Transmission error in offset toothed double helical synchronous belt drives (in quasi-static condition)

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Abstract
Some studies have shown that offset toothed double helical synchronous belts (OTDHSB) have lower transmission noise and higher fatigue life than straight toothed synchronous belts (STSB). However, the transmission errors of these belts have not been reported. The present study aims to elucidate the mechanisms of the transmission error due to the belt climbing at the beginning of the meshing and at the end of meshing. In theoretical analysis, a model was established under quasi-static conditions. The results of the model used to calculate the transmission error of the helical synchronous belt (HSB) agreed closely with the published paper results. And then, the influence of the belt width, the helical angle and the offset coefficient on the transmission error was studied using the model. Computational results revealed that the amplitude of the transmission error decreases as the increasing of the offset coefficient, belt width, and helical angle when the offset coefficient and helical ratio are less than 0.5 and 1 respectively. In addition, the amplitude of the transmission error is equal to zero when the offset coefficient and helical ratio are equal to 0.5 and an integer respectively.

Keywords : Belt width, Helical synchronous belt, Helical angle, Offset toothed double helical synchronous belt, Transmission error

1. Introduction

Synchronous belt is widely used in the power and motion transmission, such as office automation equipment, automotive engines and industrial machineries. One of the synchronous belt’s performances is to transfer the rotary motion accurately. Kagotani et al.(2000b) presented the mechanism of transmission error under quasi-static conditions. It is that the belt climbing is incurred by bending rigidity and interference between the belt and pulley teeth at the beginning and end of meshing, which would cause the tension changing on tight and slack side. Furthermore, the tension change would cause rotation angular error of driven pulley. Makita et al.(2004) studied theoretically and experimentally the transmission error in synchronous belt drives with an idler, and results showed that the belt thickness errors between the belt pitch line and back face of the belt was a main reason of the transmission error generated. Kagotani et al.(2017) studied the transmission error due to resonance between synchronous belt drive and eccentric pulley. Kagotani et al.(2012) studied the transmission errors in which the belt spans generate first mode vibration due to the resonance, and results showed that the transmission error has a period of 1/2 of one pitch of the pulley. Masanori K et al.(2015) investigated how the influence of installation tension on transmission error due to resonance in a synchronous belt.

The HSB drives resulted in an axial offset between the position of the belt on the driving pulley and the driven pulley, so that the axial displacement affected the accuracy of HSB drives. Ueda H and Kagotani M (2000a,2003,2009) studied the relationship between the axial movement of the HSB under torque and the transmission error, when the pulley was rotated in bidirectional operation using flanged pulleys under no transmitted load.

According to STSB climbing mechanism at the meshing, Kagotani et al.(2010) studied transmission error of HSB under quasi-static conditions. In their theoretical study, the HSBs can be analyzed as a set of very “narrow” offset
toothed STSB (OTSTSB) for which the helical angle is zero. Experimental and theoretical results showed that the transmission error decreased as the helical angle or the belt width increase. However, Kagotani et al. did not give a mathematical model of the transmission error.

OTDHSB drives with the function of self-guiding without flange, has high carrying capacity and low noise compared with the STSB (Gregg M J W, 1999). In the present study, the mechanisms generating the transmission error in OTDHSB drives under quasi-static conditions due to belt climbing are investigated theoretically, and the mathematical models of the transmission error are established, which have a period of one pitch of the pulley. Firstly, the calculated results of the transmission error model of HSB are compared with Kagotani’s Experimental results (Kagotani and Ueda, 2000a) and the calculation results of the model are consistent with the experimental results, which indicates that the transmission error is mainly caused by the creeping belt tooth and the effect of tooth shear deformation is little and can be ignored. In addition, the amplitudes of the transmission error of the STSB, OTSTSB, DHSB and OTDHSB are calculated at arbitrary offset coefficient. Finally, the influence of the belt width, helical angle and offset coefficient on the amplitude of the transmission error was explored.

2. The formation mechanism of Synchronous belt transmission error

When Fig.1 shows the tooth number of the belt denoted as \((j,k)\), where \(j=1\) indicated the driving pulley, \(j=2\) indicated the driven pulley, and \(k\) represents the serial number of the teeth. The first tooth before the interfering tooth at the beginning of meshing in a state of meshing was \(k=1\). The last tooth without interference at the end of meshing was \(k=n\). The angle of the pulley revolution \(\phi(j)\) was measured when the belt tooth meshed completely with the pulley tooth.

