We propose a novel scenario to explain the observed cosmological asymmetry between matter and antimatter, based on nonperturbative QCD physics. This scenario relies on a mechanism of separation of quarks and antiquarks in two coexisting phases at the end of the cosmological QCD phase transition: ordinary hadrons (and antihadrons), along with massive lumps (and antilumps) of novel color superconducting phase. The latter would serve as the cosmological cold dark matter. In certain conditions the separation of charge is C and CP asymmetric and can leave a net excess of hadrons over antihadrons in the conventional phase, even if the visible universe is globally baryon symmetric $B = 0$. In this case an equal, but negative, overall baryon charge must be hidden in the lumps of novel phase. Due to the small volume occupied by these dense lumps/antilumps of color superconducting phase and the specific features of their interaction with “normal” matter in hadronic phase, this scenario does not contradict the current phenomenological constrains on presence of antimatter in the visible universe. Moreover, in this scenario the observed cosmological ratio $\Omega_{DM} \sim \Omega_B$ within an order of magnitude finds a natural explanation, as both contributions to $\Omega$ originated from the same physics during the QCD phase transition. The baryon to entropy ratio $n_B/n_\gamma \sim 10^{-10}$ would also be a natural outcome, fixed by the temperature $T_f \sim T_{QCD}$ at which the separation of phases is completed.

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I. INTRODUCTION

The origin of the cosmological asymmetry between baryons and antibaryons, and, more specifically, the origin of the observed baryon to entropy ratio $n_B/s \sim 10^{-10}$ ($n_B$ being the net baryon number density in hadrons, and $s$ the entropy density) remains a mystery and one of the main challenges for particle-cosmology. In order to explain this number from symmetric initial conditions in the very early Universe, it is generally assumed that three criteria, first laid down by Sakharov[1], must be satisfied at the instant when the asymmetry was generated:

- C and CP are not exact symmetries.
- Baryon number violating processes exist.
- The processes take place out of thermal equilibrium.

In general, nevertheless, all three of these tight conditions can be loosened in certain scenarios. For example, C and/or CP asymmetries can be generated earlier and not necessarily at the same instant when the cosmological baryon asymmetry developed. Explicit baryon number violation is not strictly necessary; it has been shown that in certain contexts spontaneous violation is able to produce a spatial separation of baryonic charge which leaves a positive net number in our local patch of the universe. Neither out-of-equilibrium dynamics is a must: in some suggested scenarios baryogenesis develops due to spontaneous CPT breaking.

Here we argue that baryogenesis may be realized at the instant just after the cosmological QCD phase transition without explicit violation of baryon number, if at some stage in the earlier history large C and CP asymmetries have developed homogeneously over the whole visible universe. Motivated by the cosmological ratio between baryonic and dark matter densities, $\Omega_{DM} \sim \Omega_B$ within an order of magnitude, which we interpret as an indication of a close relationship between the dark matter problem and the origin of the cosmological baryonic asymmetry[2], our proposal features an scenario of baryon charge separation in a universe with zero overall baryon charge $B = 0$. This scenario relies on the idea that baryon/antibaryon charge can be stored in dense lumps/antilumps of ordinary light quarks/antiquarks condensed in color superconducting (CS) phase, which would serve as cosmological cold dark matter. In different words, we suggest that the observed cosmological excess of hadrons over antihadrons may not necessarily be expressed as a net baryon charge $B > 0$ in the universe if an equal, but negative, overall charge is accumulated in the lumps/antilumps of dense color superconducting phase. In this sense, our proposal resembles an old scenario discussed in[3] of a baryon “asymmetric” universe with zero overall baryon charge $B = 0$, in which the net excess of hadrons over antihadrons is compensated by a negative overall baryonic charge in an hypothetical new fundamental scalar field hidden in the dark sector. Our proposal does not rely on any new fundamental particle, but on a new phase of ordinary QCD matter.
This proposal follows the recent observation that chunks of condensed ordinary quarks with large baryon charge accumulated in color superconducting phase may form at temperatures $T \lesssim 0.57 \Delta$, where $\Delta \sim 100$ MeV, absolutely stable objects against decay into ordinary nuclear matter. Color superconducting (CS) phase is a theoretically motivated novel phase of QCD matter that is realized when light quarks are squeezed to a density which is a few times the nuclear density and organize a single coherent state that condense in diquark channels, analogous to Cooper pairs of electrons in BCS theory of ordinary superconductors, see original papers and recent reviews on the subject. It has been known that CS phase may be realized in Nature in neutron stars interiors and in the violent events associated with collapse of massive stars or collisions of neutron stars, so it is important for astrophysics. We argue here that if such conditions occur in the early universe during the QCD phase transition, CS phase may be important for cosmology as well.

The possibility that massive lumps of ordinary quarks in a novel state of QCD matter could form at the end of the QCD phase transition along with ordinary hadrons and antihadrons in a cosmological scenario of separation of phases was first considered in. The proposal noticed that the baryon charge hidden in these lumps would not participate in nucleosynthesis and, therefore, according to the usual definition would not contribute to $\Omega_B h^2 \simeq 0.02$ in spite of their QCD origin. These lumps would serve as “nonbaryonic” dark matter. This original proposal followed the theoretical observation that at quark densities $\mu \gtrsim 300$ MeV, larger than the strange quark mass and a few times larger than the typical nuclear density, addition of strangeness should lower the quark chemical potential and ordinary baryonic matter could prefer to settle into a novel coherent ground state of three flavours of delocalized quarks, known as ‘strange quark matter’, rather than in ordinary hadrons.

The theoretical discovery of the color superconducting phases of QCD matter has stirred renewed interest on the cosmological scenario of separation of phases at the QCD phase transition. The interest now focus on the formation of lumps of CS phase. It is important to notice that, in the strict sense, lumps of color superconducting matter are not the hypothetical ‘strangelets’ predicted in, in spite of the obvious similarity. In color superconducting phases Cooper pairing at high density, rather than the addition of a third quark flavour, is the cause responsible for lowering the energy and favouring the formation of the novel coherent state: color superconducting phases could be made of three flavours of light quarks (like CFL phase) or only two of them (like 2SC phases). In addition, diquark Cooper pairing in color superconducting phases manifest in a large gap $\Delta \sim 100$ MeV in the spectrum of single particle excitations of quark matter, which could be a crucial feature to understand the phenomenology of these balls/antiballs of condensed matter. A gapped spectrum of fermionic excitations, on the other hand, is not a necessary feature of 'strange quark matter'.

We go further with this original idea introduced in by suggesting that such dense objects could also be configurations with large negative baryon charge (antilumps), made of condensed antiquarks in the color superconducting phase. As we discuss in next sections in details, in such a form the anti-baryon charge could coexist with a net number of hadronic baryons without annihilating due to the small volume occupied by the dense lumps and due to the specific properties of the interaction of normal hadronic matter with the lumps in superconducting state. Usual baryons (hadrons) could be reflected rather than annihilated when they hit an anti-ball with the small energy typical for the present cold universe (when $e/c \sim 10^{-9}$). A similar feature is known in the interaction of conduction electrons in metals at the interface with conventional superconductors. More than that, the anti-baryon charge of the anti-ball would not change either the nucleosynthesis calculations because, locked in CS phase, it is not available to form nucleons, similar to the positive baryon charge locked in CS phase.

A complete understanding of the mechanisms that produce, in this scenario of separation of phases (and charges) at the QCD phase transition, a net excess of hadrons over antihadrons in the conventional phase should include an explanation of:

1) the mechanisms that lead to the formation at the end of the cosmological QCD phase transition of lumps and antilumps of condensed QCD matter/antimatter in color superconducting phase, coexisting with ordinary hadrons and antihadrons;

2) the mechanisms that avoid the evaporation and disappearance of the lumps and antilumps once the temperature of the universe drops well below the temperature of the phase transition and allow their survival in the later colder universe;

3) the mechanisms responsible for the C and CP asymmetric separation of the baryonic charge in the two coexisting phases that leaves a net baryon number in the ordinary hadronic phase, and how this net excess of hadrons is preserved at later times after the separation has been completed and antihadrons annihilated;

4) the phenomenological viability of the proposed scenario when confronted with currently available observational data, in particular, with constrains on large amounts of antimatter in the visible universe.

The research on the cosmological scenario of separation of phases has focused, since the birth of the idea, on the problems of formation and survival of lumps of quark matter at $T_{QCD}$ in a universe that has already developed a net baryon asymmetry in the standard sense, as it was outlined in the original proposal. In spite of all this attention, important questions about this scenario still remain open. The innovative aspect of our proposal, in
which antilumps of condensed antiquarks form along with lumps of condensed quarks and the cosmological baryon asymmetry only develops as a result of the net separation of charges, requires a re-evaluation of problems 1) and 2) plus an understanding of problem 3), which is obviously specific of our proposal. In this paper we shall focus, nevertheless, on a phenomenological study of the viability of our scenario when confronted with available observational data, problem 4), for reasons that are transparent: first, we must be convinced that the scenario that we propose is not ruled out; second, a complete study of the first three aspects is a more complicated task that can be addressed separately.

At this stage we cannot prove, let us state it clearly, that such dense objects of condensed quarks/antiquarks in color superconducting phase do form at the end of the cosmological phase transition. Neither, of course, we can precisely calculate some important phenomenological features of these droplets of condensed phase, such as temperature $T_f \lesssim T_{QCD}$ at which the formation is completed, mass spectrum of resulting balls/antiballs or their number densities. Such analysis would require a much deeper understanding of the non-equilibrium dynamics of non-perturbative QCD during the cosmological phase transition, which is not available at the present time. We understand, nevertheless, that we must present some fair evidence that issues 1), 2) and 3) can be successfully solved in order to make the proposal contained in this paper truly attractive. Indeed, we will argue below that a baryon asymmetric cosmological separation of phases can be a plausible, and even probable, output of the cosmological QCD phase transition.

Our presentation is organized as follows. In Section 2 we review the observational data related to the cosmological baryon asymmetry and argue in favour of the cosmological scenario of separation of phases as a natural framework to explain the origin of the asymmetry and solve the dark matter problem. Then we discuss the essential features of the interaction of hadronic matter with balls and antiballs of condensed quarks/antiquarks to argue that well-known constraints on absence of significative amounts of anti matter in the visible universe cannot be literally applied to the case when anti-matter is stored in dark massive droplets of the dense color superconductor phase. Moreover, we argue that under this hypothesis the observed relation $\Omega_B \sim \Omega_{DM}$ is a natural consequence of the underlying QCD physics. Section 3 is devoted to the calculation of the reflection and transmission coefficients of free quarks inciding at the interface of the color superconductor. These results are used in Section 2 devoted to the observational constraints on antimatter in the present cold Universe at temperature well below the QCD phase transition. In Section 4 we discuss some generic aspects of the charge separation mechanism during the QCD phase transition, after noticing that all three Sakharov’s criteria could be satisfied (in some looser sense) when chunks of dense color superconducting matter/antimatter form during the phase transition. Finally, we estimate the fundamental parameter $\eta \equiv (n_B - n_{\bar{B}})/n_s$ in this scenario. Section 5 contains our conclusions, where we speculate on possibilities to test the suggested scenario.
II. BARYOGENESIS VS BARYON SEPARATION

A. Observations and Phenomenology

Baryons in hadronic phase make all the directly observable astronomical objects, from gas and dust to stars and clusters of galaxies, without any significant trace of antibaryons over a spatial domain that could be as large as the present horizon, see reviews and some original papers in [25]. The origin of this asymmetric distribution of baryons and antibaryons remains one of the most fundamental open questions in cosmology in spite of the great theoretical and experimental efforts it has attracted during the last thirty five years, see e.g recent review [26].

