Constraining parity asymmetry of gravity with joint observations of space-borne gravitational-wave detectors

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Space-borne gravitational-wave (GW) detectors, including LISA, Taiji and TianQin, are able to detect mHz GW signals produced by mergers of supermassive black hole binaries, which opens a new window for GW astronomy. In this article, for the first time, we numerically estimate the potential capabilities of the future networks of multi space-borne detectors by Bayesian analysis. We modify the public package Bilby and employ the sampler PyMultiNest to analyze the simulated data of the space-borne detector networks, and test the parity symmetry of gravity by constraining the birefringence effects of GWs. We find that these detectors are expected to give a lower bound of the parity-violating energy scale of gravity $M_{\mathrm{PV}} \gtrsim \mathcal{O}(1)\text{eV}$, which is weaker than that derived from ground-based GW detectors as anticipated. Similar analysis can be applied to other tests of gravity with potential observations of various space-borne GW detectors.

I. INTRODUCTION

General relativity (GR) is one the most successful and profound physical theories, which has been tested in plenty of observations. However, with the progress in both theoretical and observational research, GR is facing difficulties such as quantization, dark matter and dark energy. Therefore, testing GR is still an important topic in physical research. Since gravitational wave (GW) was first directly detected in 2015, we now have a brand new window to investigate physics in extreme conditions. Detectable GWs are often produced by the densest objects with extremely high-energy processes (e.g. coalescence of binary black holes), and have weak interactions with matter during propagation[1,2]. Thus, GWs could carry strong and clean information from those extreme processes, and provide excellent chances of testing gravity theories.

While ground-based gravitational-wave detectors give decent probes of high-frequency GWs, low-frequency GW detection still remains blank. Several proposed space-borne gravitational-wave detectors with frequency band around millihertz, aiming at sources including Supermassive Black Hole Binaries (SMBHBs), Extreme Mass Ratio Inspirals (EMRIs) and so on, are going to launch in early 2030s[3,5]. In this paper, we shall investigate capabilities of these space-borne detectors of testing gravity. To our knowledge, this is the first attempt to study the potentials of future space-borne detector networks using a full Bayesian analysis.

Parity symmetry is an important concept in modern physics. It implies the flip in the sign of spatial coordinates does not change physical laws. Since people discovered that weak interaction is not symmetric under parity[6], more tests of parity symmetry are needed. Parity is conserved in GR, but there are some parity-violating (PV) gravity theories proposed for different motivations. For example, in string theory and loop quantum gravity, the parity violation in the high-energy regime is inevitable[7,9]. GWs probe physics in the highest energy scale, so it is nature to test parity symmetry with GWs. Parity asymmetry in gravity leads to birefringence in gravitational waves[10,11], left- and right-hand modes of GW evolves differently in the universe. Two kinds to birefringence, amplitude birefringence and velocity birefringence, and their impact on GW waveforms, are well studied in previous works[10,11], which make it possible to probe asymmetry in gravity. Recent work done with gravitational waves from solar-mass binary black holes implies that gravity is symmetric under parity at GeV scale[12]. In this article, we extend this Bayesian analysis to the space-borne GW detection by simulating the future GW signal produced by the mergers of supermassive binary black holes. With a success in recovering all GR parameters, we add parameters in parity-violating gravity to our analysis and obtain the constraints of parity asymmetry provided by space-borne detectors. Lower bound of parity-violating energy scale $M_{\mathrm{PV}}$ can be limited to $\mathcal{O}(1)\text{eV}$ by velocity birefringence and $\mathcal{O}(10^{-15})\text{eV}$ by amplitude birefringence.

This paper is organized as follows. In Sec II we give a brief introduction of parity-violating gravity, especially the GW waveform modifications. In Sec III the configuration and response of space-borne gravitational-wave detectors are presented. Our method of parameter estimation is shown in Sec IV and results are given in Sec V. In Sec VI we summarize our methodology and conclusions. Throughout this paper, we set $c = \hbar = 1$.

II. PARITY-VIOLATING GRAVITY

In this section, we briefly summarize previous work[10] on PV gravity and GW waveform with PV modification. Considering a general parity-violating gravity theory, the
action takes the form

\[ S = \frac{1}{16\pi G} \int d^4x \sqrt{g} (L_{GR} + L_{PV} + L_{\text{others}}) \tag{1} \]

where \( L_{GR} \) is the Einstein-Hilbert Lagrangian density \( R \) in GR. \( L_{PV} \) is the PV term, which depends on the gravitational theories. \( L_{\text{others}} \) represents the Lagrangian density of the other matters, the scalar field and the modification terms of gravity, which are not relevant to parity violation. In the flat Friedmann-Robertson-Walker (FRW) universe, GW is tensorial perturbation of the metric. We Denote spatial perturbation as \( h_{ij} \), which follows transverse and traceless gauge \( \delta^{ij} h_{ij} = 0 \) and \( \partial_i h_{ij} = 0 \). \( h_{ij} \) can be determined by the tensor quadratic action, which reads,

