A theoretical diagnosis on light speed anisotropy from
GRAAL experiment

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Abstract

The light speed anisotropy, i.e., the variation of the light speed with respect to the direction in an “absolute” reference frame, is a profound issue in physics. The one-way experiment, performed at the GRAAL facility of the European Synchrotron Radiation Facility (ESRF) in Grenoble, reported results on the light speed anisotropy by Compton scattering of laser photons on high-energy electrons. So far, most articles concerned with the GRAAL data have established only the upper bounds on the anisotropy parameters based on available theories. We use a new theory of the Lorentz invariance violation to analyse the available GRAAL data and obtain the stringent upper limit of the order $2.4 \times 10^{-14}$ on the Lorentz violation parameters. In the meantime, we also can reproduce the allowed light speed anisotropy appearing in the azimuthal distribution of the GRAAL experimental data, and find that the best-fit parameters are compatible with the competitive upper bounds.

Keywords: Lorentz invariance violation, light speed anisotropy, GRAAL experiment

PACS: 11.30.Cp, 12.60.-i, 14.70.Bh

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Isotropy and constancy of light speed are two basic properties of light in modern physics. Any evidence for their variation, even very tiny, will have profound significance in science. The anisotropy of the light speed in vacuum has been studied for more than 100 years, and the most famous exploration is the Michelson-Morley experiment in 1887 [1]. There are many modern analogies of experiments for the same purpose and most of them adopted round-trip or two-way path propagation of light involving averaged light speed. Therefore one-way experiments, which are sensitive to the first order of light speed variation, deserve particular attention. In this paper we provide a theoretical analysis of the results from the one-way experiment [2, 3, 4, 5] performed at the GRAAL facility of the European Synchrotron Radiation Facility (ESRF) in Grenoble.

Most papers concerned with the GRAAL data just reported the upper limits on the Lorentz violation parameters measuring the light speed anisotropy by the commonly used theory. We use a new theory [6, 7, 8] of the Lorentz invariance Violation (LV) to analyse the GRAAL data. The Lorentz violation is related with the space-time anisotropy, and then the space-time anisotropy is the source for the light speed anisotropy. Therefore one can understand the anisotropy of the propagation velocity of free photons from the Lorentz violation, and on the other hand the constraints on the light speed anisotropy put bounds on the Lorentz violation parameters of the theory. In the new theory, the Lorentz violation information is measured by a new matrix, denoted as Lorentz invariance violation matrix. With the GRAAL data, we can establish the conservative bounds on the elements of the Lorentz violation matrix of free photons. At the same time, when the GRAAL data are best-fitted, we can obtain the values of the Lorentz violation parameters to observe possible anomaly implied by the data already.

In the following, we prepare the knowledge needed first, including reviewing the experiments and presenting the new model for photons. Then we can get the upper constraints on the violation parameters and also their best-fit values by analyzing the GRAAL experimental data.

We provide a brief review on the principle [9] of the GRAAL experiment [2, 3, 4, 5], in which the highly monochromatic electrons are scattered on the laser photons, for the study of light speed anisotropy in the “absolute” inertial frame at rest defined by null dipole of the Cosmic Microwave Background (CMB) radiation. In the head-on Compton scattering of the ultra-energy electrons and the low energy photons, the energy $E$ of the scat-
tered photon is given by
\[
E = \frac{4\gamma^2 E_0}{1 + 4\gamma E_0/m_e + \theta^2\gamma^2},
\] (1)
where \(\theta\) is the angle between the scattered photon and the incident electron, \(E_0\) is the energy of the incident photon, and \(m_e\) and \(\gamma\) are the mass and Lorentz factor of the incident electron. On the other hand, the scattered electrons will separate from the main incident electrons beam, and there is a distance \(x\) between these two trajectories. The energy of the scattered photon can also be written as
\[
E = \frac{E_e x}{A + x},
\] (2)
with \(E_e\) being the energy of the incident electron and \(A\) being a constant related with the experiment set-up. The maximum energy \(E\) of the Compton scattered photons is called as the Compton Edge (CE). From Eq. (1), CE can be obtained for \(\theta = 0\). In this case, using Eqs. (1) and (2), we get
\[
x_{CE} = \frac{4A\gamma E_0}{m_e},
\] (3)
So
\[
\delta x_{CE} = \frac{4AE_0}{m_e}\gamma \delta\gamma = -\frac{4AE_0}{m_e}\beta^2\gamma^3\delta c,
\] (4)
where \(\gamma = (1 - \beta^2)^{-1/2}\) is the Lorentz factor of the incident electron. At the GRAAL facility, the mean energy \(E_e\) of the electron beam is 6.04 GeV, i.e., \(\gamma = 11820\), three UV laser lines around 351 nm and a green line at 512 nm are used, and \(A = 159.28 \pm 0.2\) mm.

