Forward dijets in high-energy collisions: evolution of QCD $n$-point functions beyond the dipole approximation

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Present knowledge of QCD $n$-point functions of Wilson lines at high energies is rather limited. In practical applications, it is therefore customary to factorize higher $n$-point functions into products of two-point functions (dipoles) which satisfy the BK evolution equation. We employ the JIMWLK formalism to derive explicit evolution equations for the 4- and 6-point functions of fundamental Wilson lines and show that if the Gaussian approximation is carried out before the rapidity evolution step is taken, then many leading order $N_c$ contributions are missed. Our evolution equations could specifically be used to improve calculations of forward dijet angular correlations, recently measured by the STAR collaboration in deuteron-gold collisions at the RHIC collider. Forward dijets in proton-proton collisions at the LHC probe QCD evolution at even smaller light-cone momentum fractions. Such correlations may provide insight into genuine differences between the JIMWLK and BK approaches.

I. INTRODUCTION

The Jalilian-Marian–Iancu–McLerran–Weigert–Leonidov–Kovner (JIMWLK) functional evolution equation describes the energy dependence of $n$-point functions of Wilson lines at small light-cone momentum fraction $x$ [1, 2]. These $n$-point functions appear in multi-particle production cross sections in hadronic (or heavy-ion) collisions. To date, our knowledge of the behavior of such $n$-point functions in QCD is very limited. Therefore, it is common to employ a Gaussian (and large-$N_c$) approximation which reduces these functions to powers of the two-point function, which at high transverse momentum corresponds to the well-known Balitsky-Fadin-Kuraev-Lipatov (BFKL) unintegrated gluon distribution [3]. The evolution of the two-point function in the dipole approximation [4] is determined by an ordinary integro-differential equation known as the Balitsky-Kovchegov (BK) equation [1, 5] which has been the subject of intense theoretical investigation in the past few years.

Evolution with energy or rapidity $y \sim \log 1/x$ occurs by (real or virtual) radiation of an additional gluon from a given $n$-point operator. Cross sections are related to expectation values of traces of such operators which project onto their physical matrix elements. If the expectation value of the $n$-point operator is split into dipoles before the radiation of the additional gluon (in order to perform the evolution step by means of the BK equation) then only the dipole from which the radiation emerged is allowed to split into two dipoles. On the other hand, if the evolution step is given by the JIMWLK equation then additional contributions arise, even at leading order in $N_c$. This is illustrated in fig. [1].
FIG. 1: Left: BK-evolution of a set of dipoles: in an evolution step, emission of a gluon can only split the parent dipole. Right: JIMWLK evolution of the $n$-point function allows for additional contributions.

To exhibit the differences between Gaussian+BK and JIMWLK evolution more clearly we focus on two specific $n$-point functions which appear in (forward) quark+gluon dijet production in pA collisions. Hadron-nucleus collisions are well suited for investigations of high gluon density QCD due to the fact that one can derive analytic relations for particle production when only the target is dense, while the projectile is dilute. Furthermore, hadron-nucleus collisions are free of the hot final state medium (“quark-gluon plasma”) produced in AA collisions which may affect the observables. Very recently, the STAR collaboration has presented two-hadron angular correlations in the forward rapidity region of deuteron-gold collisions which show a weakening/disappearance of the away side peak [6], in agreement with expectations from gluon saturation dynamics [7–10]. The purpose of the present paper is to derive explicit and complete (in terms of $N_c$ counting) evolution equations for the relevant $n$-point functions so that quantitative theoretical expectations could eventually be obtained.

