Study on the cutting mechanism of randomly deflected truncated cone shape single abrasive grain

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Abstract
Based on the random distribution characteristic of abrasive grains for the grinding wheel circumferential surface during the manufacturing process, considering the grinding parameters and deflection parameters comprehensively, a randomly deflected truncated cone shape single abrasive grain cutting simulation model was established. Then, studying its cutting mechanism, i.e., the trajectory length value by numerical method and formula method was compared, the consequence indicates that the error is extremely small; this demonstrates the accuracy of the numerical method. The calculation and analysis of the workpiece cutting surface topography and the undeformed chip shape were carried out; it shows that the cutting surface topography and the undeformed chip shape are coincident. And the calculation results of the model were compared to verify. The influence laws of each grinding process parameter on the material removal volume (MRV) and material removal rate (MRR) were investigated; the results display that the grinding method affects the changing pattern of MRV and MRR; when the grinding method was up-grinding, a high wheel speed or a small wheel diameter is able to enhance the MRR; the MRV and MRR are increments with the increasing workpiece feed speed or abrasive grain cutting depth; and the MRV and MRR decrease with the raising of the absolute value of single abrasive grain deflection angle. The situation is a similar or opposite trend in down grinding. This work can provide a theoretical reference for the selection and optimization of parameters and further lay a foundation for the research in the grinding mechanism of the grinding wheel.

Keywords Random deflection angle · Single abrasive grain cutting · Mechanism research

1 Introduction
Precision manufacturing equipment and its related frontier theory research have been a highlight area of concern in recent years. Grinding is an important precision and ultra-precision processing method in manufacturing; precision machining grinding process, generally, as the last process of machining, can effectively eliminate the processing defects that generated by the previous process, improve the quality of product processing, and play a very important role in ensuring the performance and service life of the product [1, 2]. Grinding technology, as one of the important developments of modern manufacturing technology, is widely used in aerospace, precision instruments, automobiles, and other manufacturing fields [3–7]. The grinding process can be regarded as the comprehensive effect of slipping, plowing, and cutting by a large number of discrete abrasive grains, which are distributed on the grinding wheel surface, and is a multi-edge cutting process that involves a large number of abrasive grains on the grinding wheel surface [8]. The grinding process is essentially a course in which a large number of discrete abrasive grains distributed on the surface of the grinding wheel interact with the processed component to achieve material removal; each abrasive grain on the surface of the grinding wheel makes a microscopic cutting into the worked material. Due to the characteristics of the grinding wheel manufacturing process, abrasive grains are arranged randomly on the circumferential surface of the grinding wheel. The position and angle (the normal direction which is...
Grinding is a traditional precision machining method and is one of the most critical surface finishing processes to meet desired part requirements for difficult-to-machine materials in manufacturing industries, which are carried out after the desired part requirements for difficult-to-machine materials are met. Grinding is a kind of rough machining and makes the workpiece meet the requirements for high surface quality [13–15]. Grinding is a traditional precision machining method and is one of the most critical surface finishing processes to meet desired part requirements for difficult-to-machine materials in manufacturing industries, which are carried out after the desired part requirements for difficult-to-machine materials are met. Grinding is a kind of rough machining and makes the workpiece meet the requirements for high surface quality [13–15]. Grinding is a kind of rough machining and makes the workpiece meet the requirements for difficult-to-machine materials in manufacturing industries, which are carried out after the desired part requirements for difficult-to-machine materials are met. Grinding is a kind of rough machining and makes the workpiece meet the requirements for high surface quality [13–15]. Grinding is a kind of rough machining and makes the workpiece meet the requirements for difficult-to-machine materials in manufacturing industries, which are carried out after the desired part requirements for difficult-to-machine materials are met. Grinding is a kind of rough machining and makes the workpiece meet the requirements for high surface quality [13–15]. Grinding is a kind of rough machining and makes the workpiece meet the requirements for difficult-to-machine materials in manufacturing industries, which are carried out after the desired part requirements for difficult-to-machine materials are met. Grinding is a kind of rough machining and makes the workpiece meet the requirements for high surface quality [13–15].

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Anderson et al. [10] compared the cutting action of two different single abrasive grain geometries by using experimental observations and a validated finite element model and found that both tools required approximately the same energy to shear a chip from a workpiece when friction was subtracted from the specific energy for material removal. Gu et al. [11], who conducted single abrasive grain grinding experiments on SiCp/Al composites to determine the grinding forces at different grinding process parameters, established a prediction model for the single abrasive grain grinding force to study the influence of the grinding process parameters and grinding grain angle on the grinding force of SiCp/Al composite, and the PSO-SVM algorithm–based grinding force prediction model can accurately predict the grinding force to study the influence of the grinding process parameters comprehensively. The theoretical model of cutting mechanism based on randomly deflected truncated cone shape single abrasive grain was established considering the grinding parameters and deflection parameters comprehensively. The fundamental theory research on the cutting behavior of a single abrasive grain was carried out and further laid a foundation for exploring the grinding mechanism of the grinding wheel.

At present, the methods for single abrasive grain cutting mechanism study are mainly focused on experimental research or finite element software simulation research, and the theoretical research on single abrasive grain cutting mechanism is relatively less. Based on the above, a theoretical model of cutting mechanism based on randomly deflected truncated cone shape single abrasive grain was established considering the grinding parameters and deflection parameters comprehensively. The fundamental theory research on the cutting behavior of a single abrasive grain was carried out and further laid a foundation for exploring the grinding mechanism of the grinding wheel.

Random deflection truncated cone shape single abrasive grain cutting model

Establish workpiece coordinate system

In order to describe the cutting process of the single abrasive grain with a randomly deflected angle, a coordinate system was established on the surface of the workpiece firstly, with the length of the workpiece as \( l_w \), the width as \( b_w \), and the height as \( z_w \); the subscript “\( w \)” represents the workpiece. The \( x \)-axis direction is the length direction of the workpiece, the \( y \)-axis direction is the width direction of the workpiece, and the \( z \)-axis direction is the height direction of the workpiece; the origin of the coordinate system \( O_w \) is located at the center of the upper surface of the workpiece. \( \Delta_{wx} \) denotes the discrete step size for discretization in the length direction of the workpiece; \( \Delta_{wy} \)
denotes the discrete step size for discretization in the width direction of the workpiece. \( x_w(i) \) denotes the \( x \)-coordinate of the location for the \( i \)th discrete point in the length direction of the workpiece, and \( y_w(j) \) denotes the \( y \)-coordinate of the location for the \( j \)th discrete point in the width direction of the workpiece.

