Optimization for Fatigue Pressure Testing of Metal Pressure-Containing Envelopes Based on Scenario Analysis

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Abstract. For reliability analysis of hydraulic system, it is important to achieve the fatigue characteristic (SN curve) for hydraulic component. However, it needs to improve the testing pressure level number and test samples to improve the testing accuracy. However, there is little research about to optimize the energy consumption while efficiently ensuring the testing accuracy. Moreover, uncertainty does exit for the testing accuracy estimation because of small sample size for hydraulic system. Therefore, a scenario analysis-based framework is proposed to optimize both the energy consumption and test accuracy by stochastic multi-objective programming. The case study shows the efficiency of the proposed method.

1. Introduction
Large number of mechanical equipment or components is subjected to fatigue loads during work. The hydraulic components are subjected to impact loads during operation, such as hydraulic valves and hydraulic cylinders. Fatigue failure is a common failure mode, such as fatigue fracture, fatigue pitting corrosion. To improve the reliability of components, predict and improve the service life of equipment, and reduce unnecessary losses, it is necessary to fully understand the fatigue characteristics of components and determine the fatigue life of components. A common method for analyzing fatigue characteristics is to analyze the S-N curve of the component, namely the stress-fatigue life curve. This method will also be used in this paper.

The means to determine the SN curve of components involve test method, simulation method and synthesis method. The experiment method [4] is the most reliable way and the results obtained are the most accurate. But the fatigue test method is different from the ordinary life test, and it is more difficult than ordinary life test. Owning to the long time and high energy consumption of fatigue test, it will lead to high cost. As a result, there are few studies on fatigue test. In addition, the fact that the energy consumption and test accuracy are mutually exclusive makes fatigue test more difficult, and there is no optimal test plan. Contrary to the experimental method, the simulation method [2] is conducted on the computer for simulation analysis, with short time and no energy consumption,
therefore no cost basically. However, the simulation data is assumed after all, without real fatigue life data will lead to poor accuracy of result. Hence, a synthesis method that combines a certain amount of test data with simulation analysis has been generated to obtain the SN curve. It’s a compromise between the two previous methods. Such as obtaining the S-N curve through small sample test [5]. All these efforts focused on how to get SN curve.

The energy consumption of fatigue test is enormous, so that there are many researches on energy-saving. Methods of fatigue test studied more include power recovery [6], using new energy-saving components [7], accelerated fatigue test [8] and etc. The method of energy-saving can also be used to hydraulic components. In addition, we can design test hydraulic circuit of energy-saving and hydraulic medium of energy-saving, such as parallel type energy-saving circuit [9]. Power recovery takes advantage of motor for energy transformation and recycling. The energy saving of hydraulic components and media is to reduce the energy loss on the circuit. All these paper study traditional energy-saving method, physical means, and don’t take account the accuracy of test. So this paper will offer a new perspective, test plan, to study energy-saving method, and considers the energy consumption of fatigue test and the accuracy of SN curve at the meantime.

This paper provides an optimization method of fatigue test plan in terms of energy-saving and test accuracy for Fatigue life test based on comprehensive method. In this paper, the energy consumption and test accuracy of hydraulic component fatigue test are optimized comprehensively, and the optimal solution is obtained by combining with multi-objective programming (MOP). That is to achieve the minimize energy consumption while ensuring the accuracy of test evaluation. Based on the mathematical model of each objective function and combined with relevant algorithms is the conventional multi-objective optimization process. However, due to the uncertainty of one of the objectives which named the evaluation of experimental accuracy, it’s difficult to find a definite mathematical model, so the essence of it is stochastic multi-objective programming (SMOP) problem. The commonly used methods to solve such problems include ex ante analysis and ex post analysis. ex ante analysis includes fuzzy programming and stochastic programming, and ex post analysis includes sensitivity analysis and robust optimization. In this paper, the method of scenario analysis [10] is adopted to solve the SMOP.

