Modeling of load lifting process with unknown center of gravity position

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Abstract. The article proposes a new type of lifting beams that allows one to lift loads where the position of the center of gravity is unknown beforehand. The benefit of implementing this type of traverse is confirmed by the high demand for this product from the industrial enterprises and lack of their availability on the market. In conducted studies, the main kinematic and dynamic dependencies of the load lifting process with an unknown position of the center of gravity were described allowing for design and verification calculations of the traverse with flexible slings and an adjustable bail to be carried out. The obtained results can be useful to engineers and employees of enterprises engaged in the design and manufacturing of the lifting equipment and scientists doing research in "Carrying and lifting machines".

1. Introduction

Carrying and lifting equipment is one of the important components of the production process determining its effectiveness. The most labor-intensive processes are load/unload operations, which is a major part of technological processes of any plant, warehouse, construction site and is especially important for mining enterprises where the use of large-sized structures of complex configuration is a common task [1]. This type of work is carried out at all stages of the main production processes. Lifting and transporting operations with large-sized loads with a shifted center of gravity are among the most labor-intensive tasks. An effective way to reduce labor costs in the installation and handling of the large-sized loads with the shifted center of gravity is the modernization and automation of lifting and transporting and hoisting devices [2].

Improvement of the existing and development of the new technological processes of handling operations with the use of carrying and lifting machines of the cyclic principle is possible only if modern hinged load handling devices are used as well as special devices that ensure the safety of the transported oversize loads with an unknown or shifted center of gravity.

2. Loading diagram and assumptions

Figure 1 shows the loading diagram for lifting load with flexible slings and an adjustable bail, taking into account the dynamic characteristics of the system. Beam trolley 1 moves along the traverse at constant speed \( u \), while rising vertically upwards at constant speed \( v \).

The lifting of the load will come into consideration from the moment when both slings are tightened but the load itself has not yet set in motion.

It is not practical to consider the system up to that point since the weight of the lifting beam is much less than the weight of the load and the movement of the load would not begin until both slings are tensioned.
The basic assumptions defined to solve the task are:

1. The total height of the lifting beam and the load is much less than their length ($h \ll L$). Usually, this condition is the most common. Thus, let us consider the lifting beam as a homogeneous rod with the center of gravity at its center, and the load - as an inhomogeneous rod with the center of gravity at an arbitrary point.

2. Slings are non-stretching and do not have any weight.

3. The load and the lifting beam retain their shape unchanged during the lifting.

4. The position of the centers of gravity of the lifting beam and the load remains unchanged.

5. The weight of the beam trolley is negligible compared to the weight of the lifting beam and the weight of the load ($m_1 \ll m_b$, $m_1 \ll m_l$).

3. Modeling and results

Assuming $h \ll L$ as per Figure 1, the angles of rotation of the lifting beam and the load will be identical, and therefore the angular velocities will be the same: $\omega_b = \omega_l$.

Points 1 and 4 are assumed to be stationary being the centers of rotation of the load and the lifting beam. Angular velocity $\omega_l$ through the linear velocity of point 2 can be defined as:

$$\omega_l = \frac{v_2}{L};$$

$$\ddot{x} = \omega_l.$$  (1)

Given velocities $v$ and $u$ are constant, the following equation was obtained:

$$x = x_0 - u \cdot t,$$  (2)

where $x_0$ - initial position of the beam trolley,

$t$ - current time.

As per theorem on the plane-parallel motion of a bar:

$$v_2 \cos \alpha = v_3 \cos \alpha;$$

$$\frac{v_2}{v} = \frac{v_3}{L};$$

$$\frac{x \cdot \cos (\alpha)}{L} = \frac{v_3}{L},$$

and according to equation (1), one obtains:

$$\ddot{x} = \frac{v}{(x_0 - ut) \cdot \cos (\alpha)}.$$
Dividing the variables and integrating the equation above yields:

\[
\int_0^a \cos \alpha \, d\alpha = \int_0^t \frac{v}{x_0 - ut} \, \, d\alpha;
\]

\[
\sin \alpha = \frac{v}{u} \cdot \ln \frac{x_0 - ut}{x_0}.
\]  

(3)

Thus, the dependence of angle \( \alpha \) on time for given values of \( u \) and \( t \) was determined. The equation is not valid for all possible values of the variables, since function \( y = \sin (\alpha) \) is within the following range:

\[
1 \leq \sin (\alpha) \leq 1.
\]

The physical meaning of this limitation is it would not be possible to turn the load by more than 90°. It should be noted that the logarithmic expression will be positive for any values satisfying the conditions of the problem.

Now, let us define the condition for the detachment of the load from the surface. Obviously, this will happen when the beam trolley reaches the equilibrium position which can be found from the condition that the sum of all the active forces applied to the system equals 0 [3]:

\[
(N \frac{x}{L} - \frac{1}{2} m_b g \delta \alpha - \frac{x_l}{L} m_l g) \cos \alpha \delta \alpha = 0.
\]

Two variants of the equilibrium position are obtained:

1. \( \alpha = 90^0 \) – the load flips over. Such option is obviously unsatisfactory.

