Standard emitters (clocks) and calibrated standard emitters (clocks) in spaces with affine connections and metrics

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Abstract

It is shown that the general belief that the frequency and the absolute value of the velocity of periodic signals sent by a standard emitter do not change on the world line of the emitter needs to be revised and new conditions for the existence of a calibrated standard emitter should be taken into account. The definitions of a standard clock and of a calibrated standard clock are introduced in a space with affine connections and metrics. The variation of the velocity and of the frequency of a standard clock could be compared with the constant velocity and the constant frequency of a (calibrated) standard clock along the world line of the observer. This calibrated standard clock is transported by means of a generalized Fermi-Walker transport along the same world line of the observer. Some remarks about the synchronization of standard clocks in spaces with affine connections and metrics are given.

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1 Introduction

1.1 General remarks.

1. Modern problems of relativistic astrophysics as well as of relativistic physics are related to the propagation of signals in space or in space-time. The basis
of experimental data received as results of observations of the Doppler effect or of the Hubble effect gives rise to considerations about the theoretical status of effects related to detection of signals from emitters moving relatively to observers carrying detectors in their laboratories. Nevertheless, in the last decades, there is no essential evolution of the theoretical models related to new descriptions of the Doppler and Hubble effects corresponding to the recent development of new mathematical models for the space-time. In the astronomy and astrophysics standard theoretical schemes for measuring velocities are used related to classical mechanics and / or special and general relativity [1], [2].

2. The incoming periodic signals sent by an emitter moving relatively to an observer (detector) are compared with periodic signals of an emitter (standard emitter) lying at rest with the observer. On this basis, the change of frequency and velocity of the incoming periodic signals leads to conclusions about velocities and accelerations of objects moving with respect to the observer. By that it is assumed that the periodic signals of an emitter lying at rest with the observer have constant frequency and constant absolute value of their velocity along the world line of the observer. This assumption is based on the fact that there are different emitters constructed in a way (e.g. by the use of the s.c. method of automatic frequency modulation) that their frequencies remain constant in time. In general, it is worth to be investigated how the kinematic characteristics (shear, rotation, and expansion velocities and accelerations) of the motion of the observer (respectively of the standard emitter) could influence the frequency and the absolute value of the velocity of the signals emitted by a standard emitter considered in more comprehensive models of space-time such as spaces with affine connections and metrics ([LN, g]-spaces).

3. In recent years, spaces with affine connections and metrics have received some interest related to the possibility of using mathematical models of space-time different from (pseudo-) Riemannian spaces without torsion (Vn-spaces) or with torsion (Un-spaces). It has been shown [3], [4], [5] that every differentiable manifold M (dimM = n) with affine connections and metrics ([LN, g]-spaces) could be used as models of space-time for the following reasons:

- The equivalence principle (related to the vanishing of the components of an affine connection at a point or on a curve) holds in ([LN, g]-spaces) [6] ÷ [8], [9].
- ([LN, g])-spaces have structures similar to these in (pseudo) Riemannian spaces without torsion [Vn-spaces] allowing for description of dynamic systems and the gravitational interaction [5].
- Fermi-Walker transports and conformal transports exist in ([LN, g])-spaces as generalizations of these types of transports in Vn-spaces [10], [11].
- A Lorentz basis and a light cone could not be deformed in ([LN, g])-spaces as it is the case in Vn-spaces.
- All kinematic characteristics related to the notions of relative velocity and of relative acceleration could be worked out in ([LN, g])-spaces without changing their physical interpretations in Vn-spaces [5], [12], [13].
- ([LN, g])-spaces include all types of spaces with affine connections and metrics used until now as models of space-time.

On this basis, many of the differential-geometric construction used in the
Einstein theory of gravitation (ETG) in $V_4$-spaces could be generalized for the cases of spaces with one affine connection and metrics $[(L_n, g)]$-spaces and in spaces with affine connections and metrics $[(L_n, g)]$-spaces. Bearing in mind this background a question arises about possible physical applications and interpretation of mathematical constructions from ETG generalized for $(L_n, g)$-spaces. The theory of $(L_n, g)$-spaces is worked out in details in [3]. Brief review of the properties of $(L_n, g)$-spaces is given in [4] or in [12].

4. It is well known that every classical field theory over spaces with affine connections and metrics could be considered as a theory of continuum media in these spaces [14]÷[16]. On this ground, notions of the continuous media mechanics (such as deformation velocity and acceleration, shear velocity and acceleration, rotation velocity and acceleration, expansion velocity and acceleration) have been used as invariant characteristics for spaces admitting vector fields with special kinematic characteristics.

5. If a $(L_n, g)$-space could be used as a model of space or of space-time the questions arise

- how signals propagate in a space-time described by a $(L_n, g)$-space and
- how signals could be influenced by the kinematic characteristics of the motion of an observer.

The answer of the first question is given in details in [17]. The answer of the second question is the subject of this paper.

Remark. In a previous paper [18] the variation of the velocity and the frequency of a periodic signal along the world line of the emitter has been considered without the explicit introduction of the notion of a calibrated standard emitter (clock). This fact could generate the questions with respect to what the variation of the velocity and the frequency of a standard clock could be compared and how the proper time of the frame of reference of the observer could be introduced.

6. In this paper the variation of the absolute value of the velocity and the frequency of signals sent by a standard emitter are considered in the proper frame of reference of the emitter. In Section 2 the variation of the velocity of a periodic signal along the world line of a standard emitter is considered as well as the variation of the frequency of a periodic signal along the world line of a standard emitter is determined. The variation of the velocity and the frequency of periodic signals of a standard oscillator (clock) are also found. In Section 3 the calibrated standard clocks are considered and the conditions for their existence along the world line of an observer are found. A standard emitter with constant characteristics of its emitted signals (their absolute value of velocity and their frequency) is called calibrated standard emitter. The synchronization of calibrated standard clocks on different world lines is discussed. Some concluding remarks comprise the Section 4. It is shown that the general belief that the frequency and the absolute value of the velocity of periodic signals sent by a standard emitter does not change on the world line of the emitter needs to be
revised and new conditions for existence of calibrated standard emitter should be taken into account.

