Parameter and Solar System constraint in Chameleon Brans Dick Theory

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The post Newtonian parameter is considered in the Chameleon-Brans-Dicke frame. In the first step, the general form of this parameter and also effective gravitational constant is obtained. An arbitrary function for $f(\Phi)$, which indicates the coupling between matter and scalar field, is introduced to investigate validity of solar system constraint. It is shown that Chameleon-Brans-Dicke model can satisfy the solar system constraint and gives us an $\omega$ parameter of order $10^3$, which is in comparable to the constraint which has been indicated in [10].

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1. INTRODUCTIONS

Recent cosmological and Astrophysical observations indicate that our universe is in accelerate expansion phase. These observations include type Ia supernovae data [1], Wilkinson Microwave Anisotropic Probe (WMAP) [2], X-ray [3], large scale structure [4] and etc. The combine analysis of these observations also suggest that our universe is spatially flat, and consist 73% dark energy, 23% dark matter and remanent matter is baryons [5]. Although the nature and origin of dark energy are unknown for researcher up to now; however people have introduced some proposals to describe it. Amongst the various proposals of dark energy to describe accelerate expansion of universe is a tiny positive time-independent cosmological constant, $\Lambda$, for which has the equation of state $\omega = -1$ [6, 7]. It is well-known the cosmological constant scenario has two problems: cosmological constant problem (why the cosmological constant is very smaller than its natural expectation?), and coincidence problem (why the dark energy and dark matter are comparable now?) [8]. Another alternative candidate for dark energy is the dynamical dark energy proposal. This scenario can be originate from scalar tensors concept. Whereas the Einstein’s general theory of gravity is a geometrical theory of space-time. The fundamental building block is a metric tensor field. So, the theory may be called a tensor theory. This scenario can provide a framework within which to model space-time variations of the Newtonian gravitational constant, $G$. The scalar tensor theory was conceived originally by Jordan, who started to embed a four-dimensional curve manifold in five dimensional flat space-time [9]. The scalar tensor theories feature a scalar field, $\phi$, which has non-minimal coupling with together the geometry in the gravitational action where Brans and Dicke (BD) have introduced it [10]. Brans-Dicke theory or its modifications have already proved to be useful in producing clues to the solutions for some of the outstanding problems in cosmology [11, 12].

The same mechanism that creates a scalar field non-minimal coupling to the geometry in these proposals can also lead to a coupling between the scalar and matter field. For instance two kind of scalar fields have this condition (coupling between scalar field and matter field) quintessence and chameleon [13, 14]. The quintessence mechanism is a light scalar field mass which couples to matter directly with gravitational strength, lead to undesirable large violation of the equivalence principle. Therefore authors [15] have introduced a scalar field which it has a coupling to matter of order one, named chameleon mechanism. Chameleons are scalar fields whose mass depends on the environment mass density. Indeed the chameleon proposal produces a way to an effective mass to a light scalar field via field self interaction, and interaction between matter field and scalar field. Therefore the Brans-Dicke action with non-minimal coupling between matter scalar field so called Chameleon-Brans-Dicke model [16]. Recently, Chameleon-Brans-Dicke has been used for description some models such as holographic dark energy (HDE) [17], agegraphic dark energy(ADE), and new agegraphic dark energy (NADE). All of these investigations have been worked out in the large scale, and the authors could obtain good results. Studying these works and also the works of Moffat et al. [18], where they have studied PPN-parameter in Jordan-Brans-Dicke cosmology, and recent work of Perivolaropoulos [19] motivated us to consider constraints of solar system in Chameleon-Brans-Dicke model. The motivation of this work is investigation the weak field limit and solar system constraints on parameterized post Newtonian (or Edington-Robertson) $\gamma$-parameter (PPN) in Chameleon-Brans-Dicke framework. In this case the scalar has non-minimal coupling with both of matter and geometry sector. However we are going to see how the Chameleon-Brans-Dicke theory fits the observation. Since the Chameleon-Brans-Dicke theory is a metric theory, motion of matter object can be
explained in the framework of PPN approximation. This work is organized in four sections, of which this introduction is the first. In section two, the action is introduced, and the field equation and the scalar field equation of motion are obtained, then we linearized these equations and find out the general solution for $\gamma$-post Newtonian parameter. In section three, we investigate the results for two special examples. In the first example, we suppose an exponential function for $f(\Phi)$, and the results are considered either in the presence of potential and in the absence of that. In the second example, the interaction between matter and scalar field is ignored, and we clearly see that our results are in perfect match with the results of previous works. Finally section four, which is conclusion, we summarize the results of this work.

