Exact Physical Black Hole States in Generic 2-D Dilaton Gravity*

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Abstract

The quantum mechanics of black holes in generic 2-D dilaton gravity is considered. The Hamiltonian surface terms are derived for boundary conditions corresponding to an eternal black hole with slices on the interior ending on the horizon bifurcation point. The quantum Dirac constraints are solved exactly for these boundary conditions to yield physical eigenstates of the energy operator. The solutions are obtained in terms of geometrical phase space variables that were originally used by Cangemi, Jackiw and Zwiebach in the context of string inspired dilaton gravity. The spectrum is continuous in the Lorentzian sector, but in the Euclidean sector the thermodynamic entropy must be $2\pi n/G$ where $n$ is an integer. The general class of models considered contains as special cases string inspired dilaton gravity, Jackiw-Teitelboim gravity and spherically symmetry gravity.

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I. INTRODUCTION

In order to solve fundamental problems associated with Hawking radiation [1] and black hole thermodynamics non-perturbative quantum gravity is required. Generic dilaton gravity provides a large class of models that are classically and quantum mechanically solvable. They also contain as special cases physically relevant theories: Jackiw-Teitelboim gravity (JT) [2], the string inspired model (SIG) used in the seminal paper of CGHS [3], and spherically symmetric gravity (SSG) [4]. A great deal of progress has been made towards quantizing specific models ( [5], [6], [7]) as well as the generic theory ( [8], [9]). In particular, Strobl [8] has quantized a large class of 2-D gravity theories (including generic dilaton gravity) with a Poisson-sigma model approach that generalizes the gauge theoretic formulations of JT and SIG [7]. In other work [9,10], mass eigenstates have been found in terms of a metric formulation in a WKB approximation.

The purpose of the present paper is to quantize generic 2-D dilaton gravity within a metric formulation with black hole boundary conditions. The specific boundary conditions we adopt were first considered by Louko and Whiting [11] for SSG: we restrict consideration to slices that lie on the exterior of an eternal black hole in the generic theory. One end of the slice hits the bifurcation point along a static slice, while the other approaches the asymptotic region, also along a static slice. We derive the Hamiltonian for these boundary conditions and then quantize the theory. By adopting canonical variables first used by Cangemi et al [12] for SIG we are able to find exact physical eigenstates of the mass operator and Hamiltonian. We find that while the mass spectrum is continuous in the Lorentzian sector, when the theory is quantized in the Euclidean sector, single valuedness of the wave function requires the black hole entropy to be an integer multiple of $2\pi/G$. This agrees in form, but not in detail with recent conjectures by Bekenstein and Mukhanov [13] concerning the quantization of black hole entropy in Einstein gravity.

Section II reviews some general features of generic dilaton gravity and derives the Hamiltonian surface terms required in the case of black hole boundary conditions. In Section III the theory is quantized in the Lorentzian sector, using the phase space variables of Cangemi et al. Section IV outlines the same calculation for Euclidean black holes and argues that the entropy must be quantized in this case. Finally, Section V closes with conclusions and prospects for future work.

II. GENERIC DILATON GRAVITY

The classical action for generic dilaton gravity in two spacetime dimensions is

$$I = \frac{1}{2G} \int dt dx \sqrt{-g} \left( \eta R(g) + \frac{V(\eta)}{l^2} \right),$$

(1)

where $G$ is a (dimensionless) gravitational constant, and $l$ is a fundamental constant of dimension length. Eq.(1) is the most general coordinate invariant, second order action involving a scalar and metric in two spacetime dimensions [14]. It includes as special cases spherically symmetric gravity (SSG), for which $V(\eta) = 1/\sqrt{2\eta}$, as well as string inspired
gravity ($V(\eta) = 1$). The most general solution to the field equations in the generic theory up to spacetime diffeomorphisms can be written \[15\] \[10\]:

$$ds^2 = -(j(\eta) - 2GlM)dt^2 + \frac{1}{(j(\eta) - 2GlM)}dx^2,$$

$$\eta = x/l,$$ \hspace{1cm} (2)

where $j(\eta) = \int_0^\eta d\bar{\eta}V(\bar{\eta})$ and the parameter $M$ will turn out to be the ADM mass. These solutions all have a Killing vector, $k^\mu = \epsilon^{\mu\nu}\partial_\nu \eta/\sqrt{-g}$, whose norm can be written in coordinate invariant form as

$$|k|^2 = -t^2|\nabla \eta|^2 = (2GlM - j(\eta)).$$ \hspace{1cm} (3)

