A new approach for optimality of fully fuzzy assignment problems

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Abstract. In this paper, we developed an efficient approach for the solution of fully fuzzy assignment problems involving imprecise information without transforming to equivalent precise form. Here, the imprecise information are represented as triangular fuzzy numbers and optimal solution is obtained applying branch and bound method and a new type of arithmetic operations, ranking on triangular fuzzy numbers. The proposed method is validated using a real life numerical example.

1. Introduction
A particular type of transportation problem is assignment problem, where n responsibilities (jobs) are to be assigned to an equal number of n machines (workers) in one to one basis such that the assignment cost (or profit) is minimum (or maximum). Hence, it can be considered as a balanced transportation problem in which all supplies and demands are equal, and the number of rows and columns in the matrix are equal. In real life circumstances, data obtained is not precise and hence several authors have started to work on fuzzy assignment problem as it gained more importance. They have used different methods for solving the same.

Lin et.al [1], in 2004 has used labeling algorithm to deal with a fuzzy assignment problem where the costs are imprecise in nature. Sathi Mukherjee et.al [2] showed that the effective way to handle a fuzzy assignment problem by making use of the fuzzy ranking method. Thangavelu et.al 0 has also proposed a ranking method for generalized fuzzy numbers to solve an assignment problem in fuzzy nature. Thorani et.al 0 has also used an algorithm to solve a fuzzy assignment problem with generalized fuzzy numbers. A prompting method was given by Kirubha [2] to deal with an assignment problem under fuzzy environment. Dhanasekar et.al [3] applied a ranking technique which transforms the fuzzy numbers to an ordered pair of numbers. Rajarajeswari et.al [4] has dealt with an unbalanced assignment problem in fuzzy environment and proposed a new algorithm to solve it. Narayanamoorthy et.al [5] have solved a fuzzy assignment problem using branch and bound technique. Most of the authors have solved the problem in its crisp nature and thus obtaining a crisp solution. Muruganandam et.al [6] discussed the fuzzy assignment problem using traditional arithmetic operations. Here, we considered an assignment problem with fuzzy parameters and solved the same by arriving at a solution suitable for the decision maker.
The rest of our work is organized as follows. Fuzzy set and its Basics, arithmetic operations of triangular fuzzy numbers are discussed in section 2. Algorithm for the proposed model is discussed in section 3. A real life example is dedicated to understand the fuzzy assignment problem in section 4. Conclusion part in Section 5.

2. Basic concepts

Definition 2.1.
A fuzzy set \( \tilde{a} \) defined on the set of real numbers \( R \) is said to be a fuzzy number, if its membership function \( \tilde{a} : R \rightarrow [0,1] \) has the following characteristics:
(i) \( \tilde{a} \) is convex, (i.e.) \( \tilde{a}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\tilde{a}(x_1),\tilde{a}(x_2)\}, \lambda \in [0,1], \) for all \( x_1, x_2 \in R \)
(ii) \( \tilde{a} \) is normal, (i.e.) there exists an \( x \in R \) such that \( \tilde{a}(x) = 1 \)
(iii) \( \tilde{a} \) is piecewise continuous.

Definition 2.2.
A fuzzy number \( \tilde{a} \) on \( R \) is a triangular fuzzy number if its membership function \( \tilde{a} : R \rightarrow [0,1] \) has the following characteristics:
\[
\tilde{a}(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\
\frac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \\
0, & \text{otherwise}
\end{cases}
\]
We denote this triangular fuzzy number as \( \tilde{a} = (a_1, a_2, a_3) \). We use \( F(R) \) to denote the set of all triangular fuzzy numbers defined on \( R \).

Definition 2.3.
A triangular fuzzy number \( \tilde{a} = (a_1, a_2, a_3) \in F(R) \) can also be represented as an pair \( \tilde{a} = (\underline{a}, \bar{a}) \) of functions \( \underline{a}(r), \bar{a}(r), \) for \( 0 \leq r \leq 1 \) which satisfies the following requirements:
(i) \( \underline{a}(r) \) is a bounded monotonic increasing left continuous function.
(ii) \( \bar{a}(r) \) is a bounded monotonic decreasing left continuous function.
(iii) \( \underline{a}(r) \leq \bar{a}(r), 0 \leq r \leq 1 \).
It is also represented by \( \tilde{a} = (a_0, a_+, a_-) \) where \( a_+ = (a_0 + \bar{a}), a_- = (\underline{a} - a_0) \) are called the left fuzziness index function and the right fuzziness index function respectively. For an arbitrary triangular fuzzy number \( \tilde{a} = (\underline{a}, \bar{a}) \) the number \( a_0 = \left( \frac{\underline{a}(1) + \bar{a}(1)}{2} \right) \) is said to be a location index number of \( \tilde{a} \)

2.1. Arithmetic Operations and Ranking of Triangular Fuzzy Numbers
For arithmetic operations and ranking of triangular fuzzy numbers (See [7])
3. Algorithm for Branch and Bound Method
The terminologies of branch and bound technique applied to fuzzy assignment problem are given below.

Consider $B_{\tau} = \sum_{i,j \in X} C_{ij} + \sum_{i \in X} \sum_{j \in Y} \max C_{ij}$

Which represents the upper bound of the partial assignment up to $p^n_{\tau}$. Where $n$ be the level number in the branch tree, $\tau$ be the assignment in the current node of a branching tree and assume the root node is 0. $p^n_{\tau}$ be an assignment at level $n$ of the branching tree. $A$ is the set of assigned cells up to the node $p^n_{\tau}$ from the root node.

