Numerical Investigation of the nanofluid mixed convection on two layers enclosure with rotating cylinder: High Darcy Number Effects

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Abstract. The present study, numerically investigated of mixed convection in a square enclosure with two layers, with Al\textsubscript{2}O\textsubscript{3}–water nanofluid (upper layer) and nano-porous medium (lower layer) with an adiabatic rotating circular cylinder at the center of the enclosure. The top and bottom walls are assumed adiabatic, while the left sidewall is heated, and the right sidewall kept cooled. Numerically, COMSOL code based on the Galerkin finite element method used for solved the dimensionless governing equations. The non-dimensional parameters that used in this study are: Rayleigh number (Ra) ranged from $10^3$ up to $10^6$, Darcy number (Da) equal to $10^{-3}$, the angular rotational velocity ($\Omega$) ranged (0 and 6000), the solid volume fraction ($\phi=0.06$), and the inner circular cylinder radius as ($R=0.2$). The results showed that when Rayleigh numbers increase, a noticeable increase in the flow intensity and the steep temperature gradient, while the values increase when the cylinder rotates. The value of the local Nusselt number was high in the upper half of the cavity. The effect of the cylinder rotates is greater on the value of the local Nusselt number when using the low Rayleigh Numbers.

Keywords: Nanofluid, porous media, mixed convection, rotating circular, counter-clockwise

1. Introduction
Natural convection heat transfer has been Analysis extensively made because of many engineering applications like electronics cooling, double pane windows, solar collectors, nuclear power, buildings for heating and cooling (1, 2). Ho et al. (3) show that the properties of natural convection are effected by dynamic viscosity and thermal conductivity. Also, the natural convection in porous media has been taken the researcher’s attention recently (4-8). After that, investi\textsuperscript{gate} the natural convection inside the enclosures was developed to inserted fixed bodies in the cavity with different forms and study its influence on the heat transfer and flow circulation and the values of the Nusselt number Ravnik et al. (9). For enhancement of the convection heat transfer, a lid-driven wall is used and a rotating cylinder used inside the enclosures (10). By using a rotation cylinder inside horizontal annulus Matin et al. (11) explain the influence of Reynolds number on the average Nusselt number, as well as the effect of the location rotating cylinder. Shih et al. (12) found the triangular enclosure has been shown the greatest ability to dissipate thermal energy compared to the circular enclosure. Selimefendigil et al. (13) using two rotating cylinders inside a 3-D cavity. Abdulsahi and Al-Farhany (14) investigated experimentally the mixed convection in a square enclosure with two layers.

In the current study, the effect of angular rotation velocity with Rayleigh number on natural and mixed convection inside a cavity with two layers of nanofluids and porous medium with cylinder were studied numerically.
2. Methodology:

Figure 1 presents the physical geometry of the study of 2-D mixed convection in a square enclosure having a side length (L) and includes an adiabatic rotating circular cylinder with a radius (R) at the center of the enclosure. The enclosure is filled with two layers; the nanofluid layer, represents the upper half of the enclosure, and the porous layer (solid matrix with the same nanofluid) represents the lower half of the cavity.

For the preparation of nanofluid, water is used as a base fluid and nanoparticles are (Al₂O₃). The thermophysical properties of these materials are illustrated in Table 1. The vertical left side wall is heated at constant temperature (T_h), meanwhile, the vertical right side wall kept at cooling temperature (T_c). The horizontal upper and lower walls and the rotating circular are kept thermally insulated. The nanofluid thermo-physical properties are modeled according to the Boussinesq approximation, and the flow inside the porous medium is modeled using the Darcy-Brinkman model. A permeable interface is used located between the nanofluid region and the porous region. In this study, the cylinder located at the center of the enclosure rotates at a different angular rotational velocity in a clockwise direction.

Table 1. Thermophysical properties of pure water and Nanoparticles (15)

|            | water  | Al₂O₃ |
|------------|--------|-------|
| ρ(kg/m³)   | 997.1  | 3970  |
| Cp(J/kg.K) | 4179   | 765   |
| k(W/m.K)   | 0.613  | 40    |
| β [1/ T]   | 2.1 x 10⁻³ | 8.5 x 10⁻⁶ |

In the present study, the dimensional governing equations are given by (16, 17):

**Continuity equation**

A- For nanofluid region

\[
\frac{\partial U_{nf}}{\partial X} + \frac{\partial V_{nf}}{\partial Y} = 0
\]

B- For porous/nanofluid region

\[
\frac{\partial U_p}{\partial X} + \frac{\partial V_p}{\partial Y} = 0
\]

