Single- and triple-band bandpass filters using novel perturbed isosceles right-angled triangular SIW cavities

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Abstract
Herein, a rigorous derivation of an isosceles right-angled triangular (IRT) metallic cavity is analysed. Then, the transverse field components in the IRT substrate-integrated waveguide (SIW) cavity are derived, and the expression of the resonant frequencies can also be obtained. Then, a novel perturbed IRT SIW cavity is proposed, which is based on the characteristics of resonant modes in IRT SIW cavity and the capacitive-loading technique. Three prototypes were designed to verify the validity of the derivation and analysis in this study. First, a dual-mode single passband filter was implemented using the perturbed modes \( \text{TE}_{102} \) and \( \text{TE}_{202} \). Next, two tri-mode triple-band bandpass filters (BPFs) using the perturbed modes \( \text{TE}_{101} \), \( \text{TE}_{102} \) and \( \text{TE}_{302} \) are proposed, synthesised, verified, and measured for demonstration, including a second-order direct-coupled one which operates at 8.02, 11.44 and 12.52 GHz, and a fourth-order direct-coupled one which operates at 7.7, 11.0 and 12.1 GHz. The measured curves agree well with the simulated ones. Very compact circuit sizes and excellent filtering performances have been achieved for the three prototypes.

1 | INTRODUCTION

Due to the advantages of substrate-integrated waveguide (SIW) technology, such as high Q, low insertion loss, high power handling and easy integration with planar microwave circuits, it has attracted much attention in microwave communication systems. Various bandpass filters (BPFs) and multi-band filters have been designed based on SIW technique, and one of the research hotspots is to reduce the circuit size.

Recently, multi-mode resonators (MMRs) have gained significant research interest [1–14] owing to their compact size in designing multimode SIW filters. MMRs can be used to construct a single bandpass filter based on single cavity by frequency shifting, which usually realized by using asymmetric surface feeding, perturbed by via-holes, etching slots on the surface, etc. [4,5], or by cascading high-order cavities [6,7].

In addition, MMRs can also be utilised to construct dual-band and multi-band BPFs. Compared with combining several independent single-band BPFs into a multi-band one [8] or dividing a wide passband into several ones through a method that is so-called the coupling matrix synthesis technique [9], MMRs have advantages of more degrees of freedom and more compact size [10–14]. In Refs. [13,14], dual-band filters are designed using dual modes in rectangular SIW cavity. In Ref. [10], a tri-band filter is implemented by multilayer technology, where the first band is realised by \( \text{TE}_{101} \) and the last two bands are realised by inserting transmission zeros (TZs) in the passband formed by \( \text{TE}_{201} \). However, the mechanism and structure of this design are both complicated, and the isolation between the last two passbands need to be improved. Designing triple-band BPFs (TBBPFs) by exploiting a novel type of tri-mode SIW resonator is proposed in Ref. [15] for the first time, which is perturbed by centred cross-shaped metallised via holes to adjust the frequency ratios in a certain range.

Compared with rectangular and circular SIW cavities that used most commonly, triangular SIW cavities have the advantage of a more compact size and maintain similar properties at a fixed resonant frequency. A set of filters using different types of triangular SIW cavities have been reported [16–21], especially some triangles with typical shapes, like isosceles right triangle (IRT), equilateral triangle (ET) and isosceles triangle (IT). In Ref. [17], a fourth-degree and a sixth-degree BPFs based on isosceles right-angled triangular (IRT) and ET SIW cavities are implemented, respectively, according to the calculation formula of the resonant frequency. However,
only the fundamental mode of these cavities has been analysed and utilised to design single-band BPFs. The first two modes of IRT SIW cavity are used to design bandpass filter in Ref. [20], but it has poor filtering performance with only one TZ near the high-frequency passband even if the low-temperature co-fired ceramic (LTCC) technology is combined to enhance cross coupling. There are very few works that focus on the triple-mode triangular SIW cavity. In Ref. [21], a triple-band BPF is proposed based on the triple-mode ET SIW cavity, however, TZs were obtained by loaded complementary split-ring resonators (CSRRs) which reflects the defect of its prototype circuit and this design do not refer to the field and resonant frequencies analysis of this resonator. May be due to the theoretical difficulty in analysing the field distribution, controlling the triple-mode resonant frequency and mutual coupling, no triple-band BPF has been proposed based on triple-mode IRT SIW resonator to the best of the authors’ knowledge.

Herein, a comprehensive and rigorous derivation of the IRT SIW cavity is provided carefully and thoroughly, then the resonant modes and frequencies can be calculated. After that, the perturbed IRT SIW has been proposed and applied to the design of single and TBBPFs. By introducing the capacitive patch, which is first proposed in Ref. [22] for designing a single capacitance patch, the resonant modes can be controlled as [20,23].

