Modeling and Analysis of Transverse Vibration of Traction Rope of High Speed Traction Elevator

Xiujuan Qi, Ruijun Zhang* and Qin He

School of Mechanical and Electrical Engineering, Shandong Jianzhu University, Jinan, 250101, Shandong, P.R. China

Corresponding author: zhangruijun@sdjzu.edu.cn

Abstract. Based on continuum mechanics theory and considering the influence of external excitation on high-speed traction elevator wire rope, Hamilton principle is applied to establish the transverse vibration control equation of variable-length lifting wire rope, and Galerkin method is used to discretize the solution. Finally, the model is simulated by Matlab software, and the transverse vibration law of the wire rope and the influence of damping on the transverse vibration of the wire rope are analyzed. The results show that the transverse vibration strength of the wire rope is basically the same as that of the wire rope when there is no damp effect. Increasing the damp can reduce the transverse vibration strength of the wire rope.

1. Introduction

With the rapid development of super high-rise buildings, the use of high-speed elevators has been promoted. Elevator is the transportation equipment of the vertical high-rise building nowadays. In order to improve the transportation efficiency, the elevator speed is improved. However, the safety of elevators is difficult to guarantee because of the high speed. The most important problem is the vibration of elevator lifting system. The vibration of elevator hoisting system is mainly caused by the vibration of elevator wire rope. So it is important to study the vibration of elevator wire rope for improving the vibration of lifting system.

Wire rope vibration accounts for a large proportion of the factors causing elevator vibration. Nowadays, more and more researchers are studying the vibration of wire rope. Jihu Bao [1] Considering the influence of geometric nonlinearity caused by deformation on variable length wire rope, the control equation of transverse vibration of wire rope is established by energy method, and the model is simulated by MATLAB software. Zhu [2] aiming at the problem of vibration of axially moving chords, neglects the influence of geometric deformation factors and establishes a linear differential equation for transverse vibration of horizontally moving chords with variable length. Considering the influence of balanced wire rope, Juan Wu [3] established the control equation of transverse vibration of hoisting wire rope by Hamilition principle, and analyzed the influence of guide rail unevenness and external horizontal excitation on the transverse vibration of hoisting wire rope. Hiroyuki [4] assumes that the tension and velocity of the rope are constant. The governing equation of the forced vibration of a variable length rope with linear damping is established and solved by using a virtual wave source. Yang [5] considers the coupling effect between building and elevator rope with suspension cage motion, applies Hamilton principle to establish the dynamic model of elevator rope,
and uses this model to predict the vibration of actual building and elevator rope. Jihu Bao [6] considers the coupling of the cable's axial motion and bending deformation, establishes the governing equation, and studies the nonlinear vibration of flexible hoisting wire rope with time-varying length and axial velocity. Jiang [7] takes into account the time-varying characteristics of rope mass, stiffness, damping and power, establishes the dynamic model of high-speed traction elevator, and simulates the acceleration response under given speed curve.

2. Transverse Vibration Model of Wire Rope
The model establishment and solution in this paper are based on the following four assumptions:
1. Ignore the influence of airflow and friction in wellbore;
2. Ignore the influence of balanced wire rope vibration;
3. Ignore the influence of torsion and longitudinal vibration of wire rope, and the elastic deformation caused by transverse vibration is much smaller than the length of the entire string.
4. In the operation of high-speed elevator, the linear density of wire rope is $\rho$, the cross-sectional area is $A$, and the elastic modulus is $E$ remain unchanged.

2.1. Vibration Control Equation
Figure 1 is a transverse vibration model of high-speed traction elevator lifting system. In this paper, the wire rope is regarded as a variable length string with axial motion, the linear density is $\rho$, the cross-sectional area is $A$, the elastic modulus is $E$, and the length $l(t)$ varies with time $t$. The lifting weight is simplified as a rigid weight with mass $m$ suspended at the bottom of the chord line, which is free longitudinally, Constrained by a spring with a lateral stiffness of $k$ and a damper with a damping coefficient of $c$, the existence of a rigid tank passage in a real high-speed elevator hoisting system is simulated. Assume that there is a horizontal external excitation $e_1$ at the upper end of the wire rope to simulate the eccentric rotation of the motor or other factors. The horizontal excitation $e_2$ exists at the lower end to simulate the unevenness of the rigid tank. Choose the tangent point between the traction wheel and the wire rope as the origin, the vertical downward direction of the x-axis is positive, and the horizontal left direction of the y-axis is positive.

