Abstract

Optimal detection of ultra wideband (UWB) pulses in a UWB transceiver employing multiple detector types is proposed and analyzed in this paper. We propose several fusion techniques for fusing decisions made by individual IR-UWB detectors. We assess the performance of these fusion techniques for commonly used detector types like matched filter, energy detector and amplitude detector. In order to perform this, we derive the detection performance equation for each of the detectors in terms of false alarm rate, shape of the pulse and number of UWB pulses used in the detection and apply these in the fusion algorithms. We show that the performance can be improved approximately by 4 dB in terms of signal to noise ratio (SNR) for perfect detectability of a UWB signal in a practical scenario by fusing the decisions from individual detectors.

Keywords: Neyman-Pearson test; Sensor Networks; Time of Arrival (TOA); Ultra Wideband (UWB); UWB ranging

1 Introduction

An ultra wideband (UWB) communication system is based on spreading a low power signal into wideband. There are several techniques to spread a low power signal to wideband. Impulse radio based UWB (IR-UWB) schemes are most popular as they provide better performance and complexity trade-offs compared to other UWB schemes [1–3].

IR-UWB schemes employ narrow impulse signals, which can yield fine time resolution, and hence can be used for accurate position localization and ranging. Narrow pulse duration coupled with low amplitude due to the restriction from regulatory agencies like Federal Communications Commission (FCC) makes the detection of these pulses challenging [4–6]. Generally, transmit signaling employs multiple pulses and the receiver aggregates certain characteristics from these pulses like energy, amplitude, position, etc., to make statistical inferences on the transmitted information like range (localization) or transmitted symbol value (communication) etc. [2, 7]. The performance of the receiver depends on how well the received pulse statistics are utilized for a chosen application [6, 8].

In this paper, we will consider the structure of a digital sampling receiver shown in Fig 1. The received signal is filtered by an RF band-pass filter (BPF) and is amplified using a wideband LNA. The signal is then converted into the digital domain by a high sampling rate ADC and digitally processed. The digital receiver structure offers several benefits such as flexibility in design, reconfigurability and scalability [9]. However, since IR-UWB signals occupy large bandwidth and have low time
resolution, the design of an IR-UWB digital transceiver is challenging. In order to exploit the regulatory body specifications optimally, the transceivers must operate at a $3.1 - 10\,\text{GHz}$ range. The wideband BPF design should cover the whole of the useful UWB frequency band ($3.1 - 10\,\text{GHz}$). The microwave filter’s design, based on microstrip multi-mode resonator (MMR) and hybrid coplanar waveguide/microstrip structure can cover these ranges. The work in [10–13] proposes several wideband BPF filter designs for the intended purpose. The wideband LNA amplifies the signal to the operating levels of the analog to digital converters (ADC). The work in [14,15] discusses the various design aspects of the wideband LNA for UWB radios.

The most complex and costly part of the IR-UWB digital receiver is the ADC. IR-UWB pulses are extremely narrow (order of few ns) and occupy very high bandwidth, therefore high speed ADCs are needed for faithful digital representation of the IR-UWB pulses. Typically, such high speed ADC are designed using flash ADC [16] or a bank of polyphase ADCs [17]. The recent progress in the ADC technology, as suggested by [18], indicates that such high speed ADC having good resolution with signal to noise and distortion ratio (SNDR) of higher than 30 dB can be achieved for a bandwidth of 10 GHz. This has enabled the digital designs for IR-UWB technology.

The digital samples from the ADC will be processed by a digital baseband processing block for detection. In many hardware platforms, a single UWB transceiver mounted on sensors is used for multiple applications like ranging, localization, communication, etc., each using particular statistics of the received samples for UWB pulse detection [19]. For example, large distance communication using UWB may employ energy detection at the detector over a large number of pulses; whereas short distance tracking application may use amplitude detection on a few pulses. To enable the transceiver to be used for multiple applications, the designers have multiple detectors built into the transceiver. Each detector uses a detection algorithm to infer a hypothesis from the received samples and report it to the higher layers for further processing. These are typically implemented in FPGA for faster processing, and hence, only the computed hard or soft-value decisions are available. In some applications, there are not stringent constraints to bind the usage of a particular detector; for example, demodulation of short range low rate communication data. In these situations, instead of resorting to a single detector to arrive at the hypothesis, decision information from all the detectors can be concurrently utilized to make more informed decision on the hypothesis. This will utilize transceiver infrastructure better, and since every detector decision is new information about the signaled hypothesis, it should yield better reliability and improved performance. In this paper, we formulate a binary hypothesis problem of IR-UWB pulse detection, where decisions from multiple detectors are fused using different fusion methods before deciding on the hypothesis as shown in Fig. 1.

