Photon Spheres and Spherical Accretion Image of a Hairy Black Hole

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In this paper, we first consider null geodesics of a class of charged, spherical and asymptotically flat hairy black holes in an Einstein-Maxwell-scalar theory with a non-minimal coupling for the scalar and electromagnetic fields. Remarkably, we show that there are two unstable circular orbits for a photon in a certain parameter regime, corresponding to two unstable photon spheres of different sizes outside the event horizon. To illustrate the optical appearance of photon spheres, we then consider a simple spherical model of optically thin accretion on the hairy black hole, and obtain the accretion image seen by a distant observer. In the single photon sphere case, only one bright ring appears in the image, and is identified as the edge of the black hole shadow. Whereas in the case with two photon spheres, there can be two concentric bright rings of different radii in the image, and the smaller one serves as the boundary of the shadow, whose radius goes to zero at the critical charge.

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I. INTRODUCTION

The Event Horizon Telescope (EHT) collaboration has recently achieved an angular resolution sufficient to observe the image of a supermassive black hole in the center of galaxy M87, which allows us to test gravity in strong field regime [1–6]. The major feature of the image is a shadow region surrounded by a bright ring, which results from strong gravitational lensing by the black hole [7–10]. The shadow image captured by EHT is expected to bear the fingerprint of the geometry around the black hole, and in good agreement with the predictions of the spacetime geometry of Kerr black holes. Nevertheless, the black hole mass/distance and EHT systematic uncertainties still leave some room within observational uncertainty bounds for a non-Kerr black hole. Moreover, there has been considerable debate whether the bright ring surrounding the shadow is solely determined by the photon sphere or also affected by details of the accretion flow [11, 12]. So a lot of work in progress has reported on black hole shadows for various black holes of different theories of gravity with/without considering accretion flows [13–39].

On the other hand, the observation of a black hole shadow has become a new venue to test the no-hair theorem [40–42], which states that a black hole is uniquely characterized by its mass, angular momentum and electrical charge. Various scenarios (e.g., black holes with Skyrme hairs [43, 44] and dilaton hairs [45], hairy black holes in scalar-tensor gravities [46, 47] and Gauss-Bonnet theories [48]) have been proposed to circumvent the no-hair theorem since the first hairy black hole solution was found in the context of the Einstein-Yang-Mills theory [49–51]. For a review, see

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we focus on the static spherical black hole solutions with the generic ansatz for hairy black hole solutions, it showed that the dimensionless coupling \( \alpha \) is the Ricci scalar, the scalar field \( \phi \) is minimally coupled to the metric \( g_{\mu \nu} \) and non-minimally coupled to the electromagnetic field \( A_{\mu} \), where \( R \) is the Ricci scalar, the scalar field \( \phi \) is minimally coupled to the metric \( g_{\mu \nu} \) and non-minimally coupled to the electromagnetic field \( A_{\mu} \), and \( F_{\mu \nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \) is the electromagnetic tensor field. The action (1) has scalar-free black hole solutions with the scalar field \( \phi = 0 \), corresponding to Reissner-Nordström (RN) black holes. To obtain hairy black hole solutions, it showed that the dimensionless coupling \( \alpha \) has to be larger than 1/4 \([61]\). Following \([61]\), we focus on the static spherical black hole solutions with the generic ansatz

\[
\frac{m(\varphi)}{2} \delta(\varphi_0) \delta(\varphi_h) = \delta(\varphi_0) \delta(\varphi_h) = 0, \quad V(\varphi_h) = 0,
\]

where the Misner-Sharp mass function \( m(\varphi) \) is defined through \( N(\varphi) = 1 - 2m(\varphi)/r \), and the prime denotes the derivative with respect to \( r \). The last line in Eq. (3) leads to \( V'(\varphi) = -e^{-(\delta(r)+\alpha \varphi(r))^2} Q(r)/\sqrt{r} \), in which the constant \( Q \) can be interpreted as the electric charge of the black hole. Besides, we shall implement suitable boundary conditions at the event horizon \( r_h \),

\[
m(r_h) = \frac{r_h}{2} \delta(r_h) \delta(\varphi_h) = \delta(\varphi_0) \delta(\varphi_h) = 0, \quad V(\varphi_h) = 0,
\]

and non-minimal coupling functions \([62, 63]\), dyons including magnetic charges \([64]\), axionic-type couplings \([61]\), and use this to obtain the image of a spherical accretion flow surrounding the black hole perceived by a distant observer. We conclude with a discussion in Sec. IV. We set \( 16\pi G = 1 \) throughout the paper.

