Null geodesics and observational cosmology

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Abstract

The Universe is not isotropic or spatially homogeneous on local scales. The averaging of local inhomogeneities in general relativity can lead to significant dynamical effects on the evolution of the Universe, and even if the effects are at the 1% level they must be taken into account in a proper interpretation of cosmological observations. We discuss the effects that averaging (and inhomogeneities in general) can have on the dynamical evolution of the Universe and the interpretation of cosmological data. All deductions about cosmology are based on the paths of photons. We discuss some qualitative aspects of the motion of photons in an averaged geometry, particularly within the context of the luminosity distance-redshift relation in the simple case of spherical symmetry.
1 Introduction

The averaging problem in cosmology is perhaps the most important unsolved problem in mathematical cosmology. The averaging of local inhomogeneities in general relativity (GR) can lead to very significant dynamical effects on the evolution of the Universe. We discuss the effects that averaging (and inhomogeneities in general) can have on the interpretation of cosmological data. All deductions about cosmology are based on the paths of photons. In particular, we discuss some theoretical aspects of the motion of photons in an averaged geometry.

Cosmological observations [1, 2], based on the assumption of a spatially homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) model plus small perturbations, are usually interpreted as implying that there exists dark energy, the spatial geometry is flat, and that there is currently an accelerated expansion giving rise to the so-called ΛCDM-concordance model with $\Omega_m \sim 1/3$ and $\Omega_{de} \sim 2/3$. Although the concordance model is quite remarkable, it does not convincingly fit all data [2, 3, 4, 5]. Unfortunately, if the underlying cosmological model is not a perturbation of an exact flat FLRW solution, the conventional data analysis and their interpretation is not necessarily valid. There is the related question of whether the Universe is ‘exactly FLRW’ on some large scale (the scale of homogeneity is presumably $\sim 400 \, h^{-1}\text{Mpc}$ or greater, since there is strong evidence for coherent bulk flows on scales out to at least $300h^{-1}\text{Mpc}$ [5]), or only approximately FLRW at all scales (in which case there is also the ‘fitting problem’ of which background FLRW model to take [6]).

Inhomogeneities can affect observational calculations [7]. Supernovae data (in combination with other observational data), dynamically requires an accelerating universe. However, this only implies the existence of dark matter if the Universe is (approximately) FLRW. Supernovae data can be explained without dark matter in inhomogeneous models, where the full effects of GR come into play. The apparent acceleration of the Universe is thus not caused by repulsive gravity due to dark energy, but rather is a dynamical result of inhomoge-
geneities, either in an exact solution or via averaging effects. In an exact inhomogeneous cosmological model, the inhomogeneities affect the dynamics of the model, and a special observer can indeed measure the observed effects, although the results are, of course, model dependent.

In particular, averaging effects are real, but their relative importance must be determined. In the FLRW plus perturbations approach, the (backreaction) effects are assumed small (and are assumed to stay small during the evolution of the universe). A perturbation scheme is chosen, and assumptions are made so that the approximation scheme is well defined (i.e., assumptions are made so that there are no divergences and the backreaction stays small at the level of the perturbation, and the estimated corrections are small by definition [9]). This is perhaps a cyclical argument, but is has merit since it suggests that there is a self-consistent argument that backreactions are and remain small within the FLRW plus perturbations approach. Presumably, divergences would signal a breakdown in the approximation scheme when backreaction effects could be large.

There is also the possibility that averaging effects are not small (i.e., perturbation theory cannot be used to estimate the effects, and real inhomogeneous effects must be included). This is much more difficult to study, but the results from exact inhomogeneous solutions perhaps suggest that this ought to be investigated properly. There is also the question of what it means for an effect not to be small (and hence not negligible). It has been argued that an effect at the 1% level can be regarded as significant, and could very well affect the dynamics and interpretation of observational data [10].

Averaging: The averaging problem in cosmology is of considerable importance for the correct interpretation of cosmological data. The correct governing equations on cosmological scales are obtained by averaging the Einstein equations. By assuming spatial homogeneity and isotropy on the largest scales, the inhomogeneities affect the dynamics though correction (backreaction) terms, which can lead to behaviour qualitatively and quantitatively different
from the FLRW models.

