Viscosity and secondary consolidation in one-dimensional loading

Paul G. Joseph PhD, PE
President, Engineering Solutions, Boxboro, MA, USA

Viscosity and secondary consolidation effects are observed in one-dimensional compression, but it remains unresolved if viscous effects occur during the primary consolidation phase and consequently whether the end of the primary consolidation (EOP) line is unique and the $H^2$ scaling law applies. Dynamical systems soil mechanics suggests that viscosity and secondary consolidation each have a separate physical basis. Viscous behaviour is due to strain rate dependence of the coefficients of friction at interparticle contacts as they slide against each other in simple friction, and occurs during both primary consolidation and secondary consolidation. Secondary consolidation is the continued deformation of the soil structure after excess pore pressure has dissipated, as small numbers of particles move at random shear strains, in a Poisson process, to new final positions. The near constant $C_{(w)}/C_e$ ratio is due to the form of the equation that expresses this ratio and because very few particles move to new positions during secondary consolidation. Strain rates for typical geotechnical problems being pseudo-static, viscous effects in the field are small, and the current design practice of using the EOP curve and $C_{(w)}/C_e$ to calculate compression settlements appears reasonable. Empirical evidence suggests that adsorbed water layers control strain-rate-related effects.

Notation

- $C_r$: change in void ratio per log cycle change in normal effective stress in one-dimensional (1D) compression
- $C_{(w)}$: change in void ratio per log cycle of time in 1D compression
- $C_{(w)}$: change in strain per log cycle of time in 1D compression
- $e$: Euler's number, the base of the natural log
- $c_v$: void ratio at start of test
- $H$: drainage length
- $K_0$: ratio of lateral to vertical stress for normally consolidated 1D consolidation
- $N_0$: number of particles at some particular vertical effective stress $\sigma'_v$
- $r^2$: correlation coefficient
- $t$: time since the start of the test
- $\alpha$: constant
- $\varepsilon$, $\varepsilon_{yp}$: vertical plastic strain in 1D compression
- $(\dot{\varepsilon})$: strain rate (1/s)
- $\lambda$: rate of movement of particles into the final structure per unit of vertical strain in 1D compression
- $\sigma'_v$: vertical effective consolidation stress
- $\sigma'_{(w)} = \sigma'_v$: corresponding to a strain rate of $1 \times 10^{-7}$
- $\sigma'_{p} = \sigma'_{p}$: preconsolidation stress
- $\sigma'_{(w)} = \sigma'_{p}$: corresponding to a strain rate of $1 \times 10^{-7}$

Introduction

When a saturated soil layer consisting of fine-grained soils is subjected to one-dimensional (1D) loading, the resulting deformation occurs in two successive phases, primary consolidation and secondary consolidation. During the primary consolidation phase, the soil structure deformation is restricted by the ability of the pore fluid to drain. As a consequence, pore pressures increase, and the load is transferred to the pore fluid. These excess pore pressures dissipate with time as the sample drains, and the initial total stress applied becomes effective as it transfers from the pore fluid to the soil structure. At the end of primary consolidation (EOP), excess pore pressures have dissipated, and the initial total applied stress is fully effective. Thereafter, the soil continues to deform, but at a rate that does not result in excess pore pressures, a phase referred to as secondary consolidation. During this phase, the soil continues to deform under approximately constant effective stresses. In short, primary consolidation and secondary consolidation are two successive phases; a point marking the transition between the two phases is the EOP state.

According to Terzaghi’s theory of consolidation, the time for primary consolidation is proportional to $H^2$, where $H$ is the drainage length of the soil under consolidation. Standard practice in geotechnical engineering is to use the 1D consolidation test conducted in the laboratory using thin samples in the order of 20-mm thickness to predict the in situ field settlement of the thick clay layers. For this reason, predictions of field behaviour need to
take into account the scale effects due to the differing thicknesses of the laboratory sample compared with the in situ clay deposits. However, accounting for these scale effects is complicated by the question of the effect of soil–structure viscosity, which effects the deformation of the soil skeleton, resulting in creep compression strains.

Ladd et al. (1977) and Jamiołkowski et al. (1985) have pointed out and discussed two extreme possibilities, referred to as Hypothesis A and Hypothesis B. According to Hypothesis A, creep compression strain due to viscous effects occurs only after the EOP, and consequently, that the law of $H^2$, applies, and that the strain at EOP is the same in both thick layers in situ and thin layers in the laboratory test. Hypothesis B, on the other hand, assumes that creep compression strain due to viscous effects occurs during both primary consolidation and secondary consolidation, and consequently, that the law of $H^2$ does not apply and that the compressive strain at EOP for a given stress is also a function of strain rate (Leroueil, 2006; Watabe and Leroueil, 2012).

Šuklje (1957) proposed the ‘isotache’ concept (‘iso’ for ‘equal’ and ‘tache’ for ‘rate’), according to which there is a unique relationship between effective stress, strain and strain rate, which he illustrated for 1D loading. Over the past 50 years, there has been considerable research done on Suklje’s idea (Adachi et al., 1996; Degago et al., 2011; Hawlander et al., 2003; Imai et al., 2003; Kim and Leroueil, 2001; Leroueil et al., 1986, 1988; Qu et al., 2010; Tanaka et al., 2006; Watabe and Leroueil, 2012; Watabe et al., 2008a, 2008b, 2012; Yin et al., 1994).

