Long-Range Coulomb Interaction and Frequency Dependence of Shot Noise in Mesoscopic Diffusive Contacts

K. E. Nagaev

Institute of Radiophysics and Electronics, Russian Academy of Sciences, Mokhovaya ul. 11, 103907 Moscow, Russia

The frequency dependence of shot noise in mesoscopic diffusive contacts is calculated with account of long-range Coulomb interaction and external screening. While the low-frequency noise is 1/3 of noise of classical Poisson process independently of the contact shape, the high-frequency noise tends to the full classical value for long and narrow contacts because of strong screening by the surrounding medium. In this case, the current fluctuations at opposite ends of the contact are completely independent.

I. INTRODUCTION

Recently, the shot noise in mesoscopic contacts became a subject of extensive study. One of the principal results was that in contacts with a strong elastic scattering, the low-frequency shot noise is 1/3 of the full noise of classical Poisson process. This result was obtained almost simultaneously by different authors using different methods and, more importantly, different physical assumptions. Beenakker and Büttiker [1] obtained this result using the multichannel scattering-matrix formulation and the assumption of quantum-coherent transport. In contrast to this earlier paper [2], this result was obtained using quasiclassical kinetic equation under the assumption of completely incoherent local scattering. Although all the impurity-scattering events were considered to be independent, at low frequencies the current-conservation law resulted in averaging of the random current over the contact volume, thus establishing a sort of correlation in electron transport. In a sense, this appears to be equivalent to quantum-coherent scattering, in which case the whole impurity system is considered as a single scatterer. In both approaches, the low-frequency shot noise appears to be independent of contact geometry [1], [2], [3]. However, this should not be the case for the finite-frequency noise because it should be affected by long-range Coulomb interactions. In particular, it should depend on the possibility for the charge to pile up in the contact, i.e., on its external capacity. The problem of frequency-dependent noise in diffusive mesoscopic contacts was addressed by Altshuler, Levitov, and Yakovets [4] for the case of quantum-coherent transport, where the frequency dependence of noise was determined by fermionic correlations, but no account of long-range Coulomb interactions was taken in this paper.

In the present paper, we consider the effects of contact geometry on the frequency dependence of shot noise within the semiclassical incoherent-scattering approach. We consider the case where all its dimensions are much larger than the screening length \( \lambda_0 \). The contact of length \( L \) is either a cylinder of circular section with a diameter \( 2r_0 \) or a plane-parallel layer of thickness \( d_0 \) consisting of a metal with a high impurity content (see Fig. 1). The electrodes are of the same section, yet the resistivity of their material is negligible. The contact is embedded in a perfectly conducting grounded medium, which is separated from its surface by a thin insulating film of thickness \( \delta_0 \) and the dielectric constant \( \varepsilon_d \). As will be shown below, this particular choice of contact geometry allows us to avoid solving Poisson equation in the surrounding medium and reduces the effects of environmental screening to frequency-dependent boundary conditions.

The external circuit is assumed to have a large grounding capacity, which allows accumulation of the charge in it.

Our consideration is based on the Boltzmann-Langevin approach first proposed in [5]. The nonuniform extraneous currents caused by randomness of electron-impurity scattering result in local charge-density fluctuations in the bulk of the contact. These fluctuations are effectively screened by the surface charge induced at the outer surface of the contact and in the surrounding medium (environmental screening) and at the contact-electrode interfaces (electrode screening). As will be shown below, the finite-frequency noise essentially depends on the dominating type of screening.

II. BASIC EQUATIONS

In the Boltzmann-Langevin approach, the long-range Coulomb interaction is taken into account by fluctuations of charge density \( \delta \rho \) and self-consistent fluctuations of electrical field \( \delta E \). Consider the case of strong and purely elastic...
scattering. The Boltzmann-Langevin equation for fluctuations reads
\[
\frac{\partial}{\partial t} + v \frac{\partial}{\partial r} + eEv \frac{\partial}{\partial \epsilon} \delta f + \delta I = -e \delta E \frac{\partial f}{\partial \epsilon} + \delta J^{ext}, \tag{1}
\]
where \(\delta J^{ext}\) is the random extraneous flux. The correlation function of these fluxes is given by the expression
\[
\langle \delta J^{ext}(p, r, t) \delta J^{ext}(p', r', t') \rangle = \delta(r - r')\delta(t - t')
\]
\[
\left( \delta_{pp'} \sum_q \{W(p, p + q)f(p + q)|1 - f(p)| + W(p + q, p)f(p)|1 - f(p + q)|\}
\]
\[
-\{W(p', p)f(p)|1 - f(p')| + W(p, p')f(p')|1 - f(p)|\}\), \tag{2}
where \(W(p, p')\) is the probability of scattering from state \(p'\) to state \(p\). The fluctuation of electrical field \(\delta E\) in the right hand side of Eqn. \(\ref{eq:1}\) is determined from the Maxwell equation
\[
\nabla \delta E = 4\pi \delta \rho. \tag{3}
\]