The macroscopic gravity (MG) approach is an exact approach which gives a prescription for the correlation functions that emerge in an averaging of the Einstein field equations [11]. In [12] the MG equations were explicitly solved in a FLRW background geometry and it was found that the correlation tensor (backreaction) is of the form of a spatial curvature. This result was confirmed in subsequent work in which the spherically symmetric Einstein equations were explicitly averaged [13], and is consistent with the scalar averaging approach of Buchert [14] in the Newtonian limit and with the results of averaging an exact Lemaître-Tolman-Bondi (LTB) model and the results of linear perturbation theory [15]. There is no question that the backreaction effect is real. The only question remaining is the size of the effect. However, even a small non-zero curvature \( |\Omega_k| \sim 0.01 \) would lead to significant effects on observations (for redshifts \( z \geq 0.9 \)) [16]; indeed, values of \( |\Omega_k| \sim 0.05 \) or larger have been found to be consistent with observation [17].

Curvature estimates: The Wilkinson Microwave Anisotropy Probe (WMAP) has reported \( \Omega_{tot} = 1.02 \pm 0.02 \) [2]. In \( \Lambda \)CDM models type Ia supernovae (SNIa) data alone prefers a slightly closed Universe [1]. Taken at face value this suggests \( \Omega_k = 0.02 \). Models with non-negligible curvature have been studied recently [17]. Indeed, models with no dark energy have been found to be consistent with supernovae data and WMAP data [4]. Combining these observations with Large Scale Structure (LSS) observations such as the Baryon Acoustic Oscillations (BAO) data put stringent limits on the curvature parameter in the context of adiabatic \( \Lambda \)CDM models. However, these data analyses are very model- and prior-dependent [18], and care is needed in the proper interpretation of data.

Let us discuss the observational estimates in more detail. To first order, the averaged curvature \( \Omega_k \) evolves like a constant-curvature model [19, 24, 10]. In the course of expansion the second order kinematic backreaction \( \Omega_Q \) becomes dynamically more important and can become significant when the structure formation process injects backreaction [21]. For typical
scales $\ell \sim 50 - 200 \, h^{-1} \, \text{Mpc}$ and for a Hubble radius $\ell_H \sim 3000 \, h^{-1} \, \text{Mpc}$, values of second order effects $\sim (\ell/\ell_H)^2$ are typically in the range 0.01 to 0.1. Robust estimators for intrinsic curvature fluctuations using realistically modelled clusters and voids in a Swiss-cheese model indicates that the dark energy effects can be reduced by up to 30% [20, 22]. A rough order of magnitude estimate for the variance implied by the observed density distribution of voids implies $|\Omega_Q| \sim 0.1 - 0.2$ [21]. The backreaction parameter has also been estimated in the framework of Newtonian cosmology [14]; it was found that the backreaction term can be quantitatively small (e.g., $\Omega_Q = 0.01$), but the dynamical influence of a non–vanishing backreaction, $\Omega_I = \Omega_k + \Omega_Q$, on the other cosmological parameters can, in principle, be quite large.

From perturbation theory, it might be expected that the backreaction effect $\Omega_I \sim 10^{-5} - 10^{-4}$ [19, 9] (naively it is expected that the order of the perturbation, consistent with CMB data, is $\sim 10^{-5}$ [23]). From inhomogeneous models larger effects might be expected. There is a heuristic argument that $\Omega_I \sim 10^{-3} - 10^{-2}$ [24, 10, 9], which is consistent with CMB observations [10]. Let $\Omega_k \sim \epsilon$. We can estimate $Q \sim \langle \sigma^2 \rangle_D$, where $\sigma_D$ is the fluctuation amplitude; then $\Omega_Q \sim \langle \sigma^2 \rangle_D / H_D^2 \sim \langle \delta \rangle_D \sim \epsilon^2$ (where $\delta$ is the density contrast) [19]. Consequently, $|\Omega_k| \sim 10^{-2}$. Indeed, from perturbation theory the dominant local corrections to redshift or luminosity distance go as $\nabla \Phi$, which is only suppressed as $\ell/\ell_H$ (rather than as the Newtonian potential $\Phi$ as naively expected, which is suppressed as $(\ell/\ell_H)^2$) [24]; for scales $\sim 100 \, h^{-1} \, \text{Mpc}$, $\Omega_Q \sim 10^{-5}$ [21]. A possible scenario in which $\Omega_I \sim 0.2 - 0.4$, which can occur in exact inhomogeneous models (for example, many authors have studied observations in LTB models, albeit with contradictory conclusions [25]) and in which it is possible for the observed acceleration to be explained (by backreaction) using the standard interpretation without resorting to dark energy, is probably not supported by most authors [20, 21].

