Alpha Heating and Burning Plasmas in Inertial Confinement Fusion

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Abstract.
Assessing the degree to which fusion alpha particles contribute to the fusion yield is essential to understanding the onset of the thermal runaway process of thermonuclear ignition. It is shown that in inertial confinement fusion, the yield enhancement due to alpha particle heating (before ignition occurs) depends on the generalized Lawson parameter that can be inferred from experimental observables. A universal curve valid for arbitrary laser-fusion targets shows the yield amplification due to alpha heating for a given value of the Lawson parameter. The same theory is used to determine the onset of the burning plasma regime when the alpha heating exceeds the compression work. This result can be used to assess the performance of current ignition experiments at the National Ignition Facility.

1. Introduction
In inertial confinement fusion (ICF) [1, 2] a shell of cryogenic deuterium and tritium (DT) ice is imploded at high velocities (300 - 400 km/s) and low entropy to achieve high central temperatures and high areal densities. The final fuel assembly consists of a relatively low-density (30 -100 g/cc), high-temperature (3 - 7 keV) core -the hot spot- surrounded by a dense (300 -1000 g/cc), cold (200 - 400 eV) fuel layer - the compressed shell. Fusion alphas are produced in the D+T fusion reactions with an energy of 3.5 MeV and slow down primarily through collisions with the plasma electrons. The alpha-heated electrons transfer part of their energy to the DT ions, thereby increasing the fusion reaction rate. The process of alpha-energy deposition to the hot spot of a compressed ICF capsule is called alpha heating. Ignition is a direct consequence of alpha heating and of its feedback on the thermal energy and fusion reaction rate. When this feedback process becomes unstable, it leads to a thermal runaway within the central hot spot. An ignited hot spot drives a burn wave in the surrounding dense shell, leading to fusion-energy outputs in the megajoule range that greatly exceed the thermal and kinetic energy supplied to the DT fuel by the implosion alone (∼ tens of kilojoules).

To make progress toward ignition, it is important to assess the performance of current implosions with respect to the level of alpha particle heating. Heating by the alphas enhances the fusion yield to varying degrees depending on the fraction of deposited alpha particle energy to the total hot spot energy. Here we consider the yield amplification occurring before ignition takes place and therefore limit the amplification to within a factor of about 10 folds. Yield amplification ≤ 10 are of most interest to the current experiments at the National Ignition Facility (NIF) [3]. We show that the fusion yield enhancement due to alpha heating depends...
only on the Lawson parameter $P\tau$ [4] through a universal curve valid for any ICF ignition target from small to large, both directly and indirectly driven [5, 6]. The generalized Lawson parameter [7, 8] can be easily estimated from experimental observables and the yield enhancement due to alpha heating can be inferred from the universal curves [5, 6].

Recent experiments on NIF (High-Foot targets [9, 10]) have demonstrated significant alpha heating using indirect drive. To make progress toward ignition on NIF [3], it is crucial to be able to measure the level of alpha heating and to identify intermediate plasma states where the alpha heating is the leading source of input energy (alpha dominated or burning plasmas). In magnetic confinement fusion (MCF) [11], the burning plasma regime is identified through the thermonuclear $Q = \text{fusion power output/external power input}$. Since the alpha energy is about $1/5$ of the total fusion energy, a $Q=5$ denotes the state where the alpha power equal the input power. For convenience, in this paper we use $Q_\alpha = \text{alpha power/input power}=Q/5$ and define the onset of a burning plasma at $Q_\alpha=1$.

While determining $Q_\alpha$ for a steady state MCF device is straightforward, the definition for ICF is complicated by the transient nature of an ICF implosion and by the fact that the vast majority of the input energy does not reach the DT plasma. Since this paper is only concerned with the physics of burning plasmas and not with the prospects for fusion energy, the relevant input energy is the one reaching the DT plasma where the fusion reactions occur. Therefore, the parameters $Q$, $Q_\alpha$ used here refer to the DT fuel and should not be confused with the engineering-$Q$ used for fusion reactors [11].

