Performance evaluation of cryogenic counter-flow heat exchangers with longitudinal conduction, heat in-leak and property variations

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Abstract. Counter-flow plate-fin heat exchangers are commonly utilized in cryogenic applications due to their high effectiveness and compact size. For cryogenic heat exchangers in helium liquefaction/refrigeration systems, conventional design theory is no longer applicable and they are usually sensitive to longitudinal heat conduction, heat in-leak from surroundings and variable fluid properties. Governing equations based on distributed parameter method are developed to evaluate performance deterioration caused by these effects. The numerical model could also be applied in many other recuperators with different structures and, hence, available experimental data are used to validate it. For a specific case of the multi-stream heat exchanger in the EAST helium refrigerator, quantitative effects of these heat losses are further discussed, in comparison with design results obtained by the common commercial software. The numerical model could be useful to evaluate and rate the heat exchanger performance under the actual cryogenic environment.

1. Introduction
Plate-fin heat exchangers (PFHEs), which are commonly utilized in cryogenic applications, are characterized by large ratio of total heat transfer area to volume. Different from the conventional heat exchangers, the performance of PFHEs, as key components of helium liquefaction/refrigeration systems, is sensitive to various losses including longitudinal heat conduction, heat in-leak from surroundings and property variations of fluid [1]. The commercial software, Aspen MUSE™ [2], has been successfully used to evaluate the performance of PFHEs by means of calculating the operating data for each layer independently and then determining the metal temperature profile along the parting sheet between the layers. The program can effectively take into accounts many losses such as longitudinal conduction, actual fluid physical properties, complex passage arrangements, etc., but there still exists some general and empirical inputs such as heat in-leak.

Due to the severe performance deterioration caused by these losses, many researchers have focused on them during the last several decades. Kroeger [3] concluded comprehensive, systematic analytical
results assuming constant properties and adiabatic end conditions. Narayanan and Venkatarathnam [4] discussed the parasitic heat loss through the wall for the conditions of the cold end directly connected to the evaporator in Joule–Thomson refrigerators. Hansen [5] investigated the effects of longitudinal conduction and found that there exists an optimal metal conductivity for the geometry considered in the design of heat exchangers incorporated with J–T expansion process. Krishna [6] studied the effect of longitudinal wall conduction on the performance of three-fluid cryogenic heat exchangers. Chowdhury and Sarangi [7] analysed a heat exchanger exposed to a parasitic heat transfer that is driven by a temperature difference between a fixed ambient temperature and the local fluid temperature. Gupta and Atrey [8] published the experimental results and numerical modeling considering the heat in-leak from surroundings and longitudinal conduction through the wall in the coiled tube in tube heat exchangers. Different from the assumptions of external heat transfer linearly proportional to the temperature difference between ambient and fluid, Nellis [9] considered radiative transfer as the dominant parasitic in a typical cryogenic heat exchanger.

A systematic review of the evolution and challenges in mathematical models of cryogenic heat exchangers was presented by Pacio and Dorao [10]. The differential equations derived by Kroeger [3] under conditions of constant properties could be used to study the effects of longitudinal conduction for balanced and imbalanced flow qualitatively and systematically. Gupta and Atrey [8, 11] extended the model to cover heat in-leak from surroundings and validated it with experimental results for a two-stream counter-flow recuperator. Based on this model, nonadiabatic end conditions were applied and a comprehensive research into the characterization of various losses was carried out by Aminuddin and Zubair [12]. The numerical model presented by Nellis [9] considering the effects of property variations, metal conductivity, parasitic heat load, and heat transfer coefficients was useful in the design of a two-stream counter-flow and parallel-flow heat exchangers. To take account of the transverse heat conduction for multi-stream heat exchangers, Goyal [13] presented a detailed numerical model with discretizing energy balance equations additionally for fin elements, side bars, etc.

In the present paper, in order to evaluate and rate the performance deterioration degree caused by various losses, the model early proposed is modified at the methodology of analysis and modeling. The practicability and flexibility of this model are carefully investigated. Moreover, for the intentionally shortened PFHE in the EAST helium refrigerator, loss mechanisms at low temperatures are emphatically discussed by comparing the numerical predictions with the results calculated by the common commercial software.