The perturbed mode TE_{202} can be transferred to the vicinity of the perturbed TE_{102} to construct the passband. In the design of second-order filter I and fourth-order filter III, the first three modes TE_{101}, TE_{102} and TE_{202} in the perturbed IRT SIW cavity are used to construct three passbands for the first time. Furthermore, the design parameters corresponding to the bandwidths that is the external quality factors Q_e and the internal coupling coefficients M_{ij} of the three passbands could be specified flexibly within certain ranges by employing a triple-mode coupling controlling technique. All the examples are designed in detail and experimentally verified. The measured results are agreed with the simulated ones, which further verify the design method.

Herein, Section 2 conducts the detailed analysis of the classical IRT SIW cavity while Section 3 shows the resonant characteristics of the proposed novel IRT SIW cavity and external coupling controlling technique. Then, several examples are presented based on this novel cavity. At first, a dual-mode single passband filter I is proposed in Section 4, which has a compact size and two controllable TZs. After that, a triple-band filter II is proposed, which has controllable frequency ratios and high selectivity with four TZs. In addition, the design of filter III in Section 5.2 shows that the tri-mode cavity can be used to design higher-order/enhanced-selectivity filters. Section 5 also demonstrates the comprehensive design procedures of the respective two SIW TBBPFs with different coupling topologies. Finally, Section 6 gives concise conclusions.

2 | DESIGN AND ANALYSIS

Figure 1(a) shows the cross section of the IRT SIW cavity, its top and bottom walls are formed by the metallic plates on both sides of the substrate, whereas its three sidewalls (y = 0, y = x, x = L_1, where AA’ = AO = L_1) are formed all by metallic via, which can be approximated to the conventional electric walls. Similar to solving a square or circular waveguide, the electromagnetic field inside the IRT waveguides can be expanded in terms of TE_{mn} and TM_{mn} modes. But due to the solution of transverse electromagnetic waves in an IRT SIW cannot be solved by separating variables directly. Therefore, the mode functions and related eigenvalues of such an IRT waveguide can be formulated analytically by superimposing the waveforms of the corresponding square waveguide.

2.1 | Square waveguide with electric walls

The cross section (A’ABO) in Figure 1a shows the corresponding square waveguide with electrical walls on all three sides. By solving Maxwell equations, the four transverse field components of TE_{mn} and TE_{mn} modes that characterised by E_z = 0, H_z ≠ 0, and E_z ≠ 0, H_z = 0 can be expressed as [20,23].

\[
\begin{align*}
\text{TE: } H_z &= A_{mn} \sin \frac{m \pi x}{L_1} \sin \frac{n \pi y}{L_1} e^{-j \beta z} \\
\text{TM: } E_z &= B_{mn} \cos \frac{m \pi x}{L_1} \cos \frac{n \pi y}{L_1} e^{-j \beta z}
\end{align*}
\]

where A_{mn} and B_{mn} are arbitrary amplitude constants, β is defined as the propagation constant.

By solving Maxwell equations, the four transverse field components can be expressed as [20,23].
with electrical walls on all three sides. As can be obtained by proper linear combination of these modes defined as $k_c^2 = \sqrt{k^2 - \beta^2}$.

### 2.2 | TE and TM modes of IRT waveguide

Extract transverse electric and magnetic mode solutions of IRT can be obtained by proper linear combination of these modes in square waveguide discussed above.

Figure 1a shows the cross section (AOB) of IRT waveguide with electrical walls on all three sides. As $H_z \neq 0$ in TE waveguide modes, boundary conditions that TE modes of IRT waveguide must satisfy are as follows

\[
\begin{align*}
\left( \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial y^2} \right) H_z + \left( k_c^2 - k^2 \right) H_z &= 0 \\
\frac{\partial H_z}{\partial x} \bigg|_{x=L_1} &= 0 \\
\frac{\partial H_z}{\partial y} \bigg|_{y=0} &= 0 \\
\frac{\partial H_z}{\partial x} - \frac{\partial H_z}{\partial y} \bigg|_{y=x} &= 0
\end{align*}
\]

According to the superposition principle, the TE$_{mn}$ mode of IRT waveguide can be constructed by (TE$_{mn}$ + TE$_{nm}$) of that in square waveguide. So, the solution of $H_z$ in formula above is

\[
H_z = A_{mn} \left( \cos \frac{m\pi x}{L_1} \cos \frac{n\pi y}{L_1} + \cos \frac{m\pi x}{L_1} \cos \frac{n\pi y}{L_1} \right) e^{-j\beta z}
\]

The transverse field components of the TE$_{mn}$ mode in IRT SIW can be deduced using (1)–(3) as

\[
E_x = \frac{j\omega n\pi}{k_c^2 L_1} A_{mn} \left( \cos \frac{m\pi x}{L_1} \sin \frac{n\pi y}{L_1} + \cos \frac{m\pi x}{L_1} \sin \frac{n\pi y}{L_1} \right) e^{-j\beta z}
\]

\[
E_y = \frac{j\omega n\pi}{k_c^2 L_1} A_{mn} \left( \sin \frac{m\pi x}{L_1} \cos \frac{n\pi y}{L_1} + \sin \frac{m\pi x}{L_1} \cos \frac{n\pi y}{L_1} \right) e^{-j\beta z}
\]

