Nonlinear Radiative Heat Transfer of Magnetite($Fe_3O_4$)-Water Nanofluid over Unsteady Stretching Surface with Fluid Particle Suspension

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Abstract. The exploration of the magneto hydrodynamics(MHD) flow and nonlinear radiative heat transfer of magnetite $Fe_3O_4$ – $H_2O$ nanoparticle with fluid particle suspension over an unsteady stretching sheet is the central theme of the present work. The similarity transformations were employed to transfer the governing partial differential equations into ordinary ones prior to solve numerically using Runge – Kutta – Fehlberg-fourth and fifth order method with shooting technique. The variations of the temperature and velocity distribution and coefficients of heat transfer for magnetic parameter, temperature ratio parameter nanoparticle volume fraction, and thermal radiation parameter are discussed graphically. Comparative analysis of reported ones and present work results was presented and found to be in good agreement.

Keywords. MHD, nanofluid, nonlinear radiative heat transfer, unsteady stretching sheet

1. Introduction
Improvement in heat transfer in liquid cooling process is achieved by introducing nanoparticles in the fluid base. The idea of nanotechnology started in a talk by Richard “Feynman’s There’s Plenty of Room at the Bottom”. According to him nanoparticles possess unique physical and chemical properties can be used across all the science fields like physics, chemistry, biology, material science and Engineering. Nanofluids are the fluids in which particles of nanoscale were suspended in it. This technique improves fluids thermal conductivity, moreover nanoparticles are more stable in base fluids because of its large surface area which solves the many practical problems.
In recent years, heat transfer (HT) and nanofluid flow (NF) areas have become imperative hot-topics for many researchers. Sheikholeslam [1] presented the thermal behavior of the NF in cavity with tilted elliptic inner cylinder under the influence of Lorentz force. By comparing the two different models, Ahmad et. al. [2] addressed about effective thermal conductivity of the rotating NF induced by exponential stretching. Khan et. al. [3] studied the impact of inclined magnetic field on the NF over the sheet. In order to investigate the radiation affects, Poornima et. al. have come with some mathematical solutions for magneto hydrodynamics (MHD) free convective boundary layer NFs over non-linear stretchable sheets [4].

The problem of rotating flow of silver-copper oxide/water (Ag-CuO/H2O, a hybrid NF) was investigated by Hayat et. al. [5]. The problem of micropolar NF was solved in exponentially stretchable surface by Subhani et. al. [6]. Numerical solutions to the problem of Cu-Al2O3/H2O NF over a permeable stretchable layer under the hydro-magnetic effects were addressed by Devi et. al. [7]. Prasanna et. al. and Reddy et. al. [8,9] discussed the behaviour of non-linear thermal radiation on Williamson NF under different conditions. Ibrahim et. al. [10] examined the HT and hydromagnetic boundary layer flow of NF over a porous stretchable layer in presence of thermal velocity, and solutal slip conditions. Eegunjobi et. al. [11] have reported about unsteady MHD mixed convection slip flow with chemical reaction past a porous stretchable layer medium. At stagnation point, the effects of MHD flow and double-stratification of Eyring-Powell NF past a stretched cylinder were revealed by Ramzan et al. [12]. Prasad et al. [13] numerically analyzed the HT and MHD flow in a NF over an elastic sheet. Characteristics of Al2O3, TiO2 and Fe3O4 nanoparticles in water based materials are presented by Mushtaq et al [14], Wahed [15] and Khan et al [16].

A revolutionary investigation on 3-D circulating viscous flow persuaded due to stretchable surface has been reported [17] and the its extended work for unsteady case detailed by Rajeswari et. al [18] and Nazar et. al. [19]. The influence of thermophoresis and Brownian motion over a unsteady porous shrinking layer was prescribed by Kumar et al. [20]. Numerical investigation for an unsteady MHD laminar stream of a NF carried out over a perpendicular stretchable sheet with porous medium was [21]. Sheikholeslam et. al. [22] obtained numerical results of HT and NF in circulating system by considering the effects of magnetic field. Turkyilmazoglu [23] detaild about the water based NFs flow of five different types. Bahiraei et. al. [24,25] have discussed transfer of heat efficiency of Mn-ZnFe2O4-H2O ferrofluid in counterflow double-pipe heat-exchanger as well as in annulus by applying magnetic field. Recently, unsteady magneto-hydrodynamic flow of Eyring-Powell NF over an inclined permeable stretchable layer was carried out by Bharath Kumar and Suripeddi Srinivas[26]. The thermal radiation and magnetic field influence on circular flow of MWCNTs- H2O and SWCNTs- H2O NF between rotatable stretchable disks was reported by Jyothi et. al. [27]. Elgazery [28] have considered 4 different nanoparticles to study the unsteady 2-D NFlow a vertical stretchable porous sheet under the influence of nonuniform heat sink /source and magnetic field.

