Joint Beamforming and Dynamic Relay Positioning for mmWave Urban Communications

Anastasios Dimas, Dionysios S. Kalogerias, Athina P. Petropulu

1Department of Electrical & Computer Engineering, Rutgers, The State University of New Jersey
2Department of Operations Research & Financial Engineering (ORFE), Princeton University

Abstract—The use of millimeter wave (mmWave) spectrum for commercial wireless communications is expected to offer data rates in the order of Gigabits-per-second, thus able to support future applications such as Vehicle-to-Vehicle or Vehicle-to-Infrastructure communication. However, especially in urban settings, mmWave signal propagation is sensitive to blockage by surrounding objects, resulting in significant signal attenuation. One approach to mitigate the effect of attenuation is through multi-hop communication with the help of relays. Leveraging the unique characteristics of the mmWave medium, we consider a single-source/destination 2-hop system, where a cluster of spatially distributed and reconfigurable relays is used to cooperatively amplify-and-forward the source signal to the destination. Our system evolves in time slots, during which not only are optimal beamforming weights centrally determined, but also future relay positions for the subsequent time slot are optimally selected, jointly maximizing the expected signal-to-interference-noise ratio at the destination. Optimal predictive relay positioning is achieved by formulating a 2-stage stochastic programming problem, which is efficiently approximated via a conditional sample-average-approximation surrogate, and solved in a purely distributed fashion across relays. The efficacy of the proposed near-optimal positioning policy is corroborated by against a randomized relay positioning policy, clearly confirming its superiority.

Index Terms—mmWave, Spatially Controlled Relay Beamforming, Stochastic Programming, V2I, V2V.

I. INTRODUCTION

The fifth generation (5G) of commercial wireless networks will use frequencies from the millimeter wave (mmWave) spectrum for outdoor communication. The larger bandwidth allocation will allow for data rates in the order of Gigabits-per-second (Gbps), which can support potential 5G applications such as Vehicle-to-Vehicle (V2V) or Vehicle-to-Infrastructure (V2I) communication. However, especially in urban environments, exploiting the full potential of the mmWave spectrum requires a number of challenges to be overcome, the most crucial of which is their sensitivity to blockage from surrounding objects, e.g. high-rise buildings and Line-of-Sight (LoS) directions along street canyons, reflecting off buildings constructed of concrete and glass, in a way analogous to ray optics. Moreover, since mmWaves also attenuate faster than sub-6 GHz signals, appropriate beamforming is needed to compensate for the propagation path-loss.

When the signal’s path is blocked, relaying can provide alternative paths for the signal to reach its destination. By relaying the blocked signals, a larger coverage area in the network is achieved. Determining the available and capable relays to support such a multi-hop transmission to the destination requires appropriate signaling and scheduling. The communication overhead multi-hop transmission brings, along with other drawbacks such as increased energy consumption and potential security issues, have promoted the use of 2-hop solutions when possible.

In this paper, we propose a new joint beamforming and adaptive relay positioning scheme which supports mmWave communications in an urban environment using a 2-hop cooperative relay network. More specifically, we consider a given cluster of spatially distributed relays, initially deployed on distinct road segments and at street level with the communication endpoints, that collaboratively beamform towards the destination such that the perceived Quality-of-Service (QoS), in terms of expected signal-to-interference-noise ratio (SINR), is maximized. Each individual relay of the cluster is physically constrained to be located at a discrete set of positions along a segment. The proposed dynamic positioning scheme can be viewed either as a static relay/antenna selection problem (e.g., an access point with multiple distributed antennas), or a mobile relay positioning problem (in the spatially controlled spirit of ). Actually, our adaptive positioning scheme is not tied to the particular implementation. Therefore, hereafter, we use the terms relay and antenna interchangeably.

We note that an important issue due to the cooperative nature of our scheme, is carrier synchronization between the relays and the destination. In principle, this can be mitigated using protocols proposed for sub-6 Ghz frequencies; those will have to be adapted to the mmWave regime, in order to calibrate carrier phase offsets. Synchronization can also be achieved through an optical fiber backhaul network connecting the relays with each other.

