Runaway accretion of metals from compact discs of debris on to white dwarfs

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ABSTRACT

It was recently proposed that metal-rich white dwarfs (WDs) accrete their metals from compact discs of debris found to exist around more than a dozen of them. At the same time, elemental abundances measured in atmospheres of some WDs imply vigorous metal accretion at rates up to $10^{11} \text{g s}^{-1}$, far in excess of what can be supplied solely by Poynting–Robertson drag acting on such discs of debris. To explain this observation we propose a model, in which rapid transport of metals from the disc on to the WD naturally results from interaction between this particulate disc and a spatially coexisting disc of metallic gas. The latter is fed by evaporation of debris particles at the sublimation radius located at several tens of WD radii. Because of pressure support the gaseous disc orbits the WD slower than the particulate disc. Resultant azimuthal drift between them at speed $\lesssim 1 \text{ m s}^{-1}$ causes aerodynamic drag on the disc of solids and drives inward migration of its constituent particles. Upon reaching the sublimation radius, particles evaporate, enhancing the density of the metallic gaseous disc and leading to positive feedback. Under favourable circumstances (low viscosity in the disc of metallic gas and efficient aerodynamic coupling between the discs) a system evolves in a runaway fashion, destroying the discs of debris on time-scale of $\sim 10^5 \text{ yr}$, and giving rise to high metal accretion rates up to $\dot{M}_Z \sim 10^{10}–10^{11} \text{ g s}^{-1}$, in agreement with observations.

Key words: accretion, accretion discs – protoplanetary discs – white dwarfs.

1 INTRODUCTION

Recent detections of near-infrared excesses around a number of metal-rich white dwarfs (WDs) imply the existence of warm circumstellar material reprocessing stellar radiation (Zuckerman & Becklin 1987; Graham et al. 1990; Farihi et al. 2010). Spectral modelling suggests that this material resides in an extended, compact, optically thick and geometrically thin disc (Jura 2003; Jura, Farihi & Zuckerman 2007), similar to the Saturn’s rings (Cuzzi et al. 2010). Discs have inner edges at several tens of $R_*$ (here $R_*$ is the radius of the WD), roughly consistent with those being set by particle sublimation at these locations. Their outer radii lie close to the Roche radius of the WD, $R_\text{R} \sim 1 R_\odot$, supporting the suggestion by Jura (2003) that such compact discs of debris are produced by tidal disruption of asteroid-like bodies scattered into low-periastron orbits by gravitational perturbations of massive planets, which have survived the asymptotic giant branch phase of stellar evolution (Debes & Sigurdsson 2002).

Availability of a large reservoir of high-Z elements in the form of debris discs in the immediate vicinity of some WDs naturally led to the suggestion (Jura 2003) that metal enrichment of these stars is caused by accretion from such discs. This scenario provides a promising alternative to the previously proposed interstellar accretion model of the metal pollution of WDs (Dupuis et al. 1993), which is not consistent with observations of WDs with He atmospheres.

Theoretical estimates of settling time of heavy elements in WDs imply that their observed atmospheric abundances can be maintained against gravitational settling by accretion of metals at rates $\dot{M}_Z \sim 10^6–10^{11} \text{ g s}^{-1}$ (Farihi, Jura & Zuckerman 2009; Farihi et al. 2010). If the circumstellar accretion hypothesis is correct, an evolving disc of debris must be able to supply such $\dot{M}_Z$ to the WD.

The actual transfer of metals from the disc of solids truncated at the sublimation radius $R_*$ to the WD must be accomplished in this picture via the gas disc extending from the WD surface to $R_\text{R}$ and beyond. Observational evidence of such gaseous component around several metal-rich WDs hosting compact discs of debris has been found by Gänsicke et al. (2006), Gänsicke, Marsh & Southworth (2007) and Gänsicke et al. (2008) in the form of double-peaked emission lines of Ca ii and Fe ii. These spectroscopic signatures are naturally explained as arising in a disc of metallic gas (no H or He emission lines have been detected around these WDs) in Keplerian rotation around WDs and spatially coincident with dusty discs (Brinkworth et al. 2009; Melis et al. 2010).

