International Trade and Strategic Privatization

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Abstract

The literature on mixed oligopoly does not consider the strategic interaction between governments when they decide whether to privatize their publicly-owned firms. In order to analyze this question, we consider two countries and assume that publicly-owned firms are less efficient than private firms. We obtain that when the marginal cost of the publicly-owned firms takes an intermediate value, each government wants it to be the government of the other country that privatizes its publicly-owned firm. In this case, only one government privatizes, and that government obtains lower social welfare and producer surplus than the other.

1. Introduction

One of the questions analyzed by the literature on mixed oligopoly is the decision by governments on whether to privatize their publicly-owned firms. This analysis usually considers one country and one publicly-owned firm (see, for example, De Fraja and Delbono, 1989, 1990; White, 1996; Pal and White, 1998; Willner, 1999) and there is thus no strategic interaction between governments. To fill this gap in the literature, we analyze whether publicly-owned firms are privatized when there is strategic interaction between governments.

This question is important because of the focus on privatization in policy debates, particularly in the EU. In spite of the consensus existing in Western European countries at the end of the 1980s in favor of a mixed economy including both publicly-owned and private firms (see Parker, 1998; McGowan, 1993), EU countries privatized part of their publicly-owned firms in the 1980s. In the 1990s the creation of the Single Market sparked further privatization. Privatization is a national issue and the various EU members have progressed at different rates. Simultaneously, the degree of free trade is increasing within the EU. The competitive environment created with the implementation of the Single Market has led Member States to take stock of the benefits they obtain by holding on to state ownership in some companies. Hence, the issue analyzed in this paper is a very relevant one.

The literature on mixed oligopoly (see De Fraja and Delbono, 1989, 1990) usually assumes one publicly-owned firm and a local market (i.e. there is only one country), showing that the government privatizes (does not privatize) the publicly-owned firm if the number of private firms is high (low) enough.1 Therefore, the publicly-owned firm is privatized if competition in the product market is high enough. However, in the real world there may be several publicly-owned firms competing with one another and with private firms in a world market. Therefore, there is strategic interaction between governments when they decide whether to privatize their publicly-owned firms.

In this paper, the authors consider a single market comprising two countries in which there is free trade. In each country there are one publicly-owned firm and \( n \) private...
firms. Firms have a constant marginal cost of production and the publicly-owned firm is less efficient than the private firms. Each government has to decide whether to privatize its publicly-owned firm. If a publicly-owned firm is privatized, its marginal cost of production is the same as that of private firms (i.e. there is an improvement of efficiency after privatization).

For the sake of simplicity and in order to concentrate on the decision whether to privatize when there is strategic interaction between governments, they do not consider any income other than profits which publicly-owned companies generate within their countries which might affect the decision whether to privatize (Willner, 1999). Nor do they consider decisions made by politicians to achieve political objectives (Boycko et al., 1996), the existence of economies of scale (Estrin and de Meza, 1995), differentiation in the goods produced by the companies (Cremer et al., 1991), free entry by private firms (Anderson et al., 1997), the fact that publicly-owned companies are directed by managers (Barros, 1995), the regulation of publicly-owned companies (Bös, 1998), mergers between private and publicly-owned firms (Bárcena-Ruiz and Garzón, 2003) or the partial privatization of those companies (Matsumura, 1998).

We obtain in this paper that there is strategic interaction between governments when they decide whether to privatize, if the marginal cost of publicly-owned firms takes an intermediate value. This value depends on the number of private firms existing in the world market and thus implies that the number of private firms also takes an intermediate value. In this case, only one government privatizes, and that government obtains lower social welfare and producer surplus than the other. Thus, each government wants it to be the government of the other country that privatizes its publicly-owned firm. In this case, there is strategic interaction between governments when they decide whether to privatize. If the publicly-owned firm is not inefficient enough, its profit is greater than the profit of each private firm. The output produced by the industry of the country that privatizes is lower than that of the other country.

When the marginal cost of publicly-owned firms takes an intermediate value, if one government decides whether to privatize before the other government does, the government deciding first would not privatize and the other government would privatize. The government deciding first would obtain greater social welfare than the other.

