A Taxonomy of Similarity Metrics for Markov Decision Processes

Álvaro Visús¹, Javier García¹, Fernando Fernández¹

¹Departamento de Informática, Universidad Carlos III de Madrid
Avda. de la Universidad, 30. 28911 Leganés (Madrid). Spain
avisus@pa.uc3m.es, {fjgpolo, ffernand}@inf.uc3m.es

Abstract

Although the notion of task similarity is potentially interesting in a wide range of areas such as curriculum learning or automated planning, it has mostly been tied to transfer learning. Transfer is based on the idea of reusing the knowledge acquired in the learning of a set of source tasks to a new learning process in a target task, assuming that the target and source tasks are close enough. In recent years, transfer learning has succeeded in making Reinforcement Learning (RL) algorithms more efficient (e.g., by reducing the number of samples needed to achieve the (near-)optimal performance). Transfer in RL is based on the core concept of similarity: whenever the tasks are similar, the transferred knowledge can be reused to solve the target task and significantly improve the learning performance. Therefore, the selection of good metrics to measure these similarities is a critical aspect when building transfer RL algorithms, especially when this knowledge is transferred from simulation to the real world. In the literature, there are many metrics to measure the similarity between MDPs, hence, many definitions of similarity or its complement distance has been considered. In this paper, we propose a categorization of these metrics and analyze the definitions of similarity proposed so far, taking into account such categorization. We also follow this taxonomy to survey the existing literature, as well as suggesting future directions for the construction of new metrics.

1 Introduction

Markov decision processes (MDPs) are a common way of encoding decision making problems in Reinforcement Learning (RL) tasks [Sutton and Barto, 2011]. In RL, an MDP is considered to be solved when a policy (i.e., a way of behaving for each state) has been discovered which maximizes a long-term expected return. However, although RL is known as an effective machine learning technique, it might perform poorly in complex problems, leading to a slow rate of convergence. This issue magnifies when facing realistic continuous problems where the curse of dimensionality is inevitable. Transfer learning in RL is a successful technique to remedy such a problem. Specifically, rather than learning a new policy for every MDP, a policy could be learned on one MDP, then transferred to another, similar MDP, and either used as is, or treated as a starting point from which to learn the new policy. Clearly this transfer cannot be done successfully between any two MDPs, but only in the case they are similar.

Therefore, in this context one question arises: when are two MDPs similar? We must give a better definition of what constitutes similar MDPs. In this paper, we consider the concept of similar is related to the notion of “positive transfer” [Taylor and Stone, 2009]. Formally, positive transfer happens when the knowledge in the source task contribute to the improved performance of learning in the target task, and it is considered a negative transfer otherwise, i.e., when the transfer hurts the learning performance when compared with learning from scratch. Additionally, the greater the improvement in the target task, i.e., the greater the positive transfer, the more similar the tasks has to be considered. It is important to be aware of the fact that, based on this description, the concept of similarity might not be related with the structural similarities between the MDPs. So the correct selection of metrics that allow us to measure the similarity between MDPs is a critical issue in transfer learning, precisely to avoid the negative transfer.

The literature in transfer learning has proposed different metrics to measure the level of similarity between MDPs, hence, different definitions of the concept of similarity have been considered so far. This paper surveys the existing similarity metrics and contributes a taxonomy that, in its root, classifies them into two clearly distinct categories: model-based, and performance-based metrics. We consider such a distinction as a core contribution, allowing to categorize metrics in a novel and useful way. Model-based metrics are based on the structural similarities between the MDP models. Such model-based metrics can be computed in different ways depending on what elements of the MDP models come into play to compute the similarity [Ammar et al., 2014; Taylor et al., 2008b; Milner, 1982; Castro and Precup, 2011; Svetlik et al., 2017]. Instead, performance-based metrics are computed by comparing the performance of the learning agents in the source task and the target task. Such a performance comparison can be done in two different ways: by comparing the resulting policies from learning in the
source task and the target task [Carroll and Seppi, 2005; Karimpanal and Bouffanais, 2018] or, from a transfer point of view, by measuring the reuse gain, i.e., the positive transfer [Mahmud et al., 2013; Sinapov et al., 2015; Fernández and Veloso, 2013]. Many metrics can be used to measure such a reuse gain of transfer (e.g., jumpstart, asymptotic performance, total reward) [Taylor and Stone, 2009]. In some ways, this reuse gain could be the best method for measuring similarity between two tasks [Carroll and Seppi, 2005]. Unfortunately, it is often difficult to compute all of these performance-based measures before actually solving the target task, since most of them require to be computed a posteriori, i.e., after the learning processes. However, there are some few exceptions to this rule, within which, for example, the similarity is computed on-line, i.e., during solving the target task [Fernández and Veloso, 2013].

