Equal time, point to point correlation functions for spatially separated meson currents are calculated with respect to a variational construct for the ground state of QCD. Given such an ansatz we make no further approximations in the evaluation of the correlators. Our calculations for the vector, axial vector and scalar channels show qualitative agreement with the phenomenological predictions, whereas the pseudoscalar channel does not. However, the pseudoscalar correlator, when approximated by saturating with intermediate one pion states agrees with results obtained from spectral density functions parameterised by pion decay constant and $\langle -\bar{\psi}\psi \rangle$ value obtained from chiral perturbation theory. We discuss this departure in the pseudoscalar channel, in context of the quark propagation in the vacuum.

1. Introduction

Quantum Chromodynamics (QCD) in the low energy sector is nonperturbative and the vacuum structure here is nontrivial. The vacuum structure of QCD has been studied since quite some time both with quark condensates associated with chiral symmetry breaking as well as with gluon condensates. An interesting quantity to study with such a nontrivial structure of vacuum is the behaviour of current-current correlators illustrating different physics involved at different spatial distances. This has recently been emphasized in a review by Shuryak and studied through lattice simulations. The basic point is that the correlators can be used to study the interquark interaction — its dependence on distance. In fact they complement bound state hadron properties in the same way that scattering phase shifts provide information about the nucleon-nucleon force complementary to that provided by the properties of the deuteron.

We have recently considered the structure of QCD vacuum with both quark and gluon condensates using a variational ansatz. We shall use here such an explicit construct of QCD vacuum obtained through energy minimisation to evaluate the meson correlators.

We organise the paper as follows. In section 2 we recapitulate the results of Ref. 7. We then discuss in section 3 the quark propagation in our model of the
QCD vacuum. In section 4 we define and calculate meson correlation functions. In section 5 we quote the results. Section 6 is devoted to the study of the exceptional case of pseudoscalar correlator. Finally we discuss the results in section 7.

We wish to make it clear that in this work we have calculated correlations of two currents each having the quantum numbers of the appropriate meson. The correlations are not between physical meson states.

2. QCD Vacuum with Quark and Gluon Condensates

We have considered the vacuum structure in QCD using a variational approach with both quark and gluon condensates. Here we shall very briefly recapitulate the results of the same for the sake of completeness. The trial variational ansatz for the QCD vacuum is taken as

$$|\text{vac} >= U_G U_F |0 >,$$  \hspace{2cm} (1)

obtained through the unitary operators $U_F$ and $U_G$ for quarks and gluons respectively on the perturbative vacuum $|0 >$.

For the quark sector, the unitary operator $U_F$ is of the form

$$U_F = \exp (B_F^\dagger - B_F),$$  \hspace{2cm} (2)

with the quark antiquark pair creation operator $B_F^\dagger$ given by

$$B_F^\dagger = \int \left[ h(\vec{k}) c_{iI}(\vec{k})^\dagger (\vec{\sigma} \cdot \hat{k}) \bar{c}_{iI}(\vec{-k}) \right] d\vec{k}.$$  \hspace{2cm} (3)

The operators $c^\dagger$ and $\bar{c}$ create a quark and antiquark respectively when operating on the perturbative vacuum. They satisfy the following quantum algebra in Coulomb gauge:

$$[c_{I\alpha}(\vec{k}), c_{J\beta}(\vec{k})^\dagger]_+ = \delta_{\alpha\beta} \delta^{ij} \delta(\vec{k} - \vec{k}') = [\bar{c}_{I\alpha}(\vec{k}), \bar{c}_{J\beta}(\vec{k})^\dagger]_+.$$  \hspace{2cm} (4)

Further $h(\vec{k})$ is a trial function associated with quark antiquark condensates.

Clearly Eqs. (2) and (3) correspond to operator equations which create an arbitrary number of quark antiquark pairs. In fact Eq. (2) may be interpreted as an operator to create a Bose BCS state. We shall further assume $h(\vec{k})$ to be spherically symmetric so that the non perturbative vacuum state will have the same symmetries as the perturbative vacuum i.e. zero momentum and angular momentum.

