Unflavored Leptogenesis and Neutrino Masses in Flavored SUSY SU(5) model

M.A. Loualidi and M. Miskaoui
LPHE, Modeling and Simulations, Faculty of Science, Mohammed V University in Rabat, 10090 Rabat, Morocco
E-mail: mr.medamin@gmail.com

Abstract. We propose a model with $D_4$ flavor symmetry for leptogenesis in the framework of supersymmetric SU(5) grand unified theory. Neutrino masses arise from the type I seesaw mechanism where the spontaneous symmetry breaking of the flavor symmetry leads to the well-known trimaximal mixing (TM3). We find that this model predicts the normal hierarchy (NH) for neutrino masses. We also find that the predicted range of the effective neutrino mass $m_{\beta\beta}$ can be tested at future neutrinoless double beta decay experiments. For the generation of the baryon asymmetry of the Universe (BAU), we consider the unflavored leptogenesis regime and we take into account the decay of the three right-handed neutrinos $N_i$. We conclude that the BAU originates from a correction to the Dirac operator giving rise to a new coupling and a $CP$ phase which both play an important role in generating the observed BAU.

1. Introduction
The discovery of neutrino oscillations is the first direct experimental evidence for new physics beyond the standard model (SM) [1]. One of the simplest ways to probe the origin of the small neutrino masses is by invoking the type I seesaw mechanism in which heavy right-handed neutrinos (RHNs) are added to the SM [2]. An attractive aspect of this mechanism is that it addresses three major puzzles in particle physics: the nature of neutrinos, the smallness of neutrino masses, and the matter-antimatter asymmetry problem. These three aspects provide also a motivation for leptogenesis as an approach to explain the BAU [3]. In a nutshell, this approach requires lepton number violation which emerges naturally in type I seesaw models. Then, a lepton asymmetry is created by the out of-thermal-equilibrium and $CP$ violating decays of the RHNs that is converted into a baryon asymmetry by electroweak sphalerons [4].

In this context, many models have been proposed in recent years, and since the seesaw scale is associated with the masses of RHNs which are generally assumed to be around the unification scale, supersymmetric grand unified theories (SUSY-GUTs) are one of the most appealing models to study neutrinos and their properties [5]. On the other hand, neutrino mixing can be described efficiently by using some non-Abelian discrete symmetry such as the $A_4$ group which is broadly used in $SU(5)$ models to study the patterns of neutrino mixing; see for example Refs. [6].

In this letter, based on our recent work [7], we will discuss the generation of the BAU via the leptogenesis mechanism in the framework of a SUSY SU(5) model extended by a $D_4 \times U(1)$ symmetry. The dihedral group $D_4$ is introduced to address the neutrino flavor structure while

1 Speaker
$U(1)$ controls the invariance of the superpotential of the model. We present here an update of the study done in Ref. [7] where we assume that the RH neutrino mass spectrum is not strongly hierarchical which requires taking into account the contribution of the decay of the three RHNs. In this case, we find that the $CP$ asymmetries for each Majorana neutrino $\varepsilon_{N_i}$ is dominated by the contribution of a next-to-leading (NLO) order correction to the usual type I seesaw terms. In particular, we find that the correlation between the baryon asymmetry parameter $Y_B$ and the parameters from the NLO term satisfies the experimental bound of $Y_B$ from the Planck collaboration [8]. We find also that the predicted values of the effective Majorana mass $m_{\beta\beta}$ measured in neutrinoless double beta decay ($0\nu\beta\beta$) are testable at future decay experiments.

