Acceleration of Cosmic Rays in Supernova Shocks: Elemental Selectivity of the Injection Mechanism

Adrian Hanusch1, Tatyana V. Liseykina1, and Mikhail Malkov2
1 Institut für Physik, Universität Rostock, D-18051 Rostock, Germany; adrian.hanusch@uni-rostock.de
2 CASS and Department of Physics, University of California, San Diego, La Jolla, CA 92093, USA

Abstract

Precise measurements of galactic cosmic rays revealed a significant difference between the rigidity spectral indices of protons and helium ions. This finding is a notable contrast to the commonly accepted theoretical prediction that supernova remnant (SNR) shocks accelerate protons and helium ions with the same rigidity alike. Most of the earlier explanations for the “paradox” appealed to SNR environmental factors, such as inhomogeneous p/He mixes in the shock upstream medium, variable ionization states of He, or a multi-SNR origin of the observed spectra. The newest observations, however, are in tension with most past models. In this paper, we show by self-consistent hybrid simulations that such special conditions are not vital for explaining the cosmic-ray rigidity spectra. In particular, our simulations prove that an SNR shock can modify the chemical composition of accelerated cosmic rays by preferentially extracting them from a homogeneous background plasma without additional, largely untestable assumptions. Our results confirm the earlier theoretical predictions of how the efficiency of injection depends on the shock Mach number $M$. Its increase with the charge-to-mass ratio saturates at a level that grows with $M$. We have convolved the time-dependent injection rates of protons and helium ions, obtained from the simulations, with a decreasing shock strength over the active lives of SNRs. The integrated SNR rigidity spectrum for $p$/He ratio compares well with the AMS-02 and PAMELA data.

Key words: acceleration of particles – cosmic rays – ISM: supernova remnants – methods: numerical – shock waves

1. Introduction

The PAMELA and AMS-02 measurements (Adriani et al. 2011; Aguilar et al. 2015) indicated a difference $\Delta q \approx 0.1$ between the rigidity spectral indices of protons and helium ions, put forth earlier by the balloon-borne experiment ATIC-2 (Panov et al. 2009). According to observations (Yoon et al. 2011), the scaling shown in Figure 1 is likely to continue to higher rigidities. These findings challenge the hypothesis of a cosmic-ray (CR) origin in supernova remnants (SNRs); see e.g., Bykov et al. (2018), for a recent review.

The leading CR production mechanism, the first-order Fermi, also known as diffusive shock acceleration (DSA), is electromagnetic in nature (Fermi 1949). The equations of motion of charged particles in arbitrary electric and magnetic fields can be rewritten in terms of particle rigidity $\mathcal{R} = p e / Z e$, instead of momentum $p$:

$$\frac{1}{c} \frac{d\mathcal{R}}{dt} = \mathbf{E}(r, t) + \frac{\mathbf{B}(r, t)}{\sqrt{\mathcal{R}_0^2 + \mathcal{R}^2}}, \quad (1)$$

$$\frac{1}{c} \frac{dr}{dt} = \frac{\mathcal{R}}{\sqrt{\mathcal{R}_0^2 + \mathcal{R}^2}}. \quad (2)$$

Here, $\mathcal{R}_0 = A m_p e^2 / Z e$, with $A$ being the atomic number and $m_p$ being the proton mass. The electric, $\mathbf{E}(r, t)$, and magnetic, $\mathbf{B}(r, t)$, fields here are completely arbitrary. So the equations apply not only to the acceleration of CRs in a SNR shock but also to their propagation through the turbulent interstellar medium (ISM) to an observer. Moreover, the propagation includes an eventual escape of the accelerated CRs from the Milky Way. Equations (1) and (2) show that all species with rigidities $\mathcal{R} \gg \mathcal{R}_0 = A m_p e^2 / Z e$ have nearly identical orbits in the phase space $(r, \mathcal{R})$. Hence, if different elements enter the acceleration in a time-independent ratio at some $\mathcal{R} \gg \mathcal{R}_0 = A m_p e^2 / Z e$, their rigidity spectra in this range should be identical (Malkov et al. 2012; Malkov 2017), in apparent contradiction with ATIC-2, PAMELA, AMS-02 observations, Figure 1. Note, that at reasonably low rigidities, such as 10 GV and lower, where solar modulation is also observed, the rule of equal rigidity argument does not apply. The reason is that the rest-mass rigidity, $\mathcal{R}_0 \approx 1$ GV for protons, does enter the equations of motion.