This consequently suggests a scenario in which $|\Omega_I| \sim 0.01 - 0.02$. It must be appreciated that this value for $\Omega_I$ is relatively large (e.g., it is comparable with the total contribution
of baryons to the normalized density) and may have a significant dynamical effect. Perhaps cosmological data can be explained without dark energy through a small backreaction and a reinterpretation of cosmological observations (alternatively, dark energy may still be needed for consistency with observations). However, a value of $|\Omega_I| \sim 0.01 - 0.02$ would certainly necessitate a complete reinvestigation of cosmological observations. In addition, such a value cannot be explained by inflation. From standard inflationary analysis, $|\Omega_{\text{tot}} - 1|$ would be effectively zero, so that any non-zero residual curvature might only be naturally explained in terms of a backreaction. Let us consider this in more detail.

Observations: Clearly, averaging can have a very significant dynamical effect on the evolution of the Universe; the correction terms change the interpretation of observations so that they need to be accounted for carefully to determine if a model may be consistent with cosmological data. Averaging may or may not explain the observed acceleration. However, its effects cannot be neglected. This leads us to a paradigm shift and clearly cosmological observations need to be revisited. Any observation that is based on physics in the nonlinear regime may well be influenced by the backreaction effect.

The standard analysis of SNIa and CMB data in FLRW models cannot be applied directly when backreaction effects are present, because of the different behaviour of the spatial curvature [18]. Indeed, in principle all data needs to be analysed within a particular inhomogeneous model, and not just an averaged version. Studies of the LTB model have demonstrated that the effect of inhomogeneity on the luminosity distance can mimic acceleration [25].

It is necessary to carefully identify observables actually measured by an experiment. For example, there are several different measures of the expansion rate and acceleration. In addition, there is the question of whether we are dealing with regional dynamics or global (averaged) dynamics in any particular observation. The determination of the Hubble parameter and SNIa observations (which need data at low redshift $z < 1$) are clearly based
on local measurements. However, most observations of the CMB (e.g., the integrated Sachs-Wolfe effect, which can be probed at large redshifts) and LSS are sensitive to large-scale (averaged) properties of the Universe.

Observations can clearly allow for a non-zero curvature [4, 17, 18]. In principle, in the new paradigm parameters can be implicit functions of the spatial scale; in particular, the curvature parameter can be different for different averaging domains. Thus, it is of interest to investigate the possibility of a scale dependent spatial curvature. Different observations might probe effective curvatures at different scales. In fact, SNIa data alone do not efficiently constrain curvature and the addition of other cosmological probes are necessary (at both small and large scales). It is important to test the idea of whether different quantities (valid at different scales) are measured in different experiments. In particular, the spatial curvature in SNIa experiments (small scale, $z << 2$) might be a different spatial curvature to that measured in CMB experiments (large scale, $z \sim 1100$). We could parameterize the different observations with different effective observables corresponding to different effective parameters (e.g., $\Omega_k$ for SNIa and $\Omega_{k'}$ for CMB). At first read, data from CMB and SNIa support $|\Omega_{k'} - \Omega_k| \sim 0.02$; i.e., the global (average) and local values of the curvature constant can differ by up to about 2%; that is, SNIa and CMB data allow the possibility of scale dependent spatial curvatures. It might be useful to do a comprehensive statistical analysis of cosmological data and include different scale dependent spatial curvatures as parameters (‘parameter splitting’) to be fitted to find their best fit values.
2 Null geodesics

All deductions about cosmology are based on light paths. The present assumption is that intervening inhomogeneities average out. However, inhomogeneities affect curved null geodesics [21, 20] and can drastically alter observed distances when they are sizable fraction of the curvature radius. In the real Universe, voids occupy a much larger region as compared to structures [26], hence light preferentially travels much more through underdense regions and the effects of inhomogeneities on luminosity distance are likely to be significant. Spherically symmetric models are a useful testing ground for studying the possible effects of inhomogeneities on cosmological observations. It has been shown that in spherically symmetric (LTB) spacetime model spatial variations of the expansion rate can significantly affect the cosmological observations (particularly close to the center) of a spherically symmetric model [25].

