Inventory Control Problems with Exponential and Quadratic Demand considering Weibull Deterioration

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Abstract. Supplies are goods stored for use or sale in the future. In reality, it is often found that stored goods can experience deterioration (in terms of value/quality), that is to say goods are damaged, and have a limited service life (becoming expired), such as those found in food products, medicines, chemicals, and others. These factors are important ones that the seller needs to be aware of so as to ensure that the goods he stores are in good condition. Another aspect that cannot be ignored in the management of goods inventory is the demand from consumers, which is dynamic in nature. In this paper, a mathematical model will be developed considering Weibull deterioration for exponential demand and quadratic demand with shortages of time-dependent goods. Based on the results and sensitivity analysis conducted, it was found that the total cost of inventory will increase in accordance with the level of demand and the level of deterioration of goods.

Keywords: Inventory; exponential demand; quadratic demand; time-dependent shortages; deterioration

1. Introduction

Inventory refers to goods stored for use or sale in the future. Inventory management is an important factor in a company's activities so that business activities can run smoothly. If a company does not pay attention to inventory management, it can incur losses within the company, such as the halt of production, and the tendency of consumers to turn to other companies so as to cause loss of profit for the company. The problem that companies often face in managing inventory is determining the amount of inventory, so that there is no excess or deficient inventory level ([2], [9], [10], [11]). If the available inventory is lacking, then the company is unable to meet consumer demand and loses the opportunity to make the profit it should obtain. But if the inventory is overspent, then the inventory will incur high expenses because every item stored is bound to cost money. Therefore, the inventory of goods must be determined in the right amount. Procuring a good supply of goods should certainly take into account several factors. Factors to consider include order costs, purchase costs, storage/holding costs, and deterioration in the quality of an item. When an item is stored for a certain period of time, it will usually experience a decrease in quality that may result in a decrease in the selling value of the
goods resulting in loss. Examples of frequent deterioration in the quality of goods are decay in expired fruits, vegetables, or medicines. Some researchers have developed inventory models taking into account these deterioration factors both constant ([4], [6], [7]), following certain distributions such as the Weibull model ([1], [3]), or stochastic deterioration ([8]).

Another factor affecting inventory management is consumer demand. Demand plays a salient role in the inventory model. Requests can be constant, but only for a certain period of time. In reality demand is better if a non-linear function approach is adopted due to dynamic market conditions. In addition, several inventory models have been developed with varying demand patterns, such as those depending on price ([5]), on inventory ([7]) or time-dependent ones ([8]).

In this paper, a mathematical model will be developed for inventory models with exponential and time-dependent demand taking into account deterioration factors. The model developed refers to models from [3], by making modifications to the request function in the form of quadratic functions. In addition, we found different formulations for inventory levels at the time of for a period of time of for requests that follow exponential functions. The form of quadratic function for the demand used in this paper refers to [1], but with the inventory function at the time of sale is different. Through this model, the time when the goods run out, the length of the cycle, and the amount of orders will be determined so that the minimum total cost of inventory is obtained.

2. Notation and Model

Notations used in this inventory model are:

- \( S \): order cost per order
- \( P \): purchase price of goods per unit
- \( h \): storage/holding cost per unit
- \( s \): shortage cost per unit per year
- \( l(t) \): inventory level of goods at t
- \( T_1 \): time when goods run out
- \( T \): cycle length
- \( TC \): total cost of inventory
- \( Q \): number of goods orders
- \( D(t) \): number of goods request at t
- \( \theta(t) \): Weibull distributed goods deterioration rate, i.e. \( \theta(t) = \alpha \beta t^{\beta-1}, \)
  \( 0 \leq \alpha < 1, \beta > 0 \)

2.1. Model 1: Inventory model with Exponential Demand

At the beginning, there are \( Q_1 \) units of goods. Over time \([0, T_1]\), the inventory of goods will be reduced caused by the deterioration of goods and demand for goods and until the time the \( T_1 \) of goods runs out, so:

\[
\frac{dl_1}{dt} + \theta(t)l_1(t) = -\gamma e^{\delta t} \quad 0 < t < T_1
\]  

(1)

When the goods have run out, consumer demand still persists with \( D(t) = a + bt \), thus:

\[
\frac{dl_2}{dt} = -(a + bt) \quad T_1 < t < T
\]  

