Collective excitations play an important role in probing microscopic interactions. The recently available gaseous atomic Bose-Einstein condensates (BEC), have proven to be a profitable testing ground for such studies. Atomic BEC are dilute with properties completely determined by binary interactions. Experimentally they are created at very low temperatures when the short range atom-atom (collision) interaction can be described by a single parameter: the s-wave scattering length \(a_{sc}\). All higher order partial wave collisions are suppressed at zero collision energy in a short ranged potential. This enables atoms to be modelled as hard spheres of radius \(a_{sc}\), reflecting the isotropic ground state interaction. Inside an electric or magnetic field, however, ground state atoms may be polarized, e.g. in a magnetic trap the direction of atomic spins (of the valence electron for alkali) becomes aligned. The resulting dipole interaction between condensed atom pairs may not simply average out. In this article, we investigate such spin dipole effects on shape oscillation frequencies of an atomic condensate. Although dipolar effects are typically small compared to the dominant s-wave contact interaction, our study indicates that these shifts become observable in the \(^{85}\text{Rb}\) BEC \([3, 4]\), when a Feshbach resonance is used to tune \(a_{sc}\) near zero \([3]\).

Dipolar interaction in atomic BEC leads to physics beyond the usual s-wave contact term, mainly because of the modified low energy collision threshold behavior due to the anisotropic nature of the interaction \([3]\). For magnetic spin dipoles, the net effect in the dilute gas sample is rather small, one can therefore approximate the complete two-body interaction by

\[
V(\vec{R}) = g_0\delta(\vec{R}) + g_2 \frac{1 - 3 \cos^2 \theta_R}{R^3}
\]

where \(\vec{R} = \vec{r} - \vec{r}'\), and \(g_0 = 4\pi\hbar^2 a_{sc}/M\) is the s-wave contact term. More generally, the dipole strength \(g_2\) equals \(\alpha(0)^2\vec{e}\vec{e} (\alpha(0)\text{ atomic polarizability})\) or \(\mu^2 (\mu \text{ magnetic dipole moment})\) respectively for electric or magnetic dipoles. Figure 1 illustrates the geometry for two aligned dipoles along the local magnetic field (z-axis). Several interesting properties of a dipolar condensate have already been discussed \([6, 7, 8, 9]\). In this paper, we focus on the shape oscillations of a trapped dipolar condensate assuming a tunable \(a_{sc}\).

In the standard approach, low energy collective excitations are described by the Bogoliubov theory \([10]\). The condensate wavefunction \(\phi(\vec{r}, t)\) is governed by the Gross-Pitaevskii equation, while the non-condensed atoms are described by quasi-particles \([1]\), some of which have been studied experimentally \([12, 13]\). The inclusion of the non-local dipolar interaction makes the Bogoliubov approach impractical to implement numerically. We therefore will rely on two alternative methods to study the three characteristic shape modes \([12]\) as graphically illustrated for a cylindrical symmetric trap in Fig. 2.

First we adopt the highly successful time-dependent variation approach used in Ref. \([14]\) by assuming a Gaussian ansatz

\[
\phi(x, y, z, t) = A(t) \prod_{\eta=x,y,z} e^{-\eta^2/2\nu_\eta^2 + i\eta^2 \beta_\eta(t)},
\]

For an harmonic trap \(V_t = M \sum_{\eta=x,y,z} \nu_\eta^2 \eta^2/2\) \((\nu_\eta = \omega_{\eta})\), the equations for variational parameters \(\nu_\eta\) are equivalent to the classical motion of a particle (with co-
The trapped aspect ratio \( \lambda \) affects the overall condensate mean field and the stability of the condensate with \( 10^4 \) atoms. Figure 3 shows the three mode frequencies for \( N = 10^4 \) in a spherical trap (\( \lambda = 1 \)). The solid and dashed lines denote respectively the mode frequencies without and with the dipole interaction. The vertical lines show the critical values of \( a_{sc}^C \) when the mode character switching \( b \leftrightarrow c \) occurs. This mode switching occurs whenever the overall mean-field condensate interaction changes from repulsive to attractive or vice versa. When squeezed along the radial direction, mode \( c \) (b) became predominantly excited for an overall attractive (repulsive) condensate as atoms are pulled (pushed out) along the z-direction. Without the spin dipole interaction, this mode switch always occurs at \( a_{sc} = 0 \). The anisotropic dipole interaction affects the overall condensate mean field and the stability depending on the trap aspect ratio \( \lambda \). Depending on the configuration of the dipoles, dipole-dipole interactions can be either attractive or repulsive. For two dipoles, if they were placed in a plane perpendicular to their polarization (\( \uparrow \uparrow \)), they repel each other. On the other hand, they attract each other if they were placed along the direction of their polarization (\( \rightarrow \rightarrow \)). For a cloud of trapped dipoles one therefore expects that repulsive interaction increases as one increases \( \lambda \), which leads to increased condensate stability. In Fig. 4, we display the \( \lambda \)-dependence of the critical scattering length \( a_{sc}^C \) as obtained from the variational calculation. We find that the effective scattering length \( a_{eff} \) (introduced earlier by us in Ref. [8]) provides a reasonable approximation to the sign of the overall condensate mean field. In terms of the actual values, however, the results in the figure show that \( a_{eff} \) differs significantly from \( a_{sc}^C \) when \( \lambda \) deviates from the neighborhood of unity. In Fig. 5, we show the \( \lambda \)-dependence of the dipole induced fractional changes to the mode frequencies for a condensate with \( 10^4 \) atoms at \( a_{sc} = -1 \). For strongly prolate or oblate traps, the shifts are in a few percentage range even with such small numbers of atom.
increased values of \( N \), a dipolar condensate always collapses irrespective of the sign of \( a_{sc} \). The Gaussian ansatz becomes questionable near collapse as reflected in the seemingly divergent results when \( N \) is increased.

