Combined Method of Conventional and Coherent Doppler Sonar to Avoid Velocity Ambiguity

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Abstract:

Pulse-to-pulse coherent Doppler sonar is an autonomous and powerful tool for velocity measurement in laboratory and field applications. However, coherent Doppler sonar has velocity ambiguity that limits measurable velocity. Some techniques have been developed to extend the ambiguity velocity, but they are not yet enough to eliminate it completely. On the other hand, conventional Doppler sonar, while it does not have velocity ambiguity, has larger velocity error than coherent Doppler sonar. In our research, we combine conventional and coherent Doppler sonars in a method to provide accurate and precise velocity without ambiguity. Three kinds of sonar are introduced: conventional, coherent and combined Doppler. Theoretical errors of conventional and coherent Doppler sonars are explained, and the measurement error of combined Doppler sonar is obtained theoretically. From results based on a simulation, it is clear that under certain conditions, the combined Doppler sonar can provide accurate and precise velocities without velocity ambiguity.

Classification: Signal processing, Miscellaneous (Observations, Measurements, etc.)

Keywords: velocity measurement, coherent Doppler sonar, conventional Doppler sonar, combined method, velocity ambiguity

1. Introduction

In recent years, pulse-to-pulse coherent Doppler sonar (CHDS) has been developed to measure velocity accurately over comparatively short smoothing times. It has been used effectively to explore the characters of fluids in a variety of environments. Processing techniques for CHDS have been discussed1, and a simulation model of CHDS has been provided to assist in the design of such sonar2. CHDS has also been used to explore turbulence and wave characteristics in the ocean3. However, the occurrence of range and velocity ambiguities limits more general applications of this sonar. Some approaches to expanding the ambiguity velocity have been considered. One such method extended the ambiguity velocity by introducing multiple carrier frequencies4,5. However, this method required a transducer with a wide frequency band. Another approach, based on interweaving two different pulse intervals, has been used in

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weather radar systems\(^6,7\) and sonar systems\(^8-10\). Both the multiple-frequency and dual-time-interval methods can spread the range of the velocity ambiguity, but neither can eliminate it completely.

Conventional Doppler sonar (CNDS) has usually been installed to get velocity information without ambiguity. But CNDS needs a few seconds, on average\(^11\), to obtain valuable velocity readings. As accurate velocity information plays an important role in ocean exploration, especially for devices in the water, we propose a combined method of conventional and coherent Doppler sonar (CMDS) to provide accurate velocity information with short smoothing time and without velocity ambiguity.

In this paper, CNDS and CHDS are first introduced in more detail. With the benefit of our understanding of these two methods, we then explain CMDS. According to the error analyses of CNDS and CHDS, the measurement error of CMDS is proposed. We then describe simulations carried out to measure velocities larger than the ambiguity velocity of CHDS, and evaluate the performance of CMDS compared to CNDS and CHDS. Based on the theoretical analysis and simulation results, we demonstrate that CMDS can provide the accurate and precise velocity without ambiguity under certain conditions.

2. Measurement Method
2.1 Conventional Doppler sonar

In CNDS, the frequency shift is estimated by the correlation of spectra between the transmitted and received pulse signals\(^12\). Normally the transmitted and received signals in complex domains can be expressed as:

\[ s(t) = a_0(t)e^{j(2\pi f_0 t + \phi_0)} , \]

\[ r(t) = bs(t-t_0)e^{j2\pi \Delta f(t-t_0)} , \]

where \(a_0(t)\) is the envelope of the transmitted signal;

\(f_0\) is the carrier frequency;

\(\phi_0\) is the initial phase;

\(b\) is the amplitude of the received signal;

\(t_0\) is the target round-trip time delay;

\(\Delta f\) is the Doppler frequency shift.

Then the Doppler shift can be calculated by the correlation as:

\[ \Delta f = \arg \max_{f_o \in [0,F]} \int_0^F S^*(f - f_o)R(f)df , \]

where \(S(f)\) and \(R(f)\) are the Fourier transform of \(s(t)\) and \(r(t)\), respectively, \(F\) is the frequency range of the Fourier transform, and \(S^*(f)\) is the conjugate of \(S(f)\).

