Application of Metaheuristic Algorithms in Truss Structure Sizing Optimization

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Abstract. Field studies of structural optimization have gained increased attention due to the rapid development of metaheuristic algorithms. One widely known metaheuristic algorithm, Particle Swarm Optimization (PSO), has been extensively used to solve many problems and is reported to have fast convergence behavior and good accuracy. As many problems become more complex, studies have been focused on improving PSO searching capability. This study presents the application of PSO and its variants in optimizing truss structures. The performances of PSO and several PSO variants, namely, linearly decreasing inertia weight PSO (LDW-PSO) and bare bones PSO (BB-PSO), were compared and investigated. All optimization algorithms were tested in 72-bar and 25-bar spatial truss problems. The results indicate that BBPSO was the best algorithm in terms of optimum solution, consistency, and convergence behavior.

1. Introduction
The truss structure is the most common structural component used in buildings. Steel truss structures are usually used as bracing or the main building structure. With rapid construction growth, finding more efficient structural designs through optimization are needed to minimize cost. The goal of structure optimization is to find the most efficiently sized structure without violating any engineering constraints. Structural efficiency is usually regarded as the weight of the structure [1].

Truss structure optimization has attracted recent and growing interest. Truss structures have many constraints and variables, which makes optimizing these structures complex and challenging. However, metaheuristic methods are efficient and effective in solving such large and complex problems [2]. Metaheuristic algorithms apply natural phenomena and randomization concepts to search for an optimum solution globally using trial and error [3]. Particle swarm optimization (PSO) [4] is a metaheuristic algorithm that is frequently used to solve optimization problem. PSO has a simple concept that mimics flocking birds. Despite its simplicity, PSO has some weaknesses, with one being parameters that can affect its performance. These parameters must be manually adjusted to the problem [5]. Several variants of PSO have been developed to overcome these weaknesses such as Linearly Decreasing Inertia Weight Particles Swarm Optimization (LDW-PSO) [6] and Bare Bones Particles Swarm Optimization (BBPSO) [7].
2. Particle swarm optimization (PSO)

2.1. Particle swarm optimization (PSO)
PSO mimics the behavior of flocking birds. In a manner similar to a flock of birds looking for food, the PSO searches for the optimal solution. This simple and easy to understand concept makes this algorithm popular with researchers. The weakness of this algorithm is the need to pre-set the parameters to adapt to different problems [7].

First, particle location is generated randomly in a specified range [6]. Then, each iteration particle moves to a new location using the velocity in Equation (1) and then updates its position using Equation (2). This new velocity is influenced by four factors: its initial velocity \( v_i(t) \); the best location that this particle discovers \( X_{pbest}(t) \); the best location from population \( X_{gbest}(t) \); and its current location \( X_i(t) \):

\[
X_i(t+1) = X_i(t) + v_i(t+1),
\]

\[
v_i(t+1) = wv_i(t) + r_1C_1 \left( X_{pbest}(t) - X_i(t) \right) + r_2C_2 \left( X_{gbest}(t) - X_i(t) \right),
\]

where \( v_i(t+1) \) is the next velocity; \( w \) is inertia weight; \( v_i(t) \) is the initial velocity; \( r_1 \) and \( r_2 \) are random numbers between 0 and 1; \( C_1 \) and \( C_2 \) are constants that have been set (usually 2); \( X_{pbest}(t) \) is personal best; \( X_i(t) \) is the initial location; \( X_{gbest}(t) \) is global best; and \( X_i(t+1) \) is the particle’s new location.

2.2. Linearly decreasing inertia weight particles swarm optimization (LDW-PSO)
In PSO, inertia weight is used to balance the global and local searches. A large inertia weight represents a global search while a small inertia weight represents a local search. By linearly decreasing the inertia weight, PSO should have more global search ability at the beginning of the iteration while having more local search ability near the end of the iteration [6]. The inertia weight updates with Equation (3):

\[
w = w - (w_s - w_e)(t) / t_{max},
\]

where \( w \) is inertia weight; \( w_s \) is initial inertia weight; \( w_e \) is final inertia weight; \( t \) is current iteration; and \( t_{max} \) is total iteration.

