Single-photon switch controlled by a qubit embedded in an engineered electromagnetic environment

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A single-photon switch is an important element for the building of scalable quantum networks. In the paper, we propose a feasible scheme for efficient single-photon switching. The proposed switch is controlled by a state of a qubit formed by the pair of the lowest levels of a three-level system (qutrit) coupled to a resonator. This resonator-qutrit system comprises a switching unit of the considered setup. For suppression of the Purcell relaxation of the control qubit, the switching unit is embedded into a coupled-resonator array serving as an engineered electromagnetic environment with a bandgap on a qubit transition frequency. We discuss the possible implementation of the considered single-photon switch on the microwave circuit QED architecture. We demonstrate that high switching contrasts can be attained for the parameters achievable for the state-of-the-art superconducting circuit QED setups.

I. INTRODUCTION

A quantum network is an essential ingredient necessary for the realization of scalable systems for quantum information processing (QIP) [1]. It is built of a set of nodes, where quantum information is processed and/or stored, interconnected via quantum channels, where flying qubits propagate transferring information between remote quantum nodes [2]. Photons are considered as a prime candidate for the role of flying qubits due to the ultimate propagation speed and the ability to retain the coherence over the large distances [3]. Precise and rapid control of photon propagation in quantum networks is requisite for the efficient operation of quantum networks. In this regard, various devices aimed to manipulate the transport of photons, such as quantum switches [4–11] and routers [12–15], photonic valves [16], diodes [17, 18], and transistors [19–21], were proposed.

A single-photon switch is a system that coherently controls the photonic transport on a level of individual quanta. A switch interconnects different quantum channels and represents an important component (node) of quantum networks, which motivates the studies of various schemes for switching and routing. Besides a plethora of theoretical proposals [4–6, 9–15], a number of experimental demonstrations of various schemes of single-photon switches and routers operating in both microwave and optical domains were reported [22–25].

Waveguide QED structures, such as optical nanofibers [26], photonic-crystal waveguides [27], or coplanar microwave transmission lines [28], can serve as quantum channels providing robust transport of photons. In waveguides, light is transversely confined, which gives rise to light-emitter interaction enhancement and pronounced interference between the incident and scattered fields. It was demonstrated that an individual quantum emitter embedded in a one-dimensional waveguide can act as a tunable scatterer for an incident photon [29]. By varying the light-emitter interaction [6, 30, 31] or utilizing an additional (classical [4, 10–14] or quantum [32, 33]) control signal, one can achieve either complete transmission or reflection of an incident photon. One can use this feature for the implementation of optical switches [22–24].

In the paper, we propose a scheme of an efficient single-photon switch based on a waveguide QED system which can be realized on a microwave superconducting circuit QED (cQED) hardware platform. In the scheme we consider, the pair of semi-infinite waveguides are coupled to ends of a coupled-resonator array (CRA). One of the resonators composing the CRA is coupled to a three-level system (3LS) implemented by a Josephson-junction artificial atom. This resonator-qutrit system works as an active (switching) unit in the considered scheme. The two lowest states of a 3LS constitute a qubit whose state controls whether the system transmits or reflects the input photon. Thus, there is no need in the continuous classical drive to switch the system between the reflective and transmissive states, which is required in various proposals of single-photon switches [4, 7, 11–14]. In the considered setup, one requires only short classical control pulses for preparation of the qubit state [34]. Moreover, recent theoretical [35] and experimental studies [36] suggest that one can use single-flux quantum pulses for this purpose. Such an approach allows one to integrate the control electronics along with the resonators and artificial atoms on a single chip, which reduces the length of interconnects and brings most of the setup components into the cryogenic stage.

In the considered scheme, the CRA represents an engineered electromagnetic environment with a bandgap. The frequency of the controlling qubit is tuned to fall within that bandgap, which inhibits the Purcell relaxation of the qubit and improves the performance of the switch.

We provide a fully quantum-mechanical description of the single-photon wave packet transport in the system under consideration. The dependence of the switching contrast on the system parameters is studied. A set of

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parameters of the system providing the maximal switching contrast is determined.

The paper is organized as follows. In Sec. II we describe the scheme, the principle of operation, and the possible cQED implementation of the proposed single-photon switch. The model Hamiltonian of the studied system is given as well. In Sec. III we derive the effective Hamiltonian of the system and use it to describe a single-photon transport. The results of calculations of the dependence of a switching contrast on the system parameters are demonstrated in Section IV. In Sec. V we discuss possible extensions and applications of the considered switch and summarize the results. Derivations of various equations of motion used in the main text are presented in Appendix A.

II. SETUP

A. Scheme and operational principle

We consider a realization of the single-photon switch consisting of an array (chain) of an odd number $N_{\text{res}} = 2N + 1$ of optical resonators. The terminal resonators of an array are coupled to semi-infinite one-dimensional optical waveguides marked with indices 1 and 2. In what follows, we assume that the first waveguide acts as an input dispatching the ingoing single-photon wave packet to the CRA, while the second waveguide acts as an output channeling the scattered (transmitted) photon. The central resonator of the array is coupled to a 3LS (qutrit). In practice, the latter is represented by a superconducting artificial atom. All resonators in the array, apart from the central resonator, have identical frequencies $\omega_r$. The central resonator has frequency $\omega_c$. Each resonator is coupled to its nearest neighbors with strength $J$. The schematic of the considered setup is presented in Fig. 1.

First, let us elucidate the principle of operation of the proposed single-photon switch. The switching unit, which controls the photon transport in the setup we consider, consists of a resonator coupled to a 3LS or qutrit. We use the conventional notation for the qutrit eigenstates, where $|g\rangle$ stands for the ground state, and $|e\rangle$ and $|f\rangle$ are excited states. The eigenstates form a ladder configuration implying that only $|g\rangle \leftrightarrow |e\rangle$ and $|e\rangle \leftrightarrow |f\rangle$ transitions are allowed. The transition frequency $\omega_{ef}$ between $|e\rangle$ and $|f\rangle$ levels is tuned in resonance with the resonator frequency $\omega_c$. On the contrary, the transition frequency $\omega_{eg}$ between $|g\rangle$ and $|e\rangle$ states is strongly detuned from the resonator frequency, which inhibits the excitation exchange between the $|g\rangle \leftrightarrow |e\rangle$ transition and the resonator mode. Such an interaction regime between the resonator mode and the $|g\rangle \leftrightarrow |e\rangle$ transition is referred to as the dispersive coupling regime [37].

