Multi-Antenna Jamming in Covert Communication

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Abstract—Covert communication conceals transmission of messages from Alice to Bob out of a watchful adversary, Willie, which tries to determine if a transmission took place or not. While covert communication in a basic, vanilla settings where all variables are known to Willie results in the well known square-root law, when a jammer is present and assists Alice by creating uncertainty in Willie’s decoder, this transmission may have a positive rate.

In this work, we analyze the case where the jammer is equipped with multiple antennas and obtain the optimal transmission strategy of the jammer in order to maximize his assistance to Alice, in terms of maximizing a ratio between Willie’s and Bob’s noise variance. We show that the optimal strategy of the jammer is to perform beamforming towards a single direction with all his available power. This direction though, is not trivial, since it reflects an optimal tradeoff point between minimizing the interference at Bob and maximizing the interference at Willie.

I. INTRODUCTION

In covert communication (also known as Low Probability of Detection - LPD) Alice tries to communicate reliably a message to Bob, such that a watchful adversary Willie remains unaware of the presence of the communication. To make this possible, Alice may use the fact that the channels between all participants are subject to some kind of noise, and therefore she can try to hide her communication within the margin of uncertainty at Willie’s decoder. In fact, for AWGN channels, it was shown in [1] that Alice can transmit $O(\sqrt{n})$ bits in $n$ channel uses (a.k.a the square root law). Extensions for binary symmetric, discrete memoryless and multiple access channels were done in [2]–[5], respectively.

This law essentially means that the transmission rate goes to zero with $n$; however, subsequent works showed that $O(n)$ bits in $n$ channel uses can be achieved, namely, a strictly positive rate, if Willie suffers from some kind of uncertainty in his received noise power ([6]–[8]). The uncertainty may be a result of inaccurate knowledge of Willie’s noise or a result of an active node which confuses Willie’s side (e.g., a jammer that varies his noise power randomly). The ability to achieve a strictly positive rate is of great importance since existing coding schemes can be used instead of designing special codes which are suitable only for covert communication.

The limits of covert communication in a multiple-antenna setting were first established in [9]. Therein, it was shown that in case Alice is equipped with multiple antennas, her best strategy is to perform beamforming towards Bob, which results with a constant gain to the square root law by the number of independent paths between her and Bob. However, the case where such a communication channel includes a jammer which is equipped with multiple antennas is still open and remains unclear under various settings. For example, the knowledge the jammer has on the Channel State Information (in particular the CSI of Willie), and his preference on which user to assist, may affect his strategy, and the resulting rates, significantly.

In this work, we analyze the effect of multiple antennas at the jammer on covert communication, while assuming the jammer chooses to assist Alice and Bob. For simplicity, we assume that Alice and Willie have a single antenna. Where we note that a strictly positive power (rate) can be achieved when there is a jammer with a single antenna ([8]) and that the limits for MIMO settings without a jammer was already examined ([9]). Therefore, adding more antennas at Alice and Willie won’t contribute much additional insight. The jammer’s assistance comes in the form of transmitting Artificial Noise (AN) while using all his multiple antennas. Thus, his transmission strategy is reflected by the covariance matrix for the random vector he chooses. In a way, this covariance matrix defines the power allocation and the directions for that allocation. Accordingly, we analyze the behavior of Alice’s transmission power as a function of the jammer’s strategy which, to the best of our knowledge, was not yet examined under the settings of covert communication.

The notion of AN transmission was also considered in wiretap channels problems where the AN was used to improve the secrecy rate of the system. These works can be roughly divided into two cases which differ by the source of the AN transmission. In the first case it is assumed that the transmitter (Alice) is equipped with multiple antennas and she performs simultaneously the secret transmission and the AN, e.g., [10]–[15]. In the second case, there exist an additional node(s) in the system, e.g., a relay(s) ([16]–[19]) or a dedicated jammer ([20], [21]), which output AN to degrade the eavesdropper’s channel. In the heart of the analysis of these mentioned works, one can find different but similar optimization problems depending on the models’ assumptions. Where, a major factor in all, is the CSI which is known or unknown to the nodes in the system. Regardless of this knowledge, it was shown in [11], that the transmission of AN can guarantee a minimal secrecy rate. This result is consistent with the fact that a positive rate is also achievable in covert communication problems when there exists uncertainty at Willie’s detector. Similarly to the wiretap channel problems, in this work, we are tackled also with optimization problems for the optimal AN transmission strategy of the jammer. However, these optimization problems are originated from different settings and requirements of our covert communication model.
Main Contribution: As mentioned above, in this work we explore the effect of multiple antennas of an uninformed jammer on covert communication. Where we do not consider the problem of whether a strictly positive rate can be achieved or not, since this was already shown, for the case of a single antenna jammer in [8]; thus, the same consequences follow.

Moreover, when considering the requirements of covert communication, i.e., the covertness and the ability for successful decoding by Bob, one should pay more weight for the importance of the analysis on the transmission power that achieves that covertness. This is because the sole purpose of this kind of communication is to be undetectable by Willie. Obviously, if Bob is unable to decode the message then there wouldn’t be any communication at all. However, we know that as long as positive transmission power is attainable, one can always design codes with negligible probability of error (or outage).

Accordingly, in this work, we first concentrate on the covertness-achieving transmission power of Alice. Specifically, we provide the transmission power as a function of the jammer’s transmission strategy for the cases which CSI on Willie is globally known or unknown. The transmission power promises that the system is covert; that is, Willie has nothing better to do besides guessing if communication occurred or not. We show that multiple antennas at the jammer provide additional gain for Alice’s transmission power with respect to the case of single antenna jammer. And for specific AN transmission strategies by the jammer, we may obtain a specific characterization of its behavior. Furthermore, when the CSI of Willie is unknown, we show that the transmission power of Alice is proportional to the number of antennas the jammer has.

Provided that the system is covert, assuming Alice uses the covertness achieving transmission power, we turn to analyze the covert rate as a function of the jammer’s transmission strategy. We choose to use Bob received SNR as our figure of merit since in most cases the actual rate of any coding scheme is an increasing function of the SNR. Specifically, we consider the optimization problems arises for the maximization of the received SNR according to the jammer’s strategy. It turns out that this maximization is not trivial, as the jammer’s strategy affects both in opposite directions. Nevertheless, we solve the optimization problems for the cases of known and unknown CSI on Willie. The solution is essentially describing the directions and power allocations the jammer’s transmission should take upon. When the CSI is known, the optimal transmission strategy for the jammer is to transmit in a single direction, with all of his available power. In general, this direction is not trivial since it takes into account both the channel coefficients to Bob as well as those to Willie. Thus, the transmission direction has a tradeoff between being orthogonal to Bob and in the direction of Willie. When the CSI of Willie is not known but the CSI of Bob is known to the jammer, we show that transmitting the AN diffusely into the null-space of Bob is not always optimal.

Moreover, in our model, we assume that Bob is equipped also with multiple antennas while using a linear receiver. This provides even further depth to the optimizations in question as Bob takes an active part in the decoding process. We extend the optimization problems for this case and provide insightful result accompanied by simulations.

II. SYSTEM MODEL

We consider a system in which Alice (“a” in the channel coefficients notation) wishes to communicate covertly with Bob (“b”) while Willie’s (“w”) awareness of this communication remains uncertain. Also, we assume that there is a third participant, the jammer (“j”), which may assist either Alice and Bob or Willie, depending on the side he takes and the knowledge he has. This model is depicted in Figure 1. In our settings, we assume that Alice and Willie are equipped with a single antenna, while Bob and the jammer are equipped with M and N antennas, respectively. The channel between all participates is subject to block fading and AWGN. In this setting, Willie tries to detect whether transmission by Alice was made or not, by performing a statistical hypothesis test on his received signal. The null hypothesis H₀ means that no transmission was made by Alice, while the hypothesis H₁ suggests otherwise. Throughout, lower case letters represent random variables, bold lower case represent random vectors, and bold upper case represent random matrices. Thus, under each of the hypotheses, the received signals at Bob and Willie in the i-th channel use are

\[
\begin{align*}
H_1: & \quad y_b[i] = x[i]h_{ab} + H_{jb}v[i] + n_b[i] \\
& \quad y_w[i] = x[i]h_{aw} + v[i]h_{jb} + n_w[i] \\
H_0: & \quad y_b[i] = H_{jb}v[i] + n_b[i] \\
& \quad y_w[i] = v[i]^T h_{jb} + n_w[i],
\end{align*}
\]

where \(x[i]\) is the complex symbol transmitted by Alice in the i-th channel use, with average power \(P_a\) (i.e., \(\mathbb{E}[|x[i]|^2] = P_a\)) and \(v[i] = (v_1[i], v_2[i], \ldots, v_N[i])^T\) is the vector transmitted by the jammer at the i-th channel use, with a covariance matrix \(\Sigma = \mathbb{E}[v[i]v[i]^*]\) and total power \(P_j\).

The channel coefficients between Alice, Willie and Bob are \(h_{aw} \in \mathbb{C}\) and \(h_{ab} \in \mathbb{C}^M\), respectively. \(h_{jb} \in \mathbb{C}^N\) and \(H_{jb} \in \mathbb{C}^{M \times N}\) are the channel coefficients from the jammer.
to Willie and Bod, respectively. These channel coefficients are originated from a zero-mean complex Gaussian distribution with unit variance and are considered to be fixed for the period of $n$ channel uses (a slot). We assume that Bob and the jammer know $h_{ab}$ and $H_{jb}$ and Willie knows $h_{aw}$ and $h_{jw}$. In addition, both Bob and Willie endure complex additive Gaussian noise denoted by $n_b \sim CN(0, \sigma_b^2 I_M)$ and $n_w \sim CN(0, \sigma_w^2 I_N)$. The above assumptions concerning the channel coefficients, follow similar assumptions in the covert communication literature.

