Low-energy interaction of composite spin-half systems with scalar and vector fields

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(August 19, 2018)

We consider a composite spin-half particle moving in spatially-varying scalar and vector fields. The vector field is assumed to couple to a conserved charge, but no assumption is made about either the structure of the composite or its coupling to the scalar field. A general form for the piece of the spin-orbit interaction of the composite with the scalar and vector fields which is first-order in momentum transfer $Q$ and second-order in the fields is derived.

I. INTRODUCTION

In the low-energy limit, the interaction of a composite spin-half system, such as a nucleon, with external scalar and vector fields has been shown to be governed, at least in part, by general principles of Lorentz invariance [1]. Consideration of a nucleon in a spatially-uniform, scalar field provides the simplest result: the only effect is that the mass becomes a function of the strength, $S$, of the scalar field, i.e. $M \rightarrow M(S)$. If a vector interaction coupled to a conserved charge is also present, then the energy of the composite particle may be expressed as

$$E = \sqrt{M^2(S) + p^2} + V,$$

where $p$ is the three-momentum of the particle and $V$ is the strength of the vector interaction.

Naturally, Eq. (1) may be derived using analyses based on quantum field theory. Following such an approach, Refs. [2,3] considered a specific model of a spin-half composite. In that model the composite interacts with an external scalar field which provides a pure shift of the mass, i.e. $M(S) = M + S$. A vector field is also introduced, and couples to the (conserved) baryonic charge of the model. This leads to the following expansion for the energy of the composite particle,

$$E = \epsilon(p) + \frac{M}{\epsilon(p)} S + V + \frac{p^2 S^2}{2\epsilon^3(p)} + \cdots,$$

where $\epsilon(p) \equiv \sqrt{M^2 + p^2}$ is the relativistic energy when the scalar field is zero. Of course, these are the first few terms in the expansion of Eq. (1) for the case of a simple mass shift. Since specific choices for the model of compositeness and the coupling of the scalar field to the composite were made in Refs. [2,3], the derivation of Eq. (2) is less general than that of Ref. [1].

These results provide important confirmation of effects which were discovered some time ago to be crucial ingredients in successful descriptions of spin observables in the elastic scattering of protons by nuclei at intermediate energy. In a relativistic description, the nucleon-nucleon interaction is known to possess a strong scalar attraction and a strong vector repulsion [4]. This translates into optical potentials which have the same character for nucleon-nucleus scattering. For nucleon-nucleus interactions evaluated at nuclear matter density, the terms in Eq. (2) which are linear in $S$ and $V$ end up largely canceling, with the net interaction being about 50 MeV of attraction. The term that is quadratic in $S$ then becomes significant because of the comparatively large (roughly 300 MeV) scalar attraction.

Relativistic descriptions of nucleon-nucleus scattering employ the Dirac equation with scalar and vector potentials and thus include the term in Eq. (2) which is quadratic in $S$. In such an approach this term arises from $z$-graphs. Therefore analyses based upon the Schrödinger equation with relativistic kinematics omit it. This omission has been justified by arguing that the excitation of virtual $N\bar{N}$ pairs should be suppressed due to compositeness of the nucleon [11,12]. Recent works [13,14] show that the appearance of the term which is quadratic in $S$ in Eq. (2) is a consequence of Lorentz invariance. Thus, this term will not be suppressed by compositeness in the low-energy limit.

While the analysis of Ref. [1] makes no assumptions about the composite structure or the dynamics governing its interaction with the scalar field, the quantum field theory approach of Refs. [2,3] is limited to a simple model of compositeness and a scalar field which couples to the divergence of the dilatation current of the composite model. Furthermore, both analyses apply only to spatially-uniform fields. We wish to remove these limitations by performing a quantum-field-theoretic analysis of this problem which is model independent. Such an approach allows us to explore...
whether there is a model-independent prediction for the spin-orbit interaction of the composite with the scalar and vector fields.

The first-order contribution to the spin-orbit force is a consequence of Lorentz invariance and gauge invariance (see, for example, Ref. [3]). Simple arguments (see Sec. V for details) show that the first-order spin-orbit interaction for the simple mass-shift case is, to first-order in \( Q \):

\[
V_{SO}^{(1)}(p + Q/2, p - Q/2) = \left( S - V - V_N \frac{\epsilon(p) + M}{2M} \right) \frac{i}{2\epsilon(p) \epsilon(p) + M} \sigma \cdot (p \times Q),
\]

where \( \kappa \) is the anomalous magnetic moment of the composite system under consideration. The opposite sign of the vector contribution in Eqs. (2) and (3) means that if the scalar and vector potentials \( S \) and \( V \) are large and approximately equal in magnitude but opposite in sign, then the central piece of the potential will be small and the spin-orbit force large. This ability to achieve cancellation in the central force with reinforcement in the spin-orbit force is one of the key ingredients of the success of Dirac phenomenology [4].

However, Eq. (3) tells us nothing about the piece of \( V_{SO} \) which is higher-order in the fields. In the case that the scalar interaction is zero we know that there is a model-independent result (provided the external vector field couples to a conserved charge) because of the low-energy theorem for Compton scattering \([\text{Ref. [3]}] \). The question addressed in this work is whether a similar model-independent result exists if both scalar and vector fields are present.

