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Representation of anisotropic magnetic characteristic observed in a non-oriented silicon steel sheet

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ABSTRACT
This article presents a modified Jiles–Atherton hysteresis model for a weakly anisotropic non-oriented silicon steel sheet. In a toroidal inductor, the magnetic flux density can point toward any direction compared to the sheet orientation, and the hysteresis model should take this into account. We identify the model parameters independently for unidirectional alternating $B(H)$-characteristics in seven different directions. Then, we construct an anisotropic hysteresis model, where the model parameters can depend on the magnitude and direction of the applied magnetic flux density. We demonstrate that the parameters identified in the rolling and transverse directions of the silicon steel sheet (M400-50A) are sufficient to describe the hysteresis losses in other directions.

I. INTRODUCTION

In this article, we aim to model the anisotropic magnetic behavior of a non-oriented (NO) silicon steel sheet by a modified Jiles–Atherton (J–A) hysteresis model. NO silicon steel is widely used as a magnetic core material in toroidal inductors, rotating electrical machines, and several other electromagnetic devices, and several studies confirm that the material presents a significant level of magnetic anisotropy. The anisotropy in the core affects the performance of an electromagnetic device, so it is essential to account for this effect in the magnetic hysteresis model.

The Jiles–Atherton (J–A) hysteresis model is widely used to model polycrystalline electrical steels, such as NO and grain-oriented (GO) silicon steels. Compared with the Preisach type hysteresis models, the J–A model has a simple mathematical formulation. In particular, the number of involved parameters is minimal compared to that of the Preisach model. The J–A model lacks memory properties (as opposed to the Preisach type and energy-based models). Consequently, non-symmetric minor loops are not exactly closed. In addition, it is not well understood if the J–A model can represent both the alternating and the rotational magnetic field variations simultaneously. However, if non-closed minor loops and a rotational magnetic field are not a concern, then the $B(H)$-characteristics can be modeled efficiently with the J–A model. Moreover, the model has been found to be suitable in studying the effect of external mechanical stress on the $B(H)$-characteristics of a soft-magnetic material.

The J–A model is usually presented with isotropic, fixed parameters. Several studies show that the $M$ (or $H$ or $B$)-dependent model parameters produce a better fit with the measured symmetric minor and major hysteresis loops. In an attempt to introduce anisotropy into the modified J–A model, the authors in Refs. 14 and 25 identify separate parameters for $B(H)$-loops measured in the rolling direction (RD) and transverse direction (TD), and
"interpolate" the models in the intermediate directions. Their extended J–A model, however, does not take into account the directional variation of the parameters. We claim that in devices like toroidal inductors, the anisotropic J–A model with the parameters from RD and TD alone does not accurately describe the more complicated unidirectional alternating \( B(H) \)-characteristics. As we discuss in Sec. II, the anhysteretic magnetization as well as the hysteresis losses vary according to the direction of the applied magnetic flux density.\(^\text{21}\) Thus, it is important to consider a more detailed directional variation of the J–A model parameters.

In this work, we first identify the J–A model parameters for unidirectional alternating \( B(H) \)-characteristics in seven different measurement directions. Second, we show that the identified parameters can be said to depend on both the magnitude and the direction of the input excitation. Third, we express the identified (magnitude- and direction-dependent) anisotropic parameters with analytical functions. Finally, we demonstrate that the augmented model gives a better fit with the observed \( B(H) \)-characteristics of a weakly anisotropic NO silicon steel sheet (M400-50A).

II. METHODOLOGY

A. J–A hysteresis model

The J–A model gives the relationship between \( B \) and \( H \) as a differential equation \( dB/dH \), known as the differential permeability. The original derivation of the model is presented in Refs. 7 and 8. In this work, we consider the inverse J–A model, where \( \mu(\alpha) \) represents the differential irreversible \( B(H) \)-characteristics in seven different directions. As we discuss in Sec. II, the anhysteretic magnetization as well as the hysteresis losses vary according to the direction of the applied magnetic flux density.\(^\text{21}\) Thus, it is important to consider a more detailed directional variation of the J–A model parameters.

