Multiwavelength Modeling the SED of Luminous Supersoft X-Ray Sources in Large Magellanic Cloud and Small Magellanic Cloud

Augustin Skopal ©
Astronomical Institute, Slovak Academy of Sciences, 059 60 Tatranská Lomnica, Slovakia; skopal@ta3.sk
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Abstract

Classical supersoft X-ray sources (SSSs) are understood as close binary systems in which a massive white dwarf accretes from its companion at rates sustaining steady hydrogen burning on its surface generating bolometric luminosities of \(10^{36} - 2 \times 10^{38}\) erg s\(^{-1}\). Here, we perform for the first time the global supersoft X-rays to near-infrared (NIR) spectral energy distribution (SED) for the brightest SSSs in the Large Magellanic Cloud and Small Magellanic Cloud. We test a model in which the ultraviolet–NIR is dominated by emission from a compact (unresolved) circumstellar nebula represented by the ionized gas outflowing from the SSS. The SED models correspond to luminosities of SSSs of a few times \(10^{38} - 10^{39}\) erg s\(^{-1}\), radiating at blackbody temperatures of \(\approx 3 \times 10^5\) K, and indicate a nebular continuum, whose emission measure of \(\gtrsim 2 \times 10^{60}\) cm\(^{-3}\) corresponds to a wind mass loss at rates \(\gtrsim 2 \times 10^{-5}\) \(M_\odot\) yr\(^{-1}\). Such extreme parameters suggest that the brightest SSSs could be unidentified optical novae in a post-nova SSS state sustained at a high long-lasting luminosity by resumed accretion, possibly at super-Eddington rates. New observations and theoretical multiwavelength modeling of the global SED of SSSs are needed to reliably determine their parameters, and thus understand their proper stage in stellar evolution.

1. Introduction

Luminous supersoft X-ray sources (SSSs) were discovered in the Large Magellanic Cloud (LMC) and in the Small Magellanic Cloud (SMC) by Long et al. (1981) and Seward & Mitchell (1981). They are characterized by very soft thermal spectra, emitting predominantly at energies \(\lesssim 0.5\) keV (Greiner et al. 1991). Their extremely soft spectra correspond to blackbody temperatures of \(\approx 15 - 80\) eV and bolometric luminosities of \(10^{36} - 2 \times 10^{38}\) erg s\(^{-1}\). These objects are commonly accepted as binaries that consist of a massive white dwarf (WD) radiating close to the Eddington luminosity due to a steady nuclear burning on its surface, when the hydrogen-rich material is burned as fast as it is accreted, at rates of the order of \(10^{-7}\) \(M_\odot\) yr\(^{-1}\) (van den Heuvel et al. 1992; Kahabka & van den Heuvel 1997). Following multiwavelength observations has revealed a significant excess of the radiation in the ultraviolet (UV) to near-infrared (NIR), well above the Rayleigh–Jeans tail of the SSS radiation itself (see Sections 1.1–1.4 and Section 2). To explain the high luminosity of bright SSSs, and the longer wavelength excess, Popham & di Stefano (1996) proposed a model in which a significant fraction of the WD’s radiation is converted to the UV–NIR by irradiating (i) the flared accretion disk and (ii) the donor star.

In this paper we selected four bright SSSs observed in the Magellanic Clouds, RX J0513.9-6951, RX J0058.6-7135, RX J0543.6-6822, and RX J0527.8-6954, with the aim to model their global X-ray–NIR spectral energy distribution (SED) by the method of multiwavelength modeling described by Skopal (2015). The motivation for this work is the absence of the SED model, which would uniformly fit radiation of SSSs in both the supersoft X-ray and the UV–NIR domains. Further, the extreme properties of SSSs and their spectral features indicate the presence of nebular radiation in the spectrum (see Appendix A), which could explain the observed strong UV–NIR excess as it is currently considered for the accreting nuclear-burning WDs in symbiotic binaries (Kenyon & Webbink 1984; Muerset et al. 1991; Skopal 2005). Therefore, here we are testing SED modeling, in which the UV–NIR is dominated by the nebular emission instead of the irradiated disk/companion radiation.

Section 2 summarizes the observed supersoft X-rays to the NIR SED of our targets, while Section 3 introduces the method and results of our analysis. Their discussion is found in Section 4, while Section 5 summarizes our findings, and proposes tasks for future investigation. In the following subsections, we first introduce our targets.

1.1. The LMC SSS RX J0513.9-6951

RX J0513.9-6951 is a transient binary SSS. Currently, it is thought that the system contains a somewhat evolved star and a relatively massive compact object at a 0.76 day orbit, viewed nearly pole-on (Crampton et al. 1996; Southwell et al. 1996). The system is the brightest SSS emitting a significant amount of radiation also in the far-UV (\(\gtrsim 10^{-13}\) erg cm\(^{-2}\) s\(^{-1}\) Å\(^{-1}\)).

RX J0513.9-6951 was discovered during the ROSAT All-sky Survey (1990 July–1991 January) in the LMC (Schaeidt et al. 1993). The authors found it to be variable and matched its total average spectrum by blackbody radiation (see Table 1). Pakull et al. (1993) revealed the optical counterpart as a \(B \approx 17\) mag blue star. Its spectrum was dominated by the hydrogen Balmer lines with a signature of a broad P-Cyg absorption component indicating a mass loss from the system at \(v_\infty \approx 3600\) km s\(^{-1}\). The absence of He I lines and the presence of highly ionized ions (O \(\lambda 1381\), 3834 doublet, He I) with \(I_{\text{He I} 3834} \gg I_{\text{He I} 4686}\) reflect the presence of a very hot ionizing
source in the binary. They also pointed out that the optical counterpart appears fainter during the X-ray outburst.

Pronounced optical variability of RX J0513.9-6951 was revealed by optical multicolor monitoring campaigns. Reinsch et al. (1996) measured an optical-low state from 1993 February to December lasting around 40 days with a decrease in the continuum by $\Delta V \sim 1$ mag and a factor of $\approx 10$ decrease in emission-line fluxes. The light curve obtained within the MACHO project showed the occurrence of the low stages on a timescale of $\sim 100$–200 days (Southwell et al. 1996; Alcock et al. 1996). In addition, during the high state, these authors revealed a small ($\Delta V \sim 0.05$) light modulation with a period of 0.763 day, which is consistent with variations in the radial velocities of the He II $\lambda 4686$ emission line, and thus confirms a binary nature of the system (Crampton et al. 1996).

Broad wings of the strongest emission lines (He II $\lambda 4686$) accompanied by satellite components indicated a bipolar outflow and thus the presence of a disk in the binary. The bipolar outflow in the form of jets suggested that the accretor is a WD (Crampton et al. 1996; Southwell et al. 1996).

Based on all previously published optical and X-ray data, Southwell et al. (1996) pointed out the principal feature of the variability: the anticorrelation between the supersoft X-ray and the optical fluxes. X-ray monitoring of RX J0513.9-6951 showed that the flux anticorrelation is very strict: There are cyclic changes between optical-low/X-ray-on states and optical-high/X-ray-off states (e.g., Reinsch et al. 2000; Charles et al. 2010).

Based on the Hubble Space Telescope (HST) UV spectroscopy, Gansicke et al. (1998) derived the neutral hydrogen density $N_{\text{H}} = (5.5 \pm 1.0) \times 10^{20} \text{ cm}^{-3}$. The authors found that this value is consistent with parameters determined by the model atmosphere that fits the ROSAT PSPC spectrum of 1993 July (see Table 1).

### 1.2. The SMC SSS RX J0058.6-7135 (LIN 333)

Henize (1956) labeled this object in his “Catalog of Emission Nebulae in the Small Cloud” as LH$\alpha$-115 N67. Independently, Lindsay (1961) put it in his catalog under the number 333, remarking features of a planetary nebula in its spectrum. Therefore, the identifier LIN 333 is frequently used for this object. Aller et al. (1987) analyzed its International Ultraviolet Explorer (IUE) spectra and revealed a strong nebular emission produced by the object, characterized by the emission measure of $\sim 1.4 \times 10^{60} \text{ cm}^{-3}$ (see Appendix A) and a high electron temperature of $\sim 30,000$ K. They also measured the optical position of the nebula, which helped Wang (1991) to identify LH$\alpha$-115 N67 with the strong SSS 1E0056.8-7154, which is also named “SMP SMC 22” (Sanduleak et al. 1978). Wang (1991) suggested that 1E0056.8-7154 is the single hot nucleus of the planetary nebula LH$\alpha$-115 N67. The X-ray detection of this source was first reported by Seward & Mitchell (1981) on the basis of observations with the Einstein satellite. Thereafter, X-ray observations were carried out by the ROSAT and XMM-Newton satellites; see Kahabka et al. (1994) and Mereghetti et al. (2010), respectively. Modeling of

| Object          | $N_{\text{H}}$ ($10^{20}$ cm$^{-2}$) | $R_{\text{SSS}}^\text{eff}$ ($R_\odot$) | $T_{\text{SSS}}/T_{\text{eff}}^\text{eff}$ (K) | log($L_{\text{SSS}}$) (erg s$^{-1}$) | Model | Reference          |
|-----------------|-----------------------------------|----------------------------------------|----------------------------------------|----------------------------------|-------|-------------------|
| RX J0513        | 9.4                               | $\sim 0.039$                           | 456,000$\pm$78,000                   | $\sim 38.37$                     | BB$^i$| 1                 |
| RX J0513        | 4.7                               | 0.009$-0.017$                         | $\approx 560,000$                     | 37.40$-37.95$                   | Atm$^i$| 2                 |
| LIN 333         | 5.2–16                            | 0.086                                 | 160,000$-400,000$                     | 38.28                           | BB$^v$| 9                 |
| LIN 333         | 8                                 | 0.012                                | 440,000                               | 37.30                           | Atm$^v$| 3                 |
| LIN 333         | 8.6                              | 0.072                                | 360,000                               | 38.48                           | BB$^v$| 3                 |
| LIN 333         | 2.8–3.2                           | 0.19–0.22                            | 154,000                               | 37.81$-37.93$                   | Atm$^v$| 4                 |
| CAL 83          | 5.2                              | 0.049                                | 313,000                               | 37.91                           | BB$^v$| 4                 |
| CAL 83          | >0.06                             | 302,000                               | $L_{\text{SSS}} > L_{\text{Edd}}$    | BB$^v$                          | 5                 |
| CAL 83          | 8                                 | 0.022                                | 500,000                               | $L_{\text{SSS}} \equiv L_{\text{Edd}}$ | BB$^v$| 5                 |
| CAL 83          | 8.87                             | 0.014–0.016                          | 516,000                               | 37.76                           | Atm.  | 6                 |
| CAL 83          | 6.07                             | $\geq 0.015$                         | $\leq 403,000$                        | $\geq 37.30$                   | Atm.  | 6                 |
| CAL 83          | 6.5                              | 0.012±0.003                          | 539,000$\pm$16,000                   | 37.40±0.03                     | BB$^v$| 7                 |
| CAL 83          | 6.5                              | 0.018±0.001                          | 378,000$\pm$9000                     | 37.37±0.01                     | Atm$^v$| 7                 |
| CAL 83          | 6.5±1.0                           | 0.01±0.001                           | 550,000$\pm$25,000                   | 37.54±0.13                     | Atm$^v$| 8                 |
| RX J0527        | 14                               | >0.16                                | 209,000                               | $L_{\text{SSS}} > L_{\text{Edd}}$ | BB$^v$| 5                 |
| RX J0527        | 8.5                              | 0.043                                | 348,000                               | $L_{\text{SSS}} \equiv L_{\text{Edd}}$ | BB$^v$| 5                 |

