Adaptive sliding mode control and its application in chaos control

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Liqun Shen1*, Chengwei Li1 and Mao Wang2

Abstract: The sliding motion of sliding mode control system is studied in this paper. Using the measure concept, two new quantities about the sliding motion are introduced, and a new relationship about the sliding motion is derived with the new quantities. According to this relationship, an adaptive law of the magnitude of the controller's switching part is proposed, which can minimize the chattering phenomenon according to the predefined robust margin. To verify the effectiveness of the proposed control scheme, it is applied to Rössler system with uncertain disturbances. Simulation results show that the proposed control method can stabilize Rössler system with the magnitude of the controller's switching part adjusted adaptively with the disturbances.

Subjects: Automation Control, Control Engineering, Dynamical Control Systems, Process Control - Chemical Engineering, Systems & Controls

Keywords: sliding mode control, adaptive control, measure concept, chaos control

1. Introduction

Variable structure control with sliding mode was first proposed and studied in the early 1950s in the Soviet Union and it has been developed greatly since its introduction to the world by several research papers as Utkin (1977, 1978). It is a robust control method with relatively simple controller design procedures. It can be applied in various control systems and is especially suitable for the systems where the control input is inherently switching, such as the power electronic systems.

Chattering phenomenon is a main drawback of this method, that is to say, the control input will switch at a high frequency. To alleviate chattering, this paper finds a method to reduce the amplitude of the switching part of the controller. The amplitude of the switching part can be adjusted according to the disturbances and a predefined robust factor. So, we can adjust the chattering according to a predefined robust level.
procedures. There are generally two steps to design a sliding mode controller (Edwards & Spurgeon, 1998; Hung, Gao, & Hung, 1993; Utkin, 1977, 1978; Young, Utkin, & Ozguner, 1999). In the first step, a sliding surface that determines the system performance should be designed. In the second step, a nonlinear switching controller that satisfies the reachability condition should be designed. When the sliding motion is induced, a robust control system with reduced order is obtained (Edwards & Spurgeon, 1998).

Because the sliding mode control method is a robust control method, it can be applied in nonlinear or noisy environments (Chen, Liu, Ma, & Zhang, 2012; Chen, Zhang, Ma, & Liu, 2012; Chen, Zhang, Sprott, Chen, & Ma, 2012; Chen, Zhao, Ma, & Zhang, 2011; Dadras & Momeni, 2009; Fang, Li, Li, & Li, 2013; Herterng, Chen, & Chen, 2000; Li & Chang, 2009; Roopaei, Sahraei, & Lin, 2010; Yau, 2004). However, a main drawback of sliding mode control is the chattering phenomenon, which will not only cause the wear and tear of the actuator but also deteriorate the system performance seriously. To alleviate the chattering phenomenon, there are many methods proposed, such as the boundary layer methods (Slotine, 1984; Slotine & Sastry, 1983) and the second-order sliding mode control methods (Bartolini, 1989; Bartolini, Ferrara, & Usani, 1998; Bartolini & Pydynowski, 1993). These methods utilize the spirit of sliding mode control with the control input filtered before its sending out. The main purpose of these methods is to manage the frequency of the control input and slow down the variation of it. This paper does not eliminate chattering, but deals with chattering on the other direction. If we try to reduce the magnitude of the controller’s switching part, then the chattering phenomenon will also be alleviated. In other words, the magnitude of the controller’s switching part should not be too large. If an adaptive law can be designed to make the magnitude of the controller’s switching part adapt to the disturbances of the system, the chattering phenomenon will also be alleviated. The adaptive law of the magnitude of the controller’s switching part was studied in an early time (Yoo & Chung, 1992). And till now, researchers are still doing efforts in this direction (Dadras & Momeni, 2009; Fang et al., 2013; Li & Chang, 2009; Roopaei et al., 2010; Yau, 2004). But in these adaptive schemes, there are some parameters which always have the positive derivatives. The magnitude of such parameters cannot be reduced even when the disturbances get small. Moreover, when disturbances and calculation errors are taken into account, these parameters will go to infinity in real-time algorithms. To overcome this problem, the general sliding motion is analyzed, and more information about the sliding motion is presented.

In this paper, the sliding motion of sliding mode control system is studied in detail. Two new quantities about the sliding motion are introduced using the measure concept, and a new relationship about the sliding motion is derived. According to this relationship, an adaptive law of the magnitude of the controller’s switching part is proposed, which can minimize the chattering phenomenon according to the predefined robust margin. This method can provide a new tool to minimize the chattering phenomenon.

