Radiative Mixing of the One Higgs Boson and Emergent Self-Interacting Dark Matter

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Abstract

In all scalar extensions of the standard model of particle interactions, the one Higgs boson responsible for electroweak symmetry breaking always mixes with other neutral scalars at tree level unless a symmetry prevents it. An unexplored important option is that the mixing may be radiative, and thus guaranteed to be small. Two first such examples are discussed. One is based on the soft breaking of the discrete symmetry $Z_3$. The other starts with the non-Abelian discrete symmetry $A_4$ which is then softly broken to $Z_3$, and results in the emergence of an interesting dark-matter candidate together with a light mediator for the dark matter to have its own long-range interaction.
The standard model (SM) of particle interactions requires only one scalar doublet \( \Phi = (\phi^+, \phi^0) \) which breaks the electroweak gauge symmetry \( SU(2)_L \times U(1)_Y \) spontaneously to electromagnetic \( U(1)_Q \) with \( \langle \phi^0 \rangle = v = 174 \text{ GeV} \). It predicts just one physical Higgs boson \( h \), whose properties match well with the 125 GeV particle discovered \([1, 2]\) at the Large Hadron Collider (LHC) in 2012. If there are other neutral scalars at the electroweak scale, they are expected to mix with \( h \) at tree level, so that the observed 125 GeV particle should not be purely \( h \). If future more precise measurements fail to see deviations, then a theoretical understanding could be that a symmetry exists which distinguishes \( h \) from the other scalars. For example, a \( Z_2 \) symmetry may exist under which a second scalar doublet \( (\eta^+, \eta^0) \) is odd \([3]\) and all SM particles are even. This is useful for having a viable dark-matter candidate \([4]\). It also enables the simplest one-loop (scotogenic) model \([5]\) of radiative neutrino mass through dark matter, if three neutral singlet fermions \( N_{1,2,3} \) are also added which are odd under \( Z_2 \), as shown in Fig. 1. Note that this dark \( Z_2 \) symmetry may be derived from lepton parity \([6]\).

Figure 1: One-loop \( Z_2 \) scotogenic neutrino mass.

Another important new development in the understanding of dark matter is the possibility that it has long-range self-interactions \([7]\). This may be an elegant solution to two existing discrepancies in astrophysical observations: (1) central density profiles of dwarf galaxies are flatter (core) than predicted (cusp); (2) observed number of satellites in the Milky Way is much smaller than predicted. Although (2) is less of a problem with the recent discovery of faint satellites, there is also (3) the new related problem of predicted massive subhalos which
are not observed. From a particle theory perspective, if the light mediator of dark matter is a scalar boson, then it is difficult to construct a theory such that it does not mix substantially with $h$. In this paper it will be shown how a dark-matter candidate could emerge together with its mediator, such that its mixing with $h$ is one-loop suppressed. This will be discussed later as part of my second example.

If an exact dark symmetry is imposed, then any mixing of $h$ with other neutral scalars in the dark sector would be forbidden. However, if this symmetry is softly broken, it will result in nonzero mixing, many examples of which exist. In the above scotogenic model, if the quadratic term $m_{12}^2 \Phi^\dagger \eta + H.c.$ is added thus breaking $Z_2$ softly, a nonzero vacuum expectation value $\langle \eta^0 \rangle$ would be induced. This invalidates $\eta^0$ as a dark-matter candidate. However $\langle \eta^0 \rangle$ could be naturally small as an explanation of the smallness of neutrino masses. At the same time, $h$ mixes with $\eta^0$ at tree level as expected. To obtain radiative mixing which has never been discussed before, my first example is the soft breaking of a $Z_3$ symmetry which would lead to the one-loop mixing of $h$ with two other scalars with zero vacuum expectation values.

