Adaptive virtual impedance design for three-phase inverter under unbalanced grid connection

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Abstract

As a critical part of energy conversion, the parallel inverters play an important role in the stability of distributed generation (DG) system. Although the conventional control, such as the virtual impedance-based droop control, can achieve the autonomous decoupling between active and reactive power irrespective of the condition of line impedance. However, under the unbalanced conditions, which has high occurrence due to the rural location of DG, the active and reactive power sharing are inevitably coupled so that the grid support capability is degraded, causing the stability issue to the whole system. In order to overcome this problem, this paper proposes an adaptive virtual impedance-based control strategy by purely decoupling the active and reactive power. The line impedance, i.e., the main origin of unbalances, is identified by establishing the unbalanced impedance modeling in full-dq transformation, which is further adopted as the adaptive virtual impedance generating the compensation of unbalanced voltages. Experimental results verify that the proposed method is effective to identify any unbalanced components and achieve the cancellation of unbalanced grid voltage so that the system stability is maintained.

Keywords: Parallel inverters, unbalanced conditions, virtual impedance, full-dq transformation, impedance identification

1. Introduction

The micro-grid mainly consists of distributed generation (DG) and demand-side loads, which operates in either grid-connected or islanded mode. As the energy conversion interface of DG, inverters always operate in a parallel mode at the point of common coupling (PCC) so that they should take the initiative role for power sharing and voltage support [1].

Because of the physical installation locations, the conventional droop control is a desirable option for parallel inverters, which is exempted from the communication links for PCC measurement reducing the complexity [2]. However, it becomes infeasible for the low-voltage micro-grid due to the high R/X ratio of line impedance, where its pure decoupling characteristics between active and reactive power are no longer hold, causing the degradation on performance and severe stability issue [3]. To practically overcome its dependence on the line impedance condition, the virtual impedance was further introduced at the DG terminals through the corresponding current injections, which equivalently adjusted the output impedance of the inverter to be dominantly inductive or resistive. The detailed design of virtual impedance was presented considering DG power capacity and power coupling [4]. In terms of only describing the static behavior, the static virtual impedance was extended to a frequency-domain impedance in order to facilitate transient power regulation [5]. However, the rural location of RES
inevitably brings about the other challenge on uncertain factors or unbalances in the feeder impedance, deteriorating the PCC voltage and even stability [6]. Under the unbalanced conditions, the effort by the virtual impedance becomes in vain and the pure decoupling between active and reactive power flow is contaminated. Due to the adverse effect, the 2\% limit of voltage unbalance is recommended by the International Electrotechnical Commission (IEC) [7]. And DG unit should be responsible to inject the fault clearing current for PCC voltage support [8].

The existence of unbalances in feeder impedance rudely makes the inverter model time-variant in \(dq\)-frame, causing all conventional controls inapplicable. Although some contribution has been made to clear the unbalanced voltage by injecting the negative-sequence current, the complex variable \(s\) in the negative-sequence compensation modeling was simply substituted for \(s\) [6]. The full-\(dq\) transformation, which can model the arbitrary unbalanced three-phase impedance into a combination of time-invariant basis, has pointed out that unbalances give rise to the coupling between positive- and negative-sequence in \(dq\)-frame. Therefore, the aforementioned contribution ignored the cross-coupling between positive- and negative-sequence components, making that the negative-sequence voltage cancellation was impurely achieved.

This paper designs the adaptive virtual impedance control method for parallel inverters in islanded operation in order to not only guarantee the power sharing but also achieve the PCC negative-sequence voltage cancellation under unbalanced feeder impedance condition. The arbitrary unbalanced feeder impedance is modeled in \(dq^k\) frame using full-\(dq\) transformation and the coupling between positive- and negative-sequence is addressed in detail. Based on the unbalanced impedance modeling, with the enough local and remote measurement, the arbitrary unbalance can be identified by using the pseudo-inverse matrix online, which is used to further produce the negative-sequence voltage compensation by the current injection. Experimental results are given to verify that the proposed method can effectively share the power among parallel inverters and eliminate the unbalanced voltage.

2. The Limitation of Conventional Droop Control

In order to clearly understand the deficiency of the conventional droop control, this section further addresses its coupling effect on active and reactive power under unbalanced condition using full-\(dq\) transformation.

