Exotic heavy baryons at LHC?*

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Abstract

We speculate about a heavy bottom-charm six-quark baryon. A semiclassical and a gaussian estimate reveal that the octet-octet bbb-ccc configuration can be energetically favored with respect to the singlet-singlet one. This result suggests that a confined bbb-ccc six-quark state may exist. Such objects may be produced in appreciable amount in heavy ion collisions at LHC energies.

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It has been recently considered in a study of high-energy heavy-ion collisions that multi-heavy baryons may be abundantly produced if the hadronization mechanism is described in the framework of a quark combinatoric model, which includes a phenomenological penalty factor for charm and bottom flavors [1]. In the present letter we speculate about a confined bottom-charm six-quark system (multi-heavy dibaryon), which may be in an octet-octet color state of the bbb core and ccc shell quarks, respectively. Such 'higher order' heavy baryons could not necessarily be observed up to now, since the heavy quarks decay weakly into light quarks destroying the (possible) bound state of six quarks into two 'normal' baryons. The only way to produce them seems to be a relativistic heavy-ion collision in the LHC energy range.

It is challenging also on its own right to investigate non-minimal color multiplet arrangements of several quark systems in order to gain a new insight into the nature of strong color forces in the non-relativistic (heavy quark) regime and learn so about nonperturbative and slightly collective QCD systems as well.

In this letter we estimate the energy of the six-quark bbb-ccc system treating the bbb core as one very heavy particle, which can either be in a color octet or in a color singlet state being the total six-quark system a color singlet. This way we compare the relative strength of the singlet and octet confining force as well - a problem also addressed in lattice QCD potential calculations [2] and recently in heavy meson physics [3].

The basis of our estimate is a simple non-relativistic potential model [4] of heavy quark systems, like the $J/\psi$ meson. The Hamiltonian to be used has the general form

$$H = \sum_i m_i - \sum_i \frac{1}{2m_i} \Delta_i - \sum_{i<j} \vec{q}_i \cdot \vec{q}_j \left( \tilde{\sigma} r_{ij} - \frac{\tilde{\alpha}}{r_{ij}} \right),$$

where $m_i$ is the quark rest mass (we use $m = 1.32$ GeV for the charm quark), $\Delta_i$ is the Laplace operator acting on the coordinate $\vec{r}_i$ of the $i$-th quark, $r_{ij}$ is the relative distance of the quarks $i$ and $j$ and finally $\vec{q}_i \cdot \vec{q}_j$ is a symbolic notation for an Ising-type SU(3) color-color interaction to be discussed in more detail later. $\tilde{\sigma}$ and $\tilde{\alpha}$ are phenomenological potential parameters, which we can connect to the string tension $\sigma$ and color coupling strength $\alpha$ used in heavy-meson physics by applying the above Hamiltonian (1) to the $J/\psi$ system.
Before presenting some estimates we would like to emphasize that the key physical reason to expect the existence of heavy multibaryons is that in a heavy quark system the quarks are much nearer to each other than in light quark systems and therefore they feel much stronger attractive color force than the light quarks. Furthermore, the color charges belonging to higher multiplets amplify the confining (attractive) forces between the quarks. Having possibly heavy valence quarks in mind the increase in the kinetic energy because of more degrees of freedom in the relative motion can be minor in comparison to this effect favoring so bound multibaryon states, such as a three-charm cloud around a compact bbb core would be.

Let us enlighten this reasoning by investigating some special quark configurations in this model: the \( J/\psi \) (a charm quark and an antiquark belonging to a 3 and an \( \bar{3} \) SU(3) color representation and being altogether a singlet), the \( \Omega_c \) (three charm quarks each belonging to a 3 and forming a singlet altogether) and the hypothetical bbb-ccc dibaryon. In the latter case we consider only four color charges: the compact bbb core and each of the relatively light charm quarks. The total system being color neutral and three quarks giving together either a singlet, an octet or a decuplet representation we deal with the following possibilities:

1. The bbb core is a singlet. In this case the ccc system is decoupled and must be a singlet so that a c quark (3) is attracted by a cc diquark \( \bar{3} \).

2. The bbb core is an octet. Now the ccc system is also an octet and the cc diquark can either be an antitriplet or a sextet, since both \( 3 \times \bar{3} \) and \( 3 \times 6 \) contains an octet.

