Fitting Formulae for the Effects of Binary Interactions on Lick Indices and Colours of Stellar Populations

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Received 2007 Dec. 17; accepted 2001 month day

Abstract More than about 50\% stars of galaxies are in binaries, but most stellar population studies take single star-stellar population (ss-SSP) models, which do not take binary interactions into account. In fact, the integrated peculiarities of ss-SSPs are various from those of stellar populations with binary interactions (bs-SSPs). Therefore, it is necessary to investigate the effects of binary interactions on the Lick indices and colours of populations detailedly. We show some formulae for calculating the difference between the Lick indices and colours of bs-SSPs, and those of ss-SSPs. Twenty-five Lick indices and 12 colours are studied in the work. The results can be conveniently used for estimating the effects of binary interactions on stellar population studies and for adding the effects of binary interactions into existing ss-SSP models. The results and a few procedures can be obtained on request to the authors or via \url{http://www.ynao.ac.cn/~bps/zhongmu/download.htm}.

Key words: galaxies: stellar content — galaxies: elliptical and lenticular, cD

1 INTRODUCTION

In the golden era for studying the formation and evolution of galaxies, evolutionary stellar population synthesis has been an important technique for such works, as some stellar peculiarities (e.g., stellar age and metallicity) of galaxies can be determined via this technique. Many stellar population synthesis models, e.g., [Worthey 1994] [Bruzzi (1995)] [Bressan et al. (2003)] [Vázquez & Leitherer (2005)] [Bruzual & Charlot (2003)] [Fioc & Rocca-Volmerange (1997)] [Vazdekis et al. (2003)] [Delgado et al. (2005)] and Zhang et al. (2005) were brought forward and have been widely used for stellar population studies. However, the above models except the one of Zhang et al. (2005) are single star-stellar population (ss-SSP) models that did not take the effects of binary interactions into account. This is different from the real populations of

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The real stellar populations of galaxies and star clusters consist of not only single stars, but also binary stars. Binary evolution can affect the results of stellar population synthesis significantly, especially those relating to UV bands, see e.g., Han et al. (2007). Therefore, the effects of binary evolution should be taken into account when modeling the stellar populations of galaxies and star clusters.

A few works have been tried to give some investigations about the effects of binary evolution on stellar population synthesis. For example, Zhang et al. (2005) tried to model populations via binary stars. In addition, Li & Han (2007c) built an isochrone database for quickly modeling binary star-stellar populations (bs-SSPs) and a rapid model (hereafter RPS model) for both ss-SSPs and bs-SSPs. In special, Li & Han (2007d) investigated the detailed effects of binary interactions on the results of stellar population synthesis and the results of stellar population studies. The results can help us to understand how the results obtained via ss-SSPs are different from those obtained via bs-SSPs, when taking the H$_\beta$–[MgFe] (Thomas et al. 2003) and two-colour methods. According to the results of Li & Han (2007d), when we use ss-SSP models to measure the stellar ages and metallicities of galaxies, we will obtain obviously less ages or less metallicities compared to the real values of populations, using H$_\beta$–[MgFe] and two-colour methods, respectively. However, there is no clear relation between the real metallicities and fitted (via ss-SSPs) results of populations. One please refer to Li & Han (2007d) for more details. In this case, it is difficult to get more accurate information about the stellar metallicities of galaxies via ss-SSP models, and then the chemical evolution of galaxies. Furthermore, the previous work only shows the results for H$_\beta$–[MgFe] method, when taking Lick indices for works, but some other methods and indices are also used in investigations. Thus it is necessary to investigate the effects of binary interactions on the results of stellar population synthesis obtained via various Lick indices further. The metallicity range of above bs-SSP models (Zhang et al. 2005; Li & Han 2007c) seems not wide enough (see Li et al. 2006), as it only covers the metallicity range poorer than 0.03 ($Z \leq 0.03$). If we can give the relation between the effects of binary interactions and the stellar-population parameters (age and metallicity), we will be able to understand the populations of galaxies and star clusters further, and more detailed investigations about galaxy formation and evolution will have in the future. Therefore, it is valuable to study how the effects of binary interactions on integrated peculiarities of populations change with stellar age and metallicity. We have a try in this work. As a result, a few formulae for describing the relations between the effects of binary interactions on 25 Lick indices and 12 colours, and the ages and metallicities of populations are presented.

The structure of the paper is as follows. In Sect. 2 we introduce the ss-SSP and bs-SSP models used in the paper. In Sect. 3 we show the fitting formulae for the changes of 25 Lick indices caused by binary interactions, comparing to those of ss-SSPs. In Sect. 4 we give similar investigations to 12 colours of populations. Finally, we give our discussion and conclusion in Sect. 5.

2 STELLAR POPULATION SYNTHESIS MODEL USED IN THE PAPER

The RPS model of Li & Han (2007c) is used in the investigation, because there is no more suitable model. The model calculated the integrated peculiarities (0.3 Å SEDs, Lick indices and colours) of both bs-SSPs and ss-SSPs with two widely used initial mass functions (IMFs) (Salpeter and Chabrier IMFs). Each bs-SSP contains about 50% stars that are in binaries with orbital periods less than 100 yr (the typical value of the Galaxy, see Han et al. 1995). Binary interactions such as mass transfer, mass accretion, common-envelope evolution, collisions, su-
pernova kicks, angular momentum loss mechanism, and tidal interactions are considered when evolving binaries via the rapid stellar evolution code of Hurley et al. (2002). Therefore, the RPS model is suitable for studying the effects of binary interactions on stellar population synthesis. The details about the model can be seen in the paper of Li & Han (2007c). For convenience, we take stellar populations with Salpeter IMF for our standard investigations in the work, but the results obtained via populations with Chabrier IMF are also presented.

3 FITTING FORMULAE FOR EFFECTS OF BINARY INTERACTIONS ON LICK INDICES

Lick indices are the most widely used indices in stellar population studies, because they can disentangle the well-known age–metallicity degeneracy (Worthey 1994). Making use of an age-sensitive index (e.g., Hβ) and a metallicity-sensitive index (e.g., [MgFe], see Thomas et al. 2003), the stellar age and metallicity of a population can be determined. Thus to investigate the effects of binary interactions on the Lick indices of stellar populations is important. The work of Li & Han (2007d) showed that binary interactions make the Hβ index less while some metal-line indices larger compared to those of ss-SSPs. It leads to less age estimate when we take ss-SSPs for works. However, in that work, only the results obtained via Hβ–[MgFe] method are compared to the real values of populations. Some other Lick indices, e.g., Mg2, HδA, and HγA, are also used in studies (e.g., Gallazzi et al. 2005). Therefore, it is necessary to study the effects of binary interactions on more Lick indices and give the quantitative relations between binary effects and stellar-population parameters. Here we study on 25 widely used indices and fit the relations between the changes caused by binary interactions and the stellar-population parameters (age and metallicity), via a polynomial fitting method. The results can be used to calculate the differences between 25 Lick indices, and the errors are small (typically less than 0.03 Å or mag). All Lick indices are on the Lick system (see, e.g., Worthey 1994). The changes of Lick indices caused by binary interactions can be calculated from stellar age and metallicity, by

