Panaretos, Victor M.; Tavakoli, Shahin

Fourier analysis of stationary time series in function space. (English) Zbl 1267.62094 Ann. Stat. 41, No. 2, 568-603 (2013).

Summary: We develop the basic building blocks of a frequency domain framework for drawing statistical inferences on the second-order structure of a stationary sequence of functional data. The key element in such a context is the spectral density operator, which generalises the notion of a spectral density matrix to the functional setting, and characterises the second-order dynamics of the process. Our main tool is the functional Discrete Fourier Transform (fDFT). We derive an asymptotic Gaussian representation of the fDFT, thus allowing the transformation of the original collection of dependent random functions into a collection of approximately independent complex-valued Gaussian random functions. Our results are then employed in order to construct estimators of the spectral density operator based on smoothed versions of the periodogram kernel, the functional generalisation of the periodogram matrix. The consistency and asymptotic law of these estimators are studied in detail. As immediate consequences, we obtain central limit theorems for the mean and the long-run covariance operator of a stationary functional time series. Our results do not depend on structural modelling assumptions, but only functional versions of classical cumulant mixing conditions, and are shown to be stable under discrete observation of the individual curves.

MSC:
62M15 Inference from stochastic processes and spectral analysis
65T50 Numerical methods for discrete and fast Fourier transforms
62M10 Time series, auto-correlation, regression, etc. in statistics (GARCH)
60F05 Central limit and other weak theorems

Keywords:
cumulants; discrete Fourier transform; functional data analysis; functional time series; periodogram operator; spectral density operator; weak dependence

Software:
KernSmooth; fda (R)