The fluctuations of the charge and current density are given by the expressions
\[
\delta \rho(r, t) = e \int d^3p \delta f(p, r, t), \quad \delta j(r, t) = e \int d^3p \nabla f(p, r, t). \tag{4}
\]

Now we proceed to the hydrodynamic approach and obtain a closed set of equations for macroscopic quantities \(\delta \bar{j}\) and \(\delta \bar{\rho}\). As the impurity scattering is strong, we can split the fluctuation of distribution function into the parts symmetric and antisymmetric in the momentum space. Then we separate the antisymmetric part of Eqn. \(\ref{eq:1}\) from its symmetric part. Note that the extraneous flux contains only the antisymmetric part because the electron-impurity scattering does not change the total number of electrons at a given point with a given energy. Multiply the antisymmetric part of \(\ref{eq:1}\) by \(e\nu\), integrate it with respect to \(d^3p\), and then multiply both its parts by the elastic scattering time, \(\tau\). If the characteristic times considered are much larger than \(\tau\), one obtains
\[
\delta \bar{j} = -D \frac{\partial}{\partial r} \delta \bar{\rho} + \sigma \delta \bar{E} + \delta \bar{j}^{ext}, \tag{5}
\]
where \(D = \nu^2 \tau/3\) is the diffusion coefficient, \(\sigma = e^2 N_F D\) is the conductivity of metal, and \(\delta \bar{j}^{ext} = e\tau \int d^3p \nu \delta J^{ext}\).

Integrating the symmetric part of Eqn. \(\ref{eq:1}\) with respect to momentum, one obtains just the current-conservation law
\[
\frac{\partial}{\partial t} \delta \bar{\rho} + \nabla \delta \bar{j} = 0. \tag{6}
\]

Applying the operator \(\nabla\) to both parts of Eqn. \(\ref{eq:3}\) and making use of Eqsns. \(\ref{eq:1}\) and \(\ref{eq:2}\), one obtains a closed equation for fluctuations of charge density in the form
\[
\left( \frac{\partial}{\partial t} - D\nabla^2 + 4\pi\sigma \right) \delta \bar{\rho} = -\nabla \delta \bar{j}^{ext}. \tag{7}
\]

In the left-hand side of this equation, the second term describes diffusion of electrons, and the third term describes the Coulomb screening of fluctuations. In principle, Eqns. \(\ref{eq:5}\) and \(\ref{eq:6}\) may be solved for each particular distribution of \(\delta \bar{j}^{ext}\), and then \(\delta \bar{E}\) and \(\delta \bar{j}\) may be determined from Eqns. \(\ref{eq:3}\) and \(\ref{eq:4}\), respectively. In the static limit, Eqn. \(\ref{eq:7}\) describes the screening of an extraneous charge with the standard length \(\lambda_0\) given by
\[
\lambda_0^{-2} = \frac{4\pi\sigma}{D} = 4\pi e^2 N_F. \tag{8}
\]

