Are vertical cosmic rays the most suitable to radio detection?

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Abstract

The electric field induced by extensive air showers generated by high energy cosmic rays is considered and, more specifically, its dependence on the shower incident angle. It is shown that for distances between the shower axis and the observation point larger than a few hundred meters, non-vertical showers produce larger fields than vertical ones. This may open up new prospects since, to some extent, the consideration of non-vertical showers modifies the scope of the radio-detection domain.

Key words: Radiation Mechanisms, Radio telescopes and instrumentation, Extensive air showers, Cosmic rays
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1 Introduction

The observation of ultra high-energy cosmic rays ($E \geq 10^{18}$ eV) is nowadays a question of paramount importance. They can be detected indirectly via the observation of the Extensive Air Showers (EAS) they create while interacting with the atmosphere. The induced secondary particles generate a radio-electric field whose measurements may provide an alternative to the experimental techniques consisting of counting the shower particles collected in ground detectors. It turns out that the radio signal becomes significant provided that an efficient charge separation mechanism occurs (1).

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The radio frequency component associated with EAS was studied in the 1960’s. At that time, the purpose was mainly to demonstrate the observability of such a radio-electric field \( \text{[2, 3]} \). The strategy was then to detect a radio pulse in coincidence with particles in an experimental set-up consisting of one antenna located as close as possible to some particle detectors. Such a device selects mainly vertical showers.

The purpose of this letter is to stress that, in the case of not-too-close impact parameters (i.e., large distances between the shower axis and the observation point), vertical showers produce a radio-electric signal weaker in amplitude than oblique ones. The reason is the ultra-relativistic character of most of the shower particles that leads to a strong forward enhancement of the electric field. For a given impact parameter, this means that the point at which a charge produces the largest signal along its trajectory is located far above the point of closest approach. This forward-peaked structure is counterbalanced by the rise in the number of charges as the shower develops in the dense layers of atmosphere. Quantitatively, as explained in Sect. 2, the combination of both effects will favor the observation of non-vertical showers.

Sect. 3 unfolds the consequence of this observation. The point is that, using a large array of scattered antennas in order to compensate for the scarcity of high-energy cosmic rays, there is some interest in examining the possibility of radio detection of EAS at a somewhat larger impact parameter. According to the above statement, such an apparatus, therefore, puts emphasis on the detection of non-vertical hadron-induced showers.

## 2 Effect of shower incidence on the radio signal

This section aims at studying the dependence of the radio frequency signal on the incidence angle of the air shower. For this purpose, one of the mechanisms possible for electromagnetic field generation has been chosen, namely the synchrotron radiation. The latter is associated with the deviation, due to the geomagnetic field, of the EAS charged particles. The specific choice of synchrotron radiation should not be considered as a limitation, since the main lines of argumentation are generic and can be transposed to any other mechanism. The additional choices concern the nature and the incident energy of the cosmic rays: a \( 10^{18} \) eV proton induced-shower will be considered throughout.

The first element that drives the electromagnetic field emitted during the development of the air shower is obviously the evolution of the number of charges as a function of time. The way it affects the expression of the electric field depends on the details of the mechanism considered. However, the main feature is the fact that, this charge number rises from a few units to several
$10^8$, and that, this growth takes place on a characteristic depth scale of one atmosphere \(^4\). The consequence for the time evolution of the signal is analyzed in Sect. 3 of Ref. \(^1\). At zero impact parameter, the above time sequence is Doppler contracted to almost zero and the signal duration results from the longitudinal lag and lateral spread of the shower core. On the contrary, for impact parameters larger than a few hundred meters, the Doppler contraction being less important, it is the shower growth and decay which govern the time behavior of the radio signal. In this configuration, the extension of the shower core only introduces weak time dispersions to the signal. As a consequence, it is reasonable for an estimate of the electric field at large impact parameter to modelize the shower as a point-like system moving along a straight line and containing a number of electrons and positrons varying with time.

The second element which influences the total electric field is the space-time dependence of the field of a single charge. It should be noted that the interest is not so in following a specific charge which suffers deviation in the earth magnetic field, and in addition loses energy before disappearing from the shower core. Rather, since the total field is the sum over the set of charges, the interest is in the field generated by a representative of particles of a given energy sitting where most particles sit, i.e., in the shower core. In line with the above model for the shower core, such a representative has its speed directed along the shower axis at any time.

