Abstract

We address a practical problem ubiquitous in modern industry, in which a mediator tries to learn a policy for allocating strategic financial incentives for customers in a marketing campaign and observes only bandit feedback. In contrast to traditional policy optimization frameworks, we rely on a specific assumption for the reward structure and we incorporate budget constraints. We develop a new two-step method for solving this constrained counterfactual policy optimization problem. First, we cast the reward estimation problem as a domain adaptation problem with supplementary structure. Subsequently, the estimators are used for optimizing the policy with constraints. We establish theoretical error bounds for our estimation procedure and we empirically show that the approach leads to significant improvement on both synthetic and real datasets.

1. Introduction

Learning from logged bandit feedback (Swaminathan & Joachims, 2015a) is a form of counterfactual inference given only observational data (Pearl, 2009). This problem is ubiquitous in many real-world decision making scenarios such as personalized medicine (“what would have been the treatment leading to the optimal outcome for this particular patient?”) (Xu et al., 2016) or online marketing (“which ad should have been placed in order to maximize the click-through-rate?”) (Strehl et al., 2010). We review existing approaches for solving these types of problems in Sec. 2.

In this paper, we focus on a specific flavor of learning from logged bandit feedback, which we name cost-effective incentive allocation. More precisely, we allocate economic incentives (e.g., online coupons) to customers and observe a response (e.g., whether the coupon is used or not). Each action is mapped to a cost and we further assume that the response is monotonically increasing with respect to the action’s cost. Furthermore, we incorporate budget constraints related to the global cost of the marketing campaign. This framework can be readily applied to the problem of allocating monetary values of coupons in a marketing campaign under fixed budget constraints from the management. We present the setting of batch learning from bandit feedback in Sec. 3 and the novel assumptions in Sec. 4.

Existing work in counterfactual inference using bandit feedback (Joachims et al., 2018; Shalit et al., 2017) does not make use of the supplementary structure for the rewards and is limited in practice when the cardinality of the action space is large (Lefortier et al., 2016). We therefore developed a novel algorithm which incorporates such structure. The algorithm, which we refer to as constrained policy optimization via structured incentive response estimation, has two components. First, we take into account the reward structure to estimate the incentive response. This step is based on representation learning for counterfactual inference (Johansson et al., 2016). Second, we rely on the estimates to optimize the coupon assignment policy under budget constraints. We derive error bounds for this algorithm in Sec. 5.

We benchmark our approach on simulated and real-world data against state-of-the-art approaches and show the advantages of our method in Sec. 6.

2. Related Work

Constrained Policy Optimization Safety constraints in reinforcement learning (RL) are usually expressed via the level sets of a cost function. The main challenge of safe RL is that the cost of a certain policy must be evaluated with a off-policy strategy, which is a hard problem (Jiang & Li, 2016). Achiam et al. (2017) focus on developing a local policy search algorithm with guarantees of respecting cost constraints. This approach is based on a trust-region method (Schulman et al., 2015), which allows it to circumvent the off-policy evaluation step. Our setting is more akin to that of contextual multi-armed bandits, since customers are modeled as sampled iid from a unique distribution. In this scenario, checking for the satisfaction of the cost constraints is straightforward, which makes the original prob-
lem substantially easier. Most research contributions on policy optimization with budget constraints focus on the online learning setting (Ding et al., 2013; Badanidiyuru et al., 2018; Burnetas et al., 2017; Xia et al., 2015; Wu et al., 2015). The underlying algorithms are unfortunately not directly applicable to off-line policy optimization.

**Counterfactual Risk Minimization** The problem of learning from logged bandit feedback consists in maximizing the expected rewards

\[
E_{x \sim P(x)} E_{y \sim \rho(y|x)} \delta(x, y) \frac{\pi(y | x)}{\rho(y | x)},
\]

where \(\delta\) is the reward function, \(\pi\) a parameterized policy and \(\rho\) the logging policy. All methods rely on importance sampling and it is therefore necessary to log the action probabilities along with the reward. Strehl et al. (2010) developed error bounds when the logging policy is unknown and learned from the data. Dudik et al. (2011) focus on reducing the variance of off-policy evaluation using a doubly robust estimator. Swaminathan & Joachims (2015a;b) develop learning bounds based on an empirical Bernstein inequality (Maurer & Pontil, 2009) and present a principle of Counterfactual Risk Minimization (CRM) which is based on empirical variance regularization. Wu & Wang (2018) propose a trust-region variant and further bound the empirical variance with a chi-square divergence, evaluated with an f-GAN (Nowozin et al., 2016). Joachims et al. (2018) is based on Swaminathan & Joachims (2015c) and focuses on a practical deep learning solution of equivariant estimation and stochastic optimization of Eq. (1). An advantage of this framework is that its mathematical assumptions are weak, while a disadvantage is that it is not clear how to make use of structured rewards.

