Channel feedback for multi-user (MU)-multiple-input multiple-output (MIMO) has been widely studied and some results have been got with random vector quantization scheme. However, while the low rate fixed codebook feedbacks are adopted, the performance of zero forcing (ZF) MU-MIMO will decrease as the unpredictable inter-user interference is introduced because of quantized channel state information (CSI). To decrease inter-user interference in low rate fixed codebook feedback, an enhanced user selection switch (USS) feedback scheme for ZF MU-MIMO is proposed in this article. In USS feedback, the extra USS information is added after quantized CSI and received signal-to-noise ratio feedback. The USS information indicates inter-user interference and it can be used in user selection procedure to avoid large inter-user interference. Simulation results show that the proposed USS feedback scheme is efficient to solve the problems of unpredictable inter-user interference in conventional feedback scheme with low rate codebook in ZF MU-MIMO.

Keywords: MU-MIMO, feedback, user selection, user pairing
the unit sphere. There are some disadvantages for RVQ scheme in the actual communication system:

(1) It needs a great deal feedback bits in the case of high SNR and large number of transmit antennas [16-18]. For example, while SNR is 10 dB with 4 transmit antennas, it needs about 14 bits (16,384 codebooks) and while SNR is 20 dB with 8 transmit antennas, it needs about 35 bits (34,359,738,368 codebooks).

(2) The codebook needed in RVQ scheme should randomly be generated by UE before CSI feedback, and then the codebook is sharing with BS through feedback channel. So, the large codebook number will also increase feedback overhead of codebook sharing, the computational complexity of codebook generation, and cache costs of codebook storage.

(3) RVQ needs different quantized bits for different SNR cases, so it will bring some design problems. For examples, if the feedback bits are fixed, it will cause waste for low SNR case and not enough for high SNR case. If feedback bits are flexible, new codebook will be retransmitted while SNR changed and it will decrease the effects of user selection between UEs with different SNR.

Moreover, most of the current communication system adopt small codebook size and fixed codebook structure, which both known by UE and BS, to reduce the system complexity feedback overhead. In this feedback scheme, the former performance analysis for RVQ will be not suitable. In low rate fixed codebook feedback scheme, the interference between paired users is the key problem and conventional feedback and user selection scheme have on mechanism to avoid large inter-user interference. To overcome this drawback in low rate fixed codebook feedback scheme, the reasons of large inter-user interference are analyzed detailed and an enhanced scheme named user selection switch (USS) feedback is proposed here. The USS feedback adds some extra information besides CSI and SNR to show the inter-user interference while performing ZF MU-MIMO transmission. With USS information, BS can avoid large inter-user interference in MU-MIMO transmission in user selection procedure and enhance MU-MIMO performance.

The rest of the article is organized as follows. Section 2 introduces conventional MU-MIMO transmission model and analyzes the problem of low rate fixed codebook feedback scheme. Section 3 proposes USS feedback to enhance MU-MIMO performance and gives related user selection procedure. Section 4 gives the numerical simulation to verify the performance enhancement. Section 5 provides some conclusions.

2. System model
In this article, the single cell MIMO downlink channel is considered, in which the transmitter has $M$ antennas and each UE has 1 antenna. Each user only receives one data stream, and at most $M$ users can be communicated at the same time. The system model is shown in Figure 1. In conventional feedback, only SNR and CSI are fed back to BS.

The signal received by a single user $i$ can be represented as

$$y_i = \sqrt{\frac{P_i}{\sigma^2}} g_i x_i + \sum_{j \neq i} \sqrt{\frac{P_j}{\sigma^2}} g_j x_j + n_i,$$  

where $g_i$ is pathloss between BS and UE, $H_i \in \mathbb{C}^{1 \times M}$ is the normalized channel matrix between BS and UE, $x_i$ is the transmitted signals with an average power constraint $E[|x_i|^2] = P_i$, $||\cdot||$ stands for norm operator, $P_i$ is the power constraint of each user’s data stream, $n_i$ is the additive white Gaussian noise with $\sigma^2$ variance, and $y_i$ is the signal received by UE.

