Analysis of Hardware Impairments on the Energy Harvesting Hybrid Relay Networks

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Abstract. In real communication systems, hardware impairments are often considered. When the system is suffering hardware impairments, the system performance will be worse. For this reason, in this paper, the impact of hardware impairments on the system performance for the energy harvesting hybrid relay networks is investigated. Especially, the closed-form expression of the outage performance and the optimal analysis of the instantaneous throughput for the system are derived, from the analysis we know that the impairments level leads great loss on the system performance. In addition, numerical results are derived to verify the correctness of our analysis.

1. Introduction
Now, an efficient solution to prolong the utility time of the wireless node is energy harvesting technology. In former works, if we want to change the lifetime of the wireless node, we must change battery or eliminated the cost of hard-wiring [1]-[5].

The wireless information and power transfer (SWIPT) technology was first proposed in [1] and [2]. This technology was proposed due to the wireless node can perform two functions, i.e. the node can harvest energy and decode the message at the same time. Until now, we know that it is more and more practical to analyse the energy harvesting technology, many literatures have analysed this technology [3]-[5]. Energy harvesting is needed for hybrid communication network due to the limit the power of relay, and for some conditions, we could not change the battery of the relay, hence the hybrid relay networks is badly in need of energy harvesting technology.

In many former works, the authors just assumed the hardware in wireless communication node is perfect, which is not practical in relay system. Phase noise, I/Q imbalance and the other impairments factor will affect the hardware [6]-[8]. According to [9], a general hardware impairments (HIs) model was proposed to give an easy way to analyse HIs in real communication systems.

Hardware impairments is widely analysed in single-hop systems [10]-[12]. From the literatures, we know that once the network suffers (HIs), the network will have a fundamental capacity ceiling which is fixed value just the function of impairments level, in high rate systems this scene was seriously [13]. In [14], the authors analysed the impact of HIs on two-way relay networks. In [15], two selection schemes, namely, (opportunist relay selection and partial relay selection scheme) was used in the networks to gain better system performance. Specially, the expressions for the system performance were derived. Recently, the system performance for the energy harvesting relay network and cognitive networks with hardware impairments has been analysed in [16]-[17], respectively.
In [18], a new selection scheme was used in the multi-relay networks in the presence of HIs and shadowed Rician (SR) channel. In [19], the impact of HIs on the satellite terrestrial networks was analysed, to its regret, the interference of the networks is ignored.

Only [19]-[20] researched effect of HIs on the hybrid relay networks, but the authors just analysed the HIs for regular hybrid communication system, neglecting the importance of energy harvesting on the system. The energy harvesting is a hot topic recently. Besides, taking the energy harvesting into consideration is very necessary for the hybrid networks. Hence, it is necessary for us to analyse the impact of HIs on energy harvesting hybrid relay networks. It is the first paper which analyse the effect of HIs on the energy harvesting hybrid relay networks. Secondly, we derive the expression of the outage performance of the system, as we know that the closed-form expression was first derived over hybrid networks. The paper suggests that both the fading channel parameters and HIs level affect the system performance. Finally, the optimal analysis for the instantaneous throughput is also derived, which give quick ways to estimate the impact of key parameters, i.e., the HIs level and the energy conversion efficiency on the throughput.

The structure of the rest paper is that, in Section II, the system model is introduced and the problem is formulated. In Section III, the signal-to-noise-and-distortion-rate (SNDR), the outage performance and the instantaneous throughput of the energy harvesting communication system was derived. In Section IV, the necessary simulation results are derived to verify the right of the theory results. In Section V, the conclusion is given.

2. The Illustration of the System and Channel Models
As illustrated in Figure 1, we analyse a dual-hop energy harvesting hybrid relay network with HIs. In the network, a single antenna source (S), a single antenna relay node (R) and a single antenna destination node (D) is considered. Each of them operates in the half-duplex mode, which is a scene suitable to the hybrid relay network. For the long distance, no direct link is assumed in this paper. $T$ is the total time in which the information is transmitted from S to D and $\alpha$ is the factor of the total time when relay was harvested energy from S where $0 \leq \alpha \leq 1$. In the left time, $(1 - \alpha)T/2$ is used for transmitting the information, due to R is a half-duplex relay, so $(1 - \alpha)T/2$ is used for S to R to transmit the information and the other half $(1 - \alpha)T/2$ is used for R to D to transmit the information.

