The transverse momentum spectra of the produced hadrons have been compared to a model which is based on the assumption that a nucleus-nucleus collision is a superposition of isotropically decaying thermal sources at a given freeze out temperature. The freeze-out temperature in nucleus-nucleus collisions is fixed from the inverse slope of the transverse momentum spectra of hadrons in nucleon-nucleon collision. The successive collisions in the nuclear reaction leads to gain in transverse momentum, as the nucleons propagate in the nucleus following a random walk pattern. The average transverse rapidity shift per collision is determined from the nucleon-nucleus collision data. Using these information we obtain parameter free result for the transverse momentum distribution of produced hadrons in nucleus-nucleus collisions. It is observed that such a model is able to explain the transverse mass spectra of produced pions at SPS energies. However it fails to satisfactorily explain the transverse mass spectra of kaons and protons. This indicates the presence of collective effect which cannot be accounted for by the initial state collision broadening of transverse momentum of produced hadrons, the basis of random walk model.

PACS numbers: 25.75.-q

I. INTRODUCTION

In high energy heavy-ion collisions it is observed that the average transverse momenta of the produced hadrons depends strongly on the mass of hadrons\cite{1,2}. Further it is found to be substantially larger than in nucleon-nucleon collisions at a given energy. One of the possible physical effect responsible for this is transverse flow\cite{3}. The heavier the mass of the particle more is the gain in momentum and hence more transverse flow. This has been shown by extracting the effective temperature (effective because it includes the true freeze-out temperature and the transverse flow) from transverse momentum ($p_T$) distribution of various hadrons. It is observed that protons have a larger effective temperature than kaons, which in turn has a larger effective temperature than pions. The success of hydrodynamic models in explaining the transverse momentum distributions of various hadrons at least upto lower momentum, is an indication of presence of radial flow\cite{4}. Fig.\ref{fig:1} depicts this effect for AA collisions at SPS energies. However similar effect is also observed in nucleon-nucleus (pA) collisions (also shown in Fig.\ref{fig:1}). Hence broadening of $p_T$ even in pA collisions, where one does not expect any collective effect like radial flow, indicates there exists a “normal” $p_T$ broadening observed in all heavy-ion reactions. This needs to be understood properly before making any quantitative conclusions regarding the radial flow. Such broadening of $p_T$ is thought to arise due to successive collisions in nuclear reactions. It is basically an initial state collision broadening of $p_T$. This effect in high $p_T$ (typically above 2 GeV) is referred to as Cronin effect\cite{5}. Apart from the commonly produced hadrons, the Drell-Yan dileptons and quarkonium resonances also show in pA collisions a broadening of $p_T$ compared to that observed in pp collisions. The virtual photons in Drell-Yan production do not interact in the nuclear medium and the quarkonia leave the medium before the collective effects develop. This makes the case for the $p_T$ broadening as an initial state effect stronger\cite{6,7}.

In\cite{6,8}, it has been shown that such an effect is able to explain the transverse mass ($m_T$) distribution of produced hadrons at SPS energies. Thereby questioning the presence of true collective effects...
(such as radial flow) in relativistic heavy ion collisions. The aim of the present work is to have a
detail study of the effect of $p_T$ gain through successive collisions in nuclear reaction within the framework of a random walk model \[6, 8\] (discussed in
the next section). In this paper we systematically analyze the available pp, pA and AA data and come
to the conclusion that collective effects does exists in relativistic heavy-ion collisions.

The paper is organized as follows. In the next section we discuss the random walk model. In section
III we fix the freeze-out temperature for AA collisions from the inverse slope of the $p_T$ spectra from
the pp collisions. Section IV deals with the estimation of the gain in average transverse momentum per
collision from pA collision data. We compare the results of the random walk model calculation with the
SPS AA collision data in section V. In section VI we give a discussion of the results. Finally a summary
of the work is given in section VII.

II. RANDOM WALK MODEL

Consider the nucleus-nucleus collision to be a superposition of nucleon-nucleon collision. In this picture, let us assume that in each successive interactions of a nuclear collision one creates a fire ball just like that formed in a nucleon-nucleon collision. If
a nucleon starts with zero $p_T$, after the first collision the next one will generally occur at some non-vanishing transverse velocity. Thus there is a gain in transverse momentum through successive collisions. The propagation of nucleon through successive collisions is assumed to follow a random walk pattern. It is of interest to find if such a model can explain
the $p_T$ spectra of produced hadrons in AA collisions and more specifically if it can account for the observed $p_T$ broadening with increase in mass of the hadrons. The random walk pattern is modeled by a Gaussian \[6\] in the present calculation as,

