The influence of wavy vortex in reducing the wall shear stress

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\textbf{Abstract.} A numerical study is preformed to demonstrate the influence of wavy vortex in reducing the wall shear stress for Taylor Couette flow (viscous flow in the annular gap between two concentrated cylinders). The study replicates an experimental investigation performed for Taylor Couette flow with radius ratio of 0.892. LES of WALE model is used to solve the 3D Navier-Stokes equation in OpenFOAM. Simulation was carried out for a range of Reynold’s Number (Re); and normalized torque, wall shear and velocity are compared for various flow structure. It is found that the azimuthal wall shear near the mid region drops significantly due to the appearance of wavy vortex flow. This finding may play a significant role in the study of drag reduction and can be practically applied in various fluid structure interaction systems involving rotational parts to improve frictional efficiency.

\textbf{Keywords.} Wavy vortex; Taylor Couette flow; LES OpenFOAM; Drag reduction; shear stress.

1. Introduction

The motion of viscous flow in the annular gap between two concentrated cylinders named as Taylor Couette flow has gained a lot of attention from researchers due to its complex and non-linear behaviour. In Taylor Couette flow, both cylinders can rotate in same or opposite direction; or either one cylinder is in rotation while another one stationary. Taylor Couette flow with a rotating inner cylinder and stationary outer cylinder undergoes several transition states with the increase in the angular velocity of inner cylinder and those states are named as Circular Couette flow, Taylor vortex flow, wavy vortex flow, modulated wavy vortex flow and turbulent Taylor vortex flow [1-8]. The linear stability analysis for small annular gap (i.e., annular gap $\ll$ mean radius) conducted by [1] proved that as the Reynolds number (Re) of inner wall reaches to a critical value, the circular Couette flow transforms into a toroidal vortex structure named as Taylor vortex flow due to the occurrence of imbalance between viscous and centrifugal forces. Chandrasekhar [2-3] and Donnelly [4] had carried out linear stability analysis for wide gap problems and observed aligned agreement as reported in [1]. The linear stability model developed in [1-4] had observed only Taylor vortex flow in their studies. However, [9-12] had reported a periodic wave along the azimuthal direction as Re increased beyond the critical Re. They had suggested that this periodic wave is related to the non-linear growth of non-axisymmetric disturbances because of which linear stability model [1-4] could not capture the azimuthal wave. Coles [7] had observed wavy and turbulent Taylor vortex flow experimentally where at a first critical Reynolds number, a wave along the axial direction emergence as Taylor vortex structure. As Reynolds number
approaches to a second critical point, a wave in the azimuthal direction appears with the previously occurred wave and this flow regime is named as wavy vortex regime. In a third critical value of Re, a doubly periodic wave appears with the previously indicated axial wave. Finally, at a fourth critical value of Re, Turbulent Taylor vortex appears.

Snyder & Lambert [13] had reported that the principal cause of appearing azimuthal wave is the presence of radial jet in the outflow region (i.e., fluid moving away from inner wall). In a subsequent study, Gorman and Swinney [14] also had observed travelling and modulation of azimuthal wave and they had reported that non-linear growth of non-axisymmetric disturbances are the principal cause of the occurrence of origination of these wave. Despite of several excellent efforts to better understand the physics of wavy vortex regime, the mechanism of the origination and growth of wavy vortex regime yet to be explored extensively. Several studies using direct numerical simulation (DNS) in solving Navier-Stokes equation had been reported to extract the detailed physical mechanism of wavy vortex regime. The DNS study conducted by Fasel &Booz [15] had reported that the strong radial and azimuthal jet in the outflow region play significant role in the occurrence of wavy vortex flow. The study of Marcus [16] had indicated that strong radial jet in the outflow region lead to the emergence of secondary form of instability which in turns results in the appearance of wavy and modulated wavy vortex regime. His study further illustrated that in the transition of Taylor vortex into wavy vortex regime, the speed of azimuthal wave and vortex core matches with each other. Kristiawan et al. [17] had considered flow separation regions in the outflow region of inner wall and inflow region of outer wall (i.e., Fluid moving from outer wall towards inner wall). Their study suggested that emergence of a pair of secondary vortices in the flow separation region is the fundamental cause behind the occurrence of wavy vortex regime. Similar studies [18,19] had noted the distribution of axial shear stress as the cause of the appearance of wavy vortex flow.

