Direct Simultaneous Multi-Image Registration

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Abstract. This paper presents a novel algorithm that registers a collection of mono-modal 3D images in a simultaneous fashion, named as Direct Simultaneous Registration (DSR). The algorithm optimizes global poses of local frames directly based on the intensities of images (without extracting features from the images). To obtain the optimal result, we start with formulating a Direct Bundle Adjustment (DBA) problem which jointly optimizes pose parameters of local frames and intensities of panoramic image. By proving the independence of the pose from panoramic image in the iterative process, DSR is proposed and proved to be able to generate the same optimal poses as DBA, but without optimizing the intensities of the panoramic image. The proposed DSR method is particularly suitable in mono-modal registration and in the scenarios where distinct features are not available, such as Transesophageal Echocardiography (TEE) images. The proposed method is validated via simulated and in-vivo 3D TEE images. It is shown that the proposed method outperforms conventional sequential registration method in terms of accuracy and the obtained results can produce good alignment in in-vivo images.

Keywords: direct method, simultaneous registration, bundle adjustment, 3D image.

1 Introduction

Image registration is the process of aligning two or more images of a scene into the same coordinates [¹]. It is a fundamental task for many medical image analysis problems where valuable information conveyed by different images needs to be combined and examined. In the past decades, mainstream medical imaging techniques, such as CT, MRI, and Ultrasound (US), are evolving from 2D to 3D, which not only bring in many benefits for clinicians but also propose new
challenges which has attracted a lot of attentions [2,3,4,5]. In general, images involved in the registration process may be captured by the same (mono-modal) or different (multi-modal) imaging devices. In this study, we focus on the registration tasks that involve a sequence of 3D mono-modal images in rigid scenarios.

Research works on pairwise image registration can be roughly classified into two groups: feature-based methods [6] and direct methods [7]. Feature-based methods estimate the transformations based on the extracted sparse feature points. However, there is still a lack of robust feature extraction methods for 3D images [8,9]. Thus, few works based on 3D feature extraction for registration are reported [10,11]. On the other hand, direct methods have occupied a dominant position in the field of medical registration [12,13]. The methods estimate the transformation directly based on intensities of images by maximizing the similarity between images without providing certain correspondences and therefore, direct methods can work well on 3D images [2].

Registration of a collection of images is much more complex than pairwise registration. One strategy for solving this problem is to deduce the global poses from the results of pairwise registration [14,15]. This strategy, although being intuitive, may be biased to the selected reference image and usually inevitably bring in accumulating errors in the results. The other strategy is to optimize the poses of all local frames simultaneously. [3] presented a method using multivariate similarity measures for simultaneous registration, but the overlapping areas are used to compute the objective function multiple time, thus may have the information reuse issue and increase the extra complexity in the optimization. [16] proposed a framework called congealing which uses underlying entropic information of images for alignment. But as [17] pointed out, employing entropy for congealing is problematic due to its poor optimization characteristics. High computational cost is also a challenge for original congealing, thus sampling need to be used for 3D images [18]. Bundle adjustment (BA) [19] is regarded as Gold Standard for obtaining optimal poses of multiple 2D images in computer vision and robotics community. However, most of the BA algorithms belong to the feature-based methods that require feature correspondences between images prior to BA. [20] proposed a photometric BA method which jointly refines the poses and scene structure based on 2D images. Although correspondences are not needed, it requires highly accurate initial poses (using optimized poses from feature-based BA in the study) and extracted salient 2D points to initialize 3D points and then use a visibility list to indicate the frames where the correspondences appear. These strategies uses the correspondences latently. Furthermore, fixing reference patches for every scene point may make the results have bias to the chosen reference patches.

In this paper, we propose a novel direct simultaneous registration (DSR) method which optimizes global poses of a collection of 3D images directly based on image intensities. The conventional direct method is extended from commonly used pairwise registration scenario to multi-3D images registration by introducing a panoramic image in our proposed direct bundle adjustment (DBA) framework. And then, independence of poses from intensities of panoramic is
proved so that poses can be optimized independently without considering intensities of the panoramic image to obtain exactly the same optimal results as the DBA method. For distinction, the method is named as DSR. The proposed DSR method retains the most important advantage of direct method, i.e. all intensity information of images is used, running in a simultaneous fashion and without any reference image, correspondences, or information reuse. DSR is an elegant way to obtain the optimal poses of local frames.

