Fingerprints of Majorana bound states in Aharonov Bohm geometry

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We study a ring geometry, coupled to two normal metallic leads, which has a Majorana bound state (MBS) embedded in one of its arm and is threaded by Aharonov Bohm (AB) flux $\phi$. We show that by varying the AB flux, the two leads go through resonance in an anti-correlated fashion while the resonance conductance is quantized to $2e^2/h$. We further show that such anti-correlation is completely absent when the MBS is replaced by an Andreev bound state (ABS). Hence this anti-correlation in conductance when studied as a function of $\phi$ provides a unique signature of the MBS which cannot be faked by an ABS. We contrast the phase sensitivity of the MBS and ABS in terms of tunneling conductances. We argue that the relative phase between the tunneling amplitude of the electrons and holes from either lead to the level (MBS or ABS), which is constrained to $0, \pi$ for the MBS and unconstrained for the ABS, is responsible for this interesting contrast in the AB effect between the MBS and ABS.

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Introduction :- Zero energy Majorana bound states (MBS) which appear as end states of a 1-D p-wave superconductor have been attracting a lot of interest recently\textsuperscript{1,2}, mainly due to their topological nature and relevance\textsuperscript{3} in topological quantum computation. Although serious attempts for confirming the existence of the MBS have been made experimentally\textsuperscript{4,5}, their outcome remains controversial, and it is perhaps fair to say that there still has not been a definitive experiment to verify their existence. The primary reason for this is that it is not easy to distinguish Majorana modes from other spurious zero energy modes. This has also led to considerable theoretical effort\textsuperscript{6} to look for clearly distinguishable robust signals of Majorana modes.

Many earlier theoretical studies have focussed on promising physical systems that support Majorana modes\textsuperscript{8,9}. Another focus\textsuperscript{10,11} has been understanding and extending the proto-typical model that hosts Majorana modes, which is the Kitaev model\textsuperscript{12}. There have also been generalisations which yield more than one Majorana mode at each of the edges\textsuperscript{13,14}. Floquet generation of Majorana modes\textsuperscript{15,16}, etc.

In this letter, we show that the Aharonov-Bohm (AB) effect in a ring geometry with a MBS embedded in one of its arm can provide a distinct signature which cannot be faked by an Andreev bound state(ABS). Earlier attempts to use AB flux interferometers have been in the context of teleportation\textsuperscript{17,18} or non-local conductance or persistent currents\textsuperscript{19}, but they involve the MBS at both ends of a wire. Many other recent proposals which discuss distinguishing signatures of the MBS rely on quantum noise measurements\textsuperscript{20} which are in general difficult to implement. In contrast, we propose conductance measurements which can clearly distinguish the Majorana from a spurious zero mode. Our proposed setup comprises of a two terminal ring geometry as shown in Fig.1, with direct coupling between the leads as well as coupling via a MBS/ABS hosted by a superconductor, which is the third lead and which remains grounded for our proposal.

We show that when both the normal leads are equally biased with respect to the grounded superconductor, constructive resonance for one of the normal leads is always accompanied by a destructive anti-resonance on the other normal lead. As the conductance on each lead has flux periodicity of a flux quantum ($\phi_0 = hc/e$), each normal lead goes through a resonance and an anti-resonance as the phase of the direct tunneling term, which is tunable by the AB flux, changes by $\phi_0$. On the contrary, when we replace the MBS by an ABS in the above described setup, we find that the current flowing through both the leads remains equal, irrespective of the variation of the AB flux. Hence the anti-correlation in current obtained as function of the flux can be considered as a robust and direct signature of the MBS.

