Spacetime in Semiclassical Gravity

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Abstract

This article will summarize selected aspects of the semiclassical theory of gravity, which involves a classical gravitational field coupled to quantum matter fields. Among the issues which will be discussed are the role of quantum effects in black hole physics and in cosmology, the effects of quantum violations of the classical energy conditions, and inequalities which constrain the extent of such violations. We will also examine the first steps beyond semiclassical gravity, when the effects of spacetime geometry fluctuations start to appear.

1 Introduction

This article will deal with the semiclassical approximation, in which the gravitational field is classical, but is coupled to quantum matter fields. The semiclassical theory consists of two aspects: (1) Quantum field theory in curved spacetime and (2) The semiclassical Einstein equation. Quantum field theory in curved spacetime describes the effects of gravity upon the quantum fields. Here a number of nontrivial effects arise, including particle creation, negative energy densities, and black hole evaporation. The semiclassical Einstein equation describes how quantum fields act as the source of gravity. This equation is usually taken to be the classical Einstein equation, with the source as the quantum expectation value of the matter field stress tensor operator, that is

\[ G_{\mu\nu} = 8\pi \langle T_{\mu\nu} \rangle. \]

This expectation value is only defined after suitable regularization and renormalization.

In this article, we will use units (Planck units) in which Newton’s constant, the speed of light, and \( \hbar \) are set to one: \( G = c = \hbar = 1 \). This makes all physical quantities dimensionless. Thus masses, lengths, and times are expressed as dimensionless multiples of the Planck mass, \( m_P = \sqrt{\hbar c/G} = 2.2 \times 10^{-5} \text{g} \), the Planck length, \( \ell_P = \sqrt{\hbar G/c^3} = 1.6 \times 10^{-33} \text{cm} \), and the Planck time, \( t_P = \sqrt{\hbar G/c^5} = 5.4 \times 10^{-44} \text{s} \), respectively.

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2 Renormalization of $\langle T_{\mu\nu} \rangle$

Here we will outline of the procedure for extracting a meaningful, finite part from the formally divergent expectation value of the stress tensor. More detailed accounts can be found in the books by Birrell and Davies \[1\] and by Fulling \[2\]. The first step is to introduce a formal regularization scheme, which renders the expectation value finite, but dependent upon an arbitrary regulator parameter. One possible choice is to separate the spacetime points at which the fields in $T_{\mu\nu}$ are evaluated, and then to average over the direction of separation. This leaves $\langle T_{\mu\nu} \rangle$ depending upon an invariant measure of the distance between the two points. This is conventionally chosen to be one-half of the square of the geodesic distance, denoted by $\sigma$.

The asymptotic form for the regularized expression in the limit of small $\sigma$ can be shown to be

$$\langle T_{\mu\nu} \rangle \sim A \frac{g_{\mu\nu}}{\sigma^2} + B \frac{G_{\mu\nu}}{\sigma} + (C_1 H_{\mu\nu}^{(1)} + C_2 H_{\mu\nu}^{(2)}) \ln \sigma. \quad (2)$$

Here $A$, $B$, $C_1$, and $C_2$ are constants, $G_{\mu\nu}$ is the Einstein tensor, and the $H_{\mu\nu}^{(1)}$ and $H_{\mu\nu}^{(2)}$ tensors are covariantly conserved tensors which are quadratic in the Riemann tensor. Specifically, they are the functional derivatives with respect to the metric tensor of the square of the scalar curvature and of the Ricci tensor, respectively:

$$H_{\mu\nu}^{(1)} \equiv \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}} [\sqrt{-g} R^2] = 2 \nabla_\nu \nabla_\mu R - 2 g_{\mu\nu} \nabla_\rho \nabla^\rho R - \frac{1}{2} g_{\mu\nu} R^2 + 2 R R_{\mu\nu}, \quad (3)$$

and

$$H_{\mu\nu}^{(2)} \equiv \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}} [\sqrt{-g} R_\alpha^\beta R_\alpha^\beta] = 2 \nabla_\alpha \nabla_\nu R^\alpha_{\mu} - \nabla_\nu \nabla^\rho R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R_\alpha^\beta R_\alpha^\beta + 2 R_\rho^\alpha R_{\mu\nu}. \quad (4)$$

The divergent parts of $\langle T_{\mu\nu} \rangle$ may be absorbed by renormalization of counterterms in the gravitational action. Write this action as

$$S_G = \frac{1}{16\pi G_0} \int d^4x \sqrt{-g} \left( R - 2\Lambda_0 + \alpha_0 R^2 + \beta_0 R_\alpha^\beta R_\alpha^\beta \right). \quad (5)$$

We now include a matter action, $S_M$, and vary the total action, $S = S_G + S_M$, with respect to the metric. If we replace the classical stress tensor in the resulting equation by the quantum expectation value, $\langle T_{\mu\nu} \rangle$, we obtain the semiclassical Einstein equation including the quadratic counterterms:

$$G_{\mu\nu} + \Lambda_0 g_{\mu\nu} + \alpha_0 H_{\mu\nu}^{(1)} + \beta_0 H_{\mu\nu}^{(2)} = 8\pi G_0 \langle T_{\mu\nu} \rangle. \quad (6)$$

We may remove the divergent parts of $\langle T_{\mu\nu} \rangle$ in redefinitions of the coupling constants $G_0$, $\Lambda_0$, $\alpha_0$, and $\beta_0$. The renormalized values of these constants are then the physical
parameters in the gravitational theory. After renormalization, \( G_0 \) is replaced by \( G \), the renormalized Newton’s constant, which is the value actually measured by the Cavendish experiment. Similarly, \( \Lambda_0 \) becomes the renormalized cosmological constant \( \Lambda \), which must be determined by observation. This is analogous to any other renormalization in field theory, such as the renormalization of the mass and charge of the electron in quantum electrodynamics.