Similarly, under the boundary conditions (6) that TM$_{mn}$ modes of IRT waveguide must be satisfied, the TM$_{mn}$ mode of IRT waveguide can be obtained by the superimposing the (TM$_{mn}$+TM$_{nm}$) wave modes in corresponding square waveguide.

\[
\begin{align*}
\left( \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial y^2} \right) E_x + \left( k_c^2 - k^2 \right) E_x &= 0 \\
E_{z|x=L_1} &= 0 \\
E_{z|y=0} &= 0 \\
E_{z|y=x} &= 0
\]
\]

Then $E_z$ in formula above can be expressed as

\[
E_z = B_{mn} \left( \sin \frac{m\pi x}{L_1} \sin \frac{n\pi y}{L_1} - \sin \frac{m\pi x}{L_1} \sin \frac{n\pi y}{L_1} \right) e^{-j\beta z}
\]

The transverse field components of the TM$_{mn}$ mode in IRT SIW can be deduced using (1), (6) and (7) as

\[
\begin{align*}
H_x &= \frac{j\omega m\pi}{k_c^2 L_1} B_{mn} \left( \sin \frac{m\pi x}{L_1} \cos \frac{n\pi y}{L_1} - \sin \frac{m\pi x}{L_1} \cos \frac{n\pi y}{L_1} \right) e^{-j\beta z} \\
H_y &= \frac{j\omega m\pi}{k_c^2 L_1} B_{mn} \left( \cos \frac{m\pi x}{L_1} \sin \frac{n\pi y}{L_1} - \cos \frac{m\pi x}{L_1} \sin \frac{n\pi y}{L_1} \right) e^{-j\beta z} \\
E_x &= \frac{j\omega m\pi}{k_c^2 L_1} B_{mn} \left( \sin \frac{m\pi x}{L_1} \cos \frac{n\pi y}{L_1} - \sin \frac{m\pi x}{L_1} \cos \frac{n\pi y}{L_1} \right) e^{-j\beta z} \\
E_y &= \frac{j\omega m\pi}{k_c^2 L_1} B_{mn} \left( \cos \frac{m\pi x}{L_1} \sin \frac{n\pi y}{L_1} - \cos \frac{m\pi x}{L_1} \sin \frac{n\pi y}{L_1} \right) e^{-j\beta z}
\end{align*}
\]
The cutoff wavenumbers of the TE_{mn} and TM_{mn} modes for the IRT waveguide are

\[(k_c)_{mn} = \frac{\pi}{L_1} \sqrt{m^2 + n^2} \frac{1}{\sqrt{\mu \epsilon}} \]

where \( m \neq 0, n \neq 0, m \neq n, \) and \( m, n \) are both positive integers. The resonant frequencies of TE_{mn} and TM_{mn} modes are then calculated according to the following formula:

\[ f_{cmn} = \frac{1}{2L_1 \sqrt{\mu \epsilon}} \sqrt{m^2 + n^2} \]

where \( \mu \) and \( \epsilon \) are the permeability and permittivity of the substrate, respectively. Figure 1b shows the normalised cutoff wavenumbers of the IRT waveguide with electrical walls on all three sides. The fundamental mode of the IRT waveguide is the TE_{10}/TE_{01} mode.

The above analysis of the IRT waveguide can be used to derive the resonance modes in the IRT SIW. As we know, only TE_{mn} modes can be transmitted in a substrate integrated waveguide. Therefore, the fundamental mode of the IRT SIW is the TE_{10} mode and the first pair of higher-order modes are TE_{102} and TE_{202}, which are a pair of non-degenerate modes with different resonant frequencies. The relationship between the cutoff frequencies and the structural parameters was studied, so the resonant frequency of the modes can be obtained using the following formula approximately [20, 23]:

\[ f_{cmon} = \frac{1}{2L_{1eff} \sqrt{\mu \epsilon}} \sqrt{m^2 + n^2} \]

where \( L_{1eff} \) is the effective length of SIW cavity and can be calculated by

\[ L_{1eff} = a - \frac{d^2}{0.95p} \]

\( d \) and \( p \) are the diameter of metallised vias and the spacing between adjacent via-holes, respectively.

3 | RESONANT CHARACTERISTIC OF THE PROPOSED IRT SIW CAVITY

3.1 | Configuration and field distributions

Figure 2 depicts the configuration of the proposed SIW triple-mode IRT SIW cavity perturbed by a capacitive-loaded circular metal patch. The capacitive-loaded patch was used to design compact single-mode substrate integrated waveguide (SIW) cavities [21–23] and triple-mode SIW cavities [19]. The side view of the proposed dual-mode resonator is shown in Figure 2(b). It consists of two substrate layers with thicknesses of \( b-b_1 \) and \( b_1 \), respectively. And three metal layers with a thickness of 17 \( \mu \)m. The perturbations are arranged along the diagonal, which consists of a patch with a radius of \( C_i \) and blind-via with the height of \( h-1 \). The diameter of the vias and the interval between adjacent vias are set as \( d = 0.6 \) mm and \( p = 0.85 \) mm to satisfy the condition that the array of via-holes can be equivalent to a conventional electric wall.

