Noise-Protected Gate for Six-Electron Double-Dot Qubit

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Singlet-triplet spin qubits in six-electron double quantum dots, in moderate magnetic fields, can show superior immunity to charge noise. This immunity results from the symmetry of orbitals in the second energy shell of circular quantum dots: singlet and triplet states in this shell have identical change distributions. Our phase-gate simulations, which include $1/f$ charge noise from fluctuating traps, show that this symmetry is most effectively exploited if the gate operation switches rapidly between sweet spots deep in the (3,3) and (4,2) charge stability regions; fidelities very close to one are predicted if sub-nanosecond switching can be performed.

The spin degree of freedom of the few-electron quantum dot (QD) is an excellent building block for a qubit. While a single electron spin may serve directly as a qubit\cite{1}, the difficulty of single-qubit operations makes it desirable to encode a qubit in a multi-electron state. Considerable success has been achieved with a two-electron encoding\cite{2}, in which the singlet and spinless triplet levels of the double quantum dot (DQD) define a logical qubit\cite{3}. Electric pulses, applied on the microsecond scale, permit all necessary one-\cite{4,5} and two-qubit \cite{6,7} operations when supplemented by magnetic field gradients\cite{8,9}.

This paper addresses problems with the crucial “exchange gate”, which, while it has provided a route to impressive progress in the singlet-triplet qubit, suffers considerably from low-frequency noise due to charge traps\cite{10,11}. In this gate a DQD is moved away from the left QD: electron transfer from the left QD $(2\text{, }0)$ to one having a slight bias towards double occupancy of one QD (e.g. the left QD: $(2\text{, }0)$). Only the singlet configuration permits electron transfer from $(1\text{, }1)$ to $(2\text{, }0)$, while transfer from the triplet state is blocked (Pauli spin blockade).

Here we show that, paradoxically, the exchange gate will be much less susceptible to change noise if the DQD is pulsed fully from the $(1\text{, }1)$ to the $(2\text{, }0)$ regime. Pulsing far into the $(2\text{, }0)$ region lifts the spin blockade for the triplet as well as the singlet state\cite{12}, making the singlet-triplet splitting highly protected from charge noise. We show that the fidelity of this gate will be excellent under two conditions: 1) the pulse rise and fall times should be sub-nanosecond; and 2) the electrons should be in the second shell, so that singlet and triplet states have the same charge distribution. This means that the best exchange gate is predicted to occur for the six-electron DQD with four non-participating “core” electrons, so that the desired transition is actually between $(3\text{, }3)$ and $(4\text{, }2)$.

\textbf{Model} — Our description of DQDs starts with the single-particle eigenstates of a circular QD with confining potential $V(x, y) = \frac{m_e}{2}\epsilon r^2$\cite{13,14} and out of plane magnetic field $B$. The eigenstates are the Fock-Darwin (FD) states $\psi_{\alpha,\beta}$\cite{15}, with energies $E_{\alpha,\beta} = (2\alpha + |\beta| + 1)\hbar \Omega - \beta \hbar \omega_c$, where $\omega_c = \frac{eB_c}{\hbar}$ and $\Omega^2 = \omega_0^2 + \omega_c^2$. We will consider moderate $B$ fields: the degeneracies $E_{\alpha,\beta}$ for the same $\beta$, are lifted; but $E_{\alpha,\beta}$ of different $\alpha$ do not cross $(\omega_c/\omega_0 \ll 1)$. The single-particle eigenstates are grouped into “atomic” energy shells\cite{16}. The ground state $\psi_{0,0}$ is well separated from the first two excited states $\psi_{0,\pm 1}$.

