Magnetotransport in the presence of a longitudinal barrier: multiple quantum interference of edge states

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(Dated: January 24, 2019)

Transport in a two-dimensional electron gas subject to an external magnetic field is analyzed in the presence of a longitudinal barrier. We show that quantum interference of the edge states bound by the longitudinal barrier results in a drastic change of the electron motion: the degenerate discrete Landau levels are transformed into an alternating sequence of energy bands and energy gaps. These features of the electron spectrum should result in a high sensitivity of thermodynamic and transport properties of the 2D electron gas to external fields. In particular, we predict giant oscillations of the ballistic conductance and discuss nonlinear current-voltage characteristics, coherent Bloch oscillations and effects of impurities.

PACS numbers: 75.47.-m, 05.60.Gg, 75.45.+j, 03.65.Ge

Great attention has been attracted during the last decades to study of transport properties of various mesoscopic systems, e.g. ballistic and tunnel junctions, quantum dots, etc.\textsuperscript{[1, 2]} Such systems display fascinating quantum-mechanical behavior on a macroscopic scale, which results in quantization of the conductance\textsuperscript{[1, 2]}, Coulomb blockade\textsuperscript{[2]}, weak localization\textsuperscript{[1, 2]}, mesoscopic conductance fluctuations\textsuperscript{[1]} and macroscopic quantum tunneling\textsuperscript{[3]}, just to name a few. All quantum-mechanical effects are enhanced in low-dimensional systems, such as a two-dimensional electron gas, quasi-one-dimensional quantum wires, systems of coupled small tunnel junctions. Moreover, since the application of an external magnetic field allows to transform the continuous spectrum of electrons to discrete Landau levels (in a two-dimensional electron gas), various quantum-mechanical effects like Shubnikov-de Haas oscillations\textsuperscript{[1]}, integer and fractional quantum Hall effects\textsuperscript{[1, 2]}, etc. have been observed in magnetotransport measurements in these systems.

It is clear that if a potential barrier is placed across the direction of the electron motion, the current would flow only due to tunnelling through the barrier. However, what can happen if the barrier is created along the direction of the current? To the best of our knowledge this question has not been addressed yet. Of course, the problem is not very interesting in the absence of a magnetic field but the situation drastically changes if a magnetic field is applied perpendicularly to the plane of the 2D electron gas.

In this Letter we show that the quantum-mechanical effects in the magnetotransport phenomena are enhanced and qualitatively change if such a “longitudinal” barrier is present in the system. To be specific, we consider a two-dimensional electron gas (2DEG) subject to an external magnetic field $H$. We assume that a potential barrier of a narrow width separates the systems into two parts (left and right). What is important, the barrier should be penetrable, such that electrons can tunnel from one part of the system to the other. The tunneling through the barrier can generally be characterized by a reflection amplitude $r$ that can vary from zero to one. A schematic setup is shown in Fig. 1a. Here we would like to emphasize that such a setup is realistic and similar systems have been produced by using a split-gate technique or cleaved edge fabrication method\textsuperscript{[4]}

We start our discussion with a qualitative analysis of the energy spectrum of the electrons. Effects of the external magnetic field are considered in the Landau gauge, i.e. the vector-potential $\mathbf{A} = (-Hy, 0, 0)$, where the axis $y$ is perpendicular to the barrier. In this gauge the component $p_x$ of the momentum conserves even in the presence of the longitudinal barrier along the $x$-axis.

In the limit of a completely transparent barrier, $r = 0$, all states are just Landau levels (the size of the system in the $y$-direction is assumed to be large), and the electron spectrum is $\epsilon_n(p_x) = \hbar\omega_c(n+1/2)$, where $\omega_c = eH/mc$ is the cyclotron frequency and $m$ is the electron mass. Such a spectrum is shown by dashed lines in Fig. 1b. In this limit the energy spectrum is independent of $p_x$.

