Isobaric-multiplet mass equation in a macroscopic-microscopic approach

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We study the a, b and c coefficients the isobaric-multiplet mass equation using a macroscopic-microscopic approach developed by P. Möller and collaborators [1, 2]. We show that already the macroscopic part of the finite-range liquid-drop model well describes the general trend of the a coefficients for doublets and b coefficients for all isotopic multiplets, while the staggering behavior of b coefficients for doublets and quartets can be understood in terms of difference in average proton and neutron pairing energies. The set of isobaric masses, predicted by the full macroscopic-microscopic approaches, is used to explore the general trends up to A = 100. We conclude that while the agreement for a coefficients is perfect, the global approaches have less sensitivity to predict the staggering pattern observed for b coefficients of doublets and quartets.

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I. INTRODUCTION

The concept of isospin was introduced by Heisenberg in 1939. Since that it represents a very useful paradigm in nuclear and particle physics, providing more beauty and simplification in theoretical modelling and interpretation of hadron or nuclear properties.

In particular, according to the isospin formalism, a nucleon is an isospin \( t = 1/2 \) baryon, with a neutron and a proton being assigned \( t_z = 1/2 \) and \( t_z = -1/2 \), respectively. The three Cartesian components of the isospin operator \( \mathbf{t} \) obey the \( su(2) \) commutation relations,

\[
[t_i, t_j] = i \varepsilon_{ijk} t_k
\]

and \([t^2, t_i] = 0\), \( i, j, k \) run over \( x, y, z \), and \( t^2 = t_x^2 + t_y^2 + t_z^2 \).

If we assume a nuclear Hamiltonian to be completely free from electromagnetic interactions and suppose that the proton and neutron masses are equal, such a Hamiltonian would commute with the many-body isospin operator, \( \mathbf{T} = \sum_{k=1}^{A} \mathbf{t}(k) \), and its eigenstates will represent isospin multiplets [\( |T_z\rangle \)], characterized by two quantum numbers, \( T \) and \( T_z \), with \( T_z = -T, -T+1, \ldots, T \). The members of the isobaric multiplets are called isobaric analogue states. For a given nucleus \( T_z = (N - Z)/2 \) and the isospin quantum number can take values \( T = T_z, T_z + 1, \ldots, A/2 \).

The presence of the Coulomb interaction and possible small isospin-nonconserving forces of nuclear origin breaks the isospin symmetry. Assuming their two-body nature and conservation of the isospin of a two-nucleon system, Wigner noticed that those isospin-symmetry breaking operators will be a combination of an isoscalar, an isovector and an isotensor operators. Estimating the splitting of the isobaric multiplets in lowest order perturbation theory according to the expectation value of such an operator, Wigner showed [3] that isobaric multiplets will be split according to a quadratic in \( T_z \) equation, called isobaric multiplet mass equation (IMME),

\[
M(\eta, T, T_z) = a(\eta, T) + b(\eta, T)T_z + c(\eta, T)T_z^2,
\]

(2)

Here, \( M(\eta, T, T_z) \) refers to atomic mass excess, \( \eta = (A, J^\pi, N_{exc}, \ldots) \) denotes all other quantum numbers (except for \( T \)), which are required to label a quantum state of an isobaric multiplet, whereas \( a, b \) and \( c \) are coefficients.

There has always been a lot of interest in the IMME, which proved to hold for a great number of quartets and quintets as well, serving thus as a ground for nuclear mass models [4]. The properties of the coefficients – their global trends and a specific staggering phenomenon – have been studied theoretically since 1960s [3–8] up to the present (see, e.g., Refs. [9–15] for the very recent work). A particular attention have been paid to experimental search (e.g., [16] and references therein) and interpretation of cubic or quartic terms in the IMME due to isospin mixing or to manifestation of the charge-dependent many-body forces [17–22].

Advent of modern radioactive ion beam facilities, progress in mass measurements and particle detection techniques allowed to access multiplets with more and more short-lived members. The most recent compilations of IMME coefficients contain a great number of high precision data which provides general trends of the IMME coefficients and sometimes staggering effects as a function of \( A \), giving also hints on particular shell effects.

