Dipole and Bloch oscillations of cold atoms in a parabolic lattice

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The paper studies the dynamics of a Bose-Einstein condensate loaded into a 1D parabolic optical lattice, and excited by a sudden shift of the lattice center. Depending on the magnitude of the initial shift, the condensate undergoes either dipole or Bloch oscillations. The effects of dephasing and of atom-atom interactions on these oscillations are discussed.

1. Bloch oscillations (BO) of a quantum particle in a periodic potential are one of the most fascinating phenomena of quantum physics [1]. Since the pioneering experiment [2] in 1996, this phenomenon has been intensively studied for cold atoms in optical lattices [3], with recent emphasis on quantum statistical (Fermi or Bose) and atom-atom interaction effects. In particular, the dynamics of degenerate Bose gases, on which we will focus here, was studied experimentally in [4, 5, 6]. It should be stressed from the very beginning that, when addressing this problem theoretically, one has to distinguish between quasi one-dimensional lattices (created by two counterpropagating laser beams) and truly 1D lattices (or so-called modulated quantum tubes). Indeed, in the former case the number of atoms per well of the optical lattice can be as large as $10^3 - 10^4$, and a mean field approach (based on the Gross-Pitaevskii or nonlinear Schrödinger equation) is generally justified. This is not the case of the truly 1D lattices, where only few atoms occupy a single well, and, hence, a microscopic analysis is required. For a tilted infinite lattice such analysis, based on the Bose-Hubbard model, was presented in [7, 8, 9], where the atoms are stressed from the very beginning that, when addressing this problem theoretically, one has to distinguish between quasi one-dimensional lattices (created by two counterpropagating laser beams) and truly 1D lattices (or so-called modulated quantum tubes). Indeed, in the former case the number of atoms per well of the optical lattice can be as large as $10^3 - 10^4$, and a mean field approach (based on the Gross-Pitaevskii or nonlinear Schrödinger equation) is generally justified. This is not the case of the truly 1D lattices, where only few atoms occupy a single well, and, hence, a microscopic analysis is required. 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The mean-field approach is justified in the limit of large mean-field approach \cite{13,15,16,17,18,19}. As known, atoms were studied in a number of papers, using the lattices, Bloch and dipole oscillations of the interacting atoms. In the case of quasi one-dimensional parabolic packet width only the dephasing time \[\text{through the change of the wave} \]
\[\text{function} \]

\[\Lambda = \frac{\nu}{\nu} \text{exclusively by the parameter} \]
\[\nu \text{of non-interacting atoms; (b) mean momentum of interacting} \text{parabolic lattice with parabolicity} \]
\[\nu \text{l} \text{state of the system we choose here the ground state of} \text{and vanishing microscopic} \text{interaction constant} \]
\[\lambda \text{to the discrete nonlinear Schrödinger equation,} \]
\[\lambda = \exp(-2\tilde{n}[1 - \cos(Wt/\hbar)]) . \]  

The main question we address below is the effect of atom-atom interactions on the Bloch dynamics depicted in the upper panel of Fig. 1.

First we shall discuss the initial conditions in some more detail. Throughout the paper we shall consider the ground many-body state of the atoms in a parabolic lattice as the initial wave packet (which is shifted then by the distance \(l_0\)). Clearly, along with the ratio \(J/\nu\) this state is also defined by the ratio of the interaction constant to the hopping matrix element. Namely, it is essentially given by the symmetrized product of the single-particle atomic state for \(W < J\), while it may resemble the Mott-insulator state for \(W \gg J\) \cite{13}. In what follows we restrict ourselves by a relatively weak interaction. Then the ground state of the system can be well approximated by the many-body wave function

\[|\tilde{\Psi}_0\rangle = \sum_n c_n |n\rangle , \quad c_n = \sqrt{N} \prod_i \frac{a_n^{|i|}}{\sqrt{m!}} , \]  

where \(|n\rangle = |n_1, ..., n_{N-1}, n_0, n_1, ...\rangle\) is the Fock basis and the \(a_i\) satisfy the stationary nonlinear Schrödinger equation

\[\frac{\nu}{\nu}^2 a_i^4 - \frac{J}{\nu} (a_i + a_i^+) + g|a|^2a_i = E_0a_i , \]  

For example, for \(N = 5, \nu = 0.04J\) and \(W = 0.2J\), the overlap of the state \(|\tilde{\Psi}_0\rangle\) with the exact ground state \(|\tilde{\Psi}_0\rangle\) is \(|\langle \tilde{\Psi}_0 | \tilde{\Psi}_0 \rangle |^2 = 0.97\). We note that the state \(|\tilde{\Psi}_0\rangle\) is completely coherent and is analogous to the super-fluid state in a homogeneous lattice. We shall characterize the macroscopic coherence of the given many-body state \(|\tilde{\Psi}\rangle\) by the maximal eigenvalue \(\lambda\) of the single-particle density matrix

\[\rho_{l,m} = N^{-1} \langle \tilde{\Psi}^\dagger a_i^+ a_m | \tilde{\Psi} \rangle . \]  

Then the macroscopic coherence of the state \(|\tilde{\Psi}_0\rangle\) is \(\lambda = 1\).

