Phenomenological quark-lepton mass relations and neutrino mass estimations

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Abstract

Based on the experimental data and estimations of the charged leptons and quarks masses, a close power law with exponent 3/4 has been found, connecting charged leptons masses and up quarks masses. A similar mass relation has been suggested for the masses of neutral leptons and down quarks. The latter mass relation and the results of the solar and atmospheric neutrino experiments have been used for prediction of neutrino masses. The obtained masses of $\nu_e$, $\nu_\mu$ and $\nu_\tau$ are 0.0003 eV, 0.003 eV and 0.04 eV, respectively. These values are compatible with the recent experimental data and support the normal hierarchy of neutrino masses.

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Key words: mass relation; quark-lepton symmetry; neutrino mass; normal hierarchy

1 Introduction

Decades after the experimental detection of the neutrino [1], it was generally accepted that the neutrino mass $m_{0\nu}$ was rigorously zero. The crucial experiments with the 50 kton neutrino detector Super-Kamiokande found strong evidence for oscillations (and hence - mass) in the atmospheric neutrinos [2]. The direct neutrino measurements allowed to bound the neutrino mass. The upper limit for the mass of the lightest neutrino flavor $\nu_e$ was obtained from experiments for measurement of the high-energy part of the tritium $\beta$-spectrum and recent experiments yielded $\sim 2$ eV upper limit [3, 4]. As a result of the recent experiments, the upper mass limits of $\nu_\mu$ and $\nu_\tau$ are 170 KeV [5] and 18.2 MeV [6], respectively. The Solar neutrino experiments (SNE) and Atmospheric neutrino experiments (ANE) allow to find the square mass difference $\Delta m^2_{12} = m^2_2 - m^2_1$ and $\Delta m^2_{23} = m^2_3 - m^2_2$, but not the absolute value of neutrino masses. The astrophysical constraint of the neutrino mass is $\sum m_\nu < 2$ eV [7]. The recent extensions of the Standard model lead to non-zero neutrino masses, which are within the large range of $10^{-6}$ eV – $10$ eV.

In the classical $SU(5)$ model the mass relations between charged leptons and down quark masses are simply identities: $m_e = m_d$, $m_\mu = m_s$ and $m_\tau = m_b$. The mass relations of Georgi-Jarlskog [8] ensue from the $SO(10)$ model and relate charged leptons and down quark masses: $m_e = m_d/3$, $m_\mu = 3m_s$ and $m_\tau = 3m_b$.
$m_e = m_b$. However, these mass relations deviate several times, compared to experimental data. Moreover, similar mass relations are unsuited for neutral leptons (neutrino) masses.

The seesaw mechanism naturally generates small Majorana neutrino mass $m_\nu$ from reasonable Dirac mass $m_D$ and very heavy Majorana sterile neutrino mass $M_N$, namely $m_\nu \sim \frac{m_D^2}{M_N} \ll m_D$. But there are many seesaw models that differ in the scale $M_N$ and Dirac mass. The Grand unified theories (GUT) are the main candidates for seesaw models, with $M_N$ at or a few orders of magnitude below GUT scale. Successful GUT models should essentially generate Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [9, 10] and Maki-Nakagawa-Sakata (MNS) lepton mixing matrix [11] and predict results compatible with the data from SNE and ANE. Yet, it is admitted that the predictions of the quark-lepton mass spectrum are the least successful aspect of the unified gauge theory [12, 13].

The purpose of this paper is to find simple and reliable quark-lepton mass relations, based on experimental data and estimations for quark and lepton masses. The next step is to estimate neutrino masses by means of these mass relations and data from SNE and ANE.

### 2 Power law approximation for the masses of charged leptons and up quarks

According to the Standard model, the fundamental constituents of the matter are 6 quarks and 6 leptons. The fundamental fermions group in three generations, having similar properties and increasing masses. The three generations of the fundamental fermions and their masses are presented in Table 1. The estimations of quark masses are taken from [14] and the upper mass limits of the neutrino flavors are taken from [3, 4, 5, 6].

| Fermions          | 1st generation | 2nd generation | 3rd generation |
|-------------------|----------------|----------------|---------------|
| Up quarks         | $u$ 3          | $c$ 1.25 $\times 10^3$ | $t$ 1.74 $\times 10^5$ |
| Down quarks       | $d$ 6          | $s$ 122         | $b$ 4.2 $\times 10^3$ |
| Charged leptons   | $e$ 0.511      | $\mu$ 106      | $\tau$ 1.78 $\times 10^3$ |
| Neutral leptons   | $\nu_e < 2 \times 10^{-6}$ | $\nu_\mu < 0.17$ | $\nu_\tau < 18.2$ |

