Classification of totally umbilical slant submanifolds of a Kenmotsu manifold

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Abstract

The purpose of this paper is to classify totally umbilical slant submanifolds of a Kenmotsu manifold. We prove that a totally umbilical slant submanifold $M$ of a Kenmotsu manifold $\bar{M}$ is either invariant or anti-invariant or $\dim M = 1$ or the mean curvature vector $H$ of $M$ lies in the invariant normal subbundle. Moreover, we find with an example that every totally umbilical proper slant submanifold is totally geodesic.

1 Introduction

Slant submanifolds of an almost Hermitian manifold were defined by Chen as a natural generalization of both holomorphic and totally real submanifolds [6]. On the other hand, A. Lotta [15] has introduced the notion of slant immersions into almost contact metric manifolds and obtained the results of fundamental importance. He has also studied the intrinsic geometry of 3-dimensional non anti-invariant slant submanifolds of $K$-contact manifolds [16]. Later on, Cabreroiz et. al [3] studied the geometry of slant submanifolds in more specialized settings of $K$-contact and Sasakian manifolds and obtained many interesting results.

On the other hand, in 1954, J.A. Schouten studied the totally umbilical submanifolds and proved that every totally umbilical submanifold of $\dim \geq 4$ in a conformally flat space is conformally flat [17]. After that many authors studied the geometrical aspects of these submanifolds in different settings, including those of [1, 4, 5, 8, 9, 18]. In this paper, we consider $M$, a totally umbilical slant submanifold tangent to the structure vector field $\xi$ of a Kenmotsu manifold $\bar{M}$ and obtain a classification result that either (i) $M$ is anti-invariant or (ii) $\dim M = 1$ or (iii) $H \in \Gamma(\mu)$, where $\mu$ is the invariant normal subbundle under $\phi$. We also prove that every totally umbilical proper slant submanifold is totally geodesic. To this end, we provide an example to justify our results.

2 Preliminaries

A $(2n + 1)$-dimensional manifold $(M, g)$ is said to be an almost contact metric manifold if it admits an endomorphism $\phi$ of its tangent bundle $TM$, a vector field $\xi$, called structure vector field and $\eta$, the dual 1-form of $\xi$ satisfying the following [2]:

$$\phi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \phi(\xi) = 0, \quad \eta \circ \phi = 0 \quad (2.1)$$

and

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad \eta(X) = g(X, \xi) \quad (2.2)$$

2010 Mathematics Subject Classification. 53C40, 53C42, 53B25.

Key words and Phrases. totally umbilical, totally geodesic, mean curvature, slant submanifold, Kenmotsu manifold.
for any $X, Y$ tangent to $\bar{M}$. An almost contact metric manifold is known to be Kenmotsu manifold [12] if
\[
(\bar{\nabla}_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X
\]
and
\[
\bar{\nabla}_X \xi = X - \eta(X)\xi
\]
for any vector fields $X, Y$ on $\bar{M}$, where $\bar{\nabla}$ denotes the Riemannian connection with respect to $g$.

Now, let $M$ be a submanifold of $\bar{M}$. We will denote by $\nabla$, the induced Riemannian connection on $M$ and $g$, the Riemannian metric on $\bar{M}$ as well as the metric induced on $M$, respectively and $\nabla^\perp$ the induced connection on $T^\perp M$. Denote by $\mathcal{F}(M)$ the algebra of smooth functions on $M$ and by $\Gamma(TM)$ the $\mathcal{F}(M)$-module of smooth sections of $TM$ over $M$. Then the Gauss and Weingarten formulas are given by
\[
\bar{\nabla}_X Y = \nabla_X Y + h(X, Y)
\]
\[
\bar{\nabla}_X N = -A_N X + \nabla^\perp_X N,
\]
for each $X, Y \in \Gamma(TM)$ and $N \in \Gamma(T^\perp M)$, where $h$ and $A_N$ are the second fundamental form and the shape operator (corresponding to the normal vector field $N$) respectively for the immersion of $M$ into $\bar{M}$. They are related as
\[
g(h(X, Y), N) = g(A_N X, Y).
\]

Now, for any $X \in \Gamma(TM)$, we write
\[
\phi X = TX + FX,
\]
where $TX$ and $FX$ are the tangential and normal components of $\phi X$, respectively. Similarly for any $N \in \Gamma(T^\perp M)$, we have
\[
\phi N = tN + fN,
\]
where $tN$ (resp. $fN$) is the tangential (resp. normal) component of $\phi N$.