Similar considerations are applied for constructing the gluon condensate function. Thus for the gluon sector we have,

$$U_G = \exp (B_G^\dagger - B_G) \quad ; \quad B_G^\dagger = \frac{1}{2} \int f(\vec{k}) a^\dagger_{ai}(\vec{k}) a_{ai}(\vec{-k})^\dagger d\vec{k}.$$  \hspace{2cm} (5)

Here $f(\vec{k})$ is a trial function associated with gluon condensates and $a_{ai}(\vec{k})$ the transverse gluon field creation operators.
Clearly such a structure for the vacuum eventually reduces to a Bogoliubov transformation for the operators. One can then calculate the energy density functional given as

\[ \epsilon_0 \equiv F(h(\vec{k}), f(\vec{k})). \]  

(6)

The condensate functions \( f(\vec{k}) \) and \( h(\vec{k}) \) are to be determined such that the energy density \( \epsilon_0 \) is a minimum. Since the functions cannot be determined analytically through functional minimisation except for a few simple cases, we choose the alternative approach of parameterising the condensate functions as (with \( k = |\vec{k}| \)),

\[ \tan 2h(\vec{k}) = \frac{A'}{(e^{R^2k^2} - 1)^{1/2}}. \]  

(7)

This corresponds to taking a Gaussian distribution for the perturbative quarks in the nonperturbative vacuum.

Similarly, for the function \( f(\vec{k}) \) describing the gluon condensates we take the ansatz

\[ \sinh f(\vec{k}) = Ae^{-Bk^2/2}. \]

Further one could relate the quark condensate function to the wave function of pion as a quark antiquark bound state and hence to the decay constant of pion. It is clear from Eq. (7) and the ansatz for the gluon condensate function that even for \( k = 0 \), \( f(k) \) and \( h(k) \) are non zero. Hence there is a finite probability to find low momentum states in the vacuum.

In Ref. 7 the energy density is minimised with respect to the condensate parameters subjected to the constraints that the pion decay constant \( f_\pi \) and the gluon condensate value \( \frac{2\pi}{\alpha_s} < G^\mu_\nu G^\mu_\nu > \) of Shifman Vainshtein and Zhakarov come out as the experimental value of 93 MeV and 0.012 GeV respectively.

The results of such a minimisation showed the instability of the perturbative vacuum to formation of quark antiquark as well as gluon condensates when the coupling became greater than 0.6. Further the charge radius for the pion comes out correctly \( (R_{ch} \simeq 0.65 \text{ fm}) \) for \( \alpha_s = 1.28 \). The corresponding values of \( A' \) and \( R \) of Eq. (7) are calculated to be \( A'_{\text{min}} \simeq 1 \) and \( R \simeq 0.96 \text{ fm} \).

It should be pointed out that local colour neutrality is lost in the vacuum structure we have due to the generation of mass like terms which is a limitation of our approach.

With the structure of QCD vacuum thus fixed from pionic properties and SVZ value we consider quark propagation in the next section.

3. Quark Propagation in the Vacuum

In the calculation of correlators, quark propagators enter in a direct manner and hence it is instructive to study aspects of the interacting propagator in some detail. The reason for doing this is two folds. We wish to know how it differs from a free massive propagator i.e. how good it is to have a “constituent quark” picture and further to compare and contrast with other approaches such as instanton liquid model or vacuum dominance model based on operator product expansion.
The equal time interacting quark Feynman propagator in the condensate vacuum is given as

\[ S_{\alpha\beta}(\vec{x}) = \left\langle \frac{1}{2} \left[ \bar{\psi}_{\alpha}^i(\vec{x}), \psi_{\beta}^i(0) \right] \right\rangle, \tag{8} \]

In our model this reduces to

\[ S(\vec{x}) = \frac{1}{2} \left( \frac{2}{\pi^3} \right) \int e^{i\vec{k} \cdot \vec{x}} d\vec{k} \left[ \sin 2h(\vec{k}) - (\vec{\gamma} \cdot \vec{k}) \cos 2h(\vec{k}) \right] \]
\[ = -\frac{i}{2\pi^2} \frac{\vec{\gamma} \cdot \vec{x}}{x^4} + \frac{1}{(2\pi)^{3/2}} \frac{1}{2R^3} e^{-x^2/(2R^2)} - \frac{i}{(2\pi)^2} \frac{\vec{\gamma} \cdot \vec{x}}{x^2} I(x), \tag{9} \]

where

\[ I(x) = \int_0^\infty \left( \cos kx - \frac{\sin kx}{kx} \right) \frac{k e^{-R^2 k^2}}{1 + (1 - e^{-R^2 k^2})^{1/2}} dk, \tag{11} \]

and \( x = |\vec{x}|, k = |\vec{k}|. \)

