Comparison of Energy Reconstruction Schemes and Different Granularities in the CALICE Scintillator-Steel Analogue Hadron Calorimeter

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Abstract. The CALICE collaboration develops different high-granularity hadronic calorimeter technologies for a future linear collider. These technologies differ in active material, granularity and their readout and thus their energy reconstruction schemes. The Analogue Hadron Calorimeter (AHCAL), based on scintillator tiles with Silicon Photomultiplier readout, measures the signal amplitude of the energy deposition in cells of at most $3 \times 3 \text{ cm}^2$ size. The Digital, Resistive Plate Chamber (RPC) based, HCAL (DHCAL) detects hits above a certain threshold by firing pad sensors of $1 \times 1 \text{ cm}^2$. A 2bit readout is provided by the, also RPC based, Semi-Digital HCAL (SDHCAL), which counts hits above three different thresholds per $1 \times 1 \text{ cm}^2$ cell. All three calorimeter concepts have been realised in a $1 \text{ m}^3$ prototype with interleaved Steel absorber and tested at various test beams. This study investigates the impact of the readout, granularity and active medium on the energy resolution individually by applying the reconstruction procedures on AHCAL data, that can also be processed in a way which emulates a (semi-)digital readout system. The difference in granularity is studied via simulations of an AHCAL with $1 \times 1 \text{ cm}^2$ cell sizes. Additionally, a so-called Software Compensation algorithm is developed to weight hits dependent on their energy content and correct for the difference in the response to the electromagnetic and hadronic sub-showers ($\pi^0 \neq 1$) and thus reduce the influence of fluctuations in the $\pi^0$ generation. The impact on the energy resolution will be discussed and compared to the other energy reconstruction schemes.

1. Introduction
For a future linear electron-positron collider such as ILC or CLIC, the desired jet energy resolution of $3 - 4\%$ for a wide range of jet energies can be achieved by using Particle Flow Algorithms for the jet reconstruction. Within the CALICE collaboration, several concepts for a hadron calorimeter (HCAL) optimised for Particle Flow are studied and have been tested with large, $\sim 1 \text{ m}^3$ prototypes: the so-called analogue, digital and semi-digital HCAL concepts. The concepts differ in active material for the shower detection, granularity, readout technology and reconstruction method. This makes it difficult to disentangle the influence of each of these components to the energy resolution of jets as well as of individual particles. Since the analogue HCAL prototype has a larger cell size than the other two concepts, and the digital and semi-digital HCAL prototypes do not provide analogue hit size information, it is impossible to study all different aspects in test beam data. For the data taken with the analogue HCAL prototype,
a direct comparison of the reconstruction methods is possible, and within the simulation the cell size can be altered to the S- and DHCAL cell size of $1 \times 1 \text{cm}^2$. The effect of the active media can be studied only in simulation. For reliable results from the $1 \times 1 \text{cm}^2$ AHCAL simulation it is important to validate the simulation of hadronic showers by comparing them to the measured test beam data, especially for the quantities that are relevant for the energy reconstruction.

2. Analogue Hadronic Calorimeter in the CERN 2007 test beam

For this analysis the AHCAL test beam data from 2007 with steel absorber is chosen. The 2007 CERN test beam setup at the SPS consisted of 30 layers of CALICE silicon-tungsten ECAL, 38 layers of the scintillator-steel analogue HCAL and 16 layers of the scintillator-steel tail catcher and muon tracker (TCMT). The absorber plate thickness for the HCAL was $\sim 2 \text{ cm}$. A detailed description of the test beam setup can be found in [1].

The run list and event selection follows the published analysis [2] and consists of 29 runs at 11 energies for pions between 10 and 80 GeV. Negative pion events are selected by requiring the showers to start in the second to sixth HCAL layer and by rejecting muons that have a smaller energy deposit than 150 MIP in the HCAL. One MIP is defined as the response of a minimum-ionising particle like a 0.4 GeV muon in a single AHCAL cell. The value of a MIP is measured within calibration runs in ADC counts. For a detailed description of the event selection and the whole analysis see [4]. Additionally, the test beam runs are simulated using the software packages Geant4 version 9.6 patch 1. The physics lists FTFP.BERT and QGSP.BERT from Geant4 show best performance for hadrons and were therefore chosen for comparisons in this analysis.

