Coherent state excitations and string-added coherent states in gauge-gravity correspondence

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Abstract

We analyze detailed properties of BPS coherent states and their connection to gravity. We interpret the group integral coherent state as a path integral over auxiliary variables coupled to the elementary letters of the theory. The eigenvalues of coherent state amplitudes can be viewed as collective coordinates of giant gravitons. Inspired by the above coherent states and by the integrability, we construct a new type of coherent states in the SL(2) sectors and their cousin PSU(1,1|2) sectors, analogous to the aforementioned coherent states. The large spin and small spin limits can be obtained by varying coherent state amplitudes. We add string words onto the BPS coherent states, and this gives rise to string-added coherent states. The insertions of string multi-words can be viewed as operator-insertions in this path integral. We describe BPS states and near BPS states building upon these coherent states in gauge-gravity correspondence. For example, string-added coherent states and their near BPS spectra are analyzed. This approach is particularly convenient for heavy excited states.
1 Introduction

The gauge-gravity correspondence [1, 2, 3] is a nontrivial correspondence between a quantum theory with gravity in the bulk and a different quantum system on the boundary. The correspondence allows us to perform calculations related to superstring theory and quantum gravity from working on the quantum field theory side. On the other hand, the superstring theory provides the UV completion of supergravity, and is a UV-complete quantum gravity theory. Integrability [4] has greatly increased our understanding of the gauge-gravity correspondence. The correspondence also reveals the nature of the emergent spacetime e.g. [5]-[9]. As we see, the bulk emerges dynamically from the quantum mechanical description that lives in fewer dimensions.

On the gravity side, giant graviton branes [10]-[14] are excited states. In the context of gauge-gravity correspondence, emergent backreacted geometries correspond to highly excited states in the quantum field theory side, such as the bubbling geometries [15] [13] [14]. The states in the Hilbert space of the quantum field theory are explicitly mapped to the gravity side. Analysis in the field theory side shows that these different states live in the same Hilbert space. The dual large operators and their representation bases have been illuminated [13] [14]. These heavy operators involve emergent backreacted geometries in the dual quantum gravity system.

Here we focus on states which have interesting gravitational properties. One interesting type of states are coherent states [16, 17, 18]. Gravity dual of coherent states has been analyzed. The coherent states in this paper are related to the excited states of the gravitational spacetime. These set-ups help to address the question how do curved spacetimes emerge from dual quantum theory on the boundary. Coherent states also appear very widely in many other contexts of physics, and here we concentrate on special types of coherent states.

There are various representation bases for large operators. The large operators includes those describe giant gravitons and backreacted emergent geometries. The large operators also describe further excitations on these heavy excited states. The correlation functions between light operators and large operators can also be computed. The large operators can be expanded systematically in terms of representation bases. In some sense, the larger the operators are, there are more information that can be stored with the operators. For more details along some of these ideas, see e.g. [19]-[28], [8], [18], [16]. Various bases can be transformed into each other. Operators of giant gravitons are also analyzed by various important and insightful approaches [29]-[35] and [16]. These approaches are closely related to the scenarios needed for this paper. Various ideas have been put forward, in order to make the computations with large operators more efficient and convenient.
Coherent state operators in gauge-gravity duality have also been considered in e.g., [36, 18, 37, 16, 17] and their related references. These different bases have different labelings due to their different symmetry properties of the multi-parameters of the coherent states. In particular, [16] has manifest permutation symmetry of the parameters. Mixed states and entangled states of coherent state operators in the gauge-gravity correspondence have also been considered [38]. Moreover, mixed states and entangled states of Young tableau states were considered [39]. Many interesting aspects of coherent states in relation to giant graviton states, were explored in [16, 17] and their related references. The multi-parameter in the coherent states are collective coordinates of giant gravitons.

The quantum gravitational system dual to these heavy excited state operators, involves backreacted emergent geometries. Analysis on the field theory side shows that these excited states live in the same Hilbert space of the gravity side. Since they live in the same Hilbert space, we can dynamically relate them using the Hamiltonian in the same Hilbert space. For example one can superpose states and compute transition probabilities between different states living in the gravity side, e.g. [40]–[44], [18], [37] and related discussions.

When the spacetime is dynamical and quantum, the local quantum field theory seems not adequate to describe these situations. On the other hand, string theory is able to describe these situations very well. The set-ups discussed in this paper can be put inside superstring theory, and the set-ups here are UV-complete. Somehow we have gone beyond local quantum field theory.

In Sec. 2, we analyze detailed properties of coherent states and their connection to gravity, among other things. We focus on above-mentioned heavy states. Inspired by the above coherent states and by the integrability, in Sec. 3, we construct a new type of coherent states in the $SL(2)$ sectors and their cousins such as $PSU(1,1|2)$ sectors, among other things. The large spin and small spin limits can be obtained by varying coherent state amplitudes.

In Sec. 4, we analyze string-added coherent states and their near BPS spectra, among other things. We can add some string words onto the BPS coherent states, and this gives the string-added coherent states. We identify and elaborate some classes of near BPS string-added coherent states, whose anomalous dimension energy can be extracted conveniently. The anomalous energy of the near BPS state is usually smaller than its bare dimension energy. Hence these energy are controllable on both gauge theory side and gravity side. One can expand around supersymmetric backgrounds to compute their anomalous energy from both gauge theory and gravity side. These states can be considered as excitations on the BPS background. This method is very useful for near BPS states.

Finally in Sec. 5, we make conclusions and discuss some closely related aspects. Note
Upon the completion of this work, we received the appearance of [17] which have worked out $SO(N)$ and $Sp(N)$ cases; Many analysis in the current paper here also works for these and other very interesting cases.

## 2 BPS coherent states and collective variables

We consider a new class of BPS coherent states with manifest permutation symmetries. This class of interesting coherent states in nonabelian gauge theories have been constructed in [16]. Here, we first analyze more detailed properties of the states. We use $4D \mathcal{N} = 4$ gauge theory as an important concrete example. This theory is an example of nonabelian gauge theories arising from multiple $D$-branes.

We begin by introducing some general set-ups. Operators $O$ acting on the vacuum correspond to states $|\Psi\rangle$ in the Hilbert space, i.e. $|\Psi\rangle = O |0\rangle$. The operators can be built by the fundamental fields in the theory. For example, in nonabelian gauge theories, we can have a complex matrix field $Z$, where we have some $U(N)$ gauge group. This $U(N)$ symmetry originates from the symmetry on multiple $D$-branes. The field quanta are created and annihilated by ladder operators. When acting on the operators, we have the following correspondence

$$
Z \leftrightarrow a^\dagger_Z, \quad \partial_Z \leftrightarrow a_Z \\
Z^i_j = (Z)^i_j \leftrightarrow (a^\dagger_Z)^i_j \\
(\partial_Z)^i_j = \frac{\partial}{\partial (Z)^i_j} \leftrightarrow (a_Z)^i_j 
$$

(2.1)

The action $(a_Z)^i_j, (\partial_Z)^i_j$ is equivalent to Wick contraction with $(Z)^i_j$. More detailedly, the right hand side of the correspondence contains $\frac{1}{\sqrt{N}}$ norm factor for each elementary field, and hence $\text{Tr}(a^\dagger_Z^n |0\rangle)$ corresponds to $\frac{1}{\sqrt{N}} \text{Tr}(Z^n)$. This is exact for any finite and fixed $N$. The convention here is that we have the canonical commutation relations and normalizations for $a^\dagger_Z, a_Z$.

Similarly for other complex matrix fields $Y, X$ in the $U(N)$ gauge theory, one has the correspondence

$$
Y \leftrightarrow a^\dagger_Y, \quad \partial_Y \leftrightarrow a_Y \\
X \leftrightarrow a^\dagger_X, \quad \partial_X \leftrightarrow a_X 
$$

(2.2)

In the similar way, $\text{Tr}(a^\dagger_Y^n)$ corresponds to $\text{Tr}(Y^n)$ and $\text{Tr}(a^\dagger_X^n)$ corresponds to $\text{Tr}(X^n)$. 

