Neutrino masses in the $SU(4)_L \otimes U(1)_X$ electroweak extension of the standard model

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Abstract

We study the neutrino mass generation in the $SU(4)_L \otimes U(1)_X$ electroweak extension of the standard model by considering non-renormalizable dimension five effective operators. It is shown that there exist two topologies for the realizations of such an operator at the tree-level and for one of the three-family models is explore the neutrino phenomenology after extending its particle content with an $SU(4)_L$ fermion singlet and a scalar decuplet. Constraints in the available parameters space of the model are partially discussed.

1 Introduction

The Standard Model (SM) of particle physics remains as one of the most successful theories in Nature. Despite its triumph, one of the most direct evidences that the SM is not the final theory is based on the fact that neutrinos do oscillate [1, 2, 3, 4], implying necessarily that they are massive particles. In the SM neutrinos are massless due to the absence of right-handed neutrinos, that are needed to build up a Dirac mass term in an analogous way as is done for the charged leptons. In order to accommodate neutrino masses, the model must be extended. Among the solutions to the neutrino problem, one of the simplest is given by the tree level realization of the Weinberg operator [5], which gives rise to the well-known type-I [6, 7], type-II [8, 9, 10, 11] and type-III [12] seesaw mechanism, in where, an $SU(2)$ –fermion singlet, scalar triplet and fermion triplet– are added respectively. On the other hand, the SM also lacks the explanation for the numbers of fermion generations in Nature. In the electroweak extension based on the $SU(3)_C \otimes SU(N)_L \otimes U(1)_X$ [13, 16, 17, 14, 15] (3-$N$-1 extension for short) gauge group, for $N \in \{3, 4\}$, the $SU(2)_L$ is enlarge to $SU(N)_L$. The new fermion content is accommodated into different fundamental representations, $N$ or $\overline{N}$ of $SU(N)_L$. From a theoretical point of view, the 3-$N$-1 extension can account for the number fermion generations in Nature, when the anomaly cancellation takes place between families and not family by family as in the SM [15, 14]. The electroweak $SU(4)_L \otimes U(1)_X$ also arises from little higgs [18] model, provides an explanation for the charge quantization [19], allow electroweak unification [17] and for some kind of models, the
muon anomalous magnetic moment [20, 21] is explained within the 3-4-1 framework. We focus on the 3-4-1 electroweak extension, which at low energies leads to a two higgs doublet model. In this extension neutrinos are naturally massless, and a mechanism for neutrino mass generation is explored through non-renormalizable dimension five operator (Weinberg-like operator). In this paper, we make a classification of the Weinberg-like operators in a set of four three-family models. For the so-called model F, we explain neutrino masses and mixing through the canonical seesaw mechanism and the type II-like seesaw mechanism. For the latter case, after extending model F with a scalar decuplet, the exotic neutrinos and the lightest SM neutrinos have the same mixing matrix and mass hierarchy. This model has tree-level lepton flavor violation (LFV) processes, being $\mu \to 3e$ the most sensitive, induced by doubly charged scalar $H_{1}^{++}$ and controlled by its yukawa coupling to the fermion sector $y_{\alpha \beta}$. This article is organized as follows. In section 2, the 3-4-1 electroweak extension is reviewed, in the section 3 we classified the set of non-renormalizable effective operators in different models of the 3-4-1 extension. A mechanism for neutrino mass generation in the model F is explored through seesaw-like mechanism in the section 4. Finally we summarize our main results in section 5.

2 $SU(4)_L \otimes U(1)_X$ models

In this section the 3-4-1 electroweak extension is briefly introduced. A full phenomenological study can be found in references [14, 17, 22]. We focus in the lepton sector, due that our aim is to implement higher dimensional effective operators that can account for the neutrino mass generation at the tree-level. In the electroweak $SU(4)_L \otimes U(1)_X$, the electric charge operator is a linear combination of the diagonal generators from the Cartan subalgebra.

$$Q = a T_{3L} + \frac{b}{\sqrt{3}} T_{8L} + \frac{c}{\sqrt{6}} T_{15L} + XI_{4},$$

where $a = 1$ is taken in order to reproduce the SM phenomenology. The $T_{iL}$ are the generators of $SU(4)_L$, normalize as $Tr(T_iT_j) = \delta_{ij}/2$, $X$ the hypercharge and $I_{4}$ the $4 \times 4$ identity matrix. The coefficients $b$ and $c$ remains as free parameter that need to be chosen for each model in particular. After demanding models that include particles without exotic electric charge [23], two different assignments for the free parameters are allowed. The first one, based on the selection of $b = 1 (-1)$ and $c = 1 (-1)$ which gives rise to two three-family models, Model A and Model B, and the other choice for the free parameters is $b = 1 (-1)$ and $c = -2 (2)$ that also gives rise to two three-family models, Model E and Model F$^1$.

The electroweak gauge boson sector are content in the $SU(4)_L$ adjoint representation. There are a total of 15 of them, which can be written as:

$$\frac{1}{2} \lambda_{\alpha} A^\alpha_{\mu} = \begin{pmatrix} D^0_{\mu} & W^{+}_{\mu} & K^{(b+1)/2}_{\mu} & X^{(3+b+2c)/6}_{\mu} \\ W^{-}_{\mu} & D^{0}_{2\mu} & K^{-(b-1)/2}_{\mu} & V^{(-3+b+2c)/6}_{\mu} \\ K^{-(b+1)/2}_{\mu} & K^{(b-1)/2}_{\mu} & D^{0}_{3\mu} & Y^{-(b-c)/3}_{\mu} \\ X^{-(3+b+2c)/6}_{\mu} & V^{(3-b-2c)/6}_{\mu} & Y^{(b-c)/3}_{\mu} & D^{0}_{4\mu} \end{pmatrix}.$$  

(2)

For $b = 1$ and $c = 1$ in the electric charge generator, we reach two three-family models called Model A and Model B. For the propose of this work only the lepton and scalar sector are needed, however,

