Finite element analysis of stress concentration coefficient of internal pressure conical shell

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Abstract. Based on the analysis and design idea, we use the finite element method to evaluate the dangerous position of the conical shell opening structure, select the stress concentration coefficient of the opening as the research object, analyze the influence of different opening angle and with/without a connecting pipe on it, and seeks the optimal design of the conical shell opening. The results show that the nozzle has a certain weakening effect on the stress concentration of the opening, and when the opening angle (the angle between the center line of the opening and the generatrix of the conical shell) is around 90°, the nozzle has a more significant strengthening effect on the structure. It also shows when the opening rate is fixed, the stress concentration coefficient of the opening decreases first and then increases with the increase of the opening angle, and when the opening angle is in the range of 80° to 110°, the stress concentration coefficient is the smallest between 2 to 3.

1. Introduction
The bottom of the flash chamber is a conical shell structure, and the heat is transferred through a jacket. In the design and calculation of conical shell, when the internal pressure is fixed, the maximum stress value of conical shell is directly proportional to the half vertex angle. The larger the half vertex angle is, the higher the maximum stress value is, and the thicker the wall thickness required for bearing is. Because of the particularity of its own structure, the mechanical law of conical shell opening is complex, there are many influencing factors, and the phenomenon of stress concentration is more significant. So, the analysis method is generally used to explore its law.

It is pointed out in reference [1] that it is not reasonable for GB150 to take the same wall thickness of the strengthened section and the transition section of the cylinder as the same value. The maximum value of the superposition of the primary stress and the secondary stress of the cylinder obtained by the edge stress solution and the membrane stress solution in the reference is:

$\sigma_{\text{max}} = \frac{pR}{\delta} \left(1 + \frac{R}{4r}\right) = \sigma_0 \left(1 + \frac{R}{4\delta}\right)$  \hspace{1cm} (1)

When the half apex angle of the conical head is greater than 60°, GB150 does not give a complete calculation formula to determine the wall thickness of the conical head under internal pressure, but it is calculated according to the flat cover. However, the calculation method is too conservative. In reference
The wall thickness of the conical shell with large cone angle is calculated by the strength calculation formula of the pressure vessel:

\[
\delta = \min \left\{ \max \left\{ \delta_k, \delta_r \right\} \delta_p \right\}
\]

There is no specific calculation method for the opening reinforcement of conical shell in GB150, however it has been reported in many papers. In reference [3], the opening of conical shell under external pressure can be divided into three types: the center line of opening is perpendicular to the center line of conical shell, the center line of opening is perpendicular to the curved surface of conical shell, and the center line of opening is at a small angle with the generatrix of conical shell.

Because of the complexity of the structure and the simultaneous application of various loads, the jacket conical shell structure can not be calculated by GB150, and can only be designed according to JB4732. There are many problems in the calculation of conical head by traditional SW6 software, which requires us to put forward a design optimized design method in the design.

2. Structure evaluation and stress analysis

2.1. Evaluation of cone shell opening structure

According to the requirements of fatigue design, the maximum stress value at the opening should be strictly limited.

| Design parameters | Shell | Jacket |
|-------------------|-------|--------|
| Design pressure (MPa) | 1.034 | 0.486 |
| Design temperature (℃) | 345 | 345 |
| Materials of main pressure components | SA516-Gr.70 | SA516-Gr.70 |
| Corrosion allowance (mm) in/out | 0/3 | 3/0 |

2.1.1. Finite element analysis

1. Mechanical model of structure

The conical shell, nozzle and part of cylinder are selected as the finite element model to analyze the stress of nozzle. The geometric parameters of the model are shown in Figure 1 (corrosion allowance has been considered). The nozzle size is Ø84.78 × 17.39 mm.

![Figure 1. Model geometric parameters.](image-url)
(2) Unit selection
The element type of the finite element model after meshing is solid-brick45 with 8 nodes. The solid model has 150359 units and 202402 nodes.

