Non-BPS States and Heterotic – Type I’ Duality

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Abstract

There are two families of non-BPS bi-spinors in the perturbative spectrum of the nine dimensional heterotic string charged under the gauge group $SO(16) \times SO(16)$. The relation between these perturbative non-BPS states and certain non-perturbative non-BPS D-brane states of the dual type I’ theory is exhibited. The relevant branes include a $\mathbb{Z}_2$ charged non-BPS D-string, and a bound state of such a D-string with a fundamental string. The domains of stability of these states as well as their decay products in both theories are determined and shown to agree with the duality map.
1 Introduction

Over the past couple of years stable non-BPS states and D-branes have opened a new direction to our understanding of string theory. Reviews of these developments can be found in [1, 2, 3]. Two approaches have been used to construct and analyse non-BPS D-branes. In the first approach [4, 5, 6, 7, 8] non-BPS D-branes are constructed by tachyon condensation as bound states of brane-anti-brane pairs. This construction permits for a classification of D-brane charges in terms of K-theory [9]. The second approach uses the boundary state formalism [10, 11, 12, 13, 14], to describe D-branes as coherent states in the closed string theory satisfying a number of consistency conditions [15, 16, 17, 18]. Since this latter approach provides an explicit boundary conformal field theory description of non-BPS D-branes, we use this second approach.

SO(32) heterotic string theory is conjectured to be non-perturbatively dual to type I string theory in ten dimensions [19]. It should therefore be possible to identify suitable perturbative non-BPS states of the heterotic string with non-BPS D-brane states in the type I theory. The most familiar example of stable non-BPS states are the states transforming in the spinor representation of the gauge group of the SO(32) heterotic string theory. They arise in the first excited level and are absolutely stable due to charge conservation but are not BPS as \( \mathcal{N} = 1 \) supersymmetry algebra has no central charges. The dual state in the type I theory is a stable \( \mathbb{Z}_2 \)-valued non-BPS D-particle [7, 9], which can be described as a tachyonic kink solution on the D1-D1 pair [5].

In this paper we test the S-duality between the heterotic string theory on \( S^1 \) with gauge group \( SO(16) \times SO(16) \) and the type I' theory on \( S^1 \) with the same gauge group. In such a configuration the two heterotic theories are T-dual to each other [20, 21]. The type I' theory is an orientifold of type IIA by \( \mathcal{I}_9 \Omega \), where \( \mathcal{I}_9 \) reverses the sign of \( x^9 \) and \( \Omega \) is the world-sheet parity operator. This orientifold can be thought of as two O8-planes [22] at \( x^9 = 0 \) and \( x^9 = \pi R_9 \), which in the \( SO(16) \times SO(16) \) point in moduli space has eight D8-branes, and their images placed on each of the O8-planes to cancel the tadpole locally. The positions of the D8-branes on the interval correspond in the T-dual type I theory to a Wilson line. The conserved charges of the type I string theory are Kaluza-Klein (KK) momentum and D-string winding number, and those of type I' are winding and D-particle numbers. The duality map relating various parameters between type I' and heterotic theory is given by [24]

\[
R_h = \frac{1}{\sqrt{R'_I \lambda'_I}}, \quad \lambda_h = \frac{R'_I}{\lambda'_I}, \quad G^h_{MN} = G^{I'}_{MN} \frac{R'_I}{\lambda'_I},
\]

(1.1)

where \( G_{MN} \) is the nine-dimensional metric \( (M, N = 0, \ldots, 8) \), \( \lambda_h \) and \( \lambda_I' \) are the heterotic and type I' coupling constants, and \( R_h \) and \( R'_I \) are the radii of the circle when measured in the

\[^1\text{For a non-technical review of } \mathbb{Z}_2 \text{ orientifolds of type II theories see for example [23].}\]
heterotic and type I' metric, respectively. From this relations one can see that the heterotic KK momentum is mapped to type I' winding, and heterotic winding is mapped to the type I' D-particle number.

Under the unbroken $SO(16) \times SO(16)$ gauge group the states in the spinor conjugacy class of $SO(32)$ representations decompose into two sets of bi-spinors, denoted $A$ and $B$. In the type I' theory, $A$ are shown to correspond to a $\mathbb{Z}_2$-valued non-BPS D-string stretching along the interval which is T-dual to the $\mathbb{Z}_2$-valued D-particle on $S^1$ in type I theory. We show that these bi-spinors are unstable against a decay into two single spinor states for radii less than a critical radius $R_h$. In the type I' theory, this corresponds to the appearance of a tachyon in the open string spectrum with endpoints on the $\mathbb{Z}_2$-valued D-string. For radii greater than a certain critical radius $R_I'$, the D-string decays into a D0-brane at one O8-plane and an anti-D0-brane on the other O8-plane. The $\mathbb{Z}_2$ charge of the decaying D-string is encoded in the $\mathbb{Z}_2$ choice of locations for the decay products. We determine the masses of the various states and show that at the critical radius they are equal to one another, indicating that the deformation is marginal. Further we show that the critical radii $R_h$ and $R_I'$ agree with (1.1) qualitatively. This provides a test of the type I'-heterotic duality beyond the constraints of BPS states.