2 Methodology

2.1 Direct Bundle Adjustment

Problem Statement. Suppose there are $m$ frames of 3D images taken from different viewpoints and denoted as $I = \{I_1, ..., I_i, ..., I_m\}$. And correspondingly, the rigid transformation for each frame is parameterized in Lie algebra space [21] with the pose parameters $x_\xi = [\xi_1^\top, ..., \xi_i^\top, ..., \xi_m^\top]^\top \in \mathbb{R}^{6m}$. Simultaneous registration is the process of estimating the optimal pose parameters of all local frames $\hat{x}_\xi$ at the same time that align images in a common frame.

Formulation of Direct Bundle Adjustment. In most BA problems, to obtain the optimal solution of poses, the global 3D map needs to be optimized simultaneously. In the scenario mentioned above, suppose $M$ is defined as a 3D panoramic image consisting of $n$ voxels $\{p_1, ..., p_j, ..., p_n\}, p_j \in \mathbb{R}^3$, which fuses all the images of local frames. The intensity of voxel $p_j$ in $M$ is obtained from fusing different points’ intensities in local images. Assume that the intensity of $p_j$ in $M$ and the one of its corresponding point $p_{ij}$ in local frame $I_i$ are denoted as $M(p_j)$ and $I_i(p_{ij})$, respectively. The intensity difference between $M(p_j)$ and $I_i(p_{ij})$ is

$$e_{ij}(\xi_i, M(p_j)) = M(p_j) - I_i(\omega(\xi_i, p_j)) = M(p_j) - I_i(p_{ij}),$$

(1)

where $p_{ij} = \omega(\xi_i, p_j) = T(\xi_i)p_j$ transforms $p_j$ to $p_{ij}$, and $T(\cdot) \in SE(3)$ maps the pose parameters $\xi_i$ to a 3D Euclidean transformation. Note that the coordinates of $p_j$ are integers which represent the voxel location on the 3D grid, while the transformed point $p_{ij}$ may not be integers thus not on the voxel of the grid on the local image. During the optimization process, the intensity of $p_{ij}$ in $I_i$ is obtained using interpolation in iterations to reduce the error in estimating the intensity difference $e_{ij}$ in (1).

After introducing the panoramic image $M$, we first propose a direct bundle adjustment (DBA) method that jointly refines the poses of local frames and the intensities of the panoramic image. The overall state parameters considered in the proposed DBA are $x = [x_\xi^\top, x_M^\top]^\top$, where $x_M = [..., M(p_j),...]^\top$ are the intensities of voxels in $M$ which are observed in the local frames. Then, based

\footnote{We implicitly perform the conversion between 3D Euclidean coordinates and homogeneous coordinates here for $p_j$ and $p_{ij}$.}
on direct method, we seek to obtain the optimal solution \( \hat{x} = [\hat{x}_\xi^T, \hat{x}_M^T]^T \) that minimize the sum of squared intensity differences between panoramic image and local images

\[
\hat{x} = \arg\min_{\xi, x_M} \sum_{i=1}^{n} \sum_{j=1}^{m} \sigma(p_{ij})(e_{ij}(\xi_i, M(p_j)))^2,
\]

where \( \sigma(p_{ij}) = 1 \) if the transformed point \( p_{ij} \) is within the image area of \( I_i \) (i.e. \( p_i \) is observed in \( I_i \)), otherwise \( \sigma(p_{ij}) = 0 \). In (2), although intensity optimization is not required in the registration problems, \( x_\xi \) and \( x_M \) need to be optimized jointly if we refer to conventional solutions to nonlinear least-squares problems.

**Gauss-Newton Iteration.** If we write the overall observed intensity differences as a concatenation vector \( e(x) = [\ldots, e_{ij}, \ldots]^T \) where \( \sigma(p_{ij}) = 1 \), the objective function of (2) to be optimized can be rewritten as \( f(x) = e(x)^T e(x) \). Gauss-Newton (GN) method is commonly used to solve the nonlinear least-squares problem iteratively. For (2), step change \( \Delta x \) in each iteration can be calculated from the GN equation as:

\[
J(x)^T J(x) \Delta x = -J(x)^T e(x),
\]

where \( J(x) \) is the Jacobian matrix of \( e(x) \) w.r.t. \( x \). Consider one row of \( J(x) \), denoted as \( J_{ij}(x) \) which is the Jacobian of one intensity difference \( e_{ij} \) w.r.t. \( x \). It is shown in (1) that \( e_{ij} \) is only dependent on \( \xi_i \) and \( M(p_j) \), thus,

\[
J_{ij}(x) = [0, \ldots, \frac{\partial e_{ij}}{\partial \xi_i}, \ldots, 0, \ldots, \frac{\partial e_{ij}}{\partial M(p_j)}, \ldots, 0],
\]

where

\[
\frac{\partial e_{ij}(\xi_i, M(p_j))}{\partial \xi_i} = -\frac{\partial I_i}{\partial \omega(\xi_i, p_j)} \frac{\partial \omega(\xi_i, p_j)}{\partial \xi_i}, \quad \text{and} \quad \frac{\partial e_{ij}(\xi_i, M(p_j))}{\partial M(p_j)} = 1.
\]