Nonetheless, almost four decades later, the issue remains unresolved, and the question remains whether the EOP line is truly unique and if the $H^2$ scaling law applies (Mesri, 2003; Mesri and Choi, 1985; Mesri and Godlewski, 1977). This paper suggests that the reason for this lack of resolution is that current approaches to this problem are ‘phenomenological,’ that is, have been modeled in terms of only the observed behavioural effects of secondary consolidation and viscous effects, and not in terms of the underlying physical mechanisms responsible for secondary consolidation and viscous effects. This paper proposes that issues of the EOP being unique and of the $H^2$ scaling law can be resolved by identifying the underlying physical mechanisms responsible for secondary consolidation and creep. In this context, it is important to note that the strain rates of concern in the case of either Hypothesis A or B are all pseudo-static strain rates, that is, the upper bound of the range of strain rates of interest is that which does not result in inertial effects.

Joseph (2009, 2010, 2012) showed that a Poisson process underlies soil deformation. This Poisson process governing soil shear can be described as a dynamical system in which particles move at random shear strains into the final position with stresses at interparticle contact areas acting through simple friction. As shown by Joseph (2014), this includes the case of 1D compression, which is shear along a $K$ line. The next section of this paper discusses the mechanical movement of particles in terms of this Poisson process. As the effective stress increases, particles move to the final positions. However, given interparticle stresses and the adsorbed water layers surrounding particles, for a few particles these positions are but quasi-stable; over time, they move at random shear strains to final new positions in a Poisson process. Based on this, it is possible to show that the concept of a constant $C_{sw}/C_e$ ratio (Mesri and Castro, 1987) has a rational basis.

Joseph and Graham-Eagle (2013) proposed that during shear, the static coefficient of friction dominates at small strains. Once particles mobilise, this static friction coefficient reduces to its dynamic value. Both static and dynamic coefficients of friction are strain rate dependent and responsible for the soil–structure viscosity that results in creep strains. Temperature effects are very small and act only to change the nature of this dependence on the strain rate of the interparticle friction. At present, it remains unclear if for clays the friction is between primarily particle material to particle material, or particle material to adsorbed water, or adsorbed water to adsorbed water, or some combination thereof. The effects of creep compression due to structural viscosity remain unresolved, and the question remains whether the EOP line is truly unique and if the $H^2$ scaling law applies. This is discussed in the third section of this paper, which also suggests that the viscous effect is located in the adsorbed water layers surrounding the particles. The basis of this claim is that the strain-rate-dependent behaviour observed for worldwide clays and triaxial tests appears to be approximately the same.

Variations with strain rate in the stress–strain and void-ratio-strain curves during triaxial testing, that is, viscous effects during triaxial testing, are small due to the correspondingly small dependence of the friction coefficients on the strain rate. Consequently, the expectation is that the viscous effects during 1D loadings, where the strain rates are small (primary consolidation of thick layers and secondary consolidation in general), are also small and difficult to measure. It is also possible to show on this basis that the EOP-based $e$ against $\sigma'$ relationship is unique for load-controlled tests, but is slightly strain rate dependent for constant rate of strain (CRS) tests. Hypothesis B appears to be strictly valid; though practically, the effects of the viscosity of the soil–structure are small. This is discussed in the fourth section of the paper.

The paper concludes the following: (a) the current methods of calculating settlement, based on Hypothesis A, are useful simply because viscous effects are small at the strain rates typical of field situations, and (b) a useful research program would be one targeted specifically at elucidating the role of the adsorbed layers in determining viscous effects.

Secondary consolidation is continued particle movement

Secondary consolidation is a continuation of primary consolidation in terms of the physical particle movements except that it is slow enough to where the ability of the soil to drain pore fluids is not exceeded and consequently pore pressure does not build (Mesri...
and Vardhanabhuti, 2008). Joseph (2012) showed that a Poisson process underlies particle movements during shear. Joseph (2014) generalised the dynamical system model that describes this Poisson process to include all shear paths, including that of shear along the Ks line, that is, 1D consolidation.

Specialising the model for 1D consolidation, Joseph was able to derive the standard linear relationship between void ratio and the vertical effective stress observed in 1D compression. As part of this derivation, Joseph (2014) showed that \( C_\ell \) (change in void ratio per log cycle of effective vertical stress \( \sigma'_v \)) can be written as \( C_\ell = \lambda (1 + e_0)/\lambda \), where \( e_0 \) is the void ratio at the start of the test, and \( \lambda \) is the rate at which particles move per unit strain to their final position, corresponding in the case of 1D compression to the final effective vertical stress. (Note that \( \lambda \) is not used as often in critical state soil mechanics for the slope of the 1D \( e \) against \( \log(\sigma'_v) \) line.) From this, it follows that the change in strain over one log cycle of stress, is given as \( CR = C_\ell/(1 + e_0) = 1/\lambda \).

