Quasi-multi-Regge processes with a quark exchange in the $t$-channel

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Abstract
The QCD amplitudes for particle’s production in the quasi-multi-Regge kinematics with a quark exchange in crossing channels are calculated in the Born approximation. In particular they are needed to find next-to-leading corrections to the quark Regge trajectory and to the integral kernel of the Bethe-Salpeter equation for the $t$-channel partial wave with fermion quantum numbers and a negative signature. The gauge-invariant action for the interaction of the reggeized quarks and gluons with the usual particles is constructed.

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1 Introduction

It is well known [1] [2], that gluon is reggeized in the framework of the perturbative QCD. Namely, the scattering amplitude for the process $A + B \rightarrow A' + B'$ at large energies $s^{1/2}$ ($s = (p_A + p_B)^2$) and fixed momentum transfers $q = (-t)^{1/2}$ ($t = (p_A - p_B)^2$) has the Regge form

$$f(s, t) = f_0(s, t) \Gamma_{A'}^{A}(q^2) \left(\frac{s}{q^2}\right)^{\omega(q^2)} \Gamma_{B'}^{B}(q^2),$$

(1)

where

$$f_0(s, t) = T_{A'}^{A}(c) \frac{2s}{t} T_{B'}^{B}(c), \quad [T(a), T(b)] = i f_{abc} T(c).$$

(2)

The quantities $\Gamma_{A'}^{A}(q^2)$ and $j = 1 + \omega(q^2)$ are the reggeon-particle vertex and the gluon Regge trajectory correspondingly. They are known up to the second order of the perturbation theory [3]. The BFKL Pomeron in QCD is a compound state of two reggeized gluons in the leading and next-to-leading approximations [1], [4]. With the use of non-abelian renormalization schemes and the BLM procedure for the scale setting of the running QCD coupling constant the prediction of the BFKL theory in the next-to-leading approximation turns out to be in an agreement with the experimental data of the L3 group on the hadron production in the virtual photon-photon collisions [5].

There are other colourless objects which can be constructed as composite states of several reggeized gluons in the leading logarithmic approximation (LLA). The simplest state of such type is the Odderon having the charge parity $C = -1$ and the signature $P_j = -1$. These quantum numbers are opposite in sign to that for the Pomeron. The Odderon can be constructed as a composite state of three reggeized gluons [6]. In LLA one can derive an effective field theory for the reggeized gluon interactions in the 2+1 space-time [7]. The effective lagrangian for the interaction of the reggeized gluons with the quarks and gluons is obtained from QCD [8].

The quark in QCD is also reggeized in the leading logarithmic approximation (LLA) [9] (for the case of the electron in QED it was discovered in ref. [10]). Indeed, one can show, that in the $t$-channel partial wave with the positive signature there is a Regge pole with its trajectory going through the point $j = 1/2$ for $t = M^2$. Further, for the amplitude with a negative signature in ref [9] the integral equation similar to the BFKL equation was obtained. Because all known mesons and barions are compound states of quarks, one should construct the theory of the reggeized quark interactions. In particular, it is needed to calculate next-to-leading corrections to the quark Regge trajectory using the methods which were developed earlier for the gluodynamics [1], [11]. These methods are based on an intensive use of dispersion relations and unitarity conditions starting from the Born expression for the inelastic amplitudes in the quasi-multi-Regge kinematics. An alternative method, based on the effective action, can be also used [8].

In refs. [7], [9] simple rules for finding the Born amplitudes in the multi-Regge kinematics were formulated. To calculate the loop corrections to LLA one should know inelastic amplitudes in the quasi-multi-Regge kinematics for the case, when all pair energies except
one are large in comparison with momentum transfers. Such amplitudes for the processes with the gluon exchange in the $t$-channel are known (see [11]).

This paper is devoted to the calculation of the Born amplitudes for the quasi-multi-Regge kinematics with the quark exchange in the crossing channel. It is well known, that the Regge trajectory is universal. Thus, it is enough to consider only one process with the fermion exchange: for example, the collision of quark and anti-quark. In Section 2 we calculate all amplitudes neccessary for finding two-loop corrections to the Regge trajectory of the quark and one-loop corrections to the integral kernel for the amplitude with the negative signature.

