Time Analysis of Building Dynamic Response Under Seismic Action.  
Part 2: Example of Calculation

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Abstract. The second part of the article illustrates the use of the time analysis method (TAM) by the example of the calculation of a 3-storey building, the design dynamic model (DDM) of which is adopted in the form of a flat vertical cantilever rod with 3 horizontal degrees of freedom associated with floor and coverage levels. The parameters of natural oscillations (frequencies and modes) and the results of the calculation of the elastic forced oscillations of the building's DDM – oscillograms of the reaction parameters on the time interval $t \in [0; 131,25]$ sec. The obtained results are analyzed on the basis of the computed values of the discrepancy of the DDS motion equation and the comparison of the results calculated on the basis of the numerical approach (FEM) and the normative method set out in SP 14.13330.2014 "Construction in Seismic Regions". The data of the analysis testify to the accuracy of the construction of the computational model as well as the high accuracy of the results obtained. In conclusion, it is revealed that the use of the TAM will improve the strength of buildings and structures subject to seismic influences when designing them.

1. Introduction

Earthquakes are one of the most frequent and destructive influences on buildings and structures. Annually on the whole Earth there are about one million earthquakes. Most of them are insignificant in strength and not accompanied by catastrophic consequences, however, sometimes (up to several times a year) there are earthquakes, accompanied by destruction and human casualties [1-3].

The problem of increasing the seismic resistance of buildings and structures is currently very relevant. In this area, many Russian [3-11] and foreign [12-17] specialists work.

One of the areas of development of anti-seismic measures is the development of effective methods and techniques for calculating structures for the action of seismic forces to improve the strength and survivability of buildings in an earthquake. Currently, the main methods of calculating buildings and structures for seismic actions are the normative method [18-21], the method of decomposition in terms of eigenmodes, and also the finite element method.

In this paper, the application of the time analysis method (TAM) [18] to the calculation of a 3-storey building modeled by a vertical cantilever rod with 3 degrees of freedom under the action of a seismic load is considered. The load is given by the earthquake accelerogram.
2. Numerical implementation of the problem

As an example, consider the time analysis of the elastic oscillations of a three-story building. The design dynamic model (DDM) of the building is adopted in the form of a flat cantilever rod with point masses located in floor and coverage levels (figure 1), thus the system has \( n = 3 \) dynamic degrees of freedom. The flexural rigidity of the rod is \( EJ = 19,321 \times 10^7 \) kNm².

Figure 1 also shows the calculated cross sections \( n_j \), in which the values of the bending moments are the largest.

\[
\begin{bmatrix}
9,688 & 24,219 & 38,75 \\
24,219 & 77,5 & 135,62 \\
38,75 & 135,62 & 261,55
\end{bmatrix} \times 10^{-7} \text{ m / kN}
\]

The compliance matrix \( L \), as well as the matrixes of external (mass \( M \), damping \( C \) and rigidity \( K \) matrixes) and internal (matrix \( S \)) dynamic parameters of the system have the form:

\[
M = \begin{bmatrix}
802,4 & 0 & 0 \\
0 & 802,4 & 0 \\
0 & 0 & 730,5
\end{bmatrix} \times \text{kN}^2 / \text{m}, \quad K = L^{-1} = \begin{bmatrix}
63,524 & -36,526 & 9,529 \\
-36,526 & 34,938 & -12,705 \\
9,529 & -12,705 & 5,558
\end{bmatrix} \times 10^5 \text{ kN / m}
\]

\[
S = \begin{bmatrix}
0 + 82,55i & 0 - 32,575i & 0 + 5,371i \\
0 - 32,575i & 0 + 53,382i & 0 - 19,974i \\
0 + 5,9i & 0 - 21,94i & 0 + 17,035i
\end{bmatrix} \times S^{-1}
\]

The matrix \( C \) is zero; in solving the problem, the damping was not taken into account.

The spectrum of natural frequencies, calculated on the basis of the eigenvalues of the matrix \( S \), is equal to:

\[ \omega_1 = 6,28 \text{ rad / s}; \quad \omega_2 = 40,05 \text{ rad / s}; \quad \omega_3 = 106,65 \text{ rad / s} \]

The forms of natural oscillations of the building model are shown in figure 2.

\[
\begin{align*}
\text{1st form} & \quad \text{2nd form} & \quad \text{3rd form} \\
0.557 & \quad 1,364 & \quad 2.991 \\
0.523 & \quad 1,172 & \quad 4,213
\end{align*}
\]

Figure 2. The forms of natural oscillations of a building.
The obtained values of the parameters of the natural oscillations coincide with the data given in [2].

