CrimAnalyzer: Understanding Crime Patterns in São Paulo City

Germain Garcia-Zanabria, Jaqueline Silveira, Jorge Poco Member, IEEE, Alfonso Paiva, Marcelo Batista Nery, Claudio T. Silva Member, IEEE, Sergio Adorno, Luis Gustavo Nonato Member, IEEE

Fig. 1. CrimAnalyzer system: the spatial and temporal interactive views enable the exploration of local regions while revealing their criminal patterns over time.

Abstract— São Paulo is the largest city in South America, with criminality rates as large as the city. The number and type of crimes varies considerably around the city, assuming different patterns depending on urban and social characteristics of each particular location. Previous works have mostly focused on the analysis of crimes with the intent of uncovering patterns associated to social factors, time seasonality, and urban routine activities. Therefore, those studies and tools are more global in the sense that they are not designed to investigate specific regions of the city such as particular neighborhoods, avenues or public areas. Enabling tools able to explore particular locations of the city is essential for domain experts to accomplish their analysis in a bottom-up fashion, more clearly revealing how local urban features related to mobility, passersby behavior, and presence of public infrastructures such as terminals of public transportation and schools, can influence the quantity and type of crimes. In this paper we present CrimAnalyzer, a visualization assisted analytic tool that allows users to analyze the behavior of crimes in specific regions of a city. The system allows users to identify local hotspots, their pattern of crimes, and how the hotspots and corresponding crime patterns change over time. CrimAnalyzer has been developed from the demand of a team of experts in criminology and deals with three major challenges in this context, i) flexibility to explore local regions and understand their crime patterns, ii) identification of spatial crime hotspots that might not be the most prevalent ones in terms of the number of crimes but are important enough to be investigated, and iii) understand the dynamic of crime patterns and types over time. The effectiveness and usefulness of the proposed system are demonstrated by qualitative and quantitative comparisons as well as case studies involving real data and run by domain experts. The experiments show the capability of CrimAnalyzer in identifying crime-related phenomena.

Index Terms—Crime Data, Spatio-Temporal Data, Visual Analytics, Non-Negative Matrix Factorization

1 INTRODUCTION

Since the mid-1970s, Brazilian society has experienced a known transition process from military dictatorship to democracy. With this political transition, it was expected that the conflicts would increasingly be solved, reducing the use of violence. That was not what happened. The transition was accompanied by a real explosion of conflicts, many of which associated with urban crimes. There is still no consensus among social scientists about the reasons that explain these trends in the evolution of crime and violence in Brazilian society, mainly in the big cities [1]. Among the explanations that arise more frequently is
the exhaustion of traditional security policies models. Concerning this last aspect, it is undeniable that crimes have not only grown, but also become more violent and modernized. In contrast, agencies in charge of law and order control – police and criminal justice system – did not follow these trends and has aged. The gap between the dynamics of crime and violence and the state’s ability to contain them within the rule of law has widened. Therefore, introducing modern instruments for the management of public order and crime containment is imperative to make public security policies more efficient, not only in São Paulo but also in any big city in under-development countries.

In this context, Crime Mapping, a branch of Geographic Information Systems (GIS) devoted to explaining spatio-temporal behavior of crimes, has emerged as a research field to support criminologists in their analytical process, leveraging the importance of local geography as a determinant of crime types and the rate they occur in a particular region [52]. The capability of identifying and visualizing crime hotspots and the ability to filter crime-related attributes to reveal particular information such as burglary in commercial areas or the seasonality of auto theft in certain neighborhoods are among the key components of a crime mapping approach [42]. Most existing tools developed in the context of crime mapping focused on the detection of hotspots, that is, areas with a high number of criminal incidents [15]. Although sophisticated methods have been proposed to detect hotspots [16], the search for a high prevalence of crimes ends up neglecting sites where certain types of crimes are frequent but not so intense to be considered statistically significant [55]. Moreover, most techniques enable only rudimentary mechanisms to analyze an important component of unlawful activities, the temporal evolution of crimes and their patterns. In fact, visualization resources for temporal analysis available in the majority of crime mapping systems are very restrictive, impairing users from performing elaborated queries and data exploration [3].

There is yet another important aspect to be considered in the context of crime mapping, the specificities of urban areas under analysis. São Paulo city, for example, bears one of the highest criminality rates in the world, at least one order of magnitude higher than cities such as New York and San Francisco, making glyph-based crime mapping solutions such as LexisNexis [1], NYC Crime Map [4] and CrimeMapping [3] completely unsuitable for analyzing crimes in São Paulo. Nevertheless, the pattern of crimes changes considerably around São Paulo city, even between regions that are geographically close to each other, demanding analytic solutions tailored to scrutinize local regions towards revealing their hotspots and crime patterns, while still being able to uncover their dynamics over time. Those capabilities are not currently available in most crime mapping solutions.

This work presents CrimAnalyzer, a new visualization assisted analytical tool customized to support the analysis of criminal activities in urban areas according to the characteristics of São Paulo city, that is, high criminality rates with great variability in the pattern of crimes, even in geographically close regions. CrimAnalyzer enables a number of linked visual tools tailored to reveal patterns of crimes and their evolution over time, assisting domain experts in their decision-making process and providing guidelines not only for repressive but above all preventive actions, strengthening the planning and implementation of institutional actions, especially from the police.

In collaboration with a team of domain experts, we have designed visual analytic functionalities that allow users to select and scrutinize regions of interest in terms of their hotspots, crime patterns, and temporal dynamics. Moreover, the proposed system enables resources for users to dig deeper in particular sites to understand its prevalent crimes and their behavior along time. The need for a methodology as implemented in CrimAnalyzer, which is able to operate and explore target areas, was one of the main demands raised by the domain experts. Moreover, CrimAnalyzer implements a methodology based on Non-negative Matrix Factorization [31] to identify hotspots based not only on the number of crimes but also on the rate they occur.

