Peccei-Quinn symmetry as the origin of Dirac Neutrino Masses

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Abstract

We propose a model of Dirac neutrino masses generated at one-loop level. The origin of this mass is induced from Peccei-Quinn symmetry breaking which was proposed to solve the so-called strong CP problem in QCD, therefore, the neutrino mass is connected with the QCD scale, $\Lambda_{\text{QCD}}$. We also study the parameter space of this model confronting with neutrino oscillation data and leptonic rare decays. The phenomenological implications to leptonic flavor physics such as the electromagnetic moment of charged leptons and neutrinos are studied. Axion as the dark matter candidate is one of the byproduct in our scenario. Di-photon and Z-photon decay channels in the LHC Higgs search are investigated. We show that the effects of singly charged singlet scalar can be distinguished from the general two Higgs doublet model.

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I. INTRODUCTION

Small quantities arising in physics usually requires the use of new symmetries for explanations [1]. A good example of these is the Peccei-Quinn (PQ) symmetry that plays the role for the solution of the strong CP-problem [2, 3], in which a $\theta$-angle appears in the QCD lagrangian, $L_{QCD} \in \theta g^2 S^2 \pi^2 G^a_{\mu\nu}G_{\mu\nu}^a$, and CP is violated$^1$ [4–6]. The instanton solution to the gluon field equations satisfies $n = \frac{1}{32 \pi^2} \int d^4 x G^a_{\mu\nu}G_{\mu\nu}^a$ with $n$’s being integers and representing topological charges [7]. The QCD vacuum state hence can be parametrized as $|\theta\rangle = \sum_{n=-\infty}^{n=\infty} e^{i n \theta} |n\rangle$ where $\theta$ is periodic with period $2\pi$. Furthermore for nonzero quark masses the chiral anomaly relates the weak phase in quark masses to the QCD $\theta$-term. One can parametrize the $\theta$ angle as $\bar{\theta} = \theta - \text{arg}\{\text{det} m_q\}$ [8, 9]. It induces a neutron electric dipole moment (EDM) [10–14] and the current experimental upper bound set the best constraint on $\bar{\theta}$ to be smaller than $0.6 \times 10^{-10}$ [15]. This extremely suppressed quantity is called the strong CP problem. The Peccei-Quinn solution to the strong CP problem postulates a global chiral $U(1)_{PQ}$ symmetry and makes $\bar{\theta}$ a dynamical variable, and the shift symmetry of the Nambu-Goldstone boson, axion, corresponding to $U(1)_{PQ}$ [16, 17] will set $\bar{\theta}$ zero at classical potential [18]. At one-loop level the chiral anomaly will break the shift symmetry. As a result the axion is not massless but requires a small mass $m_a \simeq \sqrt{m_u m_d} f_a m_a \sim \frac{\Lambda_{QCD}^2}{f_a} f_a$ [19–21]. Here $f_a$ is the $U(1)_{PQ}$ breaking scale and $f_\pi$ is the pion decay constant. The laboratory and outer space [22–27] searches have set $10^9$ GeV $\lesssim f_a \lesssim 3 \times 10^{11}$ GeV as the allowed regions, therefore, the axion window is $3 \times 10^{-3}$ eV $> m_a > 10^{-6}$ eV.

On the other hand, another small quantity that puzzles high energy physicists is the masses of neutrinos measured from neutrino oscillation experiments [22]. The key point to understand neutrino physics lies on whether the neutrinos are Dirac fermions or Majorana fermions. This ambiguity comes from the fact of zero electric charge carried by neutrinos. The tiny neutrino masses may be explained in terms of lepton number ($L$) symmetry which is a global $U(1)$ quantum number tagged on lepton sectors in the standard model (SM). If $U(1)_L$ is broken one can write the dimension-5 Weinberg operator to generate neutrino masses $m_\nu \propto \frac{H H L L}{\Lambda_L}$ [28], where $H$ and $L$ are the SM Higgs and the left-handed lepton fields respectively, and $\Lambda_L$ is the breaking scale of $U(1)_L$. In this case neutrinos are regarded as

$^1$ The $G^a_{\mu\nu}$ are the $SU(3)_C$ gauge fields with $a = 1, 2, \ldots, 8$ and $\tilde{G}^{\alpha\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} G^a_{\alpha\beta}$. 

2
Majorana fermions. However, the Majorana or Dirac nature of the neutrinos is unknown and is awaiting for the experimental determination from some lepton number violating processes such as the neutrinoless double beta decay. It is important to consider the possibility that Dirac neutrino masses may also connect with some global symmetry and how those small quantities we observe in physics are related to each other. In this paper we propose a simple Dirac neutrino mass model which is generated by PQ symmetry breaking, and hence the neutrino masses are closely related with axion mass.

This paper is organized as follows: in section II we propose the Dirac mass model which is embedded with PQ symmetry. Section III we consider leptonic rare decays and neutrino oscillations data to investigate the parameter space of the model. Some phenomenological implications to LHC Higgs search, dark matter, and leptonic flavor physics of this model are discussed in section IV. Then we conclude our results in section V.

II. THE MODEL

Particle content and their quantum numbers of the model are listed in Table I. The $Y$, $L$ and PQ represent the hypercharge, lepton number, and $U(1)_{\text{PQ}}$ charge respectively. Two Higgs doublets are introduced since the existence of the PQ-symmetry and one need two independent chiral transformation for the up-type and down-type fermion, that is, one scalar doublet $H_1$ couples to $d_R$ and $l_R$, while the other one $H_2$ only couples to $u_R$ by setting opposite PQ charges to doublet scalars. We consider the scenario that neutrinos are Dirac fermion, with three right-handed neutrinos $\nu_{Ri}$ ($i = 1 - 3$) assigned to our model, and hence the theory is lepton number conserved. $s_1^+$ and $s_2^+$ are $SU(2)_L$-singlet charged scalars, and $a$ is the axion field which is the Nambu-Goldstone boson of the spontaneously broken

2 The interesting models of connecting Dirac neutrino mass with leptogenesis are studied, for example, in Refs. 29, 31.

3 In Ref. 32, the PQ symmetry and Dirac neutrino masses are connected in the so-called universal seesaw model. Similar idea of linking Majorana neutrino masses with PQ symmetry was also studied in Ref. 33-42.

4 In this paper two additional global symmetries $U(1)_L$ and $U(1)_{\text{PQ}}$ are imposed in the model. It turns out the two symmetries are not independent such that $U(1)_L$ can be generated accidentally by some particular choice of $U(1)_{\text{PQ}}$ charges. One example is to take the PQ charges as $H_1 : 2$, $H_2 : -2$, $L_L : 1$, $\nu_R : 4$, $l_R : -1$, $a : 1$, $s_1 : -2$, and $s_2 : -3$. We found it is a generic feature of assigning the large ratios of the PQ charges for some particles in order to bridge the two global symmetries nontrivially.
TABLE I: Quantum numbers of $U(1)_{PQ}$ and gauge symmetries for leptons and scalars.

