MESON CORRELATORS IN QCD VACUUM - IS SATURATION THE RIGHT APPROACH?

Varun Sheel*, Hiranmaya Mishra† and Jitendra C. Parikh ‡

Theory Group, Physical Research Laboratory, Navrangpura, Ahmedabad 380 009, India

Abstract

Equal time, point to point correlation functions for spatially separated meson currents are calculated with respect to a variational construct for the ground state of QCD. With exact calculations, vector, axial vector and scalar channels show qualitative agreement with the phenomenological predictions, whereas pseudoscalar channel does not. However, the pseudoscalar correlator, when approximated by saturating with intermediate one pion states agrees with results obtained from spectral density functions parametrised by pion decay constant and \(<\bar{\psi}\psi>\) value obtained from chiral perturbation theory.

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*Electronic address: varun@prl.ernet.in
†Electronic address: hm@prl.ernet.in
‡Electronic address: parikh@prl.ernet.in
I. INTRODUCTION

Quantum Chromodynamics (QCD) in the low energy sector is nonperturbative and the vacuum structure here is nontrivial [1]. The vacuum structure of QCD has been studied since quite some time both with quark condensates associated with chiral symmetry breaking [2] as well as with gluon condensates [3][4]. An interesting quantity to study with such a nontrivial structure of vacuum is the behaviour of current-current correlators illustrating different physics involved at different spatial distances. This has recently been emphasized in a review by Shuryak [5] and studied through lattice simulations [6]. The basic point is that the correlators can be used to study the interquark interaction — its dependence on distance. In fact they complement bound state hadron properties in the same way that scattering phase shifts provide information about the nucleon-nucleon force complementary to that provided by the properties of the deuteron [6].

We have recently considered the structure of QCD vacuum with both quark and gluon condensates using a variational ansatz [7]. We shall use here such an explicit construct of QCD vacuum obtained through energy minimisation [7] to evaluate the meson correlators.

We organise the paper as follows. In section II we recapitulate the results of Ref. [7]. In section III we define and calculate meson correlation functions. In section IV we quote the results. Section V is devoted to the study of the exceptional case of pseudoscalar correlator. Finally we discuss the results in section VI.

II. QCD VACUUM WITH QUARK AND GLUON CONDENSATES

We have considered the vacuum structure in QCD using a variational approach with both quark and gluon condensates [7]. Here we shall very briefly recapitulate the results of the same for the sake of completeness. The trial variational ansatz for the QCD vacuum is taken as

\[ |\text{vac} > = U_G U_F |0 > \] (2.1)
obtained through the unitary operators $U_G$ and $U_F$ for gluons and quarks respectively on the perturbative vacuum $|0\rangle$.

For the gluon sector, the unitary operator $U_G$ is of the form

$$U_G = \exp (B_G^\dagger - B_G)$$ (2.2)

with the gluon pair creation operator $B_G^\dagger$ given by

$$B_G^\dagger = \frac{1}{2} \int f(\vec{k}) a_i^a(\vec{k})^\dagger a_i^a(-\vec{k})^\dagger d\vec{k}$$ (2.3)

In the above $a_i^a(\vec{k})^\dagger$ are the transverse gluon field creation operators satisfying the following quantum algebra in Coulomb gauge [7]

$$\left[ a_i^a(\vec{k}), a_j^b(\vec{k}')^\dagger \right] = \delta^{ab} (\delta_{ij} - \frac{k_i k_j}{k^2}) \delta(\vec{k} - \vec{k}')$$ (2.4)

with $a_i^a(\vec{k})$ annihilating the perturbative vacuum $|0\rangle$. Further $f(k)$ is a trial function associated with gluon condensates.

