Necessity of mixed kinetic term in the description of general system with identical scalar fields

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Abstract
Most general renormalizable interaction in the system with a set of scalar fields having identical quantum numbers generates naturally mixed kinetic terms in the Lagrangian. Taking into account these terms leads to modification both the renormalization group equations and the tree level analysis as compared with many published results. We obtain conditions for non-appearance of such a running mixing in some important cases.

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1. Introduction
In this paper I consider systems containing two or more scalar fields with identical quantum numbers. Such systems were studied in numerous papers (e.g. [1]-[14]). Below I consider a phenomenon that often appears and usually overlooked in analysis of such systems, sometimes – reasonably, often – unreasonably. The essence of the phenomenon is the necessity to take into account for renormalizability the mixed kinetic term in the Lagrangian. In fact, this statement can be found in ref. [5]. However the analysis there does not contain the condition when this term is not necessary or diagonalizing mixing angle is not running. Moreover, during 12 years after publication of [5] this effect is overlooked as before in many papers (for recent references see e.g. [12], [13]).

It is well known that the field mixing in these theories can be naturally accounted by transition to a new basis in \{\phi_i\}-space (general rotation plus renormalization) in which both kinetic and mass terms of the Lagrangian become diagonal, and physical particles become different from those described by basic fields. Below we consider diagonalizing mixing angles, realizing diagonalization of only kinetic term. From the naive glance, the price for the transition to this

\footnote{Such term is included in some descriptions for the system of vector gauge fields [15]}

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form is only in the redefinition of coupling constants in potential. However, the situation is more complex.

In the majority of cases a system, containing fields with identical quantum numbers, originates from the system having some extra symmetry at very small distances; at these distances our fields either are members of some more general multiplet or have quantum numbers, separating these fields. In this region the discussed field mixing either is forbidden or can be considered as some generalized rotation with non-running mixing angles. This symmetry is violated at some intermediate scale either by new (large distance) interactions or spontaneously. It results in appearance of field mixing at large distances. That is the natural theory in which, going to smaller distances, the effect of mentioned symmetry violation vanishes, system restores a primary symmetry globally, the diagonalizing mixing angles become constant. The key point of our discussion below is the classification of models with respect to their correspondence to this naturalness. We concentrate our discussion on kinetic mixing only and do not consider well known problems of diagonalization of physical states (near mass shell) – see e.g. [5] for details.

In the sect. 2 we describe considered models and show how mixed kinetic term appears. In the sect. 3 we describe some specific features of renormalization procedure for the system with fields having identical quantum numbers. The sect. 4 is devoted to classification of possible Lagrangians in respect of specific discrete $Z_2$ symmetry which appears in correspondence with classification in respect of mentioned naturalness of theory. In the sect. 5 we found condition for this naturalness – condition for non-appearance of running kinetic term. In the sect. 6 the corresponding renormalization group equations (RGE) in one-loop approximation are constructed. In the sect. 7 we consider main physical consequences of the result.

2. Models

I consider below two models. The first one is the simplest model containing two real scalar fields $\phi_1$ and $\phi_2$ (see e.g. [1],[2]). It allows to present all calculations without complex combinatorics. The essence of a phenomenon is clearly seen in this simplest model. In parallel we consider general Two Higgs Doublet Model (2HDM) which can pretend for description of physical reality.

**Simplest model.** The "most general" renormalizable Lagrangian of the system with two real scalar fields is written usually as (notation is chosen to be closer to the standard 2HDM case)

$$
\mathcal{L}_\phi = T_{0\phi} - V, \quad T_{0\phi} = \frac{\partial_\mu \phi_1 \partial_\mu \phi_1 + \partial_\mu \phi_2 \partial_\mu \phi_2}{2},
$$

$$
V = V_2 + V_4, \quad V_2 = \frac{m_{11}^2 \phi_1^2 + m_{22}^2 \phi_2^2 + 2m_{12}^2 \phi_1 \phi_2}{2},
$$