II. FRAMEWORK

In this section, we review the hairy black hole solution, discuss features of null geodesic motion, and introduce the accretion model. Specifically, we consider the asymptotically flat black hole solutions with a non-trivial scalar hair in an EMS theory with the exponential coupling \([61]\),

\[
S = \int d^4x \sqrt{-g} \left[ R - 2\partial_\mu \phi \partial_\mu \phi - e^{\alpha \phi^2} F_{\mu \nu} F^{\mu \nu} \right],
\]

where \( R \) is the Ricci scalar, the scalar field \( \phi \) is minimally coupled to the metric \( g_{\mu \nu} \) and non-minimally coupled to the electromagnetic field \( A_{\mu} \), and \( F_{\mu \nu} = \partial_\mu A_{\nu} - \partial_\nu A_{\mu} \) is the electromagnetic tensor field. The action (1) has scalar-free black hole solutions with the scalar field \( \phi = 0 \), corresponding to Reissner-Nordström (RN) black holes. To obtain hairy black hole solutions, it showed that the dimensionless coupling \( \alpha \) has to be larger than 1/4 \([61]\). Following \([61]\), we focus on the static spherical black hole solutions with the generic ansatz

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\[
m(r_h) = \frac{r_h}{2} \delta(r_h) \delta(\varphi_h) = \delta(\varphi_0) \delta(\varphi_h) = 0, \quad V(\varphi_h) = 0,
\]
and at the spatial infinity,
\[ m(\infty) = \delta(\infty) = 0, \phi(\infty) = 0, V(\infty) = \Psi, \] (5)
where \( \delta_0 \) and \( \phi_0 \) are two constant parameters, \( M \) is the ADM mass, and \( \Psi \) is the electrostatic potential. One can use the shooting method to numerically solve the non-linear differential equations (3) for hairy black hole solutions satisfying the above boundary conditions. Moreover, we focus on the fundamental state of hairy black hole solutions, for which the scalar field \( \phi(r) \) has none node.