2.2 Pulse-to-pulse coherent Doppler sonar

CHDS estimates velocities by comparing the phase of received signals from successive acoustic pulses. Given the condition of a short time interval between two pulses, the received acoustic signals can be correlated, and the change in phase will be proportional to the velocity of the moving object. When the received signals are reflected signals from the target, the relationship of the radial velocity and the change of the phase is given\(^4\) as,

\[ v = \frac{\phi c}{4\pi f\tau} , \]

with

\[ \phi = \tan^{-1}\left[ \frac{Q(t)I(t+\tau)-I(t)Q(t+\tau)}{I(t)I(t+\tau)+Q(t)Q(t+\tau)} \right] , \]

where \(\phi\) is the phase change of the adjacent received signals, \(c\) is the speed of sound in water, \(\tau\) is the time interval between adjacent pulses, and \(I(t)\) and \(Q(t)\) are the in-phase and quadrature components, respectively, of the received signal.
in the complex domain. Figure 1 shows the process of generating the in-phase and quadrature components.

Because of the time interval \( \tau \) and the detected phase change, which is limited from \(-\pi\) to \(\pi\), the ambiguity range and the ambiguity velocity are shown as:

\[
\Delta r = \frac{c\tau}{2}, \quad \Delta v = \frac{c}{4f\tau}.
\]  

(6)

By combining \( \Delta v \) and \( \Delta r \) together, we determine the “range-velocity” ambiguity as follows:

\[
\Delta v \Delta r = \frac{c\lambda}{8},
\]  

(7)

where \( \lambda \) is wavelength of the system.

According to Eq. (7), the ambiguity velocity is inversely proportional to the ambiguity range. This is the reason that a long achievable distance and a large measureable velocity range cannot be realized simultaneously.

If the radial velocity \( (v_0) \) is larger than the ambiguity velocity \( (\Delta v) \), the relationship between measured velocity \( (v_{\text{m}}) \) by CHDS without noise and the radial velocity can be expressed as:

\[
v_0 = 2n\Delta v + v_{\text{m}},
\]  

(8)

where \( n \) is the integer factor. If \( n \) can be determined, velocity ambiguity will be eliminated.

### 2.3 Combined Doppler sonar

The aforementioned CHDS can provide accurate velocity information by phase measurement, but it is seriously limited by velocity ambiguity for wider applications. Therefore, CMDS is introduced to overcome this disadvantage\(^{(12)}\). A functional block diagram of CMDS to measure the velocity \( v_0 \) is shown in Fig. 2. At the receiver block, received pulse series are detected and forwarded to CNDS and CHDS. At the start of CNDS, Doppler shift of the received pulse are calculated, and a coarse velocity \( (v_n) \) with error \( (\varepsilon_n) \) is obtained. In CHDS, phase changes between adjacent pulses are calculated, and the precise velocity \( (v_h) \) is obtained.

In order to eliminate the velocity ambiguity, the coarse velocity is used to decide the integer factor \( n \). Finally, an unlimited and accurate velocity \( (v_{\text{m}}) \) is calculated by the addition of the ambiguity velocity with the integer factor \( (2n\Delta v) \) and the velocity \( v_h \) measured by CHDS.

One of the main functions of CMDS is to calculate the integer factor \( n \). In order to carry out this calculation, two assumptions are necessary. The first is that the measurement velocity \( v_{\text{m}} \) has the same sign as the velocity \( v_{\text{m}} \) calculated by CHDS without noise. This assumption is only challenged
in situations when \( v_h \) is near to \( \pm \Delta v \). According to this assumption, the sign of \( v_{h0} \) can be estimated by \( v_h \). The second assumption is that the absolute value of coarse velocity error (\( |\epsilon_n| \)) is less than half the ambiguity velocity \( \Delta v \). From these assumptions, the following two inequalities are obtained.

\[
- \frac{1}{4} \frac{v_h}{2\Delta v} \leq n + \frac{3}{4} (v_{h0} > 0) \tag{9}
\]

\[
- \frac{3}{4} \frac{v_h}{2\Delta v} \leq n + \frac{1}{4} (v_{h0} \leq 0) \tag{10}
\]

With inequalities (9) and (10), integer \( n \) can be determined. With integer \( n \) decided and the velocity \( v_h \), the velocity measured by the CMDS can be calculated by Eq. (8).

According to Eq. (8), if the integer \( n \) is estimated correctly, CMDS can provide an accurate velocity calculation over an unlimited range.

The \( v_{h0} \) is considered to be a variable limited in the range of \((0, \Delta v]\) in inequality (9) and \((-\Delta v, 0]\) in inequality (10). However, the \( v_{h0} \) is fixed by the velocity \( v_0 \). Therefore, the range of error in the second assumption can be extended, and it can be expressed as a function of \( v_{h0} \).