\[ \Delta I = \sum_{i=1}^{5} (C_{i1} + C_{i2}Z + C_{i3}Z^2)t^{i-1}, \]

where \( \Delta I \) is the change of a Lick index caused by binary interactions, and \( Z \) is stellar metallicity while \( t \) is stellar age. The detailed coefficients for our standard investigation are shown in Tables 1 and 2. Those for populations with Chabrier IMF are shown in the Appendix. For clearly, in Figs. 1, 2, and 3, we compare the changes calculated by equation (1) with the original values obtained in the work. Note that we only show the fittings for 12 widely used Lick indices here, because the fittings for other indices are similar. As we see, for the indices shown, the values calculated by the above equation are consistent with those obtained directly by comparing the Lick indices of bs-SSPs and ss-SSPs, with typical errors of 0.03 Å or mag. Therefore, the fitting formulae presented can be used to calculate the differences of Lick indices of bs-SSPs and ss-SSPs, using the age and metallicity of populations. In addition, the results show that as what were shown in the paper of Li & Han (2007d), binary interactions make age-sensitive indices (e.g., Hβ, HδA, HεF, HγA, HγF) of a bs-SSP larger than that of an ss-SSP, which has the same age and metallicity as the bs-SSP, while the interactions make metallicity-sensitive indices (e.g., Mg or Fe indices) of a bs-SSP less than that of its corresponding (with the same age and metallicity) ss-SSP. The differences between Lick indices of bs-SSPs and ss-SSPs increase with age when stellar age is small (< about 2.5 Gyr), and they decrease with age for stellar ages larger than about 2.5 Gyr. As a whole, the values calculated via the fitting formulae obtained by the paper reproduce the evolution of the difference between Lick indices of bs-SSPs and ss-SSPs.
Fig. 1 Comparison of fitted and original values for the effects of binary interactions on four Lick indices. Circles, crosses, squares, and triangles are for the metallicities of $Z = 0.004, 0.01, 0.02,$ and $0.03$, respectively. Solid, dashed, dash-dotted, and dotted lines show the fittings for the above metallicities, respectively. The values of y-axes are calculated by subtracting the Lick indices of a bs-SSP from that of the ss-SSP which has the same age and metallicity as the bs-SSP. Panels a), b), c), and d) are for CN$_2$, H$eta$, Fe5015, and Mg$_{I1}$, respectively.

4 FITTING FORMULAE FOR EFFECTS OF BINARY INTERACTIONS ON COLOURS

Because colours can also be used for stellar population studies, we fitted the formulae for calculating the changes in colours of populations that result from binary interactions. One can refer to e.g., Li et al. (2007), Li & Han (2007a), Li & Han (2007b), Li & Han (2007d) for the application of colours in stellar population studies. Colours on Johnson system, those on the photometry system of Sloan Digital Sky Survey (hereafter SDSS-ugriz system), and some composite ones that consist of a Johnson magnitude and an SDSS-ugriz magnitude are studied. We only study the colours of populations with $Z \geq 0.004$, as it seems difficult to determine the stellar age and metallicity of metal-poor (e.g., $Z < 0.008$) populations via colours under the typical observational uncertainties (Li & Han (2007b)) and metallicity affect the colours of metal-poor populations stronger. Thus one should use the results shown here for more metal-poor populations carefully. Because it is impossible to give the formulae for all colours, we give some formulae for calculating the effects of binary interactions on 12 important colours, which are sensitive to stellar age or metallicity, according to the work of Li & Han (2007b).

As a result, The fitting formulae for these colours are obtained. The 12 colours are $(B - V)$, $(V - K)$, $(I - H)$, $(R - K)$, $(B - K)$, $(I - K)$, $(u - r)$, $(r - K)$, $(u - R)$, $(u - K)$, $(z - K)$,
Fig. 2  Similar to Fig. 1, but for Mg$_2$, Mg$_b$, Fe5270, and Fe5335.

Fig. 3  Similar to Fig. 1, but for H$\delta_A$, H$\gamma_A$, H$\delta_F$, and H$\gamma_F$. 
Fig. 4  Fitting for the effects of binary interactions on four colours of populations. Circles, crosses, squares, and triangles are for the values obtained directly from comparing the colours of bs-SSPs and ss-SSPs, for metallicities of 0.004, 0.01, 0.02, and 0.03, respectively. Solid lines show the fittings. The y-axis is obtained by subtracting the colour of a bs-SSP from that of an ss-SSP (with the same age and metallicity). The four panels are for \((B-V)\), \((V-K)\), \((I-H)\), and \((R-K)\), respectively.

\[ \Delta I' = \sum_{i=1}^{4} C_i t^{i-1}, \]  

where \(\Delta I'\) is the change of colours caused by binary interactions, and \(t\) is stellar age. The coefficients of the equation are shown in Table 3. Note that the results for populations with both Salpeter IMF (standard investigation) and Chabrier IMF are listed in the table. We can find that equation (2) does not include the metallicity of populations. The reason is that there is no clear trend for different metallicities. The fitting of the effects of binary interactions on 12 colours are shown in Figs. 4, 5, and 6. As we see, the fitting formulae can give average colour changes caused by binary interactions. However, because the results calculated using equation (2) have typical errors about 0.02 mag, some additional uncertainties may be brought into the results of stellar population studies.

1 Colours \((r-K)\), \((u-R)\), \((u-K)\), \((z-K)\), and \((g-J)\) are composite colours. The \(UBVRIJHK\) magnitudes are on Johnson system, and \(ugriz\) magnitudes are on SDSS-\(ugriz\) system.
Fig. 5 Similar to Fig. 4, but for ($B - K$), ($I - K$), ($u - r$), and ($r - K$).

Fig. 6 Similar to Fig. 4, but for ($u - R$), ($u - K$), ($z - K$), and ($g - J$).

5 DISCUSSION AND CONCLUSIONS

We present some formulae for conveniently computing the changes in 25 Lick indices and 12 colours that are caused by binary interactions, comparing to the results of single star-stellar populations (ss-SSPs). It is shown that the fitting formulae presented in the paper can calculate the changes in Lick indices caused by binary interactions with small errors and can estimate similar changes in colours. It is also found that binary interactions make age-sensitive Lick indices (not only Hβ, but also HδA, HδF, HγA, HγF) less, while metallicity-sensitive indices larger compared to those of ss-SSPs. This is useful to estimate the effects of binary evolution on the results of stellar population studies and to add the effects of binary interactions into ss-SSP models. Therefore, when an age-sensitive Lick index is used together with a metallicity-sensitive index to determine the ages and metallicities of populations, we will obtain less ages, especially for metal-poor populations, as the results of [Li & Han (2007d)]. However, only binary star-stellar populations (bs-SSPs) and ss-SSPs with four metallicities \((Z = 0.004, 0.01, 0.02,\) and 0.03) are used in the work. It makes the results more suitable for studying metal-rich \((Z \geq 0.004)\) populations, because the differences between integrated peculiarities of populations with various metallicities seem larger for metal-poor populations. In addition, although different formulae are presented for populations with various initial mass functions (IMFs), the changes calculated via two kinds of formulae (the formulae for populations with Salpeter and Chabrier IMFs) are similar for the same population. Thus the changes calculated by the formulae obtained using populations with Salpeter IMF or Chabrier IMF can give us some pictures for the effects of binary interactions. Furthermore, because the Monte Carlo technique used to generate the binary sample of stellar populations make the evolution of integrated peculiarities of populations unsmooth, some results, especially those for colours, may be somewhat rough. The additional uncertainties involved should be taken into account. If possible, we will give more detailed studies in the future.

