Transmission characteristics of simple cycloid drive with stepped planets

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Abstract. Simple cycloid drive is a type of high-sensitive gear train, by which it is possible to realize high transmission ratios in single-stage. Their advantage is also reflected in the compactness and simplicity of production, as well as achieving greater efficiency compared to conventional planetary drive train. In this paper, the basic equations for obtaining the transmission ratios, torque and efficiency of simple cycloid drive with stepped planets are shown. The dependence between basic transmission ratio and basic efficiency was also examined to find the optimum value of the transmission ratio and determine the point at which the self-locking occurs.

1. Introduction

In many industries, there is a need for using more compact and cheaper mechanical power transmissions. These targets are very difficult to achieve with conventional drive trains with fixed axes, so the need to replace these drive trains with suitable planetary drive trains is imposed. Very small dimensions of the gearbox and large transmission ratio can be achieved using planetary drive trains with cycloid gear. These gears are now mostly used in the case of single-stage cycloid reducers.

Planetary drive trains with cycloid gears have many advantages over classical planetary transmissions, but their usage is quite limited, mostly in the field of robotics. Due to the complicated and costly construction, in the past, the use of gearboxes with cycloid gears was avoided. However, with the development of the modern CNC machining centers, which enable the simple production of cycloid gears, there is the possibility of cheaper manufacturing and greater usage in commercial power transmissions.

2. Simple Cycloid Drive

In the analysis of a simple cycloid drive, it can be started from an elementary planetary mechanism with internal coupling, replacing, for example, classic involute gears with cycloid gears (figure 1). The cycloid gear $(Jp)$ meshes with a central ring gear $(I)$, which on its periphery has mounted pins and rotates around the central axis. In this case, the cycloid gear rotates around its axis and, together with the member $(S)$ - the eccentric shaft, rotates around the axis of the central gear $(I)$. The axis of rotation of the planet is at a distance from the central axis for the value $e$ (eccentricity). Thus, the cycloid gear $(Jp)$ is a planet, while the member $(S)$ is the carrier of the planet. Based on this, a complete similarity with elementary planetary mechanisms is observed, in which involute gears are used.
Members of the planetary mechanism whose axis coincides with the central axis and receive the external torques are called the basic members [1]. The members of the elementary mechanism, the central gear (1) and the carrier (S), whose axes of rotation coincide with the base axis, can’t be used in this case for the transfer of energy. In order to achieve this, one elementary member must be added to the elementary planetary mechanism. This gives a single-stage cycloid drive or simple cycloid drive. This can be accomplished in two ways:

- Adding a disc (2) with pins, this will accept the epicycle movement of the cycloid gear and transfer it to the shaft that coincides with the central axis (figure 2.a). This transfer method is used in today's cycloid reducers, and the disc (2) is called the output mechanism [2]. There are the holes on the cylinder, which allow the transfer of the rotation movement from the cycloid gear to the mechanism;
- Adding another central ring gear (2) with the pins placed on the periphery, which meshes with the second cycloid gear (2p) (figure 2.b). The cycloid gears (1p) and (2p) are tightly connected in this case. In literature, such planets are called stepped planets [3, 4].

In this paper, the transmission characteristics of the second method of obtaining a simple cycloid drive will be displayed. The equations and relations of the basic parameters defining the transmission performance will be presented.

In addition, it will be analyzed only drives with the numbers of pins of ring gear by one greater than the numbers of teeth of cycloid planet gear. For example, the number of pins of the i-th ring gear is:

\[ z_i = z_{ip} + 1, \]
where is: \( z_{ip} \) – numbers of teeth of \( i \)-th cycloid planet gear.

The gear drives shown in figure 2 have two degrees of freedom (DOF). By blocking one of the basic members, a two-shaft cycloid drive is provided which has only one DOF.

3. Transmission Ratios
If the eccentric shaft is stopped, one can be said that then the cycloid drive works as classical gearbox with a fixed axle, that is, with one degree of freedom. A simple work regime, when a carrier or eccentric shaft is stopped, can be termed as the basic mode and before that, define the transmission (speed) ratio, which is called the basic transmission (speed) ratio:

\[
i_o = i_{12} = \left( \frac{n_1}{n_2} \right)_{n_s=0},
\]

where is:  
\( n_1 \) - speed of ring gear shaft 1,  
\( n_2 \) - speed of ring gear shaft 2,  
\( n_{s} \) - speed of eccentric shaft S.