Notes.  
$^i$ ROSAT, 1990 October 30; 1 - Schaeidt et al. (1993);  
$^v$ ROSAT, 1993 July; 2 - Gansicke et al. (1998);  
$^v$ ROSAT, 1991 October 8; 3 - Heise et al. (1994);  
$^v$ XMM-Newton, dates in Table 2; 4 - Mereghetti et al. (2010);  
$^v$ ROSAT, 1990 June 20—best fit; 5 - Greiner et al. (1991);  
$^v$ As in $^i$, but for the Eddington luminosity ($L_{\text{Edd}}$);  
$^vi$ ROSAT, observation No. 500131p; 6 - Kahabka (1998);  
$^vi$ BeppoSAX, $N_{\text{H}}$ fixed, 1997 March 7–8; 7 - Parmar et al. (1998);  
$^vi$ Chandra, XMM-Newton, $N_{\text{H}}$ fixed, dates in Table 2; 8 - Lanz et al. (2005);  
$^vi$ ROSAT, 1990 June 16–25—best fit;  
$^vi$ As in $^i$, but for $L_{\text{Edd}}$;  
$^{vii}$ ROSAT, 1990 October 27–31; 9 - Kahabka et al. (1994).
The X-ray SED of LIN 333 was performed by Kahabka et al. (1994), Heise et al. (1994), and Mereghetti et al. (2010). Corresponding parameters are introduced in Table 1. Based on their models, Heise et al. (1994) found that an accreting WD in a binary is consistent with the ROSAT spectrum of 1E0056.8-7154, but also admitted a possibility that the SSS is an exceptionally hot single central star of a planetary nebula. Comparing all X-ray data and their models, spanning almost 30 yr, Mereghetti et al. (2010) did not find any apparent long-term variability in the luminosity or spectrum of LIN 333. The authors pointed out that the lack of variability, typical for other SSSs, and too low upper limit to the optical counterpart (V ~ 21, Villaver et al. 2004) do not favor a binary nature of LIN 333. Therefore, they supported the interpretation that it is a single, very hot star on its way to becoming a relatively massive (~1 M\(_{\odot}\)) WD. Using the Spitzer Space Telescope infrared spectra, Bernard-Salas et al. (2009) classified SMP SMC 22 as a high-excitation planetary nebula with high-excitation lines ([O IV] and [Ne V]). The absence of dust-related features in the spectrum was ascribed to the high temperature and luminosity of SMP SMC 22.

In spite of these arguments, we interpret the results of our multiwavelength modeling of the LIN 333 SED within a binary-star model, in which a WD accretes from its companion at a high rate (Section 4.3). The main reason for this assumption is the exceptionally high luminosity of LIN 333 compared to other planetary nebulae, an indication of a compact unresolved nebula in its spectrum—common main features of all of our targets (Figure 1, Section 4.5, Appendix A), and possible presence of the binary companion indicated by the optical fluxes (see Appendix C).

1.3. The LMC SSS RX J0543.6-6822 (CAL 83)

The bright SSS CAL 83 was discovered in the Einstein survey of the LMC (Long et al. 1981). Subsequent observations with EXOSAT from November 1983 confirmed its very soft X-ray spectrum with an upper limit of 0.3 count/s (Pakull et al. 1985). The optical spectrum from 1984 May revealed its similarity to those of the low-mass X-ray binaries LMC X-2 and Sco X-1. From radial velocity variations of the optical emission lines (H and He II \(\lambda\)4686), Crampton et al. (1987) discovered that CAL 83 is a binary system with a period of ~1 day. Orbital elements from radial velocities of the He II \(\lambda\)4686 line were improved by Schmidtke et al. (2004).

X-ray observations of CAL 83 were then carried out with the ROSAT, BeppoSAX, Chandra, and XMM-Newton satellites. Kahabka (1998) presented a 1.5 yr X-ray light curve of CAL 83 as measured with the ROSAT/HRI from 1995 July to 1996 December. The authors also indicated the first short-term X-ray-off state of CAL 83 on JD 2450200 (1996 April 26). The second and the third X-ray-off states were reported by...
Greiner & Di Stefano (2002) and Lanz et al. (2005) on the basis of Chandra observations from 1999 November and 2001 October, respectively. This demonstrates a recurrent nature of CAL 83. Modeling of the X-ray SED of CAL 83 was carried out by Greiner et al. (1991), Kabakha (1998), Parmar et al. (1998), Paerels et al. (2001), and Lanz et al. (2005). Corresponding parameters are found in Table 1.

The prototypical SSS CAL 83 was also intensively studied in the UV and optical. It was identified with a variable, blue, V ≈ 17, pointlike source with an orbital period of 1.047 days (Cowley et al. 1984; Pakull et al. 1985; Smale et al. 1988; Schmidtke et al. 2004; Rajoelimanana et al. 2013). The optical spectrum taken over a 25″ FOV; i.e., Pakull et al. (1985). CAL 83 is known to show irregular brightness variations of more than a magnitude over timescales of months. Using the MACHO V and R light curves (1993–1999), Greiner & Di Stefano (2002) indicated that CAL 83 exhibits distinct and well-defined low, intermediate, and high optical states, characterized by no color changes during transition phases. They also found that X-ray-off states were observed during optical-high states. A variability has also been found in the UV and optical. It was identified first time indicating a HeII λ4686 emission component suggested a mass outflow with velocities at 1500–2000 km s⁻¹ (Pakull et al. 1985).

Using the CLOUDY software and typical parameters of the MACHO project, Alcock et al. (1997) first suggested an optical counterpart to RX J0527.8-6954, and Cowley et al. (1997) specified it as a B8 IV star with V = 17.3 mag. Using high-resolution imaging, Oliveira et al. (2010) identified the X-ray source with a B5e V star that is associated with a bipolar Hα subarcsecond extended emission. The authors pointed out that for a massive WD and the Be V companion with M = 6 M☉, the orbital period has to exceed 21 hr, and thus the 0.39 day modulation cannot be associated with the orbit.

2. Observations

This section summarizes observations of our targets we used to model their global SED in the continuum. Spectroscopic observations with Far Ultraviolet Spectroscopic Explorer (FUSE), HST, and IUE were obtained from the satellite archives with the aid of the MAST. Other observations were taken from the literature and catalogs as referred to in the following subsections. Data obtained from the space are collected in Table 2. Stellar magnitudes in the optical–NIR were converted to fluxes according to the calibration of Henden & Kaitchuck (1982) and Bessell (1979). To correct the observed X-ray fluxes for absorptions, we used the tbabs absorption model for the ISM composition with abundances given by Wilms et al. (2000; e.g., log(A_O) + 12 = 8.69). Observations for λ > 912 Å were reddened with E(V,λ) = 0.06 and 0.08 (unless otherwise specified), and physical parameters were scaled with a distance d = 49 and 60 kpc to objects in the LMC and SMC, respectively (Mateo 1998).

2.1. RX J0513.9-6951

The observed SED of RX J0513.9-6951, used in this contribution, covers the spectral range from 1.92 to 2200 nm. It consists of the ROSAT PSPC observations (1.92–6.20 nm) as published by Schaeidt et al. (1993). The data were adopted from their Figure 2. The far-UV spectra with FUSE (B0160104, D0060102) were taken during the high state of RX J0513.9-6951 as found by Hutchings et al. (2002) for the 2001 observation, and according to the light curve of Burwitz lower than presently observed (Woods & Gilfanov 2016; Farias et al. 2020).

1.4. The LMC SSS RX J0527.8-6954

This source was discovered during the calibration phase of ROSAT pointing to the LMC field between 1990 June 16 and 25 (Greiner et al. 1991). The profile of its supersoft spectrum was very similar to that of CAL 83, but it was fainter by a factor of ∼10. The model SED was done by Greiner et al. (1991; Table 1).

Repeated observations of this source exhibited a continuous decline in the photon counts, from ∼0.2 to ∼0.01 cts s⁻¹, during 1990–1995 (Greiner et al. 1996). The authors speculated that a likely mechanism causing the X-ray variability could be a hydrogen flash on a massive WD (≥1.1 M☉) accreting at a high rate of 10⁻⁷ M☉ yr⁻¹. This view is supported by the fact that the source was not detected by the Einstein satellite 20 yr ago; thus, it appears that the source is recurrent and was in a declining phase during the ROSAT observations. Also the 4 yr light curve revealed a steady fading by ∼0.05 mag and a 0.39 day modulation with ∼0.05 mag semi-amplitude.


The observed SED of CAL 83, used in this paper, covers the spectral range from 2.07 nm to 790 nm. The calibrated superson X-ray fluxes were reconstructed from Figure 1 by Greiner et al. (1991; ROSAT) and Figure 1 by Lanz et al. (2005; Chandar, XMM-Newton). In both cases, the source was in the X-ray-on state (see Kahabka 1998, for the ROSAT observation).