Clearly, the free massless propagator is given by \( S_0(x) = -\frac{i}{2\pi^2} \frac{\vec{x}}{x^4} \) which can be derived independently or from (Eq. (11)) in the limit \( R \to \infty \) i.e. in the limit that the condensate functions vanish.

It may be useful to note that the free massive propagator is given as

\[ S_0(m_q, \vec{x}) = \frac{1}{(2\pi)^2} \frac{m_q^2}{x} \left[ K_1(m_qx) - i \frac{\vec{\gamma} \cdot \vec{x}}{x} K_2(m_qx) \right], \tag{12} \]

where \( K_1(m_qx) \) and \( K_2(m_qx) \) are the first and second order Bessel functions respectively.

In Fig. 1 we plot the two components \( \text{Tr} \, S(\vec{x}) \) and \( \text{Tr} \, (\gamma \cdot \hat{x}) S(\vec{x}) \) of the propagator for massless interacting quarks given by Eq. (11) corresponding to the chirality flip and non-flip components considered by Shuryak and Verbaarschot. The first trace is normalised to the short distance limit of the massive free quark propagator \( \text{Tr} \, S_0(m_q, \vec{x})/m_q \) which from (Eq. (12)) is \( 1/(\pi^2 x^2) \). The second trace is normalised to the free quark propagator which is \( \text{Tr} \, (\gamma \cdot \hat{x}) S_0(x) = 2 i/(\pi^2 x^3) \).

To compare with the constituent quark models with an effective constituent mass, we have also plotted the behaviour of free massive quark propagator with masses 100 MeV, 200 MeV and 300 MeV. In the chirality flip part, the propagator in the condensate medium starts from zero, consistent with zero quark mass at small distances, attains a maximum value of about 200 MeV at a distance of about 1.3 fm and then falls off gradually. Further the interacting propagator overshoots the massive propagators after about 0.8 fm. These features are qualitatively similar to that of the instanton liquid model of QCD vacuum considered in Ref. 11.

In the chirality non-flip part, the interacting propagator starts from 1, again consistent with the behaviour expected from asymptotic freedom, but at larger separation it falls rather slowly indicative of an effective mass of the order of 150 MeV. Again these features are similar to that of Ref. 11, though quantitatively there are differences.
Fig. 1. The two components of the quark propagator, \( \text{Tr}(S) \) (a) and \( \text{Tr}[(\vec{\gamma} \cdot \vec{x})S] \) (b) versus the distance \( x \) (in fm). The normalisation, indicated in the figure, have been explained in the text. The solid line corresponds to massless quark interacting propagator \( S(x) \). The three lines, short dashed, dot-short dashed and long dashed correspond to a massive free propagator with a mass of 100, 200 and 300 MeV, respectively.

It may be amusing to consider the leading behaviour of the propagator as \( x \to 0 \). In this limit, the first term of Eq. (9) is given by

\[
T_1 = \frac{1}{2} \frac{1}{(2\pi)^3} \int d\vec{k} \sin 2h(\vec{k}) + O(x^2)
\]

\[
= \frac{1}{24} \langle -\bar{\psi}\psi \rangle + O(x^2),
\]

(13)

where \( \langle -\bar{\psi}\psi \rangle = \langle \bar{u}u + \bar{d}d \rangle \) for two flavours. Similarly the leading contribution from the second term of Eq. (9) is

\[
T_2 = -i \frac{\vec{\gamma} \cdot \vec{x}}{2\pi^2} x + O(x).
\]

(15)

Thus in the small \( x \) limit the interacting propagator of Eq. (8) reduces to

\[
S(\vec{x}) = -i \frac{\vec{\gamma} \cdot \vec{x}}{2\pi^2} x + \frac{1}{24} \langle -\bar{\psi}\psi \rangle
\]

(16)
which is exactly the result of the vacuum dominance model based on operator product expansion.