**Individualized Treatment Effect Estimation** The problem of Individualized Treatment Effect (ITE) estimation aims at estimating the difference in expectation between two treatments

\[
E[r | x, y = 1] - E[r | x, y = 0],
\]

where \(x\) is a point in customer feature space and \(r\) is a random variable corresponding to the rewards. The difficulty arises primarily from the fact that the historical data do not always fall into the ideal setting of a randomized control trial. That inevitably induces an estimation bias due to the discrepancy between the empirical distributions \(P(x | y = 0)\) and \(P(x | y = 1)\). Hill (2011) presents an instrumental method for such an estimation based on Bayesian Additive Regression Trees (BART). Johansson et al. (2016); Shalit et al. (2017) cast the counterfactual question into a domain adaptation problem that can be solved via representation learning. Essentially, they propose to find an intermediate feature space \(Z\) which embeds the customers and trades off the treatment discrepancy for the reward predictability. Yoon et al. (2018) propose using generative adversarial networks to learn the counterfactual rewards and extend this framework to the multiple treatment case via a mean-square error loss. This line of work does not require knowing the logging policy beforehand. Remarkably, no previous work focuses on the setting of structured rewards.

### 3. Batch Learning from Bandit Feedback

For concreteness, we focus on the example of a marketing campaign. Let \(\mathcal{X}\) be an abstract space and \(P(x)\) a probability distribution on \(\mathcal{X}\). Let \(\mathcal{M}\) be the mediator and \((x_1, \ldots, x_n) \in \mathcal{X}^n\) a set of customers. We assume each customer is an iid sample from \(P(x)\). Let \(\mathcal{Y}\) be the set of financial incentives possibly emitted by \(\mathcal{M}\) and \(S^\mathcal{Y}\) be the space of probability distributions over \(\mathcal{Y}\). Mediator \(\mathcal{M}\) deploys a marketing campaign which we model as a policy \(\pi : \mathcal{X} \rightarrow S^\mathcal{Y}\). For simplicity, we also denote the probability of an action \(y\) under a policy \(\pi\) for a customer \(x\) using conditional distributions \(\pi(y | x)\). In response, customers can either choose to purchase the product from \(\mathcal{M}\) or from another unobserved party. As the customers might engage in interaction with other parties, this is not a fully-observable game. We therefore model our data acquisition process as a stochastic contextual bandit feedback problem. Given a context \(x\) and an action \(y\), we observe a stochastic reward \(r \sim P(r | x, y)\). In practice, the reward can be defined as any available proxy to the mediator profit (e.g., whether the coupon was used or not). From that, the mediator \(\mathcal{M}\) seeks an optimal policy:

\[
\pi^* \in \arg \max_{\pi \in \Pi} E_{x \sim P(x)} E_{y \sim \pi(y|x)} E[r | x, y].
\]

This setting is known as batch learning from bandit feedback (Swaminathan & Joachims, 2015a;bc). This problem has connections to causal and particularly counterfactual inference. As described in Swaminathan & Joachims (2015a;b), the data is incomplete in the sense that we do not observe what would have been the reward if another action was taken. Furthermore, we cannot play the policy \(\pi\) in real time; we instead only observe data sampled from a logging policy \(\rho(y | x)\) (the data generative process is described in Alg. 1). Therefore, the collected data is also biased since actions taken by the logging policy \(\rho\) are over-represented. Swaminathan & Joachims (2015c) introduce a distinction between standard overfitting which arises from performing model selection on finite-sample data and propensity overfitting which instead arises from performing risk minimization based on biased exploration data.

### 4. Cost-Effective Incentive Allocation

The setting of batch learning from bandit feedback might not be suitable when actions can be mapped to monetary
We claim that the estimation of the incentive response function \( E(y|x) \) is known. Then solving (6) reduces to a binary search over a unique Lagrange multiplier for the cost constraint. Particularly, each step of the search has time complexity \( \mathcal{O}(n) \) with \( n \) being the number of samples in the dataset.

This negative result shows that one should not rely on Incentive Response Estimation (IRE) solely in order to perform Constrained Policy Optimization (CPO). As we will see, however, the IRE problem becomes simpler under the structured reward assumption (Model Ass. 3). This is in contrast to CRM-based policy optimization methods which to not make use of the structure and therefore have the same complexity. Our approach is therefore a two-stage procedure. First, we will present a new method for IRE. That method is inspired from the ITE estimation literature but fully exploits the reward structure. Second, we solve the CPO problem based on the estimated structured reward function. We argue that exploiting the statistical structure when it exists gives advantages when compared to vanilla constrained policy optimization; this argument is similar to that comparing model-based or model-free reinforcement learning (Pong et al., 2018).

Statistical Benefits from Structured Rewards
Learning from bandit feedback is a hard problem when compared to supervised learning because of the partial feedback. On average, one single fully-supervised feedback is equivalent to \( 2^{|\mathcal{Y}|} \) instances of bandit feedback. While taking into account a structured reward, a high reward for a given incentive means that a higher incentive will also probably yield a high reward. Even though this might not be quantifiable, the benefit is to have a less-complex reward structure. Second, learning from bandit feedback is also hard due to the data collection bias. For the same reason, our structure enables one action to give clues about the consequences of other actions—which is helpful for alleviating such bias.

