Classical Control of Large-Scale Quantum Computers

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Abstract. The accelerated development of quantum technology has reached a pivotal point. Early in 2014, several results were published demonstrating that several experimental technologies are now accurate enough to satisfy the requirements of fault-tolerant, error corrected quantum computation. While there are many technological and experimental issues that still need to be solved, the ability of experimental systems to now have error rates low enough to satisfy the fault-tolerant threshold for several error correction models is a tremendous milestone. Consequently, it is now a good time for the computer science and classical engineering community to examine the classical problems associated with compiling quantum algorithms and implementing them on future quantum hardware. In this paper, we will review the basic operational rules of a topological quantum computing architecture and outline one of the most important classical problems that need to be solved; the decoding of error correction data for a large-scale quantum computer. We will endeavour to present these problems independently from the underlying physics as much of this work can be effectively solved by non-experts in quantum information or quantum mechanics.

Keywords: quantum computing, topological quantum computing, classical processing

1 Introduction

Quantum technology, specifically large-scale quantum computation, has been a significant research topic in physics since the early 1990’s. Since the publication of the first quantum algorithms [1], illustrating the computational power of quantum computers, millions of dollars has been invested worldwide and numerous technological advances have been made [2,3,4,5,6,7]. It is now routine for multiple experimental laboratories to fabricate and control small arrays of quantum bits (qubits) and perform proof of principal experiments demonstrating small quantum algorithms and protocols [8]. Quantum technology has also moved into the industrial sector via protocols such as Quantum Key Distribution (QKD) and Quantum random number generators and many non-physicists are aware of the D-Wave quantum computer, which while scientifically controversial is an attempt to build a analogue quantum computer capable of solving certain types of optimisation problems [9,10,11].
Recent experimental results in 2014 have demonstrated that two experimental systems can be built with high enough accuracy to satisfy the constraints of fault-tolerant, error corrected quantum computation [12][13]. As error rates on qubit arrays is high compared to classical nano-electronics, extensive error correction is required to successfully perform computation [11][14][16][17]. One of the most seminal results in quantum information theory is the fault-tolerant threshold theorem [18]. This theorem states that provided the fundamental error rate associated with qubits and quantum gates falls below a threshold, then arbitrarily long quantum computation is possible with a polylogarithmic overhead in physical resources. This threshold is a function of the type of quantum error correction code used for the computer [11][15][16][17] and extensive research has been performed to derive new codes, with high thresholds, that are amenable to experimental architectures. Arguably the most successful class of codes that have been developed are known as topological quantum codes [19][20][21][22][23]. Topological quantum codes are defined over a lattice (of arbitrary dimension depending on the code, but the most common are 2- and 3-dimensional) of physical qubits. The code itself can be defined over small, physically local groups of qubits while the properties of the encoded information is a global property of the entire lattice. This is what defines the code as topological. These codes are arguably preferred in quantum computer development as they exhibit comparably high fault-tolerant thresholds and they are adaptable to the physical constraints of experimental quantum systems.

Irrespective of the actual quantum code chosen to protect a quantum computer, it is well known that operating such a system requires extensive classical control infrastructure. This is not simply related to the control of the physical device hardware needed to operate a qubit (lasers, signal generators etc...), but it is also required to decode error correction information produced by the computer. This classical control software development is in its infancy and has received little attention within the fields of quantum information and classical computer science [24][25]. While there has been much work at the more abstract level of quantum algorithm design and circuit optimisation [26][27][28][29][30][31][32], we now have to go one step deeper and connect the high level work to the physical constraints of the quantum hardware.

This paper will introduce one of the main classical computer science and engineering problems associated with controlling a large scale quantum computer. We will focus on a specific form of quantum computer; namely a system that is built using an error correction code known as Topological Quantum Clusters (TQC) [33][34]. This code has received significant attention in recent years due to multiple hardware architectures utilising it in designing large scale systems [23][35][36][37][38][39][40]. We won’t discuss the details of how information to be encoded or manipulated. Instead we will focus on the basic error correction properties of the code and what this implies for classical processing of this data. In section 2 we will provide some background information on the basic definitions of qubits and quantum logic. In section 3 we will provide a brief introduction to the TQC model. This will not be an in depth introduction, but should provide
enough material to grasp the classical problems that need to be solved. Finally, in section 5 we will examine the processing that needs to be developed to perform dynamic error correction on the system and discuss the potential problems associated with the massive amount of classical data produced by the computer.

