Experimental-schemes-based non-Gaussian operations in continuous variable quantum teleportation

Chandan Kumar$^{1,*}$ and Shikhar Arora$^{1,†}$

$^1$Department of Physical Sciences, Indian Institute of Science Education and Research Mohali, Sector 81 SAS Nagar, Panjab 140306 India.

We undertake a comprehensive study of quantum teleportation using non-Gaussian entangled resource states produced by performing non-Gaussian operations on a two-mode squeezed vacuum (TMSV) state. These non-Gaussian operations include photon subtraction, photon addition, and photon catalysis and are performed via heralding schemes using photon number resolving detectors. Since these schemes produce non-Gaussian states probabilistically, it is necessary to consider the success probability. We first derive a unified expression for the Wigner characteristic function describing the photon subtracted, added, and catalyzed TMSV states. We then utilize it to obtain the fidelity of teleporting input coherent and squeezed vacuum states. Considering the success probability allows us to find the most advantageous non-Gaussian operations along with the associated optimal squeezing and beam splitter transmissivity parameters. It turns out that the symmetric one-photon subtracted TMSV state is most advantageous for teleporting input coherent and squeezed vacuum states. We expect the derived Wigner characteristic function to be useful in the state characterization and other quantum protocols.

I. INTRODUCTION

A two-mode squeezed vacuum (TMSV) state is generally employed as a resource state for various continuous variable (CV) quantum information processing (QIP) protocols including quantum teleportation [1], quantum dense coding [2], and entanglement swapping [3]. However, due to experimental limitations, it is difficult to produce high squeezed states [4], which puts an upper bound on the performance of quantum protocols. An alternative is to perform non-Gaussian operations such as photon subtraction (PS), photon addition (PA), and photon catalysis (PC) on the TMSV state, which can ameliorate the nonclassicality and entanglement content of the TMSV state. Further, non-Gaussian states including photon subtracted TMSV (PSTMSV), photon added TMSV (PATMSV), and photon catalyzed TMSV (PCTMSV) states have been shown to improve the performance of various QIP protocols such as quantum key distribution [5–10], and quantum metrology [11–15]. It has been also shown that these non-Gaussian states can improve the fidelity of quantum teleportation [16–22].

These non-Gaussian operations generated by heralding schemes using photon number resolving detectors (PNRDs) [23, 24] are probabilistic in nature [25]. The success probability represents the fraction of successful non-Gaussian operations per trial; hence, it quantifies resource utilization. When considering the enhancement in quantum features such as nonclassicality, entanglement, and teleportation fidelity, it is necessary to account for the corresponding success probability. A few research studies on non-Gaussian entanglement have considered the success probability of the involved non-Gaussian operations [25–29]. Further, preparation of non-Gaussian states such as Schrödinger cat states [30, 31] and a phase sensitivity study in Mach-Zehnder interferometer using non-Gaussian states [32] have also taken success probability into account. However, such a study in quantum teleportation has not yet been undertaken. This article strives to take the success probability into account in teleporting input coherent and squeezed vacuum states using NGTMSV states and find an optimal balance between enhancing teleportation fidelity and resource utilization. Here NGTMSV states collectively refer to PSTMSV, PATMSV, and PCTMSV states. Further, PS, PA, and PC operations shall be collectively termed “non-Gaussian operations”.

To this end, we derive the unified Wigner characteristic function of the NGTMSV states, which is then used to derive the analytical expression for the fidelity of teleporting input coherent and squeezed vacuum states. It should be noted that calculations involving non-Gaussian states are more complicated than those involving Gaussian states. Additionally, the experimental scheme based on beam splitters and PNRDs greatly increases the challenge for theoretical investigation as we have to incorporate the free parameters associated with these devices in the analytical expressions.

We optimize the transmissivities of beam splitters to maximize the fidelity of teleporting input coherent and squeezed vacuum states. While PSTMSV and PCTMSV states can teleport input coherent and squeezed vacuum states, PATMSV states can only teleport input squeezed vacuum states with large squeezing. We then study the difference between the fidelity of the NGTMSV states and that of the TMSV state, $\Delta F_{NG}$, which provides us insights about the magnitudes of the relative advantages provided by the different NGTMSV states.

Finally, we consider the success probability of non-Gaussian operations and the probability of generating multi-photon Fock states. We maximize the product

$^* $ chandan.quantum@gmail.com
$^† $ shikhar.quantum@gmail.com
of $\Delta F_{\text{NG}}$ and success probability (resource utilization) and find the optimal squeezing of the resource states and transmissivities of the beam splitters for different NGTMSV states. The analysis reveals that one PS on both the modes of the TMSV state is the most advantageous operation. This can be attributed to the fact that Fock states, which are generated probabilistically [33–37], are not required for PS operation. In contrast, they are required for PA and PC operations, which effectively decreases the success probability of PA and PC operations.

The unified Wigner characteristic function of the NGTMSV states derived in this article is of independent interest and great importance in its own right. As far as we know, this expression does not exist in the literature. We expect it will help in handling similar calculational challenges arising in various non-Gaussian CV QIP protocols. We have explicitly provided the optimal transmissivities, which shall be highly relevant to experimentalists in achieving higher performance and resource optimization in non-Gaussian quantum teleportation.

Rest of the paper is organized as follows. In Sec. II, we obtain a general expression for the Wigner characteristic function of the NGTMSV states. In Sec. III, we provide a comprehensive study of the teleportation of input coherent and squeezed vacuum states. Finally, we conclude with a discussion in Sec. IV, where we outline the implications and future aspects of the current work. In the Appendix, we briefly describe the phase space description of the CV systems relevant to this article.

II. WIGNER CHARACTERISTIC FUNCTION OF THE NGTMSV STATES

![Diagram](https://via.placeholder.com/150)

FIG. 1. Experimental setup for the preparation of non-Gaussian TMSV state. The TMSV state is mixed with ancilla modes initiated to Fock states $|m_1\rangle$ and $|m_2\rangle$ using beam splitters. Simultaneous detections of $n_1$ and $n_2$ photons in the output modes corresponding to the ancilla modes herald successful non-Gaussian operations on both the modes.

The experimental setup for the generation of NGTMSV states is shown in Fig. 1. We consider a TMSV state labeled by $A_1$ and $A_2$. We represent the modes $A_1$ and $A_2$ by the quadrature operators $(\hat{q}_1, \hat{p}_1)^T$ and $(\hat{q}_2, \hat{p}_2)^T$, respectively. We also consider two auxiliary modes labeled by $F_1$ and $F_2$ and initiated to Fock states $|n_1\rangle$ and $|n_2\rangle$, respectively. We represent the modes $F_1$ and $F_2$ by the quadrature operators $(\hat{q}_3, \hat{p}_3)^T$ and $(\hat{q}_4, \hat{p}_4)^T$, respectively. Mode $A_1$ ($A_2$) is mixed with mode $F_1$ ($F_2$) using beam-splitter of transmissivity $T_1$ ($T_2$). Prior to the beam splitter operations, the Wigner characteristic function of the four mode system is given by

$$\chi_{F_1 A_1 A_2 F_2}(A) = \chi_{A_1 A_2}(A)\chi_{|n_1\rangle}(A_3)\chi_{|n_2\rangle}(A_4),$$

