Modeling of Friction Stress in Twin Roll Modules

Sh R Khurramov

Institute of Mechanics and Seismic Stability of Structures named after M.T. Urazbaev of the Academy of Sciences of the Republic of Uzbekistan, 100125, Tashkent city, Durmon yuli street, 33, Uzbekistan
E-mail: shavkat-xurramov59@mail.ru

Abstract. The study is devoted to the analysis of the laws of the distribution of tangential friction stresses in twin roll modules. A two-roll module is considered, in which the rolls are positioned relative to the vertical by a slope to the right, have unequal diameters and elastic coatings from materials with different stiffness and friction coefficients, the material layer is sloped downward relative to the center line. Formulas are defined for calculating neutral angles in the considered two-roll module. Dependencies between the forces acting in the rolls and the stresses distributed under the influence of these forces are established. It was revealed that these dependences do not change with a change in the angle of supply of the material layer to the line of centers and the angle of inclination of the upper roll relative to the vertical.

1. Introduction

Technological processes in two-roll modules are carried out as a result of contact interaction of the rolls with the processed material. The central task of the theory of contact interaction of two-roll modules is an analytical description of the distribution laws of contact (normal and tangential) forces.

In determining the laws of distribution of contact stresses, the main factor is the model of friction stresses, taking into account the effect of friction in the contact zone of the rolls and connecting the tangential and normal stresses distributed along the contact curves of the rolls. In the study of contact interaction in two-roll modules, they mainly use the dry friction model (Amonton-Coulomb law) [1, 2]. From the point of view of the theory of interaction, this is equivalent to taking into account only slip zones on the contact surface. In fact, on the contact surface there are three zones that are different in kinematics — slip lag zone, adhesion zone, and slip advance zone [3].

However, a theoretical model of the friction stress for the adhesion zone during rolling does not exist. In studies [2, 4, 5] empirical dependencies \( t_s = \phi(x) \) are usually used. To carry out calculations on these dependences, data are required that can only be obtained during complex experimental studies. For example, for dependence \( t_s = t_0 \left( \frac{h_s - h_0}{l} \right) \), experimental data are needed on the extent of the adhesion zone \( l \), the friction stress at the beginning of the adhesion zone \( t_0 \), and the strip thickness in the neutral section \( h_0 \) [6]. Therefore, the models of friction stresses currently used in the theory of contact interaction of two-roll modules are considered approximate. For this reason, the theoretical contact stress distribution curves obtained in them are also considered approximate, therefore, they do not correspond to the experimental diagrams.
In modern roll machines, asymmetric two-roll modules are often used. In asymmetric two-roll modules asymmetric contact conditions occur during the interaction, on opposite contact surfaces of the strain zone. Quite a lot of publications are known \[7, 8, 9\] devoted to the theoretical study of contact stresses in the process of asymmetric rolling. Theoretical solutions of the differential equations of equilibrium of longitudinal forces in these works were carried out using approximate models of friction stresses. Therefore, these solutions do not provide the required accuracy in determining the patterns of contact stress distribution in two-roll modules.

It follows from the foregoing that obtaining theoretical curves of contact stress distribution corresponding to experimental diagrams is currently impossible due to the lack of correct models of friction stresses.

2. Analytical solution of the task

To systematize the studies of contact interaction in the work, first of all, based on the analysis of functional structures and classifications of roll modules, a generalized scheme for the interaction of a roll pair with the material being processed, i.e. a generalized model of two-roll modules, was chosen. In this two-roll module, the rolls are positioned relative to the vertical by tilting to the right at an angle \( \beta \), have unequal diameters \( R_1 \neq R_2 \) and elastic coatings from materials of different stiffness and friction coefficients \( f_1 \neq f_2 \), the lower shaft is a driving one and the upper one is free. The material layer has uniform thickness \( \delta_1 \) and is tilted downward relative to the line of centers at an angle \( \gamma_1 \) (figure 1).

Analyze the stress state of the contact interaction of the material layer and the lower roll, which occurs along the contact curve \( A_1A_2 \).