In summary, the main contributions of this work are:

- A new methodology to identify crime hotspots based not only on the number of crimes but also on their variation and recurrence rate.
- A visual analytics machinery that allows users to visually perform spatial and temporal queries towards understanding patterns and temporal dynamics of crimes.
- CrimAnalyzer, a visualization-assisted tool that integrates the analytical machinery in a set of linked views. CrimAnalyzer operates on target spatial regions to uncover relevant information of the region as a whole and also from its individual sites.
- A set of case studies revealing interesting phenomena about the dynamics of criminality in São Paulo city, supporting hypothesis and theories raised by domain experts and described in the literature.

2 Related Work

The literature about crime analysis is extensive, ranging from statistics and data science to visualization and GIS. Broadly speaking, crime analysis methods can be grouped into two major categories, geo-referenced and non-geo-referenced approaches. The latter, non-geo-referenced approaches, rely on mathematical and computational mechanisms such as data mining [12,60], optimization [55], machine learning [58,59], statistics [19], and data visualization [17,23,25,57,58], to identify crime patterns, criminal behavior, and also the consistency of criminal justice. In the following, though, we focus on geo-referenced techniques developed in the context of crime mapping, which are more closely related to the proposed approach. In order to better contextualize our methodology, we divide geo-referenced techniques in two groups, hotspot centered and spatio-temporal criminal pattern identification. It must be clear that there is a considerable overlap between those groups, meaning that a hotspot centered technique can also rely on spatio-temporal patterns to leverage its analysis, but the main focus of such technique is, in fact, hotspot identification.

Hotspot centered. Identifying crime hotspots is a major task in the context of crime mapping [8,15,16]. Although some works rely on Kriging [27], the most common approach for hotspot identification is a combination of Spatial Scan Statistics [29] and Kernel Density Estimation (KDE) [10], using point clouds, density map, or grid-based approaches as visualization resources [13,26,36,49]. As pointed out by Hart and Zandbergen [22], properly setting the parameters of a KDE is not easy and a loose choice of parameters can lead to erroneous or inaccurate results that overestimate or disregard hotspot locations [9]. Another issue with KDE based techniques is that locations present regular, but not intense, criminal activities are hardly pointed out as hotspots. To avoid the issues above, our approach relies on Non-negative Matrix Factorization to detect hotspots, thus avoiding parameter tuning while being able to “capture” locations difficult to be identified as hotspots by KDE based methods.

Spatio-temporal pattern identification. Besides crime hotspot identification, the analysis and visualization of temporal and spatial crime patterns are also of great importance in crime mapping [7,11,23,36,46]. Ratcliffe [45] and Townsley [51], for instance, incorporates aoristic analysis [43] in their hotspot visualization systems in order to identify important spatio-temporal patterns of crimes. Aoristic analysis takes into account the uncertainty of the exact moment that an event occurred when examining the overall incidence of crimes over time. Lukasczyk et al. [34] provide a topological perspective of the temporal evolution of hotspots based on a spatio-temporal Reeb graph built from a KDE mapping. Although interesting, techniques described above are still incipient in clearly revealing spatio-temporal crime patterns and their dynamics. Our approach, in contrast, combines a number of intuitive visual resources from which one can clearly identify crime patterns in specific locations and their temporal evolution.

There is a number of spatio-temporal techniques that rely on clustering methods to group spatially and/or temporally similar crime events in order to identify patterns. Those methods can be organized into two categories, the ones that build upon conventional clustering algorithms and the ones that rely on Self-Organizing Map (SOM) to identify pat-
terms. Clustering-based methods extract feature vectors from spatial and temporal crime attributes and cluster those attributes via k-means [2][20] or nearest neighbor clustering [27][33].

Techniques that rely on SOM have as main goal the identification of similarities among crime attributes. Chen et al. [11], in collaboration with the Tucson Police Department, proposed a spatio-temporal visualization system called COPLINK, which combines hyperbolic trees, GIS, and SOM in a unified analytical tool. Andrienko et al. [2] rely on a SOM matrix display to leverage a visual analytic framework to explore spatio-temporal similarities between the events. Hagenauer et al. [21] extended the previous approaches to explore the space-time evolution of the patterns in addition to their demographic and socio-economic attributes. In order to understand patterns between crime types, SOM has also been the basis for the spatio-temporal crime analysis system proposed by Guo and Wu [20], which builds upon a visualization infra-structure called VIS-STAMP [19] that integrates dimensionality reduction and parallel coordinates in the analysis of crime patterns. SOM has well known issues such as the proper setting of weights, number of nodes, and overfitting [30]. Moreover, SOM-based techniques described above do not integrate hotspot detection as part of the system, leaving aside an important component of analysis in the context of crime mapping.

3 Challenges, Data Set, and Analytical Tasks

In the last eighteen months, we have been interacting with two sociologists experts in social sciences with focus on criminal analysis. In particular, one of the sociologists is a researcher with a great reputation in the study of violence in South America. The other sociologist is an expert in public safety and social sciences applied to urban issues, with background in GIS and large experience in spatio-temporal analysis of crime. In partnership with the police department of São Paulo state, the team of experts built a data set (detailed in Sec. 3.2) containing seven years of criminal records in São Paulo city, and they invite us to develop a visual analytic tool to assist the understanding and analysis of the data.

Nomenclature. Before further detailing the problem, the requirements raised from the interaction with the domain experts, and the data set, lets first settle some nomenclature that will be employed throughout the manuscript.

