A First Detection of the Acoustic Oscillation Phase Shift Expected from the Cosmic Neutrino Background

Brent Follin, Lloyd Knox, Marius Millea, and Zhen Pan

University of California, Davis

The freestreaming of cosmological neutrinos prior to recombination of the baryon-photon plasma alters gravitational potentials and therefore the details of the time-dependent gravitational driving of acoustic oscillations. We report here a first detection of the resulting shifts in the temporal phase of the oscillations, which we infer from their signature in the Cosmic Microwave Background (CMB) temperature power spectrum. The magnitude of the shift is proportional to the fraction of the total radiation density in neutrinos. Parameterizing the shift via an effective number of neutrino species we find $1.9 < N_{\nu}^{\text{eff}} < 3.4$ at 68% confidence, consistent with the standard model value of $N_{\nu} = 3.046$, and inconsistent with zero.

PACS numbers: 14.60.Lm, 98.70.Vc, 95.85.Ry, 98.80.Es

Introduction: The hot, dense conditions of the early universe included a thermal background of photons. Initially scattering with a high rate off of the free electrons in the plasma, they eventually were able to stream freely as the plasma cooled to $k_B T \sim 0.3$ eV, the density of free electrons plummeted, and the mean free path became larger than the extent of the observable universe. Today we observe these photons, about 90% of which last scattered at this epoch, as the cosmic microwave background (CMB), stretched by expansion into millimeter wavelengths.

These same hot and dense conditions led to a cosmic neutrino background (CNB) that contributes nearly as much as photons to the total energy density in the early universe. The neutrinos began to freely stream at $k_B T \sim$ MeV, and continue to freely stream through the cosmos to this day. Unlike with photons, direct detection of the CNB is exceedingly difficult [1].

The CNB has been detected indirectly, through its gravitational influences on light element abundances [2] and the angular power spectrum of CMB anisotropies [3] [4] [5]. Here we report a first detection of an effect that arises from the speed-of-light propagation of neutrino perturbations – a temporal phase shift to the nearly harmonic oscillations of standing waves in the primordial plasma.

The power spectrum is $C_\ell \equiv \langle |a_{\ell m}|^2 \rangle$ where $T(\theta, \phi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \phi)$ is a decomposition of the temperature map into spherical harmonics and $\langle ... \rangle$ indicates an ensemble average. A $Y_{\ell m}$ undergoes $\ell$ oscillations every 360 degrees, so, e.g., $Y_{180m}$ are patterns with hot and cold spots separated by about 1°. We show model and measured power spectra in Fig. 1.

The series of peaks is what we expect from a collection of standing waves, all beginning their amplitude oscillations with zero initial momentum [13]. For example, the Fourier mode with spatial dependence $\delta \rho_\gamma \propto \cos(k \cdot \vec{x})$ will have a time dependent amplitude approximately given by $A \cos[k r_s(t)]$, where $r_s(t)$ is the distance a sound wave can travel by time $t$ (and we have ignored, for now, a time-dependent driving effect that alters this simple-harmonic motion solution). Modes with (comoving) wavelengths of $\sim 400$ Mpc project into $\ell \sim kd = 2\pi/300$ kpc where $d$ is the (comoving angular-diameter) distance to the last-scattering surface. These modes oscillate with a period such that they achieve their first extremum at the epoch of last scattering, $t = t_s$, and thus produce the first peak in the power spectrum. Modes with a wavelength of $\sim 220$ Mpc project into $\ell \approx 400$. They have a shorter oscillation period so that by $t = t_s$ they have gone through their first extremum and reached a null at the time of decoupling. Modes that hit their $p^{th}$ extremum at decoupling contribute to the $p^{th}$ peak.

