How quantum nonlocality without entanglement witnesses classical communication without causal order

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We show a correspondence between quantum nonlocality without entanglement and logically consistent classical processes without causal order. Three parties with access to a noncausal classical process—the AF/BW process—can perfectly discriminate the states in an ensemble—the SHIFT ensemble—exhibiting quantum nonlocality without entanglement, something impossible with local operations and classical communication (LOCC): The LOCC restriction implicitly assumes causal classical communication. This provides an operational interpretation of the AF/BW process as a noncausal information-theoretic resource. From a foundational perspective, this means that the ability to perfectly discriminate quantum states in the SHIFT ensemble via local operations and classical communication would serve as an operational witness of noncausality. We then generalize our result to show how any Boolean classical process that maximally violates causal order can be transformed to an ensemble of states that exhibits quantum nonlocality without entanglement. Since multipartite generalizations of the AF/BW process are known, this yields novel multiqubit ensembles of quantum nonlocality without entanglement, which is of independent interest.

Introduction.—The celebrated results due to Bell [1] and Kochen and Specker [2] show that physical quantities are indefinite—they are not determined prior to and independent of their measurement context. In the Bell setup, this fact is shown through space-like separated parties who appropriately measure a shared quantum system. Any strategy to observe the same correlations with classical, i.e., locally predetermined, as opposed to quantum systems fails, unless the parties communicate [3]. The creation of these Bell nonlocal correlations necessitates that the parties hold an entangled quantum state. Later, Bennett et al. [4] uncovered quantum nonlocality without entanglement—a form of nonlocality different from Bell’s. They present locally imperfectly discriminable ensembles of unentangled and otherwise perfectly distinguishable quantum states, e.g., the SHIFT ensemble, \{000, 111, |+01\}, |−01\}, |1+0, |1−0\}, |01+, |01−\}. Parties sharing an unknown state from such an ensemble cannot perfectly identify the state with local (quantum) operations and classical communication (LOCC). Their result contradicts the view “that if the states of a quantum system were limited to a set of orthogonal product states, the system would behave entirely classically and would not exhibit any nonlocality” [4].

Only recently [5] it has been suggested to regard causal order as a physical quantity sensitive to quantum indefiniteness, e.g., as in the quantum switch [6] through indefinite wires connecting quantum gates [7] or through indefinite spacetime geometries formed from matter in a superposition of locations [8]. Oreshkov, Costa, and Brukner [9] show that if, without further assumptions on causal connections, one insists that parties locally cannot detect any deviation from standard quantum theory, then indefinite causal order arises naturally: Their framework for quantum processes describes the quantum switch [10], and also exhibits noncausal correlations, i.e., correlations unattainable under a global causal order among the parties (see also Refs. [11–13]). Moreover, the authors show that the exotic causal possibilities that arise between two parties are impossible with classical processes. For three parties or more, however, logically consistent classical processes that create noncausal correlations exist [14]. The deterministic AF/BW process [15, 16] exchanges bits among three parties, Alice, Bob, and Charlie, in the following way. Each party receives a bit \(a := (y \oplus 1) z\), \(b := (z \oplus 1) x\), \(c := (x \oplus 1) z\) from the process and thereafter provides a bit of their choice \(x, y, z\) to the process. This resource allows any pair of parties to communicate to the third (e.g., Alice receives a which non-trivially depends on \(y, z\) of Bob and Charlie) in a single round: Each party acts in the causal future of the other two.

In a general-relativistic context, these instances of “acausality without quantum theory” represent mild forms of closed time-like curves (CTCs) [17, 18]: They describe reversible and linear dynamics, are logically consistent without restricting the interventions of interacting parties, do not suffer from the information paradox [19], and are computationally tame [20, 21]. While CTCs are compatible with general relativity, as was shown
most prominently by Gödel [22], extreme forms proved to be pathological [23] and lack the above natural features (see, e.g., Refs. [24–32]).

