Prediction of two-phase pressure drop in heat exchanger for mixed refrigerant Joule–Thomson cryocooler

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Abstract. The overall efficiency of a mixed refrigerant Joule–Thomson (MR J–T) cryocooler is governed by the performance of the recuperative heat exchanger. In the heat exchanger, the hot stream of the mixed refrigerant undergoes condensation at high pressure while the cold stream gets evaporated at low pressure. The pressure drop in the low pressure stream is crucial since it directly influences the achievable refrigeration temperature. However, experimental and theoretical studies related to two-phase pressure drop in mixtures at cryogenic temperatures, are limited. Therefore, the design of an efficient MR J–T cryocooler is a challenging task due to the lack of predictive tools.

In the present work, the existing empirical correlations, which are commonly used for the prediction of pressure drop in the case of pure refrigerants, evaporating at near ambient conditions, are assessed for the mixed refrigerants. Experiments are carried out to measure the overall pressure drop in the evaporating cold stream of the tube-in-tube helically coiled heat exchanger. The predicted frictional pressure drop in the heat exchanger is compared with the experimental data. The suggested empirical correlations can be used to predict the hydraulic performance of the heat exchanger.

1. Introduction
Joule-Thomson (J–T) cryocoolers are known for fast cool-down time, low cost and low vibrations at the cold end because of no moving parts in the cold section. The use of a refrigerant mixture of nitrogen-hydrocarbons as a working medium in these cryocoolers enhances their efficiency. Additionally, pressure requirements drastically get reduced up to 2 MPa in comparison to that of using a pure component as a working fluid, which allow the use of oil-lubricated compressors. However, the overall performance of such mixed refrigerant Joule–Thomson (MR J–T) cryocoolers greatly depends on the refrigerant mixture, heat exchanger and compressor. Numerous experimental and theoretical studies of MR J–T cryocoolers operating in the temperature range of 80–230 K have been reported in the literature [1-4]. These studies are mainly related to the optimization of mixtures used and the overall performance of the refrigeration systems. However, little work [5-7] is done on the performance analysis of the recuperative heat exchanger used in the MR J–T cryocooler, even though it plays a crucial role in its efficient operation. This is mainly due to the lack of experimental data available, related to flow boiling/condensation heat transfer and pressure drop of mixed refrigerants at cryogenic temperatures. The refrigerant mixture undergoes boiling and condensation heat transfer simultaneously, in the counter-flow heat exchanger. The hydraulic diameter of the heat exchangers is...
in the range of 2–3 mm to have better thermal performance. However, unfortunately, the small channel diameters lead to a higher pressure drop which may adversely influence the effectiveness of the heat exchanger. Therefore, accurate predictive tools for both pressure drop and heat transfer coefficients are necessary to design the heat exchanger for the efficient operation of the cryocooler.

A closed cycle MR J–T cryocooler mainly comprises of a compressor, an after cooler, a recuperative counter flow heat exchanger, an expansion device and an evaporator. The compressor compresses the refrigerant which is partially cooled in the after cooler and further cooled inside the tube of the heat exchanger by the return line low pressure refrigerant stream flowing through the annulus of the heat exchanger. Throttling the high pressure refrigerant in the expansion device results in the low pressure, low temperature stream. This low pressure, low temperature stream produces a cooling effect at the evaporator and then returns to the compressor through the heat exchanger.

There are many experimental studies and models available in the literature to predict the frictional pressure drop in macro and micro channels of the heat exchangers. These models are mainly developed for pure component fluids such as air, water, CFC, HCFC and HFC refrigerants and their mixtures. These methods are validated only for specific flow configurations and relatively narrow ranges of operating conditions. There is no generalized correlation for two-phase frictional pressure drop in the literature, which is applicable to a wide range of working fluids, mass velocities, pressures and channel diameters. On the other hand, studies related to two-phase pressure drop in the MR J–T heat exchanger are scarce. Baek et al. [8] measured the pressure drop in low pressure channels of the heat exchangers for a MR J–T cryocooler during cool-down. They concluded that the pressure drop of a mixed refrigerant flow can be estimated with pure fluid two-phase pressure drop correlations.

