Polarization of light and the spin state of photon

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The comparison of the polarization and spin of light is presented in the paper. It is shown that it is more easier and clearer to use the polarization of the light to explain the effect of the interaction of light and atoms than that of spin of the light. The paper also gives rise to the question whether or not the concept of spin for photon have any essence for its existence.

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I. Introduction

In the theory of quantum photon, the polarizations of the light are related with the spin angular momentum $\vec{S}$ of the photon, which can only be defined along the direction of propagation $\vec{k}$, the longitudinal direction. The longitudinal components $\vec{S} \cdot \vec{k}/|\vec{k}| = m_s \hbar$ of the spin angular momentum $\vec{S}$ can only take three values with $m_s = \pm 1$ for a right- or left-circularly polarized photon, and $m_s = 0$ for a linearly polarized one.\[1,8\]

The above statements about photon are generally accepted. Nevertheless, the polarizations of the light is different from the spin of photon, and sometimes their difference is so great that substitution of the polarization of light by its spin even will results in some error, which may be evasive in statements. We will show this by an example in the Zeeman effect (denoted as ZE or ZEs in the following). In the definition of the spin for photon, the state for the longitudinal component $m_s$ being zero is also evasive and need some clarification. These are what this paper intends to do.

The paper is organized as following. Section II gives brief introduction on Zeeman effects and the explanation of the polarization of emitted photons in ZEs by the use of the polarization vector. In section III, ZEs is explained by use of the conservation of the angular momentum and the concept. In this way, it is shown they can not give a consistent answer to the polarization concerning the $\pi$ line. Section IV give proof for the formulae used in section III, and also shows that it is favorable to use polarization vector to the spin of photon. Final section ends with conclusion.

II. Zeeman effects and its correct interpretation

ZE is one of the crucial experiments that helped in the development of the quantum theory\[1\]. It also provides a useful tool for examining the structure and hyperfine structure of the atoms and molecules. Due to the fact that magnetic fields play a more and more important role, it can also be used to to measure the strength and direction of magnetic fields in the process of star formation. So, many reference books, elemental or advanced on atom theory, contain ZE, and give explanation about it. The ZFs include the normal and anomalous ones. Even without the quantum theory, the spectrum and the polarization of the normal one could be easily explained classically. However, classical theory can not give any answer to the anomalous ZE, which needs the quantum theory, especially the orbital and the spin angular momentums of the electrons together to its interpretation.\[1\].

The peculiarity in the ZFs is the polarization of the emitted light. In the longitudinal direction of the magnetic field $\vec{B}$, only the $\sigma^\pm$ lines of the left and right circularly polarized lights are observed, but the $\pi$ line is absent. In the transverse direction, all three lines $\sigma^\pm$, $\pi$ appear linearly polarized, and $\pi$ line is polarized along the direction of the magnetic field, while $\sigma^\pm$ polarized transversely to $\vec{B}$.\[1\].

How to explain the polarization of the emitted light? There exist different ways at different levels. It can be clearly stated by the use of quantum theory of interacting atom and light system, both atom and electromagnetical field are quantized. Only quantum theory of atoms can also give explanation. Of course, even classical theory can supply the answer to those of normal ones. Most college students contact the second explanation. However, in the second explanation concerning the polarization of the $\pi$ line light, there is something misleading in some reference books, which will be picked out in the paper.\[6-8\]. The correct answer will also be given.

As stated before, ZFs are widely used in the modern physics and technology, the clear and correct explanation is absolutely needed. They also involve some fundamental concepts, like the spin and the angular momentum for photon, etc. This is the main reason for the paper to study this old problem.