In Section 3 the effective action describing the interaction of the reggeized quark with usual quarks and gluons is suggested. From this action all results of Section 2 can be derived. In the end of Section 3 the program for the calculation of next-to-leading corrections to the quark Regge trajectory and to various reggeon vertices is formulated.

\section{Inelastic processes in the quasi-multi-Regge kinematics}

We consider the Yang-Mills theory with the gauge group $SU(N_c)$ containing also the quark field in the fundamental representation. The generators $T^a$ satisfy the commutation relation $[T^a, T^b] = if^{abc}T^c$ and the normalization condition $tr(T^aT^b) = \frac{1}{2}\delta^{ab}$.

We introduce two light-cone vectors corresponding to the massless particle momenta in the c.m. system: $p_1 = (p, 0, 0, p)$ and $p_2 = (p, 0, 0,-p)$ and define light-cone components of the Lorentz vector $a_{\mu}$ as follows $a_{\pm} = a_{\mp} = a_{\mu}n_{\pm} = a^0 \pm a^3$, where $n^+ = p_2/p, n^- = p_1/p$. For the scalar products of two vectors the sum over the Lorentz indices is implied: $ab = a_\mu b_\mu \equiv a_\mu b_\mu g^{\mu\nu} = (a_+ b_- + a_- b_+)/2 + a_\perp b_\perp$.

Let us consider the annihilation of the quark and anti-quark with masses $M$ into three gluons $A + B \rightarrow A' + A'' + B'$. One can use the Sudakov decomposition for the momenta of the colliding particles

$$p_A = p_1 + \frac{M^2}{4p^2} p_2, \quad p_B = p_2 + \frac{M^2}{4p^2} p_1,$$

where $s = (p_A + p_B)^2 \simeq 4p^2 + 2M^2$.

In the quasi-multi-Regge kinematics for the gluon production in the fragmentation region of the initial quark with the momentum $p_A$ the following relations among invariants

$$u_1 = (p_{A'} - p_B)^2 \sim u_2 = (p_{A''} - p_B)^2 \sim s \gg M^2$$

$$t_1 = (p_A - p_{A'})^2 \sim t_2 = (p_A - p_{A''})^2 \sim M^2$$

$$t = (p_{B'} - p_B)^2 \sim M^2, \quad s_1 = (p_{A'} + p_{A''})^2 \sim M^2$$

are valid. The components of particle momenta in this kinematics are

$$p_B^+ \ll p_{A'}^+ \sim p_{A''}^+, \quad p_A^+ \simeq p_{A'}^+ + p_{A''}^+,$$
For the momentum transfer $q = p_{B'} - p_B = p_A - p_{A'} - p_{A''}$ we obtain
\begin{equation}
q^+ \ll p^+_A, p^+_A, p^+_A', p^+_{A''}, \quad q^- \ll p^-_B, p^-_{B'}, \quad q^2 \sim q^2_\perp \sim M^2.
\end{equation}

In the Born approximation the amplitude of annihilation of the quark $A$ and the virtual antiquark $C$ with the momentum $p_C = -q$ into two gluons $A', A''$ equals
\begin{equation}
\mathcal{A}(AC \to A'A'') = g^2 e^*_{A\mu} e^*_{A''\nu} \cdot \bar{v}_C \left( \gamma_\nu (\hat{p}_A - \hat{p}_{A'} - M)^{-1} \gamma_\mu T^{a''} T^{a'} + + \gamma_\mu (\hat{p}_A - \hat{p}_{A''} - M)^{-1} \gamma_\nu T^{a''} T^{a'} + + \gamma_{\mu\nu\sigma}(p_{A'}, p_{A''}) s_{1}\gamma_\sigma (T^{a'} T^{a''} - T^{a''} T^{a'}) \right) u_A,
\end{equation}
where $e_{A'}, e_{A''}$ are polarization vectors of the final gluons, $a', a''$ are their colour indices and $u_A, \bar{v}_C$ are the fermion spinors. The tensor
\begin{equation}
\gamma_{\mu\nu\sigma}(p, p') = (p' - p)_\sigma g_{\mu\nu} - (p + 2p')_\mu g_{\nu\sigma} + (2p + p')_\nu g_{\mu\sigma}
\end{equation}
is the triple Yang-Mills vertex.

