Time–frequency return co‑movement among asset classes around the COVID‑19 outbreak: portfolio implications

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Abstract
This study explores the time–frequency return connectedness of the four most relevant asset classes namely, equity, digital assets, commodity, and fixed income. To do so, we use the novel proxies of the S&P500 Index for equity, the S&P Cryptocurrency MegaCAP Index for digital assets, the S&P Goldman Sachs Commodity Index for commodity, and the S&P Global Developed Sovereign Bond Index for fixed income, and also employ the wavelet analysis for daily data over the period 2017: M02 to 2021: M09. In contrast to the pre-COVID-19 period, our findings indicate that the interdependence between the selected asset classes has intensified across all time scales and frequency bands during the COVID-19 crisis, proving the lack of hedging opportunities. Besides, the findings reveal that there is a significant lead-lag relationship between time series at medium and low frequencies during the research period, and the directional connectedness among asset classes is sensitive to frequencies. Especially, the co-movements among the pairs are pronounced during the COVID-19 outbreak. Remarkably, the wavelet-based Granger causality test corroborates the wavelet results and underscores there is a significant causal link between the variables during COVID compared to pre-COVID. Moreover, the results of the portfolio risk analysis by employing the value at risk (VaR) measure indicate that portfolio diversity advantages vary among frequency and across time. The results of the present study provide insight and might help foreign portfolio investors diversify their portfolios across different asset classes.

Keywords Asset classes · Commodity · Digital assets · Equity · COVID-19 · Wavelet

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1 Introduction

The relationship between various asset classes has received substantial attention from investors, portfolio managers, and policymakers, and numerous scholars have been focused to unveil the return and volatility connectedness among different asset classes especially by increasing global financial uncertainty and also market integration (e.g., globalization, the openness of markets, financialization and technological developments) (Bouri et al. 2018a). In the empirical literature, several studies have investigated the connectedness between commodity and equity markets in developed and developing economies (Mollick and Assefa 2013). Narayan and Narayan (2010) highlighted that oil shocks have a short and long-term effect on equity returns. Shahzad et al. (2020) revealed that gold is a hedge and a safe haven for most of the G7 stock indices.

Furthermore, another strand of literature on rising global financial uncertainty (e.g., global financial crisis (2008–2009), European debt crisis of (2010–13)) and also the emergence of the digital asset of Bitcoin has been shifted to investigate the co-movement between conventional (such as stocks, bonds, and currencies) and digital assets (e.g., Brière et al. 2015; Bouri et al. 2017a). Specifically, several empirical studies have attempted to examine the hedging ability of digital assets under various market conditions since commodities were less likely considered safe-haven assets and behaved as risky assets, particularly after the global crisis period 2008–2009 (Bekiros et al. 2017). This is crucial since detecting any significant spillover connectedness between Bitcoin and other asset classes provide important implications for portfolio managers, risk management decisions, and also for policymakers. Some studies found a weak relationship between Bitcoin and commodities (e.g., gold, crude oil), bonds, and equities (Bouri et al. 2018a; Ji et al. 2018), and the Bitcoin market is not influenced by those asset classes. The weak correlation implies diversification and hedging benefits of Bitcoin against traditional asset classes (Bouri et al. 2017b; Corbet et al. 2018). Consistently, the study by Brière et al. (2015) showed that the inclusion of Bitcoin can be enhanced the risk-return trade-off of well-diversified portfolios and Bitcoin is an effective diversifier for the main world asset classes.

The weak connectedness can be explained as Bitcoin prices do not mainly determine by the price factors of other asset classes (Bouoiyour et al. 2016), Bitcoin prices are less probable to depend on financial and economic factors (Ciaian et al. 2016), and Bitcoin has a specific set of characteristics including user anonymity (Ober et al. 2013), attractiveness (Kristoufek 2015), computer programming interest and prohibited activity (Yelowitz and Wilson 2015), and also energy prices (Li and Wang 2017). Remarkably, Baur et al. (2018) showed that Bitcoin is not linked with other asset classes during both calm and stress periods. The finding of prior studies also underscored that Bitcoin is a hedge against global risk (Bouri et al. 2017a) and there is a negative relationship.
between Bitcoin returns and economic policy uncertainty, implying a hedging ability of Bitcoin (Demir et al. 2018). Besides, Bouri et al. (2020a) revealed that Bitcoin is the least dependent on global stock markets, and Bitcoin has dominance diversification benefits compared to gold and commodities. Bouri et al. (2020b) also supported cryptocurrencies as a possibly valuable digital asset class against down movements in the US stock market.

