Weak Production of Strange Particles and $\eta$ Mesons off the Nucleon

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Abstract. The strange particle production induced by (anti)neutrino off nucleon has been studied for $|\Delta S| = 0$ and $|\Delta S| = 1$ channels. The reactions those we have considered are for the production of single kaon/antikaon, eta and associated particle production processes. We have developed a microscopic model based on the SU(3) chiral Lagrangian. The basic parameters of the model are $f_\pi$, the pion decay constant, Cabibbo angle, the proton and neutron magnetic moments and the axial vector coupling constants for the baryons octet. For antikaon production we have also included $\Sigma^*(1385)$ resonance and for eta production $S_{11}(1535)$ and $S_{11}(1650)$ resonances are included.

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INTRODUCTION

Neutrino physics has become one of the important field of intense theoretical and experimental studies. In spite of huge efforts made in the last few decades to understand the nature of this elusive particle, there are still many unanswered questions like the absolute masses of neutrinos, CP violation in the lepton sector, etc. It has been established that neutrinos do oscillate and efforts are being made to determine the neutrino oscillation parameters better known as elements of PMNS matrix. Some of these oscillation parameters are sensitive to $\sim 1$ GeV of neutrino energies. For this a large number of experiments like T2K [1], MiniBooNE [2], MINERvA [3], MINOS [4], NovA [5], LBNE [6], INO [7] are going on or planned. Since the neutrino-nucleon cross sections are very small, therefore, all these experiments are done using nuclear targets like $^{12}$C, $^{16}$O, $^{40}$Ar, $^{56}$Fe, $^{208}$Pb etc. In the few GeV energy region the contribution to the cross section comes from quasielastic, inelastic (like one pion, multipion, single kaon, single hyperon, associated particle production etc.) as well as from deep inelastic scattering processes. In the last two decades lots of efforts have been made to understand quasielastic scattering and one pion production processes. However, for the precise determination of neutrino oscillation parameters, it has been realized that the study of other inelastic processes like multi pion, single kaon, single hyperon, associated particle production, eta meson production etc. are also required to reduce the systematic uncertainties. The recent cross section measurements are available mostly for $\Delta S = 0$ processes in nonstrange sector with pions and/or nucleons in the final state. There are very few works available where (anti)neutrino induced strange particle production have been studied [8, 9, 10, 11]. However, with the presence of high intensity neutrino and antineutrino beam it is now possible to study reaction channels that involve the strange quark(s) and give us an opportunity to test the SU(3) flavor symmetry.

The strange particles are produced by both $|\Delta S| = 0$ and $|\Delta S| = 1$ processes. At the (anti)neutrino energies of $\sim 1$ GeV it is the single hyperon($Y$) or single kaon($K/\bar{K}$) that are produced by $|\Delta S| = 1$ reaction mechanism while $\eta$ meson and associated production of kaon accompanied by a hyperon($K + Y$) are produced by the $|\Delta S| = 0$ mechanism. The reaction cross sections are smaller than the pion production due to Cabibbo suppression in $|\Delta S| = 1$ process and due to the low phase space for $|\Delta S| = 0$ process. These processes are also important to estimate the background for the experiments performing for nucleon decay searches. Furthermore, these processes would also help in determining the various transition form factors.
In this work we have studied neutrino/antineutrino induced $1K, YK$ and $\eta$ production processes, where $K$ stands for kaon and $Y$ for hyperon, and the various processes are given as:

1. Single Kaon Production

$$\begin{align}
\nu_l + p &\rightarrow l^- + K^+ + p \\
\bar{\nu}_l + p &\rightarrow l^+ + K^- + p \\
\nu_l + n &\rightarrow l^- + K^0 + p \\
\bar{\nu}_l + p &\rightarrow l^+ + \bar{K}^0 + n \\
\nu_l + n &\rightarrow l^- + K^+ + n \\
\bar{\nu}_l + n &\rightarrow l^+ + K^- + n, \\
\end{align}$$

(1)

2. Associated Kaon Production

$$\begin{align}
\nu_l + n &\rightarrow l^- + \Lambda^0 + K^+ \\
\bar{\nu}_l + p &\rightarrow l^+ + \Lambda^0 + K^0 \\
\nu_l + n &\rightarrow l^- + \Sigma^0 + K^+ \\
\bar{\nu}_l + p &\rightarrow l^+ + \Sigma^0 + K^0 \\
\nu_l + n &\rightarrow l^- + \Sigma^+ + K^0 \\
\bar{\nu}_l + p &\rightarrow l^+ + \Sigma^- + K^+ \\
\nu_l + p &\rightarrow l^- + \Sigma^+ + K^+ \\
\bar{\nu}_l + n &\rightarrow l^+ + \Sigma^- + K^0, \\
\end{align}$$

(2)

3. Weak eta meson production

$$\nu_l + n \rightarrow l^- + \eta + p \quad \text{and} \quad \bar{\nu}_l + p \rightarrow l^+ + \eta + n. \quad (3)$$

The plan of the presentation is as follows. First we present the formalism in brief then we will discuss the results and finally we conclude our findings.

