Direction-of-arrival (DOA) estimation has been widely used in wireless communications [1], [2]. As an important DOA estimation approach, multiple signal classification (MUSIC) has gained a lot of attention due to its super-resolution property in the presence of multiple signals [3]. Massive multiple-input multiple-output (MIMO) is one of the most important enabling technologies in 5G and its beyond [4]. Due to a large number of antennas, massive MIMO is essential to millimeter-wave bands because the large array gain can compensate for the high path loss. With massive MIMO, frequency resources at millimeter-wave bands can be exploited efficiently [5], [6].

II. SYSTEM MODEL

A. Signal Model

As in Fig. 1, consider a hybrid massive MIMO system composed of a uniform linear array (ULA) with M antennas and N RF chains. Denote $y_{m,n}(t)$ to be the received signal at the $m$-th antenna of the $n$-th RF chain. Since each RF chain is connected to $\frac{M}{N}$ antennas, we have $m = 0, 1, \ldots, \frac{M}{N} - 1$ and $n = 0, 1, \ldots, N - 1$. Then, the received signal vector by the $n$-th RF chain $\mathbf{y}_n(t) = \left[ y_{0,n}(t), y_{1,n}(t), \ldots, y_{\frac{M}{N} - 1,n}(t) \right]^T$ can be represented as

$$ y_n(t) = \sum_{l=0}^{L-1} a_n(\theta_l) x_l(t) + z_n(t), $$

(1)
where $x_i(t)$’s ($t = 0, 1, \ldots, L - 1$) are $L$ narrow-band signals impinging from farfield onto the array, $\theta_i$ is the DOA of $x_i(t)$, $z_n(t)$ denotes the additive Gaussian noise vector with $E\{z_n(t)z_n^H(t)\} = N_0 I_M$, where $N_0$ is the noise power and $I_M$ is an $\frac{M}{2} \times \frac{M}{2}$ identity matrix, and $a_n(\theta_i)$ is the $\frac{M}{2} \times 1$ steering vector corresponding to the $n$-th RF chain with the $m$-th entry given by

$$a_{m,n}(\theta_i) = e^{j2\pi \frac{nM}{2} \sin \theta_i (\frac{M}{2} + m)},$$

where $d = \frac{\lambda}{2}$ denotes the antenna distance and $\lambda$ is the wave length. Take all the RF chains into account, then the overall received signal vector, the overall steering vector, and the overall additive noise vector can be represented as

$$y(t) = [y_1(t), y_1^T(t), \ldots, y_{K-1}^T(t)]^T,$$

$$\alpha(\theta_i) = [a_1^T(\theta_i), a_2^T(\theta_i), \ldots, a_{K-1}^T(\theta_i)]^T,$$

$$z(t) = [z_1^T(t), z_1^T(t), \ldots, z_{K-1}^T(t)]^T,$$

respectively. Accordingly, we have

$$y(t) = \sum_{l=0}^{L-1} \alpha_l x_l(t) + z(t).$$

Denote $R = E\{y(t)y^H(t)\}$ to be the overall SCM, then using (3), the overall SCM can be divided into

$$R = \begin{bmatrix}
R_{0,0} & \cdots & R_{0,N-1} \\
\vdots & \ddots & \vdots \\
R_{N-1,0} & \cdots & R_{N-1,N-1}
\end{bmatrix},$$

where $R_{n_1,n_2} = E\{y_{n_1}(t)y_{n_2}^H(t)\}$ is the $(n_1, n_2)$-th sub-SCM. Assuming $L$ signals are mutually independent with zero means and the power of the $l$-th signal is $E\{|x_l(t)|^2\} = \sigma_l^2$, then the $(n_1, n_2)$-th sub-SCM and the overall SCM will be

$$R_{n_1,n_2} = \sum_{l=0}^{L-1} \sigma_l^2 \cdot a_{n_1}(\theta_l)a_{n_2}^H(\theta_l) + \delta[n_1 - n_2]N_0 I_M,$$

$$R = \sum_{l=0}^{L-1} \sigma_l^2 \cdot a(\theta_l)a^H(\theta_l) + N_0 I_M,$$

respectively, where $\delta[]$ indicates Kronecker Delta function.