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$^1$The three-family models for the parameter assignments $b = -1$, $c = -1$ ($b = -1$, $c = 2$) are equivalent by hypercharge transformation to the models obtained for $b = 1$, $c = 1$ ($b = 1$, $c = -2$).
for completeness also the quark sector is displayed in table 1. The scalar sector for this set of models is given by:

\[
\begin{align*}
\langle \Phi_1^T \rangle &= \langle (\phi_1^0, \phi_1^+, \phi_1^-) \rangle = (v, 0, 0, 0) \sim (1, 4, -3/4), \\
\langle \Phi_2^T \rangle &= \langle (\phi_2^+ \phi_2^0, \phi_2^0, \phi_2^0) \rangle = (0, v', 0, 0) \sim (1, 4, 1/4), \\
\langle \Phi_3^T \rangle &= \langle (\phi_3^0, \phi_3^0, \phi_3^0) \rangle = (0, 0, V, 0) \sim (1, 4, 1/4), \\
\langle \Phi_4^T \rangle &= \langle (\phi_4^0, \phi_4^0, \phi_4^0, \phi_4^0) \rangle = (0, 0, 0, V') \sim (1, 4, 1/4).
\end{align*}
\]

For \( b = 1 \) and \( c = -2 \) in the electric charge generator we reach two three-family models called Model E and Model F, which are displayed in table 2. The scalar sector for this set of models is given by:

Table 1: Particle content for models A and B, the \( \alpha = \{1, 2, 3\} \) are the lepton generation indices, \( i \) run over the first two generations of quarks. The numbers in parentheses refer to the \((SU(3)_C, SU(4)_L, U(1)_{X})\) quantum numbers respectively.

| Model A | Model B |
|---------|---------|
| \( L_{La} = ( e^-, \nu^0, N^0, N^0 )_{La} \sim (1, \bar{4}, -1/2), \)  
\( e^\pm_{La} \sim (1, 1, 1), \)  
\( Q_{iL} = (u_i, d_i, D_i, \bar{U}_i) \sim (3, 4, 1/6), \)  
\( u^c_{iL} \sim (\bar{3}, 1, -2/3), d^c_{iL} \sim (\bar{3}, 1, 1/3), \)  
\( U_{3L} = (u_3, d_3, U_3) \sim (3, 4, -1/2), \)  
\( U^c_{3L} \sim (\bar{3}, 1, -2/3), D^c_{3L} \sim (\bar{3}, 1, 1/3), \)  | \( L_{La} = (\nu^0, e^-, E^-, \nu^0 )_{La} \sim (1, 4, -3/4), \)  
\( e^\pm_{La} \sim (1, 1, 1), E^+_{La} \sim (1, 1, 1), \)  
\( Q_{iL} = (d_i, \bar{u}_i, U_i, \bar{U}_i' ) \sim (3, 4, 5/12), \)  
\( u^c_{iL} \sim (\bar{3}, 1, -2/3), d^c_{iL} \sim (\bar{3}, 1, 1/3), \)  
\( U_{3L} = (u_3, d_3, U_3, \bar{U}_3') \sim (3, 4, -1/2), \)  
\( U^c_{3L} \sim (\bar{3}, 1, -2/3), D^c_{3L} \sim (\bar{3}, 1, 1/3), \) |

Table 2: Particle content for models E and F, the \( \alpha = \{1, 2, 3\} \) are the lepton generation indices, \( i \) run over the first two generations of quarks. The numbers in parentheses refer to the \((SU(3)_C, SU(4)_L, U(1)_{X})\) quantum numbers respectively.

| Model E | Model F |
|---------|---------|
| \( L_{La} = ( e^-, \nu^0, N^0, E^- )_{La} \sim (1, \bar{4}, -1/2), \)  
\( e^\pm_{La} \sim (1, 1, 1), E^+_{La} \sim (1, 1, 1), \)  
\( Q_{iL} = (u_i, di, D_i, U_i ) \sim (3, 4, 1/6), \)  
\( u^c_{iL} \sim (\bar{3}, 1, -2/3), d^c_{iL} \sim (\bar{3}, 1, 1/3), \)  
\( U_{3L} = (u_3, d_3, U_3, D_3) \sim (3, 4, -1/2), \)  
\( U^c_{3L} \sim (\bar{3}, 1, -2/3), D^c_{3L} \sim (\bar{3}, 1, 1/3), \)  | \( L_{La} = (\nu^0, e^-, E^-, N^0 )_{La} \sim (1, 4, -1/2), \)  
\( e^\pm_{La} \sim (1, 1, 1), E^+_{La} \sim (1, 1, 1), \)  
\( Q_{iL} = (d_i, \bar{u}_i, U_i, D_i ) \sim (3, 4, 1/6), \)  
\( u^c_{iL} \sim (\bar{3}, 1, -2/3), d^c_{iL} \sim (\bar{3}, 1, 1/3), \)  
\( U_{3L} = (u_3, d_3, U_3, D_3) \sim (3, 4, -1/2), \)  
\( U^c_{3L} \sim (\bar{3}, 1, -2/3), D^c_{3L} \sim (\bar{3}, 1, 1/3), \) |
\begin{align}
\langle \Phi^T_1 \rangle &= \langle (\phi^0_i, \phi^+_i, \phi'^+_i, \phi'^0_i) \rangle = (v, 0, 0, 0) \sim (1, \overline{4}, 1/2), \\
\langle \Phi^T_2 \rangle &= \langle (\phi^+_1, \phi^+_2, \phi'^+_2, \phi'^0_2) \rangle = (0, v', 0, 0) \sim (1, 4, -1/2), \\
\langle \Phi^T_3 \rangle &= \langle (\phi^+_3, \phi^+_3, \phi'^+_3, \phi'^0_3) \rangle = (0, 0, V, 0) \sim (1, 4, -1/2), \\
\langle \Phi^T_4 \rangle &= \langle (\phi^0_4, \phi^+_4, \phi'^+_4, \phi'^0_4) \rangle = (0, 0, 0, V') \sim (1, 4, 1/2). 
\end{align}

The pattern of the electroweak symmetry breaking (EWSB) goes as follows
\begin{equation}
SU(4)_L \otimes U(1)_X \xrightarrow{V} SU(3)_L \otimes U(1)_{X'}, \quad \xrightarrow{\nu, v'} SU(2)_L \otimes U(1)_Y \xrightarrow{\nu, v'} U(1)_Q.
\end{equation}

where \(V' \sim v' \sim v\), and \(v'^2 + v^2 = v^2_{\text{SM}} = (246 \text{ GeV})^2\).

