Opinion polarization in the Receipt-Accept-Sample model

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Abstract

The Zaller theory of opinion formation is reformulated with one free parameter $\mu$, which measures the largest possible ideological distance which can be made by a citizen in one mental step. Our numerical results show the transient effects: i) the political awareness, measured by the number of received messages, increases with time first exponentially, later linearly; ii) for small $\mu$ correlations are present between previously and newly received messages; iii) these correlation lead to a hyperdiffusion effect in the space of attitudes of messages. Citizens with small $\mu$ are more prone to extremal opinions.

1 Introduction

It is difficult to overestimate the importance of the public opinion (PO) in our life [1]. In all political systems, PO transfers the traditional receipts of solving problems to subsequent generations. Its overwhelming influence is expressed by the democratic laws, which transfer the ultimate decisions to the voters. In modern societies, the role of PO is amplified by mass media - "the fourth power", which is sometimes more authoritative that each of the remaining three. (Recently however, the list of agents at the market of information is enlarged due to the Internet, and the monopoly of the mass media is somewhat weakened [2].) The very importance of PO was not overlooked by the sociophysicists, and several models have been created to capture its dynamics - for recent reviews see [3, 4]. On the other hand, the subject has been modeled mathematically also by social scientists, and their models are of particular interest for the sociophysicists, as those models can be expected to have a firm sociological basis.

The subject of this text is the model of the opinion dynamics, formulated by the political scientist John Zaller and published for the first time in 1992 [5]. The book is cited about 1000 times [6], but only one citation [7] can be found in the arXiv database [8], where sociophysicists met. The book is devoted to the explanation and the verification of the model, which is clearly exposed on its first page [5]:

"The ideas necessary to accomplish this integration are few and surprisingly simple. The first is that citizens vary in their habitual attention to politics and
hence in their exposure to political information and argumentation in the media. The second is that people are able to react critically to the arguments they encounter only to the extent that they are knowledgeable about political affairs. The third is that citizens do not typically carry around in their heads fixed attitudes on every issue on which a pollster may happen to inquire; rather, they construct "opinion statements" on the fly as they confront each new issue. The fourth is that, in constructing their opinion statements, people make greatest use of ideas that are, for one reason or another, most immediately salient to them..."

Zaller termed his model 'Receipt-Accept-Sample', or RAS, and we shall use this abbreviation below in the text. Also we accept his term 'citizen' for a social actor. The original formulation contains at least 8 free parameters. This is convenient when we struggle for accordance with experimental data, but less handy for somebody interested in the general behaviour of the model. Our method here is to simplify the mathematical formulation of the RAS model as much as possible, preserving the sociological content of the four postulates cited above. The aim of this paper is to investigate the distribution of opinions about a model issue. Our approach to the Zaller model profits much from the continuous opinion dynamics, as described by Deffuant et al. in 2000 [9]. There, the core idea is that other opinions are taken into account by an agent if the distance from their content to his actual attitude is not larger than some threshold value. Here, the distance \( d \) is measured from a new message to the closest point on the plane of opinions, occupied by the agent in the past. This distance is compared with the threshold parameter \( \mu \); if \( d < \mu \) the message is received, but it is ignored if \( d > \mu \).

The Zaller's mathematical formulation of the RAS model is briefly reviewed in our Section 2. In Section 3 we describe its simplified version proposed in our model. Last two sections are devoted to the numerical results and their discussion.

2 The original mathematical formulation

The postulates listed above are expressed as formal axioms which constitute the RAS model [5]. On this basis, a mathematical construction is built as follows:

1. Each citizen \( i \) is endowed with the political awareness of a given value \( W_i \).
2. The probability of receipt a message relevant for the opinion formation increases with his awareness according to a sigmoidal function \( f \)

\[
f(W_i; a_0, a_1) = 1 - \frac{1}{1 + f + \exp(a_0 + a_1 W_i)}
\]

where \( f \) - floor parameter, which marks a minimum level of reception, \( a_0 \) - a coefficient which designates the intensity of a message, and \( a_1 \) - a coefficient which designates strength of a relationship between awareness and reception.

3. Provided that a message is received by \( i \), the probability of its acceptance decreases with the awareness \( W_i \) and it is given by another sigmoidal function \( g \).
where $b_0$ - a coefficient which designates the difficulty or credibility of the message, $b_1$ - a coefficient which designates the effect of awareness on resistance to persuasion, $b_2$ is a coefficient which designates the effect of the predisposition on resistance to persuasion, and $P_i$ measures the predisposition of $i$ to accept the message, and it depends on the ideological relation of $i$ to the message content.