To complete the derivation, we must obtain the correlation function of extraneous currents \(\delta \bar{j}^{ext}\). As we are restricted to the case of strong impurity scattering, the distribution function may be considered as isotropic in the momentum space and dependent only on the coordinate \(r\) and energy \(\epsilon\). Multiply Eqn. \(\ref{eq:2}\) by \(e\nu\alpha\) and \(e\nu\beta\), where \(\alpha\) and \(\beta\) label vector components, and integrate it with respect to \(d^3p\) and \(d^3p'\). As a result, one obtains the spectral density of extraneous currents in the form
\[
\langle \delta j^\alpha_{\alpha}(r) \delta j^\beta_{\beta}(r') \rangle_{\omega} = 4\sigma \delta_{\alpha\beta} \delta(r - r') \int d\epsilon f(\epsilon, r)[1 - f(\epsilon, r)]. \tag{8}
\]
Because of smallness of $\lambda_0$, the relationship between the extraneous currents and fluctuations of charge density in the bulk of the sample may be considered as local. Taking the Fourier transform of Eqn. (7) with respect to time, integrating it over the space, and making use of the Gauss theorem, one obtains:

$$
\delta \rho = -(-i \omega + 4\pi \sigma)^{-1} \nabla \delta j_{\text{ext}}^* \quad (9)
$$

Note that the quasineutrality condition does not hold for fluctuations. Introduce the fluctuating potential $\delta \phi$ that satisfies the Poisson equation

$$
\nabla^2 \delta \phi = -4\pi \delta \rho. \quad (10)
$$

Consider the boundary conditions for $\delta \phi$ at the outer insulated surface of the contact. The normal derivatives of $\delta \phi$ in the dielectric layer and inside the metal are related by the expression

$$
\varepsilon_d \frac{\partial \delta \phi}{\partial n} \bigg|_d - \frac{\partial \delta \phi}{\partial n} \bigg|_s = -4\pi \delta \sigma_s, \quad (11)
$$

where $\delta \sigma_s$ is fluctuating surface charge density induced by the extraneous currents. On the other hand, this charge density satisfies the charge-balance equation

$$
-i \omega \delta \sigma_s = -\sigma \frac{\partial \delta \phi}{\partial n} \bigg|_s. \quad (12)
$$

As the thickness of dielectric layer is much smaller than the size of the contact, the electric field across it may be considered uniform so that $\partial \delta \phi / \partial n \bigg|_d = -\delta \phi \bigg|_d$. With this condition, Eqns. (11) and (12) give the boundary condition for $\delta \phi$ in the form

$$
\left[ -i \omega \varepsilon_d \delta \phi - (i \omega + 4\pi \sigma) \frac{\partial \delta \phi}{\partial n} \right] \bigg|_s = 0. \quad (13)
$$

It is easily seen that at $\omega = 0$, Eqn. (13) takes the form $\partial \delta \phi / \partial n \bigg|_s = 0$, while at $\omega \to \infty$, it takes the form $\delta \phi \bigg|_s = 0$.

As the voltage drop across the contact is held constant, fluctuations of potential are zero at the contact-electrode interfaces:

$$
\delta \phi \bigg|_i = 0. \quad (14)
$$

As the electrodes are perfect conductors, $\partial \phi / \partial n = 0$ inside them. Equation (11) holds for contact-electrode interfaces, but the charge-balance equation takes now the form

$$
-i \omega \delta \sigma_s = -\sigma \frac{\partial \delta \phi}{\partial n} \bigg|_i - \delta j_n, \quad (15)
$$

where $\delta j_n$ is the fluctuation of current flowing into the electrodes from the contact. From Eqns. (11) and (15), it follows that the density of outgoing current is given by

$$
\delta j_n = \left( \frac{i \omega}{4\pi - \sigma} \right) \frac{\partial \delta \phi}{\partial n} \bigg|_i. \quad (16)
$$

From the standpoint of average current, the problem is purely one-dimensional, so the average distribution function $f(\epsilon, x)$ obeys the one-dimensional diffusion equation, its boundary values being zero-temperature Fermi distribution functions shifted in energy by $eV$ with respect to each other. As the contact is much shorter than the characteristic inelastic length,

$$
f(\epsilon, x) = \begin{cases} 
0, & \epsilon > eV/2 \\
1 - x/L, & eV/2 > \epsilon > -eV/2 \\
1, & \epsilon < -eV/2.
\end{cases} \quad (17)
$$

With this distribution function, the expression for the spectral density of extraneous currents (8) takes the form

$$
\langle \delta j^*_{\alpha} (r) \delta j^*_{\beta} (r') \rangle_\omega = 4 \sigma \delta_{\alpha \beta} \delta (r - r') \frac{x}{L} \left( 1 - \frac{x}{L} \right). \quad (18)
$$
III. CIRCULAR-SECTION CONTACT. ANALYTICAL RESULTS