A key length scale for understanding the response of $C_\ell$ to the CNB is the sound horizon at decoupling, $r_s(t_s)$. The sound horizon is smaller than it would be without the presence of the CNB, because the fractional expansion rate $H \propto \rho_\gamma^{1/2}$. If we scaled up the fractional expansion rate $H$ at all times by a factor $\alpha$, the decrease in time it takes for the temperature to drop to $T \approx 0.3$ eV would lead to $r_s \propto 1/\alpha$. The angle $r_s$ subtends, $\theta_s = r_s(t_s)/d$ where $d$ is the angular-diameter distance to the last-scattering surface, strongly influences the locations of the acoustic peaks such that $\delta \ell_p = \ell_p \delta \theta_s/\theta_s$. If we knew $d$, we could use this effect alone to measure the energy density in the CNB. However, $d$ depends on the (otherwise unknown) value of the cosmological constant.

In Fig. 1 we show a series of plots where we vary $N_\nu$ while holding certain other quantities fixed, in order to demonstrate the observable consequences of various effects of neutrinos. Because $\theta_s$, baryon density $\omega_b$, and the ratio of matter to radiation density $\rho_m/\rho_\gamma + \rho_\nu$ are well determined by the data, in all rows we show variations with these parameters fixed. In the top row one can see the impact of $N_\nu$, on the typical distance a photon diffuses prior to last scattering, $r_d$. This diffusion suppresses anisotropy for modes with wavelengths $\lambda \lesssim r_D$, with an approximate effect of $C_\ell \rightarrow D_\ell C_\ell$ where
rate as 1

\[ \text{diffusion is a stochastic process it scales with expansion } D, \text{ variation, one can see the subtle impact of the shifts in temporal phase } \phi. \]

The ratio of matter to radiation density \( \rho_m / (\rho_r + \rho_\nu) \) and the angular size of the sound horizon \( \theta_s \) are held fixed as these are well-determined by CMB data fairly independently of the assumed value of \( N_\nu \). In the top panel the dominant source of variation is the change in the damping scale \( \theta_D \) caused by the changes in \( N_\nu \). In the middle panel we fix \( \theta_D \) by varying the primordial fraction of baryonic mass in Helium appropriately, leaving the dominant source of power spectrum variation as the change in oscillation amplitude \( A' \). Finally in the bottom panel, with the spectra normalized to remove the effect of \( A' \) variation, one can see the subtle impact of the shifts in temporal phase \( \phi \). The data points are the 2013 Planck data.

\[ D_\ell \simeq \exp \left[ - (\ell \theta_D)^{1.18} \right] \] where \( \theta_D = r_D / d \). Because the diffusion is a stochastic process it scales with expansion rate as \( 1 / \sqrt{\alpha} \) rather than \( 1 / \alpha \) as \( r_\sigma \) does. These different scalings mean that while we adjust \( d \) to keep \( \theta_s \) fixed, we get \( \theta_D \propto \sqrt{\alpha} \). Thus for \( N_\nu = 5 \), the expansion rate is greater, leading to larger \( \theta_D \) and more damping.

To visualize more subtle effects of the CNB, we can vary the primordial fraction of baryonic mass in Helium, \( Y_p \), to keep \( \theta_D \) fixed as well [6]. Doing so in the middle panel, we can see an impact of the perturbations in the CNB. As an initially over-dense region compresses under the influence of gravity, the compression does not occur rapidly enough to prevent the gravitational potential from decaying due to the expansion-driven drop in density. By the time pressure gradients halt the compression, the gravitational potential has nearly completely decayed. The result of this temporary time-dependent gravitational driving of the acoustic oscillations is a change to the subsequent amplitude and phase so that the amplitude of a standing wave is \( A' \cos[kr_\sigma(t) + \phi] \). The values of \( A' / A \) and \( \phi \) depend on the details of the potential decay, and in particular on the fraction of the radiation that can freely stream out of over densities at the speed of light. We can see in the middle row that increasing \( N_\nu \) decreases \( A' / A \).