Tunneling into the MBS: To begin with, we consider a model where a MBS is tunnel coupled to two normal leads. We will later add a direct tunneling term (with a complex phase) between the two leads to convert it into an effective Hamiltonian describing the topological equivalent of a two path interferometer with an AB flux enclosed(see Fig.1) where the AB flux is given by the phase of the complex tunneling amplitude. The Hamiltonian for the system in the absence of direct tunneling...
is given by
\[ H_0 = \sum_{\alpha=1,2} H_{\alpha} + H_T \quad \text{with} \quad H_{\alpha} = \sum_k \epsilon_k c_{\alpha k}^\dagger c_{\alpha k} \]
and
\[ H_T = i\gamma \sum_{\alpha,k} (u_{\alpha} c_{\alpha k} + u_{\alpha}^* c_{\alpha k}^\dagger) \]
(1)

where the index \( \alpha = 1, 2 \) runs over the two leads and \( c_{\alpha k} \) corresponds to the electron operator in lead-\( \alpha \). \( u_{\alpha} \) and \( u_{\alpha}^* \) denote the complex amplitudes (matrix elements) for the coupling of the MBS to the electron and hole operators on the two leads. From general considerations of hermiticity of the Majorana operator, the tunneling can only be to an ‘equal’ linear combination of electron and hole annihilation operators on the normal lead given by \( c_{\alpha k} \) and \( c_{\alpha k}^\dagger \), and by adjusting the phases of the basis states in the leads, we can choose the tunneling amplitudes \( u_{\alpha} \) to be real. Now it is straightforward to obtain the scattering matrix at zero energy for this problem using the Weidenmuller formula\(^{21}\) given by
\[ S = 1 - 2 \frac{u_1 u_2^\dagger}{(uu)_{\dagger}} = \begin{pmatrix} s_{ee} & s_{eh} \\ s_{he} & s_{hh} \end{pmatrix}^\dagger, \quad u = \sqrt{\nu} (u_1 \ u_2 \ u_1 \ u_2) \]
(2)

where the \( s_{ee}, \) etc, are 2 \times 2 matrices in the basis of the two leads and the full \( S \)-matrix has been written in the particle-hole basis. Here, \( \nu \) is the density of states in the leads. Following Refs.\(^{22,23}\), the current and noise at zero temperature and for a finite bias on both leads can be obtained. We give the general expression for arbitrary voltages on the two leads in the supplementary section and confine ourselves to vanishingly small but equal bias on both leads here, where this reduces
\[ I_i = \frac{e^2}{h} V \frac{2u_i^2}{U}, \quad P_{11} = \frac{e^2}{h} (2V) \frac{u_1^2 u_2^2}{U^2} = P_{22} \]
and
\[ P_{12} = -\frac{e^2}{h} (2V) \frac{u_1^2 u_2^2}{U}, \quad P_{21} = P_{22} \]
(3)

where \( U = u_1^2 + u_2^2 \). An important point to note here is the fact that sum of the conductances on both leads is fixed at \( I_1/V + I_2/V = 2e^2/h \). In fact, it remains quantized at this value for any number of normal leads\(^{24}\) tunnel coupled to the Majorana. This implies that the increase in conductance in a given lead has to be compensated by a decrease in the other leads. This feature is unique to the MBS and is completely absent for the ABS.

We will show later that this anti-correlation in the conductance between the two leads can be tuned via the flux in an ABS type set up. Further, we note that the quantization of the conductance actually implies a sum rule not just for the average current but for the sum of the current operator itself given by \( \hat{I}_1 + \hat{I}_2 = 2e^2/h V \) at zero energy. This immediately implies that the fluctuation in the currents are strongly constrained, leading to the sum rule for the noise \( \sum_{ij} P_{ij} = 0 \). Once Fermi statistics is taken into account in addition to the quantization of total conductance, this automatically implies that the auto-correlated noise is positive and the cross-correlated noise is negative definite \( (P_{11} = P_{22} = -P_{12} = -P_{21}) \).

The issue of the cross-correlated noise being negative definite has been addressed earlier in the context of the MBS\(^{25}\) but its origin in the current sum rule has not been explicitly mentioned till now.

