Porosity and dissipative effects in Peristalsis of hydro-magneto nanomaterial: Application of biomedical treatment

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Abstract
The non-uniformity value which currently many applications of magneto-hydrodynamics are found in medicine where drug delivery happens through peristaltic pumping phenomena, various magnetic drugs are released to target tumor diseases and to control the drug flow movement to the desired area. Owing to these facts, the aims of this article is to examine the simultaneous influence of magneto-hydrodynamics (MHD) and slip effect on unsteady peristaltic nanofluid flow in a non-uniform porous channel of finite length. The constituent governing equations for the model have been examined under the approximation of long wave length and small Reynolds value. Keeping kerosene oil and ethylene glycol as base fluids with polystyrene chosen as nanoparticle. The current analysis is carried out for the peristaltic flow transference which carries innumerable industrialized employments. The incompressible, viscous, electrically conducting flow is studied in wave form. Here, exact method is employed to obtained closed form solutions. We have implemented computational software packages “Mathematica” as a main tool in order to obtain explicit expressions for axial velocity, temperature, stream function, pumping phenomenon and bolus formation. Obtained solutions are used for graphical analysis against different physical parameters. It is concluded that axial velocity increments for higher Hartmann number and slip parameter near the walls. The porosity effects increases the temperature whereas the temperature field shows increasing behavior for larger Brinkman number.

Keywords
Peristaltic unsteady flow, nanofluid particle, porous media, Brinkman number, unequal channel, slip effect

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Introduction
The peristalsis analysis has gained attention of engineers and investigators in many industrial and engineering processes. The mechanism of transporting dietary contents through tightening and expanding consequence of propagating wave alongside of the channel/tube is termed as the phenomenon of peristaltic impulsion. The presence of such progression is encountered in vast range of industrialized employments and physiological processes which can be seen in the bile duct,
gastrointestinal tract and further glandular channels. The urine transferal to bladder from kidney, swallowed dietary substance through gullet, transfer of lymph in lymphatic vessels, ovum gestication in fallopian tubes, moving of spermatozoa inside cervical canal and many more processes illuminate the significance of this mechanism. Further, the mechanism also parts the influential findings in diverse industrial, medical developments where we come across peristaltic pumping in roller and finger pumps, heart lung appliance, unusual pharmacological transport systems and loco movement of worms etc. Since the ground-breaking research of Latham along Shapiro et al. on peristalsis momentum emerged, extensive hypothetical and investigational methodologies were attempted for the understanding of peristaltic progression of hydrodynamic fluids in wide-ranging assumptions of extensive wavelength, small Reynolds number, trivial wave amplitude etc. Even though, urge for exploration in this direction stemmed through soundings are listed here.3–12

Various ordinary liquids like ethylene glycol, silicon oil, kerosene oil, water, etc., generally fall in as bad conductors due to their low value of thermal conductivity which reduces the amount of heat transmission. Choi in his model presented the terminology of nanofluids which is a combination of metallic nano-sized particles dispersed in adequate amount to the based fluid for the enhancement of thermal conductivity. Later his revolutionary perception was deliberated by many researchers sharing foremost contributions offering proficient models like Buongiorno, Maxwell, Hamilton and Crosser models for nanofluids. Presently nanofluid utilization has stopped at nothing less due to their inspiring features and its involvement in diverse directions like solar synthesis, biotic identifying, biochemical, nuclear reactors, the chemical engineering gas detecting, surgical procedure, protein production, field therapy, drug dispersal, cancer finding and cure, photodynamic treatment, neuro electronic interfaces, nonporous medium for size baring chromatography, shedding new light on cells, molecular motors resembling kinesis, charge centered filtration in the kidney basal membrane, and further a number of experts assumed of using nano foods in tricking the body to feel full for long thus resulting in reduced eating desire. Different studies on nanofluids over diverse geometries are also the valuable contributions of the researchers.