(3) Load
The internal pressure of the conical shell head under the design pressure is 1.034 MPa, and the internal pressure of the jacket is 0.486 MPa. The axial balance force of nozzle n23 and N22 is \(-0.259\) MPa calculated by \(-\frac{P}{k^2-1}\) (diameter ratio of cylinder). The axial balance force of cylinder shell is \(-15.66\) MPa, and the axial balance force of cylinder shell jacket is \(-25.24\) MPa.

(4) Boundary conditions
Considering the connection flange at the small end of the conical shell, the fixed end constraint is applied to the flange surface.

Stress analysis results
The results of finite element calculation under design pressure are shown in stress nephogram 2.

![Figure 2. Model stress nephogram.](image)

2.1.2. Stress intensity assessment. ① Analysis and design criteria and stress intensity limits
In consideration of the different degree of damage of various stresses to the vessel, the restrictions on them are also different. The safety criterion of stress intensity is as follows:

- Primary overall membrane stress intensity \(S_i \leq S_m\)
- Primary local membrane stress intensity \(S_{il} \leq 1.5S_m\)
- Primary film (overall or local) plus primary bending stress intensity \(S_{il} \leq 1.5S_m\)
- Primary plus secondary stress intensity \(S_{ii} \leq 3S_m\)
- Peak stress intensity \(S_v \leq S_a\)

② Strength analysis
(1) According to the stress nephogram 3 of nozzle N23 (large end), the maximum stress value of
nozzle is 146.962 MPa.

Figure 3. Stress nephogram of nozzle N23.

(2) The maximum stress value of nozzle N22 (small end) is 81.731 MPa (stress nephogram is omitted), and $S_{st}=129$ MPa, so there is no need to linearize the stress of nozzle, and the strength of nozzle is qualified.

3. Conclusion

According to the stress evaluation, the equipment is safe, and the nozzle structure of the conical shell of the flash chamber meets the ASME standard.

2.2. Stress analysis of conical shell opening structure

2.2.1. Effect of nozzle on stress concentration coefficient of opening. (1) Structural mechanical model

The opening angle $\alpha$ of the nozzle is the angle between the nozzle centerline and the conical shell generatrix. As shown in Figure 1, the opening angle of the model is $120^\circ$.

The dimensions of the first group of models are shown in Figure 1. Only the opening angle $\alpha$ is changed from $30^\circ$ to $150^\circ$ with an interval of $15^\circ$.

The second group of model size is as the first group, however the nozzle is removed.

(2). Stress analysis results

When the opening angle $\alpha$ is $120^\circ$, the stress nephogram of the two models is shown in Figure 4a and Figure 4b.
Fig. 4. (a). model group I $\alpha = 120^\circ$. (b). model group II $\alpha = 120^\circ$

3. Stress concentration analysis of nozzle

The hole opening stress concentration coefficient is $k = \frac{\sigma_{\text{max}}}{\sigma_{\theta}}$, in which $\sigma_{\text{max}}$ represents the maximum elastic stress at the conical shell and the hole opening. According to the stress nephogram, $\sigma_{\theta}$ represents the circumferential membrane stress at the hole opening when the conical shell is not opened. It is calculated by formula $\sigma_{\theta} = \frac{pr}{t \cos \alpha}$, as shown in table 3-6.
Table 3. Stress concentration coefficient of N23 opening with connecting pipe.

| $\alpha$ | $\sigma_{\max}$ | $\sigma_{\max}$ | Maximum position | $k$  |
|---------|-----------------|-----------------|-----------------|------|
| 30.00  | 272.12          | 57.44           | down            | 4.74 |
| 45.00  | 199.40          | 57.75           | down            | 3.45 |
| 60.00  | 159.77          | 57.89           | down            | 2.76 |
| 75.00  | 138.72          | 57.96           | down            | 2.39 |
| 90.00  | 126.72          | 57.98           | down            | 2.19 |
| 105.00 | 125.14          | 59.07           | up              | 2.12 |
| 120.00 | 146.96          | 59.14           | up              | 2.49 |
| 135.00 | 182.97          | 59.28           | up              | 3.09 |
| 150.00 | 254.36          | 59.59           | up              | 4.27 |

