We study the Okamoto-Nolen-Schiffer (ONS) anomaly in the binding energy of mirror nuclei at high density by adding a single neutron or proton to a quark gluon plasma. In this high-density limit we find an anomaly equal to two-thirds of the Coulomb exchange energy of a proton. This effect is dominated by quark electromagnetic interactions—rather than by the up-down quark mass difference. At normal density we calculate the Coulomb energy of neutron matter using a string-flip quark model. We find a nonzero Coulomb energy because of the neutron’s charged constituents. This effect could make a significant contribution to the ONS anomaly.

The Okamoto-Nolen-Schiffer (ONS) anomaly is the long-standing discrepancy between the calculated and measured binding-energy differences of mirror nuclei [1,2]. The anomaly is likely to arise from charge symmetry breaking (CSB) in the strong interaction [3], itself believed to originate from the up-down quark mass difference and electromagnetic effects in the Standard Model. Thus, the study of CSB is a useful tool to elucidate the structure of strongly-interacting nuclear systems.

The ONS anomaly can be calculated on several levels. Perhaps the simplest is the observation by B. A. Brown [4] that the magnitude of the anomaly is approximately equal to the Coulomb exchange energy. If one adds an extra proton to a nucleus in a simple Hartree-Fock picture, there will be both a direct (Hartree) and exchange (Fock) Coulomb interaction with the other protons. If one—arbitrarily—neglects the Fock term, one obtains a better agreement with experiment.

At a different level, Blunden and Iqbal compute the ONS anomaly by calculating the contribution from $\rho-\omega$ mixing to the CSB component of the nucleon-nucleon (NN) interaction [5]. Their CSB interaction can explain part of the anomaly. However, at present this is controversial, both in the choice of meson couplings [6] and in the momentum dependence of $\rho-\omega$ mixing [7]. There have been also a number of calculations of the anomaly based on the contribution from the up-down quark mass difference ($\Delta m$). Indeed, Nakamura and coworkers [8] calculate a CSB NN interaction using a constituent quark model where the short-range color hyperfine interaction depends explicitly on the quark masses. Moreover, the mass difference between the neutron and proton may be density dependent [9]. Finally, there are other more recent model calculations, such as the one reported in Ref. [10].

Although the observation by Brown is not a dynamical explanation, it is an interesting characterization of the size of the anomaly. Could there be something wrong with the exchange term? As the nucleon is a composite object, could it be that the exchange energy of composite objects yields results significantly different from the exchange of point nucleons? One expects identical results if the composite scale of the nucleon is much smaller than the inter-particle spacing. However these scales are similar in nuclei. Moreover, although various calculations based on the up-down quark mass difference exist, we are not aware of any calculation of electromagnetic (EM) effects between quarks to the ONS anomaly.

The neutron-proton mass difference in free space is made up from comparable contributions of $\Delta m$ and EM effects. Note that EM and $\Delta m$ terms contribute with opposite signs to the neutron-proton mass difference. However, the ONS anomaly is sensitive to the density dependence of these contributions so the relative sign is unknown. In this letter we study EM effects involving the Coulomb exchange interactions of composite nucleons.

To clarify the importance of EM and $\Delta m$ terms we consider a high-density limit of the ONS anomaly. We will show, with some mild assumptions, that in the high-density limit: (1) there is an ONS anomaly and (2) that it is dominated by EM effects with $\Delta m$ being unimportant. Further, (3) the magnitude of the anomaly is simply related to the Coulomb exchange energy and (4) its sign is the same as that observed at lower densities. Finally, we will perform model calculations to see how relevant this high-density limit is to normal-density nuclei.
Consider very high-density symmetric nuclear matter. We assume that an electron gas makes the system electrically neutral. Thus the direct Coulomb interaction vanishes. Yet Coulomb exchange effects are still present. Now add either one proton or one neutron to the system and calculate the change in energy. First, model the system as a Fermi gas of elementary nucleons. An added proton will have a Coulomb exchange energy of

\[ V_p = -e^2 k_F^2 / \pi. \quad (1) \]

Here \( k_F \) is the Fermi momentum of the proton and \( e \) is its electric charge. In contrast, an added neutron has zero Coulomb exchange energy: \( V_n = 0 \). Thus, the energy difference between an added proton and a neutron is just:

\[ E_p - E_n = V_p + M_p - M_n = -e^2 k_F^2 / \pi - \Delta M, \quad (2) \]

where \( \Delta M = M_n - M_p = 1.29 \text{ MeV} \) is the neutron-proton mass difference. Equation (2) is the simple expectation of a model with unexcited point nucleons.

