Probabilistic assessment of hydrologic retention performance of green roof considering aleatory and epistemic uncertainties

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ABSTRACT

Green roofs (GRs) are well known for source control of runoff quantity in sustainable urban stormwater management. By considering the inherent randomness of rainfall characteristics, this study derives the probability distribution of rainfall retention ratio $R_r$ and its statistical moments. The distribution function of $R_r$ can be used to establish a unique relationship between target retention ratio $R_{r,T}$, achievable reliability $AR$, and substrate depth $h$ for the aleatory-based probabilistic (AP) GR design. However, uncertainties of epistemic nature also exist in the AP GR model that makes $AR$ uncertain. In the paper, the treatment of epistemic uncertainty in the AP GR model is presented and implemented for the uncertainty quantification of $AR$. It is shown that design without considering epistemic uncertainties by the AP GR model yields about 50% confidence of meeting $R_{r,T}$. A procedure is presented to determine the design substrate depth having the stipulated confidence to satisfy $R_{r,T}$ and target achievable reliability $AR_T$.

Key words | green roof, probabilistic-based design, probability, retention ratio, uncertainty analysis

HIGHLIGHTS

- Derive the probability distribution of the rainfall retention ratio of green roof (GR) and its statistical moments.
- Present an aleatory-based probabilistic (AP) model for GR design.
- The paper shows that the design without considering epistemic uncertainties by the AP GR model yields about 50% confidence of meeting target retention ratio.
- Propose a methodology to treat epistemic uncertainty in the AP model for the uncertainty quantification of achievable reliability.
- Demonstrate the analysis procedures via a numerical example to determine GR substrate depth having the stipulated confidence to satisfy target retention ratio and target reliability.

INTRODUCTION

The use of green roofs (GRs) is becoming popular in sustainable urban stormwater management. Contributions of GRs to urban runoff control and management are primarily attributed to their retention and detention abilities, which not only reduce runoff volume but also delay and attenuate runoff peak discharge (Berndtsson 2010; Mobilia et al. 2015; Stovin et al. 2017). Other than the advantages in the hydrologic aspect, GRs can improve biodiversity in urban areas,
enhance urban runoff water quality, moderate heat island effect, improve building energy efficiency, remove pollutants in the air, and increase the life expectancy of building roof systems (Vijayaraghavan 2016).

Evaluations of the hydrologic performance of GRs have been made by field monitoring of prototype or laboratory-scale facilities over a selected period of time (e.g., Carter & Rasmussen 2006; Getter et al. 2007; Soulis et al. 2017; Johannessen et al. 2018). Also, a commonly seen approach is the use of the model of various types to evaluate hydrologic performance by simulating the runoff response of a schematized GR system under different rainfall conditions (Li & Babcock 2014; Ercolani et al. 2018; Mora-Melià et al. 2018). Depending on the performance indicators of interest, the complexity of the modeling tool may vary (Mobilia et al. 2015). To assess detention performance, one would use a physical-based hydrologic/hydraulic model that can produce time-varying runoff hydrographs from the GRs. For retention evaluation, a simple lumped hydrologic model considering event-based water balance (Carter & Jackson 2007; Starry et al. 2016; Chai et al. 2017) would generally be sufficient.

A commonly used hydrologic indicator for GR’s retention performance is the rainfall retention ratio \( R_r \):

\[
R_r = \frac{v - v_{rg}}{v} = 1 - \frac{v_{rg}}{v} \tag{1}
\]

in which \( v \), \( v_{rg} \) are the rainfall amount and the corresponding runoff volume from a GR system, respectively. The complimentary performance indicator to retention ratio \( R_r \) is the runoff production ratio \( R_p \):

\[
R_p = \frac{v_{rg}}{v} = 1 - R_r \tag{2}
\]

From the runoff control viewpoint, a GR system with a lower runoff production ratio or higher rainfall retention ratio is more desirable.

**Basic GR hydrologic model**

As the focus of performance herein is hydrologic retention, a simple lumped water balance model (Zhang & Guo 2013) for a GR system is adopted:

\[
R_c = S_l + S_c + (\theta_{fc} - \theta_i)h \tag{3}
\]

where \( R_c \) is the retention capacity of the GR system; \( S_l \) is the interception by plants; \( S_c \) is the capacity of the storage layer; \( \theta_{fc} \) is the field capacity of the substrate; \( \theta_i \) is the initial soil moisture content at the beginning of each rainstorm event; \( h \) is the depth of the substrate. The term, \( (\theta_{fc} - \theta_i)h \), in Equation (3) is the available water holding capacity (WHC) in the substrate during a rainstorm event (Allen et al. 1998; Fassman & Simcock 2012). Assuming the substrate is maintained above the plant’s wilting point, \( \theta_{wp} \), the GR system reaches its maximum retention capacity \( R_{c,max} \) when \( \theta_i = \theta_{wp} \) as:

\[
R_{c,max} = S_l + S_c + (\theta_{fc} - \theta_{wp})h \tag{4}
\]

The term \( (\theta_{fc} - \theta_{wp})h \) is the maximum WHC of the substrate.

The initial soil moisture \( \theta_i \) at the beginning of a rainfall event depends on the length of antecedence dry period \( b \), evapotranspiration (ET) rate \( E_a \), and evapotranspirable water content \( W_i \) in the GR system at the end of the preceding rainfall event. The runoff volume from a GR system can be obtained as (Zhang & Guo 2013):

\[
v_{rg} = \begin{cases} 
0, & v \leq R_{c,max}, \ b > \frac{W_i}{E_a} \\
v + Wi - R_{c,max} - E_ab, & v > R_{c,max} - Wi + E_ab, \ b \leq \frac{W_i}{E_a} \\
v - R_{c,max}, & v > R_{c,max}, \ b > \frac{W_i}{E_a} 
\end{cases} \tag{5}
\]
From Equation (5), one is able to determine runoff volume $v_{rg}$ from which the GR retention ratio can be calculated by Equation (1).

**Uncertainties in GR performance evaluation**

Referring to Equation (5), hydrologic retention assessment of a GR system involves uncertainties from various sources, which can be generally categorized into two types: aleatory and epistemic uncertainties. The former is due to the inherent natural randomness of rainfall events such as rainfall depth, duration, inter-event dry period, and temporal pattern. On the other hand, epistemic uncertainties arise from knowledge insufficiency about the rainfall-runoff transformation process in GR systems (i.e., the model), and lack of complete characterization of model parameters associated with the soil-plant-climatic system. Therefore, the assessment of the performance of a GR system in reality cannot be certain. It would be desirable to quantify the uncertainty features of the performance indicators as affected by various sources of uncertainty so that a more comprehensive analysis and design of GR systems can be made.

Probabilistic modeling of a GR system can be approached in two ways. One is an implicit approach by which a deterministic model describing involved hydrologic/hydraulic processes is coupled with long-term historically observed or stochastically synthesized climatic inputs to generate plausible realizations of the system outputs (Carson et al. 2013; Stovin et al. 2013; Locatelli et al. 2014; Cipolla et al. 2016; Chow et al. 2017). The alternative is an explicit approach to analytically derive the probabilistic features of the system performance indicators (e.g., retention ratio) for a GR system (Zhang & Guo 2013; Guo et al. 2014; Guo 2016). The implicit approach has the advantage of being able to preserve the physical features of involved processes more fully, but it is more computationally intensive. The explicit approach can provide a direct assessment of the uncertainty features of the system performance without extensive simulation, but more simplifications and idealizations of the system behaviors might be required.