Now, \(x_{CE}\) has been measured in the GRAAL experiment [2, 3, 4, 5]. With the given stable energies \(E_0\) and \(E_e\), \(x_{CE}\) varies over different directions in the space when \(\delta c\) is azimuthal dependent. The azimuthal distribution for \(x_{CE}\) measured in the experiment determines the light speed anisotropy \(\delta c\). The GRAAL results for the measured distance \(\delta x_{CE}\) related with the CE are shown in Fig. 1 for the data of the years 1998-2005 [3] and in Fig. 2 for the data in the year 2008 [4], revealing the robust CE azimuthal variation. The limits on the light speed anisotropy are reported in Ref. [5], in which the azimuthal distribution presented in Figs. 1, 2 was not discussed. We show in this paper that the GRAAL results in Figs. 1, 2 can be elegantly reproduced.
by our new theory of Lorentz violation \cite{6, 7, 8}, briefly illustrated below for the free photon sector.

Nowadays, Lorentz invariance violation has triggered more and more interests in physics (see, e.g., Refs. \cite{10, 11, 12, 13, 14, 15, 16, 17, 18} and references therein). Especially the variation of the light speed due to the effect of quantum space-time has received particular attentions together with phenomenological supports \cite{19, 20, 21, 22}. Our framework \cite{6, 7} is a new fundamental theory of Lorentz invariance violation from basic principles instead of from phenomenological considerations. We proposed a general principle of physical independence of the mathematical background manifold, i.e., the equations describing the laws of physics have the same form in all admissible mathematical manifolds. Based on such principle, we revealed the replacements $\partial^\alpha \rightarrow M^{\alpha\beta} \partial_\beta$ and $D^\alpha \rightarrow M^{\alpha\beta} D_\beta$ for the ordinary partial $\partial_\alpha$ and the covariant derivative $D_\alpha$. $M^{\alpha\beta}$ is a local matrix called Background Matrix (BM) and can be divided into the sum of two matrices, i.e., $M^{\alpha\beta} = g^{\alpha\beta} + \Delta^{\alpha\beta}$, where $g^{\alpha\beta}$ is the metric of space-time and $\Delta^{\alpha\beta}$ is a new matrix which brings new terms violating Lorentz invariance in the standard model, therefore we denote the new framework as the Standard Model Supplement (SMS). Then the Lagrangian for the free gauge particle photon reads

$$L_G = -\frac{1}{4} F^{\alpha\beta} F_{\alpha\beta} - F_{\mu\nu} \Delta^\mu_\alpha \partial_\alpha A^\nu - \frac{1}{2} \Delta^\alpha_\gamma \Delta^\nu_\beta (g_{\alpha\mu} \partial_\beta A^\rho \partial_\nu A_\rho - \partial_\beta A_\mu \partial_\nu A_\alpha).$$

Since $\Delta^{\alpha\beta}$ contains all the LV information for the space-time, we call it Lorentz invariance Violation Matrix (LVM). All the LV effects vanish when $\Delta^{\alpha\beta} = 0$. More details of this new framework and its connection with available phenomenological constraints on LV effect of photons can be found in Refs. \cite{6, 7, 8}.

We thus get the modified Maxwell equation (or motion equation)