3
The new class of the BPS coherent states is
\[ F[\Lambda] = \frac{1}{\text{Vol}_U} \int dU \exp \left( \text{Tr}(U \Lambda U^{-1} a_Z^\dagger) \right) |0\rangle. \] (2.3)

Here \( U \in U(N) \) is an unitary action on the \( N \) D-branes. The action of the unitary \( U \), originates from the nonabelian gauge symmetry of the \( N \) D-branes. The integral is over the group manifold, with the condition \( \frac{1}{\text{Vol}_U} \int dU \cdot 1 = 1 \). This defines a coherent state \( |F[\Lambda]\rangle \). The conjugate bra state is \( \langle F[\Lambda]| \) and is defined by conjugating \( a_Z^\dagger \) (respectively \( \Lambda \)) to \( a_Z \) (respectively \( \bar{\Lambda} \)).

We can also write the integral as \( \int dg \exp(\text{Tr}(\Lambda(g^{-1}a_Z^\dagger g))) |0\rangle \) and view \( g \) as auxiliary variables coupled to the \( a_Z^\dagger \) fields. We then integrate out these auxiliary variables. Here we interpret the integration as a path integral of the auxiliary variables. As a path integral, it can be performed by a saddle point method.

The inner products of the states are
\[ \langle F[\Lambda']|F[\Lambda]\rangle = \bar{F}[\bar{\Lambda}'] \ast F[\Lambda], \] (2.4)
\[ \bar{F}[\bar{\Lambda}'] \ast F[\Lambda] = \frac{1}{\text{Vol}} \int dU \exp(\text{Tr}(\bar{U}^{-1}\Lambda \bar{U} \bar{\Lambda}')). \] (2.5)

The normalizations of the states are \( \mathcal{N}_\Lambda = \langle F[\Lambda]|F[\Lambda]\rangle \), where
\[ \mathcal{N}_\Lambda = \frac{1}{\text{Vol}} \int dU \exp(\text{Tr}(U^{-1}\Lambda \bar{U} \bar{\Lambda})). \] (2.6)

For more details, see [16]. Eqn. (2.5), (2.6) are Harish-Chandra-Itzykson-Zuber (HCIZ) integrals [45, 46, 47], whose computation can be very conveniently performed by localizations [47] and by saddle point methods.

We can define an unitary displacement operator
\[ D(\Lambda) = \frac{1}{\text{Vol}_U} \int dU \exp \left( \text{Tr} \left( U \Lambda U^{-1} a_Z^\dagger - U \bar{\Lambda} U^{-1} a_Z \right) \right). \] (2.7)

Using Baker-Campbell-Hausdorff (BCH) formulas and commutation relations of the ladder operators, we write
\[ F[\Lambda] = \mathcal{N}_\Lambda^\frac{1}{2} D(\Lambda) |0\rangle \] (2.8)
and we show (2.3) and (2.8) are equivalent. \( D(\Lambda) \) is an unitary operation in \( U(\mathcal{H}) \) acting on the Hilbert space \( \mathcal{H} \) of states. In this writing, the term in the exponent is manifestly anti-Hermitian, and hence the operation \( D \) is an unitary operation. The advantage is
that it is manifestly unitary on the Hilbert space. Using BCH formulas and commutation relations of the ladder operators, we write $D$ also in the following way

$$D(\Lambda) = N - \frac{1}{2} \int dU \exp \left( \text{Tr} \left( U \Lambda U^{-1} a^\dagger Z \right) \right) \exp \left( -\text{Tr} \left( U \bar{\Lambda} U^{-1} a^\dagger Z \right) \right).$$  \hspace{1cm} (2.9)

Eqn. (2.7) and (2.9) are equivalent. The state $F(\Lambda)$ is an eigenstate of the annihilation operators and we have $\text{Tr}(a^\dagger Z F(\Lambda)) = \text{Tr}(\Lambda^n F(\Lambda))$ for gauge invariant observables.

We can also define phase shift operator

$$R(\Theta Z) = \frac{1}{Vol} \int dU \exp(\text{Tr}(iU \Theta Z U^{-1} a^\dagger Z a Z)).$$ \hspace{1cm} (2.10)

Here $\Theta Z$ is a phase matrix, whose eigenvalues $\theta^i_z, i = 1, ..., N$, are phases rotating the eigenvalues of $\Lambda Z$. Here we let $\Lambda Z, \Theta Z$ commute, i.e. $[\Lambda Z, \Theta Z] = 0$. Hence

$$\frac{1}{Vol} \int dU \exp(\text{Tr}(iU \Theta Z U^{-1} a^\dagger Z a Z)) |F[\Lambda Z]) = \frac{1}{Vol} \int dU \exp(\text{Tr}(U \Lambda Z e^{i\Theta Z} U^{-1} a^\dagger Z)) |0 \rangle$$ \hspace{1cm} (2.11)

where we used BCH formulas. The eigenvalues are rotated by $\lambda^i_z \rightarrow \lambda^i_z e^{i\theta^i_z}$, and correspondingly, $\bar{\lambda}^i_z \rightarrow \bar{\lambda}^i_z e^{-i\theta^i_z}$.

The number operator is $\hat{N} Z = a^\dagger Z a Z$. We have $\hat{J} Z = \text{Tr}(a^\dagger Z a Z)$ and $J Z = \langle \hat{J} Z \rangle$ measures the expectation value of the number of $Z$ fields in the state. The Hamiltonian operator is $\hat{H}_0 = \text{Tr}(a^\dagger Z a Z)$ and $H_0 = \langle \hat{H}_0 \rangle$ counts the excitation energy. Our Hamiltonian on the state space of the coherent states is

$$E_{F[\Lambda]} = \langle \hat{H} F[\Lambda] \rangle = N^{-1}_F \langle F(\Lambda) | \hat{H} F(\Lambda) \rangle = N^{-1}_F \hat{F}[\Lambda] \ast \hat{H} \ast F[\Lambda].$$ \hspace{1cm} (2.12)

One can use a more sophisticated Lagrangian formalism \cite{16} to obtain an effective action on coherent states derived by \cite{16}. These constructions and expressions also works for the other matrix fields $Y, X$ in the $U(N)$ gauge theory by using their ladder operators $a^\dagger_Y, a^\dagger_X$ for (2.3).

The coherent states for coulomb branches also work similarly. Consider coulomb branch gauge group $G_1 \times G_2$ inside full gauge group $G$. In that case, we need to embed $U(N_1) \times U(N_2)$ into $U(N)$, where $N_1 + N_2 = N$, and the matrix $\Lambda Z$ splits into blocks e.g. $\Lambda^{(1)}_Z, \Lambda^{(2)}_Z$ corresponding to the two gauge groups, and $\text{rk}(\Lambda^{(1)}_Z) + \text{rk}(\Lambda^{(2)}_Z) = \text{rk}(\Lambda_Z)$. Coulomb branch operators have been considered in \cite{41} which is related to Gelfand-Tsetlin patterns \cite{48}. The integration $\int_G dU$ turns into $\int_{G_1 \times G_2} dU_i$, where
$U_i \in U(N_i), i = 1,2$. In this case, the permutation symmetry $S_N$ also reduces to $S_{N_1} \times S_{N_2}$.

Ref. [16] has also constructed a new class of eighth BPS coherent states, with manifest permutation symmetries. It is constructed by enlarging $U \Lambda Z U^{-1} a_Z^\dagger$ in (2.3) to more terms, adding additional parameters $\Lambda_Y$ and $\Lambda_X$. We can take a limit case of the eighth BPS states constructed by [16], by making $\Lambda_X \equiv 0$. For $\Lambda_{(Z,Y)} \neq 0$, these are quarter BPS states. They are in the kernel of the anomalous dimension dilatation operator, and this means $[\Lambda_Z, \Lambda_Y] = 0$. This is a good advantage of using BPS coherent states.

