Hard scale dependent gluon density, saturation and forward-forward dijet production at the LHC.

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Abstract

We propose a method to introduce Sudakov effects to unintegrated gluon density promoting it to be hard scale dependent. The advantage of the approach is that it guarantees that the gluon density is positive definite and that on integrated level the Sudakov effects cancel. Besides that the method to introduce the Sudakov effects is convenient since it does not need evaluation of cross section in the process of imposing the effects. As a case study we apply the method to calculate angular correlations in production of forward-forward dijet and \( R_{pA} \) ratio for p+p vs. p+Pb collision.

Introduction

In perturbative QCD the theoretical construction that is used to evaluate cross section for hadron hadron collisions is a factorization \[1\] which allows to split the cross section into patron densities characterizing the incoming hadrons and hard subprocess. In particular the high energy factorization \[2,3\] is a prescription for such a decomposition which allows for taking into account off-shellnes of incoming partons already at the lowest order accuracy both in matrix elements \[4–9\] and parton densities. Its applications to situations where saturation effects are relevant is a phenomenologically useful model study. There are already results which generalize it in some limit of phase space to include saturation \[10\]. The basic ingredient of the formula for factorization is the unintegrated gluon density \( F(x, k^2) \) where \( x \) is the longitudinal momentum fraction while \( k \) is modulus of transversal momentum of off-shell gluon. In the high energy limit it comes from resummation of emission of gluons emitted in the \( s \) channel which are ordered in the longitudinal momentum fractions and unordered in the transversal momenta. When the \( x \) value and the splitting ratio \( z \) are small one argues that the nonlinear effects start to show up to tame the rapid power like growth of gluon density \[2\] and there are indications that indeed saturation occurs in nature \[15–18\]. Resummation of relevant contributions for introducing unitarity leads to the BK equation \[11,12\] or more general framework like the JIMWLK equation (see \[13\] and references therein). However, it turns out that in order for such framework to be applicable for processes at LHC one necessarily needs to include resummed corrections of higher orders among which the kinematical constraint \[19,20\] is the most dominant. It softens the singularities of the
BFKL kernel and therefore slows down the evolution. Its inclusion in the BFKL kernel allows for reasonable well description of $p_T$ spectra of forward-central dijet at the LHC [21,22]. Another type of effect that is beyond the BK is the angular ordering leading to dependence of the gluon density on the scale of the hard process. At the linear level inclusion of such effects leads to the CCFM evolution equation [23,24], while at the nonlinear to equations introduced in [27,29]. Importance of the hard scale dependence has been also recognized by [30,31] where the effects of coherence were introduced in the last step of evolution. The later framework is particularly interesting as it is relatively straightforward to apply since it uses parton densities which might come from collinear framework on top of which the Sudakov effects [32] in a factorized form are applied. Furthermore, it has been noticed in [33] that in order to obtain description of data in wider domain of the $\Delta \phi$ one needs to include Sudakov effects in the low-$x$ framework to ensure no emissions between the scale of the gluon transverse momentum, $k$, and the scale of the hard process, $\mu$. In the method described in [33] the Sudakov effects were imposed on the cross section level i.e. generated events were weighted with Sudakov form factor preserving unitarity, assuring that the total cross section will not be affected. Another approach to introduce Sudakov effect is to include them directly as a part of the evolution equation i.e. at all steps in the evolution. Such an approach leads to the already mentioned CCFM evolution and the Sudakov form factor gets an interpretation of object which resumes virtual and unresolved real corrections relevant when the scale of the harder process is larger than the local $k$ of gluon.

In the present paper we start directly from the gluon density resumming low-$x$ logarithms, accounting for nonlinearities and promote it to depend on the hard scale. This method is attractive since it provides gluon density which once constructed can be used in various phenomenological applications. We perform our construction for proton and lead and apply the resulting gluon density to provide estimates of relevance of coherence for nuclear modification ratio $R_{pA}$ in production of forward-forward dijet.

**Sudakov effects and unintegrated gluon density**

The solutions of evolution equation combining physics of saturation and coherence show that saturation scale gets nontrivial dependence on the scale of the hard process [36,37] and leads for instance to effect called saturation of saturation scale [38]. The equation due to its numerical complexity has not been still applied to phenomenology. Due to this below we propose a model prescription how to introduce hard scale dependence on top of gluon density obtained from the BK equation. The prescription is motivated by the method developed in [33] but reformulated in terms of unintegrated gluon density. The assumptions are the following:

- on integrated level the gluon densities obtained from hard scale dependent gluon density $A(x,k^2,\mu^2)$ and $F(x,k^2)$ are the same. This guarantees that the Sudakov formfactor just modifies the shape of the gluon density but on inclusive level the distribution is the same

- Contribution with $k > \mu$ is given by unintegrated gluon density $F(x,k^2)$ which could be obtained by solving the BK equation.

