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Thermoelastic interactions in a hollow cylinder due to a continuous heat source without energy dissipation

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Abstract
The linear generalized Green–Naghdi thermoelasticity theory without energy dissipation is employed. The study of thermoelastic interactions in a hollow cylinder under a continuous heat source is carried out. Firstly, Laplace and Hankel transforms are employed to solve the problem without the time domain. Then, the state space approach is employed to get the exact solution of the problem in the space domain. Once again, the inverse Laplace transforms is used to get the solutions in the time domain. Accurate terminologies for the temperature, thermoelastic potential, axial displacement, dilatation, and stresses are derived. Numerical outcomes for field variables are presented with the view of illustrating the theoretical results.

1. Introduction
A lot of investigators are interested to deal with the thermoelastic waves in the hollow cylinders, duty round line cylinders, thin-walled shells, telescopic cylinders, pressure vessels and pipes, cylindrical panels, etc. These structures may be undergoing various types of thermal load in the context of classical or and generalized coupled theories of thermoelasticity. In what follows the thermoelastic investigations concerning the response of the hollow cylinders will be highlighted.

Chandrasekharaiah and Srinath [1] used the model of Green and Naghdi (G–N) without energy dissipation to deal with the thermoelastic interactions due to a line heat source (One can refer to Green and Naghdi [2–4]). Allam et al [5] used the G–N theory to derive the thermal solution of an annular cylinder with axial magnetic field under harmonically varying temperature. Shao et al [6] presented the thermomechanical response of functionally graded (FG) annular cylinders under thermomechanical loads. Hosseini et al [7] and Hosseini [8] applied the finite element method (FEM) to solve the coupled thermoelastic response of inhomogeneous hollow cylinders in view of the G–N model without energy dissipation. Othman and Abbas [9, 10] derived the equations of generalized thermoelasticity of homogeneous and inhomogeneous hollow cylinders in the context of the G–N model of second and third types.

The theory of thermal stress in the context of the heat conduction equation with the Caputo time-fractional derivative of positive order greater than or equals 2 has been presented by Povstenko [11] to investigate the axisymmetric thermal stresses in a cylinder. Darabseh et al [12] used the Green and Lindsay (G–L) theory to study for the finite thermal wave speed of homogeneous and FG cylinders subjected to various kinds of loads. Shariyat [13] presented the nonlinear generalized and classical thermoelasticity for an FG cylinder with temperature-dependent material properties under thermomechanical shock at its edges. Hosseini and Abolbashari [14] presented an analytical solution to derive the thermal and mechanical waves of thick hollow cylinders using theory of coupled thermoelasticity without energy dissipation based on the G–N theory. Das et al [15] discussed the magneto-thermoelastic interactions in a hollow cylinder due to a thermal shock based on the three-phase-lag (TPL) theory. Islam et al [16] dealt with the thermoelastic interactions in an infinite hollow cylinder with traction-free at its edges. Fu et al [17] presented the coupled thermoelastic response of a multilayered or an FG hollow cylinder subjected to thermomechanical load based on the dual-phase-lag (DPL) theory.

The finite element method has been used by Zenkour and Abbas [18] to discuss the generalized thermoelastic problem of temperature-dependent hollow cylinders. Also, Zenkour and Abbas [19] discussed the
electro-magneto-thermoelastic behavior of an infinite FG hollow cylinder based on the G–N model without energy dissipation. Li et al [20] used the wave–FEM to discuss the thermoelastic waves without energy dissipation in hollow cylinders. Abbas [21] studied the exact solution for the vibration in a thermoelastic cylinder based on two-temperature G–N theory. Kumar et al [22] discussed the thermoelastic interactions in a temperature-dependent physical properties annular cylinder.