Before combining both elements, the single-charge electric field is calculated in a first step. The electric field created by an accelerated charge $e$ at a given observation point $A$ at time $t$ reads

$$E(t, A) = \frac{e}{4\pi \epsilon_0 c_0} \left[ \frac{\mathbf{R} \wedge \left[ (\mathbf{R} - n \mathbf{v}) \wedge \dot{\mathbf{v}} \right]}{|\mathbf{R} - n \mathbf{v} \cdot \mathbf{R}|^3} \right]_{t'},$$

where $\mathbf{v}$ is the particle speed in units of $c_0$, the light velocity in vacuum; $\dot{\mathbf{v}}$ is the time derivative of $\mathbf{v}$; $n$ is the refractive index of the medium and $\mathbf{R} = \overrightarrow{QA}$ where $Q$ is the location, at time $t'$, of the particle. $t$, $t'$ and $A$ are linked by

$$\frac{c_0}{n} (t - t') = R = ||\overrightarrow{QA}||.$$

A first remark about Eq. (1) is that two opposite charges $+e$ and $-e$ have opposite accelerations $+\dot{\mathbf{v}}$ and $-\dot{\mathbf{v}}$ so that their respective electric fields add up. This is the manifestation for the case of synchrotron radiation of the charge separation mechanism mentioned in the introduction.

\footnote{In the Čerenkov regime, this equation may have two solutions for $t'$. In this case, both have to be taken into account for calculating the field defined by Eq. (1).}
Fig. 1. Geometrical quantities used in Eq. (2). The dashed line is the shower axis. At time $t'$, the particle $Q$ may be located with $\theta$, the angle between $\vec{A}y$ and $\vec{A}Q$. $B$ is the point of closest approach, whereas $m$ is the point where the quantity in Eq. (2) is maximal.

The form of the field in Eq. (1) assumes a signal propagation in a medium of constant $n$. It is shown in Ref. [1] that the time variation introduced in the signal when considering refractive index effects is small at large impact parameters, i.e., well outside of the Čerenkov cone.

In order to understand the variation of $E$ in Eq. (1), a special attention has to be paid to the denominator, cubic power of the term $|R - n\,v \cdot R|^{-1}$. This factor combines the $R^{-1}$ term, related to the distance between the charge and the observation point, with the forward-angle-peaked term $|1 - n\,v \cdot R/R|^{-1}$ whose effect becomes more pronounced when the particle velocity approaches $c = c_0/n$. The dependence on the angle can be shown explicitly, using the geometrical quantities defined on Fig. 1, and reads:

$$
\frac{1}{|R - n\,v \cdot R|} = \frac{1}{b} \frac{\sin \theta}{|1 - nv \cos \theta|}.
$$

The evolution of this term as a function of $\theta$ is shown on Fig. 2. For a given impact parameter, this factor is a decreasing function of $\theta$ in the range $\theta > \theta_m$. For particle velocities $v < 1/n$ (in units of $c_0$), $\theta_m$ is defined as:

$$
\theta_m = \arccos(nv).
$$

For $v > 1/n$, it is given by:

$$
\theta_m = \arccos \left( \frac{1}{nv} \right),
$$

\[2\]
Fig. 2. \( \sin \theta / (1 - n v \cos \theta) \) as a function of \( \theta \). Two cases are represented: \( v < 1/n \) (Lorentz factor \( \gamma = 20 \)) and \( v > 1/n \) (\( \gamma = 60 \)).

and corresponds to the definition of the Čerenkov angle.\(^2\) In both cases, \( \theta_m \) is a very small angle as long as ultra-relativistic particles are concerned. The consequence is that the factor \( |R - n v \cdot R|^{-1} \) reaches its maximum value at the point \( m \), such that \( \theta(m) = \theta_m \), far above the point of closest approach \( B \) (see Fig. 1). As an example, the distance between \( m \) and \( B \) amounts to \( 23 \times b \) for a particle with \( \gamma = 20 \) and \( 56 \times b \) for a particle with \( \gamma = 60 \).

Thus, if the \( \theta \) dependence in Eq. (1) was contained only in the denominator, the point of maximal brightness for the electric field would be reached when the charge is at \( m \) (not at \( B \)). As a matter of fact, the numerator in Eq. (1) is also a function of \( \theta \). Moreover, it depends on the orientation of the particle acceleration. In the case where \( v > c \), this does not modify the fact that the electric field is a decreasing function of \( \theta \) in the range \( \theta > \theta_m \). Hence, for clarity, discussion will be restricted to this particular case. For particle velocities \( v < c \), the discussion requires a case study that will not be examined in this letter. It can nevertheless be verified that the general conclusion derived below also applies to this second case.