The assumptions above lead to the following formula:

$$A(x,k^2,\mu^2) = \theta(\mu^2 - k^2)T_s(x,\mu^2)\frac{g(x,\mu^2)}{x g_h(x,\mu^2)}F(x,k^2) + \theta(k^2 - \mu^2)F(x,k^2).$$

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1 recently fitted to $F_2$ data in [26]

2 The method used in [33] will be soon available within LxJet program [47]
corrections include for LHC we use the unintegrated gluon density with corrections formulated in [41–43]. Those coupling corrections included i.e. the rcBK [40]. Therefore to be realistic with applications to apply the BK equation to jets one necessarily has to go beyond equation with just running modulus of it’s transversal momentum. In the reference [39] it has been shown that in order and the Sudakov form factor assumes form:

\[ T_s(\mu^2, k^2) = \exp \left( - \int_{k^2}^{\mu^2} \frac{dk'^2}{k'^2} \frac{\alpha_s(k'^2)}{2\pi} \sum_{a} \int_{0}^{1-\Delta} dz' P'_{a'a}(z') \right) \]  

where \( \Delta = \frac{\mu}{\mu'k} \) and \( P'_{a'a} \) is a splitting function with subscripts \( a'a \) specifies the type of transition. In the \( gg \) channel one multiplies the \( P_{gg}(z) \) by \( z \) due to symmetry arguments [39].

The construction guarantees that at the integrated level the number of gluons does not change since after integration up to hard scale in Eq. [1] and application of Eq. (2) the terms \( xg_h(z) \) cancel and the part with \( \theta(k^2 - \mu^2) \) drops. The Sudakov form factor just makes the shape of gluon density scale dependent but does not modify its integral. In order to study properties of introduced hard scale gluon density we use the gluon density from the BK equation. At LO the BK equation reads:

\[ \mathcal{F}(x, k^2) = \mathcal{F}^{(0)}(x, k^2) + \frac{\alpha_s(k^2)}{\pi} \int_x^1 \frac{dz}{z} \int_{k_0^2}^{\infty} \frac{dl^2}{l^2} \left\{ \left( l^2 \mathcal{F}\left( \frac{x}{z}, l^2 \right) - k^2 \mathcal{F}\left( \frac{x}{z}, k^2 \right) \right) \frac{l^2}{|l^2 - k^2|} + \frac{k^2 \mathcal{F}\left( \frac{x}{z}, k^2 \right)}{|l^2 + k^2|^\frac{1}{2}} \right\} - 2\frac{\alpha_s^2(k^2)}{R^2} \left[ \left( \int_{k^2}^{\infty} \frac{dl^2}{l^2} \mathcal{F}(x, l^2) \right)^2 + \mathcal{F}(x, k^2) \int_{k^2}^{\infty} \frac{dl^2}{l^2} \ln \left( \frac{l^2}{k^2} \right) \mathcal{F}(x, l^2) \right], \]  

The linear part is given by the BFKL kernel while the nonlinear is proportional to the triple pomeron vertex [44,45] which allows for recombination of gluons. In the equation above \( x \) is the longitudinal momentum fraction of proton’s momentum carried by gluon while \( k \equiv |k| \) is modulus of it’s transversal momentum. In the reference [39] it has been shown that in order to apply the BK equation to jets one necessarily has to go beyond equation with just running coupling corrections included i.e. the rcBK [40]. Therefore to be realistic with applications for LHC we use the unintegrated gluon density with corrections formulated in [41–43]. Those corrections include

- kinematic effects limiting the \( l \) integration enforcing the virtuality of exchanged \( t \) channel gluon to be dominated by its transversal component.
- running coupling
- subleading at low-\( x \) pieces of splitting function important at larger values of splitting ratio \( z \) and contribution of sea quarks (indicated below by \( \Sigma(x, k^2) \))

The final equation assumes form:

\[ \mathcal{F}(x, k^2) = \mathcal{F}^{(0)}(x, k^2) + \frac{\alpha_s(k^2)}{\pi} \int_x^1 \frac{dz}{z} \int_{k_0^2}^{\infty} \frac{dl^2}{l^2} \left\{ \left( l^2 \mathcal{F}(x, l^2) - k^2 \mathcal{F}(x, k^2) \right) \frac{l^2}{|l^2 - k^2|} + \frac{k^2 \mathcal{F}(x, k^2)}{|l^2 + k^2|^\frac{1}{2}} \right\} + \frac{\alpha_s(k^2)}{2\pi k^2} \int_x^1 \frac{dz}{z} \left[ \left( P_{gg}(z) - \frac{2N_c}{z} \right) \int_{k_0^2}^{k^2} \frac{dl^2}{l^2} \mathcal{F}\left( \frac{x}{z}, l^2 \right) + zP_{gg}(z)\Sigma\left( \frac{x}{z}, k^2 \right) \right] - 2\frac{\alpha_s^2(k^2)}{R^2} \left[ \left( \int_{k^2}^{\infty} \frac{dl^2}{l^2} \mathcal{F}(x, l^2) \right)^2 + \mathcal{F}(x, k^2) \int_{k^2}^{\infty} \frac{dl^2}{l^2} \ln \left( \frac{l^2}{k^2} \right) \mathcal{F}(x, l^2) \right], \]  