To this end we present an analysis, based on quantum field theory, of the behavior of a spin-half composite in scalar and vector fields. The scalar interaction is arbitrary, while the vector field is assumed to couple to a conserved charge. The analysis retains terms which contribute to the spin-orbit interaction, namely those that are first-order in the spin and momentum transfer. The object is to determine the part of the spin-orbit interaction that is second-order in the strengths of the external interactions. This part comes mainly from z-graphs if the Dirac equation is used. The results extend the earlier analyses by showing that a composite system interacts at low momentum transfer in a similar fashion to a Dirac particle whose mass and anomalous magnetic moment are functions of the external scalar field.

The analysis proceeds as follows. In Sec. II we write down the composite particle Green’s function, define its couplings to scalar and vector fields, and write down the amplitude for the composite’s second-order interaction with these fields. In Sec. III we show that if one is interested only in terms to first-order in the momentum transfer then the amplitude may be evaluated with one interaction carrying the total momentum transfer, \( Q \), and the other carrying zero momentum. Section IV then derives a particularly simple relation for this amplitude using identities for the Green’s function in constant fields. This result relates the irreducible second-order interaction with one insertion carrying zero momentum to a derivative of the first-order interaction in an arbitrary scalar field. In Sec. V we write the general forms of the first-order scalar and vector vertices, correct to first-order in \( Q \), then use the result of Sec. IV to calculate the spin-orbit piece of the second-order interaction, also correct to first-order in \( Q \). Conclusions are drawn in Sec. VI.

II. THE SECOND-ORDER SCATTERING AMPLITUDE

Consider a generic model field theory which leads to a bound spin-half state with microscopic dynamics described by the Lagrangian, \( \mathcal{L}_0 \). Suppose that we construct an interpolating field for the spin-half composite, \( \psi(x) \), out of the microscopic degrees of freedom of the theory. The field \( \psi \) may be chosen to be any combination of constituent field operators with the appropriate quantum numbers. In particular, we may assume, without loss of generality, that \( \psi \) transforms as a Dirac spinor under Lorentz transformations. One example of an interpolating field with this property is the Ioffe field commonly used in QCD sum rule calculations [11]. We then examine the Green’s function,

\[
G(p) \equiv i \int d^4(x' - x) e^{ip(x' - x)} \langle 0 | T(\psi(x') \psi(x)) | 0 \rangle.
\]

The operator \( \overrightarrow{\psi} = \psi \gamma_0 \) since \( \psi \) transforms as a Dirac spinor under Lorentz transformations. If the composite has mass \( M \) then we know that this Green’s function takes the form \([2]\)

\[
G(p) = \frac{Z_2 u(p) \bar{\alpha}(p)}{p^2 - \epsilon(p) + i\eta} + \delta g(p),
\]

where \( Z_2 \) is the so-called wave function renormalization and \( \delta g \) is the piece of the Green’s function which is regular at the pole \( p_0 = \sqrt{M^2 + \mathbf{p}^2} \). The “wave function” \( u(p) \) is defined by
wavefunction renormalization, $Z$ uniform, it can change the internal structure of the composite fermion in a nontrivial way. Hence, in general, the dimensionless quantity, it follows that in the model of Refs. [2,3] it has no dependence on the scalar field whatsoever.

The vector field therefore couples to a conserved charge. If this vector field is constant, its interaction Hamiltonian commutes with the Hamiltonian describing the internal dynamics of the composite, and thus cannot produce a change in the structure of the composite; it can only shift its energy. This also implies that there is no change in the wavefunction renormalization in the presence of a uniform field.

From (6) we see that $\bar{u}(p)e^{-ipx} = \langle 0|\psi(x)|N(p)\rangle$, where $p_0 = \sqrt{M^2 + p^2}$, and $|N(p)\rangle$ is a Fock space state representing a composite with momentum $p$ and mass $M$.

Now let us couple scalar and vector fields to this composite. In this work we do not specify how the scalar field, $\sigma$, couples to the microscopic constituents. By contrast, the vector field, $\omega$, is introduced by minimal substitution, i.e. by making the replacement

$$i\partial_0 \rightarrow i\partial_0 - \omega(x).$$

The vector field therefore couples to a conserved charge. If this vector field is constant, its interaction Hamiltonian commutes with the Hamiltonian describing the internal dynamics of the composite, and thus cannot produce a change in the structure of the composite; it can only shift its energy. This also implies that there is no change in the wavefunction renormalization in the presence of a uniform $\omega$ field.

The total Lagrangian, $\mathcal{L}$, of this theory may be expressed as,

$$\mathcal{L}(x) = \mathcal{L}_0(x) - \rho_0(x)\omega(x) - \rho^{(1)}(x)\sigma(x) - \rho^{(2)}(x)\sigma^2(x) - \ldots,$$

where $\rho_0(x)$ is the vector charge density and $\rho(n)(x)$ is the scalar density through which the $n$th power of the scalar field couples. In the analysis of Refs. [3], the $\sigma$ field (which in that case is uniform) couples to the divergence of the dilatation current of the theory, and therefore $Z_2$ can only depend on powers of the scalar field. Since $Z_2$ is a dimensionless quantity, it follows that in the model of Refs. [3] it has no dependence on the scalar field whatsoever. However, general scalar couplings do not lead to this simple scaling behavior and, even if the scalar field is spatially uniform, it can change the internal structure of the composite fermion in a nontrivial way. Hence, in general, the wavefunction renormalization, $Z_2$, will be a function of the scalar field.