4. Meson Correlation Functions

Consider a generic meson current of the form

\[ J(x) = \bar{\psi}_i^\alpha(x) \Gamma_{\alpha\beta} \psi_j^\beta(x) \quad (17) \]

where \( x \) is a four vector; \( \alpha \) and \( \beta \) are spinor indices; \( i \) and \( j \) are flavour indices; \( \Gamma \) is a \( 4 \times 4 \) matrix (\( 1, \gamma_5, \gamma_\mu \) or \( \gamma_\mu \gamma_5 \)).

Because of the homogeneity of the vacuum we define the conjugate current to the above at the origin as,

\[ \bar{J}(0) = \bar{\psi}_j^\lambda(0) \Gamma'_\lambda\delta \psi_i^\delta(0) \quad (18) \]

with \( \Gamma' = \gamma_0 \Gamma^\dagger \gamma_0 \)

In general, the meson correlation function is defined as,

\[ R(x) = \langle T \bar{J}(0) J(x) \rangle_{\text{vac}}. \quad (19) \]

From now on we assume that expectation values are always with respect to the nonperturbative vacuum of our model, hence we drop the subscript \( \text{vac} \).

Hence with Eqs. (17), (18) and (19) we have

\[ R(x) = \Gamma_{\alpha\beta} \Gamma'_{\lambda\delta} \langle T \bar{\psi}_i^\lambda(x) \psi_j^\beta(x) \bar{\psi}_j^\delta(0) \psi_i^\lambda(0) \rangle. \quad (20) \]

This reduces to the identity

\[ R(x) = \Gamma_{\alpha\beta} \Gamma'_{\lambda\delta} \langle T \bar{\psi}_i^\lambda(x) \psi_j^\beta(0) \psi_j^\delta(0) \rangle \langle T \bar{\psi}_i^\lambda(0) \psi_i^\delta(0) \rangle. \quad (21) \]

The above definition of \( R(x) \) is exact since the four point function does not contribute. In fact, in the evaluation of Eq. (21) we shall have a sum of two terms. The first is equivalent to the product of two point functions which is Eq. (21). The second term arises from contraction of operators at the same spatial point, related to disconnected diagrams and thus can be discarded.

In Eq. (21) the first term can be identified as the interacting quark propagator

\[ S(x) = \langle T \bar{\psi}^j(x) \psi^j(0) \rangle \]

It can be shown using the CPT invariance of the vacuum that the second term is given as

\[ \langle T \bar{\psi}^j(x) \psi^j(0) \rangle = -\gamma_5 S(x) \gamma_5 \]

\[ = -S(-x) \quad (22) \]
Hence the correlation function of Eq. (20) becomes

$$R(x) = -Tr \left[ S(x) \Gamma' S(-x) \Gamma \right].$$

(23)

Similarly the correlator for massless noninteracting quarks can be given as

$$R_0(x) = -Tr \left[ S_0(x) \Gamma' S_0(-x) \Gamma \right].$$

(24)

Our task is now to evaluate the expression (23) with the ansatz for QCD vacuum as given in Eq. (7). Since we have evaluated the interacting quark propagator at equal time we also consider the correlation functions (Eqs. (23)-(24)) at equal time. This implies that the four vector $x$ in the above equations reduces to the three vector $\vec{x}$.

Having obtained the propagators in the earlier section, we can calculate the correlation function, Eq. (23) for a generic current of the form as in Eqs. (17) and (18).

For convenience, we will consider the ratio of the physical correlation function to that of massless noninteracting quarks and obtain (with $x = |\vec{x}|$)

$$\frac{R(x)}{R_0(x)} = \left(1 + \frac{1}{2} x^2 I(x)\right)^2 + \frac{\pi}{8} \frac{x^6}{R_0^6} e^{-x^2/R^2} \frac{x^2 Tr \left[ \Gamma' \Gamma \right]}{x^4 Tr \left[ \gamma_5 \Gamma' \gamma_5 \Gamma \right]},$$

(25)

which is then evaluated in different channels with the corresponding Dirac structure for the currents.