where $\chi_{|n_i\rangle}(A_i)$ is the Wigner characteristic function of the Fock state $|n_i\rangle$ and $\chi_{|n_j\rangle}(A_j)$ is the Wigner characteristic function of the Fock state $|n_j\rangle$. The four modes get entangled as a result of mixing of the modes by the two beam splitters collectively represented by the symplectic transformation matrix $B(T_1, T_2) = B_{A_1 F_1}(T_1) \oplus B_{A_2 F_2}(T_2)$, where $B_i(T)$ is the beam splitter operation given in Eq. (A5) of the Appendix A. The evolved Wigner characteristic function can be readily evaluated using Eq. (A16) of the Appendix A:

$$\chi_{F_1 A_1 A_2 F_2}(A) = \chi_{F_1 A_1 A_2 F_2}(B(T_1, T_2)^{-1}A).$$

PNRDs represented by the positive-operator-valued measure (POVM) $\{\Pi_{n_1} = |n_1\rangle\langle n_1|, I - \Pi_{n_1}\}$ and $\{\Pi_{n_2} = |n_2\rangle\langle n_2|, I - \Pi_{n_2}\}$, are used to measure the transformed auxiliary modes $F_1'$ and $F_2'$, respectively. The simultaneous click of the POVM elements $\Pi_{n_1}$ and $\Pi_{n_2}$ heralds successful non-Gaussian operations on both the modes. The post-measurement state corresponds to the unnormalized Wigner characteristic function of the NGTMSV states:

$$\chi_{A_1 A_2}^{\text{NG}} = \frac{1}{(2\pi)^2} \int d^2 A_3 d^2 A_4 \chi_{F_1' A_1' A_2' F_2'}(A) \chi_{|n_1\rangle}(A_3) \chi_{|n_2\rangle}(A_4).$$

We can appropriately choose input photons and measured photons in the auxiliary modes and perform different non-Gaussian operations. For instance, by choosing $m_i < n_i$, $m_i > n_i$, and $m_i = n_i$, we can perform PS, PA, and PC operations on mode $A_i$, respectively. These operations on TMSV state result in the generation of non-Gaussian states, which shall be abbreviated as PSTMSV, PATMSV, and PCTMSV states. In this work, we consider $m_1 = m_2 = 0$ and $n_1 = n_2 = 0$ for PS and PA operations, respectively. Further, we can perform asymmetric and symmetric non-Gaussian operations on TMSV state as illustrated in Table I. Here we note that we perform the asymmetric non-Gaussian operations on mode $A_2$ of the TMSV state, and therefore, only beam splitter transmissivity $T_2$ appears in relevant expressions.

Using the Wigner characteristic function of the Fock state (A9) and integrating Eq. (3), we get

$$\chi_{A_1 A_2}^{\text{NG}} = \bar{F}_1 \exp \left( A^T M_1 A + u^T M_2 A + u^T M_3 u \right),$$

(4)
TABLE I. Restrictions on the input photons (m_i) and the number of photons measured (n_i) in the auxiliary modes for various asymmetric and symmetric non-Gaussian operations on the TMSV state.

| Operations  | Input m_1 m_2 | Detected n_1 n_2 |
|-------------|---------------|------------------|
| Asym n-PS   | 0 0           | 0 n              |
| Asym n-PA   | 0 n           | 0 0              |
| Asym n-PC   | 0 n           | 0 n              |
| Sym n-PS    | 0 0           | n n              |
| Sym n-PA    | n n           | 0 0              |
| Sym n-PC    | n n           | n n              |

where the column vectors A and u are defined as 
\((\tau_1, \sigma_1, \tau_2, \sigma_2)^T\) and 
\((u_1, v_1, u_2, v_2, u_1', v_1', u_2', v_2')^T\) respectively, and the matrices \(M_1, M_2\) and \(M_3\) are given in Eqs. (B1), (B3) and (B5) of the Appendix B. Further, the differential operator \(\hat{F}_1\) is defined as

\[
\hat{F}_1 = \frac{2^{-(m_1+m_2+n_1+n_2)}}{m_1! m_2! n_1! n_2!} \frac{\partial^{m_1} \partial^{m_2} \partial^{n_1} \partial^{n_2}}{\partial u_1^{m_1} \partial u_2^{m_2} \partial v_1^{n_1} \partial v_2^{n_2}} \left( \begin{array}{c} u_1 \cdot u_2 \cdot v_1 \cdot v_2 \\ u_1' \cdot u_2' \cdot v_1' \cdot v_2' \end{array} \right)
\]  

(5)

The normalization factor corresponding to Eq. (4) represents the probability of success of non-Gaussian operations in both the modes and is evaluated as

\[
P_{\text{NG}}^{\text{Asym}} = \frac{\chi_{A_1 A_2}^{\text{NG}}}{\chi_{A_1 A_2}^{\text{NG}}} \bigg|_{\tau_1 = \tau_2 = \sigma_1 = \sigma_2 = 0} \exp \left( u^T M_3 u \right).
\]  

(6)

The normalized Wigner characteristic function \(\chi_{A_1 A_2}^{\text{NG}}\) of NGTMSV states is obtained as

\[
\chi_{A_1 A_2}^{\text{NG}}(\tau_1, \sigma_1, \tau_2, \sigma_2) = \left( P_{\text{NG}}^{\text{Asym}} \right)^{-1} \chi_{A_1 A_2}^{\text{NG}}(\tau_1, \sigma_1, \tau_2, \sigma_2).
\]  

(7)

Wigner characteristic functions of several special states can be obtained from Eq. (7) as limiting cases. By taking \(T_1 = T_2 = 1\) in the symmetric PS case, we obtain the Wigner characteristic function of the ideal PSTMSV state \(\tilde{a}_{1,2}^{n_1,2n_2}\) (TMSV). Similarly, taking \(T_1 = T_2 = 1\) in the symmetric PA case provides the Wigner characteristic function of the ideal PATMSV state \(\tilde{a}_{1,2}^{m_1,2m_2}\) (TMSV).

III. TELEPORTATION USING NGTMSV RESOURCE STATES

Having derived the Wigner characteristic function of the NGTMSV states, we proceed to derive the fidelity for teleporting input coherent and squeezed vacuum states. We follow the Braunstein-Kimble (BK) protocol for teleporting an unknown input quantum state between two distant physical systems [1]. To begin with, an entangled resource is shared between Alice and Bob. An unknown input quantum state to be teleported is provided to Alice. The density operator of the entangled resource state and the unknown input state is represented by \(\rho_A \rho'_A\) and \(\rho_{in}\), respectively. Their representation in terms of Wigner characteristic function are \(\chi_{A'_1 A'_2} (\Lambda_1, \Lambda_2)\) and \(\chi_{in}(\Lambda_{in})\), respectively.

