Numerical Study of Cosmological Relaxation of the Higgs Mass

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Abstract

In the light of no discovered new physics at the LHC, ideas managing to tackle the Hierarchy Problem without novelties around the TeV scale must be taken seriously. Such is a cosmological relaxation of the Higgs mass, proposed already in pre-LHC era, which does not rely on new physics below the Planck scale. This scenario introduces a different notion of naturalness according to which the vacuum with a small expectation value of the Higgs field corresponds to an infinitely enhanced entropy point of the vacuum landscape that becomes an attractor of cosmological inflationary evolution. In this framework we study numerically the evolution of the Higgs VEV. We model the inflationary vacuum-to-vacuum transitions that are triggered by nucleation of branes charged under three-form fields as a random walk. In particular, we investigate the impact of the number of coupled three-forms on the convergence rate of the Higgs VEV. We discover an enhanced rate with increasing number of brane charges. Moreover, we show that for late times the inclusion of more charges is equivalent to additional brane nucleations.

1 Introduction

Rooted in the quadratic sensitivity of the Higgs mass-term to the UV cutoff, the Hierarchy problem is a still unresolved puzzle in theoretical physics. (For discussion of the Hierarchy Problem and its connection to naturalness, see \cite{1}). Most of the proposed solutions such as supersymmetry, which has no quadratic divergences \cite{2}, technicolor \cite{3}, extra dimensions \cite{4,5} or large number of species \cite{6}, predict stabilizing new physics around a scale not much larger than the weak scale. However, already in pre-LHC era an alternative idea of solving the Hierarchy Problem via cosmological relaxation of the Higgs mass has been suggested \cite{7,8}. This scenario does not rely on any low energy physics as it introduces a notion of naturalness \cite{1} that is fundamentally different from the standard one by ’t Hooft \cite{9}. Here, the vacuum with the small value of the Higgs mass has infinite entropy and represents an attractor of cosmological evolution. The

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non-observation of any new physics at the LHC, gives a serious motivation for a detailed study of this scenario.

We provide a short introduction to this mechanism in section 2. A list of more recent references based on the same idea can be found in [1].

In this paper we first confirm the relaxation of the Higgs vacuum expectation value (VEV) to the attractor value by numerical simulations. Furthermore, we extend the analysis to additional three-form fields and study their affect on the convergence rate. This is motivated by embedding the attractor scenario to various fundamental theories that contain multiple forms.

The paper is structured as follows. We first briefly review the cosmic attractor model in section 2. In chapter 3 we map a simplified model to a stochastic framework. The results and interpretation of the numerical simulations are provided in section 4. We conclude the discussion in section 5.

2 Cosmic Higgs VEV Relaxation Mechanism

In this section we briefly review the Cosmic Attractor model introduced in [7] and further refined in [8] and [1]. This model exhibits a high degeneracy of vacua around a certain hierarchically-small value of the Higgs VEV $\Phi^*$. This point of enhanced entropy is reached via brane nucleations during inflationary cosmological evolution. Since inflation last eternally [10,11], the system has infinitely available time to converge to the attractor point. It is therefore natural to find oneself in the vacuum with maximal entropy and corresponding Higgs VEV $\Phi^*$.

The crux of the mechanism is to couple the Higgs to a massless three-form field which is sourced by a 2-brane with charge $Q$ set by the Higgs field itself. The fundamental nature of the brane is not important, in particular as shown in [7] and [8] it can be resolved in form of a domain wall of an heavy axion. Crossing the 2-brane (or axionic domain wall) leads to a jump in the field strength $F$ with the distance set by the brane charge:

$$\Delta F = Q(\Phi).$$

On the other hand, $F$ is back-controlling the vacuum expectation value of the Higgs field via

$$\Phi^2 = \frac{1}{\lambda} \left( \frac{F^2}{M^2} - m^2 \right),$$

where $\lambda$ is a coupling constant, $M$ is some cutoff and $m$ incorporates all other contributions to the effective Higgs mass. Using Eq. (1) and (2) the change of the Higgs VEV for small values of $Q$ when crossing a 2-brane is