This paper focuses on covert communication with a positive covert rate. Thus, we assume that the jammer assists Alice to create uncertainty at Willie’s decoder continuously, regardless of whether or not Alice transmits (Similarly to [8]). We note that the case where the jammer assists Willie is out of this paper’s scope, and may be considered as future work. The jammer’s assistance comes in the form of AN with a power allocation. We thus represent the covariance matrix $\Sigma$ as its columns. Note that under this definition, the matrix $d$ may direct his AN towards certain directions with different powers. This allocation is constructed in a way which he cannot estimate it efficiently. Therefore, following similar assumptions as in [8], [22], we assume that $P_j$ is a uniform r.v. on $[0, P_{max}]$ with the probability density function (pdf) given by,

$$ f_{P_j}(x) = \begin{cases} \frac{1}{P_{max}} & \text{if } 0 \leq x \leq P_{max} \\ 0 & \text{otherwise} \end{cases}, $$

and it’s redrawn for every $n$ channel uses independently. This way, even if Willie correctly estimates the power on the channel during $n$ channel uses, he has no way of knowing whether it is the result of solely the jammer, or does it include Alice’s transmission. The next block has independent drawings. We also note that in case Willie had not known the channel coefficients, the jammer could have used a noise distribution with constant variance since the uncertainty would arise from the random channel coefficients [8].

The jammer allocates its power $P_j$ in each slot according to the covariance matrix $\Sigma$. This allocation is constructed in a way that assists Alice as much as possible. That is, the jammer may direct his AN towards certain directions with different powers. The magnitude of this division is represented by the vector $\xi = (\xi_1, ..., \xi_d)^T$ such that $\sum_{i=1}^d \xi_i = 1$. That is, $P_j \xi$ is the singular values vector of the covariance matrix $\Sigma$, where the corresponding eigenvectors represent the directions for this power allocation. We thus represent the covariance matrix as $\Sigma = P_j V V^H$ where $X$ is a diagonal matrix with $\xi$ as its elements and $V$ is a matrix with the corresponding directions as its columns. Note that under this definition, the matrix $X$ is of dimension $d \times d$ and $V$ is of dimension $N \times d$, where $d$ is the number of directions.

We note here that since Bob knows the channel matrix $H_{jb}$, the position of the jammer does not affect the performance of Bob. Thus, this model and its analysis are also suitable, under some assumptions, for the situation where Bob is equipped with a full-duplex transceiver and he is the source of the AN.

Similar to previous works on covert communication ([1], [7], [8], [22]), we assume that Alice and Bob share a codebook which is not revealed to Willie; however, Willie knows its statistics. This codebook is generated by independently drawing symbols from a zero-mean complex Gaussian distribution with variance $P_j$ and it is assumed to be used only once. When Alice wishes to transmit, she picks a codeword and transmits its $n$ symbols as the sequence $\{x[i]\}_{i=1}^n$.

A. Covert Criteria

Upon receiving the vector $y_w$, Willie performs hypothesis testing to determine if transmission by Alice took place or not. That is, he tries to distinguish between the two hypotheses $H_0$ and $H_1$. Accordingly, Willie seeks to minimize his probability of error which is a function of the probability of miss detection $(P_{MD})$ in case a transmission occurred and the probability of false alarm $(P_{FA})$ in case a transmission did not occur. That is,

$$ P_e = p P_{MD} + (1 - p) P_{FA}, $$

where $p$ is the a priori probability that Alice transmitted a message ($H_1$). According to [8], $P_e \geq \min\{p, 1 - p\} (P_{MD} + P_{FA})$. Thus, we define the following criteria for covert communication, which eventually will define Alice’s transmission power, as

$$ P_{MD} + P_{FA} \geq 1 - \epsilon, $$

where $\epsilon > 0$ is the covertness requirement. That is, as long as $H_0$ holds for a given $\epsilon$, the transmission is considered covert. Note also that this criterion is reasonable for the following reason. Willie can easily choose a strategy with $P_{FA} = 0$ and $P_{MD} = 1$, by simply declaring $H_0$ at all times, regardless of his channel measurements. Analogously, $P_{FA} = 1$ and $P_{MD} = 0$ are achieved by always declaring $H_1$. Requiring $P_{MD} + P_{FA} \geq 1 - \epsilon$ is therefore equivalent to forcing Willie to only time-sharing between these trivial strategies.

The optimal test for Willie to distinguish between $H_0$ and $H_1$ and minimize its probability of error is to apply the Neyman-Pearson criterion, resulting in the likelihood ratio test:

$$ \frac{\prod_{i=1}^n P_0(1 - p) / N_0}{\prod_{i=1}^n P_1} < \frac{p}{1 - p}, $$

where $P_0$ and $P_1$ are the probability distributions of Willie’s observation in a single channel use under the hypotheses $H_0$ and $H_1$, respectively. Note that we may write the joint distribution as a multiplicative of the marginal distributions since the channel uses are i.i.d. In particular, under $H_0$ and given $P_j$, $P_0$ is distributed as $CN(0, \sigma_0^2)$, and under $H_1$ and given $P_j$, $P_1$ is distributed as $CN(0, \sigma_1^2)$ where,

$$ \sigma_0^2 = \sigma_w^2 + h_{jw}^\dagger \Sigma h_{jw}, $$

$$ \sigma_1^2 = \sigma_w^2 + h_{jw}^\dagger \Sigma h_{jw} + P_u |h_{uw}|^2. $$

(5)
The terms in the last line above reflect the self-noise power of Willie, the received AN power and the transmission power of Alice, respectively. Eventually, the optimal ratio test Willie performs is an energy test on the average received power. Specifically, the average received power, $P_{aw}$, is compared with a threshold $\eta$,

$$P_{aw} \triangleq \frac{1}{n} \sum_{i=1}^{n} \|y_{wi}[i]\|^2 < \eta.$$  \hspace{1cm} (6)

This was shown in [8] by using Fisher-Neyman factorization and likelihood ratio ordering techniques. One can realize that, given $P_j$, the average received power $P_{aw}$ is a Gamma r.v. with parameters $k = n$ and $\theta = \frac{a_{wi}^2}{n}$ for $i = 0, 1$, i.e. $P_{aw} \sim \Gamma(n, \frac{a_{wi}^2}{n})$.

In this work, we wish to check the effect of multiple antennas with different CSI at the jammer on covert communication. Therefore, in the next sections, we describe this effect on Alice’s transmission power, which is linked to Willie’s ability of detection, and on Bob received SNR and the jammer’s transmission strategy. Specifically, we provide a criterion for positive covert rate, discuss the received SNR at Bob while transmission strategy. Specifically, we provide a criterion for Alice’s transmission power, which is linked to Willie’s ability of detection, and on Bob received SNR and the jammer’s transmission strategy. Thus, at this stage, we assume that this information is known to Alice. Note that this does not change the main assumption of the paper - that the jammer is uninformed and does not know when Alice will transmit and when will she remain silent. In that sense, the jammer helps Alice and Bob by making noise and shares his channel knowledge with everybody, but he is not a true partner-in-crime and does not know if and when a crime is to be committed. Moreover, we note that Alice can set her transmission power regardless of the channel vector $h_{aw}$. This is because the transmission power is constrained only by the ability of Willie to detect the communication.

**Proof:** As mentioned above, Willie compares $P_{aw}$ to a threshold $\eta$; however, this threshold depends on the distribution of $P_j$ and thus may be optimized by Willie. The following analysis will show that for any optimal threshold $\tau$ that Willie set for himself, there exist a construction by Alice such that $\mathbb{H}_1$ holds. Specifically, we bound each of the probabilities $P_{MD}$ and $P_{FA}$ for a given value of $P_j$ and average it on all possible values of $P_j$ resulting with the necessary conditions for covertness. This proof’s steps are constructed similarly to arguments presented in [8] which were modified to suit the jammer’s antennas. Let us begin with the false alarm probability $P_{FA}$ given $P_j$, i.e.,

$$P_{FA}(P_j) = P_r(P_{aw}^{\delta} \geq \tau | \mathbb{H}_0, P_j).$$

Recall that $P_{aw}^{\delta} \sim \text{Gamma}(n, \frac{a_{wi}^2}{n})$, thus, the expected value of $P_{aw}^{\delta}$ is $\sigma_w^0$. Accordingly, we may describe the probability of $P_{aw}^{\delta}$ to exist around its expected value. Let $\epsilon > 0$ be a fixed small constant. Then, there exist $\delta^0(\epsilon) > 0$ such that

$$P_r(\sigma_w^0 - \delta^0 \leq P_{aw}^{\delta} \leq \sigma_w^0 + \delta^0) > 1 - \epsilon.$$

