Application of the Discrete Wavelet Transform in the Ranging Algorithm of Radio Fuze

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Abstract. Echo signal of radio fuze is a special transient signal whose wave parameters and arrival time are unknown. In this paper, an echo detection method of radio fuze based on discrete wavelet transform is introduced. The method adopts special wavelet basis function and scale factor, and obtain signal arriving time to realize distance measurement by the relationship that discrete wavelet coefficient of echo signal arrives peak at the corresponding time. Simulating results show that the method is feasible in radio fuze ranging application.

1. Introduction
Ranging of radio fuze is ascribed to a kind of testing problems of transient signals. It can realize measurement of the transient and nonsteady signal in noisy circumstances whose wave parameters and arriving time are both unknown. In some sense, it is also an important digital signal processing problem, and its measuring model of hypothesis test can be written as

\[ H_0 : x(t) = n(t) \]
\[ H_1 : x(t) = s(t) + n(t), \quad t \in [0, T] \]  

(1)

Here, \( x(t) \) refers to the echo signal, \( s(t) \) refers to the emitting signal, and \( n(t) \) refers to circumstantial noise which can usually be regarded as white Gaussian noise mixed in the echo signal. According to corresponding statistical theory, if the signal \( s(t) \) is determined, the optimal detector is Matched Filter. However, if the signal is unknown, GLRT should be utilized to perform maximum likelihood estimate for the unknown signal. When there is no any known signal or any experience, the optimal detector will become energy detector, and this is lower limit of the detector. So the detector’s performance can be greatly raised if we can effectively utilize some known experiences of the signal. Next we will expatiate on realizing the detection of unknown parameters echo signal by wavelet transform.

2. Wavelet transform analysis
Wavelet transform analysis involves decomposition of a signal function into a linear combination of approximations that are in simple, fixed building templates at different scales and positions from one mother function. The collection of these templates is called a wavelet basis, and constructed from a mother wavelet through scaling and shifting. Assume that the basic wavelet function is \( h(x) \), the scaling factor is \( a \), and the shifting factor is \( b \), then the mother wavelet can be defined as

\[ h_{a,b}(x) = |a|^{1/2} h \left( \frac{x-b}{a} \right) \]  

(2)
Moreover, the above defined mother wavelet must satisfy
\[ C_b = \int_{-\infty}^{\infty} \left| \frac{H(\omega)}{\omega} \right|^2 d\omega < \infty \]  
(3)
where \( H(\omega) \) is Fourier transform of function \( h_{a,b}(x) \), it is also be written as
\[ \int_{-\infty}^{\infty} h(x)dx = 0 \]  
(4)

For any signal function \( f(x) \in L^2(R) \), where \( L(R) \) refers to the set of all finite energy signal functions, the continuous wavelet transform can be expressed as followed
\[ W_f(a,b) = \int_{-\infty}^{\infty} f(x)h(a,x-b)dx = |a|^{-1/2} \int_{-\infty}^{\infty} f(x)h\left(\frac{x-b}{a}\right)dx \]  
(5)
According to the definition of inner product, Equation (5) can be rewritten as
\[ W_f(a,b) = \left\langle f(x), h_{a,b}(x) \right\rangle \]  
(6)

Obviously, each coefficient corresponds to \( f(x) \in L^2(R) \) decomposition on wavelet basis function \( h_{a,b}(x) \).

According to correlative conditions, the function \( h_{a,b}(x) \) can be referred as pulse response of a band-pass filter, so the continuous wavelet transform expressed by Equation (5) can be regarded as filtered signal through the band-pass filter. Because \( h(x) \) is window wavelet function, we assume that its signal width in time domain is \( \Delta h \), and the width in frequency domain is \( \Delta H \), then time domain width of the wavelet function \( h_{a,b}(x) \) is \( \Delta h_{a,b} = a|\Delta h| \) and its frequency domain width is \( \Delta H_{a,b} = \Delta H/|a| \). According to above results for \( h_{a,b}(x) \) and \( h(x) \), although their width product of time domain and frequency domain keeps unchanged, i.e. \( \Delta h_{a,b}\Delta H_{a,b} = \Delta H|a| \), their respective shape is quite different. With the increasing of the factor \( 1/|a| \), the smaller time domain width, the bigger frequency domain width, and it means that the resolution of time domain is getting higher and the resolution of frequency domain is getting lower. Similarly, there will be contrary results when the factor \( 1/|a| \) decreases. This characteristic of wavelet transform determines its special advantages in processing the abrupt signals.

3. Signal detection algorithm
Continuous wavelet transform can be discretized by sampling of the scale factor \( a \) and the shifting factor \( b \). Let \( a = a_0^m, \quad b = n a_0^m h, \quad (a_0 > 1, \quad m,n \in Z) \), then the discrete wavelet series
\[ C(m,n) = \int_{-\infty}^{\infty} h_{m,n}(t)f(t)dt \]  
(7)
Where
\[ h_{m,n}(t) = a_0^{-m/2}h\left(a_0^{-m}t-nb_0\right) \].

