Method and device for measurement of dynamic viscosity

F C Ciornei, S Alaci, D Amarandei, L Irimescu, I C Romanu and L I Acsinte
Department of Mechanics and Technologies, “Stefan cel Mare” University of Suceava, Universitatii 13, Suceava, 720229, Romania

E-mail: florina@fim.usv.ro

Abstract. The paper proposes a methodology and ensuing test rig for finding the viscosity of a liquid lubricant. The principle consists in obtaining a contact between two spherical surfaces, one concave and the other one convex. One of the surfaces is kept immobile and to the other, a rotation motion is imposed around the common normal in the contact point and then the law of motion for the mobile lens is found. The law of motion allows for estimation of friction torque, dependent on viscosity at its turn. Applying the method for mineral oils, values comparable to the ones presented by the producer were obtained.

1. Introduction

The viscosity of a fluid reflects the property of being able to transmit friction forces between particles or to exhibit resistance to shearing flows. For a quantitative characterisation of viscosity of a fluid, the laminar flow is considered [1-2] when the relative motion between two adjacent layers generates shear stresses. In the case that the magnitude of tangential stress is proportional to velocity gradient on the direction normal to flow, it is said to be a Newtonian behaviour. When the viscosity of the fluid is depending on shear rate, it is called non-Newtonian fluid. The rheological characterization of behaviour of non-Newtonian fluids is made using instruments generically named rheometers [3-8].

Next, the work refers to liquids presenting Newtonian behaviour, for which the proportionality factor between the shear stress and the magnitude of velocity gradient is the dynamic viscosity. The kinematical viscosity is obtained from dynamical viscosity, by dividing it to liquid’s density. The basis of viscometers’ design is the above viscosity definition, a viscometer being an instrument that allows finding the correlation between the characteristic parameters of a fluid and the shear stresses occurring in the fluid. Depending on the relative motion between the mobile surfaces between which the fluid is placed, there are met capillary viscometers [9-10], rotational viscometers when the relative motion between surfaces is a rotation about an axis, translation viscometers having as characteristic the fact that relative velocity between boundary surfaces keeps the same sign [11-13] oscillating viscometers, for which the relative motion presents periodical sense shifting. In [14-19] there are presented the main types of oscillating viscometers. In recent times there have been developed viscometers with special design, based on the principles mentioned but different from classical ones by the manner of estimating the relative motion and shear stresses from fluid, with modern data acquisition and processing systems [20-22]. In for the simplest case of Newtonian fluids, viscosity measurement is a difficult task due to numerous factors influencing it. Tipei [23] reveals among the parameters affecting decisively the viscosity of a lubricant: the type, temperature, pressure. Other aspects influencing viscosity are the nature and concentration of impurity and the presence of additives [24-25].
2. Viscometer proposal

The principle of the intended viscometer is based on estimation of relative velocity and friction torque occurring in an oil film placed between two spherical surfaces. The two spherical surfaces are actually two glass lenses, one concave, motionless and the other, plane-convex, mobile (figure 1). Considering an elementary volume, under the hypothesis of stationary motion, the axial symmetry of fluid motion is accepted and thus the employment of cylindrical coordinates for characterisation of relative motion is justified (figure 2).

\[ \nabla f = \frac{\partial f}{\partial \rho} u_\rho + \frac{1}{\rho} \frac{\partial f}{\partial \theta} u_\theta + \frac{\partial f}{\partial z} u_z \] (1)

Due to axial symmetry it results that \( \frac{\partial f}{\partial \theta} = 0 \). During experimental work, it can be noticed that the dimension of the region with fluid has constant value and thus one can consider that the velocity does not vary on radial direction. The velocity variation takes place only on vertical direction. As a result, accepting that the velocity gradient has component only on axial direction, the shear stress in current point \( M(\rho, \theta) \) from contact surface is given by the relation:

\[ \tau(\rho, \theta) = \eta \frac{dv}{dz} \] (2)

The boundary surfaces of the bodies between which the lubricant is placed, as shown in figure 3, are two spheres of radii \( R_1, R_2 \) expressed by the equations:

\[ x^2 + (z - R_{1,2})^2 = R_{1,2}^2 \] (3)

On vertical direction, the dimension of the gap is:

\[ \Delta z = R_1 - \sqrt{R_1^2 - x^2} - (R_2 - \sqrt{R_2^2 - x^2}) \] (4)

Since the region where there is lubricant is situated in the vicinity of tangency point between the two surfaces, the expression (4) can be approximated by a Taylor series expansion:

\[ \Delta z \approx \frac{R_2 - R_1}{R_1 R_2} \frac{x^2}{2} \] (5)
The square function is used for the approximating the distance between the two surfaces and it is noticed that the opening values $\Delta z$ are much smaller than the radii of the spheres. From figure 4 it can be seen that at distance $0.2 \max(R, R_2)$ the gap $\Delta z$ is much smaller compared to spheres radii, $\Delta z \approx 0.5\% \max(R_1, R_2)$.