In view of the above, the present study attempts to assess the existing empirical correlations for prediction of frictional pressure drop in the cold stream of the heat exchanger. Experiments are carried out to measure the overall pressure drop across the MR J–T heat exchanger. The distribution of pressure along the length of the heat exchanger is obtained using various well known existing correlations based on the homogeneous and the separated flow models. The predicted pressure drop is compared with the experimentally obtained results for two different mixture compositions.

2. Two-phase pressure drop correlations
Two-phase pressure drop is the sum of the static pressure drop, the momentum pressure drop and the frictional pressure drop. It is essential to have the knowledge of void fraction to calculate static and momentum components of the pressure drop. No study is found related to prediction of void fraction for flow boiling of mixtures at cryogenic temperatures in the open literature. In the present analysis, the static and momentum pressure drop is neglected. The correlations, commonly used to estimate frictional pressure drop, are based on either a homogeneous flow or separated flow models. The existing correlations which are evaluated in the present study are described in the following section.

2.1 Homogeneous flow model (HFM)
Homogeneous flow models (HFM) treat two-phase flow as a single phase fluid flow with averaged properties of the liquid and the vapor phase. It assumes no difference in the velocities of the two phases of the fluid. This approach is typically valid for flow regimes such as mist flow or bubbly flow where one phase is evenly dispersed into the other [9]. Whalley [10] argued that the homogeneous model is suitable to calculate the frictional pressure drop for the mass velocities greater than 2000 kg/m$^2$s. The two-phase frictional pressure drop, $\Delta P_{frict}$ is given in equation (1).

\[
\Delta P_{frict} = 4 f_{tp} \frac{L}{d_h} \frac{G^2}{\rho_{tp}}
\]  

where $L$ is length, $G$ is mass flux and $d_h$ is hydraulic diameter. The two-phase friction factor $f_{tp}$ is calculated on the basis of the two-phase Reynolds number, $Re_{tp}$ as expressed in equation (2).
where, $\mu_{tp}$ is two-phase viscosity. The effective density of two-phases, $\rho_{tp}$ is given in equation (3).

$$\frac{1}{\rho_{tp}} = \frac{x}{\rho_f} + \frac{(1-x)}{\rho_g}$$

where $\rho_f$ and $\rho_g$ is liquid and vapour phase density, respectively and $x$ is vapour quality. Table 1 gives commonly used two-phase mixture viscosity models.

### 2.2 Separated flow model (SFM)

The separated flow model (SFM) on the other hand, considers the flow of the two phases distinctly along with the interaction between them. The Lockhart-Martinelli two-phase multiplier is the most typical form of the separated flow model. The total frictional pressure gradient in the two-phase flow is calculated with a frictional multiplier, $\phi_f$ as expressed in equation (4).

$$\frac{(dp)}{dz}_{tp} = \phi_f^2 \left( \frac{(dp)}{dz}_{lf} \right)_{f}$$

where, $\left( \frac{(dp)}{dz} \right)_l$ is the frictional pressure gradient if the liquid phase flows alone in the channel. Lockhart and Martinelli [14] correlated the frictional multiplier, $\phi_f$ in terms of a parameter $X$. Later, Chisholm [15] presented the simplified form of the correlation for the two-phase multiplier as a function of the Lockhart-Martinelli parameter, $X$, and the coefficient $C$ as given in equation (5).

$$\phi_f^2 = 1 + \frac{C}{X} + \frac{1}{X^2}$$

The constant $C$ is viewed as the interaction parameter between the liquid and the vapor phases which is varied from 5 to 20, based upon whether the liquid or the vapor phase is in the laminar or turbulent regime. Following the work of Lockhart and Martinelli [14], several correlations are proposed for predicting two-phase pressure drop in large diameter tubes. The widely used empirical correlations to predict pressure drop for flows through macro channels are Lockhart and Martinelli [14], Friedel [16], Gronnerud [17], Muller-Steinhagen and Heck [18], Chisholm [19] and Sami and Duong [20] correlation, which are evaluated in the present work. OuldDidi et al. [21] assessed various correlations against the two-phase pressure drop data for different refrigerants and found that the Muller-Steinhagen and Heck [18] correlation gives the best prediction for the annular flow regime.