\[ S^{(2)} = \frac{1}{16\pi G} \int dtd^3x a^3 \left[ \frac{1}{4} h_{ij}^2 - \frac{1}{4a^2} (\partial_k h_{ij})^2 + \frac{1}{4} \left( \frac{c_1}{aM_{PV}} \varepsilon^{ijk} h_{ii} \partial_j h_{kl} + \frac{c_2}{a^3M_{PV}} \varepsilon^{ijk} \partial^2 h_{ii} \partial_j h_{kl} \right) \right] \tag{2} \]

where \( a = a(\tau) \) is the conformal scale factor and \( \tau \) is conformal time. \( dot \) means derivative with respect to the cosmic time \( t \) which obeys the relation \( dt = ad\tau \). \( c_1 \) and \( c_2 \) are dimensionless coefficients which are functions of cosmic time in general. \( M_{PV} \) is the parity-violating energy scale above which parity symmetry of gravity is broken. One can derive the equation of motion of the GW circular polarization mode \( h_A \), where \( A = \{ R, L \} \) represents right- and left- respectively:

\[ h_A'' + (2 + \nu_A) h_A' + (1 + \mu_A) k^2 h_A = 0, \quad (3) \]

where \( k \) is wave-number, \( H \) is the conformal Hubble constant. Throughout this paper, \( prime \) denotes the derivative with respect to the conformal time \( \tau \). The terms \( \nu_A \) and \( \mu_A \) represent modifications caused by the PV terms in Lagrangian, which vanish in GR. In the general PV gravity, they take the forms

\[ \nu_A = \left[ \rho_A \alpha_\nu(\tau) \right] (k/aM_{PV})' / H, \]
\[ \mu_A = \rho_A \alpha_\mu(\tau) (k/aM_{PV}) \tag{4} \]

Here \( \rho_R = 1 \) and \( \rho_L = -1 \). \( \alpha_\nu = -c_1 \) and \( \alpha_\mu = c_1 - c_2 \) are two functions that can be determined in a specific model of modified gravity. Although they are functions of time, we could treat it as constants if we consider GW events at local universe\[^{[12]}\]. In this work, we consider they are \( \sim O(1) \) by absorbing them into \( M_{PV} \). Difference in equation of motion of two circular polarization modes leads to parity asymmetry in GWs, that is to say, right- and left-hand modes have different behaviors during propagation, which is called birefringence. It has been proved that \( \nu_A \) leads to different damping rates of two polarizations in propagation, which induces the different amplitudes of GW signals. \( \mu_A \) modifies the dispersion relations of GWs, hence two polarizations have different velocities. Phenomena mentioned above are called amplitude birefringence and velocity birefringence respectively.

Birefringence in PV gravity induces phase and amplitude modifications in GW waveform. In general, GW waveform of PV gravity in frequency domain can be expressed as

\[ h_A^{PV}(f) = h_A^{GR}(f) (1 + \rho_A \delta h) e^{i\nu_A \delta \Psi}, \tag{5} \]

where \( \delta h(f) = -A_\nu \pi f, \quad \delta \Psi(f) = A_\mu (\pi f)^2 / H_0, \tag{6} \]

which are amplitude and phase modifications. Usually, they both exist in PV gravity. Note that \( \delta \Psi(f) \) is about 20 orders larger than \( \delta h(f) \), so it is safe to only take \( \delta \Psi(f) \) into consideration. However, in some special cases like Chern-Simons gravity, \( \delta h(f) \) exists while \( \delta \Psi(f) = 0 \). It is also meaningful to constrain the amplitude modification. \( A_\nu \) and \( A_\mu \) are given by

\[ A_\nu = \frac{1}{M_{PV}} [\alpha_\nu(0) - (1 + z)\alpha_\nu(z)], \]
\[ A_\mu = \frac{1}{M_{PV}} \int_0^z (1 + z') \alpha_\mu(z') \sqrt{\Omega_M(1 + z')^3 + \Omega_\Lambda}, \tag{7} \]

in which \( z \) is redshift of the GW source. One can also rewrite the waveform in plus and cross polarizations via \( h_+ = (h_L + h_R)/\sqrt{2}, h_\times = (h_L - h_R)/\sqrt{2} \):\[^{[13]}\]

\[ h_+^{PV}(f) = h_+^{GR}(f) - h_\times^{GR}(f)(\delta h - \delta \Psi), \]
\[ h_\times^{PV}(f) = h_\times^{GR}(f) + h_+^{GR}(f)(\delta h - \delta \Psi), \tag{8} \]

This is the waveform we use in this work. For the background cosmological model, we adopt a flat Planck cosmology with parameters \( \Omega_M = 0.308, \Omega_\Lambda = 0.692, H_0 = 67.8 \text{km/s/Mpc} \).

### III. SPACE-BORNE GW DETECTORS

#### A. Basic Information: Configuration and Noise

All of the three proposed space-borne GW detectors, LISA, Taiji and TianQin, are three-arm laser interferometers with heliocentric orbits, yet they have difference in some aspects.

