Nonvanishing Cosmological Constant of Flat Universe in Brane-World Scenario

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Abstract

The finite temperature effect is examined in Randall-Sundrum brane-world scenario with inclusion of the matter fields on the brane. At zero temperature it is found that the theory on the brane is conformally invariant, which guarantees AdS/CFT. At 4d effective action we derived a temperature-dependent nonvanishing cosmological constant at the flat spacetime limit of brane world-volume. At the cosmological temperature 3K the cosmological constant is roughly $(0.0004eV)^4$ which is within the upper bound of the recent experimental value $(0.01eV)^4$

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The recent astronomical observations \[1\text{-}3\] indicate that our universe is flat and has a nonvanishing positive cosmological constant. In this letter we will show that these important features of our universe can be implemented within the Randall-Sundrum(RS) brane-world scenario \[4,5\] if the effect of nonzero temperature is properly involved.

In the RS brane-world scenario our universe is treated as a three dimensional brane embedded in the higher dimensional spacetime. This picture and its variants are used to solve the various long-standing puzzles such as gauge hierarchy \[4\], Newton gravity \[5,6\], cosmological constant \[7,8\], and black holes \[9,10\]. It is also shown that the brane-world scenario supports a non-static cosmological solution \[11\text{-}13\] which leads to the conventional Friedmann equation by introducing bulk and brane cosmological constants and imposing a particular fine-tuning condition between them.

The RS bulk spacetime is two copies of $AdS_5$ glued in a $Z_2$-symmetric way along a boundary which is interpreted as the three-brane world-volume. This fact is very useful to re-interpret the RS scenario within $AdS$/CFT or holography principle \[14\]. From the fact that RS picture yields similar results to those of the Horava-Witten scenario \[15\] one can imagine that it is somehow related to the string theories. In this context it is interesting to examine whether or not the similarity of RS scenario to $AdS$/CFT is maintained at finite temperature.

It is well-known that $AdS_5$ in $AdS$/CFT is extended to the Schwarzschild-$AdS_5$ \[16\] at nonzero temperature by taking a non-extremal limit of black 3-brane solution of the string theories. Therefore the similarity of RS picture with $AdS$/CFT strongly suggests that the RS bulk spacetime at finite temperature is two copies of the Schwarzschild-$AdS_5$ attached along the boundary as follows

$$ds^2 = e^{-2k|y|} \left[ -\left( 1 - \frac{U_T^4}{k^4} e^{4k|y|} \right) dt^2 + \sum_{i=1}^3 dx^i dx^i \right] + \frac{dy^2}{1 - \frac{U_T^4}{k^4} e^{4k|y|}}$$

(1)

where $k$ is the inverse of $AdS$ radius and $U_T$ is the horizon parameter proportional to the external temperature.

However, it is shown in Ref. \[17\] that the spacetime \[1\] is not vacuum solution of $5d$
Einstein equation

\[ R_{MN} - \frac{1}{2} G_{MN} R = -\frac{1}{4M^3} \left[ \Lambda G_{MN} + v_b G_{\mu\nu} \delta_M^\mu \delta_N^\nu \delta(y) \right] \tag{2} \]

where \( \Lambda, M, \) and \( v_b \) are 5d cosmological constant, 5d Planck scale, and brane tension, although we adopt appropriate fine-tuning conditions. This means the similarity of RS scenario to AdS/CFT is not trivially extended to nonzero temperature case.

It is worthwhile noting that Schwarzschild-AdS\( _5 \) spacetime \(^1\) solves 5d Einstein equation (2) in the whole bulk except only \( y = 0 \) if one chooses \( \Lambda = -24M^3k^2 \) and \( v_b = 24M^3k(1 - U^4_7/k^4) \). Thus, there exists a possibility to make the spacetime \(^1\) to be a solution of 5d Einstein equation if the content on the brane is changed. We will show in the following that it is indeed the case when the matter fields are involved on the brane. Furthermore, inclusion of the matters on the brane naturally provides a nonvanishing cosmological constant at 4d effective action level even if the world-volume of three-brane is flat spacetime.