### 3 Dimension 5 effective operator

Neutrinos may acquire masses after the introduction of non-renormalizable dimension-five operators defined as:
\begin{equation}
\mathcal{L}_5 = \frac{\mathcal{O}_5}{\Lambda}, \quad \mathcal{O}_5 = \{ L^\dagger_{\alpha \beta} \Phi^*_i \Phi^*_j L_{\alpha \beta}, L^\dagger_{\alpha \beta} \Phi \Phi^*_i L_{\alpha \beta} \},
\end{equation}

being \(\alpha\) and \(\beta\) lepton generation indices and \(i, j\) index in the number of scalar 4-pelts. \(\Lambda\) represent the cutoff scale where new physics is expected. The operator given in Eq. (6) is the generalization of the Weinberg operator [5] for \(SU(4)_L \otimes U(1)_X\). Depending on the way as the fields transforms under \(SU(4)_L \otimes U(1)_X\), different tree-level realizations of the operator are allowed.

Table 3: Scenarios for the operator defined in Eq. (6): In the left part, the \((4(\overline{4}), X_{L(\Phi)})\) notation represents the way as the fields (either \(L_{\alpha \beta}\) or \(\Phi_i\)) transforms under \(SU(4)_L \otimes U(1)_X\). The effective operator is allowed if it is gauge invariant.

| \(L_{\alpha \beta}\) | \(\Phi_i\) | \(\mathcal{O}_5^L = L^\dagger_{\alpha \beta} \Phi^*_i \Phi^*_j L_{\alpha \beta}\) | \(\mathcal{O}_5^U = L^\dagger_{\alpha \beta} \Phi \Phi^*_i L_{\alpha \beta}\) |
|---|---|---|---|
| \((4, X_L)\) | \((4, X_\Phi)\) | \(4 \otimes 4 \otimes 4 \otimes 4 \otimes 1, 2X_L - 2X_\Phi = 0\) | \(4 \otimes 4 \otimes 4 \otimes 4 \otimes 1, 2X_L + 2X_\Phi = 0\) |
| \((4, X_L)\) | \((4, X_\Phi)\) | \(4 \otimes 4 \otimes 4 \otimes 4 \otimes 1, 2X_L - 2X_\Phi = 0\) | \(4 \otimes 4 \otimes 4 \otimes 4 \otimes 1, 2X_L + 2X_\Phi = 0\) |
| \((\overline{4}, X_L)\) | \((4, X_\Phi)\) | \(4 \otimes 4 \otimes 4 \otimes 4 \otimes 1, 2X_L - 2X_\Phi = 0\) | \(4 \otimes 4 \otimes 4 \otimes 4 \otimes 1, 2X_L + 2X_\Phi = 0\) |
| \((\overline{4}, X_L)\) | \((4, X_\Phi)\) | \(4 \otimes 4 \otimes 4 \otimes 4 \otimes 1, 2X_L - 2X_\Phi = 0\) | \(4 \otimes 4 \otimes 4 \otimes 4 \otimes 1, 2X_L + 2X_\Phi = 0\) |

For any set of fields \((\Phi, L_L)\), transforming in a general way under \(SU(4)_L \otimes U(1)_X\), different theoretical realizations of the operators are displayed in table 3. In order to allow such an operators into an effective lagrangian, we must guaranteed that the product of the irreducible representations contain the \(SU(4)\) singlet and be hyperchargeless. Since for \(SU(N)\), \(N \otimes N = [(N^2 + N)/2]_s + [(N^2 - N)/2]_A\) and \(N \otimes N^* = [N^2 - 1]_{\text{Adjoint}} + [1]\), there are only two possible main topologies for the tree-level realization of the Weinberg operator. From Eq. (6), if the intermediate particle is a scalar, it can transform as 10s and 15Adjoint\(^2\) under \(SU(4)_L\), on the other hand if it is a fermion, it can transform as 1A, and 15Adjoint under \(SU(4)_L\). In Figure 1 are displayed all the possible tree level

\(^2\)The scalar singlet does not gives rise to neutrino masses.

\(^3\) The fermion sextet is also a possible realization of the Weinger operator, however it is not allowed because after their introduction it does not give rise to neutrino masses, instead is an additional term that contribute to the masses of the charges leptons.
realization of the effective Weinberg operator in the $SU(4)_L \otimes U(1)_X$ electroweak extension. The theory reduces to a canonical seesaw, a type II-like seesaw, and a type III-like seesaw in where, for $SU(4)_L$ a -fermion singlet, scalar decuplet and fermion 15-plet - are included respectively.

To our knowledge the 3-4-1 extension with a fermion singlet (canonical seesaw mechanism) has been implemented [24], as well as with a scalar decuplet [16, 25], but the fermion 15-plet has not been proposed in the literature yet. Those new particles in case of be added should have hypercharge values that does not spoil the anomaly free structure of the model. That is why any new fermion content should have zero hypercharge or be a vector-like particle under $SU(4)_L$. In the next subsections, we display the set of effective Weinberg-like operators that can be built in the four models presented in section 2.

3.1 Model A

In this model there are a total of 9 operators, which are given by:

$$O_5 = \left\{ L_{La}^c \Phi_2 \Phi_2^{\dagger} L_{L\beta}, \ L_{La}^c \Phi_3 \Phi_3^{\dagger} L_{L\beta}, \ L_{La}^c \Phi_4 \Phi_4^{\dagger} L_{L\beta}, \ L_{La}^c \Phi_5 \Phi_5^{\dagger} L_{L\beta} \right\}. \quad (7)$$

For this model we have:

1. $\Phi_k \Phi_k^{\dagger} \Rightarrow 4 \otimes 4 = 6_A \oplus 10_S$, therefore a 10$^S$ scalar is allowed as the intermediate particle, the 6$^A$ is not allowed because of its statistic.

2. $\Phi_k^{\dagger} L_{L\beta} \Rightarrow 4 \otimes 4 = 1 \oplus 15_{\text{Adjoint}}$, then either a fermion singlet or a fermion 15-plet are allowed as intermediate particles.