### B. Review of MUSIC Algorithm

Denote $y[k] = y(kT_s)$ to be the sample of the received signal where $T_s$ denotes the sampling period. In MUSIC algorithm, $y[k]$’s are assumed to be available in the receiver. In this case, the overall SCM in (9) can be estimated using the sample average approach, that is [3], [16]

$$R \approx \frac{1}{K} \sum_{k=0}^{K-1} y[k]y^H[k],$$

where $K$ denotes the number of samples. The eigenvalue decomposition of the overall SCM can be given as $R = (U_s, U_n)\Lambda_s(U_s, U_n)^H$, where $U_s$ and $U_n$ denote the orthogonal base vectors corresponding to the signal and the noise subspaces respectively, and $\Lambda_s$ is a diagonal matrix composed of the eigenvalues. Then, unknown DOAs can be determined by searching the peak values of $p(\theta)$,

$$p(\theta) = \|U_s^H a(\theta)\|^2, \theta \in [-90^\circ, 90^\circ].$$

For sample average in (10), $y[k]$ is required to estimate SCM. In this case, the received signals at all antennas should be sent via RF chains to the digital receiver. In hybrid massive MIMO, however, Fig. 1 shows that only the combination of the entries of $y[k]$ can be sent to the digital receiver because the number of RF chains is smaller than the number of antennas. As a consequence, the sample average approach in (10) cannot be used in hybrid massive MIMO systems. In [15], we have developed an algorithm for SCM reconstruction in massive MIMO with single RF chain. In the presence of multiple RF chains, however, each RF chain is connected to only a subset of the antennas. Therefore, the overall SCM cannot be obtained directly using the algorithm in [15].

### III. BEAM SWEEPING ALGORITHM

To reconstruct the overall SCM in the presence of multiple RF chains, we can first reconstruct the sub-SCMs individually, and then the overall SCM can be obtained following (7).

To reconstruct $R_{n_1,n_2}$, define $\{\theta^{(0)}, \theta^{(1)}, \ldots, \theta^{(Q-1)}\}$ as a set of predetermined DOA angles. The analog beamformers switch the beam directions to the predetermined DOAs in turn. For the $q$-th sweeping beam, the predetermined DOA is $\theta^{(q)}$, and thus the steering vector corresponding to the $n$-th RF chain is $a_{n}(\theta^{(q)})$. The combination of the received signals on the $n$-th RF chain can be represented by

$$c_n^{(q)}(t) = a_{n}^{H}(\theta^{(q)})y_n(t).$$

From Fig. 1, the $n$-th signal combination is sampled before sent to the receiver, and thus the samples of the signal combination can be given by

$$c_n^{(q)}[k] = c_n^{(q)}(kT_s) = a_{n}^{H}(\theta^{(q)})y_n[k].$$

Denote $P_{n_1,n_2}^{(q)}$ to be the correlation between the outputs of the $n_1$-th and the $n_2$-th RF chains, that is

$$P_{n_1,n_2}^{(q)} = \frac{1}{K} \sum_{k=0}^{K-1} c_{n_1}^{(q)}[k]c_{n_2}^{(q)*}[k],$$

$$= a_{n_1}^{H}(\theta^{(q)}) \left( \frac{1}{K} \sum_{k=0}^{K-1} y_n[k]y_{n_2}^{H}[k] \right) a_{n_2}(\theta^{(q)}).$$

If the number of samples is large enough, the sample average in (14) is equivalent to the statistical average, that is

$$a_{n_1}^{H}(\theta^{(q)})R_{n_1,n_2}a_{n_2}(\theta^{(q)}) = P_{n_1,n_2}^{(q)}.$$
Using the vec(·) operator to (15), the left-hand-side of (15) can be given as
\[ \text{vec}\{a_{n_1}^H(\theta^{(q)})R_{n_1,n_2}a_{n_2}(\theta^{(q)})\} = [a_{n_2}^T(\theta^{(q)}) \otimes a_{n_1}^H(\theta^{(q)})]\text{vec}(R_{n_1,n_2}), \]  \[ (16) \]
where we have used the equation (1.10.25) in [17], that is, vec(BCD) = (D^T \otimes B)vec(C) with \( \otimes \) denoting the Kronecker product.