Acknowledgements We thank Profs. Gang Zhao, Xu Zhou, Licai Deng, Xu Kong, Tinggui Wang, and Li Zhang for useful discussions. This work is supported by the Chinese National Science Foundation (Grant Nos. 10433030, 10521001, 2007CB815406).

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This paper was prepared with the ChJAA L\TeX macro v1.0.
Appendix A: COEFFICIENTS FOR CALCULATING THE EFFECTS OF BINARY INTERACTIONS ON 25 LICK INDICES OF POPULATIONS WITH CHABRIER IMF
Table 1  Coefficients for equation (1). The coefficients are obtained via stellar populations with Salpeter IMF and can be used for populations younger than 4 Gyr (Age < 4 Gyr).

| Index | j   | C_{1j}  | C_{2j}  | C_{3j}  | C_{4j}  | C_{5j}  |
|-------|-----|---------|---------|---------|---------|---------|
| CN_1  | 1   | 0.0061004 | 0.004131 | 0.0010174 | 0.000862 |
|       | 2   | -0.7411409 | 3.674976 | -3.450761 | 0.9481077 | -0.07850 |
|       | 3   | 30.8122734 | 119.008796 | 98.2842545 | -25.5632396 | 2.0732871 |
| CN_2  | 1   | 0.0031344 | 0.0060648 | 0.0018661 | -0.0005851 | 0.0000607 |
|       | 2   | -0.3472402 | 2.6820361 | -2.8891002 | 0.8270163 | -0.0703722 |
|       | 3   | 20.0763558 | 92.9530724 | 84.0072412 | -22.6923906 | 1.8767287 |
| Ca_{4227} | 1 | 0.0031344 | 0.0060648 | 0.0018661 | -0.0005851 | 0.0000607 |
|       | 2   | -0.3472402 | 2.6820361 | -2.8891002 | 0.8270163 | -0.0703722 |
|       | 3   | 20.0763558 | 92.9530724 | 84.0072412 | -22.6923906 | 1.8767287 |
| Ca_{4455} | 1 | 0.0103725 | -0.1526486 | -0.0164836 | 0.0124241 | -0.0013244 |
|       | 2   | 11.0894652 | 9.0004221 | 12.0126956 | 4.9108205 | -0.4569935 |
|       | 3   | -75.5186140 | -252.7173536 | 549.1707675 | -171.1008580 | 14.7310943 |
| Fe_{4383} | 1 | 0.5779073 | 1.3565672 | 0.6261188 | -0.1240817 | 0.0011505 |
|       | 2   | -50.8998137 | 134.5958812 | -94.7989188 | 23.2438808 | -1.8011864 |
|       | 3   | 1740.9893518 | -4249.3881941 | 2828.2830757 | -668.7058103 | 50.9809128 |
| Ca_{4455} | 1 | 6.5364797 | -11.0259844 | 2.2737411 | 0.1733499 | -0.0013244 |
|       | 2   | 11.0894652 | 9.0004221 | 12.0126956 | 4.9108205 | -0.4569935 |
|       | 3   | -75.5186140 | -252.7173536 | 549.1707675 | -171.1008580 | 14.7310943 |
| Fe_{4531} | 1 | 0.0047432 | 0.0186972 | -0.0197387 | 0.0040785 | -0.0002709 |
|       | 2   | 0.5779073 | 1.3565672 | 0.6261188 | -0.1240817 | 0.0011505 |
|       | 3   | -50.8998137 | 134.5958812 | -94.7989188 | 23.2438808 | -1.8011864 |
| Fe_{4668} | 1 | 0.1013725 | -0.1526486 | -0.0164836 | 0.0124241 | -0.0013244 |
|       | 2   | 11.0894652 | 9.0004221 | 12.0126956 | 4.9108205 | -0.4569935 |
|       | 3   | -75.5186140 | -252.7173536 | 549.1707675 | -171.1008580 | 14.7310943 |
| H_{\beta} | 1 | -0.0313311 | -0.0500534 | -0.0035580 | 0.0037280 | -0.0007775 |
|       | 2   | 6.5364797 | -11.0259844 | 2.2737411 | 0.1733499 | -0.0013244 |
|       | 3   | -75.5186140 | -252.7173536 | 549.1707675 | -171.1008580 | 14.7310943 |
| Fe_{5015} | 1 | -0.0313311 | -0.0500534 | -0.0035580 | 0.0037280 | -0.0007775 |
|       | 2   | 6.5364797 | -11.0259844 | 2.2737411 | 0.1733499 | -0.0013244 |
|       | 3   | -75.5186140 | -252.7173536 | 549.1707675 | -171.1008580 | 14.7310943 |
| Mg_{1}  | 1   | 0.0011669 | 0.0020035 | 0.0006800 | -0.000553 | 0.000340 |
|       | 2   | 1.0977834 | -1.6954874 | 0.4725610 | -0.0265407 | -0.0021062 |
|       | 3   | -23.8411177 | 30.5518215 | -4.574311 | -1.0811535 | 0.1935160 |
| Mg_{2}  | 1   | 0.0036972 | 0.0060648 | 0.0018661 | -0.0005851 | 0.0000607 |
|       | 2   | 2.6546194 | -4.0910074 | 1.1294327 | -0.0429887 | -0.001361 |
|       | 3   | -63.2072524 | 93.1999501 | -25.0701341 | 0.5006265 | 0.2481080 |
| Mg_{b}  | 1   | 0.1159519 | 0.0564072 | 0.0367218 | 0.0207147 | -0.0023015 |
|       | 2   | 36.6456419 | -47.5077586 | 8.8070492 | 1.2388375 | -0.2750762 |
|       | 3   | -857.5383388 | 1080.0808279 | -207.5498637 | 46.6787692 | 3.503950 |
| Fe_{5270} | 1 | 0.1443967 | -0.2658498 | 0.0907173 | -0.011382 | 0.0003331 |
|       | 2   | -8.5450001 | 12.6759249 | -7.5407039 | 1.5645100 | -0.0991500 |
|       | 3   | 306.8420600 | -637.6520302 | 403.3286320 | -89.3943857 | 6.3626512 |
### Table 1 –continued.

