Entanglement concentration by ordinary linear optical devices without post-selection

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Abstract

Recently, entanglement concentrations have been experimentally demonstrated by post-selection (T. Yamamoto et al, Nature, 421, 343(2003), and Z. Zhao et al, Phys. Rev. Lett., 90,207901(2003) ), i.e., to each individual outcome state, one has to destroy it to know whether it has been purified. Here we give proposal for entanglement concentration without any post-selection by using only practically existing linear optical devices. In particular, a sophisticated photon detector to distinguish one photon or two photons is not required.

The resource of maximally entangled state(EPR state) plays a fundamentally important role in testing the quantum laws related to the non-locality [1] and in many tasks of quantum information processing [2,6] such as the quantum teleportation [3,4], quantum dense coding [3], entanglement based quantum key distribution [5] and quantum computation [6]. So far, it is generally believed that the two-photon polarized EPR state is particularly useful in quantum information processing.

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If certain type of non-maximally entangled states are shared by distant two parties Alice and Bob initially, the raw states can be distilled into highly entangled states by local quantum operation and classical communication through the entanglement concentration scheme [7]. Although research on such issues have been extensively done theoretically, the feasible experimental schemes and experimental demonstrations of the entanglement concentration are rare. So far, some schemes by linear optical devices have been proposed [8,9] and some experimental demonstrations have also been reported [10,12,13]. In all these schemes [8–12], one has to verify that each of the indicator light beams contains exactly one photon. However, by our current technology, it’s not likely to really implement a sophisticated detector [19] in the scheme to distinguish one photon or two photon in a light beam. What we can see from a normal photon detector is that whether it is clicked or not. When it is clicked, the measured light beam could contain either exactly one photon or 2 photons in the previously proposed entanglement concentration schemes [8–10]. In a recent experimental report [10], the unwanted events that one indicator beam contain two photons are excluded by post selection: Both the two indicator beams and the two outcome beams are measured. If all the 4 detectors are clicked, the maximally entangled state must have been created on the two outcome beams. Such a post selection destroys the outcome itself. That is to say, limited by the available technology of photon detector, one can only verify a maximally entangled state by totally destroying that state. This means, without a sophisticated photon detector, the set-up [10] is not supposed to really produce any maximally entangled state through the entanglement concentration, even though the requested raw states are supplied deterministically. A similar drawback also appears in [8,11,13]. The very recent experiment [12] also relies on post-selection.

So far non-post-selection entanglement concentration in polarization space with linear optical devices has never been proposed, though there are some studies on the possibility of the entanglement concentration to continuous variable states through the Gaussification scheme [14,15].

In the situation of Ref. [10,8,12], the entanglement is in the two level polarization space
and all pairs are equally shared by two remotely separated parties. This case is rather important because the polarization entanglement is easy manipulate, e.g., the local rotation operation. Also the assumption of two pair states with unknown identical parameters is reasonable in cases such as that Alice sends two halves of EPR pairs to Bob through the same dephasing channel. Note that here each group contains two identical pairs with unknown parameters, the parameters for the pair states in different groups are different. Now we go to the main result of this Letter.

We will show that the following raw state

$$|r, \phi\rangle = \frac{1}{1 + r^2}(|HH\rangle + re^{i\phi}|VV\rangle) \otimes (|HH\rangle + re^{i\phi}|VV\rangle)$$

(1)

can be probabilistically distilled into maximally entangled state without post selection, even though only normal photon detectors are used. Note that in Eq.(1) $r > 0$, both $r$ and $\phi$ are unknown parameters. In particular, the special case $r = 1$ is just the one treated in the recent experiment [10]. Since the raw state contains unknown parameters, our purification(concentration) scheme is actually to distill the maximum entanglement from the un-normalized mixed state

$$\int_0^\infty \int_0^{\pi} |r, \phi\rangle \langle r, \phi|drd\phi.$$ 

(2)

However, for simplicity we shall use the pure state notation $|r, \phi\rangle$, keeping in the mind that both parameters are totally unknown. Consider the schematic diagram, figure 1. This diagram is a modified scheme from the one given by Pan et al in Ref. [8]. However, as we shall see, the modification leads to a totally different result: Our scheme uniquely produces the even-ready maximally entangled state through entanglement purification, provided that the requested state in Eq.(2) is supplied.

