Angle dependent Gap state in Asymmetric Nuclear Matter

Xin-le Shang, Wei Zuo

Institute of Modern Physics, Chinese Academy of Science, P.O. Box 31, Lanzhou 730000

Abstract

We propose an axi-symmetric angle dependent gap (ADG) state with the broken rotational symmetry in isospin-asymmetric nuclear matter. In this state, the deformed Fermi spheres of neutron and proton increase the pairing probabilities along the axis of the symmetry breaking near the average Fermi surface. We find the state possesses of lower free energy and larger gap value than the angle-averaged gap state for large isospin asymmetry. These properties are mainly caused by the coupling of different $m_j$ components of the gap. Furthermore, we find the transition from the ADG state to normal state is of second order and the state vanish at the critical isospin-asymmetry $\alpha_c$ where the angle-averaged gap vanishes.

Key words: asymmetric nuclear matter, BCS theory, properties of superfluid

PACS: 21.65.Cd, 26.60.-c, 74.20.Fg, 74.25.-q

1 Introduction

The neutron-proton (n-p) pairing properties play an important role in the description of superfluidity of finite nuclei with $N \simeq Z$ \cite{12} and symmetric nuclear matter\cite{3,15}. In generally, the n-p pairing correlations are considered in different dominant partial-wave channels, which depend on the relevant density and temperature. For weakly isospin asymmetric systems, the isospin singlet $^3S_1 - ^3D_1$ channel dominates the pairing interaction at relatively low densities around the nuclear saturation density due to the tensor component of the nuclear force\cite{3,6,7,8,9,10}, while the $^3D_2$ channel dominates at high density well above the saturation density\cite{11,12}. In neutron star matter, the n-p pairing correlations are fatally suppressed by the isospin asymmetry. However the supernova and the hot pro-neutron star matter\cite{13,14} at sub-saturation density which have low isospin asymmetry can support $^3S_1 - ^3D_1$ channel pairing\cite{8,15}.
Since n-p pair correlations depend crucially on the overlap between the neutron and proton Fermi surface, the paring gap is suppressed rapidly as the system is driven out of isospin symmetric state. At zero temperature, a small isospin-asymmetry is enough to prevent the formation of Cooper pairs between neutrons and protons with momenta \( \vec{k} \) and \(- \vec{k}\) around the average Fermi surface where the contribution to superfluidity is dominant. Near the zero temperature the thermal excitations can reduce the suppression by smearing out the two Fermi surfaces, however it is ineffective when the separation between the two Fermi surfaces is large compared to the temperature. In isospin-asymmetric nuclear matter, the FFLO \([16,17]\) state and DFS (deformed Fermi surfaces) \([18]\) state have been studied in Refs.\([19,20]\) when the asymmetry is small. In a FFLO state, the shift of the two Fermi spheres with respect to each other, which results in the Cooper pair moving with a finite momentum, enhances the overlap between neutron and proton Fermi surfaces. And the overlap region provide the kinematical phase space for pairing phenomena to occur. While in a DFS state, the deformation of the Fermi surfaces may increase the phase-space overlap between the neutron and proton Fermi surfaces. Both in these two kinds of possible superfluid states, the quasiparticle excitation spectra are no longer isotropic since the anisotropic overlapping configurations could increase the pairing energy. On the other hand, the usually adopted angle-averaging procedure in the previous calculations\([7,19]\), which has been proved to be a quite good approximation in symmetry nuclear matter\([21]\), considers the gap as isotopic gap by ignoring the angle dependence. As the true ground state connects with the anisotropic overlapping configuration, the angle-averaging procedure may be an insufficient approximation in isospin-asymmetric nuclear matter.

In this paper we consider an angle dependent gap (ADG) solution which ensures an axi-symmetric quasiparticle spectrum and give a general and systematic comparison between the angle-averaged gap (AAG) and ADG solutions in isospin-asymmetric nuclear matter under different conditions. The paper is organized as follows: In Sec. II we briefly review the formalism for the isotropic AAG state, and derive the angle dependent gap equations from the Gorkov equations. The numerical solutions of these equations are shown in Sec. III, where we compare the AAG and ADG phases at finite temperatures. We finish in Sec. IV with a discussion of our results and present our conclusions.

2 Formalism

For isospin-asymmetric nuclear matter, the isospin singlet \( ^3S_1 - ^3D_1 \) paring channel dominates the attractive pairing force at low density. In this case we can consider \( S - D \) channel only, and the gap function is expanded according to
\[ \Delta_{\sigma_1,\sigma_2}(k) = \sum_{l,m_j} \Delta_{l}^{m_j}(k)[G_{l}^{m_j}(\hat{k})]_{\sigma_1,\sigma_2}, \]  
(1)

the elements of the spin-angle matrices are

\[ [G_{l}^{m_j}(\hat{k})]_{\sigma_1,\sigma_2} \equiv \left\langle \frac{1}{2}\sigma_1, \frac{1}{2}\sigma_2 \mid 1\sigma_1 + 1\sigma_2 \right\rangle \left\langle 1\sigma_1 + \sigma_2, l m_l \mid 1 m_j \right\rangle Y_{l}^{m_l}(\hat{k}), \]  
(2)

where \( m_j \) and \( m_l \) are the projections of the total angular momentum \( j = 1 \) and the orbit angular momentum \( l = 0, 2 \) of the pair. The \( Y_{l}^{m_l}(\hat{k}) \) denotes the spherical harmonic and \( \hat{k} \equiv k/k \). The anomalous density matrix follows the same expansion. Moreover the time-reversal invariance implies that

\[ \Delta_{\sigma_1,\sigma_2}(k) = (-1)^{1+\sigma_1+\sigma_2} \Delta_{-\sigma_1,-\sigma_2}^{\ast}(k). \]  
(3)

