Product quality control strategy of dual distribution channel structure in three-echelon supply chain

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Abstract
Based on the three-stage Stackelberg dynamic game model, we study how to make product quality control strategy in the three-echelon supply chain consisting of the manufacturer, retailer and customer under retailer dual-channel structure and manufacturer dual-channel structure. In two decision models (decentralized decision, centralized decision), we analyze the demand price elasticity, market share ratio, revenue sharing ratio and quality cost coefficient how to influence the product demand, product quality level, retail price and direct price in different channels, expected revenue functions of manufacturer and retailer, consumer surplus and product quality control strategy. We find that firstly, the retail price and direct price are positively related to product quality level, and the product quality level is negatively related to the demand price elasticity in traditional retail channel and the demand price elasticity in internet channel. What’s more, the retailer’s retail price in traditional retail channel will be higher than direct price in internet channel. Thirdly, in the case of centralized decision, the manufacturer’s product quality level, retail price, joint expected revenue and consumer surplus will all rise, but the direct price will fall. Fourthly, when the manufacturer entrusts the third-party platform to build the internet channel, the manufacturer’s product quality level, retail price, direct price, expected revenue, and consumer surplus will all decrease. Finally, we conduct the simulation analysis by MATLAB 2018, which verifies the validity and credibility of our conclusions and points the direction for the specific application of the model in practice.

Keywords] Dual-channel structure · Three-echelon supply chain · Stackelberg dynamic game · Product quality control strategy · Simulation analysis

1 Introduction
In recent years, with the rise of internet economy and e-commerce, more and more customers or consumers choose to buy products in internet channel. In 2018, e-commerce transactions in 28 major countries and regions reached 24.72 trillion USD, with online retail transactions totaling 2.97 trillion USD (Global E-commerce Data Report 2019). In the fourth quarter of 2019, U.S. e-commerce retail sales grew 16.4 percent to 187 billion USD and reached 602 billion USD for the full year (U.S. Department of Commerce on February 19, 2020). In 2019, the amount of Chinese online retail sales has exceeded 1.62 trillion USD, up 16.5 percent from 2018 (the Chinese Ministry of Commerce on June 30, 2020). On November 11, 2020, Tmall’s “Double Eleven” sales reached 76.2 billion USD, a year-on-year increase of 26%; JD Mall’s “Double Eleven” sales also reached 41.53 billion USD, a year-on-year increase of 32.8% (Chinanews.com on November 12, 2020). So, we can see that in addition to traditional retail channel, internet channel has gradually become an important way for product sales in the supply chain.

With the changing of customers or consumers’ buying behavior, more and more enterprises begin to redesign or construct their distribution channel structures. For example, HP, Nike, Lenovo, Suning and Gome have opened internet channel in addition to traditional retail channel. Dell and Xiaomi, which used to focus on internet channel,
are now beginning to sell products in traditional retail channel. While Apple and Haier have sold products in both traditional retail channel and internet channel from the beginning.

However, in the exploration of theory and practice, there are still some unresolved problems. Firstly, how to construct different distribution channels and the influence of different distribution channels on product quality decision in the three-echelon supply chain. What’s more, the influence on making product quality control strategy when the retailer establishes the dual-channel structure or the manufacturer establishes the dual-channel structure. Thirdly, when the manufacturer entrusts the third-party platform to build the internet channel or the retailer establishes the dual-channel structure, how to influence the product quality, retail price and direct price. Finally, the influence of price demand elasticity in different distribution channels on product quality decision, price decision, expected revenue functions, and consumer surplus of the customer.

In recent decades, the ever-increasing complexity and difficulty of real-world problems result in the need for more reliable optimization techniques Abualigah et al. 2018. Optimization is introduced into various fields to determine the best solution to a complex problem (Abualigah 2019); (Abualigah et al. 2021). So, based on Game Theory, we construct the three-stage Stackelberg dynamic game model and study how to make product quality control strategy in the three-echelon supply chain under the retailer dual-channel structure and the manufacturer dual-channel structure. Under decentralized decision and centralized decision, we analyze the demand price elasticity, market share ratio, revenue sharing ratio and quality cost coefficient how to influence the product demand, product quality level, retail price and direct price in different channels, expected revenue functions of manufacturer and retailer, consumer surplus and product quality control strategy.

The rest of our paper is organized as follows. In Sect. 2, we review the relevant literature, and we make hypotheses and construct the model in Sect. 3. In Sect. 4, we analyze the case that the retailer establishes the dual-channel structure under decentralized decision and centralized decision. In Sect. 5, we analyze the case that the manufacturer establishes the dual-channel structure. We use MATLAB 2018 to conduct simulation analysis in Sect. 6. Finally, the discussion is in Sect. 7, and the conclusions are in Sect. 8.

2 Literature review

Recently, many scholars have conducted a great deal of researches on how to establish different distribution channel structures in the three-echelon supply chain, and how to formulate product quality control strategy under different distribution channel structures and two decision models (decentralized decision and centralized decision), mainly in the following three aspects.

