Signatures of the neutrino mass hierarchy in supernova neutrinos

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Abstract

The undetermined neutrino mass hierarchy may leave observable imprint on the neutrino fluxes from the core-collapse supernova (SN). The interpretation of the observables, however, is subject to the uncertain SN models and the flavor conversion mechanism of neutrinos in a SN. We attempt to propose a qualitative and model-independent interpretation of the expected neutrino events at terrestrial detectors, focusing on the accretion phase of the neutrino burst. The flavor conversions due to the neutrino self-interaction, the MSW effect, and the Earth regeneration effect are incorporated in the calculation. It leads to several distinct scenarios that are identified by the neutrino mass hierarchies and the collective flavor transitions. Consequences resulting from the variation of incident angles and SN models are also discussed.

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I. INTRODUCTION

Our knowledge of the neutrino ($\nu$) flavor transition in the core-collapse supernova (SN) has encountered a paradigm shift in the recent years. It was pointed out (for an incomplete list, see, e.g., Ref. [1, 2]) that in the deep region of the core where the neutrino densities are large, the neutrino self-interaction could result in significant flavor conversion that is totally distinct in nature from the well-known Mikheyev-Smirnov-Wolfenstein (MSW) effect [3], which instead arises from the interaction of neutrinos with the ordinary stellar medium. The intense study has suggested that the coherent $\nu - \nu$ forward scatterings may lead to collective pair conversion $\nu_e \bar{\nu}_e \leftrightarrow \nu_x \bar{\nu}_x (x = \mu, \tau)$ over the entire energy range even with extremely small mixing angle. It was also pointed out that in a typical supernova, this type of flavor conversion would take place near $r \sim 10^3$ km, as contrast to that for the MSW effect at $r \sim 10^4 - 10^5$ km. In addition, the development of the collective effects depends crucially on the primary $\nu$ spectra [4, 5], as well as on the $\nu$ mass hierarchy. One would thus expect non-trivial modifications to the original SN $\nu$ fluxes as they propagate outward. In general, the self-induced flavor conversion does not alter both $\nu$ and $\bar{\nu}$ spectra under the normal hierarchy (NH). It however, leads to nearly complete spectra exchange $\bar{\nu}_e \leftrightarrow \bar{\nu}_x$ and a partial swap of the spectra, $\nu_e \leftrightarrow \nu_x$, at a critical energy under the inverted hierarchy (IH). The rich physical content, in a sense, leads to further theoretical uncertainties which complicate the interpretation of the observed SN $\nu$ bursts and the unknown properties of $\nu$ that may be revealed by the observation.

With the unique production and detection processes, the neutrino burst from a SN has long been considered as one of the promising tools for the study of neutrino parameters and the SN mechanism. A core-collapse SN emits all three flavors of neutrino with a characteristic energy range and a time scale that are totally distinct from that of the neutrinos emitted from the Sun, the atmosphere, and the terrestrial sources. It has been suggested (see, i.e., Ref.[6]) that analyzing the SN neutrino bursts may provide hints to the unknown elements of the neutrino mass and mixing matrices. With the recent pin-down of the last mixing angle $\theta_{13}$ [7, 8], the determination of the neutrino mass hierarchy may seem the next reachable goal. Recent study based on the multi-angle analysis of the SN neutrinos [11, 12], however, pointed out that the seemingly dominating collective effects may be suppressed by the dense matter during the accretion phase following the core bounce [9]. This time-dependent variation of the neutrino survival probability during the early phase (post-bounce times $t_{pb} \lesssim 0.6$ s) would give rise to a new interpretation to the observed neutrino flux and the hints to the $\nu$ properties, such as the mass hierarchy.

Focused on this issue, we analyze the expected SN $\nu$ events during the early accretion phase at Earth-bound detectors through two channels of neutrino interactions: $\bar{\nu}_e + p$ in the water Cherenkov detector (WC) or the scintillation detector (SC), and $\nu_e + Ar$ in the liquid Argon chamber (Ar). Given the uncertainties among the SN models, we propose observables that may be useful to the determination of the $\nu$ mass hierarchy. The outcomes with different incident angles at the detectors are also compared.