In the opposite limit of the zero barrier transparency, $r = 1$, the electron motion near the barrier considerably changes and can be described in terms of independent edge states in the left and right parts of the system (see Fig. 1a). In this case the degeneracy of the Landau levels is lifted, and the spectrum of the edge states near the barrier depends on $p_x$. The corresponding spectrum is represented in Fig. 1b by solid lines. A peculiar property of such a spectrum is the presence of “crossing points”, the number of which grows with an increase of the quasi-classical parameter $\alpha = \epsilon_F/(\hbar\omega_c)$, where $\epsilon_F$ is the Fermi energy of electrons in the absence of magnetic field. In-
decreased, “the distance” between the neighboring crossing points is \( \delta p_x \approx \hbar / R_c \) with \( R_c = c p_F / (eH) \) being the cyclotron radius of electron orbits. As the momentum \( p_x \) is restricted by the Fermi momentum \( p_F \), the number of the crossing points is determined by \( p_F / \delta p_x = \alpha \gg 1 \).

The case of a not complete barrier transparency (0 < \( r < 1 \)) is of the most interest. In this case, the quantum interference between the edge states lifts the degeneracy in the crossing points, and narrow “energy bands” and “energy gaps” appear in the electron spectrum \( \varepsilon_n(p_x) \). For \( r \approx 1 \) the characteristic widths of the energy bands \( \Delta \varepsilon \) and the energy gaps \( \Delta \varphi \) are of the order \( \hbar \omega_c \ll \epsilon_F \).

Notice that the electron states in the bands are delocalized, and thus, electrons move along the barrier with the velocity \( v = d\varepsilon_n(p_x) / dp_x \approx \Delta \varepsilon / \delta p_x \sim v_F \) (\( n \) is the band number).

Next, we analyze quantitatively the spectrum of the system. For this purpose we solve the Schrödinger equation for a two-dimensional electron gas (in the plane \((x, y)\)) in the presence of a magnetic field \( H \) and of the barrier described by a potential \( V(y) \):

\[
\left\{ \frac{1}{2m} \left( -i\hbar \frac{\partial}{\partial x} + \frac{eHy}{c} \right)^2 - \frac{\hbar^2}{2m} \frac{\partial^2}{\partial y^2} + V(y) - \epsilon \right\} \Psi = 0
\]

(1)

We assume that the characteristic width of the barrier \( l_b \) is much smaller than the cyclotron radius, \( l_b \ll R_c \).

In the absence of the magnetic field the scattering of electrons by the potential barrier is described by a 2 \times 2 scattering matrix

\[
\rho = ie^{i\varphi} \left( \begin{array}{cc} |r| e^{-ix} & t \\ -t^* & |r| e^{ix} \end{array} \right); \quad |r|^2 + |t|^2 = 1,
\]

(2)

where \( r \) and \( t \) are the probability amplitudes for an incident electron to be reflected back and to be transmitted through the barrier; \( \varphi \) and \( \chi \) are the scattering phases.

Solving Eq. (1) in the quasi-classical approximation at distances \(|y| \gg l_b \), where the potential barrier is negligibly small, and matching the wave functions of the electron on the left \((y < 0)\) and right \((y > 0)\) sides of the barrier with the help of the scattering matrix, Eq. (2), we come to the following dispersion equation (3):

\[
D \equiv \cos(\frac{\pi \Phi_+ (\varepsilon)}{2\phi_0} + \varphi) - r \cos(\frac{\pi \Phi_- (\varepsilon, p_x)}{2\phi_0} + \chi) = 0,
\]

(3)

where \( \phi_0 = \hbar c / 2e \) is the flux quantum, \( \Phi_\pm = HS_\pm \), \( S_1 = S_1 \pm S_2 \), and \( S_{1,2} \) are two areas bounded by the electron orbits (see Fig. 1a). Although Eq. (3) is valid for an arbitrary dispersion relation of electrons, it becomes an extremely transparent for the parabolic spectrum of electrons: the complete orbit is a circle with the radius \( R = cv\sqrt{2m\varepsilon_c / (eH)} \) and the center shifted on the distance \( cp_x / eH \) along the \( y \)-axis (see Fig. 1a).

The spectrum \( \varepsilon_n(p_x) \) obtained from Eq. (3) depends on both the electron momentum \( p_x \) and a discrete quantum number \( n \) (the band number). It displays gaps and bands with an almost periodic dependence in a wide region of \( p_x \) (see Fig. 2a). In this sense it resembles the energy spectrum of electrons in semiconducting superlattices. However, in our case the spectrum can be tuned by an external magnetic field and/or the reflection coefficient \( r \) controlled by the gate voltage. Moreover, the energy levels \( \varepsilon_n \) for a fixed value of \( p_x \) are distributed in a pseudo-random way. The typical distribution of energy levels is presented in Fig. 2b.