**MBS and the AB set up:** Now we add a direct tunneling term (not via the MBS) between the two normal leads. This converts the original Hamiltonian in Eq.1 into a Hamiltonian that describes the topological equivalent of a two path interferometer with \( AB \) flux enclosed (see Fig.[1]), where the \( AB \) flux is just given by the phase of the direct tunneling amplitude. The phase freedom of the basis states on the leads has been used to make the tunneling to the Majorana mode real. Hence, the direct tunneling amplitude between the two leads will be complex in general. We choose the gauge of the \( AB \) gauge field to identify this phase with the \( AB \) flux, and write the total Hamiltonian for the model as
\[ H = H_0 + \tau_0 e^{i\phi} \psi_1^\dagger \psi_2 + h.c. \]
(4)

where \( \phi \) has the interpretation of the \( AB \) flux enclosed and \( \tau_0 \) is a real number representing the amplitude of direct tunneling between the leads. To find the scattering matrix for the \( AB \) setup, we need to extend the standard form of the Weidenmuller formula to include the effects of the direct tunneling term which is shown explicitly in the supplementary section. This gives us the scattering matrix
\[ S(E) = S_N - i\pi \nu (1 + i\pi \nu T)^{-1} W^\dagger \times \]
\[ [E + i\pi \nu W (1 + i\pi \nu T)^{-1} W^\dagger]^{-1} W (1 + S_N) \]
where \( S_N = (1 + i\pi \nu T)^{-1} (1 - i\pi \nu T). \)
(5)

Note that in the absence of the direct tunneling term between the wires given by the \( T \) matrix defined as
\[ T = \begin{pmatrix} \tau + \tau^\dagger & 0 \\ 0 & -(\tau + \tau^\dagger)^* \end{pmatrix}, \]
(7)
the \( S \)-matrix reduces to the usual Weidenmuller formula
\[ S(E) = 1 - 2i\pi \nu W^\dagger [E + i\pi \nu W W^\dagger]^{-1} W \quad \text{for} \quad T = 0. \]
(8)

It is now straightforward to obtain the current and the noise from the scattering matrix obtained by applying Eq.[6] to the Hamiltonian for the \( AB \) set up given in Eq.[4]. We find that
\[ \hat{I}_1 = \frac{e^2}{h} V \frac{2\hat{u}_1^2}{\hat{U}}, \quad \hat{I}_2 = \frac{e^2}{h} V \frac{2\hat{u}_2^2}{\hat{U}} \]
\[ \hat{P}_{12} = -\frac{e^2}{h} (2V) \frac{\hat{u}_1^2 \hat{u}_2^2}{\hat{U}^2}, \quad \hat{P}_{21} = -\hat{P}_{12} = -\hat{P}_{21} \]
\[ \hat{u}_1 = u_1 + i\pi \nu \tau_0 e^{-i\phi} u_2, \quad \hat{u}_2 = u_2 + i\pi \nu \tau_0 e^{i\phi} u_1. \]
(9)

Here \( \hat{U} = |\hat{u}_1|^2 + |\hat{u}_2|^2 \). Firstly we note that the sum of conductance \( \hat{I}_1/V + \hat{I}_2/V = 2e^2/h \) is still quantized and is independent of \( \phi \) while the difference oscillates with the period \( \phi_0 \). Due to the sum rule, the resonance
condition for the lead-1(2) is given by $\tilde{l}_2 = 0$ ($\tilde{l}_1 = 0$). This is equivalent to having $\tilde{u}_2 = 0$ ($\tilde{u}_1 = 0$) respectively. Hence it is easy to see that the resonance condition, corresponding to having a conductance of $2e^2/h$ on a given lead, depends on both the amplitude $\tau_0$ and the phase $2\pi/\phi_0$.

In Figs. 2a and 2b, we show the conductances $G_i$ and the auto and cross-correlated noise $P_{ii}$ and $P_{ij}$ computed (see supplementary section for more details) as a function of the flux at finite temperature and finite voltage. Note that the anti-correlation remains valid even at finite temperature and voltages. However, the sum of the conductances is no longer quantised to be $2e^2/h$. Instead, it get multiplied by $F(V, T)$, a decreasing function of $V$ and $T$. The exponential fall-off of the current, and the amplitude of the cross-correlations as the bias is increased, at different temperatures, is also shown in Figs. 3a and 3b.