In the UV, 11 far-UV spectra of CAL 83 were carried out by FUSE (B0150101) on 2001 September 22 (Schmidtke et al. 2004). The continuum fluxes were estimated within the range of 93.5–118 nm. The high-resolution HST/GHRS spectrum (Z3HS0105T) covers the region from ~114.9 to ~143.5 nm. Further spectroscopic observation in the UV was carried out with the IUE satellite. The continua of the SWP spectra Nos. 24554, 32511, 39995, and 40075 and the LWP spectra Nos. 12257 and 13902 were most consistent with those of the HST and FUSE spectra. Therefore, we used their average fluxes weighted with exposure times to model the SED. Other IUE spectra had a higher continuum, in particular, the SWP30024 spectrum was a factor of >2 above the used minimum values. According to the measured variations in both the X-ray and the UV–optical fluxes and their anticorrelation (see Greiner & Di Stefano 2002), it is probable that the minimum level of the UV continuum, which we used to model the SED, corresponds to the X-ray-on phase of CAL 83.

Spectroscopic observations were supplemented with the optical UBVR broadband photometry, taken from the catalogs of Massey (2002; $U = 16.25, B = 17.18, V = 17.44$), Zaritsky et al. (2004; $U = 16.022, B = 16.864, and V = 16.976$) and the third release of the DENIS database ($R = 17.0, I = 16.538$). Compared are values reported by Crampton et al. (1987; $B = 17.19–17.34, V = 17.22–17.35$), and fluxes from the optical, 400–680 nm, spectrum published by Pakull et al. (1985) scaled with their average photometric measurements $(U - B = -1.04 \pm 0.06, B - V = 0.02 \pm 0.06, B = 17.2 \pm 0.3)$. In the NIR Kato et al. (2007) measured $J = 16.98, H = 16.81$, and $K = 16.77$, and 2MASS 6X Point Source Working
Database/Catalog reports $J = 17.433$, $H = 17.016$ and $K = 16.079$.

2.4. RX J0527.8−6954

In this case, the available observed SED in the continuum covers the spectral range from 2.65−550 nm. The calibrated supersoft X-ray fluxes were obtained by the ROSAT satellite during the pointing observations of the LMC field in 1990 (Greiner et al. 1991). According to Greiner et al. (1996), the source was at the brighter stage.

UV spectra of RX J0527.8−6954 were carried out with the IUE satellite on 1992 September 1 (SWP45499, LWP3826). The LWP23826 spectrum was underexposed; therefore, its part from the short-wavelength edge to ~240 nm was not used in the modeling (dotted line in Figure 1d).

The optical brightness of RX J0527.8−6954 was measured by Cowley et al. (1997) in the $UBV$ passbands at the Cerro Tololo Inter-American Observatory during 1992 December, 1993 December, and 1994 November. They determined $V = 17.3$, $B = 17.4$, and $U = 17.2$. On 2004 September, Oliveira et al. (2010) observed RX J0527.8−6954 using the Gemini South Telescope with the Gemini Multi-Object Spectrograph and distinguished the original star into two components. They determined the flux ratio in the $V$ band between the components to 1.43 and corrected the $V$ magnitude of the RX J0527.8−695 optical counterpart to 17.9.

3. Analysis and Results

3.1. Modeling the SED

The primary aim of this paper is to reconstruct the SED of bright SSSs in LMC and SMC from supersoft X-rays to the optical or NIR. In this way to determine physical parameters of the main components of radiation in the observed continuum. To achieve this aim we use the method of multiwavelength modeling of the composite continuum of SSSs described by Skopal (2015). Here we introduce its basic assumptions and relationships.

In our SED modeling, the observed continuum, $F(\lambda)$, is assumed to consist of a stellar component of radiation, $F_{\text{SSS}}(\lambda)$, from the WD pseudo-photosphere, and nebular continuum, $F_{\text{Ne}}(\lambda)$, from the ionized outflowing gas (see Appendix A and Section 4.4), i.e.,

$$F(\lambda) = F_{\text{SSS}}(\lambda) + F_{\text{Ne}}(\lambda). \quad (1)$$

In the model we compare the stellar continuum with the blackbody radiation at a temperature $T_{\text{BB}}$, while the nebular continuum is approximated by contributions from f-b and f-f transitions in hydrogen plasma radiating at the electron temperature $T_e$. In cases, where observations suggest a discontinuity in the continuum around $\lambda 2050$ Å, contributions from doubly ionized helium, are also included, and the $\text{He}^{++}$ abundance is set to 0.1. According to Equations (6) and (8) of Skopal (2015), Equation (1) can be expressed in the form,

$$F(\lambda) = \theta_{\text{SSS}}^2 \pi B_\lambda(T_{\text{BB}}) e^{-\sigma_{\lambda}(\text{H}) N_{\text{H}}} + k_N \times \varepsilon_\lambda(H, \text{He}^+, T_e), \quad (2)$$

where $\theta_{\text{SSS}} = R_{\text{SSS}}^{\text{eff}}/d$ is the angular radius of the SSS, given by its effective radius (i.e., the radius of a sphere with the same luminosity) and the distance $d$. Attenuation of the light in the X-ray domain is given by the total cross section for photoelectric absorption per hydrogen atom, $\sigma_{\lambda}(\text{H})$ (Cruddace et al. 1974), and the total column density of neutral atoms of hydrogen, $N_{\text{H}}$ (i.e., given by both the ISM and circumstellar matter components). The second term at the right is the nebular continuum expressed by its volume emission coefficient $\varepsilon_\lambda(H, \text{He}^+, T_e)$ scaled with the factor $k_N = EM/4\pi d^2$, where $EM = \int n_e n^+ dV$ is the emission measure of the nebula given by the electron and an ion concentration, $n_e$ and $n^+$, in volume $V$ of the ionized element under consideration. The assumption that the nebular radiation in the continuum can be characterized
by a single $T_e$ is supported by the velocity distribution of free electrons in nebulae that tends to be Maxwellian because the electrostatic encounters of free electrons are much more likely than any other inelastic scatterings (see Bohm & Aller 1947). Therefore, using a single-temperature nebula in the SED modeling expresses the measured UV–NIR fluxes quite well (see the SED models in Skopal 2005, 2015, 2019, and Figure 1). In the modeling procedure, we simultaneously fit absorbed X-ray continuum fluxes and dereddened UV to NIR fluxes searching for parameters $\theta_{\text{SSS}}$, $T_\text{BB}$, $N_\text{H}$, $T_e$, and $k_\text{SS}$ (see Skopal 2015, in detail).\(^1\) The luminosity of the SSS is calculated as

$$L_{\text{SSS}} = 4\pi d^2 \theta_{\text{SSS}}^2 \sigma_{\text{BB}} T_{\text{BB}}^4.$$

Physical parameters corresponding to our SED models are collected in Table 3.

### 3.2. $N_\text{H}$ from Rayleigh Scattering

The high-resolution HST/GHRS spectra ($\lambda$115–144 nm) show a pronounced attenuation of the continuum around the Ly$\alpha$ line that is caused by Rayleigh scattering on atomic hydrogen. This effect can be used to independently determine $N_\text{H}$ in the direction of the SSS (see Section 2.3.1 of Skopal 2015). For this purpose, we compare the function

$$F(\lambda) = \theta_{\text{SSS}}^2 \pi B(\lambda) e^{-\sigma_{\text{Ray}}(\lambda)N_\text{H}} + k_\text{N} \times \sigma_{\text{Ly}}(H, T_e)$$

(4)

to the observed spectrum. The Rayleigh cross section for scattering by hydrogen in its ground state, $\sigma_{\text{Ray}}(\lambda)$, is approximated by Equation (5) of Nussbaumer et al. (1989). Other parameters have the same meaning as in Equation (2) and are fixed according to the solution of the global SED.

### 3.3. Multiwavelength Model SED of RX J0513.9-6951

Owing to the strict soft X-ray/optical flux anticorrelation, it was possible to perform the X-ray–optical SED fitting only for the low state of RX J0513.9-6951 (Figure 1(a)), while during the high state, only the far-UV–NIR observations were available to model the corresponding SED (Figure 2). The best-fitting X-ray–optical SED model parameters, $N_\text{H} = (1.2 \pm 0.2) \times 10^{21}$ cm$^{-2}$, $\theta_{\text{SSS}} = 8.8 \times 10^{-14}$, and $T_{\text{BB}} = (350,000 \pm 10,000)$ K, correspond to a very high luminosity of the SSS source, $L_{\text{SSS}} = (1.9 \pm 0.7) \times 10^{39}$ erg s$^{-1}$, with the effective radius, $R_{\text{eff}} = (0.19 \pm 0.02) R_\odot$. The model SED also shows a significant contribution from the thermal nebula with the emission measure, $EM = (2.3 \pm 0.2) \times 10^{49}$ cm$^{-3}$ that dominates the spectrum for $\lambda > 2000$ Å.

During the optical-high state, the nebular emission increased by a factor of $\sim 3.5$ (Table 3), and the WD’s effective radius inflated to $(0.26 \pm 0.03) R_\odot$ for the same $T_{\text{BB}}$ as during the optical-low state. The former suggests that the variable nebular emission is responsible for the optical variability with $\Delta V \approx 1$ mag, while the latter reflects the increase of the far-UV fluxes of the stellar component of radiation (the blue lines in Figure 2; see Section 4.6 in detail).

Finally, fitting the far-UV HST/GHRS spectrum by Equation (4) we found that the hollow around Ly$\alpha$ corresponds to $N_\text{H} = (8 \pm 2) \times 10^{20}$ cm$^{-2}$. A comparison of the model SED and the HST spectrum constrains the reddening $E_{B-V} = 0.11 \pm 0.02$. The spectrum and Equation (4) are shown in Figure 3.

### 3.4. Multiwavelength Model SED of RX J0058.6-7135

Observations of LIN 333 span the wavelength range from supersoft X-rays to the optical. Fluxes in the near-UV and $UBV$ bands ($\sim 1 \times 10^{-15}$ erg cm$^{-2}$ s$^{-1}$ Å$^{-1}$; see Figure 1(b)) suggest a contribution from the nebula and, possibly, from a companion to the SSS in a binary (see Appendix C). In modeling the supersoft X-ray–optical SED, we fitted 13 flux-points between 24 and 62 Å and 31 flux-points between 1500 and 3600 Å by Equation (2). Observations with FUSE were not used directly in the SED-fitting procedure, because their continuum level is a factor of 1.1–1.5 above that given by the IUE and HST spectra. Nevertheless, the steep slope of their dereddened fluxes follows the short-wavelength ends of the IUE and HST spectra, which documents the dominant contribution from the SSS to the far-

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\(^1\) The corresponding software and application example are available at https://doi.org/10.5281/zenodo.6985175.