II. GENERAL FRAMEWORK

For our investigation, we consider the Brans-Dicke action as

$$ S = \int d^4x \sqrt{-g} \left( R - \frac{\omega}{\Phi} \partial_{\mu} \Phi \partial_{\nu} \Phi - V(\Phi) + f(\Phi)L_m \right), \quad (1) $$

where $g$ is the metric determinant, $R$ is the Ricci scalar constructed from the metric $g_{\mu\nu}$, and $\Phi R$ has been replaced with the Einstein-Hilbert term is such a way that $G_{\mu\nu}^{\text{eff}} = \frac{16\pi}{\Phi^2} \left[ 1 - \frac{n\phi}{\Phi_0} \right]$, where $\Phi$ is the Chameleon-Brans-Dicke scalar field, $\omega$ is the dimensionless Chameleon-Brans-Dicke constant. $V(\Phi)$ is the potential; note that since the scalar field is dependence on the local mass we need a monotically decreasing function for potential where exhibit self interaction. Therefore we consider power law potential as

$$ V(\Phi) = M^4 \left( \frac{M^2}{\Phi} \right)^n, \quad (2) $$

where we refer the reader for more review to [21]. The last term on the right hand said of (1), namely $f(\Phi)L_m$, indicates non-minimal coupling between scalar field and matter, where $f(\Phi)$ is an arbitrary function of $\Phi$. One can obtain the gravitational field equation by taking variation of the action (1) with respect to the metric $g_{\mu\nu}$ as

$$ \Phi (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) = f(\Phi) T_{\mu\nu} + \frac{\omega}{\Phi} (\partial_{\mu} \Phi \partial_{\nu} \Phi - \frac{1}{2} g_{\mu\nu} (\partial_{\alpha} \Phi)^2) + \left[ \nabla_{\mu} \nabla_{\nu} - g_{\mu\nu} \square \right] \Phi - g_{\mu\nu} \frac{V(\Phi)}{2} \quad (3) $$

where $T_{\mu\nu}$ indicates the energy-momentum tensor. Taking variation of action with respect to the scalar field $\Phi$ gives us scalar field equation of motion as

$$ (2\omega+3) \square \Phi = T(f(\Phi) - \frac{1}{2} f f_{\Phi} \Phi V_{,\Phi}(\Phi) - 2V(\Phi)). \quad (4) $$

In this work, we consider weak field approximation. Since we are in solar system, a weak field solution plays the same important role that the corresponding solutions fills in general relativity. So that we expand around a constant-uniform background filed $\Phi_0$, and a Minkowskian metric tensor $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ as

$$ \Phi = \Phi_0 + \phi, \quad (5) $$

$$ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad (6) $$

where $h_{\mu\nu}$ is computed to the linear first approximation only. The linearized solution for Eq. (3) is as

$$ - \frac{\Phi_0}{2} \left[ \square (h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h) \right] = f(\Phi) T_{\mu\nu} + \partial_{\mu} \partial_{\nu} \phi - \eta_{\mu\nu} \square \phi - (\eta_{\mu\nu} + h_{\mu\nu}) V(\Phi) \quad (7) $$

and we have below relation for scalar field equation of motion in the weak field

$$ (2\omega + 3) \square \phi = T(f(\Phi) - \frac{1}{2} f f_{\Phi}(\Phi)) + (\Phi V_{,\Phi}(\Phi) - 2V(\Phi)) = \xi T + \zeta. \quad (8) $$