This work is organized as follows. In Section II, we outline the recent progress on the measured neutrino mixing angles, the squared mass differences, and the general features of the neutrino fluxes emitted by a core-collapse supernova. The known parameters will be adopted as the input for the calculation. Section III is devoted to investigating the modification of the primary neutrino fluxes by the collective effect, the MSW effect, and the Earth regeneration effect. For illustration purpose, the calculation is based on a two-
TABLE I: Possible scenarios of the flavor conversion due to the collective effect and the mass hierarchies. Note that the time-dependent $P_\nu(t)$ of case (b) is given in Fig.1, and the step-like function $P_s$ is given in Section III.

- (a) $\bar{P}_\nu \simeq 1$ $P_\nu \simeq 1$ normal
- (b) $\bar{P}_\nu \simeq P_\nu(t)$ $P_\nu \simeq P_s$ inverted
- (c) $\bar{P}_\nu \simeq 0$ $P_\nu \simeq P_s$ inverted
- (d) $\bar{P}_\nu \simeq 1$ $P_\nu \simeq P_s$ inverted

layer model for the Earth matter. In Section IV, we propose physical observables that may provide hints to identifying the neutrino mass hierarchy and the working scenario of the flavor transition due to self-interaction. Expected trends of the event rates at the WC, SC, and Ar detectors are estimated and discussed. Results arising from varied incident angles at the detectors under different SN models are also discussed. We then summarize this work in Section V.

II. NEUTRINO PROPERTIES AND THE SN PARAMETERS

The three mixing angles in the $\nu$ mixing matrix have been determined with convincing precision \[13\]: $\sin^2 2\theta_{12} \simeq 0.857$, $\sin^2 2\theta_{23} \geq 0.95$, and $\sin^2 2\theta_{13} \simeq 0.098$. The mass-squared differences also have been measured: $\delta m^2_{21} \simeq 7.6 \times 10^{-5}$ eV$^2$, $|\delta m^2_{31}| \simeq 2.4 \times 10^{-3}$ eV$^2$. The absolute neutrino mass, the CP phase in the neutrino sector, and the neutrino mass hierarchy are yet to be determined.

The primary SN neutrino energy spectrum is typically not pure thermal, and can be modeled as a pinched Fermi-Dirac distribution \[14\]. For each neutrino flavor $\nu_\alpha$ ($\alpha = e, \mu, \tau$),

$$F^0_\alpha = \frac{L_\alpha}{T^4_\alpha g(\eta_\alpha)} \exp[(E_\nu / T_\alpha) - \eta_\alpha] + 1, \quad (1)$$

where $L_\alpha$ is the luminosity of $\nu_\alpha$, $T_\alpha$ is the effective temperature of $\nu_\alpha$ inside the respective neutrinosphere, $g(\eta_\alpha)$ is the normalization factor, and $\eta_\alpha$ is the pinching parameter. Note that another widely adopted parametrization is given by, see, e.g., Ref.\[15\].

In general, equipartition of the luminosity among the primary neutrino flavors is expected in typical SN simulations: $L^0_{\nu_e} \approx L^0_{\bar{\nu}_e} \approx L^0_{\nu_x}$, which are assumed in our calculation for the neutrino fluxes during the accretion phase. Note that, however, variations from this nearly degenerate scenario are also suggested in the literature, e.g., $L^0_{\nu_e} / L^0_{\nu_x} \sim 0.5 - 2$, $L^0_{\nu_x} = L^0_{\bar{\nu}_e}$; $L^0_{\nu_x} = L^0_{\bar{\nu}_x}$. As the input parameters, the effective temperatures are fixed in this work: $T_{\nu_e} = 3$ MeV, $T_{\bar{\nu}_e} = 5$ MeV, and $T_{\nu_x} = T_{\bar{\nu}_x} = 7$ MeV. In addition, the pinching parameters are taken to be $\eta_{\nu_e} = 3$, $\eta_{\bar{\nu}_e} = 2$, and $\eta_{\nu_x} = \eta_{\bar{\nu}_x} = 1$.