The cross section for production of a valence quark plus a gluon at forward rapidity in pA collisions was calculated in [11] (in momentum space) and in [12] (in coordinate space); we also refer the reader to refs. [13]. For completeness, we reproduce the expressions in the appendix. They involve expectation values of products of (up to) six Wilson lines in the fundamental representation (single-inclusive production involves only the 2-point function [14]). Explicitly, the following operators appear in the two parton (quark+gluon) production cross section

$$O_4(r, \bar{r} : s) \equiv \text{tr} V_r^\dagger t^a V_r t^b [U_s]^{ab} = \frac{1}{2} \left[ \text{tr} V_r^\dagger V_s \text{tr} V_r V_s^\dagger - \frac{1}{N_c} \text{tr} V_r^\dagger V_r \right]$$

(1)

and

$$O_6(r, \bar{r} : s, \bar{s}) \equiv \text{tr} V_r V_s^\dagger t^a t^b [U_s U_{\bar{s}}]^{ab} = \frac{1}{2} \left[ \text{tr} V_r V_s^\dagger V_{\bar{s}} V_s - \frac{1}{N_c} \text{tr} V_r V_r \right]$$

(2)

where $V$ ($U$) is a Wilson line in the fundamental (adjoint) representation and $r$, $s$ etc. denote two-dimensional coordinates in the transverse plane. Here, we have used the identity

$$U^{ab} t^b = V^\dagger t^a V$$

(3)

to relate matrices in the two representations.

Due to the fact that explicit evolution equations for the above combinations of Wilson lines have not been derived so far, and therefore solutions to these equations are unknown, it is common to resort to the large $N_c$ and Gaussian approximation [10, 12, 15, 16]. In this approximation, these two expectation values can be written as

$$\langle O_4(r, \bar{r} : s) \rangle \simeq \langle O_2(r - s) \rangle \langle O_2(s - \bar{r}) \rangle$$

$$\langle O_6(r, \bar{r} : s, \bar{s}) \rangle \simeq \langle O_2(r - s) \rangle \langle O_2(\bar{r} - \bar{s}) \rangle \langle O_2(s - \bar{s}) \rangle + \langle O_2(r - \bar{r}) \rangle \langle O_2(s - \bar{s}) \rangle \langle O_2(\bar{s} - s) \rangle$$

(4)

where $\langle O_2(r, \bar{r}) \rangle \equiv \langle \text{tr} V_r V_r^\dagger \rangle$. To keep track of factors of $N_c$ it is convenient to normalize expectation values of traces...
of Wilson lines, i.e. the $S$ matrices, as follows:

$$S(r, \bar{r}) = \frac{1}{C_A} \langle O_2 \rangle$$

$$S_4(r, \bar{r} : s) = \frac{1}{C_A C_F} \langle O_4 \rangle$$

$$S_6(r, \bar{r} : s, \bar{s}) = \frac{1}{C_A C_F} \langle O_6 \rangle .$$

(5)

For phenomenological applications of the Color Glass Condensate formalism to two-particle production, it is common to employ the approximations [1] to factorize the higher point functions of Wilson lines into products of two point functions. This greatly simplifies applications as it allows one to write $S_4$ and $S_6$ in terms of the well known BK two point function. Then BK evolution of the two point function $S$ is assumed to account for small-$x$ evolution of the higher point functions $S_4$ and $S_6$ [10]. We shall show below that indeed this procedure retains the leading-$N_c$ contribution to the 4-point function $S_4$. On the other hand, we also show that it misses many leading-$N_c$ contributions to the evolution of the 6-point function $S_6$.

II. EVOLUTION EQUATIONS FOR HIGHER POINT FUNCTIONS

In this section we derive explicit evolution equations for the expectation values of $O_4$ and $O_6$ which appear in the two-particle production cross section. We then apply the large-$N_c$ and Gaussian approximations to our evolution equations and compare to the “naive” result [4] where BK evolution for the two point functions is used on the right hand side of (4).

