In a recent Letter [1] we proposed an interpretation of the experimental measurement (Refs. 4-5 of [1]) of the transmission phase shift through a quantum dot (QD) in the Kondo regime, deduced from a double-slit A-B interferometer (ABI). Our starting point is the 1-D single level Anderson model (SLAM) with 2 reservoirs for which we develop a scattering theory. We distinguished between the phase shift \( \delta \) of the S-matrix responsible for the shift \( \delta_{\text{ABI}} \) in the AB oscillations \( (\delta_{\text{ABI}} = \delta) \), and the one controlling the conductance \( G \sim \sin^2 \delta G \) (with \( \delta G = \delta_\sigma \)), and claimed the following relation holds: \( \delta G = \delta_{\text{ABI}}/2 \) (or equivalently \( \delta_\sigma = \delta/2 \)). The results obtained this way are in remarkably good agreement with experimental measurements (cf. Figs.1 and 4).

In their comment, Aharony, Entin-Wohlman, Oreg and von Delft (AE-WOvD) [2] question the validity of our main assertion, claiming that it fails in some exactly known limits as the non-interacting \( (U = 0) \) Anderson model. The main point of our paper, however, was that the SLAM provides an incomplete description of the experimental device. Rather the quantum dot needs to be viewed as an artificial atom and electrons scattering off it must satisfy the generalized Levinson theorem that incorporates the Pauli principle in the many electron system. Adding this physics takes us out of the strict SLAM description. We now provide some details.

(i) First we derive Eq.(3) of JVL. The first step consists in evaluating the retarded Green’s function of one electron on the site 0 for the SLAM. Using exact results for the self-energy in an interacting Fermi liquid (see Refs.16-17 of [1] and references within) in its generalized Levinson theorem. The Levinson theorem derive Eq.(3) of JVL leading, in the case of a symmetric QD (\( \delta = 0 \)), one can show that: \( \delta = \pi(n_{0-\sigma} + n_{0\sigma}) = \pi n_0 \), in which \( n_{0-\sigma} \) is the number of bound states (Kondo singlet state in the Kondo regime), and \( n_{0\sigma} \) is the number of states excluded by the Pauli principle. As announced, the expression (1) for \( S'_{k\sigma} \) violates the generalized Levinson theorem since \( (1/2i)\ln \det S'_{k\sigma} = \delta_\sigma = \pi n_0 \), missing the other part related to \( n_{0-\sigma} \).

(ii) Our claim and we agree with the comment of AE-WOvD, is that the 1-D SLAM with 2 reservoirs is not sufficient to capture the whole physics contained in the experimental device. While it captures most of the physics, it fails to account for the many electron nature of the experimental set-up. One may try to start with a many level Anderson model (MLAM) description of the system. We have chosen another route and introduced miminally the missing ingredients through an additional multiplicative phase factor \( C_\sigma \) in front of the S-matrix of the SLAM: \( \hat{S}_{k\sigma} = C_\sigma S'_{k\sigma} \). The value of \( C_\sigma \) is determined in order to guarantee the generalized Levinson theorem. It is easy to check that \( C_\sigma = \exp(i\delta_\sigma) \) which eventually leads to Eq.(4) of JVL for \( \hat{S}_{k\sigma} \). By doing so, the total occupancy of the QD as evaluated in the 1D-SLAM is directly related to the phase shift at \( T=0 \). We believe that this is precisely the quantity measured in the quantum interferometry.

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[2] A. Aharony, O. Entin-Wohlman, Y. Oreg, and J. von Delft, Phys. Rev. Lett., comment (2005).
[3] The opposite sign convention for the definition of \( \delta \) is taken in Eq.(2) of AE-WOvD explaining the sign difference which is found.