The study of incompressible viscous flow through non uniform channels hold pronounced significance in research area having countless applications in practical, physiological, biochemical, civil, industrial, environmental, aerospace, chemical engineering along with canals and river flow pattern understanding. In human body also such flows can be seen when blood movement is observed where capillaries are linked or connect with arteries. The innovative work for the understanding of such flows were carried out by Hamel and Jeffery which later was great source of inspiration for many researchers resulting in enough investigation for various flow properties where only a few are shared in the references.26–29

The analysis of electrically conducting physiological fluids having incompressible, viscous, peristaltic flow over a divergent or convergent channel under the sway of externally acting magnetic field share fascination among theoretical study as well as applications of mathematical modeling in numerous biotic and industrialized requirements. Applications of MHD (magneto-hydrodynamics) that possibly come across in various directions are diverse cancer treatments, magnetic resonance imaging (MRI), influence on blood movement, magneto therapy, hyperthermia, blood pumps acting as carrier of cardiac processes for blood movement having arterial ailments like arteriosclerosis, engineering glitches like electromagnetic casting, incarceration of plasma and continual metal cast practice, high temperature tools like power generators, and movement of liquid metals in the cooling arrangement of innovative nuclear reactor reflecting a broadening or a shriveling flow movement in between parallel channels with variant cross-sections. Also, the giant magneto resistive (GMR) equipment which applies magnetic field with a precise delicate sensor to detect slightest motion of an object in the magnetic arena facilitating immensely in learning of peristaltic activity in tube-like configurations like fallopian pipe, bowel and possibly even in the vas deferens. Moreover, it is an essential criteria to consider flexible walls of the channel when gap in between the walls fall at small distance apart. A number of expedient learning in this topic is cited over in references.26–30

The increased amount of applications in biophysiological and industrialized flows handled with suitable analysis is deliberated diversely and extensively. Observing and keeping in focus the research literature, the aim of present study is to investigate the flow hydrodynamics under the simultaneous influences of velocity slip and transverse magnetic field on the peristaltic motion of incompressible, viscous nanomaterial over a flexible porous channel of varying cross-sectional area. The present analysis significantly shares connection with many bio-physiological and industrialized problems concerning drug deliverance, electromagnetic treatment and canal flows.

**Problem formulation and solution analysis**

Here, consider the peristaltic transference of an electrically conducting incompressible viscous nanofluid in a two-dimensional non-uniform symmetric horizontal
channel of finite-length. Schematic representation of the current consideration is shown in Figure 1. The influence of induced magnetic field is ignored with the consideration of taking in account the small value of magnetic Reynolds value. The stream movement is induced using the propagating sinusoidal wave trains having speed $c$ constantly moving down the walls of the channel. Further, there are heat absorption/generation, viscous dissipation effects and slip concepts added to the considered flow problem. The space in between the flexible walls is assumed porous and its geometric representation comprise of the form,

\[ y = h(x, t) = a_1(x) + b \sin \frac{2\pi}{\lambda} (\bar{x} - c\bar{t}) \] (1)

using $a_1(x) = a_o + m_1\bar{x}$

(2)

Now, take the flow movement in $\bar{X}$-direction and the uniform magnetic field having strength $B_0$ acting normal toward its propagation. Here, $a_1(x)$ is defined as the width of the channel taken in half at some axial space $x$ after inlet, $b$ represents the peristaltic wave amplitude, $a_o$ gives value of half width at inlet, $\lambda$ stands for wave length, $t$ is the time, $c$ denotes the velocity of propagation and $\bar{x}$ specifies the direction of wave progression. The constant $m_1$ expresses magnitude of non-uniformity which makes the volume flow rate to vary down the stream length. Indeed, keeping $m_1$ value equal to zero makes the channel uniform.

We need to improvise our choice of frame of reference to incorporate steady laminar flow. In effect fixed frame of reference $(\bar{X}, \bar{Y})$ has instability for steady flow behavior therefore we switch to option of moving frame of reference $(\bar{x}, \bar{y})$ having speed $c$ that remains constant all the way.