Having defined the type of model and given the form that the Green’s function takes in the absence of any scalar and vector fields, we next write down the Green’s functions and amplitudes for the interactions of the composite with these external fields. In general, the Green’s function for the composite moving in these fields may be written as a functional integral:

$$G(x', x; \sigma, \omega) = \int [d\zeta] \psi(x')\overline{\psi}(x) \exp(iS[\zeta, \sigma, \omega]),$$

where it is understood that the arguments after the semicolon on the left-hand side are functions, while those before it are variables. Here $\zeta$ represents all fields present in the vacuum composite Lagrangian, $\mathcal{L}_0$, and $S[\zeta, \sigma, \omega]$ is the action corresponding to the Lagrangian defined in Eq. (6). In order to obtain the interaction of the composite with the fields at a given order, the Green’s function (9) must be expanded in powers of the external fields $\omega$ and $\sigma$.

We now assume that external scalar and vector fields are present with the same spatial distribution function, $h(r)$. This assumption is made for simplicity, and is not essential to our argument. The function $h(r)$ is normalized so that $h(0) = 1$. The scalar and vector distributions are taken to have the forms,

$$\sigma(r) = S h(r); \quad \omega(r) = V h(r),$$

where $S$ and $V$ are strength parameters for the scalar and vector interactions. The first-order amplitude for the interaction of the composite with these fields may be written as $h(Q)A^{(1)}(p', p, Q)$, where $h(Q)$ is the Fourier transform of $h(r)$, and the function $A^{(1)}(p', p, Q)$ is defined via

$$G(p')A^{(1)}(p', p, Q)G(p) \equiv \int d^4x' d^4x d^4y_1 e^{ip'x' - ipx - Qy_1} \int D_{y_1} G(x', x; \sigma, \omega)_{\sigma=\omega=0},$$

where

$$D_y = S \frac{\delta}{\delta \sigma(y)} + V \frac{\delta}{\delta \omega(y)}.$$

For convenience, the strengths of the scalar and vector interactions are included in the vertex function. This allows us to define a first-order vertex function, $\Lambda$, which is a combination of scalar and vector vertex functions:
\[ A^{(1)}(p', p, Q) \equiv (2\pi)^4 \delta^{(4)}(p' - p - Q) \Lambda(p', p). \]  

The first-order potential is then found by multiplying \( \Lambda(p', p) \), with \( p \) and \( p' \) both on-shell, by \( h(Q) \). If \( h(r) \) is constant this implies that \( \Lambda(p', p) \) will be needed only at \( p' = p \), i.e. \( Q = 0 \). From Eqs. (11) and (13) we see that the \( Q = 0 \) vertex function is obtained from

\[ G(p)\Lambda(p, p)G(p) = \int d^4(x' - x)e^{ip(x' - x)} \int d^4y_1 D_{y_1}G(x', x; \sigma, \omega)|_{\sigma = \omega = 0}. \]  

In general, if \( F \) is a functional of the function \( g \), then

\[ \left. \int d^4y' \frac{\delta}{\delta g(y')} F[g] \right|_{g = 0} = \left. \frac{\partial F(g)}{\partial g} \right|_{g = 0}, \]  

where on the right-hand side, \( F \) is to be thought of as a function of the constant parameter \( g \). Thus the \( Q = 0 \) vertex function, corresponding to spatially-uniform external fields, is given by

\[ G(p)\Lambda(p, p)G(p) = D G_{\sigma,\omega}(p)|_{\sigma = \omega = 0}, \]

\[ D = S \frac{\delta}{\delta \sigma} + V \frac{\delta}{\delta \omega}, \]

where \( G^{\sigma,\omega} \) is calculated via Eq. (8), but with constant \( \sigma \) and \( \omega \) fields. Moreover, because the \( \omega \) field is introduced by minimal substitution Eqs. (6) and (7) may be rewritten as

\[ G(p)\Lambda(p, p)G(p) = D G_{\sigma}(p)|_{\sigma = 0}, \]

\[ D = S \frac{\delta}{\delta \sigma} - V \frac{\delta}{\delta p_0}, \]

and \( G_{\sigma}(p) \) is the Green’s function calculated at arbitrary constant scalar field \( \sigma \). This result will be used in Sec. IV as the basis for a straightforward calculation of the first-order vertex function.

In order to calculate the second-order scattering of the composite in scalar and vector fields we define the amplitude \( A^{(2)} \) via

\[ G(p')A^{(2)}(p', p, q_2, q_1)G(p) = \frac{1}{2} \int d^4x' d^4x d^4y_2 d^4y_1 e^{i(p'x' - px - q_2y_2 - q_1y_1)} D_{y_2}D_{y_1}G(x', x; \sigma, \omega)|_{\sigma = \omega = 0}. \]  

If \( A^{(1)}(p', p, Q; \sigma, \omega) \) is defined exactly as in Eq. (11), but with all functions retaining their dependence on the fields \( \sigma \) and \( \omega \), i.e.,

\[ G(p'; \sigma, \omega)A^{(1)}(p', p, Q; \sigma, \omega)G(p; \sigma, \omega) = \int d^4x' d^4x d^4y_1 e^{i(p'x' - px - Qy_1)} D_{y_1}G(x', x; \sigma, \omega), \]

then Eq. (20) may be rewritten as

\[ G(p')A^{(2)}(p', p, q_2, q_1)G(p) = \frac{1}{2} \int d^4y_2 e^{-iq_2y_2} D_{y_2}G(p'; \sigma, \omega)A^{(1)}(p', p, q_1; \sigma, \omega)G(p; \sigma, \omega)|_{\sigma = \omega = 0}. \]  

