Empirically Consistent Electroweak Radiative Corrections with the Two-Higgs Doublet Model

Masataka Fukugita $^{a,b}$ and Takahiro Kubota $^{b,c}$

$^a$Institute for Cosmic Ray Research, University of Tokyo, Kashiwa 277-8582, Japan

$^b$Institute for the Physics and Mathematics of the Universe, University of Tokyo, Kashiwa, 277-8582, Japan

$^c$Graduate School of Science, Osaka University, Toyonaka, Osaka 560-0043, Japan

Abstract

The electroweak radiative correction, which turned out to be marginal within the standard electroweak model having the minimal Higgs sector in view of the present experimental information, fits well the experiment when the Higgs sector is extended to have two Higgs doublets. We predict the range where the charged and CP odd Higgs boson masses would lie, taking the two CP even neutral Higgs boson masses to be degenerate which makes the analysis in multiparameter space feasible. It is shown that the mass of neutral Higgs doublet boson can arbitrarily be large consistently with the $W$ mass, if the charged Higgs boson is present and it’s mass lies in some appropriate ranges.

1 e-mail: kubota@het.phys.sci.osaka-u.ac.jp
Whereas electroweak standard model is highly successful, its structure of the Higgs sector is poorly explored and remains unconstrained. At present a uniquely useful empirical probe for the Higgs sector is a consistency test of the electroweak radiative corrections. The $W$ boson mass receives a significant electroweak radiative correction, and current data indicate that mass of the neutral Higgs boson be rather small in the standard model with the minimal Higgs sector. The current data \[1\]

$$m_W = 80.403 \pm 0.029 \text{ GeV}$$ \hspace{1cm} (1)

require that neutral Higgs boson mass $m_\phi$ be smaller than 110 GeV, while the neutral Higgs search at LEP results in

$$m_\phi \geq 114.4 \text{ GeV}$$ \hspace{1cm} (2)

which is already a marginal conflict with the radiative correction requirement, if at 1 sigma. This is particularly so, if the mass of the top quark is as small as

$$m_t = 172.6 \pm 0.8\text{(stat)} \pm 1.1\text{(syst)} \text{ GeV},$$ \hspace{1cm} (3)

as reported recently from CDF and D0 experiments \[2\], which is smaller than $m_t = 174.2 \pm 3.3 \text{ GeV}$, published in \[1\].

The situation is demonstrated in Figure 1, where radiative corrections

$$m_W^2 = \frac{1}{2} \left\{ m_Z^2 + \sqrt{m_Z^4 - \frac{4\pi \alpha}{\sqrt{2} G_\mu} m_Z^2 (1 + \Delta r)} \right\},$$ \hspace{1cm} (4)

where

$$\Delta r = \frac{1}{m_W^2} (\text{Re}A_{WW}(m_W^2) - \text{Re}A_{WW}(0)) - \frac{2\delta e}{e}$$

$$+ \frac{e^2}{s^2} \left( \frac{1}{m_Z^2} \text{Re}A_{ZZ}(m_Z^2) - \frac{1}{m_W^2} \text{Re}A_{WW}(m_W^2) \right)$$

$$+ \frac{g^2}{16\pi^2} \left\{ -8 \left( \frac{1}{n-4} + C + \log \frac{m_Z}{\mu} \right) + \frac{\log e^2}{s^2} \left( \frac{7}{2} - 6s^2 \right) + 6 \right\},$$ \hspace{1cm} (5)

are evaluated with the input data

$$m_Z = 91.1876 \text{ GeV},$$ \hspace{1cm} (6)

$$\alpha^{-1} = 137.03599911,$$ \hspace{1cm} (7)

$$G_\mu = 1.16637 \times 10^{-5} \text{ GeV}^{-2},$$ \hspace{1cm} (8)
for $m_W$ as a function of the neutral Higgs boson mass $m_\phi$ with the top quark mass shown as a varying parameter. In eq. (5) $C = (\gamma E - \log 4\pi)/2$, and $s^2$ and $c^2$ stand for $\sin^2\theta_W$ and $\cos^2\theta_W$, respectively. $A_{ZZ}(q^2)$ and $A_{WW}(q^2)$ are the $g^{\mu\nu}$ component of the gauge boson self-energy; $\delta e$ is charge renormalisation. The Figure 1 indicates that the allowed region is unnervingly restricted if the top quark mass is as low as (3). This presses us to consider some extension of the minimal Higgs structure, especially in view of the Higgs search with the LHC experiment which is the most imminent target.