Refined, Zenkour [23, 24] presented a two-temperature multi PLs model for thermomechanical response of microbeams and plane wave propagation in thermoelastic medium. Biswas et al [25] dealt with different heat source behavior in a hollow cylinder based on the TPL generalized thermoelasticity theory. Zenkour [26, 27] presented a refined multi PLs model for thermal activation and photothermal waves of gravitated semiconducting half-spaces. Also, Zenkour and Kutbi [28] dealt with the multi thermal relaxations for thermomagnetic interactions in a thermoelastic half-space. Finally, Mashat and Zenkour [29] presented the modified DPL G–N theory for thermoelastic vibration of temperature-dependent nanobeams. Zenkour [30, 31] studied the magneto-thermal shock and wave propagation of gravitated half-spaces with a refined multi DPL model.

This paper applied the Green–Naghdi model to deal with the basic equations of a thermoelastic hollow cylinder. The closed-form solutions of the temperature, dilatation, radial displacement, thermoelastic potential, radial, hoop and axial stresses, are investigated. At last, a general discussion about the picked-up outcomes is accounted together with conclusions and future perspectives. Some reported results are given to help other researchers and technicians in their applications.

2. Fundamental equations

Let us consider a thermoelastic problem of an isotropic annular cylinder in the context of the Green–Naghdi model without energy dissipation. The cylindrical coordinates system \((r, \varphi, z)\) is chosen to address this problem in which \(z\)-axis is lying along the axis of the cylinder. The outer edge of the cylinder \((r = r_{\text{oa}})\) is tracional and the inner one \((r = r_i)\) may be heated or under a heat flux.

The displacements of the present axially symmetric cylinder are reduced to

\[ u_r = u(r, t), \quad u_{\varphi}(r, t) = u_z(r, t) = 0, \]

where \(r\) and \(t\) represent the radial coordinate and the time variable.

The strains \(\varepsilon_r, \varepsilon_{\varphi}\) and the cubical dilatation \(\varepsilon\) can be expressed as

\[ \varepsilon_r = \frac{\partial u}{\partial r}, \quad \varepsilon_{\varphi} = \frac{u}{r}, \quad \varepsilon = \varepsilon_r + \varepsilon_{\varphi}. \]

The dynamic equation without body force is expressed as

\[ (2\mu + \lambda) \left( \nabla^2 u - \frac{u}{r^2} \right) - \gamma \frac{\partial \theta}{\partial r} = \rho \frac{\partial^2 u}{\partial t^2}, \]

where \(\theta\) denotes the temperature-change (above a reference temperature \(T_0\)), \(\rho\) denotes the mass density, \(\gamma = (3\lambda + 2\mu)\alpha_1\alpha_3\) is the coefficient of volume expansion, \(\lambda, \mu\) are the usual Lame’s constants of the material of the cylinder, and \(\nabla^2\) is the Laplacian operator, in which

\[ \nabla^2 u = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right). \]

The constitutive equations for coupled thermoelastic cylinder are expressed as

\[ \sigma_r = 2\mu \frac{\partial u}{\partial r} + \lambda \varepsilon - \gamma \theta, \]

\[ \sigma_{\varphi} = 2\mu \frac{u}{r} + \lambda \varepsilon - \gamma \theta, \]

\[ \sigma_z = \lambda \varepsilon - \gamma \theta, \]

where \(\sigma_r, \sigma_{\varphi}\) and \(\sigma_z\) are the normal stress tensor components. The other shear stress components are vanished, i.e., \(\sigma_\varphi = \sigma_{rz} = \sigma_{z\varphi} = 0\).

The simple Green–Naghdi heat conduction equation without energy dissipation is proposed as (one can refer to Zenkour [26, 27]).

\[ k^* \nabla^2 \theta = \frac{\partial}{\partial t} \left[ \frac{\partial}{\partial t} \left( \rho C_v \theta + \gamma T_0 e \right) - \rho Q \right], \]

where \(k^*\) is the rate of thermal conductivity, \(C_v\) denotes the specific heat at constant volume and \(Q\) denotes the strength of the internal heat source per unit volume.
3. Formulation of the problem

