Updating constraints on $f(T)$ teleparallel cosmology and the consistency with Big Bang Nucleosynthesis

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Abstract. We focus on viable $f(T)$ teleparallel cosmological models, namely power law, exponential and square-root exponential, carrying out a detailed study of their evolution at all scales. Indeed, these models were extensively analysed in the light of late time measurements, while it is possible to find only upper limits looking at the very early time behavior, i.e. satisfying the Big Bang Nucleosynthesis (BBN) data on primordial abundance of $^4$He. Starting from these indications, we perform our analysis considering both background and linear perturbations evolution and constrain, beyond the standard six cosmological parameters, the free parameters of $f(T)$ models in both cases whether the BBN consistency relation is considered or not. We use a combination of Cosmic Microwave Background, Baryon Acoustic Oscillation, Supernovae Ia and galaxy clustering measurements, and find that very narrow constraints on the free parameters of specific $f(T)$ cosmology can be obtained, beyond any previous precision. While no degeneration is found between the helium fraction, $Y_P$, and the free parameter of $f(T)$, we note that these models constrain the current Hubble parameter, $H_0$, higher then the standard model one, fully compatible with the Riess et al. measurement in the case of power law $f(T)$ model. Moreover, the free parameters are constrained at non-zero values in more than 3-$\sigma$, showing a preference of the observations for extended gravity models.
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1 Introduction

The Standard Cosmological Model, the so called $\Lambda$CDM, provides a reliable description of the Universe from some seconds after the big bang until the present epoch, under the assumptions that gravity is described by Einstein’s General Relativity (GR), the spatial sections of the Universe, at constant cosmological time, are homogeneous and isotropic, and dark matter and dark energy components exist. However, we know that the $\Lambda$CDM model is incomplete. For example, there is no final evidence of dark matter and dark energy, nor explication for matter-antimatter asymmetry or unification of gravity and the other interactions at quantum level (see Refs [1–6] and references therein). Also, new physics beyond the Standard Model has been invoked to describe the increasingly precise data of the latest generation, since several tensions have emerged between data at different scales (for a detailed discussion see Refs. [7–14] and references therein).

In this context, several assumptions have been re-considered, including the possibility of modifications and extensions of GR in order to fix the dark energy and dark matter issues due to lack of evidences of these elements on a fundamental level.

The paradigm of considering different theories of gravity, with respect to GR, comes from the fact that Einstein’s theory is proved to be not sufficient to describe dynamics of gravitational field at ultraviolet and infrared scales. According to this statement, several effective models have been proposed towards quantum gravity and cosmology with the aim to recover the agreement with the experiments and observations reached by GR but enlarging also the number of phenomena to be described at different scales and energies [15]. The debate is not only related to the possibility of adding new contributions to the Hilbert-Einstein action, like in the case of $f(R)$ gravity and analogue theories, but also to identify the correct variables describing the gravitational field. Starting from GR, any metric theory assumes the validity of Equivalence Principle in its various formulations [1]. This assumption leads to the coincidence of the geodesic and causal structure and fixes the connection which as to be Levi-Civita.

Nevertheless Einstein himself recognized that such an approach could be enlarged and improved if alternative descriptions of gravitational dynamics were considered. In particular, if tetrads, instead of metric, describe the gravitational field, dynamics is given by torsion instead of curvature and causal structure can be different from geodesic structure. In this picture, the Equivalence Principle is not the foundation of gravitational field and affinities assumes a fundamental role. These considerations led to the teleparallel formulation of GR
which, at field equations level, is equivalent to GR giving the so called Teleparallel Equivalent General Relativity (TEGR). In this perspective, also extensions of TEGR reveal interesting and then, as the straightforward extension of curvature gravity is \( f(R) \) (where \( R \) is the Ricci scalar), now \( f(T) \) extends TEGR (being \( T \) the torsion scalar).

One of the main goals to develop these alternative approach is to select self-consistent cosmological models capable of giving a realistic picture of cosmic history (see [16] for a detailed discussion). The goal is to coherently connect early (inflation) and late universe (dark energy), passing for large scale structure formation. In this program cosmography [17, 18] and Big Bang Nucleosynthesis (BBN) [19] could play a main role. In particular, BBN offers one of the most powerful methods to test the validity of cosmological models around the MeV energy scale. The precise measure of the chemical abundances of the primordial elements of BBN is one of the main efforts of the modern cosmology [20–23]. Indeed, such abundances of hydrogen, helium, lithium and deuterium are an important test for any cosmological model, being extremely sensitive to the physics of the early universe. Also, direct astrophysical observations allow to extrapolate primordial abundance. By the emission lines of nearby H\(_{II}\) regions in metal-poor star forming galaxies, the mass fraction of \(^4\)He (\(Y_P\)) has been sensitively estimated[24, 25], while the primordial \(^7\)Li abundance is determined by the atmospheres of very metal-poor stars [26, 27]. Finally, the primordial deuterium abundance can be measured using the absorption line of gas clouds [28–32]. Such a measurements allow a high precision estimate of the baryon fraction density, and has been found a concordance between the Aver(2015) analysis [25] and the Planck(2018) derived ones [33]. However, also several tensions emerged, and they are quantified in more then 2\(\sigma\), when \(\Omega_b\) is derived by different model assumption of Ref.[24] or deuterium abundance [34] (see also Ref.[33] for an updated discussion of current results).