Now, we expand the potential function (2) around $\Phi_0$, so

$$ V(\Phi) = \frac{M^{4+2n}}{\Phi_0} \left( 1 - \frac{n\phi}{\Phi_0} \right) \quad (9) $$
Also, by the same algebraic analysis we have

$$\xi = (f_0 - \frac{1}{2} \Phi_0 f_0') + \frac{\phi}{2} (f_0' - \Phi_0 f_0'') = C_1 + C_2 \phi,$$  \hspace{1cm} (10)

$$\zeta = -(n+2)M^4 \left( \frac{M^2}{\Phi_0} \right)^n \left( 1 - \frac{n \phi}{\Phi_0} \right) = -C_3 \left( 1 - \frac{n \phi}{\Phi_0} \right).$$  \hspace{1cm} (11)

where \( f_0 = f(\Phi_0), \) \( f_0' \) and \( f_0'' \) are the first and second derivative of \( f(\Phi) \) with respect to scalar field \( \Phi, \) respectively. Since our approximation related to the solar system, we consider a stationary solution corresponding to a gravitational mass such as earth. Substituting above results in the field equation \( (17) \) and scalar field equation of motion \( (8) \) we arrive at the differential equation for \( \phi \) and \( h_{\mu \nu} \) as

$$\nabla^2 \phi - \frac{C_3}{\Phi_0 (2 \omega + 3)} \phi = -\frac{C_1 + C_2 \phi}{(2 \omega + 3)} \rho - C_3,$$  \hspace{1cm} (12)

$$\nabla^2 H_{00} - \frac{2M^6}{\Phi_0^2} H_{00} = -(f_0 + \phi f_0') \rho + \frac{2M^4}{\Phi_0},$$  \hspace{1cm} (13)

$$\nabla^2 H_{ij} - \frac{2M^6}{\Phi_0^2} H_{ij} = -\delta_{ij} \left( f_0 + \phi f_0' \right) \rho + \frac{2M^4}{\Phi_0},$$  \hspace{1cm} (14)

where \( H_{00} = \Phi_0 h_{00} - \phi \) and \( H_{ij} = \phi + \Phi_0 h_{ij}, \) and note that we have taken \( n = 1. \) These equation are consistent with the results of \( [19, 23] \) even though there is a little difference in constants. This differences are resulted from non-minimal coupling between matter and scalar field and the sort of potential which we have selected. Assuming \( \rho = M_e \delta(r) \) we obtain the relations as

$$\phi(r) = \frac{C_1 M_e}{4\pi(2\omega+3)r} e^{-kr},$$  \hspace{1cm} (15)

$$h_{00} = \frac{M_e e^{-k' r}}{4\pi \Phi_0 r} \left\{ f_0 + \left( \frac{M_e f_0 e^{-k' r}}{4\pi r} + 1 \right) \frac{C_1}{2\omega+3} \frac{1}{e^{-(k-k')r}} \right\},$$  \hspace{1cm} (16)

$$h_{ij} = \frac{M_e e^{-k' r}}{4\pi \Phi_0 r} \left\{ f_0 + \left( \frac{M_e f_0 e^{-k' r}}{4\pi r} + 1 \right) \frac{C_1}{2\omega+3} \frac{1}{e^{-(k-k')r}} \right\},$$  \hspace{1cm} (17)

where \( k = \sqrt{\frac{3}{2(\omega+3)}} M^2 \) and \( k' = \frac{\sqrt{2} M^2}{\Phi_0}. \) Note that there is some constant in the above relation related to the integration constant and the last term on the right hand side of the differential equation, but they can be omitted from the above relation. Also \( h_{\mu \nu} \) is interpreted as gravitational potential, so the presence of constant is not so important because the potential difference is a physical quantity in physics. From the standard expansion of metric, namely

$$g_{00} = -1 + 2u,$$  \hspace{1cm} (18)

$$g_{ij} = (1 + 2\gamma u)\delta_{ij},$$  \hspace{1cm} (19)

where \( u \) is the Newtonian potential. The \( \gamma \)-post Newtonian parameter can be expressed as the ratio of \((ii)\) and \((00)\) component of \( h_{\mu \nu} \)

$$\gamma = \frac{f_0 + \left( \frac{M_e f_0 e^{-k' r}}{4\pi r} + 1 \right) \frac{C_1}{2\omega+3} \frac{1}{e^{-(k-k')r}}}{f_0 + \left( \frac{M_e f_0 e^{-k' r}}{4\pi r} + 1 \right) \frac{C_1}{2\omega+3} \frac{1}{e^{-(k-k')r}}},$$  \hspace{1cm} (20)