Results.—In this Letter, we show that local quantum operations assisted with classical processes allow the parties to perfectly discriminate ensembles of quantum nonlocality without entanglement: Three parties communicating through the classical AF/BW process can discriminate the SHIFT ensemble. In fact, this process allows the parties to measure quantum systems in the SHIFT basis. Conversely, we show that such a measurement implements the AF/BW process. We use the insights from these protocols to show how any Boolean n-party classical process that maximally violates causal order can be turned into an n-qubit ensemble that exhibits quantum nonlocality without entanglement. While these ensembles are of independent interest, this work also establishes an operational link between quantum nonlocality without entanglement and classical communication without causal order (see also Ref. [33] for another link with Bell nonlocality).

In light of the ongoing endeavor to reconcile quantum theory with general relativity, our results state the following. If one regards a spacetime as classical if it cannot transport quantum information nor preserve entanglement between local labs, then parties in a classical general-relativistic spacetime that realizes the quantum system is pathological [23] and lack the above natural features, most prominently by Gödel [22], extreme forms proved to be pathological [23] and lack the above natural features (see, e.g., Refs. [24–32]).

The parties Alice, Bob, and Charlie hold a quantum system in the three-qubit state $|\psi\rangle$. The following protocol implements a measurement of $|\psi\rangle$ in the SHIFT basis (consult Figure 1). First, each party receives a classical bit $a, b, c$ from the process. Then, each party applies a Hadamard transformation on their share of $|\psi\rangle$ if the received bit is 1, i.e., the parties apply $H^{(a,b,c)} := H^a \otimes H^b \otimes H^c$. Now, they measure the quantum system in the computational basis, obtain the post-measurement state $|xyz\rangle$, and forward $x, y, z$ to the AF/BW process. Finally, the parties apply $H^{(a,b,c)}$ to the post-measurement state. By this, the final state of the quantum system is

$$\sum_{x,y,z} |xyz\rangle H^{(a,b,c)}|\psi\rangle^2 H^{(a,b,c)}|xyz\rangle \langle xyz| H^{(a,b,c)}.$$  \hspace{1cm} (1)

Note that the AF/BW process determines the values of $a, b, c$ as a function of $x, y, z$.

First, we show that if $|\psi\rangle \in$ SHIFT, then this protocol returns the state $|\psi\rangle$ with certainty. If $|\psi\rangle = |000\rangle$, then the probability

$$|\langle xyz| H^{(y \oplus 1)z, (z \oplus 1)x, (x \oplus 1)y}|000\rangle|^2$$  \hspace{1cm} (2)

is one for $x = y = z = 0$, and zero otherwise: The final state is $|000\rangle$. Instead, if $|\psi\rangle = |01+\rangle$, then the only contribution with certainty arises for $x = z = 0$ and $y = 1$: $|\langle 010| H^{(0,0,1)}|01+\rangle|^2 = 1$, and the final state is $H^{(0,0,1)}|010\rangle = |01+\rangle$. By symmetry, this follows for all states of the SHIFT ensemble. In other words, for each SHIFT state there exists a unique and distinct triple $x, y, z$ that contributes to the sum; namely, $x, y, z$ encode the qubits of the SHIFT state (0 if the qubit is in the state $|0\rangle$ or $|+\rangle$, and 1 otherwise). By linearity, this analysis extends to any quantum state $|\psi\rangle$: Measuring an arbitrary state $|\psi\rangle = \sum_{k \in \text{SHIFT}} \alpha_k |k\rangle$ in the SHIFT basis yields $\sum_{k \in \text{SHIFT}} |\alpha_k|^2 |k\rangle$, which is identical to the returned state of the protocol

$$|\alpha_{|000\rangle}|^2 |000\rangle \langle 000| + |\alpha_{|01+\rangle}|^2 H^{(1,0,0)}|001\rangle \langle 001| H^{(1,0,0)} + |\alpha_{|01+\rangle}|^2 H^{(0,0,1)}|010\rangle \langle 010| H^{(0,0,1)} + \cdots.$$  \hspace{1cm} (3)

Now it is clear that if the parties communicate through the AF/BW process, then they perfectly discriminate the SHIFT ensemble. The classical data collected in the above protocol uniquely specifies the SHIFT state they were given: The bits $a, b, c$ they receive from the process specify the basis, and the bits $x, y, z$ they receive from the measurement specify the state in the corresponding
basis, e.g., \( a = 0, b = 0, c = 1, x = 0, y = 1, z = 0 \) encode the state \( |01\rangle \).

