Stability-Constrained Power System Scheduling: A Review

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ABSTRACT Power system scheduling mainly concerns economic optimization issues of the power system, which is also commonly known as the unit commitment (UC) problem. However, improper planning in the generation schedule may pose a negative impact on power system stability. Additionally, the trend of large-scale integration of renewable energy in the future power system brings critical challenges to power system stability. In consequence, it is necessary to integrate the stability constraints into power system scheduling. According to the classic classification of power system stability (i.e. voltage stability, frequency stability, and rotor angle stability), stability constraints can be constructed accordingly to guarantee system stability when solving UC problems, which ensures both the economic efficiency and technical feasibility of the UC solutions. This paper reviews typical stability constraints and how to apply these constraints in solving UC problems. Representative works are summarized to provide a guidance for addressing the stability constrained scheduling problems in the future power system operation.

INDEX TERMS Unit commitment (UC), power system stability, stability constraints.

NOMENCLATURE

A. CONSTANTS

| Symbol | Description |
|--------|-------------|
| $N_t$  | Number of planning time horizon ($N_t = 24$) |
| $N_g$  | Number of generation units |
| $t$    | $t = 1, 2, 3, \ldots, N_t$ denotes the time horizons which normally divide a day into $N_t$ hours |
| $g$    | $g = 1, 2, 3, \ldots, N_g$ represents the generation units |
| $k$    | $k$th contingency |
| $C^V_g$, $C^F_g$ | Variable and fixed cost of unit $g$ |
| $C^SU_g$, $C^SD_g$ | Start-up cost and shut-down cost of unit $g$ |
| $P^\text{min}_g$, $P^\text{max}_g$ | Minimum and maximum generation limits of unit $g$ |
| $T^\text{ON}_g$, $T^\text{OFF}_g$ | Minimum up and down time of unit $g$ |
| $N_B$  | Set of node buses |
| $m$, $n$ | Indexes of AC bus |

Conductance and susceptance between bus $m$ and $n$ |

Reactance between bus $m$ and $n$ |

Minimum and maximum voltage magnitude limits at bus $m$ |

Nominal voltage of system voltage |

Minimum and maximum power transfer limits between bus $m$ and $n$ |

Total active and reactive power demands in time horizon $t$ |

Reserve requirement in time horizon $t$ |

Small signal stability margin threshold |

Transient stability margin threshold |

Maximum limit of rate of change of frequency (RoCoF) |

Dead band of the governors |

System nominal frequency |

Specified minimum low frequency bound |

Minimum allowed negative frequency derivation.

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Industry and academia for decades. Optimal unit commitment systems, which draws considerable attention in both the power economic operation is one of the major topics in power systems. Renewable energy sources and the large-scale integration of renewable energies, make the power system hard to be controlled and more vulnerable to unintended disturbances [10], [11]. In other

\[ v_g \] is the maximum ramp rate of governor at unit \( g \).

\[ H_g \] Inertia constant of unit \( g \)

\[ M \] Total generator inertia

\[ D_L \] Load damping factor

\[ K_g \] Mechanical power gain factor of unit \( g \)

\[ P_{gmx} \] Pre-determined generation capacity of the largest unit

**B. VARIABLES**

\[ u_{gt} \] Binary variable for on/off status of unit \( g \) in time horizon \( t \)

\[ y_{gt}, z_{gt} \] Binary variables for start-up / shut-down status of unit \( g \) at the beginning of time horizon \( t \)

\[ P_{gt}, X_{ON}^{gt}, X_{OFF}^{gt} \] Active power out of unit \( g \) at time horizon \( t \)

\[ P_{mn}, X_{ON}^{mn}, X_{OFF}^{mn} \] Active power transferred between bus \( m \) and \( n \) in time horizon \( t \)

\[ \theta_{mnt} \] Voltage angle difference between bus \( m \) and \( n \) in time horizon \( t \)

\[ V_{mt}, \theta_{mt} \] Voltage magnitude and angle of bus \( m \) and \( n \) in time horizon \( t \)

\[ D_R \] Small signal stability margin in time horizon \( t \)

\[ CTS \] Set of contingencies for transient stability

\[ \tau_i^k \] Coefficient of dynamic security region for contingency \( k \)

\[ CVS \] Critical cut set for static voltage stability

\[ CVS(k) \] Set of branches for cut-set \( k \)

\[ P_{mn}^k \] Coefficient of cut-set voltage stability region for cut-set \( k \)

\[ H_{ik} \] Post-contingency system inertia that consists of total inertia of the remaining units in time horizon \( t \) for contingency \( k \)

\[ \Delta P_{ik} \] Generation loss due to contingency \( k \)

\[ R_{gtk} \] Primary reserve of unit \( g \) at time horizon \( t \) for contingency \( k \)

\[ p_{w}^t \] Positive secondary reserve of unit \( g \) during time horizon \( t \) in the scenario \( w \)

\[ f_{rec} \] System nominal frequency after starting frequency regulation reserve

\[ \eta^t \] Load frequency sensitivity index in time horizon \( t \)

\[ FRR^t \] Frequency regulation reserve in time horizon \( t \)

\[ \phi^0 \] Post fault stable equilibrium point of each line

\[ \phi^u \] Unstable equilibrium point of each line

**FIGURE 1.** Classification of power system stability [9].

**B. POWER SYSTEM STABILITY**

As another important issue of power system operation, power system stability has been widely studied since the 1920s [5]. Due to the fast development of society and increasing requirement of sustainability, the modern power system becomes extremely large and complicated. Major blackouts with tremendous loss both physically and economically have dedicated the power system stability issue to be a prominent research aspect in decades [6]–[8].

