Backflow and dissipation during the quantum decay of a metastable Fermi liquid

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The particle current in a metastable Fermi liquid against a first-order phase transition is calculated at zero temperature. During fluctuations of a droplet of the stable phase, in accordance with the conservation law, not only does an unperturbed current arise from the continuity at the boundary, but a backflow is induced by the density response. Quasiparticles carrying these currents are scattered by the boundary, yielding a dissipative backflow around the droplet. An energy of the hydrodynamic mass flow of the liquid and a friction force exerted on the droplet by the quasiparticles have been obtained in terms of a potential of their interaction with the droplet.

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I. INTRODUCTION

A first-order phase transition at a temperature close to zero is thought to proceed via quantum tunneling through an energy barrier between the metastable and stable states in the configuration space. Such a quantum decay of a metastable Fermi-liquid phase has attracted considerable attention since it was found in recent experiments of the phase separation from supersaturated $^3$He–$^4$He liquid mixtures that the observed degrees of critical supersaturation were roughly independent of temperatures below about 10 mK. Another possible example is a deconfinement phase transition in cold, dense nuclear matter as encountered in a neutron star; this may take place during compression of the nuclear matter, induced for instance by mass accretion onto a neutron star from the companion in a close binary system. As long as a homogeneous medium whose pressure is near the phase-equilibrium point adjusts adiabatically to fluctuations of a virtual droplet of the stable phase, the rate of such a quantum nucleation may be calculated by treating the droplet radius $R$ as a macroscopic variable, deriving its Lagrangian from thermodynamic quantities of the system, and describing the tunneling motion of $R$ in a semiclassical, reversible fashion.

In a real system, however, such an adjustment proceeds at a finite rate. Consequently, fluctuations of a virtual droplet are accompanied by elementary excitations in the metastable Fermi liquid. These excitations not only renormalize the mass and the potential for $R$, but also lead to energy dissipation. Such renormalization and dissipation act to alter the tunneling probability exponentially. In a hydrodynamic regime where the mean free path $l$ of the excitations is much shorter than $R$, the particle current is completely determined by the continuity equation, and the energy dissipation is described in terms of the kinematic viscosity of the liquid. When the temperature is lowered enough to turn the system into a collisionless regime ($R \ll l$), the low-lying excitations appear as quasiparticles. They are scattered by the droplet, and the resulting density fluctuation gives rise to a friction force exerted on the droplet as well as a dissipative backflow current. Here we shall investigate how the flow pattern and the energy dissipation depend on coupling of the quasiparticles with the droplet.

Backflow around a slowly moving massive impurity in a Fermi liquid has been recently calculated by Zwerger; the result has been expressed in terms of phase shifts for scattering off the impurity. Within linear response, the backflow is dipolar and proportional to the density response function. At long distances, however, the leading term is not dipolar, but is the radial contribution of at least second order in the scattering potential. It was noted that this radial backflow arises from the asymmetry in density around the fixed scatterer, induced by a finite background current. In this paper, we extend these calculations by Zwerger to a spherically symmetric backflow, surrounding a fluctuational droplet of the stable phase, in a noninteracting, neutral Fermi gas at zero temperature. We thus find that the backflow current induced by the linear response has the same radial dependence as an unperturbed current, and that these currents are governed by the particles on the Fermi surface. The friction and the dissipative backflow arising from scattering of these particles off the boundary are expressed in terms of the transport cross sections. Using the backflow pattern and friction so expressed, we derive the kinetic energy and the rate of energy dissipation of the metastable gas. Generalization to the interacting case is made within the Landau theory of Fermi liquids. Finally, the backflow and dissipation in a charged Fermi liquid relevant to dense nuclear matter are examined.
II. PARTICLE CURRENT AND DISSIPATION IN NONINTERACTING SYSTEMS