| Index | $j$   | $C_{ij}$      | $C_{ij}$      | $C_{ij}$      | $C_{ij}$      | $C_{ij}$      |
|-------|-------|---------------|---------------|---------------|---------------|---------------|
| Fe5335| 1     | 0.1121199     | -0.2143513    | 0.0776458     | -0.0095392    | 0.0002385     |
|       | 2     | -6.1989089    | 11.7530508    | -8.2144476    | 1.6722406     | -0.0990246    |
|       | 3     | 144.7440581   | -352.4412737  | 218.9898664   | -44.2594619   | 2.7694391     |
|       |       | -0.0167597    | -0.0094937    | -0.0069105    | 0.0022746     | -0.0001900    |
| Fe5406| 1     | 0.0643501     | -0.1189715    | 0.0352640     | -0.0029869    | 0.0000373     |
|       | 2     | -6.8927462    | 9.4915829     | -2.5896330    | 0.0686243     | 0.0256903     |
|       | 3     | 206.1312747   | -349.9572257  | 151.0604011   | 22.2154935    | 0.9194484     |
| Fe5709| 1     | -0.193549     | 0.0201941     | -0.0227794    | 0.0068166     | -0.0000599    |
|       | 2     | 5.8734339     | -8.3105593    | 3.3800221     | -0.6333537    | 0.0435585     |
|       | 3     | -145.6399054  | 234.0700497   | -130.6085667  | 28.1896236    | -2.0267932    |
| NaD   | 1     | -0.1041283    | 0.0782159     | -0.0131910    | 0.0033758     | 0.0000599     |
|       | 2     | 35.8174765    | -52.1821670   | 16.3021886    | -1.2441912    | 0.0404260     |
|       | 3     | -928.3347064  | 1387.5693179  | 49.3839423    | -0.301862     | -0.301862     |
| TiO1  | 1     | -0.0046972    | -0.031675     | 0.0086166     | -0.0031273    | 0.0000301     |
|       | 2     | 2.5376502     | -3.1526728    | 0.3308384     | 0.2054016     | 0.0316500     |
|       | 3     | -70.3829758   | 95.4015845    | -17.9727317   | -3.0711691    | 0.6438944     |
| TiO2  | 1     | -0.009825     | -0.0092090    | 0.0096106     | -0.0036755    | 0.0003656     |
|       | 2     | 3.9165850     | -4.9045789    | 0.8074595     | 0.209710      | -0.0375203    |
|       | 3     | -108.7948038  | 146.4687943   | -32.9218404   | 2.5493939     | 0.7759394     |
| HδA   | 1     | -0.3400174    | 0.7802154     | -0.3437893    | 0.0824056     | -0.0066930    |
|       | 2     | 36.1459627    | -178.0526116  | 166.3449941   | -45.458738    | 3.7573530     |
|       | 3     | -1381.3141000 | 5283.2867289  | -4334.062424  | 1125.344300   | -90.9465139   |
| HγA   | 1     | -0.7568478    | 1.6470466     | -0.6800223    | 0.1306246     | -0.0086650    |
|       | 2     | 58.8625436    | -210.0639058  | 174.8989489   | -45.3356756   | 3.6089852     |
|       | 3     | -2043.570337  | 6333.0522877  | -4883.069797  | 1207.4959657  | -93.9809592   |
| HδF   | 1     | -0.3060721    | 0.7125074     | -0.3205308    | 0.0683519     | -0.0050475    |
|       | 2     | 30.6677991    | -121.8747783  | 101.148311    | -26.5930774   | 2.1537733     |
|       | 3     | -1053.775807  | 3611.9972771  | -2723.952603  | 682.2344274   | -54.0667213   |
| HγF   | 1     | -0.4440136    | 0.9835898     | -0.4158944    | 0.0801404     | -0.0053903    |
|       | 2     | 35.2952498    | -120.5372324  | 95.1523223    | -24.3827679   | 1.9426707     |
|       | 3     | -1145.3060328 | 3586.2350776  | -2658.395890  | 655.6380015   | -51.2487311   |
Table 2  Similar to Table 1, but for stellar populations older than 4 Gyr (Age ≥ 4 Gyr).

| Index | \( j \) | \( C_{1j} \) | \( C_{2j} \) | \( C_{3j} \) | \( C_{4j} \) | \( C_{5j} \) |
|-------|-------|-------|-------|-------|-------|-------|
| CN1   | 1     | 0.0040188 | -0.0087338 | 0.0012365 | -0.0000628 | 0.0000011 |
|       | 2     | 0.0699149 | -0.5550663 | 0.1605598 | -0.0163788 | 0.0005280 |
|       | 3     | -4.2835098 | 11.3189732 | -2.8137761 | 0.2856955 | -0.0093980 |
| CN2   | 1     | 0.0031607 | -0.0068581 | 0.0009171 | -0.0000386 | 0.0000004 |
|       | 2     | 0.1358253 | -0.6142418 | 0.1616565 | -0.0161493 | 0.0005224 |
|       | 3     | -5.8469187 | 12.5102930 | -2.9058639 | 0.2889257 | -0.0095349 |
| Ca4227| 1     | 0.0031607 | -0.0068581 | 0.0009171 | -0.0000386 | 0.0000004 |
|       | 2     | 0.1358253 | -0.6142418 | 0.1616565 | -0.0161493 | 0.0005224 |
|       | 3     | -5.8469187 | 12.5102930 | -2.9058639 | 0.2889257 | -0.0095349 |
| G4300 | 1     | 0.0695695 | -0.1937149 | 0.0295201 | -0.0017795 | 0.0000416 |
|       | 2     | 1.5938861 | -9.3601744 | 2.3994053 | -0.2263281 | 0.006871 |
|       | 3     | -79.2726970 | 189.8780408 | -38.5276774 | 3.5614204 | -0.110842 |
| Fe4383| 1     | 0.1365771 | -0.419753 | 0.0710301 | -0.0055584 | 0.0001697 |
|       | 2     | -3.6369924 | 2.4500207 | 0.5456707 | -0.0466573 | -0.0003761 |
|       | 3     | 28.2611775 | -85.3355224 | 19.3765757 | -2.1609623 | 0.1062849 |
| Fe4455| 1     | 0.0214263 | -0.1055130 | 0.0195680 | -0.0014870 | 0.0000418 |
|       | 2     | -0.8767742 | -0.8527878 | 0.3649493 | -0.0354193 | 0.0009578 |
|       | 3     | 25.2479921 | -36.0000062 | 6.6317767 | -0.5259928 | 0.0185966 |
| Fe4531| 1     | 0.0179749 | -0.0676067 | 0.0083280 | -0.0002240 | -0.000019 |
|       | 2     | 0.1825335 | -2.1257283 | 0.6162486 | -0.0626674 | 0.0020383 |
|       | 3     | -18.3660159 | 52.8898164 | -13.1167084 | 1.3350018 | -0.0447127 |
| Hβ    | 1     | -0.0625565 | 0.2708823 | -0.0566269 | 0.0045862 | -0.0001317 |
|       | 2     | 2.8706663 | -5.7104589 | 0.9274182 | -0.0705995 | 0.0035488 |
|       | 3     | -40.1033500 | 83.2554372 | -18.0609224 | 1.5794001 | -0.0541752 |
| Fe5015| 1     | 0.0326274 | -0.1721448 | 0.0331789 | -0.0025354 | 0.0000699 |
|       | 2     | -1.1476034 | -1.5307235 | 0.5946861 | -0.0674897 | 0.0022440 |
|       | 3     | 7.2743924 | 22.9005565 | -3.9738686 | 0.4451390 | -0.0154651 |
| Mg1   | 1     | 0.0002583 | -0.0028680 | 0.0005454 | -0.0000406 | 0.0000011 |
|       | 2     | 0.0303274 | -0.1102308 | 0.0115624 | -0.0003957 | -0.000009 |
|       | 3     | -0.0290541 | 0.0201172 | 0.2573529 | -0.0302840 | 0.0011283 |
| Mg2   | 1     | 0.0003494 | -0.0047501 | 0.0010250 | -0.000857 | 0.0000026 |
|       | 2     | 0.0392009 | -0.1979893 | 0.0208337 | -0.000621 | -0.0000425 |
|       | 3     | 0.4638829 | -0.5123346 | 0.6866533 | -0.0971528 | 0.0030989 |
| Mg_b  | 1     | -0.0037660 | -0.0157034 | 0.0047387 | -0.0004621 | 0.0000160 |
|       | 2     | 0.9698109 | -4.0019894 | 0.5949829 | -0.0251483 | 0.000838 |
|       | 3     | -10.4139848 | 42.6783675 | -1.0978610 | -0.5308182 | 0.0325709 |
| Fe5270| 1     | 0.0135064 | -0.0676152 | 0.0131365 | -0.009717 | 0.000261 |
|       | 2     | -0.3787006 | -1.1509906 | 0.2995794 | -0.0297612 | 0.0009629 |
|       | 3     | 2.9257542 | 20.0893626 | -2.9644946 | 0.2873734 | -0.0103891 |
Table 2 –continued.

