Localization of Matter and Cosmological Constant on a Brane in Anti de Sitter Space

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Abstract

We study two issues, the localization of various spin fields, and the problem of the cosmological constant on a brane in five-dimensional anti de Sitter space. We find that spin-zero fields are localized on a positive-tension brane. In addition to the localized zero-mode there is a continuous tower of states with no mass gap. Spin one-half and three-half states can be localized on a brane with “negative tension”. Their localization can be achieved on the positive-tension brane as well, if additional interactions are introduced. The necessary ingredient of the scenario with localized gravity is the relation between the bulk cosmological constant and the brane tension. In the absence of supersymmetry this implies fine-tuning between the parameters of the theory. To deal with this issue we introduce a four-form gauge field. This gives an additional arbitrary contribution to the bulk cosmological constant. As a result, the model gives rise to a continuous family of brane Universe solutions for generic values of the bulk cosmological constant and the brane tension. Among these solutions there is one with a zero four-dimensional cosmological constant.

\footnote{On leave of absence from J. Stefan Institute, Ljubljana, Slovenia}
1 Introduction and results

Large extra dimensions offer an opportunity for a new solution to the hierarchy problem [1]. The crucial ingredient of this scenario is a brane on which standard model particles are localized. Field-theoretic localization mechanisms for scalars and fermions [2] as well as for gauge bosons [3] were found. In string theory, fields can naturally be localized on D-branes due to the open strings ending on them [4] (for string theory realization of the scenario of Ref. [1], see, e.g., [5, 6]).

Up until recently, extra dimensions had to be compactified, since the localization mechanism for gravity was not known. It was suggested in Ref. [7] that gravitational interactions between particles on a brane in uncompactified five-dimensional space could have the correct four-dimensional Newtonian behavior, provided that the bulk cosmological constant and the brane tension are related. Recently, it was found by Randall and Sundrum that gravitons can be localized on a brane which separates two patches of AdS$_5$ space-time [8]. The necessary requirement for the four-dimensional brane Universe to be static is that the tension of the brane is fine-tuned to the bulk cosmological constant [7, 8]. The generalization of this framework to higher dimensions [9, 10, 11, 12], as well as a number of interesting phenomenological and cosmological issues were studied in the literature [13, 14, 15, 16, 17, 18].

As we mentioned above, there is a localized graviton zero-mode on the brane worldvolume [8]. In addition, there is a continuum of graviton states in the brane background [8]. All the fields in anti de Sitter space should be given boundary conditions. This can be accomplished by adopting the holographic approach to the problem [20]. In this framework, the bulk continuum is described by a certain Field Theory on the brane. This field theory should be coupled to a four-dimensional graviton which is nothing but the localized graviton zero-mode in the original AdS$_5$ picture. In a sense, the introduction of the brane generates the coupling of the (boundary) four-dimensional field theory (which is expected to describe the bulk gravitational physics in accordance with the Maldacena conjecture [21, 22, 23]), to the four-dimensional Einstein gravity. This framework sets uniquely the boundary conditions for the fields (see detailed discussions and references in [22]).

The aim of the present paper is to study localization of spin-0, 1, 1/2 and 3/2 fields on a 3-brane in the AdS$_5$ space. The study of these issues is crucial for phenomenological model-building as well as for the holographic description of the model. Besides, this can be useful for the task of realizing this scenario in a supergravity framework.

Another, seemingly different, but in fact closely related subject which we will discuss here is the problem of the four-dimensional cosmological constant. We will

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2 Field theory in anti de Sitter space needs careful treatment [19]. In general, there are closed timelike curves in this space which can be avoided by considering the universal cover of AdS space [19]. In the present work, we will adopt the simplified terminology calling the covering space AdS.

3 This, in fact, should be Conformal Field Theory since the isometry group of AdS$_5$ (which is $SO(2, 4)$) coincides with the conformal group of the brane (boundary) Minkowski space.
argue that the necessity of the fine-tuning between the bulk cosmological constant and the brane tension can be avoided by introducing a four-form gauge field.

