Application of the polynomial filtering algorithm to identification of the narrowband process parameters

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Abstract. A polynomial filtering algorithm has been implemented to identify the central frequency and correlation interval of a narrowband process. The efficiency of the polynomial filtering algorithm is carried out by comparing it with the potential accuracy calculated using the algorithm based on multi-alternative filtering. Recurrent relations are proposed for determining the lower bound of accuracy for estimating the parameters of a narrowband process. The possibility of using less computationally expensive procedures for calculating the lower bound of accuracy is investigated.

1. Introduction

When solving problems of processing navigation information, one often has to face the need to identify the parameters of narrow-band processes.

To solve the identification problem, methods of nonlinear Bayesian filtering [1-4] are widely used, aimed at finding optimal estimates of the required parameters in the mean square sense. This approach is explored in this report. In works devoted to the problem of identifying parameters of a narrow-band process, it is usually assumed that several parameters are subject to identification, which, in particular, are the central frequency and the correlation interval. At the same time, their identification was carried out by finding the optimal Bayesian estimate obtained from the measurements of the realizations of this process, and the algorithm for finding estimates is based on optimal algorithms, often called multi-alternative filtering methods [5-7]. These algorithms can be constructed by approximately reproducing the posterior density using various numerical methods (based, for example, on the Monte Carlo method and their various modifications) [1-10]. The main disadvantage of such algorithms is significant computational complexity, which makes it necessary to develop suboptimal algorithms that require significantly less computation. Such algorithms can be constructed on the basis of the Gaussian approximation of the posterior density (Kalman-type algorithms [11-17]). In particular, in the presence of nonlinearities in the form of polynomials, so-called polynomial filtering algorithms have become widespread [13-17].

The purpose of this work is to construct and analyze the efficiency of polynomial filtering algorithms for solving the problem of identifying the parameters of a narrow-band process. The analysis of the efficiency of the polynomial filtering algorithm is performed by comparing the achieved estimation accuracy with the potential accuracy calculated using multi-alternative filtering algorithms. When analyzing potential accuracy, the Rao-Cramer inequality is also used, which sets the lower bound for the unconditional covariance matrix [18-21].
2. Problem statement

The problem of identifying the parameters of a narrow-band process can be reduced to the problem of estimating a Markov sequence whose equations of state are second-order polynomial with respect to the components of the state vector. In this case, the measurement model can also be approximately described in the form of a second-order polynomial relative to the components of the state vector. This forms the basis for the use of polynomial filtering algorithms.

When solving the problem under consideration, it is assumed that a model of Gaussian narrow-band process $z(t)$ with correlation function is given by the following relation:

$$k(\tau, \alpha, \beta) = \sigma_s^2 e^{-\alpha|\tau|} \cos \beta \tau.$$  \hspace{1cm} (1)

The reciprocal of the correlation interval $\alpha = 1/\tau_\alpha$ and the circular center frequency $\beta$ of the process $z(t)$ are unknown. The process variance $\sigma_s^2$ is assumed to be known. Scalar measurements of this process are made at discrete instants of time $t_i$

$$y(t_i) = z(t_i) + v(t_i), \quad t_i = t_{i-1} + \Delta t, \quad t_i \in [0, T],$$  \hspace{1cm} (2)

where $v_i = v(t_i)$ - measurement errors uncorrelated with $z_i$ representing Gaussian discrete white noise with known variance $\sigma_v^2$, $\Delta t$ - measurement discreteness, $T$ - measurement interval. The vector of unknown parameters is denoted as $\theta = (\alpha, \beta)^T$.

The problem is to estimate vector $\theta$ by measurements $Y_i = (y(t_1), y(t_2),..., y(t_l))^T$ within the framework of the Bayesian approach and to calculate posterior covariance matrix of its estimation errors.