This paper is organized as follows. In Section 2, the sliding motion is analyzed and a new relationship is presented using the measure concept (Dudley, 2002; Royden, 1988; Rudin, 1987). And the result is consistent with Filippov’s equivalent control theory (Filippov, 1960). Then, an adaptive law of the magnitude of the controller’s switching part is proposed in Section 3. Section 4 presents an application of the proposed control scheme in chaos control. Section 5 makes the conclusion that summarizes the contribution of this paper.

2. Sliding motion analysis

2.1. General sliding mode control system
Consider a general sliding mode control system as follows:

$$\dot{x} = f(t, x) + bu$$  (1)
where \( x \in \mathbb{R}^n \) is the state vector, \( f(t, x) \in \mathbb{R}^n \) represents a bounded continuous function, \( b = [0, \ldots, 0, 1]^T \), \( u = \begin{cases} u^+, S = C^T x > 0 \\ u^-, S = C^T x < 0 \end{cases} \) is the scalar control input, where \( C \in \mathbb{R}^n \), \( S = C^T x = 0 \) is the sliding surface, \( u^+ \) and \( u^- \) represent two bounded continuous control inputs in the right-hand side of (1) while \( S > 0 \) and \( S < 0 \), respectively.

**Assumption 1.** To focus on the sliding motion analysis, we suppose system (1) satisfies the reachability condition.

**Assumption 2.** To simplify the analysis, we assume that for all the time \( t \) during the sliding motion, the control input must be \( u(t) = u^+ \) or \( u(t) = u^- \).

By Assumption 1, the trajectory of system (1) will converge to the sliding surface, and satisfies

\[
SS < 0 \tag{2}
\]

On the sliding surface, sliding motion will take place and we have (Edwards & Spurgeon, 1998)

\[
S = 0 \tag{3}
\]

and

\[
S = 0 \tag{4}
\]

Equations 3 and 4 are two basic equations when analyzing the sliding motion.

### 2.2. Sliding motion analysis of the discretized system

If the control input of system (1) is discretized as a real-time sliding mode control system, we can get the following system

\[
\dot{x}(t) = f(t, x) + bu(k \Delta t) \tag{5}
\]

where \( u(k \Delta t) = \begin{cases} u^+, S(k \Delta t) = C^T x(k \Delta t) > 0 \\ u^-, S(k \Delta t) = C^T x(k \Delta t) < 0 \end{cases} \) \( t \in [k \Delta t, (k + 1) \Delta t) \), \( k = 0, 1, 2, \ldots \). While \( \Delta t \) approaches zero, system (5) approaches system (1). If Assumption 1 holds, then system (5) satisfies Equation 2 when \( t = k \Delta T, k = 0, 1, 2, \ldots \).

To focus on the sliding motion analysis, we suppose system (5) has already been in sliding motion while \( t = 0 \). In the following, the discrete time version of Equations 3 and 4 will be derived. Observe the dynamic of system (5) in the interval \([0, T]\), where \( T \) is a positive real number. Let \( \Delta T = \frac{T}{m} \), and \( \Delta t = \frac{\Delta T}{n} \), where \( m \) and \( n \) are two positive integers. Then the interval \([0, T]\) can be divided into \( n \) intervals as \([0, \Delta T], [\Delta T, 2 \Delta T], \ldots, [(n - 1) \Delta T, T]\), and each \( \Delta T \) interval has \( m \Delta t \) intervals. On a single point of a time interval, system (5) and system (1) are generally different. But on each \( \Delta t \) interval, system (5) and system (1) will behave similarly while \( \Delta t \) approaches zero. From the definition of the sliding surface, in each \( \Delta t \) interval we have

\[
\dot{S}(t) = C^T f(t, x) + C^T bu(k \Delta t) \tag{6}
\]

where \( t \in [k \Delta t, (k + 1) \Delta t) \), \( k = 0, 1, 2, \ldots, m \cdot n - 1 \), as well as the reachability condition is satisfied. During the sliding motion, it is obvious that

\[
|S(t)| \leq M \cdot \Delta t \tag{7}
\]
where $M = \max_{t \in [0, T)} \left\{ \left| C^T f(t, x) + C^T bu(t) \right| \right\}$.

**Remark 1.** During the sliding motion, the system trajectory will be chattering around the sliding surface. Because the reachability condition is assumed to be satisfied, the system trajectory cannot go away from the sliding surface in the successive two steps. Then, $S(t)$ can be bounded by the maximum one step out of the sliding surface, which is represented by Equation 7.

