A PROFUSION OF BLACK HOLES
FROM TWO TO TEN DIMENSIONS

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Abstract

Black holes in several dimensions and in several theories are studied and discussed. The theories are, general relativity, Kaluza-Klein, Brans-Dicke, Lovelock gravity and string theory.

1. Introduction

Black hole physics and black holes (BHs) have by now a long and interesting history since they were first predicted in 1939 by the prescient work of Oppenheimer and Snyder[1] following some hints left by Zwicky in 1934[2] that neutron stars, stars of very high densities and very small radii, could form as the end product of a supernova explosion.

It is not here the place to comment on the development of these ideas, but maybe, some would like to know that in the same year, Einstein published a paper[3] arguing forcefully that the gravitational radius, what we now call the event horizon of a BH, could never be surpassed. Einstein was, in a sense, isolated in Princeton, while Oppenheimer was on the west coast, the other side of the country, commuting with his students between Berkeley and Caltech each six months. In Caltech he could share ideas with Tolman the great relativist, and Zwicky a master of prophesying correctly (although there is no direct sign of communication between Zwicky and Oppenheimer). With hindsight, it seems that Caltech was the right place to study gravitational collapse and predict BH formation.

It is also relevant to note that 150 years before, dark stars, the Newtonian BHs, were predicted by Michell[4] in Cambridge, an idea that Laplace followed 12 years later[5]. In modern terms Michell’s idea can be put in the

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form: give a mass $M$ of an astronomical object; find its radius so that the escape velocity is the velocity of light $c$. The answer is $R = \frac{2GM}{c^2}$, where $G$ is the gravitational constant. Objects with radii below this value are dark stars. However, the argument is not strictly valid because $c$ does not have a fundamental meaning in Newton’s gravity. One could detect tachyonic particles emitted from the surface of the star, or an observer not placed at infinity, in the neighborhood of the star, say, could still see the light coming from the star. However entertaining was the dark star idea, it was dropped down for one or other reason until 1939, where it appeared in the right context, the theory of general relativity. Curiously enough, a good condition for the formation of a BH is that the radius of the star obeys Michell condition $R = \frac{2GM}{c^2}$, although now $M$ and $R$ have the corresponding relativistic meanings and $G$ and $c$ are both fundamental constants.

So, what is the picture of a star collapsing into a BH? One can best see it through a spacetime diagram. As the star collapses there is a last ray emerging from the center that can reach spatial infinity. This is the event horizon, signaling the existence of a BH, see figures 1 and 2.

![Eddington-Finkelstein diagram for the collapse of a star](image)

Figure 1. Eddington-Finkelstein diagram for the collapse of a star, (eh = event horizon). A double line in all figures represents a polynomial singularity, where curvatures and densities of infinite strength are formed.
When the BH forms there are two distinct but connected regions, the inside and the outside of the event horizon, explicitly showing that time in relativity is observer dependent. As the matter of the star continues to collapse inside the event horizon it will form a singularity where curvatures and densities of infinite strength are formed and the usual concept of spacetime is lost. Inside the event horizon light is trapped. Light not only does not escape to infinity, it cannot escape to the outside of the BH. However, to an outside observer the story is different. As the BH is being formed, the luminosity of the original star decays exponentially, \( L = L_0 e^{-\frac{t}{\tau}} \) where the characteristic time is very short, \( \tau = 3\sqrt{3}\frac{GM}{c^3} = 2.6\times10^{-5}\frac{M}{M_\odot}\) s, i.e., in a few millionths of a second the star turns totally black (\( M_\odot = \text{solar mass} \)). In addition, to an outside observer the collapse of the star results in a BH whose properties are characterized by three parameters only: mass, charge and angular momentum. One then says that BHs have no hair (in fact, they have three hairs). All the other properties, or ‘hairs’, of the matter of the star that formed the BH disappear. No observation can reveal the nature of the original star, whether it possessed anti-matter, or was made of fermions, or bosons, or whether it had any other hairs.

This picture is drastically altered if the collapse produces a singularity first, not dressed by an event horizon. BHs are well studied and their existence is highly plausible. Naked singularities do not enjoy the same status. They are a threat to the predicability power of general relativity, and for this
reason a cosmic censorship conjecture forbidding the existence of such nasty objects was formulated [3]. There are many theoretical counter-examples to the cosmic censorship conjecture [7, 8], although it is still arguable that these examples cannot occur in nature, either because they may be physically unrealistic or possibly highly unstable. One drawback to the conjecture, often invoked, is that its validity implies the impossibility of observing quantum gravity phenomena, coming out right from the singularity.

BHs formed from the collapse of stars can have masses between $3 - 100 M_\odot$. There is also the possibility that supermassive stars or the core of star clusters collapse to form BHs with masses of the order of $1000 M_\odot$. BHs with much higher masses $10^6 - 10^9 M_\odot$ may form in the center of a galaxy via gravitational collapse of a mixture of clusters of stars and gas. Primordial BHs with masses ranging up to $10^{-19} M_\odot \sim 10^{14}$ g, and the radius of a proton $10^{-13}$ cm, could have been formed in the fluctuations of the early and very early universe.

For stellar size objects, the mass is a good indicator to separate BHs from neutron stars. If the compact object has a mass $M \gtrsim 3.5 M_\odot$ then there is no equation of state, however stiff, able to support the neutron star (a cold star with a radius of $\sim 10$Km) against complete collapse. There are strong candidates in the sky to stellar BHs, the most famous of all is Cygnus X1, a binary system emitting X-rays and harboring a dark compact object with $\sim 16 M_\odot$ (see e.g. [2] for a review). There are no candidates for BHs with $\sim 1000 M_\odot$ (even the existence of supermassive stars is pure theoretical speculation). Galactic BHs should inhabit the center of active galactic nuclei, compact sources which can shine more than an entire galaxy. In some cases like quasars, the nuclei of the galaxy has a brightness equivalent to the brightness of several thousands of galaxies, in a region not bigger than the solar system. In two galaxies with active galactic nuclei the value of the central mass points to the existence of a BH: (i) in the elliptical galaxy M87 the Hubble Space Telescope measured a rotation velocity of $v \sim 550$Km/s for the gas at an orbital radius of 60 light years, which, through Kepler’s law gives $M = \frac{v^2 R}{2G} \sim 2 - 3 \times 10^9 M_\odot$; (ii) for the spiral galaxy NGC 4258 Keplerian velocities of $\sim 1000$Km/s in an inner orbit of very small radius, $R \sim 0.4$ly, have been measured through water masers which imply a central mass of $M \sim 2 \times 10^7 M_\odot$. This work is considered to provide the strongest case for a supermassive BH in the center confirming the predictions of Lynden-Bell [10], (see [3] for a review). All these methods are indirect, and to probe
directly the existence of a BH one should measure relativistic speeds of the matter circulating in the disk very near the event horizon. In addition, when the gravitational antennas are operating we should directly detect the formation of BHs either through collapse of a single star, or through the merging of binary systems. There is no observational evidence for the existence of primordial BHs.

A quantity that gives some insight to the physical processes occurring during the collapse is the average density of the collapsing matter \( \rho \) when the BH is forming, i.e., when \( R = \frac{2GM}{c^2} \), yielding \( \rho = \frac{3\pi^6}{32G^3} \left( \frac{1}{M} \right)^2 \approx 1.3 \times 10^{16} \left( \frac{M}{M_\odot} \right)^2 \text{gm/cm}^3 \). For a \( 1M_\odot \) BH this gives a density ten times larger than the nuclear density, whereas for a \( 10^8M_\odot \) BH it gives the density of water. This means the larger the mass the less uncertain is the physics at the BH formation. Even if BHs have not been produced in our cosmos, one could envisage an astronomical experiment, by assembling a very large mass in the form of dust and let it alone to collapse to form a BH. After the matter has passed its own gravitational radius, the singularity theorems \([11]\) plus theoretical models indicate that the density raises to infinity, \( \rho \to \infty \). Is it really infinity? In principle there are strong suggestions that there is a minimum scale, the Planck scale (constructed from \( G, c \) and Planck’s constant \( \hbar \)), below which the usual physical concepts break down. At the Planck scales, \( R_{pl} = \sqrt{\frac{G\hbar}{c^3}} \approx 10^{-33}\text{cm} \) and \( M_{pl} = \sqrt{\frac{\hbar}{G}} \approx 10^{-5}\text{gm} \), the density of the matter is \( \rho \approx \frac{M_{pl}}{R_{pl}} = 10^{92} \left( \frac{M}{M_{pl}} \right) \text{gm/cm}^3 \). At these scales it is expected that the topology of the spacetime breaks down in order to accomodate these large masses in such a small volume. It is interesting to note that the Planck density \( \rho_{pl} = \frac{c^5}{G^2\hbar^3} \approx 10^{92}\text{gm/cm}^3 \) is the density at which a Planck mass turns into a BH, as well as merging into the singular structure of the spacetime. General relativity provides an adequate description of BHs that are much bigger than the Planck mass. On the other hand for Planckian BHs a description in terms of general relativity breaks down and it has to be replaced by a quantum theory of gravity.

