Non-collapsing renormalized QRPA with proton-neutron pairing for neutrinoless double beta decay

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Abstract

Using the renormalized quasiparticle random phase approximation (RQRPA), we calculate the light neutrino mass mediated mode of neutrinoless double beta decay (0νββ-decay) of $^{76}$Ge, $^{100}$Mo, $^{128}$Te and $^{130}$Te. Our results indicate that the simple quasiboson approximation is not good enough to study the 0νββ-decay, because its solutions collapse for physical values of $g_{pp}$. We find that extension of the Hilbert space and inclusion of the Pauli Principle in the QRPA with proton-neutron pairing, allows us to extend our calculations beyond the point of collapse, for physical values of the nuclear force strength.

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As a consequence one might be able to extract more accurate values on the effective neutrino mass by using the best available experimental limits on the half-life of $0\nu\beta\beta$-decay.
Neutrinoless double beta decay ($0\nu\beta\beta$), which involves the emission of two electrons and no neutrinos, requires the neutrino to be a Majorana particle with non-zero mass. This process violates the lepton number conservation and occurs in some theories beyond the standard model (see e.g. [1]-[4] for reviews). The $0\nu\beta\beta$-decay is not yet observed. The experimental lower limits on the half-lives of some nuclei which undergo double beta decay, provide us with the most stringent limits on the effective mass of the electron neutrinos and the parameters of the right-handed currents after evaluating the corresponding nuclear matrix elements.

The calculation of the $0\nu\beta\beta$-decay matrix elements is based on some approximation schemes, from which the Quasiparticle Random Phase Approximation (QRPA) is the most prominent one [5]-[8], despite an uncertainty in the calculation of the nuclear matrix element, which is related to its sensitivity on the renormalization of the particle-particle component of the residual interaction. Thus in the framework of proton-neutron QRPA (pn-QRPA) using the zero range delta-force Engel et al. [8] have found a strong suppression of the $0\nu\beta\beta$-decay matrix elements by introducing short-range correlations, whereas Tomoda et al. [5] and Muto et al. [7] by using a more realistic effective NN interaction arrived at the opposite conclusion.

Recently new approaches for the calculation of the nuclear many body Green function have been proposed, which are supposed to offer more reliable results than that of pn-QRPA. The pn-QRPA has been extended to include proton-neutron pairing (full-QRPA) [9]. It has been found that proton-neutron pairing influences the $0\nu\beta\beta$-decay rates significantly [10]. Toivanen and Suhonen [11], Schwieger, Šimkovic and Faessler [12] and F. Krmpotić et al. [13] have studied the effect of the violation of the Pauli principle in the correlated ground state of the pn-QRPA introducing the renormalized pn-QRPA (pn-RQRPA). Schwieger et al. [12] have done calculations of the two-neutrino double beta decay matrix elements using this renormalized pn-QRPA but also including proton-neutron pairing (full-RQRPA). They have shown [12] that the inclusion of both p-n pairing and an improved treatment of the Pauli principle removes the difficulties arising from the strong dependence of the matrix elements.
element on the particle-particle strength $g_{pp}$ in the standard pn-QRPA.

The $0\nu\beta\beta$-decay allows to determine an upper limit of the effective Majorana electron neutrino mass $< m_\nu >$. For this process the most stringent experimental lower limit of the life-time for the $0\nu\beta\beta$-decay of $^{76}Ge$ favours especially this nucleus for extracting an upper limit of $< m_\nu >$. The purpose of this article is to study in detail the effects of the Pauli principle, the proton-neutron pairing and the truncation of the nuclear Hamiltonian in the evaluation of the nuclear matrix element governing the $0\nu\beta\beta$-decay of $^{76}Ge$. In addition the full-RQRPA will also be used to calculate $0\nu\beta\beta$-decay of $^{100}Mo$, $^{128}Te$ and $^{130}Te$. The stability of the nuclear matrix elements in respect to the renormalization of the particle-particle force will be discussed.

