Optimal decision under fuzzy uncertainty for order allocation planning with uncertain parameters: single period case

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Abstract. Inventory management on manufacturer and retail industries is commonly containing some uncertain parameters like future cost and future demand value where the decision-maker has to execute the optimal decision under these parameter’s uncertainty. While the data is available then these uncertain parameters can be assigned as a random variable and a probabilistic approach can be used to solve. But, if there is no data available for these uncertain parameters, then a fuzzy uncertainty approach can be used to solve this problem that is discussed in this paper. We have developed a mathematical model that can be used to calculate the optimal decision under fuzzy uncertainty where the decision is calculated and executed before the uncertain parameters are revealed. Numerical experiment results show that the problem was solved by using the proposed mathematical model, the optimal decision was achieved i.e. the optimal procurement volume product type and the optimal stored product volume in the inventory.

1. Introduction
There are many parameter values in supply chain which are unknown. For example, future product price and future demand value are commonly unknown. To handle some unknown parameters, usually those will be assigned as some, some historical data are needed to build the corresponding random variables and stochastic optimization is commonly used to optimize. However, to be assigned as a random variable probability distribution function. If there is no data, then, another approach has to be used in order to handle this uncertainty. As an alternative, fuzzy uncertainty can be used which will be discussed in this article.

The most used approaches to optimize existing components in supply chain management is mathematical optimization model by minimizing/maximizing some objective function(s) and taking account the conditions to be hold as the constraints. And, by using an optimization algorithm/method, the optimal decision is calculated. Due to the whole components on a supply chain is excessive, many researchers/practitioners are optimizing part by part of them. In the upper side of the chain, supplier selection and raw material/product order allocation problems are contributing significantly to the spent cost. There are many mathematical models developed in recent years to solve the supplier selection and order allocation problems. Some examples are explained as follows. A mixed-integer linear optimization model was developed in [1], [2] to determine the optimal decision for a dynamic supplier selection problem, an integer programming model for supplier selection considering discount which was developed in [3], a model for supplier selection under disruption risks [4], and a model which a quantity discount and fast service are considered in the problem [5]. Some application researches are also found in many industrial fields like rubber industry [6], logistics services [7], cement industry [8], thermal
power plant [9], and textile industry [10]. The cited models above are suitable for deterministic model where all parameters are known with certainty. For supplier selection and order allocation problem with uncertain parameters, a model was developed in [11] by using probabilistic theory approach where the solution was calculated by using stochastic mathematical programming.

In uncertainty theory, a one-step more general from probabilistic theory which was firstly developed in [12], is called as fuzzy theory and it was extended to possibility theory. Next, an concept of expected value of a fuzzy variable is developed and it was used in fuzzy programming to solve an optimization problem contained fuzzy parameter [13]. Many research articles can be found which were discussed about the application of fuzzy programming as found in [14]–[16] where it was found a discussion about data envelopment analysis based on fuzzy optimization. Other examples are also found, e.g. application of fuzzy programming in a grinding process [17], [18], portfolio optimization [19]–[21], mobile health and wellness program optimization [22], groundwater resources optimization [23], electric vehicle parking problem [24], and many more.

In this paper, we propose a newly mathematical model developed from an existing one by assigning some new parameters like cost and demand as fuzzy number/parameter and modified the model so that it could be used to determine the optimal decision under the fuzzy uncertainty on these parameters while in the previous models, these parameters were certainly known. Handling some uncertain parameters is important due to, in fact, many parameters in the real world problems are commonly uncertain. For the model proposed in this paper, we restrict ourselves to single period case so that the model would not be too complex. However, this is needed to maintain our understanding to the problem as a foundation for more complex problems. Finally, to illustrate how the optimal decision would be achieved, a computational experiment will be discussed in the numerical simulation section.

2. Problem and mathematical model

2.1. Problem definition

Let a manufacturing/retail industry face an order allocation problem with multi-product type and multi-supplier where the decision maker has to decide the amount of each product type from each supplier based on the conditions that are to be hold. First, there are demand, used for production or selling to buyer, that to be satisfied. But, in this case, the demand value is uncertain. The decision maker buys the product from supplier with some unit price where this unit price is assumed to not known yet while the computation is in process. The transporting cost from the supplier to the manufacture is also unknown. Number of products ordered from a supplier is may not fully received since there are defected product and late delivered product with some uncertain rate or percentage. These uncertain/unknown parameters are assumed as fuzzy variables with some known discrete membership function decided by the decision maker. Then, the decision maker faces a problem to determine the optimal number of each product that should be ordered from each supplier under these conditions.