**Tunneling to ABS and phase sensitivity even in the absence of direct tunneling:** Here we first consider an ABS which is tunnel coupled to two normal leads and later add a direct tunneling term (with a complex phase) between the two leads to convert the system to an AB type setup. The Hamiltonian for the system in the absence of direct tunneling is given by Eq.1, except that now the tunneling term is replaced by

$$H_T = a^\dagger \sum_{\alpha,k} (t_{\alpha} c_{\alpha k} + v_{\alpha}^* c_{\alpha k}^\dagger) + h.c.,$$

(10)

where the ABS is denoted by the resonant level creation operator $a^\dagger$ and the tunneling amplitudes to the electron and hole states on the leads are given by $t_{\alpha}$ and $v_{\alpha}$ respectively. Note that there are no terms in the Hamiltonian for the ABS itself as it is at zero energy. (A toy lattice model for the Andreev bound state leading to the above coupling has been explicitly shown in the supplementary section.) Our main aim is to contrast the results now with the earlier results where the coupling was to a MBS. The scattering matrix can be obtained as before through the Weidenmuller formula and we have

$$S = 1 - 2W^\dagger (WW^\dagger)^{-1} W \quad \text{where}$$

$$W = \begin{pmatrix} t_1 & t_2 & v_1^* & v_2^* \\ -v_1 & -v_2 & -t_1^* & -t_2^* \end{pmatrix}$$

(11)

Note that the matrix $WW^\dagger$ is singular for $t_1 = v_1$ and $t_2 = v_2$. Hence, it is not possible to obtain a comparison between the ABS and the MBS simply by ‘setting’ $a = \gamma$ and $t = v$ directly in the scattering matrix.

To understand this better, let us consider the single lead case, which can simply be written as

$$H_T = \sum_k (a^\dagger a) W \begin{pmatrix} c_{1k} \\ c_{1k}^\dagger \end{pmatrix} \quad \text{with} \quad W = \begin{pmatrix} t_1 & v_1^* \\ -v_1 & -t_1^* \end{pmatrix}$$

$$WW^\dagger = \begin{pmatrix} |t_1|^2 + |v_1|^2 & -2t_1 v_1^* \\ -2v_1^* t_1 & |t_1|^2 + |v_1|^2 \end{pmatrix}$$

and

$$\text{Det}(WW^\dagger) = (|t_1|^2 - |v_1|^2)^2$$

(12)

We can now explicitly write this Hamiltonian in terms of 2 Majorana modes by changing to the Majorana basis - $a = (\gamma + i\tilde{\gamma})/2$, $a^\dagger = (\gamma - i\tilde{\gamma})/2$ so that the Hamiltonian can be rewritten as

$$H_T = \sum_k (i\gamma \quad i\tilde{\gamma}) \tilde{W} \begin{pmatrix} c_{1k} \\ c_{1k}^\dagger \end{pmatrix}$$

with

$$\tilde{W} = \begin{pmatrix} -\frac{1}{2}(t_1 - v_1) & \frac{1}{2} (t_1^2 - v_1^2) \\ -\frac{1}{2}(t_1 + v_1) & -\frac{1}{2}(t_1^2 + v_1^2) \end{pmatrix}$$

(13)

As can seen from here, either at $t_1 = v_1$ or at $t_1 = -v_1$, one of the Majorana modes disappear from the Hamiltonian and the coupling matrix couples the lead only to a single MBS. This observation provides us with a physical picture for describing the basic difference between a MBS and an accidental zero energy ABS. An ABS tunnel coupled to a lead corresponds to having a simultaneous tunnel coupling of the lead with a pair of Majorana bound states. This is the main reason for the ABS and the MBS behaving differently when embedded in an AB ring.

