Validation studies of gyrokinetic ITG and TEM turbulence simulations in a JT-60U tokamak using multiple flux matching

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Abstract

Quantitative validation studies of flux-tube gyrokinetic Vlasov simulations on ion and electron heat transport are carried out for the JT-60U tokamak experiment. The ion temperature gradient (ITG) and/or trapped electron modes (TEM) driven turbulent transport and zonal flow generations are investigated for an L-mode plasma in the local turbulence limit with a sufficiently small normalized ion thermal gyroradius and weak mean radial electric fields. Nonlinear turbulence simulations by the GKV code successfully reproduce radial profiles of the ion and electron energy fluxes in the core region. The numerical results show that the TEM-driven zonal flow generation in the outer region is more significant than that in the core region with ITG- and ITG–TEM-dominated turbulence, leading to moderate transport shortfall of the ion energy flux. Error levels in the prediction of the ion and electron temperature gradient profiles in the core region are estimated as less than ±30\%, based on a multiple flux matching technique, where the simulated ion and electron energy fluxes are simultaneously matched to the experimental values.

Keywords: gyrokinetic simulation, ITG turbulence, TEM turbulence, validation

(Some figures may appear in colour only in the online journal)
$\rho' < 1/500$, and the evaluation of their prediction capability through quantitative comparisons with the existing plasma experiments is a crucial requirement.

In the last decade, validation studies of gyrokinetic simulation codes have been actively carried out for various tokamaks (e.g. DIII-D [9–12], ASDEX Upgrade [13], Alcator C-MOD [14], etc) and helical (e.g. LHD [15, 16]) devices. In the tokamak validation studies, a significant underprediction of the heat flux, which is the so-called ‘transport shortfall’ [10], has been identified in the outer region around $\rho = r/a \geq 0.7$, and the numerical and physical effects on the transport shortfall are investigated for the DIII-D L-mode plasma as a representative case [9, 11, 17]. Particularly, Görler et al has recently pointed out that the significant ion transport shortfall is resolved within the measurement errors of the ion temperature gradient, based on the flux matching technique [18], taking into account the stiff dependence of the turbulent heat flux on the ion temperature gradient [11]. Also, the strong impact of the mean radial electric field shearing on the simulated ion heat flux has been confirmed.

On the other hand, characteristics of the ITG and/or TEM driven zonal flows, which depend on the radial position in the L-mode plasma, and the relation with the transport shortfall, have not been investigated in earlier works. Indeed, since the transport suppression by zonal flows is expected to be more significant in future large tokamaks such as ITER than the mean radial electric field shear, which is roughly proportional to $\rho'$, gyrokinetic-simulation-based studies of the zonal flow properties are indispensable for improving the prediction capability. As a fundamental study on the nonlinear interactions of zonal flows and turbulence, the gyrokinetic entropy transfer analyses have been carried out for the ITG, TEM, and electron temperature gradient (ETG) turbulence [19–22].

In this paper, the first quantitative validation study on the ion and electron heat transport, including the zonal flow analysis for a JT-60U tokamak plasma, is presented. Here, the JT-60U L-mode discharge is carefully chosen such that the local limit condition of $\rho' = 1/300$ is well satisfied, i.e. the mean radial electric field shearing rate is negligibly small in comparison to the linear-mode growth rate. The nonlinear ITG and/or TEM turbulence simulations under the experimental conditions are performed by using an electromagnetic gyrokinetic Vlasov code GKV [23, 24]. Then, the experimental equilibrium profiles, power and particle balances are provided by an integrated tokamak transport solver TOPICS [25, 26]. Furthermore, the conventional ion heat flux matched by adjusting a single parameter of the ion temperature gradient [11] is extended to a multiple flux matching associated with the nonlinear dependence on the ion and electron temperature gradients, where the simulated ion and electron energy fluxes are simultaneously matched to the experimental values. Then, we evaluate the accuracy of the temperature-gradient profile prediction in the core region.

The rest of the paper is organized as follows. The calculation model in GKV under the experimental condition produced by TOPICS is presented in section 2. Then, plasma parameters, equilibrium profiles, and the linear instabilities of the present L-mode plasma are shown in section 3. In section 4, nonlinear simulation results on the ITG and/or TEM driven turbulent transport and zonal flows at several radial positions are presented. The quantitative comparisons with the experimental results and the multiple flux matching are also discussed. Finally, a summary of this paper is given in section 5.

2. Calculation model

2.1. Flux-tube gyrokinetic simulation model for realistic tokamak equilibria

In this section, we briefly summarize the gyrokinetic simulation model used in the electromagnetic gyrokinetic Vlasov code GKV and an interface with the integrated tokamak transport solver TOPICS providing the experimental equilibrium profiles. The detailed descriptions are also given in [24]. One of the governing equations is the electromagnetic gyrokinetic equation describing the time evolution of the perturbed gyrocenter distribution function $f'(\mathbf{x},\mathbf{v},\mu)$, where the subscript ‘s’ is the index of the particle species. The Fourier representation with the perpendicular wavenumber $k_s$ is given by

$$
\frac{\partial}{\partial t} + v_\parallel \cdot \nabla + i\omega_{Ds} - \frac{\mu}{m_s} \frac{\partial}{\partial \mu} \frac{\partial}{\partial \mu} \delta g_{k_s} = \frac{e}{B} \sum b \cdot (k_s' \times k_s) \delta \psi_k \delta \psi_{k_s} = e \frac{F_M}{T_s} \left( \frac{\partial \delta \psi_k}{\partial \mu} + i\omega_{Te} \delta \psi_k \right) + C_s(\delta g_{k_s}),
$$