2. A simple alpha-heating model

To illustrate this concept, we first consider the simplest ignition model valid for a static plasma at rest with a given pressure $P(0)$. For an ICF plasma, this represents the conditions of the central hot spot at stagnation before the alpha heating occurs. We refer to the initial conditions as $\text{no}\alpha$ conditions. We assume that the alpha particles are locally deposited at times $t > 0$, the plasma temperature is large enough that the fusion reaction rate can be approximated with $\langle \sigma v \rangle \sim T^2$ and the radiation losses are negligible compared to the alpha heating. In this case, the plasma pressure $P$ is obtained from the energy conservation

$$\frac{dP}{dt} = \frac{P}{\tau} \left( \frac{P\tau}{S_\alpha(T)} - 1 \right), \quad (1)$$

where $\tau$ is the energy confinement time (assumed constant), $S_\alpha \equiv 24T^2/\varepsilon_\alpha < \sigma v >$ and $\varepsilon_\alpha = 3.5$ MeV. Note that $S_\alpha \sim $ constant and Eq. (1) can be rewritten in the dimensionless form

$$\frac{\hat{P}}{dt} = \hat{P}[\chi_{\text{no}\alpha}\hat{P} - 1], \quad (2)$$

where $\chi_{\text{no}\alpha} \equiv P(0)/S_\alpha$, $\hat{P} = P/P(0)$, and $\hat{t} = t/\tau$. Note that the subscript $\text{no} - \alpha$ in $\chi$ is used because $P(0)$ is the pressure without alpha heating ($P_{\text{no}\alpha}$) and $\chi_{\text{no}\alpha}$ is the Lawson parameter $P_{\text{no}\alpha}\tau$ normalized with the term $S_\alpha$ (e.g. $\chi$ is the normalized Lawson parameter). The solution of Eq. (2) can be written in the following simple form

$$\hat{P} = \frac{1}{\chi_{\text{no}\alpha} - (\chi_{\text{no}\alpha} - 1)e^{\hat{t}}} \quad (3)$$

and exhibit an explosive singularity for $\chi_{\text{no}\alpha} = 1$. The singularity defines the onset of ignition and therefore $S_\alpha$ represents the minimum value of $P_{\text{no}\alpha}\tau$ required for ignition. Following Ref. [7, 8], the ignition parameter $\chi_{\text{no}\alpha}$ sets the ignition condition for the $\text{no}\alpha$ parameters (the hydrodynamic pressure in this case) and can be rewritten as

$$\chi_{\text{no}\alpha} = \frac{P_{\text{no}\alpha}\tau}{[P_{\text{no}\alpha}\tau]_{\text{ign}}^{\text{min}}} \quad (4)$$
where the denominator \( = S_α \) is the minimum requirement for ignition in terms of purely hydrodynamic parameters without the \( α \)-heating contribution. If the plasma pressure follows Eq. (1), one can easily calculate the total neutron yield \( Y \sim V_p τ P_{noa} \int_0^∞ \hat{P}^2 d\hat{t} \) where \( V_p \) is the plasma volume. Note that without \( α \)'s, the pressure decays according to Eq. (3) with \( χ_{noa} = 0 \) leading to \( P_{noa} = e^{-t} \). Therefore, one can compute a neutron yield with or without alphas and measure the effect of alpha heating through the amplification of the yield due to the alphas

\[
\hat{Y}_{amp} = \frac{Y_α}{Y_{noa}} = \frac{2}{χ_{noa}^2} \left[ ln \left( \frac{1}{1 - χ_{noa}} \right) - χ_{noa} \right].
\]

(5)

