The Rise and Fall of the Ridge

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Abstract

Recent data from heavy ion collisions at RHIC show unexpectedly large near-angle correlations that broaden longitudinally with centrality. The amplitude of this ridge-like correlation rises rapidly with centrality, reaches a maximum, and then falls in the most central collisions. In this talk we explain how this behavior can be easily understood in a picture where final momentum-space correlations are driven by initial coordinate space density fluctuations. We propose \( v_2^2/\epsilon_{n,\text{part}}^2 \) as a useful way to study these effects and explain what it tells us about the collision dynamics.

Keywords:

Introduction: The motivation for the construction of the Relativistic Heavy Ion Collider at Brookhaven National Laboratory was to collide heavy nuclei in order to form a state of matter called the Quark Gluon Plasma (QGP) [1]. These collisions deposit many TeV into a region the size of a nucleus. The matter left behind in that region is so hot and dense that hadronic matter undergoes a phase transition into a form of matter where quarks and gluons are the relevant degrees of freedom, not hadrons [2]. This is the state of matter that existed when the universe was less than a microsecond old and still very hot.

Correlations and fluctuations are an invaluable tool for probing the dynamics of heavy-ion collisions. Data from the experiments at RHIC reveal interesting features in the two-particle correlation landscape [3, 4]. Specifically, it has been found that correlation structures exist that are unique to Nucleus-Nucleus collisions. While two-particle correlations in p+p and d+Au collisions show a peak narrow in azimuth and rapidity, the near-side peak in Au+Au collisions broadens substantially in the longitudinal direction and narrows in azimuth. An analysis of the width of the peak for particles of all \( p_T \) finds the correlation extends across nearly 2 units of pseudo-rapidity (\( \Delta \eta = 2 \)) [4]. When triggering on higher momentum particles (\( p_T > 2 \text{ GeV/c} \) for example), the correlation extends beyond the acceptance of the STAR detector (\( \Delta \eta < 2 \)) and perhaps as far as \( \Delta \eta = 4 \) as indicated by PHOBOS data [4].

STAR has found that the amplitude of this correlation shows a rather rapid rise with collision centrality [3] before reaching a maximum and falling off in the most central bins. This drop in the most central bins shows up for both \( \sqrt{s_{NN}} = 200 \) and 62.4 GeV but is often overlooked. In this talk we present a geometric explanation for the centrality dependence of the ridge amplitude. We’ll use the centrality dependence of \( v_2/\epsilon_2 \), \( dN/dy \), and the third harmonic participant eccentricity \( \epsilon_{3,\text{part}}^2 \) to predict the amplitude of the near-side ridge correlation (\( A_1 \)). Given the apparent relevance of initial-state density fluctuations to the final, momentum-space correlations, we advocate the transfer function \( v_2^2/\epsilon_{n,\text{part}}^2 \) as a valuable observable for studying the length scales in heavy-ion collisions. We extract the transfer functions from intermediate \( p_T \) di-hadron correlation data where evidence has been presented for conical emission [6].

Three Premises: It has been shown by the STAR collaboration that the second harmonic component of the near-side ridge in the two particle correlations can account for nearly all of the difference between the two and four particle cumulant \( v_2 \) [7]. It was argued that \( v_2 \) fluctuations must therefore be tiny: many times smaller than the fluctuations
predicted from eccentricity models [8]. In that case, a major revision of our understanding of heavy-ion collisions would be required. We’ve argued previously, however, that eccentricity fluctuations can give rise to \( v_n \) fluctuations for more harmonics than just \( n = 2 \), and that those fluctuations could therefore be the source of the near-side ridge [9] (especially if the \( v_n \) fluctuations depend on pseudo-rapidity difference \( \Delta \eta \) i.e. \( \langle v_n(\eta)v_n(\eta + \Delta \eta) \rangle \equiv f(\Delta \eta) \)). This idea has been born out by calculations from several groups [10, 5, 11, 12]. To test this conjecture, we will attempt to explain the centrality dependence of the near-side ridge amplitude \( A_1 \) from eccentricity fluctuations. we start with three simple premises:

- the expansion of the fireball created in heavy-ion collisions converts anisotropies from coordinate-space into momentum-space,
- the conversion efficiency increases with density,
- and the relevant expansion plane is the participant plane.

The participant plane can be defined for any harmonic number and a system with a lumpy initial energy density will give rise to finite participant eccentricity at several harmonics [5,13]. This becomes conceptually clear when eccentricity is recast in terms of a harmonic decomposition of the azimuthal dependence of the initial density. Our calculation of the centrality dependence of \( A_1 \) will depend on the higher harmonic terms in an eccentricity model.

