Two Photon Distribution Amplitudes

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Abstract. The factorization of the amplitude of the process $\gamma^* \gamma \rightarrow \gamma \gamma$ in the low energy and high photon virtuality region is demonstrated at the Born order and in the leading logarithmic approximation. The leading order two photon (generalized) distribution amplitudes exhibit a characteristic $\ln Q^2$ behaviour and obey new inhomogeneous evolution equations.

Keywords: QCD factorization, Generalized distribution amplitude

PACS: 12.38.Bx,14.70.Bh,12.20.Ds

1. INTRODUCTION

The pointlike coupling to quarks of the photon enables to calculate perturbatively part of its wave function. A twist expansion generates non-leading components of the photon distribution amplitude [1], from which the lowest order one is chiralo-odd and proportionnal to the magnetic susceptibility of the vacuum. The study of the two photon state is kinematically richer and is thus a most welcome theoretical laboratory for the study of exclusive hard reactions.

The parton content of the photon has been the subject of many studies since the seminal paper by Witten [2]. A recent paper [3] extended the notion of anomalous parton distribution in a photon to the case of generalized parton distributions (GPDs) used for the factorized description of the non-diagonal kinematics of deeply virtual Compton scattering (DVCS) on a real photon target, $\gamma^* (q) \gamma \rightarrow \gamma \gamma$, namely at large energy and small hadronic momentum transfer but large photon virtuality ($Q^2 = -q^2$).

As for the two meson case [4] the two photon generalized distribution amplitudes describe the coupling of a quark antiquark (or gluon-gluon) pair to a pair of photons, and are related by crossing to the photon GPDs.

We study [6] the scattering amplitude of the $\gamma^*(q) \gamma \rightarrow \gamma \gamma$ process in the near threshold kinematics, namely at small $s$ and large $-t \sim Q^2$, at large $Q^2$ and in the leading order of the electromagnetic and strong couplings. This enables us to define and calculate perturbatively the Born approximation of the diphoton GDAs.

This is reminiscent of, but quite different from, the perturbative calculation of the rho rho GDA in terms of the rho DA [7], for which both incoming photon were chosen to be hard in order to justify this factorization of the GDA.
Two photon production in Compton scattering on a photon target
\[
\gamma^*(q)\gamma(p_1) \to \gamma(q')\gamma(p_2)
\] (1)
involves, at leading order in \(\alpha_{em}\), and zeroth order in \(\alpha_S\) six "box" Feynman diagrams with quarks in the loop.

Restricting to the threshold kinematics where \(W^2 = (p_1 + p_2)^2 = 0\) simplifies greatly the tensorial structure of the amplitude while still preserving the richness of the skewedness (\(\zeta\)) dependence of GDAs. Our conventions for the kinematics are the following:

\[
q = p - \frac{Q^2}{s} n, \quad q' = \frac{Q^2}{s} n,
\]
(2)

\[
p_1 = \zeta p, \quad p_2 = \bar{\zeta} p, \quad \bar{\zeta} = 1 - \zeta,
\]
(3)

where \(p\) and \(n\) are two light-cone vectors and \(2p \cdot n = s\). The momentum \(l\) in the quark loop is parametrized as

\[
l^\mu = z p^\mu + \beta n^\mu + l_T,
\]
(4)

with \(l_T^2 = -1^2\). The scattering amplitude is written as

\[
A = \epsilon_\mu \epsilon'_\nu \epsilon_1^* \epsilon_2^* T^{\mu\nu\alpha\beta}
\]
(5)

where the four photon polarization vectors are transverse with respect to Sudakov vectors \(p\) and \(n\).

The tensorial decomposition of \(T^{\mu\nu\alpha\beta}(W = 0)\) reads

\[
T^{\mu\nu\alpha\beta} = \frac{1}{4} g_T^{\mu\nu} g_T^{\alpha\beta} W_1 + \frac{1}{8} \left( g_T^{\mu\alpha} g_T^{\nu\beta} + g_T^{\mu\nu} g_T^{\alpha\beta} - g_T^{\mu\nu} g_T^{\alpha\beta} \right) W_2 + \frac{1}{4} \left( g_T^{\mu\alpha} g_T^{\nu\beta} - g_T^{\mu\beta} g_T^{\nu\alpha} \right) W_3,
\]
(6)

and it involves three scalar functions \(W_i, i = 1, 2, 3\).

The integration over \(l\) is performed as usual within the Sudakov representation, using

\[
d^4l = \frac{s}{2} dz d\beta d^2 l_T \to \frac{\pi s}{2} dz d\beta d^2 l_T.
\]

Let us note that in order to interpret our result in terms of factorized quantities, we will keep our expressions unintegrated with respect to \(z\), the mass of the quark playing the role of an infrared regulator.