It has been proven that the admittance matrix of arbitrary three-phase circuital components can be represented by a linear combination of five bases according to independent reciprocal bases (IRB) theory proposed in [9]. These five bases are time-invariant in \(dq\)-frame and can be expressed by

\[
\begin{align*}
\xi_{1+} &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \\
\xi_{2+} &= \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \\
\xi_{3+} &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \\
\xi_{1-} &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \\
\xi_{2-} &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \\
\xi_{3-} &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.
\end{align*}
\]

where the lower superscript “+” denotes the positive sequence, and the lower superscript “-” denotes the negative sequence. By using these bases independent to each other, the arbitrary admittance matrix \(Y\), either balanced or unbalanced one, can be decomposed into a linear combination form as follows,

\[
Y = X_1\xi_{1+} + X_2\xi_{2+} + X_3\xi_{3+} + X_4\xi_{1-} + X_5\xi_{2-}.
\]

which transforms the impedance decomposition into a linear algebra problem.

By considering the aforementioned sequence characteristics between voltages and currents, the active and reactive power of positive- and negative-sequence can be derived as in Table 1 corresponding to the five bases of the arbitrary admittance with the dominantly inductive feeder impedance. Here, \(P^+/P\) and \(Q^+/Q\) are the positive/negative-sequence active and reactive power, \(V\) is the PCC voltage, \(E\) is the output voltage amplitude of the inverter, \(X\) is the line impedance, and \(\phi\) is the power angle between the DG and PCC.
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Table 1. Positive-/negative-sequence power for base of the unbalanced conductance.

| P^+ | Q^+ | P^- | Q^- |
|-----|-----|-----|-----|
| \( \frac{3\sqrt{2}V}{X} \) | \( \frac{3\sqrt{2}V}{X} \) | \( \frac{3\sqrt{2}V}{X} \) | \( \frac{3\sqrt{2}V}{X} \) |
| \( \frac{3\text{EVX}}{2} \) | \( \frac{3\text{EVX}}{2} \) | \( \frac{3\text{EVX}}{2} \) | \( \frac{3\text{EVX}}{2} \) |
| 0 | 0 | 0 | 0 |

From Table 1, it can be found out that, for the three-phase balanced line impedance only containing \( \xi_1^+ \), the \( P^+ \) and \( Q^+ \) can be entirely decoupled by independently controlling \( \omega \) and \( V \), which is the general form of the conventional droop control. However, the unbalanced component is accompanied with the existence of \( \xi_2^+, \xi_3^+, \xi_1^-, \) and \( \xi_2^- \). Does neither the active nor reactive power generation exist on the negative-sequence basis \( \xi_1^- \), \( \xi_2^- \). But it is obvious that the remaining two positive-sequence basis \( \xi_2^+, \xi_3^+ \) cause the cross-coupling between \( P^\pm \) and \( Q^\pm \), where the resultant \( P^- \) and \( Q^- \) appears as the double-frequency ripples. Therefore, the conventional droop control is no longer valid subject to the unbalanced line impedance conditions.

3. Modeling of Unbalanced Feeder Impedance Using Full-dq Transformation

As discussed above, unbalanced conditions disable the droop control because the resultant \( P^\pm \) and \( Q^\pm \) are coupled. The proposed control for parallel inverters is based on the unbalanced component compensation, which is inspired by the virtual impedance concept. In order to realize this compensation, the precise modeling for unbalanced feeder impedance is the prerequisite to generate the opposite voltage droop.

The arbitrary unbalanced feeder impedance is shown in Fig. 1, which can be formulated by augmented matrix in positive- and negative-sequence abc frame as follows.

\[
\begin{bmatrix}
L_{abc+} & L_{abc-} & 0 \\
L_{abc-} & L_{abc+} & 0 \\
0 & 0 & L_0
\end{bmatrix}
\begin{bmatrix}
di \n_i \\
di \n_i \\
di \n_i \\
\end{bmatrix}
+ \begin{bmatrix}
R_{abc+} & R_{abc-} & 0 \\
R_{abc-} & R_{abc+} & 0 \\
0 & 0 & R_0
\end{bmatrix}
\begin{bmatrix}
i_{abc+} \\
i_{abc-} \\
i_0
\end{bmatrix}
= \begin{bmatrix}
v_{abc+} \\
v_{abc-} \\
v_0
\end{bmatrix},
\]

(3)

where \( i_{abc+} = [i_a^+ \ i_b^+ \ i_c^+]^T \) is the positive-sequence currents, \( i_{abc-} = [i_a^- \ i_b^- \ i_c^-]^T \) is the negative-sequence and \( i_0 \) is zero-sequence current. \( v_{abc+} \) and its sub-components have the identical denotation. Here, superscript \( T \) represents the transpose of matrix.

