Toward the Measurement of the Mass of Isolated Neutron Stars: Prediction of Future Astrometric Microlensing Events by Pulsars

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Abstract

The mass of single neutron stars (NSs) can be measured using astrometric microlensing events. In such events, the center-of-light motion of a star lensed by an NS will deviate from the expected nonlensed motion and this deviation can be used to measure the mass of the NS. I search for future conjunctions between pulsars, with measured proper motion, and stars in the GAIA-DR2 catalog. I identify two candidate events of stars involving lensing by a foreground pulsar in which the estimated light deflection of the background star will deviate from the nonlensed motion by more than 10 μas. PSR J185635–375435 passed ≳4″1 from a 19.4 G magnitude star on J2014.9 with an estimated deflection of 13 μas, while PSR J084606–353340 may pass ~0″2 from a 19.0 G magnitude star on J2022.9 with an estimated deflection of 91 μas. However, the proper motion of the second event is highly uncertain. Therefore, additional observations are required in order to verify this event. I briefly discuss the opposite case, in which a pulsar is being lensed by a star. Such events can be used to measure the stellar mass via pulsar timing measurements. I do not find good candidates for such events with predicted variations in the pulsar period derivative (P), divided by 1 s, exceeding 10^{-20} s^{-1}. Since only about 10% of the known pulsars have measured proper motions, there is potential for an increase in the number of predicted pulsar lensing events.

Key words: astrometry – gravitational lensing: micro – stars: neutron

1. Introduction

The mass and population mass-range of neutron stars (NSs) are fundamental properties related to their formation and evolution and to the equation of state of nuclear matter. So far, we have only obtained accurate mass measurements for NSs in binary systems (e.g., Kramer & Stairs 2008). There is some evidence that there is more than one channel to form an NS (e.g., Beniamini & Piran 2016). Therefore, measuring masses of single NSs is of great importance.

Paczynski (1995), Paczyński (1998), Miralda-Escude (1996), and Gould (2000) have suggested measuring stellar masses via the detection of astrometric microlensing events. In such events, a source is lensed by a stellar mass object in our galaxy, and the center-of-light of the source images will deviate from a uniform-rate proper motion. This motion can be more complicated if the sources are blended or if it is an astrometric binary. This deviation could be used to measure the mass of the lensing star. Sahu et al. (1998) and Salim & Gould (2000) made some predictions for future astrometric microlensing events. Harding et al. (2018) estimated the astrometric microlensing rate for known stellar remnants, and McGill et al. (2018) used the GAIA-TGAS catalog (Lindgren et al. 2016) to make predictions for astrometric microlensing events. Furthermore, Bramich (2018) and Mustill et al. (2018) made some predictions for microlensing events in the next 10 and 20 years, respectively, based on the GAIA-DR2 catalog. Finally, Lu et al. (2016) and Kains et al. (2017) presented some ongoing efforts to measure astrometric microlensing events focusing on identifying single stellar-mass black holes in our galaxy, while Sahu et al. (2017) presented the first measurements of a white dwarf mass based on an astrometric microlensing event.

Here, I search for close angular conjunctions between pulsars with a known proper motion and stars in the GAIA-DR2 catalog (Gaia Collaboration et al. 2016, 2018). I find two candidate events in the near past and future, in which there is a possibility that the pulsars pass with a small angular separation from background stars. Given the large uncertainty in the pulsar astrometry of one of the events, additional observations are crucial in order to verify this possibility.

The impact parameters of these close angular passages are hundreds of times the Einstein radius of the lens. Therefore, in most cases, these events will not result in a classical microlensing event—i.e., events that have a detectable magnification of the background star. However, if confirmed, these events may produce small astrometric shifts in the position of the background stars, relative to the linear motion at a constant angular speed expected from the proper motion component and the periodic variation expected from the parallax. Future measurements of such astrometric microlensing events may enable the first mass measurements of a single NS.

In Section 2, I describe the search for astrometric microlensing events involving pulsars, while in Section 3 I discuss the opposite case in which a star is lensing a pulsar. The candidate events are listed in Section 4, and the results are discussed in Section 5.