Under the proposed urban communication setting, this paper makes the following contributions:

1) We formulate the propagation model for the given network topology, efficiently utilizing the diversity of the alternative signal paths arriving at a relay or the destination. Our model accounts for time and space evolving path-loss, shadowing and multi-path effects, both in amplitude and phase. Specifically for the segments that contain relays, we propose a new channel correlation kernel describing the space-time evolution of the shadowing process, which effectively models the statistical dependencies of the corresponding source-relay and relay-destination channels.
2) Inspired by [12], the joint relay positioning and beamforming task is formulated as a 2-stage distributed stochastic program, executed in a sequential fashion, assuming a time-slotted system. At every time slot, the optimal beamforming weights are centrally determined, whereas each relay predicts its future position for the subsequent time slot, in a distributed fashion. We propose a near-optimal relay positioning policy, based on the Sample Average Approximation (SAA) method [15], which results in a tractable optimization problem, solved separately at each relay, formulated via conditional Monte Carlo sampling. Our numerical simulations confirm the success of the proposed approach, by showing its profound superiority against a random relay positioning policy.

II. mmWave Urban Channel Model

Our considered urban city topology is shown in Fig. 1. The circles depict the respective numbered intersections $I_i$, $i = 1, \ldots, 36$. The edges connecting two intersections represent street canyons, and are referred to as a full segment of length $d_{f\text{full}}$. The signal source of the network is located at $p_S$, the set of relays are placed at positions $p_r$, $r = 1, \ldots, N_R$, and the destination node is located at $p_D$. All nodes are equipped with a single directional antenna to compensate for the mmWave path-loss. The relays are physically constrained to be positioned only to a discrete set of possible locations in the neighborhood $\mathcal{C}(p_r)$ along the segment they are initially placed, but cannot be located on an intersection.

Due to assumed blockage by high-rise buildings, the signal sent from $p_S$ to $p_D$ is able to propagate only by traversing the street segments of Fig. 1. The set of segments traversed from $p_S$ to $p_D$ is called the propagation path of the mmWave signal. We adapt the convention of [16], where LoS and Non-LoS (NLoS) indicate the traversed order of the segments in a propagation path, such that all propagation paths are those that travel along the LoS segment in the direction towards the receiver (see Fig. 1).

Let $L_r^f$ be the number of all possible signal paths from $p_S$ to the segment of relay $r = 1, \ldots, N_R$. Under the flat fading assumption, at a point $p$ in the segment $r$, the complex channel gain from $p_S$ to $p_r$ along path $i = 1, \ldots, L_r^f$ can be decomposed as [17]

$$f_{r_i}(p, t) = f_{r_i}^{PL}(p) \cdot f_{r_i}^{SH}(p, t) \cdot f_{r_i}^{MF}(p, t),$$

where $f_{r_i}^{PL}(p)$ is the path-loss component, $f_{r_i}^{SH}(p, t)$ the large-scale fading component (shadowing), and $f_{r_i}^{MF}(p, t)$ the small-scale fading component (multi-path). A similar decomposition holds for the physical channel from $p_r$ to $p_D$, i.e., there are $L_r^p$ possible signal paths, while for path $i = 1, \ldots, L_r^p$, the channel can be decomposed as $g_{r_i}(p, t)$.

The Manhattan distance (in meters) from $p_S$ to $p_r$ is $d_{r}^f = \|p_S - p_r\|_1$, while from $p_r$ to $p_D$ it is $d_{r}^p = \|p_r - p_D\|_1$. Each propagation path from $p_S$ to $p_r$ consists of $K_r^f$ segments, i.e., a LoS segment, $K_r^f - 2$ full segments, and 1 upon which $p_r$ is located. In accordance to the free space model, every individual segment has a path-loss term. We also assume an additional loss $\Delta$ observed at every intersection, i.e. every path exhibits an intersection loss $\Delta \cdot N_r^f$, where $N_r^f$ are the number of traversed intersections from $p_S$ to $p_r$. Therefore, the overall path-loss component of the channel is expressed as

$$f_{r_i}^{PL}(p) = 10^{\frac{\Delta}{10} \cdot \sum_{j=2}^{K_r^f} d_{r_i}^{\text{full}}},$$

where $d_{r_i}^{\text{full}}$ denotes the length of segment $j$ of path $i$. With that in mind, $d_{r_i}^{\text{full}}$ is the LoS portion of the path, $d_{r_i} = d_{r_i}^{\text{full}}, j = 2, \ldots, K_r^f - 1$, and $d_{r_i}^{\text{full}} = (d_{r_i} - d_{r_i}^{\text{full}}) \mod d_{r_i}^{\text{full}}$. We assume a relay is never located on an intersection, so $d_{r_i}^{\text{full}} \neq 0$.

