Traveling Salesman Problems With Replenishment Arcs and Improved Ant Colony Algorithms

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ABSTRACT The traveling salesman problem (TSP), can be used as a typical combinatorial optimization problem, to describe a wide variety of practical engineering optimization problems in various fields. In this study, the problem of personnel and equipment utilization in the transportation industry was abstracted, a more general class of TSPs with replenishment arcs was proposed, an optimization model to minimize the total travel time was established, the ant colony optimization algorithm to solve the standard TSP was improved, and an improved ant colony algorithm based on dynamic heuristic information was designed. Simulation experiments showed that the algorithm can account for the cumulative mileage constraint and search for the shortest path, effectively solving the TSP with replenishment arcs.

INDEX TERMS Cumulative travel constraints, dynamic replenishment arc, improved ant colony algorithm, traveling salesman problem.

I. INTRODUCTION
The traveling salesman problem (TSP) is a typical combinatorial optimization problem whose general description, finding the shortest Hamiltonian cycle in a fully connected graph with \( n \) nodes, has been proven to be an NP-hard problem [1]. Because the problem is easy to state but difficult to solve, it has attracted the attention of many scholars since its introduction in 1932; however, an efficient solution has not yet been found. The TSP represents a class of combinatorial optimization problems; thus, many problems can be described as TSP in practical applications and are thereby solved by more mature methods, such as computer networking, digital mapping, traffic guidance, electrical wiring, VLSI cell placement, and ATM packet-switching networks. Given their important engineering and theoretical value, TSPs are often used as typical examples for algorithm performance studies, and the study of their approximate solutions has been an important topic in algorithm design.

After in-depth research on the TSP, its solution methods have become more diverse. Many scholars have improved and integrated the existing intelligent optimization algorithms to solve the TSP and designed more effective solution algorithms, including the discrete particle swarm optimization algorithm [1], ant colony optimization algorithm [2]–[6], two-stage genetic algorithm [7]–[9], discrete state-transition algorithm [10], mixed-integer linear programming and decomposition method [11], [12], ordered greedy algorithm [13], two-stage local search algorithm [14], simulated annealing algorithm [15], bipartite graph neural network [16], and the branch and bound method [17], [18]. From the current research results of the TSP, a variety of proven methods has been derived from the research of various experts and scholars to obtain better solutions, or optimal solutions. Therefore, a TSP-like problem with a cumulative travel time (mileage) constraint was proposed in this study by abstracting and extending the problem of personnel and equipment utilization in the transportation industry, and the improvement and application of the ant colony algorithm in solving this problem were evaluated.
II. TRAVELING SALESMAN PROBLEM WITH CONTINUOUS CUMULATIVE TRAVEL CONSTRAINTS AND RE-PLENIshMENT ARCS

In the transportation industry, many practical problems can be abstractly formulated as TSPs or their variants. Solution algorithms can then be designed according to the specific characteristics of the problems. Many scholars have described the problem of preparing the operation plan of Chinese Rail High-Speed Electric Multiple-Unit (CRH EMU) trains as a TSP (or multiple TSP) and designed corresponding algorithms to solve them. For example, Zhao and Norio [19] used a heuristic algorithm based on path exchange, Wang et al. [20] and Shi et al. [21], [22] used a simulated annealing algorithm; Zhang et al. [23], Ren [24], and Tong et al. [25] used an ant colony algorithm; Miao et al. [26] used a hierarchical optimization heuristic algorithm; Zhong et al. [27] and Li et al. [28] used an integer programming model; and Li et al. [29] used a Hungarian algorithm. For the optimization of the preparation of the locomotive turnaround chart, Tao et al. [30] transformed the problem into a multiple TSP and developed a mathematical model to design a two-stage heuristic algorithm. Chu et al. [31] described crew routing and scheduling of high-speed railways as TSPs with specific constraints and designed corresponding improved ant colony algorithms to solve them.

By summarizing the results of the abovementioned research on the problem of personnel and equipment utilization in the transportation industry, the common features of their research approaches were obtained: the tasks to be accomplished were abstracted as city nodes when the problem was modeled, and the connections between tasks were considered as intercity connections; thereby, a connected network was established. These city nodes are usually fully connected, owing to time continuity in the transportation industry. An optimal TSP tour that traverses all city nodes in the connected network was then determined under the specific constraints of each problem (utilization time or mileage of the personnel and vehicle). In some cases, for each task abstracted as a city node, the completion of that task also considered as intercity connections; thereby, a connected network was established. These city nodes are usually fully connected, owing to time continuity in the transportation industry. An optimal TSP tour that traverses all city nodes in the connected network was then determined under the specific constraints of each problem (utilization time or mileage of the personnel and vehicle). In some cases, for each task abstracted as a city node, the completion of that task also consumed the time of the personnel or the utilization mileage (and time) of the vehicle, which in turn continued to affect the scheduling of the subsequent tasks. This made solving the problem somewhat difficult. Because the travel time and mileage of a traveling salesman are interconvertible through travel speed, to avoid losing generality, the problem was studied in this study by the uniform use of travel time, requiring the consideration of both intercity and intracity travel time. The TSP with cumulative travel time constraints was referred to as the TSP with replenishment arcs (RATSP). Its modeling method was studied and an improved ant colony algorithm was designed to solve it. In some studies, only nodes were assigned weights, whereas in others, only arcs were assigned weights. To make the research problem more general, the two types of problems were integrated with the processes demonstrated in Fig. 1 and 2.

