EVIDENCE FOR DUAL SUPERCONDUCTIVITY OF QCD GROUND STATE.

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A discussion is made of the strategy to check dual superconductivity of the vacuum as a mechanism of colour confinement. Recent evidence from lattice is reviewed.

1 Introduction

No reliable analytic approach exists to QCD at large distances. The usual perturbative quantization leads to an S matrix which is not Borel summable. For reasons which are not understood the perturbative expansion works anyhow at small distances, where a few terms correctly describe experiments. It fails at large distances, where the coupling is large, in particular in describing confinement of colour.

Attempts have been made to describe the degrees of freedom relevant to confinement by effective models. Particularly attractive from the theoretical point of view, is the possibility that vacuum behaves as a dual superconductor.\cite{1,2}

Dual Meissner effect would accordingly produce confinement by constraining the chromoelectric field into Abrikosov flux tubes, with energy proportional to their length.

The mechanism is appealing because it relies on a symmetry property. Superconductivity is a Higgs mechanism, by which a charged field acquires a non-zero v.e.v., the order parameter in the Landau Ginzburg free energy. The ground state has no definite charge, the $U(1)$ related to charge conservation being spontaneously broken.

For QCD magnetic charges should condense in the confined phase, and break some magnetic $U(1)$ symmetry. A dual order parameter, a disorder parameter in the language of statistical mechanics, would then describe this change of symmetry.

Only a non-perturbative quantization, like lattice, can help in checking if the above mechanism is at work. The simplest strategy to do that consists of two steps

1. Identify the relevant magnetic $U(1)$.

2. Check by a disorder parameter if it breaks spontaneously.
2 Identifying monopoles: the abelian projection.

Monopoles in non abelian gauge theories were first discovered as solitons in the Higgs phase in a gauge theory with gauge group SO(3) coupled to a scalar field in the adjoint representation.

\[ \mathcal{L} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + (D_{\mu} \Phi)^* (D_{\mu} \Phi) - \frac{\lambda}{2} (\Phi^2 - \mu^2)^2 \]  

(1)

It was shown in ref.\textsuperscript{3,4} that monopoles exist as static solutions (solitons) in the Higgs phase of the model, i.e. for \( \mu^2 > 0, \Phi_0 = \langle \Phi \rangle \neq 0 \).

In the hedgehog gauge the monopole has the form

\[ \Phi(\vec{r}) = f(r) \Phi_0 \hat{r}, \quad A_0(\vec{r}) = 0 \quad (A_i)^a = \frac{2}{g} \varepsilon_{ia} \hat{r}^k h(r) \]  

(2)

with \( f(r), h(r) \approx 1 \) as \( r \gg 1/\mu \).

A gauge transformation to the unitary gauge, \( U(\vec{r}) \),

\[ U(\vec{r}) \Phi(\vec{r}) = \Phi_0 \equiv (0, 0, 1) \]  

(3)

is defined up to a residual \( U(1) \) gauge group of rotations around the \( z \) axis. \( U(\vec{r}) \) is singular at the zero of \( \Phi(\vec{r}) \), \( \vec{r} = 0 \). \( U(\vec{r}) \) is usually called an abelian projection. For the monopole solution the abelian field of the residual \( U(1) \) in the abelian projected gauge

\[ F_{\mu\nu} = \partial_\nu A_\mu^a - \partial_\mu A_\nu^a \]  

(4)

is the field of a Dirac monopole.

\( F_{\mu\nu} \) can be written in a gauge invariant form as

\[ F_{\mu\nu} = \Phi \tilde{G}_{\mu\nu} - \frac{1}{g} \Phi (D_{\mu} \Phi \wedge D_{\nu} \Phi) \]  

(5)

Calling \( F_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}, \quad j^M_\nu = \partial^\mu F_{\mu\nu} \), we have identically

\[ \partial^\nu j^M_\nu = 0 \]  

(6)

Eq.(5) identifies an \( U(1) \) magnetic symmetry. The corresponding charge \( Q \) is a colour singlet and is equal to two magnetic units for the monopole solution. Also \( F_{\mu\nu} \) and \( F_{\mu\nu}^* \) are colour singlets.