![Figure 1. Transverse vibration model of wire rope.](image)
The kinetic energy of the system can be expressed as:

\[
T = \frac{1}{2} P \int_{0}^{l(t)} V^2 \, dx + \frac{1}{2} m v^2 \bigg|_{x=l(t)}
\]  

(1)

The potential energy of the system can be expressed as:

\[
V = \int_{0}^{l(t)} \left( \frac{1}{2} P x^2 + \frac{1}{2} E A \varepsilon^2 \right) \, dx + \frac{1}{2} k \left[ y(l(t), t) \right]^2
\]  

(2)

where: \( P \) is the tension applied by the wire rope \( x \) at the time \( t \), and \( \varepsilon \) is the wire rope strain.

\[
P = \left[ m + \rho(l(t) - x) \right] (g - a)
\]  

(3)

where: \( a = \ddot{l}(t) \) is the overall motion acceleration of the lifting system; \( g \) is the acceleration of gravity.

The virtual work of the damping force in the system can be expressed as:

\[
\delta W = -c \frac{Dy(l(t), t)}{Dt} \delta y(l(t), t)
\]  

(4)

According to the Hamiltonian principle:

\[
\int_{t_1}^{t_2} (\delta T - \delta V + \delta W) \, dt = 0
\]  

(5)

After a series of transformations of Formula 5, the transverse vibration control equation of high-speed traction elevator hoisting wire rope can be derived.

\[
\rho (y''_x + 2v_y y'_{xx} + v^2 y''_{xx}) - (P y_{xx} + P_x y_x) - \frac{3}{2} E A y_{xx} y_x^2 + c y_x = 0, (0 < x < l(t))
\]  

(6)

The geometric and temporal boundary conditions are:

\[
y(0, t) = e_1(t), y(l(t), t) = e_2(t)
\]

The boundary condition at this time is a non-homogeneous boundary condition, and the function \( y(x, t) \) having a non-homogeneous boundary condition is transformed into a function \( y_1(x, t) \) having a homogeneous boundary condition, that is, the lateral displacement \( y(x, t) \) can be expressed as

\[
y(x, t) = y_1(x, t) + y_2(x, t)
\]  

(7)

where: \( y_1(x, t) \) is the part that satisfies the corresponding homogeneous boundary condition; \( y_2(x, t) \) is the part that does not satisfy the homogeneous boundary condition.

Substituting equation (7) into equation (6) can obtain the transverse vibration control equation of the lifting system with excitation:

\[
\rho (y''_{1,x} + 2v_y y'_{1,xx} + v^2 y''_{1,xx}) - (P y_{1,xx} + P_x y_{1,x}) - \frac{3}{2} E A y_{1,xx} y_{1,x}^2 + 2y_{1,xx} y_{1,xx} y_{1,x} + 2y_{1,x} y_{1,xx} y_{1,x} + 2y_{1,xx} y_{1,xx} y_{1,x} + y_{1,xx} y_{1,xx}^2) + 2c y_{1,x} = 0
\]  

(8)
2.2. Galerkin Method Discretization

The transverse vibration control equations of steel wire ropes are complex partial differential equations with infinite degrees of freedom, and many parameters are time-varying, so it is impossible to obtain accurate analytical solutions. Therefore, the article uses Galerkin method for discretization. An approximate numerical solution is obtained. Let the solution of equation (8) be:

\[ y_i(x,t) = \sum_{i=1}^{n} \phi_i(\xi)q_i(t) \] (9)

where: \( q_i(t) \) is the generalized coordinate, \( i = 1,2,3,\ldots,n \) is the modulus; \( \phi_i(\xi) \) is the type function.

