Stress-strain state prediction for statically indeterminate system with accounting for malfunctioning of rod elements production

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Abstract. The use of pre-tension as a regulating mechanism in structures that are statically indeterminate rod systems usually implies the absence of initial forces and stresses. It is necessary to take into account its initial stresses from probabilistic positions and adjust the pre-tension parameters accordingly. The laws of distribution of deviations of rod elements or shipment-sized sets of details from nominal sizes and shapes are usually known. The technique of determination of reliable enough value of initial effort in the certain connection of interest of the statically indeterminate system (SIS) because of deviations of rod elements or shipment-sized sets of details from their nominal sizes and forms is presented. Thus, deviations of both rod elements and dispatch brands from the nominal sizes and forms are known to be random variables with symmetric laws of distribution. The task is solved in linear statement with result of research giving the opportunity to determine the value of initial effort using just one static design calculation.

Keywords: statically indeterminate system, stress-strain state, average square deviation, random variable, linear statement, symmetric law of distribution.

1 Introduction

The use of pre-tension as a regulating mechanism in structures that are statically indeterminate rod systems usually implies the absence of initial forces and stresses. However, the actual structure is assembled from elements or shipment-sized sets of details which dimensions and shape differ from the nominal ones, which leads to the appearance of these stresses [1, 2].

To ensure the required reliability of a pre-stressed structure [3-5], it is necessary to take into account its initial stresses from probabilistic positions and adjust the pre-tension parameters accordingly.

The laws of distribution of deviations of rod elements or shipment-sized sets of details from nominal sizes and shapes are usually known [6-12]. This is true for a variety [13-16] of steel structures and constructions, including ones with a frame structure [17-19]. The problem is to determine the numerical characteristics of the law of distribution of the initial effort in the connection of the system we are interested in as a random variable [20-25]. With these characteristics, calculating a sufficiently reliable value of this initial effort is elementary [26-28]. It is quite possible to get this value in a simpler and more visual way [29-31].

In the subsequent review, the following assumptions are accepted:

• deviations of rod elements or shipment-sized sets of details from nominal sizes and shapes are random values with symmetrical distribution laws;
• values of limit deviations are determined in accordance with the selected manufacturing accuracy class, and they cannot be exceeded;
2 Materials and methods

2.1 Rod statically indeterminate system

For convenience and clarity of reasoning, we consider the hinge-rod statically indeterminate system (SIS) and note one of its properties.

We select two extra connections from SIS by definition [21]. We break one of the links, for example "m", and apply force (cause) in its direction) "Xm".

![Figure 1. Statically indeterminate hinge-rod system with variants of absent connections.](source)

From the canonical equation of the "method of forces":

\[ N_{mn} = \frac{\Delta_{mn}}{\delta_{mn}} = \frac{\sum N_{im} \bar{N}_{in} L_i}{EA_i} \]

where: \( \Sigma \) – by the connections of any possible statically-definable main system of the "method of forces"; \( \bar{N}_{in} \) – the force in the "i" connection of the main system from the action of a single force on it in the direction of the absent connection "n"; \( \bar{N'}_{in} \) – the effort in "i" connection SIS with broken links "n" and "m" from the action at its single strength in the direction of broken links "n"; \( N_{mn} \) – the force in the "i" connection of the main system from the action of the force "Xm" on it in the direction of the absent connection "m" as an external load. In order for the force sign \( N_{mn} \) to meet the usual sign rule and to be adequate for subsequent comparison:

\[ N_{mn} = -\frac{X_m \sum \bar{N}_{im} \bar{N}_{in} L_i}{EA_i} \]

where \( \bar{N}_{im} \) – the effort in the "i" connection of the main system from the action of a force on it "Xm" in the direction of the absent connection "m" as an external load;

"Xm" – unit force of the same direction as "Xm".

Similarly, we break the connection "n", and apply force (cause) in its direction "Xn". Then, as in the previous case,
Dividing (2) by (3), we get:
\[ \frac{N_{mn}}{N_{me}} = \frac{X_m \delta_{mn}}{X_m \delta_{mn} \sum \frac{N_{mn} E_{Am} L_i}{E A_i}}. \]  

(3)

Note: obviously, condition (4) is also true for the case when one of the connections is conditionally necessary by definition [8], and the other is superfluous; in addition, if one of the connections "m" or "n" is absolutely necessary by definition [8], these connections are mutually independent.