The second class of bi-spinors $B$ can be thought of as bound states of the $A$ bi-spinors and certain bi-vectors. In type I' the $B$ bi-spinors correspond to a bound state of a $\mathbb{Z}_2$-valued non-BPS D-string and F-string both stretching along the interval. We show that such a bound state does indeed exist in type I'. Although the $B$ bi-spinor (the (F,D) bound state) is stable against decay into an $A$ bi-spinor (the non-BPS D-string) and the bi-vector (the fundamental string) as we will see, it is not always stable against decay into the two different single spinor states (a D0-D0 pair) and a bi-vector (a fundamental string stretching along $x^9$). The mass of the bound state and domains of stability of the heterotic and type I' states are computed. The regimes of stability in the two theories are shown to be qualitatively the same. In the T-dual picture, the (F,D) bound state corresponds to a $\mathbb{Z}_2$-valued D-particle with a constant velocity along $S^1$ as the effect of adding a fundamental string to the D-string of type I' is to give the type I D-particle a KK momentum. We show that, unlike in the previous case, presently the non-BPS mass of the (F,D) bound state does not match with its decay product at the critical radius. This indicates that the transition from the non-BPS bound state to the D-particle pair and a F-string is not a marginal deformation.

The paper is organised as follows. In section 2 we briefly review the heterotic string on $S^1$ and point out that there are two kind of non-BPS bi-spinors arising from the unbroken gauge group $SO(16) \times SO(16)$. The duality map (1.1) is tested by comparing the masses of certain BPS states, namely bulk D-particle and fractional D-particle and anti-D-particle, in section 3. In section 4 the non-BPS bi-spinor states are analysed. In section 5 we give the type I'
analysis following boundary state approach. We conclude and raise some open problems in section 6.

2 Review of Heterotic String on $S^1$

The left- and right-moving momenta of an $SO(32)$ heterotic string compactified on $S^1$ are given by

$$ (p_L | p_R) = \left( V + A w_h, \frac{p_h}{R_h} + \frac{w_h R_h}{2}, \frac{p_h}{R_h} - \frac{w_h R_h}{2} \right), \quad (2.1) $$

where $V$ is an element of the internal $\Gamma^{16}$ lattice, $R_h$ and $p_h$ are the compactification radius and physical momentum, respectively, while $w_h \in \mathbb{Z}$ is the winding number respectively. In terms of the background gauge field, $A$ (or Wilson line) $p_h$ is given by

$$ p_h = n_h - V \cdot A - w_h A^2 / 2, \quad (2.2) $$

where $n_h \in \mathbb{Z}$ denotes the Kaluza-Klein (KK) momentum. Physical states satisfy the level matching condition

$$ p_L^2 + 2 (N_L - 1) = p_R^2 + 2 (N_R - c_R), \quad (2.3) $$

where $N_L$ and $N_R$ are left- and right-moving excitation numbers, and $c_R = 0$ and $1/2$ for the right-moving fermions in the periodic (R) and anti-periodic (NS) sectors, respectively. For BPS states all the right moving oscillators should be in the ground state: $N_R = c_R$ [27]. The heterotic mass formula is given by

$$ m_h^2 = \frac{1}{2} p_L^2 + (N_L - 1) + \frac{1}{2} p_R^2 + (N_R - c_R), \quad (2.4) $$

which using the level-matching condition (2.3) becomes

$$ m_h^2 = p_R^2 + 2 (N_R - c_R). \quad (2.5) $$

The states with $N_L = 1$ and $V^2 = 0$ are KK excitations of either the gravity multiplet or one of the vector multiplets associated with the Cartan subalgebra of the gauge group. But there are additional massless states having $N_L = 0$ and $p_L^2 = 2$. In the zero winding sector ($w_h = 0$) we have $V^2 = 2$ and $p_h = 0$, hence states (V) are roots of $SO(32)$. For a non-trivial Wilson line $A$, these states correspond to the roots of the unbroken subgroup of $SO(32)$. Without a Wilson line, the heterotic string has a $R \to 1/R$ symmetry giving rise to an enhanced $SU(2)$ gauge symmetry at the self-dual point. More generally, for a nontrivial Wilson line, $A$, the winding

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2Throughout the paper we follow the convention $\alpha'_h = 2$, $\alpha'_I = 1$. 

sectors give additional massless states at some critical radius. In this paper we consider only the Wilson line, \( A = (0^8, (1/2)^8) \), which corresponds to an \( SO(16) \times SO(16) \) gauge group and critical radius zero. In type I’ theory this is equivalent to placing eight D8-branes with mirrors at each O8-plane giving rise to a constant dilaton and metric background in the bulk \([24, 28]\).