Considering the matter fields on the brane modifies 5d Einstein equation as follows:

\[ R_{MN} - \frac{1}{2} G_{MN} R = -\frac{1}{4M^3} \left[ \Lambda G_{MN} + (v_b G_{\mu\nu} - S_{\mu\nu}) \delta_M^\mu \delta_N^\nu \delta(y) \right] \tag{3} \]

which is derived by taking a variation to the action

\[ S = \int d^4x \int dy \sqrt{-G} \left[ -\Lambda + 2M^3R + (-v_b + \mathcal{L}_m) \delta(y) \right] \tag{4} \]

where \( \mathcal{L}_m \) is Lagrangian for matter fields and \( S_{\mu\nu} \) is corresponding energy-momentum tensor derived from \( \mathcal{L}_m \);

\[ S_{\mu\nu} = \mathcal{L}_m G_{\mu\nu} - 2 \frac{\delta \mathcal{L}_m}{\delta G^{\mu\nu}}. \tag{5} \]

Inserting an ansatz

\[ ds^2 = e^{-2\sigma(y)} \left[ -f(y) dt^2 + \delta_{ij} dx^i dx^j \right] + \frac{dy^2}{f(y)} \tag{6} \]

\(^1\)Our conventions are \( M, N = 0, 1, 2, 3, 5 \), \( \mu, \nu = 0, 1, 2, 3 \) and \( i, j = 1, 2, 3 \).
into Eq. (3) Einstein equation provides the following three linear independent equations;

\[ 6 f \sigma'^2 - \frac{3}{2} f' \sigma' = -\frac{\Lambda}{4M^3} \]  

\[ 3 \sigma'' = \frac{1}{4M^3} \left[ v_b + \frac{S_{00}}{f} e^{2\sigma} \right] \delta(y) \]

\[ \frac{1}{2} f'' - 2 f' \sigma' = \frac{1}{4M^3} \left[ S_{00} \frac{S}{f} + S_{11} \right] e^{2\sigma} \delta(y) \]

where we assumed \( S_{\mu\nu} \) is a diagonalized tensor with \( S_{11} = S_{22} = S_{33} \). In fact, this assumption is consistent with the rotational symmetry of \( \text{ansatz (3)} \) in the brane coordinates \( x^i \).

As mentioned before we would like to derive the conditions for the Schwarzschild-AdS\(_5\) spacetime, \( \text{i.e.} \ f(y) = 1 - \xi e^{4k|y|} \) and \( \sigma(y) = k|y| \) where \( \xi = U_T^4/k^4 \), to be a solution of Eq. (7). For the convenience we assume that the stress-energy tensor \( S^M_N \) is \( S^M_N = \delta(y)\sqrt{1-\xi}\text{diag}(-\rho_0, p_0, p_0, p_0, 0) \) where \( \rho_0 \) and \( p_0 \) are the energy density and pressure of matters respectively. For the homogeneous property of our universe we assume that \( \rho_0 \) and \( p_0 \) are not dependent on the brane coordinate \( x^i \). Since we are considering only the static case now, we just treat \( \rho_0 \) and \( p_0 \) as some constants. When, however, we consider a non-static case later, these will be time-dependent quantities.

Inserting \( S^M_N \) into Eq. (3) Einstein equations become in the form:

\[ 6 f \sigma'^2 - \frac{3}{2} f' \sigma' = -\frac{\Lambda}{4M^3} \]  

\[ 3 \sigma'' = \frac{1}{4M^3} \left[ v_b + \sqrt{1-\xi} \rho_0 \right] \delta(y) \]

\[ \frac{1}{2} f'' - 2 f' \sigma' = \sqrt{1-\xi} \rho_0 \delta(y) \]

where \( w_0 \equiv p_0/\rho_0 \). Hence, it is easy to derive the conditions for \( f(y) = 1 - \xi e^{4k|y|} \) and \( \sigma(y) = k|y| \) to be a solution of Eq. (8);

\[ v_b + \sqrt{1-\xi} \rho_0 = 24M^3k(1 - \xi) \]  

\[ \sqrt{1-\xi}(1 + w_0)\rho_0 = -16M^3k\xi \]

with a fine-tuning condition of 5d cosmological constant \( \Lambda = -24M^3k^2 \). Therefore it is impossible to fix \( \rho_0, p_0 \) and \( v_b \) completely within Einstein equation.
In fact, one can remove the redundancy of the parameters if the 5d Einstein equation (3) is modified as follows;

\[ R_{MN} - \frac{1}{2} G_{MN} R + \frac{\Lambda}{4M^3} G_{MN} = \delta(y) \text{diag}(\alpha, \beta, \beta, \beta, 0). \]  

(10)

In this case it is easy to show that the Schwarzschild-AdS$_5$ spacetime completely solves Eq.(10) if the fine-tuning conditions

\[ \Lambda = -24M^3k^2 \quad \alpha = 6k(1 - \xi)^2 \quad \beta = -6k(1 - \frac{\xi}{3}) \]  

(11)

are imposed. However, as will be shown later our purpose is to compute the induced 4d cosmological constant at the effective action level. For this it is more convenient to treat the density, pressure, and brane tension separately.

