Globular cluster formation with multiple stellar populations: A single-binary composite scenario

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ABSTRACT

We discuss a GC formation scenario in which the first generation (1G) of single asymptotic giant branch (AGB) stars and intermediate-mass close binaries (IMCBs) eject gas, from which the second generation (2G) of stars can be formed. The two key parameters in the scenario are the fractions of binary stars ($f_b$) and the slopes ($\alpha$) of the stellar initial mass functions (IMFs) for 1G stars. Principle results derived by analytic and one-zone models of GC formation are as follows. The mass fraction of 2G stars ($f_{2g}$) can be higher than $\approx 0.4$ for $\alpha < 1.8$ and is not so dependent on $f_b$. The ratio of the initial mass of a GC to the present-day mass ($M_{gc}$) ranges from 2 to 7 depending on $\alpha$ for $0.5 \leq f_b \leq 0.9$. The differences in [Na/Fe] between 1G and 2G stars can be as large as 0.7 for a wide range of model parameters. The Li abundances of 2G stars can be as high as those of 1G even if the pristine gas from IMCBs is assumed to be Li-free. Formation histories of 2G stars show at least two peaks owing to two peaks in the total ejection rate of gas from IMCB populations. The observed correlation between $f_{2g}$ and $M_{gc}$ can be due to $\alpha$ depending on $M_{gc}$. The hypothetical long duration of 2G formation ($\approx 10^8$ yr) is possible, because massive star formation can be suppressed through frequent dynamical interaction between 1G stars and gas clouds.

Key words: galaxies: star clusters – globular clusters: general – stars:formation

1 INTRODUCTION

Most Galactic globular clusters (GCs) are observed to have chemical abundance spreads among their individual stars (see recent reviews by Gratton et al. 2019, G19; Milone and Marino 2022; MM22). This observed presence of multiple populations (MPs) in GCs is a fundamental GC characteristic that any theory of GC formation needs to explain. The MP phenomena are quite diverse, including large helium abundance spread in ω Cen (e.g., Piotto et al. 2005), anti-correlations between the chemical abundances of light elements (e.g., Carretta et al. 2009), Type I and II dichotomy (e.g., Marino et al. 2017), C+N+O abundance spreads (e.g., Yong et al. 2012) abundance spread in s-process (Marino et al. 2015) and r-process elements (e.g., Roederer & Sneden 2011 for M15), and large age and [Fe/H] differences in Terzan 5 (e.g., Ferraro et al. 2009). These observed properties of GCs can provide useful constraints on the theory of GC formation, however, previous theoretical models of GC formation with MPs appear to have potentially serious problems in explaining all of these in a self-consistent manner (e.g., Renzini et al. 2015; Bastian & Lardo 2018, BL18).

Most previous theoretical models of GC formation with MPs assumed that a GC consists of two major populations, i.e., the first generation ("1G") of stars formed within natal gas clouds, and the second generation ("2G") of stars that are formed from gaseous ejecta from 1G stars mixed with (or “diluted” by) “pristine” gas that have the same chemical abundances as those of natal clouds (i.e., 1G stars). The origin of pristine gas and the dilution processes are the two key problems in previous theoretical models of GC formation, and different models adopt different assumptions to solve the two (see BL18 for critical reviews of these models). One of the most extensively investigated models is the so-called “AGB scenario” in which 2G stars form from AGB ejecta diluted by interstellar medium (ISM) of GC-hosting gas-rich galaxies (e.g., Fenner et al., 2004; Bekki et al. 2007; D’Ercole et al. 2008, D08). Although Ventura et al. (2001) first proposed that low-mass stars can accrete AGB ejecta to show lower [O/Fe] and higher [Na/Fe] and [Al/Fe], their scenario did not assume dilution by ISM.

One of potential problems in the standard AGB scenario is the origin of pristine gas (e.g., D’Ercole et al. 2011).
Table 1. Physical meanings of acronyms and model parameters for globular clusters (GCs).

| Acronym   | Physical meaning                  |
|-----------|-----------------------------------|
| IMCB      | Intermediate-mass close binary    |
| RLOF      | Roche lobe overflow in binary stars |
| MP        | Multiple stellar population       |
| IMF       | Initial mass function of stars    |
| 1G        | First generation of stars         |
| 2G        | Second generation of stars        |
| nG        | n-th generation of stars (n=1,2,3,...) |
| m         | Mass of an individual star (M⊙)   |
| m_w       | Mass of AGB wind (M⊙)             |
| q         | Mass ratio of binary stars        |
| α         | IMF slope for 1G stars            |
| m_1       | Lower mass cut-off for the IMFs of all stars |
| m_u,1G    | Upper mass cut-off for the IMF of 1G stars |
| m_u,2G    | Upper mass cut-off for the IMF of 2G stars |
| M_t       | Total mass of a GC with m₁ ≤ m ≤ m_u,1G |
| M_gc,g    | Total mass of a GC with m₁ ≤ m ≤ 0.8M⊙ |
| M.gc       | Total gas mass of a GC            |
| M_1g       | Total mass of 1G stars with m₁ ≤ m ≤ 0.8M⊙ |
| M_2g       | Total mass of 2G stars with m₁ ≤ m ≤ 0.8M⊙ |
| f_2g       | Mass fraction of 2G stars (= M_2g/M_gc) |
| f_1o       | Mass fraction of binary stars in a GC |
| f_mcb      | IMCB fraction among intermediate-mass binaries |
| F_nb       | Mass budget factor (= M/M_1) |
| M_g,a      | Total mass of ejecta from single AGB stars |
| M_g,b      | Total mass of ejecta from IMCBs    |
| M_g        | Total gas mass from single AGB and IMCB stars |
| M'         | Gas accretion rate in a GC        |
| f_aj       | Mass fraction of ejecta from IMCBs |
| f_pr       | Mass fraction of pristine gas in the ejecta of IMCBs |
| F_iII      | Dilution factor (= f_pr M_g/a/M_g) |
| f_gb       | Fraction of gas from IMCBs among all gas |
| Δ[Na/Fe]   | Difference in [Na/Fe] between 1G and 2G stars |
| [Na/Fe]_min | Minimum [Na/Fe] in 1G stars |
| [Na/Fe]_max | Maximum [Na/Fe] in 2G stars |
| A(Li)      | Li abundance (in 2G stars)        |
| A(Li)_m    | Mass-weighted Li abundance of AGB ejecta |
| Y_Li(m)    | Li fraction from AGB stars depending on m |
| Y_Li,m     | Mean Li fraction among all AGB ejecta |
| β          | Power-law slope in the adopted SF law |
| M_g,th     | Threshold gas mass for star formation |
| ε_g        | Star formation efficiency of 2G stars |

Interstellar medium (ISM) of galaxies hosting GCs is often assumed to be pristine gas that can be mixed with AGB ejecta and subsequently converted into 2G stars (e.g., Bekki et al. 2007; D’Ercole et al. 2010; Calura et al. 2019; McKenzie & Bekki 2021, MB21; Yaghoobi et al. 2022). However, this assumption of dilution of AGB ejecta by ISM has the following potential problems. First, ISM needs to be accreted onto GCs to mix well with AGB ejecta (and to form high-density regions for star formation) when gas from massive AGB stars is accreting onto GCs. It is not so obvious that this requirement can be met in all GCs interacting with ISM in different GC host galaxies. As shown in MB21, the processes of ISM accretion onto GCs depends on a number of parameters such as the sizes of gaseous “holes” in ISM surrounding GCs generated by multiple core-collapsed supernovae (CCSNe) of 1G populations and the mass density of ISM. For example, if GCs are embedded in giant gaseous holes, then ISM accretion onto GCs can occur later than the commencement of AGB phases of intermediate-mass stars: no dilution is possible and new stars can be formed from pure AGB ejecta. This problem is referred to as the “timing” problem in the present study.

Second, a right amount of ISM needs to be accreted onto GCs to explain the observed mass fraction of 2G stars (f_2g) and the abundance differences in light elements between 1G and 2G (“amount” problem). The observed mass fraction of 2G stars for GCs with log M_gc = 5.2 is 0.4 (G19), which requires a certain range of ISM mass. Too much accretion of ISM onto GCs can end up with large f_2g that is not observed and too small differences in light elements between 1G and 2G due to dramatic dilution of AGB ejecta by pristine ISM. Accordingly, fine-tuning is required for the right amount of accreted ISM onto GCs, though not only the physical properties of GCs but also those of ISM (i.e., relative velocities between ISM and GCs) can determine the total amount of ISM accreted onto GCs.