It turns out, that with taking into account eq. (7) the amplitude of the process $A + B \to A' + A'' + B'$ can be written in in the form
\begin{equation}
\mathcal{A}(AB \to A'A''B') = g e^*_{A\mu} e^*_{A''\nu} e^*_{B'\lambda} \cdot \bar{v}_B \left( A^{a'a''\nu}_{\mu\lambda} + \delta_{\mu\nu\lambda} \right) u_A.
\end{equation}
Here we denoted
\begin{equation}
A^{a'a''\nu}_{\mu\lambda} = g^2 \gamma^{(-)}_{\nu\lambda}(0, q) T^{b'}(q - M)^{-1} \gamma^{(+)}_\nu(p_{A''}, q) \gamma^{(-)}_\mu(p_{A'} - \hat{p}_{A'} - M)^{-1} \gamma^{(-)}_{\mu\nu\lambda} T^{a''} T^{a'} + + \gamma^{(+)}_\mu(p_{A'}, q) \gamma^{(-)}_{\mu\nu\lambda} \gamma^{(+)}_\nu(p_{A''}, q) \gamma^{(-)}_{\mu\nu\lambda} T^{a''} T^{a'} + + \gamma_{\mu\nu\sigma}(p_{A'}, p_{A''}) s_{1}\gamma_{\sigma} (T^{a'} T^{a''} - T^{a''} T^{a'}) \right) u_A
\end{equation}
and introduced the effective vertices
\begin{equation}
\gamma^{(+)}_{\nu}(p, q) = \gamma_{\nu} + (\hat{q} - M) \frac{n_p^+}{p^+}, \quad \gamma^{(-)}_{\mu}(p, q) = \gamma_{\mu} + (\hat{q} - M) \frac{n^-_p}{p^+}.
\end{equation}
These vectors have the property of the transversality with respect to the gluon momentum $p_\mu \gamma^{(+)}_{\mu}(p, q) = \hat{p} + (\hat{q} - M) \to 0$, $p_\lambda \gamma^{(-)}_{\lambda}(p, q) \to 0$ providing that the colliding fermions are on their mass shell. It is important, that they differ from the usual $\gamma$-matrices only by the terms which are proportional to the factor $\hat{q} - M$ cancelling the neighbouring fermion propagator. These terms are contributions from the crossing diagrams having the additional
The additional contribution $\delta^{a'a''b'}_{\mu\nu}$ needed for the gauge invariance of the production amplitude does not have any pole in the $q^2$-channel and corresponds to the Feynman diagrams having singularities in the direct energy invariants. It is the sum of terms containing in the denominator one of the following expressions

$$e^{\ast}_{\lambda} \rho p_{A''} e_{B',\lambda} \cdot A_{\mu\nu}^{a'a''b'} = -g^{2} e^{\ast}_{B',\lambda} \gamma^{(-)}_{\lambda}(-p_{B'},q) T^{b'} \left( \frac{e^{\ast}_{A'} T^{a'\nu} T^{a'} + p^{+}_{A'} e^{\ast}_{A'} T^{a'\nu} p^{+}_{A'}}{p^{+}_{A'} p^{+}_{A}} \right) e^{\ast}_{A''} p_{B'} \cdot A_{\mu\nu}^{a'a''b'} = 0$$

(12)

The additional contribution $\delta^{a'a''b'}_{\mu\nu}$ is not gauge invariant:

$$\delta^{a'a''b'}_{\mu\nu} = g^{2} \gamma^{(-)}_{\lambda}(-p_{B'},q) \left( a \frac{p_{B_{mu}} p_{B_{nu}}}{d^{1}} + b \frac{p_{B_{mu}} p_{B_{nu}}}{d^{2}} \right) T^{b'} T^{a''\nu} T^{a'} + (p_{A'} \leftrightarrow p_{A''}, a' \leftrightarrow a'')$$

(13)

with the dimensionless constants $a, b$. To satisfy the transversality conditions one should choose the unique values for them: $a = 0, b = 1$ corresponding to the correct weights for the corresponding Feynman diagrams.