Similarly for the quark sector we have,

$$U_F = \exp (B_F^\dagger - B_F)$$ (2.5)

with

$$B_F^\dagger = \int \left[ h(\vec{k}) c_i^I(\vec{k})^\dagger (\vec{\sigma} \cdot \hat{k}) \tilde{c}_i^I(-\vec{k}) \right] d\vec{k}$$ (2.6)

Here $h(\vec{k})$ is a trial function associated with quark antiquark condensates. The operators $c^I$ and $\tilde{c}$ create a quark and antiquark respectively when operating on the perturbative vacuum. They satisfy the anticommutation relations

$$\left[ c_i^I(\vec{k}), c_j^I(\vec{k}')^\dagger \right]_+ = \delta_{rs} \delta^{ij} \delta(\vec{k} - \vec{k}') = \left[ \tilde{c}_i^I(\vec{k}), \tilde{c}_j^I(\vec{k}')^\dagger \right]_+$$ (2.7)

Clearly such a structure for the vacuum eventually reduces to a Bogoliubov transformation for the operators. One can then calculate the energy density functional given as

$$\epsilon_0 \equiv F(h(\vec{k}), f(\vec{k}))$$ (2.8)
The condensate functions \( f(\vec{k}) \) and \( h(\vec{k}) \) are to be determined such that the energy density \( \epsilon_0 \) is a minimum. Since the functions cannot be determined analytically through functional minimisation except for a few simple cases [8], we choose the alternative approach of parameterising the condensate functions as (with \( k = |\vec{k}| \)),

\[
\sinh f(\vec{k}) = Ae^{-Bk^2/2}
\]

(2.9)

This corresponds to taking a Gaussian distribution for the perturbative gluons in the non-perturbative vacuum. Similarly, for the function \( h(\vec{k}) \) describing the quark antiquark condensates we take the ansatz,

\[
\tan 2h(\vec{k}) = \frac{A'}{(e^{R^2k^2} - 1)^{1/2}}
\]

(2.10)

Further one could relate the quark condensate function to the wave function of pion as a quark antiquark bound state [9] and hence to the decay constant of pion.

In Ref. [7] the energy density is minimised with respect to the condensate parameters subjected to the constraints that the pion decay constant \( f_\pi \) and the gluon condensate value \( \frac{\alpha_s}{\pi} < G^{a}_{\mu\nu}G^{a}_{\mu\nu} > \) of Shifman Vainshtein and Zhakarov [10] come out as the experimental value of 93 MeV and 0.012 GeV\(^4\) respectively.

The results of such a minimisation showed the instability of the perturbative vacuum to formation of quark antiquark as well as gluon condensates when the coupling became greater than 0.6. Further the charge radius for the pion comes out correctly \( (R_{ch} \simeq 0.65 \text{ fm}) \) for \( \alpha_s = 1.28 \). The corresponding values of \( A' \) and \( R \) of Eq. (2.10) are calculated to be \( A'_{min} \simeq 1 \) and \( R \simeq 0.96 \text{ fm} \).

With the structure of QCD vacuum thus fixed from pionic properties and SVZ value we consider the meson correlators in the next section.

**III. MESON CORRELATION FUNCTIONS**

Consider a generic meson current of the form
\[
J(x) = \bar{\psi}^i_\alpha(x) \Gamma_{\alpha\beta} \psi^j_\beta(x) \tag{3.1}
\]

where \(x\) is a four vector; \(\alpha\) and \(\beta\) are spinor indices; \(i\) and \(j\) are flavour indices; \(\Gamma\) is a \(4 \times 4\) matrix \((1, \gamma_5, \gamma_\mu\) or \(\gamma_\mu \gamma_5\))

Because of the homogeneity of the vacuum we define the conjugate current to the above at the origin as,

\[
\bar{J}(0) = \bar{\psi}^j_\lambda(0) \Gamma'_{\lambda\delta} \psi^i_\delta(0)
\]

with \(\Gamma' = \gamma_0 \Gamma^\dagger \gamma_0\)

The meson correlation function for the above currents is defined as,

\[
R(x) = \langle T J(x) \bar{J}(0) \rangle_{\text{vac}} \tag{3.3}
\]

From now on we assume that expectation values are always with respect to the nonperturbative vacuum of our model, hence we drop the subscript \(\text{vac}\).

