Scaling the universe

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Abstract A model is presented for the origin of the large scale structure of the universe and their Mass-Radius scaling law. The physics is conventional, orthodox, but it is used to fashion a highly unorthodox model of the origin of the galaxies, their groups, clusters, super-clusters, and great walls. The scaling law fits the observational results and the model offers new suggestions and predictions. These include a largest, a supreme, cosmic structure, and possible implications for the recently observed pressing cosmological anomalies.

Keywords large scale structure, the universe, mass-radius scaling law, a supreme structure, predictions and implications, cosmic microwave background

Encomium:
Celebrating Freeman Dyson on his 90th Birthday.

Tolstoy said:
A man is like a fraction whose numerator is what he is and whose denominator is what he thinks of himself.

Freeman’s fraction is $N / \ln(N)$ where $N = 10^{76}$.

1 Introduction

One of the most compelling problems in cosmology is the origin and evolution of the large scale structure of the universe. These heavenly bodies, the galaxies and their groups, clusters, super-clusters and great walls, are as enigmatic as they are alluring.

The current scenario decrees that the tiny density fluctuations, reflected in the tiny temperature variations in the cosmic microwave background (CMB), act as the seeds which grow, under gravitational attraction as the universe expands, into these bodies. See the website [WMAP 2013] for a précis of the current picture. We’ll say no more on this save to note that there is no detailed analytical theory, akin to that for stellar structure, presently accompanying this picture.

One of the key signatures of such a model theory should be the Mass-Radius scaling law for the large scale structure. In the atomic theory of matter, taking hydrogen as the prototype atom, we know that the energy scales as the Rydberg = $\frac{1}{2} m_e c^2 (\alpha_e)^2$ and the size scales as the Bohr radius, $a_o = \lambda_e / (\alpha_e)$. Here $m_e$ and $\lambda_e$ are the electron’s mass and Compton wavelength, and $\alpha_e = e^2 / (\hbar c)$ is the fine structure constant.

In astrophysics, thanks to the seminal works of Eddington (1926), Fowler (1926) and Chandrasekhar (2010), we know that the structure of stellar matter (stars) have masses and radii that scale as shown in Table 1, where $\alpha_G = GH^2 / (hc)$ is the gravitational fine structure constant. Here, in honour of Eddington, we have chosen to represent, as he did, the mass of the nucleon as $H$.

The masses of the white dwarf and neutron stars in Table 1 are their critical masses, as obtained from the ultra-degenerate, ultra-relativistic, stellar structure

| Mass | Radius | Structure |
|------|--------|-----------|
| $\left( \frac{1}{\alpha_G} \right)^{\frac{3}{2}} H$ | $\left( \frac{1}{\alpha_G} \right)^{\frac{5}{2}} \lambda_e$ | White Dwarf |
| $\left( \frac{1}{\alpha_G} \right)^{\frac{3}{2}} a_o$ | $\left( \frac{1}{\alpha_G} \right)^{\frac{5}{2}} \lambda_H$ | Neutron Star |

Table 1 Stellar Scaling.
theory. Nowadays, we can see this simply for all three
classes of stars, where their internal pressure is \( P_i \propto h \epsilon (\rho/H)^{\frac{4}{3}} \), with \( \rho \) their mass density. When balanced
against the gravitational pressure, \( P_G \propto GM\rho/R \), this readily gives their [critical] mass, \( M \propto (1/\alpha_G)^{\frac{2}{3}} H \) as is
exhibited in Table[I].

The theory of white dwarfs owes its inception to
\( \text{Fowler (1926)} \) who established that theory in his classic
paper, following directly upon the independent discovery by Fermi and Dirac, of the quantum statistical me-
chanics of ideal fermions (as they are now called). Fol-
lowing on from Fowler, nowadays we can readily find for
ultra-degenerate, non-relativistic, white dwarf and neu-
tron stars, this utterly remarkable, beautiful, compact
Mass-Radius scaling law,

\[
\begin{align*}
\left( \frac{M}{\rho} \right)^{\frac{4}{3}} \frac{R}{H} &= C = 4.51227\ldots,
\end{align*}
\]

where \( \frac{M}{\rho} = M_o/R_o \) with \( M_o = (1/\alpha_G)^{\frac{2}{3}} H \) and \( \frac{R}{H} \) is
the Compton wavelength of the electron/neutron for the white
dwarf/neutron star. This law holds for non-relativistic
white dwarfs, requiring the Fermi momentum, \( p_F < m_e c \), meaning that

\[
\rho < \frac{1}{3\pi^2 \alpha^2_G}. \tag{2}
\]

This Fowler Scaling Law has been an abiding inspira-
tion in our search for a model of, and concomitant scal-
ing law for, the large scale structure of the universe.

