Strong final state interaction (FSI) effects can play a central role in the Standard Model prediction of weak $K \rightarrow 2\pi$ matrix elements. Here, I discuss how FSI's affect the direct CP violation parameter $\varepsilon'/\varepsilon$ by solving the Omnès problem for the necessary $K \rightarrow 2\pi$ amplitudes. The main results of this analysis have been reported in a previous paper and are further discussed in Refs. The resulting Standard Model prediction $\text{Re}(\varepsilon'/\varepsilon) = (15 \pm 5) \times 10^{-4}$ is in good agreement with the present experimental world average.

1 Introduction

In a recent paper it was pointed out that strong final state interaction (FSI) effects, when properly taken into account, produce a large enhancement (about a factor of two) in the Standard Model prediction of the direct CP violation parameter $\varepsilon'/\varepsilon$. FSI effects also play an important role in the observed $\Delta I = 1/2$ rule. At centre–of–mass energies around the kaon mass, the strong $S$–wave $\pi–\pi$ scattering generates a large phase-shift difference $(\delta_0^0 - \delta_0^2) (m_K^2) = 45^\circ \pm 6^\circ$ between the $I = 0$ and $I = 2$ partial waves. This effect is taken into account by factoring out those phases in the usual decomposition of $K \rightarrow 2\pi$ amplitudes with definite isospin $I = 0$ and $I = 2$:

$$A_I \equiv A [K \rightarrow (\pi\pi)_I] \equiv A_I e^{i\delta_I^I}. \quad (1)$$

The presence of such a large phase–shift difference also signals a large dispersive FSI effect in the moduli of the isospin amplitudes, since their imaginary and real parts are related by analyticity and unitarity. The size of the FSI effect can be already roughly estimated at one loop in Chiral

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Perturbation Theory (ChPT), where the rescattering of the two pions in the final state produces an enhancement of about 40% in the $A_0$ amplitude. However, the fact that the one loop calculation still underestimates the observed $\delta_0$ phase shift indicates that a further enhancement should be produced by higher orders.

It is then clear that FSI contributions should be included and their effect resummed in the description of $K \to 2\pi$ decays. In addition, the rescattering of the two pions in the final state is a purely long–distance mechanism, which is fully decoupled from the production mechanism of the two pions in the decay process. As a consequence, FSI effects do not introduce any dependence on short–distance quantities (i.e. they do not carry any dependence on the factorization scale in the OPE description of $K \to 2\pi$ amplitudes).

Despite the importance of FSI in $K \to 2\pi$ decays has been known for more than a decade, most of the Standard Model predictions of $\varepsilon'/\varepsilon$ did not include those contributions and failed to reproduce the experimental measurements. Lattice determinations of $K \to 2\pi$ amplitudes are still missing FSI effects, since they are obtained in a two–steps procedure where first the simpler $K \to \pi$ matrix element is measured on the lattice and second, the physical $K \to 2\pi$ matrix elements are obtained by using a lowest–order ChPT relation between $K \to \pi$ and $K \to 2\pi$. Neither the first nor the second step include FSI effects.

On the other hand, approaches based on effective low–energy models or the $1/N_c$ expansion do include some one-loop corrections and find larger values for the $A_0$ amplitude. However, the drawback in these cases could be the possible model dependence of the matching procedure with short–distance.

The approach that has been recently proposed is the Omnès approach, which permits a resummation of the universal FSI effects to all orders in ChPT. In Section the general Omnès problem is formulated, while in Section the Omnès problem is solved for the pion scalar form factor, a well known quantity in ChPT; this analysis clarifies the relevant properties of the Omnès solution. Finally, in Section we consider the Omnès solution for the $K \to 2\pi$ amplitudes which enter the prediction of $\varepsilon'/\varepsilon$. The final result of an exact matching procedure with short–distance, with the inclusion of FSI effects gives

