Flame fronts in Supernovae Ia and their pulsational stability

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The structure of the deflagration burning front in type Ia supernovae is considered. The parameters of the flame are obtained: its normal velocity and thickness. The results are in good agreement with previous work of different authors. After that the question of pulsational instability of the flame subject to plane perturbations is considered. The flame can be unstable if hydrodynamics can be ignored, e.g. in solid-body propellants. However, with account of hydrodynamics we find that the flame in type Ia supernovae is pulsationally stable with realistic parameters of reactions and thermal conduction.

I. INTRODUCTION

Supernovae explosions are among the most spectacular events in the Universe: their energy release significantly mixes the interstellar medium and acts like a driving force in gas dynamics of galaxies and production of cosmic rays. The luminosity of an exploding star becomes comparable with the luminosity of the progenitor galaxy, and allows one to make observations of processes in the most distant regions of the Universe. If a regularity of parameters from explosion to explosion is found in any subtype of supernovae, it could make possible to realize quantitative measurements of distances and to apply supernovae to cosmology.

Despite the long history of investigations of these events the complete understanding of underlying physical processes is still missing. There are several types of supernova explosions that very much differ physically and are related to different objects. Here we will consider only one subtype – thermonuclear explosions, the so-called supernovae of type Ia, SNIa. Such explosions are usually associated with degenerate stars, white dwarfs. The study of SNIa shows that the underlying physical phenomenon creating the explosion is the thermonuclear burning of premixed carbon–oxygen fuel.

The mode of the explosive nuclear burning in supernovae is still a controversial issue, in spite of many years of the research in the field. Four decades ago, Arnett [1] was the first to model the supersonic combustion, i.e. detonation, in supernovae. Later, Ivanova et al. [2] obtained a sub-sonic flame (deflagration) propagating in a spontaneous regime with pulsations and a subsequent transition to detonation, and Nomoto et al. [3] modelled the deflagration propagating due to a convective heat transfer. Both detonation and deflagration have their merits and problems in explaining the supernova phenomenon [see, e.g. 4]. It is not clear if detonation succeeds or fails to develop, but it is clear that in any case the combustion must be much faster than suggested by the analysis of the propagation of a laminar one-dimensional flame.

From the microscopic point of view the one-dimensional nuclear flame is a wave described essentially in the same way as it was done by Zeldovich and Frank-Kamenetsky [5] in spite of complications introduced by nuclear kinetics and a very high conductivity of dense presupernova matter. It is found that the conductive flame propagates in a presupernova with the speed which is too slow to explain the supernova outburst correctly: the flame Mach number is of the order of percent or less [6].

The fuel consumption can be naturally accelerated by the development of the instabilities inherent to the flame front. As it is explained in the classical paper by Landau [7, 8], the hydrodynamic instability leads to wrinkling or roughening of the front surface, and hence to an increase of its area with respect to the smooth front and consequently to acceleration of the flame propagation. In extreme cases, the instabilities can lead to a transition from the regime of slow flame propagation to the regime of detonation. Since the flame propagates in gravitational field, and the burned ashes have lower density than the unburned fuel, the Rayleigh–Taylor (RT) instability is often considered to be the dominant instability governing the corrugation of the front [9–13]. The RT instability gives birth to a turbulent cascade providing for an accel-
eration of the flame front. However it leads to additional difficulties in modeling the SN event [14–18].

It is well known [7, 8, 19, 20] that large portions of a slow planar flame front are unstable with respect to the large scale bending. This universal instability is called the Landau-Darrieus (LD) instability. For the wavelengths much longer than the flame thickness it does not depend, on complex processes which take place in the burning zone. Development of the LD instability depends only on the sign of $\Delta \rho = \rho_u - \rho_b$, where $\rho_u$ and $\rho_b$ are densities of the unburned and burned "gases" respectively. The LD instability of planar flame fronts with respect to large scale bending takes place if, and only if $\Delta \rho > 0$.

The LD instability plays an important role in many physical phenomena such as the usual chemical burning of gases, explosive boiling of liquids [21], electroweak phase transitions [22], dynamics of thermally bistable gas [23], and thermonuclear burning in supernovae [21, 24].

The detailed consideration of non-linear stage of LD instability and the calculation of the fractal dimension of the flame front for that case is given by Blinnikov and Sasorov [26]. Joulin [27] It is interesting, and somewhat puzzling, that a similar dependence of the flame fractal dimension on the density discontinuity was found in the 3D SPH simulations of the flame subject to RT instability [28].