The procedure of conventional ZF MU-MIMO is as follows [10,18].

2.1. Quantized CSI feedback
It assumed that each user knows perfect CSI and normalized it to a unit norm vector $H_i$. The quantization vector is chosen from a fixed codebook of size $N = 2^K$.

$$C = \{c_1, \cdots, c_N\}, \quad (c_j \in \mathbb{C}^{1 \times M}, N = 2^K).$$  

The codebook $C$ is designed offline and both known to the BS and UE. UE will select a vector from codebook according to the minimum distance criterion as following equation,

$$k = \arg \max_{1 \leq j \leq N} || H_i c_j^H ||.$$  

Then the index $k$ is fed back to BS, and BS treats $c_k$ as the channel matrix $H_i$ of UE.

2.2. SNR Feedback
Each user will feed back its received SNR with assumption of single user transmission. The SNR of users is

$$\text{SNR}_i = \frac{|| \sqrt{\frac{P_i}{\sigma^2}} g_i x_i ||^2}{\sigma^2} = g_i P_i / \sigma^2.$$  

UE can measure it by reference signals (RS), as the RS sequence and its power are known to UE. In the practical system, this information is quantized with small number of bits. In order to concentrate on the effect of CSI quantization and user selection, it assumes that the SNR is directly fed back without quantization.

2.3. User selection
After BS received feedback, it will select some paired users from serving user set $U = \{\text{UE}_1, \ldots, \text{UE}_K\}$, which is
correspond to all the users served by BS. The number of selected users is determined by higher layer and must be no more than \( m \) which is the number of transmit antennas. There have been many proposed user selection criteria \([20-25]\) and the basic principle is to maximize the total throughputs of the paired users. It is known that in MIMO transmission, the higher throughput will be gotten with the smaller channel correlation between paired users. So, in the simulation of conventional MU-MIMO in the article, BS will select users which have the minimal spatial channel correlation between each other. That’s means the maximum correlation between selected users will be minimal in all possible MU-MIMO user combinations. The user selection criterion can be expressed as

\[
\min_{V} \max_{i,j \in \mathcal{V}_{ij}} | H_i H_j^H |,
\]  

where \(| \cdot |\) stands for absolute value, \((\cdot)^H\) stands for Hermite transpose, \( V \) is paired user set in which the users are scheduled together to form MU-MIMO.

### 2.4. ZF precoding

After the paired user set \( V \) is determined, BS will calculate the precoding matrix for these paired users. The precoding matrix is computed by ZF methods:

\[
(p_1 \ldots p_M) = \left( \begin{array}{c} w_1 \\ \vdots \\ w_M \end{array} \right)^{*},
\]  

where \( p_i \) is precoding vector of UE \( i \), \( w_i \) is the quantized CSI of UE \( i \), \((\cdot)^{+}\) stands for pseudo-inverse operation. So, the received signals of uses in set \( V \) are

\[
\begin{bmatrix} y_1 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} \sqrt{g_1 H_1} \\ \vdots \\ \sqrt{g_M H_M} \end{bmatrix} \begin{bmatrix} p_1 \cdots p_M \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_M \end{bmatrix},
\]  

where \( a_i \) is coefficient scaling factor, \( b_i \) is power allocation factor, and \( s_i \) is transmit symbols with unit variance. The total power should be no more than max transmit power \( P_{\text{total}} \), and the constraint is

\[
\sum_{i=1}^{M} \frac{b_i^2}{a_i^2} \| p_i \| ^2 = P_{\text{total}},
\]  

where \( \| \cdot \| \) stands for Euclidean norm. The received signal of each user is

\[
y_i = \sqrt{g_i H_i} \frac{b_i}{a_i} s_i + \sqrt{g_i H_i} \sum_{j=1, j \neq i}^{M} \frac{b_j}{a_j} s_j + n_i,
\]
where $\sqrt{g_i R^H_i p_i s_i}$ is wanted signal and $\sqrt{g_i R^H_i \sum_{j=1}^{M} \beta_{i,j} p_j s_j}$ is inter-user interference.

