Event-based control and scheduling of a platoon of vehicles in VANETs

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ABSTRACT This paper investigates the vehicular platoon control problem subject to variable communication delays, packets disorder, access constraints and resource constraints in vehicular and hoc networks (VANETs). Using a novel representation of the network delays as an uncertain variable belonging to the different bounded intervals, the discrete-time variable sampling interval platoon model is established with communication constraints. An event-based control and scheduling (EBCS) codesign strategy that can robustly stabilize the platoon system is given. Based on this model and a guaranteed performance cost function, the aforementioned problem is formulated as an LMI optimization problem, which can guarantee the Global Uniform Practical Stability (GUPS). A numerous simulation and experiments with laboratory scale Arduino cars show the efficiency and practicability of the proposed methods.

INDEX TERMS Platoon control system, variable communication delays, packets disorder, access constraints, event-based control

I. INTRODUCTION
The past decade has witnessed a considerable increase of the number of cars in many metropolises, especially in china, while causing huge traffic block and air contamination, the increasing traffic accidents will also bring economic losses and casualties. An effective solution to the above problem is to increase road capacity by making cars in the same lane to run in a string (called vehicle platoon) with a very small spacing. With the quick development and deployment of unmanned vehicles and vehicular ad hoc networks, autonomous cooperative cruise control of vehicles via VANETs has become an important research topic in the field of intelligent vehicle highway systems (IVHSs) or automated highway vehicle systems (AHVS) [1-3]. Zhai [4] has proposed a cooperative optimal power split method for a group of intelligent electric vehicles travelling on a highway with varying slopes. The use of VANETs is believed to play an important role, as it involves vehicle-to-vehicle (V2V) [5,6] and/or vehicle-to-infrastructure (V2I) coordinated communications into vehicular cooperative control systems as an intrinsic component that is very flexible and efficient. As a special type of wireless communication network, VANETs are faced with several challenges. For instance, due to fast moving of vehicles, the connection time in the VANETs is usually very short. Also, the quick varying environment makes the wireless channels in the VANETs dynamic and noisy, which may result in unreliable transmissions with delay and packet disordering. Another important issue is the capacity limitation of VANETs, which may become a serious problem when there are lots of cars in a road segment (e.g., in traffic jam areas in rush hours) awaiting for network access. When the cars are large in number, they cannot be accommodated simultaneously in the VANETs for information communications, which is known as network access constraint. One cannot achieve satisfactory cooperative cruise control without effective ways to coordinate the communications for large number of cars.

There are a few research have shown that vehicular platoon control subject to communication limitations in the
the literature. The impact of the communication delay on platoon control and string stability was investigated in [7-9]. Guo [10,11] considered the influence of time-varying transmission delays on platoon control and suggested a guaranteed cost control strategy. In [12], the authors proposed an ecological cooperative adaptive cruise control (Eco-CACC) strategy for a heterogeneous platoon of heavy-duty vehicles with time delays and improved the fuel economy of heterogeneous platoons. In [13], a cooperative adaptive cruise control (CACC) method was presented in a networked control system framework to deal with the influence of time-varying communication delays. For the issue of medium access constraint or network resource constraint, a control and scheduling co-design method for vehicular platoons was proposed in Guo [14], which can effectively resolve network access constraint by scheduling some cars to await while a number of selected cars are accessing the network to exchange information. Zhang [15] proposed a centralized vehicle networks scheduling protocol based on TDMA. In [16], authors proposed a switched control strategy of heterogeneous vehicle platoon for multiple objectives with nonlinear dynamics and unidirectional information communication topologies.

It is worth noting that, research in VANETs-based vehicular cooperative control is still in a primary stage. Attentions have only been paid to fundamental issues like transmission delays, packet dropouts and medium access constraint. A systematic design method for vehicular platoon control that can deal with these communication challenges more effectively in a common framework is far more significant. One important shortcoming in the existing results is that the issue of packet disordering is ignored in the communication scheduling method, because it is rather difficult to deal with packet disordering in the time-based scheduling strategy. In [17], authors proposed an event-based control and scheduling co-design strategy for platoon system with communication constraints. In this paper, we want to investigate and research the feasibility of performing communication allocation for cooperative vehicular control by using an event-based strategy rather than the time-based scheduling method.