Both LD and RT instabilities develop on scales much larger than the flame thickness and they can be successfully studied in the approximation of the discontinuous front. This approximation is not valid for another instability, first discovered by Zeldovich [29] in his investigation of the powder combustion. This instability is a result of a strong temperature dependence of the reaction rates, so that the local fluctuations of the heating rate caused by the temperature fluctuations cannot be controlled by the thermal conduction. It can lead to a pulsating regime of the front propagation and to a renormalization of the mean front velocity [30]. This instability can develop even for one-dimensional perturbations when the plane front preserves its shape. Let us denote it as TP (thermal-pulsational) instability. After paper by [31] it was studied quantitatively in many papers [see references in 32].

The paper is organized as follows: in the Section II the physical conditions and a model of WD explosion are presented, in the Section III the stationary propagating flame is considered analytically and numerically. Some conclusions on the dependence of the results on nuclear network are presented. In Sections IV and V the stability of the flame subject to plane perturbations is considered.

II. THE MODEL

A very nice review on the subject is given by [33] [see also 34, 35]. For self contained presentation we will summarize some essential points here. There exist several scenarios of SNIa explosions. The most popular are: single–degenerate scenario, double–degenerate scenario, and sub-Chandrasekhar mass explosion. In this paper, as we think, the most reasonable one, the single–degenerate scenario, is considered. According to it a binary stellar system, which is a progenitor of the supernova, consists of a white dwarf (WD) and a non degenerate star. During accretion of matter on the WD it approaches the Chandrasekhar mass limit and at some moment becomes unstable. In the language of equation of state it happens because the adiabatic exponent approaches the critical value $\gamma \approx 4/3$. During this process the temperature in the centre of the WD rises and nuclear burning of matter (that is degenerate $^{12}$C and $^{16}$O according to evolutionary models) is ignited. The temperature of ignition depends on matter density and can be found in [36]. For $\rho \sim 10^9$ g/cm$^3$ it is about $T \sim 10^8$ K. But this burning is very slow and does not propagate outwards from the centre until its energy release is compensated for by various losses (the most significant are the neutrino losses). The dynamical stage sets in later, when $T$ rises up to $10^9$ K and the flame is born. This is the beginning of the supernova explosion. The flame starts to propagate from the centre of the star to its surface. The regime of the flame propagation is under intensive investigation and the answer is not yet found. But it is not fully unknown, the observations imply some limitations on it. There exist two types of stationary regimes: 1) deflagration when velocity is small compared to the speed of sound and the flame is driven by dissipative effects: thermo-conductivity or diffusion; 2) detonation, i.e. a supersonic wave with the shock front where the temperature jumps up drastically leading to fast burning. If the burning of the whole star proceeds in the detonation regime, it burns up to Fe-peak elements, and that contradicts observations: in a real supernova about half of the star should consist of the intermediate elements. Pure deflagration regime does not succeed too: the star expands with velocity, which is faster than the flame velocity so the temperature drastically drops down and all the burning terminates. The only feasible successful regime is the mixed one: the flame starts with deflagration in high density matter, where the expansion coefficient is small. Then it somehow accelerates at the radius, which is usually characterized by some critical density, and passes to detonation. This model theoretically succeeds in explaining all the parameters of the explosion, but has one tuning parameter, critical density, $\rho_{\text{crit}}$. From comparison of simulations with observations it is obtained that $\rho_{\text{crit}} \sim 2 \cdot 10^7$ g/cm$^3$ [see, e.g. 33, 37]. But the global problem in SNIa physics (that is surely not solved in this paper) is to construct the model of the explosion with no tuning parameters. One should calculate the critical density of the burning regime transition from the first principles, this is the problem for the future.

Let us discuss the physical conditions in the white dwarf. Suppose its mass is close to the Chandrasekhar mass limit. The density in the centre of WD is $\rho \sim 10^9$ g/cm$^3$,
The chemical composition is mainly $^{12}\text{C}$ and $^{16}\text{O}$. So the temperature at which the flame can burn under the specified conditions is about $T \sim 10^{9}$ K. Two main physical processes that matter in SN Ia explosions are thermo-conductivity and nuclear reactions (burning). In this case ions are non-degenerate and non-relativistic, electrons on the contrary are strongly or semi-degenerate and relativistic. EOS for such matter is presented in [38] and takes into account ions, all degrees of degeneracy of electrons and photons.