Third, the metallicity of ISM needs to be very similar to those of 1G stars in GCs (“metallicity” problem). Recent observational studies have revealed that Type I GCs show similar [Fe/H] between 1G and 2G, though each of the two populations exhibits [Fe/H] variations (e.g., G19; Legnardi et al. 2022). ISM can originate from different regions of GC host galaxies (e.g., MB21), and CCSNe of 1G stars can pollute the ISM of the host dwarfs to significantly increase the metallicities of ISM. Therefore, it is possible that [Fe/H] of ISM can be significantly different from 1G stars. Fourth, accretion of ISM needs to stop before stars with their original masses lower than 3M⊙ enter into their AGB phases (“truncation” problem). This is because such AGB ejecta can have enhanced C+N+O abundances (e.g., Fenner et al. 2004) and be mixed with ISM, which can lead to 2G formation with their C+N+O abundances significantly higher than those of 1G stars. These four (potential) problems related to dilution of AGB ejecta need to be solved in GC formation models based on single AGB stars. It should be noted here that Renzini et al. (2022) also pointed out possible problems of the standard AGB scenario.

Vanbeveren et al. (2012, V12) proposed an alternative model in which intermediate-mass close binaries (IMCBs) can eject fresh pristine gas that can be mixed with stellar winds of AGB stars and finally converted into 2G stars. In this model, a significant amount of gas from IMCBs can be ejected from IMCBs during the Roche lobe overflow (RLOF) and the common envelope phases, and some of the ejected gas can have chemical abundances that are almost identical to those of 1G stars (i.e., pristine gas). Accordingly, they suggested that if these pristine gas is mixed well with stellar winds from single AGB stars, then 2G stars with the observed chemical abundance patterns can be formed. Although the idea of gas from binary stars in V12 is essentially the same as the underlying assumption in “massive interacting binary” scenario by de Mink et al. (2009), V12’s scenario has not been discussed extensively so far: this scenario is referred to as the “SBC” (single-binary composite) scenario just for convenience.

Recent observational studies have shown that the initial fractions of binary pairs of intermediate-mass stars is almost 100% in stellar associations (e.g., Kouwenhoven et al. 2007, K07), though the binary fractions of the present-day GCs are not so high (e.g., Sollima et al. 2007). Also, recent numerical simulations of binary evolution in GCs have shown that the binary fractions can decrease by only 10%
within one dynamical relaxation time at half-mass radii cor-
responding to roughly 0.1 Gyr (e.g., Fig. 1 in Hong et al.
2015). Accordingly, GCs should have rather high fractions of
binary stars \( \approx 0.1 \) Gyr after their formation, when a large
amount of AGB ejecta of 1G stars are accreting onto their
central regions. Thus the SBC scenario needs to be investi-
gated by theoretical models of GC formation not only be-
cause it can possibly solve the problems related to dilution of
AGB ejecta mentioned above, but also because binary stars
dominate \( \approx 0.1 \) Gyr old GCs so that they can influences the
formation of 2G stars.

The purpose of this paper is thus to discuss the basic
characteristics of the SBC scenario using simple analytic cal-
culations and one-zone models for the physical properties of
GCs with MPs. In discussing the MP problems of GCs, we
adopt a bold assumption that GCs do not accrete gas from
ISM of their host galaxies in order to more clearly under-
stand the basic characteristics of the SBC scenario: we here
admit that such an assumption could be rather idealized,
given that recent hydrodynamical simulations demonstrated
that ISM accretion onto GCs is possible (e.g., McKenzie
& Bekki 2018; MB21). We focus exclusively on the global
properties of GCs predicted from the SBC scenario, such as
mass fractions of 2G stars, \([\text{Na}/\text{Fe}]\) differences between
1G and 2G stars, star formation histories in GCs. Accord-
ingly we will discuss their 3D structures and kinematics, de-
tailed chemical abundance patterns, and mass-dependences of
these using hydrodynamical simulations of GC formation
in our forthcoming papers.

The plan of the paper is as follows. We outline the SBC
scenario and possible advantages and disadvantages of it in
explaining the observed properties of GCs in §2. We describe
simple analytic models for the scenario and present the key
results in §3. Based on one-zone models, we investigate the
possible star formation histories of 2G populations in GCs
in §4. We discuss a number of key problems related to the
formation of MP in GCs based on the present new results.
In §5. We summarize the key characteristics of the scenario
in §6.

In this paper, we do not discuss feedback effects of vari-
ous evolved stellar populations and pulsar winds, which have
been investigated by several authors (e.g., Naiman et al.
2020). These feedback effects could remove some of the intra-
cluster gas ejected from massive AGB stars and binary stars
to end up with much less efficient star formation of 2G stars.
We do not discuss the possible anti-correlations between
light elements (e.g., Na-O anti-correlation) in the present
model either, simply because the present study does not in-
clude chemical evolution at all. Galaxy-scale physics related
to GC formation with multiple stellar populations such as
the formation of giant molecular clouds hosting young clus-
ters and cloud-cloud collisions in galaxies are also totally
ignored in the present study, though they are included in
our previous simulations on cluster formation (e.g., Bekki
et al. 2004; Williams et al. 2021). We will discuss these key
issues in our future works using more sophisticated hydro-
dynamical simulations of GC formation.

2 THE SBC SCENARIO

2.1 Outline

In this SBC scenario, massive compact stellar systems con-
sisting of 1G stars are first formed within their host gas-
rich dwarf disk galaxies with high mass densities owing to
dynamical instabilities of the disks (B19a). The GCs ini-
itially have rather high binary fractions (almost 100%) in
intermediate-mass stars, as observed in local star-forming re-

gions (e.g., K07), and thus the binary populations can start
to eject gas through RLOF about \( \sim 30 \) Myr after the initial
burst of 1G star formation. These “pristine” gas is accumu-
lated into the central regions of the GCs so that they can
form high-gas density regions within the deep gravitational
potentials. Then stellar winds from single AGB stars with
mass \( m \approx 8M_\odot \) start to be accumulated into the central
regions to mix with the pristine gas from IMCBs. Finally new
stars with peculiar abundance patterns (e.g., Na-rich, O-poor) are
formed from the mixed gas very efficiently to develop the 2G
population.

In the standard AGB scenario based on single AGB
stars only, the anti-correlation between CNO abundances can
be explained in the context of dilution of AGB ejecta
with pristine gas (e.g., D’Ercole et al. 2010). However, the
origin of pristine gas between the AGB and SBC scenarios is
quite different. The amount of pristine gas and the epoch
when the gas can be accreted onto the central regions of
pre-existing 1G populations are determined primarily by the
evolution of IMCBs within GCs in the SBC scenario. This
idea of pristine gas originating from 1G stars is essentially
the same as the proposal by Gratton & Carretta (2010) in
which less evolved 1G stars can eject a significant amount
of pristine gas (though the amount is only at most 1% of
1G stars). Given that the amount of ejecta from single AGB
stars can be determined by the properties of 1G stars in
the SBC scenario, the properties of 2G stars can be also
determined largely by GC properties themselves; it should
be noted here that the accretion rate of pristine ISM onto
GCs in the AGB scenario depends both on GC masses and
on the properties of GC host galaxies.

Secondary star formation within GCs can continue un-
til fresh supply of gas due to gas accretion from single AGB
stars and IMCBs onto their deep potential wells is severely
reduced. During this 2G formation, the maximum masses of
new stars cannot be larger than \( 8M_\odot \) so that feedback
effects of CCSNe cannot influence the 2G formation at least
\( \approx 300 \) Myr: the possible physical reasons for this are dis-
cussed later in this paper. It should be noted here that no ob-


simulations did not investigate at all the time evolution of gas ejected from IMCBs within GCs. Therefore, it is totally unknown (i) how much gas can be ejected from single and binary intermediate-mass stars, (ii) what the possible differences in chemical abundance patterns are between 1G and 2G stars, and (iii) whether or not the observed fundamental properties of GCs with MPs can be possibly explained. It is thus the main purpose of this paper to provide basic predictions of the scenario using rather idealized analytic and one-zone models of GC formation.

Gas ejected from AGB stars with different masses itself does not show the observed anti-correlation between light elements (e.g., [Na/Fe] vs [O/Fe]). However, if Na-rich and O-poor gas ejected from more massive AGB stars mix well with pristine Na-normal and O-normal gas, then 2G stars formed from such mixed gas can show a Na-O anti-correlation. We are currently investigating the anti-correlations between light elements (CNO etc) for the simulated GCs and will discuss in what physical conditions the observed Na-O anti-correlation can be reproduced well in the SBC scenario (Bekki 2022). The preliminary results suggest that the timescale of 2G formation should be less than 10$^8$ yr in order to reproduce the Na-O anti-correlation: if gas from low-mass AGB stars is converted into new stars, then the observed anti-correlation cannot be reproduced.