Even much before the quantum gravity regime starts to be important, the BH already presents a quantum mechanical behavior. Indeed following hints that a BH has an associated entropy and therefore, through the relation \( S = Q/T \), an associated temperature, Hawking \([12]\) using quantum field theory on a BH background found that BHs are not black but radiate with a blackbody spectrum at a temperature \( T = \frac{\hbar c^3}{8\pi Gk_B M} \approx 6 \times 10^{-8} \left( \frac{M}{M_\odot} \right)\text{K} \), and have an
associated entropy $S_{\text{BH}}$ given by $S_{\text{BH}} = \frac{k_B c^3 A}{4}$, where $A$ is the area of the BH and $k_B$ is the Boltzmann constant. Since so many fundamental constants enter these formulas one can say that quantum mechanics, general relativity and thermodynamics must merge together in a unified theory. For $M \sim 1M_\odot$ one has $T \sim 10^{-7}\text{K}$, whereas for a Planckian BH, $M \sim 10^{-5}\text{gm}$, $T \sim 10^{32}\text{K}$. An important unsolved problem raised by this thermal evaporation is the information paradox, which is the problem of knowing to where all the information contained inside the original star has gone after the BH has evaporated completely [13, 14].

Classically, BHs are stable objects, however quantum mechanically they are unstable. As the BH radiates energy its mass decreases, the temperature increases in a runaway process which probably ends in a final explosion. Suppose now that instead of neutral BHs one considers a charged non-rotating BH. Then, dropping the fundamental constants, $T = \frac{1}{2\pi} \frac{\sqrt{M^2 - Q^2}}{M + \sqrt{M^2 - Q^2}}$. If the charge is large enough, $|Q| = M$, then $T = 0$ and one could expect these objects to be stable. However, vacuum polarization effects will discharge the BH itself rapidly. There are two ways to stabilize the situation:

1. Take a topological charge so that there are no particles to radiate [15].

2. A charged BH will preferentially radiate away its charge, depending on the charge to mass ratio of the particles in the theory. If $\frac{Q}{m}$ is small most of the radiation will be in the form of neutral particles and $Q$ will remain constant. Take then that the lightest charged particles are heavy enough so that they cannot be created by the BH. This could be done in two instances.

   (a) For example, suppose that the BH carries magnetic charge instead of electric charge. The only way for the BH to loose this charge would be via the creation of monopoles. However, if the monopoles are heavy enough the probability of decay is heavily suppressed [16].

   (b) A variant of this scenario is to suppose that the charge arises as a central charge in a supersymmetric algebra. It is known that in $N = 2$ supergravity the bosonic sector is Einstein-Maxwell theory with a Bogomolnyi bound given by $Q \leq M$. One can then show that extreme Reissner-Nordstrom solutions $|Q| = M$ (which

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saturate the bound) are supersymmetric, in the sense that under a supersymmetric operation the metric remains invariant and the fermionic sector remains null \[7\]. These BHs have zero \( T \) and are stable.

Stable BHs can be considered as solitons of the theory and as such belong to the non-perturbative sector and should be put on the same foot as the elementary particles of the theory. To see more directly that the distinction between BHs and elementary particles can be blurred, suppose there is an elementary particle with a mass greater or equal to the Planck mass. Then its Compton wavelength is smaller or equal to its Schwarzschild radius. At these scales it is therefore hard to distinguish between what is an elementary particle from what is a BH. It is then natural to think of such particles as BHs and conversely BHs may be viewed as elementary particles \[18\]. It is expected that gravity must become the dominant field at the quantum Planck scale \( 10^{-33} \)cm, which as we have said represents the minimum scale at which spacetime can be considered smooth. BHs, viewed as elementary particles, are the objects to test this scale, through Hawking radiation. Imagine the following futuristic experiment: two incoming particles in a huge accelerator are set to collide face-on, such that, a center of mass energy of \( \sim 10^{19} \)Gev is produced. Then, one might form a Planckian BH which will evaporate quickly in a burst, allowing us to study the physics at the Planck scale. One might think that by increasing the energy the study of sub-Planckian scales would follow. However, this is not the case, since one would produce a BH with larger mass, which would decay slowly.

From all this one can see that quantum gravity plays an essential role in every theory of extremely strong gravitational fields such as BHs and singularities. One could think of reconciling general relativity with quantum mechanics, but it is known that general relativity is perturbatively unrenormalizable which is taken at face value by many people as an indication that the quantum theory does not exist. At present, the best candidate to a consistent theory of quantum gravity is string theory, a theory remarkable in some respects. The idea of string theory is to use strings as fundamental entities and treat its vibrations as manifestations of the physical world, as fields, particles, etc. The string action plus some rules (like preservation of conformal invariance at the quantum level) place strong restrictions on the possible theories and on the spacetime itself. For instance, string theories
treat the dimension of spacetime as a parameter to be settled by the theory. For the pure bosonic string theory (inconsistent at the quantum level), the dimension is $D = 26$, while $D = 10$ for the four consistent supersymmetric string theories which seem to belong to a $D = 11$ M–theory \cite{19, 20} or even a $D = 12$ F–theory \cite{21}. Although apparently incorrect, these dimensions can, in principle, be dynamically compactified into the $D = 4$ dimensions actually observed in our universe. Superstring theories can also be formulated in any dimension $D \leq 10$, with the left $10 - D$ dimensions treated as being compactified somehow \cite{22}. A remarkable feature of the theory is the presence of a bewildering variety of BH solutions in any dimension from 2 to 10. The study of BH solutions in $D \geq 4$ dimensions is not new \cite{23}, although string theory has made an important impact in their development in higher as well as lower (2 and 3) dimensions. Besides string theory, BHs in different dimensions also appear in theories like general relativity, Kaluza-Klein theory, Brans-Dicke theory, Lovelock gravity and in their corresponding supersymmetric versions. In the subsequent sections we will discuss some of these solutions and some of their properties. Some discussion of sections 2 and 5 is patterned along the lines of \cite{24}, and part of section 3 follows \cite{25}.

2. BHs in 4D

Let us start with general relativity in 4D, i.e., Einstein-Maxwell theory, characterized by the action

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - F^2),$$

where $g$ and $R$ are the determinant of the metric and the curvature scalar, respectively, and $F^2 = F_{\mu\nu}F^{\mu\nu}$, where $F_{\mu\nu}$ is the Maxwell tensor ($c = 1$). Uncharged static BHs are described by the Schwarzschild solution

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega_2^2,$$

where $d\Omega_2^2$ is the line element of the 2-sphere, $M$ is the mass of the BH, and we have put $G = 1$. The causal structure is conveniently described by the Penrose diagram of figure 3, where light rays move at $\pm 45^\circ$ and each point in the diagram represents a 2-sphere. The event horizon is located at $r = 2M$ (where $g^{rr} = 0$).
A charged static BH in general relativity is described by the Reissner-Nordstrom solution,

$$ds^2 = -(1 - \frac{2M}{r} + \frac{Q^2}{r^2})dt^2 + \frac{dr^2}{1 - \frac{2M}{r} + \frac{Q^2}{r^2}} + r^2 d\Omega_2^2,$$

(3)

where $Q$ is the charge, $F_{rt} = \frac{Q}{r^2}$ for electric $Q$, and $F_{\theta\phi} = Q \sin \theta$ for magnetic $Q$. The causal structure is richer now. There are three distinct cases depending on the charge to mass ratio. For $0 < |Q| < M$ there are two horizons (the event and the Cauchy horizon) given by the roots of $g^{rr} = 0$, $r_{\pm}$. The Penrose diagram is sketched in figure 4. For an extreme BH, $|Q| = M$, the two horizons merge in one, and for $Q > M$ the singularity is timelike and naked.

