Robust STAP Based on Training Snapshot Selection and Steering Vector Estimation

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Abstract. The presence of mismatch between the presumed and actual target direction in spatial-temporal plane and the contamination of signal of interest (SOI) in training snapshots significantly degrades the performance of space-time adaptive processing (STAP). To solve the aforementioned problems, a robust STAP by employing the training snapshot selection and target direction correction is proposed in this paper. To solve the inaccurate estimation of clutter covariance matrix (CCM), training snapshots contaminated by target are eliminated using generalized inner products (GIP) statistics, and the more accurate CCM is obtained using the updated training snapshot. Then, an accurate steering vector (SV) estimation method is introduced using the prior knowledge of SOI. Finally, we obtain the proposed robust STAP weight with the novel CCM and the corrected target space-time SV. We carried out several computer simulations to validate the performance of our proposed method.

1 Introduction

Space-time adaptive processing (STAP) makes full use of the data collected from multiple array elements and multiple pulses to suppress clutter and detect slow-moving targets, so it is one of the key technologies of airborne early warning radar [1], [2]. However, the performance of STAP degrades significantly by certain practical issues. Firstly in high resolution radar system, the presence of signal of interest (SOI) in training snapshots results in clutter covariance matrix (CCM) estimation error [3]. Secondly, the target direction error in spatial-temporal plane is present when the spatial angle and Doppler frequency error occurs, i.e., the distortion between the nominated and real target space-time steering vectors (SVs) [4]. Thus, the CCM error and target space-time SV error should be resolved to improve the target detection performance.

Various robust STAP approaches have been proposed to address the aforementioned problems [5-10]. In [5], the authors developed a robust STAP approach based on convex problem optimization. In [6], a robust STAP is proposed to mitigate the undesired mismatches in practical scenarios, which belongs to colored loading algorithms. To solve the high sidelobe and distorted mainlobe in conventional linearly constrained minimum variance (LCMV) beamformer, its all-one response vector is substituted by the optimized response vector [7]. In [8], the clutter-plus-noise subspace is employed...
to correct the mismatched target SV. However, the aforementioned methods share a common shortcoming that the performance would degrade dramatically in the high signal-to-noise ratio (SNR). This is because these methods use the sample covariance matrix (SCM) directly, which is inaccurate when the training snapshots contain the SOIs. Some other approaches are proposed to reduce the effect caused by corrupted training snapshots. In [9], a robust STAP is presented by incorporating the phase constraint which is robust against target contamination. In [10], to mitigate the CCM error and target SV error, a robust STAP by reconstructing the CCM is studied which has superior performance.

To address the inaccurate CCM estimation and target space-time SV uncertainty problem, a robust STAP is presented in this paper. We utilize the generalized inner products (GIP) statistics to remove the contaminated training snapshots, and then obtain a more accurate CCM using the updated training snapshots. In addition, with the possible sector of SOI in spatial-temporal plane, the SOI covariance matrix is constructed and an accurate target space-time SV is obtained by using the prime eigenvector of SOI covariance matrix. Finally, the proposed robust STAP weight is estimated with the obtained more accurate CCM and target SV.

2 Signal Model and Background

In general, a side-looking uniform linear array (ULA) airborne radar system is considered here. The ULA consists of \(N\) elements with inter-element spacing being \(d\). Each element emits \(K\) pulses in a coherent processing interval (CPI) with constant pulse repetition frequency (PRF) \(f_r\). The radar wavelength is \(\lambda\). The platform height and velocity are \(H\) and \(v\), respectively. Thus, the received echoes consists of the independent clutter scatters collected from the \(l\)th range cell, which is expressed as [1]

\[
x_i = \sum_{i=1}^{N_c} \alpha_i \mathbf{s}(\psi_i, f_{di}) + \mathbf{n}
\]

where \(N_c\) is the number of clutter scatters; \(\alpha_i\) is the reflection coefficient of the \(i\)th clutter scatter, \(\psi_i\) and \(f_{di}\) are the corresponding spatial cone angle, and temporal frequency; \(\mathbf{n}\) is the thermal noise vector, whose covariance matrix is \(\sigma_n^2 \mathbf{I}\), where \(\sigma_n^2\) represents the noise power, \(\mathbf{I}\) denotes the identity matrix; \(\mathbf{s}(\psi_i, f_{di})\) is the \(NK \times 1\) space-time SV, which is given by