In the case that $j(\eta)$ is monotonic, the above solutions describe Schwarzschild-like black holes \[10\]. Note, however, that Eq.(2) does not describe metrics that are in general asymptotically Minkowskian, nor is the Killing vector normalized to a constant value at spatial infinity. One can therefore define a “physical metric” for which these properties do hold by doing a conformal reparametrization $\tilde{g}_{\mu\nu} = g_{\mu\nu}/j(\eta)$. The results that follow apply equally well to the physical metric, but the present parametrization simplifies the analysis considerably.

Details of the corresponding thermodynamics can be found in \[10\]. The crucial observation for the present purposes is that event horizons are surfaces of constant $\eta = \eta_0$ for which $|k|^2 = 0$. The entropy of the corresponding black hole is proportional to the value of the dilaton at the horizon, namely

$$S = \frac{2\pi}{G} \eta_0.$$ \hspace{1cm} (4)

The Hamiltonian formulation for the geometrical theory starts with a decomposition of the metric:

$$ds^2 = e^{2\rho} \left( -u^2 dt^2 + (dx - vdt)^2 \right).$$ \hspace{1cm} (5)

A straightforward calculation yields the following form for the action, up to surface terms:

$$I = \int dt \int_{\sigma_-}^{\sigma_+} dx \left( \Pi_\rho \dot{\rho} + \Pi_\eta \dot{\eta} - u\mathcal{E} - v\mathcal{P} \right),$$ \hspace{1cm} (6)

where $\Pi_\rho$ and $\Pi_\eta$ are momenta canonically conjugate to $\rho$ and $\eta$, respectively. and we have defined the Hamiltonian and momentum constraints:

$$\mathcal{E} = -G\Pi_\rho \Pi_\eta + \frac{1}{2G} \left( 2\eta'' - 2\rho' \eta' - \frac{V(\eta)}{l^2} e^{2\rho} \right),$$ \hspace{1cm} (7)

$$\mathcal{P} = \Pi'_\rho - \Pi_\rho \rho' - \Pi_\eta \eta'.$$ \hspace{1cm} (8)

It is useful to replace $\mathcal{E}$ by the following linear combination of constraints:\[\text{1}\]

\[\text{1}\]This was first done for JT by Henneaux \[3\].
\[ \mathcal{E} \equiv l e^{-2\rho} (-\eta' \mathcal{E} + G \Pi_\rho \mathcal{P}) = \frac{d\mathcal{M}}{dx}, \]  

(9)

where:

\[ \mathcal{M} := \frac{l}{2G} \left( e^{-2\rho} (G^2 \Pi^2 - (\eta')^2) + \frac{j(\eta)}{l^2} \right) = \frac{1}{2Gl}(|k|^2 + j(\eta)). \]  

(10)

Clearly \( \mathcal{M} \) is a constant on the constraint surface. One can verify that it commutes weakly with the constraints. Furthermore, for classical solutions Eq.(2), \( \mathcal{M} = M \) is the ADM mass. Thus, as discussed in [10], the constant mode of \( \mathcal{M} \) is a physical observable, corresponding to the ADM mass of the solution. It is possible, following Kuchar [3] to do a canonical transformation in which \( \mathcal{M} \) becomes one of the phase space variables, but this will not be necessary for our purposes. We henceforth call \( \mathcal{M} \) the mass observable, to distinguish it from the total Hamiltonian.

In terms of the new constraint, the canonical Hamiltonian is:

\[ H_c = \int_{\sigma^-}^{\sigma^+} dx (-\tilde{u} \mathcal{M}' + \tilde{v} \mathcal{P}) + H_+ - H_-, \]  

(11)

where

\[ \tilde{u} = \frac{u e^{2\rho}}{l\eta'}, \]  

(12)

\[ \tilde{v} = v + \frac{uG\Pi_\rho}{\eta'}. \]  

(13)

\( H_+ \) and \( H_- \) are determined by the requirement that the surface variations vanish for a given set of boundary conditions.