The \( 2^{n-1} \)-dimensional spinor representation is the smallest representation of the ten-dimensional spinor conjugacy class of \( SO(32) \). In nine dimensions, under the Wilson line \( A = (0^8; (1/2)^8) \) with unbroken gauge group \( SO(16) \times SO(16) \), this decomposes as
\[
2^{15} \rightarrow (2^7, 2^7) \oplus ((2^7)', (2^7)'),
\]
giving two kinds of bi-spinors with
\[
(A) : \begin{cases}
\pm \frac{1}{2}, \ldots, \pm \frac{1}{2} ; \\
\text{even no. of ‘+’ even no. of ‘+’}
\end{cases} \in (2^7, 2^7),
\]
\[
(B) : \begin{cases}
\pm \frac{1}{2}, \ldots, \pm \frac{1}{2} ; \\
\text{odd no. of ‘+’ odd no. of ‘+’}
\end{cases} \in ((2^7)', (2^7)').
\]

In later sections we will show that in type I’ theory, the first set of bi-spinors corresponds to a \( \mathbb{Z}_2 \)-charged non-BPS D-string stretching along the interval with end-points at the two O8-planes. The second set corresponds to a bound state of the \( \mathbb{Z}_2 \)-charged non-BPS D-string with an F-string, both stretching along the interval.

3 Single Spinor States

In this section we describe heterotic single spinor states that are charged under one of the \( SO(16) \)'s. In type I’, they turn out to be fractional D-particles (or D0\(_{f} \)) stuck on an O8-plane. Their masses and bulk RR charges are half of those of the bulk D-particles.

A bulk D-particle is dual to a heterotic string with non-trivial winding, \( w_h = 2 \) and the physical momentum, \( p_h = 0 \). As \( V + Aw_h = 0 \) with \( V = (0^8; (-1)^8) \), these states are not charged under either of the \( SO(16) \)'s. The level-matching condition implies that \( N_R = c_R \) and \( N_L = 1 \). Hence these are BPS states and are KK excitations of either the gravity multiplet or the vector multiplets in the Cartan subalgebra. The heterotic mass formula gives
\[
m_h(D0_{\text{bulk}}) = R_h.
\]
Using (1.4) the corresponding type I’ bulk D-particle mass turns out to be
\[
m_{I'}(D0_{\text{bulk}}) = \lambda_h^{1/2}m_h(D0_{\text{bulk}}) = \frac{1}{\lambda_{I'}}.
\]
Consider next the single spinor states

\[ V_1 = \begin{pmatrix} \left( \pm \frac{1}{2} \right)^8 ; 0^8 \end{pmatrix} , \quad V_2 = \begin{pmatrix} 0^8 ; \left( \pm \frac{1}{2} \right)^8 \end{pmatrix} . \]

(3.3)

where \( V_1 \) is of the form \( V + A w_h \), with \( V = \left( \left( \pm \frac{1}{2} \right)^8; (-\frac{1}{2})^8 \right) \in \Gamma^{16} \), with an even number of ‘+’ signs, \( w_h = 1 \) and vanishing physical momentum: \( p_h = 0 \) (for KK momentum, \( n_h = -1 \)). The type I’ D-particle number, is given by \( n_{I'} = w_h/2 = 1/2 \). The level-matching condition gives \( N_L = 0 \) and \( N_R = c_R \), thus \( V_1 \) is a BPS state whose mass is

\[ m_h (V_1) = \frac{R_h}{2} . \]

(3.4)

Using the duality relations (1.1), the type I’ mass is

\[ m_{I'} (V_1) = \frac{1}{2 \lambda_{I'}} , \]

(3.5)

which is half of the D-particle’s mass. This shows that \( V_1 \) corresponds to a fractional D-particle stuck on the \( x^9 = 0 \) O8-plane.

Similarly \( V_2 \) is of the form \( V + A w_h \) where \( V = (0^8; 1^{2n}, 0^{8-2n}) \in \Gamma^{16} \) with \( w_h = -1 \), and \( n = 0, \ldots, 4 \). The level-matching condition implies that the state is BPS \( (N_R = c_R) \), with vanishing physical momentum, \( p_h = 0 \) (i.e. KK momentum, \( n_h = n - 1 \)). The mass formula gives

\[ m_h (V_2) = \frac{R_h}{2} , \]

(3.6)

which in type I’ units is

\[ m_{I'} (V_2) = \frac{1}{2 \lambda_{I'}} . \]

(3.7)

This shows that \( V_2 \) corresponds to a fractional anti-D-particle stuck at the \( x^9 = \pi R_9 \) O8-plane.

4 Bi-spinor States

In this section we discuss in some detail the bi-spinors \( A \) and \( B \). In particular we demonstrate that they are non-BPS states, which are stable in certain regions of the moduli space.

4.1 Bi-spinors in \( A \)

The lightest states in \( A \) come from the zero winding sector. Since \( A \cdot V \in \mathbb{Z} \) we choose \( p_h = 0 \). For \( N_L = 0 \) the level matching condition implies that the states are non-BPS: \( N_R = 1 + c_R \).
The mass of the states with trivial winding, vanishing momentum, \( N_L = 0 \) and \( N_R = 1 + c_R \) is
\[
m_h \left( V \in (128, 128) \right) = \sqrt{2}.
\] (4.1)

Note that there could be BPS states satisfying \( N_R = c_R \) with \( w_h = \pm 1, p_h = 0 \). But for such winding numbers the modified lattice vector \( (V + A w_h) \), does not belong to \( \Gamma^{16} \).