Alice combines her mode and the single-mode input state using a balanced beam splitter. After that, the two output modes of the beam splitter are subjected to homodyne measurement by Alice and the results are classically communicated to Bob. Based on the results, Bob displaces his mode \(A'_2\), and the resultant mode is denoted by ‘out’. The mode ‘out’ corresponds to the teleported state. The Wigner characteristic function allows us to write the teleported state as a product of the input state and the entangled resource state [38]:

\[
\chi_{out}(\tau_2, \sigma_2) = \chi_{in}(\tau_2, \sigma_2) \chi_{A'_1 A'_2}(\tau_2, -\sigma_2, \tau_2, \sigma_2). \tag{8}
\]

We define fidelity of teleportation as the overlap between the single mode input state \(\rho_{in}\) and the teleported state \(\rho_{out}\) to quantify the success of the protocol:

\[
F = \operatorname{Tr}[\rho_{in} \rho_{out}],
\]

\[
= \frac{1}{2\pi} \int d^2 A_2 \chi_{in}(A_2) \chi_{out}(-A_2). \tag{9}
\]

It has been shown that a maximum fidelity of 1/2 can be achieved without using a shared entangled state [39, 40]. Hence, successful quantum teleportation is marked by the magnitude of the fidelity rising above the classical limit of 1/2. We require an infinitely entangled resource state to achieve perfect teleportation with unit fidelity.

A. Teleporting an input coherent state

We now move on to compute the fidelity for teleporting an input coherent state via NGTMSV resource states (7). Using the Wigner characteristic function of the coherent state [Eq. (A14) of Appendix A], the fidelity can be evaluated using Eq. (9), which turns out to be

\[
F_{\text{coh}}^{\text{TMSV}} = \hat{F}_1 \exp (u^T M_4 u), \tag{10}
\]

where the matrix \(M_4\) is given in Eq. (B7) of the Appendix B. By setting \(n_1 = n_2 = 0\) and \(T_1 = T_2 = 1\) in Eq. (10), we obtain the fidelity of quantum teleportation using the TMSV resource state:

\[
F_{\text{TMSV}}^{\text{TMSV}} = \frac{1 + \tanh r}{2}. \tag{11}
\]

After deriving the analytical expression of fidelity for teleporting input coherent state, we now proceed to numerical investigation of the fidelity. We first numerically optimize the transmissivities of the beam splitters to maximize the fidelity, and the results are shown in Fig. 2.
Fig. 2. Optimized fidelity for teleporting input coherent state as a function of squeezing parameter for (a) PSTMSV states, (b) PATMSV states, and (c) PCTMSV states. The transmissivities of the beam splitters are optimized in order to maximize the fidelity.

While symmetric PSTMSV states show an advantage over the TMSV state, asymmetric PSTMSV states underperform as compared to the TMSV state. Further, the optimal transmissivity is one in this case; hence, the results correspond to ideal PSTMSV states.

We then observe that neither symmetric nor asymmetric PATMSV states improve the performance compared to the TMSV state. In this case, the optimal transmissivity also turns out to be one; therefore, these results correspond to ideal PATMSV states. We note that the quantum teleportation using ideal PSTMSV and PATMSV states as a resource has already been investigated in Refs. [18, 22].

Finally, we observe that only symmetric PCTMSV states results in the enhancement of fidelity over the TMSV state. As can be seen in Fig. 2(c), this enhancement is observed till a certain squeezing threshold beyond which the fidelity is optimized at unit transmissivity. As we can see from the schematic in Fig. 1 that the output state at unit transmissivity in the case of PC operation is simply the TMSV state. Therefore, the fidelity at unit transmissivity is equal to that of the TMSV state. This optimization of fidelity for PCTMSV states has been performed in Ref. [21]; however, their results for PCTMSV states show a lower fidelity than the TMSV state in the region where our results show equal fidelity for PCTMSV and TMSV states. Further, Asym 1,2-PCTMSV state also enhances the fidelity as compared to TMSV state. We note that Asym 1,2-PCTMSV state can be generated by setting the parameters as $n_1 = n_2 = 1$ and $m_1 = m_2 = 2$.

Fig. 3. Optimal beam splitter transmissivities as a function of squeezing parameter for different PCTMSV states. The beam splitter transmissivities have been truncated at the minimum squeezing where the TMSV state and the PCTMSV state have the same fidelity.

The optimal beam splitter transmissivities for different PCTMSV states are shown in Fig. 3. The fidelity for the symmetric PCTMSV states is maximized for $T_1 = T_2$. However, for the Asym 1,2-PCTMSV state, the fidelity is maximized when $T_1 \neq T_2$.

1. Relative enhancement in fidelity

In the previous section, we studied the absolute performance of the NGTMSV states. In order to assess the squeezing value that renders maximum enhancement in the fidelity relative to the TMSV state, we define a figure of merit as the difference in teleportation fidelity between the NGTMSV states and the TMSV state as

$$\Delta F^{NG} = F^{NG} - F^{TMSV}. \quad (12)$$
We optimize $\Delta F_{NG}$ over the transmissivity parameters and since $F_{TMSV}$ is independent of transmissivity, the optimized value of $\Delta F_{NG}$ can be simply given by

$$\Delta F_{NG}^{opt} = F_{NG}^{opt} - F_{TMSV}.$$

We plot $\Delta F_{opt}$ as a function of squeezing parameter in Fig. 4 for the symmetric PSTMSV and PCTMSV states. While $\Delta F_{opt}$ for PSTMSV states is maximized at intermediate squeezing, $\Delta F_{opt}$ for PCTMSV states is maximized in the limit of zero squeezing. The optimal transmissivity maximizing $\Delta F_{NG}$ is same as Fig. 3, which is obvious from Eq. (13). We have not considered PATMSV states as they do not provide any advantage over the original TMSV state. We shall see later in this article that PATMSV states may be advantageous over the TMSV state in the case of teleportation of an input squeezed vacuum state.

![FIG. 4. The optimized difference of teleportation fidelity between the NGTMSV and the TMSV states, $\Delta F_{opt}$, as a function of squeezing parameter for different non-Gaussian states. The transmissivities have been optimized to maximize $\Delta F_{NG}$.](image)

### 2. Relative enhancement in fidelity per trial

As discussed earlier, non-Gaussian operations are probabilistic, and their success probability, $P_{NG}$, which can be thought of as the fraction of successful non-Gaussian operations per trial, quantifies the resource utilization. It might happen that $P_{NG}$ at the optimal squeezing and transmissivity parameters maximizing $\Delta F_{NG}$ is low; which represents a low resource utilization. For instance, $\Delta F_{NG}$ is maximized at unit transmissivity for the PSTMSV resource states; but the success probability approaches zero in the unit transmissivity limit. Therefore, working at optimal squeezing and transmissivity parameters maximizing $\Delta F_{NG}$ may not represent the best scenario. In order to gain maximum advantage, we need to work at a transmissivity that renders the product of $\Delta F_{NG}$ and $P_{NG}$ maximum. This can be achieved by finding an optimal trade-off between the success probability and $\Delta F_{NG}$ by adjusting the transmissivities of the beam splitters for a given squeezing. We plot the optimized product $(P \times \Delta F)_{opt}$ as a function of squeezing in Fig. 5. The results reveal that 1-Sym PC operation provides the maximum advantage when the success probability is taken into account. Further, among the considered non-Gaussian operations in Fig. 5, all different PC operations outperform the PS operations. Figure 6 shows the optimal beam splitter transmissivities maximizing the product $P_{NG} \times \Delta F_{NG}$ as a function of the squeezing parameter.