In any case, the renormalized value of \( \langle T_{\mu\nu} \rangle \) is obtained by subtracting the terms which are divergent in the coincidence limit. However, we are free to perform additional finite renormalizations of the same form. Thus, \( \langle T_{\mu\nu} \rangle_{\text{ren}} \) is defined only up to the addition of multiples of the four covariantly conserved, geometrical tensors \( g_{\mu\nu} \), \( G_{\mu\nu} \), \( H^{(1)}_{\mu\nu} \), and \( H^{(2)}_{\mu\nu} \). Apart from this ambiguity, Wald\[3\] has shown under very general assumptions that \( \langle T_{\mu\nu} \rangle_{\text{ren}} \) is unique. Hence, at the end of the calculation, the answer is independent of the details of the regularization and renormalization procedures employed.

### 3 The Stability Problem in the Semiclassical Theory

The classical Einstein equation is a second order, nonlinear, differential equation for the spacetime metric tensor, because the Einstein tensor involves up to second derivatives of the metric. As a second order system of hyperbolic equations, it possesses a well-posed initial value formulation: if one specifies the metric and its first derivatives on a spacelike hypersurface, there exists a unique solution of the equations\[4\]. This is the usual situation in physics, where the fundamental equations can be cast as a second order system. (For example, Maxwell’s equations are equivalent to a set of second order wave equations for the vector and scalar potentials.)

There is a problem with the semiclassical Einstein equation in that it is potentially a fourth-order system of equations. This arises from terms involving second derivatives of the curvature tensor, and hence fourth derivatives of the metric. This leads to the unpleasant feature that a unique solution would require specification of the metric and its first three derivatives on a spacelike hypersurface. Even worse, it can lead to instability. The situation is analogous to that in classical electrodynamics when radiation reaction is included in the equation of motion of a charged particle\[5\]. The Abraham-Lorentz equation, which includes the radiation reaction force for a nonrelativistic particle, is third-order in time and possesses runaway solutions. In electrodynamics, the problem is partially solved by replacing the third-order Abraham-Lorentz equation by an integrodifferential equation which is free of runaway solutions, but exhibits acausal behavior on short time scales. However, this acausality is on a time scale small compared to the Compton time of the particle. As such, it lies outside of the domain of validity of classical electrodynamics.

Several authors\[6, 7, 8\] have discussed the instability problem in semiclassical gravity theory. Some of the proposed resolutions of this problem involve reformulat-
ing the theory to eliminate unstable solutions (analogous to the integrodifferential equation in electrodynamics), or regarding the semiclassical theory as valid only for spacetimes which pass a stability criterion. These are sensible approaches to the issue. Basically, one wishes to have a theory which can approximately describe the backreaction of quantum fields on scales well above the Planck scale. It is important to keep in mind that the semiclassical theory is an approximation which must ultimately fail in situations where the quantum nature of gravity itself plays a crucial role.

4 The Hawking Effect

One of the great successes of quantum field theory in curved spacetime and of semiclassical gravity is the elegant connection between black hole physics and thermodynamics forged by the Hawking effect. Classical black hole physics suffers from Bekenstein’s paradox[9]: one could throw hot objects into a black hole and apparently decrease the net entropy of the universe. This paradox can be resolved by assigning an entropy to a black hole which is proportional to the area of the event horizon. Hawking[10] carried this reasoning one step further by showing that black holes are hot objects in a literal sense and emit thermal radiation. The outgoing radiation consists of particles quantum mechanically created in a region outside of the event horizon, and carries away energy and entropy from the black hole. The resulting decrease in mass of the hole arises from a steady flux of negative energy into the horizon, and is consistently described by the semiclassical Einstein equation, Eq. [1], so long as the black hole’s mass is well above the Planck mass.

Although the Hawking effect provides an elegant unification of thermodynamics, gravity and quantum field theory, there are still unanswered questions. One is the “information puzzle”, the issue of whether information which goes into the black hole during its semiclassical phase can be recovered. Hawking[11] originally proposed that this information is irrevocably lost and that black hole evaporation is not described by a unitary evolution. This view has been disputed by several other physicists[12], who have argued that a complete quantum mechanical description of the evaporation process should be unitary. More recently, Hawking[13] has agreed with this view. However, even if the evolution is unitary, the details by which information is recovered are still unclear. One possibility is that the outgoing radiation is not exactly thermal, but contains some subtle correlations which carry the information about the details of the matter which fell into the black hole. If this suggestion is correct, it is not clear just how these correlations arise.

