Play-Hysteresis in the Joint Dynamics of Employment and Investment

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Abstract  The slow recovery of many developed economies to the recent financial crisis, and the largest fall in aggregate demand since WWII caused by the COVID-19 Pandemic with its foreseeable negative and persistent effects on the aggregate supply, has generated renewed interest in the subject of hysteresis. The presence of significant hysteresis effects has important theoretical and policy implications. First, there is no unique and predetermined long-run equilibrium level of aggregate employment, as the equilibrium is permanently changed by temporary shocks. Second, as the economic system is not self-adjusting, substantial, timely, and sustained expansionary monetary and fiscal policy should be applied to mitigate the impact of shocks, including the temporary ones. Although it is not possible to quantify hysteresis effects in real time, we can use historical data to shed some light on the possible long-term economic consequences of the COVID-19 pandemic. For that purpose, we use the linear play-hysteresis model in the context of two equation system to analyses the join hysterical dynamics of aggregate employment and investment. We implement the model empirically by means of a new algorithm for the simultaneous equations system applied to Portuguese data that separates the effects of large and small changes in aggregate demand on aggregate employment and investment using an endogenous determined switching parameter as reference.

Keywords  Play hysteresis modeling · Computational mathematics · Switching regression · Employment · Investment

Mathematics Subject Classification  62P20 · 65C60 · 91B02 · 91B40 · 91G70

1 Introduction

The slow recovery of many developed economies to the recent financial crisis, and the largest fall in aggregate demand since WWII caused by the COVID-19 Pandemic with its foreseeable negative and persistent effects on...
the aggregate supply, has generate renewed interest in the subject of hysteresis, which refers to a set of dynamical properties of a system (namely non-linearity, remanence, and selectivity) defined in terms of the reaction of a state variable to an external shock that cannot be expressed in terms of a single-valued function, and that takes the form of loops as depicted in Fig. 1c (see, e.g., [2, 3], [14], [36], [37, 38], and [52]). The presence of significant hysteresis effects has important theoretical and policy implications. First, there is no unique and predetermined long-run equilibrium level of aggregate employment, as the equilibrium is permanently change by temporary shocks. Second, as the economic system is not self-adjusting, substantial, timely, and sustained expansionary monetary and fiscal policy should be applied to mitigate the impact of shocks, including the temporary ones (see, e.g., [1], [4], [5], [13], [16], [18], and [50]). Although it is not possible to quantify hysteresis effects in real time, we can use historical data to shed some light on the possible long-term economic consequences of the COVID-19 pandemic. Therefore, the main contribution of this paper is to uncover hysteresis effects in the joint dynamics of employment and investment. For that purpose, we use the linear play-hysteresis model, also known as the friction-backlash model, in the context of two equation system. We implement the model empirically by means of a new algorithm for the simultaneous equations system applied to Portuguese data that separates the effects of large and small changes in aggregate demand on the level of employment and investment using an endogenous determined switching parameter as reference, which is theoretically a positive function of the non-convex costs of adjusting employment and the capital stock, and uncertainty.

The remainder of the paper is organized as follows. Section 2 presents a model of weak (micro) and strong (macro) hysteresis. Sect. 3 presents the empirical strategy and the data set. Section 4 presents the results, and Sect. 5 concludes.

2 Preisach Model of Hysteresis

Let us assume that the level of employment of every heterogeneous firm \( j \), \( R_{\alpha_j,\beta_j}(x_t) \), \( j = 1, 2, 3, \ldots, M \), operating in a competitive market,\(^2\) is 1 if the firm is active (i.e., in the market), and 0 if the firm is inactive (i.e., out of the market). When active, firm \( j \) uses one unit of employment, \( n_{j,t} \), paying a variable cost \( w_j \) (wage), and one unit of capital, \( k_{j,t} \), paying a variable cost \( r_j \) (the interest rate that has to be paid on the firm’s capital stock), to produce one unit of output, \( y_{j,t} \), which it sells at a price \( P_t \). The market price is specified as \( P_t = x_t f(q_{t-1}) \), where \( x_t \) is the exogenous aggregate demand shock, and \( f(q_{t-1}) \) is the deterministic component of the inverse demand function (we follow [17, 18], [23], [28], and [46]). In addition, firms face heterogeneous non-convex costs to hire and to invest in physical capital, \( H_j \), as well as to fire and to disinvest in physical capital, \( F_j \).\(^3\)

