Thermal stability studies of an experimental nuclear fusion reactor

Julio J. Martinell and Javier E. Vitela†
Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, A. Postal 70-543, Mexico D.F., Mexico
E-mail: martinel@nucleares.unam.mx

Abstract. The conditions for the operation of a tokamak fusion reactor are studied considering a fiducial operating state and analyze variations about this state. We use a volume averaged 0-D two-temperature model and consider the case of a fusion thermonuclear reactor of the ITER type which will operate with a burning plasma at an energy gain factor of $Q=10$. The possible operation states are represented in POPCON plots of density $n$ versus electron temperature $T_e$ by varying parameters such as the fusion power, the energy gain and the auxiliary heating power, but keeping the ratio $(T_e - T_i)/T_e$ fixed, in order to determine the optimal operation point. Then, the thermonuclear instability of the burning plasma around this point is studied with a linear analysis. It is found that this instability cannot be excited, due to the fact that the plasma is far from ignition.

1. Introduction
In order to optimize the performance of a fusion power plant it is necessary to minimize the auxiliary heating requirements, which will ideally be zero if it works with an ignited plasma. The optimal operation should maximize the energy gain factor $Q$ for a given fusion energy production and this operation point would be maintained by a control system. A possible control system may be based on an Artificial Neural Network that manages the fuel injection rate and the auxiliary power [1]. In this work we analyze the burn regimes of a tokamak fusion reactor with the design characteristics of ITER for different plasma parameters, using a zero-dimensional, two temperature model in which energy and particle transport are accounted for by global confinement times for energy $\tau_E$, fuel particles $\tau_{DT}$ and alpha particles $\tau_\alpha$. Although the energies of ions and electrons evolve independently, the global transport is assumed to be the same. The operating points studied in this work are constrained to keep the ratio $(T_e - T_i)/T_e$ constant.

It is also necessary to make sure that the optimal states that one chooses are stable with respect to the thermonuclear instability which is liable to appear in a burning plasma. This instability arises in certain temperature range due to the fact that the fusion reaction rate there is an increasing function of temperature, which produces more fusion power as the temperature rises, thereby providing a positive feedback. We study the stability of the operational states around the optimal points in order to make sure that the thermonuclear instability is not a threat to a safe operation. We base our analysis on POPCON (Plasma OPeration CONtours) plots which graphically show the relevant range of electron density $n_e$ and temperature $T_e$. 

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
2. The plasma model

We use a multi-fluid description for the plasma (electrons, main ions and helium ash) which can be reduced to two particle and two energy conservation equations for the total deuterium-tritium density \(n_{DT}\), alpha particle density \(n_{\alpha}\) and the electron and ion temperatures \(T_e, T_i\), assuming quasineutrality \(n_e = n_{DT} + 2n_{\alpha} + Z_{Be}n_{Be} + Z_{Ar}n_{Ar}\). Two impurities (Be and Ar) are included but we assume they are fixed. The equations are,

\[
\frac{\partial}{\partial t} n_{DT} = S_f - \frac{1}{2} n_{DT}^2 \langle \sigma v \rangle - \nabla \cdot \vec{\Gamma}_{DT},
\]

\[
\frac{\partial}{\partial t} n_{\alpha} = \frac{1}{4} (1 - f_{frac}) n_{DT}^2 \langle \sigma v \rangle - \nabla \cdot \vec{\Gamma}_{\alpha},
\]

\[
\frac{\partial}{\partial t} \left[ \frac{3}{2} n_e T_e \right] = \mathcal{P}_{aux,e} + \frac{1}{4} (1 - f_{frac}) f_e Q_{\alpha} n_{DT}^2 \langle \sigma v \rangle + \mathcal{P}_{OH} - \mathcal{P}_{brem}
- \mathcal{P}_{cycl} - \frac{3}{2} n_e (T_e - T_i) / \tau_e - \nabla \cdot \vec{\Gamma}_{E,e}
\]