### Table 1. Two-phase mixture viscosity models

| Author(s)             | Equation                                      |
|-----------------------|-----------------------------------------------|
| McAdams et al. [11]   | $\frac{1}{\mu_{tp}} = \frac{x}{\mu_f} + \frac{(1-x)}{\mu_g}$ |
| Cicchitti et al. [12] | $\mu_{tp} = x \mu_f + (1-x) \mu_g$            |
| Dukler et al. [13]    | $\mu_{tp} = \rho_{tp} \left[ \frac{x \mu_g}{\rho_f} + \frac{(1-x) \mu_f}{\rho_f} \right]$ |
Most of the available empirical correlations for the prediction of pressure drop in mini/micro-channels are based on the Lockhart-Martinelli [14] method. Several researchers focused their work on determination of accurate values of $C$ in order to improve the accuracy of separated flow models and to consider micro scale effects. In the present study, Mishima and Hibiki [22], Yu et al. [23], Lee and Mudawar [24], Li and Wu [25], Zhang et al. [26] and Kim and Mudawar [27] correlations, which are developed for micro-scale channels, are selected to predict two-phase frictional pressure drop in the heat exchanger. More details on the two-phase pressure drop models for micro-scale channels can be found in the recent review articles [28-29].

In the present work, three different correlations based on the homogeneous model from Table 1 and twelve correlations which are developed for macro-scale and micro-scale channels based on separated flow model are used to determine two-phase frictional pressure drop. These correlations are integrated numerically along the length of the heat exchanger to predict the total frictional pressure drop. The properties of the mixture are updated locally along the length of the heat exchanger.

3. Experimental set-up

Figure 1 (a) shows the schematic of the experimental set-up. It is described in detail elsewhere by the authors [7]. It mainly consists of a compressor, an after-cooler, oil filters, a heat exchanger, an expansion device, and an evaporator. The heat exchanger, the capillary tube and the evaporator are insulated with multi-layer insulation (MLI) and are placed in a stainless steel vessel in which, a vacuum of the order of $10^{-5}$ mbar is maintained using a diffusion pump. Figure 2 shows the photograph of a helically coiled heat exchanger, which is a concentric tube-in-tube arrangement. The inside and outside diameter of the inner tube is 4.83 and 6.35 mm respectively, whereas it is, 7.89 and 9.52 mm respectively, for the outer tube. The length of the concentric tubes is 15 m. The suction and the discharge pressures of the compressor are measured by two pressure gauges (Make: WIKA) located at the inlet and the outlet of the compressor, respectively. Pressures of the low and the high pressure stream are measured both at the inlet and the outlet to the heat exchanger with the help of piezo-resistive type pressure sensors (Make: Endevco, UK).

The temperature sensors (PT100) are used to measure temperatures of the hot and the cold fluid at an interval of 1.5 m along the length of the heat exchanger as shown in Figure 1 (b). All the temperature sensors are calibrated down to liquid nitrogen temperature (77 K). The temperatures of the hot and the cold fluid are recorded at the steady state using the data logging system, Data Taker-800 and are averaged over the period of a minimum 10 minutes. A rotameter is installed in the suction line to measure the volume flow rate of the refrigerant. The mass flow rate of the mixture is calculated using the density of the mixture in circulation, at the inlet conditions. The mixture composition in circulation is measured using a gas chromatograph (Make: Perkin Elmer-Clarus 500GC).

Figure 1. (a) Experimental set-up [7] (b) Locations of sensors

Figure 2. Photograph of helical heat exchanger
4. Results and discussion
Experiments are conducted on a MR J–T cryocooler to determine the total pressure drop along the length of the heat exchanger. Two specific compositions of the mixture of gases viz. nitrogen, methane, ethane, propane and iso-butane are used as a refrigerant in the system. Table 2 gives the composition of the charged mixture, and operating conditions like mass velocity, pressure and temperature for the cold fluid in the heat exchanger. The mass velocity of the cold fluid is calculated on the basis of the annulus area between the two tubes. The molar percentage of the higher boiling point components such as propane and iso-butane is nearly the same for both mixtures. However, the molar percentage of the low boiling point components such as nitrogen is significantly more in Mix#1 than that for Mix#2, while that of methane in Mix#1 is relatively less. These mixtures are designed to produce different refrigeration temperatures in the range of 100–125 K. The heat exchanger essentially operates in the two-phase region at such low temperatures. Figure 3 shows the temperature-entropy diagram for one of the mixtures used in the present work for which pressure variations are not considered. It can be noted from the figure that both the condensing and the evaporating stream remain in two-phase state over the greater portion of the heat exchanger.

