A Preference Foundation for Fehr and Schmidt’s Model
of Inequity Aversion

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Abstract

Fehr and Schmidt (1999) introduced an influential utility function for individuals in interpersonal contexts that captures self-centered inequity aversion. The power of this utility function lies in its good balance between parsimony and fit. This paper provides a preference foundation for exactly the model of Fehr and Schmidt (FS), so that the preference conditions satisfy exactly FS’s balance between parsimony and fit. Remarkably, FS is a special case of Schmeidler’s (1986, 1989) rank-dependent utility for decision under uncertainty.

Keywords: Fairness, Inequity, Rank-dependent Utility

JEL classification: D0, C91, C92, D64
1 Introduction

It has been recognized in economics that individuals often do not only care about their own payoff but also about the payoffs of others. Not only social planners, but also the members of society themselves care about the fairness and inequality of distributions of payoffs. Fehr and Schmidt (1999; from now on referred to as FS) introduced a utility function that captures concern about fairness in the sense of inequity aversion. Individuals dislike both being worse off than others, and others being worse off than themselves. Nevertheless, this inequity aversion is not too extreme so that when all others receive nothing an individual does still want to maximize his own payoff. This paper gives a preference foundation of exactly FS.

Preference foundations give behavioral conditions that are necessary and sufficient for a model to hold. These conditions state the empirical meaning of a model in terms of observables. In empirical applications the model holds if and only if the behavioral conditions hold and it fails if and only if at least one of the behavioral conditions does not hold. Normatively, the model can be justified if one agrees that behavior should follow the behavioral conditions and it can be criticized otherwise. Thus, a preference foundation allows for a verification of the empirical validity and the normative appropriateness of a model. It identifies the empirical content of a theoretical model directly in terms of observables.

Some recent studies provided preference foundations for models more general than FS (Neilson, 2006; Sandbu, 2008). From these preference foundations it does not follow how we can verify whether preferences satisfy exactly the model of FS. Although these papers provide necessary conditions for FS to hold, they do not provide sufficient conditions and therefore do not exactly identify FS’s empirical content.
Neilson (2006) considers general non-linear utility of payoff differences between one individual and the others. Neilson allows for, but does not identify a key feature of FS: that disadvantageous inequity is treated differently than advantageous inequity. Neilson’s key condition is self-referent separability, which is similar to standard separability, but which treats one component, the payoff of the decision maker, differently. This self-referent separability is not sufficient to characterize FS, because it does neither differentiate between advantageous and disadvantageous inequity, nor does it require additivity. It provides a useful starting point for obtaining generalizations of FS to reckon, for instance, with non-linear utility.

Sandbu (2008) comes close to obtaining a preference foundation of FS. He does make a difference between advantageous and disadvantageous inequity, by imposing a separability condition on rank-maintaining distributions only. His homotheticity condition implies power utility. FS is the special case of Sandbu where the power of utility equals one. Sandbu does not give a preference condition that guarantees a power of one. Thus, his conditions also are not sufficient to characterize exactly FS.

Instead of adding only few extra conditions to Sandbu’s foundation in order to provide a preference foundation of exactly FS, this paper aims at minimizing the total number of preference conditions and at keeping them as simple as possible. Thus, we start anew and focus entirely on FS. This is done to reflect the empirical content of FS best. This approach may make the preference foundation appear obvious. Still, this paper is the first providing preference conditions that exactly characterize FS.

The second part of this paper proposes the rank-dependent model of Gilboa (1987) and Schmeidler (1986, 1989) as a natural generalization of FS\(^1\). Sandbu (2008) already

\(^1\)Similarly, Ben Porath and Gilboa (1994) showed that the Gini-index in inequality measurement is a special
hinted towards this generalization, but we are the first to show exactly which weighting function in the rank-dependent model yields FS. The rank-dependent model was introduced to explain behavior for Knightean uncertainty that deviates from expected utility, and it spurred a whole stream of literature on ambiguity. Tversky and Kahneman (1992) used it to correct a theoretical mistake in the most influential theory of risk and uncertainty today: prospect theory (Kahneman and Tversky 1979). For this new version of prospect theory it was possible to establish a preference foundation (Wakker and Tversky 1993), proving its theoretical soundness. Such a preference foundation had been missing for the original prospect theory of 1979. This paper uses a refinement of Schmeidler’s comonotonic additivity axiom to characterize FS. More precisely, we use a nonmonotonic version of Schmeidler’s (1986) model, which was axiomatized by De Waegenaere and Wakker (2001). It is remarkable that Schmeidler’s idea provides the basis of a very influential model not only in decision under uncertainty, but also in interpersonal contexts.

