Rip Singularity Scenario and Bouncing Universe in a Chaplygin Gas Dark Energy Model

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Abstract

We choose a modified Chaplygin Gas Dark energy model for considering some of its cosmological behaviors. In this regards, we study different Rip singularity scenarios and bouncing model of the universe in context of this model. We show that by using suitable parameters can explain some cosmological aspects of the model.

PACS: 04.60.-m, 11.30.Cp
Key Words: Quantum Gravity, Dark Energy, Rip Singularity, Alternative Scenario, Bouncing Model.

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1 Introduction

There is growing evidence that the Universe at present era is dominated with a component by negative pressure, known as dark energy, leading to accelerated expansion for Universe. While the best candidate for this component is vacuum energy, a conceivable alternative is dynamical relation for vacuum energy [1,2]. It has been encouraged that the change of manner of the dark energy density could be managed with the change in the equation of state (EoS) of the background fluid as a replacement for the form of the potential, by means of that avoiding fine-tuning problems. This is obtained via an unusual background fluid that so-called the Chaplygin gas. Besides, the Chaplygin gas is the only fluid known to accept a supersymmetric generalization [3].

Before results of Supernova data, it seems that the Universe maybe filled with energy density which is scattered over large scales [4]. The different cosmological observations such as Cosmic Background Explorer data (COBE) [5,6,7], SNeIa data [8], large scale red-shift surveys [9], the evaluations of the cosmic microwave background (CMB) [10] and WMAP data [11] predict the Universe which is expanding with acceleration in present era. However, for a good consistency with data, A generalized Chaplygin Gas model has been proposed that known as (MCG) [12].

In other viewpoint, according to variation of energy density and scale factor with time[13], Rip singularity scenario and other cases of singularity maybe appear in Modified Chaplygin Gas model. These solutions occur when $\omega < -1$ increases rapidly. Moreover, it is possible some types of singularity, depend on energy density and scale factor how increases with time[13]. Also an interesting solution of the singularity problem in the standard Big Bang cosmology is Bouncing Universe. A bouncing universe has an initial narrow state by a minimal radius and then develops to an expanding phase [14]. This means, for the universe arriving to the Big Bang era after the bouncing, the equation of state parameter should crossing from $\omega < -1$ to $\omega > -1$. In a bouncing cosmology, what we comment to be The Big Bang scenario really is the Universe emerging from a bounce. The universe at this position has its smallest extent (smallest scale factor $a$) and largest energy density. However, we know that the cosmological model avoiding the big bang singularity within the frame of Einstein gravity has to satisfy the effective equation of state less than -1 around the bounce and then enters into regular expansion with equation of state larger than -1. This point of view was the associated scenario is usually dubbed as the Quintom scenario as comprehensively reviewed in [15]. Moreover, the idea of connecting this
scenario with bouncing cosmologies was originally carried out in [16], and its huge ideas can even be tracked backward to the cosmic duality analysis in [17]. The latest study on this respect can be found in the construction of Lee-Wick cosmology in [18] and known as “new matter bounce” cosmology in [19].

Therefore, according above discussions, we study a modified Chaplygin Gas in section 1. In section 2, we briefly discuss about general Dark Energy Model and in two last sections, we consider Rip Singularity scenario and Bouncing Universe solution in context of our model. In section 5, we summarize our results and conclusion.

2 The Modified Chaplygin Gas Model

In following, according [20], we explain a modified Chaplygin gas in Friedmann-Robertson-Walker cosmology. This model based on a equation of state as

\[ p = A\rho - \frac{B}{\rho^\alpha} \]  

(1)

where \( A, B \) and \( \alpha \) are constant parameters and usually constraint with observation data. If \( B = 0 \), we obtain usual equation of state of perfect fluid, when \( A = 0 \), it products the generalized Chaplygin gas.

The metric of \( D \)-dimensional FRW space-time given as

\[ ds^2 = -dt^2 + a^2(t)\ d\Omega_k^2 \]  

(2)

where \( a(t) \) is the scale factor and \( d\Omega_k^2 \) is the metric \((D-1)\)-space by curvature \( k = 0, \pm 1 \) (following we choose D=4).