Full Text: DOI arXiv Euclid

References:
[1] Adler, R. J. (1990). An Introduction to Continuity, Extrema, and Related Topics for General Gaussian Processes. Institute of Mathematical Statistics Lecture Notes-Monograph Series 12. IMS, Hayward, CA. · Zbl 0747.60039
[2] Anderson, T. W. (1994). The Statistical Analysis of Time Series. Wiley, New York. · Zbl 0835.62074
[3] Antoniadis, A., Paparoditis, E. and Sapatinas, T. (2006). A functional wavelet-kernel approach for time series prediction. J. R. Stat. Soc. Ser. B Stat. Methodol. 68 837-857. · Zbl 1110.62122 · doi:10.1111/j.1467-9868.2006.00569.x
[4] Antoniadis, A. and Sapatinas, T. (2003). Wavelet methods for continuous-time prediction using Hilbert-valued autoregressive processes. J. Multivariate Anal. 87 135-158. · Zbl 1030.62075 · doi:10.1016/S0047-259X(03)00028-9
[5] Benko, M., Härdle, W. and Kneip, A. (2009). Common functional principal components. Ann. Statist. 37 1-34. · Zbl 1169.62057 · doi:10.1214/07-AOS516
[6] Bloomfield, P. (2000). Fourier Analysis of Time Series: An Introduction, 2nd ed. Wiley, New York. · Zbl 0994.62093
[7] Boente, G., Rodriguez, D. and Sued, M. (2011). Testing the equality of covariance operators. In Recent Advances in Functional Data Analysis and Related Topics 49-53. Physica-Verlag/Springer, Heidelberg. · doi:10.1007/978-3-7908-2736-1_8
[8] Bosq, D. (2000). Linear Processes in Function Spaces: Theory and Applications. Lecture Notes in Statistics 149. Springer, New York. · Zbl 0962.60004 · doi:10.1007/978-1-4419-8642-1
[9] Bosq, D. (2002). Estimation of mean and covariance operator of autoregressive processes in Banach spaces. Stat. Inference Stoch. Process. 5 287-306. · Zbl 1028.62070 · doi:10.1023/A:1021279131053
[10] Bosq, D. and Blanke, D. (2007). Inference and Prediction in Large Dimensions. Wiley, Chichester. · Zbl 1183.62157
[11] Brillinger, D. R. (2001). Time Series : Data Analysis and Theory. Classics in Applied Mathematics 36 . SIAM, Philadelphia, PA. · Zbl 0983.62056 · doi:10.1137/1.9780898719246
[12] Cardot, H. and Sarda, P. (2006). Linear regression models for functional data. In The Art of Semiparametrics 49-66. Physica-Verlag/Springer, Heidelberg. · Zbl 1271.62145 · doi:10.1007/3-7908-1701-5
[13] Cuevas, A., Febrero, M. and Fraiman, R. (2002). Linear functional regression: The case of fixed design and functional response. Canad. J. Statist. 30 285-300. · Zbl 1012.62039 · doi:10.2307/3315952
[14] Dauxois, J., Pousse, A. and Romain, Y. (1982). Asymptotic theory for the principal component analysis of a vector random function: Some applications to statistical inference. J. Multivariate Anal. 12 136-154. · Zbl 0539.62064 · doi:10.1016/0047-259X(82)90088-4
[15] Dehling, H. and Sharipov, O. S. (2005). Estimation of mean and covariance operator for Banach space valued autoregressive processes with dependent innovations. Stat. Inference Stoch. Process. 8 137-149. · Zbl 1079.62084 · doi:10.1007/s11203-003-0382-8
[16] Edwards, R. (1967). Fourier Series : A Modern Introduction . Holt, Rinehart & Winston, New York. · Zbl 0189.06602
[17] Ferraty, F. and Vieu, P. (2004). Nonparametric models for functional data, with application in regression, time-series prediction and curve discrimination. J. Nonparametr. Stat. 16 111-125. · Zbl 1049.62039 · doi:10.1080/10485250310001622686
[18] Ferraty, F. and Vieu, P. (2006). Nonparametric Functional Data Analysis : Theory and Practice. Springer, New York. · Zbl 1119.62046 · doi:10.1007/0-387-36620-2
[19] Ferraty, F., Goia, A., Salinelli, E. and Vieu, P. (2011a). Recent advances on functional additive regression. In Recent Advances in Functional Data Analysis and Related Topics 97-102. Physica-Verlag/Springer, Heidelberg. · doi:10.1007/978-3-7908-2736-1_5
[20] Ferraty, F., Laksaci, A., Tadj, A. and Vieu, P. (2011b). Kernel regression with functional response. Electron. J. Stat. 5 159-171. · Zbl 1274.62281 · doi:10.1214/11-EJS600
[21] Friedl, S., Steinebach, J., Horváth, L. and Kokoszka, P. (2013). Testing the equality of covariance operators in functional samples. Scand. J. Stat. 40 138-152. · Zbl 1259.62031
[22] Gabrys, R., Horváth, L. and Kokoszka, P. (2010). Tests for error correlation in the functional linear model. J. Amer. Statist. Assoc. 105 1113-1125. · Zbl 1390.62118 · doi:10.1198/jasa.2010.tm09794
[23] Gabrys, R. and Kokoszka, P. (2007). Portmanteau test of independence for functional observations. J. Amer. Statist. Assoc. 102 1338-1348. · Zbl 1332.62322 · doi:10.1198/016214507000001111
[24] Grenander, U. (1981). Abstract Inference . Wiley, New York. · Zbl 0505.62009
[25] Grenander, U. and Rosenblatt, M. (1957). Statistical Analysis of Stationary Time Series. Wiley, New York. · Zbl 0080.12904
[26] Hall, P. and Hosseini-Nasab, M. (2006). On properties of functional principal components analysis. J. R. Stat. Soc. Ser. B Stat. Methodol. 68 109-126. · Zbl 1119.62048 · doi:10.1111/j.1467-9868.2005.00535.x
[27] Hall, P. and Vial, C. (2006). Assessing the finite dimensionality of functional data. J. R. Stat. Soc. Ser. B Stat. Methodol. 68 689-705. · Zbl 1110.62085 · doi:10.1198/jasa.2010.tm09794
[28] Hanan, E. J. (1970). Multiple Time Series . Wiley, New York. · Zbl 0211.49804
[29] Hörmann, S. and Kokoszka, P. (2010). Weakly dependent functional data. Ann. Statist. 38 1845-1884. · Zbl 1189.62141 · doi:10.1214/09-AOS768
[30] Horváth, L., Hušková, M. and Kokoszka, P. (2010). Testing the stability of the functional autoregressive process. J. Multivariate Anal. 101 352-367. · Zbl 1178.62099 · doi:10.1016/j.jmva.2008.12.008
[31] Horváth, L. and Kokoszka, P. (2012). Inference for Functional Data with Applications . Springer, New York. · Zbl 1279.62017 · doi:10.1007/978-3-7908-2736-1
[32] Horváth, L., Kokoszka, P. and Reeder, R. (2013). Estimation of the mean of functional time series and a two-sample problem. J. R. Stat. Soc. Ser. B Stat. Methodol. 75 103-122. · doi:10.1007/j.1467-9868.2012.01032.x
[33] Hunter, J. K. and Nachtergaele, B. (2001). Applied Analysis . World Scientific, River Edge, NJ. · Zbl 0981.65002
[34] Kadison, R. V. and Ringrose, J. R. (1997). Fundamentals of the Theory of Operator Algebras. Graduate Studies in Mathematics 15 . Amer. Math. Soc., Providence, RI. · Zbl 0888.46039
[35] Karlhunen, K. (1947). Über lineare Methoden in der Wahrscheinlichkeitsrechnung. Ann. Acad. Sci. Fennicae. Ser. A I Math.-Phys. 1947 79. · Zbl 0030.16502
[36] Kolmogorov, A. (1978). Stationary Sequences in Hilbert Space . National Translations Center [John Cerraz Library], Chicago. · Zbl 0063.02291
[37] Kraus, D. and Panaretos, V. M. (2012). Dispersion operators and resistant second-order functional data analysis. Biometrika 99 813-832. · Zbl 1452.62991
[38] Laib, N. and Louani, D. (2010). Nonparametric kernel regression estimation for functional stationary ergodic data: Asymptotic properties. J. Multivariate Anal. 101 2266-2281. · Zbl 1198.62027 · doi:10.1016/j.jmva.2010.05.010
[39] Ledoux, M. and Talagrand, M. (1991). Probability in Banach Spaces : Isoperimetry and Processes. Ergebnisse der Mathematik und Ihrer Grenzgebiete (3) [ Results in Mathematics and Related Areas (3)] 23 . Springer, Berlin. · Zbl 0748.60004
[40] Lévy, P. (1948). Processus stochastiques et mouvement Brownien. Suivi d’une note de M. Loève . Gauthier-Villars, Paris. · Zbl 0063.022603
[41] Liu, W. and Wu, W. B. (2010). Asymptotics of spectral density estimates. Econometric Theory 26 1218-1245. · Zbl 1294.62077 · doi:10.1017/S026646660999061X
Locantore, N., Marron, J. S., Simpson, D. G., Tripoli, N., Zhang, J. T. and Cohen, K. L. (1999). Robust principal component analysis for functional data. TEST 8 1-73. - Zbl 0980.62049 - doi:10.1007/BF02595862

