VERTICAL STRUCTURE OF NEUTRINO-DOMINATED ACCRETION DISKS AND NEUTRINO TRANSPORT IN THE DISKS

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ABSTRACT

We investigate the vertical structure of neutrino-dominated accretion disks by self-consistently considering the detailed microphysics, such as the neutrino transport, the vertical hydrostatic equilibrium, the conservation of lepton number, as well as the balance between neutrino cooling, advection cooling, and viscosity heating. After obtaining the emitting spectra of neutrinos and antineutrinos by solving the one-dimensional Boltzmann equation of neutrino and antineutrino transport in the disk, we calculate the neutrino/antineutrino luminosity and their annihilation luminosity. We find that the total neutrino and antineutrino luminosity is about \( 10^{54} \) erg s\(^{-1} \) and their annihilation luminosity is about \( 5 \times 10^{51} \) erg s\(^{-1} \) with an extreme accretion rate \( 10 M_{\odot} \) s\(^{-1} \) and an alpha viscosity \( \alpha = 0.1 \). In addition, we find that the annihilation luminosity is sensitive to the accretion rate and will not exceed \( 10^{50} \) erg s\(^{-1} \), which is not sufficient to power the fireball of most energetic gamma-ray bursts (GRBs) if the accretion rate is lower than \( 1 M_{\odot} \) s\(^{-1} \). Therefore, the effects of the spin of the black hole or/and the magnetic field in the accretion flow might play a role in powering the central engine of GRBs.

Key words: accretion, accretion disks – black hole physics – gamma-ray burst: general – neutrinos

Online-only material: color figures

1. INTRODUCTION

Gamma-ray bursts (GRBs) are extremely high-energy-relieving phenomena in the universe and are usually divided into two classes (Kouveliotou et al. 1993; Zhang & Mészáros 2004; Piran 2004; Nakar 2007): short GRBs \( (T_9 < 2 \) s) and long GRBs \( (T_9 > 2 \) s). Numerous models have been proposed to explore the central engines of GRBs, and one of the most-discussed models is neutrino-dominated accretion flows (NDAFs) with a hyperaccreting stellar massive black hole with an accretion rate of \( 0.1 \sim 10 M_{\odot} \) s\(^{-1} \). Due to the high density and high temperature in the inner part of NDAFs, the optical depth of photons is very large and photons are completely trapped; thus, neutrinos and antineutrinos become the most promising candidates to carry away thermal energy and cool the disk. The annihilation of neutrino pairs above the disk is believed to be the energy source of GRBs. Narayan et al. (1992) first proposed that neutrino pairs annihilating into electron pairs during the merger of compact object binaries may power GRBs. Afterwards, Popham et al. (1999) investigated NDAFs under the assumption that the disk is transparent to neutrinos, but they also pointed out that the assumption fails when the accretion rate is higher than \( 1 M_{\odot} \) s\(^{-1} \). Di Matteo et al. (2002) improved the model by using a simplified neutrino transport model which was believed to bridge the optically thin and optically thick neutrino limit. Chen & Beloborodov (2007) improved the model further by dealing with the neutrino emission and chemical composition in the optically thin regime, optically thick regime, and intermediate regime separately. Though many works on NDAFs have confirmed the validity of the NDAF as the central engine of GRBs (Narayan et al. 2001; Kohri & Mineshige 2002; Lee et al. 2005; Janiuk et al. 2007; Gu et al. 2006; Liu et al. 2007, 2008, 2010; Zhang & Dai 2010, 2009), there are still some uncertainties, such as: 1. The distribution of electron fraction. In many previous works, it is assumed to be of a constant value, for example, 0.5, throughout the disk. Since the electron fraction has a large effect on the emission of neutrinos and antineutrinos, as shown by Kohri & Mineshige (2002), Kohri et al. (2005), and Liu et al. (2007), we need to be more cautious when making such an assumption.

2. Neutrino transport and neutrino spectra. The most commonly used approximation was the simplified neutrino transport model introduced in Di Matteo et al. (2002). In their model, the difference between the neutrino and the antineutrino transport in the disk was neglected, but as shown in Pan & Yuan (2012), the precise spectra of neutrinos and antineutrinos sensitively determines the annihilation luminosity of neutrino pairs.

3. The annihilation of neutrino pairs. The most common method for calculating the annihilation luminosity was originally introduced by Ruffert et al. (1997) to calculate the annihilation luminosity during the merger of neutron star binaries. This method was applied to the calculation of the annihilation luminosity above NDAFs under the assumption that the emission of neutrinos and antineutrinos are isotropic and symmetric (Popham et al. 1999).

It is evident that the strictest approach to determine the neutrino/antineutrino luminosity and their annihilation luminosity above NDAFs is to build a two-dimensional disk model in which neutrino transport, vertical structure, chemical evolution, thermal evolution, and the distribution of electron fraction, mass density, and temperature are self-consistently considered. Rossi et al. (2007) first investigated the vertical structure of NDAFs by using the Eddington approximation to deal with the neutrino transport in the vertical direction, neglecting the contribution of the advection term to the disk cooling, and dealing with neutrino emission and chemical composition following a method similar to Janiuk et al. (2007) and Chen & Beloborodov (2007).

Recently, Liu et al. (2010) also investigated the vertical structure of NDAFs with many simplifications on the neutrino
transport, the equation of state (EOS), and the annihilation efficiency. The first simplification is the direct integration of the neutrino emission to calculate the neutrino luminosity by neglecting the absorption of neutrinos, which is viable in the neutrino optically thin limit (Popham et al. 1999); the second simplification is the use of a simplified EOS $p = K \rho^{2/3}$ in the vertical direction, which is viable when relativistic degenerate electrons dominate the pressure of the disk; the third simplification is the application of a toy annihilation efficiency of neutrino pairs $\eta = L_{\nu}/L_{\nu} \propto V^{-1}$, where $L_{\nu}$ is the annihilation luminosity and neutrino luminosity before annihilation, respectively, and $V_{\text{ann}}$ is the so-called annihilation volume. With all the above simplifications, this annihilation efficiency $\eta = L_{\nu}/L_{\nu}$ can even reach 100%! Obviously, this unrealistic result is caused by too many unrealistic assumptions.