(2)

where \( a > 0 \) and \( 0 < b < 1 \). With \( l_1(T_1) = 0 \) and \( l_2(0) = Q_1 \), the solution of equations (1) and (2) is
To find the components of the total cost, first we formulate the holding cost $HC$:

$$
HC = \int_0^{T_1} h \, I_1(t) \, dt
$$

$$
= h \gamma \left[ \frac{T_1^2}{2} - \alpha \delta T_1^2 + \frac{\alpha \delta^2 \beta T_1^2}{2} + \frac{\alpha^2 \delta^2 \beta^2 T_1^4}{4} - \frac{\alpha^3 \delta \beta^2 T_1^5}{4 \beta + 4} - \frac{\alpha^2 \delta \beta T_1^4}{2 \beta + 6} \right]
$$

$$
+ \frac{\alpha^2 \delta^2 \beta^3 T_1^6}{8 \beta + 12} - \frac{\alpha^2 \delta^2 \beta^3 T_1^6}{8 \beta + 12} + \frac{\alpha^2 \delta^2 \beta^3 T_1^6}{8 \beta + 12} - \frac{\alpha^2 \delta^2 \beta^3 T_1^6}{8 \beta + 12}
$$

The shortage cost $SC$ is given by:

$$
SC = s \int_{T_1}^{T} I_2(t) dt = s \left[ \frac{a}{2} (T - T_1)^2 + \frac{b}{6} (T^3 + 2T_1^3 - 3T_1^2T) \right]
$$
The order cost \( (OC) \):

\[
OC = S \tag{8}
\]

Purchase Cost \( (PC) \):

\[
PC = P Q(T) = P [Q_1(T) + Q_2(T)]
\]

with

\[
Q_2(T) = -l_2(T) = -\left[ a(T_1 - T) + \frac{b}{2} (T_1^2 - T^2) \right]
\]

Resulting in:

\[
PC = P \left[ \frac{a \delta^2 r_{2\beta+3}}{2 \beta + 6} \right] \left[ a(T_1 - T) + \frac{b}{2} (T_1^2 - T^2) \right] - \left[ a(T_1 - T) + \frac{b}{2} (T_1^2 - T^2) \right] \tag{9}
\]

Thus, the total cost for a year is the summation of (6), (7), (8), and (9) so that:

\[
TC(T, T_1) = \frac{1}{T} (OC + HC + SC + PC) \tag{10}
\]

To find the minimum total cost, the conditions of the following equations must be met.

\[
\frac{\partial TC}{\partial T_1} = 0, \quad \frac{\partial TC}{\partial T} = 0 \tag{11}
\]

### 2.2 Model 2: The Quadratic Demand Level

In the second model the inventory model with the quadratic demand will be formed, namely \( D(t) = a + bt + ct^2 \) (Begum and Sahu (2012)), where a, b, and c are constant for \( 0 < t < T_1 \). Inventory of goods decreased due to deterioration and demand, at the time of sale \( T_1 \), thus:

\[
\frac{dl_1}{dt} + 0(t)l_1 = -(a + bt + ct^2) \quad 0 < t < T_1 \tag{12}
\]

and

\[
\frac{dl_2}{dt} = -(f + gt) \quad T_1 < t < T \tag{13}
\]

where \( f > 0 \) and \( 0 < g < 1 \).

With \( l_1(T_1) = 0 \) and \( l_1(0) = Q_1 \), the solution of equations (12) and (13) is:

\[
l_1(t) = e^{-\alpha t} \left[ \left( \frac{aT_1}{\beta + 1} + \frac{a^2 r_{2\beta+1}}{4 \beta + 2} + \frac{\delta r_{2\beta+1}}{2} + \frac{bT_1^2}{\beta + 2} + \frac{b^2 r_{2\beta+2}}{4 \beta + 4} + \frac{cT_1^3}{3} + \frac{c^2 r_{2\beta+3}}{4 \beta + 6} + \right) - \left( \frac{a \delta^2 r_{2\beta+3}}{2 \beta + 6} \right) \right] \tag{14}
\]

\[
l_2(t) = f(T_1 - t) + \frac{g}{2} (T_1^2 - t^2) \tag{15}
\]
The components of the total cost are given in equations (17), (18), (19) and (20).

