Interface depinning in a disordered medium - numerical results

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Abstract: We propose a lattice model to study the dynamics of a driven interface in a medium with random pinning forces. The critical exponents characterizing the depinning transition are determined numerically in 1+1 and 2+1 dimensions. Our findings are compared with recent numerical and analytical results for a Langevin equation with quenched noise, which is expected to be in the same universality class.

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I. Introduction

In recent years there has been considerable interest in the scaling behaviour of interfaces and directed lines broadened by randomness [1] and in the critical phenomena which occur at the onset of steady-state motion [2,3]. The driven viscous motion of an interface in a medium with random pinning forces combines aspects of both of these two fields. An example is the motion of a domain wall in the random field Ising model [4,5]. The relaxation of metastable domains in a diluted antiferromagnet with applied external magnetic field provides a possible experimental realization [6]. Due to random field pinning the domain walls become rough and if the driving field is sufficiently weak compared to the random field strength, the domain walls become stuck and the domain state is frozen. Random field pinning can also be important for fluid displacement experiments in porous media [7], where an anomalous roughness in comparison with the KPZ-equation [1] was observed. Kessler, Levine and Tu [8] (KLT) and recently He, Kahanda and Wong [7] suggested that the increased roughness is due to quenched random capillary forces on the interface. (Another interesting proposal [9] is to assume that the noise fluctuates in time but follows a power-law distribution. This possibility will not be considered here.) Very similar problems arise in vortex pinning in type-II superconductors [10] and in sliding charge-density waves [2,11].

The simplest continuum description for the driven motion of an interface in a random medium is given by the following Langevin equation for a $D$-dimensional interface profile $z(x, t)$ [12,13]

$$\lambda \frac{\partial z}{\partial t} = \gamma \nabla^2 z + F + \eta(x, z)$$

(1)

where $\lambda$ and $\gamma$ are the inverse mobility and the stiffness constant, respectively, and $F$ is a uniform driving force. The random force $\eta(x, z)$ is Gaussian distributed with $\langle \eta \rangle = 0$ and correlations $\langle \eta(x_0, z_0)\eta(x_0 + x, z_0 + z) \rangle = \delta^D(x)\Delta(z)$. For the random field case the correlator $\Delta(z)$ is a monotonically decreasing function of $z$ for $z > 0$ and decays rapidly to zero over a finite distance. The difference to the Edwards-Wilkinson (EW) equation [14] is that the noise depends on the position of the interface, which makes the problem highly nonlinear. However, the noise that acts on a sufficiently
fast moving interface is fluctuating in time like in the EW equation. Therefore a
crossover to a EW regime is expected as the velocity increases to large values. Here
we are interested in the critical behaviour at the onset of motion where the external
driving force is just able to overcome the pinning forces. The velocity $v$ of the interface
scales with the reduced force $f = (F - F_c)/F_c$ as $v \sim f^\theta$ where $F_c$ is the threshold
force [2,15]. It can be shown [15] that there is a correlation length $\xi$ which diverges
with $F \to F_c$ as $\xi \sim f^{-\nu}$. Thus, at $F = F_c$, the roughness (or interface width)

$$w^2(L, t) = \langle [z(x, t) - z(x', t)]^2 \rangle$$

scales as [16]

