Statistical CSI-based Design for Reconfigurable Intelligent Surface-aided Massive MIMO Systems with Direct Links

Kangda Zhi, Cunhua Pan, Hong Ren and Kezhi Wang

Abstract—This paper investigates the performance of reconfigurable intelligent surface (RIS)-aided massive multiple-input multiple-output (MIMO) systems with direct links, and the phase shifts of the RIS are designed based on the statistical channel state information (CSI). We first derive the closed-form expression of the uplink ergodic data rate. Then, based on the derived expression, we use the genetic algorithm (GA) to solve the sum data rate maximization problem. With low-complexity maximal-ratio combination (MRC) and low-overhead statistical CSI-based scheme, we validate that the RIS can still bring significant performance gains to traditional massive MIMO systems.

Index Terms—Intelligent reflecting surface (IRS), Reconfigurable Intelligent Surface (RIS), massive MIMO, statistical CSI.

I. INTRODUCTION

Reconfigurable intelligent surface (RIS) has been recognized as a promising technology for future wireless communication systems [1], [2]. RIS is a programmable metasurface with low power consumption, and it can provide passive beamforming gains. Recently, attractive benefits of RIS have been validated in various scenarios [3]–[6]. To be specific, the authors in [3] showed that RIS has sufficient potential in future communications by jointly designing its passive beamforming and base stations (BSs)’ active beamforming. The authors in [4] further demonstrated that RIS can be deployed at the cell-edge to help boost the useful signal power as well as mitigate the inter-cell interference. Meanwhile, the benefits of RIS for simultaneous wireless information and power transfer (SWIPT) systems and Terahertz systems have been demonstrated in [5] and [6], respectively.

Different from the above instantaneous channel state information (CSI)-based contributions, extensive research attention has been shifted to exploit statistical CSI to design the phase shifts of RIS [7]–[12], since it can greatly reduce the channel estimation overhead and computational complexity. Specifically, the authors in [7] and [8] studied the RIS-aided single-user systems and [9]–[11] considered the RIS-assisted multi-user systems. Most recently, in [12], we firstly investigated the RIS-aided massive multiple-input multiple-output (MIMO) systems with statistical CSI.

However, in [12], only the special case when the direct links from BS to users are entirely blocked was studied. For massive MIMO systems deployed in the sub-6 GHz band, the direct link may exist with high probability. Therefore, it is meaningful to consider the RIS-aided massive MIMO systems with the existence of direct links. The extension to this new scenario is not straightforward. On one hand, with massive antennas, the received power from direct links could be strong, which may weaken the influence of RIS. On the other hand, with an excessive number of antennas, it is challenging to adopt complex beamforming. Instead, the maximal-ratio combination (MRC) receiver is widely used in massive MIMO systems due to its simplicity and low complexity. However, due to the shared RIS-BS link between different users, massive antennas could no longer make the multi-user interference negligible. Therefore, it is imperative to investigate whether or not the RIS would still be beneficial in enhancing the performance of massive MIMO systems.

Against the above background, an RIS-aided massive MIMO system with direct links is studied in this paper. To reduce the implementation complexity, the MRC receiver is applied at the BS and statistical CSI is used for the design of phase shifts of the RIS. We firstly derive the closed-form expression for the uplink ergodic data rate. Then, based on the derived expression, a genetic algorithm (GA) was adopted to optimize the phase shift to maximize the sum ergodic data rate. Finally, simulation results validate that statistical CSI-based RIS is capable of enhancing the performance of conventional massive MIMO systems even with simple MRC receiver.