The aim of this paper is to present an event-based control and scheduling method for vehicular cooperative control systems, which take into account the joint effect of time-varying delay, packet disordering and medium access constraint in a same framework. We developed event-based control and scheduling co-design method can robustly stabilize each of the vehicles and achieve practical vehicular platoon stability with guaranteed performance. Our contributions are different from previous works, mainly reflected in the following aspects:

i). Novel model of vehicle platoon dynamics: In modeling the vehicle platoon control problem, we take into account the VANETs induced issues like network access constraint, transmission delay and packet disordering. The transmission delay is assumed to be indeterminate, and take values in a finite set. The platoon dynamics is described a state space equation form with variable sampling interval, which is to be determined according to the transmission delay.

ii). Event-based control and scheduling collaborative design: Time-based strategies are prone to unnecessary information transmission in the case when state variation is relatively small. An event-based control and scheduling collaborative design method is proposed for the vehicular platoon control problem. The event triggering mechanism is involved in the codeign procedure of the controller and the scheduler, which is solved by formulating an optimization problem based on matrix inequalities. The resulted method can achieve uniform practical platoon stability with guaranteed control performance.

The remainder of this article is arranged as follows. In section II, the discrete-time model of vehicle dynamics and the vehicle-following control objective is presented. In section III, the event-based control and scheduling collaborative design strategy is given and the LMI optimization problem is proposed. In section IV, the numerical MATLAB simulation and experiments with Arduino cars are carried out. Section V summarizes the main conclusions and next research topic.

II. PROBLEM FORMULATION

Consider a string of $n+1$ vehicles $\Omega = \{\Omega_i, i = 0, 1, 2, ..., n\}$ driving on a level road in VANET environment (see in Fig. 1). The vehicles in the platoon are assumed to be equipped with wireless communication functionality and various onboard sensors (e.g., radar and velocimetry). Every vehicle can communicate information with some other vehicles. For platoon control, there are generally three types of strategies for information exchange: forerunner-follower strategy, leader-forerunner-follower strategy, and communications among a number of neighboring vehicles. In this paper, we will adopt the predecessor-follower strategy, i.e. each following vehicle can only receive information from its direct previous vehicle. The status information (acceleration and velocity) of the preceding vehicle is transmitted to the follower via the wireless network. The distance between two consecutive vehicles is measured by an on-board sensor.

![A platoon of autonomous vehicles](http://example.com/figure1)

**FIGURE 1. A platoon of autonomous vehicles**

A. Modeling of vehicle and platoon dynamics

Define the distance error between two consecutive vehicles as

$$e_{i}^{p}(t) = q_{i-1}(t) - q_{i}(t) - L - e_{i}^{d}, \quad 1 \leq i \leq n$$

where $q_{i}$ and $L$ is the $i$-th vehicle’s location and length, respectively. $e_{i}^{d}$ is a given minimum vehicle spacing, $1 \leq i \leq n$. The dynamics of vehicle $i$ can be represented by the following differential equations [18, 19]:

\[ \text{equation} \]
\[ \dot{q}_i(t) = v_i(t) \]
\[ v_i(t) = a_i(t) \]
\[ \dot{a}_i(t) = -\frac{1}{\eta} a_i(t) + \frac{1}{\eta} u_i(t) \]  
(2)

Note that the simplification of the system model (2) by excluding from the vehicle dynamics some characteristic parameters (e.g., the mechanical drag, the mass, and the air resistance). Among them, \( v_i \) denote the \( i \)-th vehicle’s velocity and the \( a_i \) define the \( i \)-th vehicle’s acceleration; \( \eta \) indicates the engine time constant, \( u_i \) is the input to be designed, which is given in the following form

\[ u_i(k) = k_i^q q_i^q(k) + k_i^a (v_i(1) - v_i(k)) + k_i^a (a_i(1) - a_i(k)) \]  
(3)

where \( k_i^q \) and \( k_i = [k_i^q, k_i^a]^T \) are the controller gain to be determined.

B. Event-triggered transmission mechanism and the communication delay

In spired by [20], we construct an event-based information transmission mechanism and system for each vehicle \( i \) as shown in Fig. 2. The specific details of the transmission mechanism are explained as follows. Vehicle \( i \) measures its state (i.e., spacing error \( e_i^q(k) \) and

\[ \sigma_i(k) = [v_i(k), a_i(k)] \]

by on-board sensors at sampling instant \( t_k \), \( k = 0,1,2,\ldots \). The states are saved with a time stamp \( k \) at the corresponding data packet generators (DPG), respectively. Here, for \( i = 0 \), \( \sigma_0(k) \) denotes the vehicle speed and acceleration information of the leader vehicle. At a sampling time, whether vehicle \( i \) sends its velocity and acceleration information \( \sigma_i(k) \) to its follower vehicle \( i+1 \) or not is dependent on the status of an event generator.

The transmission delays of all vehicles are assumed to be the same \( k \), which takes values in a partitioned finite set. Namely, \( \tau_k \in \tilde{S} \), where \( \tilde{S} = \{ [\tau_{\alpha(k)}, \tau_{\alpha(k)}) | \alpha(k) \in M = \{ 1,2,\ldots,M \} \} \).