– Site is the smallest territorial unity given in the spatial discretization. In our context, the sites are defined as the census units of São Paulo city, each containing from 250 to 350 residences and/or commercial establishments.

– Region is a set of sites, which can correspond to a whole neighborhood, a particular portion of a neighborhood, or even a group of sites adjacent to a street or avenue. The inset on the right shows an example of a region and its corresponding sites.

– Hotspots are sets of sites within a region with relevant criminal activity. The exact meaning of “relevant” will be clear when we present the data set. The reddish sites in the inset image correspond to hotspot sites in the given region.

– Crime type refers to the type of criminal activity, ranging from burglary to bodily injury (death, sexual, and drug-related crimes are not included in our study).

– Crime pattern accounts for the prevalence of a group of crime types in a given region or a particular set sites. In other words, if we say that the crime pattern in a set of sites is robbery, car theft, and commercial establishment attack, we mean that the three crime types are the most prevalent ones in those sites.

3.1 Problem Analysis

We run several rounds of meetings and interviews with the experts to figure out the main challenges involved in the analysis of crime data. After several interactions, we come up with the following issues:

- Analyzing the characteristics and dynamics of crimes in particular regions of the city. From their experience and interaction with officers from the police department, the experts conjecture that the type and dynamic of crimes have been changing over the years, mainly in particular regions of the city. Moreover, the type of crimes can change even in regions located close to each other depending on the urban characteristics of each region. The main difficulty to perform this analysis without a visualization assisted tool is to properly query the data set, which can be a time-consuming and exhaustive job. Many times a large number of images are generated as results, and the work of analyzing them becomes impossible. Moreover, highly prevalent crimes overshadow the presence of less frequent ones, which might also be of interest, demanding specific tools to enable a proper analysis. The difficulties, the experts have been performing their analysis focused on just one or two types of crime, considering the city as a whole or analyzing large areas that serve as administrative units within the city. Such “wide” analysis hampers the validation (or denial) of hypothesis and conjectures that have a local nature.

- Identifying crime hotspots within a particular region. The identification of crime hotspots is among the most important tasks when analyzing crimes and their dynamics. Hotspots are usually identified as locations that have a greater than the average number of criminal records [15]. However, criminal sites that are not so prevalent in terms of the number of criminal events, but bears criminal activities that deserve special analysis, tend not to be detected when a “frequentist” approach is employed to identify crime hotspots. Due to the lack of more sophisticated mechanisms, the number of criminal records has been the main mechanism employed by the experts in their identification of hotspots. Because of this, it was necessary to propose a new method for hotspot detection that meets the described restrictions. This requirement was emphasized by the domain experts.

- Understanding and comparing crime patterns. Domain experts believe that sites and hotspots within the same region can present different crime patterns. An issue in this context is to know whether the pattern of crime varies from a site (or hotspot) to another in the same region. In affirmative case, experts would like to understand how crime types are distributed an how they evolve along time. The experts were looking for a solution that would intuitively allow them to make such comparisons.

Challenges above points to a visual analytic solution endowed with functionalities to easily select regions of interest while enabling resources to assist the analysis of crime location, patterns, and temporal evolution. We followed a design process that involved the experts in most stages of the development [5], redesigning procedures, components, and functionalities according to experts feedback and demands.

3.2 São Paulo City Robbery, Burglary, and Larceny Data

The data set assembled by the experts consists of criminal records provided by the police department of São Paulo city. Only criminal acts as to robbery, burglary, and larceny were provided, leaving out murder, homicide, drug-related felony, and sexual assault. Each record contains the identification number of the census unit (site) where the crime happened, the type of crime, and the date and time of the crime.

The data set contains crime records from 2000 to 2006. In the very beginning of our studies we noticed that the information as to 2005 and 2006 was not consistent with previous years and a sanity check should be performed by the experts. Since the sanity check turned out more complex than expected, we opted to include only information from 2000 to 2004 in our studies, in total of 1,574,920 records.

Crime types range in 121 categories, and the 10% most frequent
crime types correspond to about 80% of the total crimes. The inset on the right shows the histogram of the 10% most prevalent crime types, labeling the three most frequent ones, *passerby robbery, auto theft,* and *passerby larceny.* To facilitate the analysis, experts split the original data in three independent categories, *vehicle robbery* (includes cars, motorcycles, trucks, etc.) with 295,081 instances, *larceny* in general, with 587,885 instances, and a third category with all the other types of robbery and burglary, with 691,954 instances.

Although the number of crime types are quite large, the crimes that domain experts are interested in are not that large, ranging from 3 to 5. Other crime types are sparse enough to be analyzed individually, do not demanding a sophisticated visualization tool to interpret them. Moreover, some crimes can be grouped into categories, an alternative suggested by the expert and incorporated into CrimAnalyzer. In other words, in each of the three sub-datasets, the experts ranked and grouped the crime types according to their importance.

### 3.3 Analytical Tasks

After identifying the main challenges faced by the experts and understanding how the data was structured, we conducted a new series of interviews to clearly raise questions to be investigated. It has become clear that the experts are interested in understanding the dynamics of crimes over the city by analyzing the variation of crime patterns over space and time. From the iterative processes with the experts we compiled the following list of analytical tasks:

- **Interactive selections (T1):** How can spatial regions of interest be interactively selected? Is it possible to make the interactive selection of regions flexible enough to pick from single spots to whole neighborhoods and particular avenues?
- **Crime patterns over the city (T2):** Which are the crime patterns in particular regions and sites?. How do criminal patterns change from the center to residential areas and outskirts? What about the patterns along the main avenues, streets, and highways?
- **Dynamic of crimes over time (T3):** How have crime types evolved, over time, in particular regions of the city?. More specifically, have crime patterns changed in particular regions over the years? Are crime types seasonal?
- **Crime patterns and hotspots over space (T4):** Which are the hotspots in a given region? Which are their crime patterns? How different (if the difference exists) are the crime patterns in distinct hotspots within the same region?
- **Crime patterns and hotspots over time (T5):** Have crime hotspots changed over time in a given region? Have crime patterns changed over time in a given hotspot?