We start from the JIMWLK evolution equation which determines the small-$x$ evolution of any $n$-point function $O$ from

$$\frac{d}{dy} \langle O \rangle = \frac{1}{2} \left( \int d^2x \int d^2y \frac{\delta}{\delta \alpha^b} \eta^{bd}_{xy} \frac{\delta}{\delta \alpha^d} O \right) ,$$

(6)

where

$$\eta^{bd}_{xy} = \frac{1}{\pi} \int \frac{d^2z}{(2\pi)^2} \frac{(x-z) \cdot (y-z)}{(x-z)^2(y-z)^2} \left[ 1 + U^b_x U^d_y - U^b_x U^d_y \right] .$$

(7)

We first focus on $O_4$ defined in eq. (1). The second term (the two point function) from that equation evolves according to the JIMWLK equation

$$\frac{d}{dy} \langle \text{tr} V_r^\dagger V_r \rangle = -\frac{N_c \alpha_s}{2\pi^2} \int d^2z \frac{(r-\bar{r})^2}{(r-\bar{r})^2} \left( \text{tr} V_r^\dagger V_r - \frac{1}{N_c} \text{tr} V_r^\dagger V_r V_z^\dagger V_z \right) .$$

(8)

This reduces to the BK equation in the Gaussian and large-$N_c$ approximations. On the other hand, the first term in $O_4$ involving four fundamental Wilson lines satisfies

$$\frac{d}{dy} \langle \text{tr} V_r^\dagger V_s \text{tr} V_r^\dagger V_s \rangle = \frac{1}{2} \left( \int d^2x \int d^2y \frac{\delta}{\delta \alpha^b} \frac{\delta}{\delta \alpha^d} \text{tr} V_r^\dagger V_s \text{tr} V_r^\dagger V_s \right) .$$

(9)

Using the explicit form of $\eta^{bd}_{xy}$ given by (7), we note that the derivative of $\eta$ (w.r.t. $\alpha^b$) vanishes due to the color structure, except for the last term $\sim U^b_x U^d_y$ where one does need to differentiate $U_y$. The calculation is straightforward but lengthy and involves repeated use of eq. (3) as well as the Fierz identity for the product of fundamental matrices

$$[t^a]_{ij} [t^a]_{kl} = \frac{1}{2} \left[ \delta_{il} \delta_{jk} - \frac{1}{N_c} \delta_{ij} \delta_{kl} \right] .$$

(10)
Thus, it is evident that the factorized form (4) for the 4-point function
\[ \langle \text{originate from the first and third lines of (11). Written in terms of the scattering matrix } \] 
\( S \) \( \text{of the scattering matrix for two quark-anti-quark dipoles evolving with rapidity while (multiply) scattering from the} \]
on the rhs feature at least as many Wilson lines as the original operator.

Adding the evolution equation for the two point function, eq. (8), we obtain
\[ \text{In passing, we note that to arrive at eq. (14) we have dropped 12 terms of order 1} \]
\[ \text{Using the BK equation for the 2-point function} \]
\[ \text{We now apply the large} \]
\[ \text{Equation (11) describes the small-} \]
\[ \text{Next, we derive the evolution equation for the 6-point function} \]
\[ \text{Contributions on the rhs} \]
\[ \text{The leading-} \]
\[ \text{We now apply the large} \]
\[ \text{We can rewrite eq. (12) as} \]
\[ \text{Thus, it is evident that the factorized form} \]
\[ \text{Next, we derive the evolution equation for the 6-point function} \]
\[ \text{The procedure} \]
We have checked that this equation remains finite when the internal integration variable is straightforward but very tedious, here we just quote the final result:

\[
\frac{d}{dz} \langle O_6(r, \bar{r}, s, \bar{s}) \rangle = -\frac{N_c \alpha_s}{2(2\pi)^2} \int d^2z
\]