In section 2 we study localization of various spin fields on a brane in AdS$_5$ space. We consider two different cases: a positive tension brane which localizes gravity (the Randall-Sundrum (RS) brane) and a “negative-tension” brane which gives rise to the exponentially growing warp factor. We show that a spin-0 massless field is localized on the RS brane. The emerging picture is similar to that of the localized graviton. There is a scalar zero-mode on the brane. In addition, there is a continuum of states with no mass gap. This fits well into the holographic approach mentioned above. Furthermore, we show that the “negative tension brane” with the exponentially rising warp factor does not allow to localize normalizable scalar zero-modes.

For spin-1 fields the conclusions are less promising. Neither the positive-tension nor the “negative-tension branes” are capable of localizing vector fields in the minimal setup. The way out for the phenomenological model-building would be to invoke either stringy mechanism of localization on a D-brane worldvolume, or, in the field theory framework, to use the Dvali-Shifman mechanism which is based on the bulk confinement. In either case the introduction of new bulk physics is necessary. Furthermore, we show that in the minimal setup neither spin-1/2 not spin-3/2 fermions are localized on the RS brane. Nevertheless, the localization of these fields can be achieved by introducing new interactions in the theory. This allows one to have localized chiral fermions on a brane. On the other hand, spin-1/2 and spin-3/2 fermions are localized by gravitational interactions on a brane with “negative tension” which has the exponentially rising warp factor. A peculiar feature of this mechanism is that the localized modes are not chiral.

In section 3 we discuss the fate of the four-dimensional cosmological constant. As we mentioned above, the scenarios of Refs. require a special fine-tuned relation of the bulk cosmological constant to the brane tension. Once this relation is enforced, the four-dimensional brane Universe is static, i.e., the four-dimensional cosmological constant is zero. Within the supergravity framework this relation is just the BPS condition. However, one should not expect that the BPS relation survives the quantum corrections after SUSY breaking. As a result, one eventually goes back to fine-tuning. We will show in section 3 that the necessity of the fine-tuning can be removed from the theory. This is accomplished by introducing a four-form gauge field which couples to gravity only. The four-form field does not propagate physical degrees of freedom in five-dimensional space, however, it can give rise to an arbitrary (negative or positive depending on the sign of its kinetic term) cosmological constant in the theory. Therefore, for any value of the original bulk cosmological constant, the net cosmological constant in this model is arbitrary. As a result, there are infinite number of brane Universe solutions. Each of these brane Universes is labeled by the corresponding four-dimensional cosmological constant. Among all

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4 The “negative-tension” brane is understood as a rigid non-dynamical slice of space with negative energy density.
these possible brane Universes there is one with a zero cosmological constant. This removes the fine-tuning problem from the Randall-Sundrum scenario in the sense that a static solution now exists for any value of the bulk cosmological constant and the brane tension. In addition, this framework opens an opportunity for anthropic arguments.

Note that many properties of a 3-brane in $D = 5$ are similar to those of a 2-brane in $D = 4$ which were previously studied in $[27]$ (for a review see $[28]$). These studies were recently generalized to $D - 2$ branes (domain walls) in $D$ dimensions $[29]$.

2 Localization via warp factors

We start with the five-dimensional gravity action and a cosmological constant (the $[+, -, -, -, -]$ signature will be assumed below)

$$S = -M^3 \int d^5x \left( R + 2\Lambda \right).$$

(1)

In addition one includes a static 3-brane with tension $T$ which is located at $y = 0$ $[8]$. The five-dimensional interval will be parametrized as follows:

$$ds^2 = A(y) \, dx_{3+1}^2 - dy^2.$$  

(2)

The Einstein equations in this case have two different solutions for the warp factor $A(y)$. For a positive-tension brane one finds $[8]$

$$A(y) = \exp (-H|y|),$$

(3)

while for a “negative-tension” brane the warp factor is exponentially rising $[7]$

$$A(y) = \exp (H|y|).$$

(4)

In both cases, the static solutions (3) and (4) exist if the bulk cosmological constant is fine-tuned to the brane tension

$$\sqrt{-\frac{2\Lambda}{3}} = \frac{|T|}{6M^3} \equiv H.$$  

(5)

We will show below, that the RS solution admits the localization of a spin-0 state in addition to the already known localization of gravitons. The solution (4), on the other hand, localizes spin-1/2 and spin-3/2 fermions.

