Indeterministic Quantum Gravity

V. Dynamics and Arrow of Time

Vladimir S. MASHKEVICH

Institute of Physics, National academy of sciences of Ukraine
252028 Kiev, Ukraine

Abstract

This paper is a continuation of the papers [1-4] and is devoted to the riddle of the origin of the arrow of time. The problem of time orientation reduces to that of the difference between the past and the future. The riddle escapes solution in deterministic dynamics and in the dynamics of standard indeterministic quantum theory as well. In the dynamics of indeterministic quantum gravity, the past is reconstructible uniquely, whereas the future may be forecasted only on a probabilistic level. Thus the problems of the past and the future and, by the same token, of time orientation are solved.

1E-mail: mashkevich@gluk.apc.org
Introduction

One of the most ancient riddles of physics is that of the origin of the arrow of time, or of the nature of the difference between the past and the future. It is conventional to search for a solution to this problem in dynamics, i.e., time evolution of a state of a physical system. The solution may be given by a dynamics which is asymmetric, or orientable in the sense of the sequence of states.

Deterministic dynamics does not involve such an orientability. There exists an established opinion that in standard indeterministic quantum dynamics there is no arrow of time as well [5].

The dynamics of indeterministic quantum gravity—of the theory being developed in this series of papers—features an asymmetry, which may be used for determining the arrow of time.

This paper is dedicated to a comprehensive consideration of the issues outlined above.

In Section 1, a general treatment of dynamics is given. A predynamical time, or pretime, is introduced, and the problem of the arrow of physical time consists in fixing the direction of the latter with respect to the direction of the former. Physical time is oriented from the past to the future. The idea of defining these notions is that the predeterminability of the future should be less than the reconstructibility of the past.

In Section 2, the dynamics of standard indeterministic quantum theory is examined from the standpoint of the general treatment. This dynamics is symmetric and does not give rise to a choice of the future and the past and, by the same token, to physical time orientation.

In Section 3, a new scheme for quantum jumps in indeterministic quantum gravity is introduced, and then the related dynamics is analyzed. In this dynamics, the past is reconstructible uniquely, whereas the future may be forecasted only on a probabilistic level. Thus the problems of the future and the past and, by the same token, of physical time orientation are solved.

1 Dynamics and time orientation

We introduce a treatment of dynamics which may be readily generalized for a subsequent application to cosmology.

1.1 Predynamical and physical time

Dynamics in general is time dependence of a state of a physical system. A definition of the state may involve the direction of time, which is not given a priori. Therefore we introduce a predynamical time, or pretime, for short, \( \tau \) as a point of the oriented real axis \( T \). For the direction of physical time, \( t \), there are two possibilities: \( t = \tau \) and \( t = -\tau \). Definitions of the state and dynamics should be given in terms of the pretime, and the problem of physical time orientation is to be solved on the basis of dynamics.
1.2 Dynamical process

We start with the notion of a dynamical process, which plays a central role in dynamics. Let \( \Omega \) be a set of pure states \( \omega \), \( \Delta \) be a connected subset of \( T \), i.e., an interval:

\[
\Delta = (\tau_1, \tau_2), [\tau_1, \tau_2), (\tau_1, \tau_2], [\tau_1, \tau_2], \quad -\infty \leq \tau_1 < \tau_2 \leq \infty.
\]  

A dynamical process \( P_\Delta \) on \( \Delta \) is a function from \( \Delta \) to \( \Omega \):

\[
P_\Delta : \Delta \rightarrow \Omega, \quad \Delta \ni \tau \mapsto P_\Delta(\tau) = \omega_\tau \in \Omega.
\]

In fact, it would suffice for \( P_\Delta(\tau) \) to be defined almost everywhere on \( \Delta \).

A restriction and extension of a process are defined as those of a function with regard to the fact that the domain of the process is connected.

A left (right) prolongation of a process \( P_\Delta \) to \( \Delta' \), \( \Delta' \ni \tau' < \tau \in \Delta \) (\( \Delta \ni \tau < \tau' \in \Delta' \)), is a process \( P_{\Delta \cup \Delta'} \), such that there exists a process \( P_{\Delta \cup \Delta'} \) with the restrictions \( P_\Delta \) and \( P_{\Delta'} \).

1.3 Dynamics

Dynamics on \( \Delta \), \( D_\Delta \), is a family of processes on \( \Delta \):

\[
D_\Delta = \{P_\Delta\}.
\]  

A restriction and extension of a dynamics boil down to those of corresponding processes.