$$\Pi^\gamma A_\rho = 0,$$

where $\Pi^\gamma$ is also the inverse of the photon propagator

$$\Pi^\gamma = -g^{\gamma\rho} \partial^2 + \partial^\gamma \partial^\rho + \Delta^\alpha_\gamma \partial_\alpha + \Delta^\rho_\gamma \partial_\rho \partial_\alpha + \Delta^\gamma_\beta \Delta^\mu_\nu \partial_\beta \partial_\nu - g^{\gamma\rho} (2 \Delta^\alpha_\mu \partial_\rho \partial_\alpha + g_{\alpha\mu} \Delta^{\alpha\beta} \Delta^{\mu\nu} \partial_\beta \partial_\nu).$$
Three terms $\partial^\gamma \partial^\rho$, $\Delta^\alpha \partial^\mu \partial_\alpha$ and $\Delta^\alpha \partial^\rho \partial_\alpha$ (symmetric for the indices $\gamma$ and $\rho$) can be omitted under the consideration of the Lorentz gauge condition $\partial^\alpha A_\alpha = 0$ for the gauge field. With the Fourier decomposition $A_\rho = \int dp A_\rho(p) e^{-ip\cdot x}$, we can re-write Eq. (6) as
\[ \Pi^{\gamma \rho}(p) A_\rho(p) = 0, \]
where
\[ \Pi^{\gamma \rho}(p) = g^{\gamma \rho}(p^2 + g_{\alpha \mu} \Delta^\alpha \Delta^\mu \rho \beta \rho \nu + 2 \Delta^\alpha \rho \beta \rho \nu p^\beta p^\nu) - \Delta^\gamma \rho \beta \rho \nu p^\beta p^\nu, \]
which is the inverse of the free photon propagator in the momentum space.
A general parameterization for $p_\alpha$ can be done with spherical coordinates, so $p_\alpha$ can be expressed as $(E, -|\vec{p}| \sin \theta \cos \phi, -|\vec{p}| \sin \theta \sin \phi, -|\vec{p}| \cos \theta)$, where the light speed constant is $c = 1$. We find that there is a zero eigenvalue and a corresponding eigenvector $A_\rho(p)$ for the matrix $\Pi^{\gamma \rho}(p)$. So the determinant must be zero for the existence of the solution $A_\rho(p)$
\[ \det(\Pi^{\gamma \rho}(p)) = 0. \] (8)
Then we have the equation
\[ \sum_{i=0}^{8} \lambda_i(\Delta^\alpha \beta, \theta, \phi) E^i |\vec{p}|^{8-i} = 0. \]
The coefficient $\lambda_i(\Delta^\alpha \beta, \theta, \phi)$ is a variable related to the LVM $\Delta^\alpha \beta$ and the angles $\theta$ and $\phi$. So there are 8 real solutions for $E(|\vec{p}|)$ at most, and in general there are no analytical solutions for a general high order linear equation. But for some simple cases of the LVM $\Delta^\alpha \beta$, we expect some analytical solutions for $E$. Anyway, $E$ can be solved formally as $E = f_i(\Delta^\alpha \beta, \theta, \phi) |\vec{p}|$ for $i = 1 \ldots N$, and $1 \leq N \leq 8$. $f_i(\Delta^\alpha \beta, \theta, \phi)$ is a real variable and is independent of the momentum magnitude $|\vec{p}|$ because the photon is massless in the Lagrangian of Eq. (5). So the physically free photon velocity is
\[ c_{\gamma_i} \equiv \frac{dE}{d|\vec{p}|} = f_i(\Delta^\alpha \beta, \theta, \phi), \quad \text{for} \quad i = 1 \ldots N, \quad 1 \leq N \leq 8, \] (9)
which means: (i) The free photon propagates in the space with at most 8 group velocities. (ii) For each mode, the light speed $c_{\gamma_i}$ might be azimuthal.
dependent and not a constant. As we have known, the light spreads with different group velocities for different directions in the anisotropic media in optics. In analogy, we may view the space-time as a kind of media intuitively. However, there are essential differences between the optical case and the photon case here, because all the consequences of the \( N \) modes and the light speed anisotropy are results from the Lorentz invariance violation or the space-time anisotropy suggested by the new framework.

We need to clarify some essential points concerning the light speeds, i.e., \( c_\gamma \) in our work and the conventional light speed constant \( c \). \( c_\gamma \) is determined by the Maxwell equations or the propagator in QED, and represents the real propagation speed of the photon or the Electromagnetic wave freely propagating in the space-time, whereas \( c \) is related with the Lorentz group and the space-time metric, and serves as a constant. These two speeds are regarded as the same thing generally, but we should make clear that they are two different concepts. In the natural units, \( c = 1 \). When we write it explicitly in any unit system, the metric is \( g_{\alpha\beta} = \text{diag}(1, -1/c^2, -1/c^2, -1/c^2) \), so \( g^{\alpha\beta} = \text{diag}(1, -c^2, -c^2, -c^2) \). We see that the light speed \( c \) is related with the unit definitions of the time and the space. An element \( R^{\alpha\beta} \) of the Lorentz group is defined as the one which satisfies \( g_{\beta\nu} R^{\alpha\beta} R^{\mu\nu} = g^{\alpha\mu} \) where \( c \) is invariant, so we call \( c \) Lorentz invariant constant. In this article, we do not consider the light speed \( c \) in our derivation, i.e., we set \( c = 1 \) in the natural units. Instead, the light speed implied in our arguments is actually the propagating velocity \( c_\gamma \) of the photon or the Electromagnetic wave, and generally \( c_\gamma \neq c \) here.