To explain the $p$/He rigidity paradox three ideas have been entertained: (1) shock evolution in time; (2) contributions from several SNRs with different $p$/He mixes and spectral slopes; and (3) CR spallation in the ISM that introduces particle sources and sinks in their kinetic equations. Turning to the first idea, assume that the $p$/He ratio is known at some fiducial rigidity $\mathcal{R} = \mathcal{R}_1 \gg \mathcal{R}_0$. While the shock strength naturally decreases, this ratio must increase at a rate consistent with the observed $p$/He slope in rigidity. The crucial point here is that the power-law index of shock-accelerated particles decreases with the shock Mach number $M(t)$ rather definitively:

$$q = -\frac{d \ln f}{d \ln \mathcal{R}} = \frac{4}{1 - M^{-2}}. \quad (3)$$

where $f$ is the CR distribution function. Therefore, to produce a $p$/He fixed index at $\mathcal{R} > \mathcal{R}_1$, the $p$/He ratio at $\mathcal{R} = \mathcal{R}_1$ must depend on $M(t)$ in a specific way. If this dependence is an intrinsic property of collisionless shock, it cannot be adjusted to fit the data, thus making scenario (1) fully testable. Unlike scenario (1), scenario (2) is not testable because the individual properties of contributing sources are unknown. Besides, it will...
likely fail the Occam’s razor test, especially after the AMS-02 has measured \( p/C \) and \( p/O \) ratios to be identical to those of \( p/He \) (Aguilar et al. 2017). Moreover, it would be impossible to maintain the spectral slopes in the ratios \( p/He, p/C, \) and \( p/O \) (Aguilar et al. 2017, 2018) as nearly constant over an extended rigidity range (Malkov 2017). As for the spallation effects (3), the equivalence between the He, C, and O spectra (Aguilar et al. 2018) corroborates the conclusion (Vladimirov et al. 2012) that it is insufficient to explain the observed differences between \( p \) and elements whose \( A/Z \) values are similar but higher than that of the protons. It follows that the time dependence of the subrelativistic acceleration phase, i.e., injection into DSA, option (1), is the most realistic scenario to consider.

Time dependence of particle acceleration at an SNR shock comes in two flavors. First, the natural shock weakening makes the acceleration time-dependent. Second, the medium into which the shock propagates may be inhomogeneous (an effect of SNR environment; Ohira & Ioka 2011). If also the background \( p/He \) ratio is inhomogeneous and increases outward, after the acceleration it will decrease with rigidity. This is because higher rigidities are dominated by earlier times of acceleration history when the He contribution is higher. The problem with this explanation is that not only must the He concentration decrease with growing shock radius at a specific rate (one free parameter), but so must C and O. This conclusion follows from the newest C/He and O/He AMS-02 flux ratios, which have turned out to be independent of rigidity (Aguilar et al. 2017, 2018). So, He, C, and O are likely to share their acceleration and propagation history. One natural consequence of this is that C and O are unlikely to be preaccelerated from grains, contrary to some earlier suggestions (see Ohira et al. 2016 for a recent study and earlier references). Note that it is crucial to use the rigidity dependence of the fractions of different species as a primary probe into the intrinsic properties of CR accelerators. Unlike the individual spectra, the fractions are unaffected by the CR propagation, reacceleration, and losses from the galaxy, as long as spallation is negligible.

In addition to tensions with the recent AMS-02 results, the abovementioned mechanisms require additional and untestable assumptions. To resolve these problems, (Malkov 1998) argued that in quasi-parallel shocks a specific elemental selectivity of the initial phase of the DSA (injection) occurs with no additional assumptions. Analytic calculations have established that the ion injection efficiency into the DSA depends on the shock Mach number, increases with \( A/Z \), and saturates at a level that grows with \( M \). The publication of measurements of \( p/He \) ratio by the PAMELA collaboration (Adriani et al. 2011), prompted Malkov et al. (2012) to apply analytic injection theory to the case \( A/Z = 2 \) (specifically to He\(^{++} \); it is also valid for fully stripped C and O, as accurately measured later by AMS-02), producing an excellent fit to the PAMELA data in the relevant rigidity range \( 2 < R < 200 \text{ GV} \). Moreover, the analytic results are largely insensitive to the ionization multiplicity at higher \( A/Z \) because the saturation effects become significant at \( A/Z \approx 2-4 \) (see Figure 5 in Malkov 1998). Note that lower rigidities are strongly affected by solar modulation, while at higher rigidities the PAMELA statistics was insufficient to make a meaningful comparison.