\[ \phi_i(\xi) = \sqrt{2} \sin(i\pi\xi) \]

Using the modified Galerkin method to transform the infinite dimensional transverse vibration partial differential equation of the high speed traction elevator lifting system wire rope into a finite dimensional ordinary differential equation.

\[ M\ddot{Q} + C\dot{Q} + K_1Q + K_2Q^2 + K_3Q^3 = F \] (10)

where: \( Q = [q_1(t), q_2(t), \ldots, q_n(t)]^T \) is the generalized coordinate vector; the values of the elements in the matrices \( M, C, K_1, K_2, K_3 \) and \( F \) are:

\[
m_{ij} = \rho \delta_{ij}
\]

\[
c_{ij} = \frac{2\nu \rho}{l(t)} \int_0^1 (1-\xi)\phi_i(\xi)\phi_j(\xi) d\xi + c \delta_{ij}
\]

\[
k_{ij}(t) = \rho g - a(t)/l(t) \int_0^1 (1-\xi)\phi_i(\xi)\phi_j(\xi) d\xi + \frac{2\nu \rho}{l(t)^2} \int_0^1 (1-\xi)\phi_i(\xi)\phi_j(\xi) d\xi d\xi - \frac{\nu^2(t) \rho}{l(t)^2} \int_0^1 (1-\xi)^2\phi_i(\xi)\phi_j(\xi) d\xi d\xi
\]

\[
-\frac{3EA(e_2 - e_1)^2}{2l^4(t)} \int_0^1 \phi_i(\xi)\phi_j(\xi) d\xi - \frac{c v(t)}{l(t)} \int_0^1 \xi \phi_i(\xi)\phi_j(\xi) d\xi - \frac{m g - a(t)}{l(t)^2} \int_0^1 \phi_i(\xi)\phi_j(\xi) d\xi
\]

\[
+ \frac{\rho^2 \phi_i(\xi)\phi_j(\xi) \phi_j(\xi) d\xi + \frac{\rho a(t)}{l(t)} \int_0^1 (1-\xi)\phi_i(\xi)\phi_j(\xi) d\xi \]

\[
k_{2ij}(t) = \frac{3EA}{2l^4(t)} (e_2 - e_1) \int_0^1 \phi_i(\xi) \phi_j(\xi) \phi_j(\xi) d\xi
\]

\[
k_{3ij}(t) = \frac{3EA}{2l^4(t)} (e_2 - e_1) \int_0^1 \phi_i(\xi) \phi_j(\xi) \phi_j(\xi) d\xi
\]

\[
f_j(t) = -\rho \phi_i(\xi) \int_0^1 \phi_i(\xi) d\xi - \rho \phi_i(\xi) \int_0^1 \xi \phi_i(\xi) d\xi d\xi - \frac{2\nu(t) \rho}{l(t)} (e_2' - e_1') \int_0^1 (1-\xi)\phi_i(\xi) d\xi d\xi
\]

\[
- \frac{a(t) \rho}{l(t)} (e_2 - e_1) \int_0^1 (1-\xi)\phi_i(\xi) d\xi d\xi - \frac{2\nu(t) \rho}{l(t)} (e_2 - e_1) \int_0^1 \xi \phi_i(\xi) d\xi d\xi
\]

\[
- \frac{2\nu(t) \rho}{l(t)} (e_2 - e_1) \int_0^1 \phi_i(\xi) d\xi - \frac{(\rho a(t) - \rho g + c)}{l(t)} (e_2 - e_1) \int_0^1 \phi_i(\xi) d\xi d\xi
\]
where $\delta_y$ is a delta function, and the ordinary differential equation (10) is solved, and the value $q_{j(t)}$ of the generalized coordinate can be obtained. Substituting equations (10) and (8) can obtain the real-time value of transverse vibration.