Now, let us qualitatively explain how the resonator-qutrit system provides control over the photon scattering. For this purpose, we consider a simplified version of the single-photon switch, which is represented by the resonator-qutrit system directly coupled to a pair of semi-infinite one-dimensional waveguides acting as input and output. The scheme of this setup is demonstrated in Fig. 2. In such a system, the transmission of itinerant photons from the input waveguide to the output waveguide can be controlled by manipulating the state of the qubit encoded by the pair of the lowest states of the qutrit, namely, $|g\rangle$ and $|e\rangle$. When one prepares the qubit in the ground state $|g\rangle$, the resonator-qutrit system acts effectively as just a resonator alone, since the interaction between the resonator mode and the $|g\rangle \leftrightarrow |e\rangle$ transition is dispersive. When the control qubit is prepared in the state $|e\rangle$, the transitions between $|e\rangle$ and $|f\rangle$ states can occur due to excitation exchange with the resonator,

\[ 1\] Note that the frequency of this “effective” resonator is slightly shifted compared to the frequency of the “bare” (uncoupled) resonator. This shift is induced by the dispersive interaction with the $|g\rangle \leftrightarrow |e\rangle$ transition of the qutrit (see details Sec. III.A).

FIG. 1. Scheme of the single-photon switch under analysis. A coupled-resonator array (CRA) is coupled on both sides to semi-infinite waveguides (marked with indices 1 and 2). A switching unit, highlighted by a shaded area, is composed of a resonator coupled to a 3LS (qutrit). The level structure of the 3LS is shown in the inset.
The CRA can act as such an environment. Assuming the dispersion relation $E$ that exhibits the dipole induced transparency (DIT). In this arrangement, the excited state $|e\rangle$ relaxes to the ground state $|g\rangle$, which deteriorates the performance of the waveguide-resonator-qutrit switch described above and illustrated in Fig. 2. To remedy this limitation and improve the switching efficiency, instead of coupling the resonator-qutrit system directly to the waveguides (as shown in Fig. 2), we embed the former into an engineered electromagnetic environment with a bandgap, where photons cannot propagate. The CRA can act as such an environment. Assuming that $\omega_c \approx \omega_r$, the CRA composed of $N_{\text{res}}$ resonators exhibits the dispersion relation $E_n = \omega_r - 2J \cos q_n$, where $q_n = n\pi/(N_{\text{res}}+1)$. Thus, the CRA features a pass-band of width $4J$ centered around $\omega_\text{e}$ [41]. We can harness this property and specifically design the energy levels of an artificial atom (qutrit) in a way, that the transition frequency $\omega_\text{ef}$ lies within the pass-band of the CRA, while $\omega_\text{pe}$ falls into its bandgap. In this case, the Purcell relaxation of the state $|e\rangle$ to the state $|g\rangle$ is completely suppressed [42].

### B. Circuit QED implementation

Let us briefly discuss a possible experimental realization of the proposed device within the superconducting cQED architecture. Microwave superconducting circuits provide a versatile and scalable hardware platform for the implementation of QIP devices [44]. Josephson-junction artificial atoms [45] are genuinely multilevel systems offering tunable level structure and transition frequencies. The cQED realization of the model system illustrated in Fig. 1 can be as follows. The CRA is composed of coplanar waveguide resonators (CPWR) [46] interacting via capacitive couplings, which can be made either fixed or tunable. The latter is achieved by coupling resonators via SQUIDs [47, 48], which allows one to individually control the interaction strength between the resonators by changing the external flux through each SQUID loop. However, the payoff for tunability is the increase of the setup complexity. A pair of microwave coplanar transmission lines coupled to the terminal resonators of the CRA serves as semi-infinite one-dimensional waveguides. The central resonator in the CRA is coupled to a transmon-type [49] superconducting artificial atom featuring a ladder-type structure of energy levels. This type of superconducting artificial atoms and its modifications [50, 51] offer high coherence times and tunable couplings, which makes it widely utilized for the building of various QIP devices [52]. The outlined cQED incarnation of the single-photon switch is feasible for the current technolo-

![FIG. 2. Scheme of the simplified version of the single-photon switch. The plot shows the dependence of the single-photon transmittance on the incoming photon frequency for the uncoupled resonator (solid line) and the resonator coupled to a 2LS with strength $g_{ef}$ (dashed line). The transmittance exhibits the maximum on the resonator frequency $\omega_c$ for the uncoupled resonator and the minimum for the resonator coupled to a 2LS.](image)

![FIG. 3. Schematic illustration of the possible cQED implementation of the considered single-photon switch. The CRA is comprised of capacitively coupled CPWRs. The CPWRs are arranged similarly to that in Ref. [43]. The state of the transmon is prepared using the control line (CL). Both sides of the CRA are coupled to coplanar transmission lines (TL1 and TL2). In this particular setup, all resonator frequencies and couplings are fixed and set on a fabrication stage.](image)
gies. The sketch of this cQED setup is shown in Fig. 3.

C. Model Hamiltonian

The Hamiltonian describing the model system outlined in Sec. II A reads as

\[ \hat{H} = \hat{H}_t + \hat{H}_{t-r} + \hat{H}_s + \hat{H}_w + \hat{H}_{w-r}. \]  (1)

The first term in Eq. (1) is the Hamiltonian of 2N identical resonators with frequencies \( \omega_i \):

\[ \hat{H}_t = \hbar \sum_{n=1}^{N} \omega_i \left( a_n^\dagger a_n - a_n^\dagger a_n - a_n^\dagger a_n \right), \]  (2)

where \( a_n \) is the annihilation operator of a photon in the \( n \)-th resonator of the CRA obeying the equal-time commutator \([a_n, a_{n'}^\dagger] = \delta_{n,n'}\). In what follows, the subscript \( n \) is reserved for the resonator indices running sequentially from \(-N\) to \( N \). The index \( n = 0 \) is attributed to the central resonator.