3. Energy Reconstruction

The goal of this analysis is first a direct comparison of the four reconstruction methods: analogue, digital, semi-digital and analogue software compensation, applied to the same AHCAL data and second a study of the impact of the granularity on the reconstruction performance. This includes using the same methods to extract the mean energy and the resolution. Since the distributions of the reconstructed energies from the number of hits are expected to show a non-gaussian tail, the following procedure is used to fit the distributions and to extract the mean and the width.

In a first step, the Novosibirsk function $f(x) = A \cdot \exp\left(-\frac{1}{2} \left(\ln^2[1 + \Lambda(x - \mu)] + \sigma^2\right)\right)$ with

$$\Lambda = \frac{\sin(\tau \sqrt{\ln 4})}{\tau \sqrt{\ln 4}}$$

[3] is used to fit the distributions. The fit range is limited to $\mu \pm 3 \sigma$ from a pre-fit with a Gaussian. The fit consistently provided a $\chi^2/\text{ndf}$ value better than 3. In order to extract the mean and the width of this fit function, a histogram is filled with random values generated according to the Novosibirsk function with the extracted fit parameters. The mean and RMS of the histogram are used as response and resolution for the studied energy. For each method the energy is reconstructed on an event-by-event basis.

Systematic uncertainties due to the uncertainty on the beam energy, the MIP to GeV conversion and the detector stability is taken into account. For the simulations the systematic uncertainty is estimated from the uncertainty on the light crosstalk between neighbouring cells.

3.1. Analogue

From the linear hit energy sum $E_{\text{sum}}$ above 0.5 MIP threshold in the AHCAL the analogue energy is reconstructed as

$$E_{\text{rec,analogue}} = 0.3232 \text{GeV} + \frac{e}{\pi} \cdot \omega \cdot E_{\text{sum}} \cdot c$$

(1)
with the correction for the non-compensation with $e/\pi = 1.19$, the electromagnetic calibration factor $\omega$ for the conversion from MIP to GeV scale and the scaling $c$, which is determined by a fit to $c = 1.04$ and for the $1 \times 1$ cm$^2$ AHCAL to $c_{1\times1} = 1.01$. The approximately constant contribution of the track in the ECAL is taken into account using a constant value of 0.3232 GeV.

### 3.2. Digital
Within the digital energy reconstruction the non-linear digital response is linearised as follows. The mean number of hits is fitted with a power law $\langle N_{\text{hits}} \rangle = a \cdot (E_{\text{beam}} - m)^b$. By constraining the digital reconstructed energy to show a linear behavior $E_{\text{rec,digital}} = E_{\text{beam}}$, the energy is reconstructed with the fit parameters of the power law to

$$E_{\text{rec,digital}} = m + b \sqrt{N_{\text{hits}}}. \quad (2)$$

### 3.3. Semi-Digital
The semi-digital energy reconstruction follows

$$E_{\text{rec,semi–digital}} = \alpha N_1 + \beta N_2 + \gamma N_3, \quad (3)$$

with $N_1$ the number of hits with energies above the first threshold $t_1$ and below the second $t_2$, $N_2$ the number of hits above $t_2$ and below the third threshold $t_3$ and $N_3$ the number of hits above $t_3$. These thresholds are optimised for the energy reconstruction performance [4]. $\alpha, \beta$ and $\gamma$ weight the hits depending on their energy content. Hadronic showers change their structure and compositeness with energy, which is taken into account by parameterising $\alpha, \beta$ and $\gamma$ as quadratic polynomials of the total number of hits $N_{\text{hits}} = N_1 + N_2 + N_3$. To find the best parameters a $\chi^2$ function of the form $\chi^2 = \sum_{i=1}^{N} \left( \frac{E_{\text{beam}} - E_{\text{rec}}}{E_{\text{beam}}} \right)^2$ is minimised, where $i$ runs over all events.

### 3.4. Analogue Software Compensation
The analogue software compensation algorithm follows the principle of the semi-digital energy reconstruction by using the same $\chi^2$ minimisation technique, to determine different weights $\omega$ for different classes of hits, dependent on the energy $e_j$ and the total measured energy $E_{\text{sum}}$ in the event. Thus the energy is reconstructed by

$$E_{\text{rec,SC}} = \sum_{j=0}^{N_{\text{hits}}} \omega(e_j, E_{\text{sum}}) \cdot e_j. \quad (4)$$

Hereby the hit energy spectrum is divided into 8 energy ranges. An example of the energy distributions is given in Figure 1, which shows the analogue reconstructed energy distributions in comparison to the reconstructed energies using the analogue software compensation algorithm for data and simulation.