And the pairing gap matrix \( \Delta(k) \) in spin space possesses the property

\[ \Delta(k)\Delta^{\dagger}(k) = ID^2(k), \]  
(4)
i.e., the gap function has the structure of a “unitary triplet” state \[21].\( I \) is the identity matrix and \( D(k) \) is a scalar quantity in spin space.

Once the isospin singlet \( S-D \) channel has been selected, the gap is an isoscalar and the isospin indices can be wiped off. The proton/neutron propagators follow from the solution of the Gorkov equations are present in the form \((\hbar = 1)\)

\[ G_{\sigma,\sigma'}^{(p/n)}(k, \omega_m) = -\delta_{\sigma,\sigma'} \frac{i\omega_m + \xi_k \mp \delta \varepsilon_k}{(i\omega_m + E^+_k)(i\omega_m - E^-_k)}. \]  
(5)

And the neutron-proton anomalous propagator matrix in spin space has the form

\[ F^{\dagger}(k, \omega_m) = -\frac{\Delta^{\dagger}(k)}{(i\omega_m + E^+_k)(i\omega_m - E^-_k)}. \]  
(6)

where \( \omega_m \) are the Matsubara frequencies, the upper sign in \( G_{\sigma,\sigma'}^{(p/n)} \) corresponds to protons, and the lower to neutrons. The quasiparticle spectra are determined by finding the poles of the propagators in Gorkov equations,

\[ E^\pm_k = \sqrt{\xi^2_k + \frac{1}{2}Tr(\Delta\Delta^{\dagger})} \pm \frac{1}{2}\sqrt{[Tr(\Delta\Delta^{\dagger})]^2 - 4 det(\Delta\Delta^{\dagger})} \pm \delta \varepsilon_k, \]  
(7)
where

\[
\xi_k \equiv \frac{1}{2} (\varepsilon_p^k + \varepsilon_n^k), \quad \delta \varepsilon_k \equiv \frac{1}{2} (\varepsilon_p^k - \varepsilon_n^k),
\]

and \( \varepsilon_k^{(n/p)} \) are the single particle energies of neutrons and protons. Using the “unitary” property in Eq. (7), the quasiparticle spectra are simplified to

\[
E_k^\pm = \sqrt{\xi_k^2 + D^2(k)} \pm \delta \varepsilon_k,
\]

which is separated into two branched due to the isospin asymmetry.

In the present “unitary triplet” case, the gap equation with finite temperature can be written in the standard form

\[
\Delta_{\sigma_1,\sigma_2}(k) = -\sum_{k'} \sum_{\sigma_1',\sigma_2'} < k \sigma_1, -k \sigma_2 | V | k' \sigma_1', -k' \sigma_2' > \\
\times \frac{\Delta_{\sigma_1',\sigma_2'}(k')}{2 \sqrt{\xi_{k'}^2 + D^2(k')}} [1 - f(E_{k'}^+) - f(E_{-k'}^-)],
\]

where \( f(E) = \{1 + \exp(\beta E)\}^{-1} \) is the Fermi distribution at finite temperature and \( V \) is the interaction in the \( S-D \) channel. \( \beta^{-1} = k_B T \), where \( k_B \) is the Boltzmann constant and \( T \) is the temperature. Substituting the expansion Eq.(1) into Eq.(4) and Eq.(9), one gets a set of coupled equations for the quantities \( \Delta_{IJ}^m(k) \)

\[
\Delta_{IJ}^m(k) = -\sum_{k'} \sum_{\sigma_1',\sigma_2'} < k \sigma_1, -k \sigma_2 | V | k' \sigma_1', -k' \sigma_2' > \\
\times \frac{\Delta_{\sigma_1',\sigma_2'}(k')}{2 \sqrt{\xi_{k'}^2 + D^2(k')}} [1 - f(E_{k'}^+) - f(E_{-k'}^-)]
\]

and

\[
D^2(k) = \frac{1}{2} Tr(\Delta \Delta^\dagger) = \sum_{ll'=0,2} \Delta_{IJ}^m(k) \Delta_{IJ'}^{m'}(k) Tr[G_{l}^{mJ\ast}(|k\rangle G_{l'}^{m'J}(|k\rangle)]
\]

where

\[
V_{\chi}^{ll'}(k', k) \equiv < k' | V_{\chi}^{ll'} | k \rangle = \int_0^\infty r^2 dr j_{l'}(k' r) V_{\chi}^{ll'}(r) j_l(k r)
\]
is the matrix elements of the NN interaction in different partial wave \((\lambda = T, S, l, l')\) channels. Here \(\lambda\) corresponds to the coupled \(3\)\(SD_1\) channel. Following from Eq.(5), we can get the densities of neutrons and protons,

\[
\rho^{(p/n)}(k) = \sum_{k, \sigma} n^{(p/n)}_{\sigma}(k),
\]

with the distributions

\[
n^{(p/n)}_{\sigma}(k) = \left\{ \begin{array}{ll}
\frac{1}{2}(1 + \frac{\xi_k}{\sqrt{\xi_k^2 + D^2(k)}}) f(E_k^{\pm}) \\
+ \frac{1}{2}(1 - \frac{\xi_k}{\sqrt{\xi_k^2 + D^2(k)}})[1 - f(E_k^{\mp})]
\end{array} \right. \}
\]