Next we consider a quark-gluon plasma. We assume because of asymptotic freedom, that at very high density the system is nearly a free Fermi gas of quarks. This is because the strong coupling \( \alpha_S(k_F^2) \) becomes small at the large momentum scale characterized by \( k_F \). When a proton is added it will dissociate into two up and one down quark. Therefore, the Coulomb exchange energy of these three quarks is

\[ V^{(q)}_p = -\left( \sum_{i=1}^{3} e_i^2 \right) \frac{k_F^2}{\pi}, \quad (3) \]

where \( e_i \) denotes the quark electric charge and \( k_F \) is the quark Fermi momentum. Note that there are three times as many (valence) quarks as nucleons. However the quarks have an extra color degeneracy of three. As a result, the quark Fermi momentum in Eq. (3) is the same as the proton’s Fermi momentum in Eq. (1). The sum of the squares of the valence charges in a proton is \((4/9+4/9+1/9) e^2 = e^2\). Because of this “numerical accident” the quark Coulomb exchange energy is equal to the Coulomb exchange energy of an elementary proton.

An interesting difference arises when we add a neutron. In a quark-gluon plasma the Coulomb exchange energy is no longer zero because a neutron is made up of charged constituents. Moreover, the exchange energy is always negative independent of the sign of the charges; the contributions from positive and negative charges add rather than cancel. Indeed, the sum of the squares of the valence quark charges in a neutron is \((4/9+1/9+1/9) e^2 = 2e^2/3\). Thus, the neutron Coulomb energy is fully two thirds of that of a proton: \( V^{(q)}_n = -\frac{2}{3} \frac{e^2 k_F^2}{\pi} \cdot \) The energy difference between an added proton and a neutron becomes:

\[ E^{(q)}_p - E^{(q)}_n = -\frac{e^2 k_F^2}{\pi} + \frac{2}{3} \frac{e^2 k_F^2}{\pi} = -\frac{1}{3} \frac{e^2 k_F^2}{\pi}. \quad (4) \]

We choose to define an ONS anomaly \( \Delta E_{\text{ONS}} \) as the actual energy difference, which we assume is given by Eq. (4), minus the hadronic-model expectation of Eq. (2):

\[ \Delta E_{\text{ONS}} = (E^{(q)}_p - E^{(q)}_n) - (E_p - E_n) = \frac{2}{3} \frac{e^2 k_F^2}{\pi} + \Delta M. \quad (5) \]

This anomaly arises, not because of an error in the proton’s energy but, because there is a nonzero Coulomb contribution for a (dissociated) neutron. In principle we should add to the above equation the contribution from the up-down quark mass difference. However, in the high-density limit, all contributions from \( \Delta m \) are suppressed by the large Fermi momentum. Indeed, the difference in the Fermi energy of free down and up quarks is:

\[ \sqrt{k_F^2} + m^2_d - \sqrt{k_F^2} + m^2_u = (m^2_d - m^2_u) / 2k_F. \]

Thus, in the limit of very high density the total anomaly—including contributions from \( \Delta m \) becomes dominated by Eq. (5). Moreover, the original mass difference between the neutron and proton (\( \Delta M \)) “disappears” at high density because the contributions from \( \Delta m \) are suppressed and the Coulomb self-energies of the neutron and the proton are no longer relevant, as the quarks have rearranged themselves into a uniform free Fermi gas.

In summary, we expect that at high density there will be an ONS anomaly with a magnitude that is two-thirds of the proton Coulomb exchange energy. Furthermore, EM effects dominate over the contribution from \( \Delta m \) and the sign of the anomaly is the same as that observed at normal density.

Our earlier discussion suggests that the Coulomb energy of pure neutron matter is nonzero. Below we focus on neutron matter because of the simple expectation that for point neutrons the Coulomb energy is zero. This may provide a signature of substructure.