\[ H_C = \int_0^{T_1} h \, I_1(t) \, dt \]

\[ = h \left( a T_1^2 + \frac{a T_1}{\beta + 1} + \frac{a^2 T_1}{4 \beta + 2} + \frac{b T_1^3}{2} + \frac{b a T_1}{4 \beta + 4} + \frac{b^2 T_1^2}{3} + \frac{c T_1}{\beta + 3} \right. \]

\[ + \left. \frac{c a^2 T_1}{4 \beta + 6} \right) \]  

(16)

Shortage Cost (SC):

\[ SC = s \int_{T_1}^{T} I_2(t) \, dt = s \left[ \frac{f}{2} (T - T_1)^2 + \frac{g}{6} (T^3 - 2 T_1^3 - 3 T_1^2 T) \right] \]  

(17)

Order cost (OC):

\[ OC = S \]  

(18)

Purchase Cost (PC):

\[ PC = P \, Q(T) = P \left[ Q_1(T) + Q_2(T) \right] \]  

with

\[ Q_2(T) = -I_2(T) = - \left[ f(T_1 - T) + \frac{g}{2} (T_1^2 - T^2) \right] \]  

(19)
Resulting in

\[
PC = P \left[ e^{-\alpha t_1^\beta} \left( a T_1 + \frac{a a T_1^{\beta+1}}{\beta + 1} + \frac{a a^2 T_1^{2\beta+1}}{4\beta + 2} + \frac{b T_1^2}{2} + \frac{b a T_1^{\beta+2}}{\beta + 2} + \frac{b a^2 T_1^{2\beta+2}}{4\beta + 4} + \frac{c T_1^3}{3} 
+ \frac{c a T_1^{\beta+3}}{\beta + 3} + \frac{c a^2 T_1^{2\beta+3}}{4\beta + 6} \right) - \left( \frac{f(T_1 - T)}{g(T_1^2)} + \frac{g(T_1^2 - T^2)}{2} \right) \right] \tag{20}
\]

3. Example and Sensitivity Analysis

The model sensitivity analysis is conducted to see the influence that occurs due to changes in parameter values. This chapter will discuss the effect of changes in the order cost parameters per single order (S), the level of time-dependent requests (δ and γ), the level of deterioration that follows Weibull's distribution model (α and β), the shortage cost per unit and per year (s), the time-dependent shortage level (a and b), the cost of purchase per unit of goods (P), and the cost of storage per unit (h) to the time when the goods run out (T1), cycle length (T), total cost (TC), and the amount of orders (Q). The parameter values used are as follows:

- Order cost per single order (S) = $100
- Purchase cost per unit (P) = $10
- Storage cost per unit (h) = $10
- Shortage cost per unit and per unit time (s) = $50
- \( \theta(t) \) deterioration rate = \( \alpha \beta t^{\beta-1} \) with \( \alpha = 0.7 \) and \( \beta = 4 \)

Optimal solutions for models 1 and 2 can be seen in the Table 1:

| Parameter                  | Model 1       | Model 2       |
|----------------------------|---------------|---------------|
| Time period until goods run out (T1) | 0.9128 years  | 0.3413 years  |
| Length of cycle (T)        | 1.0661 years  | 0.4323 years  |
| Amount of orders (Q)       | 117 unit      | 45 unit       |
| Total cost (TC)            | $1771.82      | $1456.75      |

In the next two subsections, we perform sensitivity analysis on both models to find the impact of parameter values’ changes to the optimal solution. We increase and decrease each values of the parameters by +/- 10% and 25% by keep the other parameters’ values as in this section.
3.1 Sensitivity Analysis of Model 1

