Matter perturbation in coupled scalar field cosmology

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Abstract. A universe containing a coupled scalar field would have different dynamical evolution compared to uncoupled cases. Significant differences are expected to emerge during matter dominated era, where scalar field density mimics how matter density evolves, by becoming subdominant. Analysis of the dynamics was investigated both analytically and numerically through phase plane method, and we obtained two attractor solutions which are compatible to a late time cosmic acceleration as the ending of matter dominated era. Ordinary scalar field triggers an acceleration with stable attractor solution which converges to \( \omega_\phi = \omega_{\text{eff}} \approx -1 \). Whereas, phantom scalar field allows negative values for its own energy \( \omega_{\text{eff}} < -1 \) and \( \Omega_\phi < 0 \); however this is not stable, therefore it will be caught by attractor solution which is stable to \( \omega_\phi = \omega_{\text{eff}} \approx -1 \). In addition, we found cosmological parameters generated from both analytical and numerical calculations, i.e. \( \omega_\phi, \Omega_\phi \) and \( \omega_{\text{eff}} \), by assuming a flat universe. The existence of a coupling constant \( Q \) between scalar field and matter induces different structure growth compared to uncoupled cases and standard cold dark matter (CDM) model. During matter dominated era, ordinary scalar field induces growth of structures so that it becomes faster than the standard model does, whereas phantom scalar field slows down the growth of structures. During scalar field dominated era (far in the future), we obtained that formed structures or density contrasts will decay in a similar way as in uncoupled cases, whereas following phantom scenario, the decay of formed structures occurs faster and more significant. This result is associated to background dynamics which shows that big rip will be the relevant fate of our universe.

1. Introduction

One of latest issues in cosmology which is yet to solve is the cosmic acceleration [1,2]. Various proposals have been put forward to explain this problem, basically by searching new kind of energy or dark energy and by modifying the theory of gravity. The simplest and most popular scenario belongs to dark energy sector is the cosmological constant, which is regarded as vacuum energy stored in spacetime. However, two serious problems rise from this model which are dubbed as fine tuning and coincidence problem [3]. Alternative models have been conceived by assuming dark energy in the form of scalar fields. The simplest model of scalar fields is quintessence with various types of potential [4,5,6,7,8]. However, the observational facts that matter-energy density nowadays is the same in order of magnitude as dark energy density suggest the possibility of coupling between them, which is mediating energy to transform. This suggestion has motivated many authors to make a generalization from uncoupled into coupled scalar field.

The existence of coupling between matter and dark energy may exhibit different signatures of the universe, especially in higher order of dynamics, i.e. inhomogeneity and anisotropy of the universe. Study of scalar field coupled to matter and its effects to structures growth have been reported well by...
different authors [9,10,11,12,13]. In this paper we adopt the model [13] and propose another technique and approach to formulate the perturbation equation analytically and find solution for its dynamics through stability analysis of its critical points. Regarding the results from [14] that allows possibility for \( \omega_\phi < -1 \), we consider here a general scalar field which includes phantom field with negative kinetic energy.

2. Background dynamics
An anzats lagrangian for scalar field coupled to matter is
\[
L(X, \phi) = X - V(\phi) - Q \rho_m \phi
\]  
where \( Q, \rho_m \) and \( \phi \) denote coupling constant, general matter density and scalar field respectively, and
\[ 
X = \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi
\]
is the kinetic component of the scalar field. For lagrangian in eq. (1) we can derive the equation of motion for scalar field in Friedmann universe,
\[
\phi'' + 2H \phi' + a^2 V_{,\phi} = -Q \rho_m \phi.
\]  
which is usually known as Klein-Gordon equation. In the above equation, the accent sign stands for the derivative with respect to conformal time \( \eta \) which is related to cosmic time \( t \) by \( dt = a d\eta \). \( a \) represents a scale factor whose value is normalized to 1 today \( (a_0 = a(t_0) = 1) \) and \( H \) is the Hubble parameter defined as \( H = \dot{a}/a \) in conformal time. The scalar field coupled to general matter satisfies the condition
\[
\nabla_\mu T^\mu = \pm Q \rho M \nabla_\nu \phi,
\]  
where the plus (+) sign refers to matter and radiation, minus (−) sign refers to scalar field, and \( \rho M \) is the trace of fluid energy-momentum tensor. This condition yields a set of continuity equations as follows
\[
\dot{\rho}_r + 4H \rho_r = 0, \\
\dot{\rho}_m + 3H \rho_m = Q \rho_m \dot{\phi}, \\
\dot{\rho}_\phi + 3H (\rho_\phi + P_\phi) = -Q \rho_m \dot{\phi},
\]  
where dot sign refers to the derivative with respect to cosmic time \( t \) and \( H = \dot{a}/a \). The dynamics analysis of the coupled scalar field cosmology can be started by investigating the existence of scaling condition, which acts as the boundary between accelerating and decelerating modes.