**FORMALISM**

The general expression for the scattering cross-section in Lab frame may be written as,

$$d^9\sigma = \frac{(2\pi)^4}{4ME} \prod_{f=1}^n \frac{d^{3}k_{f}}{2k_{f}(2\pi)^3} \delta^4(k_i - k_f) \Sigma \Sigma |\mathcal{M}|^2,$$

(4)

where $\vec{k}_f$ is the 3-momenta of the incoming and/or outgoing leptons in the lab frame with energy $k^0_f$. $E$ is the energy of incoming neutrino beam, $M$ is the nucleon mass, $\Sigma \Sigma |\mathcal{M}|^2$ is the square of the transition amplitude matrix element averaged summed over the spins of the initial(final) state. At low energies, this amplitude can be written as

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} J^{(L)}_{\mu}(H) = \frac{g}{2\sqrt{2}} I^{(L)}_{\mu} \frac{1}{M_{W}} \frac{g J^{\mu(H)}}{2\sqrt{2}},$$

(5)

where $J^{(L)}_{\mu}$ and $J^{\mu(H)}$ are the leptonic and hadronic currents respectively, $G_F = \sqrt{\frac{e^2}{8\pi m_e^2}} = 1.16639(1) \times 10^{-5}\text{GeV}^{-2}$ is the Fermi constant and $g$ is the weak gauge coupling. The Standard Model Lagrangian for leptonic current is given by,

$$\mathcal{L} = -\frac{g}{2\sqrt{2}} [W^\mu_{\mu} \bar{\nu}_l \gamma^\mu (1 - \gamma_5) l + W^\mu_{\mu} \bar{\nu}_l \gamma^\mu (1 - \gamma_5) \nu_l],$$

(6)

where, $W$ boson couples to the leptons. In the neutrino energy of $\sim 1\text{GeV}$ hadronic current may be obtained using the chiral perturbation theory [12, 13].

To obtain the hadronic current, one starts with the lowest-order SU(3) chiral Lagrangian describing the pseudoscalar mesons in the presence of an external current [14]:

$$\mathcal{L}^{(2)}_M = \frac{f_{\pi}^2}{4} \text{Tr}[D_{\mu} U(D^\mu U)^\dagger] + \frac{f_{\pi}^2}{4} \text{Tr}[(\chi U^\dagger + U \chi^\dagger)],$$

(7)

where the parameter $f_\pi = 92.4\text{MeV}$ is the pion decay constant, $U$ is the SU(3) representation of the meson fields given by

$$U(x) = \exp \left( i \frac{\Phi(x)}{f_\pi} \right),$$
\[
\phi(x) = \begin{pmatrix}
\pi^0 + \frac{1}{\sqrt{3}} \eta \\
\sqrt{2} \pi^- \\
\sqrt{2} K^-
\end{pmatrix} \begin{pmatrix}
\pi^+ \\
-\pi^0 + \frac{1}{\sqrt{3}} \eta \\
\sqrt{2} K^0
\end{pmatrix} \begin{pmatrix}
\sqrt{2} K^+ \\
\sqrt{2} K^0 \\
-\frac{1}{\sqrt{3}} \eta
\end{pmatrix},
\]
(8)

and \(D_\mu U\) is its covariant derivative

\[
D_\mu U \equiv \partial_\mu U - ir_\mu U + iU l_\mu.
\]
(9)

Here, \(l_\mu\) and \(r_\mu\) corresponds to left and right handed currents, that for the charged current (CC) case are given by

\[
r_\mu = 0, \quad l_\mu = -\frac{g}{\sqrt{2}}(W_\mu^+ T_+ + W_\mu^- T_-),
\]
(10)

with \(W^\pm\) the \(W\) boson fields and

\[
T_+ = \begin{pmatrix}
0 & V_{ud} & V_{us} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}; \quad T_- = \begin{pmatrix}
0 & 0 & 0 \\
V_{ud} & 0 & 0 \\
V_{us} & 0 & 0
\end{pmatrix}.
\]

Here, \(V_{ij}\) are the elements of the Cabibbo-Kobayashi-Maskawa matrix.

Similarly, lowest-order chiral Lagrangian for the baryon octet \(B\) in the presence of an external current is written in terms of the SU(3) matrix [14]

\[
B = \begin{pmatrix}
\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda \\
\Sigma^- \\
\Xi^-
\end{pmatrix} \begin{pmatrix}
\Sigma^+ \\
-\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda \\
\Xi^0
\end{pmatrix} \begin{pmatrix}
p \\
n \\
-\frac{1}{\sqrt{6}} \Lambda
\end{pmatrix}
\]
(11)

as

\[
\mathcal{L}_{MB}^{(1)} = \text{Tr} \left[ B (iD - M)B \right] - \frac{D}{2} \text{Tr} \left( B \gamma^\mu g_5 [u_\mu, B] \right) - \frac{F}{2} \text{Tr} \left( B \gamma^\mu g_5 [u_\mu, B] \right),
\]
(12)

