Ultrafast Modulations in Stellar, Solar and Galactic Spectra: Dark Matter and Numerical Ghosts, Stellar Flares and SETI

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Abstract: Background: From new results presented in the literature we discuss the hypothesis, presented in an our previous work, that the ultrafast periodic spectral modulations at \( f_S = 0.607 \pm 0.08 \) THz found in the spectra of 236 stars of the Sloan Digital Sky Survey (SDSS) were due to oscillations induced by dark matter (DM) cores in their centers that behave as oscillating boson stars. Two other frequencies were found by Borra in the redshift-corrected SDSS galactic spectra, \( f_1 = 9.71^{+0.20}_{-0.19} \) THz and \( f_2 = 9.17^{+0.18}_{-0.16} \) THz; the latter was then shown by Hippke to be a spurious frequency introduced by the data analysis procedure. Results: Within the experimental errors, the frequency \( f_1 \) is the beating of the two frequencies, the spurious one, \( f_2 \) and \( f_S \) that was also independently detected in a real solar spectrum, but not in the Kurucz’s artificial solar spectrum by Hippke, suggesting that \( f_S \) could actually be a real frequency. Independent SETI observations by Isaacson et al., taken at different epochs, of four of these 236 stars could not confirm with high confidence—without completely excluding—the presence of \( f_S \) in their power spectra and with the same power initially observed. Instead, the radio SETI deep-learning analysis with artificial intelligence (AI) gave an indirect confirmation of the presence of \( f_S \) through the detection of a narrowband Doppler drifting of the observed radio signals in two stars, over a sample of 7 with a high S/N. These two stars belong to the set of the 236 SDSS stars. Numerical simulations confirm that this drifting can be due to frequency and phase modulation in time of the observed frequencies (1.3–1.7 GHz) with \( f_S \). Conclusions: Assuming the DM hypothesis, the upper mass limit of the axion-like DM particle is \( m_a \approx 2.4 \times 10^3 \) µeV, in agreement with the results from the gamma ray burst GRB221009A, laser interferometry experiments, suggesting new physics with additional axion-like particle fields for the muon g-2 anomaly.

Keywords: dark matter; stellar oscillations; extraterrestrial intelligence

1. Introduction

Dark Matter and Dark Energy (DE) are among the hottest topics in astroparticle physics and modern cosmology [1,2]. While DE is thought to be the main source responsible for the accelerated expansion of the Universe, contributing with \(~ 68\%\) of the total energy [3,4], DM is supposed to be gravitationally dominating the Universe, accounting for \(~ 85\%\) of the total matter. From Concordance Cosmology DM is 26.8\% of the total mass-energy of the Universe, whilst the ordinary matter is only 4.9\% [5]. The presence of DM is usually revealed through gravitational lensing, like in the Bullet Cluster, formed after the collision of two clusters of galaxies [6,7], and explains the discrepancies observed between the
separated signatures of gravitational and luminous matter observed in the rotation of galaxies and plays a key role in the evolution of galactic structures and in the formation of clusters of galaxies [8] up to the large-scale structures of the Universe, including the anisotropies in the cosmic microwave background [9].

The Big Bang nucleosynthesis [10,11], mainly based on the Standard Model of Particles and supersymmetric (SUSY) theories suggests that DM can be mostly made up of exotic baryonic and non-baryonic matter, with fields that can be either scalar or vectorial. An example is the particle axion [12–14], introduced by Peccei and Quinn in 1977 [15] to solve the strong CP (charge–parity) problem in quantum chromodynamics. Axions are pseudo-Nambu-Goldstone bosons with mass \( m_a \), a free parameter of the Peccei–Quinn theory, defined in a wide range of masses that depend on the coupling constant of the field, \( 10^{-6} \text{eV/c}^2 < m_a < 1 \text{MeV/c}^2 \). Recent studies based on lattice QCD simulations that describe the formation of axions during the post-inflationary period within the framework of the standard model of particles, suggest a mass range \( 5 < m_a < 1500 \text{eV/c}^2 \); more recent results prefer a mass \( m_a \approx 65 \text{eV/c}^2 \) [16–18] and a recent gamma ray burst explosion GRB221009A indicates a mass in the order of \( 0.01–0.1 \text{eV/c}^2 \) [19–21]. Experimental results from CAST indicates an axion–photon coupling strength \( 0.66 \times 10^{-10} \text{GeV}^{-1} \) at 95% confidence level (c.l.) when \( m_a \lesssim 0.02 \text{eV} \) [22].

Other DM candidates are axion-like particles (ALPs), a generalization of the axion present in many extensions of the Standard Model of particles and present in string theory. Differently from the standard axion, ALPs are not always introduced in a theoretical framework to solve the strong CP-problem. WIMPs (weakly interacting massive particles), sterile or massive neutrinos and other hypothetical particles that weakly interact with the electromagnetic field are other possible candidates for DM [2,23–29].

Axions and ALPs are supposed to form halos that, in most cases, surround the visible part of the observed cosmic structures and are expected to be present in our galactic halo [30,31]. These particles can be gravitationally captured, e.g., by a star, and scatter with the nuclei of the elements present there inside, losing their kinetic energy, with the result of remaining gravitationally trapped in the stellar core. In this scenario, the axionic (or ALP) field is usually thought to behave as a classical field, stable and coherent with respect to gravity and, when trapped inside a star, it can remain gravitationally confined [32] forming DM cores with an evaporation rate that depends on the mass of the particle \( m_a \) [33]. Theory suggests that ALPs’ ultralight bosonic fields piled up in stars can form temporary stable structures, boson cores, that oscillate in time with a so-called “breathing” behavior, typical of these exotic objects known as boson stars, where both spacetime and the bosonic fields there present are described by a stress–energy tensor that presents characteristic oscillatory modes that depend on the mass of the bosonic DM particle. Thus, these structures are expected to oscillate at one or more frequencies that depend on the spectrum of masses of the boson field [34,35], with the result of making the host star oscillating. In our previous work [1] we hypothesized that the ultrafast periodic spectral modulations at \( f_S = 0.607 \pm 0.08 \text{THz} \) found in the spectra of 236 stars of the Sloan Digital Sky Survey (SDSS) [36] were due to oscillations induced by dark matter (DM) cores in their centers that behave as oscillating boson stars.

