Metric and coupling reversal in string theory

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\textbf{Abstract}

Invariance under reversing the sign of the metric $G_{MN}(x)$ and/or the sign of the string coupling field $H(x)$, where $\langle H(x) \rangle = g_s$, leads to four possible Universes denoted $1, \mathcal{I}, \mathcal{J}, \mathcal{K}$ according as $(G,H) \rightarrow (G,H), (-G,H), (-G,-H), (G,-H)$, respectively. Universe $1$ is described by conventional string/M theory and contains all M, D, F and NS branes. Universe $\mathcal{I}$ contains only D(-1), D3 and D7. Universe $\mathcal{J}$ contains only D1, D5, D9 and Type I. Universe $\mathcal{K}$ contains only F1 and NS5 of IIB and Heterotic SO(32).

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1 Introduction

1.1 Signature and coupling reversal

In a previous paper [1] we identified what class of field theory is invariant under reversing the sign of the metric tensor

$$G_{MN}(x) \rightarrow -G_{MN}(x)$$

induced by a chiral transformation on the curved space gamma matrices

$$\Gamma_M \rightarrow \Gamma M ,$$

where

$$\{ \Gamma_M(x), \Gamma_N(x) \} = 2G_{MN}(x)1 ,$$

and $\Gamma$ is the normalised chirality operator

$$\Gamma \equiv \frac{1}{\sqrt{G}} \frac{1}{D!} \epsilon^{M_1 \cdots M_D} \Gamma_{M_1 \cdots M_D} .$$

We concluded that theories must be chiral and require signature $(S, T)$ with $S - T = 4k$ in order that the Clifford algebra be symmetric. Under (2) the volume element transforms as

$$\sqrt{G} d^D x \rightarrow (-1)^{D/2} \sqrt{G} d^D x ,$$

while the curvature scalar flips sign for all $D$

$$R \rightarrow -R .$$

So for gravitational theories the requirement of invariance then selects out the dimensions

$$D = 4k + 2 , \quad k = 0, 1, 2, 3 \ldots$$

In $D=10$, for example, the chiral Type IIB supergravity is invariant while the non-chiral Type IIA supergravity is not.

In this paper we turn our attention to chiral string theory in $D=10$. It is now useful to extend the reversal symmetry to include the dilatonic string coupling field $H(x)$ whose vev is the string coupling constant

$$\langle H(x) \rangle = g_s .$$

As for the $B$-field 2-form, we shall see that its reversal is correlated with reversing $G$ and so we do not treat it as an independent transformation; the axion $C_0$

Note that we are reversing the field $H(x)$ and not $g_s$, and that the non-zero vev implies that the symmetry is spontaneously broken [1].
Table 1: Symmetries and branes in the four Universes.

| Universe | 1     | I     | J     | K     |
|----------|-------|-------|-------|-------|
| Symmetry| \((G,H)\) | \((-G,H)\) | \((-G,-H)\) | \((G,-H)\) |
| Branes   | All   | D(-1), D3, D7 | D1, D5, D9 | F1, NS5 |

transformation is similarly related to that of \(H\), so that the complex parameter \(\tau \equiv C_0 + iH^{-1}\) transforms homogeneously\(^4\).

We shall show that this leads to four possible Universes denoted \(1, I, J, K\) according as \((G,H) \rightarrow (G,H), (-G,H), (-G,-H), (G,-H)\), respectively. Universe \(1\) is described by conventional string/M theory and contains all M, D, F and NS branes. Universe \(I\) contains only D(-1), D3 and D7. Universe \(J\) contains only D1, D5, D9 and Type I. Universe \(K\) contains only F1 and NS5 of IIB and Heterotic \(SO(32)\).

First of all we show in Section 2 that at the level of supergravities, Type IIB is compatible with \(1, I, J, K\); Type I is compatible with \(1, J\); Heterotic \(SO(32)\) is compatible with \(1, K\) while Type IIA and Heterotic \(E_8 \times E_8\) are compatible only with \(1\). However, we need a finer distinction when we consider branes which we approach from several different points of view:

Section 3 Coupling to Dirac-Born-Infeld brane actions
Section 4 \(\kappa\)-symmetry
Section 5 Brane soliton solutions
Section 6 Invariance properties of string loop and \(\alpha'\) corrections, and anomalies
Section 7 Supersymmetry algebras

All these viewpoints will lead to the same \(1, I, J, K\) classification.

### 1.2 Metrics and dilatons

When we reverse the sign of the metric \(G\) and/or dilaton \(H\), it is important to know which metric and which dilaton we have in mind as there are several different ones that appear in string theory. For these purposes it is useful to consider the six strings of table 2 (there are no strings in Universe \(I\)), which are, however, related under S-duality as in table 3.

\(^4\)In the quantum theory \(\tau\) transforms under SL\((2, \mathbb{Z})\). It may seem unusual to find an action of a discrete group SL\((2, \mathbb{Z})\) on the complex parameter \(\tau\) which takes values not restricted to the upper half of the complex plane. Interestingly enough, such a situation was also recently encountered in [2]. The context was somewhat different but also involved orientation reversal.
Table 2: There are three metrics and three dilatons.

| $N = 2$ | $K$ | $J$ |
|---------|-----|-----|
| Type IIA $\rightarrow$ Type IIB$_{F1}$ $\rightarrow$ Type IIB$_{D1}$ | $G_{A} = G_{E}$ | $G_{F} = G_{O}$ | $G_{D} = G_{I}$ |
| $N = 1$ |Het$_{E_{8} \times E_{8}}$ $\rightarrow$ Het$_{SO(32)}$ $\rightarrow$ Type I | $H_{A} = H_{E}$ | $H_{F} = H_{O}$ | $H_{D} = H_{I}$ |

Table 3: S-dualities between metrics and dilatons of Universes $K$ and $J$.

| $K$ | $J$ |
|-----|-----|
| $G_{F}$ | $H_{D}^{-1}G_{D}$ |
| $H_{F}$ | $H_{D}^{-1}$ |
| $G_{O}$ | $H_{I}^{-1}G_{I}$ |
| $H_{O}$ | $H_{I}^{-1}$ |

Under T-duality, suppressing the Neveu-Schwarz 2-form and the graviphoton for simplicity,

\[(G_{A})_{MN} = \begin{cases} (G_{F})_{MN} & \text{for } M, N = \mu, \nu \\ (G_{F}^{-1})_{MN} & \text{for } M, N = m, n \end{cases}, \tag{9}\]

where $\mu, \nu$ correspond to spacetime directions and $m, n$ to the compact directions in which the T-duality is performed. The dilaton factors are related by

\[H_{A}^{-1} = \sqrt{\det(G_{F})_{mn}H_{F}^{-1}}. \tag{10}\]

There are similar T-duality formulas relating $G_{E}$ to $G_{O}$, and $H_{E}$ to $H_{O}$.

In what follows we shall take our reference metric and dilaton to be those of Type IIB$_{F}$, so $(G, H)$ shall always refer to $(G_{F}, H_{F})$.

## 2 Supergravity

### 2.1 Type IIB

The bosonic Type IIB supergravity action is given by [3]

\[S_{IIB} = S_{NS} + S_{R} + S_{CS}. \tag{11}\]
\[ S_{NS} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{G} H^{-2} \left( R + 4H^{-2}(\partial H)^2 - \frac{1}{12}|H_3|^2 \right) \]  
\[ S_R = -\frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{G} \left( |F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2 \right) \]  
\[ S_{CS} = -\frac{1}{4\kappa_{10}^2} \int C_4 \wedge H_3 \wedge F_3 \]  
\[ (12) \]  
\[ (13) \]  
\[ (14) \]

where

\[ F_{p+1} = dC_p \]  
\[ \tilde{F}_3 = F_3 - C_0 \wedge H_3 \]  
\[ \tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3 \]  
\[ (15) \]  
\[ (16) \]  
\[ (17) \]

and where we must impose the extra self-duality constraint

\[ \star \tilde{F}_5 = \tilde{F}_5 . \]  
\[ (18) \]

Since both \( S_{NS} \) and \( S_R \) contains field strengths only of odd rank, it is invariant under signature reversal of the string metric

\[ G_{MN} \longrightarrow -G_{MN} . \]  
\[ (19) \]

It is also invariant under

\[ H \longrightarrow -H . \]  
\[ (20) \]

As explained in [1] the fermionic kinetic terms

\[ \int d^Dx \sqrt{G} \Gamma^M D_M \lambda \quad \text{and} \quad \int d^Dx \sqrt{G} \Gamma^{MNP} D_N \psi_P \]  
\[ (21) \]

can be invariant in signature reversal only if the fermions are chiral. This is the case in both Type IIB and the Type I supergravities. In particular in Type IIB the complex gravitino and the complex dilatino satisfy

\[ \Gamma \psi_M = +\psi_M \]  
\[ (22) \]  
\[ \Gamma \lambda = -\lambda . \]  
\[ (23) \]

This means that their kinetic terms are invariant under signature reversal.