The most straightforward extension of the Higgs sector is to double the Higgs doublet that would make the successful Higgs effects on the gauge sector intact. With two Higgs doublets $\Phi_1$ and $\Phi_2$ having $Y = +1$, we consider the Higgs potential

$$V = \mu_1^2\Phi_1^\dagger\Phi_1 + \mu_2^2\Phi_2^\dagger\Phi_2 + \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{1}{2}(\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \lambda_5^*(\Phi_2^\dagger\Phi_1)^2).$$

We do not include those terms as $(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2)$ and $(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$, imposing the discrete symmetry under $\Phi_2 \rightarrow -\Phi_2$ for the quartic couplings, to avoid flavour-changing neutral currents [4]. The quadratic mixing terms $\Phi_1^\dagger\Phi_2$ and its hermitian conjugate that are analogous to the $\mu-$term in the minimally supersymmetric model are also omitted lest our analyses should become too clumsy.

The particle content of the two-Higgs doublet model (THDM) may be seen by putting

$$\Phi_1 = \begin{pmatrix} w_1^\dagger \\ v_1 + h_1 + iz_1 \end{pmatrix}$$

$$\Phi_2 = \begin{pmatrix} w_2^\dagger \\ v_2 + h_2 + iz_2 \end{pmatrix}$$

into the Higgs potential, where the vacuum expectation values $v_1$ and $v_2$ satisfy $v_1^2 + v_2^2 \equiv v^2 = (246 \text{ GeV})^2$. The CP-even neutral Higgs bosons, $h$ and $H$, are obtained from $h_1$ and $h_2$ by diagonalizing the mass terms with an angle $\alpha$

$$h_1 \cos \alpha - h_2 \sin \alpha \begin{pmatrix} h \\ H \end{pmatrix}.$$  

Likewise, the charged Higgs boson $H^\pm$, the CP-odd neutral Higgs boson $A^0$ and the Nambu-Goldstone bosons $w$ and $z$ are given by the rotation with the angle $\beta$, $\tan \beta = v_2/v_1$

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} z \\ A^0 \end{pmatrix},$$

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} w \\ H^\pm \end{pmatrix}. $$

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The seven parameters in the original Higgs potential \( \phi \), \( \mu_1, \mu_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \) and \( \lambda_5 \) are now replaced by the vacuum expectation value \( v \), the mixing angles \( \alpha \) and \( \beta \) and the four Higgs masses, \( m_h, m_H, m_{H^\pm}, \) and \( m_{A^0} \).

The radiative corrections in THDM has already been studied in the literature \[5\] - \[6\]. (Bertolini \[5\] has in fact computed the THDM contributions to \( A_{WW}(q^2) \) and \( A_{ZZ}(q^2) \)). The numerical analyses of \( \Delta r \) using the formulae such as those given in \[5\], however, would be awkward if we scrutinize every corner of parameter space of \( m_h, m_H, m_{H^\pm}, m_{A^0}, \alpha, \) and \( \beta, \) and it would be hard to grasp the structure of the model. To understand the structure of the Higgs sector, we want to reduce the parameter space. In fact, a considerable simplification takes place, if we set

\[
m_h = m_H
\]

with which the mixing angles, \( \alpha \) and \( \beta \) disappear in \( A_{WW}(q^2) \) and \( A_{ZZ}(q^2) \). This reduction of parameter space makes the numerical analyses significantly more transparent.

Upon requiring \((14)\), many of the terms in \( A_{WW}(q^2) \) and \( A_{ZZ}(q^2) \) coincide with those that appear in the standard model calculation. Let us denote the standard model Higgs contributions to \( A_{WW}(q^2) \) by \( \delta A_{WW}(q^2) \) and the full THDM contributions by \( \Delta A_{WW}(q^2) \), and similarly \( \delta A_{ZZ}(q^2) \) and \( \Delta A_{ZZ}(q^2) \) for \( A_{ZZ}(q^2) \), respectively. Writing \( m_\phi = m_H \) in the standard model, the relation between THDM and standard model calculations is

\[
\Delta A_{WW}(q^2) \bigg|_{m_h=m_H} = \delta A_{WW}(q^2) \bigg|_{m_\phi=m_H} + \frac{g^2}{16\pi^2} \left( \frac{2}{n-4} + 2C - 1 \right) \times \frac{1}{6} q^2
\]

