Radiation from a uniformly accelerating harmonic oscillator

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Abstract

We consider a radiation from a uniformly accelerating harmonic oscillator whose minimal coupling to the scalar field changes suddenly. The exact time evolutions of the quantum operators are given in terms of a classical solution of a forced harmonic oscillator. After the jumping of the coupling constant there occurs a fast absorption of energy into the oscillator, and then a slow emission follows. Here the absorbed energy is independent of the acceleration and proportional to the log of a high momentum cutoff of the field. The emitted energy depends on the acceleration and also proportional to the log of the cutoff. Especially, if the coupling is comparable to the natural frequency of the detector \((e^2/(4m) \sim \omega_0)\) enormous energies are radiated away from the oscillator.

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I. INTRODUCTION

It is well known that the concept of a particle depends on the motion of an observer [1]. Especially, the Minkowski vacuum is a canonical ensemble with the temperature $a/2\pi$ from the point of view of a uniformly accelerated observer with the acceleration $a$ (the thermalization theorem) [2]. This observer dependence is most easily shown if one use a particle detector model invented by Unruh [2] and DeWitt [3]. It consists of an idealized point particle with internal energy levels labeled by $E$, coupled via a monopole interaction with a scalar field $\phi$ (Unruh-DeWitt model). Following these, many works emerged in the literature. Letaw [4] exhibited the stationary world lines, on which the detectors in a vacuum have a time-independent excitation spectra. Grove and Ottewill [5] studied the problem of a non-extended detector, and clarified the radiation effect arising both from the walls of the detector and from the interaction with the field. Several authors [6,7] discussed the anisotropic nature of the thermal radiation of the accelerated detector. A full review for this thermal character was given by Takagi [8]. The vacuum noise seen by a uniformly accelerated observer in flat space-times of arbitrary dimensions was investigated and was shown to exhibit the phenomenon of the apparent inversion of statistics in odd dimensions, which was discussed precisely by Unruh [9] and Fukazawa [10]. A few years ago, the excitation rate associated with a uniformly accelerated finite-time detector interacting with the Minkowski vacuum has been analyzed in an inertial frame by Svaiter and Svaiter [11]. They found a logarithmic ultraviolet divergences on the transition amplitude, which was due to the instantaneous switching of the detector [12]. This UV divergence does not occur in lower dimensions. Grove argue that a macroscopic constantly accelerating object will emit negative energy radiation until equilibrium with the Minkowski vacuum is achieved [13].

Several years ago a new particle detector model–a harmonic oscillator coupled to a scalar field in $1 + 1$ dimensions–was introduced by Raine, Sciama, and Grove(RSG) [14]. Several aspects of this model was discussed in connection with the ‘open quantum system’ [15–17]. Hinterleitner [18] and Massar, Parentani, and Brout [19] shown that there is a polarization
cloud which surrounds the detector at all times and energy is exchanged with it locally. Audretsch and Müller [20] explored nonlocal pair correlations in accelerated detector. Recently stochastic aspects of this detector were discussed by Raval, Hu, and Anglin [21]. These works mainly interested on the asymptotic states with time independent coupling. In this paper we consider the intermediate region during the equilibrium achieved between the detector and the field. We show that this is not a simple energy absorption process but there are two main stages after the two systems in contact with. First stage is a fast absorption of energy of the oscillator from the field. This occurs shortly after the change of the coupling in a time which is much smaller than the inverse of the characteristic frequency of the oscillator. The total energy absorbed during this period is independent of the acceleration and depends on the log of a high momentum cutoff. Second stage is slow emission of radiation which exponentially decrease in a time scale of the coupling constant. The total radiated energy during this period depends on the acceleration of the oscillator. If the coupling constant is small then the total radiation is smaller than the inertial one. But if the coupling is comparable to the characteristic frequency, enormous energies are radiated away from the oscillator. In the case of a weakly coupled system, the absorbed energy during the first stage is larger than the emitted one during the second stage.

In Sec. II-A, we describe the model in Minkowski space and give the general form of the solution for the field and the oscillator. These evolutions of the operators are given by use of the inhomogeneous solution $G(\omega, t)$ of a forced harmonic oscillator. Similarly, all physical quantities like the correlation function or the stress tensor can be expressed with this single function. In Sec. II-B, the model is generalized to incorporate the uniformly accelerating oscillators. Sec. III is devoted to present two solvable models. $G(\omega, t)$ is obtained in the asymptotic region. We obtain the stress tensor in Sec. IV when the detector is turned on suddenly. Sec. V is summary and discussions. There are two appendices where we describe the details of the calculation of the stress tensor.
II. MODELS FOR THE PARTICLE DETECTOR

Let us consider a minimally coupled system of a massless real scalar field $\phi(t, x)$ in two dimensions and a detector of a harmonic oscillator $q(t)$ with mass $m$. The action for this system is

$$S = \int dx dt \frac{1}{2} \left\{ \left( \frac{\partial}{\partial t} \phi(t, x) \right)^2 - \left( \frac{\partial}{\partial x} \phi(t, x) \right)^2 \right\}$$

$$+ \int d\tau \left\{ \frac{1}{2} m \left( \frac{dq(\tau)}{d\tau} \right)^2 - \frac{1}{2} m \omega_0^2 q^2(\tau) - e(\tau) q(\tau) \frac{d\phi}{d\tau}(t(\tau), x(\tau)) \right\}.$$  

(1)

The oscillator follows the explicitly given path $(t(\tau), x(\tau))$ where $\tau$ is the proper time of the oscillator along the path. In this paper, we select two paths through which the oscillator moves: the inertial and the uniformly accelerated.

Varying Eq. (1) with respect to $\phi(t, x)$ and $q(\tau)$ we get the Heisenberg equation of motion for the field and the oscillator

$$\Box \phi(t, x) = \frac{d e(\tau) q(\tau)}{d\tau} \delta(\rho),$$

(2)

$$m \left( \frac{d}{d\tau} \right)^2 q(\tau) + m \omega_0^2 q(\tau) = -e(\tau) \frac{d\phi}{d\tau}(t(\tau), x(\tau)),$$

(3)

where $\rho$ is an appropriate space coordinate which is orthogonal to $\tau$ and the path of the oscillator can be represented as $\rho = 0$. Eq. (2) can be integrated to give

$$\phi(t, x) = \phi^0(t, x) + \frac{e(\tau_{ret})}{2} q(\tau_{ret}),$$

(4)

where $\tau_{ret}$ is the value of $\tau$ at the intersection of the past lightcone of $(t, x)$ and the detector trajectory. where we have used the explicit retarded propagator of a massless field

$$G_{ret}(t, x; 0, 0) = \frac{1}{2} \theta(t + x) \theta(t - x).$$

(5)

Substituting the solution (4) into Eq. (3), one get

$$m \ddot{q}(\tau) + \frac{1}{2} e^2(\tau) \dot{q}(\tau) + m \left( \omega_0^2 + \frac{\dot{e}^2(\tau)}{4m} \right) q(\tau) = -e(\tau) \phi^0(t(\tau), x(\tau)).$$

(6)