The electric-field distributions of the first three resonant modes TE_{101}, TE_{103}, TE_{202} of the conventional IRT SIW cavity is illustrated in Figure 3. As analysed above, mode TE_{101} has the lowest resonant frequency when the length of the right side of the IRT is determined, which should be the fundamental mode. The first pair of high-order resonant modes are non-degenerate modes TE_{102} and TE_{202}, which are a pair of

![Figure 2: Geometric configuration and side view of the proposed perturbed IRT SIW cavity](a) Geometric configuration, (b) side view

![Figure 3: Electric-field distributions of the conventional IRT SIW cavity](a) Mode TE_{101}, (b) Mode TE_{102} and (c) Mode TE_{202}
degenerate modes in square SIW. In this design, the capacitively-loaded circular metal patch is located at the strongest electric field strength of mode TE\textsubscript{202}, which can realize independent control of mode TE\textsubscript{202} and hardly affect the electric field distributions of modes TE\textsubscript{101} and TE\textsubscript{102}, and the perturbed electric field distributions of them are shown in Figure 4.

As can be seen in Figures 3 and 4, some characteristics can be derived from the electric field distributions:

1. The electric field is symmetrically distributed on both sides of the centreline of the triangle, and the field strength is the weakest at the edges and the symmetry axis in mode TE\textsubscript{102}. While in mode TE\textsubscript{202}, the electric field is distributed asymmetrically on both sides of the median line parallel to the hypotenuse, and the electric field strength is the weakest in the edge region and this line. Thus, perturbations can be used to control the resonance of the mode TE\textsubscript{202} independently.

2. The tapped feeding lines located at the common areas of dual modes TE\textsubscript{102} and TE\textsubscript{202} can excite the two modes, and the mode TE\textsubscript{101} is always excited.

### 3.2 Resonant characteristics

To demonstrate the influence that perturbations have on these three resonant modes, the simulated resonant frequency ratios \( \alpha_1 = f_2/f_1, \alpha_2 = f_3/f_1 \) versus diameter \( C_r \) and height of the blind via \( b_1 \) are provided in Figure 5. The realisable \( \alpha_1 \) and \( \alpha_2 \) would lie in the mesh area when parameters \( C_r \) and \( b_1 \) vary from 0 to 1.2 mm and 0.2 to 0.6 mm, respectively. And the three perturbed modes are denoted by modes A, B and C for convenience.

As shown in Figure 5a, wider frequency ratio ranges can be realised with larger value of the blind hole height \( b_1 \). Therefore, for easier fabrication, we choose \( b_1 = 0.508 \) mm as the preferred height of the blind hole.

Once the values of \( C_r \) and \( b_1 \) are fixed, the three frequencies can be implemented simultaneously by determining the cavity side length \( L_1 \). Figure 6 shows the simulated relationship between the frequency ratios \( \alpha_1, \alpha_2 \) and \( L_1 \) with different values of \( C_r \), that is \( P_1, P_2 \) and \( P_3 \) in Figure 5.

That is, the dimensions of the proposed IRT SIW cavity are determined as follows. Readout \( b_1 \) and \( C_r \) to obtain the required frequency ratios using Figure 5, then determine the side length \( L_1 \) of the cavity to obtain the required frequencies \( f_1, f_2 \) and \( f_3 \).

### 3.3 Extracted external quality factors

The insertion diagram in Figure 6 shows the configuration of the proposed triple-mode IRT SIW cavity coupled by a 50 \( \Omega \) microstrip feed line. The external quality factors \( Q_e \) of the three modes can be calculated by the following formula [24]:

\[
Q_e = \frac{\omega_0 \tau_{S_{11}}(\omega_0)}{4}
\]

where \( \omega_0 \) stands for the resonant frequency, \( \tau_{S_{11}}(\omega_0) \) denotes the group delay, which can be extracted by the full-wave simulation.

Taking \( P_1 \) \((i.e., \ b_1 = 0.508 \text{ mm}, \ c_r = 0.9 \text{ mm})\) as an example, the curves of corresponding \( Q_e^I, Q_e^H \) and \( Q_e^T \) against the different lengths of coupling slots \( L_{11}, L_{12} \) are extracted and depicted in Figure 6 with fixed \( w_{11} = 0.3 \text{ mm} \) and \( w_{12} = 0.4 \text{ mm} \). In addition, the \( Q_e^I, Q_e^H \) and \( Q_e^T \) have also been extracted without perturbation for comparison. As can be seen in Figure 6, the external quality factors will increase after loading the capacitive perturbation, and the tuning ranges become wider compared with the non-perturbation cavity.

