Optimal Design of Tuned Mass Damper for Base-Excited Structures

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Abstract. The tuned mass damper (TMD) is a well-established vibration control device that has been implemented in several structures worldwide. The performance of passive TMD is greatly dependent on its design parameters. A key issue for the design of the optimal TMD is the identification of the parameters of the structure to which the TMD is to be attached. Here, it is important also to consider the perturbations that may arise in the structural frequency due to different practical issues. The present work focusses on this and aims to obtain the optimum TMD parameters in a Genetic Algorithm (GA) framework. The results of the proposed optimal strategy, in terms of optimal tuning ratio and optimal damping ratio, for a given value of the mass ratio, are compared with some of the existing optimal solutions for linear TMD under base excitation. The effectiveness of all these optimally designed TMDs in reducing the structural response is further demonstrated when the structure-TMD system is subjected to recorded earthquake ground motions. It is seen that in the tuned condition, the control performance obtained by the different designs of the TMD are similar to each other, although having some minor differences in the values of the optimum parameters. Also, the effectiveness of the different TMD designs in the detuned condition is investigated, in which the present design technique is found to be the most effective.

1. Introduction
Tuned mass damper (TMD) is a passive vibration control device, extensively used in many important structures around the world for the mitigation of structural responses due to various kinds of environmental loads, such as wind, wave and earthquake excitations[1]. The performance of the passive TMD is greatly dependent on its design parameters, namely mass ratio, tuning ratio and damping ratio, out of which the first is generally fixed from practical considerations. In view of this, there has been considerable research on the determination of the optimal tuning ratio and damping ratio of the linear TMD, for different objectives of control. Den Hartog first proposed the fixed-point theory to find the optimum parameters of a TMD attached to an undamped structure [2]. For the structure with damping characteristics, the closed-form expression of the optimum TMD parameters, in terms of structural parameters is not derivable directly; however, the values of the parameters could be estimated numerically. Warburton (1982) was the first to provide optimum parameters of a TMD attached to a damped single-degree-of-freedom (SDOF) system subjected to white-noise excitation as base acceleration [3]. He focussed on the reduction of the variance in displacement response of the main structure in order to get the optimum parameters. Tsai and Lin (1993) performed a numerical search to find the optimum parameters of the TMD when the primary structure is subjected to a support motion assumed to be harmonic with fixed-acceleration amplitude [4]. The objective of the optimization procedure adopted by them was to minimise the peak displacement response of the primary structure. Sadek et al.(1997) investigated a different method of estimating the optimum parameters of the TMD, when a SDOF structure is subjected to seismic loading, the criterion used by them was to obtain the
parameters that would result in equal and larger modal damping in the first two modes of vibration [5]. Apart from this, Hoang et al. provided site-specific optimum TMD parameters when a SDOF structure is subjected to seismic loading [6]. Researchers have also studied the effectiveness of the linear TMD and found that it is not so efficient to reduce the maximum structural displacement response under seismic applications compared to that when the structure is subjected to wind excitations [7,8].

In the present paper, a procedure is proposed to optimally design the TMD, when a damped SDOF system is subjected to base excitation. A key issue for the design of the optimal TMD is the identification of the parameters of the structure to which the TMD is to be attached. Here, it is important also to consider the perturbations that may arise in the natural frequency of the primary structure, due to different practical issues such as difficulty in the accurate computation of structural frequency due to variation in structural stiffness, mass or both, from that considered in design, or problem in ensuring exact linear behaviour of the structure throughout its design life time. Keeping this in mind, the objective of the optimization procedure is set to maximise the reduction of the displacement response of a SDOF structure with a linear TMD, having a specified variation in the natural frequency of the primary structure, when the structure is subjected to a harmonic base acceleration over a range of input frequencies. The optimum parameters of the TMD are hence found using the well-known optimization procedure of Genetic Algorithm (GA), and are compared with some of the existing optimal solutions for linear TMD for seismic applications. Further, the optimally designed TMDs found using this method, as well as, those suggested by previous researchers, are checked for effectiveness in reducing the structural response when the structure-TMD system is subjected to recorded earthquake ground motions. Finally, the performance of the TMD designs is investigated when the TMDs connected to the base-excited structures are in the detuned condition.