Our scheme requires the two fold coincidence event as the indication that the maximally entangled state has been produced on the outcome beams 2’,3’, i.e. whenever both photon detectors $D_x, D_w$ click, the two outcome beams, beam 2’ and beam 3’ must be in the maximally entangled state:
\[ |\Phi^+_{2',3'} \rangle = \frac{1}{\sqrt{2}} (|H\rangle_{2'}|H\rangle_{3'} + V\rangle_{2'}|V\rangle_{3'}). \quad (3) \]

Now we show the mathematical details for the claim above. The polarizing beam splitters transmit the horizontally polarized photons and reflect the vertically polarized photons. For clarity, we use the Schrödinger picture. And we assume the non-trivial time evolutions to the light beams only takes place in passing through the optical devices.

Consider Fig.(1). Suppose initially two remote parties Alice and Bob share two pairs of non-maximally entangled photons as defined by Eq.(1), denoted by photon pair 1,2 and photon 3,4 respectively. The half wave plate HWP1 here is to change the polarization between the horizontal and the vertical. After photon 3 and 4 each pass through HWP1, the state is evolved to:

\[ \frac{1}{1 + r^2} (|HH\rangle_{12} + re^{i\phi}|VV\rangle_{12}) \otimes (|VV\rangle_{34} + re^{i\phi}|HH\rangle_{34}). \quad (4) \]

Furthermore, after the beams pass through the two horizontal polarizing beam splitters (denoted by PBS1), with perfect synchronization [16], the state is

\[ |\chi' \rangle = \frac{1}{1 + r^2} (|H\rangle_{1'}|H\rangle_{2'} + re^{i\phi}|V\rangle_{3'}|V\rangle_{4'}) \otimes (|V\rangle_{1}|V\rangle_{2} + re^{i\phi}|H\rangle_{3}|H\rangle_{4}). \quad (5) \]

This can be recast to the summation of three orthogonal terms:

\[ |\chi' \rangle = \frac{1}{1 + r^2} (|A\rangle + |B\rangle + |C\rangle) \quad (6) \]

where

\[ |A\rangle = re^{i\phi}(|H\rangle_{1'}|H\rangle_{2'}|H\rangle_{3'}|H\rangle_{4'} + |V\rangle_{1'}|V\rangle_{2'}|V\rangle_{3'}|V\rangle_{4'}); \quad (7) \]

\[ |B\rangle = |H\rangle_{1'}|V\rangle_{1'}|H\rangle_{2'}|V\rangle_{2'}; \quad (8) \]

\[ |C\rangle = r^2 e^{2i\phi}|H\rangle_{3'}|V\rangle_{3'}|H\rangle_{4'}|V\rangle_{4'}. \quad (9) \]

Obviously, term \(|B\rangle\) means that there is no photon in beam 4’ therefore this term will never click detector \(D_w\). Consequently term \(|B\rangle\) will never cause the two fold coincidence.
Similarly, term $|C\rangle$ means there is no photon in beam 1’ therefore it will never cause the required two fold coincidence either. Since neither term $|B\rangle$ nor term $|C\rangle$ will cause the required two fold coincidence, we disregard these two terms and only consider term $|A\rangle$ hereafter. The overall factor $\frac{1}{1+r^2}$ or $re^{i\phi}$ plays no role in any physical results therefore is replaced by a normalization factor hereafter.

The HWP2 takes a $\pi/4$ rotation of the beam’s polarization, i.e., it changes $|H\rangle$ into $\frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ and $|V\rangle$ into $\frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$. After the beams pass through HWP2, state $|A\rangle$ is evolved to

$$|\Phi^+\rangle_{1''4''}|\Phi^+\rangle_{2'3'} + |\Psi^+\rangle_{1''4''}|\Phi^-\rangle_{2'3'}$$

(10)

where $|\Phi^\pm\rangle_{ij} = \frac{1}{\sqrt{2}}(|H\rangle_i|H\rangle_j \pm |V\rangle_i|V\rangle_j$ and $|\Psi^\pm\rangle_{ij} = \frac{1}{\sqrt{2}}(|H\rangle_i|V\rangle_j \pm |V\rangle_i|H\rangle_j$. After pass through the two PBS2 in the figure, $|A\rangle$ is evolved to state

$$|x\rangle|w\rangle|\Phi^+\rangle_{2'3'} + |y\rangle|z\rangle|\Phi^+\rangle_{2'3'} + (|x\rangle|z\rangle + |y\rangle|w\rangle)|\Phi^-\rangle_{2'3'}$$