Summation over frequencies in Eq.(6) leads to the density matrix of the particles in the condensate,

\[
\nu(k) = \frac{\Delta(k)}{2\sqrt{\xi_k^2 + D^2(k)}}[1 - f(E_k^+) - f(E_k^-)].
\]

For the isospin-asymmetric nuclear matter, the coupled equations (10) and (13) should be solved simultaneously.

The six components \(\Delta_{l}^{m_j}(k)\) in \(\Delta(k)\) are strongly coupled due to the angle-dependent energy denominator \(\sqrt{\xi_k^2 + D^2(k)}\) in equations (10) and (13). And the equations are complicated to be solved in the full complexity, and approximation has been employed. Before introducing the angle-averaging procedure and ADG, we need to substitute \(\Delta_{l}^{m_j}(k)\) with real variables. From Eq.(3) we can find the relations

\[
\Delta_{l}^{m_j}(k) = (-1)^{m_j} \Delta_{l}^{-m_j}(k).
\]

Thus for \(3\)\(SD_1\) channel we have four independent components \(\Delta_0^0(k), \Delta_0^1(k), \Delta_2^0(k)\) and \(\Delta_2^1(k)\). Therefore, we can describe \(\Delta_{l}^{m_j}(k)\) as

\[
\begin{align*}
\Delta_0^0(k) &= i\delta_0(k), \\
\Delta_0^1(k) &= \delta_1(k) + in_1(k), \\
\Delta_2^0(k) &= i\delta_2(k), \\
\Delta_2^1(k) &= \delta_3(k) + in_3(k),
\end{align*}
\]

where the six independent variables \(\delta_0(k), \delta_1(k), n_1(k), \delta_2(k), \delta_3(k)\) and \(n_3(k)\) are real quantities. Inserting Eq.(17) into Eq.(11), we get
\[D^2(\mathbf{k}) = \frac{1}{32\pi} \left\{ 4\delta_0^2(k) - 4\sqrt{2}\delta_0(k)\delta_2(k)[3\cos^2\theta - 1] + 2\delta_2^2(k)[3\cos^2\theta - 1] + 8[\delta_2^2(k) + n_1^2(k)] + 8[\delta_3^2(k) + n_3^2(k)] + 6[\delta_2^2(k) + n_3^2(k)]\sin^2\theta + 4\sqrt{2}n_1(k)n_3(k)[3\cos^2\theta - 1] + 4\sqrt{2}\delta_1(k)\delta_3(k)[3\cos^2\theta - 1] + 12[2\delta_0(k)n_3(k) + 2\delta_3(k)n_1(k) - \sqrt{2}\delta_2(k)n_3(k)]\cos\theta \sin\theta \cos\varphi + 12[2\delta_1(k)\delta_2(k) + 2\delta_0(k)\delta_3(k) - \sqrt{2}\delta_2(k)\delta_3(k)]\cos\theta \sin\theta \sin\varphi + 6[n_3^2(k) - \delta_2^2(k) + 2\sqrt{2}\delta_1(k)\delta_3(k) - 2\sqrt{2}n_1(k)n_3(k)]\sin^2\theta \cos 2\varphi + 12[\delta_3(k)n_3(k) - \sqrt{2}\delta_1(k)n_3(k) - \sqrt{2}\delta_3(k)n_1(k)]\sin^2\theta \sin 2\varphi \right\}. \]

(18)

2.1 The angle averaging procedure

Supposing the angle-dependence can be neglected, the gap equations are simplified by substituting \(D^2(\mathbf{k})\) with its angular average values,

\[D^2(\mathbf{k}) \rightarrow d^2(k) = \frac{1}{4\pi} \int d\Omega_k D^2(\mathbf{k}) = \frac{1}{8\pi} \left[ 2\delta_0^2(k) + \delta_0^2(k) + 2n_1^2(k) + 2\delta_3^2(k) + \delta_2^2(k) + 2n_3^2(k) \right]. \]

(19)

Thereby, the energy denominator and the quasiparticle spectra are isotropic, noting the properties of \(G_{l,m_i}^{m_j}(\hat{k})\)

\[\int d\Omega_k Tr[G_{l,m_i}^{m_j}(\hat{k})G_{l,m_j}^{m_i}(\hat{k})] = \delta_{ll'}\delta_{m_im_j}, \]

