Matrix Inflation and its String Theory Origins

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Abstract. Motivated by the dynamics of $N$ coincident D3 branes in some specific flux compactifications, we construct an inflationary model in which inflation is driven by three $N \times N$ hermitian matrices $\Phi_i$, $i = 1, 2, 3$, hence the name Matrix Inflation, or M-flation for short. We show that one can consistently restrict the classical dynamics to a sector in which the $\Phi_i$ are proportional to the $N \times N$ irreducible representation of SU(2). In this sector our model behaves like an effective inflaton field, which takes super-Planckian field values, and $3N^2 - 1$ isocurvature fields. These may have the observational effects such as production of iso-curvature perturbations on cosmic microwave background. Moreover, the existence of these extra scalars provides us with a natural preheating mechanism and exit from inflation. Due to the super-Planckian excursions of the canonical effective inflaton, the model is capable of producing a considerable amount of gravity waves that can be probed by future CMB polarization experiments. Furthermore, the fine-tunings associated with unnaturally small couplings in the chaotic type inflationary scenarios are removed. We also show that even if the cutoff of the theory is lowered by the square of number of species, one can still use the effective field theory approach to justify the absence of higher dimensional operators.

1. Introduction

The inflationary paradigm, the idea that the early Universe has undergone a nearly exponential expansion phase, has appeared as the leading candidate for explaining the recent cosmological observations data [1]. The simplest and still successful model of inflation is a massive, free scalar field minimally coupled to the Einstein gravity. Nonetheless, motivated by various beyond the Standard Model particle physics or supergravity and string theory settings, many models of inflation have been constructed by introducing more non-trivial potentials for the scalar field and/or the addition of other scalar fields to the model. The latter are called multi-field models.

Cosmological perturbation in theories of multi-field inflation, in which one deals with more than one scalar field, have been studied [2]. In the multiple field inflationary models one can perform a rotation in the field space of scalar fields where the inflaton field is evolving along a trajectory while the remaining fields are orthogonal to it. These extra fields, like the inflaton itself, have quantum fluctuations which once stretched to super-Hubble scales can become classical and can therefore contribute to the power spectrum of iso-curvature as well as curvature perturbations, the details of which depends on the post inflationary dynamics and the reheating scenario.

In [3,4], inspired by the potential derived from a stringy setup in which a stack of $N$ D3-branes is exposed to an appropriate flux in a specific background, we formulated an inflationary model...
in which the inflatons are $N \times N$ hermitian matrices, hence the name “Matrix” inflation, or M-inflation. As we will see, our model has a simple but still rich dynamics for a specific truncation of the theory which is known as SU(2) truncation. We argue that Matrix inflation can solve the fine-tunings associated with the standard chaotic inflationary scenarios. Furthermore, there will be a tower of iso-curvature perturbations that except for one, whose amplitude at the Hubble scale might be observable, the rest are quite tiny and probably unobservable. The existence of the isocurvature mode can have significant observational consequences for the CMB observations [5]. Moreover, we argue that our model has an embedded efficient preheating mechanism and also observable gravity waves at the horizon scale that can be probed by future experiments. We also show that one can still use the effective field theory approach even if the UV cutoff of the theory is lowered. We should emphasize here that in calculating the potentials from the dynamics of coincident branes in an appropriate flux compactification, we did not take into account the back-reactions of the compactifications and the moduli stabilization effects on the potential.

2. String Theory Model

In the context of string theory, the world-volume theory of $N$ coincident $p$-branes is described by a (supersymmetric) $U(N)$ gauge theory. In this system, the transverse positions of the branes, $\Phi_I$, $I = p + 1, \ldots, 9$, which from the world-volume theory are scalars in the adjoint representation of $U(N)$, are hence $N \times N$ matrices. For the case of our interest, $p = 3$, there are six such scalars. The DBI action for the system of $N$ coincident D3-branes in the background RR six form flux (sourced by a distribution of D5-branes) is given by

$$S = \frac{1}{(2\pi)^3 l_s^4 g_s} \int d^4 x \text{STr} \left( 1 - \sqrt{-g_{ab}} \sqrt{|Q_{I,J}^2|} + \frac{i g_s}{2 \cdot 2\pi l_s^2} [X^I, X^J] C^{(6)}_{I,J,0123} \right).$$