Before we go further, let us make some comments on the literature. One can consider an orbifold compactification of the fifth dimension in the RS framework.

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5 As before, the “negative-tension” brane is a non-fluctuating slice of space with negative energy density. The exponentially rising warp factor was originally introduced in Ref. $[30]$ in a theory with no branes.
In this case, there are two 3-branes which are located at the orbifold fixed points \( \mathbb{R} \). One of these is a positive-tension brane and the other one is a negative-tension brane. The warp factor for one of them decreases as in \( \mathbb{R} \), while for the other one it increases exponentially \( \mathbb{R} \). Therefore, in this framework spin-0 and spin-2 states will be localized on the positive-tension brane, while spin-1/2 and spin-3/2 states will be trapped on the negative-tension one. In what follow we will be studying a single 3-brane which separates two patches of AdS\(_5\) space-time. From the point of view of the orbifold construction this corresponds to the case when one of the orbifold fixed points is removed to infinity.

### 2.1 Spin-0 fields

In this subsection we study the localization of a real scalar field in the background \( \mathbb{R} \). We will find that the solution \( \mathbb{R} \) admits a localized zero-mode.

Let us start with a massless scalar field coupled to gravity:

\[
\frac{1}{2} \int d^5x \sqrt{g} g^{AB} \partial_A \Phi \partial_B \Phi .
\]

(6)

Here \( A, B \) denote five-dimensional indices. The corresponding equation of motion takes the form

\[
\partial_A \left( \sqrt{g} g^{AB} \partial_B \Phi \right) = 0 .
\]

(7)

Using \( \mathbb{R} \) and decomposing this equation in the four-dimensional and fifth dimensional parts one finds:

\[
\frac{1}{A} \eta^{\mu\nu} \partial_\mu \partial_\nu \Phi - \frac{1}{A^2} \partial_y \left( A^2 \partial_y \Phi \right) = 0 .
\]

(8)

Let us decompose the field \( \Phi \) as follows:

\[
\Phi(x, y) = \phi(x) \chi(y) .
\]

(9)

A plane wave which propagates in a four-dimensional worldvolume satisfies the corresponding four-dimensional equation of motion:

\[
\eta^{\mu\nu} \partial_\mu \partial_\nu \phi(x) = -m^2 \phi(x) .
\]

(10)

Using this, and introducing a new variable

\[
u(y) = A(y) \chi(y) ,
\]

(11)

one gets the following Schrödinger equation for the \( y \)-dependent part:

\[
\left[ -\partial_y^2 - m^2 e^{H|y|} + H^2 - 2H \delta(y) \right] \nu(y) = 0 .
\]

(12)

This equation coincides with the one for a localized graviton \( \mathbb{R} \). The zero-mass solution \((m^2 = 0)\) to this equation takes the form:

\[
u(y) = c e^{-H|y|} ,
\]

(13)
where $c$ is a constant. It is interesting that, written in terms of the original variable $\chi$, the scalar zero-mode is just a constant
\[ \chi(y) = c. \]
(14)

At a first glance, such a solution could not be localized since it is not suppressed away from the brane. Nevertheless, the solution can still be considered as a localized mode. The presence of the exponential warp factor in the metric (2) allows one to perform the following decomposition for the zero-mode
\[ \frac{1}{2} \int d^4x dy \sqrt{g} g^{AB} \partial_A \Phi_0 \partial_B \Phi_0 = \frac{1}{2} \int_{-\infty}^{+\infty} dy \sqrt{g} \chi^2(y) \int d^4x \partial_\mu \phi(x) \partial_\nu \phi(x) \]
\[ = \frac{1}{2} \int d^4x \eta^{\mu \nu} \partial_\mu \phi(x) \partial_\nu \phi(x), \]
(15)
where we set $c = \sqrt{H/2}$.

Thus, for the zero-mode $\Phi_0(x,y) = \sqrt{H/2} \phi(x)$. Moreover, the field $\phi(x)$ is effectively “localized” on a brane worldvolume due to the exponentially decreasing warp factor. Notice that this zero-mode resembles a zero-mode of the standard Kaluza-Klein compactification. This is not surprising since the effective size of the fifth dimension, $L$, is finite, $L \sim 1/H$, in spite of the fact that this dimension is not compact [5]. Let us stress also that such a constant localized solution is possible only for the warp factor (3), but not for (4).

