Invited Comment

The long and winding road from chiral effective Lagrangians to nuclear structure

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Abstract

I review the chiral dynamics of nuclear physics. In the first part, I discuss the new developments in the construction of the forces between two, three and four nucleons which have been partly carried out to fifth order in the chiral expansion. It is also shown that based on these forces in conjunction with the estimation of the corresponding theoretical uncertainties, the need for three-nucleon forces in few nucleon systems can be unambiguously established. I also introduce the lattice formulation of these forces, which allow for truly ab initio calculations of nuclear structure and reactions. I present some pertinent results of the nuclear lattice approach. Finally, I discuss how few-nucleon systems and nuclei can be used to explore symmetries and physics within and beyond the standard model.

Keywords: effective Lagrangians, chiral symmetry, nuclear forces, nuclear structure, symmetries

1. Introduction

This contribution to celebrate the 40th anniversary of the Nobel prize to the nuclear structure investigations of Bohr, Mottelson and Rainwater reviews some work that firmly links nuclear physics to the gauge theory of the strong interactions, quantum chromodynamics (QCD), by the exploration of the symmetries of QCD and their realization. This is arguably the most important development in nuclear physics for many decades, as it puts nuclear physics on a very different level of rigor and precision than was possible before. The main ingredient in such an approach is the concept of an effective Lagrangian, that was championed for the strong interactions by Noble laureate Steven Weinberg [1] and by Gasser and Leutwyler [2, 3]. In such an approach, one is able to perform a systematic expansion in a small parameter, typically some soft external momentum or a small mass divided by a hard (large) scale. Thus scale separation is an important ingredient, and that is exactly what the spectrum of QCD for the light flavors up, down and strange exhibits. Further, such an effective Lagrangian approach allows one to estimate the uncertainty of any given calculation, an absolute must for any serious theoretical approach. Or stated more bluntly: a theoretical calculation that does not give an uncertainty is as good as any random number. In nuclear physics, matters are, however, a bit more complicated, as the very small binding energies or binding momenta seem to restrict the applicability of the effective Lagrangian approach to a very small range of energies or momenta. Again, it was Weinberg [4, 5], who laid out the framework to overcome these obstacles. His approach has been criticized by many, but so far only within the so-called Weinberg power counting scheme to be explained in detail below, nuclear structure questions can be addressed rigorously. Under certain...
circumstances, the scale separation can be used to set up other effective theories, such as the pionless nuclear effective field theory (EFT) (for reviews, see e.g. [6, 7]) or the effective theory for halo nuclei [8]. These, however, will not be discussed here. Another more direct path from QCD is the application of lattice QCD to nuclear systems. Such calculations have, however, to overcome severe obstacles, and will not be a precision tool in the next few years. Still, lots of progress is made in that approach, as witnessed e.g. by recent calculations of light nuclei and hyper-nuclei at large pion masses [9], of the magnetic moments of nuclei [10] or trying to construct a nuclear potential [11].

This paper is organized as follows. In section 2 I review the essentials of QCD, with particular emphasis on the sector of the light quarks. This is further elaborated in section 3, where chiral symmetry and its various variants of breaking are discussed. There is also a short discussion of the broken U(1)A symmetry of QCD, which can be explored to test physics beyond the standard model (SM). The next section 4 introduces the concept of the effective Lagrangian and the machinery related to it, in particular the power counting, the so-called low-energy constants and issues related to renormalization. An important ingredient to construct nuclear forces is pion energy constants and issues related to it, in particular the power counting, the so-called low-energy constants and issues related to renormalization. An important ingredient to construct nuclear forces is pion-nucleon scattering, which is discussed in section 5, including very recent results from Roy–Steiner (RS) equations matched to chiral perturbation theory, the EFT of QCD. Armed with that, nuclear forces are discussed in section 6 featuring the most recent results at fifth order in the chiral expansion for two-nucleon forces, the estimation of theoretical uncertainties and the status of three- and four-nucleon forces (4NFs). To tackle nuclei, one can either use these forces in connection with more conventional many-body approaches (shell model, coupled-cluster methods and so on) or discretize space–time and use this lattice to perform Monte Carlo simulations of nuclei. It is this latter approach which will be described in section 7, and assorted results will be presented in section 8. One of the nice features of this novel framework is that it allows one to investigate the behavior of nuclear structure and reactions under variations of the fundamental parameters, e.g. the light quark masses and the electromagnetic fine-structure constant. This is discussed in section 9 together with its consequences for our anthropic view of the Universe. The role of nuclei as precision laboratories to explore symmetries within and beyond the SM is reviewed in section 10. The final section 11 gives some perspectives and outlines future research in this exciting area of physics.

2. QCD

QCD is a fascinating theory based on a local, non-abelian SU(3)\text{color} symmetry. It embodies all of strong interaction physics essentially in one simple line\(^2\)

\[
\mathcal{L}_{\text{QCD}} = \frac{1}{2g^2} \text{Tr}(G_{\mu\nu}G^{\mu\nu}) + \bar{\psi}(i\gamma^\mu D_\mu - \mathcal{M})\psi, \quad (1)
\]

where \(G_{\mu\nu}\) is the gluon field strength tensor, and \(\psi\) is a spinor that includes the six quark flavors up, down, strange, charm, bottom and top. The forces are mediated by gluons that couple to the color not made explicit in equation (1) and we have absorbed the gauge coupling \(g\) in the definition of the gluon field. As the gluons carry color, they can interact with themselves through three- and four-point vertices. The gauge-covariant derivative \(D_\mu\) generates the quark-gluon coupling. It is important to realize that QCD can be split into two sectors, one referring to the light quarks \(u, d,\) and \(s\) and the other one is given by the heavy quarks \(c\) and \(b\). Here, light means that the current quark mass is much smaller than the typical scale of QCD, \(\Lambda_{\text{QCD}} \approx 250\text{ MeV},\) that can be inferred from the running of the strong coupling constant \(\alpha_s = g^2/(4\pi),\) whereas the mass of the heavy quarks is much larger than this scale. In the light quark sector, one can rewrite the QCD Lagrangian in terms of the three-component vector \(q\) that collects the light quark fields, \(q^i(x) = (u(x), d(x), s(x)).\) Here, the mass term can be considered as a perturbation, and to first order, one can completely ignore the mass term. This is what Heiri Leutwyler calls a ‘theoretical paradise’ [12] as one is dealing with a theory that has not a single tunable parameter as \(\Lambda_{\text{QCD}}\) is generated by dimensional transmutation. As is well known, massless fermions exhibit a chiral symmetry, that is, the theory exists in two copies, one for the left- and the other for the right-handed fields. There is no interaction between these two almost identical worlds. More on that in the next section. The heavy quarks are commonly collected in the doublet \(Q^i(x) = (c(x), b(x))\), so that the leading order Lagrangian of the heavy quark effective theory, in which the large masses \(m_c\) and \(m_b\) has been transformed into a string of \(1/m_Q\) suppressed terms, simply takes the form \(\mathcal{L}_0 = \bar{Q}(iv \cdot D)Q,\) with \(v_\mu\) the four-velocity of the heavy quark. This form obviously exhibits a SU(2)\text{flavor} symmetry as well as an SU(2)\text{spin} symmetry, combined in the SU(4) spin-flavor symmetry, as \(\mathcal{L}_0\) neither depends on the spin of the heavy quark nor on its mass. These symmetries are, of course, broken at next-to-leading order (NLO) from corrections to the kinetic energy term and the chromo-magnetic Pauli interaction. Nothing more will be said here on this intriguing part of the theory. The quarks and gluons are confined within hadrons, the strongly interacting particles. Most hadrons are simple mesons (quark-antiquark states) or baryons (three quark states), but as of today some tetraquark and maybe even pentaquark states have been established. It is still not understood why QCD mostly generates states of the simplest types and also the expected glueballs, that are made of nothing but glue, have been elusive. However, it is basically known how the massive hadrons acquire their mass. Bound states made of heavy quarks are essentially slow moving objects, with their mass largely given by the masses of the quarks they are made of. For the hadrons made of light quarks (with the exception of the Goldstone bosons to be discussed below), most of the mass is generated by the gluon field energy, beautifully realizing Wheeler’s notion of ‘mass without mass’ [13]. More formally, this can be understood

\(^3\) The top quark decays too quickly to form any strongly interacting particle.
from the breaking of the dilatation current of massless classical QCD, the so-called trace anomaly [14]. As one example, the nucleon mass can be written as follows:

\[ m_N(p) = \langle N(p) | \theta | N(p) \rangle \]

\[ = \langle N(p) | \frac{\beta_{\text{QCD}}}{2g} G^\mu_{\alpha} G^{\alpha\nu} + m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s | N(p) \rangle, \]

(2)

where \( \beta_{\text{QCD}} \) is the QCD \( \beta \)-function and \( \theta_{\mu} \) the trace of the energy-momentum tensor. The first term gives the field energy contribution, the terms proportional to \( m_u \) and \( m_d \) can be related to the pion–nucleon sigma term and the last one to the strangeness content. As discussed below, these sum up to about 110 MeV (with sizeable uncertainty for the strangeness contribution), so that the bulk of the mass is indeed coming from the gluon field energy. This scenario is also consistent with recent lattice QCD calculations of hadron masses at physical quark masses.

As noted in footnote 2, there is a bit more to QCD than just given in equation (1). I refer to the so-called \( \theta \)-term of QCD, that is a consequence of the non-trivial vacuum structure of QCD and the anomalous breaking of the \( U(1)_A \) symmetry, given by

\[ \mathcal{L}_\theta = -\frac{\theta}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G^\mu_{\alpha} G^{\nu\alpha}, \]

(3)

where \( \alpha = 1, \ldots, 8 \) are color indices. The \( \theta \)-term can be rotated into the quark mass matrix, and as long as one of the current quark masses vanishes, so does the effect of \( \theta \). However, nature does not seem to pull this option as all quark masses are non-vanishing. The \( \mathcal{G} \) term, with \( \mathcal{G}^{\alpha\beta} \sim \epsilon^{\alpha\beta\gamma\delta} G_{\gamma\delta} \), clearly has odd properties as it is proportional to the product of the chromo-electric and the chromo-magnetic fields and thus is odd under CP (charge conjugation times parity transformation), and assuming CPT to be an exact symmetry, also under T (time reversal, or, more correctly, motion reversal, as Cecilia Jarlskog often points out, see e.g. [15]). Consequently, hadrons can acquire permanent electric dipole moments (EDMs)\(^4\) that can interact with external electric fields. Measurements of the upper limit of electric dipole moment of the neutron poses a stringent limit on the value of \( \theta \) [16], where the latest determination gives \( |\theta| < 7.6 \times 10^{-11} \) [17].

3. Chiral symmetry

To start the discussion of QCD, I introduce the concept of chiral symmetry. To understand this fact, let us perform a left/right (\( L/R \))-decomposition of the fermion (spin-1/2) field

\[ \psi = \frac{1}{2} (1 - \gamma_5) \psi + \frac{1}{2} (1 + \gamma_5) \psi = P_L \psi + P_R \psi = \psi_L + \psi_R. \]

(5)

Here, the \( P_{L/R} \) are projection operators, satisfying \( P_L^2 = P_L \), \( P_R^2 = P_R \), \( P_L \cdot P_R = 0 \), \( P_L + P_R = 1 \). Further, these new fields \( \psi_{L/R} \) are helicity eigenstates

\[ \frac{1}{2} \hat{h} \psi_{L/R} = \pm \frac{1}{2} \psi_{L/R} \quad \hat{h} = \frac{\hat{\sigma} \cdot \hat{\rho}}{1 |\vec{p}|}. \]

(6)

Here, \( \hat{\rho} \) is the fermion momentum and \( \hat{\sigma} \) are the Pauli spin matrices. Expressing the Lagrangian in terms of the left- and right-handed fields, it takes the form

\[ \mathcal{L} = i \bar{\psi}_L \gamma_{\mu} \partial^\mu \psi_L + i \bar{\psi}_R \gamma_{\mu} \partial^\mu \psi_R. \]

(7)

This shows that the \( L/R \) fields do not interact. Further, as a consequence of Noether’s theorem, one finds conserved \( L/R \) currents. Clearly, a fermion mass term breaks chiral symmetry, as such a term mixes the left- and right-handed components, \( \bar{\psi}_L \mathcal{M} \psi_R = \bar{\psi}_R \mathcal{M} \psi_L \). This can be easily understood. Whereas massless fermions move with the speed of light, this is no longer the case for massive fermions. Thus, for a massive fermion with a given handedness in a certain frame, it is always possible to find a boost such that the sign of \( \hat{\sigma} \cdot \hat{\rho} \) changes. However, in the case where the mass term is sufficiently small (where ‘small’ depends on other scales in the theory), one can treat this explicit chiral symmetry breaking in perturbation theory and, consequently, this is called an approximate chiral symmetry. I will come back to this topic later.