When the marginal cost of the publicly-owned firms is high or low enough, the result is driven by efficiency reasons. Private firms are more efficient than publicly-owned firms and, thus, there is an improvement in efficiency after privatization. In these two cases, we obtain the same result as when international trade is not considered (i.e. when there is a local market). Both governments privatize their publicly-owned firms when the marginal cost of those firms is high enough since, in that case, the inefficiency of the publicly-owned firms is high enough. When the marginal cost of publicly-owned firms is low enough neither government privatizes since, in this case, these firms are efficient enough.

The paper is organized as follows. Section 2 presents the model. Section 3 shows the results and conclusions are drawn in section 4.

2. The Model

We consider a world market comprising two countries, A and B. In each country there are one publicly-owned firm and n private firms producing a homogeneous good \( n \geq 1 \). The government of each country has to decide whether to privatize its publicly-owned firm. If one government privatizes, in that country there are \( n + 1 \) private firms.
The inverse demand function for the product in country \( k \) is \( p = a - 2y_k \), where \( p \) is the price of the good in the world market and \( y_k \) is the amount of the product sold in country \( k \) (\( k = A, B \)).

The world inverse demand function for the product is \( p = a - y_A - y_B \), where \( y_A + y_B = Q_A + Q_B \) and \( Q_k = q_{k0} + \sum_{i=1}^{n} q_{ki} \) (\( k = A, B \)). We denote by \( Q_k \) the output produced by the industry of country \( k \). Therefore, \( Q_A + Q_B \) is the output produced by world industry. We denote by \( q_{k0} \) the amount of the good sold in the world market by the publicly-owned firm 0 located in country \( k \), and by \( q_{ki} \) the amount of the good produced by private firm \( i \) located in country \( k \) (\( k = A, B; i = 1, \ldots, n \)).

There is free trade and, thus, consumers from both countries can buy the product from a domestic or a foreign firm. There are no transportation costs, and no possibility of discriminating between consumers from different countries. The consumer surplus in country \( k \), denoted by \( CS_k \), is \( CS_k = (y_k)^2 \), \( k = A, B \).

Private firms have a constant marginal cost of production which is normalized to zero. The marginal cost of production of the publicly-owned firm is constant and equal to \( c \), where \( 0 < c < c^* = a/(3 + 2n) \). The publicly-owned firm is less efficient than the private firms, so if it is privatized there is an improvement in efficiency.

The profit function of firm \( i \), located in country \( k \), is:

\[
\pi_{ki} = \left( a - q_{A0} - \sum_{i=1}^{n} q_{Ai} - q_{B0} - \sum_{i=1}^{n} q_{Bi} - c_i \right) q_{ki},
\]

\( i = 0, \ldots, n; \ k = A, B; \ c_i = 0, \ \forall i \neq 0; \ c_0 = c. \)

The social welfare function considered by government \( k \) comprises the consumer surplus in country \( k \), \( CS_k \), and the producer surplus in country \( k \), \( PS_k \). As usual, the producer surplus in country \( k \) is \( PS_k = \sum_{i=0}^{n} \pi_{ki} \). Thus, the social welfare function considered by government \( k \) can be expressed as:

\[
W_k = CS_k + PS_k, \ k = A, B.
\]

The objective of this paper is to analyze whether governments privatize their publicly-owned firms when there is strategic interaction between governments. Therefore, the authors propose a two-stage game with the following timing. In the first stage, governments decide, simultaneously, whether to privatize. In the second stage, each firm chooses its output level. They solve the game by backward induction from the last stage of the game to obtain a subgame perfect Nash equilibrium.

3. Results

Given that there are two publicly-owned firms that can be privatized, one in each country, there are four subgames in the first stage that, by symmetry, can be reduced to three. These three subgames are the following: neither government privatizes (denoted by superscript \( NN \)), one government does not privatize while the other does (denoted by superscripts \( NP \) and \( PN \), respectively), and both governments privatize (denoted by superscript \( PP \)).

