Dynamics, Symmetries, and Hadron Properties

Craig D. Roberts
cdroberts@anl.gov

Physics Division
Argonne National Laboratory

http://www.phy.anl.gov/theory/staff/cdr.html
Quark and Gluon Confinement

No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon.
QCD’s Challenges

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon

- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
  - e.g., Lagrangian (pQCD) quark mass is small but ...
    - no degeneracy between $J^P=+$ and $J^P=-$
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- Neither of these phenomena is apparent in QCD’s Lagrangian yet they are the dominant determining characteristics of real-world QCD.
QCD’s Challenges

Understand Emergent Phenomena

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- QCD – Complex behaviour arises from apparently simple rules
Dichotomy of the Pion
How does one make an almost massless particle from two massive constituent-quarks?
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Not Allowed to do it by fine-tuning a potential

Must exhibit \( m_\pi^2 \propto m_q \)

Current Algebra ... 1968


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Current Algebra … 1968

The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a
- well-defined and valid chiral limit;
- and an accurate realisation of dynamical chiral symmetry breaking.
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Highly Nontrivial
What’s the Problem?
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- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
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Why problematic? Isn’t same true in quantum mechanics?
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- Differences!
What’s the Problem?

Relativistic QFT!

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Differences!

- Here relativistic effects are crucial – *virtual particles*, quintessence of Relativistic Quantum Field Theory – must be included.
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Differences!

- Here relativistic effects are crucial – *virtual particles*, quintessence of Relativistic Quantum Field Theory – must be included
- Interaction between quarks – the Interquark Potential – unknown throughout > 98% of an hadron’s volume
Intranucleon Interaction
Intranucleon Interaction

98% of the volume
The question must be rigorously defined, and the answer mapped out using experiment and theory.
Dyson-Schwinger Equations
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Well suited to Relativistic Quantum Field Theory
Dyson-Schwinger Equations

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Simplest level: Generating Tool for Perturbation Theory

Materially Reduces Model Dependence
Dyson-Schwinger Equations

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- NonPerturbative, Continuum approach to QCD
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  Hadrons as Composites of Quarks and Gluons
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  Qualitative and Quantitative Importance of:
  
  - Dynamical Chiral Symmetry Breaking
    - Generation of fermion mass from nothing
  - Quark & Gluon Confinement
    - Coloured objects not detected, not detectable?
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  ⇒ Understanding InfraRed (long-range)
  behaviour of $\alpha_s(Q^2)$
Dyson-Schwinger Equations

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NonPerturbative, Continuum approach to QCD

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- Method yields Schwinger Functions \( \equiv \) Propagators
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Cross-Sections built from Schwinger Functions
Solutions are Schwinger Functions (Euclidean Green Functions)
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- opportunity for comparisons at pre-experimental level . . . cross-fertilisation
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- Proving fruitful.
World ... DSE Perspective
Persistent Challenge

Infinitely Many Coupled Equations
Persistent Challenge

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Persistent Challenge

- **Infinitely Many Coupled Equations**
  
- Coupling between equations *necessitates* truncation
  
  - Weak coupling expansion $\Rightarrow$ Perturbation Theory
Persistent Challenge

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- Weak coupling expansion $\Rightarrow$ Perturbation Theory
  Not useful for the nonperturbative problems in which we're interested
Persistent Challenge

- Infinitely Many Coupled Equations

- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
  H.J. Munczek Phys. Rev. D 52 (1995) 4736

  *Dynamical chiral symmetry breaking, Goldstone’s theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations*
  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B 380 (1996) 7

  *Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*
**Persistent Challenge**

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- Has Enabled Proof of **EXACT** Results in QCD
- And Formulation of Practical Phenomenological Tool to
  - Illustrate Exact Results
  - Make Predictions with Readily Quantifiable Errors
Dressed-Quark Propagator
\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

**Dressed-Quark Propagator**

**Gap Equation**
\[
S(p) = \frac{Z(p^2)}{i \gamma \cdot p + M(p^2)}
\]

- Gap Equation’s Kernel Enhanced on IR domain

\[\Rightarrow \text{IR Enhancement of } M(p^2)\]
**Dressed-Quark Propagator**

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**Gap Equation’s Kernel Enhanced on IR domain**

⇒ IR Enhancement of \( M(p^2) \)

**Euclidean Constituent–Quark Mass:** \( M^E_f : p^2 = M(p^2)^2 \)

| flavour | \( u/d \) | \( s \) | \( c \) | \( b \) |
|---------|--------|--------|--------|--------|
| \( M^E/m_\zeta \) | \( \sim 10^2 \) | \( \sim 10 \) | \( \sim 1.5 \) | \( \sim 1.1 \) |
Dressed-Quark Propagator

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Predictions confirmed in numerical simulations of lattice-QCD
Hadrons

- Established understanding of two- and three-point functions
Hadrons

- Established understanding of two- and three-point functions
- What about bound states?
Without bound states, Comparison with experiment is impossible
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• They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions
• Without bound states, Comparison with experiment is impossible

• Bethe-Salpeter Equation

QFT Generalisation of Lippmann-Schwinger Equation.
• Without bound states, Comparison with experiment is impossible

• Bethe-Salpeter Equation

QFT Generalisation of Lippmann-Schwinger Equation.

• What is the kernel, $K$?
Without bound states, Comparison with experiment is impossible.

Bethe-Salpeter Equation

QFT Generalisation of Lippmann-Schwinger Equation.

