The Theory and Applications of Measuring Broad-Range and Chromosome-Wide Recombination Rate from Allele Frequency Decay around a Selected Locus

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Abstract

Recombination is the exchange of genetic material between homologous chromosomes via physical crossovers. High-throughput sequencing approaches detect crossovers genome wide to produce recombination rate maps but are difficult to scale as they require large numbers of recombinants individually sequenced. We present a simple and scalable pooled-sequencing approach to experimentally infer near chromosome-wide recombination rates by taking advantage of non-Mendelian allele frequency generated from a fitness differential at a locus under selection. As more crossovers decouple the selected locus from distal loci, the distorted allele frequency attenuates distally toward Mendelian and can be used to estimate the genetic distance. Here, we use marker selection to generate distorted allele frequency and theoretically derive the mathematical relationships between allele frequency attenuation, genetic distance, and recombination rate in marker-selected pools. We implemented nonlinear curve-fitting methods that robustly estimate the allele frequency decay from batch sequencing of pooled individuals and derive chromosome-wide genetic distance and recombination rates. Empirically, we show that marker-selected pools closely recapitulate genetic distances inferred from scoring recombinants. Using this method, we generated novel recombination rate maps of three wild-derived strains of Drosophila melanogaster, which strongly correlate with previous measurements. Moreover, we show that this approach can be extended to estimate chromosome-wide crossover interference with reciprocal marker selection and discuss how it can be applied in the absence of visible markers. Altogether, we find that our method is a simple and cost-effective approach to generate chromosome-wide recombination rate maps requiring only one or two libraries.

Key words: recombination, crossovers, pooled sequencing, allele frequency.

Introduction

T.H. Morgan first envisioned the exchange of genetic material between homologous chromosomes through physical crossovers (Morgan 1911b). After backcrossing F1 heterozygotes, he recovered novel allelic combinations, or recombinants, of markers on the same chromosome that are absent in parental lines (Morgan 1911a). At face value, this appeared to violate Mendel’s law of segregation and the chromosome theory of inheritance (Blixt 1975). A.H. Sturtevant, then a prodigious undergraduate in Morgan’s laboratory, confirmed his mentor’s theory by crossing mutant Drosophila strains with different visible X-linked markers. Noticing that different pairs of markers produced recombinants at different frequencies, Sturtevant realized that the positions of these markers can be ordered on a linear map, with the frequency of recombinants as the genetic distance between any two markers (Sturtevant 1913). This was the birth of the very first genetic map (Brush 2002).

Measured as the number of recombinants divided by the total number of offsprings, the frequency of recombinants (henceforth recombinant fraction) does not reflect the true probability of crossovers between two loci (Castle 1919). This is because only odd numbers of crossovers are observable since even numbers of crossovers produce recombinants that maintain the allelic combinations of the parents (Sturtevant 1913; Sturtevant et al. 1919). With increasing genetic distance, the probability of multiple crossovers also increases causing recombinant fraction to deviate from true genetic distance. To account for multiple crossovers, mapping functions are applied to convert recombinant fractions (D) to true genetic distances (d). Although many mapping functions have been developed to account for different rates of crossover interference (Felsenstein 1979; McPeek and Speed 1995; Tan and Fornage 2008), the two most popular are Haldane’s (Haldane 1919) and Kosambi’s mapping functions (Kosambi 1943). Haldane’s mapping function (H) assumes no crossover interference and tends to overestimate genetic distance...
(Tan and Fornage 2008). Kosambi’s mapping function \((K)\) assumes that crossover interference is inversely proportional to the genetic distance between any two loci and produces estimates that are more consistent with crossing experiments (Huehn 2011). After these functions, the genetic distance is expected to be additive such that \(H(D_{AC}) = H(D_{AB}) + H(D_{BC})\) given three loci A, B, and C, in that order.

In the century after Morgan and Sturtevant conceptualized and devised the recombinant backcross scheme, recombination has been recognized as one of the most fundamental and universal biological processes across sexually reproducing eukaryotes with vast implications in many biological fields. For example, crossover events are crucial for the fidelity of chromosome segregation (see reviews, Page and Hawley 2004; Hunter 2015; Hughes et al. 2018) and recombination rate is intimately linked with the efficacy of natural selection and genome evolution (see reviews, Cutter and Payseur 2013; Martin and Jiggins 2017; Stephan 2019). Despite rapid technological and methodological advances, methods to measure recombinant fraction, genetic distance, and recombination rate remain grounded in the approach devised by Sturtevant—tallying the number of recombinants between loci that are either phenotypically or molecularly marked. The advent of whole-genome sequencing has permitted the generation of high-resolution crossover maps, by identifying regions of the genome where parental haplotypes change in recombinant individuals (Kulathinal et al. 2008; Rockman and Kruglyak 2009; Dumont et al. 2011; Comeron et al. 2012; Miller et al. 2012). More recently, single-cell sequencing technologies have further extended such high-throughput crossover detection directly from sperm cells (Hinch et al. 2019). Although providing impressive resolution often at the level of individual bases, these approaches are difficult to scale, as each recombinant individual/cell requires a separate library preparation and/or barcode. Since each chromosome, on average, has only one to two crossovers, hundreds to thousands of individuals, and therefore libraries/barcodes, need to be sequenced for a comprehensive map.

In addition to cross-based experimental measurements, recombination rate is also frequently estimated at the population level (Langley et al. 2000; McVean et al. 2004; Wang and Rannala 2008; Chan et al. 2012). Such population estimates, in short, are inferred from the breakdown of linkage patterns from historical recombination events. Recent developments have further allowed highly cost-effective and accurate population level estimates from single diploid genomes (Barroso et al. 2019) and pooled sequencing of unrelated individuals (Adrion et al. 2020). In conjunction with experimental approaches, these methods have been instrumental in revealing the importance of recombination on population dynamics and genome evolution.

However, the lack of a scalable method to experimentally estimate recombination rate at genome scale is particularly problematic since recombination rate is a highly labile phenotype (Ritz et al. 2017). It is sensitive to environmental stresses like temperature (Stern 1926), nutrition (Neel 1941), and infection (Singh et al. 2015), and changes with age (Redfield 1966). Furthermore, recombination rate differs drastically not only between closely related species (Jensen-Seaman et al. 2004; Kulathinal et al. 2008; Smukowski and Noor 2011; Brand et al. 2018) but also between individuals of the same species (Nachman 2002; Dumont et al. 2009, 2011; Stevison and Noor 2010; Comeron et al. 2012; Kaur and Rockman 2014; Hunter et al. 2016). Therefore, a scalable and cost-efficient approach is necessary to fully capture the extent of the variability of recombination rate and enrich our understanding of this fundamental yet volatile molecular process.

Instead of scoring recombinants or identifying crossover breakpoints in individuals from a typical backcross, we demonstrate here that recombination rate can be estimated from allele frequency (AF) attenuation around loci that cause distorted AF. When the homologous alleles across all loci have equal fitness, the AF of the progeny pool is expected to be Mendelian across the chromosome, regardless of the number of recombinants produced by crossovers in the F1 parents (fig. 1A). However, when the alleles at a given locus do not have the same fitness/viability, that is, a fitness differential, the AF at the locus will deviate from the Mendelian ratio. Importantly, the deviating AF is expected to show a signature attenuation pattern whereby it peaks at the distorting locus and attenuates distally toward Mendelian due to increasing number of crossover events between the locus and gradually more distal loci. Therefore, the AF and its rate of change should be related to the genetic distance and recombination rate, respectively. Since chromosome-wide AF can be determined using pooled whole-genome sequencing of the backcross progenies (Kofler et al. 2011; Wei et al. 2017; Tilk et al. 2019), the recombination rate map can be generated with as little as one library preparation. This is, at a minimum, two orders of magnitude fewer than current approaches. Fitness differential can be easily achieved by using marker selection, therefore allowing recombination rate to be estimated from AFs in marker-selected pools (fig. 1B). Previously, AF has been utilized for fine-scale crossover rate estimates in pools where recombinants were specifically selected by a double marker selection scheme (Singh et al. 2013) but had analytical shortfalls that resulted in problematic estimates (Gilliland 2015) (also see Discussion). Our approach here requires only one marker and does not require identification and scoring of visible recombinants.