We employ a description for few electron DQDs that takes into account multiple energy levels and electron-electron interactions\cite{21,22}. As in the work of Burkard et al.\cite{17}, we use the FD single-particle eigenstates as the basis states of a Hubbard model. For two-electron DQDs, we include only the $(1\text{, }1)$, $(2\text{, }0)$ and $(0\text{, }2)$ electron configurations. The singlet (S) and $s_z = 0$ triplet (T) state can be written as the product of its spin and orbital part: $\Psi_{S/T} = \{ \phi_1, \phi_2 \}^{s/a} \otimes |(0\text{, }1\text{, }\bar{1})\rangle$. The electrons occupy states $\phi_i$, which need to be symmetrized/antisymmetrized for the S/T-state (as indicated by $\{ \bullet, \bullet \}^{s/a}$). In the $(1\text{, }1)$ configurations, $\phi_1/\phi_2$ close to the FD ground state $\psi_{0,0}^{L/R}$ on the left/right QD. In the $(2\text{, }0)$ and $(0\text{, }2)$ singlet configurations, both electrons fill the same orbital ground state, close to $\psi_{0,0}^{L/R}$ on the respective QD. For the triplet, the Pauli exclusion principle requires that the next higher orbital state $\psi_{0,1}^{L/R}$ is also occupied.

As in atoms, the first electron shell $\psi_{0,0}^{L/R}$ is completed with two electrons. We assume that in the six-electron configuration the first two electrons on each QD are paired in a singlet state. We thus adopt a frozen core approximation: The $(3\text{, }3)$ configuration for six-electron DQDs is therefore represented as a $(1\text{, }1)$ valence configuration (and similarly the $(4\text{, }2)/(2\text{, }4)$ and $(2\text{, }0)/(0\text{, }2)$ configurations). One just needs to use the appropriate orbital wave function of these “valence” electrons. The valence orbital ground state is then $\psi_{0,0}^{L/R}$, while the first excited state is $\psi_{0,\pm 1}^{L/R}$.

The DQD Hamiltonian is expressed in the basis
The time evolution of superpositions: 

\[ (1,1)_{S/T}, (2,0)_{S/T}, \text{ and } (0,2)_{S/T} : \]

\[
H = \begin{pmatrix}
0 & 0 & \tau_S & 0 & \tau_S & 0 \\
0 & 0 & 0 & \tau_T & 0 & \tau_T \\
\tau_S & 0 & U_S - \epsilon & 0 & 0 & 0 \\
0 & \tau_T & 0 & U_T - \epsilon & 0 & 0 \\
\tau_S & 0 & 0 & 0 & U_S + \epsilon & 0 \\
0 & \tau_T & 0 & 0 & 0 & U_T + \epsilon
\end{pmatrix} \tag{1}
\]

The diagonal entries describe the energy of each state. The difference between \((1,1)_S\) and \((1,1)_T\) matrix elements is neglected, since it is commonly small [15]. Unequally occupied QDs are higher in energy by \(U_{S/T}\) [24]. \(\Delta \equiv U_S - U_T\) is the energy difference between the doubly occupied states. Electrostatic bias, modeled by the parameter \(\epsilon\), influences the relative state energies of uniform and unequal electron arrangements. The off-diagonal elements in Eq. (1) describe the spin-conserving hopping process of electrons between the dots.

Fig. 1 shows the energy spectrum as a function of \(\epsilon\). Close to state degeneracies \(|\epsilon| = U_e\), the hopping process hybridizes electron configurations of the same total spin. The ground state \(E_S/E_T\) is shown in blue/red. At \(\epsilon = 0\), both energy levels are mainly in the \((1,1)\) charge configuration, and their energy difference is minimal. \(E_S\) and \(E_T\) are lowered in energy for increasing bias, due to the transfer of electrons between the QDs. For large \(\epsilon\), the ground states are close to \((2,0)_{S,T}\) with an energy difference \(\Delta\); we indicate one point deep in the \((2,0)\) region as the "high-bias" configuration \(\epsilon = \epsilon_{HB}\).