To investigate the light deflection caused by a black hole, we need to find the geodesic equation of light rays, which can be encapsulated in
\[
\frac{d^2 x^\mu}{d\lambda^2} + \Gamma^\mu_{\rho\sigma} \frac{dx^\rho}{d\lambda} \frac{dx^\sigma}{d\lambda} = 0,
\] (6)
with the affine parameter \( \lambda \) and the Christoffel symbols \( \Gamma^\mu_{\rho\sigma} \). For the static spherical black hole (2), we can confine ourselves to light rays traveling in the equatorial plane \( \theta = \pi/2 \), and introduce two conserved quantities \( E \equiv N(r)e^{-2\delta(r)} \frac{dt}{d\lambda} \) and \( L \equiv r^2 \frac{d\varphi}{d\lambda} \), which can be interpreted as the energy and the angular momentum of the light rays, respectively. Therefore, the geodesic on the equatorial plane for a light ray propagating in the metric (2) is given by
\[
\frac{dt}{d\eta} = \frac{1}{bN(r)e^{-2\delta(r)}}, \quad \frac{d\varphi}{d\eta} = \pm \frac{1}{r^2},
\] (7)
\[
\left( \frac{dr}{d\eta} \right)^2 + \frac{N(r)}{r^2} = \frac{e^{2\delta(r)}}{b^2},
\] (8)
where the new affine parameter \( \eta \) is related to the previous one by \( \eta = \lambda |L| \), the impact factor \( b \) is defined as \( |L|/E \), and \( \pm \) correspond to moving in the counterclockwise (+) and clockwise (−) along \( \varphi \)-direction, respectively. We can rewrite Eq. (9) as
\[
e^{-2\delta(r)} \left( \frac{dr}{d\eta} \right)^2 + V_{\text{eff}}(r) = \frac{1}{b^2},
\] (9)
where
\[
V_{\text{eff}}(r) = \frac{e^{-2\delta(r)} N(r)}{r^2}
\] (10)
is the effective potential. The trajectory of a light ray in the \( r-\varphi \) plane is obtained by expressing \( \varphi \) in terms of \( r \) through Eqs. (8) and (9), that is
\[
\frac{d\varphi}{dr} = \pm \frac{1}{r^2 e^{\delta(r)} \sqrt{\frac{1}{b^2} - V_{\text{eff}}(r)}},
\] (11)
Particularly, a circular null geodesic, dubbed photon sphere, occurs at extrema of the effective potential \( V_{\text{eff}}(r) \). In fact, the conditions for the presence of a photon sphere are
\[
V_{\text{eff}}(r_{ph}) = \frac{1}{b_{ph}^2} \quad \text{and} \quad V'_{\text{eff}}(r_{ph}) = 0,
\] (12)
where \( r_{ph} \) is the radius of the photon sphere, and \( b_{ph} \) is the corresponding impact parameter. Moreover, maxima/minima of \( V_{\text{eff}}(r) \) correspond to unstable/stable photon spheres. Since only unstable photon spheres play an important role in determining properties of the accretion image seen by a distant observer (e.g., the size of the black hole shadow), we focus on unstable photon spheres in the following.

In this paper, we consider a toy model of a spherical illuminating accretion flow, which is assumed to be optically transparent and statically distributed outside the black hole horizon [12]. This model is simple but enough for the purpose of this paper. On the other hand, it is noteworthy that M87 is known to contain a geometrically thick, optically thin, hot accretion flow [90]. The observed total photon intensity \( F_o \) (usually measured in ergs\(^{-1}\)cm\(^{-2}\)str\(^{-1}\))
at the celestial point \((X,Y)\) in the observer’s sky can be obtained by integrating the emissivity along the photon path \(\gamma\) [91–95],

\[
F_o(X, Y) = \int_{\nu_o} I_o(\nu_o, X, Y) d\nu_o = \int_{\nu_e} \int_{\gamma} g j_e(\nu_e) dl_{prop} d\nu_e, \tag{14}
\]

where \(g\) is the redshift factor, and \(I_o\) is the specific intensity at the observed photon frequency \(\nu_o\). Here, \(\nu_e\), \(dl_{prop}\) and \(j_e\) are the photon frequency, the infinitesimal proper length and the emissivity per unit volume measured in the rest frame of the emitter, respectively. For a photon of four-velocity \(k^\alpha = dx^\alpha/d\eta\), we have \(g = k_\alpha u^\alpha_0 / k_\beta u^\beta_0\) and \(dl_{prop} = |k_\beta u^\beta_0 d\eta|\), in which \(u^\alpha_0\) and \(u^\beta_0\) are four-velocities of the distant observer and the accretion emitter, respectively. In the model, we assume that the illuminating accretion flow is at rest, which gives \(u^\alpha_0 = (e^{\delta(r)}/\sqrt{N(r)}, 0, 0, 0)\), and the distant observer is at the spatial infinity, which gives \(u^\alpha_0 = (1, 0, 0, 0)\). Using Eqs. (7), (8) and (9), we then obtain

\[
g = \sqrt{N(r)e^{-\delta(r)}} \quad \text{and} \quad dl_{prop} = \frac{e^{\delta(r)}dr}{b\sqrt{N(r)\sqrt{\frac{e^{2\delta(r)}}{b^2} - \frac{N(r)}{r^2}}}}. \tag{15}
\]