\[
\frac{\partial}{\partial t} \left[ \frac{3}{2} (n_{DT} + n_{\alpha} + n_{Be} + n_{Ar}) T_i \right] = \mathcal{P}_{aux,i} + \frac{1}{4} (1 - f_{frac}) f_i Q_{\alpha} n_{DT}^2 \langle \sigma v \rangle
+ \frac{3}{2} n_e (T_e - T_i) / \tau_i - \nabla \cdot \vec{\Gamma}_{E,i}.
\]

where the electron energy losses and sources include bremsstrahlung \(\mathcal{P}_{brem}\) and cyclotron radiation \(\mathcal{P}_{cycl}\), ohmic heating \(\mathcal{P}_{OH}\), fusion power (in terms of the D-T reactivity \(\langle \sigma v \rangle\)) and auxiliary power \(\mathcal{P}_{aux}\). For ions there is only fusion and auxiliary power \(\mathcal{P}_{aux,i}\). There is also the electron-ion energy exchange measured by the relaxation time \(\tau_e\). The expression for the reactivity we use is from [2]. Thermalization of the alpha particles produced by fusion is assumed to be instantaneous. The birth energy of the alpha particles is \(Q_{\alpha} = 3.5\) MeV; \(f_{frac}\) is the effective fraction of alpha particles which are anomalously lost due to MHD events before they are thermalized; \(f_e\) and \(f_i\) are the fractions of the alpha particle energy \(Q_{\alpha}\) deposited to the electrons and to the ions, respectively.

The fluxes \(\Gamma_{k}\) are not considered explicitly because of the poor understanding of particle and energy plasma transport, and instead we take a volume average of the fluid equations, which reduces the transport losses to the consideration confinement times for the electron and ion energy \(\tau_{E,e}\) and \(\tau_{E,i}\), as well as for the D-T and the helium ash confinement times \(\tau_p\) and \(\tau_{\alpha}\), respectively. This operation simplifies the model to zero dimensions, since all the radial variation of the plasma quantities is washed out. In doing this we assume constant density profiles \(n_k(r) = n_{k0}\) and radial temperature profiles for both ions and electrons of the form,

\[
T_k(r,t) = T_{k0}(t)[1 - (r/a)^2]^\gamma,
\]

with \(T_{k0}\) the central temperature of each species and \(a\) the tokamak minor radius. We will consider the volume-averaged equations for steady state operation, which are expressed in terms of the central temperatures and density (we drop the 0-subscript for shortness). They take the form,

\[
S_f - \frac{1}{2} n_{DT}^2 \langle \sigma v \rangle_{\text{vol}} - n_{DT} / \tau_p = 0
\]

\[
\frac{1}{4} (1 - f_{frac}) n_{DT}^2 \langle \sigma v \rangle_{\text{vol}} - n_{\alpha} / \tau_{\alpha} = 0
\]
alpha particles and by the neutrons. Another quantity to watch is the L-H mode transition
to account the total energy produced in the DT fusion reactions, i.e. the energy carried by the
Here, $P_{\text{eff}}$ with $A$.

The factor $H$ density and temperatures for ITER\[4\], which compensates for the lack of a line radiation term
term accounts for the combined bremsstrahlung and line radiated power reported at the nominal
value. We assumed $\tau_{E,i}$ value. We assumed $\tau_{E,i}$ parameter). We have included the fudge factor
where the factor $G$ defined as the ratio of the energy generation
where $n_i = n_{DT} + n_\alpha + n_{Be} + n_{Ar}$ and the averaged power densities are given explicitly in MW/m\(^3\),
with $A_i = 5.33 \times 10^{-43}$, $A_b = 0.032$, $A_{cy}$ $A_{cy} = 1.23 \times 10^{-14}$ for $n$ and $T$ in m\(^{-3}\), keV (for ITER parameters). We have included the fudge factor $T_{\text{rad}} = 2$, such that the bremsstrahlung loss
term accounts for the combined bremsstrahlung and line radiated power reported at the nominal
density and temperatures for ITER\[4\], which compensates for the lack of a line radiation term in the energy balance equations. Likewise, the factor $F_{\text{OH}} = 7.5$ is used to match the design value. We assumed $\tau_{E,i}$, which is taken from the IPB98(y2) scaling[3],