The discretization process of \( x_w(i) \) was expressed by the formula below:

\[
x_w(i) = -\frac{l_w}{2} : \Delta w : \frac{l_w}{2}
\]  

(1)

In the length direction of the workpiece, the range of values of \( x_w(i) \) is the discrete point from the start point \(-\frac{l_w}{2}\) to the end point \( \frac{l_w}{2} \); discrete step is \( \Delta w \).

The discretization process of \( y_w(j) \) was expressed by the formula below:

\[
y_w(j) = -\frac{b_w}{2} : \Delta wy : \frac{b_w}{2}
\]  

(2)

In the width direction of the workpiece, the range of values of \( y_w(j) \) is the discrete point from the start point \(-\frac{b_w}{2}\) to the end point \( \frac{b_w}{2} \), and discrete step is \( \Delta wy \).

\( x_w(i, j) \) was denoted to express the \( x \)-coordinate of all discrete points on the upper surface of the workpiece, \( y_w(i, j) \) was denoted to express the \( y \)-coordinates of all discrete points on the upper surface of the workpiece, and \( z_w(i, j) \) was denoted to express the \( z \)-coordinates of all discrete points on the upper surface of the workpiece, and:

\[
\begin{align*}
x_w(i, j) &= x_w(i) \\
y_w(i, j) &= y_w(j) \\
z_w(i, j) &= 0
\end{align*}
\]  

(3)

Up to now, the workpiece coordinate system constructed completely, as shown in Fig. 1.

2.2 Cutting process modeling

The diameter of the grinding wheel is \( d_s \), the radius is \( r_s \), the circumferential linear velocity of the grinding wheel is \( v_s \), the circumferential angular velocity of the grinding wheel is \( \omega_s \), the center of the grinding wheel is \( O_s \), the grinding depth of the grinding wheel is \( a_e \), and the subscript “s” represents the grinding wheel; \( x_{so}, y_{so}, \) and \( z_{so} \) denote the coordinates of the center of the grinding wheel circle on the \( x \)-axis, \( y \)-axis, and \( z \)-axis, respectively. After establishing the workpiece coordinate system, the relative motion of the abrasive grain and the workpiece was transformed into the workpiece that did not move, and the grinding wheel moved; the workpiece feed speed is \( v_w \); at this time, the feed motion of the workpiece was transformed into the workpiece that did not move and the grinding wheel axis moved along the \( x \)-axis direction. The feed speed of the grinding wheel is \( v_f \), and \( v_w = -v_f \). Consequently, the abrasive grain moving trajectory was a combination in the rotational motion of the grinding wheel and the feed motion of the workpiece. The grinding wheel grinds the workpiece as shown in Fig. 2.
The abrasive grain was defined as a truncated cone shape, \( h_{\text{max}} \) was represented to the maximum cutting depth of the abrasive grain, \( h_g \) was represented to the height of the abrasive grain, and \( h_{gz} \) was represented to the height of the conical geometry (hereinafter called cone) formed by the truncated cone abrasive grain as a reference; the subscript “g” indicates the abrasive grain, and “z” the subscript indicates the cone; and \( d_{gm} \) was represented to the diameter of the abrasive grain’s large circle, \( d_{gn} \) was represented to the diameter of the abrasive grain’s small circle, the subscript “m” indicates the large circular surface of the abrasive grain, and the subscript “n” indicates the small circular surface of the abrasive grain. According to the geometric relationship of the truncated cone shape abrasive grain, \( h_{gz} \) could be expressed by \( h_g, d_{gm} \), and \( d_{gn} \); it was expressed by Eq. (4):

\[
 h_{gz} = \frac{d_{gm}h_g}{d_{gm} - d_{gn}} \tag{4}
\]

The parameter \( \beta \) was denoted to express the half-top angle of the cone (in radians) and was expressed by the following formula and is shown in Fig. 3.

\[
 \beta = \arctan\left( \frac{d_{gm}}{2h_{gz}} \right) \tag{5}
\]

The parameter \( \varphi \) was indicated to express the cut-in angle or cut-out angle of the single abrasive grain for cutting workpiece process (Fig. 3); \( \varphi \) could be expressed in Eq. (6) and is shown in Fig. 4.

\[
 \varphi = \arccos\left( \frac{z_{so}}{r_s + h_{gz}} \right) \tag{6}
\]

The parameter \( t \) was indicated to express the cutting time used by the abrasive grain from cut-in to cut-out for the workpiece, \( t_{\text{min}} \) is the time when the abrasive grain starts cutting, \( t_{\text{max}} \) is the time when the abrasive grain ends cutting, \( n_t \) denotes the number of discrete step in cutting time, \( \Delta t \) denotes the discrete step size for discrete processing of the cutting time, and \( t_i \) denotes the time corresponding to the \( i \)th discrete point in the cutting time of the abrasive grain, according to the cutting process of a single abrasive grain:

\[
\begin{align*}
 t_{\text{min}} &= 0 \\
 t_{\text{max}} &= \frac{2\varphi}{n_t} \\
 \Delta t &= \frac{t_{\text{max}} - t_{\text{min}}}{n_t} \\
 t_i &= t_{\text{min}} + \Delta t \cdot i 
\end{align*} \tag{7}
\]

At the cutting time \( t_i \) of the abrasive grain from cut-in to cut-out of the workpiece, the range of values of \( t_i \) is the discrete point from the start point \( t_{\text{min}} \) to the end point \( t_{\text{max}} \), discrete step is \( \Delta t \).

The \( v_f \) is positive when it coincides with the positive direction of the \( x \)-axis, the length traveled by the center of the wheel in the \( x \)-axis direction at the ends cutting time was expressed by \( l_{sox} \), and it was expressed by the formula below:

\[
 l_{sox} = v_ft_{\text{max}} \tag{8}
\]

At the time of \( t_i \), the coordinates of the center of the grinding wheel circle on the \( x \)-axis, \( y \)-axis, and \( z \)-axis were as follows:

\[
\begin{align*}
 x_{so} &= -\frac{l_{sox}}{2} + v_ft_i \\
 y_{so} &= 0 \\
 z_{so} &= r_s + h_g - a_e 
\end{align*} \tag{9}
\]
The trajectory of the cutting motion of the abrasive grain is shown in Fig. 5.