The Hawking temperature of static BHs can be calculated in several ways. The original calculation involves the analysis of quantum matter fields in the BH background [26]. A cleaner calculation is achieved by analytically continuing the metric in time $t$ and requiring that the resulting Riemannian space be non-singular. This implies a periodic identification in imaginary time with the temperature being equal to the inverse of the period [27]. One can then show that this BH instanton is related to a real BH in thermal equilibrium with radiation. As mentioned, for the Reissner-Nordstrom BH

$$T = \frac{1}{2\pi} \frac{\sqrt{M^2 - Q^2}}{(M + \sqrt{M^2 - Q^2})^2},$$

which for $Q = 0$ yields the familiar $T = \frac{1}{8\pi M}$. 

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Near singularities general relativity should be replaced by a quantum theory. String theory is a consistent theory that may give some clues at the Planckian scales. This raises the question of whether BHs in string theory are different from BHs in general relativity. We will see that these two theories give distinct BHs. Due to the existence of dilaton, axion and other fields in string theory there are even BHs without singularities. There are also solutions describing one-, two-, and p-dimensional objects surrounded by event horizons, i.e., black strings, black membranes and black p-branes. We will also show in the next section that general relativity also possesses these type of objects, a feature not known until recently [28, 29].

Without further details for the time being, let us consider the low energy action to heterotic string theory [22, 24]

\[
S = \frac{1}{4\pi} \int d^D x \sqrt{-g} e^{-2\phi} \left[ R - 2\Lambda + 4(\nabla \phi)^2 - F^2 - \frac{1}{12} H^2 \right],
\]

(4)

where the new fields are the dilaton scalar field \( \phi \), and the 3-form field \( H_{\mu\nu\rho} \), such that \( H^2 = H_{\mu\nu\rho} H^{\mu\nu\rho} \) and defined by \( H = dB - A \times F \) where \( B_{\mu\nu} \) is
the axion 2-form potential and $A_\mu$ is the vector potential that defines the $U(1)$ Maxwell field, $F = dA$. These fields arise naturally in string theory. The cosmological constant $\Lambda$ is set by the internal consistency of the theory and related to the dimension $D$ of the spacetime and the central charge of a possible internal conformal field theory. The constant factor $\frac{1}{4\pi}$ in front of the integral in the action (4) is somewhat arbitrary. This arbitrariness will remain throughout this article, although without loss of precision, since we are dealing mostly with classical results.

To have a full theory and not only the low energy action (4) one would have to add higher order correction terms $R^2, R^3, F^4$, etc. All the higher order terms are important for studying BHs of Planckian size and the spacetime singularities. However, using (4) one can study the properties of larger BHs away from the singularities. For $D = 4$ and in a background where $\Lambda = 0 = H$ the action simplifies to

$$S = \frac{1}{4\pi} \int d^4x \sqrt{-g} e^{-2\phi} \left[ R + 4(\nabla \phi)^2 - F^2 \right]. \quad (5)$$

Note that $\phi$ plays the role of a coupling constant, since comparing (1) and (5) roughly one has $G \sim e^{2\phi} \equiv g_s$, where $g_s$ is the string coupling constant. In order to directly compare with the Einstein-Maxwell action one rescales the string metric $g_{\mu\nu}$ (which is the metric seen by the strings) to the Einstein metric $g_E^{\mu\nu} \equiv e^{-2\phi} g_{\mu\nu}$ (the metric that puts the string action in an Einstein form) to have the action,

$$S_E = \frac{1}{4\pi} \int d^4x \sqrt{-g_E} \left[ R_E + 4(\nabla \phi)^2 - e^{-2\phi} F^2 \right]. \quad (6)$$

For $F = 0$, i.e., uncharged solutions, one deduces from (5) and the no-hair theorems [30] that uncharged BHs in the low energy string action are the same as the Schwarzschild BH of general relativity. On the other hand, for $F \neq 0$ and $\phi \neq 0$ the charged BHs in string theory are different from the Reissner-Nordstrom BHs. This could give a low energy test of string theory: if string theory is the correct one then charged BHs are not described by the Reissner-Nordstrom metric but instead by the solution [31, 32, 33]

$$\begin{align*}
  ds^2 &= -(1 - \frac{2m}{\bar{r}})(1 + \frac{2m \sinh^2 \alpha}{\bar{r}})dt^2 + \frac{\bar{r}^2}{1 - \frac{2m}{\bar{r}}} + \bar{r}^2 d\Omega_2^2 \\
  e^{-2\phi} &= 1 + \frac{2m \sin^2 \alpha}{\bar{r}} A_t = -\frac{m \sinh 2\alpha}{\sqrt{2(\bar{r} + 2m \sinh \alpha)}}, \quad (7)
\end{align*}$$
where the mass and charge are given by \( M = m \cosh^2 \alpha, \quad Q = \sqrt{2} m \sinh 2\alpha \).

For \( r = 2m \) there is an event horizon whereas for \( r = 0 \) there is a singularity. At the singularity \( g_s = e^{2\phi} \to 0 \) which might mean that in the full string theory, the string coupling remains negligible and quantum effects are suppressed. To compare with general relativity we then do the conformal rescaling mentioned above \( (ds_E^2 = e^{-2\phi} ds^2) \) and obtain

\[
ds_E^2 = -(1 - \frac{2M}{r})dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r(r - \frac{Q^2}{M})d\Omega^2 \nonumber,
\]

\[
e^{2\phi} = 1 - \frac{Q^2}{Mr}, \quad F_{rt} = \frac{Q}{r^2}, \quad \nonumber
\]

where for convenience we have defined \( r = r + \frac{Q^2}{M} \). The charged string metric is identical to Schwarzschild in the \( r - t \) plane (same Penrose diagram as in figure 1), however the spheres have smaller radii. There is the extremal limit \( |Q| = M \) given by the diagram of figure 5. For \( |Q| > M \) the singularity is naked. The string metric has the same corresponding Penrose diagrams since these diagrams are unaltered by conformal transformations.

Figure 5. Penrose diagram for the charged extreme BH in string theory. The singularity is null, or in other words, the event horizon is singular.

What about magnetic BHs? We have seen that in general relativity, electric and magnetic BHs have the same metric, i.e., neutral particles do not distinguish the two types of BHs. In string theory one can find magnetic BHs by performing an S-duality (or strong-weak) transformation, which transforms weak coupling into strong coupling and vice-versa. The transformation is

\[
F \to \tilde{F}, \quad \phi \to -\phi, \quad g_E \to g_E
\]

where \( \tilde{F} \) is the dual of \( F \), \( \tilde{F}_{\mu\nu} = \frac{1}{2} e^{-2\phi} \epsilon_{\mu\nu}^{\alpha\beta} F_{\alpha\beta} \), transforming electric into magnetic charge. Since the Einstein metric is unchanged the Penrose dia-
grams for magnetic BHs are identical to the Penrose diagrams for electric BHs. In terms of the string metric we have

\[ ds^2 = -\frac{1-\frac{2M}{r}}{1-\frac{Q^2}{r^2}}dt^2 + \frac{dr^2}{(1-\frac{2M}{r})(1-\frac{Q^2}{r^2})} + r^2d\Omega^2 \]

\[ e^{-2\phi} = 1 - \frac{Q^2}{M^2}, \quad F_{\theta\phi} = Q \sin \theta \]

The singularity happens at a finite area, when \( r = \frac{Q^2}{M} \). The extremal limit is given by \( Q^2 = 2M^2 \), for which the temperature is zero. On the other hand for the non-extreme BH given in equation (10), the temperature is \( T = \frac{1}{8\pi M} \), independent of the charge. This means that the BH radiates past beyond the extremal limit, indicating in turn that the semi-classical approximation for the calculation of the temperature breaks down.

We have only mentioned non-rotating BHs. In string theory, uncharged rotating BHs have the same metric as Kerr BHs. However the charged rotating BHs are different [34].