![Fig. 1 Transmission model of synchronous belt](image1.jpg)

The pitch difference causes interference between the teeth of driving and driven pulley and the teeth of belt at the beginning and end of meshing. As shown in Fig. 2, when the driving pulley teeth and the belt teeth start to mesh, the belt tooth, as \(k=2\), interference with pulley tooth, and belt tooth is subjected to force \(Q_{(2)}\).

![Fig. 2 Climbing tooth model in meshing area](image2.jpg)
The interference and the flexural stiffness $EA$ (the cross-sectional area of belt tooth grooves $A(A=dh \times 2W)$, $dh$ is thickness of belt tooth groove, and $2W$ is width of the belt. The modulus of elasticity of glass fiber is $E$) during meshing would cause upward excitation which would cause belt displacement away from pulley tooth. The elastic modulus of glass fiber is much larger than that of rubber, so the modulus of rubber is ignored. The belt pitch line would separate with pulley pitch and the tooth of $k=3$ would squeeze into pulley groove and have a motion of climbing. The experimental results show that the pitch line curve of the STSB is the Archimedes spiral line at the beginning and the end of meshing of driving and driven pulley (Kagotani and Ueda, 2010a).

The belt drove with reverse direction in tension and inertia. Furthermore, the Archimedes spiral line produces a reciprocating vibration on the $y$-axis. The upper-left sub-figure of Fig.2 is the schematic diagram of the position of the maximum amplitude and average value of the pitch line vibration on the $y$-axis. The amount $\delta r_{s(j,k)}$ of belt climbing was a sinusoid of time on $y$-axis.

$$\delta r_{s(j,k)} = \delta r_{am(j,k)} \sin[(\varphi_{s(j)} + \varphi_{b(j,k)}), z_{p(j)}] + \delta r_{ave(j,k)} \quad (1)$$

Where the definition of $j$ and $k$ in the subscripts $(j,k)$ and $(j)$ are same as mentioned before in this section. $\delta r_{s(j,k)}$ is the amount of belt climbing at the beginning and end of meshing of driving and driven pulley. $\delta r_{ave(j,k)}$ of belt climbing is the average deviation of belt pitch line related to the pulley pitch circle, $\delta r_{am(j,k)}$ is the climbing amplitude of belt. $\varphi_{s(j)}$ is the corresponding pulley angle in a pitch and $0<\varphi_{s(j)}, \varphi_{b(j,k)}<2\pi$, and $\varphi_{b(j,k)}$ is the phase difference at the beginning and end of meshing and $0<\varphi_{b(j,k)}, \varphi_{b(j,k)}<2\pi$. $z_{p(j)}$ is the number of pulley tooth.

In the meshing area of the driving pulley and the driven pulley, the length of the Archimedes spiral formed by the belt pitch line is longer than the corresponding arc length of pulley pitch circle. The amount of Archimedes spiral elongation is

$$\delta s_{s(j,k)} = \frac{\varphi_{s(j,k)}}{2} [\delta r_{am(j,k)} \sin[(\varphi_{s(j)} + \varphi_{b(j,k)}), z_{p(j)}] + \delta r_{ave(j,k)}] \quad (2)$$

Where $\varphi_{s(j,k)}$ is the rotation angle of pulley from the starting point with the climbing tooth to $y$-axis.

The tensions of tight $t$ and slack $s$ sides for STSB are $F(t)$ and $F(s)$ for the transmission force $F' = F(t) - F(s)$, and the span-lengths between two pulleys are $s(t)$ and $s(s)$. When the driving pulley rotates, belt’s span-length on the tight and slack side changes due to the belt teeth climbing at the driving and driven pulleys, and the tight and slack side become $s'(t) = s(t) + \delta s(t)$ and $s'(s) = s(s) + \delta s(s)$, the tensions become $F''(t) = F'(t) + \delta F'(t)$ and $F''(s) = F'(s) + \delta F'(s)$. The relationship between the amount of tension and displacement changes on the tight and slack side are, respectively:

$$\begin{align*}
\delta F(t) &= \delta s(t) \frac{EA}{s} \\
\delta F(s) &= \delta s(s) \frac{EA}{s}
\end{align*} \quad (3)$$

where $\delta s(t)$, $\delta s(s)$ are the elongation of the tight and slack side respectively. $\delta F(t)$, $\delta F(s)$ are the tensions increment the tight and slack side respectively, and $s$ is the theory span length of belt.

