Production of quarkonia and doubly heavy baryons in pp-collisions with duality approach

V.V. Kiselev∗, A.A. Novoselov†, and E.R. Tagiev‡

Institute for High Energy Physics NRC “Kurchatov Institute”, 142281, Protvino, Russia and
Moscow Institute of Physics and Technology, 141701, Dolgoprodny, Russia

In present work we discuss the production of heavy quarkonia and diquarks in pp-interactions. The aim is to take into account the production of bound states of quarks originating from independent parton scatterings. This production is regulated by constraints on the invariant mass of constituents. Such an approach leads to larger value of the diquark production cross section than traditional consideration in single parton scattering. This production mechanism of doubly heavy baryons can sooner be verified by modern experiments (e.g. LHCb). We also discuss related contributions to double and associated quarkonium production.

I. INTRODUCTION

The production of heavy quarks and their bound states in high energy hadron interactions is a perfect instrument for thorough studies of QCD. As heavy quark mass is substantially larger than the Λ_{QCD} scale the production of heavy quark-antiquark pair is described by the perturbative QCD, while its hadronization allows rigorous theoretical description due to the non relativistic nature of considered bound states.

The constituent quark model predicts the existence of the baryon multiplet, which includes baryons with more than one heavy quark. Currently only lightest of them Ξ_{cc} = ccd is claimed to be observed by the SELEX collaboration [1, 2]. However the cross section reported exceeds substantially the theoretical estimates [3]. This result has not been confirmed by latter experimental searches [4–6]. Most recent experimental search in the Ξ_{cc} \to Λ_{c}K^{-}π^{+} decay mode was performed by the LHCb collaboration [7].

Recent results on c-hadrons and quarkonium production unambiguously suggest that

∗Valery.Kiselev@ihep.ru
†Aleksey.Novoselov@ihep.ru
‡Emin.Tagiev@ihep.ru
double parton scattering (DPS) contributes greatly to multiple production of hadrons with heavy flavours. One of the first experimental indications was obtained in 2012 in the analysis of associated production of $J/\psi$ and open charm hadrons. The analysis of $\Upsilon$ and open charm clearly claimed DPS to be the main source of signal.

The natural question arises, whether DPS can contribute to the production of doubly heavy baryons. If yes, the hadronisation has to involve heavy quarks produced in different parton subprocesses. It brings us to idea of the quark-hadron duality usage. Moreover, such an approach has already been applied for the $e^+e^-$ case. However direct application of this approach leads to overestimation of related cross sections (such as $pp \to J/\psi + c\bar{c} + X$). Thus we perform a calibration consisting in matching the calculations in duality approach and calculations which are guided by the quarkonia matrix elements (ME-based). The latter are known from the non-relativistic potential models (for all the considered states) and from the decay widths (for quarkonia states).

In the following section we consider associated production of $J/\psi$ and open charm. Third section is devoted to the production of diquarks. Discussion of the results obtained is given in the last section.

II. $J/\psi$ PLUS OPEN CHARM PRODUCTION

Let us begin with calculation of $gg \to J/\psi + c\bar{c}$, $gg \to \psi(2S) + c\bar{c}$, and $gg \to \chi_c + c\bar{c}$ processes. In the leading order in $\alpha_S$ there are 36 Feynman diagrams for the $gg \to c\bar{c}c\bar{c}$ process.

To generate amplitudes of these diagrams we use the FeynArts package. Let $p$ be the quarkonium momentum and $q$ — the relative quark momentum, then heavy quark and antiquark momenta can be denoted by

$$p_1 = p/2 + q \quad \text{and} \quad p_2 = p/2 - q,$$

respectively. Following we use the following spin projector operators for the considered spin-triplet charmonium states:

$$\Pi = \sum_{\lambda_1, \lambda_2} \frac{-1}{2\sqrt{E+m}} \bar{v}(p_2, \lambda_2)\epsilon[^{\lambda_2}_\mu] \gamma^\mu \frac{\not{p} + 2E}{2E} u(p_1, \lambda_1),$$
where $m$ and $E$ are quark mass and energy, $\lambda_{1,2}$ are quark helicities. For the $S$-wave $J/\psi$ and $\psi(2S)$ states $p_1 = p_2$ and the spin projector operator simplifies as follows:

$$\Pi_S = \frac{-1}{2\sqrt{2}}(\hat{p} - 2m)\hat{\epsilon},$$

(3)

where we also substituted $m$ for $E$ due to the non relativistic treatment. Here $\epsilon$ is the spin polarization vector of the quarkonium, $\epsilon \cdot \epsilon^* = -1$ and $\epsilon \cdot p = 0$. For the $P$-wave $\chi_c$ states we have:

$$\Pi_P = \frac{-1}{8\sqrt{2}m^2}(\hat{p}/2 - \hat{q} - m)(\hat{p} - 2m)\hat{\epsilon}(\hat{p}/2 + \hat{q} + m),$$

(4)

Concerning the color part of projectors, we consider only color singlet (CS) production which is expressed by $\delta_{ij}/\sqrt{N_c} = \delta_{ij}/\sqrt{3}$ color projector operator.

For the numerical calculation of hadronic cross section we use CT14lo PDFs at the factorization scale $\mu = \sqrt{M^2 + 2m^2 + p_T^2}$, where the quarkonium mass $M$ is taken equal $2m$. Leading order expression for the strong coupling $\alpha_S(\mu)$ is used at the same scale. Numerical values of these quantities are obtained from the LHAPDF package [14].

We use $m = m_c = 1.5$ GeV as the heavy quark mass and the following values of SC long distance matrix elements:

$$\langle O(1^{3}S^{[1]}_1) \rangle = \frac{|R_{J/\psi}(0)|^2}{4\pi} = 0.0447 \text{ GeV}^3,$$

$$\langle O(2^{3}S^{[1]}_1) \rangle = \frac{|R_{\psi(2S)}(0)|^2}{4\pi} = 0.0269 \text{ GeV}^3,$$

$$\langle O(1^{3}P^{[1]}_J) \rangle = (2J + 1) \frac{3|R'_{\chi_c}(0)|^2}{4\pi} = (2J + 1) \cdot 0.0179 \text{ GeV}^3,$$

(5)

Here $|R(0)|^2$ and $|R'(0)|^2$ values are obtained from known decay widths of quarkonia.

For the $P$-wave states production we use HELAC-onia [15, 16] at the parton level. Then we weight the produced events with our PDFs and $\alpha_S(\mu)$ to get the hadronic cross sections. As HELAC-onia does not support LHAPDF6 there is no way to use it with the CT14 PDFs directly. We have checked that results for $gg \rightarrow J/\psi + c\bar{c}$, $gg \rightarrow J/\psi + J/\psi$ obtained with the HELAC-onia and our code are in agreement. For the feed-down calculation we use branching fractions from [17]. The resultant cross sections are presented in Table II.

Next we calculate the charmonium production in the duality approach. The partonic subprocess is $gg \rightarrow c\bar{c}c\bar{c}$ again, but we do not apply any spin projectors. Only CS $c\bar{c}$ pairs are selected. We take $m_c = 1.4$ GeV [3, 18] and the duality region $2m_c < M_{inv} < 2m_D + \Delta$ with $m_D = 1.86$ GeV and $\Delta = 0.5$ GeV [3]. Here $m_c$ differs from the ME-based calculation above.
Final state | $\sqrt{s_{pp}} = 7$ TeV | $\sqrt{s_{pp}} = 13$ TeV  
| No cuts | $2 < y < 4.5$ | No cuts | $2 < y < 4.5$  
| $J/\psi + J/\psi$, direct | 13.4 nb | 2.1 nb | 23.4 nb | 3.7 nb  
| $J/\psi + c\bar{c}$, direct | 0.55 $\mu$b | 74 nb | 0.95 $\mu$b | 0.13 $\mu$b  
| $2S + c\bar{c}$ | 0.33 $\mu$b | 45 nb | 0.57 $\mu$b | 79 nb  
| $\chi_{c0} + c\bar{c}$ | 0.18 $\mu$b | 24 nb | 0.31 $\mu$b | 44 nb  
| $\chi_{c1} + c\bar{c}$ | 0.19 $\mu$b | 26 nb | 0.33 $\mu$b | 46 nb  
| $\chi_{c2} + c\bar{c}$ | 0.36 $\mu$b | 49 nb | 0.63 $\mu$b | 87 nb  
| $J/\psi + c\bar{c}$ with feed-down | 0.87 $\mu$b | 0.12 $\mu$b | 1.53 $\mu$b | 0.21 $\mu$b  

