Mathematical Model of the Effect of Hyphal Death on T-W Types of fungi with Energy

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Abstract. The mathematical model is model show behavior for growth of Tip-tip anastomosis, Tip death hyphal death and we show the consumption energy. In general, To mathematical modeling to shorten the effort, time and mony to get the right result even though there is error ratio. In this paper we will study a mathematical model of branching using the solution of a system of partial equations (PDEs). The results of this solution will be describe a success or failure of the growth of the fungus species studied, and we used some codes in numerical analysis because some difficulty in direct mathematical solution.[2]

1. Introduction
The mathematical model theoretically describes an object that exists outside the field of mathematics. we note in the mathematical model three stages:
1.1 Construction, the process in which an object is transformed into a mathematical language.
1.2 Analysis or study of the prepared form.
1.3 Interpretation of the said analysis, in which the results of the study are applied to the object from which it was divided.
The benefit of such models is that they help to study how complex structures behave when faced with situations that are not easily visible in the real world. There are models working in certain cases and not accurate in other cases, as happens with mechanics Newtonian, who verified their credibility by Albert Einstein himself. The benefit of such models is that they help to study how complex structures behave when faced with situations that are not easily visible in the real world. There are models working in certain cases and not accurate in other cases, as happens with mechanics Newtonian, table(1)[3] show some branches of types of fungi and describe biological types and put these biologist phenomena as mathematical form as version, and illustrate the parameters description. In this model we mix some types of fungi [7]

2. Keywords
Tip-tip anastomosis, Tip death, Hyphal death

3. Mathematical Model
We will study a new type of branching of fungal growth with hyphal death and Consumption of whole vegetarian food, we can call it energy, this energy function lies between one and zero as.Here if means if the grow die if it is not consume energy but that mean is the growth is very good if the fungi consume all the energy.[5]
**Table 1.** illustrate Biological type, symbol of this type, version

| Biological type           | σ(p, n)   | Symbol | Parameters description                                      |
|--------------------------|-----------|--------|------------------------------------------------------------|
| Tip-tip anastomosis      | σ = −β₂n² | W      | β₁ is the rate of tip reconnections per unit time           |
| Tip death                | σ = −α₁n  | T      | α₁ is the loss rate of tips (constant for tip death)       |
| Hyphal death             | d = γ₁p   | D      | γ₁ is the loss rate of hyphal (constant for hyphal death)  |

*In our paper we will study a new type of branching of fungal growth, that’s mean[1]*

\[ \sigma(\rho, n) = WT \]  \hspace{1cm} (1)

where:

T: Tip death

W: Tip-tip anastomosis

The model system for TW is

\[ \frac{\partial \rho}{\partial t} = n \nu - \gamma \rho \]  \hspace{1cm} (2)

\[ \frac{\partial n}{\partial t} = - \frac{\partial (n \nu)}{\partial t} + e^{(-\beta_1 n^2 - \alpha_3 n)} - 1 \]

That is, energy is fully consumed, i.e. in this system \( \psi = 1 \)

To solve above system as:

3.1. Non-dimensionalisation [6][4]

Let

\( \tau \): time.

\( \bar{x} \): length scale.

\( \bar{\rho} \): “hyphal density”

\( \bar{n} \): “tip density”

\( \rho = \rho^* \bar{\rho} \), \( n = n^* \bar{n} \), \( t^* \tau = 1 \), and \( x = x^* \bar{x} \)

When put these in (2), we get:

\[ \frac{\partial \rho^*}{\partial t^*} = n^* - \rho^* \]  \hspace{1cm} (3)

\[ \frac{\partial n^*}{\partial t^*} = - \frac{\partial n^*}{\partial x^*} + e^{\left( \frac{\beta_1}{\tau} n^* + \frac{\alpha_1}{\tau} \right)} n^* - 1 \]

When put \( \bar{x} = \tau \nu \) and \( \bar{x} = \frac{\rho}{\bar{n}} \) then the system (3) becomes:

\[ \frac{\partial \rho}{\partial \tau} = n - \rho \]  \hspace{1cm} (4)

\[ \frac{\partial \rho}{\partial t} = - \frac{\partial n}{\partial x} + e^{\alpha n(1-n)} - 1 \]

Where: \( \bar{n} = \frac{\alpha_3}{\beta_1}, \) \( \alpha = \frac{\alpha_3}{\gamma} \)

To solve above system as stability solution we will show or study the stability of system (4)

3.1.1 The stability of solution

\[ n - \rho = 0 \]  \hspace{1cm} (5)

\[ e^{\alpha n(1-n)} - 1 = 0 \]

Take the in function for two sides of system (5) in above system, we get

\[ n - \rho = 0 \rightarrow f(p, n) \]  \hspace{1cm} (6)
\[ \alpha n(1 - n) = 0 \rightarrow g(p, n) \]

The system above has two uniform steady states points \((0,0), (1,1)\)

Now, we can take Jacobian and by using determinate eigenvalue of \(\lambda\), \(|A - \lambda I| = 0\) to solve the system. We get two of the value of \(\lambda\) at \((0,0)\)

We get:

We get \(\lambda_1 = -1\) and \(\lambda_2 = \alpha\).