From the above studies, it is generally accepted that the non-axisymmetric and non-periodic disturbances along azimuthal and axial direction are the principal cause of the appearance of the wavy vortex flow. The principal mechanism resulting in the appearance of wavy vortex flow had been attributed to the azimuthal and radial jet at the outflow boundary. Though several brilliant efforts had been devoted to better understand the detailed mechanism of the transition states involved in wavy vortex regime, but the comprehensive view of the physics of these transition states are remain unknown. This has motivated us to further study the wavy vortex regime to get more insights about physical phenomena involved in wavy vortex regime.

The base line of current investigation is the experimental study for Taylor Couette flow with radius ratio (inner radius/outer radius) 0.892 [8] where the characteristics of Taylor vortex flow, wavy vortex flow and modulated wavy vortex flow had been carried out with the aid of flow visualization and torque measurement. The present study concentrates towards the replication of experimental result of [8] using numerical simulation. This may help to get details understanding about mechanism and characteristics involves in transition states of wavy vortex regime.

DNS is considered as the most acclaimed numerical tools in the study of Taylor Couette flow configuration ([15], [16], [20-22]). However, huge computational cost involved in the study of wide range of Reynolds number restrict the implementation of DNS ([23,24]). Large eddy simulation (LES) with Wall-Adapting Local Eddy-viscosity (WALE) model had been used as an alternative method of numerical tools in the Taylor Couette flow configuration ([25-29]). The success of past studies has motivated us to use LES of WALE method in the present study.

2. Flow configuration

The schematic diagram presented in figure 1 illustrates the Taylor Couette flow configuration in cylindrical coordinates where both inner and outer cylinder coincides with Z axis. The radii of outer and inner cylinder are denoted as \( r_o \) and \( r_i \), respectively which translates into gap, \( d = r_o - r_i \) and the height of fluid column is presented by L. The inner cylinder rotates at a constant angular velocity \( \Omega \) and outer
cylinder remains stationary. Fluid is considered to be viscous and incompressible. The non-dimension parameter is defined as radius ratio $r_1 / r_0$ and aspect ratio $L / D$.

![Figure 1. A schematic diagram of Taylor-Couette flow configuration.](image)

The flow Reynolds number is expressed by the following equation,

$$Re = \frac{\omega r_1 d}{v}$$  

(1)

Where $v$ is kinematic viscosity.

The non-dimensional torque has been obtained using the equation below (Lim & Tan, 2004).

$$G = \frac{T}{\rho v^2 L}$$  

(2)

Where $T$ is torque and $\rho$ is density of fluid.

3. Numerical model

LES of WALE model has been implemented to solve 3D Navier-Stokes equation with the aid of pimpleFoam solver using OpenFoam. As the assumption of incompressible fluid is considered, pressure solver has been utilized. The cell face pressure has been computed using second linear interpolation schemes. In the evaluation of convective terms and velocity derivatives, the discretization has been performed using a Gauss second order linear scheme. Gauss linear schemes integrated with orthogonal correction has been implemented in the discretization of diffusive fluxes. Second order implicit method has been used for the transient discretization.

In the evaluation of pressure correction, combination of Pressure Implicit with Splitting of Operators (PISO) and Semi-Implicit Method for Pressure-Linked Equations (Simple) which is name as pimple algorithm has been used. In the solution of pressure, generalized geometric-algebraic multi-grid (GAMG) solver with Smoother Gauss-Seidel is applied. Velocity has been solved using Symmetric Gauss-Seidel smoother. Courant–Friedrichs–Lewy (CFL) number is taken as less then 0.5 to confirm the convergence of solution for whole range of Re. The mean torque coefficient reaches to steady state within 4 to 5s. Thus, the time duration of the simulation in this study has been chosen as 5 sec.