In this work, we investigate the vertical structure of NDAFs, neutrino/antineutrino luminosity, and their annihilation luminosity by self-consistently considering the neutrino/antineutrino transport, the vertical hydrostatic equilibrium, the precise EOS, the chemical equilibrium, and the thermal balance between neutrino cooling, advection cooling, and viscosity heating under the self-similar assumption of the distribution of mass density and internal energy density in the radial direction (Shakura & Sunyaev 1973; Narayan & Yi 1994, 1995). Specifically, we strictly solve the Boltzmann equation to deal with the neutrino transport, instead of using the assumption of gray body spectra (Janiuk et al. 2007) or the Eddington approximation (Rossi et al. 2007). Correspondingly, we can obtain the energy spectra of neutrino pairs precisely. Combined with the conservation of the lepton number, the distribution of chemical compositions is self-consistently and accurately determined (Chen & Beloborodov 2007; Rossi et al. 2007).

This paper is organized as follows. In Section 2, we introduce the basic equations in our calculation, including the Boltzmann equation of neutrino/antineutrino transport, angular momentum equation, hydrostatic equilibrium equation, EOS, lepton number conservation equation, and thermal evolution equation. In Section 3, we briefly introduce our numerical methods to find the steady solution of the structure of the disk. In Section 4, we list our numerical results of the disk structure, neutrino/antineutrino luminosity, and their annihilation luminosity. Conclusions and discussions are summarized in Section 5.

2. BASIC EQUATIONS

We assume a steady accretion disk with accretion rate $M = 10, 1, 0.1 M_{\odot}$ around a central black hole with mass $M = 3.3 M_{\odot}$ and adopt the standard $\alpha$ viscosity prescription of Shakura & Sunyaev (1973) with $\alpha = 0.1$ for the viscous stress of the disk. We discuss the structure of the disk and neutrino transport in cylindrical coordinates $(r, z, \phi)$ and we assume the inner boundary of the disk to be $r_{\text{in}} = 6M$ and the outer boundary to be $r_{\text{out}} = 100M$.

2.1. Boltzmann Equation

We solve the one-dimensional Boltzmann equation of neutrino and antineutrino transport in the vertical direction of the disk and obtain the energy-dependent and the direction-dependent neutrino spectrum. We define $f_{\nu}(z, \mu)$ and $f_{\bar{\nu}}(z, \mu)$ to be the distribution function for up-moving neutrinos/antineutrinos and down-moving ones, respectively, where $z$ is the vertical coordinate of the disk, $p$ is the energy of the neutrinos/antineutrinos, and $\mu = \cos(\theta)$ for up-moving neutrinos/antineutrinos and $\mu = -\cos(\theta)$ for down-moving ones, where $\theta$ is the angle of neutrinos/antineutrinos moving along the vertical direction of the disk. For the up-moving neutrinos/antineutrinos, their distribution function is determined by (Sawyer 2003; Burrows et al. 2006; Schinder & Shapiro 1982)

$$\mu \frac{\partial f_{\nu}(z, \mu)}{\partial z} = \lambda_{\nu} \left[ f_{\nu}(T(z), \mu, p) - f_{\nu}(z, \mu) \right]$$

$$+ \lambda_{\nu} \left[ -f_{\nu}(z, \mu) + \frac{1}{2} \int_{0}^{1} \left( f_{\nu}(-z, \mu) + f_{\nu}(z, \mu) \right) \right]$$

(1)

and for the down-moving ones, their distribution function is determined by

$$\mu \frac{\partial f_{\bar{\nu}}(z, \mu)}{\partial z} = -\lambda_{\nu} \left[ f_{\bar{\nu}}(T(z), \mu, p) - f_{\bar{\nu}}(z, \mu) \right]$$

$$- \lambda_{\nu} \left[ -f_{\bar{\nu}}(-z, \mu) + \frac{1}{2} \int_{0}^{1} \left( f_{\bar{\nu}}(-z, \mu) + f_{\bar{\nu}}(z, \mu) \right) \right]$$

(2)

where $\lambda_{\nu}$ is the absorption coefficient and $\lambda_{\nu}$ is the scattering coefficient of neutrinos/antineutrinos; here $f_{\nu}^{eq} = 1/(\exp((p - \mu_{\nu})/kT) + 1)$ for neutrinos, and $f_{\bar{\nu}}^{eq} = 1/(\exp((p + \mu_{\bar{\nu}})/kT) + 1)$ for antineutrinos, where $\mu_{\nu} = \mu_{\nu} - \mu_{n}$ and $\mu_{\nu}, \mu_{\bar{\nu}}, \mu_{n}$ are the chemical potential of the electron, proton, and neutron, respectively.

Because Urca processes $\nu_{e} + n \leftrightarrow e^{-} + p$ and $\bar{\nu}_{e} + p \leftrightarrow e^{+} + n$ dominate the creation and absorption of neutrinos and antineutrinos, and the neutrino/antineutrino scattering by neutrinos and protons, $(\nu_{e}, \bar{\nu}_{e}) + p \rightarrow (\nu_{e}, \bar{\nu}_{e}) + p$ and $(\nu_{e}, \bar{\nu}_{e}) + n \rightarrow (\nu_{e}, \bar{\nu}_{e}) + n$, dominates the scattering opacity (Popham et al. 1999; Janiuk et al. 2007; Liu et al. 2007), so we include no other neutrino/antineutrino processes. Thus, for simplicity, in this paper we can use $\nu$ and $\bar{\nu}$ interchangeably and it will not cause any confusion. As for the explicit expression of the absorption coefficient $\lambda_{\nu}$ and scattering coefficient $\lambda_{\nu}$ of neutrinos/antineutrinos, please refer to Pan & Yuan (2012).