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right)_{SM} = (15 \pm 5) \times 10^{-4},$$

which is in good agreement with the present experimental world average

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right)_{exp} = (19.3 \pm 2.4) \times 10^{-4}.$$  \hspace{1cm} (2)

### 2 The Omnès problem

Let us consider a generic amplitude (or form factor) $A_J^I(s)$, with two pions in the final state which have total angular momentum and isospin given by $J$ and $I$, respectively, and invariant mass $s \equiv q^2 \equiv (p_1 + p_2)^2$. The amplitude is analytic everywhere except for a cut on the real positive $s$ axis $L = [4m_{\pi}^2, \infty)$.

Below the first inelastic threshold, only the $2\pi$ intermediate state contributes to the absorptive part of the amplitude and Watson’s theorem implies that the phase of the amplitude is equal to the phase of the $\pi\pi$ partial–wave scattering amplitude, so that

$$\text{Im} A_J^I = (\text{Im} A_J^I)_{2\pi} = e^{-i\delta_J^I} \sin \delta_J^I A_J^I = e^{i\delta_J^I} \sin \delta_J^I A_J^{I*} = \sin \delta_J^I |A_J^I| = \tan \delta_J^I \text{ Re} A_J^I.$$  \hspace{1cm} (4)

Cauchy’s theorem implies instead that $A_J^I(s)$ can be written as a dispersive integral along the physical cut:

$$A_J^I(s) = \frac{1}{\pi} \int_L dz \frac{\text{Im} A_J^I(s)}{z-s-i\varepsilon} + \text{subtractions}.$$  \hspace{1cm} (5)
Inserting Eq. in the dispersion relation, one obtains an integral equation for $A_I^J(s)$ of the Omnès type, which has the well-known Omnès solution\footnote{Omnès} (for $n$ subtractions with subtraction point $s_0$ outside the physical cut):

$$A_I^J(s) = Q_{I,n}^J(s, s_0) \exp \left\{ I_{I,n}^J(s, s_0) \right\},$$

(6)

where

$$I_{I,n}^J(s, s_0) = \frac{(s - s_0)^n}{\pi} \int_{4m^2}^\infty \frac{dz}{(z - s_0)^n} \frac{\delta_I^J(z)}{z - s - i\epsilon}$$

(7)

and

$$\log \left\{ Q_{I,n}^J(s, s_0) \right\} = \sum_{k=0}^{n-1} \frac{(s - s_0)^k}{k!} \frac{d^k}{ds^k} \log \left\{ A_I^J(s) \right\} \bigg|_{s=s_0}, \quad (n \geq 1),$$

(8)

with $Q_{I,0}^J(s, s_0) \equiv 1$. The dispersive integral $I_{I,n}^J(s, s_0)$ is uniquely determined up to a polynomial ambiguity (that does not produce any imaginary part of the amplitude), which depends on the number of subtractions and the subtraction point. The simple iterative relation for the real part of $I_{I,n}^J(s, s_0)$

$$\text{Re} I_{I,n}^J(s, s_0) = \text{Re} I_{I,n-1}^J(s, s_0) - (s - s_0)^{n-1} \lim_{s \to s_0} \frac{\text{Re} I_{I,n-1}^J(s, s_0)}{(s - s_0)^{n-1}},$$

(9)

shows that only a polynomial part of $I_{I,n}^J(s, s_0)$ does depend on the subtraction point $s_0$ and the number of subtractions $n$, while the non–polynomial part of $I_{I,n}^J(s, s_0)$, the one containing the infrared chiral logarithms, is universal (i.e. $s_0$ and $n$ independent). Thus, the Omnès solution predicts the chiral logarithmic corrections in a universal way and provides their exponentiation to all orders in the chiral expansion. The polynomial ambiguity of $I_{I,n}^J(s, s_0)$ and the subtraction function $Q_{I,n}^J(s, s_0)$ can be fixed, at a given order in the chiral expansion, by matching the Omnès formula\footnote{Omnès} with the ChPT prediction of $A_I^J(s)$. It remains a polynomial ambiguity at higher orders. Notice that in the presence of a zero of the amplitude the Omnès solution can be found for the factorized amplitude $\overline{A_J^I}(s)$, such that $A_I^J(s) = (s - \zeta)^p \overline{A_J^I}(s)$, where $\zeta$ is a zero of order $p$.