\[
\mathbf{s}(\psi_i, f_{di}) = \mathbf{a}(\psi_i) \otimes \mathbf{b}(f_{di})
\]

where \(\mathbf{a}(\psi_i) = [1, e^{2\pi \cos(\psi_i)/\lambda}, \ldots, e^{2\pi(K-1)\cos(\psi_i)/\lambda}]^T\) and \(\mathbf{b}(f_{di}) = [1, e^{2\pi f_{di}/f_r}, \ldots, e^{2\pi(K-1)f_{di}/f_r}]^T\) represent the spatial and temporal SV; \(f_{di} = 2v \cos(\psi_i)/\lambda\) stands for the temporal frequency; \(\otimes\) denotes the Kronecker product.

Under the minimum variance distortionless response (MVDR) criterion, the space-time adaptive weight is obtained as

\[
\min_{\mathbf{w}} \mathbf{w}^H \hat{\mathbf{R}} \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H \hat{\mathbf{s}}_0 = 1
\]

where \(\hat{\mathbf{s}}_0 = \mathbf{s}_0(\hat{\psi}_0, \hat{f}_{d0})\) is the presumed target space-time SV with \(\hat{\psi}_0\) and \(\hat{f}_{d0}\) representing the associated spatial cone angle and Doppler frequency, \(\hat{\mathbf{R}}\) is the SCM, and it is calculated as

\[
\hat{\mathbf{R}} = \frac{1}{L} \sum_{l=1}^{L} \mathbf{x}_l \mathbf{x}_l^H,
\]

where \(\mathbf{x}_l\) is the \(l\)th training snapshot, \(L\) is the training snapshot number. The optimal STAP weight vector is
$$w_{\text{OPT}} = \frac{\hat{R}^{-1}s_0}{s_0^H\hat{R}^{-1}s_0}$$

(4)

However, when some training snapshots contain the SOI, the actual SCM \( \hat{R} \) is given by

$$\hat{R} = \hat{R}_{\text{error}} + R_{c+n}$$

(5)

where \( \hat{R}_{\text{error}} \) denotes the CCM error, and \( R_{c+n} \) is the actual clutter-plus-noise covariance matrix (CNCM). The derivation of expression (5) will be provided in equation (8).

When the spatial cone angle and Doppler frequency error are present, the actual target space-time SV can be formulated as

$$s_0 = \tilde{s}_0 + \delta$$

(6)

where \( \delta \) is the space-time SV error.

3 Proposed Robust STAP Method

In order to mitigate the aforementioned problem of CCM error and target space-time SV error, a robust STAP approach by utilizing training snapshot selection and target SV estimation is presented.

3.1 CCM Estimation With Training Snapshot selection

When target components are contained in training snapshots, the SCM is mismatched with \( R_{c+n} \), which would dramatically degrades the target detection performance. Therefore, the contaminated training snapshots must be removed from the training snapshot set to mitigate the influence of covariance matrix error \( \hat{R}_{\text{error}} \). GIP is a typical non-homogeneity detection (NHD) algorithm to detect the corrupted training snapshots [11], [12]. In GIP algorithm, training snapshots whose GIP statistics exceed the threshold are eliminated.

Denote the initial training snapshot set is \( \Omega = \{x_l, l = 1, 2, \cdots, L\} \), and then the GIP statistics are written as

$$\eta_{\text{GIP}} = x_l^H\hat{R}^{-1}x_l$$

(7)

