Evolution of a domain wall in expanding universe: inflation and after it

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It is well known that a symmetry, which is broken in vacuum, at high temperatures tends to be restored. But in general, the situation is not that simple and straightforward. It is also possible that a symmetry is broken only in a particular range of temperatures, i.e. it is restored at the highest as well as at the lowest temperatures. This is just what is needed for a matter-antimatter domain generation without domain wall problem.

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- The model with spontaneous $CP$ violation is suggested.
- $CP$ violation appears due to interaction of additional scalar field with inflaton.
- BAU is generated just after inflation due to interaction of introduced scalar field with quarks and leptons.
- This scenario leads to the generation of matter-antimatter domains in the Early Universe.
- To avoid annihilation at the domain boundary, the distance between the domains should grow exponentially fast during inflation.
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How fast the domain wall width can grow in the Early Universe?
Domain wall evolution in the de Sitter space-time

Metric:
\[ ds^2 = dt^2 - e^{2Ht} \left( dx^2 + dy^2 + dz^2 \right). \]

Scalar field:
\[ \mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{\lambda}{2} (\varphi^2 - \eta^2)^2. \]

Equations of motion:
\[ \frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} g^{\mu\nu} \partial_\nu \varphi \right) = -2\lambda \varphi (\varphi^2 - \eta^2). \]

\[ H = 0 \text{ (static universe), 1d case } (\varphi = \varphi(z)): \]
\[ \frac{d^2 \varphi}{dz^2} = 2\lambda \varphi (\varphi^2 - \eta^2). \]

Solution (wall at \( z = 0 \)):
\[ \varphi(z) = \eta \tanh \frac{z}{\delta_0}, \]
where \( \delta_0 = 1/\sqrt{\lambda \eta} \) is the wall width.
Stationary solutions for $H > 0$

Ansatz for stationary solutions ($\varphi$ depends only on $a(t) \cdot z$):

$$\varphi = \eta \cdot f(u), \quad \text{where} \quad u = H ze^{Ht}.$$ 

Equations of motion:

$$\left(1 - u^2\right) f'' - 4uf' = -2Cf \left(1 - f^2\right),$$

where $C = \frac{1}{(H \delta_0)^2} = \frac{\lambda \eta^2}{H^2} > 0$

Boundary conditions:

$$f(0) = 0,$$
$$f(\pm \infty) = \pm 1.$$
Stationary solutions

\[ \frac{\phi}{\eta} \]

\[ zH \exp(Ht) \]

C=10  C=4  C=2.5  C=2.05  C=2.0001
Beyond stationary limit

\[ \frac{\partial^2 \varphi}{\partial t^2} + 3H \frac{\partial \varphi}{\partial t} - e^{-2Ht} \frac{\partial^2 \varphi}{\partial z^2} = -2\lambda \varphi \left( \varphi^2 - \eta^2 \right). \]

Introducing dimensionless parameters \( \tau = Ht, \ \zeta = Hz, \ f(\zeta, \tau) = \varphi(z, t)/\eta, \) we get

\[ \frac{\partial^2 f}{\partial \tau^2} + 3 \frac{\partial f}{\partial \tau} - e^{-2\tau} \frac{\partial^2 f}{\partial \zeta^2} = 2Cf \left( 1 - f^2 \right), \]

where \( C = \lambda \eta^2 / H^2 = 1/(H\delta_0)^2 > 0. \)

Boundary conditions:

\[ f(0, \tau) = 0, \quad f(\pm \infty, \tau) = \pm 1, \]

Initial configuration:

\[ f(\zeta, 0) = \tanh \frac{\zeta}{\delta_0} = \tanh \sqrt{C} \zeta, \quad \frac{\partial f(\zeta, \tau)}{\partial \tau} \bigg|_{\tau=0} = 0. \]
Wall width

![Graph of Wall width](image)