Similarly, every segment of the path has its individual shadowing and multi-path terms. For both the LoS and NLoS cases, we assume a log-normal distribution for modeling the shadowing and multi-path fading [3]. So, the overall shadowing and multi-path components of the channel are respectively

$$f_{r_i}^{SH}(p, t) = s_{r_i K_r^f}(p, t) \cdot \prod_{j=1}^{K_r^f-1} s_{r_i j}(t),$$

$$f_{r_i}^{MF}(p, t) = \prod_{j=1}^{K_r^f} q_{r_i j}(t),$$

where $s_{r_i j}$ and $q_{r_i j}$ are the shadowing and multi-path terms of segment $j = 1, \ldots, K_r^f$, that belong to path $i$ from $p_S$ to $p_r$. Working in the log-scale of (1), we define

$$F_{r_i}(p, t) \triangleq a_{r_i}^F(p) + b_{r_i}^F(p, t) + c_{r_i}^F(p, t),$$

where

$$-a_{r_i}^F(p) \triangleq a_L 10^{\log_{10} d_{r_i}^{\text{full}} + \sum_{j=2}^{K_r^f-1} 10^{\log_{10} d_{r_i}^{\text{full}} + \Delta N_r^f}},$$

$$b_{r_i}^F(p, t) \triangleq \sum_{j=1}^{K_r^f-1} 10^{\log_{10} |s_{r_i j}(t)|^2} + 10^{\log_{10} |s_{r_i K_r^f}(p, t)|^2} - \sum_{j=1}^{K_r^f-1} \beta_{r_i j}(t) + \beta_{r_i K_r^f}(p, t),$$

$$c_{r_i}^F(p, t) \triangleq \sum_{j=1}^{K_r^f} 10^{\log_{10} |q_{r_i j}(t)|^2},$$

We also assume a phase term $e^{i \theta_{r_i j}}$ for each segment of the path, where each $\phi_{r_i j}$ is uniformly distributed in $[0, 1]$. Then, the channel $f_{r_i}(p, t)$ can be reconstructed as

$$f_{r_i}(p, t) = e^{-a_{r_i}^F(p) + b_{r_i}^F(p, t) + c_{r_i}^F(p, t) + \sum_{j=1}^{K_r^f} \theta_{r_i j}(t)}.$$
is itself zero mean and jointly Gaussian in time, with the correlation between time slots \( k \) and \( l \) given by

\[
B_{FF}(k, l) = \eta^2 e^{-|k-l|/\gamma},
\]

(10)

where \( \eta^2 \) is the shadowing power and \( \gamma \) the correlation time.

Likewise, the shadowing term \( \beta_F^{r Ki}(p, t) \) corresponding to the segment containing relay \( r \) is assumed to be jointly Gaussian in space and time. Specifically, we assume that \( r \) can be located at a discrete set of \( \delta \) positions within the segment. At two such positions \( p_m \) and \( p_n \), the spatial correlation of the shadowing follows Gumbandin’s model \[18], which we extend to also describe temporal correlations between time slots \( k \) and \( l \). The assumed ensemble correlation model is

\[
K_{FF}(m, n, k, l) = \eta^2 e^{-\frac{\|p_m(k) - p_n(l)\|^2}{\gamma}}.
\]

(11)

We further assume that, for a segment containing a relay, the channels \( f_r \) and \( g_r \) at positions \( p_m \) and \( p_n \) are themselves correlated between time slots \( k \) and \( l \), according to the kernel

\[
K_{FG}(m, n, k, l) =
\begin{cases}
\eta^2 e^{-\frac{\|p_m(k) - p_n(l)\|^2}{\gamma} - \frac{|k-l|}{\gamma}}, & d_o \geq d_{full}
\end{cases}
\]

(12)

where \( d_o = d_{riKi}(k) + d_{riKi}(l) \) and \( d_{max} \) is the furthest possible distance between two discrete positions in a segment. Intuitively, this kernel describes the correlation between the two channels as a result of their proportional overlap due to the relay positioning. Further details on the rationale of \[12\] will be given in an extended journal version of this work.