2.5. MU-MIMO performance with conventional feedback

The user SNR of MU-MIMO is

$$\text{MU-SNR}_i = \frac{g_i^2}{\alpha_i} \| H_i p_i \|^2 \sum_{j=1,j\neq i}^{M} \frac{\beta_{i,j}^2}{\alpha_j} \| H_j p_j \|^2 + \sigma_i^2.$$  \hfill (11)

The total throughput is

$$R_{\text{MU}} = \sum_{i=1}^{m} \log(1 + \text{MU-SNR}_i).$$  \hfill (12)

2.6. The problems of conventional feedback

In the conventional feedback scheme, BS and UE cannot know the MU_SNR clearly. For $\text{UE}_i$, it knows its channel matrix $H_i$ but does not know the channel of paired users. For BS, it knows paired users, but does not know exact channel matrix of UEs. So, the $\|H_i p_i\|^2$ cannot be known for BS and UE. Hence, the transmitting rate $R$ is evaluated in conventional user selection.

Usually $R$ is evaluated with the assumption of no inter-user interference, which means $\|H_i p_i\|^2 = 0$. But for the paired user, the inter-user interference may be very large and lead the performance decrease heavily, while $\|H_i p_i\|^2 \gg 0$. In user pairing, BS does not know the exact inter-user interference, so it has no mechanism to avoid large inter-user interference in user selection criteria.

The large inter-user interference will decrease throughput largely. For example, if the inter-user interference $\|H_i p_i\|^2$ is more than 0.0833 in the configuration of 2Tx, 2 paired UE, 10 dB SNR, the sum rate of MU-MIMO will less than SISO transmission. And the inter-user interference should be smaller in high SNR region than in low SNR region. Unfortunately, the inter-user interference usually is not small enough for MU-MIMO requirement in low fixed codebook scheme. Figure 2 shows the CDF of inter-user interference with 4 bits DFT codebook while the quantized CSI of paired user is orthogonal. It can be seen that about 50% of inter-user interference are more than 0.1; so many users are paired with large inter-user interference. Although the MU-MIMO will not work well with the large inter-user interference, the conventional feedback and user selection method cannot provide enough information to distinguish large inter-user interference and small inter-user interference.

These will cause two serious problems:

1. The performance gain of MU-MIMO will decrease, especially in high SNR case. Figure 3 shows the MU-MIMO (two paired users) performance of 4 bits feedback with DFT codebook, compared to SISO case and perfect CSI feedback. It can be seen that MU-MIMO with perfect CSI feedback has very high rate about double of that in SISO case. But for low rate quantized feedback (4 bits), the performance gain falls largely compared to perfect CSI feedback, as the CSI is the quantized version with low codebook size. The performance gain is little at high SNR region because the inter-user interference of paired users is randomly in quantized feedback with conventional user selection methods, and MU-MIMO performance is sensitive to inter-user interference in high SNR case.

2. While the quantized bits increase, the performance enhancement may not be obvious for some codebook types. Figure 4 shows the sum data rate of MU-MIMO quantized with DFT codebook of different bits. It can be seen that while the number of quantized bits increase from 2 to 3 bits the performance enhancement is obvious, and performance enhancement is little while number of quantized bits increase from 3 to 6 bits. Concluded from the growth trend, when the number quantized bit is more than 6 bits, the performance is near to case of 6 bits. So, increasing codebook size is no use to enhance MU-MIMO performance. The reason is that the increasing number of quantized bits cannot decrease the inter-user interference of paired users for fixed codebook structure unlike RVQ feedback scheme.

3. Algorithm

To decrease the bad effect of random inter-user interference in low rate fixed codebook feedback scheme, a novel USS feedback scheme is proposed. In the USS feedback, extra USS information is added after CSI feedback to show the inter-user interference. And this information is used in user selection algorithm to avoid large inter-user interference. The detailed process of the proposed scheme is elaborated as follows.

