A simple model system to study coupled photonic crystal microcavities

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In this paper, we designed and experimentally studied several systems of standard coaxial cables with different impedances which mimic the operation of so-called photonic structures like coupled photonic crystal microcavities. Using elementary cells of half-meter long coaxial cables we got resonances around 100 MHz, a range of frequencies that can be easily studied with a standard teaching laboratory apparatus. Resonant mode frequency splitting has been obtained in the case of double and triple coupled cavities. A good agreement between experimental results and transfer matrix model has been observed. The aim here is to demonstrate that standard coaxial cable is a very cheap way and an easy to implement structure to explain to undergraduate students complex phenomena that usually occur in the optical domain.

I. INTRODUCTION

Optical micro-resonators are of great interest for fundamental studies in optics and for applications in photonics such as integration of optical sources, optical filtering or bio-an chemical sensing14,15. One very popular method to integrate optical micro-resonators is to create a defect in a periodic photonic crystal.2–4 As it is the case of atomic crystals, the periodicity breaking creates resonant modes localized in the defect with a resonance frequency lying within the photonic bandgap.5–8

The coupling of optical micro-resonators gives additional degrees of freedom and enables the design of complex photonic structures, with optimized optical characteristics.9,10 As the states of photons confined in an optical microcavity are similar to confined electron states in atoms, optical micro-resonators are often referred to photonic atoms. In this physical picture, coupled resonators will support hybridized states and can be compared to photonic molecules.11–13 From a fundamental point of view these objects are still the subject of intense research efforts in quantum photonics, nonlinear optics or laser physics.14–17 The interaction of the resonant modes of two cavities leads to interesting phenomena such as frequency splitting or induced transparency and can be used for dispersion management18 or optical storage.19–22 The symmetric frequency splitting obtained in a photonic molecule composed of three coupled resonators made of whispering gallery mode resonators or photonic crystals microcavities has been used to reach the phase matching condition in the four wave mixing process.23,24 By increasing the number of coupled resonators it is even possible to obtain resonant waveguides25 or delay lines with applications in optical signal buffering.26,27

From another point of view, coaxial cable structures have been used to emulate one-dimensional photonic crystals, Bragg mirrors or quasi-periodic photonic structures in the radio-frequency domain.28–31 Doing a periodic system consisted of two sets of meter long coaxial cables, it is possible to observe a Bragg diffraction due to reflections at impedance transitions.32 Adding an extra cable in the middle of the system, a Fabry-Perot type resonance appears in the center of the stop-band; this effect has already been demonstrated in the range 10 – 50 MHz using coaxial cables.33–36 From an educational point of view, the great asset of this approach is that students can build their own mirrors or cavities without the need of complex technological facilities. This is not possible in the optical domain since in this case the involved wavelengths are very short. In this paper we propose to extend and generalize this radio-frequency (RF) analogy to coupled resonator structures made of one-dimensional photonic crystal defects cavities and show that usual laboratory equipment can be used to teach the basics of nanophotonic circuitry at the undergraduate level.

The paper is organized as follows: in section II we introduce the transfer matrix method (TMM) which will be used all along the paper to model our structures. Then, we detail the physics underlying the coupling of optical cavities focusing on the case of two and three defects. Section III is devoted to the experimental demonstration of the coupling of model photonic crystal cavities consisting of coaxial cables with two different impedances of 50 Ω and 75 Ω.

II. THEORY

A. Periodic structure

Figure. 1(a) shows a finite one-dimensional photonic crystal or Bragg mirror made of \( N \) identical cells constituted by two dielectric material layers of refractive indices \( n_1 \) and \( n_2 \) and thicknesses \( \ell_1 \) and \( \ell_2 \). \( E_{in}, E_{r}, \) and \( E_{t} \) are respectively the input, reflected and transmitted optical fields. To obtain a maximal reflection at frequency \( \nu_0 \) the phase accumulated by the wave after propagation within a unit cell have to be equal to \( \pi \) and thus the following condition must be verified:

\[
 n_1 \ell_1 + n_2 \ell_2 = \frac{\lambda_0}{2}, \tag{1}
\]

where \( \lambda_0 \) is the Bragg wavelength. The radio-frequency (RF) analog of this Bragg mirror can be obtained by a periodic system whose elementary cell consists of two coaxial cables of different length and impedance. In the RF domain it is more convenient to use voltages and we define here \( V_{in}, V_{r}, \) and \( V_{t} \).
as the input, reflected and transmitted voltages respectively. By introducing the phase velocity \( v_\varphi \) and \( v_\varphi \) of the cable of impedance \( Z_1 \) and \( Z_2 \), it is possible to write the Bragg condition as