There have been two prime motivational insights
that have guided our search. The first is the utterly
remarkable numerical coincidence, first recognised by
de Sitter and Weyl and Eddington (see the nice review in
\( \text{Gorelik (2002)} \) ), that

\[
\begin{align*}
&\text{a cosmic mass} \propto \frac{1}{\alpha_G^2} \\
&\text{a particle mass} \\
&\text{and} \\
&\text{a cosmic size} \propto \frac{1}{\alpha_G}. \tag{3}
\end{align*}
\]

This correspondence was prosecuted relentlessly by
Eddington throughout his writings \( \text{[Eddington 1931, 1948]} \) and has been a persistent inspiration to us
throughout our long search.

The second is the pioneering work of \( \text{Peebles (2001)} \)
, followed up by many since, who studied the mass
distributions of the large scale structure, by analyzing [the now extensive] data, using the 2-point, galaxy-
galaxy, correlation function, \( \xi(r) \propto r^{-P} \). There is now
very strong evidence \( \text{[Baryshev and Teerikorpi 2005]} \)
\( \text{Jones et al (2005)} \) that the exponent \( p = 1 \), thus indic-
ating a mass proportional to radius squared relationship.

These two, seemingly disparate, insights in fact are
intimately connected as will be seen in the following
sections in which we will develop a model of, and scal-
ing law for, the large scale structure of the universe.
Predictions and possible implications for the pressing
cosmological anomalies will also be given.

2 The Dawn of Structure — Part I

We turn our attention to the era just prior to recombi-
nation (decoupling). This period corresponds to tem-
perature, \( T_r \), of the universe, which given by the Saha
equation is \( T_r = 3000 \text{ K} \). The current temperature of the
universe is \( T_c = 2.725 \text{ K} \), so we know this era corre-
sponds to a red-shift \( z_r = T_r/T_c = 1100 \). [Throughout
we use \( z \) for \( z +1 \)]

We will also need the era when the matter contri-
bution to the universe’s mass density \( \rho_m \) equals \( \rho_r \), the
radiation contribution to that mass density. Cosmolog-
tical theory and experiment \( \text{[Hinshaw et al 2012]} \) both
give this as \( z_r \approx 3300 \). So at the era just before recombi-
nation, we have \( \rho_m \approx (3300/1100) \rho_r = 3 \rho_r \). The total
mass density of the universe then is \( \rho = \rho_m + \rho_r = 4 \rho_r \).
Everything now can be computed from the physical
quantities of the radiation (photons).

Proceeding, the Jeans stability condition [see also
the appendix] is

\[
w^2 = -4\pi G\rho + k^2 v^2, \tag{5}
\]

where \( \rho \) is the total mass density, \( k \) is the wave number,
and \( v \) is the speed of sound \( v^2 = (\partial P)/(\partial \rho) \), where \( P \)
is the total pressure of the universe, \( P = P_m + P_r \).

The Jeans mass, \( M_J \), and the Jeans radius, \( R_J \), at
this era are readily found. The radius is

\[
R_J^2 = \frac{5}{16\pi} \left( \frac{1}{\alpha_G} \right) \left( \frac{T_H}{T_r} \right)^4 \lambda_H^2 \tag{6}
\]

where

\[
T_H = \frac{He^2}{k} = 1.0888\ldots \times 10^{13} \text{ K}. \]

Here \( k \) is the Boltzmann constant and we use

\[
\rho = \frac{4\pi^2 (kT_r)^4}{15 \hbar^3 c^3 v} = \frac{4\pi^2}{15} \left( \frac{T_r}{T_H} \right)^4 \frac{H}{\lambda_H}. \tag{7}
\]

Readily it is shown that

\[
\frac{P_r}{P_m} \approx \frac{1}{9} \left( \frac{T_H}{T_r} \right). \tag{8}
\]
so that the total pressure is \( P = \frac{4}{3} \rho c^2 \) and \( v^2 = c^2/12 \). The Jeans mass is

\[
M_J = \frac{4\pi}{3} \rho R_J^3 = \frac{\pi}{36} \sqrt{5\pi} \left( \frac{1}{\alpha G} \right)^{\frac{2}{3}} \left( \frac{T_H}{T_r} \right)^2 H. \tag{9}
\]

Now, for the ‘pièce de résistance’, we find this utterly remarkable numerical coincidence,

\[
\frac{T_H}{T_r} = 1.0061.. \left( \frac{1}{\alpha G} \right)^{\frac{2}{3}} = 1.0061.. \left( 1.6933 \times 10^{38} \right)^{\frac{2}{3}}. \tag{10}
\]

Such numerical coincidences have solid history, having been advanced first by the insightful works of Carter (1974), of Carr and Rees (1979), and of Rees (1983). See also the fine review with references by Ellis (2011) and the nice book by Davies (1982). Of course, Eddington was already on the trail of such, several decades earlier.