5. Constrained Policy Optimization via Structured Incentive Response Estimation
We begin by making our assumptions formal.

Model Assumption 4. (Refined rewards structure) There exists a function \( f : \mathcal{X} \times \mathcal{Y} \rightarrow [0,1] \) such that
\[
\forall (x,y) \in \mathcal{X} \times \mathcal{Y}, r \mid x, y \sim \text{Bernoulli} \left( f(x,y) \right),
\]
and \( f \) satisfies the structured reward assumption
\[
\forall x \in \mathcal{X}, \forall (y,y') \in \mathcal{Y}^2, c(y) \leq c(y') \quad \implies \quad f(x,y) \leq f(x,y').
\]
In order to ensure that the causal effect is identifiable, we adopt the following classical assumptions from counterfactual inference (Rubin, 2005).

Math Assumption 1. (Overlap) The logging policy satisfies the condition
\[
\forall (x,y) \in \mathcal{X} \times \mathcal{Y}, \rho(y \mid x) \in (0,1)
\]
Math Assumption 2. (No unmeasured confounding) Let \( r^y = (r_y)_{y \in \mathcal{Y}} \) denote the vector of possible outcomes in the Rubin-Neyman potential outcomes framework (Rubin, 2005). We assume that the vector \( r^y \) is independent of the action \( y \) given \( x \).

These “strong ignorability” hypotheses are sufficient for identifying the reward function from the historical data (Shalit et al., 2017).

5.1. Bias estimation from domain adaptations bounds

We turn to the problem of estimating the function \( f \) from historical data (i.e., generated by Alg. 1). To this end, we first write the estimation problem with a general population loss. Let \( L : \mathbb{R}^2 \rightarrow \mathbb{R}^+ \) be a loss function and \( D \) a probability distribution on the product \( \mathcal{X} \times \mathcal{Y} \) (which we refer to as a domain). Let \( (\hat{f}_0, \hat{y}_0) \) be a set of functions parameterized by \( \phi \in \Phi \). We define the domain dependent population risk \( \epsilon_D^L (\hat{f}_0) \) as

\[
\epsilon_D^L (\hat{f}_0) = \mathbb{E}_{(x,y) \sim D} L (\hat{f}_0(x, y), f(x, y)).
\]

In the historical data, individual data points \((x, y)\) are sampled from the so-called source domain \( D_S = \mathcal{P}(x)\pi(y \mid x) \). However, in order to perform off-policy evaluation of a given policy \( \pi \), we would need ideally data from \( \mathcal{P}(x)\pi(y \mid x) \). The discrepancy between the two distributions will cause a strong bias in off-policy evaluation which can be quantified via learning bounds from domain adaptation (Blitzer et al., 2007). The issue is therefore that of finding a target domain \( D_T \) from which the estimated incentive response \( \hat{f}_0 \) would generalize to all policies for offline evaluation.

Intuitively, we want to find a domain which lies halfway \( \mathcal{P}(x)\rho(y \mid x) \) and \( \mathcal{P}(x)\pi(y \mid x) \) for all policies \( \pi \). We extend the work of Johansson et al. (2016); Shalit et al. (2017) for this purpose. In the ITE scenario, the domain that is central to the problem is the mixture between factual and counterfactual domain \( \mathcal{P}(x \mid t = 0)\mathcal{P}(t = 0) + \mathcal{P}(x \mid t = 1)\mathcal{P}(t = 1) \), in which the treatment assignment \( t \) (i.e., \( y \) in our setting) and the customer feature \( x \) are decoupled. In the sequel, we therefore focus on selecting a target domain \( D_T \) in the factorized form \( \mathcal{P}(x)q(y) \) where \( q(y) \) is a categorical distribution on the action space with probability \( (q_1, \ldots, q_K) \).

Following Shalit et al. (2017), we wish to derive a learning bound for the structured incentive response estimation problem that yields a principled algorithm. In particular, we wish to bound how much an estimate of \( f \) based on data from the source domain \( D_S \) can generalize to the target domain \( D_T \). To this end, we bound the discrepancy between the population risks on the source and target domains following (Blitzer et al., 2007):

\[
\left| \epsilon_{D_T}^L (\hat{f}_0) - \epsilon_{D_S}^L (f_0) \right| \\
= \left| \mathbb{E}_{x \sim \mathcal{P}(x)} \sum_{y \in \mathcal{Y}} L (\hat{f}_0(x, y), f(x, y)) (q_y - \rho(y \mid x)) \right| \\
\leq \sup_{g \in \Phi} \left| \int_x \sum_y g(x, y) \rho(x) (q_y - \rho(y \mid x)) \right|,
\]

where \( \Phi \) is the set of functions

\[
\mathcal{F} = \{(x, y) \mapsto l_\phi(x, y) = L (\hat{f}_\phi(x, y), f(x, y)) \mid \phi \in \Phi \}.
\]

We refer to the mathematical object in equation (11) as an integral probability metric (IPM) between distributions \( D_S \) and \( D_T \) with function class \( \mathcal{F} \), noted \( \text{IPM}_\mathcal{F}(D_S, D_T) \). The general problem of computing IPMs is known to be NP-hard. Shalit et al. (2017) present some specific cases for two treatments in which this IPM can be estimated, for example using the maximum mean discrepancy (MMD) (Gretton et al., 2012). Atan et al. (2018) focus on multiple treatments but relying on adversarial neural networks (Ganin et al., 2016). We focus instead on a different nonparametric measure of independence—the Hilbert-Schmidt Independence Criterion (HSIC) (Gretton et al., 2005; 2008)—which also yields an efficient estimation procedure for the IPM.