2.2 Several possible advantages of the scenario

There are the following advantages in this scenario in explaining the origin of GCs with multiple stellar populations. First, in this scenario, pristine gas from IMCBs can be ejected through RLOF and rapidly accumulated within the deep potential wells of proto GCs well before single massive AGB stars with $m \approx 8M_\odot$ eject polluted gas. Therefore, ejecta from such AGB stars can be mixed with the pristine gas and then converted into new stars, if the physical conditions for star formation can be met for the mixed gas. Thus, there is no “timing problem” (in the required dilution processes) in this scenario.

Second, the total amount of pristine and polluted gas can be determined by the physical properties of GCs themselves, such as number fraction of binary stars ($f_b$), mass fraction of pristine gas ejected from binary stars ($f_{pb}$), and initial total masses of GCs ($M_i$); there is a possibility that the “right amount” of pristine gas can be supplied from IMCBs through RLOF for a reasonable set of the above parameters. Therefore, the physical properties of GCs with multiple stellar populations, such as fractions of 2G stars ($f_{2g}$) and the mass budget factor ($F_{mb}$, i.e., the ratio of initial to present-day GC mass, defined later in detail), which is the ratio of the initial total mass of a GC to the present-day mass ($M_i/M_{gc}$), can be determined by GC properties themselves.

Third, the required cessation of star formation in 2G populations of GCs can be naturally explained in this scenario (no “truncation” problem): here secondary star formation needs to be avoided because of the formation of 2G stars with non-constant C+N+O from ejecta of low-mass AGB stars (e.g., Fenner et al. 2004). Fresh supply of gas from IMCBs for secondary star formation can be severely suppressed within $\approx 300$ Myr after GC formation, because the ejection rate of pristine gas from IMCBs become very low after $\approx 300$ Myr (see Fig. 2 in V12). Such suppression of gas supply is highly likely to end up with no/little further star formation owing to low gas densities within GCs. Fourthly, the pristine gas from IMCBs have the same [Fe/H] as the polluted gas from single AGB stars in a GC, which means that [Fe/H] of 2G stars should be the same as their 1G counterparts (no “metallicity” problem).

Fifth, 2G formation can occur only after all CCSNe are exploded. In the massive binary scenario by de Mink et al. (2009), 2G stars need to be formed sometime between (i) after the ejection of gas from massive binary stars and (ii) before massive CCSNe: only less than $\approx 3$ Myr is allowed for the gas from the binary stars to be cooled down and subsequently converted into new stars. This problem of very narrow time window for 2G formation does not exist in the SBC scenario. These advantages imply that the SBC scenario is very promising, however, the scenario has several possible problems in reproducing the observed properties of GCs, as we discuss later. Also, we need to discuss the SBC scenario in the context of well known problems of GC formation, e.g., the observed $f_{2g}$ depending on $M_{gc}$, the so-called mass budget problem, difference in Li abundances between 1G and 2G stars etc (e.g., G19, MM22) in a quantitative way.

3 BASIC CHARACTERISTICS OF THE SCENARIO

3.1 Analytic models

Using analytic models, we discuss how $f_{2g}$, $F_{mb}$, Li abundances of 2G stars (A(Li)), and [Na/Fe] differences between 1G and 2G stars depend on the model parameters of the SBC scenario. Since it is assumed that a GC consists of 1G and 2G stars, the present-day total mass of a GC ($M_{gc}$) is as follows:

$$M_{gc} = M_{1g} + M_{2g}. \quad (1)$$

The initial total mass of the GC ($M_i$) consisting only of 1G stars is significantly larger than $M_{gc}$ owing to the stellar mass loss through supernovae, stellar winds, internal dynamical evolution, and tidal stripping. The 1G stars can lose their stellar masses significantly through stellar evolution only (e.g., 17% from SNe for a canonical IMF), and they can lose much masses due to internal dynamical evolution and tidal stripping if they have more diffuse spatial distributions compared to 2G stars (e.g., Vesperini et al. 2010). The mass budget factor ($F_{mb}$) is defined as follows:

$$F_{mb} = \frac{M_i}{M_{gc}}. \quad (2)$$

The adopted power-law IMF in number for each GC is defined as follows;

$$\psi(m) = C_0 m^{-\alpha}, \quad (3)$$

where $m$ is the initial mass of each individual star and the slope $\alpha = 2.35$ corresponds to the Salpeter IMF. The normalization factor $C_0$ is a function of the initial mass of a GC ($M_i$), $m_i$ (lower mass cut-off), and $m_u$ (upper mass cut-off):

$$C_0 = \frac{M_i \times (2 - \alpha)}{m_u^{2 - \alpha} - m_i^{2 - \alpha}}. \quad (4)$$
Although we use \( \alpha = 2.3 \) (Salpeter IMF) for 2G stars, we consider that \( \alpha \) is a free parameter in the present study. We also consider that \( m_u \) can be different between the two populations: it is defined as \( m_{u,1g} \) and \( m_{u,2g} \) for 1G and 2G stars, respectively. Both 1G and 2G stars have \( m_1 (= 0.25 M_\odot) \), and \( m_{u,1g} \) is fixed at \( 50 M_\odot \).

The mass fraction of 2G stars is defined as follows:

\[
 f_{2g} = \frac{M_{2g}}{M_{6c}},
\]  

where \( M_{2g} \) and \( M_{6c} \) are estimated for low-mass stars with \( 0.25 \leq m/M_\odot \leq 0.8 \). Given that \( \alpha \) and \( m_{l,2g} \) are fixed for 2G stars, \( f_{2g} \) is determined by \( \alpha \) and \( m_{u,2g} \). We also estimate the initial total masses of single and binary stars (\( M_{i,s} \) and \( M_{i,b} \), respectively) and thereby model the time evolution of the two populations separately. We consider that the fraction of binary stars is the key parameter, which is defined as follows:

\[
 f_b = \frac{M_{i,b}}{M_i},
\]  

Using the observed properties of very young binary populations in Scorpius OB2 associations, K07 revealed that the current binary fraction of A- and B-type stars is at least 70% and also suggested that the primordial fraction can be almost 100%; see Kroupa and Jerabkova (2018) for a recent review on the binary fractions depending on stellar masses. If the distribution of orbital periods \( (P) \) of binary stars is described as \( f_b(P) \propto P^{-1} \) with the minimum and maximum \( P \) being 0.5 days and 0.15 Myr (K07) respectively, then the (original) fraction of IMCBs \( (f_{\text{imcb}}) \) among all binary populations with \( P \) less than \( 4000 \) days is 0.71: the minimum and maximum \( P \) adopted in V12 are 1.0 and 3700 days, respectively. If the maximum \( P \) of \( 10000 \) days is adopted for IMCBs that can lose gas from the systems, then \( f_{\text{imcb}} = 0.8 \). Although \( f_{\text{imcb}} \) in GC formation could be different from those of local OB associations, we consider that \( f_{\text{imcb}} \) should be rather high (likely \( f_{\text{imcb}} > 0.8 \)) even in GC formation. We assume that IMCB fraction in GC formation is 1 to avoid introducing an extra free parameter in the present model, because the results do not depend strongly on \( f_{\text{imcb}} \) for the reasonable range \( 0.7 < f_{\text{imcb}} < 1 \).

The IMFs of the single and binary (1G) populations are assumed to be the same, however, the normalization factor \( (C_0) \) needs to be estimated separately for the two using the following relations:

\[
 (1 - f_b)M_i = \int_{0.25}^{0.5} C_{0,s} m \psi(m) dm,
\]  

for the single population, and

\[
 f_b M_i = \int_{0.25}^{m_{u,2g}} C_{0,b}(1 + q(m)) m \psi(m) dm,
\]  

for the binary one. In these equations, the mass-ratio of the primary to the secondary stars in a binary pair is denoted as \( q \) and the \( q(m) \) describes how \( q \) depends on stellar masses.
We use the flat $q$ distribution (V12) and consider no initial dependence of $q$ on stellar masses. Accordingly, we adopt $q = 0.5$ in the present study. Thus, once $M_i$ and $f_b$ are given, the IMF normalization factors, $C_{0,s}$ and $C_{0,b}$ (for single and binary populations, respectively) can be determined for a given IMF slope $\alpha$. We do not consider single and binary populations in 2G stars just for simplicity in the present study.

The total mass of gas ($M_g$) that can be used for the formation of 2G stars is the sum of (i) the total mass of stellar winds from single AGB stars ($M_{g,s}$) and (ii) that of gaseous ejecta from IMCBs ($M_{g,b}$). The IMCB ejecta can be further divided into (i) pristine gas that can be used to dilute the ejecta of single AGB stars and (ii) polluted gas from EAGB and TPAGB phases of IMCBs. Following V12, we here consider that the mass fraction of pristine gas from IMCBs ($f_{pr}$) is the key for the chemical abundance patterns of 2G stars, and $f_{pr}$ is as follows:

$$f_{pr} = \frac{M_{g,b}}{M_{g,b}}.$$  \hfill (9)

Using the above equations, we can estimate $M_{g,b}$ for IMCBs with $3 \leq m_{w}/M_\odot \leq 8$ as follows:

$$M_{g,b} = f_{ej} \int_{3}^{8} C_{0,b}(1 + q(m))m\psi(m)dm,$$  \hfill (10)

where $f_{ej}$ is the average mass fraction of gas ejected from IMCBs (V12). Since both $f_{pr}$ and $f_{ej}$ are calculated in V12 for a large number of models, we can use the derived values to estimate $M_{g,pr}$ in the above equations.