To account for various forms of instability phenomena, power system stability issues, as illustrated in **FIGURE 1**, are classified as three major categories of stability problems [9]: 1) voltage stability that refers to the ability to maintain steady voltage at all buses under a disturbance; 2) frequency stability that accounts for the ability to maintain steady frequency under a contingency which induces an imbalance between power generation and demand; and 3) rotor angle stability which concerns about the ability to remain synchronism under a small disturbance (small signal stability) or a severe disturbance (transient stability).

To meet the roaring power demand and environmental requirements on sustainability, the power system evolves fast in both quantity and scale. Nowadays, the modern power system has become unprecedentedly large, complicated, and fragile. New challenges such as the phasing out of the traditional sources and the large-scale integration of renewable energies, make the power system hard to be controlled and more vulnerable to unintended disturbances [10], [11]. In other
words, the modern power system encounters more stability threats owing to the newly introduced labile power sources and variable operation conditions [12]–[14]. Consequently, the power system stability issues again become prominent.

C. RELATION BETWEEN POWER SYSTEM SCHEDULING AND POWER SYSTEM STABILITY

Power system scheduling and power system stability are generally treated as separated research topics. The former cares more about the economic operation in the steady-state condition, while the latter focuses on the secure and stable operation of the power system in the dynamic condition. Both problems have been extensively studied but in their respective fields [15], [16].

Power system scheduling seeks a feasible solution of arranging the generation units according to the practical operational limitations and aims to achieve an optimal economic objective. In some cases, the optimal solution may not always facilitate power system stability, or even violates the rules of stability, which may incur the collapse of the power system or other severe consequences [17].

Meanwhile, large-scale interconnection also makes the power system much more complicated and fragile, especially the integration of renewable energies in recent years [18], [19]. The improper scheduling of the power system possesses a threat to power system stability [20], [21]. This is because the power system scheduling problem usually considers the steady-state constraints and physical operational possibility. Whereas the power system stability, under different categories of stability, is largely associated with dynamic constraints that cannot always be considered in the scheduling problem and thus may lead to insufficient stability margin of the power system.

On the other hand, power systems traditionally hold simple and conservative stability margin limitation, which is elaborately designed to cover the worst-case situation. Given the significant amount of renewable energy in the future power system, the operational conditions will become much more divert and extreme. If the power systems are maintained at an excess level of stability margin, this may lead to extremely high operation and/or investment cost and at the same time limit its capability to accommodate renewable energy resources.

D. STABILITY CONSTRAINED POWER SYSTEM SCHEDULING IN FUTURE LOW-INERTIA POWER SYSTEM

High penetration of renewable energy and power electronic domination are two important characteristics in the future power system [22]. Such new characteristics have imposed profound challenges on both power system stability and scheduling.

The traditional large-inertia power system is dominated by conventional synchronous generators that serve as an important buffer under uncertain disturbances or contingencies. With the significant penetration of renewable energy, especially its replacement of conventional synchronous generator, the rotational inertia sources in the power system are now gradually losing the momentum in providing sufficient stability margins [23], [24]. Such stability requirements were normally not considered in power system scheduling.

Specifically, the physical characteristics of grid-connected renewable sources are substantially distinguished from that of conventional synchronous generators. As such, their interaction with the power grid is quite complex and different as well [25]–[27]. The large-mass rotating parts of the synchronous generators could act as a power buffer and contribute large inertia to the power system. Auxiliary damping devices or controllers can therefore provide sufficient damping support, which bolsters the power system oscillation stability [5]. However, it is not the case for grid-connected renewable energy. For instance, once there is a disturbance or perturbation, regarding the low-inertia nature, power electronic converters cannot slow down the power oscillation or imbalance, and thus there is very limited time for operators or controllers to take action [28]. From the perspective of power system stability, the stability margin will deteriorate with the increasing integration of low-inertia power sources. This may jeopardize the feasibility of existing power system scheduling methods, especially in the cases of high penetration of renewable energy. Therefore, there is an urgent need to take the stability constraints into account in power system scheduling for future power systems.

II. UC MODELS AND RELEVANT METHODS

Power system scheduling consists of two levels of economic management on generation units: UC level which gives the commitment solution of generating units at the optimal (minimal) operating costs with satisfied technical constraints; and the economic dispatch (ED) level that figures out the exact power output of each unit to minimize the total cost while fully supplying the power demand and complying with the transmission network constraints in every time horizon. Usually, UC and ED are considered in the same UC program, and thus it is also known as network constrained unit commitment (NCUC) [1]. Additionally, to ensure the power system with the predetermined scheduling plan to operate under a significant disturbance (e.g. the tripping of a large generating unit), security constraints are also taking into account, and the UC problem can thus be defined as the security-constrained unit commitment (SCUC) [29].