Bolstered by this most fortunate of coincidences, we use

\[
\left( \frac{T_H}{T_r} \right)^4 = \left( \frac{1}{\alpha G} \right). \tag{11}
\]

This correspondence is utterly essential in establishing a fundamental scaling law for the large scale structure of the universe. We now have at this era

\[
R_J = \sqrt{\frac{5}{16\pi}} \left( \frac{1}{\alpha G} \right) \frac{\lambda_H}{H} \tag{12}
\]

\[
\rho = \frac{4\pi^2}{15} \alpha G \frac{H}{\lambda_H^2} \tag{13}
\]

\[
M_J = \frac{\pi}{36} \sqrt{5\pi} \left( \frac{1}{\alpha G} \right)^{\frac{2}{3}} H = \sigma_J R_J^2 \tag{14}
\]

\[
\sigma_J = \frac{4\pi^2}{45} \sqrt{5\pi} \frac{H}{\lambda_H^2}. \tag{15}
\]

Most satisfying, we now have the scalings,

\[
\frac{M_J}{H} \propto \left( \frac{1}{\alpha G} \right)^{\frac{2}{3}} \tag{16}
\]

\[
\frac{R_J}{\lambda_H} \propto \left( \frac{1}{\alpha G} \right) \tag{17}
\]

\[
M_J \propto R_J^2. \tag{18}
\]

In closing this section, and leading to the next, we remark that the very important work of Chandrasekhar showed that such a giant ultra-radiation dominated structure as we have here, must be unstable and collapse. That was found to be the case independently by Feynman, and it will be discussed in Sec. 4. This collapse is required, of course, and is essential for the continuing development of our model and quest for a fundamental scaling law for the large scale structure. The Jeans surface mass density, \( \sigma_J \propto H/\lambda_H^2 \) gives the clue on how to proceed.

### 3 The Dawn of Structure — Part II

The quantity \( H/\lambda_H^2 \) is precisely the kind of mass density to be expected for an ultra-planar configuration (an ultra-collapsed 3-dimensional structure) of ultra-degenerate fermion (nucleon) matter. We envision, therefore, that the collapse of the proto-structure described in Sec. 2 resulted in such a planar structure of ultra-degenerate neutron matter; a kind of cosmic ‘pancake’. The collapse to density \( H/\lambda_H^2 \) is more than sufficient to ensure that the protons and electrons in the proto-structure will undergo inverse \( \beta \)-decay forming neutrons.

To see what the mass and radius of such an ultra-planar, ultra-degenerate neutron matter structure would be, we first look at a heuristic calculation. The statistical mechanics of such a system is elementary. In the ultra-relativistic limit, the internal pressure is \( P_i \propto \hbar c (\sigma/H)^{\frac{2}{3}} \) where the mass density \( \sigma > H/\lambda_H^2 \). Balancing this against the pressure of gravity, \( P_G \propto GM\sigma/R \), we find a critical mass \( M \simeq (1/\alpha G)^{\frac{2}{3}} H \). This is remarkably satisfying as it is comparable to \( M_J \) for the collapsed proto-system considered in Sec. 2.

In the ultra-nonrelativistic region, the internal pressure is \( P_i \propto \hbar^2 \sigma^2 / H \), and balancing against \( P_G \) results in a critical radius, in a maximum sense, of the planar structure, \( R \simeq (1/\alpha G) \lambda_H \). This is remarkably comparable to the \( R_J \) given in Sec. 2.

Thus the collapsed proto-structure fits very comfortably in such an ultra-planar, ultra-degenerate, cosmological ‘pancake’. As the surface density of the collapsed proto-structure is \( \sigma_J \simeq 3.5H/\lambda_H^2 \), see Eq. (14), the ultra-planar ‘pancake’ is an ultra-relativistic, ultra-degenerate neutron structure since that would require the Fermi momentum of the neutrons to be \( p_F > Hc \) implying \( \sigma > \frac{1}{2}\pi H/\lambda_H^2 \).