The bi-spinors \( \mathcal{A} \) are charged under both the \( SO(16)'s \). In fact these non-BPS states have the same charges as \( V_1 \) and \( V_2 \), the single spinor states discussed in section 3. This suggests that a decay process into the BPS single spinor states is possible. Since the mass of the single spinor states is radius dependent the decay too will depend on the radius. As the mass of the single spinor states is \( R h / 2 \), the bi-spinor states \( \mathcal{A} \) decay into the single spinor states \( V_1 \) and \( V_2 \) for
\[
R h < \sqrt{2}.
\] (4.2)

In the following section we construct a D-brane state which corresponds to \( \mathcal{A} \). This turns out to be a \( \mathbb{Z}_2 \) D-string stretching along the interval. This D-string is the T-dual of the \( \mathbb{Z}_2 \) D-particle of type I.

### 4.2 Bi-spinors in \( B \)

The states, \( V' \in (128', 128') \) (i.e. in sector \( B \)) have an odd number of \( +\frac{1}{2} \)'s in both the first and second eight entries. Hence they can be expressed as sum of a state, \( V \in (128, 128) \) (i.e. in sector \( \mathcal{A} \)) and a bi-vector state, \( V_{bv} \in (16, 16) \):

\[
\begin{pmatrix}
\pm\frac{1}{2}, \ldots, \pm\frac{1}{2} ; \pm\frac{1}{2}, \ldots, \pm\frac{1}{2}
\end{pmatrix}
\begin{pmatrix}
\pm\frac{1}{2}, \ldots, \pm\frac{1}{2} ; \pm\frac{1}{2}, \ldots, \pm\frac{1}{2}
\end{pmatrix}
= \pm \left( \begin{pmatrix}
1, 0^7 ; 1, 0^7
\end{pmatrix} \right).
\] (4.3)

In type I' the state \( \pm (1, 0^7 ; 1, 0^7) \) corresponds to a fundamental string stretching along the interval. The overall sign corresponds to the orientation of the string.

The lightest states in \( B \) are those with \( w_h = 0 \). As \( A \cdot V \in \mathbb{Z} + 1/2 \), these have \( p_h = \pm 1/2 \). For \( N_L = 0 \) the level matching condition implies that the states are non-BPS: \( N_R = 1 + c_R \).

Here \( 2n + 1 \) \( (n = 0, \ldots, 3) \) is the number of \( +\frac{1}{2} \)'s in the last eight entries of \( V \in (128, 128) \).

The mass formula for such states is
\[
m_h \left( V' \in (128', 128') \right) = \left( \frac{1}{4 R_h^2} + 2 \right)^{1/2}.
\] (4.4)

On the other hand, the bi-vector states, \( V_{bv} \) with \( w_h = 0, N_L = 0 \) are BPS (\( N_R = c_R \)). The
mass of these BPS states with physical momentum, \( p_h = \pm 1/2 \) (as \( A \cdot V = \pm 1/2 \)) is given by

\[
m_h (V_{bv} \in (16, 16)) = \frac{1}{2R_h}.
\]  

(4.5)

Since \( m_h^2 (V \in (128, 128)) = 2 \), we have

\[
m_h (V' \in (128', 128')) < m_h (V \in (128, 128)) + m_h (V_{bv} \in (16, 16)),
\]  

(4.6)

for a finite radius. Hence the states in \( \mathcal{B} \) are bound states of states in \( \mathcal{A} \) and the bi-vector states (4.3). Although the bi-spinor, \( \mathcal{B} \) is stable against a decay into the bi-spinor \( \mathcal{A} \) and a bi-vector state, it is not always stable against decay into the single spinor states, \( V_1 \) and \( V_2 \), and a bi-vector state. In particular this bi-vector is unstable for

\[
R_h < 1.
\]  

(4.7)

In the next section we show that in type I' theory the bi-spinor \( \mathcal{B} \) corresponds to a bound state of a non-BPS \( \mathbb{Z}_2 \) D-string with an F-string, both stretching along the interval. Equivalently, the states in \( \mathcal{B} \) describe a non-BPS D-string with a constant electric field on its world-sheet \[26\]. This \( \mathbb{Z}_2 \) D-string with constant electric field becomes under T-duality, a \( \mathbb{Z}_2 \) D-particle with constant velocity along \( S^1 \) \[37, 38\].

### 5 Type I' analysis

The states in \( \mathcal{B} \) describe a non-BPS D-string with a constant electric field on its world-sheet. Such a bound state will exist as long as the constant electric field on a \( \mathbb{Z}_2 \) D-string is invariant under \( \Omega \mathcal{I}_9 \). If the gauge field is invariant under \( \Omega \mathcal{I}_9 \) the (F,D) bound state can be stabilised from an unstable non-BPS D-string with constant electric flux in type IIA by orientifolding. The NSNS \( B \) field equation of motion has \( \mathcal{F} = F + B \) as a source term \[12, 26, 13, 33, 14\] where \( F = dA \) is the gauge field strength. Recall that in type I, the gauge field vertex operator has tangential derivatives of the form \( dX^i/d\tau \) which are odd under \( \Omega \), while the transverse scalars have normal derivatives. As a result the gauge field is projected out while the transverse scalars survive\[\text{3}\]. In type I' theory, on the other hand, the projection is \( \Omega \mathcal{I}_9 \). As a result the gauge field component along \( S^1 \) survives the projection. Similarly, while the NSNS two form \( B \) is odd under \( \Omega \) and hence projected out in type I, \( B_{\mu\nu} \) is also odd under \( \mathcal{I}_9 \) and thus survives the projection. This guarantees the gauge invariance of the bound state.