Table 4. Stress concentration coefficient of N23 opening without connecting pipe.

| $\alpha$ | $\sigma_{\max}$ | $\sigma_{\max}$ | Maximum position | $k$  |
|---------|-----------------|-----------------|-----------------|------|
| 30.00  | 284.84          | 56.69           | down            | 5.02 |
| 45.00  | 174.82          | 57.22           | down            | 3.06 |
| 60.00  | 153.48          | 57.46           | down            | 2.67 |
| 75.00  | 146.36          | 57.57           | down            | 2.54 |
| 90.00  | 140.97          | 57.60           | down            | 2.45 |
| 105.00 | 147.49          | 57.57           | down            | 2.56 |
| 120.00 | 175.01          | 57.46           | down            | 3.05 |
| 135.00 | 194.79          | 57.22           | down            | 3.40 |
| 150.00 | 251.23          | 56.69           | down            | 4.43 |

Table 5. Stress concentration coefficient of N22 with nozzle.

| $\alpha$ | $\sigma_{\max}$ | $\sigma_{\max}$ | Maximum position | $k$  |
|---------|-----------------|-----------------|-----------------|------|
| 30.00  | 143.77          | 24.21           | down            | 5.94 |
| 45.00  | 102.47          | 24.53           | down            | 4.18 |
| 60.00  | 81.41           | 24.67           | down            | 3.30 |
| 75.00  | 69.57           | 24.73           | down            | 2.81 |
| 90.00  | 65.16           | 25.83           | up              | 2.52 |
| 105.00 | 71.38           | 25.85           | up              | 2.76 |
| 120.00 | 81.73           | 25.92           | up              | 3.15 |
| 135.00 | 104.34          | 26.06           | up              | 4.00 |
| 150.00 | 145.43          | 26.37           | up              | 5.51 |

Table 6. Stress concentration coefficient of N22 opening without connecting pipe.

| $\alpha$ | $\sigma_{\max}$ | $\sigma_{\max}$ | Maximum position | $k$  |
|---------|-----------------|-----------------|-----------------|------|
| 30.00  | 159.06          | 24.00           | down            | 6.63 |
| 45.00  | 113.53          | 26.59           | up              | 4.27 |
| 60.00  | 95.54           | 26.35           | up              | 3.63 |
| 75.00  | 84.60           | 24.35           | down            | 3.47 |
| 90.00  | 81.19           | 26.21           | up              | 3.10 |
| 105.00 | 85.58           | 26.24           | up              | 3.26 |
| 120.00 | 109.72          | 26.35           | up              | 4.16 |
| 135.00 | 112.40          | 26.59           | up              | 4.23 |
| 150.00 | 188.95          | 27.12           | up              | 6.97 |
See Figure 5 for comparison of two groups of hole opening stress concentration coefficients according to the change of hole opening angle.

![Figure 5. Stress concentration coefficient of N23 and N22 opening.](image)

### 2.2.2. Effect of opening angle on opening stress concentration coefficient

1. **Structural mechanical model**
   - The first group of model dimensions are shown in Figure 1. We only change the opening angle \( \alpha \) from 30° to 150°, with an interval of 15°.
   - The size of the second group of model is as that of the first group. However the opening condition is changed, and the opening is \( \varphi 144 \times 22 \) mm.

2. **Stress analysis results**
   - The stress nephogram is omitted.

