Extracting visible $s$—channel Higgs in the lepton scattering

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Abstract
The contributions of $s$—channel Higgs in $l^+ l^- \rightarrow q \bar{q}$ processes in the general lepton collider ($l^+ l^-$ collider) within Standard Model is studied. A new idea to extract the contribution by using the data from both electron and future heavy-lepton colliders at same center-of-mass energy is proposed. Deviations due to the $s$—channel Higgs contributions are analysed and discussed for the heavy-quark final states by using the total cross-section, forward-backward asymmetry and its ratio. It is shown that significant deviations are expected for the top quark final state.

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1 Introduction

Although the development of the lepton colliders beyond the electron collider are at a very early stage, its promise for physics is clear. However, now the possibility of constructing a muon collider is coming into the limelight. The efforts to construct the heavy-lepton collider are especially motivated by the limitations of the electron collider to achieve high center-of-mass energy\[1, 2\]. High center-of-mass energy is necessary to study the behaviour of Higgs particle in detail as well as open a window of new physics \[3\]. In this letter, the Higgs particle is studied by using the total cross-section (CS), forwad-backward (FB) asymmetry and its ratio ($R$) of two jets productions in the lepton scattering, $l^+ l^- \to q \bar{q}$ within tree-level Standard Model (SM). The reason is clear, because these quantities are complementary, i.e. the terms which contribute to the total CS will be nothing in the FB asymmetry and vice-versa. On the other hand, the ratio of FB asymmetry and total CS gives a clear and may be experimentally accessible quantity, because any uncertainties in both theoretical and experimental sides are reduced. Next point of this paper is, a trial to make the $s-$channel Higgs contributions to be more visible by using the data from both electron collider and future heavy-lepton colliders together. These points are the originality of this paper.

In general, the FB asymmetry and total CS for the initial state $l$ is obtained by integrating the CS ($\sigma$) with respect to the angular variable $z (\equiv \cos \theta)$ and defining,

$$A_{FB}^l \equiv \sigma_F^l - \sigma_B^l,$$

(1)

for the FB asymmetry and

$$\sigma_T^l \equiv \sigma_F^l + \sigma_B^l,$$

(2)

for the total CS. Here, the forward and backward scattered CS are given as,

$$\sigma_F \equiv \int_0^1 \frac{d\sigma}{dz} \, dz,$$

(3)

$$\sigma_B \equiv \int_{-1}^0 \frac{d\sigma}{dz} \, dz.$$
These quantities lead to the ratio $R$ to be defined as follows

$$R^l \equiv \frac{A_{FB}^l}{\sigma_T^l}. \quad (5)$$

In the next section, the calculation of this FB asymmetry, total CS and also how the significant contributions of the $s$–channel Higgs can be extracted will be shown.

## 2 Calculation

Within tree-level SM, $l^+ l^- \rightarrow q \bar{q}$ process is realized in the vector bosons ($Z$ and photon) and also scalar Higgs mediated diagrams as depicted in Fig. 1. The related interactions are expressed as,

$$\mathcal{L}_V = g \bar{f} \gamma^\mu \left( g_{VL}^f L + g_{VR}^f R \right) f V_\mu, \quad (6)$$

$$\mathcal{L}_H = -g \frac{m_f}{2 M_W} \bar{f} f H. \quad (7)$$

Here, $f$ denotes fermions, $L$ and $R$ are the chiralities and $V = Z, A$. The couplings for the vector bosons are given as,

$$g_{A_f} \equiv g_{AL}^f = g_{AR}^f \equiv Q_f \sin \theta_W, \quad (8)$$

$$g_{ZL}^f \equiv \frac{1}{2 \cos \theta_W} \left( \pm 1 - 2 Q_f \sin^2 \theta_W \right), \quad (9)$$

$$g_{ZR}^f \equiv -\frac{\sin^2 \theta_W}{\cos \theta_W} Q_f, \quad (10)$$

with $Q_l = -1$, $Q_u = 2/3$, $Q_d = -1/3$ and $\theta_W$ is the Weinberg angle respectively. The sign $\pm$ means + for up-quarks, while − for down-quarks and leptons.
Remind that $d\sigma/dz$ is proportioned to $|M|^2$. So, in order to accomplish the FB asymmetry and total CS in Eqs. (1) and (2), one has to compute the square amplitudes of the diagrams in Fig. 1, that is

$$|M|^2 = 24 G_F^2 \left[ |M_Z|^2 + |M_A|^2 + |M_H|^2 + 2 \text{Re} \left( M_Z M_A^\dagger + M_Z M_H^\dagger + M_A M_H^\dagger \right) \right],$$

with $M_i$ denotes the amplitude of each gauge bosons and scalar Higgs. Note that color and spin averaged factors have been included. Although the calculation is quite trivial, it is better to present the analytic results for comparison and further analysis. Then, each term in Eq. (11) is given as,