The full-RQRPA, which describes the excited states of the even-even nucleus, has been studied in Ref. [12]. Therefore, here we shall present only the formulae relevant to this work.

In the notation of Ref. [12], the half life of the $0\nu\beta\beta$-decay in the case of the neutrino mass mechanism is given by,

$$ \left( T_{1/2}^{0\nu} \right)^{-1} = G_{01} (M_{mass}^{0\nu})^2 \left( \frac{< m_\nu >}{m_e} \right)^2 $$

(1)

with the effective neutrino mass

$$ < m_\nu > = \sum_j |U_{ej}|^2 m_j e^{i\alpha_j}, $$

(2)

where $\exp(i\alpha_j)$ is the CP eigenvalue of the neutrino mass eigenstate $|\nu_j>$, $U_{ei}$ is the element of the unitary neutrino mixing matrix and $m_e$ is the mass of the electron. $G_{01}$ is the integrated kinematical factor for the $0_i^+ \rightarrow 0_f^+$ transition [1]-[4]. The nuclear matrix element $M_{mass}^{0\nu}$ consists of Fermi and Gamow-Teller contributions

$$ M_{mass}^{0\nu} = M_{GT}^{0\nu} - \left( \frac{g_\nu}{g_A} \right)^2 M_{F}^{0\nu}, $$

(3)

where

$$ M_{F}^{0\nu} \left\{ \begin{array}{l} M_{GT}^{0\nu} \\ \langle H(r_{12}) \frac{1}{\sigma_1 \cdot \sigma_2} \rangle \end{array} \right\}, $$

(4)
\[ < O_{12} > = \sum_{j_{1} j_{2}, \mathcal{J}} (-)^{j_{1} j_{2} + j_{1} + j_{2}} (2 \mathcal{J} + 1) \begin{pmatrix} j_{1} & j_{2} & J \\ j_{1}' & j_{2}' & \mathcal{J} \end{pmatrix} \]

\[ \times \langle [c_{pk}^{+} \tilde{c}_{nl}]_{J} | J \rangle \]

\[ \langle J^\pi m_f | J^\pi m_i > \times \langle 0^+_f \parallel [c_{pk}^{+} \tilde{c}_{nl}]_{J} \parallel 0^+_i > . \] (5)

The short-range correlations between the two interacting nucleons are taken into account by a correlation function \( f(r_{12}) \). The neutrino-potential \( H(r_{12}) \) in the case of light neutrino exchange takes the form

\[ H(r) = \frac{2 R}{\pi r} \int_{0}^{\infty} \frac{\sin(qr)}{q + (\Omega_{j_{f} m_{f} j_{i}}^{m_{i}} + \Omega_{j_{f} m_{f} j_{i}}^{m_{i}})} \frac{1}{(1 + q^2 / \Lambda^2)^{4}} dq. \] (6)

Here, \( R = r_0 A^{1/3} \) is the nuclear radius \( (r_0 = 1.1 \text{ fm}) \) and the parameter \( \Lambda \) of the dipole shape nucleon form factor is chosen to be 0.85 GeV \[ 8,10 \]. In the full-RQRPA for the matrix elements of the one-body transition densities we obtain

\[ < J^\pi m_i \parallel [c_{pk}^{+} \tilde{c}_{nl}]_{J} \parallel 0^+_i > = \sqrt{2J + 1} \sum_{\mu, \nu = 1, 2} m(\mu k, \nu l) \left[ u_{k\mu p}^{(i)} u_{l\nu n}^{(i)} \mathcal{X}_{\mu\nu}^{m_i}(k, l, J^\pi) \\
+ v_{k\mu p}^{(i)} u_{l\nu n}^{(i)} \mathcal{X}_{\mu\nu}^{m_i}(k, l, J^\pi) \right] \sqrt{D_{i\mu\nu}^{(i)}}, \] (7)