The interacting fermionic structure of the theory is captured in the supersymmetry transformation rules. It is straightforward to check [1] that they are form invariant under signature reversal when the bosonic fields are transformed suitably as well:

The string frame type IIB supersymmetry algebra [4] turns indeed out to remain form invariant in three different transformations

\[ (G, H) \longrightarrow (\alpha G, \beta H) , \]  
\[ (24) \]
where \( \alpha, \beta = \pm 1 \), when the gauge fields are transformed according to

\[
\begin{align*}
C_0 & \rightarrow \beta C_0 \quad (25) \\
B_2 & \rightarrow \alpha B_2 \quad (26) \\
C_2 & \rightarrow \alpha\beta C_2 \quad (27) \\
C_4 & \rightarrow \beta C_4 . \quad (28)
\end{align*}
\]

It follows that the standard RR field strengths, the complex scalar \( \tau \equiv C_0 + iH^{-1} \) and the T-duality covariant matrix \( G + B \) then transform homogeneously. The supercovariant field strengths are left invariant as well under signature reversal.

We conclude that the Type IIB supergravity is invariant under change of signature in all three sectors

\[
(G, H) \rightarrow (-G, H) , \ (-G, -H) , \ (G, -H) . \quad (29)
\]

### 2.2 Type I and Type IIB

The Type I supergravity action is given by [3]

\[
S_I = S_c + S_o \quad (30)
\]

where

\[
\begin{align*}
S_c &= \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{G} \left( H^{-2} (R + 4H^{-2}(\partial H)^2) - \frac{1}{12} |\tilde{F}_3|^2 \right) \quad (31) \\
S_o &= -\frac{1}{2g_{10}^2} \int d^{10}x \sqrt{G} \cdot H \cdot \text{Tr} |F_2|^2 , \quad (32)
\end{align*}
\]

and where

\[
\tilde{F}_3 = dC_2 - \frac{\kappa_{10}^2}{g_{10}^2} \omega_3 , \quad (33)
\]

and \( \omega_3 \) is the Yang-Mills Chren-Simons 3-form

\[
\omega_3 = \text{Tr} \left( A_1 \wedge dA_1 - \frac{2i}{3} A_1 \wedge A_1 \wedge A_1 \right) . \quad (34)
\]

At first sight this seems not to be invariant under signature flip of the Type I metric

\[
G_{MN} \rightarrow -G_{MN} \quad (35)
\]

because the Yang-Mills term \( S_o \) involves field strengths of even rank. However, in contrast to \( S_c \), this term is linear in \( H \), so if we simultaneously perform a coupling flip

\[
H \rightarrow -H \quad (36)
\]

then the bosonic part of the Type I supergravity is invariant. The transformations (35) and (36) imply that the RR gauge field \( C_2 \) is left invariant under signature
reversal; it transforms therefore in the same way as the Chern-Simons term in (33) leaving $\tilde{F}_3$ invariant.

The simplest way to understand the structure of fermion interactions in Type I supergravity is to notice that the closed string sector can be understood as a truncation of the IIB supergravity theory

$$C_0 = C_4 = B_2 = 0.$$  

This means in particular that the Type I string and five-brane are the Type IIB D-string and D5-brane. The $N = 2$ doublet of real fermions $f^i = \psi^i_M, \lambda^i$ in Type IIB are already chiral, and in Type I one sets

$$(1 \pm J) f = 0.$$  

The fact that closed string sector fermionic interactions are invariant follows from the fact that Type IIB supergravity has signature reversal symmetry.

The open string sector in Type I arises in the Type IIB picture by adding a suitably symmetrised set of D9-branes. This determines, in particular, that the gauge sector including the Yang-Mills fields and the gaugino kinetic term is all multiplied by the same dilaton factor $H^{-1}$ as what played an important rôle in the bosonic action. As the dilaton factor changes sign in reversal of signature, we find the opposite chirality condition for the kinetic term of the gaugino to that of the dilatino.

We conclude that the Type I supergravity is invariant under change of signature in Universe $J$

$$(G, H) \rightarrow (-G, -H).$$  

### 2.3 Heterotic $SO(32)$ and Type IIB$_F$

The Heterotic $N = 1$ supergravity in $D = 10$ with gauge group $SO(32)$ has the bosonic action

$$S_{het} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{GH^{-2}} \left( R + 4H^{-2}(\partial H)^2 - \frac{1}{12} |\tilde{H}_3|^2 - \frac{\kappa_{10}^2}{g_{10}^2} \text{Tr} |F_2|^2 \right)$$  

where

$$\tilde{H}_3 = dB_2 - \frac{\kappa_{10}^2}{g_{10}^2} \omega_3.$$  

Since the Yang-Mills term is quadratic in $H$, Heterotic $SO(32)$ supergravity requires no flipping the sign of the heterotic metric

$$G_{MN} \rightarrow G_{MN}$$  

though it is invariant under coupling flip

$$H \rightarrow -H.$$  

The Heterotic $SO(32)$ supergravity is therefore invariant under the symmetry

$$(G, H) \rightarrow (G, -H).$$
2.4 Type IIA and Heterotic $E_8 \times E_8$

The Type IIA supergravity has the bosonic action

$$S_{IIA} = S_{NS} + S_R + S_{CS}$$  \hspace{1cm} (45)

where

$$S_{NS} = \frac{1}{2\kappa^2_{10}} \int d^{10}x \sqrt{G_A} H^{-2}_A \left( R + 4H^{-2}_A(\partial H)_A^2 - \frac{1}{12} |H_3|^2 \right)$$  \hspace{1cm} (46)

$$S_R = -\frac{1}{4\kappa^2_{10}} \int d^{10}x \sqrt{G_A} \left( |F_2|^2 + |\tilde{F}_4|^2 \right)$$  \hspace{1cm} (47)

$$S_{CS} = -\frac{1}{4\kappa^2_{10}} \int B_2 \wedge F_4 \wedge F_4 ,$$  \hspace{1cm} (48)

and where

$$\tilde{F}_4 = dC_3 - C_1 \wedge H_3 .$$  \hspace{1cm} (49)

Only even powers of the dilaton $H_A$ enter the action in $S_{NS}$, so that the Lagrangean is invariant under

$$H_A \rightarrow -H_A .$$  \hspace{1cm} (50)

However, since $S_R$ contains RR field strengths of even rank, it is not invariant under signature reversal.

As the gravitino and the dilatino have components of both chiralities, the fermionic action will not be invariant in Type IIA. Apart from the kinetic terms, this can be seen by considering supersymmetry in opposite signatures. The supersymmetry transformation rule for the graviton is in string frame

$$\delta G_{MN} = \mp \Gamma_{(M} \Psi_{N)} .$$  \hspace{1cm} (51)

Under the reversal of signature in 10D the graviton $\delta G_{MN}$ variation maps to

$$\mp \Gamma_{(M} \Psi_{N)} \rightarrow -\mp \Gamma_{(M} \Gamma \Psi_{N)} .$$  \hspace{1cm} (52)

This is consistent with $\delta G_{MN} \rightarrow -\delta G_{MN}$ only when gravitini have positive chirality in 10D. The NS 2-form has to be odd as its supersymmetry transformation reads

$$\delta B_{MN} = -\frac{\sqrt{2}}{8} \mp \Gamma_{MN} \lambda - \frac{\sqrt{2}}{4} H^{-2}_A \mp \Gamma_{N} \psi_M$$  \hspace{1cm} (53)

$$= -\frac{\sqrt{2}}{8} \mp \Gamma_{MN} \lambda - \frac{\sqrt{2}}{4} H^{-2}_A \mp \Gamma_{N} \psi_M .$$  \hspace{1cm} (54)

For the RR 3-form we get

$$\delta C_{MNP} = -\frac{\sqrt{2}}{8} \mp \Gamma_{[MN} \psi_P]$$  \hspace{1cm} (55)

$$= 0 .$$  \hspace{1cm} (56)
The Neveu-Schwarz 2-form is naturally odd in signature reversal but, given the chiral structure of the theory, the 3-form has to vanish. This can be seen by considering the gravitino transformation

$$\delta \psi_M = D_M(\hat{\Omega}) \epsilon + \frac{\sqrt{2}}{288} H_A^2 \left( \Gamma_M^{NKL} - 8 \delta_M^N \Gamma^{KL} \right) \epsilon H_{NKL}$$

$$+ \frac{\sqrt{2}}{288} \left( \Gamma_M^{NKL} - 8 \delta_M^N \Gamma^{KLP} \right) \epsilon F_{NKL} \ ,$$

where the term proportional to $F_{NKL}$ has negative chirality. The invariant sector of the theory has therefore

$$F_{NKL} = 0 \ .$$

The same results follow from considering supercovariant quantities or the fermion interactions in the Type IIA Lagrangean. The symmetric subsector is given by restricting the Type IIA fields as follows:

$$\Gamma \psi_M = + \psi_M$$

$$\Gamma \lambda = - \lambda$$

$$C_M = 0$$

$$C_{MNP} = 0 .$$

This breaks $N = 2$ to a $N = 1$ supergravity.

