Abstract: The models for soft colour interactions and colour string re-interactions, implemented in the Monte Carlo program LEPTO, are investigated regarding hadronic final states in inclusive and diffractive deep inelastic scattering.

1 Introduction

The hadronic final state in inclusive and diffractive deep inelastic scattering (DIS) can give a better understanding of the interplay between soft and hard processes in QCD. Whereas hard interactions are well described by perturbative QCD, soft interactions are not calculable within perturbation theory. Instead more phenomenological models are used to transform the perturbative partonic final state into an observable hadronic final state. It is normally assumed that the colour topology of an event is given by the planar approximation in perturbation theory, so that terms of order $1/N_C^2$ are neglected, and that this topology is not altered by soft interactions.

The models for soft colour interactions (SCI) \cite{1} and the generalised area law (GAL) \cite{2} for colour string re-interactions try to model additional soft colour exchanges which neither belong to the perturbative treatment nor the conventional hadronisation models. These soft colour exchanges can alter the colour topology and thereby produce a different final state, including such phenomena as large rapidity gaps and diffraction, as illustrated in Fig. 1.

In these models there is no sharp distinction between inclusive and diffractive events, which is the case in Regge-inspired models. Instead, there is a continuous transition between the different final states. The common assumption for the two models is that the soft colour exchanges factorises from the hard interactions which can therefore be described by standard perturbative methods, i.e. with matrix elements and parton showers. It is also assumed that compared to the perturbative interactions the momenta in the soft colour exchanges can be neglected and that their effect will be washed out by the hadronisation.

In this note we investigate the hadronic final states in inclusive and diffractive DIS resulting from the SCI and GAL models as implemented in the Monte Carlo program LEPTO \cite{3}. In section 2 we give a short review of the two models. In section 3 we show how the diffractive structure function can be used to fix the amount of soft colour exchanges in the two models and compare with data on the hadronic final state in diffractive events (the X-system). Section 4 then compares the two models with data on inclusive hadronic final states. Finally, in section 5 we summarise and conclude.

\cite{1} In proceedings ‘Monte Carlo generators for HERA physics’, DESY-PROC-1999-02, www.desy.de/\~heramc
2 Models for soft colour exchanges

The basic assumption of the soft colour interaction (SCI) model \cite{1} is that the partons produced in the hard interaction can have soft colour exchanges with the background colour field of the incoming hadron or hadrons. These exchanges can change the colour topology of the event as illustrated in Fig. 1. The probability for a soft colour exchange depends on non-perturbative dynamics and is thus not calculable at present and for simplicity it is therefore assumed to be a constant in the SCI model. Its value, $R = 0.5$, is obtained by comparing the model with the diffractive structure function in DIS. As long as the SCI model represents interactions with a colour background field, it should only be applied to reactions with initial state hadrons.

Apart from being applicable in DIS the SCI model has also been successfully used to describe the surprisingly large quarkonium cross sections observed at the Fermilab Tevatron \cite{4}. A first comparison with quarkonium photoproduction at HERA is presented in \cite{5}. In addition the model describes diffractive $W$ and jet production at the Tevatron \cite{6, 7, 8}.

The generalised area law (GAL) model \cite{2} for colour string re-interactions is similar in spirit to the SCI model in that it is a model for soft colour exchanges. The main difference is that the GAL model is formulated in terms of interactions between the strings connecting the partons produced in an event. Thus the GAL model is also applicable for hadronic final states in $e^+e^-$, since it treats string re-interactions and should apply to all interactions producing strings.

Another important feature of the GAL model is that the probability for an interaction is not constant as in the SCI model. Instead there is a dynamical suppression factor giving the probability $R = R_0 \exp(-b\Delta A)$ for a string reconnection, where $\Delta A$ is the difference between the areas in momentum space spanned by the strings in the two alternative string configurations and $b$ is one of the hadronisation parameters in the Lund model \cite{9}.