Thus, we obtain the induced vertex

$$\Gamma^{a'a''}_{\mu\nu} = -g^{2} (\hat{q} - M) \frac{n^{+}_{\mu} n^{+}_{\nu}}{p^{+}_{A}} \left( \frac{T^{a'} T^{a''}}{p^{+}_{A'}} + \frac{T^{a''} T^{a'}}{p^{+}_{A'}} \right)$$

(14)

If in the final state we have quark $A'$, anti-quark $A''$ and gluon $B'$, the production amplitude is constructed in a more simple way

$$A(AB \rightarrow A' A'' B') = g e^{\ast}_{B',\lambda} \cdot \tilde{v}_{B} T^{b'} \gamma^{(-)}_{\lambda}(-p_{B'},q) (\hat{q} - M)^{-1} \Gamma_{q\bar{q}} u_{A},$$

(15)

where

$$\Gamma_{q\bar{q}} = -g^{2} \left( \left( \bar{u}_{A'} \gamma_{\sigma} T^{c} v_{A''} \right) s^{-1} \gamma_{\sigma}^{(+)}(p_{A''} + p_{A''}, q) T^{c} + (p_{A'} - p_{A''})^{-2} T^{c} \gamma_{\sigma}^{(+)}(p_{A'} - p_{A''}, q) v_{A''} \otimes \bar{u}_{A'} \gamma_{\sigma} T^{c} \cdot \delta_{AA'} \delta_{BA''} \right)$$

and the multiplier $\delta_{AA'} \delta_{BA''}$ in the last line corresponds to the conservation of the quark flavour.

Let us consider now the more complicated process $A + B \rightarrow A' + D_{1} + D_{2} + B'$ in the quasi-multi-Regge kinematics where the gluons $D_{1}, D_{2}$ in the central rapidity region are produced with a fixed invariant mass

$$p^{+}_{B'} \ll p^{+}_{D_{1}} \sim p^{+}_{D_{2}} \ll p^{+}_{A'} \simeq p^{+}_{A},$$

$$p^{+}_{A'} \ll p^{+}_{D_{1}} \sim p^{+}_{D_{2}} \ll p^{+}_{B'} \simeq p^{+}_{B},$$

$$p_{A'\perp} \sim p_{D_{1} \perp} \sim p_{D_{2} \perp} \sim p_{B'\perp} \sim M,$$

$$s_{2} = (p_{D_{1}} + p_{D_{2}})^{2} \sim M^{2},$$

$$t_{1} = (p_{A'} - p_{A})^{2} \sim M^{2}, t_{2} = (p_{B'} - p_{B})^{2} \sim M^{2}. $$

(16)
Its amplitude has the property of the factorization

\[ A(AB \rightarrow A'D_1D_2B') = \Gamma(B \rightarrow B')(\hat{q}_2 - M)^{-1} \cdot \Gamma_{D_1,D_2} \cdot (\hat{q}_2 - M)^{-1} \Gamma(A \rightarrow A') , \]

For the vertices corresponding to the transition of the initial (reggeized) fermions A, B into the final gluons A', B' we have the known result

\[
\Gamma(A \rightarrow A') = -g e^*_{\lambda,\mu} \cdot \gamma^{(+)}(p_{A'}, q_1) T^{d'} u_A , \]
\[
\Gamma(B \rightarrow B') = -g e^*_{\mu,\lambda} \cdot \bar{v}_B T^\nu \gamma^{(-)}(-p_{B'}, q_2) \]

The effective amplitude for the gluon production in the virtual quark-anti-quark collisions is

\[
\Gamma_{D_1 D_2} = -g^2 e^*_{\mu,\nu} e^*_{\nu,\mu} \cdot \left( \gamma^{(+)}(p_{D_2}, q_2)(\hat{q}_2 + \hat{p}_{D_2} - M)^{-1} \gamma^{(-)}(-p_{D_1}, q_2) T^{d_2} T^{d_1} + + \gamma^{(+)}(p_{D_1}, q_2)(\hat{q}_2 + \hat{p}_{D_1} - M)^{-1} \gamma^{(-)}(-p_{D_2}, q_2) T^{d_1} T^{d_2} + + \gamma_{\mu,\nu,\sigma}(p_{D_1}, p_{D_2}) s_2^{-1} \gamma^{(+)}(q_1, q_2)(T^{d_1} T^{d_2} - T^{d_2} T^{d_1}) + \Delta_{\mu,\nu,\sigma}^{d_1, d_2}(q_1, q_2) \right) .
\]

Here

\[
\gamma^{(+)}(q_1, q_2) = \gamma_\sigma - (\hat{q}_1 - M) \frac{n_\sigma^-}{p_{D_1} + p_{D_2}} + (\hat{q}_2 - M) \frac{n_\sigma^+}{p_{D_1} + p_{D_2}}
\]

is the effective vertex for the transition of two reggeized quarks into one gluon.