Consider the Poisson equation with the boundary conditions (13). As the system is axially symmetric, all the quantities may be considered as independent of the azimuthal angle and dependent only on the longitudinal coordinate \( x \) and radius \( r \). In this case, the boundary condition (13) takes the form

\[
\left. \left( \frac{\partial \delta \phi}{\partial r} + \mu \delta \phi \right) \right|_{r=r_0} = 0, \quad \mu = \frac{-i \omega}{-i \omega + 4 \pi \sigma} \varepsilon_d r_0 \quad (19)
\]

Suppose first that \( \mu \) is real and positive. Then one may introduce a system of normalized eigenfunctions \( \psi_n \) satisfying the equation

\[
\frac{1}{r} \frac{d}{dr} \left( r \frac{d}{dr} \psi_n(r) \right) + k_n^2 \psi_n(r) = 0 \quad (20)
\]

with the boundary conditions (19). These functions are given by

\[
\psi_n(r) = \frac{1}{\pi^{1/2} r_0} \frac{J_0(k_n r)}{\sqrt{J_0^2(k_n r_0) + J_1^2(k_n r_0)}}, \quad (21)
\]

where \( J_0 \) and \( J_1 \) are the Bessel functions of zeroth and first order and the eigenvalues \( k_n \) are determined from the equation

\[
k_n r_0 \frac{J_1(k_n r_0)}{J_0(k_n r_0)} = \mu. \quad (22)
\]

Since functions \( \psi_n \) form an orthogonal basis, for an arbitrary charge-density fluctuation \( \delta \rho \), the solution of Poisson equation (10) with the boundary condition (19) is given by the expression

\[
\phi(x, r) = -4 \pi \sum_{n=0}^{\infty} \overline{\psi_n}(x, r) \int_0^L dx' g_n(x, x') \int dS' \psi_n(r') \rho(x', r'), \quad (23)
\]

where \( dS' = 2 \pi r' dr' \) and \( g_n(x, x') \) is the Green’s function of the equation

\[
\left( \frac{d^2}{dx^2} - k_n^2 \right) g_n(x, x') = \delta(x - x') \quad (24)
\]

with the boundary conditions \( g_n(0, x') = g_n(L, x') = 0 \), which is given by the expression

\[
\begin{align*}
g_n(x < x') &= -\frac{\sinh(k_n x) \sinh[k_n(L - x')]}{k_n \sinh(k_n L)}, \\
g_n(x > x') &= -\frac{\sinh(k_n x') \sinh[k_n(L - x)]}{k_n \sinh(k_n L)}.
\end{align*} \quad (25)
\]

Equation (23) may be analytically continued to complex values of \( \mu \) given by Eqn. (19). Substituting Eqn. (10) for \( \delta \rho \) into Eqn. (23) for \( \delta \phi \) and then (23) into (16), one obtains the expression for the fluctuation of the current flowing through the left end of the contact in the form

\[
\delta I(0) = S_0 \sum_{n=0}^{\infty} \overline{\psi_n} \int_0^L dx' \int dS' \left[ \frac{\partial^2 g_n(x, x')}{\partial x \partial x'} \right]_{x=0} \psi_n(r') \delta j_{r}^{\text{ext}} + \frac{\partial g_n(x, x')}{\partial x} \right|_{x=0} \frac{\partial \psi_n}{\partial r'} \delta j_{r}^{\text{ext}}
\]

\[
, \quad (26)
\]

where \( S_0 = \pi r_0^2 \) and \( \overline{\psi_n} \) is \( \psi_n \) averaged over the cross section of the contact:

\[
\overline{\psi_n} = \frac{1}{S_0} \int dS \psi_n(r) = \frac{2J_1(k_n r_0)}{\pi^{1/2} r_0 \sqrt{J_0^2(k_n r_0) + J_1^2(k_n r_0)}}. \quad (27)
\]
To obtain the fluctuation of the current flowing through the right end of the contact, \( \delta I(L) \), one must substitute \( x = L \) for \( x = 0 \) in Eqn. (26).