We hope our taxonomy applies to a wide range of researchers, not just those interested in transfer learning. For example, the proposed metrics could also be a critical step forward to sort the samples and tasks in Curriculum Learning [Narvekar et al., 2020], or could be used to measure the distance between simulation and the real world in a Sim-to-Real context [Zhao et al., 2020]. They could also be used to understand the similarities within a set of tasks in Multi-task Learning [Shui et al., 2019], or Automated Planning [Fernández et al., 2011].

The remainder of the paper is organized as follows. Section 2 provides some preliminary concepts required to better understand the rest of the paper. Section 3 presents our proposed taxonomy, explaining how we group and categorize the different metrics. The model-based metrics are examined in Section 4, whilst the performance-based metrics are considered in Section 5. In Section 6 we discuss the surveyed methods and in Section 7 we identify open areas of research for future work. Finally, we conclude with Section 8.

2 Background

This section introduces key concepts required to better understand the rest of the paper. First, it is introduced some background in RL (Section 2.1), then the main concepts of transfer RL are visited (Section 2.2), and finally the concepts of similarity and distance (Section 2.3).

2.1 Reinforcement Learning

Typically, RL tasks are described as Markov Decision Processes (MDPs) represented by tuples in the form $M = (S, A, T, R)$, where $S$ is the state space, $A$ is the action space, $T : S \times A \to S$ is the transition function between states, and $R : S \times A \to \mathbb{R}$ is the reward function [Sutton and Barto, 2011]. At each step, the agent is able to observe the current state, and choose an action according to its policy $\pi : S \to A$. The goal of the RL agent is to learn an optimal policy $\pi^*$ that maximizes the return $J(\pi)$:

$$J(\pi) = \sum_{k=0}^{K} \gamma^k r_k$$

where $r_k$ is the immediate reward obtained by the agent on step $k$, and $\gamma$ is the discount factor, which determines how relevant the future is (with $0 \leq \gamma \leq 1$). The interaction between the agent and the environment tends to be broken into episodes, that end when reaching a terminal state, or when a fixed amount of time has passed. With the goal of learning the policy $\pi$, Temporal Differences methods [Sutton and Barto, 2011] estimate the sum of rewards represented in Equation 1. The function that estimates the sum of rewards, i.e., the return for each state $s$ given the policy $\pi$ is called the value-function $V^\pi(s) = E[J(\pi)|s_0 = s]$. Similarly, the action-value function $Q^\pi(s, a) = E[J(\pi)|s_0 = s, a_0 = a]$ is the estimation of the value of performing a given action $a$ at a state $s$ being $\pi$ the policy followed. The Q-learning algorithm [Watkins, 1989] is one of the most widely used for computing the action-value function.

2.2 Transfer Learning for Reinforcement Learning

In the transfer learning scenario we assume there is an agent who previously has addressed a set of source tasks represented as a sequence of MDPs, $M_1, \ldots, M_n$. If these tasks are somehow “similar” to a new task $M_{n+1}$, then it seems reasonable the agent uses the acquired knowledge solving $M_1, \ldots, M_n$ to solve the new task $M_{n+1}$ faster than it would be able to from scratch. Transfer learning is the problem of how to obtain, represent and, ultimately, use the previous knowledge of an agent [Torrey and Shavlik, 2010; Taylor and Stone, 2009].