We can solve exactly for the structure in a way analogous to the classic work of Chandrasekhar for a 3-dimensional, ultra-degenerate structure. The equation of state, \( P(\sigma) \), can be calculated exactly. It is a somewhat involved algebraic entity but we require only its derivative which turns out to be pleasingly simple, viz.

\[
dP \over d\sigma = \frac{1}{2} \frac{\sigma c^2}{\sqrt{\sigma} + 1} \quad \text{where} \quad \sigma = \frac{\sigma}{\sigma_o} \quad \text{and} \quad \sigma_o = \frac{H}{2\pi \lambda_H^2}. \tag{19}
\]
The scaling density, $\sigma_o$, is that for $p_F = Hc$. The equation for hydrostatic equilibrium (HE) can be readily constructed. We use only the leading term for the gravitational potential for a planar configuration of matter as it carries the overwhelming strength of the potential. The exact equation for HE, in scaled form for $\sigma(y)$ is,

$$
\frac{1}{y} \frac{d}{dy} \left[ y^2 \frac{1}{\sqrt{\sigma(y) + 1}} \frac{d\sigma(y)}{dy} \right] = -\sigma(y)
$$

It is a modified Lane-Emden equation which can be solved numerically. The radius of the structure is $R = r_0 y_o$ where

$$
r_o = \frac{c^2}{4\pi G \sigma_o} = \frac{1}{2} \left( \frac{1}{\alpha_G} \right) \lambda_H, \tag{21}
$$

with $y_o$ being the solution of $\sigma(y_o) = 0$. The solution and $\sigma(y_o)$ are obtained for varying values of $\sigma(0)$, the central value of the scaled mass density $\sigma(y)$. The mass of the structure is

$$
M = \frac{1}{4} \left( \frac{1}{\alpha_G} \right)^2 H \left[ -y_o^2 \sigma'(y_o) \right]. \tag{22}
$$

While we solve these equations numerically for all values of the central density, $\sigma(0)$, the ultra-relativistic and ultra-nonrelativistic cases are of prime interest here; the solutions for which are easier. Taking, $\sigma(y) = \lambda \theta^n(y)$, the HE equation reduces to a Lane-Emden form,

$$
\frac{1}{y} \frac{d}{dy} \left[ y^2 \frac{d\theta}{dy} \right] = -\theta^n(y); \tag{23}
$$

$n = 1$ and 2 correspond to the ultra-nonrelativistic and ultra-relativistic cases respectively. For the former, where $\lambda/\sigma_o << 1$, we find the critical radius to be

$$
R = 1.83 \left( \frac{1}{\alpha_G} \right) \lambda_H, \tag{24}
$$

and, for the latter, where $\lambda/\sigma_o >> 1$, we find the critical mass to be

$$
M = 1.40 \left( \frac{1}{\alpha_G} \right)^2 H. \tag{25}
$$

These are the exact results confirming their forms obtained from the heuristic analysis.

Here again we have the crucial scaling we observed for the proto-structure,

$$
\frac{M}{H} \propto \left( \frac{1}{\alpha_G} \right)^2 \quad \text{and} \quad \frac{R}{\lambda_H} \propto \left( \frac{1}{\alpha_G} \right), \tag{26}
$$

and affirm that the mammoth, ultra-radiation dominated, proto-structure on collapse can be easily accommodated in a mammoth, ultra-planar structure of ultra-degenerate neutron matter.

Of course, this structure inherently is unstable and must fragment, so that, as the universe expands beyond this ($z_r = 1100$) era, the neutrons can then $\beta$-decay to free protons and electrons that will go on to form atoms, that make the stars, which ultimately assemble into the large scale structure of the universe.

How we envision the final form of our model, and are led to the ultimate goal, the scaling for the large scale structure, is the topic of Sec. 5 before which we discuss general relativistic considerations that are essential to it.

In closing this section, we reflect on some past works of ours that bear on the matters covered in this section. Some 30 years ago we gave a preliminary discussion of a planar-pancake-ultra-degenerate structure scenario of the universe (Frankel 1982), and showed that it would have a critical mass $\propto \left( \frac{1}{\alpha_G} \right)^2 H$. It was not the picture envisioned in this section. We know now just what it was telling us. It is important to emphasise that the planar structure considered here is an ultra-anisotropic 3-dimensional structure where the gravitational potential is still of the $1/r$ form. In a purely mathematical 2-dimensional space, we know from our general relativistic studies of gravitation in such a space (Cornish and Frankel 1991, 1993, 1994) that there is no Newtonian limit.