Considering that the disk is symmetric about the equatorial plane, the boundary conditions for the distribution function $f(z, \mu) = f_{\nu}(z, \mu)$ and $f_{\bar{\nu}}(z, \mu) = 0$, where $H$ is the upper boundary of the disk.

2.2. Angular Momentum Equation and Vertical Hydrostatic Equilibrium Equation

Adapting the $\alpha$ prescription, the tangential stress and angular momentum equation can be written as (Shakura & Sunyaev 1973)

$$\omega_{\varphi} \rho = \alpha \rho c_{s}^{2}$$

(3)

and

$$\rho \frac{d(\Omega r^{2})}{dt} = \rho v_{r} \frac{d(\Omega r^{2})}{dr} = \frac{1}{r} d(w_{\varphi} r^{2})$$

(4)

where $w_{\varphi}$ is the viscous stress, $\rho$ is the mass density of the disk, $c_{s}$ is the acoustic speed, $v_{r}$ is the radial drift velocity, and $\Omega$ is the Kepler angular velocity at radius $r$. Integrating Equation (4) over radius $r$ and height $z$, we obtain

$$2\pi \int_{-H}^{H} \int_{r_{m}}^{r} \rho v_{r} \frac{d(\Omega r^{2})}{dr} d\Omega dr = 2\pi \int_{-H}^{H} \int_{r_{m}}^{r} d(w_{\varphi} r^{2}) d\Omega dr$$

(5)
and by taking into consideration the steady accretion condition
\[ \dot{M} = 2\pi r \int_{H}^{0} \rho v_r dz = \text{const}, \]  
and using the torsion condition in the inner boundary \( w_r(r_m) = 0 \), the angular momentum equation (5) is transformed to
\[ \dot{M} (\Omega r^2 - (\Omega r^2)_m) = 2\pi r^2 \int_{-H}^{H} a_F \rho c_s^2 dz. \]  

The vertical hydrostatic equilibrium equation is simply:
\[ \frac{\rho GM z}{r^2} \frac{1}{r} = -\frac{dp}{dz}. \]  

2.3. Equation of State (EOS)

In our calculation, the EOS of the accreted gas including proton, neutron, and electron pairs is determined by the exact Fermi–Dirac integral; the EOS of neutrino pairs is determined by the numerical integration of their distribution functions \([f_{\nu,\bar{\nu}}(z, p, \mu)]\); the EOS of photons is simply given by Equation (15) and we do not include other kinds of particles in this work, especially helium, which was included in most of the previous works (we will check the validity of neglecting the contribution of helium in Section 4):

\[ p(\rho, Y_e, T) = p_n + p_p + p_e + p_{e^-} + p_{rad} + p_v + p_{\bar{\nu}}, \quad (9) \]

\[ u(\rho, Y_e, T) = u_n + u_p + u_e + u_{e^-} + u_{rad} + u_v + u_{\bar{\nu}}, \quad (10) \]

where \( p \) and \( u \) are the total pressure and the total internal energy density, respectively, \( Y_e \) is the electron fraction \( Y_e \equiv (n_e - n_{e^-})/(n_n + n_n) \), and \( T \) is the local temperature of the disk.

Specifically, the EOSs of gas are expressed as (Janiuk et al. 2007)
\[ p_i = \frac{2\sqrt{2} (m_i c^2)^4}{3\pi^2} \frac{\rho_i}{(\hbar c)^3} F_{5/2}(\eta_i, \beta_i) + \frac{1}{2} \beta_i F_{5/2}(\eta_i, \beta_i), \quad (11) \]

\[ u_i = \frac{2\sqrt{2} (m_i c^2)^4}{3\pi^2} \frac{\rho_i}{(\hbar c)^3} \beta_i^{5/2}[F_{3/2}(\eta_i, \beta_i) + \beta_i F_{5/2}(\eta_i, \beta_i)], \quad (12) \]

\[ n_i = \frac{\sqrt{2}}{\pi^2} \frac{(m_i c^2)^3}{(\hbar c)^3} \beta_i^{3/2}[F_{1/2}(\eta_i, \beta_i) + \beta_i F_{3/2}(\eta_i, \beta_i)], \quad (13) \]

where \( p_i, u_i, n_i \) are the pressure, internal energy density, and number density of particle \( i \), respectively \((i = n, p, e, e^-)\), \( F_k \) is the Fermi–Dirac integral of order \( k \), \( \eta_i \) is the degeneracy parameter of particle \( i \) \((\eta_i \equiv \mu_i^N/|kT| \) where \( \mu_i^N \) is the chemical potential of particle \( i \) not including the rest mass), and \( \beta_i \) is the relativity parameter of particle \( i \) \((\beta_i \equiv kT/m_i c^2)\).

The EOSs of neutrino pairs and radiation are expressed as
\[ p_{\nu,\bar{\nu}} = \frac{4\pi c^2}{15} \left( \frac{r d\Omega}{dr} \right), \]

\[ q_{\nu,\bar{\nu}} = \frac{2\pi c}{h^3} \int \rho p^3 (f_e - f_{e^-})_{\nu,\bar{\nu}} d\rho. \]
We assume the velocity distribution to be \( v_r = -\alpha c_s^2/r \Omega \), \( v_z = 0 \), and \( \Omega = \sqrt{GM/r^3} \), which is similar to the self-similar radial distribution of the mass density of the gas-pressure-dominated thin disk (Shakura & Sunyaev 1973); we also assume the distribution of mass density and energy density to be self-similar, \( \rho \sim r^{-3/2} \) and \( u \sim r^{-2} \), in the radial direction.