Where $C_{i,j}$ represents the cell entry of the profit matrix with respect to the $i^{th}$ row and $j^{th}$ column. $X$ be the set of rows which are not deleted up to the node $p^n_{\tau}$ from the node in the branching node.

The branching methodology using branch and bound method under the root nodes and upper bounds are shown in the following figure (Figure 1).

![Figure 1. Graphical representation of Branch and bound Algorithm.](image)

4. Numerical Example
Let us consider the fuzzy assignment problem discussed by Muruganandam et.al [8] with rows representing four Machines and column representing four Jobs in table 1 to 3
Find an optimal assignment of machines to jobs that will maximize the total profit. (Profit in rupees).

Converting all fuzzy numbers in parametric form, we have

In the first branch by using \( B_{11} = C_{11} + \sum_{i=2,3,4} \sum_{j=2,3,4} \text{max} \ c_{ij} \), find \( P^1_{11}, P^1_{21}, P^1_{31} \) and \( P^1_{41} \) for greatest upper bound.

That is,

\[
P^1_{11} = (3,2-2r,3-3r) + ((30, 3-3r,2-2r) + (38, 4-4r,2-2r) + (22, 3-3r,2-2r)) = (93, 3-3r, 3-3r)
\]

Similarly, we can find \( P^1_{21}, P^1_{31} \) and \( P^1_{41} \). Here the greatest upper bound is \( P^1_{41} = (136, 4-4r,2-2r) \).

In the second branch by using \( B_{22} = C_{41} + C_{12} + \sum_{i=3,4} \sum_{j=3,4} \text{max} \ c_{ij} \), find \( P^2_{12}, P^2_{22} \) and \( P^2_{32} \) for greatest upper bound and it is \( P^2_{32} = (136, 4-4r,2-2r) \).

In the third branch by using \( B_{33} = C_{41} + C_{32} + C_{33} + \sum_{i=4} \sum_{j=4} \text{max} \ c_{ij} \), find \( P^3_{13} \) and \( P^3_{23} \) for greatest upper bound and it is \( P^3_{23} = (132,4-4r,2-2r) \).

Therefore, the optimal assignment obtained by our proposed method is

\[
P_4 \rightarrow J_1, P_3 \rightarrow J_2, P_2 \rightarrow J_3, P_1 \rightarrow J_4
\]

The optimal cost = \((38, 4-4r,2-2r) + (26, 3-3r,2-2r) + (38, 4-4r,2-2r) + (30, 3-3r,2-2r)\)

= \((132,4-4r,2-2r)\)

The optimal assignment obtained by Muruganandam et.al [9] is \( P_4 \rightarrow J_1, P_3 \rightarrow J_2, P_2 \rightarrow J_3, P_1 \rightarrow J_4 \) and the optimal cost is \( (118,132,140) \).

In our proposed method, the optimal assignment is same but the optimal cost is \((128+4r, 132,134-2r)\), which is sharper than the method proposed by Muruganandam et.al [10]. The following table (Table 3) gives an additional advantage to the decision maker if he moves with our proposed method.

### Table 1. Fuzzy cost matrix.

|       | J1   | J2   | J3   | J4   |
|-------|------|------|------|------|
| M1    | (1,3,6) | (8,12,16) | (20,22,24) | (34,38,40) |
| M2    | (8,12,16) | (2,3,5) | (23,26,28) | (27,30,32) |
| M3    | (13,15,17) | (34,38,40) | (2,3,5) | (1,3,6) |
| M4    | (27,30,32) | (8,12,16) | (19,22,24) | (19,22,24) |

### Table 2. Fuzzy cost matrix in parametric form.

|       | J1   | J2   | J3   | J4   |
|-------|------|------|------|------|
| M1    | (3,2-2r,3-3r) | (12,4-4r,4-4r) | (22,2-2r,2-2r) | (38,4-4r,2-2r) |
| M2    | (12,4-4r,4-4r) | (3,1-2r,2-2r) | (26,3-3r,2-2r) | (30,3-3r,2-2r) |
| M3    | (15,2-2r,2-2r) | (38,4-4r,2-2r) | (3,1-2r,2-2r) | (3,2-2r,3-3r) |
| M4    | (30,3-3r,2-2r) | (12,4-4r,4-4r) | (22,3-3r,2-2r) | (22,3-3r,2-2r) |
Table 3. Decision maker’s choice, $r \in [0, 1]$.

| Value of $r \in [0, 1]$ | Proposed Method          |
|-------------------------|--------------------------|
| $r = 0$                 | (128, 132, 134)          |
| $r = 0.5$               | (130, 132, 132)          |
| $r = 1$                 | (132, 132, 132)          |

5. Conclusion
This paper provides a fuzzy approach to find the optimum solution for the fuzzy assignment problem. Our proposed approach using fuzziness index and location index is very simple to understand and very easy to calculate. In this proposed method there is an advantage to the decision makers, they can choose their $r \in [0, 1]$ according to their wish. This paper can be extended to find the solution of multi-objective fuzzy assignment problem using trapezoidal fuzzy number.

6. References
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