As, it has been said in the first of this section, the effective gravitational constant can be expressed by scalar field as

$$G_{eff} = \frac{1}{16 \pi} = \frac{1}{16 \pi \Phi_0} \left( 1 - \frac{\phi}{\Phi_0} \right) \frac{M_e}{4\pi(2\omega+3)r} e^{-kr},$$  \hspace{1cm} (21)

Up to now, we could attain the \( \gamma \)-post Newtonian parameter and effective gravitational constant in the general form. In the next section, we consider some special cases and compare our result with the results of previous work.

### III. TYPICAL EXAMPLE

In the previous section we could obtain the main relation in the general form, now we try to consider this relation in the special cases and compare the results with the results of previous work.

#### A. First Example

In this subsection we choose a specific function for \( f(\Phi) \) as

$$f(\Phi) = \exp(\frac{\Phi}{\Phi_0}).$$  \hspace{1cm} (22)

By making use of the definition, the constants \( C_1 \) and \( C_2 \) are obtained as

$$C_1 = \frac{1}{2} e^{\frac{1}{\Phi_0}} \quad \text{and} \quad C_2 = 0.$$  

Now we investigate the results of this case for \( M = 0 \) and \( M \neq 0. \)
• If $M = 0$
  When we take $M = 0$, it means that the potential have been omitted. For this case, as well as previous constants, the constant $G_3$ vanishes. So with attention to the definitions of $k$ and $k'$, for scalar field $\phi$ and components of $h_{\mu\nu}$, we have

$$
\phi(r) = \frac{e^1 M_e}{8\pi(2\omega + 3)r} \tag{23}
$$

$$
h_{00} = \frac{M_e e^1}{4\pi \Phi_0 r} \left\{ 1 + \frac{1}{2(2\omega + 3)} \left( \frac{M_e e^1}{4\pi \Phi_0 r} + 1 \right) \right\}, \tag{24}
$$

$$
h_{ij} = \delta_{ij} \frac{M_e e^1}{4\pi \Phi_0 r} \left\{ 1 + \frac{1}{2(2\omega + 3)} \left( \frac{M_e e^1}{4\pi \Phi_0 r} - 1 \right) \right\}. \tag{25}
$$

And for $\gamma$-post Newtonian parameter and effective gravitational constant we have respectively

$$
\gamma = \frac{(4\omega + 5) + \frac{M_e e^1}{4\pi \Phi_0 r}}{(4\omega + 7) + \frac{M_e e^1}{4\pi \Phi_0 r}} \tag{26}
$$

$$
G_{eff} = \frac{1}{16\pi \Phi_0} \left( 1 - \frac{M_e e^1}{8\pi \Phi_0 (2\omega + 3)r} \right). \tag{27}
$$

• If $M \neq 0$
  Here, in contrast to the previous case we assume that we have potential, so one can obtain the scalar field function and the component of $h_{\mu\nu}$, as follow

$$
\phi(r) = \frac{M_e}{8\pi(2\omega + 3)r} e^{1-kr} \tag{28}
$$

$$
h_{00} = \frac{M_e}{4\pi \Phi_0 r} e^{1-k'r} \left\{ 1 + \left( \frac{M_e}{4\pi \Phi_0 r} e^{1-k'r} + 1 \right) \frac{e^{-(k-k')r}}{2(2\omega + 3)} \right\}. \tag{29}
$$

$$
h_{ij} = \delta_{ij} \frac{M_e}{4\pi \Phi_0 r} e^{1-k'r} \left\{ 1 + \left( \frac{M_e}{4\pi \Phi_0 r} e^{1-k'r} - 1 \right) \frac{e^{-(k-k')r}}{2(2\omega + 3)} \right\} \tag{30}
$$