### 3. BHs in 3D

It is now known that 3D general relativity is important to study as it provides a bedtest for 4D and higher D theories [35, 36, 37]. Two features in 3D general relativity are relevant: (i) the theory has no Newtonian limit (it is still an open question which 3D theory has a Newtonian limit), (ii) there are no propagating degrees of freedom, which means that in vacuum, outside matter, spacetime is locally flat, anti-de Sitter or de Sitter depending on the value of the cosmological constant, \( \Lambda = 0, \Lambda < 0, \) and \( \Lambda > 0 \), respectively. Due to this simplicity and lack of structure it can be thought that there is no interesting object emerging from the theory. Surprisingly, from the action

\[ S = \frac{1}{2\pi} \int d^3x \sqrt{-g} (R - 2\Lambda) \tag{11} \]

and its equations of motion, Bañados, Teitelboim and Zanelli [38] found a 3D rotating BH metric known as the BTZ BH, given by

\[ ds^2 = -(\frac{r^2}{l^2} - M + \frac{J^2}{4r^2})dt^2 + \frac{dr^2}{\frac{r^2}{l^2} - M + \frac{J^2}{4r^2}} + r^2(d\phi - \frac{J}{2r^2}dt)^2, \tag{12} \]

where \( l^2 \equiv -\frac{1}{\Lambda} \), \( J \) is the angular momentum, and here \( G \equiv \frac{1}{8} \). For \( |J| < Ml \) there are two horizons \( r_\pm \) given by the zeros of \( g^{rr} \). There are also ergoregions
for $r_+ < r < r_{\text{erg}}$ where particles and observers are dragged along certain trajectories. In the extremal case, $|J| = Ml$, the two horizons merge. For $J = 0$ the BH is static. The rotating case resembles in many aspects the Kerr metric and the non-rotating case the Schwarzschild solution, although there are no polynomial singularities, only (milder) causal singularities. The maximal analytical extension of the static and rotating BHs are given in the Penrose diagrams of figures 6 and 7.

![Penrose diagram](image)

Figure 6. Penrose diagram for the 3D static BH. The line $r = 0$ in this figure and in figure 7 is a milder causal (not polynomial) singularity. Spacetime is asymptotically anti-de Sitter.

Besides the BH solution, 3D general relativity with $\Lambda < 0$ also has the anti-de Sitter (ADS) spacetime as a vacuum solution with metric given by

$$ds^2 = -(\frac{r^2}{l^2} + 1)dt^2 + \frac{dr^2}{\frac{r^2}{l^2} + 1} + r^2 d\phi^2.$$ (13)

We note that for $r \to \infty$ the BH solution (12) is asymptotically ADS. Asymptotically ADS solutions and ADS spacetime itself are interesting to study for various reasons: (i) theories of extended supergravity in which some group, like $O(N)$, is gauged have ADS as a vacuum state, and (ii) there exists a positive energy theorem, i.e., it is possible to give Witten’s proof of the positive mass theorem of Schoen and Yau to asymptotically ADS spacetimes, implying in turn that asymptotically ADS solutions are stable.

Now, in 3D there is the relation $\tilde{R}^{ab}_{\ cd} = \epsilon^{abc} \epsilon_{cdf} G^e_f$. Therefore, a solution of $G_{ab} = 0$ is flat, and a solution of $G_{ab} = -\Lambda g_{ab}$ has constant curvature. Since the BH metric and the ADS solution have both constant curvature, one concludes that patches in the BH spacetime have an isometric neighborhood.
to the ADS spacetime and the BH spacetime can be defined by a collection of such neighborhoods. Indeed, it was shown in [39] that the BH can be represented as a quotient space of the universal covering of ADS, \( \tilde{\text{ADS}} \), by some group of isometries, which provides a powerful mathematical tool in examining the BH spacetime.

\begin{center}
\textbf{Figure 7. Penrose diagram for the 3D rotating BH.}
\end{center}

3D ADS spacetime can be obtained from the plane \( R^4 \) with two time and two space coordinates \((X_1, X_2, T_1, T_2)\) (we follow [23] here). The ADS metric is then the induced metric taken from the 4D flat metric,

\[
ds^2 = -dT_1^2 -dT_2^2 + dX_1^2 + dX_2^2,
\]

restricted to the hyperboloid

\[
X_1^2 - T_1^2 + X_2^2 - T_2^2 = -l^2.
\]

From (14) and (15) the isometry group is \( SO(2, 2) \), of course. One can go further and combine \((X_1, X_2, T_1, T_2)\) in a \( 2 \times 2 \) matrix,

\[
X = \begin{pmatrix}
T_1 + X_1 & T_2 + X_2 \\
-T_2 + X_2 & T_1 - X_1
\end{pmatrix}
\]

(16)
with \( \det X = 1 \) and \( X \in SL(2, R) \). Here, the isometries can be represented as elements of the group \( SL(2, R) \times SL(2, R)/Z_2 \approx SO(2, 2) \), with each \( SL(2, R) \) acting by left and right multiplication, such that \( X' = \rho_L X \rho_R \), with \( (\rho_L, \rho_R) \sim (-\rho_L, \rho_R) \).

Now, given \( ADS \) spacetime one may cover it using three different regions parametrized by \((r, t, \varphi)\) with \(0 \leq r < \infty\), \(-\infty < t < \infty\), and \(-\infty < \varphi < \infty\). For instance, in the region \( r \geq r_+ \) we have \( X_1 = l \sqrt{\alpha(r)} \sinh(\frac{r}{l} \varphi - \frac{t}{l} t)\), \( T_1 = l \sqrt{\alpha(r)} \cosh(\frac{r}{l} \varphi - \frac{t}{l} t)\), \( X_2 = l \sqrt{\alpha(r)} - 1 \cosh(\frac{r}{l} t - \frac{t}{l} \varphi)\), and \( T_2 = l \sqrt{\alpha(r)} - 1 \sinh(\frac{r}{l} t - \frac{t}{l} \varphi)\), where, \( \alpha(r) = \frac{r^2 - l^2}{r^2 - l^2} \). This corresponds to give region I of the Penrose diagram in figure 7. Analogous transformations can be given to the regions \( r_- < r < r_+ \) and \( 0 < r < r_-\), i.e., to regions II and III of the figure 7. By repeating these regions ad infinitum one covers the entire \( ADS \) spacetime. One can pick up \( X_1, T_1, X_2, T_2 \) from these transformations, put back in the induced metric (14)-(15), and recover the form of the BH metric (12). However, note that this is not the BH spacetime since \( \varphi \) ranges from \(-\infty\) to \(+\infty\). To make \( \varphi \) an angular variable one has to indentify \( \varphi \) with \( \varphi + 2\pi \). In this construction it is easy to see that such an identification is an isometry of \( ADS \), in fact it is a boost in the \( X_1 - T_1 \) and \( X_2 - T_2 \) planes. Indeed, it leads to, \( X_1 \rightarrow X_1' = (\cosh \frac{2\pi r_+}{l}) X_1 + (\sinh \frac{2\pi r_+}{l}) T_1\), \( T_1 \rightarrow T_1' = (\sinh \frac{2\pi r_+}{l}) X_1 + (\cosh \frac{2\pi r_+}{l}) T_1\), and analogously for \( X_2 \) and \( T_2 \). This corresponds in the \( SL(2, R) \) formulation to an element \((\rho_L, \rho_R)\) given by \( \rho_L = \text{diag} \left( e^{\pi i \frac{r_+ - r}{l}}, e^{-\pi i \frac{r_+ - r}{l}} \right)\), \( \rho_R = \text{diag} \left( e^{\pi i \frac{r_+ - r}{l}}, e^{-\pi i \frac{r_+ - r}{l}} \right)\). The BTZ BH may then be viewed as a group manifold given by the quotient space \( ADS/P \), where \( P \) denotes the group generated by \((\rho_L, \rho_R)\).

This formulation has great advantages: the \( ADS \) spacetime is an extremely simple manifold and if one makes appropriate global identifications one finds a 3D BH which has inherit its own complex structure. The implications are many: (i) one can compute the Green functions in the \( ADS \) spacetime and then make a direct connection to the BH; (ii) one can find Killing spinors fairly easily, which provides an identification of the existence of supersymmetry; if the BH is embeded in a supergravity theory with vanishing gravitino field, then the existence of Killing spinors leave the metric and gravitinos invariant. It was found that Killing spinors exist for extreme BHs only \[44]\; (iii) the temperature of the BH is \( T = \frac{r^2 - r^2}{2\pi r_+ l^2} \), which for zero
rotation yields, \( T = \frac{\sqrt{M}}{2\pi} \) and an entropy \( S = 4\pi r^2 \). Unfortunately, this does not help in solving the long standing problem in 4D, to know whether or not the BH evaporates completely, since in 3D \( T \to 0 \) as \( M \to 0 \); (iv) on the other hand, one can show that the 3D BH forms from gravitational collapse of 3D matter, as in the 4D case \( [1] \); (v) 4D gravity can be written in a first order formalism as a Chern-Simons theory. Viewing the BH as an ADS space with proper identifications helps in the study of the holonomies (see \( [25] \) for a complete list of references).

