Controllable Floquet edge modes in a multifrequency driving system

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A driven quantum system was recently studied in the context of nonequilibrium phase transitions and their responses. In particular, for a periodically driven system, its dynamics are described in terms of the multidimensional Floquet lattice with a lattice size depending on the number of driving frequencies and their rational or irrational ratio. So far, for a multifrequency driving system, the energy pumping between the sources of frequencies has been widely discussed as a signature of topologically nontrivial Floquet bands. However, the unique edge modes emerging in the Floquet lattice have not been explored yet. Here, we discuss how the edge modes in the Floquet lattice are controlled and result in the localization at particular frequencies, when multiple frequencies are present and their magnitudes are commensurate values. First, we construct the minimal model to exemplify our argument, focusing on a two-level system with two driving frequencies. For the strong frequency limit, one can describe the system as a quasi-one-dimensional Floquet lattice where the effective hopping between the neighboring sites depends on the relative magnitudes of the potential for two frequency modes. With multiple driving modes, nontrivial Floquet lattice boundaries always exist from controlling the frequencies, and this gives rise to states that are mostly localized at such Floquet lattice boundaries, i.e., particular frequencies. We suggest the time-dependent Creutz ladder model as a realization of our theoretical Hamiltonian and show the emergence of controllable Floquet edge modes.

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I. INTRODUCTION

Topology is an essential concept in modern condensed matter physics [1–7]. One such example is the so-called Thouless pumping model, which is an adiabatic, time-dependent model. The model is designed to pump an integer number of electric charge, which is related to the topological winding number [8–12]. More recently, it was noted that nontrivial topological properties can also emerge in time-periodic driving systems [13–16]. Such time-periodic models are called the Floquet model, which has been extensively investigated in the context of transport, laser-controlled atoms, and electron-phonon coupled systems [17–27]. It has also been noted that a $d$-spatial-dimensional time-dependent system under $D$-frequency drives can be classified by the topology of the static Hamiltonian in $(d + D)$ spatial dimensions, and the relevant examples have been studied [28–30].

In this paper, we will uncover another phenomenon of driven quantum systems. Specifically, we will show that the multifrequency driving system can induce the localized “edge” mode in the frequency lattice, which is equivalent to the Floquet lattice (which will be defined below). These topological modes are localized at a special frequency. The physical origin of such a mode can be understood as follows. When the ratios between the frequencies are commensurate with each other, one can consider the multidimensional Floquet lattice, which repeats along a certain direction, as shown in Fig. 1. This naturally introduces an edge to the Floquet lattice, and the edge may trap an interesting mode, which depends on the topology of the driven system. As proof of this claim, we will introduce an explicit, two-level model with two commensurate driving frequencies which can explicitly demonstrate the desired physics.

The rest of this paper is organized as follows. First, we briefly review the Floquet theory in a periodically driven system. Then we introduce our model, a two-level system with two driving modes, which is transformed into a two-dimensional Floquet lattice. In the strong frequency limit, the Floquet lattice can be mapped onto a quasi-one-dimensional lattice with a nontrivial edge mode. We also discuss the possible realization of our model in the quasi-one-dimensional Creutz lattice. Finally, we also support our theoretical argument with various numerical results.

II. FLOQUET LATTICE AND FLOQUET THEORY

Here, we briefly review the basic physics of the Floquet theory [31], describing a system under multiple time-periodic driving modes. Our starting point is the Hamiltonian $\hat{H}$, which depends on the $D$-time-periodic parameters $(\theta_1, \ldots, \theta_D) = \hat{\vec{\theta}}_D$, each of which depends on one time parameter $t$. That is,
one can interpret the number

\[ n = 1 \pm \text{on-site energies on the red and blue dots have different signs, on the lattice. We will call}\]

\[ \mathbf{n} \]

\[ \text{dimensional tight-binding model defined on a lattice at sites} \]

\[ \text{Equation (5) is mathematically equivalent to the}\]

\[ \text{Note that Eq. (3) couples the frequency}\]

\[ \omega \]

\[ \text{with some constants (}\Omega_1, \ldots, \Omega_D) = \tilde{\Omega}, \]

\[ H(\theta_t) = H(\theta_t + 2\pi), \quad \theta_t(t) = \Omega t. \quad (1) \]

\[ \text{We attempt to solve the time-dependent Schrödinger equation,}\]

\[ i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H(\tilde{\theta}(t))|\psi(t)\rangle. \quad (2) \]