The redefinitions
\[ M(\tau) = m \exp \left( \int_{\tau_0}^{\tau} \frac{e^2(\tau)}{2m} \, d\tau \right), \quad (7) \]
\[ \omega^2(\tau) = \omega_0^2 + \frac{e^2(\tau)}{4m}, \quad (8) \]
\[ F(\tau) = -\frac{\dot{e}(\tau) \, d\phi_0}{m} (t(\tau), x(\tau)). \quad (9) \]

make Eq. (7) into the equation of motion of the forced harmonic oscillator with the effective mass \( M(t) \), and the frequency \( \omega^2(t) \)
\[ \ddot{q}(\tau) + \frac{d \ln M(\tau)}{d\tau} \dot{q}(\tau) + \omega^2(\tau)q(\tau) = F(\tau). \quad (10) \]

Here \( F(\tau) \) is the force density per unit effective mass. We take the normalization of the effective mass as \( M(\tau_0) = m \) at some initial time \( \tau_0 \). As one see from Eq. (10), we can arbitrarily choose the normalization of the effective mass. Note that we can rewrite this equation into quadratic form:
\[ \left[ \left( \frac{d}{d\tau} \right)^2 + \Omega^2(\tau) \right] \sqrt{M(\tau)} q(\tau) = \sqrt{M(\tau)} F(\tau), \quad (11) \]

where
\[ \Omega^2(\tau) = \omega_0^2 - \left( \frac{e^2(\tau)}{4m} \right)^2. \quad (12) \]

The behavior of a homogeneous solution of Eq. (11) changes from oscillatory to exponential decay according to the value of \( \Omega^2(t) \). We restrict our discussion into \( \Omega^2(t) \) greater than zero. If \( e(\tau) \ll \omega_0 \) then \( \Omega \) is natural positive frequency mode of the oscillator. The behavior of the homogeneous solution, in this case, is
\[ f(t) = \frac{1}{\sqrt{M(\tau)}} \exp \left[ \pm i \int_{\tau}^{\tau'} \Omega(\tau') \, d\tau' \right]. \quad (13) \]

Let the initial Heisenberg operators for the oscillator to be \( q(\tau_0) \) and \( p(\tau_0) = m\dot{q}(\tau_0) \).

Then the exact quantum motion of \( q(\tau) \) which is subjected to the external force \( F(\tau) \) in the Heisenberg picture is given by
\[ q(\tau) = q_0(\tau) + q_F(\tau) \]
\[ = q(\tau_0) \frac{\sqrt{g_-(\tau)g_+(\tau_0)}}{\omega_I} \cos [\Theta(\tau) - \chi(\tau_0)] + p(\tau_0) \frac{\sqrt{g_-(\tau)g_+(\tau_0)}}{\omega_I} \sin \Theta(\tau) \]
\[ + A_F(\tau) + A_F^*(\tau). \quad (14) \]
In this equations we use the following definitions:

\[ g_-(\tau) = f(\tau)f^*(\tau), \]
\[ g_0(\tau) = -\frac{M(\tau)}{2}\dot{g}_-(\tau), \]
\[ g_+(\tau) = M^2(\tau)\left|\dot{f}(\tau)\right|^2, \]
\[ \Theta(\tau) = \int_{\tau_0}^{\tau} d\tau \frac{\omega I}{M(\tau)g_-(\tau)}. \]

where \( f(\tau) \) is a homogeneous solution of Eq. (10) and \( \omega_I = \sqrt{g_+(\tau)g_-(\tau) - \dot{g}_0^2(\tau)} \) is invariant under the time evolution. For the definition of \( g_i(\tau) \) \((i = \pm, 0)\) see [25] and references therein. If \( \tau \) is Killing time, we can expand the free field solution into its positive solutions and negative solutions. Let us classify its solution by \( \omega \) and set the positive solution as \( u_\omega \).

Therefore

\[ \frac{\partial}{\partial \tau} u_\omega = -i|\omega|u_\omega \]

and The free field solution in two dimension can be written as

\[ \phi^0(t, x) = \int_{-\infty}^{\infty} dk[a_k u_k(\tau, \rho) + a_k^\dagger u_k^*(\tau, \rho)] \]

where \( a_k \) and \( a_k^\dagger \) is the corresponding creation and annihilation operators and \( u_k \) is proportional to \( e^{-i\omega\tau} \). With this choice, we can write the annihilation part of the inhomogeneous solution \( q_F(\tau) = A_F(\tau) + A_F^\dagger(\tau) \) as

\[ A_F(\tau) = \int_{0}^{\infty} d\omega \omega G(\omega, \tau)(a_\omega + a_{-\omega}). \]

Where \( G(\omega, \tau) \) is the classical inhomogeneous solution of the forced harmonic oscillator equation

\[ \ddot{G}(\omega, \tau) + \frac{d\ln M(\tau)}{d\tau} \dot{G}(\omega, \tau) + \omega^2(\tau)G(\omega, \tau) = -i\frac{e(\tau)}{m}u_\omega \]

with the initial condition

\[ G(\omega, 0) = 0 \quad \dot{G}(\omega, 0) = 0. \]
If we analyze $G(\omega, \tau)$, we can know all time evolutions of the operators in principle. The general solution for $G(\omega, \tau)$ can be written as

$$G(\omega, \tau) = g(\omega, \tau) - g^*(\omega, \tau),$$

(22)

where

$$g(\omega, \tau) = e^{i\Theta(\tau)} \int_{\tau_0}^{\tau} d\tau' \sqrt{g(\tau')} M(\tau') e^{-i\Theta(\tau')} u_\omega(x(\tau'), t(\tau')).$$

(23)

One can show that Eq. (14) satisfies (10) by direct substitution. The high momentum behavior of $G(\omega, t)$ is $O(1/\omega^{5/2})$ except some special case like the sudden jumping of the coupling constant, which we consider in Sec. IV. In the case of large $\omega$ the integral of (23) is approximately given by the contributions around $\tau_0$, which makes the arguments of the exponential of $u_\omega(\tau)$ vanishes. Therefore the first approximation of $g(\omega, \tau)$ is of the form $\int d\tau' f(\tau_0) e^{\pm i\omega \tau}$. But this term is canceled in $G(\omega, \tau) = g(\omega, \tau) - g^*(-\omega, \tau)$, and leaves only the $O(1/\omega^{5/2})$ and higher terms.

A. The inertial oscillator

At first let us consider the simplest inertial path: $x = 0$ and $t = \tau$. Moreover the mode solution is $u_k = 1/\sqrt{4\pi|k|} e^{-i(k(t-\tau))}$.