Under these conditions the thermo-conductivity plays the leading role in comparison with other dissipative processes. It consists of two components: radiative and electronic. The electronic thermo-conductivity, which we use in this paper, is considered in [39, 40]. An approximate equations for radiative thermo-conductivity are presented in [41]. We can estimate the value of the diffusive effects in the system. Despite the fact that in the white dwarf the matter is well mixed, let us consider the value of the diffusion coefficient. The ion diffusion can be approximated as

$$D \sim \lambda_i v_T.$$  

Here $v_T$ is the thermal speed of ions, $\lambda_i$ is the ion mean free-path, which may be crudely estimated as the inter-ion distance. So the Lewis number is about $Le = 10^4$. This result can be easily explained by the differences of speeds: relativistic particles contribute to thermo-conductivity, and non-relativistic – to diffusion. For viscosity

$$\eta = mn\lambda_i v_T.$$  

So the Prandl number is $Pr = 10^{-4}$.

At the temperatures specified earlier the nuclear reactions proceed in a branched network with a lot of isotopes. For simplification let us suppose that WD consists only of $^{12}\text{C}$. Then the first reaction in the nuclear network is $^{12}\text{C} + ^{12}\text{C} \rightarrow ^{24}\text{Mg}^*$ (here “*” means exited nuclei, it is unstable and decays in 3 channels with $p$, $n$ or $\alpha$ in final state). Its caloricty is $q = 5.6 \cdot 10^{17}$ erg/g, so it leads to significant temperature rise, up to $T \sim 10^{16}$ K, and can provoke further burning. In this paper two variants of nuclear network are considered:

1. A simplified network with only one reaction $^{12}\text{C} + ^{12}\text{C} \rightarrow ^{24}\text{Mg}^*$ [the rate can be found in [42]]. According to the fact that it is the first reaction in the network and electromagnetic nature of this reaction it can be a good approximation for the whole network at the first glance, and all dynamics will be determined by this reaction. The complete burning up to Ni could also be modeled in this framework by fixing the rate and changing the caloricty to $q = 9.2 \cdot 10^{17}$ erg/g.

2. $\alpha$-chain with 13 isotopes from [43], we will call it “APROX13”.

We will make also additional simulations with initial $^{16}\text{O}$ composition using APROX13. Because of the high matter density the degeneracy parameter satisfies $\Gamma \sim E_{\text{eon}}/E_{\text{kin}} \sim 1$, so the electron nuclear screening effects should be taken into account. The accurate screening factor in a non-ideal gas can be obtained only by Monte-Carlo simulations, and is presented, e.g., in [44]. We should emphasize also that many-order discrepancy in the screening factor can arise because of incorrect definitions, see details in [40].

Based on the above consideration we can write down the hydrodynamic system of equations which should be solved:

$$\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0,$$

$$\frac{\partial (\rho c_i)}{\partial t} + \nabla(\rho c_i v) = \sum_j R_{ij},$$

$$\frac{\partial \rho v}{\partial t} + \nabla(\rho v v) = -\nabla p + f,$$

$$\frac{\partial E}{\partial t} + \nabla(v(E + p) - \kappa \nabla T) = \sum_{ij} q_{ij} R_{ij},$$

$$p = p(\rho, c_i, \epsilon).$$

### III. ONE-DIMENSIONAL FLAME PROPERTIES

#### A. The Problem Set-up

The main questions we pose here are as follows: What is the normal velocity of flame propagation, what is the expansion coefficient of matter over the flame front, and what are the physical parameters of matter after the flame traversal. To answer these questions we should study the plane burning front microphysics taking into account the processes on the scale of the thickness of the front. Let us make some theoretical estimates. For the stationary deflagration regime the characteristic times of burning and heat transfer over the front should be:

$$\tau_{\text{nucl}} = \frac{Q}{S}, \quad \tau_{\text{cond}} = \frac{d^2}{\kappa},$$

so the flame thickness and velocity can be estimated as:

$$d = \sqrt{\frac{\kappa}{Q} S}, \quad v = \frac{d}{\tau} = \sqrt{\frac{\kappa}{S} Q}.$$  

In table [1] the estimated velocity values are presented for different conditions in WD, when the rate of burning $S$ is determined by one reaction proposed earlier, $(^{24}\text{C} \rightarrow ^{24}\text{Mg}^*)$.