(1)

where $\delta g_{k_s}$ stands for the non-adiabatic part of the perturbed gyrocenter distribution function $f'(\mathbf{x},\mathbf{v},\mu)$, i.e. $\delta g_{k_s} = \delta f_{k_s}^0 + eJ_{k_s}^0 \delta \psi_k F_M/T_s$. The particle mass, the electric charge, the equilibrium temperature, and the gyrofrequency are denoted by $m_s, e_s, T_s$ and $\Omega_s = eB/m_s c$, respectively. The parallel velocity $v_\parallel$ and the magnetic moment $\mu$ are used as the velocity-space coordinates, where $\mu$ is defined by $\mu = m_s v_\parallel^2/2B$ with the perpendicular velocity $v_\perp$. The gyro-averaged potential fluctuation is denoted by $\delta \psi_k := J_0(k_s' \delta \psi_k - v_\parallel c) \delta A_{k_s}$, where $J_0 := J_0(k_s \Omega_s)$ is the zeroth-order Bessel function, and the former and latter terms mean the electrostatic and electromagnetic parts, respectively. Since we focus here on the finite but low-\$\beta$ L-mode plasmas, the parallel magnetic field fluctuation $\delta B_{k_s}$ (so called the compression component) is ignored. The equilibrium part of the distribution function is given by the local Maxwellian distribution, i.e. $F_M = n_s(m_s/2\pi T_s)^{3/2} \exp[-(m_s v_\parallel^2 + 2\mu B)/2T_s]$, where $n_s$ represents the equilibrium density. The symbol $\sum_{k_s}$ appearing in the nonlinear term of equation (1) means the double summations with respect to $k_s'$ and $k_s''$, which satisfy the triad-interaction condition of $k_s = k_s' + k_s''$. Collisional effects are introduced in terms of a linearized model collision operator $C_c$ where a gyro-averaged Lenard–Bernstein model [27] is applied here.

The gyrokinetic equation shown in equation (1) is solved in the local flux-coordinate systems $(x, y, z)$ [28] defined as
\[ x = a(\rho - \rho_0), \quad y = a\rho_0 q(\rho_0)^{-1} \left[ q(\rho_0\theta - \zeta) \right], \quad z = \theta \] with the straight field line coordinates \((\rho, \theta, \zeta)\), where \(a\) and \(q(\rho_0)\) denote the plasma minor radius and the safety factor on the flux surface label of interest \(\rho_0\), respectively. In these coordinates, the magnetic field vector is given by \(B = B_0 \nabla x \times \nabla y\) with the field strength on the magnetic axis \(B_0\). The perpendicular wavenumber vector and the parallel derivative operator are given as \(k_\perp = k_x \nabla x + k_y \nabla y\), \(\nabla = \nabla_x \nabla y\), where \(\frac{\nabla x}{\nabla y} = \det(g^{ij})^{-1/2}\) is the Jacobian given by the contravariant components of the metric tensor \(g^{ij}\) for \((i, j) = (x, y, z)\).

The magnetic and diamagnetic drift frequencies, \(\omega_{Dx}\) and \(\omega_{T_x}\), are then given by:

\[
\omega_{Dx} = \frac{c}{eB} k_x \cdot b \times (\mu \nabla B + m_i v_i^2 b \cdot \nabla b)
= \frac{c}{eB} \left(\mu \nabla B \right)^{1/2} \left(\nabla \cdot \nabla \ln n_s\right),
\]

\[
\omega_{T_x} = \frac{c T_e}{eB} \left[ 1 + \eta_s \left(\frac{m_i v_i^2 + 2 \mu B}{2 T_e} - \frac{3}{2}\right) \right] k_x \cdot b \times \nabla \ln n_s,
\]

where \(\eta_s = L_n/L_T\) with \(L_n = -(\text{d} \ln n_s/\text{d} x)^{-1}\) and \(L_T = -(\text{d} \ln T_e/\text{d} x)^{-1}\). The geometric coefficients \(K_x\) and \(K_y\) are defined as follows:

\[
K_x = \frac{g^{xz} g^{yz} - g^{xy} g^{xz}}{B^2/B_{ax}^2} \frac{\partial \ln B}{\partial z} - \frac{\partial \ln B}{\partial y},
\]

\[
K_y = \frac{g^{xz} g^{yz} - g^{xy} g^{xz}}{B^2/B_{ax}^2} \frac{\partial \ln B}{\partial z} + \frac{\partial \ln B}{\partial x}.
\]

Note that the low-\(\beta\) approximation is applied to \(\omega_{Dx}\) so that the finite-\(\beta\) effect is ignored in the curvature drift.

The electromagnetic and Poisson–Amperé equations are determined by the Poisson and Amperé equations:

\[
(k_\perp^2 + \lambda_\perp^2) \delta \phi_k = 4\pi \sum_s e_s \int d\mathbf{v} J_{bs} \delta g_{sk},
\]

\[
k_\perp^2 \delta \phi_k = -e \int d\mathbf{v} \delta g_{sk} e^{-i k \cdot \mathbf{r}},
\]

\[
\sum_s \left[ \frac{d}{dt} T^{\text{in}}_s + T^{\text{in}}_s \frac{\delta R^{\text{in}}_s}{d\mathbf{v}} + T^{\text{in}}_s \frac{\delta J^{\text{in}}_s}{d\mathbf{v}} - T^{\text{in}}_s \frac{\delta S^{\text{in}}_s}{d\mathbf{v}} - T^{\text{in}}_s \frac{\delta D^{\text{in}}_s}{d\mathbf{v}} \right] = 0,
\]

where \(\lambda_\perp = \left(\sum_s e_s n_s T_s/\tau_e\right)^{-1/2}\) is the Debye length. The charge neutrality in the background density \(n_s\) is described as \(\sum_s e_s f_{Cs} = 1\), where \(f_{Cs} = Z_n/n_s\) means the charge-density fraction for ions with the charge number \(Z_n\). The source term for each species in the right of equations (6) and (7) are proportional to \(f_{Cs}\). In addition, the radial derivative of the above charge neutrality condition leads to another constraint for the background density gradient, i.e., \(\sum_s e_s f_{Cs} L_n = L_{n_i}\). It is also noted that, in the adiabatic electron limit with \(k_\perp \rho_0 \ll 1\), the gyrocenter density fluctuation for electrons is approximated by \(\int d\mathbf{v} \delta f^{\text{eg}}_{sk} \simeq -e n_s \delta \phi_k - ((\delta \phi_k), \delta \phi_k) T_e\), where \(\rho_s = m_s c v_{ts}/e_s \nabla T_e\) means the gyroradius evaluated with the thermal speed \(v_{ts} = (T_e/m_s)^{1/2}\), and the field-line average is defined by \(\langle X_k \rangle_L = \int dz \sqrt{g_{xy}} X_k / \int dz \sqrt{g_{xy}}\).

