Precision flavour physics with $B \to K\nu\bar{\nu}$ and $B \to Kl^+l^-$

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Abstract
We discuss how the combined analysis of $B \to K\nu\bar{\nu}$ and $B \to Kl^+l^-$ can provide us with new physics tests practically free of form factor uncertainties. Residual theory errors are at the level of several percent. This study underlines the excellent motivation for measuring these modes at a Super Flavour Factory, or, in the case of $B \to Kl^+l^-$, also at a hadron collider.

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1. INTRODUCTION

One of the best opportunities for precision tests of flavour physics will be provided by the study of $b \to s \nu \bar{\nu}$ transitions, induced by interactions at very short distances. A measurement of the inclusive decay $B \to X_s \nu \bar{\nu}$, ideal from a theory perspective, appears to be extremely challenging experimentally. More promising is the measurement of exclusive channels such as $B \to K \nu \bar{\nu}$, $B \to K^* \nu \bar{\nu}$. In this case a clean theoretical interpretation requires, however, the control of non-perturbative hadronic form factors. Direct calculations of form factors suffer from sizable uncertainties. These can be greatly reduced through a combined analysis of the rare decays $B \to K \nu \bar{\nu}$ and $B \to Kl+\bar{l}^-$. This option allows us to construct precision observables for testing the standard model and for investigating new physics effects. In particular neither isospin nor $SU(3)$ flavour symmetry are required and form factor uncertainties can be eliminated to a large extent. A detailed study has recently been given in [1], where further details can be found. Similar ideas had been considered independently in [2].

From experiment only upper limits are available for the branching ratios of the neutrino modes: [3] [4] [5] [6]

\begin{align}
B(B^- \to K^- \nu \bar{\nu}) &< 14 \cdot 10^{-6} \tag{1} \\
B(B^0 \to K^0 \nu \bar{\nu}) &< 160 \cdot 10^{-6} \tag{2}
\end{align}

The most accurate experimental results for $B \to Kl+\bar{l}^-$ are from Belle [7]. The extrapolated, non-resonant branching fraction is measured to be

$$B(B \to Kl+\bar{l}^-) = (0.48^{+0.05}_{-0.04} \pm 0.03) \cdot 10^{-6} \tag{3}$$

consistent with results from BaBar [8]. The recent paper [7] also contains information on the $q^2$-spectrum in terms of partial branching fractions for six separate bins. Similar results from CDF were reported in [9].

2. THEORY OVERVIEW

2.1. Dilepton-mass spectra and short-distance coefficients

We define the kinematic quantities $s = q^2/m_B^2$ (where $q^2$ is the dilepton invariant mass squared), $r_K = m_K^2/m_B^2$, and

$$\lambda_K(s) = 1 + r_K^2 + s^2 - 2r_K - 2s - 2r_K s \tag{4}$$

The differential branching fractions for $\bar{B} \to \bar{K} \nu \bar{\nu}$ and $\bar{B} \to Kl^+\bar{l}^-$ can then be written as

\begin{align}
\frac{dB(\bar{B} \to \bar{K} \nu \bar{\nu})}{ds} & = \tau_{\bar{B}} \frac{G_F^2 \alpha^2 m_B^5}{256\pi^3} |V_{ts}V_{tb}|^2 \cdot \lambda_K^{3/2}(s) f_L^2(s) |a(K\nu\bar{\nu})|^2 \\
\frac{dB(\bar{B} \to Kl^+\bar{l}^-)}{ds} & = \tau_{\bar{B}} \frac{G_F^2 \alpha^2 m_B^5}{1536\pi^3} |V_{ts}V_{tb}|^2 \cdot \lambda_K^{3/2}(s) f_L^2(s) \left( |a_9(K\bar{l}\bar{l})|^2 + |a_{10}(Kl\bar{l})|^2 \right)
\end{align}

(5) (6)
The coefficient \( a(K\nu\nu) \) is given by a short-distance Wilson coefficient at the weak scale, which is known very precisely. The coefficient \( a_9(Kll) \) contains the Wilson coefficient \( C_9(\mu) \) combined with the short-distance kernels of the \( B \to K\ell^+\ell^- \) matrix elements of four-quark operators evaluated at \( \mu = \mathcal{O}(m_b) \). The coefficient \( a_9(Kll) \) multiplies the local operator \( \bar{s}bV_A(\bar{l}l)V \). At next-to-leading order (NLO) the result can be extracted from the expressions for the inclusive decay \( B \to X_\ell \ell^+\ell^- \) given in \( [10,11,12] \), where also the Wilson coefficients and operators of the effective Hamiltonian and further details can be found. The coefficient \( a_{10}(Kll) \) is a short-distance quantity, which is precisely known, similarly to \( a(K\nu\nu) \).