Recent achievements in microscopic many-body theory allow to calculate IMME parameters using both phenomenological and microscopic effective nucleon-nucleon forces [11–14, 15, 22, 24]. In empirical approaches, experimental databases are often used to establish the strength in isovector and isotensor channels of an effective nuclear force. However, to provide a uniform description of the whole region of data from \( A = 5 \) to \( A \sim 100 \) nuclei is still challenging.
In this work we propose to construct theoretical $a$, $b$ and $c$ coefficients and study their properties using a global macroscopic-microscopic approach developed during a few decades by P. Möller and collaborators [1, 2]. Although phenomenological, the model has robust grounds and is optimized to describe a few thousands of nuclear masses (around 2150) with very small root-mean-square (rms) deviation of around 0.56 – 0.66 MeV. The models have been applied mainly to predict masses of (super)heavy elements and fission barriers. In this work, we exploit this approach to study the general trend and fine structure of the deduced IMME coefficients as a function of the mass number. Starting with a well-known uniformly charged liquid-drop model in Section III, we introduce a macroscopic part of the finite-range liquid-drop model (FRLDM) from Ref. [1] as a more involved and improved way of global description of IMME coefficients. We discuss the general trend provided by the model, as well as we show that the staggering effect of $b$ coefficients can be described by different proton and neutron pairing energy parameters. Finally, we explore the predictions given by full macroscopic-microscopic approaches based on FRLDM and the finite-range droplet model (FRDM) [1]. The last section summarizes conclusions and outlines perspectives of this study.

II. EXPERIMENTAL DETERMINATION OF IMME COEFFICIENTS

Starting from Eq. [2] one can express $a$, $b$ and $c$ coefficients for a given isobaric multiplet in terms of mass excesses of its members. Denoting $M(J,T,T_z) = M_r$, we get for doublets ($J^T, T = 1/2$) that

$$a = (M_{1/2} + M_{-1/2})/2, \quad b = M_{1/2} - M_{-1/2}, \quad c = M_{1/2} + M_{-1/2}$$

while $a$, $b$ and $c$ coefficients for triplets ($T = 1$) can be expressed as

$$a = M_0 \quad b = (M_1 - M_{-1})/2 \quad c = (M_1 + M_{-1})/2 - M_0.$$  

It is obvious from these expressions, that ground state mass excesses only can serve to obtain $a$ and $b$ coefficients for ground state doublets and $b$ coefficients for the lowest triplets, which are used in the present study for comparison with our theoretical results. Generally, $M_0$ involves an excited state, which is situated at a particularly high excitation energy in an $N = Z$ even-even nucleus. This prohibits derivation of $a$ and $c$ coefficients for triplets from masses only for detailed comparison with the experimental data.

For quartets ($T = 3/2$), quintets ($T = 2$) or other high $T$ multiplets, where masses of more than three members are involved, one determines $a$, $b$ and $c$ coefficients by a least-squares fit. However, for the present discussion we will estimate $b$ coefficients for $T > 1$ multiplets as Coulomb displacement energies [12, 23], divided by 2$T$: 

$$b = (M_T - M_{-T})/(2T).$$

This turns out to be sufficient for a discussion of the general trend and even a staggering phenomenon.

III. IMME COEFFICIENTS FROM A UNIFORMLY CHARGED LIQUID-DROP MODEL

Let us begin with estimations for $a$, $b$ and $c$ coefficients from the uniformly-charged sphere model [20]. The total Coulomb energy of a uniformly charged spherical nucleus of radius $R = r_0 A^{1/3}$ reads

$$E_{coul} = \frac{3e^2}{5R} Z(Z - 1)$$

$$= \frac{3e^2}{5r_0 A^{2/3}} \left[ \frac{A}{4} (A - 2) + (1 - A) T_3 + T_3^2 \right],$$

giving rise to the following contributions to the IMME $a$, $b$ and $c$ coefficients [18, 23]:

$$a = \frac{3e^2}{20r_0} \frac{A(A - 2)}{A^{1/3}},$$

$$b = \frac{3e^2}{5r_0} \frac{(A - 1)}{A^{1/3}},$$

$$c = \frac{3e^2}{5r_0} \frac{1}{A^{1/3}}.$$  

where $e^2 = 1.44$ MeV·fm and we use here the value of $r_0 = 1.27$ fm. These smooth trends are shown in Fig. [1] (labeled as LDM – liquid-drop model) in comparison with the experimental data on $b$ coefficients for doublets with $A = 17 - 61$ (middle panel) and $c$ coefficients for triplets with $A = 18 - 58$ (right panel). We observe that although the general trends are reproduced, there is a visible mismatch between the data and absence of the characteristic staggering patterns (weakly seen for $b$ coefficients of doublets and pronounced oscillating effect of $c$ coefficients for triplets). We do not discuss the trend of $a$ coefficients, since it requires all other members of the liquid-drop model (known as a Weizsäcker formula) and therefore a careful fitting of the parameters.

