Collectivity in pp from resummed interference effects?

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\section{Introduction}

Sizeable $n$-th harmonic coefficients $v_n(2s)$ of azimuthal momentum asymmetries have been observed at the LHC in nucleus-nucleus (AA), proton-nucleus (pA) and proton-proton (pp) collisions [1–6]. These asymmetries persist almost unattenuated if determined from higher order (2s)-particle cumulants, thus indicating a collective mechanism that relates all particles produced in a given collision. The dynamical origin of this collectivity continues to be sought in competing and potentially contradicting pictures:

Explanations based on final state interactions are implicit e.g. in viscous fluid dynamic simulations [7] and kinetic transport models [8–11] of nuclear collisions. They exploit that any interaction between the produced degrees of freedom implies transverse pressure gradients that translate spatial eccentricities in the overlap of the hadronic projectiles into momentum asymmetries $v_n(2s)$. In AA collisions, jet quenching phenomena provide independent evidence that isotropizing final state interactions are indeed operational, but comparable evidence is missing in the smaller pA and pp collision systems. Moreover, in marked contrast to any final state explanation of flow anisotropies $v_n$ in pp collisions, the phenomenologically successful modeling of soft multi-particle production in modern multi-purpose pp event generators [12] are based on free-streaming partonic final state distributions supplemented by independent fragmentation into hadrons. Efforts to go beyond this picture are relatively recent, see e.g. [13,14]. Therefore, two contradictory working hypotheses should be explored further: Either final state interactions are the cause for the measured $v_n(2s)$ not only in AA but also in pp and pA hadronic collision systems – this would invalidate the starting assumption of many underlying event models in pp collisions, and it would imply that quenching phenomena can be found in pp and pA on some scale. Or there are dynamical mechanisms contributing to the $v_n(2s)$ that do not involve final state interactions – these would need to be taken into account in the no-final-state interaction baseline for analyzing $v_n(2s)$ in AA collisions. The present work makes a contribution towards exploring this second working hypothesis.

Efforts to understand the measured $v_n$’s in terms of mechanisms operational in the incoming hadronic wave functions (a.k.a. initial state effects) have focussed so far mainly on parton saturation models, see e.g. Refs. [15–19] and subsequent work. In these models, non-vanishing even second order cumulants $v_2(2)$ result trivially from gluon emission of color dipoles. The calculation of higher order

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cumulants is, however, complicated, since the S-matrix is given in terms of eikonal Wilson lines $W$, and higher order cumulants involve target averages over a rapidly increasing number of $W$'s. By now, several ways of obtaining non-vanishing odd harmonics are identified, and there are calculations of fourth order and sixth order cumulants [20–25]. The most advanced model calculations [26,27] provide phenomenologically satisfactory descriptions. There is still debate on whether the modeling needed for this data comparison is quantitatively reliable [28], but the calculations per se undoubtedly indicate that there can be contributions to $v_n (2s)$ that do not invoke final state interactions.

2. Model of multi-gluon interference effects

In Ref. [29], we have proposed a simple model to study the effects of quantum interference and color flow on $v_n (2s)$ without assuming large or saturated parton densities. The strong simplification of the model consists in neglecting a dynamically explicit formulation of the scattering process: all gluons in the incoming wave function are assumed to be freed in the scattering process with the same (possibly small) probability. The model pictures the incoming hadronic wavefunction as a collection of $N$ color sources in adjoint representation distributed in transverse space according to a classical density $\rho (y_i)$. On the amplitude level, emission of a gluon of color $a$ and momentum $k_i$ from the $l$-th source is given by an eikonal factor $f (k_i) T^{a} e^{i k y_i}$ where $T^{a}$ are generators of $SU (3)$ in adjoint representation and $y_i$ is the transverse position of the $l$-th source. One example of a diagrammatic contribution to the cross section $\frac{d^3 \hat{\sigma}}{d \Omega_1 d \Omega_2 d t}$ for $m$-particle production from $N$ sources including interference effects is depicted in Fig. 1. The full result is obtained by multiplying the sum of the $N^m$ emission amplitudes with its complex conjugate and summing (averaging) over all outgoing (incoming) colors,

$$\frac{d^3 \hat{\sigma}}{d \Omega_1 d \Omega_2 d t} \propto \sum_{m=2}^{N^m} \left( N_2^m \right)^m \left( \sum_{(l_1m_1), (l_2m_2), \ldots, (l_dm_d)} \prod_{j=1}^{d} \left( 2 \cos \left( k_{l_j} \Delta y_{l_j m_j} \right) \right) \right) \cdot \left( f (k_1) T^{a} e^{i k_1 y_1} \right)^2 \cdot \left( f (k_d) T^{a} e^{i k_d y_d} \right)^2 \cdot \left( \sum_{l=1}^{N} N^m \right).$$