To illustrate the benefits of the scheme, we follow the following steps. First, we discuss the fusion algorithms for a generic set of detectors, $d_i$ (refer Fig. 1). Subsequently, we derive performance equations for the most commonly used IR-UWB detector types like matched filter (MF), amplitude detector (AD) and energy detector (ED). Here, we derive analytical expression for probability of detection, $P_D$, as a function of false alarm rate, $P_{FA}$, and signal-to-noise ratio, SNR, for a multi-pulse
UWB signal corrupted by additive white Gaussian noise (AWGN). Then, we use these expressions in the fusion algorithms to assess the performance.

The rest of the paper is organized as follows. In section 2, we will discuss the system model. Here, we will define the signal model which will be used in the rest of the paper. Section 3 discusses different fusion strategies. In section 4, we will derive analytical expression for $P_D$ as a function of $P_{FA}$, and SNR for matched filter, energy detector and amplitude detector for multi-pulse IR-UWB signal. In Section 5, we will evaluate the performance of the different fusion strategies using the performance equation of the individual detectors derived in Section 4. Finally in Section 6, we discuss the conclusions.

## 2 System Model

We consider a binary hypothesis for detection, with $H_0$ representing signal is absent and $H_1$ representing signal is present. Each of the different detectors in the UWB transceiver constructs a test statistic from the received samples, based on which inference is made about $H_0$ or $H_1$ by comparing the test statistic to some threshold $\gamma$. Depending on the chosen test statistic, we have different types of detectors. Apart from the chosen test statistic, the performance of the detector also depends on all or few of the below listed parameters.

- Probability of false alarm ($P_{FA}$),
- Signal to noise ratio (SNR),
- Number of UWB pulses used in detection ($N_p$),
- Energy of the UWB pulses, and
- Shape of the UWB pulses.

Analytical expression for probability of detection, $P_D$, for the detectors will be a function of the above parameters for different types of detectors.

The transmitted signal consists of $N_p$ frames, each frame consists of one IR-UWB pulse during hypothesis $H_1$, and during hypothesis $H_0$ nothing is transmitted ($N_p$ empty frames). Each frame is of duration $T$ and is represented by $N_s$ samples with
sampling interval of $T_s$ seconds. The transmit statistics are thus given by

\[
S(t) = \begin{cases} 
N_p^{-1} \sum_{n=0}^{N_s-1} s(t - nT - iT_s) & \text{under } H_1 \\
0 & \text{under } H_0 
\end{cases}
\]  

(1)

where, $s(t - nT - iT_s)$ is the $i$-th sample of the $n$-th frame. $N_p$ identical frames (unmodulated pulses) are used in the model. This is similar to time hopped impulse radio (TH-IR) UWB models proposed in [1,2,20], except that we are not considering time hopping, as it has no effect on the statistics collected by the receiver across multiple frames. The received test statistics are corrupted by independent Gaussian noise samples. Thus, the received signal used in the hypothesis test under both hypotheses is given by

\[
R(t) = \begin{cases} 
N_p^{-1} \sum_{n=0}^{N_s-1} \sum_{i=0}^{N_s-1} \beta s(t - nT - iT_s) + w(n, i) & \text{under } H_1 \\
N_p^{-1} \sum_{n=0}^{N_s-1} \sum_{i=0}^{N_s-1} w(n, i) & \text{under } H_0 
\end{cases}
\]  

(2)

where, $w(n, i)$ is the noise sample corresponding to the $i$-th sample of the $n$-th pulse.

Typically, the UWB channels are subject to multi-path propagation, where a large number of paths can be observed at the receiver. However, if the transceivers are in close proximity with clear line of sight (LOS), the detectors here rely on the first arriving path or LOS, this is in contrast to traditional channel measurement and modeling. Such extremely short distance high speed communication with highly directional antennas can adopt a deterministic single path channel (with pathloss $\beta$) corrupted by AWGN noise [21–23]. These short distance high speed UWB applications include transferjet and wireless USB (wUSB) [24,25]. Also, adopting a simple model proposed here will make the discussion mathematically tractable. Without loss of generality, we use $\beta = 1$.