For large $A$ values, one often uses an approximate form of $b$ coefficients, namely,

$$b = \frac{3e^2}{5r_0} A^{2/3},$$

showing that the leading order term in $b$ coefficients is proportional to $A^{2/3}$.
IV. IMME COEFFICIENTS FROM THE MACROSCOPIC PART OF THE FRLDM

A. General trend

The macroscopic part of the FRLDM proposes a more involved expression for a contribution to the atomic mass excess as compared to the liquid-drop model. We adopt the following formulation proposed by Möller et al in Refs. [1, 2]:

\[ M(Z, A) = M_H c^2 + M_n c^2 - a_v(1 - k_v I^2) A \]
\[ + a_s(1 - k_s I^2)B_1 A^{1/3} + a_0 A^0 B_W \]
\[ + c_1 Z^2 B_3 / A - c_2 Z^4 / 3 B_s / A^{1/3} \]
\[ + f(k_F r_p) Z^2 / A - c_w (N - Z) \]
\[ + E_W + E_{pair} - a_c Z^{2.39} \]  

where the first two terms are the hydrogen and neutron mass excesses, followed by the volume term in line (11), then by the surface term and the so-called \( A^0 \) energy, or a constant, can be seen in line (12). The parameter \( B_1 \) defines the generalized surface energy in a model and for a spherical nucleus can be evaluated as

\[ B_1 = 1 - \frac{3}{x_0^2} + (1 + x_0) \left( 2 + \frac{3}{x_0} + \frac{3}{x_0^2} \right) \exp(-2x_0) \]

with \( x_0 = m^3 A^{1/3} \), where \( r_0 = 1.16 \) fm and \( m = r_0/a \approx 1.706 \).

Line (13) contains the direct and exchange Coulomb term with \( c_1 = 3e^2 / (5r_0) \) and \( c_4 = 5 / 4 (3/2)^{2/3} / c_1 \). The parameter \( B_3 \) defining the relative Coulomb energy for an arbitrary shape nucleus has in leading order (for a spherical nucleus) the following expression:

\[ B_3 = 1 - \frac{5}{y_0^2} + \frac{75}{8y_0^3} - \frac{105}{8y_0^7} \]

with \( y_0 = m^3 A^{1/3} \), and \( m = r_0/a_{s} \approx 1.567 \). We suppose here that the relative surface energy parameter is \( B_s = 1 \) (a spherical nucleus).

The proton form-factor correction to the Coulomb energy, \( f(k_F r_p) \), is parameterized as

\[ f(k_F r_p) = - \frac{r_p^2 e^2}{8r_0^2} \left[ \frac{145}{48} - \frac{327}{2880} (k_F r_p)^2 \right] \]
\[ + \frac{1527}{1209600} (k_F r_p)^4 \]

with the proton radius, \( r_p = 0.8 \) fm, and \( k_F \) being the Fermi wave number

\[ k_F = \left( \frac{9 \pi Z}{4A} \right)^{1/3} \frac{1}{r_0} \]

The proton form-factor depends on \( A \) and \( T_\phi \) and varies between about \(-0.212 \) and \(-0.215 \) for the nuclei of interest, with the average value being \(-0.2138 \) MeV for \( T_\phi = 0 \) nuclei. We have checked that the resulting rms deviations for the IMME coefficients are not very sensitive to this parameter, so we kept the average value for our estimation. The charge-asymmetry term in line (13) enters with \( c_a \) contributes to the isovector term (\( b \) coefficient only).