A clear feature of the quark and charged lepton mass spectrum is the hierarchy of masses belonging to different generations:

$$m_u \ll m_c \ll m_t, \ m_d \ll m_s \ll m_b, \ m_e \ll m_\mu \ll m_\tau$$

(1)

Most likely, a similar hierarchy of masses of the neutral leptons (neutrinos) could be anticipate $m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau}$. Based on the experimental data, we search for a simple relation between the masses of charged leptons ($m_{cl}$) and the respective up quarks ($m_{uq}$) by the least squares. Although the linear
regression \( m_{cl} \approx 0.0102 m_{uq} \) eV shows close correlation, it yields electron mass many times lower than the experimental value. After examination of other simple approximations (logarithmic, exponential and power law) we found out that the power law fits best experimental data:

\[
m_{cl} = k_0 m_{uq}^\alpha \text{ eV}
\]

(2)

where \( k_0 = 9.33 \) and \( \alpha = 0.749 \approx \frac{3}{4} \).

Despite the large uncertainty of the \( u^- \) quark mass (from 1.5 MeV to 5 MeV) and \( d^- \) quark mass (from 3 MeV to 9 MeV), the slope remains within the narrow interval from 0.683 to 0.782. Although only three points make the approximation, the correlation coefficient is very high (\( r = 0.993 \)) and the maximal ratio of the predicted mass in relation to the respective experimental value is \( \Delta = m_{pr}/m_{ex} = 1.74 \) (for muon). The predicted masses of the electron and tau lepton differ from the respective experimental values with less than 40%. Therefore, the mass relation (2) could be accepted satisfactory.

3 Mass relation for neutral leptons and down quarks and estimations of neutrino masses

We suggest that a mass relation similar to (2) connects the masses of the neutral leptons \( (m_{nl}) \) and the respective down quarks \( (m_{dq}) \):

\[
m_{nl} = km_{d_q}^\alpha \text{ eV}
\]

(3)

where \( \alpha = 0.749 \approx \frac{3}{4} \) and \( k \) is an unknown constant.

For \( k = k_0 = 9.33 \) formula (3) yields \( m_{\nu_e} \approx 1.13 \text{ MeV}, m_{\nu_\mu} \approx 10.84 \text{ MeV} \) and \( m_{\nu_\tau} \approx 153.66 \text{ MeV} \). These values are several orders of magnitude bigger than the experimental upper limits of the neutrino masses (See Table 1), therefore \( k \ll k_0 \). Astrophysical constraints allow to limit more closely the value of \( k \) since they give \( m_\nu < \sum m_\nu < 2 \text{ eV} \). Thus, from equation (3) we obtain:

\[
k = \frac{m_{\nu_\tau}}{m_b^{\frac{3}{4}}} < \frac{\sum m_\nu}{m_b^{\frac{3}{4}}} \approx 1.21 \times 10^{-7}
\]

(4)

\textit{ANE} \cite{15} determine the squared mass difference:

\[
m_{\nu_\tau}^2 - m_{\nu_\mu}^2 \approx 2.2 \times 10^{-3} \text{ eV}^2
\]

(5)

Relation (5) yields:
\[
\frac{m_{\nu\mu}}{m_{\nu e}} \sim \left(\frac{m_s}{m_d}\right)^{3/4} \approx 9.60
\]  
(6)

\[
\frac{m_{\nu\tau}}{m_{\nu\mu}} \sim \left(\frac{m_b}{m_s}\right)^{3/4} \approx 14.17
\]  
(7)

Solving system (5) - (7) we obtain \(m_{\nu e} \approx 3.4 \times 10^{-4}\) eV, \(m_{\nu\mu} \approx 3.3 \times 10^{-3}\) eV and \(m_{\nu\tau} \approx 4.7 \times 10^{-2}\) eV. These results support the normal hierarchy of neutrino masses.