Let us discuss the effect of averaging on null geodesics in inhomogeneous models. We treat GR as a microscopic (classical) theory [27]. In this idealization real particles are modelled as point particles moving along timelike geodesics in the absence of external forces. The physical quantities which are observables must be identified. However, because all observations are of finite resolution, this involves a finite region of spacetime and therefore observations are extended (in the sense that they necessarily involve averages of measured quantities). In practice, only spacetime averages of field quantities have physical meaning; real observations in cosmology are invariably extended (in the sense of covering a neighbourhood of spacetime), averaging over spacetime of some characteristic length (not the same as cosmological scale though). Thus GR as a microscopic (classical) theory is incomplete and only an approximation.

The question arises of how to interpret observations in an inhomogeneous universe [30]. Only the redshift and the energy flux of light arriving from a distant source are observed, rather than expansion rate or matter density, and we need to know how to relate actual
observations to the various quantities in the equations (either averaged or non-averaged). We need to define a distance scale function and the corresponding Hubble rate; in inhomogeneous spacetime there will be different possible definitions and they will, in general, depend on both space and time.

Studying observations in inhomogeneous spacetimes is hard. Here we shall just discuss some qualitative aspects within the context of the observed luminosity distance-redshift relation. Investigating the CMB is much more difficult; in order to compute the spectrum of temperature anisotropies a full perturbation theory (e.g., in a spherically symmetric spacetime) is needed.

One problem of interpreting observations is that it is necessary, in principle, to model properties of (not only a single photon) but of a ‘narrow’ beam of photons. There is also the problem of which observations deal with macro-’geometric averages’ and which deal with statistical averages (of single observed micro-values); i.e., the question of spacetime averaging verses statistical properties. This cannot be done adequately within the Buchert [20] approach to the averaging problem, where only scalar quantities are averaged.

Real observations involve a beam or bundle of photons (a local congruence of null geodesics). From the geometric optics approximation for a test electromagnetic field we can obtain the optical scalar equations that govern the propagation of the shearing and expansion (of the cross-sectional area of the beam) with respect to the affine parameter along the congruence due to Ricci focussing and Weyl tidal focussing [29]. In FLRW models (macro-geometry), there is only Ricci focussing (the Weyl tensor is zero). However, in reality photons from distant sources have passed around massive objects along the line of sight and locally have only experienced Weyl focussing in the micro-geometry. Since the optical scalar equations (which are nonlinear) require integration along the beam, the optics for a lumpy distribution does not average and there may be important resulting effects. So it is also of importance to study the effect of averaging on a beam of photons in the optical limit.
In addition, due to the positive-definite nature of shear focussing, shearing can cause local refocussing of the beam leading to the formation of caustics (which are not present in the averaged FLRW macro-geometry), which can lead to further observational effects but which are not of primary interest here.

In addition to the question of what are the observations involved actually measuring and statistical averages verses geometrical averages, there is the question of the motion of photons in an averaged geometry. Are photons moving on null geodesics of the (averaged) macro-geometry? Are observations dealing with an (extended) beam of photons? If so, how does this relate to the motion of a beam of photons in micro/macro geometry. We shall illustrate these issues with a simple example of radial null geodesics in spherical symmetry within MG.

Null geodesics in MG: The microscopic field is to be averaged on an intermediate spatial scale, $\ell$ (which is, in principle, a free parameter of the theory, and assumed large compared to the scale on which astrophysical objects such as galaxies or clusters of galaxies have structure and is usually tacitly assumed to be a few hundred Mpc, or a fraction of the order of the inverse Hubble scale, $\ell_H$). We then obtain a macro- (averaged) geometry (using, for example, non-perturbative MG).