In the steady-state interaction process, the lower roll is affected by: pressure force of the clamping devices \( Q_1 \), horizontal reaction of the roll supports \( F_1 \), moment of resistance forces \( M_1 \), elementary

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**Figure 1.** Force scheme in a two-roll module

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Analyze the stress state of the contact interaction of the material layer and the lower roll, which occurs along the contact curve \( A_1A_2 \).

In the steady-state interaction process, the lower roll is affected by: pressure force of the clamping devices \( Q_1 \), horizontal reaction of the roll supports \( F_1 \), moment of resistance forces \( M_1 \), elementary
forces of normal pressure and friction, acting along the entire contact curve of the roll. Elementary forces in the zones of compression \((N_{1x}, T_{1x})\) and recovery \((N_{1y}, T_{1y})\) are presented separately. The friction forces at the beginning and at the end of the contact zone have opposite directions. They change their signs, turning to zero at a neutral point \(A_{13}\). It was revealed \([2, 3]\) that in the driving roll the neutral point is located to the left of the center line, that is, in the compression zone. Let the neutral point \(A_{13}\) correspond to the angle \(\theta_{1x} = -\varphi_{1x}\) (figure 1).

Considering the lower roll in equilibrium under the action of applied forces, for the compression zone we have

\[
\begin{align*}
N_{11x} + T_{11x} + F_{11x} + Q_{11x} &= 0, \\
N_{11y} + T_{11y} + F_{11y} + Q_{11y} &= 0
\end{align*}
\]

or

\[
\begin{align*}
dN_{11x} + dT_{11x} + dF_{11x} + dQ_{11x} &= 0, \\
dN_{11y} + dT_{11y} + dF_{11y} + dQ_{11y} &= 0
\end{align*}
\]  

(1)

where \(N_{11x}, N_{11y}, T_{11x}, T_{11y}\) are the projections of the main normal and tangential forces of the compression zone on the axes \(x \) and \(y\).

From the forces scheme in compression zone (figure 1) we find

\[
\begin{align*}
dN_{11x} &= dN_{11y} \sin(\theta_{1x} - \psi_{1x}), \\
dN_{11y} &= -dN_{11x} \cos(\theta_{1x} - \psi_{1x}), \\
dT_{11x} &= -dT_{11y} \cos(\theta_{1x} - \psi_{1x}), \\
dT_{11y} &= -dT_{11x} \sin(\theta_{1x} - \psi_{1x})
\end{align*}
\]

\[
\begin{align*}
F_{11x} &= dF_{11y} = F_{11y} = dQ_{11x} = 0, \\
Q_{11x} &= Q_{11y} = Q_{11},
\end{align*}
\]

(2)

where \(\psi_{1x}\) is the angle between the force \(dN_{1x}\) and the radius \(r_{1x}\).

Given these expressions from system (1), for the compression zone we have

\[
\frac{dF_{11}}{dQ_{11}} = \frac{-dT_{11} \cos(\theta_{1x} - \psi_{1x}) - dN_{11} \sin(\theta_{1x} - \varphi_{1x})}{dT_{11} \sin(\theta_{1x} - \psi_{1x}) + dN_{11} \cos(\theta_{1x} - \varphi_{1x})}.
\]

(3)

Since we are considering a steady-state process, we can assume that \(\frac{F_{1}}{Q_{1}} = C_{1}\), where \(C_{1}\) is a constant value. Hence we have

\[
\frac{d}{dQ_{1}} \left( \frac{F_{1}}{Q_{1}} \right) = \frac{Q_{1} dF_{1} - F_{1} dQ_{1}}{Q_{1}^{2}} = 0 \quad \text{or} \quad \frac{dF_{1}}{dQ_{1}} = C_{1}.
\]

Assuming that \(C_{11} = \frac{dF_{11}}{dQ_{11}}\) from equality (3) we obtain

\[
\frac{dT_{11}}{dN_{11}} = \frac{\sin(\theta_{11} - \psi_{11}) + C_{11} \cos(\theta_{11} - \varphi_{11})}{\cos(\theta_{11} - \psi_{11}) - C_{11} \sin(\theta_{11} - \varphi_{11})}.
\]

(4)

Elementary forces are connected to contact stresses by relations

\[
\begin{align*}
dN_{11} &= n_{11} \sqrt{r_{11}^{2} + r_{12}^{2}} d\theta_{11}, \\
dT_{11} &= t_{11} \sqrt{r_{11}^{2} + r_{12}^{2}} d\theta_{11},
\end{align*}
\]

(5)

where \(n_{11} = n_{11}(\theta_{11})\), \(t_{11} = t_{11}(\theta_{11})\) – are normal and shear stresses, respectively, distributed over the compression zone of the contact curve of the rolls.

