Non-factorizable contributions
in hadronic weak decays of charm mesons

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Abstract

Two body decays of charm mesons are studied by describing their amplitude in terms of a sum of factorizable and non-factorizable ones. The former is estimated by using a naive factorization while the latter is calculated by using a hard pseudo-scalar-meson approximation. The hard pseudo-scalar-meson amplitude is given by a sum of the so-called equal-time commutator term and surface term which contains all possible pole contributions of various mesons, not only the ordinary \{q\bar{q}\} but also four-quark \{qq\bar{q}\bar{q}\}, hybrid \{q\bar{q}g\} and glue-balls.

Naively factorized amplitudes for the spectator decays which lead to too big rates can interfere destructively with exotic meson pole amplitudes and the total amplitudes can reproduce their observed rates. The non-factorizable contributions can supply sufficiently large contributions to the color suppressed decays which are strongly suppressed in the naive factorization. A possible solution to the long standing puzzle that the ratio of decay rates for $D^0 \to K^+K^-$ to $D^0 \to \pi^+\pi^-$ is around 2.5 is given by different contributions of exotic meson poles.
I. INTRODUCTION

Nonleptonic weak decays of charm mesons have been studied extensively by using the so-called factorization (or vacuum insertion) prescription [1,2]. However, recent semi-phenomenological analyses [3,4] in two-body decays of $B$ mesons within the framework of the factorization suggest that the value of $a_2$ to reproduce the observed branching ratios for these decays [3,5] should be larger by about a factor 2 than the one with the leading order (LO) QCD corrections [1,2,6] where the color degree of freedom $N_c = 3$ and that its sign should be opposite to the one in the large $N_c$ limit although the phenomenological value of $a_1$ is very close to the one expected in the same approximation. $[a_1$ and $a_2$ are the coefficients of four quark operators in the effective weak Hamiltonian in the Bauer-Stech-Wirbel (BSW) scheme [1,2] which will be reviewed briefly in the next section.] The above fact implies that the large $N_c$ argument fails, at least, in hadronic weak decays of $B$ mesons. Since the large $N_c$ argument is independent of flavors, it also does not work in nonleptonic weak decays of charm mesons. Therefore dominance of the factorized amplitude in charm decays looses its theoretical support since, in charm meson decays, the large $N_c$ argument is the only one known theoretical background to support the factorization prescription [7]. In fact, a naive application of the factorization to charm decay amplitudes causes many problems, for example, too strong suppression for the color mismatched decays like $D^0 \rightarrow \pi^0 \bar{K}$ and $D^0 \rightarrow \pi^0 \pi^0$, too big rates for decays described by the so-called spectator diagrams like the Cabibbo-angle favored $c \rightarrow s + (\bar{d}u)_1$ and Cabibbo-angle suppressed $c \rightarrow d + (\bar{d}u)_1$ and $c \rightarrow s + (\bar{s}u)_1$, too small ratio (less than unity) of the rates $\Gamma(D^0 \rightarrow K^+K^-)$ to $\Gamma(D^0 \rightarrow \pi^+\pi^-)$ although the observed value is around 2.5, etc., where $(\bar{q}'q)_1$ denotes a color singlet pair of $\bar{q}'$ and $q$. To get rid of these problems, the factorization has been implemented by multiplying the factorized amplitudes by phase factors arising from final state interactions. However, the final state interactions are realized by dynamical contributions of various hadron states.

Using a hard pseudo-scalar-meson approximation, the present author has studied dynamical contributions of various hadron states to charm meson decays and has given a hint to solve the above problems in charm meson decays [8,9]. However, in these analyses, the amplitudes did not include the factorizable contributions so that the results were not necessarily satisfactory, i.e., these analyses could not satisfactorily provide an overall fit to the observed rates of Cabibbo-angle favored and suppressed decays of charm mesons. On the other hand, a recent analysis in hadronic weak decays of $B$ mesons [10] by assuming that their amplitude can be given by a sum of factorizable and non-factorizable ones suggests that, only in some processes under a particular kinematical condition (a heavy quark goes to another heavy quark plus a pair of light quark and anti-quark with sufficiently high energies like $B \rightarrow D\pi$ and $D^*\pi$), the factorization works well while non-factorizable contributions are important in the other decays, in particular, in color suppressed decays. It seems to imply that the naive factorization is not guaranteed by the large $N_c$ arguments but it works well under some special kinematical condition [11]. In this article, we reanalyze two body decays of charm mesons describing their amplitude by a sum of factorizable and non-factorizable ones. We will review briefly the naive factorization and list the factorized amplitudes for two body decays of charm mesons in the next section. Non-factorizable amplitudes in a hard pseudo-scalar-meson approximation will be presented in section III. In section IV, we will compare our result with experiments. A brief summary will be given in the final section.
II. FACTORIZED AMPLITUDES

Our starting point in this article is to describe the two body decay amplitude by a sum of factorizable and non-factorizable ones \[1\],

\[ M_{\text{total}} = M_{\text{fact}} + M_{\text{non-fact}}. \]  

The factorizable amplitude \(M_{\text{fact}}\) is evaluated by using the factorization in the BSW scheme \[1,2\] in which the relevant part of the effective weak Hamiltonian responsible for the charm decays is given by

\[ H_w^{\text{BSW}} = \frac{G_F}{\sqrt{2}} \left\{ a_1 O_1^{(s'c)H} + a_2 O_2^{(s'c)H} + (\text{penguin term}) + h.c. \right\}. \]  

It can be obtained by applying the Fierz reordering to the usual effective Hamiltonian,

\[ H_w = \frac{G_F}{\sqrt{2}} \left\{ c_1 O_1^{(s'c)} + c_2 O_2^{(s'c)} + (\text{penguin term}) + h.c. \right\}, \]

where \(c_1\) and \(c_2\) are the Wilson coefficients of the normal ordered four quark operators,

\[ O_1^{(s'c)} =: (\bar{u}d')_{V-A}(\bar{s}'c)_{V-A} :, \quad O_2^{(s'c)} =: (\bar{s}'d')_{V-A}(\bar{u}c)_{V-A} :. \]

Here \(d'\) and \(s'\) denote weak eigen states of the down and strange quarks, respectively, and \((\bar{u}d)_{V-A} = \bar{u}\gamma_\mu(1 - \gamma_5)d\), etc. We do not explicitly show the possible penguin term through which \(s\)-channel pole contributions of a scalar glue-ball can play a role in Cabibbo-angle suppressed decays, since its explicit expression will not be needed. The quark bilinears in \(O_1^{(s'c)H}\) and \(O_2^{(s'c)H}\) are treated as interpolating fields for the mesons and therefore should be no longer Fierz reordered. The coefficients \(a_1\) and \(a_2\) in Eq. (2) are given by

\[ a_1 = c_1 + \frac{c_2}{N_c}, \quad a_2 = c_2 + \frac{c_1}{N_c}. \]  

The LO QCD corrections lead to \(a_1 \simeq 1.09\) and \(a_2 \simeq 0.09\) for \(N_c = 3\) \[3\].

The factorization prescription in the BSW scheme leads to the following factorized amplitude, for example, for the \(D^+(p) \to \bar{K}^0(p')\pi^+(q)\) decay,

\[ M_{\text{fact}}(D^+(p) \to \bar{K}^0(p')\pi^+(q)) = \frac{G_F}{\sqrt{2}} U_{cs}U_{ud} \left\{ a_1 \langle \pi^+(q) | (\bar{u}d)_{V-A} | 0 \rangle \langle \bar{K}^0(p') | (\bar{s}c)_{V-A} | D^+(p) \rangle \right. \]

\[ + a_2 \langle \bar{K}^0(p') | (\bar{s}d)_{V-A} | 0 \rangle \langle \pi^+(q) | (\bar{u}c)_{V-A} | D^+(p) \rangle \}. \]

where \(U_{ij}\) is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element \[1,2\] which is taken to be real in this article since \(CP\) invariance is always assumed. Factorizable amplitudes for the other two-body decays also can be calculated in the same way.

To evaluate the factorized amplitudes, we use the following parameterization of matrix elements of currents,

\[ \langle \pi(q) | A^\mu_{\pi} | 0 \rangle = -if_{\pi}q_\mu, \quad \text{etc.}, \]

and
Table I. Factorized amplitudes for two-body decays of charm mesons. The ellipses denote neglected contributions proportional to $f_-$. 

| Decay       | $M_{\text{fact}}$                                                                 |
|-------------|-----------------------------------------------------------------------------------|
| $D^+ \to \pi^+ K^0$ | $iU_{cs}U_{ud}\frac{G_F}{\sqrt{2}}a_1f_\pi(m_D^2 - m_K^2)f_{1\pi}^K(m_K^2)^[1]$
|             | $\times \left\{ 1 + \left( \frac{m_D}{m_\pi} \right)^2 \right\} + \ldots$       |
| $D^0 \to \pi^+ K^0$ | $iU_{cs}U_{ud}\frac{G_F}{\sqrt{2}}a_1f_\pi(m_D^2 - m_K^2)f_{1\pi}^K(m_K^2) + \ldots$ |
| $D^0 \to \pi^0 K^0$ | $iU_{cs}U_{ud}\frac{G_F}{\sqrt{2}}\sqrt{2}a_2f_K(m_D^2 - m_K^2)f_{1\pi}^K(m_K^2) + \ldots$ |
| $D^+ \to \pi^0 K^0$ | $iU_{cs}U_{ud}\frac{G_F}{\sqrt{2}}a_2f_K(m_D^2 - m_K^2)f_{1\pi}^K(m_K^2) + \ldots$ |
| $D^+ \to \pi^+ \bar{K}^0$ | $-iU_{cs}U_{us}\frac{G_F}{\sqrt{2}}a_1f_\pi(m_D^2 - m_K^2)f_{1\pi}^D(m_K^2) + \ldots$ |
| $D^0 \to \pi^0 \bar{K}^0$ | $0 + \ldots$                                                                            |
| $D^+ \to \pi^+ \pi^0$ | $iU_{cs}U_{us}\frac{G_F}{\sqrt{2}}\sqrt{2}(a_1 + a_2)f_\pi(m_D^2 - m_\pi^2)f_{1\pi}^D(m_\pi^2) + \ldots$ |
| $D^0 \to \bar{K}^0 K^0$ | $0$                                                                                   |
| $D^0 \to K^+ K^-$ | $iU_{cs}U_{us}\frac{G_F}{\sqrt{2}}a_1f_K(m_D^2 - m_K^2)f_{1\pi}^K(m_K^2) + \ldots$ |
| $D^+ \to K^+ \bar{K}^0$ | $iU_{cs}U_{us}\frac{G_F}{\sqrt{2}}a_1f_K(m_D^2 - m_K^2)f_{1\pi}^K(m_K^2) + \ldots$ |
| $D_s^+ \to \pi^+ K^+$ | $-iU_{cs}U_{us}\frac{G_F}{\sqrt{2}}a_1f_\pi(m_{D_s}^2 - m_K^2)f_{1\pi}^D(m_K^2) + \ldots$ |
| $D_s^+ \to \pi^0 K^+$ | $iU_{cs}U_{us}\frac{G_F}{\sqrt{2}}\sqrt{2}a_2f_\pi(m_{D_s}^2 - m_K^2)f_{1\pi}^D(m_K^2) + \ldots$ |