\[
\left. \left[ \frac{(r-s)^2}{(z-r)^2(z-s)^2} + \frac{(r-\bar{r})^2}{(z-r)^2(z-\bar{r})^2} + \frac{(r-\bar{s})^2}{(z-r)^2(z-\bar{s})^2} + 3\frac{(s-\bar{s})^2}{(z-s)^2(z-\bar{s})^2} \right] \text{tr} V_r V_r^\dagger V_s V_s^\dagger \text{tr} V_r V_s^\dagger \right.
\]
\[
+ \frac{1}{N_c} \left[ -\left[ \frac{(r-s)^2}{(z-r)^2(z-s)^2} + \frac{(r-\bar{s})^2}{(z-r)^2(z-\bar{s})^2} - \frac{(s-\bar{s})^2}{(z-s)^2(z-\bar{s})^2} \right] \text{tr} V_r V_r^\dagger V_s V_s^\dagger \text{tr} V_r V_s^\dagger V_s V_s^\dagger \right.
\]
\[
- \frac{\left[ (r-s)^2(\bar{r}-\bar{s})^2 \right.}{(z-s)^2(\bar{s}-\bar{s})^2} \text{tr} V_r V_r^\dagger V_s V_s^\dagger \text{tr} V_r V_s^\dagger V_s V_s^\dagger \left. \left[ \frac{(r-s)^2(\bar{r}-\bar{s})^2}{(z-s)^2(\bar{s}-\bar{s})^2} - \frac{(s-\bar{s})^2}{(z-s)^2(\bar{s}-\bar{s})^2} + \frac{(s-\bar{s})^2}{(z-s)^2(\bar{s}-\bar{s})^2} \right] \text{tr} V_r V_r^\dagger V_s V_s^\dagger \text{tr} V_r V_s^\dagger V_s V_s^\dagger \right.
\]
\[
+ 2 \frac{(r-s)^2}{(z-s)^2(z-\bar{s})^2} \text{tr} V_r V_r^\dagger V_s V_s^\dagger \text{tr} V_s V_s^\dagger \right.
\]
\[
+ \left. \left[ \frac{(r-s)^2}{(z-r)^2(z-s)^2} - \frac{(r-\bar{s})^2}{(z-r)^2(z-\bar{s})^2} + \frac{(s-\bar{s})^2}{(z-s)^2(z-\bar{s})^2} \right] \text{tr} V_r V_r^\dagger V_s V_s^\dagger \text{tr} V_r V_s^\dagger V_s V_s^\dagger \right.
\]
\[
- \frac{\left[ (r-s)^2(\bar{r}-\bar{s})^2 \right.}{(z-s)^2(\bar{s}-\bar{s})^2} \text{tr} V_r V_r^\dagger V_s V_s^\dagger \text{tr} V_r V_s^\dagger V_s V_s^\dagger \left. \frac{(r-s)^2(\bar{r}-\bar{s})^2}{(z-s)^2(\bar{s}-\bar{s})^2} - \frac{(s-\bar{s})^2}{(z-s)^2(\bar{s}-\bar{s})^2} + \frac{(s-\bar{s})^2}{(z-s)^2(\bar{s}-\bar{s})^2} \right] \text{tr} V_r V_r^\dagger V_s V_s^\dagger \text{tr} V_r V_s^\dagger V_s V_s^\dagger \right.
\]
\[
- 2 \frac{(r-s)^2}{(z-s)^2(z-\bar{s})^2} \text{tr} V_r V_r^\dagger V_s V_s^\dagger \text{tr} V_s V_s^\dagger \right.
\]
\[
- \left. \left[ \frac{(r-s)^2}{(z-r)^2(z-s)^2} - \frac{(r-\bar{s})^2}{(z-r)^2(z-\bar{s})^2} + \frac{(s-\bar{s})^2}{(z-s)^2(z-\bar{s})^2} \right] \text{tr} V_r V_r^\dagger V_s V_s^\dagger \text{tr} V_r V_s^\dagger V_s V_s^\dagger \right.
\]
\[
+ \left. \left[ \frac{(r-s)^2}{(z-r)^2(z-s)^2} - \frac{(r-\bar{s})^2}{(z-r)^2(z-\bar{s})^2} + \frac{(s-\bar{s})^2}{(z-s)^2(z-\bar{s})^2} \right] \text{tr} V_r V_r^\dagger V_s V_s^\dagger \text{tr} V_r V_s^\dagger V_s V_s^\dagger \right.
\]
\[
- 4 \frac{(r-s)^2}{(z-s)^2(z-\bar{s})^2} \text{tr} V_r V_r^\dagger \right.
\]
\[
+ \frac{2}{N_c} \frac{(r-\bar{r})^2}{(z-r)^2(z-\bar{r})^2} \text{tr} V_r V_r^\dagger \right. \}
\]