III. NEUTRINO FLAVOR CONVERSION IN SN AND EARTH

Despite the variation of the SN models, the consensus seems to suggest a partial to complete modification (depending on the mass hierarchy) of the neutrino primary spectra by the collective effect, which occurs near $r \sim 10^3$ km as the neutrinos propagate outwards.
The MSW effect then takes place at $r \sim 10^4 - 10^5$ km. The two effects are considered to be independent because of the wide separation in space. The neutrino fluxes get further modification by the Earth matter before their detection. As far as the $\nu$ properties are concerned, the advantage of analyzing the $\nu$ bursts at the early stage is conspicuous. The complicated shock wave does not play a role in the flavor conversion during this early accretion phase [16]. The phenomenon, however, becomes more complicated during the later cooling phase when the shock wave is taken into consideration. Moreover, while the $\bar{\nu}_e$ and $\bar{\nu}_x$ spectra are expected to be well separated in the early phase, they tend to become indistinguishable later during the cooling phase, and the flavor conversion effect for the $\bar{\nu}_e$ channel becomes difficult to observe.

Recent simulations suggest quite diverse features about the details of the collective effects. In Table I, we summarize the possible scenarios resulting from the collective effects under both the normal (NH) and the inverted (IH) hierarchies, where $P_\nu$ and $\bar{P}_\nu$ represent the survival probabilities of the original $\nu_e$ and $\bar{\nu}_e$ fluxes, respectively, after the adjustment of the self-induced collective effects. There are four different scenarios. Under NH, both $\nu_e$ and $\bar{\nu}_e$ spectra remain unaltered, i.e., $P_\nu \simeq 1$ and $\bar{P}_\nu \simeq 1$. This corresponds to case (a). The IH leads to cases (b), (c), and (d), in which the conversion probability for the $\nu_e$ flux can be approximated by the same step-like function of energy: $P_\nu \simeq 1$ for $E < E_c$, and $P_\nu \simeq 0$ for $E > E_c$, where $E_c \simeq 8$ MeV is the critical energy [17]. The three later cases differ in the expected $\bar{\nu}_e$ flux. Case (b) represents partial matter suppression of the collective effect, and the survival probability is time-dependent: $\bar{P}_\nu = \bar{P}_\nu(t)$ [10–12]. Case (c) represents a total swap of the spectra $\nu_e \leftrightarrow \bar{\nu}_x$: $\bar{P}_{\nu_e} \simeq 0$. In this case, the self-induced effect dominates over the MSW effect, see, e.g., Ref. [2]. We add in our analysis the case (d), which indicates a complete matter suppression of the collective effect: $\bar{P}_\nu \simeq 1$. This corresponds to the traditional treatment of the SN $\nu$ flux based on the pure MSW effect, see, e.g., Ref. [18]. Note that in our calculation for scenario (b), we adopt an approximate probability function $\bar{P}_\nu(t)$ (as shown in Fig.1), which is fitted by the results of Ref. [10].

Formulating the neutrino fluxes at different stages is straightforward. After the modification of both the collective and the MSW effects, the $\nu_e$ and $\bar{\nu}_e$ fluxes take the following
forms under the normal hierarchy:

\[ F_e = P^N_{\text{MSW}} F^c_e + (1 - P^N_{\text{MSW}}) F^c_x, \]

\[ \bar{F}_e = \bar{P}^N_{\text{MSW}} \bar{F}^c_e + (1 - \bar{P}^N_{\text{MSW}}), \]

with

\[ P^N_{\text{MSW}} = |U_{e1}|^2 P_H P_L + |U_{e2}|^2 (P_H - P_H P_L) + |U_{e3}|^2 (1 - P_H), \]

\[ F^c_e = F^0_e + (1 - P_e) (F^0_x - F^0_e), \]

\[ F^c_x = (1 - P_e) F^0_e + (1 + P_e) F^0_x, \]

\[ \bar{P}^N_{\text{MSW}} = |U_{e1}|^2 (1 - P_L) + |U_{e2}|^2 \bar{P}_L, \]

\[ \bar{F}^c_e = \bar{F}_0 + (1 - \bar{P}_e) (\bar{F}^0_e - \bar{F}^0_e), \]

\[ \bar{F}^c_x = (1 - \bar{P}_e) \bar{F}^0_e + (1 + \bar{P}_e) \bar{F}^0_x, \]

where \( H \) (\( L \)) represents the higher (lower) resonance, and the quantity with a bar represents that for \( \bar{\nu} \). Note that \( F^c_e \) and \( \bar{F}^c_e \) are the fluxes after the correction of the collective effect, but before the MSW takes place.

