Maverick Examples Of Coset Conformal Field Theories

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Abstract

We present coset conformal field theories whose spectrum is not determined by the identification current method. In these “maverick” cosets there is a larger symmetry identifying primary fields than under the identification current. We find an A-D-E classification of these mavericks.
Introduction

The coset construction [1] of conformal field theories (CFTs) [2,3] has proved to be a practical method for constructing rational conformal field theories. Indeed, it may be that all rational CFTs have a coset realisation. It is of central importance to identify the spectrum of primary fields present in such theories. For an affine algebra $\hat{g}$ with subalgebra $\hat{h}$, the primary fields of the coset CFT $\hat{g}/\hat{h}$ can be labelled $\phi^\Lambda_\lambda$ where $\Lambda$ and $\lambda$ are highest weights of the Lie algebras $g$ and $h$ respectively. Not all pairs of labels $(\Lambda, \lambda)$ give genuine and distinct primary fields but some combinations do not correspond to fields present in the coset, and some combinations of labels are equivalent [4]. In examining the spectrum of primary fields in a coset CFT, the procedure, due to Schellekens and Yankielowicz [6], of introducing an “identification” current, has proved extremely useful. However, by examining the specific examples $\hat{su}(3)_2/\hat{su}(2)_8$, $(\hat{E}_6)_2/(\hat{C}_4)_2$, $(\hat{E}_7)_2/\hat{su}(8)_2$ and $(\hat{E}_8)_2/(\hat{D}_8)_2$ in detail we show that this procedure is not always applicable, contrary to various suppositions. In these examples we find that extra identifications and the vanishing of branching functions occur. In addition we demonstrate that the series of cosets $\hat{su}(N)_2/\hat{so}(N)_4$ and $\hat{so}(2N)_2/\hat{so}(N)_2 \times \hat{so}(N)_2$ also have non simple current identifications, and exhibit additional null branching functions. We shall study the series $\hat{su}(N)_2/\hat{so}(N)_4$ in detail. The list of such “Maverick” cosets we have found has an A-D-E classification.

Review of the GKO construction

Here we briefly review the Goddard, Kent and Olive (GKO) [1] construction for rational conformal field theories and the Schellekens-Yankielowicz [6] mechanism for obtaining the primary fields of coset models, through the use of an identification current.

Consider the Kac-Moody algebra $\hat{g}$ [7], associated with the Lie algebra $g$. Within the extended algebra of the Kac-Moody algebra there is a Virasoro algebra, for which the stress-energy tensor, $T_g$, is formed using the Sugawara construction [8]. The central charge, $c_g$, is related to the integer level, $k$, of the Kac-Moody algebra by

$$c_g = \frac{k \dim g}{k + \tilde{h}} \quad (1)$$

where $\tilde{h}$ is the dual Coxeter number of the Lie algebra $g$.

Suppose $g$ has a subalgebra $h$ so that correspondingly $\hat{g}$ has a subalgebra $\hat{h}$. We can then form a new stress-energy tensor [1] $T_{g/h} = T_g - T_h$ which satisfies the O.P.E. for an energy momentum tensor with central charge $c_{g/h} = c_g - c_h$. 
This construction of the energy momentum tensor is the well known GKO construction for coset conformal field theories. Using this construction it is possible to construct many CFTs with relatively small central charge, and it may be that all rational CFTs have a coset realisation. The irreducible representations of \( \hat{g} \) are labeled by the highest weights of \( g, \Lambda \). At level \( k \), only those representations satisfying \( \psi \cdot \Lambda \leq k \), where \( \psi \) is the highest root of \( g \), are allowed. The conformal weight of the associated primary field is given by

\[
h_{\Lambda} = \frac{\Lambda^2 + 2\Lambda \cdot \rho_g}{2(k + h)}
\]

where \( \rho_g \) is half the sum of the positive roots of \( g \).