\[ \text{Performing the Fourier transformation, we obtain}\]

\[ \omega |\psi(\omega)\rangle = \sum \mathcal{H}_\Omega |\psi(\omega - \tilde{\mathbf{n}} \cdot \tilde{\Omega})\rangle. \quad (3) \]

\[ \text{Here, } |\psi(\omega)\rangle \text{ and } \mathcal{H}_\Omega \text{ are the Fourier coefficients of the wave function and the Hamiltonian, respectively:}\]

\[ |\psi(t)\rangle = \int d\omega e^{-i\omega t} |\psi(\omega)\rangle, \quad \tilde{H}(\tilde{\theta}) = \sum \mathcal{H}_\Omega e^{-i\tilde{\theta} \cdot \tilde{\mathbf{n}}} |\tilde{\mathbf{n}}\rangle. \quad (4) \]

\[ \text{Note that Eq. (3) couples the frequency } \epsilon \text{ only with other frequencies } \omega = \epsilon + \tilde{\mathbf{n}} \cdot \tilde{\Omega}. \text{ Therefore, by indexing } |\tilde{n}\rangle \equiv |\psi(\epsilon + \tilde{\mathbf{n}} \cdot \tilde{\Omega})\rangle, \text{ Eq. (3) becomes}\]

\[ \epsilon |\tilde{n}\rangle = \sum \mathcal{H}_{n} |\tilde{\mathbf{n}} - \tilde{\mathbf{n}} \cdot \tilde{\Omega} \tilde{n}, \tilde{n})|\tilde{m}\rangle. \quad (5) \]

\[ \text{Equation (5) is mathematically equivalent to the } D\text{-dimensional tight-binding model defined on a lattice at sites } \tilde{n}. \text{ We call this lattice a “Floquet lattice.” In this analogy, } |\tilde{n}\rangle \text{ represents the state when a particle is exactly at site } \tilde{n} \text{ on the lattice. We will call } \epsilon \text{ the quasienergy. Physically, one can interpret the number } n_i \text{ (the } i\text{th component of the vector } \tilde{n} \text{) as the number of absorbed photons from the } i\text{th drive. Therefore, the hopping terms, } \mathcal{H}_n \text{ in Eq. (5), describe the process of absorbing and/or emitting a certain number of photons. Unlike an ordinary tight-binding model, Eq. (5) has an additional on-site potential, } -\tilde{n} \cdot \tilde{\mathbf{\Omega}}. \text{ This on-site potential term corresponds to the electric field } \tilde{\mathbf{\Omega}} \text{ in the tight-binding model; hence, we call } \tilde{\mathbf{\Omega}} \text{ the quasielectric field.}\]

\[ \text{III. THE MODEL}\]

\[ \text{Let us consider a two-frequency-driven Hamiltonian in a two-level system in the rest of this paper. The Hamiltonian is represented as}\]

\[ H(t) = B_z(k)\sigma_z + \left[ (\Delta - \delta(k)) \cos(\rho \Omega t) \right. \]

\[ + \left. \left[ (\Delta + \delta(k)) \cos(q \Omega t) \right] \sigma_z. \quad (6) \]

\[ \text{Here, } k \text{ is another parameter (periodic in } 2\pi) \text{ which can be independently tuned. This variable will be used to change parameters } B_z \text{ and } \delta, \text{ which will be shown later to play the role of staggered potential and alternating hopping in the Floquet lattice. By using a one-dimensional chain system, we note that the parameter } k \text{ can be considered a momentum along the chain. Without loss of generality, however, } k \text{ can be chosen as any time-independent control parameter, and the argument still holds. } \sigma_z \text{ and } \sigma_x \text{ are Pauli matrices, whose basis can be considered pseudospin states } |\uparrow\rangle, |\downarrow\rangle}. \text{ } t \text{ represents the time, and } \Omega \text{ is the frequency of the system with coprime integers } p \text{ and } q.\]

\[ \text{Equation (6) describes a time-periodic system with the period } T = 2\pi/\Omega. \text{ Hence, it can be Fourier transformed into the discrete frequency domain. After performing the Fourier transformation on time, we can write out the following tight-binding model on the Floquet lattice, which is equivalent to the Schrödinger equation for the following Hamiltonian } H_F:\]

\[ H_F = \sum \mathcal{H}_n \left[ B_z(k)\sigma_z^\alpha - \tilde{n} \cdot \tilde{\mathbf{\Omega}} \sigma_z^\beta \right] c_{n,\alpha}^\dagger c_{n,\beta} \]