(20)

the different \(m_j\) components \(\Delta_{l,m_j}^m(k)\) with the same \(l\) become uncoupled and all equal to each other. It follows that

\[\delta_1(k) = n_1(k) = \sqrt{\frac{1}{2}}\delta_0(k), \delta_3(k) = n_3(k) = \sqrt{\frac{1}{2}}\delta_2(k), \]

(21)

and

\[d^2(k) = \frac{3}{8\pi}[\delta_0^2(k) + \delta_2^2(k)]. \]

(22)

Taking the normalization

\[\Delta_0(k) = \sqrt{\frac{3}{8\pi}}\delta_0(k), \Delta_2(k) = -\sqrt{\frac{3}{8\pi}}\delta_2(k), \]

(23)
the set of equations in Eq.(10) reduces to two coupled equations for the \( S \) and \( D \) gap components \( \Delta_0(k) \) and \( \Delta_2(k) \), respectively. They read

\[
\begin{pmatrix}
\Delta_0 \\
\Delta_2
\end{pmatrix}(k) = \frac{-1}{\pi} \int dk' k'^2 \begin{pmatrix}
V^{00} & V^{02} \\
V^{20} & V^{22}
\end{pmatrix}(k, k') \frac{1 - f(E^+_{k'}) - f(E^-_{k'})}{\sqrt{\xi^2_{k'} + D^2(k')}} \begin{pmatrix}
\Delta_0 \\
\Delta_2
\end{pmatrix}(k'),
\]

(24)

where \( V^{00}, V^{02}, V^{20}, V^{22} \) are given in Eq.(12) with \( l, l' = 0, 2 \) and

\[
E^\pm_k = \sqrt{\xi^2_k + D^2(k)} \pm \delta \varepsilon_k, \quad D^2(k) \equiv d^2(k) = \Delta^2_0(k) + \Delta^2_2(k).
\]

(25)

Eqs. (13), (24) and (25) compose the angle-averaged gap equations and should be solved self-consistently for isospin-asymmetric nuclear matter. The quasiparticle spectra here are isotropic and the gapless excitation exists at large asymmetry near zero temperature.

2.2 The angle dependent gap

As pointed in the Sec.I, the angle dependence of quasiparticle spectra due to \( D^2(k) \) may increase the phase-space overlap of neutrons and protons near the average Fermi surface. We consider an axi-symmetric \( D^2(k) \) solution which correspond to an axi-symmetric deformation of the neutron and proton Fermi spheres. From the expression Eq.(18), the axi-symmetric solutions are restricted by

\[
\begin{align*}
2\delta_0(k) n_3(k) + 2\delta_2(k) n_1(k) - \sqrt{2}\delta_2(k) n_3(k) &= 0, \\
2\delta_1(k) \delta_2(k) + 2\delta_0(k) \delta_3(k) - \sqrt{2}\delta_2(k) \delta_3(k) &= 0, \\
n_3^2(k) - \delta_3^2(k) + 2\sqrt{2}\delta_1(k) \delta_3(k) - 2\sqrt{2}n_1(k) n_3(k) &= 0, \\
\delta_3(k) n_3(k) - \sqrt{2}\delta_1(k) n_3(k) - \sqrt{2}\delta_3(k) n_1(k) &= 0.
\end{align*}
\]

(26)

There exist only one nontrivial solution

\[
\delta_1(k) = n_1(k) = \delta_3(k) = n_3(k) = 0,
\]

(27)

which correspond to the \( m_j = 0 \) gap components of \( \Delta^{m_j}_l(k) \). In this case
D^2(k) \rightarrow D^2(k, \theta) = \frac{1}{8\pi} \left[ \delta_0^2(k) - \sqrt{2} \delta_0(k) \delta_2(k)(3 \cos^2 \theta - 1) + \delta_2^2(k) \frac{3 \cos^2 \theta + 1}{2} \right]. \quad (28)

Using the normalization

\[ \Delta_0(k) = \sqrt{\frac{1}{8\pi}} \delta_0(k), \quad \Delta_2(k) = -\sqrt{\frac{1}{8\pi}} \delta_2(k), \quad (29) \]

one gets the angle dependent gap equations

\[
\begin{pmatrix}
\Delta_0 \\
\Delta_2
\end{pmatrix}(k) = \frac{-1}{\pi} \int dk' k'^2 \begin{pmatrix}
V^{00} & V^{02} \\
V^{20} & V^{22}
\end{pmatrix}(k, k') \\
\times \int d\Omega_k' \frac{1 - f(E^+_k) - f(E^-_k)}{\sqrt{\xi_k^2 + D^2(k', \theta)}} \begin{pmatrix}
f(\theta) & g(\theta) \\
g(\theta) & h(\theta)
\end{pmatrix} \begin{pmatrix}
\Delta_0 \\
\Delta_2
\end{pmatrix}(k'), \quad (30)
\]

with the following axi-symmetric quantities,

\[
D^2(k, \theta) = \Delta_0^2(k) + \sqrt{2} \Delta_0(k) \Delta_2(k)[3 \cos^2 \theta - 1] + \Delta_2^2(k)[\frac{3 \cos^2 \theta + 1}{2}],
\]

\[
E^\pm_k = \sqrt{\xi_k^2 + D^2(k, \theta)} \pm \delta_k.
\quad (31)
\]