2.2. Mathematical notations

We define the following mathematical notations for mathematical modeling, where the fuzzy parameters are marked by upper hat:

indices:
- \( S \) : supplier's index;
- \( P \) : product type;

decision variables:
- \( X_{sp} \) : The amount (unit) of product type \( p \) allocated to be ordered to supplier \( s \);
- \( Y_s \) : Auxuliary binary decision variable (valued to be 1 if there is product ordered to supplier \( s \), 0 for otherwise);
- \( X_p \) : The amount of recourse product \( p \);
fuzzy parameters:

- \( UP_{sp} \): Fuzzy number presenting unit price supplier cost to transport ordered product at supplier \( s \);
- \( TC_s \): Fuzzy number presenting cost to transport ordered product at supplier \( s \);
- \( DE_p \): Fuzzy number presenting demand value for product \( p \);
- \( DR_{sp} \): Fuzzy number presenting rate for defected amount of product \( p \) ordered to supplier \( s \);
- \( LR_{sp} \): Fuzzy number presenting rate for late delivered amount of product \( p \) ordered to supplier \( s \);

crisp parameters:

- \( DC_{sp} \): Penalty cost per unit for defected product \( p \) ordered to supplier \( s \);
- \( LC_{sp} \): Penalty cost per unit for late delivered product \( p \) ordered to supplier \( s \);
- \( RC_p \): Recourse cost for product \( p \);
- \( SC_p \): Supplier's maximum capacity of \( s \) to supply product \( p \).

2.3. Mathematical model

The decision variable \( X_{sp} \) is executed before the uncertain parameters are revealed. It means there is possibility that the ordered product from all suppliers will not satisfy the demand after the demand value is revealed. If so, we define the decision variable \( X_{sp} \) as recourse product volume in order to satisfy the demand. This recourse product volume is procured by recourse cost although the decision maker may not to buy the recourse product volume which means that the shortage product will be ignored. The objective function is defined as the fuzzy expectation of all cost occurred in the problem. There are five cost components which are purchasing cost, transport cost, defected product cost, late delivered product cost and recourse cost. We have formulated the model to minimize the fuzzy expectation value of the total cost as follows:

\[
\min Z = E\left[ \sum_{s=1}^{S} \sum_{p=1}^{P} X_{sp} \cdot UP_{sp} + \sum_{s=1}^{S} TC_s \cdot Y_s + \sum_{s=1}^{S} \sum_{p=1}^{P} DR_{sp} \cdot DC_{sp} \cdot X_{sp} \right. \\
\left. + \sum_{s=1}^{S} \sum_{p=1}^{P} LR_{sp} \cdot LC_{sp} \cdot X_{sp} \right] + X_p \cdot RC_p
\]

subject to:

\[
\sum_{s=1}^{S} X_{sp} - \sum_{s=1}^{S} DR_{sp} \cdot X_{sp} - \sum_{s=1}^{S} LR_{sp} \cdot X_{sp} + X_p \geq DE_p, \forall p;
\]

\[
Y_s = \begin{cases} 
1, & \text{if } \sum_{p=1}^{P} X_{s, p} > 0, \forall s \in S; \\
0, & \text{else.}
\end{cases}
\]

\[
X_{sp} \leq SC_p, \forall s, \forall p;
\]

\[
X_{sp}, X_p \geq 0 \text{ and integer, } \forall s, \forall p;
\]

where the constraints are explained as follows: constraint (2) means that the total product volume ordered from all suppliers minus the defected product minus the late delivered product plus the recourse product volume satisfies the demand value for each product type, constraint (3) is used to determine whether a supplier is chosen to supply some product or not (1 means chosen, 0 means not), constraint (4) means that the ordered product volume should not exceeding the supplier’s maximum capacity to
supply the corresponding product, and constraint (5) is used to assign the non-negativity and integer value for all decision variables.

Optimization problem (1) has a linear objective function where the constraints are linear with a binary constraint, then this optimization problem is an integer linear programming. Since the feasible solution set is closed and bounded, furthermore it is finite, then the optimal solution is always exists provided by it is not empty.

2.4. Optimization method for fuzzy uncertainty based programming

Let \( \xi = [\xi_1, \xi_2, \ldots, \xi_p] \) is a vector of fuzzy variable \( \xi_i \) with known membership function. The expectation value of \( \xi_i \), denoted by \( \hat{E}[\xi_i] \) is defined by [25]

\[
\hat{E}[\xi_i] = \int_0^\infty Cr \{ \xi_i \geq r \} dr - \int_0^\infty Cr \{ \xi_i \leq r \} dr
\]

(6)

where \( Cr[\cdot] \) is credibility value and (6) is subjected to at least one of the integral terms has finite value.