The exact front parameters can be obtained only by the numerical simulations of the system [45] in 1D. For the general rates $R_{ij}$ of the reactions in system [3] the density and temperature distributions in a steady burning wave are unknown, so the initial conditions with the
TABLE I: Theoretical estimate of the front velocity and thickness with $^{21}$C→$^{24}$Mg$^*$ (Eqns. [3]–[5]):

| $\rho$, g/cm$^3$ | $v$, km/s | $\Delta x$, cm |
|-----------------|------------|----------------|
| $2 \cdot 10^8$  | 203        | $2.5 \cdot 10^{-5}$ |
| $7 \cdot 10^8$  | 742        | $3.0 \cdot 10^{-6}$ |
| $2 \cdot 10^9$  | 1300       | $1.0 \cdot 10^{-6}$ |

steady flame could not be imposed. We will ignite the flame with a warm wall and try to eliminate all interfering perturbations. We expect that the stationary flame that will appear in such conditions is a natural phenomenon. The following task for numerical calculations is considered: The region of interest is filled with a uniform matter distribution. For simplicity it will be $^{12}$C with density $\rho_0$ and temperature $T_0$, chosen so that the characteristic time of the flame burning with $T_0$ is much larger than the dynamical time in the problem under consideration. In this case the ignition is controlled by the boundaries. The right hand side boundary condition sets a constant external pressure. The left hand side boundary condition is a hard wall $v = 0$, and its temperature rises with time according to the linear law:

$$T_{\text{left wall}} = T_0 + \frac{T_1 - T_0}{\tau} t.$$  \hspace{1cm} (6)

The choice of $\tau$ will be clarified below, $T_1$ is the temperature which is larger than the temperature of the active burning. All these quantities depend on the task and are tuned "by hands" after inspection of the trial values in each case. Let $L$ be the size of the region of interest (it is smaller than the whole region of calculation). The burning leads to the temperature growth and therefore the pressure rises as well, which generates sound waves. For the sake of avoiding unnecessary perturbations we should give time for sound waves to quit the region of interest: $\tau \gg L/c_s$. In this case the pressure in the region $L$ is constant with a good accuracy.

Let us describe the numerical methods for the solutions of system [3]. Physically, the system consists of 3 parts: hydrodynamics, thermo-conductivity, and burning. For the solution we will use the method of splitting in physical processes. As an example consider the equation for energy $E$ in system [3]. At each time step the equation is split as:

$$\frac{\partial E}{\partial t} = \frac{\partial E}{\partial t}_{\text{hydr}} + \frac{\partial E}{\partial t}_{\text{nucl}} + \frac{\partial E}{\partial t}_{\text{thermocond}},$$  \hspace{1cm} (7)

$$\frac{\partial E}{\partial t}_{\text{hydr}} = -\text{div}(v(E + p)),$$  \hspace{1cm} (8)

$$\frac{\partial E}{\partial t}_{\text{nucl}} = \sum_{ij} q_{ij} R_{ij},$$  \hspace{1cm} (9)

The equation for $c_i$ is split analogously (in other equations of system [3] only one physical process in one equation is presented). Eqns. (7)–(10) are solved in 3 sub-steps: The hydrodynamic part in which $c_i = \text{const}$ and no energy generation is assumed; the nuclear part ($\rho, v = \text{const}$); and the thermo-conductivity part ($\rho, c_v, v = \text{const}$).

For the solution of hyperbolic PDE’s we will use two solvers (for cross-check): the implicit numerical scheme with Newton solver described in [15], and Kurganov–Tadmor scheme [16]. Parabolic PDE (10) is solved with Crank–Nicholson numerical scheme. The kinetic equation for $c_i$ with simplified nuclear network can be easily integrated for a small time step. The APPROX13 nuclear network has its own integrator.

Criteria that are used for determination of time and space discretization of parameters are the following:

$$\Delta t = \min_i (\Delta t_{\text{courant},i}, \Delta t_{\text{nucl},i}),$$  \hspace{1cm} (11)

where $\Delta t_{\text{courant},i}$ is the Courant condition for $i$-cell and $\Delta t_{\text{nucl},i}$ is the time step at which all reagent concentrations change not more than by 1%.