Mesri and Vardhanabhuti (2009) provide evidence that in the case of granular materials with high permeability, particles have essentially finished adjusting to the effective stress in 10–20s. Thereafter, deformations continue at a very small rate due to local, interparticle shear stresses at a few contacts, exceeding the contact’s ability to support it over the long term, and consequently, small readjustments continue to take place at a rate such that strain decreases exponentially with time. In this paper, the hypothesis is that when the vertical stress becomes fully effective, though the majority of the particles have moved to their final position, for some particles, this position is not stable. Continued slippage of these quasi-stable particles continues to occur at random strains as local interparticle contact shear stresses exceed locally available shear resistance, resulting again in a Poisson process with particles moving to new fully adjusted positions at random shear strains. Consequently, the Poisson-process-based analysis described below applies.

According to the Poisson process described by Joseph (2012, 2014), particles move to their final position at random shear strains. Per Joseph (2014), this applies to the case of 1D consolidation, where if the rate at which particles move per unit strain to their final position is \( \lambda \), and \( N_0 \) is the number of particles at some particular vertical effective stress \( \sigma'_v \), then the number of particles that remain to move to their final position at some subsequent vertical strain \( \varepsilon \), is \( N_0 e^{-\lambda \varepsilon} \), where \( \lambda = \sqrt{3/2} \varepsilon \). Over an increase of one log cycle of stress, strain (\( \varepsilon = CR = C_\ell/(1 + e_0) = 1/\lambda \)), and so \( \gamma = \sqrt{3/2} \varepsilon = (\sqrt{3/2})/\lambda \), or in other words corresponding to a change in effective vertical stress of one log cycle, that is, a stress of 10\( \varepsilon \), the number of particles that remain to move to the final position is \( N_0 e^{-\lambda \gamma} \) or \( N_0 e^{-\sqrt{3/2} \varepsilon} \). (Note that \( \varepsilon \) is the Euler number and not the void ratio.) In other words, the number of particles that moved to their final position is given by \( N_0 - N_0 e^{-\sqrt{3/2} \varepsilon} \) or \( N_0(1 - e^{-\sqrt{3/2} \varepsilon}) \).

Corresponding to this new effective vertical stress, after one log cycle of time, the change in strain is \( C_{\text{iso}} \). During this stage, particles are moving to their final position at a rate at or less than \( \lambda \), and consequently, the lower bound of the number of particles that remain to move into their final position is \( N_0 e^{-\sqrt{3/2} \varepsilon} \), or the upper bound of the number of particles that moved in one log cycle of time is \( N_0 e^{-\sqrt{3/2} \varepsilon} \) or \( N_0 e^{-\sqrt{3/2} \varepsilon} \) or \( N_0 e^{-\sqrt{3/2} \varepsilon} \) or \( N_0 e^{-\sqrt{3/2} \varepsilon} \). In other words, the upper bound of the ratio of strain during one log cycle of time to the strain during one log cycle of effective vertical stress is \( N_0 e^{-\sqrt{3/2} \varepsilon} \) or \( (N_0(1 - e^{-\sqrt{3/2} \varepsilon})) \) or \( (1 - e^{-\sqrt{3/2} \varepsilon} C_\ell) \). (Note line.) From this, \( C_{\text{iso}}/CR \) or \( C_{\text{iso}}/C_\ell \) is

\[
C_{\text{iso}}/CR = C_{\text{iso}}/C_\ell = (1 - e^{-\sqrt{3/2} \varepsilon C_\ell})/(\sqrt{3/2} \varepsilon - 1)
\]

1. \( = 1.66(1 - e^{-\sqrt{3/2} \varepsilon C_\ell}) \)

Given that this is during secondary consolidation, when deformation is proceeding at a slow rate so that the ability of the material to drain pore fluid is not exceeded and pore pressure does not develop, \( C_{\text{iso}} \) is very small, and \( \sqrt{3/2} \varepsilon C_\ell \) is near zero, that is, the numerator of Equation 1 is near zero, and is insensitive to the specific value of \( C_{\text{iso}}/\lambda \), so long as this value is small.

For example, Mesri and Castro (1987) show values of \( C_{\text{iso}}/C_\ell \) for 43 tests on Berthierville clay (a Canadian quick clay, with initial water content between 56% and 61%, liquid limit of 46% and plastic limit of 24%; Unified Soil Classification System classification CH). The average value from direct measurements of \( C_{\text{iso}}/C_\ell \), was 0.045. From their data, the value of \( C_{\text{iso}}/\lambda \) used in Equation 1 averaged 0.102, which is a small value that results in an average calculated value for \( C_{\text{iso}}/C_\ell \), from Equation 1 of 0.078. Likewise, Mesri and Castro (1987) also provide data from 36 tests of a tar sand and 24 tests of a shale \( C_{\text{iso}}/C_\ell \). From their data, the value of \( C_{\text{iso}}/\lambda \) averaged 0.044 and 0.033 respectively, again small values that in turn resulted in values of \( C_{\text{iso}}/C_\ell \), calculated using Equation 1 of 0.034 and 0.025. These compare well with their reported values from direct measurements of \( C_{\text{iso}}/C_\ell \), of 0.035 and 0.029 respectively.