The second term in Eq. (1) describes the nearest-neighbor coupling between the resonators in the array. The Hamiltonian \( \hat{H}_{t-r} \) reads

\[ \hat{H}_{t-r} = \hbar J \sum_{n=-N}^{N} (a_n^\dagger a_{n+1}^\dagger + a_{n+1} a_n). \]  (3)

The term \( \hat{H}_s \) describes the switching unit – the system composed of the central resonator coupled to the ladder-configuration 3LS. The Hamiltonian \( \hat{H}_s \) reads

\[ \hat{H}_s = \hbar \omega_c a_0^\dagger a_0 + \hbar \omega_r \sigma_{ee} + \hbar (\omega_ge + \omega_ef) \sigma_{ff} \]

\[ + \hbar g_{ge}(a_1^\dagger \sigma_{ge} + \sigma_{eg}a) + \hbar g_{ef}(a_1^\dagger \sigma_{ef} + \sigma_{fe}a). \]  (4)

The first term in Eq. (4) describes the central resonator. The second and the third terms constitute the Hamiltonian of the qutrit (3LS). The last pair of terms in Eq. (4) describe the coupling between the qutrit and the central resonator. In Eq. (4), we have introduced a ladder operator \( \sigma_{kl} = \ket{k} \bra{l} \), where \( k, l \in \{g, e, f\} \). This operator obeys the commutation relation as follows:

\[ [\sigma_{kl}, \sigma_{k'l'}] = \sigma_{k'l'} \delta_{k'l} - \sigma_{kl} \delta_{k'l'}. \]  (5)

Parameters \( g_{ge} \) and \( g_{ef} \) stand for the coupling strengths between the central resonator and \( \ket{g} \leftrightarrow \ket{e} \) and \( \ket{e} \leftrightarrow \ket{f} \) transitions, correspondingly. For the transmon, these couplings are related as \( g_{ef}/g_{ge} \approx \sqrt{2} \) [49].

The resonator-resonator and resonator-qutrit couplings are described within the rotating-wave approximation (RWA). The latter is valid provided that the following criteria are satisfied

\[ |\omega_i - \omega_i| \ll \omega_t + \omega_c, \]

\[ |\omega_{ge(ef)} - \omega_i| \ll \omega_{ge(ef)} \pm \omega_c, \]

\[ J \ll \omega_t, \omega_c, \quad g_{ge} \ll \omega_{ge}, \omega_c, \quad g_{ef} \ll \omega_{ef}, \omega_c. \]  (6a)

While there are experimental demonstrations of ultrastrong coupling [the criterion (6b) breaks down] between the resonator and the artificial atom in cQED [53, 54], the use of the RWA is well justified for the range of parameters we use in the paper.

The waveguides are described by the Hamiltonian

\[ \hat{H}_w = \hbar \int_0^\infty d\omega \sum_{j=1}^{2} b_{j,\omega}^\dagger b_{j,\omega}. \]  (7)

The bosonic operator \( b_{j,\omega} \) annihilates a photon with frequency \( \omega \) in the \( j \)-th waveguide and obeys the commutation relation \([b_{j,\omega}, b_{j',\omega'}^\dagger] = \delta(\omega' - \omega)\delta_{jj'}\).

The Hamiltonian \( \hat{H}_{w-r} \), which describes the couplings between the waveguides and the CRA, reads

\[ \hat{H}_{w-r} = \hbar \int_0^\infty d\omega f_1(\omega) \left( b_{1,\omega}^\dagger a_{N} + a_{N}^\dagger b_{1,\omega} \right) \]

\[ + \hbar \int_0^\infty d\omega f_2(\omega) \left( b_{2,\omega}^\dagger a_{N} + a_{N}^\dagger b_{2,\omega} \right), \]  (8)

where \( f_j(\omega) \) stands for the frequency-dependent coupling of the CRA to the \( j \)-th waveguide. The coupling between the \( j \)-th waveguide and the CRA gives rise to the photon exchange between them with rate \( \kappa_j = 2\pi f_j^2(\omega) \) (see details in Appendix A). The Hamiltonian \( \hat{H}_{w-r} \) in Eq. (8) is given within the RWA, assuming that \( \kappa_j \ll \omega_i \).

In our model, we do not account for the dissipation processes, assuming that they occur on timescales much longer than the coherent processes in the system. Indeed, the internal quality factor of CPWRs [55, 56] can surpass \( 10^6 \), which corresponds to the resonator dissipation rate \( \gamma_{res}/(2\pi) \lesssim 0.01 \text{ MHz} \). The probability of photon loss in an individual resonator is determined as \( \Lambda_{res} \approx \gamma_{res}\tau_{res} \), with \( \tau_{res} \sim 1/(2J) \) being a photon lifetime in a resonator.\(^3\) Since the processes of dissipation in each resonator composing the CRA are independent, the photon loss probability in the CRA is determined as \( \Lambda_{cra} \approx N_{res}\Lambda_{res} \). Taking the typical parameters \( J/(2\pi) \sim 10 \text{ MHz} \) and \( N_{res} \sim 10 \), one arrives at the estimate for the photon loss probability in the CRA \( \Lambda_{cra} \lesssim 0.01 \). Thus, the dissipation has a minor effect on photon transport through the CRA. The wave packet travel time through the CRA is \( \tau_{vcl} \approx \tau_p + N_{res}\tau_{res} \), which gives the estimate \( \tau_{vcl} \lesssim 1 \mu s \), while the coherence times of the modern transmons approach \( 100 \mu s \) [57–59], implying that the relaxation of the artificial atom can be neglected in the analysis of the photon transport.

