Interference between Coulomb and hadronic scattering in elastic high-energy nucleon collisions

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Abstract

The different models of elastic nucleon scattering amplitude will be discussed. Especially, the preference of the more general approach based on eikonal model will be summarized in comparison with the West and Yennie amplitude that played an important role in analyzing corresponding experimental data in the past.

1 Introduction

Elastic nucleon scattering at high energies represents probably the most extensive and the most precise ensemble of available experimental data enabling to perform very accurate analysis in a broad region of the four momentum transfer squared \( t \). The elastic scattering of nucleons is realized mainly due to the strong hadronic interactions. However, in the case of charged hadrons the elastic scattering is also realized due to the Coulomb interactions, which play an important role mainly at small \(|t|\).

The influence of both the strong and electromagnetic interactions in the elastic scattering of protons by a nucleus was first investigated by Bethe [1]. Using the semi classical WKB approximation he derived that the total elastic amplitude \( F^{C+N}(s,t) \) can be written in principle as the sum of hadronic amplitude \( F^N(s,t) \) and of Coulomb amplitude \( F^C(s,t) \) known from QED which were mutually bound by a relative phase; here \( s \) is the square of the CMS energy. Higher corrections to the relative phase were then obtained by Islam [2] by improving the approximations.

The first generalization of the relative phase within the relativistic theory was given by Soloviev [3] using the methods of QED and also by Rix and Thaler [4].

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Applying the methods of Feynman diagram technique (one-photon exchange) Locher [5] and mainly West and Yennie [6] derived a more general expression for the relative phase in the case of charged point-like particles as

$$
\Phi(s, t) = \pm \left[ \ln \left( - \frac{t}{s} \right) - \int_{-4p^2}^{0} \frac{dt'}{|t - t'|} \left( 1 - \frac{F^N(s, t')}{F^N(s, t)} \right) \right], \quad (1)
$$

where $p$ is the value of the momentum in the CMS.

Similar expression as (1) was derived by Franco [7] within the framework of eikonal model and by Lapidus and co-workers [8]. Franco and Varma [9] generalized this method also for the hadron-nucleus and nucleus-nucleus scattering. The eikonal model approach was used later by Cahn [10] who took into account also the form factors in order to describe the electromagnetic nucleon collisions as the extended objects with the accuracy up to the terms linear in $\alpha$. Higher order corrections to the relative phase were then calculated by Selyugin [11] and by Kopeliovich and Tarasov [12]. The influence of spins was studied within the impact parameter formalism applying the helicity amplitudes by Buttimore, Gotsman and Leader [13]. All these authors obtained the explicit analytical formulas for the relative phase describing the elastic scattering at small values of $|t|$ by introducing some further assumptions specifying the $t$ dependence of the hadronic amplitude $F^N(s, t)$. The impact parameter representation of the scattering amplitude used by the mentioned authors has been valid at very high energies and small $|t|$ only.

In all these approaches the total elastic scattering amplitude could be at the end written (after introducing some other approximations) in a very simplified form proposed already in Ref. [6]. After adding the two form factors $f_1(t)$ and $f_2(t)$ corresponding to individual colliding nucleons to the Coulomb scattering amplitude the total elastic scattering amplitude has been written as

$$
F^{C+N}(s, t) = F^C(s, t)e^{i\alpha \Phi(s, t)} + F^N(s, t) =
= \pm \frac{\alpha \sigma}{t} f_1(t)f_2(t)e^{i\alpha \Psi} + \frac{\sigma_{tot}}{4\pi} p\sqrt{s}(\rho + i)e^{Bt/2},
$$

where $\alpha = 1/137.036$ is the fine structure constant. The upper (lower) sign corresponds to the scattering of particles with the same (opposite) charges. The quantity $\rho$ and the diffractive slope $B$ in the second term are independent of $t$; together with the total cross section $\sigma_{tot}$ they can be energy dependent only and characterize the elastic hadron scattering at given energy.

It has been shown then by Locher [5] and West and Yennie [6] that the relative phase $\alpha \Phi(s, t)$ is to depend on the hadronic amplitude in a rather simple way;
they have obtained

\[ \Phi(s, t) = \mp \left[ \ln \left( \frac{-Bt}{2} \right) + \gamma \right], \]  

(3)

where \( \gamma = 0.577215 \) is Euler’s constant.

