Light composite scalar in eight-flavor QCD on the lattice

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We present the first observation of a flavor-singlet scalar meson as light as the pion in \(N_f = 8\) QCD on the lattice, using the Highly Improved Staggered Quark action. Such a light scalar meson can be regarded as a composite Higgs with mass 125 GeV. In accord with our previous lattice results showing that the theory exhibits walking behavior, the light scalar may be a technidilaton, a pseudo Nambu-Goldstone boson of the approximate scale symmetry in walking technicolor.

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Recently, a Higgs boson with mass around 125 GeV has been discovered at the Large Hadron Collider (LHC) \[1, 2\]. While the current LHC data show good agreement with the Standard model Higgs boson, there exists a possibility that the Higgs boson is a composite particle in an underlying strongly coupled gauge theory. A typical example is the walking technicolor theory, featuring approximate scale invariance and a large anomalous dimension, \(\gamma_m \approx 1\), (see also similar works \[3, 4\]). Such a theory predicts a light composite Higgs, “technidilaton” \[5\], emerging as a pseudo Nambu-Goldstone (NG) boson of the spontaneously broken approximate scale symmetry. It was shown \[5, 6\] that the technidilaton is phenomenologically consistent with the current LHC data.

Thus, the most urgent theoretical task to test walking technicolor theories would be to check whether or not such a light flavor-singlet scalar bound state exists from first-principle calculations with lattice gauge theory. Since the composite Higgs should be associated with the electroweak symmetry breaking, it must be predominantly a bound state of technifermions carrying electroweak charges, but not of technigluons having no electroweak charges (up to some mixings between them). Thus we look for a light flavor-singlet scalar meson in the correlator of fermionic operators on the lattice.

One of the most popular candidates for walking technicolor theories is QCD with a large number of (massless) flavors \(N_f\) in the fundamental representation. For the past few years, we have studied the SU(3) gauge theory with \(N_f = 4, 8, 12,\) and \(16\), in a common lattice setup \[9–11\]. (For reviews of lattice studies in search for candidates for walking technicolor theories, see \[12–13\].)

In \(N_f = 12\) QCD we actually observed \[11, 14\] a flavor-singlet scalar meson \(\sigma\) lighter than the “pion” having the quantum numbers corresponding to the NG pion \(\pi\) in the broken phase. (Recently a light flavor-singlet scalar meson consistent with ours was also observed by another group \[17\] using a different lattice action.)

We found \[9\] that \(N_f = 12\) QCD is consistent with a conformal theory. If it is a conformal theory, it should have no bound states (“unparticle”) in the exact chiral limit, and hence a light bound state can only be formed in the presence of a fermion mass \(m_f\) which explicitly (not spontaneously) breaks the scale/chiral/electroweak symmetry.

Hence such a light scalar meson in \(N_f = 12\) QCD would not be a composite Higgs associated with the spontaneous symmetry breaking. However, its presence strongly suggests that a walking theory would have a similar light scalar meson as a composite Higgs associated with the spontaneous scale/chiral/electroweak symmetry breaking, since in the walking theory the gauge coupling is similar to that of a conformal theory with the role of the explicit breaking mass \(m_f\) replaced by the dynamically generated fermion mass, \(m_D\), arising from the spontaneous chiral symmetry breaking.

In this Letter we indeed observe such a light flavor-singlet scalar fermionic bound state \(\sigma\) as light as \(\pi\) in \(N_f = 8\) QCD, which we found \[10\] is a candidate for walking technicolor, with the spontaneous breaking of chiral symmetry and large anomalous dimension near unity. Thus it can be a candidate for the composite Higgs (technidilaton) with a 125 GeV mass. The preliminary results of this work were already reported in Ref. \[18\].

