Central pseudoscalar production in $pp$ scattering and the gluon contribution to the proton spin

P. Castoldi, R. Escribano and J.-M. Frère

Service du Physique Théorique, Université Libre de Bruxelles, CP 225, B-1050 Bruxelles, Belgium

Abstract

Central pseudoscalar production in $pp$ scattering is suppressed at small values of $Q_\perp$, where $Q$ is defined as the difference between the momenta transferred from the two protons. Such a behaviour is expected if the production occurs through the fusion of two vectors. Photon exchange could provide the dominant contribution at low transferred momenta, but we argue that an extension of the experiment could probe the gluon contribution to the proton spin.
1 Introduction

The use of a glueball-$q\bar{q}$ filtering method has been recently advocated to study central hadron production in $pp$ scattering [1]. At this occasion, it was noticed that, somewhat surprisingly, pseudoscalar production (and in general $q\bar{q}$ mesons production) was suppressed at small values of $Q_\perp$ [2], where $Q$ is defined as the difference of the momenta transferred from the two protons.

We show in this note that such a behaviour is precisely expected if a pseudoscalar meson is produced through the fusion of two vector intermediaries. Furthermore, we argue that an extension of the experiment would test the gluon contribution to the proton spin.

In Sect. 2 we introduce the basic formulæ. In Sect. 3 we try to work a way around some important experimental cuts to provide suggestions for comparison to the data. In particular, we provide the $Q_\perp$ distribution at fixed $k_\perp$, $k$ being the momentum of the resonance $X$. This allows for a comparison between $\pi^0$, $\eta$ and $\eta'$ production and a test of the nature of the process. Finally, in Sect. 4 we advocate extending the study to non-exclusive channels $pp \rightarrow \tilde{p}\tilde{p}X$, where $\tilde{p}$ are jets corresponding to $p$ fragmentation, to observe the QCD equivalent of the process. We then argue that a measurement of the production cross section at $Q_\perp = 0$ would provide a test of the gluon contribution to the proton spin.

2 The basic formulæ

The WA102 and NA12 experiments [2, 3] have examined in kinematical detail the reaction $pp \rightarrow ppX$ where $X$ is a single resonance produced typically in the central region of the collision between a proton beam and an hydrogen target.

We will be more particularly interested in the case where $X$ is a $J^P = 0^-$ state, notably $\pi^0$, $\eta$ or $\eta'$. The interest of this experiment is that the kinematics are entirely determined (in fact overdetermined) since the momenta of all protons are known and the disintegration of $X$ is entirely measured (e.g. in the $\gamma\gamma$ mode).

The production cross section is affected by two distinct mechanisms: i) the emission of the intermediaries from the protons and ii) the fusion of those intermediaries into the resonance $X$. 

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We will mainly interested in the low transferred momenta régime, i.e. in
the low $t_1$ and $t_2$ region, where $t_1$ and $t_2$ are defined as the square of the
momentum exchanged at the proton vertices. For this reason, we consider as
intermediaries only the lowest-lying particles, mainly pseudoscalars, vectors
and axials. We do not consider heavier particles, in particular tensors, whose
couplings involve a large number of derivatives and are then expected to
contribute less in the kinematical region considered.

In this framework, the production of a pseudoscalar resonance through the
fusion of two intermediaries in parity conserving interactions could arise from
scalar-pseudoscalar ($SP$) or vector-axial ($VA$) fusion if no factor of momenta
is allowed, or, vector-pseudoscalar ($VP$), vector-vector ($VV$) or axial-axial
($AA$) fusion if the momentum variables can be used. Since the first
axial resonance is rather heavy, we also do not consider the case where one
or two of these resonances are involved, so we restrict our discussion to $SP$,
$VP$ or $VV$ fusion.