Now one can easily obtain the $\gamma$-post Newtonian parameter and effective gravitational constant respectively as

$$
\gamma = \frac{1 + \left( \frac{M_e}{4\pi \Phi_0 r} e^{1-k'r} - 1 \right) \frac{e^{-(k-k')r}}{2(2\omega + 3)} + \frac{1}{2(2\omega + 3)}}{1 + \left( \frac{M_e}{4\pi \Phi_0 r} e^{1-k'r} - 1 \right) \frac{e^{-(k-k')r}}{2(2\omega + 3)}} \tag{31}
$$

and

$$
G_{eff} = \frac{1}{16\pi \Phi_0} \left( 1 - \frac{M_e e^{1-kr}}{8\pi \Phi_0 (2\omega + 3)r} \right). \tag{32}
$$

In this step, one can estimate the value of background scalar field, namely $\Phi_0$ from effective gravitational constant relation. According to the relation [32], the second term in the parenthesis is the modified term for the gravitational constant. So the gravitational constant is equal to

$$
G_0 = \frac{1}{8\pi \Phi_0 \Phi_1 pl} = \frac{1}{16\pi \Phi_0}. \tag{33}
$$

As it is clear, $\Phi_1 = 2.44 \times 10^{18} \text{GeV}$ [24]. Therefore, the value of $\Phi_0$ can easily be estimated as order of $10^{38}$. Now, we try to consider allowed values of $\omega$ for $\gamma_\text{obs} - 1 = (2.1 \pm 2.3) \times 10^{-5}$ [10, 22] with the help of [31]. After some algebraic analysis, $\omega$ can be written as

$$
\omega = \frac{1}{4} \left( \frac{2\Phi_0 m_{AU}}{-\sqrt{6} M^3} W \left[ -\sqrt{6} M^3 B(m_{AU}) - \frac{A(m_{AU})}{\gamma} + 1 \right] \right)^{-2} - \frac{3}{2}, \tag{33}
$$

where $A(m_{AU}) = \frac{M_e m_{AU} e^{-k_{m_{AU}}}}{4\pi \Phi_0} e^{1-\frac{k_{m_{AU}}}{m_{AU}}} - 1$, $B(m_{AU}) = \frac{M_e m_{AU} e^{-k_{m_{AU}}}}{4\pi \Phi_0} e^{1-\frac{k_{m_{AU}}}{m_{AU}}} + 1$, and $W$ is the Lambert (or product-log) function. From the above relation,
one can realize that there is a complex value for $\omega$, for $\gamma > 1$, which is not acceptable, so $\gamma > 1$ is illegal for this model. Now, we turn our attention to the values of $\gamma$ that are smaller than one. In above figure, the $\omega$ parameter and effective gravitational constant have been plotted versus scalar field $\Phi_0$ and mass scale $m_{AU}$, which corresponds to the $r = 1AU = 10^8km$. In Fig.1, $\gamma$-post Newtonian parameter has been taken as $\gamma - 1 = 0.2 \times 10^{-5}$, and $\gamma - 1 = 2.5 \times 10^{-5}$ for Fig.2. In first case, the $\omega$ parameter has been obtained as order of $10^5$, and in the second case, it is in order of $10^4$. As we see, the $\omega$ parameter almost has a constant value for selected range of mass scale and scalar field. These value of $\omega$ satisfy the condition $\omega > 10^4$ for solar system limit. Also, the effective gravitational constant has been plotted versus the mass scale and scalar field for mentioned values of $\gamma$. For both case, $G_{eff}$ tends to a specific value, which is in order of $10^{-38}$. There is a difference between gravitational constant and effective gravitational constant.

\begin{align*}
\phi(r) &= \frac{M_\epsilon}{4\pi(2\omega + 3)r}, \quad (34) \\
h_{00} &= \frac{2(\omega + 2)}{(2\omega + 3)} \frac{M_\epsilon}{4\pi\Phi_0 r}, \quad (35)
\end{align*}

B. Second Example

In this case we set $f(\Phi) = 1$, so the model should be back to the Brans-Dicke model [19]. From the definition of constant we realize that $C_1 = 1$ and $C_2 = 0$. Like the previous subsection we investigate this case for $M = 0$ and $M \neq 0$.