Our treatment of few-electron DQD is not self-consistent; it employs energy spectra of single-particle states, which are successively filled with electrons. The FD-states are a valid ansatz for the description of few electron QDs, if the electron-electron interaction influences the single-particle energies weakly or shifts all energy levels by a fixed value. The last scenario is consistent with the calculations of Guclu et al., where the addition energy of interacting electrons has a constant offset compared to the non-interacting case [25]. This is consistent with the Constant Interaction Model, introduced by Averin and Likharev [26–27], in which the energy spectrum of QDs remains unchanged when an electron is added to or removed from a QD.

Charge Noise — Charge noise is generally modeled by a random distribution of classical two level fluctuators (TLF), which couple electrostatically to QDs [28–29]. If the occupations of the charge traps (CT) vary with a broad distribution of fluctuation rates, \(1/f\) noise is generated. The coherence of the QD decreases, as seen by the time evolution of superpositions:

\[
\langle \sigma_x \rangle (t) = \left( e^{-i \int_0^t dt' E_{\text{net}}(t') / \hbar} \right) \approx e^{-\left( \frac{t}{\tau_s} \right)^2} e_{\text{ideal}}^x(t) \tag{2}
\]

\(E_{ST}(t)\) is the time-varying energy difference of the qubit levels, which deviates from the ideal value due to the coupling to TLFs: \(\delta E_{ST} = E_{ST} - \langle E_{ST} \rangle\). \(\langle \cdots \rangle\) describes averaging over many experiments. Assuming a static environment during one run, the coherence time \(T_2\) is related to the statistics of the TLFs: \(T_2^{-1} = \sigma_{SE}^2 / 4\pi\).

We analyze the relative energy shift of the qubit levels of a DQD which couples to a CT. In first order perturbation theory the fluctuation of the singlet-triplet splitting is described by [33]:

\[
\delta E_{ST}^{(1)} = \langle \Psi_T | e \Phi_{CT} | \Psi_T \rangle - \langle \Psi_S | e \Phi_{CT} | \Psi_S \rangle \tag{3}
\]

\(\Phi_{CT}\) is the electrostatic potential of a CT. Since for QDs that are suitable for qubits, CTs are at some distance from the QD center, we make a multipole expansion of \(\Phi_{CT} = \Phi(r_0) - \mathbf{E}(r_0) \cdot \mathbf{r} - \frac{1}{2} (\partial_i E_j (r_0)) r^i r^j\). \(r_0\) is the position of the CT relative to the center of the DQD, \(\mathbf{r}\) is the QD electron coordinate. This expansion resolves the coupling of a TLF into dipole \((-\mathbf{E} \cdot \mathbf{d})\) and quadrupole \((-1/2)(\partial_i E_j) \cdot \mathbf{Q}^2\) terms; \(d^i = e \langle \Psi | r^i | \Psi \rangle\), and \(\mathbf{Q}^2 = e \langle \Psi | r^i r^j | \Psi \rangle\) are the first two electric moments of the DQD. We analyze two points in the charge stability diagram \(\epsilon = 0\) and \(\epsilon = \epsilon_{HB}\) ("sweet spots", introduced in Fig. 1 at which coupling is weak to TLFs. High couplings are obtained if the qubit states have different dipole moments, which generate energy shifts scaling like, e.g., \(1/r_0^2\) .
The eigenstates of the singlet-triplet qubit of Eq. 1 can be approximated at \( \epsilon = 0 \): \( |S/T\rangle \propto |11\rangle_{S/T} - \frac{r_{S/T}}{U_{S/T}} (|0, 2\rangle_{S/T} + |2, 0\rangle_{S/T}) \). These have equivalent dipole moments for the two qubit levels; the charge distribution of a DQD arranged in x-direction has mirror symmetry to the y-z plane. The quadrupole contribution describes the spread of the charge distributions. The unequal degree of hybridization of the singlet and the triplet state creates different variances in x-direction: 
\[
\delta E_{ST}^{(1)} \approx \left( \frac{\tau_S}{U_S} \right)^2 - \left( \frac{\tau_T}{U_T} \right)^2 = \frac{e^2}{4\pi}\frac{e^2}{4\pi} \frac{r_{S}^2 + r_{T}^2}{\tau_S^2} - \frac{1}{\tau_T^2}. 
\]

The first factor describes the hybridizations for the singlet and the triplet, the second factor involves the inter-dot distance \( d_0 \) of the DQD, and the third factor is the gradient contribution of the electric field of the TLF. It describes an energy shift proportional to the hybridization of the ground state \( \left( \frac{\tau_{S/T}}{U_{S/T}} \right)^2 \), which decays like \( 1/r_0^4 \) in the TLF-QD distance. A similar expression holds for the six-electron DQD.