The first aspect is mainly about the influence of distribution channels in the supply chain on product quality, selling price, corporate profits and consumer surplus. Chen et al. (2017a) employ two themes in terms of channel-sharing Pareto zone to characterize the impacts of channel structures on supply chain performance, including the whole system’s profit, each player’s profit, and consumer surplus. Modak and Kelle (2019) examine a dual-channel supply chain under price and delivery-time-dependent stochastic customer demand and find that uncertainty frequently arises in both retail and online channels which has an effect on the optimal order quantity and price. (Tian and Jiang 2018) study how consumer-to-consumer product sharing in the supply chain affects the strategic choice of distribution channels, as well as the impact on manufacturers’ profits and consumer surplus. (Matsui 2017) apply an observable delay game framework developed in non-cooperative game theory, investigate the timing problem concerning when a manufacturer managing dual-channel supply chains, consisting of a retail channel and a direct channel, should post its wholesale price and direct price. (Wong et al. 2019) find that manufacturers adopt generic strategic choices in decentralized channel can reduce channel efficiency losses and analyze its impact on channel profits and consumer surplus. Zhang et al. (2019a) consider the case that manufacturers establish a direct platform channel to reach customers directly and study the interrelationship between a platform’s contract choice and a manufacturer’s product quality decision.

The second aspect is about the strategic choices and decisions of different distribution channels, as well as the impact on channel coordination. Many scholars studied the distribution channel structure (Wang et al. 2017), mainly including direct channel structure (Wang et al. 2018), indirect channel structure (Luo et al. 2018) and (Dey et al. 2019) and mixed channel structure (Yan et al. 2020). (Guo and Heese 2017) investigate how the manufacturer’s optimal product variety decision differs when selling directly to customers (centralized decision) as compared to selling through a retailer (decentralized decision). (Yang et al. 2018) study the optimal distribution strategy of a supplier with limited capacity and find the supplier may adopt the supplier-only role, be the solo seller in the market, or use
the dual-channel strategy and compete with its downstream buyer. Chen et al. (2017b) examine how a direct channel added by a manufacturer can influence the decisions of the retailer and the manufacturer and further propose a retailer’s margin contract that can coordinate the dual-channel supply chain and ensure that both the retailer and the manufacturer will be more profitable. (Feng et al. 2017) investigate two problems that are comprised of designing and coordinating a reverse supply chain with a traditional and an online recycling channel and in the coordination problem, a contract with transfer and online recycling prices can coordinate the dual-recycling channel reverse supply chain but harms the dealer. (Lan et al. 2018) show that the dual-channel system benefits the manufacturer and the retailer if the level of demand uncertainty exceeds a threshold and that the competition between the two distributors leads to the coordination of the downstream supply chain (the two distributors and the retailer). Rahmani and Yavari (2019) investigate the demand disruption management in a dual-channel supply chain producing and selling green products for the first time and results reveal that when the disruption increases the market scale, or when the greening cost decreases, the optimal prices will be increased in both decision-making structures. (Song et al. 2020) study dynamic channel control and pricing of a single perishable product distributed through multiple channels and the results show that the magnitude of the opportunity cost of capacity uniquely determines the optimal channel control.

The third aspect is about how to prevent product quality risks in the strategies of supply chain distribution channels. (Liu et al. 2018) study that the uncertainty of market size will reduce the difference of product quality and determine the optimal product quality level in the centralized and decentralized distribution channel structure. Sarkar and Saren (2016) discuss how to make product quality decision in different distribution channel strategies and how to prevent channel quality risks by building product quality decision model. Zhang et al. (2019a) analyze the relationship between platform contract selection and manufacturer’s product quality decision when online sales channels exist in the supply chain and further discuss the influence of platform sales and contract design on reducing product quality risks by building a revenue sharing model. (Huang et al. 2019) develop game-theoretic models for a supply chain with a manufacturer and a PI (parallel importer), in which the manufacturer needs to determine distribution structure, product quality, and retail price and find that the advertising effect may motivate the manufacturer to improve product quality. Zhang et al. (2019b) show that manufacturer encroachment leads to a lower quality when the manufacturer’s direct selling cost is intermediate and compared to the full and no information cases, asymmetric information may increase quality when direct selling is relatively efficient while decrease quality otherwise. (Jabarzare and Rasti-Barzoki 2020) investigate how the packaging company can influence the quality of products through packaging products in a dual-channel supply chain and find that from quality-seeking customers’ perspective, the cooperation of manufacturer and packaging company under profit-sharing contract is more preferable. Zhu and You (2011), Zhu (2020) analyzes three types of distribution channels strategy (direct channel, retail channel and mixed channel) in the context of how they influence a manufacturer’s product quality decision and quality prevention strategy and focuses on how to control product quality and design quality contract in supply chain when moral hazard exists, which proposing suggestions for quality control strategy and contract design in the supply chain under the conditions of asymmetric information. Therefore, compared with previous scholars, this paper is mainly different in the following three aspects. Firstly, based on the three-stage Stackelberg dynamic game, this paper constructs the product quality control strategy model in the three-echelon supply chain and analyzes the influence of retailer dual-channel structure and manufacturer dual-channel structure in two decision models (centralized decision and decentralized decision) on product quality decision. Then, under decentralized decision and centralized decision, we discuss the demand price elasticity, market share ratio, revenue sharing ratio and quality cost coefficient how to influence the product demand, product quality level, retail price and direct price in different channels, expected revenue functions of manufacturer and retailer, consumer surplus and product quality control strategy. Finally, we use MATLAB 2018 for simulation analysis, which verifies the validity and credibility of our conclusions and points the direction for the specific application of the model in practice.