What is the kernel, $K$?
What is the light-quark Long-Range Potential?
Bethe-Salpeter Kernel
Bethe-Salpeter Kernel

Axial-vector Ward-Takahashi identity

\[ P_\mu \Gamma^l_{\bar{5}\mu}(k; P) = S^{-1}(k_+) \frac{1}{2} \lambda^l_f i\gamma_5 + \frac{1}{2} \lambda^l_f i\gamma_5 \ S^{-1}(k_-) \]

\[ -M_\zeta i\Gamma^l_{\bar{5}}(k; P) - i\Gamma^l_{\bar{5}}(k; P) M_\zeta \]

QFT Statement of Chiral Symmetry
Bethe-Salpeter Kernel

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Satisfies BSE \hspace{1cm} Satisfies DSE
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Satisfies DSE

Kernels must be intimately related
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Satisfies BSE Satisfies DSE

Kernels must be intimately related

- Relation must be preserved by truncation
Axial-vector Ward-Takahashi identity

\[ P_\mu \Gamma_{5\mu}^l (k; P) = S^{-1}(k_+) \left( \frac{1}{2} \chi_f i\gamma_5 + \frac{1}{2} \chi_f i\gamma_5 S^{-1}(k_-) \right) \]

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Satisfies BSE  \hspace{1cm} Satisfies DSE

Kernels must be \textit{intimately} related

- Relation \textit{must} be preserved by truncation
- Nontrivial constraint
Axial-vector Ward-Takahashi identity

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Satisfies BSE  Satisfies DSE

Kernels must be **intimately** related

- **Relation must** be preserved by truncation
- **Failure** \( \Rightarrow \) Explicit Violation of QCD’s Chiral Symmetry
\[ f_H \ m_H^2 = - \ \rho_H^\zeta \ M_H \]
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy
nu-th/9707003)

\[ f_H \ m_H^2 = - \ \rho_H^\zeta \ \mathcal{M}_H \]

- Mass\(^2\) of pseudoscalar hadron
Radial Excitations & Chiral Symmetry

\[ f_H \ m_H^2 = - \rho_H^H \ M_H \]

\[ M_H := \text{tr}_\text{flavour} \left[ M_{(\mu)} \ (T^H, (T^H)^t) \right] = m_{q_1} + m_{q_2} \]

- Sum of constituents’ current-quark masses
- e.g., \[ T^{K^+} = \frac{1}{2} (\lambda^4 + i\lambda^5) \]
Radial Excitations
& Chiral Symmetry

\[ f_H \rho_H^2 = - \rho_H^2 \mathcal{M}_H \]

\[ f_H p_\mu = Z_2 \int_\Lambda q \frac{1}{2} \text{tr} \left\{ (T^H)^t \gamma_5 \gamma_\mu S(q_+) \Gamma_H(q; P) S(q_-) \right\} \]

- **Pseudovector** projection of BS wave function at \( x = 0 \)
- **Pseudoscalar meson’s leptonic decay constant**

\[ \vec{\pi} \rightarrow -f_\pi k^\mu \rightarrow A_5^\mu \]

Craig Roberts: Dynamics, Symmetries, and Hadron Properties
MENU07 – Inst. Nuclear Physics, Forschungszentrum Jülich, September 10-14, 2007
Radial Excitations & Chiral Symmetry

\[ f_H \quad m_H^2 = -\rho^H_\zeta \mathcal{M}_H \]

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- Pseudoscalar projection of BS wave function at \( x = 0 \)

\[ \pi \quad -\rho_\pi \quad P_5 \]

\[ k \]

\[ \vec{\pi} \quad \vec{P}_5 \quad \vec{\Gamma}_5 \quad \vec{\pi} \]

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\[ f_H \ m_H^2 = - \rho_\zeta^H \mathcal{M}_H \]

- **Light**-quarks; i.e., \( m_q \sim 0 \)

- \( f_H \rightarrow f_H^0 \) & \( \rho_\zeta^H \rightarrow -\frac{\langle \bar{q}q \rangle_\zeta^0}{f_H^0} \), Independent of \( m_q \)

Hence \( m_H^2 = \frac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q \) \ldots GMOR relation, a corollary
Valid for ALL Pseudoscalar mesons

\[ f_H \, m_H^2 = - \, \rho_H^\zeta \, M_H \]
Valid for ALL Pseudoscalar mesons

$\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $M_H \to 0$
Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts

\[ f_H \ m_H^2 = - \ \rho_H^\zeta \ \mathcal{M}_H \]

- Valid for ALL Pseudoscalar mesons
- \( \rho_H \to \) finite, nonzero value in chiral limit, \( \mathcal{M}_H \to 0 \)
- “radial” excitation of \( \pi \)-meson,
  \[ m_{\pi_n \neq 0}^2 > m_{\pi_n = 0}^2 = 0, \text{ in chiral limit} \]
Radial Excitations & Chiral Symmetry

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  \[ \Rightarrow f_H = 0 \]
- **ALL** pseudoscalar mesons except \( \pi(140) \) in chiral limit
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**Dynamical Chiral Symmetry Breaking**

- **Goldstone’s Theorem** –
  impacts upon **every** pseudoscalar meson
Radial Excitations
& Lattice-QCD

McNeile and Michael
he-la/0607032
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**Lattice-QCD check:**
$16^3 \times 32$,
$a \sim 0.1 \text{ fm},$
	wo-flavour, unquenched

$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078 (93)$
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**Lattice-QCD check:**

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**Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)**
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The suppression of $f_{\pi_1}$ is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.
**Pion ... \( J = 0 \)**