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Even though metals are passed to the WD through the gaseous disc, the rate of mass transfer $M_Z$ is ultimately controlled by the evolution of the particulate disc. By analogy with the Saturn’s rings one expects that the collisional viscosity in discs of debris is too low to drive the non-negligible $\dot{M}_Z$. However, radiation of the WD can be quite important and Rafikov (2011, hereafter R11) demonstrated that Poynting–Robertson (PR) drag on the disc naturally drives mass accretion at the rate $\dot{M}_{PR} \gtrsim 10^8 \, \text{g s}^{-1}$. While this $M_Z$ is high it still falls short of explaining the highest observed $M_Z \sim 10^{10} - 10^{11} \, \text{g s}^{-1}$.

Here we propose a new picture of the evolution of the disc of debris, which naturally combines several physical ingredients present in the close vicinity of the disc-bearing WDs to explain high $M_Z \gtrsim \dot{M}_{PR}$.

2 DESCRIPTION OF THE MODEL

Our model of the WD–disc system is outlined below and is schematically illustrated in Fig. 1. We assume a disc of solid particles to extend from the Roche radius $R_R$ all the way into the inner radius $R_s$, which we take to coincide with the sublimation radius

$$ R_s = \frac{\sqrt{2}R_\star}{2} \left( \frac{T_s}{T_\star} \right)^2 \approx 22\sqrt{\epsilon} R_\star T_{s,28} \left( \frac{1500 \, \text{K}}{T_\star} \right)^2, $$

where $T_s$ is the sublimation temperature of solid particles (which we take to be $\sim 1500 \, \text{K}$ if particles are silicate), $T_{s,28} \equiv T_s/(10^4 \, \text{K})$ is the normalized stellar temperature, $R_\star$ is the radius of the WD, and $\epsilon$ is the ratio of particle’s emissivities for starlight and for its own thermal radiation (in the following we assume macroscopic particles and set $\epsilon = 1$). For $R_s \approx 0.01 R_\odot$ (Ehrenreich et al. 2011) one finds $R_s \approx 0.2 R_\odot$, in agreement with observations (Jura et al. 2007, 2009).

At present the size of particles $a$ constituting the disc is rather poorly constrained. Graham et al. (1990) suggested $a \approx 10–100$ cm. At the same time high-resolution infrared spectroscopy with the Infrared Spectrograph on board Spitzer reveals strong 10 μm emission feature in disc spectra indicative of a population of small, micron-sized silicate dust particles (Jura et al. 2009). But the fraction of the total disc mass locked up in such fine dust is unknown. The actual particle size is not very important for our treatment as long as the disc is optically thick. Then it is similar to the Saturn’s rings and most likely behaves as a granular flow. In the following we will treat a particulate disc as if it was a solid plate.

Solid particles brought to $R_s$ sublimate feeding a disc of metallic gas at this location. Viscous torques cause it to spread all the way to the WD surface providing means of metal transport from $R_s$ to the star. For the bulk of gas to move inward of $R_s$ its angular momentum must be transferred by viscous torques to some amount of gas viscously spreading outward of $R_s$ (Lynden-Bell & Pringle 1974). Thus, because of the conservation of angular momentum an external gaseous disc (Pringle 1991) spatially coexisting with dusty debris disc inevitably appears, naturally explaining observations of several WDs hosting both gaseous and dusty discs of debris (Brinkworth et al. 2009; Melis et al. 2010). Jura (2008) proposed collisional sputtering of small asteroids as another source of metallic gas; for the sake of clarity we do not consider this mechanism here.