Neither Government Privatizes its Publicly-Owned Firm

In the second stage of the game, private firm \( i \) located in country \( k \) chooses the output level, \( q_{ki} \), that maximizes its profit function. Publicly-owned firm 0 located in country \( k \) chooses the output level, \( q_{k0} \), that maximizes the social welfare function of government \( k \). Solving these problems simultaneously, we get the following result.
**Lemma 1.** When there is one publicly-owned firm in each country, in equilibrium, the output level of the firms, the output produced by the industry of each country, the consumer surplus, the profit of the firms, the producer surplus and social welfare in each country are, respectively:

\[
q_0^{NN} = \frac{a - c(1 + 2n)}{2}, \quad q_i^{NN} = c, \quad Q^{NN} = \frac{a - c}{2}, \quad CS^{NN} = \frac{(a - c)^2}{4}, \quad \pi_0^{NN} = 0,
\]

\[
\pi_i^{NN} = c^2, \quad PS^{NN} = nc^2, \quad W^{NN} = \frac{(a - c)^2 + 4nc^2}{4}, \quad i = 1, \ldots, n.
\]

**Only One Government Privatizes its Publicly-Owned Firm**

In the second stage of the game, each private firm chooses the output level that maximizes its profit function. Publicly-owned firm 0 located in the country that does not privatize (country \( k \)) chooses the output level, \( q_{00} \), that maximizes the social welfare function of government \( k \). Solving these problems simultaneously, we get the following result.

**Lemma 2.** When there is only one publicly-owned firm, in equilibrium, the output level of the firms, the output produced by the industry of each country, the consumer surplus, the profit of the firms, the producer surplus and social welfare in each country are, respectively:

\[
q_0^{NP} = \frac{3a - 4c(1 + n)}{5 + 2n}, \quad q_i^{NP} = q_j^{PN} = \frac{a + 2c}{5 + 2n}, \quad Q^{NP} = \frac{a(3 + n) - 2c(2 + n)}{5 + 2n},
\]

\[
Q^{PN} = \frac{(a + 2c)(1 + n)}{5 + 2n}, \quad CS^{NP} = CS^{PN} = \frac{(a(2 + n) - c)^2}{(5 + 2n)^2},
\]

\[
\pi_0^{NP} = \frac{(3a - 4c(1 + n))(a - c(3 + 2n))}{(5 + 2n)^2}, \quad \pi_i^{NP} = \pi_j^{PN} = \frac{(a + 2c)^2}{(5 + 2n)^2},
\]

\[
PS^{NP} = \frac{a^2(3 + n) - ac(13 + 6n) + 4c^2(3 + 6n + 2n^2)}{(5 + 2n)^2}, \quad PS^{PN} = \frac{(n + 1)(a + 2c)^2}{(5 + 2n)^2},
\]

\[
W^{NP} = \frac{a^2(7 + 5n + n^2) - ac(17 + 8n) + c^2(13 + 24n + 8n^2)}{(5 + 2n)^2},
\]

\[
W^{PN} = \frac{a^2(5 + 5n + n^2) + 2acn + c^2(5 + 4n)}{(5 + 2n)^2}, \quad i = 1, \ldots, n; \quad j = 0, \ldots, n.
\]

**Both Governments Privatize Their Publicly-Owned Firms**

In the second stage of the game, private firm \( i \) located in country \( k \) chooses the output level, \( q_{ki} \), that maximizes its profit function. Solving these problems simultaneously, we get the following result.

**Lemma 3.** When both governments privatize their publicly-owned firms, in equilibrium, the output level of the firms, the output produced by the industry of each country, the consumer surplus, the profit of the firms, the producer surplus and social welfare in each country are, respectively:
Once the different subgames have been solved, we have to solve stage one; i.e. we have to analyze whether each government privatizes or not.

The Decision by Governments Whether to Privatize Their Publicly-Owned Firms

By using lemmas 1, 2 and 3 we obtain the following results, which are useful in understanding the decision taken by governments.

**Proposition 1. In equilibrium:**

(i) $q_0^{NN} > q_0^{NP} > q_i^{PP} > q_i^{NP} = q_i^{PN} > q_{j}^{NN}$, $i = 1, \ldots, n; j = 0, \ldots, n$,

(ii) $Q^{NP} > Q^{NN} > Q^{PP} > Q^{PN}$, $2Q^{NN} > Q^{NP} + Q^{PN} > 2Q^{PP}$,

(iii) $CS^{NN} > CS^{NP} = CS^{PN} > CS^{PP}$.