**Higher Harmonics, Even the Odd:** Fig. 1 (left) shows the \( n^{th} \)-harmonic participant eccentricity \( \langle \epsilon_n(\text{part}) \rangle \) as defined in Ref. 5 for central Au+Au collisions from a Monte-Carlo Glauber model. Typically the participant eccentricity is calculated based on the positions of point-like participants (that is the participant is said to exist at a precise \( x \) and \( y \) position). Those results are labeled \( r_{\text{part}} = 0.0 \text{ fm} \). One can also calculate the eccentricity from a more physically realistic model with participants smeared over some region. This is done by treating each participant as many points distributed within a disk of some finite radius \( r_{\text{part}} \). Increasing \( r_{\text{part}} \) washes out the higher \( \langle \epsilon_n^2(\text{part}) \rangle \) terms. The right panel of Fig. 1 shows the ratio of \( \langle \epsilon_n^2(\text{part}) \rangle \) for a given \( r_{\text{part}} \) value divided by \( \langle \epsilon_n^2(\text{part}) \rangle \) for \( r_{\text{part}} = 0 \). The curves are labeled \( l_{\text{mfp}} \) instead of \( r_{\text{part}} \) and the ratio \( l_{\text{mfp}}/\text{ideal} \) for reasons explained below.

![Figure 1](image)

In this calculation we introduced the length scale \( r_{\text{part}} \) causing the higher terms in \( \langle \epsilon_n^2(\text{part}) \rangle \) to be washed out. That effect is more general though and we believe it is important for understanding correlations and \( v_n \) fluctuations. One can also consider what happens when particles free-stream for some amount of time \( \tau_{\text{fs}} \) before they interact; that will also introduce a length scale \( c\tau_{\text{fs}} \) which leads to a similar reduction of higher terms [12]. One can also consider the effect of a mean-free-path \( (l_{\text{mfp}}) \) on the ability of the fireball to convert higher \( \langle \epsilon_n^2(\text{part}) \rangle \) terms into \( v_n^2 \) [14,15]. If a particle on average travels for a distance \( l_{\text{mfp}} \) between interactions, it is clear that higher \( \langle \epsilon_n^2(\text{part}) \rangle \) terms will not become manifest in \( v_n^2 \). Fig. 2 shows a schematic illustration of this idea. If our probe has a \( l_{\text{mfp}} \) in the fireball, then we will be blind to features smaller than \( l_{\text{mfp}} \). All these effects will act to wash out the higher harmonics so we expect \( v_n^2 \) to drop with \( n \). Since \( v_n^2 \) is related to \( dN/d\Delta \phi \) by a Fourier transform, and since a Fourier transform of a Gaussian
is a Gaussian, all we need to reproduce the near-side Gaussian ridge in two particle correlations, is for \( v_2^n \) to drop with \( n \) with an approximately Gaussian shape. The right panel of Fig. 1 shows that this is a reasonable expectation from viscous effects.

The ridge amplitude \( A_1 \) is found by measuring \( \Delta \rho/\sqrt{\rho_{\text{ref}}} \) (the pair density \( \rho \) minus the reference pair density \( \rho_{\text{ref}} \) scaled by \( \sqrt{\rho_{\text{ref}}} \)) vs \( \Delta \phi \) and \( \Delta \eta \) [16]. A fit function is devised to describe the correlation. The fit function has a \( \Delta \phi \) independent term, a \( \cos(\Delta \phi) \) and \( \cos(2 \Delta \phi) \) term, and a near-side 2-D Gaussian. The fit function describes the data well [16]. Here we work with the conjecture that the 2-D Gaussian is a manifestation of \( \langle \epsilon_{n,\text{part}}^2 \rangle \). Based on this conjecture, we can try to calculate the centrality dependence of \( A_1 \). Our result for \( A_1 \) will be related to \( v_2^n \) so we need to know the conversion efficiency \( c \) of \( \langle \epsilon_{n,\text{part}}^2 \rangle \) into \( v_2^n \). The conversion efficiency will depend on particle density. In Fig. 3 we show \( v_2/\epsilon \) vs density \( (1/S)dN/dy \) from the STAR Collaboration [17] with a fit function to parameterize the data. The figure shows \( v_2/\epsilon \) divided by the eccentricity calculated with respect to the reaction plane from a CGC model [18]. We will take this to estimate our conversion efficiency with the understanding that the different results obtained from CGC and Glauber models gives rise to at least a 30% uncertainty in the correct conversion efficiency.

