Extension of the maximum flow-covering location and service start time problem to allow flexible consumption

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Abstract
This paper extends the maximum flow-covering location and service start time problem (MFCLSTP) by additionally imposing a minimum stay at a facility before the service can be used. The original formulation determines the locations of facilities and the start times of fixed-duration services so as to maximally cover flows of commuters who access services on the way home from work. In MFCLSTP, commuters must stay at a facility from the start of the service until the end of the service in order to consume it. Examples of such services include movies, lectures, and baseball games. There are, however, many on-demand services, which can be consumed by commuters who access the providing facility for a fixed continuous duration of \( a \) hours during any of \( c \) open hours of the facility. To deal with this situation, we extend the definition of coverage in the original model and provide an integer programming formulation of the proposed problem. The model is applied to the Tokyo metropolitan railway network, using census data of commuter traffic for railway users in this area. Exact optimal solutions are obtained by using a mathematical optimization solver with both the original and extended problems, and the characteristics of the solutions are compared. The results show that the optimal solutions for the extended problem allow a much larger flow than the optimal solutions for the original problem do, even when \( a \) is very close to \( c \). The locations selected for the optimal solutions of the proposed problem are seen to be much more spatially dispersed than those chosen for the original problem.

Key words: Facility location problem, Dynamic location model, Service start time, Flow covering, Railway network

1. Introduction

Facility location decisions arise in a wide variety of public- and private-sector problems, and researchers have developed many types of facility location models that can assist decision makers. Typical examples are classical location models such as the \( p \)-median model (Hakimi, 1964) and the maximal covering location model (Church and ReVelle, 1974). The former seeks to locate \( p \) facilities among the set of candidate sites so as to minimize the demand-weighted total distance for all demand–facility pairs, while the latter maximizes the number of demands that can be met by a facility within a pre-specified distance. See Daskin (2013) for more details about various types of location models developed thus far.

One of the distinctive characteristics of locational decisions is that they are often long-term and strategic in nature. Thus, the environment (e.g., demands, travel times, and land acquisition costs) may change considerably during the planning horizon. Focusing on this aspect, researchers have attempted to incorporate a temporal dimension into static and deterministic facility location models. For instance, models have been formulated that consider the timing of the location and relocation of facilities on the basis of temporal variation in future demands (Schilling, 1980; Gunawardane, 1982; Drezner and Wesolowsky, 1991). Review articles such as Current et al. (1998), Owen and Daskin (1998), and Snyder (2006) discuss dynamic location models in more detail.
Many of the dynamic location models regard time as a long-term planning horizon. However, another possible approach to incorporating dynamic aspects into location models is to focus on hourly variations in levels of demand. The demand for services is strongly dependent on the spatial and temporal nature of the activities of people’s daily lives. Hence, the provision of services at facilities with spatiotemporal dimensions is an important topic to explore (Tanaka, 2011; Tanaka and Furuta, 2013; Tong et. al, 2012). Thus far, however, this approach has not been widely used in existing dynamic location problems. In this paper, we focus on a model that takes this approach, the maximum flow-covering location and service start time problem (hereinafter, MFCLSTP; Tanaka, 2011), and present an extension of this model.

MFCLSTP assumes commuter flows on the way back home from work as potential sources of demand for services. Each flow element is specified not only by an origin–destination pair of nodes, but also by the departure time from the origin node, thereby incorporating temporal variations in travel demand. Each facility provides a service for a fixed number of hours, and services are required to be consumed from start to end. Examples of such services include after-work lectures at graduate schools, movies at movie theaters, and sports games at stadiums. A commuter flow is said to be \textit{covered} if it can arrive at the facility by the start time of the service, stay there until the end time, and then return home by a specified time. Under these assumptions, the model seeks to find the optimal locations of $p$ facilities, as well as the optimal time to start each service at each facility, with the aim of maximizing the number of covered flows.