### 4. Results
The resolutions obtained with the different reconstruction methods applied to the $3 \times 3$ cm$^2$ AHCAL data and the $1 \times 1$ cm$^2$ AHCAL simulations are compared in Figure 2a and 2b. In addition, the best resolution reached with AHCAL data, by applying software compensation techniques [2], is indicated. For the comparison one should keep in mind that in the earlier
analysis, the TCMT is fully included and the track in the ECAL considered in the energy reconstruction, while here a simplified treatment of the ECAL is used, the TCMT contribution is neglected and the widths are determined using a Novosibirsk function. The non-linearities of the four methods studied in this analysis are shown in the upper parts of Figure 2.

For the AHCAL with a granularity of \(3 \times 3\) cm\(^2\), the analogue and digital reconstruction procedures show rather similar resolutions at the lowest energies. For larger energies, the resolution of the analogue reconstruction method continues to decrease, while the digital resolution degrades dramatically. The semi-digital reconstruction and the software compensation both apply weights to the energy depositions in a shower depending on the hit energy. The semi-digital reconstruction achieves a resolution similar to the software compensation for the lowest energy, 10 GeV. For higher beam energies the resolution follows a similar shape as for the software compensation but with absolute values 1-2 % worse. The best resolution of all four methods for the whole energy range is found using the analogue software compensation algorithm.

The simulated AHCAL with \(1 \times 1\) cm\(^2\) cell size shows an improved resolution for the semi-digital and digital readout schemes, see Figure 2b. Compared to the classical analogue energy reconstruction the digital reconstruction shows better results for beam energies below 35 GeV. This improvement despite the reduction of information can be explained by the shape of the analogue cell signal, which follows a Landau distribution that is characterised by a long tail to high values. By counting cells above a certain signal amplitude, the signal fluctuations to high values are removed and thus the energy reconstruction can be improved. A degradation due to saturation effects of the digital resolution is observed above 25 GeV.

The increase of the number of thresholds from 1 to 3, digital to semi-digital, results in a large improvement of the energy resolution of the \(1 \times 1\) cm\(^2\) AHCAL simulation and in an even larger improvement for the AHCAL with \(3 \times 3\) cm\(^2\) cells.

In conclusion, the best resolution is achieved by applying either a weighting by the software compensation algorithm or by the semi-digital energy reconstruction. This is understood because both methods apply energy dependent weights, which are determined by a \(\chi^2\) minimisation that optimises the resolution. For both methods a decreasing resolution with increasing beam energy is observed. The \(1 \times 1\) cm\(^2\) AHCAL simulation needs the semi-digital energy reconstruction to
Figure 2: Energy dependence of the relative energy resolution of the AHCAL testbeam data in (a) and the simulation with $1 \times 1 \text{cm}^2$ granularity and the FTFP\_BERT physics list in (b), obtained using different approaches for the energy reconstruction of pions: analogue (black), digital (green), semi-digital (red) and applying the analogue software compensation algorithm (blue). The dashed and dotted curves in (a) show the resolution achieved in [2] with and without software compensation techniques, using the energy deposits in the TCMT and in the ECAL in addition to the AHCAL. The plots on the top show the residuals to the beam energy with the bands indicating the systematic and statistical uncertainties. The statistical errors are smaller than the markers.

achieve the best possible energy resolution, while the analogue software compensation algorithm has to be used for the energy reconstruction in the $3 \times 3 \text{cm}^2$ AHCAL. A first comparison [4] of the digital and semi-digital energy resolutions of the $1 \times 1 \text{cm}^2$ AHCAL simulations to the resolution achieved by the DHCAL and SDHCAL prototypes shows an advantage of the scintillator-tile calorimeter especially at low energies. However this effect can have many different explanations: the different sampling fraction of 5 mm scintillator thickness versus 1.15 mm gas gap; a different threshold setting; the difference in the signal distributions (a Landau distribution and a Polya function, that is very sensitive to high voltage variations); or the difference in the ionising energy loss in the scintillator and the gas mixture. It needs further investigations to disentangle the effects of the differences mentioned above.

References
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