It is reasonable to show the accompanying non-dimensional factors in the following parts:

\[
\{r', u'\} = \frac{1}{c_0 \eta} \{r, u\}, \quad t' = \frac{t}{\eta}, \quad \theta' = \frac{\gamma \theta}{\rho c_0^2},
\]

\[
\sigma_j' = \frac{\sigma_j}{\rho c_0^2}, \quad c_0^2 = \frac{\lambda + 2\mu}{\rho}, \quad Q' = \frac{\eta \gamma Q}{k^*}, \quad \eta = \frac{\rho c_0^2 C_{\varrho}}{k^*}.
\]  

(7)

All the governing equations, with the mentioned non-dimensional factors, are reduced to (dropping the dashed for accommodation)

\[
\sigma_r = \frac{\partial u}{\partial r} + \beta \frac{u}{r} - \theta,
\]

\[
\sigma_\varphi = \frac{u}{r} + \beta \frac{\partial u}{\partial r} - \theta,
\]

\[
\sigma_z = \beta \left( \frac{\partial u}{\partial r} + \frac{u}{r} \right) - \theta.
\]  

(8)
where

$$\beta = \frac{\lambda}{\lambda + 2\mu}, \quad \varepsilon = \frac{T_0 \gamma^2}{k^2 \rho}. \quad (11)$$

Introducing the thermoelastic potential $\phi$, defined by

$$u = \frac{\partial \phi}{\partial r}, \quad (12)$$

then the above governing equations are simplified by

$$\sigma_r = \frac{\partial^2 \phi}{\partial r^2} + \frac{\beta - 1}{r} \frac{\partial \phi}{\partial r},$$

$$\sigma_\varphi = \frac{\partial^2 \phi}{\partial r^2} + (\beta - 1) \frac{\partial^2 \phi}{\partial r^2},$$

$$\sigma_z = \frac{\partial^2 \phi}{\partial z^2} + (\beta - 1) \nabla^2 \phi. \quad (13)$$
Eliminating $\varphi$ from equations (14) and (15) yields

$$
\nabla^2 \varphi - \theta = \frac{\partial^2 \varphi}{\partial t^2},
$$

(14)

$$
\nabla^2 \theta = \frac{\partial}{\partial t} \left[ \frac{\partial}{\partial t} (\eta \theta + \varepsilon \nabla^2 \varphi) - Q \right].
$$

(15)

Eliminating $\theta$ from equations (14) and (15) yields

$$
\left( \nabla^4 - \zeta \nabla^2 \frac{\partial^2}{\partial t^2} + \eta \frac{\partial^4}{\partial t^4} \right) \varphi = - \frac{\partial Q}{\partial t},
$$

(16)

which represents a geometric function of $\phi$ where $\zeta = 1 + \eta + \varepsilon$. Once $\phi$ is controlled by illuminating this condition (under proper conditions), at that point $u$ and $\theta$ follow from equations (12) and (14) and $\sigma_r$, $\sigma_\varphi$, and $\sigma_z$ from equation (13).

4. Closed-form solution

The accompanying initial conditions should be applied to obtain the total solutions of the temperature, thermoelastic potential, dilatation, radial displacement, and stresses:
We assume that at first the cylinder is at time out in an undeformed and unstressed state at a constant reference temperature. At that point the underlying conditions for all factors are homogeneous. Additionally, we guess that the strength of the line heat source is expressed as

\[ Q(r, t) = Q_0 \delta(r) H(t), \]  

where \( \delta(r) \) represents the Dirac delta function, \( H(t) \) represents the Heaviside unit step function and \( Q_0 \) is a constant.