From (2.1) and (2.8), it is easy to observe that for each $X, Y \in \Gamma(TM)$
\[
g(TX, Y) = -g(X, TY).
\]

The covariant derivatives of the endomorphisms $\phi$, $T$ and $F$ are defined respectively as
\[
(\bar{\nabla}_X \phi)Y = \bar{\nabla}_X \phi Y - \phi \bar{\nabla}_X Y, \ \forall X, Y \in \Gamma(\bar{T}\bar{M})
\]
\[
(\bar{\nabla}_X T)Y = \bar{\nabla}_X TY - T\bar{\nabla}_X Y, \ \forall X, Y \in \Gamma(TM)
\]
\[
(\bar{\nabla}_X F)Y = \bar{\nabla}_X^\perp FY - F\bar{\nabla}_X Y \ \forall X, Y \in \Gamma(TM).
\]

Throughout, the structure vector field $\xi$ assumed to be tangential to $M$, otherwise $M$ is simply anti-invariant [15]. For any $X \in \Gamma(TM)$, on using (2.4) and (2.5), we may obtain
\[
(a) \ \bar{\nabla}_X \xi = X - \eta(X)\xi, \quad (b) \ h(X, \xi) = 0.
\]
On using (2.3), (2.5), (2.6), (2.8), (2.9) and (2.11)-(2.13), we obtain

\[ (\nabla_X T)Y = g(TX, Y)\xi - \eta(Y)TX + A_{FY}X + th(X, Y) \]  

(2.15)

\[ (\nabla_X F)Y = fh(X, Y) - h(X, TY) - \eta(Y)FX. \]  

(2.16)

A submanifold \( M \) of an almost contact metric manifold \( \bar{M} \) is said to be totally umbilical if

\[ h(X, Y) = g(X, Y)H, \]  

(2.17)

where \( H \) is the mean curvature vector of \( M \). Furthermore, if \( h(X, Y) = 0 \), for all \( X, Y \in \Gamma(TM) \), then \( M \) is said to be totally geodesic and if \( H = 0 \), then \( M \) is minimal in \( \bar{M} \).

For a totally umbilical submanifold \( M \) tangent to the structure vector field \( \xi \) of a Kenmotsu manifold \( \bar{M} \), we have

\[ g(X, \xi)H = 0, \quad \forall X \in \Gamma(TM). \]  

(2.18)

There are two possible cases arise, hence we conclude the following:

Case (i): When \( X \) and \( \xi \) are linearly dependent, i.e., \( X = \alpha \xi \), for some non-zero \( \alpha \in \mathbb{R} \), then \( g(X, \xi) = \alpha \). In this case, from (2.18), we get \( H = 0 \) with \( \dim M = 1 \), which is trivial case of totally geodesic submanifold of unit dimension.

Case (ii): When \( X \) and \( \xi \) are orthogonal, then from (2.18), it is not necessary that \( H = 0 \), which is the case has to be discussed for totally umbilical submanifolds.

In the following section, we will discuss all possible cases of totally umbilical slant submanifolds.

### 3 Slant submanifolds

A submanifold \( M \) tangent to the structure vector filed \( \xi \) of an almost contact metric manifold \( \bar{M} \) is said to be slant submanifold if for any \( x \in M \) and \( X \in T_x M - \langle \xi \rangle \), the angle between \( \phi X \) and \( T_x M \) is constant. The constant angle \( \theta \in [0, \pi/2] \) is then called slant angle of \( M \) in \( \bar{M} \). Thus, for a slant submanifold \( M \), the tangent bundle \( TM \) is decomposed as

\[ TM = D \oplus \langle \xi \rangle \]

where the orthogonal complementary distribution \( D \) of \( \langle \xi \rangle \) is known as slant distribution on \( M \). The normal bundle \( T^\perp M \) of \( M \) is decomposed as

\[ T^\perp M = F(TM) \oplus \mu, \]

where \( \mu \) is the invariant normal subbundle with respect to to \( \phi \) orthogonal to \( F(TM) \).

For a proper slant submanifold \( M \) of an almost contact metric manifold \( \bar{M} \) with the slant angle \( \theta \), Lotta [15] proved that

\[ T^2X = -\cos^2 \theta(X - \eta(X)\xi) \]  

(3.1)

for any \( X \in \Gamma(TM) \).

Recently, Cabrerizo et. al [3] extended the above result into a characterization for a slant submanifold in a contact metric manifold. In fact, they have
obtained the following theorem.