According to the mass conservation equation of steady flows \( \rho \nabla \cdot \mathbf{v} = 0 + \nabla \cdot \mathbf{q}_{\text{adv}} \), we obtain \( \nabla \cdot \mathbf{v} = 3/2(v_r/r) \), and so the advection term \( q_{\text{adv}} \) is simplified to be

\[
q_{\text{adv}} = \frac{v_r}{r} \left( \frac{3}{2} \rho - 2u \right).
\]

We will check the validity of the self-similar assumption in Section 4.

For simplicity, we adopt the value of acoustic speed at \( z = 0 \) when calculating the viscosity heating rate and radial velocity at any location, i.e., the more numerically economical distribution to be self-similar, \( \rho \sim r^{-3/2} \) and \( u \sim r^{-2} \), in the radial direction.

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\[
c_s^2 = \left[ \frac{\partial p}{\partial (\rho + u)} \right]_{\text{ad}} = \frac{1}{\rho + p + u} \left[ \rho \left( \frac{\partial p}{\partial \rho} \right)_u + (p + u) \left( \frac{\partial p}{\partial u} \right)_\rho \right].
\]

3. NUMERICAL METHODS

3.1. Two-stream Approximation

The two-stream approximation is a simplification to the full Boltzmann equation that replaces the full direction dependent distribution \( f_s(z, p, \mu) \) and \( f_-(z, p, \mu) \) by two streams at an angle \( \cos(\theta) = \pm 1/\sqrt{3} \) to the vertical direction. Under the two-stream approximation, the full Boltzmann equation is simplified to be

\[
\frac{1}{\sqrt{3}} \frac{\partial f_s(z, p)}{\partial z} = \lambda_u [f^{eq}(T(z), \mu_{eq}, p) - f_s(z, p)] + \frac{1}{2} \lambda_s [f_-(z, p) - f_s(z, p)],
\]

\[
\frac{1}{\sqrt{3}} \frac{\partial f_-(z, p)}{\partial z} = -\lambda_u [f^{eq}(T(z), \mu_{eq}, p) - f_-(z, p)] + \frac{1}{2} \lambda_s [f_-(z, p) - f_s(z, p)].
\]

The two-stream approximation has been confirmed to be valid by Sawyer (2003) and Pan & Yuan (2012), and is much easier to deal with compared with the full Boltzmann equation, and thus it is a prime choice for the calculation of the energy density \( u_{\nu,\bar{\nu}} \), the cooling rate \( q_{\nu,\bar{\nu}} \), and the lepton number flux \( F_{\nu,\bar{\nu}} \) of neutrinos and antineutrinos:

\[
u_{\nu,\bar{\nu}} \equiv \frac{2\pi}{h^3} \int p^3 (f_s + f_-)_{\nu,\bar{\nu}} dp,
\]

\[
q_{\nu,\bar{\nu}} = \frac{1}{\sqrt{3}} \frac{2\pi c}{h^3} \frac{d}{dz} \int p^3 (f_s - f_-)_{\nu,\bar{\nu}} dp,
\]

\[
F_{\nu,\bar{\nu}} = \pm \frac{1}{\sqrt{3}} \frac{2\pi c}{h^3} \int p^2 (f_s - f_-)_{\nu,\bar{\nu}} dp.
\]

3.2. Numerical Methods

Now, we start to seek for a steady solution to the disk in which the conditions for hydrostatic equilibrium, thermal balance, and chemical balance are satisfied. First, we assume an initial distribution of temperature \( T^0(r, z) = 8(r/r_m)^{1/3} \) MeV and electron fraction \( Y_e^0(r, z) = 0.4 \) (in fact, our numerical calculation has shown that the final convergent solution does not depend on the specific choice of initial conditions, which only affect the speed of convergence of the numerical calculation. Such independence proves the validity of our numerical methods in turn), then we solve the vertical hydrostatic equation (8) subject the boundary condition (7). It is not hard to obtain the zero-order mass density distribution \( \rho^0(r, z) \) and the corresponding thickness of the disk \( H^0(r) \). Then, we solve the two-stream approximation (Equations 29 and 30), and obtain the neutrino/antineutrino spectra \( f_{\nu,\bar{\nu}} \). With the neutrino/antineutrino spectra, we can solve the chemical evolution Equation (22) and thermal evolution Equation (24) with the zero-order mass density distribution \( \rho^0(r, z) \) or with the baryon number density \( n_b^0(r, z) \) fixed, until a steady state \( \partial T^1/\partial t = 0 \) and \( \partial Y_e^1/\partial t = 0 \). With the first-order distribution \( T^1(r, z) \) and \( Y_e^1(r, z) \), we solve the vertical hydrostatic equation (8) subject to the boundary condition (7) once more to get the corresponding first-order mass density distribution \( \rho^1(r, z) \). This process is iterated until the final overall convergent solution of mass density \( \rho(r, z) \), electron fraction \( Y_e(r, z) \), and temperature \( T(r, z) \) is obtained.

After that, we switch to the full Boltzmann equation, Equations (1) and (2), to solve the full energy-dependent and the direction-dependent distribution function of neutrinos and antineutrinos: \( [f_s(z, p, \mu)]_s \) and \( [f_-(z, p, \mu)]_s \). With the full energy-dependent and direction-dependent spectra of neutrinos and antineutrinos, we can calculate their annihilation luminosity precisely. The annihilation rate of neutrino pairs is expressed as (Ruffert et al. 1997; Burrows et al. 2006):

\[
Q(\nu_{\nu}\bar{\nu}_{\bar{\nu}}) = \frac{1}{4} \frac{\sigma_0 c}{(m_{\nu} c^2)^3 \langle h c \rangle^6} \left[ \frac{C_1 + C_2}{3} \int_0^\infty dp \int_0^\infty dp' (p + p') \right.
\]

\[
\times (pp')^3 \int_{4\pi} d\Omega \int_{4\pi} d\Omega' f_{\nu_{\nu}} f_{\nu_{\bar{\nu}}}(1 - \cos \Theta)^2
\]