A. REGULAR UC MODELS AND GENERAL CONSTRAINTS

The objective of SCUC for a daily-ahead UC is to minimize the total generation costs as below

$$\text{Min} \sum_{i=1}^{N_i} \sum_{g=1}^{N_g} (C^V_g p_{gt} + C^F_g u_{gt} + C^{SU}_g y_{gt} + C^{SD}_g z_{gt})$$

(1)

The general operational constraints are depicted as follows:

a) Generation limits

$$p_{gmin} \leq p_{gt} \leq p_{gmax}, \forall t$$

(2)
b) Spinning reserve limits

$$\sum_{g} R_{gt} u_{gt} \geq R_{t}^{\delta}, \forall t$$ (3)

c) Ramping limits

$$RD_{gt} u_{gt} \leq P_{gt+1} - P_{gt} \leq RU_{gt} u_{gt}, \forall g, \forall t$$
$$SD_{gt} u_{gt} \leq P_{gt+1} - P_{gt} \leq SU_{gt} u_{gt},$$ (4)

d) Minimum up and minimum down time limits

$$\left\{ \begin{array}{l}
(X_{g}^{ON} - T_{g}^{ON})(u_{gt} - u_{gt+1}) \geq 0, \forall g, \forall t \\
(X_{g}^{OFF} - T_{g}^{OFF})(u_{gt} - u_{gt+1}) \geq 0, \forall g, \forall t
\end{array} \right.$$ (5)

e) AC power balance constraints

$$\sum_{g=1}^{N_g} P_{gt} u_{gt} = P_{Dt} + V_{mt} \sum_{n=m} V_{nt}$$
$$\times (G_{mn} \cos \theta_{mnt} + B_{mn} \sin \theta_{mnt})$$
$$\sum_{g=1}^{N_g} Q_{gt} u_{gt} = Q_{Dt} + V_{mt} \sum_{n=m} V_{nt}$$
$$\times (G_{mn} \sin \theta_{mnt} - B_{mn} \cos \theta_{mnt})$$
$$\forall m, n \in N_B, \forall t$$ (6)

f) Transmission constraints

$$PT_{mnt} = V_{mt} V_{nt} \sin \theta_{mnt} / X_{mn}$$
$$PT_{mnt} \leq PT_{mnt} \leq PT_{max}^{\delta} \forall m, n \in N_B, \forall t$$ (7)

e) Bus voltage constraints

$$V_{m}^{min} \leq V_{mt} \leq V_{m}^{max} \forall m \in N_B, \forall t$$ (8)

**B. STATE-OF-ART OF GENERATION SCHEDULING METHODS**

With lots of researchers digging into this field, a large variety of scheduling methods have been proposed to solve UC problems. The current methods on UC can be classified into two main clusters: 1) direct methods that give direct solutions with resolving techniques, and 2) heuristic methods that employ artificial intelligence or heuristic natural rules.

Representative works are listed in Table 1. Because the main concern of this paper is to review stability-constrained UC problems, the details of these methods for traditional UC are not discussed here.

**III. STABILITY CONSTRAINTS AND FORMULATION METHODS**

To include the constraints of power system stability, the stability constraints should first be determined via suitable derivation of stability analysis and simplification. To cover various forms of stability issues (as illustrated in FIGURE 1), different stability constraints are formulated to tackle the corresponding categories of stability.

**A. VOLTAGE STABILITY CONSTRAINT**

To ensure voltage stability, static voltage constraints and dynamic constraints are proposed in various references. The most common static voltage constraints are the bus voltage constraints in (8) that limit the bus voltage with high and low boundaries [16], [40], [72]–[74].

A voltage derivation index is proposed to examine the voltage profile and stability [75], [76], as follows

$$M_i = \frac{(V_i - V_i^{min})(V_i^{max} - V_i)}{(V_{nom} - V_i^{min})(V_i^{max} - V_{nom})}, i \in N_B$$ (9)

From (9), it is easy to find that the voltage derivation index is one if bus voltage equals to its nominal voltage, while the index equals to zero when bus voltage operates at any of its boundaries in (8). Therefore, the larger the voltage index is, the better the voltage stability.

To further quantify the overall voltage performance, an average voltage derivation index is defined as

$$V_{ind} = \frac{1}{N_B} \sum_{i=1}^{N_B} M_i = \frac{1}{N_B} \sum_{i=1}^{N_B} \frac{(V_i - V_i^{min})(V_i^{max} - V_i)}{(V_{nom} - V_i^{min})(V_i^{max} - V_{nom})}$$ (10)

The average voltage index can be used to a static voltage stability constraint that should be maximized, viz, near to one. Meanwhile, a maximum power transmission index considering voltage stability is defined to guarantee less than 80% maximum power transmission capability, demonstrated as

$$\frac{V_{nt} V_{mt}}{X_{mn}} \sin \delta_{mnt} \leq 0.8 PT_{mnt}^{\delta}, \forall m, n \in N_B, \forall t$$ (11)

A static voltage stability margin concerning load impact is proposed in [77], in which the continuation power flow is employed to include the incremental variations in loads and
generations.
\[
\lambda_{im} = \frac{P_{\text{max}} - P_0}{P_0} \times 100\% \tag{12}
\]
where \(P_{\text{max}}\) and \(P_0\) are the maximum power and actual active power at an operation point. \(\lambda_{im}\) is the load margin that indicates the static voltage stability. A larger load margin means a better stability margin and vice versa.