Because of the belt climbing effects, the transmission force $F_b \neq F''(t) - F''(s)$ on the tight and slack side. Since the transmission force $F_b$ in the belt is a constant, driven pulley has a slight rotation $\delta \varphi_{(2)}$ and belt has a slight displacement $\delta s$. At the same time, the tight and slack tensions become $F''(t)$ and $F''(s)$, and belt lengths become $s''(t)$ and $s''(s)$ with the corresponding $F'_{b}=F''(t)-F''(s)$. In the new equilibrium, the slight displacement of belt is $\delta s = s''(t) - s''(s)> 0$ and $\delta \varphi_{(2)} = \pi/2$. As the transmission force is given by $F_b = F''(s) - F''(t) - F''(s)$, the displacement is arranged as $\delta s = 0.5(\delta \varphi_{(2)} - \delta F(t))s/EA$.

The transmission error of pulley’s one pitch is treated as one period. The relationship between the angle $\varphi_{(1)}$ of driving pulley ($0<\varphi_{(1)}<2\pi z_{p(1)}$) and the driven pulley angle error $\delta \varphi_{(2)}$ is

$$\delta \varphi_{(2)} = \frac{\delta s}{r_{p(2)}} \quad (4)$$
where $r_{p(j)}=I_2z_{p(j)}/2\pi$, $t_b$ was pitch of belt, $j=1$ indicated the driving pulley; $j=2$ indicated the driven pulley.

3. The model of offset-toothed double helical synchronous belt transmission error

For the HSB, during the meshing with the pulley in the width $W$ direction, due to the influence of the helical angle, the meshing is a gradual contact process.

The arbitrary meshing point $B$ of distance $w$ ($0<w<W$) from the belt side has the amount $x=w\tan\beta$ of a helical tooth projection in the axial direction with respect to the initial meshing point $B_0$ on the belt side, as shown in Fig.3-a. The helical tooth projection $x$ forms a rotation angle with respect to the center of rotation of the pulley. The rotation angle of the meshing point $B$ is the retard angle of the initial meshing point $B_0$ of the corresponding belt side, the phase retard angle of the helical tooth is defined as:

$$\gamma_{w(j)} = \frac{w\tan\beta}{r_{p(j)}}$$  \hspace{1cm} (5)

where $\beta$ is the helical angle of the helical tooth, $r_{p(j)}$ is the radius of the pulley, respectively.

During the process of helical tooth climbing, the amount $\delta r_{w(j,k)}$ of vibration of the belt pitch line for the arbitrary meshing point $B$ needs to consider the phase retard angle $\gamma_{w(j)}$ of the helical tooth relative to the initial position $B_0$ on the $y$-axis, so the amount of Archimedes spiral elongation for the arbitrary meshing point $B$ would be re-written by substituting Eq.(5) into Eq.(2):

$$\delta S_{i,j} = \frac{w_{(j,k)}}{2} \left[ \delta r_{w(j,k)} \sin[(\varphi_{i,j} + \varphi_{0(j,k)} + \gamma_{w(j)})z_{p(j)}] + \delta r_{w(j,k)} \right]$$  \hspace{1cm} (6)

(a). Offset toothed double helical synchronous belts (OTDHSB), $\beta\neq0^\circ, \delta\neq0$

(b). Offset-toothed straight toothed synchronous belts(OTSTSB), $\beta=0^\circ, \delta\neq0$

Fig.3. Structural model of offset-toothed double helical synchronous belt and offset-toothed straight toothed synchronous belt

For the OTDHSB, the length of the offset between the right-handed and left-handed helical tooth (Fig.3-a) is $\delta h_b$.
and the $\delta$ is a constant coefficient, 0 $<$ $\delta$ $<$ 1. $r_b$ is the pitch of belt. During the meshing process, the offset length $\delta t_b$ is divided by the radius $r_{p(j)}$ of the pulley pitch circle, defined as the offset angle:

$$
\lambda_g = \frac{\delta t_b}{r_{p(j)}} \tag{7}
$$

According to the meshing points’ order of the helical tooth, the amount of Archimedes spiral elongation of the right-handed $\delta s_{r(j),R}$ and left-handed $\delta s_{r(j),L}$ helical tooth for the OTDHSB are re-expressed as:

$$
\begin{align*}
\delta s_{r(j),R} &= \frac{\psi_{(j),(1)}}{2} \left\{ \delta r_{\text{arcA}(j,k)} \sin[(\phi_{(j)} + \phi_{(0,1)}, + \gamma_{u(j)})z_{p(j)}] + \delta r_{\text{arcB}(j,k)} \right\} \\
\delta s_{r(j),L} &= \frac{\psi_{(j),(1)}}{2} \left\{ \delta r_{\text{arcA}(j,k)} \sin[(\phi_{(j)} + \phi_{(0,1)}, + \gamma_{u(j)})z_{p(j)}] + \delta r_{\text{arcB}(j,k)} \right\}
\end{align*} \tag{8}
$$

For HSB with belt width $W$, the average elongation due to the belt climbing is integrating Eq.(6) over the belt width $W$:

$$
\delta s_{ave,(j,k)} = \frac{\psi_{(j),(1)} W}{W} \int_0^W \left[ \delta r_{\text{arcA}(j,k)} \sin[(\phi_{(j)} + \phi_{(0,1)}, + \gamma_{u(j)})z_{p(j)}] + \delta r_{\text{arcB}(j,k)} \right] dw 
\tag{9}
$$

The average elongation $\delta s_{ave}$ of the belt in tight side should be the elongation sum of the belt climbing at the beginning $\delta s_{ave,(1,3)}$ and the end $\delta s_{ave,(2,n-1)}$ of meshing of driving and driven pulley. $\delta s_{ave,(1,3)}$ and $\delta s_{ave,(2,n-1)}$ are substituted into Eq.(3). The amount of the tension increment on the tight side is:

$$
\delta F_{(j)} = \frac{E A}{2 W s} \left\{ \frac{r_{p(j)} \delta r_{\text{arcA}(1,3)}}{z_{p(1)}} \cos(\phi_{(1)} + \phi_{(0,1)}, + \gamma_{u(1)})z_{p(1)} - \cos(\phi_{(1)} + \phi_{(0,1)}, + \gamma_{u(1)})z_{p(1)} + W \delta r_{\text{arcA}(1,3)} \right\} \\
+ \frac{\psi_{(2),(1)} W}{W} \int_0^W \left[ \delta r_{\text{arcA}(2,n-1)} \cos(\phi_{(2)} + \phi_{(0,2,n-1)})z_{p(2)} - \cos(\phi_{(2)} + \phi_{(0,2,n-1)})z_{p(2)} + \delta r_{\text{arcA}(2,n-1)} \right] dw
\tag{10}
$$

The amount of the tension increment on the slack side is:

$$
\delta F_{(s)} = \frac{E A}{2 W s} \left\{ \frac{r_{p(s)} \delta r_{\text{arcA}(1,3)}}{z_{p(1)}} \cos(\phi_{(1)} + \phi_{(0,1)}, + \gamma_{u(1)})z_{p(1)} - \cos(\phi_{(1)} + \phi_{(0,1)}, + \gamma_{u(1)})z_{p(1)} + W \delta r_{\text{arcA}(1,3)} \right\} \\
+ \frac{\psi_{(0,1),(1)} W}{W} \int_0^W \left[ \delta r_{\text{arcA}(2,n-1)} \cos(\phi_{(1)} + \phi_{(0,2,n-1)})z_{p(2)} - \cos(\phi_{(1)} + \phi_{(0,2,n-1)})z_{p(2)} + \delta r_{\text{arcA}(2,n-1)} \right] dw
\tag{11}
$$

According to the Eq.(10) and Eq.(11), the unified formula of the driven pulley transmission error of the HSB is established as:

$$
\begin{align*}
\delta \phi_{\text{HSB}(2)} &= \frac{\pi}{2 W_s z_{p(2)}} \sum_{k=1}^{2} \sum_{k=1}^{n-k} (-1)^{r_{n-k}} \frac{r_{p(j)} \delta r_{\text{arcA}(j,k)}}{z_{p(j)} \tan \beta} \left[ \cos(\phi_{(j)} + \phi_{(0,j)})z_{p(j)} \\
&- \cos(\phi_{(j)} + \phi_{(0,j)})z_{p(j)} + \delta r_{\text{arcA}(j,k)} \right]
\tag{12}
\end{align*}
$$