2. Neutrino sector in $D_4 \times SU(5)$ model

The neutrino sector of the present model is enlarged by adding three gauge-singlet RHNS $N_{i=1,2,3}^c$ charged under $D_4 \times U(1)$ group. These RHNs are responsible for generating the small neutrino masses via the type I seesaw mechanism as well as the BAU through the leptogenesis mechanism. The charges under $D_4 \times U(1)$ of matter, Higgs and RHNs are listed in table (1). In this letter, the focus is put on the neutrino sector and especially the study of the BAU; however, we should mention that in Ref. [2] where the charged fermion sector is studied as well, the Higgs multiplets $H_{35}$ and $H_{24}$ were also used to produce the following ratios of the Yukawa couplings of the first and second generations $y_e/y_d = 4/9$ and $y_{\mu}/y_s = 9/2$ which are in perfect agreement with experimental data [9]. Furthermore, the scalar sector is extended by five flavons needed for flavor symmetry breaking, $U(1)$ invariance and to structure the neutrino mass matrix, see table (2) for their $D_4 \times U(1)$ charge assignments. Thus, by using the charge assignments in tables (1) and (2), the superpotential invariant under the $SU(5) \times D_4 \times U(1)$ group is given by

$$W_\nu = y_1 N_1^c F_1 H_5 + y_2 N_{3,2}^c F_{2,3} H_5 + y_3 N_1^c N_1^c \rho_1 + y_4 N_{3,2}^c N_{3,2}^c \rho_1 + y_5 N_1^c N_{3,2}^c F + y_6 N_{3,2}^c \Gamma + y_7 N_{3,2}^c N_{3,2}^c \rho_2 + y_8 N_{3,2}^c N_{3,2}^c \rho_3$$

where $y_i = 1, \ldots, 8$ are Yukawa coupling constants. The first two terms give rise to the Dirac mass matrix $m_D$ while the rest of the terms induce the Majorana mass matrix $m_M$. In particular, the couplings involving $\rho_1$ and $F$ lead to the well-known tribimaximal mixing (TBM) matrix [10], while the couplings incorporating $\rho_2, \rho_3$ and $\Gamma$ lead to $TM_2$ which is consistent with the data. We assume that the VEVs of the flavon fields point in the following directions

$$\langle \rho_1 \rangle = v_{\rho_1} \ , \ \langle \rho_2 \rangle = v_{\rho_2} \ , \ \langle \rho_3 \rangle = v_{\rho_3} \ , \ \langle F \rangle = (v_F, v_F)^T \ , \ \langle \Gamma \rangle = (0, v_\Gamma)^T$$

| $SU(5)$ | $T_1$ | $T_2$ | $T_3$ | $F_1$ | $F_{2,3}$ | $N_1^c$ | $N_{3,2}^c$ | $H_5$ | $H_{35}$ | $H_{24}$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $D_4$ | 1+,-  | 1+,-  | 1+,-  | 0+   | 1+  | 1+  | 1+  | 1+  | 1+  | 1+  |
| $U(1)$ | 6  | 12  | 4  | 13  | 13  | 5  | 5  | 8  | 4  | 16  |

Table 1. The $SU(5) \times D_4$ representations and $U(1)$ charges of the matter, RH neutrinos and Higgs superfields. See Ref. [7] for the notation of the different fields.

| Flavons | $\rho_1$ | $\rho_2$ | $\rho_3$ | $F$ | $\Gamma$ |
|---------|---------|---------|---------|-----|---------|
| $D_4$   | 1+,-    | 1+,-    | 0+     | 1+  | 1+     |
| $U(1)$  | 10      | 10      | 10      | 10  | 10      |

Table 2. The $D_4 \times U(1)$ quantum numbers of the flavons used in the neutrino sector.
while the Higgs doublet develops its VEV as is customary \( \langle H_u \rangle = v_u \). Then, using the tensor product of \( D_4 \) irreducible representations (see eqs. (C.2) and (C.3) of Ref. [7]), we find that the Dirac and Majorana mass matrices have the following forms

\[
m_D = v_u Y_D = v_u \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_2 \end{pmatrix}, \quad m_M = \begin{pmatrix} y_{3\nu_{\rho_1}} & y_{5\nu_{\rho_1}} & y_{5\nu_{\rho_1}} + y_6\nu_{\tau} \\ y_{5\nu_{\rho_1}} & y_{7\nu_{\rho_2}} - y_8\nu_{\rho_3} & 2y_{4\nu_{\rho_1}} \\ y_{5\nu_{\rho_1}} + y_6\nu_{\tau} & 2\lambda_4\nu_{\rho_1} & y_{7\nu_{\rho_2}} + y_8\nu_{\rho_3} \end{pmatrix}
\]