We have checked that this equation remains finite when the internal integration variable \( z \) approaches any of the external coordinates \( r, \bar{r}, s, \bar{s} \). Note the appearance of lower point functions on the right hand side of the equation. Unlike the similar terms in the evolution of \( O_4 \) these cannot be combined into the original operator \( O_6 \).

We may now employ the Gaussian approximation on the rhs of the equation above to exhibit the leading-\( N_c \)
contributions (from lines 1 and 7 – 13). The equation reduces to

\[ \frac{d}{dy} S_{ab} \simeq -\frac{N_c\alpha_s}{(2\pi)^2} \int d^2z \left\{ \frac{(r - s)^2}{(r - z)^2(s - z)^2} + \frac{(r - \bar{r})^2}{(r - \bar{z})^2(\bar{z} - z)^2} + \frac{(\bar{r} - s)^2}{(\bar{r} - z)^2(s - z)^2} + 3\frac{(s - \bar{s})^2}{(s - z)^2(\bar{z} - z)^2} \right\} 

\begin{align*}
&+ \left[ -\frac{(r - \bar{r})^2}{(r - \bar{z})^2(\bar{z} - z)^2} - \frac{(r - s)^2}{(r - z)^2(s - z)^2} + \frac{(s - \bar{s})^2}{(s - z)^2(\bar{z} - z)^2} \right] \left[ \left( S(z - \bar{r}) S(s - \bar{s}) + S(z - s) S(\bar{r} - \bar{s}) \right) S(r - z) S(s - \bar{s}) \right] \\
&+ \left[ \left( s - \bar{s} \right)^2 \right] \left[ \left( S(r - \bar{r}) S(s - \bar{s}) + S(r - s) S(\bar{r} - \bar{s}) \right) S(z - \bar{r}) S(s - \bar{s}) \right] \\
&- \left[ \left( r - s \right)^2 \right] \left[ \left( S(r - \bar{r}) S(s - \bar{s}) + S(r - z) S(\bar{r} - \bar{s}) \right) S(r - s) S(\bar{r} - \bar{s}) \right] \\
&+ \frac{2s}{(s - \bar{s})^2(s - z)^2} \left[ \left( S(r - \bar{r}) S(s - \bar{s}) + S(r - s) S(\bar{r} - \bar{s}) \right) S(s - z) S(\bar{r} - \bar{s}) \right] \\
&- \left[ \left( \bar{r} - s \right)^2 \right] \left[ \left( S(r - \bar{r}) S(s - \bar{s}) + S(r - \bar{z}) S(\bar{r} - \bar{s}) \right) S(r - s) S(\bar{r} - \bar{s}) \right] \\
&- \left[ \left( r - \bar{r} \right)^2 \right] \left[ \left( S(r - \bar{r}) S(s - \bar{s}) + S(r - \bar{z}) S(\bar{r} - \bar{s}) \right) S(r - s) S(\bar{r} - \bar{s}) \right] \\
&+ \frac{2s}{(s - \bar{s})^2(s - z)^2} \left[ \left( S(r - \bar{r}) S(s - \bar{s}) + S(r - s) S(\bar{r} - \bar{s}) \right) S(s - z) S(\bar{r} - \bar{s}) \right] \right\} 
\end{align*} \tag{16}