$$
V_4 = \frac{\lambda_1}{2} \phi_1^4 + \frac{\lambda_2}{2} \phi_2^4 + \lambda_3 \phi_1^2 \phi_2^2 + \lambda_6 \phi_1^3 \phi_2 + \lambda_7 \phi_1 \phi_2^3.
$$
Two Higgs Doublet Model (2HDM). One of the most important realizations of the discussed type systems presents the simplest extension of the minimal SM – the 2HDM, describing the system of two complex isodoublet scalar fields $\phi_1$ and $\phi_2$ with identical hypercharges (see e.g. [4]-[10]) with the Lagrangian

$$
\mathcal{L} = T_{0\phi} - V; \quad T_{0\phi} = \frac{\partial_\mu \phi_1^\dagger \partial_\mu \phi_1 + \partial_\mu \phi_2^\dagger \partial_\mu \phi_2}{2},
$$

$$
V = - \frac{m^2_{11} \phi_1^\dagger \phi_1 + m^2_{22} \phi_2^\dagger \phi_2 + \left( m^2_{12} \phi_1^\dagger \phi_2 + h.c. \right)}{2} + \frac{\lambda_1 (\phi_1^\dagger \phi_1)^2}{2} + \frac{\lambda_2 (\phi_2^\dagger \phi_2)^2}{2} + \lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + \lambda_4 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) +

\left[ \frac{\lambda_5 (\phi_1^\dagger \phi_2)^2}{2} + \lambda_6 (\phi_1^\dagger \phi_1) (\phi_1^\dagger \phi_2) + \lambda_7 (\phi_2^\dagger \phi_2) (\phi_1^\dagger \phi_2) + h.c. \right].
$$

(2)

Here the vacuum state with $\langle \phi_i \rangle \neq 0$ violates EW symmetry with transition to the representation of the Lagrangian via physical Higgs fields and Goldstone fields. The states of this physical Higgs representation are obtained from incident EW isodoublets by simple decomposition and rotation. This procedure does not influence the ultra-violet behavior. Therefore, one can consider our ultra-violet kinetic mixing problem in the primary EW symmetric basis with the Lagrangian e.g. in the form (2), just as it was done at obtaining positivity constraints for the potential (see e.g. [14]).

The reparameterization symmetry. The potential (1) depends on 8 real free parameters: $m^2_{11}, m^2_{22}, m^2_{12}, \lambda_1, \lambda_2, \lambda_3, \lambda_6, \lambda_7$ while potential (2) depends on 14 free parameters: real $m^2_{11}, m^2_{22}, \lambda_1, \lambda_2, \lambda_3, \lambda_4$ and complex $m^2_{12}, \lambda_7, \lambda_6, \lambda_7$.

However, the models contains two fields with identical quantum numbers. Therefore, they can be described both in terms of fields $\phi_k$ ($k = 1, 2$), used in (1), (2), and in terms of fields $\phi'_k$ obtained from $\phi_k$ by a global unitary reparametrization transformation $\mathcal{F}$ – rotation in $(\phi_1, \phi_2)$ space:

$$
\begin{pmatrix} \phi'_1 \\ \phi'_2 \end{pmatrix} = \mathcal{F} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \Rightarrow \begin{pmatrix} \lambda'_1 \\ m^2_{12} \end{pmatrix} = U \begin{pmatrix} \lambda_1 \\ m^2_{12} \end{pmatrix}.
$$

(3a)

$$
\mathcal{F} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad \text{for model (1)};
$$

$$
\tilde{\mathcal{F}} = e^{-i\rho_0} \begin{pmatrix} \cos \theta e^{i\rho/2} & \sin \theta e^{i(\tau - \rho)/2} \\ -\sin \theta e^{-i(\tau - \rho)/2} & \cos \theta e^{-i\rho/2} \end{pmatrix} \quad \text{for model (2)}.
$$

(3b)