2.3. Bare Bones Particle Swarm Optimization (BBPSO)
While LDW-PSO perfected the parameter in PSO, BBPSO eliminates all parameters. Instead of using velocity to update the location, BBPSO uses a Gaussian distribution. The particle’s next position is only calculated by its personal best position and swarm global best position. Parameter-free means the algorithm can easily adapt to different problems [7]:

\[
\mu = \frac{p_i + gbest}{2},
\]

\[
\sigma = |p_i - gbest|,
\]

\[
x(i+1) = \begin{cases} 
N(\mu, \sigma) & \text{if } \omega > 0.5 \\
p_i & \text{else}
\end{cases}
\]

where \( P = (p_1, p_2, ..., p_n) \) is the personal best position of each particle, \( gbest \) is the best position of the whole swarm, and \( \omega \) is a random number from 0 to 1.
3. Materials and methods
A combination of direct stiffness method (DSM) and metaheuristics were used for this truss optimization. Metaheuristics were used to find the optimal cross-sectional area while DSM was used to analyze the structure. DSM outputs are the displacement, axial force, and stress of each element. These outputs are used as constraints for this optimization. When a solution violates the constraints, a penalty is given to the solution.

Before conducting the research, researchers prepared a DSM program for a planar and spatial truss, and prepared three metaheuristic algorithms: PSO, LDW-PSO, and BBPSO. The DSM and metaheuristic algorithms were written using MATLAB 2017a and the results of the three algorithms were compared to determine the best performing algorithm. In general, this program randomizes the cross-section area, and iterates using trial and error until it reaches its maximum iteration. A flow chart of the truss optimization process is diagrammed in Figure 1.

![Figure 1. Truss optimization process flow chart](image-url)
4. Test problem and results
This paper compares the performance of three PSOs using a spatial 2-bar structure problem. Each structure had a load case and discrete variable, which are described next. The goal is to minimize the weight of the structure while not violating the constraints. Each algorithm was run 30 times and with 50 populations. The structures were analyzed using DSM. Algorithms and structural analyses were coded in MATLAB 2017a. Cognitive ($C_1$) and social ($C_2$) parameters for PSO and LPSO were set to 2. Inertia weight ($W$) for PSO was set to 0.8 while the LPSO’s inertia weight was linearly decreased from 0.9 to 0.1 with respect to iterations.

4.1. Spatial 25-bar truss structure
The structure model presented in Figure 2 has been previously studied [3][8]. The material density is 0.1 lb/in$^3$ and elastic modulus 10 Msi. The boundary conditions are stress and displacement. Stress limits in tension/compression is 40,000 psi and maximum nodal displacement for all free nodes in X, Y, and Z directions is ±0.35 in. Members of the structure are grouped into eight groups: (1) A1; (2) A2–A5; (3) A6–A9; (4) A10–A11; (5) A12–A13; (6) A14–A17; (7) A18–A21; and (8) A22–25.

There are two cases for this structure:
Case 1. The cross-sectional areas available are $D = [0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3.0 3.1 3.2 3.3 3.4 3.5 3.6]$ (in$^2$). (Loads are shown in Table 1.)

Case 2. The cross-sectional areas available are $D = [0.01 0.04 0.08 0.12 0.16 0.20 0.24 0.28 0.32 0.36 0.40 0.44 0.48 0.52 0.56 0.60]$ (in$^2$). (Loads are shown in Table 2.) In this case, there are two cases to be run and the cross-section used has to satisfy all boundary conditions in both cases.