As with most non-BPS configurations \[1, 31, 18\], the stability of the \( \mathbb{Z}_2 \) D-string, and its bound state with a fundamental string, depends crucially on the size of the compact

\[\text{3}\text{In fact a } \mathbb{Z}_2 \text{ subgroup of the original } U(1) \text{ survives. This describes the GSO projection on the current algebra fermions in the heterotic string in the type I-heterotic duality context } [24].\]
directions. We investigate this dependence by constructing boundary states which describe the configurations studied presently. In particular we obtain the critical radius at which the configurations become unstable and compute the tensions of these states.

The boundary state representing a $\mathbb{Z}_2$ D-string is given by

$$|B1\mathbb{Z}_2\rangle = \frac{N}{2} (|B1, +\rangle_{\text{NSNS}} - |B1, -\rangle_{\text{NSNS}}),$$

where $N$ is the normalisation obtained by factorising on an open string partition function (see for example [18] for more details on obtaining such normalisations) and

$$|B1, k, \eta\rangle_{\text{NSNS}} = \exp \left( \sum_{n=0}^{\infty} \frac{1}{n} \alpha n S_{\mu\nu} \tilde{\alpha}^{-\mu \nu} - i\eta \sum_{r \in \mathbb{N} + \frac{1}{2}} [\psi_{\mu r} S_{\mu\nu} \tilde{\psi}_{\nu r}] \right) \times \sum_{w_9} e^{i\theta w_9} |B1, k, w_9, \eta\rangle^{(0)} \otimes |B1, \eta\rangle_{\text{ghost}}. $$

(5.2)

Here $w_9$ is the winding number, $\eta = \pm 1$, $\theta$ is a Wilson line and for compactness of notation we do not write $\theta$ on the left-hand side of the above equation. The matrix $S$ encodes the boundary conditions of the D-string and is a $10 \times 10$ diagonal matrix given by

$$S = \text{diag}(1, 1, \ldots, 1, -1).$$

(5.3)

The D-string is taken to lie along directions $x^0$ and $x^9$ with $x^9$ compactified along a circle of radius $R_{I'}$, and we work in the Minkowski metric. As the $\mathbb{Z}_2$ D-string stretches in directions 0 and 9 while the O8-planes extend in directions 0, $\ldots$, 8 it is not possible to work in the light-cone gauge and ghost and superghost contributions will have to be taken into account. These are taken as in [12, 10, 32]. The ground state $|B1, k, w_9, \eta\rangle^{(0)}$ carries momentum $k$ in the directions transverse to the D-string and is unique in the NSNS sector. To obtain a localised D-brane, we have to Fourier transform the above state,

$$|Bp, y, \eta\rangle = \int \left( \prod_{\mu=1}^{8} dk^{\mu} e^{ik^{\mu} y^{\mu}} \right) \sum_{w_9} e^{i\theta w_9} |Bp, k, w_9, \eta\rangle.$$

(5.4)

Here $y$ denotes the location of the boundary state along the transverse directions, and is suppressed throughout as we take it to be $y = 0$. The normalisation $N$ is equal to that of a type IIB D-string from which it follows that the tension of the $\mathbb{Z}_2$ D-string is the same as that of a conventional type II D-string. This is a $\sqrt{2}$ bigger than the tension of a type I BPS D-string. This follows since the open string partition function for the $\mathbb{Z}_2$ D-string has no GSO projection but does have the orientifold projection $(1 + \Omega_9)/2$.

Next consider the strings that end on a D8-brane. The boundary state of a D8-brane is

$$|B8\rangle_{\text{NSNS}} = \frac{N_8}{2} (|B8, +\rangle_{\text{NSNS}} - |B8, -\rangle_{\text{NSNS}})$$
Explicitly the NSNS boundary state is as in equation (5.2) but now the matrix $S$ has eight entries equal to $-1$, with only the first and last one equal to 1. The RR sector is not discussed here as it does not enter into our analysis. Factorisation of the cylinder diagram on an annulus fixes the normalisation $N_8$. Strings that stretch between a D8-brane and the $\mathbb{Z}_2$ D-string have 9 ND and one NN boundary conditions. As a result the ground state energy in the NS sector is positive, while as always in the R sector it is zero, thus these are tachyon free.

Next we consider the Möbius strip diagrams corresponding, in the closed string channel to the exchange between the D-string and the O8-planes. A crosscap state representing an $\mathbb{N}D$ and one $\mathbb{N}N$ boundary conditions. As a result the ground state energy in the NS sector fixes the normalisation $N_{\mathbb{N}D}$ with only the first and last one equal to 1. The RR sector is not discussed here as it does not enter into our analysis. Factorisation of the cylinder diagram on an annulus fixes the normalisation $N_8$. Strings that stretch between a D8-brane and the $\mathbb{Z}_2$ D-string have 9 ND and one NN boundary conditions. As a result the ground state energy in the NS sector is positive, while as always in the R sector it is zero, thus these are tachyon free.