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**Table 3**

| Object | Supersoft X-Ray Source | Nebula |
|--------|------------------------|--------|
| $N_\text{H}$ ($10^{20}$ cm$^{-2}$) | $R_{\text{eff}}$ ($R_\odot$) | $T_{\text{BB}}$ (K) | $\log(L_{\text{SSS}})^a$ (erg s$^{-1}$) | $\log(L_{\text{X}})^b$ (erg s$^{-1}$) | $T_e$ (K) | $EM$ ($10^{49}$ cm$^{-3}$) | $\chi^2_{\text{red}}$/dof |
|--------|------------------------|--------|
| RX J0513$^a$ | 12 ± 2 | 0.19 ± 0.02 | 350,000 ± 10,000 | 39.28 ± 0.14 | 38.25 ± 0.13 | 30,000 ± 10,000 | 2.3 ± 0.2 | 2.8/40 |
| RX J0513$^b$ | ... | ... | 350,000$^b$ | 39.54 ± 0.15 | 38.52 ± 0.14 | 37,000 ± 5000 | 1.6 ± 0.2 | 2.6/39 |
| LIN 333$^d$ | 9.0 ± 0.2 | 0.17 ± 0.02 | 305,000 ± 10,000 | 38.96 ± 0.13 | 37.66 ± 0.12 | ... | ... | ... |
| LIN 333$^e$ | 7.1 ± 0.2 | 0.18 ± 0.02 | 267,000 ± 7000 | 38.75 ± 0.12 | 37.45 ± 0.11 | 30,000 ± 15,000 | 2.9 ± 0.3 | 3.8/61 |
| CAL 83$^f$ | 10.3 ± 0.5 | 0.15 ± 0.02 | 345,000 ± 15,000 | 39.04 ± 0.16 | 37.99 ± 0.15 | ... | ... | ... |
| CAL 83$^f$ | 12.6 ± 0.5 | 0.16 ± 0.02 | 357,000 ± 15,000 | 39.14 ± 0.16 | 38.15 ± 0.15 | 9000 ± 2000 | 4.3 ± 0.3 | 3.3/34 |

**Notes.**

- $^a$ Low state,
- $^b$ High state: the $(1 - 9) \times 10^7$ Å model SED for $T_e$ adapted from the low state (Figure 2),
- $^c$ Fixed value,
- $^d$ Model SED with the ROSAT data,
- $^e$ Model SED with the XMM-Newton data,
- $^f$ Model SED with the Chandra/XMM-Newton data,
- $^g$ The luminosity in the 0.2–10 keV range.
UV (selected fluxes are compared to the LiF2A channel spectrum, 1087–1181 Å).

The modeling solution with the ROSAT data has a reduced \( \chi^2 = 2.6 \), and corresponds to the fitting parameters of the SSS, \( \theta_{\text{SSS}} = (6.6 \pm 0.7) \times 10^{-14}, T_{\text{BB}} = 305,000 \pm 10,000 \) K, and \( N_H = (9.0 \pm 0.2) \times 10^{20} \) cm\(^{-2} \). These parameters imply the effective radius, \( R_{\text{eff}}^{\text{SSS}} = 0.17 \pm 0.02 (d/60 \text{ kpc}) \), and the luminosity, \( L_{\text{SSS}} = 9.1 \pm 3.0 \times 10^{36} (d/60 \text{ kpc})^2 \text{ erg s}^{-1} \), which translates into the unabsorbed X-ray luminosity \( L_X = (4.6 \pm 1.6) \times 10^{37} (d/60 \text{ kpc})^2 \text{ erg s}^{-1} \). Using the average fluxes of the three XMM-Newton spectra (see Table 2, Section 2.2, and Mereghetti et al. 2010) and the same UV-optical fluxes as above yields the following parameters of the SSS: \( N_H = (7.1 \pm 0.2) \times 10^{20} \) cm\(^{-2} \), \( T_{\text{BB}} = 267,000 \pm 7000 \) K, \( R_{\text{eff}}^{\text{SSS}} = 0.18 \pm 0.02 (d/60 \text{ kpc}) R_\odot \), and \( L_{\text{SSS}} = 5.7 \pm 1.8 \times 10^{36} (d/60 \text{ kpc})^2 \text{ erg s}^{-1} \).

The nebular component of radiation dominates the spectrum from around 2500 Å to longer wavelengths (Figure 1(b)). Its amount is scaled with \( k_N = 3.8 \pm 0.4 \times 10^{12} \text{ cm}^{-2} \), which corresponds to \( EM = (1.6 \pm 0.2) \times 10^{60} (d/60 \text{ kpc})^2 \text{ cm}^{-3} \). The nebula radiates at a high \( T_e = 37,000 \pm 5000 \) K. Both \( EM \) and \( T_e \) are in good agreement with those derived by Aller et al. (1987) from emission lines: \( EM \sim 1.8 \times 10^{60} (d/63.1 \text{ kpc})^2 \text{ cm}^{-3} \), \( T_e \sim 25,300–31,600 \) K (see their Tables 5 and 7, and Appendix A).

Finally, it is of interest to note that the long timescale of observations of LIN 333 (1984–2009, Table 2) with comparable X-ray and UV fluxes demonstrates a stability of this bright SSS (Mereghetti et al. 2010).

### 3.5. Multiwavelength Model SED of CAL 83

As in the case of RX J0513.9-6951, the high values of the near-UV–optical fluxes (a few \( \times 10^{-15} \) erg cm\(^{-2} \text{ s}^{-1} \text{ Å}^{-1} \)), their steepness (e.g., \( U–B \sim –1 \) mag), and the presence of emission lines of the hydrogen Balmer series, He II \( \lambda \lambda 4686, 11114640 \) Å, and [O III] \( 5007 \) Å in the optical spectrum (e.g., Pakull et al. 1985) indicate a strong nebular continuum that dominates the spectrum for \( \lambda > 1800 \) Å (see Figure 1(c)).

We modeled the SED of CAL 83 between 20.7 and 9000 Å by Equation (2). The best model with ROSAT data has a reduced \( \chi^2 = 3.8 \) for 61 degrees of freedom, dof and determines the variables of the SSS, \( \theta_{\text{SSS}} = (6.9 \pm 0.6) \times 10^{-14} \), \( T_{\text{BB}} = 345,000 \pm 15,000 \) K, and \( N_H = (1.03 \pm 0.05) \times 10^{21} \text{ cm}^{-2} \), which yields \( R_{\text{eff}}^{\text{SSS}} = 0.15 \pm 0.02 (d/49 \text{ kpc}) R_\odot \), and \( L_{\text{SSS}} = (1.1 \pm 0.5) \times 10^{39} (d/49 \text{ kpc})^2 \text{ erg s}^{-1} \). Model variables of the nebular component of radiation, \( k_N = (1.0 \pm 0.1) \times 10^{13} \text{ cm}^{-2} \), and \( T_e = 30,000 + 20,000/–10,000 \) K correspond to \( EM = (2.9 \pm 0.3) \times 10^{60} (d/49 \text{ kpc})^2 \text{ cm}^{-3} \) (Table 3).

The model SED with Chandra/XMM-Newton fluxes is very similar to that with the ROSAT data (see Figure 1(c), Table 3). The corresponding parameters of the SSS are \( N_H = (1.3 \pm 0.05) \times 10^{21} \text{ cm}^{-2} \), \( T_{\text{BB}} = 357,000 \pm 15,000 \) K, \( R_{\text{eff}}^{\text{SSS}} = 0.16 \pm 0.02 (d/49 \text{ kpc}) R_\odot \), and \( L_{\text{SSS}} = (1.4 \pm 1.1) \times 10^{39} (d/49 \text{ kpc})^2 \text{ erg s}^{-1} \). As in the case of LIN 333, the long timescale of observations demonstrates that CAL 83 is a stable SSS during its X-ray on-states.

Finally, the attenuation of the continuum around Ly\( \alpha \) observed on the HST/GHRS spectrum, corresponds to \( N_H = (8 \pm 2) \times 10^{20} \text{ cm}^{-2} \) (see Figure 4).

### 3.6. Multiwavelength Model SED of RX J0527.8-6954

The steepness of the SWP45499 spectrum suggests that the SSS radiation also dominates a wide range of the far-UV continuum (115–190 nm), while the LWP23826 spectrum has the opposite slope, whose profile suggests a dominant contribution from a low-temperature nebula (see Figure 1(d)). Available observations allow us to model the continuum fluxes between 26.5 and 5500 Å. The best fit to the X-ray and UV data (39 fluxes) has a reduced \( \chi^2 = 3.3 \) (for 34 dof) and determines the model variables, \( \theta_{\text{SSS}} = (7.3 \pm 0.7) \times 10^{-14}, T_{\text{BB}} = 255,000 \pm 20,000 \) K, and \( N_H = (7.1 \pm 0.2) \times 10^{20} \) cm\(^{-2} \), which yield \( R_{\text{eff}}^{\text{SSS}} = 0.16 \pm 0.02 (d/49 \text{ kpc}) R_\odot \), and \( L_{\text{SSS}} = (3.7 \pm 0.5) \times 10^{38} (d/49 \text{ kpc})^2 \text{ erg s}^{-1} \). The nebular component of radiation is determined by \( k_N = (1.5 \pm 0.1) \times 10^{13} \text{ cm}^{-2} \) and \( T_e = 9000 \pm 2000 \) K that gives \( EM = (4.3 \pm 0.3) \times 10^{60} (d/49 \text{ kpc})^2 \text{ cm}^{-3} \) (Table 3).