In short, the calculated values from Equation 1 are close to the actual measured values, and more importantly, the root cause for the insensitivity of \( C_{\text{iso}}/C_\ell \) to the non-constant term in this equation, appearing as a negative exponent of the Euler number. This value, \( \sqrt{3/2} \varepsilon C_\ell \), is the number of particles that move to the final position during one log cycle of time during secondary consolidation, a value that \( \text{de facto} \) is small so that the soil deformation can occur slowly enough to permit free drainage without buildup of excess pore pressure.

**Viscous effects**

Dynamical systems soil mechanics holds that soil deformation is driven by a Poisson process in which friction at interparticle contact areas plays a fundamental role. Joseph and Graham-Eagle (2013)
analysed strain-rate effects in soil shear in the context of dynamical systems theory. One conclusion reached was that the variation in steady-state strengths with strain rate was similar to the variation of the maximum shear strength, even though the soil structure obtained at steady state is the flow structure and different than the soil structure at maximum strength. In other words, strain-rate effects appear to be independent of structure. Again, it is important to note that the strain rates under consideration are all pseudo-static, that is, bounded at the upper limit by the maximum strain rate that does not cause inertial effects.

Based on these data, Joseph and Graham-Eagle suggested that it was the interparticle static and dynamic frictional coefficients that controlled strain-rate effects. This interparticle friction could be particle material to particle material or between adsorbed water layers, or some combination of the two. Joseph (2014) showed that the same Poisson process that applies for shear to the steady state also applies for 1D compression, and that consequently, interparticle friction also plays a key role in 1D compression. Given the lateral constraint to movement of soil particles in 1D consolidation, the strain-rate effects observed for 1D tests will be an upper bound as compared with the triaxial tests that allow for lateral movement of particles.

Sheahan (1991) conducted 28 compression and ten extension triaxial tests at various strain rates on uncemented, reworked Boston Blue Clay, a glacial outwash of illitic CL clay deposited in a marine environment. He one dimensionally ($K_v$) consolidated samples to different overconsolidation ratios (OCRs) and then sheared them undrained at various constant strain rates in monotonic compression and extension.

In general, peak shear stresses (calculated as half the difference between the vertical and horizontal effective stresses and normalised by the initial vertical consolidation stress) increased less than 2% per order of magnitude of strain rate, while the normalised effective vertical and horizontal stresses at the peak shear stress varied an average of about 4% and 2% respectively per order of magnitude of strain rate for strain rates between 0.05% and 50%. Figures 1(a) and 1(b) show respectively the variation of the normalised vertical and horizontal stresses with strain rate at different OCRs for the Sheahan data, while Table 1 shows the average values. The correlation coefficients, especially for the normalised horizontal stress data, are generally poor.

Sheahan and Watters (1997) conducted CRS tests on Boston Blue Clay at strain rates between 1% and 3% per hour (between $2.8 \times 10^{-4} \text{s}^{-1}$ and $8.4 \times 10^{-4} \text{s}^{-1}$) together with incremental loading tests. Given that for load controlled tests Mesri and Feng (1986) indicate a strain rate at EOP of $2 \times 10^{-5} \text{s}^{-1}$, the strain rates used by Sheahan and Watters are well above those typical of secondary consolidation. Sheahan and Watters found the CR to be unchanged with strain rate even though the absolute values changed. They indicate an approximately 10% increase in strain for CRS tests at a strain rate of 1% per hour ($2.8 \times 10^{-4} \text{s}^{-1}$) over tests conducted by incremental loading tests. Given that for a strain rate of 3% per hour ($8.3 \times 10^{-4} \text{s}^{-1}$) showed a similar change per log cycle of strain rate as compared with the incremental loading tests.

For Sheahan’s triaxial tests on Boston Blue Clay, the vertical and horizontal stresses increased by 3.7% and 2.3% respectively per log cycle of strain rate and compare well with the increase of 4% per log cycle of strain rates Sheahan and Watters (1997) observed for their 1D compression tests on Boston Blue Clay. They suggested that the compression behaviour for uncemented clays such as Boston Blue Clay did not have the same degree of strain-rate dependence.

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**Figure 1.** Variation of normalised (a) vertical and (b) horizontal effective stress with strain rate from Sheahan (1991) data.
as intact sedimented clays. Nonetheless, the data are sparse, and additional data are needed to determine if these values hold in shear for a wide range of clays, as was the case for the 1D compression data for worldwide clays, as reported by Watabe et al. (2012).

Diaz-Rodrigues et al. (2009) provide a concise summary of strain-rate effects observed for clayey soils in shear. Their review indicates that the undrained strength in triaxial tests increases between 5% and 15% per log cycle of strain rate. Their review also points out the contradictory results that research programs investigating strain-rate effects report, for example, Zhu et al. (1999) and Zhu and Yin (2000) report that the strain-rate effects are higher in K, consolidated samples versus isotropically consolidated samples, whereas Graham et al. (1983) saw no significant effect for anisotropic consolidation.