1.2 Abbreviations, definitions, and symbols

1. In the further considerations in this paper we will use the following abbreviations, definitions, and symbols:

\[ := \text{ means by definition.} \]

The middle point "\cdot" is used as a symbol for standard multiplication in the field of real (or complex) numbers, e.g. \[ a \cdot b \in \mathbb{R}. \]

The lower point "\cdot" is used as a symbol for symmetric tensor product, e.g. \[ u. v = \frac{1}{2} \cdot (u \odot v + v \odot u). \]

The symbol "\wedge" is used as symbol for a wedge product, e.g. \[ u, v = \frac{1}{2} \cdot (u \odot v - v \odot u). \]

\[ M \] is a symbol for a differentiable manifold with \( \dim M = n \). \( T(M) := \cup_{x \in M} T_x(M) \) and \( T^*(M) := \cup_{x \in M} T^*_x(M) \) are the tangent and the cotangent spaces at \( M \) respectively. \( T_x(M) \) and \( T^*_x(M) \) are the tangent and the cotangent spaces at a point \( x \in M \) respectively.

Remark. The notion of tangent space \( T(M) \) over a manifold \( M \) could be considered as an element of the notion of tangent bundle \( (T(M), M, \pi : T(M) \rightarrow M) \) over a manifold \( M \). A vector field in the tangent space \( T(M) \) over the manifold \( M \) could be considered as a section in the tangent space \( T(M) \).

\( (\mathcal{L}_n, g) \)-spaces are spaces with contravariant and covariant affine connections and metrics whose components differ not only by sign. In such type of spaces the non-canonical contraction operator \( S \) acts on a contravariant basic vector field \( e_j \) (or \( \partial_j \)) \( \in \{e_j \text{ (or } \partial_j) \} \subset T(M) \) and on a covariant basic vector field \( e^i \) (or \( dx^i \)) \( \in \{e^i \text{ (or } dx^i) \} \subset T^*(M) \) in the form

\[ S : (e^i, e^j) \rightarrow S(e^i, e^j) := f^i_j, \]

\[ f^i_j \in C^r(M), \quad r \geq 2, \quad \det(f^i_j) \neq 0, \]

\[ \exists f^i_k \in C^r(M), \quad r \geq 2 : \quad f^i_j \cdot f^j_k := g^i_k. \]

In these spaces, for example, \( g(u) = g_{ik} \cdot f^k_j \cdot u^j \cdot dx^i = g_{ij} \cdot u^j \cdot dx^i := u_i \cdot dx^i, g(u, u) = g_{kl} \cdot f^k_i \cdot f^l_j \cdot u^i \cdot u^j = g_{ij} \cdot u^i \cdot u^j = u_j \cdot u^j := u_i \cdot u^i, g^T_j = g^T_i \in L_n(g) \).

The components \( \delta^i_j := g^i_j \) (\( i = 0 \) for \( i \neq j \) and \( i = 1 \) for \( i = j \)) are the components of the Kronecker tensor \( K^i_j := g^i_j \cdot \delta^i_j \odot dx^j \).

\( (\mathcal{L}_n, g) \)-spaces are spaces with contravariant and covariant affine connections and metrics whose components differ only by sign. In such type of spaces the canonical contraction operator \( C := C \) acts on a contravariant basic vector field \( e_j \) (or \( \partial_j \)) \( \in \{e_j \text{ (or } \partial_j) \} \subset T(M) \) and on a covariant basic vector field \( e^i \) (or \( dx^i \)) \( \in \{e^i \text{ (or } dx^i) \} \subset T^*(M) \) in the form

\[ C : (e^i, e^j) \rightarrow C(e^i, e^j) := C(e^i, e^j) := \delta^i_j := g^i_j. \]
In these spaces, for example, \( g(u) = g_{ik} \cdot g_i^k \cdot u^i \cdot dx^i := g_{ij} \cdot u^j \cdot dx^i = u_i \cdot dx^i \), \( g(u, u) = g_{kl} \cdot g^k_l \cdot u^i \cdot u^j := g_{ij} \cdot u^i \cdot u^j = u_i \cdot u^j \).

**Remark.** All results found for \((\mathcal{T}_n, g)\)-spaces could be specialized for \((L_n, g)\)-spaces as well as for all other special cases of \((L_n, g)\)-spaces, used until now as models of space-time, by omitting the bars above or under the indices.

\( \nabla_u \) is the covariant differential operator acting on the elements of the tensor algebra \( T \) over \( M \). The action of \( \nabla_u \) is called covariant differentiation (covariant transport) along a contravariant vector field \( u \), for instance,

\[
\nabla_u v := v^i_{,j} \cdot u^j \cdot \partial_i = (v^i_{,j} + \Gamma^i_{kj} \cdot v^k) \cdot u^j \cdot \partial_i , \quad v \in T(M) ,
\]

where \( v^i_{,j} := \partial v^i / \partial x^j \) and \( \Gamma^i_{jk} \) are the components of the contravariant affine connection \( \Gamma \) in a contravariant co-ordinate basis \( \{ \partial_i \} \). The result \( \nabla_u v \) of the action of \( \nabla_u \) on a tensor field \( v \in \otimes^k(M) \) is called covariant derivative of \( v \) along \( u \). For covariant vectors and tensor fields an analogous relation holds, for instance,

\[
\nabla_u w = w_{ij} \cdot u^j \cdot dx^i = (w_{ij} + P^l_{ij} \cdot w_l) \cdot u^j \cdot dx^i , \quad w \in T^*(M) .
\]

where \( P^l_{ij} \) are the components of the covariant affine connection \( P \) in a covariant co-ordinate basis \( \{ dx^i \} \). For \((L_n, g), U_n, \) and \( V_n \)-spaces \( P^l_{ij} = - \Gamma^l_{ij} \).

