Non Trivial Effects in Heavy Nuclear Systems
due to Charge Rearrangement

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In this report we propose that a charge rearrangement within heavy ions may occur during fusion reactions, lowering the Coulomb barrier as the nuclear systems approach each other. In this situation a modified Gamow factor has been calculated, which emphasizes the importance of spatial degrees of freedom in nuclear matter.

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I. INTRODUCTION

It is commonly believed that the point-like approximation for the Coulomb potential is accurate enough when describing the nuclear dynamics during a fusion interaction [1,2]. In this context, there are a number of anomalies observed in fusion reactions near the Coulomb barrier [4,5], and a number of anomalies in the abundances of p-elements produced in supernovas [3], specially the production of rare nuclides, that do not fit well in our present understanding of supernova and nuclear dynamics.

In this work, we will consider the effect of a spatial redistribution of charge during a nuclear interaction, which may add an extra ingredient when considering the anomalies observed in fusion reactions near the Coulomb barrier [4,5]. The key idea in this model is to take each nuclei as a plasma ball (inspired in a quark-gluon plasma [6], although no calculation presented here has any relation to that specific subject), that has the ability to redistribute some of the charge it contains in the presence of the field produced by the second nuclei. As long as distances between the bodies are large enough, we can disregard the strong interaction between them, and consider only the coulomb interaction. It is important to notice that in this context the strong force plays a role within each body, making the transport of charge within the nuclei possible. Even though the dynamics of our model is developed in a quantum mechanical context, it does not consider spin effects, nor other non classical effects apart from those described by the scalar Schrödinger equation.

Our main result is that the redistribution of charge inside each ball lowers the Coulomb barrier while ions approach each other. The modification is expressed through the corrected differential cross sections and the Gamow factor, which is of special interest in astrophysical scenarios (e.g., reaction networks in supernova [6]).

II. THE MODEL

Charge rearrangement in the presence of an external electric potential is common in nature. For example, charge move within conductors in response to external electric fields, as described by distance dependent image charges.

In the case of nuclear reactions, there are a number of ways in which we can expect to obtain a charge rearrangement within a nuclei, in the presence of the field produced by a second nuclei. For example we can imagine (1) a collective excitation of the nuclei, that is a fluctuation of nuclear matter around its equilibrium state; (2) a fluctuation involving only the surface, with an unperurbed bulk, of a proton rich nuclei; (3) a local interaction affecting an unpaired proton or neutron; (4) etc. In all of these cases the specific description will be strongly dependent on the system under consideration.

For this paper, and simplicity, lets assume that this charge rearrangement inside the nuclei can be modeled, in a first approximation, as an induced electric dipole. More exact calculations can also be considered, such as higher order moments, but for simplicity will be reported elsewhere. We may imagine many others ways to induce an electric dipole moment in a nuclei, all of which have their own lifetime. Of interest is the case that involves time-reversal violation [8], but produces an effect that seems to be weaker than those coming from charge rearrangement [9]. It is important to note, that the rearrangement of charge will depend on the position and charge of the second nuclei.

For the problem at hand, we can write the electric dipole operator as

$$D = e \frac{N}{A} \sum_{p=1}^{Z} x_p - e \frac{Z}{A} \sum_{n=1}^{N} x_n,$$

where $e$ is the unit charge, $A$ is the mass number, $Z$ is the atomic number, and $N$ the neutron number. The operator $D$ has a well defined expectation value during the lifetime of the charge rearrangement, $p = (D)$. Since the strength of any multipolar field is proportional to the size of the polarized object [10], we will consider interactions of type $A + p$, where $A$ is a large ion and $p$ is a proton. We will neglect a possible dipole moment for the proton.
Suppose now that the incoming proton induce a charge rearrangement in the ion, observed externally as a dipole moment, which can be visualized as a pair of charges ±$q_e$ displaced a distance $d$ away from the ion center. The orientation of the dipole is induced in such a way as to reduce the repulsion interaction, for a given separation $r$ between the ion and the proton. Therefore, in the limit $q_e \rightarrow 2 q_e d = p_z$, we can approximate the potential energy as

$$U(r) = \frac{Z_1 Z_2 e^2}{r} - \frac{p_z Z_2 e}{r^2},$$

where $p_z \approx |p|$, and the minus sign comes from the discussion above. We are interested in the potential close to the origin, and in the limit $d \ll r$. Since, the rearrangement of charge, as described by $p_z$, will depend on the relative position of the second nucleus, we are interested in situations for which $p_z$ is finite as $r \rightarrow 0$. This definition is consistent with our above assumptions.