Here, the lower bounds \( \tau_{\alpha(k)} \) and upper bounds \( \tau_{\alpha(k)} \) are assumed to be known, but the index \( \alpha(k) \) is not known. Such modeling of the input delay is motivated by the fact that the actual communication time may vary considerably due to the changes in network load, causes the possibility of jumping from one bounded interval to another interval.

Considering the transmission delay, the control law in (3) for vehicle \( i \) becomes as follows,

\[ u_i(k) = k_i^q q_i^q(k) + k_i^a (v_i(1) - v_i(k)) + k_i^a (a_i(1) - a_i(k)) \]  
(4)

C. Medium access constraint

Due to a finite number of resources, a local event generator monitoring an event-triggering condition \( \sigma_i(k) \) is assigned to each vehicle \( \Omega_i \). The event generator determines the necessity of transmitting new status information. The following vehicles share \( n \) wireless channels to receive the velocity and acceleration for the directly vehicle in front of them, as shown in Fig. 1. Furthermore, we assume that at most one wireless channel can be transmitted to the corresponding vehicle at a time. At time instant \( t_k \), an event-based scheduler \( j(k) \in J = \{ 0,1,2,\ldots,n \} \) is introduced to decide which communication channel to be executed. Note that the scheduler has the output \( j(k) = 0 \) if all of the event generators decide that it is not necessary to transmit the state information at the instantaneous \( t_k \). Under the circumstances, the communication network can be used for other non-communication tasks or be idle. Once an event has been generated and a wireless channel has been made by the scheduler, the received information of the vehicle \( \Omega_{j(k)} \) is updated at that moment \( t_k + \tau_{k} \). For the other vehicles \( \Omega_i \neq \Omega_{j(k)} \), the last information received will be kept until the new one is delivered.

Moreover, the data packets at different instant time \( k \) may experience different communication delays, it may lead to the packet disorder. Therefore, introduce variable sampling interval \( h(k) \) to overcome the combined effects of time-varying delay and disorder. \( h(k) \) is determined based on the time-varying delay, described as

![FIGURE 2. EBCS processing mechanism of vehicle i](image-url)
where the parameter $h_0$ is to be designed. On account of $\tau_0(\alpha(k)), \alpha(k) \in M$ is different for each bounded interval, the sampling interval is time-varying and not smaller than the actual input delay.

By combining the dynamics of the vehicle (1) and (3), the state errors equation of the following vehicles can be expressed as

$$\dot{e}_i(t) = Ae_i(t) + [B - B]u_{i-1}(t) \quad (6)$$

where $e_i(t) = [e_i^T, e_i^T, e_i^T]^T$, 
$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1/\eta \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1/\eta \end{bmatrix}.$$

An augmented discrete-time state errors equation corresponding to equation (6) can be written in the form

$$e_i(k+1) = \Pi^{a(k)}_{i,j}(k)e_i(k) + \left[ \sum_{i=-j,k}^{a(k)} - \sum_{i=-j,k}^{a(k)} \right]u_{i-1}(k) \quad (7)$$

where $e_i(k) = \left[ e_{i1}^T(k), u_{i1}^T(k) \right]^T$. 

$\Pi^{a(k)}_{i,j}(k) = \left[ \Theta_i(h_{a(k)}) - \Lambda_i(h_{a(k)} - \delta_{i,j}(k))I \right] - (1 - \delta_{i,j}(k)) \Pi_{i,j}(k)$. 

$\delta_{i,j}(k) = \delta_{i,j}(k) + (1 - \delta_{i,j}(k)) h_{a(k)}$, 

$\Theta_i(t) = e^{A^T}$. 

This representation is used from a time-delay systems put forward in [21, Sec. 2.3]. Hence, the entire platoon system at time instants $t_k$ can be written as

$$e(k + 1) = \Pi^{a(k)}_{j}(k)e(k) + \sum_{j=-k,k}^{a(k)} u_{j}(k) \quad (8)$$

with 
$$e(k) = \left[ e_{1}^T(k), e_{2}^T(k), ... , e_{N}^T(k) \right]^T, \quad u(k) = \left[ u_{1}^T(k), u_{2}^T(k), ... , u_{N}^T(k) \right]^T$$

$$\Pi^{a(k)}_{j}(k) = \begin{bmatrix} \Pi^{a(k)}_{1,j}(k) & 0 & ... & 0 \\ 0 & \Pi^{a(k)}_{2,j}(k) & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \vdots & \vdots & \Pi^{a(k)}_{N,j}(k) \end{bmatrix}$$

$\sum_{i=-j,k}^{a(k)} = \begin{bmatrix} \delta_{a(k)}_{i,j}(k) & 0 & \cdots & 0 \\ \delta_{a(k)}_{i,j}(k) & \delta_{a(k)}_{i,j}(k) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \delta_{a(k)}_{i,j}(k) \end{bmatrix}$

Remark 2: Note that $\Pi^{a(k)}_{j}(k)$ and $\sum_{i=-j,k}^{a(k)}$ are a nonlinear representation due to the uncertain time-varying delay $\tau_k$, leading to a class of uncertain discrete-time models.
switched linear platoon model with non-convex uncertainty set. The above mentioned model is not general in robust control theory. Hence, we need link play the role of a bridge to application of the robust control for the platoon system, which is a polytopic uncertainty to approximate the possibly non-convex uncertainty. Through the above process, we can utilize parameter-dependent Lyapunov functions to implement LMI-based collaborative design methods.