The presence of non-convex adjustment costs is sufficient to generate a non-linear employment dynamic with the properties of path dependence and remanence, called weak hysteresis (see [2, 3],\(^4\) which can be well captured by a non-ideal relay hysteresis operator, \( R_{\alpha_j,\beta_j}(x_t) : R \mapsto \{0, 1\} \).\(^5\)

\[ R_{\alpha_j,\beta_j}(x_t) = \begin{cases} 1, & R_{\alpha_j,\beta_j}(x_{t-1}) = 0 \land x_t \geq \beta_j \lor R_{\alpha_j,\beta_j}(x_{t-1}) = 1 \land x_t > \alpha_j \\ 0, & R_{\alpha_j,\beta_j}(x_{t-1}) = 0 \land x_t < \beta_j \lor R_{\alpha_j,\beta_j}(x_{t-1}) = 1 \land x_t \leq \alpha_j \end{cases} \tag{1} \]

\(^1\) The difference between these two types of hysteresis if based on the type of remanence and memory of the system (see [2]).

\(^2\) This is a plausible assumption in many industries such agriculture, textiles, consumer electronics, and steel (21).

\(^3\) Examples of non-convex costs of employment adjustment, mainly caused by the institutional characteristics of the labor market are described in [15], [31, 32], [42], and [51]. A firm also usually incurs in sunk cost for the acquisition of physical assets like firm specific equipment or intangible assets (see, e.g., [44, 45], [21, 23], and [25]), and to disinvest (see [19]). Furthermore, to enter new markets, firms often must incur irreversible costs, e.g., for gathering information on the new market revenues, creating distribution and servicing networks, and advertising or establishing a brand name (see, [1]).

\(^4\) Aggregate demand uncertainty interacting with the non-convex adjustment costs have the potential to reinforce the hysteresis effects (see, e.g., [21, 24], and [7]).

\(^5\) \( R_{\alpha_j,\beta_j}(x_t) \) corresponds to the employment demand function of firm \( j \). This simplification does not restrict the application of the model as we can consider a firm disaggregated into single production units, each of them represented by a non-ideal relay (see [7]).
The employment demand of firm \( j \) in period \( t \), \( R_{\alpha_j,\beta_j}(x_t) \), are defined for \( R_{\alpha_j,\beta_j}(x_0) = 0 \) if \( R_{\alpha_j,\beta_j}(x_0) < \alpha_j \), for \( R_{\alpha_j,\beta_j}(x_0) = 1 \) if \( x_0 > \beta_j \), and both for \( R_{\alpha_j,\beta_j}(x_0) = 0 \) and \( R_{\alpha_j,\beta_j}(x_0) = 1 \) if \( \alpha_j < R_{\alpha_j,\beta_j}(x_0) < \beta_j \).

The difference between the two thresholds \((\beta_j - \alpha_j)\) creates an employment band of inaction (see [7,8], [12], and [22,23]). Each firm requires an aggregate positive demand shock \( x_t > \beta_j \) to hire, and an aggregate negative demand shock \( x_t < \alpha_j \) to fire (or exit the market). Demand shocks within the range \( \alpha_j < x_t < \beta_j \) do not change the level of employment. Furthermore, the current level of demand, \( x_t \), is not sufficient to determine the firm’s state of employment. The whole history of \( x_t \) must also be considered since the system exhibits path dependence and “multibranch non-linearity.” The aggregate employment hysteric dynamics (called strong hysteresis)\(^7\) can be described by the Preisach operator, which is an aggregation procedure of the elementary hysterons \( R_{\alpha_j,\beta_j}(x_t) \).\(^8\)

