Monitoring the temperature through moving average control under uncertainty environment

Muhammad Aslam1, Abdulmohsen Al Shareef1 & Khushnoor Khan1

The existing moving average control charts can be only applied when all observations in the data are determined, precise, and certain. But, in practice, the data from the weather monitoring is not exact and express in the interval. In this situation, the available monitoring plans cannot be applied for the monitoring of weather data. A new moving average control chart for the normal distribution is offered under the neutrosophic statistics. The parameters of the offered chart are determined through simulation under neutrosophic statistics. The comparison study shows the superiority of the proposed chart over the moving average control chart under classical statistics. A real example from the weather is chosen to present the implementation of the chart. From the simulation study and real data, the proposed chart is found to be effective to be applied for temperature monitoring than the existing control chart.

The monitoring of the process is done to reduce the percentage of non-conforming items during the process. The control charts are very important tools and used to watch the effect of extraneous factors on the process. The timely alert about the shift in the process is helpful to identify the reasons for this shift. The Shewhart control charts are simple to apply in the industry but detect only the larger shift in the process. Therefore, the moving average (MA), cumulative sum (CUSUM), and exponentially weighted moving average (EWMA) are the alternatives of the Shewhart control chart. These charts are sensitive to detect a small shift in the process. A little attention has been paid to design MA control charts for various situations. Chen and Yang developed an economic model for the MA chart. Wong et al. discussed the sensitivity power of the MA chart. Khoo and Wong and Areepong proposed double MA control charts. Mohsin et al. worked on a weighted MA chart using loss function. Alghamdi et al. designed the MA chart for the Weibull distribution. Ye et al. applied quality control methods for microdata. Su et al. used the machine learning technique for metrology data.

The control charts under classical statistics can be only applied when all observations from the industrial process are determined. In complex systems such as monitoring the level of water in a river, the process data may not be determined. "Fuzzy data exist ubiquitously in the modern manufacturing process". Therefore, an alternative to the existing chart is the use of a control chart using the fuzzy approach. A piece of detailed information about the fuzzy control charts can be seen in references.

The fuzzy logic that provides information about the measure of truth and falseness is the special case of neutrosophic logic, see Smarandache. The neutrosophic logic considers an additional measure is known as the measure of indeterminacy. A detailed discussion on neutrosophic logic can be seen in references. Smarandache introduced the neutrosophic statistics (NS0 using the idea of neutrosophic logic. The NS is a generalization of the classical statistics and deals with the data having Neutrosophy. Chen et al. introduced the neutrosophic numbers with applications. The np and X-bar control chart using NS were developed by Aslam et al. Aslam et al. worked on EWMA charts under NS.

A rich literature is available on control charts under classical statistics and fuzzy approach. By exploring the literature and best of our knowledge, there is no work on moving average control chart under NS. In this paper, we will introduce this neutrosophic moving average (NMA) chart and evaluate its performance over the existing MA charts under classical statistics. We will present a real example from the weather-monitoring department.

1Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah 21551, Saudi Arabia. 2Deanship of Scientific Research, King Abdulaziz University, Jeddah 21551, Saudi Arabia. *email: aslam_ravian@hotmail.com
Materials and Methods

Suppose that \( X_N = X_L + b_N I_N; I_N \in [I_L, I_U] \) be a neutrosophic random variable consists of the variable under classical statistics \( X_L \) and indeterminate part \( b_N I_N; I_N \in [I_L, I_U] \). The neutrosophic \( X_N = [X_L, X_U]; I_N \in [I_L, I_U] \) reduces to \( X \) when \( I_L = 0 \). Suppose \( n_N \in [n_L, n_U] \) presents the neutrosophic group size. Suppose that \( X_{IN} = [X_{IL}, X_{IU}] \) denotes the neutrosophic sample average for \( i \)th subgroup. Here we assume that \( X_{0N} \) follows the neutrosophic normal distribution with a population mean \( \mu N \in [\mu_L, \mu_U] \) and variance \( \sigma N^2 = [\sigma_L^2, \sigma_U^2] \) for \( i = 1, 2, \ldots, n \).