Let us consider the motion of photons in an averaged geometry, and investigate the resulting effect on cosmological observations. We assume GR is a microscopic theory on small scales, with local metric field $g$ (the geometry) and matter fields. A photon follows a null geodesic $k$ in the local geometry. After averaging we obtain a smoothed out macroscopic geometry (with macroscopic metric $\bar{g} = \langle g \rangle$) and macroscopic matter fields, valid on larger scales. But what trajectories do photons follow in the macro-geometry. Is the “averaged” vector $\langle k \rangle \equiv \bar{k}$ null, geodesic (or affinely parametrized) in the macro-geometry. How does this affect cosmological observations?
Indeed, in the micro-geometry

\[ g_{ab} k^a k^b = 0 \quad \text{(null)} \quad (2.1) \]

\[ \nabla k^a \equiv k^a_{;b} k^b = 0 \quad \text{(geodesic; affinely parametrized).} \quad (2.2) \]

After averaging, in general

\[ \langle g_{ab} k^a k^b \rangle \neq \langle g_{ab} \rangle \langle k^a \rangle \langle k^b \rangle = g_{ab} k^a k^b \]

\[ \nabla k^a = k^a_{|b} k^b \neq 0, \quad (2.3) \]

where covariant differentiation \((\nabla, |)\) is with respect to the macroscopic metric.

There is also the question of how to define the averaged matter, and whether (for example) the averaged matter moves on timelike geodesics (by the same reasoning the average of a timelike vector need not be timelike). This is not directly relevant here. In principle, it is possible to study the effects of averaging on the motion of the matter fields from the microphysics (e.g., from kinetic theory). However, this is part of the assumptions of the matter in the (macro) cosmological model and is, as in micro-GR, an “external” assumption on the matter model (and these assumptions can be probed separately). If the macro-matter is assumed to be pressure-free dust, then from the conservation equations the dust will move on timelike geodesics of the averaged macro spacetime.

As an illustration, let us consider these questions in a spherically symmetric spacetime given in volume preserving coordinates (VPC), which enables us to calculate the averaged quantities in a relatively straightforward manner. The (non-comoving) spherically symmetric metric in local VPC \((\sqrt{-g} = 1, \text{ where } g = \text{det}(g_{ab}) [13])\), is given by

\[ ds^2 = -B dt^2 + A dr^2 + \frac{1}{\sqrt{AB}} d\Sigma^2, \quad (2.5) \]

where the functions \(A\) and \(B\) depend on \(t\) and \(r\).
Let us consider an incoming radial null ray given by
\[ k = K \left( \frac{\partial}{\partial t} - \sqrt{\frac{B}{A}} \frac{\partial}{\partial r} \right). \] (2.6)

A direct calculation yields
\[ k^a \kappa^b = G^a = Gk^a, \] (2.7)
where
\[ G \equiv K \left\{ \frac{K_t}{K} + \sqrt{\frac{B}{A}} \frac{K_r}{K} + \left[ \frac{1}{2} \frac{B_t}{B} + \frac{1}{2} \frac{A_t}{A} + \sqrt{\frac{B}{A}} \right] \frac{B_r}{B} \right\}. \] (2.8)

Here \( k^a \) is automatically a null geodesic; it is affinely parametrized with affine parameter \( \lambda \), if \( G = 0 \).

The averaged metric is given by
\[ ds^2 = -B dt^2 + A dr^2 + d\Sigma^2, \] (2.9)
and we write
\[ \kappa = \kappa^a \frac{\partial}{\partial t} - \lambda^a \frac{\partial}{\partial r}. \] (2.10)

We see, in general, that from (2.4).
\[ \bar{g}_{ab} \kappa^a \kappa^b \neq 0; \] (2.11)
indeed, \( \kappa^a \) is null only when
\[ A \lambda^2 = \lambda \left( K \sqrt{\frac{B}{A}} \right)^2 = B \kappa^2. \] (2.12)
Calculating further, we obtain
\[ \nabla \kappa^a = \mathcal{H}_0^a \frac{\partial}{\partial t} + \mathcal{H}_1^a \frac{\partial}{\partial r}, \] (2.13)
where
\[ H^0 = (K_t K + K_r L) + \frac{1}{2B}(B_t K^2 + 2B_r K L + A_t L^2) \]
\[ H^1 = (L_t K + L_r L) + \frac{1}{2A}(2A_t L K + A_r L^2 + B_r K^2) \]

Clearly \( K^\alpha \) is not necessarily geodesic.