The following content will be discussed below: the second part will introduce details of the energy-saving optimization design method of pressure-bearing shell fatigue test for hydraulic components based on stochastic multi-objective programming, including the fatigue test model construction and the model solution method based on scenario analysis method. In the third part, based on the fatigue test example of pressure-bearing shell for hydraulic filter, analyze its result by the method illustrated in this paper, and the validity of the method has been proved. The fourth part is the summary and research prospects.

2. The Framework of Energy-saving Method of Fatigue Test

2.1. The Model of Accuracy Evaluation of Fatigue Test Designing Plan Based on Test-simulation Combination

The purpose of the fatigue test is to obtain the final S-N curve, and the accuracy of the curve is related to many decision variables of the test, such as the number of test pressure levels, the number of samples under each test pressure level, and the test pressure. In order to find the optimal decision
variable value, a reference is provided for the experiment. The relative percentage deviation (RPD) is introduced here to evaluate the accuracy of the obtained curve. The curve obtained by Monte Carlo simulation is compared with the curve obtained by the experiment (reference curve), and the relative percentage deviation is calculated. The smaller the value, the higher the accuracy of the curve, and the corresponding parameters of the curve are better.

The Monte Carlo simulation method was used to analyze the test. Figure 1 illustrates a flow block diagram for the method. The procedure is as follows:

Step 1: Input the parameters for SN curve, a, b, STD, all pressure level number L and the number of samples for each pressure level j (assume that the every stress level has the same number of samples). The mathematical model between test pressure and failure life is equation (1), the $S_r$ is the amplitude of cyclic pressure, a, b are the fatigue characteristic parameters of the material and can be determined by the method in Document [11].

$$ S_r = a N^b $$

(1)

Step 2: Choose a certain number of stress level $L (L \geq 2)$. Select L stress values at equal step in the interval $[S_{amin}, S_{amax}]$, as the pressure value of every stress level $S_d(i)$, $1 \leq i \leq L$. Calculate mean life at the $S_{amin}$ and $S_{amax}$ by equation (2), as the reference life, because the deviation at both ends of fitted curve is the largest.

$$ N_{av}(i) = \left( \frac{S_d(i)}{a} \right)^{1/b} $$

(2)

Step 3: It is assumed that the sample points at each pressure level are normally distributed, a random sample at each pressure level can be obtained by equation (3), and n simulation cycles are performed. j is the sample size at each pressure level, and STD is the standard deviation of the logN normal distribution.

$$ RN(i) = STD \times randn(j, 1) + \log(N_{av}(i)) $$

(3)

Step 4: For each simulation, based on all samples, fatigue characteristics were evaluated using least squares and linear regression to obtain curve parameters and b for each simulation.

Step 5: Result analysis. Calculate the RPD(i) of the upper and lower stress levels for each simulation, and take the RPD of the simulated curve with the largest absolute value, and take the maximum value in the n simulated curves as the final RPD of the simulation. The calculation formula is as follows, which $N_{ref}$ is obtained by the formula (2) and $N_{sim}$ is obtained from the simulated curve.

$$ RPD(i)(\%) = 100 \left( \frac{N_{ref}(i) - N_{sim}(i)}{N_{ref}(i)} \right) $$

(4)

Step 6: Repeat Step 3 to Step 5, until the number of cycles reaches the specified number, namely the number of scenarios $N_{scenario}$, so obtained the $RPD_{L,j,(N_{scenario})}$ corresponding to a test design.

Step 7: Select different number of test stress level L, repeat Step 3 to Step 6, obtain $RPD_{L,j,(N_{scenario})}$ of different test plan corresponding to different L.
Step 8: Select different number of specimens $j$, repeat Step 3 to Step 7, obtain $\text{RPD}_{L,j}^{(N_{\text{scenario}})}$ of different test plan corresponding to different $j$.