We carry out simulations of SU(3) gauge theory with eight fundamental fermions using two degenerate staggered fermion species with bare fermion mass \(m_f\), where each species has four fermion degrees of freedom, called tastes. We use a tree-level Symanzik gauge action and the Highly Improved Staggered Quark (HISQ) \[19\] action without the tadpole improvement or the mass correction in the Naik term \[20\]. The flavor symmetry breaking of this action is highly suppressed in QCD \[20\]. It is also true in our \(N_f = 8\) QCD simulations, where the breaking is almost negligible in the meson masses \[10\]. At the same bare coupling \(\beta = 6/g^2 = 3.8\) as in our previous work \[10\], we calculate the mass of the flavor-singlet scalar \(m_\sigma\) at five fermion masses, \(m_f = 0.015, 0.02, 0.03, 0.04,\) and \(0.06\), to investigate the \(m_f\) dependence of \(m_\sigma\). We use four volumes of spatial extent
TABLE I: Simulation parameters for $N_f = 8$ QCD at $\beta = 3.8$. $N_{\text{ev}}(N_{\text{cfg}})$ is the total number of gauge configurations (Markov chain streams). The second error of $m_\sigma$ is a systematic error coming from the fit range. The values for $m_\sigma$ and $F_\sigma$ are from Ref. [10], but the ones with (*) have been updated.

| $m_f$ | $L^3 \times T$ | $N_{\text{ev}}[N_{\text{cfg}}]$ | $m_\sigma$ | $F_\sigma$ |
|-------|-----------------|------------------------|--------|--------|
| 0.15  | 36$^3 \times 48$ | 3200[2]                | 0.155(21)(2$^1$) | 0.186(14) * |
| 0.02  | 36$^3 \times 48$ | 5000[1]                | 0.190(17)(2$^1$) | 0.220(5) * |
| 0.02  | 30$^3 \times 40$ | 8000[1]                | 0.201(21)(2$^1$) | 0.227(9) |
| 0.03  | 30$^3 \times 40$ | 16500[1]               | 0.282(27)(2$^1$) | 0.281(2) * |
| 0.03  | 24$^3 \times 32$ | 36000[2]               | 0.276(15)(2$^1$) | 0.283(14) |
| 0.04  | 30$^3 \times 40$ | 12900[3]               | 0.365(43)(2$^1$) | 0.334(9) * |
| 0.04  | 24$^3 \times 32$ | 50000[2]               | 0.332(19)(2$^1$) | 0.353(7) |
| 0.04  | 18$^3 \times 24$ | 9000[1]                | 0.228(30)(2$^1$) | 0.342(29) |
| 0.06  | 24$^3 \times 32$ | 18000[1]               | 0.467(7)(2$^1$)  | 0.429(5) |
| 0.06  | 18$^3 \times 24$ | 9000[1]                | 0.386(77)(2$^1$) | 0.431(15) |

$\chi^2$ per data point:
$$\chi^2 = \frac{1}{N_{\text{data}}} \sum_{i=1}^{N_{\text{data}}} \left( \frac{y_i - \langle y_i \rangle}{\sigma_i} \right)^2,$$

$\chi^2_\text{red} = \frac{\chi^2}{N_{\text{df}}}$

where $N_{\text{df}}$ is the degrees of freedom.

**FIG. 1:** Effective scalar mass $m_\sigma$ from correlators in Eq. (2), with the projection explained in the text, and in Eq. (1) for $L = 36$ and $m_f = 0.015$. The solid and dashed lines highlight the fit results for $m_\sigma$ with statistical error band. The dashed-dotted line represents $m_\sigma$. Effective mass of the projected connected correlator in Eq. (3) is also plotted.

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$L = 18, 24, 30,$ and $36$, with fixed aspect ratio $T/L = 4/3$, to check for finite size effects on $m_\sigma$. All the simulation parameters are tabulated in Table I. In this Letter all dimensionful quantities are expressed in lattice units.

