Berry’s Connection and $USp(2k)$ Matrix Model

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Abstract

Berry’s connection is computed in the $USp(2k)$ matrix model. In $T$ dualized quantum mechanics, the Berry phase exhibits a residual interaction taking place at a distance $m(f)$ from the orientifold surface via the integration of the fermions in the fundamental representation. This is interpreted as a coupling of the magnetic $D2$ with the electric $D4$ branes. We make a comment on the Berry phase associated with the $6D$ nonabelian gauge anomaly whose cancellation selects the number of flavours $n_f = 16$.

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Matrix models have recently received much attention as a candidate to provide a non-perturbative formulation to superstrings and \( M \) theory. The models proposed so far include the original one for \( M \) theory [1] and its heterotic counterpart [2] as well as the \( IIB \) matrix model [3] in zero dimension. This latter one may be referred to as reduced model. In ref. [4], the reduced model descending from Type \( I \) superstrings based on \( USp \) Lie algebra has been found and its connection to \( F \) theory [5] has been suggested. Physical implications and identifications of the \( USp(2k) \) matrix model have been further explored in [6]. It is a matrix model which embodies Type \( I \) superstrings as that on \( T^6/\mathbb{Z}_2 \) orientifold. The possible choice of the model has been found to be severely restricted by the condition of having eight kinematical and eight dynamical supercharges and that of the cancellation of nonabelian gauge anomalies of six-dimensional worldvolume gauge theory as well as of the nonorientability of the surface created by the Feynman diagrams. The rationales for the choice of the \( usp \) Lie algebra and that of the field contents belonging to the adjoint and antisymmetric representations have thus been given. The role played by the degrees of freedom of the hypermultiplet in the fundamental representation remains, however, relatively unexplored. They participate in the anomaly cancellation mentioned above and are responsible for creating an open string sector as the counting of planar diagrams tells us. That they do embody \( D3 \) branes is less direct to grasp.

In this letter, we consider the \( USp \) matrix model in the \( T \) dualized quantum mechanics where effects of these \( D3 \) branes could be seen as a coupling of \( D2 \) magnetic background with the quantized degrees of freedom of \( D4 \) branes. We find a residual interaction in the “effective action” for the spacetime coordinates lying in the diagonal (Cartan) components of the six adjoint matrices. This is accomplished by computing the Berry phase [7, 8] coming from each of the quantum mechanics belonging to the three fermionic sectors. We find an induced magnetic monopole background at a distance \( m(f) \) from the orientifold surface via the integrations of fermions in the fundamental representation. This is the effect which survives the cancellation of bosonic and fermionic determinants.

**Effective action for spacetime coordinates in the \( USp \) matrix model:**

The starting point is the action of the zero dimensional \( USp(2k) \) matrix model [4, 6]

\[
S = S_{\text{vec}} + S_{\text{asym}} + S_{\text{fund}} .
\]

The part \( S_{\text{vec}} + S_{\text{asym}} \) can be understood as the projection from the type \( IIB \) matrix model. Introduce a projector

\[
\hat{\rho}_{\pm} \cdot \equiv \frac{1}{2} \left( \cdot \mp F^{-1} \cdot F \right) ,
\]

which takes any \( U(2k) \) matrix (denoted by a symbol with an underline) into the matrix lying in the adjoint representation of \( USp(2k) \) and that in the antisymmetric representation respectively. We obtain

\[
S_{\text{vec}} + S_{\text{asym}} = S_{N=1}^{d=10} (\hat{\rho}_{\pm} \underline{M} ; \hat{\rho}_{\pm} \underline{N}) ,
\]

\[
S_{N=1}^{d=10} (\underline{M} ; \underline{N}) = \frac{1}{g^2} Tr \left( \frac{1}{4} [\underline{M} , \underline{N}] [\underline{M} , \underline{N}] - \frac{1}{2} \underline{N} \Gamma^M [\underline{M} , \underline{N}] \right) ,
\]

\[2 \text{ Papers dealing with related issues on fermions in matrix models include } [9, 10, 11] \]
where $\hat{\rho}_b$ is a matrix with Lorentz indices and $\hat{\rho}_f$ is a matrix with spinor indices:

$$
\hat{\rho}_b = \text{diag}(\hat{\rho}_-, \hat{\rho}_-, \hat{\rho}_-, \hat{\rho}_-, \hat{\rho}_+, \hat{\rho}_+, \hat{\rho}_+, \hat{\rho}_+),
$$

$$
\hat{\rho}_f = \hat{\rho}_{-1(4)} \otimes \begin{pmatrix} 0 & 1_{(2)} \\ 1_{(2)} & 0 \end{pmatrix} + \hat{\rho}_{+1(4)} \otimes \begin{pmatrix} 0 & 1_{(2)} \\ 1_{(2)} & 0 \end{pmatrix}.
$$