The Lagrangians, which can be obtained from each other by these transformations, describe identical physical reality. They represent the reparameterization equivalent family of the Lagrangians. These transformations form reparameterization symmetry groups, studied for 2HDM in detail, e.g. in [14]. In particular, physical observables in the simplest model depend on 7 free parameters, while in 2HDM they depend on 11 free parameters (cf. discussion in [4]-[10]).
Appearance of mixed kinetic term. Let us consider renormalization of two-point Green functions. The diagrams with two external legs appear at two-loop level, see Fig. 1. These diagrams are divergent quadratically and describe polarization operators $\Pi_{ab}$. One can choose subtraction point so that its divergent part have form 

$$\Pi_{ab} = \frac{1}{2} \left[ (Z_{3,ab} - \delta_{ab}) k^2 \phi_a \phi_b + (Z_{m,ab} - m_{ab}^2) \phi_a \phi_b \right]$$

with

$$Z_{3,11} = 1 \propto 12 \lambda_1^2 + 4 \lambda_3^2 + 3 \lambda_7^2,$$

$$Z_{3,22} = 1 \propto 12 \lambda_2^2 + 4 \lambda_3^2 + 3 \lambda_7^2,$$

$$Z_{3,12} \propto \lambda_1 \lambda_6 + \lambda_6 \lambda_3 + \lambda_3 \lambda_7 + \lambda_7 \lambda_2.$$ 

Corresponding counter-terms in the Lagrangian 

$$\left( Z_{3,aa} - 1 \right) \partial_\mu \phi_a \partial_\mu \phi_a / 2$$

and 

$$\left( Z_{m,aa} - m_{aa}^2 \right) \phi_a^2 / 2$$

have the same structure as some terms in the unperturbed Lagrangian and describe renormalization of wave functions and masses. The counter-terms

$$Z_{3,12} \partial_\mu \phi_a \partial_\mu \phi_1 \phi_2$$

and 

$$\left( Z_{m,12} - m_{12}^2 \right) \phi_1 \phi_2$$

describe the kinetic and mass field mixing respectively.

The appearance of counter-term with $Z_{3,12}$ means that the Lagrangians (1), (2) in the general case are non-renormalizable, for renormalizability the kinetic term $T_{0\phi}$ must be enlarged by the mixed kinetic term $^3$.

$$T_{0\phi} \rightarrow T_{\phi} = \left\{ \begin{array}{ll} \frac{1}{2} \partial_\mu \phi_1 \partial_\mu \phi_1 + \partial_\mu \phi_2 \partial_\mu \phi_2 + 2 \kappa \partial_\mu \phi_1 \partial_\mu \phi_2 & \text{for (1)}; \\
\frac{1}{2} \partial_\mu \phi_1^\dagger \partial_\mu \phi_1^\dagger + \partial_\mu \phi_2^\dagger \partial_\mu \phi_2^\dagger + \kappa^* \partial_\mu \phi_1^\dagger \partial_\mu \phi_2^\dagger + \kappa^* \partial_\mu \phi_2^\dagger \partial_\mu \phi_1^\dagger & \text{for (2).} \end{array} \right.$$ 

3. Renormalization procedure, case with field mixing

Let us describe in detail the renormalization procedure for the case when field mixing is possible.

We reduce each step of perturbation theory to the standard construction of $S$-matrix, in which the kinetic term have form $T_{0\phi}$, and together with diagonal

\footnote{These equations are written for the simplest model, the equations for 2HDM have the same structure but slightly more complex form.}

\footnote{This phenomenon is similar in some respects to necessity to supplement Yukawa interaction by $\phi^4$ term in the renormalizable Lagrangian, see e.g. \[15\].}
mass terms (with $m_{11}^2$ and $m_{22}^2$) it forms the unperturbed Lagrangian $L_0$, while other terms (including mixed mass term $m_{12}^2$) are treated as perturbations.  

- **Procedures $\hat{F}_B$, $\hat{Z}_B$.** Let we have basically the Lagrangian $L_B$ with non-diagonal kinetic term $T_\phi$ (7). The first stage contains reparameterization rotation $\hat{F}_B$ of form (3), diagonalizing kinetic term, and subsequent renormalization $\hat{Z}_B$, normalizing all items of kinetic term, with resulting kinetic term $T_{0\phi}$

$$L_{B;0} = \hat{Z}_B \hat{F}_B L_B.$$  

(8)

- **Procedure $\hat{P}_1$.** The calculation of radiative corrections in the first non-trivial order gives counter-terms of the Lagrangian, which lead to the renormalized Lagrangian

$$L_{R1}^B = \hat{P}_1 L_{Rd0} = \hat{P}_1 \hat{Z}_B \hat{F}_B L_B.$$  

(9)