### 2.2. Form factors

The long-distance hadronic dynamics of \( \bar{B} \to K\nu\bar{\nu} \) and \( \bar{B} \to K\ell^+\ell^- \) is contained in the matrix elements

\[
\langle \bar{K}(p')|\bar{s}\gamma^\mu b|\bar{B}(p)\rangle = f_+(s)(p+p')^\mu + [f_0(s) - f_+(s)] \frac{m_B^2 - m_K^2}{2q^2} q^\mu
\]

\( \langle \bar{K}(p')|\bar{s}\sigma^{\mu\nu} b|\bar{B}(p)\rangle = i \frac{f_T(s)}{m_B + m_K} [(p+p')^\mu q^\nu - q^\mu(p+p')^\nu] \]

which are parametrized by the form factors \( f_+, f_0 \) and \( f_T \). Here \( q = p-p' \) and \( s = q^2/m_B^2 \). The term proportional to \( q^\mu \) in (7), and hence \( f_0 \), drops out when the small lepton masses are neglected as has been done in \( [9] \) and \( [5] \). The ratio \( f_T/f_+ \) is independent of unknown hadronic quantities in the small-\( s \) region due to the relations between form factors that hold in the limit of large kaon energy \( [13,14] \).

\[
\frac{f_T(s)}{f_+(s)} = \frac{m_B + m_K}{m_B} + \mathcal{O}(\alpha_s, \Lambda/m_b)
\]

Here we have kept the kinematical dependence on \( m_K \) in the asymptotic result. In contrast to \( f_+ \) the form factor \( f_T \) is scale and scheme dependent. This dependence is of order \( \alpha_s \) and has been neglected in \( [9] \).

We remark that the same result for \( f_T/f_+ \) is also obtained in the opposite limit where the final state kaon is soft, that is in the region of large \( s = \mathcal{O}(1) \) \( [15,16,17,18,19] \). From this observation we expect \( [9] \) to be a reasonable approximation in the entire physical domain. This is indeed borne out by a detailed analysis of QCD sum rules on the light cone \( [20] \), which cover a range in \( s \) from 0 to 0.5.

The impact of the \( f_T/f_+ \) term is numerically small, about 13% of the amplitude \( a_9(Kll) \). A 15% uncertainty, which may be expected for the approximate result \( [9] \), will only imply an uncertainty of 2% for \( a_9(Kll) \) or the \( B \to K\ell^+\ell^- \) differential rate. In practice, this leaves us with the form factor \( f_+(s) \) as the essential hadronic quantity for both \( \bar{B} \to K\nu\bar{\nu} \) and \( \bar{B} \to K\ell^+\ell^- \).

We employ the parametrization proposed by Becirevic and Kaidalov \( [21] \) in the form

\[
f_+(s) \equiv f_+(0) \frac{1 - (b_0 + b_1 - a_0 b_0) s}{(1 - b_0 s)(1 - b_1 s)}
\]

The parameter \( b_0 \) is given by

\[
b_0 = \frac{m_B^2}{m_{B^*}^2} \approx 0.95 \quad \text{for} \quad m_{B^*} = 5.41 \text{ GeV} \quad (11)
\]

\( b_0 \) represents the position of the \( B^*_1 \) pole and is taken as fixed, following \( [21] \). The remaining three quantities \( a_0, b_1 \) and \( f_+(0) \) are treated as variable parameters. QCD sum rules on the light cone (LCSR) give \( [20] \)

\[
f_+(0) = 0.304 \pm 0.042, \quad a_0 \approx 1.5, \quad b_1 = b_0 \quad (12)
\]

\( f_+(0) \) only affects the overall normalization of the decay rates and cancels in appropriate ratios. Combining theoretical constraints \( [1] \), we adopt the following default ranges for the shape parameters

\[
1.4 \leq a_0 \leq 1.8, \quad 0.5 \leq b_1/b_0 \leq 1.0 \quad (13)
\]

with

\[
a_0 = 1.6, \quad b_1/b_0 = 1.0 \quad (14)
\]

as our reference values. The latter are also obtained \( [1] \) as the best fit to the shape of the measured \( B \to K\ell^+\ell^- \) spectrum in Fig. \( [1] \).
2.3. $B \to \bar{K}l^+l^-$: Nonperturbative corrections

In this section we comment on the theoretical framework for $B \to \bar{K}l^+l^-$ and on nonperturbative effects beyond those that are contained in the form factors.

