Private and Accurate Data Aggregation against Dishonest Nodes

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Abstract—Privacy-preserving data aggregation in ad hoc networks is a challenging problem, considering the distributed communication and control requirement, dynamic network topology, unreliable communication links, etc. The difficulty is exaggerated when there exist dishonest nodes, and how to ensure privacy, accuracy, and robustness against dishonest nodes remains an open issue. Different from the widely used cryptographic approaches, in this paper, we address this challenging problem by exploiting the distributed consensus technique. We first propose a secure consensus-based data aggregation (SCDA) algorithm that guarantees an accurate sum aggregation while preserving the privacy of sensitive data. Then, to mitigate the pollution from dishonest nodes, we propose an Enhanced SCDA (E-SCDA) algorithm that allows neighbors to detect dishonest nodes, and derive the error bound when there are undetectable dishonest nodes. We prove the convergence of both SCDA and E-SCDA. We also prove that the proposed algorithms are privacy-preserving and accurate DA can be achieved in the mean-square sense. However, it is more desirable and challenging to guarantee the privacy and accuracy in a deterministic manner. Furthermore, it is assumed in PPAC that there are no selfish/dishonest nodes in the networks, so if they do exist, the selfish and dishonest nodes present a real threat to many existing privacy-preserving DA solutions.

Although privacy-preserving DA has been heavily investigated, existing solutions are typically based on various cryptography techniques, requiring either secure communication channels, pre-established shared secret/keys, a trusted authority, or the combination of them. In addition to eavesdropping, some nodes may be selfish or even dishonest, so they may manipulate their data to better protect their own privacy and interest while the manipulations may pollute the aggregation results. The selfish and dishonest nodes present a real threat to many existing privacy-preserving DA solutions.

Consensus is an important distributed computing method, which has gained much attention in automatic control and signal processing areas [28–34], and has been widely used in various networking areas, e.g., time synchronization in sensor networks [35–37]. Note that an average consensus algorithm can help each node to obtain the average value of all nodes’ states in a distributed way, which is a building block of the distributed aggregation algorithm designed in this paper. Recently, Mo and Murray in [38] addressed the privacy-preserving average consensus problem, and they designed a novel Privacy Preservation Average Consensus (PPAC) algorithm to solve the problem. Using PPAC, the privacy-preserving and accurate DA can be achieved in the mean-square sense. However, it is more desirable and challenging to guarantee the privacy and accuracy in a deterministic manner. Furthermore, it is assumed in PPAC that there are no selfish/dishonest nodes in the networks, so if they do exist, the private and accurate DA problem becomes even more difficult.

To meet the above challenges of DA in ad hoc networks, in this work, we investigate the possibility of not relying on cryptography tools. To enable fully distributed additive data aggregation, we first analyze the conditions on the added noise in the consensus algorithms, which can guarantee that an average consensus can be achieved deterministically. Then, based on the given conditions, we design a secure consensus-based data aggregation (SCDA) algorithm that can achieve \((\epsilon, \sigma)\)-data-privacy and high accuracy in obtaining the sum and the
average. Given the accuracy of the aggregation, our solution can be applied to other types of aggregation such as product, variance and other high-order statistics. For systems with selfish or dishonest nodes, we further design an Enhanced-SCDA (E-SCDA) algorithm, which allows the agents to monitor each other while using a dimension expansion approach to preserve privacy. The main contributions and approaches of this paper are summarized in the following.

- First, we exploited an average consensus algorithm to solve the privacy-preserving data aggregation (DA) problem in ad hoc networks. We derived a sufficient condition and a necessary condition of the noises added to the consensus process, under which an accurate aggregation is achieved. Based on the sufficient condition, a distributed SCDA algorithm is designed without using any trusted authority, so that the aggregator can obtain the aggregated results from any participating nodes.

- Second, the privacy-preserving DA problem was considered in the systems with selfish/dishonest nodes. We proposed an E-SCDA algorithm so that neighbor nodes can fast detect the nodes with suspicious behaviors and achieve an accurate DA. We derived the error bound for the aggregation value if there exist intelligent, dishonest nodes which are undetectable.

- Finally, we proved the convergence of both SCDA and E-SCDA algorithms. To quantify the degree of the privacy protection, we introduced a novel privacy definition, named \((\epsilon, \sigma)\)-data-privacy, which means that the probability that each node can infer its neighbor nodes’ initial states in an \(\epsilon\) interval is no larger than \(\sigma\). We also proved that both of the proposed algorithms provide \((\epsilon, \sigma)\)-data-privacy, and the relationship between \(\epsilon\) and \(\sigma\) has been derived.

The remainder of the paper is organized as follows. The related works are summarized in Section II. System model and problem formulation are presented in Section III. Two DA solutions, SCDA and E-SCDA, for systems without or with selfish/dishonest nodes, are proposed and analyzed in Sections IV and V, respectively. Simulation evaluation is presented in Section VI, followed by concluding remarks and further research issues in Section VII.

II. RELATED WORK

Great efforts have been devoted to investigating privacy-preserving and accurate DA for wireless sensor networks [2]–[6], [13]–[15], smart grid [7]–[11], [16], [17], and cloud computing [18]–[20].

Privacy-preserving DA has been addressed using different cryptographic techniques. For example, Secure Multiparty Computation (SMC) was used to collaboratively compute the aggregation with privacy preservation in [9]. Considering a dishonest-but-non-intrusive (DN) adversary, a modulo addition-based encryption scheme was adopted in [16] to design differential privacy-preserving aggregation for smart metering systems. In [2], two schemes were proposed using the Shamir Secret Sharing (SSS) and Secret Splitting technique for privacy-preserving additive aggregations. Meanwhile, cryptographic schemes can also be combined with differential privacy techniques for sensitive data aggregations. [24] designed a distributed random noise generation protocol aiming at a distributed implementation of privacy-preserving statistical databases. [25] proposed a novel solution where a trusted aggregator can obtain desired statistics over participants’ data, without compromising each individual’s privacy. These protocols rely on a verifiable secret sharing scheme so secure channels and a fixed topology are required for the key allocation. Moreover, cryptographic techniques often have high computational complexity. To address the above issues, [1] proposed a novel data aggregation scheme with low complexity, and it does not rely on key distribution and trusted aggregator. Using this solution, all participants are arranged in a ring topology, which may be vulnerable to network dynamics such as node failures, link outages, and software malfunctions.

Recently, how to preserve privacy in dynamical systems has been investigated using advanced signal processing and modern control solutions, e.g., using private filters [23], or private consensus [21], [22], [38]. The idea is to add noise to the data to protect the privacy. For example, [23] designed the private filters for dynamic systems by adding white Gaussian perturbations. An independent and exponentially decaying Laplacian noise process is added to the consensus computation such that consensus can be achieved with privacy preservation [21]; however, this algorithm does not guarantee the exact average consensus convergence. More recently, Nozari et al. [22] proposed a novel linear Laplacian-based consensus algorithm, which can achieve average consensus in expectation and guarantee differential privacy. PPAC algorithm, proposed in [38], has been proven to converge in the mean-square sense. Inspired by the design of PPAC, we design privacy-preserving average consensus algorithms for ad hoc networks. Different from PPAC, our solutions can ensure exact average consensus, i.e., ensuring the deterministic convergence which is a stronger convergence than mean-square convergence. More importantly, we consider complex but realistic scenarios where there exist dishonest nodes in the networks.

III. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider an ad hoc network where nodes are self-organized into clusters (using an existing clustering algorithm [26], [40]). We focus on a connected cluster with \(n\) nodes. The data from the nodes in the cluster are aggregated, while each individual’s data should not be revealed to any other node (including the aggregator) or eavesdropper. The aggregator can poll any node in the cluster to acquire the aggregated data.

Two nodes can select each other as neighbors to exchange data with a logical link (a single-hop or multi-hop communication path) between them. Thus, an overlay network can be constructed. The overlay network is modeled as an undirected graph, \(G = (V, E)\), where \(V\) is the set of nodes and \(E\) is the set of logical links (edges) between nodes. Let \(N_i\) be the neighbor set of node \(i\), where \(j \in N_i\) iff \((j, i) \in E\). Note that the logic links are negotiated in a distributed way, thus node \(i\) knows its neighbor set \(N_i\), but does not know the full topology of the overlay network.
Let \( \mathbb{N}^+ \) be the set of positive integers. Define the infinite norm as \( \| \mathbf{x} \|_{\infty} = \max \{ |x_i| \} \), which is the maximum absolute value of all the elements of vector \( \mathbf{x} \).

### B. Problem Formulation

Denote the privacy-sensitive data of each node as \( x_i(0) \), which is also called the initial state of node \( i \). In this paper, we consider how to obtain the additive aggregation, i.e., \( \sum_{i=1}^n x_i(0) \). The main design objectives are listed below. First, the aggregation should be obtained in a distributed manner, without the knowledge of the whole network topology. Second, the initial state of each node should not be known to others (including its neighbors, the aggregator, and eavesdroppers) to preserve privacy, while the aggregation should be accurate. Third, if malfunctioning, selfish, dishonest nodes exist in the system, a distributed safeguard mechanism is needed to fast detect the suspicious behaviours and bound the error in the aggregation due to any undetectable dishonest behaviours. Lastly, computation and communication cost should be minimized.