3 Model construction and hypotheses

In this paper, we construct the three-echelon supply chain consisting of the manufacturer, retailer, third-party platform and customer and make the following hypotheses.

**H1** The manufacturer, retailer, third-party platform and customer are risk neutral.

**H2** There are two dual-channel structures which are the retailer dual-channel structure (traditional retail channel, internet channel) and the manufacturer dual-channel structure (traditional retail channel, a third-party platform internet channel).
H3 The manufacturer has two decision models to choose, which are decentralized decision and centralized decision.

H4 The manufacturer determines the quality level and wholesale price, and the retailer determines the retail price. When the retailer establishes the internet channel, it determines the direct price. The customer determines the product demand.

H5 Under the dual-channel structure, the demand price elasticity coefficient is different between traditional retail channel and internet channel.

H6 In the distribution channel strategy of supply chain, both the manufacturer and the retailer pursue the maximization of expected revenue, and the customer pursues the maximization of consumer surplus.

The relevant variables and parameters are described as follows.

\( q \): Manufacturer’s product quality level, \( q \in [0, +\infty) \).

\( w \): Manufacturer’s product wholesale price.

\( C(q) \): Manufacturer’s product production cost.

\( C(q) = kq^2/2 \), so \( C'(q) > 0 \), \( C''(q) > 0 \), and \( k \) is product quality cost coefficient.

\( p_r \): Product retail price in traditional retail channel.

\( p_d \): Product direct price in internet channel.

\( r_m \): Revenue sharing ratio between manufacturer and the third-party platform.

\( \eta_q \): Demand price elasticity in traditional retail channel and demand price elasticity in internet channel. And the customer is more price sensitive in internet channel, so \( \eta_q > \eta_r \).

\( \lambda \): Revenue sharing ratio between manufacturer and the retailer.

The customer’s consumer surplus is

\[
\text{Customer’s consumer surplus} = qv - pr - qr \sim U(0, M),
\]

and \( f(v) \) is probability density. So, the relationship of dual-channel structure constructed in this paper is shown in Fig. 1.

**Case 1** When the retailer establishes the dual-channel structure, the sequence of three-stage Stackelberg dynamic game is as follows.

Stage 1, the manufacturer determines the product quality level. Stage 2, the manufacturer determines the wholesale price. Stage 3, the retailer determines the retail price in traditional retail channel and the direct price in internet channel, respectively.

**Case 2** When the manufacturer establishes the dual-channel structure, the sequence of three-stage Stackelberg dynamic game is as follows.

Stage 1, the manufacturer determines the product quality level. Stage 2, the manufacturer determines the wholesale price in traditional retail channel and revenue sharing ratio in internet channel, respectively. Stage 3, the retailer determines the retail price, and the third-party platform determines the direct price.

### 4 The retailer dual-channel structure

#### 4.1 Decentralized decision

When the retailer establishes the dual-channel structure (traditional retail channel, internet channel), in the case of decentralized decision (manufacturer and retailer make independent decision, respectively), first of all, the manufacturer determines the product quality level, then the wholesale price. The retailer determines the retail price in traditional retail channel and the direct price in internet channel, respectively. Therefore, we construct the Stackelberg game model of manufacturer and retailer, which is as follows.

\[
\max_{q, w} \Pi_M(q, w) = (w - kq^2/2)[M - (\eta_q p_r + \eta_r r_m)/q] \\
\text{s.t. } \{p_r, p_d\} = \arg\max \Pi_R(p_r, p_d) \\
\max_{p_r, p_d} \Pi_R(p_r, p_d) = (pr - w)(\lambda M - \eta_q p_r/q) + (cr - w)[(1 - x)M - \eta_r r_m/q] 
\]

Equation (1) is the manufacturer’s expected revenue function. Equation (3) is the retailer’s expected revenue function.

**Proposition 1** When the retailer establishes the dual-channel structure under decentralized decision, the retail price in traditional retail channel and the direct price in internet channel are positively related to product quality level. The product quality level is negatively related to demand price elasticity in traditional retail channel and demand price elasticity in internet channel. So, the optimal product quality level is \( q^D = 2M/[3k(\eta_q + \eta_r)] \).
Proposition 1 indicates that when the retailer establishes the dual-channel structure, in the case of decentralized decision, the retail price in traditional retail channel and the direct price in internet channel increase with the product quality level improving; the product quality level will decrease with demand price elasticity in traditional retail channel and demand price elasticity in internet channel increasing.

Proposition 2 The retailer’s expected revenue function is a joint concave function about the retail price and direct price. And the stationary point \( \{p_r^{D*}, p_e^{D*}\} \) will make \( \PhiR \) take the maximum. \( p_r^{D*} > p_e^{D*} \), which indicates that the retail price in traditional retail channel will be higher than the direct price in internet channel.

Proof Solve the Hessian matrix of Eq. (3) about \( p_r \) and \( p_e \) and get.

\[
H = \begin{bmatrix}
\frac{\partial^2 \PhiR}{\partial p_r^2} & \frac{\partial^2 \PhiR}{\partial p_r \partial p_e} \\
\frac{\partial^2 \PhiR}{\partial p_e \partial p_r} & \frac{\partial^2 \PhiR}{\partial p_e^2}
\end{bmatrix} = \begin{bmatrix}
-2\eta_r/q & 0 \\
0 & -2\eta_e/q
\end{bmatrix}
\]

The first-order principal minor of the matrix \( H \) is less than zero and the second-order principal minor is greater than zero, which shows that \( H \) is a negative definite matrix. \( \PhiR \) is a joint concave function, and it has local maximum.