Let \( \alpha_{\text{min}} \) be the exiting threshold for the less demanding firm, and \( \beta_{\text{max}} \) the entering threshold of the most demanding firm. Full employment is represented by the area delimited by Preisach triangle, \( T \), in a \( \alpha_j - \beta_j \) plane that represents the set of the potential number of \( j \in T \) contracting firms (or units of labor) acting according to the relay \( R_{\alpha_j,\beta_j}(x_t) \), whereby entry and exit trigger values of aggregate demand are assumed to be firm specific.\(^9\) The Preisach triangle, \( T \), is defined by (see Fig. 1b).\(^{10}\)

\[
T = \{(\alpha_j, \beta_j) : \beta_j \geq \alpha_j \text{ and } \alpha_j \geq \alpha_{\text{min}} \text{ and } \beta_j \leq \beta_{\text{max}}\}
\]

Each point on \( T \) corresponds to a \( R_{\alpha_j,\beta_j}(x_t) \) operator. Given that the level of employment of every active firm is one, the aggregate employment at time \( t \), \( N_t \), is fully described by a weighted integral on elementary hysteresis operators representing the state of activity of individual firms:\(^{11}\)

\[
N_t = \int \int_{T} u(\alpha_j, \beta_j) R_{\alpha_j,\beta_j}(x_t) \, d\alpha_j \, d\beta_j ,
\]

where \( u(\alpha_j, \beta_j) \) is the weight (density) function of the individual firms in \( T \) (with \( \int \int_{T} u(\alpha_j, \beta_j) \, d\alpha_j \, d\beta_j = 1 \)) that describes the relative contribution of each relay to the aggregate hysteresis dynamics.

For cycles of variation in \( x_t \) there is a continuous macroeconomic hysteresis (counter clockwise) loop for aggregate employment as displayed in Fig. 1c. The upward branch, \( mM \), captures the aggregate employment change by increasing aggregate demand. The maximum of the upward branch (point \( M \) - positive saturation) results from the exogenous component of aggregate demand, \( x_t \), increasing from 0 to \( x_{im} \). The aggregate employment changes due to a subsequent aggregate demand decrease from \( x_{im} \) to 0 are captured by the downward branch \( Mm \).\(^{12}\) Every reversal of the direction of \( x_t \), for \( 0 < x_t < x_{im} \), leads to a structural break in the \( x_t - N_t \) relationship represented by a continuous transition between different curves. \( (x_t, N_t) \) moves into the interior of the region bounded by the major loop \( mMm \), implying that transitory changes in \( x_t \) can lead to permanent variations in \( N_t \).

The evolution of the whole system can be illustrated as follows. Consider the hypothetical dynamics of the exogenous component of the aggregate demand, \( x_t \), displayed in Fig. 1a. Let us assume the system starts in its

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\(^6\) Within the assumptions of the model the trigger values for exiting and entering in the market derived from a profit maximizing problem in discrete time with an infinite plan horizon and a discount factor \( \delta \) are \( \alpha_j = w_j + r_j - \delta F_j \) and \( \beta_j = w_j + r_j + \delta H_j \) respectively (see, e.g., [28], and [39] for a complete description of the model).

\(^7\) See [2,3].

\(^8\) The Preisach Model, with the improvements of [37,38] and [36], is one of the most promising phenomenological models used in the description of hysteresis, and is now generally recognized as a fundamental mathematical toolkit in describing a wide range of hysteretic phenomena in quite different areas.

\(^9\) The heterogeneity in the way firms react to aggregate demand shocks arises from specific cost structures including non-convex adjustment costs (that may depend on firms’ age, size, ownership, average work skill level and innovation), and from the way firms deal with uncertainty (see, [43]).

\(^10\) In the Preisach triangle the triggers for upward adjustment, \( \beta_j \), are on the ordinate, and the triggers for downward adjustment, \( \alpha_j \), are on the abscissa.

\(^11\) For a complete explanation of the Preisach model of hysteresis see, e.g., [38], and [39].

\(^12\) Assuming that firms are uniformly distributed in the Preisach triangle, \( T \), the branches of the hysteresis loop are quadratic functions.
minimum input value $x_t = x_{t_0} = 0$. Therefore, $R_{\alpha_j, \beta_j}(x_t) = 0$. If $x_t$ increases to $x_{t_1} > \beta_j$, $R_{\alpha_j, \beta_j}(x_t)$ switch to one. Then, if $x_t$ decreases to say $x_{t_2} < \alpha_j$, corresponding to the area of the Preisach triangle to the right of $\alpha_j = x_{t_2}$, all the relays with $\alpha_j > x_{t_2}$, corresponding to the area of the Preisach triangle below $\beta_j = x_{t_1}$, switch to zero.