Note that $R_p$ and $R_f$ in Equation (2) are functions of rainfall amount and the corresponding runoff volume produced which, in turn, is affected by rainstorm inter-event dry period and properties of substrate (e.g., depth, porosity, and hydraulic conductivity) and vegetation (e.g., plant type and ET). Many of these factors affecting retention (or runoff production) ratio are subject to uncertainties. Specifically, uncertainties associated with rainfall amount and the inter-event time of storm are of aleatory nature, whereas uncertainties corresponding to model parameters defining the properties of substrate, vegetation, and climate are of epistemic type. The importance of incorporating both aleatory and epistemic uncertainties in reliability evaluation has been elaborated in groundwater remediation (Hora 1996), structural systems (Der Kiureghian & Ditlevsen 2009), seismic modeling (Lambardi 2017), and detention basin capacity determination (Tung 2017).

**Outline of the study**

The overall probabilistic analysis of GR retention performance presented herein consists of two stages (see Figure 1). Stage-I, described in the ‘AP GR Model’ section, treats aleatory uncertainty from the randomness of rainfall properties. Based on the probability distribution of the GR runoff volume derived by Zhang & Guo (2013), close-form expressions for the probability distribution of retention ratio $R$ from which the corresponding statistical moments are derived. This probability distribution of $R$ can be used for the design of the GR system considering the reliability of achieving target retention ratio. Stage-II, described in the ‘Incorporating Epistemic Uncertainty in the AP GR Model’ section, further deals with epistemic uncertainties associated with the model parameters. Uncertainty features associated with the aleatory-based probabilistic (AP) GR model are quantified for assessing the confidence of meeting the target retention ratio and target achievable reliability. Through a numerical example, the ‘Illustration’ section demonstrates the probabilistic behaviors of retention ratio for a GR system and the relations between achievable reliability with substrate depth (Stage-I). Moreover, a method to systematically evaluate the statistical properties of the AP GR model considering epistemic uncertainty is presented (Stage-II). Through uncertainty analysis (UA), a reliability-based design of substrate depth for the GR system with a specific confidence of meeting target retention ratio $R_{t,T}$ and achievable reliability $AR_T$ can be implemented.
AP GR MODEL

Consider the aleatory uncertainty due to the natural randomness of rainfall amount $V$ and inter-event dry period $B$. By taking these two rainfall properties to be statistically independent random variables, each, respectively, has an exponential distribution with the probability density functions (PDFs) defined as:

Rainfall volume $(V)$: $f_V(v) = \zeta e^{-\zeta v}, \quad v \geq 0$ (6)

Inter-event dry time $(B)$: $f_B(b) = \psi e^{-\psi b}, \quad b \geq 0$ (7)
Based on the CDF and PDF of FRr in which B function (CDF) and PDF of the GR runoff volume ĊVrg as functions of the rainfall distribution parameters ζ and ψ as:

\[
\begin{align*}
\text{CDF: } F_{\text{Vrg}}(v_{\text{rg}}) &= \Pr(V_{\text{rg}} \leq v_{\text{rg}}) \\
&= 1 - \frac{e^{-\psi(v_{\text{rg}} + R_{\text{c,max}})}}{\psi + \zeta E_a} (\psi e^{\psi W} + \zeta E_a e^{-(\psi W/E_a)}) \text{ for } v_{\text{rg}} \geq 0
\end{align*}
\]

\[
\begin{align*}
\text{PDF: } f_{\text{Vrg}}(v_{\text{rg}}) &= \begin{cases} \\
1 - \frac{e^{-\psi(R_{\text{c,max}})}}{\psi + \zeta E_a} (\psi e^{\psi W} + \zeta E_a e^{-(\psi W/E_a)}), & v_{\text{rg}} = 0 \\
\frac{\zeta e^{-\psi(R_{\text{c,max}})}}{\psi + \zeta E_a} (\psi e^{\psi W} + \zeta E_a e^{-(\psi W/E_a)}), & v_{\text{rg}} > 0
\end{cases}
\end{align*}
\]

where \( F_{\text{Vrg}}(\cdot) \) and \( f_{\text{Vrg}}(\cdot) \) are, respectively, the CDF and PDF of the GR runoff volume.

Probability distribution of rainfall retention ratio

Based on the CDF and PDF of \( V_{\text{rg}} \) given in Equations (8) and (9), this section presents the distribution functions of retention ratio \( R_r \) as:

\[
\begin{align*}
\Pr(R_r = \eta') &= e^{-\psi(W/E_a + (R_{c,max}/\eta'))} \\
&\quad + \left( \frac{\psi}{\psi + (\zeta E_a/\eta')} \right) \left[ e^{-((R_{c,max} - W)/\eta'))} - e^{-((R_{c,max}/\eta') + (W/E_a))} \right]
\end{align*}
\]

\[
E(R_r) \approx 1 - E(V_{\text{rg}}) + \frac{1}{E^2(V)} \text{ Cov}(V_{\text{rg}}, V)
\]

in which \( E(\cdot) \) is the statistical expectation operator. Note that the above approximation assumes that random rainfall amount and runoff volume are statistically independent. Since \( R_r \) is non-linearly related to rainfall amount \( V \) and runoff volume \( V_{\text{rg}} \), and the latter is also affected by the former, this indicates that rainfall amount and runoff volume are correlated. However, the first-order linear approximation, given by Equation (13), does not account for dependence between rainfall amount and runoff volume. By considering the second-order approximation, \( E(R_r) \) can be estimated by (Tung & Yen 2003):

\[
\begin{align*}
E(R_r) &\approx 1 - E(V_{\text{rg}}) + \frac{1}{E^2(V)} \text{ Cov}(V_{\text{rg}}, V) \\
&\quad + \frac{E(V_{\text{rg}}) - E(V)}{E^2(V) Var(V)}
\end{align*}
\]

\[
\begin{align*}
f_{R_r}(\eta') &= \begin{cases} \\
\frac{\zeta e^{-((R_{c,max}/\eta'))}}{\eta^2} \left( \frac{e^{-\psi(W/E_a)}}{\psi + (\zeta E_a/\eta')} \right)^{\frac{R_{c,max}}{\psi + (\zeta E_a/\eta')}} - \frac{\psi E_a}{(\psi + (\zeta E_a/\eta'))^2} \right) \text{ for } 0 \leq \eta' < 1 \\
\Pr(V_{\text{rg}} = 0), & \eta' = 1
\end{cases}
\end{align*}
\]
which shows that the information about the variance of rainfall amount and its correlation with the runoff volume, represented by $\text{Cov}(V_{rg}, V)$, also play a role in estimating $E(r)$. The covariance of $V_{rg}$ and $V$ can be obtained from:

$$\text{Cov}(V_{rg}, V) = E(V_{rg} V) - E(V_{rg})E(V)$$

(15)

where $E(V) = 1/\zeta$ defined by Equation (6) and

$$E(V_{rg} V) = \int_0^\infty \int_0^\infty v_{rg} f(v_{rg}, v) dv_{rg} dv = \int_0^\infty \int_0^\infty v_{rg} f(v_{rg}, v) dv_{rg} dv = \int_0^\infty \int_0^\infty f(v_{rg}, v) dv_{rg} dv = \int_0^\infty \int_0^\infty f(v_{rg}, v) dv_{rg} dv = \int_0^\infty \int_0^\infty f(v_{rg}, v) dv_{rg} dv = \int_0^\infty \int_0^\infty f(v_{rg}, v) dv_{rg} dv = \int_0^\infty \int_0^\infty f(v_{rg}, v) dv_{rg} dv = \int_0^\infty \int_0^\infty f(v_{rg}, v) dv_{rg} dv = \int_0^\infty \int_0^\infty f(v_{rg}, v) dv_{rg} dv = \int_0^\infty \int_0^\infty f(v_{rg}, v) dv_{rg} dv$$