We theoretically, computationally, and empirically explore the series of steps to infer recombination rates from AF attenuation in marker-selected pools. First, we formally demonstrate the mathematical relationship between recombination rate and AF using the Drosophila X chromosome with the recessive white eye marker (w-) as an example. Then, we generalize the relationships allowing for applications in the presence of any locus with a fitness differential and demonstrate how additive fitness impacts from offsite loci modulate the AF. Using simulations, we show how inherent noise in the sequencing platform can be addressed statistically for robust AF estimates. For empirical validation, we show that genetic distances estimated from the AF changes closely recapitulate the distances estimated from scoring recombinant individuals.
Using this approach, we generated the third chromosome recombination map of three wild-derived strains of *Drosophila melanogaster*, producing rates that are highly correlated with previous estimates. Lastly, we extend this approach to estimate chromosome-wide crossover interference. Although this marker selection and sequencing scheme does not produce basepair resolution crossover maps since it does not infer breakpoints, it can generate a near chromosome-wide genetic and recombination rate maps in as little as one or two library preparations.

**Results**

**Recombinant Fraction Is Directly Proportional to AF in Marker-Selected Pools**

In a typical recombinant backcross, F1 heterozygotes are backcrossed to one of the inbred parents generating BC1 progenies that are scored for recombinants (fig. 1A). Although gametes of the F1s are ideal for detecting crossovers as they are direct products of meiosis (Hinch et al. 2019), there are currently very few viable and reliable means of gamete selection (Umehara et al. 2019). Marker selection in progenies entails that the half of the genome derived from the inbred backcross parent (paternal genome in flies) is uninformative despite contributing to the AF. Since the X chromosome in sons are maternally inherited, male and female pools will have different AF signatures with the male pool reflecting the gametic AF (fig. 1B). For simplicity, we will first focus on the male marker-selected pools as their X chromosomes are free of paternal contributions. For illustrative purposes, we will use the X-linked *white* gene (*w*) in *Drosophila* as the selected locus (fig. 1B); selection is based on the recessive phenotype of white eyes (*w−*) versus red eyes (*w+*). Between the selected locus *w* and any position *i*, the recombinant fraction between the two loci is *D*; the frequency that the *w−* and *w+* chromosomes had zero or even numbers of crossovers between *w* and *i* (*p_{eveni} and q_{eveni} respectively) is *p_{eveni} = (1 − D_i)/2* and *q_{eveni} = (1 − D_i)/2*. The frequency of odd numbers of crossovers is then *p_{odd} = D_i/2* and *q_{odd} = D_i/2*. In the absence of selection, the AF of the allele on the *w−* chromosome (*q_0* will therefore be *q_i = q_{eveni} + p_{odd} = (1 − D_i)/2 + D_i/2 = 0.5*. When *w−* is selected, *p_{odd} = 0*, therefore,

\[ q_i = 1 − D_i \]  

(1)

and when *w+* is selected,

\[ q_i = D_i. \]  

(2)

Thus, the AF is directly proportional to the recombinant fraction in marker-selected pools.

To illustrate this relationship between recombination and AF change in the male selected pools, we used the available X chromosome recombination rate from the Recombination Rate Calculator (Fiston-Lavier et al. 2010) which models the chromosome-wide rate by a quadratic function. We converted the recombination rate to genetic distance from the white locus, followed by transformation into the recombinant fraction using either the Haldane or Kosambi mapping functions (fig. 2A). Depending on whether *w−* and *w+* males are selected, AF peaks (*q* = 1) or troughs (*q* = 0) at the selected white locus, respectively.
and attenuates distally, approaching the expected Mendelian ratio of 0.5 (fig. 2B).

In order to select for the recessive w⁻ marker in females, they must be sired by w⁻ fathers (fig. 1B). In the homozygous female (w⁻/w⁻) marker-selected pool, the w⁻ chromosome AF with paternal contributions will therefore be

\[ q_i = \frac{1 - D_i + 1}{2}. \]  

(3)

The AF peaks \((q = 1)\) at the selected white locus and attenuates toward the expected Mendelian ratio of \(q = 0.75\) (fig. 2B). If, instead, w⁺ heterozygous females (w⁺/w⁻) are selected and pooled, the AF troughs at \(q = 0.5\) (fig. 2B). Autosomal marker selection will have the same AF attenuation pattern.

**Selection/Fitness Differential Creates AF Deviation and Decay**

The relationship between \(D\) and \(q\) can be further generalized to any locus \(l\) where the two alleles have differential fitness of \(s\) ranging between \(-1\) and \(1\), such that

\[ q_i = 0.5 + 0.5s_l. \]  

(4)

Then, in the absence of paternal contribution, the AF at position \(i\) is captured by the addition of the proportion of alleles that did not recombine between \(l\) and \(i\) and proportion in which recombination occurred between \(l\) and \(i\) (fig. 2C):

\[ q_i = (1 - D_i)(q_i) + (D_i)(1 - q_i). \]  

(5)
When \( s_l = -1 \) and \( s_i = 1, q_l \) equals 0 and 1, respectively, which recapitulates marker selection as described in equations (1) and (2). When \( s_l = 0 \), that is, no difference in fitness between alleles, the AF is Mendelian \((q = 0.5)\). With other values of \( s_i \), the AF still peaks and decays around \( l \) (fig. 2D), thus allowing for estimation of \( D \). Therefore, even in the absence of a visible and selectable marker, as long as a locus exists where alleles have differential fitness causing non-Mendelian AF, recombination rate can be estimated around it. Such a locus may be a naturally segregating deleterious or lethal alleles (McCune et al. 2002), a meiotic driver (Fishman and Willis 2005; Wei et al. 2017), or even a partially penetrant marker, allowing for wide applicability of this approach beyond model organisms.

**Accounting for Offsite Viability Effects That Modulate AF Decay**

Although the AF decay pattern is a product of crossovers between the selected and distal loci, it is also sensitive to loci with alleles that differentially affect viability, body size, and fitness (which we will collectively refer to as viability effects), as such alleles can cause non-Mendelian contributions of individuals and DNA in the final pool. To evaluate how such loci will affect AF estimates, we further expand on the mathematical relationship between AF and recombinant fraction (eqs. 1–3) by a secondary locus at site \( o \) that also modulates AF in addition to the selected locus at site \( l \). For simplicity, we are removing the paternal contribution. The two loci have respective viability effect of \( s_l \) and \( s_o \) that individually (absent of other contributing loci) cause AF \((\hat{q})\) of \( \hat{q}_l = 0.5 + 0.5s_l \) and \( \hat{q}_o = 0.5 + 0.5s_o \). Given that the distance between \( l \) and \( o \) is \( D_{lo} \), the AF at \( o \) is then

\[
q_o = \frac{(1 - D_{lo})\hat{q}_o\hat{q}_o + D_{lo}(1 - \hat{q}_l)(\hat{q}_o)}{(1 - D_{lo})\hat{q}_o + (1 - D_{lo})(1 - \hat{q}_l)(1 - \hat{q}_o) + D_{lo}(1 - \hat{q}_l)(\hat{q}_o) + D_{lo}\hat{q}_l(1 - \hat{q}_o)}.
\]

