Recurrence-based diagnostics of rotary systems

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Abstract
Rotary mechanisms are commonly used for transferring rotational movement in diverse industrial applications in mechanical engineering. Components such as gears and rolling bearings have found their purpose in various automotive, machinery or agriculture systems. During operation in mechanisms, they are subjected to defects or changes of their key parameters. This paper considers the application of recurrence plots (RP) and recurrence quantification analysis (RQA) in the detection of teeth crack in a planetary gear system and radial internal clearance (RIC) in a double row self-aligning ball bearing. Raw signals are obtained from accelerometers installed in test rigs. The analysis consists of a statistical analysis approach and calculations of basic RQA parameters i.e. recurrence rate, determinism and length of the longest diagonal. In the paper, we extract information about the fault detection in one of the rotary systems and about bearing operational parameters with nonlinear dynamics identification.

1. Introduction
Powerful tools applied to analyse nonlinear systems include recurrence plots and a recurrence quantification analysis. They have been used in a variety of areas of nonlinear science e.g. physiology, engineering, physics, astronomy. In general, the recurrence plot (RP) is an advanced technique of signal analysis. For the first time, it was proposed by Eckmann [1] as a graphical representation of a dynamical process. The RP represents a square matrix of time events, plotted with points. Black dots correspond to those time events at which a dynamical system recurs. It shows all time events when the dynamical trajectory of the system meets the same area in the phase space [2] with the assumed conditions.

The efforts of diagnostics on rotary systems are mainly focused on digital signal processing (DSP) technologies, which can be divided into the time (statistical) and frequency (spectral) domain analysis [3]. Furthermore, numerous studies consider the application of extended standard DSP methods, such as Fourier and Wavelet Transforms [4,5] or Hilbert Transform (HT) in the detection of defects and operational parameters in rotary systems [6,7]. One of the advantages of the abovementioned methods is their computation time, however, they do not allow for easy acquisition of information buried in noised or non-stationary time series. The application of recurrence plots analysis seems to be adequate to obtain additional information coming from measurements.
In this work, we present a promising approach to distinguish the particular nonlinear behaviours in two rotary systems, the planetary gear system and in the self-align ball bearing. Note that nonlinearities can be introduced to the system by defects and other operational conditions represented by a friction level, damping, and load. Their share in spectra is hard to determine unambiguously with standard methods, such as the fast Fourier transform (FFT) or signal envelope; by the application of recurrence plot analysis, however, we can obtain specific patterns in the phase space trajectory. Moreover, by application of the recurrence quantification analysis (RQA), it is possible to assign a quantificator value to describe each feature. The main advantage of the RQA is using fairly short data lengths. Besides, RQA has also a good anti-noise ability.

Up to now, several articles have considered diagnostics of rotary systems with the abovementioned method, for example, in the evaluation of a degradation level in bearings [8,9] or a fault detection of servo valves [10]. Our work investigates and compares both analyses of the fault detection in the planetary gear system with a cracked tooth [11] and the recognition of variable radial internal clearance in the self-align ball bearing [12].

2. Recurrence plot (RP) and recurrence quantification analysis (RQA)
Many dynamical systems are characterised by their property to come back to the previous state, referred to as recurrence. This technique was first proposed in seminal work of Henri Poincare in 1890 [13] and almost one hundred years later, developed by Eckmann et al. [1] formulating the recurrence plot method. In order to obtain a quantitative interpretation of the RP method, recurrence quantification analysis was introduced to obtain numeral information hidden in recurrence plots. Nowadays, the mentioned analyses are used for the detection of quantitative changes in the behaviour of dynamical systems and give satisfying results in various applications.