We first consider a case in which a noninteracting, three-dimensional Fermi gas, composed of a single component (spin 1/2, bare mass \( m \), and electric charge 0) and having an equilibrium number density \( n_1 \), is in a metastable state against a first-order phase transition at zero temperature. In this case, a spherical droplet of the stable phase, which is assumed to be incompressible and hence to be of uniform number density \( n_2 \), develops inside, under, or outside the potential barrier. Such development, for pressures so close to the phase-equilibrium point that the droplet size is macroscopic, can be reduced to fluctuations of the droplet radius \( R(t) \). Let us now assume that \( |\dot{R}| \) is far smaller than the velocity of sound \( c_s \) in the metastable Fermi gas, i.e., the gas is nearly uniform. This also means that \( |\dot{R}| \ll v_F \), where \( v_F = \hbar k_F/m \) with \( k_F = (3\pi^2 n_1)^{1/3} \) is the Fermi velocity. In the absence of the interaction of the droplet with the medium, the droplet surface acts only as a boundary condition for the unperturbed current density \( \mathbf{j}(r,t)|_{0: \text{res}} = \mathbf{j}(R,t)|_0 - n_1 \dot{R} \). Here \( r \) and \( \mathbf{r} \) are the length and the unit vector of the position vector \( \mathbf{r} \), of which the origin is set to be the droplet center. Under such a condition, the conservation law yields

\[
j(r,t)|_0 = \begin{cases} (n_1 - n_2)\dot{R} \left( \frac{R}{r} \right)^2 \mathbf{r}, & r \geq R \\ 0, & r < R. \end{cases}
\]  

(1)

The actual current density \( \mathbf{j}(r,t) \) deviates from \( \mathbf{j}(r,t)|_0 \) by the backflow due to the interaction of particles in the medium with the droplet. We describe this interaction in terms of a spherically symmetric, quasistatic potential \( V(r,t) = \int d\mathbf{r}' n_{\text{ex}}(\mathbf{r}',t) v(|\mathbf{r} - \mathbf{r}'|) \), where \( n_{\text{ex}}(r,t) = n_2 \delta[R(t) - r] \) is the density perturbation, and \( v(r) \) is the potential of interaction of a particle in the medium with that in the droplet. Within linear response in \( v(r) \), the backflow is induced by \( n_{\text{ex}} \), the Fourier transform of which is given by

\[
n_{\text{ex}}(\mathbf{q},\omega) = 2\pi n_2 \delta(\omega) 4\pi R^3 \left[ \frac{\sin qR}{(qR)^3} - \frac{\cos qR}{(qR)^2} \right]
\]

in the limit \( \dot{R} \to 0 \). The response is characterized by the density fluctuation \( n(\mathbf{q},\omega) \)

\[
n(\mathbf{q},\omega)|_{\text{res}} = v(q) \chi(\mathbf{q},\omega) n_{\text{ex}}(\mathbf{q},\omega),
\]

(2)

where \( v(q) \) is the Fourier transform of \( v(r) \), and \( \chi(\mathbf{q},\omega) \) is the density-response function. Let us assume that the range \( a \) of the interaction is short, i.e., \( a \ll R \), and set \( v(q) \) as \( v(q = 0)/(1 + a^2 q^2) \) for simplicity. Then, the macroscopic change in the number of particles near the droplet may be calculated from Eq. (2) up to \( O(a/R) \) as

\[
\Delta N|_{\text{res}} = -2\pi \frac{\partial n_1}{\partial \mu_1} n_2 v(q = 0)a R^2,
\]

(3)

where the asymptotic behavior of \( \lim_{q \to 0} \chi(\mathbf{q},0) = -\partial n_1/\partial \mu_1 \) with the chemical potential of the gas \( \mu_1 = \hbar^2 k_F^2 / 2m \) is used. By substituting Eq. (3) into the conservation law, the radial backflow at large distances is obtained as

\[
j(r,t)|_{\text{res}} = -\frac{\partial n_1}{\partial \mu_1} \frac{n_2}{n_2 - n_1} v(q = 0) \frac{a}{R} \mathbf{j}(r,t)|_0.
\]

