Light bending is one of the significant predictions of general relativity (GR) and it has been confirmed with great accuracy during the past one hundred years. In this paper, we semiclassically calculate the deflection angle for the photons that just grazing the Sun in the infinite derivative theories of gravity (IDG) which is a ghost and singularity free theory of gravity. From our calculations, we find that the deflection angle \( \theta \) only depends on \( \Lambda/E \). \( \theta \rightarrow \theta_K \) when \( \Lambda/E \rightarrow \infty \) and decrease to zero when \( \Lambda/E \rightarrow 0 \). The transition interval occurs at \( 10^4 < E/\Lambda < 10^7 \). It should be pointed out that this model can be tested by the Chandra X-ray Observatory if \( 0.01 eV < \Lambda < 0.1 eV \).

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I. INTRODUCTION

The general relativity (GR) achieved great success in the past one hundred years and has been tested through different kinds of experiments. Light bending is one important test which was first observed by Eddington and Dyson in 1919. Many measurements were made in the following years and the accuracy is greatly improved using the very long baseline radio interferometry [1, 2]. The GR theory fits the experiments very well so far.

Unfortunately, the quantum GR is not perturbatively renormalizable. The higher-derivative gravity (HDG) theory could avoid such difficulties. HDG was first introduced by Weyl [3] and Eddington [4] which includes the higher-derivative terms in the Lagrangian such as scalars \( R^2, R_{\mu \nu \alpha \beta}, R_{\mu \nu \rho \sigma} \) and so on. Such models are renormalizable [5, 6] but nonunitary at the same time and and it is unavoidable that the ghost particles emerge when the higher derivatives are introduced. The infinite derivative theories of gravity (IDG) [6, 7] is such a model that can avoid the problem of massive ghost (It should be noted that this model is also named super-renormalizable quantum gravity or super-renormalizable nonlocal quantum gravity in some other papers). More details of IDG can be found in [6]. An earlier similar theory can also be found in [10].

The significant advantages of IDG are that it could avoid the problem of massive spin-2 ghost and the divergence of gravitational potential at small distance. The gravitational action for IDG can be written as [6–8]:

\[
S = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[ R + G_{\mu \nu} \frac{a(\Box)}{\Box} R^{\mu \nu} \right],
\]

where \( \Box \) is the D’Alambertian operator, \( G_{\mu \nu} = R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R \) and \( a(\Box) = e^{-\frac{5\Box}{2\pi}} \). It should be noted that \( \Lambda \) corresponds to a non-locality scale because the gravitational interactions in IDG model is non-local. The lower limit on parameter \( \Lambda \) can be calculated by combining the result of [11, 12], which is

\[
\Lambda > 0.01 \text{ eV}.
\]

It should noted that the authors studied the much more general situation \( a(\Box) = \exp{\left(\frac{\Box}{\Lambda^2 n}\right)} \) in [11], and got much stronger lower bound on \( \Lambda \) for higher \( n \). In [12], the author found that there was evidence for a Newtonian potential with of the form in [11]. Much stronger bound on \( \Lambda \) was gotten by using IDG as an extension of Starobinsky inflation [14].

In [15–17], the authors studied the gravitational deflection within the framework of classical and semiclassical HDG and find that the deflection angle decreases to zero at \( \log_{10}|\beta| \sim 89 \) in classical HDG and \( \log_{10}|\beta| \sim 70 \) in semiclassical HDG. The deflection angle calculated in other gravitational theory can be found in [18–22]. In this paper, we calculate the gravitational deflection of photons that graze the sun within the framework of classical HDG.

This draft is organized as follows: In Sec. II we calculate the deflection angle in semiclassical IDG and our conclusions are summarized in Section III. Here we use natural units and \( \text{diag}(1, -1, -1, -1) \) as the Minkowski metric.