What’s more, it can be seen that when the parameters \( L_{11}, L_{12} \) are decreased, the \( Q_e \) of the three modes are decreased as well. \( Q_e^I, Q_e^H \) changes slowly but \( Q_e^T \) present opposite tendency which could be explained from electric field distributions in Figure 4. Moreover, the \( Q_e \) of mode C is always larger than that of modes B and \( A \), indicating that \( L_{11}, L_{12} \) are crucial impacting parameters to overall strengths. Thus, \( Q_e \) of the proposed dual-mode single-band IRT SIW cavity can be tuned by the coupling slot effectively. The EM software and substrate used herein are HFSS and Rogers 5880 which has a relative dielectric constant of 2.2 and loss tangent of 0.0009 with the thickness of 0.762 mm, respectively.

### 4 DESIGN OF THE PROPOSED DUAL-MODE IRT SIW FILTER I

#### 4.1 Coupling topology and configuration

Base on the proposed novel tri-mode IRT cavity, three design examples are demonstrated in Sections 4 and 5. According to the analysis above, the \( E \)-fields of the resonant modes in this
cavity can be perturbed by the capacitive-loaded circular metal patch, and resonances can be controlled to realize different frequencies ratios by adjusting its parameter.

The first one is a single-band filter where modes B and C are employed to generate the passband, while the non-resonant mode (fundamental mode) is designed to provide a direct input-to-output coupling. Figure 7a shows the configuration of the proposed dual-mode single-band IRT SIW filter, and its corresponding coupling scheme is shown in Figure 7b. As analysed above, the resonant frequency of three modes mainly depends on the value of $C_r$ once $h_1$ has been determined. Figure 8 gives the simulated results of the three modes against different values of diameter $C_r$.

It can be seen from Figure 8 that when the size of the patch increases, the resonant frequency of mode C can be adjusted independently and gradually approaches the resonant frequency of mode B. And because the capacitive loading patch located at weak electric fields of modes A and B, the resonances of the two modes are almost constant when the size of loading patch is changed. Since the passband has been realised by designing the dimension of perturbed circular patch. To optimise the characteristics of the passband, the simulated curves $Q_e^I$, $Q_e^{II}$ and $Q_e^{III}$ provided in Figure 6 are used to satisfy required external coupling.

According to this structure, two FTZs can be generated on both sides of the passband, and the positions of the two FTZs can be controlled. Different from etching CRSS or designing multilayer structures in Refs. [17,18] to realize and control FTZs. In our design, the first FTZ is produced by the coupling scheme with source-load (S-L) coupling, which is realised by the non-resonant modes (i.e., modes $TE_{101}$ etc.). While the other FTZ is generated by the higher-order modes. To further illustrate the controllability of the FTZs, the simulated responses with changing parameter $t$, which is the offset distance of the feedline, are presented in Figure 9. It can be concluded that when $t$ increased, the first FTZ is close to the passband and the second one is far away from the passband. In particular, when the feeding line is aligned with the strongest electric field of mode C, the two FTZs are symmetrical about the centre frequency.

4.2 Fabrication and measurement the proposed IRT SIW filter I

Based on the proposed triple-mode IRT SIW structure, a single-band filter designed and fabricated for demonstration.
Figure 7: Geometric configuration and the equivalent coupling topology of the proposed dual-mode IRT SIW filter I. (a) Geometric configuration, (b) coupling topology (S/L: source/load, B/C: resonant modes)

Figure 8: Simulated resonant frequencies against Cx with h1 and other parameters fixed

Figure 9: Simulated responses of the proposed dual-mode single-band IRT SIW filter I with controllable FTZs against parameter t

Figure 10: Comparison of the simulated and measured results and photograph of the fabricated filter I

FBWs of 2.53%. The measured return loss (RL) of the passband is better than 18.58 dB, while the minimum in-band insertion loss (IL) is 1.6 dB.

Two FTZs located at 11.41 and 12.34 GHz are measured as expected. The proposed single-band IRT SIW filter I has a more compact size, high out-band rejection and more flexible control of FTZs.

5 | DESIGN OF TRIPLE-BAND FILTERS BASED ON THE PROPOSED IRT SIW CAVITIES

5.1 | Second-order direct-coupled TBBPF

Two tri-band design examples are illustrated in Section 5 based on the proposed tri-mode IRT SIW cavity. One is a second-order direct-coupled triple-band filter II, whose geometric structure is depicted in Figure 11a. Figure 11b shows the coupling topology of the proposed filter II where I, II and III represent the three passbands generated by three pairs of modes A, B and C in the two triple-mode IRT SIW cavities, respectively. And the center frequencies (CFs) of the three passbands are denoted by f1, f2 and f3, respectively.

Different from the example I, the capacitive-loaded circular metal patch in proposed filter II is used to adjust the CFs ratios of the three passbands, as depicted in Figure 5. Therefore, a certain range of CFs ratios can be achieved by changing the radius of the perturbations.