Taking the Laplace transform, defined by

\[ \hat{f}(s) = \mathcal{L}\{f(t)\} = \int_{\infty}^{0} f(t) e^{-st} \, dt, \]  

of equation (16) under homogeneous initial conditions, with \( Q \) given by equation (18), we obtain the equation

\[ (\nabla^4 - \zeta^2 \nabla^2 + \eta s^4) \bar{\phi} = -Q_0 \frac{\delta(r)}{r}. \]  

The overbar thermoelastic potential \( \bar{\phi} \) is the Laplace transform of the function \( \phi \), and \( s \) represents the transform parameter.
Equation (20) can be taken as

\[ (\nabla^2 - m_1^2)(\nabla^2 - m_2^2)\tilde{\phi} = -Q_0 \frac{\delta(r)}{r}, \]

where \( m_1 \) and \( m_2 \) are the roots of the biquadratic equation

\[ m^4 - \xi^2m^2 + \eta s^4 = 0. \]

Once again let us consider the Hankel transform, defined by

\[ \tilde{f}(\xi) = H_0 \{ f(r) \} = \int_0^\infty r J_0(\xi r) f(r) dr, \]

in which \( J_0 \) denotes the Bessel function of the first kind and of order zero. Applying the Hankel transform to both sides of equation (21), we get the equation

\[ (\xi^2 + m_1^2)(\xi^2 + m_2^2)\tilde{\phi}(\xi, s) = -Q_0. \]

Also, the inverse Hankel transform is defined by

\[ f(r) = H_0^{-1}\{ \tilde{f}(\xi) \} = \int_0^\infty \xi J_0(\xi r) \tilde{f}(\xi) d\xi. \]
Then, taking the above inverse of equation (24), we get the following solution for $\hat{\phi} = \hat{\phi}(r, s)$:

$$\hat{\phi} = \frac{Q_0}{m_1^2 - m_2^2} \sum_{j=1}^{2} C_j K_0(m_j r),$$  \hspace{1cm} (26)

where $K_0$ denotes the modified Bessel function of second kind and of order zero and $C_j$ are integration constants.

The following recurrence relations may be used:

$$\frac{d}{dr}(K_0(m_j r)) = m_j K_1(m_j r), \quad \frac{d}{dr}(K_1(m_j r)) = -m_j K_0(m_j r) - \frac{1}{r} K_1(m_j r),$$

$$\nabla^2(K_0(m_j r)) = m_j^2 K_0(m_j r), \quad \nabla^2(K_1(m_j r)) = \left(m_j^2 + \frac{1}{r^2}\right) K_1(m_j r),$$  \hspace{1cm} (27)

where $K_i$ denotes the modified Bessel function of order one and of first kind.

Using equation (26) and the recurrence relation (27), the Laplace transform version of the axial displacement appeared in equation (12) is given by

$$\tilde{u} = - \frac{Q_0}{m_1^2 - m_2^2} \sum_{j=1}^{2} m_j C_j K_1(m_j r).$$  \hspace{1cm} (28)

Figure 6. Variation of the axial stress $\hat{\sigma}_z$ according to case 2.
Also, the corresponding dilatation in the Laplace domain is given by

$$\ddot{\epsilon} = \frac{Q_0}{m_1^2 - m_2^2} \sum_{j=1}^{m_2^2} c_j K_0(m_j r).$$

(29)

Using equation (26) and the recurrence relation (27), the Laplace transform version of equation (14) yields

$$\ddot{\theta} = \frac{Q_0}{m_1^2 - m_2^2} \sum_{j=1}^{m_2^2} (m_j^2 - s^2)c_j K_0(m_j r).$$

(30)

Also, using equation (26) and the recurrence relations (27), the Laplace transform versions of stresses appeared in equation (13) yield

$$\ddot{\sigma}_r = \frac{Q_0}{m_1^2 - m_2^2} \sum_{j=1}^{m_2^2} c_j \left\{ s^2 K_0(m_j r) - \frac{\beta - 1}{r} m_j K_1(m_j r) \right\},$$

(31)

$$\ddot{\sigma}_r = \frac{Q_0}{m_1^2 - m_2^2} \sum_{j=1}^{m_2^2} c_j \left\{ (s^2 + (\beta - 1) m_j^2) K_0(m_j r) \right.$$  

$$\left. + \frac{\beta - 1}{r} m_j K_1(m_j r) \right\},$$

(32)

Figure 7. Variation of the thermoelastic potential $\hat{\phi}$ according to case 1.
Expressions (26) and (28)–(33) determine the thermoelastic potential $\tilde{\phi}$, axial displacement $u\bar{a}$, temperature $\bar{q}$, stresses $\bar{s}_r$, $\bar{s}_z$, and $\bar{\sigma}_j$ in the Laplace transform area. By processing the inverse Laplace transforms of these terminologies, we get the field factors as elements of $r$ and $t$.