Next, observe that a contact term, corresponding to interaction within the vertex function \( A^{(1)} \) and denoted by \( C \), may be defined via

\[ \int d^4y_2 e^{-iq_2y_2} D_{y_2}A^{(1)}(p', p, q_1; \sigma, \omega)|_{\sigma = \omega = 0} = (2\pi)^4 \delta^{(4)}(p' - p - q_1 - q_2)C(p + q_1 + q_2, q_1, p). \]  

Using Eqs. (23), (11), and (13) in Eq. (20), and then taking the limit as the initial and final state poles are approached, yields the following physical amplitude for the second-order interaction of the composite with the fields at total three-momentum \( Q \):

\[ \tilde{u}(p + Q/2, A(p + Q/2, Q/2 - q, Q/2 + q, p - Q/2)u(p - Q/2) \equiv \lim_{p_0 + Q_0/2 \rightarrow c(p + Q/2)} \lim_{p_0 - Q_0/2 \rightarrow c(p - Q/2)} \left\{ \begin{array}{l} \frac{1}{2} Z_2 \tilde{u}(p + Q/2)A(p + Q/2, p + q)G(p + q)A(p + q, p - Q/2)u(p - Q/2) \\ + \frac{1}{2} Z_2 \tilde{u}(p + Q/2)A(p + Q/2, p - q)G(p - q)A(p - q, p - Q/2)u(p - Q/2) \\ + \frac{1}{2} Z_2 \tilde{u}(p + Q/2)C(p + Q/2, Q/2 - q, Q/2 + q, p - Q/2)u(p - Q/2) \end{array} \right\}. \]
Here we have made use of the overall delta function and then defined a new amplitude $A$ which does not include it. Note that the initial and final momenta both depend on $Q$, and the bars on the initial and final arguments of $A$ indicate that both external legs are on-shell, i.e. $\not{k} = (\epsilon(k), k)$. This determines $Q_0$, and implies that $p^2 = M^2$ to first-order in $Q$. Finally, in order to generate a potential, the matrix element \[24\] must be integrated against the field distribution, $h$:

$$V^{(2)}(p + Q/2, p - Q/2) = \int \frac{d^3q}{(2\pi)^3} h(Q/2 + q)h(Q/2 - q)Z_2\bar{u}(p + Q/2)A(p + Q/2, Q/2 - q, Q/2 + q, p - Q/2)u(p - Q/2). \tag{25}$$

### III. LOW-ENERGY LIMIT OF THE SECOND-ORDER AMPLITUDE

So far no approximations have been made, and the matrix element \[24\] could (in principle) be obtained by evaluating the relevant Feynman diagrams for a given composite model Lagrangian $\mathcal{L}_0$, as was done in Refs. \[2,3\] for the case $Q = 0$. However, a result which is independent of the details of both the sub-structure of the composite and its coupling to the scalar field may be obtained by taking the low-energy limit of this expression. In particular, in this section we show that in the limit of slowly-varying external fields we may restrict our analysis to the case where one of the external field interactions carries all the momentum and the other occurs at zero momentum.

First, note that the amplitude $A$ corresponds to a sum of direct and crossed Feynman diagrams and therefore is symmetric with respect to interchange of the momenta of the external fields. Defining $p_f = p + q_1 + q_2$ and $p_i = p$, this implies that

$$A(p_f, q_1, q_2, p_i) = A(p_f, q_2, q_1, p_i), \tag{26}$$

and it follows that

$$\frac{\partial A(p_f, q_1, q_2, p_i)}{\partial q_1^\mu} \bigg|_{q_1 = q_2} = \frac{\partial A(p_f, q_1, q_2, p_i)}{\partial q_2^\mu} \bigg|_{q_1 = q_2}. \tag{27}$$

Expanding the amplitude $A(p_f, Q/2 - q, Q/2 + q, p_i)$ in powers of $Q/2 + q$ and $Q/2 - q$ and using Eq. \[27\] produces

$$A(p_f, Q/2 - q, Q/2 + q, p_i) = A(p_f, 0, 0, p_i) + Q^\mu \frac{\partial A(p_f, q_1, 0, p_i)}{\partial q_1^\mu} \bigg|_{q_1 = 0} + O[(Q/2 + q)^2] + O[(Q/2 - q)^2]. \tag{28}$$

Hence, if we are interested only in terms of order $Q$, we may write

$$A(p_f, Q/2 - q, Q/2 + q, p_i) = A(p_f, Q, 0, p_i) + O[(Q/2 + q)^2] + O[(Q/2 - q)^2]$$

$$= A(p_f, 0, 0, p_i) + O[(Q/2 + q)^2] + O[(Q/2 - q)^2], \tag{29}$$

where the last equality follows from Eq. \[23\].