The baryon-antibaryon asymmetry can be quantified through the ratio \( \frac{n_B - n_{\bar{B}}}{n_\gamma} \), where \( n_B \) and \( n_{\bar{B}} \) are, respectively, the number densities of baryons and antibaryons and \( n_\gamma \) the number density of photons in the cosmic background. Theoretical models predict, and observations confirm, that baryon number \( n_B - n_{\bar{B}} \) is preserved in any comoving volume since the time of nucleosynthesis, at \( T_{eq} \approx 1 \) MeV, and probably even earlier, since the instant just after the electroweak phase transition, at \( T_{ew} \approx 100 \) GeV. Indeed, the anomalous processes which efficiently violate the baryon number in the electroweak symmetric phase are effectively suppressed soon after the instant of the phase transition when the system is in the spontaneously broken phase. Since the number of photons in a comoving volume is also preserved, the ratio \( \frac{n_B - n_{\bar{B}}}{n_\gamma} \) remains approximately fixed in physical volumes while the universe is expanding and cooling.

If \( (n_B - n_{\bar{B}}) \ll n_\gamma \), as it turns out to be the case, the early universe is approximately baryon-antibaryon symmetric until the QCD phase transition \( T_{QCD} \approx 160 \) MeV. This statement remains valid even if some asymmetry was generated at some earlier time. This is due to the strong QCD interactions in the quark-gluon plasma where massless quark-antiquark pairs can easily be produced. Therefore, \( n_B \sim n_{\bar{B}} \sim n_\gamma \) at \( T > T_{QCD} \). The universe becomes manifestly baryon asymmetric only at the temperatures \( T \lesssim T_{QCD} \), when strong QCD interactions confine quarks into heavy hadrons, with masses of the order of \( m_N \sim 1 \) GeV which, subsequently, annihilate each other leaving only a small excess of hadronic baryons as the remnants, \( n_B - n_{\bar{B}} \sim n_B \).

The ratio between the number density of this remnant of hadronic baryons and the number density of photons in the cosmic background has been recently measured with high accuracy, \( \eta \equiv n_B/n_\gamma \approx 6 \times 10^{-10} \) [27]. This parameter can be directly related to the ratio \( \eta \sim \eta \left( \frac{m_N}{T_{ew}} \right)^{-1} \) between the mass of the nucleon \( m_N \) and the temperature of matter-radiation equality, \( T_{eq} \sim 1 \) eV. Its precise observational value is in excellent agreement with the present cosmological abundances of light elements predicted by the standard nucleosynthesis scenario, see e.g [28].

In generic scenarios of baryogenesis at any time earlier than the QCD phase transition the net asymmetry \( n_B - n_{\bar{B}} = \eta n_\gamma \ll n_\gamma \) should be fine-tuned to its observational value, in the sense that \( n_B - n_{\bar{B}} \ll n_B, n_{\bar{B}} \). In most suggested scenarios the need of tuning the ratio \( \eta \) (when a natural scale for \( \eta \) is absent) manifests itself by the fact that \( \eta \) can be made either too large or too small by changing a few parameters that in many cases are only loosely constrained. In general, fine tuning is not considered as an attractive solution of a problem from a theoretical point of view when there is not a satisfactory explanation for the tuning itself. This is a very common feature of most scenarios for baryogenesis. As we shall see, such kind of fine tuning might not be required in the scenario of baryon separation advocated in the present work.

This is not the place to review all possible suggested scenarios for baryogenesis. However, we wish to briefly comment on the need of tuning of some sort in each of the current trends. For example, in supersymmetric electroweak scenarios the Higgs and stop masses must be carefully chosen to lie within narrow intervals to generate the observed value of \( \eta \). Otherwise, any asymmetry which was developed at the electroweak scale will be washed out at the end of the phase transition. Other scenarios, like leptogenesis which relies on asymmetries generated at higher scales in channels that cannot be erased by electroweak physics, need also be carefully tuned. In Affleck-Dine scenarios the common problem is that too much asymmetry is generated and must be subsequently diluted. In summary, although the general conditions under which the asymmetry could have developed are well-understood, the final word about the specific mechanism and physics involved in it still remains to be found.

B. Baryon asymmetry and QCD

It is of natural interest to explore the possibility that the baryon asymmetry may have been generated not before the electroweak phase transition (as most scenarios suggest), but later, at the instant when the universe became manifestly asymmetric, immediately after the cosmological QCD phase transition. As we have argued, this might be a viable scenario relying on nonperturbative QCD physics that, in principle, does not require the introduction of any new physics beyond the standard model, except maybe for a solution of the strong CP problem. Obviously, in absence of baryon violating interactions at \( T_{QCD} \) the only way to produce a baryon asymmetry is via charge separation. It is
to a discussion of such a mechanism in the context of a globally baryon-antibaryon symmetric universe, $n_{B}^{tot} = n_{\bar{B}}^{tot}$, to which we now turn. In such a scenario the ratio $\eta$ is fixed by the energy scale of the physics involved and the problem of fine tuning mentioned above is automatically resolved, see below.

This scenario for baryogenesis should be examined in the context of recent observations that confirm that baryonic (to be precise: the hadronic) matter contributes only a fraction of the matter content of the universe, while a much larger fraction is made of some unknown form of dark matter which is not apparent to detection through electromagnetic radiation, $\Omega_B \sim \frac{1}{4} \Omega_{DM}$ [2]. In this context, a net number of hadronic baryons $n_B - n_{\bar{B}} \neq 0$ can be generated at the QCD phase transition if some mechanism exists that separates baryon and antibaryon charges and stores an excess of the latter in compact objects of non-hadronic, color superconducting phase discussed above. The non-hadronic objects, which however carry a large baryon charge $\pm B$, have heavy mass $M_{DM}$ and would contribute, instead, to the "non-baryonic" cold dark matter of the universe, according to the standard definition of the "non-baryonic" dark matter. Once the phase transition has been completed hadrons will annihilate anti-hadrons leaving at the end only the excess of the former plus an equal negative excess of baryonic charge in the objects of condensed phase. In this case a total baryon-antibaryon annihilation after the QCD phase transition would be avoided, even though $B = 0$, as we will discuss in detail in next sections. The separation of phases must be completed before the nucleosynthesis, at $T_{nc} \sim 1 \text{ MeV}$, such that only the surviving hadronic baryon number $n_B = \eta n_{\gamma}$ participates in the composition of light nuclei.

Let us assume that this hypothetical scenario is correct and dark matter indeed consists of heavy objects made of quark matter in color superconducting phase. What phenomenological consequences can we derive from this assumption? As the total baryon number is conserved and hadronic baryons have charge $+1$, the net number density of non-hadronic QCD-balls should be

$$\tilde{n}_B - \tilde{n}_{\bar{B}} = \frac{1}{B} (n_B - n_{\bar{B}}) \simeq \frac{1}{B} n_B,$$

where we introduce notation $\tilde{n}$ describing the density of dark matter heavy particles which carry the baryon charge in a hidden form of the diquark condensate (CS phase) rather than in form of free baryons. Let then consider the ratio of dark matter number density $\equiv \tilde{n}_B + \tilde{n}_{\bar{B}}$ to baryon number density $\equiv n_B$. By definition,

$$\left( \frac{\text{dark matter number density}}{\text{baryon number density}} \right) \simeq \frac{m_N \Omega_{DM}}{M_{DM} \Omega_B}. \tag{2}$$

The dark matter number density could be naturally estimated, without any fine-tuning, to be

$$\tilde{n}_B + \tilde{n}_{\bar{B}} = \#(\tilde{n}_B - \tilde{n}_{\bar{B}}), \tag{3}$$

where $\#$ is some numerical factor $\geq 1$, if the excess $\tilde{n}_B - \tilde{n}_{\bar{B}}$ is of the same order as the number densities $\tilde{n}_B$ and $\tilde{n}_{\bar{B}}$. In fact, the excess $\tilde{n}_B - \tilde{n}_{\bar{B}}$ is indeed expected to be of order $\tilde{n}_B, \tilde{n}_{\bar{B}}$ if the universe is largely C and CP asymmetric at the onset of formation of the condensed balls. Then, the l.h.s. of the ratio [2], can be estimated to be

$$(\tilde{n}_B + \tilde{n}_{\bar{B}})/n_B = \#(\tilde{n}_B - \tilde{n}_{\bar{B}})/n_B \simeq \# \frac{1}{B}$$

according to eqs. [13]. Consequently, from [24] we obtain

$$\Omega_{DM}/\Omega_B \simeq (\#/B) \times (M_{DM}/m_N). \tag{5}$$

Now, if one demands $B \simeq (M_{DM}/m_N)$, which is a condition for the stability of the balls, one can immediately derive $\Omega_{DM}/\Omega_B \simeq (\#) \geq 1$. The point we want to make is: our assumption that the dark matter is originated at the QCD scale from ordinary quarks fits very nicely with $\Omega_{DM}/\Omega_B \geq 1$ within the order of magnitude, provided that separation of baryon charges is also originated at the same QCD scale. Generally, the relation $\Omega_B \lesssim \Omega_{DM}$, within one order of magnitude, between the two different contributions to $\Omega$ is difficult to explain in models that invoke a dark matter candidate not related to the ordinary quark/baryon degrees of freedom.
C. Cosmological scenario of separation of phases

The possibility that massive lumps of ordinary quarks in a novel state of QCD matter could form at the end of the QCD phase transition along with ordinary hadrons and antihadrons and serve as cold dark matter in a cosmological scenario of separation of phases was first considered in [9]. The idea has attracted since then a lot of effort, but crucial questions about this scenario have remained open. Namely, the mechanisms of formation of the lumps and their survival once the temperature of the universe drops well below the temperature of the phase transition are not yet completely understood. While the formation of dense lumps during the phase transition is beyond the scope of the present paper, we nevertheless want to present some arguments to show that within a broad consideration of the thermodynamical parametrical space at the onset of the cosmological QCD phase transition a separation of phases that leaves a net baryon number in the conventional hadronic phase is a plausible, and even probable, output of the transition.

In the original proposal [9], it was discussed that lumps of quark matter would form adiabatically in a first order QCD phase transition in a two stages process. First, large bubbles of quark matter would form at $T_{QCD}$ in thermal and chemical equilibrium with the expanding hadronic phase. In a second stage these bubbles of quark matter would shrink, while delivering heat and entropy to the surrounding medium. The fate of the bubbles of quark matter depends largely on the leading mechanism of heat/entropy release. If it mainly proceeds through baryon evaporation at the surface of the bubbles, they will lose their mass and finally they will disappear. On the other hand, if the heat/entropy release proceeds much faster than baryon number evaporation (for example, through emission of Goldstone modes), then a large baryon charge could get trapped within the walls of the shrinking bubble in thermal, but not chemical, equilibrium with the surrounding hadronic phase. In this case, the shrinking walls squeeze the trapped baryon charge until the mounting Fermi pressure from within stops further shrinking. At this point the quark matter density within the bubble could reach the density threshold beyond which it organizes the diquark condensate.