Although efforts are spent to reconcile these measurements [29, 34], other possible cosmological models can be explored to test if a natural agreement can be obtained between the \(\Omega_b\) value inferred from the BBN and the derived one from the Cosmic Microwave Background (CMB). For example, it is possible to bring closer the BBN and CMB predictions of the baryon density today considering extensions to the Standard Model, such as a change in the expansion rate, parameterized by the effective number of relativistic degrees of freedom, \(N_{\text{eff}}\) [8, 9, 12, 35]. Since both helium abundance and \(N_{\text{eff}}\) affect the CMB damping tail, they are partially degenerate. On the other hand, a phenomenological modeling of the current observed accelerated expansion of the universe should be ideally embedded into a more fundamental framework, i.e. deduced from first principles. It is therefore timely to test fundamental theories with a study involving all scales, from the first seconds of the universe (i.e. using BBN) to today observed accelerated expansion.

In this work, we focus on teleparallel gravity [16] and trace the observational prediction of different forms \(f(T)\) using a Boltzmann numerical resolution code. Previous studies, analysing the high temperatures characterizing the primordial Universe, constrained with upper bounds the \(f(T)\) cosmology [19], and it is timely to improve such an analysis using the wide range of available data at all scales. In this perspective, the feasibility of a teleparallel description of gravity can be realistically tested. In fact, until now, most efforts have been devoted to match late accelerated behavior by \(f(T)\) gravity but the attempt to reproduce the whole cosmic history in a teleparallel picture has to be more pursued in order to finally compare metric and tetrad descriptions. Here, in particular, we explore whether by relaxing the consistency of the BBN, it is possible that these theories are in agreement with the estimates of primordial abundances. This can be an important consistency test.
The paper is organised as follows. In Section 2, we introduce TEGR and its \( f(T) \) extension. We will derive the related background cosmology and the evolution of primordial perturbations which we will use for our analysis. In Section 3, we provide an overview of the specific models we are going to analyse, showing observational predictions and giving a state of the art of current analyses. Details of the analysis method are reported in Section 4, also indicating the data set we use to constrain the models parameters. Finally, in Section 5, we discuss the results and draw our conclusions.

2 \( f(T) \) gravity and cosmology

Let us briefly review the main features of TEGR and \( f(T) \) teleparallel gravity. First, we introduce the vierbein fields \( e_i(x^\mu), \ i = 0, 1, 2, 3 \). They forms an orthonormal basis in the tangent space at each point \( x^\mu \) of the manifold, i.e. \( e_i \cdot e_j = \eta_{ij} \), with \( \eta_{ij} = \text{diag}(1, -1, -1, -1) \) the Minkowsky metric. Denoting with \( e^i_\mu \), with \( \mu = 0, 1, 2, 3 \) the components of the vectors \( e_i \) in a coordinate basis \( \partial_\mu \), one can write

\[
e_i = e^i_\mu \partial_\mu (\text{the Latin indices refer to the tangent space, the Greek indices to the coordinates on the manifold}).
\]

The TEGR models are characterized by the fact that the curvatureless Weitzenböck connection is adopted (let us recall that, in General Relativity, one uses the torsion-less Levi-Civita connection). This allows to define the non-null torsion tensor

\[
T^\lambda_{\mu\nu} = \hat{\Gamma}^\lambda_{\nu\mu} - \hat{\Gamma}^\lambda_{\mu\nu} = e^i_\lambda (\partial_\mu e^j_\nu - \partial_\nu e^j_\mu), \tag{2.1}
\]

The action we are going to consider is of the form

\[
I = \frac{1}{16\pi G} \int d^4x \left[ T + f(T) \right] + I_m, \tag{2.2}
\]

where \( f(T) \) is a generic function of the torsion scalar \( T \), \( I_m \) is the action of matter fields, and \( e = \det(e^i_\mu) = \sqrt{-g} \) is the metric determinant. Explicitly, the torsion scalar \( T \) reads

\[
T = S^{\mu\nu}_{\rho} T^\rho_{\mu\nu}. \tag{2.3}
\]

\[
S^{\mu\nu}_{\rho} = \frac{1}{2} (K^{\mu\nu}_{\rho} + \delta^{\mu}_{\rho} T^{\theta\nu}_{\theta} - \delta^{\nu}_{\rho} T^{\theta\mu}_{\theta}), \tag{2.4}
\]

\[
K^{\mu\nu}_{\rho} = -\frac{1}{2} (T^{\mu\nu}_{\rho} - T^{\nu\mu}_{\rho} - T^{\rho\mu}_{\nu}), \tag{2.5}
\]

with \( K^{\mu\nu}_{\rho} \) the contorsion tensor which gives the difference between Weitzenböck and Levi-Civita connections.

