A New Exponential Ratio-Type Estimator with Linear Combination of Two Auxiliary Variables

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Abstract

In sample surveys, it is usual to make use of auxiliary information to increase the precision of estimators. We propose a new exponential ratio-type estimator of a finite population mean using linear combination of two auxiliary variables and obtain mean square error (MSE) equation for proposed estimator. We find theoretical conditions that make proposed estimator more efficient than traditional multivariate ratio estimator using information of two auxiliary variables, the estimator of Bahl and Tuteja and the estimator proposed by Abu-Dayeh et al. In addition, we support these theoretical results with the aid of two numerical examples.

Introduction

In the sampling theory, the use of supplementary information provided by auxiliary variables in survey sampling was extensively discussed. Such information is generally used in ratio, product and regression type estimators for the estimation of population mean of study variable. In literature, number of authors introduced many ratio and regression type estimators by using general linear transformation of the auxiliary variable [1–5]. For recent development, exponential estimators have been widely studied by several authors. Singh et al.[6] suggested the modified exponential ratio and product estimators in two phase sampling and analyzes their properties; these estimators were compared for their precision with simple mean per unit, usual double sampling ratio and product estimators. On base of the estimator of Singh et al., Ozgul and Cingi [7] suggested a class of exponential regression cum ratio estimator in two phase sampling, MSE of the proposed estimator were obtained. However, these estimators were considered using one auxiliary variate.
In this study, a new exponential ratio-type estimator using linear combination of two auxiliary variates is considered to estimate a finite population mean for the variable of interest. And we obtain mean square error (MSE) equation for the proposed estimator. We find theoretical conditions that make proposed exponential ratio-type estimator more efficient than traditional multivariate ratio estimator using information of two auxiliary variables, the estimator of Bahl and Tuteja and the estimator proposed by Abu-Dayeh et al. We compared the traditional ratio estimator using information of two auxiliary variables, the estimator of Bahl and Tuteja, the estimator proposed by Abu-Dayeh et al. and proposed exponential ratio-type estimator using two statistic data sets. And we obtained the satisfactory results.

Materials and Methods

The existed estimators

The traditional multivariate ratio estimator using information of two auxiliary variables \( x_1 \) and \( x_2 \) to estimate the population mean, \( Y \) [8], as follows:

\[
\bar{Y}_{MR} = e_1 \bar{y} \frac{\bar{x}_1}{\bar{x}_1} + e_2 \bar{y} \frac{\bar{x}_2}{\bar{x}_2}
\]  

where \( \bar{y} \) denote the sample means of the variable \( y \), \( \bar{x}_i \), and \( \bar{x}_i \) (i=1,2) denote respectively the sample and the population means of the variable \( x_i \) (i=1,2); \( e_1, e_2 \) are the weights that satisfy the condition, \( e_1 + e_2 = 1 \)

The MSE of this traditional multivariate ratio estimator is given by

\[
MSE(\bar{Y}_{MR}) \approx \frac{1 - f}{n} \left( S_y^2 + e_1^2 R_1^2 S_x^2 + e_2^2 R_2^2 S_x^2 - 2e_1 R_1 S_{yx} - 2e_2 R_2 S_{yx} + 2e_1 e_2 R_1 R_2 S_{x_1 x_2} \right)
\]  

where \( f = \frac{n}{N} \), \( n \) and \( N \) are respectively the number of units in the sample and the population; \( S_y^2, S_{x_1}^2, \) and \( S_{x_2}^2 \) are the population variances of \( Y, X_1 \) and \( X_2 \), respectively; \( S_{x_1 x_2}, S_{yx_1}, \) and \( S_{yx_2} \) are the population covariances between \( X_1 \) and \( X_2 \), \( Y \) and \( X_1 \), and \( Y \) and \( X_2 \), respectively; \( R_1 = \frac{Y}{X_1}, \quad R_2 = \frac{Y}{X_2} \).