1.4 Deterministic process and deterministic dynamics

A deterministic process \( P_\Delta \) is defined as follows: For every restriction of \( P_\Delta \) the only extension to \( \Delta \) is \( P_\Delta \) itself.

A deterministic dynamics is a family of deterministic processes.

1.5 Indeterministic point, process, and dynamics

An interior isolated indeterministic point \( \tau \in \text{int} \Delta \) of a process \( P_\Delta \) is defined as follows: There exists \( \theta > 0 \), such that:

(i) \( (\tau - \theta, \tau + \theta) \subseteq \Delta \);

(ii) left prolongations of \( P_\Delta|_{(\tau, \tau+\theta]} \) to \( (\tau - \theta, \tau) \) and right ones of \( P_\Delta|_{[\tau-\theta, \tau]} \) to \( (\tau, \tau + \theta) \) are deterministic processes;

(iii) cardinal numbers \( \text{card}^{\text{left}} \) and \( \text{card}^{\text{right}} \) of sets of those prolongations meet the condition \( \text{card}^{\text{left}} + \text{card}^{\text{right}} > 2 \).

We assume that there are only isolated indeterministic points.

An indeterministic process is that with indeterministic points. An indeterministic dynamics is one with indeterministic processes.
1.6 Orientable dynamics and time orientation: The future and the past

Let \( \tau \in \text{int} \Delta \) be an indeterministic point of a process \( \mathcal{P}_\Delta \). We introduce five dynamics related to the point as follows:

(i) \( \mathcal{D}_{(\tau-\theta,\tau)} \) and \( \mathcal{D}_{(\tau,\tau+\theta)} \) are deterministic, \( \mathcal{D}_{(\tau-\theta,\tau)} \supseteq \mathcal{P}_\Delta |_{(\tau-\theta,\tau)} \), \( \mathcal{D}_{(\tau,\tau+\theta)} \supseteq \mathcal{P}_\Delta |_{(\tau,\tau+\theta)} \);

(ii) \( \mathcal{D}_{(\tau-\theta,\tau+\theta)} = \{ \mathcal{P}_\alpha^0 |_{(\tau-\theta,\tau)} : \alpha \in A \} \),
\( \mathcal{D}_{(\tau-\theta,\tau+\theta)} |_{(\tau-\theta,\tau)} = \mathcal{D}_{(\tau,\tau+\theta)} |_{(\tau,\tau+\theta)} = \mathcal{D}_{(\tau,\tau+\theta)} \);

(iii) a graph where points are elements of \( \mathcal{D}_{(\tau-\theta,\tau+\theta)} \) and lines connecting related points are elements of \( \mathcal{D}_{(\tau-\theta,\tau+\theta)} \) is connected and complete, i.e., involves all processes associated with the indeterministic point;

(iv) \( \mathcal{D}^{\text{right}}_{(\tau-\theta,\tau+\theta)} = \{ \mathcal{P}_\alpha^0 |_{(\tau-\theta,\tau)} : \mathcal{P}_\alpha^0 |_{(\tau,\tau+\theta)} = \mathcal{P}_\alpha |_{(\tau,\tau+\theta)} \} \),
\( \mathcal{D}^{\text{left}}_{(\tau-\theta,\tau+\theta)} = \{ \mathcal{P}_\alpha^0 |_{(\tau-\theta,\tau)} : \mathcal{P}_\alpha^0 |_{(\tau,\tau+\theta)} = \mathcal{P}_\alpha |_{(\tau,\tau+\theta)} \} \),
\( \text{card}^{\text{right}} \alpha, \text{card}^{\text{left}} \alpha \) being corresponding cardinal numbers.

An indeterministic point is symmetric (asymmetric) if \( \text{card}^{\text{right}} \alpha = (\neq) \text{card}^{\text{left}} \alpha \). A symmetric dynamics is that with symmetric indeterministic points only.