4. The probability of acceptance of the message and a henceforth change of attitude is equal to the product $Pr = f(W_i; a_0, a_1)g(W_i, P_i; b_0, b_1, b_2)$.

5. The latter expression can be used to construct a kind of one-way Master equation for the probability $Prob$ of a given opinion $Opi$, provided that at $t = 0$, the baseline opinion was $Bas$

$$Prob(Opinion)_t = Prob(Bas) + (1 - Prob(Bas)) * Pr * Dum_t$$

where $Dum_t = 0$ and 1 at initial and final time, respectively. The final opinion is expected to be expressed in an opinion statement in a poll.

6. Additional consideration is due to a situation when the citizen is a subject of two streams of opposite messages. In this case, the decision on the content of the opinion statement is undertaken according to opinions most immediately accessible in a citizen’s mind. The probability to recall a previously accepted opinion is given by yet another sigmoidal function $h$

$$h(W_i; c_0, c_1) = 1 - \frac{1}{1 + \exp(c_0 + c_1 W_i)}$$

where the parameters $c_0$ and $c_1$ are analogous to $a_0$ and $a_1$.

7. To be specific, let us consider two opposite attitudes, say pro- and antiwar (P or A) ones [5]. Let us also assume that our statistical citizen expresses some opinion statement on this issue; actually, a separate expression is given in [5] for the probability of this fact. The probability of the prowar opinion statement $p_p$ expressed by citizen $i$ is supposed to be

$$p_p = \frac{S_p(i)}{S_p(i) + S_a(i)}$$

where $S_p$ ($S_a$) is the accessibility of the prowar (antiwar) messages in the citizen’s mind. These accessibilities are given as

$$S_p(i) = \sum_{i} f(W_i; a_0, a_1)g(W_i, P_i; b_0, b_1, b_2)h(W_i; c_0, c_1)$$

where $t$ is time and the sum is performed over a given time period (say, two recent years). In this sum, the constants $a_0$, $b_1$ and $b_2$ depend on time $t$, and the constants $a_0$ and $b_0$ depend on the message (prowar or antiwar).

8. Some additional assumptions (all the messages of the same intensity) allow to find all the coefficients from the fitting of the theoretical curves to the
3 Our formulation

To investigate, as we intend here, the distribution of opinions, it is necessary to postulate how the opinions vary in time. On the contrary to most sociophysical approaches, the RAS model [5] does not take into account direct interactions between citizens; it is only the influence of media what is taken into account. This does not preclude the possibility that some citizens play the role of media, which could be taken into account in future research. Here we are going to preserve the one-particle character of the Zaller model. Our interest is in the time dynamics of PO with respect to a given issue.

In particular we are interested in a possible sequence of events when a newborn citizen starts to hear political news. His initial awareness is close to zero, but he is indiscriminative in his attitude and he accepts any news as typical and normal. In this sense, his initial acceptance is large. Being a subject of a random stream of messages, initially he is not able to receive most of them, because they seem to him to be too sophisticated. There are some, however, apparently addressed to citizens like him: full of emotions, which clearly divide the world into good and bad, expressed by somebody authoritative but young. Our citizen captures such a message and learns to distinguish it from other messages. His political education just started, and his awareness increases a little bit. Simultaneously, his acceptance is strongly reduced. At least in the near future he will be willing to identify his opinion with this 'his first' message. In physics we like to term such events as 'spontaneous symmetry breaking' [10].

To capture the evolution of the citizen's understanding, we need i) a history-dependent awareness, ii) a time series of messages which vary in their content with respect to some set of issues which we consider to be salient in a given society. In fact, we do not need anything more to indicate, that there is a positive feedback between the political attitude and the character of newly received informations. On the contrary to Ref. [7], we do not need a Gaussian or any other distribution of awareness, as this distribution should be a result of the calculation. Also, we consciously do not contribute to the discussion if a given political orientation is correlated with the awareness [11]. Instead, we will show that a citizen can be randomly trapped by a series of messages close to each other till the time when his attitude is firmly established.

The mathematical formulation of our version of the RAS model is then as follows:

1. At the beginning, the political awareness of each citizen is equal to one. Later it is represented by the number \( n(t) \) of different opinions/messages received till time \( t \). Each citizen starts with one received message, placed at the centre of coordination.

