On the use of elliptical bulge tests in material characterization through inverse methodologies

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Abstract. The hydraulic bulge test is an experimental technique that can be used to identify the properties of sheet metals up to large strain. The hydraulic bulge test, usually, is a free expansion of a sheet blank through a circular hole, driven by the pressure of a fluid, where the ends of the blank are blocked using a suitable die. The state of stress during the bulge test is mainly equi-biaxial, however, using elliptical holes with different aspect ratios, it is possible to produce heterogeneous stress-strain fields that can be used to identify the properties of the material through an inverse method; in this case, the non-linear Virtual Fields Method (VFM) was adopted. With respect to the traditional tests employed with the VFM in plasticity, i.e. double notched specimens or specimens with holes subjected to tensile tension, elliptical bulge tests allow to investigate a different zone in the stress-strain space, with a higher triaxiality. The capability of this type of test to calibrate the parameters of advanced anisotropic models is here assessed using simulated experiments.

1. Introduction

The hydraulic bulge test is a well established experimental protocol generally used to investigate the mechanical response of thin bodies under multiaxial loading conditions. The test finds several examples of application for the characterization of the plastic behaviour of sheet metals thanks to the production of a biaxial stress state on the dome apex. This also allows to achieve larger plastic deformations compared to the standard uniaxial tensile test, providing important information on the material formability.

Circular bulge tests has been already used to characterize the plastic anisotropy of sheet metals [1, 2] and for the study of the fracture behaviour of advanced high strength steels [3]. The working principle of bulge test was also used to develop other multiaxial tests such as the multiaxial tube expansion in [4] or the use of elliptical dies for the bulge test [5], in order to investigate the mechanical response of the material under un-balanced biaxial conditions up to large plastic deformations.

The computation of the biaxial stress and strain curve is generally performed by using kinematic relationships between the dome height, the bulge curvature and the forming pressure: for instance, in the work of Rees [6] is presented an analytical method to predict the hoop and longitudinal stress at the ellipsoidal dome on the basis of the Hill48 anisotropy model (i.e. the method requires to know yield function coefficients), while a more general formulation based only on kinematic relations was recently proposed in [7].
2. Inverse methodology

Usually, the bulge test is used to identify the stress-strain curve in a single point of sheet metal, however, using inverse methods, it is possible to include in the identification algorithm the whole full-field measurement obtained with a suitable optical system, e.g. digital image correlation.

One of the most used inverse methods is the non-linear virtual fields method (VFM). The VFM can be used to calibrate a set of constitutive parameters $\xi$ by a minimization of a cost function $\psi(\xi)$ that represents the balance between the internal and the external virtual work, according to the principle of virtual work (PVW). Using the finite strain theory the PVW can be written in the reference configuration using the 1st Piola-Kirchhoff tensor $T^{1PK}$, so $\psi(\xi)$ becomes:

$$\psi(\xi) = \int_{\mathcal{B}_0} T^{1PK} : \delta \mathbf{F}^e \, dV_0 - \int_{\partial \mathcal{B}_0} (T^{1PK} \cdot \mathbf{n}_0) \cdot \delta \mathbf{v} \, dA_0$$

where the first integral is the internal virtual work (IVW) and the second is the external virtual work (EVW); $\delta \mathbf{v}$ is a kinematically admissible virtual fields and $\delta \mathbf{F}^e$ is the corresponding gradient. The parameters $\xi$ are used to compute the stress from the measured strain field using a suitable stress integration algorithm (e.g. radial return, backward Euler, etc.). This approach can be applied to the bulge test using the plane stress assumption. In fact, Eq. 1 is valid for any virtual field and it is written in the undeformed configuration, that, in this case, is a planar sheet. Thus, selecting virtual fields of this form:

$$\delta \mathbf{v} = \begin{bmatrix} \delta v_x \\ \delta v_y \\ 0 \end{bmatrix} \text{ and } \delta \mathbf{F}^e = \begin{bmatrix} \delta F^e_{xx} & F^e_{xy} & 0 \\ \delta F^e_{yx} & F^e_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

all components in $z$ are equal to zero and the problem reduces to two dimensions. A thorough description of the application of VFM in case of shell structures where the bending stress is negligible is given in [8].

