Analysis by Asymptotic Boundary Conditions of Strip-Grated Dielectric Pipe Over Dielectric Rod With Wide Total Band-Gap as Multifrequency Cylindrical Leaky-Wave Antennas

MALCOLM NG MOU KEHN, (Senior Member, IEEE), AND CHENG-YU WU

Institute of Communications Engineering, National Yang Ming Chiao Tung University, Hsinchu 30010, Taiwan

Corresponding author: Malcolm Ng Mou Kehn (malcolm.ng@ieee.org)

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ABSTRACT In this paper, the method of classical vector potential analysis along with the asymptotic strips boundary conditions (ASBC) is used to treat conducting strip grated dielectric pipes wrapped over core dielectric cylindrical rods. By virtue of the periodically-textured surface, this structure is able to exhibit plasmonic behaviors that facilitate strong surface field localization and wave propagation, thereby offering itself as cylindrical surface-wave waveguides or antennas. It can also be configured to exhibit electromagnetic band-gap (EBG) properties, yielding what is known as cylindrical EBG structures. As opposed to a typical counterpart whose core rod is instead conducting, the core dielectric is herein demonstrated to be vital in realizing wide total band-gaps in which neither slow surface nor fast space waves exist, something that the conducting core rod version is incapable of, being only able to offer surface-wave band-gaps, as do many others in the literature that have neglected the suppression of fast space waves as well. By coaxially connecting various such rod structures, each with its own distinct total band-gap, multi-frequency, multi-functional cylindrical antennas with mitigated inter-band interference even under simultaneous operations can be achieved when the pass-band of any one rod (in which its operating frequency lies) falls within the total band-gaps of all others, as herein investigated. Manufactured prototypes are also measured, yielding experimental results that agree well with theoretical predictions.

INDEX TERMS Asymptotic strips boundary conditions, gratings, electromagnetic band-gap, cylindrical EBG, multi-frequency antennas, leaky-wave antennas.

I. INTRODUCTION

There had been studies of periodically textured rods in the context of their abilities to support surface plasmonic modes which provide highly localized waveguiding and strong focusing of bound electromagnetic fields. These features find applications in subwavelength imaging, sensing, and detection, as well as near-field optics and spectroscopy, [1], [2] among many others.

Besides these pursuits of intense field confinement and surface wave propagation, the converse realm also constitutes an important subject of research, that being the suppression of such modal propagation along the surfaces of rods, or also termed as cylindrical electromagnetic band-gap (EBG) surfaces [3], [4], [5], [6], [7]. Throughout literature, EBGs have customarily been defined to be the frequency spans within which only slow surface waves are forbidden [8], [9], [10], whilst fast space waves have not been expressly included in the prohibition. This may be understandable because it is often very difficult to achieve designs that also disallow the presence of fast space waves. These bands are then used for the enhancements of numerous applications ranging from the reduction of mutual coupling between radiating elements, with subsequent antenna gain augmentation and sidelobe 

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mitigation, to the realization of low-profile conformal antennas. By prohibiting surface waves on their cylindrical surfaces, thereby enhancing the isolation between microwave devices, circuit elements, or radiating antennas that are placed on them, cylindrical EBG structures are important to applications such as cylindrical arrays [11], [12], [13] for base station or local area network (LAN) antennas.

The core conducting rod wrapped by a dielectric pipe that is transversely strip-grated had been studied in [14]. Although not conveyed by the results presented in that article, this configuration is, as will be shown in this paper, able to exhibit EBG phenomena by the usual definition; as in, slow surface waves are suppressed. However, as we shall also show here, such EBGs in their traditional sense, henceforth termed as “surface-wave band-gaps (SWBG), are unable to provide adequate decoupling on the cylindrical surfaces of such textured rods. Instead, this paper will demonstrate that only frequency windows in which nothing can propagate, especially not even fast space-waves, are able to strongly isolate elements on the rods. Calling this a total EBG, or an all-waves band-gap (AWBG) herein for namesake, the conducting core rod counterpart, despite the SWBG it is still able to exhibit, is unfortunately unable to display AWBG behaviors of practical bandwidths. This renders the structure incapable of providing adequate decoupling between antenna elements on the cylindrical surfaces of such rods over bands with tangible widths.

One of the main objectives of this work is then to demonstrate that the key to achieving wide AWBG properties is to do away with the core conducting rod and replace it with a dielectric one which is likewise wrapped by a dielectric pipe that is, on its outer surface, transversely strip-grated. We shall show that only with this topology can AWBG properties be produced, thereby being able to provide very strong isolation between elements on the cylindrical surface.

In addition to the two foregoing extreme-ends opposite scenarios; being the presence of localized fields by facilitated slow surface-wave modes and the absence of fields by wave suppression, one more regime in the spectrum exists, and that is where fast space waves are able to propagate out into the surrounding space, topologies by which these are supported being well known as leaky-wave antennas (LWA) [15]. The herein strip-grated pipe sheathed over a core dielectric rod will also be studied as a LWA. With the wide AWBG below the leaky-wave radiating band that it is able to provide, multfunction multi-frequency cylindrical LWAs may be realized when such coaxially-connected rod antennas occupying differently grated parts of the shared cylindrical aperture and operating at different frequencies may all simultaneously radiate with minimal mutual interference when the band for fast-wave radiation of every LWA is within the AWBGs of all other LWAs. This concept need not be limited to only multifrequency LWAs and may be extended to hybridized operations of LWAs and surface-wave antennas (SWAs), so long as the operating frequency of one antenna type within its own pass-band is in the band-gap of the other, and vice versa.

Together with the classical method of vector potentials, the asymptotic strips boundary conditions (ASBC) [14] are used here to modally treat the dielectric-cored version. Aside from the difference in topology, that prior work however did not consider fast space waves, as we shall do so here. Modal and radiated field patterns were also not studied previously, unlike the present work. These new aspects are on top of the showcasing of the present structure for use as multi-frequency cylindrical antennas with mitigated inter-band interferences even under simultaneous operations. A brief outline of the paper is as follows. After describing the geometry in Section II, details of the theory and formulation are presented in Section III. This is followed by Section IV which reports the validation with a commercial solver of the results computed by the self-developed computer code written based on this method. Differences in band-gap phenomena between the conducting core and dielectric rod versions are discussed in Sections V and VI. How the latter is advantageously used as multi-frequency antennas is presented in Section VII. Experiments conducted on manufactured prototypes are reported in Section VIII. Applications, advantages and limitations of the proposed structure are then discussed in Section IX. This is followed by parametric studies given in Section X. After laying out the features of this work in Section XI that distinguish it from a prior one of a related nature, and investigating the analogy between the conventional planar mushroom EBG structure and its rolled-up cylindrical counterpart in Section XII, key aspects of this paper are finally summarized in Section XIII.

II. DESCRIPTION OF THE GEOMETRY
A schematic of the strip-grated dielectric pipe wrapped over a core dielectric rod is shown in Fig. 1. The latter component has a radius herein symbolized by $a$ and whose material parameters are $(\mu_{in}, \varepsilon_{in})$. The outer dielectric pipe is of inner radius $a$, outer radius $b$, and material parameters $(\mu_{out}, \varepsilon_{out})$, the outer surface of which (at $\rho = b$) is grated by transverse

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**FIGURE 1.** Core dielectric rod of radius $a$ and material parameters $(\mu_{in}, \varepsilon_{in})$ wrapped by dielectric pipe of inner radius $a$, outer radius $b$ and material parameters $(\mu_{out}, \varepsilon_{out})$. The outer surface of which (at $\rho = b$) is grated by transverse conducting strips each of width $w$ and with axial period $p$. Parameters of exterior region are $(\mu_{ext}, \varepsilon_{ext})$. 

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conducting strips each of width $w$ and with axial period $\rho$. The material parameters of the exterior region are $(\mu_{ext}, \epsilon_{ext})$.

### III. THEORY AND FORMULATION

The method of classical vector potential analysis together with the use of ASBC is herein presented.