As in the case of gravitons [3], the equation (12) gives rise to a tower of continuum states with $m^2 > 0$. These states produce a nonzero contribution of order $1/r^3$ to the usual four-dimensional $1/r$ potential mediated by a massless scalar exchange on a brane. The picture is similar to that of the localized graviton [6].

2.2 Spin-1 fields

There are no localized solutions in this case. Although there is a constant solution similar to (14), nevertheless, the Lagrangian for a vector field does not yield the suppressing warp factor which was so crucial in the case of scalars. Let us see this in some details. Using the definition of the 5D field-strength
\[ F_{AB} = \partial_A V_B - \partial_B V_A, \]
choosing the gauge $V_5 = 0$, and decomposing the remaining part of the massless vector field as
\[ V_\mu(x,y) = v_\mu(x) \sigma(y), \]
(17)
we get $(f_{\mu \nu} \equiv \partial_\mu v_\nu - \partial_\nu v_\mu)$
\[ -\frac{1}{4} \int d^4x dy \sqrt{g} g^{AB} g^{CD} F_{AC} F_{BD} = -\frac{1}{4} \int_{-\infty}^{+\infty} dy \sqrt{\sigma^2(y)} \int d^4x f_{\mu \alpha} f_{\nu \beta} \]
\[ = -\frac{1}{4} \int d^4x \eta^{\mu \nu} \eta^{\alpha \beta} f_{\mu \alpha} f_{\nu \beta} \int_{-\infty}^{+\infty} dy \sigma^2(y). \]
(18)
This diverges if $\sigma(y)$ is a constant.

How about nontrivial solutions to the equation of motion for the vector field, could they give localized solutions? One can check that such normalizable solutions do not exist. Since this issue has already been studied in the orbifold version of the RS scenario in [31, 32] we skip these derivations. The net result is that neither (3) nor (4) localizes massless vector fields.

2.3 Spin-$1/2$ fermions

We start with the Lagrangian for massless spin-$1/2$ fermions

$$i\sqrt{g} \bar{\Psi} \Gamma^B D_B \Psi .$$

(19)

The corresponding equation of motion can be decomposed as follows:

$$\left( \Gamma^\mu D_\mu + \Gamma^5 D_5 \right) \Psi(x, y) = 0 .$$

(20)

The relations between the curved-space gamma matrices ($\{\Gamma^A, \Gamma^B\} = 2g^{AB}$) and the minkowskian ones ($\{\gamma^A, \gamma^B\} = 2\eta^{AB}$) read as follows:

$$\Gamma^\mu = \frac{1}{\sqrt{A}} \gamma^\mu , \quad \Gamma^5 = \gamma^5 .$$

(21)

The spin-connection and covariant derivative can also be calculated for the metric (2):

$$D_\mu = \partial_\mu + \frac{A'}{4A} \Gamma_\mu \Gamma^5 , \quad D_5 = \partial_5 .$$

(22)

After these conventions are set we can decompose the five-dimensional spinor into the four-dimensional and the fifth-dimensional parts: $\Psi(x, y) = \psi(x) \alpha(y)$. We require that the four-dimensional part satisfies the massless equation of motion $\gamma^\mu \partial_\mu \psi(x) = 0$. As a result, we obtain the following equation for the $y$ dependent part

$$\left( \partial_y + \frac{A'}{A} \right) \alpha(y) = 0 .$$

(23)

The solution to this equation reads:

$$\alpha(y) = \frac{c}{A(y)} .$$

(24)

This is not normalizable in the RS case (3), but it is normalizable if the warp factor (4) is used. To see this, one has to decompose the action (13) as follows:

$$\int d^4x \int_{-\infty}^{+\infty} dy \sqrt{g} \bar{\Psi}(x, y) i\Gamma^B D_B \Psi(x, y) =$$

$$\int_{-\infty}^{+\infty} dy \frac{A'^2}{A^2} \alpha^2(y) \int d^4x \bar{\psi}(x) i\gamma^\mu \partial_\mu \psi(x) ,$$
which is infinite for $A(y)$ defined in (3), but is finite for $A(y)$ from (4) (we choose $c = \sqrt{H/2}$ to get the correct normalization).