\[ + (\Delta - \delta(k))\sigma_z^\alpha c_{n,\alpha}^\dagger c_{n+(0,1)\alpha} + \text{H.c.} \]

\[ + (\Delta + \delta(k))\sigma_z^\beta c_{n,\beta}^\dagger c_{n+(0,1)\beta} + \text{H.c.}. \quad (7) \]

\[ \text{Here, } c_{n,\alpha}^\dagger \text{ represents the annihilation operator for a pseudospin } \alpha \text{ state with frequency } \epsilon + \tilde{n} \cdot \tilde{\Omega}, \text{ where } \epsilon \text{ is the quasienergy. Here, } \tilde{\mathbf{\Omega}} = \Omega(\rho, q).\]

\[ \text{Our primary goal here is to show that the Hamiltonian (7) can support the topological edge modes, which are localized in the corresponding Floquet lattice. We will demonstrate this in a few different ways.}\]

\[ \text{A. Instructive limit}\]

\[ \text{The nontrivial topology of Eq. (7) can be most cleanly manifested in the strong driving limit, i.e., } \Delta \gg B_z(k), \delta(k), \text{ so the dominant term is the hopping term with } \sigma_z. \text{ This implies that the distribution of the pseudospin is alternating as in Fig. 1(a). Furthermore, the strong quasielectric field } \tilde{\mathbf{\Omega}} \text{ localizes the ground state around an equipotential line by the Stark localization [32,33]. Thus, in this limit, one can choose the sites near the equipotential line and interactions between those sites to construct an effective quasi-one-dimensional lattice. Although there are many equipotential lines in the system, in the strong frequency limit, the contribution near} \]
the zero-frequency mode is dominant. (See Appendix B for details.)

To exemplify, we first focus our interest on \( q = p + 1 \). In this case, the sites (in the Floquet lattice) near the equipotential line interact with the sites connected by the alternating horizontal and vertical bonds, as shown in Fig. 1(a). Since we can tune the strength of the vertical and horizontal hoppings independently, the quasi-one-dimensional lattice becomes a one-dimensional (1D) lattice model with a unit cell which consists of the two sites. In this lattice, the pseudospin directions of the two sites are opposite because \( \Delta \gg B_z(k), \delta(k) \).

Having this in mind, the total Hamiltonian in Eq. (6) can be approximated to the following form:

\[
H_{1D}(k) = \sum_n [B_z(k)(c^\dagger_{2n}c_{2n} - c^\dagger_{2n+1}c_{2n+1}) + [\Delta - \delta(k)]c^\dagger_{2n+1}c_{2n+2} + \text{H.c.}]
\]

This introduces a "solitonic configuration" in the hopping terms and generates a topological boundary on the periodic lattice. This topological boundary exists due to the one-directional periodicity of the lattice and thus appears only when the ratios between frequencies are commensurate.

To concretely demonstrate the edge mode, we consider the easiest case, \( B_z(k) = \cos k \) and \( \delta(k) = \sin k \); and compare the \( k = \pi/2 \) and \( k = 3\pi/2 \) cases. In both cases, we have \( B_z(k) = 0 \). Hence, the 1D lattice becomes a Su-Schrieffer-Heeger (SSH) lattice with hopping terms \( \Delta \pm 1 \), and the boundary of the system can be considered the joint between the trivial and topological phases of the SSH chain. This creates the localized eigenstate near the boundary. Because each site of the Floquet lattice represents the frequency of the state, it corresponds to the state with a high occupation of the frequency which represents the boundary. Since the small perturbation does not affect the localization of the state much, the localization appears on every \( k \in [0, 2\pi] \) for \( \delta(k) \neq 0 \).

The choice of the equipotential line is arbitrary, and we may choose any equipotential line perpendicular to the quasi-electric field. With a given equipotential line, the relative hopping magnitudes \( \Delta \pm \delta \) are alternating, but there are boundaries where the same hopping magnitudes meet. At such boundaries, strong localization occurs, and we refer to them as edge modes.

Until now we have discussed only the \( q = p + 1 \) case. In this case, the position of the localized edge mode is not controllable since there is only one boundary point and the state is simply localized near it. To show the controllability of the localized mode, we introduce the \( q = p + 2 \) case, which can be considered the case with multiple boundaries.