Another important result, is that the 3D BH we have been discussing is also a solution of 3D string theory \( [12, 13] \). Using the action \( (4) \) with \( D = 3, \phi = 0 \) and \( H_{\mu\nu\rho} = \frac{2}{3}\epsilon_{\mu\nu\rho} \) one obtains the same 3D BH. This displays the versatility of string theory. One can also find a black string solution by applying a duality transformation. We have already seen the S-duality at work. There is another well known symmetry of string theory that maps any solution with a translational symmetry of the low-energy action into another solution. This symmetry is usually called T-duality or target-duality. Given a target-space solution \((g_{\mu\nu}, B_{\mu\nu}, \phi)\) which is independent of one coordinate, like \( \varphi \) in the BH solution, then there is another solution \((\tilde{g}_{\mu\nu}, \tilde{B}_{\mu\nu}, \tilde{\phi})\) related to the previous one by a T-duality \( [24] \). The T-dual solution for the 3D BH is a black string.

What else can we do with the 3D BH? It can be embedded in 4D general relativity \( [14, 15] \). One takes the product of the BTZ BH with the real line \( R \), with metric \( ds^2 = ds^2_{\text{BTZ}} + dz^2 \), and imposes that it satisfies the 4D Einstein equations derived from the action \( S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ (R - 2\Lambda) + L_{\text{matter}} \right] \). By suitably choosing the energy-momentum tensor \( T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta L_{\text{matter}}}{\delta g_{\mu\nu}} \) one finds that the 3D BH can be converted into a black string in 4D general relativity. The idea is analogous to the well-known result that point particles in 3D are related to straight infinite strings in 4D.

There is yet a different solution which relates vacuum black strings in 4D general relativity with 3D BHs of a dilaton-gravity theory. Starting with the Einstein-Maxwell action \( S = \frac{1}{16\pi} \int d^4x \sqrt{-g} (R - 2\Lambda - F^2) \) one imposes the existence of a Killing vector such that the metric can be written in the form \( ds^2 = g_{(3)}^{ab} dx^a dx^b + e^{-4\phi} dz^2 \), where \( a, b = 1, 2, 3 \) and \( g_{ab} \), and \( \phi \) are functions of \( x^a \). Then by dimensional reduction one obtains a dilaton-gravity action, \( S = \frac{1}{16\pi} \int d^3x \sqrt{-g} e^{-2\phi} (R - 2\Lambda - F^2) \). It is then easy to relate 4D and 3D solutions. In 4D general relativity there is a black string solution, with charge
and rotation, given by [46]

\[ ds^2 = - \left( \frac{\alpha^2 r^2}{\alpha r} - \frac{4M(1 - \frac{a^2 \alpha^2}{2})}{\alpha^2 r^2} \right) dt^2 + \]
\[ \frac{4aM\sqrt{1 - \frac{a^2 \alpha^2}{2}}}{\alpha r} \left( 1 - \frac{Q^2}{M(1 - \frac{a^2 \alpha^2}{2})\alpha r} \right) 2dtd\phi + \]
\[ \left( \frac{\alpha^2 r^2}{\alpha r} - \frac{4M(1 - \frac{a^2 \alpha^2}{2})}{\alpha^2 r^2} + \frac{4Q^2(1 - \frac{a^2 \alpha^2}{2})}{(1 - \frac{a^2 \alpha^2}{2})} \right)^{-1} dr^2 + \]
\[ \left[ r^2 + \frac{4Ma^2}{\alpha r} \left( 1 - \frac{Q^2}{(1 - \frac{a^2 \alpha^2}{2})\alpha r} \right) \right] d\phi^2 + \alpha^2 r^2 dz^2, \] (17)

where here \( \alpha \equiv -\frac{1}{3}\Lambda \), \( M \) and \( Q \) are the mass and charge, respectively, and \( a \) is related to the angular momentum \( J \) via \( J = \frac{3}{2}aM\sqrt{1 - \frac{a^2 \alpha^2}{2}} \), with \( 0 \leq \alpha a \leq 1 \). This solution has many similarities with the Kerr-Newman BH. For instance, the causal structure for the non-extreme BH, i.e., \( 0 < a^2 \alpha^2 < \frac{2}{3} - \frac{12\alpha^2}{81M^4(1 - \frac{a^2 \alpha^2}{2})^3} \), is given by the Penrose diagram of figure 7, with \( r = 0 \) being now a polynomial singularity. However, unlike the Kerr-Newman BH, the topology of the horizon is cylindrical or toroidal, rather than spherical, violating Hawking’s theorem [47] due to the presence of a negative \( \Lambda \). It also has implications on the hoop conjecture [48]: gravitational collapse in such a background can generate a black string even if one is not able to pass a hoop of given circumference through the matter. If there is no charge then the causal structure changes drastically, resembling the Schwarzschild-ADS BH rather than the Kerr BH [28].

The 3D BH generated through dimensional reduction of 4D general relativity, has a dilaton in addition to the metric and Maxwell fields. A study to put these black solutions in a supersymmetric context is being carried [49]. Generalizations of the 3D action to a Brans-Dicke type of action, given by \( S = \frac{1}{2\pi} \int d^3x \sqrt{-g}e^{-2\phi}(R + 4\omega(\nabla\phi)^2 - 2\Lambda) \) also yield static and stationary BH solutions [50, 51, 52]. Using a metric with two Killing vectors, one can find black membranes in general relativity, related through dimensional reduction to 2D dilatonic BHs. This is a matter for the end of the next section.

4. BHs in 2D

To analyse BHs in 2D we first return to string theory. In 2D there is less freedom for dynamics, for obvious reasons. For instance, for a compact
orientable 2D manifold of genus $g$ (e.g., sphere $g = 0$, torus $g = 1$, etc), the Einstein-Hilbert action,
$$\frac{1}{2\pi} \int d^2x \sqrt{-g} R = 2(1-g),$$ is the Euler characteristic of space, a topological invariant with no dynamics. Therefore, if one wants to go further in 2D one has to find a different action. An interesting action is provided by string theory. For understanding the appearance of BHs in 2D string theory is now important to introduce some basic concepts of the theory itself. In string theory one has to distinguish the world-sheet action for the string from the target-space or spacetime action for the usual spacetime fields. The latter follows from the former upon imposing certain restrictions related to renormalization procedures. (In particle theory there is also such a distinction but the respective actions are not inter-related a priori.) The propagation of strings in a generic curved spacetime is described by the Polyakov action

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{h} h^{\alpha\beta} \nabla_\alpha x^\mu \nabla_\beta x^\nu g_{\mu\nu}(x),$$

(18)

where $h^{\alpha\beta}$ is the world-sheet metric of the string, $x^\mu$ are the spacetime (or target-space) coordinates, $g_{\mu\nu}$ is the metric of the background, and $\alpha'$ is the string coupling constant (see figure 8). Such an action is also called a nonlinear sigma model. It is invariant under reparametrizations of the string world-sheet $\sigma \rightarrow \sigma'$ and moreover, is conformal invariant (i.e., local scale invariant), $h_{\alpha\beta} \rightarrow \Omega^2 h_{\alpha\beta}$. In principle, one should also include in the action, besides the graviton, the other massless states or fields of the (closed) bosonic string, namely, the antisymmetric tensor $B_{\mu\nu}$ and the dilaton $\phi$ (see also for the inclusion of fermionic fields and supersymmetry). The bosonic world-sheet action or $\sigma-$model is then,

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{h} h^{\alpha\beta} \nabla_\alpha x^\mu \nabla_\beta x^\nu g_{\mu\nu}(x) - \frac{1}{4\pi\alpha'} \int d^2\sigma \epsilon^{\alpha\beta\gamma} \nabla_\alpha x^\mu \nabla_\beta x^\nu B_{\mu\nu}(x) + \frac{1}{4\pi} \int d^2\sigma \sqrt{h} R_s \phi(x),$$

(19)

where $R_s$ is the curvature of $h^{\alpha\beta}$. Imposing Weyl invariance at the 1-loop level to get rid of the ultraviolet divergences translates into the requirement that the so called beta-functions associated with the background fields vanish. The beta-function associated to the metric $g_{\mu\nu}$ is

$$\beta^g_{\mu\nu} = R_{\mu\nu} - \frac{1}{4} H_{\mu}^{\lambda\sigma} H_{\nu\lambda\sigma} + 2 \nabla_\mu \nabla_\nu \phi \nabla_\nu \phi$$

which should be set to zero. The 3-form $H$ is related to $B$ through $H_{\mu\nu\lambda} = \nabla_\mu B_{\nu\lambda} + \nabla_\nu B_{\lambda\mu} + \nabla_\lambda B_{\mu\nu}$. The other beta functions are

$$\beta^B_{\mu\nu} = \nabla_\lambda H_{\mu\nu\lambda} - 2 \left( \nabla_\lambda \phi H_{\mu\nu}^{\lambda} \right) = 0,$$

$$\beta^{\phi} = R + 2\Lambda + 4 \nabla^2 \phi - 4 \left( \nabla \phi \right)^2 - \frac{1}{12} H^2 = 0.$$
The constant $\Lambda$ is connected to the dimension of spacetime. For the bosonic string $\Lambda = \frac{D-26}{6\alpha'}$, whereas for the supersymmetric string with fermions $\Lambda \propto (D - 10)$. The dimensions $D = 26, 10$ are the critical dimensions for the bosonic and supersymmetric strings, respectively, because in these dimensions the theory is free from divergences and anomalies. However, one can go away from these dimensions to the more familiar 2, 3 or 4, by considering additional internal conformal field theories with central charges to complete, so to speak, the other extra dimensions.