Using Eq. (3) we calculate the electron density of states (DOS) \( \nu (\varepsilon) \) and the conductance \( \sigma (\varepsilon_F)_n \). Of course, this gives only the contribution of the edge states.
bound to the longitudinal barrier. If the energy $\varepsilon$ is located between the Landau levels in the bulk, another well known contribution comes from the edge states on the boundaries of the sample. However, this contribution is a smooth function of the energy and is not interesting for us.

The electron DOS $\nu(\varepsilon) = \frac{2\pi}{\hbar^2} \int dp_x \sum_n \delta(\varepsilon - \varepsilon_n(p_x))$ can be written in the form $\nu(\varepsilon) = \frac{2\pi}{\hbar^2} \int dp_x |\partial D/\partial \varepsilon| \delta(D)$ (see Eq. (3)); expanding it into Fourier series in $\Phi(\varepsilon, p_x)$ and dropping terms fast oscillating in $p_x$ one finds that the main contribution to DOS comes from the zero harmonic. Calculating it one gets the electron DOS in the following form:

$$\nu(\varepsilon) = \frac{\sqrt{2m_e T_+}}{\pi^3 \hbar^2} \frac{|\sin \Phi(\varepsilon)|}{\sqrt{r^2 - \cos^2 \Phi(\varepsilon)}} \theta(r^2 - \cos^2 \Phi(\varepsilon)) \quad (4)$$

where $\Phi(\varepsilon) = 2\pi \varepsilon / (\hbar \omega_c) + \varphi$ and $T_+$ is the period of electron motion along the closed orbit for a given $\varepsilon$ and $\vartheta(x)$ is the step function. One can see from Eq. (4) that there are gaps in the DOS which can be found from the condition $\cos^2 (2\pi \varepsilon / (\hbar \omega_c + \varphi)) > r^2$.

Such a dramatic transformation of the electron spectrum has to lead to various novel effects in transport properties of 2DEG. As an example, we analyze next the ballistic transport along the $x$ direction. With the standard Landauer approach based on the relationship between the conductance and the transmission probability in propagating channels [2], and performing calculations identical to those for Eq. (4), we obtain the dependence of the linear conductance on the value of the Fermi energy level $\varepsilon_F$

$$G = \frac{e^2 \varepsilon_F}{2h} \sum_n \left( \tanh \frac{\varepsilon_n^{(t)} - \varepsilon_F}{2T} - \tanh \frac{\varepsilon_n^{(b)} - \varepsilon_F}{2T} \right)$$

where $\varepsilon_n^{(t)} = (n\pi + \pi - \arccos r)\hbar \omega_c / 2$ and $\varepsilon_n^{(b)} = (n\pi + \arccos r)\hbar \omega_c / 2$ are the top and the bottom of the $n$ energy band, respectively.

The conductance $G$, Eq. (5), becomes very sensitive to the Fermi energy $\varepsilon_F$. Experimentally, this energy can be tuned by applying a gate voltage. The typical dependence of $G$ on the gate voltage displaying giant oscillations of the conductance is shown in Fig. 3. These oscillations of the conductance reflect the presence of the bands and gaps in the spectrum of the edge states, thus proving the quantum interference of the edge states. The oscillations are smeared by temperature (compare two curves in Fig. 3).

The oscillations found here resemble those predicted [3] and observed [4] in the conductance of a junction between a superconductor and a two-dimensional electron gas. However, in that case the quantum interference between hole and electron edge states occurred due to Andreev reflection on the boundary.

FIG. 2: a) A part of the spectrum in the case of an intermediate barrier transmission, $r = 0.7$. The phases $\varphi$ and $\chi$ are obtained for the model of the $\delta$-function barrier, $V(y) = V \delta(y)$.

b) The dependence of the energy level difference $\delta \varepsilon = \varepsilon_{n+1} - \varepsilon_n$ on the energy $\varepsilon$ (energy levels distribution). We use the quasiclassical parameter $\alpha = 25$, and a particular value of $p_x/p_F = 0.3$. The points are connected by a thin line just for clarity.