II. GRAVITATIONAL DEFLECTION IN TREE-LEVEL IDG

We solve the linearized field equations of IDG for a pointlike particle using the perturbed metric

\[
g_{\mu \nu} = \eta_{\mu \nu} + \kappa h_{\mu \nu},
\]

where \( \kappa = \sqrt{16\pi G} \). The field equations derived from the action in Eq. (1) with a source component is [8]

\[
a(\Box) \Box h_{\mu \nu} - \left( \partial_\alpha \partial_\beta h^\alpha_\mu + \partial_\alpha \partial_\nu h^\alpha_\mu \right) + \left( \eta_{\mu \nu} \partial_\alpha \partial_\beta h^\alpha_\beta + \partial_\mu \partial_\nu h \right) - \eta_{\mu \nu} \Box h = -\kappa T_{\mu \nu},
\]
where $T_{\mu\nu}$ is the energy-momentum tensor of the source term. The corresponding energy-momentum tensor for a particle with mass $M$ is $M\eta_{\hat{a}\hat{a}}\delta^3(r)$. Solving the above equation with such energy momentum tensor [6–9] and we find

$$h_{\mu\nu}(r) = \frac{M\kappa}{8\pi} \left[ \frac{\eta_{\mu\nu}}{r} - \frac{2\eta_{\mu\hat{a}}\eta_{\nu\hat{a}}}{r^2} \right] \text{Erf} \left( \frac{\Lambda r}{2} \right). \quad (5)$$

In this model, the modified Newtonian potential is

$$\phi(r) = \frac{\kappa}{2} h_{00} = -\frac{GM}{r} \text{Erf} \left( \frac{\Lambda r}{2} \right). \quad (6)$$

It should be pointed out that such kind of potential was first obtained by Tseytlin in the exponential gravity motivated by string theory [24]. $\text{Erf} \left( \frac{\Lambda r}{2} \right) \to 0$ when $r \to \infty$ and we recover the Newtonian potential. $\text{Erf}(x) \sim \frac{x^2}{\sqrt{\pi}} e^{-x^2} x \sim \frac{x^2}{\sqrt{\pi}}$ when $x \to 0$ and then

$$\phi(r) \to -\frac{GM}{\sqrt{\pi}}, \quad (7)$$

So IDG model can avoid the divergence problem in GR at small distance.

Similar with [15, 22], we calculate the gravitational deflection angle within the framework of semiclassical IDG. This method provides much more information about the gravitational deflection of photons, such as the energy dependence of the deflection angle and so on. The Feynman diagram of this process that the photon scattered by the external gravity field is shown in Fig. 1 and the corresponding amplitude is given by

$$M_{rr'} = \frac{1}{2} \kappa h_{ext}^\rho(k) \left[ -\eta_{\mu\nu}\eta_{\lambda\rho}pp' + \eta_{\lambda\rho}p'_\mu p_\nu + 2(\eta_{\mu\nu}p_\lambda p'_\rho - \eta_{\mu\rho}p'_\lambda p_\nu - \eta_{\nu\rho}p_\lambda p'_\mu) \right] e^{\mu}_{\lambda'}(p)e^{\nu}_{\lambda'}(p'),$$

where $e^{\mu}_{\lambda'}(p)$ ($e^{\nu}_{\lambda'}(p)$) denotes the polarization vectors of the initial (final) photons and satisfies the following relation

$$\sum_{\nu=1}^{2} e^{\nu}_{\lambda'}(p)e^{\nu}_{\lambda'}(p) = -\eta_{\mu\nu} - \frac{p^\mu p^\nu}{(p\cdot n)^2} + \frac{p^\mu n^\nu + p^\nu n^\mu}{p\cdot n}. \quad (8)$$

where $n^2 = 1$. Here $h_{ext}^\rho(k)$ is the gravitational field in momentum space, which is

$$h_{ext}^\rho(k) = \int d^3r e^{-ik\cdot r} h_{ext}^\rho(r). \quad (9)$$

Substituting Eq. (9) into Eq. (15), we get

$$h_{ext}^{(E)\mu\nu}(k) = \kappa M \left( \frac{\eta_{\mu\nu}}{2k^2} - \frac{\eta_{\mu\hat{a}}\eta_{\nu\hat{a}}}{k^2} \right) \exp \left( -\frac{k^2}{\Lambda^2} \right). \quad (10)$$