#### A. FIELD EXPRESSIONS VIA VECTOR POTENTIALS

For $TE^z$ and $TM^z$ modes respectively, the various field components in any of the three regions are first expressed in terms of the $z$ components of the electric and magnetic vector potentials, $F_z$ and $A_z$, as follow.

\[
\begin{bmatrix}
\Omega_{reg}^{\rho} \\
\Omega_{reg}^{\phi}
\end{bmatrix} = \frac{1}{\epsilon_{reg}} \begin{bmatrix}
\pm & 1 \\
1 & 0
\end{bmatrix} \begin{bmatrix}
(\partial F_{z}^{reg} / \partial \rho) \\
(\partial F_{z}^{reg} / \partial \phi)
\end{bmatrix}
\]

\[
\begin{bmatrix}
\rho^{reg} \\
\phi^{reg}
\end{bmatrix} = \frac{1}{f_{out}^{\rho} \mu_{reg}^{\epsilon}} \begin{bmatrix}
(\partial^2 F_{z}^{reg} / \partial \rho^2) + k^2_{reg} F_{z}^{reg} \\
(\partial^2 F_{z}^{reg} / \partial \phi^2)
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Omega \\
\chi
\end{bmatrix} = \begin{bmatrix}
E \\
H
\end{bmatrix}; \quad \begin{bmatrix}
\chi \\
\mu
\end{bmatrix} = \begin{bmatrix}
TE \\
TM
\end{bmatrix}
\]

whereby the two items in the curly braces correspond to one another in any one equation. The $reg$ script signifies the region and may be either $in$, $out$, or $ext$, respectively denoting the inner core rod, the outer coating pipe, or the exterior space. The wavenumber, $k_{reg}$, of any region is defined by:

\[
k_{reg}^2 = \omega^2 \mu_{reg}^{\epsilon} \epsilon_{reg} = \left(k_{\rho}^{reg}ight)^2 + k_{z}^2
\]

The $z$ components of the vector potentials are stated for the various regions as follow.

\[
\Phi_{z}^{reg}(\rho, \phi, z) = \Psi_{m}(k_{\rho}^{reg} \rho) \left[V_{\phi}^{reg} \cos(m \phi) + W_{\phi}^{reg} \sin(m \phi)\right]
\]

\[
\Psi_{m}(k_{\rho}^{reg} \rho) = \begin{bmatrix}
J_{m}(k_{\rho}^{in} \rho); \quad \text{reg} \equiv \text{in} \\
V_{\rho}^{out} J_{m}(k_{\rho}^{out} \rho) + W_{\rho}^{out} Y_{m}(k_{\rho}^{out} \rho); \quad \text{reg} \equiv \text{out} \\
B_{m}(k_{\rho}^{ext} \rho); \quad \text{reg} \equiv \text{ext}
\end{bmatrix}
\]

\[
B_{m}(k_{\rho}^{ext} \rho) = \begin{bmatrix}
H_{m}^{(2)}(k_{\rho}^{ext} \rho); \quad k_{\rho} < k_{ext} \\
K_{m}(a_{\rho}^{ext} \rho); \quad k_{\rho} > k_{ext}
\end{bmatrix}
\]

\[
k_{ext} = \sqrt{k_{ext}^2 - k_{\rho}^2} \in \Re_{+}; \quad k_{\rho} < k_{ext}
\]

\[
a_{\rho}^{ext} = \sqrt{k_{\rho}^2 - k_{ext}^2} \in \Re_{+}; \quad k_{\rho} > k_{ext}
\]

in which $\Re_{+}$ denotes that the quantity is positive real. As usual, $J$ and $Y$ are respectively the Bessel functions of the first and second kind, whereas $H_{m}^{(2)}$ and $K_{m}$ are the Hankel and modified Bessel functions, both of the second kind and with order $m$.

### B. BOUNDARY CONDITIONS

In order for a solvable system of equations, we shall arbitrarily choose any one of the two harmonic variations in $\phi$ to assume by setting the following:

\[
W_{\phi}^{in} = W_{\phi}^{out} = W_{\phi}^{ext} = V_{\phi}^{in} = V_{\phi}^{out} = V_{\phi}^{ext} = 0
\]

With this set, the boundary conditions across the various interface boundaries are now imposed.

1) INNER DIELECTRIC ROD SURFACE AT $\rho = a$ (SMOOTH, UNGRADED)

Continuity of the tangential field components across the outer interface of the core rod at $\rho = a$ requires:

\[
\Omega_{\phi}^{in} (\rho = a) + \Omega_{\phi}^{out} (\rho = a) = \Omega_{\phi}^{ext} (\rho = a)
\]

\[
\Omega_{\phi}^{in} (\rho = a) + \Omega_{\phi}^{out} (\rho = a) = \Omega_{\phi}^{ext} (\rho = a)
\]

each of which unfolding to two equations, thus constituting four equations.

2) STRIP-GRATED OUTER SURFACE OF OUTER DIELECTRIC PIPE AT $\rho = b$: ASYMPTOTIC STRIPS BOUNDARY CONDITIONS (ASBC)

Using the asymptotic strips boundary conditions (ASBC), the pertinent relations are stated in the upcoming.

Enforcing the annulment of the $E_{\phi}$ component parallel with the circumferentially oriented conducting strips on both sides of the gratings, we have:

\[
E_{\phi}^{out} (\rho = b) = 0
\]

\[
E_{\phi}^{out} (\rho = b) = 0
\]

Continuity of the $E_{z}$ component perpendicular to the $\phi$-directed strips across $\rho = b$ require:

\[
E_{z}^{out} (\rho = b) = E_{z}^{ext} (\rho = b)
\]

Finally, enforcing the continuity of the $H_{\phi}$ component across the grated surface,

\[
H_{\phi}^{out} (\rho = b) = H_{\phi}^{ext} (\rho = b)
\]

These foregoing eight equations (10) through (15) are in terms of likewise eight unknown modal amplitude coefficients, listed as:

\[
C_{1} = V_{\phi}^{in}; \quad C_{2} = W_{\phi}^{in}; \quad C_{3} = V_{\phi}^{out}; \quad C_{4} = W_{\phi}^{out}; \quad C_{5} = V_{\phi}^{ext}; \quad C_{6} = W_{\phi}^{ext}; \quad C_{7} = V_{\phi}^{TM}; \quad C_{8} = W_{\phi}^{TM}
\]

which may be cast into a matrix equation:

\[
\begin{bmatrix}
M \\
C
\end{bmatrix} = \begin{bmatrix}
0
\end{bmatrix}
\]

of which the indexing of the row equations takes on the same sequential order of these aforementioned eight equations. The $i^{th}$ element of $\begin{bmatrix}
C
\end{bmatrix}$ is then one of those listed in (16).
Before providing the explicit expressions of the matrix elements, the following are first defined.

\[
\begin{align*}
\Psi_{\text{reg}}^\text{e} &= \left\{ \begin{array}{ll}
k_{\text{reg}}^e / s_{\text{reg}}; & \text{reg} \equiv \text{in or out} \\
s_{\text{ext}} / s_{\text{reg}}; & \text{reg} \equiv \text{ext}
\end{array} \right. \\
Q_{\text{reg}}^\text{e} &= m_{\text{reg}} \left( \omega \mu_{\text{reg}} \rho_{\text{reg}} \right)
\end{align*}
\]

(18a)

\[
K_{\text{reg}} = \left( k_{\text{reg}}^2 - k_{\text{reg}}^2 \right) / \left( \mu_{\text{reg}} \rho_{\text{reg}} \right)
\]

(18b)

\[
\Lambda_{\text{reg}}^\text{m} = \Lambda_{\text{reg}}^\text{m} \left( \beta_{\text{reg}}^\text{m} \rho_{\text{reg}} \right)
\]

(19a)

\[
\Lambda_{\text{reg}}^\text{m} = \Lambda_{\text{reg}}^\text{m} \left( \beta_{\text{reg}}^\text{p} \rho_{\text{reg}} \right)
\]

(19b)

where \( \Lambda_{\text{m}} \) may be either \( J_m, Y_m, H_m^2, \) or \( K_m \) (any of the various Bessel or Hankel functions), and the prime denotes differentiation with respect to the argument. The \( \delta \) within the triangular brace, taking on either 0 or 1 only, is thus the order of derivative. When \( \text{reg} \) is either \( \text{in or out} \), \( \beta \) is \( k \), but when \( \text{reg} \) is \( \text{ext} \), \( \beta \) is \( \kappa \). The various matrix elements are then explicitly given by the following (unspecified matrix locations are zeros).