As mentioned before, the lack of interactive mechanisms to select regions of interest combined with general-purpose analysis and visualization techniques have prevented domain experts from freely explore the data to verify hypotheses and conjectures. The first step to enable more powerful analytic resources is the design of a proper interactive selection tool, being this the goal of T1.

It has also become clear during the interviews that it is important to drill down from high-level summaries to individual analysis of sites and hotspots. Analyzing data in different scales is also essential to understand how patterns vary across space and time. For example, the pattern of crimes and hotspots can change throughout months and over different days of the week. This fact is related to T3, and demands particular data aggregation and filtering to be properly addressed.

Analytical tasks T2 and T3 are related to the problem of understanding the different patterns of crimes around the city and their evolution over time, as discussed in Sec. 3.1, while tasks T4 and T5 are associated to the problem of analyzing hotspots, also discussed in Sec. 3.1. To be properly tackled, those tasks demand specific mechanisms to detect hotspots and also visual resources to explore and understand them.

Among our goals is the integration of interactive selection methods and dedicated visual analysis tools towards allowing domain experts to accomplish both confirmatory and exploratory analysis. Moreover, some domain experts are not trained in computer science, thus the system should be as simple and intuitive as possible. However, simplicity and expressiveness must be balanced to render the system capable of supporting spatio-temporal analysis at different scales, while being able to uncover non-trivial hotspots and crime patterns.

### 3.4 The CrimAnalyzer System

Based on the raised requirements and analytical tasks, we have developed CrimAnalyzer, a system for exploring spatio-temporal crime data in particular locations. CrimAnalyzer enables simple, yet compelling, visual resources to query, filter and visualize crime data. The visual resources are supported by a mathematical and computational machinery tailored to extract and polish information so as to visually present it in an intuitive and meaningful way. The modules and system architecture are illustrated in Fig. 2. Users visually query the data set by interacting with a map and selecting a region of interest as well as by interacting with the different linked views that make up the system.

### 4 HOTSPOT IDENTIFICATION MODEL

As discussed in Sec 3, hotspot identification is one of the most important tasks for crime analysis. In our context, hotspots have a more general connotation than in previous works, corresponding to sites where criminal activity is high but also to locations where the number of crimes is not large, but frequent enough to deserve a detailed analysis. For example, a region can contain sites where the number of crimes is much larger than in any other sites, which clearly correspond to important hotspots. However, the region can also contain a particular site where crimes are frequent, but happening in much smaller magnitude if compared against the prominent ones. Moreover, the region can also contain sites where crimes are not frequent at all, but present spikes in particular time frames. If our context, we consider the three different phenomena as hotspots, seeking to identify sites where crimes are frequent and in large number, sites where crimes are frequent but do not in large number, and sites where crimes are not frequent, but happen in large numbers in particular time frames. The different crime behavior will be further discussed and illustrated in Sec. 4.

**Analysis of individual sites.** There are many alternatives to identify hotspots, as discussed in Sec 3. Although varying in terms of complexity, existing techniques typically rely on the comparison of statistical information to identify hotspots. Hotspots can be identified as particular points or as area units, depending on how the data is organized, delegating to the visualization the task of properly revealing the hotspots. The problem with this approach is that crimes happening in small magnitude or in isolated time frames tend not to be statistically significant, hardly being pointed out as hotspots.

Another issue is that several sites might be identified as hotspots, but their temporal relation remains unrelated. For example, sites can be timely correlated, meaning that crimes are committed in those sites in the same time slices. It makes sense to group timely correlated sites in a single hotspot, but computing hotspots individually and group them according to temporal matches is not easy and involves the use of thresholds to decide which sites should be grouped.

**Analysis of groups of sites.** Instead of analyzing sites individually, one can resort to techniques that directly identifies groups of sites as
hotspots. A straightforward alternative is to extract features from the sites and apply a clustering scheme to group similar sites in hotspots (see Sec. 2). However, the problem of extracting meaningful features that characterize sites spatially and temporally is quite involved, mainly due to the sparsity of the crime data. In fact, in the course of our development, we tried different alternatives to define spatio-temporal crime feature vectors, ranging from simple cumulative time windows to more sophisticated methods based on graph wavelet coefficients [33], but we could not obtain results that complied with our requirements.

To get around the difficulties pointed above, we opted to an approach based on Non-Negative Matrix Factorization [31], which worked pretty well in our context, identifying hotspots according to our requirements.