The mean runoff volume $E(V_{rg})$ in Equation (15) has been derived by Zhang & Guo (2013) as:

$$E(V_{rg}) = \frac{e^{-\zeta R_{c,\text{max}}}}{\psi + \zeta E_a} \left( \frac{2 + \zeta (R_{c,\text{max}} - W_i)}{\zeta^2} + \frac{E_a}{\zeta (\psi + \zeta E_a)} \right) (1 - e^{-\zeta W_i}) - \left( \frac{W_i}{\zeta} e^{-\zeta W_i} \right)$$

(16)

From the PDF of $R_e$, Equation (11), the statistical moments of $R_e$ of any order $m$ can be presented by:

$$E[R_e^m] = \int_0^1 (\psi)^m f_R(\psi) d\psi, \quad m = 1, 2, 3, \ldots$$

(18)

The analytical expression of $E(R_e)$ can be derived as:

$$E(R_e) = -E_1(\zeta R_{c,\text{max}} \left( \frac{E_a}{\psi R_{c,\text{max}}} \right)^2) e^{-\psi W_i/E_a}$$

$$+ E_1 \left( \left( \frac{\psi}{E_a} \right) R_{c,\text{max}} \left( \frac{E_a}{\psi R_{c,\text{max}}} \right)^2 e^{(-\psi W_i/E_a)} \right)$$

$$+ E_1 \left( \left( \frac{\psi}{E_a} \right) (R_{c,\text{max}} - W_i) \left( \frac{E_a}{\psi R_{c,\text{max}}} \right) e^{(-\psi W_i/E_a)} \right)$$

$$- E_1 \left( \left( \frac{\psi}{E_a} \right) R_{c,\text{max}} - W_i \right) \left( \frac{E_a}{\psi R_{c,\text{max}}} \right) e^{(-\psi W_i/E_a)}$$

$$+ \left( \frac{\zeta (R_{c,\text{max}} - W_i)}{\psi (R_{c,\text{max}} - W_i)} - \frac{1}{\zeta (R_{c,\text{max}} - W_i)} \right) e^{-\zeta (R_{c,\text{max}} - W_i)} + 1$$

(19)

In which $E_1(\theta)$ is the exponential integral defined as (Abramowitz & Stegun 1964):

$$E_1(\theta) = \int_\theta^\infty \frac{e^{-t}}{t} dt$$

(20)

Similarly, the analytical expression for the variance of $R_e$ can be obtained from $\text{Var}(R_e) = E[R_e^2] - E^2[R_e]$ in which, for the skew coefficient of $R_e$, it can be derived from the third-order moment $E[R_e^3]$.

$$E[R_e^2] = E_1(\zeta R_{c,\text{max}} \left( \frac{E_a}{\psi R_{c,\text{max}}} \right)^2 + \frac{E_a}{\zeta R_{c,\text{max}}} e^{(-\psi W_i/E_a)} - E_1 \left( \left( \frac{\psi}{E_a} \right) R_{c,\text{max}} \right) \left( \frac{E_a}{\psi R_{c,\text{max}}} \right)^2 e^{(-\psi W_i/E_a)}$$

$$- E_1 \left( \zeta (R_{c,\text{max}} - W_i) \right) \left( \frac{E_a}{\psi R_{c,\text{max}}} \right) e^{(-\psi W_i/E_a)}$$

$$+ \left( \zeta (R_{c,\text{max}} - W_i) \right) \left( \frac{E_a}{\psi R_{c,\text{max}}} \right) e^{(-\psi W_i/E_a)}$$

$$+ \left( \frac{\zeta (R_{c,\text{max}} - W_i)}{\psi (R_{c,\text{max}} - W_i)} - \frac{1}{\zeta (R_{c,\text{max}} - W_i)} \right) e^{-\zeta (R_{c,\text{max}} - W_i)} + 1$$

(21)
AP GR design

To quantify the probabilistic performance of a GR system by solely considering aleatory uncertainty, achievable reliability is utilized herein as a performance indicator:

$$AR(R_{r,T}; h) = Pr(R_r \geq R_{r,T}; h) = 1 - F_{R_{r,T}}(R_r, T; h) \quad (22)$$

where $AR(R_{r,T}; h)$ is the achievable reliability of meeting the target retention ratio, $R_{r,T}$, conditioned on substrate depth, $h$. As shown in Equations (10) and (22), a unique functional relation can be established between the distributional properties of $R_r$ (i.e., distribution, statistical moments, and achievable reliability) and $h$ because $R_{c,max}$, as shown in Equation (4), is a function of $h$.

However, this unique relation for substrate depth $h$, target retention ratio $R_{r,T}$, and achievable reliability $AR(R_{r,T}; h)$, defined by Equation (22), would not exist when model parameters describing the rainfall-runoff transformation processes are subject to epistemic uncertainties. Under such circumstance, treating model parameters that characterize soil, plant, and climatic properties as deterministic constants would render a GR design not achieving the target performance with desired confidence. In the following section, an analysis framework is presented to treat the epistemic uncertainties imbedded in the AP GR model and to incorporate their effects in the evaluation and design of the GR systems.

INCORPORATING EPISTEMIC UNCERTAINTY IN THE AP GR MODEL

To quantify the overall uncertainty of the GR model, the parameters subject to epistemic uncertainty, in addition to aleatory ones, that affect probabilistic features of GR performance (such as $V_n$, $R_s$, and $AR$) should also be analyzed. In this study, the five model parameters subject to epistemic uncertainties are $\theta_k$, $\theta_{wp}$, $S_i$, $E_u$, and initial solid moisture ratio, $C_i$. Their effects on the retention performance of a GR system are briefly described. Uncertainty features of these model parameters appeared in the literature are collected and analyzed in this section. The procedures to incorporate epistemic uncertainty are outlined in Stage-II of Figure 1.

Epistemic uncertainties in GR model parameters

Field capacity and wilting point of the substrate

The substrate in GR establishment provides water, nutrients, and physical sustentation to vegetation. Among the physical properties of the GR substrate, field capacity and wilting point are the two key players affecting substrate’s WHC.

Field capacity $\theta_{fc}$ indicates substrate’s ability to retain water against gravitational pull (DeNardo et al. 2005). Bengtsson (2005) showed that runoff from GR occurs when soil moisture content reaches $\theta_{fc}$. Wilting point $\theta_{wp}$ is the soil water content that is held so tightly by the soil matrix that cannot be extracted by roots. It mainly depends upon the soil moisture profile, root distribution, plant transpiration rate, and temperature (Taylor & Ashcroft 1972).

In GR establishments, engineered substrates are often used. When commercial substrates are not available or too costly, substrates from locally available materials such as pasture soils or top soils are used (Brenneisen 2006; Duszka et al. 2017; Gong et al. 2018). Like most construction materials, within the properties of the GR substrate (engineered or natural) there exist some degree of variation due to the heterogeneity of the substrate in its production and installation. In a study to compare the consistency of measuring substrate physical properties by three standard test methods in Germany, USA, and Australia, Conn et al. (2020) show that, with replicate of soil samples, not only different test methods might produce inconsistent test results but also each standard test method yielded some variation of substrate properties.