Here, $d \Omega_1 = d k_1 d \phi_1$ is the transverse phase space of the $i$-th gluon, $\Delta y_{l_i} \equiv y_i - y_m$ is the transverse size of the source dipole ($l$, $m$), and $...$ stands for many other interference terms. We focus on the terms explicitly written. The first term in (1) corresponds to incoherent emission of $m$ gluons from any of the $N$ sources, where each gluon links in the amplitude and complex conjugate amplitude to the same source (so-called diagonal gluons). As the model [29] was designed to study in a simplified setting effects present in rapidity-ordered QCD ladder-like diagrams, one neglects diagrammatic contributions from gluons that cross each other because they are expected to be suppressed by powers of logarithms of longitudinal phase space in a dynamically more complete description. For this first term, summing over initial and final color of each source leads to an adjoint color trace $Tr [1] = (N_2^m - 1)$ and each gluon emission leads to a factor $\left( f (k_{l_j}) \right)^2 T^{a} T^{a} = N_2^m \left( f (k_{l_j}) \right)^2$. Finally, as each of the $m$ gluons can be attached to any of the $N$ sources, there is an extra factor $N^m$. This explains all factors of the first term in eq. (1). As for the second term, we focus first on the contribution $d = 1$ in the sum. This contribution arises from squared amplitudes in which two gluons of color $a$ and $b$, emitted from two sources $l$ and $m$ interfere. The resulting dipole interference term is $\propto 2 \cos \left( k_{l_j} \Delta y_{l_j} \right) \cos \left( k_{a} \Delta y_{a} \right)$, and it is suppressed by one power of the adjoint trace $(N_2^m - 1)$ since the interfering gluons link the color flow between two sources $i$ and $m$ [29]. For such a contribution, $m ≥ 2$ gluons are diagonal thus leading to a factor $N^{m-2}$. As the sum $\sum_{(l_1m_1)}$ goes over $N (N-1)/2$ dipole pairs, this second term is of the same $O \left( N^m \right)$ as the first one. Analogous arguments apply to contributions with $d > 1$ dipoles in the sum of (1), as long as none of the dipoles $(l_1, m_1), ... (l_d, m_d)$ shares a source with another dipole. The correction factor $F_{(2)}$, which is the number of diagonal gluons sandwiched between the two off-diagonal ones.
In Ref. [29], we have shown that eq. (1) gives rise to momentum asymmetries $v_2[2]$ that coincide to leading $O\left(\frac{1}{(N_c^2-1)}\right)$ with results of parton saturation models, and we have obtained explicit expressions for $v_2[4]$ and $v_2[6]$ in an expansion in powers of $\frac{1}{(N_c^2-1)}$. However, corrections subleading in powers of $\frac{1}{(N_c^2-1)}$ were found to be multiplied by factors $m^2$. This seems to narrow the range of validity of the naive $\frac{1}{(N_c^2-1)}$-expansion to $m^2 < (N_c^2 - 1)$. Moreover, while $v_2[4]$ and $v_2[6]$ were found to be non-vanishing, they have parametrically different $(N_c^2 - 1)$-dependencies in an expansion in powers of $\frac{1}{(N_c^2-1)}$ [29]. Here, we show how resummation can overcome these limitations. The main result reported in the present manuscript is a closed expression that resums all leading contributions of order $\left(\frac{m^2}{(N_c^2-1)}\right)^k$ and that yields for realistic multiplicity $m$ signal strengths $v_2[2s]$ that are of the same parametric accuracy for all cumulants and whose numerical values vary mildly with the order of the cumulant.