Basic transmission ratio of simple cycloid drive with stepped planets is:

\[
i_o = i_{11p}i_{2p2} = \frac{n_1}{n_{1p}} \frac{n_{2p}}{n_2} = \frac{n_1}{n_2} \frac{z_{1p}}{z_1} \frac{z_{2}}{z_{2p}} = \frac{z_2}{z_1} \left( \frac{z_1-1}{z_2-1} \right),
\]

where is:  
\( z_{1p} \) – speed of stepped planets,  
\( z_{2p} = (z_2 - 1) \) – the number of teeth on a cycloid gear 1,  
\( z_{2p} = (z_2 - 1) \) – the number of teeth on a cycloid gear 2,  
\( i_{11p} \) – transmission ratio between ring gear 1 and cycloid gear 1,  
\( i_{2p2} \) – transmission ratio between cycloid gear 2 and ring gear 2.

With the aid of the generalized Willis equation [5]:

\[
n_1 i_o n_2 + (i_o - 1)n_3 = 0,
\]

it is possible to derive the equations for all transmission ratios of two-shaft cycloid drive with stepped gear, as shown in Table 1.

**Table 1. Equations of transmission ratios of two-shaft cycloid transmission.**

| Transmission ratio \( f(i_o) \) | Transmission ratio \( f(z_{1p}, z_{2p}) \) |
|----------------------------------|----------------------------------|
| Minimum multiplication \( i_{12} = i_o \) | \( i_{12} = \frac{z_2}{z_1} \left( \frac{z_1-1}{z_2-1} \right) \) |
| Minimum reduction \( i_{21} = \frac{1}{i_o} \) | \( i_{21} = \frac{z_1}{z_2} \left( \frac{z_2-1}{z_1-1} \right) \) |
| Maximum multiplication \( i_{15} = 1-i_o \) | \( i_{15} = \frac{z_1}{z_2} \left( \frac{z_2-1}{z_1-1} \right) \) |
| Maximum reduction \( i_{51} = \frac{1}{1-i_o} \) | \( i_{51} = \frac{z_1}{z_2} \left( \frac{z_2-1}{z_1-1} \right) \) |
| Reversible multiplication \( i_{25} = 1-\frac{1}{i_o} \) | \( i_{25} = \frac{z_1}{z_2} \left( \frac{z_2-1}{z_1-1} \right) \) |
Reversible reduction

\[ i_{s2} = \frac{i_o}{i_o - 1}, \quad i_{s2} = \frac{z_2(z_1 - 1)}{z_1 - z_2} \]

4. Torques

From the equilibrium conditions, during stationary state, the sum of all external torques on basic members shafts is equal to zero, i.e: [2, 4]:

\[ T_1 + T_2 + T_s = 0, \]

where:
- \( T_1 \) - torque of ring gear shaft \( 1 \),
- \( T_2 \) - torque of ring gear shaft \( 2 \),
- \( T_s \) - torque of eccentric shaft \( S \).

The ratio between the input and the output power is expressed through the efficiency of cycloid gear drive. The efficiency of a simple cycloid drive, in the basic mode, is called the basic efficiency:

\[ \eta_o = \eta_{z1} = \left( \frac{P_2}{P_1} \right)_{n_s = 0}, \]

where:
- \( P_1 \) - power on ring gear shaft \( 1 \),
- \( P_2 \) - power on ring gear shaft \( 2 \).

Equation (6) can also be written as [4, 6]:

\[ \frac{T_2}{T_1} = -i_o \eta_o^w, \]

\[ w = \frac{P_{R1}}{|P_{R1}|} = \frac{T_1(n_1 - n_2)}{|T_1(n_1 - n_2)|} = \pm 1, \]

where:
- \( P_{R1} \) - rolling power which is transmitted by the ring gear shaft \( 1 \).

The torque ratios are referred to as coefficients of transformation of torques or energy transmission ratios [71]. Accordingly, in order to simplified analysis, a basic energy transmission ratio or basic energy ratio can be defined, such as:

\[ i_o = i_{o0} \eta_o^w. \]

If there are losses in the transmission, then on the basis of expressions (5), (7) and (9), the ratios of the torques can be obtained as:

\[ T_1 : T_2 : T_s = 1 : -i_o \eta_o : i_o - 1. \]

The simple cycloid drive has a positive transmission ratio \( i_o > 0 \), so that \( i_o > 0 \), and accordingly to the equations (7) and (9) it follows that the torque ratio \( T_2 / T_1 < 0 \). This means that the torque of shaft \( 1 \) and shaft \( 2 \) have different signs. Therefore, one of them, which has a higher absolute value, must be a summation shaft.

