A method for analysing the jet azimuthal anisotropy in ultrarelativistic heavy ion collisions

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Abstract

The azimuthal anisotropy of jet spectra due to energy loss of jet partons in azimuthally non-symmetric volume of dense quark-gluon matter is considered for semi-central nuclear interactions at collider energies. We develop the techniques for event-by-event analysing the jet azimuthal anisotropy using particle and energy elliptic flow, and suggest a method for calculation of coefficient of jet azimuthal anisotropy without reconstruction of nuclear reaction plane.

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1 Introduction

High-$p_T$ jet production and other hard processes are considered as the promising tools for studying properties of hot matter created in heavy ion collisions at RHIC and LHC. The challenging problem is the behaviour of colour charge in quark-gluon environment associated with the coherence pattern of the medium-induced radiation. It results in interesting non-linear phenomena, in particular, the dependence of radiative energy loss per unit distance $dE/dx$ along the total distance traversed (see review [1] and references therein). In a number of papers [2, 3, 4, 5] authors analysed the azimuthal anisotropy of high-$p_T$ hadron spectra in semi-central nuclear collisions at RHIC due to partonic energy loss in azimuthally non-symmetric volume of quark-gluon plasma. At LHC energy, when the inclusive cross section for hard jet production on $E_T \sim 100$ GeV scale is large enough to study the impact parameter dependence of such processes [6], one can hope to observe the similar effect for hadronic jet itself [7]. In particular, CMS experiment at LHC [8] will be able to provide jet reconstruction and adequate measurement of impact parameter of nuclear collision using calorimetric information [9].

It is important to notice that the coherent Landau-Pomeranchuk-Migdal radiation induces a strong dependence of the radiative energy loss of a jet (but not a leading parton) on the jet angular cone size [10, 11, 12, 13]. It means that the medium-induced radiation will, in the first place, soften particle energy distributions inside the jet, increase the multiplicity of secondary particles, and to a lesser degree affect the total jet energy. On the other hand, collisional energy loss turns out to be practically independent of jet cone size, because the bulk of ”thermal” particles knocked out of the dense matter by elastic scatterings fly away in almost transverse direction relative to the jet axis [11].

The methodical advantage of azimuthal jet observables is obvious: one needs to reconstruct only azimuthal position of jet, but not the total jet energy. It can be done more easily and with high accuracy, while the reconstruction of the jet energy is more ambiguous task [9]. However the performance of inclusive analysis of jet production as a function of azimuthal angle requires event-by-event measurement of the reaction plane angle. The summarized in papers [14] present methods for determination of the reaction plane angle are applicable for studying anisotropic flow of soft and semi-hard particles in current heavy ion dedicated experiments at SPS [15] and RHIC [16], and, in principle, might be also used at LHC [7].

In this work we suggest a method to calculate the coefficient of jet azimuthal anisotropy without reconstruction of nuclear reaction plane. In some sense, it represents the development and generalization of the well-known method for measuring azimuthal anisotropy of particle
flow considered originally in a number of works (see [14, 17, 18] for instance).

2 Event-by-event analysis of azimuthal correlations

Let us remind the essence of techniques [17, 14] for measuring azimuthal elliptic anisotropy of particle distribution, which can be written in the form

$$\frac{dN}{d\varphi} = \frac{N_0}{2\pi} [1 + 2v^2 \cos 2(\varphi - \psi_R)] , \quad N_0 = \int_{-\pi}^{\pi} d\varphi \frac{dN}{d\varphi}. \quad (1)$$

Knowing the nuclear reaction plane angle $\psi_R$ allows one to determine the coefficient $v^2$ of azimuthal anisotropy of particle flow as an average (over particles) cosine of $2\varphi$:

$$\langle \cos 2(\varphi - \psi_R) \rangle = \frac{1}{N_0} \int_{-\pi}^{\pi} d\varphi \cos 2(\varphi - \psi_R) \frac{dN}{d\varphi} = v^2. \quad (2)$$