These states are

$$F[\Lambda_Y, \Lambda_Z] = \frac{1}{Vol} \int dU \exp(\text{Tr}(U \Lambda_Y U^{-1} a_Y^\dagger + U \Lambda_Z U^{-1} a_Z^\dagger)) \, |0\rangle.$$  \hspace{1cm} (2.13)

Now first, in the following, we check that the states are quarter BPS at one loop and two loop orders. The one-loop dilatation operator and effective Hamiltonian is given by

$$\Delta_2 = g_{YM}^2 \text{Tr} \left( [a_Y^\dagger, a_Y] [a_Y, a_Y] \right).$$  \hspace{1cm} (2.14)

As pointed out by [16], when we have the dilatation operator act on $F$, we get a result that is equal to zero when the parameters $\Lambda_Y, \Lambda_Z$ are commuting matrices. The $a_Y, a_Z$ appears on the rightmost. When acting on the above coherent states, we have the identification $a_Y \leftrightarrow \Lambda_Y, a_Z \leftrightarrow \Lambda_Z$, and $[a_Y, a_Z] \leftrightarrow [\Lambda_Y, \Lambda_Z] = 0$. Hence we see explicitly that the action of the one-loop dilatation operator on (2.13) is zero.

The two-loop dilatation operator is

$$\Delta_4 = -\frac{g_{YM}^2}{2} : \text{Tr} \left( [\left[ a_Y^\dagger, a_Y^\dagger \right], a_Z] [a_Y, a_Z], a_Y^\dagger \right) :$$

$$-\frac{g_{YM}^2}{2} : \text{Tr} \left( [\left[ a_Y^\dagger, a_Z^\dagger \right], a_Y] [a_Y, a_Z], a_Y^\dagger \right) :$$

$$-\frac{g_{YM}^2}{2} : \text{Tr} \left( [\left[ a_Y^\dagger, a_Y^\dagger \right], T^a] [a_Y, a_Z], T^a \right) :$$  \hspace{1cm} (2.15)

Here $g = \frac{g_{YM}^2}{8\pi^2}$ with our convention. The terms in the dilatation operators are normal ordered. The normal ordering symbols here indicate that the annihilation operators within the normal ordering symbols do not act on fields inside the normal ordering. We computed the action of the two-loop dilatation operator on the above coherent states, its action on (2.13) is again zero, due to $[\Lambda_Z, \Lambda_Y] = 0$. Then, by using nonrenormalization theorems, e.g. [49, 23], we are convinced that they are also higher-loop BPS.
The state is a generating function of single-trace and multi-trace states of the form \(s\text{Tr}(Y^{m_1}Z^{n_1}Y^{m_2}Z^{n_2}...Y^{m_l}Z^{n_l})\). The generated states are quarter BPS single-trace and multi-trace operators built by \(Z, Y\). They have been investigated in e.g. [49]. The two matrices \(Z, Y\) correspond to \(C^2\) of the transverse dimensions of \(N\) D-branes.

The unitary displacement operator is

\[
D(\Lambda Y, \Lambda Z) = \frac{1}{\text{Vol}_U} \int dU \exp(\text{Tr}(U \Lambda Z U^{-1}a^\dagger_Z + U \Lambda Y U^{-1}a^\dagger_Y) - \text{Tr}(U \bar{\Lambda} Z U^{-1}a_Z + U \bar{\Lambda} Y U^{-1}a_Y))
\]

and we write \(F = N_F \frac{1}{2} D(\Lambda Y, \Lambda Z) |0\rangle\). It is equivalent to

\[
D(\Lambda Y, \Lambda Z) = \frac{N_F^{-\frac{1}{2}}}{\text{Vol}_U} \int dU \exp(\text{Tr}(U \Lambda U^{-1}a^\dagger_Z + U \Lambda Y U^{-1}a^\dagger_Y) - \text{Tr}(U \bar{\Lambda} U^{-1}a_Z + U \bar{\Lambda} Y U^{-1}a_Y)).
\]

(2.16)

The phase shift operator is

\[
R(\Theta Y) = \frac{1}{\text{Vol}_U} \int dU \exp(\text{Tr}(i U \Theta Y U^{-1}a^\dagger_Y a_Y)).
\]

(2.17)

Here we let \(\Lambda Y, \Theta Y\) commute. The action of it rotates \(\lambda^i_y \rightarrow \lambda^i_y e^{i\theta^i_y}\).

The Hamiltonian in the state space of coherent states is \(\hat{H}_0 = \text{Tr}(a^\dagger_Y a_Y + a^\dagger_Z a_Z)\). This counts the BPS energy \(H_0 = \langle \hat{H}_0 \rangle\). On the other hand, \(\hat{J}_Y = \text{Tr}(a^\dagger_Y a_Y)\) and \(J_Y = \langle \hat{J}_Y \rangle\) measures the expectation value of the number of \(Y\) fields in the state.

Now we perform contracting the ladder operators and the inner product is

\[
N_F = \bar{F} \{\bar{\Lambda} Z, \bar{\Lambda} Y\} * F \{\Lambda Z, \Lambda Y\} = I(U, \Lambda_{\alpha}, \bar{\Lambda}_{\alpha})
\]

(2.19)

where \(\alpha = Z, Y\). The integral is

\[
I(U, \Lambda_{\alpha}, \bar{\Lambda}_{\alpha}) = \frac{1}{\text{Vol}_U} \int dU \exp \left( \Gamma_{\text{HCIZ}}(U, \Lambda_{\alpha}, \bar{\Lambda}_{\alpha}) \right).
\]

(2.20)

We denote the exponent in (2.20) \(\Gamma_{\text{HCIZ}}\). The exponent is

\[
\Gamma_{\text{HCIZ}}(U, \Lambda_{\alpha}, \bar{\Lambda}_{\alpha}) = \text{Tr}(U \Lambda Y U^{-1}a_Y + U \Lambda Z U^{-1}a_Z).
\]

(2.21)

The path integral (2.20) can be computed by a saddle point method. The saddle point equation is \(d \Gamma_{\text{HCIZ}} = 0\). Now we write \(dK = UdU^{-1}\), and we have

\[
d \Gamma_{\text{HCIZ}} = \text{Tr} \left( dK \{\bar{\Lambda} Y, U \Lambda Y U^{-1} \} \right) + \text{Tr} \left( dK \{\bar{\Lambda} Z, U \Lambda Z U^{-1} \} \right) = 0.
\]

(2.22)
The conditions for saddle points are
\[
[\Lambda_Y, U\Lambda_Y U^{-1}] + [\Lambda_Z, U\Lambda_Z U^{-1}] = 0. \tag{2.23}
\]
When \(U\) is a permutation matrix \(P \in S_N\), these are saddle points. The group integral can be viewed as a path integral of the auxiliary variables \(U\). Then the integral \(2.20\) can be computed by localization and saddle point method as described in \[16\].

We now turn to the eighth BPS case in more details. The states are
\[
F[\Lambda_Z, \Lambda_X, \Lambda_Y] = \frac{1}{Vol} \int dU \exp \left( Tr(U\Lambda_X U^{-1}a_X^\dagger + U\Lambda_Y U^{-1}a_Y^\dagger + U\Lambda_Z U^{-1}a_Z^\dagger) \right) \left| 0 \right>.
\tag{2.24}
\]
It can also be written as
\[
F[\bar{\Lambda}] = \frac{1}{Vol} \int dU \exp \left( Tr(U\bar{\Lambda} U^{-1}a_0^\dagger) \right) \left| 0 \right> , \tag{2.25}
\]
where \(\bar{\Lambda} = (\Lambda_X, \Lambda_Y, \Lambda_Z), \bar{a}_0^\dagger = (a_X^\dagger, a_Y^\dagger, a_Z^\dagger)\). This form of writing is also convenient for theories with global symmetries or flavor symmetries. As pointed out in \[16\], if the parameters \(\Lambda_X, \Lambda_Y, \Lambda_Z\) mutually commute, the states are annihilated by the one-loop dilatation operator. The effective Hamiltonian is given by
\[
\Delta = g_Y^2 M Tr([a_X^\dagger, a_Z^\dagger][a_Z, a_X]) + g_Y^2 M Tr([a_Y^\dagger, a_Z^\dagger][a_Z, a_Y]) + g_Y^2 M Tr([a_X^\dagger, a_Y^\dagger][a_Y, a_X]). \tag{2.26}
\]
When acting on the coherent states, we have the correspondence \(a_X \leftrightarrow \Lambda_X, [a_X, a_Z] \leftrightarrow [\Lambda_X, \Lambda_Z], [a_X, a_Y] \leftrightarrow [\Lambda_X, \Lambda_Y], \) etc. Hence all the three terms acting on the states are zero, and the action of the dilatation is zero. Hence \(\Lambda_X, \Lambda_Y, \Lambda_Z\) are required to commute pairwise. By using nonrenormalization theorems \[49\] \[23\], we can infer that they are in the kernel of the anomalous dilatation operator.