3. Numerical model

Nowadays, simulated experiments based on numerical simulations are becoming the state-of-art for the study of identification strategies through heterogenous test, allowing to explore different fundamental aspects of the inverse procedure such as the specimen geometry [9] or the error associated to the experimental setup [10].

In this work, numerical simulations of elliptical bulge test by using different test configurations were performed to define the mechanical states produced on the specimens. All the analysis were performed by means of the commercial implicit FE solver Abaqus/Standard®. As illustrated in Figure 1, the numerical model is composed by two main parts: the elliptic die, which drives the deformed shape of the specimen, and the sheet-metal specimen. Due to the double symmetry of the problem, only one quarter of the specimen was modelled.

Here we considered two principal elliptical die openings of the bulge, regulated through the major radius $r_x$ and the minor radius $r_y$ and expressed by means of the aspect ratio $\beta = r_y/r_x$. The specimen geometry is also defined according to $\beta$, assuming a constant thickness of 0.8 mm. All the details about the die and specimen dimensions are listed in Table 1. Note that the die’s fillet radius $r_f$ was kept constant and equal to 8 mm in all the analyses, according to the study presented in [7].

The specimen was discretized by using solid 8-node linear elements with reduced integration and hourglass control (C3D8R), employing an average mesh size of 2 mm and setting-up 6 elements along the thickness. Besides, the die was modelled as discrete rigid shell, imposing an
Figure 1. Schematic view of the numerical model adopted for the virtual experiment. The coordinate system in red indicates the material orientation of the specimen, where 1 indicate the rolling direction RD, 2 the transverse direction TD and 3 the normal direction ND.

Table 1. Elliptic bulge aspect ratios and specimen dimensions numerically simulated.

| β  | r_x | r_y | R_x | R_y |
|----|-----|-----|-----|-----|
| 0.5| 100 | 50  | 130 | 80  |
| 0.25| 100 | 25  | 130 | 55  |

average mesh size of 2 mm. To avoid a disproportionate radial sliding far from the gauge zone and to simulate the effect of the blank-holder, all nodes on the outer side of the specimen were fixed, while the pressure load was imposed on all the bottom surface. The contact between the blank specimen and the die was modelled as frictionless hard contact with finite sliding, enforced by using the Augmented Lagrange Method. Such boundary condition is an approximation of what occurs in a real test, however, it can be considered acceptable in this case because the strain maps used for the VFM identification are far from the edge of the die.

The material behaviour introduced in the FE modelling was taken from previous investigations of the authors on a BH340 bake-hardenable steel [11], adopting the hardening curve obtained from circular bulge tests on the same material and here described through the Swift law:

$$\bar{\sigma}(\bar{\varepsilon}_p) = K(\varepsilon_0 + \bar{\varepsilon}_p)^N$$  \hspace{1cm} (3)

where $\bar{\sigma}$ is the flow stress, $\bar{\varepsilon}_p$ is the accumulated equivalent plastic strain and $K$, $\varepsilon_0$ and $N$ are material parameters. Plastic anisotropy was introduced through the Hill48 material model [12], whose coefficients are listed in Table 2.
### Table 2. Material parameters adopted in the FE model.

|                            | Elastic Properties | Plastic Properties |
|---------------------------|--------------------|--------------------|
| $E$ (GPa)                 | 200                | $K$ (MPa)          |
| $\nu$                     | 0.3                | $\varepsilon_0$    |
|                           |                    | $N$                |
|                           |                    | $R_0$              |
|                           |                    | $R_{45}$           |
|                           |                    | $R_{90}$           |

|                            | Hill48 coefficients |
|---------------------------|---------------------|
| $f$                       | 0.3612              |
| $g$                       | 0.3968              |
| $h$                       | 0.6032              |
| $l$                       | 1.5                 |
| $m$                       | 1.5                 |
| $n$                       | 1.0536              |

### 4. Results and discussions

A fundamental characteristic of inverse methods based on full-field measurements is the capability to extract more information from a single test compared to standard calibration methods. Therefore, one of the basic ideas is to generate heterogeneous mechanical states on the specimen to provide a comprehensive description of the material behaviour during the identification process. An effective strategy to increase the amount of material data – especially in the case of anisotropic materials – is also to feed the identification with test on specimen obtained at different material orientations.