(8) 

$\langle \bar{K}(p')|V_\mu|D(p)\rangle = (p + p')_\mu f_{1\pi}^K(q^2) + q_\mu f_{1\pi}^K(q^2)$, etc., as usual, where $q = p - p'$. Using these expressions of current matrix elements, we obtain the factorized amplitudes for two-body decays of charm mesons listed in Table I, where terms proportional to $f_-$ are neglected since their coefficients are small or since $f_-$ is expected to be small for large values of its argument. It is seen that the factorized amplitudes for $D^0 \to \pi^0 \pi^0$ and $D^0 \to \pi^0 K^0$ described by the color mismatched diagrams, $c \to (d\bar{d})_1 + u$ and $c \to (s\bar{d})_1 + u$, respectively, are much smaller (the color suppression) than those for the spectator decays (because of $|a_1| \gg |a_2|$). The factorized amplitude for $D_s^+ \to K^+ \bar{K}^0$ is described by a sum of the color mismatched diagram and the so-called annihilation diagram in the weak boson mass $m_W \to \infty$ limit. The former is again proportional to $a_2$. The latter is proportional to $f_{1\pi}^K(m_{D_s}^2)$ and neglected. However the observed rates for these decays are not very small. The vanishing factorized amplitude for $D^0 \to K^0 \bar{K}^0$ reflects a cancellation between two possible annihilation diagrams while the measured rate for this decay is a little smaller than the ordinary ones but not extremely suppressed. To get rid of these problems, the factorization has been implemented by multiplying the amplitudes by phase factors arising from final state interactions [2]. However, the final state interactions are realized by dynamical contributions of various hadron states. Therefore, we study explicitly the dynamical contributions of various hadrons as the non-factorizable amplitudes in the next section.
III. NON-FACTORIZABLE AMPUTATIES

Now we study non-factorizable amplitudes for two-body decays of charm mesons, \( P_1(p) \rightarrow P_2(p') + P_3(q) \). As mentioned in the previous section, we assume that they are dominated by dynamical contributions of various hadron states. Then they can be estimated by using a hard pseudo-scalar-meson approximation in the infinite momentum frame (IMF, \( \text{i.e.}, \mathbf{p} \rightarrow \infty \)). It is an innovation of the old soft pion technique [13]. The non-factorizable amplitude in this approximation is given by [14, 15]

\[
M_{\text{non}-1}(P_1 \rightarrow P_2P_3) \simeq M_{\text{ETC}}(P_1 \rightarrow P_2P_3) + M_S(P_1 \rightarrow P_2P_3). 
\]  

(9)

The equal-time commutator term (\( M_{\text{ETC}} \)) and the surface term (\( M_S \)) are given by

\[
M_{\text{ETC}}(P_1 \rightarrow P_2P_3) = \frac{i}{\sqrt{2f_{P_3}}} \langle P_2|[V_{P_3}, H_w]|P_1 \rangle + (P_2 \leftrightarrow P_3) 
\]  

(10)

and

\[
M_S(P_1 \rightarrow P_2P_3) = \frac{i}{\sqrt{2f_{P_3}}} \left\{ \sum_n \left( \frac{m_{P_2}^2 - m_{P_3}^2}{m_n^2 - m_{P_3}^2} \right) \langle P_2|A_{P_3}|n \rangle \langle n|H_w|P_1 \rangle 
+ \sum_\ell \left( \frac{m_{P_2}^2 - m_{P_3}^2}{m_\ell^2 - m_{P_3}^2} \right) \langle P_2|H_w|\ell \rangle \langle \ell|A_{P_3}|P_1 \rangle \right\} + (P_2 \leftrightarrow P_3),
\]  

(11)

respectively, where \([V_{\pi} + A_{\pi}, H_w] = 0 \) has been used. (See Refs. [14] and [15] for notations.) \( M_{\text{ETC}} \) has the same form as the one in the old soft pion approximation but now has to be evaluated in the IMF. The surface term has been given by a sum of all possible pole amplitudes, \( i.e. \), \( n \) and \( \ell \) run over all possible single meson states, not only ordinary \( \{qq\} \), but also hybrid \( \{qgq\} \), four-quark \( \{qqqq\} \) and glue-balls. Since the value of wave function of orbitally excited \( \{qg\}_{L \neq 0} \) state at the origin is expected to vanish in the non-relativistic quark model, or more generally, wave function overlappings between the ground-state \( \{qg\}_{L=0} \) and their excited states are expected to be small, however, we neglect contributions of these states. In the \( u \)-channel [the second line of the right-hand-side of Eq. (11)], excited meson contributions will be not very important because of \( m_\ell^2 \gtrsim m_{P_3}^2 \gg m_{P_2}^2 \) if \( \ell \) is an excited-state meson. In contrast, in the \( s \) channel, we need to treat carefully contributions of exotic (non-\( \{qq\} \)) mesons to charm decays (if they exist) since they have been predicted around charm masses. The \( s \) channel of the color favored and mismatched spectator decays proceeds via four quark states after the weak interactions and therefore four-quark meson poles can contribute to these decays. However, in the annihilation decays in the weak boson mass \( m_W \rightarrow \infty \) limit, their \( s \) channel is given by \( \{qq\} \) state just after the weak interactions. Therefore we expect that the ground-state \( \{qg\}_0 \) and hybrid mesons can give important \( s \)-channel pole contributions to these decays. (However, we neglect contributions of scalar hybrids in this article since their masses have been expected to be considerably lower than the charm ones [16] and their contributions will be small.) The \( s \)-channel penguin can induce an \( s \)-channel pole contribution of glue-ball. In this way, the hard pseudo-scalar-meson amplitude in Eq. (9) with Eqs. (10) and (11) as the non-factorizable contribution is described in terms of asymptotic matrix elements (matrix elements taken between single
The four-quark operators \( O(a) \) creation operators or annihilation operators of quarks and anti-quarks. We associate one creation operator in the left and three annihilation operators in the right, and (d) all right, (b) three creation operators in the left and one annihilation operator in the right, (c) of products of (a) two creation operators in the left and two annihilation operators in the hadron states with infinite momentum) of charges \( V_i \) and \( A_i \), \( (i = \pi \) and \( K) \), and the effective weak Hamiltonian \( H_w \).

Asymptotic matrix elements of isospin and flavor \( SU_f(3) \) charges, \( V \pi \) and \( V_K \), are parameterized as

\[
\langle \pi^0 | V_{\pi^+} | \pi^- \rangle = \sqrt{2} \langle K^+ | V_{\pi^+} | K^0 \rangle = -\sqrt{2} \langle D^+ | V_{\pi^+} | D^0 \rangle \\
= -2 \langle K^+ | V_K^+ | 0 \rangle = -\sqrt{2} \langle D^+_s | V_K^+ | D^0 \rangle = \cdots = \sqrt{2}.
\]

(12)

The above parameterization can be obtained by applying asymptotic \( SU_f(4) \) symmetry or \( SU_f(4) \) extension of the nonet symmetry in \( SU_f(3) \) to the matrix elements or by using the quark counting. Matrix elements of axial counterpart, \( A_\pi \) and \( A_K \), of the above \( V_\pi \) and \( V_K \) are parameterized as

\[
\langle \rho^0 | A_{\pi^+} | \pi^- \rangle = \sqrt{2} \langle K^{*+} | A_{\pi^+} | K^0 \rangle = -\sqrt{2} \langle D^{*+} | A_{\pi^+} | D^0 \rangle \\
= -2 \langle K^{*+} | A_K^+ | 0 \rangle = -\sqrt{2} \langle D^{*+}_s | A_K^+ | D^0 \rangle = \cdots = h
\]

(13)

in the same way as the above. The above parameterization reproduces well the observed decay rates for \( D^* \to D\pi \) and \( D^* \to D\gamma \).