$U(1)_{PQ}$. Notice that $\nu_R$'s are complete neutral under gauge symmetries and PQ symmetry and only carry the $L$ quantum number. The Dirac neutrino mass term is forbidden by the PQ-symmetry at the tree level and is generated at one-loop level after the PQ symmetry breaking by utilizing the charged scalars $s_1^+$ and $s_2^+$. The new Yukawa interactions of the model for leptons are given by

$$\mathcal{L} = y_{\alpha\beta}(\overline{L_{\alpha}})l_{R_{\beta}}H_1 + f_{\alpha\beta}(\overline{L_{\alpha}})i\sigma_2(L_{\beta})s_1^+ + h_{\alpha\beta}(\overline{l_{R_{\alpha}}}\nu_R)s_2^+ + \text{h.c.}, \quad (1)$$

where $c$ denotes charged conjugation, $\alpha, \beta = e, \mu, \tau$, and $\sigma_i$ ($i = 1 - 3$) are the Pauli matrices. In general, $y$ and $h$ are complex matrices, and $f$ is an antisymmetric matrix due to the Fermi statistics. One can choose the basis of leptonic mass eigenstates $L_L$, $l_R$ such that $y$ is a diagonalized matrix. Also $f_{ij}$ can be chosen to be real by rephasing $L_L$ and by transferring the phases into $l_R$. Therefore only $h_{ij}$ are complex. The scalar potential can be written as

$$V = -\mu_1^2 H_1^\dagger H_1 - \mu_2^2 H_2^\dagger H_2 - \mu_\nu|\nu|^2 + \mu_1|s_1|^2 + \mu_2|s_2|^2 + \lambda_1(H_1^\dagger H_1)(H_1^\dagger H_1) + \lambda_2(H_2^\dagger H_2)(H_2^\dagger H_2) + \lambda_3(H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1)$$

$$+ (H_1^\dagger H_1)[d_1|s_1|^2 + d_2|s_2|^2 + d_\nu|\nu|^2] + (H_2^\dagger H_2)[g_1|s_1|^2 + g_2|s_2|^2 + g_\nu|\nu|^2] + h_1|s_1|^4 + h_2|s_2|^4 + h_3|s_1|^2|s_2|^2 + h_\nu|\nu|^4 + |\nu|^2(h_1|s_1|^2 + h_2|s_2|^2)$$

$$+ [h_5(H_1^\dagger H_1)a^2 + \mu s_1^+ s_2^+ a + \text{h.c.}] . \quad (2)$$

All parameters in the potential are real. Though $h_5$ and $\mu$ are in general complex parameters, one can absorb their phases by redefining $(s_1^+ s_2)$, $(H_2^\dagger H_1)$ and $a$ respectively. For the invisible axion, $d_a$, $g_a$, $h_{a1}$, $h_{a2}$, $h_a$, $\mu$ should be very small to make other scalar mass not too heavy [43-46]. The details about the scalar mass spectrum are put in the Appendix.

The leading contribution to the Dirac neutrino mass at one-loop level is shown in Fig.1 and is given by

$$(M_\nu)_{\alpha\beta} = -\frac{\mu f_a}{8\pi^2 m_\beta} f_{\alpha\beta} h_{\beta i} I(m_{s_1}^2, m_{s_2}^2, m_\beta^2) , \quad (3)$$
where \( I(m^2_{s1}, m^2_{s2}, m^2_\beta) \) is defined as

\[
I(m^2_{s1}, m^2_{s2}, m^2_\beta) = \frac{m^2_\beta}{m^2_{s1} - m^2_{s2}} \left[ \frac{m^2_{s1}}{m^2_{s1} - m^2_\beta} \log \frac{m^2_{s1}}{m^2_\beta} - \frac{m^2_{s2}}{m^2_{s2} - m^2_\beta} \log \frac{m^2_{s2}}{m^2_\beta} \right].
\] (4)

In the limit of \( m_{s1,2} \gg m_k \), the neutrino mass matrix is proportional to charged lepton mass, as given by

\[
(M_\nu)_{\alpha i} = - \frac{1}{8\pi^2} f_{\alpha\beta} h_{\beta i} m_\beta \frac{\mu f_a}{m^2_{s1} - m^2_{s2}} \log \frac{m^2_{s1}}{m^2_{s2}}
\approx - \frac{1}{8\pi^2} f_{\alpha\beta} h_{\beta i} \frac{\mu m_\beta}{m_a} \frac{\Lambda^2_{QCD}}{m^2_{s1} - m^2_{s2}} \log \frac{m^2_{s1}}{m^2_{s2}} = -C f_{\alpha\beta} m_\beta h_{\beta i}.
\] (5)

We have replaced the PQ-symmetry breaking scale by the axion mass and the QCD scale in Eq. (5). Note that there are similar construction discussed in Refs. 48, 49. However, our setup shows the interesting relation between the neutrino Dirac mass and QCD dynamics. Furthermore, the axions can be the dark matter candidate constituting of 25% of energy density in our universe. From the neutrino mass formula and scalar potential we can see that if one does not tune the couplings \( h_{a1} \) and \( h_{a2} \), then the masses of \( s_1 \) and \( s_2 \) should be of the order of \( f_a \). Combining with the value of \( \mu \) to be around the electroweak scale would directly lead to a small quantity of \( C \approx 10^{-8} \), which means one can have the observed light neutrino masses with the Yukawa couplings \( f, h \) at the order of \( 10^{-1} \). Although such

\[\text{FIG. 1: Induced Dirac neutrino mass}\]
a scenario can naturally provide a tiny neutrino mass without fine tuning the couplings, in what follows we still focus on the case in which both $m_{s_1}$ and $m_{s_2}$ are of the electroweak scale to provide richer phenomenological implications. In this case we can see that the large value of $f_a$ lifts up the mass scale of Dirac neutrino masses and in order to keep the smallness of $M_\nu$ a combination of suppressed factors will be needed such as the loop factor, the Yukawa couplings $f$, $h$, the charged lepton chirality suppression, and the parameter $\mu$. Now we roughly estimate the scale of $\mu$ which is the key parameter controlling the overall mass scale of Dirac neutrino masses and a investigation of parameter space will be discussed in section III. For $M_\nu \approx 0.1$ eV, $m_{s_{1,2}} \sim \mathcal{O}(100 - 1000)$ GeV, and $f_a \approx 10^{12}$ GeV, we have $f \sim 10^{-3}$, $h \sim 10^{-2}$, and $\mu \sim \mathcal{O}(1)$ keV. Let’s make two comments on the low scale $\mu$ and Dirac neutrino masses: 1. We can explain the small $\mu$ by implementing the Froggatt-Nielsen mechanism \cite{50} with PQ-symmetry. For example, if we assign the PQ quantum number of $a$ as $\frac{2}{n}$ in Table II, all the terms in the Lagrangian will not change except the $\mu$-coupling in the potential. One can write the effective operator as

$$\frac{1}{\Lambda^{n-2}} s_1^+ s_2^- a^n \quad \overset{U(1)_{PQ} \text{ breaking}}{\longrightarrow} \quad (\frac{\langle a \rangle}{f_a})^{n-2} \langle a \rangle s_1^+ s_2^- a, \quad (6)$$

thus $\mu = (\frac{\langle a \rangle}{f_a})^{n-2} \langle a \rangle$ at low energy scale and can be tuned to a small quantity. Here the PQ symmetry can be broken dynamically by some condensates of a new technicolor-like interaction at high scale \cite{51, 52}. This mechanism can also apply to other dimensionless couplings of non-Hermitian terms such as the $h_5$ in the potential. 2. The mass scale of Dirac neutrino masses is not necessarily small if neutrinos are Majorana fermions. Although we consider neutrinos as the Dirac fermions, the main concern in this paper is the origin of the Dirac neutrino mass, which in general does not forbidden the possibility that neutrinos are the Majorana fermions. Therefore one can still have heavy right-handed Majorana masses as inspired by the grand unification theories and obtain small neutrino masses through canonical seesaw mechanism. The goal in this paper is to point out that the Dirac neutrino mass is generated by the PQ-symmetry breaking.
III. CONFRONTING WITH NEUTRINO OSCILLATION DATA AND LEP-TONIC RARE DECAYS