Figure 3 shows an example of a quasi-one-dimensional lattice structure for \( q = p + 2 \), particularly when \( p = 3 \) and
The periodic structure of a quasi-one-dimensional Floquet lattice, with $p = 3$ and $q = 5$. The large black dots represent the points near the equipotential line, and the horizontal and vertical red and blue lines represent the interaction. The red dashed circles represent the position of the boundary on the quasi-one-dimensional lattice.

$q = 5$, i.e., $q = p + 2$. In the vicinity of the equipotential line, there are in total three different boundary points, marked by dashed red circles in Fig. 3. Specifically, when the strength of the blue bond is stronger than the red one, we expect localization at the $(0,0)$ point, which gives the same result as in the $q = p + 1$ case. On the other hand, when the strength of the red bond is stronger than the blue one, localization in the middle of the quasi-one-dimensional lattice will occur. Since the strengths of the red and blue bonds depend on the parameter $k$, we can control the localization mode of the system by tuning the $k$ value.

### C. Physical realization via the Creutz lattice

Before showing some numerical results which justify our heuristic understanding above, we present a potential realization of our model in a lattice model. So far, we have not assigned any particular physical meaning to $k$, and hence, in principle, there could be many different ways to realize the Hamiltonian in Eq. (6). Here, we consider a time-dependent 1D lattice, which is a variation of the so-called Creutz ladder [35], as a potential candidate to realize our model.

To realize the Hamiltonian in Eq. (6), we consider $k$ to be the momentum parameter. Furthermore, by considering two sublattices $A$ and $B$ in real space, we can choose the basis $\{\{\uparrow\}, \{\downarrow\}\}$ of $\sigma_z$ and $\sigma_z$. We will assign the creation operator of the momentum $k$ at $A$ and $B$ sites in a way that $|\uparrow\rangle_k = c_{k,A}^\dagger |0\rangle$ and $|\downarrow\rangle_k = c_{k,B}^\dagger |0\rangle$. This allows us to write

$$\sigma_z \rightarrow c_{k,A}^\dagger c_{k,B} + c_{k,B}^\dagger c_{k,A}, \quad \sigma_z \rightarrow c_{k,A}^\dagger c_{k,B} - c_{k,B}^\dagger c_{k,A}. \quad (10)$$

We can now write down the Hamiltonian with $B_z(k) = B_z \cos k$ and $\delta(k) = \delta \sin k$ as follows:

$$H(k, t) = B_z \cos k (c_{k,A}^\dagger c_{k,A} - c_{k,B}^\dagger c_{k,B}) + \Delta [\cos(p\Omega t) + \cos(q\Omega t)] (c_{k,A}^\dagger c_{k,B} + c_{k,B}^\dagger c_{k,A})$$
$$+ \delta \sin k [\cos(p\Omega t) + \cos(q\Omega t)] (c_{k,A}^\dagger c_{k,B} + c_{k,B}^\dagger c_{k,A}). \quad (11)$$

Using the inverse Fourier transformation, $c_{A/B,k} = \sum_x c_{A/B,x} e^{-ikx}$, we get

$$H(t) = \sum_k H(k, t) = H_h + H_v + H_d. \quad (12)$$

Here, the horizontal interaction between dimers of the ladder $H_h$, the intrainteraction of the dimers $H_v$, and the diagonal interaction between dimers of the ladder $H_d$ are represented as, respectively,

$$H_h = \frac{B_z}{2} \sum_x [c_{x+1,A}^\dagger c_{x,A} + c_{x+1,B}^\dagger c_{x,B} + H.c.],$$

$$H_v = \frac{\Delta}{2} [\cos(p\Omega t) + \cos(q\Omega t)] \sum_x [c_{x,A}^\dagger c_{x,B} + H.c.],$$

$$H_d = \frac{1}{4} [-\cos(p\Omega t) + \cos(q\Omega t)] \sum_x [c_{x,A}^\dagger c_{x,B}$$
$$+ c_{x,B}^\dagger c_{x,A} + H.c.]. \quad (13)$$

This type of ladder is the so-called the Creutz ladder. After constructing the time-dependent Creutz ladder with the coefficients given above, we may scan the system in the momentum space $k$. This process experimentally measures the localization of the state, which is the clue to the edge states in our model.

### IV. NUMERICAL DEMONSTRATIONS

In this section, we present the numerical proof of our claim above. In this numerical simulation, we solve the full Floquet problem and do not restrict ourselves to the effective 1D Floquet Hamiltonian. We find that the above understanding based on the effective 1D lattice model in Eq. (8) is indeed correct.