Another static voltage stability constraint based on the cut-set voltage stability region [78] is proposed to deal with static voltage constraint
\[
\sum_{\forall m \in \text{CVS}(k)} \beta_{mn} P_{mnt} \leq 1, \quad \forall k \in \text{CVS}, \forall t \tag{13}
\]
According to the (P, Q)-V characteristics of the multi-bus system, a voltage constraint in [16], [79], [80] is proposed to constrain the power variations as
\[
0.3p.u. \leq \{\Delta P(t), \Delta Q(t)\}, \forall t \tag{14}
\]
where \(\Delta P\) and \(\Delta Q\) are voltage stability indexes defined in [81].

Also, a voltage stability margin of the average index \(V_{asi}\) is proposed in [82],
\[
V_{asi} = (V_{azi} + V_{psi} + V_{hai})/3 \tag{15}
\]
where \(V_{azi}, V_{psi}\) and \(V_{hai}\) are equivalent impedance voltage, equivalent load, and angle index voltage, respectively.

Besides, dynamic voltage stability constraints are also drawn attention to accommodate the dynamic requirements in bus voltage. A voltage stability constraint considering the effect of the tap-changing transformer by linearizing the reactive power generation limits is proposed in [40]
\[
\Delta V^\text{min} \leq \Delta V \leq \Delta V^\text{max} \tag{16}
\]
where \(\Delta V = V^* - V^0\) is voltage shift from system voltage \(V^0\) to the desired voltage \(V^*\), \(\Delta V^\text{min}\) and \(\Delta V^\text{max}\) are the low and high boundaries of voltage shifts.

In addition to load impact, another dynamic voltage stability index considering reactive power is proposed in [83]
\[
L_{mnt} = \frac{4X_{mnt} Q_{mnt}}{V_{mt} \sin(\theta_{mnt} - \theta_{mt} + \delta_{mt})} \tag{17}
\]
If the index \(L_{mnt}\) is close to 0, branch line \(mn\) has sufficient voltage stability margin. Whereas if \(L_{mnt}\) is very close to 1, the eigenvalues related to this branch line become positive which indicates the bus voltage is unstable.

Moreover, a numerical method in [84] using polar coordinate optimal multiplier load flow is proposed to give precise voltage stability margin calculations.

**B. FREQUENCY STABILITY CONSTRAINT**

The rate of change of frequency (RoCoF) is a strong limit in power system operation and is extensively adopted in UC programs [85]–[88].

\[
\text{RoCoF} = \frac{d\Delta f(t)}{dt} = \frac{\Delta P_{\text{ik}}}{2H_k} \leq \text{RoCoF}^\text{max}, \quad \forall t, \forall k \tag{18}
\]
Together with RoCoF, the frequency nadir is also a common frequency constraint, described as
\[
R_{\text{gt}} \leq 2v_g H_k (f^0 - f_{\text{min}} - f_{\text{db}}) / \Delta P_{\text{ik}}' , \forall g, \forall t, \forall k \tag{19}
\]

The first equation in (19) ensures that the primary reserve of each unit is utilized at or before the frequency nadir occurs, while the second guarantees that all the remaining units are capable to compensate for the power balance.

In [89], a linear equation is used to characterize the frequency nadir assuming that RoCoF imposes a direct influence on the frequency nadir during the first seconds after a power disturbance, demonstrated as
\[
\Delta f = a\cdot \text{RoCoF} + b \tag{20}
\]
where \(\Delta f = f_0 - f_{\text{min}}\) and \(a, b\) are constant parameters.

Based on (18) and (20), a linear frequency expression is obtained as
\[
\Delta f (\Delta P_{\text{ik}}') = a \cdot \frac{\Delta P_{\text{ik}}'}{H_k} + b, \forall t, \forall k \tag{21}
\]
where \(a' = a f_0/2\).

Hence, a robust frequency constraint is defined as
\[
P_{gt} + \mu_{gt} \leq \frac{\Delta f_{\text{UFLS}} - b}{a'} \sum_{j \neq g} u_{jt} H_j, \forall g, \forall t \tag{22}
\]
A quasi steady-state constraint is proposed in [87] and [88] to satisfy the requirement of primary frequency response
\[
\left| \Delta f^\text{ss} \right| = \left\| \frac{\Delta P_{\text{max}} - R}{D \cdot P_D^0} \right\| \leq \Delta f_{\text{max}}^{\text{ss}} \tag{23}
\]
where \(\Delta P_{\text{max}}\) represents the loss of the largest unit, \(R\) is the total frequency response provision, \(D\) is the load damping rate (1/Hz), \(P_D^0\) is the total demand, and \(\Delta f_{\text{max}}^{\text{ss}}\) is the maximum derivation allowed at the steady-state.

Furthermore, with the aid of a simulation-based calculation method, a predefined frequency stability margin is proposed to avoid an adverse frequency nadir [90], which is defined as
\[
f_t \geq f_{\text{pre-defined}, t}, \quad \forall t \tag{24}
\]
where \(f_t\) and \(f_{\text{pre-defined}, t}\) are the actual system frequency and pre-defined minimum system frequency during period \(t\), respectively.