In order to calculate $M_{g,s}$, we use the following analytic form for the total mass of stellar wind ($m_{w}$) from a single AGB star (G19):

$$m_{w} = 0.894m - 0.434,$$  \hfill (11)

which is only slightly different from our previous works (Bekki 2011, B11). Accordingly, $M_{g,s}$ can be estimated as follows:

$$M_{g,s} = \int_{3}^{8} C_{0,s}m_{w}\psi(m)dm.$$  \hfill (12)

The sum of this $M_{g,s}$ and $(1 - f_{pr}) M_{g,b}$ is the total mass of ‘polluted’ gas. The mass fraction of pristine gas among all gas is referred to as a “dilution factor” ($F_{dil}$), which is defined as follows:

$$F_{dil} = \frac{f_{pr} M_{g,b}}{M_{g}}.$$  \hfill (13)

We investigate (i) possible differences in [Na/Fe] between 1G and 2G stars and (ii) possible Li abundance (A(Li)) of 2G stars in this SBC scenario in order to address the validity of the scenario. In order to discuss these two problems more quantitatively, we simply adopt the following assumptions. First, [Na/Fe] for 1G ([Na/Fe]$_{min}$) and polluted gas from AGB stars ([Na/Fe]$_{max}$) are −0.1 and 0.7, respectively. These are adopted as typical values of GCs with multiple stellar populations and consistent with observations, e.g., the results shown in Fig. 1 by Carretta et al. 2010. Second, pristine gas from IMCBs has no Li (“Li-free gas”), though this could lead to an underestimation of A(Li) of 2G stars. Here we follow the models by D’Antona et al. (2012, D12), who assumed Li-free gas in some of their models and thereby investigated the possible A(Li) of 2G stars in GCs. Third, the same Li yields (Y$_{Li}$) adopted by D12 are used in the present study so that the mean A(Li) of AGB ejecta can be calculated. The mean mass-weighted fraction of Li among AGB stars (Y$_{Li,m}$) is estimated as follows:

$$Y_{Li,m} = \frac{1}{M_{g,s}} \int_{3}^{8} C_{0,s}Y_{Li}(m)m\psi(m)dm$$  \hfill (14)

where $Y_{Li}(m)$ is dependent on the masses of AGB stars and adopted from D12. Based on this $Y_{Li,m}$, we estimate the mean A(Li) for AGB ejecta (A$_{m}(Li)$). The value of A$_{m}(Li)$ is 2.4 for $\alpha = 2.3$, and it does not depend on $\alpha$.

The adopted assumption of Li-free pristine gas from IMCBs would be oversimplified, because Li fraction is lower yet not zero in pre-AGB phases of intermediate-mass stars (see Fig. 9 in Karakas and Lattanzio 2014). The observed A(Li) in stars of NGC 6397 and predicted one range from $\approx 2$ to $\approx 1$ even during first dredge-up phases of intermediate-mass stars (Karakas and Lattanzio 2014). Therefore, the present model is highly likely to under-predict A(Li) of 2G stars.

### 3.2 f$_{2g}$

Using the above simple analytic models, we can discuss how f$_{2g}$ depends on the model parameters. Fig. 1 demonstrates that there is a narrow range of $\alpha$ for which f$_{2g}$ can be larger than the observed f$_{2g}$ ($\approx 0.4$) in the Galactic GCs with log($M_{gc}/M_\odot$) = 5.2 (G19). For example, the required $\alpha$ for f$_{2g} > 0.4$ in the models with $m_{u,2g} = 1M_\odot$ is $\approx 1.8$ for $f_{b} = 0.5$ and $\approx 1.7$ for $f_{b} = 0.7$, which means that IMFs of 1G stars need to be moderately top-heavy. There is no $\alpha$ that can reproduce f$_{2g} = 0.4$ in the models with $m_{u,2g} = 8M_\odot$. 

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**Figure 3.** Dilution factors ($F_{dil}$) as a function of $f_{b}$ for the models with $\alpha = 2.3$ and $f_{pr} = 0.53$. Clearly, there is almost a linear relation between $F_{dil}$ and $f_{b}$, and this relation does not depend so strongly on other parameters such as $\alpha$, $f_{ej}$, and $f_{pr}$. 

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which implies that high-mass star formation needs to be severely suppressed in the 2G formation.

Clearly, $f_{2g}$ is larger for smaller $\alpha$ (i.e., more top-heavy IMFs) for different $f_b$, mainly because the mass fractions of single AGB and IMCB stars are larger for smaller $\alpha$. Slightly larger $f_{2g}$ in smaller $f_b$ is due to the adopted assumption that the mass fractions of gaseous ejecta among single AGB stars (more than 0.8) are larger than those of gas among IMCBs ($f_{ej} = 0.53$). In these models with $\alpha = 2.3$ (standard Salpeter IMF), $f_{2g}$ cannot be larger than 0.4 for any $m_{u,2g}$ and $f_b$. This suggests that 1G stars with $\alpha = 2.3$ need to be much more efficiently lost compared with 2G through some physical processes (e.g., tidal stripping by their host dwarf galaxies) to reproduce the observed typical $f_{2g}$. Such preferential stripping of 1G stars has been already proposed in D08 and investigated by Vesperini et al. (2010) for the standard AGB scenario.

The required $\alpha \approx 1.8$ is not unrealistic, given that the IMF slopes for intermediate- and high-mass stars are inferred to be top-heavy ($\alpha < 2.0$) in 70% of the investigated 20 GCs (Marks et al. 2012, M12). However, it should be noted here that the inferred IMF slopes in M12 are based on the assumed link between the IMF slope and the gas expulsion process. The various physics in young star clusters can be possibly missed in their inference of IMF slopes from observation; see Krause et al. (2020) for a more recent review of various physical processes in young star clusters.

Fig. 1 also indicates that the observed dispersion in $f_{2g}$ can be due to the dispersion in $\alpha$ between GCs: M12 indeed showed a dispersion in $\alpha$. Necessity of top-heavy IMFs to explain the observed large $f_{2g}$ was discussed in other scenarios of GC formation (e.g., Bekki & Norris 2006; Prantzos & Charbonnel 2006), which implies that $f_{2g}$ can provide a constraint on the IMF in GC formation.

3.3 $F_{mb}$

Fig. 2 shows that $F_{mb}$ (“mass budget factor”) ranges from $\approx 2$ at $\alpha = 2.5$ to $\approx 7$ at $\alpha = 1.5$ for $m_{u,2g} = 8M_\odot$. As expected, $F_{mb}$ is larger for smaller $\alpha$ (i.e., more top-heavy IMFs), because a larger amount of gas from CCSNe is lost and a large number of massive stars are locked as compact objects (stellar mass black holes and neutron stars) for smaller $\alpha$. This $\alpha$-dependence can be clearly seen in Fig. 2 for the models with different $m_{u,2g}$, and it is confirmed to be seen in the models with different $f_b$, though the results
are not shown in Fig. 2. Based on the detailed comparison of GC properties between new Gaia DR2 data and corresponding simulations, Baumgardt et al. (2019) showed that GCs have lost about 80% of its initial mass on average (i.e., $F_{\text{inh}} \approx 5$). The derived maximum possible $F_{\text{inh}}$ ($\approx 7$) in the present study is therefore not too large (like $10 - 100$ suggested in other previous works), which implies that there would not be a serious “mass-budget” problem in this SBC scenario. Even for $\alpha < 1.7$ required for $f_{2g} > 0.4$ in Fig. 1, $F_{\text{inh}}$ is $\approx 4$ for $m_{u,2g} = 1M_\odot$.

It should be noted here that all gas ejected from single AGB stars and IMCBs are assumed to be converted into 2G stars in these models. Our recent hydrodynamical simulations of 2G formation in dense stellar systems have shown that such a 100% gas conversion efficiency is unlikely (B11, Bekki 2019, B19b). Accordingly, $F_{\text{inh}}$ in Fig. 2 can be the lower limit for each $\alpha$. In this discussion, loss of stars due to long-term internal dynamical relaxation processes and tidal stripping of stars by GC host dwarfs are not considered at all. Therefore, the initial masses of GCs can be even larger than $M_\text{i}$ estimated in the present study.