To achieve the above objectives, we choose to devise the solution based on average consensus which is a well-known distributed algorithm. Given the number of nodes \( n \), the sum is easily obtained by multiplying the average of the initial states by \( n \).

In a nutshell, distributed average consensus computes the average of the initial data by local information exchanges among neighbors (in the overlay network). The state of each node is updated iteratively by taking a weighted sum of its current state and those of its neighbors. If the weights are carefully chosen, the states of all nodes will converge to their average after a number of iterations. To preserve privacy, each state being sent to the neighbors will be added with a noise. Denote by \( x_i(k) \) the state of node \( i \) at iteration \( k \). The information being sent out at \( k \)-th iteration is designed as

\[
\bar{x}_i^+ (k) = x_i(k) + \theta_i(k), \quad i \in V,
\]

where \( \theta_i \) is the noise for privacy preservation.

In each iteration, the state is updated as follows.

\[
x_i(k+1) = w_{ii} x_i^+(k) + \sum_{j \in N_i} w_{ij} x_j^+(k) = w_{ii} (x_i(k) + \theta_i(k)) + \sum_{j \in N_i} w_{ij} (x_j(k) + \theta_j(k))
\]

for \( i \in V \), where \( w_{ij} \)'s are the weights.

To ensure that average consensus is achieved by the consensus algorithm and that the weights can be obtained in a distributed manner, we use Metropolis weights [27], given by

\[
w_{ij} = \begin{cases} 
\frac{1}{1 + \max \{ d_i, d_j \}} & , \quad j \in N_i, \\
1 - \sum_{l \in N_i} w_{il} & , \quad i = j, \\
0, & \text{otherwise},
\end{cases}
\]

where \( d_i \) and \( d_j \) are the number of neighbors of node \( i \) and \( j \). For a connected graph, a matrix with Metropolis weights is doubly stochastic.

Putting in the matrix form, we have

\[
x(k + 1) = W(x(k) + \theta(k)),
\]

where \( x, \theta \in R^{n \times 1}, W \in R^{n \times n} \) satisfying \( x = [x_1, x_2, \ldots, x_n]^T \) and \( \theta = [\theta_1, \theta_2, \ldots, \theta_n]^T \), and \( W \) is the matrix with Metropolis weights as its elements.

Define the average state as

\[
\bar{x} = \frac{1}{n} \sum_{i \in V} x_i(0).
\]

The problem is to design the noise process \( \theta(k) \) such that

\[
\lim_{k \to \infty} x_i(k) = \bar{x}, \quad i \in V.
\]

Using the Metropolis weights, \( W \) is doubly stochastic and the average consensus can be easily guaranteed when \( \theta(k) = 0 \) for all \( k \) [27]; however, non-zero noise is necessary to preserve privacy. If the aggregation can tolerate some discrepancy, we have more freedom to design the noise process \( \theta(k) \). For example, we can choose \( \theta(k) \) to be mutually independent with an exponentially decaying co-variance matrix [21]. However, to achieve the exact average consensus, the added \( \theta(k) \) has to ensure that the consensus result will not be affected and the privacy can be guaranteed, which implies that \( \theta(k) \) must be carefully designed and correlated. Mo and Murray in [38] proposed a novel algorithm, named PPAC, to guarantee the privacy and the exact average consensus, by adding and subtracting Gaussian and zero-sum noises to the consensus process. It is proved that PPAC has a mean-square convergence rate, i.e., an exact average consensus can be guaranteed by PPAC in the mean-square sense. However, the following challenging and practical problems remain open, i) what are the general conditions on the added noise that can guarantee the privacy and the exact average consensus; ii) how to guarantee the accurate and private average consensus when there are dishonest nodes in the network. In the following, we will conduct the theoretical analysis and design the algorithms to solve these problems.

### IV. Private and Accurate Data Aggregation

In this section, we first analyze the sufficient conditions and the necessary conditions on the added noise process such that a deterministic average consensus can be achieved. Then, based on the obtained conditions, we propose the SCDA algorithm and analyze its performance in terms of convergence, aggregation accuracy, privacy, and implementation complexity.

#### A. Algorithm Design

We first present a theorem, which provides a sufficient condition of deterministic average consensus and a theoretical support for our algorithm design.

**Theorem 4.1:** Considering the linear dynamic system [41], if the add noise vectors are bounded, i.e., \( \| \theta(k) \|_{\infty} \leq \alpha \rho^k \) for
some $\alpha > 0$ and $\rho \in (0, 1)$, and the sum of all added noises satisfies $\sum_{k=0}^{\infty} \sum_{i=1}^{n} \theta_i(k) = 0$, then we have
\[
\lim_{k \to \infty} x_i(k) = \bar{x}, i \in V.
\]

Meanwhile, $\sum_{k=0}^{\infty} \sum_{i=1}^{n} \theta_i(k) = 0$ is a necessary condition.

The proof of Theorem 4.1 is given in Appendix B. Based on this theorem, if the noise process $\theta(k)$ satisfies the two conditions that $\|\theta(k)\|_\infty \leq \alpha \rho^k$, i.e., exponentially decaying, and $\sum_{k=0}^{\infty} \sum_{i=1}^{n} \theta_i(k) = 0$ i.e., zero-sum, the goals of accurate and fast aggregation can be achieved. The exponentially decaying condition can ensure the convergence of the algorithm. The zero-sum condition ensures that the achieved consensus is an exact average consensus, which guarantees a fully accurate aggregation. Hence, Theorem 4.1 provides general conditions on the added noise which guarantees that an average consensus can be achieved deterministically. In [38], the noise process used in PPAC also satisfies the zero-sum condition and the variance of the noise is exponentially decaying, which explains why PPAC converges in the mean-square sense. Furthermore, from the proof of Theorem 4.1 we have the following corollaries.

**Corollary 4.2:** Consider the linear dynamic system (4). If there are $h$ sub-sequences $\theta(\ell + kh)$ of noise process $\theta(j)$ and each sub-sequence satisfies $\|\theta(\ell + kh)\|_\infty \leq \alpha \rho^k$ for some $\alpha > 0$ and $\rho \in (0, 1)$, and the noise process $\theta(\ell)$ satisfies the zero-sum condition, i.e., $\sum_{i=0}^{\infty} \sum_{i=1}^{n} \theta_i(\ell) = 0$, then $\lim_{k \to \infty} x_i(k) = \bar{x}$ for $i \in V$, where $\ell = 0, 1, ..., h - 1$.

Based on Corollary 4.2, each node can randomly divide the noise adding process into several sub-sequences, such that the correlation between any pair of adjacent adding noises is not clear to the other nodes.

**Algorithm 1:** SCDA Algorithm

1. Select each element in $\theta_i(0)$ randomly from $[-\frac{1}{2} \rho, \frac{1}{2} \rho]$.
2. Let $x_i^0(0) = x_i(0) + \theta_i(0)$ and transmit $x_i^0(0)$ to its neighbor nodes.
3. Set $\delta_i(0) = \theta_i(0)$.
4. Set $k = 1$.
5. While $k < \text{Max\_Iteration\_Number}$ do
6. Update $x_i(k)$ with (5) based on $x_i^0(k - 1)$ and $x_i^0(k - 1)$ received from all neighbor nodes ($\forall j \in N_i$).
7. Select each element of $\delta_i(k)$ randomly or autonomously from $[-\frac{1}{2} \rho^{k+1}, \frac{1}{2} \rho^{k+1}]$, i.e.,
   \[
   |\delta_i(k)| \leq \frac{\alpha}{2 \rho^{k+1}}, k \geq 1. \tag{6}
   \]
8. Set $\theta_i(k)$ according to
   \[
   \theta_i(k) = \delta_i(k) - \delta_i(k - 1). \tag{7}
   \]
9. Set $x_i^+(k)$ using (1), and then transmit $x_i^+(k)$ to its neighbor nodes.
10. $k = k + 1$.
11. End while.

We further design the SCDA algorithm for node $i$ in Algorithm 1. The Max\_Iteration\_Number in step 5 is given initially. According to our simulation, we can simply let Max\_Iteration

It should be pointed out that here we do not need $\sum_{k=0}^{\infty} \theta_i(k) = 0$ holds for each $i, i \in V$, which is different from the existing designs.

Number equal $n^2$, which is sufficiently large to guarantee an accurate aggregation. We can also let each node terminate the iteration when it finds all its neighbors’ states are sufficiently close to its own state, e.g., $|x_i(k) - x_j(k)| \leq \varepsilon$ for $\forall j \in N_i$ and a given small $\varepsilon$. SCDA is a fully distributed algorithm. Only the neighbor set $N_i$ is the input of each node $i$, and after sufficient iterations ($k \geq n^2$), all nodes’ updated states could be the output of SCDA. Based on the output, the aggregator can easily achieve the goal of DA.

**B. Convergence and Accuracy of SCDA**

For the convergence and accuracy of SCDA, we have the following theorem with the proof given in Appendix B.

**Theorem 4.3:** Using the SCDA algorithm, we have \(\lim_{k \to \infty} x_i(k) = \bar{x}\) for $\forall i \in V$, i.e., an average is achieved.