Basic transmission ratio, given in equation (3), shows that \( i_o < 1 \) if the condition \( z_1 < z_2 \) is satisfied, so that: \( 0 < i_o < 1 \) if \( w = +1 \). Since \( |T_2 / T_1| < 1 \), it can be concluded that the shaft \( 1 \) is a summation shaft.

In case \( w = -1 \), (the input shaft 2) it is possible to assume that the following three cases can occur, given the relationship between \( i_o \) and \( \eta_o \):
- \( i_o < \eta_o \), shaft \( 1 \) is summation shaft;
- \( i_o = \eta_o \), locking of the gearbox, because \( i_o = 0 \) and it follows from (10) that \( T_1 = T_2 \), \( T_s = 0 \);
- \( i_o > \eta_o \), shaft 2 become summation shaft.
Therefore, when calculating losses inside the gearbox, theoretically a locking of the gearbox and change of the summation shaft can occur, but only if the shaft 2 is the input shaft.

In order to understand the nature of the change in transmission ratios, the theoretical model of a simple cycloid drive with stepped planets will be considered, where $z_2 = 20$, while the number of teeth of ring gear $l$ changes from $z_1 = 4 \div 40$. In figure 3, the function of the change of the transmission ratios $i_{S1}$ and $i_{S2}$ is shown. If the teeth numbers are equal $z_1 = z_2 = 20$, self-locking can occur (the gearbox acts as a coupling). When $z_1 > z_2$, shaft 2 becomes a summation shaft.

![Figure 3. Change of transmission ratios $i_{S1}$ and $i_{S2}$.](image)

In figure 4, a change in the transmission ratios $i_{12}$, $i_{21}$, $i_{1S}$ and $i_{2S}$ is shown. It is obvious that there is no need for gears with a larger number of teeth $z_1$ (when the shaft 2 becomes a summation shaft), since similar transmission ratios can be achieved in the first period, or when the shaft $l$ is a summation shaft. Therefore, in order to analyze losses and self-locking, it is enough to observe the first period of changing in the number of teeth of ring gear $l$, in order to define the relationship between the efficiency and the transmission ratio.

![Figure 4. Change of transmission ratios $i_{12}$, $i_{21}$, $i_{1S}$ and $i_{2S}$.](image)
5. Efficiency

The efficiency of two-shaft cycloid drive can be presented as a function of energy ratio and speed ratio. In general case:

$$\eta_{xy} = \frac{P_x}{P_y} = \frac{T_y}{T_x} \frac{\omega_y}{\omega_x} = \frac{i_{xy}}{i_{xy}},$$  \hspace{1cm} (11)

where is:

- $\eta_{xy}$ - the efficiency when shaft $x$ is the input and shaft $y$ the output shaft,
- $i_{xy} = \left( -\frac{T_y}{T_x} \right)$ - energy ratio between shaft $x$ and $y$,
- $i_{xy}$ - speed ratio between shaft $x$ and $y$.

Thus, in the case that the shaft $I$ is the input shaft, and the shaft $S$ is the output shaft, the efficiency can be written as:

$$\eta_{IS} = \frac{P_S}{P_I} = \frac{T_S}{T_I} \frac{\omega_S}{\omega_I} = \frac{i_{IS}}{i_{IS}},$$  \hspace{1cm} (12)

and after introducing the equations from Table 1, as well as (9) and (10), the following is obtained:

$$\eta_{IS} = \frac{i_o - 1}{i_o - 1}.$$  \hspace{1cm} (13)

Since the input power is positive, then $T_i > 0$ and $n_i > 0$. Shaft $I$ is summation shaft and therefore $T_S < 0$. Because the output power is negative, so the speed $n_S > 0$. Accordingly, in order to determine the coefficient $w$ from the equation (8), it is necessary to calculate the speed ratio $i_{IS} = n_i / n_S$. From the equation in Table 1, it can be concluded that $i_{IS} < 1$, for basic transmission ratio $0 < i_o < 1$, so that $n_S > n_i$. Therefore, it is clear that the coefficient $w = -1$, because $P_{SI} = T_i (n_i - n_S) < 0$. Equation (13) can be written as:

$$\eta_{IS} = \frac{i_o \eta_o^{-1} - 1}{i_o - 1}.$$  \hspace{1cm} (14)