Since a \( \cos(\Delta \phi) \) and \( \cos(2 \Delta \phi) \) term have already been subtracted from \( \Delta \rho/\sqrt{\rho_{\text{ref}}} \) in order to obtain \( A_1 \), our estimate of \( A_1 \) can be made simpler if we predict the \( n = 3 \) component of the near-side ridge and scale that up to get the full amplitude. The \( n = 3 \) component is found from

\[
\frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\Delta \rho}{\sqrt{\rho_{\text{ref}}}} \cos(3\Delta \phi) d\Delta \phi = 0.039 A_1.
\]

The azimuthal width \( \sigma \) of the near-side Gaussian is weakly dependent on centrality so we use a typical value of \( \sigma = 0.65 \). The factor of 0.039 will be used to relate our prediction for \( \langle \epsilon_{n,\text{part}}^2 \rangle \) to the amplitude \( A_1 \). Since \( \Delta \rho/\sqrt{\rho_{\text{ref}}} \) is a per-particle measure instead of a per-pair measure, we need to include the particle density \( \rho_0 = \frac{1}{4\pi} \frac{dN}{dy} \). Putting
\[ \langle \varepsilon^2_{3,\text{part}} \rangle \text{ together with the conversion efficiency } c, \text{ particle density } \rho_0, \text{ and the factor of 0.039 to map from the } n = 3 \text{ component to a Gaussian amplitude, we find} \]

\[ A_1 \approx \rho_0 \varepsilon^2_{3,\text{part}} \times 0.039. \]  

(2)

We take \( \rho_0 \) from data, \( c \) from Fig. 3, and \( \langle \varepsilon^2_{3,\text{part}} \rangle \) from our Monte-Carlo Glauber model.

**The Rise and Fall of the Ridge:** The right panel of Fig. 4 shows our estimate of the ridge amplitude \( A_1 \) based on \( \varepsilon^2_{n,\text{part}} \) vs centrality parameter \( \nu = 2N_{\text{bin}}/N_{\text{part}} \). The left panel shows the preliminary STAR data. Our estimate for the amplitude is only approximate since we’ve applied the conversion efficiency for \( n = 2 \) to \( n = 3 \) and as discussed before, we expect the efficiency to drop with \( n \). This will lead us to overestimate the contribution from \( \langle \varepsilon^2_{3,\text{part}} \rangle \) to the ridge. On the other hand, we used a CGC based eccentricity model to extract the conversion efficiency but a Glauber model for \( \langle \varepsilon^2_{3,\text{part}} \rangle \). This should cause us to under-estimate the ridge contribution; to some extent canceling the previous overestimate. We find that our estimate of \( A_1 \) agrees quite well with data. The centrality dependence is particularly interesting: our \( A_1 \), like the data, starts at a small value and rises much faster than expectations from a Glauber Linear Superposition model (GLM) which assumes that correlations grow as \( N_{\text{bin}}/N_{\text{part}} \). The rise continues until \( A_1 \) reaches a maximum near \( \nu = 5 \), then \( A_1 \) falls again. This rise and fall is also seen in the preliminary 62.4 GeV data [16] and has no natural explanation in any of the other proposed scenarios for the ridge formation. In our picture, the rise and fall is related simply to the geometry and it’s fluctuations. \( \langle \varepsilon^2_{3,\text{part}} \rangle \) falls with \( N_{\text{part}} \) since the larger \( N_{\text{part}} \) leads to smaller fluctuations. But \( N_{\text{part}} \langle \varepsilon^2_{3,\text{part}} \rangle \) first rises then falls. This rise and fall is due the asymmetry of the overlap region which shows up even for \( \langle \varepsilon^2_{3,\text{part}} \rangle \). Since both \( c \) and \( \rho_0 \) are increasing with centrality, the product of \( \rho_0 \varepsilon^2_{3,\text{part}} \) rises until very central collisions and then falls as shown in the figure. The observation that the rise and fall shows up in the near-side ridge amplitude suggests that the near-side ridge is likely dominated by initial geometry fluctuations.

**Transfer Functions:** Our explanation for the centrality dependence of \( A_1 \) is based on the three simple premises listed earlier and provides a natural explanation for the rise and fall of the ridge. Initial estimates of the amplitude agree to within our uncertainties. This suggests that geometry fluctuations in the initial overlap region are converted into momentum space giving rise to the near-side ridge structure. We discussed that we expect the conversion efficiency to drop with \( n \) since effects like initial-state free-streaming and mean-free-path will wash out the higher harmonic terms. Measuring the conversion efficiency \( c_n = \frac{\nu_n(2)^2}{\nu_{n,\text{part}}(2)^2} \) as a function of \( n \), centrality, and particle kinematics will provide information on those effects. It will be particularly interesting to measure \( c_n \) as a function of \( \Delta \eta \) to understand how de-coherence affects manifest in the longitudinal direction.