For the OTDHSB with belt width $2W$, the transmission error is the sum of the left-handed and right-handed tooth, namely:

$$
\begin{align*}
\delta \phi_{\text{OTDHSB}(2)} &= \frac{\pi}{4 W_s z_{p(2)}} \sum_{k=1}^{2} \sum_{k=1}^{n-k} (-1)^{r_{n-k}} \frac{r_{p(j)} \delta r_{\text{arcA}(j,k)}}{z_{p(j)} \tan \beta} \left[ \cos(\phi_{(j)} + \phi_{(0,j)})z_{p(j)} \\
&- \cos(\phi_{(j)} + \phi_{(0,j)})z_{p(j)} + \delta r_{\text{arcA}(j,k)} \right]
\tag{13}
\end{align*}
$$
If the offset coefficient is \( \delta = 0 \), the OTDHSB becomes double helical synchronous belt (DHSB), and the transmission error is

\[
\delta \phi_{\text{DHSB}(2)} = \frac{\pi}{2Wb} \sum_{j=1}^{2} \sum_{k=1}^{3} \sum_{r=m-1}^{m-3} (-1)^{j+r+1} \psi_{(j,k)} \left( \frac{r_{(j,k)}}{z_{(j,k)}} \right) \left[ \cos(\varphi_{(j,k)}) + \varphi_{(j,k)} \right] z_{(j,k)} \tan \beta
\]

\[
- \cos(\varphi_{(j,k)}) + \varphi_{(j,k)} + \gamma_{W(j,k)} z_{(j,k)} + W \delta \varphi_{\text{ave}(j,k)} \right]
\]

When \( \beta = 0^\circ \), the offset toothed double helical synchronous belts (OTDHSB) would be transformed into the offset toothed straight toothed synchronous belts (OTSTSB), as shown in Fig. 3-b. Substitute \( \beta = 0^\circ \) into Eqs. (5)-(11), and the transmission error of OTSTSB is

\[
\delta \phi_{\text{OTSTSB}(2)} = \frac{\pi}{2Wb} \sum_{j=1}^{2} \sum_{k=1}^{3} \sum_{r=m-1}^{m-3} (-1)^{j+r+1} \psi_{(j,k)} \left( \frac{r_{(j,k)}}{z_{(j,k)}} \right) \left[ \sin(\varphi_{(j,k)}) + \varphi_{(j,k)} \right] z_{(j,k)}
\]

\[
+ \sin(\varphi_{(j,k)}) + \varphi_{(j,k)} + \gamma_{W(j,k)} z_{(j,k)} + 2 \delta \varphi_{\text{ave}(j,k)} \right]
\]

**4. Simulation results of the model**

In order to compare the results of the calculation model with the published paper, the experimental data and the calculation parameters of Kagotani’s literature are used: The number of driving and driven pulley teeth \( z_{(j)} \) is 22, \( t_b \) is 8 (mm), the torque \( q \) of the driving pulley is 5.2 Nm, the initial tension is 600N, and the amount \( \delta r_{(j,k)} \) of the belt climbing on \( y \)-axis is obtained by Kagotani’s experimental measurement (2000a-105-107). In this study, the transmission error is studied by calculating the amplitude of the harmonic error function.

**4.1 The transmission error of the helical synchronous belt**

The influence of helical angle \( \beta \), helical ratio \( \varepsilon \) and belt width \( W \) on the amplitude of the transmission error of the driven pulley is calculated by the model.

Figure 4 shows experimental result that the belt width \( W \) is 20mm and the helical angle \( \beta \) is from 5° to 15° by 5°. As described Eqs.(5) and (12), the phase retard angle \( \gamma_{\text{ave}(j)} \) is positively related to the helical angle \( \beta \), so the amplitude \( \delta \varphi_M \) of the transmission error \( \delta \phi_{\text{DHSB}(2)} \) decreases as the helical angle \( \beta \) increases.