However, several empirical studies highlighted a significant relationship between the commodity and the cryptocurrency markets (e.g., Hayes 2017). Bouri et al. (2018a) provided significant evidence that the Bitcoin market is not isolated fully and Bitcoin returns are linked closely to those of most of the asset classes (e.g., equities, stocks, bonds). Bouri et al. (2018b) found that there is a likelihood to estimate Bitcoin price movements according to the aggregate commodity index and gold price information. Maghyereh and Abdoh (2020) found that there is a long-term relationship between Bitcoin returns and the S&P 500 while the association between Bitcoin and commodity significantly decreases in the short term. Overall, they found that Bitcoin can provide financial diversification in certain return quantiles and time frequencies. Recently, spreading the COVID-19 outbreak that resulted in rising economic and financial uncertainties globally, the interest of some scholars triggered to investigate the connectedness across various asset classes for providing insight to investors, portfolio managers, and regulatory bodies. For example, Bouri et al. (2021) by using the TVP-VAR investigated the linkage across various assets, and they found that the total relationship spikes and the structure of the relationship changes, which coincides with the COVID-19 pandemic. Siddique et al. (2021) by using continuous wavelet transform revealed that green bonds, gold, and bitcoin have the least relationship with the equity market during the COVID-19 outbreak period, suggesting the hedge and safe haven ability of these financial assets.

Thus far, the majority of studies reviewed above focused to investigate the return and volatility connectedness between Bitcoin, commodities (e.g., oil and gold), stocks, and bonds by employing various approaches before occurring the COVID-19 outbreak. While a few only studies examined the connectedness of the inclusion of COVID-19 in their studies (e.g., Bouri et al. 2021; Siddique et al. 2021), it remains unclear what relationship exists in terms of time and frequency horizons between the various asset classes in the global market, especially during the COVID-19 outbreak. Hence, this study fills this gap and significantly contributes by selecting the unique various asset classes indices in the global market during the COVID-19 episode using the daily data between 2017: M02 to 2021: M09. More specifically, we compare the co-movement between using asset classes by splitting data before the COVID-19 crisis period (28/02/2017 to 10/03/2020) and during the COVID-19 period (11/03/2020 to 30/09/2021). Besides, the present study sheds light by answering the following questions: (i) is there any causal link between equity, digital assets, commodity, and fixed income asset classes, especially during the COVID-19 period? (ii) if yes, in which direction(s)? (iii) Does the co-movement of using asset classes help reduce portfolio risk?
To do so, this study follows the previous studies (e.g., Alam et al. 2019; Bouriet al. 2020a) and employs the wavelet coherence approach, which is rarely used in this field and has some advantages over other methods. Employing the wavelet approach has an advantage and allows us to capture the causality direction from both time and frequency dimensions. Besides, this study contributes by examining the causal linkage between asset classes using the wavelet-based Granger causality test. Moreover, the present study performs the wavelet-VaR to investigate hedging properties among using asset classes. To the best of our knowledge, this may be the first study that conducts this relationship comprehensively using wavelet analysis during the COVID-19 period.

The present study yields some consistently remarkable highlights. First, in contrast to the pre-COVID-19 period, strong relationships are found at different frequencies during the COVID-19 pandemics, multiple directions are unveiled with timescale differences, which uncovers that the co-movement and the causal association among the pairs are manifested by the difference in scale. Second, the wavelet-based Granger causality test corroborates the wavelet results and underscores there is a significant causal link between the variables during COVID compared to pre-COVID, indicating the lack of hedging opportunities. Third, the results of the value at risk (VaR) highlight that portfolio diversity advantages are variable with respect to time and frequency, which can be so useful for investors, portfolio managers, and policymakers.

The rest of this study is organized as follows. Section 2 describes the data, models, and methodologies. Section 3 is followed by presenting empirical results. Section 4 concludes the paper.

2 Data and methodology

2.1 Data

This paper examines the time–frequency co-movements among the main asset classes namely equity, digital assets, commodity, and fixed-income. The selection of four major asset classes is motivated by the fact that they are heavily used in financial markets in most countries around the globe and are likely to be widely impacted by market shocks during the COVID-19 outbreak. To do so, we use the S&P500 Index for equity (SP500), the S&P Cryptocurrency MegaCAP Index (which includes Bitcoin and Ethereum) for digital assets (CRY), the S&P Goldman Sachs Commodity Index for the commodity (GSCI), and the S&P Global Developed Sovereign Bond Index for fixed income (BOND). The daily data for all of the time series is obtained from the DataStream and the study spans the period 28/02/2017 to 30/09/2021. Remarkably, to distinguish the behaviors of these series during the non-COVID and COVID periods, the sample is divided into two periods. The COVID-19 outbreak becomes a pandemic following the official release of World Health
Organization situation reports on March 11, 2020. Therefore, we selected the pre-
COVID-19 pandemic period from 28/02/2017 to 10/03/2020 while the COVID-19
pandemic period is selected from 11/03/2020 to 30/09/2021.