### 3.4 $\Delta$[Na/Fe]

Fig. 3 describes how $F_{\text{dil}}$ (dilution factor) depends on $f_0$ for a fixed $\alpha$ and $f_{pr}$. A larger amount of pristine gas can be ejected from IMCBs for larger $f_0$, so that gaseous ejecta from single AGB stars can be diluted to a higher degree (i.e., larger $F_{\text{dil}}$). This result does not depend on $\alpha$ and $m_{u,2g}$, but it depends strongly on $f_{pr}$ and $f_0$ for IMCBs. The derived $F_{\text{dil}}$-dependence suggests that abundance differences between 1G and 2G stars can be larger for GCs with smaller $f_0$. Indeed, as demonstrated in Fig. 4, $\Delta$[Na/Fe] can be larger for smaller $f_0$ for different $f_{ej}$ and $f_{pr}$, though the $f_0$ dependence is not so strong. Clearly, $\Delta$[Na/Fe] is systematically lower in the models with larger $f_{pr}$ in which a larger amount of pristine gas can be ejected from IMCBs for a given $f_{ej}$.

The key parameter here is $f_0$ at the epoch when intermediate-mass stars enter into their AGB phases. As shown in previous numerical simulations (e.g., Hong et al. 2015), $f_0$ in GCs can be dramatically reduced due to the internal dynamical relaxation processes. If the rapidity of $f_0$ reduction can be simply scaled to the dynamical relaxation timescale at half-mass radii of GCs ($t_{\text{relax}}$), then $f_0$ at AGB phases of intermediate-mass stars can be lower for GCs with shorter $t_{\text{relax}}$. This suggests that there could be an anti-correlation between $t_{\text{relax}}$ and $\Delta$[Na/Fe] in GCs. This possible anti-correlation could disappear, if $f_{pr}$ and $f_0$ are different between GCs with different $t_{\text{relax}}$ for some physical reasons.

### 3.5 A(Li) of 2G stars

As shown in Fig. 5, the predicted dependence of A(Li) on $f_0$ is very similar to $f_0$-dependence of $\Delta$[Na/Fe]. The models with larger $f_0$ have lower A(Li) in their 2G stars owing to larger degrees of dilution of AGB ejecta by pristine gas from IMCBs. The $f_0$-dependence is rather flat for smaller $f_{ej}$ and $f_{pr}$, because only a small amount of pristine gas can be ejected from IMCBs and subsequently mixed with AGB ejecta. There is not a significant difference in A(Li) of 2G stars between models with different $f_0$ and $f_{pr}$ for $f_{pr} = 0.25$. In these models, A(Li) of 2G stars can be determined largely by the Li yields of AGB stars ($A_{\text{Li}}(Li) = 2.4$). Irrespective of $f_0$ and $f_{ej}$, the models with large $f_{pr}$ show lower A(Li).

In each model, $f_{pr}$ averaged over an IMCB population from V12 is used. However, $f_{pr}$ should be time-dependent in real GC formation, because IMCBs with different $m$, separations, and orbital periods can have different $f_{pr}$ (V12). Since pristine gas can be ejected from IMCBs earlier than polluted one, it is possible that intraccluster gas can have rather low A(Li) in the very early formation phases of GCs, e.g., $\approx 30$ Myr after GC formation, i.e., when single AGB stars with $m = 8M_\odot$ start to eject polluted gas. Accordingly, 2G stars formed earlier can have lower A(Li) and [Na/Fe] similar to that of 1G stars. Furthermore, the fractions of Li-poor stars can be larger in GCs in which 2G star formation started earlier. One of possible predictions in the SBC scenario is that there can be 2G stars with rather low A(Li) formed from almost Li-free gas. Such formation of Li-poor 2G stars can be suppressed if there is a threshold gas mass or density beyond which star formation is possible, because intraccluster gas masses/densities can significantly increase as gas from single AGB stars is accumulated within GCs.

### 4 STAR FORMATION HISTORIES OF 2G STARS

In this section, we use classic one-zone models to investigate the possible star formation histories (SFHs) of 2G stars in forming GCs. Although we used hydrodynamical simulations in our previous works on star formation rates (SFRs) within GCs (B11; Bekki 2017, B17; B19b), we consider that the present-one-zone models are enough to provide useful information on the possible SFHs in this first paper. Such more sophisticated (and time-consuming) modeling will be done in our forthcoming papers to discuss several other key issues that are not discussed in the present paper. It should be stressed here that previous observational and theoretical studies of the formation of individual stars focused exclusively on star formation within molecular clouds. Accordingly, it is totally unknown how star formation can proceed in molecular gas embedded in dense stellar systems. Therefore, the present-study makes the most of the models used in previous studies of galactic star formation and in numerical simulations of globular cluster formation (B19b).

#### 4.1 One-zone models

We investigate the time evolution of $M_{\text{GC, g}}$ (total gas mass) and SFR ($\psi(t)$) in a GC for a given accretion rate ($A(t)$) of gas from single and binary stars within GCs. The basic equations for the adopted one-zone models of SFHs in 2G stars are described as follows:

$$\frac{dM_{\text{GC, g}}}{dt} = -\psi(t) + A(t).$$  \hspace{1cm} (15)

Here we distinguish between $M_{\text{GC, g}}$ and $M_\text{g}$, because $M_{\text{GC, g}}$ is the sum of gas from all 1G stars and ISM from GC host galaxies in GC formation: ISM accretion onto GCs is not considered in the present study, however. Since this gas accretion is both from AGB winds of single stars and from
mass loss of IMCBs due to RLOF, \( A(t) \) can be defined as follows:

\[
A(t) = \frac{dM_g}{dt} = \frac{dM_{g,b}}{dt} + \frac{dM_{g,s}}{dt}.
\]

We can calculate \( dM_{g,s}/dt \) using the adopted IMF as follows:

\[
\frac{dM_{g,s}}{dt} = \frac{1}{dt} \int_{m_{10}}^{m_{10} + dt} C_{0,a} m \psi(m) dm,
\]

where \( m_{10} \) is the main sequence turn-off mass, which is a function of ages of stars (i.e., time \( t \)) and always \( m_{10}(t) > m_{10}(t + dt) \). In order to calculate \( m_{10} \) at each time \( t \), we use the following relation between stellar ages \( (t_s) \) and \( m_{10} \) by (Greggio & Renzini 2011):

\[
\log m_{10}(t_s) = 0.0434(\log t_s)^2 - 1.146 \log t_s + 7.119,
\]

where \( m_{10} \) is in solar units and \( t_s \) in years.

We use the results of the models shown in V12 (their Fig. 2) to estimate \( dM_{g,b}/dt \) for all models. Using the table kindly provided D. Vanbeveren, we first estimate \( dM_{g,b}/dt \) for each time step with \( dt = 10^6 \) yr from the table and then normalize it for a given initial mass of a GC. A key parameter here is the mass fraction \( (f_{g,b}) \) of gas ejected from IMCBs over 300 Myr and among all gas ejected from 1G stars over the same period, which is defined as follows:

\[
f_{g,b} = \frac{M_{g,b}}{M_g}.
\]

Here it should be noted that this is not simply a fraction of binary stars, but it depends both on the adopted IMFs and on a number of parameters that describe the time evolution of IMCBs (V12). Although we have investigated a number of different \( f_{g,b} \), we mainly show the results of models with \( f_{g,b} = 0.5, 0.7, \) and 0.9, because the adopted \( f_{g,b} \) values are quite reasonable.

We adopt two star formation models, because it is not clear how individual stars can be formed within such dense environments of 1G stars. One model assumes that the star formation rate \( \psi(t) \) is proportional to \( M_g^\beta \) (“mass-dependent”), where \( \beta \) is a power-law slope, and a constant star formation coefficient \( (\epsilon_{sf}) \) is adopted. Thus it is described as follows:

\[
\psi(t) = C_{sf} M_g^\beta(t)
\]

where \( C_{sf} \) is a constant that determines SFRs. This constant is determined such that most of the accreted gas can be converted into 2G stars for each model. This star formation model dependent on gas mass or gas mass fraction (density) has been adopted often in the studies of galactic chemical evolution using so-called “one-zone models”. However, it is not clear at this stage whether such star formation models can be applied for 2G formation in forming GCs. We thus suggest that the real star formation histories of 2G could be significantly different from those derived using the above model.

The other assumes that gas accretion rates can determine SFRs as follows:

\[
\psi(t) = C_{sf} A(t)^\beta = C_{sf} (dM_g/dt)^\beta.
\]

In addition to the mass-dependent model, this model is also investigated, because we consider that gas accretion rates can be important for the formation of high-density cores where star formation can occur. Again, this model is just one of possible star formation models, and accordingly, the real secondary star formation can be different from the predicted one from the model.