On the other hand, the Large mixing angle (LMA) of Mikheyev - Smirnov - Wolfenstein (MSW) solution for SNE \[16\] yields:

\[
m^2_{\nu\mu} - m^2_{\nu e} \approx 7.9 \times 10^{-5}\) eV^2
\]  
(8)

This equation, together (6) and (7), yield \(m_{\nu e} \approx 9.3 \times 10^{-4}\) eV, \(m_{\nu\mu} \approx 8.9 \times 10^{-3}\) eV and \(m_{\nu\tau} \approx 0.13\) eV. These values are almost three times bigger than the values obtained by Super Kamiokande data, therefore, they do not fit well with the latter. However, the Small mixing angle (SMA) MSW solution for SNE \[17\] yields:

\[
m^2_{\nu\mu} - m^2_{\nu e} \approx 6 \times 10^{-6}\) eV^2
\]  
(9)

This equation, together (6) and (7), yield \(m_{\nu e} \approx 2.6 \times 10^{-4}\) eV, \(m_{\nu\mu} \approx 2.5 \times 10^{-3}\) eV and \(m_{\nu\tau} \approx 3.4 \times 10^{-2}\) eV. These values differ by less than 25% from the results obtained above by Super Kamiokande data, which show that according to the suggested approach, SMA MSW solution fits better with the ANE than LMA MSW.

Thus, the obtained quark-lepton mass relations and the results of the solar and atmospheric neutrino experiments provide to estimate the masses of \(\nu_e, \nu_\mu\) and \(\nu_\tau\) of \((2.6 \div 3.4) \times 10^{-4}\) eV, \((2.5 \div 3.3) \times 10^{-3}\) eV and \((3.4 \div 4.7) \times 10^{-2}\) eV, respectively. These values are close to the neutrino masses \((2.1 \times 10^{-4}\) eV, \(2.5 \times 10^{-3}\) eV and \(5.0 \times 10^{-2}\) eV) found in \[18\] by the mass relation, connecting the masses of four stable particles and the coupling constants of the fundamental interactions.

We could calculate constant \(k\) using the most trustworthy data for neutrinos and down quarks masses, namely \(\nu_\tau\) and \(b\)-quark masses:

\[
k = \frac{m_{\nu\tau}}{m_b^{3/4}} \approx 2.42 \times 10^{-9}
\]  
(10)
Figure 1: Mass relations for the charged leptons and up quarks masses (thick solid line) and for the neutral leptons and down quarks masses (thin solid line). The dashed line shows the upper limit of the neutrino masses obtained by astrophysical constraints.

The obtained mass relations (2) and (3) are shown in Fig. 1. It shows that the neutrino masses estimated by SMA MSW are close to the neutrino masses estimated by ANE, i.e. two sets of estimations are compatible.

The attempt to relate the masses of the charged leptons with the masses of the down quarks and the masses of neutral leptons with the masses of up quarks did not yield satisfactory results since the data from SNE and ANE did not fit within the framework of the suggested approach. Besides, the respective mass relations predict the muon mass which is nearly three times less than the experimental value (see Table 2) and an electron neutrino mass less than $10^{-7}$ eV.

Table 2: Masses of charged leptons calculated by various approaches and experimental values (MeV).

| Model  | SU(5) | SO(10) | Power law (Down) | Linear (Up) | Power law (Up) | Exp. data |
|--------|-------|--------|-----------------|-------------|----------------|-----------|
| Electron | 6     | 2      | 0.905           | 0.031       | 0.663          | 0.511     |
| Muon   | 122   | 366    | 36.9            | 12.8        | 60.7           | 105.7     |
| Tau    | 4200  | 4200   | 2877            | 1775        | 2449           | 1777      |
| Δ max  | 11.74 | 3.91   | 2.86            | 16.48       | 1.74           | 1         |
Table 2 shows the masses of charged leptons calculated by different approaches and experimental values. The last row of the table shows the maximal ratio (deviation) $\Delta_{\text{max}}$ of the masses predicted by the respective approach in relation to the experimental values $\Delta = m_{\text{pr}}/m_{\text{ex}}$. Clearly, the power law relating to the masses of charged leptons and up quarks fits best experimental data.

4 Conclusions

Based on the experimental data and estimations of charged leptons and quarks masses, a power law with exponent $3/4$ has been found, connecting charged leptons masses and up quarks masses. It has been shown that this approximation is considerably better than any known approach. A similar mass relation has been suggested for neutral leptons and down quarks. The latter mass relation and the results of ANE and SNE have been used for estimations of neutrino masses. The values of neutrino masses obtained by ANE are close to the ones, obtained by the SMA MSW solution. The masses of $\nu_e$, $\nu_\mu$ and $\nu_\tau$ are estimated to $(2.6 \div 3.4) \times 10^{-3}$ eV, $(2.5 \div 3.3) \times 10^{-3}$ eV and $(3.4 \div 4.7) \times 10^{-2}$ eV, respectively, and they support the normal hierarchy of neutrino masses.

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