II. SYSTEM MODEL

As shown in Fig. 1, a typical uplink massive MIMO system is considered in this paper, where a BS equipped with $M$
antennas simultaneously communicates with \( K \) single-antenna users with the aid of an RIS. The RIS is composed of \( N \) reflecting elements, and its configuration matrix can be expressed as
\[
\Phi = \text{diag} \{ e^{j\theta_1}, e^{j\theta_2}, \ldots, e^{j\theta_N} \},
\]
where \( \theta_n \) is the phase shift of element \( n \). Then, the cascaded \( M \times K \) channels can be written as \( \mathbf{G} = \mathbf{H}_2 \Phi \mathbf{H}_1 \), where \( \mathbf{H}_1 = [\mathbf{h}_1, \ldots, \mathbf{h}_K] \) is the \( N \times K \) channels between users and the RIS, and \( \mathbf{H}_2 \) represents the \( M \times N \) RIS-BS channels. Since the BS and RIS are often deployed with certain height and the RIS is often deployed near the users, channels \( \mathbf{H}_1 \) and \( \mathbf{H}_2 \) can be line-of-sight (LoS) dominant. Thus, the Rician channel model is adopted to characterize user-\( k \)-RIS link \( \mathbf{h}_k \) and RIS-BS link \( \mathbf{h}_2 \):
\[
\begin{align*}
\mathbf{h}_k &= \sqrt{\alpha_k} \left( \sqrt{\varepsilon_k} \tilde{\mathbf{h}}_k + \sqrt{1 - \varepsilon_k} \tilde{\mathbf{h}}_k \right), \quad 1 \leq k \leq K, \\
\mathbf{h}_2 &= \sqrt{\beta} \left( \sqrt{\delta} \tilde{\mathbf{h}}_2 + \sqrt{1 - \delta} \tilde{\mathbf{h}}_2 \right),
\end{align*}
\]
where \( \alpha_k \) and \( \beta \) are distance-dependent large-scale path-loss, \( \varepsilon_k \) and \( \delta \) are Rician factors. \( \tilde{\mathbf{h}}_k \) are complex Gaussian random variables following \( \mathcal{CN}(0,1) \). By contrast, \( \tilde{\mathbf{h}}_k \) and \( \tilde{\mathbf{h}}_2 \) are deterministic LoS channel components. Under the uniform square planar array (USPA) model, \( \tilde{\mathbf{h}}_k \) and \( \tilde{\mathbf{h}}_2 \) can be respectively expressed as
\[
\begin{align*}
\tilde{\mathbf{h}}_k &= \mathbf{a}_{\Phi_k} \left( \varphi_{kr_k}, \varphi_{kr} \right), \quad 1 \leq k \leq K, \\
\tilde{\mathbf{h}}_2 &= \mathbf{a}_{\Phi_2} \left( \varphi_{kr_2}, \varphi_{kr} \right) \mathbf{a}_{\Phi}^H \left( \varphi_{kr}, \varphi_{kr} \right),
\end{align*}
\]
with
\[
\mathbf{a}_{\Phi} \left( \varphi_{kr}, \varphi_{kr} \right) = \begin{bmatrix} 1, \cos \varphi_{kr}, \ldots, \cos(N \varphi_{kr}) \end{bmatrix}^T,
\]
where \( d \) is the elements spacing, \( \lambda \) is wavelength, \( \varphi_{kr_k} \), \( \varphi_{kr} \) are respectively the azimuth and elevation angles of arrival (AoA) from user \( k \) to the RIS (from the RIS to the BS), \( \varphi_{kr_2} \), \( \varphi_{kr} \) are respectively the azimuth and elevation angles of departure (AoD) from the RIS to the BS. We assume that these angles are known since they can be computed from the locations of BS, RIS and users, and their locations can be obtained from the global position system (GPS).

Since rich scatters often exist near the ground, we use Rayleigh fading model to express the direct links between the BS and users. The channel of direct links \( \mathbf{D} \in \mathbb{C}^{M \times K} \) can be written as
\[
\mathbf{D} = [\mathbf{d}_1, \ldots, \mathbf{d}_k, \ldots, \mathbf{d}_K], \quad \mathbf{d}_k = \sqrt{c_k} \tilde{\mathbf{d}}_k,
\]
where \( c_k \) is large-scale path loss and \( \tilde{\mathbf{d}}_k \) represents the NLoS direct link for user \( k \).