From the standard formalism $M_{\nu}$ can be diagonalized by

$$M_{\nu,diag} = V_{\nu}^{\dagger} M_{\nu} V_{\nu} = -V_{\nu}^{\dagger} (C f A_{\nu} h) V_{\nu}^{\dagger},$$

where $V_{PMNS}$ is a unitary $3 \times 3$ Pontecorvo-Maki-Nakagava-Sakata (PMNS) matrix and $V_{R}$ is the transformation matrix for right-handed neutrinos. For convenience we define $F H = V_{PMNS} M_{\nu,diag}$ with $H = M_{\nu,diag} (-h V_{\nu}^{\dagger})$ and $F = C f$. Due to the anti-symmetric nature of $f$ matrix, the lightest neutrino is exactly massless in this model. Therefore, we can multiply a transformation matrix $A$ to both sides of the mass matrix to reduce one row in the left hand side of Eq. (7). One obtain

$$F' H = AV_{PMNS} M_{\nu,diag},$$

where

$$F' = AF = \begin{pmatrix} -F_{e\mu} & 0 & F_{\mu\tau} \\ F_{e\tau} & F_{\mu\tau} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ F_{\mu\tau} & -F_{e\tau} & F_{e\mu} \end{pmatrix}.$$  

Then we have

$$\begin{pmatrix} -F_{e\mu} H_{e1} + F_{\mu\tau} H_{\tau1} & -F_{e\mu} H_{e2} + F_{\mu\tau} H_{\tau2} & -F_{e\mu} H_{e3} + F_{\mu\tau} H_{\tau3} \\ F_{e\tau} H_{e1} + F_{\mu\tau} H_{\mu1} & F_{e\tau} H_{e2} + F_{\mu\tau} H_{\mu2} & F_{e\tau} H_{e3} + F_{\mu\tau} H_{\mu3} \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} m_{\nu 1} V_{\mu 1} & m_{\nu 2} V_{\mu 2} & m_{\nu 3} V_{\mu 3} \\ -m_{\nu 1} V_{\tau 1} & -m_{\nu 2} V_{\tau 2} & -m_{\nu 3} V_{\tau 3} \\ m_{\nu 1} \sum_l V_{l1} F_I & m_{\nu 2} \sum_l V_{l2} F_I & m_{\nu 3} \sum_l V_{l3} F_I \end{pmatrix}$$

with $F_I = (F_{\mu\tau}, -F_{e\tau}, F_{e\mu})$.

For the normal hierarchical spectrum with $m_{\nu 1} = 0$ we have the following relations:

$$F_{e\mu} = \frac{V_{\tau 1}^*}{V_{e 1}^*} F_{\mu\tau}, \quad F_{e\tau} = -\frac{V_{\mu 1}^*}{V_{e 1}^*} F_{\mu\tau},$$

and

$$H_{\mu 1} = \frac{V_{\mu 1}^*}{V_{e 1}^*} H_{e 1}, \quad H_{\tau 1} = \frac{V_{\tau 1}^*}{V_{e 1}^*} H_{e 1}.$$
discuss the implication to the parameter space with a hierarchy scenario.

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Similarly for the inverted hierarchical spectrum, $m_{e_3} = 0$, we obtain

$$F_{e\mu} = \frac{V_{e\mu}}{V_{e3}} F_{\mu\tau}, \quad F_{e\tau} = -\frac{V_{e\tau}}{V_{e3}} F_{\mu\tau},$$

$$H_{\mu1} = \frac{V_{\mu1}}{V_{e3}} H_{e1}, \quad H_{\tau1} = \frac{V_{\tau1}}{V_{e3}} H_{e1} + \frac{m_{e1}}{F_{\mu\tau}} V_{\mu1},$$

$$H_{\mu2} = \frac{V_{\mu2}}{V_{e3}} H_{e2}, \quad H_{\tau2} = \frac{V_{\tau2}}{V_{e3}} H_{e2} + \frac{m_{e2}}{F_{\mu\tau}} V_{\mu2},$$

$$H_{\mu3} = \frac{V_{\mu3}}{V_{e3}} H_{e3}, \quad H_{\tau3} = \frac{V_{\tau3}}{V_{e3}} H_{e3}.$$

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$$H_{\mu2} = \frac{V_{\mu2}}{V_{e3}} H_{e2}, \quad H_{\tau2} = \frac{V_{\tau2}}{V_{e3}} H_{e2} + \frac{m_{e2}}{F_{\mu\tau}} V_{\mu2},$$

$$H_{\mu3} = \frac{V_{\mu3}}{V_{e3}} H_{e3}, \quad H_{\tau3} = \frac{V_{\tau3}}{V_{e3}} H_{e3}.$$

We will use the central values of the most recently global fitting of the neutrino oscillation measurements [53] (see Table II) in our analyses. In the meantime the appearance of the new scalars will provide the extra contributions to the lepton flavor violation processes. We investigate the parameter space of the model in terms of the constraints from leptonic rare decays in the following. Before that it is worth mentioning that the size of parameter $C$ is proportional to the factor $\mu_f a / (m_{s_1}^2 - m_{s_2}^2) \log(m_{s_1}^2 / m_{s_2}^2)$. If there exists a large hierarchy between $m_{s_1}$ and $m_{s_2}$, say, $m_{s_2} = km_{s_1}$ with $k$ being a large factor, $C$ is approximately inverse proportional to $m_{s_2}^2$. However, the positive mass eigenvalue condition $m_{s_1} m_{s_2} > \mu_f a$ will also lead to the result that the upper bound of $C$ is proportional to $1/k$. In general $C$ can not be too small in order to keep the perturbativity of the Yukawa coupling $f$ and $h$. So the largest allowed hierarchy between $m_{s_1}^2$ and $m_{s_2}^2$ is around $k \simeq O(10^3)$. We will also discuss the implication to the parameter space with a hierarchy scenario.

| $\sin^2 \theta_{12}$ | $\sin^2 \theta_{23}$ | $\sin^2 \theta_{13}$ | $\Delta m_{sun}^2$ | $\Delta m_{atm}^2$ |
|----------------------|----------------------|----------------------|----------------------|----------------------|
| 0.30 $\pm$ 0.013    | 0.41$^{+0.037}_{-0.025}$ | 0.023 $\pm$ 0.0023 | $(7.50 \pm 0.185) \times 10^{-5} \text{eV}^2$ | $(2.47^{+0.069}_{-0.067}) \times 10^{-3} \text{eV}^2$ |

TABLE II: Neutrino oscillation data.
The two relevant Yukawa interactions providing the flavor violations in charged lepton sector are given by

\[ L = -2f_{\alpha\beta} \bar{l}_{\alpha} \nu_{L\beta} s_1 - h_{\alpha i} \bar{l}_{\alpha} \nu_{Ri} s_2 + \text{h.c..} \]  