2. The Search

I selected all the pulsars listed in the Australia Telescope National Facility (ATNF) pulsar database1 (version 1.58 of May 2018) that have proper motion measurements. The decl. of PSR J1856–3754 in the ATNF catalog was erroneous and I use the correct decl. from Walter & Matthews (1997).

Out of 2636 pulsars in the ATNF catalog, 277 have proper motion measurements. Figure 1 presents the distribution of errors in the proper motions of these pulsars. The typical error in the pulsars proper motion is an order of magnitude larger.

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1 http://www.atnf.csiro.au/people/pulsar/psrcat/
than that of GAIA-DR2 stars. However, a large fraction of these proper motions are good enough to predict the position of a pulsar to an accuracy of ∼0.1′ in the next 100 years. Here, I make predictions also for pulsars with poor astrometric measurements. I used the pulsars’ distances as estimated from either direct parallax measurements (if available) or from their dispersion measure.

For each pulsar, I used the catsHTM tool (Soumagnac & Ofek 2018) to query for all the GAIA-DR2 (Gaia Collaboration et al. 2016, 2018) sources within 1000 arcsec of the pulsar’s cataloged position. Given the pulsar’s and GAIA-DR2 source’s position and proper motion, I calculated the closest approach between the stars and the pulsar. For each closest approach with an angular distance below 100 arcsec, that takes place between 2000 and 2100, I calculated the expected astrometric microlensing center-of-light deflection as a function of time. I note that, when the observer-lens distance is small, conjunctions with a larger impact parameter can induce considerable light deflections. However, these deflections will typically change over timescales of decades, making them less attractive for follow-up observations.

The light deflection of the center-of-light of the source (i.e., the more distant object of the two) was calculated by taking the positions of the two images of the background star weighted by their respective magnifications, and taking into account the star and pulsar parallaxes. If the star parallax is smaller than two times the parallax error, I set the distance of the star to 10 kpc.

The search utilized the code available as part of the MATLAB astronomy and astrophysics toolbox8 (Ofek 2014).

3. Detection of Lensed Pulsars via Timing Measurements

Another interesting possibility, already proposed by Larchenko & Doroshenko (1995) and Wex et al. (1996), is that a pulsar will be lensed by a star. In principle, this offers the possibility to measure the variations in the lensing time delay via the timing observations of the pulsar. A constant time delay adds a constant phase to the pulsar timing (i.e.,

\[
\theta_\pm = \frac{1}{2}(\beta \pm \sqrt{\beta^2 + 4\theta_E^2}),
\]

and \(m_+\) and \(m_-\) are the magnifications of the two images

\[
m_\pm = \left[1 - \left(\frac{\theta_E}{\theta_\pm}\right)^4\right]^{-1}.
\]

Here, \(\theta_E\) is the angular Einstein radius, in radians, given by

\[
\theta_E = \sqrt{\frac{4GM}{c^2\,\mu_{\text{rel}}}},
\]

where \(G\) is the gravitational constant, \(M\) is the lens mass, \(c\) is the speed of light, \(\mu\) is the astronomical unit, and \(\mu_{\text{rel}}\) is the relative parallax in arcseconds

\[
\mu_{\text{rel}} = \frac{\mu_D}{\mu_s},
\]

where \(\mu_D, \mu_s,\) and \(\mu_{\text{ls}}\) are the parallaxes between the observer and the lens, the observer and the source, and the lens and the source, respectively. Note that when the image separation \((\theta_+ + \theta_-)\) becomes larger than the instrument resolution, the source deflection angle should not include the demagnified image contribution. In such a case,

\[
d = \theta_\pm - \beta.
\]

However, for practical purposes, there is no difference between Equations (1) and (6). For the case of \(\beta \gg \theta_E\), we can approximate Equations (1) and (6) using

\[
d \approx \frac{\theta_E}{\beta} = \frac{4GM}{c^2\,\mu_{\text{rel}}\,\mu_s \, \frac{1}{\mu_\star} \beta}.
\]

Finally, the timescale for the lensing phenomenon is given by \(\sim \theta_E/\mu_\star\), where \(\mu_\star\) is the lens-source relative total proper motion.