where \(M\) denotes the mass of the baryon octet, and the parameters \(D = 0.804\) and \(F = 0.463\) are determined from the baryon semileptonic decays [15]. The covariant derivative of baryon octet \(B\) is given by

\[
D_\mu B = \partial_\mu B + [\Gamma_\mu, B] ,
\]
(13)

with

\[
\Gamma_\mu = \frac{1}{2} \left[u^\dagger (\partial_\mu - ir_\mu) u + u (\partial_\mu - il_\mu) u^\dagger \right],
\]
(14)

where we have introduced \(u^2 = U\) with,

\[
u_\mu = i \left[u^\dagger (\partial_\mu - ir_\mu) u - u (\partial_\mu - il_\mu) u^\dagger \right].
\]
(15)

The next order meson baryon Lagrangian contains many new terms, which has been not considered in the present calculations and their contribution would be small. We have used the prescription given by Cabibbo. [12, 13, 15]. To include the higher order terms in the Lagrangian in this case, the coupling constants are fully determined by the proton and neutron anomalous magnetic moments. Now we shall present the amplitudes corresponding to the hadronic current for different processes given in Eqs. 1-3.

### Single Kaon Production

In the neutrino-nucleon scattering process the first process that produces strange particle in the inelastic region is the \(\Delta S = 1\) kaon/antikaon \((K/\bar{K})\) production. These processes are Cabibbo suppressed, however, in the absence of any competing process that could produce kaon and/or antikaon in the neutrino energies below 1.5GeV, they are the dominant source of \(K/\bar{K}\) production. For example, the threshold for the \(v(\bar{v})N \rightarrow l^- (\bar{l}^+)NK(\bar{K})\) is about 750MeV, as
FIGURE 1. Feynman diagrams for $\Delta S = 1$ kaon production. Top row right to left $Y^*$ resonance, direct and cross hyperon pole. Second row consists of diagrams (right to left) kaon pole, contact and kaon in flight.

compared to the next $K$-production mechanism $\nu \bar{\nu} N \rightarrow l^\pm YK$ and $\bar{K}$-production mechanism $\nu \bar{\nu} N \rightarrow l^\pm NK\bar{K}$ for which the threshold is 1.2GeV and 1.8GeV respectively. Therefore, the study of these processes could be important in giving ansatz to some important strangeness physics at low energies.

The amplitude for the hadronic current gets contribution from various Feynman diagrams shown in Fig. 1 viz. direct($Y$), cross hyperon pole($CrY$), contact($CT$) diagram, pion pole($KP$) and pion/eta in flight($\pi, \eta$). We have also included $\Sigma^*$($1385$) resonance for antikaon production. For single kaon production the amplitudes are obtained as,

$$J^\mu|_{CT} = iA_{CT}V_\sigma \frac{\sqrt{2}}{2\pi} \bar{N}(p') (\gamma^\mu + B_{CT} \gamma^\mu \gamma_5) N(p)$$

$$J^\mu|_{CrY} = iA_{Cr}V_\sigma (D - F) \frac{\sqrt{2}}{2\pi} \bar{N}(p') \left( \gamma^\mu + i\frac{\mu + 2\mu_\sigma}{2M} \sigma^{\mu\nu} q_\nu + (D - F) (\gamma^\mu - \frac{q^\mu}{m_\pi^2} \gamma_5) \right) \frac{p - p_N + M_N}{(p - p_N)^2 - M_N^2} \gamma_5 N(p),$$

$$J^\mu|_{Cr\Lambda} = iA_{Cr}V_\sigma (D + 3F) \frac{\sqrt{2}}{2\pi} \bar{N}(p') \left( \gamma^\mu + i\frac{\mu + 2\mu_\sigma}{2M} \sigma^{\mu\nu} q_\nu - \frac{D + 3F}{3} \left( \gamma^\mu - \frac{q^\mu}{m_\pi^2} \gamma_5 \right) \right) \frac{p - p_\Lambda + M_\Lambda}{(p - p_\Lambda)^2 - M_\Lambda^2} \gamma_5 N(p),$$

$$J^\mu|_{\Sigma} = iA_{\Sigma}(D - F) \bar{V}_\sigma (D + 3F) \frac{\sqrt{2}}{2\pi} \bar{N}(p') \left( \gamma^\mu + i\frac{\mu + 2\mu_\sigma}{2M} \sigma^{\mu\nu} q_\nu + (D - F) \left\{ \gamma^\mu - \frac{q^\mu}{m_\pi^2} \right\} \gamma_5 \right) \gamma_5 N(p),$$

$$J^\mu|_{KP} = iA_{KP}V_\sigma \frac{\sqrt{2}}{2\pi} \bar{N}(p') \frac{p^\mu}{q^2 - M_\pi^2} N(p),$$

$$J^\mu|_{\pi} = iA_{\pi} \frac{M_\pi \sqrt{2}}{2\pi} \bar{V}_\sigma (D + F) \frac{2p^\mu - q^\mu}{(q - p_\pi)^2 - m_\pi^2} \bar{N}(p') N(p),$$