(11)

where $|s\rangle$ denote the state of one photon in beam $s$, $s$ can be $x, y, z$ or $w$. From the above formula we can see that only the first term will cause the two fold coincidence event that both detectors are clicked. And we see that this term indicates a maximally event-ready entangle state between beam 2’ and 3’, i.e. $\frac{1}{\sqrt{2}}(|H\rangle_{2'}|H\rangle_{3'} + |V\rangle_{2'}|V\rangle_{3'})$. Note that our result here is independent of the parameters $r, \phi$. Also, the quality of the outcome is not affected by the efficiency of the photon detectors. Actually, the overall efficiency of the scheme can be increased by 4 times with two more photon detectors $D_y, D_z$, detecting the beam $y$ and beam $z$ respectively.

To really produce the event-ready entanglement through our purification scheme one need the deterministic supply of the requested raw states. This is rather challenging a task. However, even without such a deterministic supply, one can still experimentally demonstrate that our scheme can produce the event-ready maximally entangled pair if the deterministic supply is offered. In a real experiment to demonstrate our scheme, one can probabilistically produce the requested non-maxmally entangled initial state by the type II SPDC process
or other processes for the pair 1,2 and the pair 3,4 [9,17,18]. In such a case it is also possible that actually the pair 1,2 contains nothing while 3,4 contains two pairs, and vice versa. However, one may discard those cases by a post selection, i.e., by studying the 4 fold coincidence. We must point out that, this post selection method does not affect the non-post-selection nature of our scheme: Limited to the imperfection in the raw state preparation, the experimental motivation here is not the very ambitious one to really produce the event-ready maximally entangled pairs, instead, it is to verify that this set-up can produce the event-ready maximally entanglement pairs provided that the requested raw state \( |r, \phi \rangle \langle r, \phi | drd\phi \) is supplied deterministically. The post selection here is only to exclude those events where a wrong raw state had been produced, it is not used to exclude the corrupted outcome due to the imperfection of the devices, such as the yes-no photon detector. Here the manufacturer can claim safely to their customers that the set-up produces event-ready EPR pairs provided that the customers input the requested raw states.

With the SPDC process, the initial state prepared for pair 1,2 and pair 3,4 by Fig.1,2 is

\[
|in\rangle = 2|raw_1\rangle + \sqrt{3}|u_1\rangle + \sqrt{3}|u_2\rangle
\]  

(12)

where state \( |raw_1\rangle = \frac{1}{1+r^2}(a_{1H}^\dagger a_{2H}^\dagger + re^{i\phi}a_{1V}^\dagger a_{2V}^\dagger)(a_{3H}^\dagger a_{4H}^\dagger + re^{i\phi}a_{3V}^\dagger a_{4V}^\dagger)|0\rangle \) is just the requested raw state of our entanglement concentrator, \( |u_1\rangle = \frac{1}{(1+r^2)^{\frac{1}{2}}}(a_{1H}^\dagger a_{2H}^\dagger + e^{i\phi}a_{1V}^\dagger a_{2V}^\dagger)|0\rangle \) and \( |u_2\rangle = \frac{1}{(1+r^2)^{\frac{1}{2}}}(a_{3H}^\dagger a_{4H}^\dagger + e^{i\phi}a_{3V}^\dagger a_{4V}^\dagger)|0\rangle \). Note that we have omitted the overall normalization factor which plays no role here.

Consider the scheme in Fig.2. Note that R there is a phase shifter which offers a shift \( \theta \) randomly chosen from \( \{ \theta_1 = 0, \theta_2 = \pi/2, \theta_3 = -\pi/2, \theta_4 = \pi \} \). Therefore the input state is now

\[
\rho = \frac{1}{4} \sum_{j=1}^{4} |\theta_j\rangle \langle \theta_j|
\]  

(13)

and \( |\theta_j\rangle = 2e^{i\theta_j}|raw_1\rangle + \sqrt{3}e^{2i\theta_j}|u_1\rangle + \sqrt{3}|u_2\rangle \). By a straightforward calculation we have

\[
\rho = 4|raw_1\rangle \langle raw_1| + 3|u_1\rangle \langle u_1| + 3|u_2\rangle \langle u_2|.
\]  