**TABLE I.** Hadronic cross sections for the subprocesses with charmonia production. ME-based calculation. The rapidity restriction is applied to the charmonium and one of the $c$-quarks.

| Final state | $\sqrt{s_{pp}} = 7$ TeV | $\sqrt{s_{pp}} = 13$ TeV  
| $c\bar{c}c\bar{c}$ | 21 $\mu$b | 36 $\mu$b  
| $(c\bar{c})^{[1]}_{dual} + c + \bar{c}$ | 2.1 $\mu$b | 3.7 $\mu$b  

**TABLE II.** Hadronic cross sections for the duality-based approach with the SPS source ($gg \rightarrow c\bar{c}c\bar{c}$ subprocess). $(c\bar{c})^{[1]}_{dual}$ is in the CS color state and has $M_{inv}$ in the duality region.

to be consistent with other duality-based calculations. The cross sections are summarized in Table II.

Comparison with the cross sections from Table II leads to the conclusion that approximately 41% of the CS quark-antiquark pairs in the duality region transit into $J/\psi$. If we account only direct $J/\psi$ production this fraction is about 26%.

Let us now turn to the DPS production source. We have two $gg \rightarrow c\bar{c}$ partonic subprocesses. According to the phenomenological expression for the DPS cross section,

$$\sigma(pp \rightarrow 2 \times c\bar{c} + X) = \frac{1}{2} \frac{\sigma(pp \rightarrow c\bar{c} + X)^2}{\sigma_{eff.}}.$$  \hspace{1cm} (6)

The most recent measurement of $\sigma_{eff.}$ value was performed by LHCb in analysis of the $\Upsilon$ plus open charm production \cite{9}, $\sigma_{eff.} = 18$ mb. This leads to the following $pp \rightarrow 2 \times c\bar{c} + X$
The rapidity restriction is applied to the charmonium and one of the $c$-quarks.

cross sections

$$\sigma(pp \rightarrow 2 \times c\bar{c} + X, 7 \text{ TeV}) = 0.51 \text{ mb},$$

$$\sigma(pp \rightarrow 2 \times c\bar{c} + X, 13 \text{ TeV}) = 1.2 \text{ mb},$$

(7)

if the $gg \rightarrow c\bar{c}$ subprocesses is calculated in LO with $m_c = 1.4 \text{ GeV}$. We would like to notice that these DPS cross sections grow more rapidly with the $\sqrt{s_{pp}}$ increase. It is the consequence of the assumption that $\sigma_{\text{eff.}}$ depends weakly on the $\sqrt{s_{pp}}$.

From the generated DPS events we take $c$ and $\bar{c}$ quarks from the different $c\bar{c}$ pairs. We select events with $c\bar{c}$ invariant mass in the duality region and apply additional $1/9$ factor to the cross section to account only CS production. We also apply 0.41 suppression factor which we earlier found to correspond the $J/\psi$ production with feed-down from the other charmonium states. The resultant cross sections are presented in Table III.

The cross section of the $pp \rightarrow J/\psi J/\psi + X$ process obtained with our DPS plus duality approach is close to those measured by LHCb. However there should be contributions from the $gg \rightarrow J/\psi J/\psi$ subprocess and from the ordinary DPS. First one is presented in Table II. The latter is known from the measured prompt $J/\psi$ production. Currently the sum of these three contributions exceeds the measured value. Let us discuss possible reasons more in details.

The ordinal DPS cross section for double $J/\psi$ is represented by a simple expression:

$$\sigma(pp \rightarrow 2 \times J/\psi + X) = \frac{1}{2} \frac{\sigma(pp \rightarrow J/\psi + X)^2}{\sigma_{\text{eff.}}}. \quad (8)$$