The angle matrix \[
\begin{pmatrix}
f(\theta) & g(\theta) \\
g(\theta) & h(\theta)
\end{pmatrix}
\]
comes from the coupling among the different m_j components. The matrix elements are

\[
f(\theta) = Tr[G_0^0(\hat{k}') G_0^0(\hat{k}')] = \frac{1}{4\pi},
\]

\[
g(\theta) = -Tr[G_0^0(\hat{k}') G_2^0(\hat{k}')] = \frac{\sqrt{2}}{8\pi}(3 \cos^2 \theta - 1),
\]

\[
h(\theta) = Tr[G_2^0(\hat{k}') G_2^0(\hat{k}')] = \frac{1}{8\pi}(3 \cos^2 \theta + 1).
\quad (32)
\]

As a first inspection, when applying the substitution (both in the gap equations (30) and in expression (31) of E^\pm_k)

\[
\frac{3 \cos^2 \theta}{8\pi} \rightarrow \frac{1}{8\pi}, \quad (33)
\]

8
which has been used as the angle-averaged procedure for $^3PF_2$ superfluidity in Ref.\cite{22}, Eq.(30) reduce to the form of angle-averaged gap Eq.(24). At zero temperature, the paring is suppressed by the gapless excitation near the Fermi surface in AAG state. However in the ADG state, paring can exist in the interval $(0, \theta_1) \cup (\pi, \pi - \theta_1)$ of $\theta$ near the Fermi surface which contribute to superfluidity crucially, where

$$\cos^2 \theta_1 = \frac{\delta \mu^2 - \Delta_0^2(k_F) + \sqrt{2} \Delta_0(k_F) \Delta_2(k_F) - \Delta_0^2(k_F)/2}{3 \sqrt{2} \Delta_0(k_F) \Delta_2(k_F) + 3 \Delta_0^2(k_F)/2}$$

and $\delta \mu$ is the difference between the neutron and proton chemical potentials. This mechanism is consistent with the that of the FFLO state. Furthermore, the influences from the coupling of different $m_j$ components are partially taken into account via the angle matrix

$$\begin{pmatrix} f(\theta) & g(\theta) \\ g(\theta) & h(\theta) \end{pmatrix}$$

in the ADG state.

### 2.3 Thermodynamics

For asymmetric nuclear matter at fixed temperature and given neutron and proton densities, the essential quantity to describe the thermodynamics of the system is free energy defined as

$$F|_{\rho, \beta} = U - \beta^{-1} S,$$  \hspace{1cm} (34)

where $U$ is the internal energy and $S$ is the entropy. At the mean-field approximation, the entropy of the superfluid state is

$$S = -2k_B \sum_k \{ f(E_k^+) \ln f(E_k^+) + \tilde{f}(E_k^+) \ln \tilde{f}(E_k^+) \\ + f(E_k^-) \ln f(E_k^-) + \tilde{f}(E_k^-) \ln \tilde{f}(E_k^-) \},$$  \hspace{1cm} (35)

where $\tilde{f}(E_k^+) = 1 - f(E_k^+)$. The internal energy of the superfluid state reads

$$U = \sum_{\sigma k}^{(n)} \left[ \varepsilon_{k \sigma}^{(n)} n_{\sigma}^{(n)}(k) + \varepsilon_{k \sigma}^{(p)} n_{\sigma}^{(p)}(k) \right]$$

$$+ \sum_{k,k'} \sum_{\sigma_1, \sigma_2, \sigma_1', \sigma_2'} \langle k \sigma_1, -k \sigma_2 | V | k' \sigma_1', -k' \sigma_2' > \nu_{\sigma_1, \sigma_2}(k) \nu_{\sigma_1', \sigma_2'}(k').$$  \hspace{1cm} (36)

The first term in Eq.(36) includes the kinetic energy of the quasiparticles which is a functional of the paring gap. In the normal state it is reduced to
the kinetic energy of the neutrons and protons. The second term includes the BCS mean-field interaction among the particles in the condensate and can be eliminated in terms of the gap Eq.(9). Finally, the internal energy is written as

\[ U = \sum_{\sigma k} \left[ \varepsilon^{(n)}_{\sigma}(k) n^{(n)}_{\sigma}(k) + \varepsilon^{(p)}_{\sigma}(k) n^{(p)}_{\sigma}(k) \right] + \sum_{k} \frac{D^2(k)}{\sqrt{\xi^2_{k} + D^2(k)}} \left[ 1 - f(E^+_k) - f(E^-_k) \right]. \]  

(37)

A thermodynamically stable state minimizes the difference of the free energies between the superconducting and normal state, \( \delta F = F_S - F_N \), where the free energy in the normal state follows from Equations (35) and (37) when \( \Delta \to 0 \).