Generally, considering the color symmetry inherent in the QCD, we may distinguish between color configurations by grouping different quarks into a given multiplet as long as the whole system is color neutral, but we are not allowed to assign different energy to different color projections (i.e. to a quark state which is actually 'red'). Therefore we determine the color Ising factors \( \vec{q}_i \cdot \vec{q}_j \) for each case formally from the color charge square Casimir operator of the total charge and the charge of the subsystems under consideration. Doing so we use the general formula

\[
\sum T^a T^a = Q^2 \cdot 1, \tag{2}
\]
where 1 is the unit matrix, the $T^a$-s are the generators of the SU(3) algebra in the corresponding representation and the eigenvalue of the Casimir operator in a $(J,j)$ SU(3) multiplet is given by [5]

$$Q^2 = \frac{1}{9} (J^2 + j^2 - Jj + 3J). \quad (3)$$

For a singlet, triplet or antitriplet, sextet and octet we get 0, 4/9, 10/9 and 1 for $Q^2$, respectively. For the $J/\psi$ meson we obtain this way the color Ising factor

$$\vec{q}_1 \cdot \vec{q}_2 = \frac{1}{2} \left( (\vec{q}_1 + \vec{q}_2)^2 - q_1^2 - q_2^2 \right) = -\frac{4}{9}, \quad (4)$$

leading to the energy formula

$$E_{J/\psi} = 2m + \frac{P_0^2}{m} + \frac{4}{9} f(r_{12}), \quad (5)$$

with

$$f(r) = \tilde{\sigma} r - \tilde{\alpha} \frac{r}{r}. \quad (6)$$

The kinetic energy of the relative motion can be estimated semiclassically using

$$\frac{P_0^2}{m} = \frac{K}{mr^2}. \quad (7)$$

From a fit to $J/\psi$ [6], which uses $\sigma = 0.192$ GeV, $\alpha = 0.47$ and $m = 1.32$ GeV we obtain $K = 1.39$ leading to the experimental $J/\psi$ mass $E = 3.096$ GeV. This value we use for other heavy quark systems in our first estimate as well.

For the bbb-ccc system we get three different contributions to the kinetic energy (ccc’s relative motion to the bbb core, a c quark’s motion relative to the cc diquark and finally a relative motion inside the diquark) with the respective reduced masses. Assuming a symmetric configuration, $r_{12} = r_{23} = r_{31} = r$ we obtain

$$E_{\text{kin}}^{\text{bbb-ccc}} = \frac{K}{6mr^2_+} + \frac{K}{3mr^2_-} + \frac{K}{mr^2_0}. \quad (8)$$
belonging to the Jacobian coordinates [7] defined as
\[
\vec{r}_+ = \frac{1}{3} (\vec{r}_1 + \vec{r}_2 + \vec{r}_3)
\]
\[
\vec{r}_- = \vec{r}_3 - \frac{1}{2} (\vec{r}_1 + \vec{r}_2)
\]
\[
\vec{r}_0 = \vec{r}_1 - \vec{r}_2.
\]  
(9)

We consider the color Ising factors relevant for the potential energy in this system first with a color permutation assumption leading to
\[
\vec{Q} \cdot \vec{q}_i = -\frac{1}{3} Q^2
\]  
(10)

and
\[
\vec{q}_i \cdot \vec{q}_j = \frac{1}{6} \left( Q^2 - \frac{4}{3} \right)
\]  
(11)

for each corresponding bbb-c or c-c quark pair. For the singlet core $Q^2 = 0$, for the octet one $Q^2 = 1$ must be taken, so we arrive at
\[
V^1 = \frac{2}{9} (f(r_{12}) + f(r_{23}) + f(r_{31}))
\]  
(12)

for the singlet and
\[
V^8 = \frac{1}{18} (f(r_{12}) + f(r_{23}) + f(r_{31})) + \frac{1}{3} (f(r_1) + f(r_2) + f(r_3))
\]  
(13)

for the octet core.

A symmetric triangle configuration of the c quarks with a singlet bbb core $(r_+ = \infty, r_0 = r, r_- = r \sqrt{3}/2)$ leads to a minimal energy of $E^1 = 4.71$ GeV at $r = 2.15$ GeV$^{-1}$, while an equal tetraeder with an octet bbb $(r_+ = r \sqrt{2}/3, r_- = r \sqrt{3}/2, r_0 = r)$ leads to $E^8 = 4.78$ GeV at $r = 1.6$ GeV$^{-1}$, which is just slightly above the singlet energy.

This result let us be optimistic about to make a more advanced estimate. First, we distinguish now between the antitriplet $((\vec{q}_1 + \vec{q}_2)^2 = q^2 = 4/9)$ and the sextet $(q^2 = 10/9)$ cc diquark states. With this distinction and assuming that the triplet c quark is located at $\vec{r}_3$ while the diquark consists
of the quarks located at $\vec{r}_1$ and $\vec{r}_2$ respectively we arrive at the following classification of the color Ising interaction.