- To start calculation of next order radiative corrections, this Lagrangian $L_{R1}^B$ is transformed with procedures $\hat{F}_1$ and $\hat{Z}_1$, similar to $\hat{F}_B$ and $\hat{Z}_B$, in order to obtain the renormalized Lagrangian with the same kinetic term $T_{0\phi}$, as in beginning:

$$L_{Rd1} = \hat{Z}_1 \hat{F}_1 L_{R1}^B.$$  

(10)

- The subsequent iterations are described in the same manner with operators $\hat{P}_2$, $\hat{F}_2$ and $\hat{Z}_2$, etc. The new (but almost trivial) point is an appearance of new diagonalizing procedure $\hat{F}_i$ in each order of perturbation theory. Generally, *diagonalizing mixing angles* $\theta_i$ in these $\hat{F}_i$ are different — we obtain *running mixing angles* — an additional subject for renormalization, similar to coupling constants. This very phenomenon is described by introduction of mixed kinetic term in the Lagrangian, with running coefficient.

4. Different opportunities

Now we consider different opportunities with respect to $Z_2$ symmetry, i.e. invariance of the Lagrangian under transformations, which prohibit $\phi_1 \leftrightarrow \phi_2$ transitions:

$$\phi_i^2 \rightarrow \phi_i^2, \quad \phi_1 \phi_2 \rightarrow -\phi_1 \phi_2 \quad \text{for model (1)};$$

$$\phi_{1,a}^i \phi_{1,b} \rightarrow \phi_{1,a}^i \phi_{1,b}, \quad \phi_{1,a}^i \phi_{2,b} \rightarrow -\phi_{1,a}^i \phi_{2,b} \quad \text{for model (2)}$$  

(11)

($a$ and $b$ are weak isospin indices)

(A) The dimension 4 (*dim4Zs*) $Z_2$ symmetry is realized for the operator dimension 4 part of the Lagrangian, it takes place for the *dim4Zs* potential with

$$L_Q : \quad \lambda_6 = \lambda_7 = 0, \quad \kappa = 0.$$  

(12)

4In this approach the partial summation of perturbation theory series in $m_{12}^2$ allows to diagonalize mass term. We do not discuss such procedure below.
(A1) If additionally $m^2_{12} = 0$, we deal with the case of precise $Z_2$ symmetry (for the entire Lagrangian). In this case field mixing is absent in the bare Lagrangians $\mathcal{L}_1$, $\mathcal{L}_2$. The counter-terms which mix fields $\phi_a$ does not appear.

(A2) If $m^2_{12} \neq 0$, field mixing is obliged by only operator of dimension 2, transitions $\phi_1 \leftrightarrow \phi_2$ are allowed near mass shell (at large distances) and forbidden far from mass shell (at small distances) – softly violated $Z_2$ symmetry.

The mixed term $m^2_{12} \phi_1 \phi_2$ in the basic potential generates mixed polarization operator $\Pi_{12}$. However, since single $Z_2$ violating term $m^2_{12} \phi_1 \phi_2$ has operator dimension 2 (not 4!), it results in the polarization operator only logarithmic divergences, i.e. $Z_{3,12} = 0$. Therefore, in this case the mixed kinetic counter-term does not appear. In other words, the discussed mixing takes place at large distances only, it disappear at small distances (large virtualities), that is the natural theory, discussed above.

The same reasons show that the amplitudes for transitions $\phi_1 \phi_2 \rightarrow \phi_1 \phi_1$, etc., appeared in perturbations, are convergent and disappear at small distances (at large virtualities). Hence, they do not give counter-terms like $\lambda_6 \phi^3_1 \phi_2$, etc. Therefore, the Lagrangian $\mathcal{L}_1$, $\mathcal{L}_2$ is completely renormalizable.

In terms of sect. 3 in these cases the diagonalizing mixing operators $F_a \equiv 1$ in each order of perturbation theory.