When this equation is inserted into the definition \[25\], the terms of second and higher order in momentum transfer generate second and higher derivatives of the field distribution, $h(r)$. Provided that $h$ is slowly varying in space, these may be neglected and we obtain (correct to first-order in $Q$):

$$V^{(2)}(p_f, p_i) = \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} h(Q/2 + q)h(Q/2 - q)Z_2\bar{u}(p_f)A(p_f, Q, 0, p_i)u(p_i). \tag{30}$$

### IV. A GENERAL RESULT FOR THE AMPLITUDE WITH ONE ZERO-MOMENTUM INSERTION

Our next task is to evaluate the matrix element $\bar{u}(p_f)A(p_f, Q, 0, p_i)u(p_i)$. From \[24\],

$$A(p_f, Q, 0, p_i) = \lim_{p_i^0 \rightarrow \epsilon(p_i)} \frac{1}{2} Z_2\Lambda(p_f, p_i)G(p_i)\Lambda(p_i, p_i)$$

$$+ \lim_{p_j^0 \rightarrow \epsilon(p_j)} \frac{1}{2} Z_2\Lambda(p_f, p_j)G(p_j)\Lambda(p_j, p_i) + \frac{1}{2} Z_2C(p_f, Q, 0, p_i), \tag{31}$$
where a bar over a four-vector indicates that it is constrained to its mass shell. The limits here are equivalent
to taking the limit \( q \to -iQ \) in Eq. \((24)\). We wish to relate the vertices which appear in this equation to zero
momentum-transfer vertices, since we know that these may be obtained using Eq. \((18)\).

Indeed, before we begin our effort to calculate \( \bar{u}(p_f)A(p_f, Q, 0, p_i)u(p_i) \), we derive two sets of identities from
Eq. \((18)\). First, note that the Green’s function \( G_\sigma \) may be written as

\[
G_\sigma(p) = \frac{Z_2(\sigma)u_\sigma(p)\bar{u}_\sigma(p)}{p^0 - \epsilon_\sigma(p)} + \delta g_\sigma(p),
\]

where the wave function, wave function renormalization, and the regular part of the Green’s function have all become
functions of the constant scalar field \( \sigma \), while \( \epsilon_\sigma(p) = \sqrt{M(\sigma)^2 + p^2} \). Next, we insert Eq. \((32)\) into Eq. \((18)\) and
expand both sides of the result in powers of \( p_0 - \epsilon(p) \). The terms with a double pole at \( p_0 = \epsilon(p) \) yield the identity

\[
Z_2\bar{u}(p)\Lambda(\bar{p}, \bar{p})u(p) = S \left. \frac{d\epsilon_\sigma(p)}{d\sigma} \right|_{\sigma=0} - V.
\]

This agrees with the on-shell vertices obtained by Birse \([1]\). Meanwhile, the terms with a single pole at \( p_0 = \epsilon(p) \) give

\[
Z_2\bar{u}(p) \left| \frac{d\Lambda(p, p)}{dp_0} \right|_{p_0 = \epsilon(p)} u(p) + u^\dagger(p)\delta g(\bar{p})\Lambda(\bar{p}, \bar{p})u(p) + \bar{u}(p)\Lambda(\bar{p}, \bar{p})\delta g(\bar{p})\gamma_0 u(p) = \frac{DZ_2(\sigma)}{Z_2} \bigg|_{\sigma=0}
\]

\[
= S \left. \frac{dZ_2(\sigma)}{d\sigma} \right|_{\sigma=0} \gamma_0 u(p),
\]

where the last step follows since, as observed above, \( Z_2 \) does not depend upon the constant vector field. The identities
\((33)\) and \((34)\) will be crucial in our evaluation of \( \bar{u}(p_f)A(p_f, Q, 0, p_i)u(p_i) \).

We are now ready to perform this evaluation. Consider the quantity

\[
X(p_f, p_i) = \lim_{p' \to \epsilon(p_f)} \lim_{p_i' \to \epsilon(p_i)} u^\dagger(p_f) \frac{1}{2} Z_2(p_f - \epsilon(p_f))G(p_f) \times \left[ \Lambda(p_f, p_i)G(p_i)\Lambda(p_i, p_i) + \Lambda(p_f, p_f)G(p_f)\Lambda(p_f, p_i) + C(p_f, Q, 0, p_i) \right]
\]

\[
\times G(p_i)(p_i^0 - \epsilon(p_i))\gamma_0 u(p_i).
\]

This form is motivated by LSZ reduction \([13]\), but it is not exactly equal to the Feynman diagram expression \((31)\)
because in two of the three terms here there is a zero-momentum-transfer vertex. The consequent presence of two
poles at the same point in \( p_0 \) means that Eq. \((33)\) yields a different result from Eq. \((31)\). In order to develop the
relation of the vertices which appear in Eq. \((33)\) to the zero-momentum-transfer vertices that appear in \( X \), it is
necessary to expand as follows,

\[
\Lambda(\bar{p}, p) = \Lambda(p, p) + \left. \frac{\partial \Lambda(p', \bar{p})}{\partial p_0'} \right|_{p_0' = p} (\epsilon(p) - p_0) + \cdots.
\]

Using Eq. \((36)\) in order to expand the quantity \((p_0 - \epsilon(p))G(p)\Lambda(p, p)G(p)\) in powers of \((p_0 - \epsilon(p))\) leads to

\[
(p_0 - \epsilon(p))G(p)\Lambda(p, p)G(p) = Z_2u(p)\bar{u}(p)\Lambda(\bar{p}, p)G(p) + Z_2u(p)\bar{u}(p) \left. \frac{\partial \Lambda(p', \bar{p})}{\partial p_0'} \right|_{p_0' = \bar{p}} Z_2u(p)\bar{u}(p) + \delta g(\bar{p})\Lambda(\bar{p}, \bar{p})Z_2u(p)\bar{u}(p) + O(p_0 - \epsilon(p)).
\]