However, transferring knowledge is not an easy endeavour. On the one hand, we can distinguish different transfer settings depending on whether the source and target tasks share or not the state and action spaces, the transition probabilities and the reward functions. It is common to assume that the tasks share the state space and the action set, but differing the transition probabilities and/or reward functions. However, in case the tasks do not share the state and/or the action spaces, it is required to build mapping functions, $X_S(s_t) = s_s$, $X_A(a_t) = a_s$, able to map a state $s_t$ or action $a_t$ in the target task to a state $s_s$ or action $a_s$ in the source task. Such mapping functions require not only knowing if two tasks are related, but how they are related, which means an added difficulty. On the other hand, it is required to select what type of information is going to be transferred. Different types of information have been transferred so far ranging from instance transfer (a set of samples collected in the source task) to policy transfer (i.e., the policy $\pi$ learned in the source task). Nor is this a simple task, because depending on how much and how the source and target tasks are related, it could be transferred one type of information or another.

Finally, the most “similar” task among $M_1, \ldots, M_n$ to solve $M_{n+1}$ should be selected in the hope that it produces the most positive transfer. For this purpose, similarity metrics could be used, which translate into a measurable quantity of how related two tasks are.

2.3 Similarity and Distance Metrics

Similarity metrics are a very important part of transfer learning, as they provide a measure of distance between tasks.
Similarity functions, or their complementary distance functions, are mathematical functions that assign a numerical value to each pair of concepts or objects in a given domain. This value measures how similar these two concepts or objects are: if they are very similar, it is assigned a very low distance, and if they are very dissimilar, it is assigned a larger distance [Ontañón, 2020].

3 Taxonomy of Similarity Metrics for MDPs

We consider there are two tasks, $M_i$ and $M_j$, described formally by the tuples $M_i = (S_i, A_i, T_i, R_i)$ and $M_j = (S_j, A_j, T_j, R_j)$, where they could share (or not) the state space, the action space, or the transition and reward dynamics.

Definition 1. Given two tasks $M_i$ and $M_j$, we define a task distance metric as a heuristic function $d(M_i, M_j) \rightarrow [0, \infty)$, such that if $d(M_i, M_j) < d(M_k, M_l)$, then $M_i$ is considered more similar to $M_j$ than $M_k$.

Definition 1 allows us to use the function $d(\cdot, \cdot)$ to obtain an ordering between tasks in such a way that we can select the more similar one. Ideally, the concept of similarity should be related to the concept of positive transfer: the smaller the distance $d(M_i, M_j)$, the greater the positive transfer. Additionally, $d(M_i, M_j)$ should be computed before or, at least, during the transfer experiment, in order to select an adequate task to use in transfer. However, the literature proposes different ways to compute $d(M_i, M_j)$.

In this paper, we consider two main trends for the computation of the distance metric $d(M_i, M_j)$. Such trends are depicted in Figure 1. The first one measures the structural or model similarities between the given MDPs. The second measures the similarities by using the performance of the learning agent in both the source and target tasks.

4 Model-based Metrics

As regards the first, model-based metrics measure the degree of similarity between a source and a target task by using their corresponding MDP models (i.e., states, actions, transition and rewarding dynamics). There are several alternatives to this model-based metrics depending on what components of the MDPs are taken into account. In our proposed taxonomy, we categorize these metrics in five groups: (i) transition and rewarding dynamics, (ii) transitions, (iii) rewarding, (iv) state and actions and (v) states.

4.1 Transition & rewarding dynamics

They require complete knowledge of the MDP models both of the source and the target tasks. We distinguish three ways of computing similarity metrics using such a complete knowledge: (i) by a sort of metrics based on state abstraction (or state aggregation) techniques [Li et al., 2006; Ferns et al., 2004; Ferns et al., 2012; Castro, 2020], (ii) by compliance metrics [Lazaric et al., 2008; Lazár et al., 2009]; (iii) by bisimulation metrics [Fachantidis et al., 2015; Fachantidis, 2016], and (iv) by rewards based metrics [Wang et al., 2019].