By comparing the equilibrium output levels produced by each firm in the three subgames, we get that $q_0^{NN} > q_0^{NP} > q_i^{PP} > q_i^{NP} = q_i^{PN} > q_{j}^{NN}$. Given that publicly-owned firms choose the output level that maximizes the social welfare function of their governments, they are more aggressive in the product market than private firms. As a result, a publicly-owned firm produces more than a private firm, and the more publicly-owned firms there are in the market the lower the output level of the private firms will be. This implies that the output produced by the industry of a country is greater if there is a publicly-owned firm in that country than if that firm is privatized, independently whether the other country has a publicly-owned firm or not ($Q^{NI} > Q^{PI}, l = N, P$). Therefore, the quantity sold by the industry of a country in the world market depends on whether that country has a publicly-owned firm or not.

The output produced by the industry of a country when all its firms are private is greater if the other country privatizes ($Q^{PP} > Q^{PN}$) since in that case market competition decreases. Thus, the output produced by the industry of a country also depends on whether the other country has a publicly-owned firm or not.

The output level of a publicly-owned firm is greater if there are two publicly-owned firms in the market rather than one ($q_0^{NN} > q_0^{NP} > q_i^{PP} > q_i^{NP} = q_i^{PN} > q_{j}^{NN}$). This is because when one of the publicly-owned firms is privatized, the other reduces its output level (i.e. it behaves strategically) which allows the private firms to increase their output level. In this way, the profits of both the publicly-owned firm and the private firms increase.

When there is only one publicly-owned firm in the market, the output produced by the industry of the country that owns that firm is greater than the output of the industry of the other country ($Q^{NP} > Q^{PN}$), since a private firm produces less than a publicly-owned firm. If we consider as a reference the case in which there are two publicly-owned firms in the market, the industry of the country that privatizes reduces its market share, while the industry of the other country increases its market share ($Q^{NP} > Q^{NN} > Q^{PP}$). However, the output produced by the world industry is greater if there are two publicly-owned firms in the market instead of one ($2Q^{NN} > Q^{NP} + Q^{PN}$) since market competition is greater in the first case.
Given that publicly-owned firms are more aggressive in the product market than private firms, the highest world industry output is obtained when there are two publicly-owned firms in the market, and the lowest is obtained when there are none. Taking into account that the consumer surplus increases with the output of the world industry, we get that:

\[ \text{CS}_NN > \text{CS}_NP = \text{CS}_PN > \text{CS}_PP. \]

Let the value of parameter \( c \) such that \( p_0^{NP} = p_i^{PP} \). Let \( c^*_1 = \left[ \frac{a}{4(2 + n)} \right] \) the value of parameter \( c \) such that \( p_0^{NP} = p_i^{NP} \). By using lemmas 1, 2 and 3 we obtain the following lemma.

**Lemma 4.** In equilibrium:

(i) \( p_0^{NP} > p_i^{NP} = p_i^{PN} \) if and only if \( c < c^*_1, i = 1, \ldots, n; j = 0, \ldots, n \),
(ii) \( p_0^{NP} > p_i^{PP} \) if and only if \( c < c^*_2, i = 0, \ldots, n \),
(iii) \( 0 < c^*_2 < c^*_1 < c^* \).

**Proof.** See the appendix.

This lemma shows that when there is only one publicly-owned firm in the market, the profit obtained by that firm is greatest if parameter \( c \) is low enough (if \( c < c^*_2 \), both \( p_0^{NP} > p_i^{NP} = p_i^{PN} \) and \( p_0^{NP} > p_i^{PP} \)). In order to explain this result, we consider as a reference the case in which there are two publicly-owned firms in the market. If one of those firms is privatized, market competition decreases since although the output produced by each private firm increases the output of the remaining publicly-owned firm decreases and the profit of both the publicly-owned and private firms increase. Given that when there is only one publicly-owned firm in the market that firm produces more than the private firms, its profit is greater than that of the private firms if parameter \( c \) is low enough.

Let \( c^*_1 = \left[ \frac{a}{4(2 + n)} \right] \) the value of parameter \( c \) such that \( p_0^{NP} = p_i^{PP} \), and \( c^*_2 = \left[ \frac{a}{4(2 + n)} \right] \) the value of parameter \( c \) such that \( p_0^{NP} = p_i^{NP} \). By using lemmas 1, 2, 3 and 4 we obtain the following result.