\[
+ C_3 (m_{\nu} c^2)^2 \int_0^\infty dp \int_0^\infty d\Omega' p' (p + p')
\]

\[
\times (pp')^2 \int_{4\pi} d\Omega \int_{4\pi} d\Omega' f_{\nu_{\nu}} f_{\nu_{\bar{\nu}}}(1 - \cos \Theta) \left. \right],
\]

where the typical cross section of neutrino interaction is \( \sigma_0 = 1.705 \times 10^{-44} \text{ cm}^2 \); the weak interaction constants are \( C_1 + C_2 \approx 2.34; C_3 \approx 1.06; p \) and \( p' \) are the energy of neutrinos and antineutrinos, respectively; \( \Omega \) and \( \Omega' \) are the solid angle of the incident direction of neutrinos and antineutrinos, respectively; and \( \Theta \) is the angle between neutrino beams and antineutrino beams (see Figure 1).

4. RESULTS

In this section, taking the case \( M = 10 M_{\odot} \text{ s}^{-1}, \alpha = 0.1 \) as an example, we show the numerical results of the vertical structure of the accretion flow, its radial structure, the spectral energy distribution of the neutrinos/antineutrinos on the disk, and the final annihilation luminosity of neutrinos and antineutrinos.
The mass density is about $\rho \sim 10^{11} \text{ g cm}^{-3}$ and the temperature is about $T \sim 8 \text{ MeV}$ in the inner part ($r = 10M$) of the disk. The precise value of the temperature in the inner part of NDAFs is vitally important, which sensitively determines the annihilation luminosity of neutrino pairs $L_{\nu \bar{\nu}} \sim T^9$ according to Equation (34).

When the disk is in chemical equilibrium, the electron fraction cannot be described by only a constant value throughout the disk, since it varies a few times from the bottom to the surface of the disk. It is the specific distribution of the electron fraction that guarantees the chemical equilibrium; this was not included in most of the previous works, and as a result, the electron fraction had to be an artificial assumption there.

The vertical distribution of $u/p$ and electron fraction $Y_e$ are positively correlated (bottom panels of Figure 2). This implies that relativistic electrons dominate the pressure at the surface of the disk where $u/p \sim 3$, and non-relativistic protons and neutrons dominated the pressure at the bottom of the disk where $u/p \sim 3/2$. To justify the conclusion, we plot the vertical distribution of the ratio $\rho_i/p$ at radii 10$M$ and 35$M$ in Figure 3, where $p_1 \equiv p_\rho + p_\nu$, $p_2 \equiv p_e + p_{e^+}$, and $p_3 \equiv p_{\text{rad}} + p_e + p_{\bar{e}}$. Indeed so: baryons (neutrons and protons) dominate the pressure at the bottom of disk and leptons (electron pairs) dominated the pressure at the surface of the disk. So, it is not reasonable to assume a polytropic EOS $p \propto \rho^{4/3}$ (Liu et al. 2010), which is the EOS of relativistic and strongly degenerate electron gas...
whose ratio of internal energy density to pressure is \( u/p = 3 \). If a polytropic EOS is needed to do some approximation and estimation, then \( p \propto \rho^{5/3} \) is actually a better choice.

4.2. The Radial Structure of the Disk

Two main assumptions are used in the above discussion: we apply the self-similar assumption of the distribution of mass density \( \rho \) and internal energy density \( u \) in the radial direction when calculating the advection cooling term and we neglect the contribution of helium to the EOS and the contribution of helium disintegration to the cooling term. We now check their validity.

4.2.1. Self-similar Behavior in the Radial Direction

Figure 4 shows the radial distribution of mass density \( \rho \), internal energy density \( u \), pressure \( p \), and temperature \( T \) on the equator plane \( z = 0 \) and their corresponding fitting lines: \( \rho \sim r^{-1.5} \), \( u \sim r^{-2} \), and \( T \sim r^{-0.6} \). According to Figure 4, the self-similar assumption of mass density \( \rho \) and internal energy density \( u \) in the radial direction is rather a self-consistent and an accurate description.

Now we give a more physical explanation of the self-similar behavior in the radial direction: according to Figures 2 and 3, it is evident that non-relativistic protons and neutrons dominate the total pressure at the bottom of the disk and determine the surface density of the disk. Hence, we use the polytropic EOS \( p \propto \rho^{5/3} \) to estimate the vertical structure of the disk. Combining Equations (7) and (8), it is easy to obtain the scale thickness of the disk \( H \sim c_s/\Omega \) and the mass density \( \rho \sim r^{-3/2} \). In order to estimate the radial behavior of temperature and pressure, we must consider the more general EOS of non-relativistic gas \( p = \rho T \) and the thermal balance between viscosity heating and neutrino cooling \( q_c H \sim T^4 \) or \( p\Omega H \sim T^4 \), so \( T \sim p/c_s \). Consequently \( T \sim r^{-0.6} \) and \( p \sim r^{-2} \) (see Figure 4).

In order to gain insight into the nature of the disk we investigate, we also calculate the advection factor

\[
q_{\text{adv}} = \frac{\int q_{\text{adv}} dz}{\int q_+ dz},
\]  

where \( q_{\text{adv}} \) and \( q_+ \) are the advection cooling rate and heating rate defined in Section 2; the result is shown in Figure 5(a). Thus, it is evident that the disk is indeed a neutrino-cooling-dominated accretion disk as its name suggests: neutrino radiation dominates the cooling process and advection is always a minor role. The self-similar assumption is only applied in the calculation of the advection term, so even if there is some small deviation between the realistic distribution and the self-similar assumption as shown in Figure 4, it does not have much influence on the final results. Thus, the self-similar assumption is a rather reasonable simplification.