Assuming that \( K^\alpha \) is null, only if \( H^1 = -\sqrt{\frac{E}{A} H^0} \), is \( K^\alpha \) then geodesic. However, even then it is certainly not affinely parametrized (the equivalent “averaged” quantity \( G \) is not zero, and \( \lambda \) is not an affine parameter). [Alternatively, presumably we could define a new “averaged” geodesic null vector (so that equation (2.12), for example, is satisfied), but it is not necessarily affinely parametrized (and it is not clear whether such a quantity is related to observations).]

The form of the correlation tensor now depends on the assumed form for the inhomogeneous gravitational field and matter distribution. Let us assume [13]
\[ A(r, t) = \langle A(r, t) \rangle [1 + L_0 a(t) \delta_a(r, t) + O(L_0^2)] \]
\[ B(r, t) = \langle B(r, t) \rangle [1 + L_0 b(t) \delta_b(r, t) + O(L_0^2)] \]

where \( L_0 = \ell/\ell_H \sim 10^{-1} < 1 \) and \( \langle \delta_i(r, t) \rangle = 0 \). From [13], the correlation (correction) tensor in the case of a FLRW macro-geometry is of the form of a spatial curvature term with parameter \( k_0 \) (where \( k_0 \) is an integration constant depending on the actual averaging and, in particular, the averaging scale; i.e., it is not the spatial curvature parameter of the micro-geometry) [in more generality, the correlation tensor has an additional anisotropic contribution (correction), which is absent in the FLRW macro-geometry [13]]. In this case \( G^1 = -\sqrt{\frac{E}{A} G^0} + O(L_0) \), and the affine parameter is given by
\[ \lambda \approx \lambda(1 + k_0 \delta(t)). \]
Observational quantities, in terms of the luminosity distance $d_L$ and the redshift $z$ for example, can be determined from the underlying geometry [30]. For a flat FLRW background, the luminosity distance - redshift relation then becomes (for small redshift)

$$H_0 d_L \simeq z + \frac{1}{2}(1 - \bar{q}_0 + C)z^2 + \mathcal{O}(z^3),$$

where $C \simeq k_0 \epsilon(t)$. [Additional corrections, due to the fact that the trajectories are not exactly null geodesics and possible anisotropic averaging corrections (contained in $\bar{q}_0$) might be expected to be small ($\sim \mathcal{O}(L)$)–the exact values would depend on the averaging scheme, the assumed form of inhomogeneities, the averaging scale, etc.).]

That is, the averaging effectively renormalizes $\bar{q}_0$, and will include an additional contribution, $C$, from the “spatial curvature” with parameter $k_0$ that occurs from the averaging. This type of effect, $(1 - q_0) \rightarrow (1 - \bar{q}_0 + C)$, where $C$ is a dimensionless function of time characterizing the possible inhomogeneities (and perhaps depending on the density and pressure of the matter and age and expansion of the Universe), and its potential affect on cosmological observations has, in fact, been considered previously (and can be important, depending on which observations are considered) [31].

Discussion: It is clear there are a number of effects, all of order of about 1%, which will add up and perhaps produce a significant observational effect. There is a 1% effect due to the non-affine parameterization of the null geodesics in the averaged geometry, as illustrated above, together with additional corrections due to the fact that the trajectories are not exactly null geodesics, which are expected to be of a similar order of magnitude. There are corrections due to the fact that the correlation tensor is not precisely due to a spatial curvature (and might possibly contain anisotropic corrections). There are also corrections due to the fact that a beam of photons will experience expansion and shearing (and perhaps even twisting) during their evolution [29]; these effects are again expected to be at the level of about one percent [33]. There is also the effect of a special location of observers in an exact inhomogeneous cosmological model; although such an effect would influence all
observations, it might have a different quantitative effect on small scale observations (in the non-linear regime where local inhomogeneities have a more significant effect) than on large scale observations like the CMB (where linear effects might effectively cancel when integrating over scales larger than the homogeneity scale). Clearly, averaging and non-zero spatial curvature can have important effects, especially on SNIa measurements, and cannot be neglected [8].

For example, using cosmological observations to determine the equation of state of dark energy is unreliable, since this is exactly where the effects of averaging are expected to be important. Within dark energy models, with an effective equation of state parameter $w$, parameterized (for example) by $w(a) = w_0 + (1 - a)w_a[32]$, there has been an attempt to probe $w_a$. However, the effect of spatial curvature/averaging which have been neglected (and which are as much as a 1% effect [8, 10]), is as large as the effects being probed.
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