For a representation, \( \Lambda \), of the Kac-Moody algebra the restricted character is defined by

\[
\chi_{\Lambda} = \text{tr}_{\Lambda}(q^{L_0}).
\]

Since \( L_0^g = L_0^h + L_0^{g/h} \) it follows that the characters of \( g \) decompose into products of characters of \( h \) and \( g/h \),

\[
\chi_{\Lambda}(\tau) = \sum_{\lambda} \chi_{\lambda}^A(\tau) \chi_{\lambda}(\tau).
\]

The functions \( \chi_{\lambda}^A(\tau) \) are the “branching functions” of the coset CFT. Not all pairs of labels \((\Lambda, \lambda)\) give rise to non-zero branching functions and not all distinct labels give rise to distinct branching functions [4,5]. From (4) the \( h \)-values of such a field is given by

\[
h_{\Lambda}^A = h_{\Lambda} - h_{\lambda} + n
\]

where \( n \) can be calculated once (4) has been solved for \( \chi_{\lambda}^A \).

The Schellekens-Yankielowicz mechanism [6] for deciding which fields are non-zero and inequivalent is to use an “identification current” which is defined in terms of simple currents of the factors \( \hat{g} \) and \( \hat{h} \). A simple current of a general CFT, \( J \), is a primary field with the simple fusion rules [2,9] \( J \cdot \phi = \phi' \). For a rational CFT with a finite number of primary fields there must be an integer \( N \) such that \( J^N = 1 \). In general the action of \( J \) upon a primary field \( \phi \) will yield fields \( \{J^r \phi, r = 0, 1, \ldots, N - 1\} \), where \( J^{N_{\phi}} \phi = \phi \). The integer \( N_{\phi} \) must be a divisor of \( N \). When \( J \) has integer conformal weight, that is \( h(J) \) is integer. (In general it can be shown that \( h(J) = r/N \)), the CFT has a non-diagonal modular invariant (NDMI) [10,11] whose form has been suggested to be the diagonal
modular invariant of an extended algebra [4,5,10-13]. Most Kac-Moody algebras contain simple currents, some examples of which are given in following sections.

The relationship between the characters of the coset algebra and the branching functions has been the source of some confusion, but has been elegantly resolved by Schellekens and Yankielowcz [6]. This relies on the observation that the diagonal combination of characters, \( Z = \sum_{a} \chi_a \chi_a^\ast \) where the summation runs over all genuine characters of the coset CFT, must be modular invariant [14]. Expressing \( \chi_a \) as a sum of the branching functions, one should look for modular invariant combinations of the branching functions. To look for such modular invariants, note that the branching functions of \( \hat{g}/\hat{h} \) transform as the characters of \( \hat{g} \times \hat{h}^\ast \). Hence if one can find a suitable modular invariant for \( \hat{g} \times \hat{h}^\ast \) then the corresponding object for \( \hat{g}/\hat{h} \) will be modular invariant and a candidate for the diagonal modular invariant of the coset. Schellekens and Yankielowicz generate such a modular invariant using a simple current \( \phi_{J^1, J^2} \) of \( \hat{g} \times \hat{h}^\ast \). The corresponding field of \( \hat{g}/\hat{h} \), denoted \( J_I \), is called the identification current. After determining \( J_I = \phi_{J^1, J^2} \), the non-zero branching functions are those which have

\[
h(J_I \cdot \phi^\Lambda) - h(\phi^\Lambda) = 0 \mod(1)
\]

and we have the following equivalence

\[
\phi^\Lambda \equiv \phi^{J^1, J^2}.
\]

The details of the identification current, for a variety of cosets is given in [6]. The first condition is equivalent to requiring that \( \lambda \) occurs within the representation \( \Lambda \) of \( \hat{g} \). This is often referred to as a conjugacy class relation as it is equivalent to requiring \( \Lambda - \lambda' \) is a root of \( g \). (Where \( \lambda' \) is the embedding \( \lambda \).) Obviously unless this is satisfied, the character \( \chi^\Lambda \) is zero. It is an explicit and clear assumption of the identification current method that branching functions only vanish when this relationship is not satisfied. (As we shall see additional branching functions do vanish.)