In energy harvesting time, the signal received at R is expressed as

$$y_r = h_rx_r + n_r$$  \hspace{1cm} (1)
Where \( h_1 \) is the SR random variable (RV) for \( S \rightarrow R \) link, \( x \) the signal transmitted from \( S \) with power \( P \), \( n'_e \) the additive white Gaussian noise (AWGN), \( y'_r \) the signal received by \( R \) that comes from energy harvested section with the power of \( E_R \).

With the help of (1), in the harvesting period \( \alpha T \), the harvested energy, \( E_R \) is given by
\[
E_R = \mu \alpha TP |h_1|^2
\]  
(2)

Where \( 0 \leq \mu \leq 1 \) is the energy efficiency.

During an information time, in the first half \((1-\alpha)T/2\), \( S \) broads the signal to \( R \), the signal received at \( R \) is given by
\[
y_r = h_1 (x + \eta_a) + \eta_d + n_y
\]  
(3)

Where \( x \) is the sent signal from \( S \) which has the power of \( P \), \( \eta_a \) and \( \eta_d \) are the distortion noise at \( S, R \) with the power \( k_a^2 P, |h_1|^2 k_d^2 P \), respectively. \( k_a \) and \( k_d \) characterize the impairments level at \( S \) and \( R \) in the first half \((1-\alpha)T/2\). \( n_y \) is the AWGN at \( R \) with power \( n_y \sim \mathcal{CN}(0, \delta^2_y) \), \( y_r \) is the received signal by \( R \).

In the second half \((1-\alpha)T/2\), \( R \) forwards the signal received to \( D \), hence its signal received at \( D \) is expressed as
\[
y_d = h_2 (y_r + \eta_a) + \eta_d + n_d
\]  
(4)

Where \( h_2 \) is the Rayleigh RV for \( R \rightarrow D \) link, \( y_d \) is the signal received at \( D \), \( \eta_a \) and \( \eta_d \) are the distortion noise at \( R \) and \( D \) respectively, with power \( k_a^2 P, |h_2|^2 k_d^2 P \), \( k_r \) and \( k_d \) characterize the level of impairments at \( R \) and \( D \) in the second half \((1-\alpha)T/2\). \( n_d \) is the AWGN at \( D \) with power \( n_d \sim \mathcal{CN}(0, \delta^2_d) \), \( P_r \) is the power of \( y_r \).

Through our work, it is assumed that the whole harvested energy is used for transmitting its signal, hence \( P_r \) is given by
\[
P_r = \mu \alpha TP |h_1|^2 \left[ (1-\alpha) T / 2 \right]^{-1} = 2\varepsilon P |h_1|^2
\]  
(5)

Where \( \varepsilon = \mu \alpha (1-\alpha)^{-1} \), \( \mu \) is the energy conversion efficiency.

3. The Performance of The System

In this section, we derived the end-to-end signal-to-noise-and-distortion-rate (SNDR), analytical expressions of the outage performance for our network. What’s more, the optimal analysis of the instantaneous throughput is also got, in order to have a better performance, the optimal value \( \alpha \) is also given.

3.1 The End-to-End SNDR of the Considered Network

With help of (3), the SNDR of the first and second half \((1-\alpha)T/2\) are given by, respectively.
\[
y_1 = |h_1|^2 P \left[ \left( k_a^2 + k_d^2 \right) |h_1|^2 P + \delta^2 \right]^{-1}
\]  
(6)
\[
y_2 = |h_1|^2 P \left[ \left( k_r^2 + k_d^2 \right) |h_2|^2 P + \delta^2 \right]^{-1}
\]  
(7)

As AF protocol is used by \( R \), hence the SNDR of the system is expressed as
\[
y_e = |h_1|^2 |h_2|^2 \left[ \left| k_a^2 + k_d^2 \right| P \right] \left[ \left| k_r^2 + k_d^2 \right| P \right] \left( P^{(\delta_e^2 + k_a^2 \delta_e^2)} + \left| k_r^2 \right| P \left( \delta_r^2 + k_d^2 \delta_r^2 + \delta_e^2 \right) \right]^{-1}
\]  
(8)

Where \( k_a^2 = k_d^2 + k_r^2, k_r^2 = k_a^2 + k_d^2 \).