The parameters of the GAL model were obtained \cite{2} by making a simultaneous tuning to the diffractive structure function in DIS and the charged particle multiplicity distribution and momentum distribution for $\pi^\pm$ in $e^+e^-$ annihilation at the Z-resonance. This resulted in $R_0 = 0.1$, $b = 0.45$ GeV$^{-2}$ and $Q_0 = 2$ GeV, where $Q_0$ is the cut-off for initial and final state parton showers. It is not possible to have the JETSET default cut-off $Q_0 = 1$ GeV in the parton
showers and simultaneously reproduce the multiplicity distribution. One might worry that the obtained cut-off is relatively large compared to the default value. However, it is not obvious that perturbation theory should be valid for so small scales when more exclusive final states are considered. Therefore, $Q_0$ can be considered as a free parameter describing the boundary below which it is more fruitful to describe the fragmentation process in terms of strings instead of perturbative partons.

Both the SCI and GAL models have been implemented in the LSCI routine in the Monte Carlo program LEPTO \[4\]. For the GAL model one also needs a new version of subroutine LEPTO, see the GAL homepage [http://www3.tsl.uu.se/thep/rathsman/gal] for details.

### 3 Hadronic final states in diffractive DIS

The diffractive structure function in DIS was obtained from the SCI and GAL models using a subroutine from the HzTool package \[10\] and the CTEQ4 leading order parton distributions \[11\]. The results are compared with H1 data \[12\] in Fig. 2. The normalization parameters in the models, $R$ and $R_0$ respectively, were determined from this data. The default version of LEPTO was used, except for the GAL model having the modified values of the cut-off in the parton showers and the hadronisation parameter $b$. In addition, version 2 of the sea-quark treatment (see \[1\]) was used for the GAL model with the width of the mean virtuality set to 0.44 GeV. However, the result is not sensitive to this choice.

The agreement between the resulting diffractive structure function calculated from the two models and H1 data is quite good as is shown in Fig. 2, especially if one takes into account that there is only one free parameter in the models. The variables \(x_F \simeq \frac{Q^2 + M^2}{Q^2 + W^2}\) and \(\beta \simeq \frac{Q^2}{Q^2 + M_X^2}\) are defined in terms of observable invariants that do not require interpretation within a particular model. As usual, $Q^2$ is the photon virtuality and $W$ the mass of the complete hadronic system. $M_X^2 = Q^2(x_F - x_F) = Q^2 x_F$ is the mass of the diffractive system $X$.

The Regge framework requires pomeron exchange at small $x_F$ and other Regge exchanges in the transition region $0.01 < x_F < 0.1$, whereas the SCI and GAL models describes the whole region in a more economic way. The GAL model fails only for small $M_X$ which are not included in the model because of the cut-off $M_X^2 > 4 \text{ GeV}^2$ in the matrix-element. The SCI model also gives a good description of the data except for small $Q^2$ and small $M_X^2$. The reason for the SCI model overshooting the data at small $Q^2$ is probably related to the typically small number of perturbative partons produced at small $Q^2$. This in turn means that effectively the probability for a rapidity gap becomes larger. In the extreme case of only four partons in the final state the probability for a rapidity gap in the SCI model is $R = 0.5$ since there are only two possible string configurations.