At \( \omega = 0 \), all transverse modes with \( n \neq 0 \) have vanishing cross-sectional averages, \( \bar{l}_n = 0 \), and the corresponding longitudinal factors \( g_n \) exponentially decay at \( |x - x'| > r_0 \). This is quite natural because the electrical field produced by a charge inside the contact cannot penetrate through its outer surface and is uniformly distributed over the contact cross section at large distances from the source. Hence the only contribution to Eqn. (26) will be given by the lowest transverse mode with \( k_0 = 0 \) and \( \psi_0(r) = \pi^{-1/2} r_0^{-1} \). In this case, Eqn. (26) takes the form

\[
\delta I(0) = \frac{1}{L} \int_0^L dx \int dS \delta j_x^{ext}.
\]

This is just the result obtained in [2].

Consider now the case where the contact length \( L \) is much larger than its diameter \( 2r_0 \) and the frequencies are sufficiently low, i.e., \( \omega \ll 4\pi\sigma_0/\varepsilon_0 r_0 \). In this case, the corrections to the zero-frequency eigenfunctions \( \psi_n \), as well as the corrections to the products \( k_n r_0 \) with \( n \neq 0 \), are proportional to \( \mu \) and therefore small; hence the contributions to \( \delta I \) from the modes with \( n \neq 0 \) remain insignificant. However, the lowest eigenvalue is given by

\[
k_0 = r_0^{-1} (2\mu)^{1/2},
\]

and the product \( k_0 L \) may be sufficiently large. Therefore, the contribution from the lowest mode governed by \( g_0(0, x) \) may change significantly. In view of this, the expression for \( \delta I \) takes the form

\[
\delta I(0) = \int_0^L dx \frac{k_0 \cosh[k_0(L-x)]}{\sinh(k_0 L)} \int dS \delta j_x^{ext},
\]

where \( k_0 \) is given by Eqn. (29). Physically, this implies that the contact is represented as an alternating series of resistors with generators of random current and grounding capacities connecting the electrodes (see Fig. 2). Note that \( \delta I(0) \) is phase shifted with respect to the extraneous current inducing it. Multiplying Eqn. (30) by its complex conjugate, substituting the spectral density of extraneous currents (18) into the product, and performing the integration with respect to \( x \), one obtains:

\[
S_{IL}^{LL}(\omega) = 2eI \left[ 1 - \frac{1}{\gamma_\omega L} \frac{\sinh(2\gamma_\omega L) - \sin(2\gamma_\omega L)}{\cosh(2\gamma_\omega L) - \cos(2\gamma_\omega L)} \right],
\]

where

\[
\gamma_\omega = \frac{1}{2} \sqrt{\frac{\omega \varepsilon_0}{\pi \sigma_0 r_0}}.
\]

The frequency dependence of the shot noise is shown in Fig. 3. At zero frequency, we rederive the well known result \( S_{IL}^{LL} = \frac{4eI}{\gamma_\omega} \). However, at frequencies about \( \sigma_0 r_0/\varepsilon_0 L^2 \), the spectral density sharply rises and tends to the full value of classical shot noise, \( S_{IL}^{LL} = 2eI \). This suggests that the corresponding correlation function is negative at \( t \neq t' \). The anticorrelation is the consequence of the Coulomb repulsion of electrons: an entrance of an electron into the contact decreases for some time the probability for another electron to enter it, similarly to the case of a single-electron transistor [6], [5].

Along with the spectral density of noise at one end of the contact, one may also consider the cross-correlated spectral density

\[
S_{IL}^{LR}(\omega) = \frac{1}{2} \langle \delta I(0, \omega) \delta I(L, -\omega) + \delta I(0, -\omega) \delta I(L, \omega) \rangle,
\]

which describes the correlation between the currents flowing through the opposite ends of the contact. Multiplying Eqn. (30) by its complex conjugate for \( \delta I(L) \) and performing the integration with the spectral density of extraneous currents (18), one obtains:

\[
S_{IL}^{LR}(\omega) = \frac{4eI}{\gamma_\omega L} \frac{\cosh(\gamma_\omega L) \sin(\gamma_\omega L) - \cos(\gamma_\omega L) \sinh(\gamma_\omega L)}{\cosh(2\gamma_\omega L) - \cos(2\gamma_\omega L)}.
\]
The frequency dependence of $S_{I}^{LR}$ is also shown in Fig. 3. At $\omega = 0$, it also equals $4eI$. However in contrast to $S_{I}^{LL}(\omega)$, it sharply decreases with increasing frequency and tends to zero in an oscillatory way with further increase of frequency.