The Taylor series expansion method is often used to obtain the uncertainty of convex polytopic. Here, obviously, it is important to note that the over-approximation is only requested if $\delta_{i,j}(k) = 1$, i.e. $j(k) = i$. Otherwise, no approximation is requested due to the platoon model (7) was not affected by the uncertain parameter $\tau_k$.

For $\delta_{i,j}(k) = 1$, the input time-varying delay $\tau_k \in \mathcal{S}$ results in the matrix $\Lambda_i(h_{\alpha(k)} - \tau_k)$ in a nonlinear with a non-convex uncertainty set. First, a convex polytopic uncertainty set is extracted by Taylor series consists in expanding the matrix exponential included in the matrix $\Lambda_i(h_{\alpha(k)} - \tau_k)$.

Then, the Taylor series are divided into an approximation section and a remainder section

$$\Lambda_i(\delta_k) = \alpha \sum_{q=0}^{\infty} \frac{A_i^q}{q!} s^q dB = \sum_{l=1}^{L} \frac{A_l^{-1}}{l!} \delta_k^l B = \hat{\Lambda}_i(\delta_k, L) + \Delta \Lambda_i(\delta_k, L)$$

where $\delta_k = h_{\alpha(k)} - \tau_k$ is the uncertain parameter, $L$ is the order of the matrix polynomial $\hat{\Lambda}_i(\delta_k, L)$ with the tuning parameter to be designed. Note that the higher the chosen order $L$, the tighter generally the resulting polytopic uncertainty, but the codeign is more complexity. The matrix polynomial $\hat{\Lambda}_i(\delta_k, L)$ can be surrounded by a convex polyhedrons [19 Prop. 2], so

$$\hat{\Lambda}_i(\delta_k, L) = \sum_{l=1}^{L} \zeta_l(\delta_k) \Lambda_i(\delta_{\alpha(k)}), \overline{\Theta}(\alpha(k))$$

where $\zeta_l(\delta_k)$ and $\Lambda_i(\delta_{\alpha(k)}, \overline{\Theta}(\alpha(k)))$ is a non-negative real scalars with $\sum_{l=1}^{L} \zeta_l(\delta_k) = 1$ and polytope, respectively. By combining (7), (9) and (10), a discrete-time vehicle model with polytopic and additive norm-bounded uncertainty is written in the form

$$e_i(k+1) = \left( \sum_{l=1}^{L} \zeta_l(\delta_k) \Pi_{i,j}(\alpha(k)) \right) e_i(k)$$
e(k + 1) = \left[ \sum_{i=1}^{L+1} c_i^T (\bar{q}_i) \bar{v}_i^T(j || k) + \Delta \bar{v}_i^T(j || k) \right] e(k)
= \psi_j^T P e(k)
(16)

Where \( \bar{v}_i^T(j || k) = v_i^T(j || k) + \bar{v}_i^T(j || k) K_{j(k)} \),
\( \Delta \bar{v}_i^T(j || k) = \Delta v_i^T(j || k) + \bar{v}_i^T(j || k) K_{j(k)} \),
\( K_{j(k)} = \text{diag}(k_{1,j(k)}, k_{2,j(k)}, ..., k_{N,j(k)}) \).

D. The Vehicle following objective

The purpose of this research is to design a switched control law for the platoon system so that all vehicles can obtain a desired distance between two consecutive vehicles, and the following criteria needs to meet:

i). Internal stability: The entire closed-loop platoon system (16) with guaranteed performance (18) is a global uniform practical stability (GUPS).