A second mystery raised by the Hawking effect is the “trianplanckian problem”. This problem arises because the modes which will eventually become populated with the outgoing thermal radiation start out with extremely high frequencies before the black hole formed. These modes enter the collapsing body and then exit just before the horizon forms, undergoing an enormous redshift. However, as they enter and pass through the body, their frequencies are vastly higher than the Planck scale. If
one postulates that full quantum gravity will impose an effective cutoff at the Planck scale, then there seems to be a conflict; a cutoff at any reasonable frequency would eliminate the modes needed for the Hawking radiation. For a black hole of mass $M$ to evaporate, one needs to start with modes whose frequency is of order

$$\omega \approx \frac{e^{M^2}}{M}. \quad (7)$$

For a stellar mass size black hole, this corresponds to $\omega \approx 10^{10^{75}} \text{g}$, which is vastly larger than the mass of the observable universe. One possible resolution of this problem is to postulate a modified dispersion relation which allows for “mode creation”, whereby the modes would appear shortly before they are needed to carry the thermal radiation. However, this solution will require new microphysics, including breaking of local Lorentz invariance.

## 5 Quantum Effects in the Early Universe

It is likely that there is a period in the history of the universe during which quantum effects are important, but one is sufficiently far from the Planck regime that a full theory of quantum gravity is not needed. In this case, the semiclassical theory is applicable. Among the quantum effects expected in an expanding universe is quantum particle creation. Inflationary models with inflation occurring at scales below the Planck scale are plausible models for the early universe in which semiclassical gravity should hold. Indeed, such models predict that the density perturbations which later grew into galaxies had their origins as quantum fluctuations during the inflationary epoch. This leads to the remarkable prediction that the large scale structure of the present day universe had its origin in quantum fluctuations of a scalar inflaton field. More precisely, quantum fluctuations of a nearly massless scalar field in de Sitter spacetime translate into an approximately scale invariant spectrum of density perturbations. This picture seems to be consistent with recent observations of the cosmic microwave background radiation.

## 6 The Dark Energy Problem

There is now strong evidence that the expansion of the present day universe is accelerating. This evidence came first from observations of type Ia supernovae. This acceleration could be due to a nonzero value for the cosmological constant, but other possibilities are consistent with the observational data. These possibilities go under the general term “dark energy”, and require a negative pressure whose magnitude is approximately equal to the energy density. It has sometimes been suggested that the dark energy could be viewed as due to quantum zero point energy. However, there are some serious difficulties with this viewpoint. If we adopt the convention renormalization approach discussed in Sect. then the renormalized value of the
cosmological constant $\Lambda$ is completely arbitrary. At this level, quantum field theory in curved spacetime can no more calculate $\Lambda$ than quantum electrodynamics can calculate the mass of the electron. We could take a more radical approach and seek some physical principle which effectively fixes the value of the regulator parameter to a definite, nonzero value. However, for the first term on the right hand side of Eq. (2) to be the dark energy, we would have to take $\sigma \approx (0.01\text{cm})^2$. It is very hard to imagine what new physics would introduce a cutoff on a scale of the order of 0.01cm.

There is still a possibility that the dark energy could be due to some more complicated mechanism which involves quantum effects. One appealing idea is that there might be a mechanism for the cosmological constant to decay from a large value in the early universe to a smaller, but nonzero value today. Numerous authors [26, 27, 28, 29, 30, 31, 32, 33, 34] have discussed models for the decay of the cosmological constant, or models which otherwise attribute a quantum origin to the dark energy [35, 36]. However, at the present time there is no widely accepted model which successfully links dark energy with quantum processes.

7 Negative Energy Density for Quantum Fields

One crucial feature of quantum matter fields as a source of gravity is that they do not always satisfy conditions obeyed by known forms of classical matter, such as positivity of the local energy density. Negative energy densities and fluxes arise even in flat spacetime. A simple example is the Casimir effect [37], where the vacuum state of the quantized electromagnetic field between a pair of perfectly conducting plates separated by a distance $L$ is a state of constant negative energy density

$$\rho = \langle T_{tt} \rangle = -\frac{\pi^2}{720L^4}. \tag{8}$$

Even if the plates are not perfectly conducting, it is still possible to arrange for the energy density at the center to be negative [38].

Negative energy density can also arise as the result of quantum coherence effects. In fact, it may be shown under rather general assumptions that quantum field theories admit states for which the energy density will be negative somewhere [39, 40]. In simple cases, such as a free scalar field in Minkowski spacetime, one can find states in which the energy density can become arbitrarily negative at a given point.

We can illustrate the basic phenomenon of negative energy arising from quantum coherence with a very simple example. Let the quantum state of the system be a superposition of the vacuum and a two particle state:

$$|\Psi\rangle = \frac{1}{\sqrt{1 + \epsilon^2}}(|0\rangle + \epsilon|2\rangle). \tag{9}$$

Here we take the relative amplitude $\epsilon$ to be a real number. Let the energy density operator be normal-ordered:

$$\rho = :T_{tt}: = T_{tt}. \tag{10}$$
so that \( \langle 0 | \rho | 0 \rangle = 0 \). Then the expectation value of the energy density in the above state is
\[
\langle \rho \rangle = \frac{1}{1 + \epsilon^2} \left[ 2 \epsilon \text{Re}(\langle 0 | \rho | 2 \rangle) + \epsilon^2 \langle 2 | \rho | 2 \rangle \right].
\] (11)

We may always choose \( \epsilon \) to be sufficiently small that the first term on the right hand side dominates the second term. However, the former term may be either positive or negative. At any given point, we could choose the sign of \( \epsilon \) so as to make \( \langle \rho \rangle < 0 \) at that point. This example is a limiting case of a more general class of quantum states which may exhibit negative energy densities, the squeezed states.