As a simple example, for \( \tilde{su}(2)_k \) the simple current is \( (k) \) which satisfies \( (k) \cdot (l) = (k-l) \). For \( k \) odd there are no fixed points. For \( \tilde{su}(2)_k \times \tilde{su}(2)_1/\tilde{su}(2)_{k+1} \) (a realisation of the minimal models) the fields are \( \phi_{l_1, l_2}^{l_3} \). The condition for the branching function to be non-zero reduces to \( l_1 + l_2 - l_3 = 0 \mod(2) \), with the equivalence

\[
\phi_{l_1, l_2}^{l_3} \equiv \phi_{k+1-l_1, 1-l_2}^{k-l_3}
\]
which can be rearranged as the standard labelling for the fields of the minimal models.

The identification current method is elegant and applies to the majority of cosets, however, as we shall show in the next section, there exist cosets which cannot be described purely in terms of such an identification current.

**Maverick Cosets.**

In this section we present a class of coset theories whose spectrum is not described by the Schellekens-Yankielowicz procedure. In general these theories have a smaller spectrum of primary fields (h-values) than would be expected. This arises because both more branching functions are non-zero than predicted and more identifications occur. This suggests a larger symmetry than that used in the identification current method.

The simplest “maverick” coset is $\tilde{su}(3)_2/\tilde{su}(2)_8$ [16]. As was observed in ref. [16], where the characters of coset theories was studied, this model has more zero branching functions than predicted and, correspondingly, more equivalences. Since $c = 4/5$ for this model, the spectrum is entirely specified [17] and the spectrum predicted by the identification current method is clearly not viable.

The identification current is given by

$$J_I = \phi_8^{(00)}$$

and its fusion rules are

$$J_I \cdot \phi^A_\lambda = \phi^A_{8-\lambda}$$

Requiring $h^A_\lambda - h^A_{8-\lambda}$ to be an integer constrains $\lambda$ to be even. This is the only selection rule resulting from the simple current mechanism, or equivalently from conjugacy class considerations. However [16] evaluation of the characters (see table 1) gives extra identifications and nontrivial vanishing of characters, that is to say nontrivial selection rules. After, taking the extra identifications into account the spectrum matches perfectly that expected of the $c = 4/5$ minimal model in the non-diagonal modular invariant (NDMI) version. This is also the first element of the $W_3$ minimal series [18].

In the spirit of the identification current method we might expect there to be a non-trivial NDMI for the theory $\tilde{su}(3) \times \tilde{su}(2)^*$ reflecting the extra symmetry. Such a NDMI does exist, and is

$$Z = \left| \chi_0^{(00)} + \chi_8^{(00)} + \chi_4^{(11)} \right|^2 + \left| \chi_2^{(11)} + \chi_6^{(11)} + \chi_4^{(00)} \right|^2 + \left| \chi_2^{(10)} + \chi_6^{(10)} + \chi_4^{(02)} \right|^2$$

$$+ \left| \chi_2^{(01)} + \chi_6^{(01)} + \chi_4^{(20)} \right|^2 + \left| \chi_0^{(20)} + \chi_8^{(20)} + \chi_4^{(01)} \right|^2 + \left| \chi_0^{(02)} + \chi_8^{(02)} + \chi_4^{(10)} \right|^2$$

(11)
This NDMI only contains characters whose corresponding branching functions are non-zero and also reflects the identifications, for example \( \chi^{(00)}_0 \equiv \chi^{(00)}_8 \equiv \chi^{(11)}_4 \). However this NDMI is not generated by any simple current, as can be seen, for example, by looking at the component \( \chi^{00}_0 + \chi^{00}_8 + \chi^{11}_4 \). The last term is not related to the others by any simple current, and indeed is a fixed point of the identification current. If we consider the branching function \( \chi^{11}_4 \) which is equivalent to the identity, then the h-value is \( h^{(11)}_4 = h_{(11)} - h_4 + n \) where \( n \) is some integer. However the branching rule in the Lie algebra is

\[
(11) \rightarrow 2 \oplus 4
\]

so the 4 of \( su(2) \) occurs at the top level of the (11) of \( \widehat{su}(3)_2 \), and thus \( n \) is zero. This is confirmed by the computation of the characters in table 1. We thus find that \( h(\phi^{(11)}_4) = 0 \) and therefore this field must correspond to the identity. Thus it is possible to recognise \( \widehat{su}(3)_2/\widehat{su}(2)_8 \) as having a non trivial identification merely through examining the finite dimensional Lie algebra branching rules and looking for extra \( h = 0 \) currents, as the primary field corresponding to the identity in the fusion algebra must be unique. We thus avoid having to make a full evaluation of the branching functions. It is this feature which is exploited to obtain new examples.