3. Examples and Results Analysis
Taking high-speed traction elevator as an example, the model is simulated and analyzed. The maximum lifting height of the elevator parameters is $l_{\text{max}} = 150m$, the maximum speed $v_{\text{max}} = 5m/s$, the maximum acceleration $a_{\text{max}} = 1m/s^2$, and the maximum acceleration $J_{\text{max}} = 0.5m/s^3$. The car system mass $m = 400kg$, wire rope elastic modulus $E = 8 \times 10^{10} N/m^2$, wire rope cross-sectional area $A = 89.344cm^2$, wire rope wire density $\rho = 0.87kg/m$, traction wheel eccentric rotation on the wire rope upper end horizontal excitation is $e_1 = 0.01\sin(3.14t)$, rail irregularity on the wire rope lower end level The incentive is $e_2 = 0.005\sin(6.28t)$. Taking the actual running curve of the elevator as input, the numerical calculation method is used, and the simulation is carried out by using Matlab software.

![Figure 2. Running curve of the traction elevator.](image)

![Figure 3. Undamped lower acceleration vibration amplitude.](image)

From Figure 3, it can be seen that the vibration frequency and amplitude of transverse acceleration increase with the shortening of the length of the wire rope when the wire rope goes up without damp, and decrease with the length of the wire rope when the wire rope goes down, and the amplitude and peak value of transverse acceleration vibration are basically the same.
From Figure 4, it can be concluded that the vibration frequency and amplitude of transverse acceleration increase with the decrease of the length of the wire rope when the wire rope goes up under the action of damping; while the vibration frequency and amplitude of transverse acceleration decrease with the increase of the length of the wire rope when the wire rope goes down, but the vibration amplitude of the upward is larger than that of the downward, which has a greater impact on the transverse acceleration vibration of the downward wire rope. From Fig. 4 and Fig. 5, it can be seen that the vibration amplitude of lateral acceleration varies with different dampers. The average amplitude of lateral acceleration of wire rope decreases with the increase of damping.

**4. Conclusion**

(1) Aiming at the time-varying characteristics of high-speed traction elevator wire rope, considering the influence of damp on the lateral vibration of wire rope, a partial differential equation for the lateral vibration of high-speed traction wire rope is established by using Hamilton principle.

(2) Taking the actual operation curve of traction elevator as input, the vibration response of wire rope lateral acceleration under different dampers is simulated and analyzed by MATLAB software. The results show that when there is no damper, the vibration amplitude of the upward and downward lateral acceleration is basically the same. When the cable is damped, the transverse acceleration vibration intensity on the wire rope is greater than that on the wire rope. Increasing the damping can
reduce the vibration intensity of the lateral acceleration of the wire rope, which verifies the validity of this method.

(3) The establishment and analysis of the transverse vibration model of the wire rope in this paper have certain reference value for the further study and analysis of the vibration characteristics of the wire rope of the traction elevator and the control of the vibration.

**Acknowledgement**
This study was funded by the Shandong Province Nature Science Foundation, China (GRANT NO. ZR2017MEE049), the Key Research Development Project of Shandong Province (GRANT NO. 2018GSF122004).

**References**
[1] J.H. Bao, P. Zhang, Journal of Shanghai Jiaotong University. 46(3), 341-345 (2012).
[2] W.D. Zhu, J. Ni, Journal of Vibration and Acoustics. 122(3), 295-304 (2000).
[3] J. Wu, Z.M. kui, et al, Journal of Vibration and Shock. 35(2), 184-188 (2016).
[4] ITO. Hiroaki, Journal of System Design and Dynamics. 2(2), 540-549 (2008).
[5] D.H. Yang, k.y. Kim, et al, J. Sound Vibr. 390, 164-191 (2017).
[6] J.H. Bao, P. Zhang, et al, International Journal of Acoustics and Vibration. 20(3), 160-170 (2015).
[7] X.M. Jiang, L.Z. Guo, et al, 6th International Workshop of Advanced Manufacturing and Automation (IWAMA). 24, 221-224 (2016).