The descriptive statistics on the difference for the CRY, SP500, GSCI, and BOND
indexes are represented in Table 1. We can observe that the average value of the four
series is positive. In addition, the cryptocurrency index has the greatest fluctuation
according to standard deviation. Peak and thick-tail characteristics can be found in
all series in terms of skewness and kurtosis. At the same time, at a 1% significance
level, the Jarque–Bera test rejects the null hypothesis, suggesting that data does not
follow the normal distribution. Similarly, the ADF test indicates that all series are
stationary at level.

|         | CRY     | SP500   | GSCI    | BOND    |
|---------|---------|---------|---------|---------|
| Mean    | 0.318665| 0.051917| 0.027776| 0.012662|
| Standard deviation | 4.984728 | 1.252691 | 1.405311 | 0.291867 |
| Maximum | 19.88363 | 8.968316 | 7.683172 | 1.637718 |
| Minimum | -26.99658| -12.76521| -12.52372| -1.832473|
| Skeness | -0.368872| -1.130335| -1.411585| -0.413249|
| Kurtosis| 6.387979 | 24.20636 | 17.06234 | 8.456526 |
| Jarque–Bera | 579.0918*** | 21,907.18**** | 9908.833*** | 1467.001*** |
| ADF     | -34.40986*** | -10.04039*** | -34.64192*** | -19.37371*** |

The asterisks *** illustrates significance at the 1%, 5% and 10% levels, respectively.

2.2 Methodology

Existing research examines the link between two-time series variables using a vari-
essy of statistical methodologies. For instance, the bivariate cross-quantilogram
approach (Pham 2021), the DCC-GARCH model (Umar et al. 2020), rolling window
framework (Naeem et al. 2021). These approaches ignore the frequency dependen-
cies of the data in empirical estimations and fail to analyze the time and frequency
properties simultaneously. With the advent of wavelet analysis, difficulties surround-
ing simultaneous capture of time and frequency dimensions are resolved.

The current paper uses the ($\psi$) wavelet technique, which is an element of the
Morlet family wavelet. It has the equation $\psi(t) = \pi^{-\frac{i}{2}} e^{-imt} e^{-\frac{1}{2}t^2}$, $p(t), t = 1, 2, 3, ...T$.

By adding the time and frequency (represented by k and f) domain that associates with the wavelet, the transforming wavelet equation $\psi_{k,f}$ can be made, as seen
in Eq. 1.

$$\psi_{k,f}(t) = \frac{1}{\sqrt{h}} \phi \left( \frac{t-k}{f} \right), k,f \in \mathbb{R}, f \neq 0$$ (1)
Furthermore, the continuous wavelet function equation by adding the time series data $p(t)$ can be found.

$$W_p(k,f) = \int_{-\infty}^{\infty} p(t) \frac{1}{\sqrt{f}} \psi \left( \frac{t-k}{f} \right) dt$$  \hspace{1cm} (2)

By merging coefficient $\psi$ into the equation, Eqs. 3 and 4 are restored.

$$p(t) = \frac{1}{C_\psi} \int_{0}^{\infty} \left[ \int_{-\infty}^{\infty} |W_p(a,b)|^2 da \right] \frac{db}{b^2}$$  \hspace{1cm} (3)

Furthermore, the wavelet power spectrum (WPS) can be derived based on Eq. 2, to obtain information about the behavior of the time series of variables.

$$WPS_p(k,f) = \left| W_p(k,f) \right|^2$$  \hspace{1cm} (4)

Besides, we employ the cross-wavelet transform (CWT) approach to detect the time–frequency causality link between the $p(t)$ and $q(t)$ time series. The CWT is presented in Eq. (5).

$$W_{pq}(k,f) = W_p(k,f) W_q(k,f)$$  \hspace{1cm} (5)

Torrence and Compo (1998) stated the squared wavelet coherence of $W_{pq}(k,f)$ can be formed in Eq. 6.

$$R^2(k,f) = \frac{\left| S\left(f^{-1}W_{pq}(k,f)\right) \right|^2}{S \left( f^{-1} \left| W_p(k,f) \right|^2 \right) S \left( f^{-1} \left| W_q(k,f) \right|^2 \right)}$$  \hspace{1cm} (6)

where $S$ stands the smoothing process over time, with $0 \leq R^2(k,f) \leq 1$. If the $R^2(k,f)$ approaches 1, it implies the strongest co-movement between $p(t)$ and $q(t)$, bounded by a black line and depicted by the red color. In addition, if $R^2(k,f)$ approaches 0, it portrays weak evidence of co-movement between $p(t)$ and $q(t)$, and this is indicated in blue color.