**Proposition 2.** In equilibrium:

(i) if \( 0 < c \leq c^*_1^{PS} \), \( PS^{NP} \geq PS^{PP} > PS^{PN} > PS^{NN} \),
(ii) if \( c_1^{PS} < c \leq c^*_2^{PS} \), \( PS^{PP} > PS^{NP} \geq PS^{PN} > PS^{NN} \),
(iii) if \( c_2^{PS} < c < c^* \), \( PS^{PP} > PS^{PN} > PS^{NP} > PS^{NN} \),
(iv) \( 0 < c_1^{PS} < c^*_2 < c^*_1 = c_2^{PS} < c^* \).

**Proof.** See the appendix.

This proposition shows that the lowest producer surplus is obtained when there are two publicly-owned firms in the market, since in that case market competition is as great as possible.
Taking into account lemma 4, it is obtained that $PS^{NP}$ is greater than $PS^{PP}$ if parameter $c$ is low enough (if $c < c_1^{PS}$). Given that $c_1^{PS} < c_2^{PS}$, it is obtained that $\pi_0^{NP} > \pi_i^{PP}$ when $c < c_1^{PS}$, which implies that $PS^{NP}$ is greater than $PS^{PP}$ if $c < c_1^{PS}$.

When there is only one publicly-owned firm in the market, the producer surplus obtained in the country that owns that firm is greater than that obtained in the other country if parameter $c$ is low enough ($PS^{NP} > PS^{PP}$ if $c < c_2^{PS}$). This is because, as we have seen in lemma 4, $\pi_0^{NP} > \pi_i^{NP} = \pi_i^{PP}$ if and only if $c < c_1^{PS} = c_2^{PS}$.

Next we shall solve the first stage of the game. Let

$$c_1 = \frac{a(25 + 24n + 4n^2 - 2(5 + 2n)\sqrt{6 + 2n + n^2})}{5 + 104n + 84n^2 + 16n^3}$$

the value of parameter $c$ such that $W^{NN} = W^{PN}$. Let

$$c_2 = \frac{a(51 + 58n + 16n^2 - (5 + 2n)\sqrt{77 + 96n + 32n^2})}{2(3 + 2n)(13 + 24n + 8n^2)}$$

the value of parameter $c$ such that $W^{PP} = W^{NP}$. From lemmas 1, 2 and 3 we obtain the following result.

**Lemma 5.** In equilibrium:

(i) $W^{NN} > W^{PN}$ if and only if $c < c_1$,

(ii) $W^{NP} > W^{PP}$ if and only if $c < c_2$.

**Proof.** See the appendix.

In order to explain this lemma, it has to be noted that the output level of the publicly-owned firms (and, thus, the consumer surplus) decreases with their marginal cost (parameter $c$). As a result, if parameter $c$ is low enough, the consumer surplus has a greater weight in social welfare than the producer surplus since the efficiency of the publicly-owned firm is high enough. On the other hand, proposition 1 shows that $CS^{NN} > CS^{NP} = CS^{PN} > CS^{PP}$. Therefore, independently of the decision taken by one government, the other government does not privatize if parameter $c$ is low enough.

From proposition 2 and lemma 5 we obtain the following result.

**Proposition 3.** In equilibrium, neither government privatizes if $0 < c < c_1$, only one government privatizes if $c_1 \leq c < c_2$ and both governments privatize if $c_2 \leq c < c^e$, where $c^e > c_2^{PS} > c_2 > c_1 > c_1^{PS} > 0.6$

**Proof.** See the appendix.

When the marginal cost of publicly-owned firms is low enough ($0 < c < c_1$), in equilibrium, neither government privatizes. Given that parameter $c$ is low enough, the efficiency of publicly-owned firms is high enough and, thus, the consumer surplus has a greater weight than the producer surplus in social welfare. Proposition 1 shows that $CS^{NN} > CS^{NP} = CS^{PN} > CS^{PP}$ and, thus, independently of the decision taken by one government, the other government does not privatize if parameter $c$ is low enough. As a result, when $0 < c < c_1$ neither government privatizes.

