The evolution of edge waves propagated over a variable slope-shelf topography

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Abstract. A discussion is presented of the propagation of trapped waves over continental shelves having the idealized and realistic topography. The emphasis is made on the investigation of influence of the shelf shape and width on the lower modes edge wave parameters. The conclusions were based on the solution of the complete boundary-value problem using the difference approximation algorithms. A three-dimensional model was used to reproduce waves generated on the Turkish coast of Black Sea caused by the climatological wind. The model, simulated sea-level time-series, was subjected to Fast Fourier Transform to compute the power spectral density of water oscillations. It is found that the absolute maximum of the first mode wave takes place near the shore and over the edge of the shelf. Increasing the width of the continental slope leads to slower attenuation of the wave towards the open sea. The most resistant wave processes were computed on the Cape Inceburun.

1. Introduction
Barotropic long waves define most of the hydrophysical processes in sea basins or along their border. In the boundary areas, changes in of topography depth lead to the formation of horizontally modified waves, as well as to the appearance of boundary wave formations (surface and internal Kelvin waves, boundary and shelf waves [1 – 4]). These waves are important in the processes of coastal dynamics such as the transport of sedimentary material, the formation of the structure of the coastline and coastal topography, as well as the surf beats. Due to these significant impacts on coastal morphology, accurate estimates of the spatial and temporal variability of wave parameters in the nearshore zone is critical and imperative for coastal engineering.

This paper present analysis of the offshore edge waves in terms of topography of the border areas: in the area of the shelf and the continental slope. The influence of the bottom relief on the complete set of barotropic waves near the critical frequency is studied on the example of bottom profiles reflecting the continental shelf and slope topography of the Black Sea basin, and also for the real profiles along the Black Sea coasts of Turkey.

2. Problem and model description
In this section the problem of an edge wave propagating on a coast with a shelf is formulated. Theoretically, the phenomenon of wave transformation caused by bottom variation has been explained using nonlinear equations for shallow water waves.
2.1. Formulation of the edge wave problem
The mathematical model is based on a system of linear equations in the approximation of hydrostatics (approximation of long waves) for unstratified flow on a rotating $f$ plane. This yields a well-known boundary problem [5–7]

$$\frac{\partial^2 \zeta}{\partial y^2} + \frac{1}{h} \frac{\partial \zeta}{\partial y} \frac{\partial h}{\partial y} + \zeta \left( \frac{f^2 - \omega^2}{gh} - \frac{fk}{\omega} \frac{1}{h} \frac{\partial h}{\partial y} - k^2 \right) = 0,$$

subject to boundary conditions implies an exponential decay in the offshore direction ($y \to \infty$):

$$v = 0, \quad y = 0, \quad \omega \frac{\partial \zeta}{\partial y} - kf \zeta = 0, \quad h(0) \neq 0.$$

Where $y$ is the offshore coordinate, $h(y)$ is the water depth, $\zeta(y)$ is the free surface elevation, $g$ is the acceleration due to gravity, $f = 1.01 \cdot 10^{-4}$ s$^{-1}$ is the Coriolis parameter, $k$ is the alongshore wavenumber, and $\omega$ is the wave frequency. Wave solution of the boundary value problem under the condition $\omega^2 < f^2 + k^2 gH$ exponentially decays toward the open sea of depth $H$, and represents a discrete set of individual modes of the trapped waves.

The eigenvalue problem (1), (2) is solved numerically using resonance iterations over the spectral parameter $M = \frac{fk}{\omega}$ (m$^{-1}$). The solution corresponds to finding the intersection points of lines of constant phase velocity with dispersion curves of wave modes. The mode number is identified by the number of zero crossings in the fashion structure (eigenvector). The calculations are repeated with different values of $M$. The spatial resolution in the finite-difference approximation is set equal to 1 km.

2.2. Dispersion characteristics of the fundamental mode calculated for the idealized profiles
Consider how the parameters of the zero mode change depending on the geometry of the continental shelf and slope. The study of topologically modified waves is performed using a model with simple geometry, for which the parameters of the considered region depend only on one spatial variable. An idealized profile characterized by three domains is adopted. In this case, the solution to the problem of propagation of long waves in the ocean is simplified. The topography of the continental shelf and slope is approximated as a two-slope depth profile adjacent to a deep ocean of a constant depth (figure 1).