The continuous-time guaranteed performance index of the vehicle \( i \)
\[
J_i = \int_0^T \left( e_{ci}(t) \right)^T M_{ci} \left( e_{ci}(t) \right) dt
(17)
\]

We need to discretize (17) for a time-vary sampling interval \( \tau_k \) and input delay \( \tau_k \).
\[
J = \sum_{i=0}^{\infty} \left( e_i(k) \right)^T M_{ii,j} \left( e_i(k) \right)
(18)
\]

Details of the discretization process and the weighting matrices \( M_{ii,j} \), \( M_{12,j} \), and \( M_{21,j} \) were defined in [20]. Hence, the guaranteed performance index of the entire platoon system is written as
\[
J = \sum_{i=1}^{N} J_i = \sum_{k=0}^{\infty} \left( e(k) \right)^T M_{j(k)} \left( e(k) \right) = \sum_{k=0}^{\infty} e^T(k) \bar{M}_{j(k)}^T(k) e(k)
(19)
\]

where \( \bar{M}_{j(k)}^T(k) = \left[ \begin{array}{cc} I & 0 \\ K_{j(k)} \end{array} \right] \left[ \begin{array}{cc} Q_{k(k)} & 0 \\ 0 & K_{j(k)} \end{array} \right] \).

ii). Steady-state performance: For arbitrary switching sequence \( \alpha(k) \), a robust cooperative design method can make the spacing error \( e_{ci}(t) \) approaches to zero for all following vehicles.

III. EVENT-BASED SCHEDULING CONTROL CO-DESIGN

In this previous section, in spired by the work in [20], the event-based control (15) and scheduling (14) cooperative design problem of the platoon system (16) with guaranteed performance (18) can be expressed as

\[\min \max J \quad \text{subject to (14) and (16)} \]

Remark 3: The closed-loop platoon system model (16) has included the variable delays and access restrictions. The resource constraints are contained in the scheduler (14) into the optimization problem (19). It is well known that Problem 1 is a computationally intractable Minimum-Maximum optimization problem [24]. Hence, we can obtain an upper bound about objective function (19) by the following tractable optimization problem.

First, we provide the following definition and lemma, which will play an essential role in the main results.

Definition 1: The closed loop switched platoon system (16) is GUPS, if there exist a positive matrix \( P \) and a sufficiently small \( \xi > 0 \) such that
\[
\lim_{k \to \infty} e^T(k) Pe(k) \leq \xi.
\]

Lemma 1: On the basis of the event scheduler (14), the augmented error state vector \( \sigma(k) = (e^T(k) e^T(k - 1))^T \) is divided into several regions, and each region is expressed by a quadratic form
\[
\Omega_{j(k)} = \{ \sigma(k) | e^T(k) \bar{R}_{j(k)} e(k) \geq -M \sigma \}
(20)
\]

with the partitioning matrix \( \bar{R}_{j(k)} = \text{diag}(\bar{R}_{1,j(k)}, ..., \bar{R}_{j(k)}). \)

The combination of all regions \( \Omega_{j(k)} \) covers the entire error state space.

Proof: For \( j(k) = 0 \), where at least one vehicle is active at sampling instant \( k \), so
\[
\sigma_{j(k)} \geq \sigma_i(k), \quad i \neq j(k).
\]

Hence,
\[
e^T(k) diag(0, ..., R_{1,j(k)}, ..., R_{j-1,0}) e(k)
\geq \sigma^T(k - 1) diag(0, ..., R_{2,j(k)}, ..., R_{j-1,0}) e(k - 1)
(21)
\]

We can obtain (20) by summing up (21) with the block-diagonal matrices
\[
\bar{R}_{1,j(k)} = \text{diag}\left(-R_{1,0}, ..., (N - 1)R_{1,j(k)}, ..., -R_{1,N} \right)
\]
\[
\bar{R}_{2,j(k)} = \text{diag}\left(-R_{2,0}, ..., (N - 1)R_{2,j(k)}, ..., -R_{2,N} \right)
\]

For \( j(k) \neq j \), no region exists. So, the combined \( \Omega_{j(k)} \) of all regions covers the whole error state space.

The proof is completed.

Theorem 1: The closed-loop switched platoon system (16) is GUPS if
\[
\Delta V(k) < -e^T(k) M_{j(k)}^T(k) e(k) - e^T(k) \bar{R}_{j(k)} e(k)
(22)
\]

holds.

Proof: Define a parameter-dependent Lyapunov function for platoon system (16)
\[
V(k) = e^T(k) T_{10}(k)e(k) + e^T(k - 1) T_{01}(k)e(k - 1)
(23)
\]
where \( T_{1a}(k) \) and \( T_{2a}(k) \) are positive Lyapunov matrices. Hence, the difference of \( V(k) \) along the trajectories of switched platoon system (16)
\[
\Delta V(k) = V(k + 1) - V(k)
\]
\[
e^T(k)\Delta T_{a(k)}^{-1}\Delta T_{a(k)} e(k) - e^T(k - 1)T_{2a}(k)e(k - 1) \tag{24}
\]
where
\[
\Delta T_{a(k)}^{-1} = \left( T_{2a}(k + 1) - T_{1a}(k) \right)
\]
Through the lemma 1, if (22) holds, \( \Delta V(k) \) is satisfied
\[
\Delta V(k) < -e^T(k)\tilde{M}_{a(k)} e(k) + N\kappa \tag{25}
\]
Define \( e_\varepsilon = \{ e(k) | e^T(k)\tilde{M}_{a(k)} e(k) \leq N\kappa \} \).