With the selection process acting at \( l, s_l = 1 \) and \( \hat{q}_l = 1 \), so the equation reduces to

\[
q_o = \frac{(1 - D_{lo})\hat{q}_o}{(1 - D_{lo})\hat{q}_o + D_{lo}(1 - \hat{q}_o)}.
\]

From this, it can be inferred that

\[
D_{lo} = \frac{\hat{q}_o(1 - q_o)}{(1 - q_o)\hat{q}_o + q_o(1 - \hat{q}_o)}.
\]

The AF at \( i (q_i) \) will depend not only on its genetic distance from \( l (D_{li}) \) and \( o (D_{io}) \) but also on where it is positioned with respect to the two loci as its relative positions affect the recombinant allele combinations (fig. 3). First, we will consider no crossover interference. When \( l \) is between \( i \) and \( o \), that is, \( H(D_{io}) = H(D_{li}) + H(D_{io}) \) (fig. 4A),

\[
q_i = \frac{(1 - D_{li})(1 - D_{io})\hat{q}_o + (1 - D_{li})D_{io}(1 - \hat{q}_o)}{(1 - D_{li})(1 - D_{io})\hat{q}_o + (1 - D_{li})D_{io}(1 - \hat{q}_o) + D_{li}(1 - D_{io})\hat{q}_o + D_{li}D_{io}(1 - \hat{q}_o)}.
\]

This simplifies to

\[
q_i = 1 - D_{li},
\]

which indicates that as long as \( s_i = 1, s_o \) will have no effect on the AF on the distal side of \( l \). When \( i \) is between \( l \) and \( o \), that is, \( H(D_{io}) = H(D_{li}) + H(D_{io}) \) (fig. 3B),

\[
q_i = \frac{(1 - D_{li})(1 - D_{io})\hat{q}_o + (1 - D_{li})D_{io}(1 - \hat{q}_o)}{(1 - D_{li})(1 - D_{io})\hat{q}_o + (1 - D_{li})D_{io}(1 - \hat{q}_o) + D_{li}(1 - D_{io})\hat{q}_o + D_{li}D_{io}(1 - \hat{q}_o)}.
\]

When \( o \) is between \( l \) and \( i \), that is, \( H(D_{li}) = H(D_{io}) + H(D_{io}) \) (fig. 3C),

\[
q_i = \frac{(1 - D_{li})\hat{q}_o(1 - D_{io}) + (D_{io})(1 - \hat{q}_o)D_{io}}{(1 - D_{li})\hat{q}_o(1 - D_{io}) + (D_{io})(1 - \hat{q}_o)D_{io} + (1 - D_{io})(1 - \hat{q}_o)D_{io} + D_{io}\hat{q}_o(1 - D_{io})}
\]

and reduces to

\[
q_i = \frac{(1 - D_{li})\hat{q}_o(1 - D_{io}) + (D_{io})(1 - \hat{q}_o)D_{io}}{\hat{q}_o(1 - D_{io}) + (1 - \hat{q}_o)D_{io}}.
\]

To account for interference, the double crossover components are multiplied with the coefficient of coincidence (2D for the Kosambi function) with single crossovers reciprocally increased (see supplementary fig. 1, Supplementary Material online). To illustrate these relationships, we, again, used the recombination rate from the Recombination Rate Calculator (Fiston-Lavier et al. 2010) and varied the fitness differential \((s_o)\) at the offsite loci. In both the presence and absence of interference, the AF is more sensitive to negative fitness impact at the secondary loci \((s_o < 0)\). At extreme values of \( s_o \) (fig. 3D), local maximum and
minimums can be observed and identified, but at intermediate levels, the AF curves do not show obvious "kinks," suggesting that moderate loci need to be determined separately (see below). Unlike the curves in the absence of crossover interference, so modulates the AF across the selected locus (fig. 3E). Unexpectedly, at higher so, the AF is lower on the other side of l. The reason for this is that a positive so ensures more individuals lacking crossovers between l and o, which, in the presence of interference, results in increased probability of crossovers on the other side of l and therefore faster AF decline.

Given these relationships, it is then possible to solve for $D_{lo}$, $D_{o}$, and $D_{io}$ when so and qo are known. Although $D_{lo}$ can be deduced based on equation (7), we can solve for $D_{il}$ computationally using a root-finding algorithm since $D_{il}$ cannot be easily isolated in the equations (see Materials and Methods).

**Nonlinear Nonparametric Curve Fitting Robustly Estimates AF Decay in Short-Read Whole-Genome Sequences**

Because recombinant fraction and crossovers are necessarily sampled from a finite pool of individuals, larger pools will produce more representative and accurate estimates. With marker-selected pools, pool sizes can be increased with no increase to sequencing cost. To evaluate how sampling error affects recombinant fraction in marker-selected pools, we first considered pools where the AF is known exactly across the chromosome. Simulating $w/C0$ selection pools of different sizes ($n = 500$, $2,000$, and $5,000$), we generated pools of individuals with crossovers sampled based on the X chromosome recombination rate (fig. 4A, see Materials and Methods). Across all sizes, variability increases as recombinant fraction increases. This increase is partly due to the inherent variance with (binomial) sampling which is highest when $P = 0.5$. It also...

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**Fig. 3.** Effects of a secondary locus on AF decay. In the marker-selected pool, the selected locus (l), the offsite locus that affects viability (o), and any position along the chromosome (i) effectively create a three-point cross. Upper- and lowercase letters of these loci differentiate the homologous alleles. (A–C) Depending on the relative position of the three loci along the chromosome (schematics on the left), different allele combinations are produced at different frequencies (right). The frequencies of all possible allelic combinations are described by the equations which take into account of the fitness of the selected and offsite loci ($q_l$ and $q_o$, respectively). When $q_l = 1$, only the allelic combinations on the left column are incorporated in the marker-selected pool and allelic combinations on the right will all equal 0. The combinations in bold are those that contain the uppercase I allele. Equations (11)–(14) are derived from dividing the sum of the allelic combinations in bold by the sum of the allelic combinations on the left for each three-point cross. The frequencies of the different allelic combinations assume no interference. (D) AF modulation given different values of so based on the X chromosome recombination rate, Haldane’s transformation, selected locus at 5 Mb, and secondary locus at 10 Mb. Black curve represents so = 0 (no secondary offsite viability effect). (E) Same as (D), but with Kosambi’s transformation and taking into account of interference with $C = 2D$. Colors are added to differentiate between the curves with different values of $s_0$. Formula for how interference is incorporated into the AFs can be found in supplementary figure 1, Supplementary Material online.
reflects the increasing probability of double crossovers which results in nonmonotonic decay. Expectedly, these sources of noise decrease with larger pools; at their highest, the 95% confidence intervals are 0.034, 0.0106, and 0.0067 when 500, 2000, and 5,000 individuals are pooled, respectively (fig. 4A).

To evaluate whether short-read whole-genome sequencing can sensitively capture the AF signature in marker-selected pools, we simulated various average sequencing depths with a conservative single-nucleotide polymorphism (SNP) density of one differentiating site per 1 kb (see Materials and Methods). Within a reasonable range of sequencing depth of $10^5/100^5$, allele counts at individual sites are too variable to provide meaningful AF estimates (fig. 4B and supplementary fig. 2A, Supplementary Material online).

Since neighboring sites are expected to have negligible differences in their AFs, their counts and frequencies can be aggregated as if they are independent in sliding windows to better approximate the AF (Wei et al. 2017). Instead of assuming all sites in a window have the same AF, we first estimated AF using linear regression in overlapping sliding windows (Wei et al. 2017). The slope of the AF decay approximates the recombination rate. The left and right of the selected locus is expected to be increasing (positive slope) and decreasing (negative slope), respectively. The slope of the linear regressions (blue) and cubic spline fit (red) across the chromosome, and the colored areas demarcate the 95% confidence interval of the slope. The expected slope is in black (D–F) Same as (A–C), but with the heterogeneous recombination rate inferred by Comeron et al. (2012).