2. Each new message is represented as a point on a plane, with coordinates
The plane plays the role of the space of main attitudes with respect to some salient issues, for example ‘safety vs freedom’ and ‘free trade vs welfare state’. Simultaneously, each message is represented by a point at this plane. The actual value of the dimensionality of this space is of minor importance, except the condition that it is larger than one. When the dimensionality $D$ is two or larger, the size of the $(D-1)$-dimensional circumference of the occupied area increases with its size. This means, that the ability to receive new messages increases with the awareness.

3. The ability to receive a new message is a decreasing function $f$ of the distance $d$ between the point $(x, y)$ and the closest point received in the past. For simplicity we adopt the threshold function $f = \Theta(\mu - d)$ [9]. Here $\mu$ is a parameter, which can be roughly interpreted as the capacity to receive new messages. Receipt of a new message is equivalent to a visit at the point $(x, y)$ assigned to this message. The starting point is placed at the coordination center.

4. The spatial distribution of received messages can be used to calculate the probabilities of the opinion statements in a similar way, as Zaller did in Eq. (5). In our case, the probabilities are calculated from the political contents of the previously received messages, i.e. the position of the points which represents the messages. For an exemplary issue defined by the $y$ axis as the boundary between the opinions ‘pro’ and ‘anti’, the message weight is just its $x$ coordinate. Having chosen $x > 0$ as ‘pro’ and $x < 0$ as ‘contra’ we can calculate the respective probability ‘pro’ as $p$

$$p = \frac{\sum x > 0 x(t)}{\sum |x(t)|}$$

(7)

5. Other issues can be visualised as other axis, not necessarily orthogonal to the plane of salient issues, defined above. Opinions on those other issues can be formed on the basis of the projection of the new axis to the ‘salient’ plane.

6. Once a citizen has a given attitude ‘pro’ along the OX axis, i.e. $p > 0.5$, this attitude can be neutralized by receipt a given number of messages with $x < 0$. It is natural to set $p = 0$ as a neutral attitude. Then, the number of messages to neutralize $p$ can be roughly evaluated as $(2p - 1)n(t)$.

It seems to this author, that this formulation fulfills the content of the postulates of Zaller’s theory. As mentioned above, the distribution of the awareness appears as a natural consequence of the procedure listed above. As we are going to demonstrate, the area around received messages varies from one citizen to another. Simultaneously, the circumference of this area is a measure of the amount of ideas which can be accessible by the citizen in a near future. A good school is where young minds are gradually fed by new ideas, without prejudices towards this or other direction. On the contrary, if a citizen is indocrinated by only one idea, he is not able to receive anything else; in our model, such brainwashing is equivalent to the case when subsequent messages either are close to the current position, or too far from it to be received. Actually, the stream of messages we encounter in our life is not completely random, but it depends on
our intellectual environment. At the moment, however, we do not construct a
theory of the whole society, then our analysis is limited to citizens and does not
embrace the media.

We hope that the arguments given in the preceding paragraph allow to state
that the first and the second postulate of Zaller’s theory: that the citizens vary
in their awareness, and that they are able to take into account only those ideas
which they are knowledgeable about. The essence of the third and fourth pos-
tulates is captured in the point that the opinion statement is formulated on
the basis on the recent trajectory, but separately for each new issue formulated
in a poll. Here we do not concentrate much on the sampling process, when a
new issue is formulated. According to Zaller, opinions on such new issues are
formulated on the basis of other, most salient issues. To refer to an opinion
on a new issue, a relation should be determined between this new issue and
the old (salient) one. We imagine that such a relation can be constructed in a
three-dimensional space; if a new axis is orthogonal to the old ones, nothing can
be stated on a new issue. However, if their angle is different from \( \pi/2 \), infor-
mation on the distribution of opinions can be drawn by means of a geometrical
projection.

The selected form of the function \( f \) is adopted from the Deffuant model. In
general, any decreasing function \( f(x) \) fulfils the condition that messages more
distant are less likely to be received. On the other hand, one of the postulates
by Zaller indicates that some messages can be received but some others can not,
and that the ability to receive them increases with the awareness of the receiv-
ing citizen. In our formulation, the awareness increases with the area around
previously received messages. When this area is larger, more new messages can
be received.