\[
|B8\rangle_{RR} = \frac{4iN_8}{2}(|B8, +\rangle_{RR} + |B8, -\rangle_{RR})
\]

\[
|B8\rangle = |B8\rangle_{NSNS} + |B8\rangle_{RR}.
\] (5.5)

where $q = e^{-2\pi l}$, $t = 1/2l$, $\tilde{q} = e^{-\pi t}$ and $H_c$ is the closed string Hamiltonian

\[
H_c = \pi p^2 + 2\pi \sum_{\mu=0}^9 \left[ \sum_{n=1}^{\infty} (\alpha_{\mu}^n \alpha_{\mu}^n + \bar{\alpha}_{\mu}^n \bar{\alpha}_{\mu}^n) + \sum_{r \in \mathbb{N}+\frac{1}{2}} r(\psi_{\mu-r} \psi_{\mu} + \bar{\psi}_{\mu-r} \bar{\psi}_{\mu}) \right] + 2\pi C_c + \text{ghosts}.
\] (5.7)

The constant $C_c$ is $-1$.

In order to see if there are open string tachyons we expand in $\tilde{q}$ to suitable order

\[
\mathcal{A}_1 = \frac{V}{2\pi^2} \int \frac{dt}{2t} (2t)^{-1/2} \left[ \frac{1}{\tilde{q}} \left( 1 + 2\tilde{q}^2/R^2 \right) - \frac{1}{\tilde{q}} + \ldots \right].
\] (5.8)
It then follows that for $R_{I'} \leq \sqrt{2}$, the $\mathbb{Z}_2$ D-string is tachyon free and hence stable. In

equation (5.6) we have fixed the normalisation constants by requiring that the cylinder and Möbius strip diagrams factorise on suitable open string partition functions. The decay of the non-BPS D-string into a D0-D0 pair restricted to the orientifold planes is possible in the region

$$R_{I'} > \sqrt{2}.$$  

(5.9)

This is also confirmed by the fact that, in this region, the classical mass of the D-string, given below, is bigger than that of two fractional D-particles, given by (3.3) and (3.7), in the above region

$$m_{I'}(D_{1\text{nonbps}}) = \frac{R_{I'}}{\sqrt{2}\lambda_{I'}}.$$  

(5.10)

The numerical factor above follows by noting that the tension of the $\mathbb{Z}_2$ D-string is $\sqrt{2}$ bigger than the tension of a type I BPS D-string. As expected, the corresponding masses in the two theories are not related by the duality map, since for non-BPS states the masses are not protected from quantum corrections. In terms of heterotic string theory the decay corresponds to (3.3). The regimes of stability of the non-BPS state in the two dual theories, (1.2) and (5.9), are qualitatively the same, given the duality relation (1.1).

Consider next a bound state of the $\mathbb{Z}_2$ D-string with a fundamental string. We show that the tension of this state is lower than the individual tensions of the D- and F-strings, thus forming a bound state. Further we will analyse the stability of this state and find a dependence on the radius. Boundary states with gauge fields were first considered in [12] and in the context of D-branes in [13, 33, 14, 32]. In the NSNS sector the following changes occur in the boundary state description. The matrix $S$ now becomes

$$S = \begin{pmatrix}
\eta - F & \eta + F \\
-1 & -1 \\
\vdots & \vdots \\
\end{pmatrix},$$  

(5.11)

where $\eta$ is the Minkowski metric, there are still eight entries equal to $-1$ corresponding to the transverse directions and the field-strength $F$ is

$$F = \begin{pmatrix}
0 & -f \\
f & 0
\end{pmatrix}.$$  

(5.12)

We have placed the $x^0$ and $x^9$ coordinates in positions 1,2 in the matrix. In the above $f$ is given by [32]

$$f = -\frac{m}{\sqrt{n^2 + m^2}},$$  

(5.13)
where \( m = 1 \), is the number of fundamental strings, and \( n = 1 \) is the number of D-strings. The background gauge field has no effect on ghost or superghost contributions. In the presence of a gauge field the open string momentum eigenvalues on a circle become [34]

\[
p_n = \frac{n}{R} \sqrt{1 - f^2}.
\]  

The non-compact open string momentum integrals get modified in an analogous fashion. This can be viewed from the closed string channel as an extra normalisation factor

\[
\sqrt{-\det(n + F)} = \sqrt{1 - f^2},
\]

of the boundary state corresponding to the non-BPS bound state relative to the \( \mathbb{Z}_2 \) D-string [12]. We are now in a position to compute the amplitude corresponding to \( S \) the (F,D) bound state. Since \( S \) can be viewed from the closed string channel as an extra normalisation factor, the tension of a general configuration of \( m \) F-string and \( n \) D-string bound state is \( (m^2 + n^2)^{1/2} \) times the D-string tension.
Non-threshold bound states are realised if \( m \) and \( n \) are relatively prime integers. In our case the non-BPS D-string mass is given by (5.10). Hence the mass of the non-BPS D-string with one unit of electric field (with \( m = n = 1 \)) is given by

\[
m_{1'}(D_{1\text{nonbps}} + F) = \frac{R_{1'}}{\lambda_{1'}}.
\] (5.19)

Unlike the case of the non-BPS D-string, the mass of the non-BPS \((F,D)\) bound state is not same as its decay products (namely the \(D0\overline{D0}\) pair with \((8,8)\) F-string) at the critical radius (5.18). Here the F-string mass is \(R_{1'}/2\) obtained from (1.5) using duality map (1.1).