In general, engineered substrates are more homogeneous, and therefore, their properties have narrower range of variation than those of natural substrates. For natural soils, Taylor & Ashcroft (1972, p. 300) present a diagram showing the range of values for $\theta_{fc}$. Raghuwanshi & Mailapalli (2017, p. 144-2) provided a typical range of $\theta_{fc}$ and $\theta_{wp}$ for different types of natural soils; some of which are found in the literature for the GR substrate. In a study of predicting soil WHC in terms of $(\theta_{fc} - \theta_{wp})$ over Korea, Hong et al. (2013) provided the statistical features (i.e., mean and standard deviation) of $\theta_{fc}$ and $\theta_{wp}$ of different natural soils. For natural soils that are found in use for GR substrates, i.e., sand, sandy loam, and loam, data listed in...
Raghuwanshi & Mailapalli (2017) indicate that $\theta_{fc}$ has variation of 33.3, 28.6, 18.0%, respectively, whereas $\theta_{wp}$ has variation of 53.8, 33.3, 21.4%, respectively. For engineered or natural substrates used in GRs, information about the variation of $\theta_{fc}$ and $\theta_{wp}$ is relatively scarce. The great majority of the literature and commercial substrate specifications reports only the averaged or nominal values of substrate properties without offering the information about their ranges of variation. To assess the uncertainty features of $\theta_{fc}$ and $\theta_{wp}$, it is advisable to test some substrate samples used in GR installation. The practice of repeated measuring of soil samples is sometimes found in GR studies, mostly involving laboratory experiments.

In the GR literature, very few studies directly report the magnitude of uncertainty of $\theta_{fc}$ and $\theta_{wp}$, while most show variation of $WHC$ of substrate used. Young (2014) investigated the substrate in three prototype GRs in Sheffield, UK in that several test sites in each GR installation were selected and six soil samples in each test site were analyzed. The average values $WHC$ for the substrates at the three GR installations range from 0.359 to 0.590 with standard errors varying from $\pm 0.12$ to $\pm 0.43$. In a simulation study by Soulis et al. (2017) on GR test beds, a replica of three samples of the artificial substrate used yields a mean $WHC$ of 0.542 with a standard error of $\pm 0.0165$. In the study of Dusza et al. (2017) on multi-functionality of the GR plant and substrate, five samples of two different substrates yielded an average $WHC$ of 0.33 for natural sandy-loam soil and 0.41 for artificial substrate, with standard errors of $\pm 0.213$ and $\pm 0.299$, respectively.

Since the standard errors associated with the mean $\theta_{fc}$ and $\theta_{wp}$ of the substrate samples were not directly reported, a backward statistical analysis is made to estimate the coefficient of variation ($CV$) of field capacity in test substrate samples. Based on the relation of $WHC = \theta_{fc} - \theta_{wp}$, by assuming (i) $CV(\theta_{fc}) \equiv CV(\theta_{wp})$ and (ii) $\text{Mean}(\theta_{fc}) \gg \text{Mean}(\theta_{wp})$, the coefficient of variation of $\theta_{fc}$ can be estimated as $CV(\theta_{fc}) = \sqrt{n} \times \frac{SE(WHC)}{WHC}$, in which $n$ is the soil sample size, $SE(WHC)$ is the standard error of average $WHC$.

The results of the backward analysis yielded an estimation of the coefficient of variation of field capacity in substrate samples of 8.2–17.4% for the three GR installations in Young (2014); 5.3% for the artificial substrate sample in Soulis et al. (2017); and 14.3 and 16.3% for natural and artificial substrates, respectively, in Dusza et al. (2017). Unlike the great majority of commercial GR substrates only list nominal value of $WHC$, it is interesting to find a company ‘Ferm-O-Feed’ in the Netherlands, which shows the $WHC$ of its basic substrates for extensive GR is in the range of 0.30–0.50. This reveals a $\pm 0.1$ variation for $WHC$ for the substrate. Compared with the variation of field capacity and wilting point in soils occurred in nature (Hong et al. 2013; Raghuwanshi & Mailapalli 2017), the magnitude of the variation of properties of engineered or laboratory-made substrates is smaller, but not negligible.

**Interception**

The overall retention of GR systems is a combination of plant interception, internal storage capacity of the vegetation, and storage capacity of the substrate, among which the majority of the storage capacity is provided by the substrate (Martin 2008; Stovin et al. 2015; Fryer 2017a). Having said that, Martin (2008) noted that, under a smaller rainfall event, the effect of interception loss $S_i$ on GR systems cannot be ignored. Interception amount during a storm event may vary due to variations in vegetation type, density, and canopy covers. Some quantifications of interception for vegetation used in GR can be found in the literature. For example, Soulis et al. (2017) found interception for sedum ranges in 0.5–6.1 mm; Carter & Jackson (2007) used 3.1 mm based on urban forest. However, very few reports exist in the literature about the uncertainty of interception.

To authors’ limited understanding, the study by Fryer (2017a, 2017b) is the only one providing information on the variation of interception of two plant types in GR. Her laboratory measurements showed that interception for sedum has about 15% variation and 30% for meadow grasses. The 30% variation of interception for natural meadow grasses happens to be close to the study by Miralles et al. (2010) for event-based interception on a global scale, which reported an estimation of $S_i$ ranging with 1.2 ± 0.4 mm.

**ET rate**

ET rate $E_d$ encompasses the integral effect of soil, plant, and climatic components in a GR system. In general, *in situ*
measurement of ET by instruments is costly and valid data for actual ET are difficult to obtain directly (Zhao et al. 2013). Alternatively, suitable empirical and semi-empirical models are often used to indirectly estimate ET by relating it to environmental factors (such as air temperature, relative humidity, solar radiation, and wind speed). Among others, potential ET models commonly found in GR applications are Penman–Monteith equation, Thornthwaite equation, Hargreaves equation, and Priestley–Taylor equation. Marasco et al. (2014) measured the hourly $E_a$ at two extensive GRs in New York by the dynamic chamber method and compared the measured ET values with two ET estimation methods. They observed that the Penman-based method was better than the energy balance method. A literature review by Feng (2018) on ET values of green infrastructure reported that the ET rate of GRs generally falls within 0.003–11.38 mm/day. Ebrahimian et al. (2019) also provided a review concerning ET on runoff retention of green stormwater infrastructure and reported that the range of ET rate for pilot-scale GRs has a narrower range of 0.5–3.5 mm/day. Through literature review, Ebrahimian et al. (2019) also concluded that none of the available ET predictive equations, primarily derived for agricultural applications, do not accurately match observed ET data for GRs and rain gardens.

Note that the above-reported variations for GR ET are obtained from different locations of varying climatic conditions and vegetation. Therefore, they are not suitable for direct use for the uncertainty of ET at a given GR site. For in situ uncertainty quantification by any ET semi-empirical equations, measure error and random variation of model inputs/parameters reflecting local environmental conditions should be used. Although there are reports of GRs ET rates as mentioned above, information on the associated uncertainty is lacking. Nichols et al. (2004) applied the error propagation method to quantify the uncertainty of potential ET in the semi-arid region in New Mexico by three models, and the results are Penman 13%, Preistley–Taylor 18%, and Penman–Monteith 10%. McMahon et al. (2013) compared eight potential ET models and summarized their variations of estimation. Following ISO ‘Guides to the expression of Uncertainty in Measurement’ (ISO 2010), Chen et al. (2018) quantified the uncertainty of ET prediction associated with two equations for indoor cultivation due to inaccuracy of sensor measurements of environmental variables. Their examples showed that inaccuracy in sensor measurements could result in 8.4 and 17.3% of uncertainty in model-based ET prediction. Talebmorad et al. (2020) applied the bootstrap method, based on 55 years of climatic data at synoptic station in Isfahan, Iran, to evaluate the uncertainty of the mean and variance of monthly reference crop ET estimated by FAO-56 Penman–Monteith and Hargreaves–Samani models. The ranges of the variation of estimated mean monthly ET values are 0.228–0.786 mm/day by FAO-56 Penman–Monteith equation and 0.176–0.362 mm/day by Hargreaves–Samani equation, which are equivalent to 6.9–28.2% by the former equation and 5.0–14.9% by the latter. These ranges of variation in monthly ET estimate are attributed to uncertainty in climatic inputs/parameters to prediction equations. UA approaches cited above and others can be applied to quantify ET rate uncertainty when typical vegetation types in GRs are used.

