A New Methodology for the Structural Analysis of 3D Digitized Cultural Heritage through FEA

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Abstract. Finite Elements Analysis (FEA) is widely used for modelling stress behaviour in any mechanical system. The processing workflow starts from CAD 3D models representing the ideal shape of the object to be simulated. Such models are typically made of mathematical elements defining its geometrical components. Those are pre-processed before the simulation for creating a volumetric mesh out of the CAD model. Recently the use of FEA has also been extended to the simulation of ancient structures and artefacts, revealing significant potentialities for the conservation of Cultural Heritage. Unlike modern mechanical systems, heritage objects are usually altered by the time passed since their original creation, and the representation with a schematic CAD model may introduce an excessive level of approximation leading to wrong simulation results. In the last two decades, 3D documentation of CH has been developed through reality-based approaches. However, the related mesh models of the exterior surfaces are not proper for direct use in FEA. Such high-resolution surface meshes has to be converted to volumetric meshes made of tetrahedral or hexahedral elementary volumes and a limited number of external and internal nodes. The focus of this paper is on a new method aiming at generating the best possible 3D solid representation of a real artefact from its accurate reality-based surface model by reducing its number of nodes of several orders of magnitude while maintaining a geometrical coherence in the order of the measurement uncertainty of the 3D capturing technique used. The approach proposed is based on wise use of retopology procedures and a transformation of this retopologized model to a mathematical one made by NURBS surfaces, suitable for being processed by a volumetric mesh generator typically embedded in any standard FEM package. The resulting volumetric mesh allows obtaining FEA of ancient structures, providing a far better accurate simulation than those attainable by a rough CAD redrawing of the heritage asset of interest.

1. Introduction
For research, documentation, preservation and conservation issues in archaeology and Cultural Heritage, it is very important to appropriately record, document and survey artefacts and sites as an accurate and complete 3D digital documentation is a prerequisite for further analysis and interpretations.

In modern times the atmospheric agents, the urban growing, the density of constructions, the bombing during the two world wars, the negligence during the centuries and the present situation of political instability and danger in certain areas, have all contributed to damage and strongly influence stability of our Heritage. In this framework, it is fundamental to define a proper methodology to preserve our Heritage. In order to achieve diagnostic studies aiming at understanding the level of decay of Cultural Heritage for selecting the appropriate preservation methods and materials, a scientific base for allowing correct interventions would be fundamental. However, it’s always difficult to predict how a historical object or structure, built up to a few thousand years ago, will suffer for environmental agents such as earthquakes, pollution, wind and rain, or human factors like constructions in the surroundings, vehicular traffic, or heavy tourism. Conservation of Cultural Heritage is a key topic and structural changes and damages can affect the structural behaviour of Cultural Heritage artefacts and buildings. Considering that the application of technologies can help in preserving, conserving and restoring ancient structures, it is mandatory to find the best pipeline to produce the correct analysis.

The analysis of the structural behaviour of a Cultural object can be provided with the use of Finite Element Analysis, a well-known engineering technique used in modelling stress behaviour of objects and structures. The typical workflow of such analysis involves the use of CAD 3D models made by Non-Uniform Rational B-splines (NURBS) surfaces, representing the ideal shape of the object to be
simulated [1]. The major FEA packages have meshing modules capable to transform a NURBS model, made only of its exterior surfaces, to a volumetric mesh, which differently from a surface mesh typically generated with 3D capture methods, has nodes distributed both on the exterior surface and into the interior volume, connected each other by elementary volumes such as tetrahedron, pyramids, prisms or hexahedrons. This workflow is appropriate in the mechanical field, where a physical element to be simulated is very close to its ideal drawing within strict tolerances. Conversely, when applied to 3D models of Cultural Heritage (CH) objects or structures, often altered by the time passed since their original creation, the representation with a schematic CAD model may introduce an excessive level of approximation leading to wrong simulation results. Nowadays, 3D documentation of CH has been widely developed through active sensors or passive approaches like photogrammetry, but the models, formed by the exterior surfaces of the objects captured at high resolution and therefore represented by millions of polygons, are not suitable for a direct use in FEA. A few preliminary experiments have been done on real CH structures digitized with active or passive methods, whose models have been processed for simulating stress behaviour and predicting damages to artefacts considered critical within the field of conservation. The results are very promising but some issues have been made evident: a) the way for obtaining a volumetric mesh suitable for FEA from the raw 3D data is not yet clearly defined and may greatly influence the final result, b) the balance between geometric resolution and confidence level of the simulated results is often not compliant with the shape of a 3D model originated by a 3D acquisition process.