(B) Hidden dimension 4 $Z_2$ symmetric (hdim4Zs) case can be obtained from the Lagrangian with dimension 4 $Z_2$ symmetry $\mathcal{L}_1$, $\mathcal{L}_2$ with the reparametrization transformation $\hat{F}_H$, it is reparameterization equivalent to dim4Zs case:

$$\mathcal{L}_H = \hat{F}_H \mathcal{L}_Q.$$  \hspace{1cm} (13)

This Lagrangian has coefficients $\lambda'_j$ and $m'^2_{ij}$, expressed via primary values $\lambda_i$ and $m^2_{ij}$ with transformation $\mathcal{L}_1$. In particular, we have $\lambda'_0 \neq 0$, $\lambda'_7 \neq 0$. Mixed kinetic term does not appear in this stage, since our transformation is simple rotation and basic kinetic term matrix is unitary one, i.e. diagonalizing operator $\hat{F}_B = 1$.

Let us consider below the case $\lambda_1 \neq \lambda_2$, for definiteness. In terms of sect. 3 the radiatively renormalized Lagrangian

$$\mathcal{L}_H^{R1} = \hat{\mathcal{P}}_1 \mathcal{L}_H = \hat{\mathcal{P}}_1 \hat{F}_H \mathcal{L}_Q \equiv \hat{F}_H \hat{\mathcal{P}}_1 \mathcal{L}_Q.$$  \hspace{1cm} (14)

In the primary radiatively renormalized dim4Zs Lagrangian $\hat{\mathcal{P}}_1 \mathcal{L}_Q$ the field renormalization constants were different, $Z_{3,11} \neq Z_{3,22}$. Therefore, after rotation $\hat{F}_H$ the mixed kinetic counter-term $Z_{3,12}$ appears, producing mixed kinetic term in $\mathcal{L}_H^{R1}$. By construction, it is clear, that the diagonalizing operator $\hat{F}_1 = \hat{F}_H^{-1}$. Moreover, it gives simultaneously the dim4Zs form of the renormalized Lagrangian $\mathcal{L}_{Rd1}$. Beginning from the second nontrivial order of perturbation theory we come to the dim4Zs Lagrangian, without the mixing kinetic term.

(C) In the general case of true hardly violated $Z_2$ symmetry we have $\lambda_0 \neq 0$ and (or) $\lambda_7 \neq 0$ and the rotation $\mathcal{L}_1$ transforming the Lagrangian to the dim4Zs form $\mathcal{L}_2$ does not exist. In our classification, that is unnatural theory.
In this case in addition to diagonal terms with $Z_{3,aa}$ the off-diagonal term $Z_{3,12}k^2\phi_1\phi_2$ appears in the divergent part of polarization operator. It produces counter-terms $Z_{3,12}\partial_\mu\phi_1\partial_\mu\phi_2$ in the Lagrangian, which were absent in the bare Lagrangian $L_0$. It means that our theory with the kinetic term $T_{00}$ is not renormalizable. If rotation (3), transforming this potential to the dim4Zs form, does not exist, to restore renormalizability, the basic Lagrangian must be supplemented by the mixed kinetic term, i.e. the kinetic term of the Lagrangians (1), (2) must be rewritten in the form (7). As a result, the diagonalizing mixing angle is different in different orders of perturbation theory. Therefore, it is running (see discussion after (10)).

This enlargement of the kinetic term adds new degree of freedom in the tree level phenomenological analysis and makes more complex RGE. In many respects the coefficient $\kappa$ can be treated as some new coupling constant like $\lambda_i$ (see in more detail RG equations below).

5. The condition for non-appearance of running mixed kinetic term

In accordance with previous discussion, the question in the title can be rewritten in such a form: In what case the coefficient at mixed kinetic term is non-running, or how one can know whether the considered potential is true hardly $Z_2$ violating one or we deal with hdim4Zs case? To find the answer, let us remind that the $\lambda_{1-7}$ coefficients of “rotated” hdim4Zs potential are obtained from parameters of the primary dim4Zs potential (12) with transformation (3). Therefore, coefficients of hdim4Zs potential obey some relations. To obtain these relations one can express $\lambda_i'$ via $\lambda_i$ and rotation angles from ref. (3). Next, one should consider these equations as equation like $X[\cos \theta] = 0$ and find condition, at which the form $X$ is so degenerate as $X[\cos \theta] = 0$ at arbitrary $\cos \theta$.