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The inverse J–A model is written as a first-order differential equation,\(^\text{26}\)

\[
\frac{dM}{dB} = \frac{c\xi}{\mu_0[1 + c^2(1 - \alpha)]}, \quad \text{if } (M_{an} - M_{irr})dH_{eff} \leq 0,
\]

\[
\frac{1 - c\chi + c\xi}{\mu_0[1 + (1 - c)\chi(1 - \alpha) + c\xi(1 - \alpha)]} \quad \text{otherwise},
\]

where \( k \) is a pinning parameter corresponding to the average density of the pinning sites, and \( \chi \) and \( \xi \) represent the differential irreversible and differential anhysteretic susceptibilities.

Finally, the differential reluctance is obtained using the constitutive relationships (7) and (8):

\[
\frac{dH}{dB} = \frac{1}{\mu_0} - \frac{dM}{dB}.
\]

Given the initial states \( H_1 \) and \( B_0 \) and the excitation \( B_{in} \), the field strength \( H_{in} \) is solved from the differential Eq. (9) numerically by an explicit fourth-order Runge–Kutta (RK4) method.\(^\text{28}\)

B. Measurement of \( B(H) \) characteristic

The measured unidirectional alternating \( B(H) \)-characteristics used in this paper are obtained from a 0.5 mm thick NO silicon steel sheet of grade M400-50A. The quasi-static magnetic measurements are performed using a single sheet tester; for details, see Refs. 6 and 29. We utilize \( B(H) \) measurements from seven directions (0°, 15°, 30°, 45°, 60°, 75°, and 90°). For each measurement direction, the behavior of \( B(H) \) curves are measured with the peak amplitudes of \( B \) being 0.1 T, 0.2 T, 0.3 T, ..., and 1.5 T.

The measured \( B(H) \)-characteristics in seven different directions with respect to (w.r.t.) the RD are shown in Fig. 1. In Fig. 1(a), the RD (0°) and TD (90°) alternating fields are shown; Fig. 1(b) depicts alternating fields in 15° and 75° directions; Fig. 1(c) depicts alternating fields in 30° and 60° directions; Fig. 1(d) depicts alternating fields 45° direction and average \( B(H) \)-characteristics of all seven directions. The result shows that NO silicon steel requires different values of field strength \( H \) to reach the same value of flux density \( B \) in seven different directions. Hence, the measurement results verify that the NO silicon steel of grade M400-50A is magnetically anisotropic.

C. Anhysteretic magnetization

The anisotropy in the J–A model is normally introduced in the anhysteretic magnetization curve.\(^\text{30,31,34,35} \) When modeling NO silicon steel, the modified Langevin function (5) is commonly used to represent the anhysteretic magnetization.\(^\text{32,33} \) Depending on the magnetic material type, several other functions have been proposed and utilized.\(^\text{34,35} \)

In this work, we do not utilize a closed-form function to model the anhysteretic magnetization. Instead, we assume that averaging the field strength of the ascending and descending branches of the
The major hysteresis loop gives a reasonable estimate of the anhysteretic magnetization. The uncritical use of phenomenological dependence, i.e., modified Langevin function, might lead to serious problems for certain sets of model parameters. The $M(H_{avg})$ curve obtained by averaging the major hysteresis loop is expressed as a piece-wise cubic spline and used to describe the $M_{an}(H_{eff})$ relationship in the J–A model. Thus, we consider the technical saturation magnetization $M_s$ to be a fixed parameter, and it is implicitly included in the $M_{an}(H_{eff})$ magnetization curve. Based on the existing postulates made by Jiles and Atherton, we assume that the interaction field $\alpha M$ varies in accordance with the applied field strength. Therefore, the parameter that describes the inter-domain coupling $\alpha$ is assumed to be a non-constant fitting parameter [see (8) and Refs. 25 and 26].

The identified anhysteretic curves for the M400-50A NO silicon steel sheet are shown in Fig. 2. The result shows variations of the anhysteretic magnetization in seven measurement directions.
Among the seven measurement directions, the 0° (or the RD) is the magnetically easy direction, whereas 90° (or the TD) is the hard direction. As it appears to be, the \( B(H_{\text{app}}) \)-characteristics show that M400-50A possesses a significant level of magnetic anisotropy. The level of anisotropy is low for the lower values of flux density \( B \) and gradually increases until 1.5 T. At high amplitudes, the domain magnetizations coherently rotate toward the direction of the applied field strength \( H_{\text{app}} \), so the flux density \( B \) for all seven different directions converges asymptotically to a single value, the so-called technical saturation magnetization \( M_s \).