\[
\begin{align*}
M_{11} &= P_{in}^\text{e} f_{\text{in}}^{(1)\text{in}}, & M_{12} &= -Q_{in}^\text{e} f_{\text{in}}^{(0)\text{in}}; \\
M_{13} &= -P_{out}^\text{e} f_{\text{out}}^{(1)\text{out}}; & M_{14} &= -Q_{out}^\text{e} f_{\text{out}}^{(0)\text{out}}; \\
M_{15} &= Q_{out}^\text{e} f_{\text{out}}^{(0)\text{out}}; & M_{16} &= Q_{out}^\text{e} f_{\text{out}}^{(0)\text{out}}; \\
M_{22} &= R_{in}^\text{e} f_{\text{in}}^{(1)\text{in}}, & M_{25} &= -R_{out}^\text{e} f_{\text{out}}^{(0)\text{out}}; \\
M_{26} &= -R_{out}^\text{e} f_{\text{out}}^{(0)\text{out}}; & M_{31} &= Q_{out}^\text{e} f_{\text{out}}^{(0)\text{out}}; \\
M_{32} &= -P_{in}^\text{e} f_{\text{in}}^{(1)\text{in}}, & M_{33} &= -Q_{out}^\text{e} f_{\text{out}}^{(0)\text{out}}; \\
M_{34} &= -Q_{out}^\text{e} f_{\text{out}}^{(0)\text{out}}; & M_{35} &= P_{out}^\text{e} f_{\text{out}}^{(1)\text{out}}; \\
M_{36} &= P_{out}^\text{e} f_{\text{out}}^{(1)\text{out}}; & M_{41} &= -R_{in}^\text{e} f_{\text{in}}^{(1)\text{in}}; \\
M_{43} &= R_{out}^\text{e} f_{\text{out}}^{(0)\text{out}}; & M_{44} &= R_{out}^\text{e} f_{\text{out}}^{(0)\text{out}}; \\
M_{53} &= P_{in}^\text{e} f_{\text{in}}^{(1)\text{in}}; & M_{54} &= P_{in}^\text{e} f_{\text{in}}^{(1)\text{in}}; \\
M_{55} &= -Q_{out}^\text{e} f_{\text{out}}^{(0)\text{out}}; & M_{56} &= -Q_{out}^\text{e} f_{\text{out}}^{(0)\text{out}}; \\
M_{67} &= P_{ext}^\text{e} B_m^e(k_{ext}^e b); & M_{68} &= -Q_{ext}^\text{e} B_m(k_{ext}^e b); \\
M_{75} &= R_{ext}^\text{e} B_m(k_{ext}^e b); & M_{76} &= R_{ext}^\text{e} B_m(k_{ext}^e b); \\
M_{78} &= -R_{ext}^\text{e} B_m(k_{ext}^e b); & M_{83} &= Q_{out}^\text{e} f_{\text{out}}^{(0)\text{out}}; \\
M_{84} &= Q_{out}^\text{e} f_{\text{out}}^{(0)\text{out}}; & M_{85} &= -P_{out}^\text{e} f_{\text{out}}^{(1)\text{out}}; \\
M_{86} &= -P_{in}^\text{e} f_{\text{in}}^{(1)\text{in}}; & M_{87} &= -Q_{ext}^\text{e} B_m(k_{ext}^e b); \\
M_{88} &= P_{ext}^\text{e} B_m^e(k_{ext}^e b).
\end{align*}
\]

With \( \Delta(k, f) = \det \left[ M \right] \) signifying the determinant of the system matrix, which is a function of the axial propagation constant \( k \), and the frequency \( f \), the characteristic equation is then expressed as:

\[
\Delta(k_{\text{res}}, f_{\text{res}}) = 0
\]

which is satisfied by the coordinate pair: \( (k_{\text{res}}^e, f_{\text{res}}^e) \), being the roots of the equation and constituting the eigenmodal resonance which may be determined numerically (e.g., by sweeping over a search space over both coordinate variables/parameters to detect modal resonances).

IV. VALIDATION WITH COMMERCIAL SOLVER

Produced by computer codes written based on the present ASBC-based modal formulation of Section III, this section shall present results of dispersion diagrams and modal field distributions for transversely strip grated dielectric pipes wrapped over core dielectric rods, which are validated with those simulated by a commercial simulation software solver: CST Microwave Studio (henceforth just CST).

A. DISPERSION

For parameters listed as follow: \( a = 2 \text{ mm}, b = 6 \text{ mm}, \epsilon_{in} = 2.0 \epsilon_0, \epsilon_{out} = 3.8 \epsilon_0, \mu_{in} = \mu_{out} = \mu = \mu_0 \), Fig. 2 presents the dispersion diagram of a grated dielectric sheath over a dielectric rod obtained by the present analysis, the continuous traces of which are validated against the circle markers of CST. The variation with frequency of the attenuation constant \( \alpha \) of the leaky mode from 8 to 10 GHz is given as an inset diagram, which demonstrates how the fast space wave evolves into a slow surface wave mode beyond 10 GHz with associated vanishing \( a \).

B. CROSS-SECTIONAL MODAL FIELD DISTRIBUTION

By reconfiguring the system matrix \( M \) of (17) into its echelon form by Gauss elimination, the elements [those of (16)] of the eigen-vector \( C \) in (17) may all be, upon back substitution, expressed in terms of just one of them, constituting the arbitrary amplitude coefficient of that eigen-mode characterized by \( (k_{\text{res}}^e, f_{\text{res}}^e) \) at which all matrix elements are evaluated. These solved coefficients of (16) can then be substituted back into all field expressions conveyed by (1) and (2), from which the transverse \( x \) and \( y \) components of the modal field distributions over the cross sectional plane may be plotted. This is done and presented as plot-sets (a) and (c) of Fig. 3 for the \( E \) and \( H \) fields, respectively, at \( f_{\text{res}}^e = 5.5 \text{ GHz} \) with \( k_{\text{res}}^e = 132.64 \text{ rad/m}, \) for a core dielectric rod of radius \( a = 2 \text{ mm} \) and \( \epsilon_{in} = 2.0 \epsilon_0 \) wrapped by a transversely strip-grated dielectric pipe layer of outer radius \( b = 6 \text{ mm} \).
FIGURE 3. Cross-sectional fields computed by ASBC-based code at 5.5 GHz for $\beta = 132.64$ rad/m [plot sets (a) and (c)] and simulated by CST at 5.555 GHz for $\beta = 140$ rad/m [plot sets (b) and (d)]; $E$-fields [sets (a) and (b)] and $H$-fields [sets (c) and (d)], $x$ and $y$ components on left (i) and right (ii), respectively; for core dielectric rod of radius $a = 2$ mm and $\varepsilon_{in} = 2\varepsilon_0$ wrapped by transversely strip-grated dielectric pipe layer of outer radius $b = 6$ mm and $\varepsilon_{out} = 3.8\varepsilon_0$. $\mu_{in} = \mu_{out} = \mu_{ext} = \mu_0$.

$\varepsilon_{out} = 3.8\varepsilon_0$ (along with $\varepsilon_{ext} = \varepsilon_0$, $\mu_{in} = \mu_{out} = \mu_{ext} = \mu_0$). Offered together [plot sets (b) and (d) in Fig. 3] are the corresponding modal field patterns simulated by CST, at 5.555 GHz for $\beta = 140$ rad/m. Plot sets (a) and (b) convey the $E$ field patterns whereas sets (c) and (d) are for the $H$ fields. The $x$ and $y$ components of the fields are given by plots on the left (i) and right (ii) sides, respectively. The slight differences in modal resonance coordinates between the ASBC-based modal method and CST are inevitably due to numerical inaccuracies in the eigen-solver of the software and the approximations of the asymptotic method in the analysis.

V. POOR DECOUPLING BY SURFACE-WAVE BAND-GAPS IN GRATED DIELECTRIC PIPES OVER CONDUCTING RODS

In this section, we shall show that, while the core conducting rod wrapped by a transversely strip-grated dielectric pipe, with adequate searching through the parametric space, is still fairly amenable to the coaxing out of surface-wave band-gap (SWBG) properties, all-waves band-gaps (AWBG) of decent bandwidths within which neither slow surface-waves nor fast space-waves can exist are nowhere to be found.

To demonstrate this fact, a parametric study was performed for three parameters: a) the radius $a$ of the core conducting rod, b) the outer radius $b$ of the outer dielectric pipe strip-grated on its outer surface, conveyed as $b/a$, and c) the permittivity $\varepsilon_{out}$ of the latter pipe. The permeability is that of free space throughout and the exterior space is vacuum. The parametric space of $a$ spans from 3 to 5 mm whereas that of $b/a$ covers values between 1.1 and 1.4, while $\varepsilon_{out}/\varepsilon_0$ ranges from 2.0 to 5.0. Infeasible to present the many dispersion graphs for all permutations of these three parameters, just six of them, spaced more or less equally apart in the parametric space, are presented in Fig. 4. The triplet of parameters for each dispersion plot is as annotated. As observed, none shows any sign of AWBG phenomena (just possibly SWBG). Hence, core conducting rods are unamenable to the seeking out of AWBG properties.