2.1. General scaling condition
For simplification, we rewrite Friedmann equation in flat universe as
\[
H^2 = \beta \rho_T,
\]
where \( \beta = \sqrt{8\pi G/3} \) and \( \rho_T \) is the total density of the universe. The general form of \( P_\phi \) in general relativity framework is given by [15],
\[
P_\phi = Xh(X e^{\lambda \phi}),
\]  
where \( h(X e^{\lambda \phi}) \) is a function of \( Y \equiv X e^{\lambda \phi} \). During scaling era it can be proven that \( h(X e^{\lambda \phi}) = \) constant \( = Y_0 \) [13]. The yield reveals that pressure is proportional to kinetic component and only determined by its kinetic term. To construct our dynamical system of the universe, one should be recognized is the evolution of scalar field density expressed in general variables. We can write a general form of scalar field density as
\[
\rho_\phi = 2X \frac{\partial P_\phi}{\partial X} - P_\phi.
\]  
By using eq. (5), eq. (6) and their derivatives with respect to time we obtained
\[
\rho_\phi = \left[ 1 + 2X e^{\lambda \phi} \frac{\partial h(X e^{\lambda \phi})}{\partial (X e^{\lambda \phi})} \right] X h(X e^{\lambda \phi}).
\]
\[
\dot{\rho}_s = h(Y) \ddot{\phi} + 2h_Y \phi \ddot{\phi} e^{i\phi} + \frac{1}{2} h_{YY} \phi \ddot{\phi} e^{i\phi} + 2XYh_{YY} \phi \ddot{\phi} e^{i\phi} + XYh_{YY} \phi \ddot{\phi} e^{i\phi}.
\] (7)

Then, by setting \(8\pi G = 1\) and doing a little mathematics to eq. (4), and (7) we obtained
\[
\left( h(Y) + 4Yh_Y + 2Y^2 h_{YY} \right) \dot{\phi} + 3H \left( h(Y) + Yh_Y \right) \dot{\phi} + 2XY \lambda h_Y + 2XY^2 \lambda h_{YY} = -Q \rho_m,
\]
which is the scaling Klein-Gordon equation. Next, by introducing dimensionless variables [12]
\[
x = \frac{\phi}{\sqrt[3]{6}H}, \quad y = \frac{e^{-\lambda \phi^2/2}}{\sqrt[3]{5}H},
\]
and using several aforementioned equations, we can construct a system that describes our picture of the universe. However, for mathematical requirements, we propose an assumption that would simplify our analytical treatment, by assuming a scalar field which fulfil the condition
\[
\frac{1}{2} \lambda \phi^2 = -\dot{\phi}.
\]

Under this condition the dynamics of the scalar field is governed by a set of equations:
\[
\begin{align*}
\frac{dx}{dN} &= x \left[ -1 - \frac{3}{2} \frac{h(Y) + Yh_Y}{H} + \sqrt{6} \frac{\lambda x^2 \left( 3h_{Y} + 2Yh_Y + Q \right)}{2 \left( 2Y^2 h_{YY} + 5Yh_Y + h(Y) \right)} \right] , \\
\frac{dy}{dN} &= -\frac{3}{2} \left[ 1 + \omega_m + \left( 1 - \omega_m \right) x^2 h(Y) - 2\omega_m x^2 h_Y \right],
\end{align*}
\] (8)

where \(\omega_m\) and \(\omega_\phi\) are state parameters for matter and scalar field respectively, and \(\omega_{eff} = \omega_\phi \Omega_\phi + \omega_m \Omega_m\). The above set of equations is an autonomous system, whose solutions are dependent on the explicit form of \(h(Y)\).