\[ < 0^+_f \parallel [c_{pk}^{+} \tilde{c}_{nl}]_{J} \parallel J^\pi m_f > = \sqrt{2J + 1} \sum_{\mu, \nu = 1, 2} m(\mu k', \nu l') \left[ u_{k'\mu p'}^{(f)} u_{l'\nu n'}^{(f)} \mathcal{Y}_{\mu\nu}^{m_f}(k', l', J^\pi) \\
+ v_{k'\mu p'}^{(f)} u_{l'\nu n'}^{(f)} \mathcal{Y}_{\mu\nu}^{m_f}(k', l', J^\pi) \right] \sqrt{D_{i\mu\nu}^{(f)}}, \] (8)

with \( m(\mu a, \nu b) = \frac{1 + (-1)^{l_{\mu} l_{\nu}} \delta_{\mu b} \delta_{\nu a}}{(1 + \delta_{\mu b} \delta_{\nu a})^{1/2}} \). We note that the \( \mathcal{X}_{\mu\nu}^{m_i}(k, l, J^\pi) \) and \( \mathcal{Y}_{\mu\nu}^{m_f}(k, l, J^\pi) \) amplitudes are calculated by the renormalized QRPA equation only for the configurations \( \mu a \leq \nu b \) (i.e., \( \mu = \nu \) and the orbitals are ordered \( a \leq b \) and \( \mu = 1, \nu = 2 \) and the orbitals are not ordered) \[ 12 \]. For different configurations \( \mathcal{X}_{\mu\nu}^{m_i}(k, l, J^\pi) \) and \( \mathcal{Y}_{\mu\nu}^{m_f}(k, l, J^\pi) \) in Eqs. (7) and (8) are given by following the prescription in Eqs. (65) and (66) of Ref. \[ 8 \]. The index \( i \) \( (f) \) indicates that the quasiparticles and the excited states of the nucleus are defined with respect to the initial (final) nuclear ground state \( |0_i^{+} > \) \( (|0_f^{+} > ) \). The overlap between two intermediate nuclear states belonging to two different sets is given in Ref. \[ 12 \].
We note that for $D_{k\mu l\nu J} = 1$ the expressions (7) and (8) are just the one body transition densities in the full-QRPA [10]. If in addition $u_{2p} = v_{2p} = u_{1n} = v_{1n} = 0$ (i.e. there is no proton-neutron pairing), Eqs. (7) and (8) reduce to the expressions of the pn-QRPA [5,7].

We apply the pn-QRPA, full-QRPA (with proton-neutron pairing), pn-RQRPA and full-RQRPA methods to the $0^{\nu}\beta\beta$-decay of $^{76}\text{Ge}$ to study the effects of the proton-neutron pairing and the Pauli-principle violation on the nuclear matrix element $M_{\text{mass}}^{0\nu}$. In previous calculations that model space has been used, which described best the beta strength distributions for the initial and final nuclei. However, there is a substantial difference between the beta strength matrix element and the double beta decay matrix element. Unlike the case of the double beta decay, which is a second order process, meson exchange currents do not play an important role in the calculation of the single beta strength distribution. The meson exchange currents are incorporated in our calculation through the G-matrix elements of the nuclear Hamiltonian. We suppose that they can be taken into account in a proper way only if the inclusion of Pauli principle is considered, and a large enough model space is assumed. In addition we wish to stay as closely as possible to the Brueckner reaction matrix of the Bonn potential. A smaller model space requires a more drastic renormalization of the force. Since one does not know a reliable microscopic theory for the renormalization as a function of the model space, a smaller model space means more free parameters. Thus we enlarge the model space of the nuclear Hamiltonian. The model spaces used are the following:

9 – level model space: The full 3$\hbar\omega$ and 4$\hbar\omega$ major oscillator shells. This model space has been used by Tomoda et al. [5] and by Muto et al. [7].