For \((\mathcal{L}_n, g)\)-spaces, where \( S : (e^i, e_j) \rightarrow S(e^i, e_j) := f^i_{,j} \), the components \( \Gamma^i_{jk} \) of the contravariant affine connection \( \Gamma \) and the components \( P^l_{ik} \) of the covariant affine connection \( P \) should obey the condition

\[
f^i_{,j,k} = \Gamma^i_{jk} \cdot f^i_{,l} + P^l_{ik} \cdot f^l_{,j} , \quad f^i_{,j,k} = \partial_k f^i_{,j} = \frac{\partial f^i_{,j}}{\partial x^k} .
\]

\( \mathcal{L}_u \) is the Lie differential operator \( 8 \) acting on the elements of the tensor algebra \( T \) over \( M \). The action of \( \mathcal{L}_u \) is called dragging-along a contravariant vector field \( u \). The result \( \mathcal{L}_u v \) of the action of \( \mathcal{L}_u \) on a tensor field \( v \) is called Lie derivative of \( v \) along \( u \).

2. Let us now recall the physical interpretations of some of the considered mathematical structures.

**Space-time** := \((\mathcal{L}_n, g)\)-space. If necessary, \( n \) could be specified to \( n = 3, 4 \), etc.

**Space** := \( n - 1 \) sub manifold of a \((\mathcal{L}_n, g)\)-space.

**Time** := 1 dimensional sub manifold of a \((\mathcal{L}_n, g)\)-space, orthogonal to a given \( n - 1 \) sub manifold of a \((\mathcal{L}_n, g)\)-space, interpreted as a space.

**Space-time distance** := distance in a space-time := the line-element (the metric element) \( ds = \pm | g(d, d) |^{1/2} \) in a space-time with \( 14 \)

\[
ds^2 = \pm | ds |^2 = g(d, d) = g_{ij} \cdot d^i \cdot d^j = \pm | g(d, d) | ,
\]

where \( d = d^i \cdot e_i = dx^i \cdot \partial_i \) is the ordinary differential over a \((\mathcal{L}_n, g)\)-space.
Space distance = distance in a space := the line-element \( dl = \pm | g(d_\perp, d_\perp) | \)
in the space \( \mathbb{L}_n \)
\[
dl^2 = \pm | dl |^2 = g(d_\perp, d_\perp) = g_{ij} \cdot d_i^\perp \cdot d_j^\perp = \pm | g(d_\perp, d_\perp) | ,
\]
where \( d_\perp = \overline{\mathcal{F}[h_u(d)]} = l_{d_\perp} \cdot n_\perp, \)
\( g(n_\perp, n_\perp) = \pm 1, \overline{\mathcal{G}} = g^{ij} \cdot \partial_i \partial_j \)
is the contravariant metric in the \( \mathbb{L}_n \)-space,
\( h_u = g - \frac{1}{g(u, u)} \cdot g(u) \otimes g(u) \) is the projective metric,
\( u \) is a contravariant non-null (non-isotropic) vector field, orthogonal to the given space,
\( g(u, u) = \pm l_u^2 \neq 0, h_u(u) = 0. \)

Time distance = distance in time := the line-element \( d\tau = \pm | g(d_\parallel, d_\parallel) | \)
in the time \( \mathbb{T}_n \)
\[
d\tau^2 = \pm | d\tau |^2 = \frac{1}{l_u^2} \cdot g(d_\parallel, d_\parallel) = \frac{1}{l_u^2} \cdot g_{ij} \cdot d_i^\parallel \cdot d_j^\parallel = \pm \frac{1}{l_u^2} \cdot | g(d_\parallel, d_\parallel) | ,
\]
where \( d_\parallel = \frac{d\tau}{d\tau} = l_u \cdot n_\parallel, \)
\( g(n_\parallel, n_\parallel) = \pm 1, g(n_\perp, n_\parallel) = 0, \)
is a contravariant non-null (non-isotropic) vector field, orthogonal to the given space and tangential to a congruence of lines (non-intersecting curves), orthogonal to the space.

The vector field \( u \) is interpreted as a tangent vector field to a congruence of lines considered as world lines of material points (mass elements, observers, emitters, etc.) moving in space-time.

A world line is a line with a tangent vector \( u \) orthogonal to its corresponding space.

Every space-time distance \( ds \) with \( ds^2 \) could be represented by means of the space distance and the time distance in the form
\[
ds^2 = g(d, d) = \pm l_u^2 \cdot d\tau^2 \mp dl^2 .
\]

The space distance \( dl \) is the distance between two points lying in the space orthogonal to the vector field \( u \).

The space distance \( dl \) and the time distance \( d\tau \) could be related by the introduction of the notion of the absolute value of the relative velocity \( l_v \)
\[
l_v = \frac{dl}{d\tau}
\]
between two points lying in one and the same space.

Every space distance is measured on the basis of an introduced measure unit for distance (length) such as meter, inch, yards etc.

Time is the measure of a limited or periodical process. The measure unit for the time \( T_0 \) is introduced by mankind in different ways. Usually, the measure unit is related to a periodical process. The periodical process is, from its side, connected to the notion of a clock. If a clock is moving with an observer it is called standard clock of the observer. The duration of a process could be measured by different observers with different measure units for the time distance.
3. Every space-time could be decomposed (at least locally) in a pair (space, time). The decomposition is identical with the \((n - 1) + 1\) projection of a space-time \([3]\). This operation is not unique. There is a set of pairs corresponding to a given space-time. At the same time, the space-time interval \(ds\) could be decomposed in corresponding to the pairs forms. If we now consider two pairs (space, time) we could express the square of the space-time interval \(ds^2\) in two different forms

\[ds^2 = \pm t^2_{us} \cdot d\tau^2_s \mp dl^2 = \pm t^2_u \cdot d\tau^2 \mp dl^2 .\]