4 General Relativistic Considerations

Eddington developed the theory of stellar structure as presented in his seminal treatise (Eddington 1926). It is the standard model of stellar structure; one of the great achievements in modern physics. Chandrasekhar, in his set of lectures (Chandrasekhar 1982) quite rightly described Eddington as the finest astrophysicist of his time.

The standard model, with its equation of state $P \propto \rho^{4/3}$, see the start of Sec. 1, gives the mass of a star as

$$
M = 18.1 \sqrt{\frac{(1 - \beta)}{\beta^4}} M_\odot, \tag{27}
$$

where $\beta = P_m/P$, Here $P_m$ is, as in Sec. 2 the contribution of matter (particles) to the total $P = P_m + P_\gamma$, where, also as in Sec. 2 $P_\gamma$ is the contribution of radiation (photons).

It was Chandrasekhar who from the outset in his seminal work (Chandrasekhar 2010) on stellar structure, recognised and emphasized that it was the combination of fundamental constants of nature in the result, such as $\left( \frac{1}{\alpha_G} \right)^2 H$ that characterised the mass of stars.
Thus Eq. (27) can be found to be,

\[ M = 9.77 \sqrt{\frac{(1 - \beta)}{\beta^4}} \left(\frac{1}{\alpha_G}\right)^{3/2} H. \]  

(28)

For our mammoth, ultra-radiation dominated, structure of Sec. 2 where \( \beta = 9(\alpha_G)^{4/3} \), we have \( M = 0.12 \left(\frac{1}{\alpha_G}\right)^{3/2} H \), recovering the correct scaling as required for \( M_J \) given in Sec. 2.

To understand why this structure must be unstable and collapse, we turn back in time, to the work of Hoyle and Fowler (1963) who were the first to consider radiation dominated stars. They proposed stars of mass of the order of \( 10^6 M_\odot \), as a model for the then newly discovered quasars. Though they were not able to provide for exactly how such stars would appear, their study was nonetheless very interesting, providing stimulating results and suggestions.

The main reason for their nonexistence came at the same time from Chandrasekhar (1964, 1965), in his elegant calculations which showed that such massive radiation-dominated stars are unstable due to general relativistic effects, and thus the stars would collapse. Feynman, simultaneously and independently, suggested the same for the same reason. An engaging recount of all this has been given by Thorn (1990).

Due to general relativistic effects, the adiabatic exponent \( \Gamma \) in the equation of state is modified (increased) such that \( \Gamma > \frac{4}{3} \), is now the condition on \( \Gamma \) for the structure to be stable otherwise to collapse. Of course, such a collapse is what must happen for the mammoth structure set out in Sec. 2, a result that we certainly want in our model of the large scale structure.

We can see how this happens from a heuristic calculation within the Newtonian approximation which we have employed. The general relativistic effects on a massive structure are to modify (strengthen) the gravitational attraction. We can model that with a Newtonian potential modified as \( (1 + \frac{\rho M}{R^2})^{1/2} \). And it has just the right (same) radius as the \( \rho \) structuring. However, the essential results and features of the model obtained from our Newtonian calculations allow us to present a clear picture of the model’s principle properties.

We believe these main results and features would endure under general relativistic scrutiny. Such is not unusual and we can think, straight away, of two such cases. Oppenheimer and Volkoff solved the general relativistic hydrostatic equilibrium equation for a neutron star and found its mass to be approximately \( 1 M_\odot \), which is what one obtains from the Newtonian calculation. Only the pre-factor (1) is slightly bigger, but the same scaled \( M \propto (1/\alpha_G)^{4/3} H \) is found. In our study (Gailis and Frankel 2006) of the Jeans stability condition in an expanding Robertson-Walker universe, we found that the essential features of the non-relativistic Newtonian theory were preserved, e.g. the same Jeans condition, albeit slightly modified due to the expansion, that we have employed in Sec. 2.

### 5 The Model and The Scaling Law

We envision that, at the era we call the dawn of structure, just prior to \( z_t = 1100 \), the mammoth structure we presented and studied, in Sec. 2, developed. Upon its collapse, it formed into an equally mammoth, ultra-planar structure, composed of ultra-degenerate neutrons as studied in Sec. 3. As we showed, this structure has just the Mass which scales in just the right (same) way as the \( M_J \propto \left(\frac{1}{\alpha_G}\right)^{4/3} H. \) And it has just the right (same) radius as the \( R_J \propto \left(\frac{1}{\alpha_G}\right) \lambda_H. \) Furthermore, importantly, it therefore has the right scaling law, \( M_J = \sigma_J R_J^3. \) This is just the right starting point to develop the ultimate scaling law for the large scale structure of the universe.