We generate between 6400 and 100000 trajectories depending on the simulation parameters with the standard hybrid Monte Carlo algorithm using the MILC code version 7 [21] with some modifications to suit our needs, such as Hasenbusch mass preconditioning [22] to reduce the computational cost. For the thermalization we discard more than 2000 trajectories. In some parameters we make several Markov chain streams to collect thermalized configurations more efficiently. The total numbers of configurations and Markov chain streams are tabulated in Table I. For the measurement of the flavor-singlet scalar mass we use interpolating operators of the fermionic bilinear with the appropriate quantum numbers, $J^{PC} = 0^{-+}$. In this measurement we use the MILC code [21] and exploit GPGPU acceleration thanks to the QUDA library [24]. The measurements are performed every 2 trajectories. The vacuum-subtracted disconnected correlator has large statistical noise; however, it is essential to obtain $m_\sigma$. For the noise reduction, as in the $N_f = 12$ QCD calculation [11], we utilize a method [24] based on the axial Ward-Takahashi identity [25], which has been employed in the literature [24,27]. We use 64 random sources spread in spacetime and color spaces for this noise-reduction method. The statistical errors are estimated by the jackknife method, with a bin size of 200 trajectories.

Since we employ the same fermion bilinear operator as in $N_f = 12$ QCD [11], in this Letter we describe it briefly. We use the local fermionic bilinear operator of the $(1 \otimes 1)$ staggered spin-taste structure defined as

$$\mathcal{O}_S(t) = \sum_{i=1}^{N} \sum_{\vec{x}} \chi_i(\vec{x}, t) \chi_i(\vec{x}, t), \quad (1)$$

where the index $i$ runs through different staggered fermion species. The correlator of the operator is given by

$$2D(t) - C(t) = A_\sigma(t) + (-1)^t A_{\sigma SC}(t), \quad (2)$$

where $A_{\sigma}(t) = A_{\sigma}(e^{-m_H t} + e^{-m_H (T-t)})$, with $m_H$ being the mass of state $H$. Since $-C(t)$ can be regarded as a flavor non-singlet scalar correlator, it should have contributions from the non-singlet scalar ($a_0$), and its staggered parity partner, which is another flavor non-singlet (taste non-singlet and species non-singlet) pseudoscalar ($\sigma_{SC}$). When $t$ is large, we can write

$$-C(t) = A_{a_0}(t) + (-1)^t A_{\sigma SC}(t), \quad (3)$$

From Eq. (2) and Eq. (3), at large time $2D(t)$ can be written as

$$2D(t) = A_\sigma(t) - A_{a_0}(t) + (-1)^t (A_{\sigma SC}(t) - A_{\sigma SC}(t)). \quad (4)$$

If the flavor symmetry is exact, all the flavor non-singlet pseudoscalars, $\pi_{\sigma SC}$, $\pi_{SC}$, and also the NG $\pi$, are degenerate. Furthermore, in the flavor symmetric limit, their amplitudes in Eq. (4) also coincide, so that $A_{\pi SC}(t) = A_{\sigma SC}(t)$ in this limit.

After applying the positive parity projection, $C_+(t) = 2C(t) + C(t+1) + C(t-1)$ at even $t$ to minimize $A_{\sigma SC}(t)$ in Eq. (2), we evaluate the effective mass of the projected correlator $2D_+(t) - C_+(t)$. Figure I shows that the effective mass at large $t$ is almost equal to $m_\sigma$, although the error is large. We also plot the effective mass
of $2D(t)$ without the projection, which does not have an oscillating behavior. This means that the flavor symmetry breaking between $A_{RC}(t)$ and $A_{E}(t)$ in Eq. (1) is small. The effective mass plateau of $2\tilde{D}(t)$ is statistically consistent with the one of $2D_{\pi}(t) - C_{\pi}(t)$ in the large time region. Note that effective mass of $-C_{\pi}(t)$ is always larger than the one of $2\tilde{D}(t)$ in our simulations, as shown in Fig. 1. Since the plateau of $2\tilde{D}(t)$ appears at earlier time with smaller error than the one of $2D_{\pi}(t) - C_{\pi}(t)$, we choose $2\tilde{D}(t)$ to extract $m_{\sigma}$ in all the parameters. The earlier plateau might be caused by a reasonable cancellation among contributions from excited scalar states and $a_{0}$ in $2\tilde{D}(t)$. It should be noted that, because of the small $m_{\sigma}$, comparable to $m_{\pi}$, the exponential damping of $D(t)$ is slow, and this helps preventing a rapid degradation of the signal-to-noise ratio.

