Optical diagnostics of the temperature field of a hydrogen-air flame

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Abstract. A method for multi-wave optical diagnostics of phase structure and temperature fields in an axisymmetric hydrogen-air flame is discussed. An algorithm for numerical simulation of Hilbert images of phase fields reconstructed from radial temperature profiles and molar concentrations of combustion products using the Abel transform is developed and experimentally justified.

1. Introduction
The optical methods are important for contactless diagnosis in experimental hydro and gas dynamics [1, 2]. They allow one to obtain information about the thermodynamic and structural parameters of the medium under study without disturbing its state.

In [3], a method for estimating the temperature distribution in an asymmetric flame using high-contrast stereoscopic photography is described. Spectral reconstruction of temperature fields using color ratio pyrometry and interferometric tomography is reported [4]. An example adapted to the problems of flame research is optical diagnostics based on the methods of Hilbert optics and interferometry in combination with pixel-by-pixel processing of the dynamic structure of visualized phase structures induced by temperature fields [5]. However, diagnostics of the spatial optical phase structure of the flame remains an unsolved problem.

The use and improvement of optical Hilbert methods for the study of an axisymmetric hydrogen-air flame with the reconstruction of the spatial phase and temperature structure is the aim of this work. The research is aimed at solving a scientific and practical problem, which consists in finding methods for controlling the structural and thermodynamic parameters of the torch [6].

2. Experimental research method
The complex of optical diagnostics is based on the IAB–463M device (figure 1).
Figure 1. Hilbert visualizer diagram.

The probing field is formed by the system from a light source 1, a collimator lens 2, and a slit diaphragm 3. The investigated hydrogen-air flame 5 is localized in the vicinity of the front Fourier plane of the lens, which serves as an analyzer of phase disturbances induced in the probing field by this flame. Visualization of the Fourier spectrum of phase disturbances is performed by a Hilbert filter 7 placed in the frequency plane of the Fourier lens 6. The lens 8 performs the inverse Fourier transform of the Hilbert-conjugate Fourier spectrum of phase perturbations is performed. The visualized phase disturbances are registered by the camera's photo matrix 9 connected to a computer 10. The Hilbert transform performs a redistribution of signal energy from the region of low spatial frequencies to the high-frequency region. Extremes and gradients of the phase optical density of the medium under study are transformed into visualized Hilbert-bands structures the spatial distribution of which carries information about phase optical density perturbations induced by the temperature field [5].

The burner 5 is shown schematically in figure 2.a, which consists of two coaxially and vertically positioned quartz tubes (dimensions and relative positions are shown in the figure). Air is supplied through an inner tube, the fuel mixture H$_2$/N$_2$ enters through an annular channel between the inner and outer pipes (H$_2$ 36.5% by volume, 3.95% by mass). The nitrogen is added to reduce heat transfer to the burner. An example of the phase structure of a hydrogen-air flame is shown in figure 2.b. The phase structure was visualized using Hilbert optics methods. The experiments were carried out at atmospheric pressure and initial room temperature. The hydrogen-air flame research refers to areas with axial symmetry, where the choice of area is explained by the relatively simple geometry when modeling the flame.

Figure 2. (a) – a diagram of the burner; (b) – experimental Hilbert image of a hydrogen-air flame.
3. Numerical research method
According to the dispersion formula Cauchy [7]

\[ n_k(\lambda) - 1 = A_k \left( 1 + \frac{B_k}{\lambda^2} + \frac{C_k}{\lambda^4} \right), \]  

the refractive index of the \( k \)-th component of the burning mixture \( n_k \) depends on the wavelength of the radiation source \( \lambda \) and the parameters \( A_k, B_k \) and \( C_k \) (\( A_k \) and \( B_k \) are dispersion coefficients for the \( k \)-th component, \( C_k \) is the relative molar concentration of the \( k \)-th component). The refractive index of the entire mixture is defined as

\[ n = 1 + \frac{p}{p_0} \frac{T_0}{T} \sum_k A_k \left( 1 + \frac{B_k}{\lambda^2} \right) \cdot C_k, \]  

where \( p \) is the pressure inside the flame, \( p_0 \) is the pressure in the room, \( T \) is the temperature inside the flame, \( T_0 \) is the room temperature. Thus, the reconstruction of the values of molar concentrations of fuel combustion products \( C_k \) and, accordingly, the temperature distribution in the flame is possible by performing measurements at different wavelengths and using formulas (1) - (2).

An algorithm for the numerical simulation of Hilbert images by radial values of temperature and mole fraction of fuel combustion products in a hydrogen-air flame was developed according to the proposed method. This algorithm was tested on the experimental data given in [8] (temperature profiles and molar concentrations in four sections of an axisymmetric flame section).

The radial fields of molar concentrations of fuel combustion products at a height from the cut of the burner tube \( y \) from 3 to 30 mm are shown in figure 3. The corresponding radial temperature field of a hydrogen-air flame is shown in figure 4.a. The figures were obtained using piecewise-cubic Hermitian interpolation from data in sections \( y = 3, 10, 20 \) and 30 mm. The radial field of the refractive index calculated by formula (2) is shown in figure 4.b. The conditions were accepted in the calculation: \( T_0 = 27^\circ \text{C}, \lambda = 0.630 \text{ mm}, p = p_0 = 101.325 \cdot 10^3 \text{ Pa} \) (normal conditions), \( A_k \) and \( B_k \) are the values taken from the reference data.