B. Results of simulations

Now we will present the results of our computations. The flame speed is determined by the time evolution of the coordinate of the centre of the front $x(t)$ (by definition it is the point, where $c_{C_{12}} = 0.5$). An example of the time dependence, $x(t)$, is presented in Fig. 1. In Fig. 2 the coordinate dependence of the temperature for a sequence of times is shown. We see that some time after the ignition the flame stabilizes and propagates with constant velocity. Fitting the $x(t)$ dependence we obtain the velocity of flame relative to the wall, $v$. The normal flame velocity is then $v_n = \rho_0 v/\rho_u$. Table [1] presents the results of simulations for different parameters of initial matter conditions for all nuclear network variants. Comparing our results with those of [6] we can claim a good (but not excellent) agreement.

Fig. 3 shows coordinate dependence of $c_i$ for different elements at fixed time for APPROX13. The numerically obtained results for 1-step nuclear network are in good agreement with theoretical estimates (table [1]). The results with APPROX13 show that the flame speed is about an order of magnitude slower. It is an important result showing that the pure carbon burning to Mg is not a good approximation, despite the fact that it dominates at early stages of the burning (see Fig. 3).

Flame speed obtained with APPROX13 (which is supposed to be closer to the real speed) can be calculated theoretically if we substitute the rate of reaction $^{24}$Mg($\alpha, \gamma$)$^{28}$Si in Eq. [3] instead of
TABLE II: Results of the flame simulations:

| Compos. | \( \rho_0 \), g/cm\(^3\) | Calor. | \( T_{\text{max}} \), 10\(^9\) K | \( \rho_c/\rho_0 \) | \( v_\alpha \), km/s | \( \Delta x_{\text{fr}} \), cm |
|---------|----------------|--------|-----------------|--------------|----------------|-----------------|
| \(^{12}\)C | \( 2 \times 10^8 \) | Mg | 6.9 | 1.54 | 70.1 | 1.7 \( \times 10^{-4} \) |
| | | Ni | 7.9 | 1.85 | 122 | 1.2 \( \times 10^{-4} \) |
| | | APROX13 | 6.8 | 1.60 | 18.2 | 3.0 \( \times 10^{-4} \) |
| | | Mg | 9.1 | 1.33 | 302 | 1.8 \( \times 10^{-5} \) |
| | | Ni | 10.7 | 1.57 | 470 | 1.5 \( \times 10^{-5} \) |
| | | APROX13 | 8.5 | 1.35 | 55.4 | 4 \( \times 10^{-4} \) |
| | | Mg | 11.3 | 1.26 | 854 | 5.5 \( \times 10^{-6} \) |
| | | Ni | 13.8 | 1.40 | 1241 | 1.0 \( \times 10^{-5} \) |
| | | APROX13 | 9.8 | 1.23 | 134 | 1 \( \times 10^{-4} \) |
| \(^{16}\)O | \( 2 \times 10^8 \) | APROX13 | 6.1 | 1.38 | 0.94 | 2 \( \times 10^{-2} \) |
| | \( 2 \times 10^9 \) | APROX13 | 8.8 | 1.17 | 22.1 | 5 \( \times 10^{-4} \) |

FIG. 1: An example of the front coordinate dependence on time for \( \rho = 2 \times 10^9 \) g/cm\(^3\); APROX13 nuclear network is used.

FIG. 2: Sequential profiles of temperature for \( t_1 < t_2 < t_3 < t_4 \), \( \rho = 2 \times 10^9 \) g/cm\(^3\), APROX13 nuclear network

2C\( \rightarrow \)Mg. The speed should decrease by the factor \( \sqrt{R_{\text{CMB}}(T_9 \approx 10)/R_{\text{MgSi}}(T_9 \approx 10)} \sim 10 \). This result is in accordance with simulations. Reactions with \( \alpha \) particles dominate the whole burning and have more or less the same rates (the charge \( Z \) of heavy ions in the governing reactions does not vary significantly), so this estimation is accurate for all \( \alpha \)-reactions.

IV. THERMAL INSTABILITY OF NUCLEAR FLAMES

Let now consider thermal-pulsational instability with switched-off hydrodynamics. In this case, because of absence of sonic waves, this instability can be described purely without interference with other physical phenomena. Two conditions are necessary for the development of the TP instability \([30, 32]\). First, thermal diffusion must be more than an order of magnitude larger than mass diffusion. This condition is undoubtedly satisfied in presupernovae where \( \text{Le} \gg 1 \). Second, the Zeldovich number \([47]\):

\[
Ze = \left( \frac{\partial \ln q}{\partial \ln T} \right)_{P, X_i},
\]

which characterizes the temperature dependence of the heating rate \( q \), must be high enough. Here \( P \) is the pressure, \( X_i \) (\( i = 1, 2, \ldots \)) are the abundances of the reactants, and the derivative is evaluated at the temperature of the burning matter. The thermal instability takes place when \( Ze \) is higher than a certain critical value \( Ze_c \). From the Arrhenius law, and in the approximation when the reaction zone is assumed to be negligibly thin in comparison to the preheat zone of the flame, it is found \( Ze_c = 4 + 2\sqrt{5} = 8.47 \) \([32]\). The recent numerical value \([48]\), obtained by relaxing the approximation of delta function kinetics, is \( Ze_c \approx 8.24 - 8.29 \).