Therefore, the invariance of the form of the space-time interval, considered from two different pairs (space, time) leads to the relations

\[
\frac{d\tau^2}{dt^2} = \frac{t^2_{us} - t^2_{vs}}{t^2_u - t^2_v}, \quad t^2_{us} = \frac{dl^2}{d\tau^2}, \quad t^2_v = \frac{dl^2}{d\tau^2} .
\]

If \(t_{vs} = 0\) then there is a proportionality between the space-time interval \(ds\) and the time interval \(d\tau_s\)

\[ds^2 = \pm t^2_{us} \cdot d\tau^2_s .\]

The same conclusion is valid for the other pair of (space, time) if \(t_v = 0\)

\[ds^2 = \pm t^2_u \cdot d\tau^2 .\]

This proportionality is usually used for identification of the space-time interval \(ds\) with the time interval \(d\tau\) under the additional assumption that \(l_u = l_{us} = \text{const}\). There is neither a mathematical nor a physical reason for the last (above) additional assumption. Nevertheless, the condition \(l_u = l_{us} = \text{const}\) is a major assumption in general relativity (on the analogy of special relativity), where \(l_u\) is interpreted as the absolute value of the velocity of a light signal in vacuum. In general relativity the absolute value of a light signal is normalized to \(l_u = c = \text{const}\., or \(l_u = 1\). Then \(g(u, a) = g(u, a_{\parallel}) = 0\). The acceleration \(a = a_{\perp} = \mathcal{I}[u(a)]\) is orthogonal to the vector field \(u\). It is lying in the sub space orthogonal to \(u\). There are two reasons for the normalization of \(u\): one is from mathematical point of view, and the other is from physical point of view. From mathematical point of view, every non-null (non-isotropic) contravariant vector field \(u\) could be normalized by the use of the absolute value \(l_u\) of its length in the form

\[\overline{u} = \pm \frac{c_0}{l_u} \cdot u , \quad g(\overline{u}, \overline{u}) = \frac{c_0^2}{l_u^2} \cdot g(u, u) = \pm c_0^2 , \quad c_0 = \text{const.} \neq 0 .\]

From physical point of view, it is assumed that a light signal is propagating with constant absolute value \(l_u = \text{const}\. from point of view of the frame of reference of an observer. This point of view leads to its mathematical realization by means of the normalization of the vector field \(u\). The existing experimental
facts about the constancy of the speed of light cannot cover the whole range of possible decompositions of space-time in pairs (space, time) related to different frames of reference of different observers (s. the considerations below). Moreover, the existence of the proportionality between space-time intervals and time intervals is not related to a special choice of the velocity parameter \( l_u \) (respectively \( l_{us} \)). One can choose signals of different types (light signals, sound signals, periodic emission of particles etc.) with different velocities to define a time distance \( d\tau \) as well as the relation between it and the space-time distance \( ds \).

A frame of reference is determined by the set of three geometric objects [19]:

- A non-null (time like if \( \dim M = 4 \)) contravariant vector field \( u \in T(M) \).
- A tangent sub space \( T_x^{\perp u}(M) \) orthogonal to \( u \) at every point \( x \in M \), where \( u \) is defined.
- (Contravariant) affine connection \( \nabla = \Gamma \). It determines the type of transport along the trajectory to which \( u \) is a tangent vector field. \( \Gamma \) is related to the covariant differential operator \( \nabla_u \) along \( u \) [3].

Remark: The result \( \nabla_u T \) of the action of \( \nabla_u \) on a tensor field \( T \in \otimes^k l_i(M) \) is called covariant derivative of \( T \) along \( u \).

Then the definition of a frame of reference reads [19]

The set \( FR \sim [u, T^{\perp u}(M), \nabla = \Gamma, \nabla_u] \) is called frame of reference in a differentiable manifold \( M \) considered as a model of the space or of the space-time.

The notion of periodic signal could be defined from physical and from mathematical point of view.

(i) From physical point of view a periodic signal in a \((\mathbb{L}_n, g)\)-space, considered as a model of space-time, is characterized by:
- A periodic process, characterized by its direction and frequency, transferred by an emitter and received by an observer (detector).
- A periodic process with finite velocity of propagation from point of view of the observer, characterized by its absolute value of the velocity of propagation.

(ii) From mathematical point of view a periodic signal in a \((\mathbb{L}_n, g)\)-space, considered as a model of space-time, is characterized by:

- Isotropic (null) contravariant vector field \( \tilde{k} : g(\tilde{k}, \tilde{k}) = 0, \tilde{k} \in T(M) \), \( \dim M = n \), \( \text{sgn} \ g = n - 2 \) or \( \text{sgn} \ g = -n + 2 \), determining the direction of the propagation of a periodic signal in space-time. \( M \) is the differentiable manifold with dimension \( n \), provided with affine connections and metrics, \( T(M) \) is the tangent space over \( M : T(M) = \cup_{x \in M} T_x(M) \).

- Non-isotropic contravariant vector field \( u : g(u, u) = e = \pm l^2_{us} \neq 0 \). The vector field \( u \in T(M) \) is interpreted as the velocity vector field of an observer (detector).
\[ l_u^2 = \pm g(u, u) > 0, \text{ interpreted as the absolute value of the velocity of a periodic signal with respect to the proper frame of reference of an observer.} \]

The sign before \( g(u, u) \) is depending on the signature of the metric of the space-time.

\[ \text{Scalar product of } \tilde{k} \text{ and } u : g(\tilde{k}, u) = \omega > 0, \text{ interpreted as the frequency of a periodic signal with respect to the proper frame of reference of an observer (detector).} \]

\[ \text{The space direction of the propagation of a periodic signal is given by the contravariant vector field } k_{\perp} \]

\[ k_{\perp} = |h_u(\tilde{k})| = g^{ij} \cdot \bar{k}^j \cdot \partial_i, \quad (3) \]

where

\[ g(\tilde{k}, \tilde{k}) = 0, \quad \bar{g} = g^{ij} \cdot \partial_i \partial_j, \]

\[ \partial_i \partial_j = \frac{1}{2} \cdot (\partial_i \otimes \partial_j + \partial_j \otimes \partial_i), \]

\[ h_u = g - \frac{1}{g(u, u)} \cdot g(u) \otimes g(u), \]

\[ g(u, u) = \pm l_u^2 = e \neq 0. \]