The target echo signal can be denoted as
\[ f(t) = \sum_{i=1}^{I} e^{-|t-t_i|} \sin\left[\omega_i(t-t_i) + \varphi_i\right] \delta(t-t_i) \]  
(8)
where \( A_i,t_i,\omega_i,\varphi_i \) are respective attenuation coefficient, arrival time, frequency and initial phase at No. \( I \) time sample.

Single side exponent function as followed is chosen as the basic wavelet function \( h(t) \), i.e.
\[ h(t) = \begin{cases} e^{-t} & t \geq 0 \\ 0 & t < 0 \end{cases} \]  
(9)

Then the coefficients of discrete wavelet transform (DWT) of \( f(t) \) are determined by
\[
C(m,n) = \begin{cases} 
\sum_{l=1}^{\frac{t}{T}} a_0^{-m/2} e^{-a_0^{-m} n b h} \sin(\theta_l + \frac{\omega_l}{2} + \left(a_0^{-m} + A_i\right)^2)^{-1/2}, & n < \frac{t_i}{b_0 a_0^m} \\
\sum_{l=1}^{\frac{t}{T}} a_0^{-m/2} e^{-A_i \left(n b_0 a_0^m - t_i\right)} \sin\left[\omega_l \left(n b_0 a_0^m - t_i\right) + \theta_l\right] \left(\omega_l^2 + \left(a_0^{-m} + A_i\right)^2\right)^{-1/2}, & n > \frac{t_i}{b_0 a_0^m}
\end{cases}
\]  

(10)

where \( \theta_l = \arctan\left(\frac{a_0^{-m} + A_i}{\omega_l}\right) \).

From above coefficients expressions of discrete wavelet transform, we know that \( C(m,n) \) shows exponential attenuation at both sides of \( n=t_i/b_0 a_0^m \), and achieves the maximum at \( n=t_i/b_0 a_0^m \). The maximum value is determined by the following parameters \( \{a_0,m,\omega_i\} \). If wavelet transform is performed with many different scales, such as \( a_0^m \), the wavelet coefficients will always achieve a maximum at \( t_i/b_0 a_0^m \). The time exactly coincides with the arrival time of the signal. Thus, utilizing this characteristic, we can accurately distinguish the signal in adverse noisy circumstances through the proper transform.

Moreover, the wavelet transform is also multi-resolution analysis. When \( a_0^m \) is small, the time resolution will be high, and this situation is especially suitable for high frequency signal because it is easier to observe the details and accurately estimate the emerging time of the signal. Otherwise, if \( a_0^m \) is large, the frequency resolution will be high, and this situation is suitable for low frequency signal because it is easier to observe the whole domain. This make the detector based on wavelet transform have good robustness. When there are more signals at the same observing time, i.e. \( I > 1 \), the signals maybe overlap at some time domain. In this situation, we must ensure \( b_0 a_0^m \) small enough by choosing appropriate \( b_0, a_0^m \), thus, the peaks of wavelet coefficients will emerge at different \( n \), and the signal can be effectively detected.

4. Simulating results

The sampling series \( x(n) \) can be obtained through sampling the echo signal illustrated by Equation (8). The discrete wavelet coefficients \( C(m,n) \) can be determined by

\[
C(m,n) = \frac{1}{T} a^{-m/2} e^{nb} \sum_{l=1}^{N} e^{-l a^{-m}} x(l)
\]

(11)

where \( T \) is the sampling period, \([\cdot]\) is obtaining the corresponding integer.

We assume the signal \( x(t) \) comprises two transient signals whose specific parameters \( \{A_i,t_i,\omega_i,\theta_l\} \) are \( \{0.6,5,2\pi,0.5\pi\} \) and \( \{1,20,1.6\pi,0.5\pi\} \), and the scale factor and shifting factor \( (a,b) \) is \( (0.8,1.2) \). First, the echo signals are simulated in the circumstances of no noise. Let \( m=0.5,1 \), the wavelet coefficients can be obtained by the equations. So we can get the corresponding simulating results illustrated as Figure 1. From Figure 1 we can know the estimate accuracy of echo signal is determined by \( b a^m \). The smaller \( b a^m \) is, the more accurate the estimate of echo signal. Figure 2 shows typical discrete wavelet coefficients in noisy circumstances. The signal-to-noise ratio is 0 dB, and \( m=0.5,1 \). According to the simulating figures, we can see the wavelet coefficient achieve the maximum at the arrival time of the signal. So we can realize the estimate of echo signal arrival time by means of the maximum of the coefficients. Thus, we successfully realize the ranging of radio fuze by this method, and it coincides with the analysis described above.
5. Conclusion
This paper discusses the method of detecting arrival time of echo signal with discrete wavelet transform. The method adopts proper scale factor and shift factor, and obtain signal arriving time to realize ranging by the relationship that discrete wavelet coefficient of echo signal arrives peak at the corresponding time. Simulating results show that the method is feasible in radio fuze ranging application even though the signal-to-noise of the signal is low.

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