Substitute Eq. (15) into Eq. (11), and get

\[
w^{D*} = 4M^2/9k(\eta_r + \eta_e)^2
\]

Substitute Eqs. (15) and (16) into Eqs. (6) and (7), and get

\[
p_r^{D*} = \frac{aM^2}{3k\eta_r(\eta_r + \eta_e) + 2M^2/9k(\eta_r + \eta_e)^2}
\]

\[
p_e^{D*} = (1 - a)M^2/3k\eta_e(\eta_r + \eta_e) + 2M^2/9k(\eta_r + \eta_e)^2
\]

Compare Eq. (17) with Eq. (18), and get.

\[
p_r^{D*} - p_e^{D*} > 0, p_r^{D*} > p_e^{D*}
\]

Proposition 2 Indicates that the retailer’s expected revenue function is a joint concave function about the retail price and direct price. So, there are optimal retail price and optimal direct price to make the retailer’s expected revenue maximum and the retail price in traditional retail channel will be higher than the direct price in internet channel, which is consistent with the actual situation.
Corollary 2.1 

The expected revenue functions of manufacturer and retailer are negatively related to the demand price elasticity in traditional retail channel and the demand price elasticity in internet channel, respectively.

**Proof** Substitute Eqs. (15), (16), (17) and (18) into Eqs. (1) and (3), and get.

\[
E_{PM} = M^3 / 27k(\eta + \eta_e)^2
\]

\[
E_{PR} = [\eta_e x^2 + \eta_e (1 - x)^2]M^3 /[6k\eta_e(\eta + \eta_e)]
- 4M^2 / 27k(\eta + \eta_e)^2
\]

(20)

Take the first partial derivative of Eqs. (19) and (20) with respect to \(\eta, \eta_e\), respectively, and get.

\[
\frac{\partial E_{PM}}{\partial \eta} < 0, \frac{\partial E_{PM}}{\partial \eta_e} < 0
\]

(negative correlation, decreasing function)

\[
\frac{\partial E_{PR}}{\partial \eta} < 0, \frac{\partial E_{PR}}{\partial \eta_e} < 0
\]

(negative correlation, decreasing function)

\[\square\]

Corollary 2.1 Indicates that the expected revenue functions of manufacturer and retailer will decrease with the demand price elasticity in traditional retail channel and the demand price elasticity in internet channel increasing.

So, the customer’s consumer surplus is

\[CS_{PR} = \int_0^\infty (v_qD_{PR} - P_{PR}D_{PR})f(v)dv + \int_0^\infty (v_qD_{PR} - P_{PR}D_{PR})f(v)dv
\]

\[= \eta_e [\eta_e x^2 - \eta_e (1 - x)^2]M^3 / [3k\eta_e(\eta + \eta_e)] - 2M^2 / 9k(\eta + \eta_e)^2
\]

(21)

\(CS_{PR}\) is the customer’s consumer surplus when retailer establishes the dual-channel structure under decentralized decision.

4.2 Centralized decision

The manufacturer and the retailer make centralized decision, that is, the manufacturer and the retailer make joint decision, and the supply chain system composed of the manufacturer and the retailer is vertically integrated. So, we construct the supply chain system decision model composed of the manufacturer and the retailer, the model is as follows.

\[\max E_{PM}(q, p_r, p_e) = (p_r - kq^2/2)(zpdf - M - \eta q / q) + (p_e - kq^2/2)(1 - x)M - \eta e q / q
\]

(22)

Equation (22) is supply chain joint expected revenue function.

**Proposition 3** Under centralized decision, the manufacturer and the retailer make joint decision. The manufacturer’s product quality level will be higher than that under decentralized decision, i.e., \(q^{C*} > q^{D*}\).

**Proof** Use backwards induction method to solve, and take the first partial derivative of Eq. (22) with respect to \(p_r\) and \(p_e\), respectively, and get.

\[p_r = zMq/(2\eta_e) + kq^2/4
\]

\[p_e = (1 - x)Mq/(2\eta_e) + kq^2/4
\]

(23)

(24)

Solve the Hessian matrix of Eq. (22) about \(p_r, p_e\) and \(q\), and get.

\[H = \begin{bmatrix}
\frac{\partial^2 E_{PM}}{\partial q^2} & \frac{\partial^2 E_{PM}}{\partial q\partial p_r} & \frac{\partial^2 E_{PM}}{\partial q\partial p_e} \\
\frac{\partial^2 E_{PM}}{\partial q\partial p_r} & \frac{\partial^2 E_{PM}}{\partial q^2} & \frac{\partial^2 E_{PM}}{\partial q\partial p_e} \\
\frac{\partial^2 E_{PM}}{\partial q\partial p_e} & \frac{\partial^2 E_{PM}}{\partial q\partial p_e} & \frac{\partial^2 E_{PM}}{\partial q^2}
\end{bmatrix}
\]

\[= \begin{bmatrix}
-2\eta_e/q^2 & 0 & k\eta_e/2 + 2\eta_e p_r/q^2 \\
0 & -2\eta_e/q^2 & k\eta_e/2 + 2\eta_e p_r/q^2 \\
k\eta_e/2 + 2\eta_e p_r/q^2 & k\eta_e/2 + 2\eta_e p_r/q^2 & -kM - 2(\eta_e p_r^2 + \eta_e p_e^2)/q^3
\end{bmatrix}
\]

The first-order principal minor of the matrix \(H\) is less than zero, the second-order principal minor is greater than zero, and the third-order principal minor is less than zero, which shows that \(H\) is a negative definite matrix.

\[E_{PM}\] is a joint concave function.