An indeterministic dynamics is orientable if for all indeterministic points either
\[
\text{card}^{\text{right}} \alpha > \text{card}^{\text{left}} \alpha \quad \text{and} \quad \text{card} \mathcal{D}_{(\tau,\tau+\theta)} > \text{card} \mathcal{D}_{(\tau-\theta,\tau)}
\]  
(1.6.1)
or
\[
\text{card}^{\text{right}} \alpha < \text{card}^{\text{left}} \alpha \quad \text{and} \quad \text{card} \mathcal{D}_{(\tau,\tau+\theta)} < \text{card} \mathcal{D}_{(\tau-\theta,\tau)}.
\]  
(1.6.2)

An orientable dynamics is oriented as follows: The future corresponds to the greater of \( \text{card}^{\text{right}} \alpha \), \( \text{card} \mathcal{D}_{(\tau,\tau+\theta)} \) and \( \text{card}^{\text{left}} \alpha \), \( \text{card} \mathcal{D}_{(\tau-\theta,\tau)} \), i.e.,
\[
\text{card}^{\text{future}} \alpha > \text{card}^{\text{past}} \alpha, \quad \text{card} \mathcal{D}_{\text{future}} > \text{card} \mathcal{D}_{\text{past}};
\]  
(1.6.3)
so that the physical time is
\[
t = +(-) \tau \quad \text{for card}^{\text{right}} \alpha > (<) \text{ card}^{\text{left}} \alpha, \quad \text{card} \mathcal{D}_{(\tau,\tau+\theta)} > (<) \text{ card} \mathcal{D}_{(\tau-\theta,\tau)}.
\]  
(1.6.4)

This defines time orientation, or the arrow of time.

1.7 Nonpredeterminability and the question of reconstructibility

For an oriented dynamics we have
\[
\text{card}^{\text{future}} \alpha + \text{card}^{\text{past}} \alpha > 2,
\]  
(1.7.1)
\[
\text{card}^{\text{future}} \alpha > \text{card}^{\text{past}} \alpha \geq 1,
\]  
(1.7.2)
so that
\[
\text{card}^{\text{future}} \alpha > 1.
\]  
(1.7.3)
This implies that the future is not predeterminate.

If
\[
\text{card}^{\text{past}} \alpha = 1,
\]  
(1.7.4)
the past is reconstructible.

In any case, the inequality (1.6.1) implies that the reconstructibility of the past is greater than the predictability of the future. This feature is inherent in an orientated dynamics.

The phenomenon of memory should be related to dynamics orientation.
1.8 Probabilistic dynamics

Let $\tau$ be an indeterministic point of a process $\mathcal{P}_\Delta$. Time evolution implies transitions from one of the sets $\mathcal{D}_{(\tau-\theta, \tau)}$, $\mathcal{D}_{(\tau, \tau+\theta)}$ to the other: from $\mathcal{D}_{\text{initial}}$ to $\mathcal{D}_{\text{final}}$. We assume that for their cardinal numbers

$$\text{card}_{\text{final}} \geq \text{card}_{\text{initial}}$$  \hspace{1cm} (1.8.1)

holds.

Let there exist $i \to f$ transition probabilities, or conditional probabilities $w(f/i)$, where $i$ and $f$ are indexes of elements of $\mathcal{D}_{\text{initial}}$ and $\mathcal{D}_{\text{final}}$ respectively. The probabilities meet the equation

$$\sum_f w(f/i) = 1.$$  \hspace{1cm} (1.8.2)

Taking into account the relations

$$\text{card}_{\text{initial}} = \sum_i 1 = \sum_i \sum_f w(f/i) = \sum_f \sum_i w(f/i) \leq \sum_f 1 = \text{card}_{\text{final}},$$

we put

$$\sum_i w(f/i) \leq 1.$$  \hspace{1cm} (1.8.3)

By Bayes formula, the a posteriori probability is

$$w(i/f) = \frac{w(i)w(f/i)}{\sum_{i'} w(i')w(f/i')}.$$  \hspace{1cm} (1.8.5)

We put for the a priori probability

$$w(i) = \text{const},$$  \hspace{1cm} (1.8.6)

then

$$w(i/f) = \frac{w(f/i)}{\sum_{i'} w(f/i')} \geq w(f/i).$$  \hspace{1cm} (1.8.7)

Thus

$$w(f/i) \leq w(i/f) \leq 1.$$  \hspace{1cm} (1.8.8)

For a symmetric dynamics,

$$\text{card}_{\text{final}} = \text{card}_{\text{initial}}, \quad \sum_i w(f/i) = 1,$$  \hspace{1cm} (1.8.9)

so that

$$w(i/f) = w(f/i).$$  \hspace{1cm} (1.8.10)

Specifically, the increase of entropy is, on the average, the same for the future and for the past:

$$- \sum_f w(f/i) \ln w(f/i) \approx - \sum_i w(i/f) \ln w(i/f).$$  \hspace{1cm} (1.8.11)
1.9 Irreversibility and orientation

It should be particularly emphasized that irreversibility does not imply dynamics orientability and, by the same token, time orientation.