As for $S_{\text{fund}}$, we work in the original representation based on the four dimensional superfield notation with spacetime dependence dropped:

$$
S_{\text{fund}} = \frac{1}{g^2} \sum_{f=1}^{n_f} \left[ \int d^2 \theta d^2 \bar{\theta} \left( Q^*_f (e^{2\psi})_i^j Q_f^i j + \bar{Q}^*_f (e^{-2\psi})^i_j \bar{Q}_f^i j \right) + \left\{ \int d^2 \theta \left( m(f) Q^*_f \bar{Q}_f^i j Q^i j + \sqrt{2} \bar{Q}_f^i (\Phi) Q_f^i j + h.c. \right) \right\} + Q_i = Q_i + \sqrt{2} \psi \bar{Q}_i + \theta F_Q. \right)
$$

For more complete definition of the action, see [1, 3]. The mass term which we have denoted by $m(f)$ is necessary for the discussion in what follows. In the leading large $k$ (planar) limit in the sense of 't Hooft, $S_{\text{fund}}$ is ignorable. Any physical consequence coming from $S_{\text{fund}}$ must be from processes which receive a vanishing contribution from the planar diagrams.

Physical quantities of the $USp(2k)$ reduced model are obtained from the effective action for the spacetime coordinates $x_M$, (which are the diagonal elements of $v_M$),

$$
Z \left[ x_M; m(f) \right] = \int [D\tilde{v}_M] [D\Psi] [D\bar{\Psi}] \prod_{f=1}^{n_f} \left[ DQ_f(\Phi) \right] \left[ DQ^*_f(\Phi) \right] \left[ D\bar{Q}_f(\Phi) \right] \left[ D\bar{Q}^*_f(\Phi) \right] 
$$

$$
\times \left[ D\psi \bar{Q}_f(\Phi) \right] \left[ D\psi \bar{Q}^*_f(\Phi) \right] \left[ D\bar{\psi} Q_f(\Phi) \right] \exp[iS] 
$$

$$
= \int [D\tilde{v}_M] \prod_{f=1}^{n_f} \left[ DQ_f(\Phi) \right] \left[ DQ^*_f(\Phi) \right] \left[ D\bar{Q}_f(\Phi) \right] \left[ D\bar{Q}^*_f(\Phi) \right] 
$$

$$
\times \left[ \det D_{\text{fund}}(v_M) \right]^{n_f} \left[ \det D_{\text{adj}}(v_M) \right] \left[ \det D_{\text{asy}}(v_M) \right] \exp[iS_B] 
$$

and all possible operator insertions (local as well as nonlocal ones) into this object. Here $v_M = x_M + \tilde{v}_M$ and $S^B = S$ with all fermions set to zero. For simplicity in this paper we keep only the diagonal elements of six adjoint directions

$$
x_{\nu} = \text{diag}(x^{(1)}_{\nu}, \cdots, x^{(k)}_{\nu}, -x^{(1)}_{\nu}, \cdots, -x^{(k)}_{\nu}).
$$

At one-loop, it is legitimate to replace the matrix $v_{\nu} = x_{\nu} + \tilde{v}_{\nu}$ in the argument of the determinant by the diagonal matrices $x_{\nu}$.

Naively supersymmetry would tell the cancellation of the bosonic determinants against the fermionic ones. As this can be done by the 1PI Feynman diagrams, the cancellation, if true, would persist to all orders in perturbation theory. To see that this is not quite the case, we $T$ dualize the system. Recall that the $T$ duality transformation is a legitimate operation in the large $k$ limit via the recipe of [12]. We regard the fermionic integrations with
a particular set of nonlocal operator insertions as the transition amplitude of an adiabatic process. This latter process is given by the quantum mechanical systems of free fermions with external bosonic parameters \(x_\nu\) and \(m_{(f)}\). The object which we will study in what follows is