3.1. Grouping quantized codebook

In MU-MIMO transmission, the paired users are usually selected with small correlation between their channels. In USS feedback scheme, codebook $C$ is divided into several groups, and only the users whose quantized CSI from the same group can be paired together. The codebook $C$ is divided as follows:

$$C_k = \{c_{k1},...,c_{kq}\} \quad (c_{k1}^{c_k} \in C \land k \neq j \land k, j = 1,...,N),$$  \hfill (13)

where $C_k$ is subset of codebook $C$ satisfied $C = \bigcup_{k=1,...,m} C_k$ and $C_{k1} \cap C_{k2} = \emptyset(k1 \neq k2)$, $c_{ki}$ is element of
codebook $C$, $m$ is number of groups, $l$ is element number of subset, $N$ is codebook size with the relevance $N = l \times m$. $R$ is correlation threshold between code vector in subset, which means the correlation between any two paired users are no more than $R$.

Only the users which their feedback belong to same group can be paired together, so the correlation between any two paired users are no more than $R$. At most $M$ users can be transmit at same time in MU-MIMO, so lets $l \geq M$, and all the $M$ users can be selected in the same set. In the simulation of this article, the DFT codebook is adopted with setting $l = M$ and $r = 0$, as DFT codebooks are naturally separated into orthogonal groups, which has $M$ orthogonal vectors.

3.2. USS information feedback
In USS feedback scheme, $(l-1) r$ additional bits named USS information are fed back to BS besides CSI and SNR, and this information is used to indicate the MU-MIMO performance. In sub-codebook groups, user can be paired with other $(l-1)$ vector, so USS information uses $r$ bit(s) for each vector to show the MU-MIMO performance while user is paired with this vector. The feedback contents are $(\text{USS}_1, ..., \text{USS}_{i-1})$ and $\text{USS}_i$ corresponding to the $i$th vector in sub-codebook except the vector which user is fed back. For example, if $r = 1$, the user can be paired with $i$th vector while $\text{USS}_i = 1$, and the user cannot be paired with $i$th vector while $\text{USS}_i = 0$.

The value of USS information is relative to transmission and feedback configuration, such as number of paired user $m$ and USS information bits $r$. The details of the value calculation will be shown in Section 3.4 for different configurations.

3.3. User selection procedure
In USS feedback scheme, the user selection will use USS information to avoid large inter-user interference. The step is as follows:

1. BS defines three sets: serving user set $U = \{\text{UE}_1, ..., \text{UE}_K\}$, corresponding to all the users served by BS; (2) user CSI set $W = \{w_1, ..., w_K\}$, corresponding to users’ CSI; (3) paired user set MU = $\emptyset$, corresponding to the users scheduled together to adopt MU-MIMO. BS sets the number of paired users (more than 1 and no more than the number of transmit antennas).

2. BS selects first two users $(i,j)$ from set $U$. The $\text{UE}_i$ and $\text{UE}_j$ should satisfy the conditions: (a) their CSI feedback should be in the same codebook group $C_k$, that
means \( w_p, w_j \in C_k \); (b) the USS information for paired vector should not be equal to zero, that means (USS\(_{il1} > 0, \) USS\(_{jl2} > 0, \) \( c_{k1} = w_p, c_{k2} = w_j \)); (c) the summation of USS information for paired vector should be maximum in all users which satisfy conditions (a) and (b), that means \((i, j) = \max_{UE, UE} \text{satisfy (a) and (b)}(\text{USS}_{il1} + \text{USS}_{jl2}).\)

If the two users can be found, BS will put them into paired user set \( MU = \{UE_p, UE_j\} \), and remove them from serving user set \( U = U \cup \{UE_p, UE_j\} \). Otherwise, user pairing will be stopped and single user mode will be adopted.