Note that the integral of \( \rho \) over all space is the Hamiltonian, which does have non-negative expectation values:
\[
\langle H \rangle = \int d^3x \langle \rho \rangle \geq 0.
\] (12)

In the above vacuum + two particle example, the matrix element \( \langle 0 | \rho | 2 \rangle \), which gives rise to the negative energy density, has an integral over all space which vanishes, so only \( \langle 2 | \rho | 2 \rangle \) contributes to the Hamiltonian.

8 Some Possible Consequences of Quantum Violation of Classical Energy Conditions

The existence of negative energy density can give rise to a number of effects in which the predictions of semiclassical gravity differ significantly from those of classical gravity theory.

8.1 Singularity Avoidance

In the 1960’s, several elegant theorems were proven by Penrose, Hawking, and others\[41\] which demonstrate the inevitability of singularity formation in gravitational collapse described by classical relativity. These singularity theorems imply that the curvature singularities found in the exact solutions for black holes or for cosmological models are generic and signal a breakdown of classical relativity theory. However, this does not tell us whether a full quantum theory of gravity is needed to give a physically consistent, that is, singularity free, picture of the end state of gravitational collapse or the origin of the universe.

A crucial feature of the proofs of the singularity theorems is the assumption of a classical energy condition. There are several such conditions that can be used, but a typical example is the weak energy condition. This states that the stress tensor \( T_{\mu\nu} \) must satisfy \( T_{\mu\nu} u^\mu u^\nu \geq 0 \) for all timelike vectors \( u^\mu \). Thus all observers must see the local energy density being non-negative. It is not hard to understand why there could not be a singularity theorem without an energy condition: the Einstein tensor \( G_{\mu\nu} \) is a function of the metric and its first two derivatives. Thus, every twice-differentiable
metric is a solution of the Einstein equation, $G_{\mu\nu} = 8\pi T_{\mu\nu}$ for some choice of $T_{\mu\nu}$. We can also understand the role which the weak energy and related conditions play. Positive energy density will generate an attractive gravitational field and cause light rays to focus. Once gravitational collapse has proceeded beyond a certain point, the formation of a singularity is inevitable as long as gravity remains attractive. The way to circumvent this conclusion is with exotic matter, such as negative energy density, which can cause repulsive gravitational effects.

Given that quantum fields can violate the classical energy conditions, there is a possibility that the semiclassical theory can produce realistic, nonsingular black hole and cosmological solutions. This is a topic which has been investigated by several authors\[43, 44, 45\]. However, it is difficult to avoid having the curvature reach Planck dimensions before saturating. In this case, the applicability of the semiclassical theory is questionable. It is possible to avoid this difficulty with a carefully selected quantum states\[43\], a nonminimal scalar field which violates the energy conditions at the classical level\[44\], or by going to models where gravity itself is quantized\[45\].

### 8.2 Creation of Naked Singularities

There is an opposite effect which might be caused by negative energy: the creation of a naked singularity. The singularities formed in gravitational collapse in classical relativity tend to be hidden from the outside universe by event horizons. Penrose\[42\] has made a “cosmic censorship conjecture” to the effect that this must always be the case. This implies that the breakdown of predictability caused by the singularity is limited to the region inside the horizon. It is not yet known whether this conjecture is true, even in the context of classical relativity with classical matter, obeying classical energy conditions. However, unrestricted negative energy would allow a counterexample to this conjecture. The Reissner-Nordström solution of Einstein’s equation describes a black hole of mass $M$ and electric charge $Q$. However, these black hole solutions have an upper limit on the electric charge in relation to the mass of $Q \leq M$ (in our units). There are solutions for which $Q > M$, but these describe a naked singularity. Simple classical mechanisms for trying to convert a charged black hole into a naked singularity fail. If we try to increase the charge of a black hole, the work needed to overcome the electrostatic repulsion causes the black hole’s mass to increase at least as much as the charge and keep $Q \leq M$. However, unrestricted negative energy would offer a way to violate cosmic censorship and create a naked singularity. We could shine a beam of negative energy involving an uncharged quantum field into the black hole, decrease $M$ without changing $Q$, and thereby cause a naked singularity to appear\[46, 47\].
8.3 Violation of the Second Law of Thermodynamics?

If it is possible to create unrestricted beams of negative energy, then the second law would seem to be in jeopardy. One could shine the beam of negative energy on a hot object and decrease its entropy without a compensating entropy increase elsewhere. The purest form of this experiment would involve shining the negative energy on a black hole. If the negative energy is carried by photons with wavelengths short compared to the size of the black hole, it will be completely absorbed. That is, there will be no backscattered radiation which might carry away entropy. Then the black hole’s mass, and hence its entropy, will decrease in violation of the second law.[48].