For the 2HDM an elegant form of this condition is obtained with the aid of geometrical approach of [10]. In this approach it is useful to write the general potential (2) via irreducible representations of $SU(2) \times U(1)$ reparameterization symmetry group. In this way we obtain tensor and vector forms in the parameter space, constructed from coefficients of the Lagrangian,

$$a_{ij} = \frac{1}{2} \begin{pmatrix} Re\lambda_5 - a & Im\lambda_5 & Re(\lambda_6 - \lambda_7) \\ Im\lambda_5 & -Re\lambda_5 - a & Im(\lambda_6 - \lambda_7) \\ Re(\lambda_6 - \lambda_7) & Im(\lambda_6 - \lambda_7) & 2a \end{pmatrix},$$

$$a = \frac{\lambda_1 + \lambda_2 - 2\lambda_3 - 2\lambda_4}{6};$$

$$b_i = -\frac{1}{\sqrt{2}} \begin{pmatrix} Re(\lambda_6 + \lambda_7) & Im(\lambda_6 + \lambda_7) & \frac{\lambda_1 - \lambda_2}{2} \end{pmatrix}. $$

In the dim4Zs case these forms some elements of these objects becomes equal to zero. Thus, it is easy to check that in this case the vector $\Phi_k = e_{ijk}a_{ir}b_rb_j = 0$, where $e_{ijk}$ is standard Levy-Civita tensor. This identity conserves at the reparameterization transformations. Therefore, the representation independent
condition for the non-appearance of running mixed kinetic term can be written as three conditions

$$\Phi_k \equiv e_{ijk} a_i b_r b_j = 0 \quad (k = 1, 2, 3).$$

(16)

The case in which mixed kinetic term does not appear despite the explicit violation of dim4 $Z_2$ symmetry is realized at complete $\phi_1 \leftrightarrow \phi_2$ symmetry of $V_4$, i.e. at

$$\lambda_1 = \lambda_2, \quad \lambda_6 = \lambda_7, \quad \lambda_5 = \lambda_5^*.$$ 

(17)

In this case the diagonalizing operator $F_1$ of sect. 3 is degenerated to the form, proportional to 1, and reason for mixed kinetic term does not appear. One can check directly that in this case the condition (16) is also valid. Therefore, rotation to the hdim4Zs form is possible.

- **Effect of fermions, variations due to Yukawa interaction.**

The systems of scalar fields are considered usually together with fermion fields $\psi_a$, having Yukawa interaction with both our scalars. In particular, for the simplest model

$$\mathcal{L}_\psi = i \bar{\psi_a} \frac{i}{\hbar^2} \bar{\partial} \psi_a - m_a \bar{\psi_a} \psi_a + \sum_a (g_1 a \phi_1 + g_2 a \phi_2) \bar{\psi_a} \psi_a.$$ 

(18)

With this term $Z_2$ symmetry (11) takes place if only $g_2 a g_1 a = 0$ for each $a$ — in addition to $m_{12} = 0, \lambda_6 = \lambda_7 = 0$. (For 2HDM, the similar condition looks as the condition that each right-handed fermion field is coupled to only one scalar, $\phi_1$ or $\phi_2$. In particular, that are Model I or Model II in classification of [9].)

If $g_1 a g_2 a \neq 0$, the $Z_2$ symmetry is violated hardly, giving — via loop correction — counter-terms like $\lambda_6$, $\lambda_7$ and $\kappa$. And — vice versa — if initially it was, e.g. $\lambda_7 = 0, g_2 a = 0$, but $\lambda_6 \neq 0$ or (and) $\kappa \neq 0$, the loop corrections make necessary to add in the bare Lagrangian terms with non-zero $\lambda_7, g_2 a$ and $\kappa$ for renormalizability. The condition for the non-running diagonalizing mixing angles for scalar fields can be written as an existence of representation of the potential, which has softly $Z_2$ violated form in scalar sector and simultaneously $g_2 a g_1 a = 0$ in Yukawa sector. In particular, for 2HDM this property is violated in Model III for Yukawa sector (where some right fermions are coupled to both fields $\phi_1$ and $\phi_2$).