This paper proposes an extension of the original MFCLSTP allowing more flexible consumption of a service, requiring a minimum duration of stay to consume a service at a facility rather than a fixed duration of stay. The original MFCLSTP assumes that commuters can consume service when they fully enjoy that service from start to end. However, there are many types of services that may be consumed more freely. These include services that allow people to stop at a facility for a fixed length of time during hours when the facility is open. Examples of such services include fitness centers, restaurants, and other such places. Assuming such services, we present a model that incorporates flexible consumption of services by extending the coverage definition from that given in the original formulation. We call the original and proposed models the \textit{rigid consumption model} and \textit{flexible consumption model}, respectively.

In addition to the presentation of the new flexible MFCLSTP, this paper focuses on the following two aspects, which were not treated in Tanaka (2011). First, this paper uses a more realistic distribution of departure times than that assumed in the earlier paper. There, it was assumed that the distribution of departure times from origin stations follows an ideal uniform distribution over a given period. The present paper constructs the distribution from the results of a large-scale questionnaire conducted during the 2005 Japanese census, using 2010 census data for volumes. Second, this paper analyzes exact optimal solutions, in contrast with the heuristically obtained solutions developed in the previous paper.

The remainder of this paper is organized as follows. Section 2 reviews the original MFCLSTP (Tanaka, 2011) and shows that by modifying the coverage index, flexible consumption of services can be successfully incorporated into the model. Section 3 describes the Tokyo metropolitan railway network and the network flow data that will be used for the numerical examples given in later sections. In Section 4, we investigate covered flows at two fixed stations in the railway network. Then, in Section 5, we compare characteristics of optimal solutions for the rigid consumption and the flexible consumption model. Finally, in Section 6, we conclude the paper and discuss the direction of future research.

2. Model and formulation

In this section, we introduce the original version of MFCLSTP, where consumption of services is rigid, and provide an integer programming formulation of the problem. Then, the model is extended to the case where staying a duration of $a$ hours during the $c$ open hours of a facility is sufficient to consume the service.

2.1. Rigid consumption MFCLSTP

Figure 1 illustrates the situation assumed for MFCLSTP. The vertical axis represents a temporal dimension for the network. A decision maker seeks to determine the placement of $p$ identical facilities that provide a service for a fixed number of hours $c$, as shown by the bold line segments ($p = 4$ case) in Fig. 1. The objective of the model is to allow as many commuters as possible to access a service provided at any facility by optimally determining the location of $p$ facilities and the service start times.

To consume the service provided at a facility, commuters must stay at the facility from start to end. The model can be applied to various situations for planning a service in which the determination of service start times is important, such as concerts, lectures, and baseball games. To formulate MFCLSTP as an integer programming problem, we introduce several items of notation. We denote by $N$ a set of nodes, which act as both the origins and the destinations of trips, and by
a set of times of departure from origin nodes. To represent services at facilities, we let $K$ be a set of candidate locations and $S$ be a set of possible start times for services. We assume that $T$ and $S$ are both finite.

In MFCLSTP, it is assumed that a service is provided to flows of after-work commuters, where each flow is identified by an ordered triple $(i, j, t)$ of an origin node $i \in N$, a destination node $j \in N$, and a time of departure from the origin node, $t \in T$. The volume of each flow, that is, the number of commuters for flow $(i, j, t)$, is given by $f_{i,j,t}$. The service at each facility is denoted by the pair $(k, s)$ of a location $k \in K$ and a service start time $s \in S$. A commuter flow is defined to be covered when commuters can access at least one of the $p$ facilities no later than $s$, stay there until $s + c$ to consume the service for $c$ hours at the facility, and then go back home by a given time $t_{h}$ (here, the subscript “h” means home). The former condition can be written as $t + u_{ik} \leq s$ while the latter can be written as $s + c + u_{kj} \leq t_{h}$, where $u_{ij}$ is the travel time between node $i$ and node $j$. To formulate MFCLSTP, $a_{i,j,t}^{k,s}$ is given by

$$a_{i,j,t}^{k,s} = \begin{cases} 1 & \text{if flow } (i, j, t) \text{ can be covered by a service } (k, s), \\ 0 & \text{otherwise}, \end{cases}$$

(1)

where $i \in N$, $j \in N$, $t \in T$, $k \in K$, and $s \in S$.

