dot{a}_a.
\]

Following the idea of Ref. [5], we want to show that one can generalize supersymmetry in such a way that the supergenerators \(Q_a, Q_{\dot{a}}\) will become non-associative ones.

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III. THREE-POINT NON-ASSOCIATIVE GENERALIZATION OF THE SUPERSYMMETRY ALGEBRA

In Ref. [5] we have shown that it is possible to extend supersymmetry in such way that the generators $Q_a, \bar{Q}_{\dot{a}}$ become non-associative ones. It was done by introducing a four-point associator in the following way

$$[Q_a, Q_\xi, (Q_b Q_y)] = 2\zeta_0 \frac{\hbar}{\ell_0} \sigma_{\xi a}^\mu \sigma_{\xi b}^\nu M_{\mu \nu},$$

(8)

where $\zeta_0 = \pm i, \pm 1$, $\ell_0$ is some characteristic length, and the coefficient $\hbar/\ell_0^2$ has been chosen so that the right- and left-hand sides of (5) have the same dimensions [9]. Here we have introduced the associator as

$$[A, B, C] = (AB) C - A (BC).$$

(9)

Now we want to demonstrate that it is possible to introduce another generalization of supersymmetry based on a three-point associator. Let us define the following three-point associators:

$$[Q_y, Q_a, Q_\xi] = \zeta_0 \xi Q_y x_{a\xi},$$

(10)

$$[Q_\xi, Q_{\bar{a}}, Q_y] = \zeta_0 \xi x_{a\bar{a}} Q_y,$$

(11)

$$[Q_a, Q_b, Q_c] = [Q_{a\bar{a}}, Q_{b\bar{b}}, Q_{c\bar{c}}] = [Q_{a\bar{a}}, Q_{b\bar{b}}, Q_c] = [Q_{a\bar{a}}, Q_b, Q_c] = 0,$$

(12)

where the index $y = b, \bar{b}$ can be any combination of dotted and undotted indices, and

$$x_{a\bar{a}} = \sigma_{\mu a\bar{a}} \xi^\mu,$$

(13)

where $x_{a\bar{a}}, x^\mu$ are operators. The simple dimensional analysis of equations (11) and (12) shows that the dimensions of $\xi$ is

$$[\xi] = \frac{g}{\ell}.$$

(14)

Again, as in Ref. [7], we think that the relations (10) and (11) should be quantum ones and consequently have to contain the Planck constant:

$$\zeta \propto \frac{\hbar}{\ell_0^2},$$

(15)

where $\ell_0$ is some characteristic length which should be constructed from the fundamental physical constants, for example it can be done as $\ell_0^{-2} = \Lambda$ where $\Lambda$ is the cosmological constant.

IV. A WEAK VERSION OF NON-COMMUTATIVE COORDINATES

Let us calculate the anticommutator

$$\frac{1}{\zeta_0 \xi} \left\{ [Q_y, Q_a, Q_\xi], [Q_z, Q_b, Q_\xi] \right\} = (Q_y x_{a\bar{a}}) (Q_z x_{b\bar{b}}) + (Q_z x_{b\bar{b}}) (Q_y x_{a\bar{a}}).$$

(16)

Let us assume that the operators $x_{a\bar{a}}$ are in an associative subalgebra of an non-associative algebra of $Q_a, Q_\xi$. Then we can omit the brackets on the right-hand side of (16). Introducing the commutator

$$[x_{a\bar{a}}, Q_z] = \alpha_{a\bar{a}, z},$$

(17)

we obtain

$$\frac{1}{\zeta_0 \xi} \left\{ [Q_y, Q_a, Q_\xi], [Q_z, Q_b, Q_\xi] \right\} = Q_y Q_z x_{a\bar{a}} x_{b\bar{b}} + Q_z Q_y x_{b\bar{b}} x_{a\bar{a}} + \alpha_{a\bar{a}, z} + \alpha_{b\bar{b}, y}.$$

(18)

Let us choose the indices $y, z$ both either dotted or undotted. Then, using the anticommutator (2), we will have

$$Q_y Q_z \left[ x_{a\bar{a}}, x_{b\bar{b}} \right] = \frac{1}{\zeta_0 \xi} \left\{ [Q_y, Q_a, Q_\xi], [Q_z, Q_b, Q_\xi] \right\} - Q_y \alpha_{a\bar{a}, z} x_{b\bar{b}} - Q_z \alpha_{b\bar{b}, y} x_{a\bar{a}}.$$

(19)
Using the definition (13) and the inverse relations (7), we will obtain a weak version of non-commutativity for coordinates

\[ Q_y Q_z \{ x^\mu, x'^\nu \} = i \theta^\mu_\nu y z, \]

(20)

where the matrix \( \theta^\mu_\nu y z \) is defined as

\[ i \theta^\mu_\nu y z = \frac{1}{4} \sigma^{\alpha a \dot{\alpha}} \sigma^{\beta b \dot{\beta}} \left( \frac{1}{\zeta_0} \left\{ [Q_y, Q_\alpha, Q_\beta], [Q_z, Q_\beta, Q_\dot{\alpha}] \right\} - Q_y \alpha_{a \dot{a}}, z x_{b \dot{b}} - Q_z \alpha_{b \dot{b}}, y x_{a \dot{a}} \right). \]

(21)

The relation (20) have to be compared with the standard definition of non-commutativity of spacetime that can be encoded in the commutator of operators corresponding to space-time coordinates [8, 9]:

\[ [x^\mu, x'^\nu] = i \theta^{\mu \nu}, \]

(22)

where \( \theta^{\mu \nu} \) is an antisymmetric matrix. We see that the relation (20) looks like a weak version of commutator for non-commutative coordinates in the consequence of the prefactor \( Q_y Q_z \) in front of the commutator \([x^\mu, x'^\nu]\).

V. DISCUSSION AND CONCLUSIONS

Thus, here we have suggested a non-associative generalization of supersymmetry with three-point associators. The main question in this approach: is the operator \( x^\mu \) from relations (10) and (11) the same or similar to quantized space-time coordinates à la Snyder [8] and non-commutative geometry [9]? If yes, then such non-associative generalization of supersymmetry in some weak sense leads to an interesting connection with non-commutativity of coordinates. The relation is that, in our approach, the commutator of spacetime coordinates has some supersymmetric prefactor.

Acknowledgements

This work was supported by the Volkswagen Stiftung and by a grant No. 0263/PCF-14 in fundamental research in natural sciences by the Ministry of Education and Science of Kazakhstan. I am very grateful to V. Folomeev and A. Deriglazov for fruitful discussions and comments.

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