Without loss of generality here \( \nu_{Ri} \) is the mass eigenstate and we absorb the mixing matrix \(-V_R\) into the coupling \( h \). In SM + \( \nu_R \) the one-loop contribution is constrained stringently by the Glashow-Iliopoulos-Maiani (GIM) mechanism. In this model the main contribution is the one-loop diagrams with photon emission attached to the charged scalars \( s_1^+ \) or \( s_2^+ \) in the loop. Currently the latest result from MEG collaboration gives \( B(\mu \rightarrow e\gamma) < 2.4 \times 10^{-12} \). For \( l_\alpha \rightarrow l_\beta \gamma \) the effective Lagrangian can be generally written in the form

\[ L = -\frac{1}{2} \tilde{l}_\beta \sigma_{\mu\nu} (\tilde{A}_R P_R + \tilde{A}_L P_L) l_\alpha F^{\mu\nu}, \]  

where \( \tilde{A}_{L,R} \) for this model is given by

\[ \tilde{A}_R = \frac{(-1)e}{192\pi^2} \left( \sum_\gamma f_{\beta\gamma}^* f_{\alpha\gamma} \frac{m_\alpha}{m_{s1}^2} \right) \quad \text{and} \quad \tilde{A}_L = \frac{(-1)e}{192\pi^2} \left( \sum_i h_{\beta i}^* h_{\alpha i} \frac{m_\alpha}{m_{s2}^2} \right). \]  

Note that the limit \( m_\alpha \ll m_{s1,2} \) have been applied to the above formula. The decay rate of \( l_\alpha \rightarrow l_\beta \gamma \) is \( \Gamma(l_\alpha \rightarrow l_\beta \gamma) = (m_\alpha^3/16\pi)(1 - m_\beta^2/m_\alpha^2)^3(|\tilde{A}_R|^2 + |\tilde{A}_L|^2) \). One can compare it with the lepton three body decay width \( \Gamma(l_\alpha \rightarrow l_\beta \nu_\alpha \bar{\nu}_\beta) = G_F^2 m_\alpha^5/192\pi^3 \). The branching ratio \( \mu \rightarrow e\gamma \) in our model is obtained by

\[ B(\mu \rightarrow e\gamma) \equiv \frac{\Gamma(\mu \rightarrow e\gamma)}{\Gamma(\mu \rightarrow e\nu\bar{\nu})} = \frac{\alpha_e}{768\pi G_F^2} \left( \frac{16}{m_{s1}^4} \sum_\gamma |f_{e\gamma}|^2 |f_{\mu\gamma}|^2 \right) + \frac{\sum_i |h_{ei}|^2 |h_{\mu i}|^2}{m_{s2}^4}. \]  

The results are given in Fig. 2. Here we consider that \( f_{\mu\tau} \) is positive for the normal hierarchy spectrum and negative for the inverted hierarchy spectrum, respectively. The allowed regions are rather small and sensitive to the value of \( C \). For example, the parameter space will shrink to zero if we take \( C = 0.8 \times 10^{-5} \) with the same inputs.

\[ B. \, \mu \rightarrow e\gamma \]

The one loop diagrams to \( \mu \rightarrow e \) conversion include photon penguin diagrams, Z penguin diagrams, and box diagrams. Again the contribution from the diagrams involving W boson exchange is suppressed by GIM mechanism and the leading contributions come from the...
FIG. 2: The allowed region in $h_{e1} - f_{\mu\tau}$ plane in (a) the normal hierarchy, and (b) the inverted hierarchy. Different curves are generated with different $C$ (blue: $C = 10^{-4}$, green: $C = 1.1 \times 10^{-5}$, red: $C = 10^{-5}$). The other parameters are taken as $m_{s1} = m_{s2} = 500\text{GeV}$, $h_{e2} = h_{e3} = 0.5$.

penguin diagrams with the charged scalars $s_1^+$ and $s_2^+$ in the loop. In contrast, the leading $Z$ penguin contribution is suppressed by the light charged scalars. Therefore only photon penguin needs to be taken into account. The corresponding effective Lagrangian is given by

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} \frac{s_W^2}{36\pi^2} m_W^2 \bar{e} \gamma^\mu (\sum_\alpha 4 f_{e\alpha}^* f_{e\mu} \frac{1}{m_{s_1}^2} P_L + \sum_i h_{ei} h_{\mu i} \frac{1}{m_{s_2}^2} P_R) \mu \sum_q Q_q \bar{q} \gamma^\mu q.$$ (19)

In the above we used the shorthand notation $s_W \equiv \sin \theta_W$. The $\mu - e$ conversion with nucleon in atom has been calculated in details in Ref. [55], and we will adopt their notation in what follows. The general interactions associated with $\mu - e$ conversion are written as

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F e}{\sqrt{2}} [m_\mu \bar{e} \sigma^{\mu\nu} (A_R^P P_R + A_L^P P_L) \mu F_{\mu\nu} + \text{h.c.}]$$

$$-\frac{G_F}{\sqrt{2}} [\bar{e} (g_{LS}(q) P_R + g_{RS}(q) P_L) \mu \bar{q} q + \bar{e} (g_{LP}(q) P_R + g_{RP}(q) P_L) \mu \bar{q} \gamma_5 q + \text{h.c.}]$$

$$-\frac{G_F}{\sqrt{2}} [\bar{e} (g_{LV}(q) \gamma^\mu P_L + g_{RV}(q) \gamma^\mu P_R) \mu \bar{q} \gamma_\mu q + \bar{e} (g_{LA}(q) \gamma^\mu P_L + g_{RA}(q) \gamma^\mu P_R) \mu \bar{q} \gamma_\mu \gamma_5 q + \text{h.c.}]$$

$$-\frac{G_F}{\sqrt{2}} [\frac{1}{2} \bar{e} (g_{LT}(q) \sigma^{\mu\nu} P_R + g_{RT}(q) \sigma^{\mu\nu} P_L) \mu \bar{q} \sigma_{\mu\nu} q + \text{h.c.}],$$ (20)

where $A_{R,L}$ is related to the dipole interaction with photon, and $g_{S,P,V,A,T}$ indicate scalar, pseudoscalar, vector, axial vector and tensor couplings, respectively. Comparing the above
formula with Eq. [19], we have

\[ g_{LV}^{(q)} = \frac{Q_q s_W^2 v}{36\pi^2 m_W^2} (\sum_{\alpha} 4 f_{e\alpha} f_{\mu\alpha} \frac{1}{m_{s1}^2}), \quad g_{RV}^{(q)} = \frac{Q_q s_W^2 v}{36\pi^2 m_W^2} (\sum_{i} h^*_{e\mu} h_{\mu l} \frac{1}{m_{s2}^2}) \] (21a)

\[ A_R = \frac{(-1)}{192\pi^2 g_2^2} (\sum_{\alpha} 4 f_{e\alpha} f_{\mu\alpha} m_W^2 m_{s1}^2), \quad A_L = \frac{(-1)}{192\pi^2 g_2^2} (\sum_{i} h^*_{e\mu} h_{\mu l} m_W^2 m_{s2}^2) \] (21b)

and other couplings vanish. The rate of \( \mu - e \) conversion with nucleons in atom \( A \) is usually normalized to the rate of muon capture by \( A \). The conversion-to-capture ratio can be derived in the form

\[ B_{\mu \rightarrow e}^A = \frac{2G_F m_\mu^5}{\Gamma_{\text{capt}}} (|e A_R D + \bar{g}_{LV}^{(p)} V^{(p)} + \bar{g}_{LV}^{(n)} V^{(n)}|^2 + |e A_L D + \bar{g}_{RV}^{(p)} V^{(p)} + \bar{g}_{RV}^{(n)} V^{(n)}|^2) \] (22)

with \( \bar{g}_{LR}^{(u)} = 2 g_{LR}^{(u)} + g_{LR}^{(d)} \) and \( \bar{g}_{LR}^{(n)} = g_{LR}^{(u)} + 2 g_{LR}^{(d)} \). \( D \) and \( V^{(p,n)} \) are overlapped functions which can be found in Ref. [55]. The constraints from experimental results for \( \mu - e \) conversion in different nuclei [56–60] are weaker than what’s given by \( \mu \rightarrow e\gamma \).