Indeed, a reversible dynamics is defined as follows. Let $\mathcal{P}_\Delta$ be a process with a symmetric domain, i.e., $\Delta = (\tau_1, \tau_2)$ or $[\tau_1, \tau_2]$. The inverse process, $\mathcal{P}^{\text{inv}}_{\Delta}$, is defined by

$$\mathcal{P}^{\text{inv}}_{\Delta}(\tau) = \mathcal{P}_\Delta(\tau_1 + \tau_2 - \tau), \quad \tau \in \Delta. \tag{1.9.1}$$

Let $S$ be a transformation of $\Omega$, $S : \Omega \to \Omega$. The transformed process, $S\mathcal{P}_\Delta$, is defined by

$$S\mathcal{P}_\Delta(\tau) = S(\mathcal{P}_\Delta(\tau)), \quad \tau \in \Delta. \tag{1.9.2}$$

A dynamics $\mathcal{D}_{\Delta'}$ is reversible if there exists a bijection $S : \Omega \to \Omega$, such that

$S$ is an involution ($S^2$ is identity) and $\mathcal{P}_\Delta \in \mathcal{D}_{\Delta} \Rightarrow \mathcal{P}^{\text{rev}}_{\Delta} \equiv S\mathcal{P}^{\text{inv}}_{\Delta} \in \mathcal{D}_{\Delta}$ for all $\Delta \subset \Delta'$ \hspace{1cm} (1.9.3)

(rev stands for reverse).

The nonexistence of $S$ does not imply the orientability of $\mathcal{D}_{\Delta'}$.

Here is an example. Let a dynamical equation be of the form

$$\frac{d^2x}{d\tau^2} = -\alpha \frac{dx}{d\tau}. \tag{1.9.4}$$

All dynamical processes $\mathcal{P}_{(-\infty, \infty)}$ are given by

$$\mathcal{P}_{(-\infty, \infty)}(\tau) = \omega_\tau = \left(x(\tau), \frac{dx(\tau)}{d\tau}\right), \tag{1.9.5}$$

$$x(\tau) = c_1 + c_2 e^{-\alpha \tau}; \tag{1.9.6}$$

they are deterministic. The dynamics $\mathcal{D}_{(-\infty, \infty)}$ is irreversible but deterministic and, therefore, not orientable.

On the other hand, a dynamics with asymmetric indeterministic points is irreversible—in view of inequality $\text{card}^{\text{right}} \alpha \neq \text{card}^{\text{left}} \alpha$. Specifically, an orientable dynamics is irreversible.

2 Dynamics of standard indeterministic quantum theory

Let us consider the dynamics of standard, or orthodox indeterministic quantum theory from the standpoint developed in the previous section.

2.1 Standard dynamical process

In standard quantum theory, indeterminism originates from quantum jumps. A standard indeterministic dynamical process $\mathcal{P}_{(-\infty, \infty)}$ may be described as follows. Let $\tau_k$, $k \in K = \{0, \pm 1, \pm 2, \ldots\}$, be indeterministic points, i.e., points of jumps. The process is denoted by

$$\mathcal{P}_{(-\infty, \infty)}^{\{j_k, k \in K\}}, \ j_k \in J = \{1, 2, \ldots, j_{\text{max}}\}, \ j_{\text{max}} \leq \infty. \tag{2.1.1}$$
The definition of this process reduces to that of its restrictions to the intervals
\[ \Delta_k = (\tau_k, \tau_{k+1}), \quad k \in K, \]  
(2.1.2)

\[ P_{j_k}^{\Delta_k} \equiv P_{(-\infty,\infty)}^{j_k, k \in K} |_{\Delta_k}. \]  
(2.1.3)

The process \( P_{j_k}^{\Delta_k} \) is defined as follows:
\[ P_{j_k}^{\Delta_k}(\tau) = \omega_{j_k}^{\Delta_k} = (\Psi_{j_k}^{\Delta_k}(\tau), \Psi_j(\tau)), \quad \tau \in \Delta_k, \]  
(2.1.4)

where \( \Psi_{j_k}^{\Delta_k} \) is a state vector,
\[ \Psi_{j_k}^{\Delta_k}(\tau) = U(\tau, \tau_k) \Psi_{j_k}, \]  
(2.1.5)

\[ A_k \Psi_{j_k} = a_{j_k} \Psi_{j_k}, \]  
(2.1.6)

where the \( A_k \) is an observable, and \( U \) is a unitary operator of time evolution.