Table 2: Effect of Parameters Changes on Model 1

| Parameter | Change in % | \( T_1 \) (year) | \( \%\Delta T_1 \) | \( T \) (year) | \( \%\Delta T \) | \( Q \) (unit) | \( TC \) ($) |
|-----------|-------------|-------------------|------------------|--------------|----------------|-------------|-----------|
| \( S \)   | +25         | 0.9168            | 0.0044           | 1.0747       | 0.0081         | 117         | 1795.17   |
|           | +10         | 0.9144            | 0.0018           | 1.0695       | 0.0032         | 117         | 1781.18   |
|           | -10         | 0.9111            | -0.0018          | 1.0625       | -0.0032        | 116         | 1762.42   |
|           | -25         | 0.9087            | -0.0044          | 1.0572       | -0.0081        | 116         | 1748.27   |
| \( \delta \)| +25        | 0.9024            | -0.0114          | 1.0589       | -0.0068        | 117         | 1787.89   |
|           | +10         | 0.9086            | -0.0046          | 1.0631       | -0.0028        | 117         | 1778.33   |
|           | -10         | 0.9170            | 0.0046           | 1.0690       | 0.0028         | 116         | 1765.21   |
|           | -25         | 0.9236            | 0.0114           | 1.0735       | 0.0068         | 116         | 1755.15   |
| \( \gamma \)| +25       | 0.9005            | -0.0135          | 1.1250       | 0.0552         | 149         | 2129.05   |
|           | +10         | 0.9083            | -0.0049          | 1.0877       | 0.0203         | 128         | 1903.36   |
|           | -10         | 0.9179            | 0.0049           | 1.0413       | -0.0203        | 103         | 1621.79   |
|           | -25         | 0.9261            | 0.0135           | 1.0028       | -0.0552        | 83          | 1388.04   |
| \( \alpha \)| +25       | 0.8857            | -0.0297          | 1.0434       | -0.0213        | 117         | 1794.16   |
|           | +10         | 0.9012            | -0.0127          | 1.0564       | -0.0091        | 117         | 1781.14   |
|           | -10         | 0.9255            | 0.0127           | 1.0768       | 0.0091         | 116         | 1761.91   |
|           | -25         | 0.9473            | 0.0297           | 1.0953       | 0.0213         | 116         | 1745.71   |
| \( \beta \)| +25       | 0.9205            | 0.0084           | 1.0699       | 0.0036         | 116         | 1752.66   |
|           | +10         | 0.9160            | 0.0035           | 1.0676       | 0.0014         | 116         | 1763.46   |
|           | -10         | 0.9095            | -0.0035          | 1.0646       | -0.0014        | 117         | 1781.27   |
|           | -25         | 0.9045            | -0.0084          | 1.0630       | -0.0036        | 117         | 1797.65   |
| \( s \)  | +25         | 0.9147            | 0.0021           | 1.0392       | -0.0252        | 113         | 1783.18   |
|           | +10         | 0.9137            | 0.0009           | 1.0539       | -0.0114        | 115         | 1776.90   |
|           | -10         | 0.9117            | -0.0009          | 1.0807       | 0.0114         | 118         | 1765.77   |
|           | -25         | 0.9097            | -0.0021          | 1.1093       | 0.0252         | 121         | 1754.19   |
| \( a \)  | +25         | 0.9192            | 0.0070           | 1.0080       | -0.0545        | 112         | 1809.20   |
|           | +10         | 0.9158            | 0.0033           | 1.0403       | -0.0242        | 115         | 1789.53   |
|           | -10         | 0.9088            | -0.0033          | 1.0962       | 0.0242         | 118         | 1749.42   |
|           | -25         | 0.9007            | -0.0070          | 1.1532       | 0.0545         | 120         | 1704.31   |
| \( b \)  | +25         | 0.9128            | 0              | 1.0658       | -0.0003        | 116         | 1771.96   |
|           | +10         | 0.9128            | 0              | 1.0659       | -0.0002        | 116         | 1771.88   |
|           | -10         | 0.9128            | 0              | 1.0661       | 0.0002         | 116         | 1771.76   |
|           | -25         | 0.9128            | 0              | 1.0663       | 0.0003         | 116         | 1771.67   |
| \( P \)  | +25         | 0.9547            | 0.0459          | 1.1112       | 0.0423         | 117         | 2039.28   |
|           | +10         | 0.9307            | 0.0196          | 1.0855       | 0.0182         | 117         | 1880.05   |
|           | -10         | 0.8929            | -0.0196         | 1.0442       | -0.0182        | 116         | 1661.63   |
|           | -25         | 0.8583            | -0.0459         | 1.0058       | -0.0423        | 116         | 1491.97   |
| \( h \)  | +25         | 0.8637            | -0.0538         | 1.0417       | -0.0228        | 119         | 1896.08   |
|           | +10         | 0.8921            | -0.0227         | 1.0557       | -0.0098        | 117         | 1823.48   |
|           | -10         | 0.9351            | 0.0227          | 1.0775       | 0.0098         | 115         | 1717.12   |
|           | -25         | 0.9727            | 0.0538          | 1.0973       | 0.0228         | 114         | 1628.31   |