The current and noise correlations for a vanishing small but equal voltage $V$ applied to the two leads at zero temperature are given by

$$I_1 = I_2 = \frac{e^2}{h} V S(|t_1 v_2 - t_2 v_1|)^2$$

$$P_{11} = \frac{e^3}{h} V S(|t_1|^2 + |t_2|^2 - |v_1|^2 - |v_2|^2)^2(|t_1 v_2 - t_2 v_1|)^2$$

$$P_{22} = P_{12} = P_{21} \quad \text{where}$$

$$D = (|t_1|^2 + |t_2|^2 + |v_1|^2 + |v_2|^2)^2 - 4(|t_1 v_2^* + t_2 v_1^*|^2)$$

(14)

Note that here, the current on the two leads are equal, and there is no constraint on the total conductance. This can be understood as follows. The presence of a single MBS creates an anti-correlation in the current between the two leads, but the second Majorana in the ABS, which is the time reversed partner of the first one, compensates for the anti-correlation, making it completely symmetric. The conductance in each of the two leads can be tuned to its resonant value of $2e^2/h$ when the various amplitudes satisfy the condition $|t_1|^2 + |t_2|^2 = |v_1|^2 + |v_2|^2$ which can straight forwardly read off from the expression for the noise. Hence the resonant value for the sum of the conductance of the two leads for the ABS is $4e^2/h$ while it is $2e^2/h$ for the MBS. Hence any observation of total conductance exceeding $2e^2/h$ can rule out the presence of MBS.

We also note that the noise correlations on the two leads are positive, since the conductances are equal. In fact, if one could tune the relative phase between the electron and hole terms - i.e., choose $t_{\alpha} = |t_{\alpha}| e^{i\theta_{\alpha}}$ and $v_{\alpha} = |v_{\alpha}| e^{i\theta_{\alpha}}$, and tune the various $\theta_{\alpha}$, not only the noise, even the conductance would show oscillations as a function of any of the $\theta_{\alpha}$. However, the Majorana mode only couples either to the case where the phases of the couplings to the electron and hole terms are the same (the case chosen in the previous section), or when
the coupling is to the other Majorana mode, where the phases differ by \( e^{\pm i\pi} = -1 \). This phase rigidity of the couplings to the electrons and holes in the leads is again a feature of the MBS which is not shared by an accidental zero energy ABS. Hence, if this phase can be varied in a desirable fashion, it can provide a distinguishing feature between a MBS and an accidental zero energy ABS. But in general this is not possible; hence addition of the direct tunneling path with an enclosed flux discussed in this letter provides a minimal set up for accessing the above described difference between the ABS and the MBS.

Finally we also include direct tunneling between the leads to study the AB set up for the ABS. However, since there are many parameters to vary, the results are highly dependent on the parameters chosen and the results for some typical values are shown in Figs. 2c and 2d and Figs. 3c and 3d. The only general feature that we see is that the conductance on the two leads remain identical, (but the cross-correlations can be positive or negative), irrespective of details of the AB flux, hence maintaining its contrast to the MBS case. This complete our study of AB set up for the ABS.

**Discussion and conclusion:** In this paper, we have attempted to distinguish between signatures of a MBS and an accidental zero energy ABS by studying the conductance and noise correlations of two leads in a two path interferometry setup with a superconductor (giving rise to a MBS or an ABS depending on whether or not it is topological) embedded in one of its arms. By changing the phase of the direct tunneling between the leads (equivalent to the AB phase), we find that the conductances in the two leads are perfectly anti-correlated for the MBS case, with their sum quantized to \( 2e^2/h \). Furthermore, the phase of the direct tunneling can be tuned to give rise to a resonance in one of the leads, which is necessarily accompanied by an anti-resonance in the other lead. This feature is completely absent for the ABS and hence, can be used as a strong fingerprint for the existence of a MBS. We have also computed the noise correlations for both the MBS and the ABS, and attribute the negative cross-correlations in the MBS case to the strong correlation in the conductances on the two leads, coupled by the fermionic statistics of the MBS. We point out that for the coupling to the MBS, the phases between the electron and hole processes can only be either +1 or −1 (phase rigidity), whereas they can have an arbitrary phase for an ABS. This fact leads to distinguishing features in the transport across an AB set up. So the bottomline is that the distinction between MBS and ABS is achieved via just conductance measurements alone. In an AB ring geometry, the conductances in the two leads can be tuned
by the flux and the anti-correlation of the conductances in the two leads is a strong fingerprint for the MBS.