In this paper we demonstrate that the recent high-precision measurements of elemental spectra with different \( A/Z \) are not only consistent with the hypothesis of CR origin in the SNR, but also strongly support it. Although a similar stand was taken in Malkov et al. (2012) for the PAMELA findings (Adriani et al. 2011), the new AMS-02 data (Aguilar et al. 2017, 2018) and recent progress in shock simulations allow us to establish crucial missing links in the CR–SNR relation. In particular, the coincidence in the accelerated particle spectral slopes of three different elements with \( A/Z \approx 2 \) (He, C, and O) discovered by the AMS-02 experiment points to an intrinsic, \( A/Z \)-based selection mechanism and rules out incidental ones, such as particle injection from inhogeneous shock environments, preacceleration of elements locked into grains, or a variable ionization state of He (Serpico 2016). It is important to emphasize here that the latter mechanism was primarily justified by an integrated abundance of different elements, whereas the detailed rigidity spectra have only now become known. On the theoretical side, the \( p/He \) calculations (Malkov et al. 2012) are based on an analytic theory (Malkov & Völk 1995) that allows freedom in selecting seed particles for injection. Pre-energized particles evaporating from the shocked downstream plasma back upstream (Parker 1961; Quest 1988) and shock-reflected particles (Burgess & Scholer 2015) have been most often discussed. Simulations can remove this uncertainty, thus greatly improving the understanding of the \( A/Z \) selectivity mechanism.

Suprathermal protons (shot-reflected, or “evaporating” from hot downstream plasma) drive unstable Alfvén waves in front of the shock. These waves control the injection of all particles by regulating their access to those parts of the phase space from where they can repeatedly cross the shock, thus gaining more energy (Kato 2015; Marcowith et al. 2016). Furthermore, the waves are almost frozen into the local fluid. So, when crossing the shock interface, they trap most particles and prevent them from escaping upstream again, thus significantly reducing their odds for injection. As protons drive these waves, the waves also trap protons most efficiently, while, e.g., He\(^{2+} \) have somewhat better chances to escape from the proton-generated waves upstream and to get eventually injected. The trapping becomes naturally stronger, with growing wave amplitude that also grows with the Mach number. This trend is more pronounced for protons than for He ions, which is crucial for the injection selectivity.

Simulations remove another potentially important limitation of the analytic treatment (Malkov et al. 2012). Namely, He ions have not been included in the wave generation upstream and treated only as test particles. Such approximation is often
considered to be sufficient because of the large, $\sim10$, $p$/He number density ratio. However, the He ions drive resonant waves that are typically two times longer than the waves driven by the protons. In the wave–particle interaction, the resonance condition is often more important than the wave amplitude. In addition, the rational relation between the respective wavelengths is suggestive of parametric interactions between them. Such interactions should facilitate a cascade to longer waves, which are vital for the DSA, not just for particle injection. Several hybrid simulations addressing the acceleration efficiency of alpha particles have included them self-consistently (Burgess 1989; Trattner & Scholer 1991; Scholer et al. 1998, 1999); however, in some cases using dramatically reduced abundances (Caprioli et al. 2017), making them dynamically unimportant and thus completely excluding the He-driven waves. In Section 4.3 we further discuss the influence of these waves on the energy spectra of proton and He ions. Additionally, earlier fully self-consistent simulations, facing the problem of injection of different ions (Caprioli et al. 2017), did not provide sufficiently detailed Mach number scans of the $p$/He injection ratio, which are needed to test the theoretically predicted $p$/He injection bias.

2. Simulation Set-up

Full kinetic modeling of ion injection for a realistic ion-to-electron mass ratio $m_i/m_e$ is challenging because of the necessity of resolving both the electron and ion scales. In this paper we study the particle injection into the DSA using hybrid simulations (Lipatov 2002, and references therein), where only the ion plasma population is treated kinetically, while electrons are treated as a charge neutralizing massless fluid. The hybrid simulations have been proven to be a powerful tool in the investigation of the non-relativistic shocks and have been used for a variety of problems (Lipatov 2002, and references therein), including injection of protons into the DSA process (Caprioli et al. 2015) and the study of the magnetic turbulence driven by plasma instabilities (Caprioli & Spitkovsky 2014). The underlying equations and implementation details are documented in the Appendix. The electron pressure $p_e$ and the resistivity are both assumed to be isotropic quantities. The pressure $p_e$ is modeled using an adiabatic equation of state with an adiabatic index $γ_e = 5/3$. The fluid equations and the ion equations of motion are non-relativistic, as $|v| \ll c$ holds during the injection phase. In the simulations, lengths are given in units of $c/ω_p$, with $ω_p = \sqrt{4π n_0 e^2/m_p}$ being the proton plasma frequency, $n_0$ being the upstream density, and $e$ being the proton charge. Time is measured in units of inverse proton gyrofrequency, $ω^{-1}_{ce} = (e B_0/m_p c)^{-1}$. Here, $B_0$ is the magnitude of the background magnetic field.