$$J^\mu|_{\eta} = iA_{\eta} \frac{M_\eta \sqrt{2}}{2\pi} \bar{V}_\sigma (D - 3F) \frac{2p^\mu - q^\mu}{(q - p_\pi)^2 - m_\eta^2} \bar{N}(p') N(p),$$

$$J^\mu|_{\Sigma'} = -iA_{\Sigma'} \frac{\sqrt{6}}{\sqrt{6}} \bar{V}_\sigma \left( \frac{p^\mu}{P^2 - M_\Sigma'^2} + i\Gamma_{\Sigma'} \right) \bar{N}(p') \rho_{\Sigma \Lambda} (\Gamma_{\Sigma'}^\nu + \Gamma_A^\nu) N(p),$$

where, $q = k - k'$ is the four momentum transfer, $P = p + q$ is the momentum carried by the resonance and $P_{RS}^{\mu\nu}$ is the Rarita-Schwinger projection operator given by

$$P_{RS}^{\mu\nu}(P) = \sum_{spins} \psi^\mu \psi^\nu = - (P + M_\Sigma) \left[ g^{\mu\nu} - \frac{1}{3} \gamma^\mu \gamma^\nu - \frac{2}{3} \frac{P^\mu P^\nu}{M_\Sigma^2} + \frac{1}{3} \frac{P^\mu \gamma^\nu - P^\nu \gamma^\mu}{M_\Sigma^2} \right].$$

with $M_\Sigma$ the resonance mass and $\psi^\mu$ the Rarita-Schwinger spinor. The $\Sigma^* B\phi (\phi \equiv meson, B \equiv baryon)$ coupling($\delta^\prime$) is obtained from the on-shell $\Sigma^*$ width

$$\Gamma_{\Sigma'} = \Gamma_{\Sigma' \rightarrow \Lambda \pi} + \Gamma_{\Sigma' \rightarrow \Sigma \pi} + \Gamma_{\Sigma' \rightarrow \Sigma \bar{K}}.$$
TABLE 1. Constant factors appearing in the hadronic current

| Process                      | $B_{CT}$ | $A_{CT}$ | $A_{1}$ | $A_{2}$ | $A_{3}$ | $A_{4}$ | $A_{5}$ | $A_{6}$ | $A_{7}$ | $A_{8}$ |
|------------------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| $\nu n \rightarrow l^- K^+ n$ | D-F      | -1       | 0        | 0        | -1       | 0        | -1       | -1       | 0        | 0        |
| $\nu p \rightarrow l^- K^+ p$ | -F       | -2       | 0        | 0        | -1       | 1        | -2       | 1        | -1       | 0        |
| $\nu n \rightarrow l^- K^0 n$ | D-F      | -1       | 0        | 0        | $\frac{1}{2}$ | 1        | -1       | 2        | 0        | 0        |
| $\nu p \rightarrow l^- K^0 p$ | -F       | 1        | -1       | 0        | 0        | -1       | 1        | 1        | 1        | 2        |
| $\bar{\nu} n \rightarrow l^+ K^- n$ | D-F      | 1        | 0        | 0        | 0        | -1       | 1        | 1        | 1        | 1        |

where

$$
\Gamma_{\Sigma \rightarrow Y, \phi} = \frac{C_Y}{192 \pi} \left( \frac{\alpha}{f} \right)^2 \frac{(W + M_Y)^2 - m^2}{W^5} \lambda^{3/2}(W^2, M_Y^2, m^2) \Theta(W - M_Y - m). \tag{19}
$$

Here, $m, M_Y$ are the masses of the emitted meson and baryon, $\lambda(x, y, z) = (x - y - z)^2 - 4yz$ and $\Theta$ is the step function. The factor $C_Y$ is 1 for $\Lambda$ and $\frac{3}{4}$ for $N$ and $\Sigma$.

The $W^- N \rightarrow \Sigma^+$ vertex may be written in terms of a vector and an axial-vector piece given by [13]

$$
\langle \Sigma^+; P = p + q | V^\mu | N; p \rangle = V_{as} \bar{q} \gamma^\mu \gamma^5 \rho \langle \Sigma^+; P = p + q | A^\mu | N; p \rangle = V_{as} \bar{q} \gamma^\mu \gamma^5 \rho.
$$

where

$$
\Gamma^V_{\alpha \mu}(p, q) = \left[ \frac{C_V^A}{M} (g^{\alpha \mu} q - q^\mu q^\alpha) + \frac{C_V^A}{M^2} (g^{\alpha \mu} q \cdot p - q^\alpha p^\mu) + \frac{C_S^A}{M^2} (g^{\alpha \mu} q \cdot p - q^\alpha p^\mu) + C_S^A g^{\mu \alpha} + \frac{C_S^A}{M^2} q^\mu q^\alpha \right].
$$

(21)

The form factors $C_V^i, C_A^i, (i = 3, 4, 5, 6)$ are obtained using SU(3) symmetry and discussed in detail in Ref. [13].

