Distributed optomechanical fiber sensing based on serrodyne analysis: supplement

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1. THEORETICAL MODEL

The purpose of this section is to provide the detailed derivation yielding equations (2) and (3) in section 2. of the main text. The model developed here relies on the theory of linear elasticity [1, 2]. More specifically, we study the behavior of a linear and homogenous cylinder subject to viscous damping under harmonic activation. We first start by analyzing the behavior of a homogeneous linear cylindrical cavity in driven regime, then proceed to discuss the free-running case. The governing equation reads [3]:

\[
\mu \Delta U + (\lambda + \mu) \nabla (\nabla \cdot U) - \rho \frac{\partial^2 U}{\partial t^2} - 2\rho \Gamma \frac{\partial U}{\partial t} = D
\]  
(S1)

where \(\lambda\) and \(\mu\) are the first and second Lamé parameters, respectively, \(U\) is the displacement field and \(\rho\) is the material density. The term \(2\rho \Gamma \frac{\partial U}{\partial t}\) is a fictitious term accounting for acoustic dissipation [1], hence \(\Gamma\) is the acoustic damping rate. Formally speaking, acoustic dissipation in forward stimulated Brillouin scattering (FSBS) mostly originates from radiation loss at the fiber-environment boundary, hence a fully rigorous study would require to account for it by setting appropriate boundary conditions. This has recently been addressed in order to study the modal structure of a clad fiber surrounded by a fluid and precisely identify the cutoff frequencies and damping rate of the different modes under consideration [4]. Our approach is complementary, as we a priori account for internal dissipation in order to derive analytically the time-varying response of the system. The righthand-side of equation (S1) is a source term representing the contribution of the activating pulse. It is assumed to take the form

\[
D = -A_d j_1(\alpha r) \cos(\omega_d t - \theta) \hat{r}
\]  
(S2)

where \(r\) is the radial coordinate, \(\omega_d = 2\pi f_d\) and \(f_d\) is the driving frequency, \(\hat{r}\) is a unit vector oriented along the \(r\)-axis and the minus sign is introduced for later convenience. The amplitude of displacement \(A_d\) as a function of activating power is quite cumbersome to evaluate, as it depends on the spatial overlap between the acoustic mode profile and the optical mode profile [3, 4]. Its exact value or expression is however not relevant to the analysis developed here, hence we consider it as a constant. The radial function \(j_1(\alpha r)\) is assumed to automatically match the displacement profile of the targeted purely radial acoustic mode [5, 6]. Note that this is entirely valid in bare fibers only, the complete displacement profile in clad fibers exhibiting in addition a different form in the cladding. The contribution of the cladding to the overall model is neglected, as it would significantly complicate the analytical derivation performed here due to the complex coupling in terms of stress and displacement at the cladding-coating interface. The coating is however believed to act as a minor contribution to the overall behavior of the system, as attested by the excellent matching between numerical simulations based on the model developed here and experimentally acquired data (see main text). The steady-state response of the cavity \(U_d\) to the harmonic driving (S2) is assumed to take the following form

\[
U_d = A_d j_1(\alpha r) \cos(\omega_d t - \theta) \hat{r}
\]  
(S3)

where \(A\) and \(\theta\) read

\[
A = \frac{A_d}{\sqrt{(\omega_0^2 - \omega_d^2)^2 + 4\Gamma^2 \omega_d^2}}
\]  
(S4)

\[
\theta = \tan^{-1} \left( \frac{2\omega_d \Gamma}{\omega_0^2 - \omega_d^2} \right)
\]  
(S5)
where \( \omega_0 = \alpha V_d \) and \( A \) and \( \theta \) are illustrated in Fig S1, respectively. Note that these solutions correspond to the response of a simple forced damped harmonic oscillator. The magnitude response is close to a Lorentzian, although it is asymmetric. It peaks at a frequency slightly lower than the oscillator free running frequency \( \omega_0/2\pi \) yet this shift is negligible here. The phase response transits from 0 when \( f_d \ll f_0 \), that is driving and response are in phase, to \( \pi \) when \( f_d \gg f_0 \), that is driving and response are out of phase. At resonance, that is when \( f_d = f_0 \), activation and response are phase shifted by \( \pi/2 \). The response of the free-running cavity immediately after the trailing edge of the activating pulse assumes the form