In order to localize spin-1/2 fermions in the RS framework one could use the method of localization due to Jackiw and Rebbi [34]. For this, one introduces the interaction of fermions with a scalar field, $\Phi \bar{\Psi} \Psi$. The scalar should interpolate between two different vacua at different sides of the brane (one could take the kink solution, for instance). This gives an effective five-dimensional mass to fermions (constant fermion mass approximation is good enough when the scalar is heavy). However, the five-dimensional fermion mass term $M \bar{\Psi} \Psi$ would flip the sign under the reflection with respect to the brane. Under these circumstances, the chiral left(right)-handed component of the fermion can be localized on the brane [34]. The equation of motion for the fermion takes the form:

$$\left( \partial_y + \frac{A'}{A} + i\gamma_5 M \left[ \theta(y) - \theta(-y) \right] \right) \Psi(x, y) = 0 ,$$

where $\theta(y)$ is the step-function. Introducing the chiral modes as $i\gamma_5 \Psi_{L,R} = \pm \Psi_{L,R}$, one finds that the chiral solution $\Psi_L \propto \exp(H - M)$ is localized for (3) as long as:

$$M > \frac{H}{4} .$$

Likewise, in the background (4), the mode $\Psi_L$ is localized if $M < H/4$.

### 2.4 Spin-3/2 fermions

The consideration for gravitinos is similar to that of spin-1/2 fermions. Therefore, our discussions will be brief. The equation of motion for a massless gravitino reads as:

$$\Gamma^A \Gamma^B \Gamma^C D_B \Psi_C = 0 .$$

Here, the square brackets denote antisymmetrization w.r.t. all indices. We choose the gauge $\Psi_5 = 0$ and split the remaining fields as $\Psi_\mu(x, y) = \bar{\psi}_\mu(x) u(y)$. Using now the four-dimensional gauge choice $\gamma^\mu \psi_\mu = \partial^\mu \bar{\psi}_\mu = 0$, and the four-dimensional equation of motion for a massless spin-3/2 field $\gamma^{[\mu} \gamma^{\nu} \gamma^{\sigma]} \partial_\nu \bar{\psi}_\sigma = 0$, one finds the following equation for the $y$ dependent part:

$$\left( \partial_y + \frac{A'}{2A} \right) u(y) = 0 .$$

The solution to this equation is

$$u(y) = \frac{c}{\sqrt{A(y)}} .$$

\[8\]

\[6\]After we obtained these results the work [33] appeared, in which it was shown that in the orbifold RS construction massless spin-1/2 fields are not localized on a brane.
In the RS case this is not a normalizable function. The action
\[
\int d^4x \int_{-\infty}^{+\infty} dy \sqrt{g} \Psi_A(x, y)i\Gamma^A\Gamma^B\Gamma^C D_B \Psi_C(x, y) =
\int_{-\infty}^{+\infty} dy A^{1/2}(y) u^2(y) \int d^4x \bar{\psi}_\mu(x) i\gamma^{[\mu} \gamma^{\nu} \gamma^\sigma] \partial_{\nu} \psi_{\sigma}(x),
\]
diverges because of the \( y \) integration. On the other hand, for the exponentially rising warp factor (4), the \( y \) integral is finite and the 4D action is canonically normalized for \( c = \sqrt{H}/2 \). Therefore, the solution (4) admits a localized free spin-3/2 zero-mode on the 3-brane. Notice that this mode is not chiral.

Finally, let us comment on a possibility of localization of gravitinos on the background (3). One should introduce some additional interactions for this to happen (as we did for the case of spin-1/2 fields). The simplest way would be to realize the RS solution (3) in some \( N = 2 \) five-dimensional supergravity. In this case, one could hope to find a BPS 3-brane which preserves half of the original supersymmetries. Since the background (3) localizes gravitons, by supersymmetry, its SUGRA counterpart would also localize gravitinos. The dynamical reason for the localization of gravitinos could be their interactions with some other fields of the corresponding SUGRA. Detailed studies of these issues could be a subject of a separate project.

