INCLUSIVE $D^*$ HADROPRODUCTION WITH MASSIVE QUARKS

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A calculation of the next-to-leading order cross section for the inclusive hadroproduction of charm quarks including heavy quark mass effects in the hard scattering cross sections is presented. It is described how the massive hard scattering cross sections are constructed from the corresponding cross sections in a fixed order calculation where collinear pieces associated with the heavy quark are not yet subtracted. By adjusting suitable subtraction terms a massive theory with $\overline{MS}$ subtraction is established which approaches the massless theory with increasing transverse momentum. These results will allow to have a more solid comparison with recent data for the inclusive $D^{*\pm}$ cross section in $p\bar{p}$ collisions from the CDF collaboration at the Tevatron at $\sqrt{S} = 1.96$ TeV.

1 Introduction

Heavy quarks are those with masses $m_h \gg \Lambda_{\text{QCD}}$ such that $\alpha_s(m_h^2) \propto \ln^{-1}(\frac{m_h^2}{\Lambda_{\text{QCD}}^2}) \ll 1$ and hence, according to this definition, the charm, bottom and top quarks ($c, b, t$) are heavy whereas the up, down and strange quarks ($u, d, s$) are light. Apart from the fact that heavy quark production processes are fundamental elementary particle processes which take place with substantial rates at high energy colliders, and which therefore require a good phenomenological understanding, heavy quark production is theoretically interesting for at least two more reasons.

(i) Firstly, the heavy quark mass $m_h$ acts as a physical long-distance cut-off such that the heavy quark (or more precisely logarithms of the heavy quark mass) can be treated within perturbative QCD – either within fixed order perturbation theory or in resummed approaches where a heavy quark parton distribution function (PDF) is introduced using perturbatively calculated boundary conditions for the Altarelli-Parisi evolution equations. (ii) Secondly, in addition to $m_h$ another large scale, e.g. the transverse momentum $p_T$ of the heavy quark, is usually involved in the hard scattering process such that one has to deal with multi-scale processes which complicate the perturbative analysis. Obviously, terms of the order $O(m_h^2/p_T^2)$ should not be neglected in resummed approaches if $p_T$ is not much larger than the heavy quark mass. This remains also true if the perturbative boundary conditions in (i) would be abandoned in favour of experimentally determined input distributions for the heavy quark PDF.

The fixed order treatment is also called fixed flavour number scheme (FFNS) since the number of flavours in the initial state is fixed to $n_f = 3(4)$ for charm (bottom) production. On the other hand, in the parton model the number of
active flavours is increased by one unit, \( n_f \to n_f + 1 \), if the factorization scale crosses certain transition scales (which are usually taken to be of the order of the heavy quark mass)\(^a\). Accordingly, these schemes are called variable flavour number schemes (VFNS). The conventional massless parton model is usually referred to as massless or zero mass VFNS (ZM-VFNS) whereas massive parton model approaches are named massive or general mass VFNS (GM-VFNS). For details see, e.g., Refs. \[2,3\].

In this contribution I will consider open charm hadroproduction, \( p\bar{p} \to D^* X \), and describe a new calculation in a massive VFNS \[4\] guided by the factorization theorem of John Collins including heavy quark masses \[5\]. This work is along the lines of previous studies of the direct \[6\] and the single-resolved contributions \[7\] to the process \( \gamma\gamma \to D^* X \). Note also that the latter constitutes the direct contribution to \( \gamma p \to D^* X \) considered in \[5,8,3\]. The results of this new calculation can then be used also to obtain the double resolved contribution in the \( \gamma\gamma \) and the resolved contribution in the \( \gamma p \) process in order to perform a complete NLO analysis of these processes in this massive VFNS.

Recently, the CDF collaboration has published first cross section data for single inclusive \( D \) meson production in \( p\bar{p} \) collisions \[9\] obtained in Run II at the Tevatron at center-of-mass energies of \( \sqrt{S} = 1.96 \) TeV. These data have been compared with calculations \[10,11\] in two distinct theoretical approaches, the FONLL approach \[12\] and the conventional massless parton model \[13\], which both are compatible with the data within theoretical and experimental errors.\(^b\) The work presented here \[4\] extends the latter successful calculation in the massless approach by terms of the order \( O(m_c^2/p_T^2) \).

2 Basic Framework

Our theoretical basis for the calculation of differential cross sections for open charm hadroproduction, \( p\bar{p} \to D^* X \), is the familiar factorization formula [see Fig. 1], however, with heavy quark mass terms included in the hard scattering cross sections

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\(^a\)For a detailed discussion see the appendix in \[1\] and references given there.