It is well known that, because of huge backgrounds from $B \to \bar{K}\psi^{(1)} \to \bar{K}l^+l^-$, the region of $q^2$ containing the two narrow charmonium states $\psi = \psi(1S)$ and $\psi' = \psi(2S)$ has to be removed by experimental cuts from the $q^2$ spectrum of $B \to \bar{K}l^+l^-$. The overwhelming background from $\psi$ and $\psi'$ is related to a drastic failure of quark-hadron duality in the narrow-resonance region for the square of the charm-loop amplitude, as has been discussed in [23]. Nevertheless, the parts of the $q^2$ spectrum below and above the narrow-resonance region remain under theoretical control and are sensitive to the flavour physics at short distances. A key observation here is that the amplitude is largely dominated by the semileptonic operators

$$Q_9 = (\bar{s}b)_{V-A}(\bar{l}l)_V$$

$$Q_{10} = (\bar{s}b)_{V-A}(\bar{l}l)_A$$

(15)

which have large coefficients $\hat{C}_9$ and $\hat{C}_{10}$. These contributions are perturbatively calculable up to the long-distance physics contained in the form factor $f_+(s)$. The $B \to \bar{K}l^+l^-$ matrix elements of four-quark operators, such as $(\bar{s}b)_{V-A}(\bar{c}c)_{V-A}$, are more complicated, but still systematically calculable. Schematically, the $B \to \bar{K}l^+l^-$ rate is proportional to

$$|\hat{C}_9 + \Delta_{4q}|^2 + |\hat{C}_{10}|^2$$

(16)

where $\Delta_{4q}$ represents contributions from four-quark operators, for instance charm loops or weak annihilation effect. Because $\Delta_{4q}$ is numerically subleading (O(10%) of the total rate), the impact of any uncertainties in its evaluation will be suppressed.

In the low-$q^2$ region $\Delta_{4q}$ can be computed using QCD factorization [22]. This approach, which is based on the heavy-quark limit and the large energy of the recoiling kaon, should work well for the real part of the amplitude in view of the experience from two-body hadronic $B$ decays [24] and $B \to K^{*}\gamma$ [25].

In the high-$q^2$ region the appropriate theoretical framework for the computation of $\Delta_{4q}$ is an operator product expansion exploiting the presence of the large scale $q^2 \sim m_b^2$. This concept has been used in [19] in analyzing the endpoint region of $b \to sl^+l^-$, which is governed by few-body exclusive modes. A detailed treatment, including the discussion of subleading corrections, has been given in [26]. Power corrections are generally smaller than for low $q^2$. Uncertainties could still come from violations of local quark-hadron duality. The relative amplitude of oscillations in $\hat{C}_9 + \text{Re} \Delta_{4q}$ may be estimated to be of order 10 to 20%. We expect these local variations to be averaged out when the spectrum is integrated over $s$ [19] such that the residual uncertainty will be reduced. As discussed in [23], global duality in this sense cannot be expected to hold for the second order term $|\Delta_{4q}|^2$ in [16]. On the other hand, this contribution is numerically very small, at the level of few percent, and duality violations will only have a minor effect.

1Weak annihilation contributions to $B \to \bar{K}l^+l^-$ are negligibly small, although they are a leading-power effect.
2.4. $B^- \to \tau^- \bar{\nu}_\tau \to K^- \nu_\tau \bar{\nu}_\tau$

The decay $B^- \to \tau^- \bar{\nu}_\tau$ followed by $\tau^- \to K^- \nu_\tau$ produces a background for the short-distance reaction $B^- \to K^- \nu \bar{\nu}$ at the level of 15-25%, which has been discussed recently in [27]. This background has to be taken into account for a precise measurement of the short-distance branching fraction $B(B^- \to K^- \nu \bar{\nu})$. It needs to be subtracted from the experimental signal, but this should ultimately be possible with essentially negligible uncertainty [1].