The right axis of Fig. 1a is the reference for the $x_t$ dynamics. The left axis of Fig. 1a is the reference for the state of the relay $R_{\alpha_j, \beta_j}(x_t)$.

Changes in $x_t$ towards a local maximum originates the displacement of an imaginary horizontal line upwards from the position $\beta_j = \beta_{\text{min}}$ to the position corresponding to $\beta_j = x_{t_1}$, switching the relays from 0 to 1.

Changes in $x_t$ towards a local minimum originates the displacement of an imaginary vertical line to the left from the position $\alpha_j = x_{t_2}$ to $\alpha_j = x_{t_1}$, switching the relays from 1 to 0.

Figures should be read clockwise.
Adding to the sunk costs, economic uncertainty introduces an additional cost of entering (opportunity cost) that is determined by the scale effect, and the availability of employment that determines its price (input substitution effect).\(^{20}\)

For the empirical detection of hysteresis, a piecewise-linear approximation of the curved Preisach hysteresis loop by means of the play-hysteresis operator is usually adopted.\(^{18}\) The geometrical representation of the linear-play dynamics is illustrated in Fig. 1d. The model contains two steep lines: a ‘downward spurt line’, and an ‘upward spurt line’, along with flatter lines (‘play lines’) connecting the two spurt lines. The horizontal distance between the spurt lines in called the ‘play interval’, representing the employment band of inaction at the aggregate level, and which can be viewed as an indicator of the magnitude of the hysteresis effects. Starting in spurt line, the slope of a linear section changes when a local extremum of the aggregate demand shock, \(x_t\), is reached, and every change in the direction of \(x_t\) forces the aggregate employment, \(N_t\), to traverse a play interval, evolving along a play line. A strong reaction of \(N_t\) only takes place after this interval is surpassed.

Within this framework the memory effect is captured by the difference in the slopes between the play and spurt lines. Let \(\gamma_1\) represent the slope of the flatter lines (play lines), then \((\gamma_1 + \gamma_2)\) is the slope of the steeper lines (spurt lines), where \(\gamma_2\) represents the memory or remanence parameter.

Again for the purpose of illustration, assume \(x_t = x_0\) and \(N_t = N_0 = 0\) (point a in Fig. 1d). An increase in the aggregate demand shock variable to \(x_t(1)\) causes, initially, a weak response of the aggregate employment (along a play line) until a threshold value \(x_t^S\) is surpassed (the system is at point b). This is due to the presence of non-convex adjustment costs interacting with uncertainty. When the \(x_t\) increases sufficiently, i.e., when the threshold value \(x_t^S\) is surpassed, the employment start to respond strongly, increasing along the upward spurt line to \(N_t\). The system follows the sequence \(abc\). A further decrease of \(x_t\) to \(x_t(2)\) causes initially a weak decrease of the aggregate employment (along a play line) until a threshold value \(x_t^{S2}\) is reached (point d). After this point, the aggregate employment responds strongly, decreasing along the downward spurt line (the system follows the sequence \(cde\)). If \(x_t\), increases again to \(x_t(3)\), aggregate employment increases initially weakly along a play line, which was vertically shifted downward until the threshold value \(x_t^{S3}\) is reached (point f). After this point, the aggregate employment responds strongly, increasing along the upward spurt line (the system follows the sequence \(efg\)).

In our model, as \(y_{j,t} = n_{j,t} = K_{j,t}\), the dynamics of the stock of capital, \(K_{j,t}\), matches with the hysteretic dynamics of employment just described. Besides as \(K_{j,t}\) is normalized to one for entering and exiting the market it corresponds to the level of investment, \(I_{j,t}\). In fact, due to the present of sunk cost a firm usually requires a rate of return far in excess the cost of capital to invest. Conversely, if the firm has decided to invest, it will require an operational loss far in excess average variable cost to disinvest (see [44,45], [23], [20,21,24], [19], [11], and [6]).

Adding to the sunk costs, economic uncertainty introduces an additional cost of entering (opportunity cost) that is the value of the option to wait for more information, magnifying hysteresis effects (see [20,22], [44], and [24]).