A structured mesh (Hexahedral type) has been generated using Fluent Masher, and the growth rate of prism layer from the wall is 5 (figure 2). This provides a non-uniform distribution of cell size along the radial direction with the region near the wall having a finer cell size and the other regions having a relatively larger cell size. A mesh independent test was carried out for a radius ratio of 0.893. It was found that the solution for the case of radius ratio 0.893 becomes independent of the grid size after $83 \times 350 \times 200$ with 83, 350 and 200 being the number of grid points along radial direction, azimuthal direction and axial direction, respectively (figure 3). With above mesh configuration, the difference between the inner and outer wall’s torque was within 0.4%.
Figure 2. Computational grid in Taylor-Couette flow for radius ratio 0.5: (a) Grid elements in annulus region along axial direction, (b) Grid elements along azimuthal direction.

Figure 3. The variation of torque with the change in grid point along radial direction for radius ratio 0.893.

4. Results and discussion
As shown in figure 4, the normalized torque obtained in LES show’s good agreement with that of Lim & Tan [8]. Re ranges between 170 to 638 with a maximum of 3.67% error (figure 5). At a Re smaller than 170, there is an increase in error between LES and experiment is observed. This could be reported as the conversion of graph data from [8]. The critical Reynolds number at which axisymmetric Taylor vortex appears is Re 123 which is exactly same as that of Lim & Tan [8].

Figure 4. Comparison of result obtained in present LES against Lim & Tan (2004).
Figure 5. Percentage Error with CFD (LES) to result of Lim & Tan (2004) for radius ratio 0.893.

Also, figure 4 the torque increases linearly with the increase in angular velocity of inner wall up to Re 123 beyond which there is a sudden increase in the normalized torque is observed. This was reported as due to the appearance of toroidal vortex structure in the annular gap [1]. This indicates that the critical Re in our present LES is 123 which is exactly same as the experiment. Thus, the present LES and experiment exhibits a good comparison in terms of torque measurement. In the following sections, the appearance of different flow regimes and the variation of natural wavelength with the Re will be discussed.

Figure 6. The variation of flow structure in: (a) Taylor vortex flow, (b)-(c) wavy vortex flow, and (d)-(f) modulated wavy vortex flow.

As similar to the experiment, at a smaller magnitude of Re, flow structure is found to be laminar. As Re approaches to a first critical value of 123, the laminar flow is replaced by a toroidal vortex structure which was reported as Taylor vortex (figure 6 a). In a second critical value, the Taylor vortex transits into wavy vortex flow (figure 6 b-c) and following which modulated wavy vortex appears (figure 6 d-f).
The most important parameter in Taylor vortex flow, wavy vortex flow and modulated wavy vortex flow is the number of Taylor vortex cells and its corresponding natural wavelength (the axial range of two counter rotating vortices). The natural wavelength is non-dimensionalized using the following equation:

\[
\text{Normalized natural wavelength} = \frac{2L}{N\delta}
\]  \hspace{1cm} (3)

Where \( N \) is number of Taylor vortices.

Figure 7. The variation of flow structure in the annular gap for axisymmetric Taylor vortex flow, wavy vortex flow and modulated wavy vortex flow.
In Taylor vortex flow, the number of Taylor vortex cells along the axial direction is found as 42 in LES which translates to the normalized natural wavelength to be 1.85 (figure 6a). However, in experiments, the number of Taylor vortex was reported as 40 and its corresponding normalized natural wavelength was found as 1.95 (figure 6a). This flow structure is reported as axisymmetric and stationary. The deviation between LES and experiment in the appearance of number of Taylor vortex cells and its normalized natural wavelength is not well understood in this study. Further study may help to find the source of this inconsistency. The junction of two Taylor vortex cells, where fluid moves from inner wall towards outer wall is classified as outflow region and this provides jet impingement into the outer wall and flow separation region in the inner wall (figure 7a). The junction where fluid moves from outer wall towards inner wall, it exhibits jet impingement into inner wall and flow separation region in the outer wall (figure 7a). As shown in figure 7a, the axisymmetric Taylor vortex behaves like two-dimensional flow.