Following this approach, we here considered two different tests, one with the RD and the other with the TD directions aligned with the specimen major axis ($R_x$), for each elliptical geometry. Figure 2 displays the displacement field along the $Z$ direction and the corresponding equivalent plastic strain distribution at the last timestep during the bulge test with $\beta = 0.5$.

For both specimen orientations, the maximum dome height exceeded 40 mm, while the maximum equivalent plastic strain was 0.57% and 0.67% for the RD-parallel and TD-parallel configurations, respectively. It is worth mentioning that we considered as last step the last converged increment before plastic instabilities and excessive elements distortions on the dome height occurred during the analysis. The effect of anisotropy is also showed by the different maximum value of forming pressure obtained for the two tests: 92 bar for the the RD-parallel and 82 bar for the TD-parallel specimens.

Reducing the aspect ratio of the ellipse to $\beta = 25\%$ had the primary effect to drastically increase the forming pressure to 200 bar for the RD-parallel configuration and 180 bar for the TD-parallel one. Nonetheless, the maximum dome height was 25 mm and 27 mm for the two specimens, achieving a maximum plastic strain of 0.46% per the RD-parallel specimen and 0.62% for the TD-parallel specimen (Figure 3).

To evaluate the heterogeneity of the mechanical states generated by the elliptical bulge tests, Figure 4 illustrates the stress distribution on the normalised $\pi$-plane combined with the contours of the reference yield surface. Note that all the data are reported with respect the pure equi-biaxial stress state, where the yield surface is equal to 1.

The elliptical bulge with aspect ratio $\beta = 0.5$ produced large amount of mechanical states capable of covering a wide portion of the yield surface that includes the equi-biaxial stress condition, and mostly all the generalised plane strain condition for material orientations from $0^\circ$ to $90^\circ$ with respect to the RD (Figure 4a). The two specimen orientations with respect the ellipse major axes does not show significant differences in stress heterogeneity, however, a
Figure 2. Full-field maps of the out-of-plane displacement and equivalent plastic strain from elliptic bulge tests with $\beta = 0.5$: (a) RD parallel to the ellipse major axis, (b) TD parallel to the ellipse major axis.

Figure 3. Full-field maps of the out-of-plane displacement and equivalent plastic strain from elliptic bulge tests with $\beta = 0.25$: (a) RD parallel to the ellipse major axis, (b) TD parallel to the ellipse major axis.

A more dense number of points is observed on the yield surface hemisphere corresponding to the material orientation parallel to the ellipse minor axis.

On the other hand, reducing the bulge aspect ratio to $\beta = 0.25$ polarizes the stress distribution towards the uniaxial tensile condition (Figure 4b) of the material orientation parallel to the ellipse minor axis. Although there is also here a large number of datapoints in the plane strain condition, there are no information about the pure equi-biaxial stress state, moreover, only few points get close to the plane strain along the 45$^\circ$ direction with respect to the RD.

From this analysis it turns out that the elliptical bulge test with $\beta = 0.5$ represents the best candidate for the identification of the anisotropic behaviour of materials.
4.1. Parameter identification

A detailed study of the VFM application to elliptical bulge test is beyond the scope of the paper. The intent here is instead to compare the potentiality of the elliptical bulge test with respect to the classical geometries used to characterize anisotropic plasticity model with VFM, i.e. double notched specimen.