\[
Z \left[ x_\nu, x_f = 0; m_{(f)}; j_{(R)} \right] = \int \left[ D\bar{\nu}_M \right] \prod_{f=1}^{n_f} \left[ DQ_{(f)} \right] \left[ DQ'_{(f)} \right] \left[ D\bar{Q}_{(f)} \right] \left[ D\bar{Q}'_{(f)} \right] \exp[iS_B]
\]

\[
\lim_{T \to \infty} \prod_{f=1}^{n_f} \left[ \langle t = T; j_f \mid P e^{-i \int_0^T dt H_{\text{fund}}(t)} \mid t = 0; j_f \rangle \right] (f) \tag{10}
\]

\[
\langle t = T; j_{\text{adj}} \mid P e^{-i \int_0^T dt H_{\text{adj}}(t)} \mid t = 0; j_{\text{adj}} \rangle \langle t = T; j_{\text{asym}} \mid P e^{-i \int_0^T dt H_{\text{asym}}(t)} \mid t = 0; j_{\text{asym}} \rangle .
\]

Here we denote by \(H_{\text{fund}}(t), H_{\text{adj}}(t), H_{\text{asym}}(t)\), the respective Hamiltonians obtained from the fermionic part of \(S_{\text{fund}}, S_{\text{adj}}\) and \(S_{\text{asym}}\) after \(T\) duality. Their \(t\) dependence comes from that of the diagonal matrix \(x_t\) which act as external parameters on the Hilbert space of fermions. We have indicated by \(j_{(R)}\) ( \(R = \text{fund, adj, asym}\) ) quantum numbers of an adiabatic process. Let us denote by \(e^{(A)}_{(R)}\) the standard eigenbases belonging to the roots of \(sp(2k)\) and the weights of the fundamental representation and those of the antisymmetric representation respectively. We expand the two component fermions as

\[
\psi^{(R)} = \sum_A \bar{b}_A^{(R)} e^{(A)}_{(R)} / \sqrt{2}, \quad \bar{\psi}^{(R)} = \sum_A \bar{b}_A^{(R)} e^{(A)}_{(R)} / \sqrt{2},
\]

where \(N_{(\text{adj})} = 2k^2 + k\), \(N_{(\text{antisym})} = 2k^2 - k\) and \(N_{(\text{fund})} = 2k\). We find that all of the three Hamiltonians \(H_{\text{fund}}, H_{\text{adj}}\) and \(H_{\text{asym}}\) are expressible in terms of the abelian counterpart

\[
g^2 H_0 \left( x_{\ell}, \phi, \phi^*; (R), A \right) = -\bar{b}_{A\bar{\alpha}}^{(R)} \sigma^{m\bar{\alpha}} x_m b_{A\alpha}^{(R)} - d_{A\alpha}^{(R)} \sigma_{\alpha\bar{\alpha}} x_{\bar{\alpha}} d_{A\bar{\alpha}}^{(R)} + \sqrt{2} \phi b_{A\bar{\alpha}}^{(R)} \bar{d}_{A\alpha}^{(R)}
\]

\[
\tag{12}
\]

provided we replace the five parameters

\[
x_{\ell}, \quad \phi = \frac{x_A + ix_T}{\sqrt{2}}, \quad \phi^* = \frac{x_A - ix_T}{\sqrt{2}}
\]

by the appropriate ones. (See argument of \(\gamma_T\) in eq. (10) below.)

The formula for the transition amplitude of an adiabatic process is

\[
\lim_{T \to \infty} \langle t = T; j \mid P \exp \left[ -i \int_0^T dt H_0(t) \right] \mid t = 0; j \rangle = \exp \left[ -i \int_0^T E(t) dt + i \gamma_T \right],
\]

\[
\gamma_T \left[ x_m, \phi, \phi^*; j \right] = \int_0^T dt \frac{d\gamma}{dt} , \tag{14}
\]

Here \(\Gamma\) is a closed path in the parameter space. The connection one-form associated with the Berry phase \(\gamma_T\) satisfies

\[
id\gamma(t) = -\langle t \mid d \mid t \rangle \equiv -iA . \tag{15}
\]
Using this expression, the second and the third lines of eq. (10) are written as

\[
\exp(i\gamma_{\text{total}}^{(\text{total})}) \equiv \exp \left( i \sum_{j=1}^{n_f} \sum_{A=1}^{2k} \gamma_{r} \left[ w^{A} \cdot \mathbf{x}_{r}, m_{(j)}/\sqrt{2} + w^{A} \cdot \Phi, m_{(j)}/\sqrt{2} + w^{A} \cdot \Phi^\dagger \right] \right) + i \sum_{A=1}^{2k^2} \gamma_{r} \left[ R^{A} \cdot \mathbf{x}_{r}, i R^{A} \cdot \Phi, i R^{A} \cdot \Phi^\dagger \right] + i \sum_{A=1}^{2k^2} \gamma_{r} \left[ w^{A}_{\text{asym}} \cdot \mathbf{x}_{r}, w^{A}_{\text{asym}} \cdot \Phi, w^{A}_{\text{asym}} \cdot \Phi^\dagger \right].
\] (16)