The optimum values of \( e_1 \) and \( e_2 \) are given by

\[
e_1^* = \frac{S_{x_1}^2 R_2^2 - S_{x_2} R_2 + S_{yx_1} R_1 - S_{x_1 x_2} R_1 R_2}{S_{x_1}^2 R_1^2 + S_{x_2}^2 R_2^2 - 2 S_{x_1 x_2} R_1 R_2}, \quad e_2^* = 1 - e_1^*
\]

The minimum MSE of \( \bar{Y}_{MR} \) can be shown to be:
\[ \text{MSE}_{\min}(\bar{y}_{MR}) \approx \frac{1-f}{n} \left[ S_{y}^{2} + e_{1}^{2} 2R_{1}^{2}S_{x_{1}}^{2} + e_{2}^{2} 2R_{2}^{2}S_{x_{2}}^{2} - 2e_{1}^{2} R_{1} S_{y x_{1}} - 2e_{2}^{2} R_{2} S_{y x_{2}} - 2e_{1}^{2} e_{2}^{2} R_{1} R_{2} S_{x_{1}x_{2}} \right] \]  \( \text{(3)} \)

Bahl and Tuteja [9] proposed an exponential ratio-type estimator which is given by

\[ \bar{y}_{BT} = \bar{y} \exp \left( \frac{X - \bar{x}}{X + \bar{x}} \right) \]  \( \text{(4)} \)

The MSE of \( \bar{y}_{BT} \) is given by

\[ \text{MSE}(\bar{y}_{BT}) \approx \frac{1-f}{n} \left[ S_{y}^{2} + \frac{1}{4} R^{2} S_{x}^{2} - R S_{yx} \right] \]  \( \text{(5)} \)

Abu-Dayeh et al. [10] proposed the estimator using two auxiliary variables given by

\[ \bar{y}_{AD} = k_{1} \bar{y} \left( \frac{\bar{x}_{1}}{X_{1}} \right) + k_{2} \bar{y} \left( \frac{\bar{x}_{2}}{X_{2}} \right) \]  \( \text{(6)} \)

where \( k_{1} + k_{2} = 1 \).

MSE of this estimator is given as follows:

\[ \text{MSE}(\bar{y}_{AD}) \approx \frac{1-f}{n} \left[ S_{y}^{2} + k_{1}^{2} \gamma_{1}^{2} R_{1}^{2} S_{x_{1}}^{2} + k_{2}^{2} \gamma_{2}^{2} R_{2}^{2} S_{x_{2}}^{2} + 2k_{1} \gamma_{1} R_{1} S_{y x_{1}} \right. \]
\[ \left. + 2k_{2} \gamma_{2} R_{2} S_{y x_{2}} + 2k_{1} k_{2} \gamma_{1} \gamma_{2} R_{1} R_{2} S_{x_{1}x_{2}} \right] \]  \( \text{(7)} \)

The optimum values of \( k_{1} \) and \( k_{2} \) are given by

\[ k_{1}^{*} = \frac{\gamma_{2}^{2} R_{2}^{2} S_{x_{2}}^{2} - \gamma_{1} \gamma_{2} R_{1} R_{2} S_{x_{1}x_{2}} - \gamma_{1}^{2} \gamma_{2} R_{1} R_{2} S_{x_{1}x_{2}}}{\gamma_{1}^{2} R_{1}^{2} S_{x_{1}}^{2} - 2\gamma_{1} \gamma_{2} R_{1} R_{2} S_{x_{1}x_{2}} + \gamma_{2}^{2} R_{2}^{2} S_{x_{2}}^{2}}, \quad k_{2}^{*} = 1 - k_{1}^{*} \]

\[ \text{MSE}_{\min}(\bar{y}_{AD}) \approx \frac{1-f}{n} \left[ S_{y}^{2} + k_{1}^{*} \gamma_{1}^{2} R_{1}^{2} S_{x_{1}}^{2} + k_{2}^{*} \gamma_{2}^{2} R_{2}^{2} S_{x_{2}}^{2} + 2k_{1}^{*} \gamma_{1} R_{1} S_{y x_{1}} \right. \]
\[ \left. + 2k_{2}^{*} \gamma_{2} R_{2} S_{y x_{2}} + 2k_{1}^{*} k_{2}^{*} \gamma_{1} \gamma_{2} R_{1} R_{2} S_{x_{1}x_{2}} \right] \]  \( \text{(8)} \)

The proposed family of ratio estimators

We propose a new exponential ratio-type estimator using linear combination of two auxiliary variables as follows:
where $\overline{X}_k = w_1 \overline{x}_1 + w_2 \overline{x}_2$, $\overline{x}_k = w_1 \overline{x}_1 + w_2 \overline{x}_2$; $w_1, w_2$ are weights that satisfy the condition: $w_1 + w_2 = 1$.