| Y[mm] | X[mm] | a | 0 | 5 | 10 | 15 |
|-------|-------|---|---|---|----|----|
| 30    | 0.8   |   | 0.75 | 0.7 | 0.65 | 0.6 |
| 25    | 0.55  |   | 0.5 | 0.45 | 0.4  | 0.35 |

| Y[mm] | X[mm] | b | 0 | 5 | 10 | 15 |
|-------|-------|---|---|----|----|----|
| 30    | 0.2   |   | 0.16 | 0.12 | 0.08 | 0.04 |
| 25    | 0.05  |   | 0.05 | 0.03 | 0.01 | 0.005 |

| Y[mm] | X[mm] | c | 0 | 5 | 10 | 15 |
|-------|-------|---|---|----|----|----|
| 30    | 0.25  |   | 0.2 | 0.15 | 0.1 | 0.05 |
| 25    | 0.15  |   | 0.15 | 0.1 | 0.05 | 0.025 |

| Y[mm] | X[mm] | d | 0 | 5 | 10 | 15 |
|-------|-------|---|---|----|----|----|
| 30    | 0.45  |   | 0.45 | 0.35 | 0.25 | 0.15 |
| 25    | 0.55  |   | 0.55 | 0.45 | 0.35 | 0.25 |

Figure 3. The radial fields of molar concentrations of fuel combustion products (the unit of measurement – mole fraction): (a) – N2; (b) – O2; (c) – H2O; (d) – H2.

The phase structure of the probing light field, disturbed by an axisymmetric flame, in a physical experiment is determined according to the Abel integral:

\[ \Delta \psi(r, \gamma) = 2k \int_0^R \left[ n(r, y) - n_0 \right] \frac{rdr}{\sqrt{r^2 - \gamma^2}}, \]  

where \( r^2 = x^2 + z^2 \), \( R \) is the radius of the cross section of the considered zone, \( n(r, y) \) is the refractive index at a distance of \( r \) from the axis of the torch, \( k = 2\pi/\lambda \) is the wave number of the probing field, \( n_0 \) is the refractive index of the medium unperturbing by the flame. The z axis is
determined by the direction of the probe light beam, the flame torch cross section is described in \(x, z\) coordinates. The choice of the section position is determined by the \(y\) coordinate.

The relative molar concentrations of dry air components, as well as the dependence of the saturated vapor pressure on temperature, must be known to calculate the refractive index of air \(n_0\). Atmospheric pressure is equal to the sum of the partial pressures of the dry air components and the partial pressure of water vapor, which is determined by the formula for the relative humidity of air:

\[
P_W = \frac{\varphi_W \cdot P_{0W}}{100},
\]

where \(P_W\) is the partial pressure of water vapor, \(P_{0W}\) is the saturated vapor pressure at a given temperature, \(\varphi_W\) is the relative air humidity in the room.

Further, the molar concentration of water vapor in air is determined using the equation of state of an ideal gas:

\[
\nu_W = \frac{P_W}{R_f T_0},
\]

where \(R_f\) is the universal gas constant, \(T_0\) is the room temperature. The value of the molar concentration of the entire mixture (air) is calculated similarly:

\[
\nu = \frac{p_0}{R_f T_0},
\]

where \(p_0\) is atmospheric pressure (in the room). Then the relative molar concentration of water vapor in the air:

\[
C_W = \frac{\nu_W}{\nu}.
\]

The refractive index \(n_0\) of the medium unperturbing by the flame, taking into account (2) and (4) - (7), will be determined as

\[
n_0 = 1 + \frac{p_0}{p_{n.c.} \cdot T_{n.c.}} \left( \sum_k A_k \left( 1 + \frac{B_k}{\lambda^2} \right) \left[ 1 - C_W \right] + A_W \left( 1 + \frac{B_W}{\lambda^2} \right) C_W \right),
\]

where \(p_{n.c.}\) - atmospheric pressure under normal conditions \((101.325 \cdot 10^3 \text{ Pa})\); \(T_{n.c.}\) - temperature under normal conditions \((0^\circ\text{C})\); \(A_k\) and \(B_k\) - dispersion coefficients for the \(k\)th component of dry air; \(A_W\) and \(B_W\) - dispersion coefficients for water; \(\lambda\) is the wavelength of the radiation source.

The calculated inverse fields of the phase function for radius \(r\) from -15 to 15 mm and the wavelength \(\lambda = 0.630, 0.520\) and \(0.405 \mu\text{m}\), obtained by calculating the Abel integral (3), is shown in figure 6.a, 6.c and 6.e. The numerically simulated hilbertograms obtained from the calculated phase
functions are shown in figure 6.b, 6.d and 6.f. The relative air humidity in the room was taken \( \varphi_W = 60\% \) in the numerical simulation of the hilbertograms.

![Figure 5](image)

**Figure 5.** The numerically simulated hilbertograms and inverse fields of the phase functions: (a), (b) - \( \lambda = 0.630 \, \mu m \); (c), (d) - \( \lambda = 0.520 \, \mu m \); (e), (f) - \( \lambda = 0.405 \, \mu m \).

The area of combustion of the flame can be divided into zones consisting of 3 components of combustion products. Therefore, to determine all unknown parameters, it is sufficient to perform measurements at three different wavelengths of the probing field, as shown in figure 5. Thus, the results obtained confirm the feasibility of multi-wave optical diagnostics in direct and inverse problems of studying phase structure and temperature fields in an axisymmetric flame.
4. Conclusion
The possibility of studying the flame in the axisymmetric approximation by means of Hilbert optics and numerical simulation using the Abel equation is discussed in the presented work. The phase function is reconstructed from the hilbertogram in accordance with the algorithm proposed in [9], and the refractive index is determined from the solution of the Abel equation in the framework of the inverse problem (retrieving the flame temperature from the hilbertogram).

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