Nonetheless, $R^2(k,f)$ cannot offer any comprehensive facts on the sign of correlation to differentiate a positive co-movement from a negative one. Thus, Torrence & Compo (1998) proposed a process for capturing discrepancies in wavelet coherence (WTC) by utilizing deferral signs in the wavering of two-time series. Equation 7 thereby elucidates the WTC as follows;

$$\phi_{pq}(k,f) = \tan^{-1} \left( \frac{L \{ S(f^{-1}W_p(k,f)) \} }{O \{ S(f^{-1}W_p(k,f)) \} } \right)$$  \hspace{1cm} (7)

An imaginary operator and a real part operator are illustrated by L and O separately.
In this study, we perform the cross wavelet transform and wavelet coherence for testing the time–frequency linkages between the selected time series. Besides, we evaluate the interdependence by using the wavelet correlation developed by Rua (2013) and wavelet-based Granger causality.

3 Empirical results

Figure 1 presents the continuous wavelet power spectrum of a single time series. The red regions show strong fluctuations, while the yellow, green, and blue islands demonstrate weaker volatility of the examined variables. We can observe that across all plots, a common pattern appears. More accurately, there is low volatility at higher frequencies (2 and 16 days), while high volatility posits at lower frequencies (64 and 128 days) during the sample period. Specifically, we can see that strong volatility exists in the medium and long run during the COVID-19 outbreak, which illustrates that the COVID-19 crisis has a significant impact on the global market returns.

We employ cross wavelet transform and wavelet coherence methodologies between the GSCI, CRY, BOND, and SP500 return prices in the pre and during the COVID-19 crisis and the whole sample. Figures 2 and 3 demonstrate this analysis, respectively. The amount of co-movement is indicated by the color coherency, which ranges from red (high coherency) to blue (low coherency) in these illustrations. Red color denotes strong co-movements, whereas a blue color denotes...
weak co-movements. We also distinguish between causality and phase differences. The phase differences between the two assets are indicated by arrows. For example, → and ← suggest that the assets are in phase or out of phase. Being in (or out of phase) reveals a positive (or negative) relationship between the two time-series. Furthermore, ↗ and ↙ show that the first asset is leading the second one, whereas ↖ and ↘ represent that the first asset is lagging those of the second one.

Figure 2 shows several graphical examples of low co-movements between the selected market indexes across frequencies and time in the pre-COVID-19 period. This is especially true for the wavelet coherency between CRY-SP500, CRY-BOND, and GSCI-BOND, as shown by large blue islands localized in the pre-COVID-19 period in the low, medium, and high-frequency bands. However, the coherency between the SP500-GSCI and the SP500-BOND shows some significant red color regions in the high and medium frequency bands, corresponding to December 2019 and January 2020.

However, during the COVID-19 epidemic, all markets under consideration appear to react negatively to bad news when WHO officials declared the novel coronavirus. Some crucial insights emerge from an examination of the COVID-19 outbreak figure. In contrast to the pre-COVID-19 period, the cross wavelet transform and wavelet coherence analysis of the selected markets shows high co-movement across all time scales and frequency bands, implying a strong connection between the markets.
during the COVID-19 outbreak. This tendency is particularly prominent during the start of the COVID-19 epidemic pandemics.

The cross wavelet transform and wavelet coherence plots for CRY-SP500, SP500-GSCI, and GSCI-BOND show the presence of significant coherence islands between the onset of the novel coronavirus and the end of 2020, and at all frequencies, indicating significant co-movement between the markets corresponding to a constant increase in infected counts around the world. This outcome reveals that during the COVID-19 crisis, a high level of market co-movements was observed, validating the contagion hypothesis.

Figure 3 shows the cross wavelet transform (XWT) and wavelet coherence (WTC) during the whole sample period. On the left-hand side of Fig. 3, the rectified
cross-wavelet transform outcomes for the pairs CRY-SP500, CRY-GSCI, CRY-BOND, SP500-GSCI, SP500-BOND, and GSCI-BOND are shown. In particular, the covariance of the CRY-SP500, CRY-GSCI, CRY-BOND, SP500-GSCI, SP500-BOND, and GSCI-BOND pairs highlighted significant red islands between 2019 and 2020, which uncovers that a strong association exists among the pairs due to the COVID-19 outbreak. More importantly, in the cases of CRY-GSCI and SP500-GSCI, the relationship on the scale of 32–128 in about 2020 is stronger. The pairs of CRY-SP500, GSCI-BOND, and SP500-BOND show a lower connection in the 4–8 scale in 2019 and the 64–128 scale at the beginning of 2019. Even though strong
relationships are found at different frequencies during the COVID-19 pandemics, multiple directions are unveiled with timescale differences, which uncovers that the difference in scale reveals the co-movement and causal relationship between the pairs. Specifically, we observe that after 2019, the arrows point to the right for all couples at all scales, except for the GSCI-BOND pair during the research period.