The unitary displacement operator is similar to the above \(2.16\) with more terms in the exponent added. And we assume \(\Lambda_{(X,Y,Z)}\) are mutually commuting. By using BCH formulas, it is equivalent to the following, with the normalization factor,
\[
D(\Lambda_X, \Lambda_Y, \Lambda_Z)
= \frac{\mathcal{N}_F}{Vol_U} \int dU \exp \left( Tr(U\Lambda_X U^{-1}a_X^\dagger + U\Lambda_Y U^{-1}a_Y^\dagger + U\Lambda_Z U^{-1}a_Z^\dagger) \right) \exp \left( -Tr(U\bar{\Lambda} U^{-1}a_0^\dagger + U\bar{\Lambda}_X U^{-1}a_X + U\bar{\Lambda}_Y U^{-1}a_Y) \right). \tag{2.27}
\]
We write \(F = \mathcal{N}_F^2 D(\Lambda_X, \Lambda_Y, \Lambda_Z) \left| 0 \right>\). Similarly we define the phase shift operator \(R(\Theta_X)\) similar to \(2.10\) and \(2.18\).
By performing usual manipulations contracting ladder operators, the overlap $N_F$ was computed in [16]. The overlap is a HCIZ integral, where the exponent is similar to (2.21) and have three terms, with $\Gamma_{HCIZ}(U, \Lambda_\alpha, \bar{\Lambda}_\alpha) = \text{Tr}(U \bar{\Lambda} U^{-1} \bar{\Lambda})$. The conditions for saddle points are in [16]. It is in a similar form as (2.23) with three terms. The integrals can be efficiently calculated by localization and saddle point method.

The Hamiltonian in the space of coherent states is $H_0 = \text{Tr}(a^\dagger_Z a_Z + a^\dagger_Y a_Y + a^\dagger_X a_X)$, with Casimir energy subtracted. The energy in the space of coherent states is $E_F = \langle \hat{H} \rangle_F := N_F^{-1} \langle F[\Lambda(X,Y,Z)][\hat{H}]F[\Lambda(X,Y,Z)] \rangle = N_F^{-1} \hat{F} \ast H \ast F$.

The angular momentum operator is $\hat{J}_X = \text{Tr}(a^\dagger_X a_X)$ and $J_X = \langle \hat{J}_X \rangle$ measures the expectation value of the number of $X$ fields in the state. We have that $J_Z = \text{Tr}(\bar{\Lambda}_Z \Lambda_Z)$, $J_Y = \text{Tr}(\bar{\Lambda}_Y \Lambda_Y)$, $J_X = \text{Tr}(\bar{\Lambda}_X \Lambda_X)$, and

$$H_0 = J_X + J_Y + J_Z.$$  \hfill (2.29)

The vevs are

$$\langle \text{Tr}(a^\dagger_Z a_Z^n) \rangle_F = \langle \text{Tr}(\bar{\Lambda}_Z \Lambda_Z^n) \rangle_F, \quad \langle \text{Tr}(a^\dagger_Y a_Y^n) \rangle_F = \langle \text{Tr}(\bar{\Lambda}_Y \Lambda_Y^n) \rangle_F,$$

$$\langle \text{Tr}(a^\dagger_X a_X^n) \rangle_F = \langle \text{Tr}(\bar{\Lambda}_X \Lambda_X^n) \rangle_F.$$  \hfill (2.30)

Note that e.g. $\langle \bar{\Lambda}_Z \Lambda_Z \rangle^n$ is conveniently a Hermitian matrix with real eigenvalues. We then look at the eigenvalue of the parameters $\lambda^x_\alpha, \lambda^y_\alpha, \lambda^z_\alpha$. In the large $N$ limit, $\rho(\lambda) = \rho(\lambda^{(x,y,z)})$ is an eigenvalue density. The coherent state $|F[\Lambda(X,Y,Z)]\rangle$ may also be labeled as $|\rho(\lambda^{(x,y,z)})\rangle$ in the large $N$ limit. We can also calculate the right hand sides of (2.30) for the case when $n$ is not an integer, using eigenvalue formalism, e.g.

$$\langle \text{Tr}(\bar{\Lambda}_Z \Lambda_Z^n) \rangle_F = \int d^6 \lambda \rho(\lambda)(\bar{\lambda}^x \lambda^x)^n.$$  \hfill (2.31)

The methods developed in the approach of matrix eigenvalue effective models are very useful in this context [8],[50]-[54]. These vevs are dual to the multi-pole moments of the gravitational geometries in the gravity side, and can be measured in the gravity side via the methods of [55]-[61]. The $n = 1$ case of the similar quantities were analyzed in [60].

These states are heavy excited states in the gravity side. The particularly interesting regimes are $J_X, J_Y, J_Z$ of order $N^2$ where there is backreaction of the excitation onto the spacetime geometry. In these regimes, single-trace and multi-traces are more difficult and less convenient. The coherent states are especially convenient in their large amplitude regimes. The gravity waves are collective excitations of these large $N$ eigenvalue distributions. The order $N$ regimes describe multi giant gravitons. The individual
eigenvalue of the coherent state multi-parameter is the collective coordinate of giant gravitons on the gravity side \[16\], see also related observations \[18, 37\].

This new type of coherent states have been constructed by \[16, 17\]. The BPS coherent states are generating functions of single-trace and multi-trace states. The multi-trace states are multi closed string states. Hence the BPS coherent states are also generating functions of multi closed string states. It is possible to generalize to more matrices and more flavors, in other circumstances or in other types of gauge theories. A special class of quarter BPS coherent states were also constructed in \[62\], which has not made use of the manifest permutation symmetries.

3 The \(SL(2)\) sectors, their cousins, and meromorphic versions

We construct a new type of coherent states in the \(SL(2)\) sectors and their cousin \(PSU(1,1|2)\) sectors. We also construct a meromorphic version of coherent states, which can be transformed back to the full coherent states when \(\det\Lambda_Z \neq 0\). We give transformation relations between different types of coherent states, among other things.

Inspired by the coherent states in \[16\], whose aspects are analyzed in more details in Sec. 2, we construct a coherent state as generating function of

\[
\text{Tr}(D_+^n Z D_+^n Z\ldots D_+^n Z\ldots),
\]

(3.1)

\(n_\alpha = 0,1,2,\ldots\) and so on. Here \(D_+\) is a light-cone covariant derivative operator. We write \(D_+ = n^\mu D_\mu\) where \(n^\mu\) is a light-like vector. \(D_+\) is in the \((1,0)\) representation of the local Lorentz group. When \(D_+\) are dilute, they can be viewed as impurities on top of \(Z\). Similar conceptions are also raised in \[20, 63\]; see also \[64\]. For more details on \(SL(2)\) sectors and their relations to integrability and QCD, see for example \[4, 65, 66\].