**Initial soil moisture**

In the above AP GR model, Equations (8)–(11), the evapotranspirable water amount $W_i$ depends on the initial soil moisture $\theta_i$ of substrate and other system parameters as:

$$W_i = \begin{cases} R_{C,max} & \text{if } V \geq S_i + S_c + (\theta_c - \theta_i)h \\ V + (\theta_i - \theta_{ap})h, & \text{otherwise} \end{cases}$$  

(23)

in which the value of $\theta_i$ should be bounded in $[\theta_{ap}, \theta_c]$. In this study, $\theta_i$ is represented by the initial soil moisture ratio $C_i = (\theta_i/\theta_c)$ and treated as one of the model parameters. Since $\theta_i$ largely depends on the rainfall characteristics, substrate properties, and climatic factors, its value could be highly variable from one rainfall event to another. Physically, $C_i$ is bounded within $[(\theta_{ap}/\theta_c), 1]$ of which the lower bound of the bounding interval depends on two soil moisture characteristics subject to uncertainty.

**Method of UA**

When a design is based on model outputs that, in turn, are functions of several model parameters with uncertainty, the design problem becomes decision-making under uncertainty. Quantifying information about the uncertainty
features of model outputs on which the design is affected is
the objective of UA.

Methods of varying levels of sophistication have been
developed and used for UA. They can broadly be classified
into analytical methods and approximated methods. The
former can directly derive the probability distribution and/or
statistical moments of model outputs from those of
random model parameters. The latter estimate the statistical
features of model outputs through model evaluation at
selected sampling points in the parameter space. Different
approximated methods choose different sets of sampling
points according to their theoretical considerations. The
analytical methods, although mathematically elegant,
are restrictive in their applications to practical problems.
Several approximated methods (such as the
first-order variance estimation method, probabilistic point
estimation methods, and Monte Carlo simulation) are often
used in UA of practical problems. Detailed descriptions of the
various UA methods can be found elsewhere (Ayyub &
Gupta 1998; Cacuci 2003; Tung & Yen 2005). Due to the
complexity of the functional relation between the GR
model parameters and concerned model output (e.g., aleatory-based retention performance reliability), a sampling-

Generating model input-output database

In this study, the Latin hypercube sampling (LHS) scheme
(Iman & Helton 1988; McKay 1988) is used for establishing
an input-output database of the probabilistic GR model for
UA. The concerned model outputs herein are the achievable
reliability $AR(R_{c,T}, h|X)$ under a stipulated target retention
ratio $R_{c,T}$ and substrate depths $h$ as affected by the above-
mentioned five model parameters subject to the uncertainty
of epistemic nature, i.e., $X = (\theta_{fc}, \theta_{wp}, S_l, E_a, C_l)$. The LHS
scheme is chosen for its computational efficiency and accurate
estimation of model outputs’ statistical features.

LHS samples involving $K$ independent random model
parameters with sample size $M$ can be generated by
(Pebesma & Heuvelink 1999):

$$x_{km} = F_k^{-1}\left(\frac{S_{km} - U_{km}}{M}\right), \quad m = 1, 2, \ldots, M; \quad k = 1, 2, \ldots, K$$

where $x_{km}$ is the $m$th generated random variate of the $k$th
random variable $X_k$; $F_k^{-1}(\cdot)$ is the inverse CDF of the $k$th
random variable; $S_{km}$ is a random permutation of 1 to $M$
for the $k$th random variable, $X_k$; and $U_{km}$ is a uniform
random variate in $[0, 1]$, i.e., $U_{km} \sim U[0, 1]$. For a problem
involving $N$ concerned model outputs (e.g., achievable
reliability of varying target retention ratios and substrate
depths) and $K$ random model parameters, an input-output
database can be generated by the LHS scheme as:

$$\{y_{1m}, y_{2m}, \ldots, y_{Nm}\} = g(x_{1m}, x_{2m}, \ldots, x_{Nm}), \quad m = 1, 2, \ldots, M$$

where $g(\cdot)$ is the representation of the model;
$\{y_{1m}, y_{2m}, \ldots, y_{Nm}\}$ is the vector of $N$ model outputs
obtained under the condition of $K$ model parameters
$(x_{1m}, x_{2m}, \ldots, x_{Nm})$ in the $m$th LHS sample. Based on $M$
sets of model outputs, $\{y_{1m}, y_{2m}, \ldots, y_{Nm}\}_{m=1,2,\ldots,M}$, one
can easily estimate the statistical features (e.g., distribution
model and statistical moments) of the $N$ concerned model
outputs.

There are various applications of the LHS scheme to UA
of hydrosystem engineering problems which include, but are
not limited to, sediment transport modeling (Yeh & Tung
1993), water-quality modeling (Manache & Melching
2004), rainfall-runoff modeling (Yu et al. 2001; Christiaens &
Feyen 2002), storm water best management practices (Park
et al. 2011), dam overtopping (Goodarzi et al. 2013), water
erosion prediction (Ascough et al. 2013), urban drainage
modeling (Li et al. 2014), hydrological and sediment
modeling (Shen et al. 2012), and surface and subsurface hydrologic
simulations (Miller et al. 2017).

Quantification of the uncertainty of achievable reliability

To further improve the accuracy and stability in estimating
the uncertainty features of the GR’s achievable reliability,
$AR(R_{c,T}, h|X)$ in Equation (22), the LHS scheme, in
conjunction with the antithetic variates (AV) technique, are
utilized in this study for UA. The AV technique (Hammersley
& Morton 1956) attains the goal of variance reduction by
generating random variates that induce the negatively corre-
lated quantity of interest between separate simulation runs.
By the AV technique, the statistical features (e.g., moments) of concerned output, $Y$, can be estimated by computing the arithmetic average of its two unbiased estimators as:

$$\hat{\Omega}_Y = \frac{1}{2} [\hat{\Omega}_Y(U') + \hat{\Omega}_Y(U'')] \quad (26)$$

in which, $\hat{\Omega}_Y$ is the estimator of the unknown statistical features, $\Omega_Y$, of concerned model output $Y$; $\hat{\Omega}_Y(U')$, $\hat{\Omega}_Y(U'')$ are the unbiased estimators of the statistical features of model output $Y$ based on random samples generated from two uniform random variates $U' \sim U[0, 1]$ and $U'' \sim U[0, 1]$, respectively. With $U'' = 1 - U'$, the two uniform random variables $U'$ and $U''$ are negatively correlated, the two estimators $\hat{\Omega}_Y(U')$ and $\hat{\Omega}_Y(U'')$ would be unbiased, but negatively correlated estimators of $\Omega_Y$. Then, the adopted estimator $\hat{\Omega}_Y$ by Equation (26) will have smaller variance than each individual estimator, $\hat{\Omega}_Y(U')$ and $\hat{\Omega}_Y(U'')$, for estimating the statistical quantity of concerned model output, $\Omega_Y$.

**Probabilistic GR design considering aleatory and epistemic uncertainties**

According to Equation (22), $AR(R_{r,T}, h|X)$ depends on the GR system parameters $X$ that are subject to epistemic uncertainties. Hence, $AR$ is also a quantity subject to uncertainty for any stipulated $R_{r,T}$ and $h$ as schematically shown by the two distributions in Figure 2. Through UA, the statistical features of the GR performance indicator $AR$ (such as its probability distribution and statistical moments) can be quantified and their functional relations with the $R_{r,T}$ and $h$ established.