In this subsector we can define the following formal invariance:

$$\text{Vol}(M_{10}) \rightarrow + \text{Vol}(M_{10})$$

$$G_{MN} \rightarrow - G_{MN}$$

$$H_A^{-2} \rightarrow - H_A^{-2}$$

$$H_{MNP} \rightarrow - H_{MNP} .$$

That the Type IIA is invariant under this operation can be seen also by T-dualising the signature reversal symmetry of Type IIB supergravity as will be shown in section 2.1. As the dilaton field becomes imaginary in it, it is not a true symmetry of a fixed real form of the Type IIA supergravity, however.\footnote{If one were to adopt a more liberal attitude to the allowed set of transformations, for example of the kind discussed in section 1.3 of [1], one may be able to produce a finer classification of Type IIA branes and interactions beyond those of Universe 1.}

To summarise, we have found that a formally invariant sector of the Type IIA supergravity which has $N = 1$ supersymmetry. As the action of signature reversal matches the one that can be obtained from Type IIB by T-dualising, we identify the $N = 1$ theory with the Heterotic supergravity with gauge group $E_8 \times E_8$.

Therefore, neither Type IIA nor Heterotic $E_8 \times E_8$ are invariant under change of signature. The reason for this is fundamentally that the dilaton becomes imaginary in the formal operation that corresponds to signature reversal in Type IIB.
2.5 T-duality

Suppose the background has an isometry along the ninth coordinate \( x^9 \in S^1 \). This means that there is a T-dual configuration in Type IIA, with

\[
\Gamma^A_9 = (\Gamma^B_9)^{-1}.
\]  

The other gamma matrices \( \Gamma^A_\mu \) coincide with \( \Gamma^B_\mu \) when the \( B_9\mu \) components of the NS 2-form are trivial; it turns out that for the purposes of the following analysis assuming this is not a restriction. The dilatons are related by

\[
H_B = k_9 H_A,
\]

where \( k_9^{+2} = G_{99} \). The chirality operator \( \Gamma \) is invariant under T-duality.

As argued in previous sections, a change of signature in Type IIB supergravity amounts to

\[
\begin{align*}
\Gamma^B_M &\longrightarrow +\Gamma^B_M \\
H_B &\longrightarrow \pm H_B.
\end{align*}
\]

In what follows we concentrate on the transformation with the upper positive sign above.

On a circle background this defines formally a symmetry of the Type IIA supergravity as well

\[
\begin{align*}
\Gamma^A_\mu &\longrightarrow +\Gamma^A_\mu \quad \mu = 0, \ldots, 8 \\
\Gamma^A_9 &\longrightarrow -\Gamma^A_9 \\
H_A &\longrightarrow iH_A.
\end{align*}
\]

The change of sign in (73) is a consequence of (68). This is not a symmetry of the Type IIA theory because it involves a complex dilaton field (74). See, however, footnote 5.

A further duality along a Killing direction \( x^8 \in S^1 \) back to Type IIB changes one of the gamma matrices and the dilaton

\[
\begin{align*}
\tilde{\Gamma}^B_8 &= (\Gamma^A_8)^{-1} = (\Gamma^B_8)^{-1} \\
\tilde{H}_B &= k_9 \frac{k_8}{k_B} H_B,
\end{align*}
\]

where \( k_8^{-2} = G_{88} \). This induces a symmetry in the new Type IIB variables

\[
\begin{align*}
\tilde{\Gamma}^B_\mu &\longrightarrow +\tilde{\Gamma}^B_\mu \quad \mu = 0, \ldots, 7 \\
\tilde{\Gamma}^B_i &\longrightarrow -\tilde{\Gamma}^B_i \quad i = 8, 9 \\
\tilde{H}_B &\longrightarrow -\tilde{H}_B.
\end{align*}
\]

\footnote{In this section we distinguish between Type IIA and Type IIB fields by subscripts \( A \) and \( B \).}
The action of this transformation differs from a covariant operation by a reflection of \((x^8, x^9) \mapsto (-x^8, -x^9)\). Note that this operation preserves the orientation.

We are therefore back to a symmetry of a Type IIB supergravity, although with the dilaton changing sign as well.

Quite generally, \(2n\) simultaneous T-dualities take us from the standard transformation (70) – (71) to

\[
\tilde{\Gamma}_\mu \rightarrow +\tilde{\Gamma}_\mu \quad \mu = 0, \ldots, 9 - 2n \\
\tilde{\Gamma}_i \rightarrow -\tilde{\Gamma}_i \quad i = 10 - 2n, \ldots, 9 \\
\tilde{H}_B \rightarrow (-1)^n \tilde{H}_B
\]

in the new T-dual Type IIB fields.

### 3 Coupling to branes

#### 3.1 F1 and NS5 branes

The Nambu-Goto action of a Type IIB or a Heterotic SO(32) fundamental string, both with \(d = 2\), is given by

\[
S_F^2 = -T_2 \int d^2 \xi \sqrt{g},
\]

where \(g_{ij}\) is the induced metric on the brane

\[
g_{ij} = \partial_i X^M \partial_j X^N G_{MN}(X),
\]

and in the worldvolume signature \((s, t)\) we define again

\[
\sqrt{g} \equiv \sqrt{(-1)^t \det g_{ij}}.
\]

The Nambu-Goto action of an NS5-brane, with \(d = 6\),

\[
S_F^6 = -T_6 \int d^6 \xi H^{-2} \sqrt{g},
\]

depends similarly of only even powers of the dilaton factor \(H\).

Due to (84), a signature reversal in the bulk pulls back to a signature reversal on the brane

\[
g_{ij} \rightarrow -g_{ij}.
\]

The volume element on the brane \(\sqrt{g} d^d \xi\) is built out of the same pull-back metric, and a choice of reference orientation on the brane \(\epsilon^{i_1 \cdots i_d}\). To understand its behaviour under signature reversal we express it in terms of the Clifford algebra valued representation

\[
\gamma := \frac{1}{\sqrt{g} d!} \partial_{i_1} X^{M_1} \cdots \partial_{i_d} X^{M_d} \epsilon^{i_1 \cdots i_d} \Gamma_{M_1 \cdots M_d}.
\]
This object squares to $\gamma^2 = \pm 1$, and has a fundamental rôle in $\kappa$-symmetry in section 4. For $\gamma$ to have real eigenvalues, we impose a restriction on the signature of the brane worldvolume

$$s - t = 4l'$$

(89)

for some integer $l'$. The fact that in the bulk we have already required $S - T = 4k'$ guarantees that the representation of the Clifford algebra does not change in signature reversal.

Under signature reversal in the bulk, the Clifford algebra valued orientation transforms as

$$\gamma \rightarrow (-1)^t \gamma,$$

(90)

and we deduce

$$\sqrt{g} \, d^d\xi \rightarrow (-1)^t \sqrt{g} \, d^d\xi.$$  

(91)

This result is consistent with the naïve counting as

$$(-1)^t = (-1)^{d/2}$$

(92)

in these signatures.

This means that the F1 string and the NS5-brane with Minkowski signature are never invariant under signature reversal on the worldvolume, and belong to the Universe $\mathcal{K}$, where

$$(G, H) \rightarrow (G, -H).$$

(93)

### 3.2 Type I and D-branes

The coupling of Type IIB supergravity to a D-brane with worldvolume dimension $d$ is given by the action [3]

$$S^D_d = -T_d \int d^d\xi \, H^{-1} \sqrt{g}.$$  

(94)

Using the naïve rule (5) for the volume element, and without requiring that the signature should satisfy any conditions, this is invariant under changes of spacetime signature for

$$d = 4k, \quad k = 0, 1, 2, 3 \ldots$$

(95)

a multiple of four. In $d = 4k + 2$ the action can be made invariant by transforming the dilaton factor $H(x)$ as well. In order to justify this ad hoc result, we really need the $\kappa$-symmetry argument of section 4: It will turn out to lead to the same result.

As the dilaton factor in the Dirac-Born-Infeld action $H$ is a square root of the dilaton factor $H^2$ that generates the perturbation theory of the closed oriented
Type IIB string, we have the freedom to choose its transformation properties independently

\[ H \rightarrow \pm H \]  

(96)

without affecting the supergravity theory in the bulk.

D-brane worldvolume actions are symmetric under the change of signature provided the sign of \( H \) compensates for the sign coming from the volume element. We have discussed the behaviour of the volume element in detail in terms of the bulk reflection in (2), and in terms of the brane reflection in (105).

Depending on the sign (96) the D-brane theory now decomposes to two independent sectors:

\( \mathcal{I} \). For \((G, H) \rightarrow (-G, H)\) we have the D(-1), D3, and D7 branes and the double T-duals of D1, D5, and D9.

\( \mathcal{J} \). For \((G, H) \rightarrow (-G, -H)\) we have the D1, D5, and D9 branes and the double T-duals of D3, and D7.

In the former case S-duality is a symmetry of the remaining theory, whereas in the latter case it is not. S-duality exchanges Universes \( \mathcal{J} \) and \( \mathcal{K} \).

Note that, in spite of the flip in \( H \), the brane tension always remains positive.

The Nambu-Goto action for a Type I string is also of the D1 type with

\[ S_2^D = -T_2 \int d^2 \xi \ H^{-1} \sqrt{g} \]  

(97)

as may be seen from the S-duality rules in section 1.2; the Type I string inhabits, therefore, Universe \( \mathcal{J} \).