**but ...**

Orbital angular momentum is not a Poincaré invariant. However, if absent in a particular frame, it will appear in another frame related via a Poincaré transformation.
Nonzero quark orbital angular momentum is thus a necessary outcome of a Poincaré covariant description.
Pion \ldots J = 0

but \ldots

Pseudoscalar meson Bethe-Salpeter amplitude

\begin{equation}
\chi_{\pi}(k; P) = \gamma_5 \left[ i\mathcal{E}_{\pi n}(k; P) + \gamma \cdot P \mathcal{F}_{\pi n}(k; P) \right] + \gamma \cdot k \times P \mathcal{G}_{\pi n}(k; P) + \sigma_{\mu\nu} k_\mu P_\nu \mathcal{H}_{\pi n}(k; P) \right]
\end{equation}
Pseudoscalar meson Bethe-Salpeter amplitude

\[ \chi_\pi(k; P) = \gamma_5 \left[ iE_\pi_n(k; P) + \gamma \cdot P F_\pi_n(k; P) \right. \]

\[ \left. + \gamma \cdot k k \cdot P G_\pi_n(k; P) + \sigma_{\mu\nu} k_\mu P_\nu H_\pi_n(k; P) \right] \]

\( J = 0 \) \( \ldots \) but while \( E \) and \( F \) are purely \( L = 0 \) in the rest frame, the \( G \) and \( H \) terms are associated with \( L = 1 \). Thus a pseudoscalar meson Bethe-Salpeter wave function \textit{always} contains both \( S \)- and \( P \)-wave components.
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Introduce mixing angle $\theta_\pi$ such that

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\chi_\pi \sim \cos \theta_\pi |L = 0\rangle + \sin \theta_\pi |L = 1\rangle
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Introduce mixing angle $\theta_\pi$ such that

$$\chi_\pi \sim \cos \theta_\pi |L = 0\rangle + \sin \theta_\pi |L = 1\rangle$$

$L$ is significant in the neighbourhood of the chiral limit, and decreases with increasing current-quark mass.
\[ P_\mu \Gamma^a_{5\mu}(k; P) = S^{-1}(k_+) i\gamma_5 F^a + i\gamma_5 F^a S^{-1}(k_-) \]
\[ -2i M^{ab} \Gamma^b_{5}(k; P) - A^a(k; P) \]
Charge Neutral Pseudoscalar Mesons

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\[ \{ F^a | a = 0, \ldots, N_f^2 - 1 \} \text{ are the generators of } U(N_f) \]
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\{ \mathcal{F}^a | a = 0, \ldots, N_f^2 - 1 \} are the generators of \( U(N_f) \)

\( S = \text{diag}[S_u, S_d, S_s, S_c, S_b, \ldots] \)
$P_\mu \Gamma_5^a (k; P) = S^{-1}(k_+) i\gamma_5 F^a + i\gamma_5 F^a S^{-1}(k_-)$

$-2 i M^{ab} \Gamma_5^b (k; P) - A^a (k; P)$

$\{ F^a \} a = 0, \ldots, N_f^2 - 1 \}$ are the generators of $U(N_f)$

$S = \text{diag}[S_u, S_d, S_s, S_c, S_b, \ldots]$ 

$M^{ab} = \text{tr}_F \left[ \{ F^a, M \} F^b \right], \quad \mathcal{M} = \text{diag}[m_u, m_d, m_s, m_c, m_b, \ldots] = \text{matrix of current-quark bare masses}$
Charge Neutral Pseudoscalar Mesons

\[
P_\mu \Gamma_5^a (k; P) = S^{-1}(k_+) i \gamma_5 F^a + i \gamma_5 F^a S^{-1}(k_-) \\
-2i M^{ab} \Gamma_5^b (k; P) - A^a (k; P)
\]

- \{ F^a | a = 0, \ldots, N_f^2 - 1 \} are the generators of \( U(N_f) \)
- \( S = \text{diag}[S_u, S_d, S_s, S_c, S_b, \ldots] \)
- \( M^{ab} = \text{tr}_F \left[ \{ F^a, M \} F^b \right], \)
  \( M = \text{diag}[m_u, m_d, m_s, m_c, m_b, \ldots] = \text{matrix of current-quark bare masses} \)
- The final term in the second line expresses the non-Abelian axial anomaly.
Charge Neutral
Pseudoscalar Mesons

\[ P_\mu \Gamma^a_{5\mu}(k; P) = S^{-1}(k_+) i\gamma_5 F^a + i\gamma_5 F^a S^{-1}(k_-) \]

\[ -2iM^{ab}\Gamma^b_{5}(k; P) - A^a(k; P) \]

\[ A^a(k; P) = S^{-1}(k_+) \delta^{a0} A_U(k; P) S^{-1}(k_-) \]

\[ A_U(k; P) = \int d^4x d^4y e^{i(k_+ \cdot x - k_- \cdot y)} N_f \langle \mathcal{F}^0 q(x) Q(0) \bar{q}(y) \rangle \]
\[ P_\mu \Gamma_\mu^a (k; P) = S^{-1}(k_+) \gamma_5 F^a + i \gamma_5 F^a S^{-1}(k_-) \]

\[ -2i M_{ab} \Gamma_5^b (k; P) = A^a (k; P) \]

\[ A^a (k; P) = S^{-1}(k_+) \delta^{a0} A^a_U (k; P) S^{-1}(k_-) \]

\[ A^a_U (k; P) = \int d^4 x d^4 y e^{i (k_+ \cdot x - k_- \cdot y)} N_f \langle \mathcal{F}_0^a q(x) Q(0) \bar{q}(y) \rangle \]

\[ Q(x) = i \frac{\alpha_s}{4\pi} tr_C [\epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}(x)] = \partial_\mu K_\mu (x) \]

\[ \ldots \text{ The topological charge density operator.} \]

(Trace is over colour indices & \( F_{\mu\nu} = \frac{1}{2} \lambda^a F_{\mu\nu}^a \).)
Charge Neutral
Pseudoscalar Mesons

\[ P_\mu \Gamma^a_{\frac{5}{5} \mu}(k; P) = S^{-1}(k_+) i \gamma_5 F^a + i \gamma_5 F^a S^{-1}(k_-) \]

\[-2i M^{ab} \Gamma^b_5(k; P) - A^a(k; P)\]

\[ A^a(k; P) = S^{-1}(k_+) \delta^{a0} A_U(k; P) S^{-1}(k_-) \]

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\[ Q(x) = i \frac{\alpha_s}{4\pi} \text{tr}_C [\epsilon_{\mu \nu \rho \sigma} F_{\mu \nu} F_{\rho \sigma}(x)] = \partial_\mu K_\mu(x) \]

\ldots The topological charge density operator.