By using the gyrokinetic and the Poisson–Amperé equations, one can derive another important equation describing the balance and transfer of the entropy variable \(\delta S_{sk} \equiv \int d\mathbf{v} \delta (S_{sk})^2 / 2 F_{Msk}\) defined with the particle (not gyrocenter here) distribution function \(\delta f_k = -e \delta \phi_k F_{Msk}/T_e + \delta g_{sk} e^{-i k \cdot \mathbf{r}}\).

\[
\sum_s \left[ \frac{d}{dt} T^{\text{in}}_s + T^{\text{in}}_s \frac{\delta R^{\text{in}}_s}{d\mathbf{v}} + T^{\text{in}}_s \frac{\delta J^{\text{in}}_s}{d\mathbf{v}} - T^{\text{in}}_s \frac{\delta S^{\text{in}}_s}{d\mathbf{v}} - T^{\text{in}}_s \frac{\delta D^{\text{in}}_s}{d\mathbf{v}} \right] = 0,
\]
The magnetic drift is each, and the field-line-average operator, k ≡ ∑ ∑ ≠ 0. The density and temperature profiles, \( k \), and the field-line-average operator, \( k \), normalized gyroradius and collisional profiles, \( k \), is reconstructed by TOPICS. The radial profiles of the ion and electron energy fluxes are simultaneously provided by the power balance analysis for, e.g. Ohmic, NBI, and EC heatings, where the power depositions and losses are calculated by a fast-ion orbit following Monte-Carlo Fokker–Planck solver OFMC [25, 30]. After that, a flux coordinate generator IGS [24] makes high-accuracy interpolations of the flux surfaces and constructs the straight-field-line coordinates, such as the axisymmetric (or natural), Boozer, and Hamada coordinates. Using these coordinates data, the metric tensor and the Jacobian for geometry-dependent quantities and operators are calculated in GKV. It is noted that the up-down asymmetry resulting from the divertor configuration is consistently included in this framework.

The combined analyses among GKV and TOPICS via IGS enable us to make not only experimental analyses, but also predictive studies for future devices, e.g. shape optimization on the microstability and turbulent transport. Actually, in [24], GKV with realistic tokamak equilibria is numerically verified through the cross-code benchmark test. It is then applied to two types of shaped plasmas expected in the JT-60SA tokamak device, i.e. ITER-like and highly-shaped plasmas, where a decrease in the ITG-mode growth rate and enhancement of the residual zonal flow level in the highly-shaped case are identified.

3. JT-60U L-mode equilibrium and linear instability analyses

Using the framework of GKV-TOPICS, gyrokinetic simulation studies for the JT-60U tokamak experiments are carried out. Then, the prediction capability of GKV is examined through comparisons between the simulation and experimental results on the turbulent ion and electron energy fluxes. For a validation study of the local flux-tube gyrokinetic simulation, we have chosen a JT-60U deuterium plasma on the L-mode discharge with a positive magnetic shear profile [31], where

\[
\sum_{s} \left( \frac{d}{dt} T_{\delta}^{s(\text{trb})} + T_{\delta}^{s(\text{zf})} - T_{\delta}^{s(\text{trb})} - T_{\delta}^{s(\text{zf})} \right) = 0, \tag{9}
\]

where the superscripts ‘(trb)’ and ‘(zf)’ mean the non-zonal and zonal components in the wavenumber space, respectively, i.e. \( X^{(\text{trb})} = \sum_{j} \sum_{k} X_{k_{j}} \), \( X^{(\text{zf})} = \sum_{j} X_{k_{j}} \). Each term represents (i) the variation of the entropy variable, (ii) the variation of the field energy, (iii) the nonlinear entropy transfer from non-zonal to zonal modes, (iv) the entropy production by turbulent particle and heat fluxes, and (v) the collisional dissipation, respectively (see, e.g. [29] for their definitions). Note that, by definition, the turbulent-flux driven entropy production term does not appear for the zonal modes in equation (9). The entropy balance/transfer equation provides us with a good measure for the turbulence simulation accuracy as well as useful physical insights associated with the turbulence saturation mechanisms. The detailed numerical analyses of the entropy transfer and the balance processes in ITG, TEM, and ETG turbulence are shown in, e.g. [20–23].

2.2. Interface with integrated-transport solver

In this section, we present the framework of combined analyses by means of GKV and the integrated-transport solver TOPICS with experimental density and temperature profiles and magnetic fields. In GKV, effects of magnetic field geometries are incorporated into the field intensity \( B \), the parallel derivative \( \mathbf{v} \), the magnetic drift \( \omega_{D \perp} \), and the perpendicular wavenumber \( k_{\perp} \), and the field-line-average operator \( \langle \cdots \rangle \), shown in section 2.1, through the contravariant components of metric tensor \( g^{ij} \) and the Jacobian \( \sqrt{g_{ij}} \).