For a process $z(t)$ with correlation function (1), the following shaping filter can be used [1, 9]

$$\begin{align*}
\dot{x}' &= -\alpha x' + \sqrt{2\alpha \sigma_s^2} w_i, \\
\dot{x}'' &= -\alpha x'' + \sqrt{2\alpha \sigma_s^2} w_i, \\
z(t) &= x'(t) \cos(\beta t) + x''(t) \sin(\beta t),
\end{align*}$$  \hspace{1cm} (3)

where $x'(t), x''(t)$, are the so-called quadratic components, representing independent Markov processes with correlation function of $k_s(\tau) = \sigma_s^2 e^{-\alpha|\tau|}, \quad l = I, II$ type. Assuming the vector of unknown parameters $\theta$ as a random Gaussian vector with known mathematical expectation $E_\theta(\theta) = \bar{\theta} = (\bar{\alpha}, \bar{\beta})^T$ and covariance matrix $P_\theta = E_\theta((\theta - \bar{\theta})(\theta - \bar{\theta})^T) = \text{diag}(\sigma_{\alpha}^2, \sigma_{\beta}^2)$, and introducing the vector $\tilde{x}_i = (x'(t_i), x''(t_i), \theta^T) = (x_i', x_i'', \theta_i)$ ($E$ is the sign of the mathematical expectation with a subscript explaining the density of the probability distribution it is calculated from) after sampling [15] we can formulate the following filtering problem in digital form: estimation of the vector $\tilde{x}_i$

$$\begin{align*}
x_i' &= (1 - \Delta t \alpha_{i-1}) x_{i-1}' + \sqrt{2\alpha_{i-1}^2 \sigma_s^2} \Delta w_i', \\
x_i'' &= (1 - \Delta t \alpha_{i-1}) x_{i-1}'' + \sqrt{2\alpha_{i-1}^2 \sigma_s^2} \Delta w_i'', \\
\alpha_i &= \alpha_{i-1} + w_i', \\
\beta_i &= \beta_{i-1} + w_i'',
\end{align*}$$  \hspace{1cm} (5)

by measurements

$$y_i = H_i(\tilde{x}_i) + v_i = \cos(\beta_i \Delta t) x_i' + \sin(\beta_i \Delta t) x_i'' + v_i,$$  \hspace{1cm} (6)

where $w_i' \in N(0, 0.1/\Delta t), \quad l = I, II$ are random sequences independent of each other; $x_0 \in N(x_0, \bar{\theta}, P_0^I), \quad \bar{x}_0 = (\bar{x}_0', \bar{x}_0'')^T = (E_\theta(x'(0)), E_\theta(x''(0)))^T$ is a random vector describing the state.
of a narrow-band process at the initial moment of time (Hereinafter, we use the following notation: \( N(\alpha, \overline{a}, A) \) is the probability density function of a Gaussian random vector \( \alpha \) with a mathematical expectation \( \overline{a} \) and a covariance matrix \( A \)); \( \nu \in N(\nu, 0, R) \) is the Gaussian noise of measurements with known intensity \( R \).

We introduce the following notation:
- the sign \( \otimes \) denotes the Kronecker product [22].
- \( I_m \) - unity matrix of dimension \( m \times m \).
- \( w^T_i = (w_i^l, w_i^e, w_i^m, w_i^{pr}) \) - composite vector of generating noise.
- \( F_i \), \( \Gamma \) and \( F_{ss} \) - known arrays of size \( 4 \times 4 \), \( 4 \times 4 \), and \( 16 \times 4 \) accordingly.

Using this notation system (3) can be represented as follows:
\[
\bar{x}_i = (F_i + (\bar{x}_{i-1} \otimes I_4)) \bar{x}_{i-1} + \Gamma w_i,
\]
where \( F_i = I_4 \), \( \Gamma = \text{diag} \left( \sqrt{2\sigma_x^2}, \sqrt{2\sigma_y^2}, 1, 1 \right) \), \( F_{ss} \) - matrix with almost all 0 elements, except for \( \{F_{ss}\}_i = \{F_{ss}\}_{i, 1, 3} = \{F_{ss}\}_{i, 2, 4} = -0.5\Delta t \), \( \{F_{ss}\}_{i, j} \) - element of matrix \( F_{ss} \), located in row number \( i \) and \( j \)-th column. The initial state vector \( x_0 \) is a Gaussian random vector with mathematical expectation \( E_x(\bar{x}_0) = \left( \bar{x}_{01}, \bar{x}_{02}, \bar{x}_{03}, \bar{x}_{04} \right)^T \) and the covariance matrix \( P_0 = \text{diag} \left( \sigma_{x^l}, \sigma_{x^e}, \sigma_{x^m}, \sigma_{x^{pr}} \right) \); the generating noise vector \( w_i \) is a centered Gaussian with the covariance matrix \( Q = \text{diag} \left( \frac{1}{\Delta t}, \frac{1}{\Delta t}, \sigma_{w^l}, \sigma_{w^e}, \sigma_{w^m}, \sigma_{w^{pr}} \right) \), where \( \sigma_{w^a}, \sigma_{w^b} \) - the known intensities of the generating noise of the estimated parameters.