The appearance of broken symmetries is a common phenomenon in many physical systems. Particular intriguing is the appearance of spontaneous symmetry breaking. This means that the ground state of a theory has a lesser symmetry than the corresponding Lagrangian or Hamiltonian. A key ingredient in the physics of spontaneous symmetry breaking is Goldstone’s theorem [18, 19]; to every generator of a spontaneously broken symmetry corresponds a massless excitation of the vacuum. We can understand this in a nutshell (there are some subtleties like e.g. the normalization of states, however, in a more rigorous formulation one arrives at the same conclusions). Let \( \mathcal{H} \) be some Hamiltonian that is invariant under some charges \( Q^i \), so that [\( \mathcal{H}, Q^i \) = 0, with \( i = 1, \ldots, n \). Let us further assume that a certain number \( m \) of these charges, with \( m \leq n \), do not annihilate the vacuum: \( Q^j |0\rangle = 0 \), \( j = 1, \ldots, m \) Let us now define a single-particle state by \( |\psi\rangle = Q^0 |0\rangle \). This is clearly an energy eigenstate with eigenvalue zero, since \( H |\psi\rangle = H Q^0 |0\rangle = Q H |0\rangle = 0 \). Therefore, the state \( |\psi\rangle \) is a single-particle state with the properties \( E = \hat{\rho} = 0 \), i.e. it is a massless excitation of the vacuum. Such states are the famous Goldstone bosons. In the following, I will collectively denote them as pions \( \pi(x) \). These Goldstone bosons couple directly to the vacuum via the corresponding symmetry current (whereas conventional

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\(^4\) Such a permanent EDM should not be confused with an induced dipole moment. Such induced dipole moments are typically of the size or smaller than the volume of the hadron under consideration.
single particle states have zero overlap with the vacuum)
\[ \langle 0 | J^3(0) | \pi \rangle = 0. \] (8)

In fact, the non-vanishing of this matrix element is a necessary and sufficient condition for spontaneous symmetry breaking. We will later discuss the dimensionfull quantity that characterizes this matrix element.

There is another important property of Goldstone bosons to be discussed, namely the derivative nature of their couplings to themselves or matter fields. This can also be understood using some hand-waving arguments. As before, we can repeat the operation of acting with the non-conserved charge \( Q' \) on the vacuum state \( k \) times. This generates a new state of \( k \) Goldstone bosons that is, however, degenerate with the vacuum. Let us assume that the interactions of the Goldstone bosons is not vanishing at zero momentum. Then, the ground state cannot be degenerate with the state made of \( k \) Goldstone boson, and therefore the assumption must be incorrect. As before, a more rigorous form of this argument leads to the same result. In the following, the derivative nature of the pion couplings will play an important role.

Let us now consider three-flavor QCD with up, down, and strange quarks. With respect to the strong interactions, the different quarks \( u, d, s \) have identical properties, except for their masses. The quark masses are free parameters in QCD. In fact, QCD can be written down for any value of the quark masses. The quark masses are free parameters in QCD.

Let us come back to QCD. Here, one finds eight (three Goldstone bosons for chiral SU(3) (SU(2)). These particles have spin zero and negative parity. This negative parity originates from the fact that these Goldstone bosons are generated by applying the axial charges on the vacuum. As noted before, such Goldstone bosons have a non-vanishing overlap with the vacuum, see equation (8). This matrix-element is given in terms of one dimensionfull scale, which in the case of QCD is the pion decay constant (in the chiral limit)

\[ \langle 0 | A_\mu^a(0) | \pi^i(p) \rangle = i\delta^{ab} F_{\mu\nu}. \] (12)

This quantity is a fundamental mass scale of low-energy QCD. For massless quarks, the value of \( F \) differs from the physical value by terms proportional to the quark masses, \( F_* = F[1 + O(M)] \). The physical value of \( F_* \) is 92.2 MeV, determined from pion decay, \( \pi \rightarrow \mu \nu \).

However, the quark masses are not exactly zero. The quark mass term generates the already mentioned explicit chiral symmetry breaking. Therefore, the vector and axial-vector currents are no longer conserved (with the exception of
the baryon number current)

$$\partial_{\mu} V_{\alpha}^u = \frac{1}{2} i \bar{q} \{ M, \lambda_\nu \} q, \quad \partial_{\mu} A_{\mu}^u = \frac{1}{2} i \bar{q} \{ M, \lambda_\nu \} \gamma_5 q.$$  \hspace{1cm} (13)

Since the quark masses are small, it is still possible to investigate the consequences of the spontaneous symmetry violation systematically. QCD exhibits an approximate chiral symmetry so that the mass spectrum of the unperturbed Hamiltonian and the one including the quark masses are not very different. This means that the effects of the explicit symmetry breaking can be analyzed in perturbation theory. However, this perturbation leads to the remarkable mass gap of the theory. The pions (and, to a lesser extent, the kaons and the eta) are much lighter than all other hadrons. Let us consider chiral SU(2) and take a closer look at equation (13). These are Ward-identities. In particular, the second equation relates the axial current $A^\nu = \bar{d} \gamma^\nu \gamma_5 u$ to the pseudoscalar density $P = \bar{d} i \gamma_5 u$.

$$\partial_{\mu} A^\nu = (m_u + m_d) P.$$  \hspace{1cm} (14)

For on-shell pions, this matrix element takes the form

$$M^2 = (m_u + m_d) \frac{G_F}{F_\pi}.$$  \hspace{1cm} (15)

Here, the coupling constant $G_F$ is given by $\langle 0 | P(0) \pi(p) \rangle = G_F$. Some intriguing consequences follow from this equation. As demanded by Goldstone’s theorem, the pion mass vanishes in the chiral limit. Further, the ratio $G_F/F_\pi$ is a constant in the chiral limit and therefore the pion mass increases as $\sqrt{m_u + m_d}$ as the quark masses are switched on.

The quark mass term exhibits even further symmetry. In nature, hadrons appear in isospin multiplets. The intra-multiplet splittings are tiny, i.e. of the order of a few MeV. These are generated from two sources: One is the quark mass difference $m_u - m_d$, which is small with respect to the typical hadronic mass scale of a few hundred MeV. The other source is electromagnetic effects, which are typical of the same size as the strong ones. The notable exception is the charged to neutral pion mass difference which is almost entirely of electromagnetic origin. The origin of this is easily understood. For equal light quark masses, QCD is invariant under SU(2) isospin transformations:

$$q \rightarrow q' = Uq, \quad q = \begin{pmatrix} u \\ d \end{pmatrix},$$

$$U = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}, \quad |a|^2 + |b|^2 = 1.$$  \hspace{1cm} (16)

In this limit, up and down quarks are identical with respect to the strong interaction. We can make the strong isospin violation explicit by rewriting the QCD quark mass term as

$$\mathcal{H}_{\text{QCD}}^{SB} = m_u uu + m_d dd$$

$$= \frac{m_u + m_d}{2} (\bar{u}u + \bar{d}d) + \frac{m_u - m_d}{2} (\bar{u}u - \bar{d}d).$$  \hspace{1cm} (17)

The first (second) term in this equation is an isoscalar (isovector). If we also consider the strange quarks, i.e. extend these considerations to SU(3), we recover at the eightfold way of Gell-Mann and Ne’eman which was instrumental in our understanding of the quark structure of the hadrons. Note that the SU(3) flavor symmetry is also an approximate symmetry. However, as seen e.g. from the splittings in SU(3) multiplets, the breaking is much stronger than is the case for the up and down quarks. Therefore, the quark mass difference $m_s - m_d$ must be much bigger than $m_d - m_u$.

Remarkably, there is yet another source of symmetry breaking. Its origin can be best understood in terms of the path integral representation. The effective action of QCD contains an integral over the quark fields. This integral leads to the so-called fermion determinant. If we require a theory to be invariant under chiral transformations this means that not only is the action invariant, but also the fermion measure. In a very symbolic notation, this can be expressed as

$$\int [dq] [dq'] \cdots [\mathcal{J}] \int [dq] [dq'] \cdots$$  \hspace{1cm} (18)

where $\mathcal{J}$ is the Jacobian of the transformation. In the case where it is not equal to one, $|\mathcal{J}| = 1$, one speaks of an anomaly. We already encountered one example, the $\theta$-term of the QCD Lagrangian. Similar to the case of the Goldstone theorem, such a hand-waving statement has to be made more precise. The path integral requires regularization and renormalization, but after the dust has settled, the non-invariance of this Jacobian signals the appearance of an anomaly. More generally, it can be proven that three-, four-, and five-point functions with an odd number of external axial-vector sources are anomalous. The most famous example of this is the time-honored triangle anomaly of Adler, Bell and Jackiw in the decay $\pi^0 \rightarrow \gamma \gamma$. In QCD, another important example is the divergence of the singlet axial current

$$\partial_{\mu}(q^\mu q^5) = 2iq\pi^\gamma q + \frac{N_c}{8\pi} G^\mu_{\gamma a} G^{\gamma \mu a},$$  \hspace{1cm} (19)

where the second term on the right-hand side is responsible for the mass of the $q^\gamma$ (in the chiral limit). For a review of anomalies in QCD and their consequences, see e.g. [26]. In what follows, we will consider some consequences of the $\theta$-term in QCD.

4. Effective Lagrangian

To deal with systems that exhibit scale separation, one works with a properly formulated effective Lagrangian. It shares the same symmetries as the underlying theory (like in our case QCD) but is formulated in terms of the pertinent asymptotic hadronic fields (for QCD: pions and nucleons). To keep matters simple, let us first consider pions only. Pions are Goldstone bosons, therefore their interactions with themselves and other fields are of derivative nature, as discussed before. Consequently, we can write down an EFT at low energies/momenta, noting that derivatives acting on pions can be translated into small momenta. Such an EFT is
necessarily non-renormalizable, as one can write down an infinite tower of terms with increasing number of derivatives consistent with the underlying symmetries, in particular chiral symmetry. Therefore, such a theory is only valid for momenta and masses that are ‘soft’, which means that these are small compared to the so-called ‘hard’ scale, that is set by the masses of the particles not considered. In QCD, this hard scale is of the order of 1 GeV$^5$. The beauty of the EFT is the appearance of a hierarchy of terms that allows one to make precise predictions with a quantifiable theoretical order. This allows for what is called power counting. To be specific, let us study the Lagrangian

$$\mathcal{L}_{\text{eff}} = \sum_d \mathcal{L}^{(d)}_d,$$

and let us assume that the integer $d$ is bounded from below. In the case of Goldstone bosons we have $d \geq 2$. Further, $d$ is even due to Lorentz invariance or parity. The pion propagator is $D(q) = i/(q^2 - M_\pi^2)$, with $M_\pi$ the pion mass. Let us look at an $L$-loop diagram with $I$ internal lines and $V_d$ vertices of order $d$ which leads to an amplitude $Amp$ with the following scaling

$$Amp \propto \int (d^4q)^{\nu} \frac{1}{(q^2)^{L+dV_d}} \prod_d (q^d)^{V_d},$$

in terms of powers of momenta. With $Amp \sim q^\nu$, we can read of the exponent $\nu$ from equation (21), namely $\nu = 4L - 2I + \sum_d dV_d$. One can further derive a topological relation between the number of loops, the number of internal lines and vertices, which reads $L = I - \sum_d V_d + 1$. Therefore, the number of internal lines can be eliminated, leading to [1]

$$\nu = 2 + 2L + \sum_d V_d(d - 2).$$

The consequences of this simple formula are far-reaching. To lowest order (LO), one has to consider only graphs with $d = 2$ and $L = 0$. These are tree diagrams. The explicit chiral symmetry breaking is also included because the quark mass counts as two powers of $q$, see equation (15). The LO contribution has long been known as the current algebra result. It can be obtained using various but less elegant methods. The most important feature of equation (22) is that it provides a recipe to systematically construct corrections to the LO result. At NLO, one has two types of contributions. First, there are one loop graphs $L = 1$ built from the lowest order interactions. Second, we have to consider contact terms with $d = 4$. These are terms with four derivatives or with two derivatives and one quark insertion. All these terms are accompanied by parameters, the so-called low-energy constants (LECs). These LECs are not constrained by the symmetries and must be determined by a fit to data. In some cases, they can also be obtained from lattice simulations. These allow one to vary the quark masses and thus give much easier access to the operators that involve powers of quark mass insertions or mixed terms involving quark masses and derivatives. For a more detailed discussion on the determinations from lattice QCD, I refer to [27]. At next-to-next-to-leading order (NNLO), we have two-loop graphs with any number of $d = 2$ insertions, one-loop graphs with exactly one $d = 4$ insertion and various types $d = 6$ contact terms. The extension to stable matter fields like the nucleon is also possible and not too difficult. In that case one also has to deal with operators with an odd number of derivatives. Furthermore, the power counting has to be adopted properly to deal with the large nucleon mass scale, see e.g. [28] for a detailed review. For unstable states the situation is more complicated, because in such cases one must deal with the energy scales related to the decays. As an example, consider the $\Delta(1232)$-resonance. Here, a consistent power counting can be set up if we count the nucleon-delta mass difference $m_\Delta - m_N \approx 2M_\pi$ as a small parameter. Let me not further elaborate on these issues but rather refer to some early related works on the $\Delta$ and vector mesons [29–32].