The depth profile is characterized by the width of the shelf, slope, and deep ocean ($L_1$, $L_2$ and $L_3$) as well as by the depth of the coastal wall, shelf break, and deep ocean ($h_1$, $h_2$ and $h_3$). We conduct a sensitivity study of the fundamental mode dispersion characteristics by systematically varying the shelf and slope widths $L_1$ and $L_2$, as well as the shelf break. The coastal wall $h_1 = 5$ m represents a nearshore zone boundary, and also the deep-ocean depths $h_2 = 100$ m and $h_3 = 2000$ m are kept constant for all cases. Our study indicates that the dispersion curve of a zero mode is far more sensitive to the variations of a shelf width $L_1$ than to a slope $L_2$. We show that the dispersion diagram of a zero mode depends on the shelf and slope width (figure 2) while other metrics of the depth profile are held constant.
2.3. Solution to the edge wave problem for a realistic depth profile

We now examine the edge wave modes in order to understand how the wave adjusts to the changing real topography and coastline. The choice of the Turkish coast of the Black Sea is justified by the presence of continental shelves where the existence of topologically modified Kelvin waves is expected. In the Southeast at the foot of the Eastern Pontus Mountains is the coastal Pontic region. Its width is not more than 3–4 km increasing to 12 km near the Ordu, at the Cape Catly (16–20 km). Then it expands in the Sinop Bay from Ayancık and between Kandıra and Bosphorus up to 25–30 km. Its outer edge passes along isobaths of 110–130 km. The West Anatolian Shelf from the Bosphorus to Sinop has a maximum width of 25–30 km. The narrowest shelf is near Zonguldak and Karasu (3–4 km). The edge of the shelf is situated at depths of 100–110 km [8].

The initial data of the studied area is the topography of the Black Sea, represented in the file of depths (http://www.gebco.net) with spatial resolution of 0.5°. The transverse sections location and serial number (21–30, figure 3) was based on the classical scheme of the profiles placement by normal to the shore [9].

In the area along the Anatolian coast of the Black Sea is the place of the submarine terrace (profiles 23 and 24) at a 500-m depth. Profiles 25, 26 and 29 have the greatest width of shelf shallows 18.4–25 km. A sharp transition to the continental slope takes place on profile 22 (figure 4). Profiles 21–24, located in the Eastern part of the Turkish coast and profile 30 at the entrance to the Bosphorus, are the most specific. There is practically no shelf ($L_1$ is 2.1–6.7 km), the coastal shallows are narrow, there are small bottom slopes, the profile $h(y)$ is close to quasi-linear. The spatial scale of the shelf waves is determined by the width of the shelf as seen in figure 4. The points where $\zeta(y)$ has a maximum are located close to the shoreline and the edge of the shelf, the nodal line is close to the middle of the shelf.
Figure 4. Bottom topography along transects on the Anatolian coast, indicated in figure 3. The amplitudes the lower modes in that profiles accordingly.

In table 1 the values of $k$, $\omega$, and phase velocity are given. On the profile 25 in the extreme Northern point of the Anatolian coast, the longest time offshore waves are observed (23.24 h).

|   | Wavenumber (m$^{-1}$) | Wave Frequency (s$^{-1}$) | Phase Velocity (ms$^{-1}$) |
|---|-----------------------|---------------------------|-----------------------------|
| 21| 3.05-10$^{-6}$ – 9.38-10$^{-8}$ | 3.82-10$^{-8}$ – 1.36-10$^{-2}$ | 14.5 – 125.0 |
| 22| 8.51-10$^{-6}$ – 1.29-10$^{-4}$ | 1.19-10$^{-3}$ – 5.18-10$^{-3}$ | 40.0 – 140.0 |
| 23| 4.94-10$^{-6}$ – 1.10-10$^{-4}$ | 6.87-10$^{-4}$ – 7.69-10$^{-3}$ | 70.0 – 138.9 |
| 24| 3.97-10$^{-6}$ – 2.17-10$^{-4}$ | 5.44-10$^{-4}$ – 6.51-10$^{-3}$ | 30.0 – 137.0 |
| 25| 5.65-10$^{-6}$ – 1.46-10$^{-5}$ | 7.83-10$^{-4}$ – 2.93-10$^{-3}$ | 20.0 – 138.5 |
| 26| 5.49-10$^{-6}$ – 2.22-10$^{-4}$ | 7.57-10$^{-4}$ – 5.54-10$^{-3}$ | 25.0 – 138.0 |
| 27| 6.24-10$^{-8}$ – 1.48-10$^{-7}$ | 8.71-10$^{-4}$ – 8.86-10$^{-3}$ | 60.0 – 139.5 |
| 28| 3.03-10$^{-6}$ – 2.08-10$^{-4}$ | 4.19-10$^{-4}$ – 4.58-10$^{-3}$ | 22.0 – 138.6 |
| 29| 4.48-10$^{-6}$ – 2.09-10$^{-4}$ | 6.20-10$^{-4}$ – 3.76-10$^{-3}$ | 18.0 – 138.5 |
| 30| 6.93-10$^{-6}$ – 3.07-10$^{-4}$ | 9.56-10$^{-4}$ – 7.06-10$^{-3}$ | 23.0 – 138.0 |