In the case of $SP$ fusion, the only pseudoscalar which could be involved
in the $\pi^0$, $\eta$ and $\eta'$ production is the particle itself, but we still need to find a
low-lying scalar, possibly the “sigma” or a “pomeron” state. Moreover, due
to the absence of any derivative coupling, the observed suppression of the
production cross section at small $Q_\perp$ cannot occur since non trivial helicity
transfer is needed (see Ref. [4] for details). In the case of $VP$ fusion, the $VPP$
coupling involves one derivative and should obey Bose and $SU(3)$ symmetry.
For instance, a $\rho^0\pi^0\pi^0$ coupling is well-known to be forbidden. We conjecture
that the argument can be extended to $U(3)$ symmetry (in particular $\rho^0\eta\pi^0$),
which removes the discussion of $VP$ fusion from our analysis. This leaves
$VV$ fusion as the only alternative.

Vector-vector fusion is possible through the vector-vector-pseudoscalar
($VVP$) coupling

$$C_{VVP} = \epsilon_1^{\rho_\alpha_\beta} q_1^{\mu} q_2^{\nu} \epsilon_1^\alpha \epsilon_2^\beta ,$$ (1)

where $q_1$ and $q_2$ are the momenta of the exchanged vectors with polarizations
$\epsilon_1$ and $\epsilon_2$ respectively. This coupling is well known from the anomalous
decay $\pi^0 \rightarrow \gamma\gamma$. When evaluated in the $X$ rest frame with $k = q_1 + q_2$ and
$Q = q_1 - q_2$, it yields simply

$$C_{VVP} = -\frac{1}{2} m_X \vec{Q} \cdot (\vec{\epsilon}_1 \times \vec{\epsilon}_2) ,$$ (2)

where clearly the difference $\vec{Q}$ between $q_1$ and $q_2$ 3-momenta appears now as
a factor and we thus expect a suppression at small $\vec{Q}$. But this is insufficient in itself to explain the suppression observed at small $Q_\perp = |\vec{Q}_\perp|$, where $\vec{Q}_\perp$ is defined as the vector component of $\vec{Q}$ transverse to the direction of the initial proton beam. However, as seen from (2), the polarizations of the vectors play an essential role. In particular, in the $X$ rest of frame, $\vec{c}_1 \times \vec{c}_2$ must have components in the $\vec{Q}$ direction, which implies that both $\vec{c}_1$ and $\vec{c}_2$ must have components in the plane perpendicular to $\vec{Q}$, that is, the exchanged vectors must have transverse polarization (helicity $h = \pm 1$). In other terms, the production process will be proportional to the amount of intermediate vectors with $h = \pm 1$.

If we consider now the emission of a vector from a fermion, we observe that in the high-energy limit the vector only couples to $\bar{f}_L \gamma^\mu f_L$ and $\bar{f}_R \gamma^\mu f_R$, that is, the helicity of the fermion cannot change. In the $X$ rest frame, assumed to lie in the central region of the production, the colliding fermions cannot (unless they were backscattered, a situation contrary to the studied kinematical region) emit $h = \pm 1$ vectors in the forward directions, as this would violate angular momentum conservation.

We thus reach the conclusion that in the above-mentioned kinematical situation, the production of pseudoscalar mesons by two-vector fusion cannot happen if $\vec{Q}$ is purely longitudinal, but requires $\vec{Q}_\perp \neq \vec{0}$.

It is easy to write down the differential cross section for the central production of pseudoscalar resonance $X$ in the reaction $pp \to ppX$. Although it may seem daring to treat the $p$ as a pointlike particle in the process considered, this approximation of the $ppV$ ($V$ any vector) coupling seems phenomenologically more reasonable than the use of a quark parton model when strictly exclusive processes are considered (where $p$ fragmentation is not allowed for).