For the inverted hierarchy, we have

\[ F_e = P^I_{\text{MSW}} F^c_e + (1 - P^I_{\text{MSW}}) F^c_x, \]

\[ \bar{F}_e = \bar{P}^I_{\text{MSW}} \bar{F}^c_e + (1 - \bar{P}^I_{\text{MSW}}), \]

where

\[ P^I_{\text{MSW}} = P_L + (1 - 2P_L)|U_{e2}|^2, \]

\[ \bar{P}^I_{\text{MSW}} = \bar{P}_H \bar{P}_L + \bar{P}_H (1 - 2\bar{P}_L)|U_{e1}|^2. \]

Note that the indices, \( N \) and \( I \), stand for normal and inverted, respectively.

After propagating through the Earth matter, the expected \( \nu \) fluxes at the detectors then follow. For the \( \nu_e \) flux, we have

\[ F_e^D \simeq F^0_e[(1 - |U_{e3}|^2) - P_N(1 - 2|U_{e3}|^2)] + F^0_x[(1 - |U_{e3}|^2) + P_N(1 - 2|U_{e3}|^2)], \]

\[ F_e^D \simeq F^0_e[(1 - P_L) - 2P_e(1 - 2P_L)] + F^0_x[(1 + P_L) - 2P_e P_L], \]

under the normal \( (P_e = P_N) \) and inverted hierarchies \( (P_e = P_I) \), respectively. As for the \( \bar{\nu}_e \) flux, we get

\[ \bar{F}_e^D \simeq \bar{F}^0_e[(1 - \bar{P}_N) - \bar{P}_{1e}(1 - 2\bar{P}_N)] + \bar{F}^0_x[(1 + \bar{P}_N) - 2\bar{P}_{1e} \bar{P}_N], \]

\[ \bar{F}_e^D \simeq \bar{F}^0_e[(1 - \bar{P}_L) + \bar{F}^0_x(1 + \bar{P}_L)], \]

for the normal \( (\bar{P}_e = \bar{P}_N) \) and inverted \( (\bar{P}_e = \bar{P}_I) \) hierarchies, respectively. Here \( P_{1e} (\bar{P}_{1e}) \) is the probability that a mass eigenstate \( \nu_e (\bar{\nu}_e) \) is observed as a \( \nu_e (\bar{\nu}_e) \) at the detector. Note that we have used the approximation: \( P_{1e} - |U_{e3}|^2 \ll 1 \) in writing the detected \( \nu \) flux. Note also that we have set vanishing crossing probabilities in writing the above expressions: \( P_H \simeq \bar{P}_H \simeq P_L \simeq \bar{P}_L \simeq 0 \). As contrast to the matter density profile with \( \rho \sim r^{-3} \) in the traditional treatment, it has been pointed out \([19]\) that a local deviation of the uncertain
density profile from $\rho \sim r^{-3}$ may lead to significant change of the MSW crossing probabilities for certain range of $\theta_{13}$. This factor has been taken into consideration in our analysis. With the recent determination of the relatively large $\theta_{13}$, we conclude that the vanishing crossing probabilities can be adopted even if the local density profiles near the resonance become as steep as $\rho \sim r^{-8}$.