The position of the abrasive grain was changed by the rotation of the grinding wheel; the center of the large circle of the grinding grain is \( O_{gm} \); the top points of the cone are \( A_{gz} \), \( x_{gm} \), \( y_{gm} \), and \( z_{gm} \) that denoted the \( x \)-, \( y \)-, and \( z \)-coordinates of \( O_{gm} \), respectively; and \( x_{gza} \), \( y_{gza} \), and \( z_{gza} \) denoted the \( x \)-, \( y \)-, and \( z \)-coordinates of \( A_{gz} \), respectively. Then, at the time of \( t_i \), the coordinates of \( O_{gm} \) and \( A_{gz} \) were as follows:

\[
\begin{align*}
    x_{gm} &= x_{so} + r_s \sin (\varphi - \omega_i t_i) \\
    y_{gm} &= y_{so} \\
    z_{gm} &= z_{so} - r_s \cos (\varphi - \omega_i t_i) \\
\end{align*}
\]

\[
\begin{align*}
    x_{gza} &= x_{so} + (r_s + h_{gz}) \sin (\varphi - \omega_i t_i) \\
    y_{gza} &= y_{so} \\
    z_{gza} &= z_{so} - r_s + h_{gz} \cos (\varphi - \omega_i t_i) \\
\end{align*}
\]
The geometric relationship of the above formulas was as shown in Fig. 6: Fig. 6a shows up-grinding, and Fig. 6b shows down-grinding. \( O_{gm} \) was defined to express the center of the small circle of the grinding grain.

The position of the abrasive grain would change under the deflection of its axis around \( O \) of the small circle of the grinding grain. So, at the moment \( t \) to express the deflection angle was positive when the \( xOz \) plane; the angle value was positive when the \( x \) axis, \( y \) direction was the same as the positive direction of the \( x \) axis, and the range of the angle is in \( \left[ -\pi/2, \pi/2 \right] \), as shown in Fig. 7.

After deflecting angle \( \delta_1 \), \( \delta_2 \) was defined to express the deflection angle by the rotation of the abrasive grain axis projected straight line in the \( xOz \) plane around the \( y \)-axis, and the subscript “2” indicates the plane that perpendicular to the plane \( xOz \) and passing through the line \( O_{gm}O_s \). The angle value was positive when the \( \delta_2 \) deflection direction was the same as the positive direction of the \( y \)-axis, and the range of the angle is in \( \left[ -\pi/2, \pi/2 \right] \), after deflection, the projection line of the axis in the \( xOz \) plane was co-linear with the line \( O_{gm}O_s \), as shown in Fig. 7.

The position of \( A_{gzm} \) was transformed to \( A_{gzm2} \), when the deflected angle was \( \delta_2 \); \( x_{gzm2}, y_{gzm2}, z_{gzm2} \) were denoted to express the \( x \)-, \( y \)-, and \( z \)-coordinates of \( A_{gzm} \), respectively. So, at the moment \( t \), the coordinates of \( A_{gzm} \) were as follows:

\[
\begin{align*}
x_{gzm} &= x_{gm} \pm h_{gz} \cos \delta_1 \sin (\varphi - \omega t_i) \\
y_{gzm} &= y_{gm} + h_{gz} \sin \delta_1 \\
z_{gzm} &= z_{gm} - (r_s + h_{gz} \cos \delta_1) \cos (\varphi - \omega t_i)
\end{align*}
\]  

(12)

where “+” represents down-grinding, and “−” represents up-grinding.

On the basis of a deflected angle \( \delta_1 \), the position of \( A_{gzm} \) was transformed to \( A_{gzm2} \); when the deflected angle was \( \delta_2 \); \( x_{gzm2}, y_{gzm2}, z_{gzm2} \) were denoted to express the \( x \)-, \( y \)-, and \( z \)-coordinates of \( A_{gzm2} \), respectively. So, at the moment \( t \), the coordinates of \( A_{gzm2} \) were as follows:

\[
\begin{align*}
x_{gzm2} &= x_{gm} \pm h_{gz} \cos \delta_1 \sin (\varphi - \omega t_i) \pm \delta_2 \\
y_{gzm2} &= y_{gm} \\
z_{gzm2} &= z_{gm} - h_{gz} \cos \delta_1 \cos (\varphi - \omega t_i) \pm \delta_2
\end{align*}
\]  

(13)

where “+” represents down-grinding, and “−” represents up-grinding.

The workpiece topography could be represented by a matrix of points with different heights [24]. The number of discrete points in the length and width directions of the workpiece was denoted by \( n_l \) and \( n_b \), respectively. The height of each point was stored in a two dimensional matrix \( Z[n_l,n_b] \). Before grinding, the height of all the points was zero, i.e., all discrete points were on the surface of the workpiece. The distance between two adjacent points was 1/100 of the \( d_{gm} \):

\[
\begin{align*}
\Delta w_x &= \frac{1}{100} d_{gm} \\
\Delta w_y &= \frac{1}{100} d_{gm}
\end{align*}
\]  

(14)

The point numbers were as follows:

\[
\begin{align*}
n_l &= \left[ \frac{L}{\Delta w_x} \right] \\
n_b &= \left[ \frac{B}{\Delta w_y} \right]
\end{align*}
\]  

(15)

The square brackets indicate rounding.
$O_{gm}$ was selected as the reference point in the cutting process. The coordinates of $O_{gm}$ could be expressed by $(x_{gm,i}, y_{gm,i}, z_{gm,i})$. According to the abrasive grain position, the limit values $X_{\text{min}}, X_{\text{max}}, Y_{\text{min}},$ and $Y_{\text{max}}$ of the coordinates on the workpiece surface points, for heights that might be affected by the abrasive grain, could be found in the matrix $Z$ and could be easily determined by the following:

\[
\begin{align*}
X_{\text{min}} &= [x_{gm,i} - h_{g}] \\
X_{\text{max}} &= [x_{gm,i} + h_{g}] \\
Y_{\text{min}} &= [y_{gm,i} - h_{g}] \\
Y_{\text{max}} &= [y_{gm,i} + h_{g}]
\end{align*}
\]  

The square brackets indicate rounding.
The region of \([X_{\text{min}}, X_{\text{max}}] \times [Y_{\text{min}}, Y_{\text{max}}]\) was scanned in the program point-by-point to calculate.