We here newly introduce a hypothetical “threshold gas mass” \( (M_{g,th}) \) beyond which star formation is possible. This introduction of \( M_{g,th} \) is motivated by our previous simulations (B19b), which shows that star formation within GCs can be severely suppressed in lower gas mass fractions (i.e., lower \( M_{g,th}/M_{gc} \) owing to the direct collisions of cluster member stars with gas clouds. This suppression mechanism of star formation due to interaction between stars and gas clouds is very unique in dense stellar systems (not relevant to star formation within molecular clouds without existing stars). Accordingly it is better for the present study to introduce \( M_{g,th} \) at least in some models. In the model with the threshold gas mass,

\[
\psi(t) = 0
\]
We assume a high SFE of \( \epsilon \) following the adopted star formation model described in Section 2G, 3G, 4G etc due to gas ejection from SNIa. The adopted SFRs are calculated using a detailed hydrodynamical simulations of feedback effects of SNIa that can occur frequently within 0.1-1 Gyr after 1G formation can remove low-mass AGB stars. Feedback effects of SNIa that can occur during 2G, 3G, 4G etc due to gas ejection from SNIa, it would not be possible to claim that SNIa can cause such truncation of star formation. If feedback effects of SNIa that can occur during 2G, 3G, 4G etc due to gas ejection from SNIa, it would not be possible to claim that SNIa can cause such truncation of star formation. Star formation can be naturally almost truncated around 200 Myr after 1G formation in the mass-dependent models, whereas it can only slowly decline 100 Myr after 1G formation in the mass-dependent model. As pointed out for the “truncation” dilution problem in the standard AGB scenario, gas from AGB stars with \( m < 3M_\odot \) should not contribute to chemical enrichment of intracluster gas from which 2G stars are formed, because C+N+O of 2G stars can be significantly different from 1G. Therefore the SBC scenario with the mass-dependent SF has a similar truncation dilution problem. If \( \beta = 1 \) and SFRs in Bekki et al. (2017) is adopted for these SFRs, then \( m_{\text{max}} \) (i.e., \( m_{a,2g} \)) can be significantly lower than \( 50M_\odot \) (adopted for 1G), but not low enough to explain the origin of the required \( m_{a,2g} \left( = 8M_\odot \right) \) in the SBC scenario.

**Figure 7.** Time evolution of SFRs in the models with \( M_{g,\text{th}} = 10^5M_\odot \) (black solid), \( 2 \times 10^5M_\odot \) (red dashed), and \( 3 \times 10^5M_\odot \) (blue dotted). The adopted \( M_{g,\text{th}} \) are adopted so that the formation of 2G, 3G, 4G etc due to \( M_{g,\text{th}} \) can be demonstrated.

for \( M_{g,\text{th}} < M_{g,\text{th}} \). Once \( M_{g,\text{th}} \geq M_{g,\text{th}} \), gas starts to be consumed following the adopted star formation model described above. We here consider that (i) star formation efficiency (SFE, \( \epsilon_{sf} \)) is fixed and (ii) this star formation can continue until a gas mass of \( \epsilon_{sf}M_{g,\text{th}} \) (which corresponds to \( \epsilon_{sf}M_{g,\text{th}} \) when \( M_{g,\text{th}} \) becomes \( M_{g,\text{th}} \)) is all converted into new stars. We assume a high SFE of \( \epsilon_{sf} = 0.7 \) in the present models. Accordingly, if \( M_{g,\text{th}} = 10^5M_\odot \), then gas can be consumed until \( M_{g,\text{th}} \) becomes \( 3 \times 10^4M_\odot \). In the mass-dependent models, star formation needs to be truncated by some physical processes: otherwise, very low-level star formation can produce 2G stars with their chemical abundances influenced by low-mass AGB stars. Feedback effects of SNIa that can occur frequently within 0.1-1 Gyr after 1G formation can remove all of the remaining gas from forming GCs. However, without detailed hydrodynamical simulations of feedback effects from SNIa, it would not be possible to claim that SNIa can cause such truncation of star formation.

**4.2 Results**

Fig. 6 describes the time evolution of SFRs over 300 Myr in the models with different \( f_{g,b} \) in which the total gas mass ejected from single AGB stars and IMCBs is \( 6 \times 10^5M_\odot \). In these models, SFRs at each time step are calculated using the mass-dependent (SFR \( \propto M_g \) with \( \beta = 1 \)) and accretion-dependent SF models (SFR \( \propto M^2 \) with \( \beta = 2 \)) for the adopted V12 model for the time evolution rate of gas from IMCBs. Clearly, there are two peaks in SFR evolution for models with \( f_{g,b} = 0.7 \) and 0.9, which reflects the fact that there are two peaks in the time evolution of gas ejection rates for IMCBs. The two peaks are more distinct for the accretion-dependent star formation model, simply because SFR is more sensitive to \( dM_g/dt \) that is determined largely by gas ejection rates of IMCBs in the model. The two SFR peaks can end up with two stellar populations with different chemical abundances in 2G stars owing to the possible difference in chemical abundance patterns of intracluster gas at the two epochs. We will investigate this point in the context of discrete MPs in our future works.

Star formation can be naturally almost truncated around 200 Myr after 1G formation in the mass-dependent models, whereas it can only slowly decline 100 Myr after 1G formation in the mass-dependent model. As pointed out for the “truncation” dilution problem in the standard AGB scenario, gas from AGB stars with \( m < 3M_\odot \) should not contribute to chemical enrichment of intracluster gas from which 2G stars are formed, because C+N+O of 2G stars can be significantly different from 1G. Therefore the SBC scenario with the mass-dependent SF has a similar truncation dilution problem. Fig. 6 demonstrates that SFRs are rather low over the 2G formation (log SFR < −1.4M\(_\odot\) yr\(^{-1}\)). If the relation between maximum stellar masses (\( m_{\text{max}} \) and SFRs used in Bekki et al. (2017) is adopted for these SFRs, then \( m_{\text{max}} \) (i.e., \( m_{a,2g} \)) can be significantly lower than 50M\(_\odot\) (adopted for 1G), but not low enough to explain the origin of the required \( m_{a,2g} \left( = 8M_\odot \right) \) in the SBC scenario.

Fig. 7 demonstrates that multiple sharp peaks in SFR evolution with wide SFR intervals can be reproduced in the mass-dependent star formation models with \( f_{g,b} = 0.7 \) and \( M_{g,\text{th}} \geq 3 \times 10^5M_\odot \). These values of \( M_{g,\text{th}} \) are adopted so that the intervals between multiple episodes of star formation can be more clearly seen: there is little physical basis for the adopted values. The mass-ratio of \( M_{g,\text{th}} \) to \( M_\odot \) rather than simple \( M_{g,\text{th}} \) would be a more physically meaningful parameter, because self-gravity can play a role in star formation in dense stellar systems (B19b).

In these models, the number of peaks in SFR evolution depends on \( M_{g,\text{th}} \), such that it can be larger for smaller \( M_{g,\text{th}} \). For example, the formation of 2G, 3G, 4G and 5G stars is possible for \( M_{g,\text{th}} = 10^5M_\odot \) whereas only 2G formation is possible for \( M_{g,\text{th}} = 5 \times 10^5M_\odot \). Although these results imply that the origin of discrete MPs observed in some GCs can be understood in the context of \( M_{g,\text{th}} \) for 2G formation, it is theoretically unclear which value of \( M_{g,\text{th}} \) is the most reasonable and realistic.

In order for 2G stars with \( f_{g,b} \approx 0.4 \) to be formed in the SBC scenario, a long duration (at least an order of 10\(^8\) yr) of star formation is required in these models. This would be possible only if both delayed CCSNe formed from IMCBs (V12) and SNIa do not occur at all during 2G formation (or at least their formation is severely suppressed). It would be equally possible that 2G formation from gas ejected from single AGB stars and IMCBs can occur only after all delayed CCSNe from IMCBs are exploded. In this case, the total mass of 2G stars can be significantly reduced because gas ejected earlier from massive AGB stars and IMCBs can be expelled by delayed CCSNe completely from GCs: \( f_{g,b} \) can become even lower. As suggested by Krause et al. (2020), a significant number of prompt SNIa can cause steady winds and thereby remove intra-cluster gas in massive star clusters with masses more than \( 10^6M_\odot \) only 100 Myr after 1G formation. Accordingly, \( f_{g,b} \) should also depend strongly on the delay time distribution of SNIa in forming GCs.
5 DISCUSSION

5.1 Origin of \(f_{2g}\) dependent on \(M_{gc}\)

The observed mass fractions of 2G stars (\(f_{2g}\)) in GCs are larger in more massive GCs (G19, MM22). Although the origin of this \(f_{2g} - M_{gc}\) relation is yet to be fully understood, MB21 have demonstrated that this relation can be reproduced well in the simulated GCs, if accretion of ISM onto GCs and ISM is included in their GC formation models. Such results are reasonable, because accretion of ISM onto GCs depends on \(M_{gc}^2\) for the Bondi-type accretion for a given set of model parameters of ISM in GC host galaxies, and secondly because the mass of AGB ejecta depends on \(M_{gc}\) for a given IMF. However, it would be important and instructive for the present study to discuss whether the SBC scenario (without ISM accretion onto GCs) can provide an explanation for the \(f_{2g} - M_{gc}\) relation too.