\[
+ \frac{g^2}{16\pi^2} \left\{ -\frac{1}{2} K_1(m_{H^\pm}^2, m_{A^0}^2, q^2) - \frac{1}{2} K_1(m_{H^\pm}^2, m_H^2, q^2) + \frac{1}{4} m_H^2 \log \frac{m_H^2}{\mu^2} \right. \\
+ \frac{1}{2} m_{H^\pm}^2 \log \frac{m_{H^\pm}^2}{\mu^2} + \frac{1}{4} m_{A^0}^2 \log \frac{m_{A^0}^2}{\mu^2} \bigg\},
\]

\[
\Delta A_{ZZ}(q^2) \bigg|_{m_h=m_H} = \delta A_{ZZ}(q^2) \bigg|_{m_\phi=m_H} + \frac{g^2}{16\pi^2} \left( \frac{2}{n-4} + 2C - 1 \right) \times \frac{1}{12\epsilon^2} (c^2 - s^2)^2 q^2
\]

\[
+ \frac{g^2}{16\pi^2 c^2} \left\{ -\frac{1}{2} K_1(m_{A^0}^2, m_H^2, q^2) + \frac{1}{4} m_H^2 \log \frac{m_H^2}{\mu^2} + \frac{1}{4} m_{A^0}^2 \log \frac{m_{A^0}^2}{\mu^2} \right. \\
+ \frac{g^2}{16\pi^2} \left\{ -\frac{1}{2} K_1(m_{H^\pm}^2, m_{H^\pm}^2, q^2) + \frac{1}{2} m_{H^\pm}^2 \log \frac{m_{H^\pm}^2}{\mu^2} \bigg\},
\]
where $K_1$ is the integral,

$$K_1(m_1^2, m_2^2, q^2) = \int_0^1 dx \left( \frac{m_1^2 x + m_2^2 (1 - x) - q^2 x (1 - x)}{\mu^2} \right) \log \left( \frac{m_1^2 x + m_2^2 (1 - x) - q^2 x (1 - x)}{\mu^2} \right).$$

For electric charge renormalisation $\delta e$ we add the contribution from the charged Higgs boson to the two-point function of the electromagnetic currents, $\Pi^{(\text{charged})}_{\gamma\gamma}(0)$, to the leptonic and hadronic contributions in the minimal model:

$$- q^2 \Pi^{(\text{charged})}_{\gamma\gamma}(q^2) = \frac{g^2}{16\pi^2} g^{\mu\nu} s^2 \left( \frac{2}{n - 4} + 2C - 1 \right) \times \frac{1}{3} q^2$$

$$+ \frac{g^2}{16\pi^2} s^2 \left\{ - 2 K_1(m_{H^\pm}^2, m_{H^\pm}^2, q^2) + 2 m_H^2 \log \frac{m_{H^\pm}}{\mu^2} \right\}. \quad (18)$$

We write $\Delta r$ in two parts:

$$\Delta r = \Delta r^{(1)} + \Delta r^{(2)} \quad (19)$$

where $\Delta r^{(1)}$ is the term that appears in the standard model and $\Delta r^{(2)}$ is the contribution from the charged and CP-odd Higgs bosons:

$$\Delta r^{(2)} = \frac{g^2}{16\pi^2} \frac{1}{m_W} \left\{ \left( \frac{c^2 - s^2}{2s^2} - \frac{1}{2} \right) \left[ K_1(m_{H^\pm}^2, m_{A^0}^2, m_W^2) + K_1(m_{H^\pm}^2, m_{H^0}^2, m_W^2) \right] ight.$$

$$- \frac{c^2}{2s^2} K_1(m_{A^0}^2, m_{H^\pm}^2, m_{H^0}^2) + \frac{1}{2} K_1(m_{H^\pm}^2, m_{A^0}^2, 0) + \frac{1}{2} K_1(m_{H^\pm}^2, m_{H^0}^2, 0)$$

$$- \frac{c^2}{2s^2} m_{H^\pm} \log \frac{m_{H^\pm}}{\mu^2} \right\}$$

$$+ \frac{g^2}{16\pi^2} \frac{(c^2 - s^2)^2}{s^2} \frac{1}{m_Z^2} \left\{ - \frac{1}{2} K_1(m_{H^\pm}^2, m_{H^\pm}^2, m_Z^2) + \frac{1}{2} m_{H^\pm} \log \frac{m_{H^\pm}}{\mu^2} \right\}$$

$$+ \frac{g^2}{16\pi^2} \left( 1 + \log \frac{m_{H^\pm}}{\mu^2} \right) \times \left( \frac{1}{3} \frac{1}{s^2} \right). \quad (20)$$