| Index | j | C_{1j} | C_{2j} | C_{3j} | C_{4j} | C_{5j} |
|-------|---|--------|--------|--------|--------|--------|
| Fe5335 | 1 | 0.0100735 | -0.0513031 | 0.0112253 | -0.0009878 | 0.0000303 |
| 2 | -0.3764122 | -1.1131799 | -0.1092792 | 0.0392520 | -0.0018016 |
| 3 | 46.0187216 | -72.8319926 | 27.5059454 | -3.2158884 | 0.1133870 |
| Fe5406 | 1 | 0.0061912 | -0.0466338 | 0.0093712 | -0.0007327 | 0.0000206 |
| 2 | 0.0934871 | -1.3125122 | 0.1939528 | -0.0108703 | 0.0001877 |
| 3 | 1.0004186 | 10.2495446 | 1.2408277 | -0.2790750 | 0.0120963 |
| Fe5709 | 1 | 0.0084714 | -0.0287635 | 0.0050787 | -0.0003500 | 0.000089 |
| 2 | -0.4407574 | 0.0360397 | 0.0939359 | -0.0153071 | 0.0006404 |
| 3 | -0.8112771 | 19.1722212 | -4.5075237 | 0.4973342 | -0.0184540 |
| Fe5782 | 1 | -0.0002944 | -0.0149229 | 0.0032162 | -0.0003063 | 0.000099 |
| 2 | 0.2419052 | -0.2990725 | -0.1171528 | 0.0246488 | -0.0010394 |
| 3 | 20.8520931 | -52.8185426 | 16.2838692 | -1.7500469 | 0.0595642 |
| NaD | 1 | -0.0022021 | -0.0380672 | 0.0959900 | -0.0008472 | 0.0000206 |
| 2 | 0.3464177 | -0.9169326 | -0.0149337 | 0.0195706 | -0.0010391 |
| 3 | 14.6152128 | -38.0493377 | 12.0162242 | -1.3802231 | 0.0519050 |
| TiO₁ | 1 | -0.0004048 | -0.0010965 | 0.0092122 | -0.0000264 | 0.0000008 |
| 2 | 0.0098709 | -0.0060943 | -0.0015886 | 0.0001050 | -0.0000058 |
| 3 | 0.4022334 | -0.0350906 | -0.0854966 | 0.0157629 | -0.0004537 |
| TiO₂ | 1 | -0.0006406 | -0.0026907 | 0.0067050 | -0.000589 | 0.0000018 |
| 2 | 0.0411593 | 0.0211796 | -0.0159178 | 0.0015739 | -0.000518 |
| 3 | -0.4009774 | -0.3431377 | 0.1546906 | -0.0097377 | 0.0003560 |
| Hδₐ | 1 | -0.1822723 | 0.4528323 | -0.0619613 | 0.0036523 | -0.0000907 |
| 2 | -9.324935 | 26.8376168 | -7.6220614 | 0.679245 | -0.018436 |
| 3 | 273.5136304 | -38.4491796 | 87.5549531 | -6.864209 | 0.1483407 |
| Hγₐ | 1 | -0.2347224 | 0.6562770 | -0.095331 | 0.0067082 | -0.0001900 |
| 2 | -8.3702686 | 24.2585985 | -7.8246936 | 0.7094328 | -0.0186995 |
| 3 | 392.7692657 | -53.1987235 | 129.8790712 | -11.2416737 | 0.278352 |
| Hδₚ | 1 | -1.0106862 | 0.2930026 | -0.0443820 | 0.0029922 | -0.000815 |
| 2 | -3.5060331 | 10.7215430 | -3.5595302 | 0.3279227 | -0.0089577 |
| 3 | 145.3867861 | -16.5243753 | 44.0186086 | -3.7498376 | 0.0877458 |
| Hγₚ | 1 | -1.284349 | 0.3643152 | -0.0583081 | 0.0041664 | -0.0001196 |
| 2 | -1.3760493 | 7.0593899 | -2.8690465 | 0.2678504 | -0.0067945 |
| 3 | 129.2354201 | -186.3215690 | 49.9193880 | -4.3279933 | 0.0997893 |
Table 3  Coefficients for equation (2). $UBVRIJHKLMN$ magnitudes are on Johnson system, and $ugriz$ magnitudes are on SDSS-$ugriz$ system.

| IMF    | Salpeter |                          | Chabrier |                          |
|--------|----------|--------------------------|----------|--------------------------|
|        | Age $< 4.2$ Gyr |                          |          |                          |
| Colour | $C_1$  | $C_2$  | $C_3$  | $C_4$  | $C_1$  | $C_2$  | $C_3$  | $C_4$  |
| (B-V)  | -0.014222 | -0.032764 | 0.009111 | -0.000722 | -0.020059 | -0.021895 | 0.005569 | -0.000449 |
| (V-K)  | -0.080134 | -0.093961 | 0.038424 | -0.004241 | -0.086556 | -0.062010 | 0.021278 | -0.001931 |
| (I-H)  | -0.047703 | -0.049909 | 0.021701 | -0.002461 | -0.049641 | -0.033034 | 0.011729 | -0.001032 |
| (R-K)  | -0.043288 | -0.037439 | 0.016264 | -0.001812 | -0.042418 | -0.025391 | 0.008674 | -0.000701 |
| (B-K)  | -0.094328 | -0.126572 | 0.047424 | -0.004947 | -0.106087 | -0.084725 | 0.027181 | -0.002419 |
| (I-K)  | -0.054306 | -0.055596 | 0.024500 | -0.002787 | -0.055634 | -0.036844 | 0.012924 | -0.001109 |
| (u-r)  | -0.056601 | -0.042940 | 0.012347 | -0.001002 | -0.063465 | -0.023075 | 0.005139 | -0.000410 |
| (r-K)  | -0.072748 | -0.083108 | 0.035437 | -0.004014 | -0.077569 | -0.054339 | 0.019333 | -0.001782 |
| (u-R)  | -0.059219 | -0.047988 | 0.014264 | -0.001213 | -0.066913 | -0.026108 | 0.006208 | -0.000525 |
| (u-K)  | -0.129146 | -0.126222 | 0.047806 | -0.005016 | -0.140906 | -0.077425 | 0.024514 | -0.002201 |
| (z-K)  | -0.043288 | -0.037439 | 0.016264 | -0.001812 | -0.042418 | -0.025391 | 0.008674 | -0.000701 |
| (g-J)  | -0.065198 | -0.092019 | 0.035014 | -0.003704 | -0.074778 | -0.061406 | 0.020278 | -0.001863 |