3 Cosmological Constant on a Brane

In this section we study what happens if the bulk cosmological constant is not fine-tuned to its critical value which is defined by Eq. (5). This is expected to be the generic case in any realistic brane Universe models with broken SUSY. Indeed, after SUSY is broken, equation (3) can no longer be protected against corrections even though it could have been obtained as a BPS equation in some five-dimensional theory of supergravity (along the lines of [24], for instance). Therefore, in what follows we consider the case when the bulk cosmological constant differs from its critical value defined by (3). In such a case the four-dimensional brane Universe is not static any more [35, 36]. The four-dimensional cosmological constant is determined by the difference between the actual bulk cosmological constant and the critical cosmological constant satisfying (3). Generically, this difference can be big, leading to a de Sitter or anti de Sitter four-dimensional brane Universe with an unacceptably large four-dimensional cosmological constant. Thus, it seems that the fine-tuning is of vital importance. However, the fine-tuning of the parameters can be avoided by introducing into the theory a four-form gauge field \( A_{BCDE} \). This field cannot propagate any physical degrees of freedom in five-dimensions. Nevertheless, it gives rise to a dynamically generated cosmological constant [23, 26]. The value of this cosmological constant is arbitrary, since it appears as a constant of integration of the equation of motion for the \( A_{BCDE} \) field. Therefore, the net cosmological constant

\footnote{We are grateful to Gia Dvali for many useful discussions of the results of this section.}
in five-dimensions is the sum of the original cosmological constant and the dynamically generated one\footnote{Notice that it is important in our case that the four-form field does not couple to the brane itself, i.e., the brane does not carry the corresponding “Ramond-Ramond charge”, see discussions in the next section.}. Let us for simplicity assume that the original bulk cosmological constant $\Lambda$ is positive but otherwise an arbitrary number which is not necessarily fine-tuned to the brane tension (the generalization for negative $\Lambda$ is straightforward and will be given in the next section). The action for the system can be written as follows:

$$S = -M^3 \int d^5x \sqrt{g} \left( R + 2\Lambda \right) + \int d^5x \sqrt{g} \left\{ -\frac{1}{2 \times 5!} F_{ABCDE} F^{ABCDE} \right\} + S_{\text{Brane}} \ .$$

(31)

Here $F_{ABCDE}$ denotes the gauge-invariant field-strength for the field $A_{BCDE}$ (the choice of the sign of the kinetic term for this field will be discussed in the next section). The action for the brane itself in (31) is denoted by $S_{\text{Brane}}$. We will simply assume that the 3-brane is a static source which is localized at $y = 0$. It has the following energy-momentum tensor:

$$T_{AB}^{\text{Brane}} = T \delta(y) \text{ diag}(1, -1, -1, -1, 0) \ ,$$

where $T$ denotes the tension of the 3-brane.

The equations of motion of the system take the following form:

$$R_{AB} - \frac{1}{2} g_{AB} R = \frac{1}{2M^3} \left( T_{AB} + T_{AB}^{\text{Brane}} \right) + g_{AB} \Lambda \ ,$$

$$\partial_A \left( \sqrt{g} F^{ABCDE}(y) \right) = 0 \ .$$

(33)

Here $T_{AB}$ is the energy-momentum tensor for the four-form field:

$$T_{AB} = \frac{1}{4!} \left\{ -F_{ACDEG} F^CDEG_B + \frac{1}{10} g_{AB} F_{CDEGH} F^{CDEGH} \right\} \ .$$

(34)

As we discussed above, the crucial property of the four-form gauge field in five-dimensional space-time is that it does not propagate dynamical degrees of freedom. The whole dynamics is eliminated by the system of equations of motion, and the gauge constraints which emerge as a result of the gauge invariance of the action. Nevertheless, this system of equations has a constant field-strength solution which gives rise to a nonzero vacuum energy density. This is what happens in general with a d-form gauge invariant field-strength in d-dimensional space-time.