When \( Ze \) is below the critical value the front velocity relaxes quickly to a constant value, but when it is larger than \( Ze_c \) the pulsations set in. This is illustrated in Fig. \([4]\).
FIG. 3: Coordinate dependence of different elements concentrations for aprox13 nuclear network

\[
\frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) + RY_A^2.
\]  

(13)

Here, we assume the kinetic equation

\[
\frac{dY_A}{dt} = -RY_A^2,
\]

(14)

and the rate of the reaction \( R = R(T) \) is taken to be

\[
R = \exp \left( -E_a \left( \frac{1}{T} - \frac{1}{T_b + 1} \right) \right).
\]

(15)

In our units the initial temperature is \( T_0 \), ashes have temperature \( T_b = T_0 + 1 \) and the activation energy is

\[
E_a = Ze \frac{T_b}{T_b - T_0} = Ze(1 + T_0).
\]

(16)

We have used the method of lines to simulate the equation of thermal conduction, that is the space coordinate was discretized by finite differences, while integration over time was done by ODE integrators using methods of [49] and [50]. See details of the numerical techniques in more complicated situation of hydrodynamical evolution in [51]. Our simulations for this simple model give \( Ze_c \approx 8.2 \).

Timmes and Woosley [6] simulated the flame fronts of different CO mixtures using one-dimensional time-dependent hydro code. They said nothing about this instability. At first glance this means that there is no TP instability in the degenerate presupernova conditions. However, later, Bychkov and Liberman [52] and Nomoto et al. [53] have argued that C+C and O+O flame fronts should be TP-unstable. In the approximation of delta function kinetics it is found in [52] that \( Ze_c = 8 + 4\sqrt{5} = 16.9 \), i.e. exactly by factor 2 larger than that for the Arrhenius law. It is even claimed by

FIG. 4: Pulsating front velocity for \( Ze = 8.3 \) and one reaction obeying the Arrhenius law

\begin{align*}
52 & \text{ that Timmes and Woosley [6] have overlooked the instability in their simulations.} \\
52 & \text{ We believe that the latter extreme proposition should} \\
52 & \text{ be rejected and we seek a real explanation of the apparent} \\
52 & \text{ CO front stability taking into account that Ze for basic} \\
52 & \text{ reactions of CO burning can indeed be higher than the} \\
52 & \text{ critical value. The following study of stability is based in} \\
52 & \text{ part on the analytical model by [54].} \\
52 & \text{ It is known that CO nuclear burning consists of the} \\
52 & \text{ basic (C,\( \alpha \)), (C,p), (O,\( \alpha \)), ... reactions accompanied by} \\
52 & \text{ secondary (p,\( \gamma \)) and (\( \alpha \),\( \gamma \)) reactions. It is very important that:} \\
52 & \text{ 1. Ze corresponding to the secondary reactions is} \\
52 & \text{ rather low because of the lower height of the} \\
52 & \text{ Coulomb barrier;} \\
52 & \text{ 2. the amount of energy yielded by the secondary re-} \\
52 & \text{ actions is comparable to the yield of the basic ones.} \\
52 & \text{ It is very difficult to investigate analytically any mod-} \\
52 & \text{ els with many reactants. It should be made numerically} \\
52 & \text{ as it was done in the Section III and by Timmes and} \\
52 & \text{ Woosley [6]. However, it is possible to develop a simple} \\
52 & \text{ analytical model [54] with one deficient reactant which} \\
52 & \text{ possesses in part the main properties of CO burning men-} \\
52 & \text{ tioned above, and which can help us to understand the} \\
52 & \text{ qualitative behaviour of the pulsating front. Moreover,} \\
52 & \text{ it is not hard to develop a numerical model with only} \\
52 & \text{ two reactions with different values of Ze numbers, one of} \\
52 & \text{ them above and the other below the critical value. This} \\
52 & \text{ model explains how the seeming contradiction between} \\
52 & \text{...} \\
52 & \text{...}
\end{align*}
the works \cite{52,53}, and the simulations in Ref. \cite{5} can be removed.