![FIG. 5. The optimized product $(P \times \Delta F)_{opt}$ as a function of squeezing parameter. The transmissivities have been optimized to maximize the product.](image)

![FIG. 6. Optimal beam splitter transmissivities maximizing the product $P_{NG} \times \Delta F_{NG}$ as a function of squeezing parameter for different NGTMSV states. The beam splitter transmissivities have been truncated at the minimum squeezing where the TMSV state and the PCTMSV state have the same fidelity.](image)

As shown in Fig. 1, the auxiliary modes are initialized to Fock states in PA and PC operations. These Fock states can be generated by photon-number measurement on one mode of the TMSV state. The success probability of producing Fock state $|m\rangle$ is

$$P_{|m\rangle} = (1 - \lambda^2)\lambda^{2m}.$$

(14)
Therefore, the effective probability can be defined as
\[ P_{\text{eff}} = (P_{m_1} P_{m_2})^{P^{\text{NG}}}. \]  
(15)

We now optimize the product \( P_{\text{eff}}^{\text{PC}} \Delta F_{\text{opt}}^{\text{PC}} \) over the transmissivity parameters and the result is shown in Fig. 7.

For 1-Sym PC operation, the maximum value of the optimized product \( P_{\text{eff}} \Delta F_{\text{opt}} \) is \( \approx 5 \times 10^{-5} \). In contrast, for 1-Sym PS operation, the maximum value of the optimized product \( P \Delta F_{\text{opt}} \) is \( \approx 1 \times 10^{-3} \). Therefore, 1-Sym PS operation (Sym 1-PSTMSV state) turns out to be the most beneficial operation (state) for teleporting an input coherent state.

B. Teleporting an input squeezed vacuum state

We now derive the analytical fidelity expression for teleporting an input squeezed vacuum state with squeezing \( \epsilon \) using NGTMSV resource states. The expression of the Wigner characteristic function of squeezed vacuum state is given in Eq. (A15) of the Appendix A. Using the general formula for the fidelity of teleportation (9), we can evaluate the fidelity in this case, which turns out to be
\[ F_{\text{NG sqv}}^{\text{PC}} = \hat{F}_1 \exp \left( u^T M_5 u \right), \]  
(16)
where the matrix \( M_5 \) is given in Eq. (B9) of the Appendix B. Setting \( n_1 = n_2 = 0 \) and \( T_1 = T_2 = 1 \) in Eq. (16) yields the fidelity of teleporting an input squeezed vacuum state using TMSV resource state:
\[ F_{\text{TMSV}}^{\text{PC}} = \left[ \frac{\left( 1 + \tanh (r + \epsilon) \right) \left( 1 + \tanh (r - \epsilon) \right)}{2} \right]^{1/2}. \]  
(17)

On putting \( \epsilon = 0 \) in Eq. (17), we obtain the fidelity of teleporting an input coherent state using TMSV resource state (11).

We now numerically analyze the fidelity of teleporting an input squeezed vacuum state. We first optimize the transmissivities of the beam splitters to maximize the fidelity and show the results in Fig. 8. We have taken the squeezing of the input squeezed vacuum state \( \epsilon = 1.7 \). The teleportation results for an input squeezed vacuum state via PSTMSV and PCTMSV resource states show a similar trend as observed for the teleportation of an input coherent state. However, the fidelity of teleportation via symmetric PATMSV resource states outperforms the TMSV state, which contrasts with the results obtained...
for the teleportation of an input coherent state. The results for the input coherent state are the same as the input squeezed vacuum state with $\epsilon = 0$, which represent one extreme, where the PATMSV resource states provide no advantage over the TMSV state. The results shown here for $\epsilon = 1.7$, which is the maximum achievable squeezing in the lab [4], represent the other extreme. It should be noted that the fidelity via PCTMSV resource states always lies below the classical limit $1/2$. Further, for PSTMSV and PATMSV resource states, the fidelity goes above the classical limit $1/2$ at a certain threshold squeezing of the resource state.

The optimal transmissivity for the PSTMSV and PATMSV resource states is one; therefore, the results for these cases correspond to ideal PSTMSV and PATMSV resource states, which have been studied in Refs. [18, 22]. For different PCTMSV resource states, which have also been discussed in Ref. [21], the corresponding optimal beam splitter transmissivities are shown in Fig. 9. The behavior of the optimal transmissivities is similar to that of the teleportation of input coherent state [Fig. 3].

We now turn to analyze the relative enhancement in performance by considering the difference in teleportation fidelity between the NGTMSV and the TMSV states. The results for different NGTMSV states is that the fidelity is almost constant for small $\epsilon$, and the inflection in fidelity is observed for higher $\epsilon$. Therefore, the analysis for input coherent state will resemble that of input squeezed vacuum state.

We now analyze the fidelity as a function of the squeezing $\epsilon$ of the input squeezed vacuum state. While the absolute fidelity decreases as $\epsilon$ is increased for NGTMSV and TMSV resource states, an interesting observation can be made by analyzing $\Delta F_{\text{opt}}$ as a function of $\epsilon$. In Fig. 10, we plot $\Delta F_{\text{opt}}$ as a function of the squeezing $\epsilon$ of input squeezed vacuum state for different squeezing of the resource states. As $\epsilon$ is increased in the case of Sym 1-PSTMSV resource state, the optimized difference $\Delta F_{\text{opt}}^\text{PS}$ improves, attains a maximum value, and then starts decreasing. The Sym 1-PATMSV resource state outperforms the TMSV state in a region of $\epsilon$ indicated by the positive value of the optimized difference $\Delta F_{\text{opt}}^\text{PA}$. Here too, we notice that as $\epsilon$ increases, the optimized difference $\Delta F_{\text{opt}}^\text{PA}$ improves, attains a maximum value, and then starts decreasing. For Sym 1-PCTMSV resource state, the optimized difference $\Delta F_{\text{opt}}^\text{PC}$ continuously decreases as $\epsilon$ is increased. One interesting behavior observed for different NGTMSV states is that the fidelity is almost constant for small $\epsilon$, and the inflection in fidelity is observed for higher $\epsilon$. Therefore, the analysis for input coherent state will resemble that of input squeezed vacuum state for small $\epsilon$ ($<0.5$).

2. Relative enhancement in fidelity per trial

We now take into account the success probability of non-Gaussian operations as well as the probability of generation of Fock states. Since the success probability is independent of $\epsilon$, the analysis in Sec. III B 1 demonstrates the trend for quantities $(P \times \Delta F)^\text{NG}_{\text{opt}}$ or $(P_{\text{eff}} \times \Delta F)^\text{NG}_{\text{opt}}$ with respect to the squeezing $\epsilon$ of the input squeezed vacuum state.

We now take into account the success probability of non-Gaussian operations as well as the probability of generation of Fock states. Since the success probability is independent of $\epsilon$, the analysis in Sec. III B 1 demonstrates the trend for quantities $(P \times \Delta F)^\text{NG}_{\text{opt}}$ or $(P_{\text{eff}} \times \Delta F)^\text{NG}_{\text{opt}}$ with respect to the squeezing $\epsilon$ of the input squeezed vacuum state.
FIG. 11. The optimized difference of teleportation fidelity between the NGTMSV states and the TMSV state, $\Delta F_{\text{opt}}^{\text{NG}}$, as a function of squeezing parameter $\epsilon$ of the input squeezed vacuum state for different squeezing of the TMSV state. The states considered for different panels are (a) Sym 1-PSTMSV state, (b) Sym 1-PATMSV state, (c) Sym 1-PCTMSV state. The transmissivities have been optimized to maximize the product.