Suppose that $AR$ is a random variable having a CDF $F_{AR}(\theta_{AR})$, defined by its distributional parameters $\theta_{AR}$. Because $AR$ to meet a specified $R_{r,T}$ is uncertain, from design viewpoint, one would wish to determine the substrate depth $h$, such that the GR system can meet the target reliability $AR_T$ with a specified confidence level $\omega$. Referring to Figure 2, the GR system with $h$ having confidence $\omega$ of meeting the desired $R_{r,T}$ and $AR_T$ is the exceedance probability as shown by the shaded areas. The two solid curves in Figure 2 each represents confidence level, $\omega_1$ and $\omega_2$ ($\omega_2 > \omega_1$), respectively. As can be seen that, to maintain the same $R_{r,T}$ and $AR_T$ with a higher confidence $\omega$, one has to increase substrate depth $h$. In the context of design, considering aleatory and epistemic uncertainty simultaneously, the design substrate depth $h_{dsgn}$ can be determined by solving the following equation:

$$\omega_{dsgn} = Pr[AR(R_{r,T}, h_{dsgn}) \geq AR_T|\theta_{AR}] = 1 - F_{AR(R_{r,T}, h_{dsgn})}[AR_T|\theta_{AR}] \quad (27)$$

where $\omega_{dsgn}$ is the desired confidence level; $F_{AR(R_{r,T}, h_{dsgn})}[AR_T|\theta_{AR}]$ is the CDF of random $AR(R_{r,T}, h_{dsgn})$.

Since the value of $AR$ is bounded between 0 and 1, it is reasonable to postulate that the $AR$ follows a standard Beta distribution, i.e., $AR \sim f_{\text{Beta}}(\mu_{AR}, \sigma_{AR})$, with its mean and standard deviation that are related to $R_{r,T}$ and $h$. Namely, $\mu_{AR}(R_{r,T}, h)$ and $\sigma_{AR}(R_{r,T}, h)$ can be explicitly expressed in terms of $R_{r,T}$ and $h$. Under the condition of standard Beta distribution for $AR$, the design substrate depth $h_{dsgn}$ having design confidence $\omega_{dsgn}$ of meeting $R_{r,T}$ and $AR_T$ can be determined by solving:

$$F_{AR(R_{r,T}, h_{dsgn})}[AR_T|\theta_{AR}] = 1 - \omega_{dsgn} \quad (28)$$

in which, $F_{AR(R_{r,T}, h_{dsgn})}[AR_T|\theta_{AR}]$ is the CDF of random $AR$.

**ILLUSTRATION**

To illustrate the probabilistic performance of an extensive GR system considering aleatory and epistemic uncertainty, data used in Zhang & Guo (2013) are adopted herein. The two rainfall properties at Metro International Airport
Station in Detroit from April 1 through October 31 have been tested to follow exponential distributions with the mean values listed in Table 1(a). The substrate of the GR system is made of loamy soil, and the values of soil, plant, and climatic parameters listed in Table 1(b), according to Zhang & Guo (2013), are taken to be the mean values. As the initial soil moisture ratio \(C_i\) is bounded within the interval of \([\theta_{wp}, \theta_{fc}]\), 1, without monitored data available, the range of variation of \(C_i\) is set in [0.5, 1] in which the lower bound is determined by the ratio of the mean values of \(\theta_{wp}\) and \(\theta_{fc}\) for illustration purposes. Without losing generality, uniform distribution, which corresponds to maximum entropy in comparison with other unimodal distributions, is adopted herein for each random model parameter. Furthermore, the five model parameters in Table 1(b) are treated to be independent random variables, and their ranges of variation are consulted from the data in the literature presented in the ‘Epistemic uncertainties in GR model parameters’ section only for illustration purposes.

To quantify model parameter uncertainty that reflects the on-site condition, it is desirable to analyze the variation of local climatic variables and conduct tests on limited in situ GR substrate samples. It should also be noted that the statistical features of retention ratio (e.g., mean value and achievable reliability) presented in this example should be referenced to the period of analysis of rainfall record (i.e., April–October in this example).

**Behavior of the AP GR model**

By considering the AP GR model, this section examines the statistical features of \(R_r\) according to the derived analytical expressions in the ‘AP GR Model’ section. To facilitate the illustration and discussion, data for the mean values of model parameters in Table 1 are used.

**Distribution of retention ratio**

The PDF and CDF (Equations (10) and (11)) of \(R_r\) under \(W_i = R_{c,max}\) are shown in Figure 3, respectively. These two figures clearly show that \(R_r\) is a random variable with discontinuity at \(R_r = 1\) where the GR produces no runoff. As the substrate depth increases, the probability of producing zero runoff at \(R_r = 1\) gets higher and the curve corresponding to \(0 < R_r < 1\) drops lower as shown in Figure 3.

For a fixed substrate depth, \(h = 100\) mm, Figure 4 shows a comparison of the exceedance probability (1-CDF) of \(R_r\) under the conservative and optimistic conditions of evapotranspirable water. With regard to rainwater retention performance, \(W_i = R_{c,max}\) represents the conservative scenario, whereby more water in the substrate is available for ET during the dry period. Under such circumstance, the available \(WHC\) in the substrate to accommodate the incoming rainfall would be less, so is the corresponding \(R_r\). Hence, the likelihood that the GR system to have \(R_r\) exceeding a stipulated value would be lower than that of under the optimistic condition of \(W_i = 0\).

**Statistical moments of retention ratio**

Figure 5 shows the variations of mean and standard deviation of \(R_r\) with substrate depth under conservative and optimistic values of \(W_i\). The mean \(R_r\) (the two blue lines) increases with substrate depth because available \(WHC\) of the substrate becomes larger. Also, the rate of increase in mean \(R_r\) is decreasing with substrate depth.

---

**Table 1 | Statistical properties of uncertain factors in the example extensive GR**

| (a) Model inputs subject to aleatory uncertainty | (b) Model parameters subject to epistemic uncertainty |
|-----------------------------------------------|-----------------------------------------------|
| Rainfall event amount, \(V\) | ET rate, \(E_a\) (mm/h) | Field capacity, \(\theta_c\) | Intercepttion loss, \(S_l\) (mm) |
| \(E(V) = 14.35\) mm (Exponential) | 0.11 \(\pm\) 25% (Uniform) | 0.232 \(\pm\) 15% (Uniform) | 2 \(\pm\) 30% (Uniform) |
| Dry period between rainfall events, \(B\) | Wilting point, \(\theta_{wp}\) | Initial soil moisture ratio, \(C_i\) | |
| \(E(B) = 97.95\) h (Exponential) | 0.116 \(\pm\) 20% (Uniform) | 0.5–1 (Uniform) | |

Note: \(\pm\)% value defines the range of variation about the mean value.
As for the standard deviation of $R_r$ (the two red lines), Figure 5 reveals that the variability of $R_r$ is lower when the value of $W_i$ is smaller. The variability of retention ratio with substrate depth is fairly stable under $W_i = R_{c,\text{max}}$, whereas it decreases with substrate depth under $W_i = 0$. This can be explained that with a low value of $W_i$, the
corresponding initial soil moisture content at the beginning of the incoming rainfall event would be low and the available WHC of the substrate would be high to accommodate the rainfall event. The net effect would result in a higher mean and lower standard deviation of $R_r$.