As in the expression for $\sigma(pp \rightarrow 2 \times c\bar{c} + X)$ (6) we take $\sigma_{\text{eff.}}$ measured by LHCb in the associated $\Upsilon$ and open charm production [9], $\sigma_{\text{eff.}} = 18 \text{ mb}$. The extraction of $\sigma_{\text{eff.}}$ relies on

| Final state | $\sqrt{s_{pp}} = 7 \text{ TeV}$ | $\sqrt{s_{pp}} = 13 \text{ TeV}$ |
|-------------|-------------------------------|-------------------------------|
| No cuts     | $2 < y < 4.5$                 | No cuts                       |
| $c\bar{c}c\bar{c}$ | 0.51 mb                      | 1.2 mb                        |
| $J/\psi + c\bar{c}$ | 6.4 $\mu$b                    | 0.9 $\mu$b                    |
| $J/\psi + J/\psi$ | 62 nb                         | 7 nb                          |

TABLE III. Hadronic cross sections for the duality-based approach with the DPS source of $c\bar{c}$-pairs.
the assumption that all signal originates from the DPS contribution. The measured kinematic distributions support this assumption. This value of $\sigma_{\text{eff}}$ exceeds measured in other processes at same $\sqrt{s_{pp}}$. It also supports the assumption that no other sources contribute significantly to the final state considered. Taking measured cross sections of prompt $J/\psi$ production \cite{19, 20} one gets the cross sections for double $J/\psi$ presented in the first row of Table IV.

What concerns the SPS contribution, we considered the $gg \rightarrow J/\psi J/\psi$ subprocess at the LO in $\alpha_S$. There can be feed-down from the $\psi(2S)$ and $\chi_c$ states. Feed-down from the $\psi(2S)$ was considered in our earlier works \cite{21, 22} and is about $1/3$ (see Table IV). Feed-down from $J/\psi + \chi_c$ final state was also considered. In \cite{23} it was shown that it is surprisingly small and has the order of picobarns. The LO $gg \rightarrow J/\psi J/\psi$ cross section in \cite{23} differs significantly from those in Table IV because of the use of NLO CT10 PDFs for both NLO and LO calculations. Corrections due to the real gluon emission were also studied in \cite{24}. We adhere to an opinion that the LO contributions dominate for the total cross section while $\alpha_S$-corrections are crucial at high $p_T$. In earlier works we used CTEQ5L and CTEQ6L1 LO PDFs. Currently we switched to the most modern CTEQ+TEA LO set – CT14llo. These PDF updates gradually decreased the cross section prediction. The fiducial region of the LHCb detector corresponds to the $y \in [2, 4.5]$ region. The $x = x_1 x_2$ value is of order $10^{-6}$. Thus we are sensitive to the PDF values at $x_1 \approx 10^{-4}$. The refining of PDFs from CTEQ5L to CT14llo leads to factor of 2 decrease in SPS $pp \rightarrow 2 \times J/\psi + X$ cross section prediction. There are studies of gluon saturation effects in this $x$ region \cite{25}, which suggest possible further cross section decrease.
The contribution of the duality approach production from the DPS source depends on the maximum number of assumptions. It is again sensitive to the \( \sigma_{\text{eff}} \) parameter of DPS estimation. More uncertainty comes from the selection of duality region including selection of the \( m_c \). We ensure that selected duality region together with the suppression factor, which corresponds to the \( J/\psi \) production, reproduce the cross section of \( gg \to J/\psi c\bar{c} \) partonic subprocess. However there can be influence from different distribution over the \((c\bar{c})\) invariant mass in DPS and in \( gg \to c\bar{c}c\bar{c} \) subprocess. The argument in favour of such an approach is that \( J/\psi \) production in this case is correlated with the production of open charm hadrons even with DPS source. The experimental measurement \([8]\) of associated production of \( J/\psi \) and open charm hadrons indeed demonstrates that the \( p_T \) distribution of \( J/\psi \) differs from the prompt \( J/\psi \) distribution. It can be the sequence of the kinematic cut on the minimum \( p_T \) of the charm hadron produced and the correlated influence on the \( J/\psi p_T \)-spectrum. It is not the case for the \( \Upsilon \) and open charm associated production \([9]\). Cross sections in both measurements can only be interpreted if DPS source is involved.

The actual problems and results on double \( J/\psi \) production have been recently addressed in \([26]\).

III. \((cc)\)-DIQUARK PRODUCTION

As we mentioned in the introduction the \((cc)\)-diquark production studies are necessary to estimate yield of double heavy baryons in the LHC conditions. The customary consideration \([3]\) identify heavy diquark as a static color source which is surrounded by the light quark. Indeed as the mass of the charm quark is substantially larger than the \( \Lambda_{QCD} \) scale, two charm quarks form a compact diquark system, which is considerably smaller than the radius of the light quark confinement.