We may compare the front structure of the simplified models with the front structures of C(50\%) + O(50\%) and O(60\%) + Ne(30\%) + Mg(10\%) mixtures obtained in \cite{5}, comparing, e.g., the ratio of the thickness of the conductive zone to the thickness of the reaction zone which reflects the effective value of the \( Z_{E} \) number \cite{47}. Then we find that the front structure, calculated in \cite{5} is analogous to the model front structure, when \( Z_{E} \) belongs to the range 2 – 5. This range is noticeably below the critical value \( Z_{E,c} \). This fact suggests the explanation for the stability of CO flame front observed by Timmes and Woosley \cite{6}. Moreover, it shows that the secondary reactions of CO burning are very important from the viewpoint of the TP stability and that the models considered in \cite{52,53} are oversimplified.

Figs. 5-7 illustrate that for large enough \( Z_{E} \)-number the instability can develop even when the system is stabilized by the secondary reactions.

We use a simplified two-reactant system:

\[
\frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) + \left( \frac{3}{2} - \frac{1}{2} q_2 \right) R_1 Y_A^2 + q_2 R_2 Y_A Y_B. \tag{17}
\]

Here, we assume the following two kinetic equations:

\[
\frac{dY_A}{dt} = -R_1 Y_A^2 - R_2 Y_A Y_B, \tag{18}
\]

\[
\frac{dY_B}{dt} = + \frac{1}{2} R_1 Y_A^2 - R_2 Y_A Y_B, \tag{19}
\]

and the rates of the reactions \( R_i = R_i(T) \) are taken to be

\[
R_1 = \exp \left( -E_a \left( \frac{1}{T} - \frac{1}{T_0 + 1} \right) \right), \tag{20}
\]

and

\[
R_2 = \exp \left( -E_{a2} T \right) + \frac{E_a}{T_0 + 1}. \tag{21}
\]

Examples in this section were given for the case of the switched-off hydrodynamics. Now we turn to the flames with an account of hydrodynamics, which is more relevant for realistic supernovae.

V. FRONT PULSATIONS

Here we will switch on hydrodynamics again. According to the simulations in Section III with the full set of reactions the instability behaviour was not observed (see the text below). Now we will show that pulsational behaviour can exist in our simulations in the case of artificially high Zeldovich numbers and will confirm that carbon and oxygen burning are stable. To carry out this task we should change the nuclear rate function. We will treat our reaction as one-step with rate as Arrhenius law:

\[
R(T) = A e^{-B/T_0}. \tag{22}
\]

The constant \( B \) defines the Zeldovich number. \( Z_{E} \) depends on the temperature of burning matter, according to definition \cite{12}:

\[
Z_{E,\text{Arren}} = \frac{B}{T_0 \text{ burned}}. \tag{23}
\]

The value of constant \( A \) does not play any role here, but for definiteness we will determine it by the relation:

\[
\int_{T_1}^{T_2} R(T_9) dT_9 = \int_{T_1}^{T_2} R_{\text{real}}(T_9) dT_9, \tag{24}
\]

\[
A(B) = \frac{T_2}{T_1} \int_{T_1}^{T_2} R_{\text{real}}(T_9) dT_9 \left( T_9 e^{-B/T_9} dT_9 \right)^{-1}. \tag{25}
\]

By \( R_{\text{real}}(T) \) we assume here our variant with the simplified nuclear network. The nuclear caloricity is preserved from this reaction too (we will demonstrate this explicitly). From the results of the previous section we set the boundary temperatures as \( T_1 = 1, T_2 = 10 \). Fig. 8 shows different rates used, compared to the “real rate”. We see that the rate with \( B \approx 50 \) fits the physical carbon burning rate very nicely. This means that when \( T_{9,up} \approx 10 \),
Ze \approx 5, which coincides with earlier estimates. Table III presents results for different runs with \( \rho_0 = 2 \cdot 10^9 \) g/cm\(^3\). We increase \( B \) from 20 to 200 and simultaneously decrease caloricity \( q \) (from Ni to Mg) to avoid the flame acceleration. We see that for the normal caloricity (\( q = (5.6 - 9.2) \cdot 10^{17} \) erg/g) no pulsations appear for all variants of rates (upper part of the table). Zeldovich number, \( Ze \), can be additionally increased by decreasing \( T_{9\text{burned}} \). This is done by caloricity change to \( q = 5 - 7 \) MeV for one reaction (\( q = (2.0 - 2.8) \cdot 10^{17} \) erg/g). In this case pulsations appear for \( B > B_{\text{crit}} \approx 112.5 \) (lower part of the table). Example of flame coordinate dependence is depicted in Fig. 9.