The following part will demonstrate the time-invariant modeling of the arbitrary unbalanced feeder impedance using full-dq transformation. Firstly, the admittance counterpart of the impedance matrix in (3), i.e., \( L_{abc+} \) and \( R_{abc+} \), can be decomposed into the linear combination of five bases as in (2) in abc frame, where the coefficients are calculated in linear algebra as follows.

\[
\begin{bmatrix}
\xi_1 & \xi_2 & \xi_3 & \xi_4 & \xi_5
\end{bmatrix}
= \begin{bmatrix}
y_a + y_b & -y_c & y_c - y_a & y_c - y_b
\end{bmatrix}
\]

(4)
Here, \( y \) represents the admittance for either the inductance or resistance component. Therefore, \( Y_{abc+} = x_1 i_{1c+} + x_2 i_{2c+} + x_3 i_{3c+} \) and \( Y_{abc-} = x_1 i_{1c-} + x_2 i_{2c-} + x_3 i_{3c-} \).

The aforementioned unbalanced admittance in \( abc \) frame can be further transformed into \( dq^k \) frame using the \( dq \) transformation matrix \( T(\omega) \) for positive sequence and \( T(-\omega) \) for negative sequence.

The admittance sub-matrix in \( dq \) frame is,

\[
\begin{pmatrix}
A_{dq+} & A_{dq-} & 0 \\
A_{dq-} & A_{dq+} & 0 \\
0 & 0 & A_0
\end{pmatrix} = \begin{pmatrix}
B_{dq+} & B_{dq-} & 0 \\
B_{dq-} & B_{dq+} & 0 \\
0 & 0 & B_0
\end{pmatrix},
\]

(5)

where

\[
A_{dq+} = \begin{bmatrix}
\frac{R_1 + R_2 + R_3}{3} & 0 & 0 \\
0 & \frac{R_2}{2} + \frac{R_3}{6} - \frac{\sqrt{3}(R_1 - R_2)}{6} \\
0 & \frac{R_3}{6} + \frac{R_2}{2} - \frac{\sqrt{3}(R_1 - R_2)}{6}
\end{bmatrix}
\]

\[
A_{dq-} = \begin{bmatrix}
\frac{R_1 + R_2 + R_3}{3} & 0 & 0 \\
0 & \frac{R_2}{2} + \frac{R_3}{6} + \frac{\sqrt{3}(R_1 - R_2)}{6} \\
0 & \frac{R_3}{6} + \frac{R_2}{2} + \frac{\sqrt{3}(R_1 - R_2)}{6}
\end{bmatrix}
\]

(6)

In \( dq \) frame, the current derivative \( \frac{di_{dq+}}{dt} \) can be expressed by,

\[
T(-\omega) \frac{di_{dq+}}{dt} = T(-\omega) \frac{d(\Delta_{dq+0} + \frac{di_{dq+0}}{dt})}{dt} = \begin{pmatrix}
0 & -\omega & 0 \\
\omega & 0 & 0 \\
0 & 0 & 0
\end{pmatrix} \frac{di_{dq+0}}{dt} + \frac{di_{dq+0}}{dt},
\]

(7)

where \( i_{dq+0} = [i_{d+}, i_{q+}, i_{d-}, i_{q-}]^T \) is the positive (negative) sequence current represented in \( dq^k \) frame. Finally, the arbitrary unbalanced feeder impedance model can be written as follows.

\[
\begin{pmatrix}
L_{d+}^4 \Delta_{dq+0} + L_{q+}^4 \frac{di_{dq+0}}{dt} + R_{d+}^4 i_{dq+0} \\
L_{d-}^4 \Delta_{dq-0} + L_{q-}^4 \frac{di_{dq-0}}{dt} + R_{d-}^4 i_{dq-0}
\end{pmatrix} = \begin{pmatrix}
0 & -\omega & 0 \\
\omega & 0 & 0 \\
0 & 0 & 0
\end{pmatrix} \frac{di_{dq+0}}{dt} + \frac{di_{dq+0}}{dt}.
\]

(8)

where \( i_{dq+0} = [i_{d+}, i_{q+}, i_{d-}, i_{q-}]^T \) and \( v_{dq+0} = [v_{d+}, v_{q+}, v_{d-}, v_{q-}]^T \).