Thus, introducing transformation is implemented to shift from laboratory to wave frame as:

\[ \bar{x} = X - c\bar{t}, \bar{u} = \bar{U} - c, \bar{p}(\bar{x}, \bar{y}, \bar{t}) = P(X, Y, t), \bar{y} = Y, \bar{v} = \bar{V}, \] (3)

Here, considered corresponding rheological conservation laws for two-dimensional incompressible nanofluid flow in the influence of magnetic field acting transversely in a rectangular Cartesian coordinate classification may be modeled as, \(^{42,43}\)

\[ \partial u + \frac{\partial v}{\partial y} = 0, \] (4)

\[ \rho_{nf} \left( \frac{\partial u}{\partial t} + \bar{u} \frac{\partial u}{\partial x} + \bar{v} \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( \frac{2\mu_{nf}}{\partial x} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_{nf} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right) - \sigma B_0^2 u - \frac{\mu_{nf}}{k} \bar{u}, \] (5)

\[ \rho_{nf} \left( \frac{\partial v}{\partial t} + \frac{\partial v}{\partial y} \right) \]

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \] (6)

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \] (7)

where, $\bar{x}$, $\bar{y}$ signify direction continuing on and orthogonal to the horizontal channel of finite length, $\bar{u}$, $\bar{v}$ are the velocity components along $\bar{x}$ and $\bar{y}$ paths on the axis, respectively, $T$ defines local temperature of the liquid, pressure term specified by $\bar{p}$, $\sigma$ expresses electrical conductivity, $B_0$ specifies magnetic strength, kinematic viscosity is indicated by $\nu$, and symbol $k$ denotes permeability of porous medium. Further, $\rho_{nf} (\rho c_p)_{nf}$, $\mu_{nf}$, $\alpha_{nf}$ and $k_{nf}$ define effective density, heat capacitance, effective dynamic viscosity, effective thermal diffusivity and effective thermal conductivity of nanoparticle fluid are defined by:

\[ \frac{\mu_{nf}}{\mu_f} = T, \frac{k_{nf}}{k_f} = K, K = \frac{A - 2B}{A + 2B}, \] (8)
The thermo-physical physignomies of nanofluid polystyrene in two different base fluids ethylene glycol and kerosene oil are documented in Table 1.44,45

Furthermore, we use the mentioned dimensionless terms:

\[
x = \frac{x}{\lambda}, \quad u = \frac{u}{c}, \quad y = \frac{y}{a}, \quad v = \frac{v}{c}, \quad p = \frac{a^2 \bar{p}}{c \mu_f},
\]

\[
\theta = \frac{(\bar{T} - T_0)}{T_0}, \quad c = \frac{cT}{cT}, \quad \phi = \frac{b}{a},
\]

\[
\nu = \frac{\mu}{\rho}, \quad R_e = \frac{ac}{v}, \quad \rho_\text{nf}(ca) = \frac{\rho_{\text{nf}}}{\mu_{\text{nf}}}, \quad \frac{p}{\rho_{\text{nf}}}, \delta = \frac{a}{\lambda},
\]

\[
L = \frac{l}{\alpha}, M^2 = \frac{\sigma_B c a^2}{\mu_f},
\]

\[
S = L \frac{\mu_{\text{nf}}}{\mu_f}, P_t = \frac{\mu c_p}{\kappa} = \frac{\mu_{\text{nf}}(c_p)_{\text{nf}}}{k_{\text{nf}}}.
\]

\[
B_t = E_c P_t = \left(\frac{a^2}{(c_p)_{\text{nf}} T_0}\right) P_t, m = \frac{m_1 \lambda}{a}.
\]

Here, the \(x,y\) correspond to axial velocity and velocity in transverse direction, \(\phi\) represents the solid nanoparticle volume fraction, \(\delta\) the amplitude ratio, \(S\) the value of velocity slipping effect, Reynolds number is described by \(R_e\), \(P\) defines Darcy number, \(M\) the Hartmann quantity of measure, \(B_t\) the Brinkman number value, \(\theta\) presents wave number, \(\theta\) denotes dimensionless temperature.