D. Description of the J–A model parameters

The interaction between the pinning sites (imperfections, dislocations, and location of inhomogeneous strain) and the domain walls increases with the applied field strength \( H \). As a result, the hysteresis loss dissipation rises. The hysteresis losses depend on the amplitude and direction of the input excitation because the pinning sites are non-uniformly distributed in the material.\textsuperscript{19} In addition, the losses also depend on the type of input excitation, unidirectional alternating and rotational.\textsuperscript{19} Therefore, the pinning parameter \( k \) is not necessarily a constant but a function of the applied field strength \( H \) and its alternating direction.\textsuperscript{40} In other words, the pinning parameter \( k \) and the parameter describing the reversible wall bending \( c \) could be related to the coercive field strength \( H_c \) as \( k(1-c) \).\textsuperscript{41}

It is well observed that the coercive field strength \( H_c \) varies as a function of the peak amplitude of the applied field strength \( H^{\text{eff}} \) (see Fig. 3 in Refs. 42 and 43). Thus, it can be understood that either \( k \) or \( c \) varies as a function of the peak amplitude of the applied field strength \( H \). Likewise, for an inverse J–A model, the model parameters vary as a function of the peak amplitude of the flux density \( B \).

In the literature, several works related to the J–A model are presented, which utilize \( M \) (or \( H \))-dependent model parameters. We outline some of the main outcomes from the past works. In Ref. 20, the authors propose to modify the pinning parameter \( k \) of the J–A model. In their work, the pinning parameter \( k \) is allowed to vary as a function of the bulk magnetization \( M/M_s \) and two adjustable coefficients. In Ref. 10, the saturation magnetization \( M_s \), pinning parameter \( k \), and the shape parameters \( a \) and \( c \) are identified in nine different directions (0°–90° every 10° step) of the GO silicon steel sheet sample. As a result, the simulation shows a better fit with the measured unidirectional alternating \( B(H) \)-characteristics. The J–A model in Ref. 23 is supplied with the variable pinning and reversibility parameters. Both the parameters \( k \) and \( c \) vary as a function of the peak amplitude of the effective field strength \( H^{\text{eff}} \). The authors demonstrate that the simulated results with variable pinning and reversibility parameters produce a better fit with the measured \( B(H) \)-characteristics of a NO silicon steel.

Likewise, the authors in Ref. 43 estimate the pinning, reversible, and the inter-domain coupling parameters of the J–A model for all the measured symmetric minor and major hysteresis loops. Their results show that the loop-dependent parameters \( k \), \( c \), and \( a \) produce a better fit with the measured \( B(H) \)-characteristics in the RD and TD directions. The authors in Ref. 22 express the reversible parameter \( c \) as a function of the applied field strength \( |H|/H^{\text{max}} \) and two adjustable parameters. In Ref. 44, the pinning parameter \( k \) is expressed as a function of the bulk magnetization \( |M|/M_s \) and three additional adjustable parameters. Their extended J–A model is then applied to model the \( B(H) \)-characteristics of Fe\textsubscript{60}Ni\textsubscript{30}Mo\textsubscript{2}B\textsubscript{8} amorphous alloy. In Ref. 21, the interaction field \( aM \) is extended to higher-order terms. The result shows a better fit with the measured anhysteretic characteristic of a polycrystalline iron wire.

Clearly, based on the results from the past literature, we can understand that the parameters of the J–A model, especially \( k \) and \( c \), are not constants, which is in line with the postulates made by Jiles and Atherton in Ref. 7. Apart from the \( M \) (or \( H \))-amplitude dependence of the model parameters, we find that the directional dependence is somehow not addressed in the past works. Thus, through this work, we try to emphasize that both the amplitude and the directional dependence of the J–A model parameters are vital from the modeling perspective.

E. Identification of the model parameters

In the past, several techniques are demonstrated for the identification of the parameters of the J–A model.\textsuperscript{19,47} The global optimization techniques based on the heuristic methods have become very useful in determining the parameters of the J–A model.\textsuperscript{36,47} In this paper, the meta-heuristic simulated annealing optimization method is utilized to estimate the model parameters from the measured \( B(H) \)-characteristics.\textsuperscript{41} The algorithm available in Ref. 38 has been fine-tuned for this purpose. The parameters that yield the lowest value of the mean square error between the simulated and measured field strength \( H \) has been extracted. Besides, the model parameters are optimized for each of the measured symmetric minor and major hysteresis loops. The identified parameters of the J–A model for seven alternating \( B(H) \)-characteristics are shown in Figs. 3–5. The variation in the pinning parameter \( k \) w.r.t. the peak amplitude of the applied flux density \( B \) is shown in Fig. 3. Similarly, Figs. 4 and 5 show the variations of the parameters related to the reversible magnetization process and inter-domain coupling.