$$f_{pA}(\rho) = \left[\frac{4}{\pi \delta_{pA}^2}\right]^{1/2} \exp(-\rho^2/\delta_{pA}^2), \quad (1)$$

where $\rho$ is the transverse rapidity and

$$\delta_{pA}^2 = (N_A - 1)\delta^2, \quad (2)$$
denotes the kick per collision $\delta$ as determined from pA interactions. $N_A$ is the number of nucleons which
the incident proton encounters on its path through the target nucleus. It is given by \[3\]

$$N_A \sim (3/4)(2\pi r_0^2 R_A)n_0, \quad (3)$$

where $r_0 \sim 0.8$ fm is the nuclear radius, $R_A (= 1.12 A^{1/3}$ fm) is the radius of the nucleus and $n_0 = 0.17 \text{ fm}^3$ is the nuclear density.

The corresponding distribution for an $AB$ collision can be shown to of the form,

$$f_{AB}(\rho) = \left[\frac{4}{\pi \delta_{AB}^2}\right]^{1/2} \exp(-\rho^2/\delta_{AB}^2), \quad (4)$$

with

$$\delta_{AB}^2 = (N_A + N_B - 2)\delta^2, \quad (5)$$

The final expression for the transverse mass distribution is given as \[8\]

$$\left(\frac{dN}{dy dm_T dm_T}\right)_{y=0} = gV \left[\frac{4}{\pi \delta_{AB}^2}\right]^{1/2} \int d\rho \exp(-\rho^2/\delta_{AB}^2) m_T I_0 \left(\frac{p_T \sinh \rho}{T}\right) K_1 \left(\frac{m_T \cosh \rho}{T}\right). \quad (6)$$

Where $I_0$ and $K_1$ are modified Bessel functions. $T$ is the freeze-out temperature and $m_T = \sqrt{p_T^2 + m^2}$
is the transverse mass.

It should be noted that the volume in the Eqn. \[3\] refers to the volume of the system as observed in a
$pp$ collision, since each collision in the random walk produces a $pp$ type of fireball. If we now introduce
a boost-invariant distribution of fireballs along the longitudinal rapidity axis, we finally obtain by integrating over the fireball distributions
The above expression has two parameters, $T$ and $\delta$. The temperature is assumed to be same for the whole fireball. The temperature is fixed from the pp collisions and $\delta$ is calculated from pA collisions as mentioned before. Fixing these two parameters from pp and pA collisions we will attempt to describe the data for AA collisions.

**III. TRANSVERSE MOMENTUM SPECTRA IN PP COLLISIONS**

As discussed in the previous section, we would fixed the freeze-out temperature for the AA collisions from the inverse slope of the hadronic transverse momentum spectrum originating from pp collisions. At high energies the transverse momentum spectra of produced hadrons can be described by the following expression,

$$
\left(\frac{dN}{dydmTdmT}\right)_{y=0} = \frac{gV}{2\pi^2} \left[\frac{4}{\pi\delta_{AB}}\right]^{1/2} \int d\rho \exp\left(-\rho^2/\delta_{AB}^2\right) \int_{-Y_L}^{Y_L} dY m_T \cosh Y I_0 \left(\frac{p_T \sinh \rho}{T}\right) K_1 \left(\frac{m_T \cosh Y \cosh \rho}{T}\right).
$$

(7)

Using the expression given in Eq. (7) we fitted the experimental data of pp collisions of pions, kaons and protons at different centre-of-mass (CMS) energies varying from 23 GeV to 63 GeV \cite{11}. The results are shown in Fig. 4. The extracted temperature for the various hadron species are shown in Fig. 5. The error bars shown corresponds to the error on the slope parameter due to the fitting procedure adopted here. One observes that there is very little variation in $T$ for all the produced hadrons in the range of CMS energy for which data is available and analyzed. We will consider two values of temperature for our study of AA data, one corresponding to 140 MeV and the other corresponding to 150 MeV.

**IV. TRANSVERSE MOMENTUM SPECTRA IN PA COLLISIONS**

Having fixed the the freeze-out temperature for AA collisions from the inverse slope of $p_T$ spectra of produced hadrons in pp collisions, we now try to get the average transverse rapidity shift per collision, $\delta$. This we will determine empirically from transverse mass spectra of the produced hadrons in pA collisions. To get a proper understanding of this parameter, we will study pA data for various available beam energies, different species of produced hadrons and several targets. Once this parameter is fixed, along with the temperature fixed from pp collisions, we will be able to predict the initial state transverse momentum broadening in AA collisions as has been discussed in the section II. The results will be presented in the next section.