Note added: While writing up this work, the following papers [28,29] appeared. Our results are in agreement with theirs, where there is overlap.

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Supplementary material

I. MODEL FOR THE ANDREEV BOUND STATE

We consider a simple model of a superconducting double dot introduced by Leijnse et al.\textsuperscript{11} to study the ‘poor man’s Majorana bound states’, coupled to a normal lead with the Hamiltonian given by

$$ H = H_L + H_D + H_T $$

(16)

with

$$ H_L = \sum_k c_k c_k^\dagger $$

$$ H_D = e_1 d_1^\dagger d_1 + e_2 d_2^\dagger d_2 + \Delta (d_1^\dagger d_2 + d_2^\dagger d_1) $$

and

$$ H_T = \sum_k c_k^\dagger (t_1^* d_1 + t_2^* d_2) + h.c. $$

(17)

Here, $d_{1,2}$ are the dot degrees of freedom and $c_k$ are the lead fermions. The couplings of the lead to the two dots are denoted by $t_{1,2}$. $e_{1,2}$ are the energies of the quantum dot levels in the two dots respectively (we assume that the dot is represented by a single level and $\Delta$ is the pair potential induced on the dots by proximity to a common superconductor.

The Hamiltonian for the dot can be rewritten as $H_D = e_2 + \psi^\dagger H_D \psi$ with $\psi = (d_1, d_2)$ and

$$ H_D = (e_1 - \Delta \sqrt{e_1^2 + \Delta^2}) $$

(18)

We can now diagonalise this Hamiltonian which gives

$$ H_D = e_2 + \psi^\dagger D \psi = e_2 + \lambda_+ a^\dagger a + \lambda_- b^\dagger b $$

(19)

with diagonal eigenvalues

$$ \lambda_{\pm} = \frac{1}{2} [e_1 - e_2 \pm \sqrt{(e_1 + e_2)^2 + 4\Delta^2}] $$

(20)

and the diagonalising matrix (defined by $D = U^\dagger h_D U$)

$$ U = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}. $$

(21)

To obtain a zero energy state, we can choose $\lambda_+ = e_1 + \frac{\Delta}{\sqrt{e_1^2 + \Delta^2}}$, $\lambda_- = 0$ and the elements of the unitary rotation matrix to be of the form $\cos \alpha = \frac{e_1}{\sqrt{e_1^2 + \Delta^2}}$ and $\sin \alpha = \frac{\Delta}{\sqrt{e_1^2 + \Delta^2}}$.

Using the diagonalizing matrix $U$, the tunneling Hamiltonian can be rewritten in terms of the Bogoliubov operators $a, b$ as:

$$ H_T = a^\dagger \sum_k [t_1 \cos \alpha c_k - t_2^* \sin \alpha c_k^\dagger] $$

$$ + b^\dagger \sum_k [t_1 \sin \alpha c_k + t_2^* \cos \alpha c_k^\dagger] + h.c. $$

(22)

and projecting on to the zero-energy subspace spanned by the operators $b, b^\dagger$ under the constraints $e_2 = -\frac{\Delta^2}{e_1}$, $e_1 > 0$, the tunneling Hamiltonian becomes:

$$ H_T = b^\dagger \sum_k [uc_k + v^* c_k^\dagger] + h.c. $$

(23)

which is precisely of the form given in the main text in Eq.14 for tunneling into an accidental zero-energy Andreev bound state.

II. DERIVATION OF THE GENERALISED WEIDENMULLER FORMULA

We consider the Hamiltonian of $N$ normal leads coupled to each other via a resonant level (which we take here to be an ABS) as well as via direct coupling, discussed in the main letter and given in Eqs. 5 - 8.