**Associated production of strange particles**

At intermediate neutrino energies (1-2 GeV) kaon may also get produced in strangeness conserving $|\Delta S| = 0$ processes where a $\Lambda$ or $\Sigma$ are produced with kaon. However, as the hyperon mass is slightly larger than the nucleon mass, kinematically near threshold the production of kaons are favored by the $|\Delta S| = 1$ processes.

The Feynman diagrams that contribute to the $\Delta S = 0$ kaon production processes are depicted in Fig. 2. To get the coupling of the amplitudes for the above diagrams we used the same prescription as used in the case of $\Delta S = 1$ kaon production processes. However, in this case we generalize the nucleon and/or hyperon pole diagrams to incorporate form factors at the weak vertices, whereas a dipole form has been taken for the $|\Delta S| = 1$ single kaon production processes. The matrix element corresponding to the hyperon nucleon transition may be written as,

$$
J^\mu = \langle Y(k')|V^\mu - A^\mu |N(k)\rangle
$$

(22)

The amplitude thus obtained corresponding to the diagrams shown in Fig. 2 and are written as

$$
J^\mu|_S = iA_{SY} V_{ud} \sqrt{\frac{2}{f}} \bar{u}_Y(p') \gamma^\mu k^5 p + q + M \gamma^\mu u_N(p)
$$

$$
J^\mu|_U = iA_{UY} V_{ud} \sqrt{\frac{2}{f}} \bar{u}_Y(p') \gamma^\mu p - p_k + M_Y \gamma^\mu u_N(p)
$$
The axial vector coupling mass $M$ are given in Ref. [16] and are tabulated in Table 3. The appearing in Eq. 23 are summarized in Table 2.

**TABLE 2.** Constant factors appearing in the hadronic current. The upper sign corresponds to the processes with $\bar{\nu}$

| Process | $A_{CT}$ | $B_{CT}$ | $A_{SV}$ | $A_{U\Sigma}$ | $A_{U\Lambda}$ | $A_{TY}$ | $A_{\pi}$ |
|---------|---------|---------|---------|-------------|-------------|--------|--------|
| $\bar{\nu}_l \rightarrow l^+ \Lambda K^0$ | $-\sqrt{2}$ | $(D + 3F)$ | $-\sqrt{2}$ | $(D + 3F)$ | 0 | $-\frac{1}{\sqrt{2}}(D + 3F)$ | $\sqrt{2}$ |
| $\bar{\nu}_l \rightarrow l^- \Lambda K^+$ | $\frac{1}{\sqrt{2}}$ | $D - F$ | $\frac{1}{\sqrt{2}}(D - F)$ | $\frac{1}{\sqrt{2}}(D - F)$ | 0 | $\frac{1}{\sqrt{2}}(D - F)$ | $\frac{1}{\sqrt{2}}$ |
| $\bar{\nu}_l \rightarrow l^+ \Sigma^0 K^0$ | 0 | 0 | $D - F$ | $D - F$ | $\frac{1}{3}(D + 3F)$ | 0 | 0 |
| $\bar{\nu}_l \rightarrow l^+ \Sigma^- K^0$ | $-1$ | $D - F$ | 0 | $F - D$ | $\frac{1}{3}(D + 3F)$ | $D - F$ | 1 |

\[
\begin{align*}
\mathcal{A}_\mu &= iA_{TY} V_{ud} \frac{\sqrt{2}}{2f_\pi} (M + M_Y) \bar{u}_Y(p') g_s u_N(p) \frac{q^\mu - 2p_\mu^p}{(p - p')^2 - m_k^2} \\
\mathcal{A}_\mu &= iA_{CT} V_{ud} \frac{\sqrt{2}}{2f_\pi} (p'_\mu + B_{CT} \gamma^\mu) u_N(p) \\
\mathcal{A}_\mu &= iA_{\pi F} V_{ud} \frac{\sqrt{2}}{4f_\pi} \bar{u}_Y(p') (\not{q} + p_\pi) u_N(p) \frac{q^\mu}{q^2 - m_\pi^2} \\
\mathcal{A}_\mu &= f_1^P + i \frac{f_1^N}{2M} \sigma^{\mu\nu} q_{\nu} f_A \left( \gamma^\mu - \frac{q q^\mu}{q^2 - m_\pi^2} \right) \gamma^5,
\end{align*}
\]

where $\mathcal{A}_\mu$ is the transition current for $Y \rightarrow Y'$ with $Y = Y'$ Nucleon and/or Hyperon. The constant factors($A_i$) appearing in Eq. 23 are summarized in Table 2.

The form factors $f_1^P, f_1^N$ are related to the proton($f_1^P$) and neutron($f_1^N$) transition form factors, the details of which are given in Ref. [16] and are tabulated in Tab 3. The $f_{1,2}^{\mu,n}$ are parameterized with the help of electromagnetic Sach's form factors [17]. The axial vector coupling $f_A(Q^2) = \frac{f_A(0)}{1 + Q^2/M_A^2}$ is parameterized with dipole form with the axial mass $M_A = 1.05$GeV and $f_A(0) = 1.26$. 