4 \( \kappa \)-symmetry and the brane orientation

There is a fermionic gauge symmetry on the brane, \( \kappa \)-symmetry, that combines with the global supersymmetry in the bulk to induce a global supersymmetry on the brane. Branes in various signatures are discussed in [5, 6]. Kappa-symmetry is chiral, in a certain sense, as the transformation rules can be written in terms of a product structure \( \hat{\gamma} \) in the form

\[ \delta \theta = (1 + \hat{\gamma}) \epsilon. \]  

(98)

The product structure satisfies \( \hat{\gamma}^2 = 1 \) and \( \text{tr } \hat{\gamma} = 0 \).

For fundamental super \( p \)-branes such a product structure can be expressed simply in terms of the pull-back \( X^M(\xi) \) and a worldvolume permutation symbol (orientation) [7]. It is precisely the Clifford matrix

\[ \gamma := \frac{1}{\sqrt{g} d!} \partial_{i_1} X^{M_1} \cdots \partial_{i_d} X^{M_d} \epsilon^{i_1 \cdots i_d} \Gamma_{M_1 \cdots M_d} \]  

(99)
corresponding to world-volume orientation\textsuperscript{7} for both F1 and NS5.

When the twist field
\[ \mathcal{F} = F_2^{\text{CP}} - B_2 \] (100)
vanishes, the product structure simplifies to a similar form even for D-branes [8, 9]. (A nontrivial $\mathcal{F}$ introduces only an even number of gamma matrices, so our discussion generalises readily to $\mathcal{F} \neq 0$.) In even dimensions relevant to IIB we have
\[ \hat{\gamma} := \gamma \otimes \tau_d \, . \] (101)

Here we have used the notation
\[ \tau_d := \begin{cases} 
I & \text{for } s - t = 2, 3 \mod 4, \\
J & \text{for } s - t = 0, 1 \mod 4 .
\end{cases} \] (102)

The matrix $\hat{\gamma}$ is automatically traceless, and $\tau_d$ is chosen so that $\hat{\gamma}^2 = +1$. (This is because $\gamma^2 = (-1)^{\frac{1}{2}(s-t)(s-t-1)}1$ and $J^2 = -I^2 = 1$). The product structure behaves in a change of signature (2) in even dimensions precisely as $\sqrt{g} \, d^d \xi$ should in (5). The reason for this is the fact $\frac{1}{2}d(d - 1) = \frac{1}{2}d \mod 2$ for even $d$ and, as usual $\Gamma^2 = 1$.

Given the worldvolume orientation $\epsilon^{i_1 \cdots i_d}$, $\gamma$ is the volume element of the worldvolume Clifford algebra. We could not use it directly to implement a worldvolume change of signature as it does not heed the structure of the worldvolume embedding in the bulk theory in an appropriate way. Instead, we should use the product structure, and change signature on the worldvolume by
\[ \gamma^i \otimes 1_2 \longrightarrow \hat{\gamma}(\gamma^i \otimes 1_2) \, . \] (104)

As it is nilpotent $\hat{\gamma}^2 = 1$, it changes the signature in the Clifford algebra as required. In even dimensions, it changes itself only by a sign
\[ \hat{\gamma} \longrightarrow (-1)^{\frac{1}{2}d(d-1)} \hat{\gamma} \, , \] (105)
which reproduces the behaviour of $\sqrt{g} \, d^d \xi$ correctly due to the fact $\frac{1}{2}d(d - 1) = \frac{1}{2}d \mod 2$ when $d$ is even.

The rôle played by the product structure in $\kappa$-symmetry is to carry the information of how the brane is oriented in a superspace embedding. It therefore incorporates information of the orientation of the brane together with its coupling to fermions. This can be made concrete by lifting Type IIB branes into F-theory using the results of section 7.5: the extra $N = 2$ structure becomes geometric, and we see that the product structure $\hat{\gamma}$ is precisely the F-theory worldvolume volume element, as tabulated in table 4.

\textsuperscript{7}There is no extra complex phase when $p = 4l + 1, 4l + 2$. 

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| Sector | Type IIB Brane Charge | F-theory Brane Charge | Volume |
|--------|------------------------|------------------------|--------|
| \(\mathcal{I}\) | \(D(-1)\) – | \(\mathcal{F}2\) \(\tilde{Z}_{12}\) | \(-\mathcal{P} \hat{\Gamma}^{12} = \mathcal{P} \otimes I\) |
| \(D3\) | \(Z_{MNP}\) | \(\mathcal{F}5\) \(\tilde{Z}_{MNP12}\) | \(-\mathcal{P} \hat{\Gamma}^{MNPQ12} = \mathcal{P} \Gamma^{MNPQ} \otimes I\) |
| \(D7\) | \(Z_{M1\ldots M7}\) | \(\mathcal{F}9\) \(\tilde{Z}_{M1\ldots M712}\) | \(-\mathcal{P} \hat{\Gamma}^{M1\ldots M712} = \mathcal{P} \Gamma^{M1\ldots M712} \otimes I\) |
| \(\mathcal{J}\) | \(\mathcal{D}1\) \(Z^1_M\) | \(\mathcal{F}2\) \(\tilde{Z}_{M1} - \tilde{Z}_{M2}\) | \(-\mathcal{P} \hat{\Gamma}^{MN1} = \mathcal{P} \Gamma^{MN} \otimes J\) |
| \(\mathcal{D}5\) | \(Z^2_{MNPQR}\) | \(\mathcal{F}6\) \(\tilde{Z}_{MNPQR1}\) | \(-\mathcal{P} \hat{\Gamma}^{MNPQRS1} = \mathcal{P} \Gamma^{MNPQRS} \otimes J\) |
| \(\mathcal{D}9\) | \(Z^3_{M1\ldots M9}\) | \(\mathcal{F}10\) \(\tilde{Z}_{M1\ldots M91} - \tilde{Z}_{M1\ldots M92}\) | \(-\mathcal{P} \hat{\Gamma}^{M1\ldots M91} = \mathcal{P} \Gamma^{M1\ldots M9} \otimes J\) |
| \(\mathcal{K}\) | \(\mathcal{F}1\) \(Z^K_M\) | \(\mathcal{F}2\) \(\tilde{Z}_{M2} + \tilde{Z}_{M1}\) | \(-\mathcal{P} \hat{\Gamma}^{MN2} = \mathcal{P} \Gamma^{MN} \otimes K\) |
| \(\mathcal{N}5\) | \(Z^K_{MNPQR}\) | \(\mathcal{F}6\) \(\tilde{Z}_{MNPQR2}\) | \(-\mathcal{P} \hat{\Gamma}^{MNPQRS2} = \mathcal{P} \Gamma^{MNPQRS} \otimes K\) |
| \(\mathcal{F}9\) | \(Z^K_{M1\ldots M9}\) | \(\mathcal{F}10\) \(\tilde{Z}_{M1\ldots M92} + \tilde{Z}_{M1\ldots M91}\) | \(-\mathcal{P} \hat{\Gamma}^{M1\ldots M92} = \mathcal{P} \Gamma^{M1\ldots M9} \otimes K\) |
| \(\text{All}\) | \(\mathcal{P}_M\) | \(\mathcal{F}1\) \(\tilde{Z}_M\) | \(-\mathcal{P} \hat{\Gamma}^{MN} = \mathcal{P} \Gamma^{MN} \otimes 1\) |
| \(\mathcal{K}K\) | \(\tilde{Z}_{MNPQR}\) | \(\mathcal{F}5\) \(\tilde{Z}_{MNPQR}\) | \(-\mathcal{P} \hat{\Gamma}^{MNQRS} = \mathcal{P} \Gamma^{MNQRS} \otimes 1\) |

Table 4: Branes and their extensions in Type IIB and F-theory.
To summarise, we have shown how signature reversal can be implemented on brane worldvolumes in type IIB theory in an S-duality covariant way. Taking the $N = 2$ structure into account in this way, leads to the same parity (105) of the volume element as expected from a complex argument. From the F-theory point of view the distinction between the three universes boils down to whether the brane wraps the whole F-theory torus as in Universe $\mathcal{I}$; or only the direction along $x^{10}$ as in Universe $\mathcal{J}$; or $x^{11}$ as in Universe $\mathcal{K}$.

5 Brane solitons

5.1 Type I and D-branes

The solution for extremal D-branes in $D = 10$ is [10]

$$ds^2 = \pm \frac{1}{\sqrt{\mathcal{H}}} dx^2 \pm \sqrt{\mathcal{H}} dy^2$$

$$H^{-1} = g_s^{-1} \mathcal{H}^{\frac{1}{8-d}}$$

where

$$C_d = -(\mathcal{H}^{-1} - 1)g_s^{-1} dx^0 \wedge dx^1 \wedge \ldots \wedge dx^{d-1}$$

$$\mathcal{H} = 1 + \left(\frac{y_d}{y}\right)^{8-d}.$$ 

The $\pm$ sign in the metric is determined by the boundary conditions we choose to impose at infinity. Given a harmonic function $\mathcal{H} > 0$ that approaches 1 at infinity, this is done when we choose the branch of the square root in the metric.