2. State of the Art

Preliminary experiments have been done on real CH structures digitized with active or passive methods, whose models have been processed for simulating stress behaviour and predicting possible damages to improve their conservation. By analysing the previous works, different approaches have been followed: a) redrawing with a CAD modeller a new surface model following the superficial mesh originated by the acquired 3D cloud; b) using directly the triangular mesh as starting point for volumetric models; c) generating a volumetric mesh from the point cloud with no preliminary surface meshing.

The first approach has been used for simulating the behaviour of the Trajan's Markets [2], and in many other applications [3;4]. In some cases, the reconstructed mathematical model was refined with the insertion of limited patches of reality-based superficial meshes [5]. An option of using a 3D CAD model extrapolated from a reality-based model is to extract or draw cross-sections or profiles from the 3D model to produce the CAD one as shown in [6], referred to the walls surrounding Montagnana, a small ancient city near Padua. An alternative option is passing through the procedures for producing BIM/HBIM models starting from the acquired 3D data, to be used for FEA. In [10] for example the use of BIM model as a support for sites monitoring and management is explored, extrapolating thematic information for structural analysis from the model, even if the authors didn’t provide a Finite Element Analysis on the structure. The Masegra Castle, located near the city of Sondrio in Italy, was also used as a test object for a novel procedure called Cloud-to-BIM-to-FEM [11]. The BIM was turned into a finite element model with a geometric rationalization taking into consideration the preservation of irregularities and anomalies, such as verticality deviation and variable thickness. If this procedure, with all its accuracy limitations due to the re-draw of the model in a CAD software, can be applied to CH buildings because the geometry of the structure can be replicated through a CAD drawing using profiles, cannot be instead used for statues, whose geometry is more complex and that cannot be simplified through elements as bean, truss or shell, used for the modelling in FEA.

The second approach has a range of slightly different methods, such as bare simplification of the triangular surface mesh before converting it in a volumetric one, that may give significant deviations between the actual shape and the simulated one [7]. The simplified description of the shape in this case are discretized profiles giving a low-resolution representation of the interior and the exterior of the structure, from which produce a volumetric model or the fitting of the acquired model with a parametric model suitable to be converted in volumetric mesh. A recent project used the triangular mesh to perform the overall stability analysis of a marble statue when placed on its new pedestal and the mechanical
stresses generated on the statue and the pedestal materials [8]. This paper presented a new way of creating the geometrical model of the statue, by subtracting the mesh from a prismatic block shape, according to a Boolean operation performed with an open source CAD software, to preserve the spatial positioning of the statue. Then, another open source software was used to create the tetrahedral mesh.

The third strategy does not even consider the mesh, generating a volumetric approximation of the shape from the raw 3D cloud of points [9]. A further evolution of this approach [12] is called FEA \textit{in situ}, meaning that the algorithm performs the analysis directly on the 3D data without passing neither through the surface nor the volumetric mesh model. Some test on mechanical objects has been performed to validate the method.

3. Test objects
The tower of the carceres of the roman circus in Milan (fig. 1a) and two statue from the Uffizi Gallery (Figure 1a-b) have been used as test objects. The two statues were surveyed through photogrammetry using a SONY ILCE 6000 camera coupled with a 16mm lens while the tower with both photogrammetry for the interior with a Panasonic DMC-GH4 coupled with a 14mm lens (Table 1) and with laser scanning for the exterior with a FARO Focus 3D scanner.

![Figure 1. The tower of the carceres in Milan (a) and the two statues held in the Uffizi Gallery: (b) the Gladiator, (c) the Buontalenti horse.](image)

|                  | ISO | Focal Length | GSD  |
|------------------|-----|--------------|------|
| Tower of the carceres | 800 | 2.8          | 1mm  |
| Horse            | 1000| 5.6          | 0.5mm|
| Gladiator        | 1000| 5.6          | 1mm  |

Table 1. The settings of the photogrammetric surveys of the three test objects.