The first is arm length, i.e., the separation between two spacecrafts. Arm length is an important property of laser interferometer, since it determines the sensitive
frequency. A longer arm length corresponds to a lower frequency band (longer wavelength). LISA has an arm length of $2.5 \times 10^6$ km and the designed sensitive frequency is from $10^{-4}$ to 1 Hz [4]. Taiji’s arm length is $3 \times 10^6$ km, which means Taiji is more sensitive to low frequency gravitational waves [14]. TianQin’s arm length is $1.7 \times 10^5$ km [3], so it will be more sensitive at the relatively higher frequency.

These are consistent with the power spectral densities (PSDs) of these detectors. For LISA, we follow the new LISA design [15], in which PSD is given by

$$S_n(f) = \frac{4 S_{\text{acc}}(f) + S_{\text{other}}}{L} \left[ 1 + \left( \frac{f}{1.29 f_*} \right)^2 \right] + S_{\text{conf}}(f),$$

where $f_* = c/2\pi L$ is the transfer frequency of detector and $L$ is the arm length. The motion of LISA causes acceleration noise, which takes the form

$$S_{\text{acc}}(f) = \frac{9 \times 10^{-30} m^2 Hz^{-3}}{(2\pi f)^4} \left[ 1 + \left( \frac{6 \times 10^{-4} Hz}{f} \right)^2 \left( 1 + \left( \frac{2.22 \times 10^{-5} Hz}{f} \right)^8 \right) \right],$$

and other noise is

$$S_{\text{other}} = 8.899 \times 10^{-23} m^2 Hz^{-1}.$$  

The confusion noise $S_{\text{conf}}(f)$ arises from numerous GW sources in the universe. In this work, we adopt the confusion noise from unresolved binaries, which is approximated by

$$S_{\text{conf}}(f) = \frac{A}{2} e^{-s_1 f^n} f^{-7/3} \left[ 1 - \text{tanh} \left[ s_2 (f - \kappa \alpha) \right] \right],$$

where $A = 0.489975 \times 10^{-44} Hz^{4/3}$, $s_1 = 3014.3 Hz^{-\alpha}$ with $\alpha = 1.183$, $s_2 = 2957.7 Hz^{-1}$, and $\kappa = 2.0928 \times 10^{-5}$ Hz.

For Taiji and TianQin, we employ a general noise curve for space-borne GW detectors [16,17]

$$S_n(f) = \left[ \frac{S_x}{L} + \frac{4 S_n}{(2\pi f)^4 L^2} \left( 1 + \frac{10^{-4} Hz}{f} \right) \right] \times \left[ 1 + \left( \frac{f}{1.29 f_*} \right)^2 \right],$$

where $\sqrt{S_n} = 3 \times 10^{-15} m^2 Hz^{-3/2}$, $\sqrt{S_x} = 8 \times 10^{-12} m Hz^{-1/2}$ for Taiji, and $\sqrt{S_n} = 10^{-15} m^2 Hz^{-3/2}$, $\sqrt{S_x} = 10^{-12} m Hz^{-1/2}$ for TianQin.

Their noise spectra is shown in Fig.1 As discussed before, LISA and Taiji are more sensitive than TianQin at lower frequency because of their longer arm lengths, but less sensitive at higher frequency.

The second difference is orbit. LISA and Taiji have similar orbits, while TianQin’s orbit differs. For instance, LISA’s center of mass orbits around the Sun in ecliptic plane and the spacecraft orbits their center of mass. Both of the two circular motions have the period of 1 year. Three spacecrafts constitute the shape of an equilateral triangle and the plane of the detector is tilted by $60^\circ$ with respect to the ecliptic [4]. The constellation fall behind the Earth by an angle of $\sim 20^\circ$. Taiji have a similar orbit, but it is ahead of the Earth by $20^\circ$. As shown in Fig.2, LISA and Taiji are far apart (about 0.7 AU), by which GW localization could be improved [14]. Considering circular orbits, we can easily write the unit vectors along three arms in ecliptic frame. We define the $x$-$y$ plane as the ecliptic plane and $z$-axis as perpendicular to $x$-$y$ plane. Denoting $u_n$ ($n = 1, 2, 3$) as the $n$-th arm defined in Fig.2 of Ref. [18], it takes the form

$$u_n = \left( \frac{1}{2} \sin \alpha_n(t) \cos \phi(t) - \cos \alpha_n(t) \sin \phi(t), \frac{1}{2} \sin \alpha_n(t) \sin \phi(t) + \cos \alpha_n(t) \cos \phi(t), \frac{\sqrt{3}}{2} \sin \alpha_n(t) \right).$$

Here,

$$\alpha_n(t) = 2\pi t/T - \pi/12 - (n-1)\pi/3 + \alpha_0,$$

$$\phi(t) = \phi_0 + 2\pi t/T,$$
where $\alpha_0$ is a constant specifying the orientation of the arms at $t = 0$, $\phi_0$ specifies the detector’s location at $t = 0$, and $T$ equals to 1 year. These vectors will be used in next subsection to calculate instrument response.