Two types of binary variables are introduced.

$$x_{k,s} = \begin{cases} 1 & \text{if a facility is located at node } k \in K \text{ and starts its service at time } s \in S, \\ 0 & \text{otherwise}, \end{cases}$$

(2)

$$y_{i,j,t} = \begin{cases} 1 & \text{if commuter flow } (i, j, t) \text{ is covered}, \\ 0 & \text{otherwise}. \end{cases}$$

(3)

Tanaka (2011) considered two different situations in MFCLSTP. The first situation assumes that the service start time of each facility can be independently determined, while the second situation assumes that all facilities have a common start time for services. In Tanaka (2011), the former model is called the I-model (independent start time model), while the latter C-model (common start time model). Since the I-model is more flexible than the C-model, the number of covered flows for the optimal values in the I-model is larger than (or at least equal to) that in the C-model. On the other hand, the cost to manage facilities in the C-model may be lower than in the I-model because it allows the same service hours for all facilities.

Using the above definitions, the I-model version of MFCLSTP can be formulated as follows.
MFCLSTP (I-model)

maximize \[ \sum_{i \in N} \sum_{j \in N} \sum_{t \in T} f_{ij} y_{ijt} \] \hspace{1cm} (4)

subject to \[ \sum_{k \in K} \sum_{s \in S} x_{ks} = p, \] \hspace{1cm} (5)

\[ y_{ijt} \leq \sum_{k \in K} \sum_{s \in S} a_{ks} x_{ks}, \hspace{0.5cm} \forall i \in N, \forall j \in N, \forall t \in T, \] \hspace{1cm} (6)

\[ x_{ks} \in \{0, 1\}, \hspace{0.5cm} \forall k \in K, \forall s \in S, \] \hspace{1cm} (7)

\[ y_{ijt} \in \{0, 1\}, \hspace{0.5cm} \forall i \in N, \forall j \in N, \forall t \in T. \] \hspace{1cm} (8)

The objective function (4) gives the total number of covered flows, that is, the number of commuters that can access at least one of the \( p \) facilities. Constraint (5) stipulates that \( p \) facility services are provided. It should be noted that this constraint does not exclude the case where multiple facilities are located at the same node; that is, two or more co-located facilities with different start times may cover different flows. Constraints (6) require that at least one facility service, \((k, s)\), be accessible from commuter flow \((i, j, t)\) for the flow to be covered. Finally, constraints (7) and (8) are standard binary constraints on the decision variables.

The C-model, where all facilities start service at the same time, can be similarly formulated by introducing additional constraints into the I-model (Tanaka, 2011). This paper focuses on the I-model only and compare the characteristics of the original and the proposed model, which is introduced in the following.

2.2. Allowing a minimum required stay at a facility

In the original MFCLSTP, it is assumed that commuters must stay at a facility from start to end during service-providing in order to consume the service, as shown in Fig. 2 (a). However, there are many services that can be consumed by people who stop at a facility for a fixed length of time, \( a \), during any of the \( c \) hours of operation of the facility, as illustrated in Fig. 2 (b). Here, of course, \( a \leq c \). Fitness centers, restaurants, and other such facilities are examples.

As shown in Fig. 2 (b), we assume that flow \((i, j, t)\) is covered if and only if the flow can stop at a facility for \( a \) hours at any time during the \( c \) open hours that the facility is open to consume the service. As a result, there are many possible patterns to access service at the facility for \( a \) hours, which is much more flexible than cases where the entire time that the facility is open must be used.

Fig. 2 Two types of services

To formulate the above problem as an integer programming problem, we introduce a coverage index \( b_{ks}^{ijt} \) as follows:

\[ b_{ks}^{ijt} = \begin{cases} 1 & \text{if flow } (i, j, t) \text{ can stop at a facility for } a \text{ hours during the } c \text{ open hours at a facility that is located at node } k \text{ and begins service at } s, \text{ and then can get to a destination node by } t_h, \\ 0 & \text{otherwise}. \end{cases} \] \hspace{1cm} (9)

The problem is formulated similarly to the rigid-consumption problem by substituting \( b_{ks}^{ijt} \) in the original formulation of MFCLSTP with \( b_{ks}^{ijt} \).
3. Data description

We construct a railway network for which 2010 census data are available about commuter traffic in the Tokyo metropolitan area. The network is shown in Fig. 3. In the figure, Tokyo station is at the origin and each unit represents 1 km. The network has 1,969 stations along 133 lines. In the census data, the 1,969 stations are counted as distinct stations for the same stop on different lines; for example, both the JR Yamanote and the Chuo line stop at Tokyo. In the census data, these two Tokyo stations are treated separately. However, for the purpose of the present analysis, stations with the same name and stations that are very close to each other are aggregated into a single station for each set. The number of stations after aggregation is 1,469.