4.1 Non-Negative Matrix Factorization

Before presenting the details on how we have adapted NMF to operate in our context, let shortly review the main concepts and ideas involved in an NMF analysis. An $m \times n$ matrix $X$ is said non-negative if all entries in $X$ are greater or equal to zero ($X \geq 0$). The goal of NMF is to decompose $X$ as a product $W \cdot H$, where $W$ and $H$ are non-negative matrices with dimensions $m \times k$ and $k \times n$, respectively (the roles of $m, n$, and $k$ will be clear in Sec. 4.2). In mathematical terms, the problem can be stated as follows:

$$\arg\min_{W,H} \|X - WH\|^2 \quad \text{subject to} \quad W, H \geq 0$$

A solution for the minimization problem (Equation 1) provides a set of basis vector $w_i$, corresponding to the columns of $W$, and a set of coefficients $h_j$, corresponding to the columns of $H$, such that each column $x_j$ of $X$ is written as the linear combination $x_j = \sum_i h_j w_i$, (or $x_j = W h_j$). In other words, for each column in $X$ we have a corresponding column in $H$ whose entries are coefficients associated to the columns (basis vectors) of $W$. The matrix representation below (Equation 2) illustrates the relation between columns of $X$ and $H$ as well as coefficients in $H$ and basis vector in $W$.

$$\begin{bmatrix} x_1 & \ldots & x_j & \ldots & x_n \end{bmatrix} = \begin{bmatrix} w_1 & w_2 & \cdots & w_k \end{bmatrix} \begin{bmatrix} h_{1j} & h_{2j} & \ldots & h_{kj} \end{bmatrix}$$

There are two important aspects in an NMF decomposition that will be largely exploited in the context of hotspot detection, namely, low rank approximation and sparsity. Low rank approximation accounts for the fact that the basis matrix $W$ usually has much lower rank than the original matrix $X$, meaning that (the columns of) $X$ is represented using just a few basis vectors. As detailed in the next subsection, we rely on low rank approximation to define the number of hotspots, that is, by setting the rank of $W$ we also set the number of hotspots. Sparsity means the basis and coefficient matrices contain many entries equal (or close) to zero, which naturally enforces only relevant information from $X$ to be kept in $W$ and $H$. This fact is important to identify particular sites within a hotspot and the time slices where each hotspot shows up.

4.2 Identifying Hotspots with NMF

We rely on NMF to identify hotspots, their rate of occurrence and “intensity”. The matrix $X$ to be decomposed as the product $W \cdot H$ comprises crime information in a particular region of interest. Specifically, each row in $X$ corresponds to a site of the region and each column to a time slice. In order to facilitate discussion, we present the proposed approach using a synthetic example. Fig. 3(a) shows a region made up of 25 sites and we generated synthetic crime data in 60 time slices, representing months over five years. For sites denoted as $A$ and $B$, we draw 60 samples from a normal distribution with mean 8 and variance 4, ensuring that $A$ and $B$ are correlated, that is, when the number of crimes in $A$ is large the same happens with $B$ (the number of crimes in $B$ is generated by perturbing the values of $A$ using a uniform random distribution with values between $-3$ and 3). This construction is simulating two regions with high prevalence of crimes over time. Crimes in the site denoted as $C$ in Fig. 3(a) follows a normal distribution with mean 1 and variance 4, corresponding to a location where crimes are not large in number, but happening quite frequently. Finally, for site $D$ we draw 60 samples from a normal distribution with mean 0 and variance 0.25, except for time slices 35 and 47, where we set the number of crimes equal to 15 and 10 respectively, simulating a site where crimes are no frequent, but happen in large numbers in particular time slices. For all the other sites we associated 60 samples drawn from a normal distribution with mean 0 and variance 0.25. Values for all sites are rounded to the closest integer and negative values set to zero.

Fig. 3(b) illustrates the matrix $X$ built from the synthetic data described above. Notice that the simulated crime dynamics is clearly seen in $X$.

Given an $m \times n$ matrix $X \geq 0$, an NMF decomposition of $X$ results in matrices $W \geq 0$ and $H \geq 0$. In practice, the rank of $W$ is significantly less than both $m$ and $n$, i.e., $k = \text{rank}(W) \ll m, n$. In our context, the columns of $W$ correspond to hotspots while entries in the rows of $H$ indicate the “intensity” of the hotspot in each time slice. Fig. 3(c) illustrates matrices $W$ and $H$ obtained from matrix $X$ in Fig. 3(b) using a NMF decomposition with rank $k = 3$. Notice that the entries in first (leftmost) column of $W$ have values close to zero almost everywhere, except in the entries corresponding to the sites $A$ and $B$. Therefore, the hotspot derived from the first column of $W$ highlights sites $A$ and $B$ as the relevant ones. The high prevalence of crimes on those regions can clearly be seen from the first (top) row of matrix $H$, which has most of its entries with non-zero values. The second column of $W$ is mostly null, except in the entry corresponding to site $D$, where crimes are not frequent but happen with high intensity in particular time slices. Notice that the second row of $H$ has basically two entries different from zero, corresponding exactly to the time slices 35 and 47, when the site $D$ faces a large number of crimes. Finally, the last column of $W$ gives rise to a hotspot that highlights site $C$, where crimes are frequent, but in smaller magnitude when compared to $A$ and $B$. The incidence and intensity of crimes in $C$ are clearly seen in the third (bottom) row of $H$.

One can argue that the results presented in Fig. 3(c) worked so well because we wisely set the rank of $W$ equal $k = 3$ and that in practice it is difficult to find a proper value for the rank. To answer this question, Fig. 3(d) shows the result of factorizing matrix $X$ setting the rank of $W$ equal $k = 5$. Notice that the main difference between the rank $k = 3$ and rank $k = 5$ factorizations is that the first column of $W$ in Fig. 3(c) was split into two columns in the rank $k = 5$ factorization, giving rise to
Among the different versions of NMF, we opt to the sparse non-negative low p-values large z-scores. Local Indicator of Spatial Association (SSI), a well known binary data classification sim-
ges of the centroid of the census units. Since there are about 30 census units, setting the number of clusters equal to 300 tends to gen-
the ones obtained by k = 30, but slightly better for k = 3, . . . , 10, getting an average SSI greater than 0.98 for k = 3, . . . , 5, but slightly better for k = 3. This result motivated us to set k = 3 as the default value in CrimAnalyzer.