2. Model formulation

2.1. Physical model
Figure 1 schematically shows the structure of a multi-channel PFHE core. The configurations of the enhanced fins are depicted as fin height $h$, fin space $s$, fin thickness $t$ and interrupted length $l$, as shown in figure 2.
Figure 1. The schematic of a multi-channel PFHE core

Figure 2. The diagram of Offset Strip Fins

2.2. Governing equations

The computational grid consisting of the necessary nodes and elements is illustrated in Figure 3. The heat exchanger is divided into \( n \) sections. Each section consists of three elements: hot fluid side, cold fluid side and wall boundary.

Figure 3. The schematic of a multi-channel PFHE core

Considering the effects of longitudinal conduction tend to be concentrated at the ends, an exponentially distributed grid is adopted, which can be illustrated as [9]:

\[
\Delta X_i = \Delta X_{n+1-i} = \frac{\exp[-\gamma(1-\frac{2i}{n})]}{2 \sum_{i=1}^{n} \exp[-\gamma(1-\frac{2i}{n})]}
\]

(1)

where \( n, \gamma \) represents the grid number and relaxation factor, respectively.

The governing equations in the \( i^{th} \) section can be described as follows:

**Hot fluid:**

\[
\left( \frac{v_{h,i} \Delta X_i}{v_{h,i-1} \Delta X_{i-1}} \right) nu_{h,i-1} + nu_{h,i} \left( \frac{\theta_{h,i} + \theta_{h,i-1}}{2} - \theta_{w,i} \right) + 2a_{h,i} NTU_i v_{h,i} (R - \frac{\theta_{h,i} + \theta_{h,i-1}}{2}) + 1 = \frac{\mu_{h,i} + \mu_{h,i-1}}{\mu_{h,i}} (\theta_{h,i-1} - \theta_{h,i})
\]

(2)
Wall:
\[
\frac{\Delta X_i}{v_{h,i}\Delta X_{i-1}} - ntu_{h,i} + \frac{1}{v_{h,i}} ntu_{h,i} \left( \frac{\theta_{h,i} + \theta_{h,i-1}}{2} - \theta_{w,i} \right) + 2 \frac{\lambda_{e,i-1} + \lambda_{h,i}}{\Delta X_{i-1} + \Delta X_i} (\theta_{w,i-1} - \theta_{w,i}) + 2\alpha_{w,i} NTU_i (R - \frac{\theta_{w,i-1} + \theta_{w,i-1}}{2} + 1) = \frac{\Delta X_i}{v_{c,i}\Delta X_{i-1}} - ntu_{c,i} \left( \frac{\theta_{c,i} + \theta_{c,i-1}}{2} \right) + 2\alpha_{c,i} NTU_i (R - \frac{\theta_{c,i} + \theta_{c,i-1}}{2} + 1) = \frac{\Delta X_i}{\Delta X_{i-1}} (\theta_{w,i} - \theta_{w,i-1})
\]

Cold fluid:
\[
\frac{v_{c,i} \Delta X_i}{v_{c,i-1} \Delta X_{i-1}} - ntu_{c,i-1} + ntu_{c,i} \left( \frac{\theta_{c,i} + \theta_{c,i-1}}{2} \right) + 2\alpha_{c,i} NTU_i v_{c,i} (R - \frac{\theta_{c,i} + \theta_{c,i-1}}{2} + 1) = \frac{\Delta X_i}{v_{c,i-1}\Delta X_{i-1}} (\theta_{c,i} - \theta_{c,i-1})
\]

The following dimensionless parameters have been used in the above energy equations:
\[
\theta = \frac{T - T_{c,\text{in}}}{T_{h,\text{in}} - T_{c,\text{in}}} \quad R = \frac{T_{h,\text{in}} - T_{c,\text{in}}}{T_{h,\text{in}} - T_{c,\text{in}}} \quad \alpha_i = \frac{U_{o,i} A_{h,i}}{U_i A_i} \quad \lambda_i = \frac{k_{w,i} A_i}{C_{min} L}
\]

\[
v_{h,i} = \frac{C_{h,i}}{C_{h,\text{in}}} \quad v_{c,i} = \frac{C_{c,i}}{C_{c,\text{in}}} \quad \mu_{h,i} = \frac{C_{h,i}}{C_{h,\text{in}}} \quad \mu_{c,i} = \frac{C_{c,i}}{C_{c,\text{in}}}
\]

\[
ntu_{h,i} = \frac{hA}{C} v_{h,i} \quad ntu_{c,i} = \frac{hA}{C} v_{c,i} \quad NTU_i = \frac{U_i A_i}{C_{min}} \quad \Delta X_i = \frac{\Delta X_i}{L}
\]