The transverse momentum spectra for different species of produced hadrons for pA collisions can be obtained using the Eqn. (7). Where $\delta_{pA}$ is the only parameter, with $T$ being fixed from pp collisions. The results are shown in Figs. 4 and 5. Fig. 4 shows the comparison of transverse mass distribution for pion, kaon and proton at 14.6 GeV for various targets \cite{11} with those obtained from model calculations. One finds that the data is reasonably well explained by the model calculations. The values of $\delta_{pA}$ obtained for pion, kaon and proton are 0.05, 0.25 and 0.25 respectively. The corresponding $\delta$ values for the p+Cu collisions are obtained using Eqn. (7) to be 0.04, 0.22, 0.22 for pions, kaons and protons respectively. Also shown in Fig. 5 is the transverse mass spectra for pions at 14.6 GeV for various targets. We find that there is very little atomic mass dependence of $\delta_{pA}$. It was found to be 0.05 for the various targets shown in the figure. However one observes from Eqn. (8) the average transverse rapidity shift per collision, $\delta$ decreases with increase in atomic mass as we go from beryllium to gold target. Fig. 4 shows the transverse mass spectra for pion, kaon and proton for p+Pb collisions at 450 AGeV \cite{12}. The solid lines are the result of model calculation taking $T = 150$ MeV. The resulting $\delta_{pA}$, which satisfactorily fits the data as shown, turns out to be of the order of 0.45 for three hadronic species. This is a clear increase from the lower energy value, indicating that $\delta_{pA}$ has an energy dependence. The corresponding value $\delta_{pA}$ for a $T = 140$ MeV (dotted lines) lies between 0.35-0.4.

For the description of the AA data at SPS, we will consider two values of temperature, of 140 MeV and 150 MeV fixed from the pp data. Since the collisions at SPS are lead on lead target, we will use $\delta_{pPb}$ values as discussed above for pPb collisions at 450 AGeV. We will also consider one case with $\delta_{pBe}$
FIG. 2: Transverse momentum spectra of pions, kaons, protons and anti-protons for various CMS energies. The data has been scaled appropriately for clarity of presentation. The solid lines corresponds to results obtained from Eqn. 9. The fits for $\pi^-$ and $K^-$ are not shown in the figure.

FIG. 3: Temperature obtained by fitting the transverse momentum spectra presented in Fig. 2 for various hadrons. The solid lines corresponds to 140 and 150 MeV. The error bars corresponds to the error on temperature due to the fitting.

at 14.6 AGeV in order to get an idea of the effect at lower energies.

V. TRANSVERSE MOMENTUM SPECTRA IN AA COLLISIONS

Having fixed the temperature from the pp spectra and knowing the value of $\delta$ from the available pA data, we now try to see if the random walk model is able to explain the transverse mass spectra of pions, kaons and protons at SPS energies. The results of the model calculation using Eqn. 7 along with the available data [13] are shown in Fig. 6. The results for pions are shown in top panel of Fig. 6. The lines corresponds to the random walk model calculations for two different temperatures of 140 MeV and 150 MeV. One observes that the model is able to explain the pion spectra satisfactorily over the energy range of 40 AGeV to 158 AGeV for both 140 and 150 MeV temperatures. The $\chi^2/ndf$ being less than 1.0 for all the cases. Taking a $\delta$ value corresponding to pA results from AGS energies (14.3 AGeV) fails to explain the data. This is shown as dotted line for 40 AGeV beam energy. Similar results are obtained for higher beam energies and not shown in the figure for clarity of presentation. It may be mentioned that the absolute normalisation is adjusted to get the best possible fit. For the kaons (middle panel of Fig. 6), one finds that the model fails to explain the lower transverse momentum re-
Further one notices that the $\chi^2/ndf$ worsens as we go to higher beam energies. It varies from 3.5 at 40 AGeV to 10 at 158 AGeV. There is not much difference between the results for the model calculation for temperature of 140 and 150 MeV, hence the spectra corresponding to 140 MeV is not shown in the figure. The model also fails to explain the observed transverse mass spectra of protons at both lower and higher transverse momentums. The results for two different temperatures of 140 MeV and 150 MeV are shown in the bottom panel of Fig. 4. The $\chi^2/ndf$ for the three cases lies between 3.5 to 7.5.