To this purpose, the elliptical bulge test with RD along $x$ and $\beta = 0.5$ is used to identify the coefficients of the Swift law and the Hill48 criterion using a single simple virtual field, which is based on the equilibrium of the forces along the $z$-direction. The internal virtual work (IVW) and external virtual work (EVW) are, respectively:

$$IVW = p \int_{\partial A_0} F_{33}^{-T} \, dA_0$$

and

$$EVW = \int_{\partial A_0} \left( T^{1PK}_{31} \cos \alpha + T^{1PK}_{32} \sin \alpha \right) s_0 \, dl_0$$

where $p$ is the fluid pressure, $F_{ij}^{-T}$ is the inverse of the deformation gradient and $T^{1PK}_{ij}$ is the first Piola-Kirchhoff stress computed from the strain data and the constitutive parameters $\xi$; the area $A_0$ is a circle placed at the top of the elliptical bulge, in the undeformed configuration, $\alpha$ is the angle of normal vector with respect to the circumference $\partial A_0$, and $s_0$ is the metal sheet thickness. The different steps to obtain Eq. 4 and 5 are given in [8].

The 6 constitutive parameters are extracted minimizing the difference between $IVW$ and $EVW$ using a standard Nelder-Mead simplex algorithm, the results are listed in Table 3. The algorithm is able to identify correctly the hardening law and $R_{90}$, however, large errors are obtained for $R_{0}$ and $R_{45}$. To compare the elliptical bulge test with other tests, Table 4 reports the identification error obtained in [13] for the same constitutive model using a double notched specimen cut along the rolling direction. Using a single VF the level of error is much larger with respect to the elliptical bulge test, showing that, potentially, the latter has a richer information.
Table 3. Results of identification.

| Constitutive parameters | Swift law | Lankford coeff. |
|-------------------------|-----------|-----------------|
|                         | $K$/MPa   | $\varepsilon_0$ | $N$ | $R_0$ | $R_{45}$ | $R_{90}$ |
| Reference               | 709.2     | 0.0021          | 0.259 | 1.67 | 0.89 | 1.52 |
| Identified              | 717.3     | 0.0024          | 0.266 | 1.29 | 0.41 | 1.53 |
| Error                   | 1.2%      | 12.9 %          | 2.8 % | -22.5 % | -54.3% | 0.90 % |

Table 4. Error obtained with a double notched specimen in [13]

| Used VF | $K$/MPa | $\varepsilon_0$ | $N$ | $R_0$ | $R_{45}$ | $R_{90}$ |
|---------|---------|-----------------|-----|------|---------|---------|
| 1       | Error   | -20.8%          | 14.5% | 4.0% | 67.5% | 70.0% | 81.3% |
| 2       | Error   | -1.0%           | 15.5% | 3.7% | -7.4% | 4.7% | 52.0% |
| 3       | Error   | -1.7%           | 12.2% | 2.6% | 0.7% | -6.7% | 41.2% |

However, the experimental complexity of the bulge test is more critical than that of the double notched specimen, this aspect must be also taken into consideration when comparing the two methods.

Eventually, in order to reduce the error obtained with the elliptical bulge test it is necessary to implement more VFs and check the influence of the material orientation, this will be addressed in future works. Moreover, it is necessary to extend the analysis to other anisotropic yield criteria, for instance Yld2000-2D, where more constitutive parameters are involved and allow to better reproduce real materials.

5. Conclusion

This paper presents a preliminary study to assess the effectiveness of elliptical bulge tests in the inverse identification of the anisotropic behaviour of sheet metals. In particular, the study focuses on the stress distribution achievable using elliptical bulge tests with different aspect ratios and present a first identification example using a single user defined virtual field. The main outcomes are:

- the elliptical bulge test allows to cover a large portion of the stress space and the distribution is strongly affected by the aspect ratio $\beta$;
- the largest heterogeneity is obtained using an aspect ratio $\beta = 0.5$;
- the identification with the VFM, using a single user defined VF and a single test with $\beta = 0.5$, allows to retrieve correctly the hardening parameters and one of the three anisotropic coefficients;
- although the identification is not perfect, comparatively, the elliptical bulge test leads to better results with respect to the same VFM approach applied to a standard double notched specimen.
In the future it is necessary to implement for the elliptical bulge test case the routines already used to generate automatic virtual fields for the 2D case. Then the developed numerical models will be used to conduct and extensive validation and to properly quantify the accuracy of the VFM applied to elliptical bulge tests.

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