There is a loss of energy due to the interaction between the decay products in the presence of an electric field. The transition from the non-BPS bound state to the D-particle pair and a fundamental string at the critical radius does not turn out to be a marginal deformation.

6 Type I analysis

In this section we briefly outline what happens under T-duality to the analysis of the previous section. As expected T-duality remains valid when analysing non-BPS states. Under T-duality the \(\mathbb{Z}_2\) D-string becomes a \(\mathbb{Z}_2\) D-particle analysed in [7, 36]. The combined cylinder and Möbius strip diagram amplitude (5.6) undergoes only one change; the open string momentum sum \(\sum_m e^{-2t(m/R_{1'})^2}\) now becomes a winding sum \(\sum_w e^{-2t(wR_{1})^2}\), thus stabilising the D-particle for

\[
R_{1'} > \frac{1}{\sqrt{2}},
\] (6.1)

in agreement with [7, 36]. Under T-duality an electric field becomes a velocity, and so the \(\mathbb{Z}_2\) D-string with electric flux is dual to a D-particle with constant velocity in the compactified direction. The open string momentum sum in (5.16) now becomes a winding sum

\[
\sum_w e^{-2t(wR_{1})\sqrt{1-f^2})^2}.
\] (6.2)

The \(f\) dependent square-root is easily identified as a Lorentz contraction of the compactification radius as viewed by the moving \(\mathbb{Z}_2\) D-particle. This moving D-particle is stable for

\[
R_{1'} > \frac{1}{2},
\] (6.3)

a radius smaller than that of the static D-particle, which can be regarded as a consequence of Special Relativity.
7 Final Remarks and Conclusion

Let \((p, q)\) denotes a bound state of \(p\) F-strings and \(q\) D-strings with \(p, q\) any relatively prime pair in type IIB string theory. Existence of this bound state is a consequence of conjectured \(SL(2, \mathbb{Z})\) duality symmetry of the type IIB string theory [35]. The \((p, 1)\) bound states exists as a consequence of the D-string structure [29]. But the situation is different in our case with \(\mathbb{Z}_2\)-charged non-BPS D-string due to the following observation. From equation (4.3), we see that if we add even number of bi-vector states to any bi-spinor in \(\mathcal{A}\) we get a bi-spinor in \(\mathcal{A}\) itself. In type \(I'\) theory, this observation corresponds to the fact that \((p, 1)\) bound states exist only for \(p = 1\). For \(p\) odd, the bound state decays into a \(p = 1\) bound state and a number of winding states. On the other hand, if we add two \((8,8)\) F-strings to the non-BPS D-string the system is unstable against decay into the D-string itself and (probably) a full winding state or a closed string state. As far as charge conservation is concerned this is equivalent to the fact that two such D-strings together are unstable and decay into massless states. The fact that two D-strings in our example are not stable is not surprising as two spinor states can annihilate to give various massless states. For example, if we take two bi-spinor states of the form \(((\frac{1}{2})^8, (\frac{1}{2})^8)\) they together carry the same charge and same mass (at all radii) as the state \((1^{16})\) which can decay into various massless states in the adjoint representation describing \((8,8)\) F-strings with both the D8-branes on the same orientifold planes.

Since the mass of a state with unit winding is \(R_{I'}\), in the case of the D-string with two units of electric field, using (5.19), the inequality

\[
m_{I'}(\text{D-string} + 2F) > m_{I'}(\text{D-string}) + m_{I'}(\text{winding state}),
\]

holds for sufficiently small \(\lambda_{I'}\) at all radius. Hence, for a sufficiently small type \(I'\) coupling, the D-string with two units of electric field is unstable against a decay into a D-string and a closed string state. Since the configuration studied is non-BPS, it is not surprising that the above inequality only holds for small type \(I'\) coupling.

In this paper we have tested the duality between the heterotic and type \(I'\) string with \(SO(16) \times SO(16)\) gauge group beyond the BPS limit. We have found agreement between the regions of stability and decay products of non-BPS states in both theories. In particular we have found that the domains of stability are qualitatively related by the duality map between the two theories. This fact was not guaranteed \(a \text{ priori}\) as the masses of non-BPS states are not protected by supersymmetry. Unlike the case of non-BPS D-string the mass of the non-BPS \((F, D)\) bound state is not the same as its decay product at the critical radius. This indicates that the transition from the non-BPS bound state to the D-particle pair and a fundamental string is not a marginal deformation.

It would be interesting to perform a similar analysis for other Wilson lines, where th gauge
group is $SO(16 - 2N) \times SO(16 + 2N)$. This is achieved by moving $N$ D8-branes from one O8-plane to the other and to see whether there is any possible modification in string creation and gauge enhancement phenomena in the presence of these non-BPS D-branes.