During experimentation, it is observed that, the cold fluid enters the heat exchanger in the two-phase state for all the tests conducted on different mixtures. On the other hand, the cold fluid leaves the heat exchanger at a temperature greater than its dew point temperature indicating single-phase state of the fluid. The length of the heat exchanger over which the mixed refrigerant is in single-phase state depends on the mixture composition and operating conditions in the cryocooler. The temperature profile for the cold fluid flowing through the annulus is known. Therefore, the length of the single-phase region at the hot end of the heat exchanger can be calculated easily using dew point temperature of the mixture under consideration. The length of the two-phase region is obtained by subtracting the length of the single-phase region from the total length of the heat exchanger.

| Mixture | Composition (% mol) (N₂/CH₄/C₂H₆/C₃H₈/iC₄H₁₀) | Mass flux, G (kg/m²s) | Cold fluid temperature, (K) | Pressure, (bar) |
|---------|---------------------------------------------|-----------------------|----------------------------|-----------------|
|         |                                             |                       | Inlet          | Outlet          | Inlet | Outlet|
| Mix#1   | 36.0/15.0/13.0/19.0/17.0                    | 215                   | 100.2          | 293.5           | 5.61  | 2.61  |
|         |                                             | 167                   | 108.8          | 293.8           | 5.23  | 2.41  |
|         |                                             | 132                   | 112.7          | 294.7           | 4.25  | 2.01  |
| Mix#2   | 15.5/31.0/16.5/21.0/16.0                    | 151                   | 119.1          | 297.6           | 5.32  | 2.31  |
|         |                                             | 146                   | 125.4          | 300.1           | 4.94  | 2.31  |

Figure 3. Temperature–entropy diagram of Mix#1
The single-phase pressure drop is determined using the conventional Blasius correlation [10]. The experimental two-phase frictional pressure drop is obtained by subtracting the single-phase pressure drop from the measured value of the total pressure drop for the cold fluid. In the present case, the effect of helical channel on the pressure drop is neglected. The test conducted on each mixture is repeated at least three times to ensure repeatability of the results. All the properties of the mixture are calculated using aspenONE [30] considering Peng-Robinson equation of state [31]. The experimental two-phase frictional pressure drop data are compared with 15 existing correlations which are used for flows through both macro and mini/micro channels. The average absolute deviations (AAD) for the predicted results are calculated using equation (6), in order to find applicability of the correlations.

\[
AAD = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{\Delta P_{\text{exp}} - \Delta P_{\text{predicted}}}{\Delta P_{\text{exp}}} \right| \times 100
\]

where \( N \) is the number of data points, \( \Delta P_{\text{exp}} \) and \( \Delta P_{\text{predicted}} \) are the frictional pressure drop obtained from experiment and model, respectively. Table 3 gives the AAD of the predicted pressure drop from the existing correlations with the experimental frictional pressure drop. It is noted that the differences between the predictions obtained from different correlations and the experimental data are significant.

It is clear from the table that two correlations for macro channel and two correlations for micro channel show the best predictions among all the predicted results of pressure drop. Namely, Zhang et al. [26], Kim and Mudawar [27], Muller-Steinhagen and Heck [18] and Sami and Duong [20] correlation are relatively close to the experimental data. Interestingly, in the case Mix#1, these correlations over predict the experimental pressure drop for higher mass velocity of 215 kg/m\(^2\)s. It is found that the Zhang et al. [26] and Kim and Mudawar [27] correlations which are developed for micro-channels predict the experimental data within 30 % error limit. These correlations give the best prediction of the frictional pressure drop among all the 15 correlations assessed in the present work. However, some correlations are recommended for specific laminar or turbulent flow states. For example, Zhang et al. [26] correlation is not suitable when the flow is turbulent liquid and turbulent vapour flow. In the present study, it is found that the liquid flow is laminar, \( \text{(Re}_{l} < 2000) \) while the vapour flow is turbulent.