\[ \gamma_{\text{IPB98(y2)}} = 0.0562 H \left(0.93 R^{1.97} B^{0.15} M^{0.19} \kappa^{0.58} \rho^{0.78} n_e^{0.41} P_s^{0.69} \right) \]  

where the factor $H$ expresses the degree of enhancement expected over the predicted value due to
improved confinement.

The volume averaged D-T reactivity can be written as

\[ \langle \sigma v \rangle_{\text{vol}} = G(\gamma, T_i) \times \langle \sigma v \rangle \]  

where $G$ is a correction factor due to the radial profile in Eq. (5), which is fitted by a polynomial

\[ G(\gamma, T_i) = 0.249 + 0.017 T_i - 0.00011 T_i^2 - 0.13 \gamma + 0.023 \gamma^2 - 0.0077 \gamma T_i + 0.00125 T_i \gamma^2 \]

\[ +0.000067 T_i^2 \gamma - 0.0000126 T_i^2 \gamma^2 \]  

where $T_i$ is the central ion temperature in keV and $\langle \sigma v \rangle$ is the reactivity evaluated at the central
ion temperature.

For our analysis we use the energy gain factor $Q_G$ defined as the ratio of the energy generation rate in the plasma due to the fusion reactions to the total external heating power, and measures
how close a fusion reactor is to ignition conditions,

\[ Q_G = \frac{\langle P_{\text{fusion}} \rangle_{\text{vol}}}{\langle P_{\text{aux}} + P_{\text{ohmic}} \rangle_{\text{vol}}} \]  

Here, $P_{\text{aux}}$, includes both the auxiliary heating to electrons and to ions and $P_{\text{fusion}}$ takes into
account the total energy produced in the DT fusion reactions, i.e. the energy carried by the
alpha particles and by the neutrons. Another quantity to watch is the L-H mode transition
power threshold,

\[ P_{\text{threshold}} = 4.30 M_{\text{eff}}^{0.77} n_e^{0.72} R^{0.909 a^{0.975}} \]  

and we have to keep the total power above this value in order to stay in the H mode.
In the energy range covered during the alpha particles thermalization, the fraction of energy deposited to electrons and to ions is approximately 78% and 22% respectively, so $f_e = 0.78$, $f_i = 0.22$. The reference operating state for the studies presented below is $n_0 = 1.01 \times 10^{20}$ m$^{-3}$, $T_{e0} = 23.6$ keV and $T_{i0} = 23.0$ keV, for the electron density and the peak temperatures of the electrons and the ions, respectively. The radial profile parameter $\gamma$ will be taken equal to 1.85 for both electron and ions temperatures. It is assumed that 10% of the alpha particles are anomalously lost before they are thermalized, hence $f_{frac} = 0.1$ and their confinement time is $\tau_\alpha = 6.8 \tau_E$. Likewise we take $\tau_{DT} = \tau_\alpha$ for DT ions. The corresponding value of the fixed temperature ratio is $(T_e - T_i)/T_e = 0.025$ for all the operation states considered. In the following analysis we solve Eqs. (6)-(9) for the DT refueling rate, $S_f$, the fractional density of helium ash, $f_A$ and the auxiliary heating power to electrons and ions, $P_{aux,e}$ and $P_{aux,i}$, varying $n_e$ and $T_e$ over a range about the reference state.