The open string theory depends directly on the dilaton factor $H$ and in particular its sign

$$g_s H^{-1} = \begin{cases} 
\mathcal{H}^{-1}, 1, \mathcal{H} & \text{for } D(-1), D3, D7 \\
\pm(\sqrt{\mathcal{H}})^{-1}, \pm\sqrt{\mathcal{H}} & \text{for } D1, D5 
\end{cases}.$$ 

This means that a change of signature in the metric implies a change of branch of square root for D1 and D5. The signs of the dilaton factors of other D-branes D(-1), D3, and D7 do not depend on a choice of branch and are in fact all fixed given the harmonic function.

The squareroot $S \equiv \pm\sqrt{\mathcal{H}}$ is a dynamical degree of freedom that can have positive as well as negative values. Its value at infinity has to be constant and, hence, effectively either 1 or $-1$. This means that as long as we do not specify the vacuum expectation value of $\langle S \rangle$, the signature reversal symmetry is not broken. Given the dependence of the respective dilaton factors on $S$, we have therefore in the unbroken phase the symmetry

$$(G, H) \rightarrow \begin{cases} 
(-G, +H) & \text{for } D(-1), D3, D7 \\
(-G, -H) & \text{for } D1, D5 
\end{cases}.$$
This reproduces the results of section 3.2.

We noted in [1] that metric reversal alone produces the same effects as sending $x^M \rightarrow ix^M$. One may verify that this does indeed leave the metric, the dilaton and the $d$-form invariant in the D(-1), D3, and D7 solutions, where $d = 0 \mod 4$.

5.2 F1 and NS5 branes

The solution for extremal fundamental and solitonic branes in $D = 10$ is [10]

$$ds^2 = \mathcal{H}^{\frac{d-6}{4}} dx^2 + \mathcal{H}^{\frac{d-2}{4}} dy^2$$

$$H = g_s \mathcal{H}^{\frac{d-4}{4}}$$

where

$$B_d = -(\mathcal{H}^{-1} - 1) g_s^{-1} dx^0 \wedge dx^1 \wedge \ldots \wedge dx^{d-1}$$

and where $\mathcal{H}$ is a harmonic function in the space transverse to the brane

$$\mathcal{H} = 1 + \left(\frac{y_d}{y}\right)^{8-d}.$$  

Choosing $\mathcal{H}$ to tend to 1 at infinity rather than -1 involves a choice of boundary condition; as long as we do not commit ourselves to a specific signature in $dx^2$ and $dy^2$, this choice does not, as such, break signature invariance yet. The positive choice, 1, is required to satisfy (119).

Given the harmonic function $\mathcal{H} > 0$, the F1 string soliton solution is

$$ds^2 = \mathcal{H}^{-1} dx^2 + dy^2$$

$$H^2 = g_s^2 \mathcal{H}$$

and the NS5 soliton solution is

$$ds^2 = dx^2 + \mathcal{H} dy^2$$

$$H^2 = g_s^2 \mathcal{H}.$$  

The closed string effective theory depends of $g_s^2$, and of $H^2 = e^{2\Phi}$.

The choice of signature in constructing this vacuum is made when we choose the signature of the worldvolume metric $dx^2$ and that of the transverse metric $dy^2$. Changing this choice affects in no way the sign of the closed string dilaton factor $H^2$.

This choice of a fundamental string or a Neveu-Schwarz vacuum breaks the signature reversal symmetry completely. Nevertheless, as the dilaton factor is given by $H^2 = g_s^2 \mathcal{H} \geq 0$, there is the residual symmetry inherent to any closed string theory

$$(G,H) \rightarrow (G,-H)$$

that is the defining characteristic of Universe $\mathcal{K}$.
6 Quantum corrections

6.1 F1 and NS5 branes

In [1], we noted that in gravity or supergravity there will be $L$-loop counterterms of the form

\[ S_c \sim \frac{1}{2\kappa_D^2} \int d^D x \sqrt{G} \, \kappa_D^2 R^{(D-2)L+2}, \]  

(121)

where $R^n$ is symbolic for a scalar contribution of $n$ Riemann tensors each of dimension 2. These are signature reversal invariant in $D = 4k+2$. String loop corrections in $D = 10$ will be of a similar form but with $\kappa_{10}^2$ replaced by $\kappa_{10}^2 H^2$.

\[ S_c \sim \frac{1}{2\kappa_{10}^2} \int d^{10} x \sqrt{G} H^{-2} \left( \kappa_{10}^2 H^2 \right)^L R^{4L+1}, \]  

(122)

and will preserve the signature invariance. However, we noted in Section 3.1 that the coupling to a fundamental string is not invariant and so we would expect that the symmetry would not be respected by $\alpha'$ corrections. This is indeed the case. For example, at string loop tree level, $L = 0$, there are $\alpha'^3 R^4$ three-loop worldsheet corrections to the Type IIB string action. Since they involve even powers of $R$, they violate the symmetry. More generally for F1 and NS5 branes, we have on dimensional grounds [11]

\[ S_{F1} \sim \frac{1}{2\kappa_{10}^2} \int d^{10} x \sqrt{G} \, H^{-2} \left( \kappa_{10}^2 H^2 \right)^L T_2^{-l} R^{4L+l+1}, \]  

(123)

\[ S_{NS5} \sim \frac{1}{2\kappa_{10}^2} \int d^{10} x \sqrt{G} \, H^{-2} \left( \kappa_{10}^2 H^2 \right)^L \left( T_6^{-1} H^2 \right)^l R^{4L+3l+1}, \]  

(124)

where $l$ is the number of worldvolume loops. So F1 and NS5 branes are not invariant under signature reversal but only under $(G, H) \rightarrow (G, -H)$. Once again we confirm that they belong to Universe $K$.

6.2 Type I and D-branes

Under the rules of S-duality of section 1.2, however, the corresponding corrections to the action for a D-Brane or Type I string is obtained by replacing $T_d$ by $T_d H^{-1}$:

\[ S_D \sim \frac{1}{2\kappa_{10}^2} \int d^{10} x \sqrt{G} \, H^{-2} \left( \kappa_{10}^2 H^2 \right)^L \left( T_d^{-1} H \right)^l R^{4L+\frac{d}{2}+1}. \]  

(125)

Using the rule for the volume element (5), this is invariant under $(G, H) \rightarrow (-G, H)$ only for $d = 4k$, but under $(G, H) \rightarrow (-G, -H)$ in $d = 4k + 2$. So we conclude once again that D(-1), D3 and D7 belong to Universe $I$ while D1, D5, D9 and the Type I string belong to Universe $J$. 

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6.3 Chern-Simons and Green-Schwarz corrections

In the Type I theory and both of the Heterotic theories there is a Chern-Simons correction to the 3-form Bianchi identities:

$$
\begin{align*}
\text{Type I:} & \quad d \tilde{F}_3 = 2\alpha' X_4 \\
\text{Het}_{SO(32)}: & \quad d \tilde{H}_3 = 2\alpha' X_4
\end{align*}
$$

where

$$
2\alpha' = \frac{\kappa_{10}^2}{g_{10}^2} \\
X_4 \equiv \text{tr} R^2 - \text{tr} F^2
$$

in the notation of [12]. The anomaly polynomial $X_4$ is a topological term, and remains invariant under any signature reversal. As Type I belongs to the Universe $\mathcal{J}$, also the RR 3-form $\tilde{F}_3$ remains consistently invariant under metric and coupling reversal, as can be seen from equation (27). Similarly, the 3-form $\tilde{H}_3$ of the Heterotic $SO(32)$ theory remains invariant under the coupling reversal of Universe $\mathcal{K}$. In Type IIB supergravity there is no Chern-Simons correction, and the Bianchi identity is

$$
dH_3 = 0 .
$$

The absence of a Chern-Simons correction is consistent with S-duality. On the other hand, this is necessary given the transformation rule (27), and can therefore be seen also as a prediction of metric reversal invariance in Universe $\mathcal{I}$.

In Type I and both Heterotic theories there is a Green-Schwarz anomaly cancellation mechanism that can be expressed as a modification of the Bianchi identity of the dual 7-form field strengths [12]:

$$
\begin{align*}
\text{Type I:} & \quad d \tilde{F}_7 = -\frac{\beta'}{3} X_8 \\
\text{Het}_{SO(32)}: & \quad d \tilde{H}_7 = -\frac{\beta'}{3} X_8
\end{align*}
$$

where

$$
2\kappa_{10}^2 = \alpha' \beta' (2\pi)^5 \\
X_8 \equiv \text{tr} F^4 - \frac{1}{8} \text{tr} F^2 \text{tr} R^2 + \frac{1}{32} (\text{tr} R^2)^2 + \frac{1}{8} \text{tr} R^4
$$

in the notation of [12]. Once again the anomaly polynomial $X_8$ is signature reversal invariant. The 7-form field strengths are related to the 3-form field strengths by

$$
\begin{align*}
\tilde{F}_7 &= H^{-2} \ast \tilde{F}_3 \quad \text{Type I} \\
\tilde{H}_7 &= \ast \tilde{H}_3 \quad \text{Het}_{SO(32)}
\end{align*}
$$
The Hodge star operation changes as
\[ \star \Sigma_n \rightarrow (-1)^{T+n} \star \Sigma_n , \tag{135} \]
so operated on odd forms it remains invariant under signature reversal. The definitions of 7-form fluxes \( \tilde{H}_7 \) and \( \tilde{F}_7 \) contain only even powers of the dilaton factor and remain therefore invariant under coupling reversal. This means that the Bianchi identities (131) and (132) are invariant under the signature reversal of the appropriate universe. The absence of Green-Schwarz correction in the Type IIB case
\[ dH_7 = 0 . \tag{136} \]
where
\[ H_7 = H^{-2} \star H_3 \tag{137} \]
can also be seen as a consequence of metric reversal in Universe \( I \).