The frequency \( \omega \) of the periodic standard signal (the periodic signal sent by a standard emitter) is determined by the relations [17]:

\[ \omega = g(u, \tilde{k}) = l_u \cdot g(\tilde{n}_{\perp}, k_{\perp}), \quad (4) \]

where

\[ k_{\perp} = \mp l_{k_{\perp}} \cdot \tilde{n}_{\perp} = \mp \frac{\omega}{l_u} \cdot \tilde{n}_{\perp}, \quad g(\tilde{n}_{\perp}, \tilde{n}_{\perp}) = \mp 1. \quad (5) \]

By the use of the relations

\[ l_u = \lambda \cdot \nu = \lambda \cdot \frac{\omega}{2 \cdot \pi}, \quad (6) \]

\[ \omega = 2 \cdot \pi \cdot \frac{l_u}{\lambda} = l_u \cdot g(\tilde{n}_{\perp}, k_{\perp}), \quad (7) \]

the expression for the length \( \lambda \), corresponding to the frequency \( \omega \) of the periodic signal, could be found in the form

\[ \lambda = \frac{2 \cdot \pi}{g(\tilde{n}_{\perp}, k_{\perp})}, \quad (8) \]

and therefore,

\[ g(\tilde{n}_{\perp}, k_{\perp}) = \frac{2 \cdot \pi}{\lambda}. \quad (9) \]

The projection of \( k_{\perp} \) on its unit vector \( \tilde{n}_{\perp} \) has exact relation to the length \( \lambda \) of the periodic signal with frequency \( \omega \).
2 Standard periodic emitter (clock)

1. A *standard emitter* is an emitter moving together with an observer (detector) in space-time and lying at rest in the proper frame of reference of the observer (detector). The proper frame of reference of the standard emitter could be identified with the proper frame of reference \[5\] of the observer (detector).

The notion of a clock is closely related to the notion of a periodic signal. A clock is a physical device consisting of an oscillator running at some angular frequency \( \omega \) and a counter that counts the cycles. The period of the oscillator, \( T = \frac{2\pi}{\omega} \), is calibrated in some standard oscillator. The counter simply counts the cycles of the oscillator. Since some epoch, or the event at which the count started, we say that a quantity of time equal to \( NT \) has elapsed, if \( N \) cycles have been counted \[20\].

A *standard clock* is a clock moving with an observer (detector) and with his standard emitter.

2.1 Variation of the velocity of a periodic signal along the world line of a standard emitter

Let us now consider the motion of a standard emitter moving with an observer in a \((\mathcal{L}_n, g)\)-space considered as a model of space-time.

1. Let \( \tau \) be the proper time (parameter) of the world line (trajectory) \( x^i(\tau) \), \( i = 1, ..., n \), \( \text{dim} \; M = n \), \( n = 4 \), in a space-time of a standard emitter [i.e. of an emitter lying at rest with an observer (detector)]. The proper time can be introduced by the use of a calibrated standard emitter (clock). The tangent vector field along the world line of the standard emitter could be written in the form

\[
\mathbf{u} = \frac{d}{d\tau} = \frac{dx^i}{d\tau} \cdot \partial_i = u^i \cdot \partial_i , \quad u^i = \frac{dx^i}{d\tau} . \tag{10}
\]

Its absolute length \( l_u \) is defined by the expression \[3\]

\[
\pm l_u^2 = g(u, u) = g_{ij} \cdot u^i \cdot u^j , \quad g_{ij} = f^k_i \cdot f^l_j \cdot g_{kl} . \tag{11}
\]

2. The variation of the absolute value \( l_u \) of the velocity of a periodic signal of a standard emitter could be found by the use of the relations

\[
\nabla_u(l_u^2) = u(l_u^2) = 2 \cdot l_u \cdot u(l_u) = 2 \cdot l_u \cdot \frac{dl_u}{d\tau} =
\]

\[
= \pm \nabla_u[g(u, u)] = \pm [(\nabla_u g)(u, u) + 2 \cdot g(u, a)] , \tag{12}
\]

in the form

\[
l_u \cdot \frac{dl_u}{d\tau} = \pm [g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u)] . \tag{13}
\]

3. If we consider the conditions under which the absolute value \( l_u \) of the velocity of a periodic signal appears as a conserved quantity in a \((\mathcal{L}_n, g)\)-space, we can prove the following propositions:
Proposition 1. The necessary and sufficient condition for the absolute value $l_u$ of the velocity of a periodic signal to be a conserved quantity along the world line of a standard emitter with tangent vector field $u$ is the condition

$$g(u,a) = -\frac{1}{2} \cdot (\nabla_u g)(u,u),$$

equivalent to the condition

$$(\mathcal{L}_u g)(u,u) = 0.$$  

The proof is trivial. One should only take into account the relations

$$l_u \cdot \frac{dl_u}{d\tau} = \pm [g(u,u) + \frac{1}{2} \cdot (\nabla_u g)(u,u)], \quad l_u \neq 0,$$

$$\mathcal{L}_u [g(u,u)] = u [g(u,u)] = \nabla_u [g(u,u)] = (\mathcal{L}_u g)(u,u).$$

Proposition 2. A sufficient condition for the absolute value $l_u \neq 0$ of the velocity of a periodic signal of an emitter to be a constant quantity ($l_u = \text{const.} \neq 0$) along the world line of the emitter with tangent vector field $u$ is the condition

$$\mathcal{L}_u g = 0,$$

i.e. if the vector field $u$ is tangent vector field to the world line of a standard emitter then the absolute value $l_u$ of the velocity of its periodic signals is a constant quantity ($l_u = \text{const.} \neq 0$) along the world line of the emitter when $u$ is a Killing vector field.

The proof is trivial.