(14)
Therefore the input state is now in a classical probabilistic mixture of the requested raw state \( |\text{raw}_1\rangle \) and the unwanted states \( |u_1\rangle, |u_2\rangle \). This is to say, if we input the state for 1000 times, 400 of them are the requested raw state \( |\text{raw}_1\rangle \) and 300 of them are \( |u_1\rangle \) and 300 of them are \( |u_2\rangle \). We shall first observe the consequence \( C_1 \) caused by only inputting state \( |u_1\rangle \langle u_1| \) for 300 times, then observe the consequence \( C_2 \) caused by only inputting state \( |u_2\rangle \langle u_2| \) for 300 times. We finally observe the consequence caused by sending \( \rho \) for 1000 times. This consequence, with subtraction of \( C_1 \) and \( C_2 \), is the net consequence caused by the 400 times inputs of requested raw state \( |\text{raw}_1\rangle \langle \text{raw}_1| \). [20]

To only input state \( |u_1\rangle \langle u_1| \) we block beam 3,4 and input only the pair 1,2 into the set-up in Fig.2. Suppose we input 300 copies of state \( |u_1\rangle \langle u_1| \). In such a case, the only type of 4-fold clicking is \( (D_x, D_w, D_{2H}, D_{3V}) \). Suppose we have observed \( N \) times of such kind 4-fold clicking. Other types of four fold simultaneous clicking, \( (D_x, D_w, D_{2H}, D_{3H}) \) or \( (D_x, D_w, D_{2V}, D_{3V}) \) or \( (D_x, D_w, D_{2V}, D_{3H}) \) will never be observed [21]. With the same source, if we rotate both beam 2’ and beam 3’ by \( \pi/4 \) before detection, we shall find \( N/4 \) times of 4-fold events for each of the simultaneous clicking of \( (D_x, D_w, D_{2H}, D_{3V}), (D_x, D_w, D_{2V}, D_{3H}), (D_x, D_w, D_{2H}, D_{3H}) \) and \( (D_x, D_w, D_{VH}, D_{3V}) \).

Similarly, if we block beam 1 and 2 and input state \( |u_2\rangle \langle u_2| \) for 300 times, we shall observe \( N \) times of 4-fold clicking \( (D_x, D_w, D_{2V}, D_{3H}) \) and no other types of 4-fold clicking. And also, if we insert a \( \pi/4 \) HWP to both beam 2’ and beam 3’ and repeat the test, we shall find \( N/4 \) times of 4-fold events for each of the simultaneous clicking of \( (D_x, D_w, D_{2H}, D_{3V}), (D_x, D_w, D_{2V}, D_{3H}), (D_x, D_w, D_{2H}, D_{3H}) \) and \( (D_x, D_w, D_{VH}, D_{3V}) \).

Keeping these facts in the mind we now consider the test by Fig. 2 with the input state \( \rho \) given by Eq.(14). We input state \( \rho \) for 1000 times. Physically, this is equivalently to input each of state \( |\text{raw}_1\rangle, |u_1\rangle \) and \( |u_2\rangle \) for 400 times, 300 times and 300 times, respectively. We shall observe \( N \) times four fold coincidence events events of each of \( (D_x, D_w, D_{2H}, D_{3V}), (D_x, D_w, D_{2V}, D_{3H}), (D_x, D_w, D_{2H}, D_{3H}), (D_x, D_w, D_{2V}, D_{3V}) \) without inserting HWP to beam 2’ or beam 3’. However, as we have already known that, among all these 4-fold events, the 300 inputs of \( |u_1\rangle \langle u_1| \) have caused \( N \) times of 4-fold clicking \( (D_x, D_w, D_{2H}, D_{3V}); \)
the 300 inputs of $|u_2\rangle\langle u_2|$ have caused $N$ times 4-fold clicking of $(D_x, D_w, D_{2V}, D_{3H})$. This is to say, the only 4-fold events caused by the 400 times inputs of $|\text{raw}_1\rangle\langle \text{raw}_1|$ are just the $(D_x, D_w, D_{2H}, D_{3H})$ and $(D_x, D_w, D_{2V}, D_{3V})$. This shows the following conclusion:

**Conclusion 1:** We shall only observe 4-fold events of either $(D_x, D_w, D_{2H}, D_{3H})$ or $(D_x, D_w, D_{2V}, D_{3V})$, if the input is deterministically $|\text{raw}_1\rangle\langle \text{raw}_1|$