We note that the collapse time for a structure can be estimated by its free-fall time, \( t_{ff} \propto (G\rho)^{-\frac{1}{2}}. \) This
time scales like $R^2(z)$, just as the age of the universe, $t_u = H^{-1}(z)$, [H(z) here is the Hubble parameter at red-shift, $z$], does at any era of the expansion of the universe. Thus, we envision that any such ultra-radiation dominated structures that formed earlier than the $z_r = 1100$ era would have already collapsed and disappeared, with their constituent particles and photons passed back into the universe.

We continue to envision, that as this planar structure fragments, as it must, as the universe expands into the era following $z_r = 1100$, the protons and electrons are liberated by the $\beta$-decay of the neutrons, the process of atomic formation takes place, followed by the process of stellar formation, and then by the ultimate formation of the assemblies that are the large scale structure.

Now, we propose that as these assemblies are formed with the expansion of the universe, the planar nature of the structure that we have shown in Secs. 2 and 3 which we see as the progenitors of the large scale structure of the universe, persists, that is it is imprinted on the large scale structure, and that they will satisfy the same form of Mass-Radius scaling law, but now

$$M = \sigma R^2$$

(32)

where

$$\sigma = \frac{\sigma_J}{(z_r)^2} \left(\frac{M_J}{(R_J z_r)^2}\right)$$

(33)

$$=0.111... \text{ g/cm}^2$$

(34)

(35)

where $\sigma$ has scaled with the expansion of the universe and where $M$ and $R$ are the mass and radius of any of the assemblies that constitute the large scale structure.

The two prime motivating insights presented in Sec. 1 which are supported by all that emerged in the model studies in the previous sections, are embodied in this ultimate result for the proposed Mass-Radius scaling law of the large scale structure of the universe, which has been our indefatigable quest.

We can cast this scaling law in its final, elegant, form, as,

$$M = \mathcal{R}^2$$

(36)

where $M = M/M_o$, $M$ is the mass of an assembly, $M_o = M_J = 8.34... \times 10^{18} M_\odot$, and where $\mathcal{R} = R/R_o$, with $R_o = R_J(z_r) = 400$ Mpc. We note that there are no adjustable parameters in this scaling law. These $M_o$ and $R_o$ are just right to scale the current observable large scale structure. To see just how right all this is, we have chosen values of $R$ from 0.01 Mpc through to 400 Mpc and computed the $M$ values that this universal scaling law gives. All values are listed in Table 2. It is seen that all the assemblies in the large scale structure are accurately described.

We leave it for a very energetic person to scour all the catalogues and literature for the $(M, R)$ values for a legion of the observed astronomical objects in the large scale structure, and plot them carefully on a log[$M$] vs log[$R$] graph. They should, within experimental error, lie on a universal straight line with slope = 2!

The last structure in Table 2 is the ultimate Great Wall that the scaling law yields. We have christened this structure, Gigas, which is a word meaning 'Giant'. It was originally used to describe the race of Gigantes in Greek mythology. We'll discuss their possible existence along with other predictions and suggestions in the final Section, Sec. 6.

6 Predications and Suggestions

The scaling law we propose predicts that there will a largest structure - a Supreme Great Wall, the Gigas - with a mass $M = M_o$ and a radius $R_o$ as given in Sec. 5. At the beginning of 2013, the Sloan Great Wall (Gott et al. 2005) was the largest known Great Wall and it was still well within the Gigas predicted by the scaling law. Then Clowes et al. (2013) found a Huge Large Quasar Group [Huge-LQG] in the Sloan Digital Sky Survey. It has a radius of 620 Mpc and a mass of $6.1 \times 10^{18} M_\odot$.

Now there are two possibilities. One is that this H-LQG Wall is not all one structure just as Clowes et al. (2012) suggested for the Sloan Great Wall that it is a chance alignment of three smaller structures. The other is that it is really one distinct structure being now the largest we know. For the scaling law, the first possibility is readily accommodated by retaining it just as is with the Gigas, the supreme structure. Interestingly, the second can be accommodate easily by an ever so slight tweaking of the results in Sec. 5. We find for the H-LQG structure that its surface mass density is $M/R^2 = 0.0033...$. This is still remarkably close to the value of 0.011 for the proposed scaling law as seen in Sec. 5. So a very small tweaking of $\sigma$ from 0.011 to 0.0033, or in the pure scaled form by tweaking $M_o$ to 0.73 $M_o$ and $R_o$ to 1.55 $R_o$, leaves the law intact. The only other numerical effect is to change the values of $M$ in Table 2 by the small factor of 0.3. In either case [and we are naturally partial to the first possibility], the proposed scaling law stands.