2 Quantum Computers

A qubit is the quantum analogue of a bit. Its state is defined as a vector of dimension 2, where $|0\rangle = (1, 0)^T$ is the vector notation for the value corresponding to binary 0, and $|1\rangle = (0, 1)^T$ correspond to 1. The state of one qubit can be written as the linear combination $|q\rangle = a_0|0\rangle + a_1|1\rangle$, where $a_i \in \mathbb{C}$ and $\sum_i |a_i|^2 = 1$; this is a superposition of the two basis states, a concept with no analogy in classical computing. Given the principle of superposition, an array of $n$ qubits can be in an equal superposition of all binary states from $|0\rangle$ unto $|2^n-1\rangle$, i.e., $\sum_{i=0}^{2^n-1} a_i |\text{BIN}(i)\rangle$, where $a_i$ are complex numbers and $\text{BIN}(i)$ is the binary expansion of $i$.

**Measurement:** In quantum computing, measuring a state is the only way to observe results of calculation. Measuring an arbitrary quantum state $|q\rangle = a_0|0\rangle + a_1|1\rangle$ can result in two outcomes: $|0\rangle$ (with probability $|a_0|^2$), or $|1\rangle$ (with probability $|a_1|^2$). Moreover, the measurement will collapse the state leaving it in the state corresponding to the measurement result.

The goal of a quantum algorithm is to manipulate the amplitudes of each binary state, $a_i$, such that the incorrect answers have very low amplitudes, $a_j \approx 0$, $j = \text{incorrect}$ while the correct answers have amplitudes close to one, $a_j \approx 1$, $j = \text{correct}$. This will ensure that after an algorithm is completed, we have a very high probability, when we measure every qubit, to measure the correct answer. The simplest initial state is to initialise each qubit in the computer in the $|+\rangle = (|0\rangle + |1\rangle) / \sqrt{2}$ such that each $a_i = 1/2^{(n/2)}$, $\forall i$. Therefore, initially, every possible binary state will have an equal probability of being measured. The quantum algorithm will then manipulate these amplitudes to suppress the amplitudes of incorrect answers and increase the amplitude of correct ones. At any given time the state of the quantum computer is represented by a $n$-dimensional complex vector $|\psi\rangle = (a_0, a_1, a_2, ..., a_{2^n-1})^T$.

**Quantum gates:** Quantum gates act on qubits and modify their states and hence modify the amplitudes of each binary state, $a_i$. They are represented as unitary (guaranteeing a gate is reversible, a necessity in quantum theory) matrices. An $n$-qubit gate, $G$, is described by a $2^n \times 2^n$ matrix and its action on the state of the quantum computer is described by simply computing $|\psi'\rangle = G|\psi\rangle$, where $|\psi'\rangle$ is the output and $|\psi\rangle$ is the input. It has been shown that any valid operation, $G$, can be decomposed into a discrete alphabet of single qubit and 2-qubit gates and consequently we only need to realise a small set of primitive qubit operations to realise any arbitrary computation. Shown below
is an example of such an alphabet, consisting of four single qubit gates and one two-qubit gate.

\[
\begin{align*}
X &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},
Z &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
H &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix},
\text{CNOT} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix},
T &= \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\pi/8} \end{pmatrix}
\end{align*}
\]

These gates form a universal gate set (technically, \(S = \{H, T, \text{CNOT}\}\) are sufficient for universality, we include \(X\) and \(Z\) because of their relevance for QEC), i.e., arbitrary quantum gates can be decomposed into products of these gates \([1]\). (This is similar to the classical case where all gates can be represented by equivalent circuits consisting of NAND gates only.).

The properties of quantum information allow us to create certain states that have no classical analogue. These states are called entangled states. For example, if we prepare two qubits in the initial state \(|+\rangle|0\rangle\) and apply the two qubit \(\text{CNOT}\) gate (where the control qubit is the one in the \(|+\rangle\) state), we get the output \(|b\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\). This state is known as a Bell state and it has properties that no classical computational state has. Specifically if we measure one of the qubits in the \(|0\rangle\) state, the second qubits is also found to be in the \(|0\rangle\) state. Similarly for the \(|1\rangle\) state. This behaviour is unique to quantum-bits and creation and manipulation of these types of states is an identifying feature when proving, experimentally, you have a true quantum system. Entanglement is a fundamental property of quantum information and forms the basis of the TQC model we will discuss in the next section.