(4)

The proportionality of \( \mathbf{j}|_{\text{res}} \) to \( a R \dot{R} \) reflects the fact that the change \( \Delta N|_{\text{res}} \) in the number of particles due to the linear response behaves as \( a R^2 \). According to whether the interaction is repulsive or attractive, the backflow current \( \mathbf{j}|_{\text{res}} \) goes in the same or the opposite direction with respect to \( \dot{R} \). Note that Friedel-type oscillations appearing near the droplet are not influential as long as \( R \) is macroscopic, i.e., \( k_F R \gg 1 \). Then, the backflow of form (4) is expected to approximately express the flow pattern for \( r > R \). The kinetic energy \( K \) of the metastable Fermi gas can thus be calculated from the current sum \( \mathbf{j}_{\text{mac}} = \mathbf{j}|_0 + \mathbf{j}|_{\text{res}} \) as

\[
K = \frac{1}{2} \int dr m \left| \mathbf{j}_{\text{mac}} \right|^2 / n_1
\]

\[
= 2\pi mn_1 R^3 \dot{R}^2 \left( 1 - \frac{n_2}{n_1} \right)^2 \left[ 1 - 2 \frac{\partial n_1}{\partial \mu_1} \frac{n_2}{n_2 - n_1} v(q = 0) \frac{a}{R} + O(a^2/R^2) \right].
\]

(5)

It is important to note that \( K \) is directly related to the reversible motion of \( R \), since \( \mathbf{j}_{\text{mac}} \) satisfies the conservation law on a macroscopic time scale of \( R/|\dot{R}| \). In the incompressible limit, Eq. (5) reduces to the result of Lifshitz and Kagan.
The macroscopic current \( \mathbf{j}_{\text{mac}} \) in turn causes a backflow current beyond linear response through the scattering of particles in the medium off the droplet. Let us assume the droplet to be inert and characterize the scattering of the particles by the incoming momenta \( \mathbf{k} \) and the outgoing scattering states in \( V \): \( \psi_k(\mathbf{r}) = (\mathbf{r} | \mathbf{k} +) \). By noting the correspondence with a local reference frame moving with the velocity \( \mathbf{v}_{\text{mac}} = \mathbf{j}_{\text{mac}}/n_1 \), the equilibrium distribution of the particles is obtained as a shifted Fermi distribution function, \( n^0(\varepsilon_{\mathbf{k}} - \omega \mathbf{v}_{\text{mac}}) \), with \( \varepsilon_k = h^2 k^2/2m \). The actual current density may thus be calculated up to linear order in \( \dot{\varepsilon} \)

\[
\mathbf{j}(\mathbf{r}, t) = \frac{2}{(2\pi)^3} \int d\mathbf{k} \frac{\hbar}{m} \text{Im} \left[ \psi_k^*(\mathbf{r}) \frac{\partial}{\partial \mathbf{r}} \psi_k(\mathbf{r}) \right] n^0(\varepsilon_{\mathbf{k}} - \omega \mathbf{v}_{\text{mac}})
\]

\[
= \frac{2k_F^2}{(2\pi)^3} \int d\Omega_k \mathbf{v}_{\text{mac}} \cdot \hat{\mathbf{k}} \text{Im} \left[ \psi_k^*(\mathbf{r}) \frac{\partial}{\partial \mathbf{r}} \psi_k(\mathbf{r}) \right] \bigg|_{k=k_F} ,
\]