Consider the addition of a complex neutral scalar singlet $\chi$ to the SM, transforming as $\omega = \exp(2\pi i/3)$ under $Z_3$. Let the scalar potential of $\Phi$ and $\chi$ be given by

$$V = \mu^2 \Phi^\dagger \Phi + m_1^2 \chi^\dagger \chi + \frac{1}{2} m_2^2 \chi^2 + \frac{1}{2} (m_2^*)^2 (\chi^\dagger)^2 + \frac{1}{3} f \chi^3 + \frac{1}{3} f^* (\chi^\dagger)^3$$
$$+ \frac{1}{2} \lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 (\chi^\dagger \chi)(\Phi^\dagger \Phi),$$

where $Z_3$ is softly broken only by the quadratic $m_2^2$ and $(m_2^*)^2$ terms. The phase of $\chi$ may be redefined to render $m_2^2$ real, but then $f$ must remain complex in general. As $\phi^0$ acquires a nonzero vacuum expectation value $\langle \phi^0 \rangle = v$ so that $m_h^2 = 2\lambda_1 v^2$, the $2 \times 2$ mass-squared matrix for $\chi = (\chi_R + i\chi_I)/\sqrt{2}$ becomes

$$M^2_{\chi} = \begin{pmatrix} m_1^2 + \lambda_3 v^2 + m_2^2 & 0 \\ 0 & m_1^2 + \lambda_3 v^2 - m_2^2 \end{pmatrix} = \begin{pmatrix} m_R^2 & 0 \\ 0 & m_I^2 \end{pmatrix}. \quad (2)$$
The interaction Lagrangian for $h, \chi_R, \chi_I$ is then

$$-\mathcal{L}_{\text{int}} = \frac{1}{2} m_R^2 h^2 + \frac{1}{\sqrt{2}} \lambda_1 v h^3 + \frac{1}{8} \lambda_1 h^4 + \frac{\lambda_3 v}{\sqrt{2}} h (\chi_R^2 + \chi_I^2) + \frac{\lambda_3}{4} h^2 (\chi_R^2 + \chi_I^2)
+ \frac{1}{2} m_R^2 \chi_R^2 + \frac{1}{2} m_I^2 \chi_I^2 + \frac{1}{8} \lambda_2 (\chi_R^2 + \chi_I^2)^2
+ \frac{f_R}{3\sqrt{2}} \chi_R^3 - \frac{f_I}{\sqrt{2}} \chi_R \chi_I - \frac{f_R}{\sqrt{2}} \chi_R \chi_I^2 + \frac{f_I}{3\sqrt{2}} \chi_I^3. \quad (3)$$

It is clear that $h$ does not mix with $\chi_R$ or $\chi_I$ at tree level. However there will be radiative mixing in one loop as shown in Fig. 2. The $h\chi_R$ mixing is given by

$$2i\lambda_3 f_R v \int \frac{d^4k}{(2\pi)^4} \left[ \frac{1}{(k^2 - m_R^2)^2} - \frac{1}{(k^2 - m_I^2)^2} \right] = \frac{\lambda_3 f_R v}{8\pi^2} \ln \frac{m_R^2}{m_I^2}. \quad (4)$$

Similarly, the $h\chi_I$ mixing is obtained with $f_R$ replaced by $f_I$ and $m_R, I$ by $m_I, R$. This new phenomenon allows the 125 GeV particle to be essentially $h$ and yet the scalar sector is enriched with other possible consequences different from that of an exact symmetry such as dark parity.

A useful application of the above scenario is in connection with the simplest model [9] of dark matter (DM), i.e. that of a real neutral singlet $s$, which is odd under dark $Z_2$. It has been shown recently [10] that most of its parameter space has been ruled out by the LUX direct-search experiment for dark matter [11]. The tension comes from the requirement that
the $ss$ annihilation cross section to be of the correct magnitude to account for the observed DM relic density of the Universe, but its interaction with nuclei through $h$ exchange to be below the LUX bound. To satisfy the latter, the former becomes too small, hence the $s$ relic abundance would exceed what is observed. With the addition of $\chi$, the allowed $ss\chi^\dagger\chi$ interaction would add to the $ss$ annihilation cross section, but would not contribute to the direct-search constraint. In this way the parameter space for $s$ dark matter opens up to $m_s > m_\chi$. This solution also applies to the recently proposed $A_4$ model \cite{12} of neutrino mass, where $s_{1,2,3} \sim 3$ and the lightest is dark matter. Here $\chi \sim 1'$ of $A_4$.

