Transport across junctions of a Weyl and a multi-Weyl semimetal

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We study transport across junctions of a Weyl and a multi-Weyl semimetal separated by a region of thickness $d$ which has a barrier potential $U_0$. We show that in the thin barrier limit ($U_0 \rightarrow \infty$ and $d \rightarrow 0$ with $\chi = U_0/(\hbar v_F)$ kept finite, where $v_F$ is velocity of low-energy electrons and $\hbar$ is Planck’s constant), the tunneling conductance $G$ across such a junction becomes independent of $\chi$. We demonstrate that such a barrier independence is a consequence of the change in the topological winding number of the Weyl nodes across the junction and point out that it has no analogue in tunneling conductance of either junctions of two-dimensional topological materials such as graphene or topological insulators or those made out of Weyl or multi-Weyl semimetals with same topological winding numbers. We study this phenomenon both for normal-barrier-normal (NBN) and normal-barrier-superconducting (NBS) junctions involving Weyl and multi-Weyl semimetals and discuss experiments which can test our theory.

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I. INTRODUCTION

A Weyl semimetal (WSM) hosts a three-dimensional (3D) gapless topological state whose wavefunction carries a non-zero topological winding number arising out of singularity in $k$ space\cite{1}. These singularities occur at Weyl points where the conduction and the valence bands touch. The low-energy effective Hamiltonian of these WSMS around these Weyl points is given by $H = \pm \hbar v_F \vec{\tau} \cdot \vec{k}$, where $\vec{k} = (k_x, k_y, k_z)$ is the wave vector, $v_F$ is the velocity of the electrons near the Weyl point which depends on material parameters, and $\vec{\tau}$ denotes Pauli matrices. These Weyl nodes occur in pairs and are protected due to either time-reversal or inversion symmetry breaking.\cite{6} Such isotropic Weyl nodes are characterized by a topological winding number which takes values $\pm 1$ depending on the chirality of the electrons around the node. The electron around such nodes display spin momentum locking; this property along with the linear dispersion $E_{\vec{k}} = \pm \hbar v_F |\vec{k}|$ and a non-zero topological winding number distinguishes WSMs from ordinary metals. This distinction is manifested in several unconventional features associated with transport, magneto-transport and edge physics of these materials\cite{15,16}.

More recently, materials with Weyl points having anisotropic dispersion in two transverse direction (chosen to be $k_x$ and $k_y$ in this work) have been discovered.\cite{1} Such materials are termed as multi-Weyl semimetals (MSMs) since their anisotropic dispersion occurs due to merger of two or more Weyl nodes with same chirality. Such a merger is found to be topologically protected by point group symmetries (such as $C_4$ and $C_6$ rotational symmetries).\cite{19} The low energy dispersion of the electrons in MSMs remain linear in the symmetry direction (chosen to be $k_z$ in this work) but vanishes as $k^n$ (where $k = \sqrt{k_x^2 + k_y^2}$ with $n > 1$ in the transverse directions: $E(k_z = 0, k) \sim k^n$).\cite{9,11} The winding number of these anisotropic Weyl points is given by $n$ and most MSMs discovered so far has $n = 2$ or $n = 3$. The presence of a winding number different from unity modifies the helicity properties and the density of states of the electrons in these materials\cite{12}.

It is well known that transport measurement across junctions of topological materials provides access to their topological properties and unravels several unconventional features that have no analog in standard metals.\cite{19,21} In 2D topological materials such as graphene, the tunneling conductance $G$ across graphene normal metal-barrier-normal metal (NBN) junctions, display oscillatory behavior and a transmission resonance as a function of the barrier potentials.\cite{20} Similar behavior is also seen in subgap tunneling conductance of graphene NBS junctions, where superconductivity is induced in graphene via a proximate $s$-wave superconductor.\cite{13} Such an oscillatory behavior and the transmission resonance phenomenon turns out to be a signature of the Dirac quasiparticles in graphene; they do not occur in standard metals whose quasiparticles obey Schrodinger equation. Similar behavior is also seen for quasiparticles on the surface of a topological insulator.\cite{19} More recently tunneling conductance across NBN and NBS junctions of WSMs have also been studied.\cite{21,22}. In particular, it was found that the NBS junctions of Weyl semimetals may host a universal zero-bias conductance value of $e^2/h$; moreover, the subgap tunneling conductance displays oscillatory behavior as a function of the barrier strength which is expected in standard Dirac materials.\cite{22}

In this work, we study the tunneling conductance across NBN and NBS junctions between either a WSM ($n = 1$) and a MSM ($n \neq 1$) or two MSMs with $n_1 \neq n_2$ separated by a barrier of width $d$ and a potential $U_0$.\cite{9,11}
Such junctions differ from their previously studied WSM counterparts in the sense that the topological winding number of the system changes across these junctions. The main results obtained from our study are as follows. First, we show that the tunneling conductance $G$ of these junctions becomes independent of the barrier potential in the thin barrier limit where $U_0 \to \infty$ and $d \to 0$ with $\chi = U_0d/(\hbar v_F)$ being held fixed. We note that this behavior is in contrast to that found in junctions of both ordinary Schrödinger metals (where $G$ is a monotonically decaying function of $\chi$) and Dirac or WSM materials (where $G$ oscillates with $\chi$). We demonstrate that this independence is a consequence of difference of winding numbers between the WSM and MSM (or two MSMs) on two sides of the junction. Second, we find that the subgap tunneling conductance of the NBS junction depends crucially on the topological winding numbers. This is followed by a similar analysis for junctions away from the thin barrier limit. We find that the number of the system changes across these junctions.