The variation with respect to the vierbein gives the field equations [16]

\[
e^{-1} \partial_\mu (ee^i_\rho S^{\mu\nu}_{\rho}) [1 + f'] - e^i_\lambda T^{\rho}_{\mu\lambda} S^{\mu\nu}_{\rho} [1 + f'] + e^i_\lambda S^{\mu\nu}_{\rho} (\partial_\mu T) f'' + \frac{1}{4} e^\nu_i [T + f] = 4\pi G e^\nu_i \Theta^\nu_{\rho}, \tag{2.6}
\]

where we defined \( f' \equiv df/dT \) and \( S^{\mu\nu}_{\rho} = e^\rho_i S^{\mu\nu}_{\rho} \), while \( \Theta_{\mu\nu} \) is the energy-momentum tensor of perfect fluid matter.

For a flat Friedmann-Lemaître-Robertson-Walker (FLRW) background, the metric is

\[
ds^2 = dt^2 - a^2(t) \delta_{ij} dx^i dx^j, \tag{2.7}
\]
where $a(t)$ is the scale factor. The corresponding vierbien fields are $e^a_{\mu} = \text{diag}(1, a, a, a)$.

The latter and Eq. (2.3) yield the relation between the torsion $T$ and the Hubble parameter $T = -6H^2$, where $H = \frac{\ddot{a}}{a}$. Assuming that matter sector is described by a perfect fluid with energy density $\rho$ and pressure $p$, the field Eqs. (2.6) gives the cosmological equations

$$12H^2[1 + f'] + [T + f] = 16\pi G \rho, \quad (2.8)$$

$$48H^2f''\dot{H} - (1 + f')[12H^2 + 4\dot{H}] - (T - f) = 16\pi G p. \quad (2.9)$$

Moreover, the equations are closed with the equation of continuity for the matter sector $\dot{\rho} + 3H(\rho + p) = 0$. Eqs. (2.8) and (2.9) can be rewritten in terms of the effective energy density $\rho_T$ and pressure $p_T$ arising from $f(T)$

$$H^2 = \frac{8\pi G}{3}(\rho + \rho_T), \quad (2.10)$$

$$2\dot{H} + 3H^2 = -\frac{8\pi G}{3}(p + p_T), \quad (2.11)$$

where

$$\rho_T = \frac{3}{8\pi G} \left[ \frac{1}{3} \frac{T}{f'} - \frac{f}{6} \right], \quad (2.12)$$

$$p_T = \frac{1}{16\pi G} \frac{f - T\dot{f'} + 2T^2f''}{1 + \dot{f'} + 2Tf''}, \quad (2.13)$$

and define the effective torsion equation-of-state

$$\omega_T \equiv \frac{p_T}{\rho_T} = -\frac{f - T\dot{f'} + 2T^2f''}{(1 + \dot{f'} + 2Tf'')(f - 2Tf')}. \quad (2.14)$$

These effective models are hence responsible for the accelerated phases of the early or/and late Universe [16].

In order to perform our analysis, we rewrite the first FLRW equation, Eq.(2.10), making explicit the form of the torsional energy density [36–38]

$$\frac{H(a)^2}{H_0} = E(a)^2 = \left[ \Omega_{m0}a^{-3} + \Omega_{r0}a^{-4} + \frac{1}{T_0}[f - 2Tf'] \right] \quad (2.15)$$

where we define $\Omega_{i0} = \frac{8\pi G\rho_i}{3H_0^2}$, and consider the relation $T = -6H^2$. The above background evolution recovers the standard model for $\frac{1}{T_0}[f - 2Tf'] \rightarrow \Omega_{\Lambda}$. Hereafter we define such a torsional contribution as

$$y_T(a, \xi) \equiv \frac{1}{T_0}[f - 2Tf'], \quad (2.16)$$

with $\xi$ the free parameters of the $f(T)$ parameterization.