By following Montgomery\(^{36}\), MA statistic is defined as

\[
MA_{IN} = \frac{\bar{X}_{(i-1)N} + \cdots + \bar{X}_{(i-w+1)N}}{w_N}; \quad w_N \in [w_L, w_U]
\]

(1)

where \( w_N \in [w_L, w_U] \) shows the span at a time \( i \). Note here that the statistic is given in Eq. (1) is similar to the exponentially weighted moving average (EWMA) statistic. The main difference between MA statistic and the EWMA statistic is the sensitivity that each statistic shows in the calculation of the data. The EWMA gives the higher weights to the current values while the MA gives equal weight to all values in the data, see Li et al.\(^{37}\).

The \( MA_{IN} = MA_{IL}, MA_{IU} \) statistic in neutrosophic form can be written as

\[
MA_{IN} = MA = cN I_N; \quad I_N \in [I_L, I_U]
\]

(2)

where \( MA \) is the determined part and \( cN I_N; I_N \in [I_L, I_U] \) is indeterminate part of \( MA_{IN} = MA_{IL}, MA_{IU} \). The MA statistic mentioned by Montgomery\(^{36}\) is a special case of the proposed \( MA_{IN} = MA_{IL}, MA_{IU} \). The proposed \( MA_{IN} = MA_{IL}, MA_{IU} \) becomes MA statistic if \( I_L = 0 \). The neutrosophic mean and variance of \( MA_{IN} = MA_{IL}, MA_{IU} \) when the process is an in-control \( I \geq w_N; w_N \in [w_L, w_U] \) are given as follows

\[
E_N(\text{MA}_N) = \mu_{0N}; \quad MA_{IN} = MA_{IL}, MA_{IU}, \mu_{0L}, \mu_{0U}
\]

(3)

and

\[
V_N(\text{MA}_N) = \frac{\sigma^2}{n_N w_N}; \quad n_N \in [n_L, n_U], \quad w_N \in [w_L, w_U], \quad \sigma^2 \in [\sigma_L^2, \sigma_U^2]
\]

(4)

Based on the given information, the proposed control chart is mentioned as below.

**Step 1:** Select a neutrosophic sample of size \( n_N \in [n_L, n_U] \) from the production and compute \( MA_{IN} = MA_{IL}, MA_{IU} \) for specified \( I \in [I_L, I_U] \).

**Step 2:** Declare the process in-control if \( LCL_M \leq MA_{IN} \leq UCL_M \).

The operational process of the proposed control chart depends on two neutrosophic control limits, which are given as

\[
LCL_M = \mu_{0N} - k_N \sqrt{\frac{\sigma^2}{n_N w_N}}; \quad w_N \in [w_L, w_U], \quad n_N \in [n_L, n_U]
\]

(5)

\[
UCL_M = \mu_{0N} + k_N \sqrt{\frac{\sigma^2}{n_N w_N}}; \quad w_N \in [w_L, w_U], \quad n_N \in [n_L, n_U]
\]

(6)

where \( k_N \in [k_L, k_U] \) is neutrosophic control limits.

**Neutrosophic Monte Carlo simulation**

In this section, we introduce the neutrosophic Monte Carlo (NMC) simulation for the proposed NMA control chart. Suppose that be neutrosophic mean for in-control process and \( \mu_{IN} = \mu_{0N} + c_N \mu_{IN} \in [\mu_L, \mu_U] \), where \( c \) is a shift constant. Let \( r_{0N} \in [r_{0L}, r_{0U}] \) be the specified neutrosophic average run length (NARL) when the process is in-control state, for more details, the reader may refer to Li et al.\(^{38}\). The NMC simulation is stated as follows.

**Step 1:** Generate a random sample of size \( n_N \in [n_L, n_U] \) from the neutrosophic standard normal distribution with \( \mu_{0N} \in [\mu_{0L}, \mu_{0U}] \) and variance \( \sigma^2 \in [\sigma_L^2, \sigma_U^2] \). Compute \( X_{IN} = [X_{IL}, X_{IU}] \) for \( i \)th subgroup.

**Step 2:** Compute the statistic \( MA_{IN} = MA_{IL}, MA_{IU} \) and plot it on \( LCL_M [LCL_M, LCL_U] \) and \( UCL_M [UCL_L, UCL_U] \). Note the first out-of-control value, which is called the run length.

**Step 3:** Repeat the process 10,000 times and compute the neutrosophic average run length (NARL) and neutrosophic standard deviation (NSD). Choose \( k_N \in [k_L, k_U] \) for which NARL in control process, say \( ARL_{IN} \geq r_{0N} \); \( ARL_{IN} = [ARL_{0L}, ARL_{0U}] \).