### 2.3 Cutting process calculation

In the cutting process of the abrasive grain, the workpiece was defined as an ideal body (without considering the effect of deformation, residual stress, etc.). And the cutting was simply regarded as the relative motion of the abrasive grain and the workpiece, i.e., the type of contacting abrasive grain in the grinding arc was only cutting abrasive grain, and the interference part between the abrasive grain and the workpiece in the process of movement was all regarded as the removal of material. After being cut by the abrasive grain, the \(z\)-coordinate of the cut point on the workpiece surface would change, while \(x\)-coordinate and \(y\)-coordinate stay the same; \(x_w, y_w, \) and \(z_w\) denote the \(x\)-, \(y\)-, and \(z\)-coordinates of discrete points on the workpiece surface. According to the

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**Fig. 7** Schematic diagram of the grinding grain deflection angle \(\delta_1\). a Plane 1 axonometric view, b plane 1 front view

**Fig. 8** Schematic diagram of the grinding grain deflection angle \(\delta_2\). a Plane 1 axonometric view, b plane 1 front view
analysis above, it can solve the new z-axis coordinates of the cut point on the workpiece surface after abrasive grain cutting and use \( z_{wn} \) to represent. The positions of discrete points on the workpiece surface after cutting were in two cases, the first situation was located on the conical surface of the abrasive grain, and the second situation was located on the plane of small circular surface of the abrasive grain.

In the first case, the axis of the cone after deflection \( \delta_1 \) and \( \delta_2 \) is denoted by \( r_1 \) and could be expressed as follows:

\[
 r_1 = [x_{gm21} - x_{gm}, \ y_{gm21} - y_{gm}, \ z_{gm21} - z_{gm}] \tag{17}
\]

The new position and its coordinates of the cut point were expressed in terms of \( C_{gz}(x_w, y_w, z_1) \); the straight line \( C_{gz} - A_{gz12} \) is denoted by \( r_2 \) and could be expressed as follows:

\[
 r_2 = [x_{gz21} - x_w, \ y_{gz21} - y_w, \ z_{gz21} - z_1] \tag{18}
\]

The angle between the \( r_1 \) and \( r_2 \) was equal to the half-top angle of the cone, which could be expressed as follows:

\[
 \arccos \frac{r_1 \cdot r_2}{|r_1||r_2|} = \beta \tag{19}
\]

Organize Eq. (19) and obtain the following:

\[
 \cos^2 \beta = \frac{(r_1 \cdot r_2)^2}{|r_1|^2|r_2|^2} \tag{20}
\]

Since the only unknown variable in Eq. (20) was \( z_1 \); thus, the constant part in Eq. (20) could define by other characters:

\[
 \begin{align*}
 E_1 &= \cos^2 \beta |r_1|^2 \\
 E_2 &= (x_{gz21} - x_w)(x_{gz21} - x_w) + (y_{gz21} - y_w)(y_{gz21} - y_w) \\
 E_3 &= (z_{gz21} - z_{gm})z_{gz21} \\
 E_4 &= z_{gz21} - z_{gm} \\
 E_5 &= (x_{gz21} - x_w)^2 + (y_{gz21} - y_w)^2 + z_{gz21}^2 \\
 E_6 &= E_4 - E_1 \\
 E_7 &= 2E_1z_{gz21} - 2(E_2 + E_3)E_4 \\
 E_8 &= (E_2 + E_3)^2 - E_1E_5
 \end{align*} \tag{21}
\]

The formula could be transformed as follows:

\[
 E_6z_1^2 + E_7z_1 + E_8 = 0 \tag{22}
\]

The new z-axis coordinate values would be obtained after computing Eq. (22), which were named \( z_{11} \) and \( z_{12} \), due to the \( \cos^2 \beta \) having the following characteristics in Eq. (20), as shown in Fig. 9.

\[
 \cos^2 \beta = (-\cos(\pi - \beta))^2 = \cos^2(\pi - \beta) \tag{23}
\]
From Eq. (23) and Fig. 9, the angle between \( r_1 \) and \( r_2 \) was either \( \beta \) or \( (\pi - \beta) \), corresponding to the two solutions \( z_{11} \) and \( z_{12} \). Because the new position makes the angle \( \beta \) between \( r_1 \) and \( r_2 \), thus, the unqualified solution \( z_{12} \) should be discarded.

In the second case, as known from the previous case, \( r_1 \) was the normal vector of the large and small circular surfaces of the abrasive grain. \( C_{gm}(x, y, z) \) were expressed at any point and its coordinates on the large circular surface of the abrasive grain. \( L(x, y, z) \) was the plane formula of the large circular surface. From the formula of the plane equation:

\[
L(x, y, z) : (x_{gm} - x_{gz12})(x - x_{gm}) + (y_{gm} - y_{gz12})(y - y_{gm}) + (z_{gm} - z_{gz12})(z - z_{gm}) = 0
\]

(24)

Organize Eq. (24) and obtain the following:

\[
L(x, y, z) : (x_{gm} - x_{gz12})x + (y_{gm} - y_{gz12})y + (z_{gm} - z_{gz12})z = ((x_{gm} - x_{gz12})x_{gm} + (y_{gm} - y_{gz12})y_{gm} + (z_{gm} - z_{gz12})z_{gm})
\]

(25)

The new position and its coordinates of the cut point were expressed in terms of \( C_{gm}(x_w, y_w, z_2) \). The distance from the point \( C_{gm} \) to the large circular surface of the abrasive grain was equal to \( h_g \). So:

\[
h_g = \sqrt{\frac{|L(x_w, y_w, z_2)|}{(x_{gm} - x_{gz12})^2 + (y_{gm} - y_{gz12})^2 + (z_{gm} - z_{gz12})^2}}
\]

(26)

From Eq. (25):

\[
L(x_w, y_w, z_2) : (x_{gm} - x_{gz12})x_w + (y_{gm} - y_{gz12})y_w + (z_{gm} - z_{gz12})z_2 = ((x_{gm} - x_{gz12})x_{gm} + (y_{gm} - y_{gz12})y_{gm} + (z_{gm} - z_{gz12})z_{gm})
\]

(27)

Since the only unknown variable in Eq. (26) was \( z_2 \), thus, the constant part in Eq. (26) could define by other characters:

\[
\begin{align*}
F_1 &= (x_{gm} - x_{gz12})x_w + (y_{gm} - y_{gz12})y_w \\
F_2 &= (x_{gm} - x_{gz12})y_{gm} + (y_{gm} - y_{gz12})y_{gm} + (z_{gm} - z_{gz12})z_{gm} \\
F_3 &= (x_{gm} - x_{gz12})z_{gm} \\
F_4 &= \sqrt{(x_{gm} - x_{gz12})^2 + (y_{gm} - y_{gz12})^2 + (z_{gm} - z_{gz12})^2} \\
F_5 &= 2(F_1 - F_2)F_3 \\
F_6 &= (F_1 - F_2)^2 - F_4h_g^2
\end{align*}
\]

(28)

The formula could be transformed as follows:

\[
F_3z_2^2 + F_5z_2 + F_6 = 0
\]

(29)

The new z-axis coordinate values would be obtained, after computing Eq. (29), which were named \( z_{21} \) and \( z_{22} \), as shown in Fig. 10.