Since

$$P_r(P_{aw}^{\delta} \geq \sigma_w^0 - \delta^0) \geq P_r(P_{aw}^{\delta} \geq \sigma_w^0 - \delta^0),$$

for some $\delta(\epsilon)$, then for any $\tau < \sigma_w^0 - \delta(\epsilon)$ we have

$$P_r(P_{aw}^{\delta} \geq \tau | \mathbb{H}_0, P_j) > 1 - \epsilon.$$

Similarly for $P_{MD}$ given $P_j$, i.e.,

$$P_{MD}(P_j) = P_r(P_{aw}^{\delta} \leq \tau | \mathbb{H}_1, P_j),$$

$P_{aw}^{\delta} \sim \text{Gamma}(n, \frac{a_{wi}^2}{n})$ with expected value equal to $\sigma_w^1$ we have

$$P_r(\sigma_w^1 - \delta^1 \leq P_{aw}^{\delta \delta} \leq \sigma_w^1 + \delta^1) > 1 - \epsilon,$$

for some $\epsilon > 0$ and $\delta^1(\epsilon) > 0$. Again, since

$$P_r(P_{aw}^{\delta} \leq \sigma_w^1 + \delta^1) \geq P_r(P_{aw}^{\delta} \leq \sigma_w^1 + \delta^1),$$

for some $\delta(\epsilon)$, then for any $\tau > \sigma_w^1 + \delta(\epsilon)$ we have

$$P_r(P_{aw}^{\delta} \leq \tau | \mathbb{H}_1, P_j) > 1 - \epsilon.$$

Let us define the set of intervals $\mathcal{P} = \{P_j : \sigma_w^1 - \delta < \tau < \sigma_w^1 + \delta\}$ and let $\delta(\epsilon) = \max\{\delta^0(\epsilon), \delta^1(\epsilon)\}$. Thus, for all $P_j \in \mathcal{P}$ we have,

$$P_{MD}(P_j) + P_{FA}(P_j) \geq 1 - \epsilon.$$
We may compute $P_r(P)$ by rewriting (5) while using the SVD of $\Sigma$ in order to express $P_r$ as follows,

\[
\sigma_w^0 = \sigma_w^2 + P_j h^\dagger_{jw} V X V^\dagger h_{jw}, \\
\sigma_w^1 = \sigma_w^2 + P_j h^\dagger_{jw} V X V^\dagger h_{jw} + P_a |h_{aw}|^2.
\]

(8)

Since $P_j$ is a uniform r.v.

\[
P_r(P) = P_r \left( \frac{\mathbb{E} \left[ |P_j h_{jw}|^2 \right] - \mathbb{E} \left[ |h_j|^2 \right]}{P_j} \right) \\
= P_j |h_{jw}|^2 + 2 \mathbb{E} \left[ |h_j|^2 \right] \mathbb{E} \left[ |h_{jw}|^2 \right] - \mathbb{E} \left[ |h_j|^2 \right] \mathbb{E} \left[ |h_{jw}|^2 \right] \\
\leq(P_j \leq P_r \left( \frac{\mathbb{E} \left[ |P_j h_{jw}|^2 \right] - \mathbb{E} \left[ |h_j|^2 \right]}{P_j} \right)
\]

Therefore, if we set $P_a = \frac{\mathbb{E} \left[ |P_j h_{jw}|^2 \right] - \mathbb{E} \left[ |h_j|^2 \right]}{\mathbb{E} \left[ |h_{jw}|^2 \right]}$, we are left with

\[
P_r(P) = \frac{\epsilon}{2}.
\]

Considering all the above in order we have,

\[
P_{MD} + P_{FA} = \mathbb{E}_P \left[ P_{MD}(P_j) + P_{FA}(P_j) \right] \\
\geq \mathbb{E}_P \left[ P_{MD}(P_j) + P_{FA}(P_j) | P_r \right] P_r(P) \\
> 1 - \epsilon.
\]

The above shows that as long as Alice transmits with power $P_a = \frac{\mathbb{E} \left[ |P_j h_{jw}|^2 \right] - \mathbb{E} \left[ |h_j|^2 \right]}{\mathbb{E} \left[ |h_{jw}|^2 \right]}$, the system is covert. The rate of Alice can be obtained by using $P_a$ in Bob’s SNR which can be lower bounded by a constant providing a positive rate.

The result in Lemma 1 shows that the addition of multiple antennas at the jammer provides a gain of $h_{jw}^\dagger V X V^\dagger h_{jw}$ with respect to the case of a single antenna jammer as given in [8].

A. Multiple Antenna Gain Analysis

The gain to the covertness achieving power depends on the matrices $V$ and $X$. These matrices describe the transmission strategy of the jammer as will be thoroughly explained in Section V. Nevertheless, for any structure of these two matrices, the gain is upper and lower bounded according to Theorem 2.4.3, as follows,

\[
\min_{i=1,d} \xi_i \leq \frac{h_{jw}^\dagger V X V^\dagger h_{jw}}{|h_{jw}|^2} \leq \max_{i=1,d} \xi_i.
\]

Note that the gain achieves its upper bound (which is also maximized) when the rank of $V X V^\dagger$ is one. That is, $V$ is the single vector $v = \frac{h_{jw}}{|h_{jw}|}$ and $\xi$ is a scalar which is equal one. This case corresponds to the case in which the CSI of Willie is known and the jammer wishes to direct the AN towards Willie with all of its available power to maximize the gain. On the other hand, when the CSI of Willie is unknown ($h_{jw}$ and $h_{aw}$ are not known and random), then, a natural choice would be to maximize the lower bound, for example, to ensure that the gain is above a certain level with very high probability; accordingly, we would like to maximize the minimal eigenvalue. For covert communication, this may be a very important promise since it enables Alice to communicate with a higher power while remaining undetected under the non-CSI regime. We thus assume for the rest of this work that the jammer strategy is subject to maximize the minimal eigenvalue of $X$. Recall that $\sum_{i} \xi_i = 1$, thus, the lower bound can be maximized by setting $X = \frac{1}{d} I_{d \times d}$. This essentially means that the jammer transmits all his power divided equally to $d$ directions. This strategy is also compatible with other AN transmission strategies for the case of unknown CSI towards the adversary (e.g., [10], [17]). However, these did not provide an intuitive explanation for such a strategy.

Consequently, we are able to describe the random behaviour of the gain (as it depends on the random channel coefficients) as a function of the number of direction $d$ regardless the specific directions, i.e., the matrix $V$. This behaviour is given in the following theorem. To ease notation let us denote

\[
\psi = d h_{jw}^\dagger V X V^\dagger h_{jw},
\]

Theorem 1: The distribution of $\psi$, i.e., the channel’s affect on Alice’s transmission power, follows the Beta prime distribution. Specifically,

\[
\psi \sim \beta'(d, 1)
\]

for any transmission strategy of the jammer ($V$ and $X$) which divide equally all of its available power to $d$ directions (the rank of $X$).

Proof:

The proof follows from a result in [24] Theorem 2, Ch.1 for quadratic forms of a complex Gaussian vectors. Specifically, for a complex Gaussian vector $h$ and an idempotent matrix $X$, the quadratic form $h^\dagger X h \sim \chi^2_{d}$ where $d$ is the rank of $X$.

In our case, $h_{jw}^\dagger V X V^\dagger h_{jw}$ is distributed as a complex Gaussian vector with independent variables since $V$ is a unitary matrix (i.e. $V$ satisfy $VV^\dagger = I$). In addition, provided that the transmission strategy of the jammer is to divide equally all of its available power to $d$ directions, the elements of the diagonal matrix $X$ are all equal to $1/d$. Thus, $dX$ is the identity matrix which is an idempotent matrix and therefore $d h_{jw} h_j$ $V X V^\dagger h_{jw} \sim \chi^2_{d}$.

Noting that $|h_{aw}|^2 = \chi^2_{d}$, r.v. we have a proportion of Gamma r.v. which is distributed as a Beta prime distribution with parameters $a = d$ and $b = 1$ (i.e., $\psi \sim \beta'(d, 1)$).

Remark 2: Theorem 4 is correct for any transmission strategy of the jammer ($V$ and $X$) which divide equally all of its available power to $d$ directions. In any other case, the distribution of $\psi$ does not have a closed-form. We will see in the sequel that the optimal strategies of the jammer under all assumptions made in this work is to transmit his AN towards $d$ directions with equal division of power (in Theorem 4 where Willie’s CSI is known, we will show that $d = 1$).

B. Covert Criteria Compliance With no CSI

Lemma 1 provides the maximal transmit power level for Alice’s codebook such that the system is covert. This criteria requires Alice to know the CSI of Willie as it depends on $h_{jw}$ and $h_{aw}$. In case Alice does not have that knowledge, she is still able to promise a positive rate by considering the channel statistics and by knowing the jammer’s AN strategy. Which, following the previous section discussion, we assume that he transmits all
of his available power divided equally into \( d \) directions. The following theorem provides Alice’s transmission power for this case.

**Theorem 2:** Under the model of block fading AWGN channel when the CSI of Willie is unknown, as long as Alice transmits with power \( P_a^* \), such that,

\[
P_a^* = \frac{\epsilon^2 P_{\max}}{72e \ln \frac{2}{\epsilon} d},
\]

where \( d \) is the rank of the jammer’s covariance matrix \( \Sigma \) and his total transmission power was equally divided into \( d \) arbitrary directions, the system is covert and Alice can transmit with a strictly positive rate, i.e., \( \beta \) applies and Willie is unable to decided if transmission occurred.