Proposition 2. Let us assume that \( \mathcal{X} \) and \( \mathcal{Y} \) are separable metric spaces. Let \( k : \mathcal{X} \times \mathcal{X} \rightarrow \mathbb{R} \) (resp. \( l : \mathcal{Y} \times \mathcal{Y} \rightarrow \mathbb{R} \)) be a continuous, bounded, positive semi-definite kernel. Let \( \mathcal{H} \) (resp. \( \mathcal{K} \)) be the corresponding reproducing kernel Hilbert space (RKHS). Let us assume that the function space \( \mathcal{F} \) is included in the unit ball of the tensor space \( \mathcal{H} \otimes \mathcal{K} \). Let \( q^* \) be the marginal frequency of actions under the logging policy \( \rho \):

\[
\forall y \in \mathcal{Y}, q_{y}^* = \int_x \rho(y \mid x) d\mathcal{P}(x).
\]

Under these assumptions, one can identify the IPM in equation (11) as follows:

\[
\text{IPM}_\mathcal{F}(D_S, D_T) = \text{MMD}_{\mathcal{H} \otimes \mathcal{K}} (\mathcal{P}(x)\rho(y \mid x), \mathcal{P}(x)q^*(y)) = \text{HSIC}_{\mathcal{H} \otimes \mathcal{K}} (\mathcal{P}(x)\rho(y \mid x)).
\]

Proof. See Section 2.3 of Smola et al. (2007), Definition 2 from Gretton et al. (2012) and Definition 11 from Sejdinovic et al. (2013).

Notably, the HSIC can be directly estimated via samples from \( \mathcal{P}(x)\rho(y \mid x) \) (Gretton et al., 2008) in \( \mathcal{O}(n^2) \), with \( n \) number of data points.

5.2. Bias correction via feature transformation

Provided that kernels \( k \) and \( l \) are both characteristic, then the IPM in Eq. (11) is null if and only if the logging policy
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\( \rho \) is independent from the customer feature \( x \) (Gretton et al., 2008). In order to control the bias in estimating \( f \) from the historical data, we follow Johansson et al. (2016); Shalit et al. (2017) and introduce an abstract feature space \( \mathcal{Z} \) and a mapping \( \Lambda \) such that \( z = \Lambda(x) \). For technical reasons, we need to make the following assumption:

**Math Assumption 3.** \( \Lambda : \mathcal{X} \rightarrow \mathcal{Z} \) is a twice-differentiable one-to-one mapping. We let \( \Psi \) denote the corresponding inverse mapping.

Adopting the notation of Shalit et al. (2017), we can write the following bound

**Proposition 3.** Let \( \phi \in \Phi \) and \( \Lambda \) an mapping satisfying Math Ass. 3. Let us assume there exists a constant \( B \) such that \( \psi_\Lambda \) is inside the unit ball of the tensor space \( \mathcal{H} \otimes \mathcal{K} \). There exists a constant \( \kappa \) such that

\[
\epsilon_{\mathcal{D}_\Psi}^\Lambda (\hat{f}_\phi) \leq \epsilon_{\mathcal{D}_\Psi}^\Lambda (f_\phi) + \kappa \text{HSIC}(\mathcal{P}(z)\rho(y \mid z)). \tag{14}
\]

Remarkably, the first term of the right hand side is a prediction error for the historical reward (function of \( \phi \) and \( \Lambda \)) while the second one measures how much the intermediate space is informative of the logging policy (a function of \( \Lambda \) only).

### 5.3. Counterfactual policy optimization

We proceed to assessing the bias for policy evaluation as a function of the estimation error.

**Proposition 4.** Let \( \pi \) be a policy, \( \Lambda \) a mapping and \( \phi \in \Phi \). Let us assume that \( L \) is the \( \ell_1 \)-loss. Under assumptions from Prop. 3, there exists a constant \( \kappa \) such that:

\[
\mathbb{E}_{x \sim \mathcal{P}(x)} \mathbb{E}_{y \sim \pi(y \mid x)} f(x, y) \\
\geq \mathbb{E}_{x \sim \mathcal{P}(x)} \mathbb{E}_{y \sim \pi(y \mid \Lambda(x))} f_\phi(\Lambda(x), y) \\
- \frac{\mathbb{E}_{(x,y) \sim \mathcal{P}(x)\rho(y \mid x)} |f_\phi(\Lambda(x), y) - f(x, y)|}{\min_k q_k} \\
- \frac{\text{HSIC}(\mathcal{P}(z)\rho(y \mid z))}{\min_k q_k}.
\]