Also, we consider the following perturbation equations for density contrast and velocity divergence in the synchronous gauge, i.e. fixing the torsion fluid (zero acceleration) frame, valid for a perfect fluid [39–42]
\[ \dot{\delta}_i + 3H(c_{s,\text{eff}}^2 - \omega_i) \left[ \delta_i + 3H(1 + \omega_i) \frac{v_i}{k} \right] + (1 + \omega_i)kv_i + 3H\dot{w}_iv_i/k = -3(1 + w_i)\ddot{h}. \] (2.17)

\[ \dot{v}_i + H(1 - 3c_{s,\text{eff}}^2)v_i = k\delta_ic_{s,\text{eff}}^2 \frac{1 + \omega_i}{1 + \omega_i}. \] (2.18)

The derivatives are with respect the conformal time, \( H \) is the conformal Hubble parameter, \( v_i \) is the velocity, \( \omega_i = p_i/\rho_i \), the \( c_{s,\text{eff}}^2 \) is the effective sound speed in the rest frame of the \( i \)th fluid. Furthermore, in order to avoid the crossing instability problem, we use the Parameterized Post-Friedmann (PPF) approach, already implemented in the CAMB code [43, 44], the Boltzmann solver code that we use for compute the evolution of linear perturbations.

### 3 Specific \( f(T) \) models

We choose to analyze three \( f(T) \) forms well known in literature for being viable models, i.e. passing the basic observational tests [19, 36, 37]. We introduce their forms and derive their background evolution. Also, we show the theoretical observational predictions of temperature anisotropy power spectrum and the EE-mode correlation spectrum for each of them, and we summarize the current state of the art of the constraint of their free parameters.

- The first scenario is the power-law model (hereafter \( f_1 \text{CDM} \)) with

\[ f(T) = \beta (-T)^b, \] (3.1)

that recovers the GR form, \( T + f(T) = T - 2\Lambda \), for \( b = 0 \) and \( \Lambda = -\beta/2 \) [45]. Substituting this \( f(T) \) form into the first Friedmann equation at present epoch, we obtain the relation between the two parameters, \( \beta = (6H_0^2)^{1-b} \Omega_{T0}^b \), with \( \Omega_{T0} = 1 - \Omega_{m0} - \Omega_{r0} \).

The \( y_T(a,b) \), in the background evolution of Eq.(2.15), reads as \( y_T(a,b) = \Omega_{T0}E(a)^{2b} \) [38] that reduces to \( \Lambda \text{CDM} \) cosmology for \( b = 0 \), while, for \( b = 1/2 \), it gives rise to the Dvali-Gabadadze-Porrati (DGP) model [46]. At the same time, we can write the EoS Eq.(2.14) as

\[ \omega_T = \frac{b - 1}{1 - b\Omega_{T0}E(a)^{2(b-1)}} \] (3.2)

that reduces to a constant value \( \omega_T = -1 \) for \( b = 0 \). Its behavior is shown in Fig.(1), top panels, with red lines. In particular, we assume the values \( b = 0.1 \), solid line, and \( b = 0.01 \), dashed line. We can see that, in both cases, the today EoS value converges to values close to \( \omega_{T0} = -1 \), and depending on \( b \), it can assume slightly (negligible) higher values, with a variation up to \( 10^{-1} \) at small scales. We also note that \( f_1 \text{CDM} \) is the model with the behavior that most differs from the others, both in \( \omega_T \) and in \( H(a) \) evolution.

Previous results show that, using only BBN data, is possible to put an upper-limit as \( b < 0.94 \) [19], while using large scale data, it is possible to constrain \( b = 0.033^{+0.043}_{-0.035} \) by Cosmic Chronometers (CC), and \( b = 0.051^{+0.025}_{-0.019} \) when also SNe Ia and Baryonic Acoustic Oscillations (BAO) are considered [36]. The model was tested also using
Figure 1. Top: behaviour of EoS, \( \omega_T \), of Eq.(2.14) for the \( f_1 \text{CDM} \) (red), \( f_2 \text{CDM} \) (blue) and \( f_3 \text{CDM} \) (green) models assuming \( \Omega_T = 0.7 \) and \( H_0 = 70 \) and also the model free parameter at values 0.1 (solid line, left panel) and 0.01 (dashed line, right panel). Note the different scale between the two plots. Middle: Background evolution for the three models for the same choice of colors and values above, in the left panel considering \( f(T) \) parameter at values 0.1 and in the right panel for 0.01. Bottom: Background evolution for the three models in the scale factor range [0.49 - 0.5]

measurements from quasar absorption lines and radio quasars [47, 48], and also several large scale data combinations [49, 50]. Finally, including CMB by Planck (2015) data, joined with BAO and \( H_0 \) measurements, the most stringent constraint is obtained, \( \Delta = 0.005 \pm 0.002 \) [37].

Using the approach described in the previous Section, we draw the theoretical observational predictions of the temperature anisotropy and the EE correlation power spectra for this model in the top panel of Fig.(2), assuming several values for the free \( f(T) \) parameter. We can see that higher \( \Delta \) means a shift of the spectra to higher multipoles, which implies, among other things, a degeneration of this parameter with the curvature of the Universe and the current expansion. Note that our observational predictions are
fully in agreement with the ones obtained in Newtonian gauge choice [37]. Furthermore, we draw (dotted line) the case without considering the evolution of linear perturbations, i.e. calculating only the background evolution, and we see that the power at low multipoles of TT spectra is particularly affected.