4. Methodology
The starting point of the methodology proposed is the consideration of the complexity of doing structural analysis on Cultural Heritage. The difficulty lies on the geometry of the objects, compromised by the time passed, on the construction techniques and on the different materials used for buildings. Starting from the previous analysis and considering the lack of information that the methodology used since now carries on in performing structural analysis, the intrinsic characteristics of the different element involved in the pipeline and the non-linear and non-symmetric geometry of the structures, the methodology proposed took into account the use of retopology for the decimation of the reality-based 3D models. The pipeline starts from the post-processing of the acquired reality based models considering the correction of the topological errors and of the orientation of the model on a suitable reference system, usually with the XY plane corresponding to the base of the model, and the Z-axis passing from the centre of gravity of the artefact. This was considered a proper positioning for the following finite element analysis. In this
way, the high-resolution models used were closed meshes properly reoriented in order to post process them to suitable volumetric models for FEA. The second step regarded the use of retopology for generating a brand-new quad-mesh following the shape of the high-resolution models, but with an optimized arrangement of the polygons following the characteristic lines of the original model, and a strong reduction of the mesh density. The resulting quad mesh is a 3D objects lighter than the initial one and suitable for the transformation to NURBS. The conversion of a polygonal model in a NURBS model is available on some CAD packages as automatic process, that in general tends to produce a higher number of small patches when the original mesh is topologically unorganized. As a result, by rearranging the initial topology of the mesh, a preliminary condition for minimizing the number of NURBS patches of the converted model is set. In addition, the quadrangular structure of the polygons matches the intrinsic quadrangular structure of a NURBS patch, making easier the following mesh-to-NURBS transformation. Therefore, the resulting NURBS model represents a better starting point for the volumetric mesher embedded in any standard FEM packages.

After all these passages, the 3D models are finally ready for the finite element analysis.

4.1. Retopology

The retopology process is available both in open source packages such InstantMeshes [13] or Blender, or in commercial software packages such as ZBrush by Pixologic. Here, the software ZBrush was used. The main reason for the choice was the fact that this software is built specifically for rebuilding the topology of the models. Blender showed some problems in managing big files while InstantMeshes, even if powerful, gave as results sometimes inadequate meshes with holes and missing parts, especially when strongly decreasing the number of nodes. Moreover, ZBrush has the option of projecting the retopologized model on the high resolution one, increasing the adherence of the two models. The tool used for retopology was the ZRemesher that permits to strongly lower the number of polygons and reorder them on the surface, transforming also the typical triangular structure of an acquired mesh in a quad based one (Figure 2). The tool is optimized to work on all kind of structures and shapes but will by default produce better results with organic shapes. In the ZRemesher palette, there is the possibility to select the number of polygons desired for the retopology and reorder them on the surface, transforming also the typical triangular structure of an acquired mesh in a quad based one (Figure 2). The tool is optimized to work on all kind of structures and shapes but will by default produce better results with organic shapes. In the ZRemesher palette, there is the possibility to select the number of polygons desired for the retopology and reorder them on the surface, transforming also the typical triangular structure of an acquired mesh in a quad based one (Figure 2). The tool is optimized to work on all kind of structures and shapes but will by default produce better results with organic shapes. In the ZRemesher palette, there is the possibility to select the number of polygons desired for the retopology and reorder them on the surface, transforming also the typical triangular structure of an acquired mesh in a quad based one (Figure 2).

This function defines a vertex ratio based on the curvature of the mesh. A low setting will provide polygons that are as much square as possible and almost the same size, an amount of final polygons
Closely to the number set in the selection tool but can introduce topology irregularities where the geometry is much complex. A high adaptive size means obtaining polygons that are rectangular in shape to best fit the mesh’s curvature and which density can vary along the mesh surface even if the program creates smaller polygons where the geometry requires. So, increasing the value of the adapt size, the quality of retopology will increase but the program is more elastic about the target number of final polygons. This happens because when the desired number of polygons is set, the software distributes them equally on the surface and then analyses the curvature deforming the shape of the polygons or changing their density to be more adherent to the initial mesh.