Furthermore, the right-hand side of Fig. 3 shows the wavelet coherence transformation which highlights regions of significant lead-lag nexus among the selected series. For the CRY-SP500 and CRY-GSCI pairs, we observe that the significant islands cover the whole sample period at 32–128-day scales, suggesting that there
is a medium and long-run relationship between cryptocurrency, stock, and GSCI markets. In addition, the arrows point straight down, meaning that cryptocurrency leads the SP500 and GSCI markets. The increase in coherence on both medium and low scales indicates the interrelatedness between the CRY, SP500, and GSCI markets is due to both contagion and interconnection. Nevertheless, CRY experiences a lead-lag relationship with BOND in the short and medium run throughout the sample period. The arrows mostly point to the left, suggesting that there is a negative relationship between these variables, which is consistent with the previous work by Shahzad et al. (2020).

Turning to the coherence between SP500, BOND, and GSCI, SP500 seems to lead GSCI, and they have a strong relationship from 2019 until 2021 in 64–128 days. Compared to the cases of BOND-GSCI and SP500-BOND plots, the immediate increase in coherence between SP500 and GSCI is pronounced. The influence of the COVID-19 crisis is continuous from the short to the medium scale, revealing that the strong co-movement is due to pure contagion and fundamental-based contagion. Low wavelet coherence can be observed at the high time scale of 4–8 and 8–16 days in these pairs, showing that the COVID-19 outbreak spread among global stock, bond, and commodity markets.

### 3.1 Additional analysis

Following the studies by Hung (2020), Al-Rdaydeh et al. (2021), and Chien et al. (2021), we employ wavelet-based Granger causality analysis by utilizing the time–frequency band of the wavelet transform to capture the causal association between the examined indexes. In this regard, we decompose the data into eight levels spanning different holding periods using the continuous wavelet method. These levels are classified as short term (D1-D2), medium-term (D3-D4), long term (D5-D6), and very long run (S6) (Hung 2020). Tables 2 and 3 demonstrate the findings of the Granger causality test in the pre and during the COVID-19 crisis. First, we focus on the causal relationship between the selected variables in the pre-COVID-19 as shown in Table 2. In the cases of CRY-SP500 and GSCI-BOND, there is a causality running from SP500 to CRY and from GSCI to BOND in the medium and long...
run. In particular, there is bidirectional causality in the short and long term between the SP500 and the GSCI, as well as between the CRY and the BOND markets. However, our findings support the absence of a causal relationship between CRY and GSCI in both the short and long term.

Further analysis in the pairwise multi-scale bivariate causality test during the COVID-19 in Table 3 suggests that the pairs with the most statistically significant causalities are SP500-GSCI, GSCI-BOND, CRY-BOND (with 6 out of 7), CRY-SP500, CRY-GSCI, and SP500-BOND (with 5 out of 7). Before the COVID-19 crisis, there did not seem to be a preferential causality direction, whereas during the

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Fig. 3 Cross wavelet transform (XWT) and wavelet coherence (WTC) for the selected markets over the whole sample period
Fig. 3 (continued)

crisis there was bidirectional causality on the four scales. As we discussed earlier, the COVID-19 crisis might be transmitted into financial markets (BBB), which supports the hypothesis of an indirect influence. Overall, the findings uncover a bidirectional relationship between the concerned variables at short, medium, and long-term scales. This indicates that any shocks to either the S&P500 Index, S&P Cryptocurrency MegaCAP Index, the S&P Goldman Sachs Commodity Index, or the S&P Global Developed Sovereign Bond Index have short, medium, and long-term influences on each other. This is in line with the idea that there is a close linkage between financial markets and those that provide useful information for investors,
Table 2  Results of wavelet-based Granger causality test at different time scales for the selected market pairs in the pre-COVID-19