### 3. Calculating azimuthal (2s)-particle correlation functions

We want to calculate the correlation functions

$$K_{2s}^{(0)}(k_1, k_2, \ldots, k_{2s}) = \mathcal{N}^2 \int d\phi_1 \cdots d\phi_{2s} \exp \left[ \prod_{j=1}^{2s} \phi_j \right] \frac{d^{2s}N}{d\Gamma_1 d\Gamma_2 \cdots d\Gamma_{2s}},$$

(2)

where $\int \cdots = \int (\prod_{j} \rho(y_j) dy_j) \cdots$ is the average over the transverse positions of the $N$ sources, and the standard $2s$-particle spectrum reads

$$\frac{d^{2s}N}{d\Gamma_1 d\Gamma_2 \cdots d\Gamma_{2s}} = \left(\frac{m}{2s}\right) \frac{d\hat{s}}{d\Gamma_1 d\Gamma_2 \cdots d\Gamma_{2s}}.$$

(3)

In analogy to the experimental procedure of normalizing correlations by mixed event technique, the denominator in (2) is the product of one-particle multiplicity distributions. Its value $\int d\Gamma_1 \cdots d\Gamma_{2s} \prod_{j=1}^{2s} \frac{d\Gamma_j}{d\Gamma_j} \equiv m^{2s}$ is the number of unordered choices of $2s$ particles from $2s$ different events that each contain $m$ particles. The normalization in (3) is chosen such that after integration over $2s$ one-particle phase spaces $d\Gamma_i$, eq. (3) returns the number of possibilities $m^{2s}/m!$ of picking $2s$ out of $m$ particles. This fixes the normalization $\mathcal{N} = m^{2s}/(m!)$ in (2) if one requires $K_{2s}^{(0)} = 1$. We start by discussing the calculation of the numerator in (2) that can be written as

$$T^{(2s)}([k_1]) = \int_d 2\pi \prod_{i=1}^{2s} d\phi_i \exp \left[ \prod_{j=1}^{s} \phi_j - \prod_{j=s+1}^{2s} \phi_j \right] \left(\int d\Gamma_1 \cdots d\Gamma_{2s} \right) \frac{m^{2s}}{d\Gamma_1 d\Gamma_2 \cdots d\Gamma_{2s}}.$$

(4)

We first explain in which sense the terms written in (1) are the parametrically dominant interference contributions for the calculation of (2) and (4), even though there are many other contributions that are not made explicit in (1); Diagrams in which a source carries only one gluon vertex in either amplitude or complex conjugate amplitude vanish after color averaging over sources in the initial and final state, since $\text{Tr}[\Gamma^0] = 0$. Diagrams that carry more than two vertices of off-diagonal gluons on a particular source are suppressed by powers of $N$. Therefore, leading contributions in the number $N$ of sources have exactly two off-diagonal gluon lines connected with each active source, or they emit diagonal gluons only. For diagrams with $m$ gluon lines, $m - m_{\text{off}}$ lines are diagonal and $m_{\text{off}}$ are off-diagonal. In each
such diagram, the $m_{\text{off}}$ off-diagonal gluons can be grouped into a set of \( l \) non-overlapping \( n_l \)-cycles\(^1\) with $\sum_{l=1}^l n_l = m_{\text{off}}$. In general, each \( n_l \)-cycle links the color flow of \( n_l \) previously independent sources and thus, compared to diagonal gluons, leads to a suppression of $n_l - 1$ powers of the adjoint trace $\text{Tr}[1] \equiv (N_c^2 - 1)$. Since $m_{\text{off}} = \sum_{l=1}^l n_l$, the diagrams with $m_{\text{off}}$ off-diagonal gluons that are of highest power in $(N_c^2 - 1)$, are those that are organized in the maximal number \( l \) of cycles. For $m_{\text{off}}$ even, all such diagrams are therefore products of dipole emissions written explicitly in (1).