As shown in the present study, \(f_{2g}\) depends strongly on \(\alpha\) in such a way that \(f_{2g}\) is larger for more top-heavy IMFs (i.e., flatter IMFs). This is because the mass fraction of AGB stars (thus the fraction of AGB ejecta) is larger for flatter IMFs whereas the mass fraction of low-mass 1G stars is smaller for such IMFs (thus the relative fraction of 2G low-mass stars is larger). This implies that if more massive GCs have flatter IMFs during their 1G formation, then the observed \(f_{2g} - M_{gc}\) can be qualitatively reproduced at least. Young GCs with multiple stellar populations at high \(z\) are yet to be discovered. Even if the possible GC candidates in high-\(z\) dwarfs are discovered, it would be currently almost impossible for observational studies to investigate the IMFs of 1G stars for such unresolved high-\(z\) GCs. However, the IMFs inferred from mass functions of low-mass stars for the present-day GCs (e.g., De Marchi et al. 2007; M12) can be used for this discussion.

In particular, M12 showed that the IMF slope (\(\alpha_3\)) relevant to the formation of intermediate-mass and massive stars can be flatter for more massive GMCs from which GCs were formed. Although their results are not the direct observational evidence, they imply that if SFEs are not so different between different GMCs, then more massive GCs are likely to have flatter IMFs; indeed, some of more massive GCs (e.g., NGC 5139) in their Table 1 shows rather flat IMF slopes (\(\alpha_3 < 1.7\)). Although we can propose that the \(\alpha - M_{gc}\) (or more precisely, \(\alpha - M_1\)) relation can be responsible for the observed \(f_{2g} - M_{gc}\) relation, this proposal needs to be investigated more quantitatively using next-generation sophisticated numerical simulations of GC formation in our future papers.

5.2 The mass budget problem

The mass budget factor (\(F_{mb}\)) in the SBC scenario ranges from 2 to 7, depending on \(\alpha\) and \(m_{u,2g}\). As shown in Fig. 2, if \(\alpha\) is larger than 1.8, then \(F_{mb}\) can be smaller than \(\approx 5\) even for \(m_{u,2g} = 8M_\odot\). Recent N-body simulations of long-term GC evolution by Webb and Leigh (2015) have demonstrated that the initial masses of GCs are typically 4.5 times larger than their present-day masses for the adopted Kroupa IMF. Although these simulations include long-term dynamical effects on the mass loss processes of GCs, the present study did not include such effects in estimating \(F_{mb}\). These two studies cannot be simply compared each other. However, the results by Webb and Leigh (2015) strongly suggest that derived \(F_{mb}\) in the SBC scenario is quite reasonable and realistic. Accordingly, there is no serious “mass-budget” problem in the SBC scenario. However, it should be noted here that all gas from single AGB and IMCBs is assumed to be consumed by secondary star formation for the estimation of \(F_{mb}\) in the present study. Therefore, the derived \(F_{mb}\) could be underestimated.

In discussing the mass budget problem, 2G stars were assumed to form from ejecta of 1G stars in previous works. However, this assumption could be oversimplified for the following reasons. First, B19a has shown that about 40% of 2G stars in GCs are formed from ejecta of AGB stars that are not 1G stars of the GCs but are from field stars formed around the GCs at the same formation epochs of the 1G stars. Second, observations showed that star clusters can form as cluster associations (e.g., Bastian et al. 2005), which implies that GCs were also formed with other surrounding smaller star clusters. It would be possible that gas ejected from evolved stars in the smaller clusters (that do not finally become the 1G stars of the GCs but become field stars) could be trapped by the GCs to be converted into 2G stars. Therefore, gas from 1G stars of GCs is not the only source for secondary star formation: the oversimplified assumption adopted in previous studies made \(F_{mb}\) quite large (> 10). Thus, it is possible that the mass budget problem is not so serious as ever thought in previous works (e.g., BL18).

5.3 Discrete multiple stellar populations

Recent observations have revealed that some of the Galactic GCs have clearly distinct distributions of stars in the \([\text{Na}/\text{Fe}]-[\text{O}/\text{Fe}]\) and \([\text{Al}/\text{Fe}]-[\text{Mg}/\text{Fe}]\) diagrams (e.g., Carretta et al. 2012; Johnson et al. 2019). Although a few theoretical models to reproduce the observed discreteness of MP stars have been proposed so far (e.g., Bekki et al. 2017; Kim & Lee 2018; Johnson et al. 2019), its origin is yet to be fully understood. The present study has demonstrated that there can be two peaks in SFHs of 2G stars, which implies that 2G stars can have two distinct major populations. Furthermore, the present study has shown that if there is a threshold gas mass (\(M_{g,th}\)) for star formation, GCs can have multiple discrete epochs of 2G formation. Since AGB stars with different mass ranges thus different stellar yields can enrich the intracluster medium at different epochs of star formation, it is possible that 2G populations formed at different epochs can have different chemical abundances.

Previous works (e.g., Bekki et al. 2017; Kim & Lee 2018) are all based on one-zone models, for which model parameters can be fine-tuned to match the observed properties of GCs. Therefore, it is not so clear whether the ranges of physical conditions required for explaining the discreteness of MPs in GCs in these models can be achieved in real GC formation. Our previous simulations of GC formation with a model for a threshold gas density for star formation shows multiple epochs of star formation due to truncation of star formation by CCSNe (B17). However, the adopted threshold density of \(10^4 - 10^5\) cm\(^{-3}\) is yet to be justified by further theoretical investigations of star formation in dense environments of 1G stars. Furthermore, no numerical simulations have ever investigated whether or not gas from AGB stars
and massive OB stars, which should be formed at two different epochs (i.e., before and after CCSNe), can really mix well with gas left over from 1G formation to finally form new stars in the scenario proposed by Kim & Lee (2018).

The present one-zone models did not include the time evolution of chemical abundances of O, N, Mg, and Al and thus is unable to discuss the origin of the observed distinct populations on the [Na/Fe]-[O/Fe] and [Al/Fe]-[Mg/Fe] diagrams. Also so far no numerical simulations of GC formation have clearly demonstrated the distinct clumpy distribution of the stars on the [Na/Fe]-[O/Fe] and [Al/Fe]-[Mg/Fe] diagrams. Thus the main aim of our future more sophisticated simulations is to reveal (i) the physical origin of the hypothetical threshold gas mass or density for star formation in dense stellar systems and (ii) the roles of the threshold mass/density in reproducing the formation of discrete MP in the two diagrams.

5.4 Avoidance of delayed 1G CCSNe and suppression of 2G massive star formation

Like the standard AGB scenario (e.g., D08, D12), the SBC scenario requires severe suppression of massive star formation with $m \gtrsim 8M_\odot$ leading to CCSNe during 2G formation and SNIa, firstly because such energetic events can expel most gas for 2G formation (e.g., Lachin et al. 2021), and secondly because they can introduce large [Fe/H] spreads ($> 0.3$) that are not observed in Type I GCs (e.g., G19, MM22). Accordingly, $m_{\text{min},2g}$ needs to be lower than $8M_\odot$ for at least $\approx 100$ Myr in the SBC scenario. A key question is therefore whether and how such massive star formation can be almost completely suppressed in the dense environments of GCs.

Using hydrodynamical simulations of GC formation, B19b investigated how frequent dynamical interactions between (1G) stars and cold gas (“stellar bombardment”) influence the formation of gas clouds leading to star formation, and found that gas clouds more massive than $\approx 3M_\odot$ can be completely truncated by such stellar bombardment effects (e.g., Fig. 11 of B19b). The simulations, however, do not include various physical effects on star formation such as the radiation fields of 1G stars, magnetic fields, dust physics in intracluster gas of GCs. Accordingly, more sophisticated numerical simulations of 2G formation are required to confirm the low $m_{\text{min},2g}$.

De Donder & Vanbeveren (2004) showed that initially intermediate-mass stars in binaries can be transformed into massive stars that can explode as CCSNe and therefore that CCSNe can be formed even $\approx 250$ Myr after initial starbursts (corresponding to 1G formation in GC formation). This means that 2G formation can be severely suppressed for $\approx 250$ Myr after 1G formation: large reduction in the total masses of 2G stars is highly likely. Therefore, such CCSNe formation needs to be avoided or suppressed in any GC formation scenarios based on self-enrichment by AGB stars. Given that both SNIa and delayed CCSNe originate from binary stellar populations, a key question is whether delayed CCSNe can be really avoided or suppressed while feedback effects of prompt SNIa truncate 2G formation. This question needs to be addressed in our future studies in a quantitative manner.