**Table 1. Meson currents and correlation functions**

| CHANNEL   | CURRENT  | PARTICLE  | CORRELATOR $^a$ |
|-----------|----------|-----------|-----------------|
|           |          | ($J^P$,MASS in MeV) | $\frac{R(x)}{R_0(x)}$ |
| Pseudoscalar | $J^p = \bar{u} \gamma_5 d$ | $\pi^0(0^-,135)$ | $\left[1 + \frac{1}{2} x^2 I(x)\right]^2 + \frac{\pi}{8} \frac{x^6}{R_0^6} e^{-x^2/R^2}$ |
| Scalar    | $J^s = \bar{u} d$ | none($0^+$) | $\left[1 + \frac{1}{2} x^2 I(x)\right]^2 - \frac{\pi}{8} \frac{x^6}{R_0^6} e^{-x^2/R^2}$ |
| Vector    | $J_\mu = \bar{u} \gamma_\mu d$ | $\rho^+(1^-,770)$ | $\left[1 + \frac{1}{2} x^2 I(x)\right]^2 + \frac{\pi}{4} \frac{x^6}{R_0^6} e^{-x^2/R^2}$ |
| Axial     | $J_5^\mu = \bar{u} \gamma_\mu \gamma_5 d$ | $A_1(1^+,1100)$ | $\left[1 + \frac{1}{2} x^2 I(x)\right]^2 - \frac{\pi}{4} \frac{x^6}{R_0^6} e^{-x^2/R^2}$ |
5. Results

We have studied the above ratio of correlators for four channels. In each channel we associate the current with a physical meson having quantum numbers identical to that of the current. The results are shown in Table 1 and in Fig 2.

![Graphs of meson correlation functions](image)

**Fig. 2.** The ratio of the meson correlation functions in QCD vacuum to the correlation functions for noninteracting massless quarks, \( \frac{R(x)}{R_0(x)} \), Plotted vs. distance \( x \) (in fm)

We may notice some general features and relationships among the correlators. The pseudoscalar correlator is always greater than the scalar correlator and vector correlator is greater than the axial vector correlator. We may emphasize here that
these relations are rather general in the sense that they do not depend on the explicit form of the condensate function and arise due to the different Dirac structure of the currents which is reflected in the generic expression for the correlation functions as in Eq. (25). The behaviour of each channel is consistent with that predicted by phenomenology except in the pseudoscalar case where the ratio does not go as high as expected from phenomenology. We examine this in the next section.

6. Pseudoscalar Channel

The explicit evaluation of the pseudoscalar correlator gives, using Eq. (25)

$$R(x) = \frac{1}{2} (1 + \frac{1}{2}x^2 I(x))^2 + \frac{\pi}{8} \frac{x^6}{R^6} e^{-x^2/R^2}$$

which may also be read off from column 4 of Table 1. This is plotted as a function of $x$ in (Fig. 2). As may be seen from (Fig. 2) this ratio has a maximum of $\sim 1.2$ at $x \sim 1.3$ fm. Phenomenologically the peak is at $\sim 100$ at $x \sim 0.5$. In order to compare our results with other calculations we evaluate the same correlator approximately by saturating intermediate states with one pion states.

With our definition of the correlation function (Eq. (19)) we have

$$R(x) = \frac{1}{2} \int \langle \text{vac} | J^P(0) | \pi^a(p) \rangle <\pi^a(p) | \bar{J}^P(0) | \text{vac} >$$

Using translational invariance and the fact that for the pseudoscalar current $J^P = \bar{u}\gamma_5 d$ and $\bar{J}^P = -J^P$, the correlator may be written as

$$R(\vec{x}) = \frac{1}{2} \int <\text{vac} | J^P(0) | \pi^a(p) \rangle <\pi^a(p) | \bar{J}^P(0) | \text{vac} >$$

We may evaluate the above matrix element using the definition of the pion decay constant given as

$$\langle \text{vac} | J^\mu_{5a}(x) | \pi^a(p) \rangle = \frac{i f_{\pi} p^\mu}{(2\pi)^{3/2}(2p_0)^{1/2}} e^{ip\cdot x}$$

where $J^\mu_{5a} = [\bar{\psi} \gamma^\mu \gamma^5 \tau^a \psi]$ is the axial current.
It can be shown that the divergence of the axial current gives the pseudoscalar current

$$\partial_\mu [\bar{\psi} \gamma^\mu \gamma^5 \tau^a \psi] = 2i m_q [\bar{\psi} \gamma^5 \tau^a \psi]$$

(32)

where $m_q$ is the current quark mass. Thus taking divergence of both sides of Eq. (31) and using Eq. (32) we get,

$$2 m_q <\text{vac} | i J^{pa}(x) | \pi^a(p) >= \frac{-f_\pi m^2_\pi}{(2\pi)^{3/2}(2p_0)^{1/2}} e^{ip \cdot x}$$

(33)

where we have used $p^2 = m^2_\pi$.