Let us return to the discussion of CHPT with pions (and possibly nucleons). An important observation was made in [33, 34], namely that the chiral Ward identities of QCD are fulfilled in CHPT. These Ward identities connect a tower of different processes involving various numbers of pions and external sources. Therefore, once we have fixed the pertinent LECs through a fit to certain data, it is possible to make a number of testable predictions. Clearly, with increasing order in the chiral expansion, we also have to face an increase in the number of LECs. However, considering a specific process this increase is not prolific. A fine example is elastic pion–pion scattering. At one-loop order, one has to determine exactly four LECs. At two loop order, only two new LECs have to be pinned down because all other local two-loop contributions account for quark mass renormalizations of certain one-loop operators. This is a more general phenomenon as one can group the various operator structures in two classes. The first class are the so-called dynamical operators. These are local contact terms with powers of external momenta (i.e. derivatives on the hadronic fields) and no quark mass insertion. The second class are the so-called symmetry-breakers. These operators feature a certain number of quark mass insertions. Therefore, the symmetry-breakers vanish in the chiral limit. As already said, this latter type of operators can be studied using lattice simulations with varying quark masses. As stated before, the various operators connect different processes, so that the determination in one reaction leads to predictions in other ones. The best example here are the dimension-two couplings $c_i$ in the chiral effective pion–nucleon Lagrangian, as shown in figure 1 (see the review [35] for definitions and more details). These LECs can be pinned down by fitting to pion-nucleon scattering data, as indicated by the left graph in figure 1. I will come back to this topic in the next section. The same operators that appear in $\pi N$ scattering not only are an important ingredient in the two-pion exchange contribution to nucleon–nucleon scattering (middle graph in figure 1) but they also generate the long-range part of the three-nucleon interaction (right graph in figure 1). Note that one can therefore determine the $c_i$‘s directly from nucleon–nucleon scattering. This leads to values consistent...
with the ones determined from $\pi N$ scattering. In addition, the two-pion exchange contribution establishes the role of pion-loop effects (see the middle graph in figure 1) in nucleon–nucleon scattering beyond the long-established tree-level pion exchange, that was proposed by Yukawa in 1935.

There is one further issue that requires discussion, namely unitarity. According to the power counting described above, we see that imaginary parts of scattering amplitudes or form factors are only generated at subleading orders. To be more specific, the one-loop graphs generate the leading contributions to the respective imaginary parts. For most cases, this perturbative fulfillment of unitarity is not problematic. However, in all processes where the strong pion–pion final state interactions related to the low-lying and broad scalar meson appear, see e.g. [37] for an early discussion and the precise extraction of its properties from Roy equations [38], one faces a much reduced radius of convergence of the chiral expansion and resummation methods might be required to deal with this effect. A fine review of these issues has recently been given by Pelaez [39]. In fact, one can consider this topic from a different perspective. It has long been known that using analyticity and unitarity, one is able to work out the leading loop corrections without calculating a single loop diagram. I just mention two time-honored examples, namely Lehmann’s analysis of pion–pion scattering in 1972 [40] and Weinberg’s general analysis of the structure of effective Lagrangians [1]. For a pedagogical introduction to the relation between unitarity and CHPT, I refer to [41]. Finally, I remark that unitarization of chiral scattering amplitudes can generate resonances, as first stressed by Truong, see [42] (and references therein). However, such an extension of CHPT to higher energies is often violating crossing symmetry and the direct relation to the QCD Green functions is no longer easily made.

5. Pion–nucleon scattering

Pion–nucleon scattering is one of the premier reactions to test the chiral dynamics of QCD. It is also an important ingredient in the description of the forces between two nucleons, as mentioned above and will be made more explicit in a later section. The reaction $\pi^+(q) + N(p) \rightarrow \pi^0(q') + N(p')$ is best described in terms of the Mandelstam variables, with $s = (p + q)^2$, $t = (q' - q)^2$, and $u = (p - q')^2$, subject to the constraint $s + t + u = 2(M^2_N + m^2_N)$, and $a$, $b$ are isospin indices. $M_\pi(m_N)$ is the pion (nucleon) mass. The Mandelstam plane for this process is shown in figure 2. The threshold region for the $s$-channel process is the hatched area on the right side, the interior Mandelstam triangle (subthreshold region), where the scattering amplitude is real, is shown by the triangle. These are the regions where chiral perturbation theory has been applied to pin down the so important LECs $c_i$.

A first series of works, employing the heavy baryon approach, was performed at Jülich around the year 2000 [43, 44], pioneering also the matching of chiral amplitudes in the subthreshold region to a dispersive representation from the Karlsruhe–Helsinki group [45]. The resulting values for the $c_i$’s are listed in the first row of table 1. The second row gives a more recent determination from the Bochum group [46], where the range is due to the fit to various partial wave analyses, accounting also for the different counting of the nucleon mass in the chiral EFT for nuclear forces (as explained below). One notices that the errors are sizeable and that there are systematic differences. This can be partly traced back to the fact that the chiral representation of the $\pi N$ scattering amplitude does not provide sufficient curvature in the subthreshold region as first pointed out by Becher and Leutwyler [47].

Before coming back to the determination of the LECs $c_i$, another important development deserves to be mentioned. Triggered by extremely accurate measurements of the energy level shifts and widths of pionic hydrogen and deuterium at PSI [48, 49], the authors of [50, 51] used CHPT to calculate the $\pi^0d$ scattering length, where $d$ denotes the deuteron, with an accuracy of a few percent. In particular, for the first time isospin-violating corrections in the two- and three-body systems were included consistently. Using the PSI data on pionic deuterium and pionic hydrogen atoms, the isoscalar and isovector pion–nucleon scattering lengths could be extracted with high precision

\[
\begin{align*}
a^+ &= (7.6 \pm 3.1) \times 10^{-3} M^{\pi^{-1}}_\pi, \\
a^- &= (86.1 \pm 0.9) \times 10^{-3} M^{\pi^{-1}}_\pi. \quad (23)
\end{align*}
\]

This is truly a remarkable achievement. The famous LO predictions are $a^+ = 0$ and $a^- = 79.4 \times 10^{-3} M^{\pi^{-1}}_\pi$ [52, 53]. For the first time, the sign of the small isoscalar scattering
length could be fixed with 2.5 σ certainty. Using the Goldberger–Miyazawa–Oehme sum rule [54], this leads to the charged-pion–nucleon coupling constant $g_\pi^2/(4\pi) = 13.69 \pm 0.20$.

I return to the issue of determining the pion–nucleon scattering amplitude, which is best done using dispersion relations. Such a method can be applied to investigate various scattering processes, like $\pi\pi$, $\pi K$ or $\pi N$ scattering. Roy equations [55, 56] for $\pi\pi$ scattering, or RS equations [57–60] for non-totally crossing-symmetric processes, incorporate the constraints from analyticity, unitarity, and crossing symmetry in the form of dispersion relations for the partial waves. They can be shown to be rigorously valid in a certain kinematic region, in the case of $\pi N$ scattering the upper limit is $s_m = (1.38 \text{ GeV})^2$ [60]. The integral contributions above $s_m$ as well as partial waves with $l > l_m$ with $l_m$ the maximal angular momentum explicitly included in the calculation, are collected in the so-called driving terms, which need to be estimated from existing partial wave analyzes (PWAs), as do inelastic contributions below $s_m$. The free parameters of the approach are subtraction constants, which, in the case of $\pi\pi$ scattering, can be directly identified with the scattering lengths [56], while for the solution of the $\pi N$ system it is more convenient to relate them to subthreshold parameters instead. The resulting system of coupled integral equations corresponds to a self-consistency condition for the low-energy phase shifts, whose mathematical properties were investigated in detail in [61]. Following [56], the authors of [62] pursued the following solution strategy: the phase shifts are parameterized in a convenient way with a few parameters each, which are matched to input partial waves above $s_m$ in a smooth way. To measure the degree to which the RS are fulfilled, a $\chi^2$-like function is defined according to

$$
\chi^2 = \sum_{l_i, j=1}^N \frac{\left| \Phi_{l_i, j}^k (W) - F \left[ f_{l_i}^k \right] \left( W_j \right) \right|^2}{\Phi_{l_i, j}^k (W_j)},
$$

(24)

where $\Phi_{l_i, j}$ denotes a set of points between threshold and $\sqrt{s_m}$, $f_{l_i}$ are the $s$-channel partial waves with isospin $I$, orbital angular momentum $l$, and total angular momentum $j = l \pm 1/2 \equiv l_z$, and $F \left[ f_{l_i}^k \right]$ the right-hand side of the RS equations. In [62] $l_m = 1, N = 25$ (distributed equidistantly) are taken, and the number of subtraction constants are chosen in such a way as to match the number of degrees of freedom predicted by the mathematical properties of the Roy equations [61]. It should be stressed that the form of the RS equations only reduces to that of Roy equations once the $t$-channel is solved, see [60]. In the solution of the RS equations we minimize equation (24) with respect to the subtraction constants (identified with subthreshold parameters) and the parameters describing the low-energy phase shifts, while imposing equation (23) as additional constraints. The solution for the $s$-channel partial waves, expressed in terms of the phase shifts and including uncertainty estimates from these systematic studies as well as the uncertainties in the scattering lengths and the coupling constant, is shown in figure 3. Apart from low-energy phase shifts, the RS solution provides a consistent set of subthreshold parameters. In particular, this allows one to pin down the much discussed pion–nucleon $\sigma$-term:

$$
\sigma_{\pi N} = (59.1 \pm 3.5) \text{ MeV}.
$$

(25)

A crucial ingredient in this determination are the precise scattering lengths given in equation (23). By combining this information with the constraints from RS equations, the $\sigma$-term can be determined to a remarkable accuracy. From matching the results for the subthreshold parameters of pion–nucleon scattering obtained from a solution of RS equations to chiral perturbation theory up to next-to-NNLO, one can extract the pertinent low-energy constants, including a comprehensive analysis of systematic uncertainties and correlations, as shown in the third row of table 1 [63]. Results for the LECs are also presented in the counting scheme usually applied in chiral nuclear EFT, $\{p, M_z\}/m_N = \mathcal{O}(p^3)$ [5] (see below), are shown in the

---

**Table 1.** Extraction of some dimension two pion-nucleon LECs using CHPT and using Roy–Steiner equations. Units are GeV$^{-1}$. For details, see the text.

| Method                        | $c_1$     | $c_2$     | $c_3$     | $c_4$     |
|-------------------------------|-----------|-----------|-----------|-----------|
| CHPT (phase shifts + subtr.)  | $-0.9 \pm 0.2$ | $3.3 \pm 0.2$ | $-4.7 \pm 0.2$ | $3.5 \pm 0.2$ |
| CHPT (phase shifts)           | $-1.13 \ldots -0.75$ | $3.49 \ldots 3.69$ | $-5.51 \ldots -4.77$ | $3.34 \ldots 3.71$ |
| Roy–Steiner (standard counting)| $-1.11(3)$  | $3.13(3)$  | $-5.16(6)$  | $4.26(4)$  |
| Roy–Steiner (NN counting)     | $-1.10(3)$  | $3.57(4)$  | $-5.54(6)$  | $4.17(4)$  |

---

**Figure 3.** Phase shifts $\delta_{l=0}^I$ of the $s$-channel partial waves in degrees, obtained from the solution of the RS equations [62]. The dashed line indicates the central solution, the bands the uncertainty estimate. The partial waves are labeled by the spectroscopic notation $L_{2S,2J}$. 

---
fourth row of table 1. One sees that these parameters are now determined with much better precision than before.

6. Nuclear forces

In this section, I discuss the construction and status of the nuclear forces derived in chiral EFT. This is based on the ground-breaking work by Weinberg [4, 5, 64] and by van Kolck [65]. There has been much discussion about the Weinberg power counting, see e.g. the review [66] or the recent talk by Phillips [67]. Space forbids me to go into these details; I will rather stick to this framework, which has proven to be extremely successful and discuss its foundations and some very recent developments that not only allow for very precise calculations of the forces between nucleons but also supply serious uncertainty estimates, going beyond the cut-off variations mostly employed before.

For developing a systematic and model-independent theoretical framework capable of describing reactions involving several nucleons up to center-of-mass three-momenta of (at least) the order of the pion mass \( M_\pi \), one has to realize that nuclear binding is very shallow, with typical binding energies per nucleon much smaller than the pion mass. The appearance of shallow bound states can not be described in perturbation theory. The quest for an EFT that allows for the most general parameterization of the nucleon–nucleon scattering amplitude consistent with the fundamental principles such as Lorentz invariance, cluster separability and analyticity, must account for this and other basic facts of nuclear physics. The energies of the nucleons we are interested in are well below the nucleon mass, it therefore is natural and appropriate to make use of a non-relativistic expansion, that is an expansion in inverse powers of the nucleon mass \( m_N \). Accordingly, in the absence of external probes and below the pion production threshold, one is left with a potential theory in the framework of the quantum-mechanical A-body Schrödinger equation

\[
(H_0 + V)|\Psi\rangle = E|\Psi\rangle, \quad \text{with} \quad H_0 = \sum_{i=1}^A \frac{\boldsymbol{\Phi}_i^2}{2m_N} + O(m_N^{-3}).
\]

The main task then reduces to the determination of the nuclear Hamilton operator \( H_0 + V \). This can be accomplished using the framework of CHPT. It is further important to realize that the nuclear interactions feature two very distinct contributions, long-range one- and two-pion exchanges and shorter-ranged interactions, that can be represented by a tower of multi-nucleon operators. As the pion is the pseudo-Goldstone boson of the approximate chiral symmetry of QCD as discussed above, its interactions with the nucleons are of derivative nature and strongly constrained by the available data on pion–nucleon scattering and other fundamental processes. However, in harmony with the principles underlying EFT, one has also to consider operators of nucleon fields only. In a meson-exchange model of the nuclear forces, these can be pictured by the exchanges of heavier mesons like \( \sigma, \rho, \omega \), and so on, see figure 4—but such a modeling is no longer necessary and also does not automatically generate all structures consistent with the underlying symmetries. Also, in the EFT approach, the forces between three and four nucleons are generated consistently with the dominant two-nucleon forces—which could never be achieved in earlier modeling of these forces.