Since the above statements are only strictly true in the limit of very high density, we investigate their implications at normal density by performing a model calculation of neutron matter composed of valence quarks. While a model is necessary, our philosophy is to use a “minimal” one by demanding the following general features that any realistic model must posses. We require the many-quark wave function to (1) be explicitly antisymmetric for the exchange of quarks from different nucleons and (2) have cluster separability: the quark wave function of a nucleon removed to infinity must reduce to that of a free nucleon, without any unphysical long-range interactions. Finally, we demand that (3) quarks be confined and (4) for the wave function to reduce to free nucleons at low density and to a quark Fermi gas at high density. Perhaps, any model satisfying these general features can be used.

Conventional quark potential models with two-body confining interactions do not satisfy cluster separability as they generate unphysical long-range van der Waals interactions between nucleons. String-flip models on the other hand, do satisfy the four properties described
above \[11,12\]. Unfortunately, we are not aware of any other models which both satisfy these properties and allow a simple calculation. Thus, we employ the three-quark string-flip model discussed in Ref. \[13\]. The model has nonrelativistic constituent quarks of mass \(m_c\) of fixed red, green, and blue colors. A system of \(A\) nucleons is modeled with \(N = 3A\) quarks interacting via the following many-body potential: \(V = V_{RG} + V_{GB} + V_{BR}\), where each term represents the optimal pairing of quarks. For example, the “red-green” component of the potential is defined as

\[
V_{RG} = \text{Min} \left\{ \sum_{j=1}^{A} v\left( r_j^{(R)} - r_{P_i}^{(G)} \right) \right\}.
\] (6)

Here \(r_j^{(R)}\) is the coordinate of the \(j\)th red quark and \(r_{P_i}^{(G)}\) is its green partner in the neutron. The minimum is over all \(A!\) permutations \(P_i\) of the set of \(A\) green quarks. A harmonic string potential \(v(r) = kr^2/2\) is used to confine the quarks and the Hamiltonian for the model becomes

\[
H = \sum_{i=1}^{N} \frac{P_i^2}{2m_c} + V = -\sum_{i=1}^{N} \frac{\nabla_i^2}{2m_c} + V.
\] (7)

Each red quark is connected by harmonic strings to one and only one green and to one and only one blue quark. This insures that quarks will be confined into “color-neutral” clusters. For three quarks the model reduces to the well-known harmonic oscillator quark model. For neutron matter there is a very large number of permutations or ways to connect the strings. We employ an implementation of the linear sum assignment algorithm by Burkard and Derigs that efficiently finds the optimal permutation in a time proportional to \(N^3\) \[14\]. This allows Monte Carlo simulations with hundreds of quarks.

The model has two dimension-full parameters: \(k\) and \(m_c\). Yet we are only interested in the harmonic-oscillator length \(b = (km_c)^{-1/4}\) as this sets the length scale for quark confinement. The root mean square radius of a nucleon is \(\langle r^2 \rangle^{1/2} = 3^{-1/4}b\). Hence, to reproduce the experimental charge radius of the proton \(\langle r^2 \rangle^{1/2} = 0.86\) fm we choose \(b = 1.13\) fm. At the end we can rescale our results for other values of \(b\).

We are interested in simulating neutron matter. Therefore we assign to red and green quarks an electromagnetic charge of \(-e/3\) and to blue quarks a charge of \(2e/3\). For simplicity we do not include any other intrinsic degree of freedom, such as spin or isospin. The electromagnetic self-energy of an isolated neutron is \((\alpha = e^2 = 1/137)\)

\[
V_n^0 = -\sqrt{\frac{2}{9\pi}} \frac{\alpha}{\langle r^2 \rangle^{1/2}} = -0.446\text{ MeV}.
\] (8)

A simple variational wave function for the many-quark system has been constructed in Ref. \[13\]. It is given by

\[
\Psi = \exp\left(-\frac{\sqrt{3}}{kb^2}\right) \Phi,
\] (9)

with \(\Phi\) a product of Slater determinants for the red, green, and blue quarks. In Ref. \[13\] \(\lambda\) is a variational parameter characterizing the length scale for quark confinement. At low density a value of \(\lambda = 1/\sqrt{3}\) allows Eq. (6) to reproduce the gaussian wave function of a free nucleon. For simplicity we keep lambda fixed at \(\lambda = 1/\sqrt{3}\) for all densities. This insures that any change in the Coulomb energy of a neutron does not arise from an artificial change in this length scale.