Figure 2 illustrates the graphical structure of the model to be estimated. \(V = \{N_t, I_t\}\) is the set of the endogenous variables, \(U = \{X_t, W_t, R_t, \epsilon^N_t, \epsilon^I_t\}\) the set of exogenous variables, and \(F = f_N, f_I\) a two equation system that includes an employment equation \(f_N\) and an investment equation \(f_I\).\(^{19}\)

We assume that the demand for employment results from consumers’ demand for final goods and services (that determines a scale effect), and the availability of employment that determines its price (input substitution effect).\(^{20}\)

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\(^{18}\) See [52] for a general description of the model.

\(^{19}\) Here we choose not explain variables like \(X_t\), \(W_t\) and \(R_t\) (as it done in some general equilibrium economic models), keeping the model simple, as we are focusing not on the particular value of the structural parameters but on the hysterical nature of employment and investment and its interaction.

\(^{20}\) see, e.g., equations 2.22 and 3.1 in [31] pp. 30 and 64).
Regarding the investment function, \( f_I \), we consider a parsimonious specification that includes as explanatory variables the aggregate demand, \( X_t \), and the real interest rate, \( R_t \), capturing the opportunity cost of the investment.\(^{21}\) We also add a random disturbance term, \( \epsilon_I \), iid normal.

Due to the interplay of employment and physical capital in the production function it also plausible that there is endogeneity between these variables. Therefore, \( X_t \) has a direct effect on \( N_t \) and an indirect effect mediated by \( I_t \). Likewise, \( X_t \) has a direct effect on \( I_t \) and an indirect effect mediated by \( N_t \) (see Fig. 2). This dynamic is best captured by a simultaneous equation’s framework.

The graph in Fig. 2 represents the qualitative causal relationships. The best functional form of \( f_N \) and \( f_I \), specifically whether the relationship between \( X_t \) and \( N_t \) and between \( X_t \) and \( I_t \) is linear or hysterical is what we intent to uncover.

### 3 Empirical Implementation and Data

The play model can be implemented via a linear switching employment equation with an unknown splitting factor (the play interval) capturing the non-linear play hysteresis effects (see [6,9], and [40,41]).

In this equation, the change in aggregate employment, \( N_t \), induced by a change in aggregate demand, \( X_t \), is divided in a weak reaction along a play line and a strong reaction along a spurt line, for sufficient large changes in \( X_t \):

\[
N_t = \gamma_0 + \gamma_X X_t + \gamma_S S_t(\text{play}) + \gamma_W W_t + \epsilon^N_t \quad (3)
\]

where \( S_t \) (the spurt variable) is a hysteresis transformed input variable that results from the original input series \( X_t \), which accumulates all preceding and present movements on the spurt lines, originating a structural break of the relationship between \( N_t \) and \( X_t \). Since all the small changes inside the play interval (\( \Delta X_t < \text{play} \)) are filtered away, \( S_t \) can be viewed as a filtered explanatory variable.

\(^{21}\) See, e.g., [30], [35], [49], and [6].
The parameter $\gamma_X$ denotes a weak reaction of employment, $N_t$, to small variations in aggregate demand, $X_t$, along the play line, and $(\gamma_X + \gamma_S)$ the strong reaction along the spurt line to sufficiently large changes in $X_t$.\textsuperscript{22} The presence of hysteresis is corroborated if the memory or remanence parameter, $\gamma_S$, is significantly greater than zero.\textsuperscript{23}

The computation of the spurt variable, $S_t$, in a single equation is generated by means of a play algorithm originally described in [26], and a posteriori developed by [27], [8,10], and [39–41]. The innovation here is that we extend the previous algorithm to a two-equation system to estimate the spurt variables, in which the investment function has similar hysteretic structure of the employment function (see Algorithm 1).

