Optimization of a heralded single-photon source with spatial and temporal multiplexing

I Z Latypov, A V Shkalikov and A A Kalachev

1Zavoisky Physical-Technical Institute, 10/7, Sibirsky tract str., 420029, Kazan, Russia
2Kazan Federal University, 18, Kremlevskaya str., 420008, Kazan, Russia
3Institute for Informatics of Tatarstan Academy of Sciences, 36a, Levobulachnaya, 420011, Kazan, Russia

E-mail: a.a.kalachev@mail.ru

Abstract. The properties of a heralded single-photon source with temporal and spatial multiplexing are studied with the aim to maximize its efficiency for a given value of the second-order zero-time autocorrelation function. We show that the variable time delay, which is used for temporal multiplexing, can be optimized so that the mean number of photon passes through the switches and the total number of switches are respectively reduced to $\sim \log_2 N$ and $\sim (1/2)\log_2 N$, where $N$ is the temporal multiplexing degree. The total efficiency of such an optimized source is calculated for typical switching losses and the autocorrelation function is calculated in the presence of the detector dark-count noise.

1. Introduction

Generating single-photon states via spontaneous parametric down-conversion (SPDC) or spontaneous four-wave mixing (SFWM) is currently the most convenient experimental technique, which has been widely used in quantum optics and quantum information [1]. The photon number correlation between the down-conversion fields, which are usually referred to as signal and idler fields, allows one to herald the existence of one photon by detection of its partner. Such conditional preparation of single-photon states has several advantages. First, it allows one to generate a single photon into a given spatial-temporal mode of the electromagnetic field by using an appropriate filter during the detection process. Second, the nonlinear optical phenomena make it possible to generate pure single-photon states in a wide spectral range at room temperatures, provided that the nonlinear medium has specific dispersion properties. Third, the contribution of the vacuum state into the output field is removed by the heralding procedure. On the other hand, there are two significant drawbacks: nondeterministic generation of the heralded photons and nonzero contribution of multiphoton states into the output field. To approach deterministic source and simultaneously reduce the multiphoton state contribution, one can take advantage of spatial [2–6] or/and temporal [7–11] multiplexing. In both cases, several processes of photon pair generation are combined in a single source so that heralding probability can be increased without increasing generation rate in each process thereby achieving high fidelity of the source. Both methods are experimentally demonstrated [4–6, 9]. Recently a combination of spatial and...
temporal multiplexing has also been proposed [12], and different multiplexing schemes have been studied numerically [13–15].

In the present work, we show new possibilities for optimization of temporal multiplexing in a heralded single-photon source and analyze different combinations of temporal and spatial multiplexing under typical experimental conditions. In addition, we calculate the autocorrelation function of the output field in the presence of the detector dark-count noise.

2. Multiplexing

The spatial multiplexing means generation of photon pairs in a series of nonlinear sources so that heralding photon in each pair can be detected in each source and only one heralded photon can be routed to the output (figure 1). The spatial multiplexing degree $M$ is equal to the number of nonlinear sources. From the view point of losses, the optimal scheme of the router is a system of switching couplers $2 \times 1$, the number of which increases with the multiplexing degree as $\log_2 M$ [3].

![Figure 1](image1.png)

**Figure 1.** Illustrating heralded single-photon source with spatial and temporal multiplexing (above) and possible schemes of a variable optical delay line (below).

The temporal multiplexing involves repeated application of a pump pulse and a variable optical delay line (ODL) for the heralded photons. The temporal multiplexing degree $N$ is determined by the number of pump pulses that are used for generating a single photon. These pulses form a multiplexing cycle. Regarding the variable ODL, a fiber loop array (FLA) can be used. A free-space variant of such a variable time delay has recently been demonstrated [16]. Another convenient architecture is a binary division fiber network (BDFN) [10, 11], which is used in commercial devices. In the present paper, we consider the first variant and show that the system of storage loops can be optimized so that its efficiency becomes higher, while the total number of switches becomes smaller, than that of binary division network. This is possible because the ratio between the fiber loop lengths $a$ may be arbitrary and not necessarily equal to 2 or 10. The optimal value of the parameter $a$ depends on the temporal multiplexing degree.
3. Basic properties of the heralded source with multiplexing

Let us consider a heralded single-photon source where both spatial and temporal multiplexing are used. In what follows, we estimate the maximum efficiency of such a source, which can be achieved under typical experimental conditions.