MSE of this estimator can be found using Taylor series method defined as

$$f(y, x_1, x_2) \approx f(Y, X_1, X_2) + \frac{\partial f}{\partial y} (\overline{y} - Y) + \frac{\partial f}{\partial x_1} (\overline{x}_1 - X_1) + \frac{\partial f}{\partial x_2} (\overline{x}_2 - X_2)$$

where $f(y, x_1, x_2) = \overline{y}_{lcr}$

$$\overline{y}_{lcr} - \overline{Y} \approx (\overline{y} - \overline{Y}) - \frac{1}{2} w_1 R_k (\overline{x}_1 - X_1) - \frac{1}{2} w_2 R_k (\overline{x}_2 - X_2)$$

where $\frac{\overline{Y}}{w_1 \overline{x}_1 + w_2 \overline{x}_2} = R_k$.

The MSE of this new multivariate exponential ratio-type estimator is given by

$$MSE(\overline{y}_{lcr}) = E((\overline{y}_{lcr} - \overline{Y})^2) \approx \frac{1 - f}{n}$$

$$\left( s_y^2 + \frac{1}{4} w_1^2 R_k^2 s_{x_1}^2 + \frac{1}{4} w_2^2 R_k^2 s_{x_2}^2 - w_1 R_k s_{yx_1} - w_2 R_k s_{yx_2} + \frac{1}{2} w_1 w_2 R_k^2 s_{x_1 x_2} \right)$$

The optimum values of $w_1$ and $w_2$ are given by

$$w_1^* = \frac{B}{A + B}, \quad w_2^* = 1 - w_1^*$$

where $A = 2 S_{yx_2} \overline{X}_2^2 - S_{x_1 x_2} \overline{X}_1 \overline{Y} - 2 S_{yx_1} \overline{X}_1 \overline{X}_2 + S_{x_1}^2 \overline{X}_1^2 \overline{Y}$,
$$B = S_{x_2}^2 \overline{X}_2 \overline{Y} - 2 S_{yx_2} \overline{X}_1 \overline{X}_2 - S_{x_1 x_2} \overline{Y} \overline{X}_2 + 2 S_{yx_1} \overline{X}_2^2$$

The minimum MSE of $\overline{y}_{lcr}$ can be shown to be:
\[ \text{MSE}_{\text{min}}(\overline{y}_{lcr}) \leq \frac{1-f}{n} \]
\[ (S_y^2 + \frac{1}{4} w_1^2 R_{lc}^* s_{x_1}^2 + \frac{1}{4} w_2^2 R_{lc}^* s_{x_2}^2 - w_1^2 R_{lc}^* s_{y_{x_1}} - w_2^2 R_{lc}^* s_{y_{x_2}} + \frac{1}{2} w_1^2 w_2^2 R_{lc}^* s_{x_1 x_2}) \]

where \( \overline{y} = \frac{w_1^* X_1 + w_2^* X_2}{w_1^* + w_2^*} = R_{lc}^* \)

### Efficiency comparisons

We compare the MSE of the proposed exponential ratio-type estimator using linear combination of two auxiliary variables given in Eq. (12) with the MSE of traditional multivariate ratio estimator using information of two auxiliary variables given in Eq. (3) as follows:

\[ \text{MSE}_{\text{min}}(\overline{y}_{lcr}) < \text{MSE}_{\text{min}}(\overline{y}_{MR}) \]

if

\[ \left( \frac{1}{4} w_1^2 R_{lc}^* s_{x_1}^2 - w_1^2 R_{lc}^* s_{y_{x_1}} \right) + \left( \frac{1}{4} w_2^2 R_{lc}^* s_{x_2}^2 - w_2^2 R_{lc}^* s_{y_{x_2}} \right) + \left( \frac{1}{2} w_1^2 w_2^2 R_{lc}^* s_{x_1 x_2} \right) > 0 \]

We compare the MSE of the proposed exponential ratio-type estimator using linear combination of two auxiliary variables given in Eq. (12) with the MSE of the estimator of Bahl and Tuteja given in Eq. (5) as follows:

\[ \text{MSE}_{\text{min}}(\overline{y}_{lcr}) < \text{MSE}(\overline{y}_{BT}) \]

if

\[ \frac{1}{4} (w_1^2 R_{lc}^* s_{x_1}^2 + w_2^2 R_{lc}^* s_{x_2}^2 - R_{lc}^* s_{x_1 x_2}^2) + \frac{1}{2} w_1^2 w_2^2 R_{lc}^* s_{x_1 x_2} < 0 \]

where \( \overline{y}_{BT} \) denote \( \overline{y}_{BT} \) using auxiliary variable \( x_i (i=1,2) \).

