Heavy quarkonium 2S states in light-front quark model

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Abstract. We study the charmonium 2S states ψ' and η_c', and the bottomonium 2S states Υ' and η_b', using the light-front quark model and the 2S state wave function of harmonic oscillator as the approximation of the 2S quarkonium wave function. The decay constants, transition form factors and masses of these mesons are calculated and compared with experimental data. Predictions of quantities such as Br(ψ' → γη_c') are made. The 2S wave function may help us learn more about the structure of these heavy quarkonia.

PACS. 14.40.Pq Heavy quarkonia – 13.20.Gd Decays of J/ψ, Y, and other quarkonia – 11.15.Tk Other nonperturbative techniques – 13.40.Gp Electromagnetic form factors

1 Introduction

Charmonium physics has long been an interesting issue as it is related with both the perturbative and non-perturbative QCD. Charmonia not only provide us with the opportunity to investigate the interactions between the constituent quarks and the structure of quarkonia, but also the chance to learn and understand the QCD dynamics better. As exited states of charmonia, ψ' and η_c' have been studied by many authors. The decay widths of ψ' → e⁺e⁻ and η_c' → 2γ were calculated with both relativistic and QCD radiative corrections [1] and the result ψ' → e⁺e⁻ is in agreement with experimental data. The nonrelativistic potential model [2] and the Godfrey and Isgur (GI) model [3,4] have achieved much success, but their predictions of the decay widths of ψ' → γη_c (γη_b) are larger than experimental data. The lattice QCD result [5,6] of J/ψ → γη_c is consistent with experimental data, but the result of ψ' → γη_c has too large uncertainties. The intermediate meson loop contribution to the decays ψ' → γη_c (γη_b) was investigated recently [7,8], and the results are closer to experimental data. We also investigate these decays, using light-front formalism and the harmonic oscillator wave function as the approximate wave functions of the 1S and 2S quarkonia. In fact, there are still some puzzles concerning ψ', such as the well-known "ρπ puzzle" [9] [10][11][12], and the recent unanticipated small experimental value of Br(ψ' → γη_c)/Br(ψ' → γη_b') [13][14]. For the "ρπ puzzle", Ref. [15] suggested the explanation that the ψ'ρπ coupling is suppressed due to the mismatch between the nodeless wave function of the cc in the |udcc⟩ Fock state of ρ and the one-node 2S cc wave function of ψ', and our postulation of the 2S wave function of ψ' may be able to offer a numerical realization for this explanation. The BES and CLEO collaborations have conducted many experimental measurements on 1S and 2S charmonia, and the decay mode ψ' → γη_c is being studied by BES-III. It is then important to learn carefully about the structures and decay mechanisms of the 2S charmonia.

The 2S bottomonia have been studied by some experiments, but many data about them are still not available, such as the mass and decay data of η_b' [16]. In Ref. [1], the decay modes η_b' → 2γ and Υ' → e⁺e⁻ were studied by considering both relativistic and QCD radiative corrections, and predictions were made. With the same method of studying the 2S charmonia in the light-front quark model, we can study more decay modes, and calculate the masses of these bottomonia in this paper. From an experimental viewpoint, a large amount of bottomonia and their excited states could be produced at the forthcoming LHC or by the Belle experiment in the near future, and they could provide important tests of different predictions.

Moreover, heavy charmonia, especially charmonia and bottomonia, act as important diagnostic tools to probe the properties of the background QCD matter, such as of the formation of the quark-gluon plasma (QGP), in heavy ion collisions at RHIC and LHC [17]. As has been pointed in Ref. [17], the study of heavy-charmonium suppression at RHIC energy, which might be a signature for the QGP formation, calls for the knowledge of the light-front wave functions of the quarkonia: f(x_L,x'_L,τ₀) = φ(x_L)φ⁺(x'_L), where φ(x_L) can be taken as the Fourier transform of our light-front momentum space wave functions for the 1S or 2S quarkonia (Eqs. [7] and [8]), and f(x_L,x'_L,τ₀) are the essential quantities to calculate the transverse momentum distribution of quarks. Thus our wave functions can not only help us understand the structure of heavy quarkonia.

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themselves, but also be used as inputs for other physical studies.