For the sake of simplicity, we will only concern the normal ZE regarding the explanation of the polarization of the emitted light. As usual treatment, the dipolar polarization approximation is used in the following. The actual calculation of the transition rate needs the tool of time-dependent perturbation theory, however, the Fermi’s golden rule works well for our concern on the polarization of the photon. According to the golden rule,
the transition rate $\Gamma$ is proportional to the square of the matrix element of the perturbation of the interaction of the atom and electromagntial field,

$$H' = e \vec{r} \cdot \vec{E} = e E_{E,\alpha} \vec{r} \cdot \vec{e}_\alpha,$$

that is [1],

$$\Gamma \propto |e E_{E,\alpha}|^2 < f |\vec{r} \cdot \vec{e}_\alpha| i > |^2,$$  \hspace{1cm} (1)

where $| f >, | i >$ are the final and initial states of the transmitting atom. In Eq. (1), the emitted light’s electric filed is $\vec{E} = E_{E,\alpha} \vec{e}_\alpha$, and its polarization, propagation vectors are $\vec{e}_\alpha, \vec{k}$ respectively. One could write the polarization vector in the form $\vec{e}_\alpha = A_\sigma \frac{\vec{e}_x - i\vec{e}_y}{\sqrt{2}} + A_\pi \frac{\vec{e}_z}{\sqrt{2}}$. Similarly, the position vector $\vec{r}$ can be written as

$$\vec{r} = r \left[ \sin \theta \cos \phi \vec{e}_x + \sin \theta \sin \phi \vec{e}_y + \cos \theta \vec{e}_z \right]$$

$$\propto Y_{1,-1} \vec{e}_x + Y_{1,0} \vec{e}_y + Y_{1,1} \vec{e}_z - i \frac{\vec{e}_y}{\sqrt{2}}$$  \hspace{1cm} (2)

So, one obtains

$$\vec{r} \cdot \vec{e}_\alpha \propto A_\sigma Y_{1,-1} + A_\pi Y_{1,0} + A^\pi_\sigma Y_{1,1}$$  \hspace{1cm} (3)

where $\vec{r}, \vec{e}_\alpha$ are superposed as of three parts corresponding to two circularly polarized vectors and a linearly polarized one. $Y_{1,0}, Y_{1,\pm 1}$ in Eq. (2) are the spherical functions. Whenever the element $< f |\vec{r} \cdot \vec{e}_\alpha| i >$ is zero, then there is no spectral line to exist. $\sigma$-transition exists whenever $\Delta m = m_f - m_i = \pm 1$, and $\pi$-transition hold under the condition $\Delta m = m_f - m_i = 0$. Electromagnetic radiation is a transverse wave, so its electric field is at the right angle with its propagation vector $\vec{k}$. When observed along the direction of the magnetic field $\vec{B}$, there are only two $\sigma^\pm$ lines circularly polarized. $\pi$ line disappears due to its polarization is the same as the direction of $\vec{B}$. On the other hand, observed transversely, for example, along the direction of the $x$ axis, all three lines appear with $\sigma$ ones polarized along $y$ axis and $\pi$ line polarization vector along $z$ axis. This is the correct explanation for the polarization of the emitted lights in the ZFs, as already stated before. In the explanation, no concepts such as the spin for photon, etc, have been used.

III. The contradiction encounter in Explanation of Zeeman effects with the concept of spin for photon

Because the spherical wave functions $Y_{l,m}$ are the eigenfunctions of the angular momentum operator $\hat{L}^2, \hat{L}_z$ with $l, m$ quantum numbers of the total and $z$-component of the angular momentum for the atom, they extensively suggest that the conservation of the angular momentum will give somehow a better and simpler answer to the ZFs. As stated in the beginning, it is a known fact a photon has energy $\hbar \omega$, momentum $\hbar \vec{k}$ and spin angular momentum $\vec{S}$. The spin angular momentum of photon can only be defined along the direction of propagation, and its components $\vec{S} \cdot \vec{k}/|k| = m_s \hbar$ with $m_s = \pm 1, 0$. The photon with $m_s = \pm 1$ corresponds to a right- or left-circularly polarized light, but the state of $m_s = 0$ for the photon is amphibolous. In the following concerning the explanation of the polarization of the $\pi$-line in the ZFs we will give precise meaning for $m_s = 0$.

The following is the standard explanation of ZE by the conservation of angular momentum, which will result in contradiction concerning the explanation of the polarization of the $\pi$-line.