As already mentioned the general formula has been derived on the basis of impact parameter representation of the scattering amplitude for the first time by Franco [7], which has been further developed in Refs. [8,10,14]. The given approach has opened some new important questions concerning the interpretation of hadronic elastic scattering; we will explain it to a greater detail in Sect. 2. Sect. 3 will be then devoted to comparing both the approaches; especially the limitations contained in the former one and not yet sufficiently analyzed will be discussed. The importance of the latter approach for analyzing experimental data will be discussed in Sect. 4.

2 Approaches based on the impact parameter representation

The mentioned approaches using the impact parameter representation of the scattering amplitudes are based on the eikonal models. The form suggested by Glauber [15] has been used

\[ F(s, q^2 = -t) = \frac{s}{4\pi i} \int_{\Omega_b} d^2b e^{iqb} e^{2i\delta(s, b)} - 1 \]  

(4)

where \( \Omega_b \) represents the two-dimensional Euclidean space of the impact parameter \( \vec{b} \). A mathematically correct formulation of the impact parameter theory (respecting fully the existence of a finite interval of admissible \( t \) values at finite energies) was given by Adachi et al. [16] and generalized by Islam [17,18] who showed that it is valid at any \( s \) and \( t \). In the elastic scattering of two charged nucleons the corresponding eikonal can be written (due to additivity of the potentials [7,18]) as

\[ \delta^{C+N}(s, b) = \delta^C(s, b) + \delta^N(s, b), \]  

(5)

and Eq. (4) for the total elastic scattering amplitude may be rewritten as

\[ F^{C+N}(s, t = -q^2) = \frac{s}{4\pi i} \int_{\Omega_b} d^2b e^{iqb} e^{2i(\delta^C(s, b) + \delta^N(s, b))} - 1. \]  

(6)
Eq. (6) can be then transformed according to Refs. [7] and [10] (see also [14]) into the form

$$F^{C+N}(s, t) = F^C(s, t) + F^N(s, t) + \frac{s}{4\pi i} \int_{\Omega_b} d^2 b e^{i\vec{q}\cdot\vec{b}} [e^{2i\delta^C(s, b)} - 1] [e^{2i\delta^N(s, b)} - 1]$$

$$= F^C(s, t) + F^N(s, t) + \frac{i}{\pi s} \int_{\Omega_q} d^2 q' F^C(s, q'^2) F^N(s, [\vec{q} - \vec{q}']^2), \quad (7)$$

where $F^C(s, t)$ and $F^N(s, t)$ are Coulomb and elastic hadronic amplitudes defined by expression (4) with the eikonals $\delta^C(s, b)$ and $\delta^N(s, b)$. Equation (7) describes simultaneous actions of both the hadron and Coulomb forces responsible for the total elastic scattering. It includes the convolution integral of the two amplitudes defined over kinematically allowed region of momentum transfers $\Omega_q$. It means that the total elastic amplitude (7) can be expressed as the sum of both the Coulomb and hadronic amplitudes to which a function depending on both the Coulomb and hadronic amplitudes is added.

Franco and Cahn have started from Eq. (7), but they have passed to some simplifications; the impact parameter formalism used in their approaches has been valid at asymptotic energies and small momentum transfers only. They have tended mainly to re-deriving the West and Yennie formula on the basis of eikonal formalism.

The general formula (7) holds at any $s$ and $t$ and may be further reformulated [14]. Using the generalized impact parameter representation formalism introduced by Adachi and Kotani [16] and Islam [17,18] it has been possible to derive the expression for the total elastic scattering amplitude valid generally up to the terms linear in $\alpha$. It has been possible to write [14]

$$F^{C+N}(s, t) = \pm \frac{\alpha s}{t} f_1(t)f_2(t) + F^N(s, t) \left[ 1 \mp i\alpha G(s, t) \right], \quad (8)$$

where

$$G(s, t) = \int_{t_{\text{min}}}^0 dt' \left\{ \ln \frac{t'}{t} \left[ f_1(t')f_2(t') \right]' + \frac{1}{2\pi} \left[ \frac{F^N(s, t')}{F^N(s, t)} - 1 \right] I(t, t') \right\}, \quad (9)$$

and

$$I(t, t') = \int_{\Phi''}^{2\pi} d\Phi'' \frac{f_1(t'')f_2(t'')}{t''}; \quad (10)$$
here \( t'' = t + t' + 2\sqrt{tt'} \cos \Phi' \). For the case of nucleon-nucleon scattering \( t_{\text{min}} = -s + 4m^2 \). The upper (lower) sign again corresponds to the \( pp \) (\( \bar{p}p \)) scattering.