Next, we parameterize asymptotic matrix elements of \( H_w \) using quark counting \([8]\). We expect that the factorization will not be a good approximation to estimate asymptotic matrix elements of \( H_w \) since one of the external states in these matrix elements contains only light quarks and weak interactions occur in a deep sea of soft gluons where color degree of freedom of quarks will be compensated by soft gluons. Therefore, we forget the color degree of freedom of quarks for the moment and count only their flavors (and hence connected quark-line diagrams). Now we review the procedure to parameterize the asymptotic matrix elements of \( H_w \). To this, first, we rewrite the effective weak Hamiltonian in Eq.(3) as

\[
H_w = \frac{G_F}{\sqrt{2}} \{ c_- O_\pi^{(s')} + c_+ O^{(s')} + (penguin \ term) + h.c. \},
\]

(14)

where

\[
O^{(s')}_{\pm} = O^{(s')}_{1} \pm O^{(s')}_{2}.
\]

(15)

The four-quark operators \( O^{(s')}_{\pm} \) belong to \( 64 \) and \( 20'' \), respectively, of \( SU_f(4) \) in its symmetry limit. The normal ordered four-quark operators \( O_{\pm}^{(s')} \) can be expanded into a sum of products of (a) two creation operators in the left and two annihilation operators in the right, (b) three creation operators in the left and one annihilation operator in the right, (c) one creation operator in the left and three annihilation operators in the right, and (d) all (four) creation operators or annihilation operators of quarks and anti-quarks. We associate (a)–(d) with quark-line diagrams describing different types of matrix elements of \( O_{\pm}^{(s')} \). For (a), we utilize the two creation and annihilation operators to create and annihilate, respectively, the quarks and anti-quarks belonging to the meson states \( |\{q\bar{q}\}\rangle \) and \( |\{q\bar{q}\}\rangle \) in the asymptotic matrix elements of \( O_{\pm}^{(s')} \). For (b) and (c), we need to add a spectator quark or anti-quark to reach physical processes, \( |\{qq\bar{q}\} | O_{\pm}^{(s')} | \{q\bar{q}\} \rangle \) and \( |\{q\bar{q}\} | O_{\pm}^{(s')} | \{qq\bar{q}\} \rangle \), where \( |\{qq\bar{q}\}\rangle \) denotes four-quark mesons \([13]\). They can be classified into the following four types,
\{ q\bar{q}q\bar{q} \} = [q\bar{q}][q\bar{q}] \oplus (q\bar{q})(q\bar{q}) \oplus \{(q\bar{q})(q\bar{q}) \pm (q\bar{q})(q\bar{q})\}. \) Here () and [] denote symmetry and anti-symmetry, respectively, under the exchange of flavors between them. Since only the first two can have \( J^{P(C)} = 0^{+} (+) \), we here consider contributions of them. These two types of four-quark mesons are again classified into two different types with two different combinations of color degree of freedom, i.e., one consists of color singlet \( \{ q\bar{q} \} \) pairs and the other consists of color octet \( \{ q\bar{q} \} \) pairs. They can mix with each other. Their mass eigen states are listed in the tables in APPENDIX A. As seen in these tables, the predicted masses of the lighter components with relevant quantum numbers are much smaller than the charm mass while some of the heavier ones (with * in the tables) can have masses close to the charm mass. We here take into account only contributions of the latter as an approximation. (For more precise arguments, we need all contributions of these exotic mesons.)

Counting all possible quark-line diagrams, we obtain sum rules which should be satisfied by asymptotic matrix elements of \( O_{\pm}^{(s'c)} \). In this process, symmetry (or anti-symmetry) property of wave functions of external meson states under exchange of their quark and anti-quark plays an important role and therefore we have to be careful with the order of the quark(s) and anti-quark(s) in \( O_{\pm}^{(s'c)} \). Noting that the wave function of the ground-state \( \{ q\bar{q} \} \) meson is antisymmetric \([20]\) under the exchange of its quark and anti-quark, we obtain the following constraints on asymptotic matrix elements of \( O_{\pm}^{(s'c)} \) \([8]\),

\[
\langle \{ q\bar{q} \} | O_{\pm}^{(s'c)} | \{ q\bar{q} \} \rangle = 0, \tag{16}
\]

\[
\langle \{ q\bar{q} \} | O_{\pm}^{(s'c)} | \{ q\bar{q} \} \rangle = 0, \tag{17}
\]

\[
\langle \{ q\bar{q} \} | O_{\pm}^{(s'c)} | \{ q\bar{q} \} \rangle = 0, \tag{18}
\]

and then, from them, we can obtain selection rules of asymptotic matrix elements of \( H_w \). We summarize and parameterize them in APPENDIX B which have been given separately in Refs. \([8]\) and \([21]\). Inserting the above parameterizations of asymptotic matrix elements of \( A_\pi, A_K \) and \( H_w \) into \( M_S \) in Eq.\((11)\), we obtain pole amplitudes including contributions of the \( \{ q\bar{q} \}, \{ q\bar{q}\} \) and \( \{ q\bar{q}\} \) mesons.

A scalar glue-ball can give an important contribution, as an s-channel pole, to Cabibbo-angle suppressed decays through the s-channel penguin diagram. It can mix with scalar iso-singlet \( \{ q\bar{q} \} \) mesons. The glue-rich component of the mixture is described by \( S^* \). We parameterize the ratio \( \langle K | A_K | S^* \rangle \) to \( \langle \pi | A_\pi | S^* \rangle \) by

\[
Z = \frac{\langle K^+ | A_{K^+} | S^* \rangle}{\langle \pi^+ | A_{\pi^+} | S^* \rangle}, \tag{19}
\]

and then the residue of \( S^* \) meson pole as

\[
\langle K^+ | A_{K^+} | S^* \rangle \langle S^* | H_w | D^0 \rangle = -k_g \langle \pi^+ | H_w | D^+ \rangle. \tag{20}
\]

Then the glue-ball contributions to the \( D \to KK \) and \( D \to \pi\pi \) decays are given by

\[
M_S^{(\text{glue})}(D^0 \to K^0 K^0) = i \frac{\langle \pi^+ | H_w | D^+ \rangle}{f_K} \left( \frac{m_D^2 - m_K^2}{m_D^2 - m_{S^*}^2} \right) 2k_g \tag{21}
\]

\[
= M_S^{(\text{glue})}(D^0 \to K^+ K^-), \tag{22}
\]
\[ M^{(\text{blue})}_S(D^0 \to \pi^0 \pi^0) = -\frac{i}{f_\pi} \langle \pi^+ | H_w | D^+ \rangle \frac{1}{Z} \left( \frac{m_D^2 - m_\pi^2}{m_D^2 - m_S^2} \right) 2k_g \]

\[ = \sqrt{\frac{1}{2}} M^{(\text{blue})}_S(D^0 \to \pi^+ \pi^-). \tag{24} \]

An amplitude for a dynamical hadronic process can be decomposed into \textit{(continuum contribution)} + \textit{(Born term)}. In the present case, \( M_S \) is given by a sum of pole amplitudes and therefore \( M_{\text{ETC}} \) corresponds to the continuum contribution \cite{22} which can develop a phase relative to the Born term. Therefore we here parameterize the ETC terms using isospin eigen amplitudes and their phases; \textit{e.g.}, for the \( M_{\text{ETC}}(D \to \pi \bar{K}) \)'s,

\[
M_{\text{ETC}}(D^0 \to \pi^+ K^-) = \frac{1}{3} M_{\text{ETC}}^{(1)}(D \to \pi \bar{K}) e^{i \delta_3(\pi \bar{K})} + \frac{2}{3} M_{\text{ETC}}^{(1)}(D \to \pi \bar{K}) e^{i \delta_1(\pi \bar{K})}, \tag{25}
\]

\[
M_{\text{ETC}}(D^0 \to \pi^0 \bar{K}^0) = -\sqrt{\frac{2}{3}} M_{\text{ETC}}^{(3)}(D \to \pi \bar{K}) e^{i \delta_3(\pi \bar{K})} + \sqrt{\frac{1}{3}} M_{\text{ETC}}^{(3)}(D \to \bar{K}^0) e^{i \delta_1(\pi \bar{K})}, \tag{26}
\]

\[
M_{\text{ETC}}(D^+ \to \pi^+ \bar{K}^-) = M_{\text{ETC}}^{(3)}(D \to \pi \bar{K}) e^{i \delta_3(\pi \bar{K})}, \tag{27}
\]

where \( M_{\text{ETC}}^{(I)} \)'s are the isospin eigen amplitudes with isospin \( I \) and \( \delta_{2I} \)'s are the corresponding phase shifts introduced \cite{23}. \( M_{\text{ETC}} \)'s for decays into \( K \bar{K} \) and \( \pi \pi \) final states can be parameterized in a similar way. Since \( M_{\text{ETC}}(D^0 \to K^0 \bar{K}^0) = 0 \), we obtain \( M_{\text{ETC}}^{(0)}(D \to K \bar{K}) = M_{\text{ETC}}^{(2)}(D \to K \bar{K}) \) and \( \delta_0(K \bar{K}) = \delta_2(K \bar{K}) \), which lead to

\[
M_{\text{ETC}}(D^0 \to K^+ K^-) = M_{\text{ETC}}(D^+ \to K^+ K^-) = -\frac{i}{\sqrt{2 f_K}} \langle \pi^+ | H_w | D^+ \rangle e^{i \delta_2(K \bar{K})}. \tag{28}
\]

