The light scalars and the broad $\sigma(500)$ in the $U3 \times U3$ linear sigma model†

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Abstract

The lightest scalar and pseudoscalar nonets are discussed within the framework of the broken old $U3 \times U3$ linear sigma model, and it is shown that already at the tree level this model works remarkably well predicting scalar masses and couplings not far from present experimental values, when all parameters are fixed from the pseudoscalar masses and decay constants. The linear $\sigma$ model is the simplest way to implement chiral symmetry together with the broken $SU3$ of the quark model, and this, not well known, success in understanding experiment is comparable to that of the naive quark model for the heavier multiplets. It is argued that this strongly suggest that the light and very broad $\sigma$ resonance exists near 500 MeV.

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The lightest scalars, the $a_0(980)$, $f_0(980)$, $K_0^*(1430)$ and the $\sigma(400-1200)$, which we shall here call $\sigma(500)$, have remained controversial for long, since the naive quark model, without chiral symmetry and finite widths from unitarity, fails badly in trying to accomodate them. Today many authors want to give the $a_0(980)$, $f_0(980)$ and the $\sigma(500)$ other interpretations than being $q\bar{q}$ states. Popular alternative interpretations are $KK$ bound states, 4 quark states, or for the $\sigma$, a glueball.

But, in fact, there is an old chiral quark model, the linear $U3 \times U3$ sigma model in which one can treat both the scalar and pseudoscalar nonets simultaneously with chiral symmetry. Unfortunately this over 30 years old model[1] has had very few recent phenomenological applications, and therefore its success to qualitatively describe data has been forgotten.

The Lagrangian is (for more details see [2]):

$$\mathcal{L} = \frac{1}{2} \text{Tr}[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger] - \frac{1}{2} \mu^2 \text{Tr}[\Sigma \Sigma^\dagger] - \lambda \text{Tr}[\Sigma \Sigma^\dagger \Sigma \Sigma^\dagger] - \lambda' \text{Tr}[\Sigma \Sigma^\dagger]^2 + \epsilon_\sigma \sigma_{u\bar{u}+d\bar{d}} + \epsilon_{s\bar{s}} \sigma_{s\bar{s}} + \beta |\text{det} \Sigma + \text{det} \Sigma^\dagger|.$$ (1)

Here the $\epsilon$ and $\beta$ terms give the pseudoscalars mass and break the flavour and $U_A(1)$ symmetries. The stability condition, that the linear terms in the fields must vanish after the shift of the scalar fields ($\Sigma \rightarrow \Sigma + V$) determines the small parameters $\epsilon_i$ in terms of the pion and kaon masses and decay constants. One finds $\epsilon_\sigma = m_\pi^2 f_\pi$, $\epsilon_{s\bar{s}} = (2m_K^2 f_K - m_\pi^2 f_\pi)/\sqrt{2}$, while $\beta$ in the $U_A(1)$ breaking term is determined by $m_\eta'$, or by $m_\eta^2 + m_\eta'^2$.

My previous work on the scalars with the unitarized quark model (UQM)[3] is essentially a unitarization of eq. (1) with $\lambda \approx 16$ and $\lambda' = 0$, and with the main symmetry breaking generated by putting the pseudoscalar masses at their physical values.

It is an ideal problem for a symbolic program like Maple V to calculate the predicted masses, and couplings from the Lagrangian, which has 6 parameters, $\mu, \lambda, \lambda', \beta, u = d$ and $s$, of which the last two define the diagonal matrix $V$ with the flavourless meson VEV’s: $V = \text{diag}[u, d, s]$. These are at the tree level related to the pion and kaon decay constants through $u = d = \langle \sigma_{u\bar{u},d\bar{d}} \rangle / \sqrt{2} = f_\pi / \sqrt{2}$ (assuming isospin exact) and $s = \langle \sigma_{s\bar{s}} \rangle \approx (2f_K - f_\pi) / \sqrt{2}$. One finds denoting the often occurring combination $\mu^2 + 4\lambda'(u^2 + d^2 + s^2)$ by $\mu^2$:

$$m_{\pi^+}^2 = \mu^2 + 4\lambda(u^2 + d^2 - ud) + 2\beta s,$$ (2)

$$m_{K^+}^2 = \mu^2 + 4\lambda(u^2 + s^2 - su) + 2\beta d,$$ (3)

$$m_{\eta^0}^2 = \mu^2 + 4\lambda(u^2 + d^2 + ud) - 2\beta s,$$ (4)

$$m_{K^0}^2 = \mu^2 + 4\lambda(u^2 + s^2 + su) - 2\beta d.$$ (5)