**Proof:** The proof essentially is an extension for the proof given in Lemma \([1]\) where the probability \( P_r(\Psi) \) in equation \((3)\) is upper bounded while considering the channel’s statistics \( h_{aw} \) and \( h_{jw} \). Specifically, we can write

\[
P_r(\Psi) = P_r(\Psi = h_{aw}^2 > \alpha) + P_r(\Psi = h_{aw}^2 \leq \alpha) + P_r(\Psi = h_{jw}^2 > \alpha) + P_r(\Psi = h_{jw}^2 \leq \alpha).
\]

With similar technique while denoting \( \Psi = h_{jw}^T V X V^T h_{jw} \) we can further upper bound the above

\[
P_r(\Psi) = \Pr(\Psi > \alpha) + \Pr(\Psi \leq \alpha).
\]

Setting \( \alpha \) and \( \beta \) instead of \( h_{aw}^2 \) and \( \Psi \) respectively increases the interval for \( P_r \) and thus increasing the probability. Specifically we have,

\[
P_r(\Psi) \leq P_r(\Psi > \alpha) + P_r(\Psi < \beta) + P_r(\Psi \leq \alpha, \Psi \geq \beta).
\]

Thus, from here, according to the proof of Lemma \([1]\) the covertness criteria \( P_{\text{MD}} + P_{\text{FA}} \geq 1 - \epsilon \) holds.

In the following corollary, we present the covertness achieving transmission power for Alice in the case which the jammer has a single antenna (i.e., \( N = 1 \)). We note that this result is different from the one obtained in \([2]\) while considering fading channels. In \([3]\), it is assumed that Willie does not know his own CSI and thus the jammer may use AN drawn from a distribution with constant variance \( P_j = P_{\max} \) which eventually affect Alice’s power. In this work, Willie knows his own CSI and therefore we can not assume that the channel randomizes the received power at Willies. Thus, the jammer must employ AN with varying power.

**Corollary 1:** Under the model of block fading AWGN channel when the CSI of Willie is unknown and the jammer is equipped with a single antenna, as long as Alice transmits with power \( P_a^* \), such that,

\[
P_a^* = \frac{\ln \frac{6}{\epsilon}}{72e \ln \frac{2}{\epsilon}} P_{\max},
\]

the system is covert and Alice can transmit with positive rate, i.e., \( \beta \) applies and Willie is unable to decided if transmission occurred.

**Proof:** When the jammer is equipped with a single antenna, the received power at Willie, due to the AN, is \( P_j h_{jw}^2 \). Thus, in the proof of Theorem \([2]\), \( \Psi = P_j h_{jw}^2 \) which is distributed as chi-square with 2 degrees of freedom. Therefore, we may set \( \beta = 2 \ln \frac{6}{\epsilon} \) and get \( P_r(\Psi < \beta) = \frac{\epsilon}{6} \).

**IV. Detection at Bob**

In Section \([3]\) we constructed a criterion for Alice’s transmission power such that the system is covert. Accordingly, since the power is strictly positive and does not go to zero with \( n \), there exists a rate \( R \) for which Bob can decode successfully with a probability of error that goes to zero. This rate can be attained by using capacity-achieving codes for AWGN channels and is eventually a function of Bob’s SNR. In what follows we describe Bob’s received SNR; first, for the
transmission.

AN from the jammer and on the other intensify Alice's SNR. In other words, Bob projects the received vector onto the received signal which eventually increases his received receiver. In this case, Bob performs a linear operation on would like to consider also the aspect of decoder complexity scenario and all he performs is ML decoding. In this work, we

steering his antennas away from the jammer. In fact, even if Bob can easily agree on a positive rate. In section M > 1 antennas. Where we formulate a global optimization problem for both the transmission of the jammer and the detection of Bob. We start our analysis with the case that Bob has a single antenna (i.e., M = 1) for which we obtain the optimal AN transmission strategy by the jammer. We then elaborate it to the more general case while using the techniques used in the simpler case.

If the jammer knows the channel state, he may use it to assist either Alice and Bob or Willie. The jammer assistance, when using his CSI, is reflected by the covariance matrix $\Sigma$, according to the player he wishes to assist. This power allocation affects the transmission power Alice can use, the received SNR’s side at Bob and Willie’s ability to detect the communication. In this work, we assume the jammer assists Alice. Hence, the jammer should construct his covariance matrix $\Sigma$ in a way that enables Alice to increase her transmission power while still being covert, while reducing the AN at Bob to have a higher achievable rate. Recall that $\Sigma = P_J V V^H$. Thus, the jammer essentially needs to design the diagonal matrix $X$ and the matrix $V$ appropriately.

In what follows, in addition to the transmission and detection strategies we also relate to the possible CSI knowledge known on Willie by the jammer and Alice. This knowledge affects the ability of the jammer to contribute to the covert communication between Alice and Bob. We thus examine two situations; the first is that the jammer and Alice know the channel towards Willie and Bob and in the second they know only the channel towards Bob. We note that while the second case is more practical the first case can be explained by a basic setup where Willie is one of the "legitimate" players in the game, e.g., a real Warden in a prison, hence he also "transmits" from time to time (maybe to other people), and his whereabouts is not secret. Hence, the jammer (and Alice) might have the opportunity to estimate the channel to/from Willie.

Remark 4 (The jammer’s strategy knowledge): It is reasonable to assume that the jammer’s strategy is known to all since for the case of known global CSI of Willie, the optimal strategy can be computed by Alice in the same manner as the jammer preforms. And for the case which there is no CSI, the jammer can’t optimize his strategy on a specific realization with respect to CSI, so he must decide his strategy in advance. However, we note that, for both cases, the CSI of the jammer towards Bob is needed, which we assume in this work to be globally known.

A. The Case Where $M = 1$

1) Full CSI: When the jammer and Alice know $h_{jb}$ and $h_{jb}$ we may express an optimization problem for Bob’s SNR by employing (7) in to (11) since Alice has the complete

Similarly, ewe will use (12) as our target function for the maximization in Section V when we consider that $M > 1$.

V. AN TRANSMISSION AND DETECTION STRATEGIES

In this section, we provide a description for the AN transmission strategy of the jammer and the detection strategy of Bob in case he has $M > 1$ antennas. Where we formulate a global optimization problem for both the transmission of the jammer and the detection of Bob. We start our analysis with the case that Bob has a single antenna (i.e., $M = 1$) for which we obtain the optimal AN transmission strategy by the jammer. We then elaborate it to the more general case while using the techniques used in the simpler case.

If the jammer knows the channel state, he may use it to assist either Alice and Bob or Willie. The jammer assistance, when using his CSI, is reflected by the covariance matrix $\Sigma$, according to the player he wishes to assist. This power allocation affects the transmission power Alice can use, the received SNR’s side at Bob and Willie’s ability to detect the communication. In this work, we assume the jammer assists Alice. Hence, the jammer should construct his covariance matrix $\Sigma$ in a way that enables Alice to increase her transmission power while still being covert, while reducing the AN at Bob to have a higher achievable rate. Recall that $\Sigma = P_J V V^H$. Thus, the jammer essentially needs to design the diagonal matrix $X$ and the matrix $V$ appropriately.

In what follows, in addition to the transmission and detection strategies we also relate to the possible CSI knowledge known on Willie by the jammer and Alice. This knowledge affects the ability of the jammer to contribute to the covert communication between Alice and Bob. We thus examine two situations; the first is that the jammer and Alice know the channel towards Willie and Bob and in the second they know only the channel towards Bob. We note that while the second case is more practical the first case can be explained by a basic setup where Willie is one of the "legitimate" players in the game, e.g., a real Warden in a prison, hence he also "transmits" from time to time (maybe to other people), and his whereabouts is not secret. Hence, the jammer (and Alice) might have the opportunity to estimate the channel to/from Willie.

Remark 4 (The jammer’s strategy knowledge): It is reasonable to assume that the jammer’s strategy is known to all since for the case of known global CSI of Willie, the optimal strategy can be computed by Alice in the same manner as the jammer preforms. And for the case which there is no CSI, the jammer can’t optimize his strategy on a specific realization with respect to CSI, so he must decide his strategy in advance. However, we note that, for both cases, the CSI of the jammer towards Bob is needed, which we assume in this work to be globally known.

A. The Case Where $M = 1$

1) Full CSI: When the jammer and Alice know $h_{jb}$ and $h_{jb}$ we may express an optimization problem for Bob’s SNR by employing (7) in to (11) since Alice has the complete
knowledge of the covertness achieving transmission power. The optimization problem is as follows,
\[
\max_{\mathbf{v}, \mathbf{x}} \frac{\left( P_\text{max} h_{\text{uw}}^\dagger \mathbf{V} \mathbf{x}^\dagger h_{\text{wj}}^\dagger \right) |h_{ub}|^2}{P_j h_{jb}^\dagger \mathbf{V} \mathbf{x}^\dagger h_{jb}^\dagger + \sigma_b^2},
\]
\[
\text{s.t.} \quad 0 \leq \xi_i \leq 1 \quad \text{and} \quad \sum_{i=1}^{N} \xi_i = 1.
\]
(13)
Note that $\mathbf{V} \mathbf{x}^\dagger$ influence on both the numerator and the denominator differently with respect to the vectors $\mathbf{h}_{\text{uw}}$ and $\mathbf{h}_{\text{wb}}$. The knowledge of these vectors affects on the solution for the maximization problem. Note also, since the system is covert, we are only interested in maximizing Bob’s SNR as it dictates the rate eventually. Accordingly, we have the following result.