**Theorem 3.1**[3] Let $M$ be a submanifold of an almost contact metric manifold $\tilde{M}$ such that $\xi \in T M$. Then $M$ is slant if and only if there exists a constant $\lambda \in [0, 1]$ such that

$$T^2 = \lambda(-I + \eta \otimes \xi).$$

Furthermore, in such a case, if $\theta$ is slant angle, then it satisfies that $\lambda = \cos^2 \theta$.

Hence, for a slant submanifold $M$ of an almost contact metric manifold $\tilde{M}$, the following relations are consequences of the above theorem.

$$g(TX, TY) = \cos^2 \theta[g(X, Y) - \eta(X)\eta(Y)]$$

$$g(FX, FY) = \sin^2 \theta[g(X, Y) - \eta(X)\eta(Y)]$$

for any $X, Y \in \Gamma(TM)$.

In the following theorem we consider $M$ as a totally umbilical slant submanifold of a Kenmotsu manifold $\tilde{M}$.

**Theorem 3.2** Let $M$ be a totally umbilical slant submanifold of a Kenmotsu manifold $\tilde{M}$. Then at least one of the following statements is true

(i) $M$ is invariant

(ii) $M$ is anti-invariant

(iii) $M$ is totally geodesic

(iv) $\dim M = 1$

(v) If $M$ is proper slant, then $H \in \Gamma(\mu)$

where $H$ is the mean curvature vector of $M$.

**Proof.** As $M$ is totally umbilical slant submanifold, then we have

$$h(TX, TX) = g(TX, TX)H = \cos^2 \theta \{\|X\|^2 - \eta^2(X)\}H.$$

Using (2.5), we obtain

$$\cos^2 \theta \{\|X\|^2 - \eta^2(X)\}H = \nabla_T X T X - \nabla_T X T X.$$

Then from (2.8), we get

$$\cos^2 \theta \{\|X\|^2 - \eta^2(X)\}H = \nabla_T X \phi X - \nabla_T X FX - \nabla_T X TX.$$

By (2.6) and (2.11), we derive

$$\cos^2 \theta \{\|X\|^2 - \eta^2(X)\}H = (\nabla_T X \phi X + \phi \nabla_T X X + A_F X T X - \nabla_T X FX - \nabla_T X TX.$$

Using (2.3) and (2.5), we obtain

$$\cos^2 \theta \{\|X\|^2 - \eta^2(X)\}H = g(\phi TX, X)\xi - \eta(X)\phi TX + \phi(\nabla_T X X + h(X, TX))$$
From (2.8), (2.10), (2.17) and the fact that $X$ and $TX$ are orthogonal vector fields on $M$, we arrive at

$$
cos^2 \theta \{\|X\|^2 - \eta^2(X)\} H = -g(TX, TX)\xi - \eta(X)FTX + T\nabla_{TX}X
+ F\nabla_{TX}X + A_{FX}TX - \nabla_{TX}^\perp FXY - \nabla_{TX}TX.
$$

Then, using (3.2) and (3.3), we get

$$
cos^2 \theta \{\|X\|^2 - \eta^2(X)\} H = -cos^2 \theta \{\|X\|^2 - \eta^2(X)\} \xi - cos^2 \theta \eta(X)\{\xi - \eta(X)\xi\}
- \eta(X)FTX + T\nabla_{TX}X + F\nabla_{TX}X
+ A_{FX}TX - \nabla_{TX}^\perp FXY - \nabla_{TX}TX.
$$

(3.5)

Taking the inner product with $TX$ in (3.5), for any $X \in \Gamma(TM)$, we obtain

$$
0 = g(T\nabla_{TX}X, TX) + g(A_{FX}TX, TX) - g(\nabla_{TX}TX, TX).
$$

(3.6)

Now, we compute the first and last term of (3.6) as follows

$$
g(\nabla_{TX}TX, TX) = \cos^2 \theta \{g(\nabla_{TX}X, X) - \eta(X)g(\nabla_{TX}X, \xi)\}.
$$

(3.7)

Also, we have

$$
g(\nabla_{TX}TX, TX) = g(\nabla_{TX}TX, TX).
$$

Using the property of Riemannian connection the above equation will be

$$
g(\nabla_{TX}TX, TX) = \frac{1}{2} TX g(TX, TX) = \frac{1}{2} TX \{\cos^2 \theta \{g(X, X) - \eta(X)\eta(X)\}\}.
$$