The effect of rising order costs per single order does not have too much influence on the number of orders and optimal solutions. The greater the order cost per single order, the greater the total cost.
Conversely, if the order cost per single order is reduced, then the total cost that must be incurred is reduced. Changes to the order cost per single order have no meaningful effect on the length of time until the item runs out and the length of the cycle time. When the order cost per single order increases, both the length of time until the item runs out and the length of the cycle time increase. Conversely, when the order cost per single order is reduced, then the length of time until the item runs out and the length of the cycle time is reduced.

Delta parameters (δ) do not exert too much influence on the number of orders and optimal solutions. On the other hand, gamma parameters have a big influence on the number of orders and optimal solutions. The increase in the rate of demand causes the length of time until the goods run out and the length of the cycle time to increase, so the total cost increases. On the other hand, the decrease in the rate of demand leads to a smaller cycle length and length of production time so that the total cost decreases.

The changes in the α and β parameters do not greatly affect the total amount of bookings to increase and the optimal solutions. The rise in the α parameter causes a decline in both the time until the goods run out and the length of the time cycle, so that the total cost decreases, and vice versa. The rise in the β parameter extends both the length of time until the goods run out and the length of the time cycle, so that the total cost increases, and vice versa.

Changes in shortage cost per unit and per year do not greatly affect the total amount of orders and the optimal solutions. The increase in the shortage cost per unit and per year extends the length of time until goods run out and reduces the cycle time length, so that the total cost rises. Conversely, a decrease in the shortage cost per unit and per year reduces the length of time until goods run out and extends the cycle time, so that the total cost decreases as well.

The a parameter makes a considerable impact on the total amount of orders and optimal solutions. Conversely, the b parameter does not affect the total amount of orders and optimal solutions. The increase in the value of the a parameter extends the length of time until goods run out and reduces the length of the time cycle, so that the total cost rises, and vice versa.

The effect of rising purchase costs per unit has a major influence on the optimal solution, but does not affect the amount of orders. The increase in purchase cost per unit increases both the length of time until the goods run out and the length of the cycle time, resulting in increased total costs. On the other hand, the cost of purchasing per unit causes the length of time until the goods run out and the length of the cycle time to decrease, resulting in a decrease in the total cost.

The effect of rising storage costs per unit exerts considerable influence on optimal solutions, and has little effect on the amount of orders. The increase in storage cost per unit results in both a reduction in the length of time until the goods run out as well as the length of the cycle time, resulting in increased total costs. Conversely, the decrease in storage costs per unit results in an increase in the length of time until the goods run out as well as the length of the cycle time, resulting in decreased total costs.
### 3.2 Sensitivity Analysis of Model 2