In order to improve the computational efficiency and reveal the inherent relationship between these physical quantities, all the above energy balance equations are made nondimensional. Additionally, for multi-stream PFHEs with more than two passages, analogous equations can be derived according to the corresponding passage arrangement. In these expressions, the proposed by Kroeger [3] represents the dimensionless temperature of hot fluid, cold fluid and metal wall followed by the suffix, \( h \), \( c \) and \( w \), respectively. The ratio of external conductance \( U_o A_o \) to internal conductance \( U_i A_i \), heat in-leak parameter \( \alpha \), was used to determine the amount of heat in-leak from the surrounding [7]. In equation (5), \( R \) is used to measure the gap between ambient and operating temperature. The longitudinal conduction parameter, \( \lambda \), is defined as a ratio of longitudinal wall heat conduction per unit temperature difference to the heat capacity rate of the fluid per unit length [1]. With the help of previous studies, taking the same deductive strategies for these dimensionless parameters can help designers evaluate the heat exchanger performance qualitatively before calculating the model.

The core pressure drop in the \( j \)th section mainly including two components, the frictional pressure drop and the pressure drop due to the momentum rate change, can be expressed as [1]:
\[
\Delta P_j = \frac{G^2}{2g_c} \left[ \frac{4f}{D_h} \frac{\Delta X_j}{\rho} \left( \frac{1}{\rho} \right)_m + 2 \left( \frac{1}{\rho_{\text{exit}}} - \frac{1}{\rho_{\text{entrance}}} \right) \right]
\]

where \( G \), \( g_c \), \( f \) represents the mass flux, proportionality constant in Newton’s second law of motion and Fanning friction factor, respectively. The suffixes, exit and entrance, denote the inlet and outlet of the passage.

2.3. Boundary conditions
Considering every heat exchanger in the cold box gets insulted perfectly, adiabatic ends for the wall boundary are assumed. The boundary conditions specifying inlet temperatures for hot and cold fluid can be expressed as:
\[ \theta_{h,0} = 1, \theta_{w,0} = \theta_{w,1} \]  
\[ \theta_{c,0} = 0, \theta_{w,0} = \theta_{w,r+1} \]  

3. Results and discussion

3.1. Verification of the model

The proposed model can also be utilized to analyse the other heat exchangers with different structures. For the given cases about the coiled tube in tube heat exchangers [8], figure 4 illustrates the variation trend about nondimensional exit temperature of the hot fluid and the corresponding calculated time in accordance with the grid number. It may be observed that the precision and speed of the calculation results could be satisfied when using \( n = 100, \gamma = 4 \). The accuracy of this numerical model can be examined by comparing with experimental results presented previously, as shown in figure 5. It can be observed that the temperature distribution along the length shows a good match with the actual conditions. However, as suggested by the author, the general, empirically obtained heat in-leak parameter, \( \alpha \), which may be a variable along the length rather than a constant, deserves to be considered carefully. Therefore, considering the radiative heat transfer dominating the parasitic losses, the differential heat in-leak parameters, \( \alpha_i \), can be calculated based on the local heat in-leak:

\[ \alpha_i = \frac{\sigma \varepsilon_{m i} A_{i} (T_i^4 - T_i^4)}{U_i A_i} \]  

In the same way, the constant fluid properties and unchanged longitudinal conduction parameter may cause the deviation of the calculated results.

3.2. Performance rating of a cryogenic multi-stream heat exchanger

To investigate the practicability and flexibility of the proposed model, an intentionally shortened PFHE based on the prototype mounted in the 2KW helium refrigerator, EAST, is analysed. The aim is to i. Reduce the heat transfer area to increase the temperature difference on the entrances and exits, and hence examine prediction accuracy more stringently; ii. Analyse various loss mechanisms in case of limited design margin. The dimensions of the PFHE and the operation parameters are given in Table 1 and Table 2, respectively. The PFHE is covered with multi-layer insulation to keep the outside surface a low emissivity, 0.0045 and exposed to ambient temperature at 300K. Thermal-physical properties of helium and nitrogen are obtained using REFPROP [14]. The thermal conductivity of Aluminium (AL3003) at low temperature is acquired via the empirical correlation from
NIST [15]. The general accepted correlations [16] are adopted to predict heat transfer and pressure drop performance. For offset strip fins, extended surfaces are used to increase the total rate of heat transfer. The expression for fin efficiency, $\eta_f$, can be found in the literature [1]. The above high-dimensional nonlinear equations are iteratively calculated by MATLAB based on least-squares methods, as illustrated in figure 6.