It is important to remark that the \( f(T) \) power law models are selected by the existence of Noether symmetries as shown in [52]. This result can be seen as a criterion to select physical models [53, 54] which allows to reduce and, eventually, integrate the equations of motion. In particular, it is possible to find exact cosmological solutions for the form \( f_0 T^n \) which lead to the background evolution of the \( \rho_T \) density as \( a(t) = a_0 t^{2b/3} \) and \( H(t) = \frac{a}{t} = \frac{2b}{3t} \). In this case, the background evolution for the total density reads as:

\[
H(a)^2 = H_0^2 \left[ \Omega_{m0} a^{-3} + \Omega_r a^{-4} + \Omega_{T0} a^{-3b} \right]
\]  

Let us stress that this Noether solution is calculated for an action of the form \( A_T = \frac{1}{16\pi G} \int d^4 x e f(T) \).

- The second scenario is the square-root-exponential (hereafter \( f_2 \)CDM) also called \textit{Linder model} [55], with

\[
f(T) = \alpha T_0 (1 - e^{-p \sqrt{T/T_0}})
\]

where the relation between the two parameters is \( \alpha = \frac{\Omega_{T0}}{1 - \left( 1 + p \right) e^{-p}} \). The first Friedmann equation leads to \( y(a, p) = \Omega_{T0} \left[ \frac{1 - (1 + p E(a)) e^{-p E(a)}}{1 - (1 + p) e^{-p}} \right] \) [38], that reduces to \( \Lambda \)CDM cosmology for \( p \rightarrow +\infty \). The EoS for this model is

\[
\omega_T = -\frac{e^{p E(a)}(e^p - 1 - p) \left[ 2(e^{p E(a)} - 1) - p E(a)(p E(a) + 2) \right]}{(e^{p E(a)} - 1 - p E(a)) \left[ 2e^{p E(a)}(e^p - 1 - p) + \Omega_{T0} e^p p^2 \right]}
\]  

and it is drawn with blue lines in Fig.(1). Generally, instead of the parameter \( p \), its inverse is used, that is \( b \equiv 1/p \). This is because the limit \( p \rightarrow +\infty \) is equivalent to \( b \equiv 1/p \rightarrow 0^+ \), and the latter limit is considered more proper to be treated in the analyses. Previous works constrained \( b = 0.111_{-0.110}^{+0.032} \), using CC data, while the joint analysis \( CC + SNe Ia + BAO \) allows for \( b = 0.132_{-0.130}^{+0.043} \) [36] while only BBN data cannot impose constraints on the parameter value [19]. As in the previous case, we see that the use of the CMB likelihood can significantly increase the precision on the constraint of \( b \): in this case, an order of magnitude of \( 10^{-5} \) is expected.

We show the prediction for the TT and EE spectra in middle panels of Fig.(2), assuming several values for the free parameter \( b \). Also in this case, we can see a shift of the spectra to higher multipoles for increasing values of \( b \).

- The last scenario we analyze is the the exponential form (hereafter \( f_3 \)CDM) [56, 57]

\[
f(T) = \alpha T_0 (1 - e^{-p T/T_0})
\]

with \( \alpha = \frac{\Omega_{m0}}{1 - (1 + 2p) e^{-p}} \). The background evolution can be written as [38]

\[
y(a, p) = \Omega_{T0} \frac{1}{1 - (1 + 2p) e^{-p}} \left[ 1 - (1 + 2p E(a)^2) e^{-p E(a)^2} \right]
\]
which reduces to ΛCDM cosmology for $p \to +\infty$ (or $b \equiv 1/p \to 0^+$). At the same time, the EoS reads as

$$\omega_T = -\frac{e^{pE(a)^2}(e^p - 1 - 2pE(a)^2) \left[(e^{pE(a)^2} - 1 - pE(a)^2(1 + 2pE(a)^2)) \right]}{(e^{pE(a)^2} - 1 - 2pE(a)^2) \left[(e^p - 1 - 2p) - p\Omega_T e^p(1 - 2pE(a)^2) \right]} \tag{3.7}$$

and it is plotted in Fig.(1) with green lines. We note that $f_2$CDM and $f_3$CDM models show similar behaviours, indicating that the presence of the root in exponent of the exponential function does not give any observable difference in the $H(a)$ evolution, for the range of values we are considering. At the same time, looking for the TT and EE spectra predictions (shown in bottom panels of Fig.(2)) we note that a precision of $10^{-8}$ is required on the $b$ parameter to describe the observations, unlike in case $f_2$CDM.