8.4 Traversable Wormholes and Warp Drive Spacetimes

As noted above, virtually any conceivable spacetime is a solution of Einstein’s equation with some choice for the source. If the source violates the classical energy conditions, some bizarre possibilities arise. An example are the traversable wormholes of Morris, Thorne and Yurtsever.[49]. These would function as tunnels which could connect otherwise widely separated regions of the universe by a short pathway. An essential requirement for a wormhole is exotic matter which violates the weak energy condition. The reason for this is that light rays must first enter one mouth of the wormhole, begin to converge and later diverge so as to exit the other mouth of the wormhole without coming to a focal point. In other words, the spacetime inside the wormhole must act like a diverging lens, which can only be achieved by exotic matter.

The existence of traversable wormholes would be strange enough, but they have an even more disturbing feature: they can be manipulated to create a time machine.[50]. If the mouths of a wormhole move relative to one another, it is possible for the resulting spacetime to possess “closed timelike paths”. On such a path, an observer could return to the same point in space and in time, and by speeding up slightly, arrive at the starting point before leaving. Needless to say, this would turn physics as we currently understand it on its head and open the door to disturbing causal paradoxes.

An equally bizarre possibility was raised by Alcubierre.[51], who constructed a spacetime that functions as science fiction style “warp drive”. It consist of a bubble of flat spacetime surrounded by expanding and contracting regions imbedded in an asymptotically flat spacetime. The effect of the expansion and contraction is to cause the bubble to move faster than the speed of light, as measured by a distant observer, even though locally everything moves inside the lightcone. Again, negative energy is essential for the existence of this spacetime.

9 Quantum Inequalities

It is clear that unrestrained violation of the classical energy conditions would create major problems for physics. However, it is also clear that quantum field theory
does allow for some violations of these conditions. This leads us to ask if there are constraints on negative energy density in quantum field theory. The answer is yes; there are inequalities which restrict the magnitude and duration of the negative energy seen by any observer, known as quantum inequalities [11, 52, 53, 54, 55, 56, 57, 58, 59]. In four spacetime dimensions, a typical inequality for a massless field takes the form [53, 54, 58]

\[ \int \rho(t) g(t) \, dt \geq -\frac{c}{t_0^4}. \]  

(13)

Here \( \rho(t) \) is the energy density measured in the frame of an inertial observer, \( g(t) \) is a sampling function with characteristic width \( t_0 \), and \( c \) is a numerical constant which is typically less than one. The value of \( c \) depends upon the form of \( g(t) \) (e.g. Gaussian versus Lorentzian). The sampling function and its width can be chosen arbitrarily, subject to some differentiability conditions on \( g(t) \). The essential message of an inequality such Eq. (13) is that there is an inverse relation between the duration and magnitude of negative energy density. In particular, if an observer sees a pulse of negative energy density with a magnitude of order \( \rho_m \) lasting a time of order \( \tau \), then we must have \( \rho_m < 1/\tau^4 \).

Furthermore, that negative energy cannot arise in isolation, but must be accompanied by compensating positive energy. This fact, plus the quantum inequalities, place very severe restrictions on the physical effects which negative energy can create. Here is a brief summary of the implications of quantum inequalities for some of the possible effects listed above.

### 9.1 Violations of the Second Law and of Cosmic Censorship

If we were to shine a pulse of negative energy onto a black hole so as to decrease its entropy and violate the second law, the entropy decrease would have to last long enough to be macroscopically observable. At a minimum, it should be sustained for a time longer than the size of the event horizon. If the negative energy is constrained by an inequality of the form of Eq. (13), then it can be shown [58] that the resulting entropy decrease is of the order of Boltzmann’s constant or less. This represents an entropy change associated with about one bit of information, hardly a macroscopic violation of the second law.

The attempt to create a naked singularity by shining a pulse of negative energy on an extreme, \( Q = M \), charged black hole is similarly constrained. Again, any naked singularity which is formed should last for a time long compared to \( M \). However, it can be argued [16, 17] that the resulting change in the spacetime geometry may be smaller than the natural quantum fluctuations on this time scale. Thus it seems that negative energy which obeys the quantum inequality restrictions cannot produce a clear, unambiguous violation of cosmic censorship.
9.2 Constraints on Traversable Wormholes and Warp Drive

The simplest quantum inequalities, such as Eq. (13) have been proven only in flat spacetime, and hence do not immediately apply to curved spacetime. There is, however, a limiting case in which they can also be used in curved spacetime. This is when the sampling time $t_0$, as measured in a local inertial frame, is small compared to the local radii of curvature of the spacetime in the same frame. This means that the spacetime is effectively flat on the time scale of the sampling, and the flat space inequality should also apply to curved spacetime. In the special cases where explicit curved spacetime inequalities have been derived, they are consistent with this limit. That is, they reduce to the corresponding flat space inequality in the short sampling time limit.