In this context, the aim of this work is to analyse the nonlinear radiative HT of magnetite (Fe3 O4)-water NF over unsteady stretchable surface with fluid particle suspension. The similarity transformations, shooting technique and together with rkf45 method were employed to solve the controlling flow equations numerically. The effect of different significant parameters on velocity, temperature plots has been detailed through plots.

2. Mathematical Model
Here, it is considered the incompressible dusty NFs unsteady, 2-D laminar boundary flow over stretchable sheet entrenched in permeable medium. The layer is stretched in $X$- direction and perpendicular to $Y$-axis, keeping origin point fixed. The fluid flow is restricted in the region $y > 0$. It is assumed that the velocity of stretchable sheet to be $U_w(x, t) = \frac{c}{1-a}x - at$ also we have considered the flow is steady at $t < 0$ and unsteady scenario starts at $t = 0$. Along with this we have assumed the following assumptions

- Nonlinear radiation and non-uniform heat sink / source has been considered.
- Volume fraction of dust and Nanoparticles and Number density of dust-particles were considered.
- The size of dust particles assumed to be uniform.
- Shape of the dust and nano particles are considered as spherical
- Water based dusty fluid with $Fe_3O_4$ nanoparticles is taken into account.
- The nanoparticles of uniform size and shape are also considered.
- $T_f$ be surface temperature of sheet and monitored by convective HT,
- Finally, both NPs & fluid-phase are in thermal equilibrium state.

The equations which govern the flow of NF phase and dust phase under typical boundary layer approximations have become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_n}{\rho_n} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho_n} u,$$  

$$\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} = 0,$$

$$\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = K \left( u - u_p \right),$$

$$\left( \rho c_p \right)_{nf} \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = \frac{k_n}{\rho_n} \frac{\partial^2 T}{\partial y^2} + \frac{N c_{pf}}{T_T} (T_p - T) + \frac{N}{\tau_v} \left( u_p - u \right)^2 \frac{\partial q_r}{\partial y},$$

$$\frac{\partial T_p}{\partial t} + u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} = \frac{c_{pf}}{c_{mf} T_T} (T - T_p).$$
where, \((u, v)\) and \((u_p, v_p)\); the \(x\) and \(y\)-components of velocity of NF, dust phases, respectively. \(\rho_{nf}\): effective density, \(\mu_{nf}\): effective dynamic viscosity, which are expressed as [32],

\[
\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s, \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}
\]

(2.7)

where \(\phi\): solid volume fraction, \(\rho_f\) and \(\rho_s\): density of base fluid and NPs, respectively. \(\mu_f\): dynamic viscosity of base-fluid. Stokes drag constant is given by \(K = 6\pi\mu_f r\), \(N\) is number-density of particles, \(k'\) is permeability of permeable medium, \(m\) is mass concentration of dust particles, \(r\) is radius of particles.

In equations (2.5 & 2.6) and \(T_p\): fluid temperatures of NF and dust inside boundary layer. \(\tau_T\): thermal equilibrium time. \(c_{pf}, c_{mf}\): the specific heat of dust particles, \(\tau_p\): relaxation time of dust particle, \(k_{nf}\): thermal conductivity and \((\rho c_p)_{nf}\): heat capacity of NF, which are given by [32],

\[
k_{nf} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)}, \quad (\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s
\]

(2.8)

where, \(k_s\) and \(k_f\): thermal conductivity of nanoparticle and base fluid, respectively, \((\rho c_p)_f\) and \((\rho c_p)_s\): heat capacity of base-fluid & NPs, respectively.

The consequent boundary conditions are expressed as,

\[
u = U_w v = 0, \quad T = T_w \text{ at } y = 0, \quad \nu = 0, u_p \to 0, v_p \to v, T \to T_\infty, T_p \to T_\infty \text{ as } y \to \infty,
\]

(2.9)

The nonlinear Rosseland diffusion approximation was used because unlike the linearized Rosseland approximation this method will give results for either small or large differences between \(T_f\) and \(T_\infty\). Radiative heat flux is simplified using Rosseland [33,34] estimation as,

\[
q_r = -\frac{4\sigma^* T^4}{3k^*} \frac{\partial T}{\partial y}
\]

(2.10)

Further, \(q_r\) for boundary layer flow over a horizontal flat plate was simplified by Pantokratoras and Fang [30] as,

\[
q_r = \left(-\frac{16\sigma^* T_\infty^3}{3k^*}\right) \frac{\partial T}{\partial y}
\]