Hence with Eqs. (3.1), (3.2) and (3.3) we have

\[
R(x) = \Gamma_{\alpha\beta} \Gamma'_{\lambda\delta} < T \bar{\psi}^i_\alpha(x) \psi^j_\beta(x) \bar{\psi}^j_\lambda(0) \psi^i_\delta(0) >
\]

This reduces to the identity

\[
R(x) = \Gamma_{\alpha\beta} \Gamma'_{\lambda\delta} < T \bar{\psi}^i_\alpha(x) \bar{\psi}^j_\lambda(0) > < T \psi^j_\beta(x) \psi^i_\delta(0) >
\]

The above definition of \(R(x)\) is exact since the four point function does not contribute. In fact, in the evaluation of Eq. (3.4) we shall have a sum of two terms. The first is equivalent to the product of two point functions which is Eq. (3.5). The second term arises from contraction of operators at the same spatial point, related to disconnected diagrams and thus can be discarded.

In Eq. (3.5) the first term can be identified as the interacting quark propagator

\[
S(x) = \langle T \psi^j(x) \bar{\psi}^j(0) >
\]

It can be shown using the CPT invariance of the vacuum \([11]\) that the second term is given as
\[<T \bar{\psi}^i(x)\psi^i(0)> = -\gamma_5 S(x)\gamma_5\] (3.6)

\[= -S(-x)\]

Hence the correlation function of Eq. (3.4) becomes

\[R(x) = -Tr \left[ S(x)\Gamma S(-x)\Gamma \right]\] (3.7)

Similarly the correlator for massless noninteracting quarks can be given as

\[R_0(x) = -Tr \left[ S_0(x)\Gamma S_0(-x)\Gamma \right]\] (3.8)

Our task is now to evaluate the expression (3.7) with the ansatz for QCD vacuum as given in Eq. (2.10). Further we shall be interested in evaluating the equal time point to point correlation functions. With this in mind we first calculate the equal time interacting Feynman propagator in this nonperturbative vacuum given as [12]

\[S_{\alpha\beta}(\vec{x}) = \frac{1}{2} \left( \frac{\pi}{\sqrt{2}} \right)^3 \int e^{i\vec{k}.\vec{x}} d\vec{k} \left[ \sin 2h(\vec{k}) - (\vec{\gamma} \cdot \hat{k}) \cos 2h(\vec{k}) \right]\] (3.9)

In evaluating the above we have used the expectation values

\[<\psi_\alpha^i(\vec{x})^\dagger \psi_\beta^j(\vec{y})> = (2\pi)^{-3} \delta^{ij} \int \left( \Lambda_- (\vec{k}) \right)_\beta^\alpha e^{-i\vec{k}.(\vec{x}-\vec{y})} d\vec{k}\] (3.11a)

\[<\psi_\alpha^i(\vec{x})\psi_\beta^j(\vec{y})^\dagger> = (2\pi)^{-3} \delta^{ij} \int \left( \Lambda_+ (\vec{k}) \right)^\alpha_\beta e^{i\vec{k}.(\vec{x}-\vec{y})} d\vec{k}\] (3.11b)

where,

\[\Lambda_\pm (\vec{k}) = \frac{1}{2} (1 \pm \gamma^0 \sin 2h(\vec{k}) \pm (\vec{\alpha} \cdot \hat{k}) \cos 2h(\vec{k}))\] (3.12)

We next use the condensate function as given in Eq. (2.10) to evaluate Eq. (3.10). Here we shall take the parameters \(A' = 1\) and \(R = 0.96\) fm which give the correct pionic properties [7]. Then the equal time interacting quark propagator Eq. (3.10) reduces to, with \(x = |\vec{x}|\),
\[
S(\vec{x}) = -\frac{i}{2\pi^2} \frac{\vec{\gamma} \cdot \vec{x}}{x^4} + \frac{1}{(2\pi)^{3/2}} \frac{1}{2R^3} e^{-x^2/(2R^2)} - \frac{i}{(2\pi)^2} \frac{\vec{\gamma} \cdot \vec{x}}{x^2} I(x)
\]  
(3.13)

where

\[
I(x) = \int_0^{\infty} \left( \cos kx - \frac{\sin kx}{kx} \right) \frac{ke^{-R^2k^2}}{1 + (1 - e^{-R^2k^2})^{1/2}} dk
\]