Finally, in the bottom row we normalize the spectra to remove the effect of changing \( A' / A \) so that the change in

\[
\delta A' = (A'_{N_\nu=5} - A'_{N_\nu=1}) / A'_{N_\nu=1}, \quad \delta \phi = (\phi_{N_\nu=5} - \phi_{N_\nu=1}) / \phi_{N_\nu=1},
\]

values of \( \phi \) is more evident. As \( \phi \) changes, the value of \( k = k_\nu \) for which \( k_\nu r_\sigma(t_\nu) + \phi = \pi \) changes so \( \ell_\nu \simeq k_\nu d \) changes. The net result is \( \delta \ell_\nu = -\delta \phi / \theta_s \). In this Letter we show that these subtle shifts are detectable with the Planck data.

We are not the first to detect the influence of neutrino perturbations. Previous work approximated the effects of neutrino perturbations using the generalized dark matter formalism of Hu [7], where the perturbative effects of neutrinos are approximated in the parameters \( c_\text{eff}^2 \) and \( c_\text{vis}^2 \), both of which equal \( 1 / 3 \) for standard free-streaming neutrinos. The most recent constraints are consistent with these values [2] and inconsistent with zero. Here we confirm the detectability of the effects of neutrino perturbations with a more direct observational probe—through directly determining the phase shift generated by neutrino free streaming.

**Template fitting:** To quantify the sensitivity of the data to the expected phase shift we must be able to artificially increase or decrease it, independent of other effects of the CNB. Since the phase shift effects are most observable deep in the damping tail, we work with an (approximately) undamped spectrum

\[ K_\ell \equiv (\ell + 1) / (2\pi) C_\ell D_\ell^{-1}, \tag{1} \]

where \( D_\ell^{-1} \) approximately undoes the damping expected for three neutrino species and standard Big Bang Nucleosynthesis. We then define a transformation that sends

\[ \omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \] fixed, \[ \delta \theta_D \] + \[ \delta \phi / \theta_s \].

\[ + \left[ \frac{\omega_\nu}{\rho_r + \rho_\nu}, \theta_s \right] \text{ fixed}, \delta \theta_D \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]

\[ \left. \begin{array}{c}
\omega_\nu, \frac{\rho_m}{\rho_r + \rho_\nu}, \theta_s \text{ fixed }, \delta \theta_D \\
+ \theta_s \text{ fixed }, \delta \theta_D \end{array} \right\} \]
$K_{\ell} \rightarrow K_{\ell+\delta \ell_{\nu}}$, which is controlled by a new parameter $N_{\nu}^{\delta \phi}$. Following the analytic work of Bashinsky and Seljak [6] we take the amplitude of $\delta \ell_{\nu}$ to be linear in the fraction of radiation energy density in neutrinos, $R(N_{\nu})$, so that

$$\delta \ell_{\nu} = A(N_{\nu}^{\delta \phi}, N_{\nu}) f_{\ell},$$

with $N_{\nu}^{\delta \phi}$ defined such that

$$A(N_{\nu}^{\delta \phi}, N_{\nu}) \propto R(N_{\nu}^{\delta \phi}) - R(N_{\nu}),$$

with proportionality chosen to match the shift from the standard 3.046 to one neutrino specie.

The analytic result of [6] was obtained by ignoring the presence of any pressureless matter, an approximation that becomes increasingly valid as one considers modes that began oscillating deeper in the radiation-dominated era. With this approximation they find a $\delta \phi$ (and therefore $f_{\ell}$ in our language) that is independent of $k$ and therefore independent of $\ell$. To avoid this approximation we have numerically calculated $\delta \ell_{\nu}$ from the $C_{\ell}$ calculated with the Boltzmann equation solver CLASS [8] for a suite of models with varying $N_{\nu}$. To define the suite of models we compare two cosmologies: one from the ΛCDM posterior of the 2013 Planck data, and the other with nonstandard $N_{\nu}$ drawn from $1 < N_{\nu} < 6$. We isolate the effects of neutrinos from the effects of the other components contributing to gravitational potentials, as well as effects of changing $N_{\nu}$ to the background Hubble rate $H(z)$, by following Hou et al. [3] in fixing the baryon density $\omega_b$, redshift of matter-radiation equality $z_{eq}$, Silk damping scale $\theta_D$, and sound horizon scale $\theta_s$, as well as the spectral amplitude and tilt describing the adiabatic initial conditions from inflation between the two models. Since contributions to anisotropy arising after recombination project differently, we zero out contributions from the integrated Sachs Wolfe (ISW) effect.