3 Results

The numerical calculations here are focused on the effects of the angle dependence of the quasiparticle spectra and the emergence of the ADG phase in isospin-asymmetric nuclear matter. To simplify the calculations, several assumptions have been adopted. Firstly, the paring interaction is approximated by the bare interaction, i.e., the effects of the screening of the paring interaction are ignored which affect the strength of the paring interaction. Secondly, we adopt the free single particle spectrum, which may cause the density of the state at Fermi surface get larger. Both these two approximations enlarge the absolute magnitude of the paring gap. Furthermore we ignore the isospin triplet states, which is valid when the paring in the isospin singlet channel is much larger than that in the isospin triplet channel. However, the argument could be false when the first two approximations are abandoned. In the present calculations, the net density is fixed at the empirical saturation density of nuclear matter \( \rho = 0.17 fm^{-3} \) and the Argonne \( V_{18} \) potential has been adopted as the paring interaction.

Fig.1 shows the angle-averaged and angle dependent gap \( \Delta_0(k_F) \) and \( \Delta_2(k_F) \) in the \(^3SD_1\) partial-wave channel as a function of isospin asymmetry, defined as \( \alpha = (\rho^n - \rho^p)/\rho \). The temperatures are set at low-temperature regime \( \beta^{-1} = 0.5 \) MeV, 1.0 MeV, 2.0 MeV, 3.0 MeV, the critical temperature \( \beta_c^{-1} \) where the superfluid vanishes is about 7.5 MeV for symmetric case. At the temperature \( \beta^{-1} = 0.5 \) MeV, the value of \( \Delta_0(k_F) \) of ADG becomes larger than that of angle-averaged gap for \( \alpha \geq 0.07 \) and the largest difference is about 22 percent at \( \alpha = 0.23 \). Moreover the value of \( \alpha \), at which \( \Delta_0(k_F) \) in the ADG state gets larger than that of the angle-averaged gap state, increases when the temperature goes up, and the difference of \( \Delta_0(k_F) \) between the two
Fig. 1. The upper and lower curves in the figures are related to the values of \( \Delta_0(k_F) \) and \( \Delta_2(k_F) \) with varying asymmetry. The blue solid and red dashed lines correspond to the ADG and angle-averaged gap respectively.

states decreases. The critical isospin asymmetries \( \alpha_c \) at which the gaps vanish are the same in the two states and values are 0.267, 0.275, 0.30 and 0.315 for the temperatures 0.5 MeV, 1.0 MeV, 2.0 MeV and 3.0 MeV respectively. The above results imply that the thermal excitation can weaken the effect of asymmetry.

In order to have an entire inspection of the difference between the pairing gaps of the two different kinds of states, we exhibit the gap functions in Fig.2. At the temperature \( \beta^{-1} = 0.5 \) Mev, the gap functions in Eqs.(24) and (30) are almost the same except a little difference of \( \Delta_0(k) \) near the zero momentum for the asymmetry \( \alpha = 0.02 \). While the difference gets larger when the system become more asymmetric. However the curves of ADG coincide with these of the angle-averaged gap for \( \beta^{-1} = 3.0 \) Mev and \( \alpha = 0.16 \). That implies
Fig. 2. The curves marked with symbols $^3S_1$ and $^3D_1$ are related to the gap functions $\Delta_0(k)$ and $\Delta_2(k)$ in Eqs. (24) and (30). The blue solid and red dashed lines correspond to the ADG and angle-averaged gap respectively.

The larger gap functions in ADG state result in a larger energy of pair interactions in the condensate [second term in Eqs. (36) and (37)], which have important influence on the free energy of the superconducting state. Thus we calculate the difference of the free energies between the normal and superconducting states $\delta F$ and the results are shown in Fig. 3, where the temperatures are set as the same as those in Fig. 1. At the temperature $\beta^{-1} = 0.5$ MeV, $\delta F$ in the ADG state get smaller than the angle-averaged gap state when $\alpha \geq 0.06$, especially, the former is about 35 percent lower than the latter in the regime $\alpha > 0.17$. We can conclude the ADG state is more favored than the angle-averaged gap state for large asymmetry and low temperature since...
Fig. 3. The difference of the free energy between the superconducting and normal states for different temperatures. The blue solid and red dashed lines correspond to the ADG and angle-averaged gap respectively.

The angle dependence enhances the paring energy and has little effect on the kinetic energy. However, the thermal excitation weakens the influence of the angle dependence. Fig.3 also shows that the values of $\delta F$ tend to zero gently when the values of $\alpha$ approaching to $\alpha_c$ with different temperatures.

One straightforward way to understand the effect of the angle dependence is to investigate the normal and superconducting occupation probabilities [obtained from the Eqs.(14) and (15)] near the average Fermi surface (related to the average chemical potential of neutron and proton). The results are exhibited in Fig.4, where we have carried out the spin summation. We compare the neutron/proton and paring particle occupation probabilities at the average Fermi surface for temperatures $\beta^{-1} = 0.5$ MeV and 3.0 MeV with a fixed asymmetry $\alpha = 0.16$. For asymmetric nuclear matter, the large splitting neutron and
proton occupation probabilities prevent the pairing around the average Fermi surface in the angle-averaging procedure. While in the ADG configuration, the split is weakened in partial area around the average Fermi surface, e.g. in the regime $\theta \subset (0, \frac{\pi}{5}) \cup (\frac{4\pi}{5}, \pi)$ as shown in Fig.4.(a). In Fig.4.(c), as compared with the angle-averaged gap, the pairing in the ADG state is almost fully suppressed in the regime $\theta \subset (\frac{\pi}{5}, \frac{4\pi}{5})$ in ADG state. However it is obviously enhanced at $\theta$ smaller than $\frac{\pi}{5}$ and greater than $\frac{4\pi}{5}$.