Let the initial state of the combined system to be

$$|i\rangle = |n\rangle |0\rangle_M,$$

(24)

the $n$th excited state for the oscillator and the Minkowski vacuum state for the field. The correlation functions of $q(t)$ for state (24) is

$$\langle q_O(t)q_O(t') \rangle = (2n + 1) \frac{g_-(t)g_-(t')}{2\omega_I} \exp \{-i[\Theta(t) - \Theta(t')]\},$$

(25)

$$\langle q_F(t)q_F(t') \rangle = 2 \int d\omega \omega^2 G(\omega, t)G^*(\omega, t'),$$

(26)

$$\langle q_O(t)q_F(t') \rangle = 0.$$
The correlation of the homogeneous part decrease because $M(t)$ increase monotonically. Therefore for a large enough time the correlation is governed by the inhomogeneous term. If there is absent of $1/(\omega)^{3/2}$ term in $G(\omega, t)$ there is no UV contribution to the correlation function. As we see in the previous section, this is normally true. Therefore in the case of a slowly varying coupling, the main contribution to the correlation comes from the frequency region around the resonance frequency $\Omega(t)$ (See Eq. (11)). The system is symmetric about $x = 0$. Therefore, it is enough to obtain the correlations of the field in the area $x, x' < 0$.

In this area Eq. (4) becomes

$$
\phi(t, x) = \phi^0_R(u) + \phi^0_L(v) + \frac{1}{2}e(v)[q_O(v) + q_F(v)].
$$

Therefore the correlations of the field and the oscillator is for the state (24) are

$$
\langle \phi^0_R(u)q_F(v') \rangle = \langle q_F(v')\phi^0_R(u) \rangle^* = \int d\omega \omega G^*(\omega, v')u_\omega(u), \quad (29)
$$

$$
\langle \phi^0_L(v)q_F(v') \rangle = \langle q_F(v')\phi^0_L(v) \rangle^* = \int d\omega \omega G^*(\omega, v')u_\omega(v). \quad (30)
$$

From these, one get the renormalized correlation function

$$
\langle \phi(t, x)\phi(t', x') \rangle \sim \langle \phi^0(t, x)\phi^0(t', x') \rangle + e(v)e(v') \int d\omega \omega G^*(\omega, v')u_\omega(v') + \frac{1}{2}e(v)^2 \int d\omega \omega G^*(\omega, v') \left[ u_\omega(v) + u_\omega(u) \right] + \frac{1}{2}e(v) \int d\omega \omega G^*(\omega, v') \left[ u_\omega(v') + u_\omega'(u') \right].
$$

B. The uniformly accelerated oscillator

Now let us consider a uniformly accelerating trajectory $x = \frac{1}{a} \cosh a\tau, t = \frac{1}{a} \sinh a\tau$. Rindler space $(\tau, \rho)$ is given by

$$
x = \frac{1}{a} e^{\rho} \cosh a\tau, \quad t = \frac{1}{a} e^{\rho} \sinh a\tau.
$$

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In this system the retarded time is
\[ \tau_{\text{ret}} = \tau - \rho \quad \text{for} \quad \rho > 0, \]
\[ = \tau + \rho \quad \text{for} \quad \rho < 0. \]  

At the right Rindler wedge, the free field \( \phi^0(t, x) \) can be expanded with the normal modes of Rindler space-time as
\[ \phi^0(t, x) = \sum_{k=\infty}^{\infty} \left[ b_k \xi_k + H.C. \right] \]
\[ = \sum_{\lambda=0}^{\infty} \left[ b_{\lambda} \xi_{\lambda}(U) + b_{-\lambda} \xi_{\lambda}(V) + H.C. \right], \]
where \( U = \tau - \rho = -\ln(-au)/a, \) \( V = \tau + \rho = \ln(av)/a, \) and \( \xi_{\lambda} = 1/\sqrt{4\pi|\lambda|e^{-i\lambda U}} = 1/\sqrt{4\pi|\lambda|(-au)^{\lambda/a}}, \) and \( b_{\lambda}, b_{\lambda}^\dagger \) is the creation and annihilation operator in the Rindler space-time. Therefore we can set \( u_{\omega} \rightarrow \xi_{\lambda} \) and \( \omega \rightarrow \lambda \) in Sec. II.

Let us consider the initial state to be (24). The expectation value of \( q(\tau) \) for \( |i> \) is zero. The correlation functions for \( |i> \) are
\[ \langle q_{O}(\tau) \ q_{O}(\tau') \rangle = (2n + 1) \sqrt{\frac{g_{-}(\tau)g_{-}(\tau')}{2\omega_I}} \exp \left\{ -i[\Theta(\tau) - \Theta(\tau')] \right\}, \]
\[ \langle q_{F}(\tau) \ q_{F}(\tau') \rangle = 2 \int d\lambda \lambda^2 \left\{ \{1 + N(\lambda/a)\} G(\lambda, \tau)G^*(\lambda, \tau') \right\} \]
\[ + N(\lambda/a)G^*(\lambda, \tau)G(\lambda, \tau'), \]
\[ \langle \phi^0_R(U) \ q_{F}(\tau_{\text{ret}}') \rangle = \langle q_{F}(\tau_{\text{ret}}')\phi^0_R(U) \rangle^*, \]
\[ = \int d\lambda \lambda \left\{ \xi_{\lambda}(U)G^*(\lambda, \tau') \{1 + N(\lambda/a)\} + \xi_{\lambda}(U)G(\lambda, \tau_{\text{ret}})N(\lambda/a) \right\}, \]
\[ \langle \phi^0_L(V) \ q_{F}(\tau_{\text{ret}}') \rangle = \langle q_{F}(\tau_{\text{ret}}')\phi^0_L(V) \rangle^*, \]
\[ = \int d\lambda \lambda \left\{ \xi_{\lambda}(V)G^*(\lambda, \tau_{\text{ret}}') \{1 + N(\lambda/a)\} + \xi_{\lambda}(V)G(\lambda, \tau_{\text{ret}})N(\lambda/a) \right\}, \]
where \( N(\Omega) = 1/(e^{2\pi\Omega} - 1). \) We use the fact that the Minkowski vacuum is FDU thermal bath with temperature \( a/(2\pi) \) to the accelerating observer. The renormalized correlation function can be obtained with the same method of inertial case. These correlation functions can be divided into two classes: First is those of zero temperature and second is thermal contributions. The high momentum behavior of the second is cut off by the presence of the
exponential in the denominator. Therefore it is evident that the first term dominate the
correlation if there is UV divergences due to the existence of the $1/\omega$ term in $G(\lambda, \tau)$. The
sudden jump case of the coupling is exactly that case.