Substitute expressions (5) in equality (4), then transform it according to the expressions

\[
\cos \psi_{11} = \frac{r_{11}}{\sqrt{r_{11}^{2} + r_{12}^{2}}}, \quad \sin \psi_{11} = \frac{r_{12}}{\sqrt{r_{11}^{2} + r_{12}^{2}}}
\]

and obtain dependencies connecting the tangent and normal stresses at the points of the compression zone of the lower roll

\[
t_{11} = \frac{(\sin \theta_{11} + C_{11} \cos \theta_{11}) r_{11} - (\cos \theta_{11} - C_{11} \sin \theta_{11}) r_{12}'}{(\cos \theta_{11} - C_{11} \sin \theta_{11}) r_{11} + (\sin \theta_{11} + C_{11} \cos \theta_{11}) r_{12}'}\]  

\(r_{11}, -\varphi_{11} < \theta_{11} \leq 0\),

(6)
We obtain the formula connecting the tangential and normal stresses at the points of the recovery zone of the lower roll. It has the form

\[ t_{12} = \frac{(\sin \theta_{12} + C_{12} \cos \theta_{12}) r_{12} - (\cos \theta_{12} - C_{12} \sin \theta_{12}) r'_{12}}{(\cos \theta_{12} - C_{12} \sin \theta_{12}) r_{12} + (\sin \theta_{12} + C_{12} \cos \theta_{12}) r'_{12}} n_{12}, \quad 0 \leq \theta_{12} \leq \varphi_{12}. \]  

(7)

Where \( C_{12} = \frac{dF_{12}}{dQ_{12}}. \)

Note that at the point of contact curve on the line of centers the following boundary conditions are satisfied

\[ t_{11}(0) = t_{12}(0), \quad n_{11}(0) = n_{12}(0), \quad r_{11}(0) = r_{12}(0) = R_{10}, \quad r'_{11}(0) = r'_{12}(0) = 0. \]

These conditions lead to equality \( C_{11} = C_{12}. \)

Then, we have

\[ C_1 = C_{11} = C_{12} = \frac{F_1}{Q_1}. \]

So, from equations (6) and (7) we obtain a system that describes the model of friction stresses for the lower driving roll

\[
\begin{align*}
t_{11} &= \left( Q_1 \sin \theta_{11} + F_1 \cos \theta_{11} \right) r_{11} - \left( Q_1 \cos \theta_{11} - F_1 \sin \theta_{11} \right) r'_{11}, \quad -\varphi_{11} \leq \theta_{11} \leq 0, \\
&\quad \left( Q_1 \cos \theta_{11} - F_1 \sin \theta_{11} \right) r_{11}' + \left( Q_1 \sin \theta_{11} + F_1 \cos \theta_{11} \right) r'_{11}', \quad n_{11}, \quad -\varphi_{11} \leq \theta_{11} \leq 0, \\
\end{align*}
\]

(8)

In the two-roll module under consideration, the upper shaft is free. In this case, the forces acting on the upper roll \( F_2 \) and \( T_2 \) change directions [3]. Therefore, in the equations of system (8), the quantities \( t_{2j} \) (\( j = 1,2 \)) and \( F_2 \) have opposite signs. In this regard, the model of friction stresses for the upper roll has the form

\[
\begin{align*}
t_{21} &= -\left( Q_2 \sin \theta_{21} - F_2 \cos \theta_{21} \right) r_{21} - \left( Q_2 \cos \theta_{21} + F_2 \sin \theta_{21} \right) r'_{21}, \quad -\varphi_{21} \leq \theta_{21} \leq 0, \\
&\quad \left( Q_2 \cos \theta_{21} + F_2 \sin \theta_{21} \right) r_{21}' + \left( Q_2 \sin \theta_{21} - F_2 \cos \theta_{21} \right) r'_{21}', \quad 0 \leq \theta_{21} \leq \varphi_{21}, \\
\end{align*}
\]