Moreover, we obtain dimensionless formulation of the model by implementing (9) in the non-dimensional system (4)–(8) above along with use of expressions relating stream function with velocity field given by:

\[
u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad \] (10)

Under influence of low Reynolds \((R_e \rightarrow 0)\) number along with large wave length \((0 \ll \lambda \rightarrow \infty)\) assumptions, we obtain the simplified dimensionless system representations in wave frame as follows:

\[
\frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left(\frac{1}{(1-\varphi)^2} \frac{\partial^2 \psi}{\partial y^2} \right) - M^2 \left(\frac{\partial \psi}{\partial y} + 1\right)
\] (11)

Differentiating (11) we get equation free of pressure gradient term

\[
0 = \left(\frac{1}{(1-\varphi)^2} \right) \frac{\partial^4 \psi}{\partial y^4} - \left(\frac{M^2 + 1}{P}\right) \frac{\partial^2 \psi}{\partial y^2},
\]

\[
\frac{\partial p}{\partial y} = 0,
\]

\[
\left(\frac{A - 2B}{A + 2B}\right) \frac{\partial^2 \theta}{\partial y^2} + B_t \left(\frac{1}{(1-\varphi)^2} \right) \left(\frac{\partial \psi}{\partial y}\right)^2 = 0,
\]

subjected to conditions on the boundary mentioned below:

\[
\begin{align*}
\frac{\partial^2 \psi}{\partial y^2} &= 0, \quad \psi = 0, \text{at } y = 0, \\
\frac{\partial \psi}{\partial y} + S S_{\text{cy}} &= -1, \quad \psi = F, \text{at } y = h, \\
\frac{\partial \theta}{\partial y} &= 0 \quad y = 0, \quad y = h.
\end{align*}
\]

Keeping expression for,

\[
y = h = h(x) = 1 + mx + \phi \sin(2\pi x).
\]

The conditions \(\psi(0) = 0\) and \(\psi(h) = F\) imposed here have great physical significance as they maintain constant cross-sectional flow rate across the channel walls. The boundary condition \(\partial \psi/\partial y = -1 - SS_{\text{cy}}\) considered at \(y = h\) is termed as slip condition which illustrates different velocities of both fluid and wall while boundary condition \(\psi(h) = F\) corresponds to constant flow rate at each cross section of the channel. The temperature \(\theta(h) = 0\) physically represents constant temperature throughout at the wall. The velocity boundary condition \(\partial^2 \psi/\partial y^2 = 0\) at \(y = 0\) illustrates symmetry condition at the middle of the channel whereas \(\psi(0) = 0\) represents the constant velocity throughout at the middle of the channel. \(\partial \theta/\partial y = 0\) at \(y = 0\) represents symmetry in temperature throughout at middle of the channel.

The Analytical solution of the boundary value problem in (11)–(17) brings forward the closed form solutions of temperature profile, stream function, velocity field, volume flow rate, pressure gradient, pressure rise and mean flow rate decided mathematically as follows:
\[ \theta(x,y) = -\frac{B_0 e^{-\Delta t_y \nu} (F + h) A_0^{\frac{3}{2}} (P \left(e^{2A_2} - e^{2A_1} - e^{2A_3} + e^{2A_4} \right) + (R^2 - y^2)A_0) }{8K \left( h^2 P A_0^2 - (A_2)^2 + (h^2 P A_0^2 + (A_2)^2) \cosh[2A_2] + 2h A_0 \sqrt{P} A_5 \sinh[2A_2] \right)} \] (18)