One first integrates in \(\beta\) using the Cauchy theorem. The propagators induce poles in the complex \(\beta\)-plane and the pole positions depend on the values of \(z\) and \(\zeta\). The four poles lie all below the real axis for \(z > 1\) and lie all above the real axis for \(z < 0\), the only region where the amplitude may not vanish is \(1 > z > 0\). This leads to a natural interpretation of \(z\) as a partonic fraction of momentum. One then identifies different regions defined from the relative values of \(z\), \(\zeta\) and \(1 - \zeta\).
This is reminiscent of the different regions encountered in the kinematics of the
generalized parton distributions $H(x,\xi,t)$, with the boundaries controlled by the
relative values of $x$ and $\xi$, where $x$ and $\xi$ are the quantities related to our $z$ and $\zeta$
variables. For each of the six contributing diagrams the remaining integral over $l^2$
contains a UV divergent part which cancels in their sum. We get

$$W_1 = \frac{e_q^4 N_C}{2\pi^2} \int_0^1 dz \left( 2z - 1 \right) \left[ \frac{2z - \zeta}{z\zeta} \theta(z - \zeta) + \frac{2z - 1 - \zeta}{z\zeta} \theta(\zeta - z) \right] \log \frac{m^2}{Q^2} ,$$

(7)

and

$$W_3 = -\frac{e_q^4 N_C}{2\pi^2} \int_0^1 dz \left[ \frac{-\zeta}{z\zeta} \theta(z - \zeta) - \frac{\zeta}{z\zeta} \theta(\zeta - z) \right] \log \frac{m^2}{Q^2} .$$

(9)

Let us now interpret the results (7) and (9) from the point of view of QCD factorization based on the operator product expansion.

### 3. QCD FACTORIZATION AND THE $\gamma\gamma$ GDA

Let us consider two quark non local correlators on the light cone and their matrix
elements between the vacuum and a diphoton state which define the diphoton GDA
$\Phi_1$,

$$F^q = \int \frac{dy}{2\pi} e^{i(2z - 1)\frac{y}{2}} \langle \gamma(p_1)\gamma(p_2) | \bar{q}\left(\frac{-yN}{2}\right) \gamma.N q\left(\frac{yN}{2}\right) | 0 \rangle = \frac{1}{2} g^{\mu\nu} \epsilon_\mu(p_1) \epsilon_\nu(p_2) \Phi_1(z,\zeta,0)$$

(10)

where we denote $N = n/n.p$ and where we did not write explicitly the electromagnetic and the gluonic Wilson lines. We need also to define the matrix element of the
photonic correlator

$$F^\gamma = \int \frac{dy}{2\pi} e^{i(2z - 1)\frac{y}{2}} \langle \gamma(p_1)\gamma(p_2) | F^N \left( -\frac{y}{2}N \right) F^N \left( \frac{y}{2}N \right) | 0 \rangle$$

(11)

where $F^N = N_\nu F^{\nu\mu}$, which mixes with the quark correlator (10) although they
are not of the same order in $\alpha_{em}$ [2].

We regulate through the usual dimensional regularization procedure the UV
divergent quark correlator matrix elements and obtain (with $\frac{1}{\xi} = \frac{1}{\zeta} + \gamma_E - \log 4\pi$)

$$F^q = -\frac{N_C e_q^2}{4\pi^2} g_T^{\mu\nu} \epsilon_\mu(p_1) \epsilon_\nu(p_2) \left[ \frac{1}{\xi} + \log m^2 \right] F(z,\zeta) ,$$

(12)
FIGURE 1. The unpolarized anomalous diphoton GDA $\Phi_{q_1}(z, \zeta) / (N_C e_q^2/(2\pi^2) \log \frac{Q^2}{m_F^2})$ at Born order and at threshold for $\zeta = 0.1$ (dashed), 0.2 (dash-dotted), 0.4 (solid).

with $F(z, \zeta) =$

$$\frac{\bar{z}(2z - \zeta)}{\zeta} \theta(z - \zeta) + \frac{\bar{\zeta}(2\bar{\zeta} - \zeta)}{\zeta} \theta(\zeta - \bar{\zeta}) + \frac{z(2z - 1 - \zeta)}{\zeta} \theta(\zeta - z) + \frac{\bar{z}(2\bar{z} - 1 - \bar{\zeta})}{\bar{\zeta}} \theta(\bar{\zeta} - z).$$

(13)

The ultraviolet divergent parts are removed through the renormalization procedure (see for example [8]) involving quark and photon correlators $(O^q, O^\gamma)$ corresponding to $\bar{q}(-\frac{1}{2} N)\gamma q (\frac{1}{2} N)$ and $F^{N\mu}(-\frac{1}{2} N)F^{N}_{\mu} (\frac{1}{2} N)$. The renormalized operators are defined as :

$$\left( \begin{array}{c} O^q \\ O^\gamma \end{array} \right)_R = \left( \begin{array}{cc} Z_{qq} & Z_{q\gamma} \\ Z_{\gamma q} & Z_{\gamma\gamma} \end{array} \right) \left( \begin{array}{c} O^q \\ O^\gamma \end{array} \right).$$

(14)

The matrix element of the renormalized quark-quark correlator is thus equal to

$$<\gamma(p_1)\gamma(p_2)|O^q_R|0> = Z_{qq} <\gamma(p_1)\gamma(p_2)|O^q|0> + Z_{q\gamma} <\gamma(p_1)\gamma(p_2)|O^\gamma|0>$$