When solving the problem, we use an approximate measurement model, which is the sum of the first two terms of measurement function \( H(\bar{x}_i) \) Taylor expansion:
\[
y_i \approx H^{(lin)}(\bar{x}_i) = \cos(\beta_i^{(lin)} i \Delta t) x_i^{(lin)} + \sin(\beta_i^{(lin)} i \Delta t) \bar{x}_i^{(lin)} + \left( H_i^l + \left( \bar{x}_i - \bar{x}_i^{(lin)} \right)^T H_i^m \right) \left( \bar{x}_i - \bar{x}_i^{(lin)} \right) + \nu_i, \tag{8}
\]
where
\[
H_i^l = \frac{dH_i(\bar{x}_i)}{d\bar{x}_i} \bigg|_{\bar{x}_i = \bar{x}_i^{(lin)}} = \left( \begin{array}{cc} \cos(\beta_i^{(lin)} i \Delta t) & \sin(\beta_i^{(lin)} i \Delta t) \\ i \Delta t & 0 \end{array} \right) H_i^m = \left( \begin{array}{cc} 0 & -\sin(\beta_i^{(lin)} i \Delta t) i \Delta t \\ 0 & \cos(\beta_i^{(lin)} i \Delta t) i \Delta t \end{array} \right),
\]
\[
H_i^{ss} = \frac{d^2H_i(\bar{x}_i)}{d\bar{x}_i^2} \bigg|_{\bar{x}_i = \bar{x}_i^{(lin)}} = \frac{1}{2} \left( \begin{array}{cccc} 0 & 0 & 0 & -\sin(\beta_i^{(lin)} i \Delta t) i \Delta t \\ 0 & 0 & 0 & \cos(\beta_i^{(lin)} i \Delta t) i \Delta t \\ -\sin(\beta_i^{(lin)} i \Delta t) i \Delta t & \cos(\beta_i^{(lin)} i \Delta t) i \Delta t & 0 & -i \Delta t \sin(\beta_i^{(lin)} i \Delta t) x_i^{(lin)} + \cos(\beta_i^{(lin)} i \Delta t) x_i^{(lin)} \end{array} \right),
\]
\[
\bar{x}_i^{(lin)} = \left( x_i^{(lin)} \bar{x}_i^{(lin)} \alpha_i^{(lin)} \beta_i^{(lin)} \right)^T \text{ are linearization points of function } H_i(\bar{x}_i).
\]

To calculate the optimal in the mean-square sense estimate (estimate with minimum variance) of the composite vector \( \bar{x}_i \) and the corresponding covariance matrix of the estimation errors \( P(Y_i) \), it is necessary to have the posterior density \( p(\bar{x}_i Y_i) \), where \( Y_i = (y_1, ..., y_j)^T \). The calculation of \( \bar{x}_i \) and
\( P(Y_i) \) can be performed using Kalman-type algorithms based on the Gaussian approximation of the posterior density. In this work, for these purposes, the polynomial filtering algorithm [13-17] is used.

3. Polynomial filter
When constructing this filter a Gaussian approximation of the posterior density in the form of
\[
p(\hat{x}_i | Y_{i-1}) = N(\hat{x}_i; \hat{x}_{i|i-1}^{PF}, P_{i|i-1}^{PF}(Y_{i-1}))
\]
is used at each step. The forecast density parameters at each step can be determined according to the following expressions
\[
\hat{x}_{i|i-1}^{PF} = E_{p(\hat{x}_i | Y_{i-1})} (\hat{x}_i) = E_{p(\hat{x}_i | Y_{i-1})} \left( \left( F_x + \left( \hat{x}_{i-1} \otimes I_a \right) F_{x_a} \right) \hat{x}_{i-1} \right),
\]
\[
P_{i|i-1}^{PF} = E_{p(\hat{x}_i | Y_{i-1})} \left( \left( \hat{x}_i - \hat{x}_{i|i-1}^{PF} \right) \left( \hat{x}_i - \hat{x}_{i|i-1}^{PF} \right)^T \right). \tag{9}
\]