Considering the two-electron DQD for high bias (\( \epsilon = \epsilon_{HB} \)), the left QD is lower in energy than the right QD. Since the charge configuration is \( (2, 0) \), the qubit states are close to the FD-states \( \{ \psi_{0,0}, \psi_{0,1} \} \). The dipole contributions to Eq. 3 vanish. The quadrupole contribution of Eq. 3 is:
\[
\delta E_{ST}^{(1)} \approx \left( \frac{e}{4\pi}\frac{e}{4\pi} \right) \frac{r_{S}^2 + r_{T}^2}{\tau_S^2} - 2\frac{2}{\tau_T^2}. 
\]

The situation improves for six-electron DQDs. As the valence electrons’ wave functions \( \psi_{0,\pm 1} \) are complex conjugates of each other, not only the quadrupole term of Eq. 3, but all multipole contributions vanish. \( \delta E_{ST}^{(1)} \) depends on the charge density of the single-electron wave functions, as \( e\phi_{CT} \) describes interactions with only one of the two valence electrons. The second-order dipole contribution of TLFs (second-order Stark effect) vanishes accordingly, since it involves only an overall shift of the confining potential. The first non-vanishing contributions are second-order quadrupole couplings:
\[
\delta E_{ST}^{(2)} \approx - \frac{e^2}{2 \hbar^2} \sqrt{\frac{E_{0,1} - E_{0,-1}}{E_{0,1} + E_{0,-1}}} \frac{r_{S}^2 + r_{T}^2}{\tau_S^2} - 2\frac{2}{\tau_T^2}. 
\]

We note that this contribution has \( 1/r_0^4 \) scaling with the CT-QD distance, which suppresses \( \delta E_{ST}^{(2)} \) considerably. This protection criterion for six-electron DQD is strongest for perfect circular symmetry. For weakly elliptic QDs, \( V = \frac{m_o^2}{2} \rho^2 (1 + \beta \cos(2\phi)) \), the diagonal terms of the quadrupole tensor differ, weighted by the ellipticity \( \beta \):
\[
Q^{xx,yy}_{\psi_{0,1}} - Q^{xx,yy}_{\psi_{0,-1}} \approx \pm \beta \frac{m^2_o}{\omega_c^2} + \mathcal{O}\left( \frac{\lambda e^2}{\omega_c} \right), 
\]
giving a small \( 1/r_0^4 \) contribution.

A summary of \( \delta E_{ST} \) is given in Tab. 1. For \( \epsilon = 0 \), a sweet spot is present for both the \( (1, 1) \) and \( (3, 3) \) cases. \( \delta E_{ST} \) comes from a direct coupling of TLFs to the quadrupole moment of the DQD. The energy shifts are on the order of a few gigahertz, corresponding to a dephasing time of \( ns \). This time scale is consistent with experiments on QD charge qubits [35, 36]. Two sweet spots are identified at \( \epsilon = \epsilon_{HB} \). Another sweet spot is identified at \( \epsilon = \epsilon_{BH} \), where the hybridization factor \( \frac{r_{S/T}}{U_{S/T}} \) vanishes. \( T_2 \) is improved for six-electron DQDs, as the CTs modify \( E_{ST} \) coupling only to the quadrupole moment in second order.