Based on the above definitions, we can express the received signal at the BS as
\[
y = (\mathbf{G} + \mathbf{D})\mathbf{P} \mathbf{x} + \mathbf{n} = (\mathbf{H}_2 \Phi \mathbf{H}_1 + \mathbf{D}) \mathbf{P} \mathbf{x} + \mathbf{n},
\]
where \( \mathbf{P} = \text{diag} (\sqrt{p_1}, \ldots, \sqrt{p_K}) \) and \( p_k \) is transmit power of user \( k \), \( \mathbf{x} = [x_1, \ldots, x_K]^T \) denotes the information symbol vector, \( \mathbf{n} \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_N) \) is the receiver noise vector.

To reduce the implementation complexity in practical systems, we employ the low-complexity MRC technique with receiver matrix \( (\mathbf{G} + \mathbf{D})^H \). Therefore, the received signal at the BS is processed as
\[
r = (\mathbf{G} + \mathbf{D})^H (\mathbf{G} + \mathbf{D}) \mathbf{P} \mathbf{x} + (\mathbf{G} + \mathbf{D})^H \mathbf{n},
\]
and the received signal of the \( k \)-th user is
\[
r_k = \sqrt{p_k} (\mathbf{g}_k^H + \mathbf{d}_k^H) (\mathbf{g}_k + \mathbf{d}_k) x_k + \sum_{i=1, i \neq k}^{K} \sqrt{p_i} (\mathbf{g}_k^H + \mathbf{d}_i^H) (\mathbf{g}_i + \mathbf{d}_i) x_i + (\mathbf{g}_k^H + \mathbf{d}_k^H) \mathbf{n},
\]
where \( \mathbf{g}_k = \mathbf{H}_2 \Phi \mathbf{h}_k \) is the \( k \)-th column of \( \mathbf{G} \). Meanwhile, \( \mathbf{g}_k \) denotes the cascaded channel of user \( k \).

### III. Ergodic Rate Analysis

In order to design the phase shifts of the RIS with statistical CSI, the expression of ergodic data rate should be derived. To this end, we firstly present the ergodic rate expression of user \( k \):
\[
R_k = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{\| \mathbf{p}_k \| \| \mathbf{g}_k + \mathbf{d}_k \|^2}{\sum_{i=1, i \neq k}^{K} \| \mathbf{p}_i \| \| \mathbf{g}_i + \mathbf{d}_i \|^2 + \sigma^2 \| \mathbf{g}_k + \mathbf{d}_k \|^2} \right) \right\},
\]
where \( \mathbf{E}^{\text{signal}}(\Phi), \mathbf{E}^{\text{noise}}(\Phi) \) are respectively given by \( 12 \) and \( 13 \) on the top of next page. Besides, \( c_k = \sum_{n=1}^{N} \epsilon_n \), where
\[
\epsilon_n = 2 \pi d \sqrt{\frac{N}{(n-1) \sqrt{N}}} \sin \varphi_{kr_k} \sin \varphi_{kr} - \sin \varphi_{kr_k} \sin \varphi_{kr} + ((n-1) \mod \sqrt{N}) \cos \varphi_{kr_k} - \cos \varphi_{kr}.
\]

**Proof:** Please refer to Appendix A.

**Remark 1** The ergodic data rate for RIS-aided massive MIMO systems without direct links in [12] can be obtained by setting \( \gamma_k = 0, \forall \)k. Besides, the single-user case in [7] is a special case of our work with \( p_i = 0, \forall i \neq k \).