The current bounds on this model constrain $b = 0.106^{+0.006}_{-0.006}$ using CC data, while the joint analysis $CC + SNeIa + BAO$ allows for $b = 0.090^{+0.041}_{-0.080}$ [36], and also in this case, the BBN data cannot impose constraints on the parameter value [19]. These estimates are far from what is required by the TT spectrum to describe CMB observations, we can infer that analysis with the full CMB likelihood can significantly improve the constraint on this model.

Let us now modify the CAMB code [43, 44] to include changes to the background and perturbations evolution for each model, and use the CosmoMC package (where CAMB is already implemented) to perform a Monte Carlo Markov chain exploration of the parameters space [58].

4 Analysis Method

In our analysis, we consider the minimal ΛCDM model as the reference model, with the usual set of cosmological parameters: the baryon density, $\Omega_b h^2$, the cold dark matter density, $\Omega_c h^2$, the ratio between the sound horizon and the angular diameter distance at decoupling, $\theta$, the optical depth, $\tau$, the primordial scalar amplitude, $A_s$, and the primordial spectral index $n_s$. For each $f(T)$ model, we consider also one more free parameter given by the specific $f(T)$ form, by modifying the CAMB code to reflect the models described in the previous Section.

Also, we consider both the cases where the BBN consistency is considered or not. In the first case, the primordial helium fraction value is derived from the BBN consistency relation as a function of the baryon and radiation densities, and we use the PArthEnoPE fitting table\(^1\) to calculate such a primordial abundances of helium and deuterium. We refer to this case with “$f_i$CDM BBN Consistency”. Instead, in the second case, the helium fraction is considered as a free parameter of the model, and we refer to this case with “$f_i$CDM +$Y_p$”. This choice to treat $Y_p$ as a model parameter, and not derived from the BBN consistency relation, has been recently explored in the literature to resolve the so-called $H_0$ tension [11, 59–61]. Indeed, an higher radiation energy density, i.e. an higher $Y_p$ value, imply a larger expansion rate of the Universe [62, 63]. In this work, we choose to explore a free $Y_p$ to study if $f(T)$ gravity spontaneously recovers the primordial abundances predicted by the theory, and if the free parameter of $f(T)$ models shows any degeneration with the BBN abundance. It is

\(^1\)PArthEnoPE website: http://parthenope.na.infn.it/
Figure 2. TT and EE correlation CMB anisotropy power spectra for $f_1\text{CDM}$ (top panels), $f_2\text{CDM}$ (middle panels) and $f_3\text{CDM}$ (bottom panels), using several values of the $f(T)$ free parameter, $b$. For each model we draw, with dotted line, the case where only the Background (BG) evolution is considered, i.e. the linear perturbation evolution has not been included.

worth mentioning that other BBN codes are available and may give slightly different values of primordial abundances, however with an error inside $\Delta Y_p = 0.0003$ [21]. Here we use the code most widely employed and adopted also by the Planck collaboration [33]. In our analysis, we choose to work with flat priors, and consider purely adiabatic initial conditions, fixing the sum of neutrino masses to 0.06 eV. In particular, for the helium fraction $Y_p$, we explore the prior $[0.1 : 0.6]$.

We consider the joint data set of the following measurements:

- CMB measurements, through the Planck (2018) data [64], using Plik "TT,TE,EE+lowE"
likelihood by combination of temperature power spectra and cross correlation TE and EE over the range $\ell \in [30, 2508]$, the low-$\ell$ temperature Commander likelihood, and the low-$\ell$ SimAll EE likelihood. We refer to this data set as “CMB”;

- The lensing reconstruction power spectrum from the latest Planck satellite data release (2018) [64, 65], hereafter indicated with “lensing”;
- Baryon Acoustic Oscillation (BAO): we use distance measurements from 6dFGS [66], SDSS-MGS [67], and BOSS DR12 [68] surveys, as considered by the Planck collaboration;
- Hubble constant of latest Riess (2019) work (R19), $H_0 = 74.03 \pm 1.42$ km/s/Mpc [69], that is in tension at $4.4\sigma$ with CMB estimation within the minimal cosmological model. This measurement is implemented by default in the package CosmoMC by imposing a Gaussian prior for the Hubble parameter constraint.
- Pantheon compilation [70] of 1048 SNe Ia in the redshift range $0.01 < z < 2.3$, which provides accurate relative luminosity distances, hereafter indicated with “Pth”;
- Dark Energy Survey Year-One (DES) results that combine galaxy clustering and weak gravitational lensing measurements, using 1321 square degrees of imaging data [71].