**Step 4:** Generate a random sample of size \( n_N \in [n_L, n_U] \) from the neutrosophic standard normal distribution with \( \mu_{0N} \in [\mu_{0L}, \mu_{0U}] \) and variance \( \sigma^2 \in [\sigma_L^2, \sigma_U^2] \). Compute \( X_{IN} = [X_{IL}, X_{IU}] \) for \( i \)th subgroup.

**Step 5:** Compute the statistic \( MA_{IN} = MA_{IL}, MA_{IU} \) and plot it on \( LCL_M [LCL_L, LCL_U] \) and \( UCL_M [UCL_L, UCL_U] \). Note the first out-of-control value, which is called the run length for the shifted process.

**Step 6:** Repeat the process 10,000 times and compute the NARL and NSD, say \( ARL_{IN} = [ARL_{0L}, ARL_{0U}] \) at \( \mu_{1N} \in [\mu_{1L}, \mu_{1U}] \) for various values of \( c \).
Table 1. The values of NARL when \( n \in [4, 6] \) and \( w \in [3, 5] \).

| \( c \) | \( n \in [4, 6] \) and \( w \in [3, 5] \) | \( k \in [2.745, 2.68] \) | \( k \in [2.88, 2.81] \) | \( k \in [2.96, 2.89] \) |
|---|---|---|---|---|
| 200 | 300 | 370 |
| 0 | [201.71, 211.83] | [199.03, 205.38] | [289.36, 290.94] | [369.7, 372.87] | [349.46, 353.55] |
| 0.1 | [138.75, 106.01] | [137.11, 102.25] | [196.55, 137.01] | [248.96, 173.35] | [241.98, 171.21] |
| 0.2 | [368.64, 39.16] | [66.07, 35.54] | [93.49, 48.82] | [114.03, 57.83] | [111.2, 55.96] |
| 0.3 | [353.8, 18.11] | [33.46, 14.98] | [45.87, 21.63] | [54.14, 23.92] | [51.64, 20.41] |
| 0.4 | [19.48, 10.4] | [17.59, 7.13] | [24.34, 11.88] | [27.89, 12.6] | [25.69, 9.37] |
| 0.5 | [12.12, 1.83] | [10.18, 2.65] | [14.42, 7.4] | [16.03, 8.26] | [14.25, 4.86] |
| 0.6 | [8.12, 1] | [6.21, 0] | [9.31, 1] | [10.15, 1] | [8.24, 0] |

Table 2. The values of NARL when \( n \in [6, 8] \) and \( w \in [3, 5] \).

| \( c \) | \( n \in [6, 8] \) and \( w \in [3, 5] \) | \( k \in [2.745, 2.68] \) | \( k \in [2.88, 2.81] \) | \( k \in [2.96, 2.89] \) |
|---|---|---|---|---|
| 200 | 300 | 370 |
| 0.1 | [119.76, 89.24] | [119.08, 85.27] | [170.18, 113.52] | [206.34, 141.12] |
| 0.2 | [49.92, 30.56] | [48.38, 27.17] | [64.44, 33.99] | [78.47, 39.11] |
| 0.3 | [23.42, 13.69] | [21.36, 10.39] | [27.77, 16.2] | [31.75, 13.78] |
| 0.4 | [12.56, 8.33] | [10.68, 4.95] | [15.18, 8.85] | [17.7, 6.25] |
| 0.5 | [7.84, 1] | [5.87, 0] | [9.05, 1] | [7.85, 0] |
| 0.6 | [5.52, 1] | [3.53, 0] | [6.02, 1] | [4.45, 0] |

Table 3. The values of NARL when \( n \in [8, 10] \) and \( w \in [3, 5] \).