The identified parameters \( k, a, \) and \( c \) of the J–A model show a specific trend (see Figs. 3–5). Indeed, they describe the magnetic state of the material. The variation in the pinning and reversible parameters is minimal until 1.3 T and gradually rises until 1.5 T. This typical behavior can be related to the coercive field strength \( H_c \) and, specifically, to the hysteresis losses. The pinning parameters for the
high amplitude of the flux density $B$ describe the maximum interaction of the domain walls with the pinning sites. As a result, the coercive field strength $H_c$ attains the maximum value. In contrast, the reversibility parameter $c$ at high values of $B$ indicates that the differential susceptibility asymptotically approaches the value of differential anhysteretic susceptibility. In other words, further changes in the bulk magnetization $M$ are obtained by the coherent rotation of the domain magnetic moments.

The inter-domain coupling parameter $\alpha$ shows a slightly different behavior (see Fig. 5). According to the result, there seems to be weak coupling between the domain magnetizations at low flux density levels ($B \leq 0.4$ T). For instance, at low values of excitation, the interaction field is negligible. In contrast, at high field excitation, the M400-50A silicon steel is characterized by domains having large volumes, which are few in numbers; as a consequence, the interaction between the domains seems to rise. Indeed, at sufficiently high amplitude excitations, a single grain could represent a single domain in a multi-grain sample; therefore, no further increase in interaction is possible.

The average values of the model parameters, $k$, $\alpha$, and $c$, are shown in Figs. 6–8. It should be noted that the average values of the model parameters are identified from the averaged $B(H)$-characteristic [see Fig. 1(d)]. The average result could be related to the parameters identified from the $B(H)$-characteristics of the stacked silicon steel sheets, provided that the steel sheets are cut in seven different directions w.r.t. the RD.
F. Representation of the anisotropic parameters

The parameters \( k, \alpha, \) and \( c \) of the J–A model show smooth variation from the RD to TD (see Fig. 9). The result shows that the parameters depend both on the peak amplitude and on the direction of the applied flux density \( B \) (see Figs. 3–5, and 9). Therefore, based on the observation, we propose the following model to describe the anisotropic parameters of the J–A model. The parameters are expressed as

\[
k(B, \phi) = \frac{k_{RD}(B) k_{TD}(B)}{\sqrt{(k_{RD}(B) \sin \phi)^2 + (k_{TD}(B) \cos \phi)^2}} \]
\[
\alpha(B, \phi) = \frac{\alpha_{RD}(B) \alpha_{TD}(B)}{\sqrt{(\alpha_{RD}(B) \sin \phi)^2 + (\alpha_{TD}(B) \cos \phi)^2}} \]
\[
c(B, \phi) = \frac{c_{RD}(B) c_{TD}(B)}{\sqrt{(c_{RD}(B) \sin \phi)^2 + (c_{TD}(B) \cos \phi)^2}}
\]

where \( \phi \) and \( B \) are the direction and amplitude of the flux density vector, and \( k_{RD}(B), \alpha_{RD}(B), \) and \( c_{RD}(B) \) represent the identified pinning, inter-domain coupling, and reversible parameters based on the \( B(H) \)-characteristics in the RD. Likewise, \( k_{TD}(B), \alpha_{TD}(B), \) and \( c_{TD}(B) \) are the model parameters identified in the TD.

The following four cases can be considered:

(i) \( x_{RD}(B) \neq x_{TD}(B) \), where \( x = [k, \alpha, c] \) and \( M_{an} = f(H_{eff}, \phi) \);
(ii) \( x_{RD}(B) \neq x_{TD}(B) \), where \( x = [k, \alpha, c] \) and \( M_{an} = f(H_{eff}) \);
(iii) \( x_{RD}(B) = x_{TD}(B) \), where \( x = [k, \alpha, c] \) and \( M_{an} = f(H_{eff}, \phi) \);
(iv) \( x_{RD}(B) = x_{TD}(B) \), where \( x = [k, \alpha, c] \) and \( M_{an} = f(H_{eff}) \).