Here $l_s$ is the string scale and $g_s$ is the perturbative string coupling. The operator STr on a product of matrices is the trace of their symmetrized product. The induced metric on branes, $g_{ab}$, is given by $g_{ab} = G_{MN} \partial_a X^M \partial_b X^N$, where $X^M$ indicates the ten-dimensional positions of the branes and $G_{MN}$ is the ten dimensional background metric. Here the indices $I, J = 4, 5, \ldots, 9$ represent the coordinates perpendicular to the branes world-volume, the indices $a, b = 0, 1, 2, 3$ represent the brane world-volume coordinates and the capital letters $M, N = 0, 1, \ldots, 9$ indicate the ten-dimensional coordinates. The matrix $Q^I_J = \delta^{IJ} + \frac{i}{2\pi l_s^2} [X^I, X^J]$ is due to non-commutativity properties of the system [6] and $C^{(6)}_{I,J,0123}$ is a rank-6 antisymmetric Ramond-Ramond (RR) field which has two legs along the direction transverse to the D3-brane.

We consider the ten-dimensional IIB supergravity background

$$ds^2 = -2dx^+ dx^- - \hat{m}^2 \sum_{i=1}^{3} (x^i)^2(dx^i)^2 + \sum_{I=1}^{8} dx_I dx_I,$$  

$$C_{+123ij} = \frac{2\hat{\kappa}}{3} \epsilon_{ijk} x^k$$

where $i, j$ indices, which are ranging over 1, 2, 3, parameterize three out of six transverse directions to D3-brane and $x^I$ include three spatial directions along the brane and five of the transverse directions to D3-branes. With

$$\hat{m}^2 = 4g_s^2 \hat{\kappa}^2/9$$

the above background, with constant dilaton, is a solution to the supergravity equations of motion. If we turn-off fluctuations along the directions transverse to the branes and the $x^I$ directions (this may be done if we compactify six dimensions on a CY3 with two 3-cycles, one
small and one large), fix the light-cone gauge on the D3-branes, expand the action and keep up to the fourth order four $X^I$. We obtain

$$ S = \frac{1}{(2\pi)^4 l_s^4 g_s} \int d^4x \, \sqrt{-g} \, \epsilon^{IJK} \partial_I X_J \partial^K X_K - V(X) $$

$$ V = -\frac{1}{4 \cdot (2\pi l_s^2)^2} [X_i, X_j][X_i, X_j] + \frac{i g_s \kappa}{3 \cdot 2\pi l_s^2} [\epsilon_{ijk} X_i X_j X_k] + \frac{1}{2} m^2 X_i^2. $$

If we redefine

$$ X_i = \sqrt{2\pi g_s} \, l_s^2 \Phi_i, \quad \lambda = 2\pi g_s, \quad \hat{\kappa} = \frac{\kappa}{g_s \sqrt{2\pi g_s}}, \quad \hat{m}^2 = m^2, $$

the potential takes the form

$$ V = \text{Tr} \left( -\frac{\lambda}{4} [\Phi_i, \Phi_j][\Phi_i, \Phi_j] + \frac{i \kappa}{3} \epsilon_{ijk} [\Phi_k, \Phi_i] \Phi_i + \frac{\hat{m}^2}{2} \Phi_i^2 \right). $$

It is possible to show that one can consistently restrict the classical dynamics to a sector in which we are effectively dealing with a single scalar field $\tilde{\phi}$. This sector, which will be called the $SU(2)$ sector, is obtained for matrix configurations of the form

$$ \Phi_i = \tilde{\phi}(t) J_i, \quad i = 1, 2, 3, $$

where $J_i$ are the basis for the $N$ dimensional irreducible representation of the $SU(2)$ algebra

$$ [J_i, J_j] = i \epsilon_{ijk} J_k, \quad \text{Tr}(J_i J_j) = \frac{N}{12} (N^2 - 1) \delta_{ij}. $$

Since both $\Phi_i$ and $J_i$ are hermitian, we conclude that $\tilde{\phi}$ is a real scalar field.