\(^b\)However, for the central values the ratio of data/theory is about 2 at low \( p_T \) and 1.5 at high \( p_T \).
Hadroproduction with Massive Quarks

3

\[ d\sigma(p\bar{p} \to D^*X) = \sum_{i,j,k} f_i^p(x_1,\mu_f) \otimes f_j^{\bar{p}}(x_2,\mu_f) \otimes d\hat{\sigma}(ij \to kX) \otimes D_k^{D^*}(z,\mu'_f). \] (1)

Here \( f_i^p(x_1,\mu_f) \) and \( f_j^{\bar{p}}(x_2,\mu_f) \), \( i, j = u, d, s, c, g \), are universal PDFs of the proton and the anti-proton which are non-perturbative quantities in the case of gluons, \( i, j = g \), and light quarks, \( i, j = q \equiv u, d, s \). On the other hand, the charm quark PDF is usually obtained from perturbatively calculable boundary conditions. Furthermore, the fragmentation of the final state parton \( k = g, q, c, \ldots \) into the \( D^* \) meson is described by universal non-perturbative fragmentation functions (FFs) \( D_k^{D^*}(z,\mu'_f) \). Finally, the hard scattering cross sections \( d\hat{\sigma}(ij \to kX)(\mu_f,\mu'_f,\alpha_s(\mu),m_c^2) \) are perturbatively calculable and depend on the factorization scales \( \mu_f \) and \( \mu'_f \) and the renormalization scale \( \mu \). As has been shown in Ref. [5], heavy quark mass effects can be consistently included in the hard part where it is of course mandatory to use the same factorization scheme for PDFs, FFs and the hard part.

3 Hard scattering coefficients with masses

For the calculation of massive hard scattering coefficients we have adopted the following procedure [6,7,3]: (i) We have calculated the \( m \to 0 \) limit of the massive 3-FFNS coefficients only keeping \( m \) as regulator in logarithms \( \ln m^2 \). The partonic subprocesses occurring in the 3-FFNS are \( gg \to c\bar{c} \) and \( q\bar{q} \to c\bar{c} \) in LO and \( gg \to c\bar{c}g, q\bar{q} \to c\bar{c}g, \) and \( gg \to c\bar{c}q (g\bar{q} \to c\bar{c}q) \) in NLO. Here special care was required in order to recover certain distributions \( \delta(1-w), \frac{1}{2}(1-w)_+ , \frac{1}{2}(1-w)^{1/2} \) occurring in the massless \( \overline{\text{MS}} \) calculation. (ii) Subsequently, we have compared the massless limit with the corresponding short distance coefficients in the genuine massless \( \overline{\text{MS}} \) calculation in order to identify the subtraction terms by the unique prescription

\[ d\sigma_{\text{SUB}} = (\lim_{m \to 0} d\sigma(m)) - d\hat{\sigma}(\overline{\text{MS}}) \] .

(2)

The massive hard scattering cross sections have then been obtained by removing the subtraction term from the massive coefficient in the 3-FFNS:

\[ d\hat{\sigma}(m) = d\sigma(m) - d\sigma_{\text{SUB}} \] .

(3)

It should be noted that by this procedure no finite mass terms are removed from the hard part apart from the collinear logarithms \( \ln m^2 \). (iii) Contributions with charm quarks in the initial state have been included in the massless approach [16].

Note that this can be done without loss of precision [16]. Moreover, this rule is of great practical importance since the existing massless results for the hard scattering coefficients [17] can simply be used whereas their massive analogues are unknown and would require a dedicated calculation of these processes. Note that massive heavy quark initiated coefficients have been obtained for the case of deep inelastic scattering [18,19] and the results are already quite involved. Finally, this rule (scheme) has also been employed in a recent analysis of CTEQ PDFs with heavy...
quark mass effects such that it is necessary to do the same if these PDFs are to be used in a consistent analysis in the future.

4 Numerical Results

In this section numerical results for the $gg \rightarrow c + X$ channel (i.e. for $p\bar{p} \rightarrow (gg \rightarrow c) \rightarrow D^* + X$) are presented in form of the ratio

$$R_{gg} = \frac{\int_{-1}^{1} dy \frac{d\sigma^{gg}}{dy dp_T}}{\int_{-1}^{1} dy \frac{d\sigma^{gg}_{LO}}{dy dp_T} (m = 0)},$$

in dependence of the $p_T$ of the $D^{*+}$ meson. In order to investigate the size of the mass effects the cross section $d\sigma/dp_T dy$ in the numerator has been calculated in two ways: firstly, in the massive 3-FFNS using the FORTRAN code provided by I. Bojak and secondly, using the massless limit which we derived analytically as described in the preceding section. In both cases the LO cross section in the denominator has been calculated with massless charm quarks.