### 4. Discussion

#### 4.1. Amount of Absorption to the SSSs

Independently of the SED modeling, we estimated the value of \( N_H \) also from Rayleigh scattering of the SSS radiation on H atoms that creates a strong depression of the continuum around the Ly\( \alpha \) line (see Figures 3 and 4). The larger inaccuracy of this method is given by a weaker sensitivity of the Rayleigh scattering process to the number of scatterers in the Ly\( \alpha \) wings, and by the complexity of the depression profile due to the superposition of the rich absorption-line spectrum to the continuum. On the other hand, the determination of \( N_H \) from the SED modeling suffers from rather uncertain X-ray fluxes taken from previously published figures and their extreme sensitivity to the amount of absorption within the supersoft X-ray wavelengths. Another source of uncertainties in determining values of \( N_H \) is given by using nonsimultaneous X-ray and UV observations, the temporal variability of the SSS radiation, and/or the circumstellar component of \( N_H \) originating in the surrounding nebula (Section 4.6, Appendix A). Nevertheless, \( N_H \) values obtained by these two different methods are comparable (see Table 3, and Figures 3 and 4 for RX J0513.9-6951 and CAL 83). In the case of LIN 333 and CAL 83, we modeled different X-ray data together with the same UV-optical observations (Table 3, Figures 1(b) and 1(c)). Despite this shortcoming, the \( N_H \) values we found for
LIN 333 are in good agreement with the total column density $N_H \approx (7 \pm 4) \times 10^{20}$ cm$^{-2}$ determined by Wang (1991), which includes contributions in the galaxy, in the SMC, and inside the nebula. For RXJ0527.8-6954 we used X-ray fluxes from 1990 June and probably fainter far-UV fluxes from 1992 September as suggested by the X-ray and optical light curves (see Greiner et al. 1996; Alcock et al. 1997). If this is so, the true value of $N_H$ should be a little higher than that given in Table 3.

4.2. Comparison with Previous Models

Figure 5 compares our multiwavelength models and the X-ray-data models from the literature described in Sections 4.1–4.4, and summarized in Table 1. In both cases, the SED models match satisfactorily the absorbed X-ray fluxes. However, the unabsorbed models differ, often significantly, from our multiwavelength models. Mostly, they are a factor of $\sim$10–100 below the far-UV data. Such a discrepancy between the multiwavelength modeling and the X-ray-data-only modeling can be ascribed to the mutual dependence of the $N_H$, $L_{SSS}$, and $T_{BB}$ parameters, as described in Section 4.1 of Skopal (2015).

RX J0513.9-6951: The X-ray blackbody solution of Schaeidt et al. (1993) corresponds to $L_{SSS} = 2.3 \times 10^{38}$ erg s$^{-1}$, which is a factor of $\sim$15 below the far-UV fluxes. The model atmosphere fit by Gansicke et al. (1998) yields an even lower luminosity of $(2.5-9) \times 10^{37}$ erg s$^{-1}$ (see Section 1.1).

RX J0058.6-7135 (LIN 333): Blackbody and atmospheric X-ray SED models performed by Heise et al. (1994) are factors of $\sim$9 and $\sim$300 below the FUSE data, respectively. They were already discussed by Skopal (2015) as an illustrative example of the mutual dependence of the model parameters $N_H$, $L_{SSS}$, and $T_{BB}$ (see Section 4.1 and Figure 7 there). Similar results were obtained also by Meregatti et al. (2010) using the XMM-Newton observations (see Table 1). The authors questioned their atmospheric model when they found that the flux predicted in the visible band is lower by $\sim$30% than the upper limit ($V \leq 21$) obtained from the HST imaging of the central star by Villaver et al. (2004).

RX J0543.6-6822 (CAL 83): Blackbody solution at the Eddington limit ($N_H = 8 \times 10^{20}$ cm$^{-2}$, $k T_{BB} = 43$ eV, $R_{BB} = 1.5 \times 10^9$ cm; Greiner et al. 1991) is a factor of $\sim$20 below the far-UV continuum, while their softer model ($N_H = 1.7 \times 10^{21}$ cm$^{-2}$, $k T_{BB} = 26$ eV) fits the absorbed X-ray data for $\theta_{SSS} \sim 4 \times 10^{-13}$ (i.e., $R_{BB} = 6.0 \times 10^{10}$ cm, $L_{SSS} \sim 2 \times 10^{38}$ erg s$^{-1}$), which is a factor of $\sim$30 above the far-UV fluxes. Independent modeling of the BeppoSAX spectrum of CAL 83 by Parmar et al. (1998) also matches the observed X-ray data, but the unabsorbed model is a factor of $\sim$80 below the far-UV continuum.

RX J0527.8-6954: Similar results were also obtained for this SSS by Greiner et al. (1991).

Figure 6 compares our SED model for RX J0513.9-6951 and that of Popham & di Stefano (1996), who calculated models of irradiated flared accretion disks around luminous steady-burning WDs in bright LMC SSSs (CAL 83, CAL 87, and RXJ0513.9-6951) and compared the resulting disk fluxes to optical and UV observations. To increase the radiation from the disk at these wavelengths, the disk has to be flared to reproduce a sufficiently large amount (more than one-fourth) of the WD radiation into the optical and UV (see their Figure 2). Figure 6 shows their resulting model (gray line), which consists of three components of radiation: (i) from a 1.2 $M_\odot$ steady-burning WD ($M_{SSS} = 5 \times 10^{-7}$ $M_\odot$ yr$^{-1}$, $L_{WD} = 1.5 \times 10^{38}$ erg s$^{-1}$, $T_{BB} = 5 \times 10^4$ K, and $R_{WD} = 1.84 \times 10^5$ cm); (ii) from the irradiated disk, whose heated surface increases the optical and UV fluxes by factors of three to five, while the heated star–disk boundary makes the high-energy spectrum much brighter; and (iii) from the irradiated fraction of the donor star that contributes mainly to the optical spectrum. For comparison, the disk spectrum with no external heating corresponding to the above-mentioned parameters (i.e., the accretion luminosity of $2.7 \times 10^{36}$ erg s$^{-1}$) and the outer disk edge of $1.35 \times 10^{14}$ cm (Popham & di Stefano 1996) is shown by the dashed line.\(^2\) Note that a more sophisticated calculation performed by Popham & di Stefano (1996) provides a similar profile of the disk spectrum, but it is just somewhat shifted to shorter wavelengths (see their Figures 1 and 2). The Popham & di Stefano’s model is calculated for the disk inclination of 60°. If viewing the disk face-on, its flux increases twice and will match better the measured UV and optical fluxes. However, it is far below the NIR fluxes and does not reproduce their slope. The model does not match the X-ray fluxes at all.

Our multiwavelength SED models of the bright SSSs correspond to super-Eddington luminosity even for a high-mass WD accretor. The SED from the near-UV to longer wavelengths indicates a dominant contribution from the thermal nebula. Both results are fairly surprising with respect to the present picture of SSSs, whose luminosities can reach maximum in the Eddington limit for a WD accretor, and the UV–optical spectrum is thought to be dominated by the light from the accretion disk irradiated by the central steady-burning WD (Popham & di Stefano 1996). Below, we discuss these properties in detail.

4.3. Problem of the High Luminosity

4.3.1. Accreting White Dwarf?

According to van den Heuvel et al. (1992), the radiation of SSSs is generated by a stable nuclear burning of hydrogen accreted onto massive WDs. This idea is based on a theoretical conclusion that there is a small range of accretion rates, $M_{acc}$, at which the hydrogen-rich material is quasi-steady burning on the WD’s surface as it is accreting (Paczynski & Zykow 1978; Fujimoto 1982). The material is transported onto the accretor through an accretion disk. Then, the maximum luminosity of an accretion disk is given by

$$L_{max} = \frac{4 \pi R_s^2 ho \sigma T_{eff}^4}{(1 - \kappa R_s)^2} \approx 8 \times 10^{37} \frac{R_s}{10^8}\text{erg s}^{-1}$$

where $R_s$ is the radius of the accretor, $\rho$ is the density of the accreted material, $\sigma$ is the Stefan–Boltzmann constant, and $T_{eff}$ is the effective temperature of the accreted material. However, this model is not applicable to the SSSs, as their luminosities are much higher than the maximum luminosity of an accretion disk.

\(^2\) For the sake of simplicity, we adopted an optically thick disk that radiates locally, like a blackbody (Warner 1995).
SSS is given by that generated by the nuclear burning on the WD surface and the gravitational potential energy of the accretor, converted into the radiation of the disk and its boundary layer, i.e.,

\[ L_{\text{SSS}} = \eta X M_{\text{acc}} + \frac{GM_{\text{WD}} M_{\text{acc}}}{R_{\text{WD}}} \]  

where \( \eta \sim 0.007 \times c^2 \sim 6.3 \times 10^{18} \) erg g\(^{-1}\) is the energy production of 1 g of hydrogen due to the nuclear fusion of four protons, and \( X \equiv 0.7 \) is the hydrogen mass fraction in the accreted matter. \( M_{\text{WD}} \) and \( R_{\text{WD}} \) are the WD mass and radius.

However, even the extreme parameters of the WD accretor, \( M_{\text{WD}} \sim 1.4 M_\odot \), \( R_{\text{WD}} \sim 0.003 R_\odot \) (Panei et al. 2000), and the mass transferred through the disk at a high rate of a few times \( 10^{-7} M_\odot \) yr\(^{-1} \) that sustains the stable H-burning (see, e.g., Figure 2 of Shen & Bildsten 2007) generate a total luminosity \( \gtrsim 10^{38} \) erg s\(^{-1}\), which is one order of magnitude below that suggested by our multiwavelength model’s SED (Table 3). The observed luminosity would require a high accretion rate of \( M_{\text{acc}} \sim 0.1 M_{\text{Edd}} \), where \( M_{\text{Edd}} \) is the Eddington accretion rate. This result is not consistent with the model of a steady-burning WD for the brightest SSSs in the LMC and SMC unless the radiation of the source is not isotropic. Here, the emission from steady nuclear burning onto a WD should be beamed with a factor, \( L/L_{\text{sph}} \sim 0.1 \), where \( L_{\text{sph}} \) is the inferred isotropic luminosity and \( L \) is the true source luminosity.

### 4.3.2. Accreting Neutron Star?

A neutron star (NS) as the accretor in the brightest SSSs provides a somewhat better possibility to explain their high luminosities, because of a larger efficiency in converting gravitational energy into heat and radiation. The efficiency of radiative emission in units of the total mass energy accreted on a compact object is

\[ \xi = \frac{L_{\text{SSS}}}{M_{\text{acc}} c^2} = \frac{G M_{\text{acc}}}{R_{\text{acc}} c^2}. \]

For a 1.4 \( M_\odot \) NS with a radius of 10–15 km, \( \xi \sim 0.2–0.14 \), while for a WD (1 \( M_\odot \), 7000 km), \( \xi \sim 2 \times 10^{-4} \). Considering that the efficiency of turning mass energy into radiation via
nuclear fusion is of the order of $\sim 7 \times 10^{-3}$ (Equation (5)), the nuclear fusion reactions can be a vital source of energy for accreting WDs, but are negligible in converting the mass energy accreted by an NS.