In [91], the primary frequency regulation constraint is included in the upper bound of the maximum generation of units, which equals to either unit frequency regulation limit \(P_{gt} + u_{gt} R_g^{pr-\text{max}}\) or generation capacity limit \(P_{\text{max}}\), whichever is smaller, defined as
\[
\hat{f}_{\text{max}} = \min \left\{ u_{gt} P_{\text{max}} - P_{gt} + u_{gt} R_g^{pr-\text{max}} g(\forall t, \forall g) \right\} \tag{25}
\]
where \(R_g^{pr-\text{max}}\) is the primary frequency regulation reserve, which is given by
\[
R_g^{pr-\text{max}} = \begin{cases} -u_{gt} D_g f_{gt}, & \text{if } 0 \geq f_{gt} \geq f_{gt}, \\ 0, & \text{if } f_{gt} \leq \Delta f_{gt}, \forall t, \forall g \end{cases} \tag{26}
\]
It is worth mentioning that the frequency deviations in (26) must be limited to prevent load shedding by under frequency relays, i.e.

$$0 \geq \Delta f_i \geq \Delta f^\text{min}, \forall t$$  \hspace{1cm} (27)

Since frequency stability is largely contingency-based, minimum frequency constraints can be formulated with sufficient conditions that combine both system inertia and synchronous generator dynamics. Frequency containment reserve and frequency restoration reserve requirements are set up as sufficient conditions to constrain frequency [2], in which the constraints as defined as

$$E^{\text{res}}_{i,k,m} + \Delta E^{\text{rot}}_{i,k} \geq E^{\text{con}}_{i,k,m}$$

$$\sum_{j \in J} (1 - I_{j,k}^r) r_{j,k}^{\text{FCR}} \geq \sum_{j \in J} I_{j,k}^r P_{j,k}$$  \hspace{1cm} (28)

where $E^{\text{res}}_{i,k,m}$ is the energy provided by frequency containment reserve in time step $k$, $\Delta E^{\text{rot}}_{i,k}$ is the rotational energy from the system inertia in time step $k$, $E^{\text{con}}_{i,k,m}$ is the energy loss in time step $k$ during contingency $l$ at discretization point $m$, $r_{j,k}^{\text{FCR}}$ is the frequency containment reserve.

Frequency stability constraints are nonlinear constraints that are hard to be handled in MILP. Therefore, a piecewise linearization (PWL) technique is introduced based on the equivalent system frequency response model [92]. The minimum and maximum values of the variables of the model are demonstrated as

$$\min_{i \in N_t} \left\{ \frac{K_i F_i}{R_i} \right\} \leq \hat{F}_i \leq \sum_{i \in N_t} \frac{K_i F_i}{R_i}, \forall t \in N_t$$  \hspace{1cm} (29)

$$\min_{i \in N_t} \left\{ \frac{K_i}{R_i} \right\} \leq \hat{R}_i \leq \sum_{i \in N_t} \frac{K_i}{R_i}, \forall t \in N_t$$

$$\min_{i \in N_t} (2H_i) \leq \hat{M}_i \leq \sum_{i \in N_t} 2H_i, \forall t \in N_t$$  \hspace{1cm} (30)

where $\hat{F}_i$, $\hat{R}_i$, $\hat{M}_i$ are the independent variables of units.

With the PWL technique, the minimum frequency constraint is represented as

$$f^\text{min} \leq f_0 + f_0 \Delta \omega(t^r)$$

$$\Delta \omega(t^r) = -\frac{\Delta P}{R_T + D_L} (1 + e^{-\xi \omega_n t^r}) \sqrt{\frac{T(R_T - F_T)}{M}}$$  \hspace{1cm} (31)

where $f_0$ is the steady frequency right before the contingency happens, $t^r$ is the time at which $f^\text{min}$ occurs, and $\Delta \omega(t^r)$ is defined as

$$\omega_n = \frac{1}{MT} (D + R_T)$$

$$\xi = \frac{M + T(D + F_T)}{2 \sqrt{MT(D + R_T)}}$$

$$F_T = \sum_{g=1}^{N_g} \frac{K_i F_i}{R_i}$$

$$R_T = \sum_{g=1}^{N_g} \frac{K_i}{R_i}$$  \hspace{1cm} (32)

Combine equations (29), (30) and (31), the frequency constraint is mathematically derived and can be applied in the MILP program.

For an isolated power system, frequency regulation reserve (FRR) plays a crucial role in providing reserve within a very short time when a contingency occurs [93]. Hence, the FRR constraint and minimum FRR limits are given as

$$\begin{cases}
  f_{\text{rec}} = f_0 - \frac{P_{\text{gmx}} - \text{FRR}^r}{\eta^r P_D} \geq f^\text{min}, \forall t \\
  \text{FRR}^r \geq P_{\text{gmx}} - (f_0 - f^\text{min})\eta^r P_D, \forall t
\end{cases} \hspace{1cm} (33)
$$

Despite the defined frequency constraints, frequency control can also be considered as an alternative to improve the power system frequency stability in the UC problem [94]. An equivalent frequency regulation effort is defined as

$$\Delta \text{RR} = -K_f \Delta f$$

where $K_f$ is a regulation coefficient.

C. SMALL SIGNAL STABILITY CONSTRAINT

As a category of rotor angle stability, small signal stability considers the small disturbance conditions.

In [95], the critical eigenvalue constraint is proposed to ensure the small signal stability of the scheduled power system, defined as

$$\sigma_H \leq \sigma_R (\sigma_R = -0.15)$$  \hspace{1cm} (34)

where $\sigma_H$ is the real part of eigenvalue for the oscillation mode with the poorest damping for the present generation schedule at hour $H$, and $\sigma_R$ is the small-signal stability margin that prespecified based on system requirements and experiences. It is assumed that $\sigma_R = -0.15$ is suitable for a Taiwan power system.