3. PRECISION OBSERVABLES

Our predictions for the branching fractions in the standard model are

$$B(B^- \to K^- \nu \bar{\nu}) \cdot 10^6 = 4.4^{+1.3}_{-1.1} (f_+(0))$$
$$-0.8 \leq (a_0) \leq -0.7 \ (b_1) \quad (17)$$

$$B(B^- \to K^- \ell^+ \ell^-) \cdot 10^6 = 0.58^{+0.17}_{-0.15} (f_+(0))$$
$$-0.10 \leq (a_0) \leq -0.09 \ (b_1) \leq -0.04 \ (\mu) \quad (18)$$

Whereas the individual branching fractions [17] and [18] suffer from large hadronic uncertainties, we expect their ratio to be under much better theoretical control. It is obvious that the form factor normalization $f_+(0)$ cancels in this ratio. Moreover, the shape of the $q^2$ spectrum is almost identical for the two modes. This is because the additional $q^2$-dependence from charm loops in $B \to Kl^+ \ell^-$, compared to $B \to K \nu \bar{\nu}$, is numerically only a small effect outside the region of the narrow charmonium states. As a consequence, also the dependence on the form factor shape will be greatly reduced in the ratio

$$R = \frac{B(B^- \to K^- \nu \bar{\nu})}{B(B^- \to K^- l^+ l^-)}$$

Numerically we find

$$R = 7.59^{+0.01}_{-0.01} (a_0) +0.00_{-0.02} (b_1) -0.48_{+0.41} (\mu) \quad (20)$$

This prediction is independent of form factor uncertainties for all practical purposes. It is limited essentially by the perturbative uncertainty at NLO of ±6%. Using the experimental result in [4], the theory prediction [20], and assuming the validity of the standard model, we obtain

$$B(B^- \to K^- \nu \bar{\nu}) = R \cdot B(B^- \to K^- l^+ l^-)_{exp} = (3.64 \pm 0.47) \cdot 10^{-6} \quad (21)$$

With an accuracy of ±13%, limited at present by the experimental error, this result is currently the most precise estimate of $B(B^- \to K^- \nu \bar{\nu})$.

In order to obtain theoretically clean observables, the region of the two narrow charmonium resonances $\psi(1S)$ and $\psi(2S)$ has to be removed from the $q^2$ spectrum of $B \to Kl^+ \ell^-$. This leaves two regions of interest, the low-s region below the resonances, and the high-s region above. For the present analysis we define these ranges as

low $s$ : $0 \leq s \leq 0.25$

high $s$ : $0.6 \leq s \leq s_m \quad (22)$

The resonance region $0.25 < s < 0.6$ corresponds to the $q^2$ range $7 \text{GeV}^2 < q^2 < 16.7 \text{GeV}^2$. For our standard parameter set the total rate for $B \to K \nu \bar{\nu}$ or $B \to Kl^+ \ell^-$ (non-resonant) is divided among the three regions, low-s, narrow-resonance, high-s, as 35 : 48 : 17.

We first concentrate on the low-s region, where $B^- \to Kl^+ \ell^-$ can be reliably calculated. To ensure an optimal cancellation of the form factor dependence, one may restrict also the neutrino mode to the same range in $s$ and define

$$R_{25} = \frac{\int_{0.25}^{s_m} ds \ dB(B^- \to K^- \nu \bar{\nu})/ds}{\int_{0.25}^{s_m} ds \ dB(B^- \to K^- l^+ l^-)/ds} \quad (23)$$

This ratio is determined by theory to very high precision. Displaying the sensitivity to the shape parameters and the renormalization scale one finds

$$R_{25} = 7.60^{-0.00}_{+0.00} (a_0) +0.00_{-0.02} (b_1) -0.43_{+0.36} (\mu) \quad (24)$$

The form factor dependence is seen to cancel almost perfectly in $R_{25}$. The shape parameters affect this quantity at a level of only 0.5 per mille. One is therefore left with the perturbative uncertainty, estimated here at about ±5% at NLO.

The independence of any form factor uncertainties in $R_{25}$ comes at the price of using only 35% of the full $B^- \to K^- \nu \bar{\nu}$ rate. We therefore consider a different ratio, which is defined by

$$R_{256} = \frac{\int_{0.25}^{s_m} ds \ dB_{\ell \nu}/ds}{\int_{0}^{0.25} ds \ dB_{l \ell}/ds + \int_{0.06}^{s_m} ds \ dB_{l \ell}/ds} \quad (25)$$
In this ratio the fully integrated rate of $B^{-} \rightarrow K^{-} \nu \bar{\nu}$ is divided by the integrated rate of $B^{-} \rightarrow K^{-} l^{+} l^{-}$ with only the narrow-resonance region removed. This ensures use of the maximal statistics in both channels. Due to the missing region in $B^{-} \rightarrow K^{-} l^{+} l^{-}$ the dependence on the form factor shape will no longer be eliminated completely, but we still expect a reduced dependence. Numerically we obtain, using the same input as before,

$$R_{256} = 14.60^{+0.28}_{-0.38} (a_0) +^{+0.10}_{-0.02} (b_1) +^{+0.80}_{-0.62} (\mu) \quad (26)$$

This estimate shows that the uncertainty from $a_0$ and $b_1$ is indeed very small, at a level of about $\pm 3\%$. With better empirical information on the shape of the spectrum this could be further improved.