When parameter $c$ increases, the inefficiency of the publicly-owned firms increases too. When this parameter is greater than $c_1$ but lower than $c_2$, the inefficiency of the
A publicly-owned firm is great enough and, thus, one of the governments privatizes. In this interval of values of parameter $c$, the government that privatizes obtains a lower producer surplus than the other government, but both of them obtain greater producer surplus than if neither publicly-owned firm is privatized ($PS^{NN} < PS^{PN} < PS^{NP}$). On the other hand, proposition 1 shows that $CS^{NN} > CS^{PN} = CS^{NP}$. Therefore, although by privatizing one publicly-owned firm the consumer surplus of the country that privatizes decreases, the increase in its producer surplus compensates for this. It must be noted that, when $c_1 \leq c < c_2$, only one publicly-owned firm is privatized. The government that owns the remaining publicly-owned firm does not privatize since in case of privatization, as parameter $c$ takes an intermediate value, the reduction in the consumer surplus of this country ($CS^{PP} < CS^{NP}$) has a greater weight than the increase in its producer surplus ($PS^{PP} > PS^{NP}$). As a result, when $c_1 \leq c < c_2$, only one publicly-owned firm is privatized and, thus, there are two equilibria. This result is due to the strategic interaction between governments when they decide whether to privatize.

When the marginal cost of publicly-owned firms is high enough ($c_2 \leq c < c^*$), the inefficiency of these firms is also high enough and thus, in equilibrium, both governments privatize. Given that parameter $c$ is high enough, the producer surplus has a greater weight than the consumer surplus in social welfare. From proposition 2, taking into account that $c_2 > c^{PS}$ we obtain that, in this zone, $PS^{PP}$ is greater than $PS^{NP}$ and, thus, the remaining publicly-owned firm is also privatized. As a result, when parameter $c$ is high enough, both governments privatize.

Alternatively, we could assume that if a publicly-owned firm is privatized, its marginal cost does not change and, thus, there is no improvement in efficiency after privatization (see, for example, Matsumura, 1998; White, 2001). Taking into account this assumption we obtain that the main results of the paper (propositions 3 and 4) hold. We assume in the paper that when a publicly-owned firm is privatized its efficiency improves (its marginal cost is 0 instead of $c$). This assumption favors the privatization of publicly-owned firms. However, the effect of this assumption is weaker than the other effects arising in the model. If a publicly-owned firm is privatized, its efficiency does not improve, this last effect disappears and, thus, governments have lower incentives to privatize. As a result, considering this assumption, we obtain that the range of values of parameter $c$ such that neither government privatizes is greater than that obtained in proposition 3. Likewise, the range of values of parameter $c$ such that only one government privatizes is also greater. Finally, the range of values of parameter $c$ such that both governments privatize is smaller.

From lemma 2 the following result is obtained.

**PROPOSITION 4.** If $c_1 \leq c < c_2$, $W^{NP}$ is greater than $W^{PN}$.

**PROOF.** See appendix.

The intuition of this result is the following. When $c_1 \leq c < c_2$, as the marginal cost of publicly-owned firms takes an intermediate value, only one publicly-owned firm is privatized. The remaining publicly-owned firm reduces its output level (i.e. it behaves strategically) which allows private firms to increase their output level. In this way, the profits of both the publicly-owned and the private firms increase. Therefore, the country that owns the publicly-owned firm obtains a greater producer surplus than the other country ($PS^{NP} > PS^{PN}$). However, both countries have the same consumer surplus ($CS^{NP} = CS^{PN}$) since the quantity of the good sold in each country is the same.
As a result, $W^{NP}$ is greater than $W^{PN}$ in the interval $c_1 \leq c < c_2$ and, thus, each government wants the other to privatize.

If the game were sequential (i.e. one government decides whether to privatize before the other government does), the government deciding first would not privatize its publicly-owned firm and the other government would privatize. The government deciding first would obtain greater social welfare than the other government, the profit of its publicly-owned firm would be greater than that of the private firms and its country would obtain a greater producer surplus than the other country.

4. Conclusions

The literature on mixed oligopoly has analyzed the decision by governments whether to privatize their publicly-owned firms. This analysis usually considers one country and one publicly-owned firm and thus there is no strategic interaction between governments. In this paper, the authors extend this analysis by studying the case in which there is strategic interaction between governments.