12 – level model space: It consists of the full 2 – 4$\hbar\omega$ major oscillator shells.

21 – level model space: This model space extends over the 0 – 5$\hbar\omega$ major oscillator shells.

The single particle energies have been calculated with a Coulomb-corrected Woods-Saxon potential. As a realistic two-body interaction we use the nuclear matter G-matrix calculated from the Bonn one-boson-exchange potential [14]. We introduce the parameterization $d_\alpha G$ ($\alpha$=pp, nn and pn), $g_{ph} G$ and $g_{pp} G$ for the pairing, particle-hole and particle-particle interaction, respectively. The quasiparticle energies and amplitudes have been found by solving the
HFB-equation with proton-neutron pairing \[9\]. For a model space larger than two oscillator major shells we neglect the mixing of different ”n” but the same ”ljm” orbitals. We suppose that the Woods-Saxon potential is already a good potential and therefore shell mixing is not appreciably affecting the double beta decay. The renormalization of the pairing interaction has been determined to fit the empirical proton, neutron and proton- neutron pairing gaps according to Ref. \[9\]. The renormalization parameter \(d_\alpha\) together with the experimental proton (\(\Delta^\text{exp}_p\)), neutron (\(\Delta^\text{exp}_n\)) and proton - neutron (\(\delta^\text{exp}_{pn}\)) pairing gaps for \(^{76}\text{Ge}\) and \(^{76}\text{Se}\) are listed in Table I. We see that \(d_{pp}\) and \(d_{nn}\) values are close to unity. A \(d_{pn}\) value higher than unity is the price paid for the spherical symmetry of the model which excludes the treatment of the T=0 pairing. The J=0 T=0 pairs can be treated in a BCS or even HFB approach only due to deformation. The T=0 pairing is effectively taken into account by the renormalization of the T=1 J=0 n-p interaction leading to a higher value of \(d_{pn}\). We note that in the framework of the HFB method we have found that the importance of the proton-neutron pairing for the ground state properties of nuclei is decreasing significantly as isospin increases \[9\]. This fact is in agreement with the recent sophisticated study of J. Engel et al. \[15\].

Further, we fixed \(g_{ph} = 0.8\) as in our previous calculations \[10\] and only the particle-particle interaction strength \(g_{pp}\) is considered as a variable. Fig. 1 (a) shows the calculated nuclear matrix element \(M^{0\nu}_{\text{mass}}\) in the framework of the pn-QRPA for the \(0\nu\beta\beta\) of \(^{76}\text{Ge}\) as function of \(g_{pp}\). It is worthwhile to notice that by increasing the model space \(M^{0\nu}_{\text{mass}}\) becomes extremely sensitive to the \(g_{pp}\) and even within the physically acceptable region of \(g_{pp}\) collapses and crosses zero. This behaviour has its origin in the contribution of the \(J^{\pi} = 1^+\) intermediate states of the odd-odd nucleus to the many body Green function \(M^{0\nu}_{\text{mass}}\), which becomes too large because of the generation of too many ground state correlations. This feature has not been found in the previous calculations as the model spaces used there, were too small. We surmise that the different suppressions of the \(0\nu\beta\beta\)-decay matrix elements found by Tomoda et al. \[5\] and Muto et al. \[7\] on one side and Engel et al. \[6\] on the other side could have their origin not only in the different interactions but also in the different
model spaces used. Unfortunately, the model space of Ref. [11] is not specified to draw a definite conclusion. F. Krmpotić and S. Sharma [12] have calculated $0\nu\beta\beta$-decay of $^{76}$Ge by using eleven dimensional model space (The full $3\hbar\omega$ and $4\hbar\omega$ major oscillator shells plus $0h_{9/2}$ and $0h_{9/2}$ orbitals.), zero range $\delta$-force interaction and different treatment of the two-nucleon correlation function. Their results show also a strong suppression of the $M_{\text{mass}}^{0\nu}$ with increasing strength of the particle-particle force. To our knowledge, we are the first to study in details the extension of the Hilbert space for the $0\nu\beta\beta$-decay of $^{76}$Ge and showing that its solution collapse for physical value of $g_{pp}$ in the QRPA. Therefore, pn-QRPA method does not allow us to make definite $0\nu\beta\beta$-decay rate predictions.