By solving the biquadratic equation (22), we find that

\[ m_j = \kappa_j s, \quad \kappa_j^2 = \frac{1}{2} \left[ \zeta + (-1)^{j-1} \sqrt{\zeta^2 - 4\eta} \right], \quad j = 1, 2. \]  

Now, considering the following inverse Laplace transforms:

\[
\mathcal{L}^{-1}\{K_0(m_j r)\} = \bar{H}_j(r, t) = \left( t^2 - \kappa_j^2 r^2 \right)^{-\frac{1}{2}} H(t - \kappa_j r),
\]

\[
\mathcal{L}^{-1}\left\{ \frac{1}{s} K_0(m_j r) \right\} = \frac{1}{\kappa_j r} \left( t^2 - \kappa_j^2 r^2 \right)^{-\frac{1}{2}} H(t - \kappa_j r),
\]

\[
\mathcal{L}^{-1}\left\{ \frac{1}{s^2} K_0(m_j r) \right\} = \int_0^t H(\vartheta - \kappa_j r) \frac{t - \vartheta}{\sqrt{\vartheta^2 - \kappa_j^2 r^2}} d\vartheta,
\]

where $\vartheta$ is a dummy variable.
Now, substituting for $m_j$ in equations (26) and (28)–(33) and pleasing the inverse Laplace transforms of the outcomes with the help of equation (35), one obtains the following exact solution, in closed form, for all field quantities:

$$\phi(r, t) = \frac{Q_0}{\kappa_1^2 - \kappa_2^2} \sum_{j=1}^{2} C_j \hat{H}_j(r, t),$$

$$u(r, t) = -\frac{Q_0}{\kappa_1^2 - \kappa_2^2} \sum_{j=1}^{2} C_j \kappa_j \hat{H}_j(r, t),$$

$$\theta(r, t) = \frac{Q_0}{\kappa_1^2 - \kappa_2^2} \sum_{j=1}^{2} C_j (\kappa_j^2 - 1) \hat{H}_j(r, t),$$

$$e(r, t) = \frac{Q_0}{\kappa_1^2 - \kappa_2^2} \sum_{j=1}^{2} C_j \kappa_j^2 \hat{H}_j(r, t),$$

$$\sigma_r(r, t) = \frac{Q_0}{\kappa_1^2 - \kappa_2^2} \sum_{j=1}^{2} C_j \left\{ \hat{H}_j(r, t) - \frac{\beta - 1}{r} \kappa_j \hat{H}_j(r, t) \right\},$$

$$\sigma_\theta(r, t) = \frac{Q_0}{\kappa_1^2 - \kappa_2^2} \sum_{j=1}^{2} C_j \left\{ [1 + (\beta - 1) \kappa_j] \hat{H}_j(r, t) + \frac{\beta - 1}{r} \kappa_j \hat{H}_j(r, t) \right\},$$

Figure 9. Variation of the radial displacement $\hat{u}$ according to case 1.
Up to here the solution is completed. It is enough to determine the two parameters $C_j$ from the following boundary conditions:

**Case 1.**

$$
\frac{\partial \theta}{\partial r} = \theta_0 \text{ at } r = r_{in} \text{ and } \sigma_r = Q_0 \text{ at } r = r_{out},
$$

**Case 2.**

$$
\theta = \theta_0 \text{ at } r = r_{in} \text{ and } \sigma_r = Q_0 \text{ at } r = r_{out},
$$

where $r_{in}$ and $r_{out}$ are the inner and outer radii of the annular cylinder. Then, it is easily to give the stresses in terms of the radial displacement, temperature, and dilatation.
5. Validation and applications