My second example of radiative mixing of $h$ and another scalar is based on the breaking of $A_4$ to $Z_3$. Consider now three real neutral scalar singlets $\chi_{1,2,3} \sim 3$ of $A_4$. Let their scalar potential with $\Phi$ be given by

$$V = \mu^2 \Phi^\dagger \Phi + \frac{1}{2}m_1^2(\chi_1^2 + \chi_2^2 + \chi_3^2) + m_2^2(\chi_1\chi_2 + \chi_2\chi_3 + \chi_3\chi_1) + f\chi_1\chi_2\chi_3 + \frac{1}{2}\lambda_1(\Phi^\dagger\Phi)^2$$

$$+ \frac{1}{4}\lambda_2(\chi_1^2 + \chi_2^2 + \chi_3^2)^2 + \frac{1}{4}\lambda_3(\chi_1^2\chi_2^2 + \chi_2^2\chi_3^2 + \chi_3^2\chi_1^2) + \frac{1}{2}\lambda_4(\chi_1^2 + \chi_2^2 + \chi_3^2)(\Phi^\dagger\Phi),$$

(5)

where $A_4$ is broken softly to $Z_3$ by the $m_3^2$ term. Note that the trilinear $f$ term means that $\chi_{1,2,3}$ do not have the dark parity of the similar $s_{1,2,3}$ scalars discussed previously. Let \cite{13}

$$\begin{pmatrix}
\chi_0 \\
\chi \\
\chi^\dagger
\end{pmatrix} = \frac{1}{\sqrt{3}}
\begin{pmatrix}
1 & 1 & 1 \\
1 & \omega & \omega^2 \\
1 & \omega^2 & \omega
\end{pmatrix}
\begin{pmatrix}
\chi_1 \\
\chi_2 \\
\chi_3
\end{pmatrix},$$

(6)

then the real $\chi_0$ and complex $\chi$ are mass eigenstates with $m_0^2 = m_1^2 + \lambda_4 v^2 + 2m_2^2$ and $m_\chi^2 = m_1^2 + \lambda_4 v^2 - m_2^2$. Under the residual $Z_3$ symmetry, $\chi_0 \sim 1$, $\chi \sim \omega$, and $\chi^\dagger \sim \omega^2$. The interaction Lagrangian for $h, \chi_0, \chi$ is then

$$-L_{int} = \frac{1}{2}m_h^2 h^2 + \frac{1}{\sqrt{2}}\lambda_1 v h^3 + \frac{1}{8}\lambda_1 h^4 + \frac{\lambda_4}{\sqrt{2}} h(\chi_0^2 + 2\chi\chi^\dagger) + \frac{\lambda_4}{4} h^2(\chi_0^2 + 2\chi\chi^\dagger)$$

$$+ \frac{1}{2}m_0^2 \chi_0^2 + m_\chi^2 \chi\chi^\dagger + \frac{1}{4}\lambda_2(\chi_0^2 + 2\chi\chi^\dagger)^2 + \frac{f}{3 \sqrt{3}} [\chi_0^3 + \chi^3 + (\chi^\dagger)^3 - 3\chi_0 \chi\chi^\dagger]$$

$$+ \frac{1}{12}\lambda_3[\chi_0^4 - 2\chi_0 \chi^3 - 2\chi_0(\chi^\dagger)^3 + 3(\chi\chi^\dagger)^2].$$

(7)
Again there is no tree-level mixing for $h$, but radiative mixing occurs between $h$ and $\chi_0$ as shown in Fig. 3. This mixing is given by

$$m_{h\chi}^2 = \sqrt{(2/3)i\lambda_4fv} \int \frac{d^4k}{(2\pi)^4} \left[ \frac{1}{(k^2 - m_0^2)^2} - \frac{1}{(k^2 - m_\chi^2)^2} \right] = \frac{\lambda_4fv}{8\sqrt{6}\pi^2} \ln \frac{m_0^2}{m_\chi^2}. \quad (8)$$

Hence $\chi_0$ would decay to SM particles through its mixing with $h$. On the other hand, $\chi$ itself is absolutely stable and becomes a suitable DM candidate. It is an example of $Z_3$ dark matter [14, 15], which may also be connected with the two-loop generation of neutrino mass [16].