In this section we shall derive the conductance of a NBN junction between a WSM and a MSM or two MSMs with different winding numbers. The geometry of the setup is sketched in Fig. 1. The Hamiltonian of the system is given by

$$H = H_1\theta(d-z) + U_0\theta(d-z)\theta(z) + H_2\theta(z-d) \tag{1}$$

where $\theta(z)$ is the Heaviside step function. The Hamiltonians $H_1$ and $H_2$ are given by

$$H_1 = E_0(-i\partial_z r_z + \epsilon_0k_{n_1} + \cos(n_1\phi_k) r_z + \tau_y \sin(n_1\phi_k))$$

$$H_2 = \eta^{-1}E_0(-i\partial_z r_z + \epsilon_0'k_{n_2} + \cos(n_2\phi_k) r_z + \tau_y \sin(n_2\phi_k)) \tag{2}$$

where $n_1$ and $n_2$ are the topological winding numbers in regions I and II as shown in Fig. 1. $\phi_k = \arctan(k_y/k_x)$.

$\tau$ and $E_0 = \hbar v_F k_0$ is the energy scale in which all energies are measured. In the rest of this work, we shall take this energy scale to be upper cutoff up to which the low-energy continuum Hamiltonians (Eq. 2) hold. Here $v_F$ and $v'_F = v_F/\eta$ are the Fermi velocities for electrons in region I and III, $k_0$ is the momentum scale chosen to make all momenta dimensionless, and $\epsilon_0$ and $\epsilon_0'$ are material specific constants whose precise numerical value is not going to alter our main results. We shall further choose a common chemical potential $\mu_N$ across the junction.

To compute $G$, we first consider the electron wavefunction in region I. A straightforward calculation shows that the wavefunction for right (R) and left (L) moving electrons in region I in the presence of an applied voltage $eV$ is given by

$$\psi_{eR} = e^{i(k_x x + k_y y + k_z z)} e^{-i\tau_1 n_1 \phi_k/2}(\cos(\theta_1), \sin(\theta_1))^T$$

$$\psi_{eL} = e^{i(k_x x + k_y y - k_z z)} e^{-i\tau_1 n_1 \phi_k/2}(\sin(\theta_1), \cos(\theta_1))^T \tag{3}$$

where $2\theta_1 = \arcsin(\epsilon_0 k_{n_1}/|eV + \mu_N|)$, $k = \sqrt{k_x^2 + k_y^2}$, $k_{z1} = \sqrt{(eV + \mu_N)^2 - \epsilon_0^2 k^2}$ and we have measured all energies (wavevectors) in units of $E_0 = \hbar v_F k_0$ ($k_0$).

The wavefunction in region I can be written in terms
of $\psi_R$ and $\psi_L$ as
\[ \psi_1 = \psi_{1R} + r \psi_{1L} \tag{4} \]
where $r$ is the amplitude of reflection from the barrier. We note here that $\psi_{1R}(L) \sim e^{-i\tau \phi_k/2}$ leading to $\psi_1 \sim e^{-i\tau \phi_k/2}$; thus the azimuthal angle dependence of the wavefunction in region I can be interpreted as a spin rotation by an angle of $\phi_k$ about the $\hat{z}$ axis.

In region II, the electrons see an additional applied potential $U_0$. The right and the left moving electron wavefunction in this regime can be written as
\[ \begin{align*}
\psi_{2R} &= e^{ik_{xz} + e_{kx}y + kzzz} e^{-i\tau \phi_k/2} (\cos(\theta_2), \sin(\theta_2))^T \\
\psi_{2L} &= e^{ik_{xz} - e_{kx}y - kzzz} e^{-i\tau \phi_k/2} (\sin(\theta_2), \cos(\theta_2))^T
\end{align*} \tag{5} \]
where $k_{z2} = (eV + \mu_N - U_0)^2 - \epsilon^2_{0k}^2$ and $2\theta_2 = \arcsin(\epsilon_0 k_{z2} / eV + \mu_N - U_0)$. We note that $\theta_2 \rightarrow 0$ when $U_0 \rightarrow \infty$. Thus the wavefunction in region II can be written as
\[ \psi_{II} = p\psi_{2R} + q\psi_{2L} \tag{6} \]
where $p$ and $q$ denotes amplitudes of right and left moving electrons in region II. We note that $\theta_2$ and $\theta_1$ are related by
\[ \theta_2 = \frac{1}{2} \arcsin[\sin(2\theta_1) / |1 - U_0/(eV + \mu_N)|] \tag{7} \]