In the following we confirm our previous results by using also the other two frequencies \( f_{1,G} = 9.71^{+0.20}_{-0.19} \text{THz} \) and \( f_{2,G} = 9.17^{+0.18}_{-0.16} \text{THz} \) found by Borra in the cosmological redshift-corrected SDSS galactic spectra [37]. \( f_{2,G} \) is a spurious frequency introduced by the data analysis procedure, \( f_{1,G} \) is the beating between \( f_{2,G} \) and \( f_S \) was not found in the Kurucz’s [38,39] artificial solar spectrum but found in a real solar spectrum [40] and with other independent observations [41] including the radio SETI deep-learning analysis with AI [42], confirming our initial hypothesis. The axion-like particle is then expected to have mass \( m_a \approx 2.4 \times 10^3 \text{eV} \) [1], as indicated also by the results from GRB221009A, laser interferometry experiments [43]. These results suggest the presence of additional axion-like particle fields to explain the muon g-2 anomaly [44–49].
Of course, this does not mean that all stars hosting a certain quantity of DM would exhibit a stable observable oscillating behavior in time. There are many reasons to think that these oscillatory modes induced by DM could be transient. The first case is when the trapped bosonic field in the star has a multiple-valued mass spectrum or even different particles that correspond to different ALPs–DM fields to which are associated different frequencies for the ultrafast oscillation modes. These oscillations are expected, in the simplest case, to set up a spectrum of oscillatory modes that either cancel with each other dissipating the oscillations with the result of stabilizing the stellar structure, or the star experiences a stable oscillatory dynamical configuration. Another possible scenario is when there are present beatings between these frequencies characterized by relative anharmonic frequency ratios. They generate modulations that make the star to evolve from a stable state of equilibrium to a transient oscillatory motion and vice versa or to drive the stellar structure on a route to chaos [35,50], with the result that any oscillatory motion would result observable only for a finite interval of time. For a more complete description, one has to also take into consideration in the formulation of the stress–energy tensor also the properties of the standard matter present in the star that are mostly fermions, also including also the effect of viscosity damping. These models describe a mixed fermion–boson star with solutions that show a stable stellar structure or, with other types of solutions, stars with oscillatory modes that depend on the central densities of the bosons and fermions there present with a wide variety of dynamical properties, e.g., stars evolving from oscillatory to a steady dynamical state in time and vice versa [51].

Another cause that makes the oscillatory regime to be transient is the loss of ALPs from the stellar core either for the conversion of ALPs in other particles, like in the mechanism of the ALPs–photon conversion [52] or because of the evaporation of the DM core. Cores of ALPs with a very small mass are in fact expected to evaporate and leave the stellar structure without DM, if the star cannot compensate it with the gravitational capture and condensation of other DM.

In this work we will mainly focus on dark matter models based on axion and axion-like particles (or ALPs), excluding other types of DM particles such as WIMPS, dark photons etc., that are not expected to introduce oscillatory behaviors in the host star. To this aim, in Section 2, we will analyze in depth the results present in the literature that followed those by Borra and Trottier to verify whether the ultrafast spectral modulations found in the spectra of 236 SDSS stars at $f_S = 0.607 \pm 0.08 \text{ THz}$ corresponding to $1.65 \times 10^{-12} \text{ s}$ [36] and those detected after the analysis of the redshift-corrected SDSS galactic spectra, at $f_{1,G} = 9.71^{+0.20}_{-0.19} \text{ THz}$ corresponding to $1.03 \times 10^{-13} \text{ s}$ and $f_{2,G} = 9.17^{+0.18}_{-0.16} \text{ THz}$, corresponding to $1.09 \times 10^{-13} \text{ s}$, are confirmed by other observations in different epochs and with independent data analysis. The uncertainties are given by the maximum error in the data time binning, $2.1 \times 10^{-15} \text{ s}$, as reported in [37]. In Section 3, we discuss our results and draw the conclusions. In the appendix (see Appendixes A and B) we briefly review the main spectroscopic and astrophysical properties of the observed sources, stars and galaxies.

2. Real and “Ghost” Frequencies Introduced by the Data Analysis

Borra and Trottier, in 2016, interpreted their findings of periodic spectral modulations, due to ultrafast light oscillations at the frequency $f_S$, as SETI signals from a sample of 236 SDSS stars [36]. Borra in 2013 had already reported the finding of other periodic spectral modulations, characterized by the frequencies $f_{1,G}$ and $f_{2,G}$, in a subsample of 223 galaxies over a sample of 900,000 of the SDSS galactic spectra [37]. All the spectral lines and frequencies were redshift-corrected according to the concordance cosmological model [5]. These frequencies were interpreted as possible effects of ultrarapid light bursts separated by time intervals in the order of $10^{-13} \text{ s}$, originating from some hypothetical activities of their galactic black holes that were not yet observed with the new results from the observations of the fast rotating and active M87 and SgrA from EHT and other campaigns [53–55].
In our previous work, we supposed that the modulations with frequency $f_S$ could be a signature for the presence of dark matter cores in those main sequence stars for which the inner nuclei are not dominated by convective motions [1]. We now check if there is a relationship between these three frequencies, as they were revealed with the same method of data analysis from data taken with the same instrumentation, and verify if they are real frequencies associated to real physical phenomena and if the other two frequencies can fit with the DM hypothesis. The possible origin of the three frequencies from instrumental effects were already discarded in the original works by Borra and Trottier (2016) [36] and Borra (2013) [37].

2.1. Which Frequency Could Be Better from Axion Physics

We now consider for the sake of simplicity a toy model of a bosonic core, in which a permanent bosonic aggregation of DM particles with mass $m_a$ is piled up in the center of a star with spectral classes from F to K. The stellar structure is described in an oversimplified way by a polytrope with index $n$ and a pressure–density relationship, $P \propto \rho^{(n+1)/n}$ [56–58].

This aggregation of ALPs is expected to behave like a boson star, characterized by a single oscillatory behavior and a single frequency that can give rise to the ultrafast modulation observed by Borra and Trottier. In the oscillating star, the breathing behavior is present both in the dark matter core and in spacetime, due to a time dependent stress-energy tensor characterized by long-term stable oscillating geometries, characterized by a single given frequency that depends on the mass of the particle.

If the core of the star does not have convective motions and the energy transfer is radiative like in the stars from the F down to the M spectral classes, the DM core is hosted inside the star without being disturbed by the mixing of matter and can start oscillating at a single frequency transferring the oscillations to the spacetime and to the structure of the host star [59–62], oscillating at the characteristic frequency $f_S$ that depends on the mass of the axion $m_a$,

$$f_S = k \cdot 2.5 \times 10^{14} \frac{m_a c^2}{eV} \text{ Hz}, \quad (1)$$

where $k$ is a positive constant. In our previous work we indicated $f_S$ as the $k$-th harmonics of the characteristic frequency associated to the boson mass $m_a$. The spectral fluctuations with frequency $f_S$ would be caused by the breathing effects induced by axion-like particles with an upper mass value $m_a \sim 2.4 \times 10^3 \mu eV$. Lower axion masses are obtained with higher values of $k$ (see Tab 1 of Ref. [1]). The ALPs mass values $m_a$ associated to $f_S$ well overlap the mass range of axions from Lattice QCD simulations [16,17] when $2 \leq k \leq 100$ and result in agreement with the expected axion detections in Josephson junctions and solar observations [63,64] with the SMASH axion model [65] and with the recent results from laser interferometers [43].