Note that the yield amplification is only a function of the \( noa \) Lawson parameter \( χ_{noa} \) and it can be inferred from Eq. (5) if \( χ_{noa} \) is known in the experiments. To relate \( χ_{noa} \) to the stagnation properties of an ICF implosion, we follow Ref. [8] and estimate the confinement time from Newton’s law \( M_{sh}R_{hs}' = 4πPR_{ha}/τ \) applied to the dense shell of mass \( M_{sh} \) surrounding the hot spot of radius \( R_{hs} \). By ordering \( R_{hs}' \sim R_{hs}/τ^2 \), \( M_{sh} \sim (ρ_{sh}Δ_{sh})R_{ha} \) (where \( Δ_{sh} \) and \( ρ_{sh} \) are the shell thickness and density), and the yield as \( Y \sim P^2V_{ha} τ \), we can rewrite \( χ \) as

\[
χ^3 \sim (ρ_{sh}Δ_{sh})^2 Y/M_{sh} \sim (ρR)^2 Y/M_{sh}.
\]

(6)

Assuming that most of the areal density comes from the dense shell, the scaling \( ρ_{sh}Δ_{sh} \sim ρR \) has been used in Eq. (6) and \( ρR \) is the measured total area density. Except for the mass dependence, Eq. (6) has the same form of the experimental Ignition Threshold Factor ITFx [12]. The absence of the mass in the ITFx is due to the fact that the ITFx was derived by fitting the ignition condition of gain unity for targets with the same DT mass. Since the areal density is weakly dependent on alpha heating for yield amplifications \( ≤ 10 \), Eq. (6) leads to two \( χ \) parameters depending on whether the yield with or without alphas is used. The neutron yield measured in the experiments includes the contribution from the alphas and therefore the only measurable \( χ \) is \( χ_α \) that includes the effects of alpha heating on the yield. However the \( χ \) that determines the yield amplification in Eq. (5) and the ignition conditions is \( χ_{noa} \). The constant of proportionality for \( χ \) in Eq. (6) can be obtained from Ref. [5] where the ignition parameter without alphas has been fit to a large database of hydrodynamic simulations leading to

\[
χ \simeq (ρR)^{0.61} \left( \frac{0.24Y_{16}}{M_{DT}^{unab}} \right)^{0.34}
\]

(7)

where \( ρR \) is in \( g/cm^2 \), \( Y_{16} \) is the yield in \( 10^{16} \), and \( M_{DT}^{unab} \) is the unablated DT mass in mg. Equation (7) provides a way to infer the parameter \( χ_α \) from the experimental observables (neutron yield, areal density and DT mass). To find the yield amplification from Eq. (5) requires \( χ_{noa} \) that is related to the measurable parameter \( χ_α \) using the relation from Eq. (6) leading to \( χ_{noa} \sim χ_α/Y_{amp}^{1/3} \). Substituting the latter into Eq. (5) leads to the following unique relation between the yield amplification \( Y_{amp} \) and the measurable parameter \( χ_α \)

\[
\hat{Y}_{amp}^{1/3} = \frac{2}{χ_α^2} \left[ ln \left( \frac{Y_{amp}^{1/3}}{\hat{Y}_{amp}^{1/3} - χ_α} \right) - χ_α \right].
\]

(8)

The yield in the presence of alpha heating can be obtained from Eq. (3) leading to

\[
Y_α = \frac{3}{2} V_p τ P_{noa}^2 \frac{1}{S_α ε_α} \left[ ln \left( \frac{1}{1 - χ_{noa}} \right) - χ_{noa} \right].
\]

(9)
The definition of \( \chi \) given in (4) and the yield amplification in (5) are used to rewrite the yield in the following form,

\[
Y_\alpha = \frac{3}{4} \frac{P_{n\alpha} V_p}{\varepsilon_\alpha} \chi_{n\alpha} \hat{Y}_{\text{amp}}.
\]  

(10)

