Contrasting solar and reactor neutrinos with a non-zero value of $\theta_{13}$

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When solar neutrino and KamLAND data are analyzed separately one finds that, even though allowed regions of neutrino parameters overlap, the values of $\delta m^2$ and the mixing angle $\theta_{12}$ at the $\chi^2$ minima are different. We show that a non-zero, but small value of the angle $\theta_{13}$ can account for this behavior. From the joint analysis of solar neutrino and KamLAND data we find the best fit value of $\sin^2 2\theta_{13} = 0.01_{-0.01}^{+0.09}$.

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During the last decade, as solar neutrino physics moved from the discovery stage to the precision measurements stage increasingly more data became available for a critical analysis. Recent real-time high-precision solar neutrino data from Sudbury Neutrino Observatory (SNO) [1, 2, 3] and SuperKamiokande (SK) [4, 5, 6] experiments combined with data from radiochemical Homestake [7], SAGE [8], Gallex [9, 10], and Gallium Neutrino Observatory (GNO) [11] experiments pinpointed neutrino parameters, especially the value of the mixing angle usually referred to as $\theta_{12}$. (Note that when $\theta_{13}$, the mixing angle between first and third generations, is zero, $\theta_{12}$ is equal to $\theta_{12}$, the mixing angle between first and second generations [12]). This value of the mixing angle is consistent with the more recent result from the Borexino experiment [13]. In a parallel development the long-baseline neutrino oscillation experiment KamLAND, detecting reactor neutrinos, first announced a reduction of the reactor neutrino flux with distance [14], and afterwards direct evidence for spectral distortion, resulting from neutrino oscillations [15, 16]. The region of the parameter space of neutrino masses and mixings indicated by the KamLAND experiment is about the same as that was indicated by the solar neutrino experiments. Solar neutrino experiments measure neutrino flux in contrast to reactor experiments which measure antineutrino flux; hence one needs to assume that CPT is a good symmetry of the Nature to analyze them together. Assuming CPT symmetry, these experiments not only confirm one another, but also are complementary, since KamLAND is especially sensitive to $\delta m^2$. KamLAND and solar neutrino data are usually analyzed together [17, 18]. However, if they were to be analyzed separately (see e.g. Ref. [16]) one finds that, even though allowed regions of neutrino parameters overlap, $\delta m^2$ and mixing angle values at the $\chi^2$ minima are different for solar and reactor neutrinos. This is a rather small effect, but it suggests that there could be a missing ingredient in the usual analyses of the data. For example, density fluctuations in the Sun may alter the observed solar neutrino flux [19], but clearly would not change reactor neutrino spectra. Similarly although the combined effect of neutrino magnetic moment and solar magnetic field combinations are very small [20], alternative scenarios are not ruled out [21]. Other new physics beyond the Standard Model may also affect solar and reactor neutrinos differently [22].

In this paper we explore the possibility that a non-zero value of the mixing angle $\theta_{13}$, may be responsible for this effect. We show below that a non-zero, but small value of $\theta_{13}$ yields precisely the behavior observed in the analysis of solar and reactor experiments.

In our numerical calculations we first analyzed KamLAND data alone. Allowed regions of the neutrino parameter space at 95% confidence level are shown in Figure 1 for different values of $\sin^2 2\theta_{13}$. One observes that as the value of $\sin^2 2\theta_{13}$ increases the best fit value of $\delta m^2_{12}$ changes very little, but the best fit value of $\tan^2 \theta_{12}$ shifts towards the left-hand side of the panel. 95% confidence level regions shown in those panels also exhibit a similar pattern. (Furthermore, as $\sin^2 2\theta_{13}$ increases these confidence level intervals get smaller, clearly indicating that the KamLAND experiment disfavors large values of $\sin^2 2\theta_{13}$). To qualitatively understand this behavior consider the electron neutrino survival probability with three flavors in a reactor experiment

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \sim 1 - \frac{1}{2} \sin^2 2\theta_{13} - \cos^2 \theta_{13} \sin^2 2\theta_{12} \sin^2 \left(\frac{\delta m^2_{12} L}{4E}\right),$$

where, since the reactors are at a significant distance, we replaced the $\sin^2(\delta m^2_{\text{atm}} L/4E)$ term with 1/2. If $\theta_{13}$ were...
taken to be zero this probability would take the form

\[ P(\bar{\nu}_e \to \bar{\nu}_e) \sim 1 - \sin^2 2\theta_{12}^{(0)} \sin^2 \left( \frac{\delta m^2_{12} L}{4E} \right). \]  (2)

In Eq. (2) we designated the value of \( \theta_{12} \) obtained by taking \( \theta_{13} = 0 \) to be \( \theta_{12}^{(0)} \). If one requires the two fits (with Eq. (1) and with Eq. (2)) to be identical (i.e. give the same survival probability) it is easy to show that \( \delta m^2_{12} \) can be kept the same and

\[ \sin^2 2\theta_{12}^{(0)} \geq \sin^2 2\theta_{12}, \]  (3)

which is the behavior observed in Figure 1.

95% C.L. allowed regions of the neutrino parameter space when all solar neutrino experiments (chlorine, SAGE, Gallex, GNO, SK, SNO, and Borexino) are included in the analysis are shown in Figure 2 for different values \( \sin^2 2\theta_{13} \).