#### Table 3: Effect of Parameters Changes on Model 2

| Parameter | Change in % | \( T_1 \) (year) | \( %\Delta T_1 \) | \( T \) (year) | \( %\Delta T \) | \( Q \) (unit) | \( TC \) ($) |
|-----------|-------------|-----------------|-----------------|----------------|----------------|---------------|--------------|
| \( S \)   | +25         | 0.3765          | 0.1031          | 0.4784         | 0.1066         | 50            | 1511.64      |
|           | +10         | 0.3560          | 0.0431          | 0.4515         | 0.0444         | 47            | 1479.58      |
|           | -10         | 0.3255          | -0.0431         | 0.4118         | -0.0444        | 43            | 1433.06      |
|           | -25         | 0.2996          | -0.1031         | 0.3783         | -0.1066        | 39            | 1395.10      |
| \( a \)   | +25         | 0.3593          | 0.0527          | 0.4212         | -0.0257        | 56            | 1740.27      |
|           | +10         | 0.3691          | 0.0815          | 0.4304         | -0.0044        | 48            | 1546.85      |
|           | -10         | 0.3818          | 0.1187          | 0.4407         | 0.0044         | 42            | 1359.76      |
|           | -25         | 0.4079          | 0.1951          | 0.4478         | 0.0257         | 36            | 1201.05      |
| \( b \)   | +25         | 0.3328          | -0.0249         | 0.4253         | -0.0162        | 45            | 1394.75      |
|           | +10         | 0.3378          | -0.0103         | 0.4294         | -0.0067        | 45            | 1389.32      |
|           | -10         | 0.3448          | 0.0103          | 0.4352         | 0.0067         | 46            | 1381.85      |
|           | -25         | 0.3504          | 0.0249          | 0.4398         | 0.0162         | 46            | 1376.08      |
| \( c \)   | +25         | 0.3405          | -0.0023         | 0.4320         | -0.0007        | 45            | 1386.18      |
|           | +10         | 0.3410          | -0.0008         | 0.4324         | 0.0002         | 45            | 1385.84      |
|           | -10         | 0.3416          | 0.0008          | 0.4326         | 0.0007         | 45            | 1385.39      |
|           | -25         | 0.3421          | 0.0023          | 0.4330         | 0.0016         | 46            | 1385.05      |
| \( \alpha \) | +25       | 0.3395          | -0.0053         | 0.4306         | -0.0039        | 45            | 1457.24      |
|           | +10         | 0.3406          | -0.0021         | 0.4330         | 0.0016         | 45            | 1456.96      |
|           | -10         | 0.3420          | 0.0021          | 0.4330         | 0.0016         | 45            | 1456.55      |
|           | -25         | 0.3432          | 0.0053          | 0.4341         | 0.0039         | 45            | 1456.25      |
| \( \beta \) | +25       | 0.3463          | 0.0146          | 0.4370         | 0.0109         | 45            | 1455.31      |
|           | +10         | 0.3438          | 0.0073          | 0.4347         | 0.0055         | 45            | 1455.97      |
|           | -10         | 0.3377          | -0.0073         | 0.4289         | -0.0055        | 45            | 1458.03      |
|           | -25         | 0.3276          | -0.0146         | 0.4198         | -0.0109        | 44            | 1462.49      |
| \( s \)   | +25         | 0.3478          | 0.0190          | 0.4222         | -0.0233        | 44            | 1466.66      |
|           | +10         | 0.3442          | 0.0085          | 0.4277         | -0.0106        | 44            | 1461.18      |
|           | -10         | 0.3384          | -0.0085         | 0.4369         | 0.0106         | 44            | 1456.84      |
|           | -25         | 0.3348          | -0.0190         | 0.4424         | 0.0233         | 44            | 1446.52      |
| \( f \)   | +25         | 0.3699          | 0.0838          | 0.4099         | -0.0518        | 44            | 1501.00      |
|           | +10         | 0.3559          | 0.0427          | 0.4245         | -0.0180        | 45            | 1479.02      |
|           | -10         | 0.3267          | -0.0427         | 0.4401         | 0.0180         | 46            | 1459.97      |
|           | -25         | 0.3127          | -0.0838         | 0.4547         | 0.0518         | 48            | 1366.63      |
| \( g \)   | +25         | 0.3413          | 0              | 0.4322         | -0.0002        | 45            | 1385.68      |
|           | +10         | 0.3413          | 0              | 0.4322         | -0.0002        | 45            | 1385.64      |
|           | -10         | 0.3413          | 0              | 0.4323         | 0              | 45            | 1385.59      |
|           | -25         | 0.3413          | 0              | 0.4324         | 0.0002         | 45            | 1385.55      |
| \( P \)   | +25         | 0.3331          | -0.0240         | 0.4258         | -0.0150        | 44            | 1715.42      |
|           | +10         | 0.3379          | -0.0099         | 0.4296         | -0.0062        | 44            | 1560.25      |
|           | -10         | 0.3447          | 0.0099          | 0.4350         | 0.0062         | 45            | 1353.21      |
|           | -25         | 0.3501          | 0.0240          | 0.4393         | 0.0150         | 46            | 1197.80      |
| \( h \)   | +25         | 0.3449          | 0.0105          | 0.4345         | 0.0051         | 44            | 1449.91      |
|           | +10         | 0.3601          | 0.0551          | 0.4469         | 0.0337         | 46            | 1442.07      |
|           | -10         | 0.3571          | 0.0463          | 0.4480         | 0.0363         | 47            | 1435.63      |
|           | -25         | 0.3761          | 0.1019          | 0.4608         | 0.0659         | 49            | 1425.41      |
The impact of the increase in order cost per single order does not greatly affect the total amount of orders and optimal solutions. The greater the order cost per single order, the greater the total cost. Conversely, if the order cost per single order is reduced, then the total cost that must be incurred is reduced. Changes to the order cost per single order have no significant effect on the length of time until the item runs out or the length of the cycle time. When the order cost per single order increases, the length of time until the item runs out and the length of the cycle time increases. Conversely, when the order cost per single order is reduced, the length of time until the item runs out and the length of the cycle time is reduced.