**Robust Single Qubit Gating** — We have identified two points \( \epsilon = 0 \) and \( \epsilon = \epsilon_{HB} \) that are well isolated from external noise sources. It is possible to manipulate the qubit while staying mainly at these sweet spots. Changing the magnitude of \( E_{ST} \) produces a phase gate: \( U = J e^{i\sigma_z}, J = \int_0^\tau d\tau E_{ST}(\tau) \). \( E_{ST} \) is small at \( \epsilon = 0 \), while at \( \epsilon = \epsilon_{HB} \), \( E_{ST} = \Delta \). A possible gate sweep starts from \( \epsilon = 0 \), tunes the bias rapidly to \( \epsilon = \epsilon_{HB} \); after some waiting time the bias is brought back to \( \epsilon = 0 \) (cf. inset of Fig. 2). While the manipulation must be fast to avoid charge noise, it should still be adiabatic with respect to the coupling to excited states (cf. Fig. 1). The slew rate is limited by the leakage to higher states, which is approximated with the transition probability at a Landau-Zener crossing of strength \( \alpha \) which is crossed with velocity \( \frac{d\phi}{dt} \approx \frac{e^2}{2\hbar^2} \approx \frac{\pi}{2\epsilon_{slew}} \). Since the tunnel coupling enters \( P_{LZ} \) quadratically, realistic values of \( \tau \) allow very fast manipulations with permitted pulse lengths far below nanoseconds.

We show a fidelity analysis of a \( \pi \)-phase gate for a two- and six-electron DQD in Fig. 2. The slew rates are fixed through \( P_{LZ} \) to produce negligible leakage [38]. We use similar densities of the CTs for the two- and six-electron DQDs, which are positioned randomly around the DQD to generate \( 1/f \) noise; the coupling to CTs vary the parameter \( \Delta \) through electrostatic couplings to the DQD potential. We exclude a volume around the QD, where no CTs are permitted; such nearby TLFs make the DQD completely nonfunctional as a qubit. We take the ex-

| Mechanism | Coupling to electric quadrupole moment |
|-----------|----------------------------------------|
| Scaling   | \( \frac{r_{S/T}}{U_{S/T}} \) to the \( \frac{1}{r_0} \) term |
| \( T_2 \) | \( \frac{1}{r_0} \) second order |

Table I. Influence of CTs on two- and six-electron DQDs. \( E_{ST} \) is shifted, depending on the distance \( r_0 \) between CT and DQD. Two sweet spots \( \epsilon = 0 \) and \( \epsilon = \epsilon_{HB} \) are identified (cf. Fig. 1). The hybridization factor \( \frac{r_{S/T}}{U_{S/T}} \) (parameter introduced in Eq. 1) enhances the coherence time for \( \epsilon = 0 \). Increasing the magnitude of \( E_{ST} \) decreases the scaling in \( r_0 \). Note that for the six-electron DQD the \( \epsilon = \epsilon_{HB} \) the CTs and the qubit couple only in second-order perturbation theory.
tween two sweet spots. The "high bias" sweet spot gates, our approach works by going to high bias. The phase gate. Contrary to current realizations of phase manipulate singlet-triplet qubits (STQs) via a high-bias cannot completely reproduce these results.

Figure 2. Fidelity analysis for \( \pi \)-phase gate for a two/six-electron DQD, shown in red/blue. Points are from simulations involving \( \frac{1}{f} \) noise sources. The fidelities are poor for slow manipulation times, which are required by small tunnel couplings \( \tau \); cf. the transition probability at a Landau-Zener crossing \( P_{LZ} \). Increasing \( \tau \) allows faster qubit manipulations, which increases the fidelity. The fidelity of the six-electron DQD approaches 1, while it stays much lower for the two-electron system. The solid lines are fits using Eq. (2), with \( T_2 = 1.5/29.3 \) ns for the two/six-electron system. The inset describes the pulse profile of a \( \pi \)-phase gate. Starting from \( \epsilon = 0 \), the DQD is biased to \( \epsilon = \epsilon_{HB} \); we linearly increase \( \epsilon \) for a time \( t_{\text{slew}} \). The qubit stays at \( \epsilon = \epsilon_{HB} \) for \( t_{\text{wait}} \); finally the qubit is brought back to \( \epsilon = 0 \), picking up in total an odd number of \( \pi \) rotations. The overall gate time equals \( 2t_{\text{slew}} + t_{\text{wait}} \).

cluded volume for two-electron DQDs to be considerably larger than for the six-electron system. Fluctuations in the tunnel coupling or the pulse profiles are disregarded. The sweet spots, especially \( \epsilon = \epsilon_{HB} \), offer the advantage that \( E_{ST} \) does not change over a wide range of \( \epsilon \).

