Pole N-flation

with M. Dias, J. Frazer, A. Retolaza & M. Scalisi [1805.xxxxx]

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slow-roll inflation...

$\ddot{\phi} + 3H\dot{\phi} = -\partial_\phi V \equiv -V'$, $H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V\right)$

$a \sim e^{Ht}$

$\epsilon = \frac{V'^2}{2V^2} \ll 1$, $\eta = \frac{V''}{V} \ll 1$
Slow-roll inflation ...

\[ V(\phi) = \frac{m^2}{2} \phi^2 \]

\[ 3H\dot{\phi} \simeq -V', \quad H^2 \simeq \frac{1}{3}V \]

Several different regimes are possible, depending on the value of the field \( \phi \). If the potential energy density of the field is greater than the Planck density \( M_p^4 = 1 \), quantume fluctuations so strong that one cannot describe it in usual terms. Such a state is called space-time foam. At somewhat smaller energy density (for \( m \lesssim V(\phi) \lesssim 1 \), \( m^{-1}/2 \lesssim \phi \lesssim m^{-1} \)), quantum fluctuations of space-time are small, but quantum fluctuations of the scalar field \( \phi \) may be large. Jumps of the scalar field due to quantum fluctuations lead to a process so fundamental that the inflationary universe which we are going to discuss later. At even smaller values of \( V(\phi) \) (for \( m^2 \lesssim V(\phi) \lesssim m, 1 \lesssim \phi \lesssim m^{-1}/2 \)) fluctuations of the field are small; it slowly moves down as a ball in a viscous liquid. Inflation occurs for \( 1 \lesssim \phi \lesssim m^{-1} \). Finally, near the minimum of \( V(\phi) \) (for \( \phi \lesssim 1 \)) the scalar field rapidly oscillates, creates pairs of elementary particles, and the universe becomes hot.
slow-roll inflation ...

\[ V(\phi) = \frac{m^2}{2} \phi^2 \]

\[ m \sim 10^{13} \text{ GeV} \]

**Eternal Inflation**

**SPACE-TIME FOAM**

**PLANCK DENSITY**

**HEATING OF UNIVERSE**

**SCALAR FIELD**

\[ \frac{\Delta T}{T} \sim 10^{-5} \]

\[ \sqrt{\epsilon \Delta_S^2} \sim \sqrt{\Delta_T^2} \sim \frac{H}{M_P} \sim 10^{-5} \]

\[ n_s = \frac{d \ln \Delta_S}{d \ln k} = 1 - 6\epsilon + 2\eta \]

[picture from lecture notes: Linde ’07]
slow-roll inflation ...

\[ V(\phi) = \frac{m^2}{2} \phi^2 \]

Angular scale

\[ n_s = 1 - 6\epsilon + 2\eta \]
\[ \approx 0.9655 \pm 0.0062 \]
### slow-roll inflation ...

\[ V(\phi) = \frac{m^2}{2} \phi^2 \]

**Angular scale**

\[ n_s = 1 - 6\epsilon + 2\eta \]
\[ \approx 0.9655 \pm 0.0062 \]

\[ r = \frac{\Delta_T^2}{\Delta_S^2} = 16\epsilon < 0.09 \ (95\%) \]
slow-roll inflation ...

\[ V(\phi) = \frac{m^2}{2} \phi^2 \]

Angular scale

\[ n_s = 1 - 6\epsilon + 2\eta \]
\[ \simeq 0.9655 \pm 0.0062 \]

\[ r = \frac{\Delta^2_T}{\Delta^2_S} = 16\epsilon < 0.07 \ (95\%) \]
test string theory with inflation & CMB
the string theory landscape:
many isolated vacua, connected by tunneling
some mountain slopes drive \textit{inflation}

string theory's 6 compact dimensions:
\textit{strings}, \textit{branes} \& \textit{fluxes}
on large scales, allowing room for a slightly larger contribution power on small scales leads to a decrease in the scalar power lowers the value of TT this shift was caused by the 2015 analyses (about 0.9655, a key parameter for inflationary cosmology, shows one of the largest shifts of any parameter PCP13 (by about 0.002). The red contours in Fig. 21 shows the 2015 (E)E spectra also contain a signal from tensor modes coming from reionization and last scattering. However, the dip represents a failure of the second-order slow-roll consistency relation for the tensor modes. The constraints shown by the blue contours (a power-law potential for the duration of inflation. The solid black line (corresponding to a linear potential) separates concave and convex potentials.}