From this result and the analogous expression for \( G(p)\Lambda(p, p)G(p)(p_0 - \epsilon(p)) \) we find

\[
X(p_f, p_i) = \bar{u}(p_f)A(\bar{p}, Q, 0, p_i)u(p_i) + \frac{1}{2} Z_2\bar{u}(p_f)\Lambda(\bar{p}, \bar{p})u(p_i)\left( f(p_f) + \bar{f}(p_i) \right).
\]

where

\[
f(p) = \bar{u}(p) \left. \frac{\partial \Lambda(p', \bar{p})}{\partial p_0'} \right|_{p_0' = \bar{p}} Z_2u(p) + u^\dagger(p)\delta g(\bar{p})\Lambda(\bar{p}, \bar{p})u(p)\]

and

\[ \tilde{f}(p) = \bar{u}(p) \left. \frac{\partial \Lambda(p, \bar{p}')}{\partial p'_0} \right|_{p' = \bar{p}} Z_2 u(p) + \bar{u}(p) \Lambda(p, \bar{p}) \delta g(\bar{p}) \gamma_0 u(p). \]  

(40)

The result (44) may now be rewritten as

\[ f(p) + \tilde{f}(p) = \frac{D Z_2(\sigma)}{Z_2} \left|_{\sigma=0} \right. . \]  

(41)

It is sufficient for the factor \( f + \tilde{f} \) to be evaluated at \( Q = 0 \) in order for the scalar part of \( X(p_f, p_i) \) to be correct to zeroth order in \( Q \) and the spin-dependent part to be correct to first-order in \( Q \). Thus, if we are only concerned with the spin-independent forward scattering and the first-order spin-orbit interaction:

\[ \bar{u}(p_f) A(p_f, 0, \bar{p}) u(p_i) = X(p_f, p_i) + \frac{1}{2} Z_2 D \left( \frac{1}{Z_2(\sigma)} \right) \left|_{\sigma=0} \right. \frac{Z_2 \bar{u}(p_f) \Lambda(p_f, \bar{p}) u(p_i)}{2} \]  

(42)

Finally, note that Eqs. (43) and (45) imply that the contact term where one leg carries zero momentum transfer may be calculated via

\[ C(p + Q/2, 0, p - Q/2) = D \Lambda_\sigma(p + Q/2, p - Q/2) \left|_{\sigma=0} \right. . \]  

(43)

Here \( \Lambda_\sigma \) is defined to be the vertex calculated in the presence of a constant \( \sigma \) field. Using Eqs. (32), (33), (38), and (43) then produces our central result:

\[ \bar{u}(p_f) A(p_f, 0, \bar{p}) u(p_i) = \lim_{p'_j \to \epsilon(p_f)} \lim_{p'_i \to \epsilon(p_i)} u^i(p_f) \frac{1}{2} (p_j^0 - \epsilon(p_f)) D |G_\sigma(p_f) \Lambda_\sigma(p_f, p_i) G_\sigma(p_i)| Z_2(\sigma) \left|_{\sigma=0} \right. \frac{(p_i^0 - \epsilon(p_i)) \gamma_0 u(p_i)}{2}. \]  

(44)

where \( D \) acting on \( G_\sigma \) generates the zero-momentum vertices in (44), and \( D \) acting on \( \Lambda_\sigma \) produces the contact term, as shown in Eq. (43). The division by \( Z_2(\sigma) \) arises because of the effect the presence of zero momentum-transfer vertices has on the taking of the on-shell limits. Physically, it is necessary in order to ensure the correct normalization of the asymptotic states—a delicate issue in a calculation involving a constant external field which alters \( Z_2 \).

We now deal separately with the scalar and vector pieces of the \( D \) operator in Eq. (44). The vector piece, which arises from the piece of \( D \) which is proportional to \( V \), may be simplified using the decomposition (32) and Eq. (33), into the form:

\[ V \left\{ \frac{1}{2} \frac{1}{p_j^0 - \epsilon(p_f)} \bar{u}(p_f) Z_2 \Lambda(p_f, p_i) u(p_i) + \frac{1}{2} \bar{u}(p_f) Z_2 \Lambda(p_f, p_i) u(p_i) \frac{1}{p_i^0 - \epsilon(p_i)} - \frac{1}{2} \bar{u}(p_f) Z_2 \frac{d \Lambda(p_f, p_i)}{dp_i} u(p_i) \right\}. \]  

(45)

Note that in deriving this expression the \( g \) pieces of the decomposition (32) do not contribute in the limit \( p_i \to \bar{p} \), \( p_j \to p_f \). Expanding out the first two terms in powers of \( p_j^0 - \epsilon(p_f) \) and \( p_i^0 - \epsilon(p_i) \), respectively, taking the limit as the external legs are put on shell, and using Eq. (33) allows us to rewrite (45) as

\[ \frac{1}{2} \frac{1}{p_j^0 - \epsilon(p_f)} \bar{u}(p_f) \frac{1}{p_j^0 - \epsilon(p_f)} V^{(1)}(p_f, p_i) + \frac{1}{2} \frac{1}{p_i^0 - \epsilon(p_i)} \bar{u}(p_f) \frac{1}{p_i^0 - \epsilon(p_i)} V^{(1)}(p_f, p_i), \]  

(46)

where

\[ V^{(1)}(p', p) = Z_2 \bar{u}(p') \Lambda(p, \bar{p}) u(p), \]  

(47)

and \( V^{(1)}_V \) is the piece of \( V^{(1)} \) which is proportional to \( V \).