The probability $P_{2e}$ is usually written as $P_{2e} = \sin^2 \theta_{12} + f_{\text{reg}}$, with $f_{\text{reg}}$ the regeneration factor due to the Earth matter effect [20]:

$$f_{\text{reg}} = \frac{2E \sin^2 2\theta}{\delta m^2_{21}} \sin \Phi_0 \sum_{i=0}^{n-1} \Delta V_i \sin \Phi_i,$$

(18)

where $\Delta V_i \equiv V_{i+1} - V_i$ is the potential difference between adjacent layers of matter, and $\Phi_i$ is the phase acquired along the trajectories. For illustration purpose, we adopt a simple two-layer model [21] for the Earth matter, $\rho_E = 5.0 \text{g/cm}^3$ for $R_\oplus/2 < r < R_\oplus$ (mantle) and $\rho_E = 12.0 \text{g/cm}^3$ for $r < R_\oplus/2$ (core). Note that this two-layer analysis can be easily generalized to the analysis of a multi-layer model. The regeneration factor for two layers takes the form,

$$f_{\text{reg}} = \sin^2 2\theta \sin \Phi_0 [\epsilon_m \sin \Phi_0 + (\epsilon_c - \epsilon_m) \sin \Phi_1],$$

(19)

with $\epsilon_m = 2EV_m/\delta m^2_{21}$ for the mantle and $\epsilon_c = 2EV_c/\delta m^2_{21}$ for the core, and

$$\Phi_0 = \frac{L}{2} \sqrt{(\cos 2\theta - \epsilon_m)^2 + \sin^2 2\theta},$$

(20)

$$\Phi_1 = \frac{L_1}{2} \sqrt{(\cos 2\theta - \epsilon_c)^2 + \sin^2 2\theta}.$$

(21)

Here $L$ is the total path length inside Earth and $L_1$ is that inside the core.

IV. PHYSICAL OBSERVABLES IN THE DETECTORS

During the accretion phase, the detailed time evolution of the neutrino fluxes are in general model-dependent. In this section we first adopt the $\nu$ and $\bar{\nu}$ fluxes during the early phase of a $10.8M_\odot$ SN model, such as in Ref.[11] where the flux evolution during the accretion phase was given up to 0.6 seconds after the core bounce. We shall later discuss the possible impact due to different choices of the SN models.

For our purpose of analyzing the time structure of the energy-integrated fluxes,

$$\hat{N}(t) \sim \int \bar{F}^{D}_{\nu}(E,t) \cdot \bar{\sigma}_{\nu}(E) \cdot \bar{\epsilon}(E) dE,$$

$$N(t) \sim \int F^{D}_{\nu}(E,t) \cdot \sigma_{\nu}(E) \cdot \epsilon(E) dE,$$

(22)

we approximate the primary luminosity of the neutrino fluxes [11] as piece-wise functions (with arbitrary scales):

$$L^0_{\nu_e} = \begin{cases} 
  2.52 \exp\left[-\frac{(t-0.13)^2}{0.065}\right], & (0 < t \leq 0.25s) \\
  1.86 + \exp[42.95t - 13.30], & (0.25s < t \leq 0.30s) \\
  0.35 + \exp[-16.65t + 5.73], & (0.30s < t \leq 0.60s)
\end{cases}$$

(23)
FIG. 2: An estimation of $\bar{N}(t)$ (with arbitrary normalization scale) at WC or SC during the accretion phase. The four scenarios are resulted from different mass hierarchies and time structures of the neutrino fluxes. We have used $9\pi/10$ as the zenith angle at the detector.

$$L_{\bar{\nu}_e}^0 = \begin{cases} 
1.99 \exp\left[\frac{-(t-0.15)^2}{0.051}\right], & (0 < t \leq 0.25s) \\
1.54 + \exp[35.74t - 11.42], & (0.25s < t \leq 0.30s) \\
0.35 + \exp[-16.24t + 5.45], & (0.30s < t \leq 0.60s)
\end{cases}$$

and

$$F_{\nu_e}^0 = F_{\bar{\nu}_e}^0 = 0.13 + \exp[-3.46t + 0.037], \ (0 < t \leq 0.60s).$$

Note that we also assume a unity efficiency functions, $\bar{\varepsilon}(E) \sim \varepsilon(E) \sim 1$, and that the limited energy resolution of the detectors is capable of observing the qualitative time-evolution of the $\nu$ fluxes. The cross-section functions, $\bar{\sigma}_\nu(E)$ and $\sigma_\nu(E)$, will be presented later for each of the reaction channels. One may then check the qualitative behavior of the time structures that are the direct consequences of the mass hierarchies and distinct scenarios of the self-induced effects, as will be discussed in the subsections.

