The MaxEnt extension of a quantum Gibbs family, convex geometry and geodesics

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Abstract

This talk revisits Gibbs families of probability distributions and reports new results about quantum states. In 1972 Čencov has defined an extension of a log-linear model of probability distributions in order to have a maximum-likelihood estimate with probability one. His ideas have advanced in mathematical statistics in the work of Barndorff-Nielsen (1978), Csiszár and Matúš (’03, ’05, ’08), Geiger, Meek and Sturmfels (’06) and Rauh, Kahle and Ay (’11). A log-linear model is the set of Boltzmann distributions of a vector space of Hamiltonians. Its elements are results of the inference method, formulated by Jaynes (1957), to infer a distribution from expected values of the given Hamiltonians by maximizing the entropy. The norm closure is a suitable extension of a log-linear model of finite support as it consists of all maximum-entropy distributions and as it extends the maximum-likelihood estimate. Closures of log-linear model of infinite support are much more subtle.

Von Neumann has introduced Gibbs states to quantum statistical mechanics in 1927 where they play a central role every since. Gibbs states solve the inverse problem to infer a state from expected values of a set of observables. An application is the reconstruction of quantum states, see for example Bužek et al. (1999), which was also considered in the reconstruction of quantum channels by Ziman (’08).

Closure subtleties of Gibbs families begin in the quantum setting already for finite-level systems. W. and Knauf (’10) have shown that the maximum-entropy inference can be discontinuous. This has a convex geometric origin because the convex set of quantum states has curved and flat boundary portions, while the probability simplex has only flat boundary portions. The convex geometric origin becomes very transparent in the notion of openness of a restricted linear map, see the preprint cited below, which is also helpful to understand the geodesics in a Gibbs family. In information geometry, a (−1)-geodesic in a Gibbs family is a straight line in the expected value chart, see Amari and Nagaoka (’00). I report that the union of all (−1)-geodesics in a Gibbs family and their limit points equals the set of maximum-entropy states. This is remarkable since the analogous construction with (+1)-geodesics, that is straight lines in the chart of Lagrangian multipliers, can be strictly smaller. The difference has a convex geometric origin. The two constructions are equivalent for commutative observables.

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