To calculate the numerator $T^{(2)}$ of the two-particle correlation function (4), all but 2-off-diagonal momentum need to be integrated out. Since each phase space integration of an off-diagonal gluon comes with a combinatorial factor $O(m)$, there is a multiplicative factor $m_{\text{off}}^{m_{\text{off}} - 2}$ in $T^{(2)}$. On the other hand, as explained above, contributions with $m_{\text{off}}$ off-diagonal gluons are suppressed by order $1/((N_c^2 - 1)\text{mat}^2)$ or by higher powers of $(N_c^2 - 1)$. The contributions to $T^{(2)}$ that are suppressed by the least powers of $(N_c^2 - 1)$ and that are enhanced by the most powers of $m$ are therefore of order $O \left( \frac{m_{\text{off}}^{m_{\text{off}} - 2}}{(N_c^2 - 1)\text{mat}^2} \right)$ for $m_{\text{off}}$ even. The products of dipole terms written explicitly in (1) are the only contributions to that order. Contributions with $m_{\text{off}}$ odd contain at least one $n_l$-cycle with $n_l > 2$ and they will therefore give only subleading contributions to $T^{(2)}$. With an analogous line of argument, one checks that also for $s > 1$, $T^{(2s)}$ receives all parametrically leading contributions from the terms written explicitly in (1).

To write an analytic explicit expression for $T^{(2s)}(|k_i\rangle)$, we introduce a short-hand for the phase space integral over a single dipole term

$$A_2(k_a, k_b) \equiv \left| \tilde{f}(k_a) \right|^2 \left| \tilde{f}(k_b) \right|^2 \int \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \exp \left\{ i(\phi_1 - \phi_2) \right\} \cos(k_a \cdot \Delta y) \cos(k_b \cdot \Delta y)$$

$$= \left| \tilde{f}(k_a) \right|^2 \left| \tilde{f}(k_b) \right|^2 \int \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \left( J_2(k_a \cdot \Delta y) J_2(k_b \cdot \Delta y) \right) \approx \left| \tilde{f}(k_a) \right|^2 \left| \tilde{f}(k_b) \right|^2 \frac{1}{2} B k_a^2 B k_b^2 .$$

Here, the last approximation is obtained for $k_a, k_b \ll 1/\sqrt{B}$ from a Gaussian source distribution $\rho(y) = \frac{1}{\sqrt{4\pi B}} \exp[y^2/2B]$ of spatial width $\sqrt{B}$. Such distributions $\rho(y)$ arise naturally in multi-parton interaction (MPI) models of the underlying event with $B \simeq (1 - 4)\text{ GeV}^{-2}$ fixed by the measured MPI cross section in pp [29–32]. The approximation (5) is of interest since it is valid in a physically relevant range $k_a, k_b \ll 1/\sqrt{B}$, where Bessel functions $J_2$ can be expanded for small arguments and final expressions simplify considerably.

In calculating $T^{(2s)}(|k_i\rangle)$ from eqs. (1) and (4), one also finds factors that differ from $A_2$ by the absence of the phase factors $e^{2i(\phi_1 - \phi_2)}$.

$$A_0 = \int \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \left| \tilde{f}(k_a) \right|^2 \left| \tilde{f}(k_b) \right|^2 \int \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \left( J_0(k_a \Delta y) J_0(k_b \Delta y) \right) .$$

To write an explicit expression for $T^{(2s)}(|k_i\rangle)$ in terms of the shorthands (5) and (6) is now a straightforward but somewhat lengthy counting exercise:

We count \( \binom{d}{s} \) choices for attaching off-diagonal gluons to 2d out of \( N \) sources. For these 2d sources, there are \( \frac{(2d)!}{2!^{d}d!} \) different ways of combining them to dipoles.

We next count the number of choices for picking 2d -(d-s) off-diagonal gluons such that a non-vanishing contribution arises when integrating them out without phases in (4). There are \( \frac{d}{2} \binom{(d-s)}{2} \) choices for choosing these gluons, and to distribute them amongst the \( d-s \) remaining dipoles, there are \( \frac{(2d-s)!}{(2^d-s)!} \) possible ways of assigning the 2s phases in (4) to dipoles such that non-vanishing contributions arise. First, since only terms of the form (5) and (6) can arise in the calculation of $T^{(2s)}(|k_i\rangle)$, the 2s phases appearing in $T^{(2s)}(|k_i\rangle)$ must be combined to lie in $s$ out of the $d$ dipoles; for this there are \( \binom{d}{s} \) choices. Second, since exactly $s$ of the 2s phases in (4) come with a plus sign, and since each non-vanishing contribution (5) comes with one positive and one negative phase, there are $\frac{s!}{2!^d}$ choices to assign the positive phases.