$$\Phi \Delta \Phi = -\frac{F}{M^2 \lambda} Q + O(Q^2).$$

The final ingredient is the exact form of the dependence of $Q$ on $\Phi$. With the effective
brane charge given by

$$Q(\Phi) = \pm \frac{(\Phi^N - \Phi_s^N)^K}{M^{NK-2}},$$  \hspace{1cm} (4)$$

where $N$ and $K$ are positive and integer valued parameters and the sign of the charge is not fixed, the difference in the Higgs VEV for neighboring vacua vanishes for $\Phi \rightarrow \Phi_s$. Correspondingly, the density of vacua diverges at that point. The result is a probability distribution for $\Phi$ with singular peak around $\Phi_s$. This point is called an attractor since given infinite time $\Phi$ will inevitably move arbitrarily close to $\Phi_s$.

For derivation of equations (1-4) and further details we refer to the original papers [1,7,8]. In the following sections the terms brane nucleation and timestep are used interchangeably and $\Phi$ and $\lambda$ are measured in units of Planck mass $M_P$.

3 Random Walk Model

Since we are solely interested in the efficiency of the attractor mechanism we neglect all contribution to the Higgs mass other than from $F$, so we set $m = 0$. With that the change of $\Phi$ simplifies to

$$\Delta \Phi = \frac{Q}{\lambda M^2} = \pm \frac{(\Phi^N - \Phi_s^N)^K}{\lambda M^{NK}}. \hspace{1cm} (5)$$

Sufficiently close to the attractor back-reaction on the inflationary background can be ignored, therefore the probability of nucleating a brane or an anti-brane can be assumed to be equal. Correspondingly we assign to moving either in positive or negative direction the probability $P = 0.5$.

In the following we will therefore model the time evolution of $\Phi$ as a symmetric random walk. This is an extremely good approximation. Because brane nucleation is an non-perturbative process and rare, the nucleation of subsequent branes in the new vacuum is not sensitive to the original direction of nucleation. In other words, by the time that a given nucleation happens the walls of the bubble from the previous nucleation are exponentially far away and do not affect the succeeding nucleation direction.

We further simplify the original expressions slightly by assuming $\Phi_s = 0$, merging the coupling constant $\lambda$ and the cutoff $M$ into one parameter $\mu$ and keeping the exponent of this parameter independent of $\nu \equiv NK$. The change $\Delta \Phi$ at each discrete timestep (or equivalently brane nucleation) is then given in compact form by

$$\Delta \Phi(\tilde{\Phi}, \mu, \nu) = \begin{cases} +\mu \tilde{\Phi}^\nu, & P = 0.5 \\ -\mu \tilde{\Phi}^\nu, & P = 0.5 \end{cases} \hspace{1cm} (6)$$

where $P$ denotes the probability of the specific outcome and $\tilde{\Phi}$ is the current value of the variable $\Phi$. For the remaining free parameters we assume $\mu \in (0, 1)$ and $\nu \geq 2$ and integer valued.

With that the random walk can be defined iteratively by

$$\Phi_{t+1} = \Phi_t + \Delta \Phi(\Phi_t, \mu, \nu), \hspace{1cm} (7)$$
where $i$ indexes the brane nucleations and $\Phi_i$ denotes the value of $\Phi$ after the $i$th brane has nucleated.

It is straightforward to generalize this model to multiple different charges on the brane. Since these are independent from each other, the sequence in Eq. (7) can be generalized to

$$
\Phi_{i+1} = \Phi_i + \sum_{k=1}^{d} \Delta \Phi(\Phi_i, \mu_k, \nu_k),
$$

(8)

for $d$ distinct charges and possibly different and independent parameters $\nu_k$ and $\lambda_k$. So at every timestep the VEV of the Higgs is shifted by $d$ different terms where again the sign of each contribution is equally likely due to the unfixed signs of the individual charges.

## 4 Simulations

The random walk defined by Eq. (8) can be simulated on a computer. Of course, due to the statistical nature of the process, quantitative analyses can only be performed with a high number of realizations. The main focus here should lie on the question how the number of distinct brane charges $d$ affects the convergence rate of the random walk.