Consider now the high-frequency limit. In this case, the boundary condition (13) takes the form $\psi_n(r_0) = 0$, so that the quantities $k_n r_0$ are the zeros of zero-order Bessel function. In this case, functions $\psi_n$ are real and form an orthogonal system. Owing to the orthonormality conditions, the expression for the spectral density of noise may be written in the form

$$S_{I}^{LL}(\infty) = 4S_0^2 eV \sigma \sum_{n=1}^{\infty} \frac{1}{(k_n r_0)^2} \cosh(k_n L) \left[ \coth(k_n L) - \frac{1}{k_n L} \right],$$

(34)

As $J_0(k_n r_0) = 0$, Eqn. (27) reduces to $\psi_n = 2\pi^{-1/2}r_0^{-2}k_n^{-1}$. Substituting the explicit expressions for $g_n$ (25) into Eqn. (34), one obtains:

$$S_{I}^{LL}(\infty) = 8eI \sum_{n=1}^{\infty} \frac{1}{(k_n r_0)^2} \coth(k_n L) \left[ \cosh(k_n L) - \frac{1}{k_n L} \right].$$

(35)

Similarly, one obtains for the cross-correlated spectral density:

$$S_{I}^{LR}(\infty) = 8eI \sum_{n=1}^{\infty} \frac{1}{(k_n r_0)^2} \frac{1}{\sinh(k_n L)} \left[ \coth(k_n L) - \frac{1}{k_n L} \right].$$

(36)

In the limiting case of $r_0 \gg L$, both expressions take the form

$$S_{I}^{LL}(\infty) = S_{I}^{LR}(\infty) = \frac{8}{3}eI \sum_{n=1}^{\infty} \frac{1}{(k_n r_0)^2} = \frac{2}{3}eI.$$ 

(37)

In the opposite limiting case of $r_0 \ll L$, Eqn. (25) takes the form

$$S_{I}^{LL}(\infty) = 8eI \sum_{n=1}^{\infty} \frac{1}{(k_n r_0)^2} = 2eI,$$

(38)

whereas $S_{I}^{LR}(\infty)$ (36) tends to zero according to the exponential law. The $L/r_0$ dependences of $S_{I}^{LL}(\infty)$ and $S_{I}^{LR}(\infty)$ are shown in Fig. 3.

IV. PLANAR CONTACT. NUMERICAL RESULTS

Consider now a planar contact in the shape of a layer of thickness $d_0$ in the $y$ direction ($0 < y < d_0$) and of width $W$ ($W \gg \max(d_0, L)$) in the $z$ direction, the average current flowing in the $x$ direction. Because of large $W$, the effects of boundaries in the $z$ direction may be neglected and all the qualtities may be considered as independent of $z$. Introduce an orthonormal system of functions

$$\varphi_n(x) = \sqrt{\frac{2}{L}} \sin(q_n x),$$

$$q_n = \frac{\pi n}{L},$$

(39)

which obey the boundary conditions $\varphi_n(0) = \varphi_n(L) = 0$. For an arbitrary charge-density fluctuation $\delta \rho$, the potential fluctuation $\delta \phi$ induced by it may be presented in the form

$$\delta \phi(x, y) = -4\pi \sum_{n=1}^{\infty} \varphi_n(x) \int_{0}^{L} dx' \varphi(x') \int_{0}^{d_0} dy' Q_n(y, y') \delta \rho(x', y'),$$

(40)
where $Q_n(y, y')$ satisfies the equation

$$\left( \frac{\partial^2}{\partial y^2} - q_n^2 \right) Q_n(y, y') = \delta(y - y')$$  \hspace{1cm} (41)

with the boundary conditions

$$\left( \frac{-i\omega \varepsilon_d}{4\pi \sigma d_0} Q_n + \frac{\partial Q_n}{\partial y} \right) \Bigg|_{y=0} = 0, \quad \left( \frac{i\omega \varepsilon_d}{4\pi \sigma d_0} Q_n + \frac{\partial Q_n}{\partial y} \right) \Bigg|_{y=d_0} = 0. \hspace{1cm} (42)$$