Within the framework of CHPT, nuclear forces are derived from the most general effective chiral Lagrangian by making an expansion in powers of the small parameter \( q \) defined as

\[
q \in \left\{ \frac{M_\pi}{\Lambda}, \frac{|\vec{k}|}{\Lambda} \right\},
\]

where \( Q \sim |\vec{k}| \sim M_\pi \) is a typical external momentum (the soft scale)\(^6\) and \( \Lambda \) is a hard scale, sometimes also called breakdown scale. Appropriate powers of the inverse of this scale determine the size of the renormalized LECs in the effective Lagrangian. Notice that once renormalization of loop contributions is carried out and the renormalization scale is set to be \( \mu \sim M_\pi \) as appropriate in CHPT, all momenta flowing through diagrams appear to be, effectively, of the order \( \sim M_\pi \). Consequently, one can use naive dimensional analysis to estimate the importance of (renormalized) contributions of individual diagrams.

To be specific, consider a connected Feynman graph with \( N \) nucleon lines\(^7\). It is easier to count the powers of the hard scale \( \Lambda \) rather than of the soft scale \( Q \) by observing that the only way for \( \Lambda \) to emerge is through the corresponding LECs, as first pointed out by Epelbaum [68, 69]. Thus, the low-momentum dimension \( \nu \) of a given diagram can be expressed in terms of the canonical field dimensions \( \kappa_i + 4 \) of \( V \) vertices of type \( i \) via

\[
\nu = -2 + \sum_{i} V_i \kappa_i, \quad \kappa_i = d_i + \frac{3}{2} n_i + p_i - 4,
\]

where \( n_i (p_i) \) and \( d_i \) refer to the number of the nucleon (pion) field operators and derivatives or pion mass insertions, respectively. The constant \(-2\) in the expression for \( \nu \) is just a convention. The power counting can also be re-written in terms of topological variables such as the number of loops \( L \) and nucleon lines \( N \) rather than \( \kappa_i \) which are appropriate for

\[^6\] We use the small parameter \( q \) and the soft scale \( Q \) synonymously.

\[^7\] Note that nucleons cannot be destroyed or created within the non-relativistic approach.
diagrammatic approaches. For connected diagrams the above equation then takes the form
\[
\nu = -4 + 2N + 2L + \sum V_{\Delta_i}, \quad \Delta_i = d_j + \frac{1}{2} n_i - 2.
\] (29)

Chiral symmetry of QCD guarantees that the pions couple only through vertices involving derivatives or powers of \( M_\pi \).
This implies that the effective Lagrangian contains only irrelevant (i.e. non-renormalizable) interactions with \( n_i \geq 1 \)
(\( \Delta_i \geq 0 \)) which allows for a perturbative description of pion–pion and pion–nucleon scattering as well as nuclear forces. The leading interactions, i.e. the ones with the smallest possible \( \Delta_i \), that is \( \Delta_i = 0 \), have the form
\[
\mathcal{L}^{(0)} = \frac{1}{2} \partial^\mu \pi \cdot \partial^\nu \pi - \frac{1}{2} M_\pi^2 \pi^2 + N^\tau \left[ i \partial_\tau + \frac{g_A}{2F_\pi} \tau \cdot \Gamma \pi - \frac{1}{4F_\pi^2} \pi \cdot (\pi \times \pi) \right] N
- \frac{1}{2} C_5 (N^\tau N)(N^\tau N) - \frac{1}{2} C_7 (N^\tau \sigma N) \cdot (N^\tau \sigma N) + \ldots,
\] (30)

where \( \pi \) and \( N \) refer to the pion and nucleon field operators, respectively, and \( \tau \) (\( \sigma \)) denote the spin (isospin) Pauli matrices. Further, \( g_A (F_\pi) \) is the nucleon axial-vector coupling (pion decay) constant and \( C_{5,7} \) are the LECs accompanying the leading contact operators. The ellipses refer to terms involving more pion fields. It is important to emphasize that chiral symmetry leads to highly non-trivial relations between the various coupling constants. For example, the strengths of all \( \Delta_i = 0 \)-vertices without nucleons with \( 2, 4, 6, \ldots \) pion field operators are given in terms of \( F_\pi \) and \( M_\pi \). Similarly, all single-nucleon \( \Delta_i = 0 \)-vertices with \( 1, 2, 3, \ldots \) pion fields are expressed in terms of just two LECs, namely \( g_A \) and \( F_\pi \). The construction of the higher order terms is well documented in the literature [35, 66, 70–72].

The expressions for the power counting given above are derived under the assumption that there are no infrared divergences. This assumption is violated for a certain class of diagrams involving two and more nucleons (more precisely, for the two-nucleon case the so-called box diagram is the culprit) due to the appearance of pinch singularities of the kind [5]
\[
\int \frac{d q}{i (q^2 + i \epsilon)} \frac{i}{l_0 + i \epsilon} l_0 - i \epsilon.
\] (31)

Here, \( 1/(l_0 + i \epsilon) \) is the free nucleon propagator in the heavy-baryon approach (in the nucleon rest-frame) corresponding to the Lagrangian in equation (30). Clearly, the divergence is not ‘real’ but just an artifact of the extreme non-relativistic approximation for the propagator which is not applicable in that case. Keeping the first correction beyond the static limit, the nucleon propagator takes the form
\[
1/(l_0 - l_0^2/(2m_0) + i \epsilon)^{-1}
\] leading to a finite result for the integral in equation (31) which is, however, enhanced by a factor \( m_0/|\vec{q}| \) as compared to the estimation based on naive dimensional analysis. In physical terms, the origin of this enhancement is related to the two-nucleon Green’s function of the Schrödinger equation (26). The nuclear potential \( V \) we are actually interested in is, of course, well defined in the static limit \( m_0 \rightarrow \infty \) and thus not affected by the above-mentioned infrared enhancement. More precisely, the potential is defined in terms of the so-called irreducible contributions and all reducible contributions that are generated from the iteration of the potential in the Schrödinger equation.

It is now instructive to address the qualitative implications of the power counting in equation (29) and the explicit form of the effective chiral Lagrangian. First, one observes that the dominant contribution to the nuclear force arises from two-nucleon tree-level diagrams with the LO vertices. This implies that the nuclear force is dominated by the one-pion exchange potential and the two contact interactions without derivatives. Pion loops are suppressed by two powers of the soft scale. Also, vertices with \( \Delta_i > 0 \) involving more derivatives are suppressed and do not contribute at LO. One also observes the suppression of many-body forces: according to equation (29), \( N \)-nucleon forces start contributing at order \( Q^{4+2n} \). This implies the dominance of the two-nucleon force with three- and 4NFs appearing formally as corrections at orders \( Q^2 \) and \( Q^4 \), respectively. However, as pointed out by Weinberg and van Kolck, the leading irreducible contributions to the three-nucleon potential cancel, so that three-nucleon forces (3NFs) indeed start at order \( Q^5 \). All this is summarized in figure 5. The present state-of-the-art of deriving the nuclear Hamiltonian is also shown in this figure. The green (solid) boxes show the parts of the potential that have been worked out and applied, the brown (long-dashed) ones refer to contributions that are also available but are only being included in explicit calculations of observables now and the red (short-dashed) boxes refer to contributions still in the process of being worked out. As can be seen from this, calculations within the two-nucleon system are by far the most advanced, and I will therefore describe the most recent developments here. First, it is important to note that in [73] a new coordinate space regularization was introduced (see also [74–76]), that does not lead to any distortion of the long-range part of the potential as the earlier used momentum cut-off:
\[
V_{\text{long-range}}(\vec{r}) \rightarrow V_{\text{long-range}}^\text{reg}(\vec{r}) = V_{\text{long-range}}(\vec{r}) f(R/R),
\] (32)

where the regulator function \( f(x) \) is chosen such that its value goes to \( 0 \) \( (1) \) sufficiently fast for \( x \rightarrow 0 \) \( (\text{exponentially fast for} x \gg 1) \). A further advantage of this scheme is that the above choice of the regulator makes the additional spectral function regularization of the pion exchange contributions obsolete. The regulator function \( f(R/R) \) can be chosen as
\[
f\left(\frac{R}{R}\right) = \left[ 1 - \exp\left(\frac{-R^2}{R^2}\right) \right]^n,
\] (33)

where the exponent \( n \) has to be taken sufficiently large. It is necessary to choose \( n = 4 \) or larger in order to make the regularized expressions for the dimensionally regularized two-pion exchange potential at N\(^3\)LO vanish in the origin, however, larger values of \( n \) lead to more stable numerical
Contributions to the effective potential of the 2N, 3N and 4N forces based on Weinberg’s power counting. Here, LO denotes leading order, NLO next-to-leading order and so on. The various vertices according to equation (29) with $\Delta_i = 0, 1, 2, 3, 4$ are denoted by small circles, big circles, filled boxes, filled diamonds and open boxes, respectively. The boxes surrounding various classes of diagrams are explained in the text. Figure courtesy of Evgeny Epelbaum.

| Two-nucleon force | Three-nucleon force | Four-nucleon force |
|-------------------|---------------------|-------------------|
| LO ($Q^0$)        |                     |                   |
| NLO ($Q^1$)       |                     |                   |
| N$^3$LO ($Q^3$)   |                     |                   |
| N$^4$LO ($Q^4$)   |                     |                   |

Figure 5.

Fifth-order contributions to the two-pion exchange potential. Solid and dashed lines refer to nucleons and pions, respectively. Solid dots denote vertices from the lowest-order $\pi N$ effective Lagrangian. Filled rectangles, ovals and gray circles denote the order $Q^4$, order $Q^3$ and order $Q^2$ contributions to $\pi N$ scattering, respectively.

Figure 6.
complete work in [81, 82]). Although three-pion exchange formally appears at N3LO and at N4LO, it has usually been neglected, as the nominally leading 3π exchange potential at N3LO is known to be weak compared to the two-pion exchange [83, 84] and to have negligibly small effect on phase shifts. However, the subleading corrections at N4LO are enhanced due to the appearance of the LECs $c_i$ [85]. To check the assertion that the 3π exchange can still be neglected, the authors of [77] have carried out a N4LO fit for the intermediate value of the cutoff of $R = 0.9$ fm in comparison with the Nijmegen PWA [86] and the GWU single-energy PWA [87]. The bands of increasing width show estimated theoretical uncertainty at N4LO, N3LO, N2LO and NLO.

Figure 7. Results for the np S-, P- and D-waves and the mixing angles $\epsilon_1, \epsilon_2$ up to N4LO based on the cutoff of $R = 0.9$ fm in comparison with the Nijmegen PWA [86] and the GWU single-energy PWA [87]. The bands of increasing width show estimated theoretical uncertainty at N4LO, N3LO, N2LO and NLO.

Next, let us consider 3NFs. While providing a small correction to the nuclear Hamiltonian as compared to the dominant NN force, its inclusion is mandatory for quantitative understanding of nuclear structure and reactions, for recent reviews, see [88, 89]. Historically, the importance of the 3NF has been pointed out already in the 1930s [90] while the first phenomenological 3NF models date back to the 1950s. However, in spite of extensive efforts, the spin structure of the 3NF is still poorly understood [88]. Chiral EFT indeed provides a suitable theoretical resolution to the long-standing 3NF problem. As already noted, the 3NF only appears two orders after the leading NN interaction. At this order, there are only three topologies contributing, see figure 8. The two-pion exchange topology is given again in terms of the $c_i$, as discussed in detail in [91]. The so-called D-term, which is related to the one-pion exchange between a 4N contact term and a further nucleon, has gained some prominence in the first decade of this millennium, as many authors have tried to pin it down based on a cornucopia of reactions, such as $Nd \rightarrow Nd$ [94], $NN \rightarrow NN\pi$ [92, 93], $NN \rightarrow d\ell\nu$ [95–98], $d\pi \rightarrow \gamma NN$ [99–101], or the spectra of light nuclei [102], see figure 9 (here, $\gamma$ denotes a photon, $\ell$ a lepton and $\nu$ its corresponding antineutrino). This demonstrates again the power of EFT—very different processes are related through the same LECs.

Figure 8. Topologies of the leading contributions to the chiral 3NF. From left to right: Two-pion exchange, one-pion-exchange and 6N contact interaction.

Figure 9. Various reactions that all are sensitive to the D-term. Figure courtesy of Evgeny Epelbaum.

higher orders in the chiral expansion for $R = 0.9$ fm are shown. The various bands result from adding/subtracting the estimated theoretical uncertainty to/from the calculated results. Similar results are obtained for np scattering observables, see [77] for details.
thus providing many different tests of chiral symmetry (as it is also the case with the LECs $c_i$, see figure 1). The LEC $E$ related to the 6N contact interaction can only be fixed in systems with at least three nucleons, say from the triton binding energy. Indeed, the leading chiral 3NF has already been extensively explored in $ab$ initio calculations by various groups and found to yield promising results for nuclear structure and reactions [89, 103]. The first corrections to the 3NF at order $Q^2$ (N$^4$LO) have also been derived [104-106] (and are parameter-free) while the sub-subleading contributions at order $Q^3$ (N$^4$LO) are being derived [46, 107, 108]. The LENPIC collaboration [109] aims at working out the consequences of the sub- and sub-sub-leading corrections to the 3NFs in light and medium nuclei. As a first step, utilizing the fifth-order two-nucleon forces and the method for error quantification discussed before, LENPIC studied nucleon–deuteron (Nd) scattering and selected low-energy observables in $^4$He, $^8$Li, $^6$Li based on $NN$ forces only. Calculations beyond second-order differ from experiment well outside the range of the quantified uncertainties [110]. This provides truly unambiguous evidence for missing 3NFs within the employed framework, see figure 10.