3. Theoretical Error

3.1 Conventional Doppler sonar

For CNDS, the standard deviation in estimating Doppler shift for \( v_0 \) with Gaussian white noise\(^{13} \) is

\[
\sigma_{f} = \frac{1}{\tau_0 \sqrt{2E/N_0}}, \tag{11}
\]

where \( E \) is the energy of the received signal; \( N_0/2 \) is the noise power per Hertz for noise waveform;

\[
\tau_0^2 = \left( \frac{2\pi}{\gamma_b} \right)^2 \int_{-\infty}^{+\infty} |a_0(t)|^2 dt \int_{-\infty}^{+\infty} |a_0(t)|^2 dt.
\]

With the standard deviation of frequency shift, the standard deviation of measured velocity can be expressed as:

\[
\sigma_v = \frac{\sigma_f}{f_0} c. \tag{12}
\]

Because of the Gaussian white noise effect, the measurement velocity by CNDS also follows the Gaussian distribution, which is:

\[
v_0 \sim N(v_0, \sigma_v^2). \tag{13}
\]

3.2 Coherent Doppler sonar

For coherent Doppler sonar, the received signals are transferred into the complex domain, as shown in Fig. 1. In this process, since the signal passes through the low pass filter, the signal-to-noise ratio of the band-limited signal \( (\gamma_b) \) is improved significantly compared to the originally received signal. For each received signal, the probability density function of phase \( (p(\phi)) \), affected by the white Gaussian noise, can be expressed\(^{14} \) as:

\[
p(\phi) = \frac{e^{-\gamma_b}}{2\pi} \left[ 1 + \sqrt{\frac{\pi \gamma_b}{2}} \cos \phi \right] \times \left[ 1 + \text{erf} \left( \sqrt{\gamma_b} \cos \phi \right) e^{\gamma_b \cos^2 \phi} \right]. \tag{14}
\]

where \( \text{erf}(x) \) is the error function.

The probability density function of the phase difference between two adjacent pulses can then be expressed as:

\[
p_\Delta(\phi) = \int_{-\pi}^{\pi} p(\theta) p(\phi - \theta) d\theta. \tag{15}
\]

Based on the probability density function of the phase difference, the standard deviation of velocity measured by CHDS is:
\[ \sigma_h = \frac{c}{4\pi ft} \sqrt{\int_{-\pi}^{\pi} \phi^2 p_d(\phi) d\phi}. \] (16)

### 3.3 Combined Doppler sonar

For CMDS, the measurement error is decided by the accuracy of both CNDS and CHDS. In the case of inequality (9), the inequality can also be expressed as:

\[ n_e - \frac{1}{4} \frac{2n_0 \Delta v + v_{h0} + \varepsilon_n}{2\Delta v} \leq n_e + \frac{3}{4}, \] (17)

where \( v_{h0} \) is larger than 0, \( n_0 \) is the correct value of integer factor \( n \), and \( n_e \) is the estimated integer factor. Based on inequality (17), the relationship between \( \varepsilon_n \) and the estimated integer error \( \Delta n_e \) (\( \Delta n_e = n_e - n_0 \)) is obtained as follows:

\[ \Delta n_e - \frac{1}{4} \frac{v_{h0}}{2\Delta v} \leq \varepsilon_n \leq \Delta n_e + \frac{3}{4} \frac{v_{h0}}{2\Delta v}. \] (18)

In inequality (18), \( \varepsilon_n/(2\Delta v) \) is the error factor to estimate the wrong integer number. From Eq. (13), the velocity measured by CNDS follows the Gaussian distribution, so the error factor also follows the distribution shown below:

\[ \frac{\varepsilon_n}{2\Delta v} \sim \mathcal{N}\left(0, \frac{\sigma_n^2}{4\Delta v^2}\right). \] (19)

In order to obtain the theoretical error of CMDS, variable conversion from the continuous variable \( \varepsilon_n/(2\Delta v) \) to the discrete estimated integer error \( \Delta n_e \) should be carried out. With the integral scales shown in inequality (18) as the error factor, the probability of \( \Delta n_e \) occurring can be expressed as:

\[ P_{\Delta n_e} = \int_{\Delta n_e - \frac{1}{4} \frac{v_{h0}}{2\Delta v}}^{\Delta n_e + \frac{1}{4} \frac{v_{h0}}{2\Delta v}} \frac{1}{\sqrt{2\pi} \sigma_n} e^{-\frac{\varepsilon_n^2}{2\sigma_n^2}} d\varepsilon_n'. \] (20)

where

\[ \varepsilon_n' = \frac{\varepsilon_n}{2\Delta v}, \quad \sigma_n' = \frac{\sigma_n}{2\Delta v} \quad \text{and} \quad \Delta n_e \in Z. \]