\[
U = A_m l^1(\omega_m t - \theta_m)e^{-\Gamma \hat{t}}
\]

where \( A_m \) is the amplitude of the \( m^{th} \) mode and \( \theta_m \) designates its initial phase. In the considered case, the spectral purity of the activating pulse and the frequency spacing between adjacent modes ensures that the driven oscillations take place entirely along a single acoustic mode. It is crucial to emphasize here that although the acoustic cavity was activated at the driving frequency \( \omega_d \), the oscillation frequency of the free-running cavity (S6) is entirely determined by the mechanical and geometrical parameters of the fiber. Therefore, even when driven far from its resonant frequency, the free running cavity will always oscillate at the same frequency. This implies that non-uniformity in the fiber cladding diameter, acoustic-damping or density – due e.g. to strain or temperature – will result in different resonance frequencies along the fiber.

2. EXPERIMENTAL IMPLEMENTATION

The experimental setup is depicted in Fig S2. Two lasers operating at ~1550 nm while being spectrally separated by ~3 nm are respectively used for FSBS activation (orange path) and reading (red path). On the activating branch, a phase electro-optic modulator (\( \Phi \)-EOM) driven by an arbitrary waveform generator (AWG 1) programmed with a pseudo-random bit sequence dithers the phase of the light output from laser 1, thus raising the SBS threshold [7] and enabling generating a high-energy activating pulse free of any SBS induced distortion. AWG 2 delivers a phase-controlled sinusoidal pattern at a frequency \( f_d \) to an intensity electro-optic modulator (I-EOM 1) that shapes the continuous-wave (CW) light accordingly. AWG 3 then feeds I-EOM 2 with a pre-distorted gating signal designed to anticipate distortions due to the gain saturation of the erbium doped fiber amplifier (EDFA 1). The generated activating pulses exhibit a uniform power distribution of several W over a duration of 1 \( \mu \)s, ensuring steady-state activation of acoustic waves in the fiber under test (FUT) once launched into the reading path through coupler 2.

The red path depicts a conventional Brillouin optical time-domain analyzer (BOTDA). The light from laser 2 is firstly split into pump (upper) and probe (lower) branches by coupler 1. On the pump branch, I-EOM 3 is driven by a pulse generator in order to deliver 4 ns optical pulses to EDFA 2 before combination with the activating pulse. Note that both pulses require synchronization as to precisely adjust the time delay \( \Delta t \), as depicted by the black line in the figure. Before being launched into the sensing fibre, both pulses pass through a polarization scrambler, which simultaneously averages out polarization fading effects occurring in the BOTDA while alleviating any potential contributions of radial-torsional acoustic modes [8]. On the probe branch, I-EOM 4 is DC biased at extinction and intensity modulated by a radio-frequency (RF) generator.
at a frequency $f_{RF}$ in order to deliver a carrier-suppressed double-sideband (CS-DSB) modulated wave. The resulting probe signal is then amplified by EDFA 3, which ensures reaching maximum signal-to-noise ratio (SNR) at detection [9]. After running through the sensing fibre and before detection by a 350 MHz photo-detector (PD) operating just below saturation, a narrow bandwidth fiber Bragg grating (FBG) operating in reflection is used to reject one of the probe sidebands and Rayleigh scatterings from both activating and reading pulses.

**Fig. S2.** Simplified experimental setup showing the optical path of the activating pulse (orange) and the reading pulse (red). The abbreviations have the following meanings: AWG – arbitrary waveform generator, EOM – electro-optic modulator, EDFA – erbium doped fiber amplifier, RF – radio frequency, FUT – fiber under test, FBG – fiber Bragg grating, PD – photodetector.