Some application models will be displayed to place into recommendation the effect of various models on the field quantities. The material properties of the annular cylinder are stated according to the following values of parameters:

\[ \lambda = 7.76 \times 10^{10} \text{ N m}^{-2}, \quad \mu = 3.86 \times 10^{10} \text{ N m}^{-2}, \quad k^* = 1.2, \]
\[ \rho = 8954 \text{ kg m}^{-3}, \quad \alpha_L = 1.78 \times 10^{-5} \text{ K}^{-1}, \quad T_0 = 293 \text{ K}, \quad C_p = 383.1 \text{ J kg}^{-1} \text{ K}^{-1}. \]

For accommodation, the absolute values of the accompanying thermoelastic factors have been embraced to speak to the outcomes:

\[ \hat{\tau}(r) = \frac{\int \hat{\epsilon}(r) \, dr}{\rho \kappa_0} \]

Numerical results are obtained for \( \hat{\theta}_0 = 100, Q_0 = 10, r_{in} = 0.5 \) and \( r_{out} = 3 \). It is to be noted that there is no need to present the graphs of the dilatation \( \hat{\epsilon} \) since it will be similar to that of the temperature \( \hat{\theta} \). That because according to equations (38) and (39) since \( \kappa^2 \gg 1 \) then \( \epsilon(r, t) \) may be tend to \( \theta(r, t) \).

The outcomes of the variable field quantities due to the G–N theory with energy dissipation are displayed in figures 1–12 as 2D graphs through the radial direction of the hollow cylinder and as 3D graphs versus the...
distance and time. The two cases of boundary conditions are considered. In figures 1–6, we take into consideration four important positions $r_i (i = 1, 2, 3, 4)$ and their corresponding values of the time parameter $t_i$. These values are given, respectively, by $t_1 = 0.2$, $t_2 = 0.3$, $t_3 = 0.4$; and $t_4 = 2.625$, $t_5 = 0.5$.

Figures 1(a) and 2(a) show the 2D graph of the temperature $\hat{\theta}$ versus the radius of the cylinder at different values of the time parameter while figures 1(b) and 2(b) show the 3D graphs of the temperature $\hat{\theta}$ versus the time parameter and positions for the two cases. It is interested to see that the graph of the temperature $\hat{\theta}$ increases in the interval $r_{in} < r < r_i$ and vanishes identically for $r_i < r < r_{out}$ for each $t_i = t_i$. In case 1 for example, for $t = t_1 = 0.2$ the temperature $\hat{\theta}$ increases and reaches the highest value $\hat{\theta}_1 \cong 1.5411$ at the point that is scarcely overdue the $\hat{\theta}_1$-wavefront (at $r = r_1 = 1.05$), and it jumps down to the zero value past this wavefront. However, in case 2 for $t = 0.2$ the temperature $\hat{\theta}$ increases and reaches the highest value $\hat{\theta}_1 \cong 0.9031$ at the point that is scarcely overdue the $\hat{\theta}_1$-wavefront (at $r = r_1 = 1.05$), and it jumps down to the zero value past this wavefront.

Figures 3 and 4 shows that the hoop stress $\hat{\sigma}_r$ is compressive in the intervals $r_{in} < r < r_i$ and vanishes identically for $r_i < r < r_{out}$ for each $t_i = t_i$. Also, $\hat{\sigma}_r$ is no longer increasing in the interval $r_{in} < r < r_i$ and reaches the smallest value at the point that is scarcely overdue the $\hat{\theta}_1$-wavefront (at $r = r_i$), and it jumps up to the zero value past this wavefront. In case 1 for example, for $t = 0.2$ the hoop stress $\hat{\sigma}_r$ reaches the smallest value $\cong -0.78855$ at the point that is scarcely overdue the $\hat{\theta}_1$-wavefront (at $r = 1.05$). Also, for $t = 0.3$ the hoop stress $\hat{\sigma}_r$ reaches the smallest value $\cong -3.8825$ at the point that is scarcely overdue the $\hat{\theta}_2$-wavefront (at $r = 1.5756$). In addition, in case 2, for $t = 0.2$ the hoop stress $\hat{\sigma}_r$ reaches the smallest value $\cong -0.47$ at the point that is scarcely
overdue the $\hat{q}_i$-wavefront (at $r = 1.05$). Also, for $\tau = 0.3$ the hoop stress $\hat{\sigma}_z$ reaches the smallest value $\approx -0.87625$ at the point that is scarcely overdue the $\hat{q}_i$-wavefront (at $r = 1.5756$).