2.1. Recurrence plot (RP) method
Recurrence analysis is a graphical technique designed to locate hidden patterns, non-stationary and structural changes occurring in the dynamical system [14], by revealing all the times when the phase space trajectory meets the same area in the phase space. We obtain the recurrence when a state x at time i at a different time j is indicated within a two-dimensional square matrix [R] with ones (recurrence state) and zeros (no recurrence):

\[ [R_{i,j}] = \Theta(\varepsilon - ||x_i| \{-x_j\}) \quad i,j = 1, \ldots, N \]

where, N denotes the number of considered states \( x_n \), \( \varepsilon \) states for the threshold and \( \Theta(\varepsilon) \) is the Heaviside function. Recognised states on recurrence plot are represented in the following way:

\[ [R_{i,j}] = \begin{cases} 1: & \{x_i\} \approx \{x_j\} \\ 0: & \{x_i\} \neq \{x_j\} \end{cases} \quad i,j = 1, \ldots, N \]

\( \{x_i\} \approx \{x_j\} \) means equality up to an error (or distance) \( \varepsilon \). To sum up, the matrix \([R_{i,j}]\) compares states of the system at both states and if they are similar, they are marked by one in the matrix \( R_{i,j} = 1 \) or when they are different the matrix is \( R_{i,j} = 0 \). Obtained matrix creating recurrence plot, shows us when similar states of the considered dynamical system occur.

In our research for both considered systems, we have found the parameter of the embedding dimension determined with the value of the first minimum of the false nearest neighbours (FNN) algorithm [13]. On the other hand, the time delay \( \tau \) was found by the first minimum in the mutual information (MI) function.

2.2. Recurrence quantification analysis (RQA)
In order to support the interpretation of recurrence plots, recurrence quantification analysis (RQA) was introduced, which quantifies the number and the duration of recurrences for the considered dynamical system by its state space trajectory. At first, it was used for exploration of a non-stationary cardiac
signal [15], but with development of recurrence techniques, they began to be applied also in various engineering systems, such as a diesel engine [16], a bevel gearbox [17] or a car’s braking system [18]. In our work, we focus on three main quantificators i.e. recurrence rate (RR), determinism (DET) and the average length of the diagonal (L) [19,20].

- **Recurrence rate (RR)**

  The recurrence rate is a basic RQA parameter, which describes the density of recurrence points on the recurrence plot obtained by different input parameters.

  \[
  RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j} \quad \text{for } |i-j|\geq 1
  \]  

- **Determinism (DET)**

  By determinism, the percentage of recurrence points creating diagonal lines in the recurrence plot of minimal length \(l_{\text{min}}\) is expressed.

  \[
  DET = \frac{\sum_{l=l_{\text{min}}}^{l_{\text{max}}} lP(l)}{\sum_{l=1}^{l_{\text{max}}} lP(l)}
  \]  

  P(l) in formula (4) states for the frequency distribution of the lengths \(l\) of the diagonal lines.

- **Average length of the diagonal lines (L)**

  This measure refers to the predictability time of the investigated system.

  \[
  L = \frac{\sum_{l=l_{\text{min}}}^{l_{\text{max}}} lP(l)}{\sum_{l=l_{\text{min}}}^{l_{\text{max}}} P(l)}
  \]  

3. **Test rigs and measurements**

The first of analysed systems was the single-stage planetary gearbox transmission (type: 2K-H), figure 1 concerns two cases, namely, when the system works properly and with planet gear defect, figure 2. For measurements of the system’s response, we used an accelerometer mounted on the ring gear presented in figure 1 (3). In the test, only one constant rotational velocity of the sun gear was considered, in order to obtain spectra for both cases with the same characteristic rotational frequencies. We focused on the part of spectra where we got the difference between the proper and the defective system’s work.

**Figure 1.** Planetary gear system used for tests – numbers indicate eddy probe (1) and piezoelectric accelerometers (2-4).
Figure 2. One of planetary gears with a damaged tooth.

Moreover, in our research, we decided to determine if there is the possibility to detect the change of the radial internal clearance (RIC) in the double-row self-aligning ball bearing using RP and RQA. For this, we used the experimental system, figure 3, consisting of the three-phase motor, which was propelling a shaft with two above mentioned ball bearings mounted on the conical adapter sleeves and housed in the adapter with accelerometers mounted into it. We obtained the system’s response from one MEMS sensor only. In figure 3 markings L and F on the housings state for locating and fixed bearing, correspondingly. On the first one, we were changing its RIC by pressing or pulling it onto the conical adapter sleeve. The measurement procedure of bearing’s RIC is described in the following articles [21, 2]. The test was performed for three different clearance values and by one rotational velocity as in planetary gear system.