For each cluster, every node will achieve an average consensus using the SCDA algorithm, i.e., the aggregator can obtain the average state $\bar{x}$ from any node after the algorithm converges. Then, the sum can be obtained from using $n\bar{x}$, resulting in an accurate sum aggregation.

**Remark 4.4:** It should be pointed out that the proof of Theorem 4.3 only used the properties of a doubly stochastic matrix and the results given in Theorem 4.1 SCDA can also be adopted to solve the privacy of the asynchronous gossip consensus algorithms which also have the doubly stochastic matrixes in the algorithm dynamic functions, e.g., [31]–[33]. However, considering the privacy of more complicated consensus algorithms, e.g., second-order consensus, e.g., [34], it is a more challenging and open problem.

**Remark 4.5:** With SCDA, a higher accuracy of DA requires more iterations and an exact DA needs a sufficiently large number of iterations. It should be noticed that the larger communication delays will decelerate the convergence speed of SCDA. Hence, when the delays are not negligible, there is a tradeoff between convergence speed and DA accuracy, and we will discuss how to accelerate the convergence speed of SCDA at the end of this section.

**C. Privacy of SCDA**

For SCDA, node $i$ only transmits the information sequence $x_i^+(k), k = 0, 1, ..., \text{to its neighbors. For each message } x_i^+(k), there is a noise component $\theta_i(k)$ added to $x_i(k)$. Hence, any neighbor node cannot know the exact value of $x_i(0)$ based on the received information sequence from node $i$. Meanwhile, note that when $k \geq 1$, $x_i^+(k)$ is an updated state which may be quite different from the initial state $x_i(0)$, since each update is an average process among all the information received from its neighbor nodes’ states. Define for $\forall j \in N_i$, the information set which is available for node $i$ at iteration $k$ as follows,
\[
I_{ij}(k) = \{x_i(0), x_i^+(0), x_j(0), ..., x_i(k), x_i^+(k), x_j^+(k)\},
\]
Suppose that node $i$ cannot listen to all the neighbors’ information of node $j$. This assumption can be guaranteed in the overlay network construction with $N_j \not\subseteq N_i$, and it has been proved to be necessary in [39]. The added noises are assumed to be unknown to each node $i$, and the initial states of nodes are independent with each other.
Suppose that if there is no information of a variable, then the variable could be any value in $R$ for inference/estimation. When node $i$ estimates node $j$’s initial state directly (denote the estimation by $\hat{x}_j(0)$), it is reasonable to assume that
\[
\Pr\{x_j^0(0) \in [x_j(0) - \epsilon, x_j(0) + \epsilon]\} \ll \sigma,
\] (8)
where $\epsilon$ and $\sigma$ are two given small positive constants satisfying $\sigma = \max_{\nu \in [-\frac{\epsilon \rho}{\rho}, \frac{\epsilon \rho}{\rho}] \int_{V_\nu} f_{\theta_j(0)}(y)dy$. Under SCDA, the broadcast information of node $j$, i.e., $x_j^0(0), x_j^1(1), ..., x_j^l(k) \in I_{\nu_j}(k)$, is available to node $i$ to infer/estimate the initial value of neighbor node $j$. Note that the value of the broadcast information equals the value of real state plus a noise. Assume that the distributions of the added noises are available for the estimation. Hence, based on the broadcast information, node $i$ will take the probability over the space of all noises $\{\theta_j(k)\}_{k=0}^{\infty}$ (where the space is denoted by $\Theta$) to estimate the values of the added noises, and then infer the initial state of node $j$ using the difference between the broadcast information and the real initial state. That is, the estimation of $x_j(0)$ will satisfy $\hat{x}_j(0) = x_j^{out} - \theta_j$, where $x_j^{out}$ is the information output of node $j$ and $\hat{\theta}_j$ is the estimation of the added noise. Based on this estimation, we have
\[
|\hat{x}_j(0) - x_j(0)| = |\hat{\theta}_j - \theta_j|,
\]
and then
\[
\Pr\{|\hat{x}_j(0) - x_j(0)| \leq \epsilon\} = \Pr\{|\hat{\theta}_j - \theta_j| \leq \epsilon\}. \tag{9}
\]

To evaluate the privacy of SCDA, we give the definition of $(\epsilon, \sigma)$-data-privacy as follows.

**Definition 4.6:** A distributed algorithm provides $(\epsilon, \sigma)$-data-privacy, if the probability that each node can successfully estimate its neighbor node $j$’s initial value $x_j(0)$ in a given interval $[x_j(0) - \epsilon, x_j(0) + \epsilon]$ is no larger than $\sigma$, i.e.,
\[
\sigma = \max_{\nu \in [\epsilon]} \Pr\{|\hat{\theta}_j - \theta_j| \leq \epsilon\}. \tag{10}
\]
In the above definition, the parameter $\epsilon$ indicates the estimation accuracy and $\sigma$ expresses the privacy cost. Given the estimation accuracy $\epsilon$, a smaller value of $\sigma$ offers a stronger privacy guarantee.

Based on the above definition, we prove that SCDA provides $(\epsilon, \sigma)$-data-privacy, and a theorem is stated as follows.

**Theorem 4.7:** SCDA algorithm is $(\epsilon, \sigma)$-data-private, and the relationship between $\epsilon$ and $\sigma$ satisfies
\[
\sigma = \max_{\nu \in [-\epsilon, \epsilon]} \int_{V_\nu} f_{\theta_j(0)}(y)dy, \tag{11}
\]
and $\lim_{\epsilon \rightarrow 0} \sigma = 0$, where $f_{\theta_j(0)}(y)$ is the probability density function (PDF) of $\theta_j(0)$.

**Remark 4.8:** It should be noticed that Theorem 4.7 is obtained under the assumption that node $i$ cannot listen to all the neighbors’ information of node $j$. If this assumption is relaxed and node $i$ has the knowledge of $N_i$, then at any time $k \geq 1$, node $i$ can exactly calculate the value of $\theta_j(k)$ through the following equation,
\[
\theta_j(k) = x_j(k) - w_{ij}x_j(k-1) + \sum_{l \in N_i} w_{ij}x_l^l(k-1),
\]
where all the expressions on the right-hand side are known to node $i$. Hence, over the time, node $i$ can calculate all of $\theta_j(k), ..., \theta_j(1)$. Then, using the zero-sum property of the noise, node $i$ can calculate $\theta_j(0)$ by
\[
\theta_j(0) = -\sum_{k=1}^{\infty} \theta_j(k).
\]
Therefore, node $i$ knows the value of $x_j(0)$ through $x_j^0(0) = x_j^1(0) - \theta_j(0)$, i.e., $x_j(0)$ is released. This result is consistent with Theorem 4 in [59], which proved that the disclosed space of a node with $m$ neighbors is of dimension $m + 1$.

**D. Complexity of SCDA**

Since each node just calculates a weighted average at each iteration, SCDA has very low computation complexity, in $O(n)$. According to our simulation results, when the overlay network is well connected (e.g., the diameter of the graph is much smaller than $n$), the consensus can be reached in $O(n)$ iterations. Note that the logical link in the overlay network may be a multi-hop path. Thus, the communication cost is proportional to the number of iterations multiplied by the average number of hops for the logical links. Note that the number of hops is confined to the diameter of the cluster, we can also let nodes select logical neighbors within a small number of hops (e.g., 1 to 3). Thus, the communication cost is in $O(kn^2)$, where $k$ is the number of iterations which is typically smaller than $n$ for large $n$. We can further divide the network into more clusters to accelerate the convergence rate, while as a trade-off the aggregator needs to poll more nodes.

Furthermore, when the fully connected overlay network exhibits the so-called small-world phenomenon (one or more edges can be reconnected between any two nodes with a fixed probability) [41], [42], a consensus algorithm will have an ultrafast convergence rate [43]. Meanwhile, the latest consensus algorithm proposed in [30] can guarantee that an average consensus is achieved in a few iterations, or nearly linear time. All these methods can be applied to guarantee an ultrafast average consensus, which further reduce the communication cost of SCDA.

**V. DATA AGGREGATION AGAINST DISHONEST NODES**

A very challenging issue for privacy-preserving data aggregation is how to deal with malfunctioning, selfish, or dishonest nodes whose data may pollute the aggregation. For example, a dishonest node selects a positive constant noise at each iteration such that the consensus cannot be achieved. Hence, we investigate this problem and design an E-SCDA algorithm to address it.

**A. Algorithm Design**

The challenge is that we need to preserve privacy while monitoring whether nodes are misbehaving. We use two key designs to address this difficult problem. First, using the idea of dimension expansion, the initial state of each node can be divided into two parts and they will be sent with added noises to two neighbor sets. This procedure introduces additional
noises to the initial state for privacy preservation. Second, we design guidelines for nodes to monitor their neighbors to identify any misconduct. To achieve it, we design a monitoring process as a safeguard mechanism, which constrains the dishonest nodes and ensures the accurate aggregation. The detailed procedure is described below.