The expected value of a discrete fuzzy variable \( \tilde{\xi} \) with the following membership function

\[
\mu_{\tilde{\xi}} = \begin{cases} 
\mu_1, & \tilde{\xi} = \tilde{\xi}^{(1)}; \\
\mu_2, & \tilde{\xi} = \tilde{\xi}^{(2)}; \\
\vdots & \\
\mu_q, & \tilde{\xi} = \tilde{\xi}^{(q)}; \\
0, & \text{others}
\end{cases}
\]

(7)

where \( \tilde{\xi}^{(1)} < \tilde{\xi}^{(2)} < \ldots < \tilde{\xi}^{(q)} \) distinct is \( \hat{E}[\tilde{\xi}] = \sum_{i=1}^q w_i \cdot \tilde{\xi}^{(i)} \) where

\[
w_i = \frac{1}{2} \left( \max_{1 \leq j \leq i} \mu_j + \max_{i \leq j \leq m} \mu_j - \max_{1 \leq j \leq i} \mu_j - \max_{1 \leq j \leq m} \mu_j \right)
\]

for \( i = 1, 2, \ldots, q \). Furthermore, for any two independent fuzzy variables \( \xi_i \) and \( \xi_j \), and arbitrary two real numbers \( a \) and \( b \),

\[
\hat{E}[a\xi_i + b\xi_j] = a\hat{E}[\xi_i] + b\hat{E}[\xi_j].
\]

(8)

Fuzzy uncertainty-based programming discussed in this paper contains a problem of minimizing (or maximizing) an objective function of decision variable and fuzzy number as parameter subject to some constraint functions. Let \( x \in \mathbb{R}^n \) denotes vector of decision variable, \( \xi \) denotes a vector of fuzzy number. Let \( f(x, \xi) \) be the objective function and \( g_i(x, \xi), i = 1, 2, \ldots, m \) be the constraint functions. This optimization problem in general form is

\[
\left\{ \begin{array}{l}
\min f(x, \xi) \\
\text{s.t. } g_i(x, \xi) \geq 0, i = 1, 2, \ldots, m.
\end{array} \right.
\]

(9)

Fuzzy expected value based method solves (9) by minimizing the expectation value of the objective function defined as [25]

\[
\left\{ \begin{array}{l}
\min \hat{E}[f(x, \xi)] \\
\text{s.t. } \hat{E}[g_i(x, \xi)] \geq 0, i = 1, 2, \ldots, p.
\end{array} \right.
\]
3. Numerical experiment

For numerical experiment, we suppose a manufacturer that face the order allocation problem of product type P1, P2 with three supplier alternatives S1, S2, S3. Let the membership function of the fuzzy variables occurred in (1) are defined as follows:

\[
\mu_{UP_{sp}} = \begin{cases} 
\mu^{(i)}_{UP_{sp}}, & \text{if } UP_{sp} = UP^{(i)}_{sp} \\
0, & \text{otherwise}
\end{cases}
\]

\[
\mu_{TC_{i}} = \begin{cases} 
\mu^{(i)}_{TC_{i}}, & \text{if } TC_{i} = TC^{(i)}_{i} \\
0, & \text{otherwise}
\end{cases}
\]

\[
\mu_{DE_{p}} = \begin{cases} 
\mu^{(i)}_{DE_{p}}, & \text{if } DE_{p} = DE^{(i)}_{p} \\
0, & \text{otherwise}
\end{cases}
\]

\[
\mu_{DR_{sp}} = \begin{cases} 
\mu^{(i)}_{DR_{sp}}, & \text{if } DR_{sp} = DR^{(i)}_{sp} \\
0, & \text{otherwise}
\end{cases}
\]

\[
\mu_{LR_{sp}} = \begin{cases} 
\mu^{(i)}_{LR_{sp}}, & \text{if } LR_{sp} = LR^{(i)}_{sp} \\
0, & \text{otherwise}
\end{cases}
\]

(10)

where the values of \(\mu^{(i)}_{UP_{sp}}, \mu^{(i)}_{TC_{i}}, \mu^{(i)}_{DE_{p}}, \mu^{(i)}_{DR_{sp}}, \mu^{(i)}_{LR_{sp}}\), for all \(i\), and remain parameters are displayed in Table A.1 to Table A.8. The data in those all tables were randomly generated for this computational experiment purpose. In the real world problem, those data, in fact, will be generated by the decision-maker following his/her experience and his/her sense or feeling to solve the problem under uncertainties. The weight value for \(w_{UP_{sp}}, w_{TC_{i}}, w_{DE_{p}}, w_{DR_{sp}}, w_{LR_{sp}}\) are \(w^{(i)}_{UP_{sp}}, w^{(i)}_{TC_{i}}, w^{(i)}_{DE_{p}}, w^{(i)}_{DR_{sp}}, w^{(i)}_{LR_{sp}}\) respectively (shown in the appendices). Fuzzy optimization (1) was solved in LINGO® 18.0 optimization software. The optimal decision for this problem is shown in Figure 1.