Noting the Eq.(14) with the expression of $D^2(k)$ in Eq.(31) at low temperature, the quasiparticle spectra are no longer isotropic and the Fermi spheres of neutrons and protons are deformed by the pairing interaction. Since we assume an axi-symmetric solution in the ADG state, the rotational symme-
try are spontaneously broken [in terms of group theory the $O(3)$ symmetry breaks down to $O(2)$] and there exists one favored direction. The Fermi sphere of neutron possesses of an oblate deformation perpendicular to the favored direction, while the Fermi sphere of proton performs a prolate deformation along the favored direction. The two different deformations enhance the correlation between the neutrons and protons near the average Fermi surface. However, the thermal excitation reduces the angle dependence of quasiparticle spectra and the neutron/proton occupation probability in ADG becomes almost isotropic at high temperature e.g. in Fig.4.(b). In this case, the deformation of neutron/proton Fermi sphere fails to increase the phase-space overlap of neutron and proton near the average Fermi surface effectively. Thus the results of ADG state are nearly the same as that of the angle-averaged gap state i.e. the angle-averaging procedure becomes an adequate approximation at high temperatures $\beta^{-1} \geq 3$ MeV.

Fig.5 displays the entropy $(\beta^{-1}S)$ as a function of the isospin asymmetry $\alpha$ for different temperatures $\beta^{-1} = 0.5$ MeV, 1.0 MeV, 2.0 MeV and 3.0 MeV. The entropies in the superconducting states are smaller than that in the normal state near $\alpha = 0$, and get larger than that in the normal state at sufficiently large asymmetry. However around the transition point of the superfluid to the normal state, the entropies of the superconducting state approach to the value of normal state i.e. the latent heat $Q = \beta^{-1}(S_n - S_n) \to 0$ at the phase transitions from the ADG and angle-averaged gap states to normal state. Hence the transitions are of second order. At the temperature $\beta^{-1} = 0.5$ MeV, the entropy in the ADG state is nearly linear function of the asymmetry when $0.02 < \alpha < 0.22$. When the temperature rises, the linear property of the entropy curve disappears and the difference between the ADG and angle-averaged gap states gets smaller.

Comparing the gap equations (30) for the ADG state and (24) for the angle-averaged gap state, two differences appear in the ADG state, i.e. the angle dependent quasiparticle spectrum and the angle matrix

$$\begin{pmatrix}
  f(\theta) & g(\theta) \\
  g(\theta) & h(\theta)
\end{pmatrix}.$$  

The first leads to the deformation of neutron/proton Fermi sphere, and the second corresponds to the coupling among different $m_j$ gap components. Actually, the angle matrix modifies the strength of $V_{\lambda j}^{k'* l}(k', k)$ for different directions in momentum space. We replace the angle matrix by $\frac{1}{4\pi} \begin{pmatrix}
  1 & 0 \\
  0 & 1
\end{pmatrix}$ to inspect the influence of the angle matrix. The results are shown in Fig.6 for asymmetry $\alpha = 0.16$ in (b), (c), (d) and the temperature is set to be $\beta^{-1} = 0.5$ MeV. The green dash-doted lines denoted by ‘approximation in ADG’ are obtained by
Fig. 5. The entropy (scaled by $\beta^{-1}$) as a function of the isospin symmetry $\alpha$ for different temperature. The blue solid, red dashed and black dash-dotted lines correspond to the ADG, angle-averaged gap and normal state respectively.

replacing the angle matrix with $\frac{1}{4\pi} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Figs.6.(c) and (d) exhibit the neutron/proton and paring particle occupation probabilities at the average Fermi surface. The curves in ADG and ‘approximation in ADG’ are nearly the same. Whereas the gap functions in Fig.6.(b) show that the curves of ‘approximation in ADG’ behave closer to that of the angle-averaged gap state. Fig.6.(a) displays the $\Delta_0(k_F)$ and $\Delta_2(k_F)$ VS. asymmetry $\alpha$. The gaps of ‘approximation in ADG’ turn out to be smaller than both the gaps of ADG and angle-averaged gap state when $\alpha > 0.07$. Moreover, the curves of ‘approximation in ADG’ are much more approaching to that of the angle-averaged gaps. All these results indicate that the influence of the angle matrix is much more important than that of the angle dependence of quasiparticle spectrum. Furthermore, the cou-
Fig. 6. $\Delta_0(k_F)$ and $\Delta_2(k_F)$ as a function of isospin asymmetry $\alpha$ for the ADG, angle-averaged gap and approximation in ADG case in Fig.(a). Fig.(b) exhibit the gap function for the three case. The normal and superconducting occupation probabilities at the average Fermi surface for the three case are shown in Figs.(c) and (d) respectively. The blue solid, red dashed and green dash-dotted lines correspond to the ADG, angle-averaged gap and the approximation in ADG respectively. The temperature is set to be $\beta^{-1} = 0.5$ Mev, and the asymmetry $\alpha = 0.16$ in (b), (c), (d).