To proceed, denote \( r_{n_1,n_2} = \text{vec}(R_{n_1,n_2}) \) and
\[ a_{n_1,n_2}^{(q)} = a_{n_2}(\theta^{(q)}) \otimes a_{n_1}^H(\theta^{(q)}), \]  \[ (17) \]
where \( (\cdot)^\dagger \) denotes the element-wise conjugation. Apparently, both \( r_{n_1,n_2} \) and \( a_{n_1,n_2}^{(q)} \) are \( \frac{M^2}{N} \times 1 \) vectors. Then, (15) can be rewritten as
\[ (a_{n_1,n_2}^{(q)})^T r_{n_1,n_2} = p_{n_1,n_2}^{(q)}. \]  \[ (18) \]
Considering that there are \( Q \) predetermined DOAs, then (18) can be extended to a group of linear equations as
\[ A_{n_1,n_2} r_{n_1,n_2} = p_{n_1,n_2}, \]  \[ (19) \]
where \( A_{n_1,n_2} \) is a \( Q \times \frac{M^2}{N} \) matrix and \( p_{n_1,n_2} \) is a \( Q \times 1 \) vector
\[ A_{n_1,n_2} = [a_{n_1,n_2}^{(0)}, a_{n_1,n_2}^{(1)}, \ldots, a_{n_1,n_2}^{(Q-1)}]^T, \]  \[ (20) \]
\[ p_{n_1,n_2} = [p_{n_1,n_2}^{(0)}, p_{n_1,n_2}^{(1)}, \ldots, p_{n_1,n_2}^{(Q-1)}]^T. \]  \[ (21) \]

Then, the \((n_1,n_2)\)-th sub-SCM can be reconstructed by solving (19). Similar to [15], a diagonal loading coefficient can be adopted to improve the distribution of the eigenvalues, so that the ill-conditioned result can be avoided. In this case, the vector form of sub-SCM can be obtained as
\[ \hat{R}_{n_1,n_2} = \left( A_{n_1,n_2}^H A_{n_1,n_2} + \sigma^2 I \frac{M^2}{N} \right)^{-1} A_{n_1,n_2}^H p_{n_1,n_2}, \]  \[ (22) \]
where \( \sigma^2 \) denotes the diagonal loading coefficient. Then, the \((n_1,n_2)\)-th sub-SCM can be reconstructed through \( \hat{R}_{n_1,n_2} \) = unvec(\( \hat{R}_{n_1,n_2} \)). Once the sub-SCMs are obtained, the overall SCM can be reconstructed, following (7), as
\[ \hat{R} = \begin{bmatrix} \hat{R}_{0,0} & \cdots & \hat{R}_{0,N-1} \\ \vdots & \ddots & \vdots \\ \hat{R}_{N-1,0} & \cdots & \hat{R}_{N-1,N-1} \end{bmatrix}. \]  \[ (23) \]
As in [15], although matrix inversion in (22) causes a huge computational burden, the operator \( (A_{n_1,n_2}^H A_{n_1,n_2} + \sigma^2 I \frac{M^2}{N})^{-1} A_{n_1,n_2}^H \) can be pre-calculated off-line if the predetermined DOAs are fixed. In this case, matrix inversion can be avoided from on-line calculation, and the computational burden is mainly caused by the matrix-vector production in (22), which requires \(QM^2/N\) complex multiplications.

With the reconstructed SCM in (23), unknown DOAs can be obtained using (11) if classical MUSIC algorithm is adopted. In addition to the classical MUSIC algorithm, the variants of MUSIC algorithm, such as root-MUSIC [18], can be also used. To use root-MUSIC and other variants, we only need to replace sample average approach in those algorithms with the SCM reconstruction algorithm in this paper.

IV. ALGORITHM OPTIMIZATION

Although the beam sweeping algorithm in Section III can reconstruct the SCM successfully, the computation complexity is still huge due to the large dimension of matrix-vector production in (22). In this section, a low-complexity algorithm will be presented where the dimension of matrix-vector production can be reduced significantly. Based on the low-complexity algorithm, optimized selection of predetermined DOAs is further investigated. Using the optimized predetermined DOAs, the matrix inverse operation can be even avoided.

A. Low-Complexity Implementation

Low-complexity implementation of the beam sweeping algorithm is inspired by the fact that many repeated entries exist in the SCM. Although there are \( \frac{M^2}{\gamma} \) entries in \( R_{n_1,n_2} \), the number of non-repeated unknowns is only \( \frac{M^2}{\gamma} - 1 \). Therefore, the computational complexity can be reduced if we only recover the non-repeated unknowns.

