STARLIKENESS OF A CROSS–PRODUCT OF BESSEL FUNCTIONS

HUDA A. AL-KHARSANI, ÁRPÁD BARICZ AND TIBOR K. POGÁNY

Abstract. In this paper a necessary and sufficient condition is deduced for the close-to-convexity of a cross-product of Bessel and modified Bessel functions of the first kind and their derivatives by using a result of Shah and Trimble about transcendental entire functions with univalent derivatives, the newly discovered power series and infinite product representation of this cross-product, as well as a slightly modified version of a result of Lorch on the monotonicity of the zeros of the cross-product with respect to the order.

Mathematics subject classification (2010): 33C10, 30C45.
Keywords and phrases: Bessel functions of the first kind, modified Bessel functions of the first kind, close-to-convex functions, starlike functions, transcendental entire functions, zeros of cross-product of Bessel functions, infinite product.

REFERENCES

[1] M. ABRAMOWITZ, I. A. STEGUN (Eds.), Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, Dover, New York, 1972.
[2] G. E. ANDREWS, R. ASKEY, R. ROY, Special Functions, Cambridge University Press, Cambridge, 1999.
[3] Á. BARICZ, Geometric properties of generalized Bessel functions, Publ. Math. Debrecen 73 (2008), 155–178.
[4] Á. BARICZ, Generalized Bessel Functions of the First Kind, Lecture Notes in Mathematics, vol. 1994, Springer-Verlag, Berlin, 2010.
[5] Á. BARICZ, P. A. KUPÁN, R. SZÁSZ, The radius of starlikeness of normalized Bessel functions of the first kind, Proc. Amer. Math. Soc. 142 (6) (2014), 2019–2025.
[6] Á. BARICZ, S. PONNUSAMY, Starlikeness and convexity of generalized Bessel functions, Integr. Transforms Spec. Funct. 21 (2010), 641–653.
[7] Á. BARICZ, S. PONNUSAMY, S. SINGH, Cross-product of Bessel functions: monotonicity patterns and functional inequalities, Proc. Indian Acad. Sci. (Math. Sci.) (in press).
[8] Á. BARICZ, R. SZÁSZ, Close-to-convexity of some special functions and their derivatives, Bull. Malay. Sci. Soc. 39 (1) (2016), 427–437.
[9] Á. BARICZ, N. YAĞMUR, Radii of starlikeness and convexity of a cross-product of Bessel functions, Ramanujan J. (in press).
[10] R. K. BROWN, Univalence of Bessel functions, Proc. Amer. Math. Soc. 11 (2) (1960), 278–283.
[11] E. KREYSZIG, J. TODD, The radius of univalence of Bessel functions, Illinois J. Math. 4 (1960), 143–149.
[12] B. YA. LEVIN, Lectures on Entire Functions, Amer. Math. Soc.: Transl. of Math. Monographs, vol. 150, 1996.
[13] L. LORCH, Monotonicity of the zeros of a cross-product of Bessel functions, Methods Appl. Anal. 1 (1) (1994), 75–80.
[14] S. M. SHAH, S. Y. TRIMBLE, Entire functions with univalent derivatives, J. Math. Anal. Appl. 33 (1971), 220–229.
[15] R. SZÁSZ, On starlikeness of Bessel functions of the first kind, In: Proceedings of the 8th Joint Conference on Mathematics and Computer Science, Komárno, Slovakia, 2010, 9 pp.
[16] R. Szász, P. A. Kupán, *About the univalence of the Bessel functions*, Stud. Univ. Babeş-Bolyai Math. *54* (1) (2009), 127–132.

[17] G. N. Watson, *A Treatise of the Theory of Bessel Functions*, Cambridge University Press, Cambridge, 1995.