| \( c \) | \( n \in [8, 10] \) and \( w \in [3, 5] \) | \( k \in [2.745, 2.68] \) | \( k \in [2.88, 2.81] \) | \( k \in [2.96, 2.89] \) |
|---|---|---|---|---|
| 200 | 300 | 370 |
| 0 | [202.51, 211.04] | [202.12, 204.08] | [296.33, 300.31] | [371.81, 371.09] | [354.74, 353.76] |
| 0.1 | [104.63, 78.17] | [101.61, 74.31] | [149.81, 102.88] | [182.36, 122.74] | [181.09, 118.33] |
| 0.2 | [38.76, 24.34] | [36.44, 20.94] | [51.19, 29.23] | [62.12, 32.76] | [59.99, 29.49] |
| 0.3 | [17.06, 11.25] | [15.11, 8.06] | [21.25, 13.2] | [24.32, 13.62] | [22.04, 10.26] |
| 0.4 | [5.2, 1] | [3.82, 1] | [6.02, 1] | [4.44, 1] | [5.21, 0] |
| 0.5 | [1.98, 1] | [1.33, 1] | [2.22, 0] | [1.64, 0] | [1.89, 0] |

Table 4. The values of NARL when \( n \in [18, 22] \) and \( w \in [3, 5] \).

| \( c \) | \( n \in [18, 22] \) and \( w \in [3, 5] \) | \( k \in [2.745, 2.68] \) | \( k \in [2.88, 2.81] \) | \( k \in [2.96, 2.89] \) |
|---|---|---|---|---|
| 200 | 300 | 370 |
| 0 | [204.92, 208.42] | [203.64, 204.08] | [293.46, 291.44] | [371.81, 371.09] | [354.74, 353.76] |
| 0.1 | [62.86, 42.28] | [60.87, 38.62] | [85.53, 52.85] | [83.83, 49.27] | [103.05, 61.52] | [101.32, 58.7] |
| 0.2 | [17.53, 11.27] | [15.87, 8.11] | [21.25, 13.2] | [19.31, 9.79] | [24.32, 13.62] | [22.7, 10.69] |
| 0.3 | [7.31] | [5.35, 0] | [8.21] | [6.16, 0] | [8.89, 1] | [7.09, 0] |
Let values of \(171.21\), respectively. On the other hand, the values of \(\epsilon\) are [320.91, 294.43] and [313.19, 289.43], respectively. From this comparison, it is concluded that when \(c=0.1\), the values of \(ARL_{1}\) decreases as \(n_{N}\) increases. 1. For the fixed values of \(w_{N}\), the values of \(ARL_{1}\) decreases as \(n_{N}\) increases. 2. The values of \(ARL_{1}\) increases as \(n_{N}\) increases from 300 to 370.

### Results and discussion

In this section, we will compare the performance of the proposed NMA chart over the existing MA chart under classical statistics in terms of \(ARL_{1}\) and NSD and by a simulation study. For this comparison, the same values of \(r_{0N}\) and \(c\) are considered.

#### Comparison in NARL

The proposed control chart is the extension of the X-bar control chart under neutrosophic statistics proposed by Aslam and Khan\(^9\). The proposed chart reduces to Aslam and Khan\(^9\) when \(w_{N}\) is \([1, 1]\). We present the values of \(ARL_{1}\) and NSD of both charts when \(r_{0N}\) is \([370, 370]\) in Table 5.

From Table 5, it can be noted that the proposed chart has smaller indeterminacy intervals of \(ARL_{1}\) and NSD as compared to the chart proposed by Aslam and Khan\(^9\). For example, when \(c=0.1\), the value of \(ARL_{1}\) decreases as \(n_{N}\) increases. 1. The values of \(ARL_{1}\) decreases as \(n_{N}\) increases from 300 to 370.

#### Comparison by Simulation Data

Now we compare the performance of the proposed chart with the chart proposed by Aslam and Khan\(^9\) and X-bar control chart under classical statistics using the simulated data. Let \(n_{N}\) is \([6, 8]\), \(w_{N}\) is \([3, 5]\) and \(r_{0N}\) is \([370, 370]\). The 20 observations are generated from the normal distribution with \(\mu_{N}\) and variance \(\sigma_{N}^{2}\) in \([1, 1]\). Later 20 observations are generated from the process with a shift of 0.30. The values of statistic \(MAI\) and two existing charts are calculated and plotted in Fig. 1. From Fig. 1, it can be seen that the proposed chart detects shift at the 20th sample. On the other hand, the chart proposed by Aslam and Khan\(^9\) detects a shift in the process at around 30th sample while the traditional Shewhart control chart under classical statistics does not show any shift in the process. We also note that the proposed chart shows several points in the indeterminacy interval of control limits.