5 Results and Conclusions

The results of the analysis are summarized in Tab. (1), where the constraints on free parameters of the theory, and some of the derived ones, are shown. Also, in Fig.(3) we show the plane $Y_P - \Omega_b h^2$ with superimposed direct measurements of $Y_P$ by observations of helium and hydrogen emission lines from metal-poor extragalactic H II regions, combined with estimates of metallicity, $Y_P = 0.2449 \pm 0.0040$ [25], consistent with the standard BBN estimate,
Figure 4. $Y_P$-$b$ plane for our analysis. Left: $f_1$CDM model; Middle: $f_2$CDM model, Right: $f_3$CDM model.

$Y_P = 0.2477 \pm 0.0029$ [21, 23, 72]. We show that the considered $f(T)$ models are fully in agreement with direct and indirect measurements, i.e. the BBN result based on the Planck determination [33]. Noteworthy, the helium fraction parameter shows no correlation with the free parameter of the $f(T)$ gravity, avoiding the introduction of degeneration (see Fig.4).

We note that the introduction of the CMB likelihood in the analysis significantly improves the constraints on the $f(T)$ free model parameter, achieving an accuracy of $10^{-2}$, $10^{-5}$ and $10^{-9}$ respectively for the case of power law, exponential and the square-root exponential $f(T)$ gravity. Our results confirm previous analysis using the full CMB likelihood [37], and constrain the $f(T)$ gravity parameter as different from zero at more than $3\sigma$. This is particularly significant in the light of large scale data analysis results, where $f(T)$ parameters were compatible with zero in $1\sigma$ [36, 47–51]. In other words, these results show a preference of the analysed dataset for a deviation from the standard $\Lambda$CDM. We can infer that cosmic dynamics could constitute a probe for deviation with respect to GR (or TEGR). In particular, we note that $f_i$CDM models prefers higher $H_0$ values with respect to the $\Lambda$CDM one (see Fig.3) as also recently pointed out in Ref. [73], where the $f(T)$ scenario was tested to reconcile the $H_0$ measurements. We note that using the complete CMB likelihood improves the precision on the constraint of the $f(T)$ parameter and further relaxes the $H_0$ tension, even solving it in the case of $f_1$CDM model. Noteworthy, this occurs both when BBN consistency is considered and when $Y_P$ is treated as a free parameter. That is, a faster expansion is not achieved at the cost of extra amount of primordial abundances or a higher radiation density, but with a modification of gravity.

Finally, to compare $f(T)$ models with the $\Lambda$CDM, when constrained with data, we use the Deviance Information Criterion (DIC) [74]:

$$DIC := \chi^2_{\text{eff}} + 2p_D,$$

where $\chi^2_{\text{eff}}$ is the effective $\chi^2$ corresponding to the maximum likelihood and $p_D = \bar{\chi}^2_{\text{eff}} - \chi^2_{\text{eff}}$. The bar stands for the average of the posterior distribution. The DIC accounts for both the goodness of fit and the bayesian complexity of the model. In Tab.1, we indicate the

$$\Delta DIC = DIC_{f(T)} - DIC_{\Lambda \text{CDM}},$$

where we consider the convention based on Jeffreys’ scale [75, 76] for which $\Delta DIC > 10/6/2$ provides, respectively, strong/moderate/weak evidence against $f(T)$ models.
Table 1. 68% confidence limits for the $f(T)$CDM and $\Lambda$CDM analysis using CMB+lensing+BAO+R19+Pth+DES data

|                    | BBN Consistency | $\Lambda$CDM | $f_1$CDM | $f_2$CDM | $f_3$CDM |
|--------------------|-----------------|--------------|-----------|-----------|-----------|
| $100\Omega_bh^2$   |                 | 2.264 ± 0.13 | 2.251 ± 0.13 | 2.248 ± 0.14 | 2.249 ± 0.14 |
| $\Omega_c h^2$     |                 | 0.1170 ± 0.0008 | 0.1183 ± 0.0008 | 0.1189 ± 0.0010 | 0.1189 ± 0.0011 |
| $\tau$             |                 | 0.061 ± 0.008 | 0.056 ± 0.007 | 0.054 ± 0.007 | 0.054 ± 0.008 |
| ln(10$^{10}A_s$)   |                 | 3.053 ± 0.015 | 3.045 ± 0.014 | 3.041 ± 0.014 | 3.041 ± 0.015 |
| $n_s$              |                 | 0.972 ± 0.004 | 0.968 ± 0.004 | 0.967 ± 0.004 | 0.967 ± 0.004 |
| $b$                |                 | (1.4 ± 0.3) × 10$^{-2}$ | (5.6 ± 0.9) × 10$^{-5}$ | (6.8 ± 1.3) × 10$^{-9}$ |
| $Y_p$              |                 | 0.24550 ± 0.00005 | 0.24545 ± 0.00005 | 0.24543 ± 0.00005 | 0.24544 ± 0.00005 |
| $H_0$              |                 | 68.79 ± 0.36 | 73.85 ± 1.05 | 70.67 ± 0.82 | 70.14 ± 0.62 |
| $\sigma_8$         |                 | 0.806 ± 0.006 | 0.850 ± 0.010 | 0.824 ± 0.009 | 0.818 ± 0.007 |