4.2.2. The Fraction of He

The number density of helium \( n_{\text{He}} \) is expressed as

\[
n_{\text{He}} = \frac{1}{4} (1 - X_{\text{nuc}}) n_b,
\]

where the fraction of free nucleons \( X_{\text{nuc}} \) (protons and neutrons) is given by (Janiuk et al. 2007; Qian & Woosley 1996)

\[
X_{\text{nuc}} = 295.5 \rho_{10}^{-3/4} T_{11}^{9/8} \exp(-0.8209/T_{11}).
\]

where \( \rho_{10} \) is the mass density in units of \( 10^{10} \) g cm\(^{-3} \) and \( T_{11} \) is the temperature in units of \( 10^{11} \) K, and if \( X_{\text{nuc}} > 1 \), then \( X_{\text{nuc}} = 1 \).

We plot the free nucleon fraction of the gas on the equator plane \( \log(X_{\text{nuc}}) \) calculated from Equation (37) versus radius \( \log(r/M) \) in Figure 5(b). From Figure 5(b), it is easy to see that \( X_{\text{nuc}} \gg 1 \), i.e., the assumption \( X_{\text{nuc}} = 1 \) and \( n_{\text{He}} = 0 \) hold perfectly here. Thus, it is correct to neglect the contribution of helium to the EOS and the contribution of the helium disintegration to disk cooling.

It should be noted that the radius where helium dissociation becomes important depends on the viscous parameter \( \alpha \) (Chen & Beloborodov 2007). So, in the case of smaller \( \alpha \), the contribution of helium may not be negligible.

4.2.3. Profile of the Disk

In addition, we plot the radial distribution of the surface density \( \sigma \) in Figure 6(a) and the profile of the disk thickness \( H(r) \) in Figure 6(b). Here, the thickness \( H(r) \) is defined according to
Figure 4. Left panel: the radial distribution of the mass density \( \log(\rho/\text{g cm}^{-3}) \), internal energy density \( \log(u/\text{g cm}^{-3}) \), and pressure \( \log(p/\text{g cm}^{-3}) \) vs. \( \log(r/M) \). The lines are the corresponding linear fitting lines for \( \rho \sim r^{-1.5} \) (solid line), \( u \sim r^{-2} \) (dashed line), and \( p \sim r^{-2} \) (dot–dashed line). Right panel: the radial behavior of the temperature \( \log(T/\text{MeV}) \) vs. \( \log(r/M) \); the solid line is the corresponding fitting line \( T \sim r^{-0.6} \).

(A color version of this figure is available in the online journal.)

Figure 5. (a) Left panel: the radial variation of the advection factor \( f_{\text{adv}} \); (b) right panel: the radial distribution of free nucleon fraction \( \log(X_{\text{nuc}}) \) calculated from Equation (37).

Figure 6. (a) Left panel: the radial distribution of surface density \( \sigma \) in units of \( 10^{17} \text{ g cm}^{-2} \); (b) right panel: the profile of the disk thickness \( H(r) \) vs. radius \( r \).

The mass density contrast \( \rho(z = H) = \rho(z = 0)/100. H(r) \) serves as the upper boundary of the Boltzmann equation (see Section 2.1), and it is different from the definition of the usual scale thickness of the disk which is mostly used in the one-dimensional disk model. According to Figure 6, the disk we consider is a geometrically thick disk with thickness \( H \sim r \), which will contribute to higher annihilation efficiency as we will discuss in the next section.
4.3. Neutrino Spectrum

When the disk is in chemical equilibrium, the flux of the lepton number \( F_{\text{lep}} = F_{\nu} + F_{\bar{\nu}} \) vanishes, i.e.,

\[
\int p^2 (f_+ - f_-) \nu \mu d\mu d\nu = \int p^2 (f_+ - f_-) \bar{\nu} \mu d\mu d\bar{\nu}. \tag{38}
\]

Figure 7 shows the direction-averaged neutrino/antineutrino spectrum

\[
F_{\text{num}} = (p/kT)^2 \int f \mu d\mu \tag{39}
\]

at the surface of the disk at different radii, where \( f = f_+ (H, p, \mu) \), \( T = 7.4 \text{ MeV} \) for \( r = 10M \), and \( T = 3.5 \text{ MeV} \) for \( r = 35M \) (see Figure 2(b)). Then, at the surface of the disk the chemical equilibrium condition (38) is transformed to

\[
\int \nu p \mu d\nu = \int \bar{\nu} p \mu d\bar{\nu}. \tag{40}
\]

So, the chemical equilibrium only requires that the total amount of neutrino number flux and the total amount of antineutrino number flux are equal, i.e., the integration of the \( F_{\text{num}} \) of neutrinos and antineutrinos over energy \( p \) are equal. It is interesting that the form of \( F_{\text{num}} \) for neutrinos and antineutrinos almost coincide with each other.

In addition, it is necessary to explain that there is a cusp in the antineutrino spectrum in the lower energy end: it is easy to see that there is a lower energy limit \( E_{\text{min}} = m_\nu + m_\bar{\nu} - m_p \) for antineutrinos from the Urca process \( \nu_e + p \leftrightarrow e^- + n \) which gives rise to the cusp in the energy spectrum. There is no such energy limit for neutrinos from the process \( \nu_e + n \leftrightarrow p + e^- \), and so the neutrino energy spectrum is smooth as expected.

4.4. Annihilation Luminosity

In order to calculate the neutrino/antineutrino luminosity and their corresponding luminosity, we have to make an approximation on the vertical structure of the disk: we assume the disk to be lying on the equator plane, and then modify the resulting annihilation luminosity by taking the thickness of the disk into consideration. With the thin disk simplification, all the elements in Equation (34) for the annihilation rate are available (see Figure 1), so it is easy to numerically integrate Equation (34) over the entire surface of the disk and the whole energy span of neutrinos and antineutrinos.