Important that only \( A^{a=0} \) is nonzero.
Charge Neutral
Pseudoscalar Mesons

\[ P_\mu \Gamma^{a}_{5\mu}(k; P) = S^{-1}(k_+) i \gamma_5 F^a + i \gamma_5 F^a S^{-1}(k_-) \]
\[ -2i M^{ab} \Gamma^b_5(k; P) - A^{a}(k; P) \]
\[ A^a(k; P) = S^{-1}(k_+) \delta^{a0} A_U(k; P) S^{-1}(k_-) \]
\[ A_U(k; P) = \int d^4 x d^4 y e^{i (k_+ \cdot x - k_- \cdot y)} N_f \langle F^0 q(x) Q(0) \bar{q}(y) \rangle \]
\[ Q(x) = i \frac{\alpha_s}{4\pi} \text{tr}_C [\epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}(x)] = \partial_\mu K_\mu(x) \]

\[ \text{... The topological charge density operator.} \]

\[ \text{NB. While } Q(x) \text{ is gauge invariant, the associated Chern-Simons current, } K_\mu, \text{ is not } \Rightarrow \text{ in QCD no physical boson can couple to } K_\mu \text{ and hence no physical states can contribute to resolution of } U_A(1) \text{ problem.} \]
Charge Neutral
Pseudoscalar Mesons

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118
Only $A^0 \not\equiv 0$ is interesting
Only $A^0 \neq 0$ is interesting . . . otherwise all pseudoscalar mesons are Goldstone Modes!
Anomaly term has structure

$$\mathcal{A}_0^0(k; P) = \mathcal{F}_5 \gamma^5 \left[ i \mathcal{E}_A(k; P) + \gamma \cdot P \mathcal{F}_A(k; P) ight]$$

$$+ \gamma \cdot k k \cdot P \mathcal{G}_A(k; P) + \sigma_{\mu \nu} k_\mu P_\nu \mathcal{H}_A(k; P)$$
AVWTI gives generalised Goldberger-Treiman relations

\[2 f_{\eta'} E_{BS}(k; 0) = 2 B_0(k^2) - \mathcal{E}_A(k; 0),\]
\[F^0_R(k; 0) + 2 f_{\eta'} F_{BS}(k; 0) = A_0(k^2) - \mathcal{F}_A(k; 0),\]
\[G^0_R(k; 0) + 2 f_{\eta'} G_{BS}(k; 0) = 2 A'_0(k^2) - \mathcal{G}_A(k; 0),\]
\[H^0_R(k; 0) + 2 f_{\eta'} H_{BS}(k; 0) = -\mathcal{H}_A(k; 0),\]

\[A_0, B_0\] characterise gap equation’s chiral limit solution.
AVWTI gives generalised Goldberger-Treiman relations

\[
2f_{\eta'} E_{BS}(k; 0) = 2B_0(k^2) - \mathcal{E}_A(k; 0),
\]
\[
F_R^0(k; 0) + 2f_{\eta'} F_{BS}(k; 0) = A_0(k^2) - \mathcal{F}_A(k; 0),
\]
\[
G_R^0(k; 0) + 2f_{\eta'} G_{BS}(k; 0) = 2A'_0(k^2) - \mathcal{G}_A(k; 0),
\]
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H_R^0(k; 0) + 2f_{\eta'} H_{BS}(k; 0) = -\mathcal{H}_A(k; 0),
\]

\(A_0, B_0\) characterise gap equation’s chiral limit solution.

Follows that \(\mathcal{E}_A(k; 0) = 2B_0(k^2)\) is necessary and sufficient condition for absence of massless \(\eta'\) bound-state.
\[ \mathcal{E}_A(k; 0) = 2B_0(k^2) \]

Discussing the chiral limit

\[ B_0(k^2) \neq 0 \text{ if, and only if, chiral symmetry is dynamically broken.} \]

Hence, absence of massless \( \eta' \) bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.
\[ \mathcal{E}_A(k; 0) = 2B_0(k^2) \]

Discussing the chiral limit

\[ B_0(k^2) \neq 0 \quad \text{if, and only if, chiral symmetry is dynamically broken.} \]

Hence, absence of massless \( \eta' \) bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.