Fig. 10 The location of \( z_{21} \) and \( z_{22} \) on the small circular surface

From Fig. 10, the planes with distance \( h_g \) to the large circular surface of the abrasive grain were the small circular surface plane and its symmetrical mirror plane about
the large circular surface. The solution in the mirror plane was $z_{21}$, and the solution in the small circular plane was $z_{22}$. Because the new location was on the small circular surface of abrasive grain, thus, the unqualified solution $z_{21}$ should be discarded.

Based on the above, the geometric relationship between $z_{11}$ and $z_{22}$ is as shown in Fig. 11.

According to the geometric relationship represented in Fig. 11, the positions of discrete points on the workpiece surface after cutting were in two cases, located either on the conical surface or on the plane of the small circular surface, and the new point must be located on the truncated conical surface and the small circular surface. Therefore, it was clear that:

$$z_{wn} = \max(z_{11}, z_{22})$$  \hspace{1cm} (30)

The discrete points on the workpiece surface can be divided into the cut and uncut points. After the abrasive grain cutting, the $z$-coordinate of the cut point on the workpiece surface was become $z_{wn}$ while the uncut point remains the same.

### 2.4 Case model calculation and study

According to Eqs. (7)–(13), the time interval $[t_{\text{min}}, t_{\text{max}}]$ had been discretized; a series of discrete points on the abrasive grain cutting trajectory could be obtained. The adjacent time points could be expressed by $t_i$ and $t_{i+1}$, $t_{i+1} = t_i + \Delta t$. The coordinates of the corresponding $A_{gz12}$ positions were $A_{gz12,i}(x_{gz12,i}, y_{gz12,i}, z_{gz12,i})$ and $A_{gz12,i+1}(x_{gz12,i+1}, y_{gz12,i+1}, z_{gz12,i+1})$; then, the distance between the two positions was used $\Delta l_i$ to express the following:

$$\Delta l_i = \sqrt{(x_{gz12,i+1} - x_{gz12,i})^2 + (y_{gz12,i+1} - y_{gz12,i})^2 + (z_{gz12,i+1} - z_{gz12,i})^2}$$  \hspace{1cm} (31)

The length of the abrasive grain cutting trajectory was used $L_g$ to express the following:

![Fig. 11 The geometric relationship between solutions](image-url)
The following were calculation examples and verification: According to Eqs. (1)–(13), a calculation program was compiled and run with the parameters shown in Table 1. The calculation results are shown in Table 2. The calculation results in Table 2 represent verification of this work. The calculation results were compared with the theoretical model proposed by Malkin [14] for common plane surface grinding conditions. It should be noted that the grinding contact length obtained by Malkin’s formula was for an abrasive grain moving from the lowest point to the endpoint of the cut-out stage, and the shape of the abrasive grain is spherical, taking the up-grinding as an example. Therefore, the abrasive grain cutting trajectory length calculated in this work was twice that of Malkin’s formula.

Malkin’s formula:

$$L_g = \lim_{\Delta t \to 0} \sum_{j=0}^{t_{\text{max}}} \Delta l_j$$

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Malkin’s formula:

$$L_g = \left(1 \pm \frac{v_w}{v_s}\right) \sqrt{a_e d_s}$$

where “+” represents down-grinding, and “−” represents up-grinding.

Combining with Figs. 3 and 4, $A_{gz}$ was selected as the cut-in and cut-out reference position. The grinding depth $a_e$ in Eq. (33) needed to be replaced by the $A_{gz12}$’s cutting depth $a_e + h_{gz} - h_j$, because the cutting depths were different for different grains. Then, Eq. (33) becomes the following:

### Table 1 Typical grinding parameters and value ranges

| Group no. | Grinding parameters | Typical value | Value range |
|-----------|---------------------|---------------|-------------|
| Group no. 1 | Wheel speed, $v_s$ (m/s) | 20 | 10, 15, 20, 25, 30 |
| Group no. 2 | Wheel diameter, $d_s$ (mm) | 200 | 100, 150, 200, 250, 300 |
| Group no. 3 | Workpiece feed speed, $u_w$ (mm/s) | 20 | 10, 15, 20, 25, 30 |
| Group no. 4 | Grain’s cutting depth, $a_e$ (μm) | 50 | 30, 40, 50, 60, 70 |
| Group no. 5 | Deflection angle, $\delta_1$ (rad) | 0 | $-0.2, -0.1, 0, 0.1, 0.2$ |
| Group no. 6 | Deflection angle, $\delta_2$ (rad) | 0 | $-0.2, -0.1, 0, 0.1, 0.2$ |

Other parameters: $l_w = 10$ mm, $b_w = 0.3$ mm, $d_{gm} = 0.1$ mm, $d_{ga} = 0.03$ mm, $h_g = 0.1$ mm

### Table 2 Verification of the numerical method

| Group no. 1 | Grinding methods | Wheel speed, $v_s$ (m/s) | $L_g$, mm | $2L_k$, mm | Error, $(L_g - 2L_k)/ (2L_k)$, % |
|-------------|------------------|--------------------------|-----------|-----------|----------------------------------|
| Down-grinding | $d_s = 200$ mm | 10 | 8.6085 | 8.6017 | 0.079485 |
| | | 15 | 8.6143 | 8.6074 | 0.079369 |
| | | 20 | 8.6171 | 8.6103 | 0.079311 |
| | | 25 | 8.6188 | 8.6120 | 0.079276 |
| | | 30 | 8.6200 | 8.6132 | 0.079253 |
| Up-grinding | | 10 | 8.6430 | 8.6362 | 0.078791 |
| | | 15 | 8.6372 | 8.6304 | 0.078906 |
| | | 20 | 8.6433 | 8.6275 | 0.078964 |
| | | 25 | 8.6326 | 8.6258 | 0.078998 |
| | | 30 | 8.6315 | 8.6247 | 0.079021 |

| Group no. 2 | Grinding methods | Wheel diameter, $d_s$ (mm) | $L_g$, mm | $2L_k$, mm | Error, $(L_g - 2L_k)/ (2L_k)$, % |
|-------------|------------------|--------------------------|-----------|-----------|----------------------------------|
| Down-grinding | $v_s = 20$ m/s | 100 | 6.0981 | 6.0884 | 0.079485 |
| | $v_s = 20$ mm/s | 150 | 7.4646 | 7.4567 | 0.079369 |
| | | 200 | 8.6171 | 8.6103 | 0.079311 |
| | | 250 | 9.6327 | 9.6266 | 0.079276 |
| | | 300 | 10.5510 | 10.5454 | 0.079253 |
| Up-grinding | | 100 | 6.1102 | 6.0884 | 0.079485 |
| | | 150 | 7.4795 | 7.4717 | 0.079369 |
| | | 200 | 8.6343 | 8.6275 | 0.079311 |
| | | 250 | 9.6520 | 9.6459 | 0.079276 |
| | | 300 | 10.5721 | 10.5665 | 0.079253 |
The calculation results were compared under the parameters of group no. 1 and group no. 2, as shown in Table 2.