This description seemingly fixes the time orientation, namely, in view of eq.(2.1.5),
\[ t = \tau. \]  
(2.1.7)

But there is another possibility for describing the process considered.

### 2.2 Reverse description

In place of eqs.(2.1.3), (2.1.6), we may put
\[ \Psi_{j_k}^{\Delta_k}(\tau) = U(\tau, \tau_{k+1}) \Psi_{j_k}^{\text{rev}}, \]  
(2.2.1)

\[ A_{k+1}^{\text{rev}} \Psi_{j_k}^{\text{rev}} = a_{j_k}^{\text{rev}} \Psi_{j_k}^{\text{rev}}, \]  
(2.2.2)

where
\[ \Psi_{j_k}^{\text{rev}} = U(\tau_{k+1}, \tau_k) \Psi_{j_k}, \]  
(2.2.3)

\[ A_{k+1}^{\text{rev}} = U(\tau_{k+1}, \tau_k) A_k U(\tau_k, \tau_{k+1}), \]  
(2.2.4)

\[ a_{j_k}^{\text{rev}} = a_{j_k}. \]  
(2.2.5)

This description implies, in view of eq.(2.2.1), the time orientation
\[ t = -\tau. \]  
(2.2.6)

The two descriptions are completely equivalent physically.

### 2.3 Standard quantum dynamics

We have for an indeterministic point \( \tau_k \)
\[ D_k^{\text{left}} \equiv D_{(\tau_k, \tau_k-\theta)} \equiv \{ P_{\Delta_k-1}^{j_k} |_{(\tau_k-\theta, \tau_k)}, \dot{j}_{k-1} \in J \}, \]
\[ D_k^{\text{right}} \equiv D_{(\tau_k, \tau_k+\theta)} \equiv \{ P_{\Delta_k}^{j_k} |_{(\tau_k, \tau_k+\theta)}, \dot{j}_k \in J \}, \]  
(2.3.1)

\[ \text{card } D_k^{\text{left}} = \text{card } D_k^{\text{right}} = \text{card } J. \]  
(2.3.2)

Thus standard quantum dynamics is not orientable.
2.4 Standard probabilistic quantum dynamics

We have for the time orientation $t = \tau$

$$w(j_{k+1}/j_k) = w_{j_{k+1}\leftarrow j_k} = |(\Psi^{j_{k+1}}(\tau_{k+1}+0), \Psi^j(\tau_{k+1}-0))|^2 = |(\Psi_{j_{k+1}}, U(\tau_{k+1}, \tau_k)\Psi_{j_k})|^2, \quad (2.4.1)$$

$$w_{j_{k+m}\leftarrow j_{k+m-1}\leftarrow\ldots\leftarrow j_{k+1}\leftarrow j_k} = w(j_{k+m}/j_{k+m-1}) \cdots w(j_{k+1}/j_k); \quad (2.4.2)$$

for the time orientation $t = -\tau$

$$w(j_k/j_{k+1}) = w_{j_k\leftarrow j_{k+1}} = |(\Psi^{j_k}(\tau_{k+1} - 0), \Psi^{j_{k+1}}(\tau_{k+1} + 0))|^2 = w(j_{k+1}/j_k), \quad (2.4.3)$$

$$w_{j_k\leftarrow\ldots\leftarrow j_{k+m}} = w_{j_{k+m}\leftarrow\ldots\leftarrow j_k}. \quad (2.4.4)$$

The probabilities satisfy the equations

$$\sum_{j_{k+1}} w(j_{k+1}/j_k) = \sum_j w(j_{k+1}/j_k) = 1. \quad (2.4.5)$$

We obtain for $t = \tau$ by Bayes formula, under the condition $w(j_k) = \text{const}$,

$$w(j_k/j_{k+1}) = w(j_{k+1}/j_k), \quad (2.4.6)$$

which coincides with eq. (2.4.3).

2.5 Nonorientability of standard indeterministic quantum dynamics

Summing up the results of this section, we conclude that the dynamics of standard indeterministic quantum theory is nonorientable and, by the same token, does not fix the orientation of physical time.

3 Dynamics of indeterministic quantum gravity

As in standard quantum theory, in indeterministic quantum gravity indeterminism originates from quantum jumps. But the origin of the jumps in the latter theory differs radically from that in the former one.