The operators defined in Eq. (7) have two topologies at the tree-level, one in which the intermediate particle is a fermion, either singlet $N_R \sim (1,0)$ or 15-plet $\Sigma \sim (15,0)$, and the other one in which the intermediate particle is a scalar decuplet $\Delta \sim (10, X_\Delta)$ and a scalar 15-plet $\Omega \sim (15, X_\Omega)$.
3.2 Model B

For this model the operator is unique and is given by:

\[ O_5 = \left\{ \overline{L}_{L\alpha}^c \Phi_1^* \Phi_1^\dagger L_{L\beta} \right\}. \]  

(8)

1. \( \Phi_k^* \Phi_1^\dagger \Rightarrow 4 \otimes 4 = 6_A \oplus 10_S \), therefore a \( 10_S \) scalar is allowed as the intermediate particle, the \( 6_A \) is forbidden due to its statistic.

2. \( \Phi_1^\dagger L_{L\beta} \Rightarrow 4 \otimes 4 = 1 \oplus 15_{\text{Adjoint}} \), then either a fermion singlet or a fermion 15-plet are allowed as intermediate particles.

The operators given in Eq. (8) has two topologies at tree level, one in which the intermediate particle is a fermion, either singlet \( N_R \sim (1, 1, 0) \) or 15-plet \( \Sigma \sim (1, 15, 0) \), and the other one in which the intermediate particle is a scalar decuplet \( \Delta \sim (1, 10, 3/2) \). Again, to fit all the experimental neutrino oscillation parameters, at least one right-handed neutrino (15-plet fermion) per lepton generation or an scalar decuplet must be included.

3.3 Model E

For this model there are 4 operators, which are given by:

\[ O_5 = \left\{ \overline{L}_{L\alpha}^c \Phi_2^* \Phi_2^\dagger L_{L\beta}, \overline{L}_{L\alpha}^c \Phi_3^* \Phi_3^\dagger L_{L\beta}, \overline{L}_{L\alpha}^c \Phi_4^* \Phi_4^\dagger L_{L\beta}, \overline{L}_{L\alpha}^c \Phi_3^* \Phi_3^\dagger L_{L\beta} \right\}. \]  

(9)

1. \( \Phi_k^* \Phi_1^\dagger \Rightarrow 4 \otimes 4 = 6_A \oplus 10_S \), then a \( 10_S \) scalar is allowed as the intermediate particle, the \( 6_A \) is not allowed because of its statistic.

2. \( \Phi_1^\dagger L_{L\beta} \Rightarrow 4 \otimes 4 = 1 \oplus 15_{\text{Adjoint}} \), then either a fermion singlet or a fermion 15-plet are allowed as intermediate particles.

Again, each of the previous operators have two topologies at tree level, one in which the intermediate particle is a fermion either singlet \( N_R \sim (1, 1, 0) \) or 15-plet \( \Sigma \sim (1, 15, 0) \), and the other one in which the intermediate particle is a scalar decuplet \( \Delta \sim (1, 10, 1) \). In order to fit all the experimental neutrino oscillation parameters, at least two right-handed neutrinos (two fermion 15-plet) per lepton generation or an scalar decuplet must be added.

3.4 Model F

For this model there are 4 operators, which are given by:

\[ O_5 = \left\{ \overline{L}_{L\alpha}^c \Phi_1^* \Phi_1^\dagger L_{L\beta}, \overline{L}_{L\alpha}^c \Phi_1^* \Phi_4^\dagger L_{L\beta}, \overline{L}_{L\alpha}^c \Phi_4^* \Phi_1^\dagger L_{L\beta}, \overline{L}_{L\alpha}^c \Phi_4^* \Phi_4^\dagger L_{L\beta} \right\}. \]  

(10)

1. \( \Phi_k^* \Phi_1^\dagger \Rightarrow 4 \otimes 4 = 6_A \oplus 10_S \), then a \( 10_S \) scalar is allowed as the intermediate particle, the \( 6_A \) is not allowed because of its statistic.

2. \( \Phi_1^\dagger L_{L\beta} \Rightarrow 4 \otimes 4 = 1 \oplus 15_{\text{Adjoint}} \), then either a fermion singlet or a fermion 15-plet are allowed as intermediate particles.
The operators given in Eq. (10) have two topologies at tree level, one in which the intermediate particle is a fermion either singlet $N_R \sim (1, 1, 0)$ or 15-plet $\Sigma \sim (1, 15, 0)$, and the other one in which the intermediate particle is a scalar decuplet $\Delta \sim (1, 10, 1)$. Neutrino oscillation parameters are explained after the model is extended with two right-handed neutrinos (or two fermion 15-plets) per lepton generation or a scalar decuplet.

To address neutrino masses and mixing, models with fermion singlets \cite{24, 26} as well as with scalar decuplets has been constructed \cite{25}. In particular in Ref. \cite{27} not new particles were introduced, instead the $10_S$ scalar representation was build using the fundamental representation of the scalar fields content in $SU(4)_L$. Scalar decuplets also has been used to provide masses for the charged leptons in 3-4-1 models \cite{28}. In the next section we study the neutrino mass generation and mixing in the model $F$, extending with a fermion singlets, and a scalar decuplet.

4 Neutrino masses in Model F

In order to explain neutrino masses and mixing in the 3-4-1 electroweak extension, we explore the tree-level realization of the Weinberg-like operator in the model $F$ introduced\footnote{The same can be done for all the models, following the general classification given in chapter 3.} in table 2.

4.1Canonical Seesaw Mechanism

The model $F$ is extended with two right-handed neutrinos $N_{1Ri} \sim (1, 1, 0)$ and $N_{2Ri} \sim (1, 1, 0)$, being $i$ the generation index. At least three generations of $\{N_{1Ri}, N_{2Ri}\}$ are needed in order explain the neutrino masses. The most general Yukawa lagrangian for the neutral lepton sector, including the new fields reads:

$$-L_{yuk} = \left[ \lambda^{\alpha i}_1 \overline{L}_\alpha \Phi_1 N_{1Ri} + \lambda^{\alpha j}_2 \overline{L}_\alpha \Phi_1 N_{2Rj} + \lambda^{\alpha i}_3 \overline{L}_\alpha \Phi_4 N_{1Ri} + \lambda^{\alpha j}_4 \overline{L}_\alpha \Phi_4 N_{2Rj} + h.c \right] + \frac{1}{2} M_1 N_{1Ri}^C N_{1Ri} + \frac{1}{2} M_2 N_{2Rj}^C N_{2Rj} + \left[ \mu N_{1Ri}^C N_{2Rj} + h.c \right], \quad (11)$$