We now examine $P_{\text{opt}}^{\text{NG}} \times \Delta F_{\text{opt}}^{\text{NG}}$ as a function of the squeezing of resource states. The results are shown in Fig. 12. We observe that the 1-Sym PS operation provides maximum advantage when the success probability is taken into account. This contrasts with the teleportation of input coherent state, where 1-Sym PC operation provides the maximum advantage. However, we note that 1-Sym PC operation is not far behind 1-Sym PS operation for the case of input squeezed vacuum state teleportation. We have also shown the optimal beam splitter transmissivities maximizing the product $P_{\text{opt}}^{\text{NG}} \times \Delta F_{\text{opt}}^{\text{NG}}$ as a function of squeezing parameter in Fig. 13.

Finally, we plot the optimized product $(P_{\text{opt}} \times \Delta F)_{\text{opt}}^{\text{NG}}$ as a function of the squeezing parameter in Fig. 14. We see that 1-Sym PA operation outperforms 1-Sym PC operation. However, 1-Sym PS operation outperforms both 1-Sym PA and 1-Sym PC operations as we can easily verify by comparison with Fig. 12.

IV. CONCLUSION

In summary, we have provided a comprehensive analysis of the advantages of NGTMSV states, including PSTMSV, PATMSV, and PCTMSV states, when the
success probability of non-Gaussian operations and probability of generation of the Fock states are taken into account. To that end, we obtained a unified analytical expression of the Wigner characteristic function of the NGTMSV states. By choosing the input photon state and the number of detected photons in the auxiliary modes, we can subtract, add, or catalyze an arbitrary number of photons from the TMSV state. Further, the transmissivity of the beam splitters and the squeezing of the resource state appear as parameters in the expression. We then derived the fidelity of teleporting input coherent and squeezed vacuum states using the NGTMSV resource states.

In the special case, when the input Fock state equals the detected Fock state in the auxiliary modes, we obtain the PCTMSV states and the quantum teleportation using such states has been considered in Refs. [20, 21]. Further, when the input Fock state is less or greater than the detected Fock state in the auxiliary modes, we obtain the ideal PSTMSV or PATMSV states in the unit transmissivity limit, and the quantum teleportation using these states has been considered in Refs. [18, 22].

The investigation of the relative performance of NGTMSV and TMSV states by analyzing the fidelity difference between these states shows that PSTMSV states provide maximum advantage at intermediate squeezing. Further, PCTMSV states provide maximum advantage at small squeezing. Taking the success probability of non-Gaussian operations and the probability of Fock state generation into account, we find that the symmetric one-PSTMSV state is most beneficial for the teleportation of input coherent and squeezed vacuum states.

We hope that the unified Wigner characteristic function of the NGTMSV states derived in this article will be helpful in the study of nonclassicality [41], non-Gaussianity [42], and nonlocality [43] in these states and its considerations in entanglement distillation, entanglement swapping, and quantum illumination protocols.

ACKNOWLEDGEMENT

This is the second article in a publication series written in the celebration of the completion of 15 years of IISER Mohali. Both the authors thank Mohak Sharma for a careful reading of the final version of the draft. C.K. acknowledges the financial support from DST/ICPS/QuST/Theme-1/2019/General Project number Q-68.

Appendix A: Brief description of CV systems and its phase space description

Our system of interest is an $n$-mode CV system, whose $i$th mode can be expressed by a pair of Hermitian quadrature operators $\hat{q}_i$ and $\hat{p}_i$. We arrange these $n$ pairs in the form of a column vector as [44–48]

$$\hat{\xi} = (\hat{\xi}_i) = (\hat{q}_1, \hat{p}_1, \ldots, \hat{q}_n, \hat{p}_n)^T, \quad i = 1, 2, \ldots, 2n. \tag{A1}$$

This permits us to write the bosonic commutation relation between them compactly as $(h=1)$

$$[\hat{q}_i, \hat{q}_j] = i\Omega_{ij}, \quad (i, j = 1, 2, \ldots, 2n), \tag{A2}$$

where $\Omega$ is the $2n \times 2n$ matrix given by

$$\Omega = \bigoplus_{k=1}^{n} \omega = \begin{pmatrix} \omega & \cdots & \omega \\ \cdots & \cdots & \cdots \\ \omega & \cdots & \omega \end{pmatrix}, \quad \omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{A3}$$

The photon annihilation and creation operators $\hat{a}_i$ and $\hat{a}_i^\dagger$ ($i = 1, 2, \ldots, n$) are given as

$$\hat{a}_i = \frac{1}{\sqrt{2}}(\hat{q}_i + i\hat{p}_i), \quad \hat{a}_i^\dagger = \frac{1}{\sqrt{2}}(\hat{q}_i - i\hat{p}_i). \tag{A4}$$

We shall be concerned with two symplectic operations discussed below [44, 47].

**Beam splitter operation:** The two mode beam splitter operation acts on the quadrature operators $\xi = (\hat{q}_i, \hat{p}_i, \hat{q}_j, \hat{p}_j)^T$ of a two mode system as follows:

$$B_{ij}(T) = \begin{pmatrix} \sqrt{T} \mathbb{1}_2 & \sqrt{1-T} \mathbb{1}_2 \\ -\sqrt{1-T} \mathbb{1}_2 & \sqrt{T} \mathbb{1}_2 \end{pmatrix}, \tag{A5}$$

where $\mathbb{1}_2$ is the $2 \times 2$ identity matrix.

**Two mode squeezing operation:** The two mode squeezing operation acts on the quadrature operators $(\hat{q}_i, \hat{p}_i, \hat{q}_j, \hat{p}_j)^T$ as follows:

$$S_{ij}(r) = \begin{pmatrix} \cosh r \mathbb{1}_2 & \sinh r Z \\ \sinh r Z & \cosh r \mathbb{1}_2 \end{pmatrix}, \tag{A6}$$

where $Z = \text{diag}(1, -1)$. The TMSV state is obtained by the action of two mode squeezing operator on two single mode vacuum state.
1. Phase space description

Working with Wigner characteristic function turns out to be convenient in quantum teleportation. We can find the Wigner characteristic function corresponding to a density operator $\hat{\rho}$ of an $n$-mode quantum system using the relation

$$\chi(\Lambda) = \text{Tr}[\hat{\rho} \exp(-i\Lambda^T \Omega \xi)], \quad (A7)$$

where $\xi = (\hat{q}_1, \hat{p}_1, \ldots, \hat{q}_n, \hat{p}_n)^T$, $\Lambda = (\Lambda_1, \Lambda_2, \ldots, \Lambda_n)^T$ with $\Lambda_i = (\tau_i, \sigma_i)^T \in \mathbb{R}^2$. For instance, Eq. (A7) can be used to evaluate the Wigner characteristic function of a single mode Fock state $|n\rangle$:

$$\chi_{|n\rangle}(\tau, \sigma) = \exp \left[ -\frac{\sigma^2}{4} L_n \left( \frac{\tau^2}{2} + \frac{\sigma^2}{2} \right) \right], \quad (A8)$$

where $L_n(x)$ is the Laguerre polynomial. Writing the above equation in terms of exponential generating function, we get

$$\chi_{|n\rangle}(\tau, \sigma) = \exp \left[ -\frac{\sigma^2}{4} \right] \tilde{F} e^{2st + s(\tau + i\sigma) - t(\tau - i\sigma)}, \quad (A9)$$

with

$$\tilde{F} = \frac{1}{2n!} \sum_{s} \frac{\sigma^n}{\partial^n \partial^n \{s\}_{s=t=0}}. \quad (A10)$$