We can construct the ladder operators corresponding to adding derivatives \(n\) times on top of \(Z\),

\[
c_D^\dagger a_Z^\dagger \leftrightarrow D_+^n Z, \quad c_D^\dagger a_Z^\dagger \leftrightarrow D_+ Z, \quad a_Z^\dagger \leftrightarrow Z.
\]

(3.2)

We construct a type of coherent states as the generating function of (3.1),

\[
K[R_Z, \Lambda_Z] = \frac{1}{Vol} \int dU \exp(\text{Tr}(UR_ZU^{-1}c_D^\dagger a_D^\dagger a_Z)) \exp(\text{Tr}(U\Lambda_ZU^{-1}a_D^\dagger)) \mid 0 \rangle.
\]

(3.3)

Here \(R_Z, \Lambda_Z\) commute. We make sure that before acting \(c_D^\dagger\), there is already a background of \(Z\)-fields due to the right-most operator in (3.3). The angular momentum
operator and spin operator are $\hat{J}_Z = \text{Tr}(a_Z^\dagger a_Z)$ and $\hat{S} = \text{Tr}(c_D^\dagger c_D)$. Here $S = \langle \hat{S} \rangle$ measures the expectation value of the number of $D_+$. On this state (3.3), the expectation values are $J_Z = \langle \hat{J}_Z \rangle = \text{Tr}(\bar{\Lambda}_Z\Lambda_Z)$ and $S = \langle \hat{S} \rangle = \text{Tr}(R_Z R_Z \bar{\Lambda}_Z^2 \Lambda_Z^2)$.

An alternative formulation is the state

$$G[T_{DZ}, \Lambda_Z] = \frac{1}{\text{Vol}} \int dU \exp(\text{Tr}(U T_{DZ} U^{-1} c_D^\dagger a_Z)) \exp(\text{Tr}(U \Lambda_Z U^{-1} a_Z^\dagger Y)) |0\rangle.$$  (3.4)

Here $T_{DZ}, \Lambda_Z$ commute. We have that $J_Z = \text{Tr}(\bar{\Lambda}_Z\Lambda_Z)$ and $S = \text{Tr}(T_{DZ} T_{DZ} \bar{\Lambda}_Z \Lambda_Z)$.

These are heavy states in the gravity side. If the $D$ is very dilute, it is the small spin limit. This is when $\alpha = \frac{N-1}{\text{Tr}(T_{DZ} \bar{\Lambda}_Z \Lambda_Z)}$ is much smaller than one. On the other hand, if the $Z$ is dilute and $D$ is dense, it is the large spin limit. This is when $\alpha$ is much larger than one. The large spin limit has been discussed also in [26].

States of this type (3.3) can also be constructed for the operators analyzed in sections 2. We construct the following state

$$G[T_Y, \Lambda_Z] = \frac{1}{\text{Vol}} \int dU \exp(\text{Tr}(U T_Y U^{-1} a_Y^\dagger a_Z)) \exp(\text{Tr}(U \Lambda_Z U^{-1} a_Z^\dagger Y)) |0\rangle.$$  (3.5)

Here $T_Y, \Lambda_Z$ commute. The right-most operator in (3.5) is an eigenstate of $a_Z$. When acting on the coherent state background, we replace the annihilation operator by the vev. Here we replace the $a_Z$ by vev $\Lambda_Z$, when the left operator in (3.5) acts on the right-most operator. The action $a_Z^\dagger a_Z$ annihilates a $Z$ field and substitute it with a $Y$ field, e.g. $Z^2 \rightarrow Y Z$. Hence we have $J_Z = \text{Tr}(\bar{\Lambda}_Z\Lambda_Z)$ and $J_Y = \text{Tr}(T_Y T_Y \bar{\Lambda}_Z \Lambda_Z)$.

The state (3.5) is a meromorphic version of (2.13). $G[T_Y, \Lambda_Z]$ is the excitation on top of $F[\Lambda_Z]$. It is a good approximation to $F[\Lambda_Y, \Lambda_Z]$, in the regime $\text{det} \Lambda_Z \neq 0$. We see that

$$G[T_Y, \Lambda_Z]|_{T_Y = \Lambda_Z^{-1} \Lambda_Y} \sim F[\Lambda_Y, \Lambda_Z],$$  (3.6)

when $\text{det} \Lambda_Z \neq 0$ and identifying $T_Y = \Lambda_Z^{-1} \Lambda_Y$. When $\text{det} \Lambda_Z = 0$, $G[T_Y, \Lambda_Z]$ can not be transformed back to $F[\Lambda_Y, \Lambda_Z]$, since there is pole in $T_Y$. Here $T_Y$ is a meromorphic function of $\Lambda_Z$ and not a holomorphic function, hence there is pole in $T_Y$. This is when $\text{det} \Lambda_Z = 0$. $F[\Lambda_Y, \Lambda_Z]$ is a smooth resolution of $G[T_Y, \Lambda_Z]$, extending it to cover the pole locus $\text{det} \Lambda_Z = 0$.

The meromorphic version provides new understandings of the coherent states and new understanding to the important question why we have the background fields $Z$. These background fields, e.g. $Z$ fields, serve as new effective vacuum for new excitations.
Similarly, for including $X$, we have the state

$$G[T_X, T_Y, \Lambda_Z] = \frac{1}{Vol} \int dU \exp(\text{Tr}(UT_X U^{-1}a_X^\dagger a_Z + UT_Y U^{-1}a_Y^\dagger a_Z)) \exp(\text{Tr}(U\Lambda_Z U^{-1}a_Z^\dagger)) |0\rangle.$$  

(3.7)

Here $T_X, T_Y, \Lambda_Z$ mutually commute. We have that $J_Z = \text{Tr}(\bar{\Lambda}_Z \Lambda_Z)$, $J_Y = \text{Tr}(\bar{T}_Y \bar{T}_Y \bar{\Lambda}_Z \Lambda_Z)$, and $J_X = \text{Tr}(T_X \bar{T}_X \bar{\Lambda}_Z \Lambda_Z)$.

In the regime $\det \Lambda_Z \neq 0$,

$$G[T_X, T_Y, \Lambda_Z] \sim F[\Lambda_X, \Lambda_Y, \Lambda_Z],$$

(3.8)

with $T_X = \Lambda_Z^{-1} \Lambda_X$, $T_Y = \Lambda_Z^{-1} \Lambda_Y$. Hence state (3.7) is a meromorphic version of (2.24). $G[T_Y, \Lambda_Z], G[T_X, T_Y, \Lambda_Z]$ can be viewed as meromorphic versions of quarter and eighth BPS coherent states.

Here, it is very similar to having $U(2)$ and $U(3)$ flavor symmetry, or more generally $U(N_F)$ symmetry if there are $N_F$ sets of fields with global symmetry. Similar expansions also work for quiver gauge theory with global symmetry, as well as their Coulomb branches. On the other hands, there are theories with these flavor symmetries.

Turning to the $SL(2)$ sector, this sector can be enlarged to $PSU(1,1|2)$ sector. The $PSU(1,1|2)$ sector has been discussed from the point of view of integrability, e.g. [67]. Now we consider bosonic part of $PSU(1,1|2)$ sector, which is composed of $Z, Y, D_+$ and two fermions in the $(\frac{1}{2}, -\frac{1}{2})$ and $(\frac{1}{2}, \frac{1}{2})$ of the local Lorentz group. In the $PSU(1,1|2)$ sector, the bosonic part of the state is

$$G[T_{DZ}, T_{DY}, \Lambda_Z, \Lambda_Y] = \frac{1}{Vol} \int dU \exp(\text{Tr}(UT_{DZ} U^{-1}c_D^\dagger a_Z) + \text{Tr}(UT_{DY} U^{-1}c_D^\dagger a_Y)) \exp(\text{Tr}(U\Lambda_Z U^{-1}a_Z^\dagger + U\Lambda_Y U^{-1}a_Y^\dagger)) |0\rangle.$$  

(3.9)

Here $T_{DZ}, T_{DY}, \Lambda_Z, \Lambda_Y$ mutually commute. The angular momentum and spin of the state is $J_Z = \text{Tr}(\bar{\Lambda}_Z \Lambda_Z)$, $J_Y = \text{Tr}(\bar{\Lambda}_Y \Lambda_Y)$ and $S = \text{Tr}(|T_{DZ} \Lambda_Z + T_{DY} \Lambda_Y|^2)$.