Here

\[
\{ \{ w^{A} \mid 1 \leq A \leq 2k \} \} = \{ \{ \pm e^{(i)} , 1 \leq i \leq k \} \} \quad \text{and}
\{ \{ R^{A} \mid 1 \leq A \leq 2k^2 \} \} = \{ \{ \pm 2e^{(i)}, e^{(i)} - e^{(j)}, \pm \left( e^{(i)} + e^{(j)} \right) \mid 1 \leq i, j \leq k \} \} \quad \text{and}
\{ \{ w^{A}_{\text{asym}} \mid 1 \leq A \leq 2k^2 - 2k \} \} = \{ \{ \pm (e^{(i)} + e^{(j)}), e^{(i)} - e^{(j)} \mid 1 \leq i, j \leq k \} \}
\] (17)

are respectively the nonzero roots and the weights in the antisymmetric representation of \( \text{usp}(2k) \). We have denoted by \( e^{(i)} \) \( 1 \leq i \leq k \) the orthonormal basis vectors of \( k \)-dimensional Euclidean space and

\[
x_{r} = \sum_{i=1}^{k} e^{(i)} x_{r}^{(i)}, \quad \Phi = \sum_{i=1}^{k} e^{(i)} \sqrt{2} x_{r}^{(i)}, \quad \Phi^\dagger = \sum_{i=1}^{k} e^{(i)} x_{r}^{(i)} - i x_{r}^{(i)} \] .
\] (18)

**Computation of the Berry connection:**

Let us now turn to the computation of \( \gamma_{\text{r}}(t) \) associated with \( H_{0}(x_{r}, \phi, \phi^{*}) \). We define the Clifford vacuum \( \mid \Omega \rangle \) by

\[
b^{a} \mid \Omega \rangle = \tilde{d}_{a} \mid \Omega \rangle = 0.
\] (19)

We have suppressed the labels \( A \) and \( (R) \) seen in eqs. (11),(12). Any ket vector of this system can be decomposed into a set of wave functions by

\[
\mid \rangle = \left[ h_{(4)} \frac{1}{2} \tilde{a} \tilde{b}^{\dot{a}} d^a d_{\alpha} + h_{(3)} \frac{1}{2} d^a \tilde{a} \tilde{b}^{\dot{a}} + h_{(3)} \frac{1}{2} \tilde{b}_{\dot{a}} d^a d_{\alpha} + h_{(2,1)} \frac{1}{2} \tilde{b}_{\dot{a}} \tilde{b}^{\dot{a}} + h_{(2,2)} \frac{1}{2} d^a d_{\alpha} + h_{(2,3)} \frac{1}{2} d^a \sigma_{aa} \tilde{b}^{\dot{a}} + h_{(2,4)} \frac{1}{2} d^a \sigma_{aa} \tilde{b}^{\dot{a}} + h_{(1,1)} d^a + h_{(1,2)} \tilde{b}_{\dot{a}} + h_{(0)} \right] \mid \Omega \rangle ,
\] (20)

As the “particle number”, which we denote by \( n \), is conserved, the eigenvalue problem reduces to those in each sector \( n = 0, 1, 2, 3, 4 \). Both of the \( n = 0, 4 \) sectors give a zero eigenvalue trivially while \( n = 1 \) and \( n = 3 \) sectors are related to each other by \( b^{a} \leftrightarrow \tilde{b}_{\dot{a}}, \quad d_{\alpha} \leftrightarrow d^{\alpha} \). We are left to analyze