Following [91–95], we also consider a simple case in which the specific emission is monochromatic with the emitter’s rest-frame frequency \(\nu_e\), and has a radial profile as

\[
j_e(\nu_e) = \frac{\delta(\nu_e - \nu_e)}{\nu^2}. \tag{16}
\]

Putting Eqs. (15) and (16) into Eq. (14), we find that the total photon intensity measured by the distant observer can be expressed by

\[
F_o(b) = \int_\gamma \frac{N(r)^{3/2}e^{-3\delta(r)}}{br^2\sqrt{\frac{e^{2\delta(r)}}{b^2} - \frac{N(r)}{r^2}}} dr, \tag{17}
\]

with \(b^2 = X^2 + Y^2\) due to the circular symmetry of the intensity.

### III. PHOTON SPHERES AND SHADOWS

In this section, we investigate the shadows and photon spheres of hairy black hole solutions to the action (1), which are illuminated by static and spherical accretion flows. In the left panel of Fig. 1, we present the hairy black hole solutions for several representative values of charge \(Q\) with the coupling \(\alpha = 0.8\) and the mass \(M = 1\). With \(Q\) increasing, the size of event horizon of the hairy black hole becomes smaller. The right panel of Fig. 1 shows the corresponding effective potentials \(V_{\text{eff}}(r)\), which have a single maximum for \(Q = 1.04\) and \(Q = 1.045\), and two maxima for \(Q = 1.048\) and \(Q = 1.05\). In what follows, the potentials with one maximum and two maxima are dubbed single-peak and double-peak potentials, respectively. As discussed before, the maxima of \(V_{\text{eff}}(r)\) correspond to unstable photon spheres, which can be responsible for determining the size of the black hole shadow. Consequently, black hole solutions with single-peak (double-peak) effective potentials possess one (two) unstable photon sphere(s). For convenience, photon spheres refer to unstable photon spheres in the remainder of this section. In the case with two photon spheres, the photon sphere with smaller value of the impact parameter \(b\) (e.g., the photon spheres \(B\) and \(C\) for \(Q = 1.048\) and 1.05, respectively, in the right panel of Fig. 1) determines the black hole shadow size [96].

#### A. Single-peak potential

Here, we consider the shadow and photon sphere of the hairy black hole with \(\alpha = 0.8, Q = 1.04\) and \(M = 1\), which possesses a single-peak effective potential. The effective potential and the trajectories of light rays are plotted in Fig. 2. Suppose a distant observer is located to the far right of the right panel of Fig. 2. Thus, we consider a bundle of light rays traveling towards the observer, which show different behavior depending on their impact parameters \(b\). In particular, light rays with \(b = b_{\text{ph}}\) asymptotically approach the photon sphere of radius \(r_{\text{ph}}\) and revolve around a circular orbit at a constant radius \(r = r_{\text{ph}}\) by infinite times. In the parameter Region 1 with \(b < b_{\text{ph}}\), light rays (grey lines in the right panel of Fig. 2) start from the black hole horizon, and can overcome the barrier of the effective potential to propagate to infinity. On the other hand, in the parameter Region 2 with \(b > b_{\text{ph}}\), light rays (red lines in
FIG. 1. Plots of hairy black hole solutions with $\alpha = 0.8$ and $M = 1$ and the associated effective potentials $V_{\text{eff}}(r)$. **Left:** Metric functions $m(r)$ (dotted), $\phi(r)$ (dashed) and $\delta(r)$ (solid) for the hairy black holes with different electric charges $Q = 1.04$ (blue), $Q = 1.045$ (green), $Q = 1.048$ (orange) and $Q = 1.05$ (red). Vertical lines denote the corresponding event horizons. **Right:** For $Q = 1.04$ (blue) and $Q = 1.045$ (green), the black hole effective potential $V_{\text{eff}}(r)$ has a single-peak structure with a single maximum. Nevertheless, a double-peak structure with two local maxima is observed for $V_{\text{eff}}(r)$ when $Q = 1.048$ (orange) and $Q = 1.05$ (red). Note that the global maximum of $V_{\text{eff}}(r)$ is responsible for determining the size of the black hole shadow.