3 The scalar pion form factor

The scalar pion form factor is known up to two loops in ChPT\footnote{ChPT}, and it is a useful quantity to understand the details of the Omnès solution in the case $I = 0$ and $J = 0$. It is defined by the matrix element of the $SU(2)$ quark scalar density $\langle \pi^i(p')|\bar{u}u + \bar{d}d|n^k(p)\rangle \equiv \delta^{ik} F_S^\pi(t)$. At low momentum transfer, ChPT provides a systematic expansion of $F_S^\pi(t)$ in powers of $t \equiv (p' - p)^2$ and the light quark masses$,$

$$F_S^\pi(t) = F_S^\pi(0) \left\{ 1 + g(t) + O(p^4) \right\},$$

(10)

where the $O(p^2)$ correction $g(t)$ contains contributions from one-loop diagrams and tree–level terms of the $O(p^4)$ ChPT lagrangian. It is given by\footnote{Lagrangian} ($f \approx f_\pi = 92.4$ MeV):

$$g(t) = \frac{t}{f^2} \left\{ \left( 1 - \frac{M_\pi^2}{2t} \right) \bar{J}_\pi(t) + \frac{1}{4} \bar{J}_{KK}(t) + \frac{M_\pi^2}{18t} \bar{J}_{qq}(t) + 4(L_5^2 + 2L_4^2)(\mu) + \frac{5}{4(4\pi)^2} \left( \ln \frac{\mu^2}{M_\pi^2} - 1 \right) - \frac{1}{4(4\pi)^2} \ln \frac{M_K^2}{M_\pi^2} \right\}.$$

(11)
The counterterms $L_5^f, L_6^f$ from the $O(p^4)$ lagrangian are kept in the numerical analysis at the standard
reference value $\mu = M_\rho$, where $\Box \left[ L_5^f + 2L_6^f \right](M_\rho) = (0.8 \pm 1.1) \times 10^{-3}$. The functions $\tilde{J}_{\pi \pi}(t), \tilde{J}_{KK}(t)$ and $\tilde{J}_{\eta \eta}(t)$ are ultraviolet finite and, together with the logarithms, they are produced by the one-loop exchange of $\pi\pi, K\bar{K}$ and $\eta\eta$ intermediate states. Notice that at values of $t$ such that $t \ll 4M_\rho^2$

$$\tilde{J}_{PP}(t) = \frac{1}{(4\pi)^2} \left[ \frac{1}{6} \frac{t}{M_\rho} + \frac{1}{60} \frac{t^2}{M_\rho^2} + \ldots \right],$$

implying that, below the $P\bar{P}$ threshold, $\tilde{J}_{PP}(t)$ has a very smooth behaviour and it is strongly suppressed. In the case of the pion scalar form factor this means that at values of $t \ll 4M_\rho^2, 4M_\eta^2$ the one-loop functions $\tilde{J}_{KK}(t)$ and $\tilde{J}_{\eta\eta}(t)$ only give small analytic corrections, which are numerically negligible with respect to the rest of the one–loop corrections.