Figure 1: Overview of the similarity metrics considered in this survey.

Lazaric, 2008; Fachantidis et al., 2015; Fachantidis, 2016] and (iii) by metrics based on the construction of MDP graphs [Kuhlmann and Stone, 2007; Wang et al., 2019].

State abstraction

In RL, a common practice is to aggregate states in order to obtain an abstract description of the problem, i.e., a more compact and easier representation of the task to work with [Li et al., 2006]. These approaches are based on the same common principle: if a number of states are considered to be similar, they can be aggregated as a single one. This same principle can also be used to compute the similarity between two states belonging to different MDPs. In this paper, we survey two of these methods which actually have been used for transfer in RL: bisimulation [Ferns et al., 2004; Ferns et al., 2012; Castro and Precup, 2011; Song et al., 2016], and homomorphism [Ravindran and Barto, 2002; Sorg and Singh, 2009].

Definition 2. Given two MDPs $M_i$ and $M_j$, we define $d(M_i, M_j) = d(S_i, S_j)$, where $d(S_i, S_j)$ measures the distance between $S_i$ and $S_j$ by computing the bisimulation or homomorphism distances of the individual state pairs, $d(s_i, s_j), \forall s_i, s_j \in S_i \times S_j$.

Bisimulation metrics compute $d(s_i, s_j)$ by comparing the transition and rewarding dynamics of $s_i$ and $s_j$: the more similar the reward and transition structures of $s_i$ and $s_j$ are, the smaller $d(s_i, s_j)$. One of the shortcomings of the bisimulation metrics is that they require both MDPs, $M_i$ and $M_j$, to have the same action sets, $A_i = A_j$. However, such a shortcoming has been successfully addressed by the homomorphism metrics which are able to deal with different action spaces, $A_i \neq A_j$ [Ravindran and Barto, 2002; Sorg and Singh, 2009]. Once the distance between all state pairs in $M_i$ and $M_j$ is computed, it is required to compose them to compute the distance $d(M_i, M_j)$ between the
two MDPs. It could not be appropriate to simply accumulate or average the distances between all different state pairs, so it is typical to define \( d(S_i, S_j) \) as a function which measures the distance between the sets corresponding to the state spaces \( S_i \) and \( S_j \), e.g., the Hausdorff, or the Kantorovich function [Song et al., 2016], although other metrics between sets would also be possible [Conci and Kubrusly, 2018].

**Compliance**

The compliance measure is defined as the probability of a sample \((s, a, s', r)\) in a target task \(M_j\) of being generated in the source task \(M_i\) [Lazaric et al., 2008; Fachtantidis, 2016; Fachtantidis et al., 2015]. Therefore, it is easy to deduce that the compliance between the entire target task and the entire source task allows us to measure the similarity between the two tasks.

**Definition 3.** Given two MDPs, \(M_i\) and \(M_j\), and two sets of experience tuples generated \(D_{M_i}\) and \(D_{M_j}\) gathered from \(M_i\) and \(M_j\), the compliance between \(M_i\) and \(M_j\) is computed as:

\[
\Lambda = \frac{1}{n} \sum_{t=0}^{n} P(\sigma_t|D_{M_i})
\]

where \(n\) is the number of samples in \(D_{M_j}\), and \(\sigma_t\) is the \(t\)-th tuple in \(D_{M_j}\).

\(\Lambda\) is not strictly a distance metric but a probability: the more likely the samples of the target task are generated in the source task, the closer \(\Lambda\) to 1. Therefore, compliance could be used to obtain a distance metric between MDPs like the ones this survey is looking for, e.g., \(d(M_i, M_j) = 1 - \Lambda\).

**MDP graphs**

They are based on the construction of graphs that represent the transition and the reward functions both of the source task and the target task. Then, they find structural similarities between tasks based on graph-similarity or graph-matching algorithms.