In the next section, we will discuss the fusion strategies for fusing individual detector decisions (refer Fig. 1).

3 Fusion Rules for IR-UWB Signal Detection

One way to fuse the detector output shown in Fig 1, is to use $k$ out of $L$ decision approach, where the fused decision will be hypothesis $H_1$, if at least $k$ out of $L$ detectors decide on hypothesis $H_1$. Special cases of these fusion rules can be implemented by fusing individual detectors using logical functions like “AND”, “OR” and “Majority decision”. These types of fusion methods are studied in [26–29]. Here, we try to assess their performance in the IR-UWB pulse detection problems discussed earlier. The fusion rule for “AND”, “OR” and “majority decision”, will have $k = L$, $k = 1$ and $k > L/2$, respectively; where $L$ is the number of detector decisions used in the fusion. These fusion rules are depicted in the Fig. 2a, Fig. 2b and Fig. 2c.

The $k$ out of $L$ detection is biased either toward hypothesis $H_1$ (UWB pulse detection in our model), or toward $H_0$. For example, fusing using the “OR” rule will have superior detection performance, but will also have a larger false alarm.
Similarly the “AND” fusion rule is conservative in UWB pulse detection, but has superior false alarm rate performance. These aspects are further illustrated with numerical examples in the later sections. In general, if the objective is to have minimum misclassification, the decision rule based on these are sub-optimal. This is motivation to design a fusion technique that is optimal in probability of error sense. For any prior probability for \( H_0 \) and \( H_1 \), the fusion rule that reduces the probability of error is given by maximum a posteriori (MAP) formulation given below.

\[
\Pr(H_1|d)^{H_1 \geq H_0} \Pr(H_0|d) = \frac{P_1}{p(d)} \prod_{S_{H_1}} P_{D_i} \prod_{S_{H_0}} (1 - P_{D_i})
\]

We can write (3) as

\[
\log \left( \frac{\Pr(H_1|d)}{\Pr(H_0|d)} \right)^{H_1 \geq H_0} \geq 0.
\]

If \( S_{H_i} \) denotes a set for which decision, \( d_i = 1 \), \( i \) denotes the detector type, Similarly, \( S_{H_0} \) denotes the set for which decision, \( d_i = 0 \), then

\[
\Pr(H_1|d) = \frac{P_1}{p(d)} \prod_{S_{H_1}} P_{D_i} \prod_{S_{H_0}} (1 - P_{D_i})
\]

Here, we assumed that the decisions of each of the detectors are independent of each other. \( P_1 \) is the probability of hypothesis \( H_1 \). \( P_{D_i} \) is the detection probability of the \( i \)-th detector. Similarly, we can write

\[
\Pr(H_0|d) = \frac{P_0}{p(d)} \prod_{S_{H_1}} P_{F_i} \prod_{S_{H_0}} (1 - P_{F_i})
\]

\( P_0 \) is the probability of hypothesis \( H_0 \). \( P_{F_i} \) is the false alarm of the \( i \)-th detector. Substituting (5) and (6) in (4) and assuming both hypotheses are equally likely, we get the decision rule as
\[
\begin{align*}
&\log \left( \frac{P(H_1|d)}{P(H_0|d)} \right) \geq \sum_{i \in S_{H_1}} \log \left( \frac{P_{D_i}}{P_{F_i}} \right) \\
&\quad + \sum_{i \in S_{H_0}} \log \left( \frac{(1 - P_{D_i})}{(1 - P_{F_i})} \right), \\
\end{align*}
\]

or,

\[
\sum_{i \in S_{H_1}} \log \left( \frac{P_{D_i}}{P_{F_i}} \right) + \sum_{i \in S_{H_0}} \log \left( \frac{(1 - P_{D_i})}{(1 - P_{F_i})} \right) \geq 0. \\
\]

In order to assess the performance of these fusion rules, the detection performance \((P_{D,s})\) of the individual detector should be known. The most common types of IR-UWB detectors include matched filter, energy detector and amplitude detector. In the next section, we will derive the detection performance of these detectors, which will be used in the later sections to evaluate the fusion performance.