4.2. NURBS conversion

A mesh represents 3D surfaces with a series of discreet faces, more likely as pixel form an image. NURBS, on the contrary, are mathematical surfaces, able to represent complex shapes with no granularity as in the mesh. The conversion from a mesh to a NURBS in implemented in CAD software or similar (e.g. 3DMax, Blender, Rhinoceros, Maya, Grasshopper, etc.) and it transforms a mesh composed by polygons or faces to a faceted NURBS surface. In details, it creates one NURBS surface for each face of the mesh and then merge everything into a single polysurface. A NURBS surface (patch) is obtained by a series of NURBS curves in two directions (called “U” and “V”) interpolated to create a surface as the tensor product of two NURBS curves, originating a quadrangular patch. Depending on the mesh, the conversion works in different ways. If the starting point is a triangular mesh, and while, by definition, triangles are plane, the conversion creates trimmed or untrimmed planar patches. The degree of the patches is 1x1 surface trimmed in the middle to form a triangle. If the starting point is a quadrangular mesh, the conversion creates a 4-sided untrimmed degree 1 NURBS patches, meaning that the edges of the mesh are the same as the outer boundaries of the patches. Considering the theory, a quadrangular mesh is more suitable to be converted in NURBS and by rearranging the initial topology of the mesh, a preliminary condition for minimizing the number of NURBS patches of the converted model is set, and this represents a better starting point for the volumetric mesher embedded in any standard FEM package.

The use of NURBS for creating volumetric models for FEA depends on the ability of NURBS to represent exactly a given geometry at low order with a high refinement of the mesh, that allow to have a higher accuracy in the Finite Element Analysis without changing the geometry [14].

4.3. FEA analysis

The two statues were retopologiezed, compared to the high resolution one, and then converted in NURBS and imported in the FEA software for two different analysis: a static one imposing the gravitational load and a modal one, fixing 10 frequencies to determine the natural mode shapes and frequencies of the object during free vibration. The comparison on the horse between the high resolution model of 2.2M vertices gave a 2mm mean and a standard deviation of 2mm (Figure 3) on the 26K retopologized model. The conditions set for the Finite element analysis were (Figure 4):

- Density: 2500 kg/m^3,
- Young’s Modulus for marble 78000 MPa,
- Poisson Ratio 0.3, Gravity on -Z axis,
- Fixed Support under the basement of the statue as boundary condition,
- Meshing element: 10-nodes Tetrahedrons
- Element size: 50 mm;
Figure 3. A comparison between the high resolution and the retopologized model of the horse.

Figure 4. The analysis on the horse statue: static structural imposing the gravity on –Z axis, maximum principal stress (a), minimum principal stress (b) and the modal analysis (c).

The high-resolution model of statue of the gladiator had 1.1M faces and was simplified to 30K nodes. The comparison gave a mean of 0.1mm and a standard deviation of 0.4mm (Figure 5). The same condition was imposed in the structural analysis with a dimension of the volumetric element of 16mm (Figure 6).

Figure 5. Comparison between the high resolution and the simplified retopologized model of the gladiator. Values expressed in meters.
Figure 6. The static structural analysis imposing the gravity on the –Z axis (a) showing the maximum principal stress, (b) the minimum principal stress and (c) the modal analysis for the statue of the gladiator.

The high-resolution model of the tower of the carceres counted more than 7M vertices, while the retopologized one after all the topological correction was composed by 56K vertices. The comparison between the high-resolution model and the retopologized one gave a mean of 0.03mm and a standard deviation of 0.2mm, both acceptable for a structure 30 m high and considering the fact that the final model is the ensemble of two techniques (Figure 7). A static structural and a modal analysis were done with the following parameters: Young modulus for masonry 8000 MPa, Density 2000 kg/m^3 and Poisson Ratio 0.15. The dimension of the 10-nodes tetrahedral element in the meshing part of the FEA was 10mm, coherent with the dimension of the superficial mesh element and the static structural analysis was performed imposing the gravity on – Z axis (Figure 8).

Figure 7. The comparison between the high-resolution model of the tower of the carceres and the retopologized one.