(6)

where \( \hat{\mathbf{k}} \) is the unit vector of \( \mathbf{k} \), and \( d\Omega_k \) is the integration over the directions of \( \hat{\mathbf{k}} \). Equation (6) shows that the flow is governed by the particles on the Fermi surface. At large distances, by using such an asymptotic form of \( \psi_k(\mathbf{r}) \) as \( e^{i \mathbf{k} \cdot \mathbf{r}} + f_k(\hat{\mathbf{k}} \cdot \mathbf{r}) e^{i \mathbf{k} \cdot \mathbf{r}} + \ldots \), with the usual scattering amplitude \( f_k \), we obtain the backflow current \( \delta \mathbf{j}_{\text{curr}} = \mathbf{j} - \mathbf{j}_{\text{mac}} \) as

\[
\delta \mathbf{j}_{\text{curr}} = \frac{2k_F^2}{(2\pi)^3} \int d\Omega_k \mathbf{v}_{\text{mac}} \cdot \hat{\mathbf{k}} \left\{ \frac{|f_k|^2}{r^2} \mathbf{r} + \text{Im} \left[ i e^{i(k-r) \cdot \mathbf{r}} f_k \mathbf{r} + i e^{i(k_r-r) \cdot \mathbf{r}} f_k \mathbf{r} \right] \right\} \bigg|_{k=k_F} + O(r^{-3})
\]

(7)

where \( \sigma_{\text{tr}}(k) = \int d\Omega_k (1 - \hat{\mathbf{k}} \cdot \mathbf{r}) |f_k|^2 \) is the exact scattering cross section. Here we have used the asymptotic form \( e^{i(k-r) \cdot \mathbf{r}} \rightarrow i\delta(\mathbf{k} - \mathbf{r})/2kr \) as well as the optical theorem. We note that \( \mathbf{j}_{\text{mac}} \) comes from Eq. (6) via the unperturbed term \( e^{i \mathbf{k} \cdot \mathbf{r}} \). We see from Eq. (7) that the backflow \( \delta \mathbf{j}_{\text{curr}} \) is at least of order \( V^2 \) and is induced by the macroscopic current \( \mathbf{j}_{\text{mac}} \) in its opposite direction.

The physical origin of the backflow \( \delta \mathbf{j}_{\text{curr}} \) is analogous to that given by Landauer\(^{[1]} \) for a backflow around a slowly moving impurity. A portion of the metastable Fermi gas, which carries the macroscopic current \( \mathbf{j}_{\text{mac}} \) but cannot adjust adiabatically to the development of the droplet, is scattered by the boundary, leading to a density fluctuation around the droplet. This fluctuation in turn diffuses at a velocity of \( v_r \), yielding the asymptotic backflow current written by Eq. (7). This current has nonvanishing divergence, since the density fluctuation is positive or negative definite. This is a contrast to the case of a slowly moving impurity in which the density fluctuation, being positive in front of and negative behind the scatterer, is distributed in such a way that the corresponding backflow has vanishing divergence.\(^{[2]} \)

We proceed to calculate the friction force \( \mathbf{F} \) due to momentum transfer from the medium to the boundary. This force may be expressed as

\[
\mathbf{F} = \frac{2}{(2\pi)^3} \int d\mathbf{k} \left\langle \mathbf{k} + \frac{\partial V}{\partial \mathbf{r}} | \mathbf{k} + \right\rangle n^0(\varepsilon_{\mathbf{k}} - \omega \mathbf{v}_{\text{mac}}|_{r=R}) .
\]

(8)

\( \mathbf{F} \) originates from the density fluctuation induced by the macroscopic current \( \mathbf{j}_{\text{mac}} \), since it vanishes when \( \mathbf{v}_{\text{mac}}|_{r=R} = 0 \). Up to \( O(\dot{R}) \) and \( O(\alpha/R) \), we obtain

\[
\mathbf{F} = \frac{2}{(2\pi)^3} \int d\Omega_k \frac{m k_F^2}{h} \mathbf{v}_{\text{mac}}|_{r=R} \cdot \hat{\mathbf{k}} \left\langle \mathbf{k} + \frac{\partial V}{\partial \mathbf{r}} | \mathbf{k} + \right\rangle \bigg|_{k=k_F}
\]

\[
= \left[ 1 - \frac{n_1}{\partial n_2} - \frac{n_2}{n_1} v(q=0) \frac{a}{R} \right] n_1 \hbar k_F \sigma_{\text{tr}}(k_F) \left( 1 - \frac{n_2}{n_1} \right) \dot{R} \hat{\mathbf{r}} .
\]