\[
\frac{\ell_1}{v_\varphi_1} + \frac{\ell_2}{v_\varphi_2} = \frac{1}{2v_0},
\]

where \( v_0 \) is the resonant frequency. The propagation along the structure can be modeled using the TMM.\(^{37,39,40} \) The reflection and transmission at each interface between two media of impedance \( Z_i \) and \( Z_j \) is obtained via the matrix \( M_{i,j} \) defined by

\[
M_{i,j} = \frac{1}{t_{j,i}} \begin{pmatrix} 1 & r_{j,i} \end{pmatrix}
\]

where \( r_{j,i} = \frac{Z_i - Z_j}{Z_i + Z_j} \) and \( t_{j,i} = \frac{2Z_j}{Z_i + Z_j} \). At angular frequency \( \omega = 2\pi v \), the propagation in a layer of thickness \( \ell_i \) and phase velocity \( v_\varphi_i \) is taken into account thanks to matrix \( Q_i \) which reads

\[
Q_i = \begin{pmatrix} \exp(\jmath \omega \ell_i / v_\varphi_i - \kappa_i \ell_i) & 0 \\ 0 & \exp(-\jmath \omega \ell_i / v_\varphi_i + \kappa_i \ell_i) \end{pmatrix}
\]

where \( \kappa_i \) is the attenuation coefficient of the medium of impedance \( Z_i \). We define the period of the photonic crystal \( \Lambda = \ell_1 + \ell_2 \); \( V_+ (z) \) and \( V_- (z) \) are the signals propagating respectively in the forward and backward directions (see Fig. 1). Assuming that \( m \in \mathbb{N} \), the matrix \( M \) associated to the unit cell defined by

\[
\begin{pmatrix} V_+ ([m + 1] \Lambda) \\ V_- ([m + 1] \Lambda) \end{pmatrix} = M \begin{pmatrix} V_+ (m \Lambda) \\ V_- (m \Lambda) \end{pmatrix},
\]

is thus given by

\[
M = M_{2,1} Q_2 M_{1,2} Q_1.
\]

We consider now that input and output media have both an impedance \( Z_1 \), thus the matrix \( M_{\text{Bragg}} \) of a periodic structure made of \( N \) elementary cells is given by

\[
M_{\text{Bragg}} = M^N. \tag{7}
\]

In the linear regime, the voltage amplitude transmission \( t_0 \) and reflection \( r_0 \) can be deduced by using the following relation

\[
\begin{pmatrix} t_0 \\ r_0 \end{pmatrix} = M_{\text{Bragg}} \begin{pmatrix} 1 \\ r_0 \end{pmatrix}. \tag{8}
\]

The power transmission \( T \) and reflection \( R \) coefficients are thus obtained by \( T = |t_0|^2 \) and \( R = |r_0|^2 \). Figure 2 gives an example of the transmission and reflection spectra of a 20-cells periodic structure made of coaxial cables with the following parameters: \( Z_1 = 50 \, \Omega \), \( Z_2 = 75 \, \Omega \), \( v_\varphi_1 = v_\varphi_2 = \frac{\omega}{4} \) and \( \ell_1 = \ell_2 = 50 \, \text{cm} \). We consider in this section a loss-less material \( (\kappa_i = 0) \) for a sake of clarity. The propagation is forbidden within a spectral range of 27 MHz centered at \( v_0 = 100 \, \text{MHz} \), see Eq. (2), this effects manifests itself by a high reflection and a low transmission.

B. Defect modes in one-dimensional photonic crystal structures

By inserting defects or impurities in the periodic structure as shown in Fig. 3 it is possible to create localized modes whose resonant frequencies appear within the photonic bandgap.\(^{37,38} \) The defects have a length \( \ell_d \) and an impedance \( Z_d \). In this work we focus on structures with one, two or three identical defects.

1. Photonic crystal cavities

The first structure is obtained by inserting a single defect\(^{37} \). The structure is described in Fig. 3(a): it consists of a one-
The matrix $M_1 D$ associated to this structure is given by:

$$M_1 D = M_p Q_1 M_p,$$

(9)

and can be used to determine the spectral response of the defect-structure. Figure 4 shows the transmission spectrum of such a structure with $p = 10$ corresponding to the periodic structure studied in Fig. 2 with a single defect. Moreover, the inset a) of Fig. 4 gives the voltage (or field) $V(z) = V_+(z) + V_-(z)$ and impedance $Z(z)$ distributions inside the structure at $v = v_0$. An inspection of these two plots demonstrates clearly that a mode localized in the defect appears in the middle of the photonic bandgap at the frequency $v_0 = 100$ MHz. The inset b) of Fig. 4 is a zoom of the transmission peak which displays a Lorentzian profile. This resonator can also be described in the coupled mode theory (CMT) framework\textsuperscript{43,44} by writing the evolution equation of the localized mode amplitude $a(t)$ shown in Fig. 3.a):

$$\frac{da}{dt} = \left( j \omega_0 - \frac{1}{\tau} \right) a(t) + \sqrt{2 \tau} V_{in}(t),$$