Contrary to the quarkonium production diquark formation should be followed by hadronization to obtain an observable state. The recombination and fragmentation scenarios can occur. It was shown that consideration of fragmentation diagrams only is not sufficient and recombination should dominate at low \( p_T \) \([3]\). Currently there is no known way to account the probability for diquark to dissolve, which can be larger than for the quarkonium states due to the non-CS configuration. Thus the diquark production cross sections estimations presented in current section are the upper estimates for the doubly heavy baryon.
We consider production of the lightest doubly heavy baryon $\Xi_{cc}^+$. As it has two identical $c$-quarks Pauli principle restricts the diquark state to spin-triplet. For the spin projector operator we use same expression as for the $S$-wave quarkonia \(^{[3]}\). The color projector of the $3c$ diquark state is $\epsilon_{ijk}/\sqrt{2}$ with $i,j$ being color indices of quarks, $k$ — color index of diquark.

As for the $gg \rightarrow J/\psi c\bar{c}$ subprocess we start with amplitudes of the $gg \rightarrow c\bar{c}cc\bar{c}$ diagrams. We use $m = m_c = 1.5$ GeV, LO running $\alpha_S$ and CT14llo PDFs at the scale $\mu = \sqrt{M^2 + 2m^2 + p_T^2}$. Mass $M$ of the $1S$ diquark is 3.16 GeV \(^{[3]}\) and is equal to the $c$-quark mass doubled at the same level of accuracy as for the $J/\psi$, $\psi (2S)$ charmonia states. The radial wave function value in origin is taken equal $R(0) = 0.53$ GeV\(^3\) \(^{[3]}\), so

$$\langle \mathcal{O}(1^3S_1^{[3]}) \rangle = 0.022 \text{ GeV}^3.$$  \hfill (9)

The hadronic cross section results are summarized in the first row of Table V.

Next we calculate the $(cc)^{[3]}$ production in the duality approach. Subprocess $gg \rightarrow c\bar{c}cc\bar{c}$ with only $3c$ color projector operators is considered. The $1/2$ factor in amplitude is inserted to take the identity of quarks or antiquarks into account. The $(cc)^{[3]}$-state invariant mass is restricted to the duality region. As for quarkonia we take $m_c = 1.4$ GeV and the duality region $2m_c < M_{inv.} < 2m_D + \Delta$ with $m_D = 1.86$ GeV and $\Delta = 0.5$ GeV \(^{[10]}\). The cross sections are presented in the second row of Table V. The fraction of $3c$-states in the duality region that correspond to the $1^S (cc)^{[3]}_3$ is found equal 0.26.

Finally we turn to the DPS source of $c$ or $\bar{c}$-pairs. The cross sections for this source of considered final state were written down in \(^{[7]}\). From the generated DPS events we select those with invariant mass of $(cc)$ or $(\bar{c}\bar{c})$ being in the duality range. Additional $1/3$ factor accounts formation of the antitriplet states only. We also apply suppression by factor of 0.26 found above. The resultant cross sections are presented in Table VI. They are by

| Final state          | 7 TeV | 13 TeV |
|----------------------|-------|--------|
| $(cc)^{[3]}_{1S} + \bar{c} + \bar{c}$ | 0.4 $\mu$b | 0.06 $\mu$b |
| $(cc)^{[3]}_{dual} + \bar{c} + \bar{c}$ | 1.5 $\mu$b | 0.2 $\mu$b |

TABLE V. Hadronic cross sections for the $(cc)^{[3]}$ production. $(cc)^{[3]}_{dual}$ is in the antitriplet color state and has $M_{inv}$ in the duality region. The rapidity restriction is applied to the diquark only.
### Table VI

| Final state | 7 TeV | 13 TeV |
|-------------|-------|--------|
| No cuts | $2 < y < 4.5$ | No cuts | $2 < y < 4.5$ |
| $(cc)^1_3S + \bar{c} + \bar{c}$ | 3 $\mu$b | 0.5 $\mu$b | 6.6 $\mu$b | 1 $\mu$b |

The table shows the hadronic cross sections for the duality-based approach with the DPS source of $(cc)$-pairs. The rapidity restriction is applied to the diquark only.

approximately order of magnitude larger than in the ME-based approach.