Figure 2. Spatial 25-bar truss structure model
### Table 1. Loading Conditions for 25-bar Truss Problem (Case 1)

| Load Cases | Nodes | \(P_x\) (kips) | \(P_y\) (kips) | \(P_z\) (kips) |
|------------|-------|----------------|----------------|----------------|
| Case 1     | 1     | 1              | -10            | -10            |
|            | 2     | 0              | -10            | -10            |
|            | 3     | 0.5            | 0              | 0              |
|            | 6     | 0.6            | 0              | 0              |

### Table 2. Loading Conditions for 25-bar Truss Problem (Case 2)

| Load Cases | Nodes | \(P_x\) (kips) | \(P_y\) (kips) | \(P_z\) (kips) |
|------------|-------|----------------|----------------|----------------|
| Case 2     | 2     | 0              | 20             | -5             |
|            | 2     | 0              | -20            | -5             |
|            | 3     | 1              | 10             | -5             |
|            | 2     | 0              | 10             | -5             |
|            | 3     | 0.5            | 0              | 0              |
|            | 6     | 0.5            | 0              | 0              |

### Table 3. Comparison Optimization Result for 25-bar Problem (Case 1)

| Variables  | HS[8]  | PSO   | LDW-PSO | BBPSO |
|------------|--------|-------|---------|-------|
| A1         | 0.1    | 0.1   | 0.1     | 0.1   |
| A2-A5      | 0.3    | 0.3   | 0.3     | 0.3   |
| A6-A9      | 3.4    | 3.4   | 3.4     | 3.4   |
| A10-A11    | 0.1    | 0.1   | 0.1     | 0.1   |
| A12-A13    | 2.1    | 2.1   | 2.1     | 2.1   |
| A14-A17    | 1      | 1     | 1       | 1     |
| A18-A21    | 0.5    | 0.5   | 0.5     | 0.5   |
| A22-A25    | 3.4    | 3.4   | 3.4     | 3.4   |
| Best (lb)  | 484.85 | 484.85| 484.85  | 484.85|
| Average (lb)| N/A  | 488.45596 | 487.0637 | 485.7423 |
| Stdev (lb) | N/A   | 8.5055357 | 3.052439 | 1.013263 |
| No. of analyses | 13523 | 5000   | 5000    | 5000   |
| Constraint violations | None | None  | None    | None   |
Table 4. Comparison Optimization Result for 25-bar Problem (Case 2)

| Variables | HS[8] | PSO | LDW-PSO | BBPSO |
|-----------|-------|-----|---------|-------|
| A1        | 0.01  | 0.01| 0.01    | 0.01  |
| A2-A5     | 2     | 2   | 2       | 2     |
| A6-A9     | 3.6   | 3.6 | 3.6     | 3.6   |
| A10-A11   | 0.01  | 0.01| 0.01    | 0.01  |
| A12-A13   | 0.01  | 0.01| 0.01    | 0.01  |
| A14-A17   | 0.8   | 0.8 | 0.8     | 0.8   |
| A18-A21   | 1.6   | 1.6 | 1.6     | 1.6   |
| A22-A25   | 2.4   | 2.4 | 2.4     | 2.4   |
| Best (lb) | 560.59| 560.59| 560.59 | 560.59|
| Average (lb) | N/A | 567.7245 | 577.5186 | 561.1604 |
| Stdev (lb) | N/A | 11.67591 | 21.0032 | 1.475004 |
| No. of analyses | 7435 | 5000 | 5000 | 5000 |
| Constraint violations | None | None | None | None |

A comparison among the three algorithms is shown in Table 4 for Case 1 and in Table 5 for Case 2. As can be seen, there were no constraint violations for any of the algorithms. All algorithms obtained the same minimum weight (484.85 lb for Case 1 and 560.59 lb for Case 2). The BBPSO algorithm was the best PSO algorithm in terms of consistency. Figures 3 and 4 show that BBPSO demonstrated better convergence behavior. From a previous study, harmony search (HS) [8] obtained the same best results for Case 1 and Case 2 with PSO variants used in this study. However, to achieve this result, HS needed a greater number of analyses than BBPSO.