In Fig. 1 (b) we show the results obtained in the framework of the full-QRPA, which take into account proton-neutron pairing [10]. The $M_{\text{mass}}^{0\nu}$ has been calculated till the collapse of the full-QRPA, which occurs for the $J^\pi = 0^+$ or $2^+$ channel. The early collapse of the full-QRPA for the 21-level model space does not allow us to draw conclusions inside the expected physical range $0.8 \leq g_{pp} \leq 1.2$ within this approximation.

The nuclear matrix element $M_{\text{mass}}^{0\nu}$ obtained within the pn-RQRPA is shown in Fig. 1 (c). From the comparison with the Fig. 1 (a) it follows that the inclusion of ground state correlations beyond the QRPA in the calculation of $M_{\text{mass}}^{0\nu}$ removes the difficulties associated with the strong dependence of $M_{\text{mass}}^{0\nu}$ on the particle-particle strength. It means that the Pauli principle plays a prominent role in the evaluation of the nuclear matrix elements governing the second order processes. We conclude, that the rather big difference between the results of 9-level and 12-level model spaces and the small difference between the results of 12-level and 21-level model spaces favours the 12-level model space for the study of double beta decay of $^{76}$Ge.

In Fig. 1 (d) we present the results with the full-RQRPA method. This method takes into account both proton-neutron pairing and the Pauli exclusion principle and does not show a breakdown for any realistic interaction strength. A comparison with Fig. 1 (c) shows that proton-neutron pairing decreases the values of $M_{\text{mass}}^{0\nu}$ for the small model space (9-levels) and increases them for bigger, more realistic model spaces. Thus the full-RQRPA allows an
extension of the calculations beyond the point of collapse and supports more stable solutions for calculating the $M_{0\nu \text{mass}}$, with respect to the renormalization of the particle-particle force.

In order to gain more confidence in the full-RQRPA method, we have also performed calculations for $0\nu \beta\beta$-decay of $^{100}\text{Mo}$, $^{128}\text{Te}$ and $^{130}\text{Te}$. As noticed already in Ref. [10], the evaluation of $M_{0\nu \text{mass}}$ is very sensitive to the truncation of the model space in the case of $^{100}\text{Mo}$. We have found that the model spaces used in the previous calculations were not sufficient to stabilize the results and therefore we have performed calculations with the 21-level model space defined above. For the Te isotopes we have introduced a model space, which comprises 20 levels: The full 2-5 $\hbar\omega$ major oscillator shells and $0i_{11/2,13/2}$ subshells. The calculated matrix elements $M_{0\nu \text{mass}}$ are shown in Fig. 2. For all nuclei studied $M_{0\nu \text{mass}}$ is only weakly dependent on the strength of particle-particle interaction. This allows us to have more confidence in the limits for the effective electron neutrino mass $< m_\nu >$ extracted by using the best presently available experimental limits on the half-lives of the $0\nu \beta\beta$-decays. We obtain the following limits:

\begin{align*}
^{76}\text{Ge} : & \quad < m_\nu > < 1.3 \text{ eV} \quad [T_{1/2}^{0\nu-\text{exp}} > 5.6 \times 10^{24} \text{ Ref. [17]}], \\
^{100}\text{Mo} : & \quad < m_\nu > < 2.4 \text{ eV} \quad [T_{1/2}^{0\nu-\text{exp}} > 4.4 \times 10^{22} \text{ Ref. [18]}], \\
^{128}\text{Te} : & \quad < m_\nu > < 1.1 \text{ eV} \quad [T_{1/2}^{0\nu-\text{exp}} > 7.3 \times 10^{24} \text{ Ref. [19]}], \\
^{130}\text{Te} : & \quad < m_\nu > < 4.5 \text{ eV} \quad [T_{1/2}^{0\nu-\text{exp}} > 2.3 \times 10^{22} \text{ Ref. [20]}].
\end{align*}