As presented in the ‘Statistical moments of retention ratio’ section, the mean $R_r$ can be estimated by several ways. Figure 6 shows a comparison of estimated mean $R_r$ by the first-order method, Equation (15), the second-order method, Equation (14), and the analytical solution, Equation (19). In comparison with the analytical solution, Figure 6 shows that the first-order method significantly underestimate the mean $R_r$, whereas the second-order method, as expected, provides significant improvement with somewhat over-estimation as the substrate depth increases. This is because that the correlation between the rainfall amount and runoff volume plays an important role in estimating the mean $R_r$.

**GR design using the AP model**

Based on Equation (10), the CDF of $R_r$ can be used to define a unique relationship for $AR$, substrate depth, $h$, and $R_{c,T}$ as shown in Figure 7, under the conservative condition $W_i = R_{c,max}$. Clearly, for a given $h$, the $AR$ of a GR system decreases with increase in $R_{c,T}$. For a specific $R_{c,T}$, the performance reliability increases with $h$. One can also see that by increasing $h$, the higher $R_{c,T}$ would be expected while maintaining $AR$ at the same.

Since $W_i = R_{c,max}$ corresponds to a conservative condition with possible minimum WHC in the substrate, Figure 7 defines the lower bound of $AR - R_{c,T} - h$ relationship for a given $h$. The upper $AR - R_{c,T} - h$ curve can be obtained under the optimistic condition of $W_i = 0$. Under the normal condition, the reliability would be somewhere between the two curves.

**Uncertainty quantification of achievable reliability considering epistemic uncertainties**

According to LHS samples of size 50, 100, 200, 300, 500, and 1,000 for the five GR model parameters in Equation (22), it was found that the estimated values of the first three statistical moments of $AR$ did not satisfactorily converge, even under the sample size of 1,000. Hence, the AV technique for variance reduction is incorporated in the LHS scheme to enhance a stable and accurate estimation. As it turns out that the estimated statistical moments of $AR$ from the AP GR model had a quick and satisfactory convergence with sample size of only 100.

To estimate the uncertainty features of $AR(R_{c,T}, h)$, random variates of the five parameters having epistemic uncertainty are generated by the LHS scheme jointly with the AV technique as:

$$u': AR(R_{c,T}, h)_m = g(E'_{a,m}, \theta'_{fc,m}, \theta'_{wp,m}, S'_i,m, C'_{i,m}), \quad m = 1, 2, \ldots, M$$  
(29)

$$u'': AR(R_{c,T}, h)_m = g(E''_{a,m}, \theta''_{fc,m}, \theta''_{wp,m}, S''_i,m, C''_{i,m}), \quad m = 1, 2, \ldots, M$$  
(30)

in which $u'' = 1 - u'$. The LHS/AV-based statistical properties of $AR$ from the AP GR model can be computed
according to Equation (26). For example, the raw moments of any order of \( AR(R_{r,T}, h) \) by considering the epistemic uncertainty can be estimated by:

\[
E[AR(R_{r,T}, h)^s] = \frac{1}{2} \{ E[AR(R_{r,T}, h)^s] + E[AR(R_{r,T}, h)^{2s}] \},
\]

where \( s \) is the order statistical moment; and

\[
E[AR(R_{r,T}, h)^{2s}] = \frac{1}{M} \left\{ \sum_{m=1}^{M} [AR(R_{r,T}, h)_m]^{s} \right\};
\]

\[
E[AR(R_{r,T}, h)^{4s}] = \frac{1}{M} \left\{ \sum_{m=1}^{M} [AR(R_{r,T}, h)_m]^{s} \right\}.
\]

(31)

(32)

Then, the mean of LHS/AV-based estimator of \( AR(R_{r,T}, h) \) can be estimated by Equation (31) with \( s = 1 \) and the variance with \( s = 2 \) as:

\[
\sigma^2[AR(R_{r,T}, h)] = E[AR(R_{r,T}, h)^2] - E^2[AR(R_{r,T}, h)]
\]

(33)

Other than the statistical moments, it is also important to assess its probability distribution of \( AR(R_{r,T}, h) \) for reliability-based analysis and design of GR systems.

**Statistical moments of \( AR(R_{r,T}, h, X) \)**

Based on 100 LHS/AV samples, Figures 7 and 8 show the mean and standard deviation of \( AR(R_{r,T}, h|X) \), respectively. As expected, Figure 8 reveals that the mean achievable reliability, \( E_X[AR(R_{r,T}, h|X)] \) increases with \( h \), but decreases with \( R_{r,T} \). The nominal values of \( AR(R_{r,T}, h|X = \bar{x}) \) in Figure 8 are those obtained by using mean values of the model parameters \( \bar{x} \) listed in Table 1(b) without considering the epistemic uncertainties. It can be seen that the consideration of epistemic uncertainties yields slightly lower values of \( E_X[AR(R_{r,T}, h|X)] \) for thinner substrate depth, whereas for thicker substrate, the values of \( E_X[AR(R_{r,T}, h|X)] \) are higher than the nominal \( AR \).

The standard deviation of achievable reliability, \( SD_X[AR(R_{r,T}, h|X)] \) also shows an increasing trend with the \( h \) (see Figure 9). However, the rate of increase is quite different under different \( R_{r,T} \). For higher target retention ratio (\( R_{r,T} = 0.7 \) and 0.9), \( SD_X[AR(R_{r,T}, h|X)] \) monotonically increases with the substrate depth over the range of 50–150 mm, while under the lower target retention ratio (\( R_{r,T} = 0.5 \)), \( SD_X[AR(R_{r,T}, h|X)] \) starts to drop slightly from the peak value (0.0614) at \( h = 125 \) mm to 0.0603 at \( h = 150 \) mm.

**Distribution of \( AR(R_{r,T}, h|X) \)**

To identify a suitable distribution model for random \( AR(R_{r,T}, h|X) \) due to the presence of epistemic uncertainty, the chosen distribution model should reflect the properties of the collected data and could be validated through goodness-of-fit tests. In this study, 100 LHS/AV-generated random samples of \( AR(R_{r,T}, h) \) under different combinations
of $R_{r,T}$ and $h$ were tested for their goodness-of-fit to the standard Beta distribution. The standard Beta distribution was considered for being theoretically bounded in [0, 1] and versatile in shape. For illustration, Figure 10 shows the quantile-quantile (Q-Q) plot of sample values of $AR(R_{r,T}, h)$ against the Beta-based quantiles under $R_{r,T} = 0.7$ and five substrate depths $h$. The data points closely follow the $45^\circ$ line. Similar behavior of Q-Q plots are found for other combinations of $R_{r,T}$ and $h$. This indicates that the standard Beta distribution model is highly acceptable to describe the random behavior of $AR(R_{r,T}, h|X)$ induced by model parameters with epistemic uncertainty. The Kolmogorov–Smirnov (KS) test was also used to formally examine the goodness-of-fit of the standard Beta distribution to $AR$ sample data. The results reveal that the $p$-values range in $(41.6, 98\%)$, which are much higher than the commonly used significant level of 5%. Thus, the use of standard Beta distribution for random $AR(R_{r,T}, h|X)$ is validated.

