Study of Double Drell-Yan Process
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Abstract
This study focuses on the multiple parton scattering theory as a background for the new measurement of the inclusive cross section of the vector boson pair production at the LHC energies. The process under study is the double Drell-Yan annihilation. In this case, two quark-antiquark annihilations occur independently in one proton-proton annihilation. The final state with two pairs of leptons (electron or muon pair) is investigated while the intermedial vector boson can be both gamma and Z.

1. Introduction
The traditional hadron collider experiment studies are usually based on the concept where one parton from every hadron takes place in the head-on collision of the colliding particles. With larger energies, the accelerated hadron is a composite object consisting of many partons from the sea. This increases the probability that two or more independent hard parton-parton scatterings can simultaneously appear in the same hadron-hadron interaction. This mechanism is generally called multiple parton scattering (MPS).

In fact, there are two kinds of processes called multiple parton scattering, see Fig. 1. The disconnected scattering was described in the previous paragraphs. During rescattering, one parton from the first-beam hadron interacts with two or more partons from the second-beam hadron. This process is significant for heavy ion collisions but is highly suppressed at proton-proton collisions and will not be further involved.

If all the parton interactions occur independently then the number of hard scatterings \( h \) at a given impact parameter \( b \) and CMS energy \( \sqrt{s} \) of the given hadron-hadron interaction follows the Poissonian statistics:

\[
P_h(b, s) = \frac{< n(b, s) >^h}{h!} e^{-< n(b, s) >},
\]

where \( < n(b, s) > \) is the average number hard scatterings and can be later expressed as (7).

The additional parton scatterings proceed mostly via soft or semi-hard interactions at energies that have been reached up to now, but the chance of hard scattering will rise with the increase of the c.m.s. energy (like 7, 10, and 14 TeV planned for the LHC) and even faster than the classical single parton scattering production cross section.

2. Double Parton Scattering
The double parton scattering (DPS) is the simplest case of the MPS. Figure 2 shows two unspecified hard parton interactions taking part in one hadron-hadron interaction, and they are not connected or correlated in any kinematic observable. The processes are separated in transverse space by a distance of the magnitude of the hadron radius. The source of possible correlations would be an emission of virtual connecting (C) gluons by interacting partons. The C-gluons connect the hard processes (dotted lines in Fig. 2), while the non-connecting (NC) gluons contribute to each hard process separately (curly lines in Fig. 2). The CDF collaboration analyzed a 16 pb¹ sample of \( p\overline{p} \to \gamma/\pi^0 + 3 \text{jets} + X \) data and did not find any apparent evidence of correlations in parton momentum fractions \( x \), nor any evidence of kinematic correlations in mass, \( p_T \), or \( p_z \).

Fig. 2. Schematic picture of double parton scattering with possible virtual gluon connections.

The multiple parton scattering processes offer a solution for the unitarity violation of the total integrated cross section for hard collisions which is an already known fact appearing in the region near the perturbative threshold. The total cross section calculated from the parton model, assuming only single parton scattering, predicts a constant rate in the whole perturbative transverse momentum range. However, there are many processes, especially multi-jet production, contributing over the predicted value, see Fig. 3. This increased activity can be explained by assuming the MPS.

Fig. 3. The rise of the jet-production cross section over the total cross section at a low transverse momentum region.
For instance, DPS can produce the same final state
(and with a comparable production rate) as the
standard single parton process with addition of perturbative
corrections (multi-jets events) or by t-channel quark exchange
(IVB boson pair production).

The importance of the MPS lies also in its ability to
provide information on the spatial distribution of par-
tons within the proton, as well as all other possible cor-
relations among them. Its observation could bring a new
parton concept complementary to the common descrip-
tion, through the single-parton distribution functions.

The DPS cross section can be factorized by the
double parton distribution function \( \Gamma(x_1, x_2; b) \) as
a function of two momentum fractions of the inter-
acting partons and of their relative transverse distance \( b \):

\[
\sigma_D = \frac{m}{\pi} \int d^2 b \Gamma_A(x_1, x_2; b) \sigma(x_1, x'_1) \sigma(x_2, x'_2) \Gamma_B(x'_1, x'_2; b)
\times dx_1 dx'_1 dx_2 dx'_2,
\]

where \( m = 1 \) for indistinguishable partons processes and \( m = 2 \) for distinguishable partons processes.

3. Hadron Matter Distribution

The above mentioned DPS cross section formula contains
the double parton distribution functions. Current models, based on the inclusiveness of the standard par-
ton distribution functions (PDF), allow the double parton
distributions to be factorized by standard PDF’s in
convolution with the function of \( b \) expressing the spatial
distribution of partons in the transverse plane and their overlap:

\[
\Gamma(x_1, x_2; b) = f(x_1) f(x_2) F(b).
\]

The next step is to express the total cross section,
where the number of parton interactions would appear
as the index of the sum from one to infinity. Using
the optical theorem and neglecting spin effects, the total
cross section can be related to the imaginary part of the
Fourier transform of the elastic scattering amplitude in
the impact parameter \( a(b, s) \):

\[
\sigma_{\text{tot}} = 4\pi \int d^2 b Im (a(b, s)) .
\]