Even in the small $t_0$ limit, it is possible to put very strong restrictions on the geometry of traversable wormholes and warp drives[60]. The constraints on wormhole geometries vary from one model to another. In some cases, the throat of the wormhole is limited to be close to Planck dimensions, presumably outside of the domain of validity of semiclassical gravity. In other cases, the restrictions are slightly less severe, but still require some length scales to be much smaller than others, such as a band of negative energy no more than $10^{-13}$ cm thick to support a wormhole with a 1m throat. This does not quite rule out all possible wormholes based upon semiclassical gravity, but makes it hard to imagine actually constructing one. Similar, very strong restrictions are placed on warp drive spacetimes[61 62], such as the Alcubierre model.

10 Beyond Semiclassical Gravity: Fluctuations

The first extension of semiclassical gravity arises when we consider fluctuations of the gravitational field. These can be due to two causes: the quantum nature of gravity itself (active fluctuations) and quantum fluctuations of the stress tensor (passive fluctuations). The extension of the semiclassical theory to include fluctuations is sometimes called stochastic gravity[63 64 65 66]. One of the criteria for the validity of the semiclassical theory based upon Eq. (1) must be that fluctuations are small[67]. This theory can break down even well above the Planck scale if the stress tensor fluctuations are sufficiently large. A simple example is a quantum state which is a superposition of two states, each of which describe a distinct classical matter distribution (e.g. a 1000kg mass on one or the other side of a room). Equation (11) predicts a gravitational field which is an average of the fields due to the two distributions separately, (the effect of two 500kg masses on opposite sides of the room). However, an actual measurement of the gravitational field should yield that of a single 1000kg mass, but in different locations in different trials.

A treatment of small fluctuations of the gravitational field offers a window into possible extensions beyond strict semiclassical gravity. First we should be clear about the operational meaning of fluctuations of gravity. A classical gravitational field or spacetime geometry can be viewed as encoding all possible motions of test particles...
in that geometry. Consequently, fluctuations of spacetime imply Brownian motion of the test particles, which can be characterized by mean squared deviations from classical geodesics.

Test particles can include photons, and one of the striking consequences of gravity fluctuations can be fluctuations of the lightcone. Recall that the lightcone plays a crucial role in classical relativity theory. Events which are timelike or null separated from one another can be causally related, but those at spacelike separations cannot. Similarly, an event horizon is a null surface which separates causally disjoint regions of spacetime. This rigid separation cannot be maintained when the spacetime fluctuates. A simple way to have spacetime fluctuations is with a bath of gravitons in a nonclassical state, such as a squeezed vacuum state\[69, 70\]. Here the mean spacetime geometry is almost flat, apart from effects of the averaged stress tensor of the gravitons, but exhibits large fluctuations around this mean. These will include lightcone fluctuations, which will manifest themselves in varying arrival times of pulses from a source. Consider a source and a detector, which are both at rest relative to the average background and separated by a proper distance $D$, as measured in the average metric. Then the mean flight time of pulses will be $D$, but some individual pulses will take a longer time, and others a shorter time. A pulse which arrives in a time less than $D$ travels outside of the lightcone of the mean spacetime, as illustrated in Fig. 1.

As noted earlier, faster than light travel can often be used to travel backwards in time. However, there is a crucial step needed to link the two: Lorentz invariance. One must exploit the fact that one can interchange the time order of spacelike separated events by changing Lorentz frames. In the present example, Lorentz symmetry is broken by the existence of a preferred rest frame, that of the graviton bath. Thus one cannot conclude that there is any problem with causality created by these lightcone fluctuations.

Because an event horizon is a special case of a lightcone, there should be horizon fluctuations in any model with spacetime geometry fluctuations. In the case of a black hole horizon, this raises the possibility of information leaking out of the black hole, or of the horizon fluctuations drastically altering the semiclassical derivation of black hole evaporation. One estimate\[71\] of the magnitude of the effects of quantum horizon fluctuations concluded that they are too small to alter the Hawking radiation for black holes much larger than the Planck mass. However, other authors\[72, 73\] have argued for a much larger effect. It has also been suggested\[74\] that horizon fluctuations might provide the new physics needed to gracefully solve the tranplanckian problem. This is clearly an area where more work is needed.

The passive fluctuations of gravity driven by quantum stress tensor fluctuations are just one manifestation of stress tensor fluctuations. They are also responsible for fluctuation forces on macroscopic bodies, such as Casimir force fluctuations\[75, 76, 77, 78\] and radiation pressure fluctuations\[79\]. This provides the possibility of an electromagnetic analog model for passive quantum gravity. The same techniques are needed to define integrals of the stress tensor correlation function in both con-
Figure 1: The effects of lightcone fluctuations are illustrated. The dashed lines represent the average lightcone. However, pulses which are emitted at the origin can arrive at the worldline of a detector (vertical dotted line) a mean distance $D$ away at different times. A pulse detected at point $a$ travels slower than the mean speed of light, but one detected at point $b$ has traveled faster than the mean speed of light, and hence outside of the mean lightcone.
texts. In both cases, one needs to use a regularization method, such as dimensional regularization or an integration by parts. Some of the physical effect which have recently been studied using the latter technique are angular blurring and luminosity fluctuations of the image of a distant source seen through a fluctuating spacetime.