- **Three notes.**

  A. It is easy to find conditions for the non-appearance of mixed kinetic term in two-loop approximation, it follows from (11):

$$\lambda_1 \lambda_6 + \lambda_6 \lambda_3 + \lambda_3 \lambda_7 + \lambda_7 \lambda_2 = 0 \quad \text{for model(1)};$$

$$\lambda_1 \lambda_6 + \lambda_2 \lambda_7 + (\lambda_3 + \lambda_4)(\lambda_6^* + \lambda_7^*) + \lambda_5 \lambda_6 + \lambda_5 \lambda_7^* = 0 \quad \text{for model(2)}.$$ 

(19)

5This form of condition looks more useful than that obtained in — vector $b_i$ must be eigenvector of operator $a_{ij}$.
These conditions are valid only for two-loop approximation. In higher orders these conditions are supplemented by new and new conditions, having no non-trivial solution, except the case of complete \( \phi_1 \leftrightarrow \phi_2 \) symmetry in \( V_4 \) \((17)\).

**B.** One can consider \((2)\) as particular case of the Lagrangian of theory with 4 complex scalar fields which are the components of two isospinors \( \phi_1, \phi_2 \). We denote it here as 4S theory. The coefficients of this 4S Lagrangian are constrained by condition that this Lagrangian can be written via isospinors \( \phi_i \). These constraints acquire more complex form after general rotation of all fields in 4S space.

Just as it was discussed above, some terms in the potential of 4S theory generate mixed kinetic terms with different coefficients \( \kappa_{ab} (a \neq b = 1, 2, 3, 4) \). General condition for non-appearance of such terms can be written separately. However in our case (when the Lagrangian allows the existence of 2HDM form written via isospinors \( \phi_i \)) all these conditions are covered by \((16)\).

**C.** The general transformation of the Lagrangian, including reparameterization and dilations of fields, allows to eliminate the terms with \( \lambda_6, \lambda_7 \) from potential (like in the case of \( \text{dim4Zs} \)). Simultaneously in the kinetic term the diagonal terms acquire different normalizations, and the mixed kinetic term appears. This approach is useful in the analysis of tree approximation and extrema of potential (see \([11]\)) but it results in complexities in the study of perturbation theory. In particular, the counter-terms with \( \lambda_6, \lambda_7 \) appear in radiative corrections, violating the \( Z_2 \) symmetry hardly. This representation does not seem to be available for the study of the discussed problem.

6. **Modified RGE for invariant charges at** \( k^2 \gg |m^2_{ij}| \), the simplest model

The renormalization group analysis of considered systems was done e.g. in refs. \([6], [7]\). However, the mixed kinetic term was not take into account there. Here we describe main features of the RGE only for invariant charges in the ultraviolet region for the case of true hard violation of \( Z_2 \) symmetry, i.e. with the Lagrangian \((1), (7)\). (Equations for propagators, etc. can be obtained after that by standard methods.)

The discussion of RGE looks more simple if we add to our system the interaction with fermions in the form \((18)\) with \( g_1 g_2 \neq 0 \), violating \( Z_2 \) symmetry. The advantage of this case is that here the mixed kinetic term appears in one-loop approximation while in the pure scalar case it appears first in two loops only.

One can imagine two ways for RG analysis of discussed situation:

- The kinetic term \((7)\) is transformed to the diagonal form by the change of the type of \( \phi_a = A_{ab} \Phi_b \) with \( a, b = 1, 2 \) and suitable \( A_{ab} \). After this transformation the coefficients of potential are changed \( \lambda_i \to \Lambda_i \).
but the potential keeps its general form \(^{(1)}\). In accordance with the previous discussion, the perturbations produce the scale dependent mixed kinetic term. Then one must repeat diagonalization (anew at each new scale and each new iteration).

- I prefer to use another way in which the mixed kinetic term is treated as an additional contribution into \(\mathcal{L}_{\text{int}}\) with new coupling given by the coefficient of this term. The diagonalization of the kinetic term can be performed at the final stage.

In this approach, e.g., the typical tree diagrams for the process \(\phi_1\phi_1\phi_2 \rightarrow \phi_1\phi_2\phi_2\) have the form of fig. 2, where open blob \(\rho_i\) corresponds to a full vertex \(\rho_i \leftarrow \lambda_i\), dark point corresponds to a full kinetic mixing \(\tau \leftarrow \kappa\) – see \((20)\).