To make these discussions quantitative let us look for the following solution to the equations of motion (33):

$$ds^2 = A(y) \left( dt^2 - a(t) \, dx^i dx^i \right) - dy^2 \ .$$

(35)
where $a(t)$ is the scale factor of the four-dimensional brane Universe. The corresponding background for the four-form field is given by

$$F^{ABCDE} = \frac{1}{\sqrt{g}} \varepsilon^{ABCDE} k,$$  \hspace{1cm} \text{(36)}

where $k$ stands for an arbitrary integration constant (in our normalizations $\varepsilon^{12345} = 1$). With this Ansatz at hand the four-dimensional components of the energy-momentum tensor for the four-form field take the form:

$$T_{AB} = -\frac{1}{2} g_{AB} k^2.$$ \hspace{1cm} \text{(37)}

This generates an additional negative contribution to the total effective bulk cosmological constant

$$\Lambda_{\text{eff}} = \Lambda - \frac{1}{4M^3} k^2.$$ \hspace{1cm} \text{(38)}

Using these expressions the Einstein equations can be written as follows:

$$\frac{3A''}{2A} = -\Lambda_{\text{eff}} - \frac{1}{2M^3} T\delta(y), \hspace{1cm} \left(\frac{A'}{A}\right)^2 = \frac{H_0^2}{A^2} - \frac{2}{3}\Lambda_{\text{eff}},$$

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{\ddot{a}(t)}{a(t)} = H_0^2.$$ \hspace{1cm} \text{(39)}

Here, primes denote differentiation with respect to $y$, and dots denote differentiation w.r.t. the time variable. The solution to these equations reads as $^{35,36}$

$$A(y) = \text{ch} \left( \sqrt{-\frac{2\Lambda_{\text{eff}}}{3} y} \right) - \frac{T}{2M^3\sqrt{-6\Lambda_{\text{eff}}}} \text{sh} \left( \sqrt{-\frac{2\Lambda_{\text{eff}}}{3}} |y| \right).$$ \hspace{1cm} \text{(40)}

The four-dimensional Hubble constant is defined as follows:

$$H_0 = \sqrt{\frac{T^2}{36M^6} + \frac{2\Lambda_{\text{eff}}}{3}}.$$ \hspace{1cm} \text{(41)}

Therefore, the four-dimensional cosmological constant, $\Lambda_4 \equiv H_0^2$, contains an arbitrary integration constant $k$:

$$\Lambda_4 = \frac{T^2}{36M^6} + \frac{2}{3} \left( \Lambda - \frac{1}{4M^3} k^2 \right).$$ \hspace{1cm} \text{(42)}

Let us summarize the results of our discussions. The system \text{(31)} \textit{without} the $F$ field does not allow for a static solution for a generic, non-fine-tuned values of the bulk cosmological constant and the brane tension. However, when the $F$ field

\[\text{For simplicity we present here the solution when the brane worldvolume is dS}_4. \text{ The solution can be obtained for AdS}_4 \text{ as well} \text{.} \]
is switched on, there are an infinite number of brane Universe solutions. These Universes differ from each other by the value of the corresponding four-dimensional cosmological constants defined in (12). Among these solutions there is the static brane with zero four-dimensional cosmological constant. This removes the necessity of the fine-tuning in the sense that static solution now exists for any values of $\Lambda$ and $T$. Certainly, the cosmological constant problem is not solved, instead, the introduction of the $F$ field gives an opportunity for the anthropic arguments. Indeed, only those branes for which the four-dimensional cosmological constant fits into the Weinberg’s window [37]:

$$\frac{\Lambda_{4}}{8\pi G_{N}} \leq \left(10^{-3} \text{ eV}\right)^{4},$$

(43)
could accommodate the Universe similar to ours. All other brane Universes, not satisfying (13), would not be able to form the large scale structures. Concluding, the action (31) allows to avoid the necessity of fine-tuning of the bulk cosmological constant $\Lambda$ to the brane tension $T$, for any $\Lambda$ and $T$ there are an infinite number of possible brane Universe solutions of (31) and one of these solutions has zero four-dimensional cosmological constant.