Let us consider now the Omnès solution for $F^\pi_S(t)$ and fix the subtraction polynomial by performing a matching with the one-loop ChPT result. This implies at the subtraction point $t_0$

$$F^\pi_S(t) = F^\pi_S(t_0) \cdot \Omega_0(t_0, t) \approx F^\pi_S(0) \left\{ 1 + g(t_0) \right\} \cdot \Omega_0(t, t_0).$$

Thus, knowing the form factor at some low–energy subtraction point $t_0$, where the momentum expansion can be trusted, the Omnès factor $\Omega_0(t_0, t)$ provides an evolution of the result to higher values of $t$, through the exponentiation of infrared effects related to FSI. The once–subtracted solution reads:

$$\Omega^{(1)}_0(t, t_0) = \exp \left\{ \frac{t - t_0}{\pi} \int_{M_\pi^2}^\infty dz \frac{\delta_0^0(z)}{z - t_0 - z - t - i\epsilon} \right\} \equiv \Re_0^{(1)}(t, t_0) \cdot e^{i\theta_0^{(0)}(t)},$$

where the integral has been split into its real and imaginary part, making explicit that the phase of the Omnès factor is just the original phase-shift $\delta_0^0(t)$, while $\Re_0^{(1)}(t, t_0)$ is its corresponding dispersive factor. The integral has been cut at the upper edge $\bar{z}$, which represents the first inelastic threshold ($\bar{z} = 1$ GeV$^2$). In Table 1 the resulting value of $|F_\pi^S(M^2_K)/F_\pi^S(0)|$ for different subtraction points $t_0 = 0, M_\pi^2, 2M_\pi^2, 3M_\pi^2, 4M_\pi^2$ and $M_\rho^2$ and at $t = M_K^2$ is shown together with the dispersive part of the once–subtracted Omnès integral $\Re_0^{(1)}(t, t_0)$ and the one loop function $g(t_0)$. Notice that the Omnès factor is not defined for $t_0 = M_K^2$, because it lies above the threshold of the non–analyticity cut; however, by its definition in Eq. (13), $\Omega(M_K^2, t_0 = M_K^2) = 1$ holds. The ChPT calculation of $F^\pi_S(t_0)$ is obviously better at lower values of $t_0$ where the one-loop correction $g(t_0)$ is smaller. At $t_0 = 4M_\pi^2$ a sizable 26% effect is already found, while at $t_0 = M_K^2$ (the point chosen in the discussion of the Omnès factor in a recent paper) the correction is so

| $t_0$     | $g(t_0)$ | $\Re_0^{(1)}(M_K^2, t_0)$ | $|F_\pi^S(M_K^2)/F_\pi^S(0)|$ |
|-----------|----------|--------------------------|-------------------------------|
| 0         | 0        | 1.23                     | 1.45                          |
| $M_\pi^2$ | 0.042    | 1.21                     | 1.40                          |
| $2M_\pi^2$| 0.091    | 1.17                     | 1.34                          |
| $3M_\pi^2$| 0.15     | 1.12                     | 1.26                          |
| $4M_\pi^2$| 0.26     | 1.03                     | 1.11                          |
| $M_K^2$   | 0.54 – 0.46 i | $\equiv 1$ | 1.61                          |
large than one should worry about higher–order contributions. The Omnès exponential allows to predict \( F_\pi^S(M_K^2) \) in a much more reliable way, through the evolution of safer results at lower \( t_0 \) values. In Table 2 the dependence of the Omnès integral on the upper edge \( \bar{z} \) is shown. From those results one can conclude that cutting the integral at the first inelastic threshold is actually producing an underestimate of the complete FSI effect. The sensitivity of the Omnès integral to the high–energy region is reduced by considering a twice–subtracted Omnès solution. Since chiral corrections are smaller at lower values of the subtraction point, values of the Omnès factor at low \( t_0 \) should be trusted because less sensitive to higher order corrections. Based also on the twice–subtracted result one can conclude that the true value of \( |F_\pi^S(M_K^2)/F_\pi^S(0)| \) is in the range 1.5 to 1.6. Taking the experimental phase-shift uncertainties into account (see Figure (1) for the details), the result is:

\[
|F_\pi^S(M_K^2)/F_\pi^S(0)| = 1.5 \pm 0.1. \tag{15}
\]