(15)

with $Z_{qq} = 1 + O(e_2^2)$. Since the matrix element $<\gamma(p_1)\gamma(p_2)|O^q|0>$ contains a UV divergence (Eqn. 12) and since $<\gamma(p_1)\gamma(p_2)|O^\gamma|0>$ is UV finite and of order $\alpha_\text{em}^2$, one can absorb this divergence into the renormalization constant $Z_{q\gamma}$. The normalization of the renormalized correlator is fixed with the help of the renormalization condition which is chosen as $<\gamma(p_1)\gamma(p_2)|O^q_R|0> = 0$ at the renormalization scale $M_R = m$. In this way the renormalized GDA is equal to

$$F^q_R = - \frac{N_C e_q^2}{4\pi^2} g^{\mu\nu} \epsilon^*_{\mu}(p_1) \epsilon^*_{\nu}(p_2) \log \frac{m^2}{M_R^2} F(z, \zeta).$$

(16)

from which we obtain - after identifying the renormalized scale with the factorization scale, $M_R = M_F$ - that $\Phi_{q_1}(z, \zeta, 0) = \frac{N_C e_q^2}{2\pi^2} \log \frac{m^2}{M_F^2} F(z, \zeta)$. This expression together with the Born order coefficient function $C^q_{\gamma\gamma}(z) = e_q^2 \left( \frac{1}{z} - \frac{1}{\bar{z}} \right)$ leads to the quark contribution to the $\gamma^* \gamma \to \gamma\gamma$ scattering amplitude

$$W^q_1 = \int_0^1 dz C^q_{\gamma\gamma}(z) \Phi_{q_1}(z, \zeta, 0).$$

(17)
FIGURE 2. The polarized anomalous diphoton GDA $\Phi^q_q/(N_C e_q^2/(2\pi^2) \log Q^2/m^2)$ at Born order and for $\zeta = 0.1$ (dashed), 0.2 (dash-dotted), 0.4 (solid).

The contribution to $W_1$ in Eq. (7) related to the photon operator involves a new coefficient function of order $\alpha^2_{em}$ calculated at the factorization scale $M_F$ and convoluted with the zeroth order in $\alpha_{em}$ part of the photon GDA. This contribution coincides with the expression (7) in which the quark mass $m$ is replaced by the factorization scale $M_F$ playing now the role of infra-red cutoff. The sum of contributions related to the photon operator and to the quark operator (17) reproduces then the result of direct calculations (7). Let us note, that by choosing the factorization scale $M_F$ equal to the hard scale of our process $Q$, $M_F = Q$, the resulting $W_1$ comes only from the quark GDA contribution.

We have thus demonstrated that it is legitimate to define the Born order diphoton GDAs at zero $W_1$ as

$$\Phi^q_I(z, \zeta, 0) = \frac{N_C e_q^2}{2\pi^2} \log \frac{Q^2}{m^2} \left[ \bar{z}(2z - \zeta) \theta(z - \zeta) + \bar{z}(2z - \bar{\zeta}) \theta(z - \bar{\zeta}) + z(2z - 1 - \zeta) \theta(\zeta - z) + z(2z - 1 - \bar{\zeta}) \theta(\bar{\zeta} - z) \right]$$

(18)

The same procedure can be applied to the axial vector correlator defining $\Phi_3$

$$\int \frac{dy}{2\pi} e^{i(2z-1)y} \langle \gamma(p_1) \gamma(p_2) | \bar{q}(-\frac{y}{2} N) \gamma . N \gamma_5 q(\frac{y}{2} N) | 0 \rangle = -i \varepsilon^{\mu \nu \rho \sigma} e^*_\mu(p_1) e^*_\rho(p_2) \Phi^q_3(z, \zeta, 0).$$

(19)

We get

$$\Phi^q_3(z, \zeta, 0) = \frac{N_C e_q^2}{2\pi^2} \log \frac{Q^2}{m^2} \left[ \frac{z\zeta}{\zeta} \theta(z - \zeta) - \frac{z\bar{\zeta}}{\zeta} \theta(z - \bar{\zeta}) - \frac{z\zeta}{\zeta} \theta(\zeta - z) + \frac{z\bar{\zeta}}{\zeta} \theta(\bar{\zeta} - z) \right].$$

(20)

Since we focused on the logarithmic factors, we only obtained the anomalous part of these GDAs. Their $z-$ and $\zeta-$dependence are shown on Figs. 1 and 2. Note that they are discontinuous functions of $z$ at the points $z = \zeta$ and $z = \bar{\zeta}$. 

ACKNOWLEDGMENTS

We are grateful to Igor Anikin, Markus Diehl and Jean Philippe Lansberg for useful discussions and correspondance. This work is partly supported by the French-Polish scientific agreement Polonium, the Polish Grant 1 7294/R08/R09, the ECO-NET program, contract 18853PJ, the Joint Research Activity ”Generalised Parton Distributions” of the european I3 program Hadronic Physics, contract RII3-CT-2004-506078.

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