\[
\mathbb{E}_{x \sim \mathcal{P}(x)} \mathbb{E}_{y \sim \pi(y \mid x)} f(x, y) \\
\geq \mathbb{E}_{x \sim \mathcal{P}(x)} \mathbb{E}_{y \sim \pi(y \mid \Lambda(x))} f_\phi(\Lambda(x), y) \\
- \frac{\mathbb{E}_{(x,y) \sim \mathcal{P}(x)\rho(y \mid x)} |f_\phi(\Lambda(x), y) - f(x, y)|}{\min_k q_k} \\
- \frac{\text{HSIC}(\mathcal{P}(z)\rho(y \mid z))}{\min_k q_k}.
\]

\[
\max \frac{1}{n} \sum_i \pi(y \mid \Lambda(x_i)) f_\phi(\Lambda(x_i), y) \\
\text{such that} \frac{1}{n} \sum_i c(y) \pi(y \mid \Lambda(x_i)) \leq m,
\]

for which multiple solutions can be used. Options include the exact binary search procedure from Prop. 1, stochastic optimization where \( \pi \) is parameterized with a neural network, or more complex estimators such as a doubly robust estimator (Dudík et al., 2011) with our structured rewards.

We detail the practical implementation of our algorithm in Alg. 2.

### 6. Experiments

We compare our method for cost-effective incentive allocation with a CRM baseline inspired from Joachims et al. (2018) and described in Appendix B. We note that this baseline also relies on deep learning. When applicable, we also benchmark our ITE estimation against BART (Hill, 2011).

#### 6.1. Fully-simulated data

Since it is difficult to obtain a realistic dataset meeting all our assumptions and containing full information, for benchmarking purposes we first construct a synthetic dataset following Zhao et al. (2017).
Algorithm 2 Constrained Policy Optimization via Structured Incentive Response Estimation

1: Load data in the form \((x, y, r)\)
2: Fix a budget \(k \in \mathbb{R}^+\)
3: **Part 1: Incentive Effect Estimation**
4: Initialize the embedding \(\hat{\Lambda}\) and the regression \(f_\phi\)
5: Fix a Lagrange multiplier \(\kappa \in \mathbb{R}^+\)
6: Solve the following optimization problem:

\[
\hat{\phi}, \hat{\Lambda} = \arg\min_{\phi, \Lambda} \frac{1}{n} \sum_{i=1}^{n} L(f_\phi(\Lambda(x_i), y_i), r_i)
+ \kappa \text{HSIC}\left((x_i, y_i)_{i \in \{1, \ldots, n}\}}\right)
\]

(18)

7: **Part 2: Constrained Policy Optimization**
8: Solve the following optimization problem:

\[
\arg\max_{\theta} \frac{1}{n} \sum_{i=1}^{n} \sum_{y \in \mathcal{Y}} \pi_\theta(y | x_i)(f_\phi(\hat{\Lambda}(x_i), y) - \eta c(y))
\]

(19)

9: Use binary search on \(\eta\) to obtain the highest-cost-but-in-budget policy noted \(\theta^*\).
10: **output** \(\theta^*\)

Simulation framework The dataset is of the form \((x, y, r)\) and is generated following the procedure from Alg. 1. Random variable \(x\) represents the users’ features and is uniformly distributed in the cube:

\[
x \sim \mathcal{U}(0, 1)^{50}.
\]

(20)

\(y \in \{1, \ldots, 5\}\) represents the action historically assigned to the customer, the cost of which is \((0, \ldots, 4)\) respectively. Our logging policy \(\rho\) is defined as

\[
\rho(y = i | x) = \frac{x_i}{\sum_{j=1}^{5} x_j},
\]

(21)

where \(r\) represents the response after customer \(x\) receives the incentive \(y\). In order to meet Model Assumption 4, we generate the response by function \(f\) as

\[
f: (x, y) \mapsto \mathcal{S}\left(\frac{h(x) - \mu}{\sigma} + \frac{y}{5}\right),
\]

(22)

where \(\mu\) and \(\sigma\) are the mean value and standard deviation of \(h(x)\) respectively, \(\mathcal{S}\) denotes the sigmoid function and \(h\) is defined as

\[
h(x_1, \ldots, x_{50}) = \sum_{i=1}^{50} a_i^* \cdot \exp[\sum_{j=1}^{50} -b_j^* x_j - c_j^*],
\]

(23)

with \(a_i^*, b_j^*, c_j^* \in [0, 1]\). Specifically, we select a group of \(a_i^*, b_j^*, c_j^*\) for \(i, j \in \{1, 2, \ldots, 50\}\) independently according to the uniform distribution on \([0, 1]\) for each repeated experiment.