To generate the luminosity of $\sim 10^{39}$ erg s$^{-1}$, the above-mentioned parameters of an NS require the accretion rate $M_{\text{acc}} = 0.85 - 1.3 \times 10^{-7} M_\odot$ yr$^{-1}$. These values are more realistic than those required by the WD accretor. Such material can be supplied by mass transfer from a close Roche-lobe filling companion. However, these accretion rates are still above the critical values even for a high-mass NS.

The presence of an NS as the accretor in the luminous SSSs could be evidenced by, e.g., detection of strictly periodic short-term light variation on a timescale of milliseconds to seconds, or by direct measuring of the size of the X-ray emission region from eclipse analysis (Mukai 2017). However, no such observations indicating these properties of SSSs have been reported yet. In the cases where an NS probably accretes from the wind of its companion, we measure a significantly harder SSS photosphere for the super-Eddington luminosity was also documented for some extraordinary novae, e.g., FH Ser (Friedjung 1987), LMC 1988 #1 (Schwarz et al. 1998), LMC 1991 (Schwarz et al. 2001), SMC 2016-10a (Aydi et al. 2018), and nova V339 Del (Skopal 2019). The unusually long phase of hydrogen burning was indicated for the nova V723 Cas (1995) by detecting it as a bright SSS more than 12 yr after the outburst, with possible super-Eddington luminosity (depending on the spectral model; see Ness et al. 2008). The transient SSS in the nearby galaxy NGC 300, denoted as SSS1, has a bolometric luminosity of $\approx 10^{39}$ erg s$^{-1}$ (Kong & Di Stefano 2003). The source was found in outburst in 1992, 2000, 2008, and 2016, suggesting a possible recurrence period of about 8 yr, and thus could be associated with a recurrent nova (Carpano et al. 2019). Recently, Vasilopoulos et al. (2020) discovered a 30 yr long-lived post-nova SSS in the LMC undergoing residual surface nuclear burning. A large number of post-nova SSSs (250–600) was identified in M31 (Soraisam et al. 2016). The authors found that depending on the WD mass distribution in novae, their unabsorbed X-ray luminosity distribution shows significant steepening around $L_X(0.2-1.0$ keV) $\approx 10^{38}$ erg s$^{-1}$ with a maximum at $\approx 2 \times 10^{39}$ erg s$^{-1}$. According to our multi-wavelength SED modeling, $L_{\text{bol}}/L_X(0.2-1.0$ keV) $\approx 10$ (see Table 3), which would correspond to the bolometric luminosities of the brightest SSSs in M31 of $\approx 10^{39}$ erg s$^{-1}$. Note, however, that this ratio is a strong function of the temperature: $L_{\text{bol}}/L_X(0.2-1.0$ keV) $\approx 21$ or $\approx 2.4$ only for 300,000 or 600,000 K, respectively. Finally, long-term accretion at/above the nuclear Eddington limit is also indicated for symbiotic X-ray binaries RXJ0059.1-7505 (LIN 358) in the SMC.
...and the ionized hydrogen as the continuum. For example, for the measured flux in the H\(\beta\) line, \(F_{H\beta}\) we can express its luminosity as,

\[
L_{H\beta} = 4\pi d^2 F_{H\beta} = h\nu_{\beta}\alpha(H\beta, T_e) \int_{R_{SSS}}^{\infty} n_e(r) n_p(r) dV,
\]

where \(\alpha(H\beta, T_e)\) is the effective recombination coefficient for the H\(\beta\) transition, and \(n_e\) and \(n_p\) are the concentrations of electrons and protons, respectively, in the spherical H\(^+\) zone with an inner radius \(R_{SSS}\) from which the wind can become optically thin in the H\(\beta\) line. The integral in Equation (8) represents the emission measure of the H\(^+\) region (see Section 3.1). Using its expression from Equation (7), we obtain,

\[
\dot{M}_{SSS} = \left[4\pi (\mu m_H v_\infty)^2 R_{SSS}^2 \frac{L_{H\beta}}{h\nu_{\beta}\alpha(H\beta, T_e)}\right]^{1/2} g \text{ s}^{-1},
\]

which corresponds to a lower limit of \(M_{SSS}\), because in a line transition, the wind becomes optically thin above the photosphere, at distances \(\gg R_{SSS}^eff\). This problem was treated by Leitherer (1988), who found that stellar winds of luminous hot stars become optically thin in the H\(\alpha\) line from a distance of \(\sim 1.5\) times the star’s radius. Accordingly, \(F_{H\alpha} = 1.45 \times 10^{-13} \text{erg cm}^{-2} \text{s}^{-1}\) measured in the spectrum of LIN 333 by Aller et al. (1987) corresponds to \(M_{SSS} \sim 1.7 \times 10^{-6} M_\odot \text{yr}^{-1}\) for \(\alpha(H\alpha, 30,000) = 1.088 \times 10^{-14} \text{cm}^3 \text{s}^{-1}\) (Hummer & Storey 1987), \(R_{SSS} = 0.17 R_\odot\), and \(v_\infty = 1000 \text{km s}^{-1}\). These observational values of \(\dot{M}_{SSS}\) are also consistent with those suggested by the optically thick wind theory, for winds from the nuclear-burning WDs of a high mass (see Equation (23) of Kato & Hachisu 1994). It is of interest to note that wind-mass-loss rate of the order of \(10^{-6} M_\odot \text{yr}^{-1}\) was measured during the long-lasting SSS phase of the classical nova V339 Del, whose bolometric luminosity was kept at a high level of \(1-2 \times 10^{39} \text{erg s}^{-1}\) for \(\sim 150\) days (see Figure 12 of Skopal 2019).

### 4.4. Mass-loss Rate From the Nebular Emission

The high luminosities of SSSs, whose WDs probably accreted at a high rate, lead to a mass outflow in the form of wind (e.g., Kato & Hachisu 1994; Hachisu & Kato 2001, and Appendix A). According to Kato & Hachisu (1994), the optically thick/thin interface of the wind represents the SSS pseudo-photosphere. In our cases, its average value is \(R_{SSS}^eff = 0.17 R_\odot\) (Table 3). The optically thin wind above the hot and luminous pseudo-photosphere is ionized, giving rise to nebular radiation (see Appendix A). Assuming (i) that the wind flows out spherically symmetrically at a constant velocity, \(v_\infty\), (ii) it begins at the \(R_{SSS}^eff\), and (iii) its radial density distribution satisfies the mass continuity equation, then the relationship between the mass-loss rate of the ionized wind, \(\dot{M}_{SSS}\), and its emission measure, \(EM\), can be expressed as

\[
\dot{M}_{SSS} = [4\pi (\mu m_H v_\infty)^2 R_{SSS}^2 EM]^{1/2} \text{g s}^{-1},
\]

(see Equation (7) of Skopal et al. 2014), where \(\mu\) is the mean molecular weight and \(m_H\) is the mass of the hydrogen atom. This relation assumes the outer radius of the ionized wind \(\gg R_{SSS}^eff\). The average values of \(R_{SSS}^eff = 0.17 R_\odot\) and \(EM \geq 10^{60} \text{cm}^{-3}\) (Table 3) yield \(\dot{M}_{SSS} \geq 2 \times 10^{-6} M_\odot \text{yr}^{-1}\) for \(v_\infty = 1000 \text{km s}^{-1}\).

Using the same assumptions as for the nebular continuum, similar values of \(\dot{M}_{SSS}\) can also be obtained from hydrogen emission lines, because they are formed in the same volume of the ionized hydrogen as the continuum. For example, for the measured flux in the H\(\beta\) line, \(F_{H\beta}\) we can express its luminosity as,
measure (∼n²V) of a few times 10²⁰ cm⁻³ (Table 3) corresponds to the mean particle density of a few times 10⁹ cm⁻³. Such dense nebulae are characterized by a specific type of the emission-line spectrum, whose fluxes and their theoretical ratios significantly differ from those produced by the low-density planetary nebulae and/or ISM nebulae predicted to be photoionized by the central SSS (see suggestions for future work in Section 5). Here, for a comparison with a dense circumstellar nebula in the symbiotic star AX Per, Skopal et al. (2001) found the electron concentration in the H II and [O III] zone surrounding its burning WD to be of a few times 10⁹ and ∼3 × 10¹⁰ cm⁻³, respectively. Further, they suggested that the extremely steep Balmer decrement (e.g., Hα/Hβ ∼ 7–10) could be due to electron collisions in a very dense H II region.

Finally, we note that the dense unresolved circumstellar nebulae can be indicated for a given object only by the method of disentangling its composite spectrum (Section 3.1).

4.6. Origin of the X-Ray/Optical Flux Anticorrelation

The presence of strong nebular radiation in the UV to NIR spectrum of our targets (EM ≥ 2 × 10⁶⁰ cm⁻³; Table 3) and its variation (Figure 2, Section 3.3) allows us to explain the anticorrelation between their supersoft X-ray and optical fluxes by a variable mass outflow from the accreting burning WD, as in the case of the symbiotic binary AG Dra (see González-Riestra et al. 2008; Skopal et al. 2009a). It is assumed that the variable mass outflow results from a variable mass transfer. Charles et al. (2010) suggested a similar view for the SSS RX J0513.9-6951, and also estimated a mass-outflow rate of ∼10⁻⁷ M☉ yr⁻¹ for the luminosity at the Eddington limit.

An increase of the mass loss increases the particle density above the hot WD’s pseudo-photosphere, and thus the number of b-f absorptions. This leads to a decrease of the supersoft X-ray photons, and consequently to an increase of f-b and f-f transitions that causes an increase of the nebular emission, which dominates the spectrum from the near-UV to longer wavelengths.

In this way, the enhanced (wind) mass outflow increases the optical depth for the supersoft X-ray photons, and thus increases its optically thick/thin interface, which is the WD’s pseudo-photosphere. That is why we indicate a larger effective radius of the SSS during the X-ray-off/optical-high states. For RX J0513.9-6951, the R_eff SSS radius inflated from ∼0.19 R_e during the optical-low state to ∼0.26 R_e during the optical-high state (Section 3.3, Table 3). Independently, the connection between the flux anticorrelation and the WD’s radius was demonstrated for RX J0513.9-6951 by Charles et al. (2010) on the basis of the XMM-Newton simultaneous soft X-ray, UV, and optical observations (see their Figures 2 and 3). Finally, the inflated WD can also increase its radiation for λ > 912 Å (compare the blue lines in the panels (a) and (b) of Figure 2).