In region III, the right moving electrons have a wavefunction given by
\[ \psi_{3R} = e^{ik_{xz} + e_{kx}y + kzzz} e^{-i\tau \phi_k/2} (\cos(\theta_3), \sin(\theta_3))^T \tag{8} \]
where $2\theta_3 = \arcsin(\epsilon^2_{0k} k_{z3} / |eV + \mu_N|)$, $k_{z3} = (eV + \mu_N)^2 - \epsilon^2_{0k}^2$, and $\theta_3$ is the measure of the Fermi velocity mismatch across the junction. We note that $\theta_1$ and $\theta_3$, for any given voltage $eV$, are related by
\[ \theta_3 = \frac{1}{2} \arcsin \left[ |eV + \mu_N|^{n_{z2}/n_{z1}} \sin[2\theta_1]^{n_{z2}/n_{z1}} \frac{\epsilon_{0k}}{e_0} \right] \tag{9} \]
The wavefunction in region III is thus given by
\[ \psi_{III} = t\psi_{3R} \tag{10} \]
where $t$ is the transmission amplitude across the junction. We note that $\psi_3 \sim e^{-i\tau \phi_k/2}$.

To obtain the reflection and transmission amplitude across the barrier, we match the wavefunctions at $z = 0$ and $z = d$, where $d \equiv dk_0$ constitutes the width of the barrier in units of $k_0^{-1}$. This requires $\psi_I(z = 0) = \psi_{II}(z = 0)$ and $\psi_{II}(z = d) = \psi_{III}(z = d)$ and leads to
\[ \begin{align*}
\cos(\theta_1) + r \sin(\theta_1) e^{i\phi_k} &= p \cos(\theta_2) + q \sin(\theta_2) e^{i\phi_k} \\
\sin(\theta_1) e^{-i\phi_k} + r \cos(\theta_1) &= p \sin(\theta_2) e^{-i\phi_k} + q \cos(\theta_2) \\
p \cos(\theta_2) e^{ik_{z2}d} + q \sin(\theta_2) e^{i(n_{z2}+1)k_{z2}d} &= t \cos(\theta_3) e^{ik_{z3}d} \\
p \sin(\theta_2) e^{-ik_{z2}d\phi_k} + q \cos(\theta_2) e^{ik_{z2}d\phi_k} &= t \sin(\theta_3) e^{ik_{z3}d\phi_k}
\end{align*} \tag{11} \]

Solving for $r$ from these equations one obtains $r = N / D$ where
\[ \begin{align*}
N &= \left[ e^{i\phi_k} \cos(\theta_2) \sin(\theta_1 - \theta_2) + e^{i\phi_k} \cos(\theta_1 + \theta_2) \right] \\
&\quad \times \left[ \cos(\theta_1) - \left[ e^{i\phi_k} \cos(\theta_2) \sin(\theta_1) + e^{i\phi_k} \cos(\theta_2) \sin(\theta_1) \right] \right] \\
&\quad \times \cos(\theta_1 + \theta_2) + \sin(\theta_1 - \theta_2) \sin(\theta_2) \sin(\theta_3) \\
D &= \left[ e^{i(n_{z2}-2)\phi_k} \cos(\theta_2 + \theta_1) \sin(\theta_3) - e^{i\phi_k} \cos(\theta_3) \right] \\
&\quad \times \cos(\theta_1 + \theta_2) \sin(\theta_2) - \left[ e^{i(n_{z2}-2)\phi_k} \cos(\theta_3) \sin(\theta_3) \right] \\
&\quad \times \cos(\theta_1 - \theta_2) - \cos(\theta_1 - \theta_2) \cos(\theta_2) \cos(\theta_2)
\end{align*} \tag{12} \]
The expression of the transmission and hence the conductance can be obtained using Eq. 12 as $T = 1 - |r|^2$ and
\[ \begin{align*}
G &= G_0 \int_0^{k_{max}} kdk \int_0^{2\pi} d\phi_k T \\
G_0 &= \frac{e^2}{hN_1} \quad N_1 = \int_0^1 kdk \int_0^{2\pi} d\phi_k = \pi. \tag{13}
\end{align*} \]
Here $N_1 \equiv N k_0^2$ denote the total number of transverse modes around a Weyl node up to the cutoff $k_0$ for which the continuum Weyl model used here holds and $k_{max} = \text{Min}[|eV + \mu_N|^{1/n_{z1}} , |eV + \mu_N|^{1/n_{z2}}]$ is the largest momentum channel participating in current transport across the junction. Note that $k_{max}$ is determined by the condition that both $\theta_1 = \arcsin[k_{z1}^n / |eV + \mu_N|] / 2$ and $\theta_3 = \arcsin[k_{z2}^n / |eV + \mu_N|] / 2$ must be real for a particular momentum channel to conduct.