III. OPTIMAL RELAY POSITIONING WITH BEAMFORMING

We adapt the 2-stage stochastic programming approach of \[12\], which optimally selects beamforming weights and relay or antenna positions in every time slot. The rationale behind this approach is to decompose the optimization task at each time slot into a beamforming stage and a relay control stage.

A. 2-hop mmWave Cooperative Communication

Although it is possible that only a single segment between \( p_S \) and \( p_D \) exists, in general, relays are needed to assist the communication. The whole network operates for \( N_T \) time slots, or operates infinitely in \( N_T \) length windows.

At each time slot \( t = 1, \ldots, N_T \), the source at \( p_S \) transmits the signal \( \sqrt{P_S s(t)} \), where \( s(t) \) is the information symbol with \( \mathbb{E}[|s(t)|^2] = 1 \), and \( P_S > 0 \) the transmission power. The signal received at each relay is

\[
R_r(t) = \sum_{l=1}^{L_r} \sqrt{P_S} f_{r, l}(s(t)) + n_r(t),
\]

where \( n_r(t) \sim \mathcal{CN}(0, \sigma^2) \) is the noise at relay \( r \). Working in an Amplify-and-Forward (AF) fashion, each relay then modulates the received signal \( R_r(t) \) by a complex weight \( w_r(t) \) and re-transmits it. We should note that a mmWave signal arriving at \( p_D \) from a propagation path that was not forwarded from a relay is considered to be of negligible power and therefore not taken into consideration. The signal received at the destination is

\[
y_D(t) = \sqrt{P_S} \sum_{r=1}^{N_R} w_r(t) \sum_{k=1}^{L_r} g_{r, k}(f_{r, l}(s(t)) + n_D(t),
\]

\[
\sum_{r=1}^{N_R} w_r(t) \sum_{k=1}^{L_r} g_{r, k}(n_r(t) + n_D(t),
\]

(13)

where \( n_D(t) \sim \mathcal{CN}(0, \sigma_D^2) \) is the noise at \( p_D \). As explained earlier, the relays are synchronized, so their transmitted signals reach the destination at the same time.

B. Optimal Beamforming

We extend the relay beamforming schemes studied in \[19\]. \[20\], to the significantly more complex setting of urban mmWave relay networks. Here, beamforming is considered for enforcing relay cooperation, such that all individual signal paths forwarded from all relays are combined constructively at the destination.

In particular, at time slot \( t \), the goal is to obtain the relay beamforming weights \( w(t) = [w_1^T(t), \ldots, w_{N_R}^T(t)]^T \in \mathbb{C}^{N_R \times 1} \), such that the SINR at \( p_D \) is maximized subject to a total power budget of the relays. We consider the vectors

\[
f_{r_s}(p, t) \triangleq [f_{r_1}(p, t), \ldots, f_{r_{L_r}}(p, t)]^T \in \mathbb{C}^{L_r \times 1},
\]

(14)

\[
g_{r_s}(p, t) \triangleq [g_{r_1}(p, t), \ldots, g_{r_{L_r}}(p, t)]^T \in \mathbb{C}^{L_r \times 1},
\]

(15)

\[ r = 1, \ldots, N_R \]. Then, the SINR is maximized by solving \[19\]

\[
\begin{align*}
\text{maximize} & \quad \mathbb{E}\left[ w(t)^H R w(t) \right], \\
\text{subject to} & \quad \mathbb{E}[w(t)^H Q w(t)] + \sigma_D^2, \\
& \quad \mathbb{E}[w(t)^H D w(t)] \leq P_C,
\end{align*}
\]

(16)

where, after removing \( (p, t) \) for brevity,

\[
R \triangleq P_S h h^H, \quad h \triangleq [1^T g_1, 1^T g_{N_R}, \ldots, 1^T g_{N_R}, 1^T f_{N_R}, 1^T f_{N_R}]^T,
\]

(17)

\[
D \triangleq P_S \text{diag} \left[ |1^T f_1|^2, \ldots, |1^T f_{N_R}|^2 \right] + \sigma^2 I_{N_R}, \quad \text{and}
\]

(18)

\[
Q \triangleq \sigma^2 \text{diag} \left[ |1^T g_1|^2, \ldots, |1^T g_{N_R}|^2 \right].
\]