As for the scalar piece of the second-order interaction \( (i.e. \) the result generated by the action of the \( S_2/\partial \sigma \) in Eq. (44)), inserting the expression (32) in Eq. (44) and then expanding about the on-shell points, taking the appropriate limits, and using Eq. (33) and the identity

\[ u^i(p) \frac{du_\sigma(p)}{d\sigma} \left|_{\sigma=0} \right. = 0, \]  

(48)
leads to

$$\frac{1}{2} V_S^{(1)}(p_f, p_f) \frac{1}{p_f^0 - \epsilon(p_f)} V^{(1)}(p_f, p_i) + \frac{1}{2} V_S^{(1)}(p_f, p_i) \frac{1}{p_i^0 - \epsilon(p_i)} V^{(1)}(p_i, p_i)$$

$$+ \frac{1}{2} S \frac{d}{d\sigma} [\tilde{u}_\sigma(p_f) Z_2(\sigma) \Lambda_\sigma(p_f, p_i)] p_0 = \epsilon_\sigma(p) u_\sigma(p_i)]_{\sigma=0}.$$  \ (49)

where $V_S^{(1)}$ is the piece of $V^{(1)}$ which is proportional to $S$.

Observe that the sum of Eqs. (32) and (39) contains the iterate of the on-shell first-order interaction, $V^{(1)}$. In Ref. [3] it was shown that if amplitudes such as those given by (28) were to be used as a potential in a wave equation then the iterated on-shell first-order interaction had to be subtracted. Once this divergent piece of the interaction is removed and Eq. (30) invoked we have the following general result for the potential:

$$\mathcal{V}^{(2)}(p + Q/2, p - Q/2) =$$

$$\frac{1}{2} \int \frac{d^3q}{(2\pi)^3} h(Q/2 + q) h(Q/2 - q) S \frac{d}{d\sigma} [\tilde{u}_\sigma(p + Q/2) Z_2(\sigma) \Lambda_\sigma(p + Q/2, p - Q/2)] p_0 = \epsilon_\sigma(p) u_\sigma(p - Q/2)]_{\sigma=0}.$$  \ (50)

Here the value of $Q_0$ is understood to be $\epsilon(p + Q/2) - \epsilon(p - Q/2)$. The spin-independent part of (50) is correct to zeroth order in $Q$ and the spin-dependent part is correct to first-order in $Q$. We observe that at least one interaction here must be with the scalar field because the vector-vector piece of the irreducible potential is zero.

The result (50) is extremely general. It relies only on the assumption that the vector field couples to a conserved current. No assumptions about the structure of the composite or its couplings to the scalar field have been made.

Note that in the limit of constant fields ($h(Q) = \delta^{(3)}(Q)$), we may use Eq. (33) in Eq. (50) and so reproduce the result of Birse [1]:

$$\mathcal{V}^{(2)}(p + Q/2, p - Q/2) = \delta^{(3)}(Q) \frac{S^2}{2} \frac{d^2\epsilon_\sigma(p)}{d\sigma^2} \bigg|_{\sigma=0}$$

$$= \left\{ \frac{S^2}{2} \frac{d^2M}{d\sigma^2} \epsilon(p) \sigma + \frac{S^2 p^2}{2c^2(p)} \left( \frac{dM}{d\sigma} \right)^2 \right\} \delta^{(3)}(Q).$$  \ (51)

Hence Eq. (50) encompasses the result obtained in the model of Refs. [2,3] when $M$ is a linear function of the scalar interaction. More generally, the forward scattering amplitude is found to agree with that predicted by the Dirac equation when the mass is a function of the scalar interaction, $M = M(\sigma(r))$. The spin-independent interaction at second order has two components; one involving the second derivative of $M(\sigma)$, which is a quantity that depends on the internal structure of the composite system, and the other involving the square of the first-order interaction, $\frac{dM}{d\sigma}$, times a model-independent kinematical factor.

In the analysis of Refs. [2,3], contributions from off-shell vertices, off-shell propagation of the intermediate state and the contact term, all of which must be considered for a composite particle, cancel. Consequently, the term proportional to $p^2$, which in that model arises from composite p-graphs, is the complete result for $\mathcal{V}^{(2)}$ at zero momentum transfer. In the approach of this paper Eq. (50) must be applied in order to calculate $\mathcal{V}^{(2)}$ in the limit $Q \to 0$. When this is done a number of terms are generated. The derivative acting on $\Lambda_\sigma$ generates the contact interaction. Off-shell pieces of the vertices arise from $D$’s action on both the constraint $p_0 = \epsilon_\sigma(p)$ and $Z_2(\sigma)$. Furthermore, Eq. (34) shows that $DZ_2(\sigma)$ also generates contributions from off-shell intermediate-state propagation. Consequently, all the pieces that cancel in the analysis of Refs. [2,3] are generated in the more general result (50). However, in contrast to Refs. [2,3], here we see that they add to give the first term in Eq. (51). In the specific model of Refs. [2,3] this term was identically zero. Finally, the second term in Eq. (51) may be shown to arise from the parts of Eq. (50) involving derivatives of the wave function $u_\sigma(p)$ with respect to $\sigma$. Such a derivative gives a term involving the negative-energy wave function $u(-p)$, i.e.,

$$\frac{du_\sigma(p)}{d\sigma} \bigg|_{\sigma=0} = \frac{\nu(-p)\tilde{u}(-p)}{2\epsilon(p)} \frac{dM}{d\sigma} u(p).$$  \ (52)