4 Detector Performance

In this section, we will derive analytical expressions for probability of detection, \(P_D\), as a function of \(P_{EA}\) and SNR for various detector types. We define frame energy, \(E_p\), and signal-to-noise ratio, SNR, as

\[
E_p = \sum_{i=0}^{N_s-1} s^2(iT_s), \\
\]

\[
\text{SNR} = \frac{E_p}{\rho^2}, \\
\]

where \(\rho^2\) is the total noise energy in the frame.

Typical detector structure used in Fig 1 is as shown in Fig. 3. Each detector will construct a test statistic as shown in (11) from the received samples and compare it with a threshold to decide on a hypothesis. Depending on the test statistic, we have different types of detectors like matched filter, energy detector and amplitude detector.
\[ T_k = \sum_{n=0}^{N_p} \sum_{i=0}^{N_s} f(x(n,i)) \]  

(11)

4.1 Matched Filter

In matched filter, the test statistic in (11) will have

\[ f(x(n,i)) = x(t - nT - iT_s)s(t - nT - iT_s) \]  

(12)

The performance in terms of probability of detection for matched filter, \( P_{DF}^{MF} \), as a function of probability of false alarm, \( P_{FA} \), and SNR is derived in Appendix-1 and it is given by

\[ P_{DF}^{MF} = Q \left( Q^{-1}(P_{FA}^{MF}) - \sqrt{N_sN_p \text{SNR}} \right) \]  

(13)

where \( Q \) is the tail probability of the standard normal distribution.

4.2 Energy Detector

In energy detector, the test statistic in (11) will have

\[ f((x(n,i)) = x^2(t - nT - iT_s). \]  

(14)

The performance in terms of probability of detection for energy detector, \( P_{DF}^{ED} \), as a function of probability of false alarm, \( P_{FA} \), and SNR is derived in Appendix-2 and it is given by

\[ P_{DF}^{ED} = Q_{X^2(\lambda)}^{-1} \left( \sqrt{2N_pN_s}Q^{-1}(P_{FA}^{ED} + N_pN_s) \right) \]  

(15)

where \( Q_{X^2(\lambda)} \) is the tail probability of the non-central chi-square distribution with degrees of freedom \( \nu = N_pN_s \) and centrality parameter \( \lambda = N_pN_s \text{SNR} \).

4.3 Amplitude Detector

In the amplitude detector, the test statistic in (11) will have

\[ f((x(n,i)) = |x(t - nT - iT_s)|. \]  

(16)

The performance in terms of probability of detection for amplitude detector, \( P_{DF}^{AD} \), as a function of probability of false alarm, \( P_{FA} \), and SNR is derived in Appendix-3 and it is given by

\[ P_{DF}^{AD} = Q \left( Q^{-1} \left( \frac{P_{FA}^{AD}}{2} \right) - \alpha \sqrt{N_pE_p \text{SNR}} \right) \]  

\[ + Q \left( Q^{-1} \left( \frac{P_{FA}^{AD}}{2} \right) + \alpha \sqrt{N_pE_p \text{SNR}} \right) \]  

(17)
Figure 4: The performance of different detectors in simulation for a fixed $P_{FA} = 10^{-7}$. A normalized second order Gaussian pulse of width 10 ns sampled at 50 GHz is used in the simulation. 1000 random realizations are used in building the probability of detection statistic. The theoretical expressions match the simulation result for all detectors.

where $\alpha$ is defined as in (18).

$$\sum_{i=0}^{N_s-1} s(i) = \alpha E_p, \quad (18)$$

As shown by (18) and (17), the performance of the amplitude detector depends on the shape of the UWB pulse used. We have considered a normalized second order Gaussian pulse as described in [5, 6, 30]. This is given by

$$s(t) = -4\pi e^{-\frac{2\pi^2}{\tau^2}} \left( -\frac{\tau^2 + 4\pi t^2}{\tau^4} \right). \quad (19)$$

Here $\tau$ can be used to control the impulse spread. Energy normalized pulse, $E_p = 1$, with $\tau = 3.33$ ns, sampled at 50 GHz, will result in $\alpha = 14.4$. Thus, for this pulse shape the performance of the amplitude detector is given by

$$P_{AD} = Q \left( Q^{-1} \left( \frac{P_{FA}}{2} \right) - 14.4 \sqrt{N_p E_p \text{SNR}} \right) \quad (20)$$

From (13), (15), and (20) the performance of matched filter, energy detector and amplitude detector depends on environment (SNR) and on the system configuration or tuning variables like $N_p$ and $P_{FA}$. In the matched filter and energy detector, the performance is agnostic to the system specifications like pulse shape, which are fixed for a given hardware. However, in the amplitude detector, detection performance depends on the shape of the pulse as shown in (17) and (18). The variation of the
Figure 5  Transmit signal for hypothesis $H_1$, with frame duration, $T = 10$ ns, and number of frames, $N_p = 10$.