\[ \psi(x,y) = \frac{e^{-\Delta t_y \nu} \left( (e^{A_2} - e^{A_1}) \right) (F + h) A_4 - e^{A_1} \left( -F A_0^2 \left( \sqrt{P} - A_3 \right) + A_4 \right) + e^{A_3} \left( F A_0^2 \left( \sqrt{P} + A_3 \right) + A_4 \right) y )}{2 \left( h \sqrt{P} A_0^2 \cosh[2A_2] + A_0 A_5 \sinh[2A_2] \right)} \] (19)

\[ \mu(x,y) = \frac{e^{-\Delta t_y \nu} \left( (e^{A_2} - e^{A_1}) \right) (F + h) A_4 + e^{A_1} \left( F A_0^2 \left( \sqrt{P} - A_3 \right) - A_4 \right) + e^{A_3} \left( F A_0^2 \left( \sqrt{P} + A_3 \right) + A_4 \right) y )}{2 \left( h \sqrt{P} A_0^2 \cosh[2A_2] + A_0 A_5 \sinh[2A_2] \right)} \] (20)

where

\[ K = \frac{k_f}{k_{nf}} = \left( \frac{k_s + 2k_f}{k_s + 2k_f - 2\phi(k_f - k_s)} \right), \quad T = \frac{1}{(1-\phi)^2} \frac{1}{S}, \]

\[ A_0 = \sqrt{1 + M^2 P}, \quad A_1 = \frac{\sqrt{1 + M^2 P}}{\sqrt{P} \sqrt{T}}, \]

\[ A_2 = \frac{\sqrt{1 + M^2 P}}{\sqrt{P} \sqrt{T}}, \quad A_3 = \frac{\sqrt{1 + M^2 P S^2}}{\sqrt{T}}, \]

\[ A_4 = \frac{P \sqrt{1 + M^2 P \sqrt{T}}.A_5 = \left( hS + P \left( -1 + hSM^2 \right) \right) \sqrt{T} \] (21)

The volumetric rate of flow is assessed by means of

\[ F = \int_0^{h(x)} \psi dy = \int_0^{h(x)} \frac{\partial \mu}{\partial y} dy \] (22)

Axial pressure gradient is expressed by:

\[ \frac{dp}{dx} = -\frac{e^{-\Delta t_y \nu} (F + h) A_0^{\frac{3}{2}} \left( (1 + e^{2A_1} \right) \sqrt{P} + \left( -1 + e^{2A_1} \right) A_0 S \sqrt{T} \right)}{2P(h \sqrt{P} A_0^2 \cosh[2A_2] + A_0 A_5 \sinh[2A_2])} \] (23)

We can estimate the value of dimensionless pressure rise by the integral expression:

\[ \Delta P = \int_0^1 \left( \frac{dp}{dx} \right) dx \] (24)

The mean flow rate which is in relation with flow rate considered in wave frame. This relationship can be presented by:

\[ F = Q - 1. \] (25)

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**Exact results with graphical behavioral analysis discussion**

This segment reflects graphic analysis of behavioral trait followed by velocity, temperature, pumping and trapping phenomenon on the variations of pertinent physical parameters. The non-uniform permeable channel influenced by transverse magnetic field and slipping effect is deliberated generously.