Four-nucleon forces (4NFs) appear first at N$^4$LO and have been worked out some time ago [68, 69]. A rough estimate of its contribution to the $^4$He binding energy was performed in [112]. It was shown that the 4NF is attractive for wave functions with a totally symmetric momentum part and the additional binding energy provided by the long-ranged part of the 4NF is of the order of a few hundred keV. However, in heavier nuclei, the 4NFs must play a more important role, but explicit calculations need to be performed. Pioneering calculations exploring the role of chiral 4NFs in nuclear matter have been performed in [113, 114]. Strong cancellations are found between various types of contributions, but still an attractive and non-negligible contribution to the binding energy of nuclear matter at saturation density is found. The role of 4NFs in the neutron matter equation of state was investigated in [115].

7. Discretization of space–time

There are two different venues to tackle the nuclear many-body problem, that is nuclei with atomic number $A \geq 5$. Either one utilizes the forces from EFT within a conventional, well established many-body technique (no-core-shell-model, coupled cluster approach, etc) or one develops a novel scheme that combines these forces with Monte Carlo methods that are so successfully used in lattice QCD. This new scheme is termed ‘nuclear lattice simulations’ or ‘nuclear lattice EFT’ (NLEFT) and has enjoyed wide recognition in the popular press as the first ever $ab$ initio calculation of the Hoyle state in $^{12}$C has been performed, see section 8. In the following, I will give a short introduction to this novel nuclear many-body technique. The foundations of the method and its early applications are reviewed in [116]. Before discussing this new approach, let me make quite clear that the combination of well-established many-body techniques with the chiral forces is one of the forefronts of nuclear physics, as it allows one to study many aspects that are still out of the reach of nuclear lattice simulations, especially phenomena in heavy nuclei. These are, however, covered by various articles to this Fest-schrift. For the impatient reader, I give a few pertinent references as appetizers here [102, 117–122].

Space–time is discretized in Euclidean time on a torus of volume $L_x \times L_y \times L_z \times L_t$, with $L_x(L_y)$ the side length in spatial (temporal) direction. The minimal distance on the lattice, the so-called lattice spacing, is $a$ ($a_t$) in space (time). This entails a maximum momentum on the lattice, $p_{\text{max}} = \pi/a$, which serves as a UV regulator of the theory. The nucleons are point-like particles residing on the lattice sites, whereas the nuclear interactions (pion exchanges and contact terms as described before adapted to the lattice notation) are represented as insertions on the nucleon world lines using standard auxiliary field representations. The nuclear forces have an approximate spin–isospin SU(4) symmetry (Wigner symmetry) [123] that is of fundamental importance in suppressing the malicious sign oscillations that plague any Monte Carlo (MC) simulation of strongly interacting fermion systems at finite density. For this reason, nuclear lattice simulations allow access to a large part of the phase diagram of QCD, see figure 11, whereas calculations using lattice QCD are limited to finite temperatures and small densities (baryon chemical potential). In what follows, I will concentrate on the calculation of the ground state properties and excited states of atomic nuclei with $A \leq 28$. The interactions of nucleons are simulated using the MC transfer projection method. Each nucleon evolves as a single particle in a fluctuating background of pion and auxiliary fields, the latter representing the multi-nucleon contact interactions. One also performs Gaussian smearing of the LO contact interactions which is required by the too strong binding of four

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8. LENPIC stands for Low Energy Nuclear Physics International Collaboration.
nucleons on one lattice site. To leading order, one starts with a Slater determinant of single-nucleon standing waves in a periodic cube for \(Z\) protons and \(N\) neutrons (with \(Z + N = A\)) or with more correlated states, as described below. We use the SU(4) symmetric approximation of the LO interaction as an approximate inexpensive filter for the first \(t_0\) time steps—this suppresses dramatically the sign oscillations. Then, one switches on the full LO interaction and calculate the ground state energy and other properties from the correlation function.

\[
\langle \Psi_{\text{LO}} | \mathcal{H}_{\text{LO}} | \Psi_{\text{LO}} \rangle
\]

operator insertion for expectation value

\[
L_{\mu} \tau_{\lambda} \quad L_{\mu} \tau_{\lambda} \quad \text{perturbative corrections}
\]

\[
\mathcal{O}(a^4)
\]

\[
\mathcal{O}(Q^2)
\]

The world-line approach that maps the \(A\) nucleon problem on the evolution of \(A\) independent particles (except for antisymmetrization) is perfectly suitable for high-performance computer applications. In fact, the low memory and extremely parallel structure of the lattice Monte Carlo codes allow jobs on large parallel machines to run very efficiently with hundreds of thousands of processes. We are able to run very efficiently with four processes per core on the supercomputer JUQUEEN at Forschungszentrum Jülich, with very little loss in performance when compared with one process per core, thereby achieving a factor of four increase in the total performance. In figure 13 the computational time for each process on the JUQUEEN supercomputer to produce 100 hybrid Monte Carlo trajectories is shown. The time is plotted as a function of the number of parallel processes with four processes per core. One sees that the performance is entirely independent of the number of processes. The computational time for each JUQUEEN process to generate 100 hybrid Monte Carlo trajectories versus the number of nucleons \(A\) scales as \(A^{2.7+0.1}\) for these values of \(A\). For smaller values the scaling is close to linear in \(A\), while the quadratic dependence becomes more important for larger \(A\). The data wave mixing like e.g., between the \(3S_1-3D_1\) and the \(3D_3\) partial waves. Most of the results presented below have been obtained with the following lattice set-up: \(a = 1.97\) fm, \(N = 7\), \(a_t = 1.32\) fm. The forces have been obtained at NNLO, with nine two-nucleon LECs fixed from fits to \(S\)- and \(P\)-wave np phase shifts, two isospin-breaking NN LECs determined from the \(nn\) and proton–proton (pp) scattering lengths and two 3\(N\) LECs fixed from the triton binding energy and the axial-vector contribution to triton \(\beta\)-decay [98]. Further, there is some smearing required in the LO \(S\)-wave four-nucleon terms with its size parameter determined from the average \(np\) \(S\)-wave effective range [124].

The world-line approach that maps the \(A\) nucleon problem on the evolution of \(A\) independent particles (except for antisymmetrization) is perfectly suitable for high-performance computer applications. In fact, the low memory and extremely parallel structure of the lattice Monte Carlo codes allow jobs on large parallel machines to run very efficiently with hundreds of thousands of processes. We are able to run very efficiently with four processes per core on the supercomputer JUQUEEN at Forschungszentrum Jülich, with very little loss in performance when compared with one process per core, thereby achieving a factor of four increase in the total performance. In figure 13 the computational time for each process on the JUQUEEN supercomputer to produce 100 hybrid Monte Carlo trajectories is shown. The time is plotted as a function of the number of parallel processes with four processes per core. One sees that the performance is entirely independent of the number of processes. The computational time for each JUQUEEN process to generate 100 hybrid Monte Carlo trajectories versus the number of nucleons \(A\) scales as \(A^{2.7+0.1}\) for these values of \(A\). For smaller values the scaling is close to linear in \(A\), while the quadratic dependence becomes more important for larger \(A\). The data
shown is for lattice simulations of $^4$He, $^9$Be, $^{12}$C, $^{16}$O, and $^{20}$Ne in a periodic cube with length $L = 13.8$ fm and lattice spacing $a = 1.97$ fm. We are presently exploring a range of lattice spacings from $a \simeq 1$ fm to $a \simeq 2$ fm to get a better handle on the discretization errors.

8. Results from lattice simulations

Having discussed the framework of nuclear lattice simulations, we are now in the position to present some results in this approach, but without going into any details. The interested reader is referred to the original publications for more details. The works have been done under the umbrella of the NLEFT collaboration\(^9\) making use of supercomputing resources at Forschungszentrum Jülich and RWTH Aachen.

\(\textit{Ab initio calculation of the Hoyle state and its structure:}\) The excited state of the $^{12}$C nucleus with $J^P = 0^+$ known as the ‘Hoyle state’ constitutes one of the most interesting, difficult and timely challenges in nuclear physics, as it plays a key role in the production of carbon via fusion of three alpha particles in red giant stars. The first \textit{ab initio} calculation of the spectrum of $^{12}$C was performed in [125], giving its first $0^+$ excitation—the Hoyle state—at the proper energy, see figure 14. This can be considered the breakthrough investigation for the method of nuclear lattice simulations. In [126], ab initio lattice calculations were presented which unravel the structure of the Hoyle state, along with evidence for a low-lying spin-2 rotational excitation. For the $^{12}$C ground state and the first excited spin-2 state, we find a compact triangular configuration of alpha clusters. For the Hoyle state and the second excited spin-2 state, we find a ‘bent-arm’ or obtuse triangular configuration of alpha clusters. The calculated electromagnetic transition rates between the low-lying states of $^{12}$C have also been obtained at LO (higher order corrections still require improved codes).

\(\textit{Towards medium-mass nuclei:}\) We have also extended nuclear lattice simulations to the regime of medium-mass nuclei [127]. To achieve that, a method which allows one to greatly decrease the uncertainties due to extrapolation at large Euclidean time was implemented. It is based on triangulation of the large Euclidean time limit from a variety of SU(4) invariant initial interactions. The ground states of alpha nuclei from $^4$He to $^{28}$Si are calculated up to NNLO in the EFT expansion. With increasing atomic number $A$, one finds a growing overbinding as shown in figure 15. Such effects are genuine to soft $NN$ interactions and also observed in other many-body calculations, see e.g. [128–130]. While the long-term objectives of NLEFT are a decrease in the lattice spacing and the inclusion of higher-order contributions, it can be shown that the missing physics at NNLO can be approximated by an effective four-nucleon interaction. Fitting its strength to the binding energy of $^{24}$Mg, one obtains an overall excellent description as depicted in figure 15.

\(\textit{Spectrum and structure of $^{16}$O:}\) We have also performed lattice calculations of the low-energy even-parity states of $^{16}$O [131], which is another mysterious alpha-cluster type nucleus. We find good agreement with the empirical energy spectrum, see table 2, and with the electromagnetic properties and transition rates (after rescaling with the corrected charge radius as detailed in [131]). For the ground state, we find that the nucleons are arranged in a tetrahedral configuration of alpha clusters. For the first excited spin-0 state, we find that the predominant structure is a square configuration of alpha clusters, with rotational excitations that include the first spin-2 state.

\(\textit{Alpha clustering in nuclei:}\) $\alpha$-clustering is known to play an important role in the carbon nucleus $^{12}$C, see e.g. [132–135], as well as in the oxygen nucleus $^{16}$O, see e.g. [132, 136–139]. For other recent work on alpha clustering
also in heavier nuclei, see e.g. [140, 141]. In nuclear lattice simulations of these nuclei, clustering emerges naturally. This is due to the fact that the path integral samples all possible configurations, in particular also the ones with four nucleons on one lattice site, as allowed by Fermi statistics. The alpha cluster configurations in $^{12}$C and in $^{16}$O can be obtained in two ways. First, one can prepare cluster-type initial states, say three alphas for $^{12}$C or four alphas for $^{16}$O, and then investigate the time evolution of such cluster configurations and extract e.g. the corresponding energies as the Euclidean time goes to infinity. Second, one can also start with initial states that have no clustering at all, the Slater determinants of standing waves mentioned before. One can then measure the four-nucleon correlations. For such initial states, this density grows quickly with time and reaches a high level. For the cluster initial states, these correlations start out at a high level and stay large as a function of Euclidean time. This is a clear indication that the observed clustering is not built in by hand but rather follows from the strong four-nucleon correlations in the considered nucleus. Or, stated differently, if one starts with an initial wave function without any clustering, on a short time scale clusters will form and make up the most important contributions to the structure of $^{12}$C and $^{16}$O or any such type of nucleus, where $\alpha$-clustering is relevant. So the mysterious phenomenon of $\alpha$-clustering emerges naturally in this novel approach to exactly solve the nuclear $\Lambda$-body problem.

A method to go beyond alpha-cluster nuclei: So far, we have performed projection Monte Carlo calculations of nuclear lattice EFT that suffer from sign oscillations to a varying degree dependent on the number of protons and neutrons. Hence, such studies have hitherto been concentrated on nuclei where the sign oscillations are smallest. In $^{12}$C, $^{6}$He and $^{6}$Be nuclei. In the future, it will allow for studies of neutron-rich halo nuclei and asymmetric nuclear matter as well as exploring the limits of nuclear stability. For a different extrapolation method used in shell model Monte Carlo calculations about two decades ago, see [143].