After having considered a unique charge, calculations are carried out for the full set of charges, in order to take into account the shower development in the atmosphere. The field in Eq. (1) is thus multiplied by a number of charges following a given time profile. Fig. 3 displays the various quantities which have to be combined, as a function of the position along the shower axis \( y = -v \times t' \): The electric field induced by a single charge and two different

\(^2\) For \( v > c \), the factor in Eq. (2) diverges at \( \theta = \theta_m \) and the analysis should be reformulated to account for the non constancy of the refraction index. This complication is however unnecessary when limiting the discussion to large impact parameters, since \( \theta = \theta_m \) corresponds to points at high enough altitudes where the cosmic ray has not yet showered.
Fig. 3. Charge profiles along the shower axis $y$ for both vertical ($0^\circ$) and tilted ($75^\circ$) proton-induced showers. The electric field shape of a single charge is also represented as a function of $y$. Calculations have been performed for $b = 2$ km and $\gamma = 60$.

charge profiles corresponding to shower incident angles fixed at $\alpha = 0^\circ$ and $\alpha = 75^\circ$. Though this is not shown on Fig. 3, it is easy to guess that the multiplication of the $0^\circ$ profile with the single-charge electric field peaks at a much smaller amplitude than the multiplication of the $75^\circ$ profile with the same field. To be specific, the ratio of maximal values is in this case $1 : 4000$. Such a ratio depends on the parameters chosen, and on the simplifications adopted in the present derivation. However, the conclusion remains that when the single-charge electric field has its maximum at large $y$, i.e., for large impact parameters, inclined showers produce larger electric field than a vertical one.

Another way of picturing the above discussion is shown on Fig. 4. The development of air showers, with a maximum at $M_\alpha$, for either $\alpha = 0^\circ$ or $\alpha = 75^\circ$ incident angle is illustrated. Also shown in both cases is the location of the maximal single-charge field ($m_0$ and $m_{75}$) for impact parameter $b = 2$ km. The tilted shower is more efficient for radiation than the vertical one because $M_\alpha$ is closer to $m_\alpha$ in the former case. For large impact parameters, this always occurs in a non-vertical configuration. Let us be more precise about what “large” means. Fig. 4 suggests that, for a shower of incidence $\alpha$, the best configuration is obtained when $M_\alpha = m_\alpha$. Furthermore, $M_\alpha$ moves away from the point of closest approach, $B$, as $\alpha$ increases while $m_\alpha$ moves away from $B$ as the impact parameter, $b$, increases. Choosing now an intermediate angle value of $45^\circ$, large impact parameters correspond to those greater than $b_0$ defined by $m_{45}(b_0) = M_{45}$. Typically, for a $10^{18}$ eV shower, $M_{45}$ is located at 5 km ahead of $B$. This leads to $b_0 = 0.1$ km for $\gamma = 60$ and $b_0 = 0.2$ km for $\gamma = 20$. 
Fig. 4. Illustration of air shower developments in the atmosphere, for two different shower incidence angles ($\alpha = 0^\circ$ and $\alpha = 75^\circ$ defined with respect to the vertical direction).

3 Outlook

The above results put emphasis on horizontal air showers. The latter, usually considered in the study of energetic neutrino cosmic rays, appear to be also relevant for the domain of radio-detection. Thus, the perspectives of radio-detection are somewhat enlarged if (quasi-)vertical showers are not the only ones of interest.

For ground detection of secondary particles, air showers at large incident angle are the hardest to detect if only because their main components cannot reach the ground. The situation is completely different in the case of radio detection. In order to take full advantage of this fact, it is necessary to give up standard trigger methods that use particle ground detection. One should rather use the radio frequency signal itself as a trigger. Such a possibility is at the moment tested by the CODALEMA experiment $^{(5)}$. Triggering is performed on transients whose levels exceed a given threshold, and in a frequency band-width where strong steady radio transmitters are absent. Alternative attempts using coincidence between different electric pulses collected on a few antennas have also been experimented $^{(6)}$.

According to Sect. 2, such a system is more efficient for radio-detection of non-vertical showers as soon as impact parameters larger than a few hundred meters become detectable. With this proviso in mind, this means that, for a given cosmic ray incident energy, the accessible region of impact parameters is larger than expected. Conversely, the detectable cosmic ray energy is
lower, and a larger rate of triggers is expected, for non-vertical showers with a given impact parameter in this range. Calculations of Sect. 2 are, however, too limited to estimate the gain in sensitivity for the detection of such events.

Finally, some comments on antenna networks are in order. A key question in the observation of very low cosmic ray flux, is the surface achievable for a given number of antennas, and implicitly, that of antenna spacing. The present study has some relevance in terms of antenna network capabilities regarding horizontal versus vertical air showers, since the non-vertical configuration allows for a wider spacing, or vice versa, for lower incident energies and a given spacing.

The question of antenna network has been considered by the CASA/MIA collaboration (7): There could be a possibility of equipping AUGER (8) with antennas, thereby allowing for a detection in coincidence between particles collected in ground detectors and registered radio pulses. The above considerations may open up somewhat new perspectives to such a project.

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