(3.14)

We may further note that when \( h(k) = 0 \) i.e the condensates vanish, we recover the free massless propagator

\[
S_0(x) = -\frac{1}{2} \frac{1}{(2\pi)^3} \int d\vec{k} \frac{\vec{\gamma} \cdot \hat{k}}{x^4} e^{i\vec{k} \cdot \vec{x}}
\]

(3.15a)

\[
= -\frac{i}{2\pi^2} \frac{\vec{\gamma} \cdot \vec{x}}{x^4}
\]

(3.15b)

Having obtained the propagators, we can calculate the correlation function, Eq. (3.7) for a generic current of the form as in Eqs. (3.1) and (3.2). For convenience, we will consider the ratio of the physical correlation function to that of massless noninteracting quarks

\[
\frac{R(x)}{R_0(x)} = \left( 1 + \frac{1}{2} x^2 I(x) \right)^2 + \frac{\pi}{8} \frac{x^6}{R^6} e^{-x^2/R^2} \frac{x^2 Tr \left[ \Gamma' \Gamma \right]}{x^4 x^j Tr \left[ \gamma^i \gamma^j \gamma^k \sigma^l \right]}
\]

(3.16)

which is then evaluated in different channels with the corresponding Dirac structure for the currents.

**IV. RESULTS**

We have studied the above ratio of correlators for four channels. In each channel we associate the current with a physical meson having quantum numbers identical to that of the current. The results are shown in Table I and in Fig I. We may notice some general features and relationships among the correlators. The pseudoscalar correlator is always greater than the scalar correlator and vector correlator is greater than the axial vector correlator. We may emphasize here that these relations are rather general in the sense that they do not depend on the *explicit* form of the condensate function and arise due to the different Dirac structure of the currents which is reflected in the generic expression for the
correlation functions as in Eq. (3.16). The behaviour of each channel is consistent with that predicted by phenomenology except in the pseudoscalar case where the ratio does not go as high as expected from phenomenology. We examine this in the next section.

V. PSEUDOSCALAR CHANNEL

The explicit evaluation of the pseudoscalar correlator gives, using Eq. (3.16)

\[
\frac{R(x)}{R_0(x)} = \left[ 1 + \frac{1}{2} x^2 I(x) \right]^2 + \frac{\pi x^6}{8 R^6} e^{-x^2/R^2}
\]

which may also be read off from column 4 of Table I. This is plotted as a function of \(x\) in (Fig. 1). As may be seen from (Fig. 1) this ratio has a maximum of \(\sim 1.2\) at \(x \sim 1.3\) fm. Phenomenologically the peak is at \(\sim 100\) at \(x \sim 0.5\). In order to compare our results with other calculations we evaluate the same correlator approximately by saturating intermediate states with one pion states.

Using translational invariance the correlator may be written as

\[
R(x) = \frac{1}{2} \left( < J^p(0) \bar{J}^p(0) > e^{i\vec{p} \cdot \vec{x}} + < \bar{J}^p(0) J^p(0) > e^{-i\vec{p} \cdot \vec{x}} \right)
\]

Also using the fact that for the pseudoscalar current \(J^p = \bar{u} \gamma_5 d\) and \(\bar{J}^p = -J^p\) we have

\[
R(x) = \frac{1}{2} < J^p(0) \bar{J}^p(0) > \left( e^{i\vec{p} \cdot \vec{x}} + e^{-i\vec{p} \cdot \vec{x}} \right)
\]

We may evaluate the above matrix element using the definition of the pion decay constant given as \[\text{13}\]

\[
< \text{vac} | J_5^\mu(x) | \pi^a(p) > = \frac{i f_\pi p^\mu}{(2\pi)^{3/2}(2p_0)^{1/2}} e^{i p \cdot x}
\]