A quantum jump is the reduction of a state vector to one of its components. In standard quantum theory, the cause of the jump is coherence breaking between the components. In indeterministic quantum gravity, the cause is energy difference between the components, the difference occurring at a crossing of energy levels.

According to the paper [2], a jump occurs at the tangency of two levels. But level tangency imposes too severe constraints on the occurrence of the jump. Here we introduce a scheme in which the jump occurs at a simple crossing of two levels.
3.1 Level crossing

Let $\tau = 0$ be the point of a crossing of levels $l = 1, 2$; $P_{1\tau}, P_{2\tau}$ be the projectors for the corresponding states in a neighborhood of the point:

$$P_{l\tau} \leftrightarrow \omega_{ml\tau} = (\Psi_{l\tau}, \Psi_{l\tau})$$  \hspace{1cm} (3.1.1)

($m$ stands for matter), and

$$P_{\tau} = P_{1\tau} + P_{2\tau}.$$  \hspace{1cm} (3.1.2)

The part of the Hamiltonian $H_{\tau}$ related to the two levels is a projected Hamiltonian

$$H_{\tau}^{\text{proj}} = P_{\tau} H_{\tau} P_{\tau} = \epsilon_{1\tau} P_{1\tau} + \epsilon_{2\tau} P_{2\tau}$$  \hspace{1cm} (3.1.3)

($H_{\tau}^{\text{proj}}$ is $\tilde{H}_t$ in [2]). The metric tensor is

$$g = d\tau \otimes d\tau - h_{\tau}$$  \hspace{1cm} (3.1.4)

($h_{\tau}$ is $\tilde{g}_t$ in [2]).

We have

$$H_{\tau}^{\text{proj}} = H^{\text{proj}}[h_{\tau}, \dot{h}_{\tau}],$$  \hspace{1cm} (3.1.5)

where dot denotes the derivative with respect to the pretime $\tau$,

$$H_{0}^{\text{proj}} = \epsilon_{0} P_{0} = H^{\text{proj}}[h_{0}, \dot{h}_{0}], \quad \epsilon_{0} = \epsilon_{10} = \epsilon_{20}.$$  \hspace{1cm} (3.1.6)

3.2 Creation projector and creation state

We have in the first order in $\tau$

$$H_{\tau}^{\text{proj}} = H_{0}^{\text{proj}} + \dot{H}_{0}^{\text{proj}} \tau = H_{\tau}^{\text{proj}} + \dot{H}_{\tau}^{\text{proj}}[h_{0}, \dot{h}_{0}, \ddot{h}_{0}].$$  \hspace{1cm} (3.2.1)

Furthermore,

$$\ddot{h}_{0} = \ddot{h}[h_{0}, \dot{h}_{0}, P^{\text{creat}}],$$  \hspace{1cm} (3.2.2)

where $P^{\text{creat}}$ is a one-dimensional projector which creates $\ddot{h}_{0}$ and, by the same token, the Hamiltonian $H_{\tau}^{\text{proj}}$ eq.(3.2.1). This creation projector satisfies

$$P^{\text{creat}} P_{0} = P^{\text{creat}}$$  \hspace{1cm} (3.2.3)

and corresponds to a creation state $\omega_{m}^{\text{creat}}$ belonging to a state subspace determined by $P_{0}$. For the sake of brevity, we write

$$H_{\tau}^{\text{proj}} = \tilde{H}_{\tau}^{\text{proj}} + v \tau,$$  \hspace{1cm} (3.2.4)

$$v = \tilde{H}_{\tau}^{\text{proj}}[h_{0}, \dot{h}_{0}, \ddot{h}[h_{0}, \dot{h}_{0}, P^{\text{creat}}]].$$  \hspace{1cm} (3.2.5)
3.3 Diagonal Hamiltonian