There are many more terms in this equation than obtained by differentiating eq. (4). To make this more clear, we write eq. (16) in the form

\[ \frac{d}{dy} S_{ab}(r, \bar{r} : s, \bar{s}) \simeq \frac{d}{dy} \left[ S(r - s) S(s - \bar{s}) S(s - \bar{s}) + S(r - \bar{r}) S(s - \bar{s}) S(s - \bar{s}) \right] \\
- \frac{N_c\alpha_s}{(2\pi)^2} S(s - \bar{s}) \int d^2z \left\{ \left[ \frac{(r - s)^2}{(r - z)^2(s - z)^2} + \frac{(r - \bar{r})^2}{(r - \bar{z})^2(\bar{z} - z)^2} + \frac{(\bar{r} - s)^2}{(\bar{r} - z)^2(s - z)^2} + \frac{(s - \bar{s})^2}{(s - z)^2(\bar{z} - z)^2} \right] \left[ S(r - \bar{r}) S(z - \bar{r}) S(r - z) \right] S(s - \bar{s}) \\
+ \left[ \frac{(r - \bar{r})^2}{(r - \bar{z})^2(\bar{z} - z)^2} + \frac{(s - \bar{s})^2}{(s - z)^2(\bar{z} - z)^2} - \frac{(\bar{r} - s)^2}{(\bar{r} - z)^2(s - z)^2} - \frac{(r - s)^2}{(r - z)^2(s - z)^2} \right] \left[ S(r - s) S(z - \bar{s}) S(r - z) \right] S(\bar{r} - \bar{s}) \\
+ \left[ \frac{(r - \bar{s})^2}{(r - \bar{z})^2(\bar{z} - z)^2} - \frac{(r - \bar{r})^2}{(r - \bar{z})^2(\bar{z} - z)^2} + \frac{(s - \bar{s})^2}{(s - z)^2(\bar{z} - z)^2} + \frac{(\bar{r} - s)^2}{(\bar{r} - z)^2(s - z)^2} \right] \left[ S(r - s) S(z - \bar{s}) S(r - z) \right] S(\bar{r} - \bar{s}) \\
- \left[ \frac{(r - s)^2}{(r - z)^2(s - z)^2} - \frac{(r - \bar{r})^2}{(r - \bar{z})^2(\bar{z} - z)^2} + \frac{(s - \bar{s})^2}{(s - z)^2(\bar{z} - z)^2} - \frac{(\bar{r} - s)^2}{(\bar{r} - z)^2(s - z)^2} \right] \left[ S(r - \bar{r}) S(z - \bar{s}) S(z - \bar{r}) \right] \right\} \tag{17}
\]

The terms in the bracket (first line) could also be obtained from the Gaussian approximation to the 6-point function combined with BK evolution of the dipoles. All other terms would be missed. The structure of the extra terms suggests why a naive Gaussian approximation fails; to see this, consider \(O_6\) from eq. (2). The first term is a product of two traces, a trace of 4 Wilson lines times a trace of two Wilson lines. Emission of a gluon between a different

\footnote{The second term in the bracket in the first line seems to have been omitted in ref. \cite{12}.}
quark-anti-quark pair is missed by the Gaussian approximation. In other words, if the Gaussian approximation is applied to the trace of four Wilson lines in eq. (2) before the rapidity evolution step, then due to color neutrality, a radiated gluon can not end up in another dipole. On the other hand, if one performs the rapidity evolution step of the full 4-point function then the emitted gluon can end up anywhere between other quark and anti-quark pairs, including those which are not the “parents” of the radiated gluon. These are precisely the contributions which are missed by the Gaussian approximation in the case of the 6-point function. It is easy to see that this is not possible for the 4-point function since the traces involve only two Wilson lines, and only one Gaussian contraction is possible (at leading order in $N_c$). This also means that our findings here do not affect fully inclusive observables such as DIS structure functions $F_2$ and $F_L$, or single inclusive particle production in pA collisions, since those involve the two point function; its evolution is only sensitive to the 4-point function which does not receive any leading $N_c$ corrections from JIMWLK (as compared to BK).