![Figure 2](image_url)

Figure 2. Contribution of the $gg \rightarrow cX$ channel to $p\bar{p} \rightarrow D^* + X$ normalized to the LO cross section with $m = 0$. Shown are results for the massless limit (solid line), the massive calculation (dashed line), the subtracted massless limit (dotted line), and the subtracted massive calculation (dashed-dotted line). The Factorization and renormalization scales are (a) $\mu_f = \mu'_f = \mu_r = m$ (but fixed at $\mu = 2.1m$ in PDFs, FFs, and $\alpha_s$) and (b) $\mu_f = \mu'_f = \mu_r = 2m_T$.

The results in Fig. 2 have been obtained using the CTEQ6M PDFs and the FFs for $c \rightarrow D^{*+}$ from [22] (NLO OPAL version) along with the following input parameters, $m = 1.5$ GeV, $E_p = \sqrt{s}/2 = 980$ GeV, $\Lambda_{\overline{MS}}(n_f=4) = 328$ MeV (i.e., $\alpha_s(m_Z) = 0.118$). In Fig. 2 (a), the renormalization and the factorization scales...
are $\mu_r = \mu_f = \mu'_f = m$, but the scale in $\alpha_s$, in the PDFs and the FFs is chosen as $\mu = 2.1 m$, to stay above the starting scale in the PDFs and FFs. The solid lines are the massless limit and the dashed curves are the result of the massive calculation. As one can see, the massive cross section approaches the massless one very slowly at large $p_T$. At $p_T = 20$ GeV the difference between the massive and the massless result is still of the order of 6% and the massive cross section is always larger than the massless cross section ratio in the $p_T$ range between 7 and 100 GeV. From this comparison we conclude that in the $gg$ channel the terms proportional to $m^2/p_T^2$ are particularly large and lead to an increase of the massive cross section as compared to the massless approximation. This agrees with findings in [12], where the massive and the massless theory were compared as a function of the mass $m$ for fixed $p_T$. For our application we are interested in the massive cross section and its massless limit, where the subtraction terms $d\sigma_{\text{SUB}}$ are removed. This leads to the dashed-dotted (massive) and the dotted (massless) curves in Fig. 2 (a). Not shown is the genuine $\overline{\text{MS}}$ result, generated with the routine of [17,23,24], which is in exact agreement with the subtracted massless limit (dotted curve) demonstrating the correctness of the subtraction terms for $\mu_r = \mu_f = \mu'_f = m$. Fig. 2 (b) shows results for the choice $\mu_r = \mu_f = \mu'_f = 2m_T = 2\sqrt{m^2 + p_T^2}$ which can be used for small and large $p_T$. The cross sections for these scales are shown again in four curves, massless theory, massive 3-FFNS, subtracted massless theory and subtracted massive theory where the labeling of the curves is the same as in Fig. 2 (a). These curves are our final results for the $gg$ channel. As can be seen, for this different scale the cross section ratios are much flatter as a function of $p_T$ than in Fig. 2 (a). The K factor is somewhat smaller now, but it is still large, showing that for the $gg$ channel the perturbative expansion is not converging very well. As in Fig. 2 (a) we observe that the massive cross section converges from above to the corresponding massless cross sections with increasing $p_T$. The ratios for the subtracted cross sections are now negative for $p_T \leq 30$ GeV and rise up to $\approx 0.6$ at $p_T = 100$ GeV. Also for these scales we find exact agreement between the genuine $\overline{\text{MS}}$ result and the subtracted massless limit (dotted curve). Thus we can be sure that we have subtracted from the massive cross section the correct terms in order to get the massive VFNS cross section in the $\overline{\text{MS}}$ subtraction scheme, which approaches the massless limit in this scheme at large $p_T$.

In contrast to the $gg$ channel, the mass effects in the $q\bar{q}$ and $gq$ channels are quite small reducing the size of the mass effects in the sum of all three channels. Furthermore, contributions with charm quarks in the initial state and contributions from the gluon (and light quarks) fragmenting into the $D^*$ meson have to be included. These contributions are dominant and calculated with $m = 0$ as explained in Sec. 3. This further dilutes the mass effects occurring in the $gg$ channel such that quite small and positive mass effects will survive in the total result [4].

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\[\text{This can be understood already with help of Fig. 2 (b) where large subtractions are visible which are resummed in the evolved charm quark PDFs (and FFs).}\]
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