From the resultant impedance modeling above, it is obvious that this modeling technique effectively decouples the positive- and negative sequence so that the conventional stability analysis and controller design can be still applicable with this appropriate modeling. In order to guarantee the decoupling between active and reactive power, the construction of arbitrary unbalanced impedance model will be the theoretical basis for the impedance identification and further opposite voltage droop generation.

4. Identification of Unbalanced Feeder Impedance

The arbitrary feeder impedance in (8) constructs the full circuit model with the terminal voltages and circulating currents in \( dq^k \) frame. With the acquisition of three-phase voltages and currents and the sequence decomposition technique in [10], \( i_{dq+0} = [i_{d+}, i_{q+}, i_{d-}, i_{q-}]^T \) and \( v_{dq+0} = [v_{d+}, v_{q+}, v_{d-}, v_{q-}]^T \) can be measured. And the current derivate \( \frac{di_{dq+}}{dt}, \frac{di_{dq-}}{dt} \) can be computed by the following one-step backward approximation.

\[
\frac{di_{dq+}}{dt} = \frac{i_{dq+} - i_{(k-1) dq+}}{T_s},
\]

(9)

where \( k \) is the number of the sampling point at the current time instance and \( T_s \) is the sampling period.

Based on the acquisition and computation of derivate above, the independent variables of impedance model in (8) can be thus swapped to the impedance elements, i.e., \([L_a, L_b, L_c, R_a, R_b, R_c]\), as follows.
inverse method, the pseudo-inverse coefficient matrix $P$ can be obtained in the dimension of $6 \times 24$. The voltage vector on the right-hand side of (10) is identically expanded by six times as well, i.e., constituting matrix by combining six-period sampled coefficient matrix into one compact. By using the pseudo-conventional two-loop control of inverters.

The identified three-phase impedance. The resultant compensation voltage is used as the reference for the conventional two-loop control of inverters.

\[
Z = PU,
\]

where $Z$ is the impedance matrix $[L_a, L_b, L_c, R_a, R_b, R_c]^T$, $U$ is the expanded terminal voltage matrix, i.e., $[u_d, u_q, u_d, u_q]_0^T$ and $P$ is the previous pseudo-inverse coefficient matrix.

5. Control Design with Adaptive Virtual Impedance for Parallel Inverters

The adaptive virtual impedance is based on the impedance identification technique in Section 4 and is specifically designed to generate arbitrary unbalanced component compensation for the opposite unbalanced voltage droop compensation. As shown in Fig. 2, the impedance identification is integrated as the adaptive virtual impedance into the conventional droop control. With the online acquisition of arbitrary feeder impedance by (10), the unbalanced output voltage of the inverter is compensated by injecting the corresponding current circulating through the virtual impedance. This compensation voltage is obtained by

\[
\begin{bmatrix}
u_{d,\text{comp}} \\ \nu_{q,\text{comp}}
\end{bmatrix} = T(\omega) \begin{bmatrix} Z_a & 0 & 0 \\ 0 & Z_b & 0 \\ 0 & 0 & Z_c
\end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c
\end{bmatrix},
\]

where $[u_{d,\text{comp}}, u_{q,\text{comp}}]^T$ is the voltage reference for unbalanced compensation in $dq$ frame, $Z_a$, $Z_b$, and $Z_c$ are the identified three-phase impedance. The resultant compensation voltage is used as the reference for the conventional two-loop control of inverters.