As shown in Fig. 3, all nodes $v_i$ were decomposed into two corresponding virtual nodes $v_i^{in}$ and $v_i^{out}$, where $v_i^{in}$ denotes the virtual starting point of the corresponding node, $v_i^{out}$ denotes the virtual ending point of the corresponding node, $\omega_i$ denotes the weight of node $v_i$, and $\omega_{ij}$ denotes the arc weight from node $v_i^{out}$ to node $v_{j}^{in}$.

III. OPTIMIZATION MODEL

A. MODEL ASSUMPTIONS

This study begins with the following assumptions on the RATSP:

1) The traveling salesman travels at a constant speed, which is proportional to the distance between cities and the miles traveled within cities. Only the travel time is considered in the modeling process.

2) The cumulative travel time standard is a given value, $T_{\text{max}}$. When the cumulative travel time reaches $T_{\text{max}}$, a rest period must be scheduled. After the rest period, the cumulative travel time is reset and the process of visiting the remaining cities continues.

3) The traveling salesman spends a certain amount of travel time for both intercity and intracity travel, and no interruption is allowed once the travel has begun, whether it is intercity or intracity. Therefore, the travel time between any city nodes $t_{ij}$ and the intracity travel time within them $t_i$ are both less than the cumulative travel time standard $T_{\text{max}}$, satisfying $t_{ij} \leq T_{\text{max}}$ and $t_i \leq T_{\text{max}}$.

4) The traveling salesman must ensure continuity of the next journey at the start of each new journey, i.e., the travel time between the start of each new journey and the virtual starting point of the corresponding node should be less than the cumulative travel time standard $T_{\text{max}}$.
traveling salesman may only rest upon arrival in a city or right before departure from a city, but not during an intercity or intracity visit, and the rest period is of a fixed duration, \( t_{\text{rest}} \).

**B. DEFINITION OF VARIABLES**

\( N \) is the number of city nodes to be visited.

\( x_{ij} \) is a binary decision variable, which is taken as 1 when continuing to visit city \( j \) after visiting city \( i \), and 0 otherwise.

\( y_{ij}^{\text{in}} \) is a binary decision variable, which is taken as 1 if a rest period is scheduled immediately after entering the city node \( i \), and 0 otherwise.

\( y_{ij}^{\text{out}} \) is a binary decision variable, which is taken as 1 if a rest period is scheduled before leaving city node \( i \), and 0 otherwise.

\( d_{ij} \) is the distance between cities \( i \) and \( j \), which is calculated according to the coordinates of the city.

\( v \) is the average travel speed of the traveling salesman between cities.

\( t_{ij} \) is the travel time between cities \( i \) and \( j \), which is derived from the intercity distance and travel speed of the traveling salesman, \( t_{ij} = d_{ij}/v \).

\( T_{\text{max}} \) is the maximum cumulative travel time of the traveling salesman.

\( t_{\text{rest}} \) is the rest period that should be scheduled when the cumulative travel time of the traveling salesman reaches the maximum.

\( T_{ij}^{\text{in}} \) is the cumulative travel time of the traveling salesman when entering city \( i \).

\( T_{ij}^{\text{out}} \) is the cumulative travel time of the traveling salesman when leaving city \( i \).

\( t_{i} \) is the intracity travel time.

**C. OPTIMIZATION MODELING**

1) **OPTIMIZATION OBJECTIVES**

The optimization objective of the RATSP with cumulative travel time constraints in this study is to minimize the total travel time of the traveling salesman, which can be expressed by

\[
\min \ Z = \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij} t_{ij} + \sum_{i=1}^{n} t_{i} + \sum_{i=1}^{n} \left( y_{ij}^{\text{in}} + y_{ij}^{\text{out}} \right) t_{\text{rest}}
\]

(1)

In this equation, the three components denote the intercity travel time, intracity travel time, and resting time scheduled at the city node, respectively. Both the intracity travel time and resting time are related to the sequence in which the cities are visited. Each city node is visited exactly once, regardless of the visiting sequence; thus, the second term, which is the sum of the intracity visit times, is a constant value, allowing the objective function to be rewritten as

\[
\min \ Z = \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij} t_{ij} + \sum_{i=1}^{n} \left( y_{ij}^{\text{in}} + y_{ij}^{\text{out}} \right) t_{\text{rest}}
\]