A striking feature of the test programs that study rate effects is that they appear to simply observe the results of varying strain rates but have no underlying hypothesis that they seek to falsify. A useful test program would be one specifically organised for the purposes of falsifying the hypothesis that the adsorbed layers of water acting at contacts between particles control rate effects, regardless of whether shear deformation is in 1D compression or to failure and beyond in triaxial shear. Since the only difference between shear deformation in 1D compression and triaxial shear to failure is that the former has a stress path that follows a K line, it is expected that the same physical mechanisms underlie both types of deformation under loading. If one examines Figure 2 (an idealisation of 1D compression tests run at different strain rates reproduced from Watabe et al. (2013) but with the void-ratio axis horizontal), one sees that the effect of strain rate in 1D compression is qualitatively not unlike that seen in data at different strain rates from triaxial tests.

Table 1. Variation of normalised vertical and horizontal stresses with vertical strain rate from Sheahan (1991)

| OCR | Vertical stresses | Horizontal stresses |
|-----|-------------------|---------------------|
|     | Correlation coefficient | gain/log cycle of strain rate | Correlation coefficient | gain/log cycle of strain rate |
|     | $r^2$ | % | $r^2$ | % |
| 1   | 0.91 | 4.59 | 0.01 | 0.47 |
| 2   | 0.85 | 3.68 | 0.13 | 0.7 |
| 4   | 0.58 | 3.53 | 0.33 | 2.47 |
| 8   | 0.53 | 3.14 | 0.81 | 5.69 |

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**Hypothesis B is valid, but viscous effects are small**

There has been extensive debate on which of Hypotheses A and B is correct (Jamiolkowski et al., 1985; Ladd et al., 1977). The discussion below is framed in the context that the strain rates of interest are all pseudo-static.

Figure 3, which is a reproduction of Figure 10 from Watabe and Leroueil (2012), shows the compression curves obtained for a layer of clay under Kansai International Airport, located on an artificial island in the middle of Osaka Bay in Japan, during Phase 1 of its construction. Also shown in the figure are the isoache curves of Hypothesis B, corresponding to 1D compression at different strain rates for this same clay averaged from data from five tests, obtained from 24-hour incremental loading in an oedometer.

The reference strain rate is $1 \times 10^{-7} \text{s}^{-1}$, a rate chosen to normalise the strain-rate data and that is approximately the strain rate seen in long-term 24-hour load increment consolidation tests in the laboratory. This 24-hour-based curve is not the same as the EOP curve as it includes any secondary consolidation that occurred if consolidation completed in less than 24 hours. Given that for load controlled tests Mesri and Feng (1986) indicate a strain rate at EOP of $2 \times 10^{-4} \text{s}^{-1}$, the isoaches shown in Figure 2 all correspond to the secondary consolidation stage and provide no information on strain rates during primary consolidation.
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The field data shown in Figure 3 indicate that as the rate of compression decreased, the field curves traversed across the isolates. The data also show that for this clay the vertical consolidation stress corresponding to a strain rate of $1 \times 10^{-11} \text{s}^{-1}$ was 400 kPa and at a strain rate of $1 \times 10^{-7} \text{s}^{-1}$ was 500 kPa, that is, a difference of about 6% per order of magnitude of change in strain rate.

Watabe et al. (2012) and Watabe and Leroueil (2012) show the relationship between compression curves and strain rate for clays from all over the world. Figure 4 (a reproduction of Figure 7 from Watabe and Leroueil, 2012) shows this relationship. The striking feature of this curve is that it applies to a wide range of inorganic clays, regardless of variations in plasticity, mineralogy, fabric, microstructure, overconsolidation, etc. Included even is Mexico City clay, a clay noted for its exceptional characteristics as compared with other soils and with a water content of near 400%. The data also include soils from Italy, Japan, Canada and Sweden.

Figure 4 suggests that the strain-rate dependence in 1D compression is approximately the same for clays all over the world and is insensitive to the actual particle material. This seems to indicate that the adsorbed water layers surrounding particles play a fundamental role in strain-rate dependence, and factors such as actual soil particle material, fabric, structure, etc. only play secondary roles. While these latter factors may have a strong role in determining the soil’s liquid and plastic limits, the data from worldwide clays indicate that so far as the direct dependence of frictional behaviour on strain rate is concerned, such considerations are secondary. Such a hypothesis, if true, would significantly simplify research in soil strain-rate behaviour, locating it intrinsically in the adsorbed water layers.

Examination of the curve shows that between the strain rates of $1 \times 10^{-11}$ and $1 \times 10^{-7}$ considered for the Kansai International Airport clay, the normalised consolidation pressure changed by 25%, that is, again, a change of about 6% per order of magnitude change in strain rate. At higher strain rates, for example, between $1 \times 10^{-7}$ and $1 \times 10^{-4}$, the change was higher, approximately 10%.

Watabe et al. (2013) show that if the isolates follow the idealisation shown in Figure 2, then $C_{w,0}/C = (\Delta \log p)/(\Delta \log e) = \alpha$, where $\alpha$ is a constant. We have already seen that the constant ratio is due to the random movement of particles from their quasi-stable state to their final state in a Poisson process or that the variation with strain rate is independent of any substantial viscous contribution, that is, is orthogonal to either Hypothesis A or Hypothesis B.