(3)

The conditions for TBM mixing [11][13], and its deviation to TM2 require the imposition of the following assumptions on \( m_D \) and \( m_M \): \( y_1 = y_2, y_{3\nu_{\rho_1}} + y_{5\nu_{\rho_1}} = 2y_{4\nu_{\rho_1}}, \) and \( y_8\nu_{\rho_3} = -y_{7\nu_{\rho_2}} = y_6\nu_{\tau}/2 \). Now, we can apply the usual seesaw formula \( m_\nu = m_D m_M^{-1} m_D^T \) which gives rise to the total neutrino mass matrix

\[
m_\nu = \frac{m_0}{P} \begin{pmatrix} -(a + b)^2 & (a + b)(b + k) & b^2 - k^2 - b(k - a) \\ (a + b)(b + k) & -(b + k)^2 & -a^2 - ab + b^2 + kb \\ b^2 - k^2 - b(k - a) & -a^2 - ab + b^2 + kb & ak - b^2 \end{pmatrix}
\]

(4)

where we have adopted the following notations to simplify the parametrization of \( m_\nu \)

\[
m_0 = \frac{(y_1\nu_{\mu})^2}{M_R}, \quad P = (a + 2b + k)(ak - a^2 + b^2 - k^2) \\
a = \frac{y_{3\nu_{\rho_1}}}{M_R}, \quad b = \frac{y_{5\nu_{\rho_1}}}{M_R}, \quad c = \frac{2y_{4\nu_{\rho_1}}}{M_R}, \quad k = \frac{y_6\nu_{\tau}}{M_R}
\]

(5)

with \( M_R \) being the mass scale of the RH neutrinos. In order to satisfy \( CP \) violation in the lepton sector, we can take without loss of generality, only the parameter \( k \) to be complex – \( k \to |k| e^{i\phi_k} \) where \( \phi_k \) is a \( CP \) violating phase. After breaking the flavor symmetry, we find that the matrix \( m_\nu \) enjoys a remnant \( Z_2 \) symmetry called a magic symmetry referring to the equality of the sum of each row and the sum of each column of \( m_\nu \) [14]. This property implies that \( m_\nu \) is diagonalized by the trimaximal mixing matrix \( U_{TM_2} \) so that \( m_\nu^{\text{diag}} = U_{TM_2}^\dagger m_\nu U_{TM_2} \) with

\[
U_{TM_2} = \begin{pmatrix} \sqrt{\frac{2}{3}} \cos \theta & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\cos \theta & \cos \theta & \cos \theta \\ \sin \theta e^{-i\sigma} & \sin \theta e^{i\sigma} & \sin \theta e^{i\sigma} \end{pmatrix}
\]

(6)

where \( \theta \) and \( \sigma \) are respectively an arbitrary angle and a phase. The complete mixing matrix for the neutrino sector is given by \( U_\nu = U_{TM_2} U_P \) where \( U_P = \text{diag}(1, e^{i\frac{2\pi}{3}}, e^{i\frac{4\pi}{3}}) \) is a diagonal matrix that contains the two additional Majorana phases \( \alpha_{21} \) and \( \alpha_{31} \). The diagonalization of the neutrino matrix [11] by \( U_{TM_2} \) leads to the following eigenmasses

\[
|m_1| = \frac{m_0}{\sqrt{(a-b)^2 - k^2}} \frac{1}{k(a-b) \cos \phi_k + \sqrt{2} k^2/4}, \quad |m_2| = \frac{m_0}{\sqrt{(a-b)^2 + 2 k^2(a+b) \cos \phi_k + k^2/4}} \\
|m_3| = \frac{m_0}{\sqrt{(a-b)^2 - k^2(a+b) \cos \phi_k + \sqrt{2} k^2/4}}
\]