We show now that our theory suggests the light speed anisotropy with respect to the azimuthal angle in an “absolute” reference frame. To understand the azimuthal distribution of the GRAAL data, let us consider two simple forms of \( \Delta^{\alpha\beta} \). One is

\[
\Delta^{\alpha\beta} = \xi m^\alpha n^\beta,
\]

where \( m \) and \( n \) are two unit vectors in the space-time and \( \xi \) measures the magnitude of LV. When \( n \) and \( m \) are parallel, \( \Delta^{\alpha\beta} \) of Eq. (10) represents that there exists a strain along the direction \( n \) in the space-time. This case of the LVM can help us to check whether there is a preferred direction \( n \) in the space-time. When \( n \) and \( m \) are orthogonal, Eq. (10) represents a shear in the plane spanned by the two vectors \( m \) and \( n \). Another useful
parameterization for $\Delta^{\alpha\beta}$ is

$$\Delta^{\alpha\beta} = \lambda k^\alpha, \quad k^2 = \pm 1,$$  \hspace{1cm} (11)

which represents a translation along a direction $k$ and $\lambda$ measures the magnitude of LV too. Timelike unit vectors can be parameterized as $(\cosh \zeta, \sinh \zeta \sin \theta \cos \phi, \sinh \zeta \sin \theta \sin \phi, \cosh \zeta \cos \theta)$, while spacelike ones as $(\sinh \zeta, \cosh \zeta \sin \theta \sin \phi, \cosh \zeta \sin \theta \cos \phi, \cosh \zeta \cos \theta)$, where $\zeta$, $\theta$ and $\phi$ are three variables to parameterize the unit vectors.

Now, we assume that there is a preferred direction $n = m$ for the space-time. For the sake of generality, we can take this direction as the $x$-axis, i.e., $\zeta = 0$, $\theta = \pi/2$ and $\phi = 0$. So Eq. (10) reads $\Delta^{\alpha\beta} = \text{diag}(0, \xi, 0, 0)$, which is substituted into Eq. (8) and then we can obtain all the two physical solutions for the light speed $c_{\gamma i}$. $c_{\gamma 1} = \sqrt{1 - (2\xi - \xi^2) \sin^2 \theta \cos^2 \phi}$, $c_{\gamma 2} = \sqrt{1 - 2\xi \sin^2 \theta \cos^2 \phi}$. Neglecting the higher powers of $\xi$, we can get $\delta c_{\gamma a}/c_{\gamma} \equiv |c_{\gamma \text{max}} - c_{\gamma \text{min}}|/c_{\gamma} \propto |\xi|$ and $\delta c_{\gamma m}/c_{\gamma} \equiv |c_{\gamma 1} - c_{\gamma 2}|/c_{\gamma} \propto \xi^2$. $\delta c_{\gamma a}$ and $\delta c_{\gamma m}$ represent the differences resulting from the angular distribution and the mode differences respectively. So we find two interesting results: (i) The light speed difference between two modes is proportional to the square of the element of the LVM, i.e. $\delta c_{\gamma a}/c_{\gamma} \propto \xi^2$. (ii) For each mode, the light speed may be direction dependent, and this anisotropy is linearly proportional to the element of the LVM, i.e. $\delta c_{\gamma a}/c_{\gamma} \propto |\xi|$. When $\xi = 0$, the light speed $c_{\gamma i}$ is equal to the constant $c = 1$, and the angle distribution of $c_{\gamma}$ is a sphere of radius 1 in the space. But it is direction dependent now for $\xi \neq 0$. Along the direction $n$, the light speed decreases ($\xi > 0$) or increases ($\xi < 0$), and the distribution for $c_{\gamma}$ is not spherical any more.