As shown in Table 2, the comparison errors were very small (<0.16%). Thus, the proposed numerical method was verified.

According to the above analysis, after all of the points in the region were processed, the ground surface topography could be obtained. When each parameter was the same as Table 1 and the grinding method was up-grinding, Figs. 12 and 13 show the calculation results.

Figure 12a, b show the front and top views of the $A_{gz}$ moving trajectory.

$$L_k = \left( 1 \pm \frac{v_w}{v_s} \right) \sqrt{(a_e + h_{gz} - h_g) d_s} \quad (34)$$

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Figure 13a, b show the top views of the simulated 3D ground surface: (a) $\delta_1 = 0$ rad, (b) $\delta_1 = 0.1$ rad. The deeper the color is, the greater the cut depth was. It can be seen from Fig. 13a, b that the deflection angle would affect the workpiece surface topography. When the deflection angle is zero, the top view of the workpiece surface topography is segmented symmetrical up and down along the $x$-axis, symmetrical left, and right along the $y$-axis, thick in the middle, and thin at both ends. When the deflection angle is not zero, the top view of the workpiece surface topography is segmented symmetrical left and right along the $y$-axis, thick in the middle and sharp at both ends, flat at the top, and round at the bottom.

The area of the workpiece’s upper surface was denoted by $p_w$, $n_w$ was denoted to express the number of all discrete points.
on the upper surface of the workpiece, and \( p_{w_v} \) was denoted to express the area of each discrete point on the upper surface of the workpiece. Based on the cutting process model, the formula was as below:

\[
\begin{align*}
    p_{w} &= l_w b_w \\
    n_w &= n_x n_y \\
    p_{w_v} &= \frac{p_{w}}{n_w}
\end{align*}
\]

(35)

Material removal volume was denoted by MRV, which represented the sum of the cutting volume of all discrete points on the workpiece. Before grinding, the initial height value of all the points was zero; after grinding, the \( z \)-coordinate of the cut point on the workpiece surface becomes \( z_{wn} \), while the uncut point remains the same. Therefore, the MRV of the single abrasive grain cutting could be expressed as follows:

\[
\text{MRV} = \lim_{\Delta w_x \to 0} \lim_{\Delta w_y \to 0} \sum_{i=-\frac{w}{2}}^{\frac{w}{2}} \sum_{j=-\frac{w}{2}}^{\frac{w}{2}} p_{w_v} |z_{wn}(i,j) - 0|,
\]

(36)

The material removal rate was denoted by MRR, which represented the cutting volume of material removal per unit time. The MRR of the single abrasive grain cutting can be expressed as follows:

\[
\text{MRR} = \frac{\text{MRV}}{t_{\text{max}} - t_{\text{min}}}
\]

(37)

When each parameter was the same as Table 1 (without value range) and the grinding method was up-grinding, the MRV was \( 9.0023 \times 10^{-3} \text{ mm}^3 \) and the MRR was \( 20.9028 \text{ mm}^3/\text{s} \) by calculation.

In previous related studies, the cutting depth of the abrasive grain \( h \), also known as the undeformed chip thickness, is a very important parameter in the modeling of grinding processes. Many researchers have used \( h \) to study grinding quality, removal, and chip formation mechanisms. For the single abrasive grain cutting studied in this paper, \( h_{\text{max}} \), i.e., the maximum unshaped chip thickness, is equal to \( a_x \), and in the process of abrasive grain cutting, the undeformed chip thickness had a large relationship with the grinding method, MRV, and MRR. Therefore, in order to investigate the cutting mechanism of single abrasive grain in depth and improve the surface quality of the workpiece, it is necessary to calculate the undeformed chip thickness and shape of the single abrasive grain cutting model.

The workpiece was partitioned by a series of planes perpendicular to the \( x \)-axis, the distance between adjacent planes was equal to \( \Delta w_x \), and the resulting cross sections were the undeformed chip section shapes in which the thickness of the undeformed chip can be displayed. A Matlab calculation program was prepared; each parameter was the same as Table 1, and the grinding method was up-grinding; and then, the undeformed chip section shape could be obtained, as shown in Fig. 14.

Fig. 14 The undeformed chip section shapes. a \( x = 0 \) mm section, \( \delta_1 = 0 \) rad, b \( x = 0 \) mm section, \( \delta_1 = 0.1 \) rad, c \( x = 0.701 \) mm section, \( \delta_1 = 0.1 \) rad, d \( x = 1.401 \) mm section, \( \delta_1 = 0.1 \) rad, e \( x = 2.101 \) mm section, \( \delta_1 = 0.1 \) rad
The undeformed chip shape was obtained by placing all the cross sections of the workpiece in one diagram, as shown in Fig. 15.

Figure 15a, b show the $z$-axis positive direction views of the undeformed chip shapes, and Fig. 15c, d show the $z$-axis negative direction views of the undeformed chip shapes. It can be seen from Fig. 15a–d that the deflection angle would affect the undeformed chip shapes. When the deflection angle is zero, the shape of undeformed chip is segmented symmetrical up and down along the $x$-axis, symmetrical left, and right along the $y$-axis, thick in the middle, and thin at both ends. When the deflection angle is not zero, the shape of undeformed chip is segmented symmetrical left and right along the $y$-axis, thick in the middle and sharp at both ends, flat at the top, and round at the bottom. As can be seen from Figs. 13 and 15, for the single abrasive grain cutting, the undeformed chip shape and the workpiece surface topography were coincident.

2.5 Validation of the single abrasive grain cutting simulation model

Combining with the literature [25], the relevant data from the literature were substituted into the Matlab program of the single abrasive grain cutting simulation model for calculation, and the calculated results were compared with the available data in the literature. Since the shape of the abrasive grain in the literature [25] was conical, the value of $d_{gn}$ was set to zero to match the shape of the abrasive grain in the literature. Figure 16a was represented to the variation of the $MRV$ during the process of grinding wheel speed from 10 to 150 m/s. Figure 16b was
represented to the variation of the MRV during the process of half-top angle of the cone from 15 to 60°. The grinding method was up-grinding, and the results were shown below.