![FIG. 1: The Feynmann diagram of the interaction between external gravitational field and photon.](image)

Then we get the unpolarized cross-section with the following equation

$$\frac{d\sigma}{d\Omega} = \frac{1}{(4\pi)^2} \frac{1}{2} \sum_{r} \sum_{r'} \mathcal{M}_{rr'}^2 \frac{1}{\kappa^2 \exp \left( \frac{-k^2}{\Lambda^2} \right)}.$$  

where $\theta$ is the angle between $p$ and $p'$ and $E$ is the energy of the injected photon.

For small angles case, $k^2 \approx 2p^2(1 - \cos\theta) \approx E^2\theta^2$. Then the previous equation reduces to

$$\frac{d\sigma}{d\Omega} = 16G^2M^2 \left[ \frac{1}{\theta^2} \exp \left( -\frac{\theta^2}{\Lambda^2} \right) \right]^2. \quad (11)$$

where $\lambda \equiv \frac{1}{\theta}$. Obviously, the cross section only depend on $\lambda$.

From the above equation, we can see that

$$\frac{d\sigma}{d\Omega} \to \left( \frac{4GM}{\theta^2} \right)^2, \quad \text{if } \lambda \to \infty; \quad (12)$$

i.e. we recover the standard cross section of GR and the deflection angle is $1.75\theta$. And

$$\frac{d\sigma}{d\Omega} \to 0, \quad \text{if } \lambda \to 0, \quad (13)$$

which means that the deflection angle decreases to zero when $\lambda \to 0$.

We compare the classical and the tree-level cross-section formulas to get the classical particle trajectory [25, 26].

$$\frac{d\sigma}{d\Omega} = 16G^2M^2 \left[ \frac{1}{\theta^2} \exp \left( -\frac{\theta^2}{\Lambda^2} \right) \right]^2 = -\frac{r dr}{\theta d\theta}. \quad (14)$$

Performing the integration on Eq. (14), we finally get the deflection angle for the photons that just grazing the Sun, which is
We solve the above equation numerically and the result is exponential integral $y = \exp(-\frac{2\theta^2}{\lambda^2}) + \frac{2}{\lambda^2} Ei(-\frac{2\theta^2}{\lambda^2})$, \(\text{(15)}\)

where $\theta_E = \sqrt{4GM/R_\odot} = 1.75''$ is the Einstein’s deflection angle and $R_\odot$ is the radius of the sun. The exponential integral $Ei(x)$ is defined as

$$Ei(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt, \text{ (16)}$$

Defining $y \equiv \frac{2\theta^2}{\lambda^2}$ and Eq. \((15)\) becomes

$$e^{-y} \frac{y}{y} + Ei(y) - \frac{\lambda^2}{2\theta_E^2} = 0. \text{ (17)}$$

We solve the above equation numerically and the result is shown in Fig.2.

![Graph: The deflection angle as a function of log\(_{10}(\Lambda/E)\) for photons that just grazing the Sun in semi-classical IDG.](image)

FIG. 2: The deflection angle as a function of log\(_{10}(\Lambda/E)\) for photons that just grazing the Sun in semi-classical IDG.

\[ \frac{1}{\theta_E} = \frac{1}{\theta^2} \exp(-\frac{2\theta^2}{\lambda^2}) + \frac{2}{\lambda^2} Ei(-\frac{2\theta^2}{\lambda^2}), \text{ (15)} \]

\[ Ei(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt, \text{ (16)} \]

\[ e^{-y} \frac{y}{y} + Ei(y) - \frac{\lambda^2}{2\theta_E^2} = 0. \text{ (17)} \]

As $E > 10^7\Lambda$ which means there is no deflection for sufficient high energy photons. It should point out that the transition interval occurs for $10^4\Lambda < E < 10^7\Lambda$. The smallest energy to test such effect is $10^4\Lambda$ and the deflection angle decrease to $1''$ at $E = 10^5\Lambda$. If $0.01eV < \Lambda < 1eV$, the transition occurs in X-ray band which could be tested by the X-ray telescopes. And the transition occurs in hard X-ray or gamma-ray band if $1eV < \Lambda$ which could be tested by the corresponding telescopes.