One observes that as the value of \( \sin^2 2\theta_{13} \) increases the best fit values of both \( \delta m^2_{12} \) and \( \tan^2 \theta_{12} \) shift towards the upper left-hand side of the panel. 95% confidence level regions shown in those panels also exhibit a similar pattern. To qualitatively understand this behavior consider the relation between electron neutrino survival probabilities in matter calculated using two and three neutrino flavors \[ P_{3 \times 3}(\nu_e \to \nu_e) = \cos^4 \theta_{13} P_{2 \times 2}(\nu_e \to \nu_e \text{ with } N_e \cos^2 \theta_{13}) + \sin^4 \theta_{13}, \]  (4)

where \( P_{2 \times 2}(\nu_e \to \nu_e \text{ with } N_e \cos^2 \theta_{13}) \) is the standard 2-flavor survival probability calculated with the modified electron density \( N_e \cos^2 \theta_{13} \). In our calculations we numerically obtained exact solutions of the neutrino evolution equations. However, to discuss the behavior of the survival probability under parameter changes we can use the adiabatic approximation, which is rather accurate for solar neutrinos. The expression for the adiabatic survival probability is given by

\[ P(\nu_e \to \nu_e) = \frac{1}{2} + \frac{1}{2} \cos 2\theta_{12} \left[ -\frac{\varphi(x)}{\sqrt{(\delta m^2_{12} \sin 2\theta_{12}/4E)^2 + \varphi^2(x)}} \right]_{\text{source}}, \]  (5)
FIG. 2: Two-parameter 95% confidence level intervals allowed by the solar neutrino experiments for different values of $\sin^2 2\theta_{13}$.

where the last term, the matter mixing angle, is averaged over the neutrino production region in the Sun. In Eq. (5) the quantity $\varphi$ is given by

$$\varphi(x) = \frac{1}{\sqrt{2}} G_F N_e(x) - \frac{\delta m_{12}^2}{4E} \cos 2\theta_{12}. \quad (6)$$

Since $\theta_{13}$ is expected to be very small the fourth power of its sine in Eq. (4) can be ignored. Then clearly the probability with three flavors is suppressed by a factor of $\cos^4 \theta_{13}$ as compared to the probability with two flavors. To compensate for this suppression the initial matter mixing angle should increase as $\theta_{12}$ very slightly decreases. The best way to achieve this is to increase $\delta m_{12}^2$ (cf. Eqs. (5) and (6)). This is indeed what full numerical calculations give. We illustrate this behavior in Figure 3. In this figure the best fit values that correspond to $\theta_{13} = 0$ are shown with full circles and the sense of change of the best fit values for separate analyses of solar and KamLAND data as the value $\theta_{13}$ increases are indicated by arrows.

The previous discussion implies that a joint analysis of solar neutrino and KamLAND data could suggest not only new physics beyond the Standard Model, but also a non-zero value of the parameter $\theta_{13}$. We present results of such an analysis in Figures 4 and 5. In Figure 4 we show 95, 99, and 99.7% confidence level intervals for the joint analysis of the solar neutrino and KamLAND data. Projection of the global $\Delta \chi^2$ function on $\sin^2 2\theta_{13}$ is shown in Figure 5. Best fit values are indicated by dots. For $\theta_{13}$ it is at $\sin^2 2\theta_{13} = 0.01_{-0.09}^{+0.01}$.

We demonstrated that a non-zero value of $\theta_{13}$ can account for the observed difference between the best fit values of the solar neutrino and KamLAND experiments. It is worth repeating that we are only talking about the best fit values, not confidence level intervals which are more robust indicators of statistics. Clearly $\theta_{13} = 0$ is consistent with all the data. However the results are tantalizing and perhaps provide an additional motivation for attempts to measure $\theta_{13}$ directly. The current limit is $\sin^2 2\theta_{13} < 0.19$ [25], however the Double Chooz [26] and Daya Bay [27] experiments, both under construction, are expected to be able to probe lower values of $\theta_{13}$. The value of $\sin^2 2\theta_{13}$
FIG. 3: The change in the best fit values of $\delta m_{12}^2$ and $\theta_{12}$ with increasing value of $\theta_{13}$. Parameter values corresponding to $\theta_{13} = 0$ are indicated by filled circles. Both for KamLAND and solar neutrino experiments the range $0 \leq \sin^2 2\theta_{13} \leq 0.1$ is shown.

suggested by our analysis should be reachable in particular by the Daya Bay experiment.

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Note added: After the article was submitted for publication we were informed that E. Lisi and his collaborators also observed hints for a non-zero value of the mixing angle $\theta_{13}$, see http://neutrino.pd.infn.it/NO-VE2008/talks-NOVE.html.

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FIG. 4: Two parameter 95, 99, and 99.7% confidence level intervals for the joint analysis of the solar neutrino and KamLAND data with the best fit values of $\theta_{13}$ (left-hand panel) and with $\theta_{12}$ (right-hand panel). Best fit values are indicated by dots.

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FIG. 5: $\Delta \chi^2$ for the fit shown in Figure 4. $\Delta \chi^2$ shown is marginalized with the best fit values of $\tan^2 \theta_{12} = 0.47$ and $\delta m^2_{12} = 7.4 \times 10^{-5}$ eV$^2$.

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