The formation of a crater on the surface of the cathode in the explosion of micro tip

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Abstract. Present the results of numerical simulation of the formation of a crater on the surface of the cathode during the explosion of micro tip. The simulation was performed using the two-dimensional magnetohydrodynamic program, which used wide-range equation of state and the table of conductivity of the metal, based on experimental data. It is shown that the electric explosion of micro tip with parameters typical unit ecton may form on the cathode surface of the crater with a radius of a few microns.

1. Introduction
Explosive electron emission at the surface of a metal cathode is initiated at high electric field strengths and is accompanied by explosions of metal microvolumes [1, 2] as the latter acquire a high energy density on Joule heating from field emission.

The emission of electrons occurs in individual portions—ectons [3]. An ecton is generated due to overheating of metal in a microexplosion and ceases to operate as the emission zone cools off [2, 3]. The operation of an ecton is a complex process the details of which are still poorly understood. In addition to electron emission, the operation of an ecton provides the generation of multiply charged ions, liquid metal drops, etc, and on completion of its operation, craters of micron sizes can be left on the cathode surface [2, 3]. A hydrodynamic model of the crater formation and molten metal behavior on a flat surface under external pressure using equations for an incompressible fluid was considered elsewhere [4]. The aim of our study is to show that a microexplosion with parameters typical of an individual ecton can lead to the formation of a crater on the cathode surface. The model used in the study neglects the processes occurring in the vacuum between a microprotrusion and cathode, and so it is largely simplified.
2. Numerical technique

The explosion of a microprotrusion was simulated using a JULIA magnetohydrodynamic (MHD) program package [5, 6]. The software is based on the particle-in-cell method and allows one to simulate explosions of conductors in the 2D approximation. The system of MHD equations consists of equations which describe the laws of conservation of mass, momentum, and energy:

\[
\frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{v}) = 0; \quad (1)
\]

\[
\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \nabla \mathbf{v} = -\nabla p + \frac{1}{c} \mathbf{j} \times \mathbf{H}; \quad (2)
\]

\[
\frac{\partial \rho \varepsilon}{\partial t} + \nabla (\rho \varepsilon \mathbf{v}) = -p \nabla \mathbf{v} + \frac{j^2}{\sigma} + \nabla (k \nabla T); \quad (3)
\]

Maxwell equations in the quasistationary approximation (with neglect of displacement currents)

\[
\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t} = -\nabla \times \mathbf{E}; \quad \nabla \times \mathbf{H} = \frac{4\pi}{c} \mathbf{j}; \quad (4)
\]

and Ohm’s law

\[
\mathbf{j} = \sigma (\mathbf{E} - \frac{1}{c} \mathbf{v} \times \mathbf{H}). \quad (5)
\]

Here \( \rho \) is the density of matter; \( \mathbf{v} \) is the mass velocity; \( p, \varepsilon, T \) are the pressure, internal energy and temperature of matter; \( \mathbf{H} \) is the magnetic field strength; \( \mathbf{E} \) is the electric field strength in an immobile coordinate system; \( \mathbf{j} \) is the current density; \( k, \sigma \) are the heat conductivity and electrical conductivity, respectively.

The system of equations (1)–(5) was solved for a cylindrical coordinate system in the \((r, z)\) geometry. The numerical algorithm for solving the system of equations (1)–(5) was the following. The equation of motion (2) was solved for each particle, whereupon the average mass velocities and density of matter in each cell were found by summation over all particles. The equation of continuity (1) was fulfilled automatically due to the Lagrangian particle behavior. The equation of energy (3) and Maxwell equations (4) were solved on an immobile Eulerian grid, which remained unchanged during the calculations.

Solving the system of MHD equations requires boundary conditions. On the equations of motion (2), they can be imposed by specifying either the velocity or the pressure at the boundary. For integration of equation (2), the conditions at the free boundary (boundary with the vacuum) had the form \( p = 0 \), and those at the centre (at \( r = 0 \)) corresponded to axial symmetry; that is, the radial velocity component was \( v_r = 0 \).

The boundary conditions for the equation of energy (3), which included heat conduction, were imposed by specifying the heat flux at the boundaries. The heat flux throughout the boundaries was taken equal to zero, which corresponded to the absence of external heat sources and sinks.