2.2 Variation of the frequency of a periodic signal along the world line of an emitter

If an emitter is used as a standard emitter by an observer on his world line for comparison of the incoming periodic signals from another emitters then the variation of the frequency of the standard emitter is of great importance for the correct determination of the frequency of the incoming signals.

1. From the explicit form of the frequency $\omega$

$$\omega = l_u \cdot g(\tilde{n}_\perp, k_\perp),$$

where [17]

$$k_\perp = \frac{\omega}{l_u} \cdot \tilde{n}_\perp, \quad g(\tilde{n}_\perp, u) = 0,$$

it follows that

$$\nabla_u \omega = u \omega = \frac{d\omega}{d\tau} = \nabla_u [l_u \cdot g(\tilde{n}_\perp, k_\perp)] =$$

$$= u l_u \cdot g(\tilde{n}_\perp, k_\perp) +$$

$$+ l_u \cdot [(\nabla_u g)(\tilde{n}_\perp, k_\perp) + g(\nabla_u \tilde{n}_\perp, k_\perp) + g(\tilde{n}_\perp, \nabla_u k_\perp)].$$
By the use of the conditions $L_u k_\perp = 0$, $g(u, k_\perp) = 0$, (the relations are the necessary and sufficient conditions for the existence of co-ordinates curves to which $u$ and $k_\perp$ appear as tangent vectors at every point of the corresponding curve [21]), the equation for the frequency $\omega$ follows in the form

$$\frac{d}{d\tau}(\log \frac{\omega}{l_u}) = \mp [d(\bar{n}_\perp, \bar{n}_\perp) + \frac{1}{2}(\nabla_u g)(\bar{n}_\perp, \bar{n}_\perp)] \ ,$$

where $d$ is the deformation velocity tensor [22], [3]

$$d = \sigma + \omega + \frac{1}{n-1} \cdot \theta \cdot h_u \ .$$

(20)

The trace free covariant symmetric tensor $\sigma$ is the shear velocity tensor. The antisymmetric covariant tensor $\omega$ (not necessary for the further investigations) is the rotation velocity tensor. The invariant $\theta$ is the expansion velocity invariant [22], [3].

On the other side,

$$d(\bar{n}_\perp, \bar{n}_\perp) = (\sigma + \omega + \frac{1}{n-1} \cdot \theta \cdot h_u)(\bar{n}_\perp, \bar{n}_\perp) =$$

$$= \mp \left[ \frac{1}{n-1} \cdot \theta \mp \sigma(\bar{n}_\perp, \bar{n}_\perp) \right] \ ,$$

(21)

$$\frac{1}{n-1} \cdot \theta \mp \sigma(\bar{n}_\perp, \bar{n}_\perp) = H \ , \quad d(\bar{n}_\perp, \bar{n}_\perp) = \mp H \ .$$

(22)

The function $H = H(\tau)$ is the s.c. Hubble function [17]. Therefore, we can write now the equation for the frequency $\omega$

$$\frac{d}{d\tau}(\log \frac{\omega}{l_u}) = H \mp \frac{1}{2} \cdot (\nabla_u g)(\bar{n}_\perp, \bar{n}_\perp) \ .$$

(23)

Its solution follows in the form

$$\frac{\omega}{l_u} = \frac{\omega_0}{l_{u0}} \cdot \exp\left( \int [H \mp \frac{1}{2} \cdot (\nabla_u g)(\bar{n}_\perp, \bar{n}_\perp)] \cdot d\tau \right) \ ,$$

$$\omega_0 = \text{const.}, \quad l_{u0} = \text{const.}$$

2. By the use of the relations

$$l_u = \lambda \cdot \nu = \lambda \cdot \frac{\omega}{2 \cdot \pi} \ ,$$

(24)

$$\omega = 2 \cdot \pi \cdot \frac{l_u}{\lambda} = l_u \cdot g(\bar{n}_\perp, k_\perp) \ ,$$

(25)

the expression for the length $\lambda$ follows in the form

$$\lambda = \lambda_0 \cdot \exp\left\{ - \int [H \mp \frac{1}{2} \cdot (\nabla_u g)(\bar{n}_\perp, \bar{n}_\perp)] \cdot d\tau \right\} \ .$$

(26)
Therefore, the length $\lambda$ of the standard periodic signal will decrease in the proper frame of the standard emitter (observer) if

$$H \mp \frac{1}{2} \cdot (\nabla_u g)(\vec{n}_\perp, \vec{n}_\perp) > 0 , \quad \lambda < \lambda_0 , \quad (27)$$

and $\lambda$ will increase when

$$H \mp \frac{1}{2} \cdot (\nabla_u g)(\vec{n}_\perp, \vec{n}_\perp) < 0 , \quad \lambda > \lambda_0 . \quad (28)$$

If

$$H \mp \frac{1}{2} \cdot (\nabla_u g)(\vec{n}_\perp, \vec{n}_\perp) = 0 , \quad (29)$$

then

$$\lambda = \lambda_0 = \text{const.}$$

3. From the equation for $l_u$

$$\frac{1}{2} \cdot \frac{dl_u^2}{d\tau} = \pm [g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u)] \quad (30)$$

we can find the expression for $l_u$

$$\frac{dl_u^2}{d\tau} = \pm 2 \cdot [g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u)] .$$

If we now substitute the expression for $l_u$ in the expression for $\omega$ we can find the general relation for the variation of the frequency $\omega$ under the variation of the absolute value $l_u$ of a standard periodic signal with the proper time $\tau$ of the emitter

$$\omega = \omega_0 \cdot (1 \pm \frac{2}{l_{u0}} \cdot \int [g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u)] \cdot d\tau)^{1/2} \cdot \exp(\int [H \mp \frac{1}{2} \cdot (\nabla_u g)(\vec{n}_\perp, \vec{n}_\perp)] \cdot d\tau) . \quad (31)$$