Substitute Eqs. (23) and (24) into Eq. (22), and get

\[E_{PM}(q) = (zMq/2\eta_e - kq^2/4)(zM/2 - \eta qkq/4) + ((1 - x)Mq/2\eta_e - kq^2/4)(1 - x)Mq/2 - \eta qkq/4
\]

(25)

Take the first and second partial derivative of Eq. (25) with respect to \(q\), and get

\[q_1 = 2M(2 - \sqrt{\varepsilon})/[3k(\eta_r + \eta_e)]\quad \text{or} \quad q_2 = 2M(2 + \sqrt{\varepsilon})/[3k(\eta_r + \eta_e)]
\]

(26)

\[\frac{\partial E_{PM}}{\partial p_r} = 2k^2(\eta_r + \eta_e)q / 8 - Mk/2 < 0
\]

(27)

\[q < 4M/[3k(\eta_r + \eta_e)]
\]

(28)

From Eqs. (26) and (28), we get

\[q^{C*} = 2M(2 - \sqrt{\varepsilon})/[3k(\eta_r + \eta_e)]
\]

(29)

\[\varepsilon = [4\eta_r, \eta_e - 3(\eta_r + \eta_e)(\eta_e x^2 + \eta_r (1 - x)^2)]/\eta \eta_e
\]

\[0 < \varepsilon < 1, \quad 0 < (1 - \varepsilon) < 1, \quad \eta_r < \eta_e
\]

\[\varepsilon < 1
\]

Compare Eq. (29) with (15), and get

\[\square\]
respectively, and get.

\[
d^C_s = 2M(2 - \sqrt{e})/[3k(\eta_r + \eta_e)] > 2M/[3k(\eta_r + \eta_e)]
\]

\[= q^{Ds}
\]

(30)

\[\square
\]

Proposition 3 Indicates that when the retailer establishes the dual-channel structure, in the case of centralized decision, the manufacturer’s product quality level will be higher than that in the case of decentralized decision.

From proposition 3, we can get the following corollaries.

Corollary 3.1 The price in traditional retail channel will rise, i.e., \( p_r^{C_s} > p_r^{Ds} \). The price in internet channel will fall, i.e., \( p_e^{C_s} < p_e^{Ds} \).

Proof Substitute Eq. (29) into Eqs. (23) and (24), respectively, and get.

\[
p_r^{C_s} = 2M^2(2 - \sqrt{e})/[3k\eta_r(\eta_r + \eta_e)] + M^2(2 - \sqrt{e})^2/[9k(\eta_r + \eta_e)^2]
\]

(31)

\[
p_e^{C_s} = (1 - \epsilon)M^2(2 - \sqrt{e})/[3k\eta_e(\eta_r + \eta_e)] + M^2(2 - \sqrt{e})^2/[9k(\eta_r + \eta_e)^2]
\]

(32)

Compare Eqs. (31) and (32) with Eqs. (17) and (18), respectively, and get.

\[p_r^{C_s} - p_r^{Ds} > 0, \quad p_e^{C_s} - p_e^{Ds} < 0.
\]

\[p_r^{C_s} > p_r^{Ds}, \quad p_e^{C_s} < p_e^{Ds}.
\]

\[\square
\]

Corollary 3.1 Indicates that when the retailer establishes the dual-channel structure, compared with decentralized decision, centralized decision makes the price in traditional retail channel increase, but make the price in internet channel decrease.

Corollary 3.2 Under centralized decision, the supply chain system joint expected revenue will increase, compared with decentralized decision, i.e., \( EPI_{MR}^{Cs} > EPI_{MR}^{Ds} \).

Proof Substitute Eqs. (29), (31) and (32) into Eq. (22), and get.

\[
EPI_{MR}^{Cs} = M^3(2 - \sqrt{e})[\eta_e x^2 + \eta_e (1 - x)^2]/[6k\eta_e(\eta_r + \eta_e)] - M^3(2 - \sqrt{e})^2(4 + \sqrt{e})/[54k(\eta_r + \eta_e)^2]
\]

From Eqs. (19) and (20), we get

\[EPI_{M}^{Ds} + EPI_{R}^{Ds} = M^3[\eta_e x^2 + \eta_e (1 - x)^2]/[6k\eta_e(\eta_r + \eta_e)] - 6M^3/[54k(\eta_r + \eta_e)^2]
\]

\[EPI_{MR}^{Cs} - (EPI_{M}^{Ds} + EPI_{R}^{Ds}) =
\]

\[M^3(1 - \sqrt{e})[\eta_e x^2 + \eta_e (1 - x)^2]/[6k\eta_e(\eta_r + \eta_e)] + M^3[6 - 2\sqrt{e}(4 + \sqrt{e})/[54k(\eta_r + \eta_e)^2] > 0
\]

\[EPI_{MR}^{Cs} > (EPI_{M}^{Ds} + EPI_{R}^{Ds})
\]

\[\square
\]

Corollary 3.2 Indicates that under centralized decision, the supply chain system joint expected revenue will increase, because the centralized decision enables the manufacturer and retailer to make joint decision, forms the vertical integration of the supply chain system, and improves the operation efficiency.