7 Superalgebras

In this section we consider signature reversal on the level of superalgebras in 10 dimensions. The only affected algebra relations are those in which the background metric \( \eta_{MN} \) and the Clifford matrices \( \Gamma^M \) enter: these are the relations that involve the supercharges \( Q_a \)

\[ \{Q_a, Q_b\} = (\Gamma^M C)_{ab} P_M + Z_{ab} \tag{138} \]
\[ [L^N_M, Q_a] = -\frac{1}{4}(\Gamma_M^N C)^a_b Q_b , \tag{139} \]
\[ [P_M, Q_a] = 0 , \tag{140} \]
where \( Z_{ab} \) are central charges. The other commutation relations define the 10-dimensional Poincaré algebra. This is a rigid superalgebra, with a constant background metric \( \eta_{MN} \) and constant Clifford matrices \( \Gamma_M \) that satisfy the Clifford algebra

\[ \{\Gamma_M, \Gamma_N\} = +2\eta_{MN} . \tag{141} \]

We have not needed to refer to them anywhere else in the present work, or in [1].

There are in fact two possible ways of implementing signature reversal in the rigid algebra

\[ \eta_{MN} \rightarrow -\eta_{MN} \tag{142} \]
\[ \Gamma_M \rightarrow \pm \Gamma_M . \tag{143} \]

In what follows we shall investigate the symmetry properties of each algebra for both definitions of signature reversal.

As we are working in dimensions where \( \Gamma^2 = +1 \), though \( \Gamma_{MN} \) changes sign, after rising an index \( \Gamma_M^N \) does not. (This is the reason why we cannot avoid
changing the signature of $\eta_{AB}$ even in the rigid algebra.) It follows that (139) is invariant under change of signature (142) – (143), when the generators themselves are left invariant. Postulating that all generators in the superalgebra are invariant under signature reversal, we find that the only other algebra relation whose form invariance we need to check is (138).

7.1 Type I

The Type I superalgebra is common to the Type I and both Heterotic superstrings. It involves the translation generator $\hat{P}_M$, and the self-dual five-form central charge $Z_{+MNPQR}^+$

$$\{Q_a, Q_b\} = (\mathcal{P} \Gamma^M C)_{ab} \hat{P}_M + (\mathcal{P} \Gamma^{MNPQR} C)_{ab} Z_{+MNPQR}^+ .$$

(144)

There is no central element for string charges, as the charge matrix on the left has $136 = 10 + 120$ independent elements. The chirality projection

$$\mathcal{P} = \frac{1}{2} (1 - \Gamma)$$

(145)

is to be thought of as a fixed constant matrix, on equal footing with the charge conjugation matrix $C$. The minus sign in it is needed as in our conventions

$$\Gamma Z_5 = - \star Z_5 .$$

(146)

It is straightforward to check that the Type I superalgebra is invariant under the substitution

$$\Gamma^M \rightarrow - \Gamma^M$$

(147)

of (143). The fundamental reason for this is the presence of the chirality projection $\mathcal{P}$.

7.2 Type IIA

The Type IIA superalgebra in the 10D Minkowski signature contains the anticommutation relation [13]

$$\{Q_a, Q_b\} = (\Gamma^M C)_{ab} P_M +
+ (\Gamma^M \Gamma C)_{ab} Z_M + (\Gamma^{MNPQR} C)_{ab} Z_{MNPQR} +
+ (\Gamma C)_{ab} Z + (\Gamma^{MN} C)_{ab} Z_{MN} + (\Gamma^{MNPQ} \Gamma C)_{ab} Z_{MNPQ} .$$

(148)

Choosing the positive sign in (143), consistent with (147), the change of signature sends

$$P_M \rightarrow - Z_M$$

(149)

$$Z_M \rightarrow + P_M$$

(150)

$$Z_{MNPQR} \rightarrow \frac{1}{5!} \epsilon^{M'N'P'Q'R'}_{MNPQR} Z_{M'N'P'Q'R'}$$

(151)

$$Z, Z_{MN}, Z_{MNPQ} \rightarrow - Z, - Z_{MN}, - Z_{MNPQ} .$$

(152)
The Type IIA algebra is therefore not invariant under change of signature, as it induces transformations (149) – (152) on physical charges.

However, the invariant part turns out to be the Type I algebra (144) where

\[ \tilde{P}_M = \frac{1}{2}(P_M - Z_M) \]  \hspace{.5cm} (153)
\[ \tilde{Z}_M = \frac{1}{2}(P_M + Z_M) \equiv 0 \]  \hspace{.5cm} (154)

Choosing the negative sign in (143), we get the anti-chiral version of the Type I algebra.

### 7.3 Type IIB

The Type IIB superalgebra contains the anticommutation relation [13]

\[
\{Q_a^i, Q_b^j\} = (\mathcal{P}_1^M C)_{ab} \left( \delta^{ij} P M + Z_M^{(ij)} \right)
+ (\mathcal{P}_1^{MNQR} C)_{ab} \left( \delta^{ij} \tilde{Z}_{MNPQR}^+ + Z_{MNPQR}^{+(ij)} \right)
+ (\mathcal{P}_1^{MNP} C)_{ab} I^{ij} Z_{MNP},
\]  \hspace{.5cm} (155)

where \((ij)\) is the traceless symmetric representation of SO(2), and \(\mathcal{P}\) is a fixed chirality projection, to be treated on equal footing with \(C\).

When all indices \(M, N, P\) are space-like, \(Z_{MNP}\) is the D3-brane charge. The Hodge dual of \(Z_{0MN}\) can be written conveniently using the Clifford representation

\[ Z_7 = -\Gamma Z_3, \]  \hspace{.5cm} (156)

and gives the D7-brane charges \(Z_{M_1\cdots M_7}\) when \(M_1\cdots M_7\) are space-like. In the notation where \(I = i\sigma_2, J = \sigma_1\) and \(K = \sigma_3\) we can decompose the traceless symmetric representation \((ij)\) of SO(2) as

\[ (Z_n^{(ij)}) = Z_n^I J + Z_n^K K \]  \hspace{.5cm} (157)

for \(n = 1, 5\). These central charges are the D1 and the F1 string charge for \(n = 1\), and the D5-brane and the NS5 brane charge for \(n = 5\).

The Type IIB algebra is invariant under the action of the automorphism group SO(2). An example of such automorphisms is S-duality that acts on supercharges by the transformation \(S \in SO(2)\)

\[ Q_a^i \rightarrow \frac{1}{\sqrt{2}}(1 + I)^i_j Q_a^j, \]  \hspace{.5cm} (158)

thus generating an \(\mathbb{Z}_8\) subgroup of SO(2). S-duality interchanges the Ramond-Ramond and the Neveu-Schwarz central charges

\[ (Z_1^I, Z_5^I) \rightarrow (Z_1^K, Z_5^K) \]  \hspace{.5cm} (159)
\[ (Z_1^K, Z_5^K) \rightarrow (Z_1^I, Z_5^I). \]  \hspace{.5cm} (160)
Signature reversal with upper resp. lower sign in (143) induces
\[
\sigma \pm : \begin{cases} 
(Z_1^K, Z_1^J, Z_3^K, Z_3^J) & \rightarrow \pm (Z_1^K, Z_1^J, Z_3^K, Z_3^J) \\
Z_3 & \rightarrow \mp Z_3 \\
P & \rightarrow \pm P
\end{cases} .
\]

(161)

We see immediately that, choosing the upper sign, the \( \sigma_+ \) transformation is an invariance of the algebra if the central charges \( Z_{MNP} \) vanish.

Unlike the Type IIA algebra, the Type IIB algebra has a nontrivial automorphism group \( \text{SO}(2) \subset \text{GL}(2, \mathbb{R}) \). As we shall show in the next section, we can extend signature reversal \( \sigma_+ \) by including a \( \text{GL}(2, \mathbb{R}) \) action: the resulting operations \( \hat{\sigma}_+ \) preserve different subsectors of the Type IIB algebra.

The conclusion in the next section will be that the Type IIB algebra is not fully invariant under any of the extensions we can construct. However, the Type IIB algebras of a given Universe \( \mathcal{I}, \mathcal{J}, \text{or} \mathcal{K} \) are invariant under at least one such extension each.