For \( e(k) \notin e_\varepsilon \), \( \Delta V(k) < 0 \) is guaranteed. Therefore, the closed-loop platoon system in this region is asymptotically stable.

For \( e(k) \in e_\varepsilon \), \( \Delta V(k) < 0 \) is not generally guaranteed. Therefore, let the set \( e_\varepsilon \subset e_\varepsilon \) so as to the GUPS conclusion in this region will converge to \( e_\varepsilon \) and stay here all the time.

Then, the guaranteed performance index (18) of the entire platoon system (16) partitioned into two parts
\[
J = \sum_{k=0}^{K} e^T(k)\tilde{M}_{a(k)} e(k) + \sum_{k=K+1}^{\infty} e^T(k)\tilde{M}_{a(k)} e(k) \tag{26}
\]
where \( J_1 \) is the cost of the initial state \( e(0) \) to the set \( e_\varepsilon \). \( J_2 \) is the cost of the platoon system dynamics inside the set \( e_\varepsilon \). Summing up (21) over \( k = 0,1,...,K \)
\[
\max J_1 < V(0) - \sum_{k=0}^{K} e^T(k)\tilde{R}_{1a(k)} e(k) \tag{27}
\]
\[
< e^T(0)T_{1a}(0)e(0)
< tr(T_{1a}(0)e^T(0)e(0))
\]
\[
\begin{pmatrix}
\Delta_4 \\
\Pi_{a(k)} H + \Xi_{a(k)} K_{j(k)} H \\
G_{a(k)} H + G_{b(k)} K_{j(k)} H \\
M_{a(k)}^\frac{1}{2} H K_{j(k)} H
\end{pmatrix}
\begin{pmatrix}
\Delta_3 \\
\Delta_4 \\
\Delta_3 \\
\Delta_3
\end{pmatrix}
\begin{pmatrix}
\Delta_1 \\
\Delta_2 \\
\Delta_3 \\
\Delta_3
\end{pmatrix}
\begin{pmatrix}
\Pi_{a(k)} H + \Xi_{a(k)} K_{j(k)} H \\
G_{a(k)} H + G_{b(k)} K_{j(k)} H \\
M_{a(k)}^\frac{1}{2} H K_{j(k)} H \\
H^T T_{2a(k)} H + H^T \tilde{R}_{2a(k)} H
\end{pmatrix}
\geq 0 \tag{29c}
\]
where
\[
\begin{align*}
\Delta_1 &= T_{1a(k)} H - H^T \tilde{R}_{1j(k)} H - H^T T_{2a(k+1)} H \\
\Delta_2 &= H^T T_{1a(k)} H - H^T \tilde{R}_{1j(k)} H - H^T T_{2a(k+1)} H
\end{align*}
\]
**Proof:** The proof consists of two parts: i). The closed-loop switched platoon system is robustly GUPS; ii). The guaranteed performance index (27) is minimized for all possible input delay sequences of \( \tau_k \in S \).

i). Firstly, theorem 1 has given the robustly GUSP condition (22) of the closed-loop switched platoon, by (22), (24) and Schur complement, the stability condition (22) is equivalent to

An upper bound on the \( J_1 \) is giving. Where \( tr(\cdot) \) is the trace. But \( J_2 \) is unbounded due to the asymptotic stability is not generally guaranteed in the set \( e_\varepsilon \). Then, we need to ignore \( J_2 \) to analyze the stability of the platoon system. The upper bound (27) is a new constraints into the problem (19). So, the codesign problem 1 needs to transform into the following problem.

**Problem 2:** for the closed-loop switch platoon system (16) find the event-based scheduling (14) for network communication channel and the control law (15) such that for all possible input delay sequences of \( \tau_k \in S \), the cost of \( J_1 \) (27) is robustly minimized.
\[
\min_{u(k), j(k)} tr(T_{1a}(0)e^T(0)e(0)) \text{ subject to (22)} \tag{28}
\]
Here, the problem 2 can be solved equivalently as a tractable LMI optimization problem based on following lemma 2.