Denote the \((m_1,m_2)\)-th entry of \( R_{n_1,n_2} \), \( R_{n_1,n_2}[m_1,m_2] \), to be \( \gamma_{n_1,n_2}[m_1,m_2] \). If denote \( \gamma_{n_1,n_2} \) to be a \( (\frac{M^2}{\gamma} - 1) \times 1 \) column vector containing all non-repeated unknowns in \( r_{n_1,n_2} \) or \( R_{n_1,n_2} \), then \( \gamma_{n_1,n_2} \) can be given by
\[ \gamma_{n_1,n_2} = \left( \gamma_{n_1,n_2} \left[ 1 - \frac{M}{N}, \ldots, \gamma_{n_1,n_2} \left[ \frac{M}{N} - 1 \right] \right] \right)^T. \]  \[ (24) \]
Accordingly, \( r_{n_1,n_2} \) can be expressed by \( \gamma_{n_1,n_2} \) using
\[ r_{n_1,n_2} = E \cdot \gamma_{n_1,n_2}, \]  \[ (25) \]
where \( E \) is an \( \frac{M^2}{\gamma} \times (\frac{M^2}{\gamma} - 1) \) matrix
\[ E = \begin{bmatrix} O_{\frac{M^2}{\gamma}-1, \frac{M^2}{\gamma}} & I_{\frac{M^2}{\gamma}} & O_0 \\ O_{\frac{M^2}{\gamma}-2, \frac{M^2}{\gamma}} & I_{\frac{M^2}{\gamma}} & O_1 \\ \vdots & \vdots & \vdots \\ O_0 & I_{\frac{M^2}{\gamma}} & O_{\frac{M^2}{\gamma}-1} \end{bmatrix}. \]  \[ (26) \]
with \( O_{m,n} \) denoting an \( \frac{M^2}{\gamma} \times m \) all-zero matrix. By substituting (25) into (18), (18) can be rewritten as
\[ (b_{n_1,n_2}^{(q)})^T \gamma_{n_1,n_2} = p_{n_1,n_2}^{(q)}, \]  \[ (27) \]
where \( (b_{n_1,n_2}^{(q)})^T \) is a \( 1 \times (\frac{M^2}{\gamma} - 1) \) row vector given by
\[ b_{n_1,n_2}^{(q)} = (a_{n_1,n_2}^{(q)})^T E. \]  \[ (28) \]
Then, similar to (19), if we take \( Q \) predetermined DOAs into account, equation (27) can be extended to a group of linear equations as
\[ B_{n_1,n_2} \gamma_{n_1,n_2} = p_{n_1,n_2}, \]  \[ (29) \]
where \( B_{n_1,n_2} \) is a \( Q \times (\frac{M^2}{\gamma} - 1) \) matrix given by
\[ B_{n_1,n_2} = \left[ b_{n_1,n_2}^{(0)}, b_{n_1,n_2}^{(1)}, \ldots, b_{n_1,n_2}^{(Q-1)} \right]^T. \]  \[ (30) \]
Different from (22) where a diagonal loading coefficient is adopted to avoid the ill-conditioned result, equation (29) can be solved without diagonal loading if the predetermined DOAs are carefully selected, as will be discussed in the next subsection. Therefore, \( \gamma_{n_1,n_2} \) in (29) can be obtained as
\[ \hat{\gamma}_{n_1,n_2} = (B_{n_1,n_2}^H B_{n_1,n_2})^{-1} B_{n_1,n_2}^H p_{n_1,n_2}. \]  \[ (31) \]
Consequently, the sub-SCM can be reconstructed as follows
\[ \hat{R}_{n_1,n_2} = \hat{\gamma}_{n_1,n_2} [m_1, m_2]. \]  \[ (32) \]
Similar to (22), the operator \( (B_{n_1, n_2}^H B_{n_1, n_2})^{-1} B_{n_1, n_2}^H \) in (31) can be pre-calculated off-line so that the computational burden in (31) is mainly caused by the matrix-vector production. Since \( B_{n_1, n_2} \) is a \( Q \times (2M/N - 1) \) matrix, the number of complex multiplications required in (31) is only \( Q(2M/N - 1) \), which is much lower than that required in (22). In addition to the reduction of complexity, (29) also indicates that at least \( \frac{2M}{N} - 1 \) predetermined DOAs are required to achieve accurate reconstruction, that is, \( Q \geq \frac{2M}{N} - 1 \). This is because there are \( \frac{2M}{N} - 1 \) non-repeated unknowns in \( \gamma_{n_1, n_2} \), at least \( \frac{2M}{N} - 1 \) equations are required to solve (29) with each equation corresponding to one predetermined DOA.