### 7.4 Subsectors of Type IIB

One may redefine the supercharges
\[
Q_a^i \rightarrow g^i_j Q_a^j
\]
by acting on their \( N = 2 \) structure with an element \( g \in \text{GL}(2, \mathbb{R}) \). Only the subgroup \( \text{SO}(2) \subset \text{GL}(2, \mathbb{R}) \) of these transformations qualifies as actual automorphisms of the superalgebra. For instance, only \( g \in \text{O}(2, \mathbb{R}) \) preserve the translation generator \( P_M \) as the condition for this is \( g^T g = 1 \).

We are looking for an extension of \( \sigma_+ \) by combining with it an action of \( g \in \text{GL}(2, \mathbb{R}) \), such that the resulting transformation \( \hat{\sigma}_+ \) leaves at least a part of the Type IIB algebra form invariant.

Choosing an automorphism \( g \in \text{SO}(2) \) would not lead to a new operation \( \hat{\sigma}_+ \); on the other hand any element \( g \in \text{GL}(2, \mathbb{R}) \) must be orthogonal \( g^T g = 1 \). This leaves us precisely two candidates for extensions, namely using \( g = J \) or \( g = K \). The actions of these elements (including \( I \) for completeness) on the algebra are

\[
\begin{align*}
(-)^{FL} : & \begin{cases} 
Q_a^i & \rightarrow K^i_j Q_a^j \\
(Z_1^1, Z_3, Z_3^2) & \rightarrow -(Z_1^1, Z_3, Z_3^2)
\end{cases} \\
\Omega : & \begin{cases} 
Q_a^i & \rightarrow J^i_j Q_a^j \\
(Z_1^1, Z_3^1, Z_3^K) & \rightarrow -(Z_1^1, Z_3^1, Z_3^K)
\end{cases} \\
\varepsilon : & \begin{cases} 
Q_a^i & \rightarrow I^i_j Q_a^j \\
(Z_1^K, Z_3^1, Z_3^K, Z_3^1) & \rightarrow -(Z_1^K, Z_3^1, Z_3^K, Z_3^1)
\end{cases}
\end{align*}
\]
where $(-)^{F_L}$ arises in string theory as the left-handed worldsheet fermion number modulo two, and $\Omega$ as the worldsheet parity transformation, see e.g. [14].

Central charges that appear with $I$, $J$, or $K$ in the superalgebra can now be projected out by combining $\sigma_+$ with either $1$, $\Omega$, or $(-)^{F_L}$. We denote these extensions of signature reversal by

$$\hat{\sigma}_I = \sigma_+ \circ 1$$
$$\hat{\sigma}_J = \sigma_+ \circ \Omega$$
$$\hat{\sigma}_K = \sigma_+ \circ (-)^{F_L}.$$  

The corresponding projections onto invariant subsectors are $\mathcal{P}_I$, $\mathcal{P}_J$, and $\mathcal{P}_K$.

The projection procedure can be repeated. If we impose invariance under all three operations $\hat{\sigma}_I$, $\hat{\sigma}_J$, and $\hat{\sigma}_K$ simultaneously, we get the projection $\mathcal{P}_I \circ \mathcal{P}_J \circ \mathcal{P}_K = \mathcal{P}_I$, and are left with the purely gravitational Type I sector in the algebra

$$\{Q^i_a, Q^j_b\} = \left( (\mathcal{P}\Gamma^M C)_{ab} P_M + (\mathcal{P}\Gamma^{MNPQR} C)_{ab} \tilde{Z}^+_{MNPQR} \right) \delta^{ij}.$$  

This is clearly the minimal invariant subsector. The five-form charge here is associated with a Kaluza-Klein monopole, rather than a five-brane [13].

By imposing invariance only with respect to a pair of operations, we may project the algebra onto sectors that include certain subsets of brane charges. This leads to three subsectors of the theory that are separately invariant with respect to two different extensions of signature reversal:

**I.** The sector invariant under both $\hat{\sigma}_J$, and $\hat{\sigma}_K$ has the superalgebra

$$\{Q^i_a, Q^j_b\} = \left( (\mathcal{P}\Gamma^M C)_{ab} \delta^{ij} P_M + (\mathcal{P}\Gamma^{MNP} C)_{ab} I^{ij} Z_{MNP} + (\mathcal{P}\Gamma^{MNPQR} C)_{ab} \delta^{ij} \tilde{Z}^+_{MNPQR} \right).$$  

This sector involves only Ramond-Ramond D3 and D7 brane charge, apart from the gravitational charges $P_M$ and $\tilde{Z}^+_{MNPQR}$. This sector is invariant under $\varpi$.

**J.** The sector invariant under both the pure signature reversal $\sigma_+$ and its extension to $\hat{\sigma}_J$ has the superalgebra

$$\{Q^i_a, Q^j_b\} = \left( (\mathcal{P}\Gamma^M C)_{ab} \delta^{ij} P_M + Z^i_M J^{ij} + (\mathcal{P}\Gamma^{MNPQR} C)_{ab} \delta^{ij} \tilde{Z}^+_{MNPQR} + Z^i_M \tilde{Z}^+_{MNPQR} J^{ij} \right).$$  

The charges $Z^i_M$ are the D-string charge and $Z^5_M$ the D5-brane charge. The two projections onto states that are invariant under both $\sigma_+$ and $\hat{\sigma}_J$ is actually the same as the projection onto states that are invariant under $\Omega$.

The brane charges arising in this sector survive the $\Omega$ orientifold projection that projects Type IIB string theory to Type I $\simeq$ Type IIB/$\Omega$ string theory. We can identify this sector with the Type I supergravity, even if the algebra in (171) has more central elements than the actual Type I algebra.
The sector invariant under both the pure signature reversal $\sigma_+$ and its extension to $\tilde{\sigma}_K$ has the superalgebra

$$\{Q^i_a, Q^j_b\} = (P \Gamma^M C)_{ab} \left( \delta^{ij} P_M + Z^K_M K^{ij} \right) + (P \Gamma^{MNPQR} C)_{ab} \left( \delta^{ij} \tilde{Z}^+_{MNPQR} + Z^K_{MNPQR} K^{ij} \right).$$

(172)

This case is similar to $J$, except that it is the $K$-component of the symmetric central charges that survives. The surviving central charges $Z^K$ are the fundamental string charge, and $Z^K$ the Neveu-Schwarz 5-brane charge. This sector is invariant under $(-)^F_L$.

### 7.5 F-theory formulation

Let us define the 12-dimensional gamma matrices

$$\hat{\Gamma}^M = \Gamma^M \otimes 1$$

(173)

$$\hat{\Gamma}^{10} = \Gamma \otimes J$$

(174)

$$\hat{\Gamma}^{11} = \Gamma \otimes K,$$

(175)

where $M = 0, \ldots, 9$. This Clifford algebra has signature $(11, 1)$ or $(3, 9)$, the charge conjugation matrix $C = C \otimes I$, and the chirality operator $\hat{\Gamma} = -\Gamma \otimes I$. The 12D algebra is not signature reversal invariant, as it has $S - T = 2 \mod 4$. The most general ansatz for an anticommutation relation for a 12D supercharge $Q^a_a$ is

$$\{Q^a_a, Q^b_b\} = (\hat{\Gamma}^M \hat{C})_{ab} \tilde{Z}^+_{\hat{M}^+} + (\hat{\Gamma}^{\hat{M}\hat{N}} C)_{ab} Z_{\hat{M}\hat{N}}$$

$$+ (\hat{\Gamma}^{\hat{M}\hat{N}} C)_{ab} \tilde{Z}^+_{\hat{M}^+\hat{N}^+} + (\hat{\Gamma}^{\hat{M}\hat{N}\hat{P}\hat{Q}\hat{R}\hat{S}} C)_{ab} Z_{\hat{M}\hat{N}\hat{P}\hat{Q}\hat{R}\hat{S}},$$

(176)

where $\hat{M} = 0, \ldots, 11$ is the 12D index. As $64 \times 64$ matrices we have 2080 free components on both sides.

This structure does not extend to a covariant 12D Poincaré superalgebra [15]. In signature $(10, 2)$ there is a superalgebra, however, originating from the supergroup OSp$(1|32)$, but it has no translation generators $P_M$. In what follows we shall make use of (176) purely on a formal level, and no claim of its extending to a superalgebra is made. There is some evidence that this structure is not entirely accidental, however. First of all, F-theory does have the full 32 supersymmetries, and involves a fundamental 3-brane in an $(11, 1)$ background [16]. Furthermore, as we have seen in section 4, precisely this matrix structure arises in $\kappa$-symmetry.