**Lemma 2** [25, Lemma 1]: Given suitable dimensioned constant matrices \( N, M \) and \( L \), with uncertain matrix \( F \) satisfying \( \|F\| \leq 1 \), then
\[
N - \varepsilon MM^T - \varepsilon^{-1} L^T L > 0
\]
holds, when and only when there exists a positive real scalar \( \varepsilon > 0 \), satisfying
\[
N - \varepsilon MM^T - \varepsilon^{-1} L^T L > 0
\]
Hence, the LMI optimization problem can be expressed as

**Theorem 2:** The problem 2 can be solved through the following LMI optimization problem
\[
\min - \log \det \left( T_{1a(0)}^{-1} \right) > 0 \tag{29a}
\]
\[
- T_{1a(0)}^{-1} - H^T \tilde{R}_{1j(k)} H - H^T T_{2a(k)} H > 0 \tag{29b}
\]

\[
\begin{pmatrix}
\Delta_1 \\
\Delta_2 \\
\Delta_3 \\
\Delta_4
\end{pmatrix}
\begin{pmatrix}
\Pi_{a(k)} H + \Xi_{a(k)} K_{j(k)} H \\
G_{a(k)} H + G_{b(k)} K_{j(k)} H \\
M_{a(k)}^\frac{1}{2} H K_{j(k)} H \\
H^T T_{2a(k)} H + H^T \tilde{R}_{2a(k)} H
\end{pmatrix}
\begin{pmatrix}
\Delta_1 \\
\Delta_2 \\
\Delta_3 \\
\Delta_4
\end{pmatrix}
\begin{pmatrix}
\Delta_1 \\
\Delta_2 \\
\Delta_3 \\
\Delta_4
\end{pmatrix}
\begin{pmatrix}
\Pi_{a(k)} H + \Xi_{a(k)} K_{j(k)} H \\
G_{a(k)} H + G_{b(k)} K_{j(k)} H \\
M_{a(k)}^\frac{1}{2} H K_{j(k)} H \\
H^T T_{2a(k)} H + H^T \tilde{R}_{2a(k)} H
\end{pmatrix}
\geq 0 \tag{29c}
\]
where
\[
\begin{align*}
\Delta_1 &= T_{1a(k)} - \tilde{R}_{1j(k)} - \tilde{M}_{a(k)} - T_{2a(k+1)} \\
\Delta_2 &= - T_{1a(k)} - \tilde{R}_{1j(k)} - \tilde{M}_{a(k)} - T_{2a(k+1)}
\end{align*}
\]
By (16), inequality (30) is equivalent to

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Then, by (11) and lemma 2, inequality (31) is equivalent to
\[
\Delta_2 - \varepsilon^{\alpha(k)\alpha^*}_{j(k)} M^{\alpha(k)}_{j(k)} \left( M^{\alpha(k)}_{j(k)} \right)^T \left( L^{\alpha(k)}_{j(k)} - \left( L^{\alpha(k)}_{j(k)} \right)^T \right) > 0
\] (32)
where
\[
\Delta_2 = \begin{pmatrix} \Delta_1 & * & * \\ 0 & 0 & T_{2\alpha(k)} + \tilde{R}_{2j(k)} \\ 0 & 0 & T_{2\alpha(k)} + \tilde{R}_{2j(k)} \end{pmatrix} > 0
\]
\[
M^{\alpha(k)}_{j(k)} = 0, \left( D^{\alpha(k)} \right)^T, \quad L^{\alpha(k)}_{j(k)} = \left( G^{\alpha(k)} \right)^T + C^{\alpha(k)}K_{j(k)}0,0].
\]
Applying Schur complement, inequality (32) is equivalent to
\[
\begin{pmatrix} \Delta_1 & * & * \\ \Pi^{\alpha(k)}_{j(k)} + \Sigma^{\alpha(k)}_{j(k)}K_{j(k)} & \Delta_3 & * \\ 0 & 0 & I \end{pmatrix} > 0
\] (33)
where
\[
\Delta_3 = P_{1\alpha(k)}^{-1} - \varepsilon^{\alpha(k)\alpha^*}_{j(k)}D^{\alpha(k)} \left( D^{\alpha(k)} \right)^T > 0.
\]
Hence, pre- and post-multiplying the LMI (33) by diag(H^T, I, I, I, H^T), we can obtain (29c) with the full rank \( H = diag(H_1, ..., H_N) \), and its inverse always exists [26].

Due to \( \Delta_3 > 0 \), so
\[
H + H^T > -T_{1\alpha(0)}^{-1} - H^T \tilde{R}_{1j(0)}H - H^T T_{2\alpha(1)}H > 0
\] (34)
The inequality (34) implies the inequality (29b) is established.

ii. It is generally known that \( tr(\cdot) \) is the sum of the eigenvalues, and \( logdet(\cdot) \) is the sum of the logarithmized eigenvalues. Due to \( e^T(0)\varepsilon(0) \) is constant, we use \( \logdet(T_{1\alpha(0)}) = -\logdet(T_{1\alpha(0)}^{-1}) \) as the performance index (27).

The proof is completed.

IV. SIMULATIONS AND EXPERIMENTS

A. Numerical Example

Firstly, a numerical example proved the efficiency of the proposed event-based control and scheduling codesign strategy of six-vehicle platoon.