Figure 1 shows a schematic procedure to make the realistic MHD equilibrium data for turbulence simulations with GKV. First, the MHD equilibrium based on the experimentally measured density, temperature, and current profiles is reconstructed by TOPICS. The radial profiles of the ion and electron energy fluxes are simultaneously provided by the power balance analysis for, e.g. Ohmic, NBI, and EC heatings, where the power depositions and losses are calculated by a fast-ion orbit following Monte-Carlo Fokker–Planck solver OFMC [25, 30]. After that, a flux coordinate generator IGS [24] makes high-accuracy interpolations of the flux surfaces and constructs the straight-field-line coordinates, such as the axisymmetric (or natural), Boozer, and Hamada coordinates. Using these coordinates data, the metric tensor and the Jacobian for geometry-dependent quantities and operators are calculated in GKV. It is noted that the up-down asymmetry resulting from the divertor configuration is consistently included in this framework.

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\[ \rho^* \simeq 1/500 \text{ and } \rho^*_{\text{eff}} \equiv \rho_0 \sqrt{L} \simeq 1/400 \text{ for } \rho < 0.8 \text{ well satisfy the local limit condition of } \rho^* < 1/300 [5–8]. \] The configuration parameters of the experiment are summarized in table 1.

The plasma shape and equilibrium profiles are shown in figure 2. It is noted that the electron temperature is slightly higher than the ion one for \( \rho < 0.7 \). The mean toroidal rotation and the mean poloidal \( E \times B \) rotation are negligibly small in the whole radial region, i.e. \( |L_{\text{lin}}| \sim 0.1 V_\parallel \) and \( |U_{E \times B}| \sim 0.1 V_{\parallel \text{e}}/q \). One also finds almost monotonically increasing profiles of the normalized density/temperature gradient, \( R_{\text{ax}}/L_{\text{ax}} \), \( R_{\text{ax}}/L_{T_i} \), and \( R_{\text{ax}}/L_{T_e} \). The normalized collisionality \( \nu^*_{\text{ce}} \) is around 0.1 (banana-plateau regime) for both ion–ion and electron–electron collisions, where the definition is given by \( \nu^*_{\text{ce}} = q R_{\text{ax}} \tau_{\text{ce}}^{-1} (\sqrt{2} c^2/2) \) with the characteristic collision time \( \tau_{\text{ce}} \). Besides, the mean radial electric field shearing rate \( \gamma_{E_r} \) is about ten times smaller than the linear-mode growth rate \( \gamma_{\text{lin}} \). The local simulation approach is, thus, well justified for the plasma investigated here.

The wavenumber dependence of the linear instability growth rate \( \gamma_{\text{lin}}(k_y) \) calculated by the linear GKV simulations is shown in figure 3, where \( \gamma_{\text{lin}}(k_y) \) for three radial positions, \( \rho = 0.26, 0.50, 0.76 \) are plotted. Since, as shown in figure 2(g), the normalized gradients rapidly increase towards the plasma edge, the different linear microinstabilities appear at each radial position, where the ITG, ITG–TEM, and TEM instabilities are dominant in the ion-scale modes of \( k_y \rho_0 \leq 2 \) at \( \rho = 0.26, 0.50, 0.76 \), respectively. The ETG modes are also unstable at \( \rho = 0.50 \) and 0.76. The ITG- and ETG-mode growth rates calculated with adiabatic electrons or ions are also plotted. One can see that the adiabatic–electron approximation is only valid in the deep core region (\( \rho < 0.26 \)) where a fraction of the trapped electrons becomes less significant in comparison to that in the outer region. It is also noteworthy that the maximum growth rate in the ion-scale modes with \( k_y \rho_0 \leq 1 \) increases towards the outer region.

4. ITG and/or TEM driven turbulence simulations

4.1. Numerical settings and the entropy balance relations

Turbulence simulation results and comparisons with the experimental measurements are discussed in this section. The ITG and/or TEM turbulence simulations at several points in
0.26 \leq \rho \leq 0.76 are carried out. Here, \rho = 0.76 is chosen as the maximum radial position, following the baseline position of the transport shortfall in the DIII-D case considered in the earlier works [9–12]. Only ion-scale electromagnetic fluctuations of \(k_{r,max} \rho_i \leq 2\) are solved. The linearly stable region for \(k_{r,\rho_i} > 1\) and the finite collisions lead to the statistically steady turbulence state, even when any numerical dissipations are not imposed in the higher \(k_y\) region. The pure deuterium-electron plasma without any impurity ions is considered here. As for the numerical resolution, (168, 32)-mode numbers in \((k_x, k_y)\) and (64,64,32)-grid numbers in \((\varepsilon, v_i, v_e)\) are used, where the velocity-space domain is taken with \(v_{i,\max}, v_{e,\max} = (5v_{ti}, 5v_{te})\) for \(s = i, e\) at \(\varepsilon = 0\). As shown in figures 4(a) and (b), the entropy balance relations for both the non-zonal (turbulent) and zonal components are accurately satisfied in the present turbulence simulations, where the labeling number for each line corresponds to the term in the left hand side of equations (8) and (9), respectively.

4.2. Comparisons of turbulence simulation results with experimental measurements

The simulation results on the turbulent energy fluxes for ions and electrons are compared with the experimental measurements. Figure 5 show the spectro-temporal evolutions of the radial turbulent energy flux density \(Q_{ak}/Q_{GBB}\) and \(Q_{ak}/Q_{GBB}\) at \(\rho = 0.26, 0.50, 0.76\) calculated by GKV, where \(Q_{ak} = \langle c(B)/2 \rangle R e \int dv (m_n v^2/2) \delta k \delta v \delta k_{ak}^2\). Both the ion and electron energy fluxes in the quasi-steady state of \(t > 80R_{ti}/v_{ti}\) are dominated by low wavenumber modes of \(k_{r,\rho_i} < 0.5\) in the ITG–TEM and TEM cases, even with the wide unstable wavenumber region expanding beyond \(k_{r,\rho_i} \sim 1\). Indeed, the wavenumber at the spectral peak is similar to or lower than that giving the maximum linear growth rate (shown by the horizontal dashed line in the contours). It is also found that contribution of the higher-wavenumber modes is more significant for the electron energy flux, where the amplitude \(|Q_{ak}|\) for \(k_{r,\rho_i} \sim 2\).