**Figure 3.** Boundary surfaces of spherical bodies.  
**Figure 4.** Lubricant thickness and the approximation function.

This remark allows approximation of differentials from relation (2) with incremental increases:

$$\tau(\rho, \theta) = \eta \frac{\Delta v}{\Delta z}$$

(6)

For estimation of velocity augmentation, one considers that the lower surface is immobile and the upper surface rotates about vertical axis with the angular velocity $\omega$:

$$\tau(\rho, \theta) = \eta \frac{\omega \rho}{(R_2 - R_1) \rho^2}$$

(7)

In relation (7) it was taken into account the fact that, due to axial symmetry, relation (5) is valid for radial direction.

The shear stress $\tau(\rho, \theta)$ will generate a friction force:

$$dF_f = \tau \, dA = \tau \, \rho \, d\rho \, d\theta$$

(8)

that, on its turn, will produce an elementary friction torque:

$$dM_f = \rho \, dF_f$$

(9)

Considering relation (7), the elementary friction torque is:

$$dM_f = 2\eta \, \frac{R_1 R_2}{R_2 - R_1} \omega \rho \, d\rho \, d\theta$$

(10)

To obtain the total friction torque, the expression (10) should be integrated on the entire area with lubricant:

$$M_f = \int_0^{2\pi} \int_0^{r_{\max}} dM_f = \int_0^{l_{\max}} \left[ 2\eta \, \frac{R_1 R_2}{R_2 - R_1} \omega \rho \, d\rho \, d\theta \right] = 2\pi \eta r_{\max}^2 \frac{R_1 R_2}{R_2 - R_1} \omega$$

(11)

The principle of the method is based on experimental estimation of variation of mobile body’s angular velocity and of the maximum radius where liquid is placed. The two theoretical surfaces were materialized by two glass lens, biconcave the immobile one and plane-convex the mobile one. From equation (11), the friction torque is proportional to the angular velocity of the lens. To broaden the
applicability domain of the relations to be obtained an additional constant torque $M_0$ acting besides the friction torque is considered. The mobile lens is activated using an electrical drive, on the shaft of which a rubber disc is mounted, playing the part of clutch for torque transmission. The lens is set in motion to the required velocity and afterwards, let to move without restraint but in contact with the stationary lens. The mobile lens has five degrees of freedom, two translations and three rotations. It is observed from experimental tests that after a relatively short period of time, the translations and rotations around horizontal axes vanish and the only motion remaining is the rotation about vertical axis. This motion is described by the differential equation:

$$J_z \frac{d\omega}{dt} = -(M_0 + \kappa \omega)$$  \hspace{1cm} (12)

where $\kappa$ notation represents:

$$\kappa = 2\eta \tau^2_{max} \frac{R_R R_i}{R_i - R_j}$$  \hspace{1cm} (13)

The general solution of equation (12) is:

$$\frac{\ln(M_0 + \kappa \omega)}{\kappa} = -\frac{t}{J_z} + C$$  \hspace{1cm} (14)

In order to find the constant of integration $C$ the initial conditions are stated:

$$t = 0, \ \omega = \omega_0$$  \hspace{1cm} (15)

The time $t = 0$ is the moment when it is considered that the lens has only rotation around vertical axis and $\omega_0$ is the angular velocity corresponding to this moment. The particular solution of the problem is:

$$\frac{M_0 + \kappa \omega}{M_0 + \kappa \omega_0} = e^{\frac{\kappa t}{J_z}}$$  \hspace{1cm} (16)

There are two unknowns to be found, $M_0$ and $\kappa$. To this end, using a non-contact tachometer the variation of angular velocity of the lens is found. The equation (16) is written for two instants $t_1, t_2$ to which the angular velocities $\omega_1, \omega_2$ correspond and the following system results:

$$\begin{cases}
M_0 + \kappa \omega_1 = e^{\frac{\kappa t_1}{J_z}} M_0 + \kappa \omega_0 \\
M_0 + \kappa \omega_2 = e^{\frac{\kappa t_2}{J_z}} M_0 + \kappa \omega_0
\end{cases}$$  \hspace{1cm} (17)