\(^3\) The photon lifetime in the terminal resonators is estimated as \( \tau_j \sim 1/(J + \kappa_j) \) with \( j \in \{1, 2\} \). However, assuming that \( \kappa_j \) and \( J \) are of the same order of magnitude, the estimate \( \tau_j \sim \tau_{res} \) is applicable.
III. SINGLE-PHOTON TRANSPORT

A. The effective Hamiltonian

As was mentioned in Sec. II A, to suppress the excitation exchange between the control qubit and the central resonator, the frequencies of the resonator and \( |g\rangle \leftrightarrow |e\rangle \) transition of the qutrit are strongly detuned from each other. Provided that the condition

\[
|\lambda| \ll 1, \quad \lambda = \frac{g_{ee}}{\omega_{ge} - \omega_c}, \tag{9}
\]

holds [37], one can treat the interaction between the resonator and \(|g\rangle \leftrightarrow |e\rangle\) transition perturbatively and eliminate the interaction term \(a^\dagger \sigma_{ge} + \sigma_{eg} a\) in the Hamiltonian (4) using the unitary transformation [37, 60]:

\[
\hat{H} \rightarrow \hat{H}' = e^{-\lambda S} \hat{H} e^{\lambda S}, \quad \hat{S} = a_0^\dagger \sigma_{ge} - \sigma_{eg} a_0. \tag{10}
\]

For deriving the transformed Hamiltonian \(\hat{H}'\), we use Eq. (1) along with the Baker-Campbell-Hausdorff relation

\[
\hat{H}' = e^{-\lambda S} \hat{H} e^{\lambda S} = \hat{H} + \lambda [\hat{H}, \hat{S}] + \frac{\lambda^2}{2} [[\hat{H}, \hat{S}], \hat{S}] + \ldots ,
\]

where we keep only the terms contributing up to first order in \(\lambda\). The form of terms \(\hat{H}_t, \hat{H}_{t-r}, \hat{H}_{t1}\), and \(\hat{H}_{t1-r}\) are retained after applying the transformation, while the Hamiltonian of the resonator-qutrit system acquires the form \(e^{-\lambda S} \hat{H}_s e^{\lambda S} = \hat{H}'_s:\)

\[
\hat{H}'_s = \hbar (\omega_c + \chi \hat{Z}_{eg}) a_0^\dagger a_0 + \hbar (\omega_{ge} + \chi) \sigma_{ee} + \hbar (\omega_{ge} + \omega_{ef}) \sigma_{ff} + \hbar g_{ef} (a_0^\dagger \sigma_{ef} + \sigma_{fe} a_0), \tag{12}
\]

where \(\hat{Z}_{eg} = \sigma_{ee} - \sigma_{gg}\) and \(\chi = \lambda g_{eg}\). In the transformed Hamiltonian \(\hat{H}'\), we dropped \(\lambda J a_0^\dagger \sigma_{ge}, \lambda g_{eg} a_0^\dagger \sigma_{gf}\) and their conjugates, since these terms contribute in the order of \(\lambda^2\).

Since \([\sigma_{ee} + \sigma_{ff}, \hat{H}'] = 0\), it is convenient to make a transformation

\[
\hat{H} \rightarrow \hat{H} - \hbar (\omega_{ge} + \chi) (\sigma_{ee} + \sigma_{ff}), \tag{13}
\]

which turns \(\hat{H}'_s\) into the Hamiltonian as follows

\[
\hat{H}'_s = \hbar \omega_c a_0^\dagger a_0 + \hbar \omega_s \sigma_{ff} + \hbar g_{ef} (a_0^\dagger \sigma_{ef} + \sigma_{fe} a_0), \tag{14}
\]

where \(\omega_c = \omega_c + \chi \hat{Z}_{eg}\) stands for the qubit-state-dependent frequency of the “dressed” central resonator and \(\omega_s = \omega_{ef} - \chi\) denotes the frequency of the “dressed” \(|e\rangle \leftrightarrow |f\rangle\) qutrit transition. In what follows, for the description of the system dynamics, we use the Hamiltonian \(\hat{H}'\) with \(\hat{H}'_s\) expressed by Eq. (14).

B. Scattering dynamics

The probability of finding the photon at time \(t\) in the output waveguide for the control qubit prepared in one of its eigenstates \((|g\rangle \text{ or } |e\rangle)\) is determined as

\[
\mathcal{T}_q(t) = \int_0^\infty d\omega \left| \langle \psi_{2,\omega}^q(t) | \Psi_{\text{in}}^q \rangle \right|^2, \quad q \in \{g, e\}, \tag{15}
\]

where

\[
|\psi_{2,\omega}^q(t)\rangle = b_{2,\omega}(t) |\varrho_q\rangle, \\
|\varrho_q\rangle = |q\rangle |\varrho\rangle_{w1} |\varrho\rangle_{w2} \bigotimes_{n=-N}^N |\varrho\rangle_{nr}. \tag{16}
\]

The state \(|\psi_{2,\omega}^q(t)\rangle\) corresponds to the state of the system hosting a single photon of frequency \(\omega\) propagating in the second (output) waveguide, the qubit residing in the excited state \(|q\rangle\) and void of excitations in the CRA and the first (input) waveguide.

In Eq. (16), the state \(|\Psi_{\text{in}}^q\rangle\) stands for the initial (at \(t = 0\)) state of the entire system. We set that initially the single-photon wave packet propagates in the input waveguide, while the CRA and the output waveguide contain no photons. We assume that the number of thermal excitations \(n_{th}\) in the system is negligible. Superconducting cQED systems typically operate at frequencies \(\omega_s/(2\pi) \sim 3–8 \text{ GHz}\) and the temperature of the cryogenic stage \(T_s \sim 10–20 \text{ mK}\) [45]. For that frequency range and setup working temperature, the upper estimate for the thermal photon number in the system is \(n_{th} < 10^{-3}\) assuming the Bose-Einstein distribution of thermal photons \(n_{th} = \exp(-\frac{\hbar \omega_s}{k_B T_s}) - 1\) , where \(k_B\) is the Boltzmann constant. Thus, the initial state of the system \(|\Psi_{\text{in}}^q\rangle\) reads

\[
|\Psi_{\text{in}}^q\rangle = |q\rangle |1_\xi\rangle_{w1} |\varrho\rangle_{w2} \bigotimes_{n=-N}^N |\varrho\rangle_{nr}, \tag{17}
\]

where \(|\varrho\rangle_{wj}\) is a state of the \(j\)-th waveguide void of photons, and \(|\varrho\rangle_{jtr}\) is a vacuum state of the \(n\)-th resonator in the CRA. The state \(|1_\xi\rangle_{w1}\) defined as

\[
|1_\xi\rangle_{w1} = \int_0^\infty d\omega \xi(\omega) b_{1,\omega}^\dagger(0) |\varrho\rangle_{w1}, \tag{18}
\]

stands for the state of the first (input) waveguide accommodating a single-photon wavepacket characterized by the spectral distribution function [61] denoted as \(\xi(\omega)\).