### B. Selection of Predetermined DOAs

In [15], the predetermined DOAs are selected as uniformly distributed from \(-90^\circ \) to \(90^\circ\). Although simple, we will show in this subsection that the selection of the predetermined DOAs can be further optimized. Denote

\[
u^{(q)} = \frac{d}{\lambda} \sin \theta(q) = 0.5 \sin \theta(q),
\]

(33)
to be the space frequency corresponding to \( \theta(q) \). In this paper, the predetermined DOAs are selected such that \( \nu^{(q)} \) are uniformly distributed from \(-0.5\) to \(0.5\), that is, \( \nu^{(q)} = -0.5 + q/Q \). As a result, the predetermined DOAs are determined as

\[
\theta(q) = \arcsin(-1 + 2q/Q).
\]

(34)

In following, it will show \( B_{n_1, n_2}^H B_{n_1, n_2} \) can be converted to a diagonal matrix using the predetermined DOAs in (34) so that the matrix inverse operation in (31) can be avoided.

If using (17) and (26) to (28), we have

\[
(b_{n_1, n_2}^{(q)})^T = (a_{n_2}^T (\theta(q)) \otimes a_{n_1}^H (\theta(q))) \cdot E
= [a_0, a_{-1}, a_{-2}, \ldots, a_{M-1}, a_{M-2}, \ldots, a_{M-1-n_2}, a_{M-1-n_2}, \ldots]
\]

\[
\begin{bmatrix}
O_{\frac{M}{N}-1,2} & I_{\frac{M}{N}-1,1} & O_{\theta} \\
\vdots & \vdots & \vdots \\
O_{\theta} & I_{\frac{M}{N}-1,1} & O_m
\end{bmatrix}
\]

\[
= \sum_{m=0}^{M-1} a_{m, n_2}(\theta(q)) a_{n_1}^H (\theta(q)) \begin{bmatrix}
O_{\frac{M}{N}-1,1} & I_{\frac{M}{N}-1,1} & O_m
\end{bmatrix}
\]

\[
= \sum_{m=0}^{M-1} \theta_{m, n_2}(\theta(q)) a_{n_1}^H (\theta(q)) \begin{bmatrix}
O_{\frac{M}{N}-1,1} & I_{\frac{M}{N}-1,1} & O_m
\end{bmatrix},
\]

(35)

where \( \theta_{m, n_2} \) is an \( m \times 1 \) all-zero vector. If denote \( b_{n_1, n_2}^{(q)}[m_0] \) as the \( m_0 \)-th entry of \( b_{n_1, n_2}^{(q)} \), then we have \( m_0 = 0, 1, \ldots, \frac{M}{N} - 2 \) since the length of \( b_{n_1, n_2}^{(q)} \) is \( \frac{2M}{N} - 1 \). To proceed, define \( \tilde{a}_{m, n} \) as a sequence with infinite length where

\[
\tilde{a}_{m, n} = \begin{cases}
    a_{m, n}(\theta(q)), & m = 0, 1, \ldots, \frac{M}{N} - 1, \\
    0, & \text{otherwise}.
\end{cases}
\]

(36)

Then, from the last equation of (35), it is easy to verify that \( b_{n_1, n_2}^{(q)}[m_0] \) can be obtained as

\[
b_{n_1, n_2}^{(q)}[m_0] = \sum_{m=0}^{M-1} \tilde{a}_{n_2, m} \tilde{a}_{n_1, m}^*,
\]