4 Numerical results

In a generic case, the awareness of a citizen increases with time at first expo-
nentially, later it becomes a linear function of time. An example of the time
dependence of the awareness is shown in Fig. 1. At the later stage all incoming
messages appear to be at the already known area. Numerically, the distance be-
tween a new point and some already received points is shorter than the threshold
\( \mu \); then, the new point is also received. This means that the number of received
messages increases by one at each time step. At the earlier stage, however, the
time to receive a new message is longer. Namely, the probability to receive the
second message is equal to \( s = \pi \mu^2/4 \), where the factor 1/4 comes from the fact
that new messages appear as points placed randomly at the square \( 2 \times 2 \). Then
the probability \( P(n) \) to pass \( n \) new messages until the receipt of the \( n+1 \)-th one
and the increase of the awareness to \( W = 2 \) is \( P(n) = s(1 - s)^n \). The time to
receive the third message depends on where the first received point is placed,
and should be averaged over its possible positions. Similarly the distributions of
waiting time to get subsequent messages depend on the positions of the points
which refer to previously received messages. The numerical calculations indi-
icate, that these functions also decrease exponentially with \( n \), and the rate of
decrease is larger for higher values of the awareness. The plots are shown in
Fig. 2. This means in particular, that the mean time to receive a new message
decreases with the awareness.
Figure 1: Example for the time dependence of the awareness $W(t)$.

Figure 2: The probability distributions $P(n)$ to wait $n$ timesteps until an increase of the awareness $W$ to the values 3, 4, 5, 6 and 7.
Figure 3: Time dependence of the squared sum of positions of all received messages for $\mu = 0.25, 0.3, 0.4, 0.6$ and $1.5$. The slope of the curves except the one for $\mu=1.5$ is initially larger than 1.

Being (socio)physicists, we are tempted to investigate the geometrical aspect of the correlation. For this purpose we calculate the time dependence of the square $r^2$ of the sum of coordinates of the messages received till a given time against time. Note that in this way a point is taken into account several times, if no messages were received after the message which it represents. Subsequent points are not correlated in two cases: first, if the whole plane is already densely occupied by previously received messages, and second - for large $\mu$, where any new message is received, despite its position. In the latter case, the slope of $r^2$ against time should be 1 in the log-log scale in accordance with the diffusion law. Our numerical results indicate, that there are two transient times for small $\mu$: $t_1(\mu)$ and $t_2(\mu)$. For $t < t_1(\mu)$ the slope is larger than 1 and reaches 2.5. For $t_1(\mu) < t < t_2(\mu)$ the slope is smaller than 1. Finally, for $t > t_2(\mu)$ the slope is 1 for each $\mu$. When $\mu$ increases, both $t_1(\mu)$ and $t_2(\mu)$ decrease and merge. In this way, we observe a kind of cross-overs in time from the hyperdiffusion to the subdiffusion and then to the normal diffusion. These results are presented in Fig. 3. The plots are averaged over 1000 trajectories. The results are the same if we limit the area where new points appear to a circle with radius equal to 1.0.

A characteristic feature of the model is that subsequently received messages are correlated in their ideological content. In our geometrical realisation this correlation is expressed as the correlation between positions of subsequently received messages. Once a new message is selected, the area around received messages gets widened towards a given direction. This raises the possibility of correlations between subsequent messages. Then, we calculate this correlation
for the direction $x$ - the results are shown in Fig. 4. We use the expression for non-stationary correlations, i.e.

$$W_x = \frac{(x(t)x(t+1)) - \langle x(t) \rangle \langle x(t+1) \rangle}{\sqrt{\text{Var}(x(t)) \text{Var}(x(t+1))}}$$ \hspace{1cm} (8)$$

This correlation is larger for small values of $\mu$. When the whole area is covered, the correlation disappears. It is obvious, that the correlation lifetimes increase when $\mu$ decreases. The results should not depend on the selected direction; the $OX$ axis is chosen for simplicity of the parametrization. The effect of correlations is analogous to the viscous fingering in the snowflake formation $[12]$, where randomly attached molecules increase the probability of attaching further molecules at the same area, and the curvature of the surface increases.

In Fig. 5 we show the probability distribution $P(p)$ of these probabilities at a not-too-long time moment, i.e. after receipt of 500 messages. As we see, the obtained plot is close to the Gaussian distribution for $\mu > 0.4$, but its shape deviates from Gaussian for smaller $\mu$ because of the limitation of the variable $p$ to the range $(0,1)$. None of these plots shows a fat tail. Note that different trajectories contribute to the average with their different numbers of received messages. We also checked that in time, the distribution gets narrow; the time dependence of the variance of $P(p)$ decreases, as shown in Fig. 6. This narrowing can be interpreted as follows: as citizens get more complete information, their opinions are clarified. The final mean probability $< p > = 0.5$; this follows from the initial symmetry of the distribution, which is preserved by the dynamics.