In summary, we have derived explicit evolution equations for the $n$-point functions that appear in forward dijet angular correlations in pA collisions. We find that factorizing these $n$-point functions into dipoles before performing evolution in rapidity misses many leading-$N_c$ contributions; higher multipole operators obey different evolution equations which can not be reduced to BK evolution of dipoles. Moreover, a rather large number of $N_c$-suppressed terms arises in the full JIMWLK evolution equations for higher $n$-point functions which may give substantial numerical contributions, especially when $n > N_c$.

The results presented here underscore the importance of rigorous solutions of the small-$x$ evolution of high point functions of Wilson lines. These could be obtained numerically via lattice-gauge theory techniques along the lines of ref. [19,20] where the small $x$ evolution of the two point function has been studied. For the two-point function, those authors found only very minor differences between the JIMWLK and BK (Gaussian + leading $N_c$ approximation) evolution equations. One may expect that due to the many leading-$N_c$ terms missed by the Gaussian approximation the differences between the JIMWLK and BK evolution of the 6-point function should be substantial (see also [18,22] for other observables where possible differences between JIMWLK and BK were investigated). Once (numerical) solutions to these evolution equations become available, they could also be used to improve present calculations [7–10] for dijet production and angular correlations.

Appendix A: Forward $q + g$ production in pA collisions

Here, we reproduce the expression for forward $q + g$ production (with comparable rapidities) in a valence-quark nucleus collision from ref. [12]. This expression is to be convoluted with valence quark distribution functions of a proton (or deuteron) and with parton $\rightarrow$ hadron fragmentation functions in order to obtain a physical cross section, see refs. [10] [12].

The $qA \rightarrow qgX$ hard scattering cross section is given by

$$
\frac{d^3\sigma^{qA\rightarrow qgX}}{d^3k d^3q} = \alpha_s C_F \delta(p^+ - k^+ - q^+) \int \frac{d^2x}{(2\pi)^2} \frac{d^2b}{(2\pi)^2} \frac{d^2b'}{(2\pi)^2} e^{ik_\perp (x'-x) + i(q_\perp - p_\perp) \cdot (b'-b)} \sum_{\lambda\alpha\beta} \phi^\lambda_{\alpha\beta}(p, k^+, x'-b') \phi^\lambda_{\alpha\beta}(p, k^+, x-b)
$$

$$
[S_6(b, x, b', x) - S_4(b, x, b' + z(x' - b')) - S_4(b + z(x - b), x', b') + S(b + z(x - b), b' + z(x' - b'))] . \quad (A1)
$$

Here, $z = k^+/p^+$ and $\phi^\lambda_{\alpha\beta}(p, k^+, x)$ denotes the amplitude of the $|qq\rangle$ Fock state component in the wave function of a dressed quark to leading order in $\alpha_s$; the explicit expression is given in ref. [12]. Lastly, the various $n$-point functions (target averages) are given by

$$
S_6(b, x, b', x') = \frac{1}{C_F C_A} \langle \text{tr} \ V_x U_x^\dagger V_b U_b^\dagger \rangle , \quad (A2)
$$

$$
S_4(b, x, b') = \frac{1}{C_F C_A} \langle \text{tr} \ V_{b'}^\dagger U_{b'} V_b U_b^\dagger \rangle , \quad (A3)
$$

$$
S(b, b') = \frac{1}{N_c} \langle \text{tr} \ V_b V_{b'} \rangle . \quad (A4)
$$