The standard deviation of CMDS is expressed as:

\[ \sigma_m = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (v_m - v_0)^2} = \sqrt{E_1 + E_2 + E_3}, \] (21)

where

\[ E_1 = \frac{1}{N} \sum_{i=1}^{N} (v_{hi} - v_{h0})^2; \]
\[ E_2 = \frac{1}{N} \sum_{i=1}^{N} \left[4(\Delta n_{ei})^2\Delta v^2; \right) \]
\[ E_3 = \frac{1}{N} \sum_{i=1}^{N} 4\Delta n_{ei}\Delta v(v_{hi} - v_{h0}); \]
\[ N \to \infty; \]
\[ \Delta n_{ei} \] is the estimated integer error of \( i \)th calculation;
\[ v_{hi} \] is the measurement result of \( i \)th calculation by CHDS.
\[ E_1 \] is the expression of variance for CHDS.

Based on Eq. (20), and with the detail calculation referred to in Appendix B, \( E_2 \) and \( E_3 \) can be expressed as:

\[ E_2 = \sum_{\Delta n_e \in Z} P_{\Delta n_e} \left(2\Delta n_e\Delta v\right)^2, \] (24)
\[ E_3 = \sum_{\Delta n_e \in Z} P_{\Delta n_e} 4\Delta n_e\Delta v\left(v_{hi} - v_{h0}\right). \] (25)

In Eq. (25), \( \overline{v}_{hi\Delta n_e} \) is the mean value of measured velocity by CHDS, while the conventional method provides the integer factor estimation with error \( \Delta n_e \). If the number of measured data is large enough, the value of \( \overline{v}_{hi\Delta n_e} \) approximates to \( v_{h0} \) and the value of \( E_3 \) can be ignored. Consequently the standard deviation of CMDS in the case of inequality (9) is:
For the case in inequality (10), the standard deviation is deduced using the same calculation as in inequality (9), and it is referred to in Appendix C. If the absolute values of $v_{h0}$ in the cases of inequality (9) and (10) are same, the standard deviations are also same, which means the standard deviation of CMDS is independent of the velocity direction.

4. Simulation

4.1 Conditions

In our simulation, the transducer was considered as a fixed point, and the hydrophone was moving away from the transducer with a constant velocity, as shown in Fig. 3. The sign of hydrophone’s velocity was negative when the hydrophone moved away from the transducer and positive in the opposite direction.

To evaluate the performance of CMDS, some basic conditions used in the simulation are shown in Table 1. One slow-moving velocity, $-0.270 \text{ m/s}$, was selected as the half speed of our experimental moving device. Meanwhile, a fast-moving velocity, $-2.500 \text{ m/s}$, was selected as a common speed for a body moving on and under the sea. Other conditions were set to accommodate the experimental facilities in our laboratory. For CHDS, the velocity ambiguity determined by the time interval of 20 ms was 0.188 m/s. Two moving velocities, $-0.270$ and $-2.500 \text{ m/s}$, were larger than the velocity ambiguity. Therefore, CMDS was adopted to make the measurement.

In this simulation, the received signal was set as the transmitted signal with a time delay and additive white Gaussian noise (AWGN). According to a variation of AWGN, the received signals were generated in various SNRs. Specifically, there were 100 received pulses every two seconds on our simulation conditions.

4.2 Results and considerations

With the conditions shown in Table 1, the theoretical standard deviations of the three methods are shown in Fig. 4. In the figure, in the low SNR (less than $-2.0 \text{ dB}$), the difference in the standard deviation between CMDS and CNDS was small, but in the middle SNR (from $-2.0$ to $5.0 \text{ dB}$), the velocity measured by CMDS was better than CNDS. When the measurement was carried out in the high SNR (larger than $5.0 \text{ dB}$), CMDS could be as precise as

$$
\sigma_m = \sqrt{\sigma_h^2 + \sum_{\Delta n_c \in Z} P(\Delta n_c) (2\Delta n_c \Delta v)^2}.
$$

(26)
CHDS without velocity ambiguity. The standard deviations of −0.270 and −2.500 m/s measured by CMDS differed slightly in the middle range of SNR, because the two different velocities made different integration intervals in Eq. (20). However, in these simulation conditions, the standard deviations of the two velocities were not influenced a great deal. From the results shown in Fig. 4, the standard deviation of the measurement error by CNDS was about four times that of the standard deviation by CMDS in the SNR of 5 dB. In the case where the standard deviation by CNDS was the same value as by CMDS, the number of samples by CNDS required sixteen times the number by CMDS.