In (4) we consider the hysteretic relationship between investment, $I_t$, and aggregate demand, $X_t$, captured by $S^I_t$ in the similar way it was consider for employment.\textsuperscript{24}

$$\begin{align*}
\gamma & N_t = \gamma^N_0 + \gamma^N_X X_t + \gamma^N_S S^N_t + \gamma^N_W W_t + \gamma^N_I I_t + \epsilon^N_t \\
\gamma & I_t = \gamma^I_0 + \gamma^I_X X_t + \gamma^I_S S^I_t + \gamma^I_R R_t + \gamma^I_N N_t + \epsilon^I_t
\end{align*}$$

\textsuperscript{(4)}

The coefficients are identified with sub and super script indexes. The subscript identifies the associated variable. The super script $N$ and $I$ identifies whether the coefficient belongs to equation $f_N$ and $f_I$ respectively.

\begin{algorithm}
\textbf{Algorithm 1:} Linear switching employment and investment simultaneous equation estimation.

\textbf{Data:} $N_t, I_t, X_t, W_t, R_t$

\textbf{Result:} $play^N, play^I, S^N_t, S^I_t$, parameter estimates & stats

\textbf{Generation:}

- Generate grid of admissible values of $play^N$
- Generate grid of admissible values of $play^I$

\textbf{Building:}

- Built $S^N_t (play^N)$ for each admissible value of $play^N$ from the grid;
- Built $S^I_t (play^I)$ for each admissible value of $play^I$ from the grid;

(based on the algorithm in [8] and implemented in MATLAB by [41]);

\textbf{Estimation:}

- Estimate system (4) by the seemingly unrelated regression method (SUR) for each combination of $(S^N_t, S^I_t)$;
- Compute the average of the $R^2$ of $f_N$ and $f_I$

\textbf{Conclusion:}

- Chose the combination $(S^N_t, S^I_t)$ that maximizes the average of the $R^2$ of $f_N$ and $f_I$;
- Get $play^N, play^I, S^N_t, S^I_t$, parameter estimates & stats;

\end{algorithm}

In our empirical application we use quarterly data for Portugal retrieve from EUROSTAT. The series are seasonally adjusted, and the data covers the period from January 2000 to December 2019.\textsuperscript{25} This data set includes a number

\textsuperscript{22} See [9], [39], [40,41], and [6] for additional details.

\textsuperscript{23} Eq.(3) is based on a piecewise linear relationship where the segments (the plays lines and the spurt lines) are connected continuously by knots (for example in Fig. 1d the knots are points $b, c, d, e$, and $f$ when the input follows the path $x_{b_1} \mapsto x_{b_2} \mapsto x_{c_2} \mapsto x_{c_1}$). The position of the knots are a priori unknown, because they depend on the width of the play interval, which has to be estimated, and on the position of the play line that is determined by the past path of the aggregate demand shock $x_t$. Since the adjacent play and spurs segments are joined, the regression is a special case of a switching equation, and similar to a linear spline function (see [47,48]). In this case, the OLS estimator is asymptotically unbiased (i.e. consistent) and asymptotically normal distributed under the standard regression model assumptions (see [34], [33], and [48]).

\textsuperscript{24} In the employment equation, although real wages could also be a source of hysteresis, our aim is to test the existence of hysteresis caused by aggregate demand shocks. This is the reason why real wages enters as a non-hysteretic explanatory variable in the employment equation. Likewise, in the investment equation, although the interest rate can have an impact on investment through hysteresis mechanisms (see [19], [29], and [6]), we are focusing on the hysteretic effects caused by aggregate demand shocks. Therefore, $x_t$ is the common variable in both equations that operates through hysteresis mechanisms.

\textsuperscript{25} The model is estimated with the series in logarithms.
of macroeconomics shocks, comprising the recent financial crises (whose effects were special acute in Portugal), and for that reason is adequate to analyze the hysteretic joint dynamics of employment and investment. Portugal is also a good case study as the country rigidity of employment protection legislation (that is a source of employment hysteresis) ranks above the OECD average.

In equation $f_N$, the aggregate employment, $N_t$, is measured by the index of the number of employees in industry. The real production in industry adjusted by the number of working days, $X_t$, is used as the proxy of aggregate demand. Real wages, $W_t$, are measured by the index of gross wages in industry deflated by the general consumer price index.

In equation $f_I$, aggregate investment, $I_t$, is measure by the index of the gross fixed capital formation for the whole economy. We use the Portugal-Germany 10 year bond spread as a proxy for real interest rate, $R_t$.