(7)

where the denominators of \(|m_1|\), \(|m_2|\) and \(|m_3|\) corresponds to ratios of the RHN masses \(|M_1|\), \(|M_2|\) and \(|M_3|\) and their mass scale \( M_R \), respectively . Regarding the mixing angles in the case of trimaximal mixing, they are expressed as a function trimaximal mixing parameters \( \sigma \) and \( \theta \) as

\[
\sin^2 \theta_{13} = \frac{2}{3} \sin^2 \theta, \quad \sin^2 \theta_{12} = \frac{1}{3 - 2 \sin^2 \theta}, \quad \sin^2 \theta_{23} = \frac{1}{2} - \frac{3 \sin 2\theta}{2 \sqrt{3(3 - 2 \sin^2 \theta)}} \cos \sigma.
\]

(8)

\(^2\) The validity of these assumptions is discussed in appendix D of Ref. [7].
3. Leptogenesis

The key component to apply the leptogenesis mechanism is the presence of the RHNs $N_i^c$ as a means for the generation of the small neutrino masses. The matter-antimatter asymmetry is parametrized in terms of the baryon asymmetry parameter $Y_B$ which has been precisely measured by the Planck collaboration: $Y_B = (8.72 \pm 0.08) \times 10^{-11}$ [5].

In order to dynamically generate this asymmetry, the three Sakharov conditions must be fulfilled [15]: baryon number violation, $C$ and $CP$ violation and departure from thermal equilibrium. In a type I seesaw scenario, a lepton asymmetry $Y_L$ is generated through the out-of-equilibrium $CP$ violating decays of $N_i^c$ that is partially converted into the baryon asymmetry $Y_B$ via sphaleron processes [14]. In order to estimate the value of $Y_B$, we perform our study in the case of unflavored leptogenesis due to the fact that all RHN masses are above $T = 10^{12}(1 + \tan^2 \beta)$ as discussed in [7]. Moreover, since the RHN mass spectrum is not strongly hierarchical, we consider the contributions of all three RHNs to the generation of the BAU; $Y_B = \sum_{i=1}^{3} Y_{Bi}$ with

$$Y_{Bi} \approx -1.266 \times 10^{-3} \varepsilon_{N_i} \eta_{ii}$$

(9)

where $\varepsilon_{N_i}$ is the $CP$ asymmetry parameter, $\eta_{ii}$ is the efficiency factor, while the numerical value in [9] depends on the number densities of RHNs over the entropy density and on the sphaleron transitions, see [7] for the details yielding to this value. The leading order terms given in eq. (1) induce highly suppressed $CP$ asymmetry parameter $\varepsilon_{N_i}$ and thus a suppression of the value of $Y_B$. Thus, we have to rely on NLO terms to obtain an appropriate suppression of the $CP$ asymmetries. Therefore, we consider a case in which the NLO contribution arises in the neutrino sector and only corrects $Y_D$ in eq. (3). This is realized by introducing a new flavon $\omega$ which transforms as $1_+^-$ under $D_4$ with zero $U(1)$ charge

$$\delta W_D = \frac{y_0}{\Lambda} N_{i,2}^c N_{2,3}^c H_5 \omega$$

(10)

where $y_0$ is a complex coupling constant; $y_0 = |y_0| e^{i \phi_\omega}$. When the flavon field $\omega$ acquires its VEV as $\langle \omega \rangle = \nu_\omega$, we find the new Yukawa mass matrix

$$\mathcal{Y}_D = Y_D + \delta Y_D = \frac{m_D}{v_u} + \delta Y_D = \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_1 & 0 \\ 0 & 0 & y_1 \end{pmatrix} + \kappa e^{i \phi_\omega} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

