Combined analysis of the $K^+K^-$ interaction using near threshold $pp \rightarrow ppK^+K^-$ data

M. Silarski$^1$ and P. Moskal$^{1,2}$

$^1$Institute of Physics, Jagiellonian University, PL-30-059 Cracow, Poland
$^2$Nuclear Physics Institute, Research Center Jülich, D-52425 Jülich, Germany

(Dated: May 11, 2014)

The $K^+K^-$ final state interaction was investigated based on both the $K^+K^-$ invariant mass distributions measured at excess energies of $Q = 10$ and $28$ MeV and the near threshold excitation function for the $pp \rightarrow ppK^+K^-$ reaction. The $K^+K^-$ final state enhancement factor was parametrized using the effective range expansion. The effective range of the $K^+K^-$ interaction was estimated to be: $\text{Re}(b_{K^+K^-}) = -0.1 \pm 0.4_{\text{stat}} \pm 0.3_{\text{sys}}$ fm and $\text{Im}(b_{K^+K^-}) = 1.2 \pm 0.1_{\text{stat}} \pm 0.2_{\text{sys}}$ fm, and the determined real and imaginary parts of the $K^+K^-$ scattering length amount to: $|\text{Re}(a_{K^+K^-})| = 8.0 \pm 0.2_{\text{stat}}$ fm and $|\text{Im}(a_{K^+K^-})| = 0.3 \pm 0.2_{\text{stat}}$ fm.

PACS numbers: 13.75.Lb, 14.40.Aq

Keywords: final state interaction, near threshold kaon pair production

I. INTRODUCTION

The strength of the $K^+K^-$ interaction is a crucial quantity regarding the formation of a hypothetical kaon–antikaon bound state. Existence of such a state could explain the nature of the $a_0(980)$ and $f_0(980)$ scalar mesons$^1, 2$, whose masses are very close to the sum of the $K^+$ and $K^-$ masses$^3$. Among many theoretical investigations$^3, 4$ the $K^+K^-$ interaction was studied also experimentally in the $pp \rightarrow ppK^+K^-$ reaction with COSY–11 and ANKE detectors operating at the COSY synchrotron in Jülich, Germany$^{12–16, 18, 19}$. The experimental data collected systematically below$^{12–16}$ and above$^{12}$ the $\phi$ meson threshold reveal a significant enhancement in the shape of the excitation function near the kinematical threshold, which may be due to the final state interaction (FSI) in the $ppK^+K^-$ system. The indication of the influence of the $pK^-$ final state interaction was found in both COSY–11 and ANKE data in the ratios of the differential cross sections as a function of the $pK$ and the $ppK$ invariant masses,$^1$

\[ R_{pK} = \frac{d\sigma/dM_{pK^-}}{d\sigma/dM_{pK^+}}, \]

\[ R_{ppK} = \frac{d\sigma/dM_{ppK^-}}{d\sigma/dM_{ppK^+}}, \]

where a significant enhancement in the region of both the low $pK^-$ invariant mass $M_{pK^-}$ and the low $ppK^-$ invariant mass $M_{ppK^-}$ is observed$^{13, 20}$. The phenomenological model based on the factorization of the final state interaction into interactions in the $pp$ and $pK^-$ subsystems, neglecting the $K^+K^-$ potential, does not describe the whole experimental excitation function for the $pp \rightarrow ppK^+K^-$ reaction, underestimating the data very close to the kinematical threshold$^{15, 21}$. This indicates that in the low–energy region the influence of the $K^+K^-$ final state interaction may be significant$^{15, 20, 21}$. Motivated by this observation the COSY–11 Collaboration has recently estimated the scattering length of the $K^+K^-$ interaction based on the $pp \rightarrow ppK^+K^-$ reaction measured at excess energies of $Q = 10$ and $28$ MeV$^{20}$. As a result of the analysis the $K^+K^-$ scattering length was determined based on the low–energy proton–proton ($M_{pp}$) and $K^+K^-$ ($M_{K^+K^-}$) invariant mass distributions (so–called Goldhaber plot) shown in Fig. 1$^{20}$. In this article we combine the Goldhaber plot distribution established by the COSY–11 group with the experimental excitation function$^{12–15, 19}$ near threshold and determine the $K^+K^-$ scattering length with better precision compared to the previous results. We have also extracted the effective range of the $K^+K^-$ interaction.