**AF/BW process from SHIFT-basis measurement.**— Conversely, suppose three parties, Alice, Bob, and Charlie, have access to a hypothetical measurement device that upon measuring a quantum system in the SHIFT basis returns to each party the classical description of the post-measurement state. For instance, if the post-measurement state is \(|+01\rangle\), then Alice receives the label +, Bob 0, and Charlie 1. The following protocol (consult Figure 2) turns this device into an evaluation procedure for the AF/BW process, i.e., the parties start with three bits \( x, y, z \) of their choice and end up with \( a = (y \oplus 1)z, b = (z \oplus 1)x, c = (x \oplus 1)y \). Such a functionality therefore implements a communication channel from each party to the other two.

First, each party encodes their respective bit in the computational basis of a quantum state, i.e., they locally generate a quantum system in the state \( |\psi\rangle = |xyz\rangle \). In the second step, they feed \( |\psi\rangle \) into the hypothetical device and record the outcome \( \ell_A, \ell_B, \ell_C \in \{0, 1, +, -\} \), where \( \ell_A \) is Alice’s outcome and so forth. Finally, they apply the function \( f: 0 \rightarrow 0, 1 \rightarrow 1, + \rightarrow 1, - \rightarrow 1 \) to obtain the bits \( a, b, c \).

Suppose the bits \( x, y, z \) are chosen such that \( x = y = z \). The prepared quantum state \( |xyz\rangle \) is a member of the SHIFT basis. The measurement device therefore replies the labels \( \ell_A = \ell_B = \ell_C \in \{0, 1\} \), and, according to the protocol, the parties set \( a = b = c = 0 \), which is the correct value. If the bits are specified as \( x = y = 0 \) and \( z = 1 \), then the prepared state \( |xyz\rangle \) is not a member of the SHIFT ensemble and the measurement device responds probabilistically:

\[
|\langle +01|001\rangle|^2 = |\langle -01|001\rangle|^2 = 1/2.
\] (7)

In either case, however, the parties correctly end up with \( a = 1, b = c = 0 \). By symmetry, the parties compute \( a, b, c \) as desired for all inputs \( x, y, z \).

**Multiparticle quantum nonlocality without entanglement.**—The above protocols illustrate how to take a classical process and turn it into an ensemble of states that exhibits multipartite quantum nonlocality without entanglement. In this section, we show that all Boolean classical processes that violate causal order in a maximal sense—classical processes where each party can receive a signal from at least one other party—lead to such ensembles. For the sake of illustration, we define \( \omega^A \) as the function that corresponds to the AF/BW process:

\[
\omega^A : \{0, 1\}^3 \rightarrow \{0, 1\}^3
\]

\[
(x, y, z) \mapsto ((y \oplus 1)z, (z \oplus 1)x, (x \oplus 1)y).
\] (9)

The SHIFT basis is compactly expressed as

\[
\left\{ H(\omega^A)|\varphi\rangle \mid \varphi \in \{0, 1\}^3 \right\}.
\] (10)

Classical processes are neatly characterized by a unique fixed-point condition as follows (see Refs. [19, 35]). Let \( \omega^a \) be a Boolean function \( \{0, 1\}^n \rightarrow \{0, 1\}^n \), and define the set \( \mathcal{F}^n \) as the set of all functions from \( \{0, 1\}^n \) to \( \{0, 1\}^n \). The function \( \omega^a \) is a Boolean \( n \)-party classical process if and only if

\[
\forall \mu \in \mathcal{F}^n, \exists p \in \{0, 1\}^n : p = \omega^a(\mu(p)),
\] (11)

i.e., if and only if for each choice of interventions \( \mu_i \) of each party there exists a unique fixed point of \( \omega^a \circ \mu \). Here \( \mu \) should be understood as an \( n \)-tuple of local Boolean functions, i.e., \( \mu_i \) for each party \( i \in \{1, 2, \ldots, n\} \). Moreover, we say that \( \omega^a \) maximally violates causal order if and only if