B. Dimension Expansion

First, the initial state of each node \( i \) is divided into two parts, given as
\[
x_i(0) = \frac{1}{2} x_i(0) + \vartheta_i, \tag{12}
\]
and
\[
x_i(0) = \frac{1}{2} x_i(0) - \vartheta_i, \tag{13}
\]
respectively, where \( \vartheta_i \) is a random variable selected from \([-\frac{1}{2} \rho, \frac{1}{2} \rho]\). Clearly, we have \( x_i(0) = x_i^1(0) + x_i^2(0) \). Suppose that the whole network topology is available to the aggregator.\footnotemark[1] The aggregator divides the graph \( G = (V, E) \) into two undirected and connected subgraphs, denoted by \( G_1 = (V, E_1) \) and \( G_2 = (V, E_2) \), respectively, where \( E_1, E_2 \subseteq E \) (where \( E_1 \cup E_2 = \emptyset \) and \( E_1 \cup E_2 = E \) cannot be true). Define the neighbor node set \( N_i^v \) of node \( i \), where \( j \in N_i^v \) iff \( (j, i) \in E_v \) and \( v = 1, 2 \). The aggregator will let each node know the information of \( N_i^v \), and then nodes will calculate the corresponding weights \( w_{j,k} \), \( j \in N_i^v \), using (3). Then, each node will transmit \( x_i^1(k) \) and \( x_i^2(k) \) to its neighbor nodes who are in \( N_i^1 \) and \( N_i^2 \), respectively, for iteratively average updating, where
\[
x_i^1(k) = x_i^1(0) + \theta_i^1(k), \nu = 1, 2, \tag{14}
\]
for \( i \in V \). Let \( \hat{x}_i(0) \) be the estimation of \( x_i(0) \) by the aggregator for \( i \in V \). It is assumed that \( |\hat{x}_i(0) - x_i(0)| \leq E_x \), where \( E_x \) is the estimation or prediction error bound of the initial state. Define two information sets, \( I_i^v(k) \), of every node \( i \) for \( \nu = 1, 2 \) as
\[
I_i^v(k) = \{\nu, d_{i,j}, d_{i,j}', \hat{x}_i(0), E_x, x_i^v(k), x_i^v(k) : j \in N_i^v\},
\]
for \( k = 0 \) or \( k = N_i^+ \), where \( d_{i,j}^1 \) (or \( d_{i,j}^2 \)) is the number of neighbor nodes in \( N_i^v \) (\( N_i^0 \)), which is used in the information monitoring process designed in the following.

The monitoring process is to detect and constrain the dishonest nodes. Assume that the aggregator can randomly select some nodes, namely selected nodes, in each cluster to monitor their neighbor nodes, where the selected nodes are assumed to overhear the information transmitted to the nodes they monitor (which guarantees that the information \( x_i^1(k), x_i^2(k) : l \in N_i^0 \) can be listened by the selected node \( i \)). Then, the aggregator just needs to send the information of \( \nu, d_{i,j}^1, d_{i,j}^2 \), and \( N_i^0 \) (the topology information only) to the selected node only, and it can guarantee that the selected nodes have the knowledge of \( I_i^v(k) \) for \( \nu = 1 \) or \( \nu = 2 \). We thus assume that one of the information sets \( I_j^v(k) \) (it should be noticed that not both here) is available to one selected node \( i \) for \( \nu = 1 \) or \( \nu = 2 \). That is, node \( i \) can have the full knowledge of the information used for one part state update of node \( j \), and how node \( j \) updates this part at each iteration, i.e., the \( x_j^v(k) \) is available to node \( i \) for \( k \in N_i^+ \), where \( \nu = 1 \) or \( \nu = 2 \). The details of the monitoring checking process are given as follows.

C. Neighbor Monitoring

The aggregator can request a neighbor node to monitor a node at a random time instant. Once receiving such a request, a neighbor node \( i \) of node \( j \) checks the following three conditions based on the available information set, \( I_j^v(k) \), for \( \nu = 1 \) or \( \nu = 2 \) and \( k = 0 \) or \( k = N_i^+ \),
\[
c_1: |\theta_j^v(k)| \leq \frac{1}{2} \alpha \rho, \quad \text{where } \theta_j^v(k) \text{ is calculated by}
\]
\[
\theta_j^v(k) = \hat{x}_j^v(k) - |w_{j,j} x_j^v(k-1) + \sum_{i \in N_j^v} w_{ij} x_i^v(k-1)| \tag{15}
\]
and \( w_{ji} \) is calculated from (3) for \( k \in N_i^+ \).
\[
c_2: |\hat{x}_j^v(0) - \hat{x}_j^v(0)| \leq E_x + \frac{1}{2} \alpha \rho.
\]
\[
c_3: |\hat{x}_j^v(0) - \hat{x}_j^v(0)| \leq \frac{5}{4} \alpha \rho.
\]
If conditions \( c_1, c_2, \) and \( c_3 \) hold, then node \( j \) is credible. Otherwise, node \( j \) will be viewed as a dishonest node and reported to the aggregator, and then node \( j \) will be isolated so that its data will not pollute the aggregation.

In the above process, \( c_1 \) is used to guarantee that the update in each iteration is an average process and the added noise is exponentially decaying, \( c_2 \) ensures that the initial states of dishonest nodes are bounded by the estimation error, which constrains the initial state selection of each dishonest node, and \( c_3 \) is utilized to ensure that two parts dividing the initial states of nodes follow the rules of (12) and (13). Note that based on (12) and (13), one node has
\[
\begin{align*}
|x_i^v(0)| - |x_i^v(0)| & \leq \frac{1}{2} |x_i^v(0)| + \theta_j^v(k) \\
& \leq \frac{1}{2} |x_i^v(0)| + \theta_j^v(k) \\
& \leq \frac{5}{4} \alpha \rho,
\end{align*}
\]
i.e., they can satisfy \( c_3 \). The aggregator only knows the global topology information but does not know the states of nodes, which can preserve the state privacy to the aggregator. If the aggregator checks \( c_1 \) or \( c_3 \) itself, the communication costs will be higher as the aggregator needs to collect much more additional information.

Given the monitoring process, to be undetectable, a dishonest node cannot arbitrarily select the values of its noise process. For honest nodes, they can easily pass the monitoring process as they always follow the rules of SCDA which obviously satisfy \( c_1 \) to \( c_3 \). We then have the E-SCDA algorithm in Algorithm 2

For the above algorithm, the Max Iteration Number in step 7 is also given initially, and the setting of its value is the same as that in SCDA. For E-SCDA, the two neighbor sets of each node are the input, and the output is the nodes’s
Algorithm 2: E-SCDA Algorithm

1. Generate random vectors $\theta_i(0), \vartheta_i, \theta_i(1)$ and $\theta_i(2)$ where all the elements are randomly selected from $[-\frac{2}{3} \rho, \frac{2}{3} \rho]$.
2. Set $x_i^c(0)$ and $x_i^v(0)$ using (12) and (13), respectively.
3. Set $x_i^c(0)$ and $x_i^v(0)$ using (11) and (14), respectively, and transmit them to the corresponding neighbors, while $x_i^v(0)$ is transmitted to nodes in $N_i^1 \cup N_i^2$.
4. If node $i$ selected by the aggregator to monitor neighbor node $j$, it will obtain the information $\nu_i \vartheta_i^l, d_i^j, N_j^v$ from the aggregator for $\nu = 1$ or $\nu = 2$ and $l \in N_j^v$.
5. Set $\delta_i(0) = \theta_i(0)$.
6. Set $k=1$.
7. while $k < \text{Max\_Iteration\_Number}$ do
8. When node $i$ is a selected node, it uses the received $x_i^v(k-1)$ and the information set $I_j^v(k-1)$ to monitor whether node $j$’s behavior satisfies $c_1$–$c_3$.
9. Update $x_i^v(k)$ by using the following equation,
   
   $x_i^v(k) = w_i^v x_i^v(k-1) + \sum_{i \in N_j^v} w_i^v x_j^v(k-1).
   $  
10. Set $x_i(0) = x_i^c(k) + x_i^v(k)$.
11. Select $\delta_i(k)$ randomly according to
   
   $|\delta_i(k)| \leq \frac{\alpha}{2} \rho^{k+1},$  
for $k \geq 1$ and $\nu = 1,2$.
12. Set $\theta_i(k)$ by
   
   $\theta_i(k) = \delta_i(k) - \delta_i(k-1),$  
13. Set $x_i^c(k)$ using (14) and transmit $x_i^c(k)$ and $\nu$ to the corresponding neighbors.
14. Set $k=k+1$.
15. end while

updated states. Then, the aggregator also can achieve the aggregation goal with each node’s updated state. However, note that the aggregator needs the information of the whole network topology in E-SCDA, which means that the topology information should be included as the input of the aggregator.

D. Performance of E-SCDA

We next analyze the performance of E-SCDA algorithm in terms of convergence, aggregation accuracy and privacy.

1) Convergence and Accuracy: We first analyze the constraint on dishonest nodes using E-SCDA. Then, we reveal the maximum pollution from the dishonest nodes under the constraints.

Let $x_i(0)$ be the true initial state of a dishonest node $i$. Assume that the dishonest node $i$ uses $\tilde{x}_i(0)$ instead of $x_i(0)$ in the calculation of $x_i^c(0)$ and $x_i^v(0)$, i.e., $\tilde{x}_i(0)$ is the false initial state satisfying $|x_i(0) - \tilde{x}_i(0)| \leq \frac{1}{2} \alpha \rho$, and $x_i^v(0)$ is one part of the false initial state, satisfying $|x_i^v(0) - \tilde{x}_i^v(0)| \leq \frac{1}{2} \alpha \rho$ for $\nu = 1,2$. We have the following theorem.