$$w(L, t) \sim L^\zeta \Psi \left( \frac{t}{L^2} \right)$$

where $L$ is the system size, $\zeta$ is the roughness exponent, $z$ is the dynamical exponent
and $\Psi$ is a scaling function with $\Psi(y) \sim y^\beta$ ($\beta \equiv \zeta/z$) for $y \ll 1$ but becomes
constant for $y \gg 1$. In eq.(2) the overbar denotes the spatial average over $x$ and the
angular brackets mean the configurational average. It is known [12,15] that above
four interface dimensions $D$ the interface is flat, i.e. $\zeta = 0$. For $D = 4 - \epsilon$, $\epsilon \ll 1$
the critical exponents were calculated by a functional renormalization group scheme
to first order in $\epsilon$: $\zeta \simeq \epsilon/3$, $z = \zeta/\beta \simeq 2 - 2\epsilon/9$, $\nu \simeq 2/(6 - \epsilon)$ and $\theta = \nu(z - \zeta)$
[15]. The equation of motion (1) was simulated by KLT in 1+1 dimensions for a
wide range of velocities [8]. For $v \to 0$ they suggest that $\zeta \to 1$ which would be in
agreement with an Imry-Ma argument [4]. Parisi [17] proposed that the argument
$z$ of the random force $\eta$ in eq.(1) can be replaced by a constant from which follows
$\zeta = (4 - D)/2$, $\beta = (4 - D)/4$ and $\theta = 1$ for $D \leq 4$. He numerically determined $\beta$
in $D = 1, 2$ and $3$ from (1) with results consistent with his analytical arguments.
Recently, Tang [18] used the Runge-Kutta scheme to solve eq.(1) and found in 1+1
dimensions $\zeta = 1.25 \pm 0.10$, $\beta = 0.81 \pm 0.02$, $\theta = 0.4 \pm 0.05$ and $\nu = 1.1 \pm 0.1$, in
disagreement with both KLT [8] and Parisi [17].

Here we introduce a lattice model of probabilistic cellular automata [19] which
is expected to be in the same universality class as eq.(1). In 1+1 dimensions the
results are $\zeta(D = 1) = 1.25 \pm 0.01$, $\beta(D = 1) = 0.88 \pm 0.02$, and in 2+1 dimensions
$\zeta(D = 2) = 0.75 \pm 0.02$, $\beta(D = 2) = 0.475 \pm 0.015$ and $\theta(D = 2) = 0.65 \pm 0.05$. 
These values support the results of the $\epsilon$-expansion [15] because the deviations for $4 - D = \epsilon = 2$ are much smaller than for $\epsilon = 3$.

II. The model

In order to simplify the notation the model is explained for the case of 1+1 dimensions. The generalization to higher dimensions is straightforward. Consider a square lattice where each cell $(i, h)$ is assigned a random pinning force $\eta_{i, h}$ which takes the value 1 with probability $p$ and -1 with probability $q = 1 - p$. By excluding overhangs the interface is specified by a set of integer column heights $h_i(t)$, $i = 1, 2...L$. At $t = 0$ all columns have the same height $h_i(t = 0) = 0$. During the motion for a given time $t$ the value

$$v_i = h_{i+1}(t) + h_{i-1}(t) - 2h_i(t) + g\eta_{i, h}$$

is determined for all $i$, where $g$ is a parameter, $\eta_{i, h} = \pm 1$ and periodic boundary conditions are used. The interface configuration is then updated simultaneously for all $i$:

$$h_i(t + 1) = h_i(t) + 1 \quad \text{if} \quad v_i > 0$$

$$h_i(t + 1) = h_i(t) \quad \text{otherwise.}$$

The parameter $g$ measures the strength of the random force compared to the elastic force and the difference $p - q$ determines the driving force. The growth rule specified by eqs.(4) and (5) can be motivated by the continuum equation (1) because $v_i$ is the sum of the discretized Laplacian and a random force. It remains however an open question whether the use of a non-Gaussian noise and the discretization of $x, z$ and $t$ changes the universality class on accessible length scales. The relation of this model to the mechanism introduced earlier [20] for pinning by directed percolation will be discussed below.