We use the following notations: $p_1 = (E; 0, 0, p)$ is the beam proton momentum, $p_2$ the target proton momentum, $p_3$ the momentum of the outgoing proton closest to the beam kinematical area and $p_4$ the momentum of the outgoing proton closest to the target kinematical area. The transferred momenta to the intermediate vectors are $q_1 = p_1 - p_3$ and $q_2 = p_2 - p_4$ respectively, and the momentum of the resonance $X$ is then defined as $k = (W; k_\perp, k_\parallel) = q_1 + q_2$ with $k^2 = m_X^2$ and $W = \sqrt{m_X^2 + k_\perp^2 + k_\parallel^2}$. We

\textsuperscript{3}There is still a loophole: $\vec{q}_1$ and $\vec{q}_2$ must have transverse components, but in a small area of phase space we could still have $\vec{Q}_\perp = (\vec{q}_1 - \vec{q}_2)_\perp = \vec{0}$. The explicit calculation below shows this is not significant.
also define \( Q = (\omega; \vec{Q}_{\perp}, \vec{Q}_{\parallel}) = q_1 - q_2 \) as the difference between the momenta transferred from the two protons.

The differential cross section reads

\[
\frac{d\sigma}{dQ_\perp dk_1 dk_2 d\phi} = \frac{1}{(2\pi)^4} \frac{1}{128WEP} \frac{k_1 Q_\perp}{[(2p - Q_\parallel)(2E - W) - k_\parallel\omega]} |\mathcal{M}|^2, \tag{3}
\]

where we have chosen as integration variables: \( Q_\perp = |\vec{Q}_{\perp}|, k_\perp = |\vec{k}_{\perp}|, k_\parallel \) and \( \phi \) defined as the angle between the two transverse vectors \( \vec{k}_{\perp} \) and \( \vec{Q}_{\perp} \).

The averaged square of the invariant matrix amplitude \( \mathcal{M} \) at the lowest order in the vector exchange is expressed in terms of kinematical invariants as \[4\]

\[
|\mathcal{M}|^2 = \frac{(g_{ppV_1}g_{ppV_2}g_{V_1V_2}p)^2}{(t_1 - m_X^2)^2(t_2 - m_Y^2)^2} \left\{ \frac{t_1 t_2}{2} [((k \cdot Q)^2 - m_X^2 Q^2] + t_1 \left[m^2((k \cdot Q)^2 - m_X^2 Q^2) + m_X^2 (p_2 \cdot Q)^2 + Q^2 (p_2 \cdot k)^2 - 2(p_2 \cdot k)(p_2 \cdot Q)(k \cdot Q)] \right. \\
+ \left. t_2 \left[m^2((k \cdot Q)^2 - m_X^2 Q^2) + m_X^2 (p_1 \cdot Q)^2 + Q^2 (p_1 \cdot k)^2 - 2(p_1 \cdot k)(p_1 \cdot Q)(k \cdot Q)] \right. \\
+ \left. 4 \left( \epsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu k_1^\alpha Q_\parallel^\beta \right)^2 \right\}, \tag{4}
\]

where \( g_{VV'P} \) stands for the coupling constant of the \( VV'P \) interaction, \( g_{ppV} \) stands for the coupling constant of the \( ppV \) interaction, \( t_{1,2} \) are the square of the momentum transfer to each vector, \( m_V \) is the mass of the exchanged vector, \( m_X \) the resonance mass and \( m \) the proton mass. In order to write (4) in terms of the integration variables we have used the following substitutions:

\[
(p_1 \cdot k) = EW \mp pk_\parallel, \quad (p_1 \cdot Q) = E\omega \mp pQ_\parallel, \quad (k \cdot Q) = W\omega - k_\parallel Q_\parallel - k_\perp Q_\perp \cos \varphi = -2pk_\parallel + 2E\omega, \quad Q^2 = \omega^2 - Q_\perp^2 - Q_\parallel^2 = -m_X^2 + 4EW - 4pQ_\parallel
\]

and \( t_{1,2} = q_{1,2}^2 = E(W \pm \omega) - p(Q_\parallel \pm k_\parallel) \).

Due to the smallness of \( t_1 \) and \( t_2 \) we have checked that independently of the mass of the vector exchanged, the dominant contribution of (4) to the cross section (3) comes from the term proportional to \( 1/(t_1 - m_X^2)^2(t_2 - m_Y^2)^2 \).