(3) If the number of paired user is enough, start ZF procedure to compute precoding matrix. Otherwise, select the next user \( o \) from set \( U \). The \( UE_o \) should satisfy the conditions: (a) its CSI feedback should be in codebook group \( C_k \), same to users in set \( MU \), that means \( w_o \in C_k \); (b) the USS information for paired vector of \( UE_o \) and users in set \( MU \) should be more than zero, that means (USS\(_{oli} > 0, \) USS\(_{ilo} > 0, \) \( c_{kli} = w_i, c_{ilo} = w_o, UE_i \in MU \)); (c) the summation of USS information for paired vector should be maximum in all users which satisfy conditions (a) and (b), that means \((o) = \max_{UE_o, UE} \text{satisfy (a) and (b)} \sum_{UE_o \in \text{MU}}(\text{USS}_{oli} + \text{USS}_{ilo}).\)

If the user \( o \) can be found, BS will put it into paired user set \( MU = MU + \{UE_o\} \), and remove them from serving user set \( U = U \cup \{UE_o\} \). Otherwise, user pairing will be stopped and start ZF procedure to compute precoding matrix for the users in set \( MU \).

(4) If the number of paired user is enough, start ZF procedure to transmit users’ data. Otherwise, go to step 3 to select another user.

### 3.4. USS value calculation

The value of USS information is relative to the number of paired user \( m \) and USS information bits \( r \). In this section, different cases will be discussed separately.

(a) \( r = 1 \) and \( m = 2 \)

For two paired users, the SNR for each user can get from Equation (11),

\[
\text{MU SNR}_i = \frac{g_i \beta_i^2 \| H_i \beta_i \|^2}{\gamma_j + \sigma_i^2},
\]

where \( \gamma_j = \| H_j \beta_j \|^2 + \sigma_i^2 \).
where $\alpha_i$ is coefficient scaling factor, $\beta_i$ is power allocation factor. The total power should no more than max transmit power $P_{\text{total}}$, and the constraint is
\[
\frac{\beta_1}{\alpha_1} \| p_1 \|^2 + \frac{\beta_2}{\alpha_2} \| p_2 \|^2 = P_{\text{total}}.
\]

The precoding vector can be gotten from Equation (6),
\[
(w_1, w_2) = \left( w_1 \right) \left( w_2 \right)^H - \frac{1}{\| w_1 \|^2 + \| w_2 \|^2} \left( \| w_1 \|^2 \left( w_1 \right) \left( w_2 \right)^H \right.
\]
\[
\left. \left( \| w_2 \|^2 \left( w_2 \right) \left( w_1 \right)^H \right) \right)
\]
(15)

Define the correlation of vector: $w_i^H w_j^H = \sigma e^{j\phi}$. So, the precoding matrix changes to
\[
p_i = | w_j |^2 w_i^H - \sigma e^{-j\phi} w_j^H \left( | w_i |^2 | w_j |^2 - \sigma^2 \right) (i \neq j)
\]
(16)

Each user knows its channel matrix and the vector of paired user is selected in subset $C_k$. So, user can calculate the exact SNR of MU-MIMO for each vector in set $C_k$.

The equation can be simplified with following assumptions: (1) usually the codebook is normalize vector, that means $\| w_j \|^2 = 1$; (2) normalize precoding vector for each users, that means $\| p_i \|^2 = \alpha_i^2$; (3) power is equally allocated in the paired users, that means $\beta_i^2 = P_{\text{total}}/m$, where $m$ is number of paired users; (4) define correlation of CSI quantized as $H_i w_j^H = a_i e^{j\phi_{ui}}$; (5) define inter-user interference as $H_i w_j^H = b_{ij} e^{j\phi_{uj}} (i \neq j)$. By substituting Equation (16) into Equation (14), we can get
\[
\text{MUSNR}_i = \frac{g P_{\text{total}} \left| a_i e^{j\phi_{ui}} - \sigma e^{-j\phi} b_{ij} e^{j\phi_{uj}} \right|^2}{2 \| p_i \|^2 \left( 1 - \sigma^2 \right)^2} + \frac{g P_{\text{total}} \left| b_{ij} e^{j\phi_{uj}} - \sigma e^{j\phi} a_i e^{j\phi_{ui}} \right|^2}{\| p_i \|^2 \left( 1 - \sigma^2 \right)^2} + \sigma_i^2
\]
(17)

This result can be used in USS information calculation. In USS feedback scheme, a correlation threshold $R$ is used in codebook subset. It means in above equations that the correlation $\sigma$ must be no more than $R$ as the paired vector is selected from same subset. With different value of $R$, it can be divided into two categories:

(a-1) $R = 0$. In this case, it can be thought that the paired vector is orthogonal, so the correlation $\sigma$ can be
tread as zero. The precoding matrix changed to \( p_i = w_i^H \), and Equation (17) can be simplified as:

\[
\text{MU-SNR}_i = \frac{g_i P_{\text{total}} a_i^2 / 2}{g_i P_{\text{total}} b_i^2 / 2 + \sigma_i^2} = \frac{a_i^2}{b_i^2 + 2 / \text{SNR}_i},
\]

(18)

where \( \text{SNR}_i \) is the measured SNR defined in Equation (4).

So, throughput of UE, is

\[
R_i = \log(1 + \text{MU-SNR}_i) = \log \left( 1 + \frac{a_i^2}{b_i^2 + 2 / \text{SNR}_i} \right)
\]

(19)

Because user does not know the vector which BS will be schedule in user pairing, the actual transmit rate cannot be known. In USS feedback scheme, all the paired vectors are in one subcodebook \( C_k = \{c_{k1},...,c_{kL}\} \), and for one UE, the number of candidate pairing vector is \( L-1 \).

So, for each candidate pairing vector in subcodebook, user will evaluate its throughput when this vector is selected as paired vector, and the USS information is calculated based on this evaluated throughput.

User assumes that the paired user has the same correlation of quantized CSI \( a \) and the same inter-user interference level \( b \), so the evaluated sum throughput is \( R_{ij} = 2 R_i (j \neq i) \). If the sum throughput for the vector \( c_{ij} \) is more than MISO throughput \( R_{ij} = \log(1+\text{SNR}) \), set USS\(_{ij} = 1 \), which means the performance is better while UE\(_i \) paired with vector \( c_{ij} \) otherwise set USS\(_{ij} = 0 \), which means the inter-user interference is large while UE\(_i \) paired with vector \( c_{ij} \) and UE\(_j \) should avoid to pair with this vector.

(a-2) \( r > 0 \). In this case, the correlation \( \sigma \) should be considered. Equation (17) changed to

\[
\text{MU-SNR}_{ij} = \frac{a_{ij}^2}{b_{ij}^2 + \sigma_i^2 a_j^2 / 2 - 2 \sigma_i \sigma_j a_i a_j \cos(\phi_i + \phi_j)} + 2(1 - \sigma^2)/\text{SNR}_{ij},
\]

(20)

From CSI quantization criterion, it is known that \( a \) is near to 1. Usually, the correlation \( \sigma \) is set near to 0 to enhance the MU-MIMO performance and the inter-user interference \( b \) will be small guaranteed by user selection procedure. So, it can be thought that \( \sigma b \ll a \). Equation (20) can be simplified as

\[
\text{MU-SNR}_{ij} = \frac{a_{ij}^2}{b_{ij}^2 + \sigma^2 a_j^2 / 2 - 2 \sigma \sigma_j a_i a_j \cos(\phi_i + \phi_j)} + 2(1 - \sigma^2)/\text{SNR}_{ij},
\]

(21)

The USS information calculation is same to the case of \( R = 0 \). The difference is that MU\(_{\text{SNR}} \), will use Equation (21) instead of Equation (18) in USS calculation.

(b) \( r = 1 \) and \( m > 2 \)

If more than two users are paired together to form MU-MIMO, then the SNR of MU-MIMO user will be decreased compare to two paired users, as the inter-user interference is \( m-1 \) times and the power allocation of each user is also decreased. It assumes that the power is equally allocated to each user and the paired users have the same correlation of quantized CSI \( a \) and inter-user interference level \( b \) for each paired vector.