III. EXACTLY SOLVABLE MODELS

A. Constant coupling

The most easiest problem is, of course, the case of constant coupling $(e(\tau) = e)$. In this
case

$$M(\tau) = \frac{e^2}{2}(\tau - \tau_0)$$

$$f(\tau) = \frac{1}{\sqrt{m}}e^{\pm(i\Omega + e^2/(2m)(\tau - \tau_0))}$$

where $\Omega^2 = \omega_0^2 - e^2$. $G(\omega, \tau)$ satisfies the following equation:

$$\ddot{G}(\omega, \tau) + \frac{e^2}{2m} \dot{G}(\omega, \tau) + \omega_0^2 G(\omega, \tau) = -i \frac{e}{m} u_\omega$$

The general solution to this equation is given by

$$G(\omega, \tau) = a \exp(-e\tau) \exp(i\sqrt{\omega_0^2 - e^2 + \alpha}) + ie\chi(\omega)u_\omega(\tau, 0)$$

where

$$\chi(\omega) = \frac{1}{m [\omega_0^2 - \omega^2 - 2i\epsilon\omega]}$$

and $\epsilon = e^2/4m$. The coefficients $a$ and $\alpha$ must be chosen $G(\omega, \tau)$ to satisfy the initial
condition \[^1\text{2}\]. The first term exponentially decay therefore there remain only the second
term asymptotically. This is exactly the same result with Massar, Parentani, and Brout
\[^1\text{9}\].
B. Turn on of the coupling

The next example is given by the coupling

\[
2\epsilon(\tau) = \frac{\epsilon^2(\tau)}{2m} = \frac{\epsilon^2}{4m} \left(1 - \tanh \frac{\tau}{d}\right) + \frac{\epsilon^2}{4m} \left(1 + \tanh \frac{\tau}{d}\right) = \epsilon_+ \left(1 - \tanh \frac{\tau}{d}\right) + \epsilon_- \left(1 + \tanh \frac{\tau}{d}\right). \tag{44}
\]

The limit \(d \to 0\) corresponds to the sudden jump and \(d \to \infty\) to the adiabatic one. Eq. (12)

\[
\Omega^2(\tau) = \omega^2_0 - \epsilon^2(\tau) = \frac{\omega^2}{2} \left(1 - \tanh \frac{\tau}{d}\right) + \frac{\omega^2}{2} \left(1 + \tanh \frac{\tau}{d}\right) + \frac{(\epsilon_+ - \epsilon_-)^2/4}{\cosh^2(\tau/d)} \tag{45}
\]

has two limiting values \(\omega^2_\pm = \omega^2_0 - \epsilon^2_\pm\) at the positive and negative infinity. These two values define a natural positive frequency modes of the oscillator in the past and the future asymptotic region. The effective mass (7) becomes

\[
M(\tau) = m \exp \int 2\epsilon(\tau) d\tau = m \left(\frac{\cosh \tau/d}{\cosh \tau_0/d}\right)^{(\epsilon_+ - \epsilon_-) d} \exp \left[(\epsilon_+ + \epsilon_-)(\tau - \tau_0)\right]. \tag{46}
\]

From these we get the homogeneous solution for the classical equation of motion (10)

\[
f(\tau) = \frac{1}{\sqrt{M(\tau)}} e^{-i(\omega_+ + \omega_-)\tau/2} \left(\frac{\cosh \tau/d}{\cosh \tau_0/d}\right)^{-i(\omega_+ - \omega_-) d/2} 2F_1(\alpha_-, \alpha_+; 1 - i\omega_- d; y), \tag{47}
\]

where

\[
y = \frac{1 + \tanh \tau/d}{2}, \tag{48}
\]

\[
\alpha_\pm = \frac{1 \pm \sqrt{1 + (\epsilon_+ - \epsilon_-)^2 d^2}}{2} + \frac{i(\omega_+ - \omega_-) d}{2}, \tag{49}
\]

and \(2F_1\) is the hypergeometric function [23]. We choose Eq. (47) to be pure positive frequency mode at the past infinity. On the other hand, it becomes generally mixture of the positive and negative modes at the future:

\[
\lim_{\tau \to -\infty} f(\tau) = \frac{2i(\omega_+ - \omega_-) d/2}{\sqrt{M(\tau)}} e^{-i\omega_- \tau}, \tag{50}
\]

\[
\lim_{\tau \to \infty} f(\tau) = \frac{2i(\omega_+ - \omega_-) d/2}{\sqrt{M(\tau)}} \left(\alpha e^{-i\omega_+ \tau} + \beta e^{i\omega_+ \tau}\right). \tag{51}
\]
where
\[ \alpha = \frac{\Gamma(1 - i\omega_d)\Gamma(1 - i\omega_d - \alpha - \alpha_+)}{\Gamma(1 - i\omega_d - \alpha)\Gamma(1 - i\omega_d - \alpha_+)} \] (52)
\[ \beta = \frac{\Gamma(1 - i\omega_d)\Gamma(\alpha_+ + \alpha - 1 + i\omega_d)}{\Gamma(\alpha_+)} \] (53)

The absolute squares of \( \alpha \) and \( \beta \)
\[ |\alpha|^2 = \frac{1}{2} \frac{\omega_- \cosh \pi(\omega_+ + \omega_-)d + \cos 2\pi x}{\omega_+ \sinh \pi\omega_d \sinh \pi\omega_+ d}, \] (54)
\[ |\beta|^2 = \frac{1}{2} \frac{\omega_- \cosh \pi(\omega_+ - \omega_-)d + \cos 2\pi x}{\omega_+ \sinh \pi\omega_d \sinh \pi\omega_+ d} \] (55)

satisfy a Bogolubov type relation
\[ |\alpha|^2 - |\beta|^2 = \frac{\omega_-}{\omega_+}, \] (56)

where \( x = \sqrt{1 + (\epsilon_+ - \epsilon_-)^2 d^2}/2 \). The factor \( \omega_-/\omega_+ \) comes from the change of the natural frequency of the oscillator [22]. At the present problem the initial homogeneous solution for the oscillator decays by \( 1/\sqrt{M(\tau)} \) factor so the asymptotic form for large \( \tau \) is given by \( q_F \). Therefore the particle creation or other related topics must be discussed with the inhomogeneous solution \( G(\omega, \tau) \) with respect to the positive frequency mode at the future asymptotic region. Since our primary purpose is not the oscillator state but the radiation from the oscillator, we do not discuss it further. In the adiabatic limit \( \beta \) vanishes, on the other hand, in the sudden jump limit it becomes \( (1 - \omega_-/\omega_+)/2 \).

From (47) one get
\[ g_-(\tau) = f(\tau)f^*(\tau) = \frac{|_{2F1}(\alpha_-, \alpha_+; 1 - i\omega_d; y)|^2}{M(\tau)} \] (57)

and the invariant frequency \( \omega_I = \omega_- \). The integral of generalized frequency (43) is
\[ \Theta(\tau) = \omega_I \int^\tau d\tau' \frac{1}{|_{2F1}(\alpha_-, \alpha_+; 1 - i\omega_d; y)|^2} \]
\[ = \omega_I \int^\tau d\tau' \frac{1}{R^2(\tau')} - \omega_I \int^\tau d\tau' \frac{1}{R^2(\tau)} \left( 1 - \frac{R^2(\tau)}{|_{2F1}(\alpha_-, \alpha_+; 1 - i\omega_d; y)|^2} \right) \] (58)
\[ = \theta(\tau) - \theta(\tau_0) - \theta_f(\tau), \]
where \( R(\tau) \) and \( \theta(\tau) \) are the absolute value and the real phase of
\[
Re^{-i\theta(\tau)} = \alpha e^{-i\omega_+ \tau} + \beta e^{i\omega_+ \tau},
\]
(59)
and \( \theta_f(\tau) \) approaches to some finite value as \( t \to \infty \). Eqs. (14 and (57) shows \( q_O(t) \) decreases exponentially for \( \tau \gg d \). Using these and Eq. (22) we get the asymptotic form
\[
G(\omega, \tau) = ie_+ \chi(\omega) u_\omega(\tau, 0)
- \frac{1}{2m\omega_I \sqrt{M(\tau)}} \left\{ \alpha \chi_f(-\omega) - \beta^* \chi_f(\omega) \right\} e^{-i\omega_+ \tau} + \left\{ \beta \chi_f(-\omega) - \alpha^* \chi_f(\omega) \right\} e^{i\omega_+ \tau}
\]
where
\[
\chi(\omega) = \frac{1}{m[\omega_0^2 - \omega^2 - 2i\epsilon_+ \omega]},
\]
(61)
and \( \chi_f(\omega) = \lim_{\tau \to \infty} \chi_f(\omega, \tau) \).
\[
\chi_f(\omega, \tau) = -e_+ \int_{\tau_0}^\tau d\tau' \sqrt{M(\tau')} R(\tau') e^{-i\omega_+ \tau'} u_\omega(\tau', 0)
- \frac{1}{e_+} \left[ 1 - \frac{\sqrt{M(\tau') g_-(\tau') e(\tau') \sqrt{\tau' - \theta(\tau')}}}{R(\tau')} \right].
\]
(62)
This result is similar with that of the constant coupling except \( \chi_f(\omega) \) is determined by the integral.