Figure 4. In silico simulation of AF in sequenced pools. (A) Simulation of AF attenuation around white in marker-selected pools of 500, 2,000, and 5,000 males. Black line depicts the expected AF. Colored areas demarcate the 95% confidence intervals for the different pool sizes after 20,000 trials, given that AF can be exactly estimated. (B) One example of AF distribution from simulated read counts of a pool of 1,000 white males. With SNP density of one site per kb, the allele-specific read counts across the chromosome is randomly sampled to simulate fly collection, library preparation, and sequencing to a read depth of 30; the AF at each site is plotted (light blue dots) to illustrate the extent of noise from site to site. Site-specific AF chromosome-wide is either binned in overlapping 500-kb sliding windows followed by linear regression fit (dark blue dots) or fitted with cubic splines anchored at the selected w locus (orange line). Red line depicts the true AF of the simulated pool. (C) The slope of the AF decay approximates the recombination rate. The left and right of the selected locus is expected to be increasing (positive slope) and decreasing (negative slope), respectively. The slope of the linear regressions (blue) and cubic spline fit (red) across the chromosome, and the colored areas demarcate the 95% confidence interval of the slope. The expected slope is in black. (D–F) Same as (A–C), but with the heterogeneous recombination rate inferred by Comeron et al. (2012). (G) Summary of the effects of sequence depth (X-axes), pools size (colors), and resolution (panels) on the standard deviation of the slope (Y-axes). For each combination of parameters, 5,000 simulations were conducted using the Comeron et al. (2012) recombination rate, from which the standard deviation of the slope is derived at each position after curve fitting; the standard deviation averaged across the entire chromosome is plotted.
windows (see Materials and Methods). We find that 500-kb overlapping windows in 100-kb increments can roughly recapitulate the expected AF (fig. 4B) but create high variance in the slope, which approximates the recombination rate (fig. 4C). Second, rather than binning the genome into windows, we implemented a nonparametric and nonlinear curve-fitting strategy using either local regressions (LOESS) (Cleveland 1979) or cubic splines (Perperoglou et al. 2019) (fig. 4D). We find this to be more flexible and robust, as it allows for nonlinear fit, is more robust to noise (fig. 4B), and estimates the slope of the AF decay with less error than the window-based linear regressions (fig. 4C).

To further determine whether this strategy can capture realistic recombination rate distributions which can be highly heterogeneous, we simulated pools using the *D. melanogaster* X chromosome recombination rate inferred from crossover distributions by Comeron et al. (2012) (fig. 4D–F). In particular, we were interested as to whether our curve-fitting strategy can approximate the fluctuating slope (fig. 4F). As with most fitting methods, there is a tradeoff between resolution and accuracy. We fitted the AF from simulations with different numbers of individuals and sequencing depths at four different resolutions equivalent to 200 kb, 500 kb, 1 Mb, and 2 Mb and inferred the slopes of the fitted curves (fig. 4C and supplementary fig. 3, Supplementary Material online). Expectedly, increasing the numbers of individuals, sequencing depth and window sizes produces less variable slope estimates. But between the resolution of 1 and 2 Mb, the gain in accuracy becomes marginal. We note that because our simulation uses a conservatively low SNP density (1 per 1,000 bp), the extent of noise at different resolutions is likely overestimated. Indeed, simulations with a realistic SNP distribution (inferred from inbred *D. melanogaster* strains DGRP 315 and mutant strain *glass ebony*, see below and table 1) which has on average more than three times the SNP density, we find reduced noise across all resolutions (supplementary figs. 4 and 5, Supplementary Material online).

| Parents | BC1 Offspring Genotypes and Counts | Recombinant Fraction | Genetic Distance (cM) |
|---------|-----------------------------------|----------------------|----------------------|
|         | Wild-Type Strain (F) | Marker Strain (M) | Sex | Punnett Squares | Overall | Pool Specific | From AF Estimates | Overall | Pool Specific | From AF Estimates |
|          |                           |                      |     |                |         |              |                    |         |              |                    |
| Canton-S |                         |                     |     |                |         |              |                    |         |              |                    |
| e′, e′  | F e+ | e−  | 726 | 226 | 0.2384 | 0.2524 | 0.2414 | 25.95 | 27.78 | 26.33 |
|         | M e+ | e−  | 212 | 628 | 0.2476 | 0.2352 | 27.14 | 25.53 |
| Canton-S |                         |                     |     |                |         |              |                    |         |              |                    |
| e′, e′  | F e+ | e−  | 657 | 183 | 0.2370 | 0.2564 | 0.2538 | 25.76 | 28.33 | 27.98 |
|         | M e+ | e−  | 178 | 541 | 0.2776 | 0.2367 | 31.30 | 25.73 |
| Canton-S |                         |                     |     |                |         |              |                    |         |              |                    |
| e′, e′  | F e+ | e−  | 610 | 177 | 0.2370 | 0.2564 | 0.2538 | 25.76 | 28.33 | 27.98 |
|         | M e+ | e−  | 179 | 519 | 0.2776 | 0.2367 | 31.30 | 25.73 |
| Canton-S |                         |                     |     |                |         |              |                    |         |              |                    |
| e−, gl− | F gl+ | e−  | 534 | 30 | 0.0887 | 0.1339 | 0.1370 | 8.96 | 13.73 | 14.05 |
|         | M gl+ | e−  | 58 | 375 | 0.0921 | 0.1047 | 9.32 | 10.62 |
| Canton-S |                         |                     |     |                |         |              |                    |         |              |                    |
| e−, gl− | F gl+ | e−  | 485 | 41 | 0.0921 | 0.1047 | 9.32 | 10.62 |
|         | M gl+ | e−  | 46 | 404 | 0.1122 | 0.1256 | 11.42 | 12.83 |
| DGRP-360 |                     |                     |     |                |         |              |                    |         |              |                    |
| e−, gl− | F gl+ | e−  | 1,120 | 107 | 0.0885 | 0.0930 | 0.0791 | 8.95 | 9.41 | 7.97 |
|         | M gl+ | e−  | 102 | 959 | 0.0891 | 0.0745 | 9.01 | 7.51 |
| DGRP-315 |                     |                     |     |                |         |              |                    |         |              |                    |
| e−, gl− | F gl+ | e−  | 892 | 89 | 0.0940 | 0.0907 | 0.0966 | 9.52 | 9.17 | 9.78 |
|         | M gl+ | e−  | 89 | 818 | 0.0981 | 0.1000 | 9.94 | 10.14 |

Table 1. Genetic Distance Estimates between Double Markers.

Note.—Tracks of different colors represent different crosses. In each row, cells in italics are counts of selected genotypes that were pooled and sequenced.

aAfter Kosambi transformation.

bSum of all recombinants divided by sum of all individuals in the cross.

cRecombinant individuals in the pool divided by total number of individuals in the pool.

dIndividuals in this cross are genotyped and pooled into either gl− or gl+ pools.
one of the two markers followed by bulk DNA extraction and whole-genome sequencing to an average of 25.45× coverage (supplementary table 1, Supplementary Material online).

Thus, we were able to compare the genetic distance estimates between the two loci based on the de facto method of fly scoring with that from AF attenuation in the marker-selected pools within the same recombinant backcrosses (table 1). Canton-S was separately crossed to the two double marker strains in duplicates; in the crosses to Canton-S was separately crossed to the two double marker pools (positive marker selection), we also pooled the whole-genome sequencing to an average of 25.45× coverage (see Materials and Methods) (supplementary table 1, Supplementary Material online). Across replicates, and differences in pools with no marker selection (Wei et al. 2017). To emulate this, we summed the AF from the positive and negative marker selection pools for the DGRP315 × gl− e− crosses where we sequenced both positive and negative marker-selected pools. Using this method, we find a very minor AF deviation at e−, which is equivalent to s = −0.015 (supplementary fig. 11A, Supplementary Material online). Correcting for this resulted in negligible changes in the recombinant fraction (supplementary fig. 11B, Supplementary Material online).