FIG. 2. The profile of the effective potential $V_{\text{eff}}(r)$ (**Left**) and trajectories of light rays with different impact parameters $b$ (**Right**) for the hairy black hole with $\alpha = 0.8$, $Q = 1.04$ and $M = 1$. The effective potential has a single maximum of $1/b_{\text{ph}}^2$ at $r = r_{\text{ph}}$, where the photon sphere is located. The grey and red lines correspond to light rays with $b < b_{\text{ph}}$ and $b > b_{\text{ph}}$, respectively, and the dashed blue circle represents the photon sphere. The black hole is shown as a solid black disk.

the right panel of Fig. 2) start from infinity, encounter the potential barrier at turning points, and then reflect back in the outward direction towards the observer.

Using Eqs. (12) and (17), we plot the total number of light ray orbits $n$ and the observed intensity $F_o$ as functions of $b$ in Fig. 3. As shown in the left panel, the light ray with $b = b_{\text{ph}}$ orbits around the black hole an infinite number of times, and hence picks up an arbitrarily large intensity from the accretion flow. Consequently, the intensity observed by the distant observer rapidly increases when $b$ increases towards $b_{\text{ph}}$, forms a sharp peak at $b_{\text{ph}}$, and then decreases.
with increasing $b$, which is displayed in the middle panel. To present the image of the accretion flow seen by the observer, we project $F_o(b)$ to the observer’s celestial coordinates $(X, Y)$ via $b^2 = X^2 + Y^2$ in the right panel. The 2D image has a bright ring due to the peak of $F_o(b)$ at $b = b_{ph}$. The dark region inside the bright ring refers to the black hole shadow, whose intensity does not vanish since part of the radiation of the accretion flow inside the photon sphere can escape to infinity. Note that this image is quite similar to those given in [12, 30–33].

B. Double-peak potential

A novel feature of the effective potential $V_{eff}(r)$ is that $V_{eff}(r)$ can have two maxima for large enough black hole charge $Q$, corresponding to two photon spheres outside the event horizon. Nevertheless, these photon spheres do not always play a role in determining the observed image of the accretion flow around a black hole. For instance, considering the $Q = 1.048$ case (orange) in Fig. 1, light rays in the vicinity of the photon sphere associated with the peak $A$ can not escape to infinity since the effective potential $V_{eff}(r)$ at $B$ is higher than that at $A$, making this photon sphere invisible to the distant observer. As a result, it is expected that the observed accretion images of the $Q = 1.04$, $Q = 1.045$ and $Q = 1.048$ cases in Fig. 1 are quite similar. However, when the maximum of $V_{eff}(r)$ occurring at a smaller $r$ is greater than that at a larger $r$ (e.g., the $Q = 1.05$ case in Fig. 1), both of the photon spheres are responsible for the accretion image seen by the distant observer. In what follows, we focus on the hairy black hole with $\alpha = 0.8$, $Q = 1.05$ and $M = 1$ to study the effects of the two-peak structure of $V_{eff}(r)$ on the observational appearance of the accretion flow.

Using Eqs. (11) and (12), we plot the corresponding effective potential and light ray trajectories in the upper row in Fig. 4. Due to the presence of two photon spheres, the behavior of light ray trajectories is much richer than the single photon sphere case. As shown in the upper left panel of Fig. 4, the effective potential features two local maxima, $1/b_{ph1}^2$ and $1/b_{ph2}^2$, which occur at $r = r_{ph1}$ and $r = r_{ph2}$, respectively. Since $b_{ph1} < b_{ph2}$ and $r_{ph1} < r_{ph2}$, the parameter space of $b$ is divided into three regions, in which trajectories of light rays behave differently. In Region 1 with $b < b_{ph1}$, light rays coming from infinity go above the maximum of the potential and get captured by the black hole. In Region 2 with $b_{ph1} < b < b_{ph2}$, light rays from infinity first travel toward the black hole, then revolve around the smaller photon sphere until reaching the turning points, and finally scatter off to infinity. In Region 3 with $b > b_{ph2}$, light rays from infinity instead revolve around the larger photon sphere before escaping to infinity. In Fig. 4, light rays in Region 1, Region 2 and Region 3 are depicted by grey, red and orange lines, respectively.