We define the first order moments for an $n$-mode CV system as

$$d = \langle \hat{\xi} \rangle = \text{Tr}[\hat{\rho} \hat{\xi}]. \quad (A11)$$

Further, the second order moments can be written in the form of a real symmetric $2n \times 2n$ matrix, known as covariance matrix:

$$V = (V_{ij}) = \frac{1}{2} \langle \{\Delta\hat{\xi}_i, \Delta\hat{\xi}_j\} \rangle, \quad (A12)$$

where $\Delta\hat{\xi}_i = \hat{\xi}_i - \langle \hat{\xi}_i \rangle$, and $\{,\}$ denotes anti-commutator.

A special class of states, whose Wigner characteristic function is a Gaussian, are known as Gaussian states. Such states can be uniquely specified via its first and second order moments. The general formula for the Wigner characteristic function (A7) simplifies as follows for Gaussian states [47, 49]:

$$\chi(\Lambda) = \exp \left[ -\frac{1}{2} \Lambda^T (\Omega V \Omega^T) \Lambda - i(\Omega d)^T \Lambda \right], \quad (A13)$$

where $d$ and $V$ represents the displacement vector and the covariance matrix of the Gaussian state. The Wigner characteristic function of a single mode coherent state with displacement $d = (d_x, d_p)^T$ evaluates to

$$\chi_{\text{coh}}(\Lambda) = \exp \left[ -\frac{1}{4}(\tau^2 + \sigma^2) - i(\tau d_p - \sigma d_x) \right]. \quad (A14)$$

The Wigner characteristic function of a single mode squeezed vacuum state turns out to be

$$\chi_{\text{sqv}}(\Lambda) = \exp \left[ -\frac{1}{4}(\tau^2 e^{2\tau} + \sigma^2 e^{-2\tau}) \right]. \quad (A15)$$

Let $U(S)$ represent the infinite dimensional unitary representation for a homogeneous symplectic transformation $S$. Given the density operator transformation rule as $\rho \rightarrow U(S)\rho U(S)^\dagger$, the transformation of the displacement vector, covariance matrix and Wigner characteristic function turns out to be [44, 47, 49]

$$d \rightarrow Sd, \quad V \rightarrow SVS^T, \quad \text{and} \quad \chi(\Lambda) \rightarrow \chi(S^{-1}\Lambda). \quad (A16)$$

Appendix B: Matrices appearing in the Wigner characteristic function, and the fidelity of teleportation using NGTMSV states.

1. Wigner characteristic function of the NGTMSV states

Here we provide the explicit forms of the matrices $M_1$, $M_2$, and $M_3$, which appear in the Wigner characteristic function (4) of the NGTMSV states. The matrix $M_1$ is given as

$$M_1 = \frac{-1}{4a_0} \begin{pmatrix} a_1 & 0 & -a_2 & 0 \\ 0 & a_1 & 0 & a_2 \\ -a_2 & 0 & a_1 & 0 \\ 0 & a_2 & 0 & a_1 \end{pmatrix}, \quad (B1)$$

where,

$$a_0 = 6 - \alpha^2 t_1^2 t_2^2, \quad a_1 = 6 + \alpha^2 t_1^2 t_2^2, \quad a_2 = 2\alpha^2 t_1 t_2.$$  \quad (B2)

Here $t_i = \sqrt{2}t_i (i = 1, 2)$. Further $\alpha = \sinh r$ and $\beta = \cosh r$. The matrix $M_2$ is given by

$$M_2 = \frac{1}{a_0} \begin{pmatrix} b_1 & i b_1 & b_2 & -i b_2 \\ -b_1 & i b_1 & -b_2 & -i b_2 \\ b_3 & -i b_3 & b_4 & i b_4 \\ -b_3 & -i b_3 & b_4 & i b_4 \end{pmatrix}, \quad (B3)$$

where,

$$b_1 = \beta^2 r_1, \quad b_5 = -\alpha^2 r_1 t_1 t_2^2, \quad b_2 = -\alpha \beta r_1 t_2, \quad b_6 = \beta \alpha r_1 t_2, \quad b_3 = -\alpha \beta r_2 t_1, \quad b_7 = \alpha \beta r_2 t_1, \quad b_4 = \beta^2 r_2, \quad b_8 = -\alpha^2 r_2 t_1^2 t_2^2. \quad (B4)$$
Here \( r_i = \sqrt{1 - T_i} \) \((i = 1, 2)\). The matrix \( M_3 \) is given by

\[
M_3 = \frac{1}{a_0} \begin{pmatrix}
0 & c_1 & c_2 & 0 & 0 & c_3 & c_4 & 0 \\
0 & c_1 & 0 & 0 & c_2 & c_3 & 0 & c_4 \\
0 & 0 & c_1 & 0 & 0 & c_2 & 0 & c_3 \\
0 & 0 & 0 & c_1 & 0 & 0 & c_2 & c_3 \\
0 & c_2 & c_5 & 0 & 0 & c_6 & c_7 & 0 \\
0 & c_2 & 0 & 0 & c_5 & c_6 & 0 & c_7 \\
0 & c_3 & 0 & 0 & c_6 & c_8 & 0 & c_9 \\
0 & c_3 & 0 & 0 & 0 & c_6 & c_8 & 0 \\
0 & c_4 & 0 & c_7 & 0 & c_9 & 0 & c_{10} \\
0 & c_4 & c_7 & 0 & c_9 & 0 & 0 & c_{10}
\end{pmatrix}, \tag{B5}
\]

where,

\[
c_1 = \beta^2 r_1^2, \quad c_2 = \alpha \beta r_1 r_2 t_2, \quad c_3 = \beta^2 t_1 - \alpha^2 t_2^2, \\
c_4 = -\alpha \beta r_1 r_2 t_1, \quad c_5 = \beta^2 r_2^2, \\
\]

\[
\begin{align*}
M_4 &= \frac{1}{d_0} \begin{pmatrix}
0 & c_1 & d_1 & c_2 & 0 & c_3 & d_4 & 0 \\
0 & c_1 & 0 & 0 & d_1 & c_2 & 0 & c_3 \\
0 & 0 & c_1 & 0 & 0 & d_1 & 0 & c_3 \\
0 & 0 & 0 & c_1 & 0 & 0 & d_1 & c_2 \\
0 & d_1 & c_5 & 0 & 0 & c_6 & d_3 & 0 \\
0 & d_1 & 0 & 0 & c_5 & c_6 & 0 & d_3 \\
0 & d_2 & c_6 & 0 & 0 & c_8 & d_4 & 0 \\
0 & d_2 & 0 & 0 & c_6 & c_8 & 0 & d_4 \\
0 & c_4 & 0 & d_3 & 0 & c_4 & 0 & d_3 \\
0 & c_4 & d_3 & 0 & c_4 & 0 & d_3 & 0
\end{pmatrix}, \tag{B7}
\end{align*}
\]

where,

\[
\begin{align*}
d_0 &= 2\beta (\beta - \alpha t_1 t_2), \\
d_1 &= \beta^2 r_1 r_2, \\
d_2 &= \beta (2\beta t_1 - \alpha t_2 (t_2^2 + 1)), \\
d_3 &= \beta (2\beta t_2 - \alpha t_1 (t_2^2 + 1)), \\
d_4 &= \alpha r_1 r_2 (2\beta - \alpha t_1 t_2).
\end{align*}
\tag{B8}
\]

\[
2. \quad \text{Fidelity for input coherent state using NGTMSV states}
\]