Given chromosome-wide recombinant fractions (D), the genetic distance (d) can then be estimated by transformation with mapping functions (fig. 5E–G, dashed lines). However, when the recombinant fraction approaches 0.5, the genetic distance estimate approaches infinity. In practice, the genetic distance should, therefore, be limited to 50 cM which entails that two loci are effectively genetically unlinked (fig. 5E–G); this equates to recombinant fractions of <0.381 and 0.316 with the Kosambi and Haldane functions, respectively. Using the Kosambi transformation, for DGRP-315 and DGRP-360 which were under glass selection, we were able to infer recombination rate for 86.2% and 89.5% of Chr. 3, respectively (fig. 6A). The Canton-S pools were selected on both glass and ebony and the composite of the two covered 85.2% of the chromosome.

Lack of AF Decay Captures Crossover Suppression within Inversions

Interestingly, unlike the crosses with the gl− e− double marker strain, the Canton-S × se− e− crosses produced AF curves that do not have a clear peak flanked by distal decay (fig. 5B). The AF on Chr. 3L where the selected locus se resides shows minimal decay with elevated level of AF extending into 3R, where it begins to show clear attenuation. Based on the e− selected pool in the Canton-S × gl− e− crosses, the recombinant fraction between e and se is 0.368 equating to 47.0 cM using Kosambi’s map function (fig. 5E, blue line), but the estimates from both fly scoring and AF decay using the se− e− marker strain indicate a substantially lower recombinant fraction of 0.238 equating to 25.8 cM (table 1 and fig. 5F). Suspecting that the shorter genetic distance resulted from structural rearrangements on the se− e− chromosome arm causing suppression of crossovers and/or lethal rearrangements in recombinants, we used the structural variant discovery software Lumpy (Layer et al. 2014) to infer the presence of inversions. Indeed, we identified two large inversions on 3L: one from 0.25 to 7.22 Mb and another pericentric inversion from 7.81 Mb of 3L to 2.08 Mb of 3R (fig. 5B and F), confirming...
our suspicion that inversions are interfering with recombination in this region. Interestingly, we still observe minor reduction in AF across this region. This likely reflects rare double (or even number) crossover events which resolve lethal recombinant rearrangements caused by single (or odd number) crossover events within the inversion (Hughes et al. 2018).

Heterogeneity and Natural Variation in Recombination Rate

Since recombination rate is genetic distance per physical distance, for regions to the left and right of the selected locus, the recombination rate is the negative and positive slope of the genetic distance curve, respectively. We converted the genetic distance to recombination rate for all the crosses by taking the slope of d in 10-kb windows within the ±50 cM range (fig. 6A). In all three lines, the recombination rate is minimal across the highly repetitive regions and gradually increases away from the centromere, as expected. Our estimates broadly follow previous estimates based on crossover breakpoint distribution (Comeron et al. 2012) (fig. 6A, gray). Both estimates of the DGRP lines are highly and significantly correlated with each other (supplementary fig. 12, Supplementary Material online) and that of Comeron et al. (2012), as well as when it is smoothed into an equivalent resolution (fig. 6B and C). The significant positive correlations, similarly, hold at the resolution of 500 kb; but at 200 kb, the recombination rate estimates become too unreliable due to overfitting (supplementary fig. 13, Supplementary Material online). Much of the correlation appears to be driven by regions with low recombination rate; after removal of these sites the correlations, although still significant, are substantially reduced (supplementary fig. 14, Supplementary Material online).

Fig. 5. Recombinant fraction and genetic distance inferred from marker-selected pools for Chr. 3. The per site (blue) and inferred (red line) AFs across the chromosome are displayed for four pools: (A) Canton-S × gl⁻ e⁻ with gl⁻ selection (table 1, row 6), (B) Canton-S × se⁻ e⁻ with se⁻ selection (table 1, row 1), (C) DGRP-315 × gl⁻ e⁻ with gl⁻ selection (table 1, row 12), and (D) DGRP-315 × gl⁻ e⁻ with gl⁺ selection (table 1, row 13). For all marker-selected pools, see supplementary figure 6, Supplementary Material online. Note that (D) has a different Y-axis scale. The positions of the double markers are marked in the chromosome schematics above and with vertical dotted lines in the plots. For the Canton-S × se⁻ e⁻ cross, large inversions are marked in chromosome schematic above with parentheses. (E–G) Averaged recombinant fraction (solid curve) and genetic distance (dotted curve) from the selected loci. Interval around the recombinant fraction estimates represent the standard error across the replicates. Horizontal dotted lines represent recombinant fraction equivalent of 50 cM after Kosambi transformation. (E) Canton-S × gl⁻ e⁻ crosses when gl⁻ (red) or e⁻ (blue) are selected. Note here that gl⁻ pools were the female pools and e⁻ pools were the male pools. (F) Canton-S × se⁻ e⁻ crosses with e⁻ selection. (G) DGRP-315 (red) or DGRP-360 (yellow) × gl⁻ e⁻ crosses. For the DGRP-315 cross, values are averaged across gl⁺ and gl⁻ selection pools, whereas only gl⁻ pools were sequenced for DGRP-315.
Discrepancies are likely partially due to strain-specific rate variations (see below) and differences in rearing conditions as our crosses were reared in 25 °C versus 21 °C in that of Comeron et al. (2012).

Interestingly, all three lines show similar regions with elevated rates each of which spans multiple megabases and are interrupted by valleys (fig. 6A). Since these regions are also elevated in the estimates by Comeron et al. (fig. 6A), they are unlikely to be artifacts of our method, or the strains used. These broad megabase-sized blocks of elevated recombination rates and narrow valleys of low recombination are likely a general feature of the recombination landscape of the chromosome.

Despite global similarities in the recombination landscape among the different lines, recombination rate is highly variable between them. From 5.5 Mb of Chr. 3L to 29.1 Mb of Chr. 3R where recombination rate can be determined for both DGRP-315 and DGRP-360, these two lines and the Comeron et al. (2012) estimates show significantly different distribution of recombination rates (P < 0.001, paired Wilcoxon ranked sum test; fig. 6C). DGRP-315 has more regions with intermediate recombination rates (3–5 cM/Mb) than DGRP-360. The large fraction of low recombinating windows caused the DGRP-360 strain to have a shorter genetic map to the left of the selected locus (fig. 5G and supplementary fig. 15, Supplementary Material online). However, to the right of the selected locus, the genetic distance and recombination rates are comparable between the two lines (fig. 5G and supplementary fig. 15, Supplementary Material online). This suggests that the lower recombination rate and shorter distance in the DGRP-360 line is not due to global depression of crossovers, but perhaps many local modifiers to the left of the selected locus, possibly in the form of small scale inversions and TE insertions.