An alternative formulation in the bosonic part of the $PSU(1,1|2)$ sector, is

$$K[R_Z, R_Y, \Lambda_Z, \Lambda_Y] = \frac{1}{Vol} \int dU \exp(\text{Tr}(UR_Z U^{-1}c_D^\dagger a_Z^\dagger a_Z) + \text{Tr}(UR_Y U^{-1}c_D^\dagger a_Y^\dagger a_Y)) \exp(\text{Tr}(U\Lambda_Z U^{-1}a_Z^\dagger + U\Lambda_Y U^{-1}a_Y^\dagger)) |0\rangle.$$  

(3.10)

Here $R_Z, \Lambda_Z, R_Y, \Lambda_Y$ mutually commute. We see that $J_Z = \text{Tr}(\bar{\Lambda}_Z \Lambda_Z)$, $J_Y = \text{Tr}(\bar{\Lambda}_Y \Lambda_Y)$ and $S = \text{Tr}(|R_Z \bar{\Lambda}_Z \Lambda_Z + R_Y \bar{\Lambda}_Y \Lambda_Y|^2)$.
Now we turn to the quiver case. For quiver gauge theories and orbifold daughters of \( \mathcal{N} = 4 \) theory, the coherent state construction is similar. Many orbifold daughters have integrability. Many aspects of them have been discussed and worked out in e.g. \([68]-[74]\) and their related references. For example, we can make a projection of \( U(MN) \) theory to get the \( U(N)^M \) theory. The construction of coherent states for quiver gauge theories have been worked out in \([16]\). For example, we have a \( U(N_1) \times U(N_2) \) gauge group. We consider a pair of bifundamental fields \( a_{21}^\dagger, a_{12}^\dagger \) in the \((\bar{N}_1,N_2)\) and \((N_2,\bar{N}_1)\) representations. We make the same type of coherent states with their parameters \( \Lambda_{21}, \Lambda_{12} \), whose roles are similar to \( \Lambda_Z, \Lambda_Y \). The state is
\[
F[\Lambda_{21}, \Lambda_{12}] = \frac{1}{Vol} \int dU \exp \left( \text{Tr}(\Lambda_{21} U_{1} a_{12}^\dagger U^{-1} + \Lambda_{12} U_{2} a_{21}^\dagger U^{-1}) \right) |0\rangle .
\] (3.11)
Note that the conjugate of \( a_{21}^\dagger \) is \( a_{12} \), and the conjugate of \( a_{12}^\dagger \) is \( a_{21} \), with our convention. Now we construct a new state, which is the meromorphic version of (3.11),
\[
G[T_{21}, \Lambda_{12}] = \frac{1}{Vol} \int dU \exp(\text{Tr}(U_{2}^{-1} T_{21} U_{1} a_{12}^\dagger a_{12})) \exp(\text{Tr}(U_{1}^{-1} \Lambda_{12} U_{2} a_{21}^\dagger)) |0\rangle .
\] (3.12)
The right-most operator in (3.12) is an eigenstate of \( a_{12} \). We see that
\[
G[T_{21}, \Lambda_{12}] \sim F[\Lambda_{21}, \Lambda_{12}],
\] (3.13)
with \( \Lambda_{12} T_{21} = \Lambda_{21}, T_{21} = \Lambda_{12}^{-1} \Lambda_{21} \). \( G[T_{21}, \Lambda_{12}] \) is a meromorphic version of \( F[\Lambda_{21}, \Lambda_{12}] \) and can be transformed to it when \( N_1 = N_2 \) and \( \det \Lambda_{12} \neq 0 \). The inner products of (3.11) have been computed in \([16]\), where localization techniques have been implemented.

## 4 String-added coherent states

The BPS coherent state can serve as the supersymmetric background for other excitations on top of it. We can consider further excitations on the BPS coherent state background, by adding strings on top of them. One important idea is splitting and identification of fields for the background and fields for the impurities. We begin with some general settings.

### 4.1 General settings with multi-words

The background BPS coherent state is (2.23). On this background, other fields are excited. This is to add words or multi-words onto the coherent state operator. These
other excited fields may be viewed as impurities, if their numbers are much fewer than the number of background fields. We generalize the ideas of [16] from single word to multi-words, and from single pair to multi-pairs.

The BPS coherent states are parametrized by \( A = (A_X, A_Y, A_Z) \). Consider \(|v_x\rangle_i, |v_y\rangle_i, |v_z\rangle_i\) are eigenvectors, i.e.

\[
\Lambda_X |v_x\rangle_i = \lambda_x^i |v_x\rangle_i, \quad \Lambda_Y |v_y\rangle_i = \lambda_y^i |v_y\rangle_i, \quad \Lambda_Z |v_z\rangle_i = \lambda_z^i |v_z\rangle_i.
\]

When we multiply as \( UA_X U^{-1}, UA_Y U^{-1}, UA_Z U^{-1}\), the vectors are rotated to \( U |v_x\rangle_i, U |v_y\rangle_i, U |v_z\rangle_i\). The bra states are rotated to \( \langle v_x|U^{-1}\), etc. The vectors are simultaneously rotated. There is also a global symmetry \( U(3) \) acting on the set of 3 vectors.

The analysis of [16] as well as the above analysis in Sec. 2 imply that \( A_{(X,Y,Z)} \) are mutually commuting for BPS. Now we assume they mutually commute. Hence they can share the same eigenvectors. We consider there are two large eigenvalues \( \lambda_{(X,Y,Z)}^i \) and \( \lambda_{(X,Y,Z)}^j \). They share the same eigenvectors,

\[
\Lambda_X |\vec{v}\rangle_\alpha = \lambda_x^\alpha |\vec{v}\rangle_\alpha, \quad \Lambda_Y |\vec{v}\rangle_\alpha = \lambda_y^\alpha |\vec{v}\rangle_\alpha, \quad \Lambda_Z |\vec{v}\rangle_\alpha = \lambda_z^\alpha |\vec{v}\rangle_\alpha,
\]

for \( \alpha = i, j \).

Here only for two eigenvalues \( i, j \) having this property are needed. Hence \(|v_x\rangle_\alpha = |v_y\rangle_\alpha = |v_z\rangle_\alpha = |\vec{v}\rangle_\alpha\), for \( \alpha = i, j \). This means that in the two dimensional subspace involving \( i, j, A_{(X,Y,Z)} \) share the same eigenvector \(|\vec{v}\rangle_{i,j}\).

The added words on top is

\[
\langle \vec{v}_{\alpha} U^{-1} W U |\vec{v}\rangle_{\beta},
\]

and the state is

\[
\frac{1}{V \alpha \Omega} \int dU \exp \left( \text{Tr} \left( U \Lambda_X U^{-1} a_X^\dagger + U \Lambda_Y U^{-1} a_Y^\dagger + U \Lambda_Z U^{-1} a_Z^\dagger \right) \right) \langle \vec{v}_{\alpha} U^{-1} W U |\vec{v}\rangle_{\beta} |0\rangle.
\]

More generally, we add multi-words of this kind,

\[
\frac{1}{V \alpha \Omega} \int dU \exp \left( \text{Tr} \left( U \Lambda_X U^{-1} a_X^\dagger + U \Lambda_Y U^{-1} a_Y^\dagger + U \Lambda_Z U^{-1} a_Z^\dagger \right) \right) \langle \vec{v}_{i_1} U^{-1} W_1 U |\vec{v}\rangle_{j_1} \langle \vec{v}_{i_2} U^{-1} W_2 U |\vec{v}\rangle_{j_2} \ldots \langle \vec{v}_{i_n} U^{-1} W_n U |\vec{v}\rangle_{j_n} |0\rangle.
\]

The indices \((i_1, j_1, \ldots, i_n, j_n)\) form a set \( I_n \). \( W \) is a word, in other words, a string. For example, \( W \) is schematically \( D_+^{n_1} a_Z^{n_2} a_X^{n_3} \), with \( n_\alpha = 0, 1, 2, \ldots \), and so on. \( W \) can be labelled by BMN operators or spin chain like operators, and we can also include \( a_Z^\dagger \) etc.
The integral \((4.5)\) can be viewed as a path integral of the auxiliary variable \(U\) which is coupled to \(a^\dagger_Z, a^\dagger_Y, a^\dagger_X\) and \(W_\alpha\), in accord with the alternative interpretation in Sec 2. The insertions of \(W_\alpha\) can be viewed as operator-insertion in this path integral \((4.5)\). We have generalized the word insertions in \((4.4)\) to multi-word insertions, and interpreted them as operator insertions in the path integral of auxiliary variables.