2 In this respect, the effects discussed here go beyond the correlations seen in dipole chains of the form $\text{tr} \ V_y V_\gamma^\dagger$ $\text{tr} \ V_z V_\phi^\dagger$ $\text{tr} \ V_x V_\pi^\dagger \cdots$ [17]. For such operators there is no cross-dipole emission as in fig. 1.
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[1] I. Balitsky, Nucl. Phys. B 463, 99 (1996).
[2] J. Jalilian-Marian, A. Kovner, L. D. McLerran and H. Weigert, Phys. Rev. D 55, 5414 (1997); J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, Nucl. Phys. B 504, 415 (1997), Phys. Rev. D 59, 014014 (1999), Phys. Rev. D 59, 014015 (1999), Phys. Rev. D 59, 034007 (1999) [Erratum-ibid. D 59, 099903 (1999)]; A. Kovner, J. G. Milhano and H. Weigert, Phys. Rev. D 62, 114005 (2000); A. Kovner and J. G. Milhano, Phys. Rev. D 61, 014012 (2000); E. Iancu, A. Leonidov and L. D. McLerran, Nucl. Phys. A 692, 583 (2001), Phys. Lett. B 510, 133 (2001); E. Ferreiro, E. Iancu, A. Leonidov and L. D. McLerran, Nucl. Phys. A 703, 489 (2002).
[3] L. N. Lipatov, Sov. J. Nucl. Phys. 23, 338 (1976) [Yad. Fiz. 23, 642 (1976)]; E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP 45, 199 (1977) [Zh. Eksp. Teor. Fiz. 72, 377 (1977)]; I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. 28, 822 (1978) [Yad. Fiz. 28, 1597 (1978)].
[4] N. N. Nikolaev and B. G. Zakharov, Z. Phys. C 49, 607 (1991); A. H. Mueller, Nucl. Phys. B 415, 373 (1994).
[5] Y. V. Kovchegov, Phys. Rev. D 60, 034008 (1999); Phys. Rev. D 61, 074018 (2000).
[6] E. Braidot [STAR Collaboration], arXiv:1005.2378 [hep-ph].
[7] D. Kharzeev, E. Levin and L. McLerran, Nucl. Phys. A 748, 627 (2005).
[8] L. Frankfurt and M. Strikman, Phys. Lett. B 645, 412 (2007).
[9] K. Tuchin, arXiv:0912.5479 [hep-ph].
[10] J. L. Albacete and C. Marquet, arXiv:1005.4065 [hep-ph].
[11] J. Jalilian-Marian and Y. V. Kovchegov, Phys. Rev. D 70, 114017 (2004) [Erratum-ibid. D 71, 079901 (2005)].
[12] C. Marquet, Nucl. Phys. A 796, 41 (2007).
[13] N. N. Nikolaev, W. Schäfer, B. G. Zakharov and V. R. Zoller, Phys. Rev. D 72, 034033 (2005); R. Baier, A. Kovner, M. Nardi and U. A. Wiedemann, Phys. Rev. D 72, 094013 (2005).
[14] A. Dumitru and J. Jalilian-Marian, Phys. Rev. Lett. 89, 022301 (2002).
[15] J. P. Blaizot, F. Gelis and R. Venugopalan, Nucl. Phys. A 743, 57 (2004).
[16] K. Fukushima and Y. Hidaka, JHEP 0706, 040 (2007).
[17] E. Levin and M. Lublinsky, Nucl. Phys. A 730, 191 (2004) arXiv:hep-ph/0308279; Phys. Lett. B 607, 131 (2005) [arXiv:hep-ph/0411121]; R. A. Janik and R. B. Peschanski, Phys. Rev. D 70, 094005 (2004) arXiv:hep-ph/0407007; R. A. Janik, arXiv:hep-ph/0409256.
[18] A. Dumitru and J. Jalilian-Marian, Phys. Rev. D 81, 094015 (2010).
[19] K. Rummukainen and H. Weigert, Nucl. Phys. A 739, 183 (2004).
[20] Y. V. Kovchegov, J. KuoKKanen, K. Rummukainen and H. Weigert, Nucl. Phys. A 823, 47 (2009).
[21] J. Bartels, Nucl. Phys. B 175, 365 (1980); T. Jaroszewicz, Acta Phys. Polon. B 11, 965 (1980); J. Kwiecinski and M. Praszalowicz, Phys. Lett. B 94, 413 (1980); Z. Chen and A. H. Mueller, Nucl. Phys. B 451, 579 (1995).
[22] C. Marquet and H. Weigert, Nucl. Phys. A 843, 68 (2010).