Notice that \( 3/2P_{n\alpha} V_p \) is the hot spot energy from hydrodynamic compression which, in the limit of negligible radiation losses, comes from the pdV work of the imploding shell on the hot spot. It follows that the ratio of the alpha heating to the pdV work \( (Q_\alpha) \) can be determined from Eq. (10) leading to,

\[
Q_\alpha = \frac{1}{2} \chi_{n\alpha} \hat{Y}_{\text{amp}}(\chi_{n\alpha}) = \frac{1}{2} \chi_{n\alpha}(\hat{Y}_{\text{amp}}) \hat{Y}_{\text{amp}},
\]  

(11)

indicating that \( Q_\alpha \) is a unique function of the Lawson parameter \( \chi_{n\alpha} \) or alternatively a unique function of the yield amplification. A comprehensive analysis of the simplified alpha heating model is given in Ref. [6] where a more realistic expansion model is used for the hot spot.

3. Simulations and comparison with the comprehensive model

A more accurate model for the hot spot dynamics and the alpha heating process is given in Ref. [6, 5] where the temporal evolution of the hydrodynamic quantities is determined from the beginning of the shell deceleration phase up to peak compression and the following expansion phase. The fusion rate is approximated with \( \langle \sigma v \rangle \sim T^3 \) which is fairly accurate in the interesting range 3–7 keV. The hot spot model consists of the equations for conservation of mass, momentum and energy with an ideal gas equation of state. The shell is described as a compressible plasma divided into two regions by the return (or rebound) shock. The inner portion of the shell is called the “shocked shell” while the outer portion is the “free-fall shell.” The free-fall shell implodes inward with the implosion velocity while the shocked shell stagnates and converts its kinetic energy into internal energy. A fraction of the alphas produced within the hot spot escapes into the surrounding cold shell and deposits its energy near the inner shell surface thereby ablating shell material into the hot spot. This process is similar to the shell ablation driven by the heat flux leaving the hot spot. Details of the model are provided in Ref. [6] and the model is referred here as the “analytic model”. Similarly to Eq. (5), the solution of the comprehensive analytic model exhibits a singularity (ignition) when \( \chi_{n\alpha} \approx 1 \). After solving the analytic model with alphas \( (0 < \chi_{n\alpha} < 1) \) and without alphas \( (\chi_{n\alpha} = 0) \), the corresponding neutron yields with and without alphas are computed, and their ratio determines the yield amplification due to alpha heating. The solution of the analytic model is compared with the results of one- and two-dimensional hydrodynamic simulations using the codes LILAC [13] and DRACO [14], respectively. The properties with and without alphas are obtained from hydrodynamic simulations where the alpha deposition is turned on and off, respectively.

Figure 1 shows the yield amplification due to alpha heating as a function of the Lawson parameter without \( (\chi_{n\alpha}) \) and with \( (\chi_\alpha) \) alpha heating. The Lawson parameter is computed using the yield, areal densities and unablated DT mass from the simulations into Eq. (7). One- (black dots) and two-dimensional (red dots) simulations of ICF implosions are carried out with different target sizes ranging from 860 \( \mu \)m to 3.5 mm in diameter, laser energies from 30 kJ to 2 MJ, adiabats from 1 to 4 and velocities from 300 to 500 km/s. In the 2-D simulations, the level of implosion nonuniformities is varied by changing the initial DT ice roughness. Each simulation is carried out with and without alpha deposition, and the results are compared with the analytic model of Ref. [5] (solid curve). Note that the yield amplification is a strong function of the Lawson parameter \( \chi \) with or without alpha heating and it can be fitted with the simple analytic fitting formulas

\[
\hat{Y}_{\text{amp}} \approx \frac{1}{(1 - 1.04 \chi_{n\alpha})^{0.75}},
\]  