The probability of photon transmission \(\mathcal{T}_q(t)\) is governed by the evolution equation as follows (the derivation is given in Appendix A 2):

\[
\mathcal{T}_q(t) = \kappa_2 \int_0^t d\tau |\langle \varrho_q| a_N(\tau) |\Psi_{\text{in}}^q \rangle|^2. \tag{19}
\]

Let us write down the equation of motion governing the matrix element \(\langle \varrho_q| a_N(t) |\Psi_{\text{in}}^q \rangle\) standing on the rhs
FIG. 4. Dependence of the switching contrast on the interrelation between the waveguide-CRA photon exchange rates and the photon hopping rate in the CRA for different durations of the ingoing pulse: (a) \( \tau_p = 0.1 \mu s \), (b) \( \tau_p = 0.5 \mu s \), and (c) \( \tau_p = 0.9 \mu s \). Stars mark the position of the maximal contrast: (a) \( C_{\text{max}} = 0.956 \), (b) \( C_{\text{max}} = 0.989 \), and (c) \( C_{\text{max}} = 0.993 \). The rest of the parameters are the following: \( J/(2\pi) = 10 \text{ MHz}, \ g_{\text{ef}}/(2\pi) = 30 \text{ MHz} \).

of Eq. (19). Using the Heisenberg equations for the CRA variables (see Appendix A), one obtains the evolution equation for \( A_n(t) \) as follows

\[
\text{i} \hbar \dot{A}_n^q(t) = \left[ \omega_n A^q_n(t) + J \left[A^q_{n-1}(t) + A^q_{n+1}(t) \right] \right],
\]

where \( |n| \in [1,N] \). Here we introduced a notation \( A_n(t) = \langle \mathcal{S}_q |a_n(t)| \Psi_\text{in}^q \rangle \). For \( n = 0 \) (the central resonator), one has

\[
\text{i} \hbar \dot{A}_0^q(t) = [\omega_c + (2n_q - 1)\chi] A_0^q(t) + J \left[A^q_{-1}(t) + A^q_{1}(t) \right] + g_{\text{ef}} S_{\text{ef}}^q(t),
\]

where \( n_q = \langle \psi |q \rangle \). In Eq. (21), we introduced a notation \( S_{\text{ef}}^q(t) = \langle \mathcal{S}_q |\sigma_{\text{ef}}(t)| \Psi_\text{in}^q \rangle \). Using the Heisenberg equation for the ladder operator \( \sigma_{\text{ef}} \), the equation of motion governing \( S_{\text{ef}}^q(t) \) reads

\[
\text{i} \hbar \dot{S}_{\text{ef}}^q(t) = \omega_a S_{\text{ef}}^q(t) + \eta_q g_{\text{ef}} A_0^q(t).
\]

Finally, for \( n = \pm N \), one arrives at the equations of motion as follows (see Appendix A)

\[
\text{i} \hbar \dot{A}_N^q(t) = \left( \omega_t - \text{i} \frac{\kappa_2}{2} \right) A_N^q(t) + JA_{N-1}^q(t),
\]

\[
\text{i} \hbar \dot{A}_{-N}^q(t) = \left( \omega_t - \text{i} \frac{\kappa_1}{2} \right) A_{-N}^q(t) + JA_{1-N}^q(t) + f_1(\omega_c) \Xi(t),
\]

where \( \omega_t \) is the central (carrier) frequency of the ingoing wave packet. Function \( \Xi(t) \) is defined as

\[
\Xi(t) = \int_{-\infty}^{\infty} \text{d} \omega e^{-\text{i} \omega t} \xi(\omega) = \sqrt{2\pi} \varrho(-t),
\]

where \( \varrho(t) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} \text{d}\omega e^{\text{i} \omega t} \xi(\omega) \) describes the time-domain probability density amplitude of the ingoing pulse.

For computations we model the spectral distribution of the ingoing pulse \( \xi(\omega) \) by the Lorentzian function

\[
\xi(\omega) = \frac{1}{2\pi \tau_p} \left( \omega - \omega_0 \right)^{-1},
\]

which corresponds to the decaying exponent profile of the time-domain probability density amplitude

\[
\varrho(\tau) = \frac{1}{\sqrt{2\tau_p}} \exp \left( \frac{\tau}{2\tau_p} \right) \theta(-\tau),
\]

where \( \tau_p \) stands for the ingoing pulse duration and \( \theta(\tau) \) is the Heaviside step function. For convenience, we assume that the front of the ingoing pulse, which initially propagates in the first waveguide, reaches the CRA terminal resonator at instant \( t = 0 \).

We solve the system of differential equations (20)–(23b) numerically using \texttt{NDSolve} function of \textsc{Mathematica}.

IV. SWITCHING CONTRAST

As a measure of the efficiency of the considered single-photon switch, we use a quantity given by

\[
C = T_p(t_{\infty}) - T_c(t_{\infty}),
\]

which is referred to as a switching contrast by analogy with a measurement contrast employed for the characterization of the accuracy of qubit measurement [62, 63]. In Eq. (27), \( t_{\infty} \) is attributed to the time, when all scattering processes in the system are finished and the scattered photon propagates in one of the waveguide as a free excitation. It is determined by the criterion \( t_{\infty} \gg \tau_{\text{tvl}} \), where \( \tau_{\text{tvl}} = \tau_p + N_{\text{res}}/(2J) \) is a photon travel time through the CRA. For computations, we set \( t_{\infty} = 10\tau_p \).