Figure 1. The Flow Block Diagram of the Proposed Method

2.2. Energy Consumption Modeling of Fatigue Test of Pressure Shell of Hydraulic Components

For the fatigue test of pressure shell of hydraulic components, according to the principle of test, energy consumption is mainly determined by the test pressure amplitude, $S_i$, the life of test sample, $N_i$, the number of test sample $n_i$, the grade of test pressure, $L$, and the flow $Q$. The formula of energy consumption is shown below:

$$E = \sum_{i=1}^{L} n_i S_i Q N_i$$  \hspace{1cm} (5)

In this model, it is assumed that the life of the test sample $N_i$ can be expressed by the life corresponding to the pressure pulse amplitude $S_i$, and the relationship between $N_i$ and $S_i$ obeys the SN curve, that is, $N_i = (S_i/a)^{(1/b)}$. In the formula, $a$ and $b$ represent the fitting function parameters of the SN curve of the pressure shell. $L$ represents the number of pressure levels. This model assumes that the sample size $n_i$ of each pressure level is a certain value, such as $n_i = j$, $i = 1, 2, ..., L$, ($L \geq 2$). Regarding the selection of test pressure levels, in this study, it is assumed that the test pressure is evenly spaced between the hi
and lowest pressure levels.

In addition, the flow \( Q \) is determined by the total pressure volume of the tested sample. For example, for the tested component with large pressure volume (such as hydraulic cylinder), the number of samples is different, so the total pressure volume is significantly different, thus it will affect the flow rate. This paper mainly studies hydraulic components with small pressure volume, such as hydraulic valve, filter and so on. Therefore, the influence of the number of samples on the total pressure volume can be ignored, and the influence of the number of samples on the flow rate can be ignored. Therefore, it can be assumed that the flow rate corresponding to the number \( I \) of different pressure levels is the same.

So, the equation (5) of energy consumption can be simplified as:

\[
E = jQ \sum_{j=1}^{i} S_j \left( \frac{S_j}{a} \right)^{1/b}
\] (6)

From the equation above, it can be roughly seen, the increase in the number of stress levels \( L \) will significantly increase the energy consumption \( E \).

2.3. The Method of Stochastic Multi-objective Optimization Based on Scenario Analysis

Qualitatively, if the number of pressure levels increases, the RPD (relative percentage difference) of the test will decrease, and the energy consumption will be higher. In other words, the two goals are mutual exclusion between less RPD and energy consumption reduction. Therefore, the optimization for fatigue pressure test design for pressure metal pressure containing envelopes of hydraulic components can be regarded as a two-objective optimization problem. The model is as follows.

\[
\begin{align*}
[L^*, j^*] &= \arg \min_{L \in [L_{\min}, L_{\max}], j \in [J_{\min}, J_{\max}]} \{ f_1(L, j), f_2(L, j) \} \\
\end{align*}
\] (7)

Where two objective functions are:

\[
\begin{align*}
f_1(L, j) &= \text{RPD}(L, j), & f_2(L, j) &= jQ \sum_{j=1}^{i} S_j \left( \frac{S_j}{a} \right)^{1/b} \\
\end{align*}
\]

Moreover, considering the uncertainty of objective function \( f_1 \), this two-objective optimization problem is essentially a stochastic multi-objective optimization problem. Therefore, a kind of threshold based on scenario analysis method is proposed to solve the problem. On the basis of section 2.1, after obtaining \( \text{RPD}_{L,j}(\text{Scenario}) \) corresponding to various experimental configurations in multiple scenarios, the proposed scenario analysis method for stochastic multi-objective programming are as follows:

Step 1. For this step, it is assumed sample size \( j \) is determined as \( j=J \). Calculate the probability density function (PDF) of RPD for each pressure level number \( L_i \). Therefore the pdf is expressed as \( f_1 = \text{L}_{L,j} = j(\text{RPD}) \).