We now look for any possible astrophysical or cosmological indications of the model for the generation of the structure that we have presented.
3. Planck has confirmed WMAP observation, in microwaves, of the CMB Big Cold Spot. This is a mammoth spot in the CMB which is colder than the background. The diameter of this spot is huge. To date there is no explanation of this wondrous object.

Could any of these results be the tell-tale reflection of the kind of planar progeneration of structure and evolution as envisioned herein?

In closing our study, we make some rather far-flung contemplations; much more so than already proposed but which follow on from the considerations given. If the collapse of the proto-structure in Sec. 2 occurred, then might there have been an accompanying cosmical acoustical shock wave? Recall that the speed of sound in such a structure is \(c/\sqrt{12}\) and it might be associated with such an ultra-powerful effect that the acoustical vibrations may still be evident. And even further-flung yet, could such rattle the space-time fabric, rippling it so violently, as to have imposed an additional acceleration [like what is called dark energy] on the expansion of the universe?

Furthermore, as envisioned at \(z = 1100\), such ripples in the spacetime fabric -metric fluctuations- would have important ramifications for the polarization of the CMB radiation. It would imprint in a very specific way as the radiation scattering off such a gravity wave effect is quite different from that off of the density fluctuations. This is particularly relevant with respect to the very recently reported evidence of B-modes in the polarization pattern of the CMB [BICEP2 [2014]].

Finally, if following on, the ultra-degenerate, planar proto-structure of neutron matter collapse as envisioned in Secs. 3 and 5 might there not still be evident the copious spate of neutrinos that would have been released at this \(z_r = 1100\) era?

We conclude all in our study with the proclamation: 

**SI VERO NON DEBET**

**Acknowledgements** We are grateful to our long time good friend and colleague, Ken Amos, for his in-

| Table 2 | Large Scale Structure Scaling \(a: \mathcal{M} = \mathcal{R}^2\) |
|---------|-------------------------|
| \(R\) (Mpc) | \(M/\mathcal{M}_\odot\) | Structure |
| 0.01 - 0.1 | \(5.2 \times 10^9 - 5.2 \times 10^{11}\) | Galaxy |
| 0.5 - 1 | \(1.3 \times 10^{13} - 5.2 \times 10^{13}\) | Group of Galaxies |
| 1 - 5 | \(5.2 \times 10^{13} - 1.3 \times 10^{15}\) | Cluster of Galaxies |
| 10 - 50 | \(5.2 \times 10^{15} - 1.3 \times 10^{17}\) | Super-cluster of Galaxies |
| 80 | \(3.3 \times 10^{17}\) | CfA2 Great Wall (Geller and Huchra [1989]) |
| 210 | \(2.3 \times 10^{18}\) | Sloan Great Wall (Gott et al, 2005) |
| 400 | \(8.34 \times 10^{18}\) | Gigas |

\(a\)These structures are very elongated with one dimension, their length, much larger than their thickness. 

\(R\) is identified with half the length. They are planar-like structures.
valuable support in carrying out the numerical calculations of the structure equations in Sec. 3 and for his preparation of the paper for publication.
A Appendix

What kind of objects that might have formed just after the recombination era had begun, just after what we called the dawn of structure in Secs. 2 and 3 was considered by Peebles and Dicke (1968). Here the radiation has decoupled from the particles which are now free to gravitationally condense on their own. It was found that such first forming objects were globular clusters.

Their analysis relied upon the same Jeans stability condition that we employed in Sec. 2. It is, therefore, very easy to adapt our results to obtain those for this era and recover their results. What is more, we find them anew in their appropriate scaled forms.

In this era, due to the fact that the internal pressure of the condensing object is due to the particles, the radiation playing no role having decoupled, we have directly now for the speed of sound,

\[ v^2 = 9 \left( \alpha G \right)^{1/3} \left( \frac{C^2}{12} \right). \]  

(A1)

We find straight away,

\[ M = \frac{3\pi}{4} \left( \frac{5\pi}{\alpha G} \right)^{1/2} H = 1.04 \ldots \times 10^6 \ M_\odot, \]  

(A2)

and

\[ R = \frac{3}{4} \left( \frac{5\pi}{\pi} \right)^{1/2} \frac{1}{\alpha G} \lambda_H = 6.06 \ldots \times 10^{19} \ \text{cm}. \]  

(A3)

This is the globular cluster.