2 The Model

There are \( n + 1 \) individuals. Each individual \( i \in \{0, \ldots, n\} \) receives a payoff \( x_i \in \mathbb{R} \). We model preferences \( \succeq \) of individual 0 over distributions \( x = (x_0, \ldots, x_n) \). The notation \( \succ , \preceq , \prec , \text{and } \sim \) is as usual.

**Weak Ordering** Weak ordering holds if \( \succeq \) is complete (\( x \succeq y \) or \( y \succeq x \) for all distributions \( x, y \)) and transitive.

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*Case of rank-dependent utility.* Gilboa (1989) proposed a model for intertemporal choice, which is also a special case of rank-dependent utility.
For every distribution $x$ and every individual $i = 1, \ldots, n$ we define the deviation $d_i(x) = x_i - x_0$, i.e. the deviation of the payoff of individual $i$ from the payoff of individual 0. The domain of deviations is the union of positive deviations $\mathbb{R}_+$, negative deviations $\mathbb{R}_-$, and 0. A distribution $x$ is constant if $x_i = x_0$ for all $i$.

Preferences $\succeq$ can be represented by a utility function $U$ if $x \succeq y$ if and only if $U(x) \geq U(y)$. FS holds if preferences $\succeq$ can be represented by

$$U(x) = x_0 - \alpha \sum_{i=1}^{n} \max\{x_i - x_0, 0\} - \beta \sum_{i=1}^{n} \max\{x_0 - x_i, 0\}$$

with $\alpha, \beta \geq 0$. We will next discuss the properties that preferences satisfy when FS holds.

We first introduce some more notation.

For all individuals $i = 0, \ldots, n$ and all distributions $x$ the distribution $\mu_i x$ denotes the distribution $x$ with the payoff of individual $i$ replaced by $\mu$. Every payoff $\mu \in \mathbb{R}$ is identified with a constant distribution yielding payoff $\mu$ for every individual. That is, instead of $(\mu, \ldots, \mu)$ we will often write $\mu$ to denote a constant distribution. From the context it will be clear whether $\mu$ denotes a single payoff or a constant distribution.

**Constant Monotonicity** Preferences satisfy constant monotonicity if $\mu \succ \nu$ if and only if $\mu \geq \nu$.

Constant monotonicity requires that, in the absence of inequality, higher payoffs are preferred to lower ones. If for a distribution $x$ and a payoff $\mu$ we have $x \sim \mu$, then $\mu$ is a constant equivalent of $x$. Under constant monotonicity and transitivity every distribution can have at most one constant equivalent.

**Constant Equivalence** Constant equivalence holds if every distribution has a constant equivalent.
However unequal a distribution, under constant equivalence there is always a constant distribution that individual 0 finds equivalent. It can easily be verified that preferences satisfy constant monotonicity and constant equivalence if FS holds.

Two distributions $x, y$ are covalent if for every individual $i = 1, \ldots, n$ the deviation has the same sign in both distributions: $d_i(x)d_i(y) \geq 0$. Thus, if two distributions $x$ and $y$ are covalent, then it cannot be the case that an individual $i$ gets strictly more than individual 0 in $x$ and strictly less than individual 0 in $y$.

**Covalent Additivity**  
Covalent additivity holds if for all $x, y, z$ that are pairwise covalent

$$x \succeq y \iff x + z \succeq y + z.$$  

Consider distributions that make it impossible for an individual to get more than individual 0 in one distribution and less in another one. Then increasing or decreasing the payoffs of all individuals by a given amount does not affect preferences over these distributions if covalent additivity holds. Next, consider redistributing the payoffs among individuals by common absolute amounts in all distributions such that individuals who received more than individual 0 before the redistribution do not receive less than individual 0 after the redistribution. Covalent additivity also requires that these redistributions should not affect preferences between distributions. As shown in the following observation, FS implies covalent additivity. We provide the proof in the main text, because it provides more insight into FS.