Step 2. When the threshold of RPD is given, it means the RPD for the test should be not greater than \( \text{RPD}_{0b} \). Therefore, for each test pressure number, the probability can be calculated for the scenarios that \( f_i \leq \text{RPD}_{0b} \). Meanwhile, it is to find the lowest \( L \) to minimize the energy consumption. Therefore, based on the conditional probability, the probability for optimal \( L^* \) for each scenario can be
calculated as follows,

\[ p(L^* = L_i) = p(RPD^i \leq RPD_a) \]  

(8)

\[ p(L^* = L_i) = \left( \prod_{j=1}^{i-1} p(RPD^j > RPD_a) \right) \times p(RPD^i \leq RPD_a), \quad i = 2, \ldots, n \]  

(9)

Where test pressure numbers \( L_i, \ i=1,2,\ldots,n \), satisfy \( L_1<L_2<\ldots<L_n \). RPD, \( i=1,2,\ldots,n \), is the RPD when test pressure number is \( L_i \).

For an example, \( L_i, \ i=1,2,3 \) has three possible values, and \( p(RPD^1 \leq RPD_a)=40\% \), \( p(RPD^2 \leq RPD_a)=80\% \), \( p(RPD^3 \leq RPD_a)=100\% \). Then \( p(L^*=L_1)=40\% \), \( p(L^*=L_2)=60\%*80\%=48\% \), \( p(L^*=L_3)=12\% \).

Step 3. Find the two optimal test pressure levels with top two probabilities as the expected optimal test pressure levels. For the example in Step 2, the optimal test pressure levels are \( L_2 \) and \( L_1 \) with top two probabilities.

3. Example

3.1. Raw Data

According to the test data from ISO/TR [3], there are 7 levels of test data in the text. Here, the middle five pressure levels are selected, and taking the average pressure of each level, so the pressure interval is \([S_{\text{min}}, S_{\text{max}}] = [550, 1178]\). Calculate the standard deviation to log\( N \) for each level, take the mean as a fixed standard deviation, and calculate its value is 0.2482, which is STD=0.2482 in the simulation. According to the curve of \( E=0 \) in the literature, that is, the reference curve, the parameters of the curve can be found as \( a=4501.7, \ b= -0.1319 \), so the data preparation before the simulation is already available. The following will carry out simulation and multi-objective optimization according to the method described above.

The energy consumption, that is, the objective function \( f_2 \), represents the energy consumption under different pressure levels \( L \). It can be seen that the relationship between \( f_2 \) and \( L \) approximately is a straight line, so it can be replaced by a fitting straight line to simplify the process of optimization, see equation (10). (Q is a fixed value, which can be ignored first; this formula also does not consider the dimension problem)

\[ f_2 = j(0.9268L + 3.3916) \times 10^9 \]  

(10)

3.2. Optimization of the Number of the Stress Level \( L \) Based on Scenario Analysis

Firstly, analyze the RPD of the number of the tested stress level \( L \) at \( j=5 \), the number of scenarios is set as 100, get the scatter diagram of L-RPD, as shown in the Figure 2. In this case, the threshold of RPD is set to be \( RPD_a=190 \).

The results fitted of probability density function is shown in Table 1. The expectation of optimal \( L^* \) can be solved, according to formula of solving expectation of discrete point, \( (11) \), then round.

\[ E(L^*) = \sum_{i=1}^{n} L_i p(L^* = L_i) \]  

(11)
From Table 1, According to formula (11), $E(L^*) = 2 \times 0.43 + 3 \times 0.212 + \ldots + 8 \times 0.003 = 3.207$, then round to the nearest 3. It can be concluded that the expected optimal value of the pressure level is $E(L^*) = 3$, and the L of the first two places in the probability ranking is 2 and 3 respectively.