The last two terms in line (15) are the Wigner contribution, \( E_W \),

\[ E_W = W[I] + \begin{cases} \frac{1}{A}, & \text{if } Z = N \text{ odd} \\ 0, & \text{otherwise} \end{cases} \]

and the average pairing term, parameterized as

\[ E_{pair} = \begin{cases} \Delta_n + \Delta_p - \delta_{pn}, & \text{if } Z \text{ odd, } N \text{ even} \\ \Delta_p, & \text{if } Z \text{ odd, } N \text{ even} \\ \Delta_n, & \text{if } Z \text{ even, } N \text{ odd} \\ 0, & \text{if } Z \text{ even, } N \text{ even} \end{cases} \]

The average neutron and proton pairing gaps and the average neutron-proton interaction energy have been parameterized as

\[ \Delta_n = \frac{\delta_n B_s}{N^{1/3}}, \]
\[ \Delta_p = \frac{\delta_p B_s}{Z^{1/3}}, \]
\[ \delta_{np} = \frac{\delta_{np}}{B_s A^{2/3}}, \]
with $r_n = r_p = r_{\text{mac}}$ being a macroscopic pairing gap parameter. In the original work, this parameter is kept the same for protons and neutrons. In the present work we adopted two sets of numerical values of various constants and parameters from both Refs. [1, 2]. More details on the model can be found in Ref. [1].

It is obvious that Eq. (11) can be expressed in terms of $T_z$ and $A$, and thus it can be cast into form of the IMME. Not considering for the moment the Wigner term and the pairing contribution and keeping up to quadratic terms in $1/A$, one can get the following (approximate) expressions for the IMME $a$, $b$ and $c$ coefficients:

$$a = \left( \frac{M_P + M_n}{2} - a_V - \frac{5c_1}{4\alpha^2} - \frac{c_4}{24\beta} + f(k_f r_p) \right) A$$
$$+ \frac{1}{4} c_1 A^{5/3} + \left( \frac{a_S + 75}{32\alpha^3} \right) A^{2/3}$$
$$- \frac{3a_S}{\beta^2} + a_0 A^0 B_W - \frac{105}{32\alpha^5},$$

$$b = -c_1 A^{2/3} - \frac{75c_1}{8\alpha^5} A^{-1/3} - \frac{105c_1}{8\alpha^5} A^{-1}$$
$$- \frac{5c_1}{\alpha^2} + \frac{4c_4}{3(2)\beta^3} + f(k_f r_p) + 2c_a,$$

$$c = c_1 A^{-1/3}$$
$$- \left( \frac{5c_1}{\alpha^2} + \frac{4c_4}{9(2)\beta^3} - f(k_f r_p) \right) A^{-1}$$
$$+ \frac{75c_1}{8\alpha^3} A^{-4/3} - \frac{105c_1}{8\alpha^5} A^{-2},$$

which lead to the following numerical expressions:

$$a = -8.939 A + 0.1862 A^{5/3} + 21.586 A^{2/3}$$
$$- 19.420 + 6.048 A^{-1/3} \text{[MeV]},$$

$$b = -0.7448 A^{2/3} + 1.966 - 1.534 A^{-1/3}$$
$$+ 0.7823 A^{-1} \text{[MeV]},$$

$$c = 0.7448 A^{-1/3} - 1,771 A^{-1} + 1.535 A^{-4/3}$$
$$- 0.7823 A^{-2} \text{[MeV]},$$

Inclusion of Wigner and pairing energy to the $a$ and $b$ coefficients change numerical values very little. This is why we show in Fig. 11 $a$ and $b$ coefficients for doublets obtained from the full macroscopic part of FRLDM (all terms are included), in comparison with the available experimental data, while the rms deviations between the data and experiment for $a$ and $b$ coefficients can be found in Table 1.

First, we observe that the general trends of all coefficients are well reproduced. For $c$ coefficients, it is interesting to note that FRLDM predicts that the Coulomb term takes over beyond $A = 75$, where the $a$ coefficient curve reaches its minimum and starts to grow for larger values of $A$. It would be important to check these predictions by future experiments on nuclear masses along $N = Z$ line.

The macroscopic part of FRLDM predict well both the trend and the magnitude of $b$ coefficients, leading to a much better agreement with the data, than the liquid-drop model. Both models, however, show only smooth trends, without any visible staggering effect.

For $c$ coefficients, only a general trend can be extracted within the present approach in the approximation that $k_c = k_s = W = 0$ and without any pairing contribution, since no excitation energy of the lowest $T = 1$ multiplet in an $N = Z$ nucleus can be obtained within the present approach. The resulting overall trend of $c$ coefficients is shown in Fig. 11 (right panel), somewhat below the liquid-drop model predictions.

At the same time, we would be interested in getting a staggering pattern of the $b$ coefficients, which is however not provided by the formal analytical expressions of the original macroscopic part of FRLDM. We address this question in the next section, carefully considering the pairing energy parameterization.