**Observation 2.1**  
If FS holds, then covalent additivity holds.

**Proof**
Consider three distributions $x, y, z$ that are pairwise covalent. We start by observing that $x$ and $x + z$ are covalent. This can be seen as follows: if $x_i > x_0$ then $z_i \geq z_0$ by covalence of $x$ and $z$. It follows that $x_i + z_i > x_0 + z_0$. The case $x_i < x_0$ is similar. By a similar argument $y$ and $y + z$ are covalent.

Define the set of distributions $S = \{x, y, z, x + z, y + z\}$. We can split all individuals into three groups, $J^+, J^-$, and $J^0$, as follows. $J^+$ consists of those individuals that do not receive less than individual 0 in any distribution in $S$ and receive more than individual 0 in at least one distribution in $S$. $J^-$ consists of those individuals that do not receive more than individual 0 in any distribution in $S$ and receive less than individual 0 in at least one distribution in $S$. $J^0$ consists of those individuals that receive the same payoff as individual 0 in every distribution in $S$. Since all distributions in $S$ are pairwise covalent, there cannot be two distributions $x, y \in S$ with $x_i > x_0$ and $y_i < y_0$ for some $i$. Thus there cannot be an individual that belongs both to $J^+$ and to $J^-$. Thus, $J^+ \cap J^- = \emptyset$. Every individual $j$ that belongs neither to $J^+$ nor to $J^-$ satisfies $x_j = 0$ for all $x \in S$.

It follows that $U(x) \geq U(y)$, i.e.

$$x_0 - \alpha \sum_{i \in J^+} (x_i - x_0) - \beta \sum_{i \in J^-} (x_0 - x_i) \geq y_0 - \alpha \sum_{i \in J^+} (y_i - y_0) - \beta \sum_{i \in J^-} (y_0 - y_i)$$

if and only if

$$x_0 + z_0 - \alpha \sum_{i \in J^+} (x_i + z_i - (x_0 + z_0)) - \beta \sum_{i \in J^-} (x_0 + z_0 - (x_i + z_i)) \geq y_0 + z_0 - \alpha \sum_{i \in J^+} (y_i + z_i - (y_0 + z_0)) - \beta \sum_{i \in J^-} (y_0 + z_0 - (y_i + z_i)),$$

i.e. $U(x + z) \geq U(y + z)$. Thus, $x \succeq y$ if and only if $x + z \succeq y + z$. $
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If FS holds, then the utility of the sum of two covalent distributions equals the sum of the utilities of the two distributions. This follows from the proof of the previous observa-
tion. This property of the utility function is similar to Schmeidler’s (1986) comonotonic additivity and Weymark’s (1981) weak independence of income source.

**Observation 2.2** If FS holds, then $U(x + y) = U(x) + U(y)$ for all covalent $x, y$.

**Disadvantage Aversion** Disadvantage aversion holds if $\mu_i 0 \preceq 0$ for all payoffs $\mu \geq 0$ and all $i = 1, \ldots, n$.

Disadvantage aversion means that individual 0 does not like another individual receiving a positive payoff $\mu$ when all other individuals, including himself, receive nothing. If FS holds, then preferences satisfy disadvantage aversion, because $\alpha \geq 0$: for payoff $\mu \geq 0$ and for $i = 1, \ldots, n$ we have $U(\mu_i 0) = -\alpha \mu \leq 0 = U(0)$.

**Advantage Aversion** Advantage aversion holds if $\mu_i 0 \preceq 0$ for all payoffs $\mu \leq 0$ and all $i = 1, \ldots, n$.

Advantage aversion means that individual 0 does not like another individual receiving a negative payoff $\mu$ when all other individuals, including himself, receive nothing. If FS holds, then preferences satisfy advantage aversion, because $\beta \geq 0$: for payoff $\mu \leq 0$ and for $i = 1, \ldots, n$ we have $U(\mu_i 0) = \beta \mu \leq 0 = U(0)$.