More generally, an abelian projection \( U(\vec{r}) \) can be performed which is defined by eq.(3) on a generic configuration. \( U(\vec{r}) \) is singular at the zeros of \( \Phi(\vec{r}) \). Around these points the field has the topology of an abelian monopole.
We could think of a slightly more general model in which an additional Higgs field $\Phi'$ is present, e.g. with the same potential as $\Phi$

$$\mathcal{L} = -\frac{1}{4} \tilde{G}_{\mu\nu} \tilde{G}^{\mu\nu} + (D_{\mu} \Phi)^{*} (D_{\mu} \Phi) - \frac{\lambda}{2} (\Phi^{2} - \frac{\mu^{2}}{\lambda})^{2} + (D_{\mu} \Phi')^{*} (D_{\mu} \Phi') - \frac{\lambda}{2} (\Phi'^{2} - \frac{\mu^{2}}{\lambda})^{2}$$

We can define an abelian projection which brings $\Phi$ to the unitary gauge, as in the simple model above, and define the gauge invariant field $F_{\mu\nu}$, and the corresponding magnetic $U(1)$. We can play the same game with $\Phi'$, and this will in general bring to a different abelian projection and to a different $U(1)$. Both magnetic charges are gauge invariant. On a given field configuration the zeros of $\Phi$ and $\Phi'$ will not coincide in general so that the two abelian projections define different monopoles. However the theory is totally symmetric under the exchange $\Phi \leftrightarrow \Phi'$ and hence the two monopole species defined by the two abelian projections must be physically equivalent.

There is in the literature a misuse of terminology: the abelian projections are named from the abelian projected gauge, so that the two monopole species defined in the above example are called monopole in the gauge $\Phi$ and monopoles in the gauge $\Phi'$; a possible difference of physics, e.g. if the two fields have a different potential is called gauge dependence. This terminology is misleading: usually, e.g. in QED, as gauge dependent is meant a quantity which does not depend only on the physical fields, $F_{\mu\nu}$, but could depend on the choice of the gauge. Monopole charges defined by any abelian projection are instead physically well defined and gauge invariant quantities.

In QCD there is no Higgs field. However there exist infinitely many fields transforming in the adjoint representation, and each of them can define an abelian projection and with it a monopole species.

On the lattice any parallel transport along an arbitrary path $C$ coming back to the starting point defines an abelian projection and a monopole species. The corresponding monopoles are different in number and located in different sites, configuration by configuration. A possible guess is that they are all physically equivalent, in the same way as the two monopole species of the model eq.(7). For each of them it is anyhow possible to investigate condensation in the vacuum and dual superconductivity. Some results will be presented in the next section.

An alternative attitude is that some abelian projection is better than others. This attitude is popular among the practitioners of the so called maximal abelian gauge. This is an abelian projection for which the operator $\Phi$ is im-
licitly defined by maximizing numerically the quantity

\[ A = \sum_{x,\mu} Tr \left[ \Omega(x)U_{\mu}(x)\Omega^\dagger(x + \mu)\sigma_3\Omega(x + \mu)U^\dagger(x)\Omega^\dagger(x)\sigma_3 \right] \]  

with respect to the gauge transformation \( \Omega(x) \).

The numerical output is that in the new gauge all the links \( U_{\mu}(x) \) are practically aligned along \( \sigma_3 \), within 10\(-20\%\). A remarkable observation which is a consequence of this fact is the so called “abelian dominance”. Quantities like e.g. the string tension, when computed in the \( U(1) \) residual gauge, agree within 10\(-20\%\) with the exact result. In addition the abelian monopole part, corresponding to integer number of \( 2\pi \) in the abelian plaquettes, saturates the abelian approximation to within 90\% again.

Dominance is interpreted as special relevance of the specific monopoles in the long range physics.

From theoretical point of view we find more significant and anyhow necessary to investigate the symmetry of the vacuum, i.e. the condensation of different monopole species in connection with confinement.