Because the same mechanism of deformation that occurs during primary compression occurs during secondary consolidation except at a rate slow enough to permit free drainage, the same dependence on strain rate at interparticle contacts that holds during primary compression should continue to hold during secondary consolidation. However, in the case of secondary consolidation, this strain rate is very small. Nonetheless, viscous effects, though small they may be, should theoretically exist during secondary consolidation.

Watabe et al. (2008b) ran 24-hour incremental loading tests in a special oedometer to evaluate scale effects in long-term consolidation. They took particular care to account for the frictional losses arising at the interface between the sample and the oedometer ring. Their conclusions lend support to Hypothesis B, that is, viscous effects occurring both during and after primary consolidation. They found that the law of $H^2$ is essentially (i.e., not exactly) valid for laboratory specimens between 20 and 200 mm. For clay samples without a developed structure, strain variation followed Hypothesis A, but for samples with a developed structure, it followed Hypothesis B, even though in this case, at thicknesses greater than 50 mm, strain...
Variations followed Hypothesis A. They found that a structured clay will collapse at a higher strain rate than a non-structured clay and that these higher strain rates would be accompanied by larger viscous effects even during primary consolidation, that is, in accordance with Hypothesis B.

In the discussion of Watabe et al. (2008b), Mesri (2009: p. 823) quotes Mesri (2003) as follows: ‘As soon as primary consolidation begins, both $(\delta e)/(\delta \sigma)$, and $[(\delta e)/(\delta t)]_{ve}$ contribute to compression; however, only $[(\delta e)/(\delta t)]_{ve}$ contributes to compression during secondary consolidation when $(d\sigma^e)/(dt) = 0$. (Note: here $t$ is time and $e$ is void ratio.) This does not preclude viscous effects from occurring during secondary consolidation, in addition to continued Poisson-based particle movement.

Mesri in his various writings quotes the uniqueness of the EOP curve (Mesri and Choi, 1985), that is, that this curve is unique and independent of the duration of primary consolidation, that is, of layer thickness. As evidence of the uniqueness of the EOP curve and its independence on the duration of primary consolidation, he provides data from Aboshi (1973). In this paper, Aboshi provides measured values of EOP strain increments of clay layers of different thickness. The data appear to indicate that although for a given vertical stress the layers of different thicknesses took different times to finish primary consolidation, the strain increment at the EOP from the start of consolidation was the same.

Given that the smallest sample thickness for Aboshi’s tests quoted by Mesri is 60 mm, this matches the behaviour observed by Watabe et al. (2008b), where for sample thicknesses greater than 50 mm, strain variations followed Hypothesis A. It should be noted that the Aboshi data have been interpreted differently by others. Aboshi’s data claimed to show that the real behaviour lies between Hypotheses A and B. It should be noted that the Aboshi data have been interpreted differently by others. Aboshi’s data claimed to show that the real behaviour lies between Hypotheses A and B (Hawlander et al., 2003; Imai and Tang, 1992; Leroueil et al., 1985; Li et al., 2004; Watabe et al., 2009; Yasuhara, 1982). Oka et al. (1986) proposed Hypothesis C (a hypothesis midway between Hypotheses A and B) to describe Aboshi’s data. In the closure to their paper, Watabe et al. (2009) reiterate that all consolidation behaviours follow Hypothesis A when the layers are thick regardless of whether the clay has a well-developed structure or not, and that the key factor is the threshold between thick and thin clay layers, that is, the drainage path.

Small drainage paths and structured clays both result in higher strain rates for a given loading, suggesting a stronger role for viscous effects during consolidation with measurable effects of particle level viscous behaviour, that is, it would support Hypothesis B. Larger drainage paths and unstructured clays that do not collapse will exhibit smaller strain rates under loading and will result in smaller, hard to measure viscous effects during consolidation and so would appear to support Hypothesis A.

In short, particle level viscous effects occur always both during and after primary consolidation, that is, Hypotheses B is valid while Hypothesis A is not. At higher strain rates, the effects of particle level viscosity are measurable, while at lower strain rates, they are small and hard to distinguish from changes due to particle rearrangement. As per Hypothesis B, the EOP-based curve is strain rate dependent, and this strain-rate dependence can be readily observed in CRS tests. This is also in line with examples in the literature (Degago et al., 2011). However, for load-controlled tests, given that each stress increment results in its own unique strain rates through the consolidation process, the EOP-based $e$ against $\sigma'$ relationship for a load-controlled 1D consolidation test is indeed unique. This contrasts with the case for CRS tests, where the $e$ against $\sigma'$ relationship is also unique but only for the given strain rate at which the test was conducted.

**Conclusion**

Driving the viscous and secondary consolidation phenomena observed in 1D compression down to the underlying physical mechanisms furthers the understanding of these phenomena and how they determine soil behaviour in the laboratory and field. Dynamical systems soil mechanics suggests that viscosity effects and secondary consolidation each have a separate physical basis.