The important thing to notice in this model is the natural emergence of $\chi$ and $\chi_0$ as the result of $A_4$ breaking to $Z_3$ with their accompanying interactions as shown in Eq. (7). Whereas $\chi \sim \omega$ under $Z_3$ emerges as a dark-matter candidate, $\chi_0 \sim 1$ emerges automatically as its mediator. In other words, all the ingredients of self-interacting dark matter appear at once. Note that this desirable outcome is the result of $A_4$ breaking to $Z_3$. If only $Z_3$ is assumed in the beginning, then $\chi_0 \sim 1$ would have a substantial tree-level mixing with $h$. Since the mediator in self-interacting matter is required to be light, say 10 MeV, the fact that its mixing with $h$ is one-loop suppressed by Eq. (8) is an important desirable feature of this model.
Since $\chi_0$ is presumably in thermal equilibrium with all the SM particles, it would be overproduced in the early Universe if it is stable. To avoid any conflict with the very successful predictions of Big Bang Nucleosynthesis (BBN), its decay lifetime should be less than about one second. For $m_{\chi_0} = 10$ MeV, $\chi_0$ would decay to $e^-e^+$ or to $\gamma\gamma$ through its mixing with $h$. In contrast to the case of $h$ decay, the former is dominant because the latter lacks the extra enhancement of $m_h^2/m_{\chi_0}^2$. The $\chi_0 \to e^-e^+$ decay rate is easily calculated to be

$$\Gamma(\chi_0 \to e^-e^+) = \frac{m_{\chi_0}m_e^2}{16\pi v^2} \left(\frac{m_{h\chi}^2}{m_h^2}\right)^2.$$  \hspace{1cm} (9)

For $m_{\chi_0} = 10$ MeV, $m_e = 0.511$ MeV, $v = 174$ GeV, the mixing of $\chi_0$ with $h$ is constrained by $(m_{h\chi}^2/m_h^2) > 2 \times 10^{-5}$ if $\Gamma^{-1} < 1$ sec. Using Eq. (8), this translates to

$$\lambda_4 f > 0.02 \text{ GeV.}$$  \hspace{1cm} (10)

To be successful as self-interacting dark matter, the trilinear scalar interaction $(f/\sqrt{3})\chi_0\chi\chi^\dagger$ in Eq. (7) should have a magnitude of order the weak scale. Hence the above condition is easily satisfied. On the other hand, the $2 \times 2$ mass-squared matrix spanning $h$ and $\chi_0$ is given by

$$\mathcal{M}^2 = \begin{pmatrix} m_h^2 & m_{h\chi}^2 \\ m_{h\chi}^2 & m_{\chi_0}^2 \end{pmatrix}.$$  \hspace{1cm} (11)

For $m_{\chi_0}^2 << m_h^2$, the $h - \chi_0$ mixing is simply $m_{h\chi}^2/m_h^2$ but the physical mass of $\chi_0$ is also modified, i.e.

$$m_{\chi_0}^2 = m_0^2 - m_{h\chi}^4/m_h^2.$$  \hspace{1cm} (12)

For $m_{h\chi}^2/m_h^2 = 2 \times 10^{-5}$, the correction is $(2 \times 10^{-5})^2(125 \text{ GeV})^2 \simeq 6$ MeV$^2$. If $m_0 = 10$ MeV, $m_{\chi_0}$ is reduced only to 9.7 MeV. This is of course acceptable, but if $m_{h\chi}^2/m_h^2$ is too large, $m_0$ must be delicately adjusted to obtain $m_{\chi_0} \sim 10$ MeV. To avoid too much cancellation, $m_{h\chi}^2/m_h^2 < 10^{-4}$ is a reasonable condition, for which

$$\lambda_4 f < 0.1 \text{ GeV.}$$  \hspace{1cm} (13)
Given that \( f \sim 100 \text{ GeV} \) is required, \( \lambda_4 < 10^{-3} \) is implied.

The relic abundance of \( \chi \) is determined by its annihilation cross section as it decouples from other matter in the early Universe. The main interactions are shown in Fig. 4. As a result,

\[
\sigma \times v_{\text{rel}} = \frac{1}{16\pi m_{\chi}^2} \left( \lambda_2 - \frac{f^2}{4m_{\chi}^2} \right)^2 .
\]  

(14)

Setting this equal to the optimal value [17] of \( 4.4 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} \) implied by the observed relic abundance of dark matter in the Universe,

\[
\lambda_2 - \frac{f^2}{4m_{\chi}^2} = 0.0435 \left( \frac{m_{\chi}}{100 \text{ GeV}} \right)
\]  

(15)

is obtained. The final-state \( \chi_0 \) particles decay into \( e^-e^+ \) pairs through which thermal equilibrium with all other SM particles may be established.