**Definition 4.** Given two MDPs \(M_i\) and \(M_j\) and their corresponding alternative representation as graphs, \(G_{M_i}\) and \(G_{M_j}\), we define \(d(M_i, M_j)\) as inversely related to \(\Phi(G_{M_i}, G_{M_j})\), where \(\Phi(\cdot, \cdot) \rightarrow [0, \infty)\) is a function that measures the structural similarity between \(G_{M_i}\) and \(G_{M_j}\).

Wang et al. (2019) represents the MDPs as bipartite directed graphs, which permits the computation of structural similarity measures between them. Although several of these structural functions \(\Phi(\cdot, \cdot)\) could be used such as RoleSim [Jin et al., 2014] or MatchSim [Lin et al., 2012], Wang et al. are based on SimRank [Jeh and Widom, 2002], which basic idea is that two nodes are similar if their neighbors are similar. On the other hand, Kühnemann and Stone (2007) represents the MDPs as rule graphs instead of bipartite graphs for the particular problem of General Game Playing [Génesereth et al., 2005]. Such a rule graph is an accurate abstraction of the MDP problem, and can be properly compared to other rule graphs using an isomorphic function as \(\Phi(\cdot, \cdot)\) [McKay and Piperno, 2014]. Finally, Liu and Stone (2006) models the problem as a Qualitative Dynamic Bayes Network (QDBN). This assigns types to the nodes and edges, providing additional characteristics to compare.

### 4.2 Transitions

The metrics considered in this category use tuples in the form \((s, a, s', r)\) to measure the similarity between MDPs.

**Definition 5.** Given two tasks \(M_i\) and \(M_j\), and two sets \(D_{M_i}\) and \(D_{M_j}\) of experience tuples in the form \(\tau = (s, a, s')\) gathered from \(M_i\) and \(M_j\), we can define the distance between them as \(d(M_i, M_j) = d(D_{M_i}, D_{M_j})\).

Ammar et al. (2014) use \(D_{M_j}\) to build an RBM model which describes the transitions in \(M_j\) in a richer feature space. Then, they feed the tuples \(\tau_k \in D_{M_i}\) into this RBM model to obtain a reconstruction \(\tau'_k\). Afterward, they compute the Euclidean distance between \(\tau_k\) and \(\tau'_k\). Similarly, Taylor et al. (2008b) use \(D_{M_j}\) to learn a one-step transition model \(M(s, a) \rightarrow s'\). Afterward, for each tuple \(\tau_k = (s, a, s') \in D_{M_i}\), they compute the Euclidean distance between \(s'\) and \(M(s, a)\). In both cases, the distance \(d(D_{M_i}, D_{M_j})\) is computed as the average of the Euclidean distances obtained for each \(\tau_k \in D_{M_i}\).

### 4.3 Rewards

The metrics can also measure the similarity between MDPs according to the distance between their reward dynamics.

**Definition 6.** Given two MDPs \(M_i\) and \(M_j\), we define the distance between them as \(d(M_i, M_j) = d(R_i, R_j)\).

For instance, Carroll and Seppi (2005) computes \(d(R_i, R_j)\) as

\[
d(R_i, R_j) = \frac{1}{n} \sum_{s \in S} \sum_{a \in A} (R_i(s, a) - R_j(s, a))^2,
\]

where \(n\) is the total number of state-action pairs in the source and the target task. Instead, Tao et al. (2021) assumes the reward functions are a linear combination of some common features \(\phi(\cdot, \cdot)\), \(R_i(s, a) = \phi(s, a)^T w_i\) and \(R_j(s, a) = \phi(s, a)^T w_j\), and then use the cosine distance function between \(w_i\) and \(w_j\) to compute \(d(R_i, R_j)\). Finally, in the literature there are several ways of computing \(d(R_i, R_j)\) [Gleave et al., 2020].

### 4.4 State & Action Spaces

In this case, the distance between MDPs is computed as the distance between the state-action spaces.