Viscous behaviour occurs both during and after primary consolidation in accordance with Hypothesis B and is due to the strain-rate dependence of the coefficients of friction at interparticle contacts as they slide against each other in simple friction. The similarity of strain-rate effects across different soils and stress paths suggests that the physical mechanisms underlying them are located in the adsorbed water layers surrounding the soil particles. The EOP curve does depend on strain rate, but for the special case of load-controlled consolidation, where each load has its own unique strain rate, the resulting EOP-based $e$ against $\sigma'$ relationship is expected to be unique.

Secondary consolidation is the continued deformation of the soil structure after consolidation due to the small numbers of particles moving at random shear strains, in a Poisson process, to new final positions. The near constant $C_{이자}/C_{c}$ ratio is due to the form of the equation derived from the underlying Poisson process and to the fact that very few particles move to new positions during secondary consolidation. It has little to do with viscous effects given that the predictions of $C_{이자}/C_{c}$ based on this model match the empirical data. The fact that $C_{이자}/C_{c}$ is approximately constant has no direct bearing on either Hypothesis A or B.

Given that the strain rates involved in field situations are pseudo-static, viscous effects will be small during either primary or secondary consolidation, and the current design practice of using the EOP curve and $C_{이자}$ from incremental load tests to calculate compression settlements seems reasonable.

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REFERENCES
Aboshi H (1973) An experimental investigation on the similitude in the consolidation of a soft clay, including the secondary creep settlement. Proceedings of the 8th International Conference on Soil Mechanics and Foundation Engineering, Moscow, Russia, vol. 4, no. 3, p. 88.

Adachi T, Oka F and Mimura M (1996) Modeling aspects associated with time dependent behavior of soils. In Measuring and Modeling Time Dependent Soil Behavior (Sheahan TC and Kallakun VN (eds)). American Society of Civil Engineers, New York, NY, USA, Geotechnical Special Publication no. 61, pp. 61–95.

Díaz-Rodríguez JA, Martínez-Vasquez JJ and Santamarina JC (2009) Strain-rate effects in Mexico City soil. Journal of Geotechnical and Environmental Engineering, ASCE 135(2): 300–305.

Degago SA, Grimstad G, Jostad HP, Nodal S and Olsson M (2011) Use and misuse of the isotope concept with respect to creep hypotheses A and B. Geotechnical Geology 61(10): 897–908.

Graham J, Crooks JHA and Bell AL (1983) Time effects on the stress-strain behavior of natural soft clays. Geotechnical Engineering 33(3): 327–340.

Hawlander BV, Muhunthan B and Imai G (2003) Viscosity effects on one-dimensional consolidation of clays. International Journal of Geomechanics, ASCE 3(10): 99–110.

Imai G, Tanaka Y and Saeegusa H (2003) One-dimensional consolidation modeling based on the isotache law for normally consolidated clays. Soils and Foundations 43(4): 173–188.

Imai G and Tang YX (1992) Constitutive equation of one-dimensional consolidation derived from inter-connected tests. Soils and Foundations 32(2): 83–96.

Jamiokowski M, Ladd CC, Germain JT and Lancellotta R (1985) New developments in field and laboratory testing of soils. In Proceedings of the 11th International Conference on Soil Mechanics and Foundation Engineering, San Francisco, Balkema, Rotterdam, The Netherlands, vol. 1, pp. 57–153.

Joseph PG (2009) A constitutive model of soil based on a dynamical systems approach. Journal of Geotechnical and Environmental Engineering, ASCE 135(8): 1155–1158.

Joseph PG (2010) A dynamical systems based approach to soil shear. Geotechnique 60(3): 807–812.

Joseph PG (2012) Physical basis and validation of a constitutive model for soil shear derived from micro-structural changes. International Journal of Geomechanics, ASCE 13(4): 365–383, http://dx.doi.org/10.1061/(ASCE) GM.1943-5622.0000209.

Joseph PG (2014) Generalized soil deformation model based on dynamical systems theory. Geotechnical Research 1(1): 32–42, http://dx.doi.org/10.1680/geores.14.00004.

Joseph PG and Graham-Eagle J (2013) Strain-rate effects in shear highlighted by a dynamical systems model. International Journal of Geomechanics, ASCE, http://dx.doi.org/10.1061/(ASCE)GM.1943-5622.0000360.

Kim YT and Leroueil S (2001) Modeling the viscous plastic behavior of clays during consolidation: application to Berthierville clay in both laboratory and field conditions. Canadian Geotechnical Journal 38(3): 484–497.

Ladd DC, Fott R, Ishihara K, Schlosser F and Poulos HG (1977) Stress-deformation and strength characteristics. General report. In Proceedings of the 9th International Soil Mechanics and Foundation Engineering Conference, Tokyo, Kluwer-Plenum, New York, NY, USA, vol. 2, pp. 421–494.

Leroueil S (2006) The isotope approach. Where are we 50 years after its development by Professor Suklje? Prof. Suklje’s Memorial Lecture, XIII Danube-European Geotechnical Engineering Conference, Ljubljana, Slovenia. Slovenian Geotechnical Society, Ljubljana, Slovenia, pp. 55–88.

Leroueil S, Kabbaj M and Tavenas F (1988) Study of the validity of a σ’ − σ, − ε, model in situ conditions. Soils and Foundations 28(3): 3–25.