Further highlighted ... proved

\[ \langle \bar{q}q \rangle_0^0 = - \lim_{\Lambda \to \infty} Z_4(\zeta^2, \Lambda^2) \textrm{tr}_{CD} \int_q^\Lambda S^0(q, \zeta) \]

\[ = N_f \int d^4x \langle \bar{q}(x)i\gamma_5q(x)Q(0) \rangle^0. \]
AVWTI ⇒ QCD mass formulae for neutral pseudoscalar mesons
AVWTI ⇒ QCD mass formulae for neutral pseudoscalar mesons

Implications of mass formulae illustrated using elementary dynamical model, which includes Ansatz for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly
Charge Neutral Pseudoscalar Mesons

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons
- Implications of mass formulae illustrated using elementary dynamical model, which includes Ansatz for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly
- Employed in an analysis of pseudoscalar- and vector-meson bound-states
AVWTI ⇒ QCD mass formulae for neutral pseudoscalar mesons

Implications of mass formulae illustrated using elementary dynamical model, which includes Ansatz for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly

Despite its simplicity, model is elucidative and phenomenologically efficacious; e.g., it predicts

- $\eta - \eta'$ mixing angles of $\sim -15^\circ$ (Expt.: $-13.3^\circ \pm 1.0^\circ$)
- $\pi^0 - \eta$ angles of $\sim 1.2^\circ$ (Expt. $pd \to ^3\text{He} \pi^0$: $0.6^\circ \pm 0.3^\circ$)
- Strong neutron-proton mass difference . . .
  $\lesssim 75\%$ current-quark mass-difference
New Challenges

Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.
New Challenges

- **Next Steps** . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.

- Move on to the problem of a *symmetry preserving* treatment of hybrids and exotics.
Another Direction . . . Also want/need information about three-quark systems
New Challenges

- Another Direction . . . Also want/need information about three-quark systems

- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.
New Challenges

- Another Direction . . . Also want/need information about three-quark systems

- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.

- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.
Nucleon EM Form Factors: A Précis

Höll, et al.: nu-th/0412046 & nu-th/0501033
Nucleon EM Form Factors: A Précis

Höll, et al.: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
**Nucleon EM Form Factors: A Précis**

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  ⇒ Covariant dressed-quark Faddeev Equation
Nucleon EM Form Factors: A Précis

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- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons ⇒ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)

Easily obtained:

\[
\left( \frac{1}{N_H} \sum_H \frac{[M_H^{\text{exp}} - M_H^{\text{calc}}]^2}{[M_H^{\text{exp}}]^2} \right)^{1/2} = 2\%
\]

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(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)
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Höll, et al.: nu-th/0412046 & nu-th/0501033

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• But is that good?
Nucleon EM Form Factors: A Précis

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• But is that good?
  • Cloudy Bag: \( \delta M_+^{\pi-\text{loop}} = -300 \text{ to } -400 \text{ MeV}! \)
**Nucleon EM Form Factors: A Précis**

Höll, et al.: [nu-th/0412046](http://arxiv.org/abs/nu-th/0412046) & [nu-th/0501033](http://arxiv.org/abs/nu-th/0501033)

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  \( \Rightarrow \) Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)

**Easily obtained:**

\[
\left( \frac{1}{N_H} \sum_H \left[ \frac{M_H^{\text{exp}} - M_H^{\text{calc}}}{M_H^{\text{exp}}^2} \right]^2 \right)^{1/2} = 2\%
\]

- **But** is that good?
  - Cloudy Bag: \( \delta M_+^{\pi-\text{loop}} = -300 \) to \( -400 \) MeV!
  - **Critical** to anticipate pion cloud effects

Roberts, Tandy, Thomas, et al., [nu-th/02010084](http://arxiv.org/abs/nu-th/02010084)
Faddeev equation
Faddeev equation

\[ \Psi^a \rightarrow \Psi^b \]

\[ p_q \]

\[ p_d \]

\[ P \]

\[ = \]

\[ p_q \]

\[ p_d \]

\[ \Gamma^a \]

\[ \Gamma^b \]

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Faddeev equation

\[ \Psi^a \] \hspace{1cm} \Psi^b \]

\[ p_q \] \hspace{1cm} \Gamma^a \quad \Gamma^b \]

\[ p_d \] \hspace{1cm} q \]

Linear, Homogeneous Matrix equation

- Yields wave function (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon

- Scalar and Axial-Vector Diquarks . . . In Nucleon’s Rest Frame Amplitude has . . . \( s- \), \( p- \) & \( d- \)–wave correlations
Diquark correlations
Diquark correlations

Same interaction that describes mesons also generates three coloured quark-quark correlations:

- blue–red
- blue–green
- green–red

Confined . . . Does not escape from within baryon.

Scalar is isosinglet,
Axial-vector is isotriplet.

DSE and lattice-QCD

\[
\begin{align*}
    m_{[ud]}^{0+} &= 0.74 - 0.82 \\
    m_{(uu)}^{1+} &= m_{(ud)}^{1+} = m_{(dd)}^{1+} = 0.95 - 1.02
\end{align*}
\]
Results: Nucleon and $\Delta$ Masses
Results: Nucleon and $\Delta$ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and $\Delta$ masses

Set A – fit to the actual masses was required; whereas for
Set B – fitted mass was offset to allow for “$\pi$-cloud” contributions

| set | $M_N$  | $M_\Delta$ | $m_{0+}$ | $m_{1+}$ | $\omega_{0+}$ | $\omega_{1+}$ |
|-----|--------|------------|----------|----------|---------------|---------------|
| A   | 0.94   | 1.23       | 0.63     | 0.84     | 0.44=1/(0.45 fm) | 0.59=1/(0.33 fm) |
| B   | 1.18   | 1.33       | 0.79     | 0.89     | 0.56=1/(0.35 fm) | 0.63=1/(0.31 fm) |

$m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV
**Results: Nucleon and Δ Masses**

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses.

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$m_1^+ \to \infty: M_N^A = 1.15 \text{ GeV}; M_N^B = 1.46 \text{ GeV}$

Axial-vector diquark provides significant attraction
Results: Nucleon and $\Delta$ Masses

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$m_1^+ \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV

Constructive Interference: $1^{++}$-diquark + $\partial_\mu \pi$
Nucleon-Photon Vertex
Nucleon-Photon Vertex

6 terms . . . constructed systematically . . . current \textit{conserved} automatically for on-shell nucleons described by Faddeev Amplitude
6 terms . . .

constructed systematically . . . current conserved automatically
for on-shell nucleons described by Faddeev Amplitude

Nucleon-Photon Vertex
Form Factor Ratio: \( \frac{G_E}{G_M} \)

\[
\mu_p \frac{G_E^P}{G_M^P} vs. Q^2 [\text{GeV}^2]
\]

- Rosenbluth
- precision Rosenbluth
- polarization transfer
- polarization transfer
Combine these elements . . .