Next, I selected sources that have a maximum light deflection above 10 μas (relative to an event with no microlensing). One obvious contamination is that some pulsars may have companions—if all proper motion measurements are correct, such systems will have constant angular separation between the pulsar and GAIA-DR2 star as a function of time. In order to avoid selecting systems in which the astrometric deflection variations are slow, I selected only sources for which the pulsar–star separation varies by more than 0.5″ over the ±20 years of the time of closest approach. Systems with a slower relative proper motion will require observations over long timescales in order to measure any signal.

The search utilized the code available as part of the MATLAB astronomy and astrophysics toolbox8 (Ofek 2014).

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8. https://webhome.weizmann.ac.il/home/cofek/matlab/
shifting the signal in time), while the first derivative of the time delay multiplied by the pulsar period adds a constant to the pulsar periodicity. These terms cannot be measured because their intrinsic values are unknown a priori. However, the second derivative of the time delay multiplied by the pulsar period adds a constant to the first derivative of the pulsar period ($P$), and abrupt variations in $P$ (on a year timescale) can be measured and separated from the intrinsic $P$.

The time delay near a point-mass lens potential drops logarithmically with the impact parameter (see Equation (9)) and, therefore, the collective effects of the galactic potential may be important. However, when a star is passing with a small impact parameter (e.g., $\ll 1''$), it will induce variations in the time delay that are relatively abrupt and that may dominate over all other contributions to the time delay variations. The relevant formulae for the expected time delay are provided, along with an example, in the Appendix.

### 4. Candidate Events

The selection process described in Section 2 yielded four candidates for astrometric microlensing events. The candidates include (1) PSR J185635−375435, which passed $\pm 4.1''$ from a 19.4 G magnitude star on J2014.9 with an estimated deflection of 13 $\mu$as; (2) PSR J084606−353340, which may passed about $0''2$ from a 19.0 G magnitude star on J2022.9 with an estimated deflection of 91 $\mu$as; (3) PSR J221532+513536, which passed about $0''1$ from a 20.6 G magnitude star on J2051.3 with an estimated deflection of 26 $\mu$as; and (4) PSR J221532+513536, which passed about $0''08$ from a 19.2 G magnitude star on J2011.3.

The event that took place at J2011.3 is likely the companion of PSR J221532+513536 and therefore was removed. In addition, the proper motion of PSR J221532+513536 is considered to be unreliable (D. Manchester 2018, private communication). Therefore, here we consider only two candidate events involving PSR J185635−375435 and PSR J084606−353340. These events will most likely not introduce noticeable magnifications of the source. Given the poor accuracy of the proper motion of PSR J084606−353340 the event involving this pulsar is highly uncertain. The details of these two events, including the pulsar’s parameters, are listed in Tables 1 and 2.

Figures 2 and 3 present the separation and center-of-light deflection as a function of time for PSR J0846−3533 and PSR J1856−3754, respectively. Finding charts of the pulsar field are shown in Figures 4 and 5.

I do not find good candidate events for foreground stars that are lensing background pulsars, with induced variations in $P$, divided by 1 s, exceeding $10^{-15}$ s$^{-1}$.

### 5. Discussion

I present a search for conjunctions of pulsars with GAIA-DR2 stars. I find two events in which the astrometric deflection, from a constant rate motion of the background star, is expected to be $\lesssim 10$ $\mu$as. One of these events is highly uncertain.