#### Case study

A meteorologist is interested to apply the proposed control chart for the better prediction and analysis of the weather. To explain the implementation of the proposed control chart, we collected the weather data in Jeddah, Saudi Arabia. The temperature data of October 2019 at three different occasions is collected from https://www.timeanddate.com/weather/saudi-arabia/jeddah/historic. The temperature data at morning 6, afternoon and evening 6 are reported in Table 6. From Table 6, it can be seen that the weather data has neutrosophic numbers. Therefore, the existing control chart under classical statistics cannot be applied for this type of data. The use of the existing chart under classical statistics may mislead the meteorologist. The proposed control chart under neutrosophic statistics can be applied for the monitoring of the weather. The proposed control chart and existing control charts for the weather data are shown in Fig. 2.
Figure 1. The control charts using the simulation data. Left figure (proposed chart), middle figure for Aslam and Khan\textsuperscript{39} and right figure (Shewhart chart). This is generated using R version 3.2.1 (https://www.r-project.org/).
From Fig. 2, we note that several temperature values are close to the upper value of the control limits. On the other hand, the existing chart under the classical statistic shows that the weather is in-control and the meteorologist needs no action. From this comparison, it is concluded that the proposed chart indicates some issue in weather and meteorologist should be alert about it.

Table 6. The weather data.

| Weeks          | Sample | 6 AM   | 12:00 PM | 18PM   |
|----------------|--------|--------|----------|--------|
| Week 1 October 1 | 1      | [30, 35] | [33, 35] | [31, 33] |
|                | 2      | [30, 35] | [33, 35] | [31, 33] |
|                | 3      | [30, 34] | [32, 34] | [32, 33] |
|                | 4      | [30, 34] | [32, 34] | [30, 32] |
|                | 5      | [29, 34] | [32, 34] | [30, 32] |
|                | 6      | [29, 36] | [33, 36] | [32, 33] |
|                | 7      | [31, 34] | [33, 35] | [33, 38] |
| Week 2 October 8 | 1      | [31, 37] | [35, 38] | [31, 36] |
|                | 2      | [30, 37] | [33, 36] | [31, 33] |
|                | 3      | [31, 34] | [32, 34] | [31, 32] |
|                | 4      | [28, 35] | [31, 35] | [30, 31] |
|                | 5      | [29, 36] | [32, 36] | [31, 32] |
|                | 6      | [29, 35] | [35, 36] | [32, 35] |
|                | 7      | [30, 34] | [32, 35] | [31, 32] |
| Week 3 October 15 | 1      | [29, 35] | [31, 35] | [31, 32] |
|                 | 2      | [28, 36] | [33, 36] | [30, 33] |
|                 | 3      | [26, 39] | [34, 39] | [30, 32] |
|                 | 4      | [27, 37] | [32, 37] | [31, 32] |
|                 | 5      | [29, 35] | [32, 35] | [31, 32] |
|                 | 6      | [28, 34] | [31, 34] | [29, 31] |
|                 | 7      | [26, 34] | [32, 34] | [30, 32] |
| Week 4 October 22 | 1      | [28, 33] | [31, 33] | [29, 31] |
|                 | 2      | [27, 34] | [32, 34] | [30, 31] |
|                 | 3      | [27, 34] | [32, 34] | [30, 31] |
|                 | 4      | [28, 35] | [32, 35] | [30, 32] |
|                 | 5      | [29, 34] | [32, 34] | [29, 32] |
|                 | 6      | [28, 34] | [32, 34] | [28, 30] |
|                 | 7      | [26, 34] | [30, 34] | [28, 30] |

Figure 2. The control charts for real data. Left figure (proposed chart) and right figure (Shewhart chart). This figure is generated using R version 3.2.1 (https://www.r-project.org/).

From Fig. 2, we note that several temperature values are close to the upper value of the control limits. On the other hand, the existing chart under the classical statistic shows that the weather is in-control and the meteorologist needs no action. From this comparison, it is concluded that the proposed chart indicates some issue in weather and meteorologist should be alert about it.
Concluding remarks
A new moving average control chart for the normal distribution was offered under the neutrosophic statistics. The parameters of the offered chart were determined through simulation under neutrosophic statistics. The comparison study showed the superiority of the proposed chart over the moving average control chart under classical statistics. A real example from the weather was chosen to present the implementation of the chart. From the simulation study and real data, the proposed chart was found to be effective to be applied for temperature monitoring than the existing control chart. The proposed control chart can be applied in the weather department for the monitoring of the process. The proposed chart using EWMA statistics can be extended for future research.