We first confirm the localization of the modes under strong enough quasielectric field $\bar{\Omega}$, or, equivalently, in the strong frequency limit. Numerically, we show the localization near the equipotential line when the energy level of the frequency $\Omega$ is comparable with the hopping term $\Delta$. This is reasonable because if the quasielectric field is very weak compared to the hopping term, then the hopping term increases the localization length. When the quasielectric field, or, in other words, the frequency of the driving modes, is strong enough, then the occupation of the eigenstate can be restricted along the quasi-one-dimensional lattice near the equipotential line, and thus, we may project the whole Hilbert space onto the quasi-one-dimensional sublattice. Detailed numerical studies are explained in Appendix B.

Before explaining the main results, we explain the numerical method used for our calculation. Since our Hamiltonian is
a time-dependent $2 \times 2$ matrix, it is possible to consider the problem in two congruent differential equations. By solving it numerically, it is possible to find an eigenstate of the driven system. Then, by taking the Fourier transformation and plotting the amplitude of each frequency profile on the Floquet lattice, we represent our result.

Now let us explore the physics of the topological edge mode. We plot the distribution of the states on the quasi-one-dimensional Floquet lattice at $k = 0$ [Fig. 4(a)] and $k = \pi/2$ [Fig. 4(b)]. For $k = 0$, i.e., $\delta = 0$, it exhibits the extended state in addition to the localized mode at zero frequency. However, for $k = \pi/2$, i.e., $\delta \neq 0$, only the localized state at zero frequency survives, which can be easily understood from the SSH model as discussed above. Figure 4 shows the distribution of the states projected on the alternating pseudospin state, for example, $|\uparrow\downarrow\uparrow\downarrow\cdots\rangle$. Suppose that we projected the state on another alternating pseudospin configuration, $|\downarrow\uparrow\downarrow\uparrow\cdots\rangle$. Because this process changes the up state and down state, we may consider the total Hamiltonian to transform as

$$
B_z \cos(k) \rightarrow -B_z \cos(k) = B_z \cos(\pi - k),
$$

$$
\delta \sin(k) \rightarrow \delta \sin(k) = \delta \sin(\pi - k).
$$

(14)

Therefore, flipping the sign is equivalent to transforming $k$ into $\pi - k$. This shows that the $k = 0$ case in Fig. 4(a) and the $k = \pi$ case change in the different alternating pseudospin projection cases. Notice that this also implies opposite signs of the winding numbers of $(B_z, \delta)$ and $(-B_z, \delta)$, showing the boundary is created by the joint between two lattices with different topological properties.

We here emphasize the point that the difference between Figs. 4(a) and 4(b) shows that the localization indeed occurs due to the topology of the quasi-one-dimensional lattice, similar to the case of the SSH model. Indeed, the state distribution in Fig. 4(a) does not have a perfectly equidistributed extended mode but has a small preference on the one edge, even without any topological property on the lattice. This preference occurs due to the Stark localization in the direction perpendicular to the quasielectric field. However, as we can see in Fig. 4(b), the localization at the boundary due to the topological properties is much stronger than the Stark effect applied along the perpendicular direction of the quasielectric field.
appear even though we slightly change the $k$ value from $\pi/2$, as we can see in Fig. 5(c). Thus, the comparison of the two different cases, $q = p + 1$ and $q = p + 2$, supports our claim that the localization indeed occurs due to the presence of boundaries where the same hopping magnitudes meet on the Floquet lattice and that such boundaries vary depending on different frequency ratios.

In summary, we have shown that the localization modes at particular frequencies can be controlled via the multifrequency ratio and their magnitudes, and this can be explained by the transformation of our model into the Floquet SSH model on a quasi-one-dimensional lattice.

V. CONCLUSIONS

In this study, we designed a one-dimensional Creutz ladder model with two driving modes. Under strong frequency, the Floquet version of this model can be reduced to a quasi-one-dimensional model with nontrivial topological properties. Due to the construction, this quasi-one-dimensional model is mathematically equivalent to the time-driven SSH model with a boundary. Therefore, the localization on the Floquet lattice is evidence of the topological property generated by the junction of two SSH models with different topological properties, which is experimentally measurable.