**FIGURE 2.** Feynman diagrams for $\Delta S=0$ kaon production. (A)direct nucleon pole(s), (B)kaon pole(t). (C)cross hyperon pole(u), (D)Contact diagrams(CT), (E)Pion in flight($\pi\pi$)
It is well known from the studies of real/virtual photon induced $\eta$ production off a nucleon, the amplitudes corresponding to the nucleon pole and $\eta$ production dominates by $S_{11}(1535)$ resonance excitation and its subsequent decay in $\eta$. In the case of (anti)neutrino induce $1\eta$ production off a nucleon, the amplitudes corresponding to the nucleon pole and $N^*$ resonances may be written as

$$J_{\eta}^\mu_{N(s)} = \frac{g_{V_{ud}}}{2\sqrt{2}} \frac{D - 3F}{2\sqrt{3}f_{\pi}} \bar{u}_N(p') \gamma^\mu \bar{\eta} \gamma^5 \frac{p + q + M}{(p + q)^2 - M^2} \eta_{\mu N}(p)$$

$$J_{\eta}^\mu_{N(s)} = \frac{g_{V_{ud}}}{2\sqrt{2}} \frac{D - 3F}{2\sqrt{3}f_{\pi}} \bar{u}_N(p') \gamma^\mu \bar{\eta} \gamma^5 \frac{p - \bar{\eta} + M}{(p - \bar{\eta})^2 - M^2} \eta_{\mu N}(p),$$

**ETA PRODUCTION**

**TABLE 3.** Isovector $(f_1^V, f_2^V$ and axial $(f_A)$ transition form factors.)

| Sl. No. | Weak transition | $f_1^V(Q^2)$ | $f_2^V(Q^2)$ | $f_A(Q^2)$ |
|--------|-----------------|--------------|--------------|------------|
| 1      | $p \rightarrow n$ | $f_1^V(Q^2) - f_1^V(Q^2)$ | $f_2^V(Q^2) - f_2^V(Q^2)$ | $g_A(Q^2)$ |
| 2      | $p \rightarrow \Lambda$ | $-\sqrt{2} f_1^V(Q^2)$ | $-\sqrt{2} f_2^V(Q^2)$ | $-\sqrt{2} Q^2 f_A(Q^2)$ |
|        | $n \rightarrow \Lambda$ | $-\sqrt{2} f_1^V(Q^2)$ | $-\sqrt{2} f_2^V(Q^2)$ | $-\sqrt{2} Q^2 f_A(Q^2)$ |
| 3      | $\Sigma^+ \rightarrow \Lambda$ | $-\sqrt{2} f_1^V(Q^2)$ | $-\sqrt{2} f_2^V(Q^2)$ | $\sqrt{2} Q^2 f_A(Q^2)$ |
| 4      | $\Sigma^0 \rightarrow \Sigma^0$ | $\pm \frac{1}{\sqrt{2}} [2 f_1^V(Q^2) + f_2^V(Q^2)]$ | $\pm \frac{1}{\sqrt{2}} [2 f_1^V(Q^2) + f_2^V(Q^2)]$ | $\pm \sqrt{2} Q^2 f_A(Q^2)$ |
| 5      | $p \rightarrow \Sigma^0$ | $-\frac{1}{\sqrt{2}} [f_1^V(Q^2) + 2 f_2^V(Q^2)]$ | $-\frac{1}{\sqrt{2}} [f_1^V(Q^2) + 2 f_2^V(Q^2)]$ | $\frac{Q^2}{\sqrt{2}} g_A(Q^2)$ |
|        | $n \rightarrow \Sigma^0$ | $\frac{1}{\sqrt{2}} [f_1^V(Q^2) + 2 f_2^V(Q^2)]$ | $\frac{1}{\sqrt{2}} [f_1^V(Q^2) + 2 f_2^V(Q^2)]$ | $-\frac{Q^2}{\sqrt{2}} g_A(Q^2)$ |
| 6      | $n \rightarrow \Sigma^-$ | $-f_1^V(Q^2) - 2 f_2^V(Q^2)$ | $-f_2^V(Q^2) - 2 f_2^V(Q^2)$ | $\frac{Q^2}{F} g_A(Q^2)$ |
|        | $p \rightarrow \Sigma^+$ | $-f_1^V(Q^2) - 2 f_2^V(Q^2)$ | $-f_2^V(Q^2) - 2 f_2^V(Q^2)$ | $\frac{Q^2}{F} g_A(Q^2)$ |
The nucleon form factors associated strange particle production processes. The dipole form is taken for \(d_2\): \[d_2 = (10^{10})\]

\[\sigma^f(p) = 1.5 \text{ GeV}\]

\[\mu^f(p)\]

\[\text{cross section } |\Delta S| = 0 \text{ associated kaon production processes}\]

\[E_\nu = 1.5 \text{ GeV}\]

\[E_{\bar{\nu}_\mu} = 1.5 \text{ GeV}\]