Differential cross section can be then defined as

\[
\frac{d\sigma(s,t)}{dt} = \frac{\pi}{sp^2} |F^{C+N}(s,t)|^2. \tag{11}
\]

Making use of the optical theorem one can write for the total cross section

\[
\sigma_{\text{tot}}(s) = \frac{4\pi}{p\sqrt{s}} \Im F^N(s,t = 0). \tag{12}
\]

Instead of the \( t \) independent quantities \( B \) and \( \rho \), it is now necessary to define \( t \) dependent quantities

\[
B(s,t) = \frac{d}{dt} \left[ \ln \frac{d\sigma_N}{dt} \right] = \frac{2}{|F^N(s,t)|} \frac{d}{dt} |F^N(s,t)| \tag{13}
\]

and

\[
\rho(s,t) = \frac{\Re F^N(s,t)}{\Im F^N(s,t)}. \tag{14}
\]

Assuming for both the last quantities to be \( t \) independent corresponds to a fundamental limitation of formula (2) and disables practically its use for model interpretations. However, some important limitation has been contained already in Eq. (1). All problems relating to Eqs. (1) and (2) will be analyzed in the following section.

3 Limitations involved in West and Yennie approach

Leaving aside neglection of spins (assumed in both the approaches) it has been believed that the original West and Yennie integral formula for relative phase (1) does not contain practically any limitations concerning the \( t \) dependence of the hadronic amplitude \( F^N(s,t) \); and that only its simplified form expressed by Eq. (2) was based on limiting assumptions. However, it is not true as one important limitation follows already from requiring for the relative phase (1) to be real; the following relation

\[
\rho(s,t) = \frac{\Re F^N(s,t)}{\Im F^N(s,t)} = \frac{\Re F^N(s,t')}{\Im F^N(s,t')} = \rho(s) \tag{15}
\]
between the real and imaginary parts of \( F^N(s, t) \) should be fulfilled for any values of \( t \) and \( t' \). Thus the West and Yennie integral formula (1) admits only such \( t \) dependence of the hadronic amplitude \( F^N(s, t) \) which leads to constant value of the ratio \( \rho(s) = \frac{\Re F^N(s, t)}{\Im F^N(s, t)} \) in the whole region of kinematically allowed \( t \) values. As the quantity \( \rho(s) \) in the case of elastic nucleon collisions is small the West and Yennie integral formula (1) admits only the elastic hadronic amplitude with dominant imaginary part in the whole kinematically allowed region of \( t \).

The \( t \) dependence of differential cross section and diffractive slope are then given according to Eqs. (11) and (13) fully by the imaginary part of hadronic amplitude only, i.e.,

\[
\frac{d\sigma(s, t)}{dt} = \frac{\pi}{sp^2}(1 + \rho(s)^2)(\Im F^N(s, t))^2
\]  \hspace{1cm} (16)

and

\[
B(s, t) = \frac{2}{|\Im F^N(s, t)|} \frac{d}{dt}|\Im F^N(s, t)|.
\]  \hspace{1cm} (17)

Further additional assumption is then contained in the simplified West and Yennie relative phase (3). While Eq. (1) does not exclude for the diffractive slope \( B(s, t) \) to be \( t \) dependent, the Eq. (2) can hold only if \( B(s, t) = B(s) \), i.e., if the differential cross section or the modulus \(|F^N(s, t)|\) of the hadronic amplitude is purely exponential [20]. The elastic hadronic amplitude is then given by the second term of Eq. (2).