The fidelity of the gate, both for the two-electron and the six-electron systems (blue/red), is low for slow tunnel couplings \( \tau \). The fidelity increases very quickly with \( \tau \) for six-electron DQDs and reaches an ideal value very close to 1. The improvement of the fidelity for the two-electron system is much slower. We approximate the curves according to Eq. (2), yielding a coherence time of 1.5 ns for the two-electron system and 29.3 ns for the six-electron case. Steps seen for the two-electron system are generated by different waiting times in (2, 0) when constructing a \( \pi \)-phase gate; a one-parameter fit to Eq. (2) cannot completely reproduce these results.

Conclusion — We propose a fast and robust way to manipulate singlet-triplet qubits (STQs) via a high-bias phase gate. Contrary to current realizations of phase gates, our approach works by going to high bias. The qubit couples weakly to CTS; we manipulate rapidly between two sweet spots. The “high bias” sweet spot \( \epsilon_{HB} \) is not at a specific point in the charge diagram; there is a large range of parameters where \( E_{ST} \) is constant. Note that the Rabi rotation gate needed for full qubit control is envisioned to occur also at a sweet spot (at \( \epsilon = 0 \)), employing magnetic field gradients. It is worth pointing out that the proposed high-bias phase gate works also as a maximally entangling two-qubit gate for single QD qubits [1].

It would be favorable for our proposal for DQDs to have small singlet-triplet energy splitting at \( \epsilon = \epsilon_{HB} \) (\( \Delta \), cf. Fig. 1) to give comfortable electrical manipulation times (sub-nanosecond has become accessible [29]). DQDs with \( \Delta \) on the order of 30 GHz have been reported [30]. One can decrease the singlet-triplet energy splitting further by using favorable dot sizes and external magnetic field parameters. Indeed, we note that a transition from a singlet to a triplet ground state is indicated in calculations on four-electron QDs [30].

A clear prediction of our work is that the many-electron QDs, specifically those for which the valence electrons occupy the second shell, are uniquely suited to protect STQs from charge noise. The mechanism we use, resulting from the equality of singlet and triplet charge distributions in the second shell, is distinct from proposals to use screening effects in multi-electron DQDs [31, 32]. The manipulation of our six-electron STQs can be performed the same way as for the two-electron DQDs, including initialization, manipulation, and measurement. Additional noise sources, which couple in via the charge density, like pure phonon dephasing [43, 44], are also directly suppressed in our approach. We are hopeful that the prospect of an order of magnitude improvement in gate fidelity will motivate the further experimental exploration of the multi-electron regime in quantum dot qubits.

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The wave functions are described by:

\[ \psi_{\alpha,\beta}(\rho, \phi) = (m\Omega)^{|\alpha+\beta|} \sqrt{\frac{n!}{\pi^{\alpha+\beta} \Gamma(m\Omega^2)}} \cdot e^{-\frac{m\Omega^2}{2} \rho^2} \cdot e^{-i\beta \phi} \cdot e^{i\alpha \rho} L_{\alpha}^{\beta}(m\Omega \rho^2), \]

with \( \alpha = 0, 1, 2, \ldots \) and \( \beta = -\alpha, -\alpha + 1, \ldots, \alpha \). We use polar coordinates \( (\rho, \phi) \), while \( L_{\alpha}^{\beta}(x) \) are the generalized Laguerre polynomials.
1 Description of Fidelity Analysis

We model charge noise acting on double quantum dots (DQD) by a random distribution of charge traps, being either filled or empty (cf. Fig. 1). We simulate the time evolution of the DQD numerically using quantum process tomography [1]. We generate randomly a distribution of two level fluctuators (TLF), with a broad range of switching rates $\gamma$. A reasonable probability distribution is $P(\gamma) \sim 1/\gamma$ [2]. The charge distribution is constant during one run of the simulation, while the potential fluctuates between successive simulations. This scenario mimics consecutive single shot measurements, with a long time between the measurements.