By examining lists of cosets such as found in [19], and searching for examples of fields which must have, unexpectedly, \( h = 0 \) we have found the following examples

\[
\begin{align*}
\widehat{su}(N)_2/\widehat{so}(N)_4 \\
\widehat{so}(2N)_2/\widehat{so}(N)_2 \times \widehat{so}(N)_2 \\
(\hat{E}_6)_2/(\hat{C}_4)_2 \\
(\hat{E}_7)_2/(\hat{A}_7)_2 \\
(\hat{E}_8)_2/(\hat{D}_8)_2
\end{align*}
\]

(13)

In all these examples have the Lie algebra branching rule for the adjoint of \( \hat{g}, \psi_g \),

\[
\psi_g = \psi_h \oplus \lambda
\]

where there is only a single term \( \lambda \) on the r.h.s. Since this decomposition must occur at the top level in the representation we can deduce the integer shift is zero for the two branching functions \( \chi^{\psi_g}_{\psi_h} \) and \( \chi^{\psi_g}_\lambda \). With this information we can compute the h-value of the corresponding primary fields. For these examples we find \( h_{\psi_g} = h_\lambda \) implying \( h(\phi^{\psi_g}_\lambda) = 0 \) and hence

\[
\phi^{\psi_g}_\lambda \equiv \phi^{(00)}_0
\]

(15)
The three exceptional cases have central charge less than one so we can immediately check whether the identification current mechanism gives us a permissible set of weights.

For the $E_7/A_7$ case $c = 7/10$ and the spectrum of $h$-values of the fields in this minimal model is $\{0, \frac{3}{80}, \frac{1}{10}, \frac{1}{2}, \frac{7}{16}, \frac{3}{5}\}$ [17]. Applying the simple current mechanism with

$$J_I = \phi_{2\lambda_2}^{2\lambda_6}$$

we obtain the $h$-values, up to integer shifts, in table 2. The $h$-values indicated with a † are not permissible and must correspond to vanishing branching functions. Notice, that extra identification clearly exist for all fields. This model therefore is clearly a maverick with a considerably smaller spectrum (6 fields) than expected (18 fields). If one were using this model, for example, for superstring model building one would clearly be led to wrong conclusions on the spectrum using the standard identification current method.

If we examine the case $(\hat{E}_6)_2/(\hat{C}_4)_2$, we have $c = 6/7$. There appear to be numerous illegal $h$-values, as can be seen in table 3. Those $h$-values in table 3 which are permitted are $\{0, \frac{1}{21}, \frac{2}{7}, \frac{1}{3}, \frac{10}{21}, \frac{6}{7}\}$. This list corresponds exactly to that for the NDMI of the $c = 6/7$ minimal model. Hence this example is also a maverick theory. The final exceptional case $(\hat{E}_8)_2/(\hat{D}_8)_2$ has $c = 1/2$ and corresponds to the first minimal model. The case of $E_8$ is slightly trickier to deal with. In general, the identification current is valid for all values of the level $k$. However, $E_8$ has an exceptional simple current which only exists for $k = 2$ [15]. One could envisage incorporating this current into the identification current purely for $k = 2$. This is difficult to make work, however, because the extra simple current has half-integer weight and one is led to the conclusion that this model is also a maverick.

Let us now consider the sequence of theories $\hat{s\bar{u}}(N)_2/\hat{s\bar{o}}(N)_4$. Here the embedding is specified by the branching rule

$$\psi \rightarrow \psi \oplus s$$

where $s$ denotes the spinor representation of $so(N)$. We thus find $\phi^s_\psi \equiv \phi^0_0$. Similarly in $\hat{s\bar{o}}(2N)_2/\hat{s\bar{o}}(N)_2 \times \hat{s\bar{o}}(N)_2$ we find $\phi^s_\psi \equiv \phi^0_0$. Since both these series have $c = \frac{2(N-1)}{N+2}$ and $c = 1$ respectively we cannot immediately see which $h$-values, and therefore fields, are allowed. In order to do so it would be necessary to construct the modular $S$-matrix explicitly on a case by case basis. Alternatively one could attempt to identify the model involved in general. If we examine the
spectrum of h-values in the cosets $\hat{su}(N)_{2}/\hat{so}(N)_{4}$ for $N$ even then we find a large number of field which are equivalent to the identity field. Specifically