We compare the MSE of the proposed exponential ratio-type estimator using linear combination of two auxiliary variables given in Eq. (12) with the MSE of the estimator of Abu-Dayeh et al given in Eq.(8) as follows:
Numerical illustration

To examine the merits of the proposed estimator, we have considered two natural population data sets. We apply the traditional multivariate ratio estimator using information of two auxiliary variables, the estimator of Bahl and Tuteja, the estimator of Abu-Dayeh et al, given in Eqs. (1), (4), (6) and proposed exponential ratio-type estimator of a finite population mean using linear combination of two auxiliary variables, given in Eq. (9). The MSE of these estimators are computed as given in Eqs. (3), (5), (8), (12).

Example 1. In order to precisely estimate cotton output in one region, the sample size \( n = 8 \) villages were taken out from \( N = 18 \) villages using SRSWOR\[8\].

Y: Cotton output.
X\(_1\): The area of the plant.
X\(_2\): The proportion of good seed.

The statistics of example 1 are given in Table 1.

Example 2. The data set of this example can be seen in the reference \[3\].

Y: Number of 'placebo' children.
X\(_1\): Number of paralytic polio cases in the placebo group.

The statistics of example 2 are given in Table 2.

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Table 1. Data Statistics of example 1.

|          | \( N = 18 \) | \( n = 8 \) |
|----------|--------------|-------------|
| \( \bar{x}_1 \) | 38.444       | 13.797      |
| \( s_{x_1}^2 \) | 74.679       | 174.967     |
| \( s_{y_1} \) | 46.512       | 11.546      |
| \( s_{x_1} \) | 35.486       | 42.262      |

Table 2. Data Statistics of example 2.

|          | \( N = 34 \) | \( n = 10 \) |
|----------|--------------|-------------|
| \( \bar{x}_1 \) | 2.91         | 4.92        |
| \( s_{x_1}^2 \) | 10.180       | 9.732       |
| \( s_{y_1} \) | 9.990        | 6.805       |
| \( s_{x_1} \) | 24.807       | 11.642      |

---

\[
MSE_{\min}(\bar{y}_{k1}) < MSE_{\min}(\bar{y}_{AD})
\]

if

\[
\frac{1}{4} w_1^2 R_{k1}^2 - k^2 \gamma_1^2 R_{11}^2 S_{x_1}^2 + \left( \frac{1}{4} w_2^2 R_{k2}^2 - k^2 \gamma_2^2 R_{22}^2 \right) S_{x_2}^2
\]

\[
- (w_1^2 R_{k1}^2 - 2k^2 \gamma_1 R_{11}) S_{yx_1} - (w_2^2 R_{k2}^2 - 2k^2 \gamma_2 R_{22}) S_{yx_2}
\]

\[
+ \left( \frac{1}{2} w_1^2 w_2^2 R_{k1}^2 - 2k^2 \gamma_1 \gamma_2 R_{12} R_{22} \right) S_{x_1 x_2} < 0
\]
Table 3. MSE Values of Ratio Estimators of example 1.

| Estimators | MSE     |
|------------|---------|
| $\hat{y}_{MB}$ | 0.2707 |
| $\hat{y}_{BT1}$ | 27.2133 |
| $\hat{y}_{BT2}$ | 1.6963 |
| $\hat{y}_{AD}$ | 2.7847 |
| $\hat{y}_{lcr}$ | 0.2506 |

doi:10.1371/journal.pone.0116124.t003

$X_2$: Number of paralytic polio cases in the 'not inoculated' group.
The statistics of example 2 are given in table 2.

Results and Discussion

MSE values of the traditional multivariate ratio estimator using information of two auxiliary variables, the estimator of Bahl and Tuteja, the estimator of Abu-Dayeh et al and proposed exponential ratio-type estimator using linear combination of two auxiliary variables can be seen in Table 3 and Table 4.