A clear drawback of this mechanism is the requirement of a first order transition: current theoretical analysis and numerical simulations of the QCD phase transition seem to show that the transition is not first order, but a crossover, unless the strange quark mass is rather light. A different problem of this scenario has to do with the mechanisms that could avoid the evaporation of the balls of quark matter, in the case they indeed form, once the temperature of the universe drops well below the temperature of the QCD phase transition. Balls of quark matter can coexist in thermal equilibrium with the surrounding hadronic environment at temperatures close to the phase transition. It has been known that lumps of quark matter with large baryon charge $B \lesssim 10^{40}$ would also be thermodynamically stable against decay into ordinary nuclear matter at temperatures $T \sim T_s$, with $T_s = 0.57 \Delta m \sim 60$ MeV, when they settle in color superconducting phase: the typical binding energy is estimated in the literature to be of the order of $\sim 50$ MeV, per baryon. But, at intermediate temperatures $T_s \sim T \sim T_{QCD}$ the lumps of quark matter would be unstable and should evaporate before they cool off to the temperature $T_s$ at which they become stable configurations, see for example [5] and references therein. To solve these problems one can imagine a scenario involving additional fields which make the formation of such lumps much more likely to occur. One particular example would be the frustrated collapse of axion domain walls immediately after the completion of the QCD phase transition [12]. This mechanism could work even if the QCD phase transition is second order or a crossover and the stability of the droplets of CS matter formed under the bubble could reach the density threshold beyond which it organizes the diquark condensate.

Either in its original form [9] or in the elaborated version [12], the mechanism of cosmological separation of phases through squeezing of quark matter within the walls of shrinking bubbles assumes, in a subtle way, that baryogenesis in the standard sense of generation of net baryon number density $n_B/s \sim 10^{-10}$ has already happened at some earlier stage before the QCD phase transition. Hence, when the initial bubbles of quark matter form in chemical equilibrium with the surrounding hadronic phase at $T_{QCD}$ they already carry a net baryon number $n_B/s \sim 10^{-10}$, which is subsequently squeezed to higher densities $n_B/s \sim 1$ when the bubbles shrink. In order to implement a similar mechanism in our proposal, in which the average baryon number in the plasma is exactly zero $B = 0$ and the baryon asymmetry of the universe develops only as a consequence of the cosmological separation of phases, we should address, in addition, how the bubbles of quark matter can trap, while shrinking, either a positive or negative large baryon charge. This could happen, in particular, if the walls that contain the collapsing bubbles of quark matter show different penetrability for baryons and antibaryons. A typical example of such feature is the CP asymmetric axion domain walls, as suggested in [15], assuming the QCD plasma is already largely C asymmetric due to weak interactions.

After this brief review of the current status of ideas on the issues of cosmological formation and survival of lumps of quark matter, we wish to add a novel observation that could be of significative relevance for the understanding of the cosmological scenario of separation of phases at the QCD phase transition and the genesis of the cosmological baryon asymmetry and the dark matter content of the universe. First, let us notice that current lattice simulations that explore the dynamics of QCD matter at finite temperature are carried, due to analytical needs, for a plasma in which the quark chemical potential is exactly or very close to zero $\mu \approx 0$. This setup is thought to be a good
first approximation to the real QCD phase transition, because at $T \gtrsim T_{QCD}$ the universe is closely baryon 
anti-baryon symmetric, $n_B - n_{\bar{B}} \ll n_B, n_{\bar{B}} \sim n_\gamma$, even in the case when baryogenesis (in the standard sense) has already occurred. 
This complacency is, notwithstanding, unjustified. In the quark gluon plasma at temperatures above the temperature of confinement there coexist three flavours of light quarks, $u$, $d$ and $s$, and their corresponding charge conjugates. 
Each fermion can appear with two possible chiral orientations and two possible values for their helicity. That means that the quark plasma is characterized, in general, by twelve different chemical potentials $\mu_i$, as color gauge symmetry guarantees that chemical potential for the three color orientations of each sub-specie are all equal. If the average baryon number is zero or close to zero, $B \approx 0$, then the twelve different potentials are constrained to add up to zero, $\sum_i \mu_i \approx 0$. It is necessary, in addition, that $\sum_i q_i \mu_i = 0$, in order to keep electrical neutrality. Otherwise, the twelve chemical potentials can be considered as an unknown set of free parameters that fully characterize the thermodynamics of the cosmological QCD plasma. These parameters can, in principle, take values in a very broad range, only constrained by Maxwell-Boltzmann equations of thermal equilibrium and the mentioned equations on charge conservation. In the simplest, but particular, case when all these chemical potentials are zero, the cosmological QCD plasma is C and CP symmetric. In presence of some non zero chemical potential C invariance is broken. CP invariance is also broken, unless the chemical potentials for a given sub-specie and its parity conjugate are exactly opposite. In summary, even when the net baryon charge of the universe is zero $B = 0$ chemical potentials for the different quark sub-species in the cosmological QCD plasma can be very large. Indeed, large quark chemical potentials in the cosmological QCD plasma should be naturally expected, even when globally constrained to $\sum_i \mu_i \approx 0$, because C conjugation is largely violated by weak interactions. Moreover, cosmological CP violation is also generally believed to happen before the QCD phase transition, see below. For the development of non-zero chemical potentials the violating interaction must proceed in conditions of non equilibrium.

The dynamics of the QCD phase transition in presence of large quark chemical potentials $\mu_i \neq 0$ might be significantly different from the dynamics observed in current lattice simulations of the C and CP symmetric scenario when all $\mu_i \approx 0$. In fact, a quick glance at the phase diagram of QCD at non-zero $\mu_i$ suggests that in presence of large chemical potentials a direct phase transition from the quark-gluon plasma into lumps and antilumps of color superconducting phase (avoiding the transition into conventional hadronic phase), or a mix of hadrons and antihadrons coexisting with lumps and antilumps, could be a plausible outcome of the phase transition at $T_{QCD}$. In this case, the QCD phase transition could happen at temperatures as low as $T \lesssim 0.57 \Delta \approx 60$ MeV, depending on the specific values of the chemical potentials of each quark sub-specie, and the stability and survival of the formed lumps would be immediately guaranteed. In addition, we will discuss below, in subsection III D, that once a large baryon fluctuation in the condensed CS phase is formed at temperatures just below the phase transition it could tend to grow by trapping more baryons from the surrounding ”normal” phase.

Although these arguments, of course, do not prove that cosmological separation of phases indeed do happen at the QCD phase transition, they do show that a wide range of plausible cosmological scenarios could lead to a succesfull separation of phases. Obviously, our proposal is not committed, as we have said, to any of these particular mechanisms or any others that could be found suitable.

### D. Antimatter as dense color superconductor

It is important to remark here that bounds that tightly constraint the presence of significant amount of antimatter in regions of the universe of different size scales are mainly derived from the phenomenological signatures of electromagnetic matter-antimatter annihilation processes. These bounds do not strictly apply to the presence of antimatter stored in color superconducting phases if this kind of objects do not easily annihilate. Or in more precise words, if the rate of annihilation is highly suppressed. Here we want to use the physical picture of conventional superconductors to qualitatively explain why normal hadronic matter may not be annihilated, but reflected, by objects made of color superconducting antimatter. More detailed calculations which support the qualitative arguments presented in this section are presented in next Section 3.

The peculiarities of the scattering process of conducting electrons from the metal at a superconductor junction are known to be a consequence of the energy gap $\Delta$ in the spectrum of single particle excitations of the superconductor above the Fermi surface. The basic features of the interaction are discussed in a unified framework using Bogoliubov-de Gennes equations, see \cite{11}. The observed phenomena can be explained in the following simple way. The conducting electrons of the metal, modelled as a gas of free fermions, inciding at the interface with energies much smaller than the energy gap $\Delta$ cannot go through the interface simply because there is no any kinematically available state in the superconductor for a single electron. Therefore, conduction electrons from the metal inciding at the interface must, in principle, be reflected backward into the metal. This process is known in the literature as normal reflection at the metal-superconductor interface. There is, nevertheless, a peculiar way for an incident low energy electron to propagate into the superconductor known as Andreev reflection\cite{29}: the incident electron excites an additional second
electron from the metal “sea” to form a Cooper pair (which can be excited in the superconductor without surpassing any gap barrier) and leaves a hole that propagates backward into the metal. In terms of particle physics concepts, Andreev reflection is an example of particle antiparticle pair creation, in which the particle (electron) joins the incident electron and together propagate inside the superconductor as a Cooper pair, while the anti-particle (hole) propagates backward. Normal reflection of the conduction electron as an electron and Andreev reflection of the electron as a hole compete and their relative probabilities are determined by the properties of the interface. In presence of a high potential barrier \( V(x) = \nu \delta(x) \) at the interface which separates the metal and superconducting phases, classic normal reflection happens with probability close to one, \( P(e \rightarrow \nu e) \approx 1 \), while the probability for Andreev reflection is effectively zero, \( P(e \rightarrow s \rightarrow e) \approx 0 \). This limit is known in the literature on ordinary superconductors as classic interface, while the opposite situation when there is no barrier and Andreev reflection is most probable is known as metallic interface.

Now, let consider an antiparticle (hole) from the metal inciding at the interface with the superconductor. The energy gap \( \Delta \) in the spectrum of single particle excitations in the bulk of the superconductor kinematically forbids a low energy, \( E < \Delta \), incident hole to penetrate the bulk of the condensed phase and annihilate with a paired electron, because the second electron of the couple should be promoted above the energy gap. Annihilation, therefore, cannot happen in the bulk of the superconductor. It could, nevertheless, happen at the interface if the resulting unpaired electron is expelled from the superconductor into the metal. In order to estimate the probability of this annihilation process, in which an incident hole is repelled as an electron off the superconducting phase, we notice that this process is nothing but the time reversal of the Andreev reflection discussed in the paragraph above. As time reversal symmetry is preserved in common superconductors, the probability for particle-antiparticle (hole) annihilation at the metal-superconductor interface must be equal to the well-known probability for Andreev reflection (particle-hole pair production). As we have remarked before, in presence of a high potential barrier \( V(x) = \nu \delta(x) \) Andreev reflection is very suppressed at low energies \( E \ll \Delta \), which in turn implies that also electron-hole annihilation must be highly suppressed. These conditions offer us an explicit example of a normal-superconductor interface in which particle antiparticle (hole) annihilation can be avoided. To be more precise, the cross section for particle hole annihilation when the former is stored in superconductor phase can be largely suppressed when compared to the ordinary annihilation cross section in the metal. In these conditions the incident hole from the metallic phase is necessarily reflected back into the metal as a hole.

We want to use this experience gained from the analysis of conventional superconductors for a qualitative understanding of the interaction of “normal” hadronic baryons with chunks of quark matter or antimatter in CS phase, characterized by a very large gap \( \Delta \sim 100 \text{ MeV} \) in its spectrum of single particle excitations. We note that the interaction at temperatures much below \( T_{QCD} \) of hadrons inciding at the chunks of superconducting phase with kinetic energies much smaller than the gap \( E \ll \Delta \) is not asymptotically free and cannot be simply described as a quark-CS phase interface in terms of the form factors of quarks in the hadron. In spite of that it is convenient and easier to discuss first the scattering of constituent quarks by balls/antiballs of CS phase. Incident quarks should be reflected off the bulk of the color superconducting phase, either baryonic or antibaryonic, with reflection coefficient close to one, similar to the case of high potential barrier interface in ref. [11], if they do not carry enough energy to penetrate the superconductor. This discussion is presented with explicit calculations in next section. In this case, the high potential barrier at the interface is produced by the large difference in the vacuum energies, or bag constants, that characterize the two phases in contact.