However in the case when there are no other correlations of particles except those due to flow (or such other correlations can be neglected), the coefficient of azimuthal anisotropy can be determined using two-particle azimuthal correlator without the event plane angle $\psi_R$:

$$\langle \cos 2(\varphi_1 - \varphi_2) \rangle = \frac{1}{N_0^2} \int_{-\pi}^{\pi} d\varphi_1 \int_{-\pi}^{\pi} d\varphi_2 \cos 2(\varphi_1 - \varphi_2) \frac{d^2N}{d\varphi_1 d\varphi_2} \quad (3)$$

$$= \frac{1}{N_0^2} \int_{-\pi}^{\pi} d\varphi_1 \int_{-\pi}^{\pi} d\varphi_2 \cos 2(\varphi_1 - \varphi_2) \frac{dN}{d\varphi_1} \frac{dN}{d\varphi_2} = v^2_2.$$ 

Here one should to note that it is safe to neglect non-flow correlations only if the coefficient of azimuthal anisotropy $v^2$ is much larger than $1/\sqrt{N_0}$. In reality, this condition is not always satisfied. A new method, based on a cumulant expansion of multi-particle azimuthal correlations, which allows measurements of much smaller values of azimuthal anisotropies, down to $1/N_0$, has been worked out recently in paper [19]. This method automatically eliminates the major systematic errors, which are due to azimuthal asymmetries in the detector acceptance, and in principle could be used in further improvements of our approach.

Let us consider now the event with high-$p_T$ jet (dijet) production, the distribution of jets over azimuthal angle relatively to the reaction plane being described well by the elliptic form [14],

$$\frac{dN^{jet}}{d\varphi} = \frac{N_0^{jet}}{2\pi} [1 + 2v^{jet}_2 \cos 2(\varphi - \psi_R)] , \quad N_0^{jet} = \int_{-\pi}^{\pi} d\varphi \frac{dN^{jet}}{d\varphi}, \quad (4)$$

where the coefficient of jet azimuthal anisotropy $v^{jet}_2$ is determined as an average over all events cosine of $2\varphi$,

$$\langle \cos 2(\varphi - \psi_R) \rangle_{event} = \frac{1}{N_0^{jet}} \int_{-\pi}^{\pi} d\varphi \cos 2(\varphi - \psi_R) \frac{dN^{jet}}{d\varphi} = v^{jet}_2. \quad (5)$$
One can calculate the correlator between the azimuthal position of jet axis $\varphi_{jet}$ and the angles of particles, which are not incorporated in the jet(s). The value of this correlator is related to the elliptic coefficients $v_2$ and $v_{jet}^2$ as

$$\langle \langle \cos 2(\varphi_{jet} - \varphi) \rangle \rangle_{\text{event}} = \frac{1}{N_0 N_0} \int_{-\pi}^{\pi} d\varphi_{jet} \int_{-\pi}^{\pi} d\varphi \cos 2(\varphi_{jet} - \varphi) \frac{dN_{jet}^2}{d\varphi_{jet}} \frac{dN}{d\varphi}$$  \hspace{1cm} (6)$$

$$= \frac{1}{N_0} \int_{-\pi}^{\pi} d\varphi_{jet} \cos 2(\varphi_{jet} - \psi_R) \frac{dN_{jet}^2}{d\varphi_{jet}} v_2 = v_{jet}^2 v_2 .$$

Using Eq. (3) and intermediate result in Eq. (6) (after averaging over particles $\cos 2(\varphi_{jet} - \varphi)$ reduces to $v_2 \cos 2(\varphi_{jet} - \psi_R)$) we derive the formula for computing absolute value of the coefficient of jet azimuthal anisotropy (without reconstruction of sign of $v_{jet}^2$):

$$v_{jet}^2 = \left\langle \left\langle \frac{\cos 2(\varphi_{jet} - \varphi)}{\cos 2(\varphi_1 - \varphi_2)} \right\rangle \right\rangle_{\text{event}} .$$  \hspace{1cm} (7)$$

This formula does not require the direct determination of reaction plane angle $\psi_R$. The brackets $\langle \rangle$ represent the averaging over particles (not incorporated in the jet) in a given event, while the brackets $\langle \rangle_{\text{event}}$ the averaging over events.