**Corollary 1** The ergodic data rate for traditional massive MIMO systems without an RIS can be obtained by setting \( c_k = 0, \forall k \), which is given by
\[
B_k^{(w)} \approx \log_2 \left( 1 + \gamma_k \right) \approx \log_2 \left\{ 1 + \frac{\| \mathbf{p}_k \| \| \mathbf{h}_k \|}{\sum_{i=1, i \neq k}^{K} \| \mathbf{p}_i \| \gamma_i + \sigma^2} \right\}.
\]
Design the phase shifts of RIS. Besides, when both (18) on the top of this page. 

Massive MIMO systems with non-RIS-aided massive MIMO systems. When $N \rightarrow \infty$, the average data rate is approximated as

$$R_k^{(\text{NL})} \triangleq \log_2 \left( 1 + \mathcal{J}_k^{(\text{NL})} \right) \approx \log_2 \left( 1 + \frac{p_k \left( c_k^2 (MN^2 + N^2 + N) + 2c_k \gamma_k N (M + 1) + \gamma_k^2 (M + 1) \right)}{\sum_{i=1, i \neq k}^{K} p_i c_i \gamma_i \left( N^2 + 2N + c_i \gamma_i N + c_i \gamma_i N + \gamma_i \gamma_k \right) + \sigma^2 (c_k N + \gamma_k)} \right).$$

**Proof:** By solving inequality $\mathcal{J}_k^{(\text{NL})} > \mathcal{J}_k^{(w)}$, we can complete the proof after some simplifications.

Corollary 3: For a special case where cascaded channels are pure NLoS, i.e., $\delta = \varepsilon = 0, \forall k$, the ergodic rate is given by [78] on the top of this page.

It can be seen that in this special case, there is no need to design the phase shifts of RIS. Besides, when both $N \rightarrow \infty$ and $M \rightarrow \infty$, rate $R_k^{(\text{NL})}$ could grow without bound. Furthermore, [12] Fig. 4 has shown that this rich-scatter environment is favorable for RIS-aided multi-user systems since it can provide sufficient spatial multiplexing gains. Therefore, we will use this special case to investigate the gain from RIS in the presence of additional interference. To further facilitate our analysis and provide useful insights, we focus on the two-user case where users are located closely and have the same transmit power, i.e., $c_k = c, \gamma_k = \gamma, p_k = p, k = 1, 2$. Then, we have the following Corollary.

**Corollary 3** Considering an RIS deployed in the environment with rich scatters and $\gamma > 0, c > 0$, the ergodic data rate of RIS-aided massive MIMO systems is higher than non-RIS-aided massive MIMO systems when

$$p \left( c \gamma (M - 1) - \frac{1}{c} \right) \leq \frac{N + 1}{\gamma (M - 1)} + \frac{1}{\sigma^2},$$

or

$$N > \gamma \left( \frac{p \left( c \gamma (M - 1) - \frac{1}{c} \right)}{\sigma^2 (M - 1)} - 1 \right).$$

**Proof:** Using the same method as [12] Corollary 4, we can substitute terms involving $\Phi$ in [12] with their expectation. When $N \rightarrow \infty$, we can ignore the insignificant terms which are not on the order of $O(N^2)$ and then complete the proof after some simplifications.

Then, we consider a two-user case with $c_k = c, \gamma_k = \gamma, \varepsilon_k = \varepsilon, p_k = p, k = 1, 2$. By solving inequality $\mathcal{J}_k^{(\text{rl})} > \mathcal{J}_k^{(w)}$, we can obtain the following result.

**Corollary 5** When $N \rightarrow \infty$, RIS-aided massive MIMO systems with random phase shifts outperform traditional non-RIS systems if

$$\gamma \frac{p \left( c \gamma (M - 1) - \frac{1}{c} \right)}{\sigma^2 (M^2 - M)} < \frac{(2\delta^2 + 2\delta + 1) M + (2\delta + 1)}{\delta^2 (M^2 - M)}.$$
Sum user rate (bit/s/Hz) BS is dBm d is a circle centered at the RIS with radius of d otherwise, we set M based method in [12] to maximize the sum user rate the correctness of our analytical results. We use the same GA- 10 ∑ M chal lenging to meet inequality (22) under large N low-SNR regime. Besides, even when randomly, to beat non-RIS systems, it should operate in Fig. 3. Uplink ergodic rate versus user transmit power.