II. DESCRIPTION OF THE FINAL STATE INTERACTION IN THE $ppK^+K^-$ SYSTEM

As in the previous analysis$^{20}$ we use the factorization ansatz proposed by the ANKE group with an additional term describing the interaction in the $K^+K^-$ system. We assume that the overall enhancement factor originating from final state interaction can be factorized into enhancements in the proton–proton, the two $pK^-$ and the $K^+K^-$ subsystems$^4$:

\[ F_{FSI} = F_{pp}(k_1) \times F_{p-K^-}(k_2) \times F_{p-K^-}(k_3) \times F_{K^+K^-}(k_4) \]  

where $k_j$ stands for the relative momentum of particles in the corresponding subsystem$^{20}$. The proton–proton scattering amplitude was taken into account using the

---

$^*$ Electronic address: Michal.Silarski@uj.edu.pl
$^1$ Besides the standard interpretation as $q\bar{q}$ mesons$^3$, these resonances were also proposed to be $qqqq$ states$^4$, hybrid $q\bar{q}/$meson–meson systems$^3$ or even quark–less gluonic hadrons$^3$.

$^2$ In this model we neglect the $pK^+$ interaction since it is repulsive and weak$^{15}$. 
The mean of all the values from elaborations \cite{24,33} summarized in Ref. \cite{34}: $a_{pK^-} = (-0.65 + 0.78i) \text{ fm}$.

The $K^+K^-$–FSI was parametrized using the effective range expansion:

$$F_{K+K^-} = \frac{1}{a_{K+K^-} + b_{K+K^-} \frac{k^2}{2} - ik_1},$$

where $a_{K+K^-}$ and $b_{K+K^-}$ are the scattering length and the effective range of the $K^+K^-$ interaction, respectively. We have performed a fit to the experimental data treating $a_{K+K^-}$ and $b_{K+K^-}$ as free parameters. Moreover, we have repeated the analysis for every quoted $a_{pK^-}$ to check, how their different values change the result. This allowed us also to estimate the systematic error due to the $pK^-$ scattering length used in the estimation of $a_{K+K^-}$ and $b_{K+K^-}$.

It is worth mentioning, that there is a similar phenomenological model of the $K^+K^-$ final state interaction which takes into account the elastic and charge–exchange interaction allowing for the $K^0\bar{K}^0 \rightarrow K^+K^-$ transitions. This FSI should generate a significant cusp effect in the $K^+K^-$ invariant mass spectrum near the $K^0\bar{K}^0$ threshold (details can be found in \cite{33}). Another contribution to this effect may also be generated by the kaons rescattering to scalars, eg.: $KK \rightarrow f_0(980) \rightarrow KK$ and $KK \rightarrow a_0(980) \rightarrow KK$. However, the ANKE data can be described well without introducing the cusp effect \cite{33}, thus we neglect it in this analysis. We also cannot distinguish between the isospin $I = 0$ and $I = 1$ states of the $K^+K^-$ system. However, as pointed out in \cite{33}, the production with $I = 0$ is dominant in the $pp \rightarrow ppK^+K^-$ reaction independent of the exact values of the scattering lengths.

In the fit we do not take into account influence of the $f_0(980)$ and $a_0(980)$ production. There exist only very rough experimental estimates of upper limits for production of these resonances in the $N – N$ collisions. In fact, up to now in these reactions there has not been found any signal of these particles. The theoretical estimations result in negligible cross sections for the $pp \rightarrow f_0pp \rightarrow K^+K^-pp$ resonant contribution with respect to the non-resonant one \cite{37} (the upper limit of the cross section for this reaction is estimated to be about $1 \times 10^{-4} \text{ nb/MeV}$ at $Q = 5 \text{ MeV}$ and $4 \times 10^{-2} \text{ nb/MeV}$ at $50 \text{ MeV}$ \cite{37}). Also the branching ratios of $f_0(980)$ and $a_0(980)$ are very poorly known. However, according to the PDG $a_0(980)$ dominantly decays to $\eta\pi^0$ and the $\pi\pi$ channel is dominant for the $f_0(980)$ meson \cite{36}. Thus, the $f_0$ resonance contribution to the near threshold $pp \rightarrow ppK^+K^-$ reaction is expected to be negligible. Moreover, regarding the $a_0(980)$ resonance, following Ref. \cite{33} the $K^+K^-$ pairs are produced in proton–proton collisions mainly with isospin $I = 0$. Thus, $a_0(980)$ would have to decay to $K^+K^-$ through isospin violation, which is an additional suppressing factor. According to Ref. \cite{37} for energies up to $Q = 115 \text{ MeV}$ (DISTO measurement \cite{19}) the production of resonant $K^+K^-$ pairs should not produce any significant enhancement in the