11 Summary

The semiclassical theory, with quantum matter fields and a classical gravitational field, provides a crucial link between the purely classical theory and a more complete quantum theory of gravity. Any viable candidate for a full quantum theory of gravity must reproduce the predictions of semiclassical gravity in an appropriate limit. In addition, semiclassical gravity contains a rich array of physical effects which are not found at the classical level, including black hole evaporation, cosmological particle creation, and negative energy density effects. The simplest extensions of the semiclassical theory to include spacetime fluctuations provide another array of effects, including lightcone and horizon fluctuations, which will have to be better understood in the context of a more complete theory.

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References

[1] N.D. Birrell and P.C.W. Davies, Quantum Fields in Curved Space, (Cambridge University Press, 1982).

[2] S.A. Fulling, Aspects of Quantum Field Theory in Curved Space-Time, (Cambridge University Press, 1989).

[3] R.M. Wald, Commun. Math. Phys. 54, 1 (1977).

[4] The initial value problem in general relativity is actually more complicated than suggested by this sentence. This arises from the need to satisfy constraints and to have a spacetime which is globally hyperbolic. For a more detailed discussion, see, for example, R.M. Wald, General Relativity, (University of Chicago Press, Chicago, 1984), Chap. 10.

[5] See, for example, J.D. Jackson, Classical Electrodynamics, 2nd ed. (Wiley, New York, 1975), Chap. 17.
[6] G.T. Horowitz and R.M. Wald, Phys. Rev. D 17, 414 (1978).
[7] L. Parker and J.Z. Simon, Phys. Rev. D 47, 1339 (1993), gr-qc/9611064
[8] P.R. Anderson, C. Molina-Paris, and E. Mottola, Phys. Rev. D 67, 024026 (2003), gr-qc/0209075
[9] J.D. Bekenstein, Phys. Rev. D 7, 2333 (1973)
[10] S.W. Hawking, Commun. Math. Phys. 43, 199 (1975).
[11] S.W. Hawking, Phys. Rev. D 14, 2460 (1976).
[12] G. ’t Hooft, Nucl. Phys. B 335, 138 (1990).
[13] S.W. Hawking, Talk presented at GR17, Dublin, July 2004.
[14] T. Jacobson, Phys. Rev. D 44, 1731 (1991).
[15] W. G. Unruh, Phys. Rev. D 51, 2827 (1995), gr-qc/9409008.
[16] S. Corley and T. Jacobson, Phys. Rev. D 54, 1568 (1996), hep-th/9601073
[17] L. Parker, Phys. Rev. 183, 1057 (1969).
[18] V. Mukhanov and G. Chibisov, JETP Lett. 33, 532 (1981).
[19] A. Guth and S-Y Pi, Phys. Rev. Lett. 49, 1110 (1982).
[20] S.W. Hawking, Phys. Lett. B 115, 295 (1982).
[21] A.A. Starobinsky, Phys. Lett. B 117, 175 (1982).
[22] J.M. Bardeen, P.J. Steinhardt and M. S. Turner, Phys. Rev. D 28, 679 (1983).
[23] For a recent review, see M. Giovannini, astro-ph/0412601.
[24] B.P. Schmidt et al, Astrophys.J. 507, 46 (1998), astro-ph/9805200.
[25] S. Perlmutter et al, Astrophys.J. 517, 565 (1999), astro-ph/9812133.
[26] A.D. Dolgov, in The Very Early Universe, G.W. Gibbons, S.W. Hawking, and S.T.C. Siklos, eds. (Cambridge University Press, 1983).
[27] L.H. Ford, Phys. Rev. D 31, 710 (1985).
[28] L.H. Ford, Phys. Rev. D 35, 2339 (1987).
[29] S. Weinberg, Rev. Mod. Phys. 61, 1 (1989).
[30] A.D. Dolgov, M.B. Einhorn, and V.I. Zakharov, Phys. Rev. D 52, 717 (1995), gr-qc/9403056.

[31] N.C. Tsamis and R.P. Woodard, Nucl. Phys. B 474, 235 (1996), hep-ph/9602315; Ann. Phys. 253, 1 (1997), hep-ph/9602316; Phys. Rev. D 54, 2621 (1996), hep-ph/9602317.

[32] L.R. Abramo and R.P. Woodard, Phys. Rev. D 65, 063516 (2001), astro-ph/0109273.

[33] R. Schützhold, gr-qc/0204018.

[34] L.H. Ford, in On the Nature of the Dark Energy, Proceedings of the 18th IAP Astrophysics Colloquium, P. Braz, J. Martin, and J-P Uzan, eds. (Frontier Group, Paris, 2002), gr-qc/0210096.

[35] V. Sahni and S. Habib, Phys. Rev. Lett. 81, 1766 (1998), hep-ph/9808204.

[36] L. Parker and A. Raval, Phys. Rev. D 60, 123502 (1999), gr-qc/9908013.

[37] H.B.G. Casimir, Proc. K. Ned. Akad. Wet. B 51, 793 (1948).

[38] V. Sopova and L.H. Ford, Phys. Rev. D, 66, 045026 (2002), quant-ph/0204125.

[39] H. Epstein, V. Glaser, and A. Jaffe, Nuovo Cimento 36, 1016 (1965).

[40] C.-I Kuo, Nuovo Cimento 112B, 629 (1997), gr-qc/9611064.

[41] See, for example, S.W. Hawking and G.F. R. Ellis, The large scale structure of space-time, (Cambridge Univ. Press, 1973).