III. Numerical results

a) 1+1 dimensions
Since we are interested in the scaling behaviour at the depinning transition one has to find first the critical value \( p_c(g) \) of the concentration of plus cells where the infinite system gets pinned. For \( g = 1 \) we estimate \( p_c(g = 1) \approx 0.8004 \), while \( p_c(g = 2) \approx 0.8748 \). To determine the roughness exponent \( \zeta \), different system sizes with \( 8 \leq L \leq 1024 \) were simulated at \( p = p_c \) until they get pinned. The values for \( w^2(L) \) were averaged over several thousand independent runs \( n \), depending on \( L (n\sqrt{L} \approx 8 \times 10^4) \). Fig.1 shows \( w^2(L) \) in a double logarithmic plot. For \( 32 \leq L \leq 512 \) the data are well fitted by a straight line from which we get \( \zeta(D = 1) = 1.25 \pm 0.01 \). Error bars indicate only statistical uncertainties. The fluctuations are somewhat larger for \( g = 2 \) than for \( g = 1 \). For bigger \( L \) the simulations become very time consuming because it takes longer for a larger system to get pinned at \( p = p_c \). In addition, when one increases \( L \), one has to know the threshold \( p_c \) more accurately to assure \( \xi \gg L \). To see the deviations from the scaling behaviour we define an effective roughness exponent \( \zeta(L) \equiv \log[\frac{w(2L)}{w(L)}]/\log 2 \). In fig.2 \( \zeta(L) \) for \( g = 1 \) and \( p = p_c \) is compared with \( \zeta(L) \) for \( g = 2 \) and \( p = 0.8744 < p_c \). For \( L = 256 \) and 512, \( \zeta(L) \) is significantly smaller for \( p < p_c \) than for \( p = p_c \).

Besides the possibility to determine the dynamical roughness exponent \( \beta \) from the width \( w(t) \) it is also possible to consider the scaling behaviour of \( H(t) \equiv \langle \bar{h}_i \rangle \sim t^\beta \). From the latter we get \( \beta(D = 1) = 0.86 \pm 0.02 \) for \( L = 16384 \), while from the width \( w(t) \) \( \beta(D = 1) = 0.87 \pm 0.04 \). The scaling behaviour of \( H(t) \) for larger systems with \( L = 262144 \) is plotted in fig.3, while in fig.4 the width \( w^2(t) \) is shown for the same runs. The data were averaged over 40 independent runs. (This took 21.5 h CPU on one Cray Y-MP processor. The other data presented in this paper required about 2000 h CPU on a IBM RS/6000 model 320 workstation.) The effective exponents \( \beta(t) \equiv \log[\frac{w(2t)}{w(t)}]/\log 2 \) (and analogously for \( H(t) \)) are shown in fig.5a. A possibility to take into account corrections to scaling is to plot \( w^2(2t) - w^2(t) \) versus \( t \). The corresponding effective exponents are shown in fig. 5b. The exponents determined by the width remain unchanged. Neither is there a clear improvement for the scaling of the height, although the plateau of \( \beta(t) \) seems to be larger but also the fluctuations are bigger in fig. 5b. Note that the uncertainties in the last two points are much larger, so we do not expect that \( \beta(t) \) increases to larger values. Thus, we
conclude $\beta(D = 1) = 0.88 \pm 0.02$. We have checked that this value is consistent with the case $g = 2$. The height-height correlation function $C^2(r, t) = \langle [h_{i+r}(t) - h_i(t)]^2 \rangle$ is expected to scale in the same way as the width in eq.(3). Fig.6 shows a scaling plot, where $C(r, t)$ is divided by $t^\beta$ and $r$ by $t^{1/z}$. The best data collapse is achieved for $\zeta \simeq 1.25$ and $\beta \simeq 0.88$, in complete agreement with the above results.