The generation schedule in every time horizon ($H = 1 \sim 24$) should be checked to satisfy small-signal stability via Lyapunov methods. Otherwise, an area economic dispatch should be employed to modify the UC solution.

In [96] and [97], the normal vector method is employed to warrant small-signal stability by defining a robustness region, which is a finite neighborhood around the optimal operational condition. Both the continuous and Boolean parameters are included in a dynamic system as below

$$\begin{cases}
  \dot{x} = f(x, y, \alpha, s) \\
  0 = g(x, y, \alpha, s)
\end{cases} \hspace{1cm} (35)
$$

where $x, y, \alpha$, and $s$ are the state variables, algebraic variables, uncertain system parameters, and Boolean parameters, respectively. By linearizing (35), the Jacobian matrix can be reduced to

$$\tilde{f}_x = f_x - f_y \hat{g}_y^{-1} g_s$$  \hspace{1cm} (36)

Then eigenvalue analysis is performed to determine the small-signal stability of the system.
In [98], the negative impact of water network load is considered in the unit commitment by adding a load demand ratio constraint, as below
\[
\sum_{i \in I} d_i(t) \leq r(t)
\]  
where \( \sum_{i \in I} d_i(t) \) and \( D(t) \) are the total pumping load of water network loads and system load at time horizon \( t \), \( r \) is the maximum pumping power demand ratio, which is decreased in small steps (e.g., starts from 1 with a step length of 0.01) until both power flow and small-signal stability converge.

In [20], linearized small-signal stability constraints are derived, as below
\[
\begin{align*}
\Delta \lambda &= \Delta \sigma + j \Delta \omega = \psi_0^T \Delta A \varphi_0 \\
\Delta A &= \left[ \frac{\partial A}{\partial U} \right]_{(b_0, U_0)} \Delta U + \left[ \frac{\partial A}{\partial \theta} \right]_{(b_0, U_0)} \Delta \theta \\
\Delta \xi &= (\sigma_0^2 + \omega_0^2)^{-3}/2 (-\omega_0^2 \Delta \sigma + \sigma_0 \omega_0 \Delta \omega) \\
\Delta \xi &\geq \xi_T - \xi_0
\end{align*}
\]
where \( \lambda = \sigma + j \omega \) is the eigenvalue of oscillation mode with the worst damping ratio \( \xi \), \( A \) is the state space matrix, \( \xi_T \) is the prespecified damping ratio threshold (e.g., 3\%~5\%), \( U \) and \( \theta \) are bus voltage magnitude and angle respectively, the prefix \( \Delta \) represents the linearized derivation of a variable subscript, and subscript 0 denotes the present state.

In [99], the transient behavior of the natural gas network is considered. The equivalent electrical analogy of natural gas pipelines is established in the second-order differential equations, which are hard to solve and time-consuming. This is simplified by transforming the Laplace equations from the frequency domain back to the time domain.

**D. TRANSIENT STABILITY CONSTRAINT**

As another important rotor angle stability, transient stability accounts for the ability to maintain stable rotor angles in a large disturbance. Generally, transient stability analysis can be employed through two major methods [100]: 1) numerical methods that solve the differential equations following a large disturbance and determine whether all synchronous generators can maintain synchronism; and 2) direct methods that first construct the Lyapunov energy function and then compare the critical energy and transient energy.

To cover potential large disturbance contingencies, a generic transient stability constraint is defined as [78]
\[
\sum_{g=1}^{N_g} \tau_g^k P_{g, \, \text{lt}} \leq 1, \quad \forall g, \forall t, \forall k \in CTS
\]

Despite the transient stability margin \( \tau_g^k \) in (39) being accurate, it is hard to be explicitly expressed while implementing UC. In [101], a simplified transient stability model is used to tackle the transient stability constraints, which describes the electro-mechanic transient of the synchronous generator as second-order differential equations
\[
\begin{align*}
\dot{\delta}_g &= \omega_0 \omega_g \\
\dot{\omega}_g &= (P_{Gg} - P_{eg} - D_g \omega_g)/M_g
\end{align*}
\]  
where \( \omega_0 \) is synchronous angular speed, \( \delta_g \) and \( \omega_g \) are the rotor angle and rotor speed of unit \( g \), \( P_{Gg} \) and \( P_{eg} \) are active power output and the electromagnetic power of unit \( g \) and \( M_g \) are the damping coefficient and inertia constant of unit \( g \).

By discretizing (40) with the implicit trapezoidal method, the transient stability constraints are derived by the following inequality constraints
\[
\begin{align*}
\delta_{\text{min}}^g &\leq \delta_{\text{COI}}^g - \delta_{\text{COI}}^0 \leq \delta_{\text{max}}^g \\
\delta_{\text{min}}^\text{ts} &\leq \delta_{\text{COI}}^\text{ts} - \delta_{\text{COI}}^0 \leq \delta_{\text{max}}^\text{ts} \\
\delta_{\text{COI}}^g &= \sum_{i=1}^{N_g} M_i \delta_{\text{COI}}^0 / \sum_{i=1}^{N_g} M_i \\
\delta_{\text{COI}}^\text{ts} &= \sum_{i=1}^{N_g} M_i \delta_{\text{COI}}^{\text{ts}} / \sum_{i=1}^{N_g} M_i
\end{align*}
\]
where \( \text{COI} \) is defined as the center of inertia.