Multiplying all the above mentioned factors yields

$$\left( \frac{N!}{(N-2d)!} \right)^d \frac{1}{(2d-s)!} \frac{1}{(d-s)!} = \frac{N!}{(N-2d)!} \frac{s!}{(d-s)!} \frac{(m-2s)!}{(m-2d)!} \frac{1}{2^{d-s}} .$$

In addition, one has \( \binom{m}{2s} \) choices to pick 2s out of \( m \) gluons in the calculation of (4). We do not include this factor in (7), since it is taken into account in the normalization of eq. (2). Combining these factors and denoting by $U \equiv \int kdkd\phi \left| \tilde{f}(k) \right|^2$ the phase space integrals over those \( (m-2d) \) momenta that do not appear in the cosine-terms in (1), one finds for the numerator of the (2s)-particle correlation function $K^{(2s)}_{2s}( k_1, k_2, \ldots, k_{2s} )$

$$T^{(2s)}(|k_i\rangle) \equiv N_c^m \left( N_c^2 - 1 \right)^N U^m \sum_{d=s}^{N/2} \frac{F^{(2)}_{\text{corr}}(N, m)}{d-2d} \left( \frac{N!}{(N-2d)!} \right)^d \frac{1}{(N_c^2 - 1)^d} \frac{2s!}{(d-s)!} \frac{(m-2s)!}{(m-2d)!} A_{2s}^{d-s} \text{perm}_2(A_2) .$$

\(^1\)To clarify the definition of n-cycle, we note: each dipole term $\propto 2^d \cos(k_x \Delta y_{lm}) \cos(k_y \Delta y_{lm})$ can be viewed as a closed 2-cycle with an off-diagonal gluon of momentum $k_x$ connecting sources $l \rightarrow m$ and the other off-diagonal gluon of momentum $k_y$ connecting $m \rightarrow l$ and closing the cycle. Similarly, three off-diagonal gluons that connect between sources $l_1 \rightarrow l_2, l_2 \rightarrow l_3$ and $l_3 \rightarrow l_1$ form a closed 3-cycle, etc.
Here, the permanent perm$(A_2)$ of a matrix $A_2$ is defined in analogy to a determinant, but with the signs of all products of matrix elements positive irrespective of the signature of the permutation. For the $s \times s$-matrix defined by the entries $A_3(k_0, k_b)$, the permanent perm$(A_2) \equiv \text{perm}_s \{A_2\}/s!$ allows one to write an explicit expression (8) without taking recourse to the long wave-length limit $k_a, k_b \ll 1/\sqrt{B}$. In the small-$k$-approximation of (5), this simplifies to perm$(A_2)\big|_{|k^2| \ll 1} = \frac{1}{s!} \prod_{i=1}^{s^2} \left( |\tilde{f}(k_i)|^2 B k_i^2 \right)$. For the normalization of the correlation function $K^{(n)}_{2s}(k_1, k_2, \ldots, k_{2s})$, we also need to determine $\sigma$, which is the $s = 0$ term in (8),

$$\sigma = T^{(0)} = N^m \left( N^2 - 1 \right)^{-1} N^m U^m \sum_{n=0}^{N/2} \left( F_{\text{corr}}^n(N, m) \right)_d U^{-2d} \frac{1}{(N^2 - 1)^2} \frac{1}{d!} \left( \frac{N!}{(N - 2d)!} N^{2d} \right) \frac{m!}{(m - 2d)!} A_0^d. \tag{9}$$

The $(2s)$-particle correlation $K^{(n)}_{2s}(k_1, k_2, \ldots, k_{2s})$ in (2) is then given by the ratio of (8) and (9),

$$K^{(n)}_{2s}(k_1, k_2, \ldots, k_{2s}) = \frac{T^{(2s)}}{\sigma} \prod_{i=1}^{s^2} \left( |\tilde{f}(k_i)|^2 \right) = \frac{a^{-s} (m - 2s)! N^2 !}{m! \left( \frac{N}{2} - s \right)!} \prod_{i=1}^{s^2} \left( 1 - \frac{N^2}{2} \right) \left( \frac{1}{2} \right) \left( \frac{1 + m - N}{2} \right) \left( \frac{2 + m - N}{2} \right) \left( \frac{N^2 (N^2 - 1)}{16 s^2} \right) \prod_{i=1}^{s^2} \left( B k_i^2 \right). \tag{10}$$