Hereinafter, in order to simplify the notation, the \( P(Y_i) \) dependence on \( Y_i \) is not indicated. It can be shown that, taking into account the assumptions made, the expression for the assessment will be as follows [14-17]
\[
\hat{x}_{i|i-1}^{PF} = F_{x,i-1}^{PF} + \left( \left( \hat{x}_{i-1}^{PF} \otimes I_a \right) + F_{x_a} \right) \hat{x}_{i-1}^{PF} + Tr_{\otimes 4} \left( F_x \hat{x}_{i-1}^{PF} \right), \tag{10}
\]
\[
P_{i|i-1}^{PF} = 2Tr_{\otimes 4} \left( F_x \hat{x}_{i-1}^{PF} \right) + 4 \left( \left( \hat{x}_{i-1}^{PF} \otimes I_a \right) + F_{x_a} \right) \hat{x}_{i-1}^{PF} + 4 \left( \hat{x}_{i-1}^{PF} \otimes I_a \right) + F_{x_a} \hat{x}_{i-1}^{PF} + \Gamma Q^{-1}, \tag{11}
\]
\[
\hat{y}_{i|i-1} = H \left( \hat{x}_{i|i-1}^{PF} \right) + \text{tr} \left( H_{i} \otimes P_{i|i-1}^{PF} \right), \tag{12}
\]
\[
P_{y_i} = R + H_{i} \otimes P_{i|i-1}^{PF} \left( H_{i} \right)^T + 2 \text{tr} \left( H_{i} \otimes P_{i|i-1}^{PF} \right). \tag{13}
\]

In these expressions, the operator \( Tr_{\otimes a \times b} : R^{a \times n \times b} \rightarrow R^{a \times b} \) translates \( n \cdot a \times n \cdot b \) matrix
\[
M = \begin{bmatrix} M_{11} & \ldots & M_{1b} \\ \vdots & \ddots & \vdots \\ M_{a1} & \ldots & M_{ab} \end{bmatrix} \tag{14}
\]
\((M_{i,j}, i=1,\ldots,a, j=1,\ldots,b - \text{arrays of size } n \times n) \) to \( a \times b \) matrix according to the expression
\[
Tr_{\otimes a \times b} \left( M \right) = \begin{bmatrix} \text{tr} \left( M_{11} \right) & \ldots & \text{tr} \left( M_{1b} \right) \\ \vdots & \ddots & \vdots \\ \text{tr} \left( M_{a1} \right) & \ldots & \text{tr} \left( M_{ab} \right) \end{bmatrix}. \tag{15}
\]

4. Lower bound based on the Rao-Cramer inequality
To estimate vector \( \hat{x}_i \), the Rao-Cramer inequality [1,18-21] is written as \( J_i^{-1} \leq G_{i}^{opt} \), where \( G_{i}^{opt} \) is the covariance matrix corresponding to the potential accuracy of solving the problem in the framework of the Bayesian approach, and \( J_i^{-1} \) is the matrix characterizing the lower bound of accuracy [1,20-21]. This matrix \( J_i \) can be calculated recursively by the formula [2]:
\[ J_i = \left( \Gamma Q \Gamma^T \right)^{-1} + S_i + \left( \Gamma Q \Gamma^T \right)^{-1} A_i \left( J_{i-1} + \Delta A \right)^{-3} A_i^T \left( \left( \Gamma Q \Gamma^T \right)^{-1} \right)^T , \]  

(18)

where

\[ S_i = E_{\chi_4} \left( \frac{dH_i(x_i)}{dx_i} \right) R^{-1} \left( \frac{dH_i(x_{\chi_i})}{dx_i} \right), \]  