The effect of the increase in parameter $a$ has a considerable effect on the number of bookings and the total cost. While the effect of increasing parameters $a$, $b$, and $c$ does not affect the number of orders, it does have a small effect on the total cost. The increase in the value of parameters $a$, $b$, and $c$ results in a reduction of both the length of time until the goods run out and the length of the cycle time, resulting in decreased total costs.

Changes in $\alpha$ and $\beta$ parameters do not affect the number of orders and have little effect on the optimal solution. The increase in the $\alpha$ parameter causes the length of time until the goods run out as well as the length of the cycle to decrease so that the total cost increases and vice versa. The increase in $\beta$ parameters causes the length of time until the goods run out as well as the length of the cycle time to increase, resulting in decreased total cost and vice versa.

The changes in the shortage rates per unit and per year do not greatly affect the total amount of orders and the optimal solutions. The increase in the shortage cost per unit and per year reduces the length of time until the goods run out and causes a decrease in the cycle time, so that the total costs increase. Conversely, a decrease in the shortage cost per unit and per year cause the length of time until the goods run out to decrease while causing the length of the cycle time to increase, resulting in a decrease of the total costs.

Parameter $f$ has a considerable impact on the total amount of bookings and on optimal solutions. Conversely, parameter $g$ does not affect these. The increase in the value of parameter $f$ causes the length of time until the goods run out to increase, and the length of the cycle time to decrease, resulting in increased total costs and vice versa.

The effect of rising purchase costs per unit has a major influence on the optimal solution, but does not affect the amount of bookings. The increase in purchase cost per unit causes both the length of time until the goods run out and the length of the cycle time to decrease, resulting in increased total costs. Conversely, the decrease in purchase cost per unit causes both the length of time until the goods run out as well as the cycle time to increase, resulting in a decrease in the total cost.

The effect of rising storage costs per unit does not have too much effect on optimal solutions and has little effect on the amount of bookings. The increase in storage cost per unit results in both a reduction in the length of time until the goods run out and the length of the cycle time, resulting in increased total costs. Conversely, a decrease in storage costs per unit results in both an increased length of time until the goods run out and an extended length of the cycle time, resulting in a decrease in the total cost.

4. Conclusions and Suggestions
The sensitivity analysis yields the following conclusions:
1. From the development of models that have been applied, model 1 can be used for inventory that approaches exponential demand and model 2 can be used for supplies that approach quadratic demand.
2. The total cost and many optimum requests on model 1 will increase as order (S) costs increase, demand level parameters (δ and γ), parameters in the Weibull distribution (α), and storage costs (h).
3. The optimal time in one cycle in model 1 will increase as the cost of purchasing (P) increases and parameters in the Weibull distribution (β).
4. The total cost and many optimum requests on model 2 will increase as the order cost per single order (S) increases, the request level parameter (a), and the deficiency level parameter (g).
5. The optimal time in one cycle in model 2 will increase as the booking cost (S) increases as well as the parameters in the Weibull distribution (β).

The inventory model developed in this paper is a model containing one type of goods and deterministic demand. Models can be developed for different types of goods (multi-items) with probabilistic demand, by following a particular distribution. This is quite realistic considering that in the case of goods sold, these are not restricted to only one type, and furthermore the demand for such goods cannot be known for certain. These aspects may be further explored as a topic for further development.

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