We emphasize here that the two discs have an overlap in the radial distance, while in the vertical direction the gaseous disc is much more extended than the disc of particles, see Fig. 1. The latter must be very thin because inelastic collisions between particles rapidly damp any vertical random motions. We consider the debris disc to be directly exposed to the full WD radiation flux since Melis et al. (2010) found the absorption of stellar photons by the gas disc to be small.

The external gas disc has temperature different from (higher than) that of the dust disc at the same radii (Melis et al. 2010) because the different balance of heating and cooling for the two discs keeps the temperature of the gas $T_s$ above the sublimation point even outside $R_s$. Some condensation of metallic gas on the particle surfaces may be occurring (cf. Meyer & Meyer-Hofmeister 1994); however, the very fact of spatial coexistence of gaseous and particulate discs in several systems (Melis et al. 2010) strongly suggests that condensation cannot fully eliminate the gas phase above the particulate disc. We leave accurate assessment of the effects of condensation to future work.

Simultaneous existence of the two discs of high-Z elements in different phases drives their mutual evolution in the following way. Any coupling between the outer portion of the gas disc and particulate disc acts to transfer angular momentum from the faster rotating particulate disc to the slower spinning gaseous disc, causing inward motion of particles in the disc of solids (and outward motion of gas in the external gaseous disc). If the coupling is strong enough positive feedback becomes possible in the system: increasing mass of the gaseous disc leads to the increase in $M_Z$ through the disc of solids (as described below), which in turn reinforces evaporation at $R_s$ and increases gaseous mass even further.

Coupling between the gaseous and particulate high-Z discs arises because the gaseous disc orbits the WD at an angular speed, $\Omega_\star$, slightly slower than the Keplerian speed, $\Omega_K$, at which the disc of solid particles rotates. This difference is caused by the pressure support present in the gaseous disc, $\Omega_\star - \Omega_K \approx (2\pi r_\rho)^{-1} \partial P/\partial r$, and is known to cause a variety of important effects in protoplanetary discs, such as the inward migration of solids (Weidenschilling 1977). Relative azimuthal velocity between the gaseous and particulate discs at a distance $r$ from the WD is

$$ v_\varphi = \eta c_s \frac{c_s}{\Omega_K r} \approx \frac{10^2 \, \text{cm s}^{-1} T_{e,5}^3}{\mu_{28}} \left( \frac{M_\star}{0.2 R_\odot} \right)^{1/2}, $$

where $\eta \sim 1$ is a constant, $M_\star \equiv M/M_\odot$ is the normalized WD mass ($M_\star$), $\mu_{28}$ is the mean molecular weight of the metallic gas normalized by 28 $m_{\text{H}}$ (value of $m_{\text{H}}$ for pure Si), $T_{e,5} \equiv T_e/(10^5 \, \text{K})$ is the normalized gas temperature ($T_e$), and $c_s \approx 0.5 \, \text{km s}^{-1} (T_{e,5}/\mu_{28})^{1/2}$ is the sound speed of the gas (clearly $v_\varphi \ll c_s$).

This azimuthal drift gives rise to aerodynamic drag between the discs. The azimuthal drag force $f_\varphi$ per unit surface area of a
particulate disc with the surface mass density $\Sigma_g$ causes inward radial migration of the disc material at speed $v_i = 2 f_{p\theta} / (\Omega_K \Sigma_g)$. This inward particle drift gives rise to mass transport at the rate

$$M_z = 2 \pi r v_i \Sigma_g = \frac{4 \pi r f_{p\theta}}{\Omega_K}.$$  \hspace{1cm} (3)

Note that if $f_{p\theta}$ is independent of $\Sigma_g$ then the same is true for $M_z$.

The external force $f_{p\theta}$ per unit area can be generally represented in the form

$$f_{p\theta} = A \Sigma_g + B,$$  \hspace{1cm} (4)

where the first term describes the coupling between the gas disc with the surface density $\Sigma_g$ and the particulate disc, and constant $A$ determines the strength of coupling. It is natural to expect that drag scales with the surface density of the gaseous disc $\Sigma_g$; this expectation is confirmed in Section 4.