Substituting (5) into (8) and after some simplifications, \( y_e \) is given by
\[
y_e = 2\varepsilon \lambda_e \left( 2\varepsilon \lambda_e \lambda_a A + \lambda_e B + 2\varepsilon \lambda_e \lambda_d C + 1 \right)^{-1}
\]  
(9)
Where \( A = k_1^2 + k_2^2 + k_1^2 k_2^2 \), \( B = 1 + k_1^2 \), \( C = 1 + k_2^2 \). In (9), we assume that \( \delta_1^2 = \delta_2^2 = 1 \), \( \lambda_1 = P|h_1|^2 \delta_1^{-2} \) and \( \lambda_2 = P|h_2|^2 \delta_2^{-2} \).

### 3.2 The Outage Performance

Before deriving the outage performance, we first discuss the channel fading for the S \( \rightarrow \) R and R \( \rightarrow \) D links. The channel model between S and R is formed as SR channel, for R both receive the signal of LOS and the signal from multi-path, the channel model between R and D is formed as Rayleigh fading which we just consider the multi-path effect.

From [21], the probability distribution function (PDF) for \( \lambda_1 \) is given by

\[
f_λ(λ_1) = α_n \sum_{k=1}^{∞} (1-m_1)_k (−δ_1)^k (k_1)^k (λ_1)^k e^{-λ_1} \tag{10}\]

Where \( Δ_1 = (β_1 - δ_1)\lambda_1^{-1} \), \( \lambda_1 \) is the average signal-to-noise-ratio (SNR), \( α_n = \frac{2θ}{2θm + (Ω_r)^m} (2θ)^{-1} \), \( β_1 = 0.5 \times λ_1^{-1} \), \( δ_1 = Ω_1 \left[ 2θ (2θm + Ω_r) \right]^{-1} \), \( Ω_r \), \( 2θ \) and \( m \geq 0 \) correspond to the average power of the LOS component, the average power of the multi-path component and the fading severity parameter ranging from 0 to \( ∞ \), respectively, \( (.)^k \) is the Poisson symbol.

From above, we know that, the R \( \rightarrow \) D channel fading we considered is Rayleigh fading, so the PDF of \( \lambda_2 \) is expressed as

\[
f_λ(λ_2) = \lambda_2^{-\frac{1}{2}} \tag{11}\]

Where \( \lambda_2 \) is the average SNR of the R \( \rightarrow \) D link.

With the help of (9) and of (9), the outage probability is expressed as

\[
P_{out}(x_0) = \left[ \frac{C_0}{θ(x_0)^{θ}-α} \right] \cdot \left( 2λ_2 (λ_2^2 - eλ_2^2 Ax_0 - eλ_2 Cx_0) \leq λ_2 Bx_0 + x_0 \right) f_λ(λ_1) dλ_1 \tag{12}\]

Substituting (11) and (12) into (13), by using [22] and after some measures, (12) is rewritten as

\[
P_{out}(x_0) = 1 - e^{-C_0[(λ_2 - x_0)]} + \sum_{k=1}^{∞} (1-m_1)_k (−δ_1)^k (k_1)^k (λ_1)^k e^{-λ_1} \tag{13}\]

But try our best, (13) can not be solved to get a closed-from expression. To solve this hard question, we get the asymptotic mean of (13). From (9), we know that

\[
2αλ_2^2 λ_2 \left( 2αλ_2^2 λ_2 A + 2λ_2 A + 2αλ_2^2 C + B \right)^{-1} \leq 2αλ_2 λ_2 \left( 2αλ_2^2 λ_2 A + 2αλ_2 C + B \right)^{-1} \tag{14}\]

With the help of (12), (13) and (14), after doing necessary simplifications, the asymptotic expression of the outage performance is expressed (15), where \( K_v(.) \) represents the \( v \)th-order modified Bessel function of the second kind.

\[
P_{out}(x_0) = 1 - e^{-C_0[λ_2^2 - x_0]} + \frac{1}{α} \sum_{k=1}^{∞} (\frac{k}{x_0})(1-m_1)_k (−δ_1)^k (k_1)^k (λ_1)^k \cdot C_0 [λ_2^2 - x_0]^{-1} K_v \left[ \frac{Δ_2 Bx_0}{λ_2^2 [2e(2e - 2e)]} \right] \tag{15}\]