FIGURE 4. Dispersion diagrams transverse strip-grated dielectric pipe wrapped over core conducting rod, for six selected cases from parametric space of $a$ from 3 to 5 mm, $b/a$ from 1.1 to 1.4, and $\varepsilon_{out}/\varepsilon_0$ from 2.0 to 5.0. (a) $a = 3$ mm, $b/a = 1.1$, $\varepsilon_{out} = 2\varepsilon_0$; (b) $a = 3$ mm, $b/a = 1.2$, $\varepsilon_{out} = 4\varepsilon_0$; (c) $a = 3$ mm, $b/a = 1.4$, $\varepsilon_{out} = 3\varepsilon_0$; (d) $a = 3$ mm, $b/a = 1.1$, $\varepsilon_{out} = 5\varepsilon_0$; (e) $a = 5$ mm, $b/a = 1.3$, $\varepsilon_{out} = 3\varepsilon_0$; (f) $a = 5$ mm, $b/a = 1.4$, $\varepsilon_{out} = 5\varepsilon_0$.
$a = 3 \text{ mm}$ covered by a strip-grated dielectric pipe layer of outer radius $b = 3.9 \text{ mm}$, permittivity of $2.0\varepsilon_0$, permeability of $\mu_0$, and surrounded by free space. Fig. 5 presents the dispersion diagram for this configuration, generated by both the ASBC-based analytical method (dot markers) as well as CST. As seen, although there is an SWBG between 9.75 and 14 GHz, there also exists, throughout almost this entire band (about 10 to 14 GHz), a space-wave mode in the fast-wave regime. For the cell topology of this same rod but with a finite length comprising 26 unit cells each with axial period $p = 3 \text{ mm}$ and strip width $w = 0.45 \text{ mm}$, Fig. 6 presents the variation with frequency of the transmission coefficient $S_{21}$ between two wave ports placed at the two terminal ends of the finite rod, simulated in CST.

**FIGURE 5.** Dispersion for core conducting rod of radius $a = 3 \text{ mm}$ wrapped by transversely strip-grated dielectric pipe layer of outer radius $b = 3.9 \text{ mm}$, permittivity $= 2.0\varepsilon_0$, permeability $= \mu_0$, surrounded by free space. Dot markers: present ASBC-based modal approach, circle markers: CST. Axial period of 1 mm in CST. Despite presence of SWBG with decent width, there is practically no usable AWBG.

**FIGURE 6.** Simulated variation with frequency of $S_{21}$ for finitely-long core conducting rod of radius $a = 3 \text{ mm}$ wrapped by transversely strip-grated dielectric pipe layer of outer radius $b = 3.9 \text{ mm}$, permittivity $= 2.0\varepsilon_0$, permeability $= \mu_0$, comprising 26 cells each of period $p = 3 \text{ mm}$ and strip-width $w = 0.45 \text{ mm}$.

Despite this wide SWBG spanning from 9.75 to 14 GHz, the AWBG is extremely narrow, being only about from 9.75 to 10 GHz (a meager 2.5% fractional bandwidth), which accounts for the dip of the $S_{21}$ over this likewise small band in Fig. 7. Outside this tiny AWBG, the transmission becomes prohibitively strong, rising to as high as about $-3 \text{ dB}$, hardly (if at all) qualifying as a band gap. Had it instead been the SWBG by which the expectation for wave suppression is set, then the anticipation of low transmission in Fig. 6 from 9.75 to 14 GHz would end in disappointment. This demonstrates the vital role of the AWBG rather than just SWBG in attaining strong decoupling between elements on the surface of cylindrical rods, in spite of the conventionally widespread utility of the latter. Such narrow AWBGs of transversely grated conducting rods are unfortunately not so useful in most practical applications that call for even just modest bandwidths.

**VI. STRONG DECOUPLING BY AWBG OF GRATED DIELECTRIC PIPES OVER CORE DIELECTRIC RODS**

As mentioned in the Introduction, the key to achieving AWBG with relative ease, particularly as compared to the preceding structure with a conducting core rod, is to do away with the latter and replace it with a dielectric one. Typical assumptions of a core metal rod may understandably be motivated by the evolution from its planar analogue of strip-gratings imprinted on a dielectric slab grounded by a flat conductor, the latter often indeed required or is present in practice. However, there is no basis for such conventions in the cylindrical case and the core rod does not have to be metallic or of any other specific material. Contrary to the conducting core counterpart, it does not take intensive parametric studies over vast dimensional spaces of the version with a dielectric core rod to readily reveal cases with wide AWBG, attributed to the additional degree-of-freedom the latter provides. Just a partial scope of our wide-ranged parametric studies will be reported later in Section X. One among a few further examples herein is presented next.

Fig. 7 displays the dispersion diagram of a transversely strip-grated dielectric pipe wrapped over a core dielectric rod, of which the continuous traces computed by the analytical method are also validated by the circle markers of CST. For this case, the parameters are as follow: $a = 2 \text{ mm}$, $b = 8.8 \text{ mm}$, $\varepsilon_{in} = 2.25\varepsilon_0$, $\varepsilon_{out} = 3.375\varepsilon_0$, $\varepsilon_{ext} = \varepsilon_0$, $\mu_{in} = \mu_{out} = \mu_{ext} = \mu_0$. Given as an inset graph is the variation with frequency of the attenuation constant $\alpha$ of the leaky fast space wave mode from 5.6 to 6.8 GHz, which becomes a slow surface wave mode beyond 6.8 GHz with associated vanishing $\alpha$. As observed, an AWBG from 4.15 to 5.6 GHz with a large fractional bandwidth of 30% is portrayed, which is within a wider SWBG from 4.15 to 6.8 GHz that contains a passband for fast space waves from 5.6 to 6.8 GHz. For a finitely long version of this rod comprising 26 cells, each of period $p = 3 \text{ mm}$ and strip-width $w = 0.6 \text{ mm}$, it is readily seen from Fig. 8 how the simulated $S_{21}$ is indeed very low (essentially below $-30 \text{ dB}$) throughout this AWBG. But once outside of it, from 5.6 to 6.8 GHz, even though still within the SWBG, the transmission rises above $-10 \text{ dB}$, reaching as high as $-3 \text{ dB}$. As before, false expectation of low transmission over the wider SWBG (4.15 to 6.8 GHz) would have led to dismay. This once again verifies the effects of the AWBG as well as the drastically reduced decoupling strength within parts of the SWBG that permits the presence of space waves. This 30% fractional bandwidth of strong decoupling far surpasses the 2.5% (more than tenfold) offered by the case with core conducting rods of Section V.
To further exemplify the fact that AWBG characteristics are readily found in strip-grated pipes that wrap over dielectric core rods, the distinctive decoupling effects of AWBG as opposed to SWBG are further demonstrated by another set of parameters, this time for the following: $a = 1$ mm, $e_{in} = 2.0\varepsilon_0 b = 10$ mm, $e_{out} = 2.25\varepsilon_0$, $e_{ext} = \varepsilon_0$, $\mu_{in} = \mu_{out} = \mu_{ext} = \mu_0$, the dispersion diagram of which is given by Fig. 9, generated by both the ASBC-based modal method presented herein and simulations of CST. The presence of an AWBG between 4 and 5.8 GHz (36.7% fractional bandwidth) that is within SWBG from 4 to 7.5 GHz is seen. This 36.7% exceeds the 30% of the previous case and surpasses the 2.5% of the metal core version by an even greater extent (15 times larger).

In the same way as before, a finite version of this rod comprising 26 cells and with a period of $p = 5$ mm and strip-width $w = 1$ mm is simulated in CST, from which the $S_{21}$ vs. frequency graph is given in Fig. 10. A deep basin over a wide AWBG between 4 and 5.8 GHz is indeed portrayed. This once more verifies the isolation effects of the stricter AWBG in a truer way rather than the looser SWBG which would have overestimated the suppression band and given false hopes that lead to poor designs.