For the masses and mixings of isoscalar states see
Table 1. Predicted masses in MeV and mixing angles for two values of the \( \lambda' \) parameter. The asterix means that \( m_\sigma, m_K, \) and \( m_\eta, m_\eta' \) are fixed by experiment together with \( f_\pi=92.42 \) MeV and \( f_K=113 \) MeV.

| Quantity      | Model \( \lambda' = 1 \) | Experiment |
|---------------|---------------------------|------------|
| \( m_\pi \)  | 137+3                      | 137        |
| \( m_K \)    | 495*                      | 495        |
| \( m_\eta, m_\eta' \) | 547.3  | 957.8      |
| \( \Theta^{\eta'-\text{singlet}} \) | -5.0^\circ | (16.0±6.5)^\circ |
| \( m_{a_0} \) | 1028                      | 983        |
| \( m_{s_0} \) | 1123                      | 1430       |
| \( m_{a_0} \) | 651                       | 400-1200   |
| \( m_{f_0} \) | 1229                      | 980        |
| \( \Theta^{\sigma-\text{singlet}} \) | 21.9^\circ | (28±8.5)^\circ |

Ref[2].

We can fix 5 of the 6 parameters, leaving \( \lambda' \) free, by the 5 experimental quantities from the pseudoscalar sector alone: \( m_\pi, m_K, m_\eta, m_\eta', f_\pi=92.42 \) MeV and \( f_K=113 \) MeV, which are accurately known from experiment. One finds that at the tree level \( \lambda=11.57, \theta^2=0.1424 \) GeV^2, \( \beta=-1701 \) MeV, \( u=d=65.35 \) MeV, \( s=94.45 \) MeV. The remaining \( \lambda' \) parameter changes only the \( \sigma \) and \( f_0 \) masses and their trilinear couplings, not those of the pseudoscalars. It turns out that \( \lambda' \) must be small, compared to \( \lambda \), in order to fit the tri-linear couplings. By putting \( \lambda'=1 \) one gets a reasonable compromise for most of these couplings. With \( \lambda'=3.75 \) one almost cancels the OZI rule breaking coming from the determinant term, and the scalar mixing becomes near ideal (for \( \lambda'=-\beta/(4s)=4.5 \) the cancellation is exact).

As can be seen from Table 1 the predictions are not far from the experimental masses taken as \( a_0(980), f_0(980), K^0(1430), \) and \( \sigma(500) \). In particular note that one predicts a low mass for the controversial \( uu+dd \) scalar meson of 650 MeV, and which as we shall see should have a very large width (Tables 2-3 below). This is essentially a zero parameter prediction once the main parameters are fixed from the data on pseudoscalars. Considering that one expects that unitarity corrections can be up to 30\%, and should go in the right direction compared to experiment, one must conclude that these results for the other scalar masses \( (a_0, f_0(980) \) and \( K^0_\pi \) are good enough to take the model seriously.

The trilinear coupling constants follow from the Lagrangian after one has made the shift \( \Sigma \rightarrow \Sigma+V \). The predicted spp couplings at the tree-level can be expressed in terms of the predicted physical masses and mixing angles and decay constants. E.g.:

\[
g_{a_0K^0\pi^+} = (m_{a_0}^2 - m_K^2)/(\sqrt{f_K}) \ , \quad (6)
\]

\[
g_{a_0\pi^+\pi^-} = \cos \phi^{\pi^+\pi^-} f_0 (m_\pi^2 - m_{a_0}^2)/f_\pi \ , \quad (7)
\]