VI. DISCUSSION

The results indicate the following: (i) the random walk model fails to explain the transverse mass spectra as the mass of the hadron increases where the effect of possible transverse flow will be more and (ii) the $\chi^2/ndf$ indicates that the disagreement of the model calculation with data for kaons and protons increases with increase in beam energy. In this section we discuss the limitations of the model and try to estimate the relative contribution of initial state $p_T$ broadening and transverse flow.

A. Random walk pattern

The random walk pattern was chosen to be a Gaussian (Eqn. 8). In principle this can be any other type of statistical distribution, such as a Lorentzian. We have studied the sensitivity of the result to such a distribution. We find that the $p+Pb$ data for pion, kaon and proton is well explained by a $\delta_{pPb}$ of 0.2, 0.15 and 0.35 respectively. The values are lower compared to those obtained considering a Gaussian random walk pattern. Extrapolating these values to Pb+Pb collisions and taking the value of $T$ to be 150 MeV, we obtain the $m_T$ spectra for the available SPS data at the highest energy. The results are shown in Fig. 7. We observe that the model fails to explain the data satisfactorily, specially at low $m_T$ for kaons and protons. However it agrees fairly well with the observed pion spectra. It seems that the results are insensitive to the choice of random walk pattern.
FIG. 6: Transverse mass spectra for pions, kaons and protons at 40, 80 and 158 AGeV beam energy. The solid lines corresponds to calculations from the random walk model with the random pattern following a Lorentzian distribution. The model parameters $T$ and $\delta$ are not fixed from pp and p+Pb collisions respectively. The normalization is adjusted to give the best possible agreement with the experimental data.

B. Parameters of random walk model

One of the disadvantage of the present work is the absence of the pp and pA data in literature corresponding to the exact SPS energies for which AA collision data is available. However one must mention that the near constant temperature exhibited by the available pp data for a wide range of CMS energy more or less fixes the temperature parameter of the model. Contrary to this one observes a considerable variation of $\delta$ with beam energy. In view of these, it is obvious to ask, if the SPS kaon and proton transverse mass spectra can be explained within the framework of random walk model for arbitrary values of temperature and $\delta$. The results are shown in Fig. 8. One observes that the kaon spectra for the three different beam energies are well explained by the model taking a common temperature of 235 MeV and $\delta_{PbPb}$ of 0.25. The $\chi^2/ndf$ is less than 1.0 for all the three cases. It may be mentioned that this temperature although very high compared to that obtained from pp (section III), is very close to the temperature obtained taking radial flow into account [13]. For the protons the spectra is satisfactorily reproduced over a large range of transverse momentum. For this we needed a common $\delta_{PbPb}$ values of 0.15 but the temperature for 40 and 80 AGeV is 260 MeV, while that for 158 AGeV energy it is 360 MeV.
hadrons indicate that initial state $p_T$ broadening alone cannot account for experimentally measured transverse mass spectra. One observes that the model fails to explain the lower momentum part for kaons and protons. This may be attributed to the lack of chemical equilibrium, which can be ac-

FIG. 8: Transverse mass spectra for kaons and protons at SPS energies. The solid line corresponds to results obtained from random walk model. The model parameters $T$ and $\delta$ are not fixed from pp and pA collisions respectively but are adjusted to give the best possible agreement with the experimental data.

C. Transverse flow

The failure of random walk model to fully explain the transverse momentum spectra of produced hadrons may be attributed to the lack of chemical equilibrium, which can be accounted for by introducing a chemical potential for the hadrons. But the formulation of the model is such that it affects only the normalization and not the slope. The other possibility is that, there may be both initial state $p_T$ broadening and collective effect like transverse flow. In other words the resultant spectra of produced hadrons is the effect of both the above processes. In order to look at this possibility, we express the transverse mass spectra of produced hadrons as:

$$
\left( \frac{dN}{dym_Tdm_T} \right)_{total} = C_1 \left( \frac{dN}{dym_Tdm_T} \right)_{random} + C_2 \left( \frac{dN}{dym_Tdm_T} \right)_{flow}
$$

(9)