3. The optimal operating states

For all computations presented we will take the fraction of Be impurities, fixed to $f_{Be} = 0.02$, and the density of Ar fixed at $n_{Ar} = 1.21 \times 10^{17}$ m$^{-3}$ for all operating points, and thus the fractional density $f_{Ar}$ varies for the different electron density values. The resulting operating states are shown in POPCON plots (normalized $T_e$-$n_e$ space) with isolines for some relevant quantities. Fig. 1 shows the contour plots of the auxiliary heating power to electrons and ions. It is seen that for a constant electron density, the auxiliary heating power to the ions decreases when $T_e$ decreases reaching a point where $P_{aux,i} = 0$. Notice that the contour lines with $P_{aux,i} < 0$ are not included because these are not physically plausible steady states. Thus, the line $P_{aux,i} = 0$ represents a boundary for the available operating points of the reactor.

![Figure 1](image1.png)

**Figure 1.** Contour curves of auxiliary power to electrons, $P_{aux,e}$ (dashed line) and to ions, $P_{aux,i}$ labeled in MW. Only positive values are plotted

![Figure 2](image2.png)

**Figure 2.** Isolines of total fusion power in MW (dashed line) and the Q-gain factor. Lines of $P_{aux,i} = 0$ (red) and L-H transition threshold (magenta) are shown

Fig. 2 shows the contour lines of the gain factor, $Q_G$, as obtained from Eq. (13) and for constant total fusion power (including the energy of the neutrons), together with the boundary line $P_{aux,i} = 0$. We observe that for a fixed fusion power value we can decrease the electron temperature increasing simultaneously the electron density, properly moving along the corresponding $P_{fusion} = constant$ line, thus increasing the $Q_G$ monotonically until the boundary.
Almost identical to the contour line associated with Q-gain factor. We notice that the line corresponding to the L–H transition power (magenta) is threshold. The energy confinement time is presented in Fig. 3 (dashed lines), together with the operation has to be to the right of the magenta line which represents the L–H transition power (magenta) is almost identical to the contour line associated with $\tau_E \approx 5.2 \text{ s}$. Thus, we can assert that the H-mode states have $\tau_E < 5.2 \text{ s}$. Interestingly, for a given $Q_G$, in order to increase $\tau_E$ one has to reduce the density.

4. Thermal stability

Once the best steady states are determined it is necessary to look at their stability properties. The temperature fluctuations might drive the plasma away from the desired operating states, if the reactor is liable to the thermonuclear instability. This arises when the temperature rise due to the energy released by the fusion reactions increases further the DT reactivity, which provides more heat to the plasma, and has to happen in the energy range with a positive slope of the function $\langle \sigma v \rangle (T)$, i.e. $T < 50 \text{ keV}$. In order to study this instability we apply a thermal perturbation to the system and analyze its response. We consider two different cases: (a) equal temperatures, $T_i = T_e \equiv T$ and (b) two temperatures, $T_i \not= T_e$ as in the analysis of the optimal states. The first case is taken as a simple approximation, for in this case there is a single equation for the total energy $W = W_e + W_i$. Performing a linear analysis, when a perturbation of the form $T = T_0 + \delta T$ on the equilibrium value $T_0$ is applied, the total energy equation can be expanded in $\delta T$ and takes the form,

$$\frac{\partial \delta T}{\partial t} = \frac{dP(T_0)}{dT} \delta T$$

where $P(T)$ is the net power to the plasma. Clearly, the perturbation is stable when the term on the right is negative since otherwise the temperature perturbation will increase exponentially. Thus, the stability criterion is $dP(T_0)/dT < 0$. For the second case, since there are two temperatures we have to take separate perturbations $T_e = T_{e0} + \delta T_e$, $T_i = T_{i0} + \delta T_i$ and consider the electron and ion energy equations (3, 4). Linearizing again about the two equilibrium temperatures, we can find an equation for the vector $\delta \mathbf{T} \equiv (\delta T_e, \delta T_i)$, as $\delta \mathbf{T}/\partial t = \mathbf{M} \cdot \delta \mathbf{T}$ with the matrix,