where $\lambda^{\alpha i}$, for $l \in \{1, 2, 3, 4\}$ and $i \in \{1, 2\}$, are $3 \times k$ Yukawa matrix entries; $k$, the number of right-handed neutrinos per lepton generation, $M_1$ and $M_2$ are $3 \times 3$ Majorana mass matrices for the right-handed neutrinos and are assumed to be diagonal without loss of generality. $\mu$ is a mixing term, that in general is allowed by the gauge symmetry. After the electroweak symmetry breaking (EWSB), Eq. (11) becomes:

$$-L_{yuk} = \begin{pmatrix} \nu_{\alpha \alpha} & \overline{N}_{\alpha \alpha} & \overline{N}_{1Ri}^C & \overline{N}_{2Rj}^C \end{pmatrix} M \begin{pmatrix} \nu_{\alpha \alpha} \\ N_{1Ri} \\ N_{1Ri}^C \\ N_{2Rj} \end{pmatrix}, \quad (12)$$
with:

\[
\mathcal{M} = \begin{pmatrix}
0 & 0 & v\lambda_1 & v\lambda_2 \\
0 & 0 & V'\lambda_3 & V'\lambda_4 \\
v\lambda_1^\dagger & V'\lambda_3^\dagger & M_1 & \mu \\
v\lambda_2^\dagger & V'\lambda_4^\dagger & \mu & M_2
\end{pmatrix} \equiv \begin{pmatrix}
0 & 0 & m_{1D} & m_{2D} \\
0 & 0 & m_{3D} & m_{4D} \\
m_{1D}^\dagger & m_{3D}^\dagger & M_1 & \mu \\
m_{2D}^\dagger & m_{4D}^\dagger & \mu & M_2
\end{pmatrix} \equiv \begin{pmatrix}
0_{6\times6} & M_D \\
M_D^\dagger & M_R
\end{pmatrix}.
\] (13)

The mass matrix given in Eq. (13) cannot be diagonalized exactly. However for simplicity and illustrative purposes we set all elements of matrix \(\mu\) to be zero. In this model, the smallness of active neutrinos is due to the heavity of the right-handed neutrinos as happens in the SM with the type I seesaw mechanism. In the limit \(\{M_1, M_2\} \gg \{m_{1D}, m_{2D}, m_{3D}, m_{4D}\}\), the mass matrix in Eq. (13) can be diagonalized by blocks in an approximately way, and the masses for the lightest and heaviest neutrinos takes the form:

\[
\mathcal{M}^{\text{light}} = -M_R^{-1}M_DM_D^\dagger + \mathcal{O}(M_R^{-2}) \approx -\begin{pmatrix}
\alpha & \beta \\
\gamma & \delta
\end{pmatrix},
\] (14)

\[
\mathcal{M}^{\text{heavy}} = M_R + \mathcal{O}(M_R^{-1}) \approx \begin{pmatrix}
M_1 & 0 \\
0 & M_2
\end{pmatrix},
\] (15)

where

\[
\alpha = M_1^{-1}[m_{1D}m_{1D}^\dagger + m_{2D}m_{2D}^\dagger],
\]

\[
\beta = M_1^{-1}[m_{1D}m_{3D}^\dagger + m_{2D}m_{4D}^\dagger],
\]

\[
\gamma = M_2^{-1}[m_{3D}m_{1D}^\dagger + m_{4D}m_{2D}^\dagger] \equiv M_2^{-1}\beta^\dagger M_1,
\]

\[
\delta = M_2^{-1}[m_{3D}m_{3D}^\dagger + m_{4D}m_{4D}^\dagger].
\] (16)

From Eq. (14), the lightest neutrino spectrum in the physical basis is obtained as:

\[
\mathcal{M}^{\text{light}}_{\text{diag}} = U^\dagger \mathcal{M}^{\text{light}} U,
\] (17)

being \(U\) a \(6 \times 6\) matrix which mixed \([29]\) the lightest neutrinos

\[
U^{6\times6} = \begin{pmatrix}
N^{3\times3} & S^{3\times3} \\
T^{3\times3} & V^{3\times3}
\end{pmatrix}.
\] (18)

From the experimental side, oscillations between the three active SM neutrinos and exotic neutrinos have not yet being observed \([30]\), implying that new neutral leptons, if they exist, must be heavy, \(m_{N_L} > 1\ eV\). As a consequence, the mixing matrices \(S^{3\times3}\) and \(T^{3\times3}\) in Eq. (18) will be suppressed. As pointed out \([29]\), the current experimental limits on neutrinos oscillation experiments are not able to put stringent constraints in any of the new physics (NP) parameters given inside Eq. (18); however, a future generation of neutrino experiment will open the window for the exploration \([31]\).

The lepton flavor violation (LFV) processes such as \(\mu \rightarrow e\gamma\) can take place in this model at one loop level, however a full study on LFV is beyond scope of this paper. The lightest active SM neutrinos acquire masses through the canonical seesaw mechanism, as happens for the SM. Based on
the above observations, the mixing matrix in Eq. (18) is approximately diagonal\(^5\), and the masses for the lightest SM neutrinos takes the form:

\[
\mathcal{M}^{\text{diag}}_{\nu L} \approx N^\dagger \mathcal{M}_{\nu L} N ,
\]

\[
\mathcal{M}^{\text{diag}}_{\bar{\nu} L} \approx U^\dagger_{\text{PMNS}} M^{-1}_{1} [m_{1D} m_{1D}^\dagger] U_{\text{PMNS}} ,
\]

(19)

with \(U_{\text{PMNS}}\), the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix \([32]\) and \(M^{\text{diag}}_{\nu L} = \text{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3})\).