Theorem 5.1: Given the monitoring process using $c_1$–$c_3$, for each dishonest node $i$ to be undetectable, it should have

$|x_i(0) - \tilde{x}_i(0)| \leq 2 E_x + \alpha \rho,$  
and

$|\tilde{x}_i^v(0) - x_i^v(0)| \leq 2 \alpha \rho, \nu = 1,2.$  

The proof of Theorem 5.1 is given in Appendix D. This theorem implies that the dishonest nodes cannot arbitrarily select false initial states since they are bounded by (13) and (19). Then, we obtain the following theorem, which proves the convergence of E-SCDA and provides the accuracy of the aggregation.

Theorem 5.2: Suppose that the number of the dishonest nodes in a cluster is $d$. With the E-SCDA algorithm,

$\lim_{k \to \infty} x_i(k) = C, i \in V,$  
and

$|C - \bar{x}| \leq \frac{5 \alpha \rho + 2 E_x + \frac{\alpha \rho}{(1 - \rho)}}{n},$  
where $C$ is a constant.

The proof of Theorem 5.2 is given in Appendix E. From this theorem, we have that E-SCDA achieves a consensus, and the error between the consensus and the average is bounded by (21). Clearly, if the number of the dishonest nodes is larger, the error may become larger. Specifically, if there is no dishonest node, i.e., $d = 0$, from (21) the error bound is 0. This implies that an average consensus is achieved by E-SCDA according to Theorem 4.4. When the number of dishonest nodes is fixed, the accuracy of the aggregation depends on the parameters, $\alpha$, $\rho$ and $E_x$. Setting a small $\alpha \rho$ can enhance the accuracy of the aggregation, while reducing the privacy of $x_i(0)$. Increasing the accuracy of $E_x$ can also enhance the aggregation accuracy.

2) Privacy: With E-SCDA, dishonest nodes cannot know who are monitoring them and when, since the selected nodes for monitoring are chosen by the aggregator randomly. Hence, to be undetectable, the noise process used for the dishonest nodes should satisfies $c_1$–$c_3$, and thus the error due to their pollution can be bounded. Then, for the node who is monitoring node $i$, since it has the full information used for partial state update (i.e., $x_i^c(0)$, where $\nu = 1$ or $\nu = 2$), it may infer the corresponding initial state (i.e., $x_i^c(0)$ or $x_i^v(0)$ only). However, since there is a random noise $\tilde{\nu}_i$ in each part of the initial state, the monitoring node still cannot infer the exact value of $x_i(0)$. Hence, by a similar privacy analysis of SCDA, we can state the following theorem.

Theorem 5.3: E-SCDA algorithm is $(\epsilon, \sigma)$-data-private, and the relationship between $\epsilon$ and $\sigma$ satisfies

$\sigma \leq \max_{l=0}^{3} \max_{\nu \in [-2 \rho, 2 \rho]} \int_{x-l}^{x+l} f_\nu(y) dy,$

and $\lim_{\nu \to 0} \sigma = 0$, where $f_\nu(y)$ is the PDF of $2(\theta_i^0(0) \pm \vartheta_i^l)$, $\theta_i^0(0)$ and $\theta_i^2(0)$, respectively, $f_0(y) = f_1^l(y)$ and $f_1^l(y) = f_3^l(y)$.

The proof of Theorem 5.3 is given in Appendix E. Therefore, E-SCDA can preserve privacy while enabling the capability of detecting dishonest nodes, and further bound the error due to undetectable dishonest behavior.

VI. PERFORMANCE EVALUATION

In this section, we conduct extensive simulations to evaluate the performance of the proposed algorithms SCDA and E-SCDA.
A. Simulation Setup

In the simulation, there are 100 nodes randomly deployed over a 1,000 × 1,000 m² square area, where the communication range of each node is 300 m. Unless otherwise stated, the whole area is divided into 4 equal-sized sub-areas and the nodes in each sub-area are clustered, i.e., there are 4 clusters. Figs. 1 (a) and (b) show the logical links between nodes in a cluster, without or with dishonest nodes, respectively, where a blue circle denotes an honest node and a red square denotes a dishonest one.

We set α = 5 and ρ = 0.4. Define the maximum difference between nodes’ states in each cluster by

\[ V(x(k)) = \max_{i,j \in V} |x_i(k) - x_j(k)|. \]

Clearly, a consensus is achieved if \( V(x) = 0 \).

B. Evaluation of SCDA

Figure 2(a) shows the dynamics of all nodes’ states under SCDA. It is observed that the states of all 25 nodes converge exponentially to a constant state, which exactly equals the average of their initial states. This demonstrates that an average consensus can be achieved by SCDA, i.e., the aggregated sum is accurate. Figure 2(b) shows the sum of θ(k) used by the nodes, where we randomly select three of them to illustrate. Clearly, the sum of each θ_i(k) converges to 0, which satisfies the conditions in Theorem 4.1.

Then, we change the values of α and ρ and the convergence of SCDA is shown in Figs. 2(c) and 2(d). When α = 0 or ρ = 0, it means that the added noise is always 0. It is observed that the convergence rate is slightly affected by adding an exponentially decaying noise as the maximum difference is less than 10^{-4} within 20 iterations. Therefore, SCDA can preserve the privacy of each node with guaranteed accuracy of aggregation.

Next, we study the convergence of SCDA under different clustering strategies by changing the number of clusters in the network. When there are several clusters in the network, we pick one cluster randomly to illustrate in Fig. 3. From the figure, with more clusters, SCDA has a faster convergence rate as anticipated. Furthermore, note that SCDA can have a fast convergence rate even if the node number of each cluster is large, e.g., with 100 nodes for the one cluster case in Fig. 3. SCDA can converge to an acceptable accuracy (e.g., 10^{-3}) within 30 iterations.

C. Evaluation of E-SCDA

Second, we evaluate the performance of the E-SCDA algorithm. There are 5% dishonest nodes, and the elements in θ_i(k) used for dishonest node i is randomly selected from [0, αρ^k], and \( E_x = 2 \). It is observed from Fig. 4(a) that all states exponentially converge to a constant state, i.e., a consensus is achieved, while it may not be equal to the average due to the pollution introduced by the dishonest nodes.

As shown in Fig. 4(b), the sum of θ(k) used for the dishonest nodes does not converge to 0, where node 1 is dishonest. This is the main reason why the consensus is not fully accurate. Nevertheless, the gap between the consensus achieved by E-SCDA and the average consensus is small, which is bounded by (21), e.g., the gap is lower than 0.1 in Fig. 4(a).

We also vary the values of α and ρ to study the convergence of E-SCDA. The results are shown in Figs. 4(c) and 4(d) respectively. It is observed that the convergence rate is affected slightly when α changes as the maximum difference is still less than 10^{-4} within 20 iterations. Hence, setting a larger α could be an efficient approach to enhance the privacy of nodes, supporting our statement in Section V-D2. With a larger ρ, e.g., ρ = 0.8 in Fig. 4(d) the convergence rate may be decreased,
as the dishonest nodes have a higher freedom to introduce undetectable pollution. Since the privacy and accuracy depend on $\alpha \rho$, we can set a large $\alpha$ and a relatively small $\rho$ to ensure a fast convergence rate while guaranteeing the privacy and accuracy.

![Graphs showing the performance of E-SCDA](image)

Fig. 4. The performance of E-SCDA.

D. Robustness

In this subsection, we investigate the robustness of the proposed algorithms.

First, we consider the robustness of the proposed algorithm. Note that the communication delay, packet losses and the dynamic change of the topology may occur. Herein, when the delay is larger than a threshold (e.g., the interval between two iterations), the packet will be dropped. At each iteration, if a packet is lost or dropped, it is equivalent to that a logical link is broken at that iteration. In this case, the node just updates its state according to the successfully received neighbor information and adjusts the weights accordingly. In the following simulation, we randomly remove a portion of the logical links at each iteration to investigate the robustness of the proposed solutions.

As shown in Fig. 5 under different drop ratios (the percentage of links being broken in each iteration), SCDA can still converge, although the convergence rate will decrease slightly when the drop ratio becomes larger. E-SCDA demonstrates a similar performance and thus is omitted here.

![Graphs showing the robustness of SCDA](image)

Fig. 5. The robustness of SCDA.

VII. Conclusions and Further Discussions

In this paper, we have investigated the privacy-preserving data aggregation problem in ad hoc networks using the average consensus approach. We have first proposed the SCDA algorithm to solve the problem for the scenario without dishonest nodes. SCDA is simple to implement and can ensure private and accurate aggregation. SCDA does not rely on a centralized controller or a trusted aggregator, and it can be implemented in a distributed manner and robust against the network dynamics. Furthermore, considering the more challenging scenario that dishonest nodes may pollute the aggregation, we designed the E-SCDA algorithm that adopts a neighbor monitoring process to detect misbehaving nodes, and derived the error bounds due to undetectable dishonest behaviors. Simulation results have shown that the proposed algorithms have fast convergence rate and high accuracy, and they are robust against network dynamics and dishonest nodes. To the best of our knowledge, this is the first privacy-preserving data aggregation solution to have such robustness and ensure bounded error with the presence of dishonest nodes.