(2)

2) **CONSTRAINTS**

During the journey of the traveling salesman, each city node is visited exactly once; thus, each city has unique precedent and subsequent cities in the sequence of visited cities.

\[
\sum_{j=1}^{n} x_{ij} = 1 \quad j = 1, 2, \ldots n.
\]

(3)

\[
\sum_{i=1}^{n} x_{ij} = 1 \quad i = 1, 2, \ldots n.
\]

(4)

The cumulative travel time of the traveling salesman at any point during the visit does not exceed the specified time standard \( T_{\text{max}} \); that is, for any city node, the cumulative travel time of the traveling salesman entering and leaving the city does not exceed \( T_{\text{max}} \), thus, the following inequalities must hold:

\[
T_{ij}^{\text{in}} \leq T_{\text{max}}
\]

(5)

\[
T_{ij}^{\text{out}} \leq T_{\text{max}}
\]

(6)

On further analysis, there are two possible options for the traveling salesman after entering the city \( i \):

1) If the difference between \( T_{ij}^{\text{in}} \) and \( T_{\text{max}} \) is less than \( t_{i} \), then the traveling salesman must rest before continuing the visit to city node \( i \). In this case, \( y_{ij}^{\text{in}} = 1 \). The cumulative travel time after the rest period is \( t_{i} \). Hence, according to the assumptions in the study, it is natural that \( t_{i} \leq T_{\text{max}} \).

2) Conversely, if the difference between \( T_{ij}^{\text{in}} \) and \( T_{\text{max}} \) is greater than \( t_{i} \), then the traveling salesman can continue to travel within the current city node without resting. In this case, the traveling salesman must start by visiting city node \( i \) without resting to minimize the total travel time, i.e., \( y_{ij}^{\text{in}} = 0 \). The cumulative travel time after completing the visit to city node \( i \) also satisfies \( T_{ij}^{\text{in}} + t_{i} \leq T_{\text{max}} \).

The combination of these two cases can be expressed by the following equation:

\[
T_{ij}^{\text{in}} \left( 1 - y_{ij}^{\text{in}} \right) + t_{i} \leq T_{\text{max}} \quad i = 2, \ldots n.
\]

(7)

Similarly, the traveling salesman has two possible options before leaving city \( i \):

1) A new journey starts after a rest period is arranged. The cumulative travel time when reaching the next city node \( j \) is \( t_{ij} \) in this case. Correspondingly, \( t_{ij} \leq T_{\text{max}} \).

2) A traveling salesman who does not schedule a rest period before leaving city \( i \) must also have a cumulative travel time less than the cumulative travel time standard when reaching the next city node \( j \), i.e., \( T_{ij}^{\text{out}} + t_{ij} \leq T_{\text{max}} \).

Combining the two equations above and considering generalizations,

\[
T_{ij}^{\text{out}} \left( 1 - y_{ij}^{\text{out}} \right) + \sum_{j=1}^{n} x_{ij} t_{ij} \leq T_{\text{max}} \quad i, j = 1, 2, \ldots n; \ i \neq j.
\]

(8)

The following relationships between \( T_{ij}^{\text{in}} \) and \( T_{ij}^{\text{out}} \) of city \( i \) also apply:

1) If the traveling salesman does not rest in city \( i \), then \( T_{ij}^{\text{out}} = T_{ij}^{\text{in}} + t_{i} \).
2) If the traveling salesman rests in city $i$, then the following three situations can occur:

Rest upon entering the city:

$$T_i^{\text{out}} = (1 - y_i^{\text{out}}) T_i^{\text{in}} + t_i = 0 * T_i^{\text{in}} + t_i$$  \quad (9)

Rest upon leaving the city:

$$T_i^{\text{out}} = (1 - y_i^{\text{out}}) (T_i^{\text{in}} + t_i) = 0 * (T_i^{\text{in}} + t_i) = 0$$  \quad (10)

Rest upon both arrival and departure:

$$T_i^{\text{out}} = (1 - y_i^{\text{out}}) [(1 - y_i^{\text{in}}) T_i^{\text{in}} + t_i] = 0 * (0 * T_i^{\text{in}} + t_i) = 0$$  \quad (11)

By combining these three cases, regardless of whether a rest period is scheduled upon entering the city, provided a rest period is scheduled upon leaving the city, $T_i^{\text{out}} = 0$. As a result, for city $i$, the relationship between $T_i^{\text{in}}$ and $T_i^{\text{out}}$ can be expressed as

$$T_i^{\text{out}} = (1 - y_i^{\text{out}}) [T_i^{\text{in}} (1 - y_i^{\text{in}}) + t_i] \quad i, j = 1, 2, \ldots, n.$$  \quad (12)