Due to the symmetry of the problem we choose parabolic coordinates: $\xi = r - z$, $\eta = r + z$, $\varphi = \arctan(y/x)$, and seek a solution of the form

$$\psi = e^{ikz} \phi(\xi),$$

where the function $\phi$ corresponds to a spherical outgoing wave. Operating on the scalar Schrödinger equation, we obtain

$$\left[\xi \phi''(\xi) + (1 - i k \xi) \phi'(\xi) - \left(\frac{\kappa^2 - \beta^2}{\xi}\right) \phi(\xi)\right] = 0,$$

$$\kappa = \frac{Z_1 Z_2 \alpha c}{v}, \quad \beta = \frac{2}{\hbar} \frac{\sqrt{\mu p_z Z_2 e}}{\lambda}, \quad \lambda = \frac{1}{1 - \cos \theta},$$

where $k$ is the wave number, $\mu$ the reduced mass, $c$ is the speed of light, $v$ is the speed of the projectile, $\alpha$ is the fine structure constant, and $\theta$ is the azimuthal outgoing angle for the projectile (this must be small to satisfy $p_z \approx |p|$). This angle has meaning only for an elastic interaction.

This equation has two linearly independent solutions that are regular at the origin, one for each possible sign of $\beta$, so the most general solution is a linear combination of both

$$\phi(\xi) = C_+ \xi^{i\beta} F_1(-i \kappa + i \beta, 1 + 2 i \beta, i k \xi) + C_- \xi^{-i\beta} F_1(-i \kappa - i \beta, 1 - 2 i \beta, i k \xi),$$

where $F_1$ is the hypergeometric confluent function, and $C_+$ are the two constants of integration. Now we want to calculate the Gamow factor, which is by definition the quotient between the probabilities of finding close together particles with the same and opposite sign. In order to obtain a value for the constants, we normalize the flux asymptotically far and make $C_+ = C_- = 1$ for simplicity. In this way we can define the following function

$$\rho(\kappa, \beta) = \frac{2 \beta \sinh \pi(\kappa + \beta)}{(\kappa + \beta) \sinh 2 \pi \beta} e^{-\beta \pi} + \frac{2 \beta \sinh \pi(\kappa - \beta)}{(\kappa - \beta) \sinh 2 \pi \beta} e^{\beta \pi}$$

$$+ \frac{4 \beta}{\sinh 2 \pi \beta} \sqrt{\frac{\sinh \pi(\kappa - \beta)}{\sinh \pi(\kappa + \beta)}} \frac{\sinh \pi(\kappa + \beta)}{\kappa^2 - \beta^2} \cos \theta,$$

where the function $\arg$ is the argument of the complex number $(z = re^{i\theta}, \arg z = \theta)$. From this expression we can write the corrected Gamow factor

$$G_f = \left| \frac{\rho(\kappa, -i\beta)}{\rho(\kappa, +i\beta)} \right|,$$

where $\kappa \geq 0$ and $|\kappa| \gg 1$. Note that change from $\beta \rightarrow i\beta$ due to the change in the orientation of the dipole. To check this expression we take the limit $\beta \rightarrow 0$, corresponding to no rearrangement of charges, and we get

$$\lim_{\beta \rightarrow 0} G_f = e^{-2 \pi \frac{Z_1 Z_2 e^2}{\kappa}},$$

that is, we recover the usual expression. In this way we have calculated a corrected Gamow factor due to charge rearrangement within the target nuclei.

**III. ANALYSIS AND RESULTS**

With the aid of this simple model we can estimate in a first approximation how the rearrangement of charge in ions may modify the fusion process. As an example consider Fig. 1, in which we plot the Gamow window that is of interest in astrophysics processes. Here we make an estimation for a reaction $^{177}_{72}Hf + p$. In particular, this heavy ion has an E1 transition between levels with $J^\pi$: 7/2$^-$ and 9/2$^+$, energy $E_e = 214.3$(keV), and half life 1.86(ns). We take the ion in this excited state, and in order to make a simple and rough estimation, we consider a semi-classical calculation starting from eq. $\Pi$, which can be re-casted in the form