### Overcoming rotational symmetry breaking:

On the lattice, rotational invariance is broken from the full $\text{SO}(3)$ rotational group to the cubic group. Hence, observables computed on the lattice will in general be affected by rotational symmetry breaking effects. In particular, the unambiguous identification of excited states and the computation of transition amplitudes may suffer significantly due to the relatively large lattice spacings of $a \approx 2$ fm in present nuclear lattice simulations. Hence, it makes sense to carefully determine the sources of rotational symmetry breaking in actual NLEFT simulations, and search for methods that minimize their impact on physical observables. We have therefore used a simplified alpha cluster model to study the lattice matrix elements of irreducible tensor operators as a function of the lattice spacing $a$ [144, 145]. In order to minimize the effects of rotational symmetry breaking, we have introduced the ‘isotropic average’ which consists of a linear combination of the components of a given matrix element, such that each component is weighted according to the Clebsch–Gordan coefficient with the associated quantum numbers. This method, which is equivalent to averaging over all lattice orientations, enables the unambiguous computation of matrix elements even at large lattice spacings. In figure 16, we illustrate the effect of isotropic averaging on the mean square radius of $^{8}$Be within the alpha cluster model. For related work in lattice QCD, see e.g. [146].

### Lattice spacing dependence:

As stated, most of the calculations of the NLEFT collaboration have been done at the coarse lattice spacing $a \approx 2$ fm. Besides the studies of the lattice spacing dependence in alpha cluster models just discussed, in [147] we have investigated NLEFT for the two-body system for several lattice spacings ($0.5$ fm $\lesssim a \lesssim 2$ fm) at LO in the pionless as well as in the pionful theory. We find that in the pionless case, a simple Gaussian smearing allows one to demonstrate lattice spacing independence over a wide range of lattice spacings. We show that regularization methods known from the continuum formulation [73] (as discussed in section 6) are necessary as well as feasible for the pionful approach. This leads to $a$-independent observables in the two-nucleon sector for the range of lattice spacings mentioned. We are presently improving upon this by developing better smearing procedures—stay tuned.

The NLEFT collaboration is presently working out next generation lattice forces that have much reduced lattice
artifacts and show a much reduced sign problem. This will allow them to substantially improve the precision of NLEFT calculations and will give access to much larger A. Interesting times are ahead of us. Finally, let me point out that nuclear structure physics is much richer than described in these sections, and many phenomena might be out of reach of lattice simulations for quite a while. However, this approach will be able to systematically analyze how various phenomena like clustering or shell structure can be derived from the nuclear Hamiltonian, which would be a big step in nuclear theory. Further, it is always possible to develop EFTs for particular sets of phenomena, like e.g. the recent work by Papenbrock and Weidenmüller on deformed nuclei [148].

9. Fine-tuning in nuclear physics and the anthropic principle

Next, let me discuss the generation of the elements that are relevant to life on Earth. They are generated in the big bang and in stars through the fusion of protons, neutrons and nuclei. In big bang nucleosynthesis (BBN), only light elements like 4He (alpha particles) and some heavier elements are generated. The life essential elements 12C and 16O are located 7.65 MeV above the ground state. Only then, sufficient amounts of 12C and 16O are generated, because the carbon nucleus formed in the triple-alpha process fuses with an alpha particle to generate oxygen. In this step, no resonant enhancement is required. This means that there are various fine-tunings in the triple-alpha process. How can we understand these? As discussed before, all strongly interacting composites like hadrons and nuclei must emerge from the underlying gauge theory of the strong interactions. We also know that the mass of the light quarks relevant for nuclear physics is very small and thus plays little role in the total mass of nucleons and nuclei. Another important fact is that nuclei are made of protons and neutrons. To account for the Coulomb repulsion between the protons, we must account for electromagnetism, that is QED. QED is characterized by the fine-structure constant αEM ≈ 1/137. Thus, we face the following question: How sensitive is the element generation to variations in the fundamental parameters of QCD+QED? The role of the weak interactions is more subtle, see the later discussion and also the interesting paper [150].

First, let me discuss the fine-tunings related to the strong interactions. In the Weinberg scheme discussed so far, the quark mass dependence of the forces is generated explicitly (through the pion propagator) and implicitly (through the pion–nucleon coupling, the nucleon mass, and the four-nucleon couplings), see figure 17. In the following, we make use of the Gell-Mann–Oakes–Renner relation [151], $M_q^2 = B(m_u + m_d)$. Therefore, the notions pion and quark mass dependence will be used synonymously. The quark mass dependence of an observable O can be given in terms of the so-called K-factor, $\delta O_H/m_f = K_H^0 (O_H/m_f)$, with $f = u, d, s, m_f$ the corresponding light quark mass and H denotes the hadron under consideration. As discussed in detail in [152], we can obtain the pion mass dependence of pion and nucleon properties by combining results from lattice QCD with CHPT. The relevant results are: $K_{MS}^q = 0.494^{+0.009}_{-0.013}$, $K_{FS}^q = 0.048 \pm 0.012$, and $K_{MS}^q = 0.048^{+0.006}_{-0.005}$, where q denotes the average light quark mass. Matters are different for the quark mass dependence of the short-distance terms. Here, some sort of modeling using resonance saturation [153] is required, which leads to a sizeable uncertainty. In the future, more precise results might be available from lattice simulations QCD. For the NN scattering lengths, such type of modeling leads to $K_{\Sigma}^q = 2.3^{+1.9}_{-1.8}$, $K_{\Sigma}^q = 0.32^{+0.17}_{-0.18}$ and $K_{\Sigma}^q = -0.86^{+0.45}_{-0.50}$, where BE denotes the binding energy. This improves on and extends the earlier work based on EFTs and models, see e.g. [154–158]. We point out the

Figure 16. Mean square radii $\langle r^2 \rangle$ for the lowest $2^+$ multiplet of 8 Be states within a simplified alpha-cluster model calculation. The reduced lattice matrix elements all merge in the limit $a \to 0$, while at finite $a$ the matrix elements depend on the quantum number $\alpha, \beta$ and $\gamma$, which is indicative of rotational symmetry breaking. Such effects are nearly eliminated in the isotropic average, especially when $a \leq 1.7$ fm.

Figure 17. Pion (quark) mass dependence of the LO NN interactions as explained in the text. Solid (dashed) lines denote nucleons (pions).
recent work of [159], which derives low-energy theorems for nucleon–nucleon scattering at unphysical quark masses and relates to the recent lattice QCD calculations at large pion masses [160]. In view of these new results, a re-evaluation of the $K$-factors should be done. In the chiral EFT considered here, effects of shifts in $\alpha_{\text{EM}}$, that is modifications of the electromagnetic interactions, can also be calculated.

Using these results, we are now in the position to analyze what constraints on possible quark mass variations the element abundances in BBN imply. In addition to the effects already discussed, the variation of $^3$He and $^4$He with the pion mass is also needed. As pointed out in [162], the corresponding $K$-factors can be calculated from a convolution of the $2N$ $K$-factors with the variation of the three- and four-particle BEs with respect to the singlet and triplet $NN$ scattering lengths. This gives $K'_{^3\text{He}} = -0.94 \pm 0.75$ and $K'_{^4\text{He}} = -0.55 \pm 0.42$ [152], which is consistent with a direct calculation using nuclear lattice simulations, $K'_{^3\text{He}} = -0.19 \pm 0.25$ and $K'_{^4\text{He}} = -0.16 \pm 0.26$ [163]. Based on these results, the calculation of the BBN response matrix of the primordial abundances $Y_a$ at fixed baryon-to-photon ratio can be performed and one can compare to the experimentally well-known values. The most stringent limits arise from the deuteron abundance [deut/H] and the $^4$He abundance normalized to the one of protons, $^3$He/$^4$He, as most neutrons end up in alpha particles. In terms of allowed quark mass variations, we find $\delta m_q/m_q = (2 \pm 4)\%$. In contrast to most earlier determinations, we provide reliable error estimates due to the underlying EFT. This is, however, not the end of the story. In the isospin limit considered so far, the constraint from the neutron lifetime is not accounted for, see e.g. [162]. The finite neutron lifetime strongly affects the $^4$He ($Y_\beta$) abundance. Making the natural assumption that all quark and lepton masses vary with the Higgs vacuum expectation value $v$, one finds

$$\frac{\delta v}{v} = \left| \frac{\delta m_q}{m_q} \right| \leq 0.9\%.$$  \hspace{1cm} (36)

Next, let us consider the fine-tunings in the production of carbon and oxygen. Stated differently, how much can we change these parameters from their physical values to still have an habitable Earth as shown in figure 18? To answer this question, we first have to discuss which parameters we can vary. In terms of the strong interaction, one first thinks of the strong coupling constant. It is, however, tied to the nucleon mass through dimensional transmutation. As we already know, the light quark masses are external parameters. More precisely, for the reaction under consideration, only the strong isospin limit is relevant and we thus need to consider the average light quark mass. We already remarked that the quark mass contribution to the nucleon mass is small, so naively one could think that sizeable variations will not change the picture. However, the relevant scale is the average binding energy per nucleon. We know that $E/A \lesssim 8$ MeV (only a tiny fraction of the nucleon mass). As noted before, the Coulomb repulsion between protons is an important factor in nuclear binding. Consequently, we also have to account for changes in $\alpha_{\text{EM}}$. First, let us work out the fine-tunings in QCD (for details, see [164, 165]). We want to calculate the variations of the pertinent energy differences in the triple-alpha process $\delta \Delta E/\delta M_{\pi}$, which according to figure 17 boils down to

$$\frac{\partial E_i}{\partial M_{\pi}} = \frac{\partial E_i}{\partial M_{\pi}} + x_1 \frac{\partial E_i}{\partial m_{\pi}} \left|_{M_{\pi}} + \frac{1}{2} \frac{\partial E_i}{\partial m_{\pi}} \left|_{M_{\pi}} + x_2 \frac{\partial E_i}{\partial C_0} \left|_{M_{\pi}} + x_3 \frac{\partial E_i}{\partial C_0} \right|_{M_{\pi}}, \right.$$  \hspace{1cm} (37)

with the definitions

$$x_1 = \frac{\partial m_N}{\partial M_{\pi}} \left|_{M_{\pi}} + x_2 = \frac{\partial E_N}{\partial M_{\pi}} \left|_{M_{\pi}} \right.$$  \hspace{1cm} (38)

with $M_{\pi}$ the pion mass appearing in the pion-exchange potentials, and we only consider small variations around the physical value of the pion mass $M_{\pi}^0$. The various derivatives in equation (37) can be obtained precisely using our auxiliary field quantum Monte Carlo techniques. The quantities $x_{1,4}$ can be expressed in terms of the pion-mass dependence of the inverse singlet and triplet scattering lengths

$$\tilde{A}_s = \frac{\partial \alpha^{-1}}{\partial M_{\pi}} \left|_{M_{\pi}^0}, \right.$$  \hspace{1cm} (39)

This allows us to express all energy differences appearing in the triple-alpha process $\Delta \Delta E = E_8 - 2E_4$, $\Delta E_9 = E_{12} - E_8 - E_4$, $\varepsilon = E_{12} - 3E_4$, with $E_4$ and $E_8$ for the energies of the ground states of $^3$He and $^8$Be, respectively, and $E_{12}$ denotes the energy of the Hoyle state) as functions of $\tilde{A}_s$ and $\tilde{A}_t$. As shown in figure 19, the various fine-tunings in the triple-alpha process are not independent of each other but rather strongly correlated. One also observes a strong dependence on the variations of the $^8$Be. This observation can be traced back to the $\alpha$-cluster structure of the $^8$Be, $^4$He and $^6$Li ground and Hoyle states. Such correlations related to the...
production of carbon have indeed been speculated upon earlier [166, 167].

Next, we consider the reaction rate of the triple-alpha process. It is given by $r_{3\alpha} \sim N_\alpha^3 \Gamma_q \exp(-\varepsilon/k_B T)$, with $N_\alpha$ the alpha-particle number density in the stellar plasma with temperature $T$, $\Gamma_q = 3.7(5)$ meV the radiative width of the Hoyle state and $k_B$ is Boltzmann’s constant. The stellar modeling calculations of [168, 169] show that sufficient abundances of both carbon and oxygen are consistent with a variation of ±100 keV around the empirical value of $\varepsilon = 379.47(18)$ keV. From this condition one can derive a constraint on the shifts in $m_q$. It is given by (for more details, see [165]):

$$\left| 0.572(19) \bar{A}_q + 0.933(15) \bar{A}_t - 0.064(6) \left( \frac{\delta m_q}{m_q} \right) \right| < 0.15\%.$$  

(40)

The resulting constraints on the values of $\bar{A}_q$ and $\bar{A}_t$, compatible with the condition $|\delta \varepsilon| < 100$ keV are visualized in figure 20. The various shaded bands represent values of $\bar{A}_q$ and $\bar{A}_t$ that allow for carbon–oxygen based life for quark mass variations of 0.5%, 1% and 5%. The central values of $\bar{A}_q$ and $\bar{A}_t$ from [152] indicate that variations in the light quark mass of the order of 2–3% should allow for the formation of the life-essential elements carbon and oxygen. However, within the the current theoretical uncertainty in $\bar{A}_q$ and $\bar{A}_t$, a complete lack of fine-tuning cannot be ruled out. Repeating this calculation for variations in the fine-structure constant $\alpha_{\text{EM}}$, we find that carbon–oxygen based life can form if $\alpha_{\text{EM}}$ varies by ±2.5%.