3 Detecting dual superconductivity.

In the language of statistical mechanics the main issue of the problem is duality: the gauge field of monopoles presents non trivial connection or topology. A creation operator for monopoles has the form of a translation of the field in the Schrödinger representation by a monopole configuration. In \( U(1) \) gauge theory

\[ A_{0}^{\text{mon}}(\vec{x}, \vec{y}) = 0 \quad A_{\mu}^{\text{mon}}(\vec{x}, \vec{y}) = \frac{m}{g} \frac{\vec{n} \cdot (\vec{x} - \vec{y})}{|\vec{x} - \vec{y}|^2 (|\vec{x} - \vec{y}| - (\vec{x} - \vec{y}) \cdot n)} \]

\[ \mu (\vec{y}, t) = \exp \left[ i \int d^3 \vec{x} \vec{E}(\vec{x}, t) \vec{A}_{\text{mon}}(\vec{x}, \vec{y}) \right] \]  

\( \mu \) carries non zero magnetic charge.

Eq.(9) is the analog of the elementary translation

\[ e^{i\vec{p}a} |x\rangle = |x + a\rangle \]  

\( \vec{E} \) is the conjugate momentum to the field. Some technical modifications will be needed to keep the compactness of the theory into account on the lattice, and some extra care to perform the shift in the abelian projected \( U(1) \) for non abelian gauge theory.
\( \langle \mu \rangle \) is then measured. \( \langle \mu \rangle \neq 0 \) means spontaneous breaking of magnetic \( U(1) \), and hence dual superconductivity.

For different monopole species we have measured \( \langle \mu \rangle \) or better \( \rho = \frac{d}{d\beta} \ln \langle \mu \rangle \), as a function of temperature, on asymmetric lattices \( N_S \gg N_T \). \( \rho \) contains the same information as \( \langle \mu \rangle \) and has less numerical problems in its determination.

Since \( \langle \mu \rangle_{\beta=0} = 1 \),

\[
\langle \mu \rangle = \exp \left( \int_0^\beta \rho(x) dx \right)
\]  \hspace{1cm} (11)

The typical behaviour of \( \rho \) vs \( \beta = 2N_c/g^2 \) is shown in fig.1 for \( SU(2) \) and different abelian projections, and for \( SU(3) \) in fig.2, and fig.3.

**Fig.1** \( \rho \) vs \( \beta \) for different abelian projections in \( SU(2) \). The negative peak signals phase transition.

There is no practical difference between different abelian projections, in agreement with the guess of t’Hooft.

The strong negative peak occurs at the deconfining transition, and, by eq.(11), indicates a rapid drop to zero of \( \langle \mu \rangle \).

At large \( \beta \)'s \( \rho \) is computed by perturbation theory giving at the leading
order $\rho = -c_1 L_S + c_2$. As the spatial size $L_S \to \infty$, $\rho \to -\infty$ and $\langle \mu \rangle = 0$, as expected for any disorder parameter in the thermodynamical limit. For $\beta \sim \beta_c$ a finite size scaling analysis with respect to $L_S$ can be performed. Since the transition is second order for $SU(2)$ and weak first order for $SU(3)$, the correlation length $\xi$ goes large at $\beta_c$, with some effective critical index $\nu$

$$\xi \sim (\beta_c - \beta)^{-\nu}$$

(12)

By dimensional arguments

$$\mu = \mu\left(\frac{a}{\xi}, \frac{\xi}{L}\right) \sim \mu(0, \frac{\xi}{L})$$

(13)

![Fig. 2] $\rho$ for the two different monopole species in the Polyakov projection. $SU(3)$.
This implies by eq. (12)

\[ \mu = \Phi(L^{1/\nu}(\beta_c - \beta)) \]

or

\[ \frac{\rho}{L^{1/\nu}} = f(L^{1/\nu}(\beta_c - \beta)) \]  \hspace{1cm} (14)

The scaling law is obeyed for the appropriate values of \( \nu \) and \( \beta_c \). Fig. 4 shows how scaling works. The output for \( SU(2) \) is \( \nu = .62 \pm .02 \) to be compared with the expectation, the critical index of 3d Ising model \( \nu = .631(1) \).

For \( SU(3) \) we find a similar value, contrary to the expectation which should be 1/3. However our volumes are not sufficiently large, and further investigations are on the way.
4 Conclusions

A disorder parameter can be defined to investigate condensation of monopoles in the vacuum of QCD.

QCD vacuum is a dual superconductor. Different monopole species look equivalent, and condense in connection with confinement, in agreement with the conjecture of t’Hooft.

This is an important information on the symmetry of vacuum, which must be explained by any model of confinement.

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