Again by the property of Riemannian connection, we derive

$$
g(\nabla_{TX}TX, TX) = \cos^2 \theta \{g(\nabla_{TX}X, X) - \eta(X)g(\nabla_{TX}X, \xi)\}
- \cos^2 \theta \eta(X)g(\nabla_{TX}\xi, X).
$$

(3.8)

Using (2.4) and the fact that $X$ and $TX$ are orthogonal vector fields on $M$, the last term of (3.8) is identically zero, then by (2.5), we obtain

$$
g(\nabla_{TX}TX, TX) = \cos^2 \theta \{g(\nabla_{TX}X, X) - \eta(X)g(\nabla_{TX}X, \xi)\}.
$$

(3.9)

Thus, from (3.7) and (3.9), we get

$$
g(T\nabla_{TX}X, TX) = g(\nabla_{TX}TX, TX).
$$

(3.10)

Using this fact in (3.6), we obtain

$$
0 = g(A_{FX}X, TX) = g(h(TX, TX), FX).
$$

As $M$ is totally umbilical slant, then from (2.17) and (3.3), we get

$$
0 = \cos^2 \theta \{\|X\|^2 - \eta^2(X)\} g(H, FX).
$$

(3.11)

Thus, from (3.11), we conclude that either $\theta = \pi/2$, that is $M$ is anti-invariant which part (ii) or the vector field $X$ is parallel to the structure vector field.
ξ, i.e., $M$ is 1-dimensional submanifold which is fourth part of the theorem or
$H \perp FX$, for all $X \in \Gamma(TM)$, i.e., $H \in \Gamma(\mu)$ which is the last part of the thorem
or $H = 0$, i.e., $M$ is totally geodesic which is (iii) or $FX = 0$, $\forall X \in \Gamma(TM)$,
i.e., $M$ is invariant which is part (i). This proves the theorem completely. ■

Now, if we consider $M$, a proper slant submanifold of a Kenmotsu manifold $\bar{M}$, then neither $M$
is invariant nor anti-invariant (by definition of proper slant)
and also neither $\dim M = 1$. Hence, by the above result, only possibility is that
$H \in \Gamma(\mu)$ for a totally umbilical proper slant submanifold. Thus, we prove the
following main result.

**Theorem 3.3** Every totally umbilical proper slant submanifold of a Kenmotsu
manifold is totally geodesic.

**Proof.** Let $M$ be a totally umbilical proper slant submanifold of a Kenmotsu
manifold $\bar{M}$, then for any $X,Y \in \Gamma(TM)$, we have

$$\nabla_X \phi Y - \phi \nabla_X Y = g(\phi X, Y)\xi - \eta(Y)\phi X.$$  

From (2.5) and (2.8), we obtain

$$\nabla_X T Y + \nabla_X F Y - \phi(\nabla_X Y + h(X, Y)) = g(TX, Y)\xi - \eta(Y)TX - \eta(Y)FX.$$  

Again using (2.5), (2.6) and (2.8), we get

$$g(TX, Y)\xi - \eta(Y)TX - \eta(Y)FX = \nabla_X T Y + h(X, TY) - A_{FY} X$$

$$+ \nabla_X FY - T\nabla_X Y - F\nabla_X Y - \phi h(X, Y).$$

As $M$ is totally umbilical, then

$$g(TX, Y)\xi - \eta(Y)TX - \eta(Y)FX = \nabla_X T Y + g(X, TY)H - A_{FY} X + \nabla_X FY$$

$$- T\nabla_X Y - F\nabla_X Y - g(X, Y)\phi H. \quad (2.12)$$

Taking the inner product with $\phi H$ in (3.12) and using the fact that $H \in \Gamma(\mu)$,
we obtain

$$g(\nabla_X FY, \phi H) = g(X, Y)\|H\|^2.$$  

Using (2.6) and the property of Riemannian connection, the above equation takes the form

$$g(FY, \nabla_X \phi H) = -g(X, Y)\|H\|^2. \quad (3.13)$$

Now, for any $X \in \Gamma(TM)$, we have

$$\nabla_X \phi H = (\nabla_X \phi)H + \phi \nabla_X H.$$  

Using (2.3), (2.6), (2.8) and the fact that $H \in \Gamma(\mu)$, we obtain

$$-A_{\phi H} X + \nabla_X \phi H = -TA_H X - FA_H X + \phi \nabla_X H. \quad (3.14)$$