It must be remarked that a proper analysis of annihilation processes of real incident quarks (rather than holes) at the interface with the antibaryonic CS phase must include the available mass at rest of the interacting particles, which could radically change the simple arguments presented here. In ordinary superconductors slow incident positrons can annihilate in the bulk and promote the unpaired electron above the gap because the mass at rest of positrons/electrons \( \sim 0.5 \text{ MeV} \) is many orders of magnitude larger than the energy gap \( \Delta \sim 10^{-3} \text{ eV} \) in the superconductor. The situation is quite different for color superconducting quark matter. The gap \( \Delta \sim 100 \text{ MeV} \) in CS phase is of the same order of magnitude than the typical mass of constituent quarks in the hadronic phase. Moreover, the effective masses of current quarks in the bulk of the dense CS phase are largely reduced by strong interactions. On similar grounds a large suppression factor 20-30 in the annihilation rate of antimuons in dense ordinary nuclear matter (with respect to nucleon antimuon annihilation in vacuum) was predicted in [37], after noticing that the lower effective masses reduce the available phase space for annihilation. The suppression factor should be expected to be much larger in the denser CS phase where the effective masses are even lower and the available phase space is further restricted by the large energy gap \( \Delta \). A detailed calculation of these effects is out of the scope of this paper. We wish simply to highlight that annihilation cross section of incident quarks can be largely suppressed when the antimatter is stored in dense droplets of the gapped dense color superconducting phase. As the individual quarks that form an inciding hadron have different quantum numbers than the Cooper pairs in the CS phase, we argue that for hadron to penetrate/annihilate the condensed phase the individual quarks need to cooperate coherently and, in consequence, the process is kinematically further suppressed.
Therefore, QCD anti-balls and balls, if formed, could behave as very stable, massive and solid soliton-like objects that carry large, negative or positive, baryon number and reflect off usual hadrons inciding on them. This feature, as we advanced in the Introduction, plays a crucial role in the explanation of the phenomenology of these “non-baryonic” droplets and how they interact with the surrounding hadronic matter after the universe cools down to temperatures well below $T_{QCD}$. Such peculiar interaction features would imply that if the ball or antibal with small velocity $v/c \sim 10^{-3}$ enters a large massive object made of usual hadronic matter, for example the Earth, it will not necessarily decay by exploding. Rather it will go through the Earth leaving behind the shock waves. Direct searches of non-topological Q-balls, which have phenomenologically similar interaction features with ordinary matter, have been reported in [31]. At this point the search cannot rule out the existence in the present universe of a density of non-topological Q-balls, which have phenomenologically similar interaction features with ordinary matter, have necessarily decay by exploding. Rather it will go through the Earth leaving behind the shock waves. Direct searches to identify the recent seismic event with epilinear source[32] as the process which involves the dark matter particle, such as the balls of CS matter.

The physical arguments presented in this section lead to the conclusion that the cross section for annihilation between matter and antimatter segregated in different phases may be highly supressed in comparison to the standard cross section in vacuum. As we have already noticed, this point is extremely important for the explanation of the phenomenology of color superconducting QCD anti-balls in the presence of surrounding “normal” baryons in the cold universe. Moreover, if the annihilation cross section is strongly suppressed as discussed above large amounts of antimatter could exist inside the present visible universe, thus avoiding current observational constraints.

We wish to add one more comment to the last statement. The rate for matter antimatter annihilation is proportional not only to the cross section for the interaction, but also to the flux of collisions. In our previous discussions we have presented arguments which show that the cross section for matter antimatter annihilation can be largeley suppressed when the antimatter is stored in CS, non-hadronic phase. Now, we want to add that, in addition, the number of quark antiquark encounters per unit time and per unit volume when antimatter is stored in dense chunks of CS phase will be also largely reduced with respect to the case when both are in the same ordinary hadronic phase. Therefore, matter antimatter annihilation rate in the hypothetical scenario we have introduced is very supressed due to both of the two factors: suppression in the annihilation cross section and, moreover, suppression in the net flux of interactions. This favours the possibility that large amounts of antimatter could be stored in CS phase without violating current observational constraints.

Indeed, we can estimate the total number of collisions between ordinary hadrons and the dense balls of color superconducting phase in a Hubble time. The average number density of balls/antiballs in the halo of the Galaxy is, $\bar{n}_B \sim \frac{1}{1B GeV}$ with $\rho_{DM} \sim 0.3 GeV/cm^3$ and, thus, the number of collisions per hadron per unit time is given by

$$\frac{dW}{dt} = 4\pi R^2 \bar{n}_B v \simeq 4\pi R^2 \frac{1}{B GeV} \rho_{DM} v,$$

(6)

where $v \sim 10^{-3}c$ and $R \simeq 0.7 \cdot 10^{-13} B^{1/3} cm$ is the typical radius of the balls/antiballs. Even if annihilation were 100% efficient (which is not the case as argued above), the resulting lifetime for hadron annihilation is much longer, $H \frac{dW}{dt} \sim 0.2 \times B^{-1/3} \ll 1$, than the present Hubble time, $H \simeq 3 \cdot 10^{17}s$, if one uses already existing observational constraint on such kind of objects, $B \geq 10^{18}$ [0, 31].

The same result can be alternatively presented in terms of the fraction of the baryon charge of an antibal that would be annihilated in a Hubble time. The number density of hadrons in the ordinary phase is, on average, $n_B \sim \frac{0.15 \rho_{DM}}{1 GeV}$. Thus, the number of collisions per unit time in presence of a single QCD ball is given by

$$\frac{d\tilde{W}}{dt} = 4\pi R^2 n_B v \simeq 4\pi R^2 \frac{0.15 \rho_{DM}}{1 GeV} v,$$

(7)

Even if the annihilation is 100% efficient, the total (anti) baryon charge from anti QCD ball $\Delta B$ which will be destroyed by such annihilations during a Hubble time does not exceed

$$\Delta B \simeq \frac{d\tilde{W}}{dt} \cdot H \leq 0.1 B^{2/3},$$

(8)

per QCD ball with charge $B$. This represents an exceedingly small part of the QCD ball, $\Delta B/B \sim 0.1 B^{-1/3}$ for sufficiently large $B$. If one uses already existing constraint on such kind of objects, $B \geq 10^{18}$, one concludes that there is no obvious contradiction of the suggested scenario with present observations.

Obviously, complications derived from the non-uniform distribution of dark matter over astrophysical scales may modify this rough estimation. However, we do not expect that this complications will make qualitative changes to the arguments presented above. We should add, that more refined observations such as measurement of a single
511\,keV line from the bulge (as observed by INTEGRAL), or galactic diffuse spectrum in the GeV range ("GeV" excess) as observed by EGRET could be associated with annihilation of the visible matter with QCD (anti)balls. The corresponding analysis will be presented somewhere else [33].

The possibility that small domains of antimatter of astrophysical or cosmological size may exist in the universe and they do not contradict current observations was discussed earlier [34] and it was even suggested that these antimatter domains could evolve into condensed antimatter astrophysical objects [35]. Our proposal is not based on antimatter domains of such a large astrophysical size, but on droplets of macroscopic size of CS phase, and we wish to notice that when antimatter is stored in the dense phase small volumes can, nevertheless, account for a large amount of antimatter.

In summary, we have argued in this section that there might not be any phenomenological contradiction to the proposed scenario of the universe with zero total baryon charge, which is however distributed in two phases. The visible content consists of "normal baryons" in the hadronic phase with positive baryon charge, while the dark content is in the color superconducting phase (which contains both: positive as well as negative baryon charges).

### III. REFLECTION AND TRANSMISSION COEFFICIENTS

We consider in this section the scattering of quarks and antiquarks off a surface separating color superconducting (CS) and plasma phases and calculate their reflection and transmission (R&T) coefficients. This calculation involves only perturbative dynamics of quarks at the Fermi surface, \( \mu \sim 400 \, \text{MeV} \), while all nonperturbative physics is assumed to be parameterized by the diquark condensate. In order to avoid complications that may not be relevant at this stage the scattering problem is reduced to a one dimensional calculation of R&T coefficients, assuming that the typical size of the ball of condensed matter is much larger than all other scales involved in the scattering process.

Our goal is twofold. First, we want to demonstrate that under certain conditions the reflection coefficient is exactly one (complete reflection) if the energy of the incoming quark or antiquark is smaller than the energy gap \( \Delta \) in the spectrum of single particle excitations of the CS phase. This is the main goal of this section. This feature, as we have remarked in previous sections, is very important for understanding the phenomenology of balls and anti-balls as dark matter during the epoch when the universe has cooled down to temperature well below \( \Delta \sim T_{QCD} \). Such a phenomenon of total reflection of fermions off a superconducting region is well known in condensed matter physics in the interaction of conduction electrons of a metal at a high barrier junction with a conventional superconductor [11].

Our second goal is to demonstrate that at larger energies (such energies are typical when the temperature is high, \( T \sim \Delta \)) there can be a net transport of baryon number through the interface into the CS phase. This feature is important to understand the mechanism of charge separation that could lock quarks/antiquarks in the form of balls/anti-balls during the phase transition. We wish to remark again that we do not pretend to prove with our simple discussion that separation in CS objects does in fact occur during the cosmological phase transition. We only pretend to show that the conditions needed for the separation to happen can be fulfilled: this discussion must be understood only as an indication that separation can happen.

We should remark here that a similar scattering problem of particles off the interface region separating CS and hadronic phases was discussed previously in [36]. The analysis of these papers was motivated by the physics of neutron stars where CS phase is very likely to develop. Our context here is very different: we study the interactions of heavy objects made of condensed CS matter with the gas of hadrons that surround them during and after the cosmological QCD phase transition. However, the technique developed in [36] turned out to be very useful and will be widely used in the analysis which follows. The equations that describe the interface are basically the same Bogoliubov - de Gennes equations used in the classic reference [11] to describe the ordinary metal superconductor junction, with the remark that reference [36] works with relativistic Dirac equation and [11] works in the non-relativistic Schrodinger framework.

The specific features of the setup in [36] for describing the interface of phases in neutron stars match those we need for studying the process of formation of chunks of condensed color superconducting matter inside dense clouds of quark plasma during the QCD phase transition when \( T \sim \Delta \). On the other hand, the features of the setup that appropriately describes the interface of CS and hadronic matter in the cold universe at temperatures well below the phase transition \( T \ll \Delta \) are significatively different, because in such environment the density of quarks outside the "nuggets" of CS matter is much lower than the density of baryon charge inside of the "nuggets" [14].