Explicitly, $Q_n$ for $y > y'$ is given by the expression

$$Q_n(y, y') = -\frac{1}{2q_n} \frac{q_n d_0 \cosh[q_n(d_0 - y)] - i\Omega \sinh[q_n(d_0 - y)]}{q_n d_0 \cosh(q_n d_0/2) - i\Omega \sinh(q_n d_0/2)}$$

$$\times \frac{q_n d_0 \cosh(q_n y') - i\Omega \sinh(q_n y')}{q_n d_0 \cosh(q_n d_0/2) - i\Omega \sinh(q_n d_0/2)}, \hspace{1cm} (43)$$

where $\Omega = \omega \varepsilon_d d_0/4\pi \sigma d_0$ is the dimensionless frequency. The corresponding expression for $y < y'$ is obtained from Eqn. (43) by interchanging $y$ and $y'$. Substituting Eqn. (3) for $\delta p$ into Eqn. (10) and then substituting (40) into (16), one obtains the expression for the fluctuation of current flowing through the right end of the contact in the form

$$\delta I(0) = W \sum_{n=1}^{\infty} \frac{d^2 \varphi_n}{d x^2} \left|_{x=0} \right. \int_0^L dx' \int_0^{d_0} dy' \int_0^d dy \left. \int_0^{d_0} dy' \right. \right.$$

$$\left. \left[ \frac{d \varphi_n(x')}{d x'} Q_n(y, y') \delta_{j_{x}^{ext} + \varphi_n(x') \frac{\partial Q_n(y, y')}{\partial y'}} \delta_{j_{y}^{ext}} \right] \right), \hspace{1cm} (44)$$

The fluctuation of current flowing through the right end of the contact may be obtained by substituting $x = L$ for $x = 0$ in Eqn. (44). Using the correlator of extraneous currents (18), one obtains the expressions for the spectral densities $S_{LL}^I$ and $S_{LR}^I$ in the form

$$S_{LL}^I = 8\epsilon I d_0 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} k_m k_n (M_{mn} P_{mn}'' + M_{mn}'' P_{mn}'), \hspace{1cm} (45)$$

$$S_{LR}^I = 4\epsilon I d_0 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[ (-1)^m + (-1)^n \right] k_m k_n (M_{mn} P_{mn}'' + M_{mn}'' P_{mn}'), \hspace{1cm} (46)$$

In these expressions, we used the notation

$$M_{mn} = \frac{1}{d_0^3} \int_0^{d_0} dy \int_0^{d_0} dy_1 \int_0^{d_0} dy_2 Q_m(y_1, y) Q_n(y_2, y), \hspace{1cm} (47)$$

$$M_{mn}'' = \frac{1}{d_0^3} \int_0^{d_0} dy \int_0^{d_0} dy_1 \int_0^{d_0} dy_2 \frac{\partial Q_m(y_1, y)}{\partial y} \frac{\partial Q_n(y_2, y)}{\partial y}, \hspace{1cm} (48)$$

$$P_{mn} = \int_0^L dx \varphi_m(x) \varphi_n(x) \frac{x}{L} \left( 1 - \frac{x}{L} \right), \hspace{1cm} (49)$$

$$P_{mn}'' = \int_0^L dx \frac{\partial \varphi_m}{\partial x} \frac{\partial \varphi_n}{\partial x} \left( 1 - \frac{x}{L} \right), \hspace{1cm} (50)$$

7
Using the notation $t_m = \tanh(k_m d_0/2)$ and $D_m = (k_m^2 d_0^2 t_m^2 + \Omega^2)^{-1}$ and performing partial summation over the internal index, one may bring Eqns. (45) and (46) to the form

$$S_{1L}^L(\omega) = \frac{2}{3} eI + 4eI\Omega^2 \sum_{m=1}^{\infty} \left\{ \left( \frac{2}{3} - \frac{8}{k_m^2 L^2} \right) \frac{D_m t_m}{k_m d_0} - \frac{D_m}{k_m^2 L^2} (1 - t_m^2) \right\},$$

$$S_{1L}^R(\omega) = \frac{2}{3} eI + 4eI\Omega^2 \sum_{m=1}^{\infty} (-1)^m \left\{ \left( \frac{2}{3} - \frac{8}{k_m^2 L^2} \right) \frac{D_m t_m}{k_m d_0} - \frac{D_m}{k_m^2 L^2} (1 - t_m^2) \right\},$$

where the primes by the sums over $n$ show that $n \neq m$.