Let us remind that the factor $\mu$ describes the largest possible distance be-
Figure 5: The probability distribution $P(p)$ that the opinion statement of randomly selected agent is equivalent to accept a given issue with probability $p$. The curves are obtained after 500 messages are received, for $\mu=0.2, 0.25, 0.35$ and 1.5 from the widest to the most narrow curve, respectively, and averaged over $10^5$ citizens.
tween messages, from the previously to the one newly received. It is somewhat surprising that opinions of the citizens which understand less swiftly, i.e. of those with smaller $\mu$, is temporarily more wide. This effect is visible in Fig. 5, where the distribution of probabilities is more wide for smaller value of $\mu$. The reason is the correlation between subsequently received messages, which is larger for smaller $\mu$.

5 Discussion

The assumption most essential here is the geometrical-like distance between messages, determined by their political content. Next assumption correlates the distance between a new message and previously received messages with the receipt of the new message. The latter seems natural, the former can seem doubtful. However, separating out the geometrical considerations, we are left with the conjecture that messages with the content far from anything previously received are received less likely. This conjecture seems to be quite natural.

It is possible to continue this kind of reasoning. Our main goal is to demonstrate, that opinions of people endowed with smaller factor $\mu$ are more spread, i.e. there are more extremal opinions, than for the people with larger $\mu$. To falsify this theory one should demonstrate that the result should be the opposite, i.e. the capacity to receive new informations is positively correlated with the extremal opinions. It seems to this author that this is not the case. On the other hand, in our model new messages appear in a limited area. Then, the model is appropriate to a given area of knowledge (here the square $2 \times 2$), which sooner or later will be covered by received messages. We think that capable citizens spread their fields of interest beyond the boundaries which limit thoughts of less endowed people. Then in new areas everybody is a freshman at least at the beginning of their exploration.

Our result that opinions of people with smaller $\mu$ are more spread is akin to the result that opinions of people with smaller threshold parameter are more spread (Figures 1 and 2 in Ref. [9]). Although the outcome of both results is the same, their origin is different. The Authors of Ref. [9] assumed the homogeneous initial distribution of opinions. There, smaller threshold means smaller adaptation and smaller mobility in the opinion space. As a consequence, the distribution remains wide. In our case, the initial distribution is concentrated at the centre. Smaller $\mu$ means smaller adaptation and smaller mobility, but larger correlations. As a consequence, the distribution gets wider.

We found that the awareness, measured by the number of received messages, increases with time exponentially, later linearly. The seemingly exponential part reveals a substructure, which can be of interest for statistically oriented minds, but its existence relies on the assumption that messages can be represented by discrete points. It is possible that their representation as, say, extended excitations in a network of ideas could be more fruitful. Other result are indications of a hyperdiffusion in the space of messages. The effect is transient and it disappears when the considered area becomes known. In this transient time and for small $\mu$, positive correlations are found between the content of subsequently received messages. The transient time decreases with $\mu$; more capable citizens are quicker.

One of the conclusions by Zaller was that the acceptance decreases with the
Figure 6: The variance of the probability distribution $P(\mu)$ against the capacity parameter $\mu$, for various times of calculation: from 50 to 500 timesteps.

Awareness. In our formulation, the equivalent of non-acceptance is the number of messages necessary to neutralize a given attitude. For a given $p$, this number increases with the awareness $n$, as remarked in point 6 of our formulation. This point does not preclude the mean decrease of non-acceptance with time, as shown in Fig. 6. However, more detailed discussion of this question should take into account some initial distribution of attitudes. Here we concentrate on the influence of $\mu$ on the opinion spread; then we purposefully neglect this initial distribution, placing initially each citizen at the coordination centre.

We feel that the weak part of our formulation is that the messages previously received do not lead in the model to a coherent picture, but they stand by in minds till the end of the simulation. A partial remedy could be to add another parameter of forgetting old messages in time, as was done for example in the Bonabeau model [13]. Zaller proposed to take into account only some last part of the simulation. This effect is under investigation. Even more essential process disregarded here as in Ref. [5] is the contact between people. This mechanism of the opinion formation was discussed by many sociophysicists [14,15,16,17]; we recommend [4] as the current review. The next challenge is to build the interpersonal mechanism into the RAS theory. For this purpose, the Deffuant model [9,16] seems most natural. This model relies on the assumption that we react to opinions of other people if they are not too far from our initial beliefs. Our formulation of the RAS theory explores the same idea.

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