There are still many open issues worth further investigation. First, in this paper, the overlay network should be a connected, undirected graph. To ensure connectivity, a spanning tree connecting all the nodes in the cluster can be built and the links in the spanning tree should be included in the overlay network. How to deal with permanent node failures needs further investigation. The undirected graph requires bi-directional communications. In case bi-directional logical link cannot be maintained, novel consensus solutions need to be used which are much more complicated. Second, different from the SCDA case, to deal with dishonest nodes, in E-SCDA, the aggregator should have the knowledge of the topology of the overlay network, and how to relax this requirement is an interesting while challenging problem. A possible direction is to design an incentive mechanism such that all nodes are willing to be honest so as to achieve an accurate privacy-preserving aggregation at a lower cost. Third, the E-SCDA approach can help to constrain the dishonest nodes, assuming that the selected nodes will not generate false reports and there is no intrusive node in the system. How to secure the system to deal with intrusive nodes remains an open issue.

Overall, using distributed consensus can be a promising alternative to the heavily investigated privacy-preserving approaches using cryptography techniques in ad hoc networks and other distributed systems. It is also possible to combine these two powerful tools to further enhance privacy and security, or make a good tradeoff between computation and communication complexity, which beckons further research.

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APPENDIX A

THE PROOF OF THEOREM 4.1

Proof: First, we prove that each $x_i(k)$ is bounded by some constant $M$ for $i \in V$. Since $W$ is doubly stochastic, we have $\|W\|_{\infty} = 1$. Hence,

$$\| x(k+1) \|_{\infty} = \| W(x(k) + \theta(k)) \|_{\infty} \leq \| W \|_{\infty} \| x(k) + \theta(k) \|_{\infty} \leq \| x(k) \|_{\infty} + \| \theta(k) \|_{\infty} \leq \| x(0) \|_{\infty} + \sum_{\ell=0}^{k} \| \theta(\ell) \|_{\infty}. \quad (22)$$

Using the condition that $\| \theta(\ell) \|_{\infty} \leq \alpha \rho^{\ell}$, it follows

$$\| x(k+1) \|_{\infty} \leq \| x(0) \|_{\infty} + \sum_{\ell=0}^{k} \alpha \rho^{\ell} \leq \| x(0) \|_{\infty} + \frac{\alpha}{1-\rho} = M, \quad (23)$$

which implies that each $x_i(k)$ is bounded by $M$ for all $k$.

Next we prove the convergence of $V(x(k))$ as follows:

$$V(x(k)) = \max(x(k)) - \min(x(k)). \quad (24)$$

This function is nonnegative and has the property that $V(x(k)) = 0$ if and only if all the elements of $x(k)$ have the same values, i.e., $x(k) = C1$, where $C$ is a constant and $1$ is a vector with all its elements equal to $1$. 

Note that $W^\ell$ is still a doubly stochastic matrix for $\ell \in \mathbb{N}^+$, and we have $\lim_{\ell \to \infty} W^\ell = \frac{1}{n} I$. Since the topology of each cluster is assumed to be connected, we have $W^n > 0$. Then, according to Lemma 2 in [35], it follows that for any vector $y$, we have

$$\max\{W^n y\} - \min\{W^n y\} \leq (1 - \epsilon)(\max\{y\} - \min\{y\}),$$

where $\epsilon = \max_{j=1}^n \min_{i=1}^n (W^n)_{ij}, \epsilon \in (0, 1)$. Hence, we have

$$V(x(k + n)) = \max\{V(x(k + n)) - \min(x(k + n))\} \leq \max(W^n x(k)) - \min(W^n x(k))$$

$$+ \sum_{\ell=0}^n [\max(W^\ell \theta(k + n - \ell)) - \min(W^\ell \theta(k + n - \ell))]$$

$$\leq (1 - \epsilon) V(x(k)) + 2 \sum_{\ell=0}^n \rho^k (\frac{1 - \rho^{n+1}}{1 - \rho}),$$

where we used the fact of (25). From (26), one infers that

$$V(x(hn)) \leq (1 - \epsilon) V(x((h + 1)n)) + \alpha \rho^{(n-1)n}$$

$$\leq (1 - \epsilon)^2 V(x((h + 2)n)) + \alpha \rho^{(n-1)n}$$

$$= (1 - \epsilon)^2 \rho^n V(x(k)) + \alpha \rho^{(n-1)n} = (1 - \epsilon)^2 \rho^n V(x(k))$$

for $\ell = 0, 1, ..., n - 1$ and $h \in \mathbb{N}^+$, where $\alpha = \frac{2\alpha}{1 - \rho}$ is a constant. Since $\epsilon \in (0, 1)$ and $\rho \in [0, 1)$, we have

$$\lim_{h \to \infty} V(x((h + 1)n)) = 0$$

for $\ell = 0, 1, ..., n - 1$. Clearly, the above equation implies that

$$\lim_{k \to \infty} V(x(k)) = 0,$$

i.e.,

$$\lim_{k \to \infty} \max(x(k)) - \min(x(k)) = 0,$$

where we used the condition that $\sum_{\ell=0}^\infty \theta(\ell) = 0$. Combining (28) and (30) yields that

$$\lim_{k \to \infty} x(k) = C 1 = x 1,$$

i.e., an average consensus is achieved.

It notes from (29) that

$$\lim_{k \to \infty} \sum_{i=1}^n x_i(k) = \sum_{i=1}^n x_i(0) + \lim_{k \to \infty} \sum_{i=1}^n \theta_i(k).$$

If (5) holds, then $\sum_{i=1}^n x_i(\infty) = \sum_{i=1}^n x_i(0)$. It thus follows that the zero-sum condition is the necessary condition to achieve an exact average consensus with (4).

We thus have completed the proof of Theorem 4.1.

**APPENDIX B**

**THE PROOF OF THEOREM 4.3**

**Proof:** To prove the result in the theorem, we just need to prove that the SCDA algorithm can ensure the two conditions in Theorem 4.1.

First, we prove that the first condition, i.e.,

$$\lim_{k \to \infty} \sum_{i=1}^n \theta_i(k) = 0,$$

is ensured by SCDA. From step 1 and 4, one infers that $\theta_i(1) + \theta_i(0) = \delta_i(1)$ and $\theta_i(2) + \theta_i(1) + \theta_i(0) = \delta_i(2)$ for any $i \in V$. Then, by mathematical induction, one obtains

$$\lim_{k \to \infty} \sum_{k=0}^\infty \theta_i(k) = \lim_{k \to \infty} \delta_i(k).$$

From (6), one has that

$$\lim_{k \to \infty} |\delta_i(k)| \leq \lim_{k \to \infty} |\frac{\alpha}{2} \rho^{k+1}| = 0,$$

which implies that $\sum_{k=0}^\infty \theta_i(k) = 0$ for any $i \in V$. Hence, we have $\sum_{k=0}^\infty \sum_{j=1}^n \theta_i(k) = 0$.

Next we prove the added noise, $\theta(k)$, is exponentially decaying, i.e., $\|\theta(k)\| \leq \alpha \rho^k$. From (7) and (8), one infers that

$$|\theta_i(k)| = |\delta_i(k) - \delta_i(k - 1)|$$

$$\leq |\delta_i(k)| + |\delta_i(k - 1)|$$

$$\leq \frac{\alpha}{2} \rho^{k+1} + \frac{\alpha}{2} \rho^k \leq \alpha \rho^k.$$

Thus, we have $\|\theta(k)\| \leq \alpha \rho^k$. This concludes the proof of this theorem.

**APPENDIX C**

**THE PROOF OF THEOREM 4.7**

**Proof:** To prove this theorem, we need to prove that at each iteration $k$, the probability that each node $i$ can successfully infer that $x_j(0) \in [x_j(0) - \epsilon, x_j(0) + \epsilon]$ is no larger than $\sigma$ using the information set $I_{ij}(k)$. In the following, we prove this result for each iteration.

At time $k = 0$, node $i$ can estimate neighbor $j$’s initial value based on $I_{ij}(0)$ and use the fact that

$$x_j^+(0) = x_j(0) + \theta_j(0),$$

(32)
for estimation. Then, the corresponding estimation is given by

\[ x_j^+(0) = \hat{x}_j(0) + \hat{\theta}_j(0). \]  

(33)

Then, we have

\[ \Pr \{ \hat{x}_j(0) \in [x_j(0) - \epsilon, x_j(0) + \epsilon] \} = \Pr \{ |\hat{\theta}_j(0) - \theta_j(0)| \leq \epsilon \} = \Pr \{ \theta_j(0) \in [\hat{\theta}_j(0) - \epsilon, \hat{\theta}_j(0) + \epsilon] \} = \int_{\hat{\theta}_j(0) - \epsilon}^{\hat{\theta}_j(0) + \epsilon} f_{\theta_j(0)}(y)dy. \]  

(34)

Note that \( \hat{\theta}_j(0) \) is an estimation and could be any values in \([-\frac{\alpha}{2}, \frac{\alpha}{2}]\). Hence,

\[ \max_{x_j(0)} \Pr \{ \hat{x}_j(0) \in [x_j(0) - \epsilon, x_j(0) + \epsilon] \} = \max_{\nu \in [-\frac{\alpha}{2}, \frac{\alpha}{2}]} \int_{\nu - \epsilon}^{\nu + \epsilon} f_{\theta_j(0)}(y)dy, \]  

(35)

Hence, \((\epsilon, \sigma)\)-data-privacy is ensured at time \( k = 0 \) for SCDA.