Figure 8. The FEA on the tower: (a) static structural imposing gravity showing the Maximum Principal Stress; (b) the same analysis showing the Minimum Principal Stress; (c) the modal analysis.
5. Conclusions
The use of retopology and the method proposed showed great performances and presented interesting properties in the process as a consistent pre-processing step to directly use a reality-based model for Finite Element Analysis. The use of the 3D measurement uncertainty as simplification criterion, allowed to dramatically reduce the mesh size maintaining a high accuracy of the simplified model compared to the high resolution one. This was the first important point to be achieved in the research. Starting from the idea to take advantage of the high resolution and of all the geometrical details of the reality-based models for the structural analysis of complex artefacts as Cultural Heritage objects and buildings, the possibility to transform all these information into a model that can be processed in FEA software is the most important result. When dealing with complex objects, a proper topological analysis is needed and this can be easily done with specific software as for example Meshlab. The important thing is to obtain a clean, closed mesh to be converted in a closed NURBS, in order to have a volumetric model for FEA.

References
[1] Höllig, K. (2003). Finite Element Methods with B-Splines. Society for Industrial and Applied Mathematics, Stuttgart, Germany.
[2] Brune, P., Perucchio, R. (2012). Roman Concrete Vaulting in the Great Hall of Trajan’s Markets: A Structural Evaluation. J. Archit. Eng. 18, 332–340.
[3] Erkal, A., Ozhan, H.O. (2014). Value and vulnerability assessment of a historic tomb for conservation. Sci. World J. 2014, Article ID 357679.
[4] Milani, G., S. Casolo, A. Naliato, and A. Tralli. 2012. Seismic assessment of a medieval masonry tower in northern Italy by limit, non-linear static and full dynamic analyses. International Journal of Architectural Heritage 6(5):489–524.
[5] Zvietcovitch, F., Castaneda, B., Perucchio, R. (2014). 3D solid model updating of complex ancient monumental structures based on local geometrical meshes. Digit. Appl. Archaeol. Cult. Herit. 2, 12–27.
[6] Guarnieri, A., Pirotti, F., Pontin, M. & Vettore, A. 2005. Combined 3D surveying techniques for structural analysis applications. International archives of photogrammetry, remote sensing and spatial information sciences, 36, 6.
[7] Riccardelli, C., Morris, M., Wheeler, G., Soutmanian, J., Becker, L., Street, R., 2014. The Treatment of Tullio Lombardo’s Adam: A New Approach to the Conservation of Monumental Marble
[8] Bagnérès, M., Cherblanc, F., Bromblet, P., Gattet, E., Gügi, L., Nony, N., Mercurio, V., Pamart, A., A complete methodology for the mechanical diagnosis of statue provided by innovative uses of 3D model. Application to the imperial marble statue of Alba-la-Romaine (France), In Journal of Cultural Heritage, 2017, ISSN 1296-2074,
[9] Shapiro, V., Tsukanov, I. (1999). Mesh free simulation of deforming domains. CAD Comput. Aided Des. 31, 459–471.
[10] Oreni, D., Brumana, R.,Cuca, B., "Towards a methodology for 3D content models: The reconstruction of ancient vaults for maintenance and structural behaviour in the logic of BIM management," 2012 18th International Conference on Virtual Systems and Multimedia, Milan, 2012, pp. 475-482.
[11] Barazzetti, L., Banfi, F., Brumana, R., Gusmeroli, G., Previtali, M., Schiantarelli, G., Cloud-to-BIM-to-FEM: Structural simulation with accurate historic BIM from laser scans, In Simulation Modelling Practice and Theory, Vol. 57, 2015, pp 71-87, ISSN 1569-190X.
[12] Freytag, M., Shapiro, V., Tsukanov, I., Finite element analysis in situ, In Finite Elements in Analysis and Design, Volume 47, Issue 9, 2011, Pages 957-972, ISSN 0168-874X.
[13] Jakob, W., Tarini, M., 2015. Instant Field-Aligned Meshes. Siggraph Asia 34, 15
[14] Hughes, T., Cottrell, J. A., Bazilevs, Y., Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement, Comput. Methods Appl. Mech. Engrg. 194 (2005) 4135–4195, 2005