The density operator of the output field may be written as

$$\rho_{\text{out}} = \left(1 - \sum_n P(n)\right)|0\rangle\langle 0| + \sum_n P(n)|n\rangle\langle n|,$$

where $P(n)$ is the probability of observing $n$-photon state at the output by the end of a multiplexing cycle ($n > 0$). If the signal and idler photons are emitted into a single spatial-temporal mode, the state vector of the biphoton field takes the form:

$$\psi = c_s |0,0\rangle + c_i |1,1\rangle + \ldots, \quad c_s = \left[\text{th}(r)\right]^n, \quad c_i = \left[\text{ch}(r)\right]^n,$$

where the indices $s, i$ denote the modes of the signal and idler field, respectively, and $r$ is the pump parameter. Let $p_n$ be the detection probability provided that the detector interacts with the $n$-photon state. In the case of a bucket detector, which does not discriminate between photon numbers, $p_n = 1 - (1 - \eta)^n$, where $\eta$ is the detector efficiency. It is easy to show that the probability of heralding after a single pump pulse is equal to

$$P_{\text{herald}}^M = 1 - (1 - P_{\text{herald}}^M), \quad P_{\text{herald}} = p_1|1_i\rangle^2 + p_2|1_i\rangle^2 + \ldots.$$

Here $M$ is the spatial multiplexing degree. Let the cycle of temporal multiplexing consist of $N$ pump pulses separated in time by $T$. If a correlated photon pair is emitted during $i$th pump pulse, the signal photon delay should be equal to $(N - i)T$. As a result, each signal photon appearing during the cycle is emitted as it would be generated by the last pump pulse. Let us assume that the output photon in each cycle corresponds to the last heralding pulse, which leads to the minimum losses. Then

$$P(n) = (P_{\text{herald}}^M/P_{\text{herald}})\sum_{i=1}^N (1 - P_{\text{herald}}^M)\sum_{k=0}^{N-i} t^k (1 - t)^{i-k} \binom{k}{n} p_n |1_i\rangle^2,$$

where

$$t = \left(\eta_{\text{FL}}\right)^{N-i} \left(\eta_{\text{SW}}\right)^{N_{\text{sw}}(i,N)} \eta_{\text{SW}}(i,N)^{\log_2 M}.$$

Here $\eta_{\text{FL}}$ is the efficiency of a shortest fiber loop providing the delay time $T$, and $\eta_{\text{SW}}$ is the efficiency of the switching elements that are used in the FLA and in the router. The number of signal photon passes through the switching elements in the FLA for a given delay time $(N - i)T$ is denoted as $N_{\text{sw}}(i,N)$. For example, if the FLA consists of three loops delaying on time $a^0T, a^1T$ and $a^2T$ so that $N - i = A_0 a^0 + A_1 a^1 + A_2 a^2$, then $N_{\text{sw}}(i,N) = A_0 + A_1 + A_2 + 3$. The number of passes through the switching elements in the router is assumed to be equal to $\log_2 M$. The summation over $k$ in equation (4) takes into account the photon number redistribution due to the losses in the FSSL and in the router [8].