Because $f(t, x)$ and $u(t)$ are bounded, the maximum one step out of the sliding surface will approach zero while $\Delta t$ approaches zero, which is shown in Equation 7. To build a bridge between system (1) and system (5), we observe the sliding motion in small $\Delta t$ intervals. And we use the average value in $\Delta t$ interval to represent the corresponding quantities in system (1). According to Equation 7, it is obvious that

$$\lim_{\Delta t \to 0} \hat{S}_i = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{t_{i\Delta T}}^{(i+1)\Delta T} S(t) dt = 0$$

(8)

where $\hat{S}_i$ represents the average value of $S(t)$ on the $i$th $\Delta T$ interval, $i = 0, 1, \ldots, n - 1$. Then, we have

$$\lim_{\Delta t \to 0} \hat{S}_i = 0$$

(9)

$i = 0, 1, \ldots, n - 1$. This is the discrete time version of Equation 3. Also according to Equation 7, we can get the following relationship

$$\lim_{\Delta t \to 0} \hat{S}_i = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{t_{i\Delta T}}^{(i+1)\Delta T} S(t) dt = \lim_{\Delta t \to 0} \frac{1}{\Delta t} (S_{i+1} - S_i)$$

$$\leq \lim_{\Delta t \to 0} \frac{1}{\Delta t} M = 0$$

(10)

where $\Delta t \ll \Delta T$ needs to be satisfied, $\hat{S}_i$ represents the average value of $\hat{S}(t)$ on the $i$th $\Delta T$ interval, $S_i = S(i\Delta T)$, $i = 0, 1, \ldots, n - 1$. Then, we have

$$\lim_{\Delta t \to 0} \hat{S}_i = 0$$

(11)

This is the discrete time version of Equation 4. Because $f(t, x)$ is bounded and continuous, we have

$$\lim_{\Delta t \to 0} (S_{i+1} - S_i) = \lim_{\Delta t \to 0} \sum_{j=0}^{m-1} S_{ij} \Delta t$$

(12)

where $S_{ij} = S(i\Delta T + j\Delta t)$, $i = 0, 1, \ldots, n - 1$, $j = 0, 1, \ldots, m - 1$. Substitute Equation 12 into Equation 10, a new relationship can be found as

$$\lim_{\Delta t \to 0} \frac{\Delta t}{\Delta T} \sum_{j=0}^{m-1} S_{ij} = 0$$

(13)

Equations 9 and 11 are the discrete time correspondences of Equations 3 and 4. Equation 13 is a new relationship that provides additional information about the sliding motion. According to Equations 10 and 12, Equations 11 and 13 can be combined as
And Equation 14 can represent the sliding motion more exactly. This is also a relationship between the average value $\dot{\bar{S}}_i$ and the instant value $\dot{S}_{ij}$. In the following, the continuous time version of Equation 14 will be derived using the measure concept.

### 2.3. A new relationship about the sliding motion

Concerning about system (5), let $U'=(t': t' \in [0, t), u(t')=u')$ and $U=(t': t' \in [0, t), u(t')=u)$ be two sets of time to be studied.

And let $m(U'(t))$ and $m(U(t))$ represent the measure of $U'(t)$ and $U(t)$, respectively. According to Assumption 2, we have

$$m(U'(t)) + m(U(t)) = t$$

Concerning about system (5), for each $\Delta t$ interval which belongs to $[0, T)$, there must be $u'$ or $u$ in active. So, each $\Delta t$ interval must belong to $U'(T)$ or $U(T)$. Because $f(t, x)$, $u'$, and $u$ are all continuous, while $\Delta T$ approaches zero, Equation 14 can be reformulated as

$$\lim_{\Delta t \to 0} \frac{\Delta T}{\Delta t} \sum_{j=0}^{m-1} S_j = 0$$

(14)

And $U'((i+1)\Delta T)\setminus U'(i\Delta T)$ and $U'((i+1)\Delta T)\setminus U(i\Delta T)$ represent the relative complement operation. While $\Delta t$ approaches zero, Equation 5 will approach Equation 1. Now consider the general sliding mode control system as Equation 1. While $\Delta T$ approaches zero, Equation 18 can be rewritten in the differential form as

$$\dot{S}(t) = \mu^+ C f^+ + \mu^- C f^- = 0$$

(19)

where $\mu^+ = \lim_{\Delta U(t) \to 0} \frac{dU(t)}{dt}$, $\mu^- = \lim_{\Delta U(t) \to 0} \frac{dU(t)}{dt}$, $f^+ = f(t, x) + bu'(t)$, and $f^- = f(t, x) + bu(t)$. And this is the continuous time version of Equation 14. According to Equation 17, we can get the following relationship

$$\mu^+ + \mu^- = 1$$

(20)

And the important concept of equivalent control can be derived as

$$\dot{f}_{eq} = \mu^+ f^+ + \mu^- f^-$$

(21)

and

$$\dot{u}_{eq} = \mu^+ u^+ + \mu^- u^-$$

(22)
where $f_{eq}$ is the equivalent right-hand side of Equation 1, and $u_{eq}$ represents the equivalent control. Filippov’s equivalent control theory is an important result in sliding mode control theory. And the result of Equation 21 is consistent with Filippov’s equivalent control theory, which is shown in Figure 1.