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\[4\] Given the experimental condition, the diagram where the “fast” and “slow” protons are exchanged gives a negligible contribution, and it is not included in (4).
The differential cross section thus simplifies to

\[
\frac{d\sigma}{dQ_\perp dk_\perp dk_\parallel d\varphi} \simeq \frac{1}{(2\pi)^4} \frac{1}{128WEP} \frac{k_\perp Q_\perp}{|(2p - Q_\parallel)(2E - W) - k_\parallel|} \\
\times 16(g_{ppV_1}g_{ppV_2}g_{V_1V_2})^2E^2p^2 \frac{k_\perp^2 Q_\perp^2 \sin^2 \varphi}{(t_1 - m_{V_1}^2)^2(t_2 - m_{V_2}^2)^2}, (5)
\]

where now one can clearly see the suppression of the cross section at small \(Q_\perp\) (and indeed \(\lim_{Q_\perp \to 0} d\sigma = 0\), as it is seen experimentally.

Once the expression for the differential cross section is presented, we may now enter into conjectures about the nature of the vectors exchanged. The simplest candidates for elementary particles are of course photon or gluon, with the possible addition as an example of the massive vectors \(\rho, \omega\) and \(\phi\). We will first consider the case of \(t_1, t_2 \to 0\); it is then quite clear that the dominant contribution to the simplified cross section (5) comes from the exchange of massless vectors, so we neglect temporarly the possible contributions of massive vectors. Then, we are left with photons or gluons. However, in the present situation, gluon exchange seems not to be the dominant contribution, as it would lead to a large number of \(\eta'\) and \(\eta\) and no \(\pi^0\), which is clearly not the experimental situation [6]. Most probably, the selection of isolated protons in the final state is too restrictive for gluon exchange to take place significantly. So then, we conclude that a pure photoproduction hypothesis may be the main contribution to the cross section at very low transferred momenta.

Assuming the photoproduction mechanism as the main effect responsible of the pseudoscalar production, we would like to point out that very relevant information can be obtained here of the \((t_1, t_2)\) behaviour of the \(\gamma\gamma\)-pseudoscalar form factor, a question highly discussed in the literature [7].

We will see however that the experiment does not allow isolation of this low \(t_1\) and \(t_2\) kinematical region, and that at least the lowest vectors need to be included.

## 3 Working around the cuts

As we have seen in the previous section, photon-photon fusion could be the main mechanism responsible for the pseudoscalar production in the central
region for low values of exchanged momenta. Thus, we observe that the cross section tends to peak sharply at small values of $Q_\perp \simeq k_\perp$ (through the photon denominators), even though it decreases to zero when $Q_\perp$ or $k_\perp \to 0$.

This behaviour is however difficult to observe experimentally, due to the presence of experimental set-up restrictions, leading to a loss of acceptance when the transverse momenta of the outgoing protons decrease. This seems to be specially sensitive for the “slow proton”. As a result, although the theoretical cross section peaks for small non-zero values of $Q_\perp \simeq k_\perp$, this domain of parameter space is totally inadequate for a detailed comparison to experiment.

In practice, we could work at fixed $k_\perp$ in order to avoid the experimental restrictions, and explore the $Q_\perp$ dependence of the cross section. In that case, pseudoscalar production by photon-photon fusion (and in general by any vector-vector fusion) will be characterized by the vanishing of the cross section at small $Q_\perp$. Other vector exchanges provide largely enhanced contributions, retaining the low $Q_\perp$ suppression and with a high $Q_\perp$ sensitivity to the form factors used.

We give as examples, curves of the pseudoscalar photoproduction cross section of $\pi^0$, $\eta$ and $\eta'$ as a function of $Q_\perp$ for two characteristic values of the angle $\varphi$ between $\vec{k}_\perp$ and $\vec{Q}_\perp$. For $\varphi = 45^\circ$ (or in fact, any value of the angle $\varphi$ far away from $0^\circ$ and $180^\circ$), both $q_{1\perp}$ and $q_{2\perp}$ are far from zero, and we do not expect acceptance restrictions. On the contrary, for $\varphi \simeq 10^\circ$ (or in fact, any value of the angle $\varphi$ close to $0^\circ$ and $180^\circ$), the cross section turns out to be much larger, but we expect here acceptance limitations, since close to $Q_\perp = k_\perp$ we can have either $q_{1\perp} \approx 0$ or $q_{2\perp} \approx 0$.