This paper is organized as follows. In Sec. 2, we describe the light-front quark model and the 2S state wave function for the quarkonia. In Sec. 3, we present our numerical results of the decay constants, form factors and masses of these charmonia and bottomonia, and compare them with experimental data. A brief summary is given in Sec. 4.

2 Model description

Heavy quarkonia have been studied by non-relativistic treatments [18,19,20], but in some occasions related to non-perturbative scales, they have to use model dependent methods. And as the virtual photon momentum $Q^2$ increases, the relativistic effects become important. So it is useful to study quarkonia in a relativistic treatment. Several powerful non-perturbative tools have been developed to study the structure and decays of mesons, such as the QCD sum-rule technique and the lattice gauge theory. The light-front quark model is also an important model to do such studies [21,22,23], and it has a number of salient features. Light-front quark model involves some important relativistic effects that are neglected in the traditional constituent quark model, and the vacuum in the light-cone coordinate is simple because the Fock vacuum is the exact eigenstate of the Hamiltonian. Light-front quark model has been successfully applied in many investigations of hadron structures [31,32,33,34,35,36,37,38,24,25,26,27,28,29,30].

In the light-front quark model, the states of quarkonia can be described by the Fock state expansion

$$|M⟩ = ∑ |qq⟩|ψ_{qq}⟩ + ∑ |qqg⟩|ψ_{qqg}⟩ + ⋅ ⋅ ⋅ ,$$  \hspace{1cm} (1)

and to simplify the problem, we adopt the lowest order of the above expansions and take only the quark-antiquark valence states of the mesons into consideration.

The quarkonium wave function in light-front formalism is [21,22,23,24]

$$|M(P^+, P_⊥, S_z)⟩ = ∫ d^4x d^2k_⊥ \sqrt{x(1−x)}16π^3 (xP^+ + (xP_⊥ + k_⊥)^2)/(xP^+ − xP_⊥),$$

(2)

with the momentum of the struck quark being $(xP^+, |m^2 + (xP_⊥ + k_⊥)^2|)/xP^+$, $xP_⊥ + k_⊥)$, and $λ_i$ being the helicity of the $i$-th constituent quark. $ϕ(x, k_⊥)$ is the radial wave function, and $μ_M(x, k_⊥, λ_1, λ_2)$ is the light-front spin wave function, which is related to the instant-form spin wave function by the Melosh-Wigner rotation [10,11,12,13,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,24,25].

$$∥\chi_1(T)∥ = w_1[k_1^1 + m_1]|\chi_1^R(F)⟩ − k_1^R|\chi_1^L(F)⟩,$$

$$∥\chi_1(T)∥ = w_1[k_1^2 + m_1]|\chi_1^R(F)⟩ + k_1^L|\chi_1^L(F)⟩,$$

(3)

where $w_1 = 1/√[2]k_1^1$, $k_1^R = k_1^1 ± k_2$, $k_1^L = k_1^1 ± k_3 = xM$, $m_1$ is the mass of the constituent quark, and the invariant mass of the composite system $M = √[(k_1^2 + m_1^2)/(x + (k_1^2 + m_1^2)/(1−x))].$ The Melosh-Wigner rotation is an important ingredient of light-front quark model and plays an essential role in explaining the “proton spin puzzle” [21,25,26,27,28,29,30]. In the above formalism, the Drell-Yan-West $(q^2 = 0)$ frame [31,32] is used because only valence contributions are needed in this frame when studying the decay of quarkonia.

For the radial wave function $ϕ$, the harmonic oscillator wave function has been adopted to describe the 1S state mesons [39,30,31,32], and it can well explain experimental data. So we try to go further to use the 2S state harmonic oscillator wave function as the approximate wave function of the 2S quarkonia. The wave functions of the 1S and 2S states of the non-relativistic 3-dimensional isotropic harmonic oscillator in momentum space are

$$ϕ^{1S}(p) = \frac{1}{π^{3/2}(αh)^{3/2}} \exp[-p^2/2α^2h^2],$$

$$ϕ^{2S}(p) = \frac{√6}{3π^{3/4}(αh)^{7/2}} (p^2/2) \exp[-p^2/2α^2h^2].$$

(4)

(5)

where $α = √μω/\hbar$, $μ$ and $ω$ are the mass of the oscillating particle and the frequency of the corresponding classical oscillator respectively.