pling from the projection $m_j$ of the angular momentum $j$ may strengthen the paring interaction for large asymmetries at low temperature.
4 Summary and Outlook

The fermionic condensation in asymmetric nuclear matter leads to superconducting states which spontaneously break the spatial symmetries. The quasi-particle spectrum behaves as an isotropic one and the angle dependence of the gap should be reconsidered. In this work we propose an axi-symmetric angle dependent gap state in which the isotropic symmetry is broken for isospin symmetric nuclear matter, and compare with the angle-averaged gap state. The ADG state is more favored than the angle-averaged gap state at large asymmetry, the differences of both the gap values and the free energies between the two kinds of states get small when temperature rises. At low temperature $\beta^{-1} = 0.5$ MeV, the maximal differences of gap and free energy difference are about 22 and 35 percent respectively. In the ADG state, the neutron and proton deformed Fermi spheres increase the pairing probability along the axis of symmetry breaking near the average Fermi surface. The effect of the coupling among different $m_j$ gap components are also investigated in the ADG state and we find the coupling dominates the mainly contribution to the mechanism of ADG state.

The ADG state vanishes at the critical value $\alpha_c$ where the angle-averaged gap vanishes. When temperature goes up, $\alpha_c$ rises and the angle dependence of the state becomes weak. The phase transition from the ADG state to the normal state is a transition of the second order. In a certain region of $\alpha$ the latent heat has an anomalous negative sign which is consistent the results if Ref.[7]. However, this does not affect the stability of the ADG state, since its energy budget is dominated by the pair-condensation energy.

In the ADG state, the symmetry is broken spontaneously. It is different from the FFLO state where the symmetry is broken by the collective motion of the cooper pairs (the translation and rotational symmetry are both broken). The translation symmetry is maintained in the ADG state. The deformation of the Fermi sphere in the ADG state is similar to the DFS configuration, however the mechanism is different. In DFS state the symmetry breaking correspond to the deformed Fermi surface, while in the ADG state the symmetry breaking results from the angle dependence of gap. As is well known, the continuous symmetry breaking lead to collective excitations with vanishing minimal frequency (Goldstone’s theorem). The breaking of rotational symmetry which corresponds to the anisotropic $D^2(k)$ in the ADG state may imply new collective bosonic modes in asymmetric nuclear matter. However, the true ground state could be a combination of the ADG state and the FFLO state, we should consider the ADG state with the cooper pair momentum together in progress.
References

[1] A.L. Goodman, Phys. Rev. C 60, 014311 (1999), and references therein.

[2] G. Röpke, A. Schnell, P. Schuck, and U. Lombard, Phys. Rev. C 61, 024306 (2000).

[3] Th. Alm, B. L. Friman, G. Röpke, and H. Schulz, Nucl. Phys. A 551, 45 (1993).

[4] M. Baldo, U. Lombardo, P. Schuck, Phys. Rev. C 52, 975 (1995).

[5] E. Garrido, P. Sarriuguren, E. Moya de Guerra, and P. Schuck, Phys. Rev. C 60, 064312 (1999).

[6] A. Sedrakian, Th. Alm, U. Lombardo, Phys. Rev. C 55, R582 (1997).

[7] A. Sedrakian, U. Lombardo, Phys. Rev. Lett. 84, 602 (2000).

[8] U. Lombardo, P. Nozieres, P. Schuck, H. Schulze, A. Sedrakian, Phys. Rev. C. 64, 064314 (2001).

[9] Ø. Elgarøy, L. Engvik, M. Hjorth-Jensen, and E. Osenes, Phys. Rev. C. 57, R1069 (1998).

[10] A. I. Akhiezer, A. A. Isayev, S. V. Peletminsky, and A. A. Yatsenko, Phys. Rev. C. 63, 021304 (2001).

[11] A. Sedrakian, G. Röpke, T. Alm, Nucl. Phys. A 594, 355 (1995).

[12] T. Alm, G. Röpke, A. Sedrakian, and F. Weber, Nucl. Phys. A 604, 491 (1996).

[13] S. Typel, G. Röpke, T. Klähn, D. Blaschke, and H. H. Wolter, Phys. Rev. C. 81, 015803 (2010).

[14] S. Heckel, P. P. Schneider, and A. Sedrakian, Phys. Rev. C. 80, 015805 (2009).

[15] M. Stein, X.-G. Huang, A. Sedrakian, and J. W. Clark, Phys. Rev. C. 86, 062801 (2012).

[16] P. Fulde and R. A. Ferrell, Phys. Rev. 135 (1964) A550.

[17] A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 47 (1964) 1136 [translation, Sov. Phys. JETP 20 (1965) 762].

[18] H. Mütter, and A. Sedrakian, Phys. Rev. Lett. 88, 252503 (2002).

[19] A. Sedrakian, Phys. Rev. C. 63, 025801 (2001).

[20] H. Mütter, and A. Sedrakian, Phys. Rev. C. 67, 015802 (2003).

[21] M. Baldo, I. Bombaci, and U. Lombardo, Phys. Lett. B. 283, 8 (1992).

[22] M. Baldo, J. Cugnon, A. Lejeune, and U. Lombardo, Nucl. Phys. A 536 349 (1992).