The elastic scattering of \( \chi \) off nuclei proceeds through \( h \) exchange. For \( m_{\chi} = 100 \text{ GeV} \), the LUX bound of \( 10^{-45} \text{ cm}^2 \) implies [10]

\[
\lambda_4 < 0.01.
\]  

(16)

This means that direct-search experiments will need to improve by at least one or two orders of magnitude to see this effect. The \( h \to \chi_0\chi_0 \) decay rate is given by

\[
\Gamma(h \to \chi_0\chi_0) = \frac{\lambda_4^2 v^2}{16\pi m_h} = \left( \frac{\lambda_4}{0.01} \right)^2 0.5 \text{ MeV}.
\]  

(17)
This would be a contribution to the Higgs invisible width because the lifetime of $\chi_0$ will be certainly long enough for it to escape detection at the LHC.

In conclusion, I have discussed the new idea that scalars beyond the SM may only mix with the one Higgs boson $h$ (identified as the 125 GeV particle discovered at the LHC) in one loop. Two examples have been presented. One is based on the soft breaking of $Z_3$. Another is based on the soft breaking of $A_4$ to $Z_3$. In the latter, three real neutral scalars $\chi_{1,2,3}$ are added which transform as $\bar{3}$ under $A_4$. With the soft breaking, they are reorganized by Eq. (6) into a real $\chi_0 \sim 1$ and a complex $\chi \sim \omega$ under the residual $Z_3$, which is exactly conserved. Whereas $\chi_0$ mixes with $h$ radiatively, $\chi$ is a stable dark-matter candidate, with self interactions mediated by $\chi_0$. In this way, all the ingredients necessary for self-interacting dark matter emerge together from the softly broken symmetry. The relic abundance of $\chi$ is determined by the $\chi \chi^\dagger \to \chi_0 \chi_0$ annihilation cross section. The direct detection of $\chi$ from its elastic scattering off nuclei is possible through $h$ exchange. The light mediator $\chi_0$ decays mainly into $e^-e^+$, which may be the source of the observed positron excess in some astrophysical experiments [18, 19].

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References

[1] G. Aad et al. (ATLAS Collaboration), Phys. Lett. B716, 1 (2012).

[2] S. Chatrchyan et al. (CMS Collaboration), Phys. Lett. B716, 30 (2012).

[3] N. G. Deshpande and E. Ma, Phys. Rev. D18, 2574 (1978).

[4] A. Arhrib, Y.-L. S. Tsai, Q. Yuan, and T.-C. Yuan, JCAP 1406, 030 (2014).
[5] E. Ma, Phys. Rev. **D73**, 077301 (2006).

[6] E. Ma, arXiv:1502.02200 [hep-ph], PRL (in press).

[7] For a brief review, see for example S. Tulin, AIP Conf. Proc. **1604**, 121 (2014).

[8] E. Ma, Phys. Rev. Lett. **86**, 2502 (2001).

[9] V. Silveira and A. Zee, Phys. Lett. **B161**, 136 (1985).

[10] L. Feng, S. Profumo, and L. Ubaldi, JHEP **1503**, 045 (2015).

[11] D. Akerib et al. (LUX Collaboration), Phys. Rev. Lett. **112**, 091303 (2014).

[12] E. Ma, arXiv:1504.02086 [hep-ph].

[13] E. Ma and G. Rajasekaran, Phys. Rev. **D64**, 113012 (2001).

[14] G. Belanger, K. Kannike, A. Pukhov, and M. Raidal, JCAP **1301**, 022 (2013).

[15] P. Ko and Y. Tang, JCAP **1405**, 047 (2014).

[16] E. Ma, Phys. Lett. **B662**, 49 (2008).

[17] G. Steigman, B. Dasgupta, and J. F. Beacom, Phys. Rev. **D86**, 023506 (2012).

[18] O. Adriani et al. (PAMELA Collaboration), Nature **458**, 607 (2009).

[19] M. Aguilar et al. (AMS Collaboration), Phys. Rev. Lett. **110**, 141102 (2013).