\[
g_{f_0\pi^+\pi^-} = \sin \phi^{\pi^+\pi^-} f_0 (m_\pi^2 - m_{a_0}^2)/f_\pi \ , \quad (8)
\]

\[
g_{a_0\pi\eta} = \cos \phi^{\pi\eta} (m_{a_0}^2 - m_\eta^2)/f_\pi \ , \quad (9)
\]

\[
g_{a_0\pi\eta'} = \sin \phi^{\pi\eta'} (m_{a_0}^2 - m_{\eta'}^2)/f_\pi \ , \quad (10)
\]

\[
g_{a_0K^+K^-} = (m_{a_0}^2 - m_K^2)/(2f_K) \ . \quad (11)
\]

For more predictions see Ref[2]. In Table 2 several different spp couplings are compared with quoted experimental numbers.

As can be seen from Tables 2-3 most of the couplings are not far from experiment. Only the \( f_0 \rightarrow \pi\pi \) and \( a_0 \rightarrow \pi\eta \) couplings and widths come out a bit large, but these are very sensitive to higher order loop corrections due to the \( KK \) threshold, and \( f_0 \rightarrow \pi\pi \) is extremely sensitive to...
the scalar near-ideal mixing angle and $\lambda'$. If one chooses $\lambda' = 3.75$ this mixing angle nearly vanishes ($\phi^{a_0 - f_0} = -3.0^\circ$) together with the $f_0 \to \pi\pi$ coupling. From our experience with the UQM[3] the $a_0 \to K\bar{K}$ peak width, when unitarized, is reduced, because of the $K\bar{K}$ threshold, by up to a factor 5. Therefore one cannot expect that the tree level couplings should agree better with data than what those of Tables 2-3 do. After all, this is a very strong coupling model ($\lambda = 11.57$, leading to large $g_f^2/4\pi$) and higher order effects should be important.

In summary, I find that the linear sigma model with three flavours, at the tree level, works much better than what is generally believed. When the 6 model parameters are fixed mainly by the pseudoscalar masses and decay constants, one predicts the 4 scalar masses and mixing angle to be reasonably near those of the experimentally observed nonet $a_0(980), f_0(980), \sigma(500), K_0^*(1430)$. Also 8 couplings/widths of the scalars to two pseudoscalars are predicted reasonably close to their presently known, rather uncertain experimental values. The agreement is good enough considering that some of these are expected to have large higher order corrections. The model works, in my opinion, just as well as the naive quark model works for the heavier nonets. A more detailed data comparison would become meaningful, after one has included higher order effects, i.e. after one has unitarized the model, e.g., along the lines of the UQM[3]. Of course, we believe by no means that the sigma model is a fundamental theory, only that it is a reasonable effective theory at low energy, which in a compact way can incorporate constraints from symmetry and symmetry breaking. With only a few low-dimensional invariants, like in Eq.(1), the model can provide a good starting point for the inclusion of unitarity and analyticity effects.

Those working on chiral perturbation theory and nonlinear sigma models usually point out that the linear model does not predict all low energy constants correctly. However, one should remember that the energy regions of validity are different for the two approaches. Chiral perturbation theory usually breaks down when one approaches the first scalar resonance. The linear sigma model, on the other hand, includes the scalars from the start and can be a reasonable interpolating model in the intermediate energy region near 1 GeV, where QCD is too difficult to solve.

These results strongly favour the interpretation that the $a_0(980), f_0(980), \sigma(500), K_0^*(1430)$ belong to the same nonet, and that they are the chiral partners of the $\pi, \eta, \eta', K$. If the latter are believed to be unitarized $q\bar{q}$ states, so are the light scalars $a_0(980), f_0(980), \sigma(500), K_0^*(1430)$, and the broad $\sigma(500)$ should be interpreted as an existing resonance. The $\sigma$ is a very important hadron indeed, as is evident in the sigma model, because this is the boson which gives the constituent quarks most of their mass and thereby it gives also the light hadrons most of their mass. Therefore it is natural to consider the $\sigma(500)$ as the Higgs boson of strong interactions.

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