Analytical solution for two-phase flow of silica sol grouting in homogeneous fractures

Liangchao Zou¹, Heikki Sandström² and Vladimir Cvetkovic¹

¹ Division of Land and Water Resources Engineering, Department of Sustainable development, Environmental science and Engineering, Royal Institute of Technology, 10044, Stockholm, Sweden.
² Posiva Oy, 27160, Helsinki, Finland.

lzo@kth.se

Abstract. In this study, we present an analytical model for modelling two-phase flow of silica sol grouting in a single rock fracture. This model considers the time-dependent rheological property of the silica sol. The impact of groundwater flow in silica sol grouting is investigated by comparing the results of the present two-phase flow model with that of the single-phase flow model ignoring the groundwater flow. It generally shows that the groundwater flow significantly affects the pressure distribution and grouts penetration in the fracture. Ignoring the groundwater flow in single-phase flow models will underestimate the pressure at the propagation front and overestimate the propagation length. The analytical model presented in this study is able to describe the more realistic two-phase processes in rock grouting, which can be readily used in practice to reduce the potential uncertainties in the application of previous simplified analytical models.

1. Introduction
Silica sol is a grouting material that has important advantages compared to cement grouts. Since it can penetrate and seal fine fractures that traditional cement grouts cannot, it has been used in rock grouting practice for underground rock engineering projects, especially for significant underground infrastructures in the deep subsurface, such as underground radioactive waste repositories. Modeling and analysis of silica sol propagation in rock fractures are important in the design, execution, and monitoring of such rock grouting applications.

At present, modeling of rock grouting in engineering applications mainly relies on simplified analytical or numerical models due to complex rock fracture structures and grouts properties. In the literature, numerous analytical and numerical models have been developed to analyze grouting in rock fractures [1-16]. For instance, Gustafson et al. [7] developed analytical solutions for non-Newtonian cement grout (assumed as Bingham fluids) propagation in single homogeneous planar fractures, which has provided theoretical basics for the real time grouting control (RTGC) method that has been used in practice [9-10]. Funehag and Gustafson [5] proposed an analytical solution for the Newtonian silica sol grouts propagation in planar fractures. These theoretical studies assume that the rock fractures consist of smooth parallel plates and the groundwater flow is negligible. However, in reality, the natural rock fractures in deep subsurface are commonly filled with groundwater. The grouting is therefore actually a two-phase (i.e., silica sol grout and groundwater) flow process in rock fractures.
et al. [3] developed numerical models for simulation of cement grouts propagation in single fractures and regular fracture networks, with consideration of water flow. Zou et al. [11-15] developed a two-phase flow model for simulation of cement grouts propagation in homogeneous fractures and fracture networks and systematically analyzed the impact of groundwater flow on cement grouts propagation, showing that the ignoring groundwater flow may overestimate propagation rate when the grout viscosity is relatively small. However, the impact of groundwater flow on Newtonian silica sol grouts propagation remains unknown.

In this study, we aim to present a new analytical solution for two-phase flow of silica sol grouting in one-dimensional homogeneous fractures and to investigate the impact of groundwater flow on Newtonian silica sol grouts propagation. This analytical solution and results are useful for rock grouting design in practice.

2. Physical considerations

In the present study, we consider silica sol grout propagation in a single fracture represented by two smooth parallel plates, where fracture surface roughness is not considered here. Figure 1 presents the conceptual models of silica sol grouting in a single fracture with and without consideration of groundwater flow in the grouting process.

![Figure 1](image_url)

**Figure 1** Illustration of silica sol grout penetration into a single fracture: (a) without consideration of groundwater flow and (b) with consideration of groundwater flow.

The fracture aperture is $2B$. The silica sol grout is considered as a Newtonian fluid, injected from the left-hand-side of the fracture with a constant pumping pressure $P_1$. The length of the fracture is $L$. The distance between the injection boundary (on the left-hand-side) and the grout front, i.e. the interface between grout and water, represents the grout penetration length $I(t)$, which is a function of grouting time. In previous theoretical studies, flow of the groundwater is often ignored, so that the pressure on the grout front is assumed as a constant that equals to in situ groundwater pressure, $P_2$ (Figure 1a). In
fact, rock grouting process is mostly a two-phase flow process, where the grout displaces the groundwater in the fracture. With consideration of groundwater flow, the pressure on the right-hand side of the fracture is taken as the in situ groundwater pressure, $P_2$ (Figure 1b).