4 Results

Table 1 shows the results of the estimation of the linear version of system (4), i.e., without the hysteresis variables ($S_{Nt}$ and $S_{It}$) using the seemingly unrelated regression (SUR) method [53]. The residual diagnostic test for normality (Jaque-Bera) is also reported. We do not reject the null hypothesis (of normality of the residuals) for $f_N$ and $f_I$. The signs of the estimated coefficients are as expected. In the employment equation the proxy of aggregate demand, $X_t$, has a positive impact on aggregate employment, and the real wage has a negative impact. In the investment equation, $X_t$, is not significant, and the proxy of the interest rate, $R_t$, has a negative impact on aggregate investment. Base on the significance of The estimation also reveals a positive interaction between aggregate employment and aggregate investment.

In a second step we estimate system (4) with the hysteresis variables ($S_{Nt}$ and $S_{It}$). A grid search over different values of the play interval for both employment and investment is used to find the combination ($\text{play}_N$, $\text{play}_I$) with the highest average of the $R^2$ of $f_N$ and $f_I$. The absolute maximum is achieved for ($\text{play}_N = 0.108$, $\text{play}_I = 0.082$), which implies substantial hysteresis effects for both employment and investment. These values form the basis to estimate the hysteresis variables $S_{Nt}$ and $S_{It}$ according to the algorithm. Both series are displayed in Fig. 3, together with the original input variable $X_t$. The spurt variable for employment, computed for an estimated band of inaction of 0.108, does not change in 83% of the quarters. These results are in line with previous empirical studies that shows that firms do not permanently adjust the number of employees to accommodate demand shocks On the contrary their reaction is discontinuous and lumpy, with large periods of inaction interrupted by episodes of large adjustment (see e.g. [7–9,11,32,39,51]). A similar behavior is documented for investment (see e.g. [6,11,23,24,45]). Here, the spurt variable for investment, computed for an estimated band of inaction of 0.082, does not change in 78% of the quarters.

Figure 4 plots the hysteresis loop - the hysteresis variable (spurt) computed from real production in industry against the original variable. When real production varies the hysteresis variable exhibits multibranch nonlinearity that characterizes the hysterical dynamics.

Table 2 shows the estimation results of system (4) with the two hysteretic regressions.

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26 Gross fixed capital formation for the industrial sector at quarterly frequency is not available. Besides using the industrial production as a proxy of aggregate demand in an equation for aggregate investment reduces the problem of endogeneity usually present when real GDP is used as the proxy of aggregate demand in $f_I$.

27 The 10 year bond spread anchors the relative cost of borrowing face by firms. Besides, it avoids the problem of endogeneity associated to the joint determination of the quantity and the interest rate of credit contracts for investment when this last interest rate is included in $f_I$.

28 This method provides an efficient estimation of a system of equations in the case where disturbances of the system are contemporaneously correlated, as it is the case where equations have endogeneity problems, or are serially correlated, and allows the two dependent variables to have different sets of explanatory variables. The SUR method estimates the parameters of the 2 equations simultaneously, implying that the parameters of each individual equation consider the information provided by the other equation. In so being, the regression coefficient estimators are at least asymptotically more efficient than those obtained by an equation-by-equation application of least squares (see [53]: p. 348).
Table 1  SUR estimates of the Linear Version of system (4)

\[ f_N : \quad N_t = \gamma_0^N + \gamma_X^N X_t + \gamma_S^N S^N + \gamma_W^N W_t + \gamma_I^N I_t + \epsilon_t^N \]

| \( \gamma_0^N \)  | \( \gamma_X^N \)  | \( \gamma_S^N \)  | \( \gamma_W^N \)  | \( \gamma_I^N \)  | \( \tilde{R}^2 \)  | DW  | JB  |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 1.933***          | 0.388***          | -0.403***         | 0.592***          | 0.824             | 0.132             | 0.1101            | (p-value)         |
| (3.003)           | (3.332)           | (-3.185)          | (9.027)           |                   |                   |                   |                   |

\[ f_I : \quad I_t = \gamma_0^I + \gamma_X^I X_t + \gamma_S^I S^I + \gamma_R^I R_t + \gamma_N^I N_t + \epsilon_t^I \]

| \( \gamma_0^I \)  | \( \gamma_X^I \)  | \( \gamma_S^I \)  | \( \gamma_R^I \)  | \( \gamma_N^I \)  | \( \tilde{R}^2 \)  | DW  | JB  |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| -0.573            | 0.091             | -0.010***         | 1.055***          | 0.775             | 0.108             | 0.1324            | (p-value)         |
| (-0.902)          | (0.431)           | (-3.185)          | (8.310)           |                   |                   |                   |                   |