![Figure 2](image.png)  ![Figure 3](image.png)

**Figure 2.** The RPD Results for Various Scenarios for Each Pressure Level Number L when $j=5$.

**Figure 3.** The RPD Results for Various Scenarios for Various Sample Number j of Each Test Level Number when $L=5$.

**Table 1.** The probability distribution of PRD corresponding to the number of test stress levels L, and the probability corresponding to the RPD interval divided through threshold.

| Number of stress level L | PRD fitting distribution and parameters | Probability interval divided through threshold $\text{RPD}_\alpha$, $p(\text{RPD} > 190)$ and $p(\text{RPD} \leq 190)$ | Probability of optimal $L^* = L$ |
|--------------------------|----------------------------------------|-------------------------------------------------|----------------------------------|
| 2                        | N(194, 25)                             | 57%, 43%                                        | 43%                              |
| 3                        | N(199, 26)                             | 63%, 37%                                        | 21.2%                            |
| 4                        | N(192, 19)                             | 54%, 46%                                        | 16.5%                            |
| 5                        | N(183, 18)                             | 34%, 66%                                        | 12.7%                            |
| 6                        | N(179, 17)                             | 26%, 74%                                        | 4.9%                             |
| 7                        | N(173, 20)                             | 20%, 80%                                        | 1.4%                             |
| 8                        | N(167, 17)                             | 8%, 92%                                         | 0.3%                             |
### Table 2. The probability distribution of PRD corresponding to the sample size \( j \), and the probability corresponding to the RPD interval divided through threshold.

| Number of specimens \( j \) | PRD fitting distribution and parameters | Probability interval divided through threshold \( p(RPD_{th}) \) and \( p(RPD<=160) \) | Probability of optimal \( j^* = j \) |
|-----------------------------|------------------------------------------|------------------------------------------------|-------------------------------|
| 2                           | \( \text{N}(353, 59) \)                  | 99.9%, 0.1%                                      | 0.1%                          |
| 3                           | \( \text{N}(259, 34) \)                  | 99.84%, 0.16%                                    | 0.2%                          |
| 4                           | \( \text{N}(215, 26) \)                  | 98.4%, 1.6%                                      | 1.6%                          |
| 5                           | \( \text{N}(182, 16) \)                  | 91.7%, 8.3%                                      | 8.2%                          |
| 6                           | \( \text{N}(167, 20) \)                  | 63.5%, 36.5%                                     | 33%                           |
| 7                           | \( \text{N}(152, 13) \)                  | 26.2%, 73.8%                                     | 42.3%                         |
| 8                           | \( \text{N}(141, 12) \)                  | 6%, 94%                                          | 14.1%                         |

#### 3.3. Optimization of the Sample Size \( j \) Based on Scenario Analysis

Analyze the RPD of the sample size at \( L=5 \), the number of scenarios is set as 100, get the scatter diagram of \( j \)-RPD, as shown in the Figure 3.

The results fitted of probability density function are shown in Table 2. In the same way as in Table 1, it can be concluded that the expected optimal value of sample size is \( E(j^*) = 7 \). The first two \( j \) of the probability are 7 and 6 respectively.

#### 3.4. Analysis of Influence of RPD Threshold on Optimal Test Pressure Levels

This RPD\(_{th}\)-L diagram reflects the effect of the RPD threshold on the optimal L. It can be seen from the figure, the optimal number of stress level L has a short rise at the beginning and then falls, until the optimal solution \( L=2 \) tends to be stable. The optimal number of stress level L does not exceed 5 throughout the process. Referring to this figure, L closest to the desired point can be selected as the optimal number of stress levels according to different thresholds.

![Figure 4. Relationship between Different RPD Thresholds and Optimal L](image-url)
4. Conclusion and Prospects

In this paper, a framework of energy saving method for fatigue test pressure is proposed for hydraulic component. The optimization problem for both energy consumption and fatigue test accuracy is a stochastic multi-objective programming problem due to the uncertainty of fatigue test accuracy estimation. Therefore, a scenario analysis method is proposed to solve the problem with high efficiency. The case study shows the determined RPD threshold may significantly affect the optimal testing pressure level. Moreover, the proposed framework is suitable for the fatigue pressure test design for one of the hydraulic components. For future research, more work may be done for the application for other kinds of hydraulic components with both part of pressure fatigue experiments and simulations. Moreover, to increase the efficiency of fatigue test, accelerated fatigue pressure test may be designed by combining both tests and simulations.

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