**Theorem 3:** The optimal strategy for the jammer, i.e., the solution for the maximization problem in (13), when full CSI is globally known, is the following power allocation,
\[
\mathbf{\Sigma} = P_j v^* \mathbf{v}^\dagger,
\]
(14)
where
\[
v^* = \frac{\mathbf{q}}{\|\mathbf{q}\|}
\]
(15)
and $\mathbf{q}$ is the eigenvector which corresponds to the highest eigenvalue of the matrix
\[
\left( \mathbf{h}_{\text{wb}}^\dagger \mathbf{h}_{\text{wb}}^\dagger + \sigma \mathbf{I} \right)^{-1} \left( \mathbf{h}_{\text{uw}} h_{\text{wj}}^\dagger \right).
\]
(16)

**Proof:** This Proof follows similar steps as in an analytical derivation of the maximization problem performed in [25]. We can simplify (13) as follows,
\[
\max_{\mathbf{v}, \mathbf{x}} \frac{\left( P_\text{max} h_{\text{uw}}^\dagger \mathbf{V} \mathbf{x}^\dagger h_{\text{wj}}^\dagger \right) |h_{ub}|^2}{P_j h_{jb}^\dagger \mathbf{V} \mathbf{x}^\dagger h_{jb}^\dagger + \sigma_b^2} = C \max_{\mathbf{v}, \mathbf{x}} \frac{\mathbf{w}^\dagger \mathbf{x} \mathbf{w}}{\mathbf{b}^\dagger \mathbf{x} \mathbf{b} + \sigma},
\]
(17)
where $C = c|h_{ub}|^2/4|h_{uw}|^2$, $\sigma = \sigma_b^2$, $\mathbf{w} = \mathbf{V}^\dagger \mathbf{h}_{\text{uw}}$ and $\mathbf{b} = \mathbf{V}^\dagger \mathbf{h}_{\text{wb}}$. The maximization function in (17) can be written as
\[
\frac{\mathbf{w}^\dagger \mathbf{x} \mathbf{w}}{\mathbf{b}^\dagger \mathbf{x} \mathbf{b} + \sigma} = \frac{\sum_{i=1}^{N} \xi_i w_i^2}{\sum_{i=1}^{N} \xi_i b_i^2 + \sigma}.
\]

Let us assume $\xi^*$ is the optimal power allocation for fixed $\mathbf{w}$ and $\mathbf{b}$. We examine two indices $i$ and $j$ in $\xi^*$ which have power allocation $(\xi_i, \xi_j)$ such that $\xi_i + \xi_j = P_j$. We will show first that either $\xi_i = P_j$ or $\xi_j = P_j$, must occur, hence, eventually, the optimal power allocation is a unit vector (since this is true for each pair of indices). Then, we will find the corresponding direction (eigenvector) of the AN power.

The optimization problem on $\mathbf{\xi}$ can be written as follows,
\[
\max f(\mathbf{\xi}) = \max \frac{\sum_{i=1}^{N} \xi_i w_i^2}{\sum_{i=1}^{N} \xi_i b_i^2 + \sigma} = \max \sum_{i \neq j} (\xi_i b_i^2 + \xi_j b_j^2 + \xi_i + \xi_j) - \sum_{i \neq j} \xi_i b_i^2 - \xi_j b_j^2 (P_j - \xi_i) + \sigma
\]
\[
= \max \sum_{i \neq j} \frac{\xi_i w_i^2 + \xi_j w_j^2}{\xi_i b_i^2 + \xi_j b_j^2 + \xi_i + \xi_j} - \frac{w_i^2 - w_j^2}{(P_j - \xi_i) + \sigma}.
\]

Thus, for every two indices $i, j$, if $f(\mathbf{\xi})$ is monotonically increasing or monotonically decreasing with $\xi_i$ depending on the sign of $(w_i^2 - w_j^2) (\sum_{i \neq j} \xi_i b_i^2 + \sigma + b_j^2 P_j) - (b_i^2 - b_j^2) (\sum_{i \neq j} \xi_i w_i^2 + w_j^2 P_j)$.

The derivative according to $\xi_i$ shows that the function $f(\mathbf{\xi})$ is either monotonically increasing or monotonically decreasing with $\xi_i$ depending on the sign of $(w_i^2 - w_j^2) (\sum_{i \neq j} \xi_i b_i^2 + \sigma + b_j^2 P_j) - (b_i^2 - b_j^2) (\sum_{i \neq j} \xi_i w_i^2 + w_j^2 P_j)$.

In order to find this direction, which is the corresponding eigenvalue of the matrix $\mathbf{\Sigma} = P_j v^* \mathbf{v}^\dagger$, we may write the unit rank $\mathbf{\Sigma}$ as $\mathbf{\Sigma} = P_j v^* \mathbf{v}^\dagger$. Note that $\mathbf{v}$ is constrained to have a unit norm, i.e., $\mathbf{v}^\dagger \mathbf{v} = 1$. Returning to the maximization problem in (17), we have,
\[
C \max_{P_j} \frac{P_j h_{\text{uw}}^\dagger \mathbf{V} \mathbf{x}^\dagger h_{\text{wj}}^\dagger}{h_{\text{wb}}^\dagger \mathbf{V} \mathbf{x}^\dagger h_{\text{wb}}^\dagger + \sigma} = C \max_{P_j} \frac{\mathbf{v}^\dagger \mathbf{h}_{\text{uw}}^\dagger \mathbf{h}_{\text{wj}}^\dagger}{\mathbf{v}^\dagger \mathbf{h}_{\text{wb}}^\dagger \mathbf{h}_{\text{wb}}^\dagger + \sigma + \mathbf{v}^\dagger \mathbf{v}}
\]
\[
= C \max_{P_j} \frac{\mathbf{v}^\dagger \mathbf{h}_{\text{uw}}^\dagger \mathbf{h}_{\text{wj}}^\dagger}{\mathbf{v}^\dagger \mathbf{h}_{\text{wb}}^\dagger \mathbf{h}_{\text{wb}}^\dagger + \sigma + \mathbf{v}^\dagger \mathbf{v}}
\]
\[
= C \max_{P_j} \frac{\mathbf{v}^\dagger \mathbf{W} \mathbf{v}}{\mathbf{v}^\dagger \mathbf{B} \mathbf{v}},
\]
(18)
(19)
(20)
(21)
(22)

where, $\mathbf{W} = \mathbf{h}_{\text{uw}}^\dagger \mathbf{h}_{\text{uw}}$, and, $\mathbf{B} = \mathbf{h}_{\text{wb}}^\dagger \mathbf{h}_{\text{wb}} + \sigma \mathbf{I}$. The above maximization problem is also known as the Rayleigh quotient [23], which is maximized for the eigenvector that corresponds to the highest eigenvalue of the matrix $\mathbf{B}^{-1} \mathbf{W}$. Let us denote $\mathbf{q}$ to be that vector, thus the optimal $\mathbf{v}$ is,
\[
\mathbf{v}^* = \frac{\mathbf{q}}{\|\mathbf{q}\|}.
\]
(23)
We conclude that the optimal direction $\mathbf{v}^*$ of the AN depends on both channel vectors $\mathbf{h}_{\text{uw}}$ and $\mathbf{h}_{\text{wb}}$. Though it is not clear from the expression in [23] for $\mathbf{v}^*$, what is the specific
direction for the AN transmission. Alternatively, one can gain intuition on the direction from Equation (20), where, it is clearly seen that $\mathbf{v}^*$, on one hand, should be close to the direction of Willie, i.e., to maximize the projection on $\mathbf{h}_j$, while on the other hand it should be orthogonal to Bob as much as possible, i.e., minimize the projection on $\mathbf{h}_b$. Figure 2 depicts a visualization for a specific channel realization on the optimal direction of the AN transmission by the jammer for the case of $N = 3$ antennas. One can easily observe that the optimal direction is pointed to the direction of Willie while being close to 90 degrees with the direction of Bob.

2) Jammer Knows Only Channel to Bob: We now examine the case which the specific realizations of the channels towards Willie are not known globally. In this case, Alice can’t determine the specific covertness achieving transmission power and thus she transmits with a predefined power $P_a^*(\mathbf{V}, \mathbf{X})$ which takes in consideration the channel statistics and the specific AN transmission strategy by the jammer (the matrices $\mathbf{V}$ and $\mathbf{X}$). We thus may write the optimization problem as follows,

$$
\max_{\mathbf{V}, \mathbf{X}} \frac{P_a^*(\mathbf{V}, \mathbf{X}) |h_{ab}|^2}{P_j \|\mathbf{h}_b\|^2 + \sigma_b^2},
$$

s.t. $0 \leq \xi_l \leq 1$ and, $\sum_{l=1}^{N} \xi_l = 1$. (24)

As Theorem 2 suggests, the covertness-achieving power of Alice for the case of no CSI, assuming equal power allocation, only depends on the number of directions the jammer transmits into. Again, such an assumption is motivated by the desire to promise minimal value for Alice’s transmission power. Accordingly, the optimal strategy under such assumptions is given in the following theorem.