**IV. STRESS ENERGY TENSOR IN THE SUDDEN JUMP LIMIT**

Now let us study the stress tensor of the scalar field in the presence of the oscillator. We consider instant switching process (\( d \to 0 \) limit of Sec. III B). We solve this problem up to zeroth order on \( d \) or \( e^{-2|\tau|/d} \), where \( \tau \) is the proper time seen by the oscillator. We calculate \( G(\omega, \tau) \) without explicit choice of coordinates system because it is common both the inertial and the uniformly accelerating oscillator. Let us set \( \tau_0 = -\infty \) and rescale the mass to be \( M(0) = m \). Similarly we also set \( \Theta(0) = 0 \).

After carrying out the integral (62) explicitly in the limit \(|t| \gg d\) we get
\( G(\omega, \tau) = ie_-\chi_-(\omega)u_\omega(\tau, 0), \) for \( \tau \ll -d, \) \( (63) \)

\[
G(\omega, \tau) = G_\infty(\omega, \tau) + \frac{ie_+}{2\sqrt{4\pi\omega}} e^{-\epsilon_+\tau} \left\{ \chi_+^*(-\omega)e^{i\omega_+\tau} + \chi_+(\omega)e^{-i\omega_+\tau} \right\}, \quad \text{for} \quad \tau \gg d,
\]

where

\[
\chi_+(\omega) = \chi_d(\omega) + \frac{e_-}{e_+} \chi_-(\omega), \quad (64)
\]

and

\[
\chi_d(\omega) = \frac{1}{m\omega_+} \left[ \frac{1}{\omega - \omega_+ + ie_+} - \frac{1 - e_-/e_+}{+\omega - \omega_+ + i(\epsilon_+ - 2/d)} \right] - \frac{e_-}{2e_+} \left( \frac{1}{\omega - \omega_- + ie_-} + \frac{1}{\omega + \omega_- + ie_-} \right),
\]

\[
\chi_-(\omega) = \chi_-^*(-\omega) = \frac{1}{m\omega_0^2} \frac{1}{\omega^2 - 2i\omega\epsilon_-}. \quad (66)
\]

Here needs some remarks. All physical quantities like the coupling, classical solution, and effective mass must be continuous at \( \tau = 0. \) This constraints demands the second term in \( \chi_d, \) which makes \( G(\omega, \tau) \) to be quadratically decrease for large \( \omega. \) \( G_\infty(\omega, \tau) = ie_+\chi(\omega, \tau)u_\omega(\tau, 0) \) dominates the asymptotic form of \( G(\omega, \tau) \) and the second term, which is the effect of the change of the coupling, decrease exponentially on the time scale of \( 1/\epsilon_+. \) At \( \tau < 0 \) the inhomogeneous solution \( G(\omega, \tau) \) is that of the equilibrium. Therefore our system represent a system which is in equilibrium at \( \tau < 0 \) become dynamic due to the change of the coupling at \( \tau = 0. \) The solution \( G(\omega, \tau) \) describe this dynamic approaching process to equilibrium.

If we restrict the region of \( \omega \) as \( 0 < \omega < \Gamma \ll 1/d, \) we can ignore the \( d \) dependent term in \( \chi_d(\omega). \) In this limit \( \chi_d(\omega) \) becomes \( O(1/\omega) \) and gives cutoff dependent UV behaviors. On the other hands in the case of \( \Gamma \gg 1/d, \chi_d(\omega) \) is \( O(1/\omega^2) \) which makes the theory UV finite. But we cannot get a sensible theory because the asymptotic form of the stress tensor crucially depends on \( 1/d, \) which is unphysical. Therefore we restrict the cutoff \( \Gamma \ll 1/d \) and also restrict our attention to \( |\tau| > 1/\Gamma. \)
A. The Stress tensor in the presence of a inertial oscillator

In Sec. (II A) we obtain the renormalized correlation function (Eq. (27)) in the region $x, x' < 0$. In this region there is no $u, u'$ dependent terms. Therefore $T_{uu}$ component of the stress tensor vanishes. Moreover, $\langle T_{uv} \rangle = trT/4 = 0$ since we are dealing with a massless field in two dimension and there is no trace anomaly because the curvature is zero.

The stress tensor vanishes in the region $v < 0$ since $G(\omega, t)$ has the same form with the asymptotic case (60) and there is no radiation asymptotically.

In the region $t > 0$ we must calculate the stress tensor explicitly. We restrict our attention to $v > 1/\Gamma \gg d$ since we are interested in the radiation after turn on the coupling. After taking differentiation of the correlation function with respect to $v$ and $v'$ followed by the limit $v' \to v$ we get

$$T_{vv} = T_1 + \frac{e^+}{2} T_2 + \frac{e^+}{2} T_3,$$  \hspace{1cm} (67)

where

$$T_1 = (2n + 1) \frac{e^+}{8\omega I} \lim_{v' \to v} \left( \partial_v \partial' \sqrt{g_-(v)g_-(v')} \exp \left[ -i \{ \Theta(v) - \Theta(v') \} \right] \right),$$  \hspace{1cm} (68)

$$T_2 = \int d\omega \omega \left[ \partial_v G^*(\omega, v) \partial_v u_\omega(v) + \partial_v G(\omega, v) \partial_v u^*_\omega(v) \right],$$  \hspace{1cm} (69)

$$T_3 = \int d\omega \omega^2 \partial_v G(\omega, v) \partial_v G^*(\omega, v).$$  \hspace{1cm} (70)

Where we ignore terms related with $\dot{e}(v)$ which is important for $t \sim d$. Since we consider only the region $t > 1/\Gamma \gg d$, it is safe to ignore such terms.