### 3. ADDITIONAL RESULTS

In this section, we present additional results that complement the ones presented in the main text. First, we investigate further on the temporal response of harmonically activated forward stimulated Brillouin scattering (FSBS) by varying the time delay between the activating pulse and the reading pulse (see Fig.1 from main text). Finally, we address the problem of non-local effects arising from a too strong accumulated phase-modulation of the reading pulse.

**A. Time-delay and shape of the retrieved FSBS resonance**

As detailed in section 2. of the main text, the time delay $\Delta t$ between the activating and the reading is of uttermost importance due to the increasing phase shift $\theta$ of the cavity response with respect to the activation when the driving frequency $f_d$ scans over $f_0$ (S5). The experimental results presented in the main text are obtained when $\Delta t$ is set such that the reading pulse experiences a pure linear shift at resonance, i.e. when the amplitude response peaks. While this is required in order to retrieve a one-sided response $\Delta f$ with maximum amplitude, the time-delay $\Delta t$ might be seen as an additional degree of freedom. This is illustrated in Fig. S3, which depicts the measured FSBS response for different time delays $\Delta t$. The figure reproduces some of the features shown in Fig.1 of the main text, but focuses on the free-running oscillation. Notice for instance that the effect of acoustic damping is not shown here. The left-hand side of the figure shows four different reading pulses experiencing the free-running acoustic field from the cavity, while the right-hand side illustrate the corresponding retrieved responses. The red pulse (associated with the uppermost response), corresponds to the case presented in the main text, while subsequent pulses (by order blue, green and orange) are each time delayed by 2 ns, corresponding roughly to $\frac{1}{4}$ of a period ($f_0 \approx 130$ MHz). As shown in the right-hand side of the figure, the retrieved FSBS responses turns asymmetric with respect to $f_0$ when the reading pulse is delayed by a quarter of a period (2 ns) with respect to its initial delay $\Delta t$, and flips over when delayed by half a period (4 ns) with respect to $\Delta t$. 

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Fig. S3. Impact of an added time-shift between the reading pulse and the activating pulse on the retrieved FSBS resonance.

The behavior predicted in Fig. S3 is experimentally demonstrated by performing additional measurements in the section of bare fiber (see section 4.A in the main text) with reduced requirements in terms of spatial resolution. Faster acquisitions compared to the results shown in the main text were obtained by lowering the averaging down to 300 single acquisitions, and scanning the Brillouin gain spectrum (BGS) every 10 MHz instead of every 4 MHz. This yields a spatial resolution of 5 m, and the results are displayed in Fig. S4, where the colors of the different resonances shown are matching the ones displayed in Fig. S3.

More explicitly, Fig. S4(a) corresponds to the value of $\Delta t$ used in the main text (with a spatial resolution of 5 m instead of 80 cm), Fig. S4(b), (c) and (d) are achieved after further delaying the reading pulse by 2 ns, 4 ns and 6 ns, respectively. Once more, the matching between the numerical simulations and the experimental data is excellent, especially in finer features such as the sidelobes, which are induced by the finite duration of the activating pulse (see Supplement ??). We emphasize that all acquisitions are rigorously identical, except for an additional time-shift of 2 ns of the reading pulse between each case. While no specific application is targeted here, the results shown here display a high flexibility to induce a bipolar frequency shift on a pulse obtained by delaying either the activation or the reading pulse, or by operating at a frequency either below or above the FSBS resonant frequency.