The total angular momentums are conserved for the system of the atom and the photon. The atom’s angular momentum in the longitudinal direction changes as $\Delta m = \pm \hbar, 0$, so, in the conservation of angular momentums, the emitted photon should have spin with $z$-component $m_s = \mp \hbar, 0$. The photons of $z$-components $m_s = \mp \hbar$ are the $\sigma^\pm$ lines, which are circularly polarized. Of course, They appear as that in observation along the direction of magnetic field $\vec{B}$. The transverse property of the light wave makes them as linearly polarized ones detected in the direction perpendicular with $\vec{B}$. As stated before, this explanation is ok concerning the $\sigma^\pm$ lines. But concerning the $\pi$ line, The photon has an angular momentum with its $z$-component $m_s = 0$. It must be very easy to allure one to the conclusion that its angular momentum $\vec{S}$ lies in the plane perpendicular to $z$ axis [8]. Suppose $\vec{S} = |S| \vec{e}_x$, then the polarization vector corresponding to its components as $\pm \hbar$ must be the same as one of $\chi_x^\pm = \frac{\vec{e}_x \pm i \vec{e}_y}{\sqrt{2}}$. But this viewpoint is wrong because the angular momentum is not just a common vector in quantum theory as it is in classical theory. Unlike the counterparts in classical theory, the angular momentum in quantum theory is a vector operator, and its components are operators not commuting with each other. So two or more components of the angular momentum cannot be given simultaneously. As the $z$-component of the photon spin is $m_s = 0$, its other components can not be determined at all.

In the theory of quantum, the state for $z$-component $m_s = 0$ of the spin angular momentum for photon is denoted as $\chi_0$, which is an unobservable state of photon whenever the photon is detected in the direction of $z$ axis. This just shows that there is no $\pi$ line to appear in the direction of the magnetic field $\vec{B}$. However, there is really observable effect concerning the state $\chi_0$. Observed perpendicular to the magnetic field $\vec{B}$, the state $\chi_0$ can be regarded as that of linearly polarized for photon. This is also shown as it is decomposed as

$$\chi_0 = \frac{1}{\sqrt{2}}(\chi_+ \chi_+^\perp)$$  \hspace{1cm} (4)

where the superscript $\perp$ means the propagation direction of $\chi_+^\perp$ photons is at right angle with the $z$ axis. The above
formulae will be proved later in the paper.

For example, we choose \( \chi_{\pm}^{x} \) as \( \chi_{\pm}^{y} \), the above equation becomes

\[
\chi_0 = \frac{1}{\sqrt{2}}(\chi_{+}^{x} - \chi_{-}^{x})
\]

As the two circularly polarized photons being real entities are superposed in the way of Eq. (5), they will give rise to question to the explanation concerning the polarization of of \( \pi \)-line. It means for the appearance of \( \pi \)-line in the transverse direction, there must be two photons simultaneously existing to obtain the linearly polarized \( \pi \)-line in the transverse direction, for example, the \( x \) axis direction. Two photons appearing simultaneously will contradict with the conservation of energy.

If the experiment is not about the ZFs, the use of the conservation of the sum of the angular momentum of the atom and the spin of the photon to explain the experiment fact can not have more errors than the one above mentioned. When there is a magnetic field \( \vec{B} \) as in the ZFs, which will make the degenerate energy levels split, the above explanation is completely false. The photon of polarization vector \( \vec{e} \) can not exist at all. The photon of polarization vector \( \vec{e} = \frac{-\vec{e}_{\perp} + i\vec{e}_{\|}}{\sqrt{2}} \) can not exist at all.

The field \( \vec{B} \) makes the energies of states with the different \( m \) not be equal to each other, so transitions involving \( \vec{e}_{\perp} \), \( \vec{e}_{\|} \) will have different frequencies for the corresponding photons. From Eq. (6), there are three natural states for the photons, two circularly polarized ones (\( \sigma_{\pm} \)) and a linearly polarized one (\( \pi \)), and all three have different frequencies. Therefore, it is impossible for the photon to have such states of the polarization vector \( \chi_{\pm}^{x} = \frac{-\vec{e}_{\perp} + i\vec{e}_{\|}}{\sqrt{2}} \).