From Equations 19 to 22, we can see that two important quantities $\mu^+$ and $\mu^-$ are introduced. And more information can be obtained about the sliding motion. As a sum up, the following theorem can be concluded.

**Theorem 1.** By introducing two quantities as $\mu^+$ and $\mu^-$, a new relationship about the sliding motion of system (1) can be derived as

$$\dot{S}(t) = \mu^+ C^T f^+ + \mu^- C^T f^- = 0$$

And the status of system (1) can be revealed by $\mu^+$ and $\mu^-$, and the following relationship holds

$$\begin{cases} 
\mu^+ = 1, & S < 0 \\
\mu^- = 1, & S < 0 \\
\mu^+ C^T f^+ + \mu^- C^T f^- = 0, & S = 0 \text{ and } \dot{S} = 0 
\end{cases}$$

**Proof.** See the deriving process of (19).

By observing $\mu^+$ and $\mu^-$, the adaptive law of the magnitude of the controller’s switching part can be designed.

3. **Adaptive sliding mode controller design**

Consider a sliding mode control system with time-varying disturbances as follows:

$$\dot{x} = Ax + bn(t, x) + bu$$

where $x \in \mathbb{R}^n$ is the state vector, $A \in \mathbb{R}^{n \times n}$ and $b \in \mathbb{R}^n$ are constant matrices, $u$ is the control input, and $n(t, x)$ represents the time-varying disturbances.

To design a sliding mode controller, a sliding surface should be designed first as follows:

$$S = C^T x = 0$$
where \( C \in \mathbb{R}^n \) is a constant vector, which determines the system performance. Then, a controller that satisfies the reachability condition should be designed. Suppose \( C' b \) is invertible, a classical sliding mode controller can be designed as

\[
\begin{align*}
  u &= -(C^T b)^{-1} \left( C^T A x + \hat{K} \cdot sgn(S) \right) \\
  & \quad \text{where } sgn(S) = \begin{cases} 
    1, & S > 0 \\
    -1, & S < 0 
  \end{cases}
\end{align*}
\]

(25)

and make sure that its derivative along system (23) is negative definite. According to Equations 25 and 26, we have

\[
\begin{align*}
  \dot{V} &= S^T \dot{S} \\
  &= x^T C^T [A x + b n(t, x) + b u] \\
  &= x^T C^T A x + x^T C^T b n(t, x) - x^T C (C^T A x + \hat{K} \cdot sgn(S)) \\
  &\leq \left( |C^T b n(t, x)| - \hat{K} |S| \right) |S|
\end{align*}
\]

(27)

Obviously, if \( \hat{K} > |C^T b n(t, x)| \), the reachability condition will be satisfied. But to alleviate the chattering phenomenon, the magnitude of the controller's switching part \( \hat{K} \) should not be too large. We can use the following theorem as the updating law of \( \hat{K} \).

**Theorem 2.** Considering system (23) with the sliding mode controller as Equation 25, an adaptive law of \( \hat{K} \) can be designed as follows:

\[
\begin{align*}
  \hat{K} &= \frac{\gamma}{1 - |\mu - \mu|} \gamma, & S = 0 \text{ and } \dot{S} = 0 \\
  \hat{K} &= |S|, & \text{else}
\end{align*}
\]

Then, the reachability condition will be satisfied and \( \dot{\hat{K}} \) will approach the magnitude of the disturbances with the predetermined margin \( \gamma \).