C. \( \mu \rightarrow 3e \)

Penguin diagrams and Box diagrams contribute to this process. The experimental upper bounds is \( B(\mu \rightarrow eee) < 10^{-12} \) [61]. SM with right-handed neutrino singlets contribution to this processes is suppressed by the neutrino mass. The corresponding one loop diagrams in this model for \( \mu \rightarrow 3e \) are similar to those for \( \mu - e \) conversion, with the quarks replaced by electrons. In general the effective Lagrangian for \( \mu \rightarrow e\bar{e}e \) is

\[ \mathcal{L}(\mu \rightarrow e\bar{e}e) = -\frac{4G_F}{\sqrt{2}} \bar{e}_e \gamma_\mu \frac{q^e}{q^e} \bar{e}_e \sigma^{\mu\nu} (8\pi\alpha_e m_\mu) (A_R P_R + A_L P_L) \mu + \bar{e}_e \gamma^\mu (a_L P_L + a_R P_R) e e_\mu P_L \mu + \bar{e}_e \gamma^\mu (b_L P_L + b_R P_R) e \bar{e}_e \gamma_\mu P_R \mu \] (23)

The branching ratio can be calculated as

\[ B(\mu \rightarrow e\bar{e}e) \simeq (|a_R|^2 + |b_L|^2) + 2(|a_L|^2 + |b_R|^2) - 32\pi\alpha_e \text{Re}(A_R(a_R + 2a_L) + A_L(b_L + 2b_R)) + 256\pi^2 \alpha_e^2 (|A_R|^2 + |A_L|^2) \left( 4 \ln \frac{m_\mu}{m_e} - \frac{11}{2} \right) \] (24)

where the parameters \( a_{L,R} \) and \( b_{L,R} \) are given by

\[ a_{L,R} = -\frac{s_W^2}{144\pi^2 m_W^2} (\sum_{\alpha} 4 f_{e\alpha} f_{\mu\alpha} \frac{1}{m_{s1}^2}) \quad \text{and} \quad b_{L,R} = -\frac{s_W^2}{144\pi^2 m_W^2} (\sum_{i} h^*_{e\mu} h_{\mu l} \frac{1}{m_{s2}^2}) \] (25)
FIG. 3: The allowed region in $h_{e1} - f_{\mu\tau}$ plane for the case of $m_{s1} = 20 m_{s2}$ in normal hierarchy and inverted hierarchy neutrino mass spectrum respectively. Blue line corresponds to $C = 10^{-4}$ and Red line represents $C = 10^{-5}$.

from the photon penguin diagrams, while for the box diagrams the leading order of $a_{R}^{\text{box}}$ and $b_{L}^{\text{box}}$ vanishing, we have

\begin{equation}
    a_{L}^{\text{box}} = \frac{m_{W}^{2}}{32 \pi^{2} g_{2}^{2} m_{s1}^{2}} \sum_{\alpha} 4 f_{\mu\alpha} f_{e\alpha}^{*} \sum_{\alpha'} 4 f_{e\alpha'}^{*} f_{e\alpha'}
\end{equation}

and

\begin{equation}
    b_{R}^{\text{box}} = \frac{m_{W}^{2}}{32 \pi^{2} g_{2}^{2} m_{s2}^{2}} | \sum_{i} h_{\mu i} h_{e i}^{*} | \sum_{i'} h_{e i'}^{*} h_{e i'} |
\end{equation}

respectively. As shown in Fig. 2 the Yukawa coupling $f$ is in the range of $O(10^{-2})$ to $O(10^{-3})$; we can safely ignore the box diagram contributions in $\mu \rightarrow 3e$ decay. Therefore, the parameter space is looser to those of $\mu \rightarrow e\gamma$.

In the limit of mass hierarchy between $m_{s1}$ and $m_{s2}$, that is, $m_{s1} > m_{s2}$ or $m_{s2} > m_{s1}$ cases, the parameter space for Yukawa couplings $h$ and $f$ in both normal hierarchy and inverted hierarchy neutrino mass spectrum respectively are shown in Fig. 3 and Fig. 4. Here we take $m_{s1} = 20 m_{s2}$ and $m_{s2} = 5 m_{s1}$ as the reference points. In these cases, we are only able to give a severe constraint on one of the couplings, $h$ or $f$, and we illustrate our results by taking 500 GeV mass scale to the lighter scalar field.
FIG. 4: The allowed region in $h_{\epsilon 1} - f_{\mu \tau}$ plane for the case of $m_{s_2} = 5m_{s_1}$ in normal hierarchy and inverted hierarchy neutrino mass spectrum respectively. Blue line corresponds to $C = 10^{-4}$ and Red line represents $C = 10^{-5}$.

IV. PHENOMENOLOGY

A. electromagnetic moments of leptons

1. muon $g - 2$

The anomalous magnetic moment of the muon is an observable as a precision test to the SM. The current experimental results \cite{62,63} reported that the muon $g - 2$ deviation from the SM prediction is $a_\mu^{(\text{exp})} - a_\mu^{(\text{SM})} = (287 \pm 63 \pm 49) \times 10^{-11}$, which indicates some new physics contribution. Since our scenario is essentially a two Higgs doublet model (2HDM) plus two singly charged scalars $s_1^\pm$ and $s_2^\pm$. The anomalous $g - 2$ of muon in this model is given by

$$\Delta a_\mu = -\frac{m_\mu^2}{96\pi^2} \left[ 4 \sum_\alpha |f_{\mu \alpha}|^2 \frac{1}{m_{s_1}^2} + \sum_i |h_{\mu i}|^2 \frac{1}{m_{s_2}^2} + \frac{m_\mu^2}{v^2} \left( \frac{14c_\beta^2 - a}{m_h^2 \cos^2 \beta} + \frac{14s_\beta^2 - a}{m_H^2 \cos^2 \beta} - \frac{22 \tan^2 \beta}{m_A^2} - \frac{\tan^2 \beta}{m_{H^+}^2} \right) \right], \quad (27)$$

where $h$ and $H$ are scalar particles with $h$ being the SM-like Higgs, as well as the pseudoscalar field $A$ and the charged scalars $H^\pm$. $\alpha$, $\beta$ are the mixing angles ($s_{\beta - \alpha} \equiv \sin (\beta - \alpha)$ and $c_{\beta - \alpha} \equiv \cos (\beta - \alpha)$) defined in the usual 2HDM (see e.g. \cite{64,65} and references within). Comparing with the current experimental result, our model gives a rather small contribution.
2. magnetic moments of neutrinos

Neutrinos can have magnetic moment when they are massive. The present upper bound of neutrino magnetic moment from experiments is $\mu_{\nu} < 3.2 \times 10^{-11} \mu_B$ \cite{66}. The ordinary leading order contribution in SM including right-handed Dirac neutrino is the exchange of $W$, which is around $(3.2 \times 10^{-19} \mu_B)(m_{\nu}/eV)$ \cite{67} with the Bohr magneton $\mu_B \equiv e/2m_e$. In this model, the main new contribution comes from the mixing of $s_{1,2}^+$ in the loop since the same diagram also generates neutrino masses, given by

$$|\mu_{\nu}^s| = 2m_\nu e (\log \frac{m_{s_2}^2}{m_{s_1}^2})^{-1}\left(\frac{1}{m_{s_1}^2} - \frac{1}{m_{s_2}^2}\right) \approx 10^{-19} \mu_B.$$  \hspace{1cm} (28)

Note that it only depends on the neutrino and scalar masses, and this contribution is comparable with that associated with $W$ exchange. It is understandable that without imposing a symmetry \cite{68} or employing a spin suppression mechanism to $m_\nu$ \cite{69}, the generic size of the Dirac neutrino magnetic moment can not exceed $10^{-14} \mu_B$ \cite{70}. The contributions from other charged scalars such as $H^+, s_1^+, s_2^+$ in the loop are less than $\mathcal{O}(10^{-25}) \mu_B$ as a result that they do not contribute to neutrino masses.