A. $\bar{\nu}_e$ events at WC or SC

The inverse $\beta$-decay, $\bar{\nu}_e + p \rightarrow n + e^+$, dominates over other reaction channels in the water Cherenkov detectors and the scintillation detectors. We adopt the cross section $[22]$:

$$\sigma(\bar{\nu}_e p) \approx 9.5 \times 10^{-44}(E - 1.29)^2 \text{cm}^2,$$

and assume that events originate from this channel can be properly identified. The time evolution of $\bar{N}(t)$ under the four possible scenarios is shown in Fig.2, and summarized as follows.

(i) A unique, monotonically decreasing $\bar{N}(t)$, which represents case (d) in Table I, is easily singled out. An observation of this result not only suggests inverted hierarchy, but also a nearly total suppression of the collective effect: $\bar{P}_\nu \approx 1$. This result favors the scenario that MSW effect $\gg$ collective effect, which is just the traditional treatment based on the pure MSW effect without the collective effect.

(ii) An early decreasing $\bar{N}(t)$ could also arising from the time-varying survival probability $\bar{P}_\nu(t)$ as in case (b), which is due to partial suppression of the collective effect under the
FIG. 3: An estimation of $N(t)$ (with arbitrary normalization scale) at the Ar detector for cases (a) and (c) during the accretion phase. We have used $9\pi/10$ as the zenith angle at the detector.

inverted hierarchy. However, a significant peak at $t_{pb} \sim 0.3s$ distinguishes this scenario from that of case (d).

(iii) A slight increase of $\bar{N}(t)$ prior to $t_{pb} \sim 0.1s$ could represent two different scenarios: normal hierarchy with $\bar{P}_{\nu} \simeq 1$ (complete suppression of the collective effect, case (a)), or inverted hierarchy with $\bar{P}_{\nu} \simeq 0$ (total spectrum swap, case (c)). Although in general the event rates could differ by roughly 50% according to a quick estimation, a definite separation between these two scenarios may not be easy if one practically considers the model uncertainties and limitations of the experimental resolution. We therefore will not stress on the significance of this case here. However, further hints may be available from the observation of the $\nu_e$ events.

The general properties of Fig. 2 can be understood as follows. (1). For cases (a), (b), and (c), the peak and the sharp drop of the luminosity afterwards signal the onset of explosion at $t_{pb} \sim 0.3s$. This is a common feature of the primary spectra in various models. (2). With the vanishing $\bar{P}_H$ and $\bar{P}_L$, the expected $\bar{\nu}_e$ flux at the detector simply inherits the shape of the $\bar{\nu}_x$ flux after the modification of the collective effect but before that of the MSW effect: $\bar{F}_c$. Eq. (9) suggests that $\bar{F}_c$ is a combination of $\bar{F}_0^e$ and $\bar{F}_0^x$, and the weight of each component is determined by $1 - \bar{P}_{\nu}$ and $1 + \bar{P}_{\nu}$, respectively. This leads to curve (d) in Fig. 2 when $\bar{P}_{\nu} \simeq 1$, which corresponds to the situation when the collective effect is turned off and only the pure MSW effect is in action. In this case, the expected $\bar{F}_0^e$ simply follows the monotonically decreasing shape of $\bar{F}_0^x$ that is also common in different SN models. As $\bar{P}_{\nu}$ decreases, it represents the situation when the flavor transition due to the collective effect becomes more important, the fraction of $\bar{F}_0^e$ becomes larger. The direct consequence is that the peaks begin to emerge, as can be found in cases (b) and (c). (3). At the early stage, (b) and (d) are indistinguishable since the time-varying probability for case (b) also remains at $\bar{P}_{\nu} \simeq 1$ before it drops to $\sim 1/2$ near $t_{pb} \sim 0.3s$.

