Did NANOGrav see a signal from primordial black hole formation?

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We show that the recent NANOGrav result can be interpreted as a stochastic gravitational wave signal associated to formation of primordial black holes from high-amplitude curvature perturbations. The indicated amplitude and power of the gravitational wave spectrum agrees well with formation of primordial seeds for supermassive black holes.

Introduction – Strong evidence for a stochastic common-spectrum process, that can be interpreted as a stochastic GW signal, was found in the recent analysis of 12.5-year pulsar timing array (PTA) data collected by the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) [1]. NANOGrav observes a narrow range of frequencies around $f = 5.5$ nHz. The potential GW signal can be fitted by a power-law $\Omega_{GW} \propto f^\alpha$ with an amplitude $\Omega_{GW}(f = 5.5 \text{ nHz}) \in (3 \times 10^{-10}, 2 \times 10^{-9})$ and exponent $\alpha \in (-1.5, 0.5)$ at 1σ confidence level and with a small positive correlation between the amplitude and the exponent.

One possible source for a stochastic GW background at those frequencies are supermassive black hole (SMBH) binary inspirals [2], which give $\Omega_{GW} \propto f^{2/3}$. Their merger rate and therefore the amplitude of the GW signal that they will generate has, however, large uncertainties. Alternatively, instead of being astrophysical, a strong stochastic GW background at nanoHerz frequencies can originate from cosmological sources. For example, the NANOGrav result has been recently interpreted as a signal from cosmic strings [3, 4].

PTA experiments are sensitive to parts of the secondary GW background associated with the production of planetary mass or heavier primordial black holes (PBHs) from large curvature perturbations. They may therefore probe to two open problems: First, it is so far unknown whether the black hole (BH) binaries observed by the LIGO/Virgo collaboration [5–7] are of astrophysical or primordial origin. Although scenarios in which PBHs in the solar mass range comprise all of DM are heavily constrained [8–10], they might still account for the LIGO/Virgo BH mergers when they make up about 0.1%–10%. [9–13]. Second, PBH heavier than $10^9 M_\odot$ can provide a possible origin for the seeds of SMBHs [14–16] and act as generators for cosmic structures [16]. In particular, the origin of SMBH has been a long-standing problem in astrophysics as, although their existence at the center of most galaxies has been well established [17–19], their astrophysical production seems to require super-Eddington accretion [20] or direct collapse into intermediate mass BHs [21]. In the PBH scenario, even a small abundance of heavier than $10^3 M_\odot$ PBH can provide the seeds for the SMBH.

In this Letter we interpret the NANOGrav result as a stochastic GW background associated to PBH formation from high-amplitude peaks in the primordial curvature power spectrum. We consider three different well motivated shapes for a peak in the curvature power spectrum and, assuming the standard radiation dominated expansion history, we calculate the secondary GW spectrum and the corresponding PBH abundance and mass function.

Peaks in the curvature power spectrum – In order to perform our analysis in a model independent fashion, we consider three different shapes for the peak in the curvature power spectrum. First, the simplest idealized case is described by the delta function

$$P_\delta(k) = A k_D \delta_p(k-k_*) ,$$

where $\delta_D$ denotes the Dirac delta function, $k_*$ the position of the peak and $A$ its amplitude.

Second, typical peaks generated in single field inflation [22, 23] can be approximated by a broken power-law

$$P_{\text{PL}}(k) = A \frac{\alpha + \beta}{\beta(k/k_*)^{-\alpha} + \alpha(k/k_*)^\beta} ,$$

where $\alpha, \beta > 0$ describe respectively the growth and decay of the spectrum around the peak. In single field models where a peak is generated via a quasi inflection point, one typically has $\alpha \lesssim 4$ [23, 24]. Additionally, it follows that $\beta \gtrsim 0.5$, if the curvature power spectrum generated between the end of inflation and the peak obeys a power law. As a benchmark case we take in the following $\alpha = 4$ and $\beta = 0.6$.

Third, we consider a log-normal peak with an exponential UV cut-off,

$$P_{\text{LN}}(k) = A \exp \left[ \beta \left( 1 - \frac{k}{k_*} + \ln \left( \frac{k}{k_*} \right) \right) - \alpha \ln^2 \left( \frac{k}{k_*} \right) \right] ,$$

where $k_*$ is the UV cut-off and $\sigma$ the width of the log-normal peak.

1 We note that PTAs cannot probe the mass window below $10^{-10} M_\odot$ in which PBHs may constitute all dark matter, as the formation of these corresponds to much higher frequencies.