(10)

where $\omega_0 = 2 \pi v_0$, $\frac{1}{\tau}$ is the coupling rate of the mode to the external media through the Bragg mirrors and $\tau$ is the mode amplitude lifetime. Since we have neglected the losses, we have $\frac{1}{\tau} = \frac{2}{\tau_e}$. The output signal can thus be written as $V_t(t) = \sqrt{2 \tau} a(t)$. In the stationary regime at angular frequency $\omega$, $a(t) = \tilde{a} e^{j \omega t}$, $V_{in}(t) = \tilde{V}_{in} e^{j \omega t}$ and $V_t(t) = \tilde{V}_t(\omega) e^{j \omega t}$. It is straightforward to solve Eq. (10) and the transmission of the system then reads

$$T(\omega) = \left| \frac{\tilde{V}_t(\omega)}{\tilde{V}_{in}} \right|^2 = \frac{1}{1 + \frac{\omega^2}{\omega_0^2} - \frac{1}{\tau_e^2}}$$

(11)

The transmission resonance has thus a Lorentzian shape as shown in the inset b) of Fig. 4, its width $\Delta \nu$ is related to the mode amplitude lifetime and the quality factor $Q$ of the cavity by

$$Q = \frac{\omega_0 \tau}{2} = \frac{v_0}{\Delta \nu}.$$  

(12)

2. Two coupled photonic crystal cavities

Figure 2b) shows a system composed of two coupled cavities. It consists of two identical defect separated by a barrier with $q$ cells.\textsuperscript{45} The input and output coupling is obtained by tunneling through two barriers with $p$ periods. The matrix of the whole system reads

$$M_{2D} = M_p Q_1 M_p Q_1 M_p$$

(13)

Calculations of the transmission are shown Fig. 5 for $q = 13$, and $q = 15$ in the case of 10-period input and output mirrors. The transmission spectra show that the initial resonance frequency $v_0$ is split into two high and low resonant frequencies. This can be understood by writing the evolution equations of the two resonant mode amplitudes $a_1$ and $a_2$ defined in 2b) via vector $a = (a_1, a_2)^T$

$$\frac{da}{dt} = Ka(t) + \sqrt{2 \tau} V_{in}(t)$$

(14)

where matrix $K$ is given by

$$K = \begin{pmatrix}
  j \omega_0 - \frac{1}{\tau} & j \gamma \\
  j \gamma & j \omega_0 - \frac{1}{\tau}
\end{pmatrix},$$

(15)
metric mode; b) ω to the higher resonant frequency value thus ωresonant frequencies of the two split modes. 

FIG. 5. Two coupled photonic crystal cavity transmission spectra for q = 13 and q = 15. In both cases the calculations have been carried out with p = 10. v>, and v< are respectively the high and low resonant frequencies of the two split modes.

FIG. 6. Impedance Z(z) and voltage V(z) distributions for a double cavity system obtained for p = 10 and q = 13. a) v = v>, antisymmetric mode; b) v = v<, symmetric mode. z0 is the position of the center of the structure.

and vin(t) = (V in(t), 0)T. In this case τ = τc and |γ| is the coupling rate between the two cavities. The eigenvalues of K are ω0,AS = j(ω0 ± γ) − 1/2 and are associated to the symmetric and antisymmetric resonant modes of the whole system. As we consider a loss-less material, γ is real; for an even value of q we have γ > 0 and the symmetric mode corresponds to the higher resonant frequency value thus v> = Re(ω0) and v< = Re(ω0)/2π, whereas for an odd value of q, γ < 0 which leads to v> = Re(ω0) and v< = Re(ω0)/2π. This control of the sign of the coupling coefficient is similar to what has been observed in two dimensional photonic crystal molecules.10. Figure 6 gives the voltage distribution V(z) for p = 10 and q = 13 for the two frequencies v> an v< described in Fig 5. Since q is an odd number, the voltage profile is symmetric with respect to the center of the barrier for v< and antisymmetric for v> in both cases the two mode are localized at the defect locations.

FIG. 7. Transmission spectra for q = 10 and p = 10 in the case of a double (k = 2) and a triple (k = 3) coupled cavity systems. γ = ω/Q is the frequency splitting in Hz.