3. EDM of leptons

Since there is a physical CP phase among the Yukawa couplings $f$ and $h$, in general, we have the new contributions to the electric dipole moment of charged leptons. The exchange of singly charged scalars gives the EDM to the charged leptons at one-loop level as

$$d_\ell = -\frac{e}{(4\pi)^2} \sum_i \text{Im}(2h_i^*(fV)_{ii}) \frac{\mu_{\nu}}{m_\ell^2} \left[\frac{1}{m_{s_1}^2} B\left(\frac{m_{s_1}^2}{m_{s_2}^2}\right) - \frac{1}{m_{s_2}^2} B\left(\frac{m_{s_2}^2}{m_{s_1}^2}\right)\right],$$  \hspace{1cm} (29)

with the function $B(x)$ defined by

$$B(x) = \frac{\sqrt{x}}{2(1-x)^2} \left(1 + x + \frac{2x \ln(x)}{1-x}\right).$$  \hspace{1cm} (30)

Therefore we conclude that the extra contribution is smaller than $\mathcal{O}(\frac{m_{s_1}^2}{m_\nu m_{s_2}^2})$, which means $|d_\ell| \lesssim 10^{-35} e\text{ cm}$. Similarly, the neutrino EDM generated by $s_{1,2}^+$ mixing in the loop gives $|d_\nu| \lesssim 10^{-26} e\text{ cm}$. Though the new contributions to $d_\ell$ and $d_\nu$ are nonzero, both of them are unobservably small.
B. dark matter

It is known that the axion field can be a dark matter candidate constituting a significant fraction of energy density in our universe. For the purpose of completion we briefly review some aspects of this scenario in this subsection. The properties of the invisible axion are determined by the breaking scale $f_a$ where its mass and the interactions are inverse proportional to $f_a$. Hence the invisible axion is a very light, very weakly interacted and very long-lived particle. Axions with mass in the range of $10^{-5} - 10^{-6}$ eV were produced during the QCD phase transition with the average momentum of order the Hubble expansion rate ($\sim 3 \times 10^{-9}$ eV) at this epoch and hence are cold dark matter (CDM). Their number density is provided by

$$\Omega_a \simeq \frac{1}{2} \left( \frac{0.6 \times 10^{-5} \text{ eV}}{m_a} \right) \tilde{\sigma} \left( \frac{0.7}{h} \right)^2,$$

where $h$ is the current Hubble expansion rate in units of $100 \text{kms}^{-1}\text{Mpc}^{-1}$. Here we assume the ratio of the axion number density to the entropy density is constant since produced and the contribution from topological defects decay is negligible. Recently it was pointed out that the CDM axions would form a Bose-Einstein condensate due to their gravitational interactions [71]. Furthermore the rethermalization process is so fast that the lowest energy state of the degenerate axion gas consists of a nonzero angular momentum. As a result a "caustic ring" structure may form in the inner galactic halo [72–74]. The feature would make the axion a different dark matter from other CDM candidates, and we refer readers to the references [71–74] for details.

C. $h \rightarrow \gamma\gamma$

We closing our discussion on phenomenology with investigating the LHC Higgs results. Both ATLAS [75] and CMS [76] have announced the discovery of a new boson at a mass of 125 GeV which is consistent with the SM Higgs boson via the combined analyses of the $\gamma\gamma$ and $ZZ$ channels. However, the precise values of both production cross sections and decay branch ratios of the new resonance need to be measured to compare with those predictions from the SM. It was pointed out that the branching ratio of Higgs decay into two photons has excess about $1.56 \pm 0.43$ and $1.9 \pm 0.5$ times than the SM prediction in both CMS [76] and ATLAS [75] collaborations data in 2012. A updated results can be found in
Although the deviation is still within the SM expectations at 2σ level, one may consider whether there are new physics effects (see e.g. 65, 79–88 and references therein) In particular, new scalars particles have been widely treated as possible sources. We study the implications of Higgs to di-phonton and Higgs to Z-photon decay channels in our model. For simplicity we just consider one singly charged scalar \( s^\pm \) and omit the subscript in this subsection. The SM Higgs production cross section is modified by the additional doublet scalar in our scenario [65],

\[
\sigma_0 = \frac{G_F \alpha^2}{128\sqrt{2} \pi} \left| \frac{1}{2} (s_\alpha + \frac{c_\alpha}{\tan \beta}) A_{1/2}(\tau_t) + \frac{1}{2} (s_\alpha - c_\alpha \tan \beta) A_{1/2}(\tau_b) \right|^2.
\]

(32)

Notice that the bottom quark contribution is not negligible due the enhancement of large \( \tan \beta \) and \( A_{1/2}(\tau) = 2[\tau + (\tau - 1)f(\tau)]\tau^{-2} \) with \( \tau_i = \frac{M^2}{4M^2} \). The function \( f(\tau) \) is defined by

\[
f(\tau) = \begin{cases} 
(\sin^{-1} \sqrt{\tau})^2, & \tau \leq 1 \\
-\frac{1}{4} \log \left[ \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi \right]^2, & \tau > 1.
\end{cases}
\]

(33)

In type-II 2HDM the decay rate of \( h \to \gamma\gamma \) is given by

\[
\Gamma_{\gamma\gamma} = \frac{G_F \alpha^2 m_h^3}{128\sqrt{2} \pi^3} \left| (s_\alpha + \frac{c_\alpha}{\tan \beta})^4 \frac{1}{3} A_{1/2}(\tau_t) + (s_\alpha - c_\alpha \tan \beta) \frac{1}{3} A_{1/2}(\tau_b) \right|^2 + s_\alpha A_1(\tau_W) + \lambda' A_0(\tau_{H^+}) + \lambda A_0(\tau_{s^+})^2,
\]

(34)

where \( A_1(\tau) = -[2\tau^2 + 3\tau + 3(2\tau - 1)f(\tau)]\tau^{-2} \), and \( A_0(\tau) = -[\tau - f(\tau)]\tau^{-2} \). The coefficients \( \lambda \) and \( \lambda' \) are defined as \( \lambda = \nu H_{s^+s^-}/(2m_s^2) \) and \( \lambda' = \nu H_{H^+H^-}/(2m_{H^\pm}^2) \), where \( \mu_{H^+s^-} \) and \( \mu_{H^+H^-} \) are the related trilinear couplings in the Higgs potential. One can see that the effects of doublet scalar charged component \( H^+ \) and the singly charged singlet \( s^\pm \) are indistinguishable. The reason is the lack of knowledge of scalar potential. For \( h \to Z\gamma \) channel the decay width is written as [89]

\[
\Gamma_{Z\gamma} = \frac{G_F^2 m_W^2 \alpha m_h^3}{64 \pi^4} (1 - m_{Z^\pm}^2/m_{H^\pm}^2)^3 \times \left| (s_\alpha + \frac{c_\alpha}{\tan \beta}) \frac{1}{2} - \frac{4}{3} s_W^2 A_{1/2}(\eta_h, \kappa_\ell) + (s_\alpha - c_\alpha \tan \beta) \frac{1}{4} - \frac{1}{3} s_W^2) A_{1/2}(\eta_b, \kappa_b) \right|^2 + s_\alpha c_W^2 A_{1}(\eta_W, \kappa_W) + \left( \frac{1}{2} - s_W^2 \right) \lambda' A_0(\eta_{H^+, \kappa_{H^+}}) + \left( -s_W^2 \right) \lambda A_0(\eta_{s^+, \kappa_{s^+}}) \right|^2,
\]