2 This follows from $k_{\text{end}} < 10^{23} \text{Mpc}^{-1}$ and $\sigma_{P_{\text{PL}}(k_{\text{end}})} < H_{\text{inf}}^2/(8\pi M_\odot^2) < 2.5 \times 10^{-11}$ [25] and a peak with $A < 0.05$ at $k_* > 10^4 \text{Mpc}^{-1}$. 
where \( \alpha, \beta > 0 \). For example, with \( \alpha = 0.17 \) and \( \beta = 0.62 \) this shape fits well the peak obtained in two-field inflation considered in Ref. [26], and we therefore use these values as a benchmark case.

\[
\Omega_{GW}(k) = 0.387 \Omega_R \left( \frac{g_s^4 g_{s*}^3}{106.75} \right) \frac{1}{6} \int_0^1 dx \int_0^\infty dy P \left( \frac{x-y}{2} \right) P \left( \frac{x+y}{2} \right) F(x,y),
\]

where \( \Omega_R = 5.38 \times 10^{-5} \) is the radiation abundance [43], the effective number of degrees of freedom are evaluated at the moment when the constant abundance is reached, roughly coinciding with the horizon crossing moment, and

\[
F(x,y) = \frac{288(x^2 + y^2 - 6)(x^2 - 1)^2(y^2 - 1)^2}{(x-y)^8(x+y)^8} \times \left[ \left( \frac{x^2 - y^2}{2} + \frac{x^2 + y^2 - 6}{2} \log \left| \frac{y^2 - 3}{x^2 - 3} \right| \right)^2 + \frac{\pi^2}{4} (x^2 + y^2 - 6)^2 \theta(y - \sqrt{3}) \right].
\]

Examples of different SIGW spectra are shown in Fig. 1. The amplitude of the spectrum depends very weakly on \( k_* \); only through the effective number of degrees of freedom. In Fig. 1 we used \( k_* = 3.6 \text{ Mpc}^{-1} \) which corresponds to the temperature \( T \approx 0.2 \text{ GeV} \). The position of the peak on \( \Omega_{GW} \) is determined by \( k_* \) and its amplitude inherits its scaling from the curvature power spectrum peak as \( \Omega_{GW} \propto A^2 \). For a power-law curvature power spectrum \( P \propto k^{\alpha/2} \), the SIGW spectrum behaves as \( \Omega_{GW} \propto k^\delta \).

**PBH formation** – Consider a fluctuation with a density contrast \( \delta \) at a comoving scale \( k \). In a radiation dominated Universe an overdensity for which \( \delta \) is larger than a threshold value \( \delta_c \) part of the horizon mass,

\[
M_k \approx 1.4 \times 10^{13} M_\odot \left( \frac{k}{\text{Mpc}^{-1}} \right)^{-2} \left( \frac{g_s^4 g_{s*}^3}{106.75} \right)^{-1/6},
\]

where \( M_p \) is the Planck mass and \( H_k, g_s, \) and \( g_{s*} \) denote the Hubble rate and the effective number of relativistic energy and entropy degrees of freedom, collapses to a BH almost immediately when the scale \( k \) re-enters horizon [44]. The value of the critical density contrast has been studied by numerical simulations [45–47]. Following Ref. [47] we use \( \delta_c \approx 0.42 \).

Taking critical scaling of the produced PBH mass [48–50] into account, the fraction of the total energy density

\[
\frac{\Omega_{PBH}}{\Omega} = \frac{M_k}{M_*} \left( \frac{k}{k_*} \right)^{-1} \left( \frac{g_s^4 g_{s*}^3}{106.75} \right)^{-1/6} \int_0^x \frac{d\ln k}{k^2},
\]

where \( \Omega_{PBH} \) denotes the density of PBHs, \( \Omega \) the total energy density, and \( x \) the redshift of the horizon re-entry. This integration is evaluated numerically and can be approximated by

\[
\frac{\Omega_{PBH}}{\Omega} \approx 10^{-3} \left( \frac{g_s^4 g_{s*}^3}{106.75} \right)^{-1/6},
\]

where \( \Omega = \Omega_k + \Omega_{\Lambda} \) is the total energy density, and \( \Omega_k = \Omega_R + \Omega_{\Lambda} \).
that collapses into BHs with mass $M$ can be estimated using the Press-Schechter formalism [49, 51]

$$\beta_k(M) = \frac{k}{\gamma} q^{1 + 1/\gamma} P_k(\delta_k(M)), \quad (7)$$

where $q = M/cM_k$, $\delta_k(M) = \delta_c + q^{1/\gamma}$, $\kappa \simeq 3.3$ [52], $\gamma \simeq 0.36$ [53], and $P_k(\delta)$ denotes the distribution of the perturbations. We assume that the latter to be Gaussian,