One may ask whether this kind of soft colour exchange models are essentially models for the pomeron. This is not the case as long as no pomeron or Regge dynamics is introduced. The behaviour of the data on $F_2^D(\beta, Q^2)$, usually called the pomeron structure function, is in the SCI/GAL models understood as normal perturbative QCD evolution in the proton. The rise with $\ln Q^2$ also at larger $\beta$ is simply the normal behaviour at the small momentum fraction $x = \beta x_F$ of the parton in the proton. Here, $x_F$ is only an extra variable related to the gap size or $M_X$ which does not require a pomeron interpretation. The flat $\beta$-dependence of $x_F F_2^D = \frac{2}{\beta} F_2^D$ is due to the factor $x$ compensating the well-known increase at small-$x$ of the proton structure function $F_2$. For details of this and a general review of diffractive hard scattering see \[13\].
Figure 2: The diffractive structure function $x_F G^D_2(Q^2, \beta, x_F)$ versus $x_F$ for bins in $\beta$ and $Q^2$. H1 data [12] compared to the results of the GAL (full curve) and SCI (dashed) models in Lepto. The hashed plots correspond to $M_X < 2$ GeV not included properly in the models due to the matrix element cut-off.
With the free parameters of the two models fixed from the diffractive structure function the models can be tested by comparing with the hadronic final state in diffractive events. The energy flow in Fig. 3a demonstrates that both models give a reasonable description of the data, with the SCI model doing slightly better. The ‘seagull’ plot in Fig. 3b also shows that the SCI model is very close to data and that the GAL model gives a reasonable description although the transverse activity is on the high side.

Figure 3: (a) Energy flow versus pseudo-rapidity in diffractive H1 events \[14\]. (b) Seagull plot of mean transverse momentum squared versus Feynman-\(x\) in diffractive H1 events with \(18 < M_X < 30\) GeV \[14\]. The cms of the diffractive \(X\)-system is used and the curves are from the GAL (full) and SCI (dashed) models. Event selection: \(7.5 < Q^2 < 100\) GeV\(^2\), \(0.05 < y < 0.6\), \(x_F < 0.025\).

There are many other observables in diffractive events to which the models could be compared; in particular those related to the proton remnant system, such as \(t\)-dependence, momentum distribution for leading protons and neutrons etc. However, these observables are not directly related to the hadronic final state in the \(X\)-system and depend on a different part of the model contained in LEPTO. Therefore we do not study such observables here. They deserve a dedicated investigation as initiated in \[1\].
4 Inclusive hadronic final states

With both models giving a good description of the hadronic final states in diffractive events it is imperative to check that they also can describe the inclusive hadronic final states in DIS. Energy flows in the hadronic cms is an important observable which we have investigated earlier \[1\] and H1 has recently made a comprehensive comparison of their data with several models \[15\]. However, a more detailed test is obtained by looking at the $p_{\perp}$-spectrum for charged particles which is sensitive to the distribution of transverse energy and not only the average. We therefore consider this and other observables in the following.

![Figure 4: $Q^2$-dependence of scaled momentum $x_p$ of hadrons in the current region of the Breit frame in DIS.](image)

A good starting point for such an investigation is the momentum distribution of particles in the current region of the Breit frame. This part of phase-space is expected to be well described by the models since it should not be affected by the proton remnant and therefore be similar to $e^+e^-$-annihilation. The distribution of scaled momentum $x_p = 2|\vec{p}|/Q$ in this system is shown in Fig. 4. Although the overall agreement between the ZEUS data \[16\] and the models is reasonable, it is clear that the SCI model gives too many soft particles (low $x_p$) and too few
hard (high $x_p$) ones. The GAL model and also LEPTO without string topology rearrangements, describes the details of the data quite well.

The pseudo-rapidity distribution of charged particles in the detectable regions of the hadronic cms is shown in Fig. 5. Again the SCI model gives too many soft particles, whereas the GAL model is much closer to data and even better than LEPTO without reconnections.
Figure 6: Pseudo-rapidity distribution of charged particles with $p_T > 1$ GeV in the hadronic cms of inclusive DIS events.

Looking at the pseudo-rapidity distribution of charged particles with $p_T$ larger than 1 GeV changes the picture as shown in Fig. 6. Now both models as well as LEPTO without string reconnections give too few particles in the central region. Thus one should not expect either version of LEPTO to give the correct average transverse energy flow unless the lack of high-$p_T$ particles is compensated by too many soft ones. From this one might be tempted to draw the conclusion that the cascade in LEPTO gives the wrong $p_T$ distribution. However, this need not
Figure 7: The $p_{\perp}$ distribution in inclusive DIS events with large energy in central region, $E(0 < \eta < 2) > 6$ GeV.