While these curves are given as indicatives and seem to have some bearing to published data, a detailed comparison, which can only be made with an extensive simulation of the experimental detection, is clearly to be left to the experimental collaborations.

A work equivalent to the former can be done if instead of using $Q_\perp$ as the distribution variable for the curves one uses the azimuthal angle $\phi$ (not to be confused with the angle $\varphi$ introduced before) between the fast and slow proton that is related to $Q_\perp$ by $\cos \phi = \frac{Q_\perp^2 - k_\perp^2}{4 q_{1\perp} q_{2\perp}}$.

The above considerations allowed us to compute completely a simplified cross section. However the experimental difficulty to reach very small values

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5 For simplicity, we have restricted ourselves to the case $k_\parallel = 0$. 
of $t_1$ and $t_2$ invalidates largely the assumptions made. In this case indeed, similar contributions will arise from vector bosons, and their much stronger couplings will offset the mass suppression in the propagator.

We have performed such a calculation using formula (5) above. The $VVP$ coupling coefficients can be obtained along the lines of [8] and are given in Tables 1–3, while the $ppV$ ones are estimated in [9].

The behaviour obtained confirms the low $Q_{\perp}$ suppression, but the general structure of the curve and its peak value are very sensitive to vertex form factors, on which we have little independent information. These form factors, (both at the proton and pseudoscalar vertex) can be combined in a single function $f(t_1) \cdot f(t_2)$, which could of course be fitted directly from experiment, much as the traditional $\exp(-bt)$ usually is.

This offers on one hand the possibility to gather information on form factors, in particular on the $VVP$ ones [10], but as the main point of the paper is concerned (Sec. 4 below), this “background” does not affect the conclusions (since only the $Q_{\perp} \to 0$ suppression is of importance).

4 Extending the approach to gluons

In this final section, we would like to advocate for an extension of the present study to non-exclusive processes $pp \to \tilde{p}\tilde{p}X$, where $\tilde{p}$ are jets corresponding to $p$ fragmentation, in order to observe the QCD equivalent of the production mechanism (gluon-gluon fusion).

In this case indeed, we must distinguish between gluons emitted from the fermionic partons (and obeying the helicity constraints discussed at the beginning of the previous section) and “constituents” or “sea” gluons. The latter simply share part of the proton momentum and their helicity is in no way constrained. Helicity $h = \pm 1$ gluons can then be met even for $Q_{\perp} = 0$, and in that case we would expect that the production distributions in $Q_{\perp}$ could be considerably affected.

In this possible extension of the experiments, the $\eta'$ and $\eta$ now produced at small $Q_{\perp}$ are sensitive to the polarization of the individual gluons in the proton. Such polarization of the individual gluons is always present independently of the total polarization of the gluons in the proton, and is in itself not indicative of the fact that a significant proportion of the proton spin could be carried by the gluons. If such would be the case however, and
a net polarization of the gluons exists, a similar experiment conducted with polarized beams or target would lead to a difference in the production rates of $\eta'$ and $\eta$ at small $Q_\perp$, and provide a direct measurement of this polarization.

In summary, we have shown in this letter that the experimental evidence of the suppression at small $Q_\perp$ of the central pseudoscalar production in $pp$ scattering can be explained if the production mechanism is through the fusion of two vectors. We also have proposed an extension of such experiments in order to observe the QCD equivalent of the process and to provide a test for the gluon contribution to the proton spin.

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Figure 1: Differential cross section for $pp \rightarrow ppX$ ($d\sigma \times 10^{-6}$ (GeV$^{-5}$)), as a function of $Q_{\perp}$ (GeV). We have fixed $k_{\perp} = 0.5$ GeV and $k_{\parallel} = 0$, and this plot is given for $\varphi = 45^\circ$. The curves describe $X = \pi^0$ (solid line), $X = \eta$ (dashed line) and $X = \eta'$ (dashed-dotted line).
Figure 2: The same as Fig. 1 but for $\varphi = 10^\circ$.