$$\mathbf{M} = \begin{pmatrix} \frac{\partial P_e(T_{0e}, T_{0i})}{\partial T_e} & \frac{\partial P_e(T_{0e}, T_{0i})}{\partial T_i} \\ \frac{\partial P_i(T_{0e}, T_{0i})}{\partial T_e} & \frac{\partial P_i(T_{0e}, T_{0i})}{\partial T_i} \end{pmatrix},$$

where the powers $P_e(T_e, T_i)$ and $P_i(T_e, T_i)$ are the right hand sides of eqs. (3, 4) respectively. The solution is obtained in terms of the eigenvalues of $\mathbf{M}$, $\lambda_1$ and $\lambda_2$, as $\delta \mathbf{T} = \delta \mathbf{T}_0 \exp(\lambda_k t)$. The system will be unstable if the real part of at least one of the $\lambda_k$ is positive. The stability computations are made for all the operating states in the range considered before and the marginal stability curves are presented in a POPCON plot in Fig. 4 for the cases (a) and (b). It is seen that the unstable region is for low $T_e$ and high $n_e$, as found in other studies [5], for the two cases and there is actually not much difference between them. However, the region of instability falls outside the allowed operating regime since it is to the left of the green line $P_{aux,i} = 0$, thus it would not be excited for the optimal states found in Section 3, or any other allowed state.

In order to have an idea of the sensitivity of the results to the particular energy confinement scaling used, we considered a case with constant $\tau_E$. In this analysis, $\tau_E$ is kept fixed at its equilibrium value when the variations in Eqs. (15, 16) are made, instead of using (10). The results are shown in Fig. 4 with dashed lines for the two cases of equal and different temperatures. It is clear that the difference is quite noticeable, and now the unstable region covers all the range of optimal operating states. Since the constant-$\tau_E$ is not compatible with the previous calculations.
we cannot conclude that the instability will be present, but it shows that the choice of the actual confinement scaling is important.

Figure 3. Contour plots for the energy confinement time (dashed) and the gain factor $Q_G$, showing that for $Q_G$ given, $\tau_E$ increases as $n_e$ decreases

Figure 4. Marginal stability curves for cases of $T_e = T_i$ (blue) and $T_e \neq T_i$ (red). Dashed curves correspond to constant confinement time

5. Conclusions
In the present study we have used a 0D model to describe the operation of a nuclear fusion reactor, obtained by a volume average of the particle and energy equations for electrons, ions and helium ash. The steady states of operation were analyzed by taking variations of the electron density and temperature around the nominal operation point of ITER, maintaining the temperature ratio $(T_e - T_i)/T_e$ fixed. They are presented in POPCON plots showing contours of constant auxiliary power to electrons and ions, where it is seen that there are no states with $P_{aux,e} + P_{aux,i} = 0$, meaning that ignition is not possible. However, there are states with $P_{aux,i} = 0$ which represent an operational boundary and they turn out to be also the states with maximum $Q_G$ for a given fusion power. Hence we identify these operational states as the optimal ones.

The thermonuclear stability is studied using linear analysis by considering temperature perturbations about the equilibrium state. Cases with $T_e = T_i$ and $T_e \neq T_i$ were considered and in both cases we showed that the system will be stable for the possible operating states of ITER parameters. The results are sensitive to the type of $\tau_E$ scaling; thus for certain scalings the reactor may become unstable.

Acknowledgments. This work was supported by projects DGAPA-UNAM IN119408, IN109909 and Conacyt 81232-F.

References
[1] Vitela J E and Martinell J J 2001 Plasma Phys. Control. Fusion 43, 99
[2] Bosch H S and Hale G M, 1992 Nucl. Fusion 32, 611
[3] Aymar R, Barabaschi P and Shimomura Y 2002 Plasma Phys. Control. Fusion 44, 519
[4] “ITER Physics Basis” 1999 Nucl. Fusion 39 Vol. 12
[5] Mandrekas J and Stacey W M 1991 Fusion Technol. 19, 57