We use a realistic composition of the plasma consisting of ion species with number ratios corresponding to the amount of particles in the ISM. The fraction of ions respective to protons is $\sim10\%$ for helium and $\sim0.04\%$ for carbon and oxygen. Note that He ions are dynamically important and cannot be regarded as test particles. The simulations are 1D in space but 3D in velocity and field components. This setting substantially increases the particle statistics and grid resolution, lowers the noise, and improves wave description, all of which are crucial for understanding the downstream thermalization. It is more important than possible shock rippling effects not captured by 1D simulations. Additionally, shock rippling and its impact on particle reflection cannot be accurately characterized within hybrid simulations and require a full kinetic treatment (Liseykina et al. 2015; Malkov et al. 2016; Malkov 2017). For the reasons explained in detail in Sections 4.2 and 4.3 we deliberately choose the 1D treatment as a first step in a systematic study of the $A/Z$ dependence of injection efficiency. The follow up 2D simulations will be discussed elsewhere.

The simulation is initiated by sending a supersonic and superalfvénic plasma flow with velocity $v_0$ against a reflecting wall. The shock forms due to the interaction of the emerging counter-propagating flows. The background magnetic field is set parallel to the shock normal $B_0 = B_0 x$. The upstream plasma betas are $β_p = β_e = 1$. The simulation box has a length of $12-48 \times 10^3 c/ω_p$, depending on the initial velocity $v_0$. The spatial resolution of $Δx = 0.25 c/ω_p$, 100 particles per species per numerical cell are used. The time step is $Δt = 0.01/(v_0/v_δ) ω^{-1}_{ce}$ with $v_δ = B_0/\sqrt{4π n_0 m_p}$. All numerical parameters have been checked for convergence.

3. Simulation Results

3.1. $A/Z$ Dependence of Injection

We investigate the mass-to-charge dependence of the injection using self-consistent simulations for ion species with $A/Z \lesssim 16$. In addition to protons, He, C, and O ions with charge states $Z = 1$ and $Z = 2$ were included. The phase space distributions of selected ion species are shown in Figure 2 for an upstream flow velocity $v_0 = 10 v_δ$. The transition from the cold upstream flow to the hot and turbulent downstream plasma is clearly seen in the plots. The width of the particle distribution in $v_δ$ in the downstream is almost the same for all ion species, indicating higher temperatures for heavier species. The latter also thermalize further downstream, as their impact from the
The presence of ions with large \(|v_\parallel|\) in the upstream and downstream shows that some ions have already gained energy and are able to cross the shock front.

The energy spectra of the particles downstream of the shock transition at \(t = 1500 \, \omega_\perp^{-1}\), obtained using a logarithmic binning procedure, are shown in Figure 3(a). The spectra of all ion species exhibit two main features: a Maxwellian distribution and a power-law tail. The transition from the Maxwellian to the power-law tail is obscured by a contribution of suprathermal particles (Burgess & Scholer 2015). The spectra of heavier ions are shifted to higher energies, as the velocity is randomized during the shock crossing. For all species, \(\alpha\), the tail is clearly developed for energies \(E > 10 \, E^0\) with \(E^0 = \frac{1}{2} m_0 v_\perp^2\). This energy is marked in the proton energy spectrum in Figure 3(a) by the dashed gray line. After the spectra are converged (\(t \geq 2000 \, \omega_\perp^{-1}\) for \(v_0 = 10 \, v_\perp\)), we calculate the selection rate, \(\eta_{\text{sel}}\), i.e., the fraction of particles in the tail of the distribution function, as a function of mass-to-charge ratio \(A/Z\). For low \(A/Z\), Figure 3(b), \(\eta_{\text{sel}}\) grows almost linearly, a saturation occurs around \(A/Z \sim 8–12\) (in a Mach-dependent fashion, though), and at higher \(A/Z\) the selection rate decreases, recovering a physically correct \(A/Z \to \infty\) asymptotic behavior (it should tend to zero, as seen for the injection of neutrals). The preliminary 2D simulations also evident a deviation from the linear \(\eta_{\text{sel}}(A/Z)\) trend, pointing toward a saturation for higher \(A/Z\). Note that the exact position of the saturation is to some extent time-dependent, as heavier ions are accelerated at later times (Figure 3(c)). This is because the respective longer waves need to be generated by the increasing maximum energy of protons. The efficiency of these waves for injection of higher \(A/Z\) species naturally depends on their amplitudes. These amplitudes depend not only on the maximum momentum of (resonant) protons but also on the dynamics of the entire wave spectrum, i.e., spectral transfer rate, turbulent cascade, etc. Our \(\eta_{\text{sel}}(A/Z)\) scaling (Figure 3(b)) is in agreement up to \(A/Z \lesssim 8\), with an almost linear increase of the selection rate, with the mass-to-charge ratio found recently in 2D hybrid simulations, facing the problem of injection of different ions in quasi-parallel shocks with \(M > 5\) (Caprioli et al. 2017). For a more extensive discussion about the behavior of \(\eta_{\text{sel}}\) for high values of \(A/Z\), see Section 4.1.