Eq. (10) is the main result of this work. For the contribution to $K^{(n)}_{2s}$ that is leading in powers of $1/(N^2 - 1)^s$ and up to subleading orders $1/N$ in the number of sources, it resums correctly all corrections of order $\left( \frac{m!}{(N^2 - 1)^s} \right)^k$. Remarkably, this resummation is given as an analytically known expression in terms of the generalized hypergeometric functions $F_3$. Eq. (10) is written in terms of the shorthand

$$\zeta A_0 \frac{U(z)}{U(z)} = \frac{F_{\rho \kappa d k_i} \left( \tilde{f}(k_0) \right)^2 \int k_\kappa d k_i e \left( \tilde{f}(k_0) \right)^2 J_0 (k_0 \Delta y) J_0 (k_0 \Delta y)}{\left( \int \kappa d k \rho \phi \left( \tilde{f}(k_0) \right)^2 \right)^2}, \tag{11}$$

which characterizes a dipole interference of gluons that carry transverse momentum $k_0$ and $k_0$ and that were produced from sources separated by a transverse distance $\Delta y$. This term $a$ appears in $(2s)$-particle correlation functions in which the particular momenta $k_0, k_0$ (and, a fortiori, the interference effects associated with these momenta) are integrated out. For $\Delta y \equiv |\Delta y| = 0$, the interference effects in such terms are not geometrically suppressed, and thus, $a$ is maximal when the Gaussian transverse width $\sqrt{\Delta y}$ the source distribution $\rho(\Delta y)$ is negligible, $a^{1/4} \ll 1$. In the opposite limit $\Delta y \to \infty$ of a widely extended source, $a$ in eq. (11) vanishes. As seen from the definition (11), the value of $a$ depends on an interplay between the geometry and the shape of the spectrum $\zeta \tilde{f}(k_0)$. This determines to what extent the produced momenta can resolve a characteristic distance $\Delta y$. In the case of pp collisions, the differences $\Delta y$ between different sources are on sub-femtometer scale (indicating that interference terms between different sources are not negligible, $a > 0$), but some of the produced transverse momenta can resolve these distances (indicating that interference effects are not maximal, $a < 1$). To illustrate this generic situation, we choose $a = 0.1$ in the following. This is a typical value for $a$, if one uses in (11) a transverse extension of $\rho$ consistent with constraints on the size of the proton wave function and a shape $\tilde{f}(k_0)$ consistent with the slope of transverse momentum spectra.

### 4. Numerical results

From the normalized $(2s)$-particle correlations $K^{(n)}_{2s}(k_1, k_2, \ldots, k_{2s})$, we determine the higher order flow cumulants

$$v_n[2](k) = \sqrt{K^{(n)}_{2s}}, \quad \nu_n[4](k) = \left( \left( K^{(n)}_{2s} - 2 K^{(n)}_{4s} \right)^{1/4} \right), \quad \nu_n[6](k) = \left( \left( K^{(n)}_{2s} - 9 K^{(n)}_{2s} K^{(n)}_{4s} + 12 K^{(n)}_{4s}^2 \right)^{1/4} \right). \tag{12}$$

where we follow the standard practice to evaluate the $K^{(n)}_{2s}$s at $k = k$. In the limit $B k^2 \ll 1$ used to write eq. (10), the $k$-dependence of all higher order cumulants is of the form $v_2[2s](k) = v_2[2s](k = 1/\sqrt{B}) B k^2$. We note that the prefactor $v_2[2s](k = 1/\sqrt{B})$ in this equation does not only characterize the curvature of $v_2[2s](k)$ at $k = 0$, but it provides also a good proxy for the $k$-integrated value of $v_2[2s]$. This can be seen from undoing the approximation in eq. (5) with the replacement $B k^2 \to 2 \int J(2 k \Delta y)^2$. [We further note as an aside that with this replacement, one obtains a full $k$-dependence of $v_2[2s](k)$ that shares important commonalities with the experimentally observed one: it raises initially quadratically with $k$, it reaches a maximum at scale $k_{\text{max}} \sim 1/\sqrt{B} = 1 \to 2 \text{GeV}$ and it then falls off slowly with increasing $k$ [29].]