We use the connection between the equal-time wave function in the rest frame and the light-front wave function suggested by Brodsky-Huang-Lepage [21,22,39], for the quarkonia with $m_1 = m_2 = m_q$,

$$p^2 ↔ k_1^2 + m_q^2 = 4x(1−x)$$

(6)

and we use the prescription in Ref. [31] to extend the non-relativistic form wave function into a relativistic one [30]. Then we have the corresponding relativistic wave functions in light-front formalism

$$ϕ^{1S}(x, k_⊥) = 4π^3/β^3/2 \sqrt{∂k_⊥/∂x} \exp[-k^2/2β^2],$$

$$ϕ^{2S}(x, k_⊥) = 4√6π^3/β^3/2 \sqrt{∂k_⊥/∂x} (k^2 − 3/2)^2 \exp[-k^2/2β^2].$$

(7)

(8)

where $β$ is the parameter equivalent with $α$ in Eqs. (4) and (5), and its value can be chosen to fit experimental data. The longitudinal momentum $k_z = (x − 1/2)M + (m_q^2 − m_1^2)/2M$, one can easily check that this is equivalent to Eq. (6). The factor $√∂k_⊥/∂x$ in the above two equations comes from the Jacobian of the transformation $(x, k) → (k, k_z)$, and the normalization factors are from the requirement of the normalization of the total wave function [47].

Using the above formalism and wave functions, we can calculate the decay constants and transition form factors of the quarkonia [45,46].

In the $V → e^−e^+$ process, the decay constant of the vector meson $V$ is defined by

$$⟨0|j_μ|V(p, S_z)⟩ = M_V f_V e_μ(S_z),$$

(9)
and with the same method as Ref. [50], we have, for the vector quarkonium,
\[
f_V = 2\sqrt{6} \epsilon_q \int \frac{d^3k}{16\pi^3} \frac{1}{\sqrt{x(1-x)}} \phi_V(x, k)\frac{2k^2 + m_q(M + 2m_q)}{\sqrt{k^2 + m_q^2(M + 2m_q)}},
\]
where \(m_q\) and \(\epsilon_q\) is the mass and electric charge of the constituent quark of the quarkonium respectively (\(\epsilon_q = 2/3\) for charmonia, and \(-1/3\) for bottomonia).

In the \(P \rightarrow \gamma\gamma\) process, the transition form factor of the pseudoscalar meson \(P\) is defined by
\[
\langle\gamma(p-p')|J_\mu|P(p, \lambda)\rangle = ie^2 F_{P\rightarrow\gamma\gamma}(Q^2)x^{\mu\nu\rho\sigma}\epsilon_\nu(p, \lambda)p_\rho\beta_\sigma, 
\]
and we have the formula for the pseudoscalar quarkonium
\[
F_{P\rightarrow\gamma\gamma}(Q^2) = 4\sqrt{6} \epsilon_q \int \frac{d^3k}{16\pi^3} \phi_P(x, k)\frac{m_q}{x\sqrt{k^2 + m_q^2}} (\epsilon_{\mu})(\epsilon_{\nu})(\epsilon_{\rho})(\epsilon_{\sigma})\frac{2k^2 + m_q(M + 2m_q)}{\sqrt{k^2 + m_q^2}}.
\]
where \(\theta\) is the angle between \(k\) and \(q\).

The above quantities are related to the decay width of the quarkonium by [18]
\[
\Gamma(V \rightarrow e^+e^-) = \frac{4\pi\alpha^2 f_V^2}{3M_V},
\]
\[
\Gamma(P \rightarrow \gamma\gamma) = \frac{4\pi\alpha^2 M_P^2|F_{P\rightarrow\gamma\gamma}(0)|^2}{3M_V},
\]
\[
F_{V\rightarrow\gamma\gamma} = \frac{\alpha}{3} |F_{V\rightarrow\gamma\gamma}(0)|^2 \left(\frac{M_V^2 - M_P^2}{2M_V}\right)^3
\]
and we can also calculate the mass of the quarkonium, using the QCD-motivated Hamiltonian for mesons [51]
\[
H_{qq} = \sqrt{m_q^2 + k^2} + \sqrt{m_q^2 + k^2} + V_{qq},
\]
where \(k\) is the momentum of the constituent quark, and
\[
V_{qq} = a + b\gamma^2 + \frac{4\pi\alpha}{3r} + \frac{2S_q \cdot S_q}{3m_qm_q} \nabla^2 V_{coul}.
\]
with the last term being the hyperfine interaction that causes the mass splitting between vector and pseudoscalar mesons. Here we choose the confining potential (the second term) to be the harmonic oscillator potential rather than the linear potential in order to keep consistency with our harmonic oscillator wave function for the quarkonium. The values of parameters \(a, b\) and \(\gamma\) were given in Ref [51].