Figure 6  Probability of detection for matched filter, energy detector and amplitude detector for various SNR’s. Number of UWB pulses, $N_p = 10$. Number of samples used to define the pulse, $N_s = 500$.

Probability of detection, $P_D$, for different false alarm rates, for different detector types is as shown in Fig. 6. As shown in Fig. 5 (refer (1)), we use the UWB pulse shape defined in (19) with a frame duration of 10 ns and number of frames, $N_p = 10$ for hypothesis $H_1$.

4.4 Simulation Study

In deriving the energy detector performance equation (15) in Appendix-2, we assumed that a sufficiently large number of pulses are considered. Similarly, for amplitude detector performance equation (20) in Appendix-3, we assumed a particular UWB pulse shape. In this section, we will simulate the detectors and demonstrate the validity of the approximations, for a practical UWB signal setup. We define a signal model shown in Fig. 5, in which each frame is of 10 ns duration, having one normalized second order Gaussian pulse as defined in (19) with $\tau = 3.33$ ns, sampled at 50 GHz. We consider 10 such pulses (frames) for each detection. The re-
Table 1 Configuration of parameters for different detectors used in the fusion.

| Detector Type   | $P_{FA}$ | $N_p$ |
|-----------------|----------|-------|
| Matched Filter  | $10^{-7}$| 10    |
| Energy Detector | $10^{-1}$| 100   |
| Amplitude Detector | $10^{-4}$| 10    |

![Figure 7](image_url) Probability of detection for different detectors. The $N_p$ and $P_{FA}$ is taken from Table 1.

Received samples are corrupted by AWGN noise with variance $1/$SNR (since pulses are normalized, that is $E_p = 1$). Monte-Carlo simulations are done using 1000 independent realizations, with each realization consisting of $N_p = 10$ frames for detection, with a fixed $P_{FA} = 10^{-7}$. The detector performance in simulations matches the analytical expressions in (13), (15) and (20), shown in Fig. 4. This validates the derived performance expressions for a practical UWB signal configuration.

5 Performance Evaluation of Fusion Methods

When the same radio is used for multiple applications, detectors in them are tuned with different parameter values for $P_{FA}$, $N_p$, etc. For example, if the application needs a faster response, then the $N_p$ used will be small; similarly if the application needs robust detection, then it may require a larger $N_p$. In general, the parameters $N_p$, $P_{FA}$ are tuned based on the applications. As a result of this, different detectors are optimal at different SNR regions. For example, consider operating parameters such as probability of false alarm, $P_{FA}$, number of pulses, $N_p$, as defined in the Table 1. Evaluating equations (13), (15) and (17) with false alarm $P_{FA}$ and number of pulses, $N_p$ as defined in the Table 1, the probability of detection for different detectors is as shown in Fig. 7. We will consider three different types of detectors discussed earlier, i.e., matched filter, energy detector and amplitude detector. With this set of detectors, we have $L = 3$, and for “AND”, “OR” and “majority decision”, we should have $k = 3$, $k = 1$ and $k > 2$ respectively for $k$ out of $L$ fusion rule discussed in Section 3.

We performed Monte-Carlo simulations with similar signal configurations described in Section 4.4. The false alarm, $P_{FA}$, and number of pulses employed, $N_p$, for each detector type is shown in Table 1. Results for fused probability of detection, $P_D$, and probability of error, $P_e$, is as shown in Fig. 8a and Fig. 8b respectively.
Notice that for a fixed SNR, the probability of detection is high for the “OR” fusion, however, the probability of error is also high for the “OR” fusion. This indicates higher probability of false alarm and probability of miss.