For concreteness we will choose black hole boundary conditions similar to those considered recently by Louko and Whiting [11] in the case of spherically symmetric gravity. In particular we restrict consideration to slices that are static at both \( \sigma^+ \) and \( \sigma^- \). \( \sigma^+ \) will correspond to the asymptotic region of an eternal black hole, and \( \sigma^- \) will be chosen to approach the horizon bifurcation point. Thus, we require \( \tilde{v}_+ = 0 \), so that only the first term in Eq.(11) contributes a surface term at \( \sigma^+ \). Moreover if \( \sigma^+ \) lies in the asymptotic external region of the black hole, it can be shown that \( \tilde{u}_+ \to 1 \), from which it follows immediately that

\[ H_+ = \mathcal{M}_+ = \mathcal{M} \]  

(14)

For the surface at \( \sigma^- \) to approach the bifurcation point (i.e. \( k^\mu = 0 \)) along a static slice, the following conditions must be satisfied [11]: \( u_- = 0, v_- = 0, \Pi_\rho(\sigma_-) = 0 \) and \( \eta'_- = 0 \). With these boundary conditions, the constraints imply that:

\[ \text{It also corresponds to the Casimir invariant that characterizes solutions in the Poisson sigma model approach [5].} \]
\[ \tilde{u}_- = \frac{u' e^{2\rho}}{1\eta''} = \frac{2l N_0}{V(\eta_-)}. \] (15)

where we have used l’Hôpital’s rule to get the middle expression. The final expression was obtained using \( \eta'' \) from Eq.(9) with \( \eta' = 0 \) and \( \Pi_\rho = 0 \). \( N_0 := u'_- \) gives the rate of change of the unit normal to the constant \( t \) surfaces at the bifurcation two sphere [4]. For the on-shell Euclidean black hole it is proportional to the Hawking temperature. Using the fact that \( |k|^2 = 0 \) (cf Eq.(3)), one has

\[ \delta M_+ = \frac{V(\eta_-)}{2Gl} \delta \eta_-, \] (16)

so that

\[ \tilde{u} \delta M_- = \frac{N_0}{Gl} \delta \eta_. \] (17)

This can be integrated for fixed \( N_0 \), so that the total canonical Hamiltonian is:

\[ H_c = \int_{\sigma_-}^\sigma \ dx (-\tilde{u} \mathcal{M}' + \tilde{v} \mathcal{P}) + \mathcal{M} - \frac{N_0}{2\pi} S, \] (18)

where we have used Eq.(3) to define the classical thermodynamic entropy:

\[ S = \frac{2\pi}{G} \eta_- . \] (19)

Note however that \( \eta_- \) and \( M \) are not independent. In particular, Eq.(10) requires that on the constraint surface:

\[ j(\eta_-) = 2GlM. \] (20)

Eq.(18) generalizes Eq.(5.2) of [4] to the case of black holes in generic dilaton gravity. Since \( N_0 \) determines the deficit angle for the off-shell black hole, it is also consistent with the results of Teitelboim [16] who showed for SIG that \( N_0 \) is conjugate to the entropy.

### III. QUANTUM THEORY

We now introduce a generalization of the phase space variables first used by CJZ [12] in the context of SIG. These variables are also closely related to Poisson Sigma model variables [8]. We perform the canonical transformation

\[ \{ \rho(x), \Pi_\rho(x), \eta(x), \Pi_\eta(x) \} \rightarrow \{ \rho^a(x), p_a(x), \eta_-, \theta_- \} \ (a = 0, 1) \] (21)

defined by

\[ \rho^1 = e^{-\rho}(\Pi_\rho \sinh \theta - \eta' \cosh \theta), \] (22)

\[ p_1 = e^\rho \sinh \theta, \] (23)

\[ \rho^0 = e^{-\rho}(\Pi_\rho \cosh \theta - \eta' \sinh \theta), \] (24)

\[ p_0 = -e^\rho \cosh \theta, \] (25)

\[ \eta_- = \eta(\sigma_-), \ \theta_- = \theta(\sigma_-), \] (26)
where

$$\theta(x) \equiv -\int_x^{\sigma_+} d\tilde{x} \Pi_\eta(\tilde{x}).$$  \hspace{1cm} (27)$$