* \( t \)-statistics in brackets.
** ***, **, and * indicate significance at 1, 5 and 10 percent, respectively.
** Source: Authors' calculations

Fig. 3  Estimated Hysteresis Variables for both Employment and Investment
Concerning the employment equation, $f_N$, the hysteresis variable, $S^N_t$, is significant at 1%, while the original forcing variable, $X_t$, turns non-significant. This implies that the play lines in Fig. 1d are in fact horizontal. That means that employment only reacts to significant large changes in aggregate demand, which corroborates the hypothesis of hysteresis. The coefficient associated to investment remains significant at 1%, but the coefficient associated to real wages is now non-significant.

In the investment equation, $f_I$, the hysteresis variable, $S^I_t$, is significant at 5%, while the original forcing variable, $x_t$, remains non-significant. This result indicates that the presence of hysteresis is also corroborated for the case of investment. The Portugal-Germany 10 year bond spread use as the proxy for real interest rate, $R_t$, and the aggregate employment remain significant and with the expected sign.
Table 2  SUR estimates of system (4)

\[
f_N : N_t = \gamma_0^N + \gamma_X^N X_t + \gamma_S^N S_t^N + \gamma_W^N W_t + \gamma_I^N I_t + \epsilon_t^N
\]

| \(\gamma_0^N\) | \(\gamma_X^N\) | \(\gamma_S^N\) | \(\gamma_W^N\) | \(\gamma_I^N\) | \(R^2\) | DW | JB (p-value) |
|----------------|----------------|----------------|----------------|----------------|-------|------|--------------|
| 5.425***       | -0.178         | 1.209***       | -0.113         | 0.173***       | 0.906 | 0.133| 0.751       |
| (7.463)        | (-1.222)       | (7.019)        | (-1.052)       | (2.789)        |       |      |              |

\[
f_I : I_t = \gamma_0^I + \gamma_X^I X_t + \gamma_S^I S_t^I + \gamma_R^I R_t + \gamma_N^I N_t + \epsilon_t^I
\]

| \(\gamma_0^I\) | \(\gamma_X^I\) | \(\gamma_S^I\) | \(\gamma_R^I\) | \(\gamma_N^I\) | \(R^2\) | DW | JB (p-value) |
|----------------|----------------|----------------|----------------|----------------|-------|------|--------------|
| 3.685*         | -0.316         | 0.975**        | -0.013***      | 0.586***       | 0.808 | 0.141| 0.282       |
| (0.059)        | (-0.896)       | (2.380)        | (-2.968)       | (3.661)        |       |      |              |

* indicates significance at 1 percent. ** indicates significance at 5 percent. *** indicates significance at 10 percent.

Source: Authors’ calculations

5 Conclusion

This work analyzes the presence of hysteresis effects in the joint dynamics of employment and investment by means of the linear play-hysteresis model in a simultaneous equation framework.

The empirical application based on quarterly data for Portugal reveals significance hysteresis effects for both employment and investment. The estimated band of inaction used as a proxy of the magnitude of hysteresis is however larger for aggregate employment. Compared with previous analysis that focuses only on employment hysteresis, here we document a more complex and interrelated hysteretic dynamics between aggregate employment and investment. The consequence is that measures towards the deregulation of the labor market could not be effective to reduce employment hysteresis.

These results have significant consequences for macroeconomic policy. As the effects of severe negative economic shocks (even if temporary) may not be easily reversed, fiscal and monetary policy should be conducted in a more preventive way and respond in a timely, massive and sustained manner.

Concerning the response of the Covid-19 pandemic, this work find support to the idea that “now is not the time to doubt or err on the side of caution when it comes to expansionary economic policies” ([16], p. 39).

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