The parameterization of \( M_{\text{ETC}}(D \to \pi \pi) \) is taken to be compatible with that of the \( D \to \pi \pi \) amplitudes in Ref. \cite{24}. Because of the selection rule from Eq. (10), we obtain

\[
M_{\text{ETC}}^{(4)}(D \to \pi \pi) = 0, \tag{29}
\]

\[
M_{\text{ETC}}^{(6)}(D \to \pi \pi) = \frac{i}{\sqrt{2 f_\pi}} \langle \pi^+ | H_w | D^+ \rangle e^{i \delta_0(\pi \pi)}, \tag{30}
\]

\[
M_{\text{ETC}}(D_s^+ \to K^+ \bar{K}^0) = -\frac{i}{\sqrt{2 f_K}} \langle \pi^+ | H_{w} | D_s^+ \rangle e^{i \delta_2(K \bar{K})} \tag{31}
\]

since the final state \( K^+ \bar{K}^0 \) is of \( I = 1 \). For the Cabibbo-angle favored \( D \to \pi K \) and suppressed \( D_s^+ \to (\pi K)^+ \) decays, their ETC terms are given by

\[
M_{\text{ETC}}^{(1)}(D \to \pi \bar{K}) = -\frac{i}{\sqrt{2 f_\pi}} \langle \pi^+ | H_w | D_s^+ \rangle \left[ 2 - \frac{1}{2} \left( \frac{f_\pi}{f_K} \right) \right] e^{i \delta_1(\pi \bar{K})}, \tag{32}
\]

\[
M_{\text{ETC}}^{(3)}(D \to \pi \bar{K}) = -\frac{i}{\sqrt{2 f_\pi}} \langle \pi^+ | H_w | D_s^+ \rangle \left[ 1 - \frac{1}{2} \left( \frac{f_\pi}{f_K} \right) \right] e^{i \delta_1(\pi \bar{K})}, \tag{33}
\]

\[
M_{\text{ETC}}^{(1)}(D_s^+ \to (\pi K)^+) = -\frac{i}{\sqrt{2 f_\pi}} \langle \pi^+ | H_w | D^+ \rangle \left[ 2 - \frac{1}{2} \left( \frac{f_\pi}{f_K} \right) \right] e^{i \delta_1(\pi K)}, \tag{34}
\]

\[
M_{\text{ETC}}^{(3)}(D_s^+ \to (\pi K)^+) = -\frac{i}{\sqrt{2 f_\pi}} \langle \pi^+ | H_w | D^+ \rangle \left[ 1 - \frac{1}{2} \left( \frac{f_\pi}{f_K} \right) \right] e^{i \delta_2(\pi K)} \tag{35}
\]
In this way, the final state interactions are included in the non-factorizable amplitudes in the present perspective. It should be noted that ETC terms for decays into exotic final states vanish (in the $SU_f(3)$ symmetry limit, i.e., $f_K = f_\pi$).

In the approximation in which pole contributions of the ground-state $\{q\bar{q}\}_0$, scalar $[q\bar{q}][q\bar{q}]$ and $(q\bar{q})(\bar{q}\bar{q})$ mesons and a glue-ball to $M_S$ are taken into account, we obtain the non-factorizable amplitudes for the $D \to \pi\bar{K}$, $\pi\pi$ and $KK$ and $D_s^+ \to (K\bar{K})^+$ and $(\pi K)^+$ decays inserting the above parameterizations of asymptotic matrix elements of charges and the effective weak Hamiltonian and the ETC terms implemented by the phase factors into Eq.(3) with Eqs.(\[10\]) and (11). The result is listed in APPENDIX C.

IV. BRANCHING RATIOS

Now we compare our result with experiments. To this, we need to know values of parameters involved in our total amplitude (a sum of the factorized amplitude listed in Table I and the corresponding hard pseudo-scalar-meson amplitude listed in APPENDIX C). We take the following values of the CKM matrix elements and the decay constants $\sum$; $U_{us} = -U_{cd} = 0.21$, $U_{cs} = U_{ud} = 0.98$ and $f_\pi = 132$ MeV, $f_K = 159$ MeV. To calculate decay branching ratios, we use the central values of the observed lifetimes of charm mesons $\sum$: $\tau(D^+) = (1.057 \pm 0.015) \times 10^{-12}$ s, $\tau(D^0) = (0.415 \pm 0.004) \times 10^{-12}$ s and $\tau(D_s^+) = (0.467 \pm 0.017) \times 10^{-12}$ s.

The factorized amplitudes listed in Table I contain the form factors, $f_+(q^2)$’s. We put $f_+(m_\pi^2) \simeq f_+(0)$ since $m_\pi^2$ is very small. The estimated values of $f_+(0)$’s are summarized in Ref. $\sum$ as

$$f^K_D(0)_{\text{expt}} = 0.75 \pm 0.02 \pm 0.02,$$

$$\left[ \begin{array}{c} f^D_D(0) \\ f^K_D(0) \end{array} \right]_{\text{expt}} = 1.0^{+0.3}_{-0.2} \pm 0.4, \quad \text{[Mark III]},$$

$$= 1.3 \pm 0.2 \pm 0.1, \quad \text{[CLEO]}.$$}

The above result is compatible with the $SU_f(3)$ symmetry and therefore we assume that

$$f^K_D(m_\pi^2) \simeq f^D_D(m_\pi^2) \simeq f^K_D(m_\pi^2) \simeq f^D_D(m_\pi^2) \simeq f^K_D(0).$$

In this way, we can obtain the factorized amplitudes which provide the branching ratios $B_{\text{fact}}$’s in Table II. It is seen that the branching ratios for the so-called spectator decays, $D^+ \to \pi^+\pi^0$ and $D^+ \to \pi^+K^0$, are too big and that the color suppressed decays, $D^0 \to \pi^0\pi^0$ and $D^+ \to \pi^0K^0$, are too strongly suppressed as mentioned before.

Now we evaluate the non-factorizable amplitudes listed in APPENDIX C. The asymptotic matrix elements of $A_s$ and $A_K$ which have been parameterized in Eq.(\[10\]) are estimated to be $|h| \simeq 1.0$ $\sum$ by using partially conserved axial-vector current (PCAC) and the observed rate $\sum$, $\Gamma(\rho \to \pi\pi)_{\text{expt}} \simeq 150$ MeV.

We here assign $f_J(1710)$ to the glue-rich scalar $S^*$ and take $\Gamma_f = 175 \pm 9$ MeV $\sum$ as the width $\Gamma_{S^*}$ of $S^*$. Then $|\langle K|A_K|S^*\rangle|$ can be estimated from the observed width of $f_J$ and the ratio of its partial widths.
\[
\frac{\Gamma(f_J \rightarrow \pi \pi)}{\Gamma(f_J \rightarrow KK)}_{\text{expt}} = 0.39 \pm 0.14
\] (40)

as \(|\langle K^+|A_{K^+}|S^*\rangle| \lesssim 0.15\) by using PCAC. They are really small as expected. It implies that overlapping between wave functions of the ground-state-meson and the glue-ball (or glue-rich meson) is very small. Therefore size of matrix element of \(H_w\) taken between \(S^\ast\) and \(K\) will be much smaller than that between \(\pi\) and \(K\), and hence the size of \(k_y\) will be much smaller than unity. \(Z\) in Eq. (19) also can be estimated to be \(Z \sim 2.0\) \[23\] from the above value of the observed ratio of decay rates, Eq. (10), where we have taken the same sign between \(\langle K^+|A_{K^+}|S^*\rangle\) and \(\langle \pi^+|A_{\pi}^+|S^*\rangle\) as expected in the \(SU_f(3)\) symmetry.

Although existence of four-quark mesons has never been confirmed, indications of their existence are increasing \[27\]. We here take the predicted values of \([qq][\bar{q}\bar{q}]\) meson masses listed in Table A1(a) in APPENDIX A. However, the predicted \((qq)(\bar{q}\bar{q})\) meson masses listed in Table A2(a) satisfy \(m_{E_{\pi\pi}^+} < m_D < m_{E_{\pi K}^+}\). In this case, it is hard that the \((qq)(\bar{q}\bar{q})\) meson pole amplitudes for the spectator decays, \(D^+ \rightarrow \pi^+\bar{K}^0\) and \(D^+ \rightarrow \pi^+\pi^0\), interfere destructively with their factorized amplitudes, simultaneously. [The naively factorized amplitudes have lead to too big rates for the spectator decays as discussed before.] Since the predicted values of their masses still would have ambiguities, however, we here shift up the mass values of \((qq)(\bar{q}\bar{q})\) mesons by 100 MeV from the predicted ones to obtain \(m_D < m_{E_{\pi\pi}^+} < m_{E_{\pi K}^+}\) which leads to a destructive interference between the factorized amplitudes and the four-quark meson pole amplitudes for the \(D^+ \rightarrow \pi^+\bar{K}^0\) and \(D^+ \rightarrow \pi^+\pi^0\). Masses of four-quark mesons containing a charm quark listed in Tables A1(b) and A2(b) are estimated crudely by using the quark counting since our result is not very sensitive to them. Although widths of four-quark mesons are also still not known, the \([qq][\bar{q}\bar{q}]\) mesons with * are expected to be narrower than the corresponding \((qq)(\bar{q}\bar{q})\) mesons \[19\].

For the phases \(\delta_2\)'s arising from contributions of non-resonant multi-hadron intermediate states with isospin \(I\), they will be restricted in the region \(|\delta_2| < 90^\circ\). (Resonant contributions have already been extracted as pole amplitudes in \(M_S\).) We here treat them as adjustable parameters which are restricted in the region \(|\delta_2| < 90^\circ\) as mentioned above and satisfy the relations

\[\delta_0(\pi\pi) = \delta_0(K\bar{K}) = \delta_2(K\bar{K}) = \delta_1(\pi\bar{K}) = \delta_1(\pi K) \equiv \delta.\] (41)

The second equality has been obtained previously and the others are expected in the \(SU_f(3)\) symmetry limit. However the exotic phase shift \(\delta_3(\pi K)\) is treated as a parameter independent of non-exotic \(\delta_1(\pi K)\).

The remaining parameters are \(k_y^*, k_s^*\) and \(k_y\) which provide the residues of poles of \([qq][\bar{q}\bar{q}]\), \((qq)(\bar{q}\bar{q})\) and glue-ball, respectively. Since overlappings between wave functions of these exotic mesons and the ground-state ones are expected to be very small, values of these parameters will be much smaller than unity, i.e., \(|k_y|, |k_s^*|, |k_y^*| \ll 1\). It was predicted \[19\] that the couplings of the four-quark \([qq][\bar{q}\bar{q}]\) and \((qq)(\bar{q}\bar{q})\) mesons with * to the ground-state \(O^{-(+)}\) mesons would be small because of the structure of their wave functions with respect to spin and color degree of freedom and that the \((qq)(\bar{q}\bar{q})\) mesons with * would couple to the ground-state \(O^{-(+)}\) mesons much more weakly than the \([qq][\bar{q}\bar{q}]\) mesons. The above statement implies that \(|k_y^*| \ll |k_s^*|\).