We tune the frequency of the “bare” central resonator \( \omega_c \) to satisfy the relation \( \omega_c - \chi = \omega_t \). Thus, when one prepares the control qubit in its ground state \( |g \rangle \), the frequency of the “dressed” central resonator \( |\bar{g} \rangle \) is matched to the frequencies of the other resonators in the CRA. In this case, the incident photon propagates through the chain of resonators with identical couplings \( J \) and frequencies \( \omega_t \), resulting in the maximal transmission. Using that \( \chi = g_{\text{ge}}/(\omega_c - \omega_{\text{ge}}) \), one arrives
and demonstrates that the proposed single-photon switch can provide high contrasts for realistic values of $\kappa/J$ and $g_{ef}/J$ for the ingoing pulse durations (a) $\tau_p = 0.1 \mu s$ and (b) $\tau_p = 0.8 \mu s$. The dashed line marks the position of maximum $C_{\text{max}}$ for given $g_{ef}/J$. For calculations, we use the parameters as follows: $J/(2\pi) = 10 \text{ MHz}, \omega_r/(2\pi) = 7.0 \text{ GHz}$, and $\omega_{ef}/(2\pi) = 7.36 \text{ GHz}$.

at the relation between $\omega_c$ and $\omega_f$ as follows

$$\omega_c = \frac{1}{2} \left( \omega_f + \omega_{ge} + \sqrt{(\omega_f - \omega_{ge})^2 - g_{ge}^2} \right). \quad (28)$$

The frequency of the $|e\rangle \leftrightarrow |f\rangle$ transition is set in such a way that when the qubit is prepared in its excited state $|e\rangle$ one has $\langle e|\omega_c|e\rangle = \omega_c + \chi = \omega_a$. Thus, in this scenario, the qutrit transition $|e\rangle \leftrightarrow |f\rangle$ is “switched on” and its “dressed” frequency coincides with that of the central resonator that gives rise to the DIR effect leading to photon reflection. Recalling that $\omega_a = \omega_{ef} - \chi$ and using Eq. (28), one obtains

$$\omega_{ef} = \omega_f + \frac{6g_{ge}^2}{\omega_f - \omega_{ge} + \sqrt{(\omega_f - \omega_{ge})^2 - g_{ge}^2}}. \quad (29)$$

Now, let us proceed to the analysis of the performance of the proposed single-photon switch scheme. Calculations of the dependence of the switching contrast $C$ on the interrelation between the photon hopping rate $J$ and the CRA-waveguides exchange rates $\kappa_{1,2}$ shown in Fig. 4 demonstrate that the maximal contrast $C_{\text{max}}$ (for given values of $J$ and $g_{ef}$) is achieved when the CRA is equally coupled to both waveguides, i.e., $\kappa_1 = \kappa_2$. In what follows, we consider only this (symmetric) configuration of the setup. In this regard, from now on, we use a notation $\kappa \equiv (\kappa_1 = \kappa_2)$ for brevity.

Dependence of the switching contrast on the qutrit-resonator coupling and the ingoing pulse duration is shown in Figs. 5 and 6a. Computations reveal that the contrast improves with the increase of $g_{ef}/J$. The explanation is as follows. Assume that one prepares the qubit in the excited state $|e\rangle$. Since we tune the “bare” frequencies of the central resonator and the $|e\rangle \leftrightarrow |f\rangle$ transition to obtain a resonance of the “dressed” frequencies $\omega_c + \chi = \omega_a$, the single-excitation eigenfrequencies of the Jaynes-Cummings (JC) system composed of the central resonator and the 2LS formed by the qutrit levels $|e\rangle$ and $|f\rangle$, are given by $E_{\pm} = \omega _r \pm g_{ge}$. Since we set $\omega_c - \chi = \omega_r$, the eigenstates of the JC system are detuned from the frequencies of the neighbor resonators on $\delta \pm = E_{\pm} - \omega_r = \lambda g_{ge} \pm g_{ef}$. For $J < |\delta \pm|$, photon hops from the resonator with index $n = -1$ on the central resonator start to be suppressed, which leads to photon reflection. For the transmon, couplings $g_{ge}$ and $g_{ef}$ are of the same order of magnitude ($g_{ef} \approx \sqrt{2}g_{ge}$ [49]). Thus, the dominant contribution to the absolute value of the detuning $|\delta \pm|$ comes from $g_{ef}$ since $|\lambda| \ll 1$ due to Eq. (9). Therefore, the increase of $g_{ef}/J$ results in the larger probability of photon reflection and, thus, higher switching contrast.

Figure 6 demonstrates that the proposed single-photon switch can provide high contrasts for realistic values of the resonator-resonator and resonator-qutrit couplings and a wide range of the ingoing pulse durations. For $J/(2\pi) = 10 \text{ MHz}$ and $g_{ef}/(2\pi) \sim 30$–50 MHz, the con-
TABLE I. Realistic parameters of the setup to achieve high switching contrasts $C > 0.95$ for the sub-$\mu$s ingoing pulses.

| $\omega_1/2\pi$ (GHz) | $\omega_{ge}/2\pi$ (GHz) | $\omega_{rf}/2\pi$ (GHz) | $\alpha/2\pi$ (MHz) | $\omega_c/2\pi$ (GHz) | $J/2\pi$ (MHz) | $\kappa/2\pi$ (MHz) | $g_{ef}/2\pi$ (MHz) | $\tau_p$ (ns) | $C$ |
|------------------------|------------------------|------------------------|-------------------|-------------------|-------------|-------------|-------------------|-------------|-----|
| 7.000                  | 7.370                  | 6.808                  | −361.73           | 7.008             | 15.0        | 32.63       | 45.0              | 0.075       | 0.957 |
| 7.000                  | 7.340                  | 7.000                  | −332.89           | 7.010             | 10.0        | 22.05       | 40.0              | 0.250       | 0.981 |
| 7.000                  | 7.390                  | 7.000                  | −381.07           | 7.009             | 12.0        | 27.72       | 48.0              | 0.500       | 0.990 |

contrasts $C > 0.95$ can be achieved for the ingoing pulses of duration $\tau_p > 0.07\mu$s, while for the longer pulses $\tau_p > 0.55\mu$s the contrasts $C > 0.99$ can be reached.

To sum up the quantitative analysis of the performance of the proposed single-photon switch, we present Table I aggregating several sets of setup parameters for reaching $C > 0.95$ for the sub-$\mu$s ingoing pulses. All parameters presented in Table I are achievable for the state-of-the-art superconducting cQED systems.