\[
\chi^{(1,0,0,\ldots,0,0,1)} = \chi^{(1,0,0,\ldots,0,1,0)} = \chi^{(0,0,0,\ldots,0,0,2,2)} = \chi^{(0,0,0,\ldots,0,2,0,\ldots,0,0,0,\ldots,0,0,0)}
\]

This list includes those identified with the identity by the identification current procedure. This large class of fields which are equivalent to the identity is indicative of a large symmetry between the characters. For all cases when $\phi_{\Lambda}^{\alpha}$ is equivalent to $\phi_{0}^{\alpha}$ then $\Lambda$ is a member of the root lattice. In fact if we look at the set of coincidence we find that whenever $\phi_{\Lambda}^{\alpha} \equiv \phi_{0}^{\alpha}$ then we have $\Lambda - \Lambda'$ in the root lattice. If we look at the list of fields above then we find that the set of $\{\Lambda\}$ at level 2 differing from 0 is in fact saturated. Before looking at the general structure of the $\hat{su}(N)_{2}/\hat{so}(N)_{4}$ series we will look at the cases $\hat{su}(4)_{2}/\hat{su}(2)_{4} \times \hat{su}(2)_{4}$ and $\hat{su}(5)_{2}/\hat{so}(5)_{4}$. The first case is a rather special “low $N$” case which we shall examine in detail and the second shall give us the flavour of the series. For the case $\hat{su}(4)_{2}/\hat{su}(2)_{4} \times \hat{su}(2)_{4}$ the embedding of the $\hat{su}(2)_{2k} \times \hat{su}(2)_{2k}$ within $\hat{su}(4)_{k}$ is given by

\[
\begin{align*}
J_{m}^{\pm} &= (J_{m}^{\pm \alpha_{1}} + J_{m}^{\pm \alpha_{3}}); \\
H_{m} &= \left( H_{m}^{1} - \sqrt{\frac{1}{3} H_{m}^{2}} + \sqrt{\frac{2}{3} H_{m}^{3}} \right) \\
\tilde{J}_{m}^{\pm} &= (J_{m}^{\pm (\alpha_{1} + \alpha_{2})} + J_{m}^{\pm (\alpha_{2} + \alpha_{3})}); \\
\tilde{H}_{m} &= \left( \sqrt{\frac{1}{3} H_{m}^{2}} + \sqrt{\frac{2}{3} H_{m}^{3}} \right)
\end{align*}
\]

With this embedding one may calculate the branching functions for the cosets as given in ref. [16]. One finds that there are zero characters and extra equivalences beyond that expected from the identification current method. For this coset there is a pair of identification currents

\[
J_{1} = \phi_{4,0}^{(020)}, \quad J_{2} = \phi_{0,4}^{(020)}
\]

In table 4 the spectrum predicted using these identification currents is shown. Also, from a direct analysis of the branching functions [16] we have zero characters as indicated in the table. In addition there are correspondingly extra equivalences. From the branching functions alone there is a little ambiguity in regards the conjugate weights within $su(4)$. This we have resolved so that $\Lambda - \lambda$ is a root. This is consistent with the $\hat{su}(3)_{2}/\hat{su}(2)_{8}$ case where this choice was vindicated by the non-diagonal modular invariant of $\hat{su}(3)_{2} \times \hat{su}(2)_{5}$. One notable feature of the list is the branching function $\chi_{2,2}^{(101)}$. This branching function may be found by inspection to be the sum of two separate characters

\[
\chi_{2,2}^{(101)} = \chi_{1,1}^{(001)} + \chi_{2,1}^{(100)}
\]
of the coset. The branching function must contain the identity since \( h = 0 \). We find that

\[
\chi_{2,2}^{(101)} = 1 + q + 2q^2 + 5q^3 + 7q^4 + 10q^5 \cdots
\]

\[
= (1 + q^2 + 2q^3 + 4q^4 + 5q^5 \cdots) + q(1 + q + 3q^2 + 3q^3 + 5q^4 + \cdots)
\]

\[
= \chi_{0,0}^{(000)} + q\chi_{0,4}^{(000)}
\]

This is a rather surprising feature. It is usual that branching functions \( \chi_\Lambda \) correspond to irreducible characters of the coset but this provides a counter-example to this intuition. This feature will relate to the existence of fixed points of the identification current [15].