An autonomous system is a class of dynamical system that plays important role in cosmology [18] and contains two or more coupled differential equations. In eq. (8), there are two coupled differential equations in the form \(x = f(x, y, N)\) and \(y = g(x, y, N)\) and each of them do not contain explicit \(N\) - dependent terms. The critical points of the system can be found by fixing \(dx/dN = 0\) and \(dy/dN = 0\) simultaneously.

2.2. The dynamics of scalar field with \(V = V_0 e^{-\lambda \phi^2}\)

This section deals with specific scalar fields that naturally has scaling solutions. An exponential potential is a natural form of scalar field potentials, meaning that it is an exact and simpler solution of Klein-Gordon equations. Thus, we write the lagrangian as below
\[
L(X, \phi) = P_\phi = \varepsilon X - V_0 e^{-\lambda \phi^2},
\] (9)

where \(\varepsilon = +1\) for ordinary scalar field and \(\varepsilon = -1\) for phantom scalar field. Lagrangian (9) is associated with \(h(Xe^{\lambda \phi^2}) = \varepsilon - V/Xe^{\lambda \phi^2}\), consequently, the autonomous equations(8) becomes
\[
\begin{align*}
\frac{dx}{dN} &= -3x + \frac{\sqrt{6}}{2\varepsilon} \lambda V_0 y^2 + \frac{3}{2} x \left[ (1 - \omega_m) \varepsilon x^2 + (1 + \omega_m) \left( 1 - V_0 y^2 \right) \right] - \frac{\sqrt{6}}{2\varepsilon} Q \left( 1 - \varepsilon x^2 - V_0 y^2 \right), \\
\frac{dy}{dN} &= -\frac{3}{2} \lambda xy + \frac{3}{2} y \left[ (1 - \omega_m) \varepsilon x^2 + (1 + \omega_m) \left( 1 - V_0 y^2 \right) \right].
\end{align*}
\]

These give critical points, whose relations to cosmological parameter are resumed in Table 1, and in the phase diagrams that show the arrows of stability in Figure 1 and 2.
2.3. Stability and dynamics analysis

2.3.1. Ordinary scalar field (\(\varepsilon = +1\)). Based on its energy, point (a) is a kinetic dominated era, \(\omega_\phi = 1 > -1/3\) and \(\omega_{\text{eff}} > -1/3\) since the second term, i.e. \(\frac{2}{3\omega_m}Q^2 > 0\) and \(\omega_m > 0\), which corresponds to a deceleration condition. Points (b) & (c) belong also to kinetic and scalar field dominated eras, however these will never trigger an acceleration since \(\omega_{\text{eff}} = \omega_\phi = 1 > -1/3\). Points (d) and (e) are scalar field dominated solutions (\(\Omega_\phi = 1\)) with \(\omega_{\text{eff}} = \omega_\phi\), therefore acceleration occurs if \(\lambda < \sqrt{2}\) is fulfilled. Points (f) & (g) are scaling solutions. Since \(Q, \lambda\) and \(\omega_m\) are constants, hence \(\Omega_\phi = \text{constant}\) and \(\Omega_m = 1 - \Omega_\phi = \text{constant}\), therefore \(\frac{\Omega_\phi}{\Omega_m} = \text{constant}\). The requirement for \(\Omega_\phi < 1\) suggests that \(Q > 3(1 + \omega_m)/\lambda - \lambda\). However, since \(\omega_{\text{eff}}\) depends on \(Q, \lambda\) and \(\omega_m\) and the requirement for acceleration is \(\omega_{\text{eff}} < -1/3\), these points can generate acceleration if \(Q > \lambda(1 + 3\omega_m)/2\) is satisfied (see Figure 1).

![Figure 1](image1.png)

**Figure 1.** Ordinary field phase diagram for \(\lambda = 0.1\) and \(Q = 0.245\). Acceleration solution is given by point (d) (stable node), and matter dominated era by point (a) (saddle point).