The MPS model also stems from the eikonal approxima-
tion framework. Here, the elastic amplitude can be
written down in terms of the eikonal function \( \chi(b, s) \) as

\[
a(b, s) = e^{-\chi(b, s)} - 1
\]

and similarly the inelastic part of the total cross section:

\[
\sigma_{\text{hard}} = \pi \int_0^\infty d^2 b \left[ 1 - e^{-2\chi(b, s)} \right] .
\]

The assumption of the independency of individual par-
ton interactions leads to the Poissonian model, as men-
tioned above. The average number of secondary hard
scatterings is thus given by:

\[
< n(b, s) > = F(b) \sigma_{\text{hard}}(s) .
\]

The evaluation of the overlap function \( F(b) \) then de-
dpends on the model of the internal hadron structure. Figure 4 explains the parameters used in the Eq. 8
where the overlap function is expressed as a convolution
of the form factor distributions of two incoming hadrons:

\[
F(b) = \int d^2 b' \rho_A(b') \rho_B(b - b').
\]
The prediction of the effective cross section value differs according to the model of hadron structure. The first intuitive estimation uses the average hadron radius $r \approx 0.6 fm$. The $\sigma_{eff}$ would be approximately $30 mb$. The most promising model currently is the double Gaussian model that predicts $\sigma_{eff} \approx 11 mb$ [3]. Table 1, summarizes the effective cross section measurements by experiments at CERN and Fermilab [4], [5], [6], [7].

| Experiment | $\sqrt{s}$ (GeV) | Final State | $\sigma_{eff}$ (mb) |
|------------|-----------------|-------------|---------------------|
| AFS, 1986  | 63              | 4 jets      | $\approx 5$         |
| UA2, 1991  | 630             | 4 jets      | $\geq 8.3 (95\% C.L.)$ |
| CDF, 1993  | 1800            | 4 jets      | $12.1^{+10.7}_{-5.4}$ |
| CDF, 1997  | 1800            | $\gamma + 3 jets$ | $14.5 \pm 1.7^{+1.7}_{-2.3}$ |

5. Double Drell-Yan Process

![Fig. 5. The DPS mechanism of $\gamma^*/Z$ pair production.](image)

The possibility of creation of a neutral vector boson pair via the double parton scattering mechanism has already been suggested many years ago [8], [9]. In this process, the two independent s-channel quark-antiquark annihilations occur in one hadron-hadron interaction, see Fig. 5.

On the basis of the previous theoretical description, the double Drell-Yan production cross section is predicted to be approximately $\sigma_{DDY} = \frac{\sigma_{Z}^2}{\sigma_{eff}} \approx \sigma_{Z} 10^{-7} \approx 0.1 fb$, where only the electron decay channel is taken into account. The single parton production cross section was taken to be $\sigma_{Z} \approx 1.7 nb$.

The motivation for this study was the promise of the clearest signal in the detector [10] in comparison to the multi-jet processes, even at the cost of a small cross section.

The source of the data for this preliminary analysis was the Herwig++ 2.3.0 Monte Carlo event generator [11]. The model of MPS is already implemented in Herwig++. The studied process consisted of the primary hard subprocess of type $M_epqZ2ll$ and of the additional hard subprocess of the same type with the multiplicity equal to one. The full matrix element was used. The final state was created by the two electron-positron pairs. The MPS model of Herwig++ performs the calculation of the number of sub-processes independently and adds the general QCD $2 \rightarrow 2$ matrix element to them.

The simulation was performed for proton-proton collisions at $\sqrt{s} = 10 TeV$. The distribution of the total number of parton interactions for 5000 proton-proton events is shown in Fig. 6. The shape of the distribution corresponds to the Poissonian distribution with a little longer tail. The average number of interactions has been found to be four.

Other kinematical distributions seem to be similar to the single parton but are not investigated in detail in this stage of the study. There are few suspicions that the Monte Carlo implementation of the MPS model still needs to be tuned. The production cross section is not provided because the MPS is treated as the underlying event activity and can not be set as part of the production cross section calculation.

![Fig. 6. Distribution of total number of parton interactions for 5000 proton-proton events at $\sqrt{s} = 10 TeV$. There are always at least two $q\bar{q} \rightarrow \gamma^*/Z \rightarrow e^+ e^-$ interactions.](image)

6. Summary and Conclusions

Multiple parton scattering was experimentally measured by several collaborations and its existence was proven. The previous experiments aimed their attention at the multi-jet final states or at photon-plus-jets final state, where the production cross section is sufficiently large and the MPS processes contribute significantly especially at a low $p_T$ region. The motivation to search for the other processes, like vector boson pair production, is increased by the approaching LHC era.

The theoretical model describing MPS is basically developed, but it suffers from many sources of uncertainty. Especially the internal parton correlations, which is one of the unknown effects that needs to be investigated further. MPS is also a unique tool for geometrical coefficient measurement. It may resolve the size of hard/soft component of the proton, and may bring about new many-body parton distribution functions.

The preliminary predictions for the double Drell-Yan process shows a very low cross section for the lepton decay channel and it will not be measurable at LHC. For the same reason, it will not be a source of background for Higgs or for di-boson production.

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