The masses for the lightest sterile neutrinos reads,

\[
\mathcal{M}^{\text{diag}}_{\nu L} \approx V^\dagger \mathcal{M}_{N L} V ,
\]

\[
\mathcal{M}^{\text{diag}}_{\bar{N} L} \approx V^\dagger M^{-1}_{2} [m_{4D} m_{4D}^\dagger] V ,
\]

(20)

with \(M^{\text{diag}}_{N L} = \text{diag}(m_{N 1}, m_{N 2}, m_{N 3})\). The Eq. (19) and Eq. (20) were obtained after demanding \(\lambda_1 \gg \lambda_2\) and \(\lambda_4 \gg \lambda_3\). Under these assumptions the two neutrino sectors are uncorrelated. The masses for the SM neutrinos are fully determined by \(M_1, \lambda_1\) and \(U_{\text{PMNS}}\).

### 4.2 Type II-like Seesaw Mechanism

The model F displayed in table 2 is extended with a scalar decuplet \(\Delta \sim (1,10,1)\). The most general lagrangian for the neutral leptons is given by:

\[
-\mathcal{L}_{\text{yuk}} = y_{\alpha \beta} \overline{L^C_{L\alpha}} \Delta L_{\beta} + h.c. ,
\]

(21)

where, \(y_{\alpha \beta}\) is a symmetry mixing matrix, \(\overline{L^C_{L\alpha}} = L^C_{L\alpha} i\sigma \equiv (-e^C, \nu^C, -N^C, E^C)\), being

\[
\sigma = T_{2L} + T_{14L} = \frac{1}{2} \begin{pmatrix}
0 & -i & 0 & 0 \\
i & 0 & 0 & 0 \\
0 & 0 & 0 & -i \\
0 & 0 & i & 0
\end{pmatrix} .
\]

(22)

The scalar decuplet contains ten degrees of freedom, using a canonical kinetic term; those can be parametrized as:

\[
\Delta = \begin{pmatrix}
\Delta^+_{11} & \Delta^+_{12} & \Delta^+_{13} & \Delta^+_{14} \\
\Delta^+_{21} & \Delta^+_{22} & \Delta^+_{23} & \Delta^+_{24} \\
\Delta^0_{31} & \Delta^+_{32} & \Delta^+_{33} & \Delta^0_{34} \\
\Delta^+_{41} & \Delta^+_{42} & \Delta^+_{43} & \Delta^+_{44}
\end{pmatrix} \equiv \begin{pmatrix}
\frac{1}{\sqrt{2}} H^+_{1} & H^+_{1} & \frac{1}{\sqrt{2}} H^+_{2} & \frac{1}{\sqrt{2}} H^+_{3} \\
H^0_{1} & -\frac{1}{\sqrt{2}} H^+_{1} & -\frac{1}{\sqrt{2}} H^+_{2} & \frac{1}{\sqrt{2}} H^0_{3} \\
-\frac{1}{\sqrt{2}} H^0_{3} & \frac{1}{\sqrt{2}} H^3_{1} & -\frac{1}{\sqrt{2}} \omega^+ & -\kappa^0 \\
\frac{1}{\sqrt{2}} H^2_{1} & -\frac{1}{\sqrt{2}} H^2_{2} & -\frac{1}{\sqrt{2}} H^2_{3} & \rho^+ & \frac{1}{\sqrt{2}} \omega^+
\end{pmatrix} .
\]

(23)

After EWSB, the neutral components of the decuplet develop a VEV and the lagrangian in Eq. (21) becomes:

\[
-\mathcal{L}_{\text{yuk}} = y_{\alpha \beta} \left( \overline{\nu^C_{L\alpha}} \langle H^0_1 \rangle \nu_{L\beta} + \frac{1}{\sqrt{2}} \overline{\nu^C_{L\alpha}} \langle H^0_3 \rangle N_{L\beta} \right) + h.c
\]

\[
+ \frac{1}{\sqrt{2}} \overline{N^C_{L\alpha}} \langle H^0_3 \rangle \nu_{L\beta} + \overline{N^C_{L\alpha}} \langle \kappa^0 \rangle N_{L\beta} + h.c
\]

\[
= (\overline{\nu^C_{L\alpha}} \ N^C_{L\alpha}) \ M \left( \overline{\nu_{L\alpha}} N_{L\alpha} \right) ,
\]

(24)

\(^5\)There are not mixing between the sterile neutrinos and the SM ones.
with
\[
\mathcal{M} = \left( \begin{array}{cc}
y_{\alpha\beta} \langle H^0_1 \rangle & \frac{1}{\sqrt{2}} y_{\alpha\beta} \langle H^0_3 \rangle \\
\frac{1}{\sqrt{2}} y_{\alpha\beta} \langle H^0_3 \rangle & y_{\alpha\beta} \langle \kappa^0 \rangle
\end{array} \right),
\tag{25}
\]
with $\alpha$ and $\beta$ being lepton generation indices. We demand $\langle H^0_3 \rangle < 1$ GeV, in order to avoid $e - E$ large mixing. The scalar decuplet will modify the tree-level $\rho$ parameter \cite{33].

\[
\rho_{\text{tree}} \simeq 1 - \frac{2 \langle H^0_1 \rangle^2}{v^2 + v'^2 + \langle H^1_1 \rangle^2}.
\tag{26}
\]

Since, $\rho_{\text{exp}} = 1.00040 \pm 0.00024$ \cite{34], in order to satisfy the $\rho$ constraint, $\langle H^0_1 \rangle \leq 1.5$ GeV. Notice that $\langle \kappa^0 \rangle$ is not constrained by $\rho$. Assuming $\{\langle H^0_1 \rangle, \langle H^0_3 \rangle\} < \langle \kappa^0 \rangle$, the neutrino masses for the lightest SM neutrinos and the heavy ones at second order in perturbative diagonalization takes the form:

\[
\mathcal{M}_{\text{Light}} = y_{\alpha\beta} \langle H^0_1 \rangle - \frac{\langle H^0_3 \rangle^2}{\langle \kappa^0 \rangle} y_{\alpha\beta}^\dagger y_{\alpha\beta}^\dagger,
\tag{27}
\]

\[
\mathcal{M}_{\text{Heavy}} = y_{\alpha\beta} \langle \kappa^0 \rangle + \frac{\langle H^0_3 \rangle^2}{\langle \kappa^0 \rangle} y_{\alpha\beta}^\dagger y_{\alpha\beta}^\dagger.
\tag{28}
\]