3 Topological Cluster State Computation

The original formalism for quantum computation is the circuit based model \([1]\). This is where we have an array of qubits that is operated on by a pre-defined sequence of quantum gates to realise an algorithm. There is another method of performing quantum computation, known as the measurement based model (MBM) \([41]\). In this model, we pre-define what is known as a Universal Resource State (URS). A URS is a lattice of qubits where entanglement connections have been formed before any computation begins. This URS can be thought of as a graph, where each vertex represents a qubit and each edge is a two-qubit quantum gate that establishes entanglement between two vertices. Once this resource state has been prepared, quantum gates are realised by measuring individual qubits in well defined ways. As computation proceeds, qubits are consumed as they are measured. The first MBM was defined over a regular, 2-dimensional grid of qubits with nearest neighbour connections \([41]\). In this model, qubits are measured, column-by-column, to realise quantum gates. Essentially each row of qubits represented the world line of a given qubit of information and each column represented individual time steps of computation. As each column is measured, information is teleported to the next column and a quantum gate is applied during this teleportation.
Qubits measured column-by-column to perform computation

Fig. 1. A standard 2D lattice of qubits used for measurement based quantum computation. Qubits are measured from left to right and information is teleported from column to column. Processing occurs during this teleportation, applying quantum gates.

This 2-dimensional MBM showed that arbitrary computation could be achieved using a pre-defined URS, but it did not incorporate any error correction protocols to protect against noise.

The Topological Cluster State model is a MBM of quantum computation that incorporates a sophisticated topological error correction model by construction. It was derived from the seminal work of Kitaev \cite{19} and extended to a 3-dimensional entangled lattice of qubits that forms the initial URS \cite{33}. The fundamental unit cell of this lattice is illustrated in Figure 2. Again, each vertex in the image represents a physical qubit while each edge represents a two-qubit gate applied to form an entanglement bond. Preparing this state requires initialising each qubit in the $|\pm\rangle$ state, and applying a CZ gate between any two qubits connected by an edge. A CZ gate can be achieved by applying the cnot gate, interleaved by two $\text{H}$ gates on the target qubit \cite{1}. The total size of the 3-dimensional Topological cluster is dictated by the total resources needed for an algorithm. i.e. how many encoded qubits and gates does the algorithm need and how strong the error correction needs to be to successfully complete computation. For large quantum algorithms, the size of this lattice could be billions if not trillions of physical qubits \cite{42}.

3.1 Error Correction

The primary job of the TQC model is to perform error correction. The structure of the 3-dimensional lattice establishes certain symmetries that can be used to detect and correct errors that occur during the preparation and/or consumption of the state.

Arbitrary noise on a qubit can be decomposed into a series of bit-flips ($X$ gates) and phase flips ($Z$ gates). A phase flip is a gate which can convert the state $|+\rangle = (|0\rangle + |1\rangle) / \sqrt{2}$ into $|-\rangle = (|0\rangle - |1\rangle) / \sqrt{2}$ and has no classical analogue. A general error operator, $E$, acting on a single qubit can be written in the form,

$$E|\psi\rangle = k_I|\psi\rangle + k_xX|\psi\rangle + k_zZ|\psi\rangle + k_{xz}XZ|\psi\rangle$$

(2)
Fig. 2. Figure a) represents the unit cell of the lattice. Each of the Face qubits (red) are used to calculate the parity of the cell. The non-face qubits of Figure a) are face qubits on identical unit cells that are offset by half a lattice spacing along the three axes of the lattice.

where \( \{ |k_x|^2, |k_y|^2, |k_z|^2 \} \) are the probabilities that the qubit experiences an \( X \) error, a \( Z \) error or both. Therefore, to protect qubits against noise, we just need the ability to detect and correct for bit- and phase-flips.

The unit cell of the topological cluster has certain symmetries. Namely, if you measure the six face qubits of the unit cell (illustrated in red in Figure 2a) in the basis \( \{ |+\rangle, |-\rangle \} \) (known as an X-basis measurement) and you calculate the classical parity of the results (identifying the bit-value zero if we measure the qubit in \( |+\rangle \) and one if we measure it in \( |-\rangle \)), you will always get an even parity result under modulo 2 addition. i.e. while the individual measurements themselves are random, the symmetries of the quantum state of the unit cell will conspire (through the property of entanglement) to always generate an even parity result when you combine the measured values of these six qubits. Now, let us consider two of these unit cells side by side and the consequence of a \( Z \)-error on the qubit shared on a face [Figure 3a]. In quantum information the order in which you apply quantum gates is important. For example, the output of the operation \( XZ|\psi\rangle \) is not necessarily the same as the output of the operation \( ZX|\psi\rangle \), this is because the gates \( X \) and \( Z \) do not commute, i.e. \( XZ - ZX \neq 0 \). Instead, for these two operations the following holds, \( XZ = -ZX \). What does this mean when we measure our six face qubits of the unit cell when a qubit experiences an error? If no error occurs, then the six measurement, when combined modulo 2, gives us an even parity result. If one of those qubits experiences a \( Z \)-error prior to being measured in the \( X \)-basis the fact that \( XZ = -ZX \) means that the measurement of the erred qubit will flip from \( |\pm\rangle \) to \( |\mp\rangle \). Consequently, if the initial parity of the six measurements was even, it will flip to odd. Hence for the two unit cells shown in Figure 3a) when we measure the 11 face qubits and we observe a negative parity of the two sets of measurements, we can identify that a \( Z \)-error must have occurred on the qubit sharing a face