Let us now start our detailed discussion with the effective lagrangian

\[
\mathcal{L} = \bar{\psi}^a_i \left( i \partial \mu \gamma \nu - m + \mu \gamma^0 \right) \psi^a_i + \{ \Delta^{ab} \left( \psi_i^a \gamma^5 \psi_j^b \right)^\dagger + h.c. \},
\]

which describes the relevant fermionic degrees of freedom at the interface. They are represented by Dirac field operators \( \psi_i^a(x) \), with \( SU(3)_c \) color index \( a = 1, 2, 3 \), for red, green and blue, and flavour index \( i = 1, 2, 3 \), for the three u,d,s light quarks. The three flavours are assumed, for simplicity, to be degenerate in mass. The matrices \( \gamma^\mu \) are the usual...
4D Clifford matrices and \( C = i\gamma^0\gamma^2 \) is the charge conjugation matrix. The region outside of the ball is modeled, for simplicity, by a gas of free quarks. The interface region between plasma and CS phases is parameterized by an effective order parameter that is proportional to the expectation value of the diquark condensate, \( \Delta_{ij}^{ab} \propto \langle \psi_i^{aT} C\gamma^5 \psi_j^b \rangle \) in CS phase, and it is zero in the plasma phase. We keep only a single tensor structure

\[
\Delta_{ij}^{ab}(\vec{x}) = \Delta(\vec{x}) \left( \delta_i^a \delta_j^b - \delta_i^b \delta_j^a \right),
\]

relevant for CFL (Color Flavor Locking) phase. In this expression \( \Delta(\vec{x}) \) is treated as a background field. In the superconducting phase \( \Delta(\vec{x}) = \Delta_{CS} \sim 100 \text{ MeV} \) is quite large and describes the energy gap in the spectrum of excitations. Outside the ball, where the system is treated as a plasma of free quarks, there is no Bose-condensation and \( \Delta(\vec{x}) = \Delta_{QF} = 0 \).

In the limit of three massless flavours of quarks the symmetry of QCD interactions is enlarged to allow axial and vector \( SU(3)_A \times U(1)_A \times SU(3)_V \times U(1)_P \) flavour rotations, in addition to color gauge transformations \( SU(3)_c \). Notice that one of the diagonal matrices of \( SU(3)_V \) is the generator of electromagnetic gauge transformations. The diquark condensate spontaneously breaks the symmetry group \( SU(3)_c \times SU(3)_V \times SU(3)_A \) into the global \( SU(3)_c \) diagonal subgroup of symmetry of lagrangian, which locks color and flavour indices. All the gauge bosons acquire masses \( \sim |\Delta| \) via the Anderson-Higgs mechanism, except for a certain linear mix of the photon and one of the gluons that remains massless. Spontaneous breaking of the global symmetries leads to formation of nine pseudogoldstone bosons (the octet of “pions”, and \( \eta^\prime \) singlet, analogous to the pseudoscalar mesons in the hadronic phase), and a single massless scalar corresponding to superfluid collective mode of the broken \( U(1)_P \). These light, spin zero fields can play an important role in transport properties, however they do not carry baryon charge and, therefore, will be ignored in what follows.

### A. Quasiparticles in the superconducting phase.

The lagrangian density \([1] \) yields to the Dirac equation:

\[
(i\partial_\mu \gamma^\mu - m + \mu \gamma^0) \psi^a_i - \Delta_{ij}^{ab} C\gamma_5 \bar{\psi}_j^b T = 0.
\]

For \( \Delta \neq 0 \) the wavefunctions of quark fields and their hermitian conjugates couple together and it is convenient to treat the hermitian conjugate of equation \([11] \) as a second independent equation:

\[
\bar{\psi}_j^b (i \partial_\mu \gamma^\mu - m - \mu \gamma^0) - (\Delta_{ij}^{ab})^* \psi_i^a T C\gamma_5 = 0.
\]

The tensor structure \([10] \) allows to decouple the set of Dirac equations in four sectors: a) Three two-quarks channels (or 2SC sectors):

\[
\begin{pmatrix}
  u_{\text{green}} \\
  d_{\text{red}} \\
  0
\end{pmatrix},
\begin{pmatrix}
  u_{\text{blue}} \\
  0 \\
  s_{\text{red}}
\end{pmatrix},
\begin{pmatrix}
  0 \\
  d_{\text{blue}} \\
  s_{\text{green}}
\end{pmatrix}
\]

b) One three-quarks channel (or CFL sector):

\[
\begin{pmatrix}
  u_{\text{red}} \\
  d_{\text{green}} \\
  s_{\text{blue}}
\end{pmatrix}.
\]

We discuss here the CFL sector, but the analysis of the three 2SC sectors is quite similar, \([36] \). In the CFL channel the set of Dirac equations can be written:

\[
\begin{align*}
(i\partial_\mu \gamma^\mu - m + \mu \gamma^0) & u_{\text{red}} - \Delta C\gamma_5 (d_{\text{green}} + s_{\text{blue}})^T = 0, \\
(i\partial_\mu \gamma^\mu - m - \mu \gamma^0) & d_{\text{green}} - \Delta C\gamma_5 (u_{\text{red}} + s_{\text{blue}})^T = 0, \\
(i\partial_\mu \gamma^\mu - m - \mu \gamma^0) & s_{\text{blue}} - \Delta C\gamma_5 (u_{\text{red}} + d_{\text{green}})^T = 0.
\end{align*}
\]

Together with the hermitian conjugate expressions

\[
\begin{align*}
\bar{u}_{\text{red}} & (i \partial_\mu \gamma^\mu - m - \mu \gamma^0) - \Delta^* (d_{\text{green}} + s_{\text{blue}})^T C\gamma_5 = 0, \\
\bar{d}_{\text{green}} & (i \partial_\mu \gamma^\mu + m - \mu \gamma^0) - \Delta^* (u_{\text{red}} + s_{\text{blue}})^T C\gamma_5 = 0, \\
\bar{s}_{\text{blue}} & (i \partial_\mu \gamma^\mu + m - \mu \gamma^0) - \Delta^* (u_{\text{red}} + d_{\text{green}})^T C\gamma_5 = 0.
\end{align*}
\]
This set of six equations can be decoupled:

\[
(i \partial_{\mu} \gamma^\mu - m + \mu \gamma^0) \chi - 2\Delta \quad C_{55} \quad \tilde{\chi}^T = 0, \]
\[
(i \partial_{\mu} \gamma^\mu + m - \mu \gamma^0) \tilde{\chi} - 2\Delta^* \quad C_{55} = 0, \]  
(17)

\[
(i \partial_{\mu} \gamma^\mu - m + \mu \gamma^0) \omega_1 + \Delta \quad C_{55} \quad \tilde{\omega}_1^T = 0, \]
\[
(\omega_1 \partial_{\mu} \gamma^\mu + m - \mu \gamma^0) + \Delta^* \quad \omega_1^T \quad C_{55} = 0, \]  
(18)

\[
(i \partial_{\mu} \gamma^\mu - m + \mu \gamma^0) \omega_2 + \Delta \quad C_{55} \quad \tilde{\omega}_2^T = 0, \]
\[
(\omega_2 \partial_{\mu} \gamma^\mu + m - \mu \gamma^0) + \Delta^* \quad \omega_2^T \quad C_{55} = 0. \]  
(19)

for the three independent combinations of fields defined as follows, \( \chi = u_{red} + d_{green} + s_{blue}, \) \( \omega_1 = u_{red} - d_{green} \) and \( \omega_2 = u_{red} - s_{blue}. \) The three sectors are formally identical except for one important aspect that was already noticed in [30]: in the sector [17] the energy gap, that is, the energy threshold at which single particles can be excited in the superconductor, is \( 2|\Delta|, \) twice the energy gap in the other two sectors [18] and [19]. We limit our analysis to a single sector [18], so we omit the subscript for \( \omega \), and introduce the usual notation \( \alpha^i = \gamma_0 \gamma^i \) to simplify the equations,

\[
(i \partial_t + i \partial_i \alpha^i - m \gamma^0 + \mu) \omega + \Delta \quad C_{55} \quad \omega^T = 0, \]
\[
\omega^T (i \partial_t + i \partial_i \alpha^i + m \gamma^0 - \mu) + \Delta^* \quad \omega \quad C_{55} = 0, \]  
(20)

In the Dirac equations we are dealing with, the fermionic fields are quantum operators. However, in what follows, we neglect many body effects and treat \( \omega \rightarrow \varphi(t, \vec{x}) \) and \( \omega^T \rightarrow \tilde{\varphi}(t, \vec{x}) \) as c-functions describing the single particle states [35].

\[
\begin{pmatrix}
\varphi(t, \vec{x}) \\
\tilde{\varphi}(t, \vec{x})
\end{pmatrix} = \left( \sum_s u_s(\vec{q}) \alpha_s(\vec{q}) \right) \left( \sum_s v_s(\vec{q}) \beta_s(\vec{q}) \right) \exp(-iEt + i\vec{q} \cdot \vec{x}),
\]
\[
(21)

where \( u_s(\vec{q}) \) and \( v_s(\vec{q}) \) are Dirac spinors which describe particles and antiparticle (or holes), respectively, and the subindex \( s = \uparrow, \downarrow \) denotes the two possible components of spin. They obey the equations

\[
(\vec{\alpha} \cdot \vec{q} + m \gamma^0 - \mu) u_s(\vec{q}) = +\kappa_{\vec{q}} u_s(\vec{q}),
\]
\[
v_s^\dagger(\vec{q}) (\vec{\alpha} \cdot \vec{q} - m \gamma^0 - \mu) = -v_s^\dagger(\vec{q}) \kappa_{\vec{q}},
\]  
(22)

where \( \kappa_{\vec{q}} = \sqrt{\vec{q}^2 + m^2} - \mu. \) The coefficients \( \alpha_s(\vec{q}), \beta_s(\vec{q}) \) are c-numbers in this approach. They obey the Bogolubov - de Gennes equations. For \( \alpha^\dagger(\vec{q}) \) and \( \beta^\dagger(\vec{q}), \)

\[
E \alpha^\dagger(\vec{q}) = +\kappa_{\vec{q}} \alpha^\dagger(\vec{q}) + \Delta \beta^\dagger(\vec{q}),
\]
\[
E \beta^\dagger(\vec{q}) = -\kappa_{\vec{q}} \beta^\dagger(\vec{q}) + \Delta^* \alpha^\dagger(\vec{q}),
\]  
(23)

and similar equations for \( \alpha^\dagger(\vec{q}) \) and \( \beta^\dagger(\vec{q}), \)

\[
E \alpha^\dagger(\vec{q}) = +\kappa_{\vec{q}} \alpha^\dagger(\vec{q}) - \Delta \beta^\dagger(\vec{q}),
\]
\[
E \beta^\dagger(\vec{q}) = -\kappa_{\vec{q}} \beta^\dagger(\vec{q}) - \Delta^* \alpha^\dagger(\vec{q}).
\]  
(24)

There are two possible solutions of the uniform equations [21] for the superconductor, characterized by \( |\Delta| \sim 100 \text{ MeV}, \)

\[
\begin{pmatrix}
\varphi(t, \vec{x}) \\
\tilde{\varphi}(t, \vec{x})
\end{pmatrix} = \left( \frac{e^{i\xi}}{\sqrt{E^2 - |\Delta|^2}} \left( Au^\dagger(\vec{q}_1) - Bu(\vec{q}_1) \right) \right) \left( e^{-iEt + i\vec{q}_1 \cdot \vec{x}} \right) + \left( \frac{e^{i\xi}}{\sqrt{E^2 - |\Delta|^2}} \left( Cu^\dagger(\vec{q}_2) + Du(\vec{q}_2) \right) \right) \left( e^{-iEt + i\vec{q}_2 \cdot \vec{x}} \right),
\]
\[
(25)

where \( \xi = \sqrt{E^2 - |\Delta|^2} \) and \( \vec{q}_{1,2} = \pm \sqrt{(\mu \pm \xi)^2 - m^2} \) and \( \delta = \arg(\Delta/|\Delta|). \) They describe single particle excitations in the superconductor.
The qualitative features of the scattering process at the interface between CS and plasma phases can be understood without explicit solving the equations. First of all, $|\Delta|$ describes a gap in the spectrum of excitations in the CS phase. This is easy to see if we take $E < |\Delta|$. In this case $q_{1,2} = \pm \sqrt{\mu^2 - m^2 - |\xi|^2 \pm 2i|\xi|\mu}$ have imaginary parts, which implies an exponential suppression in the wavefunctions. This means that a single quark with low energy cannot propagate in the superconductor because there is no any kinematically available state for such an excitation. At such low energies the quarks from outside can only penetrate into the superconductor if they organize a Cooper pair, which can be excited without surpassing the gap energy, and leaving a hole that propagates backward in the metal. This phenomenon is known in condensed matter physics as the Andreev reflection at the interface between a metal and a superconductor [22]. Andreev reflection is an important phenomenon [11] when the potential barrier at the metal-superconductor interface is very low and the penetrability of the barrier is high. Otherwise, the normal reflection of low-energy incident electrons at the interface is the dominant process, as we will show in the coming subsection.