We now insert a complete set of intermediate states between the two currents but retain only the one pion state in the sum for the four point function. Thus,

\[
R(x) = \frac{1}{2} \int < \text{vac} | J^p(0) | \pi^a(\vec{p}) > < \pi^a(\vec{p}) | \bar{J}^p(0) | \text{vac} > \left( e^{i\vec{p} \cdot \vec{x}} + e^{-i\vec{p} \cdot \vec{x}} \right) d\vec{p}
\]
where \( J_{5}^{\mu a} = [\bar{\psi} \gamma^{\mu} \gamma^{5} \tau^{a} \psi] \) is the axial current. It can be shown \(^{14}\) that the divergence of the axial current gives the pseudoscalar current

\[
\partial_{\mu} [\bar{\psi} \gamma^{\mu} \gamma^{5} \tau^{a} \psi] = 2i m_{q} [\bar{\psi} \gamma^{5} \tau^{a} \psi] \tag{5.7}
\]

where \( m_{q} \) is the current quark mass. Thus taking divergence of both sides of Eq. (5.6) and using Eq. (5.7) we get,

\[
2 m_{q} < \text{vac} | iJ_{5}^{\mu a}(x) | \pi^{a}(p) > = \frac{-f_{\pi} m_{\pi}^{2}}{(2\pi)^{3/2}(2p_{0})^{1/2}} e^{ip \cdot x} \tag{5.8}
\]

where we have used \( p^{2} = m_{\pi}^{2} \).

In an earlier paper \(^{9}\) within our vacuum model and using the fact that pion is an approximate Goldstone mode it was demonstrated that saturating with pion states, gives the familiar current algebra result

\[
m_{\pi}^{2} = -\frac{m_{q}}{f_{\pi}^{2}} < \bar{\psi} \psi > \tag{5.9}
\]

With this result we eliminate quark mass \( m_{q} \) in Eq. (5.8) in favour of the quark condensate to get the relation

\[
< \text{vac} | J_{5}^{\mu a}(x) | \pi^{a}(\vec{p}) > = \frac{i}{2(2\pi)^{3/2}(2p_{0})^{1/2}} \frac{< \bar{\psi} \psi >}{f_{\pi}} e^{i\vec{p} \cdot \vec{x}} \tag{5.10}
\]

The expression for the pseudoscalar correlator now becomes

\[
R(x) = \frac{1}{16\pi^{3}} \left( \frac{< \bar{\psi} \psi >}{f_{\pi}} \right)^{2} \int \frac{1}{(p^{2} + m_{\pi}^{2})^{1/2}} \left( e^{i\vec{p} \cdot \vec{x}} + e^{-i\vec{p} \cdot \vec{x}} \right) d\vec{p} \tag{5.11}
\]

The above integral can be evaluated using the standard integral \(^{15}\)

\[
\int_{0}^{\infty} p(p^{2} + \beta^{2})^{-\nu/2} \sin(\alpha p) dp = \frac{\beta}{\sqrt{\pi}} \left( \frac{2\beta}{\alpha} \right)^{\nu} \cos(\nu \pi) \Gamma(\nu + \frac{1}{2}) K_{\nu+1}(\alpha \beta)
\]

for \( \alpha > 0, Re\beta > 0 \) and in the limit \( \nu \to 0 \). We then finally get for the correlator (using saturation of pion states)

\[
R(x) = \frac{1}{16\pi^{2}} \left( \frac{< \bar{\psi} \psi >}{f_{\pi}} \right)^{2} \frac{m_{\pi} K_{1}(m_{\pi}x)}{x} \tag{5.12}
\]
The correlator for free massless quarks as calculated in the earlier section for pseudoscalar is

\[ R_0(x) = \frac{1}{\pi^4 x^6} \]  \hspace{1cm} (5.13)

Hence the ratio is

\[ \frac{R(x)}{R_0(x)} = \frac{\pi^2}{16} \left( \frac{\langle \bar{\psi}\psi \rangle}{f_\pi} \right)^2 x^5 m_\pi K_1(m_\pi x) \] \hspace{1cm} (5.14)