4 Discussions and conclusions

Let us start with some comments on the form of the action (31). The four-form field in (31) reminds the Ramond-Ramond (RR) field to which a three-brane could couple. However, one can check that the sign of the kinetic term in (31) is inverse to what should have been used for the RR field. The choice of this sign is not restricted by positivity arguments in the five-dimensional theory (31), since this field does not propagate dynamical degrees of freedom. However, the negative sign is necessary in (31) if the four-form field is real and one needs to generate a negative cosmological constant in (38). This is true as long as the original cosmological constant $\Lambda$ in (31) is an arbitrary positive number. Thus, for $\Lambda > 0$ the four-form field cannot be thought of as a RR field. Let as now see what happens if $\Lambda$ is a big arbitrary negative number (in fact, without loss of generality, we take its magnitude to be bigger than the magnitude of the critical value defined by (5)). In this case one can just flip the sign of the kinetic term of the $F^{2}$ term in (31) and obtain the desired result. Indeed, in this case, the kinetic term for the four-form field is precisely equivalent to that for a RR field. This produces a positive cosmological constant in the bulk. Therefore, the expression (42) takes the form:

$$\Lambda_{4} = \frac{T^{2}}{36M^{6}} + \frac{2}{3} \left( \Lambda + \frac{1}{4M^{3}} k^{2} \right).$$

(44)

Alternatively, one could add to the action (31) another four-form field with opposite kinetic term.
Since $\Lambda$ in this case is a big negative number (its magnitude is bigger than $T^2/24M^6$), one is still able to have an infinite number of brane Universe solutions parametrized by the integration constant $k$ and one of these solutions have $\Lambda_4 = 0$. In this case (negative $\Lambda$), one could think of the four-form field as some kind of RR field of string theory. However, in our case the brane at hand should not couple to this RR field, i.e., the brane should not be a D-brane. Otherwise, the effective cosmological constant produced by the RR field would be defined by the brane RR charge. Since this is quantized, one would again need fine-tuning between the brane charge and the brane tension (as before, in the supergravity framework this is the BPS condition, however it becomes fine-tuning after SUSY is broken).

Let us now go back to the case $\Lambda > 0$ considered in the previous section. A way to think of the four-form field in this case is to recall that in theories of supergravity there are auxiliary scalar fields. These scalars usually enter an off-shell Lagrangian in a quadratic form without kinetic terms. One of these scalars, let us call it $\phi$, can be dualized into the five-form field strength via $\phi \propto \varepsilon^{ABCDE} F^{ABCDE}$. Since the original scalar was an auxiliary field and the four-form does not propagate dynamical degrees of freedom either, this dualization makes sense. As a result, the $\phi^2$ term in the original off-shell Lagrangian of supergravity, which can emerge as $\sqrt{g}(R + \phi^2)$, is replaced by the $F^2$ term in the form given in (31). This is acceptable as far as bosonic modes are concerned. The actual issue is whether it is possible to perform the dualization of the whole SUGRA supermultiplet, not only its scalar part. Such a dualization of the chiral $N = 1$ SUGRA supermultiplet into the tensor supermultiplet was shown to be possible in the four-dimensional case [39]. The issue whether the same can be done in five-dimensions remains open.

To conclude, we studied localization of various spin fields on a brane in AdS$_5$. We found that spin-0 fields are localized on a brane with positive tension which also localizes gravitons. The spectrum of spin-0 field is similar to that of the localized graviton: there is a single localized zero-mode plus a continuous tower of states. This continuum can be thought of as a bulk five-dimensional field. To fix the boundary conditions, the whole picture should be viewed in a sense of the AdS/CFT correspondence. Spin-1 fields are not localized neither on a brane with positive tension nor on a brane with “negative tension”. Within the field theory framework the Dvali-Shifman mechanism [3] should be invoked for the vector field localization. In string theory, vector fields could be localized by assuming that the brane at hand is in fact a D-brane. Spin-1/2 and spin-3/2 fields are localized due to gravitational interactions on a brane with “negative tension”. The localized modes in this case are not chiral. In order to trap spin-1/2 and spin-3/2 chiral zero-modes on a positive-tension brane additional fields should be introduced. These could be scalars of five-dimensional gauged supergravity.

Finally, we have shown that fine-tuning between the bulk cosmological constant and the brane tension can be avoided if there is a four-form gauge field in the theory. This field gives rise to an additional contribution to the bulk cosmological constant. As a result, the model with any generic $\Lambda$ and $T$ supports an infinite
number of brane Universe solutions which are parametrized by the continuous family of four-dimensional cosmological constants. Among these solutions there is one with $\Lambda_4 = 0$.

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