(11)

where $\kappa = \frac{|y_0| \nu_\omega}{\Lambda}$ is a free parameter which must be very small $-\kappa << 1 -$ to produce the correct BAU. The total Yukawa neutrino mass matrix is now defined as $\mathcal{Y}_\nu = U^\dagger \mathcal{Y}_D$. After we compute the product $\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger$, we obtain the analytic expressions of the $CP$ asymmetries

$$\varepsilon_{N_1} \approx \frac{\kappa^2 \cos^2 \phi_\omega}{9 \pi} \left[ \cos^2 (\theta) \sin^2 \left( \frac{\alpha_{21} + 4 \phi_\omega}{2} \right) f \left( \frac{\bar{m}_2}{\bar{m}_1} \right) + 2 \cos^2 \left( \frac{\alpha_{21} - 2 \sigma}{2} \right) \sin^2 (2 \theta) f \left( \frac{\bar{m}_3}{\bar{m}_1} \right) \right]$$

$$\varepsilon_{N_2} \approx \frac{\kappa^2 \cos^2 \phi_\omega}{9 \pi} \left[ \cos^2 (\theta) \sin^2 \left( \frac{\alpha_{21}}{2} \right) f \left( \frac{\bar{m}_1}{\bar{m}_2} \right) + \sin^2 \left( \frac{\alpha_{21} - \alpha_{31} + 2 \sigma}{2} \right) \sin^2 (\theta) f \left( \frac{\bar{m}_3}{\bar{m}_2} \right) \right]$$

$$\varepsilon_{N_3} \approx \frac{\kappa^2 \cos^2 \phi_\omega}{9 \pi} \left[ 2 \sin^2 (2 \theta) \sin^2 \left( \frac{\alpha_{31} - 2 \sigma}{2} \right) f \left( \frac{\bar{m}_1}{\bar{m}_3} \right) + \sin^2 \left( \frac{\alpha_{21} - \alpha_{31} + 2 \sigma}{2} \right) \sin^2 (\theta) f \left( \frac{\bar{m}_2}{\bar{m}_3} \right) \right]$$

(12)

Notice that the total light neutrino mass matrix involving the small correction $\delta Y_D$ is almost similar to the one in eq. (11) and yields approximately to the same neutrino phenomenology.
where $\tilde{m}_i$ are the washout mass parameters expressed as $\tilde{m}_i = v^2 \frac{\langle \nu_i \nu_i \rangle}{M_\nu}$. Notice here that the smallness of the parameter $\kappa << y_1$ implies that $\tilde{m}_i \approx m_i$. From the obtained expression of $\varepsilon_{N_i}$, we deduce that $Y_B$ in our model depends on the trimaximal parameters, the active neutrino masses, the Majorana phases, as well as $\kappa$ and the phase $\phi_\omega$ coming from the NLO contribution to $Y_D$. The last parameter to evaluate in order to compute $Y_B$ in eq. (9) is the efficiency factor $\eta_i$. We consider the region of RHN masses smaller than $10^{14}$ GeV, preventing possible washout effects. In this case, $\eta_i$ can be expressed as a function of the washout mass parameter $\tilde{m}_i$ as [16]

$$\eta_i \approx \left( \frac{3.3 \times 10^{-3} \text{eV}}{\tilde{m}_i} + \left( \frac{\tilde{m}_i}{0.55 \times 10^{-3} \text{eV}} \right)^{1.16} \right)^{-1}$$

(13)

4. Numerical analysis and results

From the first relation in eq. (8) and the $3\sigma$ range of $\theta_{13}$ from Ref. [17], we deduce that $\theta$ lies in the range $0.1763 \leq \theta \leq 0.1920$. Moreover, by using the $3\sigma$ ranges of $\Delta m^2_{ij} = m_i^2 - m_j^2$ in the case of the IH and the neutrino masses in eq. (7) as well as the constraint on the sum of $m_i$ from cosmological observations $\sum m_i < 0.12$ eV [8], we find that $\theta$ lies in the range $0.398 \leq \theta \leq 0.579$ which indicates that $\sin^2 \theta_{13}$ and $\sin^2 \theta_{12}$ fall far outside their allowed $3\sigma$ experimental range. For this reason, the IH pattern is excluded in our model.