The inverse transformation reads

$$e^{2\rho} = (p_0^2 - p_1^2) \equiv -p^2,$$  \hspace{1cm} (28)

$$\Pi_\rho = -\rho^0 p_0 - \rho^1 p_1,$$  \hspace{1cm} (29)

$$\eta = \eta_- + \int_{\sigma_-}^{x} d\tilde{x} (\rho^0 p_1 + \rho^1 p_0),$$  \hspace{1cm} (30)

$$\Pi_\eta = -\left[ \arctanh \left( \frac{p_1}{p_0} \right) \right]' = -\frac{p_0 p_1' - p_1 p_0'}{p^2}$$  \hspace{1cm} (31)

and generates a new symplectic form with $\theta_-$ canonically conjugate to $\eta_-.$

$$\int_{\sigma_-}^{\sigma_+} dx (\Pi_\rho \delta \rho + \Pi_\eta \delta \eta) = \int_{\sigma_-}^{\sigma_+} dx p_a \delta \rho^a + \eta_- \delta \theta_-.$$  \hspace{1cm} (32)

It would seem that the new variables form an overcomplete set because formally $\theta_-$ is not independent from the momenta $p_a:$

$$\theta_- = -\arctanh(p_1/p_0)|_{\sigma_-}. \hspace{1cm} (33)$$

However, the bifurcation point boundary conditions imply that $\rho_-^a = 0$ so that $p_a|_{\sigma_-}$ do not appear in the integral on the right hand side of Eq.(32). They appear only implicitly through the variable $\theta_-.$ Thus $\theta_-$ is conjugate to $\eta_-$ and the canonical transformation is non-singular. Another argument for including $(\eta_-, \theta_-)$ as an independent pair of conjugate variables is that variation of the resulting action with respect to $\theta_-$ leads to correct equation of motion for $\eta_-,$ $\dot{\eta}_- = 0.$

In terms of the new variables, the mass operator $\mathcal{M}$ takes a very simple form:

$$\mathcal{M} = \frac{l^2}{2G} \left( -\rho^2 + \frac{j(\eta)}{l^2} \right),$$  \hspace{1cm} (34)

where $\rho^2 \equiv (\rho^1)^2 - (\rho^0)^2.$ The momentum constraint is:

$$\mathcal{P} = -\rho_0^0 p_0 - \rho_1^1 p_1.$$  \hspace{1cm} (35)

\footnote{Note that $\theta_-$ is not analytic in terms of new momenta and the nonanalyticity point does in principle belong to their range: for the classical solution in Schwarzschild-like coordinates (Eq.(2)) $\exp[\rho(\sigma_-)] = (j(\sigma_-) - 2GM)^{-1} = \infty,$ so that from (28) and (29) $p_a(\sigma_-) = \infty.$ The singularity in the momenta at $\sigma_-$ does not however spoil the symplectic form because $p_a(\sigma_-)$ are multiplied by $\delta \rho^a(\sigma_-) = 0$ with $\rho^a(\sigma_-) = 0$ at the bifurcation point. Thus the ratio $(p_1/p_0)(\sigma_-)$ is not determined and should be taken care of by the extra variable $\theta_-$ becoming a dynamical momentum conjugate to $\eta_-.$}
One final feature of these variables that is crucial to the quantization is the fact that \( \eta \) and \( \rho^2 \) have vanishing Poisson bracket.

We now quantize the theory, in the functional Schrödinger representation, with wave functionals \( \Psi[\rho^a|\theta_-] \). The notation indicates that \( \Psi[\rho^a|\theta_-] \) is a functional of \( \rho^a(x) \), but an ordinary function of the coordinate \( \theta_- \). The momentum operators are:

\[
\hat{p}_a = -i\hbar \frac{\delta}{\delta \rho^a},
\]

\[
\hat{\eta}_- = -i\hbar \frac{\partial}{\partial \theta_-},
\]

(36)

We wish to find physical states that are eigenstates of the Hamiltonian operator:

\[
\hat{H} = \int_{\sigma_+}^{\sigma_-} dx \left( -\tilde{u}\hat{\mathcal{M}} + \tilde{v}\hat{\mathcal{P}} + \hat{\mathcal{M}} - \frac{N_0}{G}\hat{\eta}_- \right),
\]

(37)

where

\[
\hat{\mathcal{P}} = -\rho^0\hat{p}_0 - \rho^1\hat{p}_1,
\]

\[
\hat{\mathcal{M}} = \frac{l}{2G} \left( -\rho^2 + \frac{j(\eta)}{l^2} \right),
\]

\[
\hat{\eta}(x) = \hat{\eta}_- + \int_{\sigma_-}^{x} dx (\rho^0\hat{p}_1 + \rho^1\hat{p}_0).
\]

(38)