3) OPTIMIZATION MODEL

From the above analysis, the optimization model established in this study is described by the equations below.

$$\min Z = \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij} t_{ij} + \sum_{i=1}^{n} \left( y_i^{\text{in}} + y_i^{\text{out}} \right) t_{\text{rest}}$$  \quad (13)

$$\sum_{i=1}^{n} x_{ij} = 1 \quad i = 1, 2, \ldots, n.$$  \quad (14)

$$\sum_{j=1}^{n} x_{ij} = 1 \quad j = 1, 2, \ldots, n.$$  \quad (15)

$$T_i^{\text{in}} (1 - y_i^{\text{in}}) + t_i \leq T_{\text{max}} \quad i = 2, \ldots, n.$$  \quad (16)

$$T_i^{\text{out}} (1 - y_i^{\text{out}}) + \sum_{j=1}^{n} x_{ij} t_{ij} \leq T_{\text{max}} \quad i, j = 1, 2, \ldots, n.$$  \quad (17)

$$T_i^{\text{out}} = (1 - y_i^{\text{out}}) [T_i^{\text{in}} (1 - y_i^{\text{in}}) + t_i] \quad i, j = 1, 2, \ldots, n.$$  \quad (18)

$$x_{ij} \in \{0, 1\} \quad i, j = 1, 2, \ldots, n.$$  \quad (19)

$$y_i^{\text{in}}, y_i^{\text{out}} \in \{0, 1\} \quad i, j = 1, 2, \ldots, n.$$  \quad (20)

IV. SOLUTION ALGORITHM DESIGN

In contrast to the classical TSP, which is a typical NP-hard problem, it is possible to schedule rest periods when entering and leaving each city node in the RATSP considered in this work, i.e., both decision variables $y_i^{\text{in}}$ and $y_i^{\text{out}}$ can have values of 0 and 1. Therefore, the number of feasible solutions to the problem increases by a factor of $2^n$ compared to the original problem, making it more difficult to solve using a straightforward approach. When the problem under study was transformed into a TSP in [31], only intercity travel times were involved, and the travel times within the city nodes were not included. Nevertheless, the ant colony algorithm designed with dynamic heuristic information is still a very good reference for solving the RATSP in this study. The basic procedure and main parameters of the ant colony algorithm with dynamic heuristic information were presented in [31] and [32]. The improved ant colony algorithm designed in this paper based thereupon is described in the following subsections.

A. REPRESENTATION BY THE CONSTRUCTION GRAPH OF THE SOLUTION

For the RATSP with cumulative travel constraints studied herein, a route exists between any two cities. Consequently, the construction graph of the solution consists of all city nodes in a fully connected form, as shown in Fig. 4(a).

Because it takes time to travel within each city node and the time required may affect the visiting sequence of other cities, each city node $i$ is expanded into two corresponding nodes: the city entry node $i_{\text{in}}$ and city exit node $i_{\text{out}}$, and the construction graph of the solution is changed from Fig. 4(a) to Fig. 4(b), accordingly.

In Fig. 4(b), the bold line section 1$_{\text{in}}$ 2$_{\text{in}}$ 2$_{\text{out}}$ 3$_{\text{in}}$ 3$_{\text{out}}$ 4$_{\text{in}}$ 4$_{\text{out}}$ 1$_{\text{in}}$ is a feasible sequence to visit the city nodes, indicating that the visits start from city 1, then continue to cities 2, 3, 4, and finally back to city 1. The routes marked with “$\blacktriangleright$$\blacktriangleright$” at the beginning of the arrow indicate that the traveling salesman rests before starting the journey on that route, and the corresponding resting time is also added to that route, as shown in Fig. 4(b), in which the traveling salesman schedules two rest periods when leaving city 2 and arriving at city 4.

In Fig. 4, $\longrightarrow$ and $\longrightarrow$ denote the corresponding connections that were selected into the optimal solution.

B. SOLUTION CONSTRUCTION AND CONSTRAINT TREATMENT

During the process of constructing the solution, all city nodes are guaranteed to appear in the solution exactly once by introducing the set of city nodes to be visited, $S$. Once a city has been visited, it is removed from set $S$. When $S$ becomes empty, the exit node of the last visited city is connected to the entry node of the first visited city, obtaining the Hamiltonian cycle traversing all city nodes.

When the traveling salesman is at the entry node $i_{\text{in}}$ at city node $i$, as the current city node must be visited, it is only necessary to determine whether the visit within the city can be completed without rest. In other words, if the sum of the cumulative visit time when entering city $i$, $T_i^{\text{in}}$, and the visit time within that city, $t_i$, does not exceed the cumulative travel time standard $T_{\text{max}}$, the visit within city $i$ is considered to be completed directly with time $t_i$ and the cumulative travel time when leaving the city is updated to $T_i^{\text{out}} = T_i^{\text{in}} + t_i$. Otherwise, the visit to city $i$ starts after a rest period is scheduled, and
The cumulative travel time when leaving city \( i \) after visiting is \( T_{i, out}^\text{in} = t_i \).