At time \( k = 1 \), node \( i \) can estimate \( x_j(0) \) based on \( I_{xj}(1) \) and use the fact of both \( (32) \) and \( (33) \),

\[
\frac{x_j^+(1)}{w_{jj}} = x_j^+(0) + \frac{1}{w_{jj}} \left[ \sum_{l \in N_j} w_{jl} x_l^+(0) + \theta_j(1) \right] = x_j(0) + \theta_j(0) + \frac{1}{w_{jj}} \left[ \sum_{l \in N_j} w_{jl} (x_l(0) + \theta_l(0)) + \theta_j(1) \right] = x_j(0) + \theta_j(0) + \frac{1}{w_{jj}} \left[ \sum_{l \in N_j} w_{jl} (x_l(0) + \theta_l(0)) + \theta_j(1) \right].
\]

(36)

If using \( (32) \) only, we also have \( (35) \). Then, we consider the estimation with \( (36) \). Let \( f_{\theta_j(1)}(z) \) be the PDF of \( \theta_j(1) \), where

\[
\theta_j(1) = \theta_j(0) + \frac{1}{w_{jj}} \left[ \sum_{l \in N_j} w_{jl} x_l^+(0) + \theta_j(1) \right] = \theta_j(0) + \frac{1}{w_{jj}} \theta_j(1) + \sum_{l \in N_j} \frac{w_{jl}}{w_{jj}} x_l^+(0) = \theta_j(0) + \frac{1}{w_{jj}} \theta_j(1) + \theta_j''(1).
\]

(37)

Based on \( (36) \), one can make the following estimation

\[
\frac{x_j^+(1)}{w_{jj}} = \hat{x}_j(0) + \hat{\theta}_j(1).
\]

We have

\[ \Pr \{ \hat{x}_j(0) \in [x_j(0) - \epsilon, x_j(0) + \epsilon] \} \leq \Pr \{ |\hat{\theta}_j(1) - \theta_j(1)| \leq \epsilon \} \leq \Pr \{ |\hat{\theta}_j''(1) - \theta_j''(1)| \leq \epsilon \}, \]  

(38)

where we have used the independence between variables \( \theta_j(0) + \frac{1}{w_{jj}} \theta_j(1) \) and \( \theta_j'(1) \). Since node \( i \) cannot listen to all the neighbors’ information of node \( j \), there exists at least one independent variable \( x_l^+(0) \) in \( \sum_{l \in N_j} w_{jl} x_l^+(0) \) that is unknown to node \( i \) (i.e., there is no information of \( x_l^+(0) \) available to node \( i \)) to estimate the value of \( \theta_j'(1) \). From \( (8) \), it follows that

\[
\Pr \{ |\hat{\theta}_j'(1) - \theta_j'(1)| \leq \epsilon \} \leq \max_{\nu \in [-\frac{\alpha}{2}, \frac{\alpha}{2}]} \int_{\nu - \epsilon}^{\nu + \epsilon} f_{\theta_j(0)}(y)dy.
\]

(39)

Combining \( (38) \) and \( (39) \), we have

\[
\Pr \{ \hat{x}_j(0) \in [x_j(0) - \epsilon, x_j(0) + \epsilon] \} \leq \max_{\nu \in [-\frac{\alpha}{2}, \frac{\alpha}{2}]} \int_{\nu - \epsilon}^{\nu + \epsilon} f_{\theta_j(0)}(y)dy.
\]

(40)

Then, using \( (32) \) and \( (36) \) together for estimation, we have

\[
\Pr \{ \hat{x}_j(0) \in [x_j(0) - \epsilon, x_j(0) + \epsilon] \} \leq \max_{\nu \in [-\frac{\alpha}{2}, \frac{\alpha}{2}]} \int_{\nu - \epsilon}^{\nu + \epsilon} f_{\theta_j(0)}(y)dy,
\]

(41)

where \([b_1, B_1] \) \((B_1 - b_1 > 0)\) is an interval including all the possible value of \( \theta_j'(1) \). The above result means that if we combine the two facts for estimation, it will not enhance the successful estimation probability. Therefore, at time \( k = 1 \), we still have \( (35) \) and \((\epsilon, \sigma)\)-data-privacy is still ensured.

At each iteration \( k \), with similar analysis, there are \( k + 1 \) facts (equations) can be used for estimation. Based on the \((k + 1)\)-th equation, i.e.,

\[
\frac{x_j^+(k)}{W^{k}_{jj}} = \frac{1}{W^{k}_{jj}} \left[ W^{k}_{jj} x(0) + \sum_{l=0}^{k} W^{k-l}_{jj} \theta(l) \right] = x_j(0) + \hat{\theta}_j(k) + \sum_{l=0}^{k} \frac{W^{k-l}_{jj}}{W^{k}_{jj}} \theta(l) - \hat{\theta}_j(k)
\]

\[
= x_j(0) + \hat{\theta}_j(k) + \theta_j''(k) = x_j(0) + \theta_j''(k)
\]

(42)

where \( [W^{k}]_{jj} \) denotes the \( j \)-th row vector of \( W^k \), \( [W^{k}]'_{jj} \) is a vector obtained from setting \( [W^{k}]_{jj} = 0 \) for \( [W^{k}]_{jj} \), and \( \hat{\theta}_j(k) = \sum_{l=0}^{k} \frac{W^{k-l}_{jj}}{W^{k}_{jj}} \theta(l) \). The following equation always holds,

\[
\Pr \{ \hat{x}_j(0) \in [x_j(0) - \epsilon, x_j(0) + \epsilon] \} \leq \Pr \{ |\hat{\theta}_j''(k) - \theta_j''(k)| \leq \epsilon \} \leq \max_{\nu \in [-\frac{\alpha}{2}, \frac{\alpha}{2}]} \int_{\nu - \epsilon}^{\nu + \epsilon} f_{\theta_j(0)}(y)dy.
\]

(43)

where we have used the fact that \( \hat{\theta}_j''(k) \) contains the independent variables with no information available to node \( i \). Also, if we combine the equations together, we can prove that the successful estimation probability cannot be increased. That is, \( (35) \) holds and \((\epsilon, \sigma)\)-data-privacy is proved at iteration \( k \).

From the above discussion, one concludes that \( (35) \) holds and \((\epsilon, \sigma)\)-data-privacy is guaranteed by SCDA. Meanwhile,
note that \( f_{\theta_j(0)}(\nu) \) is the PDF function of \( \theta_j(0) \), it follows that \( \lim_{\nu \to 0} \sigma = 0 \). The proof is thus completed.

**APPENDIX D**

**THE PROOF OF THEOREM 5.1**

**Proof:** We first prove (18). Note that
\[
|\tilde{x}_i(0) - x_i(0)| = |\tilde{x}_i(0) - x_i^+(0) + x_i^+(0) - \tilde{x}_i(0) + \tilde{x}_i(0) - x_i(0)|
\]
\[
\leq |\tilde{x}_i(0) - x_i^+(0)| + |x_i^+(0) - \tilde{x}_i(0)| + |\tilde{x}_i(0) - x_i(0)|
\]
\[
\leq 2E_x + \alpha \rho, \tag{44}
\]
where we have used the conditions \( c_2, |\tilde{x}_i(0) - x_i^+(0)| \leq \alpha \rho \), and \( |\tilde{x}_i(0) - x_i(0)| \leq E_x \).

Next, we prove (19). Note that
\[
|\tilde{x}_i^\nu(0) - \tilde{x}_i(0)| = |\tilde{x}_i^\nu(0) - x_i^+(0) + x_i^+(0) - \tilde{x}_i(0) + \tilde{x}_i(0) - \tilde{x}_i(0)|
\]
\[
\leq |\tilde{x}_i^\nu(0) - x_i^+(0)| + |x_i^+(0) - \tilde{x}_i(0)|
\]
\[
+ |\tilde{x}_i(0) - \tilde{x}_i(0)| \leq 3\alpha \rho + \frac{5}{4} \alpha \rho \leq 2 \alpha \rho, \tag{45}
\]
for \( \nu = 1, 2 \) where we have used the conditions \( |\tilde{x}_i^\nu(0) - x_i^+(0)| \leq \frac{\alpha \rho}{2} \) and \( |\tilde{x}_i(0) - x_i(0)| \leq \frac{\alpha \rho}{2} \). Then, from condition \( c_3 \), we have
\[
|\tilde{x}_i^\nu(0) - \tilde{x}_i(0)| \leq \frac{5}{4} \alpha \rho \leq 2 \alpha \rho,
\]
for \( \nu = 1, 2 \).

