Spontaneous CP violation on the lattice

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At finite temperatures around the electroweak phase