B. $\nu_e$ events at Liquid Ar detectors

The cross section for the charged-current reaction, $\nu_e + ^{40} Ar \rightarrow ^{40} K^* + e^-$, is given by [23] $\sigma(\nu_e Ar) \simeq 3.38 \times 10^{-42}(E_{\nu}/\text{MeV})\text{cm}^2$. We show in Fig. 3 the expected time-varying behavior for the dominant $\nu_e$-induced events at the Ar detector. It is clearly seen that the
FIG. 4: The expected $\bar{\nu}_e$ flux arriving at the WC or SC detectors for case (a), and the $\nu_e$ flux arriving at the Ar detector for case (b). Three different zenith angles at the detectors are adopted: $0.01\pi$ (solid), $3\pi/5$ (dotted), and $9\pi/10$ (dashed). Despite the slight wiggles due to the Earth effect, the general trend of the curves is unaltered. Note that the curves are plotted with arbitrary scales.

case under NH with $P_\nu \simeq 1$ (case (a)) leads to a monotonically decreasing rates, while a significant bump near $t_{pb} \sim 0.3s$ appears for IH with $P_\nu \simeq 0$ (case (c)). These totally distinct time structures lift the degeneracy of the $\bar{\nu}_e$ result for case (iii) in the previous subsection, and can act as a supplement to the observation of the $\bar{\nu}_e$ flux. Note that, as discussed in subsection A, one may also understand the properties of the curves in Fig.3 by examining the details of Eqs. (2) and (10).

C. Analysis with varying incident angles

The angle of incidence at the detector is arbitrary for the SN $\nu$ fluxes, although preferred detector locations were predicted, see, e.g., Ref. [24]. A zenith angle $9\pi/10$ (mantle + core) has been adopted in our calculation. To investigate the possible deviation from our analysis, we also show the results for $3\pi/5$ (mantle) and $0.01\pi$ ($\sim$ no Earth matter) in Fig. 4. With the varying depth of the path into the Earth, the Earth effect will in principle, modify the fluxes accordingly, as indicated by the wiggles. The observability of the Earth effect has been discussed elsewhere, see, e.g., Ref. [25, 26]. Fig. 4 suggests that the angle of incidence for the $\nu$ fluxes may modify the details of the observed fluxes, but not the qualitative trend of the spectra. In our analysis, we study the general time-evolution of the expected fluxes, regardless of whether or not the detailed Earth effect can be observed.

D. Analysis with varying models

In the previous analysis, we have adopted a specific, $10.8M_\odot$ Fe-core progenitor model. The time structure of the $\nu$ flux, which is model-dependent, has been the main focus of the study. The general futures and the trend of the $\nu$ luminosity and energy, however, seem to evolve quite similarly for other models, including the $8.8M_\odot$ O-Ne-Ma-core and the $18M_\odot$ Fe-core models [27]. The uncertainties among the three models should not alter the results based on the qualitative observation of our analysis.
V. CONCLUSION

The multi-angle analysis of the SN neutrinos in the collective oscillations suggests that the dense matter during the early, accretion phase may suppress the self-induced flavor conversion and lead to time-dependent transition probabilities. Investigation of the neutrino signals that are modified by the influence of various collective effects, in addition to the usual MSW and the Earth matter effects, may shed light on the undetermined neutrino mass hierarchy. On the other hand, the time structures of the collective flavor conversion resulting from the model uncertainties and the neutrino mass hierarchies during the accretion phase may also leave signatures on the observed $\nu_e$ and $\bar{\nu}_e$ fluxes.

Intense effort has been devoted to the general study of the SN neutrino flavor conversion. Related topics, such as probing the neutrino parameters or analyzing the detectability of varied effects on the SN neutrinos, have also been widely discussed. To better clarify our motivation in this work and to distinguish our work from others, we briefly outline here the aims of similar studies in the literature:

• In Choubey et al. [26], the expected neutrino signals resulting from varied flux models were calculated and the signatures, especially the patterns of spectral split, of the collective effects are examined. The flavor conversion during the accretion phase was also discussed. However, the possibility that very dense ordinary matter may suppress the collective conversion during the accretion phase, a consequence of the multi-angle analysis, was not included. The related discussion thus corresponds to distinguishing case (a) for the NH and case (c) for the IH in our paper.