\( \partial I_i/\partial \omega = \nabla I_i \) represents the intensity gradient of \( I_i \) which can be pre-computed as the gradient space of \( I_i \) on the local 3D image before GN iterations to improve the efficiency of computation.

### 2.2 Simultaneous Registration without Intensity Optimization

**Schur Complement.** Directly sloving the linear system (3) is usually time-consuming due to the large dimension of intensities in \( M \). Here sparsity of (3) and Schur complement [22] are used. If we write Jacobian matrix of \( e(x) \) w.r.t. \( x_\xi \) and \( x_M \) separately as \( J(x) = [J_\xi(x_\xi), J_M(x_M)] \), then (3) can be rewritten as the following format:

\[
\begin{bmatrix}
H_{\xi\xi} & H_{\xi M} \\
H_{M\xi} & H_{M M}
\end{bmatrix}
\begin{bmatrix}
\Delta x_\xi \\
\Delta x_M
\end{bmatrix}
= \begin{bmatrix}
b_\xi \\
b_M
\end{bmatrix},
\]

where

\[
\begin{align*}
J_{ij}(x) & = [0, \ldots, \frac{\partial e_{ij}}{\partial \xi_i}, \ldots, 0, \ldots, \frac{\partial e_{ij}}{\partial M(p_j)}, \ldots, 0], \\
\frac{\partial e_{ij}(\xi_i, M(p_j))}{\partial \xi_i} & = -\frac{\partial I_i}{\partial \omega(\xi_i, p_j)} \frac{\partial \omega(\xi_i, p_j)}{\partial \xi_i}, \quad \text{and} \quad \frac{\partial e_{ij}(\xi_i, M(p_j))}{\partial M(p_j)} = 1.
\end{align*}
\]

\( \partial I_i/\partial \omega = \nabla I_i \) represents the intensity gradient of \( I_i \) which can be pre-computed as the gradient space of \( I_i \) on the local 3D image before GN iterations to improve the efficiency of computation.
where \( H_{\xi \xi} = J_{\xi}^T J_{\xi}, H_{\xi M} = H_{M \xi}^T = J_{\xi}^T J_{M}, H_{MM} = J_{M}^T J_{M}, b_\xi = -J_{\xi}^T e(x), \) and \( b_M = -J_{M}^T e(x). \) Then, (6) becomes:

\[
(H_{\xi \xi} - H_{\xi M} H_{MM}^{-1} H_{\xi M}^T) \Delta x_\xi = (b_\xi - H_{\xi M} H_{MM}^{-1} b_M),
\]

(6)

\[
H_{MM} \Delta x_M = b_M - H_{\xi M} \Delta x_\xi.
\]

(7)

Based on the sparse property of the Jacobian as shown in (4), \( H_{MM} \) which has the huge dimensions is diagonal and the values of diagonal elements represent the number of times that corresponding voxels of \( M \) have been observed in the local frames in the current iteration. So, the inverse of diagonal matrix \( H_{MM} \) can be easily computed by the inverse of each element on the diagonal, which makes solving (6) and (7) efficient.

**Independence of Optimizing Poses to Intensities.** In DBA, the panoramic image \( M \) contains a large number of voxels and optimizing intensities of \( M \) is not required for most of registration problems. In this paper, we further proved that the optimization of poses is independent of the intensities of panoramic image in our proposed DBA problem, which means pose parameters of local frames can be updated independently with the step changes calculated from (6) without considering the intensities in \( M \) in every GN iteration.