Now let us write down only the dominant terms of the stress tensor. (For details see appendix.) For small $v$, it is dominant the interference ($T_2$) between the oscillator and the field.

$$T_{vv} = -\frac{e^2}{8\pi m\omega_+} e^{-e_+v} \left( 1 - \frac{e_-}{e_+} \right) \frac{1}{v} \cos(\omega_+ v + \theta), \quad \text{for} \quad \frac{1}{\omega_0} \gg v > \frac{1}{\Gamma},$$  \hspace{1cm} (71)

In this region the energy is absorbed into the oscillator from the field with the amount

$$E_{absorbed} = \frac{e^2}{8\pi m} \left( 1 - \frac{e_-}{e_+} \right) \ln \frac{\Gamma}{\omega_0} + \text{smaller terms.}$$  \hspace{1cm} (72)
For $v \gg 1/\Gamma$, energy is radiated away from the oscillator and $T_3$ is dominant.

\[
T_{vv} = \frac{e^4}{8\pi} \left( \frac{\omega_0}{m\omega_+} \right)^2 e^{-2e^v \cos^2(\omega_+ v + \theta)} \left( 1 - \frac{e^{-}}{e^{+}} \right)^2 \ln \left( \Gamma/\omega_0 \right) \quad \text{for} \quad v \gg \frac{1}{\Gamma}. 
\] (73)

where $\tan \theta = e^{+}/\omega_+$. The total radiated energy in this region is

\[
E_{\text{radiated}} = \frac{e^2}{8\pi m} \left( 1 - \frac{e^{-}}{e^{+}} \right)^2 \ln \frac{\Gamma}{\omega_0} \quad \text{(74)}
\]

Therefore we can conclude that in general the absorbed energy into the oscillator is greater than the radiated one.

**B. The Stress tensor in the presence of a uniformly accelerating oscillator**

With the same reason given at the previous subsection, $T^A_{uu}$ and $T^A_{uv}$ are zero. Similarly, the stress tensor vanishes for $V < 0$.

In the region $V > 0$, we also restrict to $V > 1/\Gamma \gg 1/d$, and we ignored $\dot{e}(\tau)$ related terms. In this region, $T^A_{vv}$ components can be written as follows:

\[
T^A_{vv} = T^A_1 + \frac{e^{+}}{2}T^A_2 + \frac{e^{+}}{2}T^A_3, \quad \text{(75)}
\]

where

\[
T^A_1 = (2n + 1)\frac{e^2}{8\omega_+} \lim_{v \to v} \left( e^{-a(V+V')} \partial_V \sqrt{g_{--}(V)g_{--}(V')} \exp \left[ -i \left\{ \Theta(V) - \Theta(V') \right\} \right] \right) \quad \text{(76)}
\]

\[
T^A_2 = \int d\lambda \lambda \left\{ 1 + 2N \left( \frac{\lambda}{a} \right) \right\} \partial_V \xi(V) \partial_V G^*(\lambda, V) + \partial_V \xi^*(V) \partial_V G(\lambda, v) \right\} e^{-2aV}, \quad \text{(77)}
\]

\[
T^A_3 = \int d\lambda \lambda^2 \left\{ 1 + 2N \left( \frac{\lambda}{a} \right) \right\} \partial_V G(\lambda, V) \partial_V G^*(\lambda, V) e^{-2aV}. \quad \text{(78)}
\]

For small $V$ the stress tensor is dominated by $T^A_2$ which is given by

\[
T^A_{vv} = \frac{e^2}{8\pi m\omega_+} \frac{a}{\ln av} \left( \frac{1}{\omega_0} \right)^2 \left\{ \beta_+ (av)^{i\omega_+} + \beta^*_+ (av)^{-i\omega_+} \right\} (av)^{-2-\epsilon+/a} \quad \text{for} \quad \frac{1}{\omega_0} \gg V \gg \frac{1}{\Gamma}, \quad \text{(79)}
\]

This is exactly the same form with the inertial oscillator except the retardation factor $(av)^{-2}$ due to the acceleration and the mere coordinate change $v \to \ln av/a$. The total absorbed energy is given by
\[ E_{absorbed}^A = \frac{e^2_+}{8\pi m} \left( 1 - \frac{e_-}{e_+} \right) \ln \frac{\Gamma}{\omega_0} + \text{smaller terms.} \] (80)

This is exactly the same with Eq. (72). Physically, it is natural because there is no enough time the acceleration to act on the short time interference. For \( V \gg 1/\Gamma \), \( T_{3}^A \) is dominant.

\[ T_{\nu\nu}^A = \frac{e^2_+}{8\pi m \omega_+} \left( 1 - \frac{e_-}{e_+} \right)^2 \ln \frac{\Gamma}{\omega_0} \left\{ \beta_+(av)^{\omega_+/a} + \beta_+(av)^{-\omega_+/a} \right\}^2 (av)^{-2-2e+/a} \text{ for } V \gg \frac{1}{\Gamma}, \] (81)

This equation is quite similar to Eq. (73) except the retardation effect and the coordinate change \( (v \rightarrow \ln av/a) \) due to acceleration. This is because the main effect to the radiation comes from the high momentum region. The total energy radiated away from the oscillator is

\[ E_{radiated}^A = \frac{e^2_+}{8\pi m} \left( 1 - \frac{e_-}{e_+} \right)^2 \ln \frac{\Gamma}{\omega_0} \tan \theta \left[ \frac{1}{(x + \sin \theta) \cos \theta} + \frac{\cos 2\theta x - \tan \theta}{x^2 + 2 \sin \theta x + 1} \right], \] (82)

where \( x = a/(2\omega_0) \).

V. SUMMARY AND DISCUSSION

We have discussed the influence of a harmonic oscillator on a scalar quantum field in \( 1+1 \) dimensions. These are illustrated by calculating the radiation of the scalar field from the oscillator. The first step to do this is to express the time evolutions with the classical inhomogeneous solution \( G(\omega, t) \) of a damping forced harmonic oscillator. Then we applied this result to the sudden jumping limit of the coupling and obtain the change of the stress tensor in the presence of the oscillator.

There are two main effects on the radiation. The first is due to sudden change of the coupling which is described by the correlation between the oscillator and the field. This effect rapidly die out but the oscillator absorbs large energy from the field through this correlation. Moreover this absorption is independent of the acceleration. Subsequently, slow radiation from the oscillator take place. In case of an inertial oscillator this radiation is smaller than the absorbed energy through the first stage. The behavior of the total radiated energy become nontrivial if the detector is accelerated. In the case of a small coupling
constant \((\epsilon_+ \ll \omega_0)\), the radiated energy is maximized by \(a = 0\). But there is a peak of the radiated energy at a non-zero acceleration if \(\theta\) greater than some value \(\theta_0 \sim 1.07702\) or

\[
\left(\frac{\epsilon_+}{\omega_0}\right)^2 > \frac{-\omega_+ / \omega_0 + \sqrt{(\omega_+ / \omega_0)^2 - 8 \omega_+ / \omega_0 + 8}}{4(1 - \omega_+ / \omega_0)}
\] (83)

In this case the radiated energy can greater than the absorbed one during the first stage. Especially, if \(\epsilon \rightarrow \omega_0\) then the radiated energy becomes extremely large for non-zero acceleration.