The radial dependencies of the turbulence part \(W_{ab}\) with \(k_y = 0\) and the zonal flow part \(W_{0b}\) with \(k_y = 0\) in the generalized flow energy \(W_{total}\) are shown in figures 6(a) and (b), where \(W_{total}\) is defined by \(W_{total} = \langle \sum_k \sum_s (n_s e^2/2T_{ei})(1 – \Gamma_0) \delta F_{k,\rho_i} \delta k \delta k_{ak}^2 \rangle_i \approx \langle \sum_k \sum_s (n_s e^2/2T_{ei}) \delta F_{k,\rho_i} \delta k \delta k_{ak}^2 \rangle_i\), and the quantities are time-averaged over the steady turbulence state. For comparisons, the mixing-length estimate of diffusivity, i.e. \(max(\gamma_{\parallel} k^2)\), and the normalized entropy transfer rate for the zonal modes \(\gamma_{trl}^2 (l_{T1} T_{l1} Q_{vl})\) representing the nonlinear source for the zonal flow generation are also plotted in the figures. We see that the monotonic increase in the turbulence energy \(W_{ab}\) is well correlated with the mixing-length estimate. As for the zonal flow energy normalized by the turbulence part \(W_{0b}/W_{ab}\), a weak radial dependence is found for the core region of \(\rho < 0.50\) indicating ITG- and ITG–TEM-dominated turbulence, while more significant zonal flow generation is observed for the outer region with TEM-dominated turbulence. This is consistent with the tendency of the entropy transfer rate in figure 6(b). The understanding of the correlation among the turbulence energy, zonal flow energy, and the entropy transfer rate is useful for constructing a simplified turbulent transport model [15] which can be applied to the integrated transport simulations [32].

Figure 7 show the comparisons between the GKV simulation results and the experimental measurements on the radial profiles of the ion energy flux \(P_{ir}\), the electron energy flux \(P_{er}\), and the convective part resulting from the turbulent particle flux \((5/2)T_{ei} I_{G}\) in the unit of MW. The flux-surface-integrated radial energy and particle fluxes are defined as \(P = \int d\theta d\zeta \sqrt{g} Q_{sl} \cdot \nabla \rho = \langle Q_{sl} \rangle \nabla V_{\rho}\rangle \) and \(I_{G} = \int d\theta d\zeta \sqrt{g} \Gamma_{ir} \cdot \nabla \rho = \langle \Gamma_{ir} \rangle \nabla V_{\rho}\rangle\), where \(Q_{sl}\) and \(\Gamma_{ir}\) mean the contravariant radial components of the energy and particle flux density vectors in the unit of MW m\(^{-3}\) given by \(Q_{sl} = (c(B)) \int dv (m_n v^2 /2) \delta F_{\rho} \nabla \phi \cdot \delta \phi\), \(\Gamma_{ir} = (c(B)) \int dv \delta F_{\rho} \nabla \phi \cdot \delta \phi\), respectively. The flux-surface average is denoted by \(\langle \cdot \rangle_{G}\). The experimental values, \(P_{exp}\) and \(P_{exp}\), are evaluated by the steady power balance analysis with TOPICS. The
in comparison to that in the TGLF cases. Note that the experimental evaluation of \( \Gamma_e \) is not available due to large uncertainty in the particle source and sink depositions, which are strongly influenced by the particle recycling and the ionization/recombination processes in the plasma edge.

It is stressed that the ion energy flux in GKV decreases towards the outer region of \( \rho > 0.50 \) despite the monotonically increasing tendency of the linear growth rate shown in figure 6(a). Indeed, the stronger zonal flow generation from the TEM turbulence is associated with the transport reduction in the outer region. Consequently, we see a moderate transport shortfall with relatively larger deviations from the experimental value for \( P_i \) in comparison to those in the core region of \( \rho \leq 0.50 \). The parameter sensitivity of the transport shortfall associated with the strong zonal flows are discussed in section 4.3, as well as the flux matching evaluation of the profile prediction accuracy.

4.3. Ion and electron temperature gradient dependencies and multiple flux matching

As pointed out in [11], since the experimental plasmas often show a stiff response of temperature profiles against the auxiliary heating, the sensitivity of the calculated turbulent fluxes on the equilibrium profiles should be examined to make a more robust validation. In this section, we discuss dependencies of the turbulence simulation results on the numerical and physical parameters. Also, in the later part, the flux matching technique is applied for both \( P_i \) and \( P_e \) simultaneously to evaluate the prediction accuracy of the ion and electron temperature gradient profiles.

The numerical convergence with respect to the maximum wavenumber is shown in figure 8, where influences of the faster and finer TEM-uncstable fluctuations (figure 8(a)) are examined for the time evolution of ion and electron energy fluxes at \( \rho = 0.50 \) (figure 8(b)) and \( \rho = 0.76 \) (figure 8(c)). We see that the higher maximum wavenumber leads to faster linear growth of the TEM modes for both cases. Their growths, then, saturate at much lower levels than that after the saturation of the ITG-mode growth. Thus, the mean saturation levels of \( P_i \) and \( P_e \) in the steady state are not affected by the TEM modes with higher \( k_r \). This is also consistent with the fact that the lower wavenumber modes are dominant in the turbulent energy fluxes shown in figure 5.