For $\Delta^{\alpha\beta}$, the sum of the two above cases reads

$$\Delta^{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \xi & 0 & 0 \\ \lambda & \lambda & \lambda & \lambda \\ 0 & 0 & 0 & 0 \end{pmatrix},$$  \hspace{1cm} (12)

which means $n = m = (0, 1, 0, 0)$ in Eq. (11) and $k = (0, 0, 1, 0)$ in Eq. (11). This $\Delta^{\alpha\beta}$ represents that there is a preferred direction $n = (0, 1, 0, 0)$ and a translation along $k = (0, 0, 1, 0)$ for the space-time, meaning the space-time is not isotropic now. The equation (8) is so complicated that we can hardly solve all the eight analytical solutions for the light speed in Eq. (9). We get
two solutions. One of them is physical and its explicit form is also lengthy
\[ c_\gamma = \frac{[\sin \theta (\sin \phi + \cos \phi) + \cos \theta] \lambda^2 - \sin \theta \sin \phi \lambda - \sqrt{h}}{-1 + \lambda^2} \]  
with
\[ h = 1 + [2 \sin^2 \theta (\cos^2 \phi - \sin \phi \cos \phi - 1) - 2 \sin \theta \cos \theta \sin \phi] \lambda 
+ [\sin^2 \theta \sin \phi (\sin \phi + 2 \cos \phi) + 2 \sin \theta \cos \theta (\sin \phi + \cos \phi)] \lambda^2 
+ (1 + \lambda^2)(2\xi - \xi^2) \sin^2 \theta \cos^2 \phi. \]

Finally, Eq. (4) becomes
\[ \delta x_{\text{CE}} = -\frac{4AE_0}{m_e} \beta^2 \gamma^3 (c_\gamma - c'), \]  
where \(c'\) is an effective constant and \(c\) is close to 1. The light speed \(c_\gamma\) is the specific form of Eq. (13).

Up to now, we are ready with all materials we need. So we can establish the bounds on the violation variables in Eq. (13) now, and then use Eqs. (13) and (14) to interpret the experimental data.

The GRAAL data of the years 1998-2002 were reported in Ref. [2], and the bound \(3 \times 10^{-12}\) was given for the Lorentz violation parameters. The data of the period 1998-2005 were reported in Ref. [3], from which Fig. 1 in our paper is extracted, as representing the CE data of the years 1998-2005. In paper Ref. [4], the setup of the experiment had been improved, then the data of the year 2008 were obtained from the improved experiment, and the paper also provided the conservative bound \(1 \times 10^{-14}\) on the Lorentz violation parameters. Fig. 2 is from Ref. [4], representing the data of the experiments in 2008. The recent publication [5] provided detailed discussions on a sample of all the 2008 data, and reported the upper bound of the order of \(1.6 \times 10^{-14}\) (95% C.L.) on the Lorentz violation parameters, related to the light speed anisotropy of the Lorentz violation model therein. The upper limits in Refs. [4, 5] are almost two orders stronger in power than that of the previous work in Refs. [2, 3] with the help of experimental improvements.

Based on Eq. (14) here and Eq. (7) in Ref. [5], we can write the competitive upper limits on the violation parameters \(\lambda\) and \(\xi\) of our Lorentz violation model too
\[ \left| \sin \phi (1 - \sin \phi - \cos \phi) \lambda - (\cos^2 \phi) \xi \right| < 2.4 \times 10^{-14} \]  
(95% C.L.), (15)
Figure 1: $\delta x_{CE}$ azimuthal distribution vs angles of the GRAAL data of the years 1998-2005 on a plane ($x$-$y$ plane or $\theta = \pi/2$). $\xi = -2.89 \times 10^{-13}$, $\lambda = 6.53 \times 10^{-14}$.

Figure 2: $\delta x_{CE}$ azimuthal distribution vs angles of the GRAAL data of the year 2008 on a plane ($x$-$y$ plane or $\theta = \pi/2$). $\xi = -3.64 \times 10^{-13}$, $\lambda = 8.24 \times 10^{-14}$. 
Table 1: Constraints on the element $\xi$ of the photon LVM from some light speed anisotropy experiments.

| Experiment                  | $\delta c_{\gamma\alpha}/c_{\gamma\beta} | | \xi | |
|-----------------------------|------------------------------------------|
| Refs. [2, 3]                | $3 \times 10^{-12}$ one-way              |
| Ref. [4]                    | $1.0 \times 10^{-14}$ one-way            |
| Refs. [24, 25]             | $3 \times 10^{-9}$ one-way              |
| Ref. [26]                   | $3.5 \times 10^{-7}$ one-way            |
| Refs. [27, 28] (cf. [29, 30]) | $3 \times 10^{-17}$ two-way            |

in the case of $\theta = \pi/2$. For comparison, the upper constraints on the element $\xi$ of the photon LVM are given in Tab. 1 from some other experimental results on the light speed anisotropy.