The spatial behaviour of normalized wall shear at inner and outer walls and azimuthal, radial and axial velocities in the mean radial location is illustrated in figure 8. For both walls, the magnitude of normalized wall shear stress is found maximum in the jet impingement region and minimum in the flow.
separation region (figure 8a) and the spatial behaviour of normalized wall shear stress is found very similar to the sinusoidal distribution with uniform peak values (figure 8a). The spatial behaviour of velocity profile reveals that azimuthal velocity exhibits jet like characteristics in the outflow region (figure 8b). A strong radial jet is found in the outflow region and a weak radial jet is observed in the inflow region from the radial velocity profile (figure 8b). The axial velocity shows a shock like profile (i.e., zero velocity) in the inflow and outflow regions and approaches to its maximum value near to the vortex core (figure 8b).

In the wavy vortex flow, axisymmetric Taylor vortex transforms into periodic non-axisymmetric vortex structure (figure 6b and figure 6c). The number of Taylor vortex cells observed in LES at 42 and its corresponding natural wavelength is 1.85 at Re 170. However, in experiment, a total of 40 Taylor vortex cells with its natural wavelength 1.95 is observed. Thus, this indicates that in the wavy vortex regime, a certain deviation is observed between LES and experiment. A similar type of inconsistency has been reported in [7] at the beginning of wavy vortex regime. Thus the inconsistency observed in the number of Taylor vortex cells and normalized natural wavelength could be attributed to the error related to the experiment. The flow structure of wavy vortex regime in the annular gap presented in figure 7b shows that, in the end of cylinder, flow structure is very similar to the Taylor vortex flow. As it moves away from the ends, flow structure shows its swirling behavior (figure 7b). Two counter rotating vortices interact with each other by a subsidiary flow in the inflow region and it continues periodically (figure 7b). This indicates that the end effect may contribute in the delay of the appearance of wavy vortex.

Figure 9. The spatial behavior of normalized wall shear and velocity components along axial direction for wavy vortex flow. (a) distribution of normalized wall shear stress, (b) the variation of three velocity components at the mean radial location.
If we continue numerical study with out considering no-slip condition in the end, wavy vortex flow may appear at an earlier Reynolds number. The spatial behavior of normalized wall shear along the axial direction presented in figure 9a shows that the magnitude of normalized wall shear stress is maximum in the jet impingement and minimum in the flow separation region which is same as observed in axisymmetric Taylor vortex flow. The distribution of normalized wall shear stress is not uniform along the axial direction (figure 9a). The magnitude of normalized wall shear stress drops from ends towards the mid region of cylinder (figure 9a). It is found that the flow structure becomes wavy as it moves from the end towards the mid region of cylinder. Thus, the appearance of wavy flow structure results in the decrease in the magnitude of the peak values of wall shear stress in the mid region of cylinder (figure 9a). Three components of velocity drawn in the mean radial location along axial direction indicates that the azimuthal velocity exhibits its jet behavior in the outflow region which is same as that of axisymmetric flow structure (figure 9b). The azimuthal jet in the outflow region is found strong near to the end of cylinder and it becomes weaker as it approaches the mid of cylinder.