\[\sigma^f(p) = \frac{g_{Vf}}{2\sqrt{2}} g_{\bar{\eta}uN(p')} p_{\bar{\eta}} \frac{p + \bar{q} + M_R}{(p + q)^2 - M_R^2 + i\Gamma_R M_R} \sigma^f_{\mu}(p)\]

\[J^\mu_{R(s)} = \frac{g_{Vf}}{2\sqrt{2}} g_{\bar{\eta}uN(p')} p_{\bar{\eta}} \frac{p - \bar{q} + M_R}{(p - q)^2 - M_R^2 + i\Gamma_R M_R} \sigma^f_{\mu}(p),\]

where

\[\sigma^f_{\mu} = f^V_1(q^2) \gamma^\mu + f^V_2(q^2) i\sigma^\mu\nu q^\nu - f_A(q^2) \gamma^\mu \gamma^5 - f_P(q^2) q^\mu \gamma^5\]

\[\sigma^\mu_R = \frac{F^V_1(q^2)}{(2M)^2} i\gamma^\mu - q^2 \gamma^5 \gamma^\perp + \frac{F^V_2(q^2)}{2M} i\gamma^\mu q^\perp - F_A(q^2) \sigma^\mu - F_P(q^2) q^\mu\]

The nucleon form factors \(f^{\mu\nu}_{V,P}\) are determined in terms of the \(f^{\mu\nu}_{V,P}\) in the same way as we discussed in case of \(\Delta S = 0\) associated strange particle production processes. The dipole form is taken for \(f_A\) and \(f_P\) is related to \(f_A\) through PCAC.

The isovector form factors \(F^{V}_{V,P}\) corresponding to the \(N^*\) resonances, are given in terms of the electromagnetic transition form factors for charged \(F^V_{1,2}\) and neutral \(F^N_{1,2}\) \(N^*\) resonances:

\[F^V_1(Q^2) = F^V_1(Q^2) - F^N_1(Q^2); \quad F^V_2(Q^2) = F^V_2(Q^2) - F^N_2(Q^2).\]
The $F_{1,2}^{p,n}(Q^2)$ are obtained from the helicity amplitudes $A_{1,2}^{p,n}$ and $S_{1,2}^{p,n}$, given as

\[
A_{1/2}^{p,n} = \sqrt{\frac{2\pi\alpha_e (M_R + M)^2 + Q^2}{M}} \left( \frac{Q^2}{4M^2} F_1^{p,n}(Q^2) + \frac{M_R - M}{2M} F_2^{p,n}(Q^2) \right),
\]

\[
S_{1/2}^{p,n} = \sqrt{\frac{\pi\alpha_e (M_R - M)^2 + Q^2}{M}} \left( \frac{M_R + M)^2 + Q^2}{4M^2} \left( \frac{M_R - M}{2M} F_1^{p,n}(Q^2) - F_2^{p,n}(Q^2) \right) \right) \).}

The parameters $A_\frac{1}{2}$ and $S_\frac{1}{2}$ are generally parameterized as:

\[
A_\frac{1}{2}(Q^2) = A_\frac{1}{2}(0) \left( 1 + \alpha Q^2 \right) e^{-\beta Q^2},
\]

\[
S_\frac{1}{2}(Q^2) = S_\frac{1}{2}(0) \left( 1 + \alpha Q^2 \right) e^{-\beta Q^2}.
\]

We fitted the $A_{1/2}^p$ using the data from the MAMI Crystal Ball experiment [18] and for the $A_{1/2}^n$ and $S_{1/2}^n$ we rely on the latest parameterizations from MAID [19, 20]. For resonances $S_{11}(1535)$ and $S_{11}(1650)$ the helicity parameters used for our numerical calculations are summarized in Tab. 4.

**TABLE 4.** Parameters used for the helicity amplitude

| Resonance → | $S_{11}(1535)$ | $S_{11}(1650)$ |
|-------------|----------------|----------------|
| Helicity Amplitude | $A_{\frac{1}{2}}(0)$ | $\alpha$ | $\beta$ | $A_{\frac{1}{2}}(0)$ | $\alpha$ | $\beta$ |
| $A_{1/2}^p(Q^2)$ | 89.38 | 1.61364 | 0.75879 | 53 | 1.45 | 0.62 |
| $S_{1/2}^p(Q^2)$ | -16.5 | 2.8261 | 0.73735 | -3.5 | 2.88 | 0.76 |
| $A_{1/2}^n(Q^2)$ | -52.79 | 2.86297 | 1.68723 | 9.3 | 0.13 | 1.55 |
| $S_{1/2}^n(Q^2)$ | 29.66 | 0.35874 | 1.55 | 10.0 | -0.5 | 1.55 |

We derived Goldberger-Treiman relation $F_A(0) = g_N^{\pi^+}$ and assumed a dipole form for $Q^2$ dependence for the axial form factors.