In the following, we focus on the $s$-dependence of $v_2[2s]$, and we do not explore further the $k$-dependence. The very mild reduction of $v_2$-signals with increasing higher order cumulants is regarded as a hallmark for collectivity in pp collisions. The main numerical result of this paper is the observation that for a suitable parameter range, a similar approximate persistence of $v_2[2s]$ with increasing $s$ can arise from a physical picture that invokes solely quantum interference and color correlation effects, see left hand side of Fig. 2. Remarkably, the
resummation of all powers of \( (m^2/(N_2^2 - 1)) \) in eq. (10) implies that all higher order cumulants are parametrically of the same order (in contrast to a corresponding calculation without resummation published in [29]), and it implies that within a certain parameter range, the numerical value of higher order cumulants \( v_2(2s) \) decreases very mildly with \( s \).

As the above conclusions are limited to a certain parameter range, we now explain in some detail from where these limitations arise. To this end, we focus first on the \( N \)- and 2-dependence of \( v_2(2s)(k = 1/\sqrt{B}) \): Eq. (10) is derived in the limit of many sources, when 1/N corrections are negligible. Thus, this derivation does not provide insight into the finite \( N \)-dependence of \( v_2(2s) \). However, eq. (10) also contains an incomplete set of 1/N corrections; as numerical results are only meaningful in parameter ranges in which they are not dominated by terms of order 1/N, we have checked the stability of the results shown in Fig. 2 by setting in (10) all terms of order 1/N explicitly to zero and repeating the calculation. For \( a = 0.1 \) and the ranges plotted in Fig. 2, we confirm stability of the results against 1/N corrections. Also, while the absolute value \( v_2(2s)(k = 1/\sqrt{B}) \) changes when increasing \( a > 0.1 \), the relative s-dependence shows a very weak sensitivity to \( a \), so that all the following conclusions could be supported by a plot made for another value \( a = 0.1 \). However, for much smaller values \( a < 0.1 \), the numerical results start to become unstable since they start to be dominated by the incomplete 1/N corrections. To explain this failure in the limit \( a \rightarrow 0 \), we note first that to leading order in \( N \), eq. (10) is a resummation in powers of \( \left( \frac{m^2}{(N_2^2 - 1)} \right)^k \) which reduces for \( a \rightarrow 0 \) to the unresummed \( v_2(2s) \), which, as we know from Ref. [29], is suppressed by higher powers of \( 1/(N_2^2 - 1) \) which are not included in the calculation of (10). Therefore, the leading \( O(N^0) \) contribution to \( v_2(2s) \), \( s > 4 \) calculated here must vanish for \( a \rightarrow 0 \) (which we checked), and incomplete 1/N corrections can therefore dominate in this limit. This clarifies why the range of applicability of our calculation remains limited to \( a > 0.1 \), and it motivates the choice \( a = 0.1 \) in Fig. 2.

We next turn to the multiplicity dependence of \( v_2(2s) \). It is instructive to start this discussion with the academic limit of a system that emits a small number of gluons from a large number of sources, \( m \ll N \). In this case, the sum over the number of dipole is limited in eqs. (8), (9) by \( m/2 \) rather than by \( N/2 \). We have derived also for this limit analytical results for \( K_{2s}^{(m)} \). We find that the value of higher order cumulants decreases rapidly with the order of the cumulant and, in this sense, the case \( m \ll N \) is void of signs of collectivity. Only in the opposite case \( m \gg N \) do we observe flow cumulants \( v_2(2s) \) which decrease only mildly with increasing \( s \), see the case for \( m = 10N \) in Fig. 2. For even larger values of \( m/N \), differences between the higher order cumulants \( v_2(2s) \) become even smaller (data not shown). On the other hand, as the multiplicity \( m \) moves closer to the number of sources \( N \), first signs of the break-down of collectivity are observed: for instance, for \( m = 4N \), the eighth order cumulant \( v_2(8) \) in (11) has the wrong sign in some range of \( m \), and \( v_2(8) \) in Fig. 2 can therefore not be shown in that parameter range. As the derivation of (10) is based on an expansion in powers of \( 1/(N_2^2 - 1) \), this breakdown of collectivity can be pushed to smaller multiplicities in theoretical worlds with larger \( N_2 \), see Fig. 2. We thus find that collectivity (in the sense of a \( v_2 \)-signal that is almost independent of the order of cumulant from which it is calculated) will always be absent in the region \( m \ll N \) and it will always be approximately realized in the region \( m \gg N \). We emphasize that in the present calculation, gluons show azimuthal correlations irrespective of how far they are separated in longitudinal phase space [29]. In this sense, \( m \) is an event multiplicity and not a quantity per unit rapidity, and it is reasonable to assume that ultra-relativistic high-multiplicity pp collisions populate the range \( m \gg N \).