**Trait of velocity distribution on variant flow parameters**

Figure 2(a) to (g) focuses on analysis of the velocity profile for Hartman number $M$, nanoparticle volume fraction $\phi$, slope parameter $m$, Darcy number $P$, velocity slip $S$, flow rate $Q$ and amplitude ratio $\phi$. The graphical results reveal pathway of axial velocity along horizontal direction experiencing an increase in its magnitude near the walls with increasing $M$, $\phi$, $P$ and $S$, whereas reverse trend is noticed in the central channel area. It is witnessed that larger Hartman number $M$ leads to higher magnitude of Lorentz forces which consequently decrease the deformation in the fluid near the central region. Therefore, velocity field decrements in central region. Moreover, velocity field increases near the channel wall due to the low intensity of magnetic effects. Also, increased nanoparticle volume fraction $\phi$ amplifies the flow movement which can be noticed near the walls. In fact, higher $\phi$ increases the concentration of nanoparticles near the wall and as the result, velocity field is increased. Away from the walls, low nanoparticles concentration leads to low magnitude of velocity. It is observed in Figure 2(d) that increasing Darcy number $P$ weakens the velocity field near the walls and increment in velocity field is seen at the central region. Physically, larger Darcy number $P$ enhances the porous effect in the medium which promotes permeability which in turn lowers the resistance to flow through the porous layers. Therefore, velocity field increases away
from the walls. Furthermore, opposite trend is wit
tnessed near the channel walls. Dominant slip parameter $S$ results in decrement of velocity profile near the walls, while increased velocity is observed in central region. Physically, by the increment in velocity slip parameter, the adhesive force between fluid particles and wall reduces, so velocity profile increases near the walls.

Moreover for higher $Q$, $\varphi$ and $m$ velocity flow depicts increasing trend throughout the channel. It is justified that large $Q$ leads to more fluid deformation and hence, axial velocity is enhanced for large values of $Q$. Axial velocity field enhances by increasing amplitude ratio $\varphi$. Physically, increasing $\varphi$ corresponds to larger fluid layer amplitude which gives rise to fluid motion by

Figure 2. (a) Sway of $M$ on $u(y)$, (b) sway of $\phi$ on $u(y)$, (c) sway of $m$ on $u(y)$, (d) sway of $P$ on $u(y)$, (e) sway of $S$ on $u(y)$, (f) sway of $Q$ on $u(y)$, and (g) sway of $\varphi$ on $u(y)$. 
which fluid’s axial velocity enhances. The non-uniformity parameter along the channel shows diminishing angle values of pathway for convergent channel \((m<0)\), rising value for divergent channel \((m>0)\) and straight channel \((m=0)\) when no change. Velocity profile for channel with slope positive stands higher in magnitude when compared with slope value zero and channel having slope negative results at \(x = 0\).

**Trait of temperature distribution on variant flow considerations**

It is pragmatic in Figure 3(a) to (f) to view the temperature field illustrations experience changes when parameters Hartman number \(M\), nanoparticle volume fraction \(\phi\), slope parameter \(m\), Darcy number \(P\), velocity slip \(S\), average rate of the flow \(Q\), amplitude fraction value \(f\) and Brinkman number \(Br\) are added to the channel. Examining the graphical illustrations for temperature profile a diminishing performance is observed for increasing \(M, S\) and mounting trait prevails for parameter \(Br, \phi, m, P\) through the entire channel length. Temperature field decays for larger Hartmann number \(M\). In fact large \(M\) provides relatively low intensity of resistive forces and as a result, temperature field declines. Here, temperature field declines for higher slip parameter \(S\) due to decrease in adhesive forces which generate low heat energy and as a result, temperature field diminishes. Moreover for increasing \(Br, \phi, m, P\) graphical illustration shows that the base fluid ethylene glycol experience higher rises in temperature than kerosene oil. A significant temperature rise is observed for raised value of \(Br\), whereas fall for higher \(M\). Increasing Brinkman number which is a fraction of viscous heat generation over outside environment heating and as a result viscous dissipation produces lesser heat

![Graphical Illustrations](image)

**Figure 3.** (a) Sway of \(\phi\) on \(\theta(y)\), (b) sway of \(\phi\) on \(\theta(y)\), (c) sway of \(m\) on \(\theta(y)\), (d) sway of \(P\) on \(\theta(y)\), (e) sway of \(S\) on \(\theta(y)\) and (f) sway of \(Br\) on \(\theta(y)\).
conduction which leads to significant rise in temperature profile. The temperature profile shows divergent channel \( m > 0 \) domination to uniform and convergent channel \( m < 0 \). The temperature field of convergent channel is lower compared to the uniform channel. It is noticed that temperature field grows for greater \( P \). Physically, it is justified that intensification in Darcy number results in enhance resistance through pores which consequently increases the temperature.