**Momentum equation**

A-For nanofluid region

\[
U_{nf} \frac{\partial U_{nf}}{\partial X} + V_{nf} \frac{\partial U_{nf}}{\partial Y} = - \frac{\partial P}{\partial X} + \frac{pr}{\rho_{nf}} \frac{\rho_{nf}}{\rho_{uf}} \left( \frac{\partial^2 U_{nf}}{\partial X^2} + \frac{\partial^2 U_{nf}}{\partial Y^2} \right)
\]

\[
U_{nf} \frac{\partial V_{nf}}{\partial X} + V_{nf} \frac{\partial V_{nf}}{\partial Y} = - \frac{\partial P}{\partial Y} + \frac{pr}{\rho_{nf}} \frac{\rho_{nf}}{\rho_{uf}} \left( \frac{\partial^2 V_{nf}}{\partial X^2} + \frac{\partial^2 V_{nf}}{\partial Y^2} \right) + (\rho\beta)_{nf}(\Phi)_{nf} Ra Pr \theta
\]
B-For porous/nanofluid region

\[
U_p \frac{\partial U_p}{\partial X} + V_{nf} \frac{\partial U_{nf}}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{P_r}{(1-\Phi)^5} \frac{\partial^2 U_p}{\partial X^2} + \frac{\partial^2 U_{nf}}{\partial Y^2} - \frac{P_r}{(1-\Phi)^5} \frac{\rho_{nf} U_p}{Da} \\
U_p \frac{\partial V_p}{\partial X} + V_{nf} \frac{\partial V_{nf}}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{P_r}{(1-\Phi)^5} \frac{\partial^2 V_p}{\partial X^2} + \frac{\partial^2 V_{nf}}{\partial Y^2} - \frac{P_r}{(1-\Phi)^5} \frac{\rho_{nf} V_p}{Da} - \frac{Pr}{\rho_p \beta_f} \frac{(\rho\beta)_{nf} R_{nf} Pr \theta}{(1-\Phi)^5} (8)
\]

Energy equation

A-For nanofluid region

\[
U_{nf} \frac{\partial \theta_{nf}}{\partial X} + V_{nf} \frac{\partial \theta_{nf}}{\partial Y} = \alpha_f \left( \frac{\partial^2 \theta_{nf}}{\partial X^2} + \frac{\partial^2 \theta_{nf}}{\partial Y^2} \right) \\
B-For porous/nanofluid region

\[
U_p \frac{\partial \theta_p}{\partial X} + V_{nf} \frac{\partial \theta_{nf}}{\partial Y} = \alpha_{nf} \left( \frac{\partial^2 \theta_p}{\partial X^2} + \frac{\partial^2 \theta_{nf}}{\partial Y^2} \right)
\]

The properties of nanofluid and other physical including, the density \( \rho_{nf} \), thermal expansion coefficient \( \beta_{nf} \), thermal diffusivity \( \alpha_{nf} \), and heat capacitance \( C_{p nf} \) are calculated form the considered papers (16, 17). Besides, the dynamic viscosity is fined by using the Brinkman model and the thermal conductivity is fined by using Maxwell correlation.

The local Nusselt numbers \( \text{Nu}_L \) along the vertical hot-wall located at the left side of the enclosure are used as shown below (16, 17):

\[
\text{Nu}_L = \left( \frac{K_{nf}}{K_f} \right) \left( \frac{\partial \theta}{\partial X} \right)_{X=0}
\]

3. Enclosure boundary conditions

The boundary conditions of the enclosure, shown in Figure 1, are defined as follows:

all velocities on walls are assumed to be equal zero.

at the left and right sidewalls of the cavity: \( \theta_{nf,p} = 1, \theta_{nf,p} = 0 \) respectively.

at the upper and the lower sidewalls of the cavity: \( \partial \theta_{nf} / \partial Y = 0, \partial \theta_{p} / \partial Y = 0 \) respectively

all the properties on the interface line are isothermally equaled.

at the rotating cylinder solid surface

\[
U_{nf,p} = U_0 \frac{(Y-Y_o)}{R} = \Omega(Y-Y_o) \\
V_{nf,p} = U_0 \frac{(Xo-X)}{R} = \Omega(Xo-X)
\]

4. Solution procedure and Validation

COMSOL Multiphysics (5.4a) CFD Commercial software has been used in the present work because of its effective program for solving the scientific and engineering problems based on partial differential equations. The governing dimensionless equations are solved via the finite element method for determining the stream function, isotherms, and local Nusselt numbers inside the enclosure. The iteration number required to solve the current model is implemented when the error of the convergence criterion for each variable is less than \( 10^{-6} \).

The results are compared with published approved research papers in different cases to ensure that the program was operating properly and can be used for developing and investigating the current study.