| IMF    | Salpeter |                          | Chabrier |                          |
|--------|----------|--------------------------|----------|--------------------------|
|        | Age $\geq 4.2$ Gyr |                          |          |                          |
| Colour | $C_1$  | $C_2$  | $C_3$  | $C_4$  | $C_1$  | $C_2$  | $C_3$  | $C_4$  |
| (B-V)  | -0.069795 | 0.008838 | -0.000767 | 0.000025 | -0.062030 | 0.005088 | -0.000384 | 0.000014 |
| (V-K)  | -0.157537 | 0.014833 | -0.001013 | 0.000025 | -0.191178 | 0.028283 | -0.002893 | 0.000109 |
| (I-H)  | -0.087919 | 0.010592 | -0.000813 | 0.000020 | -0.113589 | 0.020964 | -0.002196 | 0.000076 |
| (R-K)  | -0.078362 | 0.010635 | -0.000850 | 0.000022 | -0.099112 | 0.019317 | -0.002018 | 0.000069 |
| (B-K)  | -0.222114 | 0.020910 | -0.001424 | 0.000037 | -0.245483 | 0.029974 | -0.002870 | 0.000102 |
| (I-K)  | -0.100169 | 0.012567 | -0.000978 | 0.000024 | -0.130611 | 0.024976 | -0.002622 | 0.000090 |
| (u-r)  | -0.119987 | 0.009885 | -0.001017 | 0.000042 | -0.098610 | 0.005663 | -0.000163 | 0.000019 |
| (r-K)  | -0.135641 | 0.013834 | -0.001022 | 0.000026 | -0.169588 | 0.027336 | -0.002877 | 0.000102 |
| (u-R)  | -0.126930 | 0.010203 | -0.001026 | 0.000042 | -0.106134 | 0.001228 | -0.000236 | 0.000022 |
| (u-K)  | -0.255286 | 0.027148 | -0.001696 | 0.000052 | -0.264095 | 0.024796 | -0.002561 | 0.000101 |
| (z-K)  | -0.078362 | 0.010635 | -0.000850 | 0.000022 | -0.099112 | 0.019317 | -0.002018 | 0.000069 |
| (g-J)  | -0.154783 | 0.014291 | -0.000930 | 0.000024 | -0.171114 | 0.020427 | -0.001897 | 0.000067 |
Table A.1 Coefficients for equation (1). The coefficients are obtained via stellar populations with Chabrier IMF and can be used for populations younger than 3.5 Gyr (Age < 3.5 Gyr).

| Index | j  | C_{1j} | C_{2j} | C_{3j} | C_{4j} | C_{5j} |
|-------|----|--------|--------|--------|--------|--------|
| CN    | 1  | 0.0022919 | 0.0011051 | -0.0063997 | 0.0015981 | -0.0001080 |
|       | 2  | 0.6329579 | -0.9595824 | 0.2427491 | -0.0011151 | -0.0025634 |
|       | 3  | -4.5940253 | 7.9284009 | -3.9250539 | 0.6480545 | -0.0256989 |
| CN    | 1  | 0.0012774 | 0.0032128 | -0.0069053 | 0.0016921 | -0.0001156 |
|       | 2  | 0.6334017 | -1.0656913 | 0.3400438 | -0.0309074 | -0.0000476 |
|       | 3  | -4.4675288 | 9.5105511 | -5.7521116 | 1.2233685 | -0.0747440 |
| Ca4227| 1  | 0.7660019 | -1.5947618 | 0.6186025 | -0.0971560 | 0.0052551 |
|       | 2  | 0.6334017 | -1.0656913 | 0.3400438 | -0.0309074 | -0.0000476 |
|       | 3  | -4.4675288 | 9.5105511 | -5.7521116 | 1.2233685 | -0.0747440 |
| G4300 | 1  | 0.0608788 | -0.9690479 | 0.3009590 | -0.0329757 | 0.0009352 |
|       | 2  | -72.695453 | 111.7196061 | -45.0101690 | 6.0330694 | -0.2331824 |
|       | 3  | 2100.1581312 | -3361.5113576 | 1335.715550 | -174.8630448 | 6.5371492 |
| Fe4383| 1  | 0.2564424 | -0.5328445 | 0.229634 | -0.0364862 | 0.0020371 |
|       | 2  | -7.3288591 | 13.2396365 | -7.0194851 | 1.2883776 | -0.0767611 |
|       | 3  | 258.9163092 | -510.1718798 | 260.9018384 | -46.2914894 | 2.6884347 |
| Ca4455| 1  | 0.127539 | -0.2791766 | 0.1024538 | -0.0161675 | 0.0009372 |
|       | 2  | -5.2389930 | 7.6979073 | -2.3867283 | 0.1846930 | -0.0012464 |
|       | 3  | 169.0223930 | -536.0764913 | 339.6448905 | -65.7521400 | 4.1760430 |
| Hβ    | 1  | -0.4293708 | 0.8546661 | -0.3155113 | 0.0432326 | -0.0020233 |
|       | 2  | 30.1096288 | -14.0583758 | 16.8580314 | -1.883154 | 0.0479103 |
|       | 3  | -767.8393985 | 1229.316335 | -430.0651511 | 45.5750506 | -0.8795314 |
| Fe5015| 1  | 0.2395560 | -0.3782478 | 0.1010824 | -0.011658 | 0.0005056 |
|       | 2  | -3.3348359 | 34.6665677 | 26.0424965 | -5.8877139 | 0.4132806 |
|       | 3  | 413.3734390 | 45.5824955 | -217.1527754 | 66.7394428 | -5.4107732 |
| Mg1   | 1  | 0.0057639 | -0.0122970 | 0.0048864 | -0.0007787 | 0.0000435 |
|       | 2  | -0.3355850 | 0.6283524 | -0.2850889 | 0.0319693 | -0.0005680 |
|       | 3  | 13.7239823 | -27.6095038 | 10.7662888 | -1.0488923 | 0.000013 |
| Mg2   | 1  | 0.0123326 | -0.0261803 | 0.011134 | -0.0018290 | 0.0001038 |
|       | 2  | -0.7115473 | 1.0153443 | -0.3612812 | 0.0205355 | 0.0001630 |
|       | 3  | 27.2273532 | -44.1981147 | 13.4974485 | -0.5084821 | 0.0847616 |
| Mg_b  | 1  | 0.1259541 | -0.2256908 | 0.0880777 | -0.0124308 | 0.0005828 |
|       | 2  | -8.2670312 | 5.7969056 | 0.6635323 | -0.9033180 | 0.1008251 |
|       | 3  | 311.7946049 | -348.4131106 | 36.5555001 | 20.8760850 | -2.8507546 |
| Fe5270| 1  | 0.1256957 | -0.3633561 | 0.1689542 | -0.0304730 | 0.0018686 |
|       | 2  | -16.8516642 | 33.9221551 | -18.3927017 | 3.5096217 | -0.2189850 |
|       | 3  | 587.4969012 | -1243.1886864 | 678.5522522 | -131.0104857 | 8.3026556 |
Table A.1 –continued.