![Fig. 4 The influence of the helical angle on the amplitude of the transmission error of the HSB](image-url)

Figure 5 shows the relationship between helical ratio \( \varepsilon \) and amplitude \( \delta \varphi_M \) of the transmission error \( \delta \phi_{\text{HSP}(2)} \). The helical ratio \( \varepsilon \) is the amount of a helical tooth projection at the belt side divided by the belt pitch \( t_b \), and is expressed as:
\[ \varepsilon = \frac{W \tan \beta}{t_b} \]  

(16)

Fig. 5 Influence of helical ratio \( \varepsilon \) on the amplitude of the transmission error

When the helical ratio \( \varepsilon \) is less than 1 \((\varepsilon < 1)\), the amplitude \( \delta \varphi_M \) of the transmission error \( \delta \varphi_{HSB}(2) \) decreases as the helical ratio \( \varepsilon \) increases. When the helical ratio \( \varepsilon \) is equal to the integers 1 and 2 \((\varepsilon = 1, 2)\), the amplitude \( \delta \varphi_M \) of the transmission error \( \delta \varphi_{HSB}(2) \) is equal to zero, and the helical ratio \( \varepsilon \) is greater than 1 and less than 2 \((1 < \varepsilon < 2)\), the amplitude transmission error is not monotonic.

In Fig. 4 and Fig. 5, when the helical angle \( \beta \) and the helical ratio \( \varepsilon \) are equal to zero, the calculation results are the amplitude \( \delta \varphi_M \) of the STSB transmission error \( \delta \varphi_{HSB}(2) \), which is larger amplitude than the HSB.

Figure 6 shows the relationship between belt width \( W \) and amplitude \( \delta \varphi_M \) of the transmission error \( \delta \varphi_{HSB}(2) \), when the helical angle \( \beta \) is 10° and the belt width \( W \) is from 10 (mm) to 45 (mm) by 5 (mm). The amplitude \( \delta \varphi_M \) of the transmission error \( \delta \varphi_{HSB}(2) \) of HSB decreases as the belt width \( W \) increases. However, the amplitude \( \delta \varphi_M \) of the transmission error \( \delta \varphi_{STSB}(2) \) of the STSB with helical angle 0° is larger than the HSB, and the amplitude \( \delta \varphi_M \) is constant.

When the belt width is 45 (mm), the amplitude \( \delta \varphi_M \) of the transmission error \( \delta \varphi_{HSB}(2) \) is zero because the helical ratio \( \varepsilon \) is equal to 1.

The calculation results of the error model of HSB are consistent with the Kagotani’s (2010a\(^{108}\)) literature.
4.2 The transmission error of the offset toothed double helical synchronous belt

In order to study the influence of the offset coefficient $\delta$ on the transmission error, in the calculation of the error amplitude, the offset coefficients $\delta$ are taken as 0, 0.35 and 0.5, respectively, and the other parameters are the same in section 3.

Figure 7 shows the relationship between the amplitude $\delta_{\phi M}$ of the transmission error $\delta_{\phi_{\text{OTDHSB}}}$, the parameters $\beta$ and $\delta$. In the calculation of the model, belt width $2W$ is 20mm and helical angle $\beta$ is from $0^\circ$ to $35^\circ$ by $5^\circ$. The calculation results form three curves, which are the error amplitude curves of DHSB ($\delta=0$), and OTDHSB ($\delta=0.35$ and 0.5, respectively).

![Fig. 7 Influence of the helical angle and offset coefficients on amplitude of transmission error for OTDHSB with belt width 20mm](image)

In special cases, the helical angle $\beta$ is equal to zero. Then, the amplitudes of the transmission errors become counterparts of the STSB and the OTSTSB, respectively, as shown in Fig.3. When $\delta$ is equal to 0 or 0.35, the amplitudes $\delta_{\phi M}$ of the transmission error $\delta_{\phi_{\text{OTDHSB}}}$ decreases as the helical angle $\beta$ increases. When the $\beta=0^\circ$, the error amplitude $\delta_{\phi M}$ of the STSB and the OSTSB are larger than that of the DHSB and the OTDHSB, respectively.

Under the same helical angle $\beta$ the error amplitudes $\delta_{\phi M}$ from large to small are DHSB ($\delta=0$) > OTDHSB ($\delta=0.35$) > OTDHSB ($\delta=0.5$). When the $\delta$ is 0.5, the amplitude $\delta_{\phi M}$ is equal to zero because the offset angle $\lambda_\delta$ multiplied by the number of pulley teeth $z_{\phi(0)}$ is equal to $\pi$ in the Eq.(13).