$$|M_Z|^2 = \frac{M_W^4}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \times \left[ \left( (g_{Z L})^2 + (g_{Z R})^2 \right) \left( (g_{Z L})^2 + (g_{Z R})^2 \right) \times \left( s^2 + u(s)^2 \cos^2 \theta - \frac{2}{M_Z^2} s \right) - 2 \left( (g_{Z L})^2 - (g_{Z R})^2 \right) \left( (g_{Z L})^2 - (g_{Z R})^2 \right) \times u(s) s \cos \theta \left( 1 - \frac{4}{s} \right) \right]$$

$$+ 8 m_q^2 g_{Z L} g_{Z R} \left( (g_{Z L})^2 + (g_{Z R})^2 \right) \left( s - 2 m_l^2 + \frac{s^2}{M_Z^2} \right)$$

$$+ 8 m_l^2 g_{Z L} g_{Z R} \left( (g_{Z L})^2 + (g_{Z R})^2 \right) \left( s - 2 m_q^2 + \frac{s^2}{M_Z^2} \right)$$

$$- \frac{32}{M_Z^2} m_q^2 m_l^2 g_{Z L}^2 g_{Z R}^2 g_{Z L}^2 g_{Z R}^2 s$$

$$+ \frac{4}{M_Z^2} \left( g_{Z L}^2 - g_{Z R}^2 \right)^2 \left( g_{Z L}^9 - g_{Z R}^9 \right)^2 m_q^2 m_l^2 s^2 \right], \quad (12)$$

$$|M_A|^2 = \frac{M_W^4}{s^2} \left( g_A^l g_A^q \right)^2 \left[ s^2 + u(s)^2 \cos^2 \theta + 4 s \left( m_l^2 + m_q^2 \right) \right], \quad (13)$$

$$|M_H|^2 = \frac{1}{16} \frac{m_l^2 m_q^2}{(s - M_H^2)^2 + M_H^2 \Gamma_H^2} u(s)^2, \quad (14)$$

$$\text{Re} \left( M_Z M_A^\dagger \right) = \frac{M_W^4}{s} \frac{s - M_Z^2}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} g_{A^l} g_{A^q}$$

$$\times \left\{ \left( g_{Z L}^l + g_{Z R}^l \right) \left( g_{Z L}^q + g_{Z R}^q \right) \times \left[ s^2 + u(s)^2 \cos^2 \theta + 4 s \left( m_l^2 + m_q^2 \right) \right] \right\}$$
\[ \Delta R \text{ as a function of center-of-mass energy for top (thick line) and bottom (thin line) quark final states including Higgs contributions in the electron vs muon collider case.} \]

\[ \text{Re} \left( M_Z M_H^\dagger \right) = \frac{-2 \left( g_{ZL}^l - g_{ZR}^l \right) \left( g_{ZL}^q - g_{ZR}^q \right) u(s) s \cos \theta}{\left( s - M_H^2 \right) \left( s - M_Z^2 \right) + M_H M_Z \Gamma_H \Gamma_Z} \]  
\[ \times M_W^2 m_l^2 m_q^2 \left( g_{ZL}^l + g_{ZR}^l \right) \left( g_{ZL}^q + g_{ZR}^q \right) u(s) \cos \theta, \]  
\[ \text{Re} \left( M_A M_H^\dagger \right) = \frac{M_W^2 m_l^2 m_q^2}{s} \frac{s - M_H^2}{\left( s - M_H^2 \right)^2 + M_H^2 \Gamma_H^2} g_{A}^l g_{A}^q u(s) \cos \theta, \]  

where \( \sqrt{s} \) is center-of-mass energy and \( u(s) \equiv \sqrt{(s - 4 m_l^2) (s - 4 m_q^2)} \). \( u(s) \) also expresses the boundary condition for the physical region in the process, i.e. \( s \geq 4 m_l^2 \) and \( s \geq 4 m_q^2 \) as well. From Eqs. (15), (16) and (17), it is clear that significant deviations due to the \( s \)-channel Higgs contributions would be coming out for the heavy-quark final states. Hence, further discussion will be emphasized only on the top and bottom quark final states.
Figure 3: $\Delta \sigma_T$ (left) and $\Delta A_{FB}$ (right) as a function of center-of-mass energy for top quark final state including (thick line) and excluding (thin line) Higgs contributions in the electron vs muon collider case.

3 Visible $s-$channel Higgs contribution

Further, from Fig. 1 it is clear that the $s-$channel Higgs contribution in the heavy-lepton collider would be significant, while in the electron collider is invisible due to the tiny electron mass [2]. This is the most important point, for example in the muon collider as pointed out in [3]. This simple fact leads the author to combine $e^+ e^- \rightarrow q \bar{q}$ and $l_h^+ l_h^- \rightarrow q \bar{q}$ processes at same center-of-mass energy to extract the $s-$channel Higgs contribution ($l_h$ denotes arbitrary heavy-lepton and $q = b, t$), which is expected to be significant. This task can be done simply by set off the process with initial state $l_h$ against the other one with initial state $e$, that is

$$\Delta \sigma \equiv \sigma^{l_h} - \sigma^e. \quad (18)$$

Then, all of terms in Eq. (11) which are not multiplicated by $m_l^2$ will be exactly canceled out. Despite of another non $m_l^2$ multiplicated terms from vector bosons mediated diagrams are remained, the contributions are expected to be comparable or dominated by the Higgs’ one. Hence the $s-$channel Higgs contribution would be more visible. Corresponding with Eq. (18), the FB asymmetry, total CS and ratio
Figure 4: $\Delta R$ as a function of center-of-mass energy (left) including (thick line) and excluding (thin line) Higgs contributions, and Higgs mass (right) for top quark final state in the electron vs muon collider case.