For values of the asymptotic ground-state-meson matrix elements of \(H_w\), \(|\pi^+|H_{\pi}D_{\pi}^+\), etc., we have no information. Therefore we here treat the above matrix element in addition
to the parameters mentioned above as adjustable parameters and look for overall fits to the observed branching ratios for two body decays, where the latter parameters are not perfectly free but restricted as discussed above, i.e., $|k_y| \ll 1$, $|k_y^*| \ll |k_s^*| \ll 1$ and $|\delta|$, $|\delta_{\text{exotic}}| < 90^\circ$, where $\delta$ is given in Eq. (11) and $\delta_{\text{exotic}}$ is the strong phase in the exotic $\pi K$ (or $K$) channel, $\delta_{\text{exotic}} = \delta_3(\pi K) = \delta_3(\pi \bar{K})$.

We can reproduce remarkably well the observed branching ratios for Cabibbo-angle favored and suppressed two body decays of charm mesons, simultaneously, by taking reasonable values of parameters involved, i.e., very small values of parameters providing the residues of poles of glue-ball, $[qq][\bar{q}q]$ and $(qq)(\bar{q}q)$ mesons, $|k_y| \sim 0.1$, $|k_y^*| \sim 0.1$ and $|k_s^*| \sim 0.01$, respectively, the predicted mass values of $[qq][\bar{q}q]$ mesons in Ref. [25], but $(qq)(\bar{q}q)$ meson masses larger by 100 MeV than the predicted ones, and their widths, relatively narrower $\Gamma_{[qq][qq]} \sim 0.2$ GeV and rather broader $\Gamma_{(qq)(\bar{q}q)} \sim 0.4$ GeV. These are compatible with the discussions in Ref. [19]. For the phases of the ETC term relative to the surface term in the narrow width limit, which are expected to arise from contributions of multi hadron intermediate states, rather large values ($\gtrsim 70^\circ$) of these phases in Eq. (11) are favored while our result is not very sensitive to the value of the exotic phases $\delta_3(\pi K)$ and $\delta_3(\pi \bar{K})$ since contributions of the ETC terms into exotic final states are small (vanishing in the $SU_f(3)$ symmetry limit, i.e., $f_K = f_\pi$). For the asymptotic ground-state-meson matrix element of $H_w$, Cabibbo-angle favored and suppressed ones separately satisfy the charm counterparts of (asymptotic) $SU_f(3)$ symmetry limit, i.e., $f_K = f_\pi$. For the asymptotic ground-state-meson matrix element of $H_w$, Cabibbo-angle favored and suppressed ones separately satisfy the charm counterparts of (asymptotic) $SU_f(3)$ symmetry limit, i.e., $f_K = f_\pi$. For the asymptotic ground-state-meson matrix element of $H_w$, Cabibbo-angle favored and suppressed ones separately satisfy the charm counterparts of (asymptotic) $SU_f(3)$ symmetry limit, i.e., $f_K = f_\pi$.

As an example, a typical result is shown in Table II where we have taken the follow-

\begin{equation}
\langle \pi^+|H_w|D_s^+\rangle = \frac{G_F}{\sqrt{2}} U_{ud} U_{cs} \left\{m_\pi^2 + m_{D_s}^2\right\} f_\pi f_{D_s} c_- \simeq 0.078 \times 10^{-5} \ (\text{GeV})^2
\end{equation}

obtained by using a factorization compatible with the quark counting discussed before. In the above estimate of $\langle \pi^+|H_w|D_s^+\rangle$ fact, we have taken $c_- = 0.885$ (and $c_+ = 0.375$) with the leading order QCD corrections [23] and a recent lattice result on the decay constant, $f_{D_s} \simeq 216$ MeV [24]. It may imply that our estimate of asymptotic matrix element of $H_w$ is reasonable (but it does not necessarily imply that the factorization is a reliable approximation).

As an example, a typical result is shown in Table II where we have taken the following values of unknown parameters: the asymptotic matrix element of $H_w$, $\langle \pi^+|H_w|D_s^+\rangle \simeq 0.0695 \times 10^{-5} \ (\text{GeV})^2$, which is not very far from the above $\langle \pi^+|H_w|D_s^+\rangle$ fact, the strong phases, $\delta_0(\pi \pi) = \delta_1(\pi \bar{K}) = \delta_1(\pi K) = \delta_3(\pi \bar{K}) = \delta_3(\pi K) = 85^\circ$, and the residues of meson poles, $k_0 = 0.71$, $k_a^* = -0.20$, $k_s^* = -0.02$, $k_q = 0.085$. It is seen, from Table II, that the naively factorized amplitudes for the so-called spectator decays which lead to too big rates for these decays interfere destructively with the exotic meson pole amplitudes as expected. The non-factorizable amplitudes can supply significant contributions to the so-called color suppressed $D^0 \rightarrow \pi^0 K^0$ and $D^0 \rightarrow \pi^0 \pi^0$ which are strongly suppressed in the naive factorization. To solve the well-known puzzle that the observed ratio of rates for $D^0 \rightarrow K^+ K^-$ to $D^0 \rightarrow \pi^+ \pi^-$ is around 2.5, a peculiar $SU_f(3)$ symmetry breaking may have to be introduced [29]. In the present case, such a symmetry breaking can be realized dominantly by different contributions of $[qq][\bar{q}q]$ meson poles, i.e., $m_D^2 - m_{D_s}^2 \gg m_D^2 - m_{D_s}^2$. It is known that the $D^0 \rightarrow K^0 \bar{K}^0$ decay is described by two annihilation diagrams (in the $m_w \rightarrow \infty$ limit) which cancel each other. Therefore both the factorized and the non-factorizable amplitudes for
Table II. Branching ratios (%) for two-body decays of charm mesons where $a_1 = 1.09$ and $a_2 = -0.09$ have been used. Values of parameters introduced are tentatively taken as follows; the matrix element $\langle \pi^+|H\pi|D_0^+\rangle \simeq 0.0695 \times 10^{-5}$ (GeV)$^2$, the phases $\delta_0(\pi\pi) = \delta_1(\pi K) = \delta_3(\pi K) = 85^\circ$, and the parameters providing the residues of various meson poles $k_0 = 0.71$, $k_a^s = -0.20$, $k_s^s = -0.02$, $k_g = 0.085$. $B_{\text{fact}}$, $B_{\text{non-f}}$ and $B_{\text{total}}$ include only the factorized amplitude, only the non-factorizable one (involving the $\{q\bar{q}\}_0$, $[q\bar{q}][q\bar{q}]$ and $(qq)(\bar{q}\bar{q})$ meson poles) and the sum of them, respectively. The data values are taken from Ref. [28].

| Decay          | $B_{\text{fact}}$ | $B_{\text{non-f}}$ | $B_{\text{total}}$ | $B_{\text{expt}}$ |
|---------------|------------------|------------------|------------------|-----------------|
| $D^+ \to \pi^+ K^0$ | 9.66             | 2.15             | 2.71             | 2.74 $\pm$ 0.29 |
| $D^0 \to \pi^+ K^-$  | 4.66             | 6.75             | 3.91             | 3.83 $\pm$ 0.12 |
| $D^0 \to \pi^0 K^0$  | 0.03             | 1.94             | 2.26             | 2.11 $\pm$ 0.21 |
| $D^+_s \to K^+ K^0$  | 0.06             | 3.82             | 2.93             | 3.6 $\pm$ 1.1   |
| $D^0 \to \pi^+ \pi^-$ | 0.29             | 0.68             | 0.14             | 0.152 $\pm$ 0.011 |
| $D^0 \to \pi^0 \pi^0$ | 0.00             | 0.05             | 0.05             | 0.084 $\pm$ 0.022 |
| $D^+ \to \pi^+ \pi^0$ | 0.31             | 1.11             | 0.25             | 0.25 $\pm$ 0.07 |
| $D^0 \to K^0 K^-$  | 0.32             | 0.19             | 0.41             | 0.43 $\pm$ 0.03 |
| $D^0 \to K^0 K^0$  | 0.00             | 0.12             | 0.12             | 0.13 $\pm$ 0.04 |
| $D^+ \to K^+ K^0$  | 0.81             | 1.29             | 0.98             | 0.72 $\pm$ 0.12 |
| $D^+_s \to \pi^+ K^0$ | 0.32             | 0.14             | 0.22             | $< 0.70$       |
| $D^+_s \to \pi^0 K^+$ | 0.00             | 0.02             | 0.02             | $< 0.70$       |

This decay vanish. However the $s$-channel penguin can induce a pole contribution of scalar glue-ball or glue-rich scalar meson which leads to a reasonable size of rate for this decay in consistency with the $D^0 \to \pi\pi$ and $K^+ K^-$ decays.

V. SUMMARY

Two body decays of charm mesons have been studied by describing their amplitude in terms of a sum of factorized and non-factorizable ones. The former has been estimated by using the naive factorization in the BSW scheme while the latter has been calculated by using a hard pseudo-scalar-meson approximation. It has been given by a sum of the equal-time commutator term and the surface term which contains pole contributions of various meson states, not only the ground-state $\{q\bar{q}\}_0$ mesons but also a glue-ball and exotic $\{qq\bar{q}\bar{q}\}$ mesons with $J^{P(C)} = 0^{+}(+)$. In this way, a possible solution to the long standing problems in charm meson decays has been given, at least, qualitatively. Factorizable contributions which lead to too big branching ratios $B_{\text{fact}}$’s for the spectator decays like $D^+ \to \pi^+ \pi^0$ and $\pi^+ K^0$ can interfere destructively with non-factorizable ones and a sum of these two contributions can reproduce their observed values of branching ratios. The naive factorization also leads to too strong color suppression. However, non-factorizable amplitudes can supply sufficient contributions to the color suppressed decays. The observed branching ratios for the mixed decays which
have both contributions from the spectator and the color mismatched diagrams can be reproduced by interferences between the factorizable and the non-factorizable contributions.

Two body decays of charm mesons into final states including \( \eta \) or \( \eta' \), in particular, decays into \( \pi \eta \) and \( \pi \eta' \) are interesting. However non-factorizable contributions to these decays are complicated \([30]\) because of the \( \eta-\eta' \) mixing and therefore these decays should be investigated separately.