The accelerogram of the earthquake has the form shown in figure 3. The oscillograms of seismic forces calculated from (3) in the direction of each of the three degrees of freedom of the system also have a similar form. It can be seen from the graph that in the interval $t \in [55; 60]$ s in the accelerations of the base, a noticeable burst appears – the accelerations reach values 130 m/s. In this case, seismic forces increase to 1040 kN for the masses $m_1, m_2$ and up to 950 kN for the mass $m_3$.

![Accelerogram of the earthquake.](image)

**Figure 3.** Accelerogram of the earthquake.

Using the system (5), the kinematic parameters (respectively, the displacements, velocities and accelerations of the nodes) of the DDM response, whose oscillograms are shown in figure 4.

![Oscillograms of the kinematic parameters of the response of the building.](image)

**Figure 4.** Oscillograms of the kinematic parameters of the response of the building: (a) displacements; (b) velocities; (c) accelerations.
As it can be seen from the figures, the largest displacements are related to the mass $m_3$: the maximum value of the displacement is $1,549$ cm. According to [20] it does not exceed the admissible value $h_3/500 = 24.75 \times 100 / 500 = 4.95$ cm.

To verify the correctness of the solution, the founded values of the vector functions of the force parameters (6) were substituted into the left-hand side of the equation of motion of the DDS (1). The comparison of the left and right-hand side vectors of equation (1) shows that the vector discrepancy $\Delta f(t)$ tends to zero:

$$\Delta f(t) = R(t) + F(t) + I(t) - P(t) \to 0$$

Figure 5 shows the results of this test. It can be seen that the discrepancy in absolute value does not exceed the value $4 \times 10^{-11}$ kN. This fact testifies to the high accuracy of the TAM solution of the problem.

In addition, the obtained results were compared with the results calculated on the basis of the approximate method – finite element method (FEM) implemented in the software complex “Lira-CAD 2013” [19].

Figure 5. Oscillograms of discrepancies of the equation of motion of the DDS (1).

Figure 6 compares the oscillograms of the displacements of the mass $m_3$. The solid line corresponds to the solution based on the TAM, the dotted line – based on the FEM. In the qualitative sense the solutions are quite close, quantitatively there is a discrepancy in the range from 1 to 20 %.

Figure 6. Oscillograms of the displacements of the building's mass $m_3$ calculated on the basis of the TAM (solid line) and FEM (dotted line).

In general, it is possible to say that the results indicate the accuracy of the design model of building and the accuracy of the obtained results.

Figure 7 shows the oscillograms of the bending moments arising in the calculated cross sections of the DDM elements (figure 1). According to the graph, the nature of the change in the moments in time is analogous to that of the kinematic parameters. In this case, the largest values of the moments are:
The values of the bending moments calculated by the standard method [22] are the following:

\[ M_1 = 1.65 \times 10^7 \text{kNcm}, \quad M_2 = 1.81 \times 10^7 \text{kNcm}, \quad M_3 = 1.32 \times 10^7 \text{kNcm} \]

Comparison of the results is confirmed by the fact that calculations on real accelerograms give results that exceed the results of computations for [21], more than 2 times [2].

The implementation of the mathematical model and the solution of the problem is carried out using the language of technical calculations MATLAB [22].

![Oscillograms of the bending moments in the cross sections of the DDM rods.](image)

**Figure 7.** Oscillograms of the bending moments in the cross sections of the DDM rods.

### 3. Conclusions

1. In closed form, the expressions for the kinematic and force reactions of the discrete model of the system with elastic oscillations caused by the action of seismic forces are given.
2. The time analysis of the elastic response of a discrete model of a 3-storey building is simulated by a cantilever rod with 3 degrees of freedom during an earthquake specified by the corresponding accelerogram.
3. The efficiency of the TAM of the response of the DDS in solving a similar class of problems is shown, which makes it possible to use it in the design of buildings and structures in order to improve their strength reliability.

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