As for TianQin (which is also shown in Fig. 2), the orbit is more complex. Three spacecrafts orbit around the Earth, and the normal of the detector plane points toward the direction of the vernal equinox. $\beta$ is the longitude of the perihelion. Normal of TianQin’s detector plane points to the reference source RX J0806.3+1527 whose coordinates in ecliptic frame is $(\theta_e, \phi_e)$.

where $e_{ij}^A$ in ecliptic frame are defined by a set of unit vectors $\{\hat{m}, \hat{n}, \hat{w}\}$,

$$e_{ij}^A = \hat{m}_i \hat{m}_j - \hat{n}_i \hat{n}_j, \quad e_{ij}^T = \hat{m}_i \hat{n}_j + \hat{n}_i \hat{m}_j,$$

with

$$\hat{m} = (\cos \theta_e \cos \phi_e \cos \psi_e + \sin \phi_e \sin \psi_e, \cos \theta_e \sin \phi_e \cos \psi_e - \cos \phi_e \sin \psi_e, \sin \theta_e \cos \psi_e),$$

$$\hat{n} = (- \cos \theta_e \cos \phi_e \sin \psi_e + \sin \phi_e \cos \psi_e, - \cos \theta_e \sin \phi_e \sin \psi_e - \cos \phi_e \cos \psi_e, \sin \theta_e \sin \psi_e),$$

$$\hat{w} = (- \sin \theta_e \cos \phi_e, - \sin \theta_e \sin \phi_e, - \cos \theta_e),$$

where $(\theta_e, \phi_e)$ are spherical coordinates in solar system with the ecliptic as $x$-$y$ plane and the Sun at center. $\psi_e$ is the polarization angle. $\hat{w}$ is propagation direction of GW, pointing from the source to the Sun.

The detector tensor, however, worthy of more discussions. Detector tensor is related to the tensor product of arm direction vectors. For ground-based GW detectors aiming at short-duration gravitational wave transient, arm direction vectors can be regarded as a constant during a GW event, thus detector tensor is also a constant. Nevertheless, for space-borne GW detectors whose objects are SMBHBs and EMRIs, observation often takes months to years. That is to say, detector tensor should be treated as a function of time, rather than a constant. In addition, since the wavelength of GWs is comparable to the physical arm length of detector (which is not satisfied for ground-based detectors), the GW frequency also make a difference. In this case, we have [19]

$$D^{ij}(t; f) = \frac{1}{2} \left[ \hat{u}^i(t) \hat{u}^j(t) T(f, \hat{u} \cdot \hat{w}) - \hat{v}^i(t) \hat{v}^j(t) T(f, \hat{v} \cdot \hat{w}) \right],$$

where $\hat{u}^i(t)$ and $\hat{v}^i(t)$ are unit vectors along the arms of detector which is calculated before, and $T(f, \hat{u} \cdot \hat{w})$ is transfer function defined as
where \( \text{sinc}(x) \equiv \sin x / x \). Note that in the case of low-frequency (\( f \ll f_s \)), transfer function tends to 1. The low-frequency approximation is widely used in previous works on LISA and we will also adopt this approximation. This is justified by the maximum frequency of coalescence of SMBHBs is \( \sim 10^{-3} \text{Hz} \) while \( f_s \) of these three detectors are 0.016, 0.019 and 0.28Hz. We plot GW waveform from SMBHBs of different masses in frequency domain in Fig. 3 from which we find the low-frequency approximation works well for SMBHBs with masses higher than \( 10^6 M_\odot \). In this work, we employ a higher cut-off frequency to be \( 10^{-2} \text{Hz} \), above which data is not included in analysis.

Because of the three-arm design, a single space-borne GW detector can output two independent strains. Thus, a detector corresponds to two detector tensors. In accordance with time delay interferometry, one can define two detector tensors \( D^{ij}_a, D^{ij}_p \) as

\[
D^{ij}_a = \frac{1}{6} (u_1^i u_1^j - 2u_2^i u_2^j + u_3^i u_3^j),
\]

\[
D^{ij}_p = \frac{\sqrt{3}}{6} (u_1^i u_1^j - u_2^i u_2^j + u_3^i u_3^j),
\]

(26)

where \( u_1^i, u_2^i, u_3^i \) are arm direction vectors for three arms. This formula is written under low-frequency limit.

When performing Bayesian analysis, we need instrument response in frequency domain. It is difficult to do Fourier transformation directly to Eq. (18), due to antenna pattern functions’ dependency on time. To solve this problem, we adopt stationary phase approximation (SPA). In SPA, frequency domain response can be written as

\[
\tilde{s}(f) = F_+ [t(f)] \tilde{h}_+(f) + F_\times [t(f)] \tilde{h}_\times (f),
\]

(27)

that is to say, we can change \( F_+ (t) \) into \( F_+ (t(f)) \) as a replacement of Fourier transform. The expression of \( t(f) \) is given in Appendix B Here, a tilde denotes the quantity in frequency domain.

Finally, waveform in frequency domain should include the time delay to the Sun by adding a extra phase term as follows,

\[
\tilde{h}_{+,\times} (f) = \mathcal{F} [h_{+,\times} (t)] \exp \left[ -2\pi i f \left( \frac{\hat{\varphi}}{c} + t_c - t_0 \right) \right],
\]

(28)

where \( \mathcal{F} \) means Fourier transform, \( t_c \) is coalescence time and \( t_0 \) is the start time of data.