Figure 3. Convergence curves for 25-bar problem (case 1)
4.2. Spatial 72-bar truss structure

The 72-bar structure has 20 nodes and 60 degrees of freedom in X, Y, and Z directions. It comprises four identical floors as shown in Figure 5. The material density is 0.1 lb/in³ and elastic modulus 10 Msi. The stress limit for compression/tension is 25,000 psi and displacement should not be more than ±0.35 in. Each story has a different cross-section for its vertical, horizontal, wall-bracing, and floor-bracing trusses. In total, there are 16 groups: (1) A1–A4; (2) A5–A12; (3) A13–A16; (4) A17–A18; (5) A19–A22; (6) A23–A30; (7) A31–A34; (8) A35–A36; (9) A37–A40; (10) A41–A48; (11) A49–A52; (12) A53–A54; (13) A55–A58; (14) A59–A66; (15) A67–A70; and (16) A71–A72. As in load Case 2 in the 25-bar truss structure, the 72-bar truss structure was subjected to two load cases as described in Table 5.
Table 5. Comparison Optimization Result for 72-bar Problem

| Load Cases | Nodes | Loads |
|------------|-------|-------|
|            |       |       |
|            |       |       |

Table 6. Comparison of Optimization Result for 72-bar Problem

| Variables | HS[8] | PSO   | LDW-PSO | BBPSO |
|-----------|-------|-------|---------|-------|
| A1-A4     | 1.9   | 2     | 1.9     | 1.9   |
| A5-A12    | 0.5   | 0.5   | 0.5     | 0.5   |
| A13-A16   | 0.1   | 0.1   | 0.1     | 0.1   |
| A17-A18   | 0.1   | 0.1   | 0.1     | 0.1   |
| A19-A22   | 1.4   | 1.5   | 1.4     | 1.4   |
| A23-A30   | 0.6   | 0.6   | 0.6     | 0.6   |
| A31-A34   | 0.1   | 0.1   | 0.1     | 0.1   |
| A35-A36   | 0.1   | 0.1   | 0.1     | 0.1   |
| A37-A40   | 0.6   | 0.6   | 0.6     | 0.6   |
| A41-A48   | 0.5   | 0.5   | 0.5     | 0.5   |
| A49-A52   | 0.1   | 0.1   | 0.1     | 0.1   |
| A53-A54   | 0.1   | 0.1   | 0.1     | 0.1   |
| A55-A58   | 0.2   | 0.2   | 0.2     | 0.2   |
| A59-A66   | 0.5   | 0.5   | 0.6     | 0.6   |
| A67-A70   | 0.3   | 0.5   | 0.4     | 0.4   |
| A71-A72   | 0.7   | 0.6   | 0.6     | 0.6   |
| Best      | 387.94| 412.08| 403.75  | 385.54|
| Average   | N/A   | 456.6132 | 490.91 | 390.7881|
| Stdev     | N/A   | 46.22298 | 64.38027 | 3.742455|
| No. of analyses | 16044 | 5000 | 5000 | 5000 |
| Constraint violations | None | None | None | None |

Table 6 shows that BBPSO had the best performance and the smallest standard deviation. Each algorithm ran 50,000 structural analyses. The PSO, LDW-PSO, and BBPSO obtained minimum weights of 386.81 lb, 385.54 lb, and 385.54 lb, respectively. However, LDW-PSO had 2.1% less weight than PSO while LPPO had larger standard deviation, showing less consistency. In terms of consistency, BBPSO had the best convergence behavior as shown in Figure 6. In this case study, HS[8] was outperformed by BBPSO even though HS had a greater number of analyses.
5. Conclusion
This paper tested the variance in results of three PSO algorithms: BBPSO, LDW-PSO, and original PSO. The algorithms were presented with spatial truss problems coded using direct stiffness method. The results show BBPSO to be the best algorithm of the three tested algorithms. BBPSO had exceptional performance in terms of result, consistency, and convergence behavior. This is due to BBPSO having no pre-set parameters, which means that it is more adaptable to problems, whereas PSO and LDW-PSO contain pre-set variables. LPSO returned better results than the original PSO; however, LDW-PSO tends to have poorer convergence behavior. LDW-PSO performed more focused searches at the end of iteration due to its decreasing inertia weight, whereas PSO had the same coverage through each iteration. However, this could be a problem for convergence behavior because decreasing through the iteration means that LDW-PSO needs all iterations to find the optimum solution.

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Figure 6. Convergence curves for 72-bar problem
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