5 VISUAL DESIGN
This section describes the visual components of CrimAnalyzer. Fig. 1 illustrates the web-based system, which comprises a Control Menu (a) six interactive views (b-g), and a filter widget (h). Table 1 shows the properties of each view. For instance, Map View shows the space facet and Ranking Type View the temporal and crime type facets. Columns under filter’s category show how to interact with each view. Some views allow to constraint space, time and crime types.

The design of visual resources was driven in order to satisfy the analytical tasks described in Sec. 3.3. In Table 1 Columns under task’s category indicate the relation of each view and the analytical tasks described. For instance, Ranking Type View and Radial Type View satisfy all analytical tasks.
When a time or type filter is activated by other views, our choropleth map is recalculated using the filtered data.

### Map View

The control menu has three options: dataset, time discretization (i.e., months or days) and the number of hotspots (rank of the NMF decomposition). As shown in Fig. 1(a) we are using the dataset “Roubo” with monthly discretization and three hotspots (default) in most of our analysis. The NMF decomposition is performed by the Python library called Nimfa [61], which enables resources to evaluate and automatically choose the rank value $k$. However, such an automated process is computationally costly, impairing its use in an interactive visual analytic application such as CrimeAnalyzer. Therefore, we have opted to allow the user manually set $k$.

### Hotspots View

An important component of our approach is the hotspots identification. In Sec. 4.2, we explained how Non-Negative Matrix Factorization has been used to reveal hotspots. In this view, we use multiple maps to represent the spatial distribution of each hotspot. Users can specify a region of interest by 1) clicking on the map (to select a site), 2) drawing a polyline (to select avenues or streets for example), 3) drawing a polygon (to select a whole neighborhood), or 4) provide the address of a location. Drawings can be expanded to comprise other sites in the neighborhood. Finally, CrimeAnalyzer defines the region by computing the sites that intersect the drawing. In Fig. 1(b) we can see how a region is represented.

#### Site selection:
During the exploration, when a region has already been defined, this view might be used for spatial filtering (e.g., to focus in a particular site). This operation is performed by clicking a site, which is highlighted by mapping a texture to the corresponding area.

#### Filtering:
When other views make an spatial filtering (i.e., selecting a site), the corresponding site is highlighted by changing its texture. When a time or type filter is activated by other views, our choropleth map is recalculated using the filtered data.

### 5.2 Map View

This is the first component used by users to start the analytical process, enabling resources to define the region of interest. This view is comprised of a geographical map and a choropleth map to encode the number of crimes committed at each site in the region. Also, users can perform interaction to zoom and pan the map.

#### Region selection:
Users can define a region of interest by 1) clicking on the map (to select a site), 2) drawing a polyline (to select avenues or streets for example), 3) drawing a polygon (to select a whole neighborhood), or 4) provide the address of a location. Drawings can be expanded to comprise other sites in the neighborhood. Finally, CrimeAnalyzer defines the region by computing the sites that intersect the drawing. In Fig. 1(b) we can see how a region is represented.

#### Site selection:
During the exploration, when a region has already been defined, this view might be used for spatial filtering (e.g., to focus in a particular site). This operation is performed by clicking a site, which is highlighted by mapping a texture to the corresponding area.

#### Filtering:
When other views make an spatial filtering (i.e., selecting a site), the corresponding site is highlighted by changing its texture. When a time or type filter is activated by other views, our choropleth map is recalculated using the filtered data.

### 5.3 Hotspots View

An important component of our approach is the hotspots identification. In Sec. 4.2, we explained how Non-Negative Matrix Factorization has been used to reveal hotspots. In this view, we use multiple maps to represent the spatial distribution of each hotspot. Users can specify a region of interest by 1) clicking on the map (to select a site), 2) drawing a polyline (to select avenues or streets for example), 3) drawing a polygon (to select a whole neighborhood), or 4) provide the address of a location. Drawings can be expanded to comprise other sites in the neighborhood. Finally, CrimeAnalyzer defines the region by computing the sites that intersect the drawing. In Fig. 1(b) we can see how a region is represented.

#### Site selection:
During the exploration, when a region has already been defined, this view might be used for spatial filtering (e.g., to focus in a particular site). This operation is performed by clicking a site, which is highlighted by mapping a texture to the corresponding area.

#### Filtering:
When other views make an spatial filtering (i.e., selecting a site), the corresponding site is highlighted by changing its texture. When a time or type filter is activated by other views, our choropleth map is recalculated using the filtered data.
more important, the combination of all of them allows multiple analysis simultaneously, revealing interesting crime patterns as shown in the Case Studies (Sec. 6).

6 Case Studies

This section presents three case studies that show the effectiveness of CrimAnalyzer in addressing the analytical tasks presented in Sec. 3.2. The first case study addresses analytical tasks T1, T2, and T3, while the second is focused on hotspot analysis and related to T4 and T5. The third case study is aiming to make a parallel between criminal activity in São Paulo city and some crime related phenomena reported in the literature and related to T3. In all case studies, except if explicitly stated, we used the robbery and burglary chunk of the dataset as described in Sec. 3.2, with a monthly discretization.

6.1 Comparing Crime Patterns over the City (T1, T2, T3)

The goal of this case study is to analyze pattern of crimes in different regions of the city in order to understand how they change according to urban characteristics. Moreover, we also investigate the temporal evolution of crime patterns in different regions.