The silica sol grout and groundwater are both Newtonian fluids, the rheological model is expressed as

$$\tau = \mu \dot{\gamma}$$

where $\mu$ is the dynamic viscosity (expressed as $\mu_1$ and $\mu_2$ for the silica sol grout and groundwater, respectively) and $\dot{\gamma}$ is the shear strain. For groundwater, the dynamic viscosity is constant, i.e., $\mu_2 = 0.001 \text{Pa} \cdot \text{s}$, while the dynamic viscosity of the silica sol grout is time-dependent that is often described as an exponential function, expressed as

$$\mu_1 = \mu_0 \left\{1 + \alpha \left[\exp\left(\frac{t}{t_G}\right) - 1\right]\right\}$$

where $\mu_0$ is the initial viscosity, $t$ is the time, $\alpha$ is a dimensionless parameter denoting the hardening process and $t_G$ is gel induction time representing the time when the viscosity is doubled. In the present study, we adopt that the initial viscosity $\mu_0 = 0.0068 \text{Pa} \cdot \text{s}$, parameter $\alpha = 0.582$ and gel induction time $t_G = 120 \text{s}$.

We assume that the fluids, i.e., silica sol grout and groundwater, are both incompressible, the gravitational forces and inertial effects are negligible (the flow is laminar) and the fracture aperture is much smaller than the lateral dimensions, so that the pressure gradient across the fracture aperture is negligible by adopting the lubrication approximation.

3. Analytical solutions for silica sol grouting

For comparison, analytical solutions for both cases with and without consideration of groundwater flow are presented in this section.

3.1. Analytical solution for single-phase flow

According to the assumptions presented in Section 2, the governing equations for the 1D flow can be simplified as,

$$\frac{\partial u}{\partial x} = 0$$

$$-\frac{\partial P}{\partial x} = \frac{\partial \tau}{\partial z}$$

where $u$ is the velocity across the fracture aperture and $P$ is the pressure. For Newtonian fluids flow in a homogenous fracture with a given pressure gradient $\frac{\partial P}{\partial x}$, the solution for flowrate is given by the cubic law, expressed as \[16\]

$$Q = -w \frac{2B^3}{3\mu} \frac{\partial P}{\partial x}$$

where $w$ is the width of the fracture, $B$ is half of the fracture aperture, $\mu$ is the dynamic viscosity of water. The pressure gradient is given by

$$\frac{\partial P}{\partial x} = -\frac{P_1-P_2}{L}$$

where $L$ is the fracture length, $P_1$ and $P_2$ are given pressure at the inlet and outlet, respectively.

By ignoring the flow of groundwater (as shown in Figure 1a), the mean velocity $\bar{u}$ of silica sol grout can be expressed as
\[ \bar{u} = \frac{dI(t)}{dt} = \frac{Q}{2Bw} = -\frac{B^2}{3\mu_1} \frac{\partial P}{\partial x} = \frac{B^2}{3\mu_1} \frac{(P_1-P_2)}{l(t)} \]  \tag{7}

Equation (7) is an ordinary partial differential equation, which can be separated as

\[ dI(t) \cdot I(t) = \frac{B^2(P_1-P_2)}{3\mu_0} \frac{dt}{[1+a \exp\left(\frac{t}{t_G}-1\right)]} \tag{8} \]

Integration of equation (8) yields

\[ I(t)^2 = \frac{B^2(P_1-P_2)}{3\mu_0} \frac{t_G \log\left[a \exp\left(\frac{t}{t_G}-1\right)+1\right]-t}{a-1} \tag{9} \]

Therefore, the solution of propagation length is

\[ I(t) = \left[ \frac{2B^2(P_1-P_2)}{3\mu_0} \frac{t_G \log\left[a \exp\left(\frac{t}{t_G}-1\right)+1\right]-t}{a-1} \right]^{1/2} \tag{10} \]

3.2. Analytical solution for two-phase flow

With consideration of groundwater flow as shown in Figure 1b, it is a two-phase flow problem. We denote the pressure at the interface as \( P_I(t) \). The equal flowrate in the two phases gives

\[ Q = \frac{2B^3}{3\mu_1} \frac{P_1-P_2}{I(t)} = \frac{2B^3}{3\mu_2} \frac{P_1-P_2}{L-I(t)} \tag{11} \]

where \( I(t) \) is the penetration length, \( \mu_1 \) and \( \mu_2 \) is the viscosity of the two Newtonian fluids, respectively. The pressure at the interface is given by

\[ P_I(t) = \frac{P_2 \mu_1 I(t)+P_1 \mu_2 [L-I(t)]}{\mu_1 I(t)+\mu_2 [L-I(t)]} \tag{12} \]