IV. METHODOLOGY

A. Bayesian Method

Bayesian method is one of the most widely-used ways of parameter estimation in GW astronomy. Given observed data and prior distributions of parameters, one can get the posterior distribution by

\[
 p(\vec{\vartheta}|\vec{d}(t), H) = \frac{p(\vec{d}(t)|\vec{\vartheta}, H)p(\vec{\vartheta}|H)}{p(\vec{d}(t)|H)},
\]

(29)

where \( \vec{d}(t) \) is observed data, \( \vec{\vartheta} \) is parameter set, \( H \) is a hypothesis which in our case is the model of the gravitational wave signal. The dominator evidence is often ignored since it is just a normalization constant in parameter estimation. We define inner product between two strains as

\[
\langle \hat{a}(f) | \hat{b}(f) \rangle = 4\Re \int_0^{\infty} \frac{\hat{a}(f) \hat{b}^*(f)}{S_n(f)} df,
\]

(30)

where \( \text{star} \) denotes complex conjugate. \( S_n(f) \) is the PSD of the detector. The likelihood, \( p(\vec{d}(t)|\vec{\vartheta}, H) \), takes the form

\[
p(\vec{d}(t)|\vec{\vartheta}, H) = \exp \left[ -\frac{1}{2} \sum_{i=1}^{N} \langle \hat{n}_i(f) | \hat{n}_i(f) \rangle \right],
\]

(31)

on the assumption that the noise is Gaussian. Here, the subscript \( i \) denotes the \( i \)-th data strain and \( \hat{n}_i(f) \) is

FIG. 3. \(|h_+(f)|\) of GWs from different sources. Blue, orange and green lines are generated from SMBHBs with component masses of \( 5 \times 10^5 M_\odot \times 10^6 M_\odot \times 10^7 M_\odot \), respectively. Maximum frequencies of the latter two waveforms tend to be \( \sim 10^{-3} \text{Hz} \). Thus, we claim for SMBHB with component masses larger than \( \mathcal{O}(10^6) M_\odot \), low frequency limit can be applied, and the higher cut-off frequency could be chosen under \( 10^{-2} \text{Hz} \). Here, we use the waveform template IMRPhenomXHM, which will be discussed in \[15\]
the noise. For the $i$-th strain that contains data $\tilde{d}_i(f)$, we simply have

$$\tilde{d}_i(f) = \tilde{s}_i(f) + \tilde{n}_i(f), \quad (32)$$

where $\tilde{s}_i(f)$ is detector’s response to GW signals. Thus, the likelihood can be written as

$$p(\tilde{d}(t)|\bar{\theta}, H) = \exp \left[ -\frac{1}{2} \sum_{i=1}^{N} \left( \tilde{d}_i(f) - \tilde{s}_i(f, \bar{\theta}) | \tilde{d}_i(f) - \tilde{s}_i(f, \bar{\theta})\right) \right]. \quad (33)$$

When prior probability densities are also set, we obtain the posterior distribution of parameters theoretically.

Some numerical ways are developed to get the posterior for given data and likelihood, including Markov-chain Monte Carlo (MCMC) methods and Nested sampling. Here we give a simple introduction. MCMC method was first introduced in 1953 and has been improved several times since then. In MCMC method, a set of parameters (‘walker’) undergoes a random walk, and each step has a higher odds to reach higher likelihood region in parameter space. As a result, walker’s footprints finally converge to the posterior distribution. Nested sampling was proposed by Skilling in 2004, which was originally designed to calculate the evidence. Nested sampling works with a set of live points generated from prior. After each iteration, the point with the lowest likelihood will be abandoned and the new samples with higher likelihood will be generated. In the end, those live points will be mapped to posteriors.

In this work, we employ the Multinest method, a multimodal nested sampling algorithm for its high efficiency and accuracy. For example, in our practice, a 10 dimensional parameter estimation for LIGO data will take Multinest only hours, while other samplers (dynesty, emcee, etc.) need more than one day. Several tools for Bayesian parameter estimation in GW astronomy have been developed. We adopt and modify the python toolkit Bilby in this work with sampler PyMultiNest. Codes for this paper could be found in our Github repository.

### B. Waveforms and Parameters

Bayesian theorem in Eq. (29) tells us how to obtain distributions of parameters under hypothesis $H$. In this section, we show what parameters we consider, and what hypothesis (i.e., waveform) we use in this work.

As mentioned in Eq. (3), GW waveform under parity-violating gravities is GR waveform with phase and amplitude modifications. Thus, what we need to do is just choose an appropriate GR waveform. Previous studies have shown that IMRPhenom waveform with high harmonics works better in Bayesian analysis.