B. Physics at the interface.

We proceed in this section to write down the general equations which describe the interface of a plasma of quarks and the color superconducting phase. First, as there is no spin flip in the scattering process, we limit ourselves by considering the case $B = D = 0$ corresponding to a single spin sector. Furthermore, for the sake of simplicity we assume that the background field $\Delta(z)$ depends only on the $z$ spatial coordinate. Then, the wavefunctions $\varphi(z)$ and $\zeta(z)$ are invariant over the perpendicular spatial plane and the scattering problem can be reduced to a problem in one dimension. We can further simplify the problem while still capturing its essential features by considering a step-function background $\Delta(z)$ to separate the interface between the phase of free quarks/antiquarks and the superconducting phase: $\Delta(z) = 0$, if $z \leq 0$ and $\Delta(z) = \Delta_0 \sim 100$ MeV, if $z > 0$. Then the set of equations can be solved separately in each of the two phases and the corresponding wavefunctions must be matched continuously at the interface $z = 0$. It must be noticed that the equations we need to solve are first order in the spatial derivative, and therefore we do not impose continuity conditions on the first derivatives [30]. This is a technical difference with respect to the treatment presented in the classical reference [11] of the interface between ordinary metal and superconductor, where electrons are described in the non-relativistic framework of Schrodinger’s quantum mechanics.

In Sector I, which describes the free quarks/antiquarks at $z < 0$ where $\Delta = 0$, the wave function (25) is a linear superposition,

$$
\begin{pmatrix}
\varphi_+(z) \\
\zeta_+(z)
\end{pmatrix}
= A \begin{pmatrix}
u(k_1) \\
0
\end{pmatrix} e^{ik_1z} + A' \begin{pmatrix}
u(-k_1) \\
0
\end{pmatrix} e^{-ik_1z} + B' \begin{pmatrix}0 \\
v^T(-k_2)
\end{pmatrix} e^{ik_2z},
$$

of the wave functions for an incident particle (first term, proportional to $A$), a reflected particle (second term, proportional to $A'$) and a reflected antiparticle (last term, proportional to $B'$). Obviously, an identical analysis can be done for the scattering of an incident antiparticle at the interface. The ratio $|A'|^2/|A|^2$ gives the probability for normal reflection at the interface; while $|B'|^2/|A|^2$ will give the probability for Andreev reflection. In the expression above $k_1 = \sqrt{(\mu_1 + E)^2 - m^2}$ and $k_2 = \sqrt{(\mu_1 - E)^2 - m^2}$, where we remind $E$ is the energy above the Fermi surface of the incoming quark, $\mu$ is its mass and $\mu_1$ is the quark chemical potential in the plasma. We have added a subindex to the quark chemical potential $\mu_1$ in the plasma phase to point out that it can be different from the chemical potential $\mu_2$ in Sector II, which describes the superconducting phase.

In Sector II at $z > 0$ the wave transmitted through the interface is a linear superposition

$$
\begin{pmatrix}
\varphi_+(z) \\
\zeta_+(z)
\end{pmatrix}
= a \begin{pmatrix}e^{+i\xi/2} \sqrt{\frac{E + \xi}{2E}} u(q_1) \\
e^{-i\xi/2} \sqrt{\frac{E - \xi}{2E}} u^T(-q_1)
\end{pmatrix} e^{iq_1z} + b \begin{pmatrix}e^{+i\xi/2} \sqrt{\frac{E - \xi}{2E}} u(q_2) \\
e^{-i\xi/2} \sqrt{\frac{E + \xi}{2E}} u^T(-q_2)
\end{pmatrix} e^{iq_2z},
$$

of a transmitted quasiparticle (first term, proportional to $a$) and a transmitted anti-quasiparticle (second term, proportional to $b$). In this expression $q_{1,2} = \pm \sqrt{(\mu_2 \pm \xi)^2 - m^2}$ with $\xi = \sqrt{E^2 - |\Delta|^2}$. In the superconducting phase the chemical potential is much larger than the current masses of the quarks, $m \ll \mu_2$, which will be neglected in what follows. At energies $E > |\Delta|$ larger than the gap in the spectrum of quasiparticles in the superconductor the ratios $|a/A|^2$ and $|b/A|^2$ give, respectively, the probability for an incident quark with energy $E$ to be transmitted as a quasiparticle or anti-quasiparticle into the superconductor. As we have explained in previous sections, at energies $E < |\Delta|$ lower than the gap in the spectrum of quasiparticles in the superconductor particles/antiparticles from the
plasma cannot propagate in the superconductor. This is manifest in the solution \([27]\) because the transmitted wave is not a propagating plane wave, but it decays exponentially.

The solutions in Sector I) and Sector II) must match continuously at the interface \(z = 0\), which demands

\[
Au(k_1) + A'u(-k_1) = ae^{i\delta/2} \sqrt{\frac{E + \xi}{2E}} u(q_1) + be^{i\delta/2} \sqrt{\frac{E - \xi}{2E}} u(q_2)
\]  

(28)

\[
Bv^\dagger(-k_2) = ae^{-i\delta/2} \sqrt{\frac{E - \xi}{2E}} v^\dagger(-q_1) + be^{-i\delta/2} \sqrt{\frac{E + \xi}{2E}} v^\dagger(-q_2)
\]  

(29)

The spinors \(u(p)\) and \(v^\dagger(p)\) are defined by equation \([22]\) and they can be explicitly solved in the form:

\[
u(p) = \left( \frac{(m + \sqrt{p^2 + m^2})^{1/2}}{(m + \sqrt{p^2 + m^2})^{1/2}} \chi \right),
\]

(30)

\[
v^\dagger(-p) = \left( \tilde{\chi}^\dagger(m - \sqrt{p^2 + m^2} + 2\mu)^{1/2}, \chi^\dagger(m - \sqrt{p^2 + m^2} + 2\mu)^{1/2} \right),
\]

(31)

in terms of

\[
\chi = \left( 1 \right), \quad \tilde{\chi}^\dagger = (1, 0).
\]

(32)

When these expressions are substituted in \([28], [29]\) we obtain the final set of four equations that fix the four parameters \(A', B', a\) and \(b\) in terms of the normalization constant \(A\):

\[
e^{-i\delta/2} (A + A') \left( m + \sqrt{k_1^2 + m^2} \right)^{1/2} = a \sqrt{\frac{E + \xi}{2E}} (q_1^2)^{1/4} + b \sqrt{\frac{E - \xi}{2E}} (q_2^2)^{1/4},
\]

(33)

\[
e^{-i\delta/2} (A - A') \left( m + \sqrt{k_1^2 + m^2} \right)^{1/2} = a \sqrt{\frac{E + \xi}{2E}} (q_1^2)^{1/4} + b \sqrt{\frac{E - \xi}{2E}} (q_2^2)^{1/4},
\]

(34)

\[
e^{i\delta/2} B' \left( m - \sqrt{k_2^2 + m^2} + 2\mu_1 \right)^{1/2} =
\]

\[
a \sqrt{\frac{E - \xi}{2E}} (2\mu_2 - \sqrt{q_1^2})^{1/2} + b \sqrt{\frac{E + \xi}{2E}} (2\mu_2 - \sqrt{q_2^2})^{1/2},
\]

(35)

\[
e^{i\delta/2} B' \left( m - \sqrt{k_2^2 + m^2} + 2\mu_1 \right)^{1/2} =
\]

\[
a \sqrt{\frac{E - \xi}{2E}} (2\mu_2 - \sqrt{q_1^2})^{1/2} + b \sqrt{\frac{E + \xi}{2E}} (2\mu_2 - \sqrt{q_2^2})^{1/2},
\]

(36)

where in the r.h.s of the equations, which describes Sector II, we have neglected the current masses of the quarks, as discussed above. These equations completely define the interaction at the interface.
C. Small energies, $E < \Delta$.

In this subsection we solve the set of equations (33–36) in the setup that appropriately describes the interface of a very dilute and cold plasma of quarks inciding with low energies $E \ll |\Delta| \ll \mu_2$ on the much dense superconducting medium of the balls. The analysis of this section is relevant for the understanding of the phenomenology of the balls and anti-ball in the cold universe, at temperatures much below $T_{\text{QCD}} \sim 160$ MeV. Although in this environment the CS balls and anti-balls coexist with a dilute gas of hadrons, rather than free quarks, the simpler description of the plasma phase as a diluted gas of quarks makes easier to understand the basic features of the interaction of real hadrons with the CS matter.

We specifically want to solve the equations (33–36) when the incident quark from the metal is non-relativistic $k_{1,2} \simeq \sqrt{2mE} \ll m$. This description corresponds to an incident massive constituent quark with mass $m \simeq 300$ MeV. In this case the first two equations can be simplified in the form

$$e^{-i\delta/2} (A + A') \sqrt{2m} + o(\frac{k_1}{m}) \simeq a \sqrt{\frac{-|\Delta|}{2E}} (q_1^2)^{1/4} + b \sqrt{\frac{-|\Delta|}{2E}} (q_2^2)^{1/4}$$

(37)

$$e^{-i\delta/2} (A - A') \frac{k_1}{\sqrt{2m}} + o\left(\frac{k_1}{m}\right)^2 \simeq a \sqrt{\frac{+|\Delta|}{2E}} (q_1^2)^{1/4} + b \sqrt{\frac{+|\Delta|}{2E}} (q_2^2)^{1/4}$$

(38)

which imply:

$$(A + A') \sqrt{2m} \simeq (A - A') \frac{k_1}{\sqrt{2m}} + o([k_1/m]^2)$$

(39)

and, therefore:

$$A = -A' + o([k_1/m]^2).$$

(40)

The coefficient $|A'/A|^2 \simeq 1$ is precisely the probability for normal reflection of an incident low energy quark at the interface with the superconducting medium. Total probability conservation then implies that the probability for Andreev reflection in the analyzed setup is largely suppressed, $|B'/A|^2 \simeq o([k_1/m]^2)$. As we have discussed in the previous section this result implies, due to time-reversal invariance, that the probability of low energy quarks annihilating at the interface of a ball of antiquarks in color superconducting phase is also largely supressed, $o([k_1/m]^2)$.