All simulations in this section have been performed with an initial value of the Higgs VEV given by $\phi_0 = 0.1$ and the two free parameters were set to $\lambda = 0.5$ and $\nu = 3$, respectively and equal for all three-form fields. Every simulation was calculated for $n = 10^9$ steps. For late times we plot illustrative random walks for $d = 1$ and $d = 10$ in Fig. 1. Matching the intuitive picture, both walks decrease on average as a function of the timestep $i$. Of course, the smaller $\Phi$ becomes, the slower gets the rate in accordance with Eq. (6). Thus for the Higgs VEV to relax infinitely close to zero, infinite time would be required. However, this time requirement can be accounted for by eternal inflation [10,11] as already explained in section 2.

![Figure 1](image-url) Exemplary realizations of the random walk defined in Eq. (8) with $\mu = 0.5$, $\nu = 3$ and $\Phi_0 = 0.1$ for different number of brane charges. The plot range has been set to emphasis late time behavior. In this specific example the run with higher number of charges converges faster to smaller values compared to the $d = 1$ case.
Next, we analyze the dependence of the convergence rate of $\Phi$ on the number of charges $d$. For this purpose we average over the final value $\Phi_f$ after $n = 10^9$ steps for 100 runs for various $d \in [1, 70]$. The corresponding data is plotted in Fig. 2 in blue. In accordance with Fig. 1 we clearly observe an enhanced relaxation efficiency for higher $d$.

For quantitative conclusions we fit the data in Fig. 2 with a function of the form $a \cdot x^b$ in orange. The fit parameters are calculated to $a = 0.0052$ and $b = -0.25$. The exponent matches, within statistical errors, the averaged exponent when fitting the values of $\Phi$ for individual runs. This completely matches the analytic intuition that increasing the brane nucleation channels is equivalent (up to a numerical factor) to the nucleation of more branes. The only difference that can be observed is at the beginning when $\Phi$ is still large and when the attractor’s pull is the strongest. Greater $d$ results in larger jumps within $n'$ steps in comparison with an equivalent run with $n'' = d \cdot n'$ steps and only one charge. For later times and small $\Phi$, however, the difference between jump distances for neighboring values of $\Phi$ becomes negligible.

We checked that changing the parameters $\mu$ and $\nu$ does not change this qualitative picture.

![Figure 2: Average final value $\Phi_f$ after $n = 10^9$ timesteps as a function of the number of charges $d$. For every point the average has been taken over 100 runs. For a fit function of the form $a \cdot x^b$ the parameters attain following values: $a = 0.0052, b = -0.25$. The parameters of model (8) have been set to $\mu = 0.5$ and $\nu = 3$ with an initial value given by $\Phi_0 = 0.1$.](image)

### 5 Conclusions

In this paper we have studied numerically the model first introduced in [7,8] which solves the Hierarchy Problem by cosmological relaxation of the Higgs mass towards the attractor vacuum during eternal inflation. In this scenario the Higgs mass/VEV is changed due to nucleation of branes (or axionic domain walls). At the same time

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$^1$Runs that diverged at some point ($\Phi_i > 1$ for some $i$) were discarded.
the Higgs VEV acts as a back-control parameter that directs the convergence of the relaxation progress.

Such an attractor could in principle be realized in various different models each with its own specifics. In this analysis, however, we only focused on universal key features of the Cosmic Attractor mechanism. For this we have modeled the Higgs VEV evolution as a random walk with each step mimicking a vacuum transition triggered by a brane nucleation. The observed convergence to $\Phi_* = 0$ which represents our attractor point is in accordance with analytic predictions. We then generalized this stochastic model to multiple different charges sourced by a 2-brane and studied the impact of their number on the relaxation rate of the Higgs VEV. We showed that a higher number of three-form fields leads to a faster convergence rate. That is less brane nucleation are necessary to relax the Higgs vacuum expectation value below a given value. This confirms the intuitive picture that adding brane charges is equivalent to an increase of brane nucleation channels.

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