We extract the anomalous energy from the combined state, e.g. \(F \cdot W\). The background energy are energy of BPS states. We extract the energy of non-BPS part of the excitation, corresponding to the anomalous dimension. The anomalous dimensions give rise to non-BPS energy on top of BPS energy. We works on the anomalous energy due to the quartic interaction vertices. The quartic interaction vertex connects one impurity field on the word part and one background field on the coherent state part, and they then Wick contract to their conjugates respectively. We can also use the dilatation operator.

The simplest cases are when the coherent state part \(F\) and the word part \(W\) are both BPS. The coherent state background \(F\) is BPS, hence there is no non-BPS energy from the interaction inside the coherent state part. Then the anomalous dimension energy is solely due to the interaction between the coherent state part and the word part. One of the simple cases is that \(W\) is an eighth BPS string. The entire state can be maintained as a near BPS state. Sec. 4.2 are special cases of these ideas.

### 4.2 Near BPS string-added coherent states

We first consider near BPS states of the form

\[
\frac{1}{Vol} \int dU \exp \left( \text{Tr}(U \Lambda Z U^{-1} a^\dagger_Z) \right) \cdot \\
\langle \vec{v}_{i_1} | U^{-1} W_{i_1} U | \vec{v} \rangle_{j_1} \langle \vec{v}_{i_2} U^{-1} W_{2} U | \vec{v} \rangle_{j_2} \ldots \langle \vec{v}_{i_l} U^{-1} W_{l} U | \vec{v} \rangle_{j_l} | 0 \rangle.
\]

(4.6)

The entire state can be maintained as a near BPS state. For example we add impurities corresponding to \(X\), and the word \(W\) is \(a^\dagger_X a^\dagger_Z, m = 0, 1, \ldots\) and so on. In this case, \(W\) is BPS itself, and it can be viewed as a symmetrized state in the sector of \(Z, X\). The anomalous energy is the energy corresponding to the anomalous dimension. Because \(W\) and \(F\) both has no anomalous dimension under the dilatation operator, the anomalous dimension energy is due to the interaction between the coherent state \(F\) part and the word \(W\) part.

We then consider near BPS states of the form

\[
\frac{1}{Vol} \int dU \exp \left( \text{Tr}(U \Lambda Z U^{-1} a^\dagger_Z + U \Lambda Y U^{-1} a^\dagger_Y) \right) \cdot \\
\langle \vec{v}_{i_1} | U^{-1} W_{i_1} U | \vec{v} \rangle_{j_1} \langle \vec{v}_{i_2} U^{-1} W_{2} U | \vec{v} \rangle_{j_2} \ldots \langle \vec{v}_{i_l} U^{-1} W_{l} U | \vec{v} \rangle_{j_l} | 0 \rangle.
\]

(4.7)
The word $W$ is $a_X^\dagger \{ a_Z^{\bar{m}_1} a_Y^{\bar{m}_2} \}$, $m_\alpha = 0,1, \ldots$ and so on. Here the curly bracket denotes symmetrized states in the sector of $Z,Y,X$ with $U(3)$ global symmetry, where the bracket denotes symmetrization over the global symmetry indices. See e.g. \cite{49,23}. The symmetrized states itself is BPS. We make both the coherent state $F$ part and the word $W$ part BPS, hence the non-BPS energy is coming from the interaction between the coherent state part and the word part. The above first example is a special case of the second example for $m_2 = 0$.

In the following, we compute for the case (4.7). The quartic vertices that are relevant here are $\text{Tr}[|Y, X|^2, \text{Tr}[|Z, X|^2, \text{Tr}[|Y, Z|^2]$. The anomalous dimension is due to the interaction of impurity $a_X^\dagger$ with the $a_Z^\dagger$ and $a_Y^\dagger$ respectively, on the exponent. The interaction between $a_Z^\dagger$ and $a_Y^\dagger$ gives no anomalous dimension, due to that they are in the exponent of the BPS coherent state, as calculated in Sec 2. The word $W$ is by itself BPS because it is symmetrized for $a_Z^\dagger$, $a_Y^\dagger$, $a_X^\dagger$. Hence the interaction between $Z,Y$ in this case does not lead to anomalous energy. The $Z,Y$ composite here are BPS.

The Hamiltonian involved for the excitation energy is
\begin{equation}
\Delta = g^2 \text{Tr}([a_Y^\dagger, a_X^\dagger][a_Z, a_Y]) + g^2 \text{Tr}([a_Z^\dagger, a_X^\dagger][a_Z, a_Z]).
\end{equation}
Here, the interaction leading to the anomalous energy here are quartic vertices between the $X$ in the word part and the $Z$ and $Y$, respectively in the coherent state part. For example, $X$ in the word part is attached to a quartic interaction vertex with a $Z$ in the coherent state $F$ part. And after interaction, they Wick contract to the conjugate of the word part and the conjugate of the coherent state part respectively. The coherent state part itself is BPS as explained.

The method of calculation follows from \cite{16}. One can directly work with the quartic vertices as in BMN \cite{75}, or use the dilatation operator \cite{76}. Here, $|v_y\rangle_\alpha = |v_z\rangle_\alpha$ for $\alpha = i,j$, as explained in Sec. 4.1, and we denote the pair of large eigenvalues $\vec{\lambda}_i$, $\vec{\lambda}_j$. When acting on the state, the extra annihilation operator $a_Y$ brings down $U\lambda_Y U^{-1}$, one to the left and the other to the right. These two pieces go as
\begin{equation}
g^2 \langle v_y|U^{-1}U\lambda_Y U^{-1}[a_X^\dagger, a_Y^\dagger]U |v_y\rangle_j - g^2 \langle v_y|U^{-1}([a_X^\dagger, a_Y^\dagger]U\lambda_Y U^{-1}U |v_y\rangle_j.
\end{equation}
Since $|v_y\rangle_i, |v_y\rangle_j$ are eigenstates of $\lambda_Y$, we get
\begin{equation}
g^2(\lambda^y_i - \lambda^y_j) \langle v_y|U^{-1}[a_X^\dagger, a_Y^\dagger]U |v_y\rangle_j.
\end{equation}
We now proceed the process for computing with the conjugate vector, and $a_Y^\dagger$ brings down terms of $\lambda_Y$. We obtain an integral that involves $U^{-1}$, so we get a factor $(\vec{\lambda}_i^y - \vec{\lambda}_j^y)$ from (4.10). For $a_Z$, the calculation is similar, and we get
\begin{equation}
g^2(\lambda^z_i - \lambda^z_j) \langle v_z|U^{-1}[a_X^\dagger, a_Z^\dagger]U |v_z\rangle_j.
\end{equation}
For the conjugate vector, \( q^\dagger_Z \) brings down terms of \( \bar{\Lambda}Z \). Again, we have the integral and get a factor \((\bar{\lambda}_i - \bar{\lambda}_j)\) from (4.11). Adding these two pieces from (4.10) and (4.11), the spectrum is

\[
H = \frac{g^2}{8\pi^2}|\lambda^i_z - \lambda^j_z|^2 + \frac{g^2}{8\pi^2}|\lambda^i_y - \lambda^j_y|^2 = \frac{g^2}{8\pi^2}|\bar{\lambda}_i - \bar{\lambda}_j|^2.
\]  
(4.12)

The result subsumes the simplified case (4.6).