**Fig. 6.** Recombination rate estimates in wild-derived strains. (A) Chr. 3 recombination rate estimates from Comeron et al. (2012), and marker-selected pools are depicted. Repeat content (%) and Chr. 3 schematic is plotted above. For the estimates from Comeron et al. (2012), the rates are plotted as the true rates (gray) and the rates after 1 Mb smoothing (red). For the marker-selected pools (DGRP-315, DGRP-360, and Canton-S), rate estimates are restricted to ±0.381 recombinant fraction (equating to 50 cM after Kosambi’s transformation) from the marker. Area around the line demarcates the standard error estimated across the different pools. (B) Correlation of recombination rates of DGRP-315 (red) and DGRP-360 (blue) with unsmoothed (left) and 1-Mb-smoothed estimates (right) from Comeron et al. (2012). Each point is the recombination rate estimate of a 100-kb window. The red and blue lines are the least square regressions. (C) Distribution of the recombination rate of estimates from Comeron et al. (2012) (gray) and of DGRP-315 (red) and DGRP-360 (blue).
Inferring Chromosome-Wide Crossover Interference from Reciprocal Selection of Double Markers

Crossover interference, usually expressed as $1 - C$ where $C$ is the coefficient of coincidence (Muller 1916), is typically estimated from three-point crosses where double recombinants can be scored. $C$ is the fraction of observed double crossovers over expected, and, given three loci ABC in that order, is expressed as

$$D_{AC} = D_{AB} + D_{BC} - 2D_{AB}D_{BC}.$$  \hspace{1cm} (10)

Therefore, if the recombinant fraction between three loci is known, $C$ can be inferred by

$$C = \frac{D_{AB} + D_{BC} - D_{AC}}{2D_{AB}D_{BC}}.$$  \hspace{1cm} (11)

In our Canton-S × gl− e− crosses, we sequenced pools of both gl− and e− selection (table 1), generating chromosome-wide recombinant fractions with respect to these two loci. With these two curves, we can estimate the coefficient of coincidence, as for any site $i$, we have the genetic fractions between gl and $i$ ($D_{i, gl}$), e and $i$ ($D_{i, e}$), and gl and e ($D_{gl,e}$). Therefore, to the left of gl, between the two, and right of e, we can solve for $C$ with the following, respectively:

$$C = \frac{D_{i, gl} + D_{gl,e} - D_{i, e}}{2D_{i, gl}D_{gl,e}},$$  \hspace{1cm} (12)

$$C = \frac{D_{i, e} + D_{gl,e} - D_{i, gl}}{2D_{i, e}D_{gl,e}}.$$  \hspace{1cm} (13)

For this proof-of-principle analysis, we used the broad window size of 2 Mb to minimize error in estimates of $D$, since these crosses have a low density of informative sites. Because of the denominator, regions close to loci with small $D$ will yield nonsensical results. We, thus, only estimate the interference beyond ±2.5 Mb from the markers, which also precludes estimating between gl and e, as they are only 2.8 Mb apart (fig. 7). To the right of e, C increases distally toward the 3R telomere. To the left of gl, C drops sharply and remains low around the pericentomere, indicating high interference. Across the pericentomere of 3L, C increases rapidly toward the 3L telomere. In comparison to the C assumed by the Kosambi function, the estimated C increases more precipitously, reaching 1 (i.e., no interference) before that of Kosambi’s function. Historically, the centromere has been assumed to act as a barrier to interference, disrupting the propagation of interference across it (Mather 1939; Colombo and Jones 1997). Similar to later studies (Colombo and Jones 1997), our chromosome-wide estimates of $C$ are inconsistent with this assumption and show that interference is instead high around the centromere (Hultén 2011), thus further reducing the already low probability of crossovers within the pericentromere.

Discussion

Broad-Range Recombination Rate Estimation Using AF in Marker-Selected Pools

We demonstrated theoretically and empirically that AF decay around a locus with selection differential in a recombinant backcross can be used to infer near chromosome-wide recombination rates. Instead of genotyping individuals for recombinant breakpoints, this approach relies on pooled sequencing of large numbers of marker-selected individuals from recombinant backcrosses. Since the AF surrounding the selected locus decays proportionally with the recombinant fraction, the genetic distance can be determined using mapping functions, and the slope will then approximate the recombination rate. Since each pool can provide broad-range recombination rate estimates, this method substantially reduces the number of library preparations needed for chromosome-wide estimates. Previously, hundreds to thousands of individuals needed to be sequenced or genotyped to capture a comprehensive spread of recombinant breakpoints from which chromosome-wide recombination rates are inferred, but here we used as little as two libraries for near chromosome-wide estimates. Recently, crossover detection has been made possible in bulk with “linked read sequencing,” where large molecular weight DNA molecules are effectively barcoded in high throughput by 10× Genomics (Sun et al. 2019). Nevertheless, the sequencing depth required for this...
approach is still substantially greater than the 30× used here. Not accounting for fly upkeep, each library of a marker-selected pool costs <500 to generate and sequence. Using this approach, we inferred the recombination rate of Chr. 3 of three wild-derived lines and demonstrated that how it can be used to infer crossover interference.

Previously, Singh et al. (2013) also used AF to estimate fine-scale crossover frequency; recombinants between garnet and scalloped (two phenotypic maker on the D. melanogaster X chromosome) that carry either one of the two markers were pooled to estimate the fine-scale recombination rate between the two loci. The method presented here differs from Singh et al. (2013) in that recombinants are not explicitly targeted for selection, and thus the AF decay is directly reflecting the recombinant fraction (eqs. 1–3). When only recombinants between two loci are selected, the AF decay is effectively restricted to estimates of recombinant fraction between the two loci. As demonstrated by Gilliland (2015), the approach used by Singh et al. (2013) to measure AF and recombination rate produces rates that are strongly anticorrelated with the heterogeneous window sizes used for estimation. As shown by our simulations and pointed out by Gilliland (2015), the variability of read counts at individual sites is too large for fine-scale estimates. Instead, we use curve-fitting methods leveraging all informative sites across the chromosome to avoid the inherent noise associated with read counts in small window sizes.

Methodological Limitations and Workarounds

Given that crossover breakpoints are not identified in marker-selected pools, recombination rate is estimated from changes in recombinant fraction, which is itself inferred from AF. The accuracy of AF estimation is dependent on the sequence coverage, SNP distribution, and the curve-fitting strategy. However, whether the AF decay (even if perfectly estimated) accurately reflects the recombinant fraction depends on the number of individuals in the pool (see fig. 4A and D). With smaller pool sizes, sampling error of genotypes leads to greater levels of noise in the AF decay; this is akin to inferring crossover breakpoint distribution from a small number of individuals. Although increasing the number of individuals necessitates increasing the number of DNA extractions, library preparations, and sequencing to infer crossover breakpoints, the number of individuals in marker-selected pools can be easily increased with no increase to sequencing cost, especially for highly fecund invertebrates like worms and flies. Therefore, although the resolution of this method is markedly lower than the rates from crossover breakpoints, it can potentially generate more representative maps with very large numbers of individuals at a fraction of the sequencing cost.

There are several additional limitations to sequencing marker-selected pools, although some of the issues are not specific to this method of estimating recombination rates. First, the conversion from recombinant fraction to genetic distance with mapping functions requires assumptions about crossover interference. The Haldane, Kosambi, and other mapping functions have explicit assumptions about the extent (or lack) of interference, which, accordingly, affect the genetic distance conversion, with higher crossover interference producing shorter genetic maps. On the other hand, we presented an extension of our method to infer crossover interference (fig. 7); by reciprocally selecting for markers in a double marker strain, we were able to estimate the coefficient of coincidence across Chr. 3 with respect to the two markers. Improved understanding of crossover interference in a genomic context will allow for more accurate conversion between the recombinant fraction to genetic distance in future investigations. Alternatively, since the mapping functions have negligible effects for smaller values of recombinant fractions (<0.10), multiple markers (if available) along the chromosome can be used to estimate recombination rate in smaller intervals. Although this significantly increases the number of crosses, the number of libraries generated is still at least one order of magnitude less than sequencing individuals. As a corollary, mapping functions break down when the recombinant fraction approaches 0.5. Here, we explicitly restrict the Kosambi mapping function to a genetic distance of 50 cM on either side of the selected loci, which translates to a total measurable recombinant fraction of 0.762 (0.381 on either side). Again, to extend the genetic map, multiple markers strategically chosen along the chromosome may be needed to encompass the entirety of the chromosome.