The performance is also evaluated using the MAP fusion rule (8), for a detector set, \( d = \{MF, ED, AD\} \), with the configuration shown in Table 1. The probability of detection and probability of error are as shown in Fig. 9a and Fig. 9b, respectively. Notice that the MAP fusion method (defined by (8)) is close to “OR” fusion in detection performance, with superior probability of error performance as shown in Fig. 9b. Comparing the performance of energy detector alone with the MAP fusion rule for multiple detectors in Fig. 9a and Fig. 9b, indicates that significant performance improvement can be achieved by using the fusion of multiple detector types.

6 Conclusion

In this paper, we analyzed the UWB detection performance of matched filter, energy detector and amplitude detector. We derived the analytical expression for proba-
bility of detection, $P_D$, for each of the detectors as a function of SNR. These are shown in (13), (15) and (17). We verified these expressions in simulations, this is shown in Fig. 4. We analyzed the performance in terms of detection probability and probability of error for different fusion methods like “AND”, “OR” and “majority decision”. This is shown in Fig. 8a and Fig. 8b. Using Bayes rule, we derived an optimal fusion rule (8) for UWB detection, which is optimal in probability of error sense and compared its performance. This is shown in Fig. 9a and Fig. 9b.

Results indicate that by making a suitable choice of fusion rule, a trade off between detection and false alarm can be achieved. For example, Fig. 8a, shows that OR fusion is more biased towards detection, however, it also results in higher errors (due to false alarms, refer Fig. 8b) compared to AND fusion. If the error performance is critical for the UWB application, then MAP fusion formulation gives superior performance in terms of errors as shown in Fig. 9b. In general, if there are multiple detectors available in the UWB transceiver platform, then decision information from these detectors can be concurrently utilized and intelligently fused based on the application criteria to make a more informed decision on the hypothesis.

Appendix-1

In matched filter, the test statistics and hypothesis test are given by

$$T_{MF} \begin{cases} H_1 \\ H_0 \end{cases} \gamma,$$  \hspace{1cm} (21)

where $T_{MF}$ is given by

$$T_{MF} = \sum_{n=0}^{N_p-1} \sum_{i=0}^{N_s-1} x(t - nT - iT_s) s(t - nT - iT_s)$$  \hspace{1cm} (22)

If we assume transmitted UWB frames are identical with no modulation, then $s(t - nT - iT_s) = s(iT_s)$. Therefore, equation (12) can be written as

$$T_{MF} = \begin{cases} N_p E_p + \sum_{n=0}^{N_p-1} \sum_{i=0}^{N_s-1} s(i) w(n, i) & \text{under } H_1 \\ \sum_{n=0}^{N_p-1} \sum_{i=0}^{N_s-1} s(i) w(n, i) & \text{under } H_0 \end{cases}$$  \hspace{1cm} (23)

Since the total noise energy in the frame is $\rho^2$, we can assume that noise samples within a frame are independent and identically distributed with $\mathcal{N}(0, \sigma^2/N_s)$. The distribution of the test statistic given by

$$p(T_{MF}) = \begin{cases} \mathcal{N} \left( N_p E_p, \frac{N_p \rho^2 E_p}{N_s} \right) & \text{under } H_1 \\ \mathcal{N} \left( 0, \frac{N_p \rho^2 E_p}{N_s} \right) & \text{under } H_0 \end{cases}$$  \hspace{1cm} (24)

Suppose if we fix the false alarm rate $P_{FA}$ to $\gamma$, then detection probability is given by
\[ P_{FA}^{MF} = \Pr(T_{MF} > \gamma; H_0) = Q \left( \frac{\gamma}{\sqrt{N_p\sigma_w^2E_p}} \right) \]  
\[ P_{D}^{MF} = \Pr(T_{MF} > \gamma; H_1) = Q \left( \frac{\gamma - N_pE_p}{\sqrt{N_p\sigma_w^2E_p}} \right) \]  

Where \( Q \) is the tail probability of the Gaussian distribution. By solving for \( \gamma \) in (25) for a fixed false alarm and substituting it in (26), we get the analytical form for the probability of detection. It is given by

\[ P_{D}^{MF} = Q \left( Q^{-1}(P_{FA}) - \sqrt{N_sN_pSNR} \right) \]  

SNR is defined in (10).