Next, we note that in contrast to junctions between
WSMs with $n_1 = n_2 = 1$ or two similar MSMs with $n_1 = n_2 \neq 1$, $|r|^2$, and hence $T$ possess non-trivial $\phi_k$ dependence for $n_1 \neq n_2$. To understand this phenomenon better, we now move to the thin barrier limit. In this limit, it is easy to see that $\theta_2, k_{z3}d \to 0$, and $k_{z2}d \to \chi$. The boundary conditions can then be written as

$$\psi_1(z = 0^-) = e^{-i\chi x} \psi_{III}(z = 0^+) \quad (14)$$

We note that this implies that the dimensionless barrier potential induces a rotation by $2\chi$ in spin space about the $z$ axis. For $n_1 = n_2$, this leads to oscillatory dependence of the conductance on $\chi$. In contrast, for $n_1 \neq n_2$, since $\psi_{III} \sim \exp[-i\tau_3 n_1 |n_2| \phi_k/2]$, the rotation induced by the barrier can be offset by changing $\phi_k \to \phi_k + \delta \phi$, where $\delta \phi = 2\chi/(n_1 - n_2)$. Thus the junction conductance, which involves a sum over all azimuthal angles, is expected to become barrier independent in the thin barrier limit.

To verify this expectation, we first substitute $\theta_2, k_{z3}d \to 0$, and $k_{z2}d \to \chi$ in Eq. (12) and obtain, after a few lines of algebra,

$$T_{ib} = \frac{A}{B - C \cos[(n_1 - n_2) \phi_k + 2\chi]}$$

$$A = \cos(2\theta_1) \cos(2\theta_3), \quad C = \sin(2\theta_1) \sin(2\theta_3)/2$$

$$B = \sin^2(\theta_1) \sin^2(\theta_3) + \cos^2(\theta_1) \cos^2(\theta_3) \quad (15)$$

From Eq. (15) we find that in the presence of a change in winding number across region I and III ($n_1 \neq n_2$), $\chi$ appears as a phase shift to the azimuthal angle $\phi_k$. Since $A$, $B$, and $C$ are independent of $\phi_k$, the integration over $\phi_k$ in Eq. (13) is straightforward and yields

$$\int_0^{2\pi} d\phi_k T_{ib} = T_1, \quad G = G_0 \int_0^{k_{max}} dk k T_1$$

$$T_1 = \frac{4\pi \cos(2\theta_1) \cos(2\theta_3)}{[\cos(2\theta_1) + \cos(2\theta_3)]^2} \quad (16)$$

Thus $G$ becomes independent of $\chi$ in the thin barrier limit according to our earlier expectation. This independence is a direct consequence of $\phi_k$ dependence of $T$ which happens for $n_1 \neq n_2$. Thus such a barrier independence of $G$ requires a change in the topological winding number across the junction; consequently, this effect would not show up in junctions between WSMs or MSMs with $n_1 = n_2$. We would like to point out that this phenomenon can only occur in $d > 2$ where there are more than one transverse directions; thus it does not have an analogue in 2D topological materials.

Next, we provide numerical support to our finding. To this end, we first obtain $G/G_0$ by numerically integrating $T_{ib}$ over $k$ and $\phi_k$. For all numerical plots we shall choose $\eta = \epsilon_0 = \epsilon_0 = 1$; we have checked that the numerical values of these quantities do not alter qualitative nature of the results presented. The corresponding results are shown in Fig. (2) and (3). In Fig. (2), we show the variation of $G/G_0$ as a function of the applied voltage $eV/E_0$ in the thin barrier limit for $n_1 - n_2 = -1$ with $n_1 = 1$ and $n_2 = 2$ and for two representative values of $\chi = 0, \pi/4$. We have checked that the behavior of $G$ is identical for $n_1 - n_2 = 1$ and qualitatively similar for $n_1 - n_2 = \pm 2$ for same $n_1$. The different behavior of $G$ as a function of $eV/E_0$ for $n_1 = 1$ and $n_1 = 2$ can be understood as follows. We note from Eq. (15) and (16) that for small $eV$, $\theta_3 \ll \theta_1$ (Eq. (9)). Consequently one may approximate

$$T_1 \simeq \frac{4\pi \cos(2\theta_1)}{[1 + \cos(2\theta_1)]^2} \quad \frac{4\pi \sqrt{1 - k_{max}^2/(eV + \mu_0)^2}}{1 + \sqrt{1 - k_{max}^2/(eV + \mu_0)^2}} \quad (17)$$

The integral $T_1$ over $k$ can then be analytically performed and leads to $G/G_0 \sim k_{max}^2 = c(eV + \mu_0)^{2/\alpha_1}$, where $c$ is
a constant. Thus $G/G_0$ is a parabolic (linear) function of the applied voltage for $n_1 = 1(2)$ and $\mu_N = 0$. An exactly similar behavior emerges when $n_1 > n_2$ since $T_1$ is symmetric under the interchange of $\theta_3$ and $\theta_1$. Note that for finite $\mu_N/E_0 < 1$ and $eV \ll \mu_N$, $G/G_0$ will always vary linearly with $eV$.