The algebra (176) should be understood as convenient 12D notation for the
following 10D central charges:

\begin{align}
P_M &= \tilde{Z}_M + 6 \mathcal{Z}_{M12} \quad \text{(177)} \\
Z^{(ij)}_M &= 2 \left( (\tilde{Z}_{Ma}1 + \mathcal{Z}_{Ma} I) \cdot \tau^a \right)^{ij} \quad \text{(178)} \\
Z_{MNP} &= \mathcal{Z}_{MNP} - 20 \tilde{Z}_{MNP12} \quad \text{(179)} \\
\check{Z}_{MNPQR} &= \tilde{Z}_{MNPQR} \quad \text{(180)} \\
Z^{(ij)}_{MNPQR} &= 6 \mathcal{Z}_{MNPQR} (\tau^a I)^{ij} \quad \text{(181)} \\
Z_{M_1\ldots M_7} &= \mathcal{Z}_{M_1\ldots M_7} - 72 \tilde{Z}_{M_1\ldots M_712} \quad \text{(182)} \\
Z^{(ij)}_{M_1\ldots M_9} &= 10 \left( (\tilde{Z}_{M_1\ldots M_9\alpha} 1 + \mathcal{Z}_{M_1\ldots M_9\alpha} I) \cdot \tau^a \right)^{ij} \quad \text{(183)}
\end{align}

where $M, N, P, Q, R$ are 10D indices, $\alpha, \beta = 1, 2$, and $\tau^1 = J, \tau^2 = K$, and we set other central charges to trivial values. With these restrictions in effect, the negative chirality part of (176) is the Type IIB superalgebra

\[ \{Q_a, Q_b\} = \left( \mathcal{P}\Gamma^M C \otimes 1 P_M + \mathcal{P}\Gamma^M C \otimes Z_M \right)_{ab} + \left( \mathcal{P}\Gamma^{MNPQR} C \otimes (\tilde{Z}_{MNPQR} 1 + Z_{MNPQR}) \right)_{ab} + (\mathcal{P}\Gamma^{MNP} C \otimes I)_{ab} Z_{MNP} . \quad \text{(184)}\]

In $D$ dimensions an $n$-form central charge $Z_{M_1\ldots M_n}$ is related to a $D - n$-form charge $Z_{M_1\ldots M_{D-n}}$ by Hodge duality

\[ \tilde{Z}_n \Gamma \equiv Z_{D-n} . \quad \text{(185)}\]

This provides a relationship between $\tilde{Z}_1, \mathcal{Z}_3$ and $\mathcal{Z}_9, \tilde{Z}_7$ in 10D, respectively, and $\tilde{Z}_5, \mathcal{Z}_2, \tilde{Z}_2$ and $\mathcal{Z}_7, \tilde{Z}_{10}, \mathcal{Z}_{10}$ in 12D. We have made use of this structure in identifying the central charges (182)–(183).

Implementing the 10D signature reversal in this algebra keeps central charges $Z_n$ invariant when $n = 4k + 2$ and reverses the central charge when $n = 4k$. The action of $O(2)$ can now be understood in terms of reflections in the two new directions $x^{10}$ and $x^{11}$, so that the three involutions $(-)^F I, \Omega$, and $\varpi$ can be lifted to geometric actions on this 12D algebra. The sector $\mathcal{I}$, for instance, will not involve all of the 12D charges but components not involving indices in both of the $x^{10}$ and $x^{11}$ directions get projected out

\begin{align}
\mathcal{Z}_{MNP} &= 0 \quad \text{(186)} \\
\check{Z}_{M_1\ldots M_7} &= 0 \quad \text{(187)}
\end{align}

only $\tilde{Z}_{MNP12}$ and $\tilde{Z}_{M_1\ldots M_712}$ survive the projection. Similarly in the sectors $\mathcal{J}$ and $\mathcal{K}$, central charges corresponding to branes wrapping around the direction $x^{10}$ and $x^{11}$ will be projected out.
8 U-duality

We have seen that under \((G, H) \to (-G, H), (-G, -H), (G, -H)\), Type IIB breaks up in three different Universes \(\mathcal{I}, \mathcal{J}, \mathcal{K}\). T-duality interchanges \(\mathcal{I}\) and \(\mathcal{J}\), S-duality interchanges \(\mathcal{J}\) and \(\mathcal{K}\).

In the usual Universe \(\mathcal{I}\), for which \((G, H)\), compactification to \(D = 4\) on \(T^6\) results in a U-duality group \([17, 18, 19]\) \(E_7(7)\) under which the charges transform as a \(56\). Decomposing under the S and T-duality groups,

\[ E_7(7) \to \text{SL}(2, \mathbb{Z}) \times \text{SO}(6, 6; \mathbb{Z}) \] (188)

yields

\[ 56 \to (2, 12) \oplus (1, 32) \] (189)

where the \((2, 12)\) refers to NS charges and the \((1, 32)\) to RR. These charges have an interpretation in terms of branes wrapped around \(T^6\) [19]. The \((2, 12)\) come from the F1 and NS5 while the \((1, 32)\) come from the D-branes. In order to see how these are divided between the different branes, we decompose further

\[ \text{SO}(6, 6; \mathbb{Z}) \to \text{SL}(6, \mathbb{Z}) \times \mathbb{Z}_2 \] (190)

under which

\[ 32 \to 1^{+3} \oplus 1^{-3} \oplus 15^{1+1} \oplus 15^{-1} \] (191)

\[ 32 \to 6^{+2} \oplus 6^{-2} \oplus 20^0 \] (192)

for IIA and IIB, respectively. Here \(1^{\pm 3}\) refers to D0 and D6, \(15^{\pm 1}\) to D2 and D4, \(6^{\pm 2}\) to D1 and D5 and \(20^0\) to D3.

This gives rise to the U-duality groups for the various sectors shown in the table:

| \(1\) | \(\mathcal{I}\) | \(\mathcal{J}\) | \(\mathcal{K}\) |
|-------|-------|-------|-------|
| \(E_7(7)\) | \(\text{SL}(6, \mathbb{Z})\) | \(\text{SL}(6, \mathbb{Z})\) | \(\text{SL}(2, \mathbb{Z}) \times \text{SO}(6, 6; \mathbb{Z})\) |
| \(56\) | \(20\) | \(6 \oplus 6\) | \((2, 12)\) |

9 Yang-Mills: bulk versus brane

The existence of signature reversal invariant Yang-Mills theories in signature \((3, 1)\) is remarkable given the negative conclusions we reached in [1]. There, we recall, the kinetic term in pure Yang-Mills

\[ S_{YM} = \frac{1}{4g_D^2} \int d^D x \sqrt{G} \text{Tr} |F_2|^2 \] (193)

contains two contractions with the background metric. Invariance then requires that the volume form should not change sign under signature reversal. Consequently pure Yang-Mills theory is invariant only in dimensions \(D = 4k\). If the theory is coupled to fermions, and we require \(S - T = 4k'\), this leads to \(D = 4k' + 2T\)
so that there would have to be an even number of time-like dimensions. In particular, \( D = 10 \) super Yang-Mills is ruled out. However, the Yang-Mills theory that appears in AdS/CFT is the one appropriate to Dirac-Born-Infeld D3-brane action

\[
S_D^4 = -T_4 \int d^4 \xi \, H^{-1} \sqrt{\det(g_{ij} + F_{ij})}, \tag{194}
\]

where the pull-back metric is defined in (84) and the twist field on the brane \( F_{ij} \) in (100). As we have seen in section 3.2, this is invariant in (3, 1), because the relevant gamma matrices are those of \( D = 10 \) rather than \( d = 4 \).

There are two other ways in which Yang-Mills in (3, 1) is allowed: first, as shown in [1], although forbidden in \( D = 4k + 2 \), Maxwell and Yang-Mills terms can arise after compactification to lower dimensions. Secondly, their absence in \( D = 4k + 2 \) applies in pure Yang-Mills theory only. In the Yang-Mills sector of Type I supergravity the kinetic term is multiplied by a dilaton factor \( H(x) \) that may also change sign to compensate for the change in sign of the volume element [1]. It would be worthwhile to investigate the residual effects of \( D = 10 \) signature reversal in 4-dimensional string and M-theory phenomenological models.

10 Conclusions

We have seen that combined effects of metric and coupling reversal give rise to four different universes. Curiously, only Universe 1 is M-friendly in the sense of admitting M2 and M5-branes, and their descendants in Type IIA and Heterotic \( E_8 \times E_8 \). On the other hand, Universe \( \mathcal{I} \) has no strings at all, but only D(-1), D3, and D7-branes. It is worth pursuing what such a universe would be like: for example, we still have AdS/CFT on \( \text{AdS}_5 \times S^5 \), but not on \( \text{AdS}_4 \times S^7 \) or \( \text{AdS}_7 \times S^4 \). It could be argued, of course, that there is an advantage in singling out Yang-Mills theory in (3, 1) rather than less phenomenologically desirable theories in (2, 1) or (5, 1). This universe \( \mathcal{I} \) may also have some interesting cosmological features: for example, the 3-brane and 7-brane cosmology of [20].

Finally, we recall [1] that although the flipping of the sign of the metric tensor may leave the equations of motion invariant, a choice has to be made when choosing the boundary conditions. A metric vacuum expectation value

\[
\langle G_{MN}(x) \rangle = \eta_{MN} \tag{195}
\]

breaks the reversal symmetry spontaneously. Similarly the dilaton field expectation value

\[
\langle H(x) \rangle = g_s \tag{196}
\]

breaks spontaneously the sign reversal \( H \rightarrow -H \). So one might entertain the possibility of different domains within a given universe.
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