Suppose intensity differences \( e(x) \) are decomposed into two components \( e(x) = A - B, \) where \( A = [..., M(p_i),...]^T \) and \( B = [..., I_i(p_{ij}),...]^T \) represent observed intensities in the panoramic image and their corresponding intensities in local frames, respectively. If only the right hand side of (6) is considered, we have:

\[
\begin{align*}
  b_\xi - H_{\xi M} H_{MM}^{-1} b_M &= -J_{\xi}^T (A - B) + J_{\xi}^T J_{M} (J_{M}^T J_{M})^{-1} J_{M}^T (A - B) \\
  &= -J_{\xi}^T (A - J_{M} (J_{M}^T J_{M})^{-1} J_{M}^T A) - J_{\xi}^T (J_{M} (J_{M}^T J_{M})^{-1} J_{M}^T B - B).
\end{align*}
\]

(8)

It is shown from \( J_{ij} \) in (4) that there is only one nonzero element 1 in each row of \( J_{M} \). This nonzero element means the voxel \( p_{ij} \) is observed in the local frame \( i \).

One voxel, e.g. \( p_{ij} \), may be observed in different local frames, thus there may be more than one element whose values are equal to 1 in each column of \( J_{M} \). It is easy to know that the columns of the matrix \( J_{M} \) are linearly independent from its structure so that \( J_{M} (J_{M}^T J_{M})^{-1} J_{M}^T \) is a projection matrix [23]. According to the observed status of panoramic image in the local frames which is indicated by the structure of \( J_{M} \), we have \( J_{M} x_M = A \), i.e. \( A \) is in the column space [23] of \( J_{M} \). Therefore, the first term on the right side of (8) becomes:

\[
-J_{\xi}^T (A - J_{M} (J_{M}^T J_{M})^{-1} J_{M}^T x_M) = 0.
\]

(9)

Then, (6) becomes:

\[
(H_{\xi \xi} - H_{\xi M} H_{MM}^{-1} H_{\xi M}^T) \Delta x_\xi = -J_{\xi}^T (J_{M} (J_{M}^T J_{M})^{-1} J_{M}^T B - B),
\]

(10)

which is independent of intensities \( x_M \) of panoramic image \( M \). Note that \( B \) is not in the column space of \( J_{M} \), thus \( J_{M} (J_{M}^T J_{M})^{-1} J_{M}^T B \neq B \). Therefore, we
can solve the poses using (10) in the proposed DBA algorithm only, which is equivalent to solving the complete DBA problem in (2). For distinction, we call this method as Direct Simultaneous Registration (DSR).

Fig. 1. Accuracy assessment of the proposed method.

Fig. 2. Left: Comparison of the fused images using poses from DSR and Sequential results; Right: AS and LAA before (upper row) and after (bottom row) multi-view fusion (viewed in three slices).

3 Experiments and Results

Registration of US image is usually more challenging than other imaging modality like CT and MRI due to its relatively low signal-to-noise ratio and small field of view (FoV). Thus, in this section, validation of the proposed DSR algorithm is performed using the simulated and in-vivo 3D TEE image data.

3.1 Simulated Experiments

We run experiments based on the simulated 3D TEE images for quantitative validation of the proposed DSR algorithm in terms of accuracy. Five sequences of 3D images are generated from a 3D CT image (used as ground truth) by using different poses of simulated US probe. The FoV of simulated TEE images are
the same as the one in actual 3D TEE image (see last column in Table 1) to get similar imagery information as real volumes. Each sequence contains 11 frames of 3D images and the magnitude of transformations between consecutive frames varied between \( \pm 12 \) degrees in three dimensions (rotating around X-Y-Z axes), and \( \pm 15 \) pixels in three dimensions (translating along X-Y-Z axes). Studies show that, after logarithmic compression, multiplicative speckle noise [24] can be transformed as a kind of additive noise in the US images and is close to Gaussian distribution [24,25,26]. Thus, Gaussian noise with std 25 is generated randomly and added to these five sequences.

To verify the equivalence of proposed DSR and DBA for pose estimation, the two methods are performed respectively from the same initial guess and the objective functions in each iteration are compared. Note that in the proposed DSR method, the intensities of panoramic image are not needed. Panoramic image is computed in DSR method deliberately only for comparing the objective function. It is found that the values of the objective functions from the two methods are always the same at every iteration, which indicates the estimated poses from the two algorithms at every step are always the same.

Then, comparative experiments are performed by using the sequential registration method [15] on the same datasets. Sequential registration adapts the pairwise registration method by registering one image to the panoramic image at a time, then, fusing the current image into the panoramic image, and use this panoramic image as the reference for the next frame. Sum of squared difference (SSD) is used as the similarity measures for sequential registration. In Fig. 1, mean absolute errors (MAE) of translation and Euler angles w.r.t. the ground-truth poses are compared for the five groups of comparative experiments, respectively. It is clear that the DSR method has better accuracy and is more robust in terms of image noise than the sequential method. For the accuracy, the MAE of the results obtained from the sequential method is within 0.5 pixels for translations and within \( 10^{-3} \) for rotations. While, the MAE for both translations and rotations from the sequential method are 2-4 times larger than those from the proposed DSR method.