Figure 12 illustrates several groups of theoretical frequency responses of the triple-band filter II. As observed, the larger the diameter Cx, the third passband get closer to the second passband, while keeping the CF of the second passband almost unchanged. Meanwhile, the CF of the first passband is slightly shifted, so that different values of CFs ratios can be achieved.

After the CFs have been determined, the main consideration here is to acquire specified specifications, including the external quality factor Qe, and the internal coupling coefficient Me. In the design, the external couplings are controlled by lengths of the coupling slots which have been analysed in
Section 3.3, while the internal couplings are realised through two offset coupling windows opened on common sidewall.

Considering the influence of perturbation on internal coupling, the coupling coefficients of three passbands are all extracted in the presence of the perturbation. As displayed in Figure 11a, \( t_x \) and \( g_y \) represent the respective offset and coupling window width of the lower window, while \( t_y \) and \( g_x \) denote the respective offset and window width of upper one. From the electric field field analysis above in Section 3.1, it can be known that the electric field in the IRT SIW cavity is symmetrical about the centreline. Thereinto, the electric field of Mode A is concentrated in the middle of the common wall, the electric field of Mode B is distributed on both sides of it, while the electric field of Mode C is distributed along the whole common wall.

In general, the coupling coefficient of the two coupled cavities can be extracted by full-wave simulations and can be evaluated by [24],

\[
M_{ij} = \frac{f_{p2}^2 - f_{p1}^2}{f_{p2}^2 + f_{p1}^2} \quad (14)
\]

where \( f_{p1} \) and \( f_{p2} \) are the lower and higher resonant frequencies when the two synchronously tuned coupled cavities weakly coupled by input and output ports.

As analysed above, the location and length of the two coupling windows have different effects on the coupling coefficients of modes A, B and C (represented by \( M_{12}^I \), \( M_{12}^{II} \) and \( M_{12}^{III} \), respectively). Therefore, at first, the extracted curves of \( M_{12}^{III} \) and initial \( M_{12}^{III} \) (which is only contributed by the upper window while the lower one is closed) versus \( t_x \) and \( g_x \) are shown in Figure 13(a). Then the extracted curves of \( M_{12}^{II} \) and final \( M_{12}^{II} \) (which is contributed by both the upper window and the lower one) versus \( t_y \) and \( g_y \) is shown in Figure 13(b). It can be seen that \( M_{12}^{II} \) increase significantly with \( g_y \) but shows the opposite trend with \( t_x \). And except for \( t_x > 10 \text{ mm}, M_{12}^{III} \) hardly changes with \( t_x \) but increases greatly when \( g_x \) becomes larger. The change degree of \( M_{12}^{III} \) versus \( t_y \) and \( g_y \) different under different values of \( t_y \) and \( g_y \). These variations are consistent with the electric field distributions in Figure 4.

Hence, the values of \( M_{12}^{III}, M_{12}^{II} \) and \( M_{12}^{III} \) can be determined by the length and locations of the two coupling windows, indicating that the 3-dB FBWs of the third passband can also be flexibility controlled within a certain range.

Since the structure parameters corresponding to the required values of \( Q_e^{I}, Q_e^{II} \) and \( Q_e^{III} \) can be obtained by Figure 6. Then the final coupling parameters can be determined by fine-tuning the dimensions of the filter II.
After the dimensional parameters have all been determined, the performance of the proposed filter II has been optimised to approach the specifications. Then, the final dimensions have been listed in Table 1. The insertion diagram in Figure 14 shows the photograph of the fabricated prototype with circuit size excluding the microstrip transition is 28.2 × 28.2 mm² (1.11 λg × 1.11 λg), where λg is the guided wavelength in the dielectric substrate at f1. Figure 14 presents the simulated and measured S-parameters of the filter II. The measured results agree well with simulated results. As can be seen, total four TZs are obtained located at 8.17, 11.81, 13.07 and 13.42 GHz in the vicinity of the passbands, which can improve the frequencies selectivity highly. The generation mechanism of the TZs is caused by multipath coupling, so it cannot be adjusted by changing the position of the feedline like filter I. The measured CFs of the three passbands are 8.02, 11.44 and 12.52 GHz with 3-dB FWBs of 1.35%, 3.58% and 3.39%, respectively. The minimum ILs measured in the three passbands are 1.91, 1.35 and 1.43 dB, and the measured RLs of the three passbands are better than 16 dB. The difference between simulated and measured results may come from machining and measurement errors.