One can obtain the Friedmann equation as

\[ H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{2\rho}{(D-1)(D-2)} - \frac{k}{a^2} \]  

(3)

where \( H \) is the Hubble parameter.

Now the conservation law for a fluid with an energy density \( \rho \) and a pressure \( p \) can write as

\[ \dot{\rho} + (D-1)H(\rho + p) = 0. \]

(4)

By using equations (3) and (4) we have

\[ \frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{(D-1)p + (D-3)\rho}{(D-1)(D-2)}. \]  

(5)
If we assume $\Gamma = a^{(D-1)(A+1)}$ and rewrite density $\bar{\rho} = \rho \Gamma$, the equation (4) gives
\[
\dot{\bar{\rho}} - \frac{B}{A+1} \frac{\Gamma^{\alpha}}{\bar{\rho}^{\alpha}} = 0 .
\] (6)
Now with integrated from equation (6), we have
\[
\frac{\bar{\rho}^{\alpha+1}}{\alpha + 1} = \frac{B}{A+1} \frac{\Gamma^{\alpha+1}}{\alpha+1} + \frac{C}{\alpha+1}
\] (7)
where $C$ is an integration constant.

The energy density will obtain
\[
\rho = \left( \frac{B}{A+1} + \frac{C}{\Gamma^{\alpha+1}} \right)^{\frac{1}{\alpha+1}} .
\] (8)
If $C$ expressed in terms of the cosmological scale $a_0$, ( in form of $\Gamma_0 = a_0^{(D-1)(A+1)}$) and when the fluid has a vanishing pressure, therefore we obtain this parameter in form
\[
C = \frac{B}{A+1} \frac{\Gamma_0^{\alpha+1}}{A} .
\] (9)
Now we rewrite the energy density $\rho$ as
\[
\rho = \left( \frac{B}{A+1} \right)^{\frac{1}{\alpha+1}} \left( 1 + \frac{1}{A \Gamma^{\alpha+1}} \right)^{\frac{1}{\alpha+1}}
\] (10)
where $\Gamma_r = \Gamma/\Gamma_0$.

Here, one can take some limits for parameter $\Gamma$ and obtain interesting cosmological results[20].

2.1 Modified Chaplygin Gas as a Scalar Field

Following [21,22], we study the modified Chaplygin gas cosmological model by introducing a scalar fields with a potential $U(\varphi)$ and the Lagrangian in form of
\[
\mathcal{L}_\varphi = \frac{\dot{\varphi}^2}{2} - U(\varphi) .
\] (11)
We notice both the energy density and the pressure of the modified Chaplygin gas depend on the scalar $\varphi$ in the following transformation equations
\[
\rho_\varphi = \frac{\varphi^2}{2} + U(\varphi) = \rho,
\]
\[
p_\varphi = \frac{\varphi^2}{2} - U(\varphi) = A\rho - \frac{B}{\rho^\alpha} .
\] (12)
The kinetic energy of the scalar field given as
\[ \dot{\phi}^2 = (1 + \omega_\phi) \rho_\phi \]
\[ U(\phi) = \frac{1}{2} (1 - \omega_\phi) \rho_\phi \] (13)

where \( \omega_\phi = p_\phi/\rho_\phi \).

We know \( \dot{\phi} = \varphi' \Gamma_r \) where the prime denotes derivation with respect to \( \Gamma_r \) and \( \dot{\Gamma}_r = (D - 1)(A + 1)H \Gamma_r \), hence we have
\[ \varphi'^2 = \frac{D - 2}{2(D - 1)(A + 1)^2} \frac{1 + \omega_\phi}{\Gamma_r^2}. \] (14)

However, if we use equation (3) for a flat universe, \( k = 0 \), we have
\[ \varphi' = \sqrt{\frac{D - 2}{2(D - 1)A(A + 1)} \frac{1}{\Gamma_r^{\frac{D-2}{2}} \sqrt{1 + \frac{1}{A}\Gamma_r^{\alpha+1}}}} \]
\[ U(\phi) = \frac{1}{2} \left( \frac{B}{A + 1} \right)^{\frac{1}{2}} \frac{2 + \frac{1-A}{A^{\frac{D-2}{2}}}}{\sqrt{1 + \frac{1}{A}\Gamma_r^{\alpha+1}}} \] (15)

With integrated from the first equation, we have
\[ A\Gamma_r^{\alpha+1} = \frac{1}{\sinh^2 (\xi(\alpha+1)\Delta \varphi))} \] (16)

where \( \xi = \sqrt{(D-1)(A+1)/(2(D-2))} \) and \( \Delta \varphi = \varphi - \varphi_0 \).