- **If $M = 0$**

In this state, from the definition of constants we have

$$C_2 = C_3 = k = k' = 0.$$ 

The scalar field get the simple form

\begin{equation}
\phi(r) = \frac{M_\epsilon}{4\pi(2\omega + 3)r}, \quad (34)
\end{equation}

and for $h_{\mu\nu}$ we have

\begin{equation}
h_{00} = \frac{2(\omega + 2)}{(2\omega + 3)} \frac{M_\epsilon}{4\pi\Phi_0 r}, \quad (35)
\end{equation}
\[ h_{ij} = \delta_{ij} \frac{2(\omega + 1)}{2(\omega + 3)} \frac{M_e}{4\pi\Phi_0 r}. \] (36)

In this case as we expected, \( \gamma \)-post Newtonian parameter takes its familiar form \cite{25}, namely

\[ \gamma = \frac{h_{ij}|_{i=j}}{h_{00}} = \frac{\omega + 1}{\omega + 2}, \] (37)

and we have effective gravitational constant as

\[ G_{\text{eff}} = \frac{1}{16\pi\Phi_0} \left( 1 - \frac{M_e}{4\pi\Phi_0(2\omega + 3)r} \right). \] (38)

- **If \( M \neq 0 \)**

In this situation, the constant \( C_3 \) is not vanished, and if we attend to the action we realize that we have come back to the BD action, so we should be back to the results of \cite{19}; however note that we have pick out a different potential function. For this case, we have

\[ \phi(r) = \frac{M_e}{4\pi(2\omega + 3)r} e^{-kr}, \] (39)

and also for the component of \( h_{\mu\nu} \), we have

\[ h_{00} = \frac{M_e e^{-k'r}}{4\pi\Phi_0 r} \left( 1 + \frac{e^{-(k-k')r}}{(2\omega + 3)} \right), \] (40)

\[ h_{ij} = \delta_{ij} \frac{M_e e^{-k'r}}{4\pi\Phi_0 r} \left( 1 - \frac{e^{-(k-k')r}}{(2\omega + 3)} \right). \] (41)

From the above relation we can express \( \gamma \)-post Newtonian parameter as

\[ \gamma = \frac{1 - \frac{e^{-(k-k')r}}{2\omega + 3}}{1 + \frac{e^{-(k-k')r}}{2\omega + 3}}. \] (42)

And also from \cite{21} the effective gravitational constant can be obtained as

\[ G_{\text{eff}} = \frac{1}{16\pi\Phi_0} \left( 1 - \frac{M_e e^{-kr}}{4\pi\Phi_0(2\omega + 3)r} \right), \] (43)

when we compare these result with the result of \cite{19}, we see that they are in a perfect match, although there is a difference which appears in \( k' \). This difference is resulted from the sort of potential function we have selected.

From above discussion we see that our results is compatible with the results of previous work \cite{19, 25} and this match can be a conformation for the validity of Chameleon-Brans-Dicke theory model.

**IV. CONCLUSION**

In this work, we tried to obtain \( \gamma \)-post Newtonian parameter in the Chameleon-Brans-Dicke model. The gravitational constant is promoted to a dynamical scalar field in this model, therefore we could predict the effective gravitational constant as well. In section two, we found out the general form of \( \gamma \)-post Newtonian parameter, and as we saw it is a complicated function. In the first part of third section, an exponential function for \( f(\Phi) \) has been suggested, and \( \gamma \)-post Newtonian parameter and effective gravitational constant has been acquired in the presence of potential and without potential either. When we plotted the \( \omega \) parameter for two observational values of \( \gamma \), we realized that \( \omega \) took the values of order \( 10^4 \) which satisfy the solar system constraint. This is an achievement of Chameleon-Brans-Dicke model in solar system limit. Also, we could plot the effective gravitational constant as well. We found out that there is a small difference between gravitational constant and effective gravitational constant. In the last part of this work, we ignored the coupling between matter and scalar field, by setting \( f(\Phi) = 1 \), to consider the compatibility of our results with the previous works; after some algebraic analysis, we arrived at this result that they are match. The achievements of Chameleon-Brans-Dicke model in large scale, in description of agegraphic, new egegraphic, and holographic models, and also satisfying solar system constraint indicate advantage of this model.

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