We conclude that ratios such as those in (23) and (25), or similar quantities with modified cuts, are theoretically very well under control. They are therefore ideally suited for testing the standard model with high precision.

4. NEW PHYSICS

The branching fractions of $B \rightarrow K \nu \bar{\nu}$ and $B \rightarrow K^0 \pi^0$ are sensitive to physics beyond the standard model. In general, nonstandard dynamics will have a different impact on $B \rightarrow K \nu \bar{\nu}$ and $B \rightarrow K^0 \pi^0$. The excellent theoretical control over the ratios $R_{25}$ or $R_{256}$ will help to reveal even moderate deviations from standard model expectations.

One example is the scenario with modified Z-penguin contributions [28], if these contributions interfere destructively with those of the standard model. In that case the ratios $R_{25}$ or $R_{256}$ could be significantly suppressed. The modified Z-penguin scenario may be realized, for instance, in supersymmetric models [28,29].

Another class of theories that do change the ratios are those where $B \rightarrow K l^{+} l^{-}$ remains standard model like while $B \rightarrow K \nu \bar{\nu}$ receives an enhancement (or a suppression). Substantial enhancements of $B(B \rightarrow K \nu \bar{\nu})$ are still allowed by experiment, in fact much more than for $B \rightarrow K l^{+} l^{-}$.

A first example are scenarios with light invisible scalars $S$ contributing to $B \rightarrow K S S$, which has been suggested in [30,31] as an efficient probe of light dark matter particles. $B \rightarrow K S S$ is also discussed in [29]. This channel adds to $B \rightarrow K \nu \bar{\nu}$, which is measured as $B \rightarrow K + \text{invisible}$. If the scalars have nonzero mass, $B \rightarrow K S S$ could be distinguished from $B \rightarrow K \nu \bar{\nu}$ through the missing-mass spectrum. On the other hand, if the mass of $S$ is small, or the resolution of the spectrum is not good enough, a discrimination of the channels may be difficult. The corresponding increase in $B(B \rightarrow K \nu \bar{\nu})$ could be cleanly identified through the ratios $R_{25}$ and $R_{256}$. Similar comments apply to the case where the invisible particles are light (or massless) neutralinos $\tilde{\chi}_1^0$, which are still allowed in the MSSM [32]. Substantial enhancements of $B \rightarrow K + \text{invisible}$ over the standard model expectation through $B \rightarrow K \tilde{\chi}_1^0 \tilde{\chi}_1^0$ are possible in the MSSM with non-minimal flavour violation [33].

A further example is given by topcolor assisted technicolor [34]. A typical scenario involves new strong dynamics, together with extra $Z'$ bosons, which distinguishes the third generation from the remaining two. The resulting flavour-changing neutral currents at tree level may then predominantly lead to transitions between third-generation fermions such as $b \rightarrow s \tau \bar{\nu}_\tau$. An enhancement of $B(B \rightarrow K \nu \bar{\nu})$ would result and might in principle saturate the experimental bound [1]. An enhancement of 20%, which should still be detectable, would probe a $Z'$-boson mass of typically $M_{Z'} \approx 3 \text{ TeV}$. A similar pattern of enhanced $B \rightarrow K \nu \bar{\nu}$ and SM like $B \rightarrow K l^{+} l^{-}$ is also possible in generic $Z'$ models [29].

The subject of new physics in $b \rightarrow s \nu \bar{\nu}$ transitions has been discussed in [35] and more recently in [29,30,31,33]. New physics in $B \rightarrow K l^{+} l^{-}$ has been studied in [36].

5. CONCLUSIONS

The strategy discussed here puts $B \rightarrow K \nu \bar{\nu}$ as a new physics probe in the same class as $K \rightarrow \pi \nu \bar{\nu}$, the ‘golden modes’ of kaon physics. Suitable ratios of (partially integrated) $B \rightarrow K \nu \bar{\nu}$ and $B \rightarrow K l^{+} l^{-}$ decay rates are essentially free
of form factor uncertainties, while retaining sensitivity to interesting New Physics scenarios. $B \to K\nu\bar{\nu}$ together with $B \to Kl'^{-}$ thus hold exciting opportunities for $B$ physics in the era of Super Flavour Factories.

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