The temperature-gradient dependencies of the ion energy flux \( P_i \), the electron energy flux \( P_e \), the turbulence energy \( W_{\text{trb}} \), and the zonal flow energy \( W_{\text{zf}} \) (normalized by the total one \( W_{\text{total}} = W_{\text{trb}} + W_{\text{zf}} \)) in the ITG (\( \rho = 0.30 \)), ITG-TEM (\( \rho = 0.50 \)), and TEM (\( \rho = 0.76 \)) dominated regimes are summarized in figures 9(a)–(c). Note that the variable \( L_{\text{exp}}^{-1} \) in the horizontal axis is normalized by the experimental mean value \( L_{\text{exp}}^{-1} \), as shown in figure 1(g), and \( X_{\text{ref}} \) means the reference simulation results of \( X = \{ P_i, P_e, \Gamma_e, W_{\text{trb}}, W_{\text{zf}}/W_{\text{total}} \} \) in terms of \( L_{\text{exp}}^{-1} \). Reflecting the interplay of turbulence, zonal flows and the resultant fluxes, each temperature-gradient dependence is somewhat complicated. However, we can see a

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**Figure 6.** Radial profiles of (a) the mixing-length estimate of diffusivity \( \max[\gamma_{\text{ad}}/k^2] \) and the turbulence energy \( W_{\text{trb}} \) in the gyro-Bohm unit, (b) the normalized entropy transfer rate for zonal modes \( J_i^{(z)} (L_T^{-1} Q_i/T_i) \) and the zonal flow energy normalized by the turbulence one \( W_{\text{zf}}/W_{\text{trb}} \).
clear overall tendency that the magnitude of the temperature-gradient dependence becomes weak towards the outer region. Actually, $|\partial \rho \over \partial L_T^{-1}\partial T|_{\rho=0.76}$ is much smaller than that in the ITG-dominated region of $\rho=0.30$. It is also stressed that $P_i$ and $P_e$ indicate different $-LT^{-1}_i$- and $-LT^{-1}_e$-dependencies, which are crucial in order to apply a multiple flux matching technique shown below.

In the present local flux-tube delta-f approach with the fixed background gradients, the steady temperature and density profiles in the power balanced state can be predicted by using the so-called flux matching technique. [11, 18].

A way of evaluating the prediction accuracy is to measure the deviation between actual gradients observed in the experiment and the input gradient values reproducing the experimental turbulent fluxes in the simulation. For instance, let $P_{(\text{sim})} = P_{(\text{sim})}(L_T^{-1}, L_{\rho}^{-1}, L_{\phi}^{-1}, L_{\theta}^{-1})$, and $\Gamma_{(\text{sim})} = \Gamma_{(\text{sim})}(L_T^{-1}, L_{\rho}^{-1}, L_{\phi}^{-1}, L_{\theta}^{-1})$ be the calculated ion and electron energy and particle fluxes, respectively, as nonlinear functions of the ion-, electron-temperature, and density gradients, where the other equilibrium parameters are fixed. Then, the temperature- and density-gradient variations ($\Delta L_T^{-1}$, $\Delta L_{\rho}^{-1}$, $\Delta L_{\phi}^{-1}$) are determined by the following flux matching relations for the ions and electrons,
Figure 9. $L_T^{-1}$ and $L_E^{-1}$-scans for the ion energy flux ($P_i$), the electron energy flux ($P_e$), the turbulence energy ($W_{\text{rb}}$), and the zonal flow energy normalized by the total one ($W_{\text{rh}}/W_{\text{total}}$) at (a) $\rho = 0.30$, (b) $\rho = 0.50$, and (c) $\rho = 0.76$, where the other parameters are fixed in each scan.

\[ P_{\text{sim}}(L_T^{-1}, L_E^{-1}, L_{\text{rh}}^{-1}) = P_{\text{EXP}} \]

where the subscript ‘(EXP)’ means the nominal experimental values. Note that equations (10)–(12) are a set of nonlinear coupled equations for ($\Delta L_T^{-1}$, $\Delta L_E^{-1}$, $\Delta L_{\text{rh}}^{-1}$), but one can reduce them to a linearized form, i.e.

\[
\begin{pmatrix}
\Delta P_{\text{sim}} \\
\Delta L_T^{-1} \\
\Delta L_E^{-1} \\
\Delta L_{\text{rh}}^{-1}
\end{pmatrix} =
\begin{pmatrix}
\frac{\partial P_{\text{sim}}}{\partial P_{\text{EXP}}} & \frac{\partial P_{\text{sim}}}{\partial L_T^{-1}} & \frac{\partial P_{\text{sim}}}{\partial L_E^{-1}} & \frac{\partial P_{\text{sim}}}{\partial L_{\text{rh}}^{-1}} \\
\frac{\partial P_{\text{EXP}}}{\partial P_{\text{sim}}} & \frac{\partial P_{\text{EXP}}}{\partial L_T^{-1}} & \frac{\partial P_{\text{EXP}}}{\partial L_E^{-1}} & \frac{\partial P_{\text{EXP}}}{\partial L_{\text{rh}}^{-1}} \\
\frac{\partial \Gamma_{\text{sim}}}{\partial \Gamma_{\text{EXP}}} & \frac{\partial \Gamma_{\text{sim}}}{\partial L_T^{-1}} & \frac{\partial \Gamma_{\text{sim}}}{\partial L_E^{-1}} & \frac{\partial \Gamma_{\text{sim}}}{\partial L_{\text{rh}}^{-1}}
\end{pmatrix}^{-1}
\begin{pmatrix}
\Delta P_{\text{EXP}} \\
\Delta L_T^{-1} \\
\Delta L_E^{-1} \\
\Delta L_{\text{rh}}^{-1}
\end{pmatrix}
\]

where $\Delta P \equiv P_{\text{EXP}} - P_{\text{sim}}$, $\Delta L_T \equiv L_T^{-1}_{\text{EXP}} - L_T^{-1}_{\text{sim}}$, and $\Delta \Gamma \equiv \Gamma_{\text{EXP}} - \Gamma_{\text{sim}}$. Actually, the coefficient matrix is evaluated in a similar way to figures 9(a)–(c). In the previous work in [11], the ion heat flux is adjusted by only the ion temperature gradient parameter, so that the electron heat flux still deviates from the experimental one. For more precise treatments, one needs to consider the $3 \times 3$ matrix approach shown in equations (10)–(13), resulting from the coupling among the heat and particle fluxes at each radial position, but the accurate experimental evaluation of the particle flux is necessary.