We will show now to a greater detail the approximations being included in derivation of all previous formulas and assertions. Let us denote the integral in Eq. (1) by \( A \). Then

\[
A = \int_{-4p^2}^0 \frac{dt'}{t' - t} \left[ 1 - \frac{F^N(s, t')}{F^N(s, t)} \right] = \int_{-4p^2}^0 \frac{dt'}{|t' - t|} \left[ 1 - e^{B(t' - t)/2} \right] =
\]

\[
= \int_0^{(4p^2 + t)B/2} dy \frac{1 - e^{-y}}{y} + \int_{-Bt/2}^0 dy \frac{1 - e^y}{y} =
\]

\[
= E_1(B/2(4p^2 + t)) + \ln(B/2(4p^2 + t)) + \gamma - Ei(-Bt/2) + \ln(-Bt/2) + \gamma,
\]  \hspace{1cm} (18)

where formulas (5.1.39) and (5.1.40) from Ref. [21] have been applied to. The exponential integrals \( E_1(z) \) and \( Ei(x) \) are defined, e.g., by Eqs. (5.1.1) and
Introducing further two other simplifications (asymptotic energies and small $|t|$) which may be applied to (as West and Yennie did; see Ref. [6]) the integral $A$ equals

$$A = \ln(Bs/2) + \gamma,$$

(19)

which gives finally the simplified West and Yennie formula (3) for the relative phase; the asymptotic expansions for the involved exponential integrals (Eqs. (5.1.50) and (5.1.51) from Ref. [21]) being applied to.

Thus the simplified formula (3) involves a series of limiting assumptions: (i) constant $\rho$; (ii) purely exponential $t$ dependence of the modulus; (iii) asymptotic energies; (iv) small $|t|$. It means that it is in principle inconsistent with actual experimental data.

Some important discrepancies concerning the experimental characteristics have followed already from $t$ independence of $\rho$. If $\rho$ is constant at the given energy its derivative in the $t$ variable is zero. Then it follows

$$\frac{d}{dt} \Re F^N(s, t) \quad \Im F^N(s, t) = \Re F^N(s, t) \quad \frac{d}{dt} \Im F^N(s, t)$$

(20)

for all admissible values of $t$. On the other hand the existence of diffractive minimum observed in all elastic nucleon collisions leads to a condition that the first derivative of differential cross section should be zero. It holds then

$$\Re F^N(s, t_D) \quad \frac{d}{dt} \Re F^N(s, t_D) = -\Im F^N(s, t_D) \quad \frac{d}{dt} \Im F^N(s, t_D);$$

(21)

the corresponding diffractive minimum being at $t_D$. It follows from Eqs. (20) and (21) that both the real and imaginary parts of $F^N(s, t_D)$ should equal zero, which contradicts the experimental data as the differential cross section does not vanish in the diffractive minimum. The existence of diffractive minimum observed in all diffractive hadron collisions is, therefore, in a clear contradiction to $\rho$ being constant.

As to the experimental data it is, of course, also the assumption concerning the purely exponential $t$ dependence of the modulus $|F^N(s, t)|$ at all kinematically allowed values of momentum transfers that is in contradiction to the present high energy elastic nucleon scattering data. It has been observed experimentally that for the $pp$ elastic scattering at the ISR energies the region of approximately exponential $t$ dependence of differential cross section is only for $t$ running from the forward direction to the diffractive minimum. The values of $\frac{d\sigma}{dt}$ change here within 8 or 9 orders of magnitude. And this
region becomes narrower and the range of the corresponding magnitudes becomes smaller when the collision energy increases, e.g., for $\bar{p}p$ scattering at the Collider energy $541$ GeV the magnitude change is only $5$ orders. And the model predictions for $pp$ scattering at the LHC energy $14$ TeV tell us that this change will be approximately only $3$ orders of magnitude (see, e.g., Ref. [22]). At higher $|t|$ the secondary maximum appears that clearly indicates the modulus not to exhibit purely exponential behavior in $t$ in the interval of all admissible $t$ values.

However, in the past (before ISR experiments) nothing was known about the existence of diffractive minima in elastic hadron collisions. The differential cross section data exhibited only purely exponential $t$ dependence in the rather narrow regions of studied momentum transfers. Therefore, the use of simplified West and Yennie amplitudes for analysis of elastic scattering data at lower energies could be regarded as justified at that time.

Having been aware of great discrepancies of formula (2) in the region of higher $|t|$ people started to use two different formulas: Eq. (2) for the region of very small $|t|$ and some phenomenological formula for the whole other interval of $|t|$: for detail see, e.g., Refs. [24,25].