2.3.2. Phantom scalar field (\(\varepsilon = -1\)). Point (a) is a kinetic dominated solution (\(y = 0\)). The condition \(\varepsilon = -1\) in the second term keeps this term always negative. For matter with \(\omega_m = 0\) we get \(-2Q^2/3 < -1/3\), meaning that acceleration can be reached if \(Q < \sqrt{1/2}\) is satisfied. However, if we are considering

![Figure 2](image2.png)

**Figure 2.** Phantom field phase diagram for \(\lambda = 0.1\) and \(Q = 0.245\). Acceleration solution is given by point (d) (stable node) and matter dominated era by point (a) (saddle point).
energy fraction, acceleration will not occur for $0 \leq \Omega_\phi \leq 1$. Points (b) & (e) are irrelevant because they are not real solutions. Points (d) & (e) are scalar field dominated solutions ($\Omega_\phi = 1$). Cosmic acceleration will always be satisfied by these points because $\lambda^2$ will always be positive, so that $\lambda^2/3\epsilon$ will always be negative, and $\omega_{\text{eff}} = \omega_\phi < -1/3$. So to speak, this is a special property of the phantom field, i.e. $\omega_\phi = \omega_{\text{eff}} \leq -1$. Points (f) & (g) just exist if $2Q(Q + \lambda) > 3(1 - \omega_m^2)$ is fulfilled. Cosmic acceleration occurs if $Q > \lambda(1 + 3\omega_m)/2$ is satisfied (See Figure 2).

| Table 1. Critical points and cosmological parameters generated from the models |
| --- |
| Point | $x$ | $y$ | $\Omega_\lambda$ | $\omega_\lambda$ | $\omega_\phi$ |
| (a) | $-\frac{\sqrt{3}Q}{3\epsilon(1 - \omega_\phi)}$ | 0 | $\frac{2}{3\epsilon} \left( \frac{Q}{1 - \omega_\phi} \right)$ | 1 | $\omega_{\text{eff}} + \frac{2}{3\epsilon} \left( \frac{Q'}{1 - \omega_\phi} \right)$ |
| (b) | $\frac{1}{\sqrt{\epsilon}}$ | 0 | 1 | 1 | 1 |
| (c) | $-\frac{1}{\sqrt{\epsilon}}$ | 0 | 1 | 1 | 1 |
| (d) | $\frac{\lambda}{\sqrt{6\epsilon}}$ | $\left[ \frac{1}{\sqrt{3}} \left( 1 - \frac{\lambda^2}{6\epsilon} \right) \right]^{1/2}$ | 1 | $-1 + \frac{\lambda^2}{3\epsilon}$ | $-1 + \frac{\lambda^2}{3\epsilon}$ |
| (e) | $\frac{\lambda}{\sqrt{6\epsilon}}$ | $\left[ \frac{1}{\sqrt{3}} \left( 1 - \frac{\lambda^2}{6\epsilon} \right) \right]^{1/2}$ | 1 | $-1 + \frac{\lambda^2}{3\epsilon}$ | $-1 + \frac{\lambda^2}{3\epsilon}$ |
| (f) | $\frac{\sqrt{6}(1 + \omega_\phi)}{2(\lambda + Q)}$ | $\left[ \frac{2Q(\lambda + Q) + 3\epsilon(1 - \omega_\phi)}{3V_\phi(\lambda + Q)} \right]^{1/2}$ | $Q(\lambda + Q)/3\epsilon + 3\epsilon(1 + \omega_\phi)$ | $Q(\lambda + Q)/3\epsilon + 3\epsilon(1 + \omega_\phi)$ | $Q(\lambda + Q)/3\epsilon + 3\epsilon(1 + \omega_\phi)$ |
| (g) | $\frac{\sqrt{6}(1 + \omega_\phi)}{2(\lambda + Q)}$ | $\left[ \frac{-2Q(\lambda + Q) + 3\epsilon(1 - \omega_\phi)}{3V_\phi(\lambda + Q)} \right]^{1/2}$ | $-Q(\lambda + Q)/3\epsilon + 3\epsilon(1 + \omega_\phi)$ | $-Q(\lambda + Q)/3\epsilon + 3\epsilon(1 + \omega_\phi)$ | $-Q(\lambda + Q)/3\epsilon + 3\epsilon(1 + \omega_\phi)$ |