We sample 100 different cosmology pairs, and find that in the region of parameter space explored by these models, the phase shift is well captured by a linear response proportional to the fraction of radiation density in free-streaming neutrinos. The multipole dependence is well described by a logarithmic template, which is jointly sampled with cosmology against both the March 2013 Planck temperature data and the measured phase shifts in the 100 cosmology pairs. Posterior samples of the template are shown in Fig. 2, along with the measured phase shifts $\delta \ell_{\nu}$ at the peaks in the ISW-less temperature power spectrum $K_{\ell}^{TT}$, as well as, for visualization, the locations of the corresponding peaks in the polarization spectrum $K_{\ell}^{EE}$, which is not used in the fit.

To apply this template to models we confront with the data, we first decompose the temperature power spectrum $C_{\ell}$ into ISW and ISW-less components: $C_{\ell} = C_{\ell}^{\text{no ISW}} + C_{\ell}^{\text{ISW}} + C_{\ell}^{\text{cross}}$, with $C_{\ell}^{\text{cross}}$ the (ISW) × (no ISW) contribution. The artificial $\ell$–space shift we introduce is given by

$$K_{\ell} \rightarrow K_{\ell+\delta \ell_{\nu}} = K_{\ell}^{\text{no ISW}} + K_{\ell}^{\text{ISW}} + K_{\ell}^{\text{cross}}$$

with the implied definitions for the various $K_{\ell}$ components. The artificial $\ell$–space shift alters the power spectrum away from that of the physical model which always has $N_{\nu} = N_{\nu}^{\delta \phi}$, for which $A = 0$. To vary just the phase shift effect, we can set $N_{\nu} = 3.046$, the fiducial value, while varying $N_{\nu}^{\delta \phi}$.

Results from Planck: We use the publically available likelihood code click [9] to determine constraints from the 2013 Planck temperature power spectrum measurements, with the polarization constraints approximated as a Gaussian prior on the optical depth to last scattering $\tau$ for simplicity. We place uniform (flat) priors on the parameters $N_{\nu}$ and $N_{\nu}^{\delta \phi}$, which results in a flat prior for the physical case where $N_{\nu} = N_{\nu}^{\delta \phi}$. We explore the model space using the MCMC routines provided by the Python library CosmoSlik [14].

In Fig. 3 we show the constraint on $N_{\nu}^{\delta \phi}$ from the ΛCDM + $N_{\nu}^{\delta \phi}$ model. For comparison, we include the constraints on the ΛCDM + $N_{\nu}$ model space, with the phenomenological amplitude $A(N_{\nu}^{\delta \phi}, N_{\nu}) \equiv 0$. We find best-fit values of $N_{\nu} = 3.3^{\pm 0.4}_{-0.2}$ [15] and $N_{\nu}^{\delta \phi} = 2.3^{\pm 0.4}_{-0.1}$. In the ΛCDM + $N_{\nu}^{\delta \phi}$ model space, we can exclude $N_{\nu}^{\delta \phi} = 0$ at greater than one part in $10^7$, corresponding to a detection of the neutrino phase shift at greater than 4.5$\sigma$. We explore the model
While the constraint on neutrino number from the phase shift is much weaker than that due to more pronounced effects of neutrinos on the CMB, such as Silk damping, importantly the phase shift effect is largely decoupled from other $\Lambda$CDM parameters.

