Medium Effects in Reactions with Rare Isotopes

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Abstract. We discuss medium effects in knockout reactions with rare isotopes of weakly-bound nuclei at intermediate energies. We show that the poorly known corrections may lead to sizable modifications of knockout cross sections and momentum distributions.

1. Introduction

Most practical studies of the medium corrections of nucleon-nucleon scattering are done by considering the effective two-nucleon interaction in infinite nuclear matter, or G-matrix, as a solution of the Bethe-Goldstone equation

\[ \langle k | G(P,\rho_1,\rho_2) | k_0 \rangle = \langle k | v_{NN} | k_0 \rangle - \int \frac{d^3k'}{(2\pi)^3} \frac{\langle k | v_{NN} | k' \rangle Q(k',P,\rho_1,\rho_2) \langle k' | G(P,\rho_1,\rho_2) | k_0 \rangle}{E(P,k') - E_0 - i\epsilon} \]

(1)

with \( k_0, k, \) and \( k' \) the initial, final, and intermediate relative momenta of the NN pair, \( k = (k_1 - k_2)/2 \) and \( P = (k_1 + k_2)/2 \). If energy and momentum is conserved in the binary collision, \( P \) is conserved in magnitude and direction, and the magnitude of \( k \) is conserved. \( v_{NN} \) is the nucleon-nucleon potential. \( E \) is the energy of the two-nucleon system, and \( E_0 \) is the same quantity on-shell. Thus \( E(P,k) = e(P+k) + e(P-k) \), with \( e \) the single-particle energy in nuclear matter. It is also implicit in Eq. 1 that the final momenta \( k \) of the NN-pair also lie outside the range of occupied states.

We have performed a study of medium effects in knockout reactions which include different methods to treat medium effects [1]. To test the influence of the medium effects in nucleon knockout reactions, we consider the removal of the \( l = 0 \) halo neutron of \( ^{15}\text{C} \), bound by 1.218 MeV, and the \( l = 0 \) neutron knockout from \( ^{34}\text{Ar} \), bound by 17.06 MeV. The reaction studied is \( ^9\text{Be}(^{15}\text{C},^{14}\text{C}_{gs}) \). The total cross sections as a function of the bombarding energy are shown in figures 1. The solid curve is obtained with the use of free nucleon-nucleon cross sections. The dashed curve includes the geometrical effects of Pauli blocking. The dashed-dotted curve is the result using the Brueckner theory, and the dotted curve is the phenomenological parametrization of the free cross section.

In figure 2 we plot the longitudinal momentum distributions for the reaction \( ^9\text{Be}(^{11}\text{Be},^{10}\text{Be}) \), at 250 MeV/nucleon [1]. The dashed curve is the cross section calculated using the NN cross section from the Brueckner theory and the solid curve is obtained the free cross section. One sees that the momentum distributions are reduced by 10%, about the same as the total cross
Figure 1. Total knockout cross sections for removing the $l = 0$ halo neutron of $^{15}\text{C}$, bound by 1.218 MeV, in the reaction $^9\text{Be}(^{15}\text{C},^{14}\text{C}_{gs})$. The solid curve is obtained with the use of free nucleon-nucleon cross sections. The dashed curve includes the geometrical effects of Pauli blocking. The dashed-dotted curve is the result using the Brueckner theory, and the dotted curve is a phenomenological parametrization.

Figure 2. Longitudinal momentum distribution for the residue in the $^9\text{Be}(^{11}\text{Be},^{10}\text{Be})$, reaction at 250 MeV/nucleon. The dashed curve is the cross section calculated using the NN cross section from the Brueckner theory and the solid curve is obtained the free cross section.

sections, but the shape remains basically unaltered. If one rescales the dashed curve to match the solid one, the differences in the width are not visible.

| Projectile | Residue | $nlj$ | $C^2S_{\text{NuShell@MSU}}$ | $C^2S_{\text{Ref[4]}}$ | $\text{Sn(Sp)(MeV)}$ |
|------------|---------|------|-----------------|-----------------|------------------|
| $^{14}\text{O}$ | $^{13}\text{O}$ | 0$p_{3/2}$ | 4.97 | 3.7 | 23.176 |
| $^{13}\text{N}$ | 0$p_{1/2}$ | 1.83 | 1.8 | 4.628 |
| $^{36}\text{Ca}$ | $^{35}\text{Ca}$ | 1$s_{1/2}$ | 2.16 | 19.113 |
| $^{35}\text{K}$ | 0$d_{3/2}$ | 3.54 | 2.559 |

Table 1. Spectroscopic factors ($C^2S$).