In our numerical exploration in what follows we consider only the case where the CP-even Higgs bosons, $h$ and $H$, are degenerate. There are then no mixing parameters $\alpha$ and $\beta$ in appearing, leaving us with only three parameters, $m_H$, $m_{H^\pm}$, and $m_{A^0}$. Note that the Higgs-gauge coupling depends only on the difference $\alpha - \beta$ from the structure of the mixing (11) - (13) and the dependence on $\alpha$ and $\beta$ disappears simultaneously by setting (14).
Figure 2 shows the correction $\Delta r$ for the minimal model and an example of the THDM as a function of the neutral Higgs boson mass. The empirically viable region is indicated by shading. For the THDM radiative correction we take, as an example, $m_{H^\pm} = 200$ GeV and $m_{A^0} = 150$ GeV. The figure shows that the contributions of the charged and/or CP odd neutral Higgs bosons cancel partly the correction from the CP even neutral Higgs bosons, making the radiative correction consistent with the empirical $W$ boson mass even with a large CP even neutral Higgs mass that would be unfavoured in the minimal Higgs model.

Figure 3 shows the region favoured for the charged and CP odd neutral Higgs boson masses for a given CP even neutral Higgs mass, which is taken to be 120, 150, 180 and 210 GeV. We see that there are two separate regions for any $m_H$ allowed for $(m_{H^\pm}, m_{A^0})$, one along the oblique line $m_{H^\pm} \approx m_{A^0}$ for large mass and the other in a lower plane with $m_{H^\pm}$ increasing only little with $m_{A^0}$. The four panels (a)–(d) show that the allowed regions are generally shifted upwards as $m_H$ increases, but the increments depend on specific branches. The smallest mass of the upper branch with respect to $m_H$ is nearly constant, $m_{H^\pm}/m_H \approx 1.1 - 1.2$. For the large mass region of the same branch the upward shift with $m_H$ is modest, and the asymptotics seem converging to $m_{H^\pm} = m_{A^0}$. The asymptotics of the lower branch also increase with $m_H$, but the solution includes the charged Higgs mass staying at 60 – 180 GeV, irrespective of the neutral Higgs boson masses. There is always a solution with $m_{H^\pm} < 100$ GeV, which would yield the required radiative correction. We emphasise that the Higgs mass ratios $m_{H^\pm}/m_H$ and $m_{A^0}/m_H$ would give a useful indicator of the THDM for the Higgs sector through the electroweak radiative correction.

The regions favoured by radiative corrections can be seen in a simple argument. Noting that for large $M$, $K_1$ behaves as

$$\lim_{M \to \infty} K_1(M^2, M^2, q^2) \sim \frac{1}{2} M^2 \log\left(\frac{M^2}{\mu^2}\right) - \frac{1}{4} M^2 + O(q^2),$$

(21)

$$\lim_{M \to \infty} K_1(M^2, m^2, q^2) \sim M^2 \log\left(\frac{M^2}{\mu^2}\right) + O(q^2, m^2),$$

(22)

we find for the limiting case $m_{H^\pm} = m_{A^0} = M \to \infty$ along $m_{H^\pm} \approx m_{A^0}$,

$$\Delta A_{WW}(q^2)_{m_h=m_H} \sim \frac{g^2}{16\pi^2} \times \frac{1}{8} M^2, \quad \text{as} \quad m_{H^\pm} = m_{A^0} = M \to \infty,$$

(23)

$$\Delta A_{ZZ}(q^2)_{m_h=m_H} \sim \frac{g^2}{16\pi^2} \times \frac{1}{8e^2} M^2, \quad \text{as} \quad m_{H^\pm} = m_{A^0} = M \to \infty,$$

(24)

which lead to a cancellation in $\text{Re} A_{WW}(m_W^2) - \text{Re} A_{WW}(0)$ and also in $m_Z^2 \text{Re} A_{ZZ}(m_Z^2) -$
\( m_{h^2}^2 \text{Re} A_{WW}(m_{W^2}) \) in the expression of \( \Delta r \). This means that allowed regions exist even for a very large \( m_H \) in the vicinity of the line \( m_{H^\pm} = m_{A^0} \).