**VII. MULTI-FREQUENCY OPERATION**

Before commencing this subsection, it is first stated that the design parameters adopted here were obtained by rigorous investigations over the parametric space spanned by all four parameters that characterize the grated rod (namely the radiiues and permittivities of the inner rod and outer pipe), a partial scope of which is presented later in Section X.

As the case of Fig. 2 has shown, there is a surface-wave pass-band up to 6 GHz, an AWBG from 6 to 8 GHz, followed by a pass-band (for both surface and space waves) from 8 to 11 GHz. Whereas, that of Fig. 7 displays a deep basin over wide AWBG (4 to 5.8 GHz) indeed portrayed.

![FIGURE 9. Dispersion for core dielectric rod of radius $a = 1$ mm and $e_{in} = 2.0\varepsilon_0$ wrapped by transversely strip-grated dielectric pipe layer of outer radius $b = 10$ mm and $e_{out} = 2.25\varepsilon_0$, $e_{ext} = \varepsilon_0$. Dot markers: present ASBC-based modal approach, circle markers: CST (axial period 5 mm, strip width $w = 1$ mm). AWBG between 4 and 5.8 GHz (36.7% fractional bandwidth) is within SWBG from 4 to 7.5 GHz. Cross markers: $\beta = k_0 \cos (\theta_{CST})$ where $\theta_{CST}$ = beam angle for 2 connected rods under port 1 excitation, this rod's end as port 1; the other rod (whose end is port 2) is that of Fig. 2.](image1)

![FIGURE 10. Variation with frequency of $S_{21}$ for finitely-long core dielectric rod of radius $a = 1$ mm and $e_{in} = 2.0\varepsilon_0$ wrapped by transversely strip-grated dielectric pipe layer of outer radius $b = 10$ mm and $e_{out} = 2.25\varepsilon_0$, comprising 26 unit cells each of axial period $p = 5$ mm, strip width $w = 1$ mm; $e_{ext} = \varepsilon_0$, $\mu_{in} = \mu_{out} = \mu_{ext} = \mu_0$. Deep basin over wide AWBG (4 to 5.8 GHz) indeed portrayed.](image2)
surface-wave mode up to 4.15 GHz followed by an AWBG from 4.15 to 5.6 GHz, a pass-band from 5.6 to 8 GHz, and another AWBG from 8 to 9 GHz. For the sake of convenience, let the cylindrical structure of the latter (Fig. 7) be referred to as leaky-wave antenna 1 while that of the former (Fig. 2) as antenna 2. Upon connecting finitely-long versions of these two grated rods together coaxially, we get the schematic in the inset diagram of Fig. 11, the respective ends of which being the associated ports; i.e., antennas 1 (thicker rod) and 2 (thinner rod) are excited from their terminal ends by ports 1 and 2.

Now, if the operation frequency for space-wave radiation of antenna 1 is set as, e.g., 6.2 GHz, which is of course within its own pass-band (5.6 ~ 8), but also within the AWBG (6 ~ 8) of antenna 2, while that of antenna 2 is set at, say 8.6 GHz, being inside its own pass-band (8 ~ 11) as well as in the AWBG (8 ~ 9) of antenna 1, the operation of any one antenna will then have mitigated interference with that of the other one. This facilitates an effective dual-frequency cylindrical coaxial antenna, each operation band responsible for a certain function.

The simulated variation with frequency of $|S_{11}|$ and $|S_{22}|$ for these two coaxially connected finite rods (port 1 is that of Fig. 7 while port 2 is that of Fig. 2) are presented in Fig. 11. Going by the stipulated operation frequencies of $f_1 = 6.2$ GHz and $f_2 = 8.6$ GHz respectively for antennas 1 and 2 (fed by ports 1 and 2), it is observed from Fig. 11 that $|S_{11}|$ and $|S_{22}|$ are indeed low and high respectively at $f_1 = 6.2$ GHz, whereas at $f_2 = 8.6$ GHz, they are expectedly high and low, again respectively, differing in levels in both cases by at least 20 dB.

From the dispersion diagram of Fig. 7 for the rod structure as antenna 1, the axial propagation phase constant at $f_1 = 6.2$ GHz is about $\beta_{z,1} = 95$ rad/m, which translates to an expected elevation beam angle of $\theta_1 = \cos^{-1}[\beta_{z,1}/(2\pi f_1/c)] = 43^\circ$ made by the beam direction with the axial $z$ axis, with $c = 3 \times 10^8$ m/s, the speed of light in vacuum. This beam direction indeed shows up in Fig. 12(a), which conveys the simulated gain pattern in a principal $\phi = 90^\circ$ plane cut of the two coaxially connected rod antennas under port 1 excitation. Conversely, from the dispersion diagram of Fig. 2 for rod antenna 2, its $\beta_{z,2}$ is about $155$ rad/m at $f_2 = 8.6$ GHz, pertaining to an anticipated $\theta_2 = \cos^{-1}[\beta_{z,2}/(2\pi f_2/c)] = 30^\circ$. This is likewise manifested in the simulated gain pattern of Fig. 12(b) for the same structure in the same azimuth plane, but this time with port 2 being excited, appearing as a beam towards $153^\circ$, which closely approximates $\theta'_2 = 180^\circ - \theta_2 = 150^\circ$, this deduction from $180^\circ$ being due to the reversal of the excitation wave direction (+z and −z traveling waves are launched from ports 1 and 2 respectively). Presented in Figs. 13(a) and 13(b) are the corresponding simulated three-dimensional (3D) directivity patterns portraying conical beams as expected of our rod structures, and of which Figs. 12(a) and (b) are planar plots in the $\phi = 90^\circ$ plane, for those same two coaxially connected finite rods, as of Fig. 11, i.e., Fig. 13(a) is with port 1 excited at $f_1 = 6.2$ GHz, while Fig. 13(b) is for port 2 excited at $f_2 = 8.6$ GHz. In both Figs. 12 and 13, the 'nice and clean' beams without the presence of any other severely distortive lobes in any one radiation pattern are indicative of the effective isolation of either one operation mode from the other, as intended by the designated AWBGs.

As an additional illustration, this time with one of them as an SWA, let us refer back to Fig. 9, which has displayed a surface-wave mode up to 4 GHz followed by an AWBG from 4 to 5.8 GHz and a pass-band beyond. By replacing leaky-wave antenna 1 of the previous study with the configuration of this Fig. 9 while that of Fig. 2 remains as antenna 2 (but now as an SWA), then if this new antenna 1 operates at say $f_1 = 6.6$ GHz within its own fast-wave pass-band...
(5.8 ~ 7.5 GHz) and also in the AWBG (6 ~ 8 GHz) of antenna 2, whereas the latter is now operated at say \( f_2 = 5 \) GHz, being within its surface-wave pass-band (below 6 GHz) and in the AWBG (4 ~ 5.8 GHz) of antenna 1, we may then expect that the leaky-mode fast space-wave radiation by antenna 1 will be well isolated from the slow wave propagation supported by antenna 2, operating simultaneously as a surface-wave antenna.

S-parameter simulations of this pair of coaxially connected grated rods, each comprising 26 unit cells of axial period \( p = 3 \) mm and strip width \( w = 0.6 \) mm, were carried out, results of which are presented in Fig. 14 as graphs of \(|S_{11}|\) and \(|S_{22}|\) versus frequency. According to the prescribed operation frequencies of \( f_1 = 6.6 \) GHz and \( f_2 = 5 \) GHz, it is evident from Fig. 14 that \(|S_{11}|\) and \(|S_{22}|\) are predictively low and high at \( f_1 = 6.6 \) GHz, whereas they are at \( f_2 = 5 \) GHz high and low as anticipated, all respectively. The absolute dB difference between \(|S_{11}|\) and \(|S_{22}|\) at either of the two operating frequencies is in excess of 15 dB, indicative of strong decoupling between both functions.