The 95% limits on r0, these parameter constraints in the Planck TT+lowP plane, are model-dependent and extensions to many isolated vacua, connected by tunneling some mountain slopes drive inflation. The string theory landscape: string theory’s 6 compact dimensions: strings, branes & fluxes
D3’s

string theory’s 6 compact dimensions: 
strings, branes & fluxes

moduli & axions: light scalars ...

strings!

convex/concave

$n_s$

$r = T/S$

Planck TT+lowP+lensing+ext +BK14

$V \propto \phi$

$V \propto \phi^2$

$V \propto \phi^{2/3}$

cosmological CMB data

Planck Collaboration 2015 XII, ArXiv e-prints (2015), http://www.cosmos.esa.int/web/planck/pla
\[ M^2_P = \frac{1}{\alpha'} \]

\[ m^2_{KK}, m^2_{c.s.}, m^2_{\tau} \]

\[ m^2_{KK, warped}, m^2_{\nu}, m^2_\rho \]

\[ H^2, m^2_\phi \]

\[ SM + axions \]
Pole $N$-flation

One way of building inflation:

- Find a scalar field with kinetic term:
  \[ \mathcal{L}_{\text{kin.}} = \frac{1}{2} \left( \partial \phi \right)^2 \]

- Give an approximate shift symmetry from UV, dominantly broken by inflation scalar potential $V_0(\phi)$ itself:
  \[ \Delta \text{ corrections like } \frac{V_0''}{M_P^2}, \frac{V_0}{M_P^4} \]
  and with powers of shift away breaking couplings to other sectors $\partial_\mu \phi$, so better $\partial_\mu \phi \ll 1$

- Or, give UV theory with finite \( \Delta \) dim-6 corrections $\delta V(\phi)$, and fine-tune an inflation point

Another way, shift responsibility to a non-canonical kinetic term:

- Either $\mathcal{L}_{\text{kin.}}$ is higher-derivative, e.g. DBI, $V_0(\phi)$ can be steep here

- Or provide $\mathcal{L}_{\text{kin.}}$ with a 3rd-order pole in the inflation:

  \( i) \quad \mathcal{L}_{\text{kin.}} = \frac{1}{2 M_P^2} (\partial_\mu \phi)^2 \)
  \[ V = V_0 - \alpha, \phi + ... \]
  pole at $\phi = 0$

  \( ii) \quad \mathcal{L}_{\text{kin.}} = \frac{2}{(1-c^2 \phi^2)^2} (\partial_\mu \phi)^2 \)
  \[ V = \alpha \phi^2 + ... \]
\( \phi \) and \( X \) related by Cayley transform:

\[
\phi = \frac{1 - e^X}{1 + e^X}
\]

both various produce after canonical normalization:

\[
V(\phi) = V_0 - V_1 e^{-c\phi} + \delta V(\phi - 2\pi p)
\]

and all higher positive powers in \( \phi \) or \( X \) in \( V \) wash out...

\( X \)-attractor / pole inflation mechanism

2 crucial underlying assumptions:

i) really, it is:

\[
2\text{kin} = \left( \frac{\phi^2}{4^2} + \frac{\phi^4}{4^4} + \ldots \right) (\partial \phi)^2
\]

and assume: \( c_2 \gg c_p \) \( \forall p \geq 3 \)

\( \sim \) demand of approximate shift symmetry in different frame

ii) absence of \( \phi \)-powers of both signs in \( V(\phi) \):

\( \sim \) otherwise:

\[
V(\phi) = V_0 - a_1 \phi + \delta V_a \frac{1}{\phi} + \ldots
\]

\[
\rightarrow V(\phi) = V_0 - V_1 e^{-c\phi} + \delta V e^{c_p \phi}(\phi)
\]