In this way, the ionization/recombination process in the variable wind from the SSS causes the simultaneous presence of the X-ray-on/off and optical-low/high states. Corresponding changes in the nebular emission can explain ∼1 mag changes in the optical brightness (Section 3.3).

5. Conclusions and Future Work

In this paper, we performed the multiwavelength modeling of the supersoft X-ray–NIR SED for the brightest Magellanic Cloud SSSs, RX J0513.9-6951, RX J0543.6-6822, RX J0058.6-7135, and RX J0527.8-6954. The global SED models satisfactorily fit the measured multiband fluxes. The main results of the modeling can be summarized as follows.

1. The SED models revealed that their fundamental parameters, L_{SSS}, T_{eff}, and R_{eff SSS} are ≥ 10³⁸–10³⁹ erg s⁻¹, ∼ 3 × 10²⁵ K, and ∼0.17 R_e, respectively. Their supersoft radiation is attenuated with the total N_{H} ∼ (7–12) × 10²⁰ cm⁻². The modeling identified the nebular component of radiation with EM ≥ 2 × 10⁶⁰ cm⁻³ (Table 3). Luminosities of our targets differ significantly from those inferred previously from modeling the X-ray data only (Section 4.2, Figure 5).

2. In spite of super-Eddington luminosities for a 1.4 M☉ compact object, the accretor has to be a WD, because the X-ray spectrum profile and a low speed of jets are not consistent with those of an accreting NS or BH (see sections 4.3.2 and 4.3.3).

3. The high luminosities lead to mass loss at high rates. Assuming that the nebular component of the radiation is produced by the ionized wind from SSSs, its emission measure corresponds to a mass-loss rate ≥ 2 × 10⁻⁶ M☉ yr⁻¹ (Section 4.4). The ionized wind represents a dense unresolved circumstellar nebula in SSSs (Section 4.5, Appendix A).

4. Accordingly, we suggest that the brightest SSSs in the LMC and SMC could be unidentified optical novae in a post-nova SSS state, whose lifetime and luminosity are enhanced by the resumed accretion, possibly at super-Eddington rates (Section 4.3.4).

5. The observed anticorrelation between the supersoft X-ray and optical fluxes can be caused by a variable mass outflow from the SSS due to variations in the mass transfer. A higher mass loss absorbs more X-ray photons, which are re-emitted in the form of the nebular emission, and vice versa. As a result, we observe simultaneously the X-ray-off/on and optical-high/low states (Section 4.6).

For future investigation, we suggest the use of multiwavelength modeling, including simultaneously obtained fluxes from both the ascending and descending parts of the SSS spectrum. In this way, parameters more reliable than those obtained by modeling only one part of the spectrum can be determined. To test the proposed model, and develop a more perfect version, we suggest that new observations be obtained and better theoretical modeling used. In particular, we recommend the following.

1. A flux-calibrated low-resolution optical/NIR spectrum (λ ≥ 350 nm) should verify the presence of the nebular continuum in the spectrum by its specific profile and the presence of the Balmer jump in emission. Moreover, for LIN 333, such a spectrum should confirm or refute its binary nature proposed in Appendix C.

2. High-resolution spectra should directly indicate a high-velocity mass outflow by (presumably) broad wings of the strongest lines, such as Hα, Hβ, and He II λ4686. In addition, for RX J0513.9-6951, the presence of satellite components to these lines would indicate a collimated bipolar outflow resulting from a very high accretion rate onto the WD (see Southwell et al. 1996).

3. In accordance with the study of Nielsen & Gilfanov (2015), we recommend using the photoionization code CLOUDY to determine the critical value of the mass-loss
rate, $M_{\text{crit}}$, from the burning WD, which causes the obscuration of the central X-ray source for parameters determined by our modeling. The corresponding EM should be comparable with that obtained from the SED modeling during the X-ray-off states. In this way, one can test the origin of the X-ray/optical flux anticorrelation by a variable wind from the SSS proposed in Section 4.6.

Furthermore, the calculated emission-line spectrum produced by the ionized wind from the X-ray source should provide additional constraints on the SSS environment. Analogously with symbiotic stars, a dense (unresolved) nebula, which we also suggested for SSSs by our SED modeling (Section 4.5; Appendix A), should be characterized by an anomalous Balmer decrement (e.g., $H\alpha/H\beta \gg 3$). It should also be characterized by a very low rate of nebular [O III] lines to [O III] $\lambda4363$ — in strong contrast to values produced by the low-density planetary nebulae (e.g., Gurzadyan 1997), and/or nebulae predicted to exist around SSSs imbedded in the low-density ISM (Nielsen & Gilfanov 2015, Section 4.5).

4. In modeling the global SED, we recommend one should also consider the component of radiation from the irradiated accretion disk proposed by Popham & di Stefano (1996). Its possible contribution in the far-UV will reduce the radiation from the burning WD here and, thus, its bolometric luminosity, and the amount of required absorption.

These tasks present new challenges for further observations and theoretical modeling, which should help us in understanding the nature and evolutionary status of the most luminous SSSs.

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Appendix A

Justification of Nebular Continuum in the Spectrum of the Supersoft X-Ray Sources

The nebular component of radiation has not yet been considered in modeling the SED of SSSs. Specifically, we are referring to the spatially unresolved source of nebular emission, the presence of which in the spectrum is indicated by multiwavelength modeling of the SED (Section 3.1 and Figure 1), as in the case of dense nebulae in symbiotic binaries and/or classical novae (see Skopal 2005, 2015, 2019, and Section 4.5). Its presence in the spectrum of SSSs can also be justified as follows.

1. It is probable that extremely luminous and hot SSSs will generate an outflow of material driven by radiation pressure, as is observed and theoretically justified for luminous hot stars (e.g., Castor et al. 1975; Klein & Castor 1978), accreting WDs in symbiotic binaries (e.g., Vogel & Nussbaumer 1994; Skopal 2006; Skopal et al. 2017), classical novae (e.g., Friedjung 1966; Bath & Shaviv 1976; Kato & Hachisu 1994; Skopal 2019), and in SSSs (e.g., Pakull et al. 1985, 1993; Oliveira et al. 2010; Charles et al. 2010). Simultaneously, the hot and luminous central WD and the boundary layer of accretion disk generate a large flux of hydrogen ionizing photons ($\sim 10^{38}$ s$^{-1}$ for $T_{\text{BB}} \sim 300,000$ K and $L_{\text{SSS}} \sim 10^{38}$ erg s$^{-1}$), which ionizes the outflowing material, giving rise to nebular emission. This reprocessed radiation dominates the Rayleigh–Jeans tail of a hot source radiating at $T_{\text{BB}} \gtrsim 10^5$ K, usually from the near-UV to longer wavelengths (see Skopal 2005; Nielsen & Gilfanov 2015). This nebular component of radiation is aimed at explaining the strong UV–NIR excess observed for bright SSSs.

2. Below we summarize indications of the nebular emission observed in the spectrum of our targets.

(a) Mass-loss from the accreting nuclear-burning WD is indicated by P-Cyg profiles of H lines, their often strong flux and broad emission wings, and/or satellite components to main H $\beta$ and He II $\lambda4686$ emission cores as for RX J0513.9-6951 (see references above and those in Sections 1.1–1.4). Therefore, the H $\alpha$ line flux, as a tracer of mass loss from hot stars, has been usually used to estimate its rate (e.g., Klein & Castor 1978; Leitherer 1988; Lamers & Cassinielli 1999; Skopal et al. 2002; Skopal 2006). The outflowing material ionized by the central hot source then gives rise to nebular emission.
Figure 7. Broad wings of the resonance OVI λ1032, λ1038 doublet observed in the FUSE spectra of LMC SSS RX J0513.9-6951 (top panel) and of the X-ray symbiotic binary AG Dra with the model of the scattering of line photons on free electrons (bottom panel; adopted from Sekeráš & Skopal 2012). The similarity of these profiles also suggests the presence of a dense circumstellar nebula in RX J0513.9-6951, showing mass outflow (see the text). Note that the spectra of RX J0513.9-6951 and AG Dra are shifted in wavelength due to their space velocity by ~+500 and ~−150 km s^{-1}, respectively (Fekel et al. 2000; Hutchings et al. 2005).

(b) During the optical-high state of RX J0513.9-6951, the resonance OVI λ1032, λ1038 doublet shows broad wings due to electron scattering, with the superposed central P Cyg-type of the profile (see Hutchings et al. 2005). By comparison with accreting nuclear-burning WDs in symbiotic stars, such wings are created in the layer of electrons, throughout which the line photons are transferred, with an electron optical depth τ_e ∼ 0.05–0.6, depending on the system and its activity (see Sekeráš & Skopal 2012, and Figure 7 for illustration). As there is no resolved ionization nebula surrounding RX J0513.9-6951, the layer of free electrons has to be spread in the vicinity of the ionizing source, which indicates the presence of a compact (unresolved) circumstellar nebula in RX J0513.9-6951, independently to our SED model. For comparison, in the case of the X-ray symbiotic binary AG Dra, the layer of scattering electrons extends to ∼10–100 R_*, which corresponds to mean electron concentrations of 10^{10}–10^{12} cm^{-3} to satisfy the measured τ_e (see Skopal et al. 2009a).

(c) The presence of emission lines directly indicates the presence of the corresponding nebular continuum, because both the continuum and lines originate in the same ionized volume. An example is the case of RX J0058.6-7135 (LIN 333), for which Aller et al. (1987) revealed the presence of a strong nebular emission radiating at a high T_e ∼ 30,000 K. In particular, the measured (extinction-free) flux in the Hβ line, F_{H\beta} = 1.45 \times 10^{-13} \text{ erg cm}^{-2} \text{s}^{-1}, corresponds to the emission measure,

$$EM = 4\pi d^2 \frac{F_{H\beta}}{h\nu_{H\beta} \alpha(H\beta)} = 1.4 \times 10^{60}(d/60 \text{ kpc})^2 \text{ cm}^{-3}$$

(A1)

which agrees well with our value determined from the nebular continuum (see Table 3). In Equation (A1), we used an effective recombination coefficient for the Hβ transition and T_e = 30,000 K, α(Hβ) = 1.088 \times 10^{-14} \text{ cm}^3 \text{s}^{-1} (Hummer & Storey 1987).