Because the photons of the states of \( \chi_{\pm}^{x} \) can not exist at all, Eq. (6) is no longer having any meaning. The appearance of the wrong states \( \chi_{\pm}^{x} \) for a photon comes from the belief in the concept of the spin for it, which is taken for granted just as the well-known concepts of its energy \( h\omega \) and linear momentum \( \hbar \vec{k} \).

This in turn put the concept of the spin of photon in question. Though Beth shows the photon has spin one in 1936 [9]. Unlike the spin of ordinary particles, like electron, proton, etc, the spin of the photon must not be used without great care. It is not unreasonable to suggest its supersetion by the simple concept of the polarization vector for the photon. This also is favored from the following consideration.

**IV. Connection of polarization vector and the state for the spin**

The following will give some comments on the real meaning of the state of photon with zero longitudinal component of its spin and the proof of Eq. 5.

In quantum electrodynamics, the spin vector operator of the photon is \( \vec{s} \),

\[
s_{x} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad s_{y} = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}
\]

\[
s_{z} = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad s_{2} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}
\]

The corresponding spin vector \( \chi_{\mu} \) satisfy the following equations

\[
s_{2}^{2}\chi_{\mu} = 2\chi_{\mu}, \quad s_{z}\chi_{\mu} = \mu\chi_{\mu}
\]

That is

\[
\chi_{0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \chi_{1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}, \quad \chi_{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}
\]

The physical meaning of the vectors \( \chi_{1}, \chi_{-1} \) is clear, the right and the left circular polarization. Equivalently one says that the angular spin of the photon is \( \pm 1 \) along the direction \( z \).

The state \( \chi_{0} \) shows that \( s_{z}\chi_{0} = 0 \). However, the state \( \chi_{0} \) is not defined as the state of zero of the \( z \)-component of the spin for the photon, which is defined as the state of

\[
\chi_{1} \pm \chi_{-1}
\]

Actually, it is generally accepted that the state \( \chi_{0} \) is physically unobservable. However, the statement must be taken with care. It is observed in the direction of \( z \) that the state \( \chi_{0} \) is physically unobservable. Whenever we make observation in other direction, we could observe it. The following is the explanation.

The spin vector operator in the direction \( \vec{k} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \) is

\[
s_{k} = \begin{pmatrix} \sin \theta \cos \varphi & \sin \theta \sin \varphi & \cos \theta s_{x} \\ i \cos \theta & 0 & i \sin \theta \sin \varphi \\ -i \sin \theta \sin \varphi & 0 & i \cos \theta \cos \varphi \end{pmatrix}
\]

The corresponding spin vector \( \chi_{k}^{\mu} \) satisfy the following equations

\[
s_{2}^{2}\chi_{k}^{\mu} = 2\chi_{k}^{\mu}, \quad s_{k}\chi_{k}^{\mu} = \mu\chi_{k}^{\mu}
\]

That is

\[
\chi_{0}^{k} = \begin{pmatrix} \sin \theta \cos \varphi \\ \sin \theta \sin \varphi \end{pmatrix}
\]

\[
\chi_{1}^{k} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sin \varphi + icos\theta \cos \varphi \\ -\cos \varphi + icos\theta \sin \varphi \end{pmatrix}
\]

\[
\chi_{-1}^{k} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sin \varphi - icos\theta \cos \varphi \\ -\cos \varphi - icos\theta \sin \varphi \end{pmatrix}
\]
The spin state of the photon related to the direction $\vec{k}$ is

$$\chi = C_0^k \chi_0^k + C_1^k \chi_1^k + C_{-1}^k \chi_{-1}^k$$ (17)

Of course, spin state of the photon related to the direction $z$ is

$$\chi = C_0 \chi_0 + C_1 \chi_1 + C_{-1} \chi_{-1}$$ (18)

The relation between $C_i$, $C_i^k$, $i = 0, \pm 1$ is

$$C_0^k = C_0 \cos \theta + \frac{\sin \theta}{\sqrt{2}} (C_1 e^{-i \phi} + C_{-1} e^{i \phi})$$ (19)

$$C_1^k = \frac{i \sin \theta}{\sqrt{2}} C_0 + \frac{1 - \cos \theta}{2} C_1 e^{-i \phi} - i \frac{1 + \cos \theta}{2} C_{-1} e^{i \phi}$$ (20)

$$C_{-1}^k = - \frac{i \sin \theta}{\sqrt{2}} C_0 + \frac{1 + \cos \theta}{2} C_1 e^{-i \phi} - i \frac{1 - \cos \theta}{2} C_{-1} e^{i \phi}$$ (21)