Figure 10. Flux-matching results on (a) the ion energy flux $P_i$ and (b) the electron energy flux $P_e$ with the modified $L_T$ and $L_E$, where the open diamond and star symbols show the cases where the flux matching is prevented by the weak $L_T$ and $L_E$ dependencies.

In the present study, since there are no experimental data for $\Gamma_e$ as mentioned in section 4.2, the multiple flux matching technique is applied to the GKV simulation results (shown in figure 7) such that the radial profiles of $P_i$ and $P_e$ simultaneously match the experimental ones, i.e.
Table 2. Temperature-gradient variations in multiple flux matching for $P_i$ and $P_e$.

| $\rho$ | $\Delta L_{Ti}^{-1}L_{Texp}^{-1} (%)$ | $\Delta L_{Te}^{-1}L_{Eexp}^{-1} (%)$ | $P_{i, matched}$ | $P_{i, EXP}$ | $P_{e, matched}$ | $P_{e, EXP}$ |
|--------|---------------------------------|---------------------------------|-----------------|-----------|-----------------|-----------|
| 0.30   | $-15$                           | $+15$                           | 0.299           | 0.274     | 0.177           | 0.208     |
| 0.40   | $-10$                           | $+0$                            | 0.464           | 0.490     | 0.348           | 0.338     |
| 0.50   | $+10$                           | $-30$                           | 0.735           | 0.743     | 0.545           | 0.503     |
| 0.58   | $+20$                           | $-30$                           | 0.823           | 0.891     | 0.681           | 0.645     |
| 0.66   | $+20$                           | $-20$                           | 0.581           | 0.972     | 0.783           | 0.789     |
| 0.66   | $+15$                           | $+0$                            | 0.516           | 0.972     | 0.929           | 0.789     |
| 0.76   | $+0$                            | $+30$                           | 0.625           | 0.973     | 1.044           | 0.986     |
| 0.76   | $+15$                           | $+0$                            | 0.613           | 0.973     | 0.695           | 0.986     |
| 0.76(w/-15% in $L_{ni}^{-1}$) | $+0$                            | $+0$                            | 0.399           | 0.973     | 0.581           | 0.986     |
| 0.76(w/+15% in $L_{ni}^{-1}$) | $+0$                            | $+0$                            | 0.668           | 0.973     | 1.002           | 0.986     |
| 0.76(w/o ZF) | $+0$                            | $+0$                            | 1.192           | 0.973     | 1.353           | 0.986     |

Note that although the linearized form with a matrix coefficient in equation (13) provides one with the lowest order evaluation of $(\Delta L_{Ti}^{-1}, \Delta L_{Te}^{-1})$ as shown in figure 9, the present multiple flux matching based on equations (14) and (15) takes into account their nonlinear dependencies. Figures 10(a) and (b) show the flux matching results for the ion and electron energy fluxes, respectively. The temperature-gradient variations for each radial position are summarized in table 2, where $P_{i,e, matched}$ means the GKV simulation results with $(L_{Ti}^{-1}, L_{Te}^{-1})$. One finds that the experimental fluxes are well reproduced except for the ion flux in the outer region of $\rho > 0.58$, where the prediction error for the ion and electron-temperature gradients for $\rho \leq 0.58$ is evaluated as less than ±30%. In the outer region, the weak dependence on the temperature-gradient shown in figure 9(c) prevents the flux matching within ±30% range of $(\Delta L_{Ti}^{-1}, \Delta L_{Te}^{-1})$. Besides, the density-gradient validations of ±15% from the experimental value (shown in table 2) indicate that the ion and electron energy fluxes are roughly proportional to $L_{ni}^{-1}$ (or $\eta_{ci}^{-1}$), e.g. $P_i = 0.399$ at $-15\% L_{ni}^{-1}$ and $P_i = 0.668$ at $+15\% L_{ni}^{-1}$ (shown by the 9th and 10th rows in table 2), where $\eta_c \simeq 2$ at $\rho = 0.76$ in the present case. A similar characteristic in the density-gradient dependence of $\eta_c > 1$ has also been found in [34].

Since it has been confirmed that the significant zonal flow generation in the TEM turbulence is related to the transport suppression in the outer region (see figure 6(b)), the turbulence simulation without zonal flow generations can provide us with useful insights, where the result is shown in the last row in table 2. Then, we see that the simulation results on both the ion and electron energy fluxes are strongly affected by the existence of zonal flows, and the transport shortfall in the ion flux becomes less significant. This is an artificial treatment, but clearly suggests the importance of the accurate zonal flow treatment in the simulation model. Actually, the generation of TEM-driven zonal flows and its impact on the turbulent transport show quite strong dependence on the equilibrium parameters ($R/L_{Tc}, T_c/T_i$ etc) as shown in [35, 36], and have not been fully clarified yet. Also, collisions with impurities from the wall and divertor and/or the toroidal magnetic field ripple, which are ignored in the present simulation model, may lead to relatively stronger zonal flow damping in the outer region. Some global effects of the radial propagation of the heat flux and the turbulence intensity [8, 37, 38] can also influence the zonal flow dynamics.