Similar analysis can be used to obtain equations for the efficiency of other variants of the two-shaft cycloid drive, as shown in Table 2.

| Table 2. The efficiency equations of two-shaft cycloid drive. |
|---------------------------------------------------------------|
| $w$ | $\eta_{12}$ | $\eta_{21}$ | $\eta_{1S}$ | $\eta_{S1}$ | $\eta_{2S}$ | $\eta_{S2}$ |
|----------------------------------------------------------------|
| $f(i_o, i_o)$ | $\bar{\eta}_o$ | $\frac{i_o}{i_o}$ | $\frac{i_o - 1}{i_o - 1}$ | $\frac{i_o - 1}{i_o - 1}$ | $\frac{i_o (1-i_o)}{i_o (1-i_o)}$ | $\frac{i_o (1-i_o)}{i_o (1-i_o)}$ |
| $f(i_o, \eta_o)$ | $\eta_o$ | $\eta_o$ | $\frac{i_o \eta_o^{-1} - 1}{i_o - 1}$ | $\frac{i_o - 1}{i_o - 1}$ | $\frac{i_o - \eta_o}{i_o - 1}$ | $\frac{i_o - 1}{i_o - \eta_o}$ |

From Table 2, it can be concluded that in cases where the eccentric shaft is output shaft ($\eta_{S1}, \eta_{S2}$) and for $i_o \geq \eta_o$, the two-shaft cycloid drive is blocked (self-locking occurs), because $\eta_{IS} \leq 0$ and
\( \eta_{2S} \leq 0 \). In addition, this shows that it is impossible for shaft 2 to become a summation shaft. This is possible only in the case of the basic mode, when shaft 2 is the input shaft.

If a constant value of the basic efficiency of \( \eta_b = 99.6\% \) is assumed, a change in the efficiency of the other two-shaft cycloid drive modes can be displayed. In Figure 5, a change in the efficiency for the modes where the shaft 2 is locked can be observed, in the function of changing the number of gear teeth \( (z_i) \). The value of the efficiency, in case the shaft 1 is stopped, is slightly different from the change of the efficiency shown in Figure 5. Therefore, only cases when the shaft 2 is locked will be considered. It is obvious that the cycloid drive achieves a higher efficiency when working as a reducer.

**Figure 5.** Change in the efficiency for the mode where the shaft 2 is locked.

It is very important to examine the dependence between the transmission ratio and the efficiency, in order to find the optimum value of the transmission ratio and determine the point at which the self-locking occurs. In figure 6, the relationship between the transmission ratio and the efficiency is shown when the shaft 1 is input and the shaft S is output shaft. The self-locking occurs when the number of teeth is \( z_i = 19 \), since the value of the efficiency is \( \eta_{1S} = -44.6\% \).

**Figure 6.** Relationship between the transmission ratio \( i_{1S} \) and the efficiency \( \eta_{1S} \).
In figure 7, the relationship between the transmission ratio and the efficiency is shown, when the shaft $S$ is the input and shaft $1$ is the output shaft. It is obvious that the increase in the transmission ratio is proportional to the decrease in the efficiency. But even so, it can be concluded that in the particular area of transmission ratios, this kind of simple cycloid drive provides a high degree of efficiency than they are for conventional planetary drives.

For example, Wolfrom two-stage planetary drive, with transmission ratio 97:1, has the efficiency $\eta = 45\%$ [8]. A simple cycloid drive, with a slightly higher transmission ratio ($z_1=16$, $z_2=20$, $i_{S1}=108$, $\eta_o=99.6\%$), has an efficiency of $\eta = 70.1\%$.

![Figure 7. Relationship between the transmission ratio $i_{S1}$ and the efficiency $\eta_{S1}$.](image)

6. Conclusion

By theoretical analysis, it can be noticed that a simple cycloid drive with stepped gear can achieve very high transmission ratio, with great efficiency. In addition, their advantage is reflected in the compactness and simplicity of production, as well as achieving greater efficiency compared to conventional planetary drive train. It is recommended to design these gearboxes with a smaller number of teeth, as this results in smaller dimensions of the gearbox while preserving a large transmission ratio.

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