To gain a better understanding of two photon spheres, we consider light rays with impact parameters in the vicinity of $b_{ph1}$ and $b_{ph2}$, as shown in the lower row of Fig. 4. For $b = b_{ph1} - 10^{-5}$ and $b = b_{ph1} + 10^{-5}$, both incident light rays move towards the black hole and circle around the smaller photon sphere at $r_{ph1}$ by many times. However, the light ray with $b = b_{ph1} - 10^{-5}$ eventually falls into the black hole, whereas that with $b = b_{ph1} + 10^{-5}$ is reflected back to infinity. The light rays with $b = b_{ph2} - 10^{-5}$ and $b = b_{ph2} + 10^{-5}$ both revolve around the larger photon sphere at $r_{ph2}$ by many times before escaping to infinity. Nevertheless, the former can go over the peak at $r_{ph2}$ and approach

FIG. 3. Plots of the total number of light ray orbits $n$, and the intensity $F_o$ and accretion image seen by a distant observer for the hairy black hole with $\alpha = 0.8$, $Q = 1.04$, and $M = 1$. **Left:** The total number of orbits $n \equiv \Phi/(2\pi)$ as a function of $b$, where $\Phi = \Delta \varphi$ is the total change of the azimuthal angle of light rays traveling outside the event horizon. The red (grey) segment is determined by light rays with $b > b_{ph}$ ($b < b_{ph}$). **Middle:** The total photon intensity $F_o(b)$ as a function of $b$, which has a sharp peak at $b = b_{ph}$. Light rays with $b > b_{ph}$ ($b < b_{ph}$) contribute to the red (grey) segment. **Right:** The 2D image of the accretion flow viewed in the observer’s sky. The bright ring is determined by the photon sphere, and serves as the boundary of the black hole shadow.
FIG. 4. Upper: Plots of the effective potential and trajectories of light rays for the hairy black hole with $\alpha = 0.8$, $Q = 1.04$ and $M = 1$. The effective potential has two maxima at $r = r_{ph1}$ and $r = r_{ph2}$, corresponding to two photon spheres. Two dashed blue concentric circles of radii $r_{ph1}$ and $r_{ph2}$ denote the two photon spheres. The grey, red and orange lines represent light rays in Region 1, Region 2 and Region 3, respectively. Lower: Plots of the fractional number of photon orbits $\varphi(r)/(2\pi)$ and the corresponding trajectories for four light rays with different $b$, which come from infinity and revolve around the photon spheres many times before escaping. The dashed blue vertical lines in the left panel correspond to the photon spheres.

the smaller photon sphere at $r_{ph1}$ while the latter can not. Moreover, the closer $b$ of a light ray is to $b_{ph1}$ or $b_{ph2}$, the more times it will revolve around the corresponding photon sphere. Thus it is expected that light rays with $b = b_{ph1}$ or $b = b_{ph2}$ will asymptotically approach the corresponding photon sphere and orbit around it by infinite times.