Form Factor Ratio: GE/GM

\[ \frac{\mu_p G_E^p}{G_M^p} \]
Combine these elements . . .

Dressed-Quark Core

Form Factor Ratio: $\frac{G_E}{G_M}$

![Graph showing $\mu_p G_E^p / G_M^p$ vs $Q^2$]

- Rosenbluth
- precision Rosenbluth
- polarization transfer
- polarization transfer
Combine these elements . . .

- Dressed-Quark Core
- Ward-Takahashi Identity preserving current

Form Factor Ratio: \( \frac{G_E}{G_M} \)
Combine these elements . . .

- Dressed-Quark Core
- Ward-Takahashi Identity preserving current
- Anticipate and Estimate Pion Cloud’s Contribution

Form Factor Ratio: \( \frac{G_E}{G_M} \)
Combine these elements . . .

- Dressed-Quark Core
- *Ward-Takahashi*
  Identity preserving current
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**Form Factor Ratio:**

\[ \frac{G_E}{G_M} \]
Combine these elements . . .

- Dressed-Quark Core
- Ward-Takahashi Identity preserving current
- Anticipate and Estimate Pion Cloud’s Contribution

All parameters fixed in other applications . . . Not varied.
Combine these elements . . .

- Dressed-Quark Core
- *Ward-Takahashi* Identity preserving current
- Anticipate and Estimate Pion Cloud’s Contribution

All parameters fixed in other applications . . . **Not** varied.

Agreement with Pol. Trans. data at $Q^2 \gtrsim 2\text{ GeV}^2$
Combine these elements . . .

- Dressed-Quark Core
- *Ward-Takahashi* Identity preserving current
- Anticipate and Estimate Pion Cloud’s Contribution

All parameters fixed in other applications . . . Not varied.

- Agreement with Pol. Trans. data at $Q^2 \gtrsim 2$ GeV$^2$
- Correlations in Faddeev amplitude – quark orbital angular momentum – essential to that agreement
Combine these elements . . .

- Dressed-Quark Core
- Ward-Takahashi Identity preserving current
- Anticipate and Estimate Pion Cloud’s Contribution

All parameters fixed in other applications . . . Not varied.

- Agreement with Pol. Trans. data at \( Q^2 \gtrsim 2 \text{ GeV}^2 \)
- Correlations in Faddeev amplitude – quark orbital angular momentum – essential to that agreement
- Predict Zero at \( Q^2 \approx 6.5 \text{ GeV}^2 \)
Quark Distribution Functions

DIS

\[ \ell' \rightarrow k', s' \]

\[ \ell \rightarrow k, s \]

SI–DIS

\[ \ell' \rightarrow k', s' \]

\[ \ell \rightarrow k, s \]

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- p. 29/42
Three twist-2 parton distributions \((k_\perp = 0)\):

- Spin-Independent: \(q(x)\)
- Helicity: \(\Delta q(x)\)
- Transversity: \(\Delta_T q(x)\)

All distributions have probability interpretation.

By definition, contain essentially non-perturbative information about a given process.
**Definition and Sum Rules**

- Light-cone Fourier transforms:
  \[
  \Delta_T q(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} \langle p, s | \overline{\psi}_q(0)\gamma^+\gamma^1\gamma_5\psi_q(\xi^-) | p, s \rangle_c
  \]
  \[
  q(x) = \langle \gamma^+ \rangle, \quad \Delta q(x) = \langle \gamma^+\gamma_5 \rangle
  \]
  - Related to the nucleon axial & tensor charges via
    \[
    g_A = \int dx [\Delta u(x) - \Delta d(x)], \quad g_T = \int dx [\Delta_T u(x) - \Delta_T d(x)],
    \]
  - Must satisfy: positivity constraints and Soffer bound
    \[
    \Delta q(x), \Delta_T q(x) \leq q(x), \quad q(x) + \Delta q(x) \geq 2 |\Delta_T q(x)|
    \]
Once more on the one that got away.
Model predictions

Cloët, Bentz, Thomas
arXiv:0708.3246 [hep-ph]
Model predictions

- Simplified Faddeev equation

\[ Q^2 = 2.4 \text{ GeV}^2 \]

\[ x u(x) \quad x \Delta u(x) \quad x \Delta_T u(x) \]

- Satisfy: Soffer bound, baryon & momentum SRs.
Model predictions

- **Simplified Faddeev equation**

- **Satisfy**: Soffer bound, baryon & momentum SRs.

- **Moments at** $Q^2 = 0.16$ GeV$^2$:
  \[ \Delta u = 0.97, \quad \Delta d = -0.30 \quad \Rightarrow \quad g_A = 1.267 \]
  \[ \Delta_T u = 1.04, \quad \Delta_T d = -0.24 \quad \Rightarrow \quad g_T = 1.28 \]
Simplified Faddeev equation

- Satisfy: Soffer bound, baryon & momentum SRs.
- Moments at $Q^2 = 0.16 \text{ GeV}^2$:
  \[ \Delta u = 0.97, \quad \Delta d = -0.30 \implies g_A = 1.267 \]
  \[ \Delta_T u = 1.04, \quad \Delta_T d = -0.24 \implies g_T = 1.28 \]

\[ \Delta q(x) \sim \Delta_T q(x) \text{ in valence region for } Q^2 \lesssim 10 \text{ GeV}^2 \]
Epilogue
Epilogue
DCSB exists in QCD.
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- It is manifest in dressed propagators and vertices
- It impacts dramatically upon observables.
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Confinement
Epilogue

- DCSB exists in QCD.
  - It is manifest in dressed propagators and vertices
  - It impacts dramatically upon observables.
- Confinement
  - Expressed and realised in dressed propagators and vertices associated with elementary excitations
  - Observables can be used to explore model realisations
Epilogue

- DCSB exists in QCD.
  - It is manifest in dressed propagators and vertices
  - It impacts dramatically upon observables.