(19)
The scalar $P_C > 0$ is the available transmission power of all the relays and $1$ the all ones vector. The optimal value of \( V \) can be expressed as \[ V(t) = \sum_{r=1}^{N_R} \frac{P_C P_S |\mathbf{T}_r|^{2} |\mathbf{g}_r|^{2}}{P_S \sigma_D^2 |\mathbf{T}_r|^{2} + P_C \sigma_D^2 |\mathbf{g}_r|^{2} + \sigma_D^2} \]
\[ = \sum_{r=1}^{N_R} V_f(S_r(p_r, t)), \tag{20} \]
where $S_r(p_r, t)$ is a vector of all random variables describing the shadowing, multi-path, and phase terms of the unique segments traversed from all paths through $p_r$. The optimal beamforming vector that achieves \( V \) is \[ \hat{\mathbf{w}}^*(t) = \sqrt{P_C \mathbf{D}^{-\frac{1}{2}}} \times \lambda_{max} \left( (\sigma_D^2 \mathbf{I}_{N_R} + P_C \mathbf{D}^{-\frac{1}{2}} \mathbf{Q} \mathbf{D}^{-\frac{1}{2}})^{-1} \mathbf{D}^{-\frac{1}{2}} \mathbf{R} \mathbf{D}^{-\frac{1}{2}} \right)^{\frac{1}{2}}, \tag{21} \]
where $\lambda_{max}$ is the normalized principal eigenvector operator.

**C. Optimal Myopic Relay Positioning**

At time slot $t$, optimal (in some sense) relay positions at $t+1$ should be decided, such that $V(t+1)$ is maximized. However, exact knowledge of $V(t+1)$ would require knowledge of the channel state information (CSI) at $t+1$, for all feasible relay positions. As this information is not available, a reasonable causal criterion for optimal relay positioning is to maximize the MMSE predictor $E[V(t+1)|C_r(T_t)]$. From \( \text{V} \), this means that each relay $r$ independently solves
\[
\begin{align*}
\text{maximize}_{p} & \quad E[V_f(S_r(p, t+1)|C_r(T_t))] \\
\text{subject to} & \quad p \in \mathcal{C}(p_r(t))
\end{align*}
\tag{22}
\]
where $p_r(1)$ is the initial positioning decision at $t = 1$, and $\mathcal{C}(p_r(\cdot))$ constitutes the neighborhood around $p_r(\cdot)$, within which the relay is physically constrained to be positioned.

At every $p \in \mathcal{C}(p_r(t))$, the objective of \( \text{V} \) can be expressed as
\[
E[V_f(S_r(p, t+1)|C_r(T_t))] = \int V_f(s)p_S_r(p, t+1)|C_r(T_t)(s) \, ds, \tag{23}
\]
where $p_S_r(p, t+1)|C_r(T_t)$ constitutes the joint pdf of all random variables of vector $S_r(p, t+1)$, conditioned on $C_r(T_t)$. These consist of Gaussian random variables corresponding to the sum of the shadowing and multi-path channel components, with mean zero and covariance matrix derived from \( \text{I}, \text{II}, \text{III} \), as well as independent uniform in $[0, 2\pi]$ random variables corresponding to the phase components.

Since a closed-form representation of the objective of \( \text{V} \) is impossible to derive, we resort to an approximate, near-optimal approach. In particular, we rely on the SAA method, which replaces \( \text{V} \) by an easily computable surrogate, based on conditional Monte Carlo sampling.

For every individual relay $r$, the SAA method proceeds as follows. First, a total of $N_S$ SAA scenarios are generated for vector $S_r(\cdot, t+1)$, drawn from the distribution $p_S_r(p, t+1)|C_r(T_t)$. Then, at every candidate position $p \in \mathcal{C}(p_r(t))$, each scenario $S_r^{(i)}, i = 1, \ldots, N_S$ is used to evaluate $V_f(S_r^{(i)})$. The SAA of \( \text{V} \) is formulated as
\[
\text{maximize}_{p} \quad \frac{1}{N_S} \sum_{i=1}^{N_S} V_f(S_r^{(i)}), \tag{24}
\]
subject to $p \in \mathcal{C}(p_r(t))$

which may be solved by enumeration. The optimal solution of \( \text{V} \) corresponds to the relay position chosen at $t+1$. The proposed 2-stage scheme is described in Algorithm 1.