Second, this approach is sensitive to the SNP density. Based on simulations, our method is robust even when the SNP density is as low as one site per 1,000 bp. Although intraspecific strain differences are likely higher in flies, polymorphisms are not evenly distributed across the chromosome. The SNP density drops rapidly around the pericentromeric and telomeric regions, which resulted in increased error rates. The decrease in SNP density is particularly problematic in window-based AF estimates, but our usage of nonlinear non-parametric curve fitting alleviates this issue on at least two fronts: The smoothing is conducted based on the number of sites instead of genomic windows and the reduced pericentromeric SNP density for metacentric chromosomes (e.g., Chromosome 3) is flanked by high density on either sides thus producing more robust estimates across the pericentromeric region. However, we note that reduced SNP density also poses a challenge when inferring breakpoints, since the precise location of haplotype changes will be difficult to pinpoint.

Lastly, we demonstrated that offsite viability effects can modulate the AF decay around the selected locus. Such viability effects can result from alleles that induce lethality or reduce body size, both of which will change the AF in the DNA pool to be sequenced. Lethality is similarly problematic for genotyping and breakpoint inference in individuals, since it changes the number of recoverable recombinants at specific loci, but are typically ignored. We analytically showed how additive offsite viability effects can be accounted for, provided that the extent of the fitness reduction (s_f) can be determined. To estimate s_f, without additional experiments, we simply summed the AF of both the positive and negative marker selection pools that originated from the same cross. This effectively removes the peak and the effect of the selected locus and the remaining elevations and drops.

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that deviate from Mendelian ratios are then regions with viability effects.

**Applications beyond Drosophila, Model Genetic Organisms, and Phenotypic Markers**

Although this study focused on *D. melanogaster*, this method is readily applicable to other organisms, particularly for model species with a wealth of phenotypic markers readily available like *Caenorhabditis elegans* (Greenwald 2016) or mice (Singh and Coppola 2014). As discussed above, for species with longer genetic maps, multiple markers are needed to capture recombination rate along the entirety of the chromosome, thus increasing the number of crosses and libraries. Since each marker-selected pool only reflects recombination rate on one chromosome, species with higher chromosome numbers will require more libraries and crosses overall. Notably, markers from different chromosomes can be selected at the same time in any given cross allowing for estimates on different chromosomes in parallel. However, this can introduce potential epistatic effects between markers which may modulate AFs of the chromosomes. As with typical recombinant crosses, the sensitivity of the method depends on the number of BC1 individuals (fig. 3A). Although we recommend over 1,000 individuals to be collected, for organisms where large numbers of offspring are unfeasible, the estimates from marker-selected pools will be similarly underpowered as counting or genotyping the number of recombinant individuals between markers, but with the added benefit of requiring only one marker. For species like mice where pulverizing individuals is impractical, care must be taken when pooling tissues to minimize variation in tissue size and the resulting DNA contribution per individual.

Although the abundance of visible markers make our method particularly suitable for model genetic organisms, visible polymorphisms and mutations can be found across wide ranges of species and taxa enabling application of this method; within the *Drosophila* genus, many species, even those distantly related to *D. melanogaster*, have marker stocks readily available (Clark et al. 2007, and see The National Drosophila Species Stock Center). Similarly, many nonmodel or emerging model organisms including *Apis mellifera* (Schulte et al. 2014), *Anopheles gambiae* (Bernardini et al. 2018), *Bombyx mori* (Yasukochi 1998), *Musca domestica* (McDonald et al. 1975; Meisel et al. 2017), *Gerris buenoi* (Armisén et al. 2018), *Daphnia magna* (Ismai et al. 2018), and *Parhyale hawaiensis* (Ramos et al. 2019) have not only selectable phenotypes permitting this approach but also high quality reference genomes.

Moreover, we showed that the signature AF attenuation enabling estimation of recombination rate is generated at any locus with a fitness differential (eq. 5 and fig. 2C). However, AF attenuation will be easier to estimate given stronger fitness differentials. The closer $s_i$ is to 0, the recombinant fraction will be more difficult to tease apart from the noise introduced by sample pooling and sequencing. Systems with segregating recessive lethals (McCune et al. 2002) and strong meiotic drivers (Fishman and Willis 2005) are therefore prime candidates for this method. Even without a source of lethality or segregation bias, loci with associated traits, even if not fully penetrant, can also be used to generate the fitness differential required. Additionally, advances in gene editing with CRISPR-Cas9 (Russell et al. 2017; Adli 2018) and ease of genome assembly with long-read sequencing (Miller et al. 2018; Bracewell et al. 2019) will continue to increase the catalog of organisms in which marker-selected pools can be applied. Therefore, our theoretical, statistical, and empirical investigations here set the stage for wide application of this cost-effective and scalable method to estimate recombination rate.

**Materials and Methods**

MarSuPial

The analytical and statistical methods described below are implemented in the MarSuPial package found in KW's github page (https://github.com/weikevinhc/Marsupial). It is an R package with tools to analyze and simulate read count data from marker-selected pools.

**Conversion of Recombinant Fraction from Recombination Rate Function**

To convert recombination rate function (from Recombination Rate Calculator [Fiston-Lavier et al. 2010]) to genetic distance centered at the selected locus (e.g., w), we integrated the quadratic formula from the selected locus to every position on the chromosome. For example, the genetic distance between w (at position X : 2,684,632) and position $i$ in megabases is

$$\int_{2,684,632}^{i} (-0.03x^2 + 0.6x + 1.15) \, dx.$$ 

As per Fiston-Lavier et al. (2010), positions <1.22 Mb and >21.21 Mb have rates of 0. To convert genetic distance to recombinant fraction with the Haldane and Kosambi mapping functions, we used their inverse functions:

$$D = -\ln\frac{1 - 2d}{2}$$

(Haldane 1919) and

$$D = \frac{1}{4} \ln\frac{1 + 2d}{1 - 2d}$$

(Kosambi 1943), respectively.

Since the formula from the Recombination Rate Calculator are based on r5 of the D. melanogaster genome, we used the r5 instead of r6 coordinates for the genes in the simulations. The remaining analyses were all based on the r6 reference.

**Simulation of Recombinant Fraction and Pooled Sequencing of Marker-Selected Pools**

To estimate the variance of recombinant fraction resulting from sampling errors, we first simulated crossover events based on the recombination rate function of Chr. X. For each 1,000-bp window, the probability of a crossover is determined by integrating over the recombination rate of that window.
To simulate crossover events for one chromosome, a Bernoulli draw is conducted at each window, with successes denoting crossovers. Starting from the selected locus with an allele state of TRUE, every Bernoulli success causes a change in allele state (represented in Boolean as either TRUE or FALSE) of all subsequent windows. To simulate \( n \) individuals, this process is repeated \( n \) times. The AFs across the windows are then determined based on the proportion of 0 and 1 allele states in the pool of \( n \) individuals. The AF of the one allele equals the recombinant fraction. This process is repeated 20,000 times to determine the error rate of recombinant fraction given different \( n \). This simulates crossovers without interference as each crossover is independent. For read count and AF simulations, the AF \( q \) (or \( q + 1 \)/2 if there is paternal contribution) from the simulated pools (above) is used for two random draws. A Poisson draw is first conducted with the desired read depth as the mean. A second binomial draw is then conducted with probability of \( q \) and the Poisson-drawn read depth as the number of trials, to simulate allele-specific read counts at the site.