To compute the coverage index (9) that is used in the formulation, we first calculate the inter-station travel-time matrix. To this end, we set the cost of each link in the network in the following way. There are two types of links: physical links and transfer links. Physical links connect adjacent stations along the same railway line. The cost of each physical link is set as the average time for a train to move between the two adjacent stations in the railway network, regardless of direction. Second, we also create transfer links that connect pairs of stations where transfer is possible. In the present analysis, pairs of stations within 2 km are selected as allowing transfer links, which can be considered as close enough to walk between them. The time cost of each transfer link is set, following Taguchi (2005), as 2 min plus the time to walk between the stations at a speed of 55 m/min. Using the network thus created, we obtained the travel-time matrix by calculating the shortest paths for all station pairs.

Next, we explain how to construct dynamic flow data. First, using the census data for 2010, we construct (spatial) origin–destination (OD) traffic flow matrix, in which each element represents the number of commuters traveling from the origin station represented by the row to the destination station represented by the column. The number of distinct OD flows is about 77,000, and the total flow volume is about 7.89 million.

To create time-dependent flow data, we focus on the departure time from the origin station. Using the census data for 2005, we first extracted departure time from origin stations for all commuter traffic. Here, we use the 2005 census data because the 2010 census data do not contain departure times from origin stations. It is desirable to use the latest available census data (i.e., 2010) to construct the railway network, and the general trend for departure-time distribution of origin stations, provided by 2005 census data, can be considered to reasonably approximate the situation in 2010.

Figure 4 shows the histogram of departure times of origin stations for all commuter traffic (2005 census data). The width of each bin of the histogram is 30 min. Table 1 shows additional values of some of the bins. As can be seen from Fig. 4, the histogram has a strong peak around 6 p.m.

For the purpose of the present analysis, we focus on three intervals: around 5 p.m. (4:30–5:29), around 6 p.m. (5:30-6:29) and around 7 p.m. (6:30-7:29). (All times used in this paper are p.m.) Then, we assume that commuters in each class depart exactly at 5 p.m., 6 p.m. and 7 p.m. These three aggregated departure times account for approximately 10% (at 5 p.m.), 23% (at 6 p.m.), and 18% (at 7 p.m.) of the total flow volume of 7.89 million, and the combined volume of the three classes is about 51% (4.04 million). Using this aggregated departure time distribution, we create dynamic OD data.

![Fig. 3 Tokyo railway network.](image-url)
by allocating each OD volume to one of the three classes $T = \{5:00, 6:00, 7:00\}$, based on the corresponding rates: 10%, 23%, and 18%, respectively. Throughout the paper, we use this aggregated departure time distribution with three distinct elements. This approximation allows a mathematical optimization solver to generate optimal solutions within a reasonable amount of time.

Fig. 4 Departure-time distribution of origin station.

Table 1 Departure-time distribution of origin station as constructed from 2005 census data

| range      | frequency | range      | frequency |
|------------|-----------|------------|-----------|
| 4:00–4:29  | 3.104%    | 7:00–7:29  | 8.499%    |
| 4:30–4:59  | 3.326%    | 7:30–7:59  | 6.285%    |
| 5:00–5:29  | 6.996%    | 8:00–8:29  | 6.834%    |
| 5:30–5:59  | 10.745%   | 8:30–8:59  | 4.676%    |
| 6:00–6:29  | 12.352%   | 9:00–9:29  | 4.659%    |
| 6:30–6:59  | 9.293%    | 9:30–9:59  | 3.493%    |

4. Analysis of covered flows at specific stations

Using the data explained in the previous section, we calculated covered flows for various service start times when only one facility is located at two fixed stations: Shinjuku and Hachioji. Shinjuku is one of the busiest terminal stations in the network, and Hachioji is a large suburban station. In the following analysis, values of $c = 3.0h$ and $t_h = 11:00$ are used. As for the minimum required time to stay at a facility, $a$, we examine three cases: $a = 3.0h$, $a = 2.75h$, $a = 2.5h$. It should be noted that the case of $a = 3.0h$ is equivalent to the original rigid consumption model.