When the traveling salesman is at the exit node \( i_{out} \) of city node \( i \), the cumulative travel time after connecting to any unvisited city \( j \) is calculated tentatively. If the cumulative travel time \( T_{i, in}^\text{in} \) is less than the cumulative travel time standard \( T_{\text{max}} \), it means that the traveling salesman can reach city \( j \) without resting and the cumulative travel time is \( T_{i, in}^\text{in} = T_{i, out}^\text{in} + t_{ij} \); otherwise, the travel time should be modified to be the sum of the original travel time and resting time, which means that a rest period must be arranged before leaving city \( i \) to start a visit to city \( j \). In this case, the cumulative travel time when reaching city \( j \) is \( T_{j, out}^\text{in} = t_{ij} \).

### C. REPRESENTATION, INITIALIZATION, AND UPDATING PHEROMONES

There are two types of connections in the construction graph of the solution presented in this study: one type is the connection between the city entry and exit nodes, which does not require the introduction of pheromones owing to the uniqueness of its connection relationship; the other type is the connection between the city exit and entry nodes, with the corresponding pheromone \( \tau_{ij} \) denoting the probability of selecting city \( j \) as the next city to be visited from the exit node \( i_{out} \) of city \( i \), and its initial value is set as the reciprocal of the intercity travel time while its update rule is similar to that of the ant colony algorithm for solving the standard TSP, which will not be repeated here.

### D. SELECTION STRATEGY

When the traveling salesman is at the entry node of a certain city, only that city can be visited next, so it is only necessary to determine whether to schedule a rest period.

When the traveling salesman is at the exit node, \( i_{out} \), of a certain city, the next city node to be visited, \( j \), is chosen based on probability \( p_{ij}^\text{k} \) using the roulette method. \( p_{ij}^\text{k} \) is a function of pheromone \( \tau_{ij} \) and heuristic information \( \eta_{ij} \). \( \alpha \) represents the importance factor of the pheromone, and \( \beta \) represents the importance factor of the heuristic information.

\[
p_{ij}^\text{k} = \begin{cases} \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{l \in N_i^k} [\tau_{il}]^\alpha [\eta_{il}]^\beta}, & \text{if } j \in N_i^k; \\ 0, & \text{otherwise.} \end{cases}
\]

\( \eta_{ij} \) is the heuristic information, which is calculated according to the following rule.

\[
\eta_{ij} = \begin{cases} 1/t_{ij}, & \text{if } T_{i, out}^\text{in} + t_{ij} < T; \\ 1/(t_{ij} + t_{\text{rest}}), & \text{otherwise.} \end{cases}
\]

According to the construction process of the above solution, when the traveler is at the exit node of city \( i \), two situations can occur when connecting to the subsequent city. The first is to visit the subsequent city directly when the cumulative travel time constraint has not yet been reached, which does not require scheduling an additional resting time period. The second is to continue the visit after the compulsory scheduling of the rest period because of the cumulative travel time constraint, thus adding additional resting time.

If there is still a portion of cities that can be visited by the traveling salesman without resting, then a rest period is scheduled only when the upper limit of the cumulative travel time is almost reached to most effectively use the travel time. These cities should have a greater probability of being selected than those that can only be visited after resting. Hence, the heuristic information of the two types of city nodes to be visited should be computed differently.

### E. SELECTION OF EVALUATION FUNCTIONS

In a class of the traveling salesman problem with supply arcs, the objective function is the shortest total travel time. The total travel time includes three items: the first is the travel time between cities, the second is the travel time within cities, and the third is the repair time arranged on city nodes. In the third term of the objective function (1), the supply arc \( y_{i, out}^\text{in} + y_{i, out}^\text{in} \)
is equal to the total supply times, which is the quantity for decision-making. Therefore, we directly use the total travel time as the fitness function.

Equation (1) is used as the evaluation function to measure the quality of the optimal path searched by the ants. The visiting sequence of the city nodes with the shortest total travel time is the optimal solution.

**F. IMPROVED ANT COLONY ALGORITHM PROCESS**

The overall flow chart of the improved ant colony algorithm is shown in Fig. 5, and the algorithm calculation steps are as follows:

1. **Step 1**: Calculate the mutual distance between cities based on coordinate information (X,Y)
2. **Step 2**: Initialization parameters: the pheromone importance $\alpha$, heuristic information importance $\beta$, pheromone volatility coefficient, ant population $m$, number of city $n$ and number of iterations $\text{iter}_{\text{max}}$, pheromone matrix Tau;
3. **Step 3**: The starting point for all ants is set as city 1; $T_{i}^{\text{out}}$ is recorded before visiting the city j. The probability of choosing the next city using (21) and (22) is then calculated. The next city to visit is determined based on the roulette method, until all ants have traversed all the cities;
4. **Step 4**: Calculating the path time of each ant and finding the shortest time;
5. **Step 5**: Updating the pheromone matrix Tau;
6. **Step 6**: The number of iterations plus one. If the number of iterations is equal to the maximum number of iterations $\text{iter}_{\text{max}}$, the optimal solution is output. Otherwise, return to step 3.