### 3.2. Elemental Selectivity: Proton-to-helium Ratio

In the following we investigate the elemental selectivity of the injection by focusing on the \(p/\text{He}\) ratio. To extract this quantity we calculate the injection efficiency of \(p\) and \(\text{He}^{2+}\) separately. The direct measurement of the injection efficiency is difficult, because the transition from the Maxwellian distribution to the power-law tail is not sharp. Therefore, we fit a thermal distribution \(f_{\text{th}} \propto E^{1/2} \exp(-E/T)\) as well as a power law with a cutoff, \(f_{\text{pow}} \propto E^{-q} \exp(-E/E_{\text{cut}})\) to the low-energy and high-energy parts of the downstream spectrum. Here, \(T\) is the downstream temperature of the respective ion species and \(E_{\text{cut}}\) is the cutoff energy. In Figure 3(a) the dotted line denotes the fitted Maxwellian, while the dashed line is the power-law fit to the energy spectrum of protons. With \(E_{\text{inj}}\) defined for each species from \(f_{\text{th}}(E_{\text{inj}}) = f_{\text{pow}}(E_{\text{inj}})\) the injection efficiency is calculated as

\[
\eta_{\text{inj}} \propto \left. \left( \frac{dN}{dE} \right) \right|_{E=E_{\text{inj}}} = \frac{f_{\text{th}}(E_{\text{inj}})}{\int_0^{\infty} f_{\text{th}}(E) \, dE}. \tag{4}
\]

Figure 4 shows the value of \(\eta_{\text{inj}}(M)\) obtained from a series of simulations with different initial upstream flow velocities \(v_0\). The corresponding Alfvénic shock Mach numbers are...
\[ \mathbf{M} = (v_0 + v_s)/v_\infty. \] Here, \( v_s \) is the shock velocity in the downstream rest frame.

The \( M \)-dependence of \( \eta_{\text{inj}} \) is similar for \( p \) and \( \text{He}^{2+} \). It increases for \( M \lesssim 5 \) for protons, \( M \lesssim 7 \) for \( \text{He} \) ions and decreases at higher \( M \), tending to the predicted (Malkov 1998) \( \eta_{\text{inj}}(M) \sim \ln M/M \) asymptotics.

Two aspects are important here. First, the injection of protons dominates for low \( M \) with \( \eta_{\text{inj}}^p \), exceeding the value of \( \eta_{\text{inj}}^{\text{He}} \) by an order of magnitude. Second, the maximum of \( \eta_{\text{inj}}^p \) is shifted toward smaller \( M \) compared to \( \text{He}^{2+} \). The prevalence of proton injection at weak shocks is also noticeable in the downstream temperature ratio \( T_{\text{he}}/T_p \), which for \( M < 15 \) exceeds the expected ratio of \( T_{\text{he}}/T_p = 4 \).

### 3.3. Rigidity Spectra

To model the time-dependent CR acceleration, we combine the Mach-number-dependent injection efficiency obtained from simulations (Figure 4) with the theoretical spectral slope, \( q = 4/(1 - M^{-2}) \) which allows us to extend the simulation spectra further in rigidity than any simulation may possibly reach. The extension is justified by simulation spectra reaching the asymptotic DSA power law, Figure 3(a).

During the Sedov–Taylor phase of the SNR evolution, the shock radius increases with time as \( R_s \sim C_{\text{ST}} t^{2/5} \), while the shock velocity decreases as \( V_s = (2/5)C_{\text{ST}} t^{-3/5} \), with \( C_{\text{ST}} \approx (2 E_c/\rho_0)^{1/3} \). Here, \( E_c \) is the ejecta energy of the supernova, and \( \rho_0 \) is the ambient density. The number of CR species \( \alpha \), deposited in the shock interior, as the shock radius increases from \( R_{\text{min}} \) to \( R_{\text{max}} \), amounts to

\[
N_{\alpha}(p) \propto \int_{R_{\text{min}}}^{R_{\text{max}}} f_{\alpha}(R, M(R)) R^2 dR \propto \int_{M_{\text{min}}}^{M_{\text{max}}} f_{\alpha}(R, M) dM^{-2}.
\]

The spectra \( f_{\alpha} \) are

\[
f_{\alpha} \propto \eta_{\text{inj}}^\alpha(M) (R/R_{\text{inj}})^{-q(M)}
\]

with \( q(M) = 4/(1 - M^{-2}) \). Equations (5) and (6) are accurate for the most interesting sub-TV particles that are accelerated quickly. Equation (5), however, tacitly implies an unimpeded release of accelerated particles into the ISM, which is poorly known. The key to our approach is that the \( p/\text{He} \) ratio is still independent of the release mechanism and even ensuing propagation across the ISM, simply because the underlying equations of motion are identical for \( p \) and \( \text{He} \).