\[ \text{(5)} \]
The plots of the gluon density obtained from solving of (5) and its extension for Pb target is shown on Fig. (1). The blue lines corresponds to situation when the hard scale effects are not taken into account. The kink at $k_0 = 1 GeV^2$ is an artifact of matching condition between model extension below to $k < k_0 = 1 GeV$. The maximum of the distribution signals the emergence of the saturation scale. For details on the input gluon distribution and method of solution and extension for the Pb target we refer the Reader to [21] and references therein. The gluon density from equation (5) has been successfully applied to description of $F_2$ structure function data [21] and after accounting for Sudakov effects (at cross section level) for description of azimuthal angle correlations of forward-central dijet in inclusive and inside jet tag scenario [33].

Applications

In this section we apply the hard scale dependent gluon density to study angular correlations of forward-forward dijet and to calculate $R_{pA}$ ratio for p+Pb collision [3]. As argued in [39] this observable is particularly interesting for testing low $x$ effects since the kinematical configuration of two jets probes gluon density at $x \approx 10^{-5}$. Furthermore, the distance in rapidity of produced jets is small therefore the phase space for emission of further jets is suppressed. In order to calculate the cross section we are after we use the hybrid high energy factorization [34]:

$$\frac{d\sigma}{dy_1dy_2dp_{t1}dp_{t2}d\Delta\phi} = \sum_{a,c,d} \frac{p_{t1}p_{t2}}{8\pi^2(x_1x_2S)^2} |M_{ag \rightarrow acd}|^2 x_1 f_{a/A}(x_1, \mu^2) \mathcal{F}_{g/B}(x_2, k^2, \mu^2) \frac{1}{1 + \delta_{cd}}, \quad (6)$$

with $k^2 = p_{t1}^2 + p_{t2}^2 + 2p_{t1}p_{t2} \cos \Delta\phi$ and

$$x_1 = \frac{1}{\sqrt{S}} \left( p_{t1} e^{y_1} + p_{t2} e^{y_2} \right), \quad x_2 = \frac{1}{\sqrt{S}} \left( p_{t1} e^{-y_1} + p_{t2} e^{-y_2} \right),$$

In the formulas above $S$ is the squared energy in the center of mass system of the incoming hadrons (for p+p energy is 7 TeV while for p+Pb it is 5.02 TeV) the matrix elements correspond to processes $qg^* \rightarrow qg, gg^* \rightarrow gg, gg^* \rightarrow qg$ and $f(x_1, \mu^2)$ is a collinear parton density while the

\footnote{The calculation has been done within Mathematica package MATH4JET available form the author on request.}
Figure 2: Left: cross section for decorrelations in production of forward-forward dijet in p+p collision at 7 TeV gluon density. The rapidities of produced jets satisfy $p_{t1} > p_{t2} > 20$ GeV. The continuous red line corresponds to situation with Sudakov effects included while the blue dashed line omits Sudakov effects. Right: the $R_{pA}$ ratio for p+p v. p+Pb. The continuous red line corresponds to situation with Sudakov effects included while the blue dashed line omits Sudakov effects, the brown line just helps to see the deviation from unity.

Figure 3: Left: ratio of gluon density of lead to gluon density of proton evaluated at $x = 10^{-3}$ at hard scale $\mu^2 = 25 GeV^2$ (green dotted line), $\mu^2 = 45 GeV^2$ (purple dashed line), $\mu^2 = 80 GeV^2$ (magenta dotted line), $\mu^2 = 400 GeV^2$ (red continuous line), no hard scale dependence (blue dashed line). Right: ratio of gluon density of lead to gluon density of proton evaluated at $x = 10^{-5}$ at hard scale $\mu^2 = 25 GeV^2$ (green dotted line), $\mu^2 = 45 GeV^2$ (purple dashed line), $\mu^2 = 80 GeV^2$ (magenta dotted line), $\mu^2 = 400 GeV^2$ (red continuous line), no hard scale dependence (blue dashed line).

To finalize our study we investigate the $R_{pA}$ i.e. ratio of the cross section for decorrelations of dijet produced in p+p and p+Pb. We see that the hard scale dependence leads to the ratio...
of considered cross sections to be smaller where the saturation effects play a role i.e. at values of large $\Delta \phi$. By inspecting the plots of unintegrated gluon densities and their ratios in Fig. 3 we see that the gluon density of proton is more affected by Sudakov effects than the lead gluon density therefore the ratio is smaller than one in wider range of $k$ and therefore in larger range of $\Delta \phi$. This is because in case of the lead the saturation effects are larger and the suppression of low $k$ region is more significant already for hard scale independent gluon density.

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