Corollary 3.3 Under centralized decision, the consumer surplus of the customer will be higher than that under decentralized decision, i.e., \( CS^{C_s} > CS^{Ds} \).

Proof From Eqs. (29), (31) and (32), we get.

\[CS^{C_s} = M^2(2 - \sqrt{e})[\eta_e x^2 - \eta_e (1 - x)^2]/[3k\eta_e(\eta_r + \eta_e)] - M^2(2 - \sqrt{e})^2/[9k(\eta_r + \eta_e)^2]
\]

(34)

Compare Eq. (34) with (21), and get

\[CS^{C_s} - CS^{Ds} = M^2(1 - \sqrt{e})[\eta_e x^2 - \eta_e (1 - x)^2]/[3k(\eta_r + \eta_e)] + M^2 \sqrt{e}/[9k(\eta_r + \eta_e)^2] > 0
\]

\[CS^{C_s} > CS^{Ds}
\]

\[\square
\]

Corollary 3.3 Indicates that under centralized decision, the consumer surplus of the customer will be higher than that under decentralized decision, which shows that centralized decision is also beneficial to the customer.

5 The manufacturer dual-channel structure

When the manufacturer establishes the dual-channel structure, the manufacturer builds its own traditional retail channel and entrusts the third-party platform to build the internet channel. The manufacturer and the third-party platform determine the revenue sharing ratio \( \delta \) (revenue sharing contract). The manufacturer determines the product quality level and wholesale price. The retailer determines the retail price, and the third-party platform determines...
internet direct price. Therefore, we construct the Stackelberg game model between the manufacturer, retailer and third-party platform, which is as follows.

\[
MaxEPI_M(q, w) = (w - kq^2/2)(zm - \eta p_r/q) + (z p_r - kq^2/2)((1 - z)m - \eta p_c/q)
\]

(35)

s.t. \( p_r = \arg MaxEPI_R \)

\[
p_c = \arg MaxEPI_T
\]

\[
MaxEPI_R(p_r) = (p_r - w)(zm - \eta p_r/q) \tag{36}
\]

\[
MaxEPI_T(p_c) = (1 - z)p_c[(1 - z)m - \eta p_c/q] \tag{37}
\]

Equation (35) is the manufacturer’s expected revenue function, Eq. (36) is the retailer’s expected revenue function, and Eq. (37) is the third-party platform’s expected revenue function.

**Proposition 4** When the manufacturer establishes the dual-channel structure, that is, entrusts the third-party to build the internet channel, the manufacturer’s product quality level is positively related to the revenue sharing ratio. And its product quality level is lower than that when the retailer establishes the dual-channel structure under decentralized decision and centralized decision, that is, \( q^{D_s} < q^{D_u} < q^{C_s} \).

**Proof** Use backwards induction method to solve and take the first partial derivative of Eqs. (36) and (37) with respect to \( p_r \) and \( p_c \), respectively, and get.

\[
p_r = zm/(2\eta_r) + w/2 \tag{38}
\]

\[
p_c = (1 - z)Mq/(2\eta_c) \tag{39}
\]

Substitute Eqs. (38) and (39) into Eq. (35), and get

\[
EPI_M(q, w) = (w - kq^2/2)(zm/2 - \eta_r w/2q) + (1 - z)m/2(1 - z)Mq/2\eta_c - kq^2/2 \tag{40}
\]

Take the first partial derivative of Eq. (40) with respect to \( w \), and get

\[
w = zm/2\eta_r + kq^2/4 \tag{41}
\]

Substitute Eq. (41) into Eq. (40), and get

\[
EPI_M(q) = (zm/2\eta_r - kq^2/4)(zm/4 - k\eta_r q/8) + (1 - z)m/2(1 - z)Mq/2\eta_c - kq^2/2 \tag{42}
\]

Take the first and second partial derivative of Eq. (42) with respect to \( q \), and get

\[
q_1 = [4(2 - z)M - 2M\sqrt{\eta}/3k\eta_r] \text{ or } q_2 = [4(2 - z)M + 2M\sqrt{\eta}/3k\eta_r] \tag{43}
\]

\[
\eta = 4(2 - z)^2 - 3x^2 - 6\eta_r\eta^{-1}x(1 - z)^2
\]

\[
\partial^2 EPI_M(q)/\partial q^2 = 3k^2\eta_r q/16 - (2 - z)MK/4 < 0 \tag{44}
\]

(Concave function)

\[
q < 4(2 - z)M/3k\eta_r
\]

From Eqs. (43) and (44), we get

\[
q^{T_s} = [4(2 - z)M - 2M\sqrt{\eta}/3k\eta_r] \tag{45}
\]

From Eq. (45), we get

\[
\partial q^{T_s}/\partial \lambda = 2M(1 - x)^2/kn_r \sqrt{\eta}
\]

(increasing \( g \) function, positive correlation)

\[
\lambda \in [0, 1], q^{T_s}(\lambda) \text{ is the increasing function.}
\]

\[
q^{T_s}(\lambda = 1)/q^{D_s} = (\eta_r + \eta_c)(4 - 2x - \sqrt{\eta})/\eta_r < 1
\]

\[
q^{T_s}(\lambda = 1) < q^{D_s}
\]

So, when \( \lambda \in [0, 1], q^{T_s} < q^{D_s} \)

\[
q^{D_s} < q^{C_s} \text{ (Proposition 3 has proved)}
\]

\[
q^{T_s} < q^{D_s} < q^{C_s} \tag{48}
\]

**Proposition 4** indicates that when the manufacturer establishes the dual-channel structure entrusting the third-party platform to build the internet channel, the manufacturer’s product quality level is positively related to the revenue sharing ratio. And its product quality level is lower than that when the retailer establishes the dual-channel structure under decentralized decision and centralized decision.