5. Summary

In this paper, quantitative comparisons of the ion and electron heat transport between gyrokinetic simulation and the JT-60U tokamak experiment are carried out by using a local gyrokinetic code GKV incorporating realistic magnetic geometry and fully-gyrokinetic electrons, where the ITG- and/or TEM-driven turbulent transport and zonal flow generations are investigated. In order to examine the prediction capability of the flux-tube gyrokinetic simulations, an L-mode plasma with sufficiently small $\rho'(\sim 1/500)$ is chosen for the quantitative comparisons, where the mean radial electric field and its shearing effect are also negligibly small. Nonlinear ITG and/or TEM simulations by GKV with kinetic electrons successfully reproduce the radial profiles of the ion and electron energy fluxes, which are relevant to the experimental values in the core region, whereas the adiabatic-electron case indicates relatively larger deviations. It is revealed that the zonal flow generation in the outer region ($\rho > 0.58$) with TEM-dominated turbulence is much more significant than that in the core region with ITG- and ITG–TEM-dominated turbulence ($\rho \leq 0.58$). Then, the transport shortfall for the ion energy flux appears in the outer region.

Extending the conventional ion heat flux matching by adjusting a single parameter of the ion temperature gradient, we performed a multiple flux matching for both the ion and electron energy fluxes. The temperature-gradient variations giving the matched energy fluxes in the core region are simultaneously determined for ions and electrons with a prediction error less than ±30%, while the weak temperature-gradient dependence of turbulent transport and zonal flows in the outer region prevents the flux matching. The
turbulence simulation without the TEM-driven zonal flows indicates the strong influence of both the ion and electron energy fluxes, where the ion transport shortfall becomes less significant.

To further improve the prediction accuracy of gyrokinetic simulations, and also to construct a credible reduced transport model, one needs the quantitative comparison of the particle flux between detailed experimental measurements and turbulence simulations. Also, more detailed analyses of the zonal flow dynamics are required, including its generation and damping mechanisms associated with the collisions with impurities, the toroidal magnetic field ripple, and some global effects, such as the radial propagation of the heat flux and the turbulence intensity. As the fluctuation measurements in JT-60U L-mode plasmas are available only for the very edge region, direct comparisons of the turbulence spectrum with the experimental measurement is out of scope at present, but they will be addressed for other appropriate experimental set-ups in future works.

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References

[1] Brizard A.J. and Hahm T.S. 2007 Rev. Mod. Phys. 79 421
[2] Sugama H. and Horton W. 1998 Phys. Plasmas 5 2560
[3] Garbet X., Idomura Y., Villiard L. and Watanabe T.-H. 2010 Nucl. Fusion 50 043002
[4] Idomura Y., Watanabe T.-H. and Sugama H. 2006 C. R. Phys. 7 650
[5] Candy J. and Waltz R.E. 2003 Phys. Rev. Lett. 91 045001
[6] Waltz R.E., Candy J. and Petty C.C. 2006 Phys. Plasmas 13 072304
[7] McMillan B.F. et al 2010 Phys. Rev. Lett. 105 155001
[8] Nakata M. and Idomura Y. 2013 Nucl. Fusion 53 113039
[9] Holland C. et al 2009 Phys. Plasmas 16 055201
[10] Rhodes T.L. et al 2011 Nucl. Fusion 51 063022
[11] Görler T. et al 2014 Phys. Plasmas 21 122307
[12] Chowdhury J. et al 2014 Phys. Plasmas 21 112503
[13] Tell D. et al 2013 Phys. Plasmas 20 122312
[14] Howard N.T. et al 2013 Phys. Plasmas 20 032510
[15] Nunami M., Watanabe T.-H. and Sugama H. 2013 Phys. Plasmas 20 092307
[16] Ishizawa A. et al 2015 Nucl. Fusion 55 043024
[17] Bravenec R.V., Candy J., Barnes M. and Holland C. 2011 Phys. Plasmas 18 122505
[18] Candy J. et al 2009 Phys. Plasmas 16 060704
[19] Waltz R.E. and Holland C. 2008 Phys. Plasmas 15 122503
[20] Nakata M., Watanabe T.-H. and Sugama H. 2012 Phys. Plasmas 19 022303
[21] Maeyama S. et al 2015 Phys. Rev. Lett. 113 255002
[22] Asahi Y. et al 2015 Plasma Fusion Res. 10 1403047
[23] Watanabe T.-H. and Sugama H. 2006 Nucl. Fusion 46 24
[24] Nakata M. et al 2014 Plasma Fusion Res. 9 1403029
[25] Hayashi N. and JT-60 Team 2010 Phys. Plasmas 17 056112
[26] Honda M. et al 2013 Nucl. Fusion 53 073050
[27] Lenard A. and Bernstein I.B. 1958 Phys. Rev. 112 1456
[28] Miyato N., Kishimoto Y. and Li J.Q. 2006 Plasma Phys. Control. Fusion 48 A335
[29] Sugama H., Watanabe T.-H. and Nunami M. 2009 Phys. Plasmas 16 112502
[30] Tani K., Azumi M., Kishimoto H. and Tamura S. 1981 J. Phys. Soc. Japan 50 1726
[31] Yoshida M. et al 2006 Plasma Phys. Control. Fusion 48 1673
[32] Toda S. et al 2014 J. Phys.: Conf. Ser. 561 012020
[33] Staebler G.M., Kinsey J.E. and Waltz R.E. 2007 Phys. Plasmas 14 055909
[34] Ernst D.R. et al 2009 Phys. Plasmas 16 055906
[35] Lang J., Chen Y. and Parker S.E. 2007 Phys. Plasmas 14 082315
[36] Lang J., Parker S.E. and Chen Y. 2008 Phys. Plasmas 15 055907
[37] Idomura Y., Urano H., Aiba N. and Tokuda S. 2009 Nucl. Fusion 49 065029
[38] Yi S., Kwon J.M., Diamond P.H. and Hahm T.S. 2015 Nucl. Fusion 55 092002