The angular resolution should be better than $1.75''$ to test the deviation of deflection angle from $\theta_E$. The performance of the current(or planed) X-ray(or $\gamma$-ray) detectors is shown in Table I From Table IV we can see that only Chandra X-ray Observatory (Chandra)\(^{[27]}\) satisfies such requirement. Chandra works in the photon energy range of 0.2-10 keV. So IDG model can be tested if $0.01eV < \Lambda < 1eV$. It is a big challenge to avoid the damage of detectors when doing such measurements because the sun is the brightest X-ray source in the sky. A large part of the X-ray are sheltered by the moon when the total solar eclipse occurs. So it may be possible to measure the X-ray deflection angle during the total solar eclipse with Chandra to test IDG model.

### III. SUMMARY

In this draft, we calculate the deflection angle of photons that graze the sun within semiclassical IDG model. We find that the deflection angle only depends on $\Lambda/E$. When $\Lambda/E \to \infty$, $\theta \to \theta_E$. In other words, we recover the prediction of GR for low energy photons. $\theta \to 0$ when $\Lambda/E \to 0$, which means that there is no deflection for sufficiently high energy photons. The transition occurs at range $10^4 < E/\Lambda < 10^7$.

The deviation of deflection angle from $\theta_E$ occurs at X-ray and gamma ray range because $\Lambda > 0.01eV$. It can be tested by X-ray or gamma ray telescopes with good enough angular resolution. However, only Chandra can be possibly used to test this effect. It is interesting to measure the deflection angle of high energy photons and such measurement has never been done before.

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| Detectors     | Energy Range | Angular Resolution |
|---------------|--------------|-------------------|
| Chandra\(^{[27]}\) | 0.1 – 10 keV | 0.5''             |
| Swift-XRT\(^{[28]}\) | 0.2 – 10 keV | 18''HPD@1.5keV   |
| Swift-BAT\(^{[28]}\) | 15 – 150 keV | 17''             |
| FOXSI\(^{[29]}\)    | 5 – 15 keV  | 12''             |
| XMM-Newton\(^{[30]}\) | 0.1 – 12 keV | 5'' ∼ 14''       |
| NuSTAR\(^{[31]}\)   | 3 – 79 keV  | 9.5''            |
| IXO\(^{[32]}\)      | 1 – 250 keV | < 5'             |
| Einstein Probe\(^{[33]}\) | 0.5 – 4 keV | < 5'             |
| Athena\(^{[34]}\)   | 0.3 – 12 keV | 10''            |
| Fermi-LAT\(^{[35]}\) | 10 – 3 x 10^6 MeV | ∼ 0.1°    |
| DAMPE\(^{[36]}\)    | 5 – 10^4 GeV | ∼ 0.1°          |
| CALET\(^{[37]}\)    | 5 – 10^5 GeV | ∼ 0.1°          |