V. CALCULATION OF THE SPIN-ORBIT INTERACTION

In this section we calculate the actual value of the irreducible piece of the second-order spin-orbit potential, $\mathcal{V}^{(2)}(p + Q/2, p - Q/2)$. In order to use Eq. (50), the general form of the first-order vertex $\Lambda_\sigma(p + Q/2, p - Q/2)$ must be
written down. In the scalar case, up to first-order in $Q$ the most general function $\Lambda_S$ which is consistent with Lorentz invariance, is

$$
\Lambda_S(p + Q/2, p - Q/2) = S \{ a_S + b_S(p^2) \not p + c_S(\not q) [\not p, \not q] + d_S(p^2) p \cdot Q \}.
$$

(53)

However, as discussed in Sec. 4, $u(p)$ is proportional to a Dirac spinor corresponding to mass $M$. Thus

$$
\bar{u}(p + Q/2) p \cdot Q u(p - Q/2) = 0
$$

(54)

and

$$
\bar{u}(p + Q/2) [\not p, \not Q] u(p - Q/2) = O(Q^2).
$$

(55)

Therefore, to first-order in $Q$ we have

$$
V^{(1)}_S(p + Q/2, p - Q/2) = \frac{dM}{dQ} \frac{i}{2\epsilon(p)} \frac{1}{\epsilon(p) + M} \not \sigma \cdot (p \times Q).
$$

(57)

Furthermore, the most general form for the vector vertex, consistent with the use of minimal substitution as in (3), is:

$$
V^{(1)}_V(p + Q/2, p - Q/2) = \bar{u}(p + Q/2) V[F_1(Q^2)\gamma_0 + F_2(Q^2)\frac{i}{2M} \not \sigma \not q] u(p - Q/2),
$$

(58)

with $F_1(0) = 1$ and $F_2(0) = \kappa$, the anomalous magnetic moment. Thus, to first-order in $Q$ the most general form of the total first-order spin-orbit potential is

$$
V^{(1)}_{SO}(p + Q/2, p - Q/2) = \left( \frac{dM}{dQ} S - V \frac{\epsilon(p) + M}{2M} \right) \frac{i}{2\epsilon(p)} \frac{1}{\epsilon(p) + M} \not \sigma \cdot (p \times Q).
$$

(59)

In the presence of a scalar field the mass $M$, and hence $\epsilon$, as well as the anomalous magnetic moment $\kappa$, all become dependent on $\sigma$. So, from (58) and (59) we derive:

$$
\nabla^{(2)}_{SO}(p, Q) = \frac{1}{2} \int d^3 q \left[ h(Q/2 + q)h(Q/2 - q) \right] \times
$$

$$
\left[ -(S\frac{dM}{d\sigma} - V)S \frac{dM}{d\sigma} \frac{1}{2\epsilon^2(p)} + S^2 \frac{d^2 M}{d\sigma^2} \frac{1}{2\epsilon(p)} + \frac{1}{2\epsilon(p)} + SV \frac{dM}{d\sigma} \kappa \frac{\epsilon^2(p) + M^2}{4\epsilon^2(p)M^2} - SV \frac{d\kappa}{d\sigma} \frac{1}{4\epsilon(p)M} \right] \not \sigma \cdot (p \times Q),
$$

where all quantities are understood to be evaluated at $\sigma = 0$.

VI. CONCLUSION

As claimed in the Introduction, both the spin-independent and spin-dependent second-order interactions agree with those obtained from the Dirac equation at low momentum transfer, provided that the scalar potential is introduced to the Dirac equation through a replacement of the mass by $M(\sigma(r))$, and the anomalous magnetic moment also is made a function of the scalar potential, $\kappa(\sigma(r))$. This result must hold for any composite-particle model and any scalar interaction. The vector interaction is limited to one which couples to a conserved charge of the composite system. The choice of the interpolating field made in Eq. (14) is, apart from its transformation properties, entirely arbitrary and should have no effect on the results because amplitudes are examined only at the composite-particle pole. This paper suggests that when the compositeness of the nucleon is considered, two differences from the practices followed in Dirac phenomenology for nucleon-nucleus scattering appear. First, the introduction of the scalar interaction though a function $M(\sigma(r))$ implies that nonlinear terms such as $\frac{1}{2} \frac{d^2 M}{d\sigma^2} \kappa$ arise. Such terms depend upon the internal structure of the nucleon and are model dependent. Similar nonlinear terms are implied in the interaction between two free nucleons. However, introducing such terms into the Dirac phenomenology is not entirely straightforward. Since the impulse approximation is used to construct the Dirac potentials from the empirically determined $NN$ interaction, the contribution this nonlinear effect makes to the two-body interaction is already included in Dirac phenomenology calculations. However, the nonlinear terms also imply three-body and higher interactions and these are not usually included. Second, while the anomalous magnetic moment typically is included in Dirac phenomenology, the dependence of $\kappa$ on $\sigma$ is model dependent, and is generally omitted.
ACKNOWLEDGMENTS

We thank J. A. McGovern for useful conversations. D. R. P. thanks J. A. Tjon for challenging him to think further about the meaning of Eq. [14]. We are grateful to the U.S. Department of Energy for its support under contract no. DE-FG02-93ER-40762. M. C. B. acknowledges support from the EPSRC.

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