Subsequent works emphasize that high harmonics play an important role in Bayesian analysis for space-borne GW detectors. In consideration of this, we choose IMRPhenomXHM, a frequency domain model for the GW from non-precessing black-hole binaries in which high harmonics are available. One can decompose waveform into spherical harmonic modes. Here we give a simple introduction. MCMC method was first introduced in 1953 and has been improved several times since then. In MCMC method, a set of parameters (‘walker’) undergoes a random walk, and each step has a higher odds to reach higher likelihood region in parameter space. As a result, walker’s footprints finally converge to the posterior distribution. Nested sampling was proposed by Skilling in 2004, which was originally designed to calculate the evidence. Nested sampling works with a set of live points generated from prior. After each iteration, the point with the lowest likelihood will be abandoned and the new samples with higher likelihood will be generated. In the end, those live points will be mapped to posteriors.

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$$h_+ = \sum_{\ell,m} h_{\ell m,+} = \frac{1}{2} \sum_{\ell,m} (2Y_{\ell m}h_{\ell m} + 2Y_{\ell m}^*h_{\ell m}^*),$$

$$h_\times = \sum_{\ell,m} h_{\ell m,\times} = \frac{i}{2} \sum_{\ell,m} (2Y_{\ell m}h_{\ell m} - 2Y_{\ell m}^*h_{\ell m}^*),$$

where $2Y_{\ell m}$ is spin-weighted spherical harmonics. Except for the dominant term $(\ell, m) = (2, 2)$, we also adopt higher modes including $(\ell, m) = (2, 1), (3, 3), (4, 4), (5, 5)$ in our analysis. Note that different modes correspond to different frequency components of GW, thus the function $t(f)$ from SPA differs from modes to modes. We have

$$t_{\ell m}(f) = t_{22}(2\ell/m)$$

and Eq. (27) should be rewritten as

$$s(f) = \sum_{\ell,m} F_+[t_{\ell m}(f)]h_{\ell m,+}(f) + F_\times[t_{\ell m}(f)]h_{\ell m,\times}(f).$$

In general, GW waveform from compact binary black holes has 15 parameters: masses of two black holes, spins of two black holes which has six components, luminosity distance $d_L$, coalescence time $t_c$, coalescence phase $\phi$, inclination angle $i$, polarization angle $\psi$, and source direction which in our work is $(\phi_s, \theta_s)$. There are another two parameters in parity-violating gravities which specify velocity and amplitude birefringence respectively. To investigate the constraint of parity asymmetry, we consider two cases. (1) GW waveform with only velocity birefringence. We ignore amplitude modification because it is too small compared to phase modification and it may result in some bad behaviors in sampling. (2) GW waveform with only amplitude birefringence. The significance of this case is stated in Figure 1. In other words, we investigate velocity and amplitude birefringence individually.
In PV gravities, $\delta h$ and $\delta \Psi$ are the two modification terms. In this work, we choose $A_p/H_0$ and $-A_v$ as additional parameters in waveforms, and denote them as “A” and “B”, respectively. Thus, we have

$$\delta h(f) = B(\pi f),$$
$$\delta \Psi(f) = A(\pi f)^2. \quad (37)$$

The posterior distribution of $A$ and $B$ can be easily converted to $\mathcal{M}_{PV}$ by Eq. (7).

Unfortunately, a 16-dimensional nested sampling is extremely computationally expensive, especially when higher modes are taken into consideration and several data strains are included. To lessen computation burden, we only consider zero-spin black holes, which means we have 9 parameters in GR and 1 additional modification parameter. Since the major effects of velocity and amplitude birefringence take place during propagation, ignoring spins will not produce significant influence on our conclusions. Further simplification is needed when it comes to a three-detector network, which will be discussed in next section.

V. RESULTS AND DISCUSSIONS

A. Recovering GR Parameters

To make sure our estimation is reliable, we first try to recover 9 parameters in GR. We inject a GW signal of a SMBHB with masses at order of $10^6 M_\odot$ and luminosity distance at 20Gpc. Here, we should emphasize that, in order to fasten the calculation speed, we assume the duration of the signal is $2^{18}$s (about 3 days) and sampling frequency is 1/16 Hz. Other parameters are chosen randomly. It takes PyMultiNest one week to generate about 8000 posterior samples. We find that if the higher modes are not included, sampling can be done much faster but not all parameters could be recovered because the parameter degeneracies will corrupt the result. Corner plot in Fig. 5 is the result of single detector LISA, in which we observe that all parameters are correctly recovered. The coalescence between SMBHB produces a large signal-to-noise ratio (SNR) in space-borne GW detectors. In our 3-day-long simulated data, the matched filter SNR is larger than 400, which enables detector to give a precise estimation.

We also make a comparison between detector network and single detector by comparing their localization ability. Different detectors are in different positions, so a network could provide more triangulation information which is critical in GW event localization. By using the Fisher matrix analysis, recent work on LISA and Taiji[14] has shown that the network can improve the localization by several orders. Our result shows a consistent results by adopting the Bayesian analysis. As shown in Fig. 6 posterior samples form detector network are much more precise and accurate. 90% confidence area is $\mathcal{O}(10^{-2})$ square degree for LISA-Taiji network, while for single LISA 90% confidence area is $\mathcal{O}(10^{-1})$. Note that the improvements in localization increase with observation time[14]. If signal duration is longer than 3 days, network localization will be more distinct from single detector. The order of confidence area is consistent with previous work[18], which confirms the reliability of our analysis.