**Inequity Aversion** Preferences $\succeq$ satisfy inequity aversion if they satisfy disadvantage aversion and advantage aversion.

**Anonymity** Preferences $\succeq$ satisfy anonymity if $c_i 0 \sim c_j 0$ for all $c$ and all individuals $i, j \in \{1, \ldots, n\}$.
Anonymity means that individual 0 does not favor one other individual over the other. FS implies anonymity, because the coefficients $\alpha$ and $\beta$ are independent of the individuals, i.e. they are independent of $i$. The following theorem characterizes FS. Its proof is given in the Appendix.

**Theorem 2.3** The following two statements are equivalent.

(i) FS holds (Eq.1).

(ii) Preferences $\succeq$ satisfy

(a) Weak ordering;

(b) Constant monotonicity;

(c) Constant equivalence;

(d) Covalent additivity;

(e) Inequity aversion;

(f) Anonymity.

**Self-Rationality** individual 0 is self-rational if $\mu_00 \succ 0$ for all payoffs $\mu > 0$.

Self-rationality means that even if individual 0 is advantage averse, he still prefers receiving a positive payoff to receiving nothing if all other individuals receive nothing. Thus, keeping the payoff of others fixed, individual 0 will always want to maximize his own payoff.

**Preference for advantages over disadvantages** individual 0 prefers advantages over disadvantages if $(-\mu)_i0 \succeq \mu_i0$ for all $\mu > 0$ and all $i = 1, \ldots, n$. 
Corollary 2.4 If FS holds, then

(i) individual 0 is self-rational if and only if $\beta < \frac{1}{n}$.

(ii) individual 0 prefers advantages over disadvantages if and only if $\alpha \geq \beta$.

3 FS as a case of Rank-Dependent Utility

From the preference foundation of FS as given in the previous section, and from the formulation of Eq.1 deviations seem to play a crucial role in FS. What an individual cares about is his own payoff and the deviations of his own payoff from the payoffs of others. An individual values his own payoff and the deviations and then aggregates the obtained values to obtain the value of the distribution.

FS can also be viewed as a special case of Schmeidler’s (1986, 1989) rank-dependent utility with non-monotonic weighting function, as will be shown next. In that case, what an individual cares about is his own payoff and the payoffs of others, not necessarily the deviations. Thus, he values his own payoff and the payoff of every other individual, and then aggregates the obtained values to obtain the value of the distribution. Rank-dependent utility captures the fact that individuals are concerned about their rank in an income distribution (Rablen, 2008). Under rank-dependent utility the weight that an individual attaches to the payoff of another individual depends on the rank of this other individual’s payoff in the distribution of payoffs. It can, for instance, be the case that he weights the payoff of the individual with the lowest payoff more than the payoff of the individual with the largest payoff. Under FS an individual weights the payoffs of other individuals who are better off differently from the payoffs of other individuals who are worse off. The payoffs
of all others who are worse off or all others who are better off are weighted equally, as we will show later.

We first define the *weighting function* \( W \), which assigns a value to every subset of individuals. We assume that \( W(\emptyset) = 0 \) and \( W(\{0, \ldots, n\}) = 1 \). Consider a distribution \( x \). First we order the payoffs from best to worse, i.e. we take a permutation \( \rho \) on the set of individuals \( \{0, \ldots, n\} \) such that \( x_{\rho(0)} \geq \cdots \geq x_{\rho(n)} \). The *decision weight* for individual \( \rho(j) \) is then defined by

\[
\pi_{\rho(j)} = W(\{\rho(0), \ldots, \rho(j)\}) - W(\{\rho(0), \ldots, \rho(j-1)\})
\]

for all \( j \). Then *rank-dependent utility (RDU)* is given by

\[
\text{RDU}(x) = \sum_{i=0}^{n} \pi_i u(x_i),
\]

where \( u \) is a *utility function*. The weighting function is *monotonic* if \( A \supseteq B \) implies \( W(A) \geq W(B) \) for all sets of individuals \( A, B \), and *non-monotonic* otherwise.