We proceed with the dynamics. The middle panel in Fig. 1 shows the mean momentum of \(N = 5\) interacting atoms \((W = 0.2J)\). In comparison with the noninteracting case (upper panel), a qualitative change is noticed. This change can be understood by analyzing the macroscopic coherence of the system, shown in the lower panel. It is seen that the macroscopic coherence oscillates with some characteristic period \(T_W\). In the case of a homogeneously tilted lattice these oscillations were studied in Ref. \cite{5}. The origin of the oscillations was shown to be the Stark localization of the single-particle wave functions which, together with the discreteness of the atom number, leads to the following expression for the macroscopic coherence,

\[\lambda = \exp(-2\tilde{n}[1 - \cos(Wt/\hbar)]) . \]
In Eq. (11) $\bar{n}$ is the mean number of atoms per lattice site and the limit $Fd \gg J$ is implicitly assumed. Since for the considered local static force $Fd = \nu l_0 = 3.2J$ Stark localization is not complete, the oscillations of the macroscopic coherence decay in time. Nevertheless, if this irreversible decay of coherence is slow on the time scale of the dephasing time, one can observe the revival of BO of the interacting atoms – an effect which attracts much attention because it provides an independent and accurate method for measuring the microscopic interaction constant $W$.

5. Let us now turn to the case $l_0 < l^*$. Here we meet dipole oscillations of a BEC with a characteristic frequency given by the frequency of small pendulum oscillations $\omega_0 = (\nu J)^{1/2}/h$. (We recall in passing that the frequency of BO was given by $\omega_B = \nu l_0/h \approx 2\omega_0 l_0/\nu$, $l_0 \gg l^*$. For vanishing atom-atom interactions these dipole oscillations are shown in the upper panel of Fig. 2, where $l_0 = l^*/2 = 5$, and the other parameters are the same as in Fig. 1. The dephasing time $\tau_\gamma$ is again given by Eq. (5) but with the parameter $\nu$ substituted by the nonlinearity parameter $\tilde{\nu} = \nu/8$ [21]. (The latter parameter also defines the revival time.) The middle and lower panels in Fig. 2 refer to interacting atoms. An exponential decay of the macroscopic coherence is noticed. The other point to which we want to draw the attention of the reader is that a moderate interaction stabilizes the dipole oscillations against dephasing. Within the mean-field approach (which reduces the Bose-Hubbard model to the discrete nonlinear Schrödinger equation), this phenomenon is discussed in Ref. [13].

6. In conclusion, we have shown that the dynamics of cold atoms in parabolic lattices is governed by the relation between two characteristic times – the dephasing time $\tau_\gamma$ and the decoherence time $\tau_W$.

The dephasing time is inversely proportional to the width $\gamma$ of the initial wave packet and the nonlinearity $\tilde{\nu}$, which, in turn, is defined by the initial shift $l_0$ of the wave packet relative to the separatrix $l^* = 2(J/\nu)^{1/2}$. Namely, $\tilde{\nu} = \nu/8$ for $l_0 \ll l^*$, and $\tilde{\nu} = \nu$ for $l_0 \gg l^*$. It is interesting to estimate the dephasing time in a typical laboratory experiment. Taking, as an example, the recent experiment [10] with rubidium atoms in an array of axially modulated quantum tubes, we have $\nu = 0.0014E_R$ and $J = 0.38E_R$ for the modulation amplitude (depth of the optical lattice) of one recoil energy. This gives a separatrix position $l^* = 33$, and a period $T_0 = 12.1$ ms of small dipole oscillations. Assuming a dilute gas (which, in fact, is not the case realized in the cited experiment) the width of the initial wave packet is $\gamma \approx (J/4\nu)^{1/4} = \sqrt{T_0}/2 \approx 3$ and, hence, the dephasing time $\tau_\gamma = 85$ ms for dipole oscillations, and $\tau_\gamma = 10.6$ ms for BO. Note that these are upper estimates for the dephasing times, and for the initial shift $l_0$ closer to the separatrix the dephasing times are essentially smaller. It is also worth noting that there is a maximal shift $l_0$ above which the single band approximation (used throughout the paper) is not valid. The crucial parameter here is the energy gap between the Bloch bands ($\Delta = 0.5E_R$ for the specified parameters). The analysis of BO in a parabolic lattice beyond the single-band approximation will be subject of a separate paper.

The decoherence time $\tau_W$ is defined by the characteristic density of the atomic gas $\bar{n}$ and by the value of the microscopic interaction constant $W$. The latter, in turn, is defined by the $s$-wave scattering length and by the degree of confinement of the atoms in the wells of the optical potential. In particular, in the experiment [10], the quantum tubes were created by two crossing quasi 1D optical lattices with an amplitude $V = 30E_R$. For the axial modulation with $V = E_R$ this gives $V = 0.73E_R$. For this relatively high value of the interaction constant few atoms per one tube are enough to destroy the dipole/Bloch oscillations on a very short time scale. This qualitatively explains the results of the experiment [10], where the number of atoms per one quantum tube was around 20. To observe the effects discussed in this paper one has to decrease either the atomic density or the interaction constant (e.g. by use of a Fishbach resonance), as compared to those of Ref. [10].

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