\[
M_{3} \begin{pmatrix} h_{(3)}^{a} \\ h_{(3)}^{\dot{a}} \end{pmatrix} = g^{2} E \begin{pmatrix} h_{(3)}^{a} \\ h_{(3)}^{\dot{a}} \end{pmatrix} , \quad \text{and} \quad M_{2} \begin{pmatrix} h_{(2,1)} \\ h_{(2,2)} \\ h_{(2,3)} \\ h_{(2,4)i} \end{pmatrix} = g^{2} E \begin{pmatrix} h_{(2,1)} \\ h_{(2,2)} \\ h_{(2,3)} \\ h_{(2,4)i} \end{pmatrix} .
\] (21)
Here $M_3$ and $M_2$ are representation matrices for $g^2H_0$ of an appropriate size:

$$M_3 = \begin{pmatrix}
-x_3 & -x_1 - i x_2 & \sqrt{2}\phi^* & 0 \\
-x_1 + i x_2 & x_3 & 0 & \sqrt{2}\phi^* \\
\sqrt{2}\phi & 0 & x_3 & x_1 + i x_2 \\
0 & \sqrt{2}\phi & x_1 - i x_2 & -x_3
\end{pmatrix}.$$ \hspace{1cm} (22)

$$M_2 = \begin{pmatrix}
0 & 0 & \sqrt{2}\phi^* & 0 \\
0 & 0 & -\sqrt{2}\phi & 0 \\
\sqrt{2}\phi & -\sqrt{2}\phi^* & 0 & -2x^i \\
0 & 0 & -2x_i & 0
\end{pmatrix}.$$ \hspace{1cm} (23)

The eigenvalues of $M_3$ are

$$\pm \sqrt{x_3x_3 + 2\phi\phi^*} \equiv \pm \lambda_0,$$

each being two-fold degenerate. The eigenvalues of $M_2$ are

$$0, \pm 2\lambda_0,$$

where the zero eigenvalue is four-fold degenerate. Alternatively one can show these by constructing the eigenmode operators of $H_0$.

We now analyze the bundle structure associated with the first one of eq. (21). Let us write

$$\left( \frac{h_{(3)}^\alpha}{\bar{h}_{\dot{\alpha}}^{(3)}} \right) \equiv h_{A,a},$$ \hspace{1cm} (26)

where the first subscript of the right hand side refers to $h_{(3)}$ or $\bar{h}_{(3)}$ and the second one to the spinor indices. Introducing five dimensional spherical coordinates

$$x_2 = r \sin \phi_1 \sin \theta_1 \cos \phi_2,$$

$$x_1 = r \cos \phi_1 \sin \theta_1 \cos \phi_2,$$

$$x_3 = r \cos \theta_1 \cos \phi_2,$$

$$x_4 = r \sin \theta_2 \cos \phi_2, \quad 0 \leq \phi_2 \leq 2\pi,$$

$$x_7 = r \sin \theta_2 \sin \phi_2, \quad 0 \leq \theta_2 \leq \pi,$$

we find

$$\mathcal{M}(\theta_2, \phi_2, \theta_1, \phi_1) \equiv \frac{1}{r} M_3 = \left( \begin{array}{cc}
-\cos \theta_2 U(\theta_1, \phi_1) & \sin \theta_2 e^{-i\phi_2} \mathbf{1}_{(2)} \\
\sin \theta_2 e^{i\phi_2} \mathbf{1}_{(2)} & \cos \theta_2 U(\theta_1, \phi_1)
\end{array} \right),$$ \hspace{1cm} (28)

where

$$U(\theta_1, \phi_1) = \begin{pmatrix}
\cos \theta_1 & \sin \theta_1 e^{i\phi_1} \\
\sin \theta_1 e^{-i\phi_1} & -\cos \theta_1
\end{pmatrix}.$$ \hspace{1cm} (29)

Note that

$$\mathcal{M}(\theta_2, \phi_2, \theta_1, \phi_1)^2 = \mathbf{1}_{(2)} \otimes \mathbf{1}_{(2)} \equiv \mathbf{1}_4,$$ \hspace{1cm} (30)

and

$$[ \mathcal{M}(\theta_2, \phi_2, \theta_1, \phi_1), \mathbf{1}_{(2)} \otimes U(\theta_1, \phi_1) ] = 0.$$ \hspace{1cm} (31)
The four eigenstates are specified by the eigenvalues ±1 of \( M(\theta_2, \phi_2, \theta_1, \phi_1) \) and those of 
\( 1_{(2)} \otimes U(\theta_1, \phi_1) \), which are obtained by the projection operators 
\[
\frac{1}{2} (1_{(4)} \pm M) \frac{1}{2} (1_{(4)} \pm 1_{(2)} \otimes U) h_{A,a}
\]
up to a numerical constant.