\( V \)...

for \( V_1 \gg \delta V \)
idea: use moduli with

\[ K = -3 \alpha \cdot \ln \left( 1 - \sum_i \Phi_i \bar{\Phi}_i \right) \]

N-flation like structure inside

one log

\[ K = -\sum_i \alpha_i \ln \left( 1 - \Phi_i \bar{\Phi}_i \right) \]

has been studied by people but abstract

charge variables:

\[ \phi_i = \frac{R}{\sqrt{\lambda_i}} \Sigma_i (\psi_i) e^{i \Theta_i} \]

angles between each \( \phi_i \)'s

Canonical normalization:

\[ R = \tanh \left( \frac{\varphi}{\sqrt{\beta}} \right) \approx 1 - e^{-K \varphi} \equiv 1 - \varepsilon, \quad R = \sqrt{2} \]

\[ V = V_0 (\Theta_i, \psi_i) - V_1 (\Theta_i, \psi_i) e^{-K \varphi} + O(e^{-2K \varphi}) \]
$W$-fication case:  
\[ \sum \delta_i^2 = 1 \text{ and } \sum \delta_i^2 = \frac{\delta_i^2}{N} \]

Normalized angles $(\theta_1, \theta_2)$ at fixed $\theta$ to \((\delta_1, \delta_2)\):

\[ C_{\delta \theta} \sim \frac{3\delta^2}{N} \cdot \frac{N-1}{N^2} \delta \theta \text{ at large } \delta \]

$G_{ij}$ diagonal! axion!

\[ f_{a} \sim \frac{3\delta^2}{N^2} \quad (a = 1, \ldots, N-1), \quad f_{\theta} \sim \frac{3\delta^2}{N^2} \]

Assume, as tending to suppress $\theta_1, \theta_2$, dependence of $V_\theta$ has been done:

\[ \implies V_{\theta \theta} \sim V_{\theta j} \sim V_\theta \]

\[ \implies V_{\phi \phi} \sim V_\phi, \quad e = \frac{N}{N-1} \]

\[ V_{\phi \phi} \sim V_\phi \cdot N, \quad V_{\phi \phi} \sim V_\phi \cdot N \cdot e^2 \]

while:

\[ V_{\phi \phi} \sim V_{\phi \phi} \sim V_\phi \cdot e \]

so, $\phi$ much lighter than $\phi$ at $\phi \gg \text{MeV}$, while $V_{\phi}, \theta$

have similar mass to $\phi$.

$V$: same situation as

AtRARO, Kibble, Linde & Witten '84 have done for case of one such

complex field $\phi$.

$\Rightarrow$ discuss \(3D\) plots & mass spectrum graph
Figure 2. Mass spectrum of pole $N$-flation. The left-hand side shows the mass hierarchies of eq. (3.17) in the deep plateau limit, and large $N$ limit. The right-hand side shows the mass hierarchies when the proximity to the moduli boundary is constrained by weak gravity arguments. As discussed in §4, imposing consistent couplings to weak gravity limits $\frac{1}{pN}$; in this regime the field $\hat{\#}$ has generically a mass of order $H$. 

The fate of inflationary plateaus - weak gravity strikes back

Effective field theories arising from a theory of quantum gravity are constrained by consistency conditions such as the Weak Gravity Conjecture (WGC) or the Swampland Distance Conjecture (SDC). The SDC states that whenever one moves an infinite distance in moduli space, an infinite tower of states becomes massless causing the break of the effective description. While we will make the connection between the SDC and $\epsilon$-attractor models later in this section, for now we focus on enforcing consistency arguments coming from the weak gravity conjecture.

The WGC arose as a proposal to argue that black holes should always be able to evaporate via Hawking radiation, such that the final state of any charged black hole would be able to decay and leave no remnant. For this to happen it is necessary that any theory of quantum gravity has at least a fundamental object fulfilling certain condition on its charge to mass ratio, such that the decay process is possible for any black hole. A number of attempts were made to further constrain the particular object fulfilling the WGC condition. These have given rise to the multiple current versions of the WGC. It is still unknown which version of the conjecture (if any) is the right one and it is not our intention to provide any new insight in this direction, so we just refer the interested reader to [31–34, 50–74] for extensive discussion. Our purpose, instead, will be to apply the WGC constraints to the $\epsilon$-attractor models as a consistency requirement to allow for a string theory embedding of these effective field theories.