be the case. The $p_{\perp}$ distribution in Fig. 7 for events with large energy in the central region is well described by the GAL model and essentially also by LEPTO without reconnections. Thus the $p_{\perp}$ distribution is well reproduced by the cascade but there are too few events with large energy in the forward region. For the SCI model, on the other hand, more forward energy is made up of soft particles from 'zig-zag' shaped strings resulting in a too soft $p_{\perp}$ distribution.
Another instructive observable is the energy-energy correlation which in $e^+e^-$ annihilation has been useful to study the internal structure of jets. In DIS one defines the transverse energy-energy correlation $\Omega(\omega) = 1/N_{\text{event}} \sum_{\text{events}} \sum_{i \neq j} E_\perp i E_\perp j/Q^2 (1 - y)$ between pairs $(ij)$ of hadrons separated a distance $\omega_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$. Fig. 8 shows this correlation in the two models and without reconnections compared to data from H1 [18]. The SCI model has the wrong shape since the correlation is smeared out due to the formation of ‘zig-zag’ shaped strings. The suppression of such ‘long’ strings in GAL avoids this and produces a reasonably good description of the data.

5 Summary and conclusions

We have shown that both the SCI and GAL models give satisfactory descriptions of the diffractive structure function and of more detailed hadronic properties of the $X$-system such as the energy flow and the seagull plot. However, when comparing with detailed properties of inclusive DIS final states it is clear that the SCI model fails in some respects, whereas the GAL model gives a description which is as good as or better than LEPTO without string reconnections. Specifically, the SCI model gives too many soft particles both in current and target regions in the Breit frame whereas the GAL model gives a good description of soft particles but has too few particles with large $p_\perp$, just as when having no reconnections, which results in the average transverse energy flow being too low compared to data [15]. At the same time the GAL model gives a reasonable description of the $p_\perp$-distribution in events with large energy in the central region. Thus it is too few events with high-$p_\perp$ emissions that is the problem and not the modelling of the fragmentation process. In other words it is the cross-section for hard emissions that is too small in the model. This may be partly cured by adding resolved photon contributions as in RAPGAP [19]. From the energy-energy correlations it is also clear that the SCI model smears out the energy-energy correlations by making the string go ‘zig-zag’, whereas GAL only has minor effects on the energy-energy correlation.
One may consider whether the shortcomings of the SCI model are genuine or can be tuned away. The problems of giving too many soft particles is related to events where the string after SCI goes back-and-forth producing a zig-zag shape, *i.e.* a longer string. Hadronisation will then produce more, but softer hadrons. This helps to reproduce the inclusive transverse energy flow [1, 15], but make the agreement with some of the above observables worse. In principle one may be able to tune the hadronisation parameters to recover a good description of the data. We have chosen not to attempt this, since that would be against the principle of having a universal hadronisation model, with the same parameter values in DIS and $e^+e^-$. A possible way out for the SCI model could be to think of it not as interactions with a background field, but taking place generally between all partons in any type of event. Then it should also apply to $e^+e^-$ annihilation and the modified string topologies would require a retuning of the hadronisation parameters in JETSET in order to fit data. Although this might improve the ability of the SCI model to describe DIS data, we have not embarked on such a road because it has no substantial theoretical justification. Another possibility would be to extend the SCI model with some dynamics that suppresses the probability to get longer strings, similarly to the GAL model.

The problem of too many soft hadrons is solved in the GAL model by suppressing the probability for long and thereby ‘zig-zag’ strings. At the same time the problem with too few particles with $p_T > 1$ GeV remains and thus the average transverse energy flow is below the data [15]. However, as already mentioned, the source of this problem is to be found in the matrix elements and parton showers describing the hard interactions and not in the soft hadronisation model.

In conclusion, it is far from easy to construct a single Monte Carlo model, based on reasonably physics input and few parameters, that can well describe all kinds of hadronic final states in all interactions. Nevertheless, this should be the goal.

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