For the case of \( R = 0 \), the evaluated MU-MIMO SNR changed to

\[
\text{MU-SNR}_{ij} = \frac{a_{ij}^2}{(m-1)b_{ij}^2 + m/\text{SNR}_{ij}}.
\]

(22)

While \( R > 0 \), the evaluated MU-MIMO SNR changed to

\[
\text{MU-SNR}_{ij} = \frac{a_{ij}^2}{(m-1)b_{ij}^2 + m/\text{SNR}_{ij}} + 2(1 - \sigma^2)/\text{SNR}_{ij},
\]

(23)

The evaluated sum rate changed to

\[
R_{ij} = m R_i = m \log(1 + \text{MU-SNR}_{ij}).
\]

(24)

The USS information calculation is same to the case of \( m = 2 \). The difference is that MU\(_{\text{SNR}} \), uses Equations (22) and (23) for different cases and the sum throughput \( R_{ij} \) uses Equation (24).

\( (c) r > 1 \)

If each USS is more than 1 bit, it should be quantized by \( 2^r \) rank. The sum throughput \( R_{ij} \) is evaluated and it is mapped into region from \( R_{\text{lower}} \) to \( R_{\text{upper}} \) with \( r \) bits. The sum rate \( R_{ij} \) is calculated same to cases (a) and (b). The lower bound is defined as single user performance \( R_{\text{lower}} = \log(1+\text{SNR}) \), as sum rate of MU-MIMO should be more than single user transmission. The upper bound is defined as the users paired with orthogonal vectors with no inter-user interference:

\[
R_{\text{upper}} = m \log \left( 1 + \frac{a_{ij}^2}{(m-1)(\sigma^2 a_j^2 / 2) + m(1 - \sigma^2)/\text{SNR}_{ij}} \right).
\]

(25)

The quantization is performed as follows:

(1) if \( R_{ij} \leq R_{\text{lower}} \), set USS\(_{ij} = 0 \).

(2) if \( R_{ij} \geq R_{\text{upper}} \), set USS\(_{ij} = \sigma \).

(3) if \( R_{\text{lower}} < R_{ij} < R_{\text{upper}} \), set USS\(_{ij} = -1 + \lfloor (R_{ij} - R_{\text{lower}})/(R_{\text{upper}} - R_{\text{lower}}) \rfloor \), where \( \lfloor \cdot \rfloor \) is floor function.

### 3.5 Feedback overhead

In USS feedback scheme, the extra USS information is added after quantized CSI, and the feedback overhead is changed. So, the overhead of USS feedback, conventional feedback, and RVQ feedback is analyzed in this section. As discussed above, it assumed that (1) the codebook size is \( N = 2^R \); (2) the quantization vector \( c_j \in C^* \); (3) UE will feed back one quantized CSI in each feedback period.

For conventional feedback, only a quantized CSI is fed back to BS in each feedback period, so the feedback overhead is \( B \) bits in a feedback period.

For USS feedback, in each feedback period, the extra USS information is fed back to BS besides the quantized
CSI. As discussed in Section 3.4, it has \( l \) elements in subset and \( r \) bits USS information for each element in subset. So, the feedback overhead is \( B + (l-1)r \) bits in a feedback period.

For RVQ feedback, a quantized CSI is fed back to BS in each feedback period. Besides, the random codebook should be shared between BS and UE, and this codebook is randomly generated by UE then fed back to BS through feedback channel. It is assumed the random codebook can be used in \( q \) periods and the 16 bits quantization with short floating point number is adopted for each complex element of codebook. So, the initialization overhead is \( N^*M^*16^*2 \), and this overhead cover to each period is \( N^*M^*16^*2/q \). The totally feedback bits in a feedback period is \( N^*M^*32^*/q + B \).

The overhead comparison of the three methods is list in Table 1.

| Table 1 The overhead comparison |
|----------------------------------|
| **Scheme** | **Initialization** | **Quantized CSI** | **Additional** | **Totally feedback bits/period** |
| Conventional | 0 | B | 0 | B |
| USS | 0 | B | (l-1)r | B+(l-1)r |
| RVQ | \( N^*M^*16^*2/q \) | B | 0 | \( N^*M^*32^*/q + B \) |

4. Simulation

In this section, a MIMO system with \( M = 4 \) transmit antennas at the BS and single antenna at the UE is considered. The DFT codebook with different size is used in simulation. DFT codebook has orthogonal vector groups, so each orthogonal vector group is treated as one subcodebook. Hence, the correlation threshold \( R \) is equal to zero.