The situation we just described corresponds to the interface of color superconducting balls (or anti-balls) and a system of free constituents quarks. As we mentioned, the relevant degrees of freedom outside the ball are not really the constituent quarks, but rather, hadrons made of confined quarks. However, the arguments we presented above should convince the reader that total reflection happens at the hadron-superconductor interface due to kinematic reasons not very sensitive to the confinement phenomena. Moreover, quantum numbers of individual quarks in a hadron do not exactly match those needed to form Cooper pairs in the superconductor, which implies that Andreev reflection of the hadron would demand a suppressed coherent collaboration of many quarks. This discussion explains why we claimed in the previous section that usual baryons colliding with the ball or antball with a small energy $v/c \sim 10^{-3}$, will be mainly reflected by the ball/antiball.

Many-body effects, which were completely ignored in these calculations, are not expected to change significatively the result of complete reflection of quarks and antiquarks off balls that we have described. Indeed, the annihilation of a quark with an antiquark coupled in a Cooper pair could be successful if the energy released from the annihilation is immediately transmitted to the second quark from the Cooper pair. In this case the second quark can receive sufficient energy to overcome the gap and propagate as a quasiparticle in the superconductor. The probability of this to happen is expected to be quite small because it involves interactions between many particles and, furthermore, the total energy available from the current masses of the annihilating quark and antiquark in the dense CS phase is just above or maybe even below the gap threshold as we argued in Section II D. The precise estimate of this contribution requires the use of quantum field theory methods where the many body effects are properly taken into account. Such estimates are beyond the scope of the present work, and shall not be considered here. Indeed these details will only slightly change the phenomenology of the balls/antiballs of CS matter/antimatter, but they will not change the most profound implications of the cosmological scenario introduced here. We also do not consider in this work, for the same reason, the physical interface region which is probably a mixture of nuclear and color superconducting phases.
The analysis of scattering in this case could be a very complicated problem. However, we expect that the noticeable suppression of the rate of matter antimatter annihilation still happens when the latter is stored in compact and dense objects of CS phase due to both a reduction in the cross section of the interaction and the reduction in the total flux of particle antiparticle collision events.

### D. Large energies, \( E \gtrsim \Delta \)

In the text below we present some results on physics at the interface of superconducting and normal matter at large energies comparable to the gap. Such a study is not directly relevant for the the main purpose of this paper, which is the understanding of the phenomenology of the QCD phase transition. At present we do not have much to add to this difficult subject on the dynamics of the QCD phase transition. It is not our goal to describe the physics of the QCD phase transition in quark matter originally introduced in [9]. Large baryon number fluctuations could also occur due to the axion related physics when the domain wall network with strong CP violation decays, provided that the universe is already CP asymmetric due to weak interactions. This effect could also lead to the separation of baryon/antibaryon charges. Finally, we have observed in sub-section II C that even in a closely baryon symmetric \( B \approx 0 \), but largely \( C \) and CP asymmetric, universe at the onset of the QCD phase transition there can exist large chemical potentials \( \mu_i \) of different quark subspecies in the asymptotically free plasma (only constrained to \( \sum_i \mu_i \approx 0 \)). In this context lumps of quark matter would naturally form at the end of the phase transition in quark sectors with large positive chemical potential \( \mu_i > 0 \), while antilumps of quark antimatter would form in the sectors of quarks with large negative chemical potential \( \mu_i < 0 \).

We do not have much to add to this difficult subject on the dynamics of the QCD phase transition. It is not our goal to describe the nature of the fluctuations of baryon number which eventually may seed the formation of balls. Our remark is the following: all Sakharov’s conditions are satisfied when a fluctuations with a large baryon number in CS phase is formed, see below, and, thus, the condensed region could continue to grow in size by accumulating the quarks coming from the plasma of free quarks. Indeed, one can suggest a simple model to account for this phenomenon by considering the ball surrounded by the dense plasma of quark matter with relatively large \( \mu \). As we already mentioned, the set of equations that describe such a situation of the scattering of quarks at the interface has been solved previously in [36], in the context of the physics of neutron stars. There the transmission and reflection coefficients of quarks and holes (at high densities holes, rather than antiquarks, are the relevant quasiparticles) were calculated and it was shown that at energies \( E \gtrsim |\Delta| \) there would be a net current of baryon charge through the interface,

\[
j_z = 2\mu_2 \frac{2\sqrt{E^2 - |\Delta|^2}}{E + \sqrt{E^2 - |\Delta|^2}}.
\]

Indeed, it was shown that in these conditions incident quarks from the surrounding plasma can only be transmitted through the interface or Andreev reflected. Both processes contribute to the net transport of baryon number into the condensed phase: \( \Delta B = +1 \) when the incident quark is transmitted into the bulk of the superconducting phase and \( \Delta B = +2 \) when the incident quark is reflected into the plasma as a hole with baryon charge \( B = -1 \). Therefore, Andreev reflection (largely suppressed at energies \( E \ll |\Delta| \)) could be an important mechanism that leads to the growth of balls/anti-balls, which eventually leads to matter antimatter separation, at temperatures (energies) \( T \lesssim |\Delta| \).
IV. MECHANISM OF SEPARATION OF BARYONIC CHARGES

The main point of this work is that formation of the cosmological dark matter and baryon asymmetry are closely related phenomena and originated from the same physics during the QCD phase transition. Therefore, the mechanism of formation of the dark matter is essentially the same physical process which produces the baryon-antibaryon separation. The mechanism how a chunk of dense matter (which is identified with the dark matter) is formed during the QCD phase transition might include new particles or fields or might require a strong first order phase transition, but those are questions that shall not be addressed in detail here. We simply assume that such kind of objects made of condensed quark matter can be formed. Our goal here is to discuss some general requirements which should be satisfied to have a successful separation mechanism of the baryon charges which leaves a net positive baryon number in the ordinary hadronic phase.

A. Sakharov’s Criteria

All three Sakharov’s criteria are satisfied (in a looser sense) during the formation of the balls of CS phase, without the need to introduce any new physics beyond the standard model (except for, maybe, a solution for the strong CP problem in QCD). Indeed,

1. The diquark condensate \( \langle \psi^T C \gamma^5 \psi \rangle \neq 0 \) forms in a local spatial region, or in a sector of quarks, where the chemical potential \( \mu_i \) is large. The net quark density breaks C and CP symmetries.

2. The diquark condensate \( \langle \psi^T C \gamma^5 \psi \rangle \neq 0 \) formed in the CS phase spontaneously breaks the baryon symmetry.

3. The dynamics of the QCD phase transition may provide the non-equilibrium conditions when the balls of the condensed phase are formed, if the transition does not proceed adiabatically; furthermore, the diquark condensate \( \langle \psi^T C \gamma^5 \psi \rangle \neq 0 \) form within a local region or quark sector of large chemical potential, which violates CPT symmetry.

It has been known for a while that these three ingredients (notice in particular spontaneous, rather than explicit, breaking of the baryon symmetry) can be responsible for a mechanism of charge separation (see e.g. [3] and review paper [11], where some simple toy models were discussed to explain the phenomenon of charge separation), which do not generate a net baryon number.

Given these three ingredients, lumps of condensed quarks, as well as, antilumps of condensed antiquarks can both form in cosmologically large comoving volumes. In order to produce a separation of charges that hides a net excess of antibaryons over baryons in these balls and antiballs, and leaves an equal but positive excess of hadrons over antihadrons, one more ingredient is necessary:

4. Cosmological dynamics before or at the onset of the QCD phase transition must develop large C and CP asymmetries homogeneously over the whole comoving visible universe.

The large cosmological C, CP asymmetries are stored in large chemical potentials \( \mu_i \) in the different quark sectors (constrained to \( \sum_i \mu_i = 0 \), if the net baryon number of the universe is zero \( B = 0 \)). The development of these net quark densities can proceed before the QCD phase transition due to independent processes, see below. Obviously, in order to develop net quark densities these C and CP violating processes must proceed in out-of-equilibrium conditions.

B. Hierarchy of spatial scales

We must make some specific emphasis on the spatial correlation scales of the sources of violation of the different symmetries due to the formation of the condensate. We will discuss four fundamentally different spatial scales: the QCD scale, \( \sim T_{QCD}^{-1} \), determined by the fundamental physics involved in this scenario; the typical scale of the balls, \( \sqrt{\mathcal{B}} T_{QCD}^{-1} \sim 10^{-3} \text{ cm} \), where \( B \sim 10^{18} \) up to \( B \leq 10^{40} \) for the configurations discussed in [3, 5, 6]; the comoving Hubble horizon \( H_{QCD}^{-1} \sim 30 \text{ km} \) at the QCD phase transition; and, finally, the present Hubble horizon \( H^{-1} \ll H_{QCD}^{-1} \).

In the picture we advocate in this work, the universe carries zero total baryon number \( B = 0 \) but it is not invariant under C or CP transformations, because it has an excess of baryons in the hadronic phase (visible matter) and an excess of antibaryon number stored in the CS phase (dark matter). Such cosmological C and CP asymmetries have a very large correlation length \( \sim H^{-1} \) comparable to the size of the present horizon, which is the typical scale where it is observed a homogeneous excess of hadronically baryonic \( n_B = B/n_B - B = 10^{-10} n_B \). On the other hand, the size of the objects where baryon symmetry is spontaneously broken by the Bose-condensate is of the order of the size of the balls \( \sim \sqrt{\mathcal{B}} T_{QCD}^{-1} \). This scale, although macroscopically large in comparison with the QCD scale \( \sim T_{QCD}^{-1} \), is still very small in comparison with the comoving Hubble horizon \( H_{QCD}^{-1} \) and, therefore, much smaller than Hubble horizon at the present time \( H^{-1} \). Therefore, if the universe was globally C and CP symmetric on the Hubble scale at the onset of the cosmological QCD phase transition, equal number of balls and antiballs would be formed in the
comoving Hubble volume leaving no net baryon number in the hadronic phase. On the contrary, in presence of large and homogeneous C and CP asymmetries over cosmological scales (condition 4 in the previous subsection), the excess in the number density of anti-solitons over the number density of solitons is naturally of order one, $\tilde{n}_B - \bar{n}_B \sim n_B - \bar{n}_B$, as we assumed in our estimation of the ratio $\Omega_{DM}/\Omega_B$ in Section 2. This excess fixes in fact the number density of remnant baryons which are left in the hadronic phase, without any need of fine tuning.