\[3 \] This is not a completely general description of a noise channel, but introducing the formalism for a general channel would require us to delve more into the mathematics of qubits.
between the two cells. Similarly errors on the other five face qubits are detected by parity flips with the other unit cells bordering the five other faces [Figure 3].

Fig. 3. A single error on a face qubit of a unit cell will cause two parity flips on the cells which share the qubit [Figure a)]. The six neighbouring cells bordering a given cell allows us to uniquely determine which qubit experienced an error [Figure b)].

An obvious question arises. We have so far only considered the six qubits on each of the faces of the unit cell. What about the other remaining qubits lying on edges? If we stack together eight unit cells into a cube, at it’s centre is an identical unit cell. The face qubits associated with this unit cell correspond to the qubits on the edges of the eight cells in the cube. The topological lattice embeds two self similar lattices, one which we call the primal lattice and the other which we call the dual. Face qubits on primal unit cells correspond to edge qubits on dual cells and visa versa. These two self similar lattices also explains how X-errors are corrected. In the previous paragraph we only considered Z errors because the Z-gate didn’t commute with the X-basis measurement of each face qubit and consequently the parity of the six face measurements flipped when an error occurred. Again, without going into the mathematical detail, the symmetries of the topological lattice allows us to convert X-errors on a qubit into Z-errors on other qubits. If an X-error occurs on a given qubit, the entanglement bonds connecting qubits can convert this X-error into Z-errors on all the qubits it is connected to. If you examine the structure of the unit cell [Figure 2a)] you will note that a given face qubit is only connected to qubits on the edge of a unit cell. Therefore an X-error occurring on a face qubit will be converted to Z-errors on edge qubits (which correspond to face qubits on dual cells). Therefore, all errors can be converted to Z-errors in either the primal or dual lattices and detecting these parity flips in both spaces is sufficient for correcting arbitrary errors on each individual qubit.

We discussed how single errors can be corrected by examining the parity of neighbouring cells, the next issue is what happens when multiple errors occur.

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Which is primal and which is dual is arbitrary.
This is shown in Figure 4. As the parity condition for a unit cell is calculated modulo two we only see an odd parity if an odd number of errors have occurred. If an even number occur then the parity will remain even. Therefore, if there is a chain of errors we will only see a parity flip for the two unit cells at the endpoint of the error chain. In the case of isolated errors, endpoints are of neighbouring cells. Hence decoding the error correction information requires us to match up the endpoints (which we detect via the calculation of a cells parity) with the actual physical sets of errors that occurred (which are not directly detected).

In quantum information we assign a probability, \( p \), that a given qubit will experience a bit (\( X \)) and/or phase (\( Z \)) error over some time interval, \( t \). This probability encapsulates the physical sources of noise such as environmental decoherence and control that could effect the operation of the qubit. Provided that \( p < 1 \), increasing numbers of errors occurring in a given time interval become exponentially less probable. Consequently, the most probable event that gives rise to the observed set of parity flips in the topological cluster is the one with the fewest number of errors. Given a set of parity flips measured in the topological cluster we connect them in a pairwise fashion such that the total length of all connections is minimised. This is a well known classical problem and was solved by Edmonds in 1967 [44] who developed a classical algorithm for minimum weight perfect matching who’s runtime scales polynomially with the number of nodes (which in our case corresponds to the number of parity flips we observe).

### 4 Physical Data Flow in an Operational Computer

What occurs in a physical quantum computer built using this model? For the TQC model, the physical quantum hardware is responsible for preparing the
lattice. If we assume that the physical qubits in the quantum computer are single particles of light (photons), then each photon is prepared from a source and sent through the quantum computer to be entangled with its neighbours \[36\]. Each 2-dimensional cross-section of the lattice is prepared sequentially as photons "flow" through the quantum hardware.

![Architecture for an optical quantum computer](image)

**Fig. 5.** From Ref. [24]. Architecture for an optical quantum computer. Single photons are prepared, sent through a preparation network which is responsible for creating the topological lattice. After the lattice is prepared it flows into detector arrays which performs measurement to perform computation.