(37)

which is essentially a linear convolution between \( \tilde{a}_{m, n_1}^* \) and \( \tilde{a}_{M-1-m_0, n_2} \). Using (37), the \((m_1, m_2)\)-th entry of \( B_{n_1, n_2}^H B_{n_1, n_2} \) with \( m_1, m_2 = 0, 1, \ldots, \frac{M}{N} - 2 \) is given by

\[
[B_{n_1, n_2}^H B_{n_1, n_2}](m_1, m_2) = \sum_{q=0}^{Q-1} (b_{n_1, n_2}^{(q)})^* (b_{n_1, n_2}^{(q)})^T (m_1, m_2)
= \sum_{q=0}^{Q-1} \sum_{i_1=0}^{M-1} \sum_{i_2=0}^{M-1} \sum_{q=0}^{Q-1} \tilde{a}_{n_1, i_1}^* \tilde{a}_{n_2, i_2} \tilde{a}_{n_1, i_1} \tilde{a}_{n_2, i_2}^* \tilde{a}_{n_1, i_1} \tilde{a}_{n_2, i_2}.
\]

(38)

Since the sequence \( \tilde{a}_{m, n} \) has non-zero values only when \( m = 0, 1, \ldots, \frac{M}{N} - 1 \), \( i_1 \) in (38) should satisfy

\[
0 \leq i_1 \leq M/N - 1,
\]

(39)

\[
0 \leq M/N - 1 - m_1 + i_1 \leq M/N - 1,
\]

(40)
simultaneously. Therefore, we can obtain

\[
U^-_1 \leq i_1 \leq U^+_1,
\]

(41)
where \( U^-_1 = \max[0, m_1 - (\frac{M}{N} - 1)] \) and \( U^+_1 = \min[m_1, \frac{M}{N} - 1] \). Similarly, we have

\[
U^-_2 \leq i_2 \leq U^+_2,
\]

(42)
where \( U^-_2 = \max[0, m_2 - (\frac{M}{N} - 1)] \) and \( U^+_2 = \min[m_2, \frac{M}{N} - 1] \).

With the constraints of \( i_1 \) and \( i_2 \), (38) can be rewritten as

\[
[B_{n_1, n_2}^H B_{n_1, n_2}](m_1, m_2) = \sum_{i_1=U^-_1}^{U^+_1} \sum_{i_2=U^-_2}^{U^+_2} \sum_{q=0}^{Q-1} e^{j2\pi \nu^{(q)}(m_1-1-m_2)},
\]

(43)

where we have used the identity

\[
\tilde{a}_{M-1-m_1+i_1, n_2}^* (\theta(q)) a_{n_1, m_1} (\theta(q)).
\]

(44)
If applying the predetermined DOAs in (34) to (43), we have

$$
\sum_{q=0}^{Q-1} e^{j2\pi q(1+q)/Q} (m_1 - m_2) = \begin{cases} Q, & m_1 = m_2, \\ 0, & m_1 \neq m_2. \end{cases}
$$

(45)

As a result, we have \([B^H_{m_1,m_2} B_{m_1,m_2}] (m_1,m_2) = 0\) for \(m_1 \neq m_2\) and when \(m_1 = m_2 = 0, 1, \ldots, 2 \frac{M}{Q} - 2,\)

$$
[B^H_{m_1,m_2} B_{m_1,m_2}] (m_1,m_2) = Q(U_1^+ - U_1^- + 1)^2
$$

$$
= \begin{cases} Q(m_1 + 1)^2, & m_1 \leq \frac{M}{Q} - 1, \\ Q \left(2 \left(\frac{M}{Q} - 1\right) - m_1 + 1\right)^2, & m_1 > \frac{M}{Q} - 1. \end{cases}
$$

(46)

Apparently, \(B^H_{m_1,m_2} B_{m_1,m_2}\) has been converted into a diagonal matrix by selecting predetermined DOAs as in (34), and correspondingly, the matrix inverse operation in (31) can be avoided completely.

V. SIMULATION RESULTS

Computer simulation is adopted in this section to investigate the proposed algorithms. We consider a ULA with \(M = 64\) antennas and the distance between antennas is 0.5\(^\circ\). 32 signals are impinging onto the ULA where DOAs of the signals are uniformly distributed from \(-90^\circ\) to \(90^\circ\). The arrival signals are assumed independent with zero means and unit powers. Without specification, the signal-to-noise ratio (SNR) is \(-5\) dB, and the predetermined DOAs are as in (34). Similar to [15], normalized-square-error (NSE) is used in the simulation to evaluate the accuracy of reconstructed SCM, that is NSE = \(\|R - \hat{R}\|^2_2 / \|R\|^2_2\). To demonstrate the effectiveness of reconstructed SCM, classical MUSIC algorithm is adopted in the simulation for DOA estimation. Accordingly, means-square-error (MSE) is used to evaluate the accuracy of DOA estimation, that is MSE = \(E[(\hat{\theta} - \theta)^2]\). For the basic reconstruction beam sweeping algorithm as in (22), the diagonal loading coefficient is fixed as \(\sigma^2 = 1\).