Incentive Response Estimation via HSIC Three common metrics for the estimation errors—Precision in Estimation of Heterogeneous Effect (PEHE), Average Treatment Effect (ATE) and Individualized Treatment Effect (ITE) (Johansson et al., 2016)—are compared. For the multiple treatments experiments, we use the Mean Squared Error (MSE) to evaluate our method following Yoon et al. (2018). Each experiment with the same HSIC penalty \(\kappa\) is repeated 50 times. We use stochastic gradient descent as a first-order stochastic optimizer with a learning rate of 0.01, an RBF kernel and a three-layer neural network with 512 neurons for each hidden layer. Experimental results on the synthetic dataset for a binary treatment (resp. multiple treatments) are shown in Fig. 1(a) (resp. Fig. 1(b)). For the binary treatments experiments, the dataset described above is modified so that only the first two actions are considered as treatments. In the binary treatments experiments of Fig. 1(a), we compare our approach with the ITE error of BART while the other two error metrics are also shown. The error curves are flat for \(\kappa \leq 10^2\). In the regime \(10^3 \leq \kappa \leq 10^4\), the HSIC improves the performance on three indexes, which shows that HSIC helps in improving the treatment estimation. For \(\kappa \geq 10^4\), the errors increase rapidly, which shows that HSIC may hurt representation learning if over-optimized. In the case of the multiple treatments experiments of Fig. 1(b), we can draw the conclusion that the HSIC also improves the incentive response estimation and outperforms BART.

Constrained Policy Optimization results Finally, we perform constrained policy optimization via the incentive response estimation according to Alg. 2. Our results in Fig. 1(c) are averaged across 10 repeated experiments. The performance of the constraint policy optimization shows a similar trend as the incentive response estimation, with respect to \(\kappa\). For suitable values of \(\kappa\), we outperform the CRM baseline.

6.2. Simulating Structured Bandit Feedback from Nested Classification

Standard contextual bandits algorithms can be evaluated by simulating bandit feedback in a supervised learning setting (Agarwal et al., 2014). We propose a novel approach to evaluate cost-effective incentive allocation algorithms. To this end, we use a model of nested classification with bandit-type feedback that we describe in detail in Appendix C.

Experiment We randomly select images with labels “Animal,” “Plant,” and “Natural object” from ImageNet (Ima, 2018), and we focus on the special nested structure “Animal” ⊃ “Arthropod” ⊃ “Invertebrate” ⊃ “Insect”.

A logging policy, with the exact same form as in the fully-simulated experiment, is used to assign one of these four labels for each image. Feedback \(r\) is 1 if the label is correct,
or 0 otherwise. The corresponding costs for selecting these labels are \{3, 2, 1, 0\}.

The dataset is randomly split into training and testing datasets with ratio 8:2. The ratio of the positive and negative samples is equal to 1:10. Images are uniformly preprocessed, cropped to the same size and embedded into $\mathbb{R}^{2048}$ with a pre-trained convolution neural network (VGG-16 from Simonyan & Zisserman (2014)).

Results We perform constrained policy optimization via structured incentive response estimation according to Alg. 2. Our estimation errors are reported in Fig. 2(a). We compare our learned policy with the CRM baseline in Fig. 2(b). We average all results across ten repeated experiments.

We span the same range of the HSIC penalty $\kappa$ as in the previous experiments. All the results are obtained using the same parameters except the number of neurons for the hidden layers that is doubled. In Fig. 2(a), the dashed line represents the MSE error for $\kappa = 0$. Consistently with the simulations, the error decreases first with $\kappa$ but then increases. This demonstrates again that the HSIC improves the treatment estimation.

Correspondingly, the performance of the constrained policy optimization based on feature representation is also influenced by value of $\kappa$. As shown in Fig. 2(b), with suitable $\kappa$ we can also obtain better policies comparing to the CRM baseline. Notably, it is comforting that the same $\kappa$ leads to the smallest estimation error as well as the best performance for policy optimization. The superiority reaches a higher level of significance when the average budget constraint becomes smaller, due to the fact that policy optimization becomes harder with smaller budgets.

Moreover, after adopting the structured response assumption, both the incentive response estimation and the constraint policy optimization perform much better, for which proper values of $\kappa$ still help. Specifically, we show the result from an experiment with structured incentive response estimation but without the HSIC penalty. The incentive

Figure 1. Benchmarking on the fully-simulated dataset. Comparison for incentive response estimation (IRE) and corresponding results for constrained policy optimization (CPO). (a) Estimation error with respect to $\kappa$ in the binary treatments experiment. (b) Estimation error with respect to $\kappa$ in the multiple treatments experiment. (c) Performance of CPO under a budget constraint of 3 with respect to $\kappa$.

Figure 2. Benchmarking on the simulated structured bandit feedback for structured incentive response estimation (SIRE) and corresponding results for constrained policy optimization (CPO). (a) Estimation error with respect to $\kappa$. (b) Performance of CPO under a budget constraint of 2 with respect to $\kappa$. (c) Performance of CPO under a budget constraint of 1 with respect to $\kappa$. 
response in this experiment fits the structured assumption perfectly, which is the main reason why the structured estimation helps so much.

7. Discussion

We have presented a novel framework for counterfactual inference based on the batch learning from bandit feedback scenario but with additional structure on the reward distribution as well as the action space. For this specific setting, we have proposed a novel algorithm based on domain adaptation which effectively trades off prediction power for the rewards against estimation bias. We obtained theoretical bounds which explicitly emphasize this trade-off and we presented empirical evaluations that show that our algorithm outperforms state-of-the-art methods based on the counterfactual risk minimization principle.