Null Hypothesis

| Time domain | Result       | Market i does not cause market j | Market j does not cause market j |
|-------------|--------------|----------------------------------|----------------------------------|
|             |              | F-test   | P-value | F-test   | P-value |
| CRY-SP500   |              |          |         |          |         |
| D1          | No Causality | 0.29668 | 0.9819  | 0.80650 | 0.6225  |
| D2          | No Causality | 0.09585 | 0.9086  | 1.06638 | 0.3448  |
| D3          | $SP500 \rightarrow CRY$ | 1.36042 | 0.2572  | 5.36209 | 0.0049  |
| D4          | No Causality | 1.93580 | 0.1451  | 0.80650 | 0.6225  |
| D5          | $SP500 \rightarrow CRY$ | 0.78251 | 0.4577  | 2.93490 | 0.0538  |
| D6          | $SP500 \rightarrow CRY$ | 2.14985 | 0.1173  | 2.55979 | 0.0780  |
| S6          | $SP500 \rightarrow CRY$ | 1.55087 | 0.2128  | 5.36917 | 0.0048  |
| CRY-GSCI    |              |          |         |          |         |
| D1          | No Causality | 0.75713 | 0.4694  | 0.35967 | 0.6980  |
| D2          | $CRY \rightarrow GSCI$ | 2.98486 | 0.0512  | 1.04484 | 0.3523  |
| D3          | $CRY \rightarrow GSCI$ | 2.77351 | 0.0631  | 1.52671 | 0.2180  |
| D4          | $GSCI \rightarrow CRY$ | 0.79551 | 0.4518  | 2.50500 | 0.0824  |
| D5          | No Causality | 0.19546 | 0.4518  | 2.50500 | 0.0824  |
| D6          | No Causality | 1.09638 | 0.3346  | 1.47966 | 0.2284  |
| S6          | No Causality | 0.30253 | 0.7390  | 1.14886 | 0.3176  |
| CRY-BOND    | $BOND \rightarrow CRY$ | 1.74562 | 0.1753  | 3.63268 | 0.0269  |
| D2          | $CRY \leftrightarrow BOND$ | 3.09740 | 0.0458  | 4.68667 | 0.0095  |
| D3          | No Causality | 0.04027 | 0.9605  | 1.94377 | 0.1439  |
| D4          | $CRY \leftrightarrow BOND$ | 3.05605 | 0.0477  | 3.09692 | 0.0458  |
| D5          | $CRY \rightarrow BOND$ | 4.82552 | 0.0083  | 1.49015 | 0.2283  |
| D6          | $CRY \leftrightarrow BOND$ | 11.0327 | 0.000   | 2.37374 | 0.0939  |
| S6          | $CRY \leftrightarrow BOND$ | 22.3935 | 0.000   | 17.3224 | 0.000   |
| SP500-GSCI  | $SP500 \rightarrow GSCI$ | 2.85214 | 0.0584  | 10.6269 | 0.000   |
| D2          | $GSCI \rightarrow SP500$ | 0.52469 | 0.5920  | 3.88161 | 0.0211  |
| D3          | $GSCI \rightarrow SP500$ | 2.00183 | 0.1359  | 3.23998 | 0.0397  |
| D4          | $GSCI \rightarrow SP500$ | 0.84036 | 0.4320  | 5.37055 | 0.0048  |
| D5          | $GSCI \rightarrow SP500$ | 2.09486 | 0.1238  | 7.19991 | 0.0008  |
| D6          | $GSCI \rightarrow SP500$ | 0.93394 | 0.3935  | 4.66447 | 0.0097  |
| S6          | $SP500 \rightarrow GSCI$ | 6.20187 | 0.0021  | 20.8167 | 0.0000  |
| SP500-BOND  | $SP500 \rightarrow BOND$ | 3.95462 | 0.0196  | 2.25828 | 0.1053  |
| D2          | No Causality | 0.96778 | 0.3804  | 1.46085 | 0.2327  |
| D3          | $SP500 \leftrightarrow BOND$ | 4.16345 | 0.0159  | 3.47621 | 0.0315  |
| D4          | $BOND \rightarrow SP500$ | 0.39638 | 0.39638 | 2.90688 | 0.0553  |
| D5          | No Causality | 1.16338 | 0.3130  | 0.57884 | 0.5608  |
policymakers, and market participants. These results are consistent with the evidence provided by Bouri et al. (2020a, b) and Siddique et al. (2021).

### 3.2 Robustness check

The robustness test is carried out using the wavelet cohesion method, which is a time–frequency methodology established by Rua (2013) that measures the cross wavelet transform correlation to provide more information about the co-movement of two variables. He developed the correlation intensity measure $\rho_{x,y}$ as the real number on [-1,1] by considering the wavelet cross spectrum as follows:

$$\rho_{x,y} = \frac{\Re \left( W_n^x W_n^y \right)}{\sqrt{|W_n^x|^2 |W_n^y|}}$$

Figure 4 shows interdependence linkage by using the wavelet correlation test. A color code indicates the intensity of causal links between the series, ranging from dark blue (no causal effects) to dark red (high causal effects). As indicated in Fig. 4, the red region at the bottom of the wavelet in Rua’s plot shows the strongly positive co-movement at medium and low frequencies. Nevertheless, at high frequencies (less than 16 days), the blue regions reveal a weak dependence or negative correlation between the markets under consideration. These findings indicate a variable frequency pattern of co-movement between the CRY, SP500, GSCI, and BOND markets. More importantly, short-run horizon investment provides the benefit of diversification to mitigate risk, whereas there is no diversification benefit among