[42] R. Penrose, in General Relativity: An Einstein Centenary Survey, S.W. Hawking and W. Israel, eds. (Cambridge Univ. Press, 1979).

[43] L. Parker and S.A. Fulling, Phys. Rev. D 7, 2357 (1973).

[44] A. Saa, E. Gunzig, L. Brenig, V. Faraoni, T.M. Rocha Filho, and A. Figueiredo, Int. J. Theor. Phys. 40, 2295 (2001), gr-qc/0012105.

[45] P. Hajicek and C. Kiefer, Int. J. Mod. Phys. D 10, 775 (2001), gr-qc/0107102.

[46] L.H. Ford and T.A. Roman, Phys. Rev. D 41, 3662 (1990).

[47] L.H. Ford and T.A. Roman, Phys. Rev. D 46, 1328 (1992).

[48] L.H. Ford, Proc. R. Soc. London A 364, 227 (1978).

[49] M. Morris, K. Thorne, and U. Yurtsever, Phys. Rev. Lett. 61, 1446 (1988).
[50] See, for example, M. Visser, *Lorentzian Wormholes; From Einstein to Hawking*, (AIP, Woodbury, N. Y., 1995).

[51] M. Alcubierre, Class. Quantum Grav. 11, L73 (1994).

[52] L.H. Ford, Phys. Rev. D 43, 3972 (1991).

[53] L.H. Ford and T.A. Roman, Phys. Rev. D 51, 4277 (1995), gr-qc/9410043

[54] L.H. Ford and T.A. Roman, Phys. Rev. D 55, 2082 (1997), gr-qc/9607003

[55] E.E. Flanagan, Phys. Rev. D, 56, 4922 (1997), gr-qc/9706006.

[56] M.J. Pfenning and L.H. Ford, Phys. Rev. D 55, 4813 (1997), gr-qc/9608005

[57] M.J. Pfenning and L.H. Ford, Phys. Rev. D 57, 3489 (1998), gr-qc/9710055

[58] C.J. Fewster and S.P. Eveson, Phys. Rev. D 58, 084010 (1998), gr-qc/9805024

[59] C.J. Fewster, Class. Quantum Grav. 17, 1897 (2000), gr-qc/9910060.

[60] L.H. Ford and T.A. Roman, Phys. Rev. D 53, 5496 (1996), gr-qc/9510071

[61] M.J. Pfenning and L.H. Ford, Class. Quantum Grav., 14, 1743 (1997), gr-qc/9702026.

[62] A.E Everett and T.A. Roman, Phys. Rev. D 56, 2100 (1997), gr-qc/9702049.

[63] J.M. Moffat, Phys. Rev. D 56, 6264 (1997), gr-qc/9610067

[64] R. Martin and E. Verdaguer, Phys. Rev. D 60, 084008 (1999), gr-qc/9904021

[65] B.L. Hu and E. Verdaguer, Class. Quant. Grav. 20, R1 (2003), gr-qc/0211090

[66] B.L. Hu and E. Verdaguer, Living Rev. Rel. 7, 3 (2004), gr-qc/0307032.

[67] C.I. Kuo and L.H. Ford, Phys. Rev. D 47, 4510 (1993), gr-qc/9304008

[68] B.L. Hu and N.G. Phillips, Int. J. Theor. Phys. 39, 1817 (2000), gr-qc/0004006

[69] L.H. Ford, Phys. Rev. D 51, 1692 (1995), gr-qc/9410047

[70] L.H. Ford and N.F. Svaiter, Phys. Rev. D 54, 2640 (1996), gr-qc/9604052

[71] L.H. Ford and N.F. Svaiter, Phys. Rev. D 56, 2226 (1997), gr-qc/9704050

[72] R.D. Sorkin, *Two Topics concerning Black Holes: Extremality of the Energy, Fractality of the Horizon*, gr-qc/9508002; *How Wrinkled is the Surface of a Black Hole?*, gr-qc/9701056
[73] A. Casher, F. Englert, N. Itzhaki, and R. Parentani, Nucl. Phys. B 484, 419 (1997), hep-th/9606106.

[74] C. Barrabes, V. Frolov, and R. Parentani, Phys. Rev. D 62, 04020 (2000), gr-qc/0001102.

[75] G. Barton, J. Phys. A 24, 991 (1991); 24, 5563 (1991).

[76] C. Eberlein, J. Phys. A 25, 3015 (1992); A 25, 3039 (1992).

[77] M.T. Jaekel and S. Reynaud, Quantum Opt. 4, 39 (1992); J. Phys. I France 2, 149 (1992); 3, 1 (1993); 3, 339 (1993).

[78] C.-H. Wu, C.-I Kuo and L.H. Ford, Phys. Rev. A 65, 062102 (2002), quant-ph/0112056.

[79] C.-H. Wu and L. H. Ford, Phys. Rev. D 64, 045010 (2001), quant-ph/0012144.

[80] L.H. Ford and R.P. Woodard, Class. Quant. Grav. 22, 1637 (2005), gr-qc/0411003.

[81] J. Borgman and L. H. Ford, Phys. Rev. D 70, 064032 (2004), gr-qc/0307043.