While letting $N_\nu$ and $N_\nu^{\delta \phi}$ vary independently (top of Fig. 3), the width of the constraint on $N_\nu$ is dominated by an $n_\nu$--$N_\nu$ degeneracy: fixing $n_\nu$ results in roughly a halving of the characteristic width of the posterior in the $N_\nu$ direction. For the $N_\nu^{\delta \phi}$ direction, no such correlations exist, so while our constraints on $N_\nu$ depend somewhat on the characterization of initial conditions due to inflation (as well as, in extended models such one with the helium fraction $Y_p$ free), our constraint on $N_\nu^{\delta \phi}$ is due to a feature in the data that is difficult to mimic with other cosmological degrees of freedom. In addition, we find the constraints on $N_\nu$ and $N_\nu^{\delta \phi}$ are nearly uncorrelated. This lack of correlation follows from the fact that the response of $C_l$ to changing the phase shift is essentially orthogonal to the response due to other observable effects of the cosmic neutrino background in the CMB.

Finally, we note that there is a slight dependence on priors for the 2D posterior shown in Fig. 3. If we switch from uniform priors on the number of neutrino species $N_\nu$ and $N_\nu^{\delta \phi}$ to their corresponding neutrino fractions $R(N_\nu)$ and $R(N_\nu^{\delta \phi})$, the average value of the posterior shifts down by $(\Delta N_\nu = 0.3, \Delta N_\nu^{\delta \phi} = 0.5)$, a shift of slightly more than 0.5$\sigma$ in both directions. This is predominantly due to a contraction in the high probability region at high $N_\nu$ or $N_\nu^{\delta \phi}$. Regardless of prior, $N_\nu^{\delta \phi} = 0$ is heavily disfavored.

**Conclusions:** In this letter, we present the first detection of the temporal phase shift generated by neutrino perturbations during the acoustic oscillation phase of cosmological evolution, and find an effect consistent with the standard value associated with the three known neutrino species. We show this feature, and its degeneracy with other effects of neutrinos, plays an important role in the CMB constraints on the effective number of neutrino species $N_\nu$. Importantly, the constraints on the number of neutrino species is effectively decoupled from other $\Lambda$CDM parameters, providing an unambiguous signature of free streaming neutrinos from the CMB.

Were we to have found $N_\nu \neq N_\nu^{\delta \phi}$ then we would be compelled to look for a physical explanation, such as $\nu$-$\nu$ scattering which could inhibit free streaming [10]. With current data, we see consistency with the standard model. Future experiments, such as *Planck* polarization data [11] and CMB stage-4 [12] will provide even stronger constraints on the phase shift, either providing a signature of new physics or increasing confidence in the standard cosmological model.

---

[1] S. Betts, W. R. Blanchard, R. H. Carnevale, C. Chang, C. Chen, S. Chidzik, L. Ciebiera, P. Clossner, A. Cocco, A. Cohen, J. Dong, R. Klemmer, M. Komor, C. Gentile, B. Harrop, A. Hopkins, N. Jarosik, G. Mangano, M. Messina, B. Osherson, Y. Raitses, W. Sands, M. Schaefer, J. Taylor, C. G. Tully, R. Woolley, and A. Zwicker, arXiv:1307.4738 [astro-ph, physics:physics] (2013) arXiv: 1307.4738.

[2] G. Steigman, Advances in High Energy Physics **2012**, 1 (2012) arXiv: 1208.0032.

[3] Z. Hou, R. Keisler, L. Knox, M. Millea, and C. Reichardt, Physical Review D **87**, 083008 (2013)

[4] The Planck Collaboration, arXiv:1502.01589 [astro-ph] (2015) arXiv: 1502.01589.