(35)

where \( A_{1/2}(\eta, \kappa) = -4(I_1(\eta, \kappa) - I_2(\eta, \kappa)) \), \( A_1(\eta, \kappa) = -4(2 - \frac{4}{7}) I_2(\eta, \kappa) - [(1 + \frac{2}{7}) (\frac{4}{7} - 1) - (5 + \frac{2}{7})] I_1(\eta, \kappa) \), and \( A_0(\eta, \kappa) = 2I_1(\eta, \kappa) \) with the parameters \( \eta = \frac{4M_{H^\pm}^2}{M_h^2} \) and \( \kappa = \frac{4M_{H^\pm}^2}{M_Z^2} \). Functions
FIG. 5: Solutions of $s^+$ effects in terms of $\cos \alpha$ by assuming $A_0(\eta, \kappa)/A_0(\tau) = 1.00$ with $R_{\gamma\gamma} = \frac{\sigma_{\gamma\gamma}}{\sigma_{\gamma\gamma}^{SM}} = 1.5$ and $R_{Z\gamma} = \frac{\sigma_{Z\gamma}}{\sigma_{Z\gamma}^{SM}} = 1$. $\sigma_{ii}$ and $\sigma_{ii}^{SM}$ are the production cross sections for the Higgs to $ii$ channel in our model and SM respectively.

$I_1(\eta, \kappa)$ and $I_2(\eta, \kappa)$ are given as

$$I_1(\eta, \kappa) = \frac{\eta \kappa}{2(\eta - \kappa)} + \frac{\eta^2 \kappa^2}{2(\eta - \kappa)^2} \left[ f(\eta^{-1}) - f(\kappa^{-1}) \right] + \frac{\eta^2 \kappa}{(\eta - \kappa)^2} \left[ g(\eta^{-1}) - g(\kappa^{-1}) \right]$$

and

$$I_2(\eta, \kappa) = -\frac{\eta \kappa}{2(\eta - \kappa)} \left[ f(\eta^{-1}) - f(\kappa^{-1}) \right]$$

with

$$g(x) = \begin{cases} \sqrt{x^{-1}} - 1 \sin^{-1} \sqrt{x}, & x \leq 1 \\ \frac{1-x^{-1}}{2} \left[ \log \frac{1+\sqrt{1-x^{-1}}}{1-\sqrt{1-x^{-1}}} - i\pi \right], & x > 1 \end{cases}$$

respectively. Again we see from the Eq. (35) that the singly charged singlet effect is hidden in the 2HDM. In order to extract the information of $s^+$ from the decays we found the ratio of $A_0(\eta, \kappa)/A_0(\tau)$ lies in the range of $1.00 - 1.07$ with charged scalar masses above 100 GeV. We then take the ratio as a constant and subtract the $H^+$ contributions in $\Gamma_{\gamma\gamma}$ and $\Gamma_{Z\gamma}$. Other SM Higgs decaying channels are calculated from HDECAY [90]. The results of the $s^+$ effects in terms of the parameter $\cos \alpha$ with $\tan \beta = 10, 50$ are shown in Fig. 5. Here we take the values $R_{\gamma\gamma} = \frac{\sigma_{\gamma\gamma}}{\sigma_{\gamma\gamma}^{SM}} = 1.5$ and $R_{Z\gamma} = \frac{\sigma_{Z\gamma}}{\sigma_{Z\gamma}^{SM}} = 1$ with $\sigma_{ii}$ and $\sigma_{ii}^{SM}$ being the production cross sections for the Higgs to $ii$ channel in our model and SM respectively. Since the new physics effects can have both constructive and destructive interferences with the SM $W^\pm W^\pm$ amplitude, there are four lines exist in Fig. 5. The results of $s^+$ effects with different values of $R_{Z\gamma}$ and the ratio of $A_0(\eta, \kappa)/A_0(\tau)$ for the fixed $R_{\gamma\gamma} = 1.5$ are shown in
FIG. 6: Effects of singly charged singlet scalar with values of $R_{Z\gamma} = 0.5, 1, 2$ as indicated on each curves, and $A_0(\eta, \kappa)/A_0(\tau) = 1.07$ (solid lines), 1.00 (dashed lines) respectively for the fixed value $R_{\gamma\gamma} = 1.5$. Blue lines correspond to $\tan \beta = 10$ and Red lines represent $\tan \beta = 50$. Note that each figure corresponds to one of the lines exhibited in Fig. 5.

Fig. 6 Here $R_{Z\gamma} = 0.5, 1, 2$ and $A_0(\eta, \kappa)/A_0(\tau)$ is taken to be 1.07 and 1.00 respectively. It shows that taking the ratio of $A_0(\eta, \kappa)/A_0(\tau)$ as a constant is a good approximation in the regions of parameters we are interested in and is useful for the deviation of $\Gamma_{Z\gamma}$ from the SM prediction for the singly charged singlet scalar $s^+$. Finally, it is worth to note that the charged singlet $s_1^+$ and $s_2^+$ can be produced by the quark annihilation through gauge boson $\gamma$ and $Z^0$ mediating. This process could have the final states including two charged leptons, the same as that of $pp \rightarrow h \rightarrow WW^*$, for which final states of $e$ or $\mu$ are tagged by LHC. The related signal strength observed from ATLAS is $1.3 \pm 0.5$ [91], and the $1\sigma$ deviation of it corresponds to around 20pb. This deviation can be regards as the allowed space for new contribution beyond SM. The singly charged scalar production cross section with parton
distribution function given from CTEQ6 [92] is also around 20pb with \( m_s \simeq 150 \text{GeV} \), which can be regarded as the lower bounds on \( m_s \) if we assume \( \text{Br}(s \to e(\mu)\nu) \simeq 100\% \). Finally, we estimate the background contribution for \( h \to WW^* \to e\nu\mu\nu \) channel from \( s^\pm s^\mp \) production. Both singly charged scalars are off-shell and we found that the partial decay width for the center of mass in one of virtual scalar to be around \( m_W \) is negligible compared to the SM prediction.