(9)

The systems of equations (8) and (9) determine the models of friction stresses in the considered two-roll module. They show that the models of friction stresses in two-roll modules are independent of the inclination of the material layer feed to the center line and of the inclination of the upper roll relative to the vertical. An analysis of these models showed that they describe stress models of all cases of the two-roll module under consideration.

We transform the formula (8) taking into account the expression \( \tan \xi_1 = \frac{F_1}{Q_1} \) and \( C_{11} = C_1 \)

\[ t_{11} = \frac{(tg \theta_{11} + C_1) - (1 - C_1 tg \theta_{11}) tg \psi_{11}}{(1 - C_1 tg \theta_{11}) + (tg \theta_{11} + C_1) tg \psi_{11}} n_{11}, \]

Assuming now \( C_1 = \tan \xi_{11} \), we pass to the expression

\[ t_{11} = \tan (\theta_{11} - \psi_{11} + \xi_1) n_{11}. \]

(10)

Transforming also \( t_{12} \), we rewrite system (8) in the form

\[
\begin{align*}
t_{11} &= \tan (\theta_{11} - \psi_{11} + \xi_1) n_{11}, \quad -\varphi_{11} \leq \theta_{11} \leq 0, \\
\end{align*}
\]

(11)
At the neutral point \( t_1(\varphi_{13}) = 0 \) and \( n_1(\varphi_{13}) \neq 0 \). Therefore, according to expression (11), the condition for determining the neutral angle can be represented as

\[
\varphi_{13} + \varphi_{11}^* = \xi_1,
\]

where \( \varphi_{11}^* = \arctg \frac{n_{11}'(\varphi_{13})}{n_{11}(\varphi_{13})} \).

It follows from the equations of system (9) and expression (12) that the models of friction stresses and the values of neutral angles of two-roll modules depend on the external forces acting on the rolls and the curve shape of roll contact.

Calculate \( t \) and \( f = \frac{t}{n} \) from the formulas of system (8) when rolling a deformable layer of material according to the harmonic law of normal stresses distribution

\[
n_{11} = \frac{n_{\max}}{2} \left( 1 + \cos \left( \frac{\theta_{11}}{\varphi_{11}} \pi \right) \right), \quad n_{12} = \frac{n_{\max}}{2} \left( 1 + \cos \left( \frac{\theta_{12}}{\varphi_{12}} \pi \right) \right).
\]

The results of calculations of the distribution patterns of tangential stresses and the changes in contact stress ratio are shown in figures 2 and 3. The obtained patterns correspond to experimental distribution diagrams [2, 3].

**Figure 2.** Patterns of tangential stresses distribution at: \( 1 - F_1 = 0; \ 2 - F_1 = 0.025Q_1; \ 3 - F_1 = 0.075Q_1 \).

**Figure 3.** Patterns of changes in \( f = \frac{t}{n} \) at: \( 1 - F_1 = 0; \ 2 - F_1 = 0.025Q_1; \ 3 - F_1 = 0.075Q_1 \).

3. Conclusions
1. For the first time, models of friction stress were found in a two-roll module, in which: the rolls are located relative to the vertical slope to the right, have unequal diameters and elastic coatings from materials with different stiffnesses and friction coefficients; lower roll drive, upper - free; the material layer is tilted downward relative to the center line.
2. It was revealed that the obtained models are general in the sense that they are applicable for partial cases of interaction in two-roll modules.
3. For the first time, the relationships between the forces acting in the rolls and the stresses distributed under the effect of these forces are established. It was revealed for the first time that these dependences do not change with a change in the angle of supply of the material layer to the line of centers and the angle of inclination of the upper roll relative to the vertical.

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