Fig. 2 shows the reconstruction accuracy with different numbers of predetermined DOAs. The number of samples is fixed as \(K = 5000\). For the basic algorithm, when \(Q\) is small, the reconstruction accuracy can be improved significantly as the rising of \(Q\). When \(Q > \frac{2M}{K} - 1\), there have been sufficient predetermined DOAs and thus the NSE can be hardly reduced further by increasing \(Q\). For the low-complexity algorithm as in (31), when \(Q < \frac{2M}{K} - 1\), \(B^H_{m_1,m_2} B_{m_1,m_2}\) is ill-conditioned, and thus the low-complexity algorithm is only available when \(Q \geq \frac{2M}{K} - 1\). It shows that the low-complexity algorithm can achieve minimum NSE when \(Q = \frac{2M}{K} - 1\). Therefore, the reconstruction procedure can be accomplished within \((\frac{2M}{K} - 1)K\) samples. For comparison, Fig. 2 also presents the existing algorithm for the single RF chain case [15]. Different from (34), the predetermined DOAs are selected as uniformly distributed from \(-90^\circ\) to \(90^\circ\). In this situation, more predetermined DOAs are required to achieve the performances of the proposed algorithms in this paper, leading to a much longer procedure for SCM reconstruction.

Fig. 3(a) shows the reconstruction accuracy with different number of samples. The number of predetermined DOAs is fixed as \(Q = \frac{2M}{K} - 1\). For the low-complexity algorithm, the NSE can be reduced as the increasing of the number of samples. For the basic algorithm (\(N = 4\) in particular), however, the reconstruction accuracy can be hardly improved when \(K\) is large enough. This observation actually coincides with the result in Fig. 2. Fig. 3(a) also shows that better accuracy can be obtained with a smaller number of RF chains. This is because \(Q \cdot K\) samples are adopted in overall to reconstruct the SCM. As \(Q = \frac{2M}{K} - 1\), more samples will be adopted when \(N\) is small, and thus the NSE will be improved. For comparison, the traditional sample average algorithm is also included. As expected, the sample average algorithm has the worst performance because it adopts only \(K\) samples.

Fig. 3(b) shows the performance of MUSIC algorithm based on the reconstructed SCM with different numbers of samples (\(K = 500\) and \(K = 1000\)) and SNR is from \(-10\) dB to 5 dB. The sample-average based MUSIC algorithm [3] and the H-MUSIC algorithm [10] are also included for comparison. For the proposed algorithm, the number of predetermined DOAs is fixed as \(Q = \frac{2M}{K} - 1\). Since the NSE can be reduced as the reduction of the number of the RF chains, the DOA estimation accuracy can be also improved for massive MIMO systems with small number of RF chains. It also shows that the proposed algorithm can achieve even better performance than sample-average based MUSIC algorithm. This is because the sample average approach is not as accurate as the proposed SCM reconstruction algorithm, as shown in Fig. 3(a). Although H-MUSIC algorithm can be also used for DOA estimation in hybrid massive MIMO, the estimation accuracy is even worse than the classic MUSIC algorithm. This observation coincides with the result in [10].
A comparison on the computation complexity for the basic beam sweeping algorithm and the low-complexity algorithm is shown in Fig. 3(c). As expected, the low-complexity algorithm in this paper can reduce the computational burden due to the reduction of dimension of matrix-vector production. It also shows that the complexity reduction is more significant when the number of RF chains is smaller.

VI. CONCLUSION

In this paper, a beam sweeping approach has been proposed for SCM reconstruction in order to enable MUSIC algorithm in hybrid massive MIMO systems with multiple RF chains. We have presented the basic algorithm in the case with multiple RF chains. Then, a low-complexity algorithm has been also introduced by removing the repeated entries in the SCM. Furthermore, the selection of the predetermined DOAs has also been optimized and the matrix inversion in the low-complexity algorithm can be even avoided. Simulation results have shown that the proposed approach can achieve better performance than existing baselines.

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