The second term $B$ represents forces acting even in the absence of the coupling between the gas disc and the particulate disc. PR drag is an example of such force and one can easily show (R11) that for the coupling between the gas disc and the particulate disc, $PR$ drag mass transport due to the $PR$ drag alone (when $\alpha = 0$) at $R_s$ is (R11)

$$M_{PR} = \frac{4 \pi R_s f_{PR}}{\Omega_K} = \frac{4 \phi_i R_s L_s}{3 \pi R_s c^2} \approx 10^8 \text{ g s}^{-1} \phi_i \frac{L_s}{10^{-3} L_\odot} \frac{20}{R_s/R_\odot}.$$  \hspace{1cm} (5)

In the following we will assume that $B = B_{PR}$.

### 3 COUPLED EVOLUTION OF THE PARTICULATE AND GASEOUS DISCS

Here we present a simple local model of the coupled evolution of the two discs and investigate the conditions under which rapid metal accretion becomes possible. This model provides motivation and qualitative support for the more detailed global calculations presented in Metzger et al. (in preparation).

Particle sublimation at $R_s$ increases the mass of the gaseous disc at the rate $M_z$. Assuming that the surface density of the gaseous disc $\Sigma_g$ varies on scale $\sim R_s$, we can write that sublimation increases $\Sigma_g$ at the rate $\Sigma_g \sim M_z / (\pi R_s^2) = 4 f_{p\theta} / (\Omega_K R_s)$. At the same time, viscous spreading reduces $\Sigma_g$ at the rate $\Sigma_g \sim t \delta \Sigma_g / t_s$, where $t_s$ is the characteristic viscous time in the disc:

$$t_s \sim \frac{R_s^2}{\nu} \approx 10 \text{ yr} \alpha^{-1} \frac{M_{28}}{L_\odot} \left( \frac{R_s}{0.2 R_\odot} \right)^{1/2},$$  \hspace{1cm} (6)

assuming $\alpha$-parametrization of viscosity $\nu = \alpha c_s^2 / \Omega_K$ (Shakura & Sunyaev 1973). This time-scale can be very short if $\alpha$ is not very small.

We can now describe the evolution of $\Sigma_g$ in the vicinity of $R_s$ with the following heuristic equation:

$$\frac{\partial \Sigma_g}{\partial t} = \Sigma_g - \Sigma_g = \frac{4}{\Omega_K R_s} (A \Sigma_g + B) - \frac{\Sigma_g}{t_s}.$$  \hspace{1cm} (7)

A solution of this equation satisfying the initial condition $\Sigma_g(t = 0) = 0$ (no gas disc initially) is

$$\Sigma_g(t) = \frac{M_{PR} t_s}{\pi R_s^2} \left( 1 - \frac{t}{t_s} \right)^{-1} \left\{ \exp \left[ \frac{t}{t_s} \left( 1 - \frac{t}{t_s} \right) \right] - 1 \right\}.$$  \hspace{1cm} (8)

![Figure 2](https://example.com/figure2.png)

**Figure 2.** Time evolution of (a) $M_z$ and (b) total sublimated mass $M_s$. Evolutionary tracks are shown for different values of the feedback parameter $t_s/t_s \equiv \tau$ and demonstrate runaway behaviour for $t_s/t_s > 1$ and saturation of $M_z$ for $t_s/t_s < 1$. Note that for $t \lesssim t_s$ the behaviour of both $M_z$ and $M_s$ is independent of $t_s/t_s$.

where we used equation (5) and

$$t_s = \frac{\Omega_K R_s}{4A},$$  \hspace{1cm} (9)

is the sublimation time $\Sigma_g / \Sigma_g$ on which the mass of the gaseous disc increases if $B_{PR} = 0$. From equations (3), (4) and (8) the rate at which particles sublimate is

$$\frac{M_z}{M_{PR}} = \left( 1 - \frac{t}{t_s} \right)^{-1} \left\{ \exp \left[ \frac{t}{t_s} \left( 1 - \frac{t}{t_s} \right) \right] - 1 \right\},$$  \hspace{1cm} (10)

and the total mass lost by the disc of debris to sublimation $M_s$ can be easily obtained by integrating this expression.