**Trait of pumping characteristics on variant flow controls**

The contents of this section exhibits the influence of emerging parameters Hartman number \( M \), nanoparticle volume fraction \( \phi \), slope parameter \( m \), Darcy number \( P \), velocity slip \( S \), flow rate \( Q \), amplitude ratio value \( f \), axial pressure gradient \( \frac{dp}{dx} \) along with dimensionless pressure change per wavelength \( \nabla p \). Wider the channel

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**Figure 4.** (a) Sway of \( M \) on \( \frac{dp}{dx} \), (b) sway of \( \phi \) on \( \frac{dp}{dx} \), (c) sway of \( m \) on \( \frac{dp}{dx} \), (d) sway of \( P \) on \( \frac{dp}{dx} \), (e) sway of \( S \) on \( \frac{dp}{dx} \), (f) sway of \( Q \) on \( \frac{dp}{dx} \) and (g) sway of \( \phi \) on \( \frac{dp}{dx} \).
pathway lesser the pressure gradient which means the fluid will move without imposing large $\frac{dP}{dx}$ where on contrary a narrow channel to maintain same flux will need flow to exert much amplified $\frac{dP}{dx}$ value. It is noticed in Figure 4(a) to (g) that the pressure gradient $\frac{dP}{dx}$ profile faces decline on maximizing parameters value of $M$, $\phi$, $Q$ and elevates in the case of increasing $P$, $S$. The graphical representation of $\frac{dP}{dx}$ for divergent channel $m>0$ faces decreasing trend and increases in regard to convergent channel $m<0$. Moreover greater the value of $\phi$ considered the graph of $\frac{dP}{dx}$ results in a sinusoidal wave with amplitude diminishing amplitude. Figure 5(a) to (e) reflects dimensionless pressure rise $\Delta p$ to dimensionless flow rate $Q$ graphical analysis for parameters Hartman number $M$, nanoparticle volume fraction $\phi$, slope parameter $m$, Darcy number $P$, velocity slip $S$. The peristaltic pumping is defined for three regions that are peristaltic pumping ($\Delta p > 0$), free pumping ($\Delta p = 0$) and augmented pumping ($\Delta p < 0$) region. It can be seen that increasing $M$, $\phi$ value $\nabla p$ shares inverse relation in the augmented pumping region and directly proportional affect for the peristaltic pumping area. Reverse analysis is seen for the case with greater values of $m$, $P$, $S$ we notice increment in augmented pumping section and decrement in peristaltic pumping, hence sharing direct relation in augmented and inverse relation in pumping region. A linearized relation is witnessed throughout the illustration for pressure rise ($\Delta p$) along with volumetric drift rate $Q$.

**Trait of bolus trapping on variant flow factors**

Trapping is an important fascinating happening in the process of peristalsis in which the formed bolus within move along the velocity of streams and bolus that get trapped are then pressed in the pathway of propagating wave with the same speed as of the wave in motion. Figure 6(a) to (d) engrosses on mechanism of bolus
Figure 6. (a) Sway of slope parameter $m$ on the stream function $\psi$, (b) sway of Hartman number $M$ on the stream function $\psi$, (c) sway of Darcy number $P$ on the stream function $\psi$ and (d) sway of velocity slip effect $S$ on the stream function $\psi$. 
trapping for various pertinent parameters Hartman number $M$, slope parameter $m$, Darcy number $P$, velocity slip $S$ and helping in the understanding of pressure relationship with streamlines. The variation impact observed for the slope parameter $m$ reveals that the convergent channel has maximized streamlines flow when compared to uniform channel and divergent channel. Whereas divergent channel has lesser strength of flow that the uniform channel case. Illustration of the pattern followed by the streamlines for $M$ increased MHD parameter results in decreasing stream line strength throughout the channel and trapping vanishes in the center region. It is observed for raised parameter values $P$, $S$ the phenomena of trapping reduces size and dies out whereas streamlines in the center experience upsurge. No bolus formation is seen by the walls for all chosen parameter values and symmetrical graphical results are also noticed.