Now by using the above relations in equation (12), we can obtain energy density, pressure \( \rho_\phi, p_\phi \) and equation of state relation \( \omega_\phi \) in terms of the scalar field \( \varphi \) as
\[ \rho_\phi = \left( \frac{B}{A + 1} \right)^{\frac{1}{\alpha+1}} \cosh^{\frac{2}{\alpha+1}} (\xi(\alpha + 1)\Delta \varphi) \]
\[ p_\phi = \left( \frac{B}{A + 1} \right)^{\frac{1}{\alpha+1}} \left[ A \cosh^{\frac{2}{\alpha+1}} (\xi(\alpha + 1)\Delta \varphi) - \frac{A + 1}{\cosh^{\frac{2}{\alpha+1}} (\xi(\alpha + 1)\Delta \varphi)} \right] \]
\[ \omega_\phi = - \frac{1 - A \sinh^2 (\xi(\alpha + 1)\Delta \varphi))}{\cosh^2 (\xi(\alpha + 1)\Delta \varphi)} \] (17)

For a flat universe \( k = 0 \), using Eq. (3), (12) and (13) we have
\[ \Delta \varphi = \varphi - \varphi_0 = \pm \frac{2}{\sqrt{(D - 1)(1 + A)(1 + \alpha)^2}} \sinh^{-1} \left[ \frac{1}{A} \frac{a_0^{(D-1)(A+1)(\alpha+1)}}{a_0^{1.5(D-1)(A+1)(\alpha+1)}} \right] \] (18)
At last, we obtain the potential which has a form as

\[ U(\varphi) = \frac{1}{2} \left( \frac{B}{A + 1} \right)^{\frac{1}{a + 1}} \left[ \frac{1 + A}{\cosh^{\alpha + 1} (\xi (\alpha + 1) \Delta \varphi)} + (1 - A) \cosh^{\alpha + 1} (\xi (\alpha + 1) \Delta \varphi) \right] \]. (19)

Equations (17) are dynamic in term of time. In following, to get an explicit form of the energy density and pressure according to the scalar field, we assume a phenomenologically reliable power law expansion of the scale factor \( a(t) \) \([14,23]\) as, \( a(t) = ((t - t_0)^2 + t_0^\frac{2}{1-\beta})^{\frac{1}{1-\beta}} \) so that, for \( \beta < 1 \) we get accelerated expansion of the Universe thus satisfying the observational constrains. In following, we use this setup for consideration of cosmological aspects of MCG model.

In this section, we considered the realization of the generalized Chaplygin gas as a scalar field with a nontrivial potential. However, this model construction cannot realize the background equation of state across -1 due to a proof of No-Go theorem. This No-Go theorem states that, any dark energy models realized by a single scalar field or single perfect fluid within standard Einstein gravity is not allowed to give rise to equation of state across -1, otherwise the model suffers severe gradient instability at perturbation level(for the detailed see[24]). In order to break the No-Go theorem consistently, there are two simple mechanisms, which can accommodate with the present study very well. One mechanism is to introduce a higher derivative term nonlinearly, such as the string theory inspired dark energy model proposed in [25]. The other mechanism is to take the spinor field instead such as proposed in [26]. Both two models are able to give rise to the generalized Chaplygin gas behavior. Especially, the explicit realization of Chaplygin gas was already discussed in the second mechanism. This means, as authors proposed in [26] a model could constructed by Spinor Quintom which combines the feature of a Chaplygin gas. The generic expression of the potential is given by \( U = \sqrt[1+\gamma]{f(\varphi \bar{\varphi})} + B \) where \( f(\varphi \bar{\varphi}) \) is an arbitrary function of \( \varphi \bar{\varphi} \). By Choosing \( f(\varphi \bar{\varphi}) \) to be \( f(\varphi \bar{\varphi}) = U_0(f(\varphi \bar{\varphi}) - b)^2 \), where \( U_0, b, c \) are undetermined parameters, the crossing over -1 takes place when \( \varphi \bar{\varphi} = b \) (for the detailed see[26]).