In Fig. 5 we show the conversion efficiency \( \nu_n(2)^2/\nu_{n,\text{part}}(2)^2 \) for intermediate \( p_T \) di-hadron correlations from STAR [6].
Figure 5: The transfer function \( v_2(2) / \epsilon_{n,\text{part}}^2 \) for intermediate \( p_T \) di-hadron correlations from 20-60% central 200 GeV Au+Au collisions [6].

The panels show various combinations of trigger particle and associated particle \( p_T \) selections. Intermediate \( p_T \) di-hadron data rather than showing an away-side Gaussian, show two peaks shifted to either side of \( \Delta \phi = \pi \). It has been suggested that this is evidence of conical emission on the away-side of the higher \( p_T \) trigger particle [19, 20].

The previously ignored effects of \( v_3^2 \) which were, however, could explain some or all of these novel structures [9].

To investigate these structures, we look at the \( p_T \) dependence of \( c_n(p_{T_{\text{TRIG}}}, p_{T_{\text{ASSOC}}}) \). By plotting \( v_2(2)^2 / \epsilon_{n,\text{part}}^2 \) from these correlations, we hope to see what portion of those correlations can be explained by geometric effects. For relatively lower momentum cuts, we find that \( c_2(2.5 \text{ GeV}, 0.5 \text{ GeV}) \) behaves much as we expect from mean-free path effects for example; the higher terms drop off monotonically similar to Fig. 1 (right). But for higher \( p_{T_{\text{ASSOC}}} \) cuts \( c_3 \) shows a pronounced peak at \( n = 3 \). See \( c_3(2.5 \text{ GeV}, 2.5 \text{ GeV}) \) for example. This feature disappears again however when a large enough \( p_{T_{\text{TRIG}}} \) cut is applied. For \( p_{T_{\text{TRIG}}} > 6 \text{ GeV} \) for example, we see only effects due to the presence of correlations from jets. It remains interesting to speculate about the possible source of that local maximum at \( n = 3 \) for intermediate \( p_T \) di-hadron correlations (whether it’s due to a suppression of the lower harmonic super-horizon modes [21], mach-cones [20] or some other acoustic effects [22]). It seems to be larger than trivially expected from \( \epsilon_{n,\text{part}}^2 \). We note however that a complete investigation of the systematic errors on our Glauber Model has not been carried out.

We include the oblate shape measured for the Au nucleus. That oblateness leads to a large enhancement of the \( n = 2 \) component of \( \epsilon_{n,\text{part}}^2 \). Reducing the oblateness of the Au nucleus could therefore reduce or eliminate the prominence of the \( n = 3 \) term in \( c_n \). This remains for further investigation.

Conclusions: We presented the participant eccentricity vs harmonic when the participants are treated as point-like or smeared over a radius \( r_{\text{part}} \). The larger values of \( r_{\text{part}} \) wash out the higher harmonic eccentricities. We argued that, similarly, a large mean-free-path should wash out higher harmonics of \( v_n \). Such an effect could lead to a Gaussian peak in two particle correlations at \( \Delta \phi = 0 \). We’ve calculated the contribution to the near-side Gaussian peak that we expect from initial density fluctuations. We based our calculation on three premises 1) that the expansion of the fireball created in heavy-ion collisions converts anisotropies from coordinate-space into momentum-space, 2) the conversion
efficiency depends on particle density, and 3) the relevant expansion plane is the participant plane. Following these premises, we find that the near-side peak from density fluctuations should rise rapidly, reach a maximum just before the most central events, then fall. Our estimate of the magnitude is in agreement within our uncertainties with the available data and the shape matches that seen in the data. This is the only calculation to correctly describe the rise and fall of the ridge amplitude. We conclude therefore that density fluctuations are likely the dominant source for the low $p_T$ ridge-like correlations. We have also shown the ratio of the final momentum space correlations $v_n(2)^2$ to the initial coordinate-space eccentricities $e_{n,\text{part}}^2$ for intermediate $p_T$ di-hadrons. We find that for intermediate $p_T$ correlations the $n = 3$ term is larger than $n = 2$. This suggests that even after taking into account initial density fluctuations, some interesting signal may exist in the intermediate $p_T$ di-hadron data.

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