### Table 1. Geometry of the heat exchanger

| Core dimensions (width × length × height, mm) | Side bar width (mm) | Separating plate thickness (mm) | Cap sheet thickness (mm) | Stream number | Fin type | Fin height (mm) | Fin space (mm) | Fin thickness (mm) | Interrupted length (mm) |
|---------------------------------------------|---------------------|-------------------------------|--------------------------|---------------|----------|----------------|----------------|-------------------|----------------------|
| 350 × 1200 × 598                           | 25                  | 1                             | 6                        | A             | Serrated | 4.7            | 2              | 0.3               | 3                    |
|                                             |                     |                               |                          | B             | Serrated | 9.5            | 1.4            | 0.2               | 3                    |
|                                             |                     |                               |                          | C             | Serrated | 9.5            | 1.4            | 0.2               | 3                    |
|                                             |                     |                               |                          | D             | Serrated | 9.5            | 1.4            | 0.2               | 3                    |

### Table 2. Operation conditions

| Stream number | Working fluid | Mass flow rate (g s⁻¹) | Inlet temperature (°C) | Inlet pressure (bar) | Number of layers | Layer pattern               |
|---------------|--------------|-------------------------|------------------------|---------------------|------------------|-----------------------------|
| A             | Helium       | 59.2                    | 300                    | 19.5                | 22               | A(BCA/4)D(BCA/4)D(BAC/4)    |
| B             | Helium       | 41.3                    | 77.8                   | 1.1                 | 20               | D(ACB/4)D(ACB/4)A           |
| C             | Helium       | 13.4                    | 77.8                   | 0.37                | 20               |                            |
| D             | Nitrogen     | 15.9                    | 78.8                   | 1.15                | 4                |                            |

![Flow chart for the calculation procedure](image-url)
Table 3 illustrates the comparison results of the outlet temperature and pressure drop calculated by the numerical model and the commercial software Aspen MUSE™. It can be observed that the prediction results from the model show a lower effectiveness and higher pressure drop due to considering radiation parasitics and actual local metal properties additionally. Figure 7 shows the temperature distribution for four streams. The variation trends of longitudinal heat conduction per unit length through the separating plates and cumulative radiation parasitics along the flow length are plotted in figure 8. The results indicate that the longitudinal conduction rate varies significantly with the existence of different temperature gradients through the wall rather than keeps a constant. Similarly, the radiative heat in-leak variations don’t exhibit linear relationship with nondimensional flow length and the final cumulative value reaches 3.12 W. In addition, the local heat load imbalance due to the non-ideal channel arrangement causes temperature cross and heat loss, the results of zig-zag chart about cumulative layer heat load are compared in figure 9, characterized by the same trend and different heat load in each layer.

| Stream number | Outlet temperature (K) | Pressure drop (kPa) |
|---------------|------------------------|---------------------|
|               | Aspen MUSE             | Numerical predictions | Aspen MUSE | Numerical predictions |
| A             | 90.26                  | 92.60               | 0.15       | 0.18                   |
| B             | 291.81                 | 289.37              | 0.88       | 1.09                   |
| C             | 292.28                 | 291.38              | 0.64       | 0.81                   |
| D             | 290.94                 | 286.71              | 0.24       | 0.29                   |

Figure 7. The temperature distribution

Figure 8. Longitudinal metal conduction rate and cumulative radiation parasitics along the flow length

Figure 9. Zig-zag chart of cumulative layer heat load
4. Conclusions
The numerical model taking into account of various loss mechanisms including longitudinal heat conduction, heat in-leak from surroundings and property variations for counter-flow heat exchangers was investigated. Through establishing the local differential governing equations for every passage, the model based on distributed parameter methods could calculate the performance parameters, evaluate longitudinal wall conduction and predict heat in-leak in each finite section. Great deviations from the original design condition might be caused by various losses occurred in the actual operating environments. It’s worth noting that the model could also be applied in the condition of flow mal-distribution, while it appeared weak in case of side by side streams in one layer. However, for the high efficient PFHE used in helium liquefaction/refrigeration systems, the method could be useful to evaluate and rate the heat exchanger performance under the actual cryogenic conditions.

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Acknowledgements
The authors express their great appreciation to the EAST cryogenic group for their help.