From now on, we focus on the (+, +) case without losing generality. The normalized 
eigenfunction is actually a section. Indicating its local forms around \((\theta_2, \theta_1) = (0, 0), (0, \pi), \) \((\pi, 0), \) and \((\pi, \pi)\) by \((N,N)\), \((N,S)\), \((S,N)\) and \((S,S)\) respectively, we find 
\[
\begin{align*}
  h_{A,a}^{(N,N)} &= \left( \sin \frac{\theta_2}{2} e^{-i \phi_2} \right)_A \otimes \left( \cos \frac{\theta_1}{2} e^{i \phi_1} \right)_a, \\
  h_{A,a}^{(N,S)} &= \left( \sin \frac{\theta_2}{2} e^{-i \phi_2} \right)_A \otimes \left( \cos \frac{\theta_1}{2} e^{-i \phi_1} \right)_a, \\
  h_{A,a}^{(S,N)} &= \left( \sin \frac{\theta_2}{2} e^{i \phi_2} \right)_A \otimes \left( \cos \frac{\theta_1}{2} e^{i \phi_1} \right)_a, \\
  h_{A,a}^{(S,S)} &= \left( \sin \frac{\theta_2}{2} e^{i \phi_2} \right)_A \otimes \left( \cos \frac{\theta_1}{2} e^{-i \phi_1} \right)_a.
\end{align*}
\]

This is obviously a simplest generalization of the original problem discussed in \[7\]. (See also \[8\].) We obtain the connection one-form 
\[
\begin{align*}
  A^{(N,N)} &= -\frac{i}{2} (1 - \cos \theta_2) d\phi_2 + \frac{i}{2} (1 - \cos \theta_1) d\phi_1, \\
  A^{(N,S)} &= -\frac{i}{2} (1 - \cos \theta_2) d\phi_2 - \frac{i}{2} (1 + \cos \theta_1) d\phi_1, \\
  A^{(S,N)} &= +\frac{i}{2} (1 + \cos \theta_2) d\phi_2 + \frac{i}{2} (1 - \cos \theta_1) d\phi_1, \\
  A^{(S,S)} &= +\frac{i}{2} (1 + \cos \theta_2) d\phi_2 - \frac{i}{2} (1 + \cos \theta_1) d\phi_1.
\end{align*}
\]

We have also determined the normalized eigenfunctions for the second eq. of (21), namely, 
the two-particle case, which we now describe only briefly and qualitatively. In contrast to eq. 
(33), the eigenfunctions belonging to zero or ±2\( \lambda_0 \) are described by ordinary functions, 
not developing into expressions involving half angles indicative of singularities. For the nonzero 
eigenvalues, we find the vanishing connection while the states with zero eigenvalue give a 
pure gauge configuration and are gauged away. We conclude that the \( n = 1,3 \) sectors, or 
equivalently, the first and the third excited states give rise to the nontrivial connection of 
the form of eq. (34) in the parameter space while the remaining states including the ground 
state (with \( \lambda_0 = -2 \)) do not.

**Brane interpretation:**

Let us now apply the formula (eq. (34)) we obtained to our original problem. Due to 
the symmetry of the roots and the weights under \( e^{(i)} \leftrightarrow -e^{(i)} \), the contributions from the
adjoint and antisymmetric representations (the second and the third terms in the exponent of eq. (16)) cancel when summed over $A$. The cancellation occurs as well to the part from the fundamental representation which does not involve $\phi^{(i)}$ or $\phi^{(i)*}$. We find that $\gamma^{(\text{total})}_T$ in eq. (16) is written as

$$
\gamma^{(\text{total})}_T = \sum_{f=1}^{n_f} \sum_{i=1}^k (\gamma^{(\text{Berry})}_T) \left[ x^{(i)}_3, m_f/\sqrt{2} + \phi^{(i)}, m_f/\sqrt{2} + \phi^{(i)*} \right] + \sum_{f=1}^{n_f} \sum_{i=1}^k (\gamma^{(\text{Berry})}_T) \left[ x^{(i)}_3, m_f/\sqrt{2} - \phi^{(i)}, m_f/\sqrt{2} - \phi^{(i)*} \right].
$$

(35)

Here

$$
\gamma^{(\text{Berry})}_T [x'_3, \phi, \phi^*] = \int A^{(\text{Berry})}
$$

(36)

$$
A^{(N)}(\text{Berry}) = -i \frac{1}{2} (1 - \cos \theta_2) d\phi_2, \quad A^{(S)}(\text{Berry}) = +i \frac{1}{2} (1 + \cos \theta_2) d\phi_2.
$$

(37)

and $x^{(i)2}_3 = x^{(i)2}_1 + x^{(i)2}_2 + x^{(i)2}_3$.