The induced vertex coming from the diagrams without particles in q_1^2 or q_2^2 channels is

\[
\Delta_{\mu,\nu,\sigma}^{d_1, d_2}(q_1, q_2) = (\hat{q}_2 - M) \frac{n_\mu^+ n_\nu^-}{p_{D_1} + p_{D_2}} \left( \frac{a}{p_{D_1}^2} + \frac{b}{p_{D_2}^2} \right) T^{d_2} T^{d_1} + + (\hat{q}_1 - M) \frac{n_\mu^- n_\nu^+}{p_{D_1} + p_{D_2}} \left( \frac{c}{p_{D_1}^2} + \frac{d}{p_{D_2}^2} \right) T^{d_1} T^{d_2} + + (q_1 - p_{D_1})^{-2} \frac{n_\mu^+}{p_{D_2}^2 + q_2^2} \left( \frac{T^{d_2} T^c}{q_2^2} + \frac{T^c T^{d_2}}{p_{D_2}^2} \right) \right) ,
\]

where \( \gamma^+ \equiv \gamma_\sigma n_\sigma^+ \) and c is the colour index of the virtual (reggeized) gluons.

At last the effective amplitude for the transition of the virtual (reggeized) quark (with flavour \( F_1 \)) and anti-quark (with flavour \( F_2 \)) into the real quark \( D_1 \) and anti-quark \( D_2 \) is

\[
g^2 \left( (\bar{u}_{D_1} \gamma_\sigma T^c v_{D_2}) s_2^{-1} \gamma^{(+)}(q_1, q_2) T^c \cdot \delta_{F_1, F_2} \delta_{D_1, D_2} + + (q_1 - p_{D_1})^{-2} T^c \gamma^{(+)}(q_1, q_2) v_{D_2} \otimes (\bar{u}_{D_1} \gamma_\sigma^{-})(p_{D_1} - q_1, q_1) T^c \cdot \delta_{F_1, D_1} \delta_{F_2, D_2} \right) .
\]
3 Effective action for the reggeized gluon and quark interactions

The effective action for the reggeized gluon interactions local in the rapidity interval \((y_0 - \frac{\eta}{2}, y_0 + \frac{\eta}{2})\) \((\eta \ll \ln \frac{s}{M^2})\) can be written as follows [8]

\[
S_{G_{\text{eff}}} = \int d^4x \left( L_{\text{QCD}} + \text{tr} \left( (A_+(v_+) - A_+) \partial^2 \partial_\mu A_+ + (A_-(v_) - A_-) \partial^2 \partial_\mu A_- \right) \right),
\]
where the usual QCD lagrangian is well known

\[
L_{\text{QCD}} = \frac{1}{2} \text{tr} G_{\mu\nu}^2 + \bar{\psi} (i \slashed{D} - M) \psi, \quad G_{\mu\nu} = \frac{1}{g} [D_\mu, D_\nu], \quad D_\mu = \partial_\mu + gv_\mu.
\]

It is expressed in terms of the anti-hermitian gluon field \(v_\mu = -iv_\mu^a T^a\) \((T^a \text{ is the colour group generator in the fundamental representation})\) and the quark field \(\psi\). The composite field \(A_\pm(v_\pm)\) contains an infinite number of terms

\[
A_\pm(v_\pm) = \sum_{n=0}^{\infty} (-g)^n v_\pm (\partial_\pm^{-1} v_\pm)^n
\]
and can be written in terms of the covariant derivative

\[
A_\pm(v_\pm) = -\frac{1}{g} \partial_\pm \frac{1}{D_\pm} \partial_\pm 1,
\]
where 1 is a unit matrix.