3. **Stress concentration analysis of nozzle**
   - The calculation summary of nozzle stress concentration factor is shown in table 7 to 10.

| \( \alpha \) | \( \sigma_\theta \) | \( \sigma_{\text{max}} \) | \( k \) | Opening rate |
|--------------|----------------|----------------|-----|-------------|
| 30.00        | 57.44          | 272.12         | 4.74|             |
| 45.00        | 57.75          | 199.40         | 3.45|             |
| 60.00        | 57.89          | 159.77         | 2.76|             |
| 75.00        | 57.96          | 138.72         | 2.39|             |
| 90.00        | 57.98          | 126.72         | 2.19|             |
| 105.00       | 59.07          | 125.14         | 2.12|             |
| 120.00       | 59.14          | 146.96         | 2.49|             |
| 135.00       | 59.28          | 182.97         | 3.09|             |
| 150.00       | 59.59          | 254.36         | 4.27| 0.0313      |
Table 8. N22 stress concentration coefficient of the first group of model nozzles.

| α   | σ_θ | σ_max | k     | Opening rate |
|-----|-----|-------|-------|--------------|
| 30.00 | 24.21 | 143.77 | 5.94  |              |
| 45.00 | 24.53 | 102.47 | 4.18  |              |
| 60.00 | 24.67 | 81.41  | 3.30  |              |
| 75.00 | 24.73 | 69.57  | 2.81  |              |
| 90.00 | 25.83 | 65.16  | 2.52  | 0.0723       |
| 105.00 | 25.85 | 71.38  | 2.76  |              |
| 120.00 | 25.92 | 81.73  | 3.15  |              |
| 135.00 | 26.06 | 104.34 | 4.00  |              |
| 150.00 | 26.37 | 145.43 | 5.51  |              |

Table 9. Stress concentration coefficient of the second group of model nozzle N23.

| α   | σ_θ | σ_max | k     | Opening rate |
|-----|-----|-------|-------|--------------|
| 30.00 | 56.36 | 276.50 | 4.91  |              |
| 45.00 | 56.99 | 189.15 | 3.32  |              |
| 60.00 | 57.27 | 147.27 | 2.57  |              |
| 75.00 | 57.40 | 125.91 | 2.19  |              |
| 90.00 | 57.44 | 116.60 | 2.03  | 0.053        |
| 105.00 | 57.40 | 115.31 | 2.01  |              |
| 120.00 | 57.27 | 121.91 | 2.13  |              |
| 135.00 | 60.04 | 143.47 | 2.39  |              |
| 150.00 | 60.67 | 201.69 | 3.32  |              |

Table 10. N22 stress concentration coefficient of the second group of model nozzles.

| α   | σ_θ | σ_max | k     | Opening rate |
|-----|-----|-------|-------|--------------|
| 30.00 | 23.13 | 150.49 | 6.51  |              |
| 45.00 | 23.77 | 98.82  | 4.16  |              |
| 60.00 | 24.05 | 75.27  | 3.13  |              |
| 75.00 | 24.18 | 63.70  | 2.63  |              |
| 90.00 | 26.37 | 61.54  | 2.33  | 0.123        |
| 105.00 | 26.41 | 67.22  | 2.55  |              |
| 120.00 | 26.54 | 80.18  | 3.02  |              |
| 135.00 | 26.82 | 106.34 | 3.96  |              |
| 150.00 | 27.45 | 167.30 | 6.09  |              |

3. Conclusion
Through the finite element software ANSYS, the open hole structure of the conical shell of the flash chamber is modeled and analyzed. Taking the open hole stress concentration coefficient as the research object, the open hole structure is changed, the influence of the opening angle and whether there is a nozzle on the open hole stress concentration coefficient is studied, and the law that the stress concentration coefficient changes into parabola with the opening angle is obtained. The optimal opening angle is in the range of 80 ° to 110 °, and the stress concentration coefficient is the smallest, and the reinforcement effect of nozzle is the most significant. The general rule is that the closer the opening is to the big end, the smaller the stress concentration coefficient is. But this does not mean that small end openings are more dangerous than large end openings. Because the membrane stress at the small end is much smaller than that at the large end, even though the stress concentration coefficient at the small end
is larger than that at the large end, the stress concentration value at the small end is still much smaller than that at the large end, and the stress intensity assessment result of the hole at the small end is much better than that at the large end.

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