As a final result, for the variations of the absolute value $l_u$, the frequency $\omega$, and the length $\lambda$ of a standard periodic signal with the proper time $\tau$ of a standard emitter we obtain the relations

$$l_u = (l_{u0}^2 \pm 2 \cdot \int [g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u)] \cdot d\tau)^{1/2} , \quad (A)$$

$$l_{u0}^2 = \text{const.} > 0 ,$$

$$\omega = \omega_0 \cdot (1 \pm \frac{2}{l_{u0}} \cdot \int [g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u)] \cdot d\tau)^{1/2} \cdot \exp(\int [H \mp \frac{1}{2} \cdot (\nabla_u g)(\vec{n}_\perp, \vec{n}_\perp)] \cdot d\tau) , \quad (B)$$

$$\lambda = \lambda_0 \cdot \exp\{- \int [H \mp \frac{1}{2} \cdot (\nabla_u g)(\vec{n}_\perp, \vec{n}_\perp)] \cdot d\tau\} , \quad (C)$$

$$\lambda_0 = \text{const.}$$
2.3 Variation of the velocity and the frequency of periodic signals of a standard oscillator (clock)

2.3.1 Variation of the period of a standard clock

Let us now consider the change of the frequency of periodic signals of a standard oscillator used as a standard clock in the frame of reference of an observer. The period of the oscillator, $T = \frac{2 \cdot \pi}{\omega}$, is calibrated in some standard oscillator. If the frequency $\omega$ is changing along the world line of the oscillator then the period $T$ is also changing in the corresponding form

$$T = \frac{2 \cdot \pi}{\omega} = T_0 \cdot (1 \pm \frac{2}{l_u} \cdot \int [g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u)] \cdot d\tau)^{-1/2}.$$  

$$T = \frac{2 \cdot \pi}{\omega} = \text{const.}$$

Therefore, a standard oscillator (clock) would not change its frequency and period if and only if the following conditions are fulfilled

$$g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u) = 0,$$  

$$H = 0.$$  

These conditions are in generally not fulfilled even in the Einstein theory of gravitation, where the above relations should also be valid in their special forms

$$g(u, a) = 0,$$  

$$H = 0.$$  

In all cosmological models in general relativity with Hubble function $H = H(\tau)$ different from zero and $g(u, a) = 0$ a standard clock will move in space-time with a period $T$ obeying the condition

$$T = \frac{2 \cdot \pi}{\omega} = T_0 \cdot \exp(- \int H \cdot d\tau).$$

Furthermore, the condition $g(u, a) = 0$ is considered as a corollary of the assumption of the constant value $l_u = c = \text{const.}$ or $l_u = 1$ of the speed of light. There is no unique physical argument for the last assumption if a theory of gravitation has to describe the behavior of physical systems including the speed of propagation of their interactions on the basis of its own structures.

3 Calibrated standard emitter (clock)

If a standard emitter (clock) changes its frequency and the absolute value of the velocity of its signals along the world line of the observer the question arises how
these changes could be registered and compared to the changes of other standard emitters. The emitter (clock) to which the data of the standard clock should be compared should be again a standard emitter but with constant frequency and constant absolute value of the velocity of its signals along the world line of the observer.

A standard emitter (clock) with constant frequency and constant absolute value of the velocity of its signals along the world line of the observer is called a calibrated standard emitter (clock).

Because of its properties, a calibrated standard emitter (clock) should be transported along the world line of the observer by a transport, different from the covariant transports of the standard emitters along the world line of the observer. This type of transport should preserve the signals’ characteristics of the emitted signals (frequency, length, absolute value of the velocity of the emitted signals). Therefore, we should find such type of transports along the world line of the observer. From mathematical point of view these transports should preserve the length of the vector field $u$ (related to the absolute value of the velocity of the signals) as well as the length of the vectors, orthogonal to $u$ (related to the frequency and the length of the signals).

Transports preserving the lengths of vector fields, orthogonal to each other in a $(\mathcal{L}_n, g)$-space, are called generalized Fermi-Walker transports (FWT). The theory of FWT is worked out in details in [3]. Brief reviews of the properties of FWT are given in [10] and in [11].

In general, if the transport along the world line is determined by a generalized type of a Fermi-Walker transport [3], [10], [11] the length of the vector field $u$ remains a constant quantity along its trajectory as well as the length of the vectors orthogonal to it. In this case, the following conditions are fulfilled

\[ l_u = l_{u0} = \text{const.} \neq 0, \]
\[ k_\perp = \mp \frac{\omega}{l_u} \cdot \tilde{n}_\perp = \mp l_{k_\perp} \cdot \tilde{n}_\perp, \]
\[ l_{k_\perp} = \text{const.} = \frac{\omega}{l_u}. \]

Since $l_u = \text{const.} \neq 0$, it follows that

\[ \omega = \omega_0 = \text{const.}, \]
\[ \lambda = \lambda_0 = \text{const.}. \]

A calibrated standard clock could be now used as a device for comparison and determination of the changes of the frequency and the absolute value of the velocity of signals, emitted by other (non-calibrated) standard emitters (clocks). The same conclusion is also valid for non-standard emitters (clocks).

### 3.1 Synchronization of calibrated standard clocks

In the Einstein theory of gravitation the synchronization of calibrated standard clocks [23], [24] means the use of curves on which the absolute value $l_u$ of the velocity of a light signal remains constant. Such types of curves are the geodesic
trajectories of observers in space-time. But the existence of geodesic trajectories is only a sufficient but not a necessary condition.

In the general relativity as well as in (pseudo-) Riemannian spaces without torsion ($V_n$-spaces) the necessary and sufficient condition for $l_u = c = \text{const.} \neq 0$ is the condition $g(u, a) = 0$, i.e. the orthogonality between $u$ and $a = \nabla_u u$. On the other side, this condition does not lead to the condition $l_u = c = \text{const.} \neq 0$ in Weyl’s spaces. Therefore, if we search for a condition for $u$ leading to the condition $l_u = c = \text{const.} \neq 0$ in a $(T_n, g)$-space we should consider other possible preconditions.