The scale of neutrino masses can be probed by studying the $0\nu\beta\beta$. The effective Majorana mass $|m_{\beta\beta}|$ associated with this process is defined as $|m_{\beta\beta}| = |\sum_i U^2_{ei} m_i|$ where $m_i$ are the neutrino masses and $U_{ei}$ are the elements of the first row of the mixing matrix. In this numerical analysis, we consider the current limits on $|m_{\beta\beta}|$ from KamLAND-Zen, CUORE, GERDA and EXO experiments corresponding to $|m_{\beta\beta}| < (0.061 - 0.165)$ eV, $|m_{\beta\beta}| < (0.075 - 0.35)$ eV, $|m_{\beta\beta}| < (0.079 - 0.180)$ eV and $|m_{\beta\beta}| < (0.078 - 0.239)$ eV, respectively [18]. The left panel of

![Figure 1](image_url)

Figure 1. Left panel: $|m_{\beta\beta}|$ as a function of $m_1$. Right panel: $Y_B$ as a function of $\kappa$ while the high energy phase $\phi_\omega$ is shown in the colored palette.

Figure 1 shows the correlation between $|m_{\beta\beta}|$ and the lightest neutrino mass $m_1$ for the NH. This plot is obtained by varying the oscillation parameters in their $3\sigma$ range while the Majorana phases $\alpha_{21}$ and $\alpha_{31}$ are varied in the interval $[0 \rightarrow 2\pi]$. The horizontal dashed lines represent the limits on $|m_{\beta\beta}|$ from current $0\nu\beta\beta$ decay experiments while the vertical gray region is disfavored by the Planck+BAO data. From this figure, we extract our range of $|m_{\beta\beta}|$

$$0.000715 \lesssim |m_{\beta\beta}| (\text{eV}) \lesssim 0.022028$$

(14)

The next-generation experiments such as GERDA Phase II, CUPID, nEXO and SNO+-II will cover the values of $|m_{\beta\beta}|$ in eq. (14) as they aim for sensitivities around $(0.01 - 0.02)$ eV, $(0.006 - 0.017)$ eV, $(0.008 - 0.022)$ eV and $(0.02 - 0.07)$ eV, respectively [19].
As discussed above, $Y_B$ depends on the parameters $\kappa$ and $\phi_\omega$ which are responsible for the generation of the observed value of $Y_B$, thus we use as their input ranges $[-0.1 \to 0.1]$ and $[0 \to 2\pi]$, respectively. Then, we plot in the right panel of figure 1 the correlation between $Y_B$ and $\kappa$ while the color palette shows $\phi_\omega$. We find that the observed value of $Y_B$ is predicted for the following ranges of $\kappa$ and $\phi_\omega$: $\kappa \in [-0.1 \to -0.0085] \cup [0.009 \to 0.1]$ and $0 \lesssim \phi_\omega \lesssim 6.279$. We find also that the $CP$ conserving values $\phi_\omega = \frac{\pi}{2}$ and $\phi_\omega = \frac{3\pi}{2}$ as well as the regions around them are excluded. Thus, this source of $CP$ violation plays a crucial role in generating the baryon asymmetry in the present model. A detailed study concerning the correlation between $Y_B$ and the other oscillation parameters has been performed in ref. [7].

5. Summary and conclusion
We have studied leptogenesis and $0\nu\beta\beta$ in the framework of a SUSY $SU(5)$ GUT supplemented by a $D_4$ symmetry. We have adopted a neutrino mixing matrix of trimaximal form, and through numerical analysis we found that the IH scheme is excluded. Interestingly, we found that the predictions for $m_{\beta\beta}$ are accessible in future experiments. On the other hand, by ignoring flavor effects and taking into account that the three RH neutrinos $N_i$ are not strongly hierarchical, we computed the baryon asymmetry parameter $Y_B$ in the NH case. This latter depends mainly on the parameters $\kappa$ and $\phi_\omega$ emerged from the correction added to the Dirac mass matrix. We have shown through plots that generating the baryon asymmetry in the present model calls for non conserving values of the high energy $CP$ phase $\phi_\omega$.