We can chose different definitions for the integral operators \(\partial_\pm^{-1}\) to express \(A_\pm(v_\pm)\) through \(P\)-ordered Wilson exponents with the integration contours displaced along light cone lines. The Feynman diagram prescription corresponds to the symmetric form of \(A_\pm(v_\pm)\)

\[
A_\pm(v_\pm) = -\frac{1}{g} \partial_\pm U(v_\pm),
\]

\[
U(v_\pm) = \frac{P}{2} \exp \left( -\frac{g}{2} \int_{-\infty}^{\infty} dx^\pm v_\pm(x') \right) + \frac{\bar{P}}{2} \exp \left( \frac{g}{2} \int_{-\infty}^{\infty} dx^\pm v_\pm(x') \right), \quad U^+(v_\pm) = U^{T*}(v_\pm).
\]

The anti-hermitian fields \(A_\pm = -iA_\pm^a T^a\) describe the production and annihilation of the reggeized gluons. Their bare propagator is

\[
\int d^4x e^{-ipx} < A_{\pm}^{\alpha}(x) A_{\pm}^{\beta}(0) > = q_\pm^{-2} \theta(y' - y - \eta) \delta^{\alpha\beta}.
\]

The reggeon fields satisfy the kinematical constraints [8]

\[
\partial_+ A_- = \partial_- A_+ = 0.
\]
Taking also into account the above prescription for \( U(v_{\pm}) \) it leads to the following expression for the reggeon-gluon interaction term \([2]\)

\[
S_{G}^{\text{int}} = -\frac{1}{g} \text{tr} \int d^2 \vec{k} \left( \int_{-\infty}^{\infty} d x^+ T(v_-) \partial_{\mu}^2 A_+ + \int_{-\infty}^{\infty} d x^- T(v_+) \partial_{\mu}^2 A_- \right),
\]

and provides the hermicity property of the effective action.

Under the gauge transformations of the gluon and quark fields

\[
v_{\mu} \to \frac{1}{g} e^{\chi(x)} D_{\mu} e^{-\chi(x)}, \quad \psi \to e^{\chi(x)} \psi, \quad \bar{\psi} \to \bar{\psi} e^{-\chi(x)}
\]

we have

\[
U(v_{\pm}) \to e^{\chi} U(v_{\pm}), \quad U^+(v_{\pm}) \to U^+(v_{\pm}) e^{-\chi}, \quad T(v_{\pm}) \to T(v_{\pm}).
\]

Using the requirement, that for the gauge parameter falling at large distances \((\chi(x) \to 0, \ x \to \infty)\) the reggeon fields \( A_{\pm} \) are invariant

\[
A_{\pm} \to A_{\pm},
\]

one can verify, that \( S_{G,\text{eff}} \) is also gauge invariant \([3]\). Because \( A_{\pm} \) belong to the adjoint representation of the gauge group, the action is invariant also under the global gauge transformations.

The action describing the gauge-invariant interaction of the gluon and quarks with the reggeized quarks within the fixed interval of rapidities \( \eta \) can be written as follows

\[
S_{Q,\text{eff}} = \int d^4x \left( \bar{a}_{-} (i \partial - M)(a_{+} - U^+(v_{\pm})\psi) + \bar{a}_{+} (i \partial - M)(a_{-} - U^+(v_{\pm})\bar{\psi}) + \right.
\]

\[
\left. + (\bar{a}_{-} - \bar{\psi} U(v_{\pm}))(i \partial - M)a_{+} + (\bar{a}_{+} - \bar{\psi} U(v_{\pm}))(i \partial - M)a_{-} \right),
\]

where \( a_{+}, (\bar{a}_{+}) \) are the fields describing the production of (reggeized) quarks (anti-quarks) in the \( t \)-channel and \( a_{+}, (\bar{a}_{+}) \) are the corresponding fields for their annihilation (cf. \([7]\)).

The reggeon fields satisfy the kinematical constraints

\[
\partial_{+} a_{-} = \partial_{-} a_{+} = 0, \quad \partial_{+} \bar{a}_{-} = \partial_{-} \bar{a}_{+} = 0,
\]

\[
\gamma_{+} a_{-} = \gamma_{-} a_{+} = 0, \quad \bar{a}_{-} \gamma_{+} = \bar{a}_{+} \gamma_{-} = 0.
\]

The first constraint is similar to the analogous condition for \( A_{\pm} \) and means, that the light-cone components \( q^{+} \) and \( q^{-} \) of the reggeon momentum \( q \) are much smaller than the corresponding components of the particles emitting and absorbing the reggeon. The second constraint is related with analogous relations for the \( \gamma \)-matrix light-cone components.
The reggeon fields are assumed to be gauge invariant

\[ a_\pm \rightarrow a_\pm, \quad \bar{a}_\pm \rightarrow \bar{a}_\pm, \quad (36) \]

if the parameters \( \chi(x) \) decrease at large \( x \). They are transformed according to the fundamental representations under the global colour rotations. It means, that \( S_{Q_{\text{eff}}} \) is invariant under local and global gauge transformations.