In order to fix them completely we need to derive an additional condition which is linearly independent to conditions (3). Furthermore, the additional condition should fix $w_0 = 1/3$ at $\xi = 0$ to guarantee AdS/CFT at zero temperature. We will show in the following that the additional condition is derived if we assume that our finite temperature solution discussed here is a static limit of RS cosmological solution.

RS cosmological solution is non-static solution of Einstein equation (3), which is solved by ansatz

\[ ds^2 = -n^2(t, y) dt^2 + a^2(t, y) \delta_{ij} dx^i dx^j + b^2(t, y) dy^2. \]  

(12)

As shown in Ref. [11–13] in order to derive a Friedmann-type equation it is enough to solve Eq.(3) with the ansatz (12) in the vicinity of brane located at $y = 0$. We also introduce time-dependent stress-energy tensor on the brane

\[ S^M_N = \frac{\delta(y)}{b} \text{diag}(-\rho, p, p, p, 0) \]  

(13)

where $\rho$ and $p$ are assumed to be dependent only on the time due to homogeneous property of our universe. Following Ref. [11] we also introduce several notations for convenience:

\[ 5 \]
\begin{align*}
\Delta f &= f(0^+) - f(0^-) \\
\bar{f} &= \frac{f(0^+) + f(0^-)}{2} \\
f'' &= \hat{f}'' + \Delta f' \delta(y)
\end{align*}

where \( \hat{f}'' \) is non-distributional part of \( f'' \).

Then, it is straightforward to show that the distributional parts of \((0,0)\) and \((1,1)\) components of Einstein equation yield

\begin{align*}
\frac{\Delta a'}{a_0 b_0} &= -\frac{1}{12M^3} (\rho + b_0 v_b) \\
\frac{\Delta n'}{n_0 b_0} &= \frac{1}{12M^3} [(3p + 2\rho) - b_0 v_b] 
\end{align*}

where \( a_0, b_0 \) and \( n_0 \) are \( a, b, \) and \( n \) at \( y = 0 \) and prime denotes a differentiation with respect to \( y \). Also it is easy to derive

\[ \dot{\rho} + 3(\rho + p) \frac{\dot{a}_0}{a_0} + v_b \dot{b}_0 = 0 \]

from \((0,5)\) component of Einstein equation whose explicit expression is

\[ \frac{\dot{a} n'}{n a} + \frac{\dot{a}' b}{a b} - \frac{\dot{a}}{a} = 0 \]

where dot denotes a differentiation with respect to \( t \). Eq.(14) is nothing but the conservation condition of the stress-energy tensor, \( i.e. \) \( S^M_{N;M} = 0 \). Finally, the jump of the \((5,5)\) component of Einstein equation yields

\[ \frac{\bar{a}'}{a_0} (\rho - b_0 v_b) = \frac{\bar{n}'}{3n_0} (\rho + b_0 v_b) \]

and the mean value of the same equation gives the Friedmann-like equation;

\[ \left( \frac{\dot{a}_0}{a_0} \right)^2 + \frac{\dot{a}_0}{a_0} = \frac{1}{12M^3} \left[ \Lambda - \frac{(\rho + b_0 v_b)(\rho + 3p - 2b_0 v_b)}{48M^3} \right] . \]

Of course, when deriving Eq.(19) we have used the conditions (16), (17), and (18).

If we assume that our static finite temperature solution is a static limit of the RS cosmological solution, we can derive the additional condition by taking a static limit of the Friedmann-type equation (19);
\[(\rho_0 + b_0 v_b)(1 + 3w_0)\rho_0 - 2b_0 v_b] = 48 M^3 \Lambda. \tag{20}\]

If one inserts, however, \(\Lambda = -24 M^3 k^2\) and \(b_0 = 1/\sqrt{1 - \xi}\), Eq.\,(20) becomes

\[
\left[ \rho_0 + \frac{v_b}{\sqrt{1 - \xi}} \right] \left[ (1 + 3w_0)\rho_0 \frac{2v_b}{\sqrt{1 - \xi}} \right] = -1152 M^6 k^2 \tag{21}\]

which is not linearly independent to the previous conditions \(9\). Therefore we have to find another linearly independent condition.