Corollary 5 shows that when RIS’s phase shifts are set randomly, to beat non-RIS systems, it should operate in low-SNR regime. Besides, even when N → ∞, it is still challenging to meet inequality (22) under large M.

IV. NUMERICAL RESULTS

In this section, numerical simulations are presented to verify the correctness of our analytical results. We use the same GA-based method in [12] to maximize the sum user rate R = ∑Kk=1 Rk.

Our simulation setup is shown in Fig. 2. Unless stated otherwise, we set M = N = 49, σ = −104 dBm, p0 = 30 dBm, εk = 1, ∀k, δ = 10. Four users are equally located on a circle centered at the RIS with radius of d1 = 20 m. RIS-BS distance is dIB = 1000 m and the distance between user k and BS is (d2 k)2 = (d1B − d1U1 sin (ξ k))2 + (d1U1 cos (ξ k))2. All the AoA and AoD are generated randomly from [0, 2π]. 4, 5. The distance-based path-loss are αk = 10−3d1U1 2, β = 10−2.5, γk = 10−3 (d1B)−3, ∀k.

Fig. 3 shows that even with a simple MRC receiver, statistical CSI-based RIS can still effectively improve the rate performance in massive MIMO systems in the low-SNR regime. However, due to the multi-user interference, as SNR increases, conventional massive MIMO systems will outperform random phase shifts-based RIS systems. Finally, with extremely high SNR, it could even outperform the optimal phase shifts-based RIS systems. These results agree with our analysis in Corollary 3 and Corollary 5.

Fig. 4 and Fig. 5 plot the ergodic rate versus N and M with dIB = 1000 and dIB = 700, respectively. Note that smaller dIB means stronger direct links. We can see that the RIS with optimal phase shifts bring a significant rate improvement to traditional massive MIMO systems in both figures, and this improvement can hold with quite large M. However, when the direct links are strong or when M is large, massive MIMO systems without RIS will have a better performance than RIS-aided systems with random phase shifts. This result is consistent with our analysis in Corollary 5. Besides, these observations indicate that to fully take advantages of RIS with a simple MRC receiver, it is better to use RIS with a large number of elements to serve cell-edge users, and the RIS has ability to play a significant role in the low-SNR regime.

V. CONCLUSION

In this paper, we have studied an RIS-aided massive MIMO system with direct links. A closed-form ergodic rate expression has been derived. Then, based on the expression, we have found that with low-complexity MRC beamforming, RIS-aided massive MIMO systems can outperform non-RIS systems in the low-SNR regime. Finally, our analytical results have been verified by the simulations.

APPENDIX A

To begin with, by applying [13] Lemma 1, ergodic data rate in (11) can be approximated as

\[ R_k \approx \log_2 \left( 1 + \frac{p_k}{\sum_{i=1}^{K} \epsilon_k \sigma_k \mathbb{E} \left[ \|g_k + d_k\|^2 \right]} + \sigma^2 \mathbb{E} \left[ \|g_k + d_k\|^2 \right] \right) \]  (23)

To derive a closed-form expression of (23), we need to derive signal term \( \mathbb{E} \left[ \|g_k + d_k\|^2 \right] \), interference
that

where $\mathbb{E}\left\{\|g_k\|^2\right\}$ has been given in [12] Lemma 1. 