In an earlier paper within our vacuum model and using the fact that pion is an approximate Goldstone mode it was demonstrated that saturating with pion states, gives the familiar current algebra result

$$m^2_\pi = -f^2_\pi <\bar{\psi} \psi >$$

(34)

With this result we eliminate quark mass $m_q$ in Eq. (33) in favour of the quark condensate to get the relation

$$<\text{vac} | i J^{pa}(0) | \pi^a(p) >= \frac{i}{2(2\pi)^{3/2}(2p_0)^{1/2}} \frac{<\bar{\psi} \psi >}{f_\pi}$$

(35)

Using this, the expression for the pseudoscalar correlator now becomes

$$R(\vec{x}) = \frac{1}{64\pi^3} \left(\frac{<\bar{\psi} \psi >}{f_\pi}\right)^2 \int \frac{1}{(p^2 + m^2_\pi)^{1/2}} (e^{i\vec{p} \cdot \vec{x}} + e^{-i\vec{p} \cdot \vec{x}}) d\vec{p}$$

(36)

The above integral can be evaluated using the standard integral

$$\int_0^\infty p(p^2 + \beta^2)^{\nu - 1/2} \sin(\alpha p) dp = \frac{\beta}{\sqrt{\pi}} \left(\frac{2\beta}{\alpha}\right)^\nu \cos(\nu \pi) \Gamma(\nu + \frac{1}{2}) K_{\nu + 1}(\alpha \beta)$$

for $\alpha > 0$, $Re\beta > 0$ and in the limit $\nu \to 0$. We then finally get for the correlator (using saturation of pion states and with $x = |\vec{x}|$)

$$R(x) = \frac{1}{16\pi^2} \left(\frac{<\bar{\psi} \psi >}{f_\pi}\right)^2 \frac{m_\pi K_1(m_\pi x)}{x}$$

(37)

The correlator for free massless quarks as calculated in the earlier section for pseudoscalar is

$$R_0(x) = \frac{1}{\pi^4 x^6}$$

(38)

Hence the ratio is

$$\frac{R(x)}{R_0(x)} = \frac{\pi^2}{16} \left(\frac{<\bar{\psi} \psi >}{f_\pi}\right)^2 x^5 m_\pi K_1(m_\pi x)$$

(39)
We have plotted in Fig. 3 this ratio for our value of $< -\bar{\psi}\psi >$ and that used by Shuryak.\textsuperscript{5, 19} Note that our value of $< -\bar{\psi}\psi >$ is an output of the variational calculation consistent with low energy hadronic properties.\textsuperscript{7} We thus observe that the approximate calculation of the pseudoscalar correlator due to saturation with one pion states (Fig. 3(a)) yields higher values (≃ 15 times more) as compared to the calculations without saturation as an approximation (Fig. 2). Thus the fermionic condensate model for QCD vacuum\textsuperscript{7} does not give as high values for the pseudoscalar correlator as required by phenomenological results. The value we have used for $< -\bar{\psi}\psi >$ is smaller than Shuryak’s value of (307.4 MeV)$^3$\textsuperscript{b} which appears in the parameterisation of the physical spectral density through the coupling constant.\textsuperscript{19} With his value of $< -\bar{\psi}\psi >$ the ratio $R(x)/R_0(x)$ is shown in (Fig. 3(b)) which agrees with phenomenology.

7. Summary and Discussions

We have evaluated the mesonic correlators in this paper using a variational construct for the QCD vacuum. Except for the pseudoscalar channel the results show qualitative agreement with phenomenological results.\textsuperscript{5} We have also shown that the quark propagation in our construct for the QCD vacuum is almost identical to that in the instanton model.