Note that, while the globular cluster is a 3-dimensional object, it also scales in a most surprising way with its surface density, namely \( M/R^2 = 0.66 \), which is not the 0.011 given in the scaling law for the large scale structure of the universe. It is remarkably close and as such quite suggestive, not only in the light of our model, but also in light of the fact that the surface mass density of any star, a natural 3-d object, is many orders of magnitude greater as evident in Table 1. For example, the surface mass density of a star like the sun is typically \( \left( \frac{1}{\alpha G} \right)^{1/2} H/a_o^2 \approx 10^{12} \); a massively different value to put it mildly!

References

Baryshev Y. and Teerikorpi, P. 2005, Bull. Spec. Astrophys. Obs. Russian Academy of Science, 59
BICEP2 Collaboration (2014), [arXiv:1403.3985] [astro-ph.CO]
Carr B. J. and Rees, M. J. 1979, Nature 278, 605
B. Carter, B. 1974, IAU Symposium 63: Confrontation of Cosmological Theories with Observational Data, Reidel, Dordrecht and arXiv: 0710.3543v1 [hep-th]
Chandrasekhar, S. 1964, Phys. Rev. Lett. 12, 114, 437(E); Astrophys. J.140, 417
Chandrasekhar, S. 1965, Phys, Rev. Lett. 14, 241
Chandrasekhar, S. 1983, Eddington: The Most Distinguished Astrophysicist of His Time, Cambridge University Press
Chandrasekhar, S. 2010, An Introduction to the Study of Stellar Structure, Dover Publications
Clowes, R. G. et al. 2012, Mon. Not. R. Astron. Soc.419, 556
Clowes, R. G. et al. 2013, Mon. Not. R. Astron. Soc.429, 2910
Cornish, N. J. and Frankel, N. E. 1991, Phys. Rev. D 43, 2555
Cornish, N. J. and Frankel, N. E. 1993, Phys. Rev. D 47, 714
Cornish, N. J. and Frankel, N. E. 1994, Class. Quantum Grav. 11, 723
Davies, P. C. W. 1982, The Accidental Universe, Cambridge University Press
Eddington, A. S. 1926, The Internal Constitution of the Stars, Cambridge University Press
Eddington, A. S. 1931, Proc. Cambridge Philos. Soc. 27, 15; Proc. Roy. Soc. (London) 133, 605
Eddington, A. S. 1948, Fundamental Theory, Cambridge University Press
Ellis, G. F. R. 2011, Gen. Rel. Grav. 43, 3213
Fowler, R. H. 1926,Mon. Not. R. Astron. Soc.87, 114
Frankel, N. E. 1982, Phys. Lett. A90, 323
Gallis, R. M. and Frankel, N. E. 2006, J. Math. Phys. 47, 062505, 062506
M. J. Geller, M. J. and Huchra, J. P. 1989, Science 246, 897
G. Gorelik G. 2002, *Herman Weyl and large numbers in relativistic cosmology*, in Einstein Studies in Russia, Yuri Balashov and Vladimir Vizgin, Einstein Studies, Vol. 10, Boston Birkhauser

Gott J. R. et al. 2005, *Astrophys. J.*624, 463

Hinshaw G. et al. 2012, [arXiv:1212.8226](http://arxiv.org/abs/1212.8226) [astro-ph.CO]

Hoyle, F. and Fowler, W. A. 1963, *Mon. Not. R. Astron. Soc.*125, 169

Ibata, R. A. et al. 2013, *Nature* 493, 62

Jones B. J. T. *et al.* 2005, *Rev. Mod. Phys.*, 1211

Hinshaw G. *et al.* 2012, [arXiv:1307.6210](http://arxiv.org/abs/1307.6210) [astro-ph.CO]

Hoyle, F. and Fowler, W. A. 1963, *Mon. Not. R. Astron. Soc.*125, 169

Ibata, R. A. et al. 2013, *Nature* 493, 62

Jones B. J. T. *et al.* 2005, *Rev. Mod. Phys.*, 1211

Rees, M. E. 1983, *Phil. Trans. Roy. Soc. A310*, 311

Shaya, E. J. and Tully, R. B. 2013, *Mon. Not. R. Astron. Soc.*436, 2096

Thorne, K. S. 1990, “Forward” in S. Chandrasekhar Selected Papers, Vol. 5, Chicago University Press

WMAP Formation of Universe Structures, [http://map.gsfc.nasa.gov/universe/bb_cosmo_struct.html](http://map.gsfc.nasa.gov/universe/bb_cosmo_struct.html)

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