When the marginal cost of publicly-owned firms takes an intermediate value only one government privatizes. That government obtains lower social welfare and producer surplus than the other, and the output produced by its industry is also lower. In this case, there is strategic interaction between governments when they take the decision whether to privatize their publicly-owned firms.

We obtain that when the marginal cost of publicly-owned firms is high enough both governments privatize, since the producer surplus has a greater weight in social welfare than the consumer surplus. When the marginal cost of publicly-owned firms is low enough, neither government privatizes. In both cases the result is driven by efficiency reasons.

One possible extension of the paper would be to consider that multiple governments (such as local and central governments) interact with one another in a local market. A local government takes into account only the consumers and producers of its region, while a central government considers the consumers and producers of the whole country (which includes the consumers and producers considered by the local government). This issue is left for further research.

Appendix

Proof of Lemma 4

From lemma 2 we obtain that:

$$\pi^{NP}_0 - \pi^{NP}_i = \frac{(2a - c(1 + 2n))(a - 4c(2 + n))}{(5 + 2n)^2} > 0 \quad \text{if and only if } c < c^*_i.$$  

$$\pi^{NP}_0 - \pi^{NP}_i = \frac{4c^2(1 + n)(3 + 2n)^3 - ac(13 + 10n)(3 + 2n)^2 + 2a^2(1 + 8n + 4n^2)}{(3 + 2n)^2(5 + 2n)^2} > 0$$

if and only if $c < c^*_2$,

where

$$c^*_i - c^*_2 = \frac{a(5 + 2n)(-3(3 + 2n)(4 + 3n) + (2 + n)\sqrt{(3 + 2n)(19 + 18n)})}{8(1 + n)(2 + n)(3 + 2n)^2} > 0 \quad \text{for all } n.$$
Proof of Proposition 2

From lemmas 1, 2 and 3 we obtain that:

\[ PS^{PN} - PS^{NN} = \frac{a^2(1+n) + 4ac(1+n) - c^2(-4 + n(21 + 20n + 4n^2))}{(5 + 2n)^2} > 0 \quad \text{for all} \ c < c^*, \]

\[ PS^{NP} - PS^{NN} = \frac{(a - c(3 + 2n))(a(3 + n) - c(4 - 3n - 2n^2))}{(5 + 2n)^2} > 0 \quad \text{for all} \ c < c^*, \]

\[ PS^{NP} - PS^{PP} = \frac{2a^2 - c(3 + 2n)^2(a(13 + 6n) - 4c(3 + 2n(3 + n)))}{(3 + 2n)^2(5 + 2n)^2} > 0 \quad \text{if and only if} \ c < c_1^{PS}, \]

\[ PS^{NP} - PS^{PN} = \frac{(2a - c(1 + 2n))(a - 4c(2 + n))}{(5 + 2n)^2} > 0 \quad \text{if and only if} \ c < c_2^{PS}, \]

where

\[ c_2^{PS} - c_1^{PS} = \frac{a((3 + 2n)(13 + 10n) - (5 + 2n)\sqrt{(3 + 2n)(19 + 18n)})}{8(1 + n)(3 + 2n)^2} > 0 \quad \text{for all} \ n, \]

\[ c_2^{PS} - c_1^{PS} = \frac{1}{8(1 + n)(3 + 2n)^2(3 + 2n(3 + n))} \left( a(75n + 140n^2 + 84n^3 + 16n^4) - (5 + 2n) 
\right. \]

\[ (3 + 2n(3 + n))\sqrt{(3 + 2n)(19 + 18n)} + (1 + n)(3 + 2n)(5 + 2n)\sqrt{57 + 12(n(7 + 3n))} > 0 \quad \text{for all} \ n. \]

Proof of Lemma 5 and Proposition 3

From lemmas 1, 2 and 3 we obtain that:

\[ W^{NN} - W^{PN} = \frac{5a^2 - 2ac(25 + 4n(6 + n)) + c^2(5 + 4n(2 + n)(13 + 14n))}{4(5 + 2n)^2} > 0 \]

if and only if \( c < c_1. \)

\[ W^{PP} - W^{NP} = \frac{ac(3 + 2n)^2(17 + 8n) - a^2(13 + n(14 + 4n)) - c^2(3 + 2n)^2(13 + n(24 + 8n))}{(3 + 2n)^2(5 + 2n)^2} > 0 \]