In Fig. 5, we show the total number of light ray orbits $n(b)$, the intensity profile $F_o(b)$ and the image of the static spherical accretion flow seen by a distant observer in the hairy black hole with $\alpha = 0.8$, $Q = 1.05$ and $M = 1$. Unlike the single-peak case, there are two sharp peaks of the total orbit number $n(b)$ at $b = b_{ph1}$ and $b = b_{ph2}$, which result from the two photon spheres at $r = r_{ph1}$ and $r = r_{ph2}$. Consequently, the observed intensity $F_o(b)$ has two peaks at $b = b_{ph1}$ and $b = b_{ph2}$, which lead to two bright concentric rings of the accretion image. As shown below, the bright ring with the larger radius ($b_{ph2}$), which is quite noticeable, is reminiscent of the bright ring of radius $b_{ph1}$ in the the accretion image in the single-peak case. However, due to the sharpness of the peak of $F_o(b)$ at $b = b_{ph1}$, the smaller bright ring, which locates at the edge of the shadow, is barely visible. Note that, similar to the single-peak case, the intensity of the dark shadow does not vanish completely due to the accretion radiation inside the photon sphere.
Here, we turn to the dependence of the size of the hairy black hole shadow on the black hole charge. In Fig. 6, we plot the impact parameters of light rays traveling at the photon spheres (red lines), the radii of the photon spheres (blue lines) and the event horizon radius (black line) as functions of the black hole charge for hairy black holes with $\alpha = 0.8$ and $M = 1$. In [61], it showed that the hairy black holes exist for some range of the black hole charge $Q$, $Q_{\text{exi}} \leq Q \leq Q_{\text{cr}}$. Here, $Q_{\text{exi}}$ is the existence charge, for which a hairy hole solution bifurcates from a RN black hole solution, and $Q_{\text{cr}}$ is the critical charge, for which the black hole horizon radius vanishes with the black hole mass and charge remaining finite. In Fig. 6, the vertical black dashed and dotted lines represent $Q = Q_{\text{exi}}$ and $Q = Q_{\text{cr}}$, respectively. When $Q$ increases from $Q_{\text{exi}}$, we find that the hairy black holes possess a single photon sphere, and the
Light of a shadow has a minimum value of 4 holes can capture some key features of astrophysical black hole shadows. Considering a RN black hole of mass may shed light on astrophysical observations of magnetic black holes. Therefore, our results under the electromagnetic duality transformation, to the paucity of magnetic monopoles. Finding astrophysical observations of such objects can greatly expand our insights into astrophysical black hole images released in current and future EHT experiments.

Finally, we end with a few comments. For a Schwarzschild black hole, another spherical accretion model, where the radiating gas radially moves in towards the black hole, was also considered in [12]. The authors showed that the image of infalling accretion flow is quite similar to that of static accretion flow except that the shadow region in the infalling model is significantly darker than that in the static model due to the inward gas motion. Likewise, it is reasonably expected that, for the hairy black hole considered in this paper, the image of the infalling accretion flow closely resembles that of the static accretion flow, except with a darker shadow. Thus, the static accretion model suffices to illustrate effects of single- and double-peak structures of the effective potential. On the other hand, real accretion flows are not spherically symmetric, and hence considering more realistic accretion models would gain more insight into astrophysical black hole images released in current and future EHT experiments.

Electric charges of astrophysical black holes are usually neglected since charged accretion flows can lead to prompt discharging. On the other hand, magnetically charged black holes are possible exotic astrophysical objects due to the paucity of magnetic monopoles. Finding astrophysical observations of such objects can greatly expand our understanding of the universe [100–104]. It is worth noting that under the electromagnetic duality transformation, the electric hairy black hole solution (2) can also describe magnetically charged black holes [64]. Therefore, our results may shed light on astrophysical observations of magnetic black holes.

For an astrophysical black hole with a nonzero spin, while the image of the accretion flow is expected to be strongly spin dependent, the size of the shadow is rather insensitive to the value of the spin. So studying spherical black holes can capture some key features of astrophysical black hole shadows. Considering a RN black hole of mass \( M \), the shadow radius has a minimum value of 4\( M \) in the extremal limit when the black hole is 'backlit' from a distant
uniform source or illuminated by the above-discussed spherical accretion flows [12]. Note that other accretion models may lead to a bigger black hole shadow [11]. In contrast, the shadow radius of the hairy black hole considered in this paper can become zero at the critical charge (see Fig. 6). The observation of a black hole shadow with a radius much smaller than 4M may indicate modified/alternative theories of gravity. Our results can provide a viable scenario for this.

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