It has been assumed that the triple of quantities $\sigma_{tot}, \rho, B$ may characterize an actual hadronic amplitude in the region around $t \sim 0$. However, neither this assumption may be regarded as justified as Eq. (2) has been derived by integrating over the whole kinematically allowed interval of $t$ under the conditions differing drastically from reality. Consequently, only the general formula (without all mentioned simplifications and limitations) may be efficient in interpreting available experimental data.

We should like to repeat that the simplified West and Yennie amplitude (2) is used in an inconsistent way even if it is applied to the interference region only. This discrepancy cannot be removed even if a more general shape is applied to other regions of the measured differential cross section. It is also this application of two different formulas (one for interference region and a different one for hadronic region - based on some phenomenological approach; for detail see, e.g., Refs. [24,25]) that represents an important deficiency. All shortages and discrepancies may be removed, however, by using one common eikonal formula, only.

Several authors [10,11,12] tried to improve formula (2) by calculating the next-to-leading or next-to-next-leading order terms to it which might be important at higher $|t|$. The mentioned authors wanted to derive the analytical expressions for the relative phase between the Coulomb and hadronic elastic amplitudes. In tending to it they have had to assume the exponential $t$ dependence of hadronic amplitude and of the form factors at all kinematically allowed $t$
values (in order to perform the analytical calculation of the corresponding integrals). However, such assumptions do not correspond to the actual behavior of the experimental data and can depress the importance of higher corrections and their influence on the relative phase. This has been confirmed by recent analysis of experimental data (see Ref. [23]) when the used different formulas for the relative phase gave approximately the same results.

4 General formula and different interpretations of experimental data

There is not any actual theory of elastic hadronic nucleon scattering and the shape of elastic hadronic amplitude $F^N(s, t)$ must be derived from experimental data concerning the $t$ dependence of differential cross section. However, the complex elastic hadronic amplitude

$$F^N(s, t) = i |F^N(s, t)| e^{-i \zeta^N(s, t)}, \quad (22)$$

is characterized by a pair of the real functions, i.e., by the modulus $|F^N(s, t)|$ and the phase $\zeta^N(s, t)$; it holds

$$\rho(s, t) = \frac{\Re F^N(s, t)}{\Im F^N(s, t)} = \tan \zeta^N(s, t). \quad (23)$$

However, for this pair of functions only one experimentally determined function $\frac{d\sigma}{dt}$ is available. Thus, the complete form of scattering amplitude cannot be derived from experimental data and some other arguments for its definite form must be looked for.

It is the distribution of processes in the impact parameter space that depends on the function $\zeta^N(s, t)$ in a decisive way. This distribution $D(s, b), b \geq 0$ is given by the Fourier-Bessel transformation

$$h_{el}(s, b) = \frac{1}{4p\sqrt{s}} \int_{t_{\text{min}}}^{0} dt \ F^N(s, t) J_0(b\sqrt{-t}); \quad (24)$$

it holds

$$D(s, b) = |h_{el}(s, b)|^2. \quad (25)$$

Assuming for $\rho(s, t)$ to hold $\rho(s, 0) \ll 1$ in Coulomb and interference regions and to increase monotony for higher values of $|t|$ in such a way that
\( \Im F^N(s,t) \) vanishes in the diffractive minimum, e.g., \( \zeta^N(s,t_D) = \frac{\pi}{2} \), the amplitude \( F^N(s,t) \) may be fully derived from experimental data. In such a case the function \( D(s,b) \) has a Gaussian shape with the maximum at \( b = 0 \). And we can speak about the central behavior of elastic hadronic collisions, which attributes, e.g., to proton the structure differing fundamentally from that required normally for diffractive production collisions.

In such a diffractive case the function \( D(s,b) \) should have the maximum at \( b > 0 \) and we can speak about peripheral behavior. The corresponding function \( D(s,b) \) may be easily obtained if the function \( \zeta^N(s,t) \) having a small non-zero value at \( t = 0 \) increases quickly with rising \( |t| \). The more detailed description of such a case can be found in Refs. [26,27,28,29]. Similar behavior of the phase \( \zeta^N(s,t) \) was considered by Franco and Yin [30] for nuclear collisions.