if and only if \( c > c_2, \)

where:

\[ c_1 - c_2 = \frac{a(5 + 2n)}{2(3 + 2n)(13 + 8n(3 + n))(5 + 4n(2 + n)(13 + 4n))(339 + 100n - 396n^2)} \]

\[ - 304n^3 - 64n^4 - 4(3 + 2n)(13 + 8n(3 + n))\sqrt{6 + n(2 + n)} 
\]

\[ + (5 + 4n(2 + n)(13 + 4n))\sqrt{77 + 32n(3 + n)}), \]

which is positive for all \( n. \) Given that: (i) \( W^{NN} - W^{PN} > 0 \) if and only if \( c < c_1, \)

(ii) \( W^{PP} - W^{NP} > 0 \) if and only if \( c > c_2, \) and (iii) \( c_2 > c_1, \) it is straightforward to obtain

the result shown in proposition 3.

Comparing \( c_1 \) with \( c_1^{PS} \) and \( c_2^{PS} \) with \( c_2, \) respectively, we obtain that:

\[ c_1 - c_1^{PS} = \frac{1}{8(3 + 2n)(3 + 2n(3 + n))(5 + 4n(2 + n)(13 + 4n))} \]

\[ (a(5 + 2n)((3 + 2n)(107 + 36n - 28n^2 - 16n^3) - 16(3 + 2n)(3 + 2n(3 + n))\sqrt{6 + n(2 + n)} 
\]

\[ + (5 + 4n(2 + n)(13 + 4n))\sqrt{57 + 12n(7 + 3n))} > 0 \quad \text{for all} \ n. \]

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Proof of Proposition 4
From lemma 2 we obtain that:

\[ c_2^{PS} - c_2 = \frac{-a(5+2n)((3+2n)(11+4n) - 2(2+n)\sqrt{77 + 32n(3+n)})}{4(2+n)(3+2n)(13+8n(3+n))} > 0 \quad \text{for all } n. \]

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Notes

1. This analysis has been extended to study whether governments privatize their publicly-owned firms when there is international trade (see, for example, Fjell and Pal, 1996; Pal and White, 1998). However, although all these papers consider international trade in a framework of mixed oligopoly, they assume that there is only one publicly-owned firm and that firms sell their product only in a local market. Thus, there is no strategic interaction between governments when deciding whether to privatize their publicly-owned firms.

2. It can be shown that the main results of the model hold when a convex cost function is assumed (see note 6).

3. If we assume that privatization of publicly-owned firms does not improve their efficiency, the main results of the paper hold.

4. With no loss of generality, we assume that \( c < c^* \) to simplify the exposition of the results. This assumption ensures that publicly-owned firms produce more than private firms, a result usually obtained in the literature on mixed oligopoly (see, for example, De Fraja and Delbono, 1989, 1990). This assumption does not affect the main result of the model, and it permits us to simplify the comparison of the output of the firms under the different market structures.

5. This assumption is usually employed in mixed oligopoly literature to avoid a trivial solution. If the publicly-owned firm is at least as efficient as the private firms the publicly-owned firm will produce a quantity such that the market price equals its marginal cost, resulting in a publicly-owned monopoly (see Pal, 1998; Estrin and de Meza, 1995). Although there is empirical evidence that shows both the superior efficiency of private firms relative to comparable publicly-owned firms (Mueller, 1989; Vining and Boardman, 1992), and the improvement in efficiency after privatization (Kikeri et al., 1992; Megginson et al., 1994), empirical evidence also shows that differences in efficiency can go either way (Martin and Parker, 1997; Willner, 2001).

6. If firms have a convex production cost function and publicly-owned firms are more inefficient than private firms the main result of proposition 3 holds. For example, if we assume \( C(q_{ik}) = zq_{ik}^2 \), where \( z = 1/3 \) if the firm is publicly owned and \( z = 1/4 \) if the firm is privately owned, the following equilibrium is obtained: if \( n < 2 \), neither government privatizes; if \( 2 \leq n < 17 \), only one government privatizes; lastly, if \( n \geq 17 \), both governments privatize. If publicly-owned firms are as efficient as private firms we obtain, assuming that \( z = 1/4 \), that if \( n \leq 5 \) neither government privatizes; if \( n > 5 \) only one government privatizes.