From Fig.2, we can straightforwardly see that $\theta \sim \theta_E$ as $E < 10^4\Lambda$, which recovers the result of GR and $\theta \sim 0$
[1] D. Lebach et al., Phys. Rev. Lett. **75**, 1439 (1995).
[2] E. Fomalont, S. Kopeikin, G. Lanyi, and J. Benson, Astrophys. J. **699**, 1395 (2009).
[3] H. Weyl, *Space-Time Matter* (Dover, 1952).
[4] A. Eddington, *The Mathematical Theory of Relativity*, 2nd ed. (Cambridge University Press, 1924).
[5] K. Stelle, Phys. Rev. D **16**, 953 (1977).
[6] L. Modesto, Super-renormalizable Quantum Gravity, Phys. Rev. D **86**, 044005, [arXiv:1107.2403v1 [hep-th]].
[7] T. Biswas, E. Gerwick, T. Koivisto and A. Mazumdar, Towards singularity and ghost free theories of gravity, Phys. Rev. Lett. **108**, 031101, [arXiv:1110.5249 [gr-qc]].
[8] L. Modesto, Super-renormalizable Higher-Derivative Quantum gravity, [arXiv:1202.0008v1 [hep-th]]. L. Modesto, T. de Paula Netto and Ilya. L. Shapiro, On Newtonian singularities in higher derivative gravity models, High Energ. Phys. (2015) **2015**: 98, [arXiv:1412.0740 [hep-th]].
[9] L. Buoninfante, [arXiv:1610.08744[gr-qc]].
[10] T. Biswas, A. Mazumdar, W. Siegel, JCAP 0603 (2006) 009, [hep-th/0508194].
[11] J. Edholm, A. Koshelev and A. Mazumdar, Universality of testing ghost-free gravity, 2016, [arXiv:1604.01989 [gr-qc]].
[12] D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle and H. E. Swanson, Phys. Rev. Lett. **98** (2007) 021101.
[13] Leandros Perivolaropoulos, [arXiv:1611.07293].
[14] J. Edholm, A. S. Koshelev and A. Mazumdar, Behavior of the Newtonian potential for ghost-free and singularity-free gravity, Phys. Rev. D **94**, no. 10, 104033 (2016) doi:10.1103/PhysRevD.94.104033 [arXiv:1604.01989 [gr-qc]].
[15] A. Accioly, J. Helayel-Neto, B. Giacchini and W. Herdy, Phys.Rev. D**91** (2015) no.12, 125009
[16] A. Accioly et al., [arXiv:1604.07348]
[17] A. Accioly, B. L. Giacchini and I. L. Shapiro, [arXiv:1610.05260]
[18] R. Paszko and A. Accioly, Class. Quantum Grav. **27**, 145012 (2010).
[19] A. Accioly and R. Paszko, Int. J. Mod. Phys. D **18**, 2107 (2009).
[20] A. Accioly and R. Paszko, Adv. Stud. Theor. Phys. **3**, 65 (2009).
[21] A. Accioly and R. Paszko, Phys. Rev. D **78**, 064002 (2008).
[22] A. Accioly, R. Aldrovandi, and R. Paszko, Int. J. Mod. Phys. D **15**, 2249 (2006).
[23] Ya-Peng Hu et al., Adv.High Energy Phys. **2014** (2014) 604321.
[24] A.A. Tseytlin, Phys. Lett. B **363** (1995) 223 [hep-th/9509050].
[25] R. Delbourgo and P. Phocas-Cosmetatos, Phys. Lett. B **411**, 533 (1997).
[26] F. Berends and R. Gastmans, Ann. Phys. (N. Y.) **98**, 225 (1976).
[27] http://chandra.harvard.edu/
[28] https://swift.gsfc.nasa.gov/
[29] S. Krucker et al., The Focusing Optics X-ray Solar Imager (FOXSI), in Optics for EUV, X-Ray, and Gamma-Ray Astronomy IV, ser. Proc. SPIE, S. L. ODell and G. Pareschi, Eds., vol. 7437, 2009, p. 743705.
[30] http://sci.esa.int/xmm-newton/
[31] http://www.nustar.caltech.edu/
[32] http://www.hxmt.org/
[33] Yuan W. et al., 2015, ArXiv e-prints:1506.07735
[34] Barcons, X., Barret, D., Decourchelle, A., et al. 2012, ArXiv e-prints:1207.2445
[35] https://fermi.gsfc.nasa.gov/
[36] J. Chang, Dark Matter Particle Explorer: The First Chinese Cosmic Ray and Hard-ray Detector in Space, Chin. J. Spac. Sci. **34** (2014) 550
[37] http://calet.phys.lsu.edu/