The formula (7) can be generalized by introducing as weights the particle momenta,

$$v_{2(p)} = \left\langle \left\langle \frac{\cos 2(\varphi_{jet} - \varphi) p_T(\varphi)}{\cos 2(\varphi_1 - \varphi_2) p_{T1}(\varphi_1) p_{T2}(\varphi_2)} \right\rangle \right\rangle_{\text{event}} .$$  \hspace{1cm} (8)$$

In this case the brackets $\langle \rangle$ denote the averaging over angles and transverse momenta of particles. The other modification of (8),

$$v_{2(E)} = \left\langle \left\langle \frac{\cos 2(\varphi_{jet} - \varphi) E(\varphi)}{\cos 2(\varphi_1 - \varphi_2) E_1(\varphi_1) E_2(\varphi_2)} \right\rangle \right\rangle_{\text{event}} .$$  \hspace{1cm} (9)$$

($E_i(\varphi_i)$ being energy deposition in calorimetric ring $i$ of position $\varphi_i$) allows one using calorimetric measurements (9) for the determination of jet azimuthal anisotropy, in particular, under condition of CMS experiment at LHC.

### 3 Numerical simulation and discussion

In order to illustrate the applicability of method presented for real physical situation, we consider the following model.

\footnote{The other possibility is to fix the azimuthal position of a leading particle in the jet. In this case calculating azimuthal correlations can provide the information about azimuthal anisotropy of high-$p_T$ particle spectrum.}
Jets. The initial jet distributions in nucleon-nucleon sub-collisions at $\sqrt{s} = 5.5$ TeV have been generated using PYTHIA_5.7 generator [20]. We simulated the rescattering and energy loss of jets in gluon-dominated plasma, created initially in nuclear overlap zone in Pb–Pb collisions at different impact parameters. For details of this model one can refer to our previous papers [3,4]. Essentially for our consideration here is that in non-central collisions the azimuthal distribution of jets is approximated well by the elliptic form (4). In the model the coefficient of jet azimuthal anisotropy increases almost linearly with the growth of impact parameter $b$ and becomes maximum at $b \sim 1.2R_A$, where $R_A$ is the nucleus radius. After that $v_2^{jet}$ drops rapidly with increasing $b$: this is the domain of impact parameter values, where the effect of decreasing energy loss due to reducing effective transverse size of the dense zone and initial energy density of the medium is crucial and not compensated more by stronger non-symmetry of the volume). Other important feature is that the jet azimuthal anisotropy decreases with increasing jet energy, because the energy dependence of medium-induced loss is rather weak. Finally, the kinematical cuts on jet transverse energy and rapidity has been applied: $E_T^{jet} > 100$ GeV and $|y^{jet}| < 1.5$. After this dijet event is superimposed on Pb–Pb event containing anisotropic flow.

Particle flow. Anisotropic flow data at RHIC [16] can be described well by hydrodynamical models for semi-central collisions and $p_T$ up to $\sim 2$ GeV/c [21], while the majority of microscopical Monte-Carlo models underestimate flow effects (however, see [22]). One can expect that the hydrodynamical model can be applied to estimation of particle flow effects at LHC, may be extending this approach to even higher $p_T$ values. On the other hand, at collider energies one more reason for anisotropic flow in relatively high-$p_T$ domain can arise: the sensitivity of semi-hard particles to the azimuthal asymmetry of reaction volume under the condition that the major part of them are the products of in-medium radiated gluons or parent hard partons [2,3]. Of course, for more detailed simulation, one has to take into account the interplay between hydro flow and semi-hard particle flow. However here we restrict our consideration to using the simple hydrodynamical Monte-Carlo code [23] giving hadron (charged and neutral pion, kaon and proton) spectrum as a superposition of the thermal distribution and collective flow. For the fixed in the model ”freeze-out” parameters — temperature $T_f = 140$ MeV, collective longitudinal rapidity $Y_L^{max} = 5$ and collective transverse rapidity $Y_T^{max} = 1$ — we get average hadron transverse momentum $< p_T^h > = 0.55$ GeV/c. We set the Poisson multiplicity distribution and take into account the impact parameter dependence of multiplicity in a simple way, just suggesting that the mean multiplicity of particles is proportional to the nuclear
overlap function \( \langle dN/dy \rangle (b) \propto T_{AA}(b) \). In the framework of this model, anisotropic flow can be introduced on the assumption that the spatial ellipticity of "freeze-out" region,