![Figure 1. Optimal amount of the product ordered from all suppliers for each product](image-url)
The optimal objective function value i.e. the fuzzy expected total cost was 15439. The number of the core decision variables is 16 where the number of the deterministic equivalent model has 50000 decision variables. The constraint number for the core model is 22 linear constraints whereas the deterministic equivalent model has 93742 constraints. The feasible solution set is non-empty and thus the optimal solution exists.

The optimal order allocation, \( X_{sp} \) for all product types to all suppliers, is shown in Fig. 1. It can be seen that the decision maker shall order 500 units of product P1 and 400 units of product P2 from supplier S1, 118 units of product P1 from supplier S2, 18 units of product P2 from supplier S3. There are no units of product P2 ordered from supplier S2 and there are no units of product P1 ordered from supplier S3. These allocations will give the minimal fuzzy expected value of the total cost occurred in the problem.

4. Concluding remarks and future research direction

In this article, a mathematical optimization model for product order allocation with fuzzy parameters was considered. The objective function was formulated as a fuzzy expected value of the total cost whereas the main constraint is the demand satisfaction. The model can be used to determine the optimal product amount of each product type ordered from each supplier alternatives. Numerical experiments were conducted and from the results the order allocation was achieved.

Our next works will develop the model so that it can be used for multi-period case where the decision maker has warehouse system to store the product that can be used for future periods. Furthermore, some metaheuristics optimization methods will be employed to solve the model in a large scale problem.

Appendices

**Table A1. Parameter value for \( UP_{sp} \)**

| \( i \) | \( UP_{sp} \) | Membership value | \( W_{UP_{sp}}^{(i)} \) |
|---|---|---|---|
| 1 | 12 | 0,2 | 0,1 |
| 2 | 13 | 0,4 | 0,1 |
| 3 | 14 | 1 | 0,4 |
| 4 | 15 | 0,8 | 0,35 |
| 5 | 16 | 0,1 | 0,05 |

**Table A2. Parameter value for \( DR_{sp} \)**

| \( i \) | \( DR_{sp}^{(i)} \) | Membership value | \( W_{DR_{sp}}^{(i)} \) |
|---|---|---|---|
| 1 | 0,02 | 0,1 | 0,05 |
| 2 | 0,04 | 0,5 | 0,2 |
| 3 | 0,06 | 0,8 | 0,15 |
| 4 | 0,08 | 1 | 0,55 |
| 5 | 0,10 | 0,1 | 0,05 |
Table A3. Parameter value for $LR_{sp}^{(i)}$

| $i$ | $LR_{sp}^{(i)}$ | Membership value | $LR_{sp}^{(i)}$ |
|-----|-----------------|------------------|-----------------|
| 1   | 0.01            | 0.8              | 0.4             |
| 2   | 0.02            | 1                | 0.15            |
| 3   | 0.03            | 0.9              | 0.25            |
| 4   | 0.04            | 0.4              | 0.05            |
| 5   | 0.05            | 0.3              | 0.15            |

Table A4. Parameter value for $TC_{s}^{(i)}$

| $i$ | TCs  | Membership value | $TC_{s}^{(i)}$ |
|-----|------|------------------|-----------------|
| 1   | 50   | 0.4              | 0.2             |
| 2   | 55   | 0.5              | 0.05            |
| 3   | 60   | 1                | 0.4             |
| 4   | 65   | 0.7              | 0.1             |
| 5   | 70   | 0.5              | 0.25            |

Table A5. Parameter value for $DE_{p}^{(i)}$

| $i$ | $DE_{p}^{(i)}$ | Membership value | $DE_{p}^{(i)}$ |
|-----|----------------|------------------|----------------|
| 1   | 400            | 0.4              | 0.2            |
| 2   | 450            | 0.5              | 0.05           |
| 3   | 500            | 1                | 0.5            |
| 4   | 550            | 0.5              | 0.15           |
| 5   | 600            | 0.2              | 0.1            |

Table A6. Penalty cost for defected product per unit

| Supplier | Products | P1 | P2 |
|----------|----------|----|----|
| S1       |          | 0.50| 0.50|
| S2       |          | 0.75| 1.00|
| S3       |          | 0.75| 0.25|

Table A7. Penalty cost for late delivered product per unit

| Supplier | Products | P1 | P2 |
|----------|----------|----|----|
| S1       |          | 0.75| 0.1 |
| S2       |          | 0.5 | 0.85|
| S3       |          | 0.85| 1   |
Table A8. Supplier maximum capacity

| Supplier | Products |   |   |
|----------|----------|---|---|
|          | P1       | P2 |   |
| S1       | 500      | 400|   |
| S2       | 400      | 600|   |
| S3       | 800      | 1000|  |

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