The $bbb$ core - $c$ quark interaction energy is nonzero only if the core is in an octet state. In this case either

$$V_{0}^{8,6} = \frac{5}{12}f(r_1) + \frac{5}{12}f(r_2) + \frac{1}{6}f(r_3),$$

if the diquark is in a sextet or

$$V_{0}^{8,3} = \frac{1}{4}f(r_1) + \frac{1}{4}f(r_2) + \frac{1}{2}f(r_3),$$

if the diquark is in an antitriplet state. The $c$ - cc interaction energy is

$$V_{3}^{8,6} = \frac{5}{36}(f(r_{23}) + f(r_{31}))$$

and

$$V_{3}^{8,3} = -\frac{1}{36}(f(r_{23}) + f(r_{31}))$$

for an octet core and

$$V_{3}^{1,3} = \frac{2}{9}(f(r_{23}) + f(r_{31}))$$

for a singlet core, respectively. Finally the intra-diquark interaction is either

$$V_{12}^{3} = \frac{2}{9}f(r_{12})$$

for an antitriplet or

$$V_{12}^{6} = -\frac{1}{9}f(r_{12})$$

for a sextet diquark state. For the sake of comparison we give here the interaction energy inside the $J/\psi$ meson, where the $c - \bar{c}$ system is in a singlet state, as well

$$V_{12}^{1} = \frac{4}{9}f(r_{12}).$$

Besides noting the color hyperfine splitting of the interaction energies an interesting frustration phenomenon known from spin glasses can be observed:
in the four color charge system with an octet bbb core cannot be all pairwise interaction attractive (and hence confining), although the whole system is confined.

The second improvement on the semicalssical estimate is to use a gaussian wave function ansatz,

$$\Psi \propto \exp \left(-\frac{9r_+^2/R_+^2 + r_-^2/R_-^2 + r_0^2/4R_0^2}{4}\right), \quad (22)$$

with the Jacobian coordinates defined in eq.(9). A gaussian ansatz, being with some calculational tricks always factorizable in the very coordinate needed to evaluate the interaction energy, leads to the generic result

$$\frac{1}{<r>^2} = \frac{4}{\pi} \frac{1}{<r>}, \quad (23)$$

so we need to present the gaussian averages of c-c and c-bbb quark distances only. They are

$$<r_{12}> = 4\sqrt{\frac{2}{\pi}} R_0$$

$$<r_{23}>=<r_{31}> = \frac{2}{\sqrt{\pi}} \left(R_0^2 + R_-^2\right)^{1/2}$$

$$<r_3> = \frac{4}{3} \sqrt{2\pi} \left(R_+^2 + 4R_-^2\right)^{1/2}$$

$$<r_1>=<r_2> = \frac{4}{3} \sqrt{2\pi} \left(R_+^2 + R_-^2 + 9R_0^2\right)^{1/2}. \quad (24)$$

Minimizing the energy numerically in terms of the ansatz parameters $R_+$, $R_-$ and $R_0$ we obtain the following energies using the standard parameters of ref. [6], $\sigma = 0.192$ GeV$^2$, $\alpha = 0.47$:

\begin{align*}
J/\psi: & \quad 3.09 \text{ GeV} \\
\Omega_c: & \quad 4.54 \text{ GeV} \\
\text{bbb} - \text{ccc}(8, 6): & \quad 4.57 \text{ GeV} \\
\text{bbb} - \text{ccc}(8, 3): & \quad 4.74 \text{ GeV}
\end{align*}
not counting the 15 GeV bbb rest mass in the two letter cases. Although this
estimate slightly disfavors the confined bbb-ccc system we do think that this
problem is worth of further investigation, because there is some freedom in
choosing the parameters used in this estimate. In fact varying the coupling
strength $\alpha$ at fixed string tension $\sigma = 0.192$ GeV$^2$ a critical range around
$\alpha = 0.5$ can be identified, where the bbb-ccc system may or may not be energetically favored (fig.1).

On the other hand, if we speculate about an altered string constant in
many-quark systems, using for example $\sigma = 0.22$ GeV$^2$ and $\alpha = 0.54$ we obtain

$J/\psi$: 3.097 GeV
$\Omega_c$: 4.544 GeV
$bbb - ccc(8, 6)$: 4.515 GeV
$bbb - ccc(8, 3)$: 4.716 GeV

Concluding this letter we investigated whether the heavy bbb-ccc six-
quark system can be energetically favored with respect to two separate heavy
baryons. Although we have not yet found a decisive quantitative result, a
semiclassical and a gaussian estimate show that this possibility cannot be
excluded on the basis of phenomenological knowledge about heavy meson
masses, like $J/\psi$. Since such exotic heavy baryons are in principle producable
in relativistic heavy ion collisions at LHC energies and they may be observed
through a measurement of like-charge muon triplets [8], we do think that a
concise future investigation of this problem with more advanced calculations
is desirable.

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**Figure caption**

Fig.1. The phenomenological energy of a $c\bar{c}$ and a $ccc$ heavy quark system in a singlet or in an octet state bounded to an octet $bbb$ core.