### 3 General Dark Energy Models

In present work, we consider a MCG model as a candidate for dark energy, but according to [28], there are many models that explain dark energy. Hence, In this section for a
eligible review, in summary we point out them.
As we know several observational data (SNe Ia, CMB, LSS, BAO, WMAP, SDSS) imply that the expansion of the universe is accelerating at the present era. For explain this result, there are two representative categories: 1. To add “dark energy” in the right-hand side of the Einstein equation in general relativity. 2. To modify the left-hand side of the Einstein equation, known as a modified gravitational theory, (for example $F(R)$ gravity). In this regards, the $\Lambda$ cold dark matter ($\Lambda$CDM) model has been studied for playing a role of dark energy in framework of general relativity. However, the theoretical derivation of the cosmological constant $\Lambda$ has not been sorted out. In other hand, other different models for dark energy without the cosmological constant has been offered. For example, a canonical scalar field, a non-canonical scalar field called as phantom, tachyon scalar field and holographic dark energy.

Most important parameter to describe the aspects of dark energy models is the equation of state (EoS) $w_{DE} = \frac{P}{\rho}$. In the framework of Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology, there are two ways to describe dark energy models. One is a fluid and the other is to describe the action of a scalar field theory. In the fluid proposal, we state the pressure as a function of energy density $\rho$. But in the scalar field theory we induce the phrases of the energy density and pressure of the scalar field from the action of theory. In both proposals, we can imply the gravitational field equations, this means that a cosmological model may be described equally by different model illustrations. In this article, we explicitly show that a cosmology with a fluid description, also to describe in form of a scalar field theory. In other words, the main goal of this article is to represent that one dark energy model can exhibited as other dark energy models, therefore, such a outcome unified illustration of dark energy models might applied to any particular cosmology. Moreover, one can show that degeneration among parameter of models can be removed by precise data analysis of large data.

However, in following, we study a description of dark energy universe. In this regards, we introduce some types of the finite-time future singularities as well as the energy conditions in the MCG model.

### 4 Rip Singularity Scenario

In this section, we will consider the modified Chaplygin gas model which contain finite-time, future singularities. We know that depending on energy density and pressure con-
ditions of model such singularities can lead to different ways. We classify these situations in the following items[29,30]:

- **Type I ("Big Rip")**: For \( t \to t_s, a \to \infty, \rho \to \infty \) and \(|p| \to \infty\)
- **Type II ("sudden")**: For \( t \to t_s, a \to a_s, \rho \to \rho_s \) and \(|p| \to \infty\)
- **Type III**: For \( t \to t_s, a \to a_s, \rho \to \infty \) and \(|p| \to \infty\)
- **Type IV**: For \( t \to t_s, a \to a_s, \rho \to 0, |p| \to 0\)

where \( t_s, a_s \) and \( \rho_s \) are constants with \( a_s \neq 0 \). The type I, known as the Big Rip singularity which appears for a phantom equation of state: \( w < -1 \). The type II point out the sudden future singularity which \( a \) and \( \rho \) be finite but \( p \) diverges.

In figures (1-4), we show variation scalar field \( \phi \), energy density \( \rho \), pressure \( p \) and equation of state \( \omega \) by choosing arbitrary parameter of space as: \( A = 0.66, B = 0.1, \alpha = 0.5 \) and \( \beta = -4, t_0 = 1 \) in \( \Delta \phi < 0 \), for MCG model.

If we choose different values for parameters of space of relations, we can obtain different evolution of universe, these results summarize in table (1). First, we consider the case of the Big Rip singularity, Our calculation implies a Big Rip singularity occurs just in \( \beta > 3 \). If \( \beta \) parameter be less 1, this type of singularity never occurs. Second, as we shown in table (1) just type IV singularity can appears in this setup of model and other types of singularity can not produce.