The mean velocity \( \bar{u} \) of silica sol grout front is

\[ \bar{u} = \frac{dI(t)}{dt} = \frac{dI(t)}{dt} = \frac{Q}{2Bw} = \frac{B^2}{3} \frac{(P_1-P_2)}{\mu_2 L+(\mu_1-\mu_2)I(t)} \tag{13} \]

Therefore, the solution for silica sol grout propagation length is

\[ I(t) = \int_0^t \frac{B^2}{3} \frac{(P_1-P_2)}{\mu_2 L+(\mu_1-\mu_2)I(t)} dt = \int_0^t \frac{B^2}{3} \frac{(P_1-P_2)}{\mu_2 L+\mu_0 [1+a \exp\left(\frac{t}{t_G}-1\right)]I(t)-\mu_2 L} dt \tag{14} \]

Although the explicit expression of the solution is not available, equation (14) can be easily solved numerically. In the present study, equation (14) is solved by using the Runge-Kutta method [17].

4. Results

In this Section, we illustrate the two solutions for silica sol grouting with and without consideration of the flow of groundwater and therefore demonstrate the importance of groundwater flow on the grout penetration process. In this illustration example, we adopt that the fracture aperture is \( 2B = 1 \) mm, the fracture length is \( L = 20 \) m, and the pressure difference is \( P_1 - P_2 = 10^5 \) pa.

4.1. Pressure distributions

Figure 2 presents the pressure distributions at different injection time for the cases without and with consideration of the groundwater flow. For two-phase flow cases with consideration of groundwater flow, there are pressure gradients in both the grout and groundwater (see solid lines in Figure 2), while pressure gradient presences only in the grout (see dash lines in Figure 2) once the groundwater flow is ignored. Pressure gradients reduce with the increasing propagation length. With consideration of groundwater flow, the pressure at the propagation front is generally higher than that when the
groundwater flow is ignored, even though the difference reduces with the increasing propagation length and injection time. This result shows that ignoring groundwater will generally underestimate the pressure at the propagation front.

![Figure 2](image.png)

**Figure 2** Comparison of the pressure distributions along the fracture for the cases with and without consideration of groundwater flow.

4.2. Propagation length

Figure 3 shows the evolution of grout propagation length with injection time for the two cases with and without consideration of the groundwater flow.

In general, the silica sol grout propagate in the fracture with the increasing injection time and stops at around 480s for both cases, due to the hardening of silica sol grout and the reducing pressure gradient. The penetration length is generally larger for the case without considering groundwater flow than that with considering groundwater flow. This result indicates that ignoring groundwater flow will overestimate the propagation length.
In order to quantitatively demonstrate the overestimation of propagation length by ignoring groundwater flow, the differences between propagation lengths of the cases are presented in Table 1.

Table 1. Comparison of propagation lengths at different injection time for the two cases with and without consideration of groundwater flow.

| Injection time [s] | Propagation length [m] (w/o groundwater flow) | Propagation length [m] (w/ groundwater flow) | Difference [m] | Relative differences |
|-------------------|-----------------------------------------------|-----------------------------------------------|----------------|----------------------|
| 15                | 5.95                                         | 3.71                                         | 2.24           | 60.13%               |
| 30                | 8.26                                         | 6.10                                         | 2.16           | 35.49%               |
| 60                | 11.24                                        | 9.28                                         | 1.96           | 21.16%               |
| 120               | 14.69                                        | 13.02                                        | 1.68           | 12.87%               |
| 240               | 17.77                                        | 16.34                                        | 1.42           | 8.68%                |
| 480               | 19.28                                        | 17.97                                        | 1.30           | 7.26%                |

The relative difference is defined by the ratio between the difference of the propagation lengths and the propagation length with consideration of groundwater flow. At the initial stage of injection, i.e., t < 15s, ignoring groundwater flow will overestimate the propagation length over 60%. With increases of propagation length and injection time, such overestimation gradually reduces. However, it remains over 7% of overestimation when the grout stops when t = 480s. The comparison result indicates that it is important to consider the groundwater flow in modeling of silica sol grouting for rock fractures.
5. Concluding remarks
In this work, we present a two-phase flow solution for modeling of silica sol grouting in single rock fractures with consideration of groundwater flow that is often ignored in previous studies. Using the two-phase flow solution, we investigated the importance of groundwater flow in modeling of silica sol grouting.