**Definition 7.** Given two MDPs \(M_i\) and \(M_j\), and their corresponding state-action spaces, \(S_i \times A_i\) and \(S_j \times A_j\), we define the distance as \(d(M_i, M_j) = d(S_i \times A_i, S_j \times A_j)\).

For instance, Narayan and Leong (2019) compute this distance as the averaged difference between the corresponding state-action transition distributions of the two tasks. Instead, Taylor et al. (2008) compute the Euclidean distance between state-action pairs in the source and target tasks. This work is not focused on the construction of a distance measure between MDPs, but such Euclidean distances between all the state-action pairs could be composited to obtain a sort of similarity metric.

### 4.5 States

Finally, the metrics in this category use the state space in both the source and target tasks to compute the similarity between them.
Definition 8. Given two MDPs \( M_i \) and \( M_j \), we can define the distance between them as \( d(M_i, M_j) = d(S_i, S_j) \), where \( d(S_i, S_j) \) measures the distance between \( S_i \) and \( S_j \).

Svetlik et al. (2017) propose to compute \( d(S_i, S_j) \) as the relation between the applicability of the value function (measured as the number of states in \( M_i \) that also appears in \( M_j \)), and the experience required to learn in \( M_j \) (measured as the difference of size between \( S_j \) and \( S_i \)). Another area of research that is relevant in this category is that of case based reasoning (CBR) [Aamodt and Plaza, 1994]. RL approaches based on CBR use a similarity function between the states in the target task and the states stored in a case base corresponding to a previous source task [Celiberto et al., 2011]. Such similarity function could be used to measure the similarity between the state spaces, hence, the similarity between the two tasks.

5 Performance-based Metrics

As regards the second major category in the proposed taxonomy, they are based on the performance of the agents in the source task and the target task, where this performance can be related to the policies themselves learned by the agents in these tasks, or to the reuse gain an agent obtains reusing the knowledge of a source task in a target task. So, we distinguish two different approaches to overcoming the problem of computing such a performance-based similarities: (i) by the policy similarity and (ii) by the reuse gain obtained transferring the knowledge from a source task to the target task.

5.1 Policy similarity

They are based on the use of the learned value function \( V^\pi \) or the action-value function \( Q^\pi \), or equivalently, on the behavioral policies \( \pi \) obtained in the source task and the target task. Therefore, these metrics require the full (or partial) learning of these policies before the computation of the similarity between tasks. Such a comparison can be conducted in two different ways depending on what is being compared: (i) the policy values, or (ii) the policy parameters.

Policy values

In this case, the comparison is conducted by observing the specific \( q \)-values or \( v \)-values of the \( Q^\pi \)-function or the \( V^\pi \)-function of the source and target tasks. Therefore, in this case, it is really being measured the degree of similarity of the policies obtained in both tasks.

Definition 9. Given two tasks \( M_i \) and \( M_j \) and the \( q \)-values of \( Q^\pi_i \) and \( Q^\pi_j \) learned in these tasks, we define \( d(M_i, M_j) = d(Q^\pi_i, Q^\pi_j) \). This metric is transformed into \( d(M_i, M_j) = d(V^\pi_i, V^\pi_j) \) if \( V^\pi_i \) and \( V^\pi_j \) are computed instead.

Carroll and Seppi (2005) propose to compute \( d(V^\pi_i, V^\pi_j) \) as the number of states with identical maximum \( v \)-value, and \( d(Q^\pi_i, Q^\pi_j) \) as the mean squared error of the \( q \)-values in \( Q^\pi_i \) and \( Q^\pi_j \). Instead, Zhou and Yang (2020) compute \( d(Q^\pi_i, Q^\pi_j) \) deriving latent structures of tasks and finding matches between \( Q^\pi_i \) and \( Q^\pi_j \).

Policy Approximation Parameters

In RL, the value function \( V^\pi \) or the action-value function \( Q^\pi \) usually are represented or approximated with a parameter vector \( \theta \). Intuitively, the metrics within this category compare the particular weights of the parameter vectors corresponding to the value functions of the source task and the target task in order to measure the similarity between them.