Figure 15 shows the electric field distributions in this TBBPF at f1, f2 and f3 at a cross section of h = 0.5 mm. (The height h1 of perturbation via is fixed as 0.508 mm as in the filter I). As can be seen, TE101 modes in the proposed filter II dominate the first passband, while modes TE201 and TE202 in the two IRT SIW cavities construct the second and third passbands as expected.

| TABLE 1 Dimensions of the second-order direct-coupled TBBPF II |
|---------------------------------------------------------------|
| Symbol | Value (mm) | Symbol | Value (mm) |
|-------|------------|-------|------------|
| L1    | 28.2       | s1    | 0.9        |
| C1    | 0.6        | s2    | 0.72       |
| g1    | 4.71       | l1    | 4.6        |
| g2    | 4.23       | ωs   | 0.3        |
| t1    | 10.2       | D     | 4.2        |
| t2    | 10.2       | C2    | 6.48       |

5.2 Fourth-order direct-coupled TBBPF

To verify the potential of the proposed resonator in the design of higher-order and enhanced-selectivity filters, a Chebyshev-type fourth-order direct-coupled TB-BPF. Figure 16 shows the geometric configuration and coupling topology of this TBBPF, where superscripts A, B and C denote the three resonant modes that construct the three passbands I, II and III, respectively. Similar to the filter II, the values of C1 and L1 are determined as C1 = 0.5 mm, L1 = 28.0 mm to achieve acquired frequency ratios according to the progress described in Section 3.2. Then, the main challenge here is to acquire the required design parameters, that is, Qe and Mij of the three passbands. According to the synthesis approach described in [25], Qe and Mij of the three passbands corresponding to the designed specifications can be obtained as

\[
Q_e^I = 22.70 \quad M_{12}^I = M_{34}^I = 0.0339 \quad M_{23}^I = 0.0249
\]

\[
Q_e^{II} = 24.58 \quad M_{12}^{II} = M_{34}^{II} = 0.0313 \quad M_{23}^{II} = 0.0230
\]

\[
Q_e^{III} = 23.76 \quad M_{12}^{III} = M_{34}^{III} = 0.0324 \quad M_{23}^{III} = 0.0238
\]

Figure 17 shows the simulated curves of Q_e^I, Q_e^{II} and Q_e^{III} versus the offset position D2 under different values of l4. To meet the requirement of specifications, the values of D2 and l4 are evaluated as 6.5 and 5.8 mm, respectively. Then by fine-tuning the value of l4, we determine the parameters of the external coupling structure.

The internal coupling coefficients M12 = M34 and M23 of the three passbands can be obtained according to Figure 18. It can be seen from Figure 16a that, unlike Filter II, which achieves internal coupling through hypotenuse, the internal coupling of Filter III is achieved through right-angle edges. And because the electric field is strong in the upper half of this cavity but very weak in the lower half for Mode (b) Consequently, the internal coupling of Mode B is mainly controlled by the upper window, while couplings of Modes A and C are controlled by both windows. Figure 18a shows the extracted curves of M_{12}^{II} and M_{12}^{III} against g2 and t2 when the lower windows closed. Then,
Figure 18b shows $M_{III,i}^{i+1}$ and $M_{I,i}^{I,i+1}$ against $g_x$ and $t_x$ when $g_y$ and $t_y$ have been determined according to Figure 18a that is both windows are open. As mentioned above, the initial $M_{II,i}^{I,i+1}$ can be chosen arbitrarily to satisfy the required $M_{II,i}^{I,i+1}$ to get the initial value of $g_y$ and $t_y$ after that, adjust the values of $g_x$ and $t_x$ simultaneously to meet the required values of $M_{III,i}^{I,i+1}$ and $M_{I,i}^{I,i+1}$. At last, the final coupling parameters can be obtained by the design procedures introduced above.

After fine-tuning, the filter parameters have been optimised to acquire the specifications. Table 2 gives the final dimensions of the proposed filter III. The insertion diagram in Figure 19 shows the photograph of the fabricated prototype with circuit size excluding the microstrip transition is $39.6 \times 39.6 \text{mm}^2$ ($1.52 \lambda_g \times 1.52 \lambda_g$), where $\lambda_g$ is the guided wavelength in the dielectric substrate at $f_1$.

The photo of the fabricated fourth-order SIW filter III is illustrated in Figure 19a. The simulated and measured results of the filter III are shown in Figure 19b. As can be seen, the measured results agree well with simulated results. The measured CFs of the three passbands are 7.7, 11.0 and 12.1 GHz with 3-dB FWBs of 4.93%, 2.90% and 3.14%, respectively. The minimum ILs measured in the three passbands are 1.98, 2.01 and 2.15 dB, and the measured RLs of the three passbands are better than 19 dB. What’s more, the out-of-band suppression level can reach 60 and 50 dB between passbands I/II and II/III, respectively, which demonstrate high selectivity of the proposed filter. The difference between

---

**Table 2** Dimensions of the fourth-order direct-coupled TBBPF III

| Symbol | Value (mm) | Symbol | Value (mm) |
|--------|-----------|--------|-----------|
| $g_{x1}$ | 5.2 | $w_{0}$ | 2.2 |
| $g_{x1}$ | 4.6 | $l_{1}$ | 3.8 |
| $t_{x1}$ | 8.9 | $w_{11}$ | 0.4 |
| $t_{x2}$ | 17.3 | $l_{2}$ | 2.0 |
| $g_{x2}$ | 5.0 | $w_{x2}$ | 0.2 |
| $g_{x2}$ | 4.2 | $D_{2}$ | 6.2 |
| $t_{x2}$ | 8.7 | $C_{r}$ | 0.55 |
| $t_{y2}$ | 19.1 | $C_{y}$ | 6.56 |
| $L_{1}$ | 27.5 | $L_{2}$ | 27.8 |
simulated and measured results may come from machining and measurement errors.