(2.11)

In view to equation (2.11), the equation (2.5) becomes

\[
(\rho c_p)_{nf} \left[\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right] = \frac{\partial}{\partial y} \left(k_{nf} + \frac{16\sigma^* T_\infty^3}{3k^*}\right) \frac{\partial T}{\partial y} + \frac{N c_{pf}}{\tau_T} (T_p - T) + \frac{N}{\tau_p} (u_p - u)^2.
\]

(12.2)

Following similarity transformations have incorporated to convert derived equations to a set of similarity equations:

\[
\begin{align*}
    u &= \frac{cx}{1 - a} f'(\eta), \quad v = -\frac{cv_f}{(1 - at)} f(\eta), \quad \eta = \frac{c}{v_f (1 - at)} y, \\
    u_p &= \frac{cx}{1 - at} F'(\eta), \quad v_p = -\frac{cv_f}{1 - at} F(\eta), \\
    \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty}, \quad \theta_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty}
\end{align*}
\]

(2.13)

with \(T = T_f(1 + (E - 1)\theta)\)and \(E = \frac{T_w}{T_\infty}, E > 1\) the temperature ratio parameter. In view of equation (2.13), the equations (2.1 & 2.3) are automatically holds good and (2.2), (2.4), (2.6) and (2.12) will takes the form ,

\[
f''' + (1 - \phi)^{2.5} \left[(1 - \phi) + \phi \frac{\rho_s}{\rho_f}\right] \left(ff'' - f'^2 - \frac{a}{2}(\eta f''' + f'')\right)
\]

(2.14)
The consequent boundary conditions are now become,

\[ f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \theta'(\infty) = 0. \]

Where \( \alpha = \frac{a}{c}, \beta = \frac{1-a}{\tau_v c} \) and \( \beta_T = \frac{1-a}{\tau_v c} \), \( k_p = \frac{\nu f (1-a)}{k c} \), \( M = \frac{\sigma B^2_0}{\rho_f} \), \( N_r = \frac{16 \sigma B^3_0}{3 \kappa c f^2} \), respectively are the unsteady-parameter, fluid-particle-interaction parameter for velocity and temperature, permeability parameter, magnetic parameter, radiation parameter. \( \eta = \frac{m N}{\rho_f} \) is particles mass concentration, \( \tau_v = \frac{m}{k} \) is relaxation time, \( \gamma = \frac{c_{pf}}{c_{mf}} \) is specific heat ratio and \( Pr = \frac{(\mu c_p f)}{k_f} \) is Prandtl number, \( Ec = \frac{u_r^2}{(T_f - T_\infty)c_{mf}} \) is Eckert number.

The local Nusselt number \( (Nu_x) \) is expressed as,

\[ Nu_x = \frac{\chi q_w}{k_f (T_f - T_\infty)} \frac{\partial T}{\partial y} \bigg|_{y=0}. \]

### 3. Numerical Solution

Runge-Kutta-Fehlberg45 order approach usually abbreviated as RKF45 coupled with shooting technique is considered to tackle the system of nonlinear equations (2.14) to (2.17), subjected to the boundary conditions (2.18). Shooting method incorporated in Maple software used to identify appropriate initial conditions by an iterative process to obtain a precise and accurate solution to original boundary-value problem which is employed to crack a variety of non-linear problems successfully. The thermo-physical properties of H2O-Fe3O4 nanoparticles are tabulated in Table 1. While, Table 2 collects the all comparative analysis of the obtained results with reported ones [29,31] for derived Nusselt number. From the above analysis, it is concluded that obtained results have shown good agreement with the reported ones.

| Pure H2O | 997.1 | 4179 | 0.613 |
|----------|-------|------|-------|
| Magnetite(Fe3O4) | 5180 | 670 | 9.1 |

**Table 1. Thermo-physical properties of H2O - Fe3O4 nanoparticles.**

| Pr | Elbashbeshy et al. [29] | Prasannakumara et al. [31] | Present work |
|----|------------------------|---------------------------|--------------|
| 0.72 | 0.76728 | 0.7672701 | 0.767290 |
| 1 | 0.95478 | 0.9547788 | 0.957888 |
| 2 | 1.47146 | 1.4714572 | 1.471472 |
| 3 | 1.86907 | 1.8690619 | 1.869317 |
| 5 | 2.50013 | 2.5001321 | 2.500134 |
| 10 | 3.66037 | 3.6603723 | 3.660384 |

**Table 2. Comparative analysis of \(-\theta'(0)\) when \( \beta = \phi = N_r = E = \alpha = l = 0 \)**
4. Results and Discussion

The nature of velocity and temperature distribution for both phases on changing unsteady parameter ($\alpha$) is shown in Fig. 2 and 3. It is seen that both temperature and velocity plots increases with $\alpha$ Fig. 4 and 5 depict the effect of fluid-particle interaction parameter for velocity ($\beta$) on velocity and temperature distribution. It is observed that velocity plot of momentum boundary layer decreases and temperature plot of thermal boundary layer augments as $\beta$ increases.