The behaviour of $M_z$ and $M_s$ is shown in Fig. 2 for different values of $t_s/t_s$, which clearly demonstrates that for $t \lesssim t_s$ (i.e. as long as $M_z \lesssim M_{PR} t_s$) the evolution of the discs of debris is insensitive to $t_s/t_s$ and $M_z \approx M_{PR}$. This is because initially the disc of metallic gas is not dense enough for the drag it produces on the disc of debris to compete with the PR drag – it takes certain time to accumulate enough mass of the gas by particle sublimation.

This means, in particular, that if the disc of debris starts with mass $M_d$, which is lower than the critical mass $M_{PR} t_s$, then its evolution is determined only by the PR drag and coupling to the gaseous disc is never effective – the disc of debris is eroded before the gas disc grows massive enough. Lifetime of the disc of debris is then

$$t_{PR} = \frac{M_d}{M_{PR}} \approx 3 \text{ Myr} \frac{M_d}{10^{22} \text{ g}} \left( \frac{M_{PR} t_s}{10^8 \text{ g s}^{-1}} \right)^{-1},$$  \hspace{1cm} (11)

independent of the relation between $t_s$ and $t_s$.

In the opposite case of a massive initial disc – $M_d \gtrsim M_{PR} t_s$ – evolution does depend on $t_s/t_s$. Whenever $t_s \lesssim t_s$ the action of viscosity is so effective at removing gas released by particle sublimation at $R_s$ that the gas does not accumulate there. Then $\Sigma_g$ simply saturates at the constant low level, the accretion rate tends
to \( M_Z \approx M_{PR}(1 - t_r/t_s)^{-1} \sim M_{PR} \) for \( t \gtrsim t_s \) and the disc gets exhausted on time-scale \( t_{PR} \) given by equation (11). This evolutionary path clearly cannot explain the highest observed values of \( M_Z \).

However, in the case of a massive disc and \( t_r \gtrsim t_s \) sublimation supplies gas to the \( r \sim R \), region faster than viscous diffusion removes it, and \( \Sigma_g \) grows exponentially on time-scale \( \sim t_s \) (as long as the particle disc has enough mass to provide the source). This means that \( M_Z \) also increases exponentially for \( t \gtrsim t_s \) (see Fig. 2).

This runaway stops and the gaseous disc wanes only when the mass of the particulate disc \( M_d \) is exhausted. The latter happens on the runaway time-scale

\[
\tau_{run} \approx t_s \left(1 - \frac{t_r}{t_s}\right)^{-1} \ln \left(\frac{M_d}{M_{PR} t_s}\right),
\]

(12)
determined by the condition \( M(t_{run}) \equiv \int_{t=0}^{t_{run}} M_Z dt \approx M_d \). Clearly, \( \tau_{run} \ll t_{PR} \) if \( M_d/(M_{PR} t_s) \gg 1 \).

Thus, the ratio

\[
\mathcal{F} \equiv \frac{t_r}{t_s} = \frac{4AR_s}{\alpha c_s^2}
\]

(13)
is the critical parameter determining the strength of positive feedback for massive discs: if \( \mathcal{F} > 1 \) a particulate disc gets rather rapidly (within several \( t_s \)) converted into metallic gas at \( R_s \), with its subsequent viscous accretion on to the WD on time-scale of several \( t_s \), whereas for \( \mathcal{F} < 1 \) feedback is not strong enough to reinforce gas supply at \( R_s \) and the disc slowly evolves on time-scale \( t_{PR} \) due to PR drag. As equation (13) shows, runaway evolution and high \( M_Z \) require (1) strong coupling between the gaseous and particulate discs and (2) low viscosity.