B. Constraining PV Parameters

In Sec. IV we defined two parameters $A$ and $B$ in parity-violating GW waveforms and explained two cases to consider. Here, we inject GW signals from SMBHBs with the same masses as in GR case, and set $A = B = 0$ in our fiducial model. In the Bayesian analysis, we add the PV parameters in addition to the parameters in GR. Note that, the expectation values of these PV parameters are zero. The results are as follows.

Corner plots in the left bottom of Figs. 5 and 6 are the sampling results of two cases. Fig. 5 shows the constraint of velocity birefringence parameter, while Fig. 6 shows that of the amplitude birefringence parameter. In both of the two corner plots, we consider the case with single LISA detector. We find that all parameters are correctly recovered, yet PV parameters show correlations with some GR parameters. Constraints on $\mathcal{M}_{PV}$ are also given in Fig. 5 and Fig. 6 where we also compare results given by single detector and detector networks. Nevertheless, a ten dimensional analysis for multi-detector is too computational expensive. When we add a space-borne detector to analysis, we are actually adding two data strains, and computational burden increases fast. Due to limited time, we lower the dimensionality by setting some parameters’ priors as delta function located at injected value and compare three cases under this simplification. We do this to $m_1$, $m_2$, $\theta_c$, $\phi_e$, $\phi$, $\epsilon$ in velocity birefringence, and to $m_1$, $m_2$, $\theta_c$, $\phi_e$, $\phi$, $t_e$, $\psi$ in amplitude birefringence. That is to say, parameters that have correlation with PV parameter are preserved, so that detectors’ ability of constraining PV parameters is not significantly influenced.

As in top right of Figs. 5 and 6 we present the posterior distributions of the effective PV parameters and constraints on $\mathcal{M}_{PV}$. From the violin plots on the upper panel, we find that compared to case with a single detector, detector networks could effectively reduce the impact of the correlations and give a more precise estimation, while the number of detectors in network (2 or 3) does not make a difference. Since TianQin is less sensitive at the low frequency that we consider in this work, adding TianQin to detector network does not improve the constraint. On the lower panel lies the posterior distribution of $\mathcal{M}_{PV}$. Different cases are shown in different colors and the 90% confidence level are labeled with dashed vertical lines. In velocity birefringence case, the lower limits of confidence level are labeled with dashed vertical lines. In the Bayesian analysis, we find that all parameters are correctly recovered, yet PV parameters show correlations with some GR parameters. Constraints on $\mathcal{M}_{PV}$ are also given in Fig. 5 and Fig. 6 where we also compare results given by single detector and detector networks. Nevertheless, a ten dimensional analysis for multi-detector is too computational expensive. When we add a space-borne detector to analysis, we are actually adding two data strains, and computational burden increases fast.
fringence they are $6.42 \times 10^{-16}$ eV, $1.95 \times 10^{-15}$ eV and $1.44 \times 10^{-15}$ eV respectively. It is reasonable that velocity birefringence gives a stronger constraint since the effect of amplitude birefringence is much weaker. Detector network can improve the constraint on $M_{\text{PV}}$ by at most 125% in amplitude birefringence case.

Finally, we give a comparison of the constraints on PV gravity from different tests in Table I, in which we can see the detection of GWs greatly improved the constraints on PV gravity. Space-borne GW detectors could give constrain on $M_{\text{PV}}$ at $O(1)$ eV, which is not a strong constraint in comparison with that provided by ground-based GW detectors. It is understandable because the phase modification is proportional to $f^2$, and space-borne detectors have lower frequency bands. That is to say, GW has a smaller phase modification which is more difficult to detect. However, this is the first chance for testing parity symmetry of gravity with supermassive black holes. Probes into this extreme process still have exciting prospects.

VI. CONCLUSIONS AND DISCUSSIONS

The gravitational-wave signals, produced by the coalescence of compact binaries, provide an excellent opportunity to test the fundamental properties of gravity in the strong gravitational fields. In addition to various ground-based GW detectors, several space-borne detectors, including LISA, Taiji and TianQin, are expected to be launched in the near future. They are sensitive to the GW signals at a lower frequency bands, and will open a new window for the GW astronomy. In this article, we extend our previous works on testing the parity symme-
FIG. 5. Parameter estimation with velocity birefringence. Left bottom: Corner plot generated by a single detector LISA, in which we can see all parameters are correctly recovered, yet it shows correlations between $A$, $\psi$, and $t_c$. Top right: Constraints on parity asymmetry of gravity from different detector networks. Upper panel is violin plot for PV parameter $A$ and lower panel is posterior distribution of $M_{PV}^{-1}$ from different detector networks. Note that $M_{PV}^{-1} \rightarrow \infty$ returns to GR, so we choose $M_{PV}^{-1}$ as $x$ axis. Red, green, blue lines correspond to LISA, LISA+Taiji, LISA+Taiji+TianQin respectively. Dashed lines are 90 percentiles confidence levels, which equal to 1.50eV, 2.66eV and 2.14eV for these three cases.