We fit $2\tilde{D}(t)$ in the region $t = 6-11$ by a single cosh form assuming only $\sigma$ propagating in this region to obtain $m_{\sigma}$ for all the parameters. The fit result on $L = 36$ at $m_{f} = 0.015$ is shown in Fig. 1. In this parameter it is possible to fit $2\tilde{D}(t)$ with a longer fit range, while in some parameters the effective mass of $2\tilde{D}(t)$ in the large time region is unstable with large error in the current statistics. Thus, we choose this fixed fit range in all the parameters. In order to estimate the systematic error coming from the fixed fit range, we carry out another fit in a region at larger $t$ than the fixed one, with the same number of data points. An example of this fit is shown in Fig. 1. We quote the difference between the two central values as the systematic error.

The values of $m_{\sigma}$ and also $m_{\pi}$ for all the parameters are summarized in Table I. Figure 2 presents $m_{\sigma}$ as function of $m_{f}$ together with $m_{\pi}$. These are our main results. The data on the largest two volumes at each $m_{f}$, except for $m_{f} = 0.015$, agree with each other, and suggest that finite size effects are negligible in our statistics. We find a clear signal that $\sigma$ is as light as $\pi$ for all the fermion masses we simulate. This property is distinctly different from the one in usual QCD, where $m_{\sigma}$ is clearly larger than $m_{\pi}$ [28, 29], while it is similar to the one in $N_f = 12$ QCD observed in our previous study [11]. Thus, this might be regarded as a reflection of the approximate scale symmetry in this theory, no matter whether the main scale symmetry breaking in the far infrared comes from $m_{f}$ or $m_{d}$, as we noted before. The figure also shows that our simulation region is far from heavy-fermion limit, because the vector meson mass obtained from the $m_{\pi}$ operator, denoted by $\rho(PV)$, is clearly larger than $m_{\pi}$.

Although the accuracy of our data is not enough to make a clear conclusion for a chiral extrapolation, we shall report some results below. While in the previous paper [10] we found that the data for $m_{\pi}$ and $F_{\pi}$, $\pi$ decay constant at each $m_{f}$, are consistent with chiral perturbation theory (ChPT) in the region $m_{f} \leq 0.04$, the updated data [20], tabulated in Table I show consistency with ChPT in a somewhat smaller region $m_{f} \leq 0.03$. Thus, we shall use the lightest three data with the smallest error at each $m_{f}$, i.e., the two data on $L = 36$ and the lightest data on $L = 24$, in the following analyses.

The validity of ChPT is intact even when the light $\sigma$ comparable with $\pi$ is involved in the chirally broken phase: the systematic power counting rule as a generalization of ChPT including $\sigma$ as a dilaton was established in Ref. 31 ("dilaton ChPT (DCChPT)") including computation of the chiral log effects. At the leading order we have $m_{\pi}^{2} = 4m_{f}(\bar{\psi}\psi)/F^{2}$ (Gell-Mann-Oakes-Renner relation) and