So the result is that in “real” system (physical EOS and thermoconductivity) pulsations can exist, but only in the case of artificial parameters: the Zeldovich number should be increased by a factor of \( \sim 4 \). Table IV presents the critical values of \( Ze \) number found in our numerical experiments for different densities of matter.

In the case of complex nuclear network (many reactions) \( Ze \) number cannot be obtained in such straightforward way, because it depends on the whole history of burning \( q(T) \). The front is stable in simulations with APROX13, presented in Section III for all runs. The only run must be considered carefully: That with initial \( ^{16}\text{O} \) and \( \rho = 2 \cdot 10^8 \) g/cm\(^3\) demonstrates some irregular structure in \( x(t) \) dependence (see Fig. 11) compare with Fig. 9). This case is the best candidate for pulsations because of low \( T_b \) and strong temperature dependence. But the period of these pulsations, \( \tau_1 \sim 3 \cdot 10^{-10} \) s, is close to \( \tau_2 = \Delta x \rho_b / (u_n \rho_a) = 8 \cdot 10^{-10} \) s, so it is probably a numerical effect. Moreover, it does not contribute to physical effects like changing the flame velocity.

VI. CONCLUSIONS

The one-dimensional flame propagation in presupernova white dwarf has been considered. Flame properties for different densities were obtained. It is shown that when only one nuclear reaction in the nuclear network is considered the flame velocity differs strongly from
FIG. 9: Pulsational regime of flame with artificially increased Zeldovich number

FIG. 10: Flame coordinate for run with $^{16}\text{O}$, $\rho = 2 \cdot 10^8$ g/cm$^3$, APROX13 network

TABLE III: Results of simulations with Arrhenius law:

|    | A      | B   | q, $10^{17}$ erg/g | $T_{\text{burned}}$ | comm. |
|----|--------|-----|-------------------|---------------------|-------|
| 1  | 3.68 $\cdot 10^6$ | 20.0 | 9.2               | 13.5                | flame |
| 2  | 1.38 $\cdot 10^8$ | 50.0 | 9.2               | 13.5                | flame |
| 3  | 7.62 $\cdot 10^{12}$ | 150.0 | 5.6               | 11                  | flame |
| 4  | 1.47 $\cdot 10^{15}$ | 200.0 | 5.6               | 14                  | deton. |
| 5  | 7.62 $\cdot 10^{12}$ | 150.0 | 2.8               | 10                  | puls.  |
| 6  | 3.60 $\cdot 10^{10}$ | 100.0 | 2.8               | 8                   | flame  |
| 7  | 5.32 $\cdot 10^{11}$ | 125.0 | 2.8               | 8                   | puls.  |
| 8  | 1.39 $\cdot 10^{11}$ | 112.5 | 2.8               | 8.2                 | flame  |
| 9  | 1.39 $\cdot 10^{11}$ | 112.5 | 2.0               | 7.8                 | puls.  |
| 10 | 1.39 $\cdot 10^{11}$ | 112.5 | 2.4               | 8                   | puls.  |

TABLE IV: Critical Zeldovich number for carbon burning at different densities:

| $\rho$, g/cm$^3$ | $Z_{\text{cr}}$ |
|------------------|-----------------|
| $2 \cdot 10^8$   | 18.4 < $Z_e$ < 21.4 |
| $7 \cdot 10^8$   | 15.3 < $Z_e$ < 16.7 |
| $2 \cdot 10^9$   | 13.7 < $Z_e$ < 14.1 |

We have studied also the one-dimensional pulsational instability. In “real” simulations no pulsation regime was obtained, so we used an artificial Arrhenius law to make it possible to change Zeldovich number, $Z_e$. For high $Z_e$ numbers the pulsations do exist. By means of numerical simulations we have obtained critical Zeldovich numbers for densities in the range $\rho = [2 \cdot 10^8, 2 \cdot 10^9]$ g/cm$^3$. These values are larger than those in real flame, which proves the one-dimensional stability of realistic flame fronts in supernovae.

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