For quasi two body \( D \to VP \) decays, the mixing between iso-singlet mesons in the final states is rather simple, \( i.e., \) the \( \omega-\phi \) mixing is known to be approximately ideal. However, in these decays, all the four types of four-quark mesons can contribute except for annihilation decays. Fortunately, the decays, \( D^0 \to \bar{K}^0\phi \), \( D^+_s \to \pi^+\rho^0 \) and \( \pi^+\omega \), are described approximately by annihilation diagrams and four-quark meson contributions can be neglected. Here the first one has been observed with a substantial rate but the last two are suppressed. Since the factorized amplitudes for these decays are always suppressed \([1]\), the observed rates should be dominantly supplied by non-factorizable dynamical contributions of various hadrons in the present approach. The \( D^0 \to \bar{K}^0\phi \) amplitude is dominantly given by a sum of its ETC term describing contributions of multi hadron intermediate states and the \( \bar{K}^0 \) meson pole amplitude \([41]\). The observed suppression of the \( D^+_s \to \pi^+\rho^0 \) suggests that a hybrid pseudo scalar meson \( \pi_H \) with a mass very close to \( m_{D_s} \) and with a rather narrow width exists (a recently observed pseudo scalar hybrid meson \( \pi(1800) \) \([32]\) may be assigned to this one although its mass \( \sim 1.8 \text{ GeV} \) is not sufficiently close to \( m_{D_s} \)) and that its pole contribution cancel a sum of the ETC term and the pion pole amplitude for this decay \([33]\). A strange component \( \pi_H \) belonging to the same multiplet as \( \pi_H \) does not disturb our good result on the \( D^0 \to \bar{K}^0\phi \) decay. A recent observation of the \( D^+_s \to \pi^+\omega \) with a small rate \([34]\), \( B(D^+_s \to \pi^+\omega) = (2.7 \pm 1.2) \times 10^{-3} \), suggests that an iso-triplet hybrid meson with \( J^{PC} = 1^{+-} \) exists but is not very close to \( m_{D_s} \) (probably much lower than \( m_{D_s} \) as expected \([16]\)) or couples very weakly to \( \pi\omega \). Quasi two body decays of charm mesons including factorizable contributions will be investigated more extensively elsewhere.

Finally, hadronic weak interactions of charm mesons are intimately related to hadron spectroscopy. More informations of hadron spectroscopy will be needed to find a more quantitative solution to the puzzles in hadronic weak interactions.

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APPENDIX A: FOUR-QUARK MESONS

Scalar four-quark mesons with charm quantum number \( C = 0 \) and 1 are listed, where \( S \) and \( I \) denote the strangeness and isospin. Particles with superscript \( s \) contain an \( s\bar{s} \) pair. Mass values of non-charm (\( C = 0 \)) mesons are given in Ref. [19]. Masses of four-quark charm (\( C = 1 \)) mesons estimated by using the quark counting (with \( m_u = m_d = 0, m_s = 0.2 \) GeV, \( m_c = 1.5 \) GeV and the predicted mass values of \( C = 0 \) mesons mentioned above) are given between parentheses ( ). Particles containing double (\( s\bar{s} \)) pairs and (\( c\bar{c} \)) pair(s) are dropped since they do not contribute in this paper.

Table A1(a). Ideally mixed scalar \([qq][\bar{q}\bar{q}]\) mesons with \( C = 0 \).

| \( S \) | \( I = 1 \) | \( I = \frac{1}{2} \) | \( I = 0 \) | Mass(GeV) |
|------|-------|------|-----|-----|
| 1    | \( \hat{\kappa} \) | \( \hat{\kappa}^* \) | 0.90 |
| 0    | \( \hat{\delta}^s \) | \( \hat{\delta}^* \) | 0.65 |
|      | \( \hat{\delta}^{ss} \) | \( \hat{\delta}^{ss*} \) | 1.00 |

Table A1(b). Ideally mixed scalar \([qq][\bar{q}\bar{q}]\) mesons with \( C = 1 \).

| \( S \) | \( I = 1 \) | \( I = \frac{1}{2} \) | \( I = 0 \) | Mass(GeV) |
|------|-------|------|-----|-----|
| 1    | \( \hat{F}_I \) | \( \hat{F}_I^* \) | \( \hat{F}_0 \) | (2.4) |
|      | \( \hat{\bar{D}} \) | \( \hat{\bar{D}}^* \) | (2.2) |
|      | \( \hat{\bar{D}}^s \) | \( \hat{\bar{D}}^{ss*} \) | (2.6) |

Table A2(a). Ideally mixed scalar \((qq)(\bar{q}\bar{q})\) mesons with \( C = 0 \).

| \( S \) | \( I = 2 \) | \( I = \frac{3}{2} \) | \( I = 1 \) | \( I = \frac{1}{2} \) | \( I = 0 \) | Mass(GeV) |
|------|-------|------|-----|-----|-----|-----|
| 2    | \( E_{KK} \) | \( E_{KK}^* \) | 1.55 |
| 1    | \( E_{\pi K} \) | \( C_K \) | 1.35 |
|      | \( E_{\pi K}^* \) | \( C_K^* \) | 1.95 |
|      | \( C_\pi \) | \( C_\pi^* \) | 1.75 |
|      | \( C_\pi \) | \( C_\pi^* \) | 2.20 |
| 0    | \( E_{\pi\pi} \) | \( C_\pi \) | 1.15 |
|      | \( C_\pi \) | \( C_\pi^* \) | 1.80 |
|      | \( C_\pi \) | \( C_\pi^* \) | 1.55 |
|      | \( C_\pi^* \) | \( C_\pi \) | 2.10 |


Table A2(b). Ideally mixed scalar \((qq)(\bar{q}\bar{q})\) mesons with \(C = 1\).

| \(S\) | \(I = 2\) | \(I = \frac{3}{2}\) | \(I = 1\) | \(I = \frac{1}{2}\) | \(I = 0\) | \(\text{Mass(GeV)}\) |
|---|---|---|---|---|---|---|
| 2 | | \(E_{K_F}\) | \(E_{K_F}^*\) | \(C_F\) | \(C_F^*\) | \(2.9\) |
| 1 | \(E_{\pi F}\) | \(E_{\pi F}^*\) | \(C_F\) | \(C_F^*\) | \(C_F^*\) | \(3.5\) |
| 0 | \(E_{\pi D}\) | \(E_{\pi D}^*\) | \(C_D\) | \(C_D^*\) | \(C_D^*\) | \(3.1\) |
| –1 | \(E_{KD}\) | \(E_{KD}^*\) | \(C_D\) | \(C_D^*\) | \(C_D^*\) | \(3.7\) |

APPENDIX B: ASYMPTOTIC MATRIX ELEMENTS OF \(H_w\)

Constraints on asymptotic matrix elements of \(H_w\) have been previously derived by using an algebraic method based on commutation relations between charges and currents (and hence the effective weak Hamiltonian) and, then, by counting all possible connected quark-line diagrams. We here summarize a part of them which are useful in this paper. We here describe the Cabibbo-angle favored \((\Delta C = -1, \Delta S = -1)\) and suppressed \((\Delta C = -1, \Delta S = 0)\) weak Hamiltonians as \(H_w(-, -)\) and \(H_w(-, 0)\), respectively, for convenience’ sake.

(i) Constraints on asymptotic ground-state-meson matrix elements of \(H_w\):

\[
\begin{align*}
\langle \bar{K}^0 | H_w(-, -) | D^0 \rangle &= -\langle \pi^+ | H_w(-, -) | D_s^+ \rangle, \\
\langle K^{*0} | H_w(-, -) | D^0 \rangle &= -\langle \rho^+ | H_w(-, -) | D_s^+ \rangle \\
&= \langle \bar{K}^0 | H_w(-, -) | D^{*0} \rangle = -\langle \pi^+ | H_w(-, -) | D^* \rangle = (\sqrt{2}k_0/h)\langle \bar{K}^0 | H_w(-, -) | D^0 \rangle,
\end{align*}
\]

\[
\begin{align*}
\langle \pi^+ | H_w(-, 0) | D^+ \rangle &= -\langle K^+ | H_w(-, 0) | D_s^+ \rangle, \\
\langle \pi^+ | H_w(-, 0) | D^{*+} \rangle &= -\langle K^+ | H_w(-, 0) | D_s^{*+} \rangle \\
&= \langle \rho^+ | H_w(-, 0) | D^+ \rangle = -\langle K^{*+} | H_w(-, 0) | D_s^+ \rangle = (\sqrt{2}k_0/h)\langle \pi^+ | H_w(-, 0) | D^+ \rangle,
\end{align*}
\]

where \(k_0 = \sqrt{\frac{3}{2}}h\) with \(h = \langle \rho^0 | A_{\pi^+} | \pi^- \rangle\) has been obtained by using an algebraic method [35]. It will be understood more intuitively since all the external states in the above matrix elements of \(H_w\) are of helicity = 0 states of the ground-state \(\{qq\}_0\) mesons and the difference of spins will be not very important in the IMF.
(ii) Constraints on asymptotic matrix elements of $H_w$ between the ground-state meson and $[qq][\bar{q}q]$ meson states:

\[
\begin{align*}
\langle \tilde{\kappa}^0 | H_w(-, -) | D^0 \rangle &= -\langle \delta^{*+} | H_w(-, -) | D^+_s \rangle = (k_a^*/2A^*_a) \langle \tilde{K}^0 | H_w(-, -) | D^0 \rangle, \\
\langle \tilde{K}^0 | H_w(-, -) | \tilde{D}^0 \rangle &= -\langle \tilde{K}^0 | H_w(-, -) | \tilde{D}^{*+} \rangle = \sqrt{2}(\pi^+ | H_w(-, -) | \tilde{F}^{*+}) = \sqrt{2}(\pi^0 | H_w(-, -) | \tilde{F}^{*0}) \\
&= \sqrt{2}(\pi^+ | H_w(-, -) | \tilde{F}^{*+}) = (k_a^*/2A^*_a) \langle \tilde{K}^0 | H_w(-, -) | D^0 \rangle, \\
\langle \pi^+ | H_w(-, -) | \tilde{F}^{*+} \rangle &= 0, \\
\end{align*}
\]

(B3)

where $A^*_a$ is the invariant matrix element of axial charge defined by $A^*_a = -\frac{1}{2}(\tilde{\kappa}^{*+} | A^*_a | K^0)$.