Based on these results we have developed a new version of the code MOMDIS [2] in order to treat medium effects an Coulomb recoil properly. In the next sections we report a few preliminary results, for reactions which are being planned at RIKEN.
Table 2. Bound state potential parameters.

| Projectile | Residue | $V_0$(MeV) | $R_0$(fm) | $a$(fm) |
|------------|---------|------------|-----------|---------|
| $^{14}$O   | $^{13}$O | 81.46      | 2.50      | 0.6     |
| $^{13}$N   |         | 43.10      | 3.00      | 0.8     |
| $^{36}$Ca  | $^{35}$Ca| 63.19      | 3.93      | 0.6     |
| $^{35}$K   |         | 47.63      | 3.93      | 0.6     |

2. Proton and neutron knockout of numerous systems

In this work, $^{9}$Be($^{14}$O,$^{13}$O), $^{9}$Be($^{14}$O,$^{13}$N) at 50 and 300 AMeV, and $^{9}$Be($^{36}$Ca,$^{35}$Ca), $^{9}$Be($^{36}$Ca,$^{35}$K) at 70.5 and 300 AMeV cases have been analyzed. The ground state spins for nuclei $^{13}$O, $^{13}$N, $^{35}$Ca and $^{35}$K are respectively $3/2^-$, $1/2^-$, $1/2^+$ and $3/2^+$ are taken from Ref. [3], except for $^{35}$Ca. The ground state spin for $^{35}$Ca is not defined. Nevertheless, we have assumed in our calculations it has $J^\pi = 1/2^+$ ground state spin.

Table 3. Total cross-sections.

| Projectile | Residue | $E$(MeV) | $\sigma_{\text{MOMDIS}}$(mb) | $\sigma$(mb) | $\sigma_{\text{exp}}$(mb) |
|------------|---------|----------|-------------------------------|--------------|---------------------------|
| $^{14}$O   | $^{13}$O | 57       | 9.61                          | 51.60        | 13.4±1.4                  |
|            |         | 300      | 16.04                         | 86.10        |                           |
| $^{13}$N   |         | 57       | 29.96                         | 59.21        | 67±6                      |
|            |         | 300      | 31.50                         | 62.26        |                           |
| $^{36}$Ca  | $^{35}$Ca| 70.5     | 6.72                          | 15.39        |                           |
|            |         | 300      | 8.07                          | 18.48        |                           |
| $^{35}$K   |         | 70.5     | 8.96                          | 33.62        |                           |
|            |         | 300      | 10.56                         | 39.63        |                           |

A single nucleon removal cross section $\sigma$ from the $J^\pi = 0^+$ the ground state (g.s.) of projectile to the g.s. of the knockout residue is given by

$$
\sigma = \left(\frac{A}{A-1}\right)^n C^2 S \sigma_{\text{MOMDIS}}
$$

(2)

where $\sigma_{\text{MOMDIS}}$ is calculated using a modified version of the MOMDIS code [2]. The A-dependent term is a center-of-mass correction to the shell-model spectroscopic factors ($C^2 S$) where $n$ is major oscillator shell number ($n = 1$ for sp shell nucleus $^{14}$O and $n = 2$ for sd shell nucleus $^{36}$Ca) [4, 5].

The spectroscopic factors which are used in the present work are calculated with the shell model code NuShell@MSU [6]. We have calculated $C^2 S$ considering the WBT interaction for $^{14}$O [7] and USDA interaction for $^{36}$Ca [8]. Our calculation is in agreement with Ref. [4] for $^{14}$O proton removal case, but the neutron removal spectroscopic factor $C^2 S$ for the present calculation is 34% bigger than in Ref. [4].

The bound state wave functions are calculated with Woods-Saxon potentials and the parameters are tabulated in Table 5. The total cross sections obtained with these parameters are shown in Table 6.
Table 4. Separation energies from [3].

| Projectile (g.s.) | Residue (g.s.) | nlj | Sp (MeV) |
|-------------------|----------------|-----|----------|
| $^9$C, 3/2$^-$    | $^8$B          | 0p3/2 | 1.301    |
| $^{17}$F, 5/2$^+$ | $^{16}$O       | 0d5/2 | 0.600    |
| $^{27}$P, 1/2$^+$ | $^{26}$Si      | 1s1/2 | 0.860    |

Table 5. Bound state potential parameters.

| Projectile | Residue | $V_0$ (MeV) | $R_0$ (fm) | $a$ (fm) |
|------------|---------|-------------|------------|----------|
| $^9$C      | $^8$B   | 47.69       | 2.50       | 0.6      |
| $^{17}$F   | $^{16}$O| 71.25       | 2.80       | 0.6      |
| $^{27}$P   | $^{26}$Si| 19.76     | 3.30       | 0.5      |

Table 6. Nuclear densities.

| Nucleus | Model       | $a$ (fm) | $\alpha$ (fm) |
|---------|-------------|----------|---------------|
| $^8$B   | LDM         |          |               |
| $^{16}$O| Gaussian[9] | 1.833    | 1.544         |
| $^{26}$Si| LDM       |          |               |
| $^{12}$C| Gaussian[9]| 1.73     | 1.38          |

The $^{14}$O proton removal result is in agreement with data from Ref.[4], but for neutron removal case there is a huge difference, probably due to the very big value of $C^2S = 4.97$. This is probably the reason why $^{14}$O cases have different bound state potential parameters, which we have searched for best matching to the data of Ref. [4].

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