Conclusions

Magneto nanomaterial flow in non-uniform porous channel having finite length is investigated here. The mixture of polystyrene with base fluid ethylene glycol and other base fluid kerosene oil is observed in combined form under the slip phenomenon. The outcomes are described and also graphically illustrated using Mathematica10 command. The findings of the present work analysis on the pertaining parameters Hartman number $M$, nanoparticle volume fraction $\phi$, slope parameter $m$, Darcy number $P$, velocity slip $S$, flow rate $Q$, amplitude ratio $\phi$ and Brinkman number $B_r$ added to the horizontal channel is mentioned as:

1. The velocity faces diverse relation for enhanced parameter value of $M, S, \phi$ and $P$ having its maximum value of flow at the center also fall happens for rising parameter $M, S, \phi$. On the contrary reverse scenario occurs in case of rising values for porosity $P$.
2. Increasing $Q, \phi$ parameter values lessens magnitude of pressure gradient whereas for velocity profile it shares a growing trend.
3. Temperature field is directly proportional to increasing behavior of parameters $B_r, P, \phi$ whereas inversely proportional to growing parameters $M, S$.
4. An enhancement in magnitude of pressure gradient $\nabla p_{s}$ side by side of streamlines is noticed for amplified values of permeability, slip $(P, S)$ and reduction takes over for both when Hartman number $M$ is raised higher. Moreover, pressure field is weakened with increased nanoparticles volume fraction $\phi$.
5. Pressure rise $\nabla p$ graphs climb up in the augmented pumping region on rising values of $m, P, S$ and take a decline in the pumping area whereas reverse trend is noted for $M, \phi$.
6. The non-uniformity parameter for values of $m>0$ share the most amplified graphical views for axial velocity, temperature, axial pressure gradient $\frac{\partial p}{\partial x}$ and streams $\psi$. Whereas, results for $m = 0$ and $m < 0$ fall behind $m > 0$ with $m < 0$ always staying with lesser attained values than $m = 0$.

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**Appendix**

**Notation**

| Symbol | Description |
|--------|-------------|
| $a_1$  | Channel width |
| $b$    | Peristaltic wave amplitude |
| $\lambda$ | Wave length |
| $c$    | Velocity of propagation |
| $a_0$  | Half width value |
| $m_1$  | Magnitude of non-uniformity |
| $\bar{u}$, $\bar{v}$ | Velocity components |
| $\rho_{nf}$ | Effective density |
| $\rho_f$ | Density of base fluid |
| $\rho_{nf}$ | Effective mass density |
| $\mu_{nf}$ | Effective dynamic viscosity |
| $\sigma$ | Electric conductivity |
| $B_0$  | Magnetic strength |
| $k$    | Permeability of porous medium |
| $(c_p)_{nf}$ | Effective heat capacity |
| $k_{nf}$ | Effective thermal conductivity |
| $\phi$ | Nanoparticle volume fraction |
| $k_f$  | Thermal conductivity of base fluid |
| $T$    | Fluid temperature |
| $M$    | Hartmann number |
| $\psi$ | Stream function |
| $P$    | Darcy number |
| $B_r$  | Brinkman number |
| $S$    | Slip parameter |
| $F$    | Volumetric flow rate |
| $Q$    | Flow rate |
| $\theta$ | Dimensionless temperature |
| $R_e$  | Reynolds number |
| $\Delta P$ | Pressure rise |
| $Ec$   | Eckert number |
| $Pr$   | Prandtl number |
| $m$    | Slope parameter |
| $\varphi$ | Amplitude ratio |
| $(c_p)_{f}$ | Heat capacity of base fluid |