**Figure 5** shows one example of the velocity −0.270 m/s measured by means of CNDS, CHDS and CMDS in the SNR of −1.0 dB. The velocity

![Figure 5](image_url)

Fig. 5 Simulation results of −0.270 m/s with three methods in −1.0 dB SNR.

**Figure 6** shows simulation and theoretical errors of three methods: (a) CNDS (−0.270 m/s); (b) CHDS (−0.270 m/s); (c) CMDS (−0.270 m/s); (d) CNDS (−2.500 m/s); (e) CHDS (−2.500 m/s); (f) CMDS (−2.500 m/s).

![Figure 6](image_url)

Fig. 6 Simulation and theoretical errors of three methods: (a) CNDS (−0.270 m/s); (b) CHDS (−0.270 m/s); (c) CMDS (−0.270 m/s); (d) CNDS (−2.500 m/s); (e) CHDS (−2.500 m/s); (f) CMDS (−2.500 m/s).
of CHDS was precise and stable, but it deviated a couple of ambiguity velocity (2∆ν) from the true velocity (ν₀). The velocity of CMDS included one large impulsive behavior. This impulsive behavior was generated due to the wrong estimation of the integer factor n. Except for this impulse, the velocity of CMDS could be as stable as CHDS, and became accurate. From Eq. (20), in order to decrease the impulses, two methods were considered. One would have decreased the standard deviation of CNDS (σₙ), but in practice it was too difficult to achieve. The other method was to enlarge the ambiguity velocity of CHDS (Δν), which could be accomplished through use of the multi-frequency⁵) or the dual pulse repetition rate⁶).

Figure 6 (a)–(f) show the theoretical and simulation errors of the three methods in different SNR at −0.270 and −2.500 m/s velocities. The results of the comparison between the theoretical and simulated errors at two different velocities show that these two methods correspond well to each other; three theoretical equations (Eq. (12), Eq. (16), Eq. (26)) are used in the figures to evaluate each measurement error and can be used to access the performance of these measurement methods.

5. Summary

In this paper, we proposed a method that combines conventional and coherent Doppler sonar to avoid velocity ambiguity, and three theoretical error equations are derived to access the performance of measurement methods. Simulations were carried out to evaluate the derived equations correctly. From the theoretical results, in the low SNR, the proposed method has the same performance level as the conventional method, but can be used to measure accurate and precise velocity without velocity ambiguity in the high SNR.

In future research, we would like to develop a more effective processing method to improve the performance level of the proposed method.

Appendix A

| Symbol | Definition |
|--------|------------|
| b      | Amplitude of received signal |
| c      | Sound speed in water |
| r      | Time interval between adjacent pulses |
| λ      | Wave length |
| n      | Integer factor used to fold the velocity into the measurable range of the CHDS |
| a₀(t)  | Envelope of transmitted signal |
| φ₀     | Initial phase |
| f₀     | Carrier frequency |
| t₀     | Target round-trip time delay |
| ν₀     | True velocity of target |
| n₀     | Correct value of integer factor n |
| Δf    | Doppler frequency shift |
| s(t)   | Transmitted pulse |
| r(t)   | Received pulse |
| S(f)   | Fourier transform of s(t) |
| R(f)   | Fourier transform of r(t) |
| I(t)   | In-phase of the received signal |
| Q(t)   | Quadrature of the received signal |
| Δv    | Ambiguity velocity |
| Δr    | Ambiguity range |
| vₙ    | Velocity measured by CNDS |
| v₀    | Velocity measured by CHDS |
| vₘ    | Velocity measured by CMDS |
| vₙ₀   | Velocity measured by CHDS without noise |
| σₙ    | Standard deviation of velocity in CNDS |
| σₙₐ   | Standard deviation of velocity in CHDS |
| σₙₘ   | Standard deviation of velocity in CMDS |
| σₙ₁   | Standard deviation of Doppler shift |
| wₙ    | Error of the velocity measured by CNDS |
| n₂    | Estimated value of integer factor n |
| Δnₑ   | Estimated integer error of integer factor n |
| Δnₑᵢ | Estimated integer error of i-th calculation |