The explicit form of matrix \( M_5 \) appearing in the fidelity of teleportation of input coherent state using NGTMSV states (10) is

\[
M_5 = \frac{1}{e_0} \begin{pmatrix}
e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \\
e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 \\
e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} \\
e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} & e_{11} \\
e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} & e_{11} & e_{12} \\
e_6 & e_7 & e_8 & e_9 & e_{10} & e_{11} & e_{12} & e_{13} \\
e_7 & e_8 & e_9 & e_{10} & e_{11} & e_{12} & e_{13} & e_{14} \\
e_8 & e_9 & e_{10} & e_{11} & e_{12} & e_{13} & e_{14} & e_{15} \\
e_9 & e_{10} & e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} \\
e_{10} & e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} & e_{17} \\
e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} & e_{17} & e_{18} \\
e_{12} & e_{13} & e_{14} & e_{15} & e_{16} & e_{17} & e_{18} & e_{19} \\
e_{13} & e_{14} & e_{15} & e_{16} & e_{17} & e_{18} & e_{19} & e_{20} \\
e_{14} & e_{15} & e_{16} & e_{17} & e_{18} & e_{19} & e_{20} & e_{21} \end{pmatrix}, \tag{B9}
\]

where,

\[
\begin{align*}
& e_0 = 2 (a_0 \delta + a_1), \\
& e_1 = -\beta^2 r_1^2, \\
& e_2 = \beta^2 r_2^2 \left( \frac{a_0}{a_3} + \delta \right), \\
& e_3 = \beta r_1 r_2 \left( \delta \left( \frac{a_0}{\sqrt{a_3}} + \alpha t_1 t_2 \right) + \frac{a_1}{\sqrt{a_3}} - \alpha t_1 t_2 \right), \\
& e_4 = \beta^2 \gamma r_1 r_2, \\
& e_5 = -\alpha \beta \gamma r_1 r_2 t_1, \\
& e_6 = \frac{e_0}{a_0} \left( 1 + \alpha^2 t_1^2 - \alpha^2 r_2^2 \left( \frac{a_0}{a_3} + \alpha \gamma r_1 r_2 \right) \right), \\
& e_7 = -\alpha \beta r_1 r_2 t_1 \left( \frac{a_0}{a_3} + \delta \right), \\
& e_8 = \alpha \beta \gamma r_1 r_2 t_1, \\
& e_9 = -\beta^2 \gamma r_1 t_2, \\
& e_{10} = \beta^2 r_2^2 \left( \frac{a_0}{a_3} + \delta \right), \\
& e_{11} = -\alpha \beta r_1 r_2 t_1 \left( \frac{a_0}{a_3} + \delta \right), \\
& e_{12} = \alpha \beta \gamma r_1 r_2 t_1, \\
& e_{13} = -\alpha \beta \gamma r_1 r_2 t_1, \\
& e_{14} = \frac{e_0}{a_0} \left( 1 + \alpha^2 \gamma r_1 t_1^2 - \alpha^2 r_2^2 \left( \frac{a_0}{a_3} + \alpha \gamma r_1 r_2 \right) \right), \\
& e_{15} = -\alpha \beta \gamma r_1 r_2 t_1, \\
& e_{16} = \alpha \gamma r_1 t_2 \left( \frac{a_0}{a_3} + \delta \right), \\
& e_{17} = \alpha \gamma r_1 r_2 t_1 \left( \frac{2\beta^2}{\sqrt{a_3}} - \alpha t_1 t_2 \frac{a_0}{a_3} + \frac{a_1}{\sqrt{a_3}} + \beta \right), \\
& e_{18} = \alpha \gamma r_1 r_2 t_1 t_2, \\
& e_{19} = -\alpha \gamma r_1 r_2 t_1 t_2, \\
& e_{20} = \alpha \gamma r_1 r_2 t_1 t_2 \left( \frac{a_0}{a_3} + \delta \right),
\end{align*}
\tag{B10}
\]