In addition, the neutrino/antineutrino energy flux \( \tilde{F}_{\nu,\bar{\nu}} \), luminosity at the surface of the disk \( L_{\nu}, L_{\bar{\nu}} \), and their annihilation luminosity \( L_{\nu,\bar{\nu}} \) are expressed as

\[
\tilde{F}_{\nu,\bar{\nu}} = \frac{2\pi c}{h^3} \int \int p^3 f_{\nu,\bar{\nu}} \mu d\mu d\nu, \tag{41}
\]

\[
L_{\nu} = 2 \int_{r_{\text{in}}}^{r_{\text{out}}} 2\pi r \tilde{F}_{\nu} dr, \tag{42}
\]

\[
L_{\bar{\nu}} = 2 \int_{r_{\text{in}}}^{r_{\text{out}}} 2\pi r \tilde{F}_{\bar{\nu}} dr, \tag{43}
\]

\[
L_{\nu,\bar{\nu}} = 2 \int_{H(r)}^{\infty} \int_{0}^{\infty} \tilde{F}_{\nu,\bar{\nu}} dz dr, \tag{44}
\]

where \( r_{\text{in}} = 6M, r_{\text{out}} = 100M \) in our calculation, and \( H(r) \) is the thickness of the disk at radius \( r \) (see Figure 6(b)).

The results are as follows: \( L_{\nu} \approx 5.2 \times 10^{53} \text{ erg s}^{-1} \), \( L_{\bar{\nu}} = 1.66 \times 10^{51} \text{ erg s}^{-1} \), and the annihilation efficiency \( \eta \equiv L_{\nu,\bar{\nu}}/(L_{\nu} + L_{\bar{\nu}}) = 0.16\% \). While it is not the final result, we must take into consideration the thickness of the disk \( H \approx r \) (see Figure 6(b)) to correct the thin disk simplification.

Figure 8 shows the concrete distribution of the annihilation rate \( Q_{\nu,\bar{\nu}} \) calculated with the thin disk simplification in the \( r, z \) plane. The distribution of the annihilation rate is nearly isotropic aside from the directions occupied by the disk and the half open angle of the empty funnel along the central axis of the disk, which is about 45° and is determined by the ratio of the thickness to the radius of the disk, \( H/r \approx 1 \). The major contribution to the total annihilation luminosity is concentrated at the zone near the central black hole of the disk. The solid angle that the assumed thin disk lying on the equator plane subtends to the zone is about \( \Omega_{\text{thin}} \approx 2\pi \), while solid angle that the realistic thick disk subtends to that zone is about \( \Omega_{\text{thick}} \approx 2\pi(1+\cos(45°)) \approx 1.7\Omega \).

Thus, the final annihilation luminosity should be multiplied by a modification factor \( (\Omega_{\text{thick}}/\Omega_{\text{thin}})^2 \approx 3 \).

Hence, the final results are as follows: the neutrino/antineutrino luminosity is about \( L_{\nu} = 5.2 \times 10^{53} \text{ erg s}^{-1} \), annihilation luminosity is about \( L_{\nu,\bar{\nu}} = 5 \times 10^{51} \text{ erg s}^{-1} \), and annihilation efficiency is about \( \eta = 0.48\% \).
different accretion rates $10, 1, 0.1 \ M_{\odot} \text{s}^{-1}$, the thickness $H(r)$ we defined in Section 4.2.3 is roughly equal to the radius $r$.

It is obvious that the accretion rate sensitively determines the annihilation luminosity of neutrino pairs, and that the annihilation luminosity will not exceed $10^{50}$ erg s$^{-1}$ when the corresponding accretion rate is lower than $1 \ M_{\odot} \text{s}^{-1}$. So, NDAFs with accretion rates lower than $1 \ M_{\odot} \text{s}^{-1}$ are unlikely to serve as central engines of GRBs.

Neutrino and antineutrino spectra are the second major factors that determine the annihilation luminosity. The resulting neutrino and antineutrino spectra obtained by solving the Boltzmann equation show that, when the disk is in chemical equilibrium, the emission of neutrinos and antineutrinos is almost symmetric with nearly identical energy spectra, but the spectra is in the form of neither a black body nor a gray body. As shown in Pan & Yuan (2012), the black body spectra of neutrino and the neutrino spectra based on the most commonly used simplified model of neutrino transport (Di Matteo et al. 2002), or equivalently the gray body spectra (Janiuk et al. 2007), can overestimate the annihilation luminosity by nearly one order of magnitude. In the following, we will check the validity of the previous assumption of the neutrino transport.

It is easy to estimate the mean optical depth of neutrinos in the inner part of the disk. From Figure 6(a) and Figure 2(b), at $r = 10M$, the surface density is $\sigma = 3 \times 10^{17}$ g cm$^{-2}$ and the temperature is $T = 7.4$ MeV. According to Di Matteo et al. (2002), the neutrino opacities of absorption $\tau_a$ and scattering $\tau_s$ are expressed as

$$\tau_a = 2.7 T_{11}^2 \sigma_{17},$$

$$\tau_s = 4.5 T_{11}^2 \sigma_{17},$$

where $T_{11}$ is the temperature in units of $10^{11}$ K and $\sigma_{17}$ is the surface density in units of $10^{17}$ g cm$^{-2}$. Therefore, the optical depth of neutrinos that we obtain is $\tau_a = 10$, $\tau_s = 6$, so the disk is optically thick for neutrinos at $r = 10M$. However, as Pan & Yuan (2012) have shown in the quasi optically opaque case ($\tau_a = 0.1 \sim 1$), the neutrino spectra are neither the black body spectra $f_{\text{black}}$ (Popham et al. 1999) nor the gray body spectra $f_{\text{gray}}$ (Di Matteo et al. 2002; Janiuk et al. 2007):

$$f_{\text{black}} = \frac{1}{\exp(p/kT)+1},$$

$$f_{\text{gray}} = \frac{b}{\exp(p/kT)+1},$$

and the block factor $b$ is given by

$$b = \frac{1}{(3/4)\tau_{a}(\tau/2+1/\sqrt{3}+1/3\tau_{a})},$$

where $\tau = \tau_a + \tau_s$, so here $b = 0.16$.