**Table 3.** Assessment of existing two-phase frictional pressure drop correlation

| Correlation                  | Mass velocity, \( G \) (kg/m\(^2\)s) | AAD, %  |
|-----------------------------|---------------------------------------|---------|
|                             |                                       | Mix#1   | Mix#2   |
| Homogeneous model (macro)   |                                       | 215     | 167     |
| McAdams et al. [11]         |                                       | 21.4    | 44.4    |
| Cicchitti et al. [12]       |                                       | 126.8   | 23.0    |
| Dukler et al. [13]          |                                       | 21.5    | 44.3    |
| SFM: Macro-scale model      |                                       | 143.2   | 102.0   |
| Lockhart-Martinelli [14]    |                                       | 62.4    | 7.9     |
| Friedel [16]                |                                       | 125.4   | 59.8    |
| Gronnerud [17]              |                                       | 31.6    | 13.6    |
| Muller-Steinhagen and Heck [18] |                                   | 143.6   | 160.4   |
| Chisholm [19]               |                                       | 143.6   | 160.4   |
| Sami and Duong [20]         |                                       | 14.2    | 20.4    |
| SFM: Micro-scale model      |                                       | 103.5   | 37.5    |
| Mishima and Hibiki [22]     |                                       | 76.0    | 82.3    |
| Yu et al. [23]              |                                       | 135.9   | 38.7    |
| Lee and Mudawar [24]        |                                       | 134.7   | 45.4    |
| Li and Wu [25]              |                                       | 22.0    | 9.2     |
| Zhang et al. [26]           |                                       | 27.5    | 16.3    |
The Muller-Steinhagen and Heck correlation [18] is ranked as the second best correlation. It may be due to the fact that the correlation is derived from very large database, 25000 data points and covers a wide range of refrigerants. Also, with this correlation, the two phase pressure drop reduces to single-phase liquid pressure drop at vapour quality, $x = 0$ and single-phase vapour pressure drop at vapour quality, $x = 1$. Ribatski et al. [28] assessed this correlation at micro scale and found that it gives the best predictions among the twelve different correlations tested. The Sami and Duong correlation [20] is ranked as the third best suitable correlation since the AAD of the predicted results for Mix#2 is slightly higher for this correlation. All other correlations do not predict the results consistently and scatter of the data is significant. Therefore, only four best correlations are used for further analysis of the pressure drop characteristics in the heat exchanger.

Figure 4 shows the variation in pressure drop with respect to mass velocity for Mix#1. It also compares the predicted variation in pressure drop using four correlations against the experimental values. It is found that the pressure drop increases with the increase in mass velocity. However, it should be noted here that the inlet conditions of pressure and temperature of the cold fluid entering the heat exchanger are not exactly same for all the cases. The comparison of pressure drop for both the mixtures revealed that the mixture composition affects the pressure drop significantly due to significant changes in the thermo-physical properties with the mixture composition.

Figure 5 compares the pressure profiles of the evaporating stream in the heat exchanger predicted by the four recommended correlations. The measured values of pressure at the inlet and the exit of the heat exchanger are shown in the figure, for Mix#1. It can be seen from the figure that the pressure profile obtained by the Zhang et al. [26] correlation is relatively linear in nature, while the pressure gradient for the Muller-Steinhagen and Heck correlation is more towards the exit of the heat exchanger. The pressure profiles predicted by Kim and Mudawar [27] and Sami and Duong [20] are almost similar over the entire length of the heat exchanger.

The recommended empirical correlations can be used with care to predict the frictional pressure drop in a MR J–T heat exchanger. However, for more accurate predictions of two-phase pressure drop in the heat exchanger for a MR J–T cryocooler, further experimentation is necessary towards development of a generalized correlation suitable to a wide range of mixture compositions and operating conditions.
5. Conclusions
In the present work, an extensive evaluation of the existing two-phase frictional pressure drop correlations is presented. A total of 15 widely used correlations based on the homogeneous and separated flow models developed for both macro-scale and micro-scale channels are selected to study their applicability for the mixed refrigerants at cryogenic temperatures. The experiments are conducted to measure overall pressure drop in the evaporating stream of the MR J–T heat exchanger for two different mixture compositions. It is observed that the mass flux and the mixture composition have a significant influence on the pressure drop in the heat exchanger. The comparison of the predicted pressure drop with the experimental data shows that the Zhang et al. [26] and Kim and Mudawar [27] correlation, which are developed for micro-channels give the best prediction of pressure drop among all the correlations assessed. These correlations can be used for predicting the hydraulic performance of the heat exchanger for a MR J–T cryocooler.

6. References
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