Moreover, by increasing q the coupling between the two cavities is reduced which leads to a smaller frequency splitting as illustrated in Fig. 5

3. Three coupled photonic crystal cavities

This reasoning can be generalized to k defects, in this case the transfer matrix Mkd is given by

\[
M_{kd} = M^p Q_1 (M^p Q_1)^{k-1} M^p
\]

(16)

In Fig. 3c) we have sketched a coupled cavity system with k = 3 identical defects. The spectrum of such a system with p = 10 and q = 10 is shown in Fig. 7. The initial mode is now split in three modes. A more quantitative study of this splitting can be carried out using the CMT using Eq. (14) where a = (a1, a2, a3)T (see Fig. 3c) and vin(t) = (V in(t), 0, 0)T. For k = 3, matrix K is now given by

\[
K = \begin{pmatrix}
    j(ω0) - 1/2 & jγ & 0 \\
    jγ & j(ω0) & jγ \\
    0 & jγ & j(ω0) - 1/2
\end{pmatrix}
\]

(17)

If the coupling rate is large enough (|γ| ≪ 1) the new resonant frequencies are the eigenvalues of K which read

\[
\begin{pmatrix}
    j(ω0) - 1/2 - γ\sqrt{2}/2π \\
    j(ω0 + γ\sqrt{2}) - γ\sqrt{2}/2π
\end{pmatrix}
\]

(18)

Hence, the split resonances have a bandwidth which is half of that of the central resonance and the frequency splitting is increased by a \sqrt{2} factor in comparison with the two cavity system. These effects are summarized Fig. 7 where we also represent the transmission of a double cavity obtained for p = 10 and q = 10.
III. EXPERIMENTS

A. Experimental setup

Experiments have been carried out using two sets of 50 cm long RG-58/U and RG-59/U coaxial cables whose nominal impedances are respectively 50 Ω and 75 Ω. Attenuation of cables are taken into account in the TMM by using the following values for the attenuation coefficients:

\[
\kappa_1(v) = 4.61 \times 10^{-12}v + 2.29 \times 10^{-6}\sqrt{v} \quad (19)
\]
\[
\kappa_2(v) = 4.61 \times 10^{-12}v + 1.46 \times 10^{-7}\sqrt{v}, \quad (20)
\]

where \(v\) is given in Hz. For both cable sets the phase velocity is \(v_{\phi 1} = v_{\phi 2} = 0.66c\). The experimental setup is described Fig. 8 the electromagnetic waves with frequency around 100 MHz are produced by an arbitrary waveform generator SIGLENT SDG6022X and analyzed using an oscilloscope Tektronix DPO4104. The transmission spectra are obtained by measuring the RMS values of the voltage at the input and at the output of the coaxial cable structures at several frequencies.

B. Bragg mirror and photonic crystal cavity

The first experiment consisted in measuring the transmission of a periodic structure (see Fig. 1(a) made of \(N = 10\) unit cells. The experimental results are given Fig. 2(a). The actual length \(\ell_1 = \ell_2 = 51.9\) cm of the cables has been deduced by measuring the Bragg frequency \(v_0 = 96.4\) MHz and by using Eq. (2). The difference between the nominal and the actual cable lengths comes from the additional length due to the BNC adapters. The calculations shown in Fig. 2(a) has been carried out using the TMM and varying \(Z_1\) and \(Z_2\). In the rest of this work we will use the impedance values \(Z_1 = 54\) Ω and \(Z_2 = 70.5\) Ω obtained from the best fit of the periodic structure transmission experimental data. The photonic crystal cavity or single defect resonant structure is obtained by adding an extra cable of impedance \(Z_1\) (see Fig. 2(a)). The associated experimental transmission spectrum is given Fig. 2(b). At the center of the photonic bandgap \((v_0)\), the transmission spectrum shows an attenuation-limited resonance with a spectral width \(\Delta v = 1.6\) MHz corresponding to a quality factor \(Q = 61.5\). The calculations (full line) which have been carried out using the length and impedance values obtained from the previous fit without adjusting any parameter are in good agreement with the experimental data.

C. Two coupled photonic crystal cavities

In this section we give the experimental results obtained for the system made of two cavities shown in Fig. 3(b) and analyzed from a theoretical point of view at section 3B.2. The measurements have been done for several values of the number of unit cells separating the two cavities: \(q \in \{1, 4\}\). For \(q = 4\) (Fig. 10(a)), the cavity coupling is weak and thus the two split frequencies are not well separated. By decreasing \(q\), the cavity coupling is increased leading to a stronger frequency splitting. In particular, for \(q = 2\) (Fig. 10(c)), we can measure a frequency splitting \(\Delta v = 10.8\) MHz much larger than the...
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