$\gamma\gamma P \; \; g_{\gamma\gamma P}$

| $\gamma\gamma\pi^0$ | $g_{\gamma\gamma\pi^0} \equiv \frac{\sqrt{2}}{4\pi f_{\pi}}$ | $\approx 0.27 \text{ GeV}^{-1}$ |
|----------------------|-----------------|-----------------|
| $\gamma\gamma\eta$ | $g_{\gamma\gamma\eta} \left( \frac{c\theta}{\sqrt{3}} \frac{f_{\eta}}{f_{s}} - 2\sqrt{\frac{2}{3}} s\theta \frac{f_{s}}{f_{0}} \right)$ | $\approx 0.26 \text{ GeV}^{-1}$ |
| $\gamma\gamma\eta'$ | $g_{\gamma\gamma\eta'} \left( \frac{s\theta}{\sqrt{3}} \frac{f_{\eta}}{f_{s}} + 2\sqrt{\frac{2}{3}} c\theta \frac{f_{s}}{f_{0}} \right)$ | $\approx 0.34 \text{ GeV}^{-1}$ |

Table 1: $\gamma\gamma P$ couplings. We have used the following values: the pion decay constant $f_{\pi} = 0.132 \text{ GeV}$, the $\eta - \eta'$ mixing angle $\theta = -16.5^\circ$ and the $\eta_8$ and $\eta_0$ decay constants $f_8 = 1.04 f_{\pi}$ and $f_0 = 1.12 f_{\pi}$. 

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\[ \begin{align*} 
V \gamma P \quad & g_{V \gamma P} \\
\rho^0 \gamma \pi^0 & \equiv \frac{g}{4\pi f_s} \quad \sim 0.82 \text{ GeV}^{-1} \\
\omega \gamma \pi^0 & \quad \sim 2.46 \text{ GeV}^{-1} \\
\phi \gamma \pi^0 & \quad \sim 0.15 \text{ GeV}^{-1} \\
\rho^0 \gamma \eta & \quad \left( \sqrt{3} c \theta \frac{f_s}{f_0} - \sqrt{6} s \theta \frac{f_s}{f_0} \right) \quad \sim 1.82 \text{ GeV}^{-1} \\
\rho^0 \gamma \eta' & \quad \left( \sqrt{3} s \theta \frac{f_s}{f_0} + \sqrt{6} c \theta \frac{f_s}{f_0} \right) \quad \sim 1.33 \text{ GeV}^{-1} \\
\omega \gamma \eta & \quad \left[ c \theta \frac{f_s}{f_0} \left( \frac{c \varphi_V}{\sqrt{3}} - 2 \sqrt{\frac{2}{3}} s \varphi_V \right) - s \theta \frac{f_s}{f_0} \left( \sqrt{\frac{2}{3}} c \varphi_V + \frac{2}{\sqrt{3}} s \varphi_V \right) \right] \quad \sim 0.55 \text{ GeV}^{-1} \\
\omega \gamma \eta' & \quad \left[ s \theta \frac{f_s}{f_0} \left( \frac{c \varphi_V}{\sqrt{3}} - 2 \sqrt{\frac{2}{3}} s \varphi_V \right) + c \theta \frac{f_s}{f_0} \left( \sqrt{\frac{2}{3}} c \varphi_V + \frac{2}{\sqrt{3}} s \varphi_V \right) \right] \quad \sim 0.51 \text{ GeV}^{-1} \\
\phi \gamma \eta & \quad \left[ c \theta \frac{f_s}{f_0} \left( \frac{c \varphi_V}{\sqrt{3}} + 2 \sqrt{\frac{2}{3}} c \varphi_V \right) - s \theta \frac{f_s}{f_0} \left( \sqrt{\frac{2}{3}} s \varphi_V - \frac{2}{\sqrt{3}} c \varphi_V \right) \right] \quad \sim 1.03 \text{ GeV}^{-1} \\
\phi \gamma \eta' & \quad \left[ s \theta \frac{f_s}{f_0} \left( \frac{c \varphi_V}{\sqrt{3}} + 2 \sqrt{\frac{2}{3}} c \varphi_V \right) + c \theta \frac{f_s}{f_0} \left( \sqrt{\frac{2}{3}} s \varphi_V - \frac{2}{\sqrt{3}} c \varphi_V \right) \right] \quad \sim -1.15 \text{ GeV}^{-1} 
\end{align*} \]