Appendix B

In order to convert E₂ and E₃, expressed in Eqs. (22) and (23), to the expressions with the probability of the estimated integer error, the count of the case Δnₑ=q is set as M_q, e.g. M_q=1 if the number of Δnₑ satisfying Δnₑ=q is equal to 1. Then Eq. (22)
and Eq. (23) are changed to be:

\[ E_2 = \sum_{q \in Z} \frac{M_q}{N} q^2 (2\Delta v)^2 \]  

(27)

\[ E_3 = -\frac{1}{N} \sum_{q \in Z} \sum_{z: \Delta n_q = q} 4q\Delta v (v_{hi} - v_{h0}) \]

\[ = \frac{M}{N} 4q\Delta v \left\{ \frac{1}{M_q} \sum_{z: \Delta n_q = q} (v_{hi} - v_{h0}) \right\} \]

\[ = \frac{M}{N} 4q\Delta v (\bar{v}_{h0} - v_{h0}), \]  

(28)

where the summation of \( M_q (q \in Z) \) is \( N \); \( \bar{v}_{h0} \) is the mean value of measured velocity by CHDS, while the conventional method provides the integer estimation error \( q \). When \( N \to \infty \), \( M_q/N \) is the probability of estimated integer error \( q \). Therefore, with the probability shown in Eq. (20), Eq. (27) and Eq. (28) can be expressed as:

\[ E_2 = \sum_{q \in Z} P_q (2q\Delta v)^2 \]  

(29)

\[ E_3 = \sum_{q \in Z} P_q 4q\Delta v (\bar{v}_{h0} - v_{h0}). \]  

(30)

With the integer \( q \) replaced by \( \Delta n_e \) in Eq. (29) and Eq. (30), Eq. (24) and Eq. (25) can be obtained.

Appendix C

For the case in inequality (10), the inequality can also be expressed as:

\[ \Delta n_e - \frac{3}{4} \leq \frac{2n_0\Delta v + v_{h0} + e_n}{2\Delta v} \leq \Delta n_e + \frac{1}{4}, \]  

(31)

where \( v_{h0} \) is not larger than 0. Then inequality about the error factor \( e_n/(2\Delta v) \) is obtained as:

\[ -\Delta n_e - \frac{1}{4} + \frac{v_{h0}}{2\Delta v} < -\frac{e_n}{2\Delta v} \leq -\Delta n_e + \frac{3}{4} + \frac{v_{h0}}{2\Delta v}. \]  

(32)

Because \( e_n/(2\Delta v) \) follows a Gaussian distribution, \(-e_n/(2\Delta v)\) has the same distribution as \( e_n/(2\Delta v) \) shown in Eq. (19). Therefore, the probability of the discrete estimated integer error \( \Delta n_e \) can be expressed as:

\[ P_{\Delta n_e} = \int_{-\infty}^{\Delta n_e} \frac{1}{\sqrt{2\pi} \sigma_n} e^{-\frac{e_n^2}{2\sigma_n^2}} \, de_n'. \]  

(33)

Based on Eq. (21), Eq. (33) and Appendix B, the standard deviation of CMDS in the case of \( v_{h0} \leq 0 \) is shown as:

\[ \sigma_m = \sqrt{\sigma_h^2 + \sum_{\Delta n_e \in Z} P_{\Delta n_e} (2\Delta n_e \Delta v)^2}. \]  

(34)

Compared with Eq. (33) and Eq. (20), if the absolute values of \( v_{h0} \) in different directions are same, the relationship between \( P_{\Delta n_e} \) and \( P_{-\Delta n_e} \) can be expressed as:

\[ P_{-\Delta n_e} = P_{\Delta n_e}. \]  

(35)

According to Eq. (35), the relation between of \( \sigma_m^- \) and \( \sigma_m \) can be calculated as:

\[ \sigma_m^- = \sqrt{\sigma_h^2 + \sum_{\Delta n_e \in Z} P_{\Delta n_e} (2\Delta n_e \Delta v)^2} \]

\[ = \sqrt{\sigma_h^2 + \sum_{\Delta n_e \in Z} P_{-\Delta n_e} (2\Delta n_e \Delta v)^2} \]

\[ = \sigma_m. \]  

(36)

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