Now we are able to calculate the basic parameters of the single-photon source. In particular, the total efficiency, i.e., the probability of generating a single-photon state in a single multiplexing cycle, is equal to $E = P(1)$. Another important parameter is the heralding efficiency $E_{\text{herald}} = P(1)/P_{\text{herald}}^M$, which illustrates the efficiency of the optical path from the nonlinear media to the output. Here $MN$ is the total multiplexing degree. The theoretical value of the second-order zero-time autocorrelation function $g^{(2)}(0)$ is calculated as

$$g^{(2)}_{\text{shear}}(0) = \left(\sum P'(n)n\right)^2 \sum P'(n)n(n-1),$$
where \( P'(n) = P(n)/\sum P(n) \). The corresponding experimental value, which is measured using a 50/50 beam splitter and two detectors \( A \) and \( B \), is calculated as

\[
g^{(2)}(0) = \left( \sum P'(n) P'_A \right)^2 \sum P'(n) P'_{AB},
\]

where

\[
P'_{AB} = 1 + (1 - \eta_A)^n - 2(1 - \eta_A/2)^n - 4 p_{dc} \left[ (1 - \eta_A)^n - (1 - \eta_A/2)^n \right]
\]

is the coincidence count probability with the \( n \)-photon state at the input of the beam splitter, and

\[
P'_A = 1 - (1 - p_{dc})(1 - \eta_A/2)^n
\]

is the count probability of a single detector. Here \( p_{dc} \) is the dark count probability, which is assumed to be equal for the detectors. So is the efficiency \( \eta_A \).

4. Basic results

Numerical simulations on the basis of equation (4) show that the maximum efficiency of the single-photon source is achieved at the optimal value of the parameter \( a \), which depends on the multiplexing degree and not necessarily equal to 2. To minimize the number of photon passes through the switches we can take \( a = \sqrt{N+1} \), where \( s \) is the number of storage loops. The optimal number of sources for given losses is found numerically by finding the multiplexing degree which maximizes the total efficiency for a given value of \( g^{(2)}(0) \). It should be noted that such optimization is different from the usual approach which does not take into account the value of the autocorrelation function. In order to compare different combinations of temporal and spatial multiplexing, we consider only multiplexing degrees which are powers of 2. To be more specific, we take \( \eta_{SW} = 0.891 \) (0.5 dB insertion loss), \( \eta_L = 0.999 \) (\( T = 100 \text{ ns} \) in a fiber loop with 0.2 dB/km loss), and \( g^{(2)}(0) = 0.05 \). In this case, the maximum efficiency is achieved when \( MN = 256 \) with the optimum value of \( s \) depending on the temporal multiplexing degree \( N \) (table 1).

| \( M \) | \( N \) | \( s = 1 \) | \( s = 2 \) | \( s = 3 \) | \( s = 4 \) |
|---|---|---|---|---|---|
| 1 | 256 | 0.12 | 0.31 | 0.38 | **0.38** |
| 2 | 128 | 0.20 | 0.36 | **0.40** | 0.39 |
| 4 | 64 | 0.30 | 0.40 | **0.41** | 0.37 |
| 8 | 32 | 0.38 | **0.42** | 0.41 | 0.38 |
| 16 | 16 | **0.43** | 0.42 | 0.40 | 0.36 |

Table 1. Total efficiency \( E \) for different combinations of temporal and spatial multiplexing. The result of numerical simulation on the basis of equation (4) with the following values of parameters: \( \eta = 0.5 \), \( \eta_L = 0.999 \), \( \eta_{SW} = 0.891 \).

The dependence can be approximated as \( s_{opt} = \left[ (1/2) \log_2 N \right] \). Under such conditions, the mean number of photon passes through the switches proves to be about \( \log_2 N + 1 \). On the other hand, when using the BDFN, we obtain \( E = \eta_{SW}^{M+1} \log_2^{M+1} \approx 0.35 \), which never exceeds the efficiency of the optimized FLA. In addition, the total number of switches in the optimized FLA is two times smaller than \( \log_2 N + 1 \) which is needed for the BDFN. Figure 2 illustrates the basic parameters as functions of the mean photon number \( \langle n \rangle \approx r^2 \approx P(1) \) under expected experimental conditions.
Regarding the autocorrelation function, the value $p_{dc}=10^{-5}$ corresponds to the dark count rate 1 kHz and detection window 10 ns. It can be seen from figure 2 that such a dark count rate reveals itself only for high fidelity, when $g^{(2)}(0)<0.01$. In this case, a measured value of the autocorrelation function may differ significantly from an expected theoretical one. However, increasing $p_{dc}$ up to $10^{-4}$ gives an error 50% when the second-order zero-time autocorrelation function is equal to 0.01.