(9)

Here we have used the relation\(^{[2]} \left\langle \mathbf{k} + \frac{\partial V}{\partial \mathbf{r}} | \mathbf{k} + \right\rangle = (h^2 k^2/m) \sigma_{\text{tr}}(k_F) \hat{\mathbf{k}} \), derived from the Lippmann-Schwinger equation. The rate of energy dissipation due to this friction may then be calculated as \( \dot{E} = -4\pi \mathbf{F} \cdot \mathbf{v}_{\text{mac}}|_{r=R} \). From Eqs. (1), (4), and (9), we obtain

\[
\dot{E} = -4\pi^2 \alpha \left[ 1 - 2 \frac{n_1}{\partial n_2} - \frac{n_2}{n_1} v(q=0) \frac{a}{R} + O(\alpha^2/R^2) \right] n_1 \hbar k_F \left( 1 - \frac{n_2}{n_1} \right)^2 R^2 \dot{R}^2 ,
\]

(10)

with \( \alpha = \sigma_{\text{tr}}(k_F)/\pi R^2 \). We thus find that Eq. (10) is similar to the result obtained by Burmistrov and Dubovskii\(^{[3]} \) using a dimensional analysis based on the expression for \( \dot{E} \) in the hydrodynamic regime. Their result, however, has left unknown the coefficient of \( n_1 \hbar k_F (1 - n_2/n_1)^2 R^2 \dot{R}^2 \), which is now expressed in terms of the potential \( v \).
III. EXTENSION TO INTERACTING AND CHARGED SYSTEMS

Short-range interaction \( \hat{v} \) between particles in the medium plays a role in modifying the backflow current and the rate of energy dissipation, calculated for a noninteracting Fermi gas. This role can be examined within the Landau theory of Fermi liquids,\(^7\) which allows one to construct a quasiparticle state \( \psi_p \) with momentum \( p \) from the corresponding noninteracting state via adiabatic switching on of the interaction \( \hat{v} \). In order to obtain the distribution \( n_p \) of the quasiparticles, it is sufficient to notice that a transport equation for the quasiparticles is identical with the corresponding equation for a noninteracting system, except that the velocity term is determined by the quasiparticle energy \( \varepsilon_p \) and that the force \(-\partial \varepsilon_p / \partial r\), exerted by other quasiparticles, emerges. The transport equation thus reads

\[
\frac{\partial n_p}{\partial t} + \frac{\partial n_p}{\partial r} \frac{\partial \varepsilon_p}{\partial p} - \frac{\partial n_p}{\partial p} \frac{\partial (\varepsilon_p + V)}{\partial r} = 0 .
\] (11)