\[
\forall i \exists k, \varphi \in \{0, 1\}^n : \omega^a(\varphi) \neq \omega^a(\varphi^{(k)}),
\] (12)

where \( \varphi^{(k)} = (x_1, \ldots, x_{k-1}, x_k \oplus 1, x_{k+1}, \ldots, x_n) \) is the same as \( \varphi \) but where the \( k \)-th bit is flipped, and where \( \omega^a \) is the \( i \)-th component of \( \omega^a \). This condition states that every party \( i \) can receive a signal through the process from at least one other party \( k \); no party lies in the global past of all other parties.

**Theorem.**—If \( \omega^a \) is a Boolean \( n \)-party classical process that maximally violates causal order, then

\[
\omega_n := \left\{ H(\omega^a)|\varphi\rangle \mid \varphi \in \{0, 1\}^n \right\}
\] (13)

is a basis of orthonormal states that exhibits quantum nonlocality without entanglement.

**Proof.**—The states in the set \( \omega_n \) with cardinality \( 2^n \) are obviously normalized. Now we show that they are orthogonal, i.e.,

\[
\forall \varphi \neq \varphi' : \langle \varphi | H(\omega^a(\varphi) \oplus \omega^a(\varphi')) \rangle = 0,
\] (14)

where the ‘\( \oplus \)’ in \( \omega^a(\varphi) \oplus \omega^a(\varphi') \) above is bitwise addition modulo two. Pick two \( n \)-bit strings \( \varphi \neq \varphi' \) and suppose without loss of generality that they differ in the first \( k \) positions only. The above condition states that there exists some \( i \leq k \) with \( \omega^a(\varphi_i) = \omega^a(\varphi'_i) \). Towards a contradiction, however, assume \( \forall i \leq k : \omega^a(\varphi_i) \neq \omega^a(\varphi'_i) \). Since \( \omega^n \)}
is a classical process, the reduced function $\tilde{\omega}^n : \{0,1\}^k \to \{0,1\}^k$ with
\[z \mapsto (\omega^n_k(z, x_{k+1}, \ldots, x_n), \ldots, \omega^n_k(z, x_{k+1}, \ldots, x_n)) \quad (15)\]
is a classical process as well (see, e.g., Ref. [17, Lemma A.3]). To simplify notation, let $\bar{z}'$ be the first $k$ bits of $\bar{z}$, and similarly for $\bar{y}'$, and define
\[a := \tilde{\omega}^n(z'), \quad b := \tilde{\omega}^n(y'). \quad (16)\]
Now, $a$ and $b$ are the fixed points under the following two interventions $\alpha$ and $\beta$, respectively:
\[
\alpha, \beta : \{0,1\}^k \to \{0,1\}^k
\]
\[
\alpha : w \mapsto x' \oplus a \oplus w
\]
\[
\beta : w \mapsto y' \oplus b \oplus w
\]
\[
\bar{a} = \tilde{\omega}^n(\alpha(a)) \quad (20)
\]
\[
\bar{b} = \tilde{\omega}^n(\beta(b)). \quad (21)
\]
However, because $\forall i \leq k : \bar{z}' = \bar{y}' \oplus 1 \wedge a_i = b_i \oplus 1$, the function $\tilde{\omega}^n \circ \alpha$ has at least two fixed points—$a$ and $b$—and therefore $\omega^n$ is not a classical process:
\[
\tilde{\omega}^n(\alpha(b)) = \tilde{\omega}^n(z' \oplus a \oplus b)
\]
\[
= \tilde{\omega}^n(y' \oplus b \oplus b)
\]
\[
= \tilde{\omega}^n(\beta(b)) \quad (24)
\]
\[
\bar{b} = \tilde{\omega}^n(\beta(b)). \quad (25)
\]
This proves that the set $S_{\omega^n}$ forms a basis of orthonormal states. What remains to show is that this set exhibits quantum nonlocality without entanglement. This follows directly from the assumption that $\omega^n$ maximally violates causal order. For each party $i$ there exist two bit-strings $\bar{z}, \bar{y}$ such that the $i$-th qubit of $H(\omega^n(\bar{z})|\bar{z})$ is in a mutually unbiased basis compared to the $i$-th qubit in $H(\omega^n(\bar{y})|\bar{y})$, as follows from Eq. (12).