A necessary and sufficient condition is that the following be satisfied:

\[
\hat{\mathcal{P}} \Psi[\rho^a|\theta_-] = 0,
\]

\[
\hat{\mathcal{M}} \Psi[\rho^a|\theta_-] = M \Psi[\rho^a|\theta_-],
\]

\[
\hat{\eta}_- \Psi[\rho^a|\theta_-] = \eta_- \Psi[\rho^a|\theta_-],
\]

(39)

where \( M \) and \( \eta_- \) are (constant) c-numbers. Inspired by the solution of [12] to SIG, we try the following ansatz:

\[
\Psi[\rho^a|\theta_-] = \exp \left( \frac{i}{\hbar} \int_{\sigma_-}^{\sigma_+} dx \left( \omega(\rho^2)(\rho^0\rho^1 + \rho^1\rho^0) \right) \right) \exp \left( \frac{i}{\hbar} \eta_- \theta_- \right).
\]

(40)

This is invariant under spatial diffeomorphisms because the integrand is the product of a scalar and a density. (The \( \rho^i \)'s are all scalars and their derivatives densities). It can be shown that [4]

\[
\hat{\eta} \Psi[\rho^a|\theta_-] = (\eta_- + \omega \rho^2) \Psi[\rho^a|\theta_-],
\]

(41)

\[\text{We are assuming the boundary condition at } \sigma_+ \text{ that } (\rho^0_+/\rho^1_+) \text{ is also fixed. Otherwise the variation of } \Psi[\rho^a|\theta_-] \text{ would require the addition of a boundary term to the phase. This boundary condition naturally arises in the asymptotically static case. From the classical solution } (2) \text{ and } \theta_+ = 0 \text{ it follows that } (\rho^0/\rho^1)_+ = -\Pi_{\rho}/\eta'|_+ = 0.\]
where we have used black hole boundary condition $\rho_2^2 = 0$. Now we use the fact that $[\hat{\eta}, \rho^2] = 0$ (there is no anomaly in this commutator because $\rho^2 = (\rho^1)^2 - (\rho^0)^2$ is a 2D indefinite quadratic form and anomalies coming from $\rho^1$ and $\rho^0$ sectors cancel out [12]): the action of $j(\hat{\eta})$ on $\Psi[\rho^a|\theta_-)$, for any $j(\eta)$ that has a Taylor expansion, is simply:

$$j(\hat{\eta})\Psi[\rho^a|\theta_-) = j(\eta_+ + \omega \rho^2)\Psi[\rho^a|\theta_-).$$

(42)

Thus, the state will be an eigenstate of the ADM mass operator $\hat{M}$ with eigenvalue $M$ if:

$$j(\eta_+ + \omega \rho^2) - l^2 \rho_2^2 = 2GlM.$$  

(43)

The eigenvalue $\eta_+$ is fixed by Eq.(43) and the boundary condition $\rho_2^2 = 0$ to be:

$$\eta_+ = j^{-1}(2GlM).$$

(44)

The solution for $\omega$ is therefore:

$$\omega(\rho^2) = \frac{1}{\rho^2} \left( j^{-1}(2GlM + l^2 \rho^2) - j^{-1}(2GlM) \right).$$

(45)

With $\omega$ and $\eta_+$ given above the wave functional Eq.(40) represents a physical eigenstate of both the ADM mass operator $\hat{M}$ and the entropy operator $\hat{\eta}_-$. For a given mass $M$, the eigenvalue of the entropy is fixed to be its classical value, and the corresponding eigenvalue of the Hamiltonian is:

$$E = M - \frac{N_0}{G} j^{-1}(2GlM).$$

(46)

Note that for SIG, $j(\eta) = \eta$, which gives

$$\omega = l^2$$

(47)

in agreement with the result of [12].

IV. EUCLIDEAN BLACK HOLES

We now briefly describe the result of quantizing the black holes in the Euclidean sector. The Euclidean version of the theory originates from the Wick rotation to imaginary (Euclidean) time: $t = -it_E$, $v = iv_E$ (the latter redefines the shift to absorb an extra factor of $i$ in the metric so that everything is real) and boils down to imaginary values of momenta $\Pi_{\rho} = -i\Pi^E_{\rho}$, $\Pi_{\eta} = -i\Pi^E_{\eta}$, so that the mass operator in terms of the Euclidean momentum acquires a negative kinetic term