These results (Figs. 3-6) clearly show that dipolar effects are detectable in the current \(^{85}\text{Rb} \) BEC setup, given the extraordinary capability of 0.1% frequency measurement. To confirm the validity of the variational calculations, we have invested considerable effort in an exact numerical method based on the time dependent Gross-Pitaevskii equation. By applying external periodic forcing terms described by the potential

\[
V_F(\vec{r}) = \sum_{\eta=x,y,z} V_\eta \left\{ e^{-\frac{(\eta + \Delta_\eta(t))^2}{2w^2}} + e^{-\frac{(\eta - \Delta_\eta(t))^2}{2w^2}} \right\},
\]

with \( \Delta_\eta(t) = \eta_0 + \delta_\eta \sin(\Omega_\eta t + \phi_\eta) \) to the condensate ground state, selected shape oscillations become predominantly excited, as first numerically implemented by Ruprecht et al. [17].

After a selected duration \( T \), typically several periods of the trap radial oscillation, the forcing term \( V_F \) is tuned off. The free propagation of the time dependent Gross-Pitaevskii equation is continued, and the dynamic condensate width \( \sqrt{\langle \eta^2(t) \rangle} \) is sampled. The shape oscillation frequencies are then identified by taking the Fourier transformation of \( \langle \eta^2(t) \rangle \). A typical result from this calculation is given in Fig. 7, which shows remarkably clear signal. By varying \( V_\eta, \eta_0, \delta_\eta, \) and \( \Omega_\eta \), our results are self-consistently checked, i.e. to be independent of all parameters involved as they should be in the small amplitude oscillation limit and to be numerically accurate. For the experimental parameter of \( \nu_r = 17.35 \) (Hz) and \( \nu_z = 6.8 \) (Hz). We have computed the shape oscillation frequencies numerically for \( a_{sc} = 0, 1, -2 \) \((a_0)\), and compared them with the variational results in Table I. The quoted errors for the numerical results shown in the tables mainly come from the limited sampling window in time. Further improvement is hard as the calculations are time consuming and numerical accuracies become difficult to control at long times. We find that
TABLE I: Mode frequencies (in unit of $\nu_r$) with dipolar interaction

| $a_{sc}(a_0)$ mode | numerical | variational |
|-------------------|-----------|-------------|
| $-2$              | $b$       | 2.0010 ± 0.0040 | 2.0016 |
|                   | $c$       | 0.8578 ± 0.0085 | 0.8657 |
| $0$               | $b$       | 1.9995 ± 0.0028 | 1.9964 |
|                   | $c$       | 0.7874 ± 0.0022 | 0.7895 |
| $+1$              | $b$       | 0.7705 ± 0.0060 | 0.7675 |
|                   | $c$       | 1.9974 ± 0.0032 | 1.9974 |

The variational results are consistent with the exact numerical results. In order to affirm such mode frequency shifts are indeed from the dipolar interaction, rather than a mis-calibration of $a_{sc}$, we also need to validate the variational approach in the absence of the dipole interaction. This was explored earlier in Ref. [14], where the numerical and variational results of the condensate dynamics were compared. Our results are presented in Table I. It convincingly proves that dipolar efforts reported in the Table I is due to the physics of spin dipole interaction.

In conclusion, we have studied low energy shape oscillations of a trapped dipolar condensate. Using a magnetic field dependent Feshbach resonance to tune the s-wave scattering length to around zero, we have shown that the weak spin dipole interaction becomes detectable as shifts to shape oscillation frequencies under currently available experimental conditions [3, 4]. These shifts grow with the number of trapped atoms, and are around 1% level with as few as $10^4$ atoms. We have also independently verified the accuracy of the variational calculation by developing a rigorous numerical approach for the three predominant shape oscillation modes. Near a Feshbach resonance, significant atom loss might occur as found in the Na experiment [18]. The subsequent damping could lead to a broadening of the shape oscillation thus masking the direct observation of the proposed spin dipole effects. Fortunately for $^{85}\text{Rb}$ atoms, the condition of $a_{sc}(B) = 0$ corresponds to far off resonance on the high B-field side, where impressive controls have been demonstrated without any significant loss [3]. Over the last few years, mean field theory has proven to be remarkably successful when applied to BEC physics. The s-wave $g_0$ contact pseudo-potential has made the concept of scattering length $a_{sc}$ widely popular. It is often said that scattering length is the only relevant atomic parameter since the net interaction effect scales as $Na_{sc}/\hbar \omega_0$ for a harmonically trapped atomic condensate, where $a_{sc}$ is the trap size. Dipole interactions as discussed in this article points to interesting physics beyond the s-wave. Within the current experimental regime of dilute atomic gases, mean field theory remains applicable and allows for the calculation of dipolar induced shifts of collective excitation frequencies. Successful experimental verification of our predictions will shed new light on atomic BEC.

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