For large $p = 0.95$ and 0.97 the velocity is about $v = 0.78$ and 0.88, respectively ($g = 2$). From the interface width $w(t)$ we find $\beta(p \gg p_c) = 0.25 \pm 0.01$, which is in agreement with the expected EW [14] regime far away from the depinning transition. We were not able to estimate the exponent $\theta$. Due to the large roughness the fluctuations of $v$ become too large to fix the exponent $\theta$ with a reasonable accuracy.

b) 2+1 dimensions

As in 1+1 dimensions we determine $\zeta$ from the simulation of interfaces with different sizes until they get pinned. Fig.7 shows the effective exponents $\zeta(L)$ for $g = 4$, $p = p_c \simeq 0.6416$ and $g = 6$, $p = p_c \simeq 0.74448$ from which we estimate $\zeta = 0.75 \pm 0.02$. The scaling behaviour of the interface width for a system of size $L^2 = 1024^2$ is shown in fig.8 from which follows $\beta(D = 2) = 0.475 \pm 0.015$. (The data were averaged over 15 independent runs.) The scaling of $H(t)$ has to be considered with caution as will become clear from fig.9 where the effective exponents $\beta(t)$ are plotted. For $t \gg t_* = AL^{z}$, $H(t)$ goes to a constant for $p < p_c$ or grows linearly for $p > p_c$, while the scaling $H(t) \sim t^\beta$ is expected for $t_a \ll t \ll t_*$ where $t_a$ is the time where $\beta(t)$ achieves its asymptotic value. The plateau of $\beta(t)$ determined by $H(t)$ for $p > p_c$ in fig.9 (triangles) for $32 \leq t \leq 128$ may be attributed to a very slow crossover because $t_a$ is of the same order as $t_*$. Also for $p = p_c$, $\beta(t)$ (circles) never achieves its asymptotic value. Thus, also the value for $\beta(t)$ determined from the scaling of the interface height $H(t)$ in 1+1 dimensions has to be considered with caution. For $g = 6$ we find the same value $\beta = 0.475 \pm 0.015$. (For $g = 1$ the velocity is nonzero even for $p < q$ which would correspond to "negative driving forces". This strange situation is perhaps due to the fact that the growth rule (5) does not allow $h_i$ to decrease. Thus we choose $g$ such that $p_c > 1/2$.) The best scaling plots of $C(r, t)$ are achieved for the parameters $\zeta = 0.74$ and $\beta = 0.47...0.48$ consistent with the values given above.
In fig. 10 the velocity is plotted versus $p - p_c$ and with $v \sim (p - p_c)^\theta$ we get $\theta = 0.65 \pm 0.05$. Although the fluctuations of $v$ are smaller in $D = 2$ than in $D = 1$ the uncertainties remain rather big. For $g = 4$ and very small $v$, $\theta$ seems to be about 0.68 but it then crosses over to $\theta = 0.64$. The exponent $\theta$ is very sensitive to the value of $p_c$ for small $v$. For larger $v$, on the other hand, it is not clear whether the scaling law $v \sim (p - p_c)^\theta$ still holds. It is known from the charge-density waves that the critical region is quite narrow [2]. So the error bar on $\theta$ is larger than those on the roughness exponents $\zeta$ and $\beta$.

IV. Discussion

From the two roughness exponents $\zeta$ and $\beta$ we can calculate the dynamical exponent $z \equiv \zeta/\beta$: $z(D = 1) = 1.42 \pm 0.03$ and $z(D = 2) = 1.58 \pm 0.04$, i.e. the dynamics at the depinning transition is superdiffusive. The correlation length exponent $\nu$ can be found from the general scaling relation $\nu = \theta/(z - \zeta)$ [15]. In 2+1 dimensions we get $\nu(D = 2) = 0.8 \pm 0.05$. Simple arguments suggest [15] that there is a Harris criterion [21] for a sharp 2nd order depinning transition: $1/\nu \leq (D + \zeta)/2$. In our case this relation is indeed fulfilled as an inequality.