2. Modeling of the structure-TMD system

The structure is modelled as a linear SDOF system with mass, $m_1$, stiffness, $k_1$, and damping coefficient, $c_1$, and is subjected to ground acceleration, $\ddot{x}_g$. A tuned mass damper of mass, $m_2$, is connected to the structure with a spring of stiffness, $k_2$, and a linear viscous damper of coefficient, $c_2$. The displacement of the structure with respect to the ground is $x_1$, and the displacement of the TMD with respect to the structure is $x_2$. A schematic diagram of the model of the structure-TMD system is shown in figure 1.

![Figure 1. Schematic model of the structure-TMD system.](image)

3. Optimal design of TMD using Genetic Algorithm

For the system described in figure 1, the equations of motion for $m_1$ and $m_2$ are given by the following two equations respectively:

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 - c_2\dot{x}_2 - k_2x_2 = -m_1\ddot{x}_g$$

$$m_2\ddot{x}_2 + m_2\dot{x}_2 + c_2\dot{x}_2 + k_2x_2 = -m_2\ddot{x}_g$$

Normalization of Eqns. (1) and (2) w.r.t $m_1$ and $m_2$ respectively leads to

$$\ddot{x}_1 + 2\xi_1\omega_1\dot{x}_1 + \omega_1^2x_1 - 2\mu\xi_2\omega_2\dot{x}_2 - \mu\omega_2^2x_2 = -\ddot{x}_g$$

$$\ddot{x}_2 + \dot{x}_2 + 2\xi_2\omega_2\dot{x}_2 + \omega_2^2x_2 = -\ddot{x}_g$$
where, $\xi_1$ and $\omega_1$ are the damping ratio and natural frequency of the primary structure, respectively, and, $\xi_2$ and $\omega_2$ are the damping ratio and the natural frequency of the TMD, respectively. $\mu$ and $\gamma$ are the mass ratio and the tuning ratio of the TMD, respectively, expressed as

$$\mu = \frac{m_2}{m_1} \quad \text{and} \quad \gamma = \frac{\omega_2}{\omega_1}$$ (5 a, b)

For the optimal design of the TMD, the different parameters of the TMD are to be optimised, namely, the mass ratio, $\mu$, the tuning ratio, $\gamma$, and the damping ratio, $\xi_2$. Generally, the mass ratio is fixed according to the restrictions due to practical considerations. Therefore, the optimization problem here, assumes the mass ratio to be a preset parameter, while the other parameters are optimised for the best design of the TMD, under the stated objective.

Here, an optimization problem is framed for the design of the TMD, which will be effective over a wide range of input frequencies, because, seismic excitations are usually broad-banded in nature. With this, the ground acceleration is assumed to be harmonic with constant amplitude, $a$, and input frequency, $\bar{\omega}$, varying over a specified range. The ground excitation is thus expressed as

$$x_g = a \sin(\bar{\omega}t)$$ (6)

Moreover, to make the design more robust, a variation in the structural frequency is incorporated in the design procedure. Hence, the objective here would be to maximise the volume under the surface plot of the reduction in peak structural displacement over a specified variation of the natural frequency of the primary structure and over a range of excitation frequencies.

The optimization is carried out by using the well-known optimization procedure, Genetic Algorithm (GA). This search algorithm is based on the principles of natural selection and genetics. GA has been proved to be useful in the optimization of complex problems where the objective function is not known explicitly. The problem is solved using the GA solver provided in the MATLAB (R2018a) optimization toolbox. The following parameters are used to characterise the GA in the solver.

- Population size of 50 individuals
- Fitness scaling according to rank
- Crossover function scattered with a crossover fraction of 0.8
- Stopping criteria of 200 generations or function tolerance of 1e-6

4. Numerical study and comparison with existing optimal solutions

The design of the optimal TMD using GA is numerically illustrated in two example structures with a natural frequency of 0.5 seconds each, and damping ratios of 2% and 5%. Two different TMD mass ratios are considered as 1% and 3%. The structure-TMD system is subjected to a base acceleration assumed to be harmonic, given by Eqn. 6, with peak ground acceleration of 0.02g, and external frequency, $\bar{\omega}$ varied over a range of 0.85$\omega_1$ to 1.15$\omega_1$. A variation of $\pm$5% in the time period of the main structure is considered for the design procedure.