We now turn to a simpler case, \( \hat{su}(5)/\hat{so}(5)_4 \). The spectrum of \( h \)-values determined using the identification current method is given in table 5. In this case we will not calculate the branching function explicitly but shall ‘deduce’ or postulate the spectrum. Since we know \( \phi^{(1001)}_{(20)} \) is equivalent to the identity definately, plus various other equivalences we postulate that only the fields indicated are non-zero and basically each field in the coset has three different labelings (in addition to those for the identification current.) For example, \( \phi_{(00)}^{(0000)} \equiv \phi_{(02)}^{(1001)} \equiv \phi_{(20)}^{(0110)} \) and \( \phi_{(20)}^{(0200)} \equiv \phi_{(12)}^{(0001)} \equiv \phi_{(02)}^{(1010)} \). In the second case we had a choice of identifications due to the conjugacy (i.e. to take \( (1010) \) or \( (0101) \)). We have been able to do so by requiring that \( \Lambda - \Lambda' \) is a root in all cases. For this example we could have identified the spectrum of this coset by selecting the fields determined by the identification current and then ordering the set of \( \{ \Lambda \} \) into conjugacy classes under the root lattice. The entire spectrum could then be determined by only using a single \( \{ \Lambda \} \) from each conjugacy class. With this procedure we obtain a spectrum of \( h \)-values \( (0, (2/35)^2, (3/35)^2, (1/5)^2, (2/7)^2, (17/35)^2, (23/35)^2, (4/5)^2, (6/7)^2, ) \) where the \( h \)-values are quoted up to an integer part. Have we any confidence that this spectrum is genuine?. In fact this spectrum matches precisely that of the first element of the minimal \( W_5 \) extended algebra. In general the \( \hat{su}(N)_2/\hat{so}(N)_4 \) has the same \( c \)-value and, after making the postulated changes to the procedure for obtaining the spectrum, spectrum of \( h \)-values as the first element of the \( W_N \) minimal series, \( (\hat{su}(N)_1 \times \hat{su}(N)_1)/\hat{su}(N)_2) \). When one looks at the examples \( \hat{su}(3)_2/\hat{su}(2)_8 \) and \( \hat{su}(4)_2/\hat{su}(2)_4 \times \hat{su}(2)_4 \) where we have calculated the branching functions we find the characters match exactly those for the first elements of the minimal series of \( W_3 \) and \( W_4 \) algebras respectively.

The examples we have found have a very simple A-D-E classification These cosets are \( \hat{g}_2/\hat{h}_{2k'} \), one for each \( \hat{g} \) in the A-D-E series. These models are postulated to be equivalent to the first elements of the minimal series \( \hat{g}_1 \times \hat{g}_1/\hat{g}_2 \).
Although a proof is lacking, we have studied carefully models outside this classification. For example, we have take $\hat{G}/\hat{h}$ for all possible maximal $\hat{h}$ ( $\hat{su}(3)$, $\hat{su}(2)$, $\hat{su}(2) \times \hat{su}(2)$) and have found the spectrum generated by the identification current methods to be correct. We have no examples where $\hat{g}$ is other than simply laced and at level 2.

**Conclusions**

In examining the spectrum of primary fields of coset theories, we have found that coset conformal theories have a more subtle structure than previously understood and have found a class of cosets with a “maverick” spectrum. In these theories, there is a smaller spectrum of primary fields than predicted by the identification current method indicating a larger symmetry than used in that method. The maverick behaviour only occurs for the level equal to two; for higher levels the spectrum being described correctly by the identification current method. The smaller spectrum arises firstly, from the vanishing of branching functions other than that expected purely from conjugacy class selection rules and, secondly, from the extra equivalences amongst fields.