At first glance, the relative value of $a$ to $c$ seems large, compared with real world cases. However, there are several real world situations where this setting can be applied. For example, it is important for fitness centers to determine desirable service hours for night-time members. Because night-time members can use the fitness center during designated hours only (an example of a typical plan is 3 hours from 7:00 to 10:00), members on such plans receive a discount. Centers’ main target for night-time members is full-time workers who work until evening and come to the center after work. In this situation, potential users of the facility are those who can stay at the facility (here, the fitness center) for a substantial amount of time, such as two and a half hours (including the time for preparation), during the designated three hours. Thus, the problem is to optimally determine which three hours (e.g., 7:00 to 10:00 or 7:30 to 10:30) will cover as many potential commuters as possible, assuming that they will demand access of a certain length, such as 2.5 h, during the three hours.

We calculated the number of commuters that can access a service when only one facility is located at two fixed stations: Shinjuku and Hachioji. Fig. 5 shows the objective value for a facility at Shinjuku when service start times are chosen at 19 candidates times, which are in 10 min increments from 5 p.m. to 8 p.m. The analogous figure for Hachioji is shown as Fig. 6. In these figures, the objective values are measured by the proportion with respect to the total targeted flow of about 4.04 million who depart their origin station at 5, 6, or 7 p.m. The circles with “S” and “H” marks within them represent the locations of Shinjuku and Hachioji, respectively. From these figures, we see that the service-start time
strongly affects the number of covered flows. When service starts too early, few commuters can access a facility because the majority of commuters cannot arrive in time for the start of the service after departing their origin stations. In contrast, when service starts too late, many commuters cannot arrive home by $t_h$ after consuming a service at the facility.

These figures show that Shinjuku can attract many more commuters than Hachioji can. In addition, it can be seen that the value of $a$ strongly affects the objective value. This large difference in the objective value is mainly for the following two reasons. First, for fixed $c$, it becomes easier to access service as the minimum required time to stay at a facility, $a$, becomes smaller. Second, it is much more flexible to access a service of $a$ hours (here $a \leq c$) since the $a$ hours of stay can be accomplished during the $c$ hours that the facility is open. It is interesting to see that by comparing the cases of $a = 3.0h$ and $a = 2.75h$ for both Fig. 5 and Fig. 6, even a 15-min difference in $a$ strongly affects the objective values.

5. Analysis of optimal solutions

In this section, we apply the models to dynamic flow data for the railway network introduced in Section 3 and investigate optimal solutions when multiple facilities are simultaneously located. Tanaka (2011) devised heuristic methods and analyzed approximate solutions obtained by applying the methods to the railway network covered by stations included in 2000 census data; these size of that network is similar to the size of the present network. One purpose of the present paper is to investigate solving for exact optimal solutions when using the network shown in Section 3. For this goal, we selected 108 large stations, shown in Fig. 7 by black disks, as candidate stations for introducing services.

Each candidate station has at least three railway lines passing through it. These railway lines may share the same physical railway line or may run in parallel. Because of this, in Fig. 7, some of the candidate stations have only two links. As can be seen from Fig. 7 (b), candidate stations are sufficiently dense in the central area of Tokyo, and im-
portant large stations in the suburban area are also covered as candidate locations, as Fig. 7 (a) illustrates. For possible service-start times, we assume that a decision maker has five choices, from 5:30 to 7:30 in increments of 30 min: $K = \{5:30, 6:00, 6:30, 7:00, 7:30\}$. We set $t_h = 11:00$ as before. For a given pair of $c$ and $a$, we calculated for each flow $(i, j, t)$ where it can be covered by each service $(k, s)$, using the definition of coverage index in Eq. (9). Using $b_{i j t k s}$, we formulated the model and obtained exact optimal solutions by applying the mathematical optimization solver Gurobi Optimizer version 5.6.2. We present these solutions in the following and analyze their characteristics.