For both $n_1 = 1$ and $n_1 = 2$, from Fig. 2 we find that $G/G_0$ is independent of $\chi$. This independence can be more directly seen from Fig. 3(a). We also note that such a barrier independence is absent if $n_1 = n_2$; this is easily seen from Fig. 3(b), where $G$ oscillates with $\chi$ for a junction between two WSMs ($n_1 = n_2 = 1$) or MSMS ($n_1 = n_2 = 2,3$). We note that these numerical results confirm our earlier analytical expectation that the $\chi$ independence of $G$ is a consequence of the change in topological winding number across the junction.

Next, we investigate the fate of $G$ as a function of $V_0$ away from the thin barrier limit for several representative values of $d$. To this end, we numerically compute $T = 1 - |r|^2$ from Eq. 12 and use Eq. 13 to obtain $G$. Fig. 4 shows a plot of $G/G_0$ as a function of $U_0$ for several representative values of $d$. We note that $G/G_0$ has small oscillatory dependence on $U_0$; the amplitude of these oscillations decay as $U_0$ is increased and $G/G_0$ becomes independent of $U_0$ for large $U_0$. This is consistent with our earlier results in the thin barrier limit.

III. NBS JUNCTIONS

In this section, we study the transport through a NBS junction between a WSM and a MSM or two MSMS with different topological winding numbers. Throughout this section we shall work in the regime where the chemical potentials on the normal and superconducting regions (\(\mu_N\) and \(\mu_S\)) are large compared to the applied volt-

![Figure 4](image-url)  
**FIG. 4:** Plot of $G/G_0$ as a function of $U_0$ (in units of $E_0$) for $dk_0 = 1$ (red line), 0.1 (blue line) and 0.05 (green line). For all plots $eV/E_0 = 0.5$, $n_1 = 1$, and $n_2 = 2$. Here $d$ is measured in units of $k_0^{-1}$ and all other parameters are same as in Fig. 2.

![Figure 5](image-url)  
**FIG. 5:** A schemtaic representation of a NBS junction between two MSMS or a WSM and a MSM (characterized by topological winding numbers $n_1$ and $n_2$ as shown) in regions I and III separated by a barrier in region II ($-d \leq z \leq 0$). The barrier region II constitutes the same material as in region I but has an additional potential $U_0$. Superconductivity is induced in region III via a proximate $s$-wave superconductor. The reflection, Andreev reflections and transmission amplitudes, $r$, $r_A$, $t_1$ and 2, for an electron approaching the barrier from region I, is shown schematically.
where \(\tan(2\theta_3^s[\phi_k]) = \frac{e^0k_x}{e^0k_{z,2}[2]}\). In Eq. 19, \(k_{z,2}[2]\) correspond to electron-[-hole-]like quasiparticles and are given by (for \(\mu_S \gg eV, \Delta_0\))

\[
k_{z,2}[2] = +[-\sqrt{(\mu_S + i\zeta)^2y^2 - (\epsilon_0k_{n,2})^2}] = 1[i]\sqrt{\Delta_0^2 - (\epsilon V)^2}, \quad \text{for } eV \leq |\Delta_0|
\]

where we have scaled all energy scales by \(E_0\). We note that the wavefunctions of the quasiparticles and quasi-holes retain the property \(\psi_{3e(h)} \sim \exp[-i\tau_{z,2}n_0\phi_k/2]\).

The computation of tunneling conductance for such a junction follows the standard BTK formalism applied to topological materials. To this end, we consider a right moving electron in region I approaching the barrier. Upon reflection (Andreev) from the barrier, a left moving electron (hole) propagates to the left. The wavefunctions of these electron and holes are given by

\[
\psi_{1eR} = e^{(k_x+k_y+k_{z,1})e^{-i\tau_{z,1}n_0\phi_k/2}}(\cos(\theta_1), \sin(\theta_1), 0, 0)^T
\]

\[
\psi_{1eL} = e^{(k_x+k_y-k_{z,1})e^{-i\tau_{z,1}n_0\phi_k/2}}(\sin(\theta_1), \cos(\theta_1), 0, 0)^T
\]

\[
\psi_{1hR} = e^{(k_x+k_y+k_{z,1})e^{-i\tau_{z,1}n_0\phi_k/2}}(0, 0, -\sin(\theta_1'), \cos(\theta_1'))^T
\]

\[
\psi_{1hL} = e^{(k_x+k_y-k_{z,1})e^{-i\tau_{z,1}n_0\phi_k/2}}(0, 0, \cos(\theta_1'), -\sin(\theta_1'))^T
\]