![Fig. 8 The amplitude of transmission error for both the helical ratio and offset coefficient](image)
Figure 8 shows the effect of the helical ratio \( \varepsilon \) and offset coefficient \( \delta \) on the amplitude \( \delta \phi_M \) of transmission error \( \delta \phi_{OTDHSB(2)} \). In addition to the helical ratio \( \varepsilon \) is equal to the integer, the error amplitude \( \delta \phi_M \) of the three offset coefficient curves are in descending order: \( \delta \phi_{M(\delta=0)} > \delta \phi_{M(\delta=0.35)} > \delta \phi_{M(\delta=0.5)} = 0 \), and their changes are the same as in Fig.7.

When the helical ratio \( \varepsilon \) is equal to an integer, the error in Eq.(13) and Eq.(14) is equal to zero due to the phase retard angle \( \gamma_{W(j)} \) multiplied by the number of pulley teeth \( z_{W(j)} \) equal to \( 2\pi \).

According to Eq.(13) and Eq.(14), when the helical ratio \( \varepsilon \) is less than 1, the error amplitude \( \delta \phi_M \) decreases as the helical ratio \( \varepsilon \) increases. If the helical ratio \( \varepsilon \) is greater than 1, there is a larger belt width or helical angle, so the error amplitude \( \delta \phi_M \) will be smaller. The calculation result is the same as that of \( \beta=0^\circ \), when the helical ratio is \( \varepsilon=0 \), as shown in Fig.7.

Figure 9 shows the effect of belt width \( 2W \), the helical angle \( \beta \) and offset coefficients \( \delta \) on the amplitude \( \delta \phi_M \) of the transmission error \( \delta \phi_{OTDHSB(2)} \), the belt width \( 2W \) is from 16 (mm) to 32 (mm), and the increment is 4mm.

For DHSB \( (\delta=0) \) and OTDHSB \( (\delta=0.35) \) with \( \beta=30^\circ \), when the belt width is less than 28 (mm), the amplitude \( \delta \phi_M \) of the transmission error would decrease as the belt width increasing. According to Eq.(5), the belt width \( W \) and helical angle \( \beta \) is positively related to the phase retard angle \( \gamma_{W(j)} \). Therefore, according to Eq.(13), the increase in the phase retard angle \( \gamma_{W(j)} \) would decrease the amplitude of the transmission error. When the belt width is 28 (mm), the transmission error amplitude \( \delta \phi_M \) would be equal to zero due to the helical ratio \( \varepsilon \) being equal to one. For OTDHSB \( (\beta=30^\circ, \delta=0.5) \), the error harmonic function formed by the left and right helical belt tooth has the same amplitude and opposite phase, so the transmission error is zero.

For STSB \( (\delta=0) \) and OTSTS B \( (\delta=0.35 \text{ and } 0.5) \) with \( \beta=0^\circ \). The amplitude of the error is independent of the belt width and is constant. The order of amplitude from large to small is STSB \( (\delta=0) \), OTSTS B \( (\delta=0.35) \) and OTSTS B \( (\delta=0.5) \). According to Eq.(15), the transmission error of STSB \( (\delta=0) \) is composed of a single harmonic curve. The transmission error of OTSTS B \( (\delta=0.35) \) is the superposition of two different phase harmonic curves, which will reduce the amplitude of the error curve. The transmission error of OTSTS B \( (\delta=0.5) \) is composed of two harmonic curves with the same amplitude and opposite phase, so the error is zero. As shown in Fig.9, the error curves of OTDHSB \( (\beta=30^\circ, \delta=0.5) \) and OTSTS B \( (\beta=0^\circ, \delta=0.5) \) are coincident.

![Fig. 9 Shows the influence of the belt width and offset coefficient on the amplitude of transmission error.](image)

5. Conclusions

The models of the transmission error of the HSB and OTDHSB based on quasi-static conditions and the belt climbing characteristics are established. The model calculation results are as follows:

1) The influence of the belt width, the helical angle and the helical ratio of the HSB on the transmission error calculated by the model is consistent with the experimental results of the published papers, indicating that the established transmission error models based on the belt climbing are feasible.

2) The amplitudes of the transmission error with different offset coefficients are calculated by the OTDHSB error...
model respectively. The amplitudes of transmission error decrease with the increasing of the offset coefficient (less than 0.5). Moreover, when the offset coefficient is 0.5, the amplitude of transmission error is equal to zero.

3) The amplitudes of the transmission error decreases as the helical ratio increases due to belt width and helical angle are positively related to the helical ratio. When the helical ratio is equal to an integer, the error amplitudes is equal to zero. The error amplitudes of STSB and OTSTSB are respectively greater than DHSB and OTDHSB.

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