The optimum parameters of the TMD obtained from this method are listed in table 1, and they are compared with the optimum parameters obtained from the previous studies on the optimal design of TMD for seismic applications. Here, three previously found optimal solutions of the TMD parameters are referred, as follows, firstly, the optimum parameters of a TMD attached to a SDOF system subjected to white-noise excitation as base acceleration provided by Warburton (1982)[3]. Secondly, the optimum parameters of TMD for support-excited structures given by Tsai and Lin (1993)[4], and thirdly, when a SDOF structure is subjected to seismic loading, the optimum parameters of the TMD presented by Sadek et al.(1997) [5] are considered for the performance study. All the optimum parameter sets obtained from the previous studies and from this method are listed in table 1.

From table 1, it is clear that the tuning ratios of the optimum TMDs obtained by optimization using GA are considerably lesser than the optimum tuning ratios obtained from the other methods. Further, the damping ratios of the TMDs obtained from this optimization procedure is close to the optimum damping ratios obtained by Tsai and Lin (1993). However, unlike Tsai and Lin (1993), the values of the optimum damping ratio of the TMDs obtained from this method have very slight differences due to the change in the structural damping ratio. Further, the optimum parameters obtained by Sadek et al. (1997) is seen to have slightly greater values than the other three optimal design parameter sets for all the
example structure-TMD systems. The whole design procedure using GA was repeated for some different amplitudes of the harmonic base acceleration, and all the designs yielded the same optimum parameters, which validates that the optimum parameters of the linear TMD do not depend on the intensity of the load applied.

Table 1. Optimum TMD parameters obtained from different studies.

| Structural Damping Ratio | Mass ratio of TMD | Optimum TMD parameters | Methods used in TMD design |
|--------------------------|-------------------|-------------------------|----------------------------|
|                          |                   | γ                       | Warburton (1982)           |
|                          |                   | ξ2                      | Tsai and Lin (1993)        |
|                          | 1%                | 0.9819                  | Sadek et al. (1997)        |
|                          |                   | 0.0498                  | Current study              |
|                          | 3%                | 0.9545                  |                            |
|                          |                   | 0.0857                  |                            |
|                          | 5%                | 0.9704                  |                            |
|                          |                   | 0.9380                  |                            |

The effect of these differences in the optimum parameters obtained from the four different designs of the TMD is studied through the time-domain displacement response analysis of these structure-TMD systems when subjected to three recorded seismic accelerograms, as described in table 2.

Table 2. Classification of earthquakes.

| Event name       | Year | Station name     | PGA   |
|------------------|------|------------------|-------|
| Loma Prieta      | 1989 | Golden Gate Bridge | 0.23g |
| Imperial Valley-01 | 1940 | El Centro        | 0.35g |
| Kobe, Japan      | 1995 | Abeno             | 0.22g |

Table 3 shows the reductions obtained in the variance of the displacement responses of the primary structures by the four different designs of the TMD. It is observed that the reductions obtained from the four TMD designs are close to each other; however, the TMD designed by optimization using GA has performed better for the Imperial Valley-01 earthquake. TMD design given by Warburton (1982) has worked the best for Kobe earthquake, and design given by Tsai and Lin (1993) has proved to be the most effective for Loma Prieta earthquake. Reductions obtained by the TMD designed according to Sadek et al. (1997) are always seen to be slightly lesser than the reductions obtained from the other designs.