The examples given all have the following list of properties in common:

1) $G/H$ is a symmetric space.
2) $\hat{h}_{k'} \subset \hat{g}_1$ is a conformal embedding.
3) $g$ is simply laced.
4) $\hat{g}$ is at level $k = 2$.
5) An additional $h = 0$ field $\phi_{\lambda}^\psi$ is uniquely determined by the Lie algebra branching rule specifying the embedding.
6) $\phi_{\lambda}^\psi$ is a fixed point of the identification current.

Exactly how these properties are related to the existence of a sufficient number of null states in the representations of $\hat{g}$ that we have additional null branching functions is not apparent. It would be significant if one could find an example which does not satisfy the above criteria. The extra symmetry responsible for maverick behaviour is perhaps related to the $W_N$ and related algebras.

The majority of coset models have, of course, a spectrum determined by the identification current. The models we have found (by an exhaustive search of cosets in [19]) have a classification in terms of the A-D-E series, which appears in so many diverse areas of physics. For each element, $g$, of the A-D-E series we have precisely one example $\hat{g}/\hat{h}$ which is a maverick. This classification is very interesting and may lead to an understanding of the occu-
rance of Maverick coset and eventually a better understanding of coset theories in general.

This work was supported by a S.E.R.C advanced fellowship a NATO grant CRG-910285 (D.C.D) and a S.E.R.C studentship (K.G.J). We thank John Gracey and David Olive for helpful discussions.
### Equivalences of Character

| Character Value | Character Value |
|-----------------|-----------------|
| \( \chi^{(00)}_0 \equiv \chi^{(00)}_8 \equiv \chi^{(11)}_4 \) | 1 + \( q^2 + 2q^3 + 3q^4 + 4q^5 \cdots \) |
| \( \chi^{(11)}_2 \equiv \chi^{(11)}_6 \equiv \chi^{(00)}_4 \) | \( 1 + 2q + 2q^2 + 4q^3 + 5q^4 + 8q^5 \cdots \) |
| \( \chi^{(10)}_2 \equiv \chi^{(10)}_6 \equiv \chi^{(02)}_4 \) | \( 1 + q + 2q^2 + 3q^3 + 5q^4 + 7q^5 \cdots \) |
| \( \chi^{(01)}_2 \equiv \chi^{(01)}_6 \equiv \chi^{(20)}_4 \) | \( 1 + q + 2q^2 + 2q^3 + 4q^4 + 5q^5 \cdots \) |
| \( \chi^{(02)}_0 \equiv \chi^{(02)}_8 \equiv \chi^{(10)}_4 \) | 0 |
| \( \chi^{(10)}_0 \equiv \chi^{(10)}_6 \equiv \chi^{(00)}_4 \) | 0 |
| \( \chi^{(01)}_0 \equiv \chi^{(01)}_6 \equiv \chi^{(20)}_4 \) | 0 |
| \( \chi^{(00)}_0 \equiv \chi^{(00)}_8 \equiv \chi^{(11)}_4 \) | 0 |

### Table 1.

For the coset theory \( \hat{su}(3)/\hat{su}(2)_8 \), we show the extra equivalences and vanishing of characters beyond that expected by the identification current method. Using the identification current method all characters shown are expected to be non-zero and only the first equivalences are expected.

| \( A_7 \) weight | \( E_7 \) weight label for orbit |
|------------------|---------------------------------|
| 0                | 0 \( \lambda_1 \) \( \lambda_6 \) \( \lambda_7 \) |
| 0                | 0 \( \frac{9}{10} \) \( \frac{9}{5} \) |
| 2\( \lambda_2 \) | \( \frac{1}{2} \) \( \frac{3}{5} \) |
| \( \lambda_4 \)  | \( \frac{1}{10} \) \( 0 \) |
| \( \lambda_2 + \lambda_6 \) | \( \frac{3}{5} \) \( \frac{1}{2} \) |
| \( \lambda_1 + \lambda_3 \) | \( \frac{7}{10} \) \( \frac{3}{5} \) |
| \( \lambda_3 + \lambda_5 \) | \( \frac{3}{5} \) \( \frac{1}{2} \) |
| \( \lambda_2 \)   | \( \frac{3}{80} \) \( \frac{51}{80} \) |
| \( \lambda_1 + \lambda_5 \) | \( \frac{7}{16} \) \( \frac{3}{80} \) |
| 2\( \lambda_1 \)  | \( \frac{67}{80} \) \( \frac{7}{16} \) |
Table 2. The $h$-values for the coset $(\hat{E}_7)_2/(\hat{A}_7)_2$ after calculating the spectrum using the identification current. Only the non-zero inequivalent fields are shown. Those shown with a $\dagger$ are not part of the spectrum of the $c = 7/10$ minimal model and hence must vanish.