where \(2\theta_1' = \arcsin[\epsilon_0k_{n,1}/|eV - \mu_N|]\), \(k_{z,1} = -\sqrt{(eV - \mu_N)^2 - \epsilon_0^2k_{n,1}^2}\), and we shall choose \(\mu_N = \mu_S\) for all numerical evaluations. In region I, the wavefunction can then be written as

\[
\psi_{1eR} = \psi_{1eR} + r\psi_{1eL} + r_A\psi_{1hL}
\]

where \(r\) and \(r_A\) denotes amplitude or ordinary and Andreev reflections respectively. We note that \(\sin(\theta_1') = -\sin(\theta_1)(eV + \mu_N)/|(eV - \mu_N)|\) so that \(\theta_1' \rightarrow -\theta_1\) for \(\mu_N \gg eV\). Moreover, we find that \(\psi_1 \sim e^{-i\tau_{z,1}n_0\phi_k/2}\); thus for both electrons and holes, one can interpret \(\phi_k\) dependence of the wavefunctions as a rotation in spin space about the \(z\) axis.

In region II, the wavefunctions of right/left moving electrons and holes are given by Eq. 21 with \(k_{z,1} \rightarrow k_{z,2}, \quad \theta_1 \rightarrow \theta_2, \quad k_{z,1}' \rightarrow k_{z,2}', \quad \text{and } \theta_1' \rightarrow \theta_2'\). The wavefunctions of left and right-moving electrons and holes are therefore given by

\[
\psi_{2eR} = e^{(k_x+k_y+k_{z,2})e^{-i\tau_{z,1}n_0\phi_k/2}}(\cos(\theta_2), \sin(\theta_2), 0, 0)^T
\]

\[
\psi_{2eL} = e^{(k_x+k_y-k_{z,2})e^{-i\tau_{z,1}n_0\phi_k/2}}(\sin(\theta_2), \cos(\theta_2), 0, 0)^T
\]

\[
\psi_{2hR} = e^{(k_x+k_y+k_{z,2}')e^{-i\tau_{z,1}n_0\phi_k/2}}(0, 0, -\sin(\theta_2'), \cos(\theta_2'))^T
\]

\[
\psi_{2hL} = e^{(k_x+k_y-k_{z,2}')e^{-i\tau_{z,1}n_0\phi_k/2}}(0, 0, \cos(\theta_2'), -\sin(\theta_2'))^T
\]

where \(2\theta_2' = \arcsin[\epsilon_0k_{n,1}/|eV - \mu_N|]\), and \(k_{z,2}' = -\sqrt{(eV - \mu_N + U_0)^2 - \epsilon_0^2k_{n,1}^2}\). We note that \(\theta_2'\) is related to \(\theta_1'\) by the relation \(2\theta_2' = \arcsin[\sin(2\theta_1')/|1 - U_0/(eV - \mu_N)|]\). The wavefunction
in region II is thus given by
\[ \psi_{II}^{nbs} = p_1 \psi_{2eR} + p_2 \psi_{2LR} + p_3 \psi_{2hL} + p_4 \psi_{2hR} \] (24)

In region III, the wavefunctions constitutes a superposition of electron-like and hole-like quasiparticles are given by
\[ \psi_{III}^{nbs} = t_1 \psi_{3e}^{s} + t_2 \psi_{3h}^{s} \] (25)
where \( \psi_{3e(h)}^{s} \) are wavefunctions of electron- and hole-like quasiparticles given by Eq. (19). We note that one can express \( \theta_3^{s} \) and \( \theta_3^{s} \) in terms of \( \theta_1 \) and \( \theta'_1 \) as
\[
\sin(2\theta_3^{s}) = \frac{(\sin(2\theta_1)|eV + \mu N|^{-1})^{n_2/n_1}}{\eta(\mu S + i\zeta)}
\]
\[
\sin(2\theta_3^{s}) = -\frac{(-\sin(2\theta_1')|eV - \mu N|^{-1})^{n_2/n_1}}{\eta(\mu S - i\zeta)}
\]

Also, we find that \( \psi_{3}^{nbs} \sim e^{-i\tau z n_1 \phi_k/2} \).

To compute the conductance, we solve for \( r \) and \( r_A \) numerically using Eqs. 27 and 28. One can then obtain
\( T_s = (1 - |r|^2 + |r_A|^2) \) and obtain the tunneling conductance of the junction using
\[ G_s = G_{0s} \int_{k_{max}^{-}}^{k_{max}^{+}} dk \int_{0}^{2\pi} d\phi_k k T_s \] (29)

where \( k_{max}^{+} = \text{Min}(|eV + \mu N|, |eV - \mu N|) \) and \( G_{0s}(eV + \mu N)^{2/n_1^2/4\pi h} \) is the normal state conductance of region I. Note that the expression of \( k_{max}^{+} \) follows from the requirement that both \( \sin(\theta_1) \) and \( \sin(\theta'_1) \) be real.