FIG. 3: The giant oscillations of the conductance as a function of the Fermi energy (the gate voltage). The influence of temperature is shown: $k_B T_1 = 0.03(\hbar \omega_c)$, $k_B T_2 = 0.2(\hbar \omega_c)$ and $k_B T_3 = 0.5(\hbar \omega_c)$. We use the quasiclassical parameter $\alpha = 25$ and the reflection amplitude $r = 0.7$. 
As the main features of the electron motion under consideration are due to the quantum interference, thermodynamics and transport properties of the system are extremely sensitive to the influence of weak external fields. Thus, in the ballistic regime an increase of the transport voltage should lead to "giant steps" in the current-voltage characteristics (CVC). The width of the voltage steps is determined by the width of the gaps in the electron spectrum, i.e. $\Delta V \approx \Delta_0/e$.

In the diffusive regime, using an analogy with the electron transport in metals under magnetic break-down, a twinned plate and semiconducting superlattices one can expect coherent Bloch oscillations, and hence, an $N$-type non-linear CVC under relatively weak electric fields. Indeed, in the presence of an electric field $E$ satisfying an inequality $E > \hbar \omega_c/(e l_0)$, where $l_0$ is the electron mean free path, the periodicity of $E_n(p_x)$ as a function of $p_x$ results in localization of electrons along the $x$-direction. The localization length can be estimated as $L_{loc} = v_F \tau_{loc} \sim v_F \delta p_x/(e E) \sim h v_F/(e E R_e)$. The conductivity $G$ is obtained as $\sigma = n_e e^2 u$ where $n_e$ is the density of the charge carriers and the mobility $u$ is determined by the Einstein relation $u = D/\varepsilon_F$ ($D$ is the diffusion constant). In the case under consideration the particle moves over the distance $L_{loc}$, during the mean free time $\tau_0$, and hence $L_{loc}^2 \sim D \tau_0$. Therefore, the current carried by the localized electrons is

$$j = B\sigma_0 \left( \frac{\hbar}{\tau_0 e \varepsilon R_e} \right)^2 \mathcal{E} = B\sigma_0 \left( \frac{\hbar}{e \tau_0 R_e} \right)^2 (6)$$

where $B$ is a constant of order unity and $\sigma_0$ is the Drude conductivity at $H = 0$.

Finally, note that in the conventional situation (in the absence of the longitudinal barrier), a smooth (on the scale of the Fermi wave-length $\lambda_F \sim \hbar/p_F$) random scattering potential $U(x, y)$ does not change quasi-classical transport properties of 2DEG and is not usually seen in experiments. In contrast, in the presence of the longitudinal barrier the same smooth potential can qualitatively change the electron motion along the barrier. Arguments analogous to those presented for the Bloch oscillations lead to the conclusion that the electrons are localized by this potential (or by an inhomogeneity of the magnetic field) at a localization length $L_{trap} \sim (\Delta \varepsilon/U_0)L_{rand}$ ($U_0$ and $L_{rand}$ are the characteristic value and the correlation radius of the random potential $U$, accordingly) as soon as $\Delta \varepsilon \ll U_0$. Therefore, the giant oscillations (Fig. 3) can be observed if $U_0 \ll \Delta \varepsilon \sim \hbar \omega_c$. Actually, this is the condition for the observation of Schubnikov-de Haas oscillations and the integer quantum Hall effect.

In conclusion, we demonstrated that a two-dimensional electron gas in the presence of a magnetic field and a longitudinal barrier is a very interesting object. A crucial property of such a system is the quantum interference of electron edge states propagating along the barrier that gives rise to narrow energy bands and gaps in the electron spectrum. The spectrum is characterized by a quasi-periodic dependence of $\varepsilon_n(p_x)$ (with the period $\sim h/R_e \ll p_F$) in a wide region of $p_x$. The widths of the bands and the gaps can be tuned by the magnetic field and the gate voltage. Many interesting novel effects in the electron transport such as giant oscillations of the ballistic conductance of 2DEG as a function of the gate voltage, non-linear CVC, etc. are possible in such a system.

The authors thank the financial support of SFB 491 and A. M. Kadirobov gratefully acknowledges the hospitality of the TP III Institute, Ruhr-Universität, Bochum.

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$$V \sqrt{\frac{\hbar p}{p_F}} D_c(p_x \sqrt{\frac{R_e}{p_F}})D_c(-p_x \sqrt{\frac{R_e}{p_F}}) = 1 ,$$

where $D_c(x)$ is the parabolic cylinder function, and $V$ is determined by the properties of the barrier.
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