It is easy to see that the state $\chi = \chi_0$ could be written as

$$\chi_0 = \cos \theta \chi_0^k + \frac{i \sin \theta}{\sqrt{2}} \chi_1^k - i \frac{\sin \theta}{\sqrt{2}} \chi_{-1}^k$$ (22)

From the above equation, we will obtain Eqs. (4), (5) under the assumption $\theta = \frac{\pi}{2}$.

Of course, it is the state $\chi_0^k$ that is unobservable in the direction $\vec{k}$. Hence, one still could observe the state $\chi_0$ as

$$\frac{i \sin \theta}{\sqrt{2}} (\chi_1^k - \chi_{-1}^k) = \begin{pmatrix} - \sin \theta \cos \theta \cos \varphi \\ - \sin \theta \cos \theta \sin \varphi \\ \sin^2 \theta \end{pmatrix}$$ (23)

$$= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - \cos \theta \begin{pmatrix} \sin \theta \cos \varphi \\ \sin \theta \sin \varphi \\ \cos \theta \end{pmatrix}$$ (24)

$$= \vec{e}_z - \cos \theta \vec{k}.$$ (25)

From the above equation, it is easy to see that the state $\chi_0$ will be observed as $\frac{i \sin \theta}{\sqrt{2}} (\chi_1^k - \chi_{-1}^k)$, linearly polarized, in the direction $\vec{k}$. Therefore, $\chi_0$ is not observed only in the direction of $z$ axis, and will be observed in other directions. Eq. (25) shows that $\frac{i \sin \theta}{\sqrt{2}} (\chi_1^k - \chi_{-1}^k)$ is the vector of the polarization projected from the vector $\vec{e}_z$ of the photon in the direction orthogonal to the vector $\vec{k}$. So, it is reasonable to assume that $\chi_0$ state is one that the photon is polarized along the $z$ axis direction. Furthermore, as already pointed, the spin concept will give rise to controversies for the explanation concerning the polarization of $\pi$ line in ZE. Therefore, we consider that the polarization of light is more physically favorable than the concept of spin of photon.

V. Conclusion

Conclusion comes as following. Though it may give somehow simpler interpretation such as in the case of that concerning the $\sigma$-lines, the spin of the photon might give some wrong information as in the case related that of the $\pi$-lines in ZEs. All ZE experiments will be ok with the polarization vector to replace it. Therefore, we are more favorable in substitution of the spin of photon by its polarization vector in the explanation of ZEs.

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[1] C. J. Foot, *Atomic Physics* (Sciencecep, Beijing, China, This adaption is published by arrangement with Oxford University Press, U.K. and is for sale in the Mainland of P.R.C., 2005), pp29-31, pp
[2] W. Heitler, *Quantum theory of radiation*, 3rd (Clarendon: Oxford, 1953), pp401-404
[3] Berestetski, V. B.; Lifshits, E. M.; Pitaevskii, L. P. Quantum Electrodynamics; 2nd ed; (Interscience Publishers, 1965), pp17-31
[4] L. Mandel and E. Wolf *Optical Coherence and Quantum Optics*, (Cambridge University Press, NewYork, 1995), pp484-491
[5] H. Haken and H.C. Wolf, The physics of Atoms and Quanta Sixth edition, (Spring-Verlag, Berlin, 2000), pp212-214
[6] F. J. Yang *Atomic Physics*, 3rd,( Higher Education Press, Beijing, 2000), pp176-180
[7] G. G. Liu *The polarization of the light in Zeeman effects*, College of Physics, 13(11), 1994 year (in chinese)
[8] H. B. Cui *tomic Physics*,(Press of University of Chinese Science and technology 2009. HeFei, China, 2009), pp.270-271
[9] R. A. Beth, "Detection and Measurement of the angular momentum of light", Phys. Rev. 50, 115-125 (1936).