\[
\epsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle},
\]

is directly related to the initial spatial ellipticity of nuclear overlap zone, \( \epsilon_0 = b/2R_A \). Such "scaling" allows one to avoid using additional parameters and, at the same time, results in introducing elliptic anisotropy of particle and energy flow due to dependence of effective transverse size of "freeze-out" region \( R_f(b) \) on azimuthal angle of "hadronic liquid" element \( \Phi \):

\[
R_f(b) = R_f(b = 0) \min \{ \sqrt{1 - \epsilon_0^2 \sin^2 \Phi + \epsilon_0 \cos \Phi}, \sqrt{1 - \epsilon_0^2 \sin^2 \Phi - \epsilon_0 \cos \Phi} \}. \tag{11}
\]

Obtained in such a way azimuthal distribution of particles is described well by the elliptic form \( \epsilon \) for the domain of reasonable impact parameter values. Note that in the framework of present paper we do not aim at detailed description of azimuthal hadronic spectra and its comparison with experimental data (in principle, fit of the data could be performed using, for example, \( \epsilon_0 \) in (11) as a parameter). In order to investigate the reliability of the method, we just need here to introduce the elliptic anisotropy of energy flow in Monte-Carlo event generator.

**Energy flow.** To be specified, we consider the example of CMS detector at LHC collider \[8\]. The central ("barrel") part of the CMS calorimetric system will cover the pseudo-rapidity region \(|\eta| < 1.5\), the segmentation of electromagnetic and hadron calorimeters being \( \Delta \eta \times \Delta \phi = 0.0174 \times 0.0174 \) and \( \Delta \eta \times \Delta \phi = 0.0872 \times 0.0872 \) respectively \[8\]. In order roughly to reproduce the real experimental situation (not including real detector effects, but just assuming calorimeter hermeticity), we apply formula (11) to integrated over rapidity energy deposition \( E_i(\varphi) \) of generated particles in 72 rings (according to the number of rings in the hadron calorimeter) covering full azimuth.

Figure 1 shows the \( b \)-dependence of "theoretical" value of \( v_2^{jet} \) (calculated including collisional and radiative energy loss), and \( v_2^{jet} \) determined by the method (11) for the two estimated values of the input parameter, number of charged particles per unit rapidity at \( y = 0 \) in central Pb–Pb collisions, \( dN^\pm/dy = 3000 \) and 6000. One can see that the accuracy of \( v_2^{jet} \) determination is close to 100% for semi-central collisions and becomes significantly worse in very peripheral collisions. In fact, the influence of initial nuclear effects like shadowing or saturation can result in a weaker impact parameter dependence of \( \langle dN/dy \rangle \) (see for example \[2\]). Thus, if the particle density in central collisions is fixed as a parameter, we are getting here the most "pessimistic", minimal estimation of mean multiplicity of non-central events. Since the accuracy of the method used improves with the increasing multiplicity, for weaker \( b \)-dependences one can obtain even better resolution for \( v_2^{jet} \).
collisions \((b \sim 2R_A)\), when decreasing multiplicity and azimuthal anisotropy of the event results in large relative fluctuations of energy deposition in a ring \(^1\).