If we apply the DM hypothesis to the other two frequencies $f_{G,1}$ and $f_{G,2}$, from Equation (1) the additional DM fields would have upper mass values $m_{G,1} = 36 \text{ meV}$ and $m_{G,2} = 2 \text{ meV}$. These values are beyond the upper limit expected from Lattice QCD simulations, even if some different theoretical constructions may admit the extension of the mass window for the axion QCD model to those values and beyond [66]. In conclusion, lattice QCD favors the frequency $f_S$ as the best candidate for DM signature.

2.2. Further Results from Other Observations and Data Analysis

In the following years, other independent studies that followed those by Borra and Trottier were made to verify in a few sub-samples of these 236 stars the presence of these spectral modulations in the search of signals of extraterrestrial intelligence. As explained more in detail below, in certain stars $f_S$ was re-observed but with different values of the spectral power. The other two frequencies $f_{1,G}$ and $f_{2,G}$ were re-observed as well and $f_{2,G}$ was proven to be artificially introduced by the data analysis.

Let us analyze point by point the three frequencies: $f_S, f_{1,G}$ and $f_{2,G}$. They were found by Borra and Trottier in both the SDSS catalogues always using the dubbed Spectral Fourier
Transform (SFT) technique, a method developed to detect and characterize the presence of ultrashort pulses. What is immediately evident is that, within the experimental errors, \( f_S \sim f_{1,G} - f_{2,G} \). This is clearly a frequency-beating relationship. By definition, the beat frequency is equal to the absolute value of the difference in frequency of two waves. Thus, either \( f_S = |f_{1,G} - f_{2,G}| \) or \( f_{1,G} = |f_{2,G} - f_S| \) or \( f_{2,G} = |f_{1,G} - f_S| \). This implies that either \( f_{1,G} \) is the beating between the frequency \( f_{2,G} \) and \( f_S \) or \( f_S \) is the beating between the frequencies \( f_{1,G} \) and \( f_{2,G} \) and so on. This combination of frequencies and their slow drift and modulation in time is similar to what occurs in acoustics when playing two strings simultaneously in a violin generating the Tartini’s third tone [67] whose real existence was scientifically demonstrated by Hermann von Helmholtz [68] and which was recently discussed for the presence of many other tones with different frequencies that also drift with frequency in time as the violin is played [69,70].

Now that we have the three frequencies and the beating relationship together with a toy model to play with, as the first step, we exclude the hypothesis that \( f_S, f_{1,G} \) and \( f_{2,G} \) are introduced by the windowing caused by the upper and lower values of the instrumental windows in the SDSS frequency spectrum or caused directly by a combination of them. The windowing origin of all the three frequencies is excluded as the two spectrographs worked simultaneously in the SDSS frequency spectrum or caused directly by a combination of them. The frequencies \( f_{1,G} \) and \( f_{2,G} \) are the lines of the Ca II doublet, which are influenced by strong departures from the local thermodynamic equilibrium, three-dimensional radiative transfer and partially coherent resonance scattering of photons in the outer layers of the Sun. Of course, as they are easy to understand, the possible effects of DM particles or still unknown ultrafast phenomena were included in the synthetic solar spectrum. That is why finding \( f_{2,G} \) there is a smoking-gun proof that it cannot be a real physical frequency, as this artificial solar spectrum cannot have neither SETI signals nor DM signatures reported inside, being built by a database of known solar spectral lines and does not include spectral variations due to peculiar solar activities like those observed by Borra and Trottier. The frequency \( f_{2,G} \) obtained from the SFT data analysis is then an artificial product of the line spacing in the artificial solar spectrum, as claimed by Hippke.

To explain this first result more in detail, peering into the lines of the power spectra, the SFT analysis of the synthetic solar spectrum by Hippke shows evidence of a high peak with power \( \text{pow}_{2,G} \sim 22.4 \), corresponding to \( f_{2,G} \) \( (1.09 \times 10^{-13} \text{ sec}) \) but no power is found in correspondence with \( f_S \), for which the power is about \( \text{pow}_S \sim 0.6 \) on the order of the background noise, embedded in the noise near the deepest region between two peaks, \( a \) and \( b \), located at the time intervals and with power \( (t_a = 1.49 \times 10^{-12} \text{ s}, \text{pow}_a = 2.21) \) and \( (t_b = 1.70 \times 10^{-12} \text{ s}, \text{pow}_b = 2.34) \). The term “pow” represents the power of the peak in the Fourier spectrum.

To summarize, only the “fake” ghost frequency \( f_{2,G} \) is observed by Hippke’s analysis of the synthetic solar spectrum, but neither \( f_{1,G} \) nor \( f_S \) are present because the physical causes that originate \( f_S \) and its beating, \( f_{1,G} \), are simply not included in the construction...
of the artificial solar spectrum, and thus, not detected by the SFT analysis. In the position of the Fourier spectrum where \( f_{1,G} \) is expected to be, instead, is found the deepest point with \( \text{pow} < 2.65 \), located between the peak of \( f_{2,G} \) and another at \( t = 1.02 \times 10^{-13} \text{ s} \), with \( \text{pow} = 8.07 \). This would suggest that only one of the two frequencies, either \( f_S \) or \( f_{1,G} \), can be associated to real phenomena and the other one is the result of the beating with the spurious frequency \( f_{2,G} \).

\( f_{2,G} \) was artificially introduced by the SFT technique with a probability \( \sim 0.05% \) of happening by coincidence, generated by the SFT technique from to the non-random spacings of certain spectral absorption lines that are in common to most of the observed spectra in these campaigns (see the discussion in the Appendixes A and B for more details about the galactic spectra). The spectral lines present in both the galactic and synthetic solar spectra form a comb-like structure characterized by a regular set of distances between each spectral line and SFT erroneously identifies this periodicity as a real modulation due to ultrashort pulses in the spectra with frequency \( f_{2,G} \). More precisely, \( f_{2,G} \) emerges from the comb-like structures of the spectral lines that are common in all the spectra up to now analyzed (stellar spectral classes F to K) such as the Balmer hydrogen series or the one time ionized calcium lines that are also present in the synthetic solar spectrum, the Sun being a G2V-type star. Of course, \( f_{2,G} \) is also observed in the SDSS galactic spectra because any galactic spectrum is the result of the convolution of all the spectra of the stars present in that galaxy, mainly stars with spectral classes from F to K.