Data availability
The data is selected from https://www.timeanddate.com/weather/saudi-arabia/jeddah/historic.

Received: 13 April 2020; Accepted: 19 June 2020
Published online: 22 July 2020

References
1. Chen, Y.-S. & Yang, Y.-M. An extension of Banerjee and Rahim's model for economic design of moving average control chart for a continuous flow process. Eur. J. Oper. Res. 143, 600–610 (2002).
2. Wong, H., Gan, F. & Chang, T. Designs of moving average control chart. J. Stat. Comput. Simul. 74, 47–62 (2004).
3. Khoo, M. B. & Wong, V. A double moving average control chart. Commun. Stat. Simul. Comput. 37, 1606–1708 (2008).
4. Areepong, Y. Optimal parameters of double moving average control chart. World Acad. Sci. Eng. Technol. Int. J. Math. Comput. Phys. Electr. Comput. Eng. 7, 1283–1286 (2013).
5. Moinih, M., Aslam, M. & Jun, C.-H. A new generally weighted moving average control chart based on Taguchi’s loss function to monitor process mean and dispersion. Proc. Inst. Mech. Eng. Part B J. Eng. Manuf. 230, 1537–1547 (2016).
6. Alghamdi, S. S. D., Aslam, M., Khan, K. & Jun, C.-H. A time truncated moving average chart for the Weibull distribution. IEEE Access 5, 7216–7222 (2017).
7. Ye, X., Kan, Y., Xiong, X., Zhang, Y. & Chen, X. A quality control chart based on an improved kernel regression algorithm for surface air temperature observations. Adv. Meteorol. 2020, 604592 (2020).
8. Su, A., Li, H., Cui, L. & Chen, Y. A convection nowcasting method based on machine learning. Adv. Meteorol. 2020, 5124274 (2020).
9. Khadem, M. & Amirzadeh, V. Fuzzy rules for fuzzy Shewhart control chart. SSSS, 55–66 (2014).
10. Faraz, A. & Moghadam, M. B. Fuzzy control chart a better alternative for Shewhart average chart. Qual. Quant. 41, 375–385 (2007).
11. Zarandi, M. F., Aalaeddin, A. & Turkser, I. A hybrid fuzzy adaptive sampling—Run rules for Shewhart control charts. Inf. Sci. 178, 1152–1170 (2008).
12. Faraz, A., Kazemzadeh, R. B., Moghadam, M. B. & Azadian, A. Constructing a fuzzy Shewhart control chart for variables when uncertainty and randomness are combined. J. Qual. Quant. 44, 905–914 (2010).
13. Wang, D. & Hryniewicz, O. A fuzzy nonparametric Shewhart chart based on the bootstrap approach. Int. J. Appl. Math. Comput. Sci. 25, 389–401 (2015).
14. Kahraman, C., Gulbey, M. & Boland, M. F. A fuzzy Statistical Decision-Making 263–280 (Springer, Berlin, 2016).
15. Kaya, I., Erdogan, M. & Yildiz, C. Analysis and control of variability by using fuzzy individual control charts. Appl. Soft Comput. 51, 370–381 (2017).
16. Khan, M. Z., Khan, J. F., Yum, M., Niazi, S. T. A. & Mughal, A. R. A fuzzy EWMA attribute control chart to monitor process mean. Information 9, 312 (2018).
17. Smarandache, F. Neutrosophic Statistical Probability, Set, and Logic Vol. 105, 118–123 (ProQuest Information & Learning, Ann Arbor, 1998).
18. Wang, H., Storandt, F., Sunderraman, R. & Zhang, Y.-Q. Interval Neutrosophic Sets and Logic: Theory and Applications in Computing: Tools and Applications in Computing Vol. 5 (Infinite Study, New York, 2005).
19. Hamzeh, I., Salama, H. & Mahfouz, K. Neutrosophic classical events and its probability. Int. J. Math. Comput. Appl. Res. 3, 171–178 (2013).
20. Guo, Y. & Jeng, J. ACM. Neutrosophic evidential c-means clustering algorithm. Neural Comput. Appl. 26, 561–571 (2015).
21. Abdel-Basset, M., Manogaran, G., Gamal, A. & Smarandache, F. A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria. Des. Autom. Embed. Syst. 22, 257–278 (2018).
22. Loharud, R., Rana, M. M., Farah, H. & Salama, A. Some neutrosophic probability distributions. In Neutrosophic Sets and Systems (eds Smarandache, F. & Broumi, S.) 30–38 (University of New Mexico, Albuquerque, 2018).
23. Kuroki, S., Bakali, A., Talea, M. & Smarandache, F. Bipolar neutrosophic minimum spanning tree (Infinite Study, New York, 2018).