![Experimental Goldhaber plots for the $pp \rightarrow ppK^+K^-$ reaction. The solid lines of the triangles show the kinematically allowed boundaries. Individual events are shown in (a) and (b) as black points. The superimposed squares represent the same distributions but binned into intervals of $\Delta M = 2.5 \text{ MeV/c}^2 (\Delta M = 7 \text{ MeV/c}^2)$ widths for an excess energy of $Q = 10 \text{ (28) MeV}$, respectively. The area of the square is proportional to the number of entries in a given interval. The figure was adapted from \cite{24}.](image_url)
III. DETERMINATION OF THE $K^+K^-$ SCATTERING LENGTH AND EFFECTIVE RANGE

In order to estimate the strength of the $K^+K^-$ interaction the experimental Goldhaber plots, determined at excess energies of $Q = 10$ and 28 MeV, together with the total cross sections measured near the threshold were compared to the results of the Monte Carlo simulations treating the $K^+K^-$ scattering length $a_{K^+K^-}$ and effective range $b_{K^+K^-}$ as unknown complex parameters. To determine $a_{K^+K^-}$ and $b_{K^+K^-}$ we have constructed the following $\chi^2$ statistics:

$$\chi^2(a_{K^+K^-}, b_{K^+K^-}, \alpha) = \sum_{i=1}^{8} \frac{(\sigma_i^{\text{exp}} - \alpha \sigma_i^{\text{m}})^2}{(\Delta \sigma_i^{\text{exp}})^2} + 2 \sum_{j=1}^{10} \sum_{k=1}^{5}\beta_j N_j^k - N_e^k + N_e^k \ln\left(\frac{N_e^k}{\beta_j N_j^k}\right),$$

where the first term was defined following the Neyman’s $\chi^2$ statistics, and accounts for the excitation function near the threshold for the $pp \rightarrow ppK^+K^-$ reaction. $\sigma_i^{\text{exp}}$ denotes the $i$th experimental total cross section measured with uncertainty $\Delta \sigma_i^{\text{exp}}$ and $\sigma_i^{\text{m}}$ stands for the calculated total cross section normalized with a factor $\alpha$ which is treated as an additional parameter of the fit. $\sigma_i^{\text{m}}$ was calculated for each excess energy $Q$ as a phase space integral over five independent invariant masses $\cite{39}$:

$$\sigma^{\text{m}} = \int \frac{\pi^2 |M|^2}{8s\sqrt{-B}} dM_{pp}^2 dM_{K^+K^-}^2 dM_{pK^-}^2 dM_{ppK^+}^2.$$  

Here $s$ denotes the square of the total energy of the system determining the value of the excess energy, and $B$ is a
The statistical uncertainties in this case were taken in the effective range expansion $Q = \pm 28$ MeV $(j = 2)$ using COSY–11 data [24]. $N_{jk}$ denotes the number of events in the $k$th bin of the $j$th experimental Goldhaber plot, and $N_{jk}$ stands for the content of the same bin in the simulated distributions. $\beta_j$ is a normalization factor which is fixed by values of the fit parameters $\alpha$ and $a_{K+K^-}$. It is defined for the $j^{th}$ excess energy as the ratio of the total number of events expected from the calculated total cross section $\sigma_{ppK^+K^-}^m$ and the total luminosity $L_j$ [14], to the total number of simulated $pp \rightarrow ppK^+K^-$ events $N_j^{gen}$:

$$\beta_j = \frac{L_j \sigma_{ppK^+K^-}^m}{N_j^{gen}}.$$  

The $\chi^2$ distributions (after subtraction of the minimum value) for $F_{K+K^-}$ taken in the effective range expansion are presented as a function of the real and imaginary parts of $a_{K+K^-}$ and $b_{K+K^-}$ in Fig. 2. The best fit to the experimental data corresponds to

$$\text{Re}(b_{K+K^-}) = -0.1 \pm 0.4_{\text{stat}} \pm 0.3_{\text{sys}} \, \text{fm}$$

$$\text{Im}(b_{K+K^-}) = 1.2 \pm 0.1_{\text{stat}} \pm 0.2_{\text{sys}} \pm 0.2_{\text{sys}} \, \text{fm},$$