The boundary conditions on Maxwell equations (4) had the following form. The radial vector component of the electric field strength was \( E_r = 0 \) at \( r = 0, z = 0 \), and \( z = Z_{\text{max}} \). The azimuthal vector component of the magnetic field strength was \( B_\phi = 0 \) at \( r = 0 \) and \( B_\phi = 2I(t)/(cR_{\text{max}}) \) at \( r = R_{\text{max}} \), where \( Z_{\text{max}} \) and \( R_{\text{max}} \) are the maximum values of the coordinate \( z \) and radius \( r \) of the Eulerian grid; \( I(t) \) is the current through a microprotrusion, which was calculated in integrating circuit equations.

In the program, wide-range semiempirical equations of state [7] were used with taking into account high-temperature melting and evaporation effects. Electrophysical characteristics and the heat conductivity of a metal were calculated using data from the experiment and simulation [8] by means of an experimental-computational technique [9, 10].
3. Simulation results
The problem was solved in the following statement. It was assumed that on the surface of a flat Cu cathode there is a microprotrusion shaped as a cylinder of radius 0.3 µm and length 1.5 µm. The diode is connected in a circuit described by the equation

\[ U_{\text{out}}(t) = I(t)R_{\text{out}} + U_{\text{load}}(t), \]

where \( U_{\text{out}}(t) \) is the external voltage, which rises up to \( U_0 = 3200 \) V in 0.1 ns and then remains constant; \( R_{\text{out}} = 1000 \) Ohm is the external resistance, the value of which was chosen so that the circuit current in short-circuit mode was 3.2 A; \( U_{\text{load}}(t) \) is the diode voltage. The current \( I(t) \) served as a boundary condition for solving Maxwell equations (4).

The maximum current equal to 3.2 A was chosen from the following reasoning. For copper cathode, the minimum current, at which an ecton can operate, is about 1.6 A, and as the current is increased two times, a second ecton is formed at the cathode surface [2]. Consequently, the current 3.2 A is the maximum current through an individual ecton. The radial size of the microprotrusion was chosen so that the current density through the microprotrusion would be about \( 10^9 \) A/cm\(^2\), i.e. would be close to the limiting current density of field emission [1].

Figure 1 shows time dependences of the current and voltage obtained by numerically solving the system of equations (1)–(6). The peak of the voltage within the first nanoseconds of the discharge is due to an electrical explosion of the microprotrusion. The fact is that the metal, when heated and transformed form the solid to plasma state, loses its conductivity, and when the microprotrusion explodes a conducting dense plasma with a temperature of several electron volts is formed near the cathode.

As the microprotrusion explodes (figure 2), a region of increased pressure ranging to several tens of kilobars is formed near the cathode surface. As a result, a crater with a radius of several microns is produced on the cathode surface (figure 3). The most intense crater formation takes place during the first nanoseconds of the discharge.
Figure 2. Density distribution of matter at $t = 2$ ns.

Figure 3. Density distribution of matter at $t = 8$ ns.

Thus, the magnetohydrodynamic simulation shows that a microexplosion with parameters typical of an individual ecton can lead to the formation of a crater with a radius of several microns on the cathode surface.
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References
[1] Mesyats G A and Proskurovsky D I 1971 Pis’ma Zh. Eksp. Teor. Fiz. 13 7–10
[2] Mesyats G A 2000 Ectons in Vacuum Discharge: Breakdown, Spark, Arc (Moscow: Nauka)
[3] Mesyats G A 1995 Phys. Usp. 38 567–590
[4] Mesyats G A and Uimanov I V 2015 IEEE Trans. Plasma Sci. 43 2241–2246
[5] Oreshkin V I, Baksht R B, Ratakhin N A, Shishlov A V, Khishchenko K V, Levashov P R and Beilis I I 2004 Phys. Plasmas 11 4771–4776
[6] Oreshkin V I, Chaikovsky S A, Ratakhin N A, Grinenko A and Krasik Ya E 2007 Phys. Plasmas 14 102703
[7] Fortov V E, Khishchenko K V, Levashov P R and Lomonosov I V 1998 Nucl. Instr. Meth. Phys. Res. A 415 604–608
[8] Oreshkin V I, Baksht R B, Labetsky A Yu, Rousskikh A G, Shishlov A V, Levashov P R, Khishchenko K V and Glazyrin I V 2004 Tech. Phys. 49 843–848
[9] Bakulin I D, Kuropatenko V F and Luchinskii A V 1976 Zh. Tekh. Fiz. 46 1963–1969
[10] Oreshkin V I, Khishchenko K V, Levashov P R, Rousskikh A G and Chaikovskii S A 2012 High Temp. 50 584–595