To verify this expectation, we first write out the above-mentioned boundary condition equations explicitly. This leads to a set of four equations for \( r, r_A, t_1 \) and \( t_2 \) given

To make further analytical processing, we now resort to the thin-barrier limit, for which \( U_0 \to \infty \) and \( d \to 0 \) with \( \chi = U_0d/\hbar v_F \) held fixed. As in Sec. 11, in this limit \( \theta_2, \theta'_2, k_{z1d}, k'_{z1d} \to 0 \) and \( k_{z2d}, k'_{z2d} \to \chi \). Consequently, it is easy to eliminate \( p_1, p_2, p_3 \) and \( p_4 \) from Eqs. 27 and 28. The boundary condition in the thin barrier limit can again be written as \( \psi_{I}^{nbs}(z = 0^-) = \psi_{II}^{nbs}(z = 0) = \psi_{III}^{nbs}(z = 0) \). The boundary condition at \( z = -d \) leads to

While that at \( z = 0 \) yields
\[
\begin{align*}
    t_1 \cos(\theta_3^{s}) e^{i\mu_0} + t_2 \cos(\theta_3^{s}) e^{-i\mu_0} &= p_1 \cos(\theta_2) + p_2 \sin(\theta_2) e^{-i\phi_k} \\
    t_1 \sin(\theta_3^{s}) e^{i\mu_0} + t_2 \sin(\theta_3^{s}) e^{-i\mu_0} &= p_1 \sin(\theta_2) e^{-i\phi_k} + p_2 \cos(\theta_2) \\
    t_1 \cos(\theta_3^{s}) e^{i\mu_0} + t_2 \cos(\theta_3^{s}) e^{-i\mu_0} &= p_4 \cos(\theta'_2) - p_3 \sin(\theta'_2) e^{-i\phi_k} \\
    t_1 \sin(\theta_3^{s}) e^{i\mu_0} + t_2 \sin(\theta_3^{s}) e^{-i\mu_0} &= -p_4 \sin(\theta'_2) e^{i\phi_k} + p_3 \cos(\theta'_2)
\end{align*}
\]

(28)
The denominator, written in terms of $z$, substitution for the subsequent discussion. In fact, from Eq. 32, we expect that

$$t_1 \cos(\theta_1^*) e^{i\mu_0} + t_2 \cos(\theta_3^*) e^{-i\mu_2} \phi_k$$

$$= e^{i\chi} \left[ \cos(\theta_1) + r \sin(\theta_1) e^{-i\mu_1} \phi_k \right]$$

$$= e^{-i\chi} \left[ \sin(\theta_1) e^{i\mu_1} + r \cos(\theta_1) \right]$$

$$= t_1 \cos(\theta_1^*) e^{i\mu_0} + t_2 \cos(\theta_3^*) e^{-i\mu_2} \phi_k$$

$$= e^{i\chi} \left[ \cos(\theta_1) + r \sin(\theta_1) e^{-i\mu_1} \phi_k \right]$$

Solving for $r$ and $R_A$ one obtains, in the thin barrier limit, we obtain $R_{tb} = |r|^2 = |Y/Z|^2$ and $R_A = |\cos(2\theta_1^*) \cos(2\theta_1) \sin(\theta_3 - \theta_3^*)/Z|^2$, where $Y$ and $Z$ are given by

$$Y = e^{i\mu_0} \left[ \sin(\theta_1) \cos(\theta_3^*) - e^{-i[|n_1 - n_2|\phi_k + 2\chi]} \cos(\theta_1) \sin(\theta_3^*) \right] \left[ \cos(\theta_1) \sin(\theta_3^*) + e^{i[|n_1 - n_2|\phi_k + 2\chi]} \sin(\theta_1) \cos(\theta_3^*) \right]$$

$$Z = e^{i\mu_0} \left[ \cos(\theta_1) \cos(\theta_3^*) + e^{i[|n_1 - n_2|\phi_k + 2\chi]} \sin(\theta_1) \cos(\theta_3^*) \right] \left[ -\cos(\theta_1) \cos(\theta_3^*) + e^{-i[|n_1 - n_2|\phi_k + 2\chi]} \sin(\theta_1) \sin(\theta_3^*) \right]$$

We note that both $R$ and $R_A$ displays a non-trivial $\phi_k$ dependence if $n_2 \neq n_1$. Further, in the thin barrier limit, the dimensionless barrier strength $\chi$ always appear as a phase shift to $(n_1 - n_2)\phi_k$. One can now aim to compute the transmission $T_{tb}^3$ and perform the $\phi_k$ integral. To this end, we find, after a cumbersome calculation,
oscillatory behavior is found.