with \( \gamma = \sinh(2\epsilon) \), \( \delta = \cosh(2\epsilon) \) and \( a_3 = (a_1 - a_2) \).
[1] S. L. Braunstein and H. J. Kimble, Teleportation of continuous quantum variables, Phys. Rev. Lett. 80, 869 (1998).
[2] M. Ban, Quantum dense coding via a two-mode squeezed-vacuum state, Journal of Optics B: Quantum and Semiclassical Optics 1, L9 (1999).
[3] P. van Loock and S. L. Braunstein, Unconditional teleportation of continuous-variable entanglement, Phys. Rev. A 61, 010302 (1999).
[4] H. Vahlbruch, M. Mehmert, K. Danzmann, and R. Schnabel, Detection of 15 db squeezed states of light and their application for the absolute calibration of photoelectric quantum efficiency, Phys. Rev. Lett. 117, 110801 (2016).
[5] P. Huang, G. He, J. Fang, and G. Zeng, Performance improvement of continuous-variable quantum key distribution via photon subtraction, Phys. Rev. A 87, 012317 (2013).
[6] H.-X. Ma, P. Huang, D.-Y. Bai, S.-Y. Wang, W.-S. Bao, and G.-H. Zeng, Continuous-variable measurement-device-independent quantum key distribution with photon subtraction, Phys. Rev. A 97, 042329 (2018).
[7] Y. Guo, W. Ye, H. Zhong, and Q. Liao, Continuous-variable quantum key distribution with non-gaussian quantum catalysts, Phys. Rev. A 99, 032327 (2019).
[8] W. Ye, H. Zhong, Q. Liao, D. Huang, L. Hu, and Y. Guo, Improvement of self-referenced continuous-variable quantum key distribution with quantum photon catalysis, Opt. Express 27, 17186 (2019).
[9] C. Kumar, J. Singh, S. Bose, and Arvind, Coherence-assisted non-gaussian measurement-device-independent quantum key distribution, Phys. Rev. A 100, 052329 (2019).
[10] L. Hu, M. Al-amri, Z. Liao, and M. S. Zubairy, Continuous-variable quantum key distribution with non-gaussian operations, Phys. Rev. A 102, 012608 (2020).
[11] R. Birrittella, J. Mimih, and C. C. Gerry, Multiphoton quantum interference at a beam splitter and the approach to heisenberg-limited interferometry, Phys. Rev. A 86, 063828 (2012).
[12] R. Carranza and C. C. Gerry, Photon-subtracted two-mode squeezed vacuum states and applications to quantum optical interferometry, J. Opt. Soc. Am. B 29, 2581 (2012).
[13] D. Braun, P. Jian, O. Pinel, and N. Treps, Precision measurements with photon-subtracted or photon-added gaussian states, Phys. Rev. A 90, 013821 (2014).
[14] Y. Ouyang, S. Wang, and L. Zhang, Quantum optical interferometry via the photon-added two-mode squeezed vacuum states, J. Opt. Soc. Am. B 33, 1373 (2016).
[15] H. Zhang, W. Ye, C. Wei, Y. Xia, S. Chang, Z. Liao, and L. Hu, Improved phase sensitivity in a quantum optical interferometer based on multiphoton catalytic two-mode squeezed vacuum states, Phys. Rev. A 103, 013705 (2021).
[16] T. Opatrný, G. Kurizki, and D.-G. Welsch, Improvement on teleportation of continuous variables by photon subtraction via conditional measurement, Phys. Rev. A 61, 032302 (2000).
[17] A. Kitagawa, M. Takeoka, M. Sasaki, and A. Cheffes, Entanglement evaluation of non-gaussian states generated by photon subtraction from squeezed states, Phys. Rev. A 73, 042310 (2006).
[18] F. Dell’Anno, S. De Siena, L. Albano, and F. Illuminati, Continuous-variable quantum teleportation with non-gaussian resources, Phys. Rev. A 76, 022301 (2007).
[19] Y. Yang and F.-L. Li, Entanglement properties of non-gaussian resources generated via photon subtraction and addition and continuous-variable quantum-teleportation improvement, Phys. Rev. A 80, 022315 (2009).
[20] X.-x. Xu, Enhancing quantum entanglement and quantum teleportation for two-mode squeezed vacuum state by local quantum-optical catalysis, Phys. Rev. A 92, 012318 (2015).
[21] L. Hu, Z. Liao, and M. S. Zubairy, Continuous-variable entanglement via multiphoton catalysis, Phys. Rev. A 95, 012310 (2017).
[22] S. Wang, L.-L. Hou, X.-F. Chen, and X.-F. Xu, Continuous-variable quantum teleportation with non-gaussian entangled states generated via multiple-photon subtraction and addition, Phys. Rev. A 91, 063832 (2015).
[23] M. J. Fitch, B. C. Jacobs, T. B. Pittman, and J. D. Franson, Photon-number resolution using time-multiplexed single-photon detectors, Phys. Rev. A 68, 043814 (2003).
[24] M. Mićuda, O. c. v. Haderka, and M. Ježek, High-efficiency photon-number-resolving multichannel detector, Phys. Rev. A 78, 025804 (2008).
[25] T. J. Bartley and I. A. Walmsley, Directly comparing entanglement-enhancing non-gaussian operations, New Journal of Physics 17, 023038 (2015).
[26] T. J. Bartley, P. J. D. Crowley, A. Datta, J. Nunn, L. Zhang, and I. Walmsley, Strategies for enhancing quantum entanglement by local photon subtraction, Phys. Rev. A 87, 022313 (2013).
[27] L. Hu, M. Al-amri, Z. Liao, and M. S. Zubairy, Entanglement improvement via a quantum scissor in a realistic environment, Phys. Rev. A 100, 052322 (2019).
[28] Y. Mardani, A. Shafiei, M. Ghadimi, and M. Abdi, Continuous-variable entanglement distillation by cascaded photon replacement, Phys. Rev. A 102, 012407 (2020).
[29] J. Liu, Y. Maleki, and M. S. Zubairy, Optimal entanglement enhancing via conditional measurements, Phys. Rev. A 105, 062405 (2022).
[30] N. Quesada, L. G. Helt, J. Izac, J. M. Arrazola, R. Shahrokhshahi, C. R. Myers, and K. K. Sabapathy, Simulating realistic non-gaussian state preparation, Phys. Rev. A 100, 022341 (2019).
[31] K. Takase, J.-i. Yoshikawa, W. Asavanant, M. Endo, and A. Furusawa, Generation of optical schrödinger cat states by generalized photon subtraction, Phys. Rev. A 103, 013710 (2021).
[32] C. Kumar, Rishabh, and S. Arora, Realistic non-gaussian-operation scheme in parity-detection-based mach-zehnder quantum interferometry, Phys. Rev. A 105, 052437 (2022).
[33] M. D. Eisaman, J. Fan, A. Migdall, and S. V. Polyakov, Invited review article: Single-photon sources and detectors, Review of Scientific Instruments 82, 071101 (2011).
[34] A. Ourjoumtsev, R. Tualle-Brouri, and P. Grangier, Quantum homodyne tomography of a two-photon fock state, Phys. Rev. Lett. 96, 213601 (2006).
[35] M. Cooper, L. J. Wright, C. Söller, and B. J. Smith, Experimental generation of multi-photon fock states, Opt. Express 21, 5309 (2013).

[36] M. Bouillard, G. Boucher, J. F. Ortas, B. Kanseri, and R. Tualle-Brouri, High production rate of single-photon and two-photon fock states for quantum state engineering, Opt. Express 27, 3113 (2019).

[37] J. Tiedau, T. J. Bartley, G. Harder, A. E. Lita, S. W. Nam, T. Gerrits, and C. Silberhorn, Scalability of parametric down-conversion for generating higher-order fock states, Phys. Rev. A 100, 041802 (2019).

[38] P. Marian and T. A. Marian, Continuous-variable teleportation in the characteristic-function description, Phys. Rev. A 74, 042306 (2006).

[39] S. L. Braunstein, C. A. Fuchs, and H. J. Kimble, Criteria for continuous-variable quantum teleportation, Journal of Modern Optics 47, 267 (2000).

[40] S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and P. van Loock, Quantum versus classical domains for teleportation with continuous variables, Phys. Rev. A 64, 022321 (2001).

[41] W. Ge and M. S. Zubairy, Evaluating single-mode non-classicality, Phys. Rev. A 102, 043703 (2020).

[42] J. Park, J. Lee, K. Baek, and H. Nha, Quantifying non-gaussianity of a quantum state by the negative entropy of quadrature distributions, Phys. Rev. A 104, 032415 (2021).

[43] C. Kumar, G. Saxena, and Arvind, Continuous-variable clausner-horne bell-type inequality: A tool to unearth the nonlocality of continuous-variable quantum-optical systems, Phys. Rev. A 103, 042224 (2021).

[44] Arvind, B. Dutta, N. Mukunda, and R. Simon, The real symplectic groups in quantum mechanics and optics, Pramana 45, 471 (1995).

[45] S. L. Braunstein and P. van Loock, Quantum information with continuous variables, Rev. Mod. Phys. 77, 513 (2005).

[46] G. Adesso and F. Illuminati, Entanglement in continuous-variable systems: recent advances and current perspectives, J. Phys. A 40, 7821 (2007).

[47] C. Weedbrook, S. Pirandola, R. García-Patrón, N. J. Cerf, T. C. Ralph, J. H. Shapiro, and S. Lloyd, Gaussian quantum information, Rev. Mod. Phys. 84, 621 (2012).

[48] G. Adesso, S. Ragy, and A. R. Lee, Continuous variable quantum information: Gaussian states and beyond, Open Syst. Inf. Dyn. 21, 1440001, 47 (2014).

[49] S. Olivares, Quantum optics in the phase space, The European Physical Journal Special Topics 203, 3 (2012).