Table 2: \( V \gamma P \) couplings. We have used the following values: the \( SU(3) \) symmetric coupling constant \( g = 4.28 \) determined by neutral \( \rho \) decay, the departure from \( \omega-\phi \) ideal mixing parametrized by the angle \( \varphi_V = 3.4^\circ \), as well as the values introduced in Table [1].
\[
\begin{array}{l}
\rho^0 \omega \pi^0 & \frac{3\sqrt{2} g^2}{4 \pi^2 f_s} c \phi_V \\
\rho^0 \phi \pi^0 & \frac{3\sqrt{2} g^2}{4 \pi^2 f_s} s \phi_V \\
\rho^0 \rho^0 \eta & \frac{6\sqrt{2} g^2}{4 \pi^2 f_s} \left( \frac{c\theta f_s}{2 \sqrt{3}} f_8 - s\theta f_8 \right) \\
\rho^0 \rho^0 \eta' & \frac{6\sqrt{2} g^2}{4 \pi^2 f_s} \left( s\theta f_8 f_0 + c\theta f_8 \right) \\
\omega \omega \eta & \frac{6\sqrt{2} g^2}{4 \pi^2 f_s} \left[ c\theta f_8 \left( \frac{c^2 \phi_V}{2 \sqrt{3}} - \frac{s^2 \phi_V}{\sqrt{3}} \right) - s\theta f_8 \right] \\
\omega \omega \eta' & \frac{6\sqrt{2} g^2}{4 \pi^2 f_s} \left[ s\theta f_8 \left( \frac{s^2 \phi_V}{2 \sqrt{3}} - \frac{c^2 \phi_V}{\sqrt{3}} \right) + c\theta f_8 \right] \\
\phi \phi \eta & \frac{6\sqrt{2} g^2}{4 \pi^2 f_s} \left[ c\theta f_8 \left( \frac{s^2 \phi_V}{2 \sqrt{3}} - \frac{c^2 \phi_V}{\sqrt{3}} \right) - s\theta f_8 \right] \\
\phi \phi \eta' & \frac{6\sqrt{2} g^2}{4 \pi^2 f_s} \left[ s\theta f_8 \left( \frac{c^2 \phi_V}{2 \sqrt{3}} - \frac{s^2 \phi_V}{\sqrt{3}} \right) + c\theta f_8 \right] \\
\omega \phi \eta & \frac{3\sqrt{2} g^2}{4 \pi^2 f_s} \left[ c\theta f_8 \frac{\sqrt{3}}{2} c \phi_V s \phi_V \right] \\
\omega \phi \eta' & \frac{3\sqrt{2} g^2}{4 \pi^2 f_s} \left[ s\theta f_8 \frac{\sqrt{3}}{2} c \phi_V s \phi_V \right]
\end{array}
\]

\[\varepsilon \approx 14.86 \text{ GeV}^{-1}\]

\[\varepsilon \approx 0.88 \text{ GeV}^{-1}\]

\[\varepsilon \approx 11.00 \text{ GeV}^{-1}\]

\[\varepsilon \approx 8.06 \text{ GeV}^{-1}\]

\[\varepsilon \approx 10.92 \text{ GeV}^{-1}\]

\[\varepsilon \approx 8.08 \text{ GeV}^{-1}\]

\[\varepsilon \approx -12.68 \text{ GeV}^{-1}\]

\[\varepsilon \approx 15.07 \text{ GeV}^{-1}\]

\[\varepsilon \approx 0.70 \text{ GeV}^{-1}\]

\[\varepsilon \approx -0.21 \text{ GeV}^{-1}\]

Table 3: VVP couplings. We have used the values introduced in Tables [1] and [2].