We see, that by increasing significantly the model space for $^{100}\text{Mo}$ the value of $M_{0\nu \text{mass}}$ is much bigger as in our previous calculations [10]. The rather big energy release for the $0\nu \beta\beta$-decay of $^{100}\text{Mo}$ and the large value of the matrix element $M_{0\nu \text{mass}}$ favour this nucleus for further experimental study.

The above results suggest, that in the calculation of the nuclear matrix element $M_{0\nu \text{mass}}$, it is necessary to take into account the Pauli principle and to consider a large enough model space in addition to pn-pairing. As it was noticed already by J. Hirsch, P. Hess and O. Civitarese [21] and by F. Krmpotić, A. Mariano, E.J.V. Passos, A.F.R. de Toledo Piza and T.T.S. Kuo [13], the price which is paid for taking into account the ground state correlation
beyond the QRPA, is the violation of the Ikeda Sum Rule. In the framework of the full-RQRPA we have

\[ S^+ - S^- = \sum_{k\mu l\nu} | \langle k \| \sigma \| l \rangle |^2 D_{k\mu l\nu} \]
\[ \times \left( v_{k\mu n}^2 - v_{k\mu p}^2 + v_{k\mu p} u_{k\mu n} v_{l\nu p} u_{l\nu m} - u_{k\mu p} v_{k\mu n} u_{l\nu p} v_{l\nu n} \right), \]

which yields \(3(N-Z)\) only when \(D_{k\mu l\nu} = 1\) (quasiboson approximation). If the usual HFB constraint on the particle number is used and if the model space contains both states \(j_{1/2, -1/2} = l \pm 1/2\) of the spin-orbit splitting, the QRPA fulfills the Ikeda Sum Rule independent of the chosen model space, which is not a realistic feature of the model. In the RQRPA we necessarily face its violation because of \(D_{k\mu l\nu} \neq 1\). By using the full-RQRPA for \(^{76}\text{Ge}\) and \(^{76}\text{Se}\) we have found \(S^+ - S^-\) to be \(0.8 - 0.9 \times 3(N - Z)\) in the physical acceptable region of the parameter \(g_{pp}\). It is supposed that the omission of the scattering terms in the operators for \(\beta^+\) and \(\beta^-\) transitions, could be the reason for this small mismatch [13,21]. It is worthwhile mentioning that the Ikeda Sum Rule is not satisfied even if the BSC equations are solved within the RQRPA with the condition that the average particle number in the correlated ground state is conserved [13]. Nevertheless we expect that a small violation of the Ikeda Sum Rule wont alter our main results.