To conclude this section, let us discuss the influence of different factors on results for \(v_{2}^{\text{jet}}\) determined with the present method. First of all, note that the theoretical absolute value of \(v_{2}^{\text{jet}}\) and its \(b\)-distribution are, of course, very model-dependent. For example, the effect of jet energy loss and corresponding azimuthal anisotropy are sensitive to the defined jet cone size \([10,11,12,13]\). Since we obtained here the energy loss only on the partonic level, for the finite jet cone size \(\theta_0 \neq 0\) the effect should be less pronounced. Another reason for reducing jet azimuthal anisotropy can be sharp transverse collective expansion in events with large impact parameter values \([5]\). However, the relative accuracy of \(v_{2}^{\text{jet}}\) determination does not show any significant dependence on its absolute value. The reason for this is the following. The relative error for the method \([7]\) using particle flow can be roughly estimated as a sum of two terms, which are proportional to \((v_{2}^{\text{jet}}N_{\text{event}})^{-1}\) and \((v_{2}^{\text{part}}N_{\text{part}})^{-1}\) respectively \((v_{2}^{\text{part}}\text{ is the coefficient of particle azimuthal anisotropy and }N_{\text{part}}\text{ is the mean multiplicity in the event})\). If the number of events is large enough, \(N_{\text{event}} \gg 1\), the main influence would be expected due to the second term, which does not depend on \(v_{2}^{\text{jet}}\). This is also the reason why the relative error becomes significant at low values of both the coefficient of particle azimuthal anisotropy and the particle multiplicity. The same conclusion will be valid for the method \([9]\) using energy flow, if the condition \(N_{\text{part}} \gg N_{\text{ring}} \gg 1\) is fulfilled.

Let us also note that in a real experimental situation, the pattern of azimuthal anisotropy can be more complicated due to non-flow correlations, finite accuracy of impact parameter determination, detector resolution effects, etc. On the other hand, one can try to improve accuracy of this technique considering, for example, the correlation between two equal multiplicity sub-events \([14]\), or using results of work \([19]\).

\(^3\)Note that the applicability of hydrodynamical model to very peripheral collisions is unclear. Moreover, the edge effects near the surface of the nucleus, impact parameter dependence of nuclear parton structure functions, early transverse expansion of the system and other potentially important phenomena for such collisions are beyond our consideration here.

\(^4\)As it has been shown in recent work \([4]\), in the extreme scenario assuming instant transverse expansion of the medium, the geometrical anisotropy is strongly reduced, and the anisotropy of particles (at \(p_T \gtrsim 2 - 10\) GeV/c under RHIC conditions) may drop below the observable level. In principle, this effect can have influence also on jet anisotropy at LHC conditions. Let us just mention that the possibility for realization of such scenario for transverse expansion is not obvious. Within original Bjorken approach \([25]\) the rarefaction wave front moves with the sound velocity \(c_s = \sqrt{dp/d\varepsilon}\) and the whole volume of the fluid appears to be involved in three-dimensional expansion in the time of order \(\tau \sim \tau_0 + R_A/c_s\).
4 Conclusions

The strong interest is springing up to the azimuthal correlation measurements in ultrarelativistic heavy ion collisions. One of the main reasons is that the rescattering and energy loss of hard partons in azimuthally non-symmetric volume of dense quark-gluon matter can result in visible azimuthal anisotropy of high-$p_T$ hadrons at RHIC and high-$E_T$ jets at LHC.

In jet case, the methodical advantage of azimuthal observables is that one needs to reconstruct only azimuthal position of jet without measuring total jet energy. One of the ways to perform the inclusive analysis of jet production as a function of azimuthal angle is event-by-event determination of the nuclear reaction plane angle. In the present paper we suggest the method for measurement of jet azimuthal anisotropy coefficients without reconstruction of the event plane. This technique is based on the calculation of correlations between the azimuthal position of jet axis and the angles of particles (not incorporated in the jet), azimuthal distribution of jets being described by the elliptic form. The method is generalized by introducing as weights the particle momenta or energy deposition in the calorimetric rings. In the latter case, we have illustrated the reliability of the present method in real physical situation under LHC conditions. The accuracy of the method improves with increasing multiplicity and particle (energy) flow azimuthal anisotropy, and is practically independent of the absolute values of azimuthal anisotropy of the jet itself.

To summarize, we believe that the present techniques may be useful for future data analysis in heavy ion collider experiments.

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Figure 1: The impact parameter dependence of "theoretical" value of $v_2^{jet}$ including collisional and radiative energy loss (solid curve), and $v_2^{jet}$ determined by the method (9) for $dN^\pm/dy(y = 0, b = 0) = 3000$ (dotted curve) and 6000 (dashed curve).