Our framework involves the use of a nonparametric measure of dependence to unbias the estimation of rewards. Penalizing the HSIC as we do for each mini-batch implies that no information is aggregated during training about the embedding \( z \) and how it might be biased with respect to the logging policy. On the one hand, this is positive since we do not have to estimate more parameters, especially if the joint estimation would require a minimax problem as in Atan et al. (2018); Yoon et al. (2018). On the other hand, that approach could be harmful if the HSIC could not be estimated with only a mini-batch. Our experiments show this does not happen in a reasonable set of configurations. Trading a minimax problem for an estimation problem does not come for free. First, there are some computational considerations. The HSIC is computed in quadratic time but linear-time estimators of dependence (Jitkrittum et al., 2017) or random-feature approximations (Pérez-Suay & Camps-Valls, 2018) should be used for non-standard batch sizes.

Following up on our work, a natural question is how to properly choose the optimal \( \kappa \), the regularization strength for the HSIC. In previous work such as Shalit et al. (2017), such a parameter is chosen with cross-validation via splitting the datasets. However, in a more industrial setting, it is reasonable to expect the mediator \( M \) to have tried several logging policies which once aggregated into a mixture of deterministic policies enjoy effective exploration properties (see, e.g., Strehl et al., 2010). In particular, an interesting methodological development would include deriving a new notion of Counterfactual Cross-Validation (which informally would reduce the variance when compared against a randomized CV by preventing the propensity overfitting).

Another scenario in which our framework could be extended is the case of continuous treatments. That extension would be natural in the setting of financial incentive allocation and has already been of interest in recent research work (Kallus & Zhou, 2018). The HSIC would still be an adequate tool for quantifying the selection bias since kernel are flexible tools for continuous measurements.

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A. Reduction of incentive estimation to policy optimization

Proof. Let us denote the reward function as \( f(x, y) = \mathbb{E}[r \mid x, y] \). We use the following estimator for the objective as well as the cost in problem 6. Let \( n \in \mathcal{N} \) and \((x_1, \ldots, x_n) \in \mathcal{X}^n \). An empirical estimate of the expected reward can be written as

\[
\max_{\pi \in \Pi} \frac{1}{n} \sum_{i=1}^n \sum_{y} \pi(y \mid x_i) f(x_i, y)
\]

such that \( \frac{1}{n} \sum_{i=1}^n c(y) \pi(y \mid x_i) \leq m \).

There exists a Lagrange multiplier \( \lambda^* \) such that this problem has same optimal set than the following

\[
\max_{\pi \in \Pi} \frac{1}{n} \sum_{i=1}^n \sum_{y} \pi(y \mid x_i) \left( f(x_i, y) - \lambda^* c(y) \right)
\]

In this particular setting, we can directly solve for the optimal action \( y_i^* \) for each customer, which is given by

\[
\arg \max_{y \in \mathcal{Y}} f(x_i, y) - \lambda^* c(y)
\]

To search for the ad-hoc Lagrange multiplier \( \lambda^* \), a binary search can be used to make sure the resulting policy saturates the budget constraint.

B. Counterfactual Risk Minimization baseline

We consider again the constrained policy optimization problem in Eq. (6). We propose to use as a baseline the self-normalized IPS (Swaminathan & Joachims, 2015c) estimator and a novel algorithmic procedure adapted from Joachims et al. (2018) to solve the constrained policy optimization problem with the stochastic gradients algorithm:

\[
\max_{\theta} \sum_{i=1}^n \delta_i \frac{\pi_{\theta}(y_i \mid x_i)}{\hat{p}(y_i \mid x_i)}
\]

s.t. \( \sum_{i=1}^n c(y_i) \pi_{\theta}(y_i \mid x_i) \leq k \)

Let us assume for simplicity that there exists a unique solution \( \theta^*(k) \) to this problem for a fixed budget \( k \). This solution has a specific value \( S^*(k) \) for the normalization covariate \( \frac{1}{n} \sum_{i=1}^n \frac{\pi_{\theta^*(k)}(y_i \mid x_i)}{\hat{p}(y_i \mid x_i)} \). If we knew the covariate \( S^*(k) \), then we would be able to solve for:

\[
\max_{\theta} \sum_{i=1}^n \delta_i \frac{\pi_{\theta}(y_i \mid x_i)}{\hat{p}(y_i \mid x_i)}
\]

s.t. \( \sum_{i=1}^n c(y_i) \pi_{\theta}(y_i \mid x_i) = S^*(k) \)

and \( \frac{1}{n} \sum_{i=1}^n c(y_i) \pi_{\theta}(y_i \mid x_i) \leq k \)

The problem, as explained in Joachims et al. (2018), is that we do not know \( S^*(k) \) beforehand but we know it is supposed to concentrate around its expectation. We can therefore search for an approximate \( S^*(k) \), for each \( k \), in the grid \( S = \{S_1, \ldots, S_p\} \) given by standard non-asymptotic bounds. Also, by properties of Lagrangian optimization, we can turn the constrained problem into an unconstrained one and keep monotonic properties between the Lagrange multipliers and the search of the covariate \( S^*(k) \) (see Algorithm 3).