We calculate the total Coulomb energy

\[
V_{\text{Coul}}^{\text{tot}} = \sum_{i<j} \frac{e_i e_j}{|\textbf{r}_i - \textbf{r}_j|},
\] (10)

of a system of \(N = 3A\) quarks in a box of volume \(V\) with antiperiodic boundary conditions. To minimize finite size effects we use periodic distances to compute the quark separation. The neutron density of the system is \(\rho_n = A/V\). We use standard Metropolis Monte Carlo techniques to calculate the expectation value of the total Coulomb energy for the wave function given in Eq. (6).

Figure 1 shows the change in the Coulomb energy per neutron

\[
\Delta V \equiv \frac{1}{A} (V_{\text{Coul}}^{\text{tot}}) - V_n^0,
\] (11)

as a function of density for systems with \(N = 96\) and 264 quarks. We have subtracted the neutron self-energy \(V_n^0\) of Eq. (8) because this is included in the experimental neutron-proton mass difference. We find \(\Delta V\) to be nonzero.

![FIG. 1. Change in Coulomb energy per neutron as a function of baryon density for pure neutron matter. The insert compares the model to a free Fermi gas (solid line) at high density.](image-url)

At normal density \(\rho_n = 0.08\text{ fm}^{-3}\) and \(N = 96\): \(\Delta V = -78 \pm 1\text{ keV}\). The scale of this result suggests that
changes in the Coulomb energies of quarks can make a significant contribution to the ONS anomaly. More refined models may give results which are of the same order of magnitude, given the ratio of the nucleon size to interparticle spacing. Furthermore, we expect an additional contribution from the up-down quark mass difference $\Delta m$. Our result is somewhat smaller than the total observed anomaly of the order 200 keV in mass 15 and 300 keV in mass 39 \cite{15}. Note that, for simplicity, we have calculated the average Coulomb energy per neutron rather than the self-energy of a single valence neutron. These quantities are expected to be similar. Indeed, in a free Fermi gas the average Coulomb energy per proton is three fourths of that of Eq. (1).

Figure 2 shows $\Delta V$ as a function of the nucleon root mean square radius at the fixed density of $\rho_n = 0.08$ fm$^{-3}$. Making the quark core of a nucleon smaller reduces $\Delta V$, but not by much. Further, as the oscillator length is made very small the scale of the neutron self-energy $V_0^n$ grows and this can increase $\Delta V$. Of course, if the nucleon core is small one must use a large meson cloud to account for the full proton charge radius. This meson cloud, which we have not included, could further increase $\Delta V$.

One should extend our results by using more elaborated quark models. It is important to study models with more intrinsic spin and flavor degrees of freedom along with more complete treatments of color. However, in these more complete models we still expect an exchange or dynamical correlation between quarks associated with the nucleon’s hard core. This correlation could lead to a nonzero Coulomb energy for neutrons. Note that we have used harmonic oscillator confining strings. Thus, our wave function has gaussian tails. Linear confinement may increase the tails and this should enhance the Coulomb exchange energy.

In conclusion, we have considered a high-density limit of the Okamoto-Nolen-Schiffer anomaly to clarify the role of electromagnetic interactions (EM) and of the up-down quark mass difference $\Delta m$. We have added a single neutron or proton to a quark gluon plasma. In this high-density limit we find that: (1) there is an ONS anomaly, (2) it is dominated by EM interactions rather than by $\Delta m$, and (3) its magnitude is two-thirds of the proton Coulomb exchange energy. We find an attractive Coulomb exchange energy for an added neutron because of the neutron’s charged constituents. This suggests that the ONS anomaly could be closely related to the nucleon substructure. We use a minimal string-flip quark model to calculate the Coulomb energy of pure neutron matter. The model wave function is fully anti-symmetric and satisfies cluster separability and quark confinement. At normal density, we find a nonzero Coulomb energy for neutron matter that could make a significant contribution to the ONS anomaly.

This work was supported in part by DOE grants DE-FG02-87ER40365, DE-FC05-85ER250000, and DE-FG05-92ER40750.

\[\text{FIG. 2. Change in the Coulomb energy per neutron as a function of the nucleon root mean square radius at the fixed neutron density of } \rho_n = 0.08 \text{ fm}^{-3}.\]

\[\text{\cite{15}}\]