Figure 20 provides the electric field distribution at 7.7 GHz ($f_1$), 11.0 GHz ($f_2$) and 12.1 GHz ($f_3$) in the proposed filter III.

It can be intuitively observed that Modes A, B and C in the four cavities construct the first, second and third passbands, respectively.

5.3 | Comparisons

Table 3 lists the comparison of our presented second-order/ fourth-order direct-coupled TBBPF II and III with other reported SIW designs. As can be seen, the proposed TBBPF can provide flexibly ratios of CFs and FBWs of the three passbands, better selectivity and relatively lower ILs with very simple configuration and compact size compares with the most of the other ones based on the novel triple-mode IRT SIW cavity.

6 | CONCLUSION

A novel multi-mode IRT SIW cavity is presented, which is realised by loading a capacitive patch and employed to design single- and TBBPFs. The resonant characteristics of the cavity have been analysed first to show the resonant modes and the range of realisable frequency ratios. Then, a dual-mode single-band filter I based on a single IRT SIW cavity is realised, which has the characteristics of compact size, high selectivity and controllable TZs. Afterwards, two examples of triple-mode TBBPF are proposed based on the novel IRT SIW cavity for the first time. One is a second-order direct-coupled triple-band filter II by combining two IRT SIW cavities, which generates four TZs in the vicinity of three passbands. The other prototype of Chebyshev-type fourth-order tri-band filter III is proposed, analysed and verified to show the usefulness of this novel cavity in higher-order designs. In the design of the two tri-band filters, by determining the parameters of coupling slots and coupling windows, the required $Q_i$ and $M_{ij}$ for every passband can be achieved. That is, the FBWs of the three passbands can be specified and allocated flexibly over a range of ratios. The measured results of these three devices are all in good agreement with the simulated ones. Compact circuit sizes and good filtering performances have been achieved, showing the great potential of these IRT SIW single- and multi-band

| Table 3 | Compared with other reported SIW triple-band BPFs |
|---|---|---|---|---|---|
| Reference | Structure | Shape | $f_1/f_2/f_3$ (GHz) | Order | 3-dB FBW (%) | IL (dB) | RL (dB) | Size ($\lambda_e \times \lambda_g$) |
| [2] | CSRR-loaded SIW | Rectangular | 3.27/4.75/6.3 | 3 | 3.0/2.5/2.6 | 3.23/3.69/1.67 | >14 | 0.17 $\times$ 0.14 |
| [6] | Planar SIW | Rectangular | 20.1/20.9/21.9 | 2 | 1.99/2.15/2.97 | 1.6/0.9/0.85 | >19 | 2.10 $\times$ 1.96 |
| [10] | LTCC SIW | Rectangular | 29.9/34.8/36.81 | 6 | 4.33/3.16/3.27 | 1.65/1.68/1.79 | 11.8/10 | 1.35 $\times$ 1.13 |
| [12] filter I | Planar SIW | Square | 13/14/15 | 3/3/3 | 4.06/3.31/2.82 | 1.71/1.80/2.20 | 16/18/18 | 3.38 $\times$ 1.19 |
| [12] filter II | Planar SIW | Square | 11/12/13 | 4/4/4 | 2.69/1.92/2.25 | 2.02/2.72/2.57 | 20/18/18 | 2.20 $\times$ 2.20 |
| [18] | CSRR-loaded SIW | ET | 5.55/7.5/8.6 | 2/2/2 | 5.58/5.20/4.65 | 1.39/1.33/1.53 | >14 | 1.79 $\times$ 0.63 |
| This work filter I | Planar SIW | IRT | 8.02/11.44/12.52 | 2/2/2 | 1.35/3.58/3.39 | 1.91/1.35/1.43 | >16 | 1.11 $\times$ 1.11 |
| This work filter II | Planar SIW | IRT | 7.7/11.0/12.1 | 4/4/4 | 4.63/3.40/3.14 | 1.98/2.01/2.15 | >19 | 1.52 $\times$ 1.52 |
BPFs in miniaturisation and integration of multifunctional and multi-standard transceivers.

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**How to cite this article:** Zhang J, Liu Q, Zhou D, Zhang D. Single- and triple-band bandpass filters using novel perturbed isosceles right-angled triangular SIW cavities. *IET Microw. Antennas Propag.* 2021;15:241–252. [https://doi.org/10.1049/mia2.12034](https://doi.org/10.1049/mia2.12034)