| $C_4$ weight | $E_6$ weight label |
|--------------|--------------------|
| 0            | $0^*$              |
| $\lambda_4$ | $\frac{1}{7}$     |
| $\lambda_1 + \lambda_3$ | $\frac{6}{7}^*$   |
| $2\lambda_1$ | $\frac{2}{7}^*$  |
| $2\lambda_2$ | $\frac{5}{7}$    |

Table 3. The $h$-values (up to integer shift) for the coset $(\hat{E}_6)_2/(\hat{C}_4)_2$. Those field indicated $\ast$ appear in the $c = 6/7$ minimal model. All other branching functions must vanish.

| $SU(2) \times SU(2)$ weight | $SU(4)$ weight label |
|------------------------------|---------------------|
|                              | (000)   | (100)   | (001)   | (200)   | (101)   | (010)   |
| 0, 0                         | 0       | .       | .       | $\frac{3}{4}$ | $\frac{8}{12}$ | $\frac{5}{12}$ |
| 0, 2                         | $\frac{2}{3}^*$ | .       | .       | $\frac{5}{12}$ | $\frac{1}{3}$ | $\frac{1}{12}$ |
| 0, 4                         | 0       | .       | .       | $\frac{3}{4}$ | $\frac{8}{12}$ | $\frac{5}{12}$ |
| 1, 1                         | .       | $\frac{1}{16}$ | $\frac{1}{16}$ | .       | .       |
| 1, 3                         | .       | $\frac{9}{16}$ | $\frac{9}{16}$ | .       | .       |
| 2, 0                         | $\frac{2}{3}^*$ | .       | .       | $\frac{5}{12}$ | $\frac{1}{3}$ | $\frac{1}{12}$ |
| 2, 2                         | $\frac{1}{3}$ | .       | .       | $\frac{1}{12}$ | 0       | $\frac{3}{4}$ |

Table 4. The $h$-values for the coset $(\hat{su}(4))_2/(\hat{su}(2)_4 \times \hat{su}(2)_4)$. The values shown correspond to the spectrum predicted by the identification current method. Those
values indicated by † are in fact zero when explicitly evaluated. Only one of each conjugate pair of \( \widehat{su}(4) \) weights is shown.
| SO(5) weight | SU(5) weight label |
|--------------|--------------------|
|              | (0000) | (1001) | (0110) | (2000) | (1100) | (0100) | (0200) | (1000) | (1010) |
| (0, 0)       | 0 *    | 5/7    | 1/7    | 4/9    | 3/35   | 18/35  | 1/5    | 12/35  | 32/35  |
| (1, 0)       | 5/7    | 3/7    | 6/7 *  | 18/35  | 23/35  | 8/35   | 32/35  | 2/35   | 22/35  |
| (2, 0)       | 2/7 *  | 0 *    | 3/7    | 3/35   | 8/35   | 28/35  | 17/35  | 22/35  | 1/5 *  |
| (0, 2)       | 4/7    | 2/7 *  | 5/7    | 13/35  | 18/35  | 3/35   | 27/35  | 32/35  | 17/35 *|
| (1, 2)       | 1/7    | 6/7 *  | 2/7 *  | 33/35  | 3/35   | 23/35  | 12/35  | 17/35  | 2/35   |
| (0, 4)       | 6/7 *  | 4/7    | 0 *    | 23/35  | 28/35  | 13/35  | 2/35   | 1/5 *  | 27/35  |

**Table 5.** The $h$-values for the coset $\hat{su}(5)_{2}/\hat{so}(5)_{4}$. All values shown are allowed by the identification current procedure. Those indicated by * are those postulated to correspond to genuine non-zero branching functions.
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