**V. SIMULATION EXPERIMENTS**

A. **CASE CALCULATION**

It was assumed that a traveling salesman visits 40 cities by car. The city coordinates and intracity travel times are as listed in Table 1. The average travel speed of the traveling salesman is 60 km/h, the cumulative travel time does not exceed 10 h, and the rest time after each cumulative travel time is reached is 10 h. Because the calculation of the cumulative travel time of the traveling salesman departing from different city nodes and the arrangement of rest periods are different, all city nodes were visited from city 1 using an improved ant colony algorithm. The resulting optimal sequence of cities is presented in Table 2.

In Table 2, each cell indicating the visiting sequence of cities consists of three parts, namely, the middle part, which indicates the city node number; the upper part, which indicates the cumulative travel time when entering the current city (CTTE); and the lower part, which indicates the cumulative travel time when leaving the current city (CTTL). An asterisk before the node number indicates that a rest period was scheduled immediately after entering the city, and an asterisk after the node number indicates that a rest period was arranged before leaving the city. The traveling salesman departed from city 1 and returned; thus, there are two values in the upper part: the initial travel time, 0, and the final cumulative travel time, 561.1132 min, upon returning. Moreover, because the journey was completed upon finally returning to city node 1 and the cumulative travel time did not exceed 600 min, no rest period was scheduled.

After several experiments, the final parameters selected for the ant colony algorithm in this study are as follows: the pheromone importance, heuristic information importance, pheromone volatility coefficient, ant population, and number of iterations, which were 2, 5, 0.3, 30, 40, respectively. The convergence process is shown in Fig. 6 (taking the iterative cycle for obtaining the optimal solution as an example). In the optimal solution obtained by this algorithm, eight rest periods, which accounted for a total of 4800 min, were scheduled during the entire journey, and the total travel time was 10020.6036 min. The specific arrangement of the rest periods and the respective cumulative travel time of each rest period are enumerated in bold in the table.

According to the calculations described herein, the minimum intercity travel time was 1994.685 min, whereas the intracity travel time was 3218.63 min. Consequently, the minimum number of rest periods to be scheduled (excluding the last part of the journey back to the city node, where the entire journey was started) was $\left[ \frac{1994.685 + 3218.63}{60} \right] = 8$ times; thus, the theoretical minimum value of the total travel time was $1994.685 + 3218.63 + 4800 = 10013.315$ min. This value was reached only when the traveling salesman traveled along the shortest TSP tour and the number of scheduled rest periods was 8.
TABLE 1. Information of cities to be visited.

| City Number | City coordinates (km) | Intracity travel time (min) |
|-------------|-----------------------|----------------------------|
|             | X                     | Y                          |
| 1           | 127.23                | 63.13                      | 62.57 | 21 | 115.56 | 454.21 | 103.17 |
| 2           | 53.80                 | 108.88                     | 43.32 | 22 | 172.40 | 245.87 | 52.26 |
| 3           | 279.58                | 316.76                     | 76.05 | 23 | 72.31  | 275.51 | 113.34 |
| 4           | 912.44                | 54.57                      | 51.55 | 24 | 447.86 | 278.12 | 117.64 |
| 5           | 824.17                | 531.46                     | 71.66 | 25 | 673.20 | 48.47  | 91.10  |
| 6           | 893.25                | 473.29                     | 44.83 | 26 | 262.00 | 125.58 | 101.96 |
| 7           | 698.07                | 331.21                     | 61.26 | 27 | 502.87 | 410.90 | 71.95  |
| 8           | 121.66                | 141.22                     | 60.75 | 28 | 170.50 | 396.27 | 113.90 |
| 9           | 996.59                | 48.76                      | 107.23| 29 | 796.54 | 443.90 | 108.33 |
| 10          | 235.29                | 262.26                     | 81.58 | 30 | 875.33 | 385.67 | 83.50  |
| 11          | 645.05                | 120.17                     | 103.06| 31 | 944.41 | 161.25 | 89.30  |
| 12          | 921.24                | 442.32                     | 65.51 | 32 | 143.88 | 195.20 | 97.13  |
| 13          | 218.66                | 403.09                     | 46.21 | 33 | 243.37 | 54.21  | 45.58  |
| 14          | 806.53                | 79.26                      | 102.79| 34 | 513.45 | 31.05  | 44.08  |
| 15          | 634.82                | 406.97                     | 65.28 | 35 | 496.01 | 334.26 | 81.80  |
| 16          | 160.98                | 120.20                     | 71.11 | 36 | 557.68 | 447.50 | 47.50  |
| 17          | 584.34                | 75.56                      | 99.02 | 37 | 791.28 | 249.51 | 106.05 |
| 18          | 67.15                 | 32.63                      | 76.03 | 38 | 475.71 | 463.91 | 108.60 |
| 19          | 529.72                | 152.74                     | 82.44 | 39 | 205.41 | 177.98 | 73.70  |
| 20          | 608.94                | 463.15                     | 91.73 | 40 | 984.91 | 255.60 | 103.96 |