Substitute Eq. (45) into Eq. (41), and get

\[
w^{T_s} = [(16 + \eta - 2x^2 - 4x) - (8 - x)\sqrt{\eta}]M^2/9k\eta_r^2 \tag{49}
\]

Substitute Eqs. (45) and (49) into Eqs. (38) and (39), respectively, and get

\[
p_r^{T_s} = [(16 + \eta - 14x^2 + 20x) - (8 + 5x)\sqrt{\eta}]M^2/18k\eta_r^2 \tag{50}
\]

\[
p_c^{T_s} = [2(1 - x)(2 - x) - (1 - x)\sqrt{\eta}]M^2/3k\eta_r\eta_c \tag{51}
\]

Substitute Eqs. (45), (49), (50) and (51) into Eqs. (35), (36) and (37), respectively, and get

\[
EPI^{T_s}_{M} = [(10x^2 - 28x - \eta - 16) - (7x - 8)\sqrt{\eta}]M^2/18k\eta_r^2(24 - 2x) - 12\sqrt{\eta} \tag{52}
\]

\[
EPI^{T_s}_{R} = [2(1 - x)(2 - x) - (1 - x)\sqrt{\eta}^2(1 - \lambda)M^2/3k\eta_r\eta_c[4(2 - x) - 2\sqrt{\eta}] \tag{53}
\]
The retailer dual-channel structure (centralized decision) is the description of the retailer dual-channel structure (centralized decision), and Eqs. (45) and (49–55) are the description of the manufacturer dual-channel structure (entrusting the third-party platform to build internet channel.

Equations (15–21) are the description of the retailer dual-channel structure (decentralized decision), Eqs. (29–34) are the description of the retailer dual-channel structure (centralized decision), and Eqs. (45) and (49–55) are the description of the manufacturer dual-channel structure (entrusting the third-party platform to build internet channel.

Table 2 The retailer dual-channel structure (centralized decision)

| $\eta_r$ | $\eta_e$ | $q^{CE}$ | $p_r^{CE}$ | $p_e^{CE}$ | $\eta^{CE}_{MR}$ | $\eta^{CE}_{MR}$ | $CS^{CE}$ |
|--------|--------|--------|--------|--------|--------|--------|--------|
| 1.000  | 2.000  | 3.333  | 22.222 | 36.111 | 23.611 | 59.028 | 145.519 |
| 1.100  | 2.200  | 3.030  | 18.365 | 29.844 | 19.513 | 48.783 | 45.914 |
| 1.200  | 2.400  | 2.778  | 15.432 | 25.077 | 16.397 | 40.992 | 101.055 |
| 1.300  | 2.600  | 2.564  | 13.149 | 21.368 | 13.762 | 34.928 | 86.106 |
| 1.400  | 2.800  | 2.381  | 11.338 | 18.424 | 10.337 | 30.116 | 74.244 |
| 1.500  | 3.000  | 2.222  | 9.877  | 16.049 | 8.494  | 26.325 | 64.675 |
| 1.600  | 3.200  | 2.083  | 8.681  | 14.106 | 6.233  | 23.058 | 56.843 |
| 1.700  | 3.400  | 1.961  | 7.689  | 12.495 | 4.810  | 20.710 | 50.353 |
| 1.800  | 3.600  | 1.852  | 6.859  | 11.145 | 3.787  | 18.218 | 47.356 |
| 1.900  | 3.800  | 1.754  | 6.156  | 10.003 | 2.586  | 15.389 | 31.741 |
| 2.000  | 4.000  | 1.667  | 5.556  | 9.028  | 1.500  | 14.757 | 28.646 |
channel), we conduct the simulation analysis by MATLAB 2018, and the results are shown in Tables 1, 2 and 3 and Figs. 2, 3 and 4.

From Table 1, we can see that when the retailer establishes the dual-channel structure under decentralized decision, with the demand price elasticity in traditional retail channel and the demand price elasticity in internet channel increase, product quality level, wholesale price, retail price, direct price, expected revenue and consumer surplus will all decrease.

From Table 2, we can see that when the retailer establishes the dual-channel structure under centralized decision, compared with decentralized decision, product quality level, retail price, joint expected revenue and consumer surplus will increase. But direct price will decrease.