The diagonalization of the Hamiltonian $H^{\text{proj}}_\tau$ eq. (3.2.4) gives

$$H^{\text{proj}}_\tau = (\epsilon_0 + \epsilon^+_\tau)P^+ + (\epsilon_0 + \epsilon^-_\tau)P^-, \quad (3.3.1)$$

where

$$\epsilon^\pm_\tau = \frac{\tau v_{11} + v_{22}}{2} \pm |\tau| \sqrt{\frac{(v_{11} - v_{22})^2}{4} + |v_{12}|^2}, \quad (3.3.2)$$

$$\Psi^+ = e^{i\beta} \cos \vartheta \Psi_1 + \sin \vartheta \Psi_2, \quad \Psi^- = -e^{i\beta} \sin \vartheta \Psi_1 + \cos \vartheta \Psi_2, \quad (3.3.4)$$

$$\tau \rightarrow -\tau \Rightarrow e^{i\beta} \rightarrow -e^{i\beta}, \quad \tan \vartheta \leftrightarrow \cot \vartheta, \quad \sin \vartheta \leftrightarrow \cos \vartheta, \quad \Psi^+ \leftrightarrow \Psi^- , P^+ \leftrightarrow P^-, \quad (3.3.9)$$

3.4 Germ projector, germ state, and germ process

A germ projector $P^{\text{germ}}$ is one of the two projectors $P^\pm$ eq. (3.3.3); it gives rise to a germ process $\mathcal{P}^{\text{germ}}$—a process in a proximity of the point $\tau = 0$; $\mathcal{P}^{\text{germ}}_{(0,0)}$ and $\mathcal{P}^{\text{germ}}_{(-\theta,0)}$ are defined by

(i) $\mathcal{P}^{\text{germ}}$ is deterministic;
(ii) $\lim_{\tau \rightarrow +0} \mathcal{P}^{\text{germ}}_{(0,0)}(\tau) = \omega^{\text{germ right}}_{m} \leftrightarrow \mathcal{P}^{\text{germ right}}$;
(iii) $\lim_{\tau \rightarrow -0} \mathcal{P}^{\text{germ left}}_{(-\theta,0)}(\tau) = \omega^{\text{germ left}}_{m} \leftrightarrow \mathcal{P}^{\text{germ left}}$.

For $\Psi \in \mathcal{H}_0^{(2)}$ we put

$$\Psi = e^{i\alpha} \cos \varphi \Psi_1 + \sin \varphi \Psi_2, \quad (3.4.1)$$

so that

$$P^{\text{creat}} \leftrightarrow (\alpha, \varphi). \quad (3.4.2)$$

The operator $v$ eq. (3.2.5) is a function of $(\alpha, \varphi)$, so that $\beta, \theta$ eqs. (3.3.5), (3.3.6), (3.3.7) are such functions as well,

$$\beta, \theta \rightarrow (\alpha, \varphi). \quad (3.4.3)$$

We assume that there exist the inverse functions,

$$\beta, \theta \rightarrow (\alpha, \varphi). \quad (3.4.4)$$
so that there exists a bijection
\[(\alpha, \varphi) \leftrightarrow (\beta, \theta).\] (3.4.5)
As
\[P^+ + P^- = P_0,\] (3.4.6)
so that
\[P^+ \leftrightarrow P^-\] (3.4.7)
and
\[P_{\text{germ}} \rightarrow \{P^+, P^-, \} \leftrightarrow (\beta, \theta),\] (3.4.8)
we have
\[P_{\text{germ}} \leftrightarrow P_{\text{germ}} \rightarrow (\beta, \theta) \leftrightarrow (\alpha, \varphi) \leftrightarrow P_{\text{creat}}.\] (3.4.9)
Thus
\[P_{\text{germ}} \rightarrow P_{\text{creat}}.\] (3.4.10)

3.5 Regular crossing

Let for \(\tau < 0\)
\[
\omega_m^{\text{creat left}} = \omega_m^{\text{germ left}} \tag{3.5.1}
\]
hold. Then it is natural to put for \(\tau > 0\)
\[
\omega_m^{\text{creat right}} = \omega_m^{\text{creat left}} = \omega_m^{\text{creat}} \tag{3.5.2}
\]
and, in view of eqs.(3.3.9), (3.3.10),
\[
\omega_m^{\text{germ right}} = \omega_m^{\text{germ left}}. \tag{3.5.3}
\]
Thus, there exists a germ process \(P_{\text{germ}}^{(-\theta, \theta)}\), such that \(P_{\text{germ left}}^{(-\theta, 0)} \) and \(P_{\text{germ right}}^{(-\theta, 0)}\) are its restrictions,
\[
\lim_{\tau \to -0} P_{\text{germ left}}^{(-\theta, \theta)}(\tau) = \lim_{\tau \to +0} P_{\text{germ right}}^{(-\theta, \theta)}(\tau) = P_{\text{germ}}^{(-\theta, 0)}(0) = \omega_m^{\text{creat}}, \tag{3.5.4}
\]
and there is no jump. The point \(\tau = 0\) and the process \(P_{\text{germ}}^{(-\theta, \theta)}\) are deterministic.