Figure 3. Test rig used for the experiment (a) and its cross-sectional view (b) with indicated distances.

4. Results and discussion

One of the proper approaches to start recurrence analysis is the application of basic statistical tools like a mean or a standard deviation for obtained signals. This is the first step in which we could identify numerically the difference between each system’s operational state. In our analysis, we calculated the standard deviation for signals and their values are aggregated in Table 1. The discrepancies of SD values in each system are the result of applying different accelerometers and their montage.

| Table 1. Standard deviation values for time courses of all considered cases. |
|--------------------------------|
| Planetary Gear System | Healthy | Planet Gear Crack |
|------------------------|---------|-------------------|
| SD (m/s²)              | 0.1392  | 0.2918            |
| RIC (µm)               | 7       | 34                |
|                        |         | 46                |
| SD (m/s²)              | 2.8727  | 3.9073            |
|                        |         | 4.6406            |

The next step is to find values of three crucial input parameters for drawing recurrence plot i.e. embedding dimension – m, time delay – τ and threshold – ε. Two first parameters were found with the approach described in the last paragraph of sub-section 2.1, but there is still no unambiguous method
for the threshold determination. Because two different rotary systems are compared, we decided to use the same threshold value $\varepsilon = 0.5$ (in standard deviation units) for all considered cases, in order to ensure the consistency of obtained data. Our approach we can name as the empirical one and all the values are collected in Table. 2. That value we assumed based on recurrence rate results, figure 4a, where we can observe the clear separation between each recurrence rate line for both systems.

|                  | m | $\tau$ | $\varepsilon$ |
|------------------|---|--------|--------------|
| Planetary Gear System | 3 | 2      | 0.5          |
| RIC              | 2 | 3      |              |

Table 2. Values of recurrence plots input parameters, $\tau$ is represented in sampling intervals.

The last step of analysis is RQA, which we performed for three recurrence quantificators described in subsection 2.2. All results are presented for adopted signal intervals in figure 4. It needs to be emphasised that with the increase in standard deviation value we obtain higher values of each quantificators in $\varepsilon$ domain. In both systems, each considered state is recognisable on all plots, that what makes the systems different from each other is the sensitivity on gear damage and clearance value in different $\varepsilon$ range.

Figure 4. RQA results for the planetary gear system (in the left panel) and the radial internal clearance (in the right panel). The order of quantificators is the following: a) Recurrence rate, b) Determinism, c) Length of the longest diagonal.
Representative examples of studied RP are presented in figure 5 and figure 6. For all considered cases they are reproducing dynamic system trajectories returning to the same area in the embedded space, thus giving a recurrence point marked with a black dot. Analysing results obtained for planetary gear system, we can determine that RP during planetary gear crack is characterised with small fluctuations indicating periodicity in the system. The fluctuations in question are strongly connected with increased nonlinearity, including phase couplings between main and super- and sub-harmonics during rotation of the cracked gear, which brings a more regular pattern of the RP. In the studied bearing system, we noticed that the clearance, which causes a nonlinear effect can lead to the observation of lines splitting in the monotonic pattern.

**Figure 5.** Recurrence plots for the state with planet gear crack (left) and for the proper operation (right).

**Figure 6.** Recurrence plots for RIC=7µm (top left), RIC=34µm (top right) and RIC=46µm (centre bottom).
5. Conclusions
Based on RPs and various RQAs, we detected planetary gear faults and clearance level effects. In the considered systems, we focused on the increasing nonlinearities reflected in the system responses. This response can become non-harmonic, non-periodic or intermittent by large nonlinearity. It is proved by isolated areas on the RP consisting of recurrence points localised closely to each other. Additionally, non-harmonicity causes RP’s pattern to be more compact (improving contrast between black and white regions) with increasing RIC. In light of this study, we are convinced that RP and RQA statistics are useful tools for faults and operational parameter identification.

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