[5] E. Komatsu, K. M. Smith, J. Dunkley, C. L. Bennett, B. Gold, G. Hinshaw, N. Jarosik, D. Larson, M. R. Nolta, L. Page, D. N. Spergel, M. Halpern, R. S. Hill, A. Kogut, M. Limon, S. S. Meyer, N. Odegard, G. S. Tucker, J. L. Weiland, E. Wollack, and E. L. Wright, Astrophys. J. **192**, 18 (2011) arXiv:1001.4538 [astro-ph.CO].

[6] S. Bashinsky and U. Seljak, Physical Review D **69** (2004), 10.1103/PhysRevD.69.083002 arXiv: astro-ph/0310198.

[7] W. Hu, The Astrophysical Journal **506**, 485 (1998) arXiv: astro-ph/9801234.

[8] D. Blas, J. Lesgourgues, and T. Tram, arXiv:1104.2933

---

FIG. 3: **Top:** 2D constraints on the jointly varying $\Lambda$CDM+$N_\nu+N_\nu^{\delta \phi}$ parameter space. The constraints on $N_\nu$ (damping) and $N_\nu^{\delta \phi}$ (phase shift) are essentially orthogonal. **Bottom:** Constraints from March 2013 *Planck* temperature power spectrum measurements on the number of neutrino species from (1) blue/solid: varying $N_\nu^{\delta \phi}$ while holding $N_\nu$ fixed at three and (2) green/dashed: varying along the physical direction $N_\nu = N_\nu^{\delta \phi}$. The constraints assume a Gaussian $\tau$ prior of mean $\mu = 0.085$ and width $\sigma = 0.015$.
[9] The Planck Collaboration, Astronomy & Astrophysics 571, A1 (2014), arXiv: 1303.5062.

[10] F.-Y. Cyr-Racine and K. Sigurdson, Phys. Rev. D 90, 123533 (2014) [arXiv:1306.1536].

[11] The Planck Collaboration, [arXiv:astro-ph/0604069] (2006) arXiv: astro-ph/0604069.

[12] K. N. Abazajian, K. Arnold, J. Austermann, B. A. Benson, C. Bischoff, J. Bock, J. R. Bond, J. Borrill, E. Calabrese, J. E. Carlstrom, C. S. Carvalho, C. L. Chang, H. C. Chiang, S. Church, A. Cooray, T. M. Crawford, K. S. Dawson, S. Das, M. J. Devlin, M. Dobbs, S. Dodson, O. Dore, J. Dunkley, J. Errard, A. Fraisse, J. Gallicchio, N. W. Halverson, S. Hanany, S. D. Hildebrandt, A. Hincks, R. Hlozek, G. Holder, W. L. Holzapfel, K. Honscheid, W. Hu, J. Hubmayr, K. Irwin, W. C. Jones, M. Kamionkowski, B. Keating, R. Keisler, L. Knox, E. Komatsu, J. Kovac, C.-L. Kuo, C. Lawrence, A. T. Lee, E. Leitch, E. Linder, P. Lubin, J. McMahon, A. Miller, L. Newburgh, M. D. Niemack, H. Nguyen, H. T. Nguyen, L. Page, C. Pryke, C. L. Reichardt, J. E. Ruhl, N. Sehgal, U. Seljak, J.Sievers, E. Silverstein, A. Slosar, K. M. Smith, D. Spergel, S. T. Staggs, A. Stark, R. Stompor, A. G. Vieregg, G. Wang, S. Watson, E. J. Wollack, W. L. K. Wu, K. W. Yoon, and O. Zahn, arXiv:1309.5383 [astro-ph, physics:hep-ph] (2013) arXiv: 1309.5383.

[13] These are the “initial conditions” generated by the simplest models of inflation.

[14] https://github.com/marius311/cosmoslik.git

[15] This approximation results in a $\sim 10\%$ widening of the constraint on $N_\nu$ in the standard model extended to arbitrary number of neutrino species when compared to the full CMB likelihood, which includes WMAP9 polarization.