It is found that the groundwater flow is important in the silica sol grouting for rock fractures. Ignoring groundwater flow will underestimate the pressure at the grout front and overestimate the propagation length. Specifically, ignoring groundwater flow may cause over 60% of errors in prediction of the propagation length at the initial stage, and over 7% when the grout stops. Therefore, in practice, it is very important to consider the groundwater flow in design of silica sol grouting for rock fractures to predict the propagation lengths more precisely.

We considered one-dimensional two-phase flow in single idealized planar fractures analogy to previous analytical studies. However, natural rock fractures consist of complex rough surfaces, which will affect the flow process in realistic rock fractures [18-20]. The impact of surface roughness on silica sol grouting remains an important topic for the future study.

References
[1] Hässler L. 1991 Grouting of rock - Simulation and Classification. PhD Thesis. Royal Institute of Technology, Stockholm, Sweden.
[2] Hässler L., Håkansson U, Stille H. 1992 Computer-simulated flow of grouts in jointed rock. Tunnelling and Underground Space Technology. 7: 441–446.
[3] Eriksson M., H. Stille, J. Andersson, 2000 Numerical calculations for prediction of grout spread with account for filtration and varying aperture. Tunnelling and Underground Space Technology. 15:353-364.
[4] Lombardi G. 1985 The role of cohesion in cement grouting of rock. Commission International, Lusanne, Switzerland.
[5] Funehag J., Gustafson G., 2008 Design of grouting with silica sol in hard rock – New methods for calculation of penetration length, Part I, Tunnelling and Underground Space Technology, 23(1): 1-8.
[6] Funehag J, Thörn J. 2014 Fundamentals around grouting and penetration of grout. In: Proceedings of the Rock Mechanics Meeting - Rock Engineering Research Foundation, p. 105–114.
[7] Gustafson G, Claesson J, Fransson Å. 2013 Steering Parameters for Rock Grouting. Journal of Applied Mathematics. p. 1-9.
[8] Gustafson G, Stille H. 2005 Stop criteria for cement grouting. Felsbau. 25:62–68.
[9] Kobayashi S, Stille H, Gustafson G, Stille B. 2008 Real time grouting control method: development and application using Åspö HRL data. R-08-133, Swedish Nuclear Fuel and Waste Management Company, Stockholm, Sweden.
[10] Stille H. 2015 Rock Grouting-Theories and Applications. Vulkan Förlag Stockholm.
[11] Zou L, Håkansson U, Cvetkovic V 2018 Modeling of rock grouting in saturated variable aperture fractures, In: Proceedings of Bergdagarna 2018, Stockholm, Sweden.
[12] Zou L, Håkansson U, Cvetkovic V. 2018 Two-phase cement grout propagation in homogeneous water-saturated rock fractures. International Journal of Rock Mechanics and Mining Sciences, 106: 243–249.
[13] Zou L, Håkansson U, Cvetkovic V. 2019 Cement grout propagation in two-dimensional fracture networks: Impact of structure and hydraulic variability. International Journal of Rock Mechanics and Mining Sciences, 115: 1-10.
[14] Zou L, Håkansson U, Cvetkovic V. 2020 Yield-power-law fluid propagation in water-saturated fracture networks with application to rock grouting, Tunnelling and Underground Space Technology 95, 103170.
[15] Zou L, Håkansson U, Cvetkovic V. 2020 Radial propagation of yield-power-law grouts into water-saturated homogeneous fractures. International Journal of Rock Mechanics and Mining Sciences, 115: 1-11.
[16] Snow DT. 1965 A parallel plate model of fractured permeable media. PhD thesis. Univ. of Calif., Berkeley.

[17] Dormand JR and Prince PJ, 1980 A family of embedded Runge-Kutta formulae. J. Comp. Appl. Math., 6: 19–26.

[18] Zou L, Häkansson U, Cvetkovic V 2019 Non-Newtonian grout flow in single rough-walled rock fractures, In: Proceedings of Bergdagarna 2019, Stockholm, Sweden.

[19] Zou L, Jing L., Cvetkovic V. 2015 Roughness decomposition and nonlinear fluid flow in a single rock fracture. International Journal of Rock Mechanics and Mining Sciences, 75: 102-118.

[20] Zou L, Jing L., Cvetkovic V. 2017 Shear enhanced nonlinear flow in rough-walled rock fractures. International Journal of Rock Mechanics and Mining Sciences, 97: 33-45.