For a more precise definition of the notion of synchronization we should assume that a calibrated standard clock should not change its frequency $\omega$ and the absolute value $l_u$ of the velocity of its periodic signals if it could be synchronized with other standard clocks.

The notion of synchronized calibrated standard clocks could be then defined in the form:

Definition. Two calibrated standard clocks (lying on their world lines) are called to be synchronized if they do not change their frequency $\omega$ and the absolute value $l_u$ of the velocity of their periodic signals along their own (proper) world lines, i.e. $l_u = l_{uo} = \text{const.} \neq 0$ and $\omega = \omega_0 = \text{const.}$ on every world line on which a standard clock is moving in space-time. This means that the length $l_u$ of the vector field $u$ does not change along its curve to which it is a tangent vector. In general, if a curve is determined by a generalized type of a Fermi-Walker transport [9], [10], [11] the length of the vector field $u$ remains a constant quantity along its trajectory as well as the length of the vectors orthogonal to it. In this case, the following conditions are fulfilled

\[
l_u = \text{const.} \neq 0 , \quad k_\perp = \mp \frac{\omega}{l_u} \cdot \tilde{n}_\perp = \mp l_{k_\perp} \cdot \tilde{n}_\perp , \quad l_{k_\perp} = \text{const.} = \frac{\omega}{l_u} .
\]

Since $l_u = \text{const.} \neq 0$, it follows that $\omega = \omega_0 = \text{const.}$

Therefore, calibrated standard clocks could be synchronized if they are transported by generalized Fermi-Walker transports along the world lines of the corresponding observers (detectors).

4 Conclusions

In the present paper the variation of the absolute value and the frequency of a periodic signal sent by a standard emitter is considered. The obtained results contradict with the general belief that a standard emitter does not change the frequency of its periodic signals on its world line considered also as the world
line of an observer (detector) moving together with the standard emitter. Calibrated standard emitters (clocks) could exist if they are transported along the world line of the observer by a generalized Fermi-Walker transport. Calibrated standard emitters (clocks) should exist if an exact comparison with other standard emitters or with incoming signals is required. The synchronization of calibrated standard clocks is also possible in spaces with affine connections and metrics under the condition that these clocks are transported by a generalized Fermi-Walker transports along their world lines.

References

[1] Lindegren, L. and Dravins, D. (2003). Astronomy and Astrophysics **401**, 1185-1201.

[2] Pauli, E. M., Napiwotzki, R., Altman, M., Heber, U., Odenkirchen, M., and Kerber, F. (2003). Astronomy and Astrophysics **400**, 877-890.

[3] Manoff, S. (2002). *Geometry and Mechanics in Different Models of Space-Time: Geometry and Kinematics* (Nova Science Publishers, New York).

[4] Manoff, S. (1999). *Physics of Elementary Particles and Nuclei (Particles and Nuclei)* [Russian Edition: **30**, 5, 1211-1269], [English Edition: **30**, 5, 527-549].

[5] Manoff, S. (2002). *Geometry and Mechanics in Different Models of Space-Time: Dynamics and Applications* (Nova Science Publishers, New York).

[6] Iliev, B. Z. (1996). *J. Phys. A: Math. Gen.* **29**, 6895-6901.

[7] Iliev, B. Z. (1997). *J. Phys. A: Math. Gen.* **30**, 4327-4336.

[8] Iliev, B. Z. (1998). *J. Geom. Phys.* **24**, 209-222.

[9] Hartley, D. (1995). *Class. and Quantum Grav.* **12**, L103-L105.

[10] Manoff, S. (1998). *Class. Quantum Grav.* **15**, 2, 465-477.

[11] Manoff, S. (1998). *Intern. J. Modern Phys.* **A 13**, 25, 4289-4308.

[12] Manoff, S. (1995). In *Complex Structures and Vector Fields*, St. Dimiev and K. Sekigawa (Eds.) (World Scientific, Singapore) pp. 61-113.

[13] Manoff, S. and Dimitrov B (2002). *Class. Quantum Grav.* **19**, 16, 1111-1125. *(Preprint: ArXiv gr-qc/00 11 045).*

[14] Schmutzer, E. (1968). *Relativistische Physik (Klassische Theorie)* (B G Teubner Verlagsgesellschaft, Leipzig)

[15] Hehl, F. W. and Kerlick, G. D. (1978). *Gen. Rel. and Grav.* **9**, 8, 691.
[16] Manoff, S. and Lazov, R. (1999). In *Aspects of Complex Analysis, Differential Geometry and Mathematical Physics*, S. Dimiev, K. Sekigava (Eds.) (World Scientific, Singapore), pp. 289-314 (Extended version: Preprint: ArXiv gr-qc/99 07 085).

[17] Manoff, S. (2004). *Physics of elementary particles and nuclei (Particles and Nuclei)* 35, 5, 1185-1258 (Preprint: ArXiv gr-qc/03 09 050).

[18] Manoff, S. (2004). Variation of the velocity and the frequency of a periodic signal along the world line of the emitter (Preprint: ArXiv gr-qc/0406058).

[19] Manoff, S. (2001). *Class. Quantum Grav.* 18, 6, 1111-1125. (Preprint: ArXiv gr-qc/99 08 061).

[20] Bahder, T. B. (2004). Clock Synchronization and Navigation in the Vicinity of the Earth (Preprint: ArXiv gr-qc /04 05 001).

[21] Bishop, R. L. and Goldberg, S. I. (1968). *Tensor Analysis on Manifolds* (The Macmillan Company, New York).

[22] Stephani, H. (1977). *Allgemeine Relativitaetsbeorie* (VEB Deutscher Verlag d. Wissenschaften, Berlin) pp. 75-76.

[23] Tucker, R. W. and Teyssandier, P. (2004). Gravity, Gauges and Clockes (Preprint CNRS).

[24] Rizzi, G., Ruggiero, M. L., and Serafini, A. (2004). Synchronization Gauges and the Principle of Special Relativity (Preprint: ArXiv gr-qc/04 09 105).