- **C=10**
- **C=4**
- **C=2.5**
- **C=1**
- **C=0.1**
- **C=0.01**
Let us consider a simple model of inflation with quadratic inflaton potential $U = \frac{m^2 \Phi^2}{2}$, then the Hubble parameter is

$$H = \sqrt{\frac{8\pi \rho}{3m_{pl}^2}} \approx \sqrt{\frac{8\pi}{3m_{pl}^2}} \frac{m^2 \Phi^2}{2} = \sqrt{\frac{4\pi}{3}} \frac{m}{m_{pl}} \Phi,$$

and the equation of motion of the inflaton in the slow-roll regime is the following:

$$\dot{\Phi} \approx -\frac{m^2 \Phi}{3H} \approx -\frac{m_{pl} m}{\sqrt{12\pi}},$$

where $m_{pl}$ is the Planck mass, $m$ is the inflaton mass.

$$\Phi(t) = \Phi_i - \frac{m_{pl} m}{\sqrt{12\pi}} t,$$

where $\Phi_i$ is the initial value of inflaton field.
The Hubble parameter and the scale factor can also be easily found:

\[ H(t) = \sqrt{\frac{4\pi}{3} \frac{m}{m_{pl}} \Phi_i - \frac{1}{3} m^2 t}, \]

\[ a(t) = a_0 \cdot \exp \left( \sqrt{\frac{4\pi}{3} \frac{m}{m_{pl}} \Phi_i t - \frac{1}{6} m^2 t^2} \right). \]

These formulas are valid only till the end of inflation, \( t < t_e \equiv \frac{\sqrt{12\pi \Phi_i}}{m_{pl}} m^{-1} \).

It is convenient to use \( 1/m \) units in equation of motion:

\[ \frac{\partial^2 f}{\partial (t \cdot m)^2} + m(t_e - t) \frac{\partial f}{\partial (t \cdot m)} - \frac{1}{a^2(t)} \frac{\partial^2 f}{\partial (z \cdot m)^2} = \frac{2}{(m \cdot \delta_0)^2} f (1 - f^2). \]

In numerical calculations the following parameters were used:

\[ \Phi_i = 2 m_{pl}, \ t_i = 0, \ a_0 = 1. \]
Inflation: $C(t)$ dependence

$$C(t) = \frac{1}{(H(t)\delta_0)^2}.$$  

Time $t_C$ at which $C(t_C) = 2$:

$$m \cdot t_C = m \cdot t_e - \frac{3\sqrt{2}}{2m\delta_0}.$$  

Parameter $C(t)$ can be equal 2 only if $t_C \geq 0$:

$$m \cdot \delta_0 \geq \frac{3\sqrt{2}}{2mt_e} = \frac{\sqrt{3}m_{pl}}{2\sqrt{2}\pi\Phi_i} \approx 0.173.$$
\[ \delta_0 = 0.025 \cdot m^{-1}. \]

\[ \delta_0 = 0.05 \cdot m^{-1}. \]

\[ \delta_0 = 0.1 \cdot m^{-1}. \]

\[ \delta_0 = 0.125 \cdot m^{-1}. \]
\[ \delta_0 = 0.15 \cdot m^{-1}. \]

\[ \delta_0 \approx 0.173 \cdot m^{-1}. \]

\[ \delta_0 = 0.2 \cdot m^{-1}. \]

\[ \delta_0 = 0.4 \cdot m^{-1}. \]
\[
\delta_0 = 10 \cdot m^{-1}.
\]

For the parameter \(\delta_0 = 100 \cdot m^{-1}\), the behavior of the domain wall evolves similarly to the previous case.
Universe with $p = w \rho$

\[ a(t) = a_0 \cdot \left( \frac{t}{t_i} \right)^\alpha, \]

\[ H(t) = \frac{\dot{a}}{a} = \frac{\alpha}{t}, \text{ where } \alpha = \frac{2}{3(1 + w)} > 0, \]

The values $w = 0$ ($\alpha = 2/3$) and $w = 1/3$ ($\alpha = 1/2$) correspond to the matter-dominated and radiation-dominated universe, respectively.