3. Perturbation dynamics

Matter perturbation can be regarded as a small deviation of Einstein field equation from its background value. In the linear regime perturbed gravity can be written as follows

$$\delta \left[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right] = \delta \left[ 8\pi G T_{\mu\nu} \right]. \tag{10}$$

Metric tensor of perturbed universe in a conformal-Newtonian gauge is given by

$$g_{\mu\nu} = a^2 \begin{pmatrix} -(1 + 2\Phi) & 0 \\ 0 & (1 - 2\Psi) \delta_{ij} \end{pmatrix}. \tag{11}$$

For our necessity, in this section we assume that the universe contains only matter and scalar field, which are both perfect fluids and obey energy momentum tensor $T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}$. By solving eq. (10) with eq. (11) and defining $c_s^2 = \dot{\rho}/\rho$ and $\delta = \delta\rho/\rho$, and then expanding eq. (3) up to linear perturbation, we obtained

$$3 \dot{\Phi} - \dot{\Phi} - \dot{\Phi} - \Phi = 4\pi G a^2 \dot{\delta}\rho,$$

$$k^2 (\dot{\Phi}' + \Phi) = 4\pi G a^2 \left( 1 + \omega \right) \rho \theta,$$

$$\Phi'' + 3H \dot{\Phi} + \left( 2H^2 - H_0^2 \right) \Phi = 4\pi G a^2 c_s^2 \delta\rho,$$
\[ \delta' = 3Q\phi'\left(\omega - c_s^2\right)\delta + 3H\left(\omega - c_s^2\right)\delta - (1 + \omega)(\theta + 3\Phi') + Q\left(1 - 3\omega\right)\left(\dot{\phi}\right)', \]  

where \( \dot{\phi} \equiv \delta \phi \). Here we have transformed the equations into Fourier space, where \( k \) denotes the wavenumber of perturbation. Eq. (12) is reduced to uncoupled case for \( Q = 0 \). From the last term of the above equations, we see that the existence of coupling \( Q \) connects scalar field perturbation to matter perturbation. As a result, the deviation of scalar field will affect the evolution of the matter perturbation. The explicit form of \( \dot{\phi} \) can be obtained by perturbing eq. (2), and for sub-horizon scale it yields

\[ \dot{\phi} = -3\lambda_k^2 Q\Omega_m (1 - \Omega_s)\delta_m', \]

where \( \lambda_k^2 \equiv aH/k \). In \( N \equiv \ln \alpha \) coordinate and applying \( \omega \approx c_s^2 \ll 1 \) eq. (12) takes a simpler form

\[ \frac{d\delta_m}{dN} = -3\frac{d\phi}{dN} + Q\frac{d\dot{\phi}}{dN}. \]  

To solve eq. (13) we need an equation of velocity perturbation, and it has been given by [16]. In our notation it is written as

\[ \frac{d\dot{\theta}_m}{dN} = -\left(2 + \frac{1}{2H} + \sqrt{6}Q\right)\dot{\theta}_m + \frac{1}{\lambda_k^2}\left(\Phi + Q\dot{\phi}\right). \]  

From eq. (13) and (14) we finally obtained the perturbation equation

\[ \frac{d^2\delta_m}{dN^2} + \left(1 - 3\omega_{\text{eff}} + \sqrt{6}Q\right)\frac{d\delta_m}{dN} - \frac{3}{2} (1 - \Omega_s)\left[1 + 2Q_\text{eff} (1 - \Omega_s)^2\right] \delta_m = 0. \]  

Eq. (15) is a second-order non-linear differential equation, which is difficult to solve analytically because of its complexity in terms of time. However, we can still approximate its solutions by recalling our results in the previous section.