It is satisfying to see a pair of magnetic monopoles sitting at $x^{(i)}_4 = \pm m_f$ from the orientifold surface for $i = 1 \sim k$. (See 14 for the presence of orientifold surfaces in the USp matrix model.) These monopoles live in the parameter space, which is the spacetime coordinates generated by the matrix model. Coming back to eq. (14), we conclude that the Berry phase generates an interaction

$$
Z [x_I, x_I = 0; \ldots] = \int [D\bar{v}_M] \prod_{f=1}^{n_f} [DQ^{(f)}_T] [DQ^{*(f)}_T] [DQ^{(f)}_T] [DQ^{*(f)}_T] \exp[iS_B + i\gamma^{(\text{total})}_T].
$$

(38)

Let us give this configuration we have obtained a brane interpretation [13] first from the six dimensional and subsequently from the ten dimensional point of view. It should be noted that the two coordinates which the connection $A^{(\text{Berry})}$ does not depend on are the angular coordinates $\theta_1, \phi_1$, so that $x_1, x_2$ are not quite separable from the rest of the coordinates $x_3, x_4, x_7$ in eq. (37). Only in the asymptotic region $|x'_3| >> |x_3|$, there exists an area of size $\pi |x'_3|^2$ transverse to the three dimensional space where the Berry phase is obtained. In this region, the magnetic flux obtained from the $b(= 1)$-form connection embedded in $d(= 6)$-dimensional spacetime looks approximately as is discussed in 14: the flux no longer looks coming from a pointlike object but from a $d - b - 3(= 2)$ dimensionally extended object. The magnetic monopole obeying the Dirac quantization behaves approximately like a magnetic $D2$ brane [13] extending to the $(1,2)$ directions, which are perpendicular to the orientifold surface. In fact, the presence of this object and its quantized magnetic flux have been detected by quantum mechanics of a point particle (electric $D0$ brane) obtained from the $n = 1$ and $n = 3$ particle states of the fermionic sector in the fundamental representation. The induced interaction is a minimal one. We conclude that the $D0$ represented by the first and the third excited states of the quantum mechanical problem given above is under the magnetic field created by $D2$. To include the four remaining coordinates $(x_5, x_6, x_8, x_9)$ of the antisymmetric directions, we appeal to the translational invariance which is preserved.
in these directions. The simplest possibility is that they appear in the coupling through the derivatives

\[ \int A^{(\text{Berry})} = \int \prod_{I=5,6,8,9} dX^I dX^\nu A_{\nu 5689} \]. \tag{39}\]

With this assumption, the D0 brane is actually a D4 brane extended in (5, 6, 8, 9) directions while the magnetic D2 still occupies (1, 2): the quantization condition is preserved in ten dimensions as well.

We have exhibited a residual interaction due to the fermions in the fundamental representation. In the situation we have dealt with as quantum mechanics, however, the magnetic flux escapes to infinity. As a result, there is no conservation law which fixes the number of flavors \( n_f \). Let us finally make a brief comment on the case in which we compactify all six adjoint directions. As is shown in [6], the number of flavour is determined to be \( n_f = 16 \) by the cancellation of the nonabelian anomaly of 6d worldvolume gauge theory obtained from the USp matrix model via the recipe of [12]. We can apply to this case the same line of treatment given above based on the Berry phase. In fact, following the works of ref. [15], we find that the residual interaction is given by the two-parameter integral of the anomalous commutator of the Gauss law generators. This is a two cocycle \( w_2 \) of the gauge group on \( R^5 \) obtained from

\[ w_{-1} = \frac{1}{4!(4\pi i)^4} \text{Str} (F \wedge F \wedge F \wedge F) \] \tag{40}\]

via descent equations

\[ w_{-1} = dw_0 , \quad \delta w_0 = dw_1 , \quad \delta w_1 = dw_2 \]. \tag{41}\]

The cancellation of the strength of the anomalous commutator fixes \( n_f = 16 \).

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3 Similar reasoning is seen in [9].
4 For other applications of anomalies and anomalous interactions to D branes, see, for example, [16].
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