This indicates that appearance of wavy vortex flow results in the dampening of the strength of azimuthal jet (figure 9b). Similar behavior is observed for the radial jet in the inflow and outflow regions (figure 9b). The magnitude of axial velocity is found very smaller near to the end of cylinder and its increases within the wavy vortex flow structure which is mid region of the cylinder due to the influence of axial subsidiary flow (figure 9b). The average value of axial velocity is far greater than the zero which indicates that in the wavy vortex flow, there is resultant magnitude of axial velocity along the axial direction which could be directly related to the drops of wall shear stress. This finding may play a significant role in the drag reduction studies. In the wavy vortex regime, the number of Taylor vortex cells decreases with the increase in Re (figure 6c) and this results in the increase of natural wavelength. At Re 271, the number of Taylor vortex cells and natural wavelength obtained in LES exactly matches with that of experiment (figure 6c).

In modulated wavy vortex flow, two dimensional doubly periodic wave appears with the wavy vortex flow (figure 6 d-f). In this flow regime, the number of Taylor vortex cells and natural wavelength obtained in LES is exactly same as that of experiment. The number of Taylor vortex cells decreases with Re which in turns results in the increase in natural wavelength. As it was described in the previous section that two counter rotating vortices interact with each other by a subsidiary flow in the inflow region. In this flow regime, the flow structure extracted from the annular gap indicates that more than a pair of vortices interact with each other by periodic subsidiary flow along the axial direction (figure 7d-f). Under the influence of this subsidiary flow, Taylor vortex becomes weaker and starts to dissipate into the walls. In the same time, secondary vortices appear in the flow separation region and that magnifies to Taylor vortices. Again, newly formed Taylor vortices dissipates into the wall under the influence of subsidiary flow. The dissipation of Taylor vortices and its formation occurs periodically. As Re increases, this subsidiary flow becomes more dominant due to which Taylor vortex cells tilts, stretches and deforms (figure 7 d-f). The spatial behaviour of normalized wall shear stress along the axial direction in modulated wavy vortex flow at Re 482 has been illustrated in figure 10a. As shown in Taylor vortex flow and wavy vortex flow, the spatial behaviour of normalized wall shear exhibits repetitive behaviour along the axial direction.
However, in the modulated wavy vortex flow, the spatial behaviour of wall shear stress is no more repetitive along the axial direction. It shows its maximum magnitude at the jet impingement and minimum at the flow separation region which is somehow similar to the behaviour of wall shear stress at the flow separation and jet impingement region of Taylor vortex and wavy vortex flow (figure 10a). However, the region between jet impingement and flow separation, the behaviour of normalized wall shear stress is distorted due to the influence of subsidiary flow. In the region where secondary vortices appear, a drop in the normalized wall shear stress is observed (figure 10a). As it was described in the previous flow regime that there is a strong influence of ends of the cylinder in the waviness of flow structure. However, in this flow regimes, the ends effect of the cylinder is not found effecting the spatial distribution of normalized wall shear stress (figure 10a). As shown in figure 10b, the radial and azimuthal velocity shows prominent characteristics of jet in the outflow region near to the end of cylinder. As it moves away from ends, the radial and azimuthal velocity behave like jet flow near to the core of vortices where the influence of subsidiary flow is minimum. In Taylor vortex and wavy vortex flow, axial velocity exhibits shock like profile in the inflow and outflow region. However, in modulated wavy vortex flow, axial velocity does not show any shock like behaviour in those regions (figure 10b).
5. Conclusion
A numerical model is developed to replicate the Taylor vortex, wavy vortex and modulated wavy vortex flows in the Taylor Couette flow configurations. The model is then compared with experimental study and reasonable agreement is achieved. Parametric study performed using this model over a range of Reynold’s number reveals that with the increase of Reynold’s number, the flow structure becomes wavy as it moves from the end towards the middle of the cylinder. The magnitude of normalized wall shear stress also drops near the mid region of the cylinder. This phenomenon clearly reveals the influence of wavy flow structure on the reduction of wall shear stress. However, further study is required to explain the physics behind the transition among the various types of wavy vortex regions and why such regions are generated. A better understanding of these facts will help to reduce drag in various applied field of Engineering including, aviation, marine and hydraulics.

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