(19)

\[ \Delta A = E_{\chi_4} \left( \frac{\partial \left( F_{\chi_4} x_{\chi_i} + \left( x_{\chi_i} \right)^T \otimes I_4 \right) \Gamma_{\chi_4} x_{\chi_i} - \left( x_{\chi_i} \right)^T \otimes I_4 \Gamma_{\chi_4} x_{\chi_i} }{\partial x_{\chi_i}} \right) \left( \Gamma Q \Gamma^T \right)^{-3} \left( \frac{\partial \left( F_{\chi_4} x_{\chi_i} + \left( x_{\chi_i} \right)^T \otimes I_4 \right) \Gamma_{\chi_4} x_{\chi_i} - \left( x_{\chi_i} \right)^T \otimes I_4 \Gamma_{\chi_4} x_{\chi_i} }{\partial x_{\chi_i}} \right)^T , \]  

(20)

\[ A_i = E_{\chi_4} \left( \frac{\partial \left( F_{\chi_4} x_{\chi_i} + \left( x_{\chi_i} \right)^T \otimes I_4 \right) \Gamma_{\chi_4} x_{\chi_i} - \left( x_{\chi_i} \right)^T \otimes I_4 \Gamma_{\chi_4} x_{\chi_i} }{\partial x_{\chi_i}} \right) \left( \Gamma Q \Gamma^T \right)^{-3} \left( \frac{\partial \left( F_{\chi_4} x_{\chi_i} + \left( x_{\chi_i} \right)^T \otimes I_4 \right) \Gamma_{\chi_4} x_{\chi_i} - \left( x_{\chi_i} \right)^T \otimes I_4 \Gamma_{\chi_4} x_{\chi_i} }{\partial x_{\chi_i}} \right)^T , \]  

(21)

From expression (18) it follows that, to find matrix \( J_i \), it is necessary to determine the value of integrals (19) - (21), which can be calculated approximately, for example, using the Monte Carlo method [23].

As a result, the following relations were obtained to calculate integrals (19) - (21), as applied to the problem under consideration:

\[ S_i = R^{-1} \begin{bmatrix} \{ S_i \}_{1,1} & \{ S_i \}_{1,2} & 0 & \{ S_i \}_{1,4} \\ \{ S_i \}_{1,2} & \{ S_i \}_{2,2} & 0 & \{ S_i \}_{2,4} \\ \{ S_i \}_{1,4} & \{ S_i \}_{2,4} & 0 & \{ S_i \}_{4,4} \end{bmatrix} , \]  

(22)

\[ \{ S_i \}_{1,1} = 0.5 \left( \cos \left( 2 \{ \chi_i \}_4 \ i \Delta t \right) e^{2 \left( \Delta t \right)^2 \left[ \phi \right]_{1,4} + 1 \right), \]  

\[ \{ S_i \}_{1,2} = 0.5 \sin \left( 2 \{ \chi_i \}_4 \ i \Delta t \right) e^{2 \left( \Delta t \right)^2 \left[ \phi \right]_{1,4}}, \]  

(23)

\[ \{ S_i \}_{1,4} = e^{2 \left( \Delta t \right)^2 \left[ \phi \right]_{1,4}} \left( \left( \{ \chi_i \}_4 \ i \Delta t \right) \left( \{ \chi_i \}_1 \ sin \left( 2 \{ \chi_i \}_4 \ i \Delta t \right) - \{ \chi_i \}_4 \ sin \left( 2 \{ \chi_i \}_4 \ i \Delta t \right) \right) + \frac{i \Delta t}{2} \left( \left( \{ \chi_i \}_2 \ cos \left( 2 \{ \chi_i \}_4 \ i \Delta t \right) - \{ \chi_i \}_1 \ sin \left( 2 \{ \chi_i \}_4 \ i \Delta t \right) \right) \right) \]  

(24)

\[ \{ S_i \}_{2,1} = e^{2 \left( \Delta t \right)^2 \left[ \phi \right]_{1,4}} \left( \left( \{ \chi_i \}_4 \ i \Delta t \right) \left( \{ \chi_i \}_1 \ sin \left( 2 \{ \chi_i \}_4 \ i \Delta t \right) - \{ \chi_i \}_4 \ sin \left( 2 \{ \chi_i \}_4 \ i \Delta t \right) \right) + \frac{i \Delta t}{2} \left( \left( \{ \chi_i \}_2 \ cos \left( 2 \{ \chi_i \}_4 \ i \Delta t \right) - \{ \chi_i \}_1 \ sin \left( 2 \{ \chi_i \}_4 \ i \Delta t \right) \right) \right) \]  