From Table 3 and 4, we notice that our proposed exponential ratio-type estimator using linear combination of two auxiliary variables is more efficient than traditional multivariate ratio estimator using information of two auxiliary variables, the estimator of Bahl and Tuteja and the estimator of Abu-Dayeh et al.

We examine the conditions, determined in paper, for two data sets,

The examining of condition (13), (14) and (15) about example 1 can been seen as follows.

(i) condition (13)

$$\left(\frac{1}{2} w_1^2 R_{lc}^2 - e_1^2 R_1^2\right) S_{x_1}^2 + \left(\frac{1}{4} w_1^2 R_{lc}^2 - e_1^2 R_1^2\right) S_{x_2}^2
- (w_1^2 R_{lc}^2 - 2e_1^2 R_1) S_{x_1 x_2} - (w_2^2 R_{lc}^2 - 2e_2^2 R_2) S_{x_1 x_2}
+ \left(\frac{1}{2} w_1^2 w_2^2 R_{lc}^2 - 2e_1^2 e_2^2 R_1 R_2\right) S_{x_1 x_2} = -0.289 < 0$$

(ii) condition (14)

$$\left(\frac{1}{4} w_1^2 R_{lc}^2 * 2S_{x_1}^2 + w_2^2 R_{lc}^2 * 2S_{x_2}^2 - R_{lc}^2 S_{x_1}^2 - R_{lc}^2 S_{x_2}^2\right) - (w_1^2 R_{lc}^2 * S_{x_1} + w_2^2 R_{lc}^2 * S_{x_2} - R_1 S_{x_1})
+ \frac{1}{2} w_1^2 w_2^2 R_{lc}^2 * S_{x_1 x_2} = -388.263 < 0$$
Table 4. MSE Values of Ratio Estimators of example 2.

| Estimators | MSE    |
|------------|--------|
| y_{MR}    | 0.9062 |
| y_{BT1}   | 0.8383 |
| y_{BT2}   | 1.0497 |
| y_{AD}    | 0.9804 |
| y_{Xr}    | 0.8216 |

\[
\begin{align*}
\frac{1}{4}(w_1^{*2}R_{lc}^{*2}S_x^{2} + w_2^{*2}R_{lc}^{*2}S_x^{2} - R_2^{2}S_x^{2}) - (w_1^{*}R_{lc}^{*}S_{yx1} + w_2^{*}R_{lc}^{*}S_{yx2} - R_2S_{yx2}) \\
+ \frac{1}{2}w_1^{*}w_2^{*}R_{lc}^{*2}S_{x1x2} = -20.819 < 0
\end{align*}
\]

(iii) condition (15)

\[
\begin{align*}
\frac{1}{4}(w_1^{*2}R_{lc}^{*2} - k_1^{*2}R_1^{2})S_x^{2} + \frac{1}{4}(w_2^{*2}R_{lc}^{*2} - k_2^{*2}R_2^{2})S_x^{2} \\
- (w_1^{*}R_{lc}^{*} - 2k_1^{*}R_1)S_{yx1} - (w_2^{*}R_{lc}^{*} - 2k_2^{*}R_2)S_{yx2} \\
+ \frac{1}{2}w_1^{*}w_2^{*}R_{lc}^{*2} - 2k_1^{*}k_1^{*}R_1R_2)S_{x1x2} = -36.492 < 0
\end{align*}
\]

The examining of condition (13), (14) and (15) about example 2 can be seen as follows.

(iv) condition (13)

\[
\begin{align*}
\frac{1}{4}(w_1^{*2}R_{lc}^{*2} - e_1^{*2}R_1^{2})S_x^{2} + \frac{1}{4}(w_2^{*2}R_{lc}^{*2} - e_2^{*2}R_2^{2})S_x^{2} \\
- (w_1^{*}R_{lc}^{*} - 2e_1^{*}R_1)S_{yx1} - (w_2^{*}R_{lc}^{*} - 2e_2^{*}R_2)S_{yx2} \\
+ \frac{1}{2}w_1^{*}w_2^{*}R_{lc}^{*2} - 2e_1^{*}e_2^{*}R_1R_2)S_{x1x2} = -1.198 < 0
\end{align*}
\]