\[
F_A(Q^2) = F_A(0) \left( 1 + \frac{Q^2}{M_A^2} \right)^{-2},
\]

\[
F_P(Q^2) = \frac{(M_R - M)M}{Q^2 + m_\pi^2} F_A(Q^2).
\]
is the energy at resonance rest frame, which for on-mass shell reduces to the mass of resonance i.e. form factor and almost all the contribution comes from the Born terms. We have multiplied the hadronic current by a global dipole that the contact term is dominant in both cases. In the case of antineutrino induced kaon production the contribution of almost the same. In this figure, we have presented the contribution from each term of the Feynman amplitude. We find The off-shell effects are taken for the presented in Fig. 3. It is interesting to note that the cross sections for the neutrino and antineutrino induced reactions are where the partial decay width $\Gamma_{R\Phi}$ are calculated using the relation,

$$\Gamma_{S_{11}}(\Phi) = C_{S} \left( \frac{g_{S}}{g_{S}} \right)^{2} \frac{\left| \vec{p}_{CM} \right|}{8\pi} \frac{(W^{2} - M^{2})^{2} - m_{S}^{2}(W^{2} + M^{2} - 2MM_{R})}{W^{2}}$$

where $C_{S}$ is 3 for pion and $C_{S}$ is 1 for eta meson and

$$\left| \vec{p}_{CM} \right| = \frac{1}{2W} \sqrt{[W^{2} - (M + m_{S})^{2}] [W^{2} - (M - m_{S})^{2}]}.$$  

$W$ is the energy at resonance rest frame, which for on-mass shell reduces to the mass of resonance i.e. $W_{on-mass} = M_{R}$. The quantities in the brackets in Eq.29, are the weightage of partial decay widths of the various channel. Comparing the decay width with the available PDG values enable us to fix the various couplings involved.

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$S_{11}^{+}(P^{+})$

$S_{11}^{+}(P^{+})$

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of the cross section on $M_d$. The effect of 10% variation on $M_d$ changes the cross section by 10% and are shown by the shaded area.

In Fig. 4, we have presented the results for the total scattering cross section $\sigma$ for the $\Delta S = 0$ associated(Y+K) strange particle production processes for all the reactions shown in Eq. 2. In this case also we find(not shown here) that the contribution from the contact term to be dominant one followed by the direct and cross Born term diagrams. Furthermore, we find that the cross sections for the reaction channels with $\Lambda$ in the final state are the largest. This can be understood from the relative strength of the coupling $g_{NKA} = \sqrt{3}(D+3F)/(6f_x)$ vs $g_{NKS} = -3(D-F)/(6f_x)$. Apart from the larger coupling, the $\Lambda$ production is favored by the available phase space due to its small mass relative to $\Sigma$ baryons. For $\nu\mu n \to \mu^-\Sigma^+K^0$ and $\bar{\nu}_\mu p \to \mu^+\Sigma^-K^+$ there is no contribution from the contact term and hence the cross sections are relatively lower.

In Fig.5, we have presented the results for the kaon energy distribution near threshold at 1.5 GeV for (anti)neutrino induced $\Delta S = 0$ processes. It may be observed that the $\Lambda$ in the final state has the highest peak. However, the nature of the peaks are little different. For example, in the case of neutrino induced process the peak lies around 200MeV of kinetic energy of $\Lambda$ while in the case of antineutrino induced process it lies around 50MeV.

Before doing the numerical calculations for (anti)neutrino induced $\eta$ production off nucleon, first we have obtained the total scattering cross section for photo production of $\eta$-meson and compared them with the experimental results recently obtained from MAMI Crystal Ball experiment [18]. These results are shown here in Fig.6. The vector form factors of the $N-S_{11}$ transition has been obtained using the helicity amplitude extracted in the analysis of world pion photo- and electro- production data with the unitary isobar model MAID [19, 20].

In Fig. 7, we have presented the results for neutrino and antineutrino induced $\eta$ production from nucleon for the processes given in Eq. 3. Here we have also given the contribution coming from the individual term like $s$- and $u$-channel Born diagram and $s$- and $u$-channel $S_{11}(1535)$ and $S_{11}(1650)$ resonance contributions. We find that like in the photo- and electro- induced $\eta$ production process, in the case of weak interaction induced process also there is $S_{11}(1535)$ dominance. The contribution of $S_{11}(1650)$ is negligible. This can be understood easily as $S_{11}(1535)$ is lighter in mass and has a relatively larger branching ratio into $\eta N$ than $S_{11}(1650)$. The contribution of the non-resonant diagrams in the case of neutrino induced charged current process is higher than the corresponding antineutrino process. We also observe that in the neutrino mode the contribution of $u$-channel diagram is slightly larger than the corresponding $s$-channel diagram. Furthermore, we have studied flux integrated $p_{\eta}$ dependence corresponding to (anti)neutrino flux at T2K and atmospheric neutrinos. The results are presented in Fig. 8.

To conclude, in this work we have studied single kaon/antikaon, eta and associated particle production processes in the (anti)neutrino induced reactions from nucleon targets. These results may be helpful in the analysis of MINERvA experiment as well as the experiments looking for proton decay.

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