Elimination of $M_0$ between the equations of the system gives the following equation:

$$(\omega_0 - \omega_1) e^{\frac{\kappa t_1}{J_z}} + (\omega_0 - \omega_2) e^{\frac{\kappa t_2}{J_z}} = \omega_2 - \omega_1$$  \hspace{1cm} (18)

The equation (18) is a transcendental equation and a numerical method is required for solving it (for instance bipartition or Newton-Raphson [27]). With the assumption:

$$\frac{\kappa}{J_z} \ll 1$$  \hspace{1cm} (19)
the approximation relation is used:

\[ \exp(\alpha x) = 1 + \alpha x + O(x^2) \]

and a linear equation with \( \kappa \) unknown is obtained, having the solution:

\[ \kappa = \frac{\omega_1 - \omega_2}{(\alpha_1 - \alpha_2)I_1 + (\alpha_0 - \alpha_2)I_2}J_z \]

In the case when condition (19) is not satisfied, the value obtained by (21) can be used as “guess value” for the numerical algorithm employed in solving equation (18).

\[ M_0 = -\kappa \frac{\omega_2 e^{-\frac{\omega_2}{J_1}} - \omega_1 e^{-\frac{\omega_1}{J_2}}}{e^{-\frac{\omega_1}{J_2}} - e^{-\frac{\omega_2}{J_1}}} \]  

3. Experimental set-up and results
The method proposed is experimentally tested using lubricant between two glass lenses: (1) plane-convex mobile and (2) bi-concave immobile (figure 5). The mineral oil (3) was placed between the two lenses and the mobile lens was set in motion but it was noticed that the motion rapidly ceased. To increase the motion duration, to the mobile lens was attached a metallic ring (4), coaxial to the lens. Thus, the moment of inertia of the assembly lens-ring is greater and a considerable kinetic energy increase is obtained. The lens is set in motion by hand. Using a digital non contact tachometer the variation in time of lens angular velocity is found. The tachometer allows saving the data in numerical format and then graphical representation using a computer.

![Figure 5](image-url)  

**Figure 5.** Experimental test-rig: general view (left) and detail (right).

Figure 6 presents the curves of rotation velocity variation of the lens for several successive launchings. From plots it can be noticed that two domains are visible, on any of the curves, namely a region with rapid velocity decrease followed by a second region where the rotation velocity decreases more slowly. The transition from the first domain is made at a value slightly inferior to \( n = 150 \text{rot/min} \). The two domains correspond to spatial motion and rotation around a fixed axis respectively, for mobile lens. In figure 7 and figure 8 the image of the lubricated region between the two lenses is presented. In the case of spatial motion, the participating oil performs a planetary motion around the axis of immobile lens (figure 7) while after rotation motion around the axis of immobile lens is settled, the oil mark is stationary.

The disadvantage of using the tachometer for analysing the rotation motion of the lens after the stabilization, as observed from figure 6, consists in the fact that for small angular velocity values, the instrument’s indications are not relevant. To eliminate this drawback, a mark was applied to the
surface of the collar-ring attached to the mobile lens. After actuating the lens, the motion was recorded using a video camera having the image acquisition performance of $120 \text{frames/sec}$; after that, the movie was decomposed into frames using a software and the number of frames corresponding to a complete rotation was determined.

![Digital Tachometer Graphic Display (rpm 1000)](image)

**Figure 6.** Experimental angular velocity variation.

![Planetary motion of oil mark for spatial motion of mobile lens.](image)

**Figure 7.** Planetary motion of oil mark for spatial motion of mobile lens.

![Oil mark settles after the stabilization of lens’ motion.](image)

**Figure 8.** Oil mark settles after the stabilization of lens’ motion.

![Time period versus rotation number.](image)

**Figure 9.** Time period versus rotation number.

In figure 9 and figure 10 there are presented the values of time duration corresponding to complete rotation and the variation of rotation angle of the lens, respectively. In figure 10 it was considered that $t = 0$ instant corresponds to the moment afterwards the stationary motion of the lens was obtained and for which the rotation angle is considered $\theta = 0$.
To apply the data from figure 10 in equation (17), it was considered that $\omega_0 = 2\pi/ (t_1 - t_0)$. The initial angular velocity is regarded as constant during a complete rotation. With imposed initial conditions $\theta (t = 0) = 0$ the equation was integrated once more and the form of the dependency is:

$$\theta(t) = \frac{J_{01} \omega_0}{\kappa} + \frac{J_c - \kappa T M_0 - \frac{J_{01}}{\kappa^2} (M_0 + \kappa \omega_0) \exp \left( \frac{-\kappa T}{J_{01}} \right)}$$  \hspace{1cm} (23)