Finally, let me discuss these results in connection with the anthropic view of the Universe. As already stated, the Hoyle state is responsible for a dramatic increase in the reaction rate of the triple-alpha process. The corresponding resonant enhancement strongly depends on the exact value of $\varepsilon$, which can be considered as the principal control parameter of this reaction. Because of this, the Hoyle state has been nicknamed the ‘level of life’ [170], and it is is often claimed to be the best example of the so-called anthropic principle. In a nutshell, the anthropic principle can be formulated as follows: the observed values of the fundamental physical and cosmological parameters are restricted by the requirement that life can form to determine them, and that the Universe be old enough for that to occur [171, 172]. A detailed thorough historical discussion of the Hoyle state and its relation to the anthropic principle is given in [173]. The prominent role of the anthropic principle in cosmology and string theory has recently been reviewed in [174]. Note, however, that the allowed variations in $\varepsilon$ are not that small [167]. Indeed, $|\delta \varepsilon/\varepsilon| \approx 25\%$ still allows for carbon–oxygen based life. Thus one might conclude that the anthropic principle is not required for explaining the various fine-tunings in the triple-alpha process. However, as I just discussed, the relevant shifts are the ones in the light quark mass and the fine-structure constant. The allowed variations of these are only a few percent, so the rather mild fine-tuning in the energy difference $\varepsilon$ leads to a much stronger restriction for the variations of the fundamental parameters. Beyond these modest changes in $m_q$ and $\alpha_{\text{EM}}$, it appears necessary to invoke the anthropic principle to explain the observed abundances of $^{12}\text{C}$ and $^{16}\text{O}$. A more

\[ \frac{\delta m_q}{m_q} = \frac{0.005}{0.01} \]

\[ \frac{\delta m_q}{m_q} = 0.05 \]
detailed account of these considerations is given in the review [175].

10. Nuclei as precision laboratories

In this section, I show how chiral EFT can be used to test physics within and beyond the SM. I focus here on hadronic parity violation (PV), the calculation of light ion EDMs to test CP violation, and the use of chiral EFT currents to perform better calculation for WIMP\(^{11}\) scattering off nuclei, that is used for direct dark matter detection. The first two topics have recently been reviewed [177], so I only discuss the underlying physics and results obtained in the last few years.

The observation of PV in the weak interaction is one of the pillars on which the SM of particle physics was built. In the SM, PV is induced because only left-handed quarks and leptons participate in the (charged current) weak interaction. At the fundamental level, PV originates from the exchange of the charged (and neutral) weak gauge bosons. For low-energy (hadronic) processes, the heavy gauge bosons decouple from the theory leading to effective PV four-fermion interactions. The effective interactions resulting from the exchange of charged gauge bosons induce, for example, the beta-decay of the muon and the neutron, while the exchange of both charged and neutral gauge bosons gives rise to various PV four-quark operators. Although PV induced by the weak interaction is well understood at the level of elementary particles, its manifestation at the hadronic and nuclear level is not well understood. This holds particularly true for the strangeness-conserving part of the weak interaction which induces PV in hadronic and nuclear systems. The SM predicts PV forces between nucleons, however, their forms and strengths are masked by the non-perturbative nature of QCD at low energies. In order to circumvent this problem, the NN interaction has been parametrized in the past through PV meson exchanges with adjustable strengths, the so-called DDH-framework [178]. Given enough experimental input the unknown couplings can be determined and other processes can then be predicted. However, the extractions of the DDH coupling constants from different experiments seem to be in disagreement [179, 180].

In recent years, we have developed a framework for hadronic and nuclear PV based on chiral EFT and applied this to calculate PV hadronic and nuclear observables. This approach has a number of big advantages over the more traditional DDH model. First of all, there is a clear link to the underlying theory, i.e., QCD supplemented with PV four-quark operators. Second, the EFT approach makes it possible to calculate the P-even and -odd NN potentials within the same framework. The resulting potentials can then be treated on the same footing. Third, the chiral Lagrangian can be improved by going to higher orders in the expansion. In fact, we have developed the PV potential up to N\(^2\)LO. Fourth, the chiral approach can be extended to other systems, such as reactions involving photons, which require the calculation of PV currents. These currents can be evaluated within the same framework as the potential, something which is not possible in the DDH model where the currents need to be modeled separately (in principle, using the method of unitary transformations, see e.g., [181], this could be achieved but that approach has never been applied to this problem).

Already some work in the past has been done on deriving a chiral EFT PV potential. At leading order in the power counting, the only term appearing in the chiral Lagrangian is the weak pion–nucleon vertex

\[ \mathcal{L}_{PV} = \frac{h}{\sqrt{2}} \bar{N} (\bar{\pi} \times \bar{\pi})^j N, \]

proportional to the LEC \( h \). Together with the usual pseudovector P-conserving (PC) pion–nucleon interaction, the LO PV potential follows as

\[ V_{1\pi} = -\frac{g_A h}{2\sqrt{2} F_\pi} (\bar{\pi}_1 \times \bar{\pi}_2)^j (\bar{\sigma}_1 + \bar{\sigma}_2) \cdot \vec{q} \frac{M_N^2 + q^2}{M_N^2}, \]

in terms of the nucleon spin \( \bar{\sigma}_{1,2} \) and the momentum transfer flowing from nucleon 1 to nucleon 2: \( \vec{q} = \vec{p}_1 - \vec{p}_2 \) (\( q = |\vec{q}| \), where \( +p' \) and \( +p'' \) are the momenta of the incoming and outgoing nucleons in the center-of-mass frame. The LO OPE potential changes the total isospin of the interacting nucleon pair, and at low energies, dominantly contributes to the \( ^3S_1 \leftrightarrow ^1P_1 \) transition. The isospin change ensures that the LO potential vanishes for pp and nn scattering. At NLO the number of LECs proliferates. First of all, five NN short-range contact interactions appear. In addition, there appear two-pion-exchange (TPE) diagrams that are proportional to \( h \) just as the OPE potential [183, 184]. However, in contrast to the OPE potential, the TPE potential does contribute to PV in pp scattering. This leads to a dependence on \( h \) on PP pp observables which had not been taken into account in earlier studies. In fact, one of the main goals of experiments on hadronic and nuclear PV is to measure the size of \( h \). At the moment the size of \( h \) is unknown despite several measurements of nuclear PV observables. Several theoretical estimates exist in the literature: The most simple one is the use of naive-dimensional analysis which gives the following estimate \( h \sim \frac{C}{G_F F_\pi A_{\pi}} \sim 10^{-6} \), in terms of the Fermi coupling constant \( G_F \). In the original DDH paper [178], the authors have attempted to estimate \( h \) using SU(6) symmetry arguments and the quark model finding the reasonable range \( 0 \leq h \leq 1.2 \times 10^{-9} \), and a ‘best’ value of \( h \approx 4.6 \times 10^{-7} \), consistent with the naive-dimensional analysis estimate. The authors of [185] have calculated several PV meson–nucleon vertices in a framework of a nonlinear chiral Lagrangian where the nucleon emerges as a soliton. They obtained significantly smaller values for \( h \approx 0.2 \times 10^{-7} \). In [186], the calculation was sharpened based on a three-flavor Skyrme model calculation with the result \( h \approx 1 \times 10^{-7} \) in agreement with a recent large \( N_c \) analysis [187]. Recently, the first lattice QCD calculation [188] has been performed for \( h \) using a lattice size of 2.5 fm and a pion mass \( M_\pi \approx 389 \text{ MeV} \), finding the result

\(^{11}\) WIMP stands for weakly interacting massive particle. For an introduction to the field of dark matter searches and possible candidate particles, see e.g. [176].
$h_x = (1.1 \pm 0.5 \text{ (stat)} \pm 0.05 \text{ (sys)}) \times 10^{-7}$, which is also rather small with respect to the DDH range. It should be stressed that this calculation does not include disconnected diagrams nor has it been extrapolated to the physical pion mass. The smaller estimates seem to be in better agreement with data. Experiments on $\gamma$-ray emission from $^{18}\text{F}$ set the rather strong upper limit [189], $h_x < 1.3 \times 10^{-7}$, although it must be stressed that calculations for nuclei with this many nucleons bring in additional uncertainties. On the other hand, the Cesium anapole moment prefers a much larger value $h_x \approx 10^{-6}$ although the involved uncertainties are also larger.

Considering the uncertain status of the leading-order LEC $h_x$, it is clearly crucial to get a better handle on its size. In the first step, one therefore aims at extracting $h_x$ from data on PV in $pp$ scattering. The observable we are interested in is the so-called longitudinal analyzing power (LAP) which vanishes in the limit of $P$ conservation. It is defined as the difference in cross section between an unpolarized target and a beam of positive and negative helicity, normalized to the sum of these cross sections. The LAP has been measured for several beam energies and results have been reported for 13.6 [190], 45 [191], and 221 [192] MeV (lab energy). Traditionally, it was assumed that the $pp$ LAP does not depend on $h_x$ [193] as the OPE potential does not contribute due to its isospin-violating nature. However, this is not true for the TPE potential. In [194] the goal was to include the TPE potential and extract $h_x$ from the $pp$ data. However, at the order of the TPE diagrams the $pp$ data also depends on one combination of PV short-range parameters that we defined as $C$. In particular, the goal was to perform a fully systematic analysis within chiral nuclear EFT. Technically the task is to solve the Lippmann–Schwinger (LS) equation in the presence of a potential $V$ which is the sum of the strong, PV, and Coulomb potentials. For the strong potential the $N^2\text{LO}$ potential [195] was used, for various values of the cut-off appearing in the LS equation to get a handle on the theoretical uncertainty. We then fitted $h_x$ and $C$ to the $pp$ data. The data point at 221 MeV corresponds to a transmission experiment which means that it is sensitive to small forward scattering angles where the Coulomb potential diverges. This difficulty has been carefully taken into account in the analysis and plays an important role in the extraction of the LECs. Unfortunately, because only three data points exist with significant uncertainties, the fits allow a rather large range of parameters. The allowed range for the LECs, at the total $\chi^2 = 2.71$ level, is approximately $h_x = (1.1 \pm 2) \times 10^{-6}$, $C = (-6.5 \pm 8) \times 10^{-6}$, and the couplings are heavily correlated. The large uncertainty is dominated by experimental errors and the lack of data points. Our findings have been confirmed in the analysis of [196].

Additional experiments are needed to reduce the uncertainties in the fits. One of the conclusions therefore is that a transmission experiment around 125 MeV would be very beneficial in reducing the uncertainties on the LECs. Such an experiment could be performed at COSY at the Forschungszentrum Jülich. Unfortunately, due to the large allowed range of $h_x$ we cannot confirm nor rule out small values of $h_x$. However, the suggested smallness of $h_x$ by the $^{18}\text{F}$ data and several theoretical estimates, indicate that possible higher-order corrections to the PV potential might be relevant. It was thus necessary to derive these NNLO corrections to the potential which might become crucial in understanding hadronic PV in case $h_x$ turns out to be very small. The corrections have been systematically studied in [197] where it was found that five new LECs appear. These LECs describe new PV pion–nucleon and pion–pion–nucleon interactions. Only two combinations of these five LECs contribute to PV in $pp$ scattering. The combinations contribute via OPE and TPE diagrams, however, a detailed study indicates that the TPE diagrams are by far dominant and the OPE diagrams can be neglected. The TPE diagrams themselves can be divided into two parts. One part involves no new LECs and is proportional to $h_x c_4$, where $c_4 \simeq 3.4 \text{ GeV}^{-1}$ is a well-known PC LEC which takes on a large value due to underlying $\Delta$ and $\rho$ dynamics. The second part involves a combination of new LECs $h_{\text{PE}}^N$. Due to the large size of $c_4$ we can expect the $h_x c_4$ term to dominate the $N^2\text{LO}$ potential. Because this combination involves no new LECs compared to the NLO potential studied above, we have been able to fit $h_x$ to the $pp$ data by including the dominant $N^2\text{LO}$ correction. This leads to a slightly improved fit and a somewhat smaller value for $h_x$

$$h_x = (0.8 \pm 1.5) \times 10^{-6}, C = (-5.5 \pm 7) \times 10^{-6}. \quad (43)$$

The $N^2\text{LO}$ correction does not affect the values of the LECs by a large amount. This indicates that the expansion of chiral EFT is converging well. Unfortunately, due to a lack of data it is not possible to extract a value for $h_x$ and $h_{\text{PE}}^N$ at the same time. However, from an analysis of corrections proportional to $h_{\text{PE}}^N$, one can draw a few conclusions. Unless $h_x$ is very small, the $N^2\text{LO}$ TPE corrections are dominated by terms proportional to $h_x$. This would imply that the dominant part of the $N^2\text{LO}$ potential contains no new LECs. Finally, if it turns out that $|h_x| < 10^{-7}$, the $N^2\text{LO}$ corrections calculated in [197] might need to be included.