5.5 Pristine gas from 1G PMS stars

We have assumed that pristine gas required in the SBC scenario originates only from IMCBs. We here suggest that gas from low-mass PMS stars can be possibly mixed with AGB ejecta to be converted into 2G stars. Low-mass 1G stars ($m < 1.5M_\odot$) are still in the PMS phases [10 – 100] Myr after 1G formation (e.g., Stahler & Pallia 2004). If these PMS stars with surrounding gas disks interact violently with other (more massive) stars and consequently lose the significant mass fraction of the gas disks, then the gas can be dispersed into intracluster medium and become pristine gas for 2G formation. This is a very speculative idea, but, Tailo et al. (2015) have already shown that stellar encounters within GCs can possibly destroy the disks around 2G PMS stars: such destruction should be possible for 1G too.

Suppose that (i) the present-day GC has $1.2 \times 10^5M_\odot$ in its 1G stars ($0.25 \leq m/M_\odot \leq 0.8$) and (ii) the GC initially had $3.6 \times 10^5M_\odot$ in their 1G low-mass stars (i.e., lost $\approx 70\%$ of the original 1G mass via tidal stripping etc), how much pristine gas can be ejected from 1G PMS stars? The observed mass-ratios of circumstellar disks to PMS stars for $\log m \leq 0.2$ range from $\approx 0.003$ to $\approx 0.3$ (e.g., Natta 2004). Accordingly, if just $5\%$ of the gas disks (in mass) in 1G PMS stars with $0.25 \leq m/M_\odot \leq 1.5$ can be lost by stellar encounters, then the total mass of the pristine gas can be $2.7 \times 10^3M_\odot$. This is roughly $30\%$ of the total mass of the present-day 2G stars with $0.25 \leq m/M_\odot \leq 0.8$ ($8 \times 10^3M_\odot$ for $f_{2g} = 0.4$). Therefore, the amount of this pristine gas is not negligible at all. However, the above assumption of $5\%$ loss from 1G PMS stars could be just an overestimation.

In the above discussion, the disk around PMS stars cannot be destroyed completely before 2G stars start to form. If the two-body dynamical relaxation timescales ($t_{\text{relax}}$) of GCs correspond to disk-star interaction timescales, then most PMS disks of low-mass stars should survive at least $\approx 10^8$ yr owing to $t_{\text{relax}} \approx$ several $10^8$ yr. This disk-star interaction, however, should be investigated using hydrodynamical simulations to discuss the survival of the disks around PMS stars. In order to discuss this idea in our future papers, we will need to better quantify the total mass lost from low-mass PMS stars in a more quantitative manner using realistic numerical simulations of dynamical interaction between gas disks around PMS stars and 1G stars.

6 CONCLUSIONS

We have investigated the mass fractions of 2G stars ($f_{2g}$), ratios of initial GC masses to total masses of low-mass 1G and 2G stars ($F_{\text{dyn}}$, i.e., the mass budget factors), dilution factors ($F_{\text{dil}}$), differences in [Na/Fe] between 1G and 2G stars, and Li abundances of 2G stars in GCs (A(Li)) using analytic and one-zone models of the SBC (“single-binary-composite”) scenario. In this scenario, gaseous ejecta from single AGB stars and IMCBs can be well mixed to be converted into new 2G stars. The fractions of binary stars ($f_b$) and the slopes of IMFs in 1G ($\alpha$) are the two key parameters that can control the basic characteristics of GCs. In order to discuss how the abovementioned physical properties of GCs depend on $f_b$ and $\alpha$, we have used the results of V12, which predicted the mass fractions of gas
ejected from IMCBs \((f_{\text{ej}})\) and those of pristine gas in the ejecta \((f_{\text{pr}})\). The principle results are described as follows.

1. If \(\alpha\) is smaller than 1.7 for \(m_{\text{u,2G}} = 1M_{\odot}\) and \(f_{b} = 0.7\), then \(f_{\text{2G}}\) can be as large as the observed value of \(\approx 0.4\) for the typical Galactic GCs with \(\log(M_{b}/M_{\odot}) = 5.2\). However, \(f_{\text{2G}}\) can be always lower than 0.4 for \(m_{\text{u,2G}} = 8M_{\odot}\) for a reasonable range of \(\alpha\). This suggests that for such IMFs with larger \(m_{\text{u,2G}}\), 1G stars need to be more efficiently removed from GCs during their long-term evolution to reduce \(f_{\text{2G}}\) to the observed level. Irrespective of \(f_{b}\) and \(m_{\text{u,2G}}\), \(f_{\text{2G}}\) is higher for lower \(\alpha\).

2. The original GC masses should be by a factor of 2 – 7 larger than the present-day total GC (1G+2G) masses with \(0.25 \leq m/M_{\odot} \leq 0.8\) for \(1.5 \leq \alpha \leq 2.5\), and this result does not depend on \(f_{b}\). \(F_{\text{lab}}\) is larger for smaller \(\alpha\) (more top-heavy IMFs), however, it is still \(\approx 7\) for \(\alpha = 1.7\) required for \(f_{\text{2G}} = 0.4\). The derived \(F_{\text{lab}}\) is not so large (i.e., not like \(F_{\text{lab}} > 10\)), which implies that there is no mass budget problem in the SBC scenario. It should be noted, however, that all gas from single AGB and IMCBs is assumed to be converted into 2G stars in the present study. Accordingly, the derived \(F_{\text{lab}}\) is the lower limit.

3. Since the total amount of pristine gas from IMCBs depends strongly on \(f_{b}\). \(F_{\text{lab}}\) depends strongly on \(f_{b}\) too. As a results of this, [Na/Fe] differences between 1G and 2G stars (\(\Delta[\text{Na/Fe}]\)) depend on \(f_{b}\) such that \(\Delta[\text{Na/Fe}]\) is smaller for larger \(f_{b}\) for a given \(f_{b}\) and \(f_{pr}\). For a reasonable range of model parameters (e.g., \(\alpha\) and \(f_{b}\)), \(\Delta[\text{Na/Fe}]\) can be consistent with observations. Therefore, the “right” amount of pristine gas required to dilute the AGB ejecta to the “right” degree is naturally explained in the SBC scenario.

4. A(Li) in 2G stars can be as high as those in 1G (e.g., A(Li) \(\approx 2.2\) for low-metallicity GCs), if 2G formation occurs only after Li-free gas from IMCBs is mixed well with Li-rich AGB ejecta. 2G stars can have lower A(Li), if they are formed early when Li-free gas from IMCBs is not yet fully mixed with an enough amount of polluted gas from single AGB stars. A(Li) of 2G stars depends on \(f_{b}\), though other parameters have no/little effects on A(Li).

5. Formation histories of 2G stars show two weak peaks if \(f_{b}\) is higher \((> 0.5)\) for \(M_{b,\text{th}} = 0\), mainly because there are two peaks in the time evolution of gas ejection from IMCBs predicted from V12. The models with a threshold gas mass/density show discrete multiple peaks of star formation, which implies that the observed discrete populations in GCs could be due to a threshold gas mass or density for the formation of 2G stars in dense stellar systems. Since no accretion of ISM from GC host dwarf galaxies is assumed in the present study, formation histories of 2G stars is determined by \(f_{b}\) for a given IMF.

6. Although the formation of 2G stars with low A(Li) could naturally explain the observed low A(Li) of 2G stars in some GCs (e.g., NGC 6752; Pasquini et al. 2005), it should be avoided for most GCs with MPs: it can be a potentially serious problem in the scenario. The hypothetical threshold gas mass or density for 2G formation can alleviate this potential problem, because it can allow 2G formation to occur only after both single AGB stars and IMCBs have ejected a significant amount of gas (thus after they have mixed together).

7. The four dilution problems (i.e., timing, amount, metallicity, and truncation) in the standard AGB scenario are not serious in the SBC scenario. However, CCSNe should be severely suppressed during 2G formation that can last for an order of \(\approx 10^{8}\) yr. Multiple dynamical interactions between 1G stars and gas within GCs can lower the maximum possible mass for individual star formation (i.e., \(m_{\text{u,2G}} < 8M_{\odot}\)) so that CCSNe cannot occur. This avoidance of supernovae is a fundamental requirement for the scenario to be viable. Therefore, more sophisticated hydrodynamical simulations need to be done to understand what physical processes determine \(m_{\text{u,2G}}\).

8. IMCBs can provide an enough amount of pristine gas for the formation of 2G stars in GCs. However, they can also possibly provide a significant number of delayed CCSNe, which are likely to suppress star formation within GCs even 250 Myr after 1G formation. Therefore, incorporating IMCBs in GC formation models could be a double-edge sword.

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8 DATA AVAILABILITY

The data used in this paper (outputs from computer simulations) will be shared on reasonable request to the corresponding author.

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