$$ m_{\sigma}^{2} = d_{0} + d_{1}m_{\pi}^{2}, \quad (5) $$

where $d_{0} = m_{\pi}^{2}|_{m_{f}=0}$ and $d_{1} = (3 - \gamma_{m})(1 + \gamma_{m})/4 \cdot (N_{f}F^{2}/F_{\sigma}^{2})$, with $\gamma_{m}$ being mass anomalous dimension in the walking region, $F$ and $F_{\pi}$ being the decay constants of $\pi$ and $\sigma$, respectively, in the chiral limit. $(F/\sqrt{2})$ corresponds to 93 MeV for the usual QCD $\pi$.) In the following fit, we ignore higher order terms including chiral log. We plot $m_{\sigma}^{2}$ as a function of $m_{\pi}^{2}$ in Fig. 8. The extrapolation to the chiral limit based on Eq. (5) gives a reasonable $\chi^{2}/d.o.f. = 0.27$, with a tiny value in the chiral limit, $d_{0} = -0.019(13)(\rho(PV))$ where the first and second errors are statistical and systematic, respectively. It agrees with zero with 1.4 standard deviation and shows a consistency with the NG nature of $\sigma$. Although errors are large at this moment, it is very encouraging for obtaining a light technidilaton to be identified with a composite Higgs with mass 125 GeV, with the value very close to $F/\sqrt{2} \approx 123$ GeV of the one-family model with 4 weak-doublets, i.e., $N_{f} = 8$. The value of $F$ from our data is estimated as $F = 0.0202(13)(\rho(PV))$, which is updated from the previous paper [10] using more statistics and a new smaller $m_{f}$ data. (If this scalar is to be identified with a composite Higgs, we expect $d_{0} \sim F^{2}/2 \sim 0.0002$.)
From the value of $d_1$, we can read $F_\sigma$, because the factor $(3 - \gamma_m)(1 + \gamma_m)/4$ is close to unity when we use $\gamma_m = 0.6 - 1.0$ [10]. The value of $F_\sigma$ is important to make a prediction of the couplings of the Higgs boson from the chiral limit to clarify. Open square symbols are slightly shifted for clarity. Result of a chiral extrapolation by the DChPT fit in Eq. (3) is plotted by the solid line and full circle. Linear fit result, $m_\sigma = c_0 + c_1 m_f$, is also plotted by dashed curve and full square. Dotted line denotes $m_\sigma^2 = m_{\sigma'}^2$.

FIG. 3: Mass squared of the flavor-singlet scalar $m_\sigma^2$ as function of $m_{\sigma'}^2$. Outer error represents the statistical and systematic uncertainties added in quadrature, while inner error is only statistical. From the value of $d_1$, we can read $F_\sigma$, because the factor $(3 - \gamma_m)(1 + \gamma_m)/4$ is close to unity when we use $\gamma_m = 0.6 - 1.0$ [10]. The value of $F_\sigma$ is important to make a prediction of the couplings of the Higgs boson from the chiral limit to clarify. Open square symbols are slightly shifted for clarity. Result of a chiral extrapolation by the DChPT fit in Eq. (3) is plotted by the solid line and full circle. Linear fit result, $m_\sigma = c_0 + c_1 m_f$, is also plotted by dashed curve and full square. Dotted line denotes $m_\sigma^2 = m_{\sigma'}^2$.

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In summary, using the same calculation techniques as in the study of $N_f = 12$ QCD [11], we have observed clear signals of a flavor-singlet scalar as light as the pion in $N_f = 8$ QCD, which was shown to be a candidate for the technicolor boson mass of 125 GeV, although the errors on the extrapolated values are large.

Obviously, an important future direction is to obtain a more precise value of $m_\sigma$ in the chiral limit to clarify whether this theory can really reproduce the Higgs boson mass of 125 GeV, and is really a candidate of theory beyond the standard model. To do this, we should observe $m_\sigma > m_\pi$ discussed above, which could be regarded as another signal of walking behavior.