3.1. Matter dominated era

This regime is associated with point (a) in Table 1. For matter with \( \omega_m = 0 \) the parameters are: \( x = -\sqrt{6}/3\epsilon, \Omega_\phi = 2Q^2/3\epsilon \), and \( \omega_{\text{eff}} = 2Q^2/3\epsilon \). The solution is \( \delta(a) = C_1a^{\gamma_1} + C_2a^{\gamma_2} \), where

\[ \gamma_{1,2} = \frac{1}{\epsilon} \left[ \frac{3Q^2}{2} - \frac{\epsilon}{4} - \frac{1}{12} \left(192Q^6 - 576\epsilon^2 Q^4 + 432\epsilon^2 Q^2 + 3324Q^2 - 252\epsilon Q^2 + 225\epsilon^2\right) \right]. \]

For the ordinary field \( \epsilon = +1 \), \( Q \neq 0 \) implies \( \gamma > 1 \), which means that for this mode structures grow faster than in uncoupled cases, since \( \omega_{\text{eff}} \) is not exactly equal to zero, but there exists a small amount of energy which is transformed into matter via the coupling \( Q \). In contrast, phantom field \( \epsilon = -1 \) makes structures grow slower than in uncoupled cases, due to its negative energy that inverts the flow of the energy current.

3.2. Matter scaling era

This regime is associated with point (f). For matter scaling with \( \omega_m = 0 \), it is easily obtained that \( Qx = -\sqrt{6}\omega_{\text{eff}}/2 \). The solution is similar to that in subsection 3.1, with

\[ \gamma_{1,2} = \frac{1}{4} \left[ 9\omega_{\text{eff}} - 1 \pm \sqrt{96Q^2\Omega_\phi^2 - 144Q^2\Omega_\phi^2 + 48Q^2 + 81\omega_{\text{eff}}^2 + 48\Omega_\phi^2 - 18\omega_{\text{eff}} + 25} \right]. \]  

Growth of matter perturbation in this regime is determined by three quantities: \( Q, \omega_{\text{eff}} \) and \( \Omega_\phi \). Eq. (16) shows that \( \gamma > 1 \) for \( Q \neq 0 \). However, we do not expect an excessive value of \( \gamma \), because it has not been expected by observation of Integrated Sachs-Wolfe (ISW) effect [13]. Phantom field corresponds to the condition \( \Omega_\phi + \omega_{\text{eff}} < 0 \), where the solutions are negative and complex with negative real parts. The complex solutions with negative real part cause perturbation to decays and oscillates harmonically. In [17] this phenomenon is called phantom damping.
3.3. Scalar field dominated era

This regime is associated with point (d) in Table 1. The solution is similar to that in previous subsection, with

\[
\gamma_{1,2} = \frac{1}{2} \left[ -5 + \frac{3 \lambda^2}{2 \varepsilon} \pm \sqrt{\frac{25 - 9 \lambda^4}{4 \varepsilon^2}} \right].
\]  

(17)

\(\gamma_1\) will always be zero for all \(\lambda\), which leads to constant mode solutions. Whereas, \(\gamma_2\) depends only to \(\lambda\), i.e. \(\gamma_2 = -5 + 3 \lambda^2/2 \varepsilon\). As explained in the previous section, point (d) will trigger acceleration solutions if \(\lambda < \sqrt{2}\) is satisfied, therefore it leads the universe towards the condition \(\gamma_2 < -5 + 6/2 \varepsilon\). In the eq. (17) we see that at point (d) the coupling effect is vanished, since the matter fraction in the universe is nearly zero. An ordinary field (\(\varepsilon = +1\)) implies \(\gamma_2 < -2\), meaning that density contrast will decay in time, whereas phantom field implies \(\gamma_2 < -8\). The decay of density contrasts following phantom scenario will occur faster than that of ordinary field does. This indicates that background dynamics in phantom model expands more explosively compared to that in ordinary models. This result also suggests that big rip will be the relevant fate of our universe if phantom field is the dark energy we are searching for.

4. Conclusions

We have shown that a universe containing a coupled scalar field would have different dynamical evolution and effects to structures growth compared to uncoupled cases. By investigating the dynamics through stability analysis of its critical points we have found the critical points of the universe that related to physical regimes such as radiation dominated and its scaling era, matter dominated and its scaling era, and finally the solutions will be caught by stable solutions, i.e. scalar field dominated solutions. During matter dominated era, following ordinary field scenario structures grow faster than in phantom field and uncouple cases, whereas following phantom field scenario structures grow slower than both uncoupled and ordinary cases. Far in the future, the decay of structures following phantom scenario occurs drastically faster compared to that of ordinary field scenarios. This is related to background dynamics, which shows that big rip will be the relevant fate of our universe.

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