The suppression of $G_s$ for $eV \leq \Delta_0$ and $n_2 > n_1$ in Figs. 6(a) and 6(b) can be qualitatively understood from Eq. 26. We first note that our numerical results are presented for $\mu_s = \mu_N \gg eV, \Delta_0$ and $\epsilon_0 = \epsilon'_0 = \eta = 1$. In this limit, one finds, from Eq. 26, $\sin(2\theta_s) \simeq (\sin(2\theta_1))^{n_2/n_1} \mu_N^{n_2/m_1-1}$. Thus $\theta_s^{(3)}$ has no real solution for a majority of the transverse channels for which $(\sin(2\theta_1))^{n_2/n_1} > \mu_N^{-1}$. For these transverse modes, $\sin(\theta_s^{(3)}) \rightarrow i \exp[(\sin(2\theta_1))^{n_2/n_1} \mu_N^{n_2/m_1-1}]/2$ for large $\mu_N$. A similar behavior is found for $\theta_s^{(1)}$. Thus from Eq. 31 one finds that for these modes $Z \sim \exp[2(\sin(\theta_1))^{n_2/n_1} \mu_N^{n_2/m_1-1}]$. Thus $R_A \sim \sin^2(\theta_3^{(3)} - \theta_s^{(3)})/|Z|^2 \sim \exp[-2 \mu_N^{n_2/m_1-1}(\sin(2\theta_1))^{n_2/n_1}]$ and vanishes exponentially for these modes. The number of such modes constitute a majority of the total available transverse modes for large $\mu_N$; consequently $G_s \rightarrow 0$ in this limit. We note that the suppression of the subgap tunneling conductance for large $\mu_N$ and $\mu_S$ is completely controlled by the change of the topological winding numbers $n_1$ and $n_2$ across the junctions. In contrast, for $n_2 < n_1$, $\theta_3^{(3)} \rightarrow 0$, since $\mu_N^{n_2/m_1-1} \ll 1$ in this regime. Similarly, from Eq. 26 we find that $\theta_s^{(3)} \rightarrow \pi/2$ in this limit. Thus $R_A$ remain finite and one finds finite subgap $G_s$ as can be seen in Fig. 5. Thus we conclude that the subgap tunneling conductance of these junctions depends crucially on the ratio $n_2/n_1$. We note that for $n_1 = 1, n_2 = 1$, our results reproduces those in Ref. 28 for $\mu_N, \mu_s \gg \Delta_0$ as special case.

IV. DISCUSSION

In this work, we have studied the tunneling conductance between junctions of a WSM and a MSM (or two MSMs) where the topological winding numbers of the Weyl nodes change across the junction. We have shown that the tunneling conductance of such junctions exhibits several unconventional features which are absent both in junctions involving 2D topological materials such as graphene or topological insulators surfaces and in those made out of 3D topological materials such as WSMs or MSMs with $n_1 = n_2$. The most striking of such features is the barrier independence of $G$ and $G_s$ in the thin barrier limit. We note that such a feature is in sharp contrast to both Schrodinger materials (where $G$ decays monotonically with increasing $\chi$) and previously studied topological materials (where $G$ oscillates with $\chi$). We demonstrate that such barrier independence is a consequence of the change in topological winding number of the Weyl nodes across the junction. Moreover, for NBS junctions with $\mu_N, \mu_S \gg eV, \Delta_0$, the subgap tunneling conductance $G_s(eV \leq \Delta_0)$ vanishes when $n_2 > n_1$; however, it is finite when $n_1 > n_2$. Thus the subgap tunneling conductance of such NBS junctions depend crucially on the ratio of the topological winding numbers of the WSMs/MSMs forming the junction.

The simplest experimental verification of our work would constitute formation of a junction between a WSM and MSM. The longitudinal direction of such junctions needs to be the symmetry axis of the MSM (taken to be $\hat{z}$ in our work). The barrier regions can be simulated by putting an additional local gate voltage $U_0$ on the WSM in a region of width $d$. For large $U_0$, we predict that $G(eV)$ will be independent of the dimensionless barrier strength $\chi$. Another, experimentally more challenging, possibility would be to study the subgap tunneling conductance of such junctions when superconductivity is induced either on the WSM or the MSM. We predict that the subgap tunneling conductance $G_s(eV \leq \Delta_0)$ in these two cases will show qualitatively different behavior for $\mu_N, \mu_S \gg eV, \Delta_0$. For the case, when superconductivity is induced in the MSM, $G_s$ will vanish; in contrast it will be finite, if superconductivity is induced in the WSM. However, in both cases, $G_s$ will be independent of $\chi$ for large $U_0$. We note that such features can also be observed in a junction constructed out of two multi-Weyl semimetals of similar material provided one applies a sufficiently large strain on one of them. This would split the Weyl nodes leading to $n = 1$ in that region while the other part will have $n \neq 1$. This will lead to the crucial jump in topological winding number across the junction and lead to predicted the barrier independent transport.

In conclusion, we have studied transport in NBN and NBS junctions between a WSM and a MSM or two MSMs with different topological winding numbers. We have shown demonstrated barrier independence of tunneling conductance for such junctions in the thin barrier limit and analyzed the role of the topological winding numbers in shaping the applied voltage dependence of their tunneling conductance. We have discussed experimental signatures of these phenomena.

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