In a multifrequency system with commensurate frequency ratios, our work suggested a different method to build the boundary on the Floquet lattice. It offers insight into how multifrequency driving systems can be analyzed in the Floquet lattice with topological properties and the relevant localized modes. This localization can be measured experimentally with comparable frequency scales and amplitudes of the driving modes if one can prepare the eigenstate of the driven system. Because we have shown the topological property which occurs due to the interplay between the spatial dimension and frequency modes, our system can be extended not only by adding driving modes but also by increasing the spatial dimension. This makes possible a topological system in a spatial multidimension with multifrequency drives, suggesting different kinds of topological Floquet systems.

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APPENDIX A: RICE-MELE MODEL

The Rice-Melee model is a time-periodic model with adiabatic charge pumping, with a nontrivial topological property. As a one-dimensional lattice with a unit cell containing two sublattices, its Hamiltonian can be written as follows:

$$H(t) = \sum_n \left[ \delta h(t)(c_{n,1}^\dagger c_{n,1} - c_{n,2}^\dagger c_{n,2}) + \frac{v + \delta v(t)}{2} c_{n,1}^\dagger c_{n,2} + \text{H.c.} \right] + \frac{v - \delta v(t)}{2} c_{n,2}^\dagger c_{n+1,1} + \text{H.c.}. \quad (A1)$$

Here, we modulate a staggered potential $\delta h(t)$ and an alternating hopping $\delta v(t)$ as the time $t$ changes, while the constant hopping $v$ never changes. The creation and annihilation operators on the $n$th unit cell, with sublattices 1 and 2, are written as $c_{n,1/2}^\dagger$ and $c_{n,1/2}$, respectively. The visualized structure of the Rice-Melee model is shown in Fig. 6(a).

In the case when $|\delta h|,|\delta v| < |v|$ and at half filling, as the path of the parameter $(\delta h, \delta v(t))$ winds around the origin of the parameter space, this model pumps charges with the time period. For example, when $\delta h(t)$ and $\delta v(t)$ are given as

$$\delta h(t) = \cos \left( \frac{2\pi t}{T} \right), \quad \delta v(t) = \sin \left( \frac{2\pi t}{T} \right), \quad (A2)$$

the charge pumping occurs in period $T$ since the parameter set $(\delta h, \delta v)$ winds around the origin as in Fig. 6(b).

The number of charges pumped by the Rice-Melee model coincides with the winding number of the parameter space, or, equivalently, the Chern number of the Hamiltonian,

$$C = \frac{1}{2\pi} \sum_{n} \int_0^T dt \int dk F_{1,k}^n. \quad (A3)$$

FIG. 6. (a) Visualized structure of the Rice-Melee model. The green rectangles show unit cells which contains two sublattices with staggered potential $\pm \delta h$, marked by blue and red dots. Blue and red lines connecting two sites represent the alternating hopping $v \pm \delta v$. (b) Trace of the parameters $(\delta h, \delta v)$ when $\delta h(t) = \cos(2\pi t/T)$ and $\delta v(t) = \sin(2\pi t/T)$. In a period, the trace of parameters $(\delta h, \delta v)$ winds the origin once in the counterclockwise direction.
Here, the summation \( \sum \) runs around the eigenstates below the energy gap, and \( F_{1,k}^n \) represents the Berry curvature of the \( n \)th eigenstate at time \( t \) and momentum \( k \). Because the Chern number is a topological property of the system, this shows why the amount of charge is quantized.

**APPENDIX B: LOCALIZATION NEAR THE EQUIPOTENTIAL LINE**

The relation between the strength of the frequency and the localization on the equipotential line is a key point of the research. In this Appendix we show that the localization indeed occurs when the frequency is high enough.

For the parameter of localization, we choose the variation of the distance from the equipotential line \( \Delta x \). Specifically, we choose an equipotential line and define \( x \) as an operator measuring the distance from the equipotential line to each site. By calculating the variance \( \Delta x^2 = \langle x^2 \rangle - \langle x \rangle^2 \) of the state, we get the dispersion of the state around the equipotential line. Notice that the variance \( \Delta x^2 \) does not depend on the position of the equipotential line we take.

As we can see in Fig. 7, the variation \( \Delta x \) gets smaller as the frequency gets higher. This indicates that the state has a contribution from more sites near the equipotential line. This corresponds to the fact that the localization on the equipotential line occurs due to the quasielectric field, whose strength depends on the frequency. As the frequency increases, the strength of the quasielectric field on the Floquet lattice also increases, which makes the Stark effect even stronger.