Mas, A. (2000). Estimation d’opérateurs de corrélation de processus linéaires fonctionnels: lois limites, déviations modérées. Ph.D. thesis, Université Paris VI.

Panaretos, V. M., Kraus, D. and Maddocks, J. H. (2010). Second-order comparison of Gaussian random functions and the geometry of DNA minicircles. J. Amer. Statist. Assoc. 105 670-682. - Zbl 1392.62162 - doi:10.1198/jasa.2010.tm09239

Panaretos, V. M. and Tavakoli, S. (2013). Cramér-Karhunen-Loève representation and harmonic principal component analysis of functional time series. Stochastic Process. Appl. To appear, available at http://www.sciencedirect.com/science/article/pii/S0304414913000793 . - Zbl 1285.62109 - doi:10.1016/j.spa.2013.03.015

Panaretos, V. M. and Tavakoli, S. (2013). Supplement to “Fourier analysis of stationary time series in function space.” . - Zbl 1267.62094

Peligrad, M. and Wu, W. B. (2010). Central limit theorem for Fourier transforms of stationary processes. Ann. Probab. 38 2009-2022. - Zbl 1206.60026 - doi:10.1214/10-AOP530

Pollard, D. (1984). Convergence of Stochastic Processes . Springer, New York. - Zbl 0544.60045

Priestley, M. B. (2001). Spectral Analysis and Time Series , Vol. I and II . Academic Press, San Diego.

Ramsay, J. O. and Silverman, B. W. (2005). Functional Data Analysis , 2nd ed. Springer, New York. - Zbl 1079.62006

Rice, J. A. and Silverman, B. W. (1991). Estimating the mean and covariance structure nonparametrically when the data are curves. J. Roy. Statist. Soc. Ser. B 53 233-243. - Zbl 0800.62214

Rosenblatt, M. (1984). Asymptotic normality, strong mixing and spectral density estimates. Ann. Probab. 12 1167-1180. - Zbl 0545.62058 - doi:10.1214/aop/1176993146

Rosenblatt, M. (1985). Stationary Sequences and Random Fields . Birkhäuser, Boston, MA. - Zbl 0597.60095

Sen, R. and Klüppelberg, C. (2010). Time series of functional data. Unpublished manuscript. Available at .

Shao, X. and Wu, W. B. (2007). Asymptotic spectral theory for nonlinear time series. Ann. Statist. 35 1773-1801. - Zbl 1147.62076 - doi:10.1214/0090536060000001479

Wand, M. P. and Jones, M. C. (1995). Kernel Smoothing. Monographs on Statistics and Applied Probability 60 . Chapman & Hall, London. - Zbl 0854.62043

Weidmann, J. (1980). Linear Operators in Hilbert Spaces. Graduate Texts in Mathematics 68 . Springer, New York. - Zbl 0434.47001

Wheeden, R. L. and Zygmund, A. (1977). Measure and Integral : An Introduction to Real Analysis. Pure and Applied Mathematics 43 . Dekker, New York. - Zbl 0362.26004

Yao, F., Müller, H.-G. and Wang, J.-L. (2005). Functional linear regression analysis for longitudinal data. Ann. Statist. 33 2873-2901. - Zbl 1084.62096 - doi:10.1214/009053605000000669

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.