### 3.3 The Instantaneous Throughput Performance

We now analyse the instantaneous throughput, and try to get the optimal \( α \) for the system. With help of [5], the instantaneous throughput is given by
\[ R(\alpha) = (1-\alpha)2^{-1} \log \left( 1 + 2\alpha \lambda^2_1 \lambda_2 \left( 2\alpha \lambda^2_1 \lambda_2 A + \lambda_1 B + 2\alpha \lambda_1 \lambda_2 C + 1 \right) \right) \]  

(16)

The optimal \( \alpha \) can be derived for solving this problem.

\[ \alpha^* = \arg \max_{\alpha} R(\alpha) \]

\[ s.t. 0 \leq \alpha \leq 1 \]  

(17)

Given the fact that \( R(\alpha) \) is a concave function of \( \alpha \), the optimal value \( \alpha^* \) can be derived through the answer of this question \( \frac{dR(\alpha)}{d\alpha} = 0 \). However, a closed-from answer of this problem is not available.

In order to solve this problem, a numerically evaluated value \( \alpha^* \) is given.

Substituting \( \varepsilon = \frac{2\alpha}{1-\alpha} \) into (16), (16) can be repressed as

\[ R(\alpha) = 2^{-1}(1-\alpha)\log \left( 1 + (c_1\alpha + c_2) \right) \]  

(18)

Where \( c_1 = 2\mu_2 \lambda_2^2 \), \( c_2 = 2\mu_2 \lambda_2^2 A - \lambda_1 B + 2\mu_1 \lambda_2 C - 1 \) and \( c_3 = \lambda_1 B + 1 \).

Substituting (18) into \( \frac{dR(\alpha)}{d\alpha} = 0 \), we have the following expression

\[ \frac{dR(\alpha)}{d\alpha} = -\frac{1}{2} \log \left( 1 + (c_1\alpha + c_2) \right) + c_1(1-\alpha) \frac{2}{(c_1 + c_3)\alpha + c_1} = 0 \]  

(19)

The answer for (19) is the optimal value, but trying out best we can not solve this equation, hence in the numerical results we will give the numerical solution.

4. Numerical Results

In this section, some necessary simulations are given to show the right of the theory results. The simulation tool we used is MATLAB. In the following, we set the parameters \( \mu = 1 \) for the energy conversion efficiency.

- case1: \( k_1 = k_2 = 0 \);
- case2: \( k_1 = k_2 = 0.2 \);
- case3: \( k_1 = k_2 = 0.3 \);

![Figure 2](image1.png)

**Figure 2.** Outage probability of the system with ideal hardware and hardware impairments.

![Figure 3](image2.png)

**Figure 3.** Outage probability of the system versus different \( x_0 \).

In Figure 2, \( \alpha = 1/3 \) is assumed. From Figure 2, we know that the outage performance will be significant reduced due to the self-interference on the network when the system has HIIs. Moreover, it also shows that the system has a worse performance when the channel is under heavy fading.

From Figure 3, we know that the system has an outage threshold when the system has HIIs. The larger \( k_1, k_2 \) is, the lower outage threshold is. But if the system is under perfect hardware, there is no outage threshold, the outage probability will be 1 as the outage threshold increase to infinite. Figure 3
also depicts that, when $\alpha$ grows larger, the outage probability will become lower, which is easy to know, when $\alpha$ becomes larger, the power of relay becomes larger simultaneously.

![Figure 4](image-url)

**Figure 4.** The instantaneous throughput versus different $\alpha$.

Figure 4 illustrates the instantaneous throughput versus different $\alpha$, from Figure 4, we know that $\alpha$ has an optimal value when the system has the largest instantaneous throughput value. It also depicts that the optimal $\alpha$ not only relates to $k_1, k_2$ but also relates to the channel fading. The heavy fading is, the lower $\alpha$ is; the larger $k_1, k_2$ are, the lower $\alpha$ is.

5. Conclusions

Summarized from our work, we studied the performance of the energy harvesting hybrid relay network with HIs. At first, we got the SNDR of the provided network. Secondly, the analytical expressions of the system performance which gave fast ways to estimate the effect of the HIs on the provided network. It is also found that the impairments level had great effects on the system performance. Moreover, the optimal analysis of the instantaneous throughput gave us the way to choose an optimal value $\alpha$. We found it that the optimal value not only had the relationship with the impairments level but also with the channel fading parameters, which guided us how to balance the impairments level and the channel fading.

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