References
[1] T. Kajita, Rev. Mod. Phys. 88 (2016) 030501, A.B. McDonald, Rev. Mod. Phys. 88 (2016) 030502.
[2] P. Minkowski, Phys. Lett. B 67 (1977) 421, T. Yanagida, Conf. Proc. C 7902131 (1979) 95, T. Yanagida, Prog. Theor. Phys. 64 (1980) 1103, M. Gell-Mann, P. Ramond and R. Slansky, Conf. Proc. C 790927 (1979) 315, R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. 44 (1980) 912.
[3] M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45.
[4] S.Y. Khlebnikov and M.E. Shaposhnikov, Nucl. Phys. B 308 (1988) 885.
[5] S. Raby, Lect. Notes Phys. 939 (2017) 1.
[6] S.F. King, Prog. Part. Nucl. Phys. 94 (2017) 217, F. Björkeroth, F.J. de Anda, I. de Medeiros Varzielas and S.F. King, JHEP 06 (2015) 141, I.K. Cooper, S.F. King and C. Luhn, JHEP 06 (2012) 130, S. Antusch, S.F. King and M. Spinrath, Phys. Rev. D 87 (2013) 096018, R. Ahl Laamara, M.A. Loualidi, M. Miskaoui and E.H. Saidi, Phys. Rev. D 98 (2018), R. Ahl Laamara, M.A. Loualidi, M. Miskaoui and E.H. Saidi, Nucl. Phys. B 916 (2017) 430-462, P. Ciafaloni, M. Picariello, E. Torrente-Luican and A. Urbano, Phys. Rev. D 79 (2009) 116010, G. Altarelli, F. Feruglio and C. Hagedorn, JHEP 03 (2008) 052, S. Antusch, S.F. King and M. Spinrath, Phys. Rev. D 83 (2011) 013005, I.K. Cooper, S.F. King and C. Luhn, Phys. Lett. B 690 (2010) 396.
[7] M. Miskaoui and M. A. Loualidi, JHEP 11 (2021) 147.
[8] Planck Collaboration, N. Aghanim et al., Astron. Astrophys. 641, A6 (2020).
[9] A. Masiero, D.V. Nanopoulos, K. Tamvakis and T. Yanagida, Phys. Lett. B 115 (1982) 380.
[10] P.F. Harrison, D.H. Perkins and W.G. Scott, Phys. Lett. B 530 (2002) 167.
[11] T. Fukuyama and H. Nishiura, (1997) [arXiv:9702253].
[12] C.S. Lam, Phys. Lett. B 507 (2001) 214.
[13] P.F. Harrison and W.G. Scott, Phys. Lett. B 547 (2002) 219.
[14] C.S. Lam, Phys. Rev. D 78 (2008) 113004.
[15] A.D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. 5 (1967) 32 [JETP Lett. 5 (1967) 24] [Sov. Phys. Usp. 34 (1991) 392], [Usp. Fiz. Nauk 161 (1991) 61].
[16] G.F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, Nucl. Phys. B 685 (2004) 89.
[17] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, and A Zhou, JHEP 09 (2020) 178.
[18] KamLAND-Zen Collaboration, Phys. Rev. Lett. 117 (2016) 082503 [Addendum ibid. 117 (2016) 109903].
[19] CUORE Collaboration, Phys. Rev. Lett. 120 (2018) 132501, GERDA Collaboration, Phys. Rev. Lett. 111 (2013) 122503, EXO-200 Collaboration, Phys. Rev. Lett. 123 (2019) 161802.
[20] M. Agostini et al., Nature 544 (2017) 47, CUPID Collaboration, [arXiv:1504.03599], NEXO Collaboration, J.B. Albert et al., Phys. Rev. C 97 (2018) 065503, SNO+ Collaboration, Adv. High Energy Phys. 2016 (2016) 6194265.