The simulated three-dimensional (3D) far-field directivity patterns of these two coaxially attached finite rods of Fig. 14 are presented in Figs. 15(a) and 15(b) for excitations of ports 2 and 1 at 5 GHz and 6.6 GHz, respectively. With port 2 being excited and at \( f_2 = 5 \) GHz, the strong radiation towards the \(-z\) direction along the axis due to surface-wave propagation is clearly conveyed by Fig. 15(a), whereas the 3D pattern at \( f_1 = 6.6 \) GHz under port 1 excitation displaying a fast space wave beam emitted towards \( \theta_{\text{beam}} = 45^\circ \) is seen in Fig. 15(b) and more clearly shown by its 2D pattern of Fig. 15(c) in \( \phi = 0 \) plane. From the dispersion diagram of Fig. 9 for the rod as antenna 1, the axial propagation phase constant at \( f_1 = 6.6 \) GHz is about \( \beta_1 = 100 \) rad/m, translating to an expected \( \theta_{\text{disp}} = \cos^{-1}[\beta_1/(2\pi f_1/c)] = 44^\circ \) which virtually equals \( \theta_{\text{beam}} = 45^\circ \). Indicated as cross markers in Fig. 9, being the dispersion diagram of the rod

**FIGURE 13.** Simulated 3D directivity patterns portraying conical beams, of which Figs. 12(a) and (b) are planar plots in \( \phi = 90^\circ \) plane, for two coaxially connected finite rods, as of Fig. 11, (a) port 1 excited at \( f_1 = 6.2 \) GHz, and (b) port 2 excited at \( f_2 = 8.6 \) GHz.

**FIGURE 14.** Simulated variation with frequency of \(|S_{11}|\) and \(|S_{22}|\) for two coaxially connected finite rods (inset diagram); port 1: core dielectric rod of radius \( a = 1 \) mm and \( \varepsilon_{\text{in}} = 2\varepsilon_0 \) wrapped by transversely strip-grated dielectric pipe layer of outer radius \( b = 10 \) mm and \( \varepsilon_{\text{out}} = 2.25\varepsilon_0 \); port 2: \( a = 2 \) mm, \( \varepsilon_{\text{in}} = 2\varepsilon_0 \), \( b = 6 \) mm, \( \varepsilon_{\text{out}} = 3.8\varepsilon_0 \); each of both comprising 26 unit cells of axial period \( p = 3 \) mm, strip width \( w = 0.6 \) mm, and with \( \varepsilon_{\text{ext}} = \varepsilon_0 \), \( \varepsilon_{\text{in}} = \varepsilon_{\text{out}} = \varepsilon_{\text{ext}} = \varepsilon_0 \).

**FIGURE 15.** Simulated 3D directivity patterns of two coaxially connected finite rods of Fig. 14: (a) at \( f_2 = 5 \) GHz with port 2 excitation as SWA, and (b) at \( f_1 = 6.6 \) GHz with port 1 excitation as LWA. In (c), planar plot in \( \phi = 0 \) plane of (b).

**FIGURE 16.** Photographs of the two manufactured prototypes of strip-grated dielectric pipe wrapped over core dielectric rod (the latter invisible). (a) \( a = 2 \) mm, \( \varepsilon_{\text{in}} = 2\varepsilon_0 \), \( b = 6 \) mm, \( \varepsilon_{\text{out}} = 3.8\varepsilon_0 \), and (b) \( a = 2 \) mm, \( \varepsilon_{\text{in}} = 2.25\varepsilon_0 \), \( b = 8.8 \) mm, \( \varepsilon_{\text{out}} = 3.375\varepsilon_0 \).
attached to port 1 in Fig. 15, are the phase constants translated via $\beta = k_0 \cos (\theta_{\text{beam}})$ from the simulated beam directions of the pair of coaxially connected finite rods (as of Fig. 15) under port 1 excitation.

VIII. EXPERIMENTATION ON PROTOTYPES

Two prototypes of the transverse strip-grated dielectric pipe wrapped around a core dielectric rod were manufactured. One of them comprises a core dielectric rod of radius $a = 2$ mm and $\varepsilon_{\text{in}} = 2.0\varepsilon_0$ wrapped by a transversely strip-grated dielectric pipe layer of outer radius $b = 6.0$ mm and $\varepsilon_{\text{out}} = 3.8\varepsilon_0$, thus pertaining to the case of Fig. 2. The second one is that of Fig. 7, bearing the following parameters: $a = 2$ mm, $\varepsilon_{\text{in}} = 2.25\varepsilon_0$, $b = 8.8$ mm, $\varepsilon_{\text{out}} = 3.375\varepsilon_0$. For both structures, $\mu_{\text{in}} = \mu_{\text{out}} = \mu_0$, and with common axial period $p = 6$ mm and strip-width $w = 2$ mm. Photographs of these two grated rods, in the above respective order, are shown in Figs. 16(a) and 16(b).

Three types of measurements were carried out, namely those of modal dispersion, the $S_{21}$ transmission coefficient, and far-field radiation patterns.

A. MODAL DISPERSION

For the measurement of the modal dispersion, the setup of the experiment is schematized by Fig. 17(a), in which a feed horn antenna that launches waves onto the rod and a coaxial probe that is oriented perpendicular to the cylindrical surface of the structure and moved along it with the help of a precision platform are each connected to one of the two ports of a vector network analyzer. The axes of the horn and rod are aligned with each other. Absorbers are placed around the structures to mitigate interferences by reflected and scattered waves. By sliding the probe along the surface of the rod, the waveform of the fields manifested via the cyclical transmission signal picked up by the probe can be measured, from which the surface wavelength of the oscillation is determined. By dividing $2\pi$ by this latter, the propagation constant along the surface of the rod is obtained, quantifying the amount of phase progression per unit distance along the rod. Extracted at various frequencies, the dispersion can be measured.

For the second rod structure, the measured $S_{21}$ is plotted against frequency in Fig. 20 and compared with the corresponding simulated transmission spectrum. As seen, the basin range from about 4.1 to 5.5 GHz representative of the theoretically predicted AWBG is indeed well portrayed by the practical experiment.
C. FAR-FIELD DIRECTIVITY PATTERNS

The far-field radiation patterns of the two manufactured grated rods coaxially connected to each other were also measured in an anechoic chamber. Photographs of the scenario and setup of the experiment are given in Fig. 21, which show the connected rods and feed horn antenna placed on a rotating platform on one end of the chamber, with a standard gain horn mounted on another pedestal at the other end.

![Photograph of measurements in an anechoic chamber](image)

By laying the interconnected coaxial rods flat on the platform as shown in the photographs, the axis of the structure is thus swept horizontally upon rotation of the platform, thereby measuring the radiation pattern in the plane that contains the rod axis, i.e., variation with theta.

With port 1 deemed as the end-side of the rod of Fig. 7 while port 2 as that of the rod of Fig. 2, the measured directivity patterns at 6.2 GHz and 8.6 GHz when the former and latter are excited by a feed horn are presented in Figs. 22(a) and 22(b), all respectively, the excited port-end of the interconnected rod being placed right in front of the horn and oriented perpendicular to its aperture (and as before, the horn and rod are coaxial). Given within each pattern is the corresponding simulated patterns, as those of Figs. 12(a) and 12(b). As can be seen, good agreement of the experimental results with predictions by theory has been achieved. Importantly, the distinct beam in any of the two patterns that is attributed to one of the two connected rods is uninterrupted by the presence of the other one. The effective simultaneous dual-band (bifunctional) operation expected of this design by virtue of the non-overlapping all-wave total band-gaps of the two rods is thus successfully verified by experimentations on a manufactured prototype.

![Graphs of directivity patterns](image)

FIGURE 22. Measured directivity patterns compared with simulations, for two coaxially connected finite rods, as of Fig. 11, (a) port 1 excited at $f_1 = 6.2$ GHz, and (b) port 2 excited at $f_2 = 8.6$ GHz.

IX. APPLICATIONS, ADVANTAGES, AND LIMITATIONS

This section discusses the applications, advantages and limitations of the proposed strip-grated rod in the context of the investigations of it as well as its use as leaky or surface wave antennas and cylindrical EBG structures.

A. APPLICATIONS

The strip-grated rod presented in this work serves well as antennas with omnidirectional radiation patterns in the azimuth plane and directive ones in the elevation plane, which are useful for modern wireless communications systems. Such radiation properties find applications such as wireless local area networks (WLAN) in indoor as well as outdoor environments [3], [6]. The use of cylindrical EBGs is to enhance the directivity in the elevation plane [4]. It is also able to fulfill the needs of applications such as high-gain millimeter-wave antennas required by consumer devices and multimedia services that demand high data rates and channel capacities. The nature of its topology also makes our rod compatible with integrated circuits with limited platform real estate. It can also be used for many radar, imaging, and probing applications, through its implementation with or as dielectric rod antennas and surface waveguides.

Being cylindrical rather than planar, our strip-grated rod is beneficial to applications which involve elongated structures that take on the forms of masts, poles, beams, struts, or the likes of them, as well as those with curved surfaces, such as aircraft fuselage, missiles, base station antennas, cylindrical arrays, conformal antennas, and many more, all of which the more common planar versions are instead unsuitable for. Our structure is also advantageous in applications with scarce
real estate (footprint area) on the platform or base panel but
permitting protrusion from it, such as base station and air
traffic control tower antennas.