As shown in Ref.\,[12,13] the Friedmann-type equation \(19\) is transformed into the conventional Friedmann equation

\[
\left( \frac{a_0}{a_0} \right)^2 + \frac{\ddot{a}_0}{a_0} = \frac{1}{576 M^6} [v_b b_0 (\rho - 3p) - \rho \rho + 3p)] \tag{22}\]

when the fine-tuning condition

\[
\Lambda + \frac{v_b^2 b_0^2}{24 M^3} = 0 \tag{23}\]

is satisfied. Therefore, another candidate for the additional condition is either one of Eq.\,(23) or static limit of Eq.\,(22). Both with the previous conditions \(9\) yield identical results;

\[
\Lambda = -24 M^3 k^2 \tag{24}\]

\[
v_b = 24 M^3 k \sqrt{1 - \xi} \]

\[
\rho_0 = -24 M^3 k (1 - \sqrt{1 - \xi}) \]

\[
w = \frac{(3 - \xi) - 3\sqrt{1 - \xi}}{3\sqrt{1 - \xi}(1 - \sqrt{1 - \xi})}
\]

which is our main result in this letter. It is worthwhile noting that \(\lim_{\xi \to 0} w = 1/3\) which guarantees AdS/CFT at zero temperature.

Now, let us compute 4d effective action to show that the nonzero temperature effect makes a nonvanishing cosmological constant at flat universe by considering a small fluctuation around Schwarzschild-AdS_5

\[
ds^2 = e^{-2\sigma(y)} g_{\mu \nu}(x, y) dx^\mu dx^\nu + \frac{dy^2}{f(y)} \tag{25}\]
where

\[ g_{\mu\nu}(x, y) = \bar{g}_{\mu\nu}(x) + (1 - f(y))\bar{g}_0^0 \bar{g}_0^0. \]  

(26)

Here, \( \bar{g}_{\mu\nu}(x) \) represents a physical gravity in the effective theory and will be replaced by flat metric \( \eta_{\mu\nu} \) at final stage. The curvature scalar \( R \) computed from the metric (25) is

\[ R = e^{2\sigma} \bar{R} + \Delta R_1 + \Delta R_2, \]  

(27)

where \( \bar{R} \) is the 4d curvature scalar derived from \( \bar{g}_{\mu\nu} \) and

\[ \Delta R_1 = 8f \sigma'' - 20f \sigma'^2 + \frac{\bar{g}^{00}ff'' + [4(1 + \bar{g}^{00}) - 9\bar{g}^{00}]f'\sigma'}{1 + (1 - f)\bar{g}^{00}} + \frac{\bar{g}^{00}(1 + \bar{g}^{00})}{2[1 + (1 - f)\bar{g}^{00}]^2}f'^2 \]  

(28)

\[ \Delta R_2 = \frac{(1 - f)e^{2\sigma}}{1 + (1 - f)\bar{g}^{00}} \left[ \left( \bar{g}^{\mu\nu}\Pi^{(1)}_{\mu\nu} - \bar{R}^{00} \right) + \frac{1 - f}{1 + (1 - f)\bar{g}^{00}} \left( \bar{g}^{\mu\nu}\Pi^{(2)}_{\mu\nu} - \bar{g}^{\mu0}\bar{g}^{00}\Pi^{(1)}_{\mu\nu} \right) \right] - \left( \frac{1 - f}{1 + (1 - f)\bar{g}^{00}} \right)^2 \bar{g}^{\mu0}\bar{g}^{00}\Pi^{(2)}_{\mu\nu}. \]

In Eq. (28) \( \Pi^{(1)}_{\mu\nu} \) and \( \Pi^{(2)}_{\mu\nu} \) are quantities dependent on the intrinsic geometry of brane world-volume as follows:

\[ \Pi^{(1)}_{\mu\nu} \equiv \nabla_\nu \omega^\rho_\mu - \nabla_\rho \omega^\nu_\mu, \]

(29)

\[ \Pi^{(2)}_{\mu\nu} \equiv 2\omega^\rho_\mu \omega^\sigma_\nu - \omega^\rho_\nu \omega^\sigma_\mu - \omega^\sigma_\mu \omega^\rho_\nu \]

where \( \nabla_\mu \) is a covariant derivative and

\[ \omega^\rho_\nu = \frac{1}{2} \bar{g}^{\rho0}\bar{g}^{\nu0}(\partial_\nu \bar{g}_{\rho\sigma} + \partial_\rho \bar{g}_{\nu\sigma} - \partial_\sigma \bar{g}_{\nu\rho}). \]

(30)