In the MATLAB simulation, we take into account four vehicles running a horizontal road. Without losing universality, the initial velocity of the leader vehicle is 0, the acceleration for the leader vehicle is described as
\[
a_0 = \begin{cases} 2 \text{ m/s}^2 & 0 \leq t \leq 5 \\ -2 \text{ m/s}^2 & 5 \leq t \leq 7 \\ 0 & \text{others} \end{cases}
\] (35)

It is supposed that the uncertain time-varying communication delay \( \tau_k \in S = [(0.11),(1.5.2),[2.1.2.5)] \) ms. The length of vehicle \( L = 5m \) and the desired vehicle spacing \( e_0^p = 4m \). The engine time constant \( \eta = 0.2 \). The weighting matrices of guaranteed performance index (17) of vehicle \( i \) are \( M_{ci} = diag(1000,0.1) \) and \( H_{ci} = 0.01 \).

Based on the given platoon parameters and communication delay parameters, the parameters of the discrete-time switched platoon system (11) is obtained. Then, the LMI optimization problem (29) with the chosen parameters are settled via the MATLAB toolbox YALMIP [27] with the SeDuMi solver [28]. By solving of the LMI optimization problem (29), an event-based control and scheduling collaborative design control gain \( K_{j(k)} \) can be obtained as shown in Fig. 4, which can robustly stabilize the platoon system, can efficiently handle the problems of variable communication delays, packets disorder, access constraints and resource constraint for the networked platoon system.

\[\text{FIGURE 4. The scheduling sequence } j(k)\]

In Fig. 5, the graph shows the tracking distance error response of the 5 following vehicles. The Fig. 5 indicates that the codesign controller can robustly stabilize with communication constrain, and the inter-vehicle spacing error \( e_1^p, e_2^p, e_3^p, e_4^p, e_5^p \) satisfy the performance requirements. The Fig. 6, 7 and 8 show the acceleration, position and velocity response of the following vehicle with the leader vehicle to accelerate or slow down, respectively.

\[\text{FIGURE 5. The spacing distance error between adjacent}\]
B. Experiment with Arduino Cars

The experiment with four Arduino cars (in Fig. 9) shows the practicability of the proposed event-based control and scheduling codesign strategy. The Arduino car is driven and steered by two nose wheels. The spacing distance between two consecutive vehicles is measured by two infrared sensors, the actual spacing distance employs the averaged value of two sensors. Hence, the longitudinal speed and acceleration of the Arduino car through the incremental encoder sensor installed on the rear wheel axle and the acceleration sensor installed on the top of the Arduino car, respectively. The purpose of the cameras mainly keep each car traveling a straight line. In each car, an Arduino processor can perform the real-time calculation and control task.

The vehicles for the network control of the platoon system (in Fig. 10) communicate via wireless module APC220, whose transmission distance is 1000 meters and operating frequency is 418 MHz to 455 MHz. The experiment adopts the single-packet transmission strategy. In the platoon, the leader vehicle 0 needs to send the velocity and acceleration pack data packet $\sigma_0(k)$ to vehicle 1. Similarly, the vehicle 1 and 2 also send the $\sigma_1(k)$ and $\sigma_2(k)$ to the following vehicle 2 and 3, respectively.

The 4 vehicles share 3 wireless channels, but the access constraints and resource constraint are inevitable. At most one wireless channel can be transmitted to the corresponding vehicle at a time. In the experiments, suppose the leader vehicle 0 is driving on a horizontal road with $20 \text{ cm/s}$ and accelerate at the time instantaneous of 2 and decelerate at the time instantaneous of 5. The expected distance between two consecutive vehicles $e_i^p = 15 \text{ cm}$. In the Fig. 11 and 12, the spacing error and velocity response of the four vehicles is shown, which the vehicular platoon control in the vehicular ad hoc networks is subject to variable communication delays, packets disorder, access constraints and resource constraint.
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V. CONCLUSIONS

In this article, we have investigated and researched an event-based control and scheduling codesign strategy for the platoon system subject to variable communication delay and packets disorder, access constraints and resource constraints. The variable communication delay belongs to different bounded intervals, which results in the variable sampling interval for the platoon system. The networked of the platoon system is written as a discrete-time switched platoon system. Based on the new platoon control modeling, an event-based control and scheduling (EBCS) collaborative design strategy that can robustly stabilize the platoon system is given. Global consistency practical stability with guaranteed performance is guaranteed via formulating as LMI optimization problems. A numerous simulation and experiments with laboratory scale Arduino cars show the effectiveness and practicability of the proposed methods.

In the future research, the vehicular network with fading channels will plan to consider. Furthermore, seeking for the new control and scheduling co-design strategy for vehicular networks with other communication constrains. For a more general platoon system, another important issue is to propose an event-based control and scheduling collaborative design strategy to deal with physical constraints, i.e. fueling delay and the throttling/braking delay.

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