(12)
Figure 1. Yield amplification from alpha heating versus the Lawson parameter $\chi$ [Eq. (7)] without (left) and with (right) alpha-particle energy deposition. The black and red dots are from 1D and 2D simulations respectively. The simulations use directly driven targets with laser energies from 30 kJ to 2 MJ. The solid curves are from the analytic alpha-heating model of Ref. [5]. NIF high-foot implosion N140520 [10] exhibits $\chi_\alpha \simeq 0.93$, $\chi_{\text{no } \alpha} \simeq 0.66$ and a yield amplification of about $2.5 \times$.

$$\hat{Y}_{\text{amp}} \approx \exp(\chi_\alpha^{1.2}).$$

(13)

When compared to the results of Spears and Lindl [15] for the NIF indirect-drive-ignition target ($M_{DT} \approx 0.18$ mg), the yield amplification curves are in good agreement with the data points from the simulation database of that specific target. In this paper, the analysis is extended to all targets, large or small, directly or indirectly driven as long as the ignition parameter $\chi_\alpha$ is calculated using Eq. (7). For the high-foot shot N140520 [10] that achieved a yield of about $9 \times 10^{15}$ neutrons, areal density of $\approx 0.8$ g/cm$^2$, with $M_{DT} \approx 0.18$ mg, we find that $\chi_\alpha \approx 0.93$ and the yield amplification is $\sim 2.5$ (close to the value of 2.2 quoted in Ref. [10]). The corresponding $\chi_{\text{no } \alpha} \approx 0.66$ is inferred from the left plot and the yield amplification.

Following the discussion in section 2 [Eq. (11)], the implosion performance with respect to the burning plasma parameter $Q_\alpha$ is assessed through Fig. 2 showing the yield amplification as a function $Q_\alpha$. Here $Q_h^{hs} = 1/2E_\alpha/W_{hs}$ where $W_{hs}$ is the pdV work done by the shell on the hot spot and $E_\alpha$ is the total alpha particle energy. The factor 1/2 is due to the transient character of the implosion. Since the pdV work on the hot spot is positive only up to stagnation, we exclude the contribution from the alphas that are produced after stagnation (about 1/2 of the total). In addition to the hot-spot burning-plasma parameter $Q_h^{hs}$, Fig. 2 also shows the value of the total $Q_\alpha$ defined as the ratio of the alpha heating to the total compression work, $Q_{\alpha}^{\text{tot}} = 1/2E_\alpha/W_{\text{tot}}$ (where $W_{\text{tot}} = \text{pdV work to the hot spot and to the shell}$). The black dots are the results of 1D simulations using the code LILAC [13], the blue solid curve is the analytic model and the red curve is for a steady-state plasma. The red dot represents the High-Foot implosions N140520 [10] that achieved $Q_\alpha \approx 0.6$ and $Q_{\text{tot}} \approx 0.3$. The yellow dots in Fig. 2 show possible improvements of the High-Foot implosion achieving burning plasma conditions within the hot spot ($Q_h^{hs} \approx 1$) with about 50 kJ of total fusion-energy yield, as well as the more
Figure 2. Yield amplification from alpha heating versus the burning plasma parameters $Q_{\alpha}^{hs}$ and $Q_{\alpha}^{tot}$. The black dots are from 1D simulations. The solid blue curve is from the analytic model of Ref. [5]. The red curve is the steady state plasma result. The red dot represents the HF implosion 140520 [10]. The yellow dots show hypothetical improvements of the HF implosion to achieve the hot-spot burning plasma regime ($Q_{\alpha}^{hs} = 1$) and the state where the alpha heating equal the total input work to the hot spot and shell ($Q_{\alpha}^{tot} = 1$).

advanced burning plasma state where the alpha heating exceeds the total pdV work ($Q_{\alpha}^{tot} = 1$) with about 120 kJ of fusion yield. This material is based upon work supported by the DOE under Cooperative Agreements DE-FC02-04ER54789 (OFES) and under Award Number DE-NA0001944 (NNSA), the University of Rochester, and the New York State Energy Research and Development Authority. The support of DOE does not constitute an endorsement by DOE of the views expressed in this article.

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