Plugging these into the action (5) and adding the four-dimensional Einstein gravity, we obtain

$$ S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R + \frac{\lambda}{4} \epsilon_{ijkl} \partial_I \partial^l \tilde{\phi} + \frac{\lambda}{2} \tilde{\phi}^2 + \frac{2 \kappa}{3} \tilde{\phi}^3 - \frac{m^2}{2} \tilde{\phi}^2 \right], $$

where $\text{Tr} J^2 = \sum_{i=1}^3 \text{Tr}(J_i^2) = N(N^2 - 1)/4$. Interestingly enough, this represents the action of chaotic inflationary models with a non-standard kinetic energy. Upon the field redefinition

$$ \tilde{\phi} = (\text{Tr} J^2)^{-1/2} \phi = \left[ \frac{N}{4} (N^2 - 1) \right]^{-1/2} \phi, $$

the kinetic energy for the new field $\phi$ becomes standard, while the potential takes the form of a symmetry-breaking potential, taking into account the condition (4)

$$ V_0(\phi) = \frac{\lambda_{eff}}{4} \phi^4 - \frac{2 \kappa_{eff}}{3} \phi^3 + \frac{m^2}{2} \phi^2 = \frac{\lambda_{eff}}{4} \phi^2 \left( \phi - \mu \right)^2 $$

where

$$ \lambda_{eff} = \frac{2 \lambda}{\text{Tr} J^2} = \frac{8 \lambda}{N(N^2 - 1)}, \quad \kappa_{eff} = \frac{\kappa}{\sqrt{\text{Tr} J^2}} = \frac{2 \kappa}{\sqrt{N(N^2 - 1)}}, \quad \mu = \sqrt{2m}/\sqrt{\lambda_{eff}}. $$

The minimum $\phi = \mu$ for the potential (12) corresponds to the supersymmetric background where $N$ D3-branes blow-up into a giant D5-brane [7]. It is worth noting that in the brane theory setting the $U(N)$ symmetry appears as a gauge symmetry.

As a result of motion of D3-branes in the background $C^{(6)}$ flux, two of the directions transverse to D3-branes blow-up into (fuzzy) two sphere, which in the large $N$ limit behaves as a D5-brane with world-volume $R^4 \times S^2$. In this geometric picture our inflaton field $\phi$ is nothing but the radius of this two-sphere.
3. Mass spectrum of \( \Psi_i \) modes in Matrix Inflation

We notice that there are \( 3N^2 - 1 \) fields denoted as \( \Psi_i \) perpendicular to the SU(2) direction that even though do not contribute to the classical inflationary dynamics, do have quantum fluctuations and will hence affect the cosmological perturbation analysis. To compute these effects we need to have the mass spectrum of the \( 3N^2 - 1 \) modes coming from the \( \Psi_i \).

Assuming \( \Psi_i = \Phi_i - \hat{\phi} J_i \) where \( \text{Tr}(\Psi_i J_i) = 0 \), we expand the action up to the second order in \( \Psi_i \). The kinetic term readily takes the standard form \( \frac{1}{2} \text{Tr}(\partial_\mu \Psi_i \partial^\mu \Psi_i) \). After a slightly lengthy but straightforward computation the potential to second order in \( \Psi_i \) is obtained as

\[
V_{(2)} = \text{Tr} \left[ \frac{\lambda}{2} \phi^2 \Omega_i \Omega_i + \frac{m^2}{2} \Psi_i \Psi_i + \left( -\frac{\lambda}{2} \phi^2 + \kappa \phi \right) \Psi_i \Omega_i \right]
\]

where

\[
\Omega_k \equiv i\epsilon_{ijk}[J_i, \Psi_j].
\]

From the above form we see that if we have the eigenvectors (eigen-matrices) of the \( \Omega_i \) we can compute the spectrum of \( \Psi_i \) in terms of \( \phi \)-field (to be viewed as the inflaton). Finding the eigenvectors of \( \Omega_i \) is mathematically the same problem as finding the vector spherical harmonics. (For a detailed discussion see e.g. [8], section 5.2.) If we denote the \( \Omega \) eigenvalues by \( \omega \), i.e. \( \Omega_i = \omega \Psi_i \), we obtain

\[
V_{(2)} = \left( \frac{\lambda_{eff}}{4} \phi^2 (\omega^2 - \omega) + \kappa_{eff} \omega \phi + \frac{m^2}{2} \right) \text{Tr} \Psi_i \Psi_i.
\]

If we have the possible values of \( \omega \) we can read off the effective (\( \phi \)-dependent) mass of the \( \Psi_i \) modes

\[
M^2 = \frac{\lambda_{eff}}{2} \phi^2 (\omega^2 - \omega) + 2\kappa_{eff} \omega \phi + m^2
\]

Following the analysis of [8], we find that \( \omega \) can take three values:

- **“The zero modes”** \( \omega = -1 \). This happens for modes of the form \( \Psi_i = [J_i, \Lambda] \), with \( \Lambda \) being an arbitrary matrix. Since U(N) symmetry is a gauge theory, zero modes can be removed by the gauge transformations. Noting that \( \Lambda \) is an arbitrary matrix there are \( N^2 \) of such modes, all with the same mass.
- **“The \( \alpha \) modes”:** \( \omega = -(l+1), l \in \mathbb{Z}, 0 \leq l < N \). Each mode for a given \( l \) has a multiplicity of \( 2l + 1 \). Therefore, there are \( N^2 \) of \( \alpha \)-modes.
- **“The \( \beta \) modes”:** \( \omega = l, l \in \mathbb{Z}, 0 < l < N \). Each mode for a given \( l \) has a multiplicity of \( 2l + 1 \). Therefore, there are \( N^2 - 1 \) of \( \beta \)-modes.

As expected, there are altogether \( 3N^2 - 1 \) zero, \( \alpha \), and \( \beta \) modes.

4. Power spectra in the presence of \( \Psi_i \) modes

With the mass spectrum for \( \Psi_i \) modes computed in the previous section we can compute the power spectra of the adiabatic and the iso-curvature perturbations.

There are two distinguishable branches for the model to inflate, \( \phi > \mu \) and \( \mu/2 < \phi < \mu \), that we will analyze them separately here.

(a) \( \phi > \mu \)

To match the observational constraints from WMAP, the above parameters have to take the following values:

\[
\lambda_{eff} \simeq 4.91 \times 10^{-14}, \quad m \simeq 4.07 \times 10^{-6} M_P, \quad \kappa_{eff} \simeq 9.57 \times 10^{-13} M_P
\]
\[ \phi_i \simeq 43.57 M_P, \quad \phi_f \simeq 27.07 M_P, \quad \mu \simeq 26 M_P, \]  

(19)

where \( \phi_i \) and \( \phi_f \) are respectively the values of the inflaton at the end of inflation and when the current Hubble scale left the horizon. The scalar spectral index for the adiabatic perturbations is \( n_R \simeq 0.959 \).

The lowest mass in the tower of \( \Psi_{r,lm} \) iso-curvature modes belongs to the \( l = 0 \alpha \)-mode, whose mass is \( M^{2}_{\alpha,0}(\phi) = \lambda \phi^2 - 2\kappa \phi^4 + m^2 \). The amplitudes and spectral index of the iso-curvature spectrum are respectively \( P_{S_{\alpha,0m}} \simeq 1.162 \times 10^{-11} \) and \( n_{\alpha,0m} \simeq 0.981 \). The next ones in the tower of iso-curvature modes have negligible amplitudes at Hubble scale (i.e., \( \gtrsim 10^{-15} \)) and therefore their contributions could be ignored. The amplitude of tensor spectrum at Hubble scale is \( P_T(k_{\text{inf}}) \simeq 4.84 \times 10^{-10}, r \simeq 0.2 \) with the spectral index \( n_T \simeq -0.025 \). Planck should be able to verify this model.

(b) \( \mu/2 < \phi < \mu \)

Here to satisfy the constraints from the amplitude and spectral index from WMAP5, one has to adjust the the parameters as follows:

\[ \phi_i \simeq 23.5 M_P, \quad \phi_f \simeq 35.03 M_P, \quad \mu \simeq 36 M_P. \]  

(20)

\[ \lambda_{\text{eff}} \simeq 7.187 \times 10^{-14}, \quad m \simeq 6.824 \times 10^{-6} M_P, \quad \kappa_{\text{eff}} \simeq 1.940 \times 10^{-12} M_P. \]  

(21)

The index of the adiabatic spectrum for such values of parameters is \( n_R \simeq 0.961 \).

Again the least massive iso-curvature mode is the \( l = 0 \alpha \)-mode. Its amplitude and spectral index are, respectively, \( P_{S_{\alpha,0m}} \simeq 1.46 \times 10^{-11} \) and \( n_{\alpha,0m} \simeq 0.987 \). Next iso-curvature modes have larger masses and therefore their amplitudes are negligible in comparison with the adiabatic one (\( \gtrsim 10^{-12} \)). The amplitude of tensor spectrum at Hubble scale is \( P_T(k_0) \simeq 1.307 \times 10^{-11} \), i.e. \( r \simeq 0.048 \) with the spectral index \( n_T \simeq -0.006 \). Such gravity wave spectrum could be detected by CMBPOL [9]. The tensor spectrum is very close to being scale-invariant in this case.