| Time domain | Result | Market i does not cause market j | Market j does not cause market j |
|-------------|--------|----------------------------------|----------------------------------|
|             |        | F-test  | P-value   | F-test  | P-value   |
| D6          | $BOND \rightarrow SP500$ | 1.58853 | 0.2050    | 2.53006 | 0.0804    |
| S6          | $SP500 \leftrightarrow BOND$ | 3.85134 | 0.0217    | 11.9546 | 0.0000    |
| GSCI-BOND   | D1     | No Causality | 2.16525 | 0.1155    | 1.55896 | 0.2111    |
|             | D2     | No Causality | 1.84044 | 0.1595    | 0.36907 | 0.6915    |
|             | D3     | $BOND \rightarrow GSCI$ | 1.26761 | 0.2821    | 6.27074 | 0.0020    |
|             | D4     | No Causality | 1.49728 | 0.2244    | 0.15396 | 0.8573    |
|             | D5     | $BOND \rightarrow GSCI$ | 1.92044 | 0.1473    | 2.90643 | 0.0553    |
|             | D6     | $BOND \rightarrow GSCI$ | 0.56342 | 0.5695    | 4.67407 | 0.0096    |
|             | S6     | No Causality | 3.01990 | 0.0494    | 1.71015 | 0.1816    |
Table 3  Results of wavelet-based Granger causality test at different time scales for the selected market pairs during COVID-19

Null Hypothesis

| Time domain | Result          | Market i does not cause market j | Market j does not cause market j |
|-------------|----------------|----------------------------------|----------------------------------|
|             | F-test | P-value | F-test | P-value |
| CRY-SP500   |         |         |        |         |
| D1          | No Causality | 0.27234 | 0.7617 | 0.25563 | 0.7745 |
| D2          | No Causality | 0.23359 | 0.7917 | 1.84545 | 0.1587 |
| D3          | SP500 → CRY | 0.99916 | 0.3687 | 2.38968 | 0.0924 |
| D4          | SP500 ↔ CRY | 5.27555 | 0.0053 | 6.71758 | 0.0013 |
| D5          | SP500 → CRY | 1.27701 | 0.2795 | 6.56152 | 0.0015 |
| D6          | SP500 → CRY | 1.27903 | 0.2789 | 17.0900 | 0.0000 |
| S6          | SP500 ↔ CRY | 1.63098 | 0.1965 | 3.18237 | 0.0421 |
| CRY-GSCI    |         |         |        |         |
| D1          | No Causality | 0.20801 | 0.8122 | 0.31037 | 0.7333 |
| D2          | No Causality | 2.04888 | 0.1110 | 0.85635 | 0.4251 |
| D3          | GSCI → CRY | 0.47240 | 0.6237 | 5.02534 | 0.0068 |
| D4          | CRY ↔ GSCI | 3.98923 | 0.0189 | 4.92180 | 0.0075 |
| D5          | GSCI → CRY | 0.00335 | 0.9967 | 4.18722 | 0.0156 |
| D6          | GSCI → CRY | 0.33791 | 0.7134 | 9.24800 | 0.0001 |
| S6          | GSCI → CRY | 0.77727 | 0.4600 | 4.28301 | 0.0142 |
| CRY-BOND    |         |         |        |         |
| D1          | CRY → BOND | 2.79589 | 0.0617 | 0.00811 | 0.9919 |
| D2          | CRY → BOND | 4.13470 | 0.0164 | 1.15542 | 0.3155 |
| D3          | CRY ↔ BOND | 3.42332 | 0.0331 | 2.77324 | 0.0631 |
| D4          | BOND → CRY | 0.89374 | 0.4096 | 7.37151 | 0.0007 |
| D5          | No Causality | 0.53018 | 0.5887 | 0.96433 | 0.3817 |
| D6          | CRY → BOND | 6.36543 | 0.0018 | 21.2713 | 0.0000 |
| S6          | BOND → CRY | 1.73534 | 0.1771 | 3.72262 | 0.0246 |
| SP500-GSCI  |         |         |        |         |
| D1          | SP500 ↔ GSCI | 19.3287 | 0.0000 | 28.5048 | 0.0000 |
| D2          | SP500 ↔ GSCI | 17.1327 | 0.0000 | 65.6497 | 0.0000 |
| D3          | SP500 ↔ GSCI | 22.8176 | 0.0000 | 10.9594 | 0.0000 |
| D4          | SP500 ↔ GSCI | 6.91717 | 0.0011 | 24.6670 | 0.000 |
| D5          | SP500 ↔ GSCI | 15.2759 | 0.0000 | 2.33928 | 0.0971 |
| D6          | SP500 ↔ GSCI | 8.39847 | 0.0000 | 41.8959 | 0.0000 |
| S6          | SP500 ↔ GSCI | 5.55127 | 0.0041 | 27.7414 | 0.0000 |
| SP500-BOND  |         |         |        |         |
| D1          | No Causality | 1.01394 | 0.3633 | 2.14365 | 0.1180 |
| D2          | No Causality | 0.98675 | 0.3733 | 0.81580 | 0.4427 |
| D3          | SP500 → BOND | 3.99427 | 0.0188 | 0.33567 | 0.7150 |
| D4          | SP500 ↔ BOND | 3.98827 | 0.0189 | 3.17818 | 0.0423 |
| D5          | SP500 ↔ BOND | 8.94266 | 0.0001 | 35.9453 | 0.0000 |
markets in the long run. These outcomes are consistent with the findings of the cross wavelet transform, wavelet coherence, and wavelet-based Granger causality.