V. DISCUSSIONS AND CONCLUSIONS

We investigate a model that neutrinos are Dirac fermions and their masses are generated from the Peccei-Quinn symmetry breaking at one-loop level. As a result the neutrino mass is related to \( \Lambda_{\text{QCD}} \) and axion is appeared to be a good candidate of dark matter in explaining the missing energy density of our universe. Leptonic rare decays constrain the model parameters severely, and therefore, the model can be tested in the near future. We also studied the implications of the new scalars to the electromagnetic moments of leptons and recent Higgs signals at LHC, specifically the \( h \to \gamma\gamma \) and \( h \to Z\gamma \) decays. Finally we would like to make a brief comment on the Majorana extension of our scenario. Lepton number symmetry is one of the key to understand the underlying neutrino physics. The smoking gun signals to resolve the question are \( 0\nu\beta\beta \) decays and some lepton number violating processes at LHC. Without the direct observations Majorana neutrinos and Dirac neutrinos are equally good in many aspects to describe the phenomena such as neutrino oscillations, leptogenesis, and Big Bang Nucleosynthesis, etc. Although we discuss the Dirac neutrino in this model and argue that the Dirac neutrino mass is originated from the Peccei-Quinn symmetry breaking which is well-motivated in QCD field theory, a Majorana masses of the right-handed neutrinos, in general, can be formed without violating any principle except the lepton number symmetry. The main modifications are to include some terms in the scalar potential such as \( H_1H_2s_1^\dagger \) and \( H_1H_2as_2^\dagger \). Therefore, our discussions can be easily embedded the scenario of Majorana neutrino and diagonalize the neutrino mass matrix via the seesaw mechanism.
Appendix A: Scalar Mass Spectrum

We briefly analyze the scalar potential given in Eq. (2) and their particle spectrum in this appendix. If the neutral components of $H_1 = (h_1^+, (R_1 + i A_1)/\sqrt{2})^T$, $H_2 = (h_2^+, (R_2 + i A_2)/\sqrt{2})^T$ and $a = (R_a + i A_a)/\sqrt{2}$ acquire the vacuum expectation values (VEVs) $v_1$, $v_2$, and $v_a$, respectively, and the related tadpole conditions can be expressed as

\[-\mu_1^2 = -[\lambda_1 v_1^2 + \frac{1}{2}(\lambda_3 + \lambda_4)v_2^2 + \frac{1}{2}(d_a + h_5\frac{v_2}{v_1})v_a^2],\]

\[-\mu_2^2 = -[\lambda_2 v_2^2 + \frac{1}{2}(\lambda_3 + \lambda_4)v_1^2 + \frac{1}{2}(g_a + h_5\frac{v_1}{v_2})v_a^2],\]

\[-\mu_a^2 = -[h_a v_a^2 + h_5 v_1 v_2 + \frac{1}{2}(d_a v_1^2 + g_a v_2^2)].\]  

(A1)

(A2)

$v_{1,2}$ are responsible for the electroweak symmetry breaking and $v_a \approx 10^{12}\text{GeV}$ is aimed for the PQ symmetry breaking scale. Notice that the sizes of $\mu_1$, $\mu_2$ and $\mu_a$ are not required to be in electroweak scale, while the TeV-scale charged singlets $s_{1,2}$ will bound the couplings $h_{a1,2}$ to be around $v^2/v_a^2$ and the value of $\mu$ to be less than $(10^3\text{GeV})^2/v_a$. Furthermore, the trilinear couplings $d_a$ and $g_a$ for the scalar fields and the axion will be constrained to be larger than $v/v_a$. The $3 \times 3$ scalar neutral mass matrix elements $M_{ij}$ in the basis \{R$_1$, R$_2$, R$_a$\} are given as

\[M_{11}^2 = 2\lambda_1(\cos \beta)^2 v^2 - \frac{h_5}{2} \tan \beta v_a^2,\]

\[M_{22}^2 = 2\lambda_2(\sin \beta)^2 v^2 - \frac{h_5}{2} \cot \beta v_a^2,\]

\[M_{33}^2 = 2h_a v_a^2,\]

\[M_{12}^2 = \frac{1}{2}[(\lambda_3 + \lambda_4)(\sin 2\beta)v^2 + h_5 v_a^2],\]

\[M_{23}^2 = (g_a \sin \beta + h_5 \cos \beta)v v_a,\]

\[M_{13}^2 = (f_a \cos \beta + h_5 \sin \beta)v v_a.\]  

(A3)

(A4)

(A5)

(A6)

(A7)

(A8)

The relations $\tan \beta = v_2/v_1$ and $v = (v_1^2 + v_2^2)^{1/2}$ are used in the formulae. The mixing angle between $R_{1,2}$ and mass eigenstates $H, H_t$ is defined as $\tan(\beta - \alpha)$, the same as the definition in [65]. The discovery of 126 GeV scalar implies that the magnitude of $h_5$ should be of the order $v^2/v_a^2$. Similarly, the masses of pseudo-scalar and charged Higgs bosons are given by

\[m_A^2 = -h_5(\sin 2\beta v^2 + \frac{1}{\sin 2\beta} v_a^2),\]

\[m_{H^\pm}^2 = -\frac{\lambda_4}{2} v^2 - h_5 \frac{1}{\sin 2\beta} v_a^2.\]  

(A9)
From the above mass formulae we get $h_5 < 0$, and $m_A^2 - m_{H^\pm}^2 = (\lambda_4/2)v^2$ by applying $v_a \gg v$. Finally, the charged particles $s_1$ and $s_2$ will not mix with $H^\pm$. The mass matrix elements $M_{sij}$ are given by

\begin{align*}
M_{s_{11}}^2 &= \mu_{s_1}^2 + h_{a1}v_a^2 + \frac{1}{2}[f_1(\cos \beta)^2 + g_1(\sin \beta)^2]v^2, \\
M_{s_{22}}^2 &= \mu_{s_2}^2 + h_{a2}v_a^2 + \frac{1}{2}[f_2(\cos \beta)^2 + g_2(\sin \beta)^2]v^2, \\
M_{s_{12}}^2 &= \frac{\mu v_a}{\sqrt{2}}.
\end{align*}

Notice that $h_{a1, a2}$ are chosen in the order of $(v/v_a)^2$ to ensure the electroweak scale $m_{s_1}$ and $m_{s_2}$. For simplicity if we take the mixing terms $M_{12}^2, M_{23}^2,$ and $M_{13}^2$ to be vanished, we obtain

$$\lambda_3 = -\lambda_4 - \frac{h_5 v_a^2}{\sin 2\beta v^2} \quad \text{and} \quad g_a = d_a = 0.$$  \hspace{0.5em} (A13)

Then $\lambda_3$ is also fixed once the values $\tan \beta, h_5,$ and $\lambda_4$ are given, and at the moment $\sin \alpha = \sin \beta$. The trilinear couplings of SM Higgs $H$ with charged scalars $H^+, s_1,$ and $s_2$ respectively are given by

\begin{align*}
\mu_{HH^+H^-} &= \sin \beta[2\lambda_2(\cos \beta)^2 + \lambda_3(\sin \beta)^2 - \lambda_4(\cos \beta)^2]v, \\
\mu_{Hs_1^+s_1^-} &= g_1 v \sin \beta, \quad \mu_{Hs_2^+s_2^-} = g_2 v \sin \beta.
\end{align*}

(A14)

As an illustration, taking $\tan \beta = 10, \lambda_1 = 0.5, \lambda_2 = 0.125, \lambda_4 = 0, \lambda_5 = -0.3$ as input values, then the related scalar masses are $m_h = 126 \text{ GeV}, m_H \simeq m_A = m_{H^\pm} = 303 \text{ GeV}$. On the other hand, for $\tan \beta = 50, \lambda_1 = \lambda_2 = 0.13, \lambda_4 = 0, \lambda_5 = -0.1$, we have $m_H \simeq m_A = m_{H^\pm} = 389 \text{ GeV}$. Both cases satisfy the experimental constraints. In summary, having the scalar with 126 GeV mass is insensitive to the value of $h_5$ due to the $\cos \beta$ suppression. The current constraint on neutral scalar mass would give $|h_5(v_a^2/v^2)| \gtrsim 0.3$ for $\tan \beta = 10$ and $|h_5(v_a^2/v^2)| \gtrsim 0.06$ for $\tan \beta = 50$, by setting $\lambda_4 = 0$.

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21
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