To establish the parameters $M_0$ and $\kappa$ it is imposed the condition that the function (23) interpolates the experimental data $(t_k, \theta_k)$, plotted in figure 11, with minimum squared error. To this end, the following function is constructed:

$$\Phi(M_0, \kappa) = \sum_{k=1}^{n} [\theta(t_k) - \theta_k]^2$$  \hspace{1cm} (24)

and the system is proposed to be solved:

$$\begin{cases}
\frac{\partial \Phi(M_0, \kappa)}{\partial M_0} = 0 \\
\frac{\partial \Phi(M_0, \kappa)}{\partial \kappa} = 0
\end{cases}$$  \hspace{1cm} (25)

To resolve the system (25), a numerical procedure is required, and there is necessary précising the guess value for algorithm initialization. It is easily noticed that the solution of system (25) is strongly dependant on the guess values. $M_0$ and $\kappa$ found with the aid of relations (21) and (22) may be such a solution. For the present case, in order to establish a guess value the function $\theta(t)$ from relation (23) was constrained to pass through two of the experimental points from the graph given in figure 10. The experimental data (from figure 10) and the interpolation curve are presented in figure 11.

Figure 11. Interpolation curve for experimental data.

The characteristic parameters of the experimental set-up also used in the quantitative evaluation are: mass of the ring $M_1 = 0.894 \text{kg}$; moment of inertia of the ring $J_{c1} = 2 \cdot 10^{-4} \text{kg} \cdot \text{m}^2$; mass of mobile lens $M_2 = 0.264 \text{kg}$; moment of inertia of the lens $J_2 = 2.37 \cdot 10^{-4} \text{kg} \cdot \text{m}^2$; oil mark radius $r_{\text{max}} = 0.02 \text{m}$; convex lens radius $R_1 = 0.07 \text{m}$; concave lens radius $R_2 = 0.2 \text{m}$;
\[ \kappa = 3.74 \cdot 10^{-5} \text{N}\cdot\text{m}\cdot\text{s} \] density of oil tested [28] \[ \rho_{\text{oil}} = 1030 \text{kg/m}^3 \]. Replacing in relation (13) the values for the parameters given above, it results a kinematical viscosity \( \nu = 482 \text{ mm}^2/\text{s} \).

4. Conclusions

The paper proposes a method for establishing the oil viscosity. The main goal was designing a most simple structure of the device so that only the viscous friction force should act upon the mobile body as dissipative force.

The principle of the method consists in obtaining the contact between two spherical surfaces, one concave and one convex. The concave surface is kept immobile and the convex surface is movable. Between the two surfaces, namely glass lenses- a concave one and plane-convex the other one, the lubricant to be tested is introduced. The radii of the two lenses have comparable values, thus the maximum oil thickness is less than 1mm. Under this assumption it can be accepted the hypothesis that the velocity variation along a vertical segment joining two points from the two lenses is linear, increasing from zero to a maximum value.

Another advantage of the proposed device consists in the fact that regardless of the actuation motion of the mobile lens, at a certain time, the motion of the lens becomes a rotation about an immobile vertical axis passing through the lowest point of the stationary lens.

A theoretical model is proposed, expressed by an equation in which the viscous friction is described using a damping coefficient dependant on the fluid viscosity and constant friction torque. The latter was introduced to ensure a finite time phase of mobile lens’ motion.

After the stationary motion is obtained, the law of motion is experimentally found and, damping coefficient and in the end, the viscosity are found by imposing that the theoretical law of motion interpolate with minimum error the experimental data.

Concerning the experimental establishing of the law of motion, initially, the angular velocity variation was estimated using a non contact tachometer. This option was removed since at the end of the motion, the errors of the tachometer increased substantially. As ultimate solution, video recording of the lens’ motion was chosen. Splitting the film into frames makes possible establishing the variation in time of the rotation angle during stationary phase. The experimental data are excellent interpolated by the theoretical curve. The tests were done using an oil of known viscosity

The viscosity obtained experimentally via the proposed methodology is comparable to the values from technical literature.

One has given up determining the angular velocity by tachometer but the method proved to be useful as it allows emphasising the transition angular velocity from spatial motion to stationary motion.