Also, for any $X \in \Gamma(TM)$, we have

$$g(\nabla_X^H, FX) = g(\nabla_X H, FX)$$

$$= -g(H, \nabla_X FX).$$
Using (2.8), we get
\[ g(\nabla^\perp_X H, F X) = -g(H, \nabla_X \phi X) + g(H, \nabla_X P X). \]
Then from (2.5) and (2.11), we derive
\[ g(\nabla^\perp_X H, F X) = -g(H, (\bar{\nabla}_X \phi) X) - g(H, \phi \bar{\nabla}_X X) + g(H, (\bar{\nabla}_X \phi) X). \]
Using (2.3) and (2.17), the first and last term of right hand side of the above equation are identically zero and hence by (2.2), the second term gives
\[ g(\nabla^\perp_X H, F X) = g(\phi H, \bar{\nabla}_X X). \]
Again, using (2.5) and (2.17), finally we obtain
\[ g(\nabla^\perp_X H, F X) = g(\phi H, H)\|X\|^2 = 0. \]
This means that
\[ \nabla^\perp_X H \in \Gamma(\mu). \] (3.15)
Now, taking the inner product in (3.14) with \( FY \), for any \( Y \in \Gamma(TM) \), we get
\[ g(\nabla^\perp_X \phi H, F Y) = -g(F A_H X, F Y) + g(\phi \nabla^\perp_X H, F Y). \]
Using (3.15), the last term of the right hand side of the above equation will be zero and then from (3.4), (3.13), we obtain
\[ g(X, Y)\|H\|^2 = \sin^2 \theta \{g(A_H X, Y) - \eta(Y)g(A_H X, \xi)\}. \] (3.16)
Hence, by (2.7) and (2.17), the above equation reduces to
\[ g(X, Y)\|H\|^2 = \sin^2 \theta \{g(X, Y)\|H\|^2 - \eta(Y)g(h(X, \xi), H)\}. \] (3.17)
Since, for a Kenmotsu manifold \( \bar{M} \), \( h(X, \xi) = 0 \), for any \( X \) tangent to \( \bar{M} \), thus we obtain
\[ g(X, Y)\|H\|^2 = \sin^2 \theta g(X, Y)\|H\|^2. \]
Therefore, the above equation can be written as
\[ \cos^2 \theta g(X, Y)\|H\|^2 = 0. \] (3.18)
Since, \( M \) is proper slant, thus from (3.18), we conclude that \( H = 0 \) i.e., \( M \) is totally geodesic in \( \bar{M} \). This completes the proof of the theorem. \( \blacksquare \)

Now, we give the following counter example of totally geodesic submanifold of \( R^5 \).

**Example 3.1** Consider a 3–dimensional proper slant submanifold with the slant angle \( \theta \in [0, \pi/2] \) of \( R^5 \) with its usual Kenmotsu structure
\[ x(u, v, t) = 2(u \cos \theta, u \sin \theta, v, 0, t). \]
If we denote by \( M \) a slant submanifold, then its tangent space \( TM \) span by the vectors
\[ e_1 = \frac{\partial}{\partial u} + 2 \cos \theta y^1 \frac{\partial}{\partial t} + 2 \sin \theta y^2 \frac{\partial}{\partial t}, \]
\begin{align*}
e_2 &= \frac{\partial}{\partial v} = 2 \frac{\partial}{\partial y^1}, & e_3 &= \frac{\partial}{\partial t} = \xi.
\end{align*}

Moreover, the vector fields
\begin{align*}
e_1^* &= -2 \sin \theta \left( \frac{\partial}{\partial x^1} + y^1 \frac{\partial}{\partial t} \right) + 2 \cos \theta \left( \frac{\partial}{\partial x^2} + y^2 \frac{\partial}{\partial t} \right),
\end{align*}
\begin{align*}
e_2^* &= 2 \frac{\partial}{\partial y^2},
\end{align*}
form the basis of $T^\perp M$. Furthermore, using Koszul’s formula, we get $\nabla_{e_i} e_i = -e_3 = -\xi$, $i = 1, 2$ and when $i \neq j$, then $\nabla_{e_i} e_j = 0$, for $i, j = 1, 2, 3$. Also, $\nabla_{e_3} e_3 = 0$, thus, from Gauss formula and (2.14), we obtain
\begin{align*}
h(e_1, e_1) &= 0, & h(e_2, e_2) &= 0, & h(e_1, e_2) &= 0, & h(e_2, e_3) &= 0
\end{align*}
and hence we conclude that $M$ is totally geodesic.

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