Since string theory contains fundamental charged objects of different dimension, first note that the above argument can be extended to other black objects of different dimensionality. This implies one is not completely free concerning the assignment of charges and tensions of the fundamental objects of any theory. For our purposes we will be interested – 10 –
can be written as

Appendix

and the inflationary dynamics is determined by their collective motion. In this maximally

were already noted for a real 2-disk

with $d_i$

the model makes valleys in the potential bundle-up. While $d_i$

Figure 1

minimized. The top-left plot shows the potential eq. $(3.13)$

Also in this regime, close to the boundary, the metric for the axionic fields

$A_{ij}$

is the all-ones matrix (a square matrix with all entries equal to 1). The eigenvalues

where

which ranges from 3

multifield case, this corresponds to the choice

$p_i$

Different radial directions have plateaus

$2 \gamma$

for any

$\mathcal{N}$

has the same signature for all initial conditions,

$p_r$

has rank one, with one single non-zero

$J_i$

$A_1 = 1$ and $A_2 = 1$

$A_1 = 1$ and $A_2 = 0.2$
\[ K = -2 \ln \left( \sqrt{\frac{1}{2} \tau_1 \tau_2} - \frac{\tau_s^3}{2} + \xi \right) \]

\[ W = W_0 + e^{-2\pi T_s} \]

\[ \mathcal{L}_{kin} = \frac{1}{4\tau_1^2} \left( \partial \tau_1 \right)^2 + \frac{1}{4\tau_2^2} \left( \partial \tau_2 \right)^2 \]

\[ T_j = \tau_j + ic_j \]

The 'swiss cheese Calabi-Yau' is an \( \alpha \)-attractor!
Fibre inflation

[Cicoli, Burgess & Quevedo ‘08]

\[ K = -2 \ln (\sqrt{\tau_1 \tau_2} - \tau_s^{3/2} + \xi) \]

\[ W = W_0 + e^{-2\pi T_s} \]

\[ \mathcal{L}_{\text{kin}} = \frac{1}{4\tau_1^2} (\partial \tau_1)^2 + \frac{1}{4\tau_2^2} (\partial \tau_2)^2 \]

flat in \( \tau_1 \) at tree-level - add string loops:

\[ \tau_1 = e^{-\kappa \varphi} \]

\[ V = V_0(\langle \mathcal{V} \rangle, \langle \tau_s \rangle) + \frac{1}{\langle \mathcal{V} \rangle^2} \left( 1 - e^{-\kappa \varphi} + \ldots \right) \]
**Pole N-flation in Fibred Calabi-Yau spaces**

\[
K = -2 \ln \left( \sqrt{\tau_1} (\tau_2 - R^2(\phi_i, \bar{\phi}_i)) - \tau_s^{3/2} + \xi \right)
\]

\[
= -\ln \tau_1 - 2 \ln \left( \langle \tau_2 \rangle - R^2(\phi_i, \bar{\phi}_i) \right) + \ldots
\]

**pole N-flation**

\[
K = -3\alpha \ln(1 - R^2)
\]

\(\phi_i\): e.g D3-brane positions — potential from instantons:

\[
W = W_0 + f(\phi_i)e^{-2\pi T_2}
\]
there are ... axions — many axions in this model:

\[
\frac{1}{4\tau_1^2} (\partial c_1)^2 \quad \frac{1}{2\tau_2^2} (\partial c_2)^2 \\
\frac{1}{N\epsilon} (\partial \theta_a)^2 \quad \frac{1}{N\epsilon^2} (\partial \psi)^2
\]

inflationary plateau — go to pole:

\[
\tau_i \to 0 \quad , \quad \epsilon \to 0
\]

\[
\Rightarrow \quad f^2 \rightarrow \infty
\]
... we’ve got a problem — there is something called:

Weak Gravity Conjecture (WGC)
weak gravity conjecture

• ‘gravity should be weak’ - so should have state in U(1) gauge theory:
\[
\frac{q}{m} > 1
\]

• reasons:
  - no global symms. in QG (no \( q \to 0 \) limit) …
  - extremal BHs should decay …

• varieties:
  - mild form: ’a’ state has \( q > m \)
  - strong form: lightest state has \( q > m \)
axions & WGC

- generalization to axions:

\[ \frac{m}{q} < M_P \rightarrow T_{p-1} < g_{A_p} M_P \rightarrow \]

\[ S_{\text{inst.}} < \frac{M_P}{f} \Rightarrow f < M_P \]

- constrains multi-axion (N-flation/alignment) models

- question:
  - is this the dominant instanton?
but we got many axions …

$$\prod_{a=1}^{N} U(1)_{a}$$

particles $i$ with charges $q_{ia}$ and masses $m_{i}$

$$\vec{q}_{i} = q_{ia}$$

$$\vec{z}_{i} = q_{ia} \frac{M_{P}}{m_{i}}$$

black hole: $\vec{Q}$, $M$, $\vec{Z} = \vec{Q} \frac{M_{P}}{M}$
extremal black hole must decay ...

\[ BH \rightarrow \tilde{\mathcal{Q}} = \sum_i n_i \tilde{q}_i \land M > \sum_i n_i m_i \]

\[ \tilde{Z} \text{ subunitary weighted average of } \tilde{z}_i \]

\[ \text{... convex hull of } z_i \text{ must contain unit ball!} \]
The quadratic divergence of $\mathcal{R}$ is large. If one of these new states satisfies the condition of the WGC is satisfied. How-ever, this is insu-bstantial. The WGC places a constraint on the charge-to-mass ratios, but after a time of order the pho-netic field theory to break down at a cut-off.

Consider a model of two Abelian factors and two symmetric groups and two subunitary weighted averages of these vectors.

**axion N-flation**

\[
\mathcal{L} = \frac{1}{2} \sum_i f^2 (\partial \theta_i^2) - \sum_i A^4 (1 - \cos \theta_i)
\]

\[
\theta_i = \theta \implies \tilde{\theta} = \sqrt{N} \theta, \quad \tilde{f} = \sqrt{N} f > 1 ?
\]

\[
\tilde{z}_1 = \left( \frac{1}{f}, 0 \right) \quad \tilde{z}_2 = \left( 0, \frac{1}{f} \right)
\]

\[
\Rightarrow \tilde{f} < 1
\]
axions & weak gravity conjecture

• many works:

[Cheung/Remmen; de la Fuente/Saraswat/Sundrum … ’14]

[Rudelius; Ibanez, Montero, Uranga, Valenzuela;
Brown, Cottrell, Shiu, Soler; Bachlechner, Long, McAllister;
Hebecker, Mangat, Rompineve, Witkowski; Junghans; Palti; Kaplan, Rattazzi;
Heidenreich, Reece, Rudelius; Kooner, Parameswaran, Zavala; Kaloper et al.; Harlow;
Hebecker, Rompineve, AW; … ’15]

[Conlon/Krippendorf … ’16]

… and more (lattice WGC, AdS/CFT …) !!

… discussion ongoing !!
Pole $N$-flation \quad $N = 3$

$V(\varphi)$

$\varphi [M_P]$

$f_a^2 \sim \frac{1}{N \epsilon} = \frac{1}{N} \frac{1}{1 - e^{-\kappa \varphi}}$

$f_\varphi^2 \sim \frac{1}{N \epsilon^2} = \frac{1}{N} \frac{1}{(1 - e^{-\kappa \varphi})^2}$
Pole N-flation \( N = 100 \)

\[ V(\varphi) \]

\[ \varphi [M_P] \]

\[ \frac{1}{f_\varphi} \]

\[ \frac{1}{f_a} \]
all forms of positive vacuum energy in string theory flatten below linearly adding up sources!

Muchas Gracias!
Funding acknowledgement:

This work is supported by the ERC Consolidator Grant STRINGFLATION under the HORIZON 2020 grant agreement no. 647995.