Definition 10. Given two tasks \( M_i \) and \( M_j \) and the parameterize functions \( Q^\theta_i \approx Q^\pi_i \) and \( Q^\theta_j \approx Q^\pi_j \) learned in these tasks, we consider \( d(M_i, M_j) = d(\theta_i, \theta_j) \).

The distance \( d(\theta_i, \theta_j) \) can be computed using the cosine similarity between two non-zero vectors [Karimpanal and Bouffanais, 2018]. Such an approach opens the door to the comparison of other policy representations [Ferrante et al., 2008].

5.2 Reuse gain

In these techniques, the level of similarity is an approximation to the advantage gained by using the knowledge in one source task to speed up the learning of another target task [Carroll and Seppi, 2005; Carroll, 2005; Taylor and Stone, 2009].

Definition 11. Given two tasks \( M_i \) and \( M_j \), and \( g(M_i, M_j) \) which denotes the gain of transferring the knowledge learned in \( M_i \) to \( M_j \), we can consider \( d(M_i, M_j) \) as inversely related to \( g(M_i, M_j) \).

Therefore, it is important to bear in mind that these metrics actually require that the transfer experiment be entirely or partially run before measuring the degree of similarity between tasks. However, regardless of the particular technique used to compute such a reuse gain, the higher the reuse gain, the greater the similarity between the tasks. In this paper, we distinguish two approaches within this category depending on whether the reuse gain is computed after or during the transfer process: (i) off-line reuse gain, and (ii) on-line reuse gain.

Off-line reuse gain

In this case, the reuse gain \( g(M_i, M_j) \) is estimated as the difference in performance between the learning process with and without transfer, and once the learning processes are considered to be finished [Carroll, 2005; Mahmud et al., 2013; Sinapov et al., 2015; Zhan et al., 2016]. Such gain \( g(M_i, M_j) \) can be computed as the jumpstart [Sinapov et al., 2015; Carroll, 2005], the time to convergence [Carroll, 2005], the asymptotic performance [Mahmud et al., 2013], although other metrics such as the total reward or the transfer ratio could be also used [Taylor and Stone, 2009].

On-line reuse gain

On the contrary, in these approaches, the reuse gain is estimated on-line at the same time that the policy in the target task is computed [Fernández and Veloso, 2013; Azar et al., 2013; Li and Zhang, 2017]. It is important to bear in mind that the on-line computation of this gain only makes sense if during the learning process we have several transfer sources to choose from. At the beginning of the learning
process, these approaches have at its disposal the knowledge learned in solving a set of previous tasks \( \{M_1, \ldots, M_n\} \) to learn the new task \( M_j \). During learning, they compute \( g(M_i, M_j) \) of each past task \( M_i \in \{M_1, \ldots, M_n\} \). To do that, they transfer the knowledge acquired solving \( M_i \) to \( M_j \) during a limited number of episodes \( m \). Then, \( g(M_i, M_j) \) is computed as the average reward obtained during those \( m \) episodes [Fernández and Veloso, 2013; Azar et al., 2013]. Once all gains are computed, it is possible to decide on-line which is the closest task to \( M_j \) within \( \{M_1, \ldots, M_n\} \), so that the knowledge of the selected closest task can have a greater influence on learning about the policy in \( M_j \).

6 Discussion

From a transfer point of view, the ultimate goal of all similarity metrics is in some way to predict the relative advantage that would be gained by using a source task in a target task. The more similar the source and target tasks are, the greater the positive transfer. The correct selection of a distance metric should be carried out attending to four dimensions: nature of the state-action space, amount of information available on the tasks, computation moment, and transfer technique.

Obviously, the selection of the metric depends on the nature of the state-action space. Some approaches typically require enumerating all the states both in the source and the target tasks [Castro, 2020; Zhou and Yang, 2020], but such full state enumeration is impractical for large state spaces, and impossible for continuous state spaces. Other methods require that both tasks have the same state-action space [Lazaric et al., 2008; Azar et al., 2013], which is not true in most of the cases. However, the latter can be partially alleviated by the construction of inter-task mapping functions, \( X_S \) and \( X_A \).