**Theorem 4:** The optimal strategy for the jammer, i.e., the solution for the maximization problem in (24), when the CSI of Willie is not known and the jammer is restricted to transmit with equal power to all directions, is the following power allocation (for $N \geq 2$ antennas),

$$
\Sigma = P_j \mathbf{V}^* \mathbf{X}^* \mathbf{V}^\dagger,
$$

where in case $\frac{\sigma^2}{P_j^2} < \frac{N-1}{N} \|\mathbf{h}_b\|$. 

$$
\mathbf{V}^* = \text{span}\{\mathbf{Q}\} \text{ and, } \mathbf{X}^* = \frac{1}{N-1} \mathbf{I}_{(N-1)\times(N-1)}, \quad (26)
$$

Otherwise,

$$
\mathbf{V}^* = \text{span}\{\mathbf{Q}\} \cup \frac{\mathbf{h}_b}{\|\mathbf{h}_b\|} \text{ and, } \mathbf{X}^* = \frac{1}{N} \mathbf{I}_{N\times N}, \quad (27)
$$

where $\mathbf{Q} = \text{nullspace}\{\mathbf{h}_b\}$, and $\text{span}\{\mathbf{Q}\}$ is a matrix with $N-1$ orthogonal vectors from $\mathbf{Q}$.

**Proof:** Setting the transmission power of Alice from Theorem 2 in the optimization problem on Bob’s SNR as given in (24), gives the following optimization problem,

$$
\max_{\mathbf{V}, \mathbf{X}} \frac{c^2 P_{\text{max}}}{2e \ln 2} d |h_{ab}|^2
$$

$$
\text{s.t. } \xi_l = \frac{1}{d} \text{ for, } l = 1, ..., d.
$$

The above can be simplified to

$$
C_1 \frac{P_{\text{max}}}{P_j} \max_{d, \mathbf{b}} \frac{\sum_{l=1}^{d} \frac{1}{d} b_l^2 + \sigma}{\|\mathbf{h}_b\|^2 + \sigma},
$$

$$
C_1 \frac{P_{\text{max}}}{P_j} \min_{d, \mathbf{b}} \frac{\sum_{l=1}^{d} \frac{1}{d} b_l^2 + \sigma}{\|\mathbf{h}_b\|^2 + \sigma}.
$$

where $C_1 = \frac{c^2 h_{ab}}{2e \ln 2}$, $\sigma = \frac{\sigma_b^2}{P_j}$ and $\mathbf{b} = \mathbf{V}^\dagger \mathbf{h}_b$. Note that we are able to change the parameters in the optimization due to the structure of $\mathbf{X}$ and the quadratic form in the denominator. Let $\mathbf{Q} = \text{nullspace}\{\mathbf{h}_b\}$, and let $\mathbf{V}^*$ to be a matrix with $d$ orthogonal vectors from $\mathbf{Q}$ as its columns. Obviously, the maximal columns degree of $\mathbf{V}^*$ is $N-1$ and for such $\mathbf{V} = \mathbf{V}^*$ we have $b_l^2 = 0$ for $l = 1, ..., N-1$. Taking less element in the sum, i.e., $d < N-1$, will result in a lower value in the target function. The case for which $d = N$ will attain a higher value only when $\sigma > \frac{\sigma_b^2}{\|\mathbf{h}_b\|^2}$. Thus, in this case, $\mathbf{V}$ must span the whole space (i.e. $\mathbf{V}$ must be of rank $N$). So, by setting $\mathbf{V} = \mathbf{V}^* \cup \frac{\mathbf{h}_b}{\|\mathbf{h}_b\|}$ we have $b_N = \|\mathbf{h}_b\|$ which completes the proof.

The above result implies that the jammer should transmit his AN isotropically to all direction or to the null space of Bob. This choice depends on the magnitude of the channel $\|\mathbf{h}_b\|$. That is, for some fixed $N, \sigma_b^2$, and $P_j$ if $\|\mathbf{h}_b\|$ is high the optimal strategy would be to transmit only to the null space. On the other hand, when $\|\mathbf{h}_b\|$ is low, transmitting to all direction including Bob’s would be better in the sense of maximizing the covert rate since we add another DoF to the confusion of Willie. Another way to understand this is by considering the parameter $\sigma$ which reflects the ratio between the "bad" noise power that Bob endures due to its antenna noise and the "good" AN power which helps Alice and consequently helps Bob’s SNR. So, when $\sigma$ has
a low value, then Bob endures more "bad" noise and thus the jammer should avoid adding more noise power in the direction of Bob. We note also that Theorem 4 refer only to the case of multiple antenna jammer (i.e. $N \geq 2$), which is considered in this work. The case which $N = 1$ is not considered since no optimization can be performed and besides, not transmitting AN is not an option as it results with zero covert rate (square root law). In Figure 3 one can observe the turning point of the average SNR value when comparing the strategy of transmitting the AN isotropically against transmitting only in the null space. The simulation was performed for $N = 4$ antennas at the jammer; and as described in Theorem 4 we can see in the figure that around the point of $\sigma = \frac{\sqrt{2}}{\sqrt{N}} ||h_{jb}|| = \frac{\sqrt{2}}{\sqrt{4}}(15) \approx 2.05$ the jammer should switch strategy in order to maximize the SNR.

B. The Case Where $M > 1$

When Bob is equipped with multiple antennas, the AN transmission and the detection strategies of the jammer and Bob are coupled and depend on the joint information both have. In this subsection, we present the global optimization problem to maximize the covert rate for these strategies and we analyze it with respect to the knowledge Willie and Bob have.

1) Full CSI: Recall that Bob employs a linear receiver and thus he preforms a linear function on the received signal. Accordingly, following the SNR expression in (12), and assuming that the system is covert (i.e. Alice is transmitting with the power given in (7)), we may write the global optimization problem for maximizing Bob’s SNR by employing (7) in to (12) as follows,

$$\max_{\mathbf{V}, \mathbf{X}, \mathbf{c}} \left| \mathbf{c}^H \mathbf{h}_{ab} \right|^2 \frac{\mathbf{e}^H \left( \frac{\mathbf{P}_{\text{max}}}{\mathbf{I}_{\text{w}}^H} \right) \mathbf{h}_{bw} \mathbf{V}^H \mathbf{h}_{bw}}{\mathbf{e}^H \left( \mathbf{P}_j \mathbf{H}_{jb} \mathbf{V}^H \mathbf{H}_{jb}^H + \sigma_b^2 \mathbf{I} \right) \mathbf{c}}$$

$$s.t. \quad \mathbf{0} \leq \mathbf{\xi}_l \leq 1 \text{ and, } \sum_{l=1}^{N} \mathbf{\xi}_l = 1.$$  

Note that $\mathbf{c}$ should be optimized together with $\mathbf{\Sigma}$ (i.e. with $\mathbf{V}$ and $\mathbf{X}$) together. That is, to determine the optimal strategies for the jammer and Bob, both must know the realizations of the random factors in the SNR expression. In the following lemma, we first show that in the optimal solution for the optimization above the jammer should direct his AN to a single direction.

**Lemma 2:** The optimal strategy for the jammer, i.e., the optimal solution for the maximization problem in (28), when full CSI is globally known, is to transmit all his available power towards a single direction.

**Proof:** Let us examine the maximization problem in (28), which can be written as follows using definitions from Theorem’s 3 proof,

$$\max_{\mathbf{V}, \mathbf{X}, \mathbf{c}} \left| \mathbf{c}^H \mathbf{h}_{ab} \right|^2 \frac{\mathbf{e}^H \left( \frac{\mathbf{P}_{\text{max}}}{\mathbf{I}_{\text{w}}^H} \right) \mathbf{h}_{bw} \mathbf{V}^H \mathbf{h}_{bw}}{\mathbf{e}^H \left( \mathbf{P}_j \mathbf{H}_{jb} \mathbf{V}^H \mathbf{H}_{jb}^H + \sigma_b^2 \mathbf{I} \right) \mathbf{c}}$$

$$\max_{\mathbf{V}, \mathbf{X}, \mathbf{c}} \left| \mathbf{c}^H \mathbf{h}_{ab} \right|^2 \frac{\mathbf{e}^H \left( \frac{\mathbf{P}_{\text{max}}}{\mathbf{I}_{\text{w}}^H} \right) \mathbf{h}_{bw} \mathbf{V}^H \mathbf{h}_{bw}}{\mathbf{e}^H \left( \mathbf{P}_j \mathbf{H}_{jb} \mathbf{V}^H \mathbf{H}_{jb}^H + \sigma_b^2 \mathbf{I} \right) \mathbf{c}}$$

$$\max_{\mathbf{V}, \mathbf{X}, \mathbf{c}} \left| \mathbf{c}^H \mathbf{h}_{ab} \right|^2 \frac{\mathbf{e}^H \left( \frac{\mathbf{P}_{\text{max}}}{\mathbf{I}_{\text{w}}^H} \right) \mathbf{h}_{bw} \mathbf{V}^H \mathbf{h}_{bw}}{\mathbf{e}^H \left( \mathbf{P}_j \mathbf{H}_{jb} \mathbf{V}^H \mathbf{H}_{jb}^H + \sigma_b^2 \mathbf{I} \right) \mathbf{c}}$$

where $C_2 = \epsilon/4||\mathbf{h}_{aw}||^2$, $\sigma = \frac{\epsilon}{\sqrt{P}_j}$, $\mathbf{w} = \mathbf{V}^H \mathbf{h}_{bw}$ and $\mathbf{b}' = \mathbf{V}^H \mathbf{H}_{jb} \mathbf{c}$. Following the same analysis performed in Theorem’s proof it can be shown that for any fixed $\mathbf{w}, \mathbf{b}'$ and $\mathbf{c}$ the optimal power allocation $\mathbf{\xi}^*$ (i.e., the diagonal of $\mathbf{X}$) is a unit vector.