Figure 8. Spacetime diagram showing the nomenclature for the propagation of strings.

The equations for the three $\beta-$functions are the field equations of first order string theory, which can be derived from a spacetime effective action given by

$$S_{\text{eff}} = \frac{1}{4\pi} \int d^D x \sqrt{-g} e^{-2\phi} \left( R - 2\Lambda + 4(\nabla \phi)^2 + \frac{1}{12} H^2 \right).$$

(20)

The Maxwell field $F_{\mu\nu}$ has been left out in this discussion (compare (20) and (4)), as well as other fields like the tachyon $T$ of the bosonic string, but they can be included consistently. Putting $D = 2$ and $H = 0$ in the equations of motion derived from (20) one finds a 2D BH solution in [53] given by

$$ds^2 = -(1 - e^{-2\lambda r})dt^2 + \frac{dr^2}{1 - e^{-2\lambda r}}, e^{-2\phi} = e^{-\lambda r},$$

(21)

where $\lambda^2 \equiv -\frac{\Lambda}{2}$. This solution has horizons at $r_+ = 0$ and a singularity at $r = -\infty$. The Penrose diagram is identical to the Schwarzschild diagram in
Since this is a solution of the low-energy action it is only valid as long as the curvature is small compared to the Planck curvature. Is there a way to find an exact solution of the full action, i.e., of the world-sheet action, without resorting to perturbation theory? Yes, and the idea was initiated in [54]. One starts with the Wess-Zumino-Novikov-Witten (WZNW) model described by the action

\[ S_{\text{WZNW}}[g] = \frac{k}{8\pi} \int d^2\sigma \sqrt{h} \varepsilon^{\alpha\beta} \text{tr} \left( \nabla_\alpha g^{-1} \nabla_\beta g \right) + ik \Gamma(g), \tag{22} \]

where \( g \) is an element of some group, function of a field \( x^\mu \), \( k \) is a real and positive number (called the level of the Kac-Moody algebra) and the last term is the Wess-Zumino term which guarantees conformal invariance of the action and for the purposes used here is of no importance. The motivation for this model comes from the need to simplify the background in order to find solutions. One good simplification is to assume string propagation in a group manifold of a Lie group \( G \) with elements \( g \). Note the analogy of (22) with the world-sheet action (18), where the trace has the role of a metric.

Now, if one supposes that \( g \in SL(2,\mathbb{R})/U(1) \) one can parametrize it by

\[ \left( \begin{array}{cc} a & u \\ -v & b \end{array} \right) \tag{23} \]

with \( ab + uv = 1 \). Since \( SL(2,\mathbb{R}) \) has dimension 3, and \( U(1) \) has dimension 1, the quotient space group manifold \( SL(2,\mathbb{R})/U(1) \) has dimension 2, which, in turn, can be parametrized by the coordinates \( u, v \). After imposing that the action (22) is gauge invariant and solving the equations of motion one finds [54]

\[ S_{\text{WZNW}}[g] = \frac{k}{4\pi} \int d^2\sigma \sqrt{h} \varepsilon^{\alpha\beta} \frac{\nabla_\alpha u \nabla_\beta v}{1 - uv}. \tag{24} \]

Comparing with the world-sheet action (18) one immediately finds that the target space metric is

\[ ds^2 = \frac{du dv}{1 - uv}, \tag{25} \]

which upon further coordinate transformation can be put in the form (21).

The dilaton can also be made to enter in this picture, see [54]. Since one has to solve the classical equations of motion this treatment is semiclassical. The full
treatment was attempted in [55] where it was found without approximations that the metric and dilaton are given by

\[
    ds^2 = 2(k - 1) \left[ -\left( \frac{x+1}{x-1} - \frac{2}{k} \right)^2 dt^2 + \frac{dx^2}{4(x^2-1)} \right] \\
    e^{-2\phi} = \frac{x^2-1}{\left( \frac{x+1}{x-1} - \frac{2}{k} \right)^2},
\]

(26)

where \( x \) is a new radial coordinate. In the semiclassical approximation, when \( k \to \infty \) one recovers Witten’s result. The causal structure is given in figure 9 [56], the novel feature being that in the exact solution of the full theory the BH has no singularities! This indicates that string theory has indeed new things to show at the singularities.

\[
    x = \frac{8}{x^2-1},
\]

Figure 9. Penrose diagram for the non-singular 2D BH in string theory.

Having this exact solution and using the tools of string theory, namely, conformal field theory, one can in principle know how strings propagate in the BH background, calculate the latest stages of the BH evaporation and solve the information paradox. However, in practice the problem is still out of reach [57]. Extensions to 4D of the idea of using a WZNW model to find exact solutions with associated conformal field theories have been tried with some interesting but limited progress [58].

We have just seen that the dilaton gives non-trivial dynamics to 2D. This has been known since the works of Teitelboim [59] and Jackiw [60] where the power of 2D theories was first understood. They proposed the theory

\[
    S = \frac{1}{2\pi} \int d^2 x \sqrt{-g} e^{-2\phi} (R - 2\Lambda),
\]

(27)
with $\Lambda < 0$. Although spacetime has constant and negative curvature it is possible to find a BH solution which is asymptotically ADS [61, 62, 63]. The thermodynamics of this BH has been studied (see this volume [64] and [65]).

In trying to find meaningful 2D actions one can look for connections with 4D general relativity, as it was done for 3D theories (see last section). Starting with the Einstein-Hilbert action $S = \frac{1}{16\pi} \int d^4x \sqrt{-g} (R - 2\Lambda)$ and imposing planar symmetry (two-killing vectors), with a metric given by $ds^2 = g_{ab} dx^a dx^b + e^{-2\phi} (dx^2 + dy^2)$, one finds upon dimensional reduction the following 2D action [29]

$$S = \frac{1}{2\pi} \int d^2x \sqrt{-g} e^{-2\phi} \left( R + 2(\nabla\phi)^2 - 2\Lambda \right).$$

This theory also possesses a BH which, when reinterpreted in 4D yields a black membrane in general relativity [29]. An obvious generalization of these three 2D theories is given by the Brans-Dicke action [66]

$$S = \frac{1}{2\pi} \int d^2x \sqrt{-g} e^{-2\phi} \left( R + 4\omega (\nabla\phi)^2 - 2\Lambda \right),$$

where $\omega$ is a free parameter, and $\omega = -1, -\frac{1}{2}, 0$ corresponding to string theory, planar general relativity and the Teitelboim-Jackiw theory, respectively. When $\omega \to \infty$ one obtains the 2D analogue of general relativity [57], also called the $R = T$ theory [68]. The BH in this case is a massless BH as has been shown in [56]. The BHs of action (28) for all rational $\omega$s have been analysed in detail in [53] and the quantum version in [59]. What about the temperature of these BHs? Usually the temperature goes with some power of the mass $M$, $T \propto M^\gamma$, where for instance for $\omega = 0$, $\gamma = \frac{1}{2}$ [34, 35]. Thus, these 2D theories cannot tell much about the latest stages of the BH evaporation. A notable exception is string theory ($\omega = -1$) for which $\gamma = 0$ and $T \propto$ constant, independent of the mass. Thus, following this result, the BH radiates indefinitely, which cannot be correct. In order to remedy the situation one has to make a full quantum treatment of the backreaction (see e.g. [70, 71]). For further extensions and interests on lower dimensional BHs see, e.g., [72].