| Index | j    | C1j  | C2j  | C3j  | C4j  | C5j  |
|-------|------|------|------|------|------|------|
| Fe5335 1 | 0.2076133 | -0.4791791 | 0.2222679 | -0.0377229 | 0.0021281 |
|       2 | -32.4556974 | 65.1973802 | -31.8891247 | 5.4057781 | -0.2980594 |
|       3 | 935.2591701 | -1881.6098576 | 872.5249261 | -142.2695305 | 7.5855211 |
| Fe5406 1 | 0.0953714 | -0.2161430 | 0.0970755 | -0.0171741 | 0.0010419 |
|       2 | -3.9285567 | 6.4393293 | -3.6907976 | 0.6355834 | -0.0343046 |
|       3 | 174.845638 | -345.4890330 | 172.7005100 | -28.2035654 | 1.4789489 |
| Fe5709 1 | 0.0665533 | -0.1333817 | 0.0460766 | -0.0056987 | 0.0002133 |
|       2 | -8.4184254 | 11.6155577 | -2.8870447 | 0.0168183 | 0.0277514 |
|       3 | 229.4380018 | -346.7249858 | 109.9625205 | -7.5674339 | -0.2639790 |
| Fe5782 1 | 0.0742796 | -0.1553226 | 0.0682675 | -0.0111777 | 0.0006135 |
|       2 | -12.3652716 | 23.1281644 | -10.711540 | 1.7517677 | -0.0935373 |
|       3 | 379.6596477 | -665.9564600 | 268.9228630 | -38.448161 | 1.7703055 |
| NaD 1 | 0.0714688 | -0.1651977 | 0.0664805 | -0.0103341 | 0.0005679 |
|       2 | -1.5605587 | -4.9385375 | 5.3394247 | 0.1304109 | 0.0010630 |
|       3 | 148.3896060 | -24.0899505 | -130.0698652 | 49.4638805 | -4.4865514 |
| TiO1 1 | -0.0115706 | 0.0112891 | -0.0010864 | -0.0006067 | 0.0000840 |
|       2 | 2.0705973 | -2.9503555 | 0.8324854 | -0.0401368 | -0.0003084 |
|       3 | -44.5370053 | 67.5680594 | -20.7546877 | 1.4636658 | 0.0469295 |
| TiO2 1 | -0.0125496 | 0.0097243 | -0.0001968 | -0.0008541 | 0.0001063 |
|       2 | 2.4479466 | -3.6429429 | 1.1610628 | -0.0929928 | -0.014161 |
|       3 | -49.2619296 | 79.6645168 | -27.329096 | 2.6274999 | -0.0183345 |
| HδA 1 | -0.1701616 | 0.1253346 | 0.2496936 | -0.0688490 | 0.0048300 |
|       2 | -22.721592 | 34.6318625 | -16.939841 | 2.8138566 | -0.141402 |
|       3 | 35.0832106 | -330.3012402 | 581.7572135 | -167.8682957 | 12.8850332 |
| HγA 1 | 180.2526812 | -282.6477080 | 118.3329534 | -17.2376118 | 0.7715612 |
|       2 | -5010.462342 | 7614.1515475 | -2881.3416065 | 351.0737254 | -11.0363129 |
|       3 | 53.441919 | -86.753473 | 36.9363160 | -5.49198 | 0.2510974 |
| HδF 1 | -0.1251870 | 0.1470139 | 0.1086295 | -0.0343830 | 0.0025515 |
|       2 | -17.3098016 | 30.7053467 | -17.0765539 | 3.2519080 | -0.1978084 |
|       3 | 137.358335 | -435.852796 | 470.4948769 | -124.2942197 | 9.3212486 |
| HγF 1 | -0.6254725 | 1.1213832 | -0.3612095 | 0.0484349 | -0.0022475 |
|       2 | 53.441919 | -86.753473 | 36.9363160 | -5.49198 | 0.2510974 |
|       3 | -1532.477541 | 2361.4227142 | -884.2807841 | 102.2525869 | -2.6516141 |
### Table A.2

Similar to Table A.1, but for stellar populations older than 3.5 Gyr (Age $\geq 3.5$ Gyr).