The above result agrees with Theorem’s conclusion of a single direction and show that the fact that Bob is equipped with several antennas is irrelevant on that matter. Using Lemma 2 the maximization problem in (28) can be reduced as follows,

$$\max_{\mathbf{V}, \mathbf{X}, \mathbf{c}} \left| \mathbf{c}^H \mathbf{h}_{ab} \right|^2 \min_{\mathbf{w}, \mathbf{b}'} \frac{\mathbf{w}^H \mathbf{X} \mathbf{w}}{\mathbf{b}'^H \mathbf{X} \mathbf{b}'+ \sigma||\mathbf{c}||^2}$$

which is, perhaps, more intuitive for understanding, but very complicated to solve due to the coupled expressions. Instead, inspired by existing works on MIMO communication, we suggest and analyze sub-optimal solutions for the above optimization problem. Specifically, we consider the null steering (also known as "masked beamforming") and Maximal Ratio Combiner (MRC) techniques (26), (27) which Bob and the jammer may perform independently to simplify the optimization problem. That is, instead of solving the global optimization for both strategies simultaneously, we fix one and perform optimization on the other. In a way, this suboptimal schemes decentralized the global optimization problem by forcing the jammer and Bob to rely only on themselves and their ability to enhance the covert rate.

The transmission strategies for Bob and the jammer should cancel the interference caused by the AN at Bob, or they should try to maximize the received power from Alice’s transmission. Canceling the AN can be done by projecting the AN (by the jammer in advance, i.e., precoding) onto a
null-space for which Bob can retrieve the message without $i$. Alternatively, Bob himself can perform the projection by designing his linear filter appropriately. Where depending on the number of antennas both have, they may agree in advance upon the responsibility for the cancelation task. This is due to the fact that finding a null-space requires rank deficiency of the matrix $H_{jb}$ or $H_{jb}^T$. On the other hand, maximizing the received power from Alice’s transmission can be done by the jammer in directing the AN towards Willie or by designing the linear filter to match the channel between Alice and Bob while ignoring the AN. For all the above, once the vectors $v$ or $c$ are fixed, that is either the jammer or Bob fix their strategy, the other can optimize his strategy.

In what follows we present analysis for the suboptimal strategies described above. Specifically, we solve the optimization problem in (30) for a fixed $c$ and for a fixed $v$ and then we present the different strategies discussed herein. That is, the MRC by Bob, the direction of the AN toward Willie by the jammer and lastly, depending on the number of antennas, the cancelation of the AN at Bob’s receiver by the jammer or Bob.

**Fixed $c$:**
For any fixed $c$ we have the following theorem,

**Theorem 5:** The optimal strategy for the jammer, i.e., the solution for the maximization problem in (30) when full CSI is globally known and Bob fixes a certain linear filter $c$, is the following power allocation,

$$\Sigma = P_j v^* v^\dagger,$$  \hspace{1cm} (31)

where $v^*$ is the (normalized to one) eigenvector which corresponds to the highest eigenvalue of the matrix $B^{-1}W$, where $W = h_{jb}h_{jb}^\dagger$ and, $B = H_{jb}v v^\dagger + \sigma I$. We have the following theorem,

**Proof:** Assuming $c$ is fixed the optimization problem in (30) is reduced to,

$$\max_{v} \frac{c^\dagger \left( H_{jb}v v^\dagger H_{jb}^\dagger + \sigma I \right) c}{v^\dagger h_{jb}v},$$

which is the Rayleigh quotient maximization problem. Similarly to Theorem 3 proof, setting $W = h_{jb}h_{jb}^\dagger$ and, $B = H_{jb}v v^\dagger + \sigma I$, the optimal $v$ is the eigenvector which corresponds to the highest eigenvalue of the matrix $B^{-1}W$.

**Fixed $v$:**
For any fixed $v$ we have the following theorem,

**Theorem 6:** The optimal strategy for Bob, i.e., the solution for the maximization problem in (30) when full CSI is globally known and the jammer fixes a certain $v$, is the linear filter $c^*$ which is the eigenvector which corresponds to the highest eigenvalue of the matrix $B^{-1}W$, where $W = h_{ab}h_{ab}^\dagger$ and, $B = H_{jb}v v^\dagger H_{jb}^\dagger + \sigma I$.

**Proof:** Assuming $v$ is fixed the optimization problem in (30) is reduced to,

$$\max_{c} \frac{|c^\dagger h_{ab}|^2}{c^\dagger \left( H_{jb}v v^\dagger H_{jb}^\dagger + \sigma I \right) c},$$

which is the Rayleigh quotient maximization problem. Similarly to Theorem 3 proof, setting $W = h_{ab}h_{ab}^\dagger$ and, $B = H_{jb}v v^\dagger H_{jb}^\dagger + \sigma I$, the optimal $c$ is the eigenvector which corresponds to the highest eigenvalue of the matrix $B^{-1}W$.

As a consequence of Theorems 3 and 5 we now present the sub-optimal strategies in the following corollaries.

**Corollary 2:** When Bob employs a MRC for the channel between him and Alice while ignoring the AN, then, $c = \frac{h_{ab}}{\|h_{ab}\|}$. Then, the optimal direction for the jammer is $v^* = \frac{h_{jb}}{\|h_{jb}\|}$ where $v$ is the eigenvector which corresponds to the highest eigenvalue of the matrix $B^{-1}W$, where $W = h_{jb}h_{jb}^\dagger$ and, $B = H_{jb}v v^\dagger H_{jb}^\dagger + \sigma I$.

**Corollary 3:** When the jammer directs his AN towards Willie, i.e., $v = \frac{h_{wh}}{\|h_{wh}\|}$, the optimal linear filter $c^*$ for Bob is the eigenvector which corresponds to the highest eigenvalue of the matrix $B^{-1}W$, where $W = h_{ab}h_{ab}^\dagger$ and, $B = H_{jb}v v^\dagger H_{jb}^\dagger + \sigma I$.

As mentioned above, the cancelation of the AN at Bob’s receiver depends on the number of antennas that Bob and the jammer have. If Bob has more antennas than the jammer, i.e., $M > N$, then he can find a null-space for the columns of $H_{jb}$ such that the linear filter $c$ will satisfy $c^\dagger h_{jb} = 0$ for $c \in Q = \text{nullspace}(H_{jb})$. In other words, Bob projects the received vector $y_b[i]$ at the $i$-th channel use onto a subspace spanned by the null space $Q$. That is, we have

$$c^T y_b[i] = x[i]c^T h_{ab} + c^T H_{jb}v[i] + c^T n_b[i] = x[i]c^T h_{ab} + c^T n_b[i].$$

The above implies that the vector $\tilde{h}$ in Theorem 5 is equal to zero. Note that, Bob may choose $c$ to improve the SNR of the above while being restricted to be in $Q$. That is, to choose $c$ such that,

$$c = \arg \max_{c \in Q} \frac{|c^\dagger h_{ab}|^2}{\|c\|^2}.$$  \hspace{1cm} (32)

Consequently, we have the following corollary.

**Corollary 4:** The optimal strategy for the jammer, when Bob cancels the AN and chooses $c$ according to (32) is the following power allocation,$$
\Sigma = P_j v^* v^\dagger,$$  \hspace{1cm} (33)
where,

\[ \mathbf{v}^* = \frac{\mathbf{h}_{jw}}{\|\mathbf{h}_{jw}\|}. \]  

**Proof:** Since Bob cancels the received AN from the jammer, following the notations of Theorem 5, we have \( \mathbf{B}^{-1} = \mathbf{B} = \sigma \|\mathbf{c}\|^2 \mathbf{I} \) and \( \mathbf{W} = \mathbf{h}_{ab} \mathbf{h}_{ab}^\dagger \). Since the matrix \( \mathbf{B} \mathbf{W} \mathbf{B} \) is of rank 1, it has only a single non-zero eigenvalue which his corresponding eigenvector is \( \mathbf{h}_{jw} \).

**Corollary 4** essentially shows that when Bob can cancel the AN in any way, the jammer can transmit his AN towards Willie without degrading the covert rate between Alice and Bob.

On the other hand, if the jammer has more antennas than Bob, i.e., \( N < M \), then, the jammer may construct his covariance matrix such that the received AN power at Bob will fall first in the null-space of \( \mathbf{H}_{jb}^\dagger \), and then as close as can be the direction of Willie. This strategy is different than the optimal one obtained in Theorem 4 since here the direction is orthogonal to Bob. Thus, letting \( \mathbf{Q}' = \text{nullspace}(\mathbf{H}_{jb}^\dagger) \) and setting \( \mathbf{v} \) such that it satisfies \( \mathbf{H}_{jb} \mathbf{v} = 0 \) for \( \mathbf{v} \in \mathbf{Q}' \), the jammer is able to cancel the AN at Bob. Similarly, we note that the jammer may choose \( \mathbf{v} \) also to be as close as can be to the direction of Willie to improve further the SNR while being restricted to be in \( \mathbf{Q}' \). That is, to choose \( \mathbf{v} \) such that,

\[ \mathbf{v} = \arg \max_{\mathbf{v} \in \mathbf{Q}' } \left( \mathbf{h}_{jw}^\dagger \mathbf{v} \mathbf{v}^\dagger \mathbf{h}_{jw} \right). \]  

Accordingly we have,

**Corollary 5:** The optimal strategy for Bob, when the jammer cancels the AN and chooses \( \mathbf{v} \) according to (35), to perform a MRC for the channel between Alice and himself. That is,

\[ \mathbf{c}^* = \frac{\mathbf{h}_{ab}}{\sigma c}. \]  

**Proof:** Since the jammer cancels the AN in advance, following the notations of Theorem 6, we have \( \mathbf{B}^{-1} = \mathbf{B} = \sigma \|\mathbf{I}\| \) and \( \mathbf{W} = \mathbf{h}_{ab} \mathbf{h}_{ab}^\dagger \), which we can write again as the following maximization problem,

\[ \max_{\mathbf{c}} \frac{|\mathbf{c}|^2 \mathbf{h}_{ab}^\dagger}{\sigma c^2}, \]

The above is the MRC maximization problem (also known as the matched filter [27]) which is maximized when \( \mathbf{c} = \frac{\mathbf{h}_{ab}}{\sigma c} \).