What could be then the source of the cosmological C and CP asymmetries? The important observation needed here to answer this question is that cosmological C and CP asymmetries must not necessarily be produced at the same instant of the QCD phase transition but could, instead, have been generated at some earlier stages by any other independent mechanism. In particular, the source of the cosmological C asymmetry is very natural because C is largely violated by weak interactions and, therefore, there is no reason to expect that the universe would be C invariant at the onset of the QCD phase transition. The origin of the large scale CP asymmetry of the universe, on the other hand, is still unknown. It is thought it must rely on physics beyond the standard model during the very early history of the universe, because the only known CP phase in the CKM matrix seems to be unable to do the work. We remark that any new CP source in extensions of the standard model (say, supersymmetric phases, neutrino interactions,...) could be fit to generate a large cosmological CP asymmetry before the QCD phase transition in the scenario that we are introducing. Other possibility to consider in this context is that cosmological CP asymmetry may be related to the same mechanism which eventually solves the so-called strong CP problem. To this day the preferred solution to the strong CP problem is the dynamics of the hypothetical axion field. At temperature $T \sim T_{QCD}$ the strong CP phase has not yet relaxed to its ground state and thus might be of order unity, $\theta(T_{QCD}) \sim 1$, so that CP is largely violated. If this mechanism is supposed to generate an homogeneous asymmetry over the whole visible universe it is specifically required that the initial value $\theta(T_{QCD})$ is the same everywhere in the entire observed Universe. This will occur, for example, if the Universe underwent inflation during or after the Peccei-Quinn symmetry breaking at $T \sim 10^{15}$ GeV, which is the standard assumption in the axion-related physics. This explicit source of CP violation relaxes to zero $\theta(T = 0) = 0$ soon after the QCD phase transition is completed and the chiral condensate is formed. At this point the axion field is settled at its minimum. In conclusion: a wide variety of possible mechanism could drive the universe to become C and CP globally asymmetric before or during the QCD phase transition while the total baryon number remains equal to zero. Our proposal of charge asymmetric cosmological separation of phases is not committed to any of these or other possible source of CP violation.

C. Estimation of the baryon excess $n_B/n_\gamma$

We want to introduce the discussion of this section with a paragraph from the textbook The Early Universe, by E. Kolb and M. Turner [38] where the idea of baryon-antibaryon separation at the QCD scale (rather than generation of a net baryon number) is explicitly mentioned,  

"In a locally baryon symmetric universe nucleons and antinucleons remain in chemical equilibrium down to a temperature of $\sim 22$ MeV, when $n_B/n_\gamma = n_B/n_\gamma \simeq 7 \times 10^{-20}$, a number that is 9 orders of magnitude smaller than the observed value of $n_B/n_\gamma$. In order to avoid the annihilation catastrophe an unknown physical mechanism would have to operate at a temperature greater than 38 MeV, the temperature when $n_B/n_\gamma = n_B/n_\gamma \simeq 8 \times 10^{-11}$ and separate nucleons and antinucleons." [40]

If there is a mechanism of segregation of quarks and antiquarks (into hadronic and color superconducting phases) during or immediately after the QCD phase transition, and the universe is largely C and CP asymmetric already at that time, it leaves an excess of antiballs over balls of order one $\tilde{n}_B - \bar{n}_B \sim n_B - \bar{n}_B$. The same mechanism produces an excess of hadrons over anti-hadrons of order $n_B - n_\bar{B} = B(n_B - \bar{n}_B)$. This excess is preserved until today as a net remnant density of hadronic baryons once the hadron antihadron annihilation has been completed when the temperature reaches $22$ MeV, $n_B - n_\bar{B} \simeq n_B$. Therefore, the theoretical calculation of the present ratio $n_B/n_\gamma$ in the hypothetical scenario that we are discussing is reduced to the calculation of the corresponding time (temperature $T_{form}$) when the balls/antiballs complete their formation. This temperature would be determined by many factors: transmission/reflection coefficients, evolution of the balls, expansion of the universe, cooling rates, evaporation rates, maybe dynamics of the axion domain wall network, etc. All these effects are, in general, of the same order of magnitude. Therefore, a precise theoretical calculation of $T_{form}$ in this context is a very difficult task.

To reproduce the precise observational value of the baryon number in the hadronic phase

$$\eta \equiv \frac{n_B - n_\bar{B}}{n_\gamma} \simeq \frac{n_B}{n_\gamma} \sim 10^{-10},$$

within this scenario, the magnitude $T_{form}$ must be precisely

$$T_{form} \simeq 41 \text{ MeV} \quad \text{(observation)},$$

(42)
according to the relation $n_B \sim \exp \left( -\frac{m_n}{\eta T_{form}} \right)$ and the methods described long ago. It should be noticed that the ratio is very sensitive to the precise value of the temperature $T_{form}$: small variations in the value by a few MeV would change the ratio by several orders of magnitude. This should not be interpreted as a need for fine tuning. Indeed, it is only a consequence of the fact that the cosmic time scale, measured by the temperature of the universe, is very long compared with the characteristic QCD time scale.

With this point in mind, it is clear that any theoretical estimation of $T_{form}$ must be obtained with high precision if we want to confront it with the precise observational data. Unfortunately, such theoretical calculation is not feasible at this time. A very rough estimation of $T_{form}$ is possible, it can be easily obtained by noticing that the baryons/antibaryons become completely opaque for the baryon charge in both directions for incident particles with energies much lower than the gap $\Delta$. Independently, we know that the BCS type phase transition from quark gluon phase to color superconductivity takes place at temperature $T_c \approx 60 \text{ MeV}$. For the standard value of the energy gap $\Delta = 100 \text{ MeV}$, $T_c \approx 60 \text{ MeV}$. The typical energies of particles at this temperature will be also of the same order. Therefore, one should expect that when temperature drops by some factor $\sim 1/2$ or so, the number of particles with relatively high energy capable to overcome the gap barrier will be tiny. Then, most of the particles will be reflected at such temperature. We expect that at this point the balls complete their formation period, and the thermodynamical equilibrium of the balls with the environment will be settled. Therefore, the temperature of formation $T_{form}$ can be roughly estimated to lie in the interval,

$$30 \text{ MeV} \lesssim \frac{1}{2} T_c \lesssim T_{form} \lesssim T_c \approx 60 \text{ MeV},$$

which should be compared with the “observational” value. The factor $1/2$ in this estimate is, of course, a quite arbitrary numerical factor which accounts for a suppression of the density of particles with sufficient energy to surpass the energy gap $\Delta$. Even this rough and quite arbitrary estimation can only predict a value for $\eta$ within a window of several orders of magnitude.

Notwithstanding the lack of precission of our estimation, there is an essential feature that makes this scenario physically attractive: the baryon to photon ratio is fixed in this scenario by the scale $T_{form}$ at which the mechanism of separation operates, while in most other suggested scenarios the models generally lack a natural scale in the problem that fixes the ratio $\eta$.

V. DISCUSSION

We have discussed a cosmological scenario where the universe is largely C and CP asymmetric, but carries zero baryon number. Large amounts of quarks and antiquarks would be stored in very heavy chunks of matter or antimatter in the color superconducting phase that would have formed during the cosmological QCD phase transition. We argue that the process of formation can leave an excess of baryons over antibaryons in the hadronic phase.

We have studied possible phenomenological constraints on this scenario and have concluded that the scenario is not ruled out and even not tightly constrained by available data. In particular, current constraints on antimatter in the universe would not directly apply to our scenario because chunks of antimatter do not easily annihilate with the normal (hadronic) matter. This is a consequence of the well-known features of the interaction of normal matter at the interface with a superconductor and, essentially, the small volume occupied by the balls/antiballs and their extremely low number density.

The chunks of dense matter would contribute to the “nonbaryonic” dark matter of the universe, in spite of their QCD origin, because the baryon charge stored in the diquark condensate would not be available for nucleosynthesis. Therefore the baryon charge locked in the chunks of dense matter does not contribute to $\Omega_B h^2 \approx 0.02$. Direct searches of of non-topological solitons carrying net baryon number has been reported in [31]. These observational data neither can rule out the range of parameters $B \gtrsim 10^{17}$ and $M \sim B m_N$, in which the balls of condensed quarks/antiquarks would be thermodynamically stable.

The most profound consequences of the scenario are formulated in section II B and section IV C:

- We have shown in section II B that the ratio $\Omega_{DM}/\Omega_B \gtrsim 1$ can be naturally understood as a direct consequence of the underlying QCD physics, and it is related to the fact that both contributions are originated at the same instant during the QCD phase transition. As it is known this ratio is very difficult to understand if both contributions to the energy density of the universe do not have the same origin.

- In section IV C we have shown that the fundamental ratio can be naturally understood without any fine tuning parameters as a direct consequence of the underlying QCD physics. This ratio is determined by the temperature $T_{form}$ when the balls complete their formation. This temperature falls exactly into the appropriate range of values where the baryon density can assume its observed value.
Therefore, the “exotic”, dense color superconducting phase in QCD, might be a much more common state of matter in the Universe than the “normal” hadronic phase we know. In conclusion, qualitative as our arguments are, they suggest that baryogenesis can proceed at the QCD scale, and might be tightly connected with the origin of the dark matter in the Universe. The scenario proposed in this paper offers a framework wherein to carry out calculations of cosmological parameters $\Omega_{DM}/\Omega_B$ as well as $\eta \equiv n_B/n_\gamma$. Both parameters offer well-defined testable predictions, although the present lack of theoretical and experimental knowledge on the QCD phase transition makes impossible for now to lay down this predictions more precisely than we have done in this paper.

In spite of the intrinsic difficulties to test directly the ideas introduced in this paper, it would be very interesting to check how can they be probed by a laboratory type experiment in the spirit of the Program Cosmology in the Laboratory (COSLAB). Over the last few years several experiments have been done to test ideas drawn from cosmology and astrophysics (see [40] and web page [41] of the latest COSLAB meeting for further details). Ideally, the first test of the ideas introduced in this paper should be the confirmation of the features of the interaction of hadrons at the interface with a color superconducting phase discussed in section 3. We suggest that these features might be tested in the scattering of low energy positrons and electrons by a conventional superconductor in some regime which would be analogous to the cosmological environment.

This scenario with no doubt leads to important consequences for cosmology and astrophysics, which have not been explored yet. In particular, the recent detection [32] of the seismic event with epilinear (in contrast with a typical epicentral ) sources may be related to a soliton-like dark object, which could be the very dense balls. Also, the cuspy halo problem in dwarf galaxies might be related to some kind of interacting [39] or annihilating cold dark matter in the denser regions of galaxies and clusters, which could be related to the balls discussed in this work. Last, unexpected excess of photons measured by a number of collaborations may also be related to the balls. In particular, 511 KeV line emission from the galactic bulge [42, 43] could be related to the excess of positrons (electrons) surrounding the QCD anti-balls (balls). Also, the excess of 100 MeV photons could be related to the decay of goldstone modes propagating in the bulk of the balls. We expect many other consequences which have not been explored yet.

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The value of the baryon to photon ratio stated in this paragraph is, in fact, an old estimation. The value reported by the mechanism of baryon separation should operate need to be somewhat larger than 38 MeV than the temperature at which the inverse width of the interface region is much larger than any other scales of the problem.

Of course, there is no any physical jumps in chemical potentials between these two phases. However, we model the interface region (which probably includes a mixture of hadronic matter as well as nuclear matter) as a sharp

region (which probably includes a mixture of hadronic matter as well as nuclear matter) as a sharp
It is a simple exercise to estimate the number of collisions between ordinary hadrons and balls/antiballs of CS phase in a Hubble time as a function of the temperature $T \lesssim T_{\text{form}}$, as we did in Section II D for the average number densities in the Galaxy at present time, and show that annihilation events are already suppressed immediately after the balls complete their formation.