Figure 2 illustrates the validation of the isotherms and streamlines of the present model with Abdulkadhim et al. (18) who investigated the convection of natural heat within a cavity having a square shape with an inner circular cylinder filled with Cu-water nanofluid at \( Ra=10^6 \).
5. Results
Numerical simulations are performed for the two-dimensional mixed convection in a cavity filled with two layers; the lower half filled with porous medium and the upper half filled with nanofluid with an adiabatic rotating cylinder. The non-dimensional parameters that used in this study are: Rayleigh number (Ra) ranged from $10^3$ up to $10^6$, Darcy number (Da) equal to $10^{-3}$, the angular rotational velocity ($\Omega$) ranged (0 and 6000), and the inner circular cylinder radius as $(R = 0.2)$. The nanofluid chosen in this study is Al$_2$O$_3$-water and the solid volume fract ($\phi=0.06$).

Describes the effect of Rayleigh number on stream function and isotherm line in Figure 3 for $\Omega=0$, $\phi=0.06$, $Da=10^{-3}$, and $R=0.2$. From the figures, it can be noticed that the Ra increase from $10^3$ to $10^6$ with natural convection due to the different temperatures between the left hot vertical wall and the cold right wall, the flow field inside the enclosure by the buoyancy force. Therefore, the flow rotation begins from the left hot vertical wall to rise to the upper horizontal wall that is thermally insulated to complete its cycle down due to the impacted by the cold right vertical wall then it completes its turn to the insulated lower horizontal wall.

From the left column of the figure at $Ra=10^3$ it can be observed that there are two small vortices in the upper of the cylinder, due to used small Rayleigh number and the fluid affected by the insulated circular cylinder, and the maximum value of the stream function very small was (0.1). When the Rayleigh number increases to $10^5$ one vortex remain on the right side of the cylinder due to the fluid movement affected by the cold wall and because increase the Rayleigh number noticed that the high value of the stream function was (6). When Rayleigh numbers increase to $10^6$, a noticeable increase in the intensity of the flow up to (17) due to the increase in temperature, which leads to increase the buoyancy force.

The right columns show that the isotherms line also affected by the different values of the Rayleigh numbers. At $Ra=10^3$ the isotherm lines are in general have a vertical line because of weak convection and in the porous medium region, the conduction is dominant while in $Ra=10^5$ the steep temperature gradient in the upper half increase. The isotherms lines change to horizontal lines and a more steep temperature gradient in the upper half at $Ra=10^6$ due to convection heat transfer.

When the cylinder rotates $\Omega=6000$, the heat transfer by mixed convection as illustrates in Figure 4 because it becomes there are two effects on the production of flow vortices inside the cavity, the first as mentioned is the buoyancy force and the second is because of the sheer force of the rotational
circular cylinder. So the buoyancy force and cylinder rotation force work together on the fluid rotation with a clockwise direction.

From the left column in the figure, notice that the noticeable increase in the stream function value due to the cylinder rotation. Also, an observation that the highest value of the stream was obtained at Ra=10^6 where it reaches (55) around the cylinder wall.

The right columns show that the isotherms lines that are also affected by the variation of the angular rotation velocity. The distribution of isotherm lines is higher at the top half of the enclosure when the circular cylinder rotates in clockwise. This shows a clear effect of the cylinder rotation on the heat transfer.

Figure 5 shows the effect of shear force from Rayleigh number on the local Nusselt number at Ω =0, φ = 0.06, Da =10^{-3}, and R=0.2. In general, we noticed that the value of the local Nusselt number was higher at the upper half of the cylinder. The reason is that because of the influence of buoyancy force at the nanofluid layer. The figure shows the clear effect of the Rayleigh number, where the highest value of local Nusselt number (1.2) at Ra=10^3, while it rises to (18.5) at Ra=10^6.

While Figure 6 Describes the influence of shear force from Rayleigh number on the local Nusselt number when the cylinder rotates Ω=6000. Where the apparent change in the value of the local Nusselt number along the hot wall is observed at the low values of the Rayleigh number. Concludes that the effect of the cylinder rotates was greater on the value of local Nusselt number when using the low Rayleigh Numbers.

**Figure 3.** Effect of Rayleigh number on stream function (left columns) and patterns of Isotherm (right columns) for Ω = 0
Ra  Stream function ($\Psi$)  Isotherms lines($\theta$)

$10^3$

$10^5$

$10^6$

Figure 4. Effect of Rayleigh number on stream function (left columns) and patterns of Isotherm (right columns) for $\Omega = 6000$

Figure 5. Local Nusselt number for different values of Rayleigh number at $\Omega = 0$

Figure 6. Local Nusselt number for different values of Rayleigh number at $\Omega = 6000$

6. Conclusions

1. When Rayleigh numbers increase, a noticeable increase in the intensity of the flow and the steep temperature gradient due to the increase in temperature, which leads to increase the buoyancy force. While the values increase when the cylinder rotates because it effects together with buoyancy force on the production of flow vortices inside the cavity.

2. The value of the local Nusselt number was higher at the top half of the cavity because the influence of the buoyancy force was greater at the nanofluid layer.
3. The influence of the cylinder rotates is greater on the value of the local Nusselt number when using the low Rayleigh Numbers.

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