**Algorithm 1: Joint Beamforming & SAA-based Relay Control**

1. for $t = 1 : N_T$
2. at relay $r$:
3. Beamforming (2nd stage of $t$-th slot)
4. Compute optimal $\hat{\mathbf{w}}(t)$ from \( \text{V} \)
5. Relay Positioning (1st stage of $(t+1)$-th slot)
6. for each $p \in \mathcal{C}(p_r(t))$
7. Generate $S_r^{(i)} \sim p_S_r(\cdot, t+1)|C_r(T_t), i = 1, \ldots, N_S$
8. Compute $V_f(p)$ from \( \text{V} \)
9. end for
10. Choose $p_r(t+1) = \arg\max_{p \in \mathcal{C}(p_r(t))} V_f(p)$
11. end for

**IV. SIMULATIONS**

In this section, we examine the performance of our 2-stage relay positioning scheme using synthetically generated CSI. For our simulations, we assume the topology and relay placement that is depicted in Fig. 1. All segments are of length $d_{\text{rel}} = 100m$. The segments containing relays are divided into $\delta = 50$ discrete positions upon which the relay can be located. The channel parameters are $a_L = a_N = 1.5, n^2 = 100, \gamma = 15, \beta = 20, \sigma_D = 1$. The transmission power is $P_S = 100dB$ and the total relay power budget is $P_C = 100dB$.

To assess the long term system performance using the SAA approach, we compare it against two benchmark relay positioning policies. The first is the oracle policy, where at time slot $t$ each relay is also aware of its immediate future CSI at $t+1$, and chooses, from all possible candidate relay positions along its segment, the one that maximizes the quantity $V_f(S_r(\cdot, t+1))$. Clearly, this policy is not implementable in reality, but nevertheless provides an upper bound on the performance of any admissible policy. The second benchmark policy is the agnostic one, where the relay randomly chooses its subsequent position within the assumed neighborhood, irrespective of the observed CSI. If the neighborhood is unconstrained (that is, the whole segment), this policy provides a lower bound for any relay positioning policy.

All three relay policies were executed for $N_T = 50$ time slots, with $N_S = 500$ SAA scenarios. This constitutes one trial of the whole communication task. Initially, we consider the case of an unconstrained neighborhood (for the SAA and agnostic policies). For each positioning policy, the QoS achieved in each time slot was averaged over 10000 trials, as shown in Fig. 2. The superiority of the SAA policy
is evident, as it achieves on average 9.76dB higher SINR than the agnostic policy, while also performing on average 3.52dB worse than the oracle. In Fig. 2b, a similar experiment is shown, where now a smaller positioning neighborhood is considered, in the range \([p_r(t) - 16m, p_r(t) + 16m]\). We observe that, on average, the SAA performs 8.16dB better than the agnostic, as well as it being 5.92dB away from the oracle. Note that, in this case, the agnostic policy is no longer a universal lower bound, as it is possible its neighborhood is different from that of SAA.

V. CONCLUSION

This paper considered a set of spatially distributed relays that cooperatively enhances mmWave communications in an urban environment. The assumed 2-hop network operates in a time slotted fashion. We have proposed a dynamic relay positioning scheme, guaranteeing maximal QoS in every observed time slot. This was achieved by solving a 2-stage stochastic programming problem, where relay beamforming weights and positions are optimally updated, on a per time slot basis. Relay positions are determined using a conditional SAA-based surrogate approach, resulting in a near-optimal positioning policy, which is implemented in a distributed fashion, locally at each relay. The effectiveness of our approach is established via numerical simulations, which corroborate its superiority against an CSI-agnostic, purely randomized relay positioning policy, as well as its efficiency relative to a reference benchmark oracle policy.