Table 3. Reduction in variance of structural displacement response using the different optimal designs, when subjected to the three earthquakes.

| Structural Damping Ratio | Mass ratio of TMD | Earthquake name     | Reduction in variance of displacement response achieved by |
|--------------------------|-------------------|---------------------|----------------------------------------------------------|
|                          |                   |                     | Warburton (1982) | Tsai and Lin (1993) | Sadek et al. (1997) | Current study |
|                          |                   | Loma Prieta         | 38.42          | 39.83              | 37.90              | 37.49          |
|                          |                   | Imperial Valley-01  | 32.50          | 33.90              | 39.10              | 33.93          |
|                          |                   | Kobe, Japan         | 54.08          | 51.69              | 39.92              | 53.25          |
|                          |                   | Loma Prieta         | 49.55          | 51.54              | 51.40              | 50.00          |
|                          |                   | Imperial Valley-01  | 50.82          | 51.32              | 45.20              | 51.34          |
|                          |                   | Kobe, Japan         | 70.84          | 68.65              | 56.87              | 68.33          |
|                          |                   | Loma Prieta         | 17.70          | 22.62              | 26.12              | 21.69          |
|                          |                   | Imperial Valley-01  | 16.11          | 17.67              | 13.85              | 18.13          |
|                          |                   | Kobe, Japan         | 32.98          | 28.63              | 16.80              | 29.70          |
|                          |                   | Loma Prieta         | 21.23          | 23.90              | 23.60              | 22.92          |
|                          |                   | Imperial Valley-01  | 32.95          | 33.14              | 25.66              | 33.43          |
|                          |                   | Kobe, Japan         | 47.64          | 43.80              | 31.20              | 44.51          |
Although all the TMD designs are seen to provide similar reductions in structural displacement responses due to the recorded seismic excitations in the tuned condition, it is worth reviewing their performance in the detuned condition. A sample structure-TMD system from the above examples is chosen with the primary structure's time period of 0.5 seconds and 2% structural damping, having a TMD of 3% mass ratio. Next, this structure-TMD system is used to illustrate the performance of the TMD designs in the detuned condition. Figure 2 provides contours showing the reduction in peak structural displacement response against a variation of the natural frequency of the primary structure and with the change in the excitation frequency, when the structure is subjected to a harmonic base excitation. From this figure, it is clear that the TMD performance in the detuned condition deteriorates by a considerable amount. In the worst detuned condition, Warburton’s (1982) TMD design slightly amplifies the displacement response; Tsai and Lin’s (1993) design gives no reduction at the same configuration of the system; however, the designs proposed by the current study and by Sadek et al. (1997) still give a small reduction to the tune of 5% in this condition of the TMD. The design of the TMD given by Sadek et al. (1997), even though works fine in detuned condition, in the tuned condition the reductions achieved by this design is almost 10% lesser than those achieved by the other three designs. Therefore, from figure. 2, it may be inferred that the TMD designed by optimization using GA works more efficiently than the other three designs, when there is a perturbation in the natural frequency of the main structure.

Figure 2. Reduction in peak displacement response of the primary structure with the variation in structural frequency and variation in excitation frequency, when the TMD is designed according to (a) Warburton (1982), (b) Tsai and Lin (1993), (c) Sadek et al. (1997), and (d) Current study.

5. Conclusion
Many authors have provided optimal design strategies to find the parameters of a linear TMD which is to be attached to a structure subjected to base excitation, where different performance criteria are optimised for the best performance of the TMD. However, it must be noted that the optimum parameters of the TMD are designed for a certain set of structural parameters, and a variation in these structural parameters adversely affects the performance of the linear TMD. This paper proposes a new design
strategy, where a specified variation in the natural frequency of the primary structure is considered in the design optimization problem, and the parameters of the optimal TMD are numerically searched. The optimum parameters found using this method is then compared with the optimum parameters of a TMD designed for similar applications given by some previous researchers. Further, to check the effectiveness of the TMD, time-history analysis of some example structure-TMD systems are studied under three actual recorded seismic accelerograms. It is found that all the TMD results yield similar reductions in the variance of structural displacement responses under these three earthquakes. Thereafter, to study the effectiveness of the TMD designs in the detuned condition, the contours showing the reduction in peak structural displacement response are plotted against the variation of structural frequency over a range of input frequencies. From this study, it is confirmed that the TMD designed by this method, exceeds the level of performance achieved by the other three TMD designs when the structure is subjected to a harmonic load as base excitation.

6. References
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