Fig. 2. The control diagram of Adaptive virtual impedance.
6. Control Design with Adaptive Virtual Impedance for Parallel Inverters

The experiment platform is built for verification purpose, which can be divided into the main circuit and control hardware. The main is one three-phase inverter circuit, where mainly consists of an intelligent power module (IPM), an optocoupler isolation drive circuit, a filtering inductor, and a DC bus absorption capacitor. The control circuit is composed of Digital Signal Processing (DSP) and Field-Programmable Gate Array (FPGA) as the core units. Two identical three-phase inverter circuits are connected in parallel.

6.1. Experiment with unbalanced line impedance for standalone inverter

This part firstly verifies the proposed method to compensate the unbalanced terminal voltage by the proposed adaptive virtual impedance under standalone operation. The unbalanced feeder impedance is connected in series between the inverter 1 and three-phase balanced loads. The resistance element of phase A is 5 Ω, phase B is 3 Ω, and phase C is not added. The load voltage under this standalone connection is shown in Fig. 3(a). It can be seen that due to the inclusion of the unbalanced line resistance, the voltage droop across the line resistance causes the PCC voltage unbalanced. In order to cancel the unbalanced PCC voltage and get recovery to the balanced rating, the compensation voltage is added to the reference of voltage and current dual-loop so that the required output current can be generated and injected through the identified unbalanced impedance. Its physical principle is that the voltage droop caused by unbalanced feeder impedance is compensated by raising the corresponding three-phase output voltage of the inverter, such that the PCC voltage can recover to the rated voltage and become balanced again. The compensated PCC voltage is shown in Fig. 3(b).

![Fig. 3. Load voltage with unbalanced three-phase line impedance: (a) no compensation and (b) with compensation.](image)

6.2. Experimental with unbalanced line impedance for parallel operation

When the feeder impedance of parallel inverters becomes unbalanced, the output power of inverters will fluctuate greatly, causing severe stability problem as shown in Table 1. For the parallel operation, the series resistance of 5 Ω is connected on phase A between the inverter 1 and the common three-phase

![Fig. 4. Load voltage experimental waveform and Matlab drawing point diagram of parallel inverters.](image)
resistive load. In the control implementation of the inverter 1, the virtual impedance of 5 Ω is identified and its compensation is added to the voltage reference of phase A. The PCC voltage of parallel inverters with unbalance compensation is demonstrated in Fig. 4(a), where the gain of oscilloscope is 20 times. In order to clearly present the PCC voltage, the voltage data captured in the oscilloscope is stored and re-plotted by Matlab, which is drawn in Fig. 4(b). Here, the time reference in Fig 4 is marked by 0 s due to the oscilloscope triggering mode.

It can be seen that the PCC voltage precisely remains at the rated value when the adaptive virtual impedance control is enabled. After disabling the adaptive control, the unbalanced PCC voltage immediately arises. Due to the balanced line impedance of the parallel-connected inverter 2, the PCC voltage is clamped so that the degree of unbalance doesn’t exceed that for the standalone operation. For the clear demonstration of PCC voltage’s variation tendency, the voltage unbalance factor formulated by

\[
VUF = \frac{\sqrt{u_q^2 + u_d^2}}{\sqrt{u_q^2 + u_d^2}}
\]  

is calculated using the sampled PCC voltage data, which is depicted in Fig. 5.

![Fig. 5. The change of voltage unbalanced factor when removing unbalance compensation.](image)

As can be seen that the voltage unbalance at PCC suddenly rises when disabling the adaptive virtual impedance control at 0 s and diverge at last, verifying that the virtual impedance compensation method is effective and has the positive effect on the unbalanced PCC voltage under unbalanced grid connection.

7. Conclusion

In this paper, the full-\(dq\) transformation is applied to model the arbitrary unbalanced feeder impedance, based on which the arbitrary impedance identification technique is designed to create the adaptive virtual impedance. The impedance identified is used for unbalanced voltage compensation by injecting the corresponding currents through the virtual impedance. The experiment verifies that the proposed adaptive virtual impedance can effectively reduce the unbalances on the PCC voltage, which has great benefits to the stable operation of parallel inverters.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

You Hu conducted the research, Lini Zheng and Ruixuan Wang analyzed the experimental data, Zhen Li, Xi Chen and Guoqing He provided advice about the experiment; You Hu wrote the paper, Lini Zheng and Zhen Li helped improve the paper; all authors had approved the final version.
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