$\Delta DIC$        | strongly preferred | moderately preferred | moderately preferred |

$Y_p$ free

|                    | $\Lambda$CDM+$Y_p$ | $f_1$CDM+$Y_p$ | $f_2$CDM+$Y_p$ | $f_3$CDM+$Y_p$ |
|--------------------|--------------------|----------------|----------------|----------------|
| $100\Omega_bh^2$   | 2.270 ± 0.017      | 2.247 ± 0.016 | 2.242 ± 0.020 | 2.250 ± 0.017 |
| $\Omega_c h^2$     | 0.1170 ± 0.0008    | 0.1184 ± 0.0008 | 0.1190 ± 0.0010 | 0.1190 ± 0.0010 |
| $\tau$             | 0.062 ± 0.008      | 0.056 ± 0.007 | 0.053 ± 0.007 | 0.053 ± 0.007 |
| ln(10$^{10}A_s$)   | 3.055 ± 0.016      | 3.044 ± 0.014 | 3.038 ± 0.015 | 3.039 ± 0.015 |
| $n_s$              | 0.974 ± 0.006      | 0.967 ± 0.005 | 0.965 ± 0.007 | 0.966 ± 0.006 |
| $b$                | (1.4 ± 0.3) × 10$^{-2}$ | (5.7 ± 0.9) × 10$^{-5}$ | (7.1 ± 1.3) × 10$^{-9}$ |
| $Y_p$              | 0.250 ± 0.011      | 0.243 ± 0.010 | 0.239 ± 0.014 | 0.243 ± 0.010 |
| $H_0$              | 68.87 ± 0.41       | 73.86 ± 1.09 | 70.68 ± 0.79 | 70.21 ± 0.59 |
| $\sigma_8$         | 0.807 ± 0.007      | 0.851 ± 0.011 | 0.823 ± 0.009 | 0.818 ± 0.008 |

$\Delta DIC$        | strongly preferred | moderately preferred | moderately preferred |

We find that the analysed $f(T)$ models are always preferred over the standard one. This result is to be read as a preference of the data, especially of R19 Gaussian prior, for models with a current scale-dependent evolution. We detail, in Fig.(5), the $\chi^2$ density posterior distributions of each dataset considered, which allows us to understand why the $f_1$CDM model is preferred over others. Indeed, $f_1$CDM minimizes the $\chi^2_{R19}$, i.e it is more in agreement with the estimate of R19, as it can also be seen in Fig.(3). Also, we note that the high-$\ell$ CMB likelihood, the $\chi^2_{plik}$, also shows lower values in the case of $f(T)$ models compared to the $\Lambda$CDM. The combination of these two effects brings a $\chi^2$ value about 25 points lower than the standard model for the $f_1$CDM. It is clear that the result would be different if the prior of R19 was removed in the choice of the dataset, since there would be no difference in $\chi^2_{R19}$ shown in Fig.(5). Furthermore, we would expect different evidences if instead of the $\Lambda$CDM model, we consider more general models like the $w$CDM model as reference, with $w$ different from the standard EoS of $\Lambda$CDM (see [18]).

In conclusion, in this paper we have studied $f(T)$ extensions of teleparallel gravity intended as corrections to TEGR where only the torsion scalar $T$ is considered. In particular, we studied power law and exponential corrections, where the standard $\Lambda$CDM can be easily recovered. Specifically, we draw both the background and the linear perturbation evolution for three $f(T)$ models, implementing in a Boltzman solver code the theory and studying the
Figure 5. Posterior distribution density of the $\chi^2$ values of the several data used in the analysis.

Theoretical predictions in the light of both large and small scale data. Our analysis constrain the free parameters of the theory with unprecedented precision, noting that the recovery of GR is out of more than $3\sigma$. Also, when the helium fraction is treated as a free parameter of the models, its constrained value is fully compatible with both direct measurements of primordial abundance and the standard Big Bang Nucleosynthesis estimate, also allowing for a higher $H_0$ value than the standard cosmological model. Noteworthy, this allows to significantly relax the tension on the value of the today observed Hubble constant.

Future CMB experiments, as COre [77], Stage IV CMB experiment [78–80] and SPT-3G [81], will better constrain the primordial abundances [82]. Also Euclid mission [83], combining it with the latest Planck data and with the future COre mission, clearly will help in breaking the degeneracy between the cosmological parameters, with a significant reduction of the error on $Y_P$. Finally, Square Kilometre Array (SKA) mission is proved to be a promising tool to test gravity over a large range of scales and redshifts. [84].

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