The presented structure can also be used as plasmonic rods which support the modal propagation of surface plasmon polaritons (SPPs) that strongly confine fields and waves to the
cylindrical surfaces [14], [17], [18]. Besides being suitable for use as surface waveguides or surface-wave end-fire anten-
as, the localized electromagnetic energies and signals that it facilitates enables intense focusing at the terminal end that
may span minuscule regions, thus finding usage in probing and imaging applications requiring pinpoint precision, as well as in the medical arena such as for diagnostics and ablative
technologies.

B. ADVANTAGES

The strengths of the present study and design of our grated rod are multifaceted, as categorized in this subsection.

1) STRUCTURAL AND COST BENEFITS

The presented rod device is lightweight but yet structurally robust in nature, thus being resistant against damage. It is also simple and inexpensive to manufacture as compared to competing cylindrical EBG structures that are structurally more complicated. For instance, there are many which are composed of metal cylinders or other elements, arranged in circular array formations with radial and/or circumferential periodicities [5], [7], [19], [20], [21], [22], [23], [24], [25], [26], [27], and [28], thus being bulky, heavy, volumetrically cumbersome, and are easily damaged. Several others are in the form of longitudinal or axially-oriented corruga-
tions [29], [30], [31], which are also more complex, fragile, and costlier to fabricate. There are also others which are cilyndrically multilayer (multiple dielectric pipes wrapped over other pipes) [32], [33], which are more complex than ours, entailing just a single core rod wrapped by one grated pipe, being far simpler and cheaper to manufacture. Yet still, there are others that are just excessively complicated in structural and/or electrical nature; e.g., the use of magnetic springs, rubber rings, electrodes, spring-mass resonators, piezoelectric, etc. [34], [35], [36], [37], and [38].

2) TRUE & PROPER EBG QUANTIFICATION OR
CHARACTERIZATION

Our work here has performed a rigorous, complete, and thus, true quantification and characterization of the bandgap of the strip-grated cylindrical EBG rod, by studying the modal dispersion band diagrams (obtained by solving the eigenvalue problem formulated by the presented modal analysis entailing the ASBC) as well as substantiating with the transmittance spectra. Importantly, in addition to the slow surface-wave regions of the dispersion diagrams, we have also considered the fast leaky-wave regimes through complex wave analysis. This has herein been proven vital in ascertaining the AWBG, as opposed to just surface-wave EBGs assumed by most other works, which lack the true and complete characterization of wave suppression. The following subsections showcase the various ways, to varying degrees, that contemporary works in literature are lacking in these abovementioned aspects but which our paper possesses.

a: ABSENCE OF MODAL DISPERSION DIAGRAMS IN OTHERS

Unlike our work here, most other papers in the literature on
EBGs have not studied the modal dispersion diagrams for
conveying their bandgaps. Among these, many have not even presented anything that properly quantifies or conveys the bandgaps (widths and ranges) of their EBG structures, such as those of [3], [4], [6], [22], [24], [38], [39], [40], [41], and [42]. For the rest (of those without dispersion diagrams) that have at least exhibited some form of EBG characterization, they were through the spectra of transmission/transmittance or mutual coupling [20], [23], [26], [30], [31], [43], [44], [45], [46], [47], [48], [49], [50], [51], [52], [53], [54], and [55], all of which lack the theoretical rigor and scientific depth of the dispersion approach. Such spectra could at best be validations of the latter but should not supersede it as the sufficient portrayal of EBG phenomena in the first place.

b: EVEN IF PRESENCE OF DISPERSION STUDIES IN OTHERS

i) LACK OF CONSIDERATION OF FAST-WAVE REGIMES

For those few papers out there that have presented modal dispersion band diagrams, only slow surface-wave traces were considered in most of them, leaving out the study of modes within the fast-wave regimes [56], [57], [58], [59], [60], [61], [62], which is vital in determining whether or not AWBG is present.

ii) NARROWER AWBG BANDWIDTH

And for those even scarcer works that have presented dispersion band diagrams and also taken the fast-wave regime into account, most of them have narrower fractional AWBG bandwidths than ours, which is as high as 37% (as shown earlier above). For instance, the AWBG fractional bandwidth (FBW) reported in [34] is about 10% whereas the widest one presented in [37] is only around 16% while that of [63] is 16.5%. Those that came closer but still falling short of ours are [64] and [65] with 29% and 31% AWBG FBWs, respectively. These are summarized in Table 1.

| Ref. | FBW (%) |
|------|--------|
| [34] | 10     |
| [37] | 16     |
| [63] | 16.5   |
| [64] | 29     |
| [65] | 31     |

3) STUDY OF ATTENUATION CONSTANT

(ABSENT IN ALL OTHERS)

This article has rigorously studied the complex nature of the
wavenumbers (complex propagation constants along the rod axis) in the fast-wave regime, typically pertaining to leaky
wave modes, and the imaginary parts of which representing the attenuation constants. Such an investigation of this theoretical aspect is something that is lacking in most other works in the literature, despite its importance. The only two papers that have done so which could be found, at least as far as the extensive survey that we conducted, are just those of [32] and [37].

4) ANALYTICAL CLOSED-FORM MATHEMATICAL FIELD FUNCTIONS

Through the modal analysis presented herein, our paper possesses the advantage of the provision of analytical closed-form mathematical modal field functions, by first solving the eigenvalue problem at any given frequency for the resonant wavenumbers and then solving for the eigenvector containing the unknown amplitude coefficients by back substitution via Gauss-Jordan elimination for recasting the system matrix into a row-echelon form. Not only do these provide theoretical elegance and completeness, these field expressions also offer conceptual insights into the electromagnetic phenomena of the structure. The ASBC-based formulation here is simple to follow, easy to code up in a computer program, and the latter is rapid but yet accurate. Whereas, not only do most papers out there lack such modal theories, those few that do exhibit field analyses [19], [29], [32], [33], are however excessively complex (involving unclosed infinite summations and integrals), programmatically cumbersome, algorithmically difficult, and also likely computationally less efficient.

Moreover, the analytical field functions provided here enable the plotting of modal fields (as we have done so as given earlier), something which most other works also fail to match up with. For those few that do present field plots, they were mostly simulated by commercial field solver software rather than via closed-form mathematical expressions [25], [26], [31], [34].

5) NOVEL APPLICATION AS MULTIFREQUENCY LWA ROD ANTENNA

Last but not at all the least, we are the first and thus only ones (at least to the best of our knowledge) who have studied the use of such grated rods with wide AWBGs for beneficial use as cylindrical LWAs in high-performance multifunction, multifrequency applications with mitigated mutual coupling among the ports of the various simultaneous operations.

A summary of the strengths and novelties of our analysis work and proposed design is offered in Table 2.

C. LIMITATION

A limitation of the presented modal analysis treatment of the strip grated dielectric rod is that the periodicity of the grating is assumed to be much smaller than the wavelength, typically by a factor of around 10, thus being a prerequisite for validity or applicability of the method. The very fabric of the entailed ASBC hinges on the homogenization of the grated surface into a new one that has shed the microscopic grating details of the former, preserving only the macroscopic effects which the homogenized surface have on the tangential fields via the boundary conditions. As such, the cell size and the strip-width do not ‘enter the picture’ of the formulation and the technique thus is unable to discern between grated rods of different periodicities and strip-widths, and subsequently the varying effects they lead to. This shortcoming inflicts not only the present cylindrical rod but all other forms of strip-grated surfaces treated by the ASBC.

TABLE 2. Strengths, advantages, novelty of our analysis and design.

| Structural & cost benefits                  | True and proper EBG characterization |
|---------------------------------------------|--------------------------------------|
| Wide FBW of AWBG (latter often neglected elsewhere) | Includes study of attenuation (rarely done) |
| Theoretical elegance and completeness       | Simple to follow and formulate, easy to code up, computationally rapid, but yet accurate |
| Novel application as multifrequency LWA rod antennas | |

X. PARAMETRIC STUDIES

Carried out in this section are investigations of the effects which variations of the parameters have on the dispersion phenomena. Four parameters of the grated rod shall be considered, namely the 1) radius \( a \) of the core rod, 2) outer radius \( b \) of the outer pipe (via \( b/a \)), 3) relative permittivity of the core rod \( \varepsilon_{in}/\varepsilon_0 \), and 4) permittivity of the outer pipe conveyed as \( \varepsilon_{out}/\varepsilon_0 \).