Figure 5 shows the throughput among SISO, perfect SCI feedback, conventional feedback with 7 bits CSI quantization, 4 bits RVQ feedback, and 7 bits USS feedback (4 bits CSI quantization + 3 bits USS information). RVQ feedback needs to share random codebook between UE and BS through uplink feedback channel and this one-off overhead should be converted into each feedback period, assumed it is equal to 3 bits per feedback period (the lifetime of random codebook is about 650 feedback periods). So, the totally feedback overhead of the three feedback scheme is same. In simulation, 2 paired users are selected from total 20 users. It can be seen that the performance of RVQ feedback is higher than conventional feedback and USS feedback in low SNR region. In high SNR region, the performance gain of RVQ feedback and conventional feedback compared to SISO is decreased. Unlike the conventional feedback, the performance gain of USS feedback is nearly constant with SNR increase, so the performance gain is not decreased in high SNR region and the performance gain is about 2 bits/Hz. From this result, it can be seen that the proposed USS feedback scheme has better performance enhancement in high SNR region.

Figure 6 shows the throughput of two paired users which are selected from different number of users in conventional feedback with 7 bits CSI quantization, 4 bits RVQ feedback, and 7 bits USS feedback (4 bits CSI quantization + 3 bits USS information). It can be seen that the performance of RVQ feedback and conventional feedback changes very small with different number of users. This is because the user selection for conventional feedback and RVQ feedback cannot avoid large inter-user interference brought by channel quantization. For the USS feedback, the performance is increased with the number of users increasing. The increasing is obvious for small number of users and little for large number of users. It is because the user pairing procedure usually cannot find proper paired users for MU-MIMO transmission in small user number case. While the number of users increases, it has more users with little inter-user interference, and then the user pairing procedure for USS feedback is easily to find proper paired users for MU-MIMO.

Figure 7 shows the throughput with different CSI quantization bits in USS feedback, RVQ feedback, and conventional feedback with 7 bits CSI quantization. As shown in Figure 4, the performance is almost same while the feedback bits is more than 3, so the performance of conventional feedback with 7 bits is showed here to stand for performance of conventional feedback with different feedback bits. It can be seen that the performance is increased with the feedback bits increase in RVQ feedback, as the quantization accuracy is increased. For the USS feedback, the performance is increased with feedback bits increased because of the enhanced quantization accuracy. While the CSI quantization bits increase to 6 bits, there is a performance decreasing in high SNR region, that is, because with the CSI quantization bits increasing, the number of codebook subset is increase, so little users will be in one subset than lower CSI quantization bits. Hence, the user pairing procedure cannot find proper users to form MU-MIMO in this region, and performance will be decreased when SU transmission is adopted. From this result, it can be seen that the proposed USS feedback scheme is suitable for very low CSI quantization bits.

Figure 8 shows the throughput with case of \( r > 1 \) in USS feedback scheme. It can be seen that the
Figure 5 Data rate comparison.

Figure 6 Throughput with different number of users.
Throughputs with different CSI quantization bits

Figure 7 Throughput with different CSI quantization bits

Throughputs with different USS bits

Figure 8 Throughput with different USS bits
performance enhancement is larger while USS information increase from 1 bit to multiple bits. If the number of bits is more than 2, the enchantment is little. So, 2 bits (four orders) quantization of USS information is enough.

5. Conclusion
In this article, a novel USS feedback scheme and relative user selection procedure are proposed to avoid large inter-user interference in downlink ZF MU-MIMO for low rate fixed codebook feedback. The inter-user interference will largely decrease the MU performance gain in high SNR region and leads to the MU-MIMO throughput does not increase with the codebook size increasing. With the help of additional information, the proposed USS feedback scheme can avoid large inter-user interference in ZF MU-MIMO transmission, and it can be used in various configurations such as different codebook type, different number of antennas, and different paired users. Simulation results show that the proposed USS feedback scheme is efficiency for users with very low CSI quantization bits and paired other users at high SNR region.

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Competing interests
The authors declare that they have no competing interests.

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