The contour plots of $S_{1L}^L$ and $S_{1L}^R$ vs. logarithms of frequency and contact length are shown in Figs. 5 and 6. Qualitatively, their behavior is similar to that in the case of a circular-section contact. At low frequencies and small contact lengths, both quantities tend to $2eI$. At high frequencies and large contact lengths, $S_{1L}^L$ and $S_{1L}^R$ tend to $2eI$ and zero, respectively. It is also clearly seen that at $L/d_0 \geq 2.86$, the frequency dependences of $S_{1L}^R$ exhibit negative portions.

V. CONCLUSION

Both circular and planar contacts exhibit qualitatively similar noise properties. At small length-to-width ratios, when the screening of charge fluctuations by the electrodes is more efficient than the screening by the ambient medium and pile-up of the charge in the contact is forbidden, the effects of long-range Coulomb interaction reduce to averaging the extraneous currents over the volume of the contact at arbitrary frequencies. The situation is different, however, for long and narrow contacts, where the charge fluctuations are mostly screened by the ambient medium and pile-up of charge in the contact is allowed. At sufficiently high frequencies, the correlation length of fluctuations becomes smaller than the length of the contact. In this case, the fluctuations of current at the ends of the contact, which are observed in the external circuit, are dominated by extraneous currents in the narrow adjacent layers. The corresponding spectral densities are equal to that of the classical shot noise, $2eI$, while the fluctuations at different contact ends are completely independent.

This work was supported by DOE’s Grant #DE-FG02-95ER14575 and by the Russian Foundation for Basic Research (project #96-02-16663-a).

The author acknowledges a fruitful discussion with G. B. Lesovik.

[1] C. W. J. Beenakker and M. Büttiker, Phys. Rev. B 46, 1889 (1992).
[2] K. E. Nagaev, Phys. Lett. A 169, 103 (1992).
[3] Yu. Nazarov, Phys. Rev. Lett. 73, 134 (1994).
[4] Sh. M Kogan and A. Ya. Shul’man, Zh. Eksp. Teor. Fiz. 56, 862 (1969) [Sov. Phys. JETP 29, 467 (1969)].
[5] B. L. Altshuler, L. S. Levitov, and A. Yu. Yakovets, Pis’ma Zh. Eksp. Teor. Fiz., 59, 821 (1994) [JETP Lett. 59, 857 (1994)].
[6] A. N. Korotkov, Phys. Rev. B 49, 10381 (1994).
[7] A. N. Korotkov, D. V. Averin, K. K. Likharev, and S. A. Vassenko, in Single-Electron Tunneling and Mesoscopic Devices, edited by H. Koch and H. Bübbig (Springer-Verlag, Berlin, 1992), p. 45.

FIGURE CAPTIONS
FIG. 1. Longitudinal cross section of the contact. The dotted rectangle is the metal with impurities, thin solid lines show the contact-electrode interfaces, thick lines show the dielectric layers of thickness $\delta_0$, and the hatched areas show the grounded ambient medium.

FIG. 2. Physical model of noise in a long and narrow contact. Each section of the $R-C$ line contains a generator of random current.

FIG. 3. Dependences of the normalized spectral density of noise at one of the contact ends $S_{LL}^{1L}/2\varepsilon I$ (solid line) and the cross-correlated spectral density $S_{LR}^{1L}/2\varepsilon I$ (dashed line) on the dimensionless frequency $\omega L^2 \varepsilon d/4\pi \delta_0 r_0$ for a long narrow contact.

FIG. 4. Dependences of the normalized spectral density of noise at one of the contact ends $S_{LL}^{1L}/2\varepsilon I$ (solid line) and cross-correlated spectral density $S_{LR}^{1L}/2\varepsilon I$ (dashed line) on the length-to-radius ratio $L/r_0$ in the high-frequency limit.

FIG. 5. Contour plots of $S_{LL}^{L}$ vs. logarithms of normalized frequency $\Omega = \omega d \varepsilon_0 / 4\pi r_0$ and normalized length $L/d_0$ for the planar contact.

FIG. 6. Contour plots of $S_{LR}^{L}$ vs. logarithms of normalized frequency $\Omega = \omega d \varepsilon_0 / 4\pi r_0$ and normalized length $L/d_0$ for the planar contact.