Figure 8 summarizes the optimal values for different numbers of facilities ($p = 1$ to $p = 10$) under four different scenarios. Three black line graphs show the optimal values for $c = 3.0h$ and three different minimum required times, $a$, to stay at a facility: $a = 3.0h$, $a = 2.75h$, and $a = 2.5h$. The case of $a = 3.0h$ corresponds to the rigid-consumption model. By comparing these three cases, we see that the value of $a$ strongly affects optimal values. In particular, the optimal values with $a = 3.0h$ and $a = 2.75h$ are quite different. Also, it can also be seen that $a = 2.5h$ captures almost all flows when more than five facilities are used.

The large difference in the optimal values with $a = 3.0h$ and $a = 2.5h$ is mainly due to two facts: (1) it becomes easier to consume a service of $a$ hours as $a$ becomes smaller, and (2) $a$ hours of stay can be any time during the $c$ hours that the facility is open, thus allowing flexibility of access time to a service. It is interesting to analyze the impact of the latter, that is, the degree to which the flexibility of access time affects the number of covered flows. To examine this, we also calculated the optimal values for $c = 2.75h$, $a = 2.75h$. These are indicated by the gray graph in the figure. This scenario corresponds to the case where the access time is smaller than 3.0 h, but there is no flexibility in timing to consume the service; that is, $a = c$. By comparing the optimal values of this case to those of $a = 3.0h$ and $a = 2.75h$ with $c = 3.0h$, it can be seen that the impact of this flexibility is very large. For example, the optimal value in the $p = 3$ case for $a = 3.0h$, $a = 2.75h$ is larger than that of even the $p = 5$ case for $a = 2.75h$, $a = 2.75h$; the former value is 3,536,210 (87.49%) and while the latter 3,521,474 (87.12%).

Next, we analyze optimal solutions. In Figs. 9–12, the set of selected stations in the optimal solutions are shown in the railway network for both the rigid model ($a = 3.0h$) and the flexible model ($a = 2.75h$) with $c = 3.0h$ in both cases. In addition, Tables 2 and 3 list names of selected stations for each solution together with optimal service start times. Optimal objective values and its percentage of the total flow volume of 4.04 million are shown in the last row of Tables 2 and 3. The numbers for each selected station in the leftmost column correspond to the stations with the same number in Figs. 9–12.

Before comparing optimal solutions between two models, it is worthwhile to briefly mention computational time to obtain optimal solutions in Figs. 9–12 by the mathematical optimization solver Gurobi Optimizer version 5.6.2. Our hardware was a PC with an Intel Core i7-4770 (3.40 GHz) and 32 GB of RAM. For the rigid model, it took 2,457 s, 2,887 s, 2,229 s and 2,531 s respectively, and for the flexible model it took 6,353 s, 4,524 s, 4,203 s and 3,928 s respectively.

In the following, we compare characteristics of the rigid model ($a = 3.0h$) with those of the flexible model ($a = 2.75h$) for each value of $p$. 
Optimal solutions for $p = 2$ The optimal solution for the rigid model is (Shinjuku, 7:30) and (Shinagawa, 6:30), while that of the flexible model is (Shinjuku, 7:00) and (Tokyo, 7:30). In both models, large terminal stations in the central area of Tokyo are selected. It is interesting to note that for the rigid model the service start time for the two facilities are separated by one hour. This result arises because we take temporal factors into account as well as spatial factors. If we focus on only the spatial location of facilities, locating facilities close to each other is not a desirable choice since they will look similar to potential customers. However, two spatially close facilities can capture different dynamic flows when the service-start times of each facility are sufficiently separated.