Let us divide \( n_p \) into the equilibrium part \( \langle n_p \rangle \) and the fluctuating part \( \delta n_p \). For the translationally invariant system considered here and \( \partial n_p / \partial t = 0 \), we obtain \( \langle n_p \rangle = n^0(\varepsilon_k, \varepsilon_p - \varepsilon_{\text{mac}}) \), where \( \varepsilon_k = h^2 k^2 / 2m^* \) with the quasiparticle effective mass \( m^* \), and \( \delta n_p = \langle n_p \rangle (|\psi_p|^2 - 1) \). Note that \( \langle n_p \rangle \) and \( \delta n_p \) associated with the current densities \( j_{\text{mac}} \) and \( j_{\text{curr}} \), reduce to the similar expressions \( n^0(\varepsilon_k, \varepsilon_p - \varepsilon_{\text{mac}}) \) and \( n^0(\varepsilon_k, \varepsilon_p - \varepsilon_{\text{mac}}) (|\psi_p/h|^2 - 1) \) for \( \hat{v} = 0 \). Consequently, \( j_{\text{mac}}, j_{\text{res}}, \) and \( j_{\text{curr}} \) are still described by Eqs. (1), (4), and (7) in which the scattering amplitude \( f_k \) and the thermodynamic quantity \( \partial \mu_1 / \partial n_1 \) are modified by the interaction \( \hat{v} \). Here, the interaction potential \( v \) is assumed to be unchanged by \( \hat{v} \). This holds if the potential \( v \) is a scalar field. By using the obtained current densities and distribution of the quasiparticles, we can calculate the kinetic energy \( K \), the friction force \( F \), and the rate of energy dissipation \( \dot{E} \). The results are given by Eqs. (5), (9), and (10) in which the scattering amplitude \( f_k \) and the thermodynamic quantity \( \partial \mu_1 / \partial n_1 \) are those calculated for an interacting system, and the momentum of unit volume \( n_1 h k_F \) is multiplied by \( m / m^* \). Note that the generalization to the interacting case mentioned above is valid for a Fermi liquid at the low temperatures that ensure \( l \gg R \).\(^9\) At higher temperatures, where quasiparticle collisions should be built into the transport equation as a collision integral, the current and the rate of energy dissipation remain to be examined.

Next, with reference to dense nuclear matter that is neutralized by a gas of electrons and muons and is metastable against deconfinement,\(^1\) we consider a case in which a metastable Fermi liquid consists of particles of charge \( q_1 e \) that interact mainly via the short-range potential \( \hat{v} \) and are embedded in a uniform neutralizing background, and a stable phase is immersed in the same background. Then, a droplet of the stable phase has nonvanishing charge as long as \( n_1 \neq n_2 \). This charge is in turn screened by the charged particles in the medium. The resulting particle migration is characterized by the Thomas-Fermi screening length \( \lambda_{\text{TF}} = (\partial \mu_1 / \partial n_1) / 4 \pi (q_1 e)^2 \) since \( |R| \ll c_s \), being assumed in the medium, is consistent with the condition \( \lambda_{\text{TF}} \gg R \), appropriate for the linear Thomas-Fermi approximation of the charge response of the medium. Let us take account of the electrostatic potential \( \phi \) of interaction between the particles inside and outside the droplet and the charge density perturbation \( \rho_{\text{ex}} = q_1 e (n_2 - n_1) \theta[R(t) - r] \), in addition to the short-range potential \( v \) and the density perturbation \( n_{\text{ex}} \). Accordingly, the Coulomb-induced backflow \( j_{\text{res}}^{(C)} \) of linear order arises along with the backflow \( j_{\text{res}} \) given by Eq. (4). When \( r \gg \lambda_{\text{TF}} \) is satisfied, one obtains \( j_{\text{res}}^{(C)} = -j_0 \) since perfect screening ensures cancellation between the unperturbed current \( j_0 \) and the backflow current \( j_{\text{res}} \). At distances \( r \lesssim \lambda_{\text{TF}} \), however, the screening action of the particles gives rise to slight inhomogeneity of order \( (R/R_{\text{TF}})^2 \) in the density distribution of the medium, which we have calculated within the linear Thomas-Fermi approximation.\(^12\)