$R$ become

$$\Delta A_{FB} = \Delta \sigma_F - \Delta \sigma_B,$$  \hspace{1cm} (19)

$$\Delta \sigma_T = \Delta \sigma_F + \Delta \sigma_B,$$  \hspace{1cm} (20)

$$\Delta R = \frac{\Delta A_{FB}}{\Delta \sigma_T}.$$  \hspace{1cm} (21)

These equations are the main points in the paper, and will be analysed further. The author points out that Eqs. (19) ~ (21) are sensitive to the $s$-channel Higgs contribution as shown below. In other words, by using the data from the electron and heavy-lepton collider at same center-of-mass energy, considering the FB asymmetry and total CS in both colliders should be a reliable way to confirm the existence of $s$-channel Higgs.

In the numerical calculations, the parameters have been put as $[4]$, $m_e = 0.51$ (MeV), $m_\mu = 105.66$ (MeV), $m_b = 4.3$ (GeV), $m_t = 180$ (GeV), $M_Z = 91.19$ (GeV), $M_W = 80.33$ (GeV), $\Gamma_Z = 2.49$ (GeV) and $\sin^2 \theta_W = 0.231$. For the Higgs decay width, approximately only the decays to fermionic final states except top quark, within tree-level SM are considered here $[4]$. Note that the higher order corrections or more complete results should be seen in some references $[4]$. However, this rough
approximation for the Higgs decay width is not so important for our interest in the present letter.

Because the recent study on the heavy-lepton collider is focused in the muon collider, let us consider only the muon case in the present letter. The results for the electron vs muon collider case are presented in Figs. 2 ∼ 4. As mentioned before, the physical region is from $\sqrt{s} \geq 10$ (GeV) for bottom and $\sqrt{s} \geq 360$ (GeV) for top quark final states. In Fig. 2, the ratio $\Delta R$ for bottom and top quark final states is presented. It seems that observing the bottom quark final state process is better due to its higher rate. However, the author has checked that in the bottom quark final state case, the $s$-channel Higgs contributions are tiny and negligible, i.e. no visible discrepancy when the $s$-channel Higgs are included or not. Hence further analysis will be done only for top quark final state case. The (unnormlized) FB asymmetry and total CS for top quark final state are then presented in Fig. 3 with varying $\sqrt{s}$. In Fig. 4, the Higgs mass and center-of-mass energy are fixed to be $M_H = 200$ (GeV) for the left figure, and on the other hand $\sqrt{s} = 400$ (GeV) for the right one.

4 Discussion

From Fig. 2, the deviation due to different flavor of quark final states is significant, and is going to be larger for larger mass difference between them. It have also been checked that the deviations will be larger as considering electron vs heavier-lepton collider, like tauon collider. Especially in the very heavy quark final state case, like top quark, the deviation in all quantities seems large, as depicted in Figs. 3 and 4. It can be concluded that in general, FB asymmetry is more sensitive than the total CS for any quark final states in electron vs any heavier-lepton colliders. The reason is, in the FB asymmetry most of the vector-bosons contributions are canceled out, while in the total CS the contributions are still remained. This result leads to the importance of $\Delta R$ which is newly defined in the previous section. However, the visibility will be reduced when one considers any light quark final states which are
lighter than bottom quark, even for FB asymmetry.

Finally, a rough estimation for required luminosity to observe $\Delta R$ can be given under assumptions that $\sqrt{s} \geq 400$ (GeV) and the event numbers is 10 at one year running of machines. For the most optimistic case, the integrated luminosity is required to be larger than 100 (fb)$^{-1}$. Remark that $\sqrt{s} = 400$ (GeV) with the integrated luminosity $\geq$ few (fb)$^{-1}$ is considered to be available in the muon collider (First Muon Collider, FMC) [2]. However, any analysis of the possible background effects in the lepton collider under consideration (both electron and muon colliders in the current case) must be studied further. The details of study for the background effects will be published elsewhere.

Lastly, although the idea that there might be two lepton (with different flavor) colliders with the same center-of-mass energy seems farfetched, the current paper points out and shows an example that there may be any interesting physics by constructing a heavy-lepton collider with same center-of-mass energy as the present electron collider. The author also hopes the study will encourage the interest in the possibility of constructing the heavy-lepton collider like the muon collider which should complement the present electron collider to examine the SM as well as open a window for new physics.

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