\textsuperscript{5}CGZ obtained only two states (as opposed to the continuous spectrum found here) because they imposed stronger boundary conditions. We are grateful to E. Benedict for conversations on this point.
\[ M = -\frac{l}{2G} e^{-2\rho((\eta')^2 + \Pi^2)} + \frac{j(\eta)}{2Gl}. \] (48)

From (27) it is obvious that \( \theta \) is also subject to Wick rotation \( \theta = i\theta_E \) whence it follows that in the canonical transformation to CJZ variables the hyperbolic sine and cosine go over into trigonometric functions of an angular variable \( \theta_E \) and, moreover, \( \rho^0 \) and \( p_1 \) become imaginary: \( \rho^0 = i\rho_0^0, \rho_1 = i\rho_1^1 \). Thus the coordinates \( \rho^a \) under Euclideanization transform as the Wick rotation of a “timelike” coordinate, emphasizing their geometric nature as embedding variables in the Lorentzian and Euclidean 2D spacetimes. For Euclidean variables the CJZ canonical transformation

\[
\begin{align*}
  e^{2\rho} &= (p_0^E)^2 + (p_1^E)^2, \\
  \Pi_\rho &= -(\rho_0^E p_0^1 + \rho_1^E p_1^0), \\
  \eta(x) &= \eta_\pi - \int_0^x d\tilde{x}(\rho_0^0 p_0^1 - \rho_1^1 p_1^0), \\
  \Pi_\eta &= -\left[\arctan\left(\frac{p_1^E}{p_0^E}\right)\right]'
\end{align*}
\] (49)

generates the same symplectic form (52) with \( \theta_\pi = -\arctan(\rho_1^E/\rho_0^E) |_- \) and yields a mass operator \( 2GM = -l \rho_0^2 + j(\eta)/l, \rho_0^2 \equiv (\rho_0^E)^2 + (\rho_1^E)^2 \) for which one can find exact quantum eigenstates precisely as before. Now however the angular variable \( \theta_\pi \) determines the orientation of the vector \( p_0^E \) in the 2D Euclidean momentum plane, and it is natural to demand that the physical state is periodic in this variable. Therefore, single valuedness of the factor \( \exp(i\eta_\pi \theta_\pi / \hbar) \) in the wavefunction requires the eigenvalue \( \eta_\pi \) of the dilaton operator to be an integer in \( \hbar \) units. The black hole entropy is therefore quantized: \( S(M) = 2\pi \hbar n/G \).

Remarkably this gives the same spectrum for Euclidean black holes as reduced quantization [17].

V. CONCLUSIONS

We have quantized generic dilaton gravity theory exactly in terms of geometrical variables and derived physical black hole states that are eigenstates of the Hamiltonian, mass operator and entropy operator. In previous work [15], the constraints were first linearized in the momenta and then solved exactly at the quantum level. The resulting states were only eigenstates of the mass operator in the WKB approximation. Louis-Martinez [18] has been able to show that the states presented here are equivalent to those presented in [15], thus generalizing Benedict’s proof [19] for SIG. In retrospect this is not surprising. In the present parametrization we have found that the WKB approximation is exact. To WKB order it is always possible to find the transformation relating states derived in two different parametrizations. What the present analysis shows is that there exists a factor ordering for the mass operator for which the WKB states found in [15] are also exact.

It is clearly of interest to explore the properties of these wave functionals in more detail and, in particular, to understand the meaning of the quantization of the theory in the Euclidean sector. It should be emphasized that the procedure we used was not the analytic continuation of the Lorentzian quantum state to the Euclidean section of phase space, but first the Euclideanization of the theory with its subsequent quantization. An important
difference of this procedure from the usual transition to Euclidean spacetime in nongravitational theories is a complexification of the phase space resulting here in a Wick rotation of the “timelike” CJZ coordinate $\rho^0$. With the other choice of variables for quantization this complexification might look very involved and geometrically vague, while in CJZ parameterization this Euclideanization has a geometrically covariant form. Clearly, Euclidean 2D dilaton gravity should be related to the physical theory in Lorentzian spacetime just like in ordinary theories: it describes thermodynamical and underbarrier penetration properties or serves as a powerful calculational tool for quantum transition amplitudes. Understanding this relation might give an insight into the quantized nature of the black hole mass and entropy and explain the statistical mechanical origin of the latter. Finally, given the utility of the CJZ variables in the pure gravity case, it is likely that the quantization of the generic theory with matter would also be possible using these techniques. This is currently under investigation.

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