(iii) Constraints on asymptotic matrix elements of $H_w$ between the ground-state-meson and $(qq)(\bar{q}q)$ meson states:

\[
\begin{align*}
\sqrt{2}(E_{\pi K}^* | H_w(-, -) | D^+) &= \sqrt{2}(E_{\pi K}^* | H_w(-, -) | D^0) \\
&= 3(C_K^* | H_w(-, -) | D^0) = \sqrt{3}(C_{s+}^* | H_w(-, -) | D^+_s) \\
&= (k_a^*/A^*_a) \langle \tilde{K}^0 | H_w(-, -) | D^0 \rangle, \\
\langle \pi^+ | H_w(-, -) | E_{\pi F}^0 \rangle &= \sqrt{2}(\pi^0 | H_w(-, -) | E_{\pi F}^0) \\
&= -\left(\frac{3}{2}\right)(\tilde{K}^0 | H_w(-, -) | E_{\pi D}^0) = \sqrt{3}(K^- | H_w(-, -) | E_{\pi D}^*) \\
&= 3(\tilde{K}^0 | H_w(-, -) | C_{D}^*) = \sqrt{3}(K^- | H_w(-, -) | E_{KF}^*) = \sqrt{3}(K^- | H_w(-, -) | E_{KF}^*) \\
&= (k_a^*/A^*_a) \langle \tilde{K}^0 | H_w(-, -) | D^0 \rangle, \\
\langle \pi^+ | H_w(-, -) | C_{D}^* \rangle &= 0.
\end{align*}
\]

(B5)

where $A^*_a$ is the invariant matrix element of axial charge defined by $A^*_a = (C_K^* | A^*_a | K^0)$.

In the above, $k_a^*$ and $\tilde{k}_a^*$ ($k_a^*$ and $\tilde{k}_a^*$) are not generally equal to each other. However, use of the commutation relation, $[[H_w(-, -), V_{D^-}], V_{D^-}] = [[H_w(-, -), V_{\bar{q}^-}], V_{\bar{q}^-}]$, with asymptotic
SU_f(4) symmetry (or an SU_f(4) extension of the nonet symmetry in the flavor SU_f(3) with respect to asymptotic matrix elements of charges) leads to \( k^*_a = k^*_a \) (and \( k^*_s = k^*_s \)).

Matrix elements of \( H_w(-, -) \) and \( H_w(-, 0) \) can be related to each other, for example, as

\[
\langle \pi^+ | H_w(-, -) | D^+_s \rangle = \frac{V_{es}}{V_{cd}} \langle \pi^+ | H_w(-, 0) | D^+ \rangle, \quad \text{etc.} \tag{B7}
\]

by using the commutation relations, \([O_{\pm}(-, -), V_K] = O_{\pm}(-, 0)\) and \([O_{\pm}(-, 0), V_{\bar{K}0}] = 2O_{\pm}(-, -)\), where \( O_{\pm}(-, -) \) and \( O_{\pm}(-, 0) \) are four quark operators in \( H_w(-, -) \) and \( H_w(-, 0) \), respectively, and contributions of the QCD induced penguin term have been neglected.

**APPENDIX C: NON-FACTORIZABLE AMPLITUDES**

We here list hard pseudo-scalar-meson amplitudes as the non-factorizable ones which include the ETC term describing continuum contributions and the surface term containing pole contributions of the ground-state \( \{q\bar{q}\}_0 \), scalar \( [qq][\bar{q}\bar{q}] \) and \( (qq)(\bar{q}\bar{q}) \) mesons and a glueball. They are revised from the ones given in Ref. [8] in which the amplitudes involved some misprints and the insufficient parameterization of phases of the ETC terms.

(i) Cabibbo-angle-favored decays:

\[
M_{\text{non-}f}(D^+ \to \pi^+ K^0) \simeq -i \frac{1}{\sqrt{2} f_\pi} \langle \pi^+ | H_w | D^+_s \rangle \times \left\{ (1 - \frac{f_\pi}{f_K}) e^{i\delta_3} + \left[ \frac{m_D^2 - m_K^2}{m_D^2 - m_K^2} - \frac{m_D^2 - m_\pi^2}{m_D^2 - m_\pi^2} \left( \frac{f_\pi}{f_K} \right) \right] k_0 \right. \\
\left. + \left[ \frac{m_D^2 - m_K^2}{m_D^2 - m_K^2} - \frac{m_D^2 - m_\pi^2}{m_D^2 - m_\pi^2} \left( \frac{f_\pi}{f_K} \right) \right] k^*_a \right. \\
\left. + \left[ 2 \left( \frac{m_D^2 - m_K^2}{m_D^2 - m_K^2} \right) + 2 \left( \frac{m_D^2 - m_\pi^2}{m_D^2 - m_\pi^2} \left( \frac{f_\pi}{f_K} \right) \right) \right] k^*_s \right\}, \tag{C1}
\]

\[
M_{\text{non-}f}(D^0 \to \pi^+ K^-) \simeq \frac{i}{\sqrt{2} f_\pi} \langle \pi^+ | H_w | D^+_s \rangle \left\{ -\frac{1}{3} \left( 1 - \frac{f_\pi}{f_K} \right) e^{i\delta_3(\pi K)} + \frac{1}{3} \left( 4 - \frac{f_\pi}{f_K} \right) e^{i\delta_1(\pi K)} \right. \\
\left. + \left[ \frac{m_D^2 - m_K^2}{m_D^2 - m_K^2} - \frac{m_D^2 - m_\pi^2}{m_D^2 - m_\pi^2} \left( \frac{f_\pi}{f_K} \right) + \left( \frac{m_D^2 - m_\pi^2}{m_D^2 - m_\pi^2} \left( \frac{f_\pi}{f_K} \right) \right) \right] k_0 \right. \\
\left. + \left[ \frac{m_D^2 - m_K^2}{m_D^2 - m_K^2} + \left( \frac{m_D^2 - m_\pi^2}{m_D^2 - m_\pi^2} \left( \frac{f_\pi}{f_K} \right) \right) \right] k^*_a \right. \\
\left. - \left[ \left( \frac{m_D^2 - m_K^2}{m_D^2 - m_K^2} \right) + \left( \frac{m_D^2 - m_\pi^2}{m_D^2 - m_\pi^2} \left( \frac{f_\pi}{f_K} \right) \right) \right] k^*_s \right\}, \tag{C2}
\]
\[ M_{\text{non}-f}(D^0 \to \pi^0 \bar{K}^0) \]
\[ \simeq -\frac{i}{\sqrt{2}f_\pi} \left\langle \pi^+ | H_w | D^+_s \right\rangle \sqrt{\frac{1}{2}} \left\{ \frac{-\sqrt{2}}{3} \left( 1 - \frac{f_\pi}{f_K} \right) e^{i\delta_5(\pi K)} - \frac{\sqrt{2}}{3} \left( 2 - \frac{1}{2} \frac{f_\pi}{f_K} \right) e^{i\delta_1(\pi K)} \right\} k_0 \]
\[ + \left[ \left( \frac{m^2_{D^*} - m^2_{K^*}}{m^2_{D^*} - m^2_{K^*}} \right) - \left( \frac{m^2_D - m^2_{\pi^+}}{m^2_D - m^2_{\pi^+}} \right) \left( \frac{f_\pi}{f_K} \right) + \left( \frac{m^2_D - m^2_{K^*}}{m^2_D - m^2_{K^*}} \right) \right] k_a \]
\[ + \left[ \left( \frac{m^2_{D^*} - m^2_{K^*}}{m^2_{D^*} - m^2_{K^*}} \right) + \left( \frac{m^2_D - m^2_{\pi^+}}{m^2_D - m^2_{\pi^+}} \right) \left( \frac{f_\pi}{f_K} \right) \right] \]
\[ + \left( \frac{m^2_D - m^2_{K^*}}{m^2_D - m^2_{K^*}} \right) - 2 \left( \frac{m^2_D - m^2_{\pi^+}}{m^2_D - m^2_{\pi^+}} \right) \left( \frac{f_\pi}{f_K} \right) \right\} k_s^* \}, \quad (C3) \]

\[ M_{\text{non}-f}(D^+_s \to K^+ \bar{K}^0) \]
\[ \simeq -\frac{i}{\sqrt{2}f_K} \left\langle \pi^+ | H_w | D^+_s \right\rangle \left\{ \frac{1}{2} \left\{ \frac{-\sqrt{2}}{3} \left( 1 - \frac{f_\pi}{f_K} \right) e^{i\delta_2(\pi \bar{K})} + \left( \frac{m^2_{D^*} - m^2_{K^*}}{m^2_{D^*} - m^2_{K^*}} \right) k_0 \right\} \right\} \]
\[ + \left[ \frac{2 \left( \frac{m^2_{D^*} - m^2_{K^*}}{m^2_{D^*} - m^2_{\delta^*}} \right) - \left( \frac{m^2_D - m^2_{\pi^+}}{m^2_D - m^2_{\pi^+}} \right) \right] k_a^* \]
\[ + 2 \left[ \left( \frac{m^2_{D^*} - m^2_{K^*}}{m^2_{D^*} - m^2_{\delta^*}} \right) + \left( \frac{m^2_D - m^2_{\pi^+}}{m^2_D - m^2_{\pi^+}} \right) \left( \frac{f_\pi}{f_K} \right) \right] \]
\[ + \left( \frac{m^2_D - m^2_{K^*}}{m^2_D - m^2_{K^*}} \right) - 2 \left( \frac{m^2_D - m^2_{\pi^+}}{m^2_D - m^2_{\pi^+}} \right) \left( \frac{f_\pi}{f_K} \right) \right\} k_s^* \}, \quad (C4) \]