Following Shuryak\textsuperscript{5, 19} we also see that using current algebra approach the pseudoscalar correlator rises sharply with spatial separation. Let us recall that the condensate value differs from the standard one by a factor of $2^{1/2}$.

\textsuperscript{b}Our definition of the condensate value differs from the standard one by a factor of $2^{1/2}$.
rent algebra result also follows from the approximation of saturating by one pion states in the normalisation of the pion state.\[1\]

It might appear that by suitably changing the value of $\langle -\bar{\psi}\psi \rangle$ in our calculations without saturation one might be able to reproduce all the phenomenological results. Actually we find that it is not so. In fact, it adversely affects the correlators in the other channel which can be seen in the expressions given in column 4 of Table 1. Also the results of our calculations are not very sensitive to the parameter $R$.\[2\]

In view of these findings, it is not clear whether saturation of intermediate states by one pion states only in the evaluation of the correlator, is sufficiently well justified. We therefore think that a unified treatment of correlation functions in all the channels is still not available.

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References

1. E.V. Shuryak, The QCD vacuum, hadrons and the superdense matter (World Scientific, Singapore, 1988).
2. Y. Nambu, Phys. Rev. Lett. 4, 380 (1960); Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); ibid, 124 246 (1961); J.R. Finger and J.E. Mandula, Nucl. Phys. B199, 168 (1982); A. Amer, A. Le Yaouanc, L. Oliver, O. Pene and J.C. Raynal, Phys. Rev. Lett. 50, 87 (1983); ibid, Phys. Rev. D28, 1530 (1983); S.L. Adler and A.C. Davis, Nucl. Phys. B244, 469 (1984); R. Alkofer and P. A. Amundsen, Nucl. Phys. B306, 305 (1988); A.C. Davis and A.M. Matheson, Nucl. Phys. B246, 203 (1984); S. Schramm and W. Greiner, Int. Jour. Mod. Phys. E1, 73 (1992).
3. D. Schutte, Phys. Rev. D31, 810 (1985).
4. T. H. Hansson, K. Johnson, C. Peterson, Phys. Rev. D26, 2069 (1982).
5. E.V. Shuryak, Rev. Mod. Phys. 65, 1 (1993)
6. M.-C. Chu, J. M. Grandy, S. Huang and J. W. Negele, Phys. Rev. D48, 3340 (1993); ibid, Phys. Rev. D49, 6039 (1994).
7. A. Mishra, H. Mishra, S.P. Misra, P.K. Panda and Varun Sheel, Int. J. Mod. Phys. E5, 93 (1996).
8. H. Mishra, S.P. Misra and A. Mishra, Int. J. Mod. Phys. A3, 2331 (1988); S.P. Misra, Phys. Rev. D35, 2607 (1987).
9. A. Mishra, H. Mishra and S. P. Misra, Z. Phys. C57, 241 (1993); H. Mishra and S.P. Misra, Phys Rev. D48, 5376 (1993); A. Mishra and S.P. Misra, Z. Phys. C58, 325 (1993).
10. M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147, 385 (1979), 448 and 519.
11. E. V. Shuryak and J. J. M. Verbaarschot, Nucl. Phys. B410, 37 (1993).
12. See e.g. in J. D. Bjorken and S. D. Drell, Relativistic Quantum Fields (McGraw-Hill, New York, 1965) p. 155 and 213.
13. M. G. Mitchard, A. C. Davis and A. J. Macfarlane, Nucl. Phys. B325, 470 (1989).
14. T. D. Lee, Particle Physics and introduction to Field Theory (Harwood Academic, 1982) p. 791.
15. J. J. Sakurai, *Currents and Mesons* (Chicago Lectures in Physics, 1969) p. 18.
16. *Table of Integrals, Series and Products* edited by I. S. Gradshteyn and I.M. Ryzhik (Academic Press, 1980) p. 427.
17. E. V. Shuryak and J. J. M. Verbaarschot, *Nucl. Phys.* B410, 55 (1993).
18. T. Schäfer, E. V. Shuryak and J. J. M. Verbaarschot, *Nucl. Phys.* B412, 143 (1994).
19. E.V. Shuryak, *Nucl. Phys.* B319, 541 (1989).