In [102], the minimum accelerating power is set as the transient stability constraint, as below
\[
\eta_a = -P_a(\min(t_{\text{min}})) = -\min(P_a(t)) < 0, \forall t > t_{\text{cl}}
\]
where \( t_{\text{min}} \) denotes the moment that the accelerating power reached a minimum after fault clearance, while \( t_{\text{cl}} \) is the moment that the fault is cleared.

Transient instability usually occurs in the form of synchronization mismatch between different coherent groups of synchronous generators when the large disturbance contingency cannot meet the requirement of critical clearing time (CCT). Therefore, by comparing the transient energy function and the potential energy, the transient stability can be assessed and set as a constraint in UC [103, 104].

\[
E_{TS} \leq E_{cr}
\]
\[
E_{TS} = \frac{M_A M_B}{2(M_A + M_B)} (\omega_A - \omega_B)^2 + \sum_{m,n \in N_B} X_{mn} V_m V_n h \int_{\theta_{mn}^0}^{\theta_{mn}} (\sin \theta - \sin \theta_{mn}) d\theta
\]
\[
E_{cr} = \sum_{m,n \in N_B} X_{mn} V_m V_n \int_{\theta_{mn}^0}^{\theta_{mn}} (\sin \theta - \sin \theta_{mn}) d\theta
\]
where \( E_{TS} \) is the transient energy which consists of kinetic energy between two coherent groups A and B and penitential energy of transmission lines; \( E_{cr} \) is the critical energy of transmission lines when reaches the approximate unstable equilibrium points.

Time-domain simulation-based transient stability constraints are also proposed in [21], and the transient stability constraints are included below
\[
0 \leq \eta_{t,k} \leq \varepsilon, \forall t, \forall k
\]
where $\eta_{i,t}$ is a variable that is obtained from a rigorous time-domain simulation with a transient stability assessment procedure. The improved extended equal area criterion (EEAC) is used to investigate the transient stability via time-domain simulations.

While in [105] and [106], a more detailed framework to constrain transient stability is depicted in FIGURE 2. In the proposed framework, the results of SCUC are examined and shaped in the hybrid differential-algebraic equations (HDAE) based on the time domain simulation platform. This method can also be used to examine small signal stability since it is a simulation-based method.

In [107], a stability region is constructed with Lyapunov linearization techniques and optimal control methods. The objective stability margin is designed as

$$W_i(x) = -(x - x_i)^T Q_i (x - x_i)$$

(45)

where $Q_i = Q_c + P_i B_u R_c^{-1} B_u^T P_i$.

In [108], an objective function to maximize the coherency between the committed units is proposed to guarantee a transient stability margin. The coherency constraints are defined as

$$DV_{i,s} - 1 \leq L_{i,s} - L_{i,s}^{-1} \leq 1 - DV_{i,s}, \forall i \in \Omega_g, \forall t \in \Omega_T, \forall s \in \Omega_s$$

$$0 \leq L_{i,s} \leq DV_{i,s}, \forall i \in \Omega_g, \forall t \in \Omega_T, \forall s \in \Omega_s$$

$$\sum_{s \in \Omega_s} DV_{i,s} = 1, \forall i \in \Omega_g, \forall t \in \Omega_T$$

$$CED_{i,s}^f = L_{i,s}^f \cdot SED_{i,s}^f, \forall i \in \Omega_g, \forall t \in \Omega_T, \forall s \in \Omega_s$$

(46)

and the objective function is

$$CCF = \sum_{r \in \Omega_T} \sum_{i \in \Omega_g} \sum_{s \in \Omega_s} CED_{i,s}^f, \forall i \in \Omega_g, \forall t \in \Omega_T, \forall s \in \Omega_s$$

(47)

IV. REPRESENTATIVE WORKS PERTAIN TO STABILITY CONSTRAINED UNIT COMMITMENT

Compared with the static security constraints in unit commitment, stability constraints are dynamic, which assure the stable operation of the power system under the scheduled generation plan. The main concern is to examine the feasibility of the unit commitment solution. Since stability constraints are used to impose restrictions on dynamic state variables of the power system, it is not easy to set clear constraints and applied them in the regular unit commitment program. In this section, typical examples of four major stability constrained unit commitment are discussed.

A. VOLTAGE STABILITY CONSTRAINED UNIT COMMITMENT

Voltage stability is the ability to maintain bus voltages after being subjected to a disturbance and is closely related to the equilibrium between load demand and load supply.

Simple voltage constraints such as high and low voltage references are conservative network constraints that satisfy network static security requirements and cannot reveal or assure the voltage stability conditions. A precise voltage stability margin based on the polar co-ordinate optimal multiplier load flow method and the continuation method is proposed for unit commitment.

A widely used predictor-corrector type continuation method is implemented to calculate the voltage stability margin. For a scheduled generation plan, the voltage stability margin can be defined as the margin from the current base load to the extreme load condition. The voltage stability margin is defined as

$$L_v(t) = L_{0d} + t_L L_d$$

(48)

where $L_{0d}$ is the basic load/generation and $L_d$ is the load/generation pattern. $t_L$ represent the demand growth.