Optimal solutions for $p = 3$ The optimal solution of the rigid model adds (Shinjuku, 7:00) to the $p = 2$ solution. In this case, we have two facilities at Shinjuku whose service start times are separated by half an hour. This result demonstrates the very high potential of Shinjuku to capture flows. In the case of the flexible model, the optimal solution is (Shinjuku, 7:00), (Ikebukuro, 7:30), and (Yokohama, 7:30). As Table 3 shows, only one service, (Shinjuku, 7:00), is common to $p = 2$ while the other two are different. This solution has two stations at the center of the network and the other one, Yokohama, is the largest station in the southern part of the network. The accessibility of Yokohama is very high for most commuters whose destination stations are located in the southern half of the covered area. This result is in clear contrast with the three central facilities of the rigid model. As we have already seen, the flexible model provides much more flexibility to commuters in terms of access time for services. This characteristic allows a facility to capture flows with more highly variable departure times from the origin station, resulting in more spatially dispersed distribution of services than in the rigid model.

Optimal solutions for $p = 4$ The rigid model has two facilities at Shinjuku—(Shinjuku, 7:30) and (Shinjuku, 7:00)—plus (Tokyo, 6:30) and (Yokohama, 6:30). The $p = 4$ solution does not contain the $p = 3$ solution as a subset. Instead of (Shinagawa, 6:30), a service at (Tokyo, 6:30) is realized. This can be understood as reflecting that Tokyo and Yokohama are more spatially distant than Shinagawa and Yokohama are, resulting in capturing a larger amount of dynamic flows. The solution of the flexible model is (Shinjuku, 7:00), (Ikebukuro, 7:30), (Yokohama, 7:30), and (Funabashi, 7:30), the same solution for $p = 3$ plus a service at Funabashi. The selected stations are much more spatially dispersed than those in the solution for the rigid model, as shown in Fig. 11.

Optimal solutions for $p = 5$ In the case of $p = 5$, the optimal solutions of the rigid and flexible models are quite different. The solution of the rigid model is the solution for $p = 4$ solution plus (Akihabara, 7:30). The solution for the flexible model is (Tokyo, 6:30), (Akabane, 7:30), (Yokohama, 7:30), (Funabashi, 7:30), and (Kokubunji, 7:30). The former solution has four facilities at the center of the network and only one facility, Yokohama, outside the center. For five solutions of the rigid model (see Table 2), Yokohama is the only station that is outside central Tokyo. In addition, each solution of the flexible model for $p = 3$, $p = 4$, and $p = 5$ has a facility at Yokohama (Table 3). These results indicate the very high potential of Yokohama to cover flows. As Fig. 12 (b) shows, the selected stations are spatially dispersed over...
the network for the flexible model. This is a large contrast with Fig. 12 (a), where four stations are selected in the central area. In Fig. 12 (b), the only facility in the center is located at Tokyo; the four other facilities are located some distance away from the center, with each of them located on different lines in different directions from the center. This spatially dispersed solution is obtained because a single facility has a larger ability to capture various dynamic flows than a single facility in the rigid model. Another interesting aspect is that the service start time of Tokyo is 6:30 while all others are 7:30. Around Tokyo, many workplaces exist. This enables the facility at Tokyo to capture flows departing origin stations relatively early. The other four stations are large stations some distance away from the center, around which commuters’ homes (destination stations) are densely distributed, making the facilities easier to capture flows whose destinations are close to them.
Fig. 11 Comparison of optimal solutions for \( p = 4 \)

Fig. 12 Comparison of optimal solutions for \( p = 5 \)

Table 2 Optimal solutions and optimal values for \( a = 3.0 \)h

| \( p = 2 \) | \( p = 3 \) | \( p = 4 \) | \( p = 5 \) |
|-----------|-----------|-----------|-----------|
| 1         | Shinjuku  | Shinjuku  | Shinjuku  |
|           | 7:30      | 7:30      | 7:30      |
| 2         | Shinagawa | Shinagawa | Tokyo     |
|           | 6:30      | 6:30      | 6:30      |
| 3         | Shinjuku  | Shinjuku  | Shinjuku  |
|           | 7:00      | 7:00      | 7:00      |
| 4         | Yokohama  | Yokohama  |
|           | 6:30      | 6:30      |
| 5         | Akihabara |
|           |          |          | 7:30      |

optimal value

\[ \begin{align*}
&\text{2,580,759} & \text{2,836,147} & \text{3,000,223} & \text{3,116,804} \\
&\text{(63.848\%)} & \text{(70.167\%)} & \text{(74.226\%)} & \text{(77.110\%)}
\end{align*} \]
Table 3 Optimal solutions and optimal values for $a = 2.75h$