As can be seen from Fig. 16, with the increase of the grinding wheel speed, the value of the MRV by model calculated and the existing value of the MRV in the literature [25] both gradually decreased, and the higher the grinding wheel speed, the smaller the change of the MRV. As the cone half-top angle of the grinding grain increases, the value of the MRV by model calculated and the existing value of the MRV in the literature [25] both gradually increased, and the larger the half-top angle, the greater the variation of the MRV. The calculated results were consistent with the data in the literature, and the error was small, which verified the accuracy of the calculated values for the single abrasive grain cutting model.

3 Influence of grinding process parameters on the MRV and MRR

Material removal has always been a big deal during the manufacturing process and the material removal rate has a direct connection with the period and cost of manufacturing. During the cutting process of randomly deflected truncated cone shape single abrasive grain, the grinding process parameters (wheel speed, wheel diameter, workpiece feed speed, cutting depth of the abrasive grain, δ₁ and δ₂) are very closely related to MRV and MRR and play a very important role in the cutting process. Therefore, it is meaningful and important to analyze the MRV and MRR under different grinding parameters and deflection parameters, discuss the rational selection and optimization of each grinding process parameter in depth, and study the influence law of each parameter in the cutting process of single abrasive grain in order to obtain higher grinding efficiency.

According to the previous subsection analysis, a Matlab calculation program was run under the parameters in Table 1, as shown in Fig. 17.

As can be seen from Fig. 17, the related results obtained were consistent with the findings in the relevant literature [25]. That is, when the grinding method is down-grinding, the MRV decreases with the increment of the wheel speed; however, the MRR increases with the wheel speed. The MRV and MRR are increment with the increasing grain’s cutting depth of abrasive grain. The MRV and MRR have increased with the workpiece feed speed. All conclusions are as follows:

1. When the grinding method was down-grinding, the MRV increased with the increment of wheel speed, the growth rate decreased gradually, and the MRR also increased with the increment of wheel speed. When the grinding method was up-grinding, the changing way of MRV was the opposite of that in the down-grinding, while the MRR was the same. This indicated that a high wheel speed was able to enhance the MRV; however, when wheel speed increased to a certain value, its effect on MRV was not distinct. According to Eqs. (7) and (37), \( \omega_s = v_s/r_s \), it can be seen that the MRR was both positively correlated with the wheel speed and the MRV, the variation range size of the MRV was not in the same order of magnitude as the variation range size of the wheel speed, which was negligible in comparison, so the MRR in the figure showed a linear increment with the increase of the wheel speed.

2. The MRV increased with the growth of wheel diameter; the larger the wheel diameter, the slower the
Fig. 17 The values of $MRV$ and $MRR$ versus 

- a wheel speed, 
- b wheel diameter, 
- c workpiece feed speed, 
- d grain’s cutting depth, 
- e deflection angle $\delta_1$, and 
- f deflection angle $\delta_2$
Fig. 17 (continued)
change rate, based on the same reason as the first one: the MRR was both positively correlated with the wheel diameter and the MRV; and thus, the MRR decreased linearly with the growth of wheel diameter. In addition, it was not affected by the pattern of the grinding method. So, increasing the grinding wheel diameter can increase the MRV, but it would decrease the MRR.

3. When the grinding method was down-grinding, the MRV and MRR decreased with the increment of workpiece feed speed, and when the grinding method was up-grinding, the MRV and MRR increased with the increment of the workpiece feed speed. Therefore, in the up-grinding condition, increasing the workpiece feed speed would help to improve the MRR.

4. The MRV and MRR increased with the increment of the grain’s cutting depth, and the changing pattern was the same under different grinding methods. Therefore, increasing the grain’s cutting depth would help to improve the MRR, but the depth of the grain’s cutting cannot be increased indefinitely to avoid affecting the surface quality of the workpiece after cutting.

5. For the deflection angle δ1, regardless of the down-grinding or up-grinding, when the deflection angle δ1 increased from small to large, the MRV and MRR increased first and then decreased, and there was a maximum value when the deflection angle was 0 rad. Thus, increasing the absolute value of the abrasive grain deflection angle δ1, the MRV and MRR would be reduced.

6. For the deflection angle δ2, when the grinding method was down-grinding, the MRV and MRR increased first and then decreased with the deflection angle δ2 increased from small to large, and the increase rate was less than the decrease rate. When the grinding method was down-grinding, and the deflection angle δ2 increased from small to large, the MRV and MRR increased first and then decreased; however, the increase rate was larger than the decrease rate. This indicates that decreasing the absolute value of the deflection angle δ2 could help to improve the MRV and MRR, and its change pattern was related to the grinding method.

4 Conclusion

In the present study, based on the random distribution characteristic of abrasive grains on the grinding wheel circumferential surface during the manufacturing process, in order to reflect the random distribution for the abrasive grains’ position and angle, considering the grinding parameters and deflection parameters comprehensively, a theoretical model of cutting mechanism based on randomly deflected truncated cone shape single abrasive grain was established. The main contributions of this paper and the new findings are as follows:

1. Considering the grinding parameters and deflection parameters comprehensively, a mathematical case model of the moving motion trajectory of a randomly deflected truncated cone shape single abrasive grain was established. The trajectory length value calculated by the numerical method was verified, the obtained trajectory length error was extremely small between the numerical method and Malkin’s formula, and the maximum error was less than 0.16%.

2. The single abrasive grain of a randomly deflected truncated cone shape cutting process model was also established; the grain moving trajectory, ground surface topography, and undeformed chip shape were obtained and found that the undeformed chip shape and the workpiece surface topology were coincident for single grain cutting.

3. The influence of the grinding parameters and deflection parameters on the MRV and MRR were studied. When the grinding method was up-grinding, the MRV decreases with the increasing wheel speed, while the MRR increases with wheel speed. The MRV increased with the growth of wheel diameter, while the MRR decreases with wheel diameter indicating that the high wheel speed and small wheel diameter are able to enhance the MRR. Both the MRV and MRR increase with the increment of cutting depth of abrasive grain or workpiece feed speed, while the MRV and MRR decrease with the raising of the absolute value of single abrasive grain deflection angle. The influence law on the MRV and MRR of down-grinding was similar or opposite trend to up-grinding.