Table 2: Dependence of \( \Re_0^{(1)}(M_K^2, t_0) \) on the upper edge \( \bar{z} \) (in GeV\(^2\) units) of the dispersive integral, for various choices of \( t_0 \). The fit to the experimental data for \( \delta_0^0 \) has been used

| \( \bar{z} = 1 \) | \( \bar{z} = 2 \) | \( \bar{z} = 3 \) |
|---|---|---|
| \( t_0 = 0 \) | 1.45 | 1.58 | 1.62 |
| \( t_0 = M_\pi^2 \) | 1.40 | 1.51 | 1.55 |
| \( t_0 = 2M_\pi^2 \) | 1.34 | 1.44 | 1.47 |
| \( t_0 = 3M_\pi^2 \) | 1.26 | 1.35 | 1.38 |
| \( t_0 = 4M_\pi^2 \) | 1.11 | 1.86 | 1.21 |

4 Final State Interactions in \( \epsilon'/\epsilon \)

The analysis of the Omnès solution for the scalar pion form factor has clarified two main points. First, the Omnès solution given by the product of the polynomial amplitude times the Omnès
factor is independent of the subtraction point as it has to be. Second, the Omnès exponential allows to predict the physical quantity in a much more reliable way, through the evolution of safer results at lower values of the subtraction point.

In analogy to the scalar pion form factor, one can derive the Omnès solution for any \( K \to 2\pi \) amplitude, both CP–conserving and CP–violating (the presence of a CP–violating weak phase does not modify the Omnès derivation, being such a phase of short–distance origin). At lowest order in the chiral expansion, the most general effective bosonic Lagrangian, with the same amplitude, both CP–conserving and CP–violating (the presence of a CP–violating weak phase corresponding to the pion scalar form factor \( F_\pi \)).

\[
L_2^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ g_8 \left( \lambda L_\mu L^\mu \right) + g_{27} \left( L_{\mu 23} L_{11}^\mu + \frac{2}{3} L_{\mu 21} L_{13}^\mu \right) \right\} + 2e^2 f^6 g_{EM} \langle \lambda U^\dagger QU \rangle + \text{h.c.} \tag{16}
\]

The flavour–matrix operator \( L_\mu = -if^2 U^\dagger D_\mu U \) represents the octet of \( V – A \) currents at lowest order in derivatives, where \( U = \exp i\sqrt{2}\phi/f \) is the exponential representation of the light pseudoscalar meson field with \( \phi \) the flavour octet matrix. \( Q = \text{diag}(2, -1, -1) \) is the quark charge matrix, \( \lambda \equiv (\lambda^6 – i\lambda^7)/2 \) projects onto the \( \bar{s} \to \bar{d} \) transition \( [\lambda_{ij} = \delta_{i3}\delta_{j2}] \) and \( \langle A \rangle \) denotes the flavour trace of \( A \).

At generic values of the squared centre–of–mass energy \( s = (p_{\pi 1} + p_{\pi 2})^2 \), the \( I = 0, 2 \) amplitudes generated by the lowest–order lagrangian in Eq. 16 are given by

\[
\begin{align*}
A_0 &= -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sqrt{2} f \left\{ g_8 + \frac{1}{9} g_{27} \right\} (s - M_\pi^2) - \frac{4}{3} f^2 e^2 g_{EM} \ , \\
A_2 &= -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \frac{2}{9} f \left\{ 5 g_{27} (s - M_\pi^2) - 6 f^2 e^2 g_{EM} \right\} . \tag{17}
\end{align*}
\]

In the absence of \( e^2 g_{EM} \) corrections, the tree–level isospin amplitudes have a zero at \( s = M_\pi^2 \), because the on-shell amplitudes should vanish in the SU(3) limit. This is not the case for the matrix elements of the electroweak penguin operator \( Q_8 \), whose lowest–order realization is given by the term proportional to \( e^2 g_{EM} \). Those matrix elements are constant (of order \( e^2 p^0 = p^2 \) in the chiral power counting) at the lowest order in ChPT.