For comparison, we plot the direction-averaged spectra $F_{\text{num}}$ of the black body spectra, the gray body spectra, and the more realistic spectra obtained by solving the Boltzmann equation (see Figure 9). From Figure 9, it is obvious that the first assumption of neutrino transport overestimates the neutrino luminosity by about $30\%$ and the corresponding annihilation luminosity by about $(1.3^2 - 1) \approx 70\%$, while the second assumption underestimates the neutrino luminosity by about 5 times and the corresponding annihilation luminosity by about

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**Table 1: Luminosity of Neutrino Annihilation with Different Accretion Rates**

| $M$ ($M_{\odot} \text{s}^{-1}$) | $T_{10M}$ (MeV) | $L_{\nu} + L_{\bar{\nu}}$ (erg s$^{-1}$) | $L_{\nu,0}$ (erg s$^{-1}$) | $\eta = L_{\nu,0}/(L_{\nu} + L_{\bar{\nu}})$ |
|-----------------------------|-----------------|-------------------------------------|-----------------|----------------------------------|
| 10                          | 7.4             | $1.04 \times 10^{54}$                | $5.0 \times 10^{51}$ | $4.8 \times 10^{-3}$               |
| 1                           | 4.2             | $0.92 \times 10^{53}$                | $3.3 \times 10^{50}$ | $3.6 \times 10^{-4}$               |
| 0.1                         | 3.3             | $0.30 \times 10^{52}$                | $4.0 \times 10^{46}$ | $1.3 \times 10^{-6}$               |

---

5. CONCLUSIONS AND DISCUSSIONS

The temperature of the inner part of NDAFs sensitively determines the final luminosity of neutrino annihilation to be $L_{\nu,0} \propto T^9$. In the extreme model of this paper ($M = 3.3 \ M_{\odot}$, $M = 10 \ M_{\odot}$, and $\alpha = 0.1$), it is found that the temperature in the inner part of the disk is about $T \approx 8$ MeV, and the corresponding annihilation luminosity is about $L_{\nu,0} \approx 5 \times 10^{53}$ erg s$^{-1}$ which roughly satisfies the energy demand of the most energetic GRBs. If the temperature of the disks changes from 8 MeV to 4 MeV, the annihilation luminosity is reduced about 500 times, which is not enough to power the fireball of the most energetic GRBs with isotropic luminosity of about $10^{52}$ erg s$^{-1}$. In the standard model of GRBs, the duration of the energy injection is the duration of the GRB prompt emission, which could be about 2 s for short GRBs and about 100–1000 s for long bursts. Assuming an extreme constant accretion rate of $10 \ M_{\odot} \text{s}^{-1}$ therefore means that an enormous total mass consumption powers GRBs, especially long bursts, which might be unrealistic. Therefore, the simple NDAF model investigated in this work cannot produce sufficient energy to power GRBs; the effects of the spin of black hole or/and the magnetic field in the accretion flow might be introduced to produce the central engine of GRBs (see Li 2002; Chen & Beloborodov 2007; Lei et al. 2009, 2010; Janiuk & Yuan 2010, for instance).

It is very easy to understand that the temperature of the inner disk mainly depends on the accretion rate of the flow. Besides the extreme model, we also investigate models with lower accretion rate, such as $M = 0.1 \ M_{\odot} \text{s}^{-1}$ and $1 \ M_{\odot} \text{s}^{-1}$. The results, including the resulting temperature $T$ in the inner part of disk ($r = 10M$), neutrino and antineutrino luminosity $L_{\nu} + L_{\bar{\nu}}$, annihilation luminosity $L_{\nu,0}$, and the corresponding annihilation efficiency $\eta$, are listed in Table 1. For all the three different accretion rates $10, 1, 0.1 \ M_{\odot} \text{s}^{-1}$, the thickness $H(r)$
The thickness of the disk is the third factor that affects the final annihilation luminosity: a larger ratio of thickness to radius means the disk subtends a larger solid angle to the annihilation zone. In the case we consider, the thick disk (\( H \approx r \)) will enhance the annihilation luminosity by about three times than that of a thin disk.

The significance of the distribution of electron fraction in NDAFs has been discussed in many previous works (Kohri & Mineshige 2002; Lee et al. 2005; Liu et al. 2007; Kohri et al. 2005). In this present work, the distribution of electron fraction is a natural result of chemical equilibrium, thermal balance, and hydrostatic equilibrium, rather than an artificial assumption as in most previous works, so it should be the most reliable.

As shown in Figure 5, helium is completely absent within 100M of NDAFs, so it is not needed to consider the contribution of helium to the total pressure and internal energy or the contribution of helium disintegration to the cooling of the disk.

It should be emphasized that our calculation also has its own limitations. First, in this work, the disk is thick and has a two-dimensional structure, while the neutrino transport is treated in one dimension. Second, the effects of the motion of accretion flow and curved spacetime on the vertical transport of neutrinos are neglected. These effects are significant in the inner part of the disk; for example, at \( r = 10M \), the special relativity correction to the neutrino energy could be \( v/c \approx 30\% \), and the general relativity correction to the energy could be \( M/r \sim 10\% \).

Third, in the zone close to the event horizon of the central black hole where the annihilation is concentrated, the trajectories of neutrinos are severely bent by the central black hole and a considerable amount of neutrinos will undoubtedly be captured by the hole. In addition, gravitational instability of the disk was proposed by Perna et al. (2006) to explain the energetic X-ray flares after prompt emission in GRBs. The outer region of NDAFs with extremely high accretion rate is gravitationally unstable (Chen & Beloborodov 2007), will fragment, and cause a variable accretion rate in the inner region.

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