The first case, (i), is the most general case, as it describes the anisotropic J–A model (with anisotropic parameters). The second case, (ii), describes anisotropy in loss dissipation but isotropic anhysteretic magnetization. On the contrary, Case (iii) describes isotropic characteristics in loss dissipation and anisotropic anhysteretic magnetization. Accordingly, Case (iv) describes isotropic magnetic characteristics. Based on the results shown in Figs. 2–5, and 9, the unidirectional alternating \( B(H) \)-characteristic observed in M400-50A can be described by Case (i).
The anisotropy introduced by the rolling of the silicon steel results in better anhysteretic characteristics in the RD\textsuperscript{42,43} (see Figs. 1 and 2); therefore, Case (ii) may seldom occur in NO silicon steel. Case (iii) is utilized under the application of external stress.\textsuperscript{18} It is a common practice to use the isotropic J–A model based on Case (iv) to model the measured $B(H)$-characteristics obtained from the standard Epstein-frame device and ring core samples.\textsuperscript{13} Moreover, Case (iv) is preferred in numerical magnetic field computations of rotating electrical machines.\textsuperscript{48} The results produced by (10)–(12) are shown in Figs. 9(a)–9(c), respectively. It can be observed that for the low and medium amplitudes of the flux density $B$, the proposed analytical equations produce a good fit with the identified parameters of the J–A model; however, at high amplitude levels, some discrepancy can be seen.

III. RESULTS

The simulations of the field strength $H$ are performed using the isotropic (average parameters) and anisotropic (identified parameters in seven directions, and proposed analytical functions)
parameters in the modified J–A model. Figures 10–12 show the simulated and measured hysteresis loops for 45° direction w.r.t. the RD. It should be noted that the measured hysteresis loops have rotational symmetry w.r.t. the origin, so, only the upper half of the B(H)-loop is shown. The result shows that the anisotropic parameters applied to the modified J–A model produce a good fit with the measured data (see Figs. 10 and 11). In contrast, the J–A model that utilizes the average parameters shows a significant level of disagreement with the measurement, particularly for the major B(H)-loop. As depicted in Fig. 12(c), the simulated B(H)-characteristic shows poor fitting for the medium and high amplitudes of the magnetic-flux density B > 1.1 T.

Figure 13 shows the simulated and measured hysteresis losses for seven different directions. Besides, the losses are simulated from

![Graphs showing hysteresis loops and iron losses](image-url)
six hysteresis loops with the peak amplitudes of the flux density $B$ being 0.5 T, 0.7 T, 1.0 T, 1.2 T, and 1.5 T. The simulation results shown in Fig. 13(a) are obtained from the modified J–A model that utilizes anhysteretic magnetization shown in Fig. 2 and the parameters depicted in Figs. 3–5, respectively. The comparison between the simulated and measured hysteresis losses shows that the identified parameters produce sufficiently accurate results [see Fig. 13(a)]. However, at high amplitude excitation ($B = 1.5$ T), the simulated losses are slightly higher than the measured ones.

Figure 13(b) shows the losses simulated from the J–A model that uses the parameters described by (10)–(12). The result shows a good agreement with the measured losses. In contrast, the J–A model with the averaged parameters produces identical $H$ loci in all seven directions. Therefore, the simulated losses depicted in Fig. 13(c) are not in good agreement with the measured losses. Besides, it can be observed that the isotropic model overestimates the losses for 0°, 15°, and 30°, whereas it underestimates them for 60°, 75°, and 90°.

IV. CONCLUSION

The pinning, reversible, and the inter-domain coupling parameters of the J–A model depend on the amplitude and direction of the applied magnetic flux density. A suitable analytical function is applied to describe the model parameters ($k$, $a$, and $c$). It is apparent from the simulation results that the model based on the parameters in the RD and TD of the NO silicon steel sheet (M400-50A) is sufficient to describe the anisotropically magnetic characteristic in other directions. Apart from the model parameters, the anhysteretic magnetization varies in different measurement directions. The results based on the average (isotropic) parameters of the J–A model show a significant amount of disagreement with the measurement data. Alternatively, the results based on the proposed (modified) J–A model show a good agreement with the measured data.

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DATA AVAILABILITY

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available.

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