All numerical results demonstrated in Figs. 4–6 and Table I were obtained for the CRA composed of $N_{\text{res}} = 7$ resonators. To satisfy the criterion (9) of the dispersive regime of interaction between the resonator and the $|g\rangle \leftrightarrow |e\rangle$ qutrit transition, we keep $\lambda < 0.1$ for all computations unless stated otherwise. The relative anharmonicity $\alpha_{\text{rel}} = (\omega_{ef} - \omega_{ge})/\omega_{ge}$ of energy levels of the typical transmon artificial atom is around $−0.05$ [49]. Thus, in all calculations we choose the setup parameters in such a way that the relative anharmonicity of the qutrit is $−0.06 \leq \alpha_{\text{rel}} \leq −0.04$.

V. DISCUSSION AND SUMMARY

Having analyzed the performance of the proposed single-photon switch, let us discuss its possible applications. By inserting a circulator into the first (input) waveguide of the switch, one can implement a two-port quantum router. A non-reciprocal element (circulator) is required for the separation of the input and reflected signals into the different channels. Since in this scheme, both the signal and control information are quantum, one can regard the considered router as genuinely quantum [64]. The multi-port routing can be achieved by connecting a number of those two-port single-photon routers in a cascade configuration as proposed, e.g., in Refs. [15, 23]. The scheme of this multi-port router is shown in Fig. 6. Recent advances in the demonstration of on-chip microwave circulators [65, 66] paves the way to entirely on-chip realization of the multi-port quantum router for microwave photons.

Besides the multi-port single-photon router, one can harness the proposed switch to implement the high-fidelity readout of superconducting artificial atoms using the single-photon probe pulses. As it was pointed out in Ref. [63], the use of the single-photon probe allows one to avoid the readout errors arising from the nonorthogonality of the probe state, which is always the case for the coherent-state readout pulses. The high-efficient on-demand sources of microwave single-photon pulses are readily available [67, 68]. Moreover, one can employ a detector of itinerant photons [69] attached to the output waveguide of the switch to provide a “click” for a particular state of the qubit [63]. When one prepares the qubit in the ground state, the probe photon is transmitted through the switch to the output waveguide and the photodetector clicks. Conversely, when the qubit is prepared in the excited state, the switch is reflective, and the probe photon can not reach the detector. In this case, the latter gives no click. Promising theoretical proposals [70–75], as well as recent experimental demonstrations [76–80] of itinerant microwave photon detectors, allow us to be optimistic about the perspectives of the near-term realization of the scheme for the superconducting qubit readout outlined above.

To summarize, we have proposed a scheme of an efficient single-photon switch and examined its performance in detail. The possible superconducting cQED realization of the considered single-photon switch was outlined. We have demonstrated that parameters of the setup required for achieving high switching contrasts are feasible for the state-of-the-art superconducting cQED devices. A few

![FIG. 7. Scheme of a five-port single-photon router composed of a series of two-port routers. A two-port router is built by embedding a circulator into the input waveguide of a proposed single-photon switch. When the control qubit is prepared in the ground state $|g\rangle$, the photon is routed to the output waveguide (port 2). The photon is routed to port 1 when one prepares the qubit in the excited state $|e\rangle$.](image-url)
applications of the proposed switch, namely, a multi-port quantum router and a scheme for a single-photon readout of a qubit state, were considered as well. Study of the effect of the system inhomogeneities (e.g., random variations of frequencies and couplings) on the efficiency of the considered switching scheme may be of interest. Besides, assessing the efficiency of the proposed switch in the regime of the multi-photon input constitutes the possible direction for the follow-up research.

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Appendix A: Derivation of equations of motion

1. Heisenberg equations

The effective Hamiltonian \( \hat{H}' \) generates the following Heisenberg equations for the CRA variables:

\[
\dot{i}a_n = \omega_r a_n + J(a_{n-1} + a_{n+1}). \tag{A1}\]

For the central resonator variable \( a_0 \), one has

\[
\dot{i}a_0 = (\omega_c + \chi Z_{eq}) a_0 + g_{ef} \sigma_{ef} + J(a_{-1} + a_1). \tag{A2}\]

The Heisenberg equations for the annihilation operators of a photon in the terminal resonators read as

\[
\dot{i}a_N = \omega_r a_N + Ja_{N-1} + \int_0^\infty d\omega f_2(\omega)b_{2,\omega}, \tag{A3a}\]

\[
\dot{i}a_{-N-1} = \omega_r a_{-N-1} + Ja_{-N-1} + \int_0^\infty d\omega f_1(\omega)b_{1,\omega}. \tag{A3b}\]

The waveguides variables \( b_{1,\omega} \) and \( b_{2,\omega} \) obey the equations of motion

\[
\dot{i}b_{1,\omega} = \omega b_{1,\omega} + f_1(\omega)a_{-N}, \tag{A4a}\]

and

\[
\dot{i}b_{2,\omega} = \omega b_{2,\omega} + f_2(\omega)a_N. \tag{A4b}\]

The formal solutions of these equation read

\[
b_{1,\omega}(t) = \tilde{b}_{1,\omega}(t) - i f_1(\omega) \int_0^t d\tau e^{-i\omega(t-\tau)} a_N(\tau), \tag{A5a}\]

\[
b_{2,\omega}(t) = \tilde{b}_{2,\omega}(t) - i f_2(\omega) \int_0^t d\tau e^{-i\omega(t-\tau)} a_{-N}(\tau), \tag{A5b}\]

where \( \tilde{b}_{1,\omega}(t) = b_{1,\omega}(0)e^{-i\omega t} \) denotes the annihilation operator of a free-propagating photon in the \( j \)-th waveguide.