• The paper by Borriello et al. [25] aimed at the observability of the Earth matter effect for the SN neutrinos. The time-integrated spectra during part of the accretion phase and the cooling phase were taken as benchmark for the calculation. For flavor conversion, the complete matter suppression of the collective effect was considered, and the neutrinos underwent only the dominant MSW conversion in SN. The related discussions correspond to separating case (a) and case (d) in our analysis.

• In Serpico et al. [16], the analysis was focused on probing the neutrino mass hierarchy with the neutrino signals at IceCube Cherenkov detector. Complete matter suppression of the collective effect during the early era of the accretion phase ($t_{pb} < 0.2s$) was considered. In this analysis the MSW conversion also plays the dominant role. The related discussion thus corresponds to distinguishing case (a) and case (d) as in part of our analysis ($t_{pb} < 0.2s$), but based on the outcome of a different detector.

Our approach differs from the above analyses mainly in that, as far as the influence of the collective effect is considered, we focus on the time-varying collective flavor conversion probability and the resultant time-dependent neutrino fluxes during the accretion phase, up to $t_{pb} \sim 0.6s$. Thus, the possible consequences due to (i) a complete suppression of the collective effect, (ii) a partial suppression of the collective effect, and (iii) a large collective effect are all discussed. Given the current situation that uncertainty still remains in the mechanism of the collective effect, our discussion covers the scope that allows model uncertainties in the time structures of the flavor conversion and the fluxes. Furthermore, our analysis was established based on a conservative assumption that the Earth matter effect is beyond detectable. More precisely, observing a specific effect, such as the collective effect or the Earth matter effect, is not the main issue in our paper and is irrelevant to our aim, although
it could be crucial to other analyses. In fact, it was suggested \cite{25} that observing the Earth effect under a general situation may be more challenging than expected, and the optimistic view toward an identification of the neutrino mass hierarchy may need to be reevaluated. In a sense, this may also suggest that one should try to avoid as many as possible the factors that may complicate the interpretation of the signals in probing the neutrino mass hierarchy. Regardless of the observability of the Earth matter effect, our analysis seems in agreement with this logic by establishing the qualitative properties that are unaffected by whether or not the Earth matter effect can be singled out.

It should be pointed out that the details of the time-structure for the primary $\nu$ fluxes and their relative magnitude are, of course, model-dependent. However, the general qualitative features do not vary much among models, as discussed in Sec. IV. As far as our purposes are concerned, the qualitative study should be able to provide moderate hints as an addition to those of the numerical studies that may suffer from model uncertainties and the experimental details involved. In addition, out of the three independent effects included in our calculation, the time-dependent collective effect plays a key role in our analysis because the variation of the expected outcome is determined nearly all by the not-so-well understood collective effect. In this regard, we further analyze the time structure of the consequences resulting from different models. The results may also help shed light on identifying the working scenario of the collective effect in SN.

We have assumed equipartitioned luminosity for the neutrino flavors, as mentioned in Section II. Note that the relative magnitude of the luminosity may not remain the same during the later cooling phase. Even if in the case that a slight deviation from the equipartition occurs during the accretion phase, our qualitative analysis, which focuses on the trend of event rate in time, remains valid since the numerical details involved in the calculation do not alter the general pictures of the physical observables.

Our results suggest that not only the mass hierarchy, but also the SN mechanism related to the collective effect may, in principle, be resolved to certain extent by the observation of both $\nu$ and $\bar{\nu}_e$ fluxes. However, this physics potential of observing SN neutrinos rely heavily on the resolution power of a detector and the proper numerical analysis. The more detailed numerical study, which takes into consideration the possible consequences due to different models, variation of the luminosity strength, and the experimental details, would certainly be in high demand and shall be discussed elsewhere. In this work, we only estimate the qualitative properties of the observables. Nevertheless, it is still hoped that analysis along this line would provide an alternative approach toward a better understanding of the SN neutrinos, especially under the current situation that the model uncertainties exist and a comprehensive knowledge of the SN physics is still lacking.

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11