### B. Staggering pattern of $b$ coefficients and average proton and neutron pairing gaps

The $r_{\text{mac}}$ parameter of the macroscopic part of FRLDM is an average between proton and neutron pairing energies. The magnitude is not important for the full model, since it is the microscopic part which is added and optimized to provide with the exact strength. However, precise numbers in the macroscopic part can help us to understand the staggering phenomenon. In the context of different models, this effect has been discussed in Refs. [7, 11, 12]

From the study of proton and neutron pairing gaps, however, we know that that those proton and neutron energies may be different. Following the work of Möller and Nix [22], we performed a least-squares fit of the $r_n$ and $r_p$ parameters separately to the experimental pairing gaps of nuclei along $N = Z$ line from $A = 12$ to $A = 74$, which are of interest in our work. The resulting values are $r_n = 6.83$ MeV and $r_p = 6.63$ MeV, which results in a difference $r_n - r_p = 0.02$ MeV. The $b$ coefficients obtained from the FRLDM with new $r_n$ and $r_p$ values for $T = 1/2$, 1, 3/2 multiplets are shown in comparison with experimental data in Fig. 21. Indeed, we notice staggering which is in accord with the experimental data. The resulting rms deviations are summarized in Table 1 (FL-MAC-NP). The inclusion of different proton and neutron pairing energy strength reduces the rms deviations for $a$ and $b$ coefficients, as seen from the first two lines of the table.

The results on staggering can be understood analytically. The contribution of the pairing term to $b$ coefficients of $A = 4n + 1$ and $A = 4n + 3$ doublets are

$$b_{\text{pair}}^{T=1/2} = \begin{cases} (r_n - r_p) \frac{2^{1/3} B_n}{(A + 1)^{1/3}}, & A = 4n + 1 \\ (r_p - r_n) \frac{2^{1/3} B_n}{(A - 1)^{1/3}}, & A = 4n + 3 \end{cases}$$

which results in a staggering amplitude $\Delta b = b(A) - b(A - 1)$.
\[
\Delta b^{T=1/2} \approx 2(r_n - r_p) \frac{2^{1/3} B_s}{A^{1/3}} 
\]

At the same time, for quartets we get the following contributions to the \( b \) coefficients:

\[
b_{\text{pair}}^{T=3/2} = \begin{cases} 
(r_p - r_n) \frac{2^{1/3} B_s}{3(A + 3)^{1/3}}, & A = 4n + 1 \\
(r_n - r_p) \frac{2^{1/3} B_s}{3(A - 3)^{1/3}}, & A = 4n + 3 
\end{cases}
\]

which results in a staggering amplitude of

\[
\Delta b^{T=3/2} \approx -2(r_n - r_p) \frac{2^{1/3} B_s}{3A^{1/3}} 
\]

This is about of 1/3 of the amplitude of staggering for doublets in absolute value and is out of phase. The resulting trends of \( \Delta b \) can be seen in Fig. 3 for \( T = 1/2 \) on the first panel and for \( T = 3/2 \) multiplets on the third panel in comparison with available experimental data.

Estimation of the staggering effect for \( b \) coefficients of triplets shows that

\[
b_{\text{pair}}^{T=1} = \begin{cases} 
0, & A = 4n + 2 \\
2(r_p - r_n) \frac{2^{1/3} B_s}{3A^{1/3}}, & A = 4n 
\end{cases}
\]

which results in a staggering amplitude of

\[
\Delta b^{T=1} \approx 2(r_n - r_p) \frac{2^{1/3} B_s}{A^{4/3}}, 
\]

which is \( 1/A \) hindered as compared to the staggering in \( b \) coefficients for doublets or quartets. As we can observe from Fig. 3 second panel, the staggering is indeed hardly visible on the scale of the figure, while the general trend is in robust agreement with experiment.

Similarly, going further, we get that the staggering effect for \( b \) coefficients of quintets is negligible, as given by the analytical expression

\[
\Delta b^{T=2} \approx 2(r_n - r_p) \frac{2^{1/3} B_s}{A^{4/3}}, 
\]

while in \( T = 5/2 \) multiplets the staggering is again stronger and in phase with \( T = 1/2 \) pattern, being 5 times smaller in amplitude, according to

\[
\Delta b^{T=5/2} \approx 2(r_n - r_p) \frac{2^{1/3} B_s}{5A^{1/3}}. 
\]

The resulting trends can be seen in Fig. 3 (last two panels). These patterns are expected to hold for higher half-integer and integer \( T \) multiplets. Future experiments will verify these predictions.
TABLE I: The rms deviations of the IMME a and b coefficients calculated within macroscopic part of FRLDM (denoted as FL-MAC), as well as from the full versions of FRDM and FRLDM as given in Ref.[2] (1995) and [1] (2016). Calculations labeled as FL-MAC-NP mean the different neutron and proton pairing gaps as explained in Section IV.B, while FL-MAC-NP, no FF, refers to the calculation without any proton form-factor. See text for details.

| Model                  | $T = 1/2$ ($A=17\text{–}67$) | $T = 1$ ($A=18\text{–}50$) | $T = 3/2$ ($A=19\text{–}39$) |
|------------------------|-------------------------------|-----------------------------|-------------------------------|
|                        | rms a (MeV)                  | rms b (MeV)                 | rms b (MeV)                  |
| FL-MAC                 | 3.009                        | 0.189                       | 0.191                        |
| FL-MAC-NP              | 2.446                        | 0.180                       | 0.191                        |
| FL-MAC-NP (no FF)      | 2.472                        | 0.112                       | 0.078                        |
| FRLDM (1995)           | 1.210                        | 0.454                       | 0.416                        |
| FRLDM (2016)           | 1.109                        | 0.434                       | 0.469                        |
| FRDM (1995)            | 1.145                        | 0.321                       | 0.250                        |
| FRDM (2016)            | 1.062                        | 0.246                       | 0.234                        |

V. IMME COEFFICIENTS FROM FRLDM AND FRDM

In this section we deduce the IMME a and b coefficients using the full mass predictions from FRDM and FRLDM for multiplets with $A > 16$ and up to 101 for a coefficients and up to $A = 71$ for b coefficients. The results obtained from the values of mass compilation [1] are shown in Fig. 1 while the rms deviations are summarized in Table I for both mass compilations, the one from 1995 [2] and the one from 2016 [1]. The overall rms deviations are smaller with the latest set.

First, it is very well seen that both FRDM and FRLDM perfectly describe the a coefficients for doublets, reproducing the end of the strong down slopping trend beyond $A = 57$. The predicted a coefficients for heavier nuclei along $N = Z$ line is expected to stay roughly flat up to about $A = 100$ and start to decrease in their absolute value beyond this value. No data exists yet to compare.

For b coefficients, the agreement between the two models (FRDM and FRLDM) and the data is less convincing compared to what we could obtained from the macroscopic part of FRLDM only (c.f. with Fig. 2). The corresponding rms deviations support this conclusion. The staggering effect is plotted in Fig. 3 for doublets, triplets and quartets and is seen to be too much exaggerated compared to the data. It is well probable, that the fitting procedure, intended to describe a great number of masses, is not sensitive enough to this specific isovector component. This eventually worsens the description of the b coefficients.

VI. CONCLUSIONS AND SUMMARY

In the present work we have applied a macro-microscopic approach of Möller and collaborators, FRDM and FRLDM, to estimate the IMME a and b coefficients in nuclei in the vicinity of $N = Z$ until $A \sim 100$. We found out that the macroscopic part of FRLDM, representing a refined version of the well-known liquid-drop model, is very well suited to describe the general trends of the coefficients. In particular, introducing adjusted to experiment proton and neutron pairing energy parameters allows to reproduce the staggering effect of b coefficients in a very good agreement with the data.

The full FRDM and FRLDM approaches prove themselves to be very advantageous in providing the general trend of the a coefficients towards heavier nuclei along $N = Z$ line which would be interesting to measure experimentally. However, the description of b coefficients is not satisfactory. This may hint on a possible opportunity to improve on the parameterization from specific consideration of the isovector part of the total energy.

The results presented here can be used to predict the masses of proton-rich partners based on the masses of the neutron-rich mirrors and calculated values of the b coefficients. The work along this line is in progress and will be published elsewhere. The masses of very neutron-deficient nuclei up to $A = 100$ are very important for simulations of the astrophysical rp-process.

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FIG. 4: (Color online) Theoretical IMME $a$ and $b$ coefficients obtained from the full FRDM and FRLDM, as well as those obtained from the macroscopic part of the FRDLM only, in comparison with experimental data.

FIG. 5: (Color online) Differences of IMME $b$ coefficients, $\Delta b = b(A) - b(A - 2)$, for doublets (left), triplets (middle) and quartets (right) as obtained from the FRDM and FRDLM in comparison with the experimental data.

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