If the acceleration is large enough, the radiation decrease according to the inverse of the acceleration. The following two points can help to understand this phenomena. First, the radiation is not due to the acceleration but due to the change of the coupling. Second, as acceleration grows the unit proper time of an accelerating oscillator correspond to a larger coordinate time to the Minkowski observer. Therefore the coordinate time which takes to vary the coupling becomes larger for the larger acceleration.
Total radiated energy is plotted according to the acceleration. The acceleration of each time is given by $a/(2\omega_0) = \{\pi/6, \pi/3, \pi/2 - 0.3, \pi/2 - 0.2, \pi/2 - 0.1\}$ from the below. The unit for energy is

$$E_{\text{radiated}} = \frac{e^2}{8\pi m} \left(1 - \frac{e}{e^*}\right)^2 \ln\left(\frac{\Gamma}{\omega_0}\right).$$

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APPENDIX
A. The Stress Energy Tensor of the field in the sudden jump of the coupling – Inertial Case

In this appendix we obtain the stress tensor for the model of Sec. III B in the sudden jump limit. It is easy to know that the stress tensor simply vanishes for \( v < 0 \), from (63). Therefore we calculate it only for \( v \geq 0 \) in the left hand side of the oscillator. The stress tensor (67) is composed of three terms.

The first term can be evaluated easily to become

\[
T_1 = \frac{\epsilon_+}{4\omega_{-\omega_+}^2} e^{-2\epsilon_+ v} \left[ (\omega_+^2 + \omega_-^2)\omega_0^2 + (\epsilon_+^2 - \omega_+^2)(\omega_+^2 - \omega_-^2) \cos 2\omega_+ v \right. \\
\left. + 2\epsilon_+ \omega_+ (\omega_+^2 - \omega_-^2) \sin 2\omega_+ v \right].
\] (84)

\( T_2 \) is sum of two terms which are mutually complex conjugate. One of these is

\[
\int d\omega \omega \partial_v G(\omega, v) \partial_v u_\omega^*(v) = \int d\omega \omega \partial_v G_\infty(\omega, v) \partial_v u_\omega^*(v) + \frac{i\epsilon_+}{8\pi} e^{-\epsilon_+ v} T_{2a},
\] (85)

where

\[
T_{2a} = -\beta_+ e^{i\omega_+ v} \int d\omega \omega \chi_+^*(\omega) e^{i\omega v} + \beta_+^* e^{-i\omega_+ v} \int d\omega \omega \chi_+(\omega) e^{i\omega v},
\] (86)

and we define the constant

\[
\beta_\pm = \omega_\pm + i\epsilon_\pm.
\] (87)

Therefore

\[
T_{2a} - T_{2a}^* = -\frac{1}{m\omega_+} \beta_+ e^{i\omega_+ v} \left[ \left( 1 - \frac{e_-}{e_+} \right) \frac{2i}{v} + 2\beta_+ e^{-i\beta_+ v} Ei(i\beta_+ v) \right. \\
\left. - \frac{e_-}{e_+} \left( \left( 1 + \frac{\omega_+}{\omega_-} \right) \beta_- e^{-i\beta_+ v} Ei(i\beta_- v) - \left( 1 + \frac{\omega_+}{\omega_-} \right) \beta_-^* e^{i\beta_- v} Ei(-i\beta_-^* v) \right) \right]
\] (88)

\[
= C.C.
\]

Rather than use this complex form, lets us extract only its limiting form for small and large \( v \). In the region \( d \ll 1/\Gamma < v \ll 1/\omega_0 \)

\[
T_2 = \frac{e_+}{4\pi m\omega_+} e^{-\epsilon_+ v} \left( 1 - \frac{e_-}{e_+} \right) \frac{\omega_0}{v} \cos(\omega_+ v + \theta)
\] (89)
and for large $v \gg 1/\omega_0$, $T_2 = O(e^{-2\epsilon_+ v})$. Where $\tan \theta = \epsilon_+ / \omega_+$.

Finally, let us evaluate $T_3$. If we define the following integrals

$$I_1(v) = \int_0^\Gamma d\omega \omega^2 \chi(\omega) \chi_d(-\omega)e^{-i\omega v},$$

$$I_2(v) = \int_0^\Gamma d\omega \omega^2 \chi(\omega) \chi_d^*(\omega)e^{-i\omega v},$$

$$J(v) = \int_0^\Gamma d\omega \omega^2 \chi(\omega) \chi_-(\omega)e^{-i\omega v},$$

then $T_3$ becomes

$$T_3 = \lim_{v' \to v} \int d\omega \omega^2 \partial_\omega G_\infty(\omega, v) \partial_\omega G_\infty^*(\omega, v')$$

$$+ \frac{\epsilon_+^2}{8\pi} \left( e^{-i\beta_+ v} \beta_+^* T_{3a} + e^{i\beta_+ v} \beta_+ T_{3a}^* \right) + \frac{\epsilon_+^2}{16\pi} e^{-2\epsilon_+ v} T_{3b}.$$  

where

$$T_{3a} = -I_1(v) + I_2^*(v) + \frac{\epsilon_+}{\epsilon_+} [-J(v) + J^*(v)]$$

$$T_{3b} = \omega_0^2 \int_0^\Gamma d\omega \omega \left( |\chi_+(\omega)|^2 + |\chi_-(\omega)|^2 \right)$$

$$- \beta_+^2 e^{2i\omega_+ v} \int_0^\Gamma d\omega \omega \chi_+^*(\omega) \chi_+^*(\omega) - \beta_+^2 e^{-2i\omega_+ v} \int_0^\Gamma d\omega \omega \chi_+(\omega) \chi_+(\omega).$$

where we have introduced explicit high momentum cut-off $\Gamma$ to regularize the UV behaviors. The first term of $T_3$ is canceled by the $\lim_{v' \to v} \int d\omega \omega (\partial_\omega G_\infty \partial_\omega G_\infty^* + \partial_\omega u^* \partial_\omega G_\infty)$ term of $T_2$ (The detail of the calculation can be consulted in Ref. [19].) As one can see in $T_{3b}$ major contribution to the stress tensor comes from the ultra-violet region. As one can easily see $T_{3b}$ don’t have UV contribution. Therefore the major contribution comes from $T_{3b}$. Let us examine $T_{3b}$ in detail. $\chi_-$ is of order $O(1/\omega^2)$ for large $\omega$, therefore only the first term of the integral

$$\int d\omega |\chi_+(\omega)|^2$$

$$= \int d\omega |\chi_d|^2 + \frac{\epsilon_+}{\epsilon_+} \int d\omega \omega \left( \chi_d \chi_d^* + \chi_- \chi_-^* \right) + \frac{\epsilon_+^2}{e_+^2} \int d\omega |\chi_-|^2$$

can have important ultra-violet contribution. If one try to extract only the high momentum part it is ease to show that
\[ \int d\omega \omega |\chi_d|^2 \approx \frac{1}{m^2 \omega_+^2} \left( 1 - \frac{e_-}{e_+} \right)^2 \int^\Gamma d\omega \frac{1}{\omega} \]

\[ = \frac{1}{m^2 \omega_+^2} \left( 1 - \frac{e_-}{e_+} \right)^2 \ln (\Gamma/\omega_0). \]

Therefore

\[ T_3 = \frac{e_+^2}{4\pi} \left( \frac{\omega_0}{m\omega_+} \right)^2 e^{-2e_+v \cos^2(\omega_+ v + \theta)} \left( 1 - \frac{e_-}{e_+} \right)^2 \ln (\Gamma/\omega_0). \]  

(98)

where \( \tan \theta = \frac{e_+}{\omega_+} \).