To perform the study we selected four areas in São Paulo, two in the center of the city, denoted as C1 and C2 in Fig. 6, and two in residential areas, pointed as R1 and R2 in Fig. 6. C1 is a financial district, hosting the headquarter of important banks and financial institutions, while C2 is a commercial area with many stores, an important metro terminal, and also several touristic places. Both C1 and C2 have a huge flow of people during the whole year. Residential areas R1 and R2 differ in terms of the economic level of residents, R1 is a middle class neighborhood while R2 is a richer area, with luxurious buildings and houses.

Fig. 6 (bottom right) depicts region C1, selected by drawing a polylines along the main avenue of the financial district (T1), and highlights the radial type view (C1-c) of the three most prevalent crime types of two sites in C1 (indicated by the arrows). The ranking type view (C1-d) on the bottom shows how the incidence of the five most frequent crimes varies along the time. Two crime types lead the ranking along the years (the beige and pinkish curves on top), passerby robbery and auto burglary. By analyzing the radial type view (C1-b and C1-c) of the highlighted sites, one can notice that those two crime types are indeed the prevalent ones in those regions (encoded by the color). Inspecting other sites by simply clicking on them on the map view, we concluded that passerby robbery and auto burglary are the prevalent crime types in almost all sites in C1.

Performing the same analysis in region C2 (top right in Fig. 6), which was selected by clicking and expanding the central site of the region (the brownish one), we observe a different behavior. The ranking type view (C2-d) shows that there is one crime type that has been grown over the years (green curve), cargo theft. Selecting cargo theft from the radial type view in the CrimAnalyzer interface (Fig. 1(h)), the map view (Fig. 1(b)) reveals that cargo theft is not prevalent in the whole region, but it is concentrated in just a few sites, being the dark brown site in the center of the region. Notice that cargo theft became the third most common crime type in that region over time, being behind only of passerby robbery and document theft. Other sites present a more uniform behavior, having passerby robbery, auto burglary, and commercial establishment burglary as the main crime types.

Moving from the city center to more residential areas, the analysis reveals a substantial change in crime patterns, as one can observe on the left of Fig. 6, where the crime pattern in R1 and R2 is summarized. In the residential region R1, for example (top left in Fig. 6), passerby robbery remains the most common crime type, followed by document theft. However, some sites in R1 have bus robbery (passengers and/or drivers of public bus service are robbed) as the second most common criminal activity (R1-d). The orange site pointed out by the top arrow is an example (R1-b). Site-by-site crime pattern analysis is easy to perform with CrimAnalyzer in this case, since the number of sites is mild and users need only to select the site on the map to make its crime pattern revealed. The importance of bus robbery in R1 is easily noticed in the ranking type view (R1-d) depicted on the bottom, where the blue curve (bus robbery) reaches high rank levels in several opportunities.

Similarly to what happen in C1, C2, and R1 (and also in most of the city), region R2 (bottom left in Fig. 6) does have passerby robbery as the predominant crime type, what can clearly be seen from the ranking type view (R2-d). However, crime patterns vary considerably among the sites and some of them do not even have passerby robbery as the prevalent crime, as the two highlighted sites, which have passerby robbery as second in importance (R2-c and R2-d). Moreover, home burglary is the most relevant crime in one of those regions. In fact, home burglary is a relevant crime in R2 as a whole, as indicated by the reddish curve in the ranking type view (R2-d). Notice that home burglary has increased in importance over the years.
The discussion above shows that the visual analytic functionalities implemented in CrimAnalyzer are able to sort out analytical tasks T1, T2, and T3 in a simple, intuitive, and effective way. The flexibility to handle spatially complex neighborhoods with different shapes allows users to scrutinize set of blocks as well as regions along avenues and streets (analytical task T1). The combination of the ranking type view and the radial type view allows users to understand crime pattern in each region and also in particular sites, making evident how crime patterns change around the city and even from site to site in a particular region (analytical task T2), a task difficult to be performed with direct searches in the crime database. In particular, ranking type view and radial type view turn out quite effective in revealing the temporal behavior of crime patterns, making clear that patterns have changed along the years (analytical task T3). With non-visual resources, this analysis could be an arduous process, demanding the implementation of multiple filters and sophisticated numerical and computational tools.

6.2 Hotspot Analysis and Cargo Theft (T1, T4, T5)

This case study has been driven by the domain experts, they were interested on a particular type of crime, cargo theft. Although cargo theft does not figure among the most prominent crime types in São Paulo city, it is of great interest due to its spatial characteristics, the high values involved, and the engagement of violent gangs in this type of criminal activity. It is well known that robbery (or theft) of high valuable cargo commodities tends to happen close to the main highways connecting São Paulo to other regions of Brazil. Therefore, domain experts focused their analysis in two important highways, SP230, which connects São Paulo to states in the South of Brazil, and BR116, which connects São Paulo to Rio de Janeiro.

In order to perform their analysis, domain experts relied on the polyline selection tool to select a considerable number of sites along the highways and avenues that connect the city to the highways. The number of regions involved in these analysis renders a site-by-site investigation tedious, making hotspots a better alternative. Fig. 7 shows three hotspots obtained from the regions selected along BR116 and SP230 and nearby avenues. The highways are highlighted in red and the nearby avenues in blue in the hotspot maps depicted in Fig. 7. The ranking type view reveals the crime patterns in each hotspot (considering only the 5 most relevant crime types). Notice that in BR116, cargo theft figures among the most relevant crimes (green lines), becoming the second most relevant crime in several moments. In SP230, though, cargo theft is not predominant, not appearing among the five most relevant crimes in the ranking type view in any hotspot. In SP230, the predominant pattern is passersby robbery, vehicle burglary, and commercial establishment burglary. CrimAnalyzer makes clear which sites are relevant in each hotspot, their crime patterns, and how crime patterns evolve, thus properly tackling analytical tasks T4 and T5.