The GIP statistics are considered as the square of the amplitude of the whitening filter output, and its expectation is given by \( E(\eta_{\text{GIP}}) = E\{x_l^H\hat{R}^{-1}x_l\} = E\{\hat{R}^{-1/2}x_l\}^2 = NK \), where \( E(\bullet) \) represents the expectation. In addition, the performance of GIP algorithm relies on the CCM accuracy \( \hat{R} \), and the CCM error \( \hat{R}_{\text{error}} \) would degrades the performance of conventional GIP algorithm. Assume that \( L_0 \) training snapshots are contaminated by target, and \( \tilde{x}_l = s_0 + x_l \) represents the corrupted training snapshot. Then, the SCM can be rewritten as

$$\hat{R} = \frac{1}{L} \left( \sum_{l=1}^{L_0} \tilde{x}_l\tilde{x}_l^H + \sum_{l=L_0+1}^{L} x_lx_l^H \right) = \frac{1}{L} \sum_{l=1}^{L_0} s_0s_0^H + \frac{1}{L} \sum_{l=1}^{L} x_lx_l^H = \hat{R}_{\text{error}} + R_{c+n}$$

(8)

It is seen from (8) that the larger of \( L_0 \), there is a larger mismatch between \( \hat{R} \) and \( R_{c+n} \). On the other hand, when \( L_0 \) is small, the gap between \( \hat{R} \) and \( R_{c+n} \) is also small. Thus, in this paper, suppose that the number of contaminated training snapshots is small, i.e. \( L_0 \ll L \). It is appropriate to remove the corrupted training snapshots by using GIP algorithm under the condition that \( L_0 \ll L \). However, the computational load is heavy when calculating the GIP statistics of all training snapshots.
Assume that the possible sector of SOI is known, that is, the possible range where the SOI locates is known. This can be obtained by using the range estimation techniques. Hence, the contaminated training snapshots are in the prior training snapshot set \( \Omega_0 \) and \( \Omega_0 \) is much smaller than \( \Omega \). Thus, we can calculate the GIP statistics of training snapshots in the search region \( \Omega_0 \), which significantly reduces the search region.

Therefore, the main steps of the proposed CCM estimation method are summarized as follows.

1) Design the prior training snapshot set \( \Omega_0 \) by using the prior knowledge of SOI, i.e., possible range of target.

2) Calculate the GIP statistics of training snapshots using (7) which are in the prior training snapshot set \( \Omega_0 \).

3) Discard the training snapshots that may be corrupted by target according to their GIP statistics compared with threshold.

Calculate the CCM according to SCM using the remaining training snapshots which are not corrupted by SOI, and the accurate CCM \( \hat{R}_s \) can be obtained.

### 3.2 Target Space-time SV Estimation

Here, we estimate the target space-time SV to alleviate the mismatch between the presumed and actual target SV. Suppose that the possible sector of SOI \( \Phi \) is obtained, where the spatial cone angle region \( \Phi_s \) and Doppler frequency region \( \Phi_d \) in the whole angle-Doppler plane are known. The possible clutter region in the whole two dimension plane can be denoted as \( \Phi \). In this paper, \( \Phi \) and \( \Phi \) are distinguishable from each other, i.e., \( \Phi \cap \Phi = \emptyset \). Therefore, we can compute the SOI covariance matrix as

\[
R_{ROI} = \iint_{\Phi} s(\psi, f_d) s^H(\psi, f_d) \hat{R} \, d\psi \, df_d
\]

In order to efficiently calculate the integral, we uniformly sample the continuous spatial angle region \( \Phi_s \) and Doppler frequency region \( \Phi_d \) to a discrete region. Thus, \( P \) space-time grid points are obtained, that is, \( \{ s_1(\psi_1, f_{d1}), s_2(\psi_2, f_{d2}), \ldots, s_p(\psi_p, f_{dp}) \} \). Replacing the integral with summation, the SOI covariance matrix can be approximately estimated as

\[
R_{ROI} \approx \sum_{i=1}^{P} s_i(\theta_i, f_{di}) s_i^H(\theta_i, f_{di}) \hat{R} s_i(\theta_i, f_{di})
\]

We eigen-decompose the SOI covariance matrix as

\[
R_{ROI} = \sum_{i=1}^{NK} \lambda_i \mu_i^H \mu_i
\]

where \( \lambda_i \) is the \( i \)th eigenvalue of \( R_{ROI} \), \( \mu_i \) is the eigenvector associated with \( \lambda_i \). Therefore, the actual target SV can be estimated with the primary eigenvector of \( R_{ROI} \). Similar to [10], the target space-time SV can be estimated as

\[
\hat{s}_0 = \sqrt{NK} \mu_1
\]