Next, the signal term $\mathbb{E}\left\{\|g_k + d_k\|^2\right\}$ can be expanded as

$$
\mathbb{E}\left\{\|g_k + d_k\|^2\right\} = \mathbb{E}\left\{\|g_k\|^2 + 2 \text{Re}\{d_k^H g_k\} + \|d_k\|^2\right\} = \mathbb{E}\left\{\|g_k\|^4\right\} + 4 \mathbb{E}\left\{\text{Re}\{d_k^H g_k\}\right\}^2 + \mathbb{E}\left\{\|d_k\|^4\right\} + 2 \mathbb{E}\left\{\|g_k\|^2 \|d_k\|^2\right\},
$$

(25)

where $\mathbb{E}\left\{\|g_k\|^4\right\}$ has been given in [12] Lemma 1. Assuming that $g_k = v_m + j w_m$, $d_k = s_m + j t_m$, where both $s_m$ and $t_m$ independently follow $\mathcal{CN}(0, \frac{\gamma_k}{2})$, we have

$$
\mathbb{E}\left\{\text{Re}\{d_k^H g_k\}\right\}^2 = \mathbb{E}\left\{\sum_{m=1}^{M} s_m v_{m} - t_m w_{m}\right\}^2 = \mathbb{E}\left\{\sum_{m=1}^{M} (s_m v_{m} - t_m w_{m})^2\right\} = \sum_{m=1}^{M} (v_{m}^2 + w_{m}^2)^2 = \frac{2 \gamma_k}{2} \mathbb{E}\left\{\|g_k\|^2\right\}.
$$

Then, the remaining two terms in (25) can be obtained as

$$
\mathbb{E}\left\{\|d_k\|^4\right\} = \mathbb{E}\left\{\sum_{m=1}^{M} \|d_k\|^2\right\}^2 = \mathbb{E}\left\{\sum_{m=1}^{M} \|d_k\|^2\right\}^2 + \mathbb{E}\left\{\sum_{m=1}^{M} \sum_{m' = 1, m' \neq m}^{M} \|d_k\|_{m} \|d_k\|_{m'}\right\}^2 = 2M \gamma_k^2 + M(M-1) \gamma_k^2 = (M^2 + M) \gamma_k^2,
$$

(27)

and

$$
\mathbb{E}\left\{\|g_k\|^2 \|d_k\|^2\right\} = \mathbb{E}\left\{\|g_k\|^2\right\} \mathbb{E}\left\{\|d_k\|^2\right\} = M \gamma_k \mathbb{E}\left\{\|g_k\|^2\right\}.
$$

(28)

Substituting (26), (27) and (28) into (25), the expression of signal term is given by

$$
\mathbb{E}\left\{\|g_k + d_k\|^4\right\} = \mathbb{E}\left\{\|g_k\|^4\right\} + 2(M + 1) \gamma_k \mathbb{E}\left\{\|g_k\|^2\right\} + (M^2 + M) \gamma_k^2.
$$

(29)

Finally, the interference term can be written as

$$
\mathbb{E}\left\{\|g_k^H d_k\|^2\right\} \mathbb{E}\left\{\|g_k + d_k\|^2\right\} = \mathbb{E}\left\{\|g_k^H g_k\|^2\right\} + \mathbb{E}\left\{\|g_k^H d_k\|^2\right\} + \mathbb{E}\left\{\|g_k^H d_k\|^2\right\} + \mathbb{E}\left\{\|d_k^H d_k\|^2\right\} + \mathbb{E}\left\{\|d_k^H d_k\|^2\right\},
$$

(20)

where $\mathbb{E}\left\{\|g_k\|^2\right\}$ has been given in [12] Lemma 1, and

$$
\mathbb{E}\left\{\|d_k\|^2\right\} = \mathbb{E}\left\{\|g_k\|^2\right\} + \mathbb{E}\left\{\|g_k\|^2\right\} + \mathbb{E}\left\{\|g_k\|^2\right\} + \mathbb{E}\left\{\|g_k\|^2\right\},
$$

$$
\mathbb{E}\left\{\|d_k^H d_k\|^2\right\} = \mathbb{E}\left\{\|g_k^H d_k\|^2\right\} + \gamma_k \mathbb{E}\left\{\|g_k\|^2\right\},
$$

$$
\mathbb{E}\left\{\|d_k^H d_k\|^2\right\} = \mathbb{E}\left\{\|g_k^H d_k\|^2\right\} + \gamma_k \mathbb{E}\left\{\|g_k\|^2\right\}.
$$

(31)

Therefore, combining (24), (29) and (30) with (23) and after some simplifications, we can complete the proof.