This search should be regarded as a preliminary search for candidates. The pulsar’s and star’s proper motions and distances should be verified with future observations, and it will also be useful to obtain high-resolution imaging of these fields.

| Parameter                         | Value   |
|-----------------------------------|---------|
| **Pulsar data**                   |         |
| Name                              | PSR J084606.060−353340.64 |
| J2000.0 R.A.                      | 08:46:06.060 $\pm$ 0.5$''$ |
| J2000.0 decl.                     | $-35:33:40.64$ $\pm$ 0.5$''$ |
| Proper motion in R.A.             | 93 $\pm$ 72 mas yr$^{-1}$ |
| Proper motion in decl.            | $-15$ $\pm$ 65 mas yr$^{-1}$ |
| Coordinates epoch                 | MJD 48719 |
| Distance (Dispersion Measure)     | 0.54 kpc |
| Period                            | 1.1161 s |
| Period derivative                 | $1.60 \times 10^{-15}$ |
| Characteristic age                | $1.1 \times 10^{7}$ years |

| Star data                         |         |
|-----------------------------------|---------|
| J2000.0 R.A.                      | 08:46:06.291901 $\pm$ 0.15 mas |
| J2000.0 decl.                     | $-35:33:41.34052$ $\pm$ 0.18 mas |
| Coordinates epoch                 | MJD 2015.5 |
| Proper motion in R.A.             | $-3.17 \pm 0.32$ mas yr$^{-1}$ |
| Proper motion in decl.            | $2.80 \pm 0.35$ mas yr$^{-1}$ |
| Parallax                          | 0.10 $\pm$ 0.23 mas |
| R.A./decl. correlation            | 0.15 |
| Astrometric excess noise          | 0.38 mas |
| Mag G                             | 19.027 $\pm$ 0.002 |
| Mag BP                            | 19.806 $\pm$ 0.048 |
| Mag RP                            | 18.197 $\pm$ 0.018 |
| Time of minimum separation        | J2022.9 |
| Minimum angular separation        | $-0''22$ (uncertain) |
| Estimated Einstein radius         | 4.5 mas |
| Maximum astrometric deviation     | 91 $\mu$as |

Note. Due to the large uncertainty in the pulsar coordinates and proper motion, the minimum angular separation, and expected light deflection, are highly uncertain. Pulsar proper motions are adopted from Zou et al. (2005). The current errors in proper motion are larger (in absolute value) than the proper motion itself, and hence this event is uncertain. Given the importance of such events, additional observations are required in order to decrease the errors in the position and proper motion of the pulsar.

An important requirement for such a program is the ability to measure small astrometric shifts, preferably on the level of 1–10 $\mu$as. Current state-of-the-art ground-based observations deliver 100 $\mu$as astrometric precision (e.g., Tendulkar et al. 2012; Lu et al. 2016), and new instruments are likely to improve this considerably (e.g., Perraut et al. 2018). Therefore, new techniques and methodologies are required in order to improve the currently available level of precision. I note that the current limitations of ground-based optical astrometric measurements likely result from systematic errors (e.g., Service et al. 2016). Therefore, with a better understanding of these sources of noise there is a realistic potential for improvement.

A related important question is how well do we need to measure the light deflection and distances to the lens and source in order to measure the lens mass to some accuracy (see also Gould 2000). For $\beta \gg \theta_E$, the mass depends linearly on $\beta$, $\delta$, and $1/\pi_{rel}$ (Equation (7)). Specifically, the relative error in the mass estimate will be

$$\frac{\sigma_M}{M} \propto \frac{\sigma_M}{\beta} \left( \frac{\sigma_M}{\delta} \right)^2 + \left( \frac{\sigma_M}{\pi_{rel}} \right)^2.$$  

(8)

Here, $\sigma_X$ is the uncertainty in variable $X$.  