\( f_S \) is real and \( f_{1,G} \) is a beating. To understand which of the two frequencies \( f_S \) and \( f_{1,G} \) is the “real” one we recall that lattice QCD simulations and the DM hypothesis would favor \( f_S \) as real frequency [1]. This frequency has also been revealed in some of the galactic spectra presented in [37] but is mainly found through the beating of \( f_{1,G} \) with \( f_{2,G} \), because \( f_{2,G} \) has always a high spectral power, being artificially produced by the SFT analysis and \( f_S \) is more easily found through its beating with \( f_{2,G} \), viz., \( f_{1,G} \). We will explain this better through the analysis of the probabilities of their detections. As it is known from spectral analysis, when the fake frequency \( f_{2,G} \) is found in the galactic spectra, it has a high power in the spectrum because it is originated by the spacing of the convolutions of many spectral lines together. \( f_S \) instead is originated by a sub sample of these stars and, as commonly seen in spectral analysis when the stellar spectra are translated in time series with the SFT technique, this frequency it is better revealed through the beating with \( f_{2,G} \) that creates the beating frequency \( f_{1,G} \) with a spectral power given by about the average of the powers of \( f_S \) and \( f_{2,G} \): from the experimental data, the probability of finding the peak at \( f_S \) in the sample of stars of our galactic halo is much lower, 0.009%, versus the much higher probability of having \( f_{2,G} \) which is about \( \sim 0.05% \). A more detailed discussion about the properties of the synthetic, real solar spectra and stellar and galactic spectra is reported in the Appendixes A and B.

In favor of \( f_S \) again: this frequency was observed with the SFT analysis in the real solar spectrum taken with the solar spectrometer installed at the International Scientific Station of the Jungfraujoch by the University of Liège [71–73], together with \( f_{1,G} \), by using the dubbed SFT technique. Peering more in detail in the real solar spectrum, we notice that the SFT analysis of the real solar spectrum clearly shows the presence of a small bump in correspondence to \( f_S \) with the following properties: \( t_S = 1.64 \times 10^{-12} \text{ s}, \text{pow}_S = 6.47 \). This peak is well above the background noise and is present the peak \( f_{1,G} \) is present too \( t_{1,G} = 1.03 \times 10^{-13} \text{ s}, \text{pow}_{1,G} = 8.04 \), confirming that the frequency \( f_{1,G} \) is the beating between the highest peak of the Fourier spectrum, \( f_{2,G} \), and \( f_S \). The detection of \( f_S \) (and, of course, \( f_{1,G} \)) in the real solar spectrum only and not in the synthetic one, implies that most likely the synthetic solar spectrum analysis does not take in account all the features of the solar lines including their variations or fast oscillations like those due to DM or to flare or plasma activities that are still not well known: not all the transient phenomena of the Sun are included there [38,39].

The physical cause of \( f_S \), as we found ultrafast modulations also in our Sun, would discard the presence of SETI signals in the neighborhoods of our star, favoring the idea
that $f_S$ may be related to physical processes occurring in the Sun like the DM effects or still not completely known high energy emissions from microflares/flare activities involving plasma and magnetic fields. These energy emissions would introduce a beating frequency from hypothetic flare activities that are known to present pulsating emissions. Similar behaviors have been observed in our Sun with frequencies from 0.5 to 1 pulses per second in the sub-THz (up to 405 GHz) and in the 30 THz band with a positive slope in the spectral component at the highest observable frequencies [74–78]. A THz spectrometer in space could give a better perspective to understand these still unknown phenomena.

2.3. Frequency Drift and $f_S$ from Different Observations

Following this line of clues, if the frequency $f_S = 0.607$ THz is present in the spectrum of these stars, the presence of the oscillations induced by DM can be one of the causes of the frequency drifting observed in the stars HIP 62207 (10 Canum Venaticorum—HD 110897), a debris disc star [79] and HIP 54677 (HD 97233). This frequency drifting was revealed by the deep-learning search for techno-signatures in 820 nearby stars looking for narrowband Doppler drifting of radio signals (Table 1). These stars represent the actual intersection of the set of stars observed by Borra and Trottier and the sample of the 820 stars where possible SETI signatures have been found by Xiangyuan Ma and collaborators [42]. Numerical simulations confirm this hypothesis: when we superimpose $f_S$ with the frequencies there observed, $f_O$, in the periodograms and is varied the phase change between $f_S$ and $f_O$ in time, one obtains a set of frequency drifting of $-0.4998$ up to $-0.9996$ Hz/s. In Figure 1 is reported the periodogram of the beating between $f_S$ and a sample frequency $f_O = 1.5$ GHz with a slow phase modulation of 1 Hz from Equation (2) that agrees with what is observed in these two stars. (A debris disc star is a star with an observable relic of the planetesimal formation process it, which is analogous to the Edgeworth-Kuiper belt of our Solar system, making it a good candidate for hosting exoplanets).

![Figure 1. Periodogram of the beating between $f_S$ and a typical flare activity with varying phase at $f_O = 1.5$ GHz (numerical simulation). We obtain the same time drift as observed in the deep learning SETI search for extraterrestrial technosignatures (Ma et al., 2023 [42]) from Equation (2).]
The resulting beat signal can be described as the product of the two signals when two signals with different frequencies interact with each other, they can produce a phenomenon is known as “beat frequency” or “beat phenomenon”. A classical example is two notes also change, a frequency drift similar to a trill can rise up. The third tone can be present also at higher or intermediate frequencies and is used also in organ playing to obtain loer vibrating tones with shorter organ pipes. 

In our case, observing a low-frequency signal with frequency \(f_{\text{low}} \sim 1.5 \text{ GHz}\) superimposed with a very high-frequency signal with frequency \(f_S = 0.607 \text{ THz}\) will cause a modification on the low frequency signal: the high-frequency signal is significantly higher than the low-frequency signal, the beat frequency will be the difference between the two, and when a modulation in phase is present, this will cause the low-frequency signal to appear to drift slowly in frequency. 

If the phase of the low-frequency signal is also variable in time with respect to that with higher frequency, it will introduce additional complexity to the beat phenomenon. The interaction between signals of different frequencies, along with varying phase relationships, can lead to interesting effects in the resulting beat signal, the Frequency Shifting and Drifting: varying phase relationships can cause the beat frequency to shift and drift over time. The apparent frequency of the beat signal will not remain constant but will change as the phase difference between the two signals changes.