From Table 3, we can see that when the manufacturer establishes the dual-channel structure, that is, entrusts the third-party platform to build internet channel, compared with the retailer dual-channel structure under decentralized decision and centralized decision, the product quality level, wholesale price, retail price, direct price, retailer’s expected revenue, manufacturer’s expected revenue, joint expected revenue, and consumer surplus will all decrease.

| \( \eta_r \) | \( \eta_c \) | \( q^{T_x} \) | \( w^{T_x} \) | \( p^{T_x}_1 \) | \( E\Pi_{T_x}^{R} \) | \( E\Pi_{T_x}^{T} \) | \( E\Pi_{T_x}^{M} \) | \( E\Pi_{T_x}^{S} \) | \( CS^{T_x} \) |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1.000  | 2.000  | 1.940  | 16.500 | 22.800 | 7.275  | 20.459 | 27.280 | 38.949 | 59.408 |
| 1.100  | 2.200  | 1.764  | 13.636 | 18.843 | 6.012  | 22.545 | 32.289 | 49.098 | 14.027 |
| 1.200  | 2.400  | 1.617  | 11.458 | 15.833 | 5.052  | 14.208 | 19.444 | 24.048 | 41.256 |
| 1.300  | 2.600  | 1.492  | 9.763  | 13.491 | 4.305  | 12.106 | 16.142 | 23.047 | 35.153 |
| 1.400  | 2.800  | 1.386  | 8.418  | 11.633 | 3.712  | 10.438 | 13.918 | 19.872 | 30.310 |
| 1.500  | 3.000  | 1.293  | 7.333  | 10.133 | 3.233  | 9.093  | 12.124 | 17.311 | 26.404 |
| 1.600  | 3.200  | 1.213  | 6.445  | 9.066  | 2.842  | 7.992  | 10.656 | 15.214 | 23.206 |
| 1.700  | 3.400  | 1.141  | 5.709  | 7.889  | 2.517  | 7.079  | 9.439  | 13.477 | 20.556 |
| 1.800  | 3.600  | 1.078  | 5.093  | 7.037  | 2.245  | 6.315  | 8.420  | 12.021 | 18.336 |
| 1.900  | 3.800  | 1.021  | 4.571  | 6.316  | 2.015  | 5.667  | 7.557  | 10.789 | 16.457 |
| 2.000  | 4.000  | 0.970  | 4.125  | 5.700  | 1.819  | 5.115  | 6.820  | 9.737  | 14.852 |

Fig. 2 Comparison of product quality level under three situations

Fig. 3 The expected revenue functions under three situations
with the demand price elasticity in internet channel increasing, the third-party platform’s expected revenue will decrease.

We can find that the product quality level under centralized decision will be higher than that under decentralized decision, and will also be higher than when the manufacturer entrusts the third-party platform to build internet channel.

We can find that the manufacturer’s expected revenue and retailer’s expected revenue under decentralized decision will be higher than that when the manufacturer entrusts the third-party platform. The joint expected revenue under centralized decision will be higher than that under decentralized decision, and also higher than that when the manufacturer entrusts the third-party platform.

From Fig. 4, we can see that the customer’s consumer surplus under centralized decision will be higher than that under decentralized decision, and also higher than that when the manufacturer entrusts the third-party platform.

7 Discussion

Based on the three-stage Stackelberg dynamic game, we study how to make product quality control strategy in the three-echelon supply chain composed of the manufacturer, retailer and customer under retailer dual-channel structure and manufacturer dual-channel structure. And, we can get the following findings.

Frist of all, when the retailer establishes the dual-channel structure under decentralized decision, with the demand price elasticity in traditional retail channel and the demand price elasticity in internet channel increase, product quality level, wholesale price, retail price, direct price, expected revenue and consumer surplus will all decrease. When the retailer establishes the dual-channel structure under centralized decision, compared with decentralized decision, product quality level, retail price, joint expected revenue and consumer surplus will increase, but direct price will decrease.

Secondly, when the manufacturer establishes the dual-channel structure that is entrusts the third-party platform to build internet channel, compared with the retailer dual-channel structure under decentralized decision and centralized decision, the product quality level, wholesale price, retail price, direct price, retailer’s expected revenue, manufacturer’s expected revenue, joint expected revenue, and consumer surplus will all decrease. With the demand price elasticity in internet channel increasing, the third-party platform’s expected revenue will decrease.

What’s more, the product quality level under centralized decision will be higher than that under decentralized decision, and higher than that when the manufacturer entrusts the third-party platform to build the internet channel. The manufacturer and retailer’s expected revenues under decentralized decision will be higher than that when the manufacturer entrusts the third-party platform. The joint expected revenue and customer’s consumer surplus under centralized decision will be higher than that under decentralized decision, and also higher than that when the manufacturer entrusts the third-party platform.

8 Conclusions

Through model solving and simulation analysis, we analyze the demand price elasticity, market share ratio, revenue sharing ratio and quality cost coefficient how to influence the product demand, product quality level, retail price and direct price in different channels, expected revenue functions of manufacturer and retailer, consumer surplus and product quality control strategy. On the one hand, the paper enriches the theoretical basis of product quality control strategy in three-echelon supply chain. On the other hands, it points the direction for the specific application of the model in practice.

However, the model only considers the situation of one manufacturer and one retailer or only entrusting one third-party platform, and it is the Stackelberg dynamic game under the condition of complete information. In future research, we will consider how to formulate product quality control strategy in different distribution channels under the condition of asymmetric information and try to establish a multi-stage, dynamic and repeated game between the manufacturer and retailer or third-party platform to analyze the impact on product quality decision, expected revenue function, customer’s consumer surplus and social welfare.

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Data availability All the data are available and within the manuscript, no supplement materials data.

Declarations

Conflict of interest The authors declare that no competing interests exist.

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