We then insert a $\pi/4$ HWP to both beam 2' and beam 3'. This time we shall find $N/2$ four fold events of $(D_x, D_w, D_{2H}, D_{3V})$, $N/2$ four fold events of $(D_x, D_w, D_{2V}, D_{3H})$, $3N/2$ times of four fold events of $(D_x, D_w, D_{2H}, D_{3H})$ and $3N/2$ times of four fold events of $(D_x, D_w, D_{2V}, D_{3V})$. However, as we have already known from the previous paragraphs, in such a case the 300 times of inputs of $|u_1\rangle\langle u_1|$ and $|u_2\rangle\langle u_2|$ have caused $N/2$ times of four fold coincidence events for each of the four types of four fold simultaneous clicking. Therefore the net result caused by the 400 time inputs of $|\text{raw}_1\rangle\langle \text{raw}_1|$ is only the four fold events of $(D_x, D_w, D_{2H}, D_{3H})$ and $(D_x, D_w, D_{2V}, D_{3V})$, each appearing $N$ times. This shows

the following conclusion:

**Conclusion 2:** After inserting a $\pi/4$ HWP to both beam 2' and beam 3', one shall only observe 4-fold events of either $(D_x, D_w, D_{2H}, D_{3H})$ or $(D_x, D_w, D_{2V}, D_{3V})$, if the input is $|\text{raw}_1\rangle\langle \text{raw}_1|$.

Combining conclusion 1 and conclusion 2 we conclude that no matter whether we insert $\pi/4$ HWP to beam 2', 3', the net four fold events caused by $|\text{raw}_1\rangle\langle \text{raw}_1|$ are only $(D_x, D_w, D_{2H}, D_{3H})$ and $(D_x, D_w, D_{2V}, D_{3V})$. Therefore the non-post-selection entanglement concentrator is demonstrated. In carrying out the experiment, one should make sure $N$ large enough so that $\sqrt{\frac{N}{2}} < < 1$, to reduce the statistical error. This requires that the set-up has to be stable for several hours [12].

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[20] Consider an analog classical story: The fake coin and the good coin cannot be distinguished by eyes. Suppose we have produced a machine and we want to demonstrate the claim that whenever you insert a good coin, it produces a bottle of wine. However, the machine itself has a property that if you insert a fake coin, it produces a bottle of water. We don’t have any supply of pure good coins. But we have pure supply of fake coins, supply \( B \) and mixed supply of half good coins and half fake coins, supply \( A \). We input 200 coins from supply \( A \) and find 100 bottles of wine and 100 bottles of water. We then input 100 coins from supply \( B \) and find 100 bottles of water. Our claim is verified by the combination of these two facts. In our main text the role of \( \rho \) is similar to supply \( A \) while \( |u_{1,2}\rangle\langle u_{1,2}| \) is similar to supply \( B \).

[21] With beam 3 and beam 4 being blocked, the input state is now \( |u_1\rangle \) and the state of beam 1’’,2’,3’ and 4’’ is \( \frac{1}{\sqrt{2}}[|2H\rangle_1\nu|2H\rangle_2' + r^2 e^{i2\phi}|2V\rangle_3'|2V\rangle_4' + \frac{1}{2}r e^{i\phi}(|H\rangle_1\nu + |V\rangle_1\nu)|H\rangle_2'|V\rangle_3'(|H\rangle_4' - |V\rangle_4') \] the four fold event is uniquely caused by the term \( \frac{1}{2}r e^{i\phi}(|H\rangle_1\nu + |V\rangle_1\nu)|H\rangle_2'|V\rangle_3'(|H\rangle_4' - |V\rangle_4') \). Whenever we see a four fold event here, it must be \((D_x, D_w, D_{2H}, D_{3V})\).
FIG. 1. Non-post-selection quantum entanglement concentration by practically existing devices of linear optics. The two fold coincidence event of both detector $D_x$ and detector $D_w$ being clicked indicates that a maximally entangled state is produced on beam 2 and 3. PBS: polarizing beam-splitter. Here HWP1 rotates the polarization by $\pi/2$, HWP2 rotates the polarization by $\pi/4$.

FIG. 2. A schematic scheme to verify that all the four fold coincidence events caused by the state $|raw_1\rangle\langle raw_1|$ are $D_x, D_w, D_{2H}, D_{3H}$ or $D_x, D_w, D_{2V}, D_{3V}$, no matter whether we take a $\pi/4$ polarization rotation to beam 2’ and beam 3’ before the detection. $R$ is a phase phase shift randomly chosen from $\{0, \pi/2, -\pi/2, \pi\}$. The square box of “Entanglement Concentrator” is just the set-up given by Fig.1 with the two PBS2 and two photon detectors being cut off.