**Proof.** While the trajectory of system (23) is out of the sliding surface, the control scheme should make sure the reachability condition be satisfied. Let \( K_{max} = \max |C^T b n(t, x)| \), we can choose a Lyapunov function as

\[
V = \frac{1}{2} S^2 + \frac{1}{2} (\hat{K} - K_{max})^2
\]

(28)

According to Equations (23), (25), (28), and Theorem 2, we have

\[
\begin{align*}
  \dot{V} &= S^T \dot{S} + (\hat{K} - K_{max}) \dot{\hat{K}} \\
  &= S^T C^T [A x + b n(t, x) + b u] + (\hat{K} - K_{max}) |S| \\
  &= S^T C^T A x + S^T C^T b n(t, x) - S^T C^T A x + \dot{\hat{K}} |S| + (\hat{K} - K_{max}) |S| \\
  &= C^T b n(t, x) \cdot S - K_{max} \cdot |S| \\
  \leq 0
\end{align*}
\]

(29)

We can see that the reachability condition is satisfied.
Next, the robust margin $\gamma$ should be studied. According to Theorem 1 and the definition of the controller, a relationship between the system behavior and the two introduced quantities $\mu^+$ and $\mu^-$ can be built as

$$
\begin{cases}
\mu^+ = 1, & S > 0 \\
\mu^- = 1, & S < 0 \\
\dot{K}(\mu^+ - \mu^-) = C^T b n(t, x), & S = 0 \text{ and } S = 0
\end{cases}
$$

(30)

According to Equation 30, we have

$$
\dot{K} \| (\mu^+ - \mu^-) \| = \| C^T b n(t, x) \|
$$

(31)

during the sliding motion. As stated in Theorem 2, $\dot{K} = \frac{\gamma}{1 - |\mu^+ - \mu^-|}$ during the sliding motion. Eliminating $|\mu^+ - \mu^-|$ in Equation 31, we have

$$
\dot{K} = \| C^T b n(t, x) \| + \gamma
$$

(32)

where $\gamma$ is the predefined robust margin.

Remark 2. In real-time algorithms, $\mu^+$ and $\mu^-$ can be approximated as in Equation 18 by setting $\Delta T$ a fixed small value and $\Delta t$ the sampling interval.

4. Adaptive sliding mode control of Rössler system

To show the effectiveness of the proposed control scheme, Rössler system is used as an example in the simulation. The state equation of Rössler system can be described as

$$
\dot{x} = Ax + bf(x) + bd(t) + bu
$$

(33)

where $x = [x_1, x_2, x_3]'$ is the state vector, $A = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ 0 & 0 & -5.7 \end{bmatrix}$ and $f(x) = x_1 x_3 + 0.2$ represent the dynamic structure of Rössler system, $d(t)$ stands for the external disturbances, $b = [0, 0, 1]'$, $u$ is the control input. It has been proved that system (33) presents chaos when $d(t) = 0$ and $u = 0$. 

Figure 2. The state response of Rössler system.
Figure 3. The trajectory of Rössler system.

Figure 4. The response of the sliding surface.

Figure 5. The magnitude of the controller's switching part.
In the simulation, the initial condition of Equation 33 is set as \( x(0) = [5 6 7]^T, \dot{K}(0) = 0 \), and \( \gamma = 2 \). Let \( d(t) = 3x(t-40)\sin(t) + 10x(t-80)\sin(0.2t) \), where \( x(t) \) represents the step function. After a stable sliding surface is determined by \( C = [-3.2 - 1.64 1]^T \), the controller formed as Equation 25 with the adaptive law in Theorem 2 can stabilize Rössler system in Equation 33. Simulation results are shown in Figures 2–6, where the control input is turned on at \( t = 10 \) s.

Figures 2 and 3 show that Rössler system can be stabilized by the proposed controller. Figure 4 shows the response of the sliding surface. Figures 5 and 6 show that the magnitude of the controller's switching part can adapt with the external disturbances. From Figure 5, we can see that the information of disturbances can be abstracted, which may be useful for further applications.

If we only use the conventional adaptive law \( \dot{K} = [S] \), the switching part can only increase as shown in Figure 7, which is not acceptable in real applications.
5. Conclusions

This paper was motivated by the demand to reduce the amplitude of the switching part of the sliding mode controller. A new adaptive sliding mode control scheme is proposed. Through the discretization of the sliding mode controller, several corresponding discrete time version equations about the sliding motion are derived. And a new relationship about the sliding motion is obtained. This relationship is consistent with Filippov’s equivalent control theory. With this relationship, an adaptive scheme of the magnitude of the controller’s switching part is proposed based on the two new quantities μ⁺ and μ−, which are first introduced in this paper. Then, the magnitude of the controller’s switching part can be adjusted adaptively with a predefined robust margin according to the disturbances. Simulation results of Rössler system show the effectiveness of the proposed control scheme.

In the future, the application in chaos synchronization and secure communication may also be studied. This adaptive method may be more suitable to be applied in digital control systems, where μ⁺ and μ− can be obtained easily.

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