![Figure 2](image.png)

**Figure 2.** Heralding efficiency (a), total efficiency (b), experimental $g^{(2)}(0)$ (c) and theoretical $g^{(2)}(0)$ (d) as functions of the mean photon number $\langle n \rangle$. The result of numerical simulation on the basis of equation (4) with the following values of parameters: $N=64$, $M=4$, $\eta=0.5$, $\eta_s=0.2$, $\eta_e=0.999$, $\eta_{SW}=0.891$, $a=4$ $(s=3)$, $p_{dc}=10^{-5}$.

### 5. Conclusion
We have studied basic properties of a heralded single-photon source with temporal and spatial multiplexing. We found that the efficiency of a variable optical delay line, which is used for temporal multiplexing, can be optimized so that the total number of switches is reduced twice compared with the binary division network, while the efficiency becomes higher. In this respect, the optimized optical delay line seems to be the easiest one for implementation. According to the numerical simulations, a single-photon source with the efficiency about 40% and the second-order zero-time autocorrelation function 0.05 may be realized under typical experimental parameters.

### Acknowledgments
This research was supported by the Russian Foundation for Basic Research (Grant No. 14-12-0719214-ofi).

### References
[1] Eisaman M D, Fan J, Migdall A and Polyakov S V 2011 *Rev. Sci. Instrum.* **82** 071101
[2] Migdall A L, Branning D and Castelletto S 2002 *Phys. Rev.* A **66** 053805
[3] Shapiro J H and Wong F N 2007 *Optics Letters* **32** 2698
[4] Ma X S, Zotter S, Kofler J, Jennewein T and Zeilinger A 2011 Phys. Rev. A 83 043814
[5] Collins M J, Xiong C, Rey I H, Vo T D, He J, Shahnia S, Reardon C, Krauss T F, Steel M J, Clark A S and Eggleton B J 2013 Nature Communications 4 2582
[6] Meany T, Ngah L A, Collins M J, Clark A S, Williams R J, Eggleton B J, Steel M J, Withford M J, Alibart O and Tanzilli S 2014 Laser and Photonics Reviews 8 L42
[7] Pittman T B, Jacobs B C and Franson J D 2002 Phys. Rev. 66 042303
[8] Jeffrey E, Peters N A and Kwiat P G 2004 New Journal of Physics 6 100
[9] Peters N A, Arnold K J, VanDevender A P, Jeffrey E R, Rangarajan R, Hosten O, Barreiro J T, Altepeter J B and Kwiat P G 2006 Proc. SPIE 6305 630507
[10] Mower J and Englund D 2011 Phys. Rev. A 84 052326
[11] Schmiegelow C T and Larotonda M A 2013 Appl. Phys. B 116 447
[12] Glebov B L, Fan J and Migdall A 2013 Appl. Phys. Lett. 103 031115
[13] Mazzarella L, Ticozzi F, Sergienko A V, Vallone G and Villoresi P 2013 Phys. Rev. A 88 023848
[14] Adam P, Mechler M, Santa I and Koniorczyk M 2014 Phys. Rev. A 90 053834
[15] Bonneau D, Mendoza G J, O’Brien J L and Thompson M G 2014 Effect of loss on multiplexed single-photon sources Preprint quant-ph/1409.5341
[16] Christensen B G, McCusker K, Goggin M, Crimmins K and Kwiat P 2011 Free-space photon storage with variable time delays CLEO:2011 – Laser Applications to Photonic Applications, OSA Technical Digest (CD) (Optical Society of America, 2011) paper QThJ2