Algorithm 3 Counterfactual Risk Minimization baseline

1: Load data in the form \((x, y, r)\)
2: Fix a budget \( k \in \mathbb{R}^+ \)
3: Approximate the logging policy \( \hat{p}(y \mid x) \approx p(y \mid x) \) using a logistic regression or mixture density networks.
4: Fix a list of Lagrange multipliers \((\lambda_1, \ldots, \lambda_p) \in \mathbb{R}^p \).
5: for \( j \in [p] \) do
6: Solve the following optimization problem with stochastic optimization for a fixed \( \eta \) :

\[
\theta_j(\eta) = \arg \max_{\theta} \frac{1}{n} \sum_{i=1}^n \left[ (\delta_i - \lambda_j) \frac{\pi_{\theta}(y_i \mid x_i)}{\hat{p}(y_i \mid x_i)} - \eta \sum_{i} c(y) \pi_{\theta}(y_i \mid x_i) \right]
\]

7: Use binary search on \( \eta \) to obtain the highest-cost-but-in-budget policy noted \( \theta_j \).
8: Get the \( S_j \) corresponding to \( \lambda_j \) via

\[
\frac{1}{n} \sum_{i=1}^n \frac{\pi_{\theta_j}(y_i \mid x_i)}{\hat{p}(y_i \mid x_i)} = S_j
\]

9: end for
10: Examine the convex envelop for \( S \) and make sure it overlaps enough with the Chernoff announced bound. Otherwise refine the set of \( \lambda \) and rerun the procedure.
11: Select the adequate Lagrange multiplier \( \lambda \) by following the rule:

\[
\theta^*(k), S^*(k) = \arg \max_{(\theta, S_j)} \frac{1}{n} \sum_{i=1}^n \delta_i \frac{\pi_{\theta}(y_i \mid x_i)}{\hat{p}(y_i \mid x_i)}
\]

12: output \((\theta^*(k), S^*(k))\)

C. Nested classification to structured bandit feedback

Vanilla supervised learning aims at discriminating between very different entities (dogs, cats, humans, etc.) and is not straightforwardly compatible with our structured feedback.
The vanilla setting lacks an ordering between the actions. We propose to simulate this ordering by using data with nested labels \( Y_i \) \( \in \mathcal{I} \). In classification problems, the sets \( Y_i \) are disjoint. In our setting, they are monotonic with respect to the set inclusion. One concrete example can be constructed from the ImageNet dataset (Ima, 2018). We focus on the nested labels “Animal” \( \supset \) “Arthropod” \( \supset \) “Invertebrate” \( \supset \) “Insect”.

Fully-supervised feedback would give what is the optimal class to describe a given image. Classical bandit feedback would reveal whether or not the guess from the learner is valid. A structured bandit says only if the guess is semantically valid. In this particular example of nested classification, say we observe a dataset \( (x_i, y_i^*) \sim P \) where \( x_i \) is the pixel value a single image \( i \) and \( y_i^* \in [K] \) is the perfect descriptor for this image among the \( K \) labels. The vanilla supervised learning problem refers to finding a labeling function \( \Psi_{SL} : \mathcal{X} \rightarrow [K] \) that maximizes the classification error

\[
E_{(x,y^*)\sim P}[\mathbb{1}_{\Psi_{SL}(x)=y^*}]. \tag{29}
\]

Here, we are interested in a different setting where labels are nested and partial reward should be given when the labels is correct but not optimal. To motivate the incentive allocation problem, we give to the agent full reward whenever the label is correctly assigned. The corresponding loss can be written as (after adequately permuting the labels):

\[
E_{(x,y^*)\sim P}[\mathbb{1}_{\Psi(x)\supseteq y^*}]. \tag{30}
\]

As in the marketing problem, a trivial labeling function constant equal to \( K \) can maximize this.

**Cost constraints and analogy to incentive estimation**

As a consequence, a vanilla policy optimization problem will learn to label all the dataset as “Animal” since this will give a maximal reward. Therefore, we need again to add cost constraints and a budget so that the learner will have to guess what is the optimal decision to take (i.e., with an optimal ratio of reward / cost). The problem becomes:

\[
\max_{\Psi} E_{(x,y^*)\sim P}[\mathbb{1}_{\Psi(x)\supseteq y^*}] \quad \text{such that} \quad E_{(x,y^*)\sim P}[c(\Psi(x))] \leq k. \tag{31}
\]

Remarkably, it is easy to characterize when the problem is trivial or not.

**Proposition 5.** Let us fix a cost function \( c \).

1. If the budget \( k \) is greater than the cost of \( \Psi_{SL} \), than \( \Psi_{SL} \) is feasible and optimal. Other solutions will make choices that are correct but suboptimal.

2. If the budget is exactly equal to the cost of \( \Psi_{SL} \), then the only solutions to the nested classification are optimal also for the supervised learning case.