Before proceeding, it is first asserted that the parametric study presented herein spans only a subspace of a vaster domain of the multidimensional parametric space that was investigated (from which the design parameters of Section VII were obtained), of which the complete results would be too voluminous to present here. Starting off, the so-called control parameters assumed in this partial scope selected for presentation are first defined. They refer to attributes that are kept common throughout all categories of parametric investigations, of course except the one that is being varied. These shared control parameters are stated as follow: \( a = 2.0 \text{ mm}, b/a = 3.0, \text{ period } \varepsilon_{in}/\varepsilon_0 = 2.5, \text{ and } \varepsilon_{out}/\varepsilon_0 = 1.75 \). About these values are various ones of the four parameters considered in the parametric studies conducted herein, which are tabulated in Table 3, in which the row of control values is highlighted.

TABLE 3. Parametric values.

| \(a (\text{mm})\) | \(b/a\) | \(\varepsilon_{in}/\varepsilon_0\) | \(\varepsilon_{out}/\varepsilon_0\) |
|------------------|--------|-------------------------------|-------------------------------|
| 1.0              | 2.0    | 2.0                           | 1.25                          |
| 1.5              | 2.5    | 2.25                          | 1.5                           |
| 2.0              | 3.0    | 2.5                           | 1.75                          |
| 2.5              | 3.5    | 2.75                          | 2.0                           |
| 3.0              | 4.0    | 3.0                           | 2.25                          |

With these laid out, the dispersion diagrams for the four sets of parametric variations are presented in the four panels in Fig. 23, each containing the modal traces for the various
values of the investigated parameter, as indicated in the legends, with all the rest of the parameters taking on their control values as tabulated in Table 3 and specified in the title at the top of each plot. Parametric variations for \(a, b/a, \varepsilon_{in}/\varepsilon_0 \), and \( \varepsilon_{out}/\varepsilon_{in} \) are given respectively in Figs. 23(a) to 23(d). Straightaway, it is seen that AWBGs are present in all investigated cases, despite the randomness of the choice of this regime within the wider parametric space (that we have investigated) for showcasing here. This once again proves the ease of obtaining AWBGs by the present topology entailing a core dielectric rod as opposed to a conducting one, the difficulty of which already shown by Fig. 4.

Observing further, as the investigated parameter increases in value, there is a general leftward shift of the modal traces. This would be expected of the first two dimensional parameters, associated with frequency scaling (structural upsizing generally leads to frequency downshifting). The same can also be said of the other two material parameters, since in order to maintain a certain modal resonance pertaining to a particular waveform oscillation, a lower frequency would be entailed to match up or compensate for the reduction in the effective wavelength within the medium that arises from an increase in permittivity.

### XI. DISTINCTION FROM RELATED ANALYSIS REPORTED IN PRIOR WORK

The previous ASBC treatment of [14] had the strip-grated dielectric pipe wrapped over a core electric conducting rod, whereas it is now a dielectric rod. This entails different mathematical analysis. Stating just a couple of them, Bessel functions of only the first kind within the core dielectric rod are herein used to describe the fields within the core dielectric rod (without the second kind), whereas previously no field descriptions were needed for the core conducting rod; and different boundary conditions on the surface of the core dielectric rod are now entailed, namely the continuities of both the tangential \( E \) and \( H \) fields, whereas just the vanishing of the tangential \( E \)-fields over the former metal core rod that was entailed.

That earlier work of [14] also considered only slow surface-waves whereas the present one studies both slow surface and fast leaky space waves, the complex wave-numbers of the latter also manifesting a very important quantity: the attenuation constant, which was absent previously. To include the investigation of the fast-wave regime calls for substantially more analytical and programming work requiring deeper theoretical knowledge than just treating the slow-wave regime. And only through the proper consideration of the fast-wave modal traces in the dispersion diagrams can AWBGs be ascertained (vital for truly strong wave suppression), as we have done here. Furthermore, no far-field radiation patterns were studied there but are presented here, both computationally and experimentally. Moreover, as compared to that of [14], the presented formulation in the paper is more elegant, concise, but yet self-sufficient and complete.

Ultimately, the primary value of this herein configuration lies more with the discovery of its ability to provide AWBGs with large FBWs and its vital application as multifrequency cylindrical LWA with mitigated interband interferences under simultaneous operations, rather than its ASBC-based modal treatment, which takes on a secondary but nonetheless still significant contribution. This aforementioned main value is thus applicative in nature (innovation that solves or relieves existing engineering challenges or shortcomings), whereas that of [14] offers no improvement or support of any practical technology.
is based upon are listed as: patch size = 15 mm, gap = 1 mm (for a period of 16 mm), slab thickness = 1.6 mm, and slab permittivity = 4.5$\varepsilon_0$. As observed, despite the slight translational shift along the horizontal frequency axis of the modal traces between the two configurations, which is only expected and understandable considering the difference in structural form, the surface-wave bandgaps of both versions are almost equal, which is what matters; being about 3.5 to 7 GHz for Fig. 24(a) and around 2.2 to 3.3 GHz for Fig. 24(b). The close analogy between the conventional mushroom EBG structure and its rolled-up cylindrical counterpart is thus established.

XIII. CONCLUSION

The asymptotic strips boundary conditions (ASBC) along with classical vector potential analysis have been used to study transverse strip-grated dielectric pipes sheathed over core dielectric rods. This approach is swifter and less memory-intensive than either software solvers or full-wave modal techniques, but yet not inferior in accuracy. On top of these, ours is also much simpler to formulate and code up in computer programs than full-wave methods, thus reaching further out to readers that are not well within the inner circles of this subject. The herein technique thus constitutes a sweet-spot that is not burdened by the main drawbacks of its contemporaries but yet preserves the benefits of them all.

Compared to the typical conducting core rod counterpart which is incapable of realizing all-wave band-gaps (AWBG) that forbid the existence of both fast and slow waves, the additional degree-of-freedom provided by our proposed dielectric core version allows parameters that yield AWBGs with wide bandwidths to be found fairly easily. This property of total suppression of waves has been shown to be essential in ensuring that both ends of the rod (or generally any two points on it) are adequately decoupled, giving rise to a truly effective cylindrical EBG structure. An important application of this feature is in achieving highly effective multi-frequency multi-function cylindrical antennas that operate at different bands simultaneously with mitigated inter-band interferences, as has been showcased here.

Manufactured prototypes were measured for modal dispersion and transmission, yielding results that concur well with theoretical predictions.

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MALCOLM NG MOU KEHN (Senior Member, IEEE) received the B.Eng. (Hons.) degree from the National University of Singapore, Singapore, in 2001, and the Licentiate and Ph.D. degrees from the Chalmers University of Technology, Gothenburg, Sweden, respectively, in February 2004 and December 2005, all in electrical engineering. From 2006 to 2008, he was a Postdoctoral Fellow with the Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg, MB, Canada. Following this, he proceeded to Concordia University, Montreal, QC, Canada, for another year of postdoctoral research. In 2009, he joined the Department of Electrical and Computer Engineering, National Chiao Tung University (NCTU), Hsinchu, Taiwan, now renamed as the National Yang Ming Chiao Tung University (NYCU), where he is currently a Professor. He has been actively involved in research projects funded by the Swedish Defense Research Agency, from 2002 to 2006. In Autumn 2004, he spent several months at the University of Siena, Italy, for a Research Visit. From 2006 to 2009, he worked extensively on numerous projects supported by Canadian industry and national research bodies. Since August 2009, he has been securing research project grants funded by the Ministry of Science and Technology, Taiwan. Over the past years, he has served numerous international symposia under various capacities, such as being part of the organizing and technical program committees, taking on the roles of technical program chairs as well as special session and workshop chairs. He was a recipient of the Union Radio-Scientifique Internationale (URSI) Young Scientist Award, in 2007. He received the NCTU Meritorous Teaching Award, in 2012 and 2013, and the NCTU Meritorous Mentor Award, in 2020. He is an Associate Editor of IET Microwaves, Antennas and Propagation.

CHENG-YU WU received the B.Eng. and master’s degrees in electrical engineering from the National Yang Ming Chiao Tung University, Taiwan, in 2021 and 2022, respectively. He majored in the fields of computer communications and robotic bionic technology. His research interests include antennas, applied electromagnetics, and microwave engineering.

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