Let us evaluate the integrals

\[
\mathcal{I}_j(t) = \int_0^\infty d\omega f_j(\omega)b_{j,\omega}(t), \quad j \in \{1, 2\},
\]

standing in Eqs. (A3). Using Eqs. (A5), one obtains

\[
\mathcal{I}_2(t) = \int_0^\infty d\omega f_2(\omega)\tilde{b}_{2,\omega}(t)
- i \int_0^t d\tau \int_0^\infty d\omega f_2(\omega)e^{-i\omega(t-\tau)} a_N(\tau), \tag{A6}\]

Consider the second term on the right-hand side of the above equation. It follows from Eq. (23b), that one can write \( a_N(t) = a_N(0)e^{-i\omega t} \), where \( a_N(t) \) represents a slowly-varying component of the operator \( a_N(t) \). Due to integration over \( \tau \), only the narrow region of frequencies in the vicinity of \( \omega \) gives the dominant contribution to the integral. Thus, one can assume \( f_2(\omega) \approx f_2(\omega) \). The lower boundary of integration over \( \omega \) can be extended to \(-\infty\). Using these approximations along with the property \( \int_{-\infty}^\infty d\omega e^{-i\omega(t-\tau)} = 2\pi\delta(t-\tau) \), one obtains

\[
\mathcal{I}_2 = \tilde{\mathcal{I}}_2 - \frac{\kappa}{2} a_N, \quad \mathcal{I}_1 = \tilde{\mathcal{I}}_1 - \frac{\kappa}{2} a_{-N}, \tag{A7}\]

where \( \kappa = 2\pi f_2(\omega) \) and \( \tilde{\mathcal{I}}_j \) is defined as

\[
\tilde{\mathcal{I}}_j(t) = \int_0^\infty d\omega f_j(\omega)\tilde{b}_{j,\omega}(t). \tag{A8}\]

For evaluation of \( \mathcal{I}_1(t) \), we employed the analogous reasons as those used for evaluation of \( \mathcal{I}_2(t) \). Finally, substituting Eq. (A7) into Eqs. (A3), one arrives at the result

\[
\dot{i}a_N = \left( \omega_r - \frac{\kappa}{2} \right) a_N + Ja_{N-1} + \tilde{\mathcal{I}}_2, \tag{A9a}\]

\[
\dot{i}a_{-N} = \left( \omega_r - \frac{\kappa}{2} \right) a_{-N} + Ja_{-N-1} + \tilde{\mathcal{I}}_1. \tag{A9b}\]

It follows from the above equations that parameter \( \kappa \) stands for the rate of the photon exchange between the CRA and the \( j \)-th waveguide.

Using the Hamiltonian (14) and the commutation relation (5), one obtains

\[
\dot{i}a_{\sigma_{ef}} = \omega_{\sigma_{ef}}a_{\sigma_{ef}} - g_{\sigma_{ef}} \tilde{Z}_{\sigma_{ef}}a_0, \tag{A10}\]

where \( \tilde{Z}_{\sigma_{ef}} = \sigma_{\sigma_{ef}} - \sigma_{\sigma_{ef}} \).

2. Equation of motion for \( T_q(t) \)

Let us derive the evolution equation for the transmission probability \( T_q(t) \) given by Eq. (15). Using Eq. (16) along with Eq. (A5), one obtains
\[ T_\eta(t) = \int_0^\infty d\omega |\langle \varphi_\eta | b_{2,\omega}(t) | \Psi_{in}^q \rangle|^2 = \int_0^t d\tau \int_0^t d\tau' f_2^r(\omega) e^{i\omega(\tau-\tau')} \langle \Psi_{in}^q | a_{N'}(\tau') | \varphi_\eta \rangle \langle \varphi_\eta | a_N(\tau) | \Psi_{in}^q \rangle, \]  

(A11)

where we used that \( \hat{b}_{2,\omega}(t)|\Psi_{in}^q\rangle = 0 \), which follows from Eqs. (17) and (18). Next, using the similar consideration that led us to Eqs. (A7), we extend the lower limit of integration over photon frequencies to \( -\infty \) and make an approximation \( f_2(\omega) \approx f_2(\omega_t) \). Now, integration over \( \omega \) gives \( 2\pi \delta(\tau-\tau') \) leading to Eq. (19).

### 3. Derivation of Eqs. (23)

Using Eqs. (A3) and (A8), one derives the equation of motion for \( A_{\pm N}^q(t) = \langle \varphi_\eta | a_{\pm N}(t) | \Psi_{in}^q \rangle \) as follows

\[ i\partial_t A_{N}^q(t) = \left( \omega_t - i\frac{K_q}{2} \right) A_{N}^q(t) + JA_{N-1}^q(t) + \int_0^\infty d\omega e^{-i\omega t} f_2(\omega) \langle \varphi_\eta | b_{2,\omega}(0) | \Psi_{in}^q \rangle. \]  

(A12a)

\[ i\partial_t A_{-N}^q(t) = \left( \omega_t - i\frac{K_1}{2} \right) A_{-N}^q(t) + JA_{-N+1}^q(t) + \int_0^\infty d\omega e^{-i\omega t} f_1(\omega) \langle \varphi_\eta | b_{1,\omega}(0) | \Psi_{in}^q \rangle. \]  

(A12b)

Let us consider the last terms on the rhs of the above equations. Employing Eq. (17) in Eq. (A12a), one obtains \( \langle \varphi_\eta | b_{2,\omega}(0) | \Psi_{in}^q \rangle = 0 \), which immediately leads to Eq. (23a). In Eq. (A12b), one has \( \langle \varphi_\eta | b_{1,\omega}(0) | \Psi_{in}^q \rangle = \xi(\omega) \) resulting in

\[ \int_0^\infty d\omega e^{-i\omega t} f_1(\omega) \xi(\omega) \approx f_1(\omega_0) \int_{-\infty}^\infty d\omega e^{-i\omega t} \xi(\omega), \]

where the above approximation is made under the assumption that the spectral distribution of the ingoing wave packet is strongly localized in the vicinity of its central frequency \( \omega_0 \), i.e., the wave packet is narrowband \( \tau_{pr}^{-1} \ll \omega_0 \). Thus, following the lines of derivation of Eqs. (A9), one can set \( f_1(\omega) \approx f_1(\omega_0) \) and extend the lower limit of integration on \( \omega \) to \( -\infty \). Combining this result with Eq. (A12b) and recalling the definition of \( \Xi(t) \) given by Eq. (24), one arrives at Eq. (23b).

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