**B. The stress tensor of the field in the sudden jump limit of the coupling – Accelerating Case**

The stress tensor for \( V < 0 \) vanishes. In the region \( V > 0 \), the term \( T_1^A \) is

\[ T_1^A(v) = \frac{\epsilon_+}{4\omega_- \omega_+^2} \left[ (\omega_+^2 + \omega^2) \omega_0^2 \right. \]

\[ - \frac{1}{2} (\omega_+^2 - \omega^2) \left( \beta^2 (av)^{-2i\omega/a} + \beta_+^2 (av)^{2i\omega/a} \right) \left( av \right)^{-2(1+\epsilon/a)}. \]  

(99)

The integral for \( T_2^A \) are of the form \( \int d\lambda \lambda (1 + 2N(\lambda/a)) f(\lambda, V) \). One can separate the ultra-violet (UV) 2 \( \int d\lambda \lambda f(\lambda, V) \) from its thermal contributions \( \int d\lambda \lambda N(\lambda/a) f(\lambda, V) \). Let us look at each term more closely.

The UV term of \( T_3^A \) is

\[ T_{3UV} = \int^\Gamma d\lambda \lambda \partial_\lambda G_\infty(\lambda, V) \partial_\lambda G_\infty^*(\lambda, V) \]

\[ + \frac{e_+^2}{8\pi} e^{-2aV} \left( e^{-i\beta^*_+ V} \beta^*_+ T_{3a} + e^{i\beta^*_+ V} \beta^*_+ T_{3a}^* \right) + \frac{e_+^2}{16\pi} e^{-2(\epsilon_+/a)V} T_{3b}^A, \]  

(100)

where \( T_{3a} \) and \( T_{3b}^A \) are given by Eqs. (94) and (95) if one replace \( \omega \to \lambda, v \to V \). The first term of \( T_3^A \) is canceled by the \( \int d\lambda \lambda \{ \partial_\lambda G_\infty(\lambda, V) \partial_\lambda \xi_\lambda^* (V) + \partial_\lambda \xi_\lambda (V) \partial_\lambda G_\infty^*(\lambda, V) \} \) term of \( T_2^A \) \([9\). The dominant term for this UV contribution is

\[ \frac{e_+^2}{4\pi} \left( \frac{1 - e_-/e_+}{m\omega_+} \right)^2 \ln \frac{\Gamma}{\omega_0} \omega_0^2 \cos^2(\omega_+ V + \theta)e^{-2(\epsilon_+/a)V} \]

\[ = \frac{e_+^2}{16\pi} \left( \frac{1 - e_-/e_+}{m\omega_+} \right)^2 \ln \frac{\Gamma}{\omega_0} \left[ 2\omega_0^2 + \beta_+^2 (av)^{2i\omega/a} + \beta_+^2 (av)^{-2i\omega/a} \right] (av)^{-2(1+\epsilon/a)} \]

(101)
There are thermal contributions in $T^A_3$ but we can argue that it does not give comparable contribution to the UV term. The general form of the integral of the thermal part is

$$\int d\lambda \frac{2\lambda^2}{e^{\lambda/a} - 1} \partial_V G(\lambda, V) \partial_V G^*(\lambda, V).$$

As one can easily see, there is no UV divergence because of the thermal factor in the denominator. Moreover there is no IR contributions which comes from $\lambda \sim 0$. Therefore it do not give terms depends on the cutoff $\Gamma$ which is the main contribution of the $T^A_{3UV}$.

In case of $T^A_2$ the situation is much different to $T^A_3$ because there are no UV contributions and the main contribution of it is only for small $V$. So we must calculate it exactly in the region $\frac{1}{\Gamma} < V \ll 1/\omega_0, 2/a$. $T^A_2$ is sum of two terms which are mutually complex conjugate. One of these is

$$\int d\lambda \lambda \coth(2\pi \lambda/a) \partial_V G(\lambda, V) \partial_V \xi^*(\lambda, V) e^{-2aV}$$

$$= \int d\lambda \lambda \coth(2\pi \lambda/a) \partial_V G_\infty(\lambda, V) \partial_V \xi^*(\lambda, V) e^{-2aV} + \frac{ie^\pm e^{-(\epsilon + 2a)v} T^A_{2a}}{8\pi}$$

where

$$T^A_{2a} = -\beta^a e^{i\omega_+ V} \int d\lambda \lambda \coth 2\pi \lambda/a \chi^*_+(-\lambda) e^{i\lambda V}$$

$$+ \beta^*_a e^{-i\omega_+ V} \int d\lambda \lambda \coth 2\pi \lambda/a \chi^+_+ (\lambda) e^{i\lambda V}.$$ 

Therefore we must calculate $T^A_{2a} - T^A_{2a}$. After change of variable and using the fact $\lambda \coth 2\pi \lambda/a$ is even function on $\lambda$, we get

$$T^A_{2a} - T^A_{2a} = -\beta^a e^{-i\omega_+ V} \int_{-\infty}^{\infty} d\lambda [\lambda \coth 2\pi \lambda/a] \xi^*_+(-\lambda) e^{i\lambda V}$$

$$- C.C. (105)$$

Now we use

$$\coth \pi x = \frac{1}{\pi x} + \frac{2\pi}{\pi} \sum_{k=1}^{\infty} \frac{1}{x^2 + k^2}$$

and do the residue integral along the upper half plane of the $\lambda$ plane, then we get
\[ T_{2a}^A - T_{2a}^{A*} = i a \beta_+ e^{-i \omega_+ V} \left[ S(-2i \beta_+) - \frac{e_-}{2e_+} \left\{ \left( 1 + \frac{\omega_+}{\omega_-} \right) S(-2i \beta_-) + \left( 1 - \frac{\omega_+}{\omega_-} \right) S(2i \beta^*_-) \right\} \right] - C.C \]  

where

\[ S(\beta) = \sum_{k=1}^{\infty} \frac{ak e^{-akV/2}}{ak + \beta}. \]  

If we restrict \( V \) to \( V \ll 2/a, 1/\omega_0 \), we get

\[ S(\beta) \approx \frac{2}{aV} + \frac{\beta}{a} \ln \left( \frac{V}{\omega_0} \right). \]  

The second term is much smaller than the first. Therefore we can write

\[ T_2^A = -\frac{e_+}{4\pi} \left( 1 - \frac{e_-}{e_+} \right) e^{-(\epsilon_+ + 2a)V} \frac{\omega_0}{V} \cos(\omega_+ V + \theta) \]  

\[ = -\frac{e_+}{4\pi m \omega_+} \left( 1 - \frac{e_-}{e_+} \right) \frac{a}{\ln av} \left[ \beta_+(av)^{i \omega_+} + \beta_+^*(av)^{-i \omega_+} \right] (av)^{-(2+\epsilon_+/a)}. \]
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