Confinement

- Expressed and realised in dressed propagators and vertices associated with elementary excitations
- Observables can be used to explore model realisations
- DSEs ... contemporary tool that describes and explains these phenomena, and connects them with prediction of observables
Quenched-QCD

Dressed-Quark Propagator

$M(p)$

$Z(p)$
**Quenched-QCD**

**Dressed-Quark Propagator**

2002

\[ M(p) \]

\[ Z(p) \]

“data:” Quenched Lattice Meas.

– Bowman, Heller, Leinweber, Williams: [he-lat/0209129](http://arxiv.org/abs/he-lat/0209129)
“data”: Quenched Lattice Meas.
– Bowman, Heller, Leinweber, Williams: [he-lat/0209129](http://arxiv.org/abs/he-lat/0209129)
current-quark masses: 30 MeV, 50 MeV, 100 MeV
**Quenched-QCD**

**Dressed-Quark Propagator**

- **“data”:** Quenched Lattice Meas.
  - Bowman, Heller, Leinweber, Williams: [he-lat/0209129](https://arxiv.org/abs/he-lat/0209129)
  - current-quark masses: 30 MeV, 50 MeV, 100 MeV

- **Curves:** Quenched DSE Cal.
  - Bhagwat, Pichowsky, Roberts, Tandy [nu-th/0304003](https://arxiv.org/abs/nu-th/0304003)
Quenched-QCD
Dressed-Quark Propagator

$M(p)$

$Z(p)$

“data:” Quenched Lattice Meas.

– Bowman, Heller, Leinweber, Williams: [he-lat/0209129](https://arxiv.org/abs/he-lat/0209129)

Current-quark masses: 30 MeV, 50 MeV, 100 MeV

Curves: Quenched DSE Cal.

– Bhagwat, Pichowsky, Roberts, Tandy [nu-th/0304003](https://arxiv.org/abs/nu-th/0304003)

Linear extrapolation of lattice data to chiral limit is inaccurate
Kernel of Gap Equation: $D_{\mu\nu}(p - q)\Gamma_\nu(q)$

Dressed-gluon propagator and dressed-quark-gluon vertex

Reliable DSE studies of Dressed-gluon propagator:

- R. Alkofer and L. von Smekal, *The infrared behavior of QCD Green’s functions...*, Phys. Rept. **353**, 281 (2001).
QCD & Interaction Between Light-Quarks

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Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:
- D. B. Leinweber, J. I. Skullerud, A. G. Williams and C. Parrinello [UKQCD Collaboration], *Asymptotic scaling and infrared behavior of the gluon propagator*, Phys. Rev. D 60, 094507 (1999) [Erratum-ibid. D 61, 079901 (2000)].

Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex
\[ D_{\mu \nu}(k) = \left( \delta_{\mu \nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{Z(k^2)}{k^2} \]

Suppression means \( \exists \) IR gluon mass-scale \( \approx 1 \text{ GeV} \)

Naturally, this scale has the same origin as \( \Lambda_{\text{QCD}} \)
Dressed-gluon Propagator

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Dynamical chiral symmetry breaking and a critical mass
Lei Chang, Yu-Xin Liu, Mandar S. Bhagwat, Craig D. Roberts and Stewart V. Wright . . . nucl-th/0605058
Phys. Rev. C 75 (2007) 015201 (8 pages)
Realistic models of QCD’s gap equation simultaneously admit two inequivalent DCSB solutions & solution connected with realisation of chiral symmetry in Wigner mode.
Critical Mass for Chiral Expansion

Realistic models of QCD’s gap equation simultaneously admit two inequivalent DCSB solutions & solution connected with realisation of chiral symmetry in Wigner mode.

Wigner solution and one DCSB solution are destabilised by current-quark mass & both disappear when that mass exceeds a critical value, $m_{cr}$.

The zeros of $G(M)$ characterise the different solutions of the gap equation. Solid curve: obtained with $m^{bm} = 0$, in which case $G(M)$ is odd under $M \rightarrow -M$; long-dashed curve: $m^{cm} = 0.01$; short-dashed curve: $m^{cm} = m^{bm} = 0.033$; dotted curve: $m^{bm} = 0.05$. 
Critical Mass for Chiral Expansion

- Realistic models of QCD’s gap equation simultaneously admit two inequivalent DCSB solutions & solution connected with realisation of chiral symmetry in Wigner mode.

- $m_{cr}$ also bounds domain on which surviving DCSB solution possesses a chiral expansion: $m_{cr} = \lim_{n \to \infty} \left( \frac{1}{|a_n|} \right)^{1/n}$
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- For a pseudoscalar meson constituted of equal mass current-quarks, it corresponds to a mass $m_{cr}^0 \sim 0.45$ GeV.
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- For a pseudoscalar meson constituted of equal mass current-quarks, it corresponds to a mass $m_{cr}^{0-} \sim 0.45$ GeV.

- Entails lattice-QCD simulations must have many results at $m_\pi < m_{cr}^{0-} \sim 0.45$ GeV for reliable extrapolation via EFT.
Realistic models of QCD's gap equation simultaneously admit two inequivalent DCSB solutions & solution connected with realisation of chiral symmetry in Wigner mode.