The simulations of the continuum equation (1) by Tang [18] yield a dynamical roughness exponent $\beta(D = 1) = 0.81 \pm 0.02$ which was determined from the scaling of the height $H(t)$. This value is somewhat smaller than our result $\beta = 0.88 \pm 0.02$. However, we have seen that it is rather difficult to settle $\beta$; the fluctuations of the width are large and the height shows slow crossover phenomena, especially for smaller systems. Another possibility is that due to the special discretization the dynamics of the lattice model is not described by the continuum equation (1), e.g. according to the growth rule eq.(5) there are only two velocities, zero and one, whereas in the continuum model the velocity depends continuously on the force. On the other hand the static roughness exponent $\zeta \simeq 1.25$ agrees with the simulation [18] of eq.(1) but not with the Imry-Ma result [8] ($\zeta = (4 - D)/3$ for all $D \leq 4$) nor with the conjecture of Parisi [17] that $\zeta = (4 - D)/2$, $\beta = (4 - D)/4$, $\theta = 1$. Our numerical values for the critical exponents support the perturbative renormalization group expansion in $D = 4 - \epsilon$, $\epsilon \ll 1$ [15], because the extrapolation to $\epsilon > 1$ gives
a better estimate for $\epsilon = 2$ than for $\epsilon = 3$. It is interesting to note that the scaling relations $z = \zeta/\beta = 2 - (\epsilon - \zeta)/3$ and $\theta = 1 - \frac{1}{3}(\epsilon - \zeta)/(2 - \zeta)$ which were derived for small $\epsilon$ [15] are perfectly fulfilled by our numerical results, although we have no arguments why they should be exact. By inserting our $\zeta$ in the scaling relations for $D = 1$ we obtain $\theta(D = 1) \simeq 0.22$ and $\nu(D = 1) \simeq 1.33$.

Our roughness exponents are much larger than those of the EW model and the KPZ equation [22], i.e. quenched noise roughens an interface more than thermal noise. This difference corresponds to the observed crossover from the behaviour at the threshold ($\beta(D = 1) \simeq 0.88$) to that of large velocities ($\beta(D = 1) \simeq 0.25$) where the noise is short-range correlated in time. Next, we discuss the relation to the growth mechanism "pinning by directed percolation" [20], where the dominating noise is also quenched. There, if the height difference between two neighbouring columns exceeds a certain small value, the lower column grows, independent of random pinning forces. Thus, the difference between neighbouring columns is kept small and the growth is side-ways. It can be shown [18] that the model is in the same universality class as eq.(1) if a KPZ-term $F(\nabla z)^2/2$ is added, which favours lateral growth. In the model presented here, however, only the discretized Laplacian, i.e. the sum of the height difference to all nearest neighbours enters the growth rule (5). Therefore, the motion is not side-ways, which corresponds to the absence of the gradient-squared term $F(\nabla z)^2/2$ in eq.(1), and arbitrary large slopes are possible which can occur when the height difference to two neighbours has opposite sign. Indeed, our roughness exponents are larger than those of an interface pinned by a directed percolation cluster $(\zeta(D = 1) = \beta(D = 1) \simeq 0.63$ at the threshold [20]). In 1+1 dimensions the exponent $\zeta \simeq 1.25 > 1$ is even unphysical. If one derives eq.(1) from a Hamiltonian [1,4] the gradient terms have to be assumed to be small. If however $\zeta \geq 1$ the neglect of overhangs and higher order gradients is no longer justified. It is therefore questionable to apply this simple model to real 1+1 dimensional systems. For the equilibrium case of the random field Ising model it is known [4] that $d = D + 1 = 2$ is the lower critical dimension $d_l$. At and below $d_l$ no long range order can be established and the domain walls are convoluted in contradiction with the assumption of no overhangs.

In 2+1 dimensions a recent simulation [5] of the motion of a domain wall in the
random field Ising model yields a roughness exponent $\zeta = 0.67 \pm 0.03$ which was obtained by a plot of the height-height correlation function $C(r)$ for systems up to linear size 300. In our measurements $C(r)$ never reaches its asymptotic scaling with $\zeta \simeq 0.75$ due to finite time and finite size effects but gives effective exponents about $2/3$. It would be therefore very interesting to measure the width of a domain wall for different system sizes to check whether the domain walls in the random field Ising model of ref.[5] are correctly described by the model presented in this paper.