(25)

\[ \{ S_i \}_{2,2} = e^{2 \left( \Delta t \right)^2 \left[ \phi \right]_{1,4}} \left( \left( \{ \chi_i \}_4 \ i \Delta t \right) \left( \{ \chi_i \}_1 \ sin \left( 2 \{ \chi_i \}_4 \ i \Delta t \right) - \{ \chi_i \}_4 \ sin \left( 2 \{ \chi_i \}_4 \ i \Delta t \right) \right) + \frac{i \Delta t}{2} \left( \left( \{ \chi_i \}_2 \ cos \left( 2 \{ \chi_i \}_4 \ i \Delta t \right) - \{ \chi_i \}_1 \ sin \left( 2 \{ \chi_i \}_4 \ i \Delta t \right) \right) \right) \]  

(26)
\[ S_3 = \frac{1}{2} e^{-2(j\omega t)\{P\}_{1,1}} \cos(2\beta_j j\Delta t) \left( \left( \tilde{x}_j^T + 2 \{ P \}_{1,2} j\Delta t \right)^2 + \{ P \}_{1,1} - 8 \left( \{ P \}_{1,2} j\Delta t \right)^2 \right) - \]
\[ \left( \tilde{x}_j^T + 2 \{ P \}_{1,2} j\Delta t \right)^2 - \{ P \}_{1,1} + 8 \left( \{ P \}_{1,2} j\Delta t \right)^2 \right) - \]
\[ \frac{1}{2} e^{-2(j\omega t)\{P\}_{1,1}} \sin(2\beta_j j\Delta t) \left( \tilde{x}_j \tilde{x}_j'' + \{ P \}_{1,2} - 4 \{ P \}_{1,2} \left( \{ P \}_{1,2} j\Delta t \right)^2 - 4 \{ P \}_{1,2} \tilde{x}_j j\Delta t + 4 \{ P \}_{1,2} \tilde{x}_j'' j\Delta t \right) \]
\[ A_0 = \left( F_y + 2 \left( \tilde{x}_T \otimes I_n \right) F_{ab} \right) \left( \Gamma Q \Gamma^T \right)^{-1} \left( F_y + 2 \left( \tilde{x}_T \otimes I_n \right) F_{ab} \right) + \]
\[ 4T_{res} \left( \{ \tilde{P} \otimes I_n \} F_{ab} \right) \left( \Gamma Q \Gamma^T \right)^{-1} \frac{\Delta A}{T} \]
The lower bound calculated in this way is denoted as $\left(J^{-1}_k\right)^{-1}$.

5. Simulation results

During the simulation it was assumed that $\bar{\alpha}_0 = (0,0)^T$, $\bar{\alpha} = 60 \, s^{-1}$, $\bar{\beta} = 165500 \, s^{-1}$, $\sigma_\alpha^2 = 1$, $\sigma_\alpha = 10 \, s^{-1}$, $\sigma_\beta = 100 \, s^{-1}$, $R = 1.1$, $\sigma_\alpha^2 = 0.0001 \, s^{-2}$, $\sigma_\beta^2 = 0.0005 \, s^{-2}$.

Process observation time is 1 s, sampling step is $\Delta t = 1/300$, s. Figures 1 and 2 show the graphs of the calculated and actual characteristics of accuracy - the mean square errors in determining the value of the inverse correlation interval (Fig. 1) and the circular center frequency (Fig. 2). The purple and blue curves correspond to the calculated and actual estimates of the accuracy for the algorithm based on the multi-filtering method [8-9] (using 50 alternative hypotheses). The red and green curves correspond to the calculated and actual estimates of the accuracy for the polynomial filtering algorithm. The averaging was carried out over 100 realizations. The black and yellow curves correspond to the lower boundary $\left(J^{-1}_i\right)^{-1}$ and $\left(J^{-1}_k\right)^{-1}$ respectively $\left(J^{-1}_i\right)^{-1}$ built on 500 iterations of the Monte Carlo method.

![Figure 1. Low bounds and calculated and real RMS of estimation $\alpha$.](image-url)
6. Conclusion
An algorithm for polynomial filtering and simplified expressions for calculating the estimation accuracy lower bound for the quantity inverse to the correlation interval and circular center frequency in the problem of identifying the parameters of a Gaussian narrow-band random process is described. The possibility and expediency of their application is illustrated by a methodological example.

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