Photons are continuously injected into the rear of the preparation network. Each passes through a network of quantum devices, which act to link them together into the topological lattice. Each quantum device operates on a fundamental clock cycle, \(T\), and each device operates in a well-defined manner. Once a given photon has been connected to its relevant neighbours, it does not have to wait until the rest of the lattice is constructed, it can be measured immediately. This is exactly how the actual computer will operate. The lattice is consumed at the same rate at which it is created, hence in the third dimension there only exists a small number of 2D cross-sections at any given time.

As one dimension of the topological lattice is identified as simulated time, the total 2D cross section defines the actual size of the quantum computer. The lattice is built such that when each 2D cross-section is measured, all encoded information is teleported to the next successive layer along the direction of simulated time allowing an algorithm to be implemented (in a similar manner to standard cluster state computation \[41\]).

In Figure 6 we illustrate the structure of the detection system. A given unit cell flows through a set of nine optical fibres which carry the individual photons that have been linked together in the lattice. As they flow into the detectors the parity of the cell is calculated as,

\[
P(i, j, T) = (s^{T-1}_{(i,j)} + s^T_{(i-1,j)} + s^T_{(i,j-1)} + s^T_{(i,j+1)} + s^T_{(i+1,j)} + s^{T+1}_{(i,j)}) \mod 2 \quad (3)
\]
where $s_{i,j}^T$ is the detection result (1, 0) of detector $(i, j)$ at time $T$.

Error decoding and correction must occur in real-time as the computer is operating in order to ensure the system will operate correctly. Hence the classical data processing must be done efficiently, fast and in a highly parallel way.

5 The decoding problem

The error correction decoding problem is a classical software and hardware optimisation problem to effectively perform the minimum weight perfect matching algorithm to an arbitrarily large topological lattice running at high speeds. Resource estimates for topological quantum computing has shown that to successfully implement fully error corrected, large-scale algorithms would require an enormous topological lattice [42]. The results of Ref. [42] indicate that a lattice of the order of a billion cells in cross-section, running for a year at 10 nanoseconds per cross-sectional sheet is necessary to factor a 1024-bit number using Shor’s algorithm. At 6-bits of raw data per cell, we would need to classically process on the order of $(6 \times 10^9)/(30 \times 10^{-9}) = 2 \times 10^{17}$ bits/second of data to perform error correction decoding for the entire computation.

This clearly is a phenomenal amount of data that needs to be processed while the computer is running. Clearly we require a large amount of parallel processing and a modular classical processing framework to decode error correction data for a full-scale machine. There has been work attempting to address this problem which falls into two categories. The first is further optimisation of the minimum weight perfect matching algorithm. The Blossom V algorithm is currently used when performing simulations of the topological cluster state model [45] and we can examine its performance for large lattices [Figure 7]. From this figure (which was produced by running the algorithm on a standard laptop) shows that Blossom V runs far too slowly to handle the processing of error correction data for
a large-scale computer. This necessitates further optimisation of the algorithm. Work by Fowler and others attempts to rectify this problem, but at this stage no benchmarking has been performed using this package. The second category is dedicated hardware implementations of the decoding operations. There are several steps which is illustrated in Figure 8.

The raw data is the bit streams coming directly from the quantum hardware. Parity filtering is the first step, where the co-ordinates of unit cells that have experienced a parity flip are retained and all other data is disregarded. This can reduce the amount of information as the probability that a unit cell of the lattice will experience a parity flip is of the order of the error rate of each qubit, $p$, which will be approximately 0.1%. The next step is to convert the collection of co-ordinates into a graph which is used as input for the minimum weight matching
algorithm. This data will produce a lookup table associating a vertex number for the graph with the co-ordinate of the relevant cell. The matching algorithm comes next and will produce a list of bi-partite connections telling us which nodes in the graph are connected. Output processing then converts these nodes back into the cell co-ordinates allowing us to correct the actual errors.

Each of these stages will have to be handled by dedicated circuits, built primarily for speed. This has not currently been done and we do not have evidence if current technology is sufficient to achieve fast enough speeds for quantum computing systems. For various physics related reasons, we do not wish to slow down the operational speed of the quantum hardware to accommodate slow classical processing. The speed of the classical system must be commensurate with the quantum system (which can vary between 10ns and 10ms depending on the underlying technology). The first generation of quantum computers will be slow, so the demands on the classical hardware should not be too significant in the short term. But more futuristic technology is being developed and will run at much higher clock rates. Designing the classical system with these faster systems in mind should ensure that quantum computer development is not bottlenecked by the necessary classical systems being underdeveloped.

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