**Appendix-2**

In energy detector, the test statistic and hypothesis test are given by

\[ T_{ED}^{H_1} \geq \gamma, \]  

where \( T_{ED} \) is given by

\[ T_{ED} = \sum_{n=0}^{N_p-1} \sum_{i=0}^{N_s-1} x^2(t - nT_i) \]  

Under hypotheses \( H_0 \) and \( H_1 \), (14) can be written as

\[ T_{ED} = \begin{cases} 
\sum_{n=0}^{N_p-1} \sum_{i=0}^{N_s-1} (s(i) + w(n, i))^2 & \text{under } H_1 \\
\sum_{n=0}^{N_p-1} \sum_{i=0}^{N_s-1} w^2(n, i) & \text{under } H_0 
\end{cases} \]  

Considering only \( H_0 \) we have

\[ (T_{ED}; H_0) = \sum_{n=0}^{N_p-1} \sum_{i=0}^{N_s-1} w^2(n, i) \]  

\( w(n, i) \) is independent and identically distributed with \( N(0, \sigma^2/N_s) \), we can modify the test statistic as below

\[ \left( \frac{T_{ED}N_s}{\sigma^2}; H_0 \right) = \sum_{n=0}^{N_p-1} \sum_{i=0}^{N_s-1} l^2(n, i) \]  

\( l(n, i) \) is independent and identically distributed with \( N(0, 1) \). The probability density of \( (T_{ED}N_s)/\sigma^2 \) under hypothesis \( H_0 \) is given by

\[ p \left( \frac{T_{ED}N_s}{\sigma^2}; H_0 \right) = \chi^2_{N_sN_p}(0). \]
In (33) the additive nature of chi-square distribution is utilized. Under hypothesis \( H_1 \),

\[
(T_{ED}; H_1) = \sum_{n=0}^{N_p-1} \sum_{i=0}^{N_s-1} (x(n,i))^2.
\]

(34)

Where \( x(n,i) \) is normally distributed as \( \mathcal{N}(s(i), \sigma^2/N_s) \). The modified test statistic of \( (T_{ED}N_s)/\sigma^2 \) under hypothesis \( H_1 \) is given by

\[
\left(\frac{T_{ED}N_s}{\sigma^2}; H_1\right) = \sum_{n=0}^{N_p-1} \sum_{i=0}^{N_s-1} (l(n,i))^2.
\]

(35)

where \( l(n,i) \) is normally distributed as \( \mathcal{N}(s(i), N_s/\sigma^2) \). Therefore, the probability density function of \( (T_{ED}N_s)/\sigma^2 \) under hypothesis \( H_1 \) is given by

\[
p\left(\frac{T_{ED}N_s}{\sigma^2}; H_1\right) = \chi^2_{N_s} \left( \frac{\sum_{i=0}^{N_s-1} N_s s(i)^2}{\sigma^2} \right)
\]

(36)

\[
= \chi^2_{N_s} (N_s N_p, \text{SNR}).
\]

(37)

In (37), the additive nature of non-central chi-square distribution is exploited and SNR is defined as in (10). Equations (37) and (33) can be compactly written as

\[
p\left(\frac{T_{ED}N_s}{\sigma^2}; H_1\right) = \begin{cases} 
\chi^2_{N_s} (N_p N_s, \text{SNR}) & \text{under } H_1 \\
\chi^2_{N_s} (0) & \text{under } H_0
\end{cases}
\]

(38)

From (32), we have

\[
\left(\frac{T_{ED}N_s}{\sigma^2}; H_0\right) = \sum_{n=0}^{N_p-1} \sum_{i=0}^{N_s-1} l^2(n,i)
\]

(39)

\[
= \sum_{p=0}^{N_L} z_p^2.
\]

(40)

\( z_p \) is an independent and identically distributed random variable with \( \mathcal{N}(0,1) \) and \( N_L = N_p N_s \). For large \( N_L \) probability distribution can be approximated using central limit theorem. Therefore, we can write

\[
p\left(\frac{T_{ED}N_s}{\sigma^2}; H_0\right) \sim \mathcal{N}(NE[z_p]^2, N\text{var}(z_p^2))
\]

(41)

since \( z_p \sim \mathcal{N}(0,1) \) we get

\[
E(z_p^2) = 1
\]

(42)

\[
\text{var}(z_p^2) = E(z_p^4) - (E(z_p^2))^2
\]