## 4 AERODYNAMIC COUPLING

Relative azimuthal motion between gaseous and particulate discs at speed \( v_\phi \) induces the aerodynamic drag between them. The drag produces a force per unit area of the disc (Schlichting 1979)

\[
f_\phi = Re^{-1} \rho_s v_\phi \nu,
\]

(14)
where \( \rho_s = \Omega_s \Sigma_g/c_s \) is the gas density and \( Re^{-1} \) is a proportionality constant. There is a significant spread of opinions regarding the value of \( Re \), characterizing the drag by the turbulent flow on a smooth solid plate, with numbers ranging between \( Re \approx 20 \) (Dobrovolskis, Dacles-Mariani & Cuzzi 1999) to \( Re \approx 500 \) (Goldreich & Ward 1973). It is also likely that a smooth plate approximation underestimates the drag (overestimates the value of \( Re \)) for the particulate disc of debris, which does not have continuous surface and may interact with gas more like a rough plate (Schlichting 1979) or even as a combination of individual particles, in which case smaller \( Re \) is more appropriate.

According to equation (14) the aerodynamic drag force can be written in the form \( f = A \Sigma_g \) with

\[
A = \frac{\Omega_s v_\phi}{Re c_s} = \frac{\eta^2}{\eta^2} c_s^2 / \frac{Re}{\Omega_s 2\pi R_s^2},
\]

(15)
where equation (2) was used. From equation (9) the sublimation time at \( R_s \) is

\[
t_s = \frac{Re \ g M_f}{4\eta^2 c_s^2} \approx 10^3 \ \mathrm{yr} \ \frac{Re \ M_{PR} t_s}{\eta^2 c_s^2},
\]

(16)
where \( c_s \approx c_s/(1 \ \mathrm{km} \ \mathrm{s}^{-1}) \). The critical mass separating low- and high-mass discs (see Section 3) is

\[
M_{PR} t_s \approx 3 \times 10^{-8} \frac{\eta^2 \ c_s^2}{\eta^2} \frac{L_s}{\eta^2} \frac{20}{M_{PR} t_s} R_s/R_.
\]

(17)
The feedback parameter for the aerodynamic coupling is

\[
\mathcal{F} = \frac{4\eta^2}{\alpha Re \ \Omega_s \ R_s} = \frac{\eta^2}{\alpha Re} \left(\frac{T_{PR} R_s}{R_\odot} \right)^{1/2},
\]

and it depends on both the viscosity parameter \( \alpha \) and the strength of the aerodynamic coupling, parametrized by \( Re \).

Equation (18) shows that runaway evolution with \( \mathcal{F} \gtrsim 1 \) requires rather low viscosity in the gaseous disc, at the level of \( \alpha \sim 10^{-1} - 10^{-4} \), and for higher \( Re \), smaller \( \alpha \) is needed. Viscosity is most likely provided by the magneto-rotational instability (MRI), which requires a certain level of ionization to operate effectively. In the ideal magnetohydrodynamic (MHD) limit simulations with no net flux typically produce \( \alpha \sim 10^{-2} \) (Hawley, Gammie & Balbus 1995). However, in our scenario the gaseous disc exists in immediate contact with the particulate disc, which is observationally known to contain a population of micron size dust grains (Jura et al. 2009).

MRI-driven turbulence will mix some of this fine dust with the gas, lowering the ionization fraction (small grains have large surface area and are very efficient charge absorbers), and giving rise to the non-ideal MHD effects, e.g., via the increased resistivity (Balbus 2011). The latter are known from simulations to decrease effective \( \alpha \) substantially, down to \( \alpha \sim 10^{-4} \) or lower (Fleming, Stone & Hawley 2000; Bai & Stone 2011). As a result, it is conceivable that \( \alpha \) in our model can be much lower than in the ideal MHD limit of MRI.