In the limit $\langle H^0_3 \rangle \ll \langle \kappa^0 \rangle$, the Eq. (27) and Eq. (28) are diagonalized by the same $U_{\text{PMNS}}$ mixing matrix \footnote{The same conclusion is draw for Eq. (27) and Eq. (28) forcing $y_{\alpha\beta}$ to be real, assumption which is not general, and only will be valid for a real $U_{\text{PMNS}}$.}

\[
\mathcal{M}_{\nu}^{\text{diag}} = U_{\text{PMNS}}^\dagger \mathcal{M}_{\text{Light}} U_{\text{PMNS}} = \langle H^0_1 \rangle U_{\text{PMNS}}^\dagger Y U_{\text{PMNS}},
\tag{29}
\]

\[
\mathcal{M}_N^{\text{diag}} = U_{\text{PMNS}}^\dagger \mathcal{M}_{\text{Heavy}} U_{\text{PMNS}} = \langle \kappa^0 \rangle U_{\text{PMNS}}^\dagger Y U_{\text{PMNS}},
\tag{30}
\]

where leptonic indices has been suppressed in matrix $Y$. Since both matrices; $\mathcal{M}_{\text{Heavy}}$ and $\mathcal{M}_{\text{Light}}$ are diagonalized by the same matrix, then the heavy neutral leptons (exotics) and the lightest (SM ones) has the same mass hierarchy. Therefore,

\[
\mathcal{M}_N^{\text{diag}} = \frac{\langle \kappa^0 \rangle}{\langle H^0_1 \rangle} \mathcal{M}_{\nu}^{\text{diag}},
\tag{31}
\]

\[
\begin{pmatrix}
m_{N1} & 0 & 0 \\
0 & m_{N2} & 0 \\
0 & 0 & m_{N3}
\end{pmatrix} = \frac{\langle \kappa^0 \rangle}{\langle H^0_1 \rangle} \begin{pmatrix}
m_{\nu 1} & 0 & 0 \\
0 & m_{\nu 2} & 0 \\
0 & 0 & m_{\nu 3}
\end{pmatrix}.
\]

Using the data from neutrino oscillation \cite{30], the lightest of the sterile neutrino satisfies $M_{N1} > 1$ eV. From this we derived the next constraints on the VEV of the scalar decuplet.

\[
\langle \kappa^0 \rangle > \frac{1 \text{ eV} \langle H^0_1 \rangle}{m_{\nu 1}}
\tag{32}
\]

Assuming for instance $m_{\nu 1} \simeq \sqrt{\Delta m^2_{12}} \simeq 8.717 \times 10^{-3}$ eV, which is the maximum possible value for $m_{\nu 1}$ in the normal hierarchy (NH) scenario\cite{35], then $\langle \kappa^0 \rangle > 114.707 \langle H^0_1 \rangle$, is a lower bound on $\langle \kappa^0 \rangle$. 
Figure 2: $\text{BR}(\mu^- \to e^+ e^-)$ as a function of $y_{ee}$. The vertical dashed line represents the point where couplings of order $\sim 4\pi$ are expected, and the horizontal dashed line is the upper limit for $\text{BR}(\mu^- \to e^+ e^-)$ process.

derived from neutrino physics. In this model, LFV processes such as $\mu^- \to e^+ e^- e^-, \tau^- \to e^+ e^- e^-$, $\tau^- \to \mu^+ \mu^- \mu^-$ are mediated by $H_1^{++}$ at the tree-level. These processes are controlled by $y_{\alpha\beta}$ and also depends on the new scalar sector spectrum.

$$\text{BR}(\mu^- \to e^+ e^- e^-) \approx \frac{\Gamma(\mu^- \to e^+ e^- e^-)}{\Gamma(\mu^- \to e^+ \nu_\mu \nu_e)} = \frac{1}{(M_{H_1^{++}})^4 G_F^2 |y_{\mu e}|^2 |y_{ee}|^2}. \quad (33)$$

$\text{BR}(\mu^- \to e^+ e^- e^-)$ is constrained [36] to satisfy $\text{BR}(\mu^- \to e^+ e^- e^-) < 1.0 \times 10^{-12}$, which is the most severe limit. In figure 2 is displayed the $\text{BR}(\mu^- \to e^+ e^- e^-)$ as a function of $y_{ee}$. The vertical dashed line are the points with yukawa couplings of order $\sim 4\pi$, which represents the perturbative limit. To the left of that line neutrino masses and mixing are explained. The points with $y_{ee} > 4\pi$ are ruled out by perturbativity. The horizontal dashed line represents the upper limit on $\text{BR}(\mu^- \to e^+ e^- e^-)$, above that limit the points are ruled out. All the points in the plot were obtained performing a scan of the following parameters in the range

$$100 \text{ GeV} < m_{H_1^{++}} < 100 \text{ TeV},$$
$$10^{-9} \text{ GeV} < \langle H_1^0 \rangle < 1.5 \text{ GeV},$$
$$10^{-9} \text{ GeV} < \langle H_3^0 \rangle < 1 \text{ GeV},$$
$$10^{-7} < y_{\alpha\beta} < 2 \times 10^1.$$
All the points in figure 2 satisfy the neutrino mixing and masses constraints \cite{35} at 2σ. On the other hand, notice that model $F$ account for neutrino masses and mixing extending it with two fermion 15-plet per lepton generation. Since the fermion 15-plet mixes with the charged 4-plet leptons, then tree-level LFV processes mediated by the neutral gauge bosons (Z, $Z'$ and $Z''$) are present. The model will also have restrictions coming from colliders constraints on heavy exotic leptons. This model is very interesting, its phenomenology is more richer than the two other realizations shown before, but is beyond scope this work and will be considered in a future work.