(v) condition (14)

\[
\begin{align*}
\frac{1}{4}(w_1^{*2}R_{lc}^{*2}S_x^{2} + w_2^{*2}R_{lc}^{*2}S_x^{2} - R_1^{2}S_x^{2}) - (w_1^{*}R_{lc}^{*}S_{yx1} + w_2^{*}R_{lc}^{*}S_{yx2} - R_1S_{yx1}) \\
+ \frac{1}{2}w_1^{*}w_2^{*}R_{lc}^{*2}S_{x1x2} = -0.235 < 0
\end{align*}
\]
\[
\frac{1}{4}(w_1^2 R^2_{k_1} S^2_{x_1} + w_2^2 R^2_{k_2} S^2_{x_2} - R^2_2 S^2_{y_2}) - (w_1^2 R^2_{k_1} S_{y_1 x_1} + w_2^2 R^2_{k_2} S_{y_2 x_2} - R^2_2 S_{y_2 x_2}) \\
+ \frac{1}{2} w_1^2 w_2^2 R^2_{k_c} S_{x_1 x_2} = -3.230 < 0
\]

(vi) condition (15)

\[
\left(\frac{1}{4} w_1^2 R^2_{k_1} - k^2_1 \gamma^2_1 R^2_1\right) S^2_{x_1} + \left(\frac{1}{4} w_2^2 R^2_{k_2} - k^2_2 \gamma^2_2 R^2_2\right) S^2_{x_2} \\
- (w_1^2 R^2_{k_1} - 2k^2_1 \gamma_1 R_1) S_{y_1 x_1} - (w_2^2 R^2_{k_2} - 2k^2_2 \gamma_2 R_2) S_{y_2 x_2} \\
+ \frac{1}{2} w_1^2 w_2^2 R^2_{k_c} - 2k^2_1 k^2_2 \gamma_1 \gamma_2 R_1 R_2 S_{x_1 x_2} = -2.249 < 0
\]

The result shows that the condition (13), (14) and condition (15) are satisfied. Therefore, we suggest that we should apply the proposed estimator to example 1 and 2.

Conclusions

We develop a new exponential ratio-type estimator of a finite population mean using two auxiliary variables and find theoretical conditions that make proposed estimator more efficient than traditional multivariate ratio estimator using information of two auxiliary variables, the estimator of Bahl and Tuteja and the estimator proposed by Abu-Dayeh et al. These theoretical conditions are also satisfied by the results of two numerical examples.

Author Contributions

Conceived and designed the experiments: JL. Performed the experiments: JL. Analyzed the data: JL ZY XP. Contributed reagents/materials/analysis tools: JL ZY XP. Wrote the paper: JL.

References

1. Lu J (2013) The Chain Ratio Estimator and Regression Estimator with Linear Combination of Two Auxiliary Variables. Plos one 8(11): e81085.
2. Lu J, Yan Z (2014) A Class of Ratio Estimators of a Finite Population Mean Using Two Auxiliary Variables. Plos one 9(2): e89538.
3. Choudhury S, Singh BK (2012) A Class of Product-cum-dual to Product Estimators of the Population Mean in Survey Sampling Using Auxiliary Information. Asian J Math Stat 6.
4. Kadilar C, Candan M, Cingi H (2007) Ratio estimators using robust regression. Hacet J Math Stat 36: 181–188.
5. Al-Omari Al, Jemain AA, Ibrahim K (2009) New ratio estimators of the mean using simple random sampling and ranked set sampling methods. Revista Investigacion Operacional 30: 97–108.
6. Sing HP, Vishwakarma GK (2007) Modified exponential ratio and product estimators for finite population mean in two phase sampling. Austrian Journal of Statistics 36(3): 217–225.

7. Ozgul N, Cingi H (2014) A new class of exponential regression cum ratio estimator in two phase sampling. Hacettepe Journal of mathematics and statistics 43(1):131–140.

8. Feng SY, Ni JX, Zou GH (1998) The Theory and Methods of Sampling Survey. Beijing: China Statistics Press, 145–150p. (in Chinese)

9. Bahl S, Tuteja RK (1991) Ratio and product type exponential estimators. J Inform Opti Scien 12: 159–163.

10. Abu-Dayeh WA, Ahmed MS, Ahmed RA, Muttlak HA (2003) Some Estimators of a Finite Population Mean Using Auxiliary Information. Appl Math Comput 139: 287–298.