The present paper is only a starting point. The first solution in improving the methodology consists in the use of lenses with highly controlled curvature radii and of close values of curvature radii, the latter leads to an increase in velocity gradient.

Acknowledgement

The infrastructure used for this work was partially supported by the project “Integrated Center for research, development and innovation in Advanced Materials, Nanotechnologies, and Distributed Systems for fabrication and control”, Contract No. 671/09.04.2015, Sectorial Operational Program for Increase of the Economic Competitiveness co-funded from the European Regional Development Fund.

References

[1] Gupta S V 2014 Viscometry for liquids (Springer) chapter 1 pp 1-17
[2] Nakayama Y, Boucher R F 2000 Introduction to Fluid Mechanics (Butterworth-Heinemann Oxford) chapter 2 pp 9-12
[3] Macosko C W 1994 Rheology Principles, Measurements and Applications (Wiley-VCH NY) chapter 5 pp 181-251
[4] Barnes H A 2000 A Handbook of Elementary Rheology (The University of Wales Aberystwyth)
Barnes H A, Maia J M accessed 2016 Rheometry, Rheology vol. I (www.EOLSS.net) pp 1-10
Barnes H A, Hutton J E, Walters 1989 An Introduction to Rheology (Elsevier) pp 9-10
Morrison FA 2001 Understanding rheology (Oxford University Press) chapter 10 pp 382-433
Viswanath D S, Ghosh T, Prasad D H L, Dutt N V K, Rani K Y 2007 Viscosity of Liquids: Theory, Estimation, Experiment, and Data (Springer) chapter 2 pp 65-67
***ASTM D446-07 Standard specifications and operating instructions for glass capillary kinematic viscometers
***ISO 3105-1994 Glass capillary kinematic viscometers—Specif and operating Instructions
Flowers A E 1914 Viscosity measurement and a new viscometer. Proc Am Soc Test Mater 14 565–616
Forgacs R L 1965 Improved Torsion viscometer for Polymer studies. Rev Sci Instrum 36 307–313
***HAAKE Instruction Manual Falling Ball Viscometer C
Kestin J, Newell G F 1957 Theory of oscillating type viscometers I: the oscillating cup. Z Angew Math Phys 8 433–449
Beckwith D A, Newell G F 1957 Theory of oscillating type viscometers, the oscillating cup part II. Z Angew Math Phys 8 450–465
Azeitia A G, Newell G F 1958 Theory of oscillating type viscometers III, a thin disc. Z Angew Math Phys 9a 97–118
Azeitia A G, Newell G F 1959 Theory of oscillating type viscometers IV, a thick disc. Z Angew Math Phys 10 15–34
Newell G F 1959 Theory of oscillating type viscometers V, disc between fixed plates II. Z Angew Math Phys 10 450–465
Newell G F 1959 Theory of oscillating type viscometers: the oscillating cup part II. Z Angew Math Phys 10 160–174
Sheen S H, Chien H T, Raptis A C 1997 Ultrasonic methods for measuring liquid viscosity and volume percent of solids Argonne National Laboratory (Argonne Illinois) Report ANL-97/4 chapter v2 pp 3-10
Hager H E 1986 Fluid property evaluation by piezoelectric crystals operating in the thickness shear mode. Chem Eng Common 43 25–38
Kanazawa K K, Gordon J G II 1985 The oscillation frequency of a quartz resonator in contact with a liquid. Anal Chem Acta 175 99–105
Tipei N, Costantinescu, V N, Nica Al, Biţă O 1961 Lagăre cu alunecare (Ed Acad Bucureşti) chapter IX pp 297-302
Cate W E, Deming M E 1970 Effect of impurities on density and viscosity of simulated wet-process phosphoric acid J. Chem. Eng. Data 15 (2) 290–295
Krantz T L., Kahraman A 2005 An Experimental Investigation of the Influence of the Lubricant Viscosity and Additives on Gear Wear (Nat Aeron and Space Adm Glenn Research Center) pp 11-14
Spiegel M 2009 Schaum’s Outline of Theory and Problems of Vector Analysis and an Introduction to Tensor Analysis (McGraw Hill) chapter 7 pp 157-188
Hamming R W 1987 Numerical Methodss for Scientists and Engineers (Dover Publications) chapter II pp 427-443
REMARK Co. 2008 AGIP BLASIA S (F.T. TI 6/2008) available at www.lubrifianti.com/documente-up/fisa-tehnica_47736.pdf