Another issue that needs to be addressed is how much information we have about the tasks to solve. Most model-based approaches require prior full information (or accurate approximations) about the transition and reward dynamics [Castro and Precup, 2010; Lazaric et al., 2008; Wang and Liang, 2019], or about the size of the state and action spaces [Svetlik et al., 2017; Carroll, 2005]. In this sense, performance-based metrics have a clear advantage over model-based ones: in general, performance-based metrics require less a priori information about the task to be solved, although as a counterpoint they need to fully or partially run the transfer experiment to obtain the distance measurement.

This leads us to the third issue: the computation moment. Ideally, the computation of the similarity metric should be before or, at least, during the transfer. Off-line reuse gain approaches are undoubtedly the best method for measuring similarity between two tasks: they produce such a measure after the transfer experiment has been run, in such a way that we can compute the real gain. However, if the point is to use the task similarity measure to choose a task to use in transfer, these metrics are useless. In this case, model-based metrics take advantage over performance-based metrics: they allow to compute the metrics before the transfer process. These metrics can be used to choose the most similar MDP before transfer, but as far as we know there is no theoretical guarantees that the most similar MDP is similar enough to produce a positive transfer. By contrast, the metrics based on the on-line reuse gain are at the point half-way between both. They allow to compute the similarity metric during transfer, so that depending on the similarity of the source task, it will introduce a greater or lesser exploration bias in the learning process of the new task.

Finally, the success of a metric is intimately tied with the transfer technique that uses it [Carroll, 2005; Carroll and Seppi, 2005]. This means that there is probably no one best universal metric that works with all transfer techniques and problems, in the same way that there is no one best universal transfer technique. Since each metric can capture different types of similarity and each transfer technique induces different bias in the learning process, the question of selecting the best metric turns into finding the correct metric for a transfer technique to be applied to a particular task.

7 Future Directions

The previous discussion points out several future directions. On the one hand, since there is not a best metric, it would be useful to use several of them. For instance, model-based metrics can be used to return a useful approximation of task similarity before the tasks are learned, although this measure can be adapted on-line during the learning process so that the bias that the source task induces in the exploration process is adjusted dynamically. On the other hand, given that different metrics compute different types of similarity (i.e., model-based metrics measure structural similarities, whilst performance-based metrics measure performance similarities), the agent can be equipped with the ability not only to determine which source task to use, but also which transfer technique to use given the type of similarity between the source and target tasks.

Another interesting line of research is that based on building semantic representations of the tasks through domain-dependent features. For instance, we can define a particular Pac-Man task from features like the number of ghosts, behavior of the ghosts, or the type of the maze, and use these features to build a similarity metric between different Pac-Man tasks. In fact, one may heuristically combine structural, performance, but also semantic similarity aspects into a same metric. Thus, it could be obtained a metric more aligned with the way in which humans decides what is similar, since humans analyze the similarity between concepts or objects from different perspectives [Kemmerer, 2017].

Finally, transferring learned models from simulation to the real world remains one of the hardest problems in control theory [Zhao et al., 2020]. In this case, similarity metrics can help to answer how similar simulations and the actual world are. They could be used to provide theoretical guarranties that ensure the learned policies transferred from simulation to the actual world will perform as required, or to define mechanisms to tune/modify the simulated environments, so the gap between the simulated world and the actual one decreases.
8 Concluding Remarks

This paper contributes a compact and useful taxonomy of similarity metrics for Markov Decision Processes. The leaves of the taxonomy have been used to provide a literature review that surveys the existing work. We differentiated between model-based and performance-based metrics, depending on whether a structural or performance criterion has been used in its creation. The proposed taxonomy permits to organize clearly the different similarity metrics, or find commonalities between them. This can help the reader to choose similarity metrics for their tasks, or even define their own. We also discussed different selection criteria and some promising future research directions.

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