Fluid particle interaction parameter for temperature $\beta_T$ on velocity plots is presented in Fig. 6. It is noticed that the velocity curve of the fluid decreases whereas velocity profile of the dust phase increases as increasing the values of temperature fluid particle interaction parameter.

The effect of mass concentration of particle parameter ($l$) on both phases of velocity profile is depicted in Fig 9 and effect of $l$ on both phases of temperature is plotted in Fig. 10. From these plots, it is examined that, the velocity and temperature profiles decreases as mass concentration of particles increases, which is true for NF as well as dust phases.

The Lorentz force produced due to magnetic field perpendicular to flow in conducting fluid acts against the flow, due to this the increasing values of magnetic parameter ($M$) decreases velocity profile in the boundary layer region. It is revealed in Fig. 11. But on the thermal boundary layers magnetic field influences positively which is shown in Fig. 12. Physically, it can be concluded that effect of magnetic field increases the thickness of thermal boundary layer and reduces the momentum boundary layer.

Radiation parameter (Nr) exhibits the property that temperature profiles for both phases increases as Nr increases, which is illustrated in Fig. 13. Increasing in solid volume fraction parameter ($\phi$), velocity and temperature profiles of the both fluid & particle phases increases, which is graphically illustrated in Fig. 14 and 15.

Fig. 16 demonstrates the effects of Prandtl number (Pr) on $\theta$ profiles. It is observed that temperature profiles of both phases decreases as increasing in the values of Pr. Lower the Prandtl number, thicker in the thermal boundary layer structures because lesser Prandtl number implies the fluids with greater thermal conductivity.

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**Figure 2.** Effect of $\alpha$ on $f'(\eta)$, $F(\eta)$

**Figure 3.** Effect of $\alpha$ on $\theta(\eta)$, $\theta_p(\eta)$
Figure 4. Effect of $\beta$ on $f'(\eta), F(\eta)$

Figure 5. Effect of $\beta$ on $\theta(\eta), \theta_p(\eta)$

Figure 6. Effect of $\beta_T$ on $f'(\eta), F(\eta)$

Figure 7. Effect of $E$ on $\theta(\eta), \theta_p(\eta)$

Figure 8. Effect of $Ec$ on $\theta(\eta), \theta_p(\eta)$

Figure 9. Effect of $l$ on $f'(\eta), F(\eta)$
Figure 10. Effect of $l$ on $\theta(\eta), \theta_p(\eta)$

Figure 11. Effect of $M$ on $f'(\eta), F(\eta)$

Figure 12. Effect of $M$ on $\theta(\eta), \theta_p(\eta)$

Figure 13. Effect of $Nr$ on $\theta(\eta), \theta_p(\eta)$

Figure 14. Effect of $\phi$ on $f'(\eta), F(\eta)$

Figure 15. Effect of $\phi$ on $\theta(\eta), \theta_p(\eta)$
Figure 16. Effect of $Pr$ on $\theta(\eta), \theta_p(\eta)$

5. Conclusion
The problem of an unsteady non-linear radiative & transfer of heat flow features of dusty NF over linear stretchable sheet is studied by considering volume fraction of dust and nanoparticles. Here, $Fe_3O_4$-water entrenched with dust particles was considered. A set of similarity transfer is presented to alter boundary layer equations into self-similar form and then numerically solved by RKF-45 approach coupled with shooting-technique. Impact of non-dimensional governing parameters on temperature and velocity plots for dust and fluid phases have been detailed through plots. It is finally summarized that:

- As volume fraction of the nanoparticles increases, an enhancement in friction factor and rate HT were observed.
- As the fluid particle interaction parameter rises, there was an increment in rate of HT and reduction in friction factor.
- Higher value of Prandtl number rapidly decreases the thermal boundary layer thickness and temperature.
- Effect of temperature ratio parameter, unsteadiness parameter and thermal radiation parameter is enhanced the temperature profile for both phases.
- An appreciable cooling performance was observed for dusty fluid conducting with $Fe_3O_4$ nanoparticles.
- Higher thermal conductivity has been noticed for dusty NFs, compared to NF and dusty fluid.

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