5. Particle creation and preheating scenario

Even though one can argue that during the slow-roll inflation particle creation and hence its back-reaction on the dynamics of the \( \phi \) field is not large, it becomes important when \( \epsilon, \eta \) are of order one. To show this schematically, we approximate the potential with a quartic potential and follow the analysis performed in [10] for massless preheating, using the potential

\[ V(\phi, \chi) = \frac{\lambda}{4} \phi^4 + \frac{1}{2} g \phi^2 \chi^2 \]  

(22)

As shown in [10], the \( \phi \) equation of motion toward the end of inflation is an (unharmonic) oscillator around \( \phi = 0 \), the amplitude of the solution is decreasing as \( t^{-1/2} \). The equation for the \( \Psi \) modes, can be also solved explicitly in this case. One can see that the equation for all three zero, \( \alpha \) and \( \beta \) modes is of the form of \( g^2/\lambda = n(n+1)/2 \) (with \( n = 1, l, l - 1 \) respectively for zero, \( \alpha \) and \( \beta \) modes). As discussed in [10] for these specific values of \( g^2/\lambda \) we have the significant property that there is an enhancement in the parametric resonance leading to considerable creation of zero, \( \alpha \) and \( \beta \) modes. For the odd \( n \) (i.e. for our zero modes, odd \( l \) \( \alpha \)-mode, and even \( l \) \( \beta \)-mode) the particle creation is peaked around zero momentum \( k \) modes. For the even \( n \) modes, however, the particle creations is peaked around momenta \( k^2 = \frac{3}{2} H_{\text{inf}}^2 \epsilon \sqrt{\frac{g^2}{2\lambda}} \), where \( \epsilon \) is computed at the beginning of the slow-roll inflation and \( H_{\text{inf}} \) is the Hubble during inflation. For low \( n \), the Floquet index \( \mu_k \propto \ln n_k \) (\( n_k \) is the number density of the produced particles at momentum \( k \)) is around 0.15 for odd \( n \) and around 0.5 for even \( n \). Therefore, among the low \( n \) modes the main contribution to preheating is coming from odd \( n \) [10]. As discussed in [10] the bigger \( k \) is, the more energy can be transferred from the inflationary sector to the \( \Psi \).
sector and a more efficient preheat mechanism. This means that $\alpha$ and $\beta$ modes with large $l$, $l$ of order $N$, make the biggest contribution. Of course, one should note that zero modes are not physical in the stringy motivated model matrix inflation, where they are gauged. All in all, due to the existence of the large $l$ modes and for large $N$ in our model, we expect to have a very efficient preheating model.

6. UV behavior

In a theory with many particles the scale where quantum gravity effects become large is lowered to

$$\Lambda^2 = \frac{M_p^2}{N_{cl}}$$

(23)

where $N_{cl}$ counts all the species with mass below the cutoff $\Lambda$ [11], here in our case $N_{cl} = 3N^2$. Here we check that the amount of excursions for the effective inflaton $\phi$ is less than this cutoff for our model. This will guarantee that the higher dimensional operators do not affect the dynamics. We first focus on the branch in which the setup inflates in the region $\phi > \mu$. To find the number of species in our case, we assume that $\lambda = 1$. To obtain the value of $\lambda = 4.91 \times 10^{-14}$, one needs $N = 5.46 \times 10^6$ D3 branes. The amount of excursion in the physical field space, $\Delta \phi$, is

$$\left( \frac{\Delta \phi}{\Lambda} \right)^2 \simeq 0.06$$

for the mass term and $\left( \frac{\Delta \phi}{\Lambda} \right)^3 \simeq 0.014$ (and smaller) for the cubic (and quartic) term. The other interesting thing that is worthwhile mentioning is that one obtains the mass parameter for the model to be close to the cutoff, $m \simeq 4.07 \times 10^{-6} M_p$, and one would only need a tuning of order $\simeq 50\%$ to achieve the value required to fit the observational data. Repeating the same analysis for the branch in which $\mu/2 < \phi < \mu$, one realizes that that the higher dimensional operators are suppressed at least by factors of $\left( \frac{\Delta \phi}{\Lambda} \right)^2 \simeq 0.02$, therefore the UV behavior is under much better control. One can conclude that at least the stringy picture of matrix inflation is safe from the UV criticism of [12].

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