**Predicting and Smoothing the AF with Linear Regression, LOESS, and Cubic Splines**

Predicting and smoothing the AF with linear regression, LOESS, and cubic splines once AF at differentiating sites is determined from read counts (simulated or real) across the chromosome, sites within 500-kb windows are then used for a weighted linear regression with the R function `lm()` where the weight of each site is the coverage of that site. The AF of the window is then determined by the midpoint of the window using the slope and \( y \)-intercept of the linear regression. Given the linear regression, AF can also be predicted for any point within the window. AF in 500-kb windows are determined every 100 kb, resulting in overlapping sliding windows. For LOESS and cubic splines fitting, we implemented the R function `loess()` and `smooth.splines()` in MarSuPial, respectively, to fit two curves on either side of the selected locus, which prevents smoothing of the expected peak/trough. The AF at each site is weighted by the coverage at the site. To “anchor” the functions, we include an additional point at exactly the selected locus with the expected AF (0, 0.5, or 1, depending on the selection procedure and/or chromosome), with a weight of 1,000,000, ensuring that the curves intersect the expected AF at the selected locus. The predict() function is used on these curve-fitting objects to estimate the AF and standard error for any position across the chromosome. For cubic splines, we used degrees of freedom (df) as a proxy for resolution. Since \( df = 4 \) number of knots are evenly spaced across the sites for cubic splines, window size = chromosome size/(df – 4). With cubic splines, the first derivative (slope) of the fitted curves is determined using the predict() function in R.

**Genetic Distance and Recombination Rate from AF**

Given the LOESS-predicted AF chromosome wide in 5-kb windows for each cross, we removed the paternal contribution and inferred the recombinant fraction using equation (3). The only exception is the DGRP315 × gl− e− pool where the gl+ individuals were selected (negative marker selection); instead, we used the formulae:

\[
q_i = \frac{D_i + 1}{2}.
\]

To get the genetic distance, we then applied either Haldane’s or Kosambi’s mapping functions:

\[
d = -\frac{1}{2} \ln (1 - 2D)
\]

and

\[
d = \frac{1}{4} \ln \frac{(1 + 2D)}{(1 - 2D)},
\]

respectively. Recombination rate \( r \) at position \( i \) bp was then derived taking the positive or negative slope between the 5-kb windows:

\[
r_i = \frac{d_{i+2,500} - d_{i-2,500}}{5,000},
\]

depending on whether \( i \) was to the left or right of the selected locus, respectively. Summary statistics for genetic distances and recombination rates (median, standard deviation, etc.) were estimated from replicate and/or different sexed pools from the same cross. For the Canton-S × gl− e− crosses, the reciprocal marker selection produced two different sets of genetic distances from either glass or ebony. However, once converted into recombination rate, the reciprocals were then treated as replicates, since recombination rate is expected to be unaffected by the marker selected. To convert the publicly available recombination rate which is in 100-kb windows Comeron et al. (2012) to genetic distance, we multiplied the recombination rate in each window by 100 kb.

**Fly Stocks, Maintenance, and Collection**

The Canton-S and double marker lines were ordered from Bloomington Drosophila Stock Center, stock numbers BL64349 (Can-S), BL1669 (e− se−), and BL507 (gl− e−). Notedly, the source of the elevated heterozygosity in the Canton-S strain is due to stock center contamination, which BDSC has now acknowledged under the strain’s listing. DGRP-315 and DGRP-360 are gifts from Dr Yuh Chwen G. Lee. All stocks and crosses were raised on standard molasses food at 25 °C. For the crosses, 4–8-day-old virgin females were mated with the marker strain males and F1 virgins were then collected. For each recombinant cross, over 40 F1 virgins were collected and backcrossed to the marker males in vials of five to eight virgin females to eight to ten males. To avoid overcrowding in the subsequent generation, backcrosses were transferred every 2–3 days to new vials and after adults began to emerge, flies were cleared and scored daily. Every vial is collected for 10 days to ensure that genotypes that may
introduce developmental delays will not be underrepresented. Sexed and genotyped flies were maintained in fresh vials for 3–5 days prior to freezing in the −20 °C freezer.

**DNA Extraction and Library Preparation**

Frozen flies of the desired genotypes were pooled, pulverized with sterilized mortar and pestle that are chilled in liquid nitrogen, and then transferred to 50-ml falcon tubes. After adding 15 ml of Cell Lysis Solution from Qiagen (Catalog No.158908), samples were incubated at 65 °C for 4 h and vigorously shaken every hour. In total, 75 μl of ProteinaseK (Catalog No. 19131) was then added and incubated at 55 °C overnight; 200 μl of EtOH was added into 400 μl of the sample then passed through the columns from the DNeasy kit (Catalog No. 69506). The columns were then processed in accordance with the kit protocol. DNA for parental lines were extracted from five females using the DNeasy kit. The resulting DNA was fragmented to 550 bp using the Covaris sonicator and libraries were made with the Illumina Truseq DNA Nano kit. Library quality was determined with the Bioanalyzer at the Functional Genomics Laboratory at UC Berkeley and samples were pair-end sequenced using the Illumina HiSeq 4000 machine at the Vincent J. Coates Genomics Sequencing Laboratory at UC Berkeley. Coverage of each cross can be found in supplementary table 1, Supplementary Material online.

**Predicting and Smoothing the AF with Linear Regression, LOESS, and Cubic Splines**

Demultiplexed paired-end reads were mapped to the D. melanogaster genome r6.12 downloaded from Flybase (Thurmond et al. 2019), using bwa mem (v0.7.15) on default settings (Li and Durbin 2009). Raw reads for the two DGRP strains were downloaded from SRA under SRX006143 for DGRP-315 and SRX155999 for DGRP-360. We removed duplicates using Picard tools (v2.18.14) and merged the parental strains with the crosses using Samtools (v1.5) (Li et al. 2009) to allow them to be genotyped together with GATK HaplotypeCaller (v3.8) (McKenna et al. 2010). By default, HaplotypeCaller only outputs sites where at least one sample has a nonreference variant. This is particularly problematic if samples were genotyped individually, since many of the sites will be deemed as homozygous reference and unreported, particularly around the selected locus in our selected pools where the AF is close to either 0 or 1. By genotyping the crosses together with the parents, we are ensuring that all informative sites are reported, because at least one of the two parents will be homozygous for the nonreference allele at informative sites. To filter for informative sites, we used bcftools (Li 2011) (v1.6) to first isolate the parental strains and retained only SNP sites where the parental strains have different homozygous alleles with both genotype quality of ≥30 (supplementary table 2, Supplementary Material online). In the crosses, all sites other than the informative sites are removed. No genotyping filter is applied on the crosses since many of the sites in the crosses will have intermediate AF that are difficult to genotype. Sites within 100 bp of repeats are removed to avoid copy number variants. The coverage and number of SNP sites after filtering can be found in supplementary table 1, Supplementary Material online. The allele-specific read counts can be determined from the AD field in the vcf files (Danecek et al. 2011).

**Inversion Identification**

For structural variant calls, we used the smoove wrapper for Lumpy (v0.2.13) after aligning e se to the reference genome (Layer et al. 2014). We identified large structural variants over 1 Mbp within and between Chr. 3L and 3R.

**Removing Offsite Viability Effects with Root-Finding Algorithm**

Given the complex equations for the offsite viability effects, we use root-finding algorithms to solve for D_{ij} instead of isolating it from the equation. Since q_f and D_{ij} can be predetermined, and D_{ij} can be substituted with D_{ij} in accordance with equation (9), we are left with formulae with only D_{ij} as the variable to solve. We use the root-finding function uniroot.all() from the package rootSolve in R to closely approximate the solution (Soetaert and Herman 2009). Note in some instances, more than one solution is possible, but usually only one is within reasonable range (between 0 and 0.5).

**Data Availability**

Intermediate files can be found on Dryad at https://doi.org/10.6078/D10D8F.

**Supplementary Material**

Supplementary data are available at Molecular Biology and Evolution online.

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