5. BHs in higher D

We have been considering BHs in general relativity, Brans-Dicke and string theories in 4 and lower dimensions. However, higher dimensional
BHs are also important to study since they may shed some light on the understanding of non-perturbative effects in quantum gravity (such as the compactification scheme), as well as expose which of the features of the usual four-dimensional BH solutions remain in higher dimensions. Let us then go on to higher dimensions and consider, for a change, the original Kaluza-Klein theory in 5D. This is simply 5D general relativity in which the fifth dimension is a Killing direction, i.e., the fields are independent of the 5th dimension, \( x^5 \), say. The theory has two descriptions, the first given by the action

\[
S = \frac{1}{16\pi} \int d^5x \sqrt{-g} R ,
\]

and metric components \( g^{(5)}_{\mu\nu}, g^{(5)}_{\mu5} \) and \( g^{(5)}_{55} \), \( \mu, \nu = 0, 1, 2, 3 \). In the other description the action takes the form

\[
S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left( R - 2(\nabla\phi)^2 - e^{2\sqrt{3}\phi} F^2 \right) ,
\]

with the 5D metric related to the 4D fields by the usual Kaluza-Klein ansatz,

\[
g^{(5)}_{\mu\nu} = e^{\frac{2\phi}{\sqrt{3}}} \left( g^{(4)}_{\mu\nu} + e^{-2\sqrt{3}\phi} A_\mu A_\nu \right) ,
\]

\[
g^{(5)}_{\mu5} = e^{-\frac{4\phi}{\sqrt{3}}} A_\mu ,
\]

\[
g^{(5)}_{55} = e^{-\frac{4\phi}{\sqrt{3}}} .
\]

Due to this connection, one can generate with little effort static non-vacuum solutions from static vacuum solutions. Given a static vacuum 4D metric one can take its product with the real line \( \mathbb{R} \), 4D solution \( \times \mathbb{R} \), to obtain a 5D solution with two symmetry directions \( (t, x^5) \). If one boosts this 5D solution in the 5th direction it still satisfies the 5D equations. However, when reinterpreted in 4D one obtains a solution with non-zero Maxwell and dilaton fields. In other words, given a 4D metric \( g_{\mu\nu} \) one obtains a new solution \( (\tilde{g}_{\mu\nu}, \tilde{A}_\mu, \tilde{\phi}) \) given by the transformations,

\[
\tilde{g}_{tt} = \frac{g_{tt}}{(\cosh^2 \alpha + g_{tt} \sinh^2 \alpha)^{\frac{1}{2}}} ,
\]

\[
\tilde{g}_{ij} = g_{ij} (\cosh^2 \alpha + g_{tt} \sinh^2 \alpha)^{\frac{1}{2}} ,
\]

\[
\tilde{A}_t = \frac{1 + g_{tt} \sinh 2\alpha}{2(\cosh^2 \alpha + g_{tt} \sinh^2 \alpha)} ,
\]

\[
e^{-\frac{4\phi}{\sqrt{3}}} = \cosh^2 \alpha + g_{tt} \sinh^2 \alpha
\]

where \( \alpha \) is the boost parameter and \( i, j = 1, 2, 3 \). Example: given the Schwarzschild solution (2) one obtains after performing the above transformations, the following \[73, 74, 75\]

\[
ds^2 = -\frac{1-r_s}{\sqrt{1-\frac{r_s}{\tau}}} dt^2 + \frac{dr^2}{(1-\frac{r_s}{\tau})(1-\frac{r}{\tau})} + r^2 \left( 1 - \frac{r_s}{r} \right) d\Omega_2^2 ,
\]

24
\[ A_t = \sqrt{\frac{r_+ r_-}{r}}, \quad e^{-\frac{2\phi}{\sqrt{3}}} = 1 - \frac{r}{r}, \quad (33) \]

where we have redifined the Schwarzschild radial coordinate \((r_S, \text{say})\) in (2) to \(r_S = r \left(1 - \frac{r_-}{r}\right)\), and put \(r_- = 2m \sinh^2 \alpha, r_+ = 2m \cosh^2 \alpha, m\) being the Schwarzschild mass. The ADM mass and electric charge are \(M = m \sinh 2\alpha, Q = m \cosh 2\alpha\), respectively. There are horizons at \(r = r_\pm\) and the singularity is at \(r = 0\). Another type of transformation, called Harrison transformation [76], transforms metrics within general relativity, taking for instance, the Schwarzschild metric into the Reissner-Nordstrom metric. Now, in string theory there is the analogue of these boost transformed solutions. In a simple case, one starts with a static solution \((g_{\mu\nu}, \phi)\), with \(B_{\mu\nu} = 0\) and \(A_\mu = 0\). Then one gets a new solution \((\tilde{g}_{\mu\nu}, \tilde{A}_\mu, \tilde{\phi})\) by making the following transformations \([77] \)

\[
\begin{align*}
\tilde{g}_{tt} &= \frac{g_{tt}}{(\cosh^2 \alpha + g_{tt} \sinh^2 \alpha)^2}, \\
\tilde{A}_t &= \frac{1 + g_{tt} \sinh 2\alpha}{2\sqrt{2}(\cosh^2 \alpha + g_{tt} \sinh^2 \alpha)}, \\
e^{-2\tilde{\phi}} &= e^{-2\phi} \cosh^2 \alpha + g_{tt} \sinh^2 \alpha. \quad (34)
\end{align*}
\]

Recalling that the Schwarzschild solution (2) is a solution of string theory, one can apply (34) to obtain the electric charged BHs given in equation (7). But we are still discussing 4D BHs.

To obtain charged BHs in higher \(D\), one starts with a \(D\)-dimensional uncharged BH [23],

\[
ds^2 = -(1 - \frac{cm}{r^n}) dt^2 + \frac{dr^2}{1 - \frac{cm}{r^n}} + r^2 d\Omega_{n+1}^2, \quad (35)
\]

where \(n = D-3\) and \(c\) is a constant. This is a solution of both \(D\)-dimensional general relativity and string theory. By using the transforming equations (34) one can obtain the \(D\)-dimensional electrically charged BHs in string theory [32],

\[
ds^2 = -\left(1 - \frac{cm}{r^n}\right) \left(1 + \frac{cm \sinh^2 \alpha}{r^n}\right) dt^2 + \frac{dr^2}{1 - \frac{cm}{r^n}} + r^2 d\Omega_{n+1}^2, \\
A_t = -\frac{cm \sinh 2\alpha}{2\sqrt{2}(r^n + cm \sinh^2 \alpha)}, \\
e^{-2\phi} = 1 + \frac{cm}{r^n} \sinh^2 \alpha. \quad (36)
\]
The ADM mass and charge are given by $M = m (1 + \frac{2m}{n+1} \sinh^2 \alpha)$ and $Q = \frac{cmn \sinh 2\alpha}{\sqrt{2}}$. The event horizons are at $r = (cm)^{\frac{1}{n}}$, and the singularities at $r = 0$. In contrast with 4D we have that in the extremal limit the singularity is timelike rather than null, and the temperature of the extreme BH is zero. There are no higher $D$ magnetically charged BHs because there are no Maxwell magnetic charges (one cannot integrate a 2-form $F$ over a $D-2$ sphere). However, using a magnetic charge associated with the 3-form field $H$, one can find magnetically charged BH solutions in string theory [78].

From BHs in $D-$dimensions one can find straightforwardly black strings in $(D+1)-$dimensions. It is only necessary to take the product of the BH with $R [78]$.

$$ds^2 = -(1 - \frac{cm}{r^n})dt^2 + \frac{dr^2}{1 - \frac{cm}{r^n}} + r^2d\Omega_{n+1}^2 + dx^2. \quad (37)$$

If one takes the product of the BH with $R^2$, $R^3$, $R^p$, one obtains a black membrane, a black 3-brane, and a black p-brane. These branes are simple products. For instance, to get a black string that is not a simple product one performs, after Lorentz boosting to get charge, a T-duality transformation on the simple product black string to obtain

$$ds^2 = -\frac{(1 - \frac{cm}{r^n})}{(1 + \frac{cm \sinh^2 \alpha}{r^n})} dt^2 + \frac{dr^2}{1 - \frac{cm}{r^n}} + r^2d\Omega_{n+1}^2 + \frac{dx^2}{1 + \frac{cm \sinh^2 \alpha}{r^n}},$$

$$B_{xt} = -\frac{cm \sinh 2\alpha}{2(r^n + cn \sinh^2 \alpha)},$$

$$e^{-2\phi} = 1 + \frac{cm}{r^n} \sinh^2 \alpha. \quad (38)$$

The causal structure is identical to Schwarzschild. In the extremal limit the metric field is given by

$$ds^2 = -\frac{dt^2 + dx^2}{1 + \frac{cm}{r^n}} + dr^2 + r^2d\Omega_{n+1}^2, \quad (39)$$

where $\overline{c}$ is a redefinition of $c$. There are two novel features in this solution (39): (i) an extra symmetry has appeared, the metric is now boost-invariant in the $(x,t)$ plane, and (ii) the solution is the same solution found in [79] for a straight fundamental macroscopic string. These objects appear as stable extended states of closed-string theories and are distinct from the cosmic
strings of string theory. This means that fundamental strings are extreme black strings. There is no such analogue in general relativity. The electron, a fundamental particle is not an extreme BH.