**Examples.**—The following is an ensemble of states exhibiting quantum nonlocality without entanglement for four parties. It is constructed from the classical process of Ref. [33] which produces, due to its close relationship to the Ardehali-Svetlichny nonlocal game [36, 37], the strongest possible genuinely multipartite noncausal correlations with binary variables:
\[
\begin{align*}
\{0000, & 0\pm01, \pm01+, \pm01-, & 001-, & 01+0, & 1+-01, & 01-0, & 0111, & 1+0+, & 1+++, & 1+-01+, & 1+++-, & 1-00, & 1+-01, & 1111, & 1-00, & 1+-01, & 1111,
\end{align*}
\]
\[
\begin{align*}
& 0000, & 0\pm01, & 0\pm11, & 0\pm10, & 0\pm101, & 0\pm110, & 0\pm111, & 001+, & 001-, & 01+0, & 01-0, & 0111, & 1+0+, & 1+++, & 1+-01+, & 1+++-, & 1-00, & 1+-01, & 1111, & 1-00, & 1+-01, & 1111.
\end{align*}
\]

**Conclusions.**—We have shown here that Boolean $n$-party classical processes maximally violating causal order are in one-to-one correspondence with a family of $n$-qubit ensembles exhibiting quantum nonlocality without entanglement. We illustrated this connection explicitly for the tripartite case of the SHIFT ensemble introduced in Ref. [4] with respect to the AF/BW process [15, 16].

Several open questions arise from our results. We have, in particular, not discussed bipartite instances of quantum nonlocality without entanglement, e.g., the two-qutrit domino states of Ref. [4]. This is because in the bipartite case, logically consistent classical processes cannot violate causal inequalities, as shown by Oreshkov et al. [9]. To be sure, this instance of quantum nonlocality without entanglement can be interpreted as an instance of classical communication without causal order, but this seems to require a relaxation of the constraint of logical consistency which is central to the process-matrix framework [9]. In particular, such a relaxation leads to logical paradoxes. In the multipartite case, our results allow us to reinterpret the phenomenon of quantum nonlocality without entanglement as an operational witness of noncausality that has a qualitatively different character than the violation of causal inequalities. This opens up several potential connections with the wider literature on quantum nonlocality without entanglement and calls for a deeper understanding of its connection with noncausality. Indeed, as we have demonstrated, these results also offer a route to constructing new instances of quantum nonlocality without entanglement. We also know that in standard quantum theory, multiqubit instances of quantum nonlocality without entanglement are incapable of witnessing a strong form of nonclassicality, i.e., logical proofs of the Kochen-Specker theorem [39], and it would be interesting to investigate the implications of this fact for (non)causality in the process-matrix framework [9]. Similarly, higher-dimensional generalizations of multipartite quantum nonlocality without entanglement [40] could also inspire new types of noncausal classical processes. The bipartite case, together with other generalizations of our results in the multipartite setting, will be taken up in forthcoming work.

Let us also remark that whether the AF/BW process arises in classical general relativity would affect the interpretation of the noncausality witnessed via the perfect state discrimination task we have considered: If the process does arise in a classical general-relativistic spacetime, then perfect discrimination of the SHIFT ensemble under the assumption of local operations and (in general, noncausal) classical communication via this process would demonstrate the classical noncausality of the surrounding spacetime. If, on the other hand, the process is somehow forbidden in classical general relativity, then success in the discrimination task would imply that the AF/BW process has been realized in a manner that is intrinsically non-classical, i.e., it would certify intrinsi-
ernly non-classical noncausality arising from some theory of gravity that admits the AF/BW process, e.g., quantum gravity. To be sure, in such a situation, the communication between the labs would still be classical but the physical conditions for achieving this communication would be outside the realm of possibilities afforded by classical general-relativistic spacetimes. The latter possibility could have interesting implications for how one might interpret time-delocalized realizations of the AF/BW process [34].

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