REFERENCES

[1] C. Pan, H. Ren, K. Wang, J. F. Kolb, M. Elkashlan, M. Chen, M. D. Renzo, Y. Hao, J. Wang, A. L. Swindlehurst, X. You, and L. Hanzo, “Reconfigurable intelligent surfaces for 6G and beyond: Principles, applications, and research directions,” 2020. [Online]. Available: https://arxiv.org/abs/2011.04350

[2] Q. Wu, S. Zhang, B. Zheng, C. You, and R. Zhang, “Intelligent reflecting surface aided wireless communications: A tutorial,” 2020. [Online]. Available: https://arxiv.org/abs/2007.02759

[3] Q. Wu and R. Zhang, “Intelligent reflecting surface enhanced wireless network via joint active and passive beamforming,” IEEE Transactions on Wireless Communications, vol. 18, no. 11, pp. 5394–5409, 2019.

[4] C. Pan et al., “Multicell MIMO communications relying on intelligent reflecting surfaces,” IEEE Transactions on Wireless Communications, vol. 19, no. 8, pp. 5218–5233, 2020.

[5] C. Pan, H. Ren, K. Wang, M. Elkashlan, A. Nallanathan, J. Wang, and L. Hanzo, “Intelligent reflecting surface aided MIMO broadcasting for simultaneous wireless information and power transfer,” IEEE Journal on Selected Areas in Communications, vol. 38, no. 8, pp. 1719–1734, 2020.

[6] Y. Pan, K. Wang, C. Pan, H. Zhu, and J. Wang, “UAV-assisted and intelligent reflecting surfaces-supported terahertz communications,” 2020. [Online]. Available: https://arxiv.org/abs/2010.14223

[7] Y. Han, W. Tang, S. Jia, C.-K. Wen, and X. Ma, “Large intelligent surface-assisted wireless communication exploiting statistical CSI,” IEEE Transactions on Vehicular Technology, vol. 68, no. 8, pp. 8238–8242, 2019.

[8] Y. Jia, C. Ye, and Y. Cui, “Analysis and optimization of an intelligent reflecting surface-assisted system with interference,” IEEE Transactions on Wireless Communications, 2020.

[9] Z. Peng, T. Li, C. Pan, H. Ren, W. Xu, and M. Di Renzo, “Analysis and optimization for IRS-aided multi-pair communications relying on statistical CSI.” [Online]. Available: https://arxiv.org/abs/2007.11704

[10] X. Hu, C. Zhong, Y. Zhang, X. Chen, and Z. Zhang, “Location information aided multiple intelligent reflecting surface systems,” IEEE Transactions on Communications, 2020.

[11] L. You, J. Xiong, Y. Huang, D. W. K. Ng, C. Pan, W. Wang, and X. Gao, “Reconfigurable intelligent surfaces-assisted multiuser MIMO uplink transmission with partial CSI.” [Online]. Available: https://arxiv.org/abs/2003.13014

[12] K. Zhu, C. Pan, H. Ren, and K. Wang, “Power scaling law analysis and phase shift optimization of IRS-aided massive MIMO systems with statistical CSI,” 2020. [Online]. Available: https://arxiv.org/abs/2010.13525

[13] Q. Zhang, S. Jin, K. Wong, H. Zhu, and M. Matthaiou, “Power scaling of uplink massive MIMO systems with arbitrary-rank channel means,” IEEE Journal of Selected Topics in Signal Processing, vol. 8, no. 5, pp. 966–981, 2014.