The two DCSB solutions of the gap equation enable a valid definition of $\langle \bar{q}q \rangle$ in the presence of a nonzero current-mass.
Critical Mass for Chiral Expansion

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- The two DCSB solutions of the gap equation enable a valid definition of $\langle \bar{q}q \rangle$ in the presence of a nonzero current-mass.

- The behaviour of this condensate indicates that the essentially dynamical component of chiral symmetry breaking decreases with increasing current-quark mass.
Consituent-quark $\sigma$-term

Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark $\sigma$-term

$$\sigma_f := m_f(\zeta) \frac{\partial M^E_f}{\partial m_f(\zeta)} , \quad (M^E)^2 := s \mid s = M(s)^2.$$
**Consituent-quark $\sigma$-term**

- Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark $\sigma$-term

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\sigma_f := m_f(\zeta) \frac{\partial M_f^E}{\partial m_f(\zeta)}, \quad (M^E)^2 := s \mid s = M(s)^2.
\]

- Renormalisation-group-invariant and determined from solutions of the gap equation
Consituent-quark $\sigma$-term

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Unambiguous probe of impact of explicit chiral symmetry breaking on the mass function
Consituent-quark $\sigma$-term

Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark $\sigma$-term

\[
\sigma_f := m_f(\zeta) \frac{\partial M^E_f}{\partial m_f(\zeta)}, \quad (M^E)^2 := s \mid s = M(s)^2.
\]

Ratio

\[
\frac{\sigma_f}{M^E_f} = \frac{\text{EXPLICIT}}{\text{EXPLICIT} + \text{DYNAMICAL}}
\]

measures effect of EXPLICIT chiral symmetry breaking on dressed-quark mass-function
cf. SUM of effects of EXPLICIT AND DYNAMICAL CHIRAL SYMMETRY BREAKING
Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark $\sigma$-term

$$\sigma_f := m_f(\zeta) \frac{\partial M_f^E}{\partial m_f(\zeta)}, \quad (M^E)^2 := s \mid s = M(s)^2.$$
Constituent-quark $\sigma$-term

Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark $\sigma$-term

\[
\sigma_f := m_f(\zeta) \frac{\partial M^E_f}{\partial m_f(\zeta)}, \quad (M^E)^2 := s \mid s = M(s)^2.
\]

Obvious: ratio vanishes for light-quarks because magnitude of their constituent-mass owes primarily to DCSB. On the other hand, for heavy-quarks it approaches one.
Consituent-quark $\sigma$-term

Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark $\sigma$-term

$$\sigma_f := m_f(\zeta) \frac{\partial M^E_f}{\partial m_f(\zeta)}, \quad (M^E)^2 := s \mid s = M(s)^2.$$ 

Essentially dynamical component of chiral symmetry breaking, and manifestation in all its order parameters, vanishes with increasing current-quark mass.
Crude estimate based on magnitudes $\Rightarrow$ probability for a $u$-quark to carry the proton’s spin is $P_u^{\uparrow} \sim 80\%$, with $P_u^{\downarrow} \sim 5\%$, $P_d^{\uparrow} \sim 5\%$, $P_d^{\downarrow} \sim 10\%$.

Hence, by this reckoning $\sim 30\%$ of proton’s rest-frame spin is located in dressed-quark angular momentum.
Neutron Form Factors

\[ \mu_n G_E^n / G_M^n \]

\[ Q^2 \quad [\text{GeV}^2] \]

Craig Roberts: Dynamics, Symmetries, and Hadron Properties
MENU07 – Inst. Nuclear Physics, Forschungszentrum Jülich, September 10-14, 2007
Neutron Form Factors

Expt. Madey, et al. nu-ex/0308007
Neutron Form Factors

Expt. Madey, et al. nu-ex/0308007

Calc. Bhagwat, et al. nu-th/0610080

\[ \mu_p \frac{G^n_E(Q^2)}{G^n_M(Q^2)} = -\frac{r_n^2}{6} Q^2 \]

Valid for \( r_n^2 Q^2 \lesssim 1 \)
Expt. Madey, et al. nu-ex/0308007

Calc. Bhagwat, et al. nu-th/0610080

\[
\mu_p \frac{G^n_E(Q^2)}{G^n_M(Q^2)} = -\frac{r_n^2}{6} Q^2
\]

Valid for \( r_n^2 Q^2 \lesssim 1 \)

No sign yet of a zero in \( G^n_E(Q^2) \), even though calculation predicts \( G^p_E(Q^2 \approx 6.5 \text{ GeV}^2) = 0 \)

Data to \( Q^2 = 3.4 \text{ GeV}^2 \) is being analysed (JLab E02-013)
Contemporary Reviews

Dyson-Schwinger Equations: Density, Temperature and Continuum Strong QCD
C.D. Roberts and S.M. Schmidt, nu-th/0005064,
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The IR behavior of QCD Green's functions: Confinement, DCSB, and hadrons . . .
R. Alkofer and L. von Smekal, he-ph/0007355,
Phys. Rept. 353 (2001) 281

Dyson-Schwinger equations: A Tool for Hadron Physics
P. Maris and C.D. Roberts, nu-th/0301049,
Int. J. Mod. Phys. E 12 (2003) pp. 297-365

Infrared properties of QCD from Dyson-Schwinger equations.
C. S. Fischer, he-ph/0605173,
J. Phys. G 32 (2006) pp. R253-R291

Nucleon electromagnetic form factors
J. Arrington, C.D. Roberts and J.M. Zanotti, nucl-th/0611050,
J. Phys. G 34 (2007) pp. S23-S52.