5 Conclusions

In this paper, we provide a mechanism to explain the origin of neutrino masses and mixing in the $SU(4)_L \otimes U(1)_X$ electroweak extension of the SM through the tree level realization of the Weinberg-like operator. For the model $F$, we construct two of these realization and show how the masses are generated. For the canonical seesaw, even when the model predicts the existence of a mixing between the SM neutrinos and the heavy ones, those sector are uncorrelated due to the absence of significantly data on the neutrino sector. Implying that heavy neutrinos, if they exist, must be heavy. For the type II like seesaw model, the introduced decuplet account for the mixing and masses of the SM neutrinos and predicts that the exotic neutral leptons has the same mass hierarchy and mixing pattern than the lightest neutrinos. In this scenario, the neutral components of the scalar decuplet (except $\langle \kappa^0 \rangle$) potentially modified the $e - E$ mixing and the tree level $\rho$ parameter. The lower experimental limit established for the mass of the lightest exotic neutrino give us a lower bound on $\langle \kappa^0 \rangle$. It is worth to mention that the study done in this paper does not take into account the analysis of the full scalar sector of the model, mainly because of the complexity of the scalar potential. The model give rises to tree level LFV processes, being $\mu \rightarrow 3e$ the most sensitive, which is mediated by $H_1^{++}$. Since we do not evaluate neither the full scalar potential nor the scalar spectrum, then, there are not considerations regarding collider signatures. However, the model posses signatures worth of exploring. In the best case scenario ($m_{H_1^{++}}$ being the lightest of the exotic scalars and small mixing in the full scalar potential) the signal $p p \rightarrow H_1^{++} H_1^{--} \rightarrow l^+ l^+ l^- l^-$ will be the promising channel \cite{37} to find the $H_1^{++}$. The phenomenology done through this paper for model $F$, shall be analogous to the model $E$. For model $A$, when the tree level realization is the canonical seesaw mechanism, requires the introduction of three right handed neutrinos (15-plet fermions) per lepton generation, the model will contain a total of 18 neutral leptons, 3 of them being the SM neutrinos and 15 of them being exotic. Neutrino masses and mixing are explained within this model.

The model $B$ is trivial, in the sense that after extending it with a fermion singlet, scalar decuplet and a fermion 15-plet we reach the same phenomenology of the SM when the type-I, the type-II and the type-III seesaw mechanism are considered.

References

\[1\] Y. Fukuda \textit{et al.} [Super-Kamiokande Collaboration], Phys. Rev. Lett. \textbf{81}, 1562 (1998) [hep-ex/9807003].

\[2\] Q. R. Ahmad \textit{et al.} [SNO Collaboration], Phys. Rev. Lett. \textbf{89}, 011301 (2002) [nucl-ex/0204008].

\[7\] we only consider the case for the NH scenario.
[3] Q. R. Ahmad et al. [SNO Collaboration], Phys. Rev. Lett. **89**, 011302 (2002) [nucl-ex/0204009].

[4] M. H. Ahn et al. [K2K Collaboration], Phys. Rev. D **74**, 072003 (2006) [hep-ex/0606032].

[5] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).

[6] P. Minkowski, Phys. Lett. B **67**, 421 (1977).

[7] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[8] M. Magg and C. Wetterich, Phys. Lett. B **94**, 61 (1980).

[9] J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980).

[10] T. P. Cheng and L. F. Li, Phys. Rev. D **22**, 2860 (1980).

[11] G. B. Gelmini and M. Roncadelli, Phys. Lett. B **99**, 411 (1981).

[12] R. Foot, H. Lew, X. G. He and G. C. Joshi, Z. Phys. C **44**, 441 (1989).

[13] F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992) [hep-ph/9206242].

[14] D. A. Gutierrez, W. A. Ponce and L. A. Sanchez, Eur. Phys. J. C **46**, 497 (2006) [hep-ph/0411077].

[15] W. A. Ponce, Y. Giraldo and L. A. Sanchez, AIP Conf. Proc. **623**, 341 (2002) [hep-ph/0201133].

[16] R. Foot, H. N. Long and T. A. Tran, Phys. Rev. D **50**, 34 (1994) [hep-ph/9402243].

[17] Riazuddin and Fayyazuddin, Eur. Phys. J. C **56**, 389 (2008) [arXiv:0803.4267 [hep-ph]].

[18] K. Y. Lee and S. h. Nam, J. Phys. G **42**, no. 12, 125003 (2015) [arXiv:1412.1541 [hep-ph]].

[19] J. M. Cabarcas and J.-A. Rodriguez, Mod. Phys. Lett. A **29**, no. 06, 1450032 (2014) [arXiv:1303.5332 [hep-ph]].

[20] D. Cogollo, Physics International, 2015, Volume 6, Issue 1,Pages 42-50 [arXiv:1411.2810 [hep-ph]].

[21] D. Cogollo, Int. J. Mod. Phys. A **30**, no. 09, 1550038 (2015) [arXiv:1409.8115 [hep-ph]].

[22] A. Palcu, Int. J. Mod. Phys. A **24**, 4923 (2009) [arXiv:0902.3756 [hep-ph]].

[23] W. A. Ponce and L. A. Sanchez, Mod. Phys. Lett. A **22**, 435 (2007) [hep-ph/0607175].

[24] A. Palcu, Mod. Phys. Lett. A **24**, 2589 (2009) [arXiv:0908.1636 [hep-ph]].

[25] H. N. Long, L. T. Hue and D. V. Loi, arXiv:1605.07835 [hep-ph].

[26] A. Palcu, arXiv:1510.06717 [hep-ph].

[27] A. Palcu, Phys. Rev. D **85**, 113010 (2012) [arXiv:1111.6262 [hep-ph]].
[28] F. Pisano and V. Pleitez, Phys. Rev. D **51**, 3865 (1995) [hep-ph/9401272].

[29] F. J. Escrihuela, D. V. Forero, O. G. Miranda, M. Tortola and J. W. F. Valle, arXiv:1505.01097 [hep-ph].

[30] A. A. Aguilar-Arevalo et al. [MiniBooNE Collaboration], Phys. Rev. Lett. **110**, 161801 (2013) [arXiv:1207.4809 [hep-ex], arXiv:1303.2588 [hep-ex]].

[31] M. Wurm et al. [LENA Collaboration], Astropart. Phys. **35**, 685 (2012) [arXiv:1104.5620 [astro-ph.IM]].

[32] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[33] P. Langacker, Phys. Rept. **72**, 185 (1981).

[34] K. A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014).

[35] D. V. Forero, M. Tortola and J. W. F. Valle, Phys. Rev. D **90**, no. 9, 093006 (2014) [arXiv:1405.7540 [hep-ph]].

[36] U. Bellgardt et al. [SINDRUM Collaboration], Nucl. Phys. B **299**, 1 (1988).

[37] Z. L. Han, R. Ding and Y. Liao, Phys. Rev. D **91**, 093006 (2015) [arXiv:1502.05242 [hep-ph]].