To derive the $S$-matrix, we use the equation of motion (EOM) method\textsuperscript{26,27} and write

$$ i\partial_t b(t) = [b, H](t) $$

$$ = \sum_\alpha \{ t_\alpha \psi_\alpha(0,t) + v_\alpha^* \psi_\alpha^\dagger(0,t) \} $$

(24)

In terms of

$$ t = (t_1, t_2, \ldots t_N) $$

$$ v = (v_1, v_2, \ldots v_N) $$

$$ W = \begin{pmatrix} t & v^\dagger \\ -v^\dagger & -t \end{pmatrix}. $$

(25)

and writing the wave-functions in the particle-hole basis as

$$ \psi(0, E) = (\psi_F(0, E), \psi_H(0, E))^T $$

$$ = (\psi_1(0, E), \psi_2(0, E), \ldots \psi_N(0, E), \psi_1^\dagger(0, E), \psi_2^\dagger(0, E), \ldots, \psi_N^\dagger(0, E))^T $$

(26)

we obtain

$$ \psi(0+, E) = S(E) \psi(0-, E) $$

with

$$ S(E) = [E(1 + i\pi N T) + i\pi N W^\dagger W]^{-1} $$

$$ = [E(1 - i\pi N T) - i\pi N W^\dagger W]^{-1} $$

and

$$ v = \begin{pmatrix} \tau & \tau \dagger \\ -\tau \dagger & -\tau \end{pmatrix}. $$

(27)
Note here that $\nu = 1/2\pi v_F$. We now make contact with the usual Weidenmuller formula by rewriting the scattering matrix as

$$S(E) = S_N - i\pi\nu(1 + i\pi\nu T)^{-1} W^\dagger$$

$$[E + i\pi\nu W(1 + i\pi\nu T)^{-1} W^\dagger]^{-1} W (1 + S_N)$$

where

$$S_N = (1 + i\pi\nu T)^{-1}(1 - i\pi\nu T)$$

so that in the absence of the direct tunneling term between the wires given by the $T$ matrix, the $S$-matrix reduces to the usual Weidenmuller formula - i.e.,

$$S(E) = 1 - 2i\pi\nu W^\dagger [E + i\pi\nu W W^\dagger]^{-1} W$$

On the other hand, when there is no coupling to the resonant level, the $S$-matrix is just the usual tunneling matrix $T$ written in the particle-hole basis -

$$S(E) = S_N \text{ for } W = 0$$

For tunneling to a MBS, we replace $b$ by the Majorana operator $\gamma$. Furthermore, the matrix $W$ is replaced by $W = (u, u^*)^T$.

## III. Expressions for Finite Bias Subgap Current and Noise

The expressions for the average current and the zero-frequency noise at normal leads for a junction of multiple normal leads connected to a grounded superconductor are given by:

$$I_p = \frac{e}{\hbar} \sum_{k\in\{N,S\},\alpha\gamma} sgn(\alpha) \int_0^\infty dE A_{k\gamma,k\gamma}(p, \alpha, E)$$

$$f_{k\gamma}(E)$$

$$A_{pq} = \frac{e^2}{\hbar} \sum_{k,l\in\{N,S\},\alpha\beta\gamma\delta} sgn(\alpha) sgn(\beta) \int_0^\infty dE A_{k\gamma,l\delta}(p, \alpha, E) A_{l\delta,k\gamma}(q, \beta, E)$$

$$f_{k\gamma}(E)(1 - f_{l\delta}(E))$$

$$A_{k\gamma,l\delta}(p, \alpha, E) = \delta_{pk} \delta_{pl} \delta_{\alpha\gamma} \delta_{\alpha\delta} - \langle s_{pk}^\alpha \rangle (E) s_{pl}^{\alpha\delta}(E)$$

$$f_{k\gamma}(E) = [1 + e^{\frac{E - sgn(\gamma)}{\hbar k_B T}}]^{-1}$$

$$sgn(\alpha) = \begin{cases} 1 & \alpha = e \\ -1 & \alpha = h \end{cases}$$

These were the equations used to obtain the figures in Figs. 2 and 3 of the main text.