Figure 5 plots the squared VaR ratio considering an equally weighted portfolio of the four markets with and without co-movements. The ratio of the portfolio’s deviations with and without assuming co-movement provides insight into the degree of connectivity between the assets in the portfolio. The wavelet coherence and wavelet-based Granger causality graphs are analogous to this representation. Cooler colors have lower wavelet VaR, while hotter colors have higher wavelet VaR. The VaR ratio is clearly above 0.5, indicating that these assets are a good diversification in the short

Table 3 (continued)

| Null Hypothesis | Market i does not cause market j | Market j does not cause market j |
|-----------------|---------------------------------|---------------------------------|
| Time domain     | Result                          | F-test | P-value | F-test | P-value |
| D6              | $BOND \rightarrow SP500$       | 0.98515 | 0.3739 | 15.5340 | 0.0000 |
| S6              | $BOND \rightarrow SP500$       | 2.18214 | 0.1136 | 16.7566 | 0.0000 |
| GSCI-BOND       | D1 $GSCI \leftrightarrow BOND$ | 26.4959 | 0.0000 | 3.20968 | 0.0410 |
|                 | D2 $GSCI \leftrightarrow BOND$ | 26.3320 | 0.0000 | 3.26759 | 0.0387 |
|                 | D3 $GSCI \leftrightarrow BOND$ | 11.9477 | 0.0000 | 5.09670 | 0.0063 |
|                 | D4 $GSCI \leftrightarrow BOND$ | 33.4724 | 0.0000 | 7.86651 | 0.0004 |
|                 | D5 $GSCI \leftrightarrow BOND$ | 10.9560 | 0.0000 | 26.4684 | 0.0000 |
|                 | D6 $GSCI \leftrightarrow BOND$ | 31.2275 | 0.0000 | 56.7349 | 0.0000 |
|                 | S6 $GSCI \leftrightarrow BOND$ | 14.8123 | 0.0000 | 11.0244 | 0.0000 |

Fig. 4 Wavelet correlation
run. On the other hand, the findings co-vary on relatively higher scales. Overall, the plot is dominated by cool colors, suggesting that the selected series are weakly connected during the period shown, which reveals that the assets under study have a safe-haven property. Put differently, the ratio is higher than one demonstrating that the co-movements between CRY, SP500, GSCI, and BOND markets imply a higher VaR whatever the frequency or the moment in time. It is important to note that co-movements have various effects on portfolio risk at various times and frequencies. As a result of this finding, investors should change their portfolio structure regularly. The ratio, on the other hand, is higher at low frequencies during the whole sample period, indicating that prospective portfolio losses are greater at low frequencies than at high frequencies. This result also demonstrates how the benefits of portfolio diversity fluctuate over time.

4 Conclusion

This study investigates the time–frequency return connectedness of the four most relevant asset classes namely, equity, digital assets, commodity, and fixed income. To do so, we use the novel proxies of the S&P500 Index for equity, the S&P Cryptocurrency MegaCAP Index for digital assets, the S&P Goldman Sachs Commodity Index for commodity, and the S&P Global Developed Sovereign Bond Index for fixed income, and also employ the wavelet analysis for daily data over the period 2017: M02 to 2021: M09. Overall, the results underscore that the interdependence between the selected asset classes has been intensified across all time scales and frequency bands during the COVID-19 compared to pre-COVID-19, indicating the lack of hedging opportunities. Besides, the findings reveal that there is a significant lead-lag relationship between time series at medium and low frequencies during the research period, and the directional connectedness among asset classes is sensitive to frequencies. Especially, the co-movements among the pairs are pronounced during the COVID-19 outbreak. Moreover, portfolio diversity advantages vary among frequency and across time, according to a portfolio risk analysis employing the VaR measure and portfolio losses are larger in low frequencies than in high frequencies.
For investors, the findings of this study have a variety of new and noticeable policy consequences because they should consider the changing dynamic risk spillover among major asset classes. Statistics clearly show that during the COVID-19 crisis, the CRY, SP500, GSCI, and BOND markets had a strong causal relationship in the medium and long run. The dependence on risk spillovers over different times and frequencies suggests that the portfolio should be adjusted in the medium and long run. To achieve maximum risk reduction, investors should include BOND and CRY among all assets and build a low-risk portfolio. Furthermore, investors can use the CRY, SP500, GSCI, and BOND markets to hedge against extreme market volatility. For further study, it would be interesting to examine the volatility connectedness between the various asset classes, particularly during COVID-19.

Data availability The data that support the findings of this study are available from the corresponding author upon request.

Declarations

Competing interests The authors have no relevant financial or non-financial interests to disclose.

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