As usual (see \([16]\)), we introduce five 1PI 4-scalar-vertexes \(\Delta_{abcd}\) with \(a, b, c, d = 1, 2\), two 1PI fermion-scalar vertexes \(\Gamma_a\), nominator of fermion propagator and matrix numerator of boson propagator \(s\) and \(d_{ab}\), defined in the considered region via complete propagators as \(S = \hat{k}s(k^2)/k^2\) and \(D = \frac{1}{k^2}(d_{11} \  d_{12} \  d_{21} \  d_{22})\). The typical definitions for invariant charges are similar to well known ones \([16]\), for example

\[
\rho_4 = d_{11}^{3/2} d_{22}^{1/2} \Delta_{1112}; \quad \sigma_1 = s d_{11}^{1/2} \Gamma_1; \quad \tau_{12} = d_{11}^{1/2} d_{22}^{1/2} d_{12}. \tag{20}
\]

In the considered simplest case the new invariant charge \(\tau_{12} = \tau_{21}\) and we omit label at \(\tau\).

In calculations below we assume, for definiteness, \(1 \gg \kappa \gg \lambda_i \sim -g_i^2\).

Now, for example, the typical 4-scalar vertex diagrams in one loop approximation have the form of fig. 3. Similar correction must be included in fermion polarization operator. Corresponding RGE and \(\beta\)-functions are calculated easily via known loop integrals with new simple combinatorics. In the one-loop approximation, for example (\(L = \ln(k^2/\mu^2)\)),

\[
\frac{d\rho_1}{dL} \equiv \beta_{\lambda_1} = 9\rho_1^2 + \rho_3^2 - 4\sigma_1^4 + 4\sigma_1^2\rho_1 + \frac{9}{4} \rho_2^2 + 18\kappa\rho_1\rho_4 + 9\kappa^2\rho_1\rho_3 + \frac{9}{2} \kappa^3 \rho_4^2;
\]

\[
\frac{d\sigma_1}{dL} \equiv \beta_{\sigma_1} = \frac{5}{2} \sigma_1^3 + \frac{1}{2} \sigma_1 \sigma_2^2 + \kappa \sigma_1^2 \sigma_2;
\]

\[
\frac{d\tau}{dL} \equiv \beta_{\tau} = 2\sigma_1 \sigma_2 + \kappa (\sigma_1^2 + \sigma_2^2) + c \sum_{k=0}^{3} C_k \kappa^k.
\]

\(C_0 = 12(2\rho_1\rho_4 + \rho_4\rho_5 + \rho_3\rho_5 + 2\rho_5\rho_2); \quad C_3 = 16\rho_1\rho_2 + 4\rho_3^2 + 10\rho_4\rho_5.\)

In the sum \(\sum C_k \kappa^k\) we add two-loop terms, related to the scalar self-interaction, Fig. 1 to remind that the modification of RGE takes place even in the system without fermions, \(c\) is the numerical coefficient and \(C_k\) are easily calculated (we present \(C_0\) and \(C_3\)) but these values are not interesting for our discussion.
Other equations can be written easily in the similar way. Combination of results of \[2\] with simple combinatorics allows to write the complete system of RGE in two-loop approximation.

7. Discussion

• We prove that for the description of the most general system with a number of scalar fields, having identical quantum numbers, an additional mixed kinetic term is necessary. The obtained RGE like \[21\] demonstrate that in the case of hard violation of $Z_2$ symmetry the fields $\phi_1$ and $\phi_2$ are mixed at small distances and the mixing parameters (angle and normalizations) vary with variation of distance (renormalization scale) $\mu$. In other words, the scalar fields cannot be separated from each other even at very small distances (unnatural theory).

Taking into account this mixed kinetic term makes more complex both RGE and phenomenological analysis.

• We find conditions \[16\], at which the coefficient at this additional kinetic mixing is not running. These conditions describe models with hdim4$Z_s$ potential. In this respect, the reparameterization equivalent dim4$Z_s$ representation of this potential is preferable for detail studies. In addition, for this representation also in the Yukawa interaction (for 2HDM) each right fermion is coupled to only one Higgs boson, i.e. $g_1g_2 = 0$ for each right fermion field.

If mentioned conditions are valid, the scalar field mixing at small distances does not vary with the change of distance. Such theory seem natural for the description of reality.

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