|   | $p = 2$ | $p = 3$ | $p = 4$ | $p = 5$ |
|---|---------|---------|---------|---------|
| 1 | Shinjuku | Shinjuku | Shinjuku | Tokyo   |
|   | 7:00    | 7:00    | 7:00    | 6:30    |
| 2 | Tokyo   | Ikebukuro | Ikebukuro | Akabane |
|   | 7:30    | 7:30    | 7:30    | 7:30    |
| 3 | Yokohama | Yokohama | Yokohama |         |
|   | 7:30    | 7:30    | 7:30    |         |
| 4 | Funabashi | Funabashi |         |         |
|   | 7:30    | 7:30    |         |         |
| 5 |         |         |         | Kokubunji |
|   |         |         |         | 7:30    |

optimal value 3,342,752 3,536,210 3,654,464 3,732,515
(82.700%) (87.486%) (90.412%) (92.343%)

6. Conclusion

This paper extended MFCLSTP, which was proposed in Tanaka (2011), by introducing a minimum required time to stay at a facility to consume a service. MFCLSTP determines the location of facilities and the corresponding start times of services of fixed duration so as to maximize coverage for flows on the way back home from work. In MFCLSTP, it is assumed that commuters must stay at a facility from start to end during service-providing hours to consume its service. However, there are many services that can be consumed by people stopping at a facility for a fixed length of time, $a$, during the $c$ hours that the facility is open.

We provided an integer programming formulation similar to the original formulation by extending the definition of coverage. The proposed formulation was applied to the railway network of the Tokyo metropolitan area, and we obtained exact optimal solutions. The results showed that the optimal values for the flexible model are much larger than those for the flexible model, even when $a$ is very close to $c$. In the optimal solutions of the flexible model, the selected locations are much more spatially dispersed than those of the original model.

The flexible model presented in this paper assumes that services can be consumed when people stop at a facility for $a$ hours during the hours of operation of the facility; otherwise, the flow is not covered at all. In many situations, however, services are increasingly attractive as the amount of time that can be stayed increases. This variable level of coverage can be mainly interpreted in one of the following two manners.

(1) When the time people can stay at a facility becomes longer, people can enjoy the service more.

(2) When the time people can stay at a facility becomes longer, it becomes more flexible for people to choose when to consume $a$ hours of service at the facility.

Thus, it can be assumed that the probability of commuters becoming an actual consumer of a service increases as the time possible to stay at a facility increases. The multiple-coverage model proposed in Tanaka and Furuta (2013) can be applied to introduce different levels of coverage. In the original formulation of that paper, services are more attractive when they allow the commuter to return home earlier. The model of that paper attached greater weight to whose commuters that can arrive home earlier after consuming a service at a facility than those arriving home later. Similarly, the scenario assumed in this paper can be incorporated into the model by attaching higher weight to those commuters that can spend larger amount of time at a facility.

To apply the model to real-world instances, it is important to focus on how a covered commuter actually becomes an user of a facility. For example, commuters may be discouraged to consume a service if they have to wait very long until the start of the service after departing their origin station. Various scenarios such as this can be effectively incorporated into the coverage index to model realistic choice behavior of commuters.

As shown in section 5, it required more than one hour for a mathematical optimization solver to obtain solutions for the flexible model shown in Figs. 9–12. To solve problem instances with a larger number of variables, efficient solution methods should be developed. One possible approach is to devise heuristic algorithms for solving the flexible model.
Another possibility is to apply special techniques developed for basic facility location problems. Since the proposed problem can be regarded as an extension of the maximal covering location model (MCLM), the column reduction rule developed for MCLM may be effectively applied. The basic idea of the rule is to delete a set of candidate locations that are dominated by other candidate locations, thereby reducing the size of the problem. If candidate location $j$ covers every demand node covered by candidate location $k$ in addition to at least one other demand node, then we can eliminate candidate location $k$. In this case, candidate location $k$ is said to be dominated by candidate location $j$. See Daskin (2013) for more details about the method.

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