The equation of motion

\[ \frac{\partial^2 f}{\partial t^2} + 3H(t) \frac{\partial f}{\partial t} - \frac{1}{a^2(t)} \frac{\partial^2 f}{\partial z^2} = \frac{2}{\delta_0^2} f (1 - f^2), \]

where $f(z, t) = \varphi(z, t)/\eta$. 
Feature of the $p = w\rho$ universe

Since

$$H(t)\delta_0 = H\left(\frac{t}{\delta_0}\right),$$

after the substitution $\tau = t/\delta_0$, $\zeta = z/\delta_0$ we get:

$$\frac{\partial^2 \tilde{f}}{\partial \tau^2} + \frac{3}{\sqrt{C(\tau)}} \frac{\partial \tilde{f}}{\partial \tau} - \frac{1}{\tilde{a}^2(\tau)} \frac{\partial^2 \tilde{f}}{\partial \zeta^2} = 2\tilde{f} \left(1 - \tilde{f}^2\right),$$

where $\tilde{f}(\zeta, \tau) = f(\zeta \cdot \delta_0, \tau \cdot \delta_0)$, $\tilde{a}(\tau) = a(\tau \cdot \delta_0) = a_0 \cdot (\tau/\tau_i)^\alpha$, and

$$C(\tau) = \left(H(\tau \cdot \delta_0) \cdot \delta_0\right)^{-2} = H^{-2}(\tau).$$

No explicit dependence on $\delta_0$!
The parameter $C(t)$ increases as

$$C(t) = \frac{1}{(H(t)\delta_0)^2} = \frac{t^2}{(\alpha\delta_0)^2} \propto t^2.$$  

The time $t_C$ at which $C(t_C) = 2$:

$$\frac{t_C}{\delta_0} = \sqrt{2}\alpha.$$  

We obtain that $t_C > t_i$ for

$$w < \frac{2\sqrt{2}}{3} \frac{\delta_0}{t_i} - 1.$$
\[ t_i / \delta_0 = 0.5 \]
\( t_i/\delta_0 = 1.0 \)

Graph showing the evolution of a domain wall in an expanding universe, with various lines indicating the time at which the wall collapses for different values of the parameter \( w \). The graph includes lines at specific values of \( t/\delta_0 \) for different values of \( w \):

- Blue line: \( w = 7/8 \)
- Orange line: \( w = 1/3 \)
- Green line: \( w = 0 \)
- Red line: \( w = -1/3 \)
- Purple line: \( w = -2/3 \)
- Brown line: \( w = -7/9 \)

The graph illustrates the time evolution of the universe from inflation to the present day.
Conclusions

The evolution of the domain walls was considered in the following cases:

- **de Sitter universe**
  - For $C = \lambda \eta^2 / H^2 = 1 / (H \delta_0)^2 > 2$ the solutions tend to the stationary ones.
  - For $C = \lambda \eta^2 / H^2 = 1 / (H \delta_0)^2 < 2$ the wall width grows rapidly. For $C \lesssim 0.1$ the growth is practically exponential, so the wall expands with the universe.

- **during inflation:**
  - For $m \cdot \delta_0 \lesssim 0.173$ the deviation of the wall width from $\delta_0$ is small.
  - For $0.173 \lesssim m \cdot \delta_0 \lesssim 1$ the wall width can reach cosmologically large values, but then it quickly diminishes and reaches $\delta_0$.
  - For $m \cdot \delta_0 \gg 1$ the wall width grows with the scale factor and by the end of inflation it reaches cosmologically large size.

- **$p = \omega \rho$ universe:**
  - Domain walls with cosmologically large width can exist only in the beginning of this phase.
  - For $t/\delta_0 \gg \sqrt{2} \alpha$ the wall width is close to $\delta_0$. 