The resulting density distribution yields the backflow \( j_{\text{res}}^{(C)} \) via the continuity equation:

\[
j_{\text{res}}^{(C)} = \{-1 + [1 - \kappa R + O(\kappa^2 R^2)](1 + kr)e^{-\kappa(r-R)}\}j_0 ,
\] (12)

with \( \kappa = \lambda_{\text{TF}}^{-1} \). Up to \( O(\kappa R) \) and \( O(a/R) \), the kinetic energy \( K \), the dissipative backflow \( \delta j_{\text{curr}} \), the friction force \( F \), and the dissipation rate \( \dot{E} \) can be calculated following a line of argument for a neutral Fermi liquid and by incorporating Eq. (12) in the macroscopic current \( j_{\text{mac}} \) and modifications by \( \phi \) in the scattering amplitude \( f_k \). Since \( j_{\text{res}}^{(C)} \sim O(\kappa^2 R^2) \) at \( r = R \), the resultant expressions for \( F \) and \( \dot{E} \) are identical with Eqs. (9) and (10), respectively, except that \( n_1 h k_F \) is replaced by \( (m/m^*) n_1 h k_F \). On the other hand, \( K \) and \( \delta j_{\text{curr}} \) have corrections due to \( j_{\text{res}}^{(C)} \); the expression for \( \delta j_{\text{curr}} \) is given by Eq. (7) in which \( j_{\text{mac}} = j_0 + j_{\text{res}} + j_{\text{res}}^{(C)} \), and that for \( K \) is obtained as

\[
K = 2 \pi mn_1 R^3 R^2 \left(1 - \frac{n_2}{n_1}\right) \frac{2}{1 - 2 \frac{\partial \mu_1}{\partial n_1} n_2 - n_1} \frac{1}{\kappa} (q = 0) \frac{a}{R} - \frac{3}{2} \kappa R .
\] (13)

In Eq. (13), the term of order \( \kappa R \) reflects the role played by \( j_{\text{res}}^{(C)} \) in cancelling the unperturbed current \( j_0 \) at \( r \gtrsim \lambda_{\text{TF}} \).
IV. CONCLUSION

We have evaluated the flow pattern and the friction induced during nucleation of the stable phase in a metastable Fermi liquid. The kinetic energy $K$ and the dissipation rate $E$ of the liquid, which are relevant to the calculations of the rate of quantum nucleation, have been expressed in terms of the interaction potential $v$. In order to make practical estimates, one needs information as to $v$, which is generally hard to obtain from experiments. The corresponding scattering potential $V$ may have an inelastic property if the droplet undergoes excitations due to the scattering of quasiparticles in the medium off the boundary. It is important to recall that the results for the current and dissipation as obtained above are based on the assumption of the liquid being nearly incompressible. The effect of compressibility not only develops the nonlinear response contribution to the macroscopic current $j_{\text{mac}}$, but also yields a reduction in the kinetic energy $K$ via the emission of sound. The latter, which stems from the finite $\omega$ behavior of $j_{\text{mac}}$, plays a role in increasing the nucleation rate exponentially.

Finally, we consider a case in which the liquid is in a superfluid state. The superfluidity has consequence in reducing the available quasiparticle states with momenta close to the Fermi surface and hence in weakening the friction force $F$. This effect may be estimated by changing, in the Fermi distribution in Eq. (8), the shifted quasiparticle energy $\varepsilon_k^0 - \hbar v_{\text{mac}}$ into the quantity $\sqrt{((\varepsilon_k^0 - \hbar^2 k^2_F/2m^*))^2 + \Delta_k^2} - (m/m^*)\hbar k \cdot v_{\text{mac}} + \hbar^2 k^2_F/2m^*$, with the pairing gap $\Delta_k$. This indicates that, when the macroscopic current flows slowly compared with the critical velocity that makes $\Delta_k$ vanish, i.e., $|v_{\text{mac}}| < (m^*/m)\Delta_k/\hbar$, and the temperature is zero, the friction does not occur. This is because Cooper pairs sustain $j_{\text{mac}}$ as a supercurrent. With increasing temperature and/or flow velocity, however, quasiparticle excitations become prevailing, giving rise to the dissipation of energy. We thus find that the nucleation rate depends strongly on the temperature and the flow velocity, a feature expected to be ascertained by future helium experiments at ultralow temperatures.

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