The coupling strength of DQD and TLF is determined by their distance. As described in the main text, we take the shift in the singlet-triplet splitting $\delta E_{ST}$ as the only dynamic variable. For the $(2,0)$ and $(4,2)$ configurations, the energy shifts are:

$$\delta E_{ST}^{(2,0)} = \left( \frac{e \hbar}{4m\omega_0} \right) \cdot \frac{e}{4\pi\varepsilon_0\varepsilon_r} \left( \frac{x_0^2 + y_0^2}{r_0^2} - 2 \frac{z_0^2}{r_0^2} \right) + \mathcal{O} \left( \frac{\omega_c}{\omega_0} \right)^2$$

(1)

$$\delta E_{ST}^{(4,2)} = \frac{9}{16} \left( \frac{\hbar}{m^2\omega_0^2} \right) \left( \frac{\omega_c}{\omega_0} \right) \cdot \frac{e^2}{4\pi\varepsilon_0\varepsilon_r} \left( \frac{x_0^2 + y_0^2}{r_0^2} \right)^2 + \mathcal{O} \left( \frac{\omega_c}{\omega_0} \right)^2$$

(2)

The excessively occupied QD is positioned in the x-y plane at the coordinate origin, while charge traps are randomly occupying the space around the DQD.

We use material parameters of GaAs. The confining strength $\hbar\omega_0 = 3$ meV is a common approximation for QDs [3]. $\omega_c/\omega_0 = 0.1$ describes moderate external magnetic fields of 0.7 T. The singlet-triplet splitting $\Delta$ is rather small, consistent with Dial et al. [4]. All parameters are summarized in Tab. 1.

The electron distribution can be approximated by the spread of the ground state wave function: $a_B \equiv \sqrt{\frac{\hbar}{m\omega_0}} \approx 20$ nm. We use 250 TLF with a distance [2.5, 15] $a_B$ from the coordinate origin for the six-electron system. For the two-electron system, we need to exclude a larger volume around the DQD. Otherwise the energy shifts due to Eq. (1) destroy the qubit fidelity completely. To generate the same density of TLFs around the DQD, we include 196 charge traps with a distance [15, 25] $a_B$ from the origin.

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Figure 1: Model of charge traps which couple electrostatically to a DQD. The charge traps fluctuate between being filled with one electron or being empty. The charge traps are randomly generated for a single run of the simulation. They fluctuate slowly; during one gate simulation their electrostatic potential is constant.

| Name                          | Value             |
|-------------------------------|-------------------|
| energy difference between     | $U_S$             |
| (1, 1) / (2, 0) or (4, 2) /  |
| (3, 3)                        | 0.5 meV           |
| singlet-triplet splitting     | $\Delta$         |
|                               | 10 GHz            |
| dielectric constant           | $\epsilon_r$     |
|                               | 12.5              |
| effective mass                | $m$               |
|                               | $0.067 \cdot m_e$|
| confining energy              | $\hbar \omega_0$ |
|                               | 3 meV             |
| magnetic energy               | $\hbar \omega_c$ |
|                               | $0.1 \hbar \omega_0$ |

Table 1: Parameters used for the simulation of the STQs. $U_S$ and $\Delta$ are chosen to describe the DQD dynamics according to Eq. (1) in the main text. The dielectric constant $\epsilon_r$ and effective mass $m$ corresponds to GaAs; $\omega_0$ and $\omega_c$ mimic common confining strengths and magnetic fields of 0.7 T.