TABLE 2. Optimal visiting sequence of cities and its cumulative travel time.

| Serial number | CTTE | City | CTTL |
|---------------|------|------|------|
| 1             | 0561.1132 | 1    | 62.57 | 21 | 146.3787 | 7   | 207.6387 |
| 2             | 96.6035 | 18   | 172.6335 | 22 | 284.007 | 29  | 392.337 |
| 3             | 211.7289 | 2    | 255.0489 | 23 | 437.7525 | 5   | 509.4125 |
| 4             | 293.0241 | 8    | 353.7741 | 24 | 555.039 | 6   | 599.869 |
| 5             | 383.2621 | 32   | 480.3921 | 25 | 21.0843 | 12  | 86.5943 |
| 6             | 512.6668 | 39*  | 586.3668 | 26 | 123.4302 | 30  | 206.9302 |
| 7             | 45.1654 | 10   | 126.7454 | 27 | 287.7605 | 37  | 393.8105 |
| 8             | 159.567 | 22   | 211.827 | 28 | 491.669 | 40* | 595.629 |
| 9             | 264.5566 | 23   | 377.8966 | 29 | 51.8661 | 31  | 141.1661 |
| 10            | 470.7808 | 21*  | 573.9508 | 30 | 203.8119 | 9   | 311.0419 |
| 11            | 40.3404 | 28   | 154.2404 | 31 | 353.6534 | 4   | 405.2034 |
| 12            | 183.9671 | 13   | 230.1771 | 32 | 460.1375 | 14* | 562.9275 |
| 13            | 295.1722 | 3    | 371.2222 | 33 | 69.1241 | 25  | 160.2241 |
| 14            | 458.4422 | 24*  | 576.0822 | 34 | 199.1448 | 11  | 302.2048 |
| 15            | 37.3522 | 35   | 119.1522 | 35 | 340.2666 | 17  | 439.2866 |
| 16            | 158.0265 | 27   | 229.9765 | 36 | 481.5678 | 34  | 525.6478 |
| 17            | 260.0575 | 38   | 368.6575 | 37 | 587.665 *19 | 82.44 |
| 18            | 410.8892 | 36   | 458.1892 | 38 | 218.3742 | 26  | 320.3342 |
| 19            | 485.2701 | 20*  | 577.0001 | 39 | 357.6017 | 33  | 403.1817 |
| 20            | 31.245 | 15   | 96.525 | 40 | 456.5088 | 16  | 527.6188 |

periods was exactly eight. However, the cumulative travel time constraint is not considered when simply solving for the shortest TSP tour. Consequently, arranging the rest periods based on the shortest TSP tour usually cannot fully utilize the cumulative travel time standard, resulting in additional rest periods and making it difficult to obtain the theoretical minimum. For the example in this study, if the optimal TSP tour was first solved and then the rest periods were scheduled,
the total travel time was 10613.315 min with nine rests scheduled, for a total of 5400 min, which is one more rest period than the theoretical minimum. The results of the comparison between the two calculation methods are presented in Table 3.

It can be observed from Table 3 that when the improved ant colony algorithm designed in this study was used to solve the above RATSP, although the total intercity travel time was 7.2886 min more than that when simply solving the TSP, the method successfully coordinated the cumulative travel time and the travel time between (or within) the city nodes to be visited during the selection of the city nodes, effectively reducing the number of scheduled rest periods and reducing the total travel time to 592.7114 min.

**B. REVERSE TRANSFORMATION**

We transform the problem of personnel and equipment with the operation mileage and time limitation in the transport industry into a traveling salesman problem with a supply arc. After transformation, the solution of the TSP problem can be easily converted into the actual transportation problem. The following is an illustration of the reverse transformation of the EMU problem. Under the condition of the high-speed railway network operation in China, the EMU operation planning is the most important railway equipment scheduling problem. Some high-speed railway networks in China are shown in Fig. 7.

In the EMU operation network in Fig. 7, there is a train operation line between Beijing Shanghai high-speed railway (Beijing south—Shanghai Hongqiao), which needs to be acted by train units. Generally, there are EMU operation stations at the starting and terminal stations of the high-speed railway lines. For the EMUs acting as the train operation line, the EMUs must meet the maintenance constraints, i.e., after the EMU undertakes a series of tasks, it must be arranged for a certain level maintenance when the EMU reaches its specified accumulated running mileage or accumulated running time.