## IV. DISCUSSION AND CONCLUSION

This report is devoted to phenomenological analysis of the doubly heavy baryon production in $pp$-interaction. As it is stated in [27], this process is closely connected to the associated $J/\psi$ and open charm production. If SPS processes were the dominant source of the $J/\psi + c\bar{c}$ signal the comparison with experiment could eliminate uncertainties connected with $\alpha_S(\mu)$, PDFs and $m_c$ selection. However recent observations claim that DPS processes dominate in the production of many final states, which include several heavy hadrons. This is also the case for the $J/\psi + c\bar{c}$ production. Currently the SPS and DPS contributions to this final state production can not be separated. The work is more active for the $J/\psi + J/\psi$ case, where kinematic correlations provide better basis for separation.

Thus a dramatic difference between $J/\psi + c\bar{c}$ and $(cc) + \bar{c}\bar{c}$ production processes arises. Due to the quark composition only first final state cross section can be estimated with the customary DPS treatment. Succeeding with the interpretation of measured cross section, this treatment fails to account for the difference in $p_T$-spectra of associated and prompt $J/\psi$-mesons. Interesting to mention that there is no such discrepancy in production of $\Upsilon$ plus open charm. We suppose that the reason for the discrepancy in the $J/\psi + c\bar{c}$ case is the existence of kinematic cut on the transverse momentum of the open charm hadron. The $p_T$-spectrum of $J/\psi$ is affected by it if there are correlations between production of the $J/\psi$-meson and the accompanying $c\bar{c}$-pair. Such an interinfluence can arise from correlations between momenta of initial partons, which are not taken into account in the simple DPS model. In such a case it should be observable through the alteration of the $p_T$-spectrum of $\Upsilon$-mesons with application of more rough $p_T$-cut on the open charm hadrons in $\Upsilon$ and $c\bar{c}$ associated production. Another possibility is the involvement of quarks produced in different
partonic subprocesses to the quarkonia formation. The $\Upsilon + c\bar{c}$ production is not influenced in this scenario.

In our analysis we consider second scenario more in details. Apart from the influence on the $J/\psi + c\bar{c}$ production features it can give rise to the doubly heavy baryon cross section. For the bound state formation from quarks originating from different parton subprocesses one can not control the quantum numbers of quark pairs. We suggest using the duality approach for this case. Apart from requirement for the invariant mass to be in the duality region one needs to apply some suppression factors to the cross section to correspond with the formation of specific states. The correspondence is achieved by matching results obtained in the duality approach with those obtained in the standard ME-based formalism. It can be done in SPS production case as in it both approaches describe the single phenomenon. Then we apply duality-based calculation to the DPS source. The resultant cross section predictions for diquark production are about order of magnitude larger than those obtained in the ME-based calculation.

All calculations of heavy diquark production cross section are to be considered as upper estimates for the doubly heavy baryon production. The interaction between diquark and gluons is not suppressed in contrary to the CS $c\bar{c}$ pairs, where the quarkonium dissociation supposes an exchange by two hard gluons with the quark-gluon sea. The gluon virtuality is to be of same order or greater than the inverse size of quarkonium. In principle one can imagine that study of this phenomenon is possible by measuring suppression of doubly heavy baryon yield in the low-$p_T$ range with respect to the quarkonium plus open flavour yield but in practice the uncertainties involved ruin this opportunity. Indeed better understanding of parton shover in proton is needed to obtain more rigorous predictions in the DPS formalism.

We discussed uncertainties brought by the duality usage by the example of $J/\psi$ pair production. Apart from the uncertainties connected with $\alpha_S(\mu)$, PDFs and $m_c$ values one faces with the dependence on the duality region selection. Even with introduction of the suppression factor which provides matching with the ME-based calculation there is dependence on the invariant mass distribution shape, which is different for SPS and DPS production sources.

Despite mentioned difficulties we consider the mechanism discussed as quite feasible. DPS is also known to give increase to the cross sections of many processes with multiple heavy quark production. The cross sections predicted give hope the $\Xi_{cc}^+$-baryons will soon
be observed by the LHC experiments. Additional insights will be provided by updated measurements of double and associated quarkonia production.

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