The mass of the meson is obtained as \(M_{qq} = \langle\phi|H_{qq}|\phi\rangle\) [51].

For the 2S quarkonium, we have
\[
M_{qq} = \frac{16}{3\sqrt{\pi}\beta} \int_0^\infty \left(\sqrt{m_q^2 + p^2}(2\frac{3}{2}F^2)^2e^{-p^2/\beta^2}dp + a + \frac{7b}{2\beta^2} + \frac{20a\alpha\beta}{9\sqrt{\pi}} \right)
\]
\[
+ \left\{\begin{array}{ll}
\frac{4\alpha\beta^2}{\sqrt{\pi}}(\text{vector quarkonia}), \\
-\frac{4\alpha\beta^2}{\sqrt{\pi}}(\text{pseudoscalar quarkonia}),
\end{array}\right.
\]
and such formula of the mass of the 1S quarkonium can be found in Ref [52].

### 3 Numerical results

In our numerical calculation, the parameter \(\beta\) in the wave function and the mass of the constituent quark \(m_q\) were chosen to fit experimental data. Since the only difference between the vector and pseudoscalar quarkonia that share the same energy quantum number \(n\) is the hyperfine interaction term in this model, we choose the same \(\beta\) for them. For charmonia, \(m_c\) and \(\beta_{J/\psi}(\beta_{b\psi})\) were fixed by Refs. [17, 52], and their results are in good agreement with experimental data, so we use their values of \(m_c\) and \(\beta_{J/\psi}(\beta_{b\psi})\), and we only have to fix the parameter \(\beta_{\psi}(\beta_{b\psi})\).

The parameters of the charmonia are fixed as
\[
m_c = 1.8 \text{ GeV}, \quad \beta_{J/\psi}(\beta_{b\psi}) = 0.6998 \text{ GeV},
\]
\[
\beta_{\psi}(\beta_{b\psi}) = 0.630 \text{ GeV},
\]
and our numerical results of 2S charmonia are listed in Table 1. The numerical results of 1S charmonia can be found

| \(F_{\psi\rightarrow\gamma\gamma}(0)\) | 0.0392 \pm 0.0031 | 0.0402 |
| \(f_{\psi}\) (\(\psi \rightarrow e^+e^-\)) | 0.1910 \pm 0.0057 | 0.2474 |
| \(M_{\psi}\) | 3.686093 \pm 0.000034 | 3.778 |
| \(M_{\psi}\) | 3.637 \pm 0.04 | 3.637 |
| \(F_{\psi\rightarrow\gamma\gamma}(0)\) | < 0.9006 | 0.6292 |
| \(F_{\psi\rightarrow\gamma\gamma}(0)\) | < 0.0590 | 0.0271 |

The transition form factors \(F_{\psi\rightarrow\gamma\gamma}(0)\) and \(F_{\psi\rightarrow\gamma\gamma}(0)\) are our predictions, and we see from the table that they are well below experimental upper limits.
We can also obtain the branching ratios of the two decay modes using Eqs. (16) and (17) and the total widths of \( \psi' \) and \( \eta_b' \):

\[
\text{Br}(\psi' \to \gamma \eta_b') = 3.9012 \times 10^{-4}, \quad \text{Br}(\eta_b' \to 2\gamma) = 1.0555 \times 10^{-4}.
\]

BES-III collaboration reported very recently the first measurement of the branching ratio \( \text{Br}(\psi' \to \gamma \eta_b') = (4.7 \pm 0.9_{\text{stat}} \pm 3.0_{\text{syst}}) \times 10^{-4} \) [53], and our prediction in Eq. (22) is in agreement with the preliminary data within error bars. The very recent theoretical study in Ref. [8] gives \( \Gamma(\psi' \to \gamma \eta_b') = 0.08^{+0.03}_{-0.01} \text{keV} \), and converted into the branching ratio using the total width of \( \psi' \) [16], it is \( \text{Br}(\psi' \to \gamma \eta_b') = (2.7972 \pm 1.1) \times 10^{-4} \). Other theoretical predictions for \( \text{Br}(\psi' \to \gamma \eta_b') \) fall in a range of \((0.1 \text{ to } 6.2) \times 10^{-4} \) [54].