However, the experts are particularly interested in cargo theft. To center the analysis in a single crime type users only need to select that crime type in the radial type view, filtering the data such that hotspots and all the views are updated to depict only information related to the selected crime type. Fig. 8 shows the hotspots associated to cargo theft only. The gauge widgets show that the number of cargo theft in the BR116 is one order of magnitude larger than in SP230, also presenting a higher rate of occurrence. The temporal evolution (radial type view) on the center right of each grid shows the temporal behavior of cargo theft in each hotspot. It is clear that the number of cargo theft in SP230 has lessened over the years, while in BR116 no reduction is observed. The histograms below the gauge widget show the intensity of cargo theft (the short dark bars) in each month, comparing them against the total number of crimes in the region.

Another interesting aspect pointed out by the experts is the capability of revealing hotspots associated with sparse criminal activities, as the one depicted in Fig. 8(b) (see the spikes in the radial view). Sparse hotspots are relevant and deserve to be investigated. Notice that these finds could hardly be made without the visual resources enables by CrimAnalyzer.

6.3 Seasonality and the Temporal Element of Crime (T3)

This case study corroborates if some criminal behaviors described and validated in previous works also take place in São Paulo. Seasonality An important aspect related to criminal activities is seasonality. There is a number of studies in the literature that support the hypothesis that certain crime types are seasonal while others are not. For instance, van Koppen and Jansen [54] argue that, in Netherlands, commercial establishment burglary (robbery) increases during the winter due to the increased number of dark hours during the day. In South America, winter usually starts in mid June and last until mid September, during this period, especially in July and August, the number of dark hours is higher than in the rest of the year. An interesting question related to analytical task T3 is whether the findings of van Koppen and Jansen is valid in São Paulo city. To search an answer, we relied on CrimAnalyzer to explore six major commercial areas in São Paulo city, three commercial districts and three popular commercial streets. Fig. 9 shows the cumulative temporal view of each of the analyzed regions. The overlaid darker histograms correspond to the number of commercial establishment burglary and robbery in each month. The overlaid histogram is generated by simply selecting
After playing with CrimAnalyzer and run a number of experiments, which varies in terms of the frequency of crimes and the number of

Near Repeat Victimization
Near repeat victimization theory claims that when a home is burgled, the risk of recidivism in a short period of time is not only higher

The experts were quite enthusiastic about CrimAnalyzer, as it allowed them to understand and create new hypotheses about the behavior of the crime type cargo theft. Specifically, one mentioned “With the use of this tool, we can put in the hands of the decision takers clear evidence of how public policies should be defined with respect to car thefts”.

8 Discussion and Limitations
CrimAnalyzer was developed in close cooperation with domain experts. The current version satisfies their the requirements, however, some limitations and future work leaped out of this collaboration.

NMIF stability. Our approach for identifying hotspots is not stable, this is because the Non-Negative Matrix Factorization technique depends on the initial conditions of the optimization procedure. To counteract this effect, some implementations, like the one we are using in the presented system, enables us to run the method a number of times, keeping the solution with the smallest error. Although the results got quite stable after enabling the multiple run alternative, a more robust approach could be sought to mitigate possible effects.

Space Discretization. The space discretization used in CrimAnalyzer is the census units in São Paulo city, we adopted this measure because our collaborators had interest in seeing the analysis in this level of detail. However, we are aware of the modifiable areal unit problem (MAUP), census units do not represent “natural units” of analysis and the result of certain analysis can change by modifying the aggregation unit. An immediate future work would be to extend and make more flexible our space discretization. In this way, we should be able to apply our tool in other contexts.

Multiple data sources. Crime events by their own rarely tell the whole story. Additional data that can be used to enhance the understanding of the crime layer. For example, the presence of bars and pubs, distance to parks, vacant land and buildings, weather, among other information might have a relation with certain criminal activities. Given the increasing number of initiatives to make data publicly available, we are considering to combine that information to further understanding crimes in urban areas. An interesting mathematical tool in this context is tensor decomposition, a generalization of matrix decomposition able to extract patterns from multiple data sources. Developing visual analytical tools to map tensor decomposition information into visual content is an important problem that has barely been approached in the context of crime analysis.

Global vs Local approach. CrimAnalyzer uses a local-based approach to explore and analyze crime patterns. Even though this was a requirement from the domain experts, and we agree that it was the correct approach to this problem, mainly because domain experts have prior knowledge and hypothesis regarding crime behaviors in particular locations, in some of our interviews with domain experts we discussed the option of having a global-based technique that might process the whole space and propose interesting locations to be explored.
alternative was accepted by the experts but as a complementary technique. As future work, we are also interested in tackling this problem from both perspectives (global and local).

**Multiple cities and different scenarios** Finally, we intend to apply and validate our system in other cities and countries. Currently, we are in the process of collecting crime data from multiple locations, and in a short time, we expect to release the system to analyze multiples cities in Brazil. In addition, our approach can be extended to other scenarios than crime analysis. For instance, one can use the system to analyze the dynamics of traffic-accidents in particular locations of the city, making possible to uncover how the number of car-car crashes, car-bus crashes, run overs, etc. evolve over time.

**9 Conclusion**

We introduced a visualization assisted analytic tool to support the analysis of crimes in local regions. We developed CrimAnalyzer in close collaboration with domain experts and translated their analytical into the visualization system. We also propose a technique based on NMF to identify hotspots. Our system was validated by qualitative and quantitative comparisons, and case studies using real data and with feedback from the domain experts. Moreover, we verified two crime behavior facts (i.e., seasonality and near repeat victimization) using São Paulo crime data.

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