If the voltage stability margin is violated, a penalty cost is added to the transition constraints and affects the unit commitment solution. With dynamic regulations using a proper penalty cost, the voltage stability margin can be met in the final schedule. The overall procedure of unit commitment is demonstrated in FIGURE 3.

A representative reference in [84] has adopted the optimal multiplier continuation method that is efficient in calculating the voltage security margin. On this basis, a transition constraint regarding voltage stability is defined and included in the dynamic programming process while determining the short-term generation scheduling.

B. FREQUENCY STABILITY CONSTRAINED UNIT COMMITMENT

Frequency stability largely relies on the ability to recovery from the frequency nadir after a frequency contingency such as large disturbance or load variation.

The frequency nadir is closely related to system inertia, damping, and load, which is included as

$$\frac{H}{f_0} - \frac{R_s T_s}{4\Delta f_{\text{max}}} R_G \geq \alpha - \beta P_D$$

(49)
where
\[
\alpha = \frac{(P_L - R_s)^2 T_g}{4\Delta f_{\text{max}}}, \quad \beta = \frac{(P_L - R_s) T_g D}{4}
\]  (50)

A linearization technique is used to linearize the above nonlinear constraints by dividing them into two continuous variables on the left and quartic term on the right.

By using the standard big-M technique [109], the linearized frequency nadir constraint is expressed as
\[
\sum_{l \in \mathcal{L}} m_l i_l^2 - \frac{T_s}{4\Delta f_{\text{max}}} \sum_{l \in \mathcal{L}} k_l i_l^4 - a_P P_L + b_P R_S + c_P - \frac{(P_L - R_S) T_g D}{4} P_D \geq 0
\]  (51)

Therefore, the nonlinear frequency stability constraint, which contains both system parameters and load conditions, is transformed into linear constraints that can be handled in the MILP program.

A typical example in [86] has demonstrated how to simplify the nonlinear frequency stability constraints and integrate them into the unit commitment model.

### C. SMALL SIGNAL STABILITY CONSTRAINED UNIT COMMITMENT

Small signal stability is the ability to maintain stability after being subjected to a small disturbance and mainly focuses on the synchronism of synchronous generators. It can be evaluated by small signal stability analysis via linearization techniques.

Small signal stability analysis is to assess whether a specific operational condition is robust and stable when a small disturbance occurs. Generally, there are two methods to check small signal stability: 1) time domain simulation that induce a small disturbance and examine whether oscillation can be suppressed and back to the original steady-state; or 2) frequency domain analysis in which the state-space equations are established and modal analysis are conducted to identify critical oscillation modes. The second method can clearly reveal the root cause of oscillation and help design control strategies, and thus is widely used for small signal stability analysis.

The overall procedure is illustrated in FIGURE 4.

In [95], the small-signal stability constraint is calculated by the second method. Eigenvalue analysis is carried out to identify the critical mode (i.e. the worst damped mode), and the stability criterion in (34) is used for stability check. If the constraint in (34) is not met, then the present generation schedule needs to be modified by reducing the inter-area line flow until the stability constraint is satisfied.
D. TRANSIENT STABILITY CONSTRAINED UNIT COMMITMENT

Transient stability constraints aim to assure the scheduled generation plan can endure transient stability under certain large disturbance contingencies. A classical criterion to quantify transient stability is the equal area criterion (EAC) by calculating the accelerating area and decelerating area in a large disturbance. An improved EAC, i.e. extended equal area criterion (EEAC) is proposed to transform the multimachine system to an equivalent single machine infinite bus system and provide a quantitative criterion to determine transient stability [110]. EEAC can be integrated into time domain simulations and used to calculate the transient stability margin $\eta_t$ [111].

If the transient stability margin is not sufficient, the power shift between critical machines (CMs) and non-critical machines (NMs) should be deployed to increase the decelerating area. The overall procedure is illustrated in FIGURE 5.

As a fine example in [21], the improved EEAC is used to investigate the transient stability by transforming the multimachine trajectories to an equivalent one machine infinite bus trajectory. On this basis, EAC is feasible to employ on the equivalent one machine infinite bus system. A decomposition strategy is then implemented by solving the master problem (i.e. unit commitment) while checking the slave problem (i.e. transient stability constraint).

According to the representative works for stability constrained unit commitment, the major constraints and the corresponding methodology are compared and summarized in Table 2.

V. SUMMARY

Due to the rapid growth in energy demand and mismatched infrastructure development in power systems, the replacement of traditional generation units with low-inertia power sources has introduced control problems as well as deteriorated the inertia response in the power system. As a result, the stability conditions have become more severe since the power system may have to operate near its stability limit under the complex operational conditions and rigid physical constraints. Traditional UC programs cannot guarantee power system stability and thus more stability constraints should be considered while performing power system scheduling. In this paper, four typical stability constrained UC problems are reviewed and summarized to present the state of the art for stability constrained power system scheduling.

Furthermore, with the current trend of power electronics domination and large scale integration of renewable energy in modern power systems [22], more uncertainties have
been imported and threatened power system stability. New classifications of stability issues such as resonance stability and converter-driven stability have been drawn attention recently [112] and may become a prominent stability bottleneck in future power system scheduling. To this end, the corresponding stability constraints have not been clearly defined and may become a research focus in prospective stability constrained UC programs.

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