$$p_z = e^{N Z A d},$$

where $d$ is the distance between the center of mass for proton and neutron in the heavy ion. Now we take the energy associated to this separation $d$ equal to $E_e$, as the work done in a Yukawa type potential. This estimation give us $p_z = 0.122(e\text{-fm})$, which is not too far from experimental measurements in similar ions. Now we need to estimate the azimuthal outgoing angle $\theta$, and in the same spirit, we use the Rutherford relation between impact parameter $s$ and $\theta$.
where \( E \) is the energy of the incident particle. Taking \( E = kT \), and for \( s \) the diameter of the proton in order to maintain \( \theta \) small, we have that for \( kT = 3(\text{keV}) \) we obtain \( \theta = 0.00012(\text{rad}) \), while for \( kT = 10(\text{keV}) \) we obtain \( \theta = 0.00039(\text{rad}) \). As can be seen in the figure, the impact of this factor is strongly dependent not only on the specific system considered but also on thermodynamic variables.

Reaction cross section, which includes Gamow factor through the astrophysical factor, is proportional to the amplitude of this curve. Therefore, the rearrangement of charge could be important in reaction networks in stellar interiors, permitting the burning process of heavy elements to start with less energy than in the case in which there is no charge rearrangement.

At this point, it is important to notice that this charge rearrangement effect is being considered in a general framework, and any detail in subsequent products will depend on the specific microscopic nuclear model used. For example, if we choose the Hauser-Feshbach formalism, the mechanism here proposed only affects the pre-compound nucleus stage, changing the number of such systems for the same incident energy but no its characteristics, leaving any subsequent nuclear evolution qualitatively unaltered.

Within the present model, the charge fraction that participates in the charge rearrangement, and hence the dipole moment \( p_z \), can not be derived and we need to estimate it from other considerations (parameters \( p_z \) and \( \theta \) given in the figures should be considered for example purposes only). We are developing a technique similar to molecular dynamic simulations (nucleon clusters) which may permit us not only estimate the charge fraction that participates in the charge rearrangement, but also it may provide us with the shape of the observed heavy ions as explained through charge rearrangement. The inclusion of the spin effects, which have not been considered in this analysis, may become relevant when taking into account selection rules due to angular momentum conservation. These considerations will be presented elsewhere.

Now, a comment on the approximation \( p_z \approx |\mathbf{p}| \) is necessary. In eq. (1), the position operators \( \mathbf{x}_k \) are subject to fluctuations [13] as reflected in the vector dipole moment, therefore, the dipole vector is not always pointing in the exact direction of the incoming projectile. This allows for the possibility of a finite \( \lambda \), implying an increase in the Gamow factor. On the other hand, it is possible that there are other contributions to the dipole moment, besides the induction effect. For example the nuclei could absorb electromagnetic radiation before the encounter with the proton, generating a dipole vector that will tend to align to the field of the incoming particle, but not exactly because of the fluctuations. Note that the requirement of a small \( \theta \neq 0 \) is compatible with the assumptions made in the calculation of the scattering cross section for the Coulomb field [11].

In the scope of nuclear astrophysics, the electric dipole strength in the energy region close to the neutron threshold has shown a strong influence on various processes of nucleosynthesis because in relevant scenarios even sub percent contributions to the E1-energy weighted sum rule in this region may lead to completely different abundance patterns [13] [10]. Of course, this has an strong connection to p-process [10].

It is not too far fetched that the proposed idea may also contribute in the longstanding puzzle observed in laboratory measurements, in which the electron screening effect is surprisingly larger than theoretical prediction based on an atomic physics model [21].

![Gamow window](image)

**FIG. 1: Gamow window:** The dashed line corresponds to the non corrected case and the solid line represents the corrected case for \( p_z = 0.122(\text{cm}) \) and \( \theta = 0.00012(\text{rad}) \). We use in this example the interaction \(^{179}\text{Hf} + p\), for \( kT = 3(\text{keV}) \). At the bottom the same reaction but for \( \theta = 0.00039(\text{rad}) \) and \( kT = 10(\text{keV}) \). (\( f = g_f(E, \beta) e^{-G_f}, \) where \( g_f \) is \( G_f \) as a function of \( E \) instead of \( \kappa \), and have been scaled by \( 10^{98} \) and \( 10^{65} \) respectively).

IV. CONCLUSION

We have found that approximating the source of the coulomb field by a point may not always provide an accurate description in a fusion interaction, because it hides spatial degrees of freedom, and may be one source of disagreement between the theory and experimental mea-
measurements. The rearrangement of charge in ions may tend to increase the reaction cross section, and could provide an extra ingredient to the understanding of some anomalies in reactions near the coulomb barrier without modification of the nuclear potential.

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