When analyzing expression (4), we should emphasize one circumstance that is important for further reasoning.

In a statically indeterminate system, a pair of connections is characterized by the fact that the relation of the effect in the first of them, when the cause acts in the direction of the absent second connection, to the effect in the second connection, when the cause acts in the direction of the absent first connection, has the same sign as the relation of causes. Otherwise these connections are mutually independent.

This circumstance makes it possible to offer the following method for determining a sufficiently reliable value of the initial force in the SIS connection we are interested in due to deviations of the core elements or shipment-sized sets of details from their nominal sizes and shapes.

It is proposed to search for this value in two stages.

The first step is to determine the maximum possible value of the initial effort $N_{\text{max}}$ in the SIS connection of interest in the following sequence:

- break the connection we are interested in and apply a single force action of the desired direction;
- determine the signs of effort in the remaining links of the system or in the links between shipment-sized sets of details from the action of this reason;
- enter the maximum possible deviations in these relations $\Delta \xi(\phi)$ as determined with signs opposite to the previous ones;
- determine the maximum possible effort in the connection we are interested in $N_{\text{max}}$ from deviations entered in the system.

However, the initial effort in the connection we are interested in from the inaccuracy of manufacturing core elements or shipment-sized sets of details is a random value as a result of the action of the sum of independent random variables, which are deviations. In this case, the relationship between causes and effects is linear, i.e.
\[ N = Y = \sum A_i X_i, \]  

(5)

where $N$ – initial effort in the connection we are interested in; $A_i$ – coefficient of influence of a random variable $X_i$ (deviation in size or shape from the nominal value) of $Y$. Since $X_i$ – a normally distributed random variable, then it is characterized by a mathematical expectation $M_i$ and the average square deviation $\sigma_i$ (the distribution law is normal truncated). Values of each $X_i$ in truncation points:
\[ X_i = X \pm 3 \sigma. \]  

(6)

It should be expected that $Y$ is a random variable distributed normally with a normal truncated distribution law. According to the above, the limit values of the value $Y$ at the truncation points are determined in the only way through the truncation points $X_i$ and the relationship here is not probabilistic, i.e.
\[ N_{\text{max}} = Y_{\text{lim}} = \sum A_i (M_i \pm 3 \sigma_i). \]  

(7)

On the other hand, if you know the numerical characteristics of the distribution law of a random variable $Y$, the limit value at the truncation point:
\[ N_{\text{max}} = Y_{\text{lim}} = M \pm K \sigma, \]  

(8)

where $M_i$ is the mathematical expectation of the random variable $Y$; $\sigma_j$ is the average square deviation of the random variable $Y$. Thus:
\[ N_{\text{max}} = Y_{\text{lim}} = M \pm K \sigma = \sum A_i (M_i \pm 3 \sigma_i). \]  

(9)
In our case, the desired sign of the value $Y$ coincides with the signs $A_i(\pm 3\sigma_i)$ in the first case and with the sign $\pm K\sigma_j$ in the second. Therefore, it can be assumed that:

$$M_i=\pm K\sigma_j = \Sigma A_i(M_i \pm 3\sigma_i). \quad (10)$$

Since for the problem under consideration all random variables are centered, $M_i=0$:

$$K\sigma_j = 3\Sigma A_i\sigma_i. \quad (11)$$

Since $\sigma_j = \sqrt{\sum A_i'\sigma_i'^2}$, then $K\sqrt{\sum A_i'\sigma_i'^2} = 3\Sigma A_i\sigma_i$.