\section{\textit{M}öbius Strip Diagram}

In this appendix, we explicitly derive the M"{o}bius strip part of equation (5.16). The term we are then interested in is

$$\mathcal{M} = \langle C8| e^{-iH_c} | B1_{\mathbb{Z}_2}, F \rangle.$$  \hspace{1cm} (A.1)

The contributions from the ghosts, superghosts and directions transverse to the $F$ are as in the case of $F = 0$. We focus here on the matter contributions in the directions $x^0$ and $x^9$. We write

$$\eta - F \over \eta + F = \begin{pmatrix} \cosh(2\nu) & \sinh(2\nu) \\ \sinh(2\nu) & \cosh(2\nu) \end{pmatrix},$$  \hspace{1cm} (A.2)

since the matrix is orthogonal relative to the inner product defined by the Minkowski metric $\eta$. The bosonic contribution to $\mathcal{M}$ is given by

\[
\langle C8| e^{-iH_c} | B_{\mathbb{Z}_2}1, F \rangle_{0,9} = \sum_{n=1}^{\infty} \sum_{k,l,j,p,s,t=0}^{\infty} \frac{(\frac{(-1)^n}{n} \alpha_n \tilde{\alpha}_n)^k (\frac{(-1)^n}{n} \alpha_n \tilde{\alpha}_n)^l (\frac{-1}{n} \alpha_n \tilde{\alpha}_n q^{2n} \sinh(2\nu))^j}{k! l! j!} \times \frac{(\frac{-1}{n} \alpha_n \tilde{\alpha}_n q^{2n} \cosh(2\nu))^p (\frac{-1}{n} \alpha_n \tilde{\alpha}_n q^{2n} \sinh(2\nu))^s}{p! s!} \frac{(\frac{-1}{n} \alpha_n \tilde{\alpha}_n q^{2n} \sinh(2\nu))^t}{t!} [0] \\
= \sum_{n,k,l,j} \frac{(k + j)! (l + j)! (iq)^{2n(k+l+2j)} \cosh^{k+l}(2\nu) \sinh^{2j}(2\nu)}{k! l! n! (j!)^6} \times [0] \\
= \sum_{n,k,l,j} (1 + (iq)^{2n} \cosh(2\nu))^{-j-1} (1 - (iq)^{2n} \cosh(2\nu))^{-j-1} ((iq)^{4n} \sinh^2(2\nu))^j
\]
$$= \prod_n (1 - (iq)^{2n})^{-1} (1 + (iq)^{2n})^{-1}. \quad (A.3)$$

In the above we have used the following

$$\sum_{k=0}^{\infty} \binom{k+j}{j} a^k = (1 - a)^{-j-1}. \quad (A.4)$$

This demonstrates that the bosonic non-zero modes contribute the same factor to the Möbius strip amplitude with or without a gauge field. The fermions are substantially easier as the coherent state exponentials terminate, due to the anti-commutative nature of the $\psi^\mu$. Explicitly we have

$$\langle C8|e^{-iH_{c}}|B_{\mathbb{Z}_2}1, F \rangle_{0,9}$$

$$= \langle 0| e^{\sum_{s \in \mathbb{Z}_2} (1 - i)^{s} (\psi^{\mu}_{r} \bar{\psi}^{\nu}_{r} + \bar{\psi}^{\nu}_{r} \psi^{\mu}_{r})} e^{-iH_{c}}$$

$$\times \prod_{r \in \mathbb{N} + \frac{1}{2}} \langle 0| (1 + i(-1)^{r} \bar{\psi}^{0}_{r} \bar{\psi}^{9}_{r})(1 + i(-1)^{r} \psi^{9}_{r} \bar{\psi}^{0}_{r})(1 + iq^{2r} \cosh(2\nu) \psi^{0}_{r} \bar{\psi}^{9}_{r})$$

$$\times (1 - iq^{2r} \cosh(2\nu) \psi^{9}_{r} \bar{\psi}^{0}_{r})(1 + iq^{2r} \sinh(2\nu) \psi^{0}_{r} \bar{\psi}^{9}_{r})(1 - iq^{2r} \sinh(2\nu) \psi^{9}_{r} \bar{\psi}^{0}_{r})|0\rangle$$

$$= \prod_{r \in \mathbb{N} + \frac{1}{2}} \langle 0| (1 + i(-1)^{r} \psi^{0}_{r} \psi^{9}_{r} + i(-1)^{r} \psi^{9}_{r} \psi^{0}_{r} - (-1)^{2r} \psi^{0}_{r} \psi^{9}_{r}) \psi^{0}_{r} \psi^{9}_{r} |0\rangle$$

$$\times (1 + iq^{2r} \cosh(2\nu) \psi^{9}_{r} \bar{\psi}^{0}_{r} - iq^{2r} \cosh(2\nu) \psi^{0}_{r} \bar{\psi}^{9}_{r} + q^{4r} \cosh^{2}(2\nu) \psi^{0}_{r} \psi^{9}_{r} \psi^{0}_{r} \psi^{9}_{r})$$

$$\times (1 + q^{4r} \cosh^{2}(2\nu) \psi^{9}_{r} \bar{\psi}^{0}_{r} \bar{\psi}^{9}_{r} \bar{\psi}^{0}_{r})|0\rangle$$

$$= \prod_{r \in \mathbb{N} + \frac{1}{2}} \left(1 - (iq)^{4r} (\cosh^{2}(2\nu) - \sinh^{2}(2\nu))\right)$$

$$= \prod_{n=1}^{\infty} (1 - (iq)^{2n-1})(1 + (iq)^{2n-1}), \quad (A.5)$$

which is indeed the same as for the case with no gauge field.

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