$$\text{Re}(a_{K+K^-}) = 8.0 \pm 0.2_{\text{stat}} \pm 0.4_{\text{sys}} \, \text{fm},$$

$$\text{Im}(a_{K+K^-}) = 0.0 \pm 0.0_{\text{stat}} \pm 0.0_{\text{sys}} \, \text{fm},$$

with a $\chi^2$ per degree of freedom of: $\chi^2/\text{ndof} = 1.30$. The statistical uncertainties in this case were determined at the 70% confidence level taking into account that we have varied five parameters $[\alpha, \text{Im}(a_{K+K^-}), \text{Re}(a_{K+K^-}), \text{Im}(b_{K+K^-}), \text{Re}(b_{K+K^-})]$. Here uncertainties correspond to the range of values for which the $\chi^2$ of the fit is equal to: $\chi^2 = \chi^2_{\text{min}} + 6.06$ [40]. Systematic errors due to the uncertainties in the assumed $pK^-$ scattering length were instead estimated as a maximal difference between the obtained result and the $K^+K^-$ scattering length determined using different $a_{pK^-}$ values quoted in Refs. [31] and [34]. One can see, that the fit is in principle sensitive to both the scattering length and effective range, however, with the available low statistics data the sensitivity to the scattering length is very weak.

Results of the analysis with inclusion of the interaction in the $K^+K^-$ system described in this article are shown as the solid curve in Fig. 3. One can see that the experimental data are described quite well over the whole energy range.

$^3$ Due to the fact that in the case of scattering length the systematic uncertainties are much smaller than the statistical ones we neglect them in the final result.

**FIG. 3.** (Color online) Excitation function for the $pp \rightarrow ppK^+K^-$ reaction. Triangle and circles represent the DISTO and ANKE measurements, respectively [13, 17, 19]. The squares are results of the COSY–11 [12, 13, 20] measurements. The dashed curve represents the energy dependence obtained assuming that the phase space is homogeneously and isotropically populated, and there is no interaction between particles in the final state. Calculations taking into account the $pp$ and $pK^-$ FSIs are presented as the dashed–dotted curve. The dashed and dashed–dotted curves are normalized to the DISTO data point at $Q = 114$ MeV. Solid curve corresponds to the result obtained taking into account $pp$, $pK$, and $K^+K^-$ interactions parametrized with the effective range approximation. These calculations were obtained using the scattering length $a_{K+K^-}$ and effective range $b_{K+K^-}$ as obtained in this work. The latest data point measured by the ANKE group was published recently [17], and thus it was not taken into account in the fit.

**IV. CONCLUSIONS**

We have performed a combined analysis of both total and differential cross section distributions for the $pp \rightarrow ppK^+K^-$ reaction in view of the $K^+K^-$ final state interaction. In the analysis we have used a factorization proposed by the ANKE group with an additional term describing interaction in the $K^+K^-$ system, without distinguishing between the isospin $I = 0$ and $I = 1$ states. We have also neglected a possible charge exchange interaction leading to a cusp effect in the $K^+K^-$ invariant mass spectrum, since taking it into account would require much more precise data [35]. The $K^+K^-$ enhancement factor was parametrized using the effective range expansion. Fit to experimental data is very weakly sensitive to $a_{K+K^-}$, but allows us to estimate for the first time the effective range of the $K^+K^-$ FSI.
All studies of the \( pp \rightarrow ppK^+K^- \) reaction near threshold \([12–16]\) reveal that in the \( ppK^+K^- \) system the interaction between protons and the \( K^- \) meson is dominant, and \( \sigma_{K^+K^-} \) is relatively small. It seems that this reaction is driven by the \( \Lambda(1405) \) production \( pp \rightarrow K^+\Lambda(1405) \rightarrow ppK^+K^- \) rather than by the scalar mesons \([32]\), which may, however, contribute to the observed cusp effect by rescattering of kaons. Thus, for precise determination of the kaon–antikaon scattering length we will need higher statistics, which can be available at, for example, the ANKE experiment at COSY \([41]\) or less complicated final states like \( K^+K^-\gamma \) or \( K^0\bar{K}^0\gamma \), where only kaons interact strongly. These final states can be studied for example via the \( e^+e^- \rightarrow K^+K^-\gamma \) or \( e^+e^- \rightarrow K^0\bar{K}^0\gamma \) reactions with the KLOE–2 detector operating at the DAΦNE \( \phi \)–factory \([42]\).