V. Summary

We have proposed a lattice model for the driven motion of an interface in a random medium. The critical exponents characterizing the depinning transition are determined in one and two interface dimensions. We believe that the model is in the same universality class as the continuum equation (1). Our results (e.g. $\zeta(D=1) \simeq 1.25$, $\zeta(D=2) \simeq 0.75$) are inconsistent with the validity of an Imry-Ma argument [8] ($\zeta = (4 - D)/3$, $D \leq 4$) and with the approximation in eq.(1) to neglect the $z$-dependence of the random force $\eta$ [17] ($\zeta = (4 - D)/2$), but they support the analytical results of the renormalization group for $D = 4 - \epsilon$, $\epsilon \ll 1$ [15] ($\zeta = \epsilon/3$), because the deviations are smaller for $\epsilon = 2$ than for $\epsilon = 3$. It is of course desirable to investigate whether this difference decreases further when $\epsilon = 1$.

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[22] For comparison: the EW eq. has $\zeta = 1/2$, $\beta = 1/4$ in $D = 1$, while for $D = 2$ the roughness diverges only logarithmically [14]; the KPZ exponents are $\zeta = 1/2$, $\beta = 1/3$ in $D = 1$ and $\zeta \simeq 0.39$, $\beta \simeq 0.24$ in 2+1 dimensions [1].

Note added:

We determined the velocity exponent $\theta$ in 1+1 dimensions by a simulation of large systems (up to $L = 262144$) to $\theta = 0.25 \pm 0.03$. (This took 30 h CPU on one Cray Y-MP processor.)
Figure captions

Fig.1 The width $w^2(L)$ for $g = 1$, $p = p_c \simeq 0.8004$ (squares) and for $g = 2$, $p = p_c \simeq 0.8748$ (circles). The statistical uncertainties are smaller than the size of the symbols.

Fig.2 The effective roughness exponent $\zeta(L)$ for $g = 1$, $p = p_c$ (squares) and $g = 2$, $p = 0.8744 < p_c$ (circles). For $p < p_c$ an interface is pinned at earlier times and one is able to simulate larger systems than for $p = p_c$.

Fig.3 Scaling of $H(t)$ for $L = 262144$ and $p = p_c = 0.8004$ ($g = 1$).

Fig.4 The width $w^2(t)$ for the same runs as in fig.3.

Fig.5 The effective exponents $\beta(t)$ determined from the height $H(t)$ (circles) and from the width $w^2(t)$ (squares). In fig. 5b $\beta(t)$ is determined from a plot $w^2(2t) - w^2(t)$ (and analogous $H(2t) - H(t)$), whereas in fig 5a the exponents are calculated from a simple plot $w^2(t)$ and $H(t)$.

Fig.6 Best scaling plot of the height-height correlation function $C(r, t)$, which is scaled by $t^\beta$, $\beta = 0.88$ versus $r/t^{3/\zeta}$, $\zeta = 1.25$. The same plotting symbol is used for data at a given time $t$. In fig. 6a all data are plotted and in fig. 6b the crossover region is shown only.

Fig.7 The effective exponents $\zeta(L)$ for $g = 4$, $p = p_c \simeq 0.6416$ (squares) and $g = 6$, $p = p_c \simeq 0.74448$ (circles).

Fig.8 The interface width $w^2(t)$ for $L^2 = 1024^2$, $g = 4$, $p = p_c = 0.6416$.

Fig.9 The effective exponents $\beta(t)$ determined from the plot of $w^2(t)$ (squares) and from $H(t)$ (circles) for $g = 4$, $p = p_c = 0.6416$. In addition, $\beta(t)$ from the height for the case $p = 0.643 > p_c$ (triangles) is plotted for comparison (see text).

Fig.10 Double logarithmic plot of the velocity versus $p - p_c$ for $g = 4$ (squares) and $g = 6$ (circles).