$$P_k(\delta) = \frac{1}{\sqrt{2\pi}\sigma_k} \exp \left( -\frac{\delta^2}{2\sigma_k^2} \right), \quad (8)$$

with the variance $\sigma_k^2$ is given by the smoothed density contrast,

$$\sigma_k^2 = \left( \frac{4}{9} \right)^2 \int_0^\infty \frac{dk'}{k'} e^{-k'^2/k} \left( \frac{k'}{k} \right)^4 P(k'). \quad (9)$$

At the present day the mass function of PBHs normalised to the total PBH abundance, $\int d\ln M \psi(M) = \Omega_{\text{PBH}}$ is

$$\psi(M) \simeq \frac{2 \times 10^{-12}}{\gamma} \frac{M}{M_\odot} \frac{M}{\text{Mpc}^3} \sigma_k^2. \quad (10)$$

By numerical fits we find that the PBH mass function for the curvature power spectra (1), (2) and (3) is roughly of the form

$$\psi(M) \propto M^{1 + 1/\gamma} e^{-c_1 (M/\bar{M})^{c_2}}, \quad (11)$$

where $c_1$ is fixed such that $\bar{M} = \langle M \rangle$, and $c_2 \simeq 2$ depending mildly on the amplitude of the peak. The low mass tail is dominated by PBHs forming close to the threshold and is thus determined by the details of the critical collapse [49]. The heavier tail gets exponentially suppressed as density perturbations capable of producing heavier PBHs become exponentially more unlikely. The abundance of PBHs and their mean mass are

$$\Omega_{\text{PBH}} \simeq c_0 A e^{-c_A/A} k_s/\text{Mpc}^{-1}, \quad \langle M_{\text{PBH}} \rangle \simeq c_M A^{c_M} M_k, \quad (12)$$

where $c_0 \simeq 10$, $c_A \simeq 1$, $c_M \simeq 3$ and $c_M \simeq 1/3$. In the following we show the PBH abundance relative to the observed dark matter abundance, $f_{\text{PBH}} \equiv \Omega_{\text{PBH}}/\Omega_{\text{DM}}$, where $\Omega_{\text{DM}} = 0.26$ [43].

**Results** – The first five bins of the NANOGrav analysis, for which a power-law fit is provided in [1], are in the narrow frequency range $f/\text{Hz} \in (2.5 \times 10^{-5}, 1.2 \times 10^{-8})$. Therefore, we expand the predicted spectrum around $k_0 = 2\pi \times 5.5 \text{ nHz} = 3.6 \times 10^7 \text{ Mpc}^{-1}$ as

$$\Omega_{\text{GW}} \simeq \Omega_{\text{GW},0} (k/k_0)^\zeta \quad (13)$$

and compare the experimental ranges for the parameters $\Omega_{\text{GW},0}$ and $\zeta \equiv d\ln \Omega_{\text{GW}}(k_0)/d\ln k$ with the theoretical predictions for SIGW from given primordial curvature spectra.

The SIGW spectrum has a flat region around $k \sim k_s$ and can thus provide a decent fit for the shape of the NANOGrav signal. Around the peak $\Omega_{\text{GW}} \simeq 10^{-5} A^2$, while from Eq. (12) we see that for $k_s = k_0$ PBHs will comprise a significant fraction of DM if $A \simeq 0.05$. This implies $\Omega_{\text{GW}} \simeq 10^{-8}$ which is in more than 2$\sigma$ tension with the NANOGrav analysis, as can be seen from Fig. 2. For that we have fixed $k_s$ and $A$ such that the expansion (13) holds around $k = k_0$, which, for a given shape of the curvature power spectrum peak, fix the PBH abundance and mass function. The thick contours instead show the 1$\sigma$ and 2$\sigma$ confidence level regions indicated by the NANOGrav results, obtained by a simple transformation [3] from the power-law fit to the five lowest frequency bins presented in [1]. We see that the PBH abundance in the 2$\sigma$ region is very small $f_{\text{PBH}} \ll 1$, and
at most $f_{\text{PBH}} \sim 10^{-6}$ is reached at the boundary of the $2\sigma$ region for a $\delta$-function spectrum.