To better describe the beat phenomenon with variable phase, we build a toy model with two sinusoidal signals, a high-frequency signal with frequency \(f_{\text{high}}(t) = f_{\text{high}} + \delta f_{\text{high}}(t)\), where \(f_{\text{high}}\) is the nominal frequency, and \(\delta f_{\text{high}}(t)\) represents small variations in frequency over time and a low-frequency one with \(f_{\text{low}}(t) = f_{\text{low}} + \delta f_{\text{low}}(t)\) with modulating phase. The resulting beat signal can be described as the product of the two signals,

\[
B(t) = A_{\text{high}}(t)A_{\text{low}}(t) \cos[2\pi(f_{\text{high}}(t) - f_{\text{low}}(t)) t + \phi(t)]
\]

where \(A_{\text{high}}(t)\) and \(A_{\text{low}}(t)\) are the amplitudes of the high-frequency and low-frequency signals, respectively. The two quantities \(f_{\text{high}}(t)\) and \(f_{\text{low}}(t)\) are the time-varying frequencies of the high-frequency and low-frequency signals, respectively. The varying phase difference between the two signals at time \(t\) is \(\phi(t)\). The phase difference \(\phi(t)\) will determine how the two signals align with each other, and its variation over time can introduce amplitude modulation to the beat signal. The beat signal will present a frequency equal to the absolute
difference between the time-varying frequencies: $|f_{\text{high}}(t) - f_{\text{low}}(t)|$, which can shift and drift over time due to the variations in the two input frequencies.

2.4. Stability in Time of the Ultrafast Oscillations

As discussed before, these oscillatory models can be transient because of DM core evaporation or other causes like the transient oscillatory regimes expected in certain models of mixed matter and DM cores.

Analyzing the results started from the SDSS datasets then analyzed by Borra [37] to those here discussed [40–42] we span an interval of observation epochs of about 10–18 years, which is considered very short for stable stellar structures but, assuming the DM hypothesis valid, can provide additional information about the evaporation process of these cores. The fact that $f_S$ was not always re-observed with the same spectral power indicates that the DM particles are ultralight and thus evaporate in a very short time. The presence in the sample of a M5V fully convective star, would suggest the presence of an efficient axion cooling mechanism due to the evaporation of the DM core with very low mass that inhibits the convection mechanisms. Another hypothesis is that there is a mechanism of axion conversion in other particles that reduces the axion core.

If this short stability or variation of the power in the spectra of $f_S$ at different epochs does not depend on the data analysis procedure, this would suggest that DM is clustered in knots at the galactic scale [80,81] and between galaxies in galactic clustering [82]. In this scenario, stars would cross these knots of DM during their galactic motion and act like gravitational vacuum cleaners capturing with their gravitational fields quantities of DM until they reach enough concentration to start these oscillatory motions for a very short time with respect to the time life of a star. As suggested by these observations, one can observe some differences in the oscillatory regime of a star after a few tens of years because of the DM core evaporation. Of course, if the star is surrounded by DM, the accretion can balance the evaporation and conversion in other particles of the DM quanta and the oscillatory regime remains stable.

In favor of the transience of these physical phenomena there are also the results obtained during the SETI campaign with the Automated Planet Finder and high-resolution optical Levy Spectrometer. Three stars of the 236 SDSS sample were re-observed without finding the same power at $f_S$. While for TYC2041-872-1, a F9V type star no signal is present at all, for TYC2037-1484-1, a G2V star, and TYC3010-1024-1, F9V, $f_S$ is there present, even though there is not a high power peak in correspondence of the frequency 0.607 THz. In any case, one can notice the presence of a smooth small wide bump with power emerging from the continuum with a power 1% with respect to the continuum intensity, which is about the sensitivity of the instrument and thus at the limit to be considered an acceptable result. One can suppose that $f_S$ is there present after the results by presented in Ref. [41], where similar features can be seen in the simulations obtained by superimposing a sinusoid with frequency $f_S$ to the real spectra. The results obtained by varying the power of $f_S$ around $\sim 1\%$ in the power spectrum of the stars result comparable to the experimental data, limited by the sensitivity of the instrument. The SFT re-analysis by Hippke of the same spectrum of the F9V star TYC 2041-872-1 at the different epoch corresponding to the SDSS survey, shows instead a peak in the periodogram significantly above the noise floor with power at 0.607 THz, confirming the existence of $f_S$ at that observational epoch: the star shows some residuals of the presence of the spurious frequency $f_{2,G}$, as expected, and a spectral power pow = 5.57, for $f_S$, which is almost one unit bigger than what is detected in the real solar spectrum. This suggests that some physical changes have been occurred in the source if the DM hypothesis is valid. Following these results, one can conclude that the variation of the power spectra at $f_S$ are actually due to the fact that the oscillatory modes are transient or the SFT data reduction process that was specifically developed for the detection and characterization of the presence of ultrashort pulses, has a better efficiency to give evidence to these ultrafast phenomena and different noise levels of the different
instrumentation with the caveat that spurious frequencies like $f_{2,G}$ can arise from regular geometric properties present in the spectra.

The transient behavior observed can be attributed to an axion core evaporation or to the mechanism of axion-photon coupling and production of different particles as hypothesized in massive stars, including the conversion mechanisms of axions with photons such as photon/matter interactions [83], even if high magnetic fields are required [84,85]. Anyway, depending on the properties of the axion, some authors think that even the coupling with the stellar magnetic fields that permeate down to the radiative zone should be able to produce photons reducing the axion core [86], with a tiny extra production of energy from the star. In this case the Sun could be one of the brightest sources and detector of axions from the axion-photon coupling that produces axions with large mass through the inverse Primakoff scattering of thermal photons embedded in the electromagnetic field of the solar plasma [87] favoring $f_{1,G}$ instead of $f_S$. The Majorana Demonstrator experiment sets new limits on the axion-photon coupling, $g_{a\gamma} < 1.45 \times 10^{-9}$ GeV$^{-1}$ with a c.l. of 95% for massive axions with mass in the range $1.4 < m_a < 100$ eV/c$^2$, whose upper limits go outside from our mass range [88], finding agreement in its lower values with the mass ranges here found.

Another cause for the transient behavior of the oscillations could be due to the onset of additional frequencies or instabilities inside the stellar core due to multiple states of the same field or multiple fields [89] that can temporarily reduce the power associated to $f_S$ that can be explained by particle theory. This also recalls the behavior of models of fermion-boson stars that can have transitions from stable to unstable configurations and vice versa [51] and both amplitude, oscillations and stability of the oscillatory regime depend on the mixture of matter/DM present in the core of the star. This model agrees with most of the stars sample which corresponds to the $\sim 1\%$ MS stars in the halo, population II and metal-poor [90] and almost all belong to F-K classes known to have a radiative stable nucleus. In this case oscillatory regimes due to DM can occur. In favor of this DM scenario is that F9 is the dominant spectral type in which the distribution is peaked, with mass $M \sim 1.2$ $M_\odot$ and the energy transfer is almost totally radiative, like in the ideal model described by a polytrope. Thus, a more realistic model of DM core made with a mixture of matter (fermions) and DM (bosons) could solve this problem as it presents a set of transient regimes from stable to oscillatory to a route to chaos depending on the properties of the matter/DM mixture [35], including additional mechanisms of axion conversion in other particles but this goes beyond the purpose of the present work.