In fact, \( \Delta R_2 \) is not important in this letter because it goes to zero at flat limit. Inserting \( f(y) = 1 - \xi e^{4k|y|} \) and \( \sigma(y) = k|y| \) into Eq. (28) one can show that \( \Delta R_1 \) is reduced to

\[ \Delta R_1 = 16k \left[ 1 - \frac{\xi}{2} + \frac{(1 + \xi)\bar{g}^{00}}{1 + \xi\bar{g}^{00}} \right] \delta(y) - 20k^2 + \frac{4k^2 \xi(1 + \bar{g}^{00})e^{4k|y|}[1 + 3\xi\bar{g}^{00}e^{4k|y|}]}{(1 + \xi\bar{g}^{00}e^{4k|y|})^2}. \]

(31)

Using Eq. (27) and

\[ \sqrt{-G} = \sqrt{-\bar{g}_4}e^{-4\sigma} \sqrt{\frac{1 + (1 - f)\bar{\mu}}{f}} \]

(32)
where $\bar{g}_4 = \text{det} \bar{g}_{\mu\nu}$, $\bar{g}_3 = \text{det} \bar{g}_{ij}(i, j = 1, 2, 3)$, and $\bar{\mu} \equiv \bar{g}_3/\bar{g}_4$, one can calculate a 4d effective action whose form is

$$S_{\text{eff}} = \int d^4x \sqrt{-\bar{g}_4} \mathcal{L}_{\text{eff}}$$

where

$$\mathcal{L}_{\text{eff}} = \sqrt{\frac{1 + \bar{\mu} \xi}{1 - \xi}} (-v_b + \mathcal{L}_m)$$

$$+ \int dy e^{-4\sigma} \sqrt{\frac{1 + (1 - f)\bar{\mu}}{f}} \left[ -\Lambda + 2M^3(e^{2\sigma} \bar{R} + \Delta R_1 + \Delta R_2) \right].$$

As indicated earlier modern astronomical observations show that our universe is flat in spite of its non-vanishing cosmological constant. In order to examine whether or not it is realized at brane-world scenario we take a flat limit by choosing $\bar{\mu} = -1$, $\Delta R_1 = 16k(1 - \xi/2)\delta(y) - 20k^2$, and $\Delta R_2 = 0$. Then one can show easily the effective Lagrangian contains a nonvanishing cosmological constant

$$\mathcal{L}_{\text{eff}} = 2M^2_{\text{pl}} \bar{R} + \mathcal{L}_m - \lambda_4$$

where $M^2_{\text{pl}} = M^3/k$ and

$$\lambda_4 = 24M^3k \left[ \frac{2}{3} \xi + \sqrt{1 - \xi} - 1 \right].$$

The constant $\lambda_4$ is a quantity combined by the brane tension and the quantity arising from the contribution of the fifth dimension. In this sense it can be referred as an induced 4d cosmological constant. Actually, however, in general there would be a contribution from the matters $\mathcal{L}_m$ to the 4d cosmological constant. For example, let us consider the scalar matter fields

$$\mathcal{L}_m = -\frac{1}{2} \partial_{\mu}\phi^i\partial^{\mu}\phi^i - V(\phi).$$

Due to homogeneity and isotropy of our universe we assume the scalar fields $\phi^i$ are dependent only on time, i.e. $\phi^i = \phi^i(t)$. Then, Eq.(5) and Eq.(24) yield
\[ S_{ii} = \mathcal{L}_m = -24M^3k \left(1 - \frac{\xi}{3} - \sqrt{1 - \xi}\right). \tag{38} \]

Thus the 4d effective Lagrangian (35) is reduced to \( \mathcal{L}_{\text{eff}} = 2M_{\text{pl}}^2 \bar{R} - \Lambda_4 \) where the 4d cosmological constant \( \Lambda_4 \) is

\[ \Lambda_4 = 8M^3k\xi. \tag{39} \]

As expected \( \Lambda_4 \) becomes zero at zero temperature limit. At the cosmological temperature 3K with \( M \approx k \approx 1\text{TeV} \), we obtain roughly \( \Lambda_4 \approx (0.0004\text{eV})^4 \) which is within the upper bound of the experimental value \( (0.01\text{eV})^4 \).

In summary, we examined the nonzero temperature effect in RS brane-world scenario. The assumptions that the RS bulk spacetime is two copies of the Schwarzschild-\( AdS_5 \) and finite temperature solution is a static limit of the RS cosmological solution enable us to derive a temperature dependence of the 4d cosmological constant at the effective action. The most interesting one is that the scenario presented in this letter yields a small positive cosmological constant which is smaller than the upper bound of the recent experimental value. This means that the modern astronomical observations can be realized within the RS brane-world scenario by inclusion of temperature. It seems to be interesting to examine how the temperature effect modifies the Newton power law determined by the zero mode and higher Kaluza-Klein excitation in the fluctuation spectrum, which will be discussed elsewhere.
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