Table 1: Summary of the behavior of MCG model depending specific value of parameters.

| Type             | \( \alpha \) | \( A \) | \( B \) | \( \beta \) | Appear in MCG |
|------------------|--------------|--------|--------|------------|---------------|
| Type I ("Big Rip") | \( 0 \leq \alpha \leq 1 \) | *any*  | \( B > 0 \) | \( > 3 \)   | Yes           |
| Type II ("sudden") | \( 0 \leq \alpha \leq 1 \) | *any*  | \( B > 0 \) | \( < 1 \)   | No            |
| Type III         | \( 0 \leq \alpha \leq 1 \) | *any*  | \( B > 0 \) | \( < 1 \)   | No            |
| Type IV          | 0.01         | 0.99   | 0.01   | \(-0.5\)   | Yes           |

5 Bouncing Universe

An interesting solution of the singularity problem of the standard Big Bang cosmology known as Bouncing Universe. A bouncing universe model has an initial narrow state by
Figure 1: Variation of the scalar field $\varphi(t)$.

Figure 2: Variation of the energy density $\rho(t)$.

Figure 3: Variation of the pressure $p(t)$.

Figure 4: Variation of the equation of state $\omega(t)$. 
a non-zero minimal radius and then develop to an expanding phase [14]. For a successful bouncing model in the standard cosmology, the null energy condition is broken for a period of time near the bouncing point. Moreover, for the universe going into the hot Big Bang era after the bouncing, the equation of state parameter of the universe should crossing from $\omega < -1$ to $\omega > -1$.

In following, we study necessary conditions needed for a successful bounce in a model of universe with modified Chaplygin gas. We know that in the contracting phase, the scale factor $a(t)$ is decreasing, this means, $\dot{a}(t) < 0$, and in the expanding phase, scale factor $\dot{a}(t) > 0$. Finally in the bouncing point, $\dot{a}(t) = 0$, and near this point $\ddot{a}(t) > 0$ for a period of time. In other view, in the bouncing cosmology the Hubble parameter $H$ passes across zero ($H = 0$) from $H < 0$ to $H > 0$.

Before bouncing point, we see that $\omega < -1$ and after the bounce, the universe requires to enter into the hot Big Bang era, else the universe filled with the matter by an equation of state $\omega < -1$ and occurs the big Rip singularity[31]. We can see from figure (5,6) that in our setup a bouncing happens with the Hubble parameter $H$ going across zero and a minimal non zero scale factor $a$. At the bouncing point $\omega$ has a finite negative value. So it is possible to realize bouncing solutions in a Chaplygin gas dark energy model.

Therefore, we shown that by a suitable choose of parameter space, this model can explain

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure5.png}
\caption{Crossing of the phantom divide line by equation of state parameter in a Chaplygin gas dark energy model for $A = 0.62, B = 0.001, \alpha = 0.6$ and $\beta = -3, t_0 = 1$.}
\end{figure}

a Bouncing solution. We emphasize range of parameter that used in numerical analyze
Figure 6: Variation of the scale factor $a$ relative to cosmic time $t$ (left), and Hubble parameter $H$ (right) for $t_0 = 1$ and $\beta = -3$.

adjust with observational constraints [20,27,32]. Overall, observational signatures in cosmological surveys related to the cosmological perturbation theory widely studied in the literature. For theoretical aspect, the corresponding perturbation theory within bouncing cosmology was developed in [33,34,35]. And for observational aspect, the comparison of bouncing cosmology with data can be found in [36,37].

6 Summary

In the present article, we have considered finite-time future singularities in modified Chaplygin Gas model. We have shown this model realizing a crossing of the phantom divide line. Also it has been shown that some types of singularity may appear in MCG model. This means, in this framework, there is the possibility of a Rip singularity by suitable tuning in the parameters. Moreover, our results show that such a model universe with Chaplygin Gas as dark energy component avoids the problem of the Big Bang singularity with a bouncing scenario.
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