Excluding the hypothesis of SETI signals—that they cannot be always be detected back as they are not always transmitting and may be doing something else after the first transmission—an alternative explanation of this transient regime could be instead due to the presence of flares and/or microflare activities. A campaign of observations in the sub-THz regime of the Sun at the frequencies $f_S$ and $f_{1,G}$ could clarify better this scenario.

3. Discussion and Conclusions

Following our previous work [1], we analyzed the common features of the stellar and galactic spectra of the SDSS survey that presented an oscillatory regime corresponding to the frequency $f_S = 0.607 \pm 0.08$ THz and to the two other frequencies, $f_{1,G} = 9.71^{+0.20}_{-0.19}$ THz and $f_{2,G} = 9.17^{+0.18}_{-0.16}$ THz. These frequencies were revealed with the dubbed Spectral Fourier Transform (SFT) technique, developed to detect and characterize the presence of ultrashort pulses [36,91,92]. A clear relationship of frequency beating between the three frequencies is evident: $f_{1,G}$ results to be the beating of $f_S$ with $f_{2,G}$.

An important clue for the real physical origin of $f_S$ in favor of the DM hypothesis is given by the comparison of the results of the analysis of the artificial solar spectrum by Kurucz and a real solar spectrum using the same SFT technique. From SFT analysis of the Kurucz synthetic solar spectrum, Hippke demonstrated that the peak observed at 9.71 THz ($f_{2,G}$) was artificially introduced by the SFT procedure. The pattern of absorption lines in SDSS spectra is characterized by a regular comb-like structure formed by chemical
elements present in the Sun and in most stars, including galaxies. The spacing between some spectral lines was erroneously interpreted by the SFT algorithm as a modulation due to $f_{2G}$ [40]. Noteworthy, the other two frequencies were not detected in the artificial spectrum. Instead, in the real solar spectrum obtained with the solar spectrometer at the International Scientific Station of the Jungfraujoch, the SFT procedure revealed, together with $f_{2G}$, the presence of $f_{1G}$ and $f_S$, giving a clear indication that something related to the latter frequencies was missing in the artificial solar spectrum even if the main solar spectral lines and, above all, their spacings, are correctly reported. One can conclude that, if $f_S$ is real, as suggested by the comparison of the two solar spectra, it must be originated by the same physical phenomenon occurring in the SDSS galaxies, stars and in the Sun and do not depend from instrumental effects.

Other observations with different data reduction techniques showed in the Fourier spectrum the complete lacking of $f_{2G}$ and the presence of a low peak in correspondence with $f_S$, just below the 1%, the value to which corresponds the smallest detectable value due to the sensitivity of the instrument. In any case, the results of numerical simulations of the presence of $f_S$ in the spectrum at the observed value confirm the presence of this frequency with a lower power [40,41]. These results suggest that either the SFT technique is not reliable or its efficiency and results must be taken in better consideration as we are doing here, to exclude the presence of artificially produced frequencies, or to conclude that we are dealing with transient phenomena in the stellar structures that in the DM hypothesis can be due to the formation and evaporation of the axion bosonic cores.

An alternative explanation for the transitory characteristics of the DM phenomena could be due to a different structure of the DM core. Theory suggests that the presence in the stars of a core made with a mixture of bosonic and fermionic matter and dark matter can give rise to transient oscillatory regimes and describe also temporarily stable structures that evolve from a static, steady state of a quiet core to one dominated by oscillating regimes and vice versa or even the core experiences a route to chaos. Assuming $f_S$ real for the DM hypothesis, then one finds an agreement with the expected axion masses from lattice QCD as already discussed in Ref. [1], and also with the axion masses expected from the recent results of the gamma ray burst GRB221009A, for which $m_a = O(0.01 - 0.1) \, \mu eV$ [19–21], for $k > O(10^3)$.

From our results, if the DM is made with axions only, one finds a discrepancy between this mass range with that of the axion masses expected from certain particle theory models that involve the Standard Model of particles and the muon g-2 anomaly. In fact, the upper mass value found by us is in contrast with certain models that explain the anomalous magnetic dipole moment of a muon, also known as muon g-2 anomaly [44,45] involving the axion [46]. This discrepancy would suggest the presence of new physics involving the muon or around or below the weak scale and a better study of the effects of vacuum polarization of the Standard Model. In certain scenarios these models would require couplings between the Standard Model leptons and the axion, on the order of (25–100) GeV and a heavy axion with mass from the 10 MeV up to the 10 GeV scale, which is much beyond of the axion model here assumed. The models for the muon g-2 anomaly that consider the loop effects of an axion that couples to leptons and photons would mostly need need large axion masses and couplings [47] including massive fermion fields with large axion-lepton couplings in contrast with our hypothesis. If these results will be confirmed by future experiments, a new set of axion, ALP particles and fermionic fields are expected to contribute to the DM scenario [48]. An ultralight axion mass like discussed by us and lattice QCD then is expected to imply slightly new physics to explain this anomaly, such as the presence of at least another family of the axion with extremely high mass values, taking in account all the possible theoretical [49] and experimental issues related to this challenging and key measurement for particle physics.

Another possible alternative explanation to the onset of ultrafast oscillations could be related to the presence of plasma activities and magnetic fields or the presence of transient flare/microflare activity, as it could be expected to occur in our Sun and in the M5V star.
present in the SDSS sample of 236 stars. To prove the flare/microflare hypothesis we would need further and deeper experimental investigations of the high-energy processes in solar and stellar flares occurring during the impulsive phase of the flares. New observations in the Sun in the THz and sub-THz bands and in the solar emission around 0.607 THz may provide better insights.

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Appendix A. Properties of the Spectra of the Source Samples

Let us discuss the main features present in the spectra where the three frequencies were detected and the stellar spectral classes of the stars considered. The stars analyzed by Borra and Trottier and in the other following works [40–42] are main sequence (MS) stars for which the energy flux in the inner core is carried out by radiative transfer and the matter there inside behaves like a static fluid. If ALPs pile up in the stellar core [34,35], then spacetime and the local density of the radiative stellar core start an oscillatory regime with one or more periods that depend from the properties of the scalar field. The oscillatory regime remains stable in time if the main DM core remains stable with enough particle concentration to modify the relativistic stress-energy tensor and initiate the ultrafast oscillation regime of matter and spacetime [34,35]. Axion-photon conversion, DM core evaporation and other different interactions between axions and the fermions that makes the matter of the star will modify the stress-energy tensor with the result that the oscillatory regime will be either damped or disappear or experience transitions from static to oscillatory with a possible route to chaos as already discussed in the main text. The only exception is the M5V star which is expected to be fully convective, either with a DM dominated energy transfer or suggesting that these ultrafast oscillations can be originated by flare activities, that may occur in this spectral class.