For all the cases one can write a once–subtracted Omnès solution for the on–shell physical amplitudes in the form

\[
A_I(M_\pi^2) = a_I(M_\pi^2, s_0) \Omega_I^{(1)}(M_\pi^2, s_0) . \tag{18}
\]

The amplitude \( a_I(M_\pi^2, s_0) \) depends on which weak effective operator is mediating the corresponding \( K \to 2\pi \) amplitude, while the once–subtracted Omnès factor \( \Omega_I^{(1)}(M_\pi^2, s_0) \) is universal, i.e. the same for the pion scalar form factor \( F_\pi(s) \) and the amplitude \( A_0(s) \) (this is in principle no more true for the twice–subtracted factor that depends on the derivative of the function itself). Moreover, taking a low subtraction point where higher–order corrections are very small, one can just multiply the tree–level formulae in Eq. 17 with the experimentally determined Omnès exponential. The results are

\[
\Re_0(M_\pi^2, 0) = 1.5 \pm 0.1 \quad \Re_2(M_\pi^2, 0) = 0.92 \pm 0.06 , \tag{19}
\]

where the subtraction point \( s_0 = 0 \) has been chosen. How the proper inclusion of FSI effects drastically modify the Standard Model prediction of \( \varepsilon'/\varepsilon \) has been already explained.\(^6\)

\(^6\)These values update earlier estimates of \( \Re_0^{(1)}(M_\pi^2, M_\rho^2) \). The correction factors \( \Re_I^{(1)} \) were also considered in an estimate of \( \varepsilon'/\varepsilon \) within the \( SU(2) \otimes SU(2) \otimes U(1) \) model of CP violation.\(^7\)
To obtain a complete Standard Model prediction for $\varepsilon'/\varepsilon$ an exact matching procedure has been proposed\cite{3}. It is inspired by the large--$N_c$ expansion, but only at scales below the charm quark mass $\mu \leq m_c$ (where the logarithms that enter the Wilson coefficients are small). FSI effects, which are next--to--leading in the $1/N_c$ expansion but numerically relevant, are taken into account through the multiplicative factors $\Re I(M^2_K,0)$ while avoiding any double counting. The general formula for $\varepsilon'/\varepsilon$ can be written as follows

$$\text{Re} \frac{\varepsilon'}{\varepsilon} = \frac{\omega}{\sqrt{2\varepsilon}} \text{Re} A_0 \left( \frac{1}{\omega} \text{Im} A_2 - \text{Im} A_0 \right),$$

(20)

where $\omega = \text{Re} A_2/\text{Re} A_0 = 1/22.2$ is the inverse of the CP--conserving $\Delta I = 1/2$ ratio. The CP–conserving quantities $\omega$ and $\text{Re} A_0$ are taken from the experiment, while the CP–violating terms $\text{Im} A_0$ and $\text{Im} A_2$ are calculated theoretically. The indirect CP violation parameter $\varepsilon$ can be either taken from the experiment or evaluated theoretically. Each contribution to $\text{Im} A_0$ and $\text{Im} A_2$ will now contain the FSI effect through the corresponding dispersive factor $\Re I(M^2_K,0)$ with $I = 0$ or 2 which was not taken into account in the previous Standard Model predictions of $\varepsilon'/\varepsilon$. The result

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right)_{SM} = (15 \pm 5) \times 10^{-4},$$

(21)

has an uncertainty which is dominated by the large--$N_c$ based matching procedure. The prediction is within one sigma respect to the present experimental world average\cite{3}

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right)_{\text{exp}} = (19.3 \pm 2.4) \times 10^{-4},$$

(22)

and provides an enhancement of about a factor of two respect to the previous short–distance based Standard Model predictions of $\varepsilon'/\varepsilon$\cite{9,10}.

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