Thus, the presence of the dusty disc of debris in close contact with the gaseous disc can quite naturally lengthen viscous time-scale \( t_{PR} \) and facilitate gas accumulation, making possible runaway evolution due to the aerodynamic drag. We note that after the particulate disc completely sublimates, the value of \( \alpha \) in the resultant gaseous disc will go up since the source of small dust particles lowering ionization has disappeared.

## 5 DISCUSSION

Our calculations suggest that compact discs of debris around WDs are self-destructive whenever there is a strong dynamical (frictional) coupling between the gaseous and solid components. Adopting for illustration \( Re = 20 \) and setting all other dimensionless constants to unity we find from equations (16)–(18) \( t_r \approx 2 \times 10^2 \ \mathrm{yr} \approx 6 \times 10^9 \ \mathrm{g} \) and that \( \alpha \lesssim 10^{-4} \) is needed for \( \mathcal{F} > 1 \).

The typical lifetime of a massive disc with \( M_f = 10^{22} \ \mathrm{g} \) (mass of a 200 km asteroid) in the runaway scenario (if the viscosity is low) is of the order of several sublimation time-scales, i.e. about \( 10^3 \ \mathrm{yr} \). According to equations (10) and (11) in this case the maximum \( M_Z \) for this \( M_d \), achieved right before the disc of debris completely disappears, is \( \max(M_Z) \approx M_d/t_s \sim 10^{10} \ \mathrm{g} \). These numbers agree with observations (Farihi et al. 2009, 2010) quite well.

However, if positive feedback is not strong enough to drive runaway (e.g. because \( \alpha > 10^{-5} \) or higher \( Re \)), then \( M_Z \approx M_{PR} \) and it takes 3 Myr for the same disc to be exhausted, as equation (11) demonstrates. In this case the surface density of the gas at \( R_s \) satisfies at the low level \( \Sigma_g \sim M_{PR} t_s/(\pi R_s^2) \sim 0.1 \ \mathrm{g} \ \mathrm{cm}^{-2} \), see equation (8).

Equation (11) also shows that a low-mass disc with \( M_f = 10^{19} \ \mathrm{g} \approx M_{PR} t_s \) (mass of a 20 km asteroid) is destroyed by the PR drag very rapidly, within several thousands of years (as long as the disc is optically thick to incoming stellar radiation, see R11).

Our model of the evolution of the disc of debris naturally explains coexisting gaseous and particulate discs of debris reported in Melis et al. (2010). Systems with reported IR excesses but lacking
emission lines of high-Z elements in the gas phase may simply possess gaseous sublimation of solids at \( R_s \). Compositional variations between different WDs may also explain such systems.

Based on modelling the Ca II and Fe II emission lines arising in gaseous discs around metal-rich WDs, Melis et al. (2010) inferred that these discs do not extend much inward of the inner radii of the discs of debris in the same system. This observation appears to be in conflict with the idea of metal transport from the sublimation radius to the WD surface via the gaseous disc accretion, which is a part of our model. However, it is plausible that the metal gas is in fact present interior to \( R_s \) but does not produce detectable emission in the same line as the gas at larger separation. Other mechanisms may exist to explain the apparent lack of gas emission originating from this part of the gaseous discs. The magnetic field of the WD, if strong enough, may also help clear out the gas between \( R_s \) and the WD surface.

Calculations presented in this work are rather simple and local in nature. We studied only one coupling mechanism – aerodynamic drag, while other possibilities may be available as well (e.g. induction interaction (Drell, Foley & Ruderman 1965; Gurevich, Krylov & Fedorov 1978) between the MRI-generated B-field in the gaseous disc and the debris disc particles). Also, here we did not consider low surface density discs (we use only the thin plate approximation), non-trivial initial radial distributions of the debris’ surface density, fate of the angular momentum lost by the discs of debris and absorbed by the gas disc, and so on. Future global models of coupled evolution of gaseous and particulate discs of debris (Metzger et al., in preparation) will take these issues into account to provide a more accurate description of the WD pollution with circumstellar high-Z material.

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