Synthetic solar spectrum. The synthetic solar spectrum is artificially built with data from spectral libraries that provide lists of the atomic and molecular lines and other information needed to best model the solar spectrum. As it is known, synthetic spectra still do not reproduce all the features of the observed lines; several spectral transitions and some minor lines may be missing and the profile of certain lines of the computed synthetic spectra can have, in the worst case, errors up to 200% [93] but not in their positions in the spectrum. In any case most of the fundamental lines such as hydrogen and calcium lines in the solar spectra [94], also observed in the galactic spectra are present. Here no ultrafast oscillations can be present and were not detected.

Galaxies: the galactic spectra are the result of an integration of all the stellar spectra and emission mechanisms that can occur in a galaxy; all the stellar spectral classes, nebulae and other sources contribute to the profile of a galaxy’s spectrum in proportion with their population. The spectrum of a galaxy is the convolution of the spectra of the stars, each of which can be seen as a tiny signal overwhelmed by noise of the light of all the other sources there present but, by adding many stellar spectra together, the main spectral lines will unavoidably emerge and build up the “spectral comb” of spectral lines for \( f_{2,G} \) and sum up also the other effects that generate the other two frequencies \( f_{5} \) and \( f_{1,G} \). This comb-correlation effect in the spectral lines remains valid also when the frequencies are redshifted: the redshift effect of the galactic spectral lines is a rigid translation, with a
probability 0.025% (223 on a sample of 0.9 million galaxies) as some of the lines that can be held responsible for the presence of $f_{2,G}$ can be recovered from the noise by summing up many spectra of the stars in each of the galaxies. Because of this, the argumentation used to discard $f_{2,G}$ does not apply to $f_S$ and $f_{1,G}$ is nothing but the beating with the spurious frequency $f_{2,G}$. Of course we can conclude that when $f_{1,G}$ and $f_S$ are detected in the galactic spectra, differently from the synthetic solar spectrum, the galactic spectra are real spectra and $f_S$ can be supposed to be a real frequency.

Interestingly, the integrated spectrum of the other galaxies presented similar features but the bump at $f_S = 0.607$ THz was not always reported in the Fourier galactic spectra even if put in evidence by $f_{1,G}$ that inherits most of the power from $f_{2,G}$, a spurious frequency caused by the data analysis procedure. The most common spectral lines that are found in the observed integrated galactic spectra are absorption and emission lines that present a smooth progression with the Hubble type classification of galaxies. Most spiral and irregular galaxies exhibit detectable emission lines of hydrogen, $H_\alpha$, the strongest emission line generated in ionization-bounded HII regions and by stars accompanied by the nebular lines like the doublet [OII], related to the internal kinematics of the ionized gas and found also in B0 spectral type stars. Other nebular multiplets like [NII] and [SII] can be found. These lines are followed by $H_\beta$ with [OIII] $95\AA$. Nebular multiplets are not so frequent in F to K stellar spectral classes.

From this, one can find a whole set of common frequencies in all the data analyzed with SFT, from the artificial solar spectrum to the SDSS stars and galaxies. Each single star has its own spectrum with its peculiar lines that depend on the spectral class. Mostly the signals arise in many luminous celestial bodies having spectra with high S/N ratio in which absorption lines are present. The pattern of the absorption lines in SDSS spectra, due to the most common chemical elements present in the Sun and stars, is characterized by a regular comb-like structure. The distance between certain spectral lines is misinterpreted by the SFT procedure as due to the presence of a fake frequency, the spurious frequency $f_{2,G}$.

From SFT analysis of the Kurucz synthetic solar spectrum, Hippke demonstrated that the peak observed at 9.71 THz ($f_{2,G}$) was artificially introduced by the SFT procedure. The pattern of absorption lines in SDSS spectra is characterized by a regular comb-like structure formed by chemical elements present in the Sun and in most stars, including galaxies. The spacing between some spectral lines was erroneously interpreted by the SFT algorithm as a modulation due to $f_{2,G}$ [40].

Amongst these lines, which can be recovered from the noise by summing up many spectra, there are the Balmer H (mainly $H_\alpha$ and $H_\beta$) and Ca II lines, with CaII H & K and the CaII IR triplet in common with the MS stars and the solar spectrum (artificial and real).

Appendix B. Common Features of Spectral Classes and Spectral Lines

We now analyze the main characteristics of the stellar spectra per spectral class and of the galactic spectra in which a signal was detected to give evidence to the similarities in the spectra and in the distribution of the spectral lines as stars of different stellar class and with different metallicity do exhibit a set of different absorption lines that generates $f_{2,G}$ as spurious line. The spectra of those stars, even if different, should have common features that form stable comb structures of spectral lines that generate not only $f_{2,G}$: the power of the corresponding pulse spacings in the Fourier periodogram will vary as well for any different set of spectra but should preserve some power in certain frequencies associated to artificial frequencies like $f_{2,G}$.

In the sample of the 236 stellar spectra of our galaxy the $f_S = 0.607$ THz oscillation was found mainly in F and G stars, in a range of spectral classes from A to K and one sample with M5V. This sample presents a first selection effect due to the SEGUE survey which targeted stars mostly of the F to K-type, cutting off the O and B classes and lowering the percentage of the A spectral class, where hydrogen lines are very strong and dark with a maximum strength for A0, being H lines strong in the temperature range from 4000 °K to 12,000 °K. The other relevant lines are those of one time ionized Calcium, Ca II, which start
becoming more evident in the late subclasses of F stars to M7 stars, being very weak in A stars. Other ionized metals such as Mg II and Fe II are present.

These MS stars have masses in the interval 0.3–1.2 $M_\odot$, spectral classes from F to M and are in the halo, which means that they are mainly of population II, metal-poor with low metallicity less affecting the spectral lines than in our Sun which is of population I. When the stellar mass $M \approx 1.2–1.3 M_\odot$, stars are almost totally radiative and for smaller values of mass, the stellar core starts being surrounded by a convective envelope, like our Sun. In these stars ideally a bosonic core could form and start its oscillatory motion. This may favor the hypothesis of DM oscillations. Instead, MS stars like the M5V star in our sample ($M < 0.3 M_\odot$) are totally convective and with flare/microflare activities. This, instead, may favor the hypothesis that the oscillations could be due to flare activities or dominated by axion cooling. In addition to the M5V star, there are five of them that have spectral class A, supposed to have a convective nucleus. In principle, these types of stars do not fit with the stable oscillating bosonic core model, unless astrophysical effects like axion cooling and conversion were temporarily stabilizing the structure of the inner core. Some of the mechanisms that can be invoked to stabilize the inner core are stellar rotation, DM influencing the stellar evolution processes such as the energy transport in the inner core [56–58,90]: the energy generated by the nuclear reactions is transported away in a very efficient manner [2]. These non-standard axion cooling mechanisms may explain the discrepancies between the observational data and stellar evolution [2,96], with the result that their lifetime is shortened and the oscillation phenomena may be transient. These exceptions might be caused by other astrophysical effects or even by the presence of DM fields that may alter their structure.

