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Stability of entropic optimal transport and Schrödinger bridges. (English) Zbl 07573798
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Summary: We establish the stability of solutions to the entropically regularized optimal transport problem with respect to the marginals and the cost function. The result is based on the geometric notion of cyclical invariance and inspired by the use of c-cyclical monotonicity in classical optimal transport. As a consequence of stability, we obtain the wellposedness of the solution in this geometric sense, even when all transports have infinite cost. More generally, our results apply to a class of static Schrödinger bridge problems including entropic optimal transport.

MSC:
90C25 Convex programming
49N05 Linear optimal control problems

Keywords: entropic optimal transport; Schrödinger bridge; stability; Sinkhorn’s algorithm

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