B. Non-local effects

In this section, we mention the presence of non-local effects that might seriously deteriorate the retrieved FSBS response if not properly tackled. Note that this effect is particularly detrimental when the accumulated phase shift reaches a certain threshold, hence it is more likely to cause problems in fully distributed FSBS sensing rather than remote distributed FSBS sensing. To illustrate this, we simulate a section of fiber similar to the polyimide fiber presented in section 4.B of the main text on fully distributed FSBS sensing. The fiber, which is 500 m long in total, exhibits a uniform FSBS resonance over its first 490 m, while the remaining 10 m shows a similar resonance but with a slightly lower resonance frequency (200 kHz lower). The results, showing the effect of four different activating pulse power (minimum power in blue, maximum power in black), are shown in Fig. S5. Fig. S5(a) shows the accumulated frequency shift $\Delta f_{acc}$ at the resonant frequency of the first 490 m of fiber (here $f_0 = 130.5$ MHz). The blue curve represents a scenario very similar
Fig. S4. Measured FSBS response (x-markers) in bare fiber with a spatial resolution of 5 m compared to numerical simulations (solid lines). The time delay between the activating and the reading pulses $\Delta t$ is the same as in the main text in (a), and is extended by 2 ns (b), 4 ns (c) and 6 ns (d) with respect to (a).

to the experimental data shown in the main text, as the accumulated shift at resonance reaches 150 MHz at the fiber end. The effect of the activating pulse is stronger in subsequent cases, reaching an overall accumulated shift of 200 MHz (red), 250 MHz (green) and 300 MHz (black). The retrieved FSBS resonance with a spatial resolution of 2 m is shown at three fiber locations in Fig. S5(b), (c) and (d), corresponding to 100 m, 400 m and 500 m, respectively. At the fiber beginning, i.e. when the total phase modulation experienced by the reading pulse is low, the FSBS resonance may be accurately retrieved in all cases, as all curves shown in Fig. S5(b) are identical apart from a scaling factor. Towards the fiber end (400 m), the retrieved FSBS resonance starts to exhibit noticeable distortions, although the fiber was simulated to be perfectly uniform. Notice for instance how the upper part of the black curve in Fig. S5(c) is broadened and peaks at a higher value compared to the corresponding curve shown in Fig. S5(b), although in ideal conditions both curves should be identical. The distortions are even more pronounced in Fig. S5(d), which corresponds to a section of fiber (10 m) which FSBS central resonance is downshifted by 200 kHz with respect to the previous 490 m. In this case, the resonance turns strongly asymmetric due to accumulated non-local effects. These non-local effects are a pure consequence from a too severe phase modulation impinged on the reading pulse, which should be maintained in all cases below a certain level. Whilst this was estimated here thanks to the use of numerical simulations, this problem ought to be addressed in depth, in order to identify safe operating conditions, similar to other distributed optical fiber sensors [10–12]. This study, which falls out of the scope of this paper, is expected to be complex, as the phase modulation not only depends on the activating pulse power, but also on and the FSBS resonance profile and the reading pulse width. For instance, a shorter reading pulse will behave better in terms of phase modulation, as it will experience a narrower section of the sinusoidal refractive index modulation (hence the Taylor expansion presented in section 2. of the main text gets more accurate), but will also yield a poorer signal-to-noise ratio due to the reduced energy that it carries.

The results presented hereabove indicate that while in remote distributed FSBS sensing (section 4.A in the main text, where the sensing fiber is only ~30 m long) the activating pulse power did not require any limitation imposed by non-local effects; however, in fully distributed FSBS sensing (section 4.B in the main text, where the sensing fiber is ~500 m long), the activating pulse...
Pulse power: 0.5*\(P_{\text{max}}\)  
Pulse power: 0.66*\(P_{\text{max}}\)  
Pulse power: 0.8*\(P_{\text{max}}\)  
Pulse power: \(P_{\text{max}}\)

Fig. S5. (a) accumulated frequency shift \(\Delta f_{\text{acc}}\) at resonance \(f_d = 130.5\) MHz. Retrieved FSBS resonance with a spatial resolution of 2 m at 100 m (b), 400 m (c) and at fiber end (d).

power required proper reduction in order to avoid too strong accumulated phase modulation of the reading pulse.

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