In summary, we have studied the effects of Pauli principle violation and proton-neutron pairing on the calculation of the \(0\nu\beta\beta\)-decay matrix element. We have shown that the renormalised QRPA allows extending the calculations beyond the collapse and thus that the simple quasiboson approximation is not sufficiently accurate to calculate reliable the nuclear many-body Green function governing the \(0\nu\beta\beta\)-decay process. This becomes evident, if a larger, more realistic, Hilbert space is considered. We have found that the \(0\nu\beta\beta\)-decay matrix element calculated via the full-RQRPA, which includes the Pauli effect of fermion pairs, avoids the collapse for physical values of the nuclear force, and therefore we can argue that we have some evidence that the Pauli effect violation inherent in the traditional QRPA affects the \(0\nu\beta\beta\)-decay matrix element significantly. As a consequence, to our opinion, the results presented in the text for the effective neutrino mass for \(g_{pp} = 1\) are more accurate.
Note added. After completing this work, we received preprints by J. Hirsch et al. [22] and by J. Engel et al. [23] who studied the validity of the renormalized QRPA within a schematic exactly solvable models, which are not intended to reproduce actual nuclear properties. They confirmed that the renormalized QRPA offers advantages over the QRPA. However, they founded some discrepancies between the exact and the RQRPA solutions after the point of collapse of the QRPA. It is questioned whether these discrepancies should hold in more realistic calculations. If they would, it just supports our conclusion that we need a theory in which the Pauli principle is explicitly incorporated. In the renormalized QRPA the Pauli principle is considered in an approximate way. We note that there is no exactly solvable realistic model.
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TABLE I. Experimental proton ($\Delta_{p}^{\text{exp}}$), neutron ($\Delta_{n}^{\text{exp}}$) and proton - neutron ($\delta_{pn}^{\text{exp}}$) pairing gaps and renormalization constants of the proton - proton ($d_{pp}$), neutron - neutron ($d_{nn}$) and proton - neutron ($d_{pn}$) pairing interactions for studied nuclei.

| Nucleus ( $\Delta_{p}^{\text{exp}}, \Delta_{n}^{\text{exp}}, \delta_{pn}^{\text{exp}}$ ) | model | $d_{pp}$ | $d_{nn}$ | $d_{pn}$ |
|---------------------------------|-------|----------|----------|----------|
| $^{76}_{32}Ge_{44}$ (1.561, 1.535, 0.388) | 9 level | 1.063 | 1.238 | 2.093 |
|                                  | 12 level | 0.988 | 1.150 | 1.777 |
|                                  | 21 level | 0.899 | 1.028 | 1.506 |
| $^{76}_{34}Se_{42}$ (1.751, 1.710, 0.459) | 9 level | 1.135 | 1.255 | 1.678 |
|                                  | 12 level | 1.027 | 1.182 | 1.524 |
|                                  | 21 level | 0.934 | 1.059 | 1.325 |
| $^{100}_{42}Mo_{58}$ (1.612, 1.358, 0.635) | 21 level | 0.980 | 0.923 | 1.766 |
| $^{100}_{44}Ru_{56}$ (1.548, 1.296, 0.277) | 21 level | 1.002 | 0.945 | 1.568 |
| $^{128}_{52}Te_{76}$ (1.127, 1.177, 0.149) | 20 level | 0.873 | 0.942 | 1.780 |
| $^{130}_{52}Te_{78}$ (1.043, 1.180, 0.090) | 20 level | 0.835 | 0.945 | 1.87 |
| $^{128}_{54}Xe_{74}$ (1.307, 1.266, 0.199) | 20 level | 0.920 | 0.972 | 1.530 |
| $^{130}_{54}Xe_{76}$ (1.299, 1.243, 0.190) | 20 level | 0.914 | 0.974 | 1.614 |
FIGURES

FIG. 1. The calculated nuclear matrix element $M_{\text{mass}}^{0\nu}$ for the $0\nu\beta\beta$-decay of $^{76}\text{Ge}$ as a function of the particle-particle interaction strength $g_{pp}$. In (a), (b), (c) and (d) $M_{\text{mass}}^{0\nu}$ have been calculated within the pn-QRPA, full-QRPA, pn-RQRPA and full-RQRPA, respectively. The dotted line corresponds to the 9-level model space, the dashed line to the 12-level model space and the solid line to 21-level model space, respectively.

FIG. 2. The calculated nuclear matrix element $M_{\text{mass}}^{0\nu}$ for the $0\nu\beta\beta$-decay of $^{76}\text{Ge}$ within the full-RQRPA as function of particle-particle interaction constant $g_{pp}$. In the case of $^{76}\text{Ge}$ and $^{100}\text{Mo}$ ($^{128}\text{Te}$ and $^{130}\text{Te}$) the 21-level (20-level) model space has been used.