One may wonder on general grounds how the present calculation could yield non-vanishing higher order cumulants given that only products of dipole terms are kept in the starting point (1). Technically, the reason is that for any finite \( a = A_b/U \), the \( a \)-dependence of \( \tau^{(2s)} \) in (8) and of \( K_{2s} \) in (10) cannot be written as an \( a \)-dependent factor raised to the power \( 2s \). As a consequence, the connected \( 2s \)-point correlation functions constructed from \( K^{(2s)} \) cannot vanish since the different terms in the brackets of eq. (12) have different \( a \)-dependencies. Physically, what leads to deviations from an exact Gaussian statistical distribution is the fact that we consider events of finite multiplicity with quantum interference effects between particles. We note that other cases are known [34] in which seemingly small deviations from Gaussian statistical distributions due to a fixed finite multiplicity can yield to non-vanishing cumulants of arbitrarily high order, even though particles are apparently uncorrelated since drawn from a unique one-particle distribution.

In summary, we have demonstrated in a simple model that resummed quantum interference effects can lead to azimuthal flow signals \( v_n(2s) \) that persist almost unattenuated in higher order cumulants. This effect arises from dipole correlations that are “integrated out” and whose strength is parametrized by the parameter \( a \) in (11). They are always subleading in a naive expansion in powers of \( \frac{1}{N_2^2 - 1} \), but they enter cumulants at leading order if powers of \( \frac{m^2}{N_2^2 - 1} \) are resummed. We caution that the simple model studied here does not capture all observed flow phenomena. For instance, the \( v_2 \)'s in Fig. 2 are seen to decrease with increasing \( m \) while the opposite qualitative trend is seen in the data.\(^2\) Our conclusion is therefore limited to the statement that the model calculation presented here provides a proof of principle that quantum interference can contribute to flow-like multi-particle correlations even if both final state rescattering effects and effects of parton saturation are absent.

We finally comment on the interpretation of our model calculation and on the relation of our results to calculations in parton saturation models. As we learnt from discussions with A. Kovner, the starting point of [29] coincides with setting in CGC calculations (such as eq. (1) of Ref. [33]) the target averages over Wilson lines to unity and taking the color charge densities in projectile averages as locally correlated in transverse space. We note that the model studied here does not arise as the limiting case of varying a physical condition formulated in CGC calculations. Rather it is a model whose simplicity allows one to study a subset of the physics effects present in the CGC calculations (namely quantum interference and color flow) in greater detail at the expense of using an ad hoc simplified description of the scattering process. This simplification made it possible here to perform the parametrically important resummation of all powers of \( \left( \frac{m^2}{(N_2^2 - 1)} \right)^k \). It is an interesting question whether an analogous resummations can be performed in the more complete CGC formalism, and whether it would yield to a qualitatively similar conclusion. Physically, Ref. [29] can then either be interpreted as a model for final state gluon production based on the simplified assumption that all gluons in the initial state are freed with the same (possibly small) probability. This

\(^2\) More precisely: the particular \( m \)-dependence seen in Fig. 2 follows from a Glauber-like ansatz \( m \ll N \). Varying this ansatz will yield a modified \( m \)-dependence. A signal \( v_n(2s) \) that increases with \( m \) is therefore not excluded by our study, but it is not a natural prediction of the framework explored here.
is the point of view taken throughout this manuscript and in Ref. [29]. Alternatively, the same calculation may be viewed as characterizing initial state effects: the calculation would indicate then that quantum interference and color flow in the in-state can give rise to significant asymmetries in the intrinsic $k_T$-distribution of the incoming hadronic wave function. As one supplements this initial state interpretation with the assumption that the scattering process maps asymmetries in the intrinsic $k_T$-distribution linearly to the final state, one regains the above-mentioned final state interpretation. We close by repeating that the simplicity of the model studied here has allowed us to perform explicitly a resummation of $O(m^2/(N_c^2 - 1))$ that is required on physical grounds. Our calculation provides a proof of principle that momentum asymmetries that persist in higher order cumulants can arise from quantum interference and color flow alone.

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