We have plotted in Fig. 2 this ratio for our value of \( \langle \bar{\psi}\psi \rangle \) and that used by Shuryak [5,17]. Note that our value of \( \langle \bar{\psi}\psi \rangle \) is an output of the variational calculation consistent with low energy hadronic properties [7]. We thus observe that the approximate calculation of the pseudoscalar correlator due to saturation with one pion states (Fig. 2(a)) yields higher values (\( \simeq 15 \) times more) as compared to the exact calculations (Fig. 1). Thus the fermionic condensate model for QCD vacuum [9] does not give as high values for the pseudoscalar correlator as required by phenomenological results. The value we have used for \( \langle \bar{\psi}\psi \rangle \simeq (190 \text{ MeV})^3 \) is smaller than Shuryak’s value of \( (307.4 \text{ MeV})^3 \) [16] which appears in the parameterisation of the physical spectral density through the coupling constant [17]. With his value of \( \langle \bar{\psi}\psi \rangle \) the ratio \( R(x)/R_0(x) \) is shown in (Fig. 2(b)) which agrees with phenomenology.

VI. SUMMARY AND DISCUSSIONS

We have evaluated the mesonic correlators in this paper using a variational construct for the QCD vacuum. Except for the pseudoscalar channel the results show qualitative agreement with phenomenological results [5]. Following Shuryak [5,17], we also see that using current algebra approach the pseudoscalar correlator rises sharply with spatial separation. Let us recall that the current algebra result also follows from the approximation of saturating by one pion states in the normalisation of the pion state [9].

It might appear that by suitably changing the value of \( \langle \bar{\psi}\psi \rangle \) in the exact calculation one might be able to reproduce all the phenomenological results. Actually we find that it is
not so. In fact, it adversely affects the correlators in the other channel which can be seen in the exact expressions given in column 4 of Table I.

In view of these findings, it is not clear whether saturation of intermediate states by one pion states only in the evaluation of the correlator, is sufficiently well justified. We therefore think that a unified treatment of correlation functions in all the channels is still not available.

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TABLE I. Meson currents and correlation functions

| CHANNEL   | CURRENT                  | PARTICLE  | CORRELATOR $^a$ |
|-----------|--------------------------|-----------|-----------------|
|           | ($J^P$, MASS in MeV)     |           | $\left[ \frac{R(x)}{R_0(x)} \right]$ |
| Pseudoscalar | $J^p = \bar{u}\gamma_5d$ | $\pi^0(0^-, 135)$ | $\left[ 1 + \frac{1}{2}x^2I(x) \right]^2 + \frac{\pi}{8} \frac{x^6}{R^6} e^{-x^2/R^2}$ |
| Scalar    | $J^s = \bar{u}d$         | none($0^+$) | $\left[ 1 + \frac{1}{2}x^2I(x) \right]^2 - \frac{\pi}{8} \frac{x^6}{R^6} e^{-x^2/R^2}$ |
| Vector    | $J_\mu = \bar{u}\gamma_\mu d$ | $\rho^\pm(1^-, 770)$ | $\left[ 1 + \frac{1}{2}x^2I(x) \right]^2 + \frac{\pi}{4} \frac{x^6}{R^6} e^{-x^2/R^2}$ |
| Axial     | $J^5_\mu = \bar{u}\gamma_\mu\gamma_5 d$ | $A_1(1^+, 1100)$ | $\left[ 1 + \frac{1}{2}x^2I(x) \right]^2 - \frac{\pi}{4} \frac{x^6}{R^6} e^{-x^2/R^2}$ |

$^a$The integral I(x) is defined in Eq. (3.14)
FIG. 1. The ratio of the meson correlation functions in QCD vacuum to the correlation functions for noninteracting massless quarks, $\frac{R(x)}{R_0(x)}$, Plotted vs. distance $x$ (in fm)
FIG. 2. The pseudoscalar correlator plotted for our value of $\langle \bar{\psi}\psi \rangle = (190 \text{ MeV})^3$ in (a) and that of Shuryak $\langle \bar{\psi}\psi \rangle = (307.4 \text{ MeV})^3$ in (b)