ACKNOWLEDGMENTS

We are grateful to M. Hartmann, J. Ritman, and A. Wirzba for reading the earlier version of the manuscript and for their comments and corrections. We acknowledge the support by the FFE grants of the Research Center Jülich, by the Foundation for Polish Science and by the Polish National Science Center through the Grants No. 2011/03/N/ST2/02652 and 2011/03/B/ST2/01847.

[1] D. Lohse, J. W. Durso, K. Holinde, J. Speth, Nucl. Phys. A 516, 513 (1990).
[2] J. D. Weinstein, N. Isgur, Phys. Rev. D 41, 2236 (1990).
[3] D. Morgan, M. R. Pennington, Phys. Rev. D 48, 1853 (1993).
[4] R. L. Jaffe, Phys. Rev. D 15, 267 (1977).
[5] E. Van Beveren, et al., Z. Phys. C 30, 615 (1986).
[6] R. L. Jaffe, K. Johnson, Phys. Lett. B60, 201 (1976).
[7] R. Kaminski, L. Lesniak, Phys. Rev. C 51, 2264 (1995).
[8] V. Baru, et al., Phys. Lett. B586, 53 (2004).
[9] S. Teige et al., Phys. Rev. D 59, 012001 (2001).
[10] D. V. Bugg, V. V. Anisovich, A. Sarantsev, B. S. Zou, Phys. Rev. D 50, 4412 (1994).
[11] Z. Fu, Eur. Phys. J. 72, 2159 (2012).
[12] M. Wolke, Ph.D. thesis, IKP Jülich (3532), 1997 (unpublished).
[13] C. Quentmeier et al., Phys. Lett. B515, 276 (2001).
[14] P. Winter et al., Phys. Lett. B635, 23 (2006).
[15] Y. Maeda et al., Phys. Rev. C 77, 015204 (2008).
[16] Q. J. Ye et al., Phys. Rev. C 85, 035211 (2012).
[17] Q. J. Ye et al., Phys. Rev. C 87, 065203 (2013).
[18] P. Moskal et al., J. Phys. G 29, 2235 (2003).
[19] F. Balestra et al., Phys. Lett. B428, 7 (1999).
[20] M. Silarski et al., Phys. Rev. C 80, 045202 (2009).
[21] C. Wilkin, Acta Phys. Polon. Suppl. 2, 89 (2009).
[22] P. Moskal et al., Phys. Lett. B482, 356 (2000).
[23] H. P. Noyes, H. M. Lipinski, Phys. Rev. C 4, 995 (1971).
[24] H. P. Noyes, Annu. Rev. Nucl. Part. Sci. 22, 465 (1972).
[25] J. P. Naisse, Nucl. Phys. A 278, 506 (1977).
[26] J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001).
[27] A. N. Ivanov et al., Eur. Phys. J. A 21, 11 (2004).
[28] B. Borasoy, R. Nissler, W. Weise, Phys. Rev. Lett. 94, 213401 (2005).
[29] A. N. Ivanov et al., Eur. Phys. J. A 21, 11 (2004).
[30] B. Borasoy, R. Nissler, W. Weise, Phys. Rev. Lett. 94, 213401 (2005).
[31] N. V. Shevchenko, A. Gal, J. Mares, Phys. Rev. Lett. 98, 082301 (2007).
[32] Z. -H. Guo and J. A. Oller, Phys. Rev. C 87 (2013) 035202 arXiv:1210.3485 [hep-ph].
[33] B. Borasoy, U. -G. Meissner, R. Nissler, Phys. Rev. C 74, 055201 (2006).
[34] M. Wolke, Ph.D. thesis, IKP Jülich (3532), 1997 (unpublished).
[35] A. D. Martin, Nucl. Phys. B179, 33 (1981).
[36] Y. Yan, arXiv:0905.4818.
[37] A. D. Martin, Nucl. Phys. B179, 33 (1981).
[38] S. Baker and R. D. Cousins, Nucl. Instrum. Meth. 221, 437 (1984).
[39] P. Nyborg et al., Phys. Rev. 140, 914 (1965).
[40] F. James, Comput. Phys. Commun. 20, 29 (1980).
[41] M. Hartmann (private communication).
[42] G. Amelino-Camelia et al., Eur. Phys. J. C 68, 619 (2010).