**Corollary 5** essentially state that once the jammer is able to cancel his AN in advanced at Bob’s receiver, Bob only performs maximal ratio combining to maximize his received SNR.

**Remark 5** (The case which \( M = N \)). In case the jammer and Bob have the same number of antennas, the sub-optimal schemes suggested herein are not applicable due to the fact that \( \mathbf{H}_{jb} \) and \( \mathbf{H}_{jb}^\dagger \) are full rank matrices with probability 1. Therefore, other types of schemes must be performed. We note that in any case, the jammer or Bob can always degenerate one or more antennas to apply the suggested schemes.

Simulation results which compare the average SNR value, as a function of \( \sigma \), are depicted in Figure 4 for the cases of \( M < N, M = N \) and \( M > N \). The figures compare the SNR for the global optimization problem with the suggested sub-optimal schemes given in Corollaries 4, 5 and 6. Specifically, in Figure 4(a) where \( M < N \) we consider the sub-optimal scheme given in Corollary 4 which cancels the AN at bob’s receiver and in Figure 4(c) we consider Corollary 5. We note also that when \( M = N \) not Bob or the jammer can cancel the AN as the matrices \( \mathbf{H}_{jb} \) and \( \mathbf{H}_{jb}^\dagger \) are full rank.

In the simulation results, one can observe several interesting insights. The first is that when \( M < N \), choosing the strategy that fixes \( \mathbf{c} \) to be the MRC and optimize only on \( \mathbf{v} \) performs very well compared to the global optimization. On the other hand when \( M > N \), it is better to choose the strategy which fixes \( \mathbf{v} \) to be in the direction of Willie and optimize only on \( \mathbf{c} \). This can be explained by the number DoF which are left for the optimization. That is, for example, when \( M < N \) and we fix \( \mathbf{c} \) we are left with \( N \) variables in the optimization problem which can provide a more accurate search for the optimum. A piece of additional evidence for that is the fact that the two curved unite when \( M = N \).

The second observation is that as \( \sigma \) grows, i.e., the “bad” noise is increasing which we may refer it to low SNR situation, the sub-optimal schemes, with the exception of the AN cancelation scheme, coincide with the optimal SNR making
them asymptotically optimal schemes.

Finally, one can observe that the strategy of canceling the AN is always worse than MRC or directing the noise towards Willie. The reason lies in the fact that once the AN is canceled, the inner products in the numerator are restricted to lie only in the specific null-space of Bob (as can be seen in equations (32) and (35)). This evidently, restricts much the SNR value as the figures depict. Moreover, even when \( \sigma^2 \rightarrow \infty \) this restriction remains and thus there is no conversion to the optimal SNR.

2) Jammer Knows Only Channel to Bob: Similar to the case which Bob has a single antenna when the CSI of Willie is unknown, Alice transmits with the power given in Theorem 2 in order to guarantee a covert transmission. Accordingly, we have the following global optimization problem for Bob and the jammer,

\[
\max_{\mathbf{v}, \mathbf{x}, \mathbf{c}} \quad \frac{c^2 \mathbf{P}_{\text{max}}^d}{2 \mathbb{E} \ln 2} |\mathbf{c}^\dagger \mathbf{h}_{ab}|^2 \\
\text{s.t.} \quad \xi_l = \frac{1}{d} \quad \text{for} \quad l = 1, ..., d.
\]

As in the previous subsection, we are facing a very complicated global optimization problem which does not appear to have a closed solution. Therefore, we suggest again sub-optimal solutions for the above optimization problem while considering only the cancelation of the AN at Bob’s receiver. Specifically, in case Bob has \( M > N \) antennas he cancels the AN by projecting it onto the null-space of \( \mathbf{H}_{jb} \) and choosing \( \mathbf{c} \) which is closest to \( \mathbf{h}_{ab} \). The jammer’s strategy for this case is given in the following theorem.

**Theorem 7:** The optimal strategy for the jammer, i.e., the solution for the maximization problem in (37), when the jammer has no CSI of Willie and is restricted to transmit with equal power to all directions while Bob uses a linear filter \( \mathbf{c} = \frac{\mathbf{h}_{ab}}{\sigma} \) is the following power allocation,

\[
\Sigma = \mathbf{P}_j \frac{1}{N} \mathbf{I}_{N \times N}.
\]

**Proof:** Once the interference term in the denominator of (37) is gone, the jammer should maximize the number of direction \( d \) which is \( N \) regardless the specific directions.

The above theorem implies that once Bob can cancel the noise, the jammer should try to impair Willie’s detection as much as possible as we do not know his CSI (which eventually comes in the form of a higher power for Alice). This is different from the result in Theorem 2 since now Bob takes an active part in the reception which enables the jammer to concentrate on Willie and to transmit in every direction. Note that according to Theorem 2 such strategy is also optimal even when Bob is passive however it depends on certain channel conditions.

When the jammer has more antennas than Bob (\( M < N \)), we are facing a more complicated problem since now the SNR gains from the number of direction \( d \) (as it appears in the numerator). So, to completely null the channel from the jammer to Bob and still maximize \( d \), the jammer must have at least \( 2M \) antennas if he does not know the linear filter \( \mathbf{c} \) that Bob has chosen. According to Corollary 5 we know that in case the jammer can completely remove the AN interference, Bob optimal linear filter is the MRC \( \mathbf{c}^* \) as given in the corollary. We thus suggest a scheme for which the jammer relay upon this choice of Bob for the linear filter \( \mathbf{c}^* \) and can null the channel even for \( N < 2M \) antennas. This strategy is given in the following theorem.

**Theorem 8:** The optimal strategy for the jammer, i.e., the solution for the maximization problem in (37), when the jammer has no CSI of Willie and is restricted to transmit with equal power to all directions while Bob uses a linear filter \( \mathbf{c}^* = \frac{\mathbf{h}_{ab}}{\sigma} \) is the following power allocation,

\[
\Sigma = \mathbf{P}_j \mathbf{V}^\dagger \mathbf{X}^\dagger \mathbf{V}^\dagger,
\]

where in case \( \sigma^2 \mathbf{P}_j < \frac{N-1}{N} \| \mathbf{h}_{jb} \|_{\infty}^2 \),

\[
\mathbf{V}^* = \text{span} \{ \mathbf{Q} \}
\]

and,

\[
\mathbf{X}^* = \frac{1}{N-1} I_{(N-1) \times (N-1)}.
\]

Otherwise,

\[
\mathbf{V}^* = \text{span} \{ \mathbf{Q} \} \bigcup \frac{\mathbf{h}}{\| \mathbf{h} \|},
\]

and,

\[
\mathbf{X}^* = \frac{1}{N \times N}.
\]

where \( \mathbf{Q} = \text{nullspace} \{ \mathbf{h} \} \) for \( \mathbf{h} = \mathbf{H}_{jb} \mathbf{c}^* \), and \( \text{span} \{ \mathbf{Q} \} \) is a matrix with \( N-1 \) orthogonal vectors from \( \mathbf{Q} \).

**Proof:** Setting \( \mathbf{c}^* = \frac{\mathbf{h}_{ab}}{\sigma} \) and using notations from Theorem 4 proof, the maximization problem (37) can be reduced to,

\[
\max_{\mathbf{v}, \mathbf{x}, \mathbf{c}} \quad \frac{d}{c^\dagger \mathbf{H}_{jb} \mathbf{V}^\dagger \mathbf{V}^\dagger \mathbf{H}_{jb}^\dagger \mathbf{c}^* + \sigma \| \mathbf{c}^* \|^2}
\]

\[
\text{subject to } \mathbf{d} = \mathbf{v}^\dagger \mathbf{H}_{jb} \mathbf{V}^\dagger \mathbf{V}^\dagger \mathbf{H}_{jb}^\dagger \mathbf{c}^* + \sigma \| \mathbf{c}^* \|^2
\]

where \( \mathbf{h} = \mathbf{H}_{jb} \mathbf{c}^* \). The above can be simplified to

\[
\max_{\mathbf{d} : \mathbf{d}, \mathbf{v}, \mathbf{h}} \quad \frac{d}{\sum_{l=1}^{d} \frac{1}{2} \mathbf{h}^\dagger \mathbf{h} + \sigma \| \mathbf{c}^* \|^2}
\]

\[
\min_{\mathbf{d} : \mathbf{d}, \mathbf{v}, \mathbf{h}} \quad \frac{\sum_{l=1}^{d} \frac{1}{2} \mathbf{h}^\dagger \mathbf{h} + \sigma \| \mathbf{c}^* \|^2}{d}
\]

which is similar to the expression in Theorem 4 proof and thus the rest follows similarly.

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