Or

$$\hat{E} = 3\sqrt{\frac{\sum A_i'\sigma_i'^2}{\sum A_i'\sigma_i'^2}} = 3\sqrt{\frac{\sum A_i\sigma_i A_j'\sigma_j}{\sum A_i'\sigma_i'^2}}. \quad (12)$$

But the value of $2\sum A_i\sigma_i A_j\sigma_j$, according to the above, is always positive, so that means that $K > 3$. For practical calculations it is enough to know $Y$ in points $M_i \pm 3\sigma_i$ of the distribution law. Therefore, at the second stage, a sufficiently reliable value of the initial force in the connection $[N_{max}]$ of interest is determined from the deviations $\Delta i (\phi_i)=XI$ of the rod elements or shipment-sized sets of details from their nominal sizes and shapes.

$$[N_{max}] = \frac{2\sum A_i\sigma_i A_j\sigma_j}{1 + \sum A_i'\sigma_i'^2}. \quad (13)$$

2.2 Example for calculation

The 6 m high and 12 m span frameless frame is assembled from an I-beam with non-parallel faces of shelves № 36 and is manufactured according to the fourth class of accuracy. All components of the frame are rigid using flanged connections. Construction tolerances: for columns in length $\pm 1.6$ mm, on the top flange $\pm 1.768/1000$ rad, on the bottom flange of $\pm 2/1000$ rad; for bolt – on length $\pm 2$ mm, flange $\pm 1.768/1000$ rad; for base plate – on the span $\pm 7.5$ mm, on height $\pm 1.5$ mm, on inclination $\pm 0.667/1000$ rad.

We need to determine: A sufficiently reliable value of the bending moment in the support section of the left post with the stretched fiber on the right.

In a separate figure (Ошибка! Источник ссылки не найден.) the results of static calculation of the frame for the impact of $M = 1$ of the necessary direction and the most unfavorable distribution of deviations in the system are shown.

![Figure 2. Results of static calculation for a single impact.](image-url)
The table below (table 1) shows the influence coefficients and the average square deviations of random variables – inaccuracies in the manufacture of rod elements in length and shape.

According to (14):

$$[M_{\text{max}}] = \frac{8,296}{\sqrt{2,625184 \times 6,8916}} = \frac{8,296}{2,7653} \approx 3(\mu)$$

*Figure 3. Results of the final calculation to determine the maximum possible value of the bending moment of interest.*

| №  | Type and location of a random variable inaccuracy | $A_i \sigma_i$ (tm) | $(A_i \sigma_i)^2$ | Note |
|----|-------------------------------------------------|---------------------|-------------------|------|
| 1  | Inclination of the left base plate              | 0,20567             | 0,0423            |      |
| 2  | Height offset of the left base plate            | 0,01474             | 0,0002            | The impact is negligible |
| 3  | Horizontal offset of the left base plate        | 0,35083             | 0,1231            |      |
| 4  | Angular deviation of the lower flange of the left post | 0,61672 | 0,3803 |                  |
| 5  | Length deviation of the left post               | 0,015632            | 0,0002            | The impact is negligible |
| 6  | Angular deviation of the upper flange of the left post | 0,05154 | 0,0027 | The impact is negligible |
| 7  | Angular deviation of the left flange of the crossbar | 0,05154 | 0,0027 | The impact is negligible |
| 8  | Length deviation of the crossbar                | 0,09309             | 0,0087            |      |
| 9  | Angular deviation of the right flange of the crossbar | 0,16797 | 0,0282 |                  |
| 10 | Angular deviation of the upper flange of the right post | 0,16797 | 0,0282 |                  |
| 11 | Length deviation of the right post              | 0,015632            | 0,0002            | The impact is negligible |
| 12 | Angular deviation of the lower flange of the right post | 0,38082 | 0,1450 |                  |
| 13 | Horizontal offset of the right base plate       | 0,35083             | 0,1231            |      |
| 14 | Height offset of the right base plate           | 0,01520             | 0,0002            | The impact is negligible |
| 15 | Inclination of the right base plate             | 0,12700             | 0,0161            |      |

$$\Sigma A_i \sigma_i = \Sigma (A_i \sigma_i)^2 \approx 2,625184 \times M_{\text{max}}/3 \approx 0,9012$$
4 Conclusions
In connection with the above results it can be concluded that a fairly narrow range of values for the denominator of the formula (14) can be set for frequently used statically indeterminate structural forms (frames, arches, etc. – with a small degree of static indeterminability). This gives the opportunity to determine \[N_{\text{max}}\] using just one static design calculation.

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