Only additional PV data can tell us which of the above scenarios is realized in nature. For example, a measurement of PV in the reaction $n\bar{p} \rightarrow d\gamma$ could shed light on the size of $h_x$. In contrast to $pp$ scattering, the longitudinal asymmetry $a_\gamma$ in this process does depend on the leading-order OPE potential and is therefore much more sensitive to $h_x$. For a long time, however, there was no non-zero measurement of $a_\gamma$. Recently a first preliminary result for $a_\gamma$ was reported [198]

$$a_\gamma = (-7.14 \pm 4.4) \times 10^{-8}. \quad (44)$$

This result is based on a subset of the full data taken and an improved result with an uncertainty at the $10^{-8}$ level is expected in the near future. The goal of [199] was to combine the above-described analysis of $pp$ scattering with this recent data on $n\bar{p} \rightarrow d\gamma$ in order to extract a value of $h_x$. In contrast to $pp$ scattering, the analysis of $n\bar{p} \rightarrow d\gamma$ requires the inclusion of PC and PV electromagnetic currents. These can be derived from chiral EFT in the same way as the potentials have been derived. The PC currents that we included in the analysis arise from the nucleon magnetic moments, a recoil correction to the nucleon charges, and currents associated
with a single pion exchange. With these currents we calculate a $P$-even cross section within 3\% of the experimental value. The PV currents arise from a single pion exchange where one of the vertices is $h_\gamma$. Summing up all LO contributions, we find that each of the individual contributions only has a minor dependence on the cut-off value, but the sum suffers from a larger dependence due to mutual cancellations. In total we find

$$a_\gamma = (0.11 \pm 0.05) \times 10^{-7}. \quad (45)$$

We can now combine the $a_\gamma$ analysis with that of $pp$ scattering. Including the theoretical uncertainty from cut-off variations we obtain the following ranges for the LECs

$$h_\pi = (1.1 \pm 1.0) \times 10^{-6}, \quad C = (-6.5 \pm 4.5) \times 10^{-6}. \quad (46)$$

The fits indicate that small values of $h_\pi \sim 10^{-7}$ are barely consistent with the data, with values of $h_\pi \sim (5 - 10) \times 10^{-7}$ being preferred. Such larger values disagree with the upper limit from $^{19}$F gamma-ray emission, $h_\pi \leq 1.3 \times 10^{-7}$, and lattice and model calculations of $h_\pi \approx 10^{-7}$. The increasing increase in sensitivity of the $a_\gamma$ measurement will significantly improve the fit and tell whether small values of $h_\pi$ are consistent with few-body experiments. We have estimated the uncertainties of the fits due to experimental uncertainties, variation of cut-off parameters in the LS equation, and higher-order corrections and find the first of these to be dominant.

All this work has thus significantly advanced the understanding of hadronic and nuclear CP violation. The application of chiral EFT to this longstanding problem has proven to be crucial in combining the various processes in a single unitary framework. The Pisa group has picked up our approach and applied it to various PV observables in systems with three and four nucleons, such as $n-d$ scattering or the charge exchange reaction $^3$He ($n$, $p$)$^4$He [196]. So we can summarize the status of $h_\pi$ as follows: if the upcoming data on $np \rightarrow d\gamma$ confirms the preliminary number, equation (44), this strongly indicates that $h_\pi$ has a value significantly larger than the upper bound from $^{19}$F data. There are then two options. Either $h_\pi$ is actually that small and higher-order corrections calculated already can explain the large value of $a_\gamma$. This is, however, not a satisfying explanation, as this requires higher-order LECs that are significantly larger than expected from the power counting and resonance saturation. The other option is that something is missing in the theoretical analysis of the $^{18}$F data. Nuclear lattice simulations will eventually be able to do a clear-cut calculation of the parity-mixing in $^{18}$F—stay tuned.

Next to the study of PV, the closely related subject of hadronic and nuclear CP violation has been investigated in recent years. The SM contains two sources of CP violation (CPV), one in the phase of the quark-mass matrix and one in the strong interactions, the QCD $\theta$ term. The former manifests itself in CPV flavor-changing interactions and leads only to very small EDMs. On the other hand, the QCD $\theta$ term is flavor conserving and gives rise to an, in principle, large neutron EDM. As noted before, the non-observation of the latter then forces $\theta \leq 10^{-10}$. This extreme smallness is known as the strong CP problem. In addition, EDMs can obtain contributions from physics beyond the SM. In fact, large EDMs are generated in various popular extensions of the SM such as supersymmetric and left–right symmetric models. The extreme accuracy of low-energy EDM measurements probe high-energy scales comparable to the large hadron collider. The above considerations have led to a large experimental endeavor to measure EDMs of leptons, hadrons, nuclei, atoms, and molecules [200]. The main motivation for our work in this field is the plans to measure the neutron EDM with higher accuracy and to measure for the first time the EDMs of light nuclei in storage rings. It has been proposed that storage rings can be used to measure the EDMs of the proton and deuteron with a precision of $10^{-29}$ e cm, three orders of magnitude better than the current neutron EDM limit. EDMs of other light ions, such as the helium ($^4$He nucleus), are candidates as well. Any finite signal in one of the upcoming experiments would be due to physics not accounted for by the phase in the quark-mass matrix. Such a signal would either be caused by physics beyond the SM (BSM) or by an extremely small, but non-zero, $\theta$ term. An interesting and important problem is therefore to investigate whether it is possible to trace a non-zero $\theta$ with EDM experiments. That is, can we confidently disentangle the $\theta$ term from possible BSM sources? As will be shown, some progress has been made to answer this question.

Let us first stay within the SM. Once the QCD Lagrangian is supplemented by a non-zero QCD $\theta$ term, CPV interactions between the low-energy degrees of freedom appear. Since the $\theta$ term breaks chiral symmetry like the quark masses, CHPT can be easily extended to include such interactions. In particular, this extension gives rise to CPV couplings between pseudo-Goldstone bosons (pion, kaon, eta) and baryons (in particular nucleons) whose strengths can be related to known baryon mass splittings and sigma terms [16, 201]. It then becomes possible to calculate the EDM of the nucleon (and heavier baryons) with CHPT [202, 203]. The divergences appearing in these loops are absorbed by counter terms whose sizes cannot be obtained from CHPT directly. Lattice-QCD simulations and non-physical quark masses and including a non-zero $\theta$ term have been performed to calculate these unknown counter terms. By using the CHPT expressions of the nucleon EDMs [202, 203], the results of the simulations can be extrapolated to the physical point and infinite volume [205]. In this way, the proton and nucleon EDM have been calculated in terms of $\theta$ directly [17, 203]

$$d_n = - (3.8 \pm 1.0) \times 10^{-16} \ e\ cm, \quad d_p = (2.1 \pm 1.2) \times 10^{-16} \ e\ cm. \quad (47)$$

These calculations provide an important contribution to the study of hadronic and nuclear EDMs. Once non-zero nucleon EDMs are measured the above results can be used to test whether the $\theta$ term is responsible or some other source of CPV.

Considering the success of the SM of particle physics, it is likely that any additional physics appears at a scale...
considerably higher than the electroweak scale $\sim 100$ GeV. This scale separation makes it possible to treat the SM as the dimension-four and lower part of a more general EFT containing higher-dimensional operators. For the study of EDMS it can be shown that the first operators appear at dimension six and are suppressed by two powers of the scale where the additional CPV appears. BSM CPV can be studied in a model-independent way by adding all possible CP-odd dimension-six operators at the high-energy scale. The great advantage is that it is not necessary to choose a specific SM extension. Nevertheless, as discussed below, the approach can be matched to specific high-energy models to study their low-energy consequences. EDM experiments take place at very low energies such that the dimension-six operators must be evolved to lower energies taking into account QCD and electroweak renormalization-group evolution [206]. Once the dust settles, only a relatively small set of operators remain at a scale $\sim 1$ GeV consisting of quark EDMS and chromo-EDMS, the gluon chromo-EDM, and several four-quark operators. At lower energies, QCD becomes non-perturbative and to proceed further we have extended CHPT to include the dimension-six operators. The resulting CHPT Lagrangians have been built in great detail in [204, 207]. All dimension-six operators (and the $\theta$ term) break CP symmetry, however they all break chiral symmetry in different ways leading to different CHPT interactions. The different interactions, in turn, lead to different hierarchies of EDMS. Thus, given enough measurements it becomes possible to unravel the underlying source of CPV. This hierarchy of EDMS can be best studied for the EDMS of light nuclei. In principle the nucleon EDM induced by BSM sources can be calculated within CHPT in the same way as for the $\theta$ term. However, in contrast to the $\theta$ term, the associated counter terms that appear have not been calculated with lattice QCD. Such calculations are significantly more difficult than for $\theta$. Furthermore, even if they had been calculated, measuring only nucleon EDMS will not be enough to unravel the sources.

The plans to measure the EDMS of light nuclei in storage rings with high accuracy make it attractive to focus on these observables. Similar, to the research program on PV described above, this can be done by combining CP-even chiral EFT potentials with CP potentials that are calculated for each possible source of CPV. The power counting shows that EDMS of light nuclei are dominated by a small set of hadronic interactions. These interactions are the EDMS of the constituent nucleons ($d_n$ and $d_p$), two CPV pion–nucleon couplings ($g_0$ and $g_1$), one CP-odd three-pion vertex ($\Delta$), and two CPV nucleon–nucleon couplings ($C_1$ and $C_2$) [209]. For instance, the deuteron EDM $d_D$ at N$^3$LO is found to be [208]

$$d_D = 0.9 \ d_n + 0.92 \ d_p - [(0.18 \pm 0.02)g_1 + (0.75 \pm 0.15) \ \Delta] \ e\ fm. \quad (48)$$

Note that the leading CPV 4N couplings do not contribute here. For sources such as quark chromo-EDMS or certain four-quark operators, the contributions from pion exchange can be significantly larger than the single nucleon EDMS. On the other hand, for the $\theta$ term, due to its isospin-conserving nature, the contributions from $g_1$ and $\Delta$ are only a fraction of the neutron EDM

$$d_D - 0.9 \ d_n - 0.92 \ d_p = -(0.9 \pm 0.3) \cdot 10^{-16} \ \theta \ e\ cm \ll d_n. \quad (49)$$

These calculations show that measurements of both the neutron and the deuteron EDM can provide strong hints for BSM physics. A large hierarchy between $d_D$ and $d_n$ would point towards sources of CPV not in the SM. The above calculations have been extended to the EDMS of the triton and helium. In particular the latter is interesting as it can be probed in a storage-ring experiment as well. The calculations show that the $^3$He EDM is, in contrast to $d_D$, also sensitive to $g_0$ and therefore complementary. In [210] a comprehensive study of the EDM signature of several popular BSM models was performed. It was shown that the measurements of the EDMS of a few light nuclei could be enough to unravel several high-energy BSM models. This study shows that low- and high-energy searches for BSM physics are complementary and that EDM measurements are able to probe the highest energy scales.

As the last topic of this section I discuss recent work by the Darmstadt group, that has made considerable progress in the calculations for dark matter particles/WIMPs scattering off nuclei, utilizing state-of-the-art nuclear structure calculations combined with currents derived from chiral nuclear EFT. First, they worked out the structure factors for elastic spin-dependent WIMP scattering off nuclei relevant to dark matter detection experiments, namely $^{129}$Xe, $^{131}$Xe, $^{127}$I, $^{75}$Ge, $^{85}$Rb, $^{23}$Na, $^{27}$Al and $^{29}$Si. For the first time, the spin-dependent WIMP-nucleus currents were based on chiral EFT, and uncertainty bands due to nuclear uncertainties where supplied [211]. An important further step was taken in [212], where inelastic scattering was explored. It is assumed that the
dark matter particle excites the nucleus to a low-lying state with an excitation energy of 10–100 keV followed by a prompt de-excitation. It is found that for momentum transfers of the order of the pion mass, which can typically be reached in such processes, the inelastic channel is comparable or can even dominate the elastic one. This can have a very distinct effect on the integrated spectra as shown in figure 21. Instead of the expected exponential fall-off from the elastic reaction, one observes a double-plateau structure, depending of course on the mass of the dark matter particle and other assumptions specified in [212]. The precise location of these plateaus will thus allow one to constrain the mass of the dark matter particle scattering off the nucleus. Matters are different for spin-independent WIMP scattering off Xe, where the structure factors for inelastic scattering are suppressed by about four orders of magnitude compared to the coherent elastic factors for inelastic scattering are suppressed by about four orders of magnitude compared to the coherent elastic response [213]. Finally, in [214] a power counting scheme for scalar, pseudoscalar, vector and axial-vector WIMP–nucleon interactions was presented and all one- and two-body currents to third order in the chiral expansion were derived. It is also shown that chiral symmetry predicts a hierarchy between the various operators. Further, the relevance of two-body currents is stressed, which is nothing but a reflection of the importance of meson-exchange currents in the nuclear response to external probes. The intriguing field is certainly only at its beginning and more work on improving the nuclear structure aspects is called for.

11. Perspectives

Nuclear physics has entered a new area and is now firmly rooted in the underlying gauge theory of the strong interactions, QCD. This is a remarkable achievement since after the Nobel prize to Bohr, Mottelson and Rainwater, many had announced the end of nuclear physics. On the contrary, we are just at the beginning of an exciting period in nuclear physics in which the importance of the underlying quark and gluon degrees of freedom is becoming clearer. This is a non-trivial exercise, as renormalizability poses severe constraints that can most easily be accounted for using tailor-made unitary transformations, see e.g. [219, 220]. For a status review, the reader is referred to [221] and recent work on the axial currents is found in [222].

- Lattice QCD attempts to derive nuclear properties from the underlying quark and gluon degrees of freedom. This is a very ambitious but potentially very rewarding program. In my opinion, this will require much more work and time. Clearly, one would like to see calculations at or close to the physical quark masses, as has become available in the meson and baryon sector.

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