| Index | $j$ | $C_{1j}$ | $C_{2j}$ | $C_{3j}$ | $C_{4j}$ | $C_{5j}$ |
|-------|-----|---------|---------|---------|---------|---------|
| CN$_1$ | 1   | $0.0050070$ | $-0.0101123$ | $0.0016866$ | $-0.0001101$ | $0.0000026$ |
|       | 2   | $-0.7387795$ | $-0.0265996$ | $0.0467357$ | $-0.0067710$ | $0.0002559$ |
|       | 3   | $1.8158227$ | $8.1027523$ | $-2.3010584$ | $0.2598589$ | $-0.0027777$ |
| CN$_2$ | 1   | $0.0048267$ | $-0.0086639$ | $0.0014493$ | $-0.0000926$ | $0.0000021$ |
|       | 2   | $0.0253654$ | $-0.0268996$ | $0.0467357$ | $-0.0063150$ | $0.0002435$ |
|       | 3   | $1.8158227$ | $8.1027523$ | $-2.3010584$ | $0.2598589$ | $-0.0027777$ |
| Ca4227 | 1   | $0.0048267$ | $-0.0086639$ | $0.0014493$ | $-0.0000926$ | $0.0000021$ |
|       | 2   | $-0.6713808$ | $1.5319737$ | $-0.4799447$ | $0.0598457$ | $-0.0022187$ |
|       | 3   | $2.4347829$ | $7.4817292$ | $-1.9820797$ | $0.2264804$ | $-0.0083295$ |
| Ca4455 | 1   | $0.0048267$ | $-0.0086639$ | $0.0014493$ | $-0.0000926$ | $0.0000021$ |
|       | 2   | $-6.6685854$ | $1.4457768$ | $-0.0535448$ | $-0.0104817$ | $0.0006266$ |
|       | 3   | $114.6093932$ | $-5.2478791$ | $-4.3355245$ | $0.8004669$ | $-0.008992$ |
| Fe4383 | 1   | $0.0048267$ | $-0.0086639$ | $0.0014493$ | $-0.0000926$ | $0.0000021$ |
|       | 2   | $-12.6431474$ | $5.9250743$ | $-0.6822331$ | $0.0345448$ | $-0.007249$ |
|       | 3   | $256.6237954$ | $70.1814681$ | $-29.9674944$ | $4.616691$ | $-0.195432$ |
| Fe4455 | 1   | $0.0048267$ | $-0.0086639$ | $0.0014493$ | $-0.0000926$ | $0.0000021$ |
|       | 2   | $-6.6685854$ | $1.4457768$ | $-0.0535448$ | $-0.0104817$ | $0.0006266$ |
|       | 3   | $114.6093932$ | $-5.2478791$ | $-4.3355245$ | $0.8004669$ | $-0.008992$ |
| H$_\beta$ | 1   | $0.0048267$ | $-0.0086639$ | $0.0014493$ | $-0.0000926$ | $0.0000021$ |
|       | 2   | $5.2527398$ | $-5.1845420$ | $0.5595370$ | $-0.022041$ | $0.0000535$ |
|       | 3   | $-6.8462900$ | $-4.4966224$ | $9.4442175$ | $-1.335820$ | $0.0000483$ |
| Fe5015 | 1   | $0.0048267$ | $-0.0086639$ | $0.0014493$ | $-0.0000926$ | $0.0000021$ |
|       | 2   | $-16.5223730$ | $0.8351887$ | $0.0980695$ | $-0.0229461$ | $0.0009453$ |
|       | 3   | $369.7815953$ | $-72.0823837$ | $-1.2940653$ | $1.1387862$ | $-0.054915$ |
| Mg$_1$ | 1   | $0.0048267$ | $-0.0086639$ | $0.0014493$ | $-0.0000926$ | $0.0000021$ |
|       | 2   | $0.0681456$ | $-0.2239214$ | $0.0461259$ | $-0.0037007$ | $0.0001017$ |
|       | 3   | $-1.7505218$ | $4.5445049$ | $-1.1536134$ | $0.1112986$ | $-0.0034731$ |
| Mg$_2$ | 1   | $0.0048267$ | $-0.0086639$ | $0.0014493$ | $-0.0000926$ | $0.0000021$ |
|       | 2   | $-0.3140984$ | $-0.1102010$ | $0.0232351$ | $-0.0016550$ | $0.0000414$ |
|       | 3   | $3.8491416$ | $2.2473119$ | $-0.6944256$ | $0.0739731$ | $-0.0024430$ |
| Mg$_b$ | 1   | $0.0048267$ | $-0.0086639$ | $0.0014493$ | $-0.0000926$ | $0.0000021$ |
|       | 2   | $-8.0187674$ | $0.1621416$ | $0.0561989$ | $-0.0091935$ | $0.0004596$ |
|       | 3   | $158.6306124$ | $-15.8632521$ | $0.7547564$ | $0.1990004$ | $-0.0142351$ |
| Fe5270 | 1   | $0.0048267$ | $-0.0086639$ | $0.0014493$ | $-0.0000926$ | $0.0000021$ |
|       | 2   | $-6.2888460$ | $0.8351887$ | $0.0980695$ | $-0.0229461$ | $0.0009453$ |
|       | 3   | $106.4686559$ | $11.8581491$ | $-9.6007405$ | $1.2154258$ | $-0.055243$ |
Table A.2 –continued.

| Index | \( j \) | \( C_{1j} \) | \( C_{2j} \) | \( C_{3j} \) | \( C_{4j} \) | \( C_{5j} \) |
|-------|--------|-------------|-------------|-------------|-------------|-------------|
| Fe5353 | 1 | -0.0299036 | -0.0168634 | 0.0034824 | -0.0003504 | 0.0000128 |
|       | 2 | -0.8764299 | -2.3631112 | 0.3216860 | -0.0034947 | -0.0004859 |
|       | 3 | -19.1581245 | 20.3079448 | 1.5735482 | -0.7362636 | 0.0365434 |
| Fe5406 | 1 | -0.0273466 | -0.0191557 | 0.0032597 | -0.0002321 | 0.0000069 |
|       | 2 | -1.6859288 | -1.4089294 | 0.3578634 | -0.0309966 | 0.0008859 |
|       | 3 | 3.2750120 | 49.7275784 | -12.4265243 | 1.1493382 | -0.0351427 |
| Fe5709 | 1 | 0.0058300 | -0.0239914 | 0.0038273 | -0.0002377 | 0.0000055 |
|       | 2 | -3.1693551 | 1.3275974 | -0.1282929 | 0.0015248 | -0.0005652 |
|       | 3 | 62.3172407 | -5.6986762 | -1.2147039 | 0.3089610 | -0.0142722 |
| NaD | 1 | -0.0351759 | 0.0072524 | -0.0009035 | -0.0000295 | 0.0000040 |
|       | 2 | 2.9123930 | -2.1632509 | 0.2392269 | 0.0009764 | -0.0005629 |
|       | 3 | -87.2972968 | 23.4730212 | 0.8426133 | -0.6095760 | 0.0321487 |
| TiO1 | 1 | -0.0019580 | -0.0001078 | 0.0001071 | -0.0000133 | 0.0000005 |
|       | 2 | 0.1861284 | -0.0836849 | 0.0091382 | -0.0007762 | 0.0000260 |
|       | 3 | -4.2466215 | 3.0328182 | 0.2498990 | -0.0856260 | -0.0005149 |
| TiO2 | 1 | -0.0055418 | 0.0002662 | 0.0008389 | -0.0000138 | 0.0000006 |
|       | 2 | 0.1861284 | -0.0836849 | 0.0091382 | -0.0007762 | 0.0000260 |
|       | 3 | -4.2466215 | 3.0328182 | 0.2498990 | -0.0856260 | -0.0005149 |
| Hδ\( A \) | 1 | -0.4834854 | 0.6747627 | -0.1136504 | 0.0081802 | -0.0002208 |
|       | 2 | 48.2162849 | -8.6259083 | -0.5680772 | 0.1330894 | -0.0069865 |
|       | 3 | -391.8235480 | -44.3395429 | 41.4350353 | -5.4239881 | 0.1828454 |
| Hγ\( A \) | 1 | -0.5340026 | 0.8499248 | -0.1407715 | 0.0102053 | -0.0002837 |
|       | 2 | 57.5218630 | -12.4432398 | -1.0323349 | 0.2184516 | -0.0069865 |
|       | 3 | -381.7703663 | -237.374815 | 109.5206537 | -12.9300524 | 0.4258356 |
| Hδ\( F \) | 1 | -0.2328216 | 0.3790464 | -0.0645870 | 0.0047630 | -0.0001318 |
|       | 2 | 28.4618861 | -8.4343118 | 0.2882612 | 0.0224536 | -0.0007588 |
|       | 3 | -256.8427724 | 41.7473144 | 12.0094480 | -2.0067178 | 0.0654486 |
| Hγ\( F \) | 1 | -0.2081433 | 0.4185126 | -0.070583 | 0.0052155 | -0.0001461 |
|       | 2 | 26.6970903 | -8.4741360 | 0.0074237 | 0.0631139 | -0.020772 |
|       | 3 | -176.6172339 | -82.1781696 | 45.6384532 | -5.4871418 | 0.1779060 |