The analyses of experimental data for \( pp \) scattering at the energy of 53 GeV and for \( \bar{p}p \) scattering at energy of 541 GeV based on different additional assumptions may be found in Ref. [14]. The following conclusion should be done from the given analysis:

- the diffractive slope \( B(s,t) \) and the \( \rho(s,t) \) quantity are \( t \) dependent in the whole measured \( t \) region in the peripheral as well as in the central behavior of elastic hadron scattering
- the influence of Coulomb scattering can be hardly neglected at higher \( |t| \) values
- the peripheral feature of elastic nucleon scattering at high energies seems to be slightly statistically preferred
- the use of the total elastic amplitude (8) should be strongly supported

In the end of this section we should like to mention yet one recent attempt of solving the interference problem on the basis of simplified formula (2); see Ref. [31]. It is possible to write in such a case

\[
\frac{\pi}{sp^2} \frac{d\sigma}{dt} = \left[ \left( F^C + \Re F^N \right)^2 + \left( \alpha \Phi F^C + \Im F^N \right)^2 \right].
\]  

In the case of \( pp \) elastic scattering at high energies the \( \Re F^N \) is small and the first term on the right hand side should tend to zero already in interference region; this zero value being reached at \( t_{\text{min}} \). And the differential cross section at this point should be given by the other term. Combining experimental data with remaining theoretical values the value of \( t_{\text{min}} \) may be derived. As the remaining part of the equation has not been fulfilled authors of Ref. [31] have introduced an approach of bringing it to zero and claimed that they established final values of \( \sigma_{\text{tot}}, \rho \) and \( B \). However, it does not represent any actual solution of the collision pattern as the formula used is burdened by large approximations. In such a case one should actually write
The expression of the right hand side of Eq. (28) exhibits some deep minima (even if never equal zero). The behaviors corresponding to peripheral and central cases of the $pp$ elastic scattering at energy of 53 GeV are shown in Fig. 1; results being based on the analysis of data given in Ref. [14]. There are pronounced minima, their positions being different for central and peripheral behaviors. The full line corresponds to the case of peripheral behavior while the other curves correspond to the central behavior of elastic scattering. The dashed line corresponds to the central phase used in Ref. [14] while the dotted line corresponds to the central behavior given by a slightly modified phase of hadronic amplitude, i.e., \( \zeta_{N}(s, t) = \arctan(\zeta_{1} - \frac{\zeta_{0}}{1 - |t_{diff}|}) \). The two cases with central behavior exhibit different numbers of minima.

However, let us go back to Fig. 1. The first minimum for peripheral behavior and those for the central solution lies in the neighborhood of $|t| \sim 0.02$ GeV$^2$ as shown in Fig. 2. They correspond to the condition $\Re F^N = - F^C$, while all other minima in Fig. 1 relate to $\Re f^N = 0$ (changes of sign).
5 Conclusion

In the conclusion let us return once more to the general formula (8) where the expression in the last bracket may be regarded as the first term in the Taylor series expansion of the exponential $e^{-iaG}$; then one can write within the same precision

$$F^{N+C}(s, t) = F^C(s, t) + F^N(s, t)e^{-iaG(s,t)},$$

the form being practically identical with original formula of West and Yennie. However, the $G(s, t)$ (being complex) cannot be interpreted as a mere phase. The reality required for $G(s, t)$ would be equivalent to the condition that the quantity $\rho(s, t)$ is constant and vice versa.

Thus, the approach of West and Yennie has been burdened by a significant simplification from the beginning, tending immediately to some unjustified conclusions. A definite answer may be obtained only if the general formulas (8) or (29) are used for experimental data interpretation.

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Figure captions:

Fig. 1: Representation of the $t$ dependence of $\Delta^2_R(t)$ (i.e., of the right hand side of Eq. (28)) for $pp$ collisions at 53 GeV; (i) for peripheral behavior - full line; (ii) for central behavior: dashed line for the phase used in Ref. [14], dotted line for the phase mentioned in the text.

Fig. 2: Representation of the $t$ dependence of Eq. (28) $\Delta^2_R(t)$ for $pp$ collisions at 53 GeV. Zoomed Fig. 1 for small values of $|t|$.
pp 53 GeV

Figure 1
Fig. 2

\( \Delta^2(t) \)

pp 53 GeV