**APPENDIX E**

**THE PROOF OF THEOREM 5.2**

**Proof:** According to the condition \( c_1 \) in the information checking process, one infers that \( \theta_j^\nu(k) \) used for dishonest node \( i \) should satisfy
\[
|\theta_j^\nu(k)| \leq \frac{1}{2} \alpha \rho^k, k \in \mathbb{N}^+, \nu = 1, 2,
\]
which means that the added noise processes is exponentially decaying. Then, one infers that there exists
\[
\lim_{k \to \infty} x_i^\nu(k) = C^\nu, i \in V, \nu = 1, 2,
\]
where \( C^\nu \) is a constant vector. Then, from step 10, we have
\[
\lim_{k \to \infty} x_i(k) = \lim_{k \to \infty} (x_i(0) + x_i^2(k)) = C^1 + C^2 = C, i \in V,
\]
which means that (20) holds.

Since \( W \) is still a doubly stochastic matrix, we have \( \sum(Wx) = \sum(x) \) for any a vector \( x \). Then, we have
\[
\sum(x^\nu(k)) = \sum[W(x^\nu(k - 1) + \theta^\nu(k - 1))]
\]
\[
= \sum(x^\nu(k - 1) + \theta^\nu(k - 1))
\]
\[
= \sum(x^\nu(0) + \sum_{\ell=0}^{k-1} \theta^\nu(\ell)), \tag{46}
\]
for \( \nu = 1, 2 \). Note that for each cluster \( c \), the initial state vector used for each honest node, say \( i \), is \( x_i^c(0) \), and for each dishonest node, say \( j \), is \( \tilde{x}_i^c(0) \), for \( \nu = 1, 2 \). And, the added noise process for each honest node \( i \) satisfies \( \sum_{\ell=0}^{\infty} \theta_j^\nu(\ell) = 0 \). Let \( V^h \) be the set of honest nodes and \( V^d \) be the set of dishonest nodes in each cluster. Then, taking limiting of both sides of (46), we have
\[
\lim_{k \to \infty} \sum(x^\nu(k)) = \sum(x^\nu(0) + \sum_{\ell=0}^{\infty} \theta_j^\nu(\ell))
\]
\[
= \sum_{i \in V^h} x_i^\nu(0) + \sum_{j \in V^d} \tilde{x}_j^\nu(0) + \sum_{j \in V^d} \sum_{\ell=0}^{\infty} \theta_j^\nu(\ell). \tag{47}
\]
Clearly, we have \( \lim_{k \to \infty} \sum(x^\nu(k)) = nC^\nu \) and \( x_i(0) = x_i^1(0) + x_i^2(0) \) for honest nodes. Since the added noise process of every dishonest node is exponentially decaying, one infers that \( \sum_{\ell=0}^{\infty} \theta_j^\nu(\ell) \leq \frac{\alpha \rho}{2(1-\rho)} \). Then, from (18) and (19), one follows that
\[
\left| \sum_{j=1}^{n} \sum_{\ell=0}^{\infty} \theta_j^\nu(\ell) \right| \leq \sum_{j \in V^d} \left| x_j^\nu(0) + \sum_{\ell=0}^{\infty} \theta_j^\nu(\ell) \right|
\]
\[
\leq \sum_{j \in V^d} \left| \sum_{\ell=0}^{\infty} \theta_j^\nu(\ell) \right|
\]
\[
\leq \sum_{j \in V^d} \left( 2\alpha \rho + 2\alpha \rho + 2E_x + \alpha \rho \right)
\]
\[
\leq d(5\alpha \rho + 2E_x), \tag{48}
\]
and from (47), one further infers that
\[
n \sum_{\nu=1}^{2} C^\nu = \sum_{\nu=1}^{2} \left[ \sum_{i \in V^h} x_i^\nu(0) + \sum_{j \in V^d} \tilde{x}_j^\nu(0) + \sum_{j \in V^d} \sum_{\ell=0}^{\infty} \theta_j^\nu(\ell) \right]
\]
\[
= \sum_{i \in V} x_i(0) + \sum_{j \in V^d} \left[ \sum_{\nu=1}^{2} (x_j^\nu(0) - x_j^\nu(0)) + \sum_{\ell=0}^{\infty} \theta_j^\nu(\ell) \right].
\]
Since \( \sum_{i \in V} x_i(0) = n\bar{x} \) and \( \sum_{\nu=1}^{2} C^\nu = C \), from the above equation, we have
\[
|n(C - \bar{x})| = \left| \sum_{j \in V^d} \sum_{\nu=1}^{2} \left[ x_j^\nu(0) - x_j^\nu(0) + \sum_{\ell=0}^{\infty} \theta_j^\nu(\ell) \right] \right|
\]
\[
\leq \left| \sum_{j \in V^d} \sum_{\nu=1}^{2} (x_j^\nu(0) - x_j^\nu(0)) \right| + \sum_{j \in V^d} \sum_{\nu=1}^{2} \sum_{\ell=0}^{\infty} \theta_j^\nu(\ell)
\]
\[
\leq d(5\alpha \rho + 2E_x) + \sum_{j \in V^d} \sum_{\nu=1}^{2} \sum_{\ell=0}^{\infty} \theta_j^\nu(\ell)
\]
\[
\leq d(5\alpha \rho + 2E_x) + d\frac{\alpha \rho}{(1-\rho)}. \tag{49}
\]
Hence, it follows that
\[ \| C - \hat{x} \|_\infty \leq \frac{d \left[ 5\alpha \rho + 2E_x + \frac{\alpha \rho}{(1-\rho)} \right]}{n}. \]
Therefore, we have completed the proof.

**APPENDIX F**

**THE PROOF OF THEOREM 5.3**

*Proof:* Compared with the SCDA algorithm, the information set which can be used for one node to estimate or infer the initial state of its neighbor node is changed in E-SCDA. Based on the different available information set, we analyze the privacy considering the following three cases, respectively.

Case 1: the information set \( I_{ij}^1(k) = \{ x_j^{v+} (0), \ldots, x_j^{v+}(k) \} \), where \( \nu = 1 \) or \( \nu = 2 \), is available to node \( i \) for estimation, i.e., node \( i \) belongs to one of the neighbor set \( N_j^\nu \) (not both). In this case, it holds that
\[ x_j(0) = 2(x_j^*(0) \pm \vartheta_j). \] (50)

Then, following the same steps as the proof of Theorem 4.1 and combining (50) to estimate \( x_j(0) \), we obtain
\[ \max_{x_j(0)} \Pr \{ \hat{x}_j(0) \in [x_j(0) - \epsilon, x_j(0) + \epsilon] \} \leq \max_{z \in [-2\alpha \rho, 2\alpha \rho]} \int_{z-\epsilon}^{z+\epsilon} f_1(y)dy. \] (51)

Case 2: both \( I_{ij}^1(k) \) and \( I_{ij}^2(k) \) are available to node \( i \) for estimation, i.e., node \( i \) belongs to both of the neighbor set \( N_j^\nu \) for \( \nu = 1, 2 \). In this case, except the similar estimation as in Case 1, we can still use the following fact for estimation,
\[ x_j(0) = x_j^{1+} (0) + x_j^{2+} (0) - (\theta_j^1(0) + \theta_j^2(0)). \] (52)

Based on the above fact, it infers that
\[ \max_{x_j(0)} \Pr \{ \hat{x}_j(0) \in [x_j(0) - \epsilon, x_j(0) + \epsilon] \} \leq \max_{z \in [-\alpha \rho, \alpha \rho]} \int_{z-\epsilon}^{z+\epsilon} f_2(y)dy, \] (53)
where \( f_2(y) \) is the PDF of \( \theta_j^1(0) + \theta_j^2(0) \). Therefore, in this case, we have
\[ \max_{x_j(0)} \Pr \{ \hat{x}_j(0) \in [x_j(0) - \epsilon, x_j(0) + \epsilon] \} \leq \max_{\ell = 0}^{2} \max_{z \in [-2\alpha \rho, 2\alpha \rho]} \int_{z-\epsilon}^{z+\epsilon} f_\ell(y)dy. \] (54)

Case 3: the information set \( I_{ij}^\nu(k) \), where \( \nu = 1 \) or \( \nu = 2 \), is available to node \( i \) for estimation, i.e., node \( i \) is one of the selected nodes. In this case, based on \( I_{ij}^\nu(k) \), we can use (15) to obtain \( \theta_j^\nu(k) \) for \( k \in \mathbb{N}^+ \), and then infer \( \theta_j^\nu(0) \) using the fact that \( \sum_{k=0}^{\infty} \theta_j^\nu(k) = 0 \). It means that \( x_j^*(0) \) is known and available for estimation. Then, with (50), we obtain
\[ \max_{x_j(0)} \Pr \{ \hat{x}_j(0) \in [x_j(0) - \epsilon, x_j(0) + \epsilon] \} \leq \max_{z \in [-\alpha \rho, \alpha \rho]} \int_{z-\epsilon}^{z+\epsilon} f_3(y)dy. \] (55)

Also, (54) could hold in this case since both \( I_{ij}^1(k) \) and \( I_{ij}^2(k) \) may be available to a selected node.

Combine the above three cases, we conclude that
\[ \max_{x_j(0)} \Pr \{ \hat{x}_j(0) \in [x_j(0) - \epsilon, x_j(0) + \epsilon] \} \leq \max_{\ell = 0}^{3} \max_{z \in [-2\alpha \rho, 2\alpha \rho]} \int_{z-\epsilon}^{z+\epsilon} f_\ell(y)dy, \] (56)
which completes the proof.

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