Ultimately, one would like to get a BH solution of 10D string theory, suitably dimensionally reduced to 4D. One starts with the 10D action

\[ S = \frac{1}{16\pi} \int d^{10}x \sqrt{-G} \left[ R_{G} + \nabla_{M} \Phi \nabla^{M} \Phi - \frac{1}{12} H^{2} - \frac{1}{4} F^{I2} \right] \]  

where \( H^{2} = H_{MNP} H^{MNP}, F^{I2} = F_{MN}^{I} F^{I MN}, \) capital letters denote 10D fields and indices, and \( I \) is an internal index. Through a Kaluza-Klein reduction to 4D, one can find an effective 4D action, with the other dimensions compactified on a six torus. One writes the ansatz,

\[ G_{MN} = \begin{pmatrix} e^{2\phi} g_{\mu\nu} + G_{mn} A^{m}_{\mu} A^{n}_{\nu} & A^{m}_{\mu} G_{mn} \\ A^{n}_{\nu} G_{mn} & G_{mn} \end{pmatrix} \]  

with the 4D spacetime indices \( \mu \nu = 0, 1, 2, 3, m, n = 1, \ldots, 6, \) and \( \phi \) and \( A \) are the 4D dilaton and Kaluza-Klein \( U(1) \) fields, respectively. The action then turns into

\[ S = \frac{1}{16\pi} \int d^{4}x \sqrt{-g} \left( R - \frac{1}{2} \nabla_{\mu} \phi \nabla^{\mu} \phi - \frac{1}{2} e^{2\phi} \nabla_{\mu} \psi \nabla^{\mu} \psi - \frac{1}{4} e^{-\phi} F_{\mu\nu} F^{\mu\nu} + \frac{1}{8} \text{tr} \left( \nabla_{\mu} M \nabla^{\mu} M \right) \right), \]  

where \( M \) is a \( O(6, 22) \) matrix of the scalar (moduli) fields appearing in the reduction process and \( \psi \) is the axion related to \( H_{\mu\nu\lambda} \) by \( H_{\mu\nu\lambda} = \frac{e^{2\phi}}{\sqrt{-g}} \epsilon^{\mu\nu\lambda\rho} \nabla_{\rho} \psi, \) see [80] for all details. This is quite complicated to solve, but applying a generalized boosting procedure and using all the symmetries it is possible to find the most general BH solution with all charges [80]. An important consequence brought from this 4D analysis is that the extreme BH solutions correspond to massive excitations of 4D superstrings, suggesting that BHs are simple string states [81] and confirming the idea that elementary particles (represented here by those string states) might behave like BHs. These BHs saturate the Bogomolniy-Prasad-Somerfield bound of the underlying supersymmetric theory and are called extreme BPS BHs.

There are also studies on black p-branes in string theory (e.g. [82, 83]) motivated by their importance in the non-perturbative dynamics of the 11D
M-theory \[19\], a theory not explicitly formulated, but known to agglutinate the four consistent (heterotic, type I, type IIA and B) superstring theories.

We have been presenting higher dimensional BH solutions in Kaluza-Klein theory, string theory and general relativity. Yet, although pure general relativity can be formulated in other dimensions, when one goes to dimensions higher than four it is not anymore unique. The natural generalization is given by the Lovelock action \[84\] so that the field equations for the metric remain of second order. The theory can also be considered as a dimensional continuation of the Euler densities of lower dimensions \[85, 86, 87\]. In four dimensions one has to take in consideration two Euler densities. The Euler density of the 0-dimensional space which is proportional to \(\sqrt{-g}\), and the Euler density of the 2-dimensional space, proportional to \(\sqrt{-g}R\), where \(g\) is the determinant of the metric and \(R\) the Ricci curvature scalar. Thus Lovelock gravity in four dimensions reduces to Einstein gravity, with action

\[
\frac{1}{16\pi G} \int d^4 x \sqrt{-g} (-2\Lambda + R).
\]

A similar construction and action is obtained for three dimensions. In six dimensions one has still to add the Euler characteristic of four dimensional space, i.e. the Gauss-Bonnet term, to have the Lanczos action, given by,

\[
\frac{1}{16\pi G} \int d^6 x \sqrt{-g} \left(-2\Lambda + R + \alpha_2 (R_{\alpha\beta\gamma\sigma} R^{\alpha\beta\gamma\sigma} - 4 R_{\alpha\beta} R^{\alpha\beta} + R^2)\right),
\]

where \(\alpha_2\) is a new constant. A similar construction and action can be obtained for five dimensions. For each two new dimensions there exists a new constant \(\alpha_p\). These constants do not seem to have a direct physical meaning. In order to find a meaningful set of constants in any dimension \(D\), it was proposed in \[88, 89\] a method which restricts drastically the number of independent constants to two, \(G\) and \(\Lambda\), thus yielding a restricted Lovelock gravity. This method separates, in a natural manner, theories in even dimensions \((D = 2n, n = 1, 2,..)\) from theories in odd dimensions \((D = 2n + 1)\). The BH solutions are given by \[89\]

\[
ds^2 = - \left[1 - \left(\frac{2sM}{r^p} + q\right)^{\frac{1}{n-1}} + (\frac{q}{r})^2\right] dt^2 + \frac{dr^2}{1 - (\frac{2sM}{r^p} + q)^{\frac{1}{n-1}} + (\frac{q}{r})^2} + r^2 d\Omega_D^{2},\tag{43}
\]

where for odd \(D\) one puts \((s = \frac{1}{2}, p = 0, q = 1)\), and for even \(D\) one has \((s = 1, p = 1, q = 0)\). There are horizons at \(r = r_+\) given by the zeros of \(g^{rr}\) and the singularity is at \(r = 0\). Note that there is no restriction in the dimension of spacetime, it can be any natural number from 3 to \(\infty\). Since in general relativity BHs appear as the final state of gravitational collapse
it is important to know if the BH solutions found in Lovelock gravity can, in an analogous manner, form from gravitational collapse. It was shown that, indeed, Lovelock BHs form from regular initial data [90, 91]. The collapsing matter is modelled by a Friedmann type metric, and the solution can be viewed as a dimensional continued Oppenheimer-Snyder gravitational collapse. A possible scenario for the occurrence of this collapse in $D$ dimensions, would be in the very early universe, before the $(D-4)$ extra dimensions have been compactified. In turn, these newly formed higher dimensional BHs could play a role in the compactification process. It is interesting to note that these BH and collapsing solutions show that some important features of classical general relativity are preserved and carried into Lovelock gravity in any dimension.

6. Conclusions

We have investigated BH, black string and black membrane solutions in several dimensions and in several theories (general relativity, Kaluza-Klein, Brans-Dicke, Lovelock gravity and string theory). We have seen that new properties come into play. For instance, in string theory there are BHs without singularities. It was also shown that the existence of a negative cosmological term can be important in producing black solutions, as was the case of black strings in $4D$ general relativity. We have also seen that some features appearing in general relativity remain in other theories, like in Lovelock gravity, where the BHs also form from gravitational collapse of matter. Other important developments not discussed here are solutions of BHs with both electric and magnetic charges, rotating BHs in several $D$, duality between charge and angular momentum, and multi-BH solutions in the various theories, to name a few.

With such a profusion of BHs in all these gravity theories, one could hope to understand in some detail the BH evaporation process, at least, in one of those solutions. However, the problem of calculating Hawking radiation of BHs, black strings, black membranes or black p-branes, through the latest stages of the evaporation process, remains.

A remarkable property of BHs is that they appear in all scales, from the Planck length to astronomical dimensions. This seems to be unique. Electrons, molecules, stars and galaxies have well defined scales, BHs do not.
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