(d) Often, a measured flux ratio F(HeII λ4686)/F(H β) > 1 suggests the presence of a nebular continuum from doubly ionized helium, which we can indicate as a discontinuity at ∼2050 Å (see Figure 1(c), Figure 2(a), and Figure 8). Despite that the He^{++} region around strong SSSs should be especially prominent as compared to other astrophysical nebulae (Rappaport et al. 1994), its presence within the resolved ionization nebula around CAL 83 had not been detected for a long time (Remillard et al. 1995). The first evidence of an HeII λ4686 region around CAL 83 was presented by Gruyters et al. (2012) as a faint (relative to its bright stellar component) asymmetrically distributed zone confined to one side of CAL 83. In addition, the authors found that the H II emission did not fit in with model predictions. This unusual HeII λ4686 nebular emission could be explained by the presence of a compact (unresolved) circumstellar nebula around the burning WD of CAL 83, whose He^{++} zone is almost ionization-bounded, i.e., almost all photons with λ < 228 Å are absorbed within this zone. For comparison, in the case of the eclipsing symbiotic star AX Per, the He^{++} region extends up to ∼50 R_⊙ from the WD within its dense wind escaping at a rate of ∼2 \times 10^{-6} M_⊙ yr^{-1} and generating a total emission measure of a few times

Footnote 3: Note that values of parameters derived from the emission lines are usually smaller than those derived from the continuum because nebulae are usually more opaque in the lines than in the continuum.
10^{59} \text{ cm}^{-3}$ (from the eclipse profile; see Skopal et al. 2011).

(ii) Similarly to the above point, the circumstellar nebulae in the brightest SSSs as suggested by our SED models could also attenuate the flux of H-ionizing photons to such an extent that it would not allow us to detect the corresponding extended ISM nebulae in the optical. From this point of view, the strange absence of ionization ISM nebulae surrounding the brightest SSSs in LMC and SMC, up to CAL 83 (Remillard et al. 1995; Woods & Gilfanov 2016; Farias et al. 2020) could be explained by the presence of compact (unresolved) circumstellar nebulae in these objects.

Appendix B

CAL 83: The SED of the Central Source and the Surrounding Nebula

Figure 8 shows the UV–optical part of the CAL 83 SED compared with that of the surrounding inner $7.5 \times 7.5 \text{ pc}^2$ ionization nebula. The fluxes of the latter (red crosses in the figure) were taken from Figure 2 of Gruyters et al. (2012). The spectrum of the outer nebula can be matched with the same type of nebular continuum (dotted green line) as the circumstellar nebula (solid green line), but scaled with a factor of $\sim 17$ times larger. This suggests a similar electron temperature ($\sim 30,000 \text{ K}$), but $\sim 17$ times larger emission measure (i.e., $\sim 4.9 \times 10^{64} \text{ cm}^{-3}$) in comparison with the dense circumstellar nebula (see Table 3, Section 4.5).

As noted above, there is a lack of doubly ionized helium in the surrounding resolved nebula (Appendix A). Therefore, we also compared its fluxes with the net hydrogen nebular continuum that corresponds to $EM \sim 8.0 \times 10^{61} \text{ cm}^{-3}$ and the same $T_e$ (gray line in Figure 8). Such a high $EM$ requires a high flux of the ionizing photons. Assuming that all H-ionizing photons from CAL 83 balance the rate of recombinations in both nebulae, we can estimate the corresponding $L_{\text{SSS}}$ for the temperature of the ionizing source $T_{\text{BB}}$ according to expression,

$$L_{\text{SSS}} = \alpha_B(H, T_e)EM \frac{\sigma_{\text{BB}}T^4_{\text{BB}}}{f(T_{\text{BB}})} \tag{B1}$$

(see Skopal et al. 2017), where the function $f(T_{\text{BB}})$ determines the flux of ionizing photons emitted by the $1 \text{ cm}^2$ area of the ionizing source, and $\alpha_B(H, T_e)$ is the recombination coefficient to all but the ground state of hydrogen (i.e., Case B). Then the total $EM \sim 8.29 \times 10^{61} \text{ cm}^{-3}$ (i.e., of both nebulae), $T_{\text{BB}} = 345,000 \text{ K}$, $f(T_{\text{BB}}) = 6.0 \times 10^{-27} \text{ cm}^2 \text{ s}^{-1}$, and $\alpha_B(H, 30,000) = 9.97 \times 10^{-14} \text{ cm}^3$ (Nussbaumer & Vogel 1987) correspond to $L_{\text{SSS}} \sim 1.1 \times 10^{50} \text{ erg s}^{-1}$. This value is close to that given by the SED models, but an order of magnitude higher than those derived from the emission lines in previous works (see Section 1.3). Consequently, the very high $EM$ ($8.0 \times 10^{61} \text{ cm}^{-3}$) of the $7.5 \times 7.5 \text{ pc}^2$ nebula surrounding CAL 83 corresponds to an average particle density $\sim 39 \text{ cm}^{-3}$ and a mass of $\sim 1700 M_\odot$, which are factors of $\geq 4$ and $\sim 11$, respectively, larger than those previously derived from emission lines (e.g., Remillard et al. 1995; Gruyters et al. 2012). Such a large difference in the surrounding nebula properties can be explained by a large difference in the opacity of the emitting medium in the line and the continuum transitions, respectively.

Appendix C

LIN 333: Planetary Nebula or a Binary System?

Here we present two reasons suggesting that LIN 333 could be a binary system.

(i) First, we compare the nebular spectrum, both in the continuum and emission lines, of LIN 333 with that observed in symbiotic binaries containing an accreting nuclear-burning WD. Panels (a) and (b) of Figure 9 show a comparison with SY Mus in the 1200–2200 Å part of the spectrum. Both LIN 333 and SY Mus contain the same emission lines in the far-UV spectrum, except for the low-excitation O I $\lambda\lambda 1302–1306$ lines$^4$, which are characteristic for symbiotic stars. Strong resonance emission lines, N V $\lambda\lambda 1238,1242$, C IV $\lambda\lambda 1548,1550$, high-ionization line He II $\lambda 1640$, and the intersystem lines of O IV $\lambda\lambda 1397–1405$, N IV $\lambda\lambda 1483–1486$, O III $\lambda\lambda 1661,1666$, N II $\lambda\lambda 1746–1753$, Si III $\lambda\lambda 1892$, and C II $\lambda\lambda 1907,1909$ reflect the circumstellar nebula conditions of symbiotic stars (Meier et al. 1994), and thus also of SSS LIN 333.

Additionally, the nebular continuum in the LIN 333 spectrum is unambiguously indicated by our method as for other targets (green lines in Figure 1). As there is no resolved ionization nebula around LIN 333, the indicated nebular continuum suggests the presence of a compact unresolved nebula in the system, the origin of which is similar to that of dense symbiotic nebulae, i.e., resulting from the interaction between the binary components, given by the ionized mass outflows (e.g., Boyarchuk et al. 1966; Seaquist et al. 1984; Nussbaumer & Vogel 1987; Skopal 2005). Here, it is of interest to note that the first studies of symbiotic stars pointed to the similarity of their hot components to the central star of planetary nebulae (e.g., Berman 1932), and this possibility was often considered until the first satellite observations, which unambiguously revealed the binary nature of symbiotic stars (e.g., Kenyon & Webbink 1984, and references therein).

Table 4 compares the luminosities of the strongest emission lines in the UV spectrum, $EM$, and $T_e$ of the compact nebulae of LIN 333 and SY Mus.

(ii) According to the $UBV$ photometry and the HST spectroscopy of LIN 333 (Section 2.2), the measured fluxes in the visual band lie above our two-component model (see Figure 1(b)), which suggests a contribution from the companion in a binary system. To match the optical continuum of the HST spectrum$^5$ with a minimum effect on the near-UV region, where the original two-component model is satisfactory (see Figure 1(b)), we selected a synthetic spectrum of a yellow giant from a grid of models made by Hauschildt et al. (1999), calculated for $T_{\text{eff}} = 5000 \text{ K}$, $\log g = 3.5$, and [M/H] = −1. Its scaling in Figure 9(c) corresponds to a luminosity of 278 $L_\odot$, and a radius of $22 R_\odot$—parameters similar to a G8 II giant (e.g., van Belle et al. 1999). If this were the case, LIN 333 would belong to the so-called yellow symbiotic stars. However, new optical/NIR observations are highly desirable to confirm or refute the binary nature of LIN 333.

$^4$ The absence of O I and very faint resonance of the O VI $\lambda\lambda 1032,1038$ doublet probably reflect a low abundance of oxygen in the LIN 333 nebula suggested by Leisy & Dennefeld (1996).

$^5$ We ignored the $U$, $B$, and $V$ fluxes, because they are not corrected for emission lines and, thus, lie well above the true continuum.
The distance-dependent parameters are scaled to 1.0 kpc.

The similarity of the nebular spectra of both systems suggests the same origin—in a compact circumstellar nebula (see the text). Right: the UV–NIR SEDs of these objects. The case of LIN 333 allows for the presence of a yellow giant companion (orange dashed line) to the SSS, suggesting a binary nature of LIN 333 (see Appendix C). Panels (a) and (c) represent details of Figure 1(b), while panels (b) and (d) were adapted according to Figure 18 of Skopal (2005).

**Table 4**

Luminosities of the Strongest Permitted lines in the UV Spectrum (in erg s$^{-1}$), Emission Measure EM (cm$^{-3}$), and Electron Temperature $T_e$ (K) of LIN 333 and SY Mus

| Object   | $L_{\text{N} \lambda 1240}$ | $L_{\text{C} \lambda 1550}$ | $L_{\text{He} \lambda 1660}$ | EM        | $T_e$ (K) |
|----------|-----------------------------|-----------------------------|-----------------------------|-----------|----------|
| LIN 333  | $2.7 \times 10^{33}$        | $5.4 \times 10^{44}$        | $2.9 \times 10^{45}$        | $1.6 \times 10^{10}$ | $\sim 37,000$ |
| SY Mus   | $2.1 \times 10^{34}$        | $2.2 \times 10^{44}$        | $1.1 \times 10^{45}$        | $3 \times 10^{10}$  | $\sim 18,500$ |

**Note.** The distance-dependent parameters are scaled to 1.0 kpc (Skopal 2005) and 60 kpc for SY Mus and LIN 333, respectively. The corresponding spectra and models are shown in Figures 9(a) and (b).

**ORCID iDs**

Augustin Skopal https://orcid.org/0000-0002-8312-3326

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