REFERENCES

[1] Federal Communications Commission, “Spectrum frontiers report and order and further notice of proposed rulemaking: FCC16-89,” 2016.
[2] S. Rangan, T. S. Rappaport, and E. Erkip, “Millimeter-wave cellular wireless networks: Potentials and challenges,” *Proceedings of the IEEE*, vol. 102, no. 3, pp. 366–385, 2014.
[3] H. Zhao, R. Mayzus, S. Sun, M. Samimi, J. K. Schulz, Y. Azar, K. Wang, G. N. Wong, F. Gutierrez, and T. S. Rappaport, “28 GHz millimeter wave cellular communication measurements for reflection and penetration loss in and around buildings in New York city,” in *Communications (ICC), 2013 IEEE International Conference on*. IEEE, 2013, pp. 5163–5167.
[4] A. F. Molisch, A. Karttunen, R. Wang, C. U. Bas, S. Hur, J. Park, and J. Zhang, “Millimeter-wave channels in urban environments,” in *IEEE 10th European Conference on Antennas and Propagation*, 2016, pp. 1–5.
[5] T. S. Rappaport, G. R. MacCartney, S. Sun, H. Yan, and S. Deng, “Small-scale, local area, and transitional millimeter wave propagation for 5G communications,” *IEEE Transactions on Antennas and Propagation*, vol. 65, no. 12, pp. 6474–6490, 2017.
[6] T. S. Rappaport, S. Sun, R. Mayzus, H. Zhao, Y. Azar, K. Wang, G. N. Wong, J. K. Schulz, M. Samimi, and F. Gutierrez Jr, “Millimeter wave mobile communications for 5G cellular: It will work!” *IEEE access*, vol. 1, no. 1, pp. 335–349, 2013.
[7] S. Biswas, S. Vuppala, J. Xue, and T. Ratnarajah, “On the performance of relay aided millimeter wave networks,” *IEEE Journal of Selected Topics in Signal Processing*, vol. 10, no. 3, pp. 576–588, 2016.
[8] T. Bai and R. W. Heath, “Coverage and rate analysis for millimeter-wave cellular networks,” *IEEE Transactions on Wireless Communications*, vol. 14, no. 2, pp. 1100–1114, 2015.
[9] X. Lin and J. G. Andrews, “Connectivity of millimeter wave networks with multi-hop relaying,” *IEEE Wireless Commun. Letters*, vol. 4, no. 2, pp. 209–212, 2015.
[10] Q. Hu and D. M. Blough, “Relay selection and scheduling for millimeter wave backhaul in urban environments,” in *14th International Conference on Mobile Ad Hoc and Sensor Systems*. IEEE, 2017, pp. 206–214.
[11] S. Wu, R. Atat, N. Mastronarde, and L. Liu, “Improving the coverage and spectral efficiency of millimeter-wave cellular networks using device-to-device relays,” *IEEE Transactions on Communications*, vol. 66, no. 5, pp. 2251–2265, 2018.
[12] D. Kalogerias and A. Petropulu, “Spatially controlled relay beamforming,” *IEEE Transactions on Signal Processing*, 2018.
[13] R. Mudumbai, D. R. B. Iii, U. Madhow, and H. V. Poor, “Distributed transmit beamforming: challenges and recent progress,” *IEEE Communications Magazine*, vol. 47, no. 2, pp. 102–110, 2009.
[14] “Deutsche Telekom, Ericsson set out to prove mmWave suitable for fiber-like backhaul,” https://www.fiercewireless.com/wireless/deutsche-telekom-ericsson-set-out-to-prove-mmWave-suitable-for-fiber-like-backhaul, Last access, Feb. 2019.
[15] A. Shapiro, D. Dentcheva, and A. Ruszczyński, *Lectures on stochastic programming: modeling and theory*. SIAM, 2009.
[16] Y. Wang, K. Venugopal, R. W. Heath, and A. F. Molisch, “MmWave vehicle-to-infrastructure communication: Analysis of urban microcellular networks,” *IEEE Transactions on Vehicular Technology*, 2018.
[17] R. W. Heath, *Introduction to Wireless Digital Communication: A Signal Processing Perspective*. Prentice Hall, 2017.
[18] M. Gudmundson, “Correlation model for shadow fading in mobile radio systems,” *Electronics letters*, vol. 27, no. 23, pp. 2145–2146, 1991.
[19] V. Havary-Nassab, S. Shahbazpanahi, A. Grami, and Z.-Q. Luo, “Distributed beamforming for relay networks based on second-order statistics of the channel state information,” *IEEE Trans. Signal Processing*, vol. 56, no. 9, pp. 4306–4316, 2008.
[20] G. Zheng, K.-K. Wong, A. Paulraj, and B. Ottersten, “Collaborative-relay beamforming with perfect CSI: Optimum and distributed implementation,” *IEEE Signal Processing Letters*, vol. 16, no. 4, pp. 257–260, 2009.