We also see a similar cancellation in the limit \( m_{A^0} \to \infty \) with \( m_H \) and \( m_{H^\pm} \) fixed. We obtain

\[
\Delta A_{WW}(q^2) \bigg|_{m_h = m_H} \sim \frac{g^2}{16\pi^2} \times \left( -\frac{1}{4} \right) M^2 \log \left( \frac{M^2}{\mu^2} \right), \quad (\text{as } m_{A^0} = M \to \infty) \tag{25}
\]

\[
\Delta A_{ZZ}(q^2) \bigg|_{m_h = m_H} \sim \frac{g^2}{16\pi^2} \times \left( -\frac{1}{4c^2} \right) M^2 \log \left( \frac{M^2}{\mu^2} \right), \quad (\text{as } m_{A^0} = M \to \infty) \tag{26}
\]

This causes a similar cancellation in \( \Delta r \), making the second region where \( m_{H^\pm} \approx \text{constant} \) allowed in the figure.

We also study the case for both \( m_H \) and \( m_{H^\pm} \) being large. The limiting case can also be seen with a limiting case similar to the above, \( m_{H^\pm} = m_H = M \to \infty \). We then arrive at expressions for \( \Delta A_{WW}(q^2) \) and \( \Delta A_{ZZ}(q^2) \) the same as eqs. (25) and (26) but merely by a factor of 2 larger. This shows a cancellation of the radiative corrections, which means the presence of a preferred region in the vicinity of the line in agreement with the conclusion we derived for Figure 3. In a similar way we see a preferred region along \( m_{H^\pm} = \text{const} \). We show the preferred region of \( m_{H^\pm} \) that is correlated with \( m_H \) in Figure 3 for a specified \( m_{A^0} \).

In the present paper we show that the electroweak radiative correction, which is marginally consistent with the present experiment in the minimal Higgs model in that the Higgs mass preferred by the radiative correction appears too low compared with the lower limit from the Higgs search, is well relaxed if the Higgs sector is endowed with two Higgs doublets. We explored the mass region where the charged and CP odd neutral Higgs bosons may reside, albeit in a restricted model where two CP even Higgs bosons are degenerate in mass, which made the systematic exploration feasible. The mass ratios among Higgs bosons to be looked for at LHC would serve as useful indicators to explore the Higgs sector via electroweak radiative corrections.
Acknowledgement

One of us (T.K.) thanks Luis Alvarez-Gaume for his hospitality at CERN Theory Division. MF is supported by Grant in Aid of the Ministry of Education.

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Figure 1: $W$-boson mass after the electroweak radiative correction as a function of the standard model Higgs boson mass $m_{\phi}$. The shade shows an empirically allowed region, $80.432 \text{ GeV} > m_W > 80.374 \text{ GeV}$ and $m_\phi > 114.4 \text{ GeV}$. The solid lines correspond to $m_t = 178(a), 176(b), 174(c)$, and $172(d) \text{ GeV}$, respectively.

Figure 2: Radiative correction to the $W$ boson mass, $\Delta r$, in the on-shell renormalisation scheme for the standard model (SM) with minimal Higgs sector (thick line) and for an example of the THDM (thin line), where $m_{A^0} = 150 \text{ GeV}$ and $m_{H^\pm} = 200 \text{ GeV}$ are adopted. The top quark mass is assumed to be $172 \text{ GeV}$.
Figure 3: The region expected from the radiative correction for the \((m_{A^0}, m_{H^\pm})\) plane when (a) \(m_H = 120\) GeV, (b) 150 GeV, (c) 180 GeV and (d) 210 GeV. We fix top quark mass at \(m_t = 172\) GeV. The curves correspond to (A) \(m_W = 80.46\), (B) 80.43, (C) 80.40, (D), 80.37, and (E) 80.34 GeV. The region empirically allowed by the \(W\) mass is shown by shading.
Figure 4: Region favoured from the radiative correction for $m_H^\pm$ as a function of $m_H$ in the THDM. The CP odd neutral Higgs boson is taken to have $m_{A_0} = 150$ GeV. The curves correspond to (A) $m_W = 80.46$, (B) 80.43, (C) 80.40, (D) 80.37, (E) 80.34 GeV, and the region with hatched is allowed. We fix top quark mass at $m_t = 172$ GeV.