The stability of the steady state on the value of \(\alpha\)

If \(\alpha > 0\)

Then \(\lambda_1\) is negative, \(\lambda_2\) is positive, we get the steady state saddle point.

if \(\alpha < 0\)

Then \(\lambda_1\) is negative, \(\lambda_2\) is negative, we get the steady state stable node.

When \(\lambda\) at \((1,1)\)

We get \(\lambda_1 = -1\) and \(\lambda_2 = -\alpha\)

The stability of the steady state,

If \(\alpha > 0\)

Then \(\lambda_1\) is negative, \(\lambda_2\) is negative, we get the steady state stable node.

If \(\alpha < 0\)

Then \(\lambda_1\) is negative, \(\lambda_2\) is positive, we get the steady state saddle point.

3.1.2 Traveling Wave Solution

We take the model

\[ \rho(x, t) = P(z) \quad (7) \]

\[ n(x, t) = N(z) \]

Here

“\(P(z)\), \(N(z)\)” represent density profiles

\(c\) can be interpreted as the rate of propagation of the colony edge for these to be biologically meaningful we should \(P\) and \(N\) to be bounded nonnegative functions of \(z\)
Then \( p(x, t) \) and \( n(x, t) \) are a travelling wave, and it moves at a constant speed \( c \) in the positive \( x \) direction if \( c \) positive.

Clearly if \( x - ct \) is constant, so are \( p(x, t) \) and \( n(x, t) \) coordinate system moves with speed \( c \).

The wave speed \( c \) generally has to be determined. The dependent variable \( z \) is sometimes called the wave variable. When we look for travelling wave solutions of an equation or system of equations in \( x \) and \( t \) in the form (4)

\[
\begin{align*}
\frac{\partial p}{\partial t} &= -c \frac{dp}{dz} \\
\frac{\partial n}{\partial t} &= -c \frac{dn}{dz} \\
\frac{\partial n}{\partial x} &= \frac{dn}{dz}
\end{align*}
\]

The system (4) becomes:

\[
\frac{dp}{dz} = \frac{-1}{c} [N - P] \quad \frac{dn}{dz} = \frac{1}{1-c} \text{Exp}[\alpha N(1 - N)] \quad c \neq 1, \quad \infty < z < \infty
\]

The system above has two uniform steady states points (0,0) (1,1). Now, we can take Jacobian and by using determinate eigenvalue of \( \lambda \), \( |A - \lambda I| = 0 \) to solve the system. We get two of the value of \( \lambda \) at (0,0)

We get:

\[
\begin{align*}
\lambda_1 &= \frac{1}{c}, \quad c \neq 0 \\
\lambda_2 &= \frac{-\alpha}{c - 1}, \quad c \neq 1
\end{align*}
\]

The stability of the steady state

If \( c > 1 \)

Then \( \lambda_1 \) is positive, \( \lambda_2 \) is negative we get the steady state saddle point, where all the value \( \alpha = 1 \).

If \( 0 < c < 1 \)

Then \( \lambda_1 \) is positive, \( \lambda_2 \) is positive we get the steady state unstable node, where all the value \( \alpha = 1 \).

If \( c < 0 \)

Then \( \lambda_1 \) is negative, \( \lambda_2 \) is positive we get the steady state saddle point, where all the value \( \alpha = 1 \).

When \( \lambda \) at (1,1)

\[
\begin{align*}
\lambda_1 &= \frac{1}{c}, \quad c \neq 0 \\
\lambda_2 &= \frac{\alpha}{c - 1}, \quad c \neq 1
\end{align*}
\]

The stability of the steady state

If \( c > 1 \)

Then \( \lambda_1 \) is positive, \( \lambda_2 \) is positive we get the steady state unstable node, where all the value \( \alpha = 1 \).

If \( 0 < c < 1 \)

Then \( \lambda_1 \) is positive, \( \lambda_2 \) is negative we get the steady state saddle point, where all the value \( \alpha = 1 \).

If \( c < 0 \)

Then \( \lambda_1 \) is negative, \( \lambda_2 \) is negative we get the steady state stable node, where all the value \( \alpha = 1 \).
3.1.3 Numerical Solution
To show this system (4), we will use pdepe code in Matlab. To show behavior branch and tip.

Figure 3. "The initial condition of solution to the system (4) with the parameters $\alpha=0.5$,"
4. The Conclusions
In this paper it was concluded that there is a relationship between traveling wave solution (c) and parameter \( \alpha \) where traveling wave (c) increase whenever the values of \( \alpha \) increase.
Through the following relation $\alpha = \frac{\alpha_3}{\gamma}$, we see that the value of ($\alpha$) is directly proportional to the value of ($\alpha_3$) and inversely to ($\gamma$), which means that the growth increases by increasing the value of ($\alpha$).[3]

5. References

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