For the traveling salesman problem with the supply arc proposed in this study, when it is inversely transformed into the EMU operation planning problem, the city nodes can be transformed into the train operation lines that need to be undertaken, and the businessmen can be transformed into the utilized EMUs. When the cumulative mileage and time of EMUs are considered simultaneously, we can use the method proposed in this study to describe and solve the EMU operation problem.

**VI. CONCLUSION**

In this paper, the personnel and equipment utilization problem in the transport industry with cumulative working time or utilization mileage constraints was abstracted and extended to obtain a class of RATSP with cumulative travel constraints and dynamic replenishment arcs. The constraints that should be satisfied in the journey of the traveling salesman were provided by analyzing the cumulative travel time of the traveling salesman upon entering and leaving the city node. Under the assumption that the travel time and travel mileage are interchangeable, an optimization model with the
objective of minimizing the total travel time was developed. To solve the established RATSP, this study began from the algorithmic approach of the construction solution, and an ant colony algorithm was used to solve the problem. Additionally, some parameters of the ant colony algorithm were redesigned considering the characteristics of the problem, such that the calculation method of the heuristic information could be dynamically adjusted during the solution construction process according to the cumulative travel time of the traveling salesman. The subsequent city nodes conducive for reducing the total travel time could then be selected. The traveling process of the traveling salesman to 40 city nodes was calculated in the simulation example. The results showed that the model developed and the algorithm designed in this study are effective for solving the RATSP. Both the intercity and intracity travel time were considered in the TSP studied herein. In practical applications, if only the interior or intracity travel time is involved, the corresponding node or arc weights can be set to zero, and the algorithm in this study remains applicable. Additionally, the cumulative travel time as the main factor considered by the traveling salesman was evaluated in this study. If the traveling process of the traveling salesman is constrained by the cumulative travel mileage or by both the utilization mileage and time, only simple improvements to the model and algorithm are required to ensure the applicability of the existing foundation.

The rapid development of China’s high-speed railways puts forward higher requirements on the organization of high-speed railway transport. As an important part of the daily operation plan, the flight attendant plan requires a more efficient process and humanized results. From the actual point of view of the compilation of China’s high-speed railway crew plan, the current crew plan is mostly prepared manually, which has the defects of low efficiency and difficult adjustment of results. Therefore, optimizing the method for preparing high-speed railway passenger crew schedule planning is of great significance for improving the efficiency of the high-speed railway operation organization and service quality.

This study examines the optimization of high-speed railway passenger crew scheduling. The specific research content is as follows:

1. It explains the classification of high-speed railway cabin crew plans and passenger cabin crew work processes, analyzes the influencing factors of preparing cabin crew plans, outlines the cabin crew plan preparation process, and compares and analyzes the advantages and disadvantages of several methods of preparing cabin crew plans. It lays a theoretical foundation for establishing the optimization model of the flight attendant scheduling plan.

2. It examines the individualized scheduling of flight attendants. By considering the crew’s satisfaction with the type of shift, the condition of the individual needs of the crew is added on the basis of the traditional scheduling method, and a general weight distribution method is designed to indicate that the crew is different in the planning cycle. The degree of preference work for different classes of rest days and daily schedules. Moreover, by considering constraints such as work tasks, working hours, shift types, and shift intervals, a multi-objective optimization model for flight attendant scheduling considering individual needs is constructed, and the number of flight attendants is minimized. Individualized requirements are satisfied to the greatest extent, and the rationality of the optimization model is verified through calculation examples.

3. The problem of high-speed railway passenger crew planning under multi-period work balance is studied. By considering the flight attendant’s handing preference and rest day preference, the traffic mode satisfaction, rest day satisfaction, scheduling mode satisfaction, maximum possible satisfaction, satisfaction rate and other parameters are generated, and the degree of satisfaction of the flight crew’s preferences based on these parameters is measured. Considering the task of flight attendants such as the number of flight attendants as constraints, we finally establish an optimization model with the maximum satisfaction and maximum satisfaction rate of all flight attendants, and obtain specific solution steps.

4. A calculation example is constructed to verify the rationality of the model in this study. Different optimization objectives are then used to solve the schedule plan separately, and make a relevant analysis of the results. The result of the calculation example proves the feasibility of the study, which has practical significance for improving the efficiency of high-speed railway crew planning and realizing the automation and informatization of crew planning.

The above-mentioned research not only has practical significance for improving the level of preparation of high-speed railway passenger crew scheduling plans, and realizing the intelligentization of crew scheduling planning, but also is a reference for the research and design of China’s high-speed railway passenger crew scheduling automation system.

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