In this sample there are no giants, collapsed objects or stars away from the main sequence. The selection of MS stars from the other types of stars in the same spectral class occurs via their spectra. MS stars are mainly characterized by the ratios of the magnitudes of the H spectral lines at 4045 Å ($H_\delta$) and 4226 Å ($H_\gamma$). The most sensitive criterion of absolute magnitude is at all effects the Spectrum–Luminosity Indicator in late-type stars with the ratio between the Sr lines at 4077 Å and the luminosity at 4226 Å of Ca I that result different for normal giants and dwarfs. For supergiants and giants, instead, one uses the ratios 4077 Å with that of $H_\delta$ and 4171–4173 Å with 4226 Å allowing a very accurate luminosity classification [97].

Excluding the only one A-class star present in the sample, which is supposed to have a convective nucleus (see [1] for more details), the main spectral classes analyzed by the SDSS where these frequencies were found, independently from the metal content and stellar population are the following:

**F stars:** what we notice in Table A1 is that the SEGUE selection effect is not ruling the percentage per spectral class of the stars with $f_S$, the dominant spectral class with that signal is the late subclasses of F type stars (61%), F8-F9 of solar type (F8V and F9V), where there is the limit to the onset of turbulence in their inner cores that may be mitigated by the axion cooling. In this spectral class hydrogen lines (mainly the series from Lyman $H_\alpha$ to $H_\delta$) are dominant and are present other ionized metal lines that start becoming evident.

**G and K stars:** just after F9 stars one finds the G-type stars like our Sun (G2V), in which the signal was present, and K-class. Both the classes have similar percentages of stars with $f_S$ present, (17.4 % and 14.8%), but their respective population ratio in the Milky Way is quite different, as the ratio between the number of G stars with respect to K stars is 2/3. We can deduce that the percentages of stars with $f_S$ in these two populations is not due by instrumental selection effects or by their populations, indicating that the oscillation at 0.607 THz needs additional explanations. In G stars and, in particular in the subclass G2V, the H and CaII lines are still present together with other absorption lines of neutral metallic atoms and ions that grow in strength as one moves in the lower spectral classes.

**M5V, wholly convective, with microflare activity.** In the M stars the spectrum is dominated by molecular bands, especially of titanium oxide present together with those of neutral atoms and Ca II, much stronger if the star is active, with flares and microflare
activity. Differently from these three spectral classes, the only M5V star (id. 2MASS J12160751+3003106—Low-mass Star from the Two Micron All Sky Survey [98]) in the sample that presents $f_S$ is a MS red dwarf. M5 star is supposed to be totally convective and an oscillating boson core would result perturbed and quite unstable, unless the stellar structure is modified by the presence of axions with a resulting cooler stellar structure. Microflare activity is present in these stars and it would be a possible candidate for the line $f_S$, even if observations in the sub-THz band are still lacking. The active fraction of stars of the M class peaks at spectral type M8, where 73% of stars are active and one cannot tell the active stars from non active ones by using the color index only. The activity is revealed by their optical spectra that present interesting features in the $H_\alpha$ emission line, CaII and the higher-energy hydrogen Balmer lines from the chromosphere. With its $g-r = 1.826 \pm 0.064$ index the star is redder than a typical M5V star, for which $g-r = 1.52 \pm 0.13$, indicating that the star may be active and presents Ca II lines as in the other spectral classes. More precisely the activity is measured with the ratio of the luminosity emitted in $H_\alpha$ to the bolometric luminosity, $L_{H_\alpha}/L_{\text{bol}}$ [99]. The decline in mean activity strength begins at spectral types M5–M6. Even if these differences are not statistically significant, for an active star one finds a slightly bluer u-g color index and the g-r color index slightly redder when compared to inactive stars. This does not apply for the star 2MASS J12160751+3003106 we are considering. The star is at 168.53 pc and the fraction of M5 active star of the SDSS sample is quite high, 0.55, with a temperature $T_{\text{eff}} = 3228 \pm 166 \degree K$. To conclude, if the DM hypothesis is correct, one of the possible explanations is the axion cooling and conversion mechanism or the presence of transient oscillations in mixed matter and dark matter cores.

**Table A1.** Spectral classes, temperature and spectral lines, (S) = strongest, (s) = strong, (W) = weak, (w) = weaker, (M) = metals, i.e., chemical elements heavier than hydrogen and helium (data from Ref. [100–102]). At 0.607 THz, there is no clear correlation between the percentage of stars in our galaxy and that of the detected signal per spectral class, indicating that the selection effect of SEGUE survey is not dominant.

| Spectral Class | Temperature $10^3 \degree K$ | Absorption Lines | % Stars | % w/Signal 236 |
|----------------|-------------------------------|------------------|---------|----------------|
| A              | 7.5–10                        | H(S), Ca II, Mg II, Fe II | 0.6%    | 1.7 %          |
| F              | 6–7.5                         | H(w), Ca II ionized M | 3%      | 61.9%          |
| G              | 5.2–6                         | H(w), Ca II ionized + neutral M | 7.6%    | 17.4%          |
| K              | 3.7–5.2                       | H(W), Ca II (S) neutral M (s) | 12.1%   | 14.8%          |
| M5             | 2.4–3.7                       | Neutral atoms (s), TiO | 76%     | 0.4%           |

**Common spectral lines:** $H$ and Ca II lines. In the three classes, F, G, K, the most dominant spectral absorption lines are those of Hydrogen, with weak to very low effects in the spectrum with respect to the earlier spectral classes. Other relevant spectral lines are those of one time ionized calcium, Ca II. In the solar spectrum the two strongest lines are the resonance doublet of Ca II, the H line at 3968.469 Å and the K line at 3933.663 Å. Ca II presents also a triplet of three strong lines at 8498.018Å, 8542.089Å and 8662.140 Å formed in the chromosphere [103] in the infrared region. There are some similarities between the F and G classes because of the presence of ionized metals, whilst G and K spectral classes have in common the presence of absorption lines due to neutral atoms with an effect of presence of spectral lines varying from weak (G) to strong (K).
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