FIG. 9: Transverse mass spectra for pions, kaons and protons at 158 AGeV Pb+Pb Collisions. The solid line corresponds to results obtained from a combination random walk model and transverse flow. The model parameters $T$ and $\delta$ are fixed from pp and p+Pb collisions respectively and the transverse flow velocity $\beta_T$ is taken to be $\sim 0.45$. The normalization is adjusted to give the best possible agreement with the experimental data.
where, the contribution from flow \[13, 14\] is defined as, \[m_T I_0 \left( \frac{m_T \sinh \beta}{m_T} \right) K_1 \left( \frac{m_T \cosh \beta}{m_T} \right) \sim \frac{dN}{dy m_T \text{flow}}\]

is the available nucleon-nucleon collision data. There is a gain in transverse momentum through successive collisions, the propagation of which is assumed to follow a random walk pattern. The average transverse rapidity shift per collision or the gain in transverse momentum per collision is obtained from the nucleon-nucleon collision data. Having fixed the two parameters of the model, we then apply it to nucleus-nucleus collisions. It is observed that, although the model fairly well explains the transverse mass spectra of pions at SPS energies, it fails to do so for the higher mass hadrons like kaons and protons. We find the choice of different distribution of random walk pattern yields the same result. This indicates the presence of true collective effects in the nucleus-nucleus collisions. However it seems a considerable portion of the gain in transverse momentum comes from initial state effects.

We tried to see if such a parametrization can explain the SPS data at 158 AGeV beam energy and get the values of \(C_1\) and \(C_2\) which will tell the relative contribution of the two effects. The results are shown in the Fig. 9. Since pion data is very well explained by the random walk model, we tried not to fit it with the above parametrization. The aim here is to see how much contribution of transverse flow in addition to initial state \(p_T\) broadening through the random walk model can explain the data. From the figure we find that the results for the kaons and protons shows that such a parametrization works. We have taken a value of \(\beta_T\) of 0.45 for kaons and 0.48 for protons. The values of \(C_1\) and \(C_2\) are 0.5 for kaons and protons, \(C_1 = 0.2\) and \(C_2 = 0.8\). The results indicate the flow effects dominates with increase in hadron mass.

VII. SUMMARY

In summary, a systematic study has been carried out to understand the transverse mass spectra of the produced hadrons in nucleus-nucleus collisions within the framework of a simple random walk model. The model is based on the assumption that the nucleus-nucleus collision is a superposition of nucleon-nucleon collision where for each successive interactions of a nuclear collision one creates a fire ball. The temperature of the fire ball is fixed from

Acknowledgments

I am grateful to the Board of Research on Nuclear Science and Department of Atomic Energy, Government of India for financial support. I would like to thank Michele Murray and Subrata Bhattacharyya for providing me the experimental data. I would like to thank Jan-e Alam for critical reading of the manuscript and for many helpful discussions.

[1] I.G. Bearden et al. (NA44 collaboration), Phys. Rev. Letters 78, 2080 (1997).
[2] B. Mohanty, J. Alam, S. Sarkar, T.K. Nayak and B.K. Nandi, nucl-th/0304023.
[3] U. Heinz, K.S. Lee and E. Schnedermann, in “Quark-Gluon Plasma”, pp. 471, Ed. R.C. Hwa, World Scientific, Singapore (1992); Jean-Paul Blaizot and Jean-Yves Ollitrault, Adv. Ser. Direct. High Energy Phys. 6, 393 (1990).
[4] C.M. Hung and Edward V. Shuryak, Phys. Rev. C 57, 1891 (1998).
[5] J. W. Cronin et al., Phys. Rev. Lett. 31, 1426 (1973); J. W. Cronin et al., Phys. Rev. D 11, 3105 (1975).
[6] A. Leonidov, M.Nardi and H. Satz, Nucl. Phys. A 610, 124c (1996) and Z. Phys. C 74, 535 (1997).
[7] H. Satz, Proceedings of the International Conference on the Physics and Astrophysics of the Quark-Gluon Plasma (ICPA-QGP’97), Jaipur, India, March 15-21, 1997, hep-ph/9706342.
[8] Jan-e Alam, J. Cleymans, K. Redlich and H. Satz, nucl-th/9707042.
[9] F. Becattini, Z. f. Physik C 69, 485 (1996); F. Becattini and U. Heinz, Z. f. Physik C 76, 269 (1997).
[10] B. Alper et al., Nucl. Phys. B 87, 19 (1987) and Nucl. Phys. B 100, 237 (1975).
[11] E-802 Collaboration, T. Abbott et al., Phys. Rev. D 45, 3906 (1992).
[12] NA44 Collaboration, I.G. Bearden et al., Phys. Rev. C 57, 837 (1998); H. Boggild et al., Phys. Rev. C 59, 328 (1999).
[13] NA49 Collaboration, S.V. Afanasiev, et al. Phys.
Rev. C 66, 054902 (2002); M. van Leeuwen et. al. (NA49 collaboration), nucl-ex/0208014

[14] E. Schnedermann, J. Sollfrank and U. Heinz, Phys. Rev. C 48, 2462 (1993).