3.6 Singular crossing and quantum jump

Now let
\[
\omega_m^{\text{creat left}} \neq \omega_m^{\text{germ left}}. \tag{3.6.1}
\]
There is no possibility for a continuous process \(P_{\text{germ}}^{(-\theta, \theta)}\). Since \(\omega_{m0}\) is not determined by the process \(P_{\text{germ left}}^{(-\theta, 0)}\), we put
\[
\omega_{m0} = \lim_{\tau \to -0} \omega_{m\tau} = \omega_m^{\text{germ left}}. \tag{3.6.2}
\]
Furthermore, it is natural to put
\[
\omega_m^{\text{creat right}} = \omega_{m0}, \tag{3.6.3}
\]
so that
\[
\omega_m^{\text{creat right}} = \omega_m^{\text{germ left}} = \lim_{\tau \to -0} \omega_{m\tau} = \omega_{m-0}. \tag{3.6.4}
\]
We have, by eqs. (3.4.2), (3.4.3), (3.4.8), (3.6.4), a quantum jump

\[ P_{-0}^\text{left} \leftrightarrow \omega_{m-0} \xrightarrow{\text{jump}} \omega_{m+0}^l \leftrightarrow P_{+0}^\text{right}\ l = \pm, \]

(3.6.5)

with a transition probabilities related to it

\[ w\left(P_{(0,\theta)}^\text{germ right}\ l / P_{(-\theta,0)}^\text{germ left}\right) = \text{Tr}\{P_{+0}^\text{right}P_{-0}^\text{left}\}, \ l = \pm. \]

(3.6.6)

In the case of a regular crossing, eq. (3.6.6) gives \( w = 1 \) or \( 0 \); this case is an idealized limiting one.

Thus, a singular crossing gives rise to a quantum jump.

### 3.7 Orientability of dynamics and arrow of time

A point which corresponds to a singular crossing is indeterministic. We have for the cardinal numbers related to it

\[ \text{card}_{\text{future}}\ \alpha = \text{card}_{\text{right}}\ \alpha = 2 > 1 = \text{card}_{\text{left}}\ \alpha = \text{card}_{\text{past}}\ \alpha, \ \alpha = l = \pm. \]

(3.7.1)

Thus the dynamics of indeterministic quantum gravity is orientable; it determines the arrow of time given by

\[ t = \tau. \]

(3.7.2)

We find for the probabilities of subsection 1.8

\[ w(f/i) = \text{Tr}\{P_{+0}^\text{right}P_{-0}^\text{left}\}, \ i = 1, \ f = l = \pm, \]

(3.7.3)

\[ \sum_f w(f/i) = \text{Tr}\{P_0^\text{right}P_{-0}^\text{left}\} = \text{Tr}\{P_{-0}^\text{left}\} = 1, \]

(3.7.4)

\[ \sum_i w(f/i) = w(f/1) \leq 1, \]

(3.7.5)

\[ w(i/f) = 1, \]

(3.7.6)

so that

\[ w(f/i) \leq w(i/f) = 1. \]

(3.7.7)

### 3.8 Nonpredeterminability of the future and reconstructibility of the past

The dynamics developed is indeterministic, therefore the future is not predeterminate and may be forecasted only on a probabilistic level. On the other hand, in view of eqs. (3.4.10), (3.6.4), we have

\[ P_{(0,\theta)}^\text{germ future} \rightarrow \omega_m^\text{create right} = \omega_m^\text{germ left} \leftrightarrow P_{(-\theta,0)}^\text{germ past}, \]

(3.8.1)

so that

\[ P_{(0,\theta)}^\text{germ future} \rightarrow P_{(-\theta,0)}^\text{germ past}. \]

(3.8.2)

Thus, the past is reconstructible uniquely.
References

[1] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity* (gr-qc/9409010, 1994).

[2] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity II. Refinements and Developments* (gr-qc/9505034, 1995).

[3] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity III. Gravidynamics versus Geometrodynamics: Revision of the Einstein Equation* (gr-qc/9603022, 1996).

[4] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity IV. The Cosmic-length Universe and the Problem of the Missing Dark Matter* (gr-qc/9609035, 1996).

[5] R. Penrose, *Singularities and Asymmetry in Time, in: General Relativity, ed. S.W. Hawking and W. Israel* (Cambridge University Press, 1979).