It can easily be verified that FS is the special case of RDU with utility \( u(x_i) = x_i \) and a weighting function given by \( W(I) = 1 - (n - |I| + 1)\beta \) if \( 0 \in I \) and \( W(I) = -|I|\alpha \) if \( 0 \notin I \), where \( |I| \) gives the number of individuals in the set \( I \). Since \( \alpha, \beta \geq 0 \) the weighting function is non-monotonic. De Waegenaere and Wakker (2001) provided a preference foundation for RDU with non-monotonic weighting function and linear utility. Thus, an alternative preference foundation of FS could be obtained by taking the conditions of De Waegenaere and Wakker (2001) as a point of departure and then adding the extra conditions needed to specify FS. Moreover, rank-dependent utility may provide useful generalizations of FS.
4 Conclusion

This paper provided a preference foundation for the influential utility function that was introduced by Fehr and Schmidt (1999). This utility function captures self-centered inequity aversion in the sense that individuals dislike both being worse off than others, and others being worse off than themselves. Remarkably, this utility function is a special case of Schmeidler’s (1986, 1989) rank-dependent utility. Thus, rank-dependent utility is a natural generalization of FS, which may prove useful in modeling behavior in interpersonal contexts.

5 Appendix

Proof of Theorem 2.3

It was already shown in the main text that (i) implies (ii). We will now show that (ii) implies (i). Assume (ii).

Let \( e(x) \) denote the constant equivalent of distribution \( x \). For all \( b, c \in \mathbb{R}_+ \) and all \( i = 1, \ldots, n \) we have, by covalent additivity,

\[
e((b + c)_i0) \sim (b + c)_i0
= b_i0 + c_i0
\sim b_i0 + e(c_i0)
\sim e(b_i0) + e(c_i0).
\]

Thus,

\[
e((b + c)_i0) = e(b_i0) + e(c_i0)
\]
for $i = 1, \ldots, n$. By disadvantage aversion and constant monotonicity $-e(b_i0)$ is non-negative for small positive $b$. Thus, we can apply Theorem 1 of section 2.1.1 in Aczél (1966). It follows that there is an $\alpha_i \geq 0$ such that

$$e(b_i0) = -\alpha_i b$$

for all $b \in \mathbb{R}_+$ and all $i = 1, \ldots, n$. By a similar argument, and advantage aversion we have a $\beta_i \geq 0$ such that

$$e(b_i0) = \beta_i b$$

for all $b \in \mathbb{R}_-$ and all $i = 1, \ldots, n$. By anonymity we have $\alpha_j = \alpha_i \equiv \alpha$ and $\beta_j = \beta_i \equiv \beta$ for all $i, j$.

For all $a \in \mathbb{R}$, all distributions $x$, and all individuals $i$ we have that $a_i0$ is covalent with $(x_0)x$. Therefore, by repeated application of covalent additivity we have

$$e(x) \sim x = x_0 + \sum_{i=1}^{n}(d_i(x))_i0$$

$$\sim x_0 + e((d_1(x))_10) + \sum_{i=2}^{n}(d_i(x))_i0$$

$$\vdots$$

$$\sim x_0 + \sum_{i=1}^{n}e((d_i(x))_i0)$$

$$= x_0 - \alpha \sum_{i=1}^{n} \max\{x_i - x_0, 0\} - \beta \sum_{i=1}^{n} \max\{x_0 - x_i, 0\}.$$ 

It follows that

$$e(x) = x_0 - \alpha \sum_{i=1}^{n} \max\{x_i - x_0, 0\} - \beta \sum_{i=1}^{n} \max\{x_0 - x_i, 0\}.$$ 

From constant monotonicity we know that $e$ represents preferences. \qed
Proof of Corollary 2.4

If individual 0 is self-rational then for all $\mu > 0$.

$$\mu - n \beta \mu = U(\mu_0) > U(0) = 0.$$ 

Thus, $\beta < 1/n$.

Individual 0 prefers advantages over disadvantages if $\mu_i0 \preceq (-\mu)_i0$ for all $\mu \geq 0$. We have $\mu_i0 \preceq (-\mu)_i0$ if and only if $-\alpha \mu \leq -\beta \mu$ if and only if $\alpha \geq \beta$. \hfill \Box

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