24. Peng, J. X. & Dai, J. Approaches to single-valued neutrosophic MADM based on MARAC, TOPSIS and new similarity measure with score function. Neural Comput. Appl. 29, 933–954 (2018).
25. Shahin, A., Amin, K., Sharawi, A. A. & Guo, Y. A novel enhancement technique for pathological microscopic image using neutrosophic similarity score scaling. Optical 161, 84–97 (2018).
26. Abdel-Basset, M., Mohamed, M., Elhoseny, M., Chichana, F. & Zaied, A.-E.-N.-H. Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases. Artif. Intell. Med. 101, 101735 (2019).
27. Jana, C. & Pal, M. A robust single-valued neutrosophic soft aggregation operators in multi-criteria decision making. Symmetry 11, 281 (2019).
28. Nabeeth, N. A., Abdel-Basset, M., El-Ghareeb, H. A. & Aboelhelou, A. Neutrosophic multi-criteria decision making method for IoT-based enterprises. IEEE Access 7, 59559–59574 (2019).
29. Smarandache, F. Introduction to neutrosophic statistics (Infinite Study, New York, 2014).
30. Chen, J., Ye, J. & Du, S. Scale effect and anisotropy analyzed for neutrosophic numbers of rock joint roughness coefficient based on neutrosophic statistics. Symmetry 9, 208 (2017).
31. Chen, J., Ye, J., Xu, S. & Yong, R. Expressions of rock joint roughness coefficient using neutrosophic interval statistical numbers. Symmetry 9, 123 (2017).
32. Aslam, M., Bantan, R. A. & Khan, N. Design of a new attribute control chart under neutrosophic statistics. Int. J. Fuzzy Syst. 21, 433–440 (2019).
33. Aslam, M., Khan, N. & Khan, M. Monitoring the Variability in the Process Using Neutrosophic Statistical Interval Method. Symmetry 10, 562 (2018).
34. Aslam, M., Al-Marshadhi, A. H. & Khan, N. A new X-bar control chart for using neutrosophic exponentially weighted moving average. Mathematics 7, 957 (2019).
35. Aslam, M., Bantan, R. A. & Khan, N. Design of S2N—NEWMA control chart for monitoring process having indeterminate production data. *Processes* **7**, 742 (2019).
36. Montgomery, D. C. *Introduction to Statistical Quality Control* (Wiley, Hoboken, 2007).
37. Li, Z., Wang, Z. & Wu, Z. Necessary and sufficient conditions for non-interaction of a pair of one-sided EWMA schemes with reflecting boundaries. *Statistics & probability letters* **79**, 368–374 (2009).
38. Li, Z., Zou, C., Gong, Z. & Wang, Z. The computation of average run length and average time to signal: an overview. *J. Stat. Comput. Simul.* **84**, 1779–1802 (2014).
39. Aslam, M. & Khan, N. A new variable control chart using neutrosophic interval method-an application to automobile industry. *Journal of Intelligent & Fuzzy Systems* **36**, 2615–2623 (2019).

**Acknowledgements**
The author is deeply thankful to the editor and the reviewers for their valuable suggestions to improve the quality of this manuscript. We are also very thankful Dr. Nasrullah Khan (UVAS, Lahore Pakistan) for his valuable suggestions. This work was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah, under grant No. (130-265-D1440). The author, therefore, gratefully acknowledge the DSR technical and financial support.

**Author contributions**
Design and conduct of the study (M.A); collection of data (M.A., AAS. and K.K.); management, analysis, and interpretation of data (M.A).

**Competing interests**
The authors declare no competing interests.

**Additional information**
Correspondence and requests for materials should be addressed to M.A.

**Reprints and permissions information** is available at www.nature.com/reprints.

**Publisher’s note** Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

[Open Access] This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this license, visit http://creativecommons.org/licenses/by/4.0/.

© The Author(s) 2020.