| Parameter                             | Value |
|---------------------------------------|-------|
| **Table 1**                           |       |
| PSR J0846−3533 Conjunction Parameters |       |
| Name                                  |       |
| PSR J084606.060−353340.64             |       |
| J2000.0 R.A.                          | 08:46:06.060 $\pm$ 0.5$''$ |
| J2000.0 decl.                         | $-35:33:40.64$ $\pm$ 0.5$''$ |
| Proper motion in R.A.                 | 93 $\pm$ 72 mas yr$^{-1}$ |
| Proper motion in decl.                | $-15$ $\pm$ 65 mas yr$^{-1}$ |
| Coordinates epoch                    | MJD 48719 |
| Distance (Dispersion Measure)         | 0.54 kpc |
| Period                               | 1.1161 s |
| Period derivative                    | $1.60 \times 10^{-15}$ |
| Characteristic age                   | $1.1 \times 10^{7}$ years |
| Star data                            |       |
| J2000.0 R.A.                          | 08:46:06.291901 $\pm$ 0.15 mas |
| J2000.0 decl.                         | $-35:33:41.34052$ $\pm$ 0.18 mas |
| Coordinates epoch                    | MJD 2015.5 |
| Proper motion in R.A.                 | $-3.17 \pm 0.32$ mas yr$^{-1}$ |
| Proper motion in decl.                | $2.80 \pm 0.35$ mas yr$^{-1}$ |
| Parallax                             | 0.10 $\pm$ 0.23 mas |
| R.A./decl. correlation                | 0.15 |
| Astrometric excess noise              | 0.38 mas |
| Mag G                                | 19.027 $\pm$ 0.002 |
| Mag BP                               | 19.806 $\pm$ 0.048 |
| Mag RP                               | 18.197 $\pm$ 0.018 |
| Time of minimum separation            | J2022.9 |
| Minimum angular separation            | $-0''22$ (uncertain) |
| Estimated Einstein radius             | 4.5 mas |
| Maximum astrometric deviation         | 91 $\mu$as |
For systems with reliable predictions (i.e., \( \sigma_\beta = \beta \)) the first term is expected to be negligible. In such systems, the second (\( \delta \)) and third (\( \pi_{\text{rel}} \)) terms will likely dominate the errors. Therefore, measuring the mass of the NS to better than about 10% accuracy will require controlling the systematic errors in the astrometry to better than 10\( \mu \)as, and measuring the relative parallax (which depends on \( \pi_l \) and \( \pi_s \)) to better than 10%.

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Appendix

The Second Derivative of the Shapiro Time Delay

Larchenkova & Doroshenko (1995) and Wex et al. (1996) suggested that pulsar time variations can be used to detect an unseen mass or measure the mass of stars.

In the limit that the impact parameter is much larger than the Einstein radius, the geometric time delay can be neglected and the gravitational time delay is given by the Shapiro time delay

\[ \Delta t \approx - \sum l \frac{2GM_l}{c^3} \ln[1 - \cos(\beta_l)]. \]  

(9)

Here, \( M_l \) is the mass of the \( l \)th lens positioned at an angular distance \( \beta_l \) from the source.

The logarithmic dependence of the time delay on \( \beta \) suggests that the stochastic background (e.g., the Galactic potential) is typically the dominant contributor to the time delay. However, when a star is passing with a small impact parameter (e.g., \( \beta < 1 \)) from a pulsar, then the star gravitational potential may dominate the time delay.

For pulsar observations, the Shapiro time delay induces a phase shift to the pulsar observations. The first derivative of the Shapiro time delay multiplied by the pulsar period, adds a constant to the measured periodicity. The second derivative of the Shapiro time delay, multiplied by the pulsar period, adds a constant to the measured \( \dot{P} \). Therefore, we are interested in the second derivative of the Shapiro time delay, \( \ddot{\Delta}t \). For two sources moving with a relative proper motion \( \mu_{e} \) (ignoring parallax), the angular distance \( \beta \) as a function of time \( t' \) is

\[ \beta = \sqrt{\beta_m^2 + \left[\mu_e(t' - t_0)\right]^2}, \]

(10)

where \( t_0 \) is the time of minimum separation, and \( \beta_m \) is the minimum impact parameter. Denoting \( t = t' - t_0 \), we get

\[ \dot{\Delta}t = \frac{GM}{c^3} \frac{\mu_e^2 \beta_m^2 \sin(\sqrt{\mu_e^2 + \beta_m^2}) - \dot{\mu}_e^2 \beta_m^2 \sqrt{\mu_e^2 + \beta_m^2}}{\left(\dot{\mu}_e^2 + \beta_m^2\right)^{3/2} \left(\cos(\sqrt{\mu_e^2 + \beta_m^2}) - 1\right)}. \]

(11)

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