The extra energy is perturbation energy around the eigenvalue background, for a single pair of eigenvalues. We insert multiple words of this kind in (4.7). Hence for inserting multiple words of this kind,

\[
H = \sum_{(i, j)} \frac{g^2}{8\pi^2}|\bar{\lambda}_i - \bar{\lambda}_j|^2,
\]  
(4.13)

where \((i, j) \in I_s\). This is in agreement with alternative observations [77, 78, 79]. The spectrum of this type of Hamiltonian was also studied by [79, 78, 77, 80, 81]; see also related methods [82].

In this case, by the method of centrally extended algebra [83, 84], the excitation energy are written in a square-root form, whose expansion gives (4.13). Hence the full square-root form is

\[
\sum_{(i, j)} \left( \sqrt{1 + \frac{g^2}{4\pi^2}|\bar{\lambda}_i - \bar{\lambda}_j|^2} - 1 \right).
\]  
(4.14)

Here we subtract the bare dimension out in (4.14) to obtain the anomalous energy. The subtractions are the anomalous piece of the energy. The centrally extended algebra gives important insights in [84, 85, 72] and these are in agreement with the observations here.

These phenomena have very interesting dual interpretations on the gravity side. This is string spectra in backreacted excited state spacetime geometry. This analysis not only works for AdS background but also for backreacted backgrounds, in the context of string excitations in backreacted geometries. Eqn. (4.14) is in complete agreement with the analysis and observations [63, 86, 87, 88] on both the gauge theory side and the gravity side. This spectrum is in agreement with observations in the gravity side, in terms of magnon energy in backreacted geometries, e.g. magnon energy in annularly shaped droplet geometries.

The Eqn. (4.13) can be viewed as in the nonrelativistic regimes of (4.14) and it was considered as the near BPS excitation of heavy giant gravitons [78], in their near BPS limit. Eqn. (4.14) is the relativistic version of (4.13). This near BPS limit would be the same as the one in spin-matrix theory [89, 90] and they are hence closely related.
5 Discussion

The coherent state representation facilitates the calculation of ladder operators and hence the dilatation operator. For instance, it converts dilatation operator manipulations to algebraic manipulations. The coherent state representation of the operators have the advantage of simplifying the action of the above involved dilatation operators. We also checked higher loop dilatation operators on quarter BPS coherent states and the action is again vanishing as expected. By using nonrenormalization theorems, e.g. [49, 23], we infer that the BPS coherent states are in the kernel of the anomalous dilatation operator. This construction also works for coulomb branches.

In its original invention, [16] constructed coherent states averaged over group orbit, for the purpose of gauge invariance. Here we alternatively interpret this group average as a path integral of auxiliary variables coupled to the elementary letters of the theory. The insertions of string words on the coherent state can be viewed as operator-insertion in this path integral. We have generalized the word insertion to multi-word insertions, by conveniently interpreting them as operator insertions in this path integral. Hence, the group integral can be viewed as a path integral of the auxiliary variables $U$ which are coupled to the elementary letters and added string words.

We also constructed a meromorphic version of coherent states, which can be transformed back to the full coherent states when $\det \Lambda_Z \neq 0$. Here $G[T_Y, \Lambda_Z]$ is a meromorphic version of $F[\Lambda_Y, \Lambda_Z]$. The meromorphic versions are particularly useful when there are background $Z$ fields. We also constructed meromorphic version of coherent states for product gauge groups and quivers. We give transformation relations between different types of coherent states, among other things.

We constructed new type of coherent states in the $SL(2)$ sectors and bosonic part of $PSU(1, 1|2)$ sectors. These are operators with derivative insertions. The large spin limit and small spin limit can be reached by varying the coherent state amplitudes. We have also discussed other new states related to them.

The different coherent states differ by the symmetry properties of the multi-parameters of the coherent states. The coherent states constructed in [16] have manifest permutation symmetries among the eigenvalues. Other coherent states have gauge-fixed these symmetries [37], and can be viewed as gauge-fixed versions. Although different construction of coherent states may have different norms, there are many norm-independent features and observables of coherent state construction, for example, when calculating the vev of $a_Z^{j n} a_Z^l$, the norm factor of the numerator and of the denominator cancels each other and is hence norm-independent.

We have used these coherent states to describe BPS and near BPS states, in gauge-string correspondence. We analyzed string-added coherent states, in general a product
of a coherent state and a word, and calculated anormalous dimension energy. The string-added coherent state captures a class of near BPS spectra. One can use more general words, by using BMN like operators or spin chain like operators. The method of [16] gives great insights for computing excitations on supersymmetric backgrounds. Magnon excitations have also been computed for both gauge theory side and gravity side in related works. The agreement to these calculations is astounding, because the underlying methods and physical perspectives of these different approaches are different and alternative to each other. This approach is also intimately related to the effective eigenvalue model. In these involved circumstances, the eigenvalues provide some string theory background, and the off-diagonal fluctuations provide string excitations [8]. Non-BPS excitations are added as a perturbation of the BPS excitations. These observations pointed out to a way to solve the strong coupling dynamics of these near-BPS systems, and we have encountered some new strong coupling phenomena in those regimes. On a very different perspective, the approach of adding excitations may be relevant for [91] [92].

On the gravity side, these coherent states are related to giant gravitons and backreacted geometries. The individual eigenvalue of coherent state is the collective coordinate of giant gravitons [16], see also related discussions [18] [37]. The coherent state construction is an important step in this regime. These are regimes where the single-trace and multi-trace operators are very difficult to handle efficiently. The effective action of the coherent state collective coordinate in the Lagrangian formalism has been constructed [16], where properties of giant gravitons and geometric quotient and projection of the transverse space in the dual gravity geometry, in the case of quiver theory, have been observed by this approach [16]. These are important for giant gravitons and backreacted geometries and related to various important discussions of e.g. [93] [97].

Our approach shield new lights to near BPS states and near BPS sectors. We conveniently look at eighth BPS states and near BPS states, in this set-up. Another way to understand near BPS states is by spin-matrix theory e.g. [89] [90], which is closely related to the approaches in this paper. The spin-matrix theory is very insightful for both near BPS string states and near BPS giant gravitons. This is intimately related to the consistent reductions. The related consistent reductions are also very interestingly discussed in [98] to fewer dimensions.

The involved path integral can be computed by saddle point method and it is an exact computation in the context of localization. The localization technique is also very useful for Wilson operators. Localization methods have occurred remarkably in Wilson loops in e.g. [99] and related references. Moreover, emergent geometries are also dual to large Wilson loop operators, e.g. [100] [104] as some examples. It would be good to have an unified understanding together with these circumstances.
The coherent state operators, Young tableau operators, and large Wilson operators are heavy operators. They have gravitational dual descriptions. The gravitational dual system to these very heavy operators, involves backreacted emergent geometries, e.g. [105]-[114] as some examples, and see their related references. These heavy states induce backreactions in the dual quantum gravity system. The coherent states are also heavy excited states. The coherent state approach leads to higher multi-pole moments in the gravity side, which can be measured on the boundary of the gravity side, as discussed in Sec. 2. Our scenarios are closely related to fuzzball geometries and their related discussions, e.g. [115]-[121] and [122].

Near BPS states can also describe near BPS black holes, which are related to various giant configurations and intersecting giants. The string configurations [123] on giants are relevant. A deeper understanding of the quarter and eighth BPS sector as well as near BPS sector will lead to implications for physics of extremal and near-extremal black holes e.g. [124, 125].

The SO($N$) and Sp($N$) cases of this approach have been worked out in [17]. The related Young tableaux basis have been addressed and developed in details in e.g. [27, 126, 127] and their related references. The case of other gauge groups and product gauge groups have also been considered [17, 16], as well as in this paper. This approach also works for SO/Sp theories, quiver theories, and coulomb branches. Although the analysis of the current paper is mainly for $U(N)$ and $SU(N)$ case, many features can be similar in the case of these other gauge groups.

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