In analysis, we first simulate GW signals with the waveform template IMRPhenomXHM, and inject them into various detectors. Then, employing the modified Bilby package, we use the Bayesian method to estimate the physical parameters of the compact binaries and constrain the parameters which quantify the velocity birefringence and amplitude birefringence effects in parity-violating gravities. We find that the future space-borne GW detectors are able to give the lower bound of the parity-violating energy scale $M_{PV} \gtrsim O(1) eV$ by constraining the velocity birefringence effect of GWs. Since the space-borne detectors are sensitive to the GW signal of lower frequencies, this bound is weaker than that derived from the observations of ground-based GW detectors.

At the end of this paper, we should mention that, as the first attempt to numerically investigate the potentials of future space-borne detector networks, we have to simplify the calculation in the following aspects: First, we adopt only three-day GW signals for analysis, which is much less than the realistic duration of future GW detection. Second, in Bayesian analysis, we have reduced the number of parameters by setting them as known values, including the spin of black holes. Third, in order to transfer the responses of detectors from time domain to frequency domain, we adopt the SPA to simplify our calculation. We should emphasize, these are common
FIG. 6. Same with Fig. 5 but here we consider the case with the effect of amplitude birefringence, instead of the velocity birefringence.

| Method                                      | Lower Limit of $M_{PV}$ |
|--------------------------------------------|-------------------------|
| LIGO-VIRGO events Bayesian analysis [12]   | 0.07 GeV                |
| space-borne GW detector events Bayesian analysis | ~ 1 eV                  |
| waveform-independent constraint from GWs [31] | $1.4 \times 10^4$ eV  |
| GW speed [32, 33]                                      | 10 eV                   |
| solar system tests [34]                                  | $2 \times 10^{-13}$ eV |
| binary pulsar [35, 36]                                | $5 \times 10^{-10}$ eV |

TABLE I. Constraint on parity-violating energy scale from different tests.

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Appendix A: TianQin Orbit

Orbit of TianQin in ecliptic frame is given as follows [3]:

problems in community when it comes to Bayesian analysis of GW signal of space-borne detectors, which should be overcome by various techniques in future works.
where $R = 1$ AU and $e = 0.0167$ are the semi-major axis and the eccentricity of the geocenter orbit around the Sun; $R_1 = 1.0 \times 10^5$ km and $e_1$ are the semi-major axis and the eccentricity of the spacecraft orbit around the Earth. $\theta_s = -4.7^\circ$, $\psi_s = 120.5^\circ$ is the ecliptic coordinates of RX J0806.3+1527. $f_m$ equals to $1$/(oneyear) and $\alpha(t) = 2\pi f_m t + \kappa_0$ is the mean ecliptic longitude of the geocenter in the heliocentric-ecliptic coordinate system. $\kappa_0$ is the mean ecliptic longitude measured from the vernal equinox at $t = 0$. $\beta$ is the longitude of the perihelion. $\alpha_n$ represents orbit phase of the n-th spacecraft. A specific introduction of the orbit can be found in [3].

Appendix B: $t(f)$ in stationary phase approximation

In stationary phase approximation, the relation $t(f)$ mentioned in Sec. III takes the form[37]

$$t(f) = t_c - \frac{5}{256 (GMc)^{8/3}} (\pi f)^{-8/3} \sum_{i=0}^{7} \tau_i \pi f G m^{i/3},$$

with coefficients

$$\tau_0 = 1, \quad \tau_1 = 0,$$

$$\tau_2 = \frac{743}{252} + \frac{11}{3} \eta,$$

$$\tau_3 = -\frac{32}{5} \pi,$$

$$\tau_4 = \frac{3058673}{508032} + \frac{5429}{504} \eta + \frac{617}{72} \eta^2,$$

$$\tau_5 = -\left(\frac{7729}{252} - \frac{13}{3} \eta\right) \pi,$$

$$\tau_6 = -\frac{10052469856691}{23471078400} + \frac{128 \pi^2}{3} + \frac{6848 \gamma}{105} + \left(\frac{3147553127}{3048192} - \frac{45 \pi^2}{12}\right) \eta,$$

$$-\frac{15211}{1296} \eta^2 + \frac{25565}{1296} \eta^3 + \frac{3424}{105} \ln \left[16(\pi mf)^{2/3}\right],$$

$$\tau_7 = \left(-\frac{15419335}{127008} - \frac{75703}{378} \eta + \frac{14809}{378} \eta^2\right) \pi,$$
where $\gamma = 0.5772$ is the Euler-Mascheroni constant, $m$ is total mass $m_1 + m_2$ of binary. $\eta = m_1 m_2 / (m_1 + m_2)^2$ is the symmetric mass ratio and $M_0 = \eta^{3/5} m$ is chirp mass.

Note that the time-frequency relation $t(f)$ defined by Eq. (B1) is for the dominant term. For other modes, we have

$$t_\text{tm}(f) = t(2f/m). \quad (B3)$$
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