(ii) Cabibbo-angle suppressed decays:

\[ M_{\text{non}-f}(D^0 \to \pi^+ \pi^-) \]
\[ \simeq \frac{i}{\sqrt{2}f_\pi} \left\langle \pi^+ | H_w | D^+_s \right\rangle \left\{ \frac{1}{2} \left\{ \frac{-\sqrt{2}}{3} \left( 1 - \frac{f_\pi}{f_K} \right) e^{i\delta_3(\pi \pi)} + \left( \frac{m^2_D - m^2_{\pi^+}}{m^2_D - m^2_{\pi^+}} \right) k_0 \right\} \right\} \]
\[ + \left[ \frac{2 \left( \frac{m^2_D - m^2_{\pi^+}}{m^2_D - m^2_{\pi^+}} \right) - \left( \frac{m^2_{D^*} - m^2_{\pi^+}}{m^2_{D^*} - m^2_{\pi^+}} \right) \right] k_a^* \]
\[ - \left[ \frac{2 \left( \frac{m^2_D - m^2_{\pi^+}}{m^2_D - m^2_{\pi^+}} \right) - \left( \frac{m^2_{D^*} - m^2_{\pi^+}}{m^2_{D^*} - m^2_{\pi^+}} \right) \right] k_s^* - \frac{2}{Z} \left( \frac{m^2_D - m^2_{\pi^+}}{m^2_D - m^2_{\pi^+}} \right) k_g \}, \quad (C5) \]

\[ M_{\text{non}-f}(D^0 \to \pi^0 \pi^0) \]
\[ \simeq \frac{i}{\sqrt{2}f_\pi} \left\langle \pi^+ | H_w | D^+_s \right\rangle \left\{ \frac{1}{2} \left\{ \frac{-\sqrt{2}}{3} \left( 1 - \frac{f_\pi}{f_K} \right) e^{i\delta_4(\pi \pi)} + \left( \frac{m^2_D - m^2_{\pi^+}}{m^2_D - m^2_{\pi^+}} \right) k_0 \right\} \right\} \]
\[ + \left[ \frac{2 \left( \frac{m^2_D - m^2_{\pi^+}}{m^2_D - m^2_{\pi^+}} \right) - \left( \frac{m^2_{D^*} - m^2_{\pi^+}}{m^2_{D^*} - m^2_{\pi^+}} \right) \right] k_a^* \]
\[ + \left[ \frac{2 \left( \frac{m^2_D - m^2_{\pi^+}}{m^2_D - m^2_{\pi^+}} \right) - \left( \frac{m^2_{D^*} - m^2_{\pi^+}}{m^2_{D^*} - m^2_{\pi^+}} \right) \right] k_s^* - \frac{2}{Z} \left( \frac{m^2_D - m^2_{\pi^+}}{m^2_D - m^2_{\pi^+}} \right) k_g \}, \quad (C6) \]
\begin{align}
M_{\text{non-f}}(D^+ \to \pi^+ \pi^0) & \simeq \frac{i}{\sqrt{2}f_\pi} \langle \pi^+ | H_w | D^+ \rangle \left\{ \left[ 2 \left( \frac{m_D^2 - m_{\pi}^2}{m_D^2 - m_{E^*_\pi}^2} \right) - \left( \frac{m_D^2 - m_{\pi}^2}{m_{E^*_\pi}^2} - m_{\pi}^2 \right) \right] \sqrt{2} k_s^* \right\}, \tag{C7} \\
M_{\text{non-f}}(D^0 \to K^+ K^-) & \simeq -\frac{i}{\sqrt{2}f_K} \langle \pi^+ | H_w | D^+ \rangle \left\{ e^{i\delta_2(K\bar{K})} + \left( \frac{m_D^2 - m_{\pi}^2}{m_D^2 - m_{K}^2} \right) k_0 \\
& + \left[ 2 \left( \frac{m_D^2 - m_{K}^2}{m_D^2 - m_{F^*}^2} \right) - \left( \frac{m_D^2 - m_{K}^2}{m_{F^*}^2} - m_{K}^2 \right) \right] k_s^* \\
& - \left[ 2 \left( \frac{m_D^2 - m_{K}^2}{m_D^2 - m_{C^*}^2} \right) - \left( \frac{m_D^2 - m_{K}^2}{m_{C^*}^2} - m_{K}^2 \right) \right] k_a^* \right\}, \tag{C8} \\
M_{\text{non-f}}(D^0 \to K^0 \bar{K}^0) & \simeq -\frac{i}{\sqrt{2}f_K} \langle \pi^+ | H_w | D^+ \rangle \left\{ e^{i\delta_2(K\bar{K})} + \left( \frac{m_D^2 - m_{\pi}^2}{m_D^2 - m_{K}^2} \right) k_0 \\
& + \left[ 2 \left( \frac{m_D^2 - m_{K}^2}{m_D^2 - m_{F^*}^2} \right) - \left( \frac{m_D^2 - m_{K}^2}{m_{F^*}^2} - m_{K}^2 \right) \right] k_s^* \\
& - \left[ 2 \left( \frac{m_D^2 - m_{K}^2}{m_D^2 - m_{C^*}^2} \right) - \left( \frac{m_D^2 - m_{K}^2}{m_{C^*}^2} - m_{K}^2 \right) \right] k_a^* \right\}, \tag{C9} \\
M_{\text{non-f}}(D^+ \to K^0 \bar{K}^0) & \simeq -\frac{i}{\sqrt{2}f_K} \langle \pi^+ | H_w | D^+ \rangle \left\{ e^{i\delta_2(K\bar{K})} + \left( \frac{m_D^2 - m_{\pi}^2}{m_D^2 - m_{K}^2} \right) k_0 \\
& + \left[ 2 \left( \frac{m_D^2 - m_{K}^2}{m_D^2 - m_{F^*}^2} \right) - \left( \frac{m_D^2 - m_{K}^2}{m_{F^*}^2} - m_{K}^2 \right) \right] k_s^* \\
& - \left[ 2 \left( \frac{m_D^2 - m_{K}^2}{m_D^2 - m_{C^*}^2} \right) - \left( \frac{m_D^2 - m_{K}^2}{m_{C^*}^2} - m_{K}^2 \right) \right] k_a^* \right\}, \tag{C10} \\
M_{\text{non-f}}(D_s^+ \to \pi^+ \pi^0) & \simeq \frac{i}{\sqrt{2}f_\pi} \langle \pi^+ | H_w | D_s^+ \rangle \left\{ \left[ 2 \left( \frac{f_\pi}{f_K} \right) - 1 \right] e^{i\theta_1(\pi K)} \\
& - \left[ \left( \frac{m_{D_s}^2 - m_{K}^2}{m_{D_s}^2 - m_{K}^*} \right) - \left( \frac{m_{D_s}^2 - m_{\pi}^2}{m_{D_s}^2 - m_{K}^*} \right) \left( \frac{f_\pi}{f_K} \right) \right] k_0 \\
& + \left[ \left( \frac{m_{D_s}^2 - m_{\pi}^2}{m_{D_s}^2 - m_{K}^*} \right) + \left( \frac{m_{D_s}^2 - m_{\pi}^2}{m_{D_s}^2 - m_{K}^*} \right) \left( \frac{f_\pi}{f_K} \right) \right] k_s^* \\
& - \left[ \left( \frac{m_{D_s}^2 - m_{K}^2}{m_{D_s}^2 - m_{K}^*} \right) + \left( \frac{m_{D_s}^2 - m_{\pi}^2}{m_{D_s}^2 - m_{K}^*} \right) \left( \frac{f_\pi}{f_K} \right) \right] \left( \frac{f_\pi}{f_K} \right) \right\}, \tag{C11} \\
\end{align}
\[ M_{\text{non}-f}(D_s^+ \to \pi^0 K^+) \approx \frac{i}{\sqrt{2}f_{\pi}} \langle \pi^+ | H_w | D_s^+ \rangle \sqrt{\frac{1}{2}} \left[ 2 \left( \frac{f_{\pi}}{f_K} \right) - 1 \right] e^{i\delta_{1}(\pi K)} \]

\[ -\left[ \left( \frac{m^2_{D_s} - m^2_K}{m^2_{D_s} - m^2_{K*}} \right) - \left( \frac{m^2_{D_s} - m^2_\pi}{m^2_{D_s} - m^2_{\pi*}} \right) \left( \frac{f_{\pi}}{f_K} \right) \right] k_0 \]

\[ + \left[ \left( \frac{m^2_{D_s} - m^2_K}{m^2_{D_s} - m^2_{K*}} \right) + \left( \frac{m^2_{D_s} - m^2_\pi}{m^2_{D_s} - m^2_{\pi*}} \right) \left( \frac{f_{\pi}}{f_K} \right) \right] \]

\[ - 2 \left( \frac{m^2_{D_s} - m^2_\pi}{m^2_{\pi} - m^2_K} \right) + \left( \frac{m^2_{D_s} - m^2_\pi}{m^2_{\pi} - m^2_\pi} \right) \left( \frac{f_{\pi}}{f_K} \right) \]

\[ k_0^* \]

\[ + \left[ \left( \frac{m^2_{D_s} - m^2_K}{m^2_{D_s} - m^2_{E*_{K}}} \right) + \left( \frac{m^2_{D_s} - m^2_\pi}{m^2_{D_s} - m^2_{E*_{\pi}}} \right) \left( \frac{f_{\pi}}{f_K} \right) \right] \]

\[ - \left( \frac{m^2_{D_s} - m^2_\pi}{m^2_{E*_{D}} - m^2_\pi} \right) \left( \frac{f_{\pi}}{f_K} \right) k^*_a \]
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