Instead of feeding the simulation data for \( \eta_{\text{inj}}^\alpha(M) \) to the convolution given by Equations (5) and (6), we first fit the following simple function, \( \eta_{\text{inj}}(M) = a(M - b)M^{-c} \) in the range \( M_{\text{min}} = 3, 5 < M < M_{\text{max}} = 100 \), to the data extracted from the simulations (Figure 4) and then calculate the \( p/\text{He} \) ratio, \( N_p/N_{\text{He}} \), according to Equation (5), as a function of rigidity. The resulting \( p/\text{He} \) spectrum (red line), shown in Figure 5, compares well in the high-rigidity range with the AMS-02 and PAMELA data (shadow areas).

### 4. Discussion

#### 4.1. \( A/Z \) Trend of the Selection Rate

Our simulations show (Figure 3(b)) that the selection rate \( \eta_{\text{sel}} \) growth with the mass-to-charge ratio saturates in a Mach-dependent fashion around \( A/Z \sim 8-12 \), and then decreases for higher \( A/Z \) values, recovering a physically correct \( A/Z \to \infty \) asymptotic behavior, as expected for neutral particles. However, this result contradicts the findings reported in (Caprioli et al. 2017), where the authors obtained in 2D hybrid simulations the quadratic growth with \( A/Z \) of the chemical enhancement for mass-to-charge ratios as high as \( A/Z = 56 \). The striking contradiction is that the quadratic growth of the chemical enhancement implies the linear growth of the selection rate with \( A/Z \) without a saturation trend up to at least \( A/Z = 56 \). The question to address is whether the injection rate saturates and vanishes with growing \( A/Z \), or, on the contrary, the accelerated protons generate such strong and long waves and/or magnetized eddies downstream that they scatter and inject species with \( A/Z \gg 1 \) more efficiently than (reductio ad absurdum) the protons themselves. We stand by the statement that the unlimited growth of the selection rate with \( A/Z \) is unphysical or at minimum, imposes quite unusual constraints on the scattering turbulence. It is worth mentioning that the chemical enhancement of heavier elements with \( A/Z > 8 \) in Caprioli et al. (2017) is determined in the upstream plasma because at the time of measurement these ion species have not yet developed the universal downstream DSA spectrum. Whether this approach is justified in the first place is controversial. The question of the exact position of the maximum of \( \eta_{\text{sel}} \) as a function of \( A/Z \) is debatable and there is indeed not yet a consensus. Physically, its position should also depend on the current maximum energy of protons because the resonant waves they produce may scatter particles with larger \( A/Z \). But the particle scattering rate in general decays with the growing wave length, so this effect should not be overestimated.

CR abundances of heavier elements, such as iron, have a weaker comparative potential for the verification of the \( A/Z \) scaling of injection efficiency than the rigidity spectra of \( p/\text{He}, p/\text{O}, p/\text{C} \) used in this paper. There are two reasons for this. First, there are not yet rigidity spectra for the ratios of heavier elements that are comparable in quality to the recently published AMS-02 data for the above ratios. The integrated abundances are available, but they are affected by many factors that are either unrelated to the microphysics of injection selectivity in collisionless shocks or highly uncertain. These include, but are not limited to, CR spallation effects during the propagation to the Earth, the possible contribution from the disintegration of dust grains, and the uncertainty in the ionization state during the injection process.
4.2. Do 2D Simulations Produce More Credible Results?

Although most fundamental aspects of shocks are one-dimensional, there are indeed essential phenomena that cannot be fully understood if two coordinates or even just one coordinate are ignored. Obviously, realism demands trading fully 3D simulations for an adequate resolution. In the case of hybrid simulations this requirement concerns both the particle statistics and parameters of the numerical grid. Despite the progress in computational performance, 3D simulations meeting such conditions are unfeasible now, therefore 2D modeling is a computationally expensive but plausible compromise. With progress in computational performance, 3D simulations meet this requirement concerns both the particle dimensional, there are indeed essential phenomena that cannot

First, an inverse cascade in 2D fluid leads to coherent structures that may become responsible for an excessive particle scattering and reflection, i.e., injection. Although the difference between 2D and 3D dynamics is not so explicit in the MHD, the conditions and the character of an inverse cascade in 3D MHD are not as robust as those in the 2D case (Pouquet et al. 1976).