Once again, figures 5 and 6 shows that the axial stress $\hat{\sigma}_z$ is compressive in the intervals $r_{in} \leq r < r_{i}$ and vanishes identically for $r_{i} < r \leq r_{out}$ for each $\tau = \tau_i$. Also, $\hat{\sigma}_z$ is directly decreasing in the interval $r_{in} \leq r < r_{i}$ and reaches the smallest value at the point that is scarcely overdue the $\hat{q}_i$-wavefront (at $r = r_i$), and it jumps up to the zero value past this wavefront. In case 1 for example, for $\tau = 0.2$ the axial stress $\hat{\sigma}_z$ reaches the smallest value $\approx -0.7685$ at the point that lies just overdue the $\hat{q}_i$-wavefront (at $r = 1.05$). In case 2, for $\tau = 0.2$ the axial stress $\hat{\sigma}_z$ reaches the smallest value $\approx -0.45$ at the point that is scarcely overdue the $\hat{q}_i$-wavefront (at $r = 1.05$).

For the sack of completeness and comparisons the time parameters and positions of the $\hat{q}_i$-wavefront and the corresponding values of the temperature $\hat{\theta}$, the hoop stress $\hat{\sigma}_{z}$, and the axial stress $\hat{\sigma}_z$ are reported in table 1 for both cases.

Table 1. Values of the temperature $\hat{\theta}$, the hoop stress $\hat{\sigma}_z$, and the axial stress $\hat{\sigma}_z$ according to the $\hat{q}_i$-wavefront in terms of $\tau_i$ and $\tau_i$.

| $\tau_i$ | $\tau_i$ | $\tau_i$ | $\tau_i$ |
|---------|---------|---------|---------|
| 1.05    | 1.5756  | 2.1     | 2.625   |
| 0.2     | 0.3     | 0.4     | 0.5     |

Cases 1 $\hat{\theta}$ $\hat{\sigma}_z$ $\hat{\sigma}_z$ $\hat{\sigma}_z$
| 1.54097 | 7.76587 | 43.89725 | 18.82375 |
| -0.78855 | -3.88226 | -21.89725 | -9.39429 |
| -0.76850 | -3.87290 | -21.89192 | -9.38854 |
| 0.90290 | 1.73925 | 5.27185 | 1.41634 |
| -0.47016 | -0.87625 | -2.63407 | -0.70968 |
| -0.45028 | -0.86738 | -2.62911 | -0.70634 |

6. Conclusions

The aim of this article has been to contemplate some trademark highlights of the Green–Naghdi theory without energy dissipation by considering the cylindrical wave propagation through the hollow cylinder due to a continuous heat source as a test-example. The modified Green–Naghdi heat conduction equation without energy dissipation is proposed for this purpose. The system of differential coupled equation is solved, and all field variables are obtained for the hollow cylinder subjected to different edge conditions. The Hankel and Laplace integral transforms are successively used to obtain the solution of the problem. 2D and 3D sample plots are illustrated along the radial direction of the hollow cylinder. Some results are tabulated to serve as benchmark results for future comparisons with other investigators and to help for practical purpose. The outcomes of this article show that all field quantities, and especially, the temperature, the hoop and axial stresses are very sensitive to some positions at what so called the temperature-wavefront. This perception confirms that the Green–Naghdi theory is undoubtedly a generalized thermoelasticity theory. Therefore, this work is intended to show whatever outcomes for a commonplace issue of reasonable intrigue.

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