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[1] ATLAS (ATLAS Collaboration), Phys.Lett. B716, 1 (2012), 1207.7214.
[2] CMS (CMS Collaboration), Phys.Lett. B716, 30 (2012), 1207.7235.
[3] K. Yamawaki, M. Bando, and K.-i. Matumoto, Phys.Rev.Lett. 56, 1335 (1986).
[4] B. Holdom, Phys.Lett. B150, 301 (1985).
[5] T. Akiba and T. Yanagida, Phys.Lett. B169, 432 (1986).
[6] T. W. Appelquist, D. Karabali, and L. Wijewardhana, Phys.Rev.Lett. 57, 957 (1986).
[7] S. Matsuzaki and K. Yamawaki, Phys.Rev. D86, 115004 (2012), 1209.2017.
[8] S. Matsuzaki and K. Yamawaki, Phys.Lett. B719, 378 (2013), 1207.5911.
[9] Y. Aoki, T. Aoyama, M. Kurachi, T. Maskawa, K.-i. Nagai, H. Ohki, A. Shibata, K. Yamawaki, and T. Yamazaki (LatKMI Collaboration), Phys.Rev. D86, 054506 (2012), 1207.3060.
[10] Y. Aoki, T. Aoyama, M. Kurachi, T. Maskawa, K.-i. Nagai, H. Ohki, A. Shibata, K. Yamawaki, and T. Yamazaki (LatKMI Collaboration), Phys.Rev. D87, 094511 (2013), 1302.6859.
[11] Y. Aoki, T. Aoyama, M. Kurachi, T. Maskawa, K.-i. Nagai, H. Ohki, E. Rinaldi, A. Shibata, K. Yamawaki, and T. Yamazaki (LatKMI Collaboration), Phys.Rev.Lett. 111, 162001 (2013), 1305.6006.
[12] L. Del Debbio, PoS LATTICE2010, 004 (2010), 1102.4066.
[13] E. T. Neil, PoS LATTICE2011, 009 (2011), 1205.4706.
[14] J. Giedt, PoS LATTICE2012, 006 (2012).
[15] J. Kuti, PoS LATTICE2013, 004 (2013).
[16] Y. Aoki, T. Aoyama, M. Kurachi, T. Maskawa, K.-i. Nagai, H. Ohki, E. Rinaldi, A. Shibata, K. Yamawaki, and T. Yamazaki (LatKMI Collaboration) (2013), 1302.4577.
[17] Z. Fodor, K. Holland, J. Kuti, D. Nogradi, and C. H. Wong, PoS LATTICE2013, 062 (2014), 1401.2176.
[18] Y. Aoki, T. Aoyama, M. Kurachi, T. Maskawa, K. Miura, K.-i. Nagai, H. Ohki, E. Rinaldi, A. Shibata, K. Yamawaki, and T. Yamazaki (LatKMI Collaboration), PoS LATTICE2013, 070 (2013), 1309.0711.
[19] E. Follana et al. (HPQCD Collaboration, UKQCD Collaboration), Phys.Rev. D75, 054502 (2007), hep-lat/0610092.
[20] A. Bazavov, T. Bhattacharya, M. Cheng, C. DeTar, H. Ding, et al., Phys.Rev. D85, 054503 (2012), 1111.1710.
[21] Milc public lattice gauge theory code, URL http://physics.indiana.edu/~sg/milc.html
[22] M. Hasenbusch, Phys.Lett. B519, 177 (2001), hep-lat/0107019.
[23] M. Clark, R. Babich, K. Barros, R. Brower, and C. Rebbi, Comput.Phys.Commun. 181, 1517 (2010), 0911.3191.
[24] L. Venkataraman and G. Kilcup, Phys.Rev.D (1997), hep-lat/9711006.
[25] C. McNeile, A. Bazavov, C. Davies, R. Dowdall, K. Hornbostel, et al., Phys.Rev. D87, 034503 (2013), 1211.6577.
[26] E. B. Gregory, A. C. Irving, C. M. Richards, and C. McNeile, Phys.Rev. D77, 065019 (2008), 0709.4224.
[27] X.-Y. Jin and R. D. Mawhinney, PoS LATTICE2011, 066 (2011), 1203.5855.
[28] T. Kunihiro et al. (SCALAR Collaboration), Phys.Rev. D70, 034504 (2004), hep-ph/0310312.
[29] C. Bernard, C. E. DeTar, Z. Fu, and S. Prelovsek, Phys.Rev. D76, 094504 (2007), 0707.2402.
[30] Y. Aoki, T. Aoyama, M. Kurachi, T. Maskawa, K. Miura, K.-i. Nagai, H. Ohki, E. Rinaldi, A. Shibata, K. Yamawaki, and T. Yamazaki (LatKMI Collaboration), in preparation.
[31] S. Matsuzaki and K. Yamawaki (2013), 1311.3784.
[32] Y. Aoki, T. Aoyama, M. Kurachi, T. Maskawa, K. Miura, K.-i. Nagai, H. Ohki, E. Rinaldi, A. Shibata, K. Yamawaki, and T. Yamazaki (LatKMI Collaboration), PoS LATTICE2013, 073 (2013), 1310.0963.