The two figures, Fig. 1 and Fig. 2, show the consistency between the two periods of the experiments in the years 1998-2005 and the year 2008. Due to the potentially systematic errors from the experiments for the CE data in Fig. 1, the error bars may be underestimated, and the data of 2008 in Fig. 2 may have the same problem. So the evidential regularity of oscillation shown in Figs. 1 and 2 still needs to be confirmed by more precise experiments in the future. Nevertheless, it is still enlightening for us to obtain the best-fit values of the Lorentz violation coefficients too, besides the upper constraints on the violation parameters $\lambda$ and $\xi$ we have gotten.

We fit Eq. (14) with the experimental results presented in Figs. 1 and 2, in which the solid curves represent the GRAAL data from Refs. 3, 4. The dashed curves are the calculated results of Eq. (14), and they are obtained to fit the experimental curves. The bright-color solid curves are the calculated results averaged over 90 degrees to fit the experimental curves too, and they are completely allowed within error bars by the GRAAL data. In Fig. 1, the best-fit parameters are: $\xi = -2.89 \times 10^{-13}$, $\lambda = 6.53 \times 10^{-14}$, and in Fig. 2, $\xi = -3.64 \times 10^{-13}$, $\lambda = 8.24 \times 10^{-14}$. We also find that the best-fit occurs when $\xi \simeq -4\lambda$. In this article, we can take the average and get $\xi = -3 \times 10^{-13}$ and $\lambda = 7 \times 10^{-14}$. So the best-fit LVM for photons can be approximated by

$$\Delta^{\alpha\beta} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & -3 \times 10^{-13} & 0 & 0 \\
7 \times 10^{-14} & 7 \times 10^{-14} & 7 \times 10^{-14} & 7 \times 10^{-14} \\
0 & 0 & 0 & 0
\end{pmatrix}$$
and $\delta c_{\gamma a}/c_{\gamma} \simeq 10^{-14}$-$10^{-13}$. The magnitudes of these LVM elements are consistent with the constraints of various experiments [8]. Finally, we find that if the tiny anomaly exits for the light speed isotropy in GRAAL experiments, the corresponding best-fit violation parameters $\lambda$ and $\xi$ are still allowed by the conservative upper bounds Eq. (15).

Under the strong upper constraint at the level $10^{-14}$, we should be serious to treat even tiny anomaly for the light speed anisotropy. The available GRAAL results of the azimuthal distribution have been explained by the light speed anisotropy suggested by the new theory of the Lorentz invariance violation. The best-fit violation parameters $\xi$ and $\lambda$ are compatible with the upper limits shown in Eq. (15), so more experiments are needed to determine whether the evidential light speed anisotropy exists or not.

Finally, we present a summary of our findings:

(i) Even if for a given type of particles (photons here), different Lorentz violation models have their own Lorentz parameters, but the same GRAAL experiment provides the upper limits of the same order for the different Lorentz parameters from different models. We have seen that the GRAAL data in Refs. [4, 5] give the constraints at the level $10^{-14}$ for the Lorentz parameters of the model therein, and same data also give same constraints for $\lambda$ and $\xi$ of our model too.

(ii) The regularity implied by the GRAAL data in Figs. 1 and 2 is stimulating for us to suggest an anomaly for the light speed anisotropy. In our framework, the Lorentz invariance violation or the space-time anisotropy for the photon is the source of the light speed anisotropy. At the same time, we also get the upper limits on the Lorentz violation parameters of the new model. It is a surprise that our new model calculations can reproduce the possible azimuthal oscillation in the reported GRAAL data of the years 1998-2008 in an elegant manner. As the oscillation of the light speed anisotropy extracted from Figs. 1 and 2 is allowed by the corresponding upper constraints also, it is unclear whether the anomaly for the light speed anisotropy really exists or not. Therefore more experiments are highly demanded to clarify the situation.

(iii) This work not only manifests the elegant application of the new theory to fit the experimental results, but also suggests new chances to test the theoretical predictions from the new framework and to constrain the newly introduced Lorentz invariance violation matrix by future experiments.

Acknowledgements: This work is partially supported by National Natural Science Foundation of China (Grants Nos. 11021092, 10975003, 11035003, 111
and 11120101004) and by the Research Fund for the Doctoral Program of Higher Education of China.

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