Due to the experimental uncertainties in the slope, it is possible that the peak of the SGWB spectrum lies away from the NANOGrav range, especially if the SIGW spectrum has relatively flat tails. For the $\delta$-function and log-normal benchmark curvature spectra the tails are too steep and the slopes compatible with the NANOGrav range are near the peak of the spectrum. However, as can be seen from Fig. 1 for the broken power-law benchmark case the slope of the high frequency tail, $\xi = -2\beta = -1.2$, is within the $1\sigma$ region. Therefore, we can find $k_*$ far enough of $k_0$ such that the amplitude of the curvature power spectrum at the peak is sufficiently high to produce a large abundance of PBHs but at $k_0$ the amplitude of the induced GW spectrum is within the NANOGrav range.

We demonstrate this in Fig. 3 from which we see that in this case $f_{\text{PBH}} = 1$ can be reached within the $1\sigma$ region and the produced PBHs, for which $f_{\text{PBH}}$ is sizeable, are heavy, $\langle M_{\text{PBH}} \rangle \gtrsim 100M_\odot$. The CMB $\mu$ distortion observations \cite{54, 55} exclude the region left from the red line, bounding the PBH mass to $\langle M_{\text{PBH}} \rangle \lesssim 10^4M_\odot$.

As was outlined in the introduction, there are two PBH scenarios with a particular phenomenological relevance:

- The primordial origin for the LIGO/Virgo BH mergers requires a distribution of PBH with \cite{9}
  
  
  
  \[ f_{\text{PBH}} \sim 0.01, \quad \langle M_{\text{PBH}} \rangle \approx 20M_\odot \]  

  
  
  \[ \text{and a narrow width of the PBH mass function. As indicated by the red star in Fig. 3, we find that scenarios with broken power-law peaks would require a SIGW background that is too strong to be consistent with the NANOGrav signal.} \]

  
  
  We remark, that changing the shape of the primordial curvature power spectrum is unlikely to relieve this tension. Since the allowed values of $A$ decrease with $\xi$, picking a bigger value for $\beta$ would not increase the scale $k_*$ sufficiently so that a sizeable $f_{\text{PBH}}$ would be within the $2\sigma$ region. Therefore, and because very flat peaks in the curvature power spectrum are not easily obtained from the model building point of view, we find it unlikely that the NANOGrav result could be related to the PBH scenario for the LIGO/Virgo events.

- For the primordial origin for SMBH seeds we assume a mass range of $M_{\text{PBH}} \in (10^3, 10^6)M_\odot$ \cite{14}. To roughly estimate the required seed abundance, we assume that the SMBHs comprise about 0.025% of the stellar mass in their host galaxies \cite{56}, while stars make up a fraction of about 1% of the Universe's matter content \cite{57}, implying that the total SMBH density is about a factor of $10^6$ smaller than the DM density. Using $10^7$ as a representative value for the SMBH mass, we then find that the primordial seeds can be characterized by

  
  \[ f_{\text{PBH}} \sim 10^{-13}\langle M_{\text{PBH}} \rangle /M_\odot, \quad \langle M_{\text{PBH}} \rangle > 10^3M_\odot. \]  

  
  
  As seen from Fig. 3, there is a range that is consistent with the NANOGrav signal and the constraints from $\mu$-distortion of the CMB, and implies production of a sufficiently large abundance of primordial SMBH seeds. We remark that this scenario can be tested by future PIXIE like experiments \cite{58}.

Conclusions – We showed that the NANOGrav result can be interpreted as a signal from PBH formation from peaks in the curvature power spectrum. We found that the secondary GW backgrounds consistent with NANOGrav will, in general, correspond to the production of a negligible amount of PBH dark matter and cannot be related to the PBH scenario for LIGO/Virgo merger events. Nevertheless, the NANOGrav signal agrees well with scenarios in which PBHs provide the seeds of supermassive black holes.

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\begin{figure}
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\includegraphics[width=\textwidth]{fig3}
\caption{The thick black solid and dashed lines indicate the $1\sigma$ and $2\sigma$ ranges for the amplitude $\Omega_{\text{GW}}(f = 5.5\text{ nHz})$ at $\xi = -1.2$. The thin solid, dashed and dotted lines show the PBH abundance $f_{\text{PBH}} \equiv \Omega_{\text{PBH}}/\Omega_{\text{DM}}$, and the color coding shows the mean mass of the PBH mass function, assuming a power-law peak in the curvature power spectrum (2) with $\alpha = 4$ and $\beta = 0.6$. The region left of the red line is excluded by the COBE/Firas results on CMB $\mu$-distortions. The green band indicates the values compatible with SMBH seed formation and the red star shows the PBH scenario for the LIGO/Virgo GW events.}
\end{figure}