The operators \( 1/\partial_\pm \) correspond to the propagators of the virtual particles emitting the gluons within the given interval of rapidities. Indeed, the factor \( i\hat{\partial} - M \) standing in front of the Wilson exponents in the action cancels in the Feynman diagrams the neighbouring propagator of the reggeon fields \( a_\pm, \bar{a}_\pm \), which gives a possibility to interprete \( 1/\partial_\pm \) as the colour particle propagators. Therefore the analytic structure of the Feynman diagrams in the effective field theory is the same as in the initial QCD. According to the equations of motion for the quark field

\[ a_\pm = U^+(v_\pm) P_\pm \psi, \quad P_\pm = \frac{1}{2} \gamma_\mp \gamma_\pm, \quad (37) \]

the reggeon field \( a_\pm \) can be considered as a classical component of the corresponding quark field coinciding with it in the perturbation theory \( g \rightarrow 0 \) or in the light-cone gauge. Moreover, the independence of the physical results from the intermediate rapidity parameter \( \eta \) gives a possibility to go in a continuous way from the usual QCD corresponding to \( \eta = \ln s \) with the absence of the phase space for \( A_\pm \) to the reggeon field theory where \( \eta \) is small (cf. [2]).

After the shift of the fermion fields \( \psi \rightarrow \psi + a_+ + a_-, \quad \bar{\psi} \rightarrow \bar{\psi} + \bar{a}_+ + \bar{a}_- \) the bare propagators for the reggeized fermions can be written as follows

\[ \int d^4x e^{-iqx} <a'_y(x)a''_0(0)> = P_+(\hat{q}_\perp - M)^{-1} \cdot \theta(y' - y - \eta), \]

\[ \int d^4x e^{-iqx} <a''_y(x)a'_{y'}(0)> = P_-(\hat{q}_\perp - M)^{-1} \cdot \theta(y' - y - \eta). \quad (38) \]

The total gauge-invariant action for reggeized gluon and quark interactions is

\[ S_{\text{eff}} = S_{G_{\text{eff}}} + S_{Q_{\text{eff}}}, \quad (39) \]

This action is local in the rapidity interval \( (y_0 - \eta, y_0 + \eta) \). The physical results do not depend on \( \eta \) due to the cancelation of this dependence in the two-dimensional integrals of the reggeon field theory. The effective action allows to reproduce easily the results of calculations of all known reggeon-particle amplitudes.

### 4 Conclusion

In this paper we constructed the Born amplitudes for the processes of the particle production in the quasi-multi-Regge kinematics. To calculate two-loop corrections to the quark Regge trajectory one can use the \( t \)- or \( s \)-channel unitarity conditions (cf. [3], [11]). In both cases
two- and three-particle intermediate states should be taken into account. For two-particle contributions one should find the elastic amplitudes in the one-loop approximation, which can be done with the use of the $t$-channel unitarity (see [3], [11]). For three-particle intermediate states in the $s$-channel there are three kinematical regions of integration corresponding to the multi-Regge and quasi-multi-Regge kinematics. In the first region one can use the results of LLA for the production amplitudes and for two other regions the production amplitudes are calculated in this paper. After the integration over the phase space of the intermediate particle momenta and taking into account two-particle contributions we should subtract the leading term proportional to $\ln s$ and appearing from LLA for the Regge trajectory. The contribution to the Regge trajectory from the three-particle intermediate state in the $t$-channel does not contain the divergency, corresponding to LLA. It appears only in the two-particle contribution and should be subtracted to avoid the double-counting.

To calculate the one-loop correction to the kernel of the integral Bethe-Salpeter equation for the $t$-channel partial wave with meson quantum numbers, apart from the two-loop correction to the quark Regge trajectory and the reggeon-reggeon-particle-particle amplitude, calculated in this paper, one should know also one-loop correction to the reggeon-reggeon-particle, which can be found with the use of the crossing channel unitarity relations and the results of this paper (cf. [3], [11]).

Another method of calculations of the next-to-leading corrections to the quark Regge trajectory and to the integral kernel for the Bethe-Salpeter equation is based on the above constructed effective action for interactions of the reggeized quark and gluons with the usual QCD partons (cf. [2]). We hope to return to these problems in our subsequent publications.

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