By adopting the standard Beta distribution for $AR(R_{r,T}, h)$, one can determine the quantile values of achievable reliability as:

$$AR(q|R_{r,T}, h) = F_{\text{Beta}}^{-1}(q | E_X[AR(R_{r,T}, h|X)], SD_X[AR(R_{r,T}, h|X)])$$

(34)

where $AR(q|R_{r,T}, h)$ is the $q$th-order quantile of uncertain $AR(R_{r,T}, h)$; $F_{\text{Beta}}^{-1} [\cdot]$ is the inverse CDF of the standard Beta variable; $E_X[AR(R_{r,T}, h|X)]$, $SD_X[AR(R_{r,T}, h|X)]$ are the mean and standard deviation of $AR(R_{r,T}, h)$, respectively, both are functions of $R_{r,T}$ and $h$. Figure 11 illustrates a series of quantile curves of achievable reliability under $R_{r,T} = 0.7$ and different $h$ that can be established by Equation (34). One can also establish a confidence band of estimated $AR(R_{r,T}, h)$. For instance, the 90% confidence band can be defined by using $q = 95\%$ quantile curve (blue solid line) as the upper bound and $q = 5\%$ quantile curve (blue dash line) as the lower bound. Figure 11 also shows that, without considering epistemic uncertainties, the nominal achievable reliability is slightly above the 50% quantile curve for shallow substrate and becomes lower than the 50% curve with thicker $h$. This means that, by considering only aleatory uncertainty, the use of substrate depth with $h \leq 75\text{ mm}$ for an extensive GR, its corresponding nominal achievable reliability $AR(R_{r,T}, h|X = \bar{x})$ to meet $R_{r,T} = 0.7$ has about 50% confidence. On the other hand, using thicker substrate depth with $h > 75\text{ mm}$, the confidence that the nominal achievable reliability to meet $R_{r,T} = 0.7$ is less than 50%. This means that the nominal achievable reliability of the system to meet the $R_{r,T}$ tends to be conservative because it underestimates the actual median achievable reliability. Furthermore, to maintain on the same level of non-exceedance probability, $q$, the required $h$ to meet a $R_{r,T}$ would increase with achievable reliability.
Reliability-based GR design considering both aleatory and epistemic uncertainties

The reliability-based GR design requires the establishment of functional relationships between the statistical properties of standard Beta distribution and the two design parameters \((R_{r,T} \text{ and } h)\). The two parameters of standard Beta distribution are related to the first two moments as:

\[
\alpha_{AR} = (1 - \mu_{AR}) \left(\frac{\mu_{AR}}{\sigma_{AR}}\right)^2 - \mu_{AR} \tag{35}
\]

\[
\beta_{AR} = \mu_{AR} \left(\frac{1 - \mu_{AR}}{\sigma_{AR}}\right)^2 - (1 - \mu_{AR}) \tag{36}
\]

In which \(\alpha_{AR}, \beta_{AR}\) are the parameters of standard Beta distribution; \(\mu_{AR}, \sigma_{AR}\) are the mean and standard deviation of \(AR(R_{r,T}, h)\), respectively. Based on 100 LHS/AV-generated samples, the empirical functional relations between the mean \(\mu_{AR}\) and standard deviation \(\sigma_{AR}\) of \(AR(R_{r,T}, h)\) with \(R_{r,T}\) and \(h\) are established, respectively, through regression analysis as:

\[
\mu_{AR}(R_{r,T}, h) = 0.7413 - 0.9883 R_{r,T} + 0.0325 h
+ 0.3871 R_{r,T}^2 - 0.00079 h^2
+ 0.001659 R_{r,T} h \tag{37}
\]

\[
\sigma_{AR}(R_{r,T}, h) = 0.02573 + 0.004612 h - 0.03304 R_{r,T}^2
- 0.000223 h^2 + 0.003187 R_{r,T} h \tag{38}
\]

In Equations (37) and (38), range of data in \(R_{r,T}\) and \(h\) used are 0.4–0.9 and 5–15 cm, respectively. The coefficient of determination corresponding to the above two empirical equations are both 0.999. Utilizing Equations (37) and (38), the mean and standard deviation of \(AR(R_{r,T}, h)\) for a pair of \((R_{r,T}, h)\) can be computed which, in turn, can be used to determine the corresponding Beta distribution parameters defined by Equations (35) and (36). Then, the design substrate depth, \(h_{dsgn}\), corresponding to \(R_{r,T}, AR_T\), and design confidence, \(\omega_{dsgn}\), can be obtained by solving

\[
F_{AR|R_{r,T}, h}(AR_T | \alpha(AR| h_{dsgn}, R_{r,T}), \beta(AR| h_{dsgn}, R_{r,T})) = 1 - \omega_{dsgn} \tag{39}
\]

where \(\alpha(\cdot)\) and \(\beta(\cdot)\) are the parameters of standard Beta distribution describing random achievable reliability. The design confidence \(\omega_{dsgn}\) is the probability that the random achievable reliability exceeds the stipulated target \(AR_T\).

Figure 12 shows an example design diagram obtained from solving Equation (39) that defines the relationship between \(h_{dsgn}\) with confidence \(\omega_{dsgn}\) to meet target \(R_{r,T}\) and \(AR_T\). It is clear that, for the fixed value of \(R_{r,T}\), \(h_{dsgn}\) increases with \(\omega_{dsgn}\) and \(AR_T\).

CONCLUSION AND DISCUSSION

By considering the inherent randomness of rainfall amount of individual rainstorm event and inter-event dry period, this study extends the work of Zhang & Guo (2013) to derive the PDFs of the GR retention ratio \(R_r\), based on which the analytical expression for the exact mean and variance of the retention ratio are derived. The analytical expression allows direct calculations of relevant statistical characteristics of \(R_r\) to rapidly assess the hydrological retention performance of a GR system without intensive simulation.

This study also evaluates the accuracy of estimating the mean \(R_r\) by two approximations with respect to the exact solution. The simplistic first-order approximation shows under-estimation of the true mean \(R_r\). The second-order approximation significantly improves the first-order estimation because the variance of the rainfall amount and its correlation with the runoff volume play an important role.
Physically, runoff volume from a GR system is caused by rainfall, and therefore, the two random quantities should be positively correlated. This paper shows that such a correlation indeed is quite strong.

From the distribution function of random \( R_t \), the relationship between target retention ratio \( R_{T, T} \), achievable reliability \( AR \), and substrate depth \( h \) for the AP GR model can be established. For illustration, the study shows a unique relation between the design substrate depth \( h_{dsgn} \) and \( AR \). Nonetheless, there exists non-rain factors describing soil, plant, and climatic properties that affect the rainfall-runoff transformation process in the GR system. These non-rain factors are model parameters subject to the uncertainty of epistemic nature induced by knowledge deficiency. When epistemic uncertainty is taken into consideration, the \( AR \) obtained from the AP model is no longer certain. Thus, in order to have a comprehensive reliability assessment of the GR design, epistemic uncertainties are further incorporated in the AP model.

This study presents a systematic framework to assess the influence of the epistemic uncertainty on the performance of a GR system. Due to highly nonlinearity of the model parameter-output relations, the AV technique was jointly implemented with the LHS scheme in UA to obtain fast, stable, and accurate estimations of the statistical features of \( AR \). Furthermore, the standard Beta distribution was found to fit the distribution of \( AR \) satisfactorily. One can easily construct the quantile curves and confidence intervals of \( AR \) according to its estimated moments from the LHS/AV procedure.

This study shows that the design of a GR system by considering only aleatory uncertainty due to the natural randomness of rainfall characteristics would roughly have 50% confidence to achieve the desired \( R_t \). To determine a substrate depth for achieving the target reliability \( (AR_T) \) with 50% confidence or higher, when epistemic uncertainties are considered, one would have to use a thicker substrate depth \( h \). The incremental depth beyond the nominal \( h \) (under aleatory uncertainty only) depends on the degree of epistemic uncertainty and the design confidence level \( (0_{dsgn}) \). This incremental depth can be viewed as the safety margin to account for the presence of epistemic uncertainties. The proposed analysis framework leads to a more comprehensive and complete analysis/design of a GR system.

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DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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