Leroueil S, Kabbaj M, Tavenas F and Bouchard R (1986) Closure to ‘Stress-strain-strain-rate relation for the compressibility of sensitive natural clays’. Geotechnique 36(2): 288–290.

Leroueil S, Kabbaj M, Tavenas F and Bouchard R (1985) Stress-strain-strain rate relation for the compressibility of sensitive natural clays. Geotechnique 35(2): 159–180.

Li S, Shirako H, Sugiyama M and Akaishi M (2004) Time effects on one-dimensional consolidation analysis. Proceedings of the School of Engineering Tokai University 29: 1–8.

Mesri G (2003) Primary and secondary compression. In Soil Behavior and Soft Ground Construction (Germaine JT, Sheahan TC and Whitman RV (eds)). American Society of Civil Engineers, Reston, VA, USA, Geotechnical Special Publication no. 119, pp. 122–166.

Mesri G (2009) Discussion on effects of friction and thickness on long-term consolidation behavior of Osaka Bay Clays. Soils and Foundations 49(5): 823–824.

Mesri G and Feng TW (1986) Stress strain strain rate relation for the compressibility of sensitive natural clays. Géotechnique 36(2): 283–287.

Mesri G and Castro A (1987) The Cc/C, concept and K0 during secondary compression. Journal of Geotechnical Engineering, ASCE 113(3): 230–247.
Mesri G and Choi YK (1985) The uniqueness of the end-of-primary (EOP) void ratio-effective stress relationship. In Proceedings of the 11th ICSMFE, San Franciscisco. Balkema, Rotterdam, the Netherlands, vol. 2, pp. 587–590.

Mesri G and Godlewski PM (1977) Time- and stress-compressibility interrelationship. Journal of Geotechnical and Geoenvironmental Engineering, ASCE 103: 417–430.

Mesri G and Vardhanabhuti B (2008) Compression of granular materials. Canadian Geotechnical Journal 46: 369–392.

Oka F, Adachi T and Okano Y (1986) Two-dimensional consolidation analysis using an elasto-viscoplastic constitutive equation. International Journal for Numerical and Analytical Methods in Geomechanics 10(1): 1–16.

Qu G, Hinchberger SD and Lo KY (2010) Evaluation of the viscous behavior of clay using generalized overstress viscoplastic theory. Geotechnique 60(10): 777–789.

Sheahan TC (1991) An Experimental Study of the Time-dependent Undrained Shear Behavior of Resedimented Clay Using Automated Stress Path Triaxial Equipment. ScD thesis, Massachusetts Institute of Technology, Cambridge, MA, USA.

Sheahan TC and Watters PJ (1997) Experimental verification of CRS consolidation theory. Journal of Geotechnical and Geoenvironmental Engineering, ASCE 123(5): 430–437.

Šuklje L (1957) The analysis of the consolidation process by the isotache method. In Proceedings of the 4th International Conference on Soil Mechanics and Foundation Engineering, London. Butterworths, London, UK, vol. 1, pp. 200–206.

Tanaka H, Udaka K and Nosaka T (2006) Strain-rate dependency of cohesive soils in consolidation settlement. Soils and Foundations 46(3): 315–322.

Watabe Y, Udaka K and Morikawa Y (2008a) Strain-rate effects on long-term consolidation of Osaka Bay clay. Soils and Foundations 48(4): 495–509.

Watabe Y, Udaka K, Kobayashi M, Tabata T and Emura T (2008b) Effects of friction and thickness on long-term consolidation behavior of Osaka Bay clays. Soils and Foundations 48(4): 547–561.

Watabe Y, Udaka K, Kobayashi M, Tabata T and Emura T (2009) Closure to effects of friction and thickness on long-term consolidation behavior of Osaka Bay clays. Soils and Foundations 49(5): 824–825.

Watabe Y, Udaka K, Nakatni Y and Leroueil S (2012) Long-term consolidation behavior interpreted with isotache concept for worldwide clays. Soils and Foundations 52(3): 449–464.

Watabe Y, Udaka K, Nakatni Y and Leroueil S (2013) Correspondence related to Watabe, Y., Udaka, K., Nakatni, Y. and Leroueil, S. 2012. Soils and Foundations 53(2): 360–362.

Watabe Y and Leroueil S (2012) Modeling and implementation of isotache concept for long-term consolidation behavior. International Journal of Geomechanics, http://dx.doi.org/10.1061/(ASCE)GM.1943-5622.0000270.

Yasuhara K (1982) A practical model for secondary compression. Soils and Foundations 48(4): 45–56.

Yin JH, Graham J, Clark JL and Gao L (1994) Modeling unanticipated pore-water pressures in soft clays. Canadian Geotechnical Journal 31(5): 773–778.

Zhu J, Yin J and Luk S (1999) Strain-rate effects on stress-strain strength behavior of soft clay. Proceedings of the Eleventh Asian Regional Conference on Soil Mechanics and Geotechnical Engineering. Balkema, Rotterdam, the Netherlands, pp. 61–64.

Zhu J and Yin J (2000) Strain-rate dependent stress-strain behavior of over consolidated Hong Kong marine clay. Canadian Geotechnical Journal 37(6): 1272–1282.

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