Assuming that \( \eta_c \) and \( \eta_c' \) have equal branching fractions to \( K^0 \bar{K} \pi \), Ref. [55] obtained the experimental data \( \Gamma_\gamma(\eta_c') = 1.3 \pm 0.6 \text{keV} \), using the total width of \( \eta_c' \) [16], the branching ratio is \( \text{Br}(\eta_c' \to 2\gamma) = (0.9286 \pm 0.63) \times 10^{-4} \). The theoretical prediction in Ref. [1], converted into branching ratio, gives \( \text{Br}(\eta_c' \to 2\gamma) = 1.4286 \times 10^{-4} \). We see that our prediction in Eq. (22) is close to these data. However, the accurate experimental data for this decay mode is still not available, and only the upper limit \( \text{Br}(\eta_c' \to 2\gamma) < 5 \times 10^{-4} \) is given [16]. Future experimental measurements at BES and CLEO may provide tests for these predictions of \( \text{Br}(\psi' \to \gamma \eta_c') \) and \( \text{Br}(\eta_c' \to 2\gamma) \).

For bottomonia, the parameters are fixed as

\[
m_b = 5.1 \text{ GeV}, \quad \beta_T(\eta_b) = 1.1656 \text{ GeV}, \quad \beta_T(\eta_b) = 1.1050 \text{ GeV},
\]

and our numerical results of the 1S and 2S bottomonia are listed in Table 2. Our results give the prediction \( \Gamma(\eta_b' \to 2\gamma) = 0.1494 \text{ keV} \), compared with the prediction given in Ref. [1], \( \Gamma(\eta_b' \to 2\gamma) = 0.21 \text{ keV} \). Future experiments at LHC or by the Belle experiment on \( \eta_b' \) can not only test these predictions, but also help us learn more about this meson by providing more experimental information about it.

The small values of the experimental data for the branching ratio of the mode \( V(2S) \to \gamma P(1S) \) in Table 1 and Table 2 can be easily understood with our wave functions, as the 2S and 1S wave functions are orthogonal to each other and their overlap in Eq. (13) is suppressed. Although nonrelativistic models have provided efficient and powerful theoretical tools to handle various problems related to hadron structure, the relativistic models have been successful in many investigation of hadron structures [31,32,33,34,35,36,37,38,24,25,26,27,28,29,30]. It is thus necessary to make an estimate of the effect due to nonrelativistic to relativistic treatments [50]. For simplicity, we assess the non-relativistic to relativistic effects by letting \( k_s^2 \) of \( \sqrt{k_s^2 + m_q^2} \) in the expressions of the decay constants and form factors to be zero. For examples, after this procedure, we have \( F_{\psi' \to \eta_c',\gamma,\gamma}(0) = 0.1167 \text{ GeV} \), compared with \( F_{\psi' \to \eta_c',\gamma,\gamma}(0) = 0.0402 \text{ GeV/0.0392 GeV} \) from the relativistic treatment / experimental data, and \( f_T(T' \to e^+ e^-) = 0.2159 \text{ GeV} \) compared with \( f_T(T' \to e^+ e^-) = 0.1944 \text{ GeV/0.1657 GeV} \) from the relativistic treatment / experimental data. We see that the relativistic treatment is needed to describe the experimental data well.

4 summary

In this work, we studied the 2S quarkonia \( \psi', \eta_c', T' \) and \( \eta_b' \) in light-front quark model. Similar with the 1S harmonic oscillator wave function that was commonly used as the 1S quarkonium wave function in light-front quark model studies, we tried to use the 2S harmonic oscillator wave function as the 2S quarkonium wave function. The decay constants and transition form factors of these quarkonia are calculated. Using the QCD-motivated Hamiltonian for mesons, we also calculated masses of these quarkonia. Our numerical results of these quantities are in agreement with experimental data. Predictions of transition form factors and masses of these quarkonia are made, and these predictions can be tested by future experiments. The 1S and 2S wave functions could also be used as inputs to study other problems such as the “\( \rho \pi \) puzzle” and the suppression of heavy-quarkonia at RHIC energy.

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