### 3.2 In-vivo Experiments

In the in-vivo experiments, datasets of 3D TEE images from six different patients are collected using a 2D array transducer with the assistance of the ECG-gating technique [27] so that registration of these images can be considered as rigid. The details of the datasets are listed in the first four columns of Table 1.

First, the equivalence of the proposed DSR and DBA methods for pose estimation are verified again, which shows that calculated poses at each iteration are always the same. Analyzing the accuracy of results of the proposed algorithm based on in-vivo datasets is complex because ground-truth pose is usually not available. Visually we can confirm that the transition areas of the fused image will be smooth if the poses are estimated accurately and there will not be ghosting in the fused image. Thus, in-vivo 3D TEE images are fused using the estimated poses to provide qualitative results of experiments. We manually
Table 1. Details of In-vivo Datasets

| Patient | No. frames | Volume size (voxel) | Resolution (mm/voxel) | FoV w.r.t. Orig. |
|---------|------------|---------------------|-----------------------|-----------------|
| # 1     | 11         | 236×224×208         | 0.69×0.72×0.77        | 2.18            |
| # 2     | 9          | 240×160×208         | 0.69×0.98×0.73        | 2.10            |
| # 3     | 6          | 240×160×208         | 0.77×1.11×0.82        | 2.02            |
| # 4     | 6          | 277×208×208         | 0.58×0.87×0.63        | 2.01            |
| # 5     | 6          | 240×160×208         | 0.64×0.92×0.68        | 1.80            |
| # 6     | 8          | 277×208×208         | 0.63×0.90×0.68        | 2.06            |

select areas which contain sharp boundaries like left atrium (LA) wall in the fused images for evaluation because generally, misalignment/ghosting caused by poses with low accuracy can be easily found in these areas. The selected regions are shown in Fig. 3 with three orthogonal slices and the boundaries of two registered images in the fused images are highlighted in colors. By observing the LA walls which are indicated by white arrows in Fig. 3, it is found that the stitching areas have smooth transition and no misalignment/ghosting is found in the image, which suggests that good quality alignments have been obtained by the proposed algorithm.

As an example, the advantages of the proposed DSR method over the sequential method can be illustrated using Fig. 2(left), where the fusing results using the poses obtained by DSR and the poses by sequential method are both shown. It is clear that the result by sequential method has misalignment. We also compare the objective function value (2) for the all six in-vivo datasets. The objective function values are (4.04, 3.35, 4.57, 2.37, 2.46, 9.56)×10^9 and (5.64, 4.60, 4.81, 2.47, 3.45, 11.76)×10^9, for DSR and sequential method respectively, showing the results by sequential method are local minima.

Simultaneous registration has many important clinical applications. For transcatheter left atrial appendage (LAA) occlusion, it is essential to visualize the atrial septum (AS) and the LAA in a single volume to facilitate transseptal puncture at specific site of the AS in relation to the LAA position and orientation. After we fuse the multi-view images using the estimated poses from DSR, complete structures of AS and LAA which cannot be fully observed in a single volume (shown in upper row in Fig. 2(right)) are recovered (shown in the bottom row in Fig. 2(right)). By counting the number of voxels, it is found that the FoV of the fused image is enlarged to 2.18, 2.10, 2.02, 2.01, 1.80, and 2.06 times as compared with the original single frame of TEE image of # 1 to # 6 respectively, which are listed in the fifth column of Table 1.

4 Conclusion

Starting from the framework of direct bundle adjustment, a novel direct simultaneous registration algorithm for 3D image is proposed in the paper. The method can optimize the poses of a sequence of local frames simultaneously without any
Fig. 3. Fused 3D TEE images using registration results from DSR for six in-vivo datasets. LA walls which have sharp structures in the images are indicated by white arrows in selected areas. Colored frames are the boundaries of two registered volumes.

information loss or information reuse. Simulated and in-vivo experiments on 3D Transesophageal Echocardiography datasets are performed to demonstrate that the proposed algorithm can obtain more accurate results compare to the conventional sequential registration method. In-vivo experiments also shows accurate structures and extended field of view, indicating a good quality of registration and a significant potential clinical value of the proposed algorithm.

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