With the accurate estimates of CCM \( \hat{R} \) and target space-time SV \( \hat{s}_0 \), we can present the proposed robust STAP weight as

\[
w_{\text{Proposed}} = \frac{\hat{R}^{-1} \hat{s}_0}{s_0^H \hat{R}^{-1} \hat{s}_0}
\]
The computational burden of the proposed approach is mainly in the target space-time SV estimation. Specifically, the target space-time SV estimation involves $O \left( \max \left( P(NK)^2, (NK)^3 \right) \right)$. So the proposed method has the computational burden of $O \left( \max \left( P(NK)^2, (NK)^3 \right) \right)$. Compared with the Reconstruction-Estimation method in reference [10], which needs to reconstruct the CCM in near the whole two dimension plane, the computational burden of the proposed method is reduced dramatically.

4 Simulation Results

Computer simulations are implemented to evaluate the effectiveness of the proposed method in this section. The main parameters of airborne radar system are as follows: element number $N = 10$, pulse number $K = 10$, $d = 0.15 \text{m}$, $\lambda = 0.15 \text{m}$, PRF $f_r = 2000 \text{Hz}$, clutter-to-noise ratio (CNR) is 50 dB, $v = 140 \text{m/s}$, $H = 8000 \text{m}$, actual and presumed target spatial cone angle are $0^\circ$ and $3^\circ$, actual and presumed target Doppler frequency are $-420 \text{Hz}$ and $-400 \text{Hz}$. Assume that the SOI is $\Phi_S = \left[ -5^\circ, 5^\circ \right]$, $\Phi_D = \left[ -450 \text{Hz}, -390 \text{Hz} \right]$, $P = 18$. The LSMI-MVDR [4], Worst-case optimization method [13], Reconstruction-Estimation method [10] are also presented for comparison. The loading factor for LSMI-MVDR is 8 and the parameter is $\varepsilon = 0.9\sqrt{NK}$ for Worst-case. All of the results are obtained through 100 Monte-Carlo runs.

Fig. 1 presents the different output SCNR curves with the variation of SNR. The number of snapshots is set 400. It is seen that the performance of LSMI-MVDR and Worst-case degrades when the input SNR is high. The reason is that these two methods use the SCM directly, which causes the target signal self-nulling effects. The Reconstruction-Estimation and the proposed methods have the similar output SCNR curve to the optimal one. However, it is observed that the proposed method performs a little worse than the Reconstruction-Estimation method when the input SNR is small. This is because the proposed method cannot remove the corrupted training snapshots at small input SNRs. The optimal SCNR curve is also plotted.

Fig. 2 depicts the output SCNR with the variation of snapshot number. The input SNR is set 20 dB. As seen in Fig. 2, all the methods except the proposed method has a dramatic output SCNR curves at the small snapshot number. The Reconstruction-Estimation and the proposed methods exhibit superior performance to the LSMI-MVDR and Worst-case at large snapshot number. In addition, the proposed method performs better than Reconstruction-Estimation at large snapshot number.

The SCNR loss performance is employed to validate the aforementioned approaches with SNR being 20 dB, as shown in Fig. 3. As expected, the SCNR loss curves of LSMI-MVDR and Worst-case approaches are much lower in the sidelobe clutter region, whereas Reconstruction-Estimation and the proposed methods performs better because the estimated CCM of them is more accurate. In addition, the proposed method outperforms Reconstruction-Estimation method at very small Doppler frequency. This means that the proposed method can detect very slowly moving target.
5 Conclusion

The contaminated training snapshots and mismatched target steering vector would cause severe performance degradation of STAP. To tackle this problem, we present a robust STAP approach by utilizing training snapshot selection and target SV estimation. Instead of reconstruct the CCM, we use the GIP algorithm to remove the corrupted training snapshots and acquire a more accurate CCM. Then, a computational efficient method is introduced to obtain the target space-time SV. Several computer simulations are implemented to illustrate the effectiveness of the proposed method. Furthermore, the proposed method can be utilized in the practical STAP environment since its computational complexity is low.

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