2. \( N \frac{x}{L} = \frac{1}{2} m_b g - \frac{x_l}{L} m_l g = 0 \) – static equilibrium position

Using equation (2), time \( t_p \), during which the beam trolley reaches the equilibrium, the following can be obtained:

\[
t_p = \frac{1}{u} \cdot \left( x_0 - \frac{m_l x_l + m_b L}{m_b + m_l} \right).
\]  

(4)

As a result of previous studies, it was found that for the absence of slippage of the load over the surface, it is necessary that the limiting angle of its inclination \( \alpha \) is not wider than 23.7° [4, 5]:

\[
\alpha_{lim} \leq 23.7^0.
\]  

(5)

Slippage of the load can lead to its damage, which is unacceptable [2].

Using the Matlab software package [6] allows one to visualize the obtained equations as graphs.

Equation \( \alpha = \alpha(t) \) is demonstrated in (Figure 2).

Substituting equations (4) and (5) in (3), the following equation of dependence of the beam trolley velocity on the rate of lifting is obtained:

\[
u = \frac{v}{\sin \alpha_{lim}} \ln \frac{(m_l + m_b) x_0}{m_l x + m_b L}. \]

The corresponding graphical representations are shown in Figure 3.

The resulting equations are linear. The angle of inclination of the resulting straight lines is directly proportional to the deviation of the initial position of the beam trolley from the equilibrium position.

When the beam trolley position is the same as the position of equilibrium of the system, the required speed of the beam trolley will be zero.

Under the given conditions corresponding to the parameters of the test bench \( m_l = 20.65 \text{ kg}, m_b = 5 \text{ kg}, L = 0.6 \text{ kg} \), the equilibrium position is at the point with coordinate \( x = 0.22 \text{ m} \).
To determine the equation for the tension force of the lifting block vs. time, it is necessary to compose a differential equation of the system in motion. For example, this can be done with the help of Lagrange equation of the 2nd kind [3]:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\alpha}} \right) - \frac{\partial T}{\partial \alpha} = Q,$$

(6)

where $T$ – kinetic energy of the system,

$Q$ – generalized active force.

$$T = \left( J_b + J_l \right) \dot{\alpha}^2,$$

where $J_b$ and $J_l$ are moments of inertia of the lifting beam and load relative to points 4 and 1 respectively.

$$J_b = \frac{1}{3} m_b L^2;$$

$$J_l = \frac{1}{12} m_1 L^2 + m_1 x_i^2.$$

Generalized force $Q$ is found from the condition:

$$Q = \frac{\delta A}{\delta \alpha} = \left( N \frac{x}{L} - \frac{1}{2} m_b g - \frac{x_i}{L} m_1 g \right) \delta \alpha;$$

$$Q = \frac{N x}{L} - \frac{1}{2} m_b g - \frac{x_i}{L} m_1 g.$$

(7)

From formulas (2, 6, 7), the following equation was obtained:

$$\left( \frac{1}{12} m_1 L^2 + m_1 x_i^2 + \frac{1}{3} m_b L^2 \right) \ddot{\alpha} = N \cdot \frac{x_0 - u \cdot t}{L} - \frac{1}{2} m_b g - \frac{x_i}{L} m_1 g.$$

The angular acceleration of load $\ddot{\alpha}$ is found from formula (6) by double differentiation with respect to time:
Let us denote that:
\[ J = \frac{1}{12} m_1 l^2 + m_1 x_1^2 \frac{1}{3} m_0 l^2; \]
and express \( N \):
\[ N = \frac{Jv}{\cos \alpha (x_0 - u \cdot t)^2} \left( \frac{v \sin \alpha}{\cos^2 \alpha} + u \right) + \frac{L}{x_0 - u \cdot t} \left( m_0 g - m_1 l g \right). \] (8)

To obtain the tension force of the lifting block at the moment of detachment of the load from the surface, value \( \alpha = \alpha_{\text{lim}} \) and \( t = t_p \) must be substituted into the expression (8).

Fig. 4 demonstrates the change in normal component of force \( N \) vs. time \( t \) for different positions of the center of gravity of the load.

The set of formulas (7) and (8) for given parameters of the system determines the equation for the tension force vs. time and, since the necessary speed of lifting the load is already known, it is possible to calculate the power consumed by the engine:
\[ P = kNv, \] (9)

where \( k \) is the safety factor that takes into account the possible deviation of the angle of the lifting block from the vertical axis:
\[ k = \frac{1}{\cos \theta}. \]

According to recommendations [7-11], the deviation angle of the lifting block does not exceed 20°, therefore it equals 1.07°.

4. Conclusion
As a result of the conducted studies, a mathematical model was developed that takes into account the dynamics of lifting the load with a shifted center of gravity with the help of a lifting beam with flexible slings and allows one to determine the following:
1. minimum allowable beam trolley speeds depending on the speed of lifting of the load;
2. the angle of inclination of the load to the supporting surface in the dynamics of the lifting process;
3. the equation for the change in the tension force of the lifting block vs. time.

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