(43)

\[
= 2
\]

(44)
Substituting (42) and (44) in (41) and using the fact $N_L = N_pN_s$ we get

$$p \left( \frac{T_{ED}N_s}{\sigma^2}; H_0 \right) \sim \mathcal{N}(N_pN_s, 2N_pN_s).$$  \quad (45)

False alarm rate can be written as

$$P_{FA}^{ED} = \Pr (T_{MF} > \gamma; H_0).$$  \quad (46)

From (45) and (46) solving for $\gamma$ will result in

$$\gamma = \frac{\sigma^2}{N_s} \sqrt{2N_pN_s} Q^{-1}(P_{FA}^{ED} + N_pN_s)$$  \quad (47)

The probability of the detection for a fixed threshold $\gamma$ is given by

$$P_{D}^{ED} = \Pr (T_{MF} > \gamma; H_1).$$  \quad (48)

From (38), (47) and (48) we get

$$P_{D}^{ED} = \frac{1}{\sqrt{2N_pN_s} Q^{-1}(P_{FA}^{ED} + N_pN_s)}$$  \quad (49)

Where $Q_{x^2(\lambda)}$ is the tail probability of the non-central chi-square distribution with degrees of freedom $\nu = N_pN_s$ and centrality parameter $\lambda = N_pN_sSNR$. The SNR is defined as in (10).

**Appendix-3**

The amplitude detector, the test statistic and hypothesis test are given by

$$T_{AD} \overset{H_1}{\geq} \gamma; \quad (50)$$

where $T_{AD}$ is given by

$$T_{AD} = \sum_{n=0}^{N_p-1} \sum_{i=0}^{N_s-1} |x(t - nT - iT_s)|.$$  \quad (51)

The test statistic under both hypotheses is given by

$$T_{AD} = \begin{cases} 
\sum_{n=0}^{N_p-1} \sum_{i=0}^{N_s-1} |s(i) + w(n, i)| & \text{under } H_1 \\
\sum_{n=0}^{N_p-1} \sum_{i=0}^{N_s-1} |w(n, i)| & \text{under } H_0
\end{cases}$$  \quad (52)

Assuming all transmitted frames as identical and noise samples are independent and identically distributed with $\mathcal{N} (0, \sigma^2/N_s)$, the probability density under the null hypothesis is given by

$$p(T_{AD}; H_0) = \begin{cases} 
2\mathcal{N} (0, N_p\sigma^2) & \text{when } T_{AD} > 0 \\
0 & \text{otherwise}
\end{cases}$$  \quad (53)
For a fixed false alarm rate $P_{FA}^{AD}$, the positive threshold ($\gamma$) is given by

$$\gamma = \sqrt{N_p \sigma^2} Q^{-1} \left( \frac{P_{FA}^{AD}}{2} \right).$$

(54)

The probability of detection, $P_D$, for a fixed threshold, $\gamma$, is given by

$$P_{FA}^{AD} = p(T_{AD} > \gamma; H_1)$$

$$= Q \left( \frac{\gamma - N_p \sum_{i=0}^{N_s-1} s(i)}{\sqrt{N_p \sigma^2}} \right)$$

$$+ \left[ 1 - Q \left( \frac{- \gamma - N_p \sum_{i=0}^{N_s-1} s(i)}{\sqrt{N_p \sigma^2}} \right) \right]$$

(55)

If we set

$$\sum_{i=0}^{N_s-1} s(i) = \alpha \sum_{i=0}^{N_s-1} s^2(i),$$

$$= \alpha E_p,$$

(56)

(57)

we can express the probability of detection $P_{D}^{AD}$ in terms of SNR. The probability of the detection for a fixed threshold $\gamma$ is given by

$$P_{D}^{AD} = \Pr (T_{AD} > \gamma; H_1).$$

(58)

From (52), (54), (55) and (56) we get

$$P_{D}^{AD} = Q \left( Q^{-1} \left( \frac{P_{FA}^{AD}}{2} \right) - \alpha \sqrt{N_p E_p \text{SNR}} \right)$$

$$+ Q \left( Q^{-1} \left( \frac{P_{FA}^{AD}}{2} \right) + \alpha \sqrt{N_p E_p \text{SNR}} \right)$$

(59)

Competing interests

The authors declare that they have no competing interests.

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