The main features of the zero-mode wave profile are determined by the shape of the continental shelf. The role of slope geometry increases for the second mode waves. In this case, the wavelength and phase velocity are increased both by increasing the width of the shelf and the width of the slope. The waves of the second mode of the first nodal line $\zeta$ are located above the shelf, while the second –
over the continental slope near the edge of the shelf. In the case where the width of the shelf exceeds the width of the slope, the maximum $\zeta$ is shifted from the coastline. As a result, the waves of the second mode are more sensitive to changes in the geometry of the shelf compared to the waves of the first mode.

3. Three-dimensional analysis
The simulations are performed with a three-dimensional regional modeling system of the Black and Azov seas of the Marchuk Institute of Numerical Mathematics of RAS [10]. The model based on the primitive equations of ocean, recorded in approximations of incompressibility, hydrostatics and Boussinesq. Spatial resolution by longitude and latitude is $0^\circ 3' \times 0^\circ 2'^24''$, the time calculation step – 5 min. We used this model to reproduce the generation of edge waves by the climatological wind. Climatological atmospheric influence was used according to the data of CORE [11] with spatial resolution on longitude 1.825° and uneven step on latitude. The integration of motion equations in experiments were performed from zero initial conditions for two years. The simulation results were analyzed for 10 profiles located on the Turkish coasts of the Black Sea (figure 3).

3.1. Analysis of the sea level spectra
The simulated values of sea-level oscillations were then subjected to Fast Fourier Transform [12] for the 50 stations located on the Anatolian coast profiles. Figure 5 presents the simulation data during 10 months from second year computation.

![Figure 5. Power spectral density of sea-level oscillations.](image-url)
The right panel shows power spectral density of water level oscillations on the shelf part and in the deep-sea water part (left), shown in figure 5. Spectral estimates were computed for 45-minute sea-level records. We used a spectral window with a selected window length of 7200 hours (300 days) with half-overlap. Accordingly, the spectral resolution was ≈ 0.0033 cycle/day. The peak of the power spectral density of water level fluctuations is released at 6, 12, and 24 hours. The 12 hour peak is more pronounced at the deep-water stations. Fluctuations in sea-level on a semidiurnal period for all spectra have a maximum amplitude of all mesoscale sea-level fluctuations, including those exceeding the value of the spectral amplitude of inertial oscillations.

4. Conclusion
Analysis of numerical calculations showed that the absolute maximum of the profile of the first mode wave takes place near the shore and over the shelf break. Increasing the width of the continental slope leads to slower decay of the wave towards the open sea. On the profiles of the bottom, in which the width of the shelf is comparable to or less than the width of the slope, the maximum is observed at the shore. According to the results of calculations for climatological wind at the 50 stations located on these profiles, we calculated spectra of sea level fluctuations with the help of Fast Fourier Transform. We have obtained that sea level fluctuations of the semidiurnal period for all spectra have the maximum amplitude. We concluded that the most resistant wave processes occur in the area of Cape Inceburun, where they are associated with the manifestation of zero fashion trapped edge waves.

Acknowledgments
I wish to express my heartfelt gratitude to Professor of South Carolina’s University Alexander E. Yankovsky for insightful comments and suggestions. Financial support was provided in the framework of the Marine Hydrophysical Institute (theme No. 0827-2018-0004).

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