In summary, the basic theory of cutting mechanism based on randomly deflected truncated cone shape single abrasive grain was established. The obtained conclusions have great significance for the selection and optimization of grinding parameters and deflection parameters and for further study in the grinding mechanism of the grinding wheel.

Author contribution Conceptualization, Jingliang Jiang; methodology, Dexiang Wang and Shuteng Sun; investigation, Lansheng Zhang; data curation, Lansheng Zhang; writing—original draft preparation, Lansheng Zhang; writing—review and editing, Dexiang Wang; supervision, Jingliang Jiang.

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Declarations

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References

1. Sun C, Hong Y, Xi S, Yao Y (2021) Grain refinement mechanism of metamorphic layers by abrasive grinding hardening. J Manuf Processes 69:125–141. https://doi.org/10.1016/j.jmapro.2021.07.040
2. Fan Z, Tian Y, Zhou Q, Shi C (2020) A magnetic shear thickening media in magnetic field-assisted surface finishing. Proc Inst Mech Eng B J Eng Manuf 234(6–7):1069–1072. https://doi.org/10.1177/095440541986119
3. Agarwal S, Rao PV (2010) Modeling and prediction of surface roughness in ceramic grinding. Int J Mach Tools Manuf 50:1065–1076. https://doi.org/10.1016/j.ijmachtools.2010.08.009
4. Wang Y, Xi S, Zhang S (2021) Microstructure evolution and crystallographic slip modes during grind hardening in TC21 titanium alloy. Surf Coat Technol 417:127211. https://doi.org/10.1016/j.surfcoat.2021.127211
5. Oliveira JFG, Silva EJ, Gomes JIF, Klacke F, Friedrich D (2005) Analysis of grinding strategies applied to crankshaft manufacturing. CIRP Ann - Manuf Technol 54:269–272. https://doi.org/10.1016/S0268-3768(05)60100-0
6. Stephenson DJ, Sun X, Zervos C (2006) A study on ELID ultra precision grinding of optical glass with acoustic emission. Int J Mach Tools Manuf 46:1053–1063. https://doi.org/10.1016/j.ijmachtools.2005.08.013
7. Caggiano A, Teti R (2013) CBN grinding performance improvement in aircraft engine components manufacture. Procedia CIRP 9:109–114. https://doi.org/10.1016/j.procir.2013.06.177
8. Dai C, Yu T, Ding W, Xu J, Yin Z, Li H (2019) Single diamond grain cutting-edges morphology effect on grinding mechanism of Inconel. Precis Eng 55:119–126. https://doi.org/10.1016/j.precisioneng.2018.08.017
9. Ren J, Hao M, Lv M, Wang S, Zhu B (2018) Molecular dynamics research on ultra-high-speed grinding mechanism of monocrystalline nickel. Appl Surf Sci 455:629–634. https://doi.org/10.1016/j.apsusc.2018.06.042
10. Anderson D, Warkentin A, Bauer R (2012) Comparison of spherical and truncated cone geometries for single abrasive-grain cutting. J Mater Process Tech 212(9):1946–1953. https://doi.org/10.1016/j.jmatprotec.2012.04.021
11. Gu P, Zhu C, Tao Z, Yu Y (2020) A grinding force prediction model for SiCp/AI composite based on single-abrasive-grain grinding. Int J Adv Manuf Technol 109:1563–1581. https://doi.org/10.1007/s00170-020-06538-7
12. Yin J, Xu J, Ding W, Su H (2012) Effects of grinding speed on the material removal mechanism in single grain grinding of SiCf/SiC ceramic matrix composite. Ceram Int 47(9):12795–12802. https://doi.org/10.1016/j.ceramint.2021.01.140
13. Tian Y, Li L, Han J, Fan Z, Liu K (2020) Development of novel high-shear and low-pressure grinding tool with flexible composite. Mater Manuf Process 36(4):479–487. https://doi.org/10.1080/10426914.2020.1843673
14. Malkin S (2008) Grinding technology: theory and application of machining with abrasives. Industrial Press Inc., New York
15. Tian Y, Li L, Liu B, Han J, Fan Z (2020) Experimental investigation on high-shear and low-pressure grinding process for Inconel718 superalloy. Int J Adv Manuf Technol 107(7–8):3425–3435. https://doi.org/10.1007/s00170-020-05284-z
16. Chen X, Rowe WB (2014) Analysis and simulation of the grinding process. Part I: generation of the grinding wheel surface. Eur Heart J 35:2264–2265. https://doi.org/10.1093/eurheartj/ehu271
17. Zhang Y, Li C, Ji H, Yang X, Yang M, Jia D, Zhang X, Li R, Wang J (2017) Analysis of grinding mechanics and improved predictive force model based on material-removal and plastic-stacking mechanisms. Int J Mach Tools Manuf 122:81–97. https://doi.org/10.1016/j.ijmachtools.2017.06.002
18. Aurich JC, Kirsch B (2012) Kinematic simulation of high-performance grinding for analysis of chip parameters of single grains. CIRP J Manuf Sci Technol 5:164–174, https://doi.org/10.1016/j.jmst.2012.07.004
19. Bhushan B (2013) Introduction to tribology. Wiley
20. Annadita S, Mote RG, Singh R (2017) Stochastic analysis of microgrinding tool topography and its role in surface generation. J Manuf Sci Eng Trans ASME 139:1–14. https://doi.org/10.1115/1.4038056
21. Koshy P, Jain VK, Lal GK (1997) Stochastic simulation approach to modelling diamond wheel topography. Int J Mach Tools Manuf 37:751–761. https://doi.org/10.1016/S0890-6955(96)00086-7
22. Zhang X, Yao B, Feng W, Shen Z, Wang M (2015) Modeling of a virtual grinding wheel based on random distribution of multi-grains and simulation of machine-process interaction. J Zhejiang Univ Sci A 16:874–884. https://doi.org/10.1631/jzus.A1400316
23. Cooper WL, Lavine AS (2001) Grinding process size effect and kinematics numerical analysis 122:59–69
24. Jiang J, Sun S, Wang D, Y Y, Liu X (2020) Surface texture formation mechanism based on the ultrasonic vibration-assisted grinding process. Int J Mach Tool Manuf 156:0890–6955. https://doi.org/10.1016/j.ijmachtools.2020.103595
25. Wang C, Chen J, Fang Q, Liu F, Liu Y (2016) Study on brittle material removal in the grinding process utilizing theoretical analysis and numerical simulation. Int J Adv Manuf Technol 87:2603–2614. https://doi.org/10.1007/s00170-016-8647-8

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