Second, the high computational demands of injection studies force elongation of simulation box, especially in the shock normal direction, which is unnatural for a shock alignment along its front. The small transverse box size renders the 2D simulations quasi-one-dimensional, but the artificial scale introduced by it can cause an artificial periodicity for ions with large Larmor radii and is likely to determine the size of the scattering structures (Ngan et al. 2005; Celani et al. 2010), shock corrugation scale, and the inverse cascade anisotropy. Structures that appear to be strongly influenced by the box geometry are seen in some advanced 2D simulations, e.g.,

These structures would perhaps be acceptable for a shock tube setting, but are problematic for a freely propagating shock front.

Third, in addition to their exaggerated magnetic strength, coherent structures in the 2D downstream turbulence are highly consequential for the particle injection for another reason as well: as the particle motion is considered three-dimensional, these structures, being extended along the ignorable coordinate, dramatically increase the effective scattering cross section for particles. The particles are scattered by these structures regardless of their velocity projection on the ignorable coordinate. In a square box of size $L^2$ and the typical scale of the scattering structure $\sim a$, this enhancement is a factor of $L/a > 1$ compared to the 3D box $L^3$. Such particle dynamics may indeed result in an excessive return upstream of particles with high $A/Z$, which would in a 3D pass through these structures, not to mention the uncertainty of their formation in the 3D.

4.3. Importance of the He-driven Waves

As already stated, the peculiarity of our study is in the interaction of the shock with a large number of different species for which adequate particle statistics is vital, especially when exploring the high-energy tails of the distribution functions. As we briefly discussed in the Section 1, the He-driven waves significantly enrich the wave spectrum through their parametric interaction with the proton-driven waves, thus facilitating particle thermalization downstream. Because of the high computational demand in the 2D hybrid numerical studies of injection, the heavier ions, including He, are either treated as test particles, or are included quasi-self-consistently with low statistics and extremely low abundance. The latter protects the hybrid simulations from excessive numerical heating, which otherwise would unavoidably represent a serious problem, but at the same time completely excludes the generation of He-driven waves by making the He component dynamically unimportant. In Figure 6(a) the spectrum of the transverse magnetic field $\mathcal{F}(B_{\perp})(k)$ in two-ion species (90% protons and 10% He$^{2+}$) plasma is shown for $\tau = 500\omega^{-1}$ in comparison with the corresponding spectrum in a pure hydrogen plasma. If the abundance of the He$^{2+}$ component is high, a component at lower wavenumber $k$ appears in the spectrum (shown by an arrow). The critical role of the self-consistent, as opposed to test-particle, treatment of the He$^{2+}$ population is confirmed by the enhanced number of downstream protons with high energies and an increase in the number of helium ions near the cutoff (Figure 6(b)). In general, DSA is a bootstrap process in which the particles with high energy drive the longest waves that help to accelerate them. Here, helium paves the way for protons. The well-established part of the He spectrum is in turn dominated by more abundant protons in both cases.

5. Summary

We investigate the particle injection into the DSA using self-consistent hybrid simulations. We provide sufficiently detailed Mach number scans of the $p$/He injection ratio that is needed to test the injection bias. It should be emphasized that the rigidity spectra of the fractions of different species do not depend on the relation, in which these elements are in the most productive SNRs, as protons are considered to be dynamically the most important species. The reduced spatial dimensionality of the simulations allows us to increase the particle statistics and grid resolution dramatically. Our simulations show that the selection rates of different ion species increase with $A/Z$, saturate, and peak as a function of Mach number. They correctly predict the decrease in proton-to-helium ratio with increasing rigidity (Figure 5) at almost exactly the rate $\Delta q \approx 0.1$, measured in the experiments for $R \gtrsim 10$ GV. At lower rigidities, the difference between the data and our predictions is significant. Based on the discussion in the introduction, the difference $\Delta q$ occur because the equations of motion (Equations 1 and 2) for protons and helium ions deviate toward lower rigidities. The most likely cause of this deviation is particle interaction with the turbulent solar wind in the heliosphere, but the interaction with the ISM turbulence may also contribute, again, because the equations of motion are different for $p$ and He in the low-rigidity range. In contrast, the deviation from the AMS-02 data in the high-rigidity range, where the equations of motion for $p$ and He become identical, are insignificant as expected. This deviation is much less than the difference $\Delta q \approx 0.1$. Whether it comes from a simplified integration over the SNR in Equations 5 and 6 or it is a mixing effect from different SNRs or spallation in the ISM, remains unclear. The difference is small enough to be accounted for by any of these phenomena. Except for this uncertainty, the suggested mechanism for the $A/Z$ dependence of the injection fully explains the measured $p$/He ratio. Our
interpretation of the elemental “anomaly” is therefore intrinsic to collisionless shock mechanisms and does not require additional assumptions, such as contributions from several different SNRs, their inhomogeneous environments, or acceleration from grains.

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Appendix

In hybrid modeling the evolution of the ion distribution function $f$ is governed by the kinetic Vlasov equation:

$$\frac{\partial}{\partial t} f + \mathbf{v} \nabla f + \frac{q_i}{m_i} \left( \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} - \eta \mathbf{J} \right) \frac{\partial}{\partial \mathbf{v}} f = 0. \quad (7)$$

Here, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields and $\mathbf{J} = e(n_e \mathbf{v} - n_i \mathbf{v}_i) \approx e n_e (\mathbf{v} - \mathbf{v}_i)$ is the current density. Furthermore, $q_i = Z e$ and $m_i = A m_p$ are the ion charge and mass and $\eta$ denotes a scalar resistivity.

The plasma electrons are treated as a charge neutralizing massless fluid,

$$n_e \frac{d \mathbf{v}_e}{dt} = 0 = -e n_e \left( \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) - \nabla p_e + e n_e \eta \mathbf{J}. \quad (8)$$

Here, $p_e$ denotes the electron pressure, which can be calculated as $p_e = n k_B T_e$. In order to close the set of equations we assume an adiabatic equation of state:

$$\frac{T_e}{T_0} = \left( \frac{n_e}{n_0} \right)^{\gamma - 1} \quad \text{with:} \quad \gamma = \frac{5}{3}. \quad (9)$$

We found that the use a polytropic equation of state instead of an adiabatic one, as in, e.g., Caprioli et al. (2015), does not change the energy spectra significantly and the injection rate behavior for large $A/Z$ is not affected by the prescription for the electron equation of state. Furthermore, an effective adiabatic index, based on the assumption of equilibration of electron and ion temperatures in the downstream, might be justified for weak shocks, as Vink et al. (2015) predicted a thermalization between electrons and ions at low M shocks. However, at higher Mach numbers ($5 < M_e < 60$) the same authors predicted a behavior of $T_e/T_i \propto M_e^{-2}$. Hence, the assumption of electron and ion temperature equilibration might not hold.

Furthermore, we use the low-frequency magnetostatic model, neglecting the displacement current in the Ampère’s law.

A.1. Implementation

The particle-in-cell method is used for the ion plasma components. A first-order weighting is applied to interpolate the fields to the particle position, as well as to obtain ion current and charge density from the known positions of the ion-particles relative to the grid points. The Boris algorithm (Boris 1970) is used to update the particle positions and velocities. The magnetic field evolves according to Faraday’s law, while the electric field is calculated from the electron momentum equation, Equation (8). The equations of the evolution of the fields are discretized using second-order finite difference stencils:

$$B^{n+1/2} = B^n - \frac{\Delta t}{2} \nabla \times E^n$$

$$E^{n+1/2} = F(B^{n+1/2}, n_i^{n+1/2}, J_i^{n+1/2})$$

$$B^{n+1} = B^{n+1/2} - \frac{\Delta t}{2} \nabla \times E^{n+1/2}.$$

Technically, the problem of the time evolution of the fields reduces the problem of calculating $E^{n+1}$, which cannot be calculated directly because $J_i^{n+1}$ is not known. Several different methods have been proposed in the literature (see Winske et al. 2003 for a review). We use a predictor-corrector method, which is simple and has good energy conserving properties. The algorithm has the following steps:
1. Advance $B^n$ and $E^n$ to the time step $n + 1/2$

$$B^{n+1/2} = B^n - \frac{\Delta t}{2} \nabla \times E^n$$
$$E^{n+1/2} = F(B^{n+1/2}, n_i^{n+1/2}, J_i^{n+1/2}).$$

2. Predict fields $B^{n+1}$ and $E^{n+1}$

$$E^{n+1} = 2E^{n+1/2} - E^n$$
$$B^{n+1} = B^{n+1/2} - \frac{\Delta t}{2} \nabla \times E^{n+1}.$$

3. Advance particles using the predicted fields and obtain $n_i^{n+3/2}$ and $J_i^{n+3/2}$.

4. Calculate predicted fields $B^{m+3/2}$ and $E^{m+3/2}$

$$B^{m+3/2} = B^{m+1} - \frac{\Delta t}{2} \nabla \times E^{m+1}$$
$$E^{m+3/2} = F(B^{m+3/2}, n_i^{m+3/2}, J_i^{m+3/2}).$$

5. Determine $B^{n+1}$ and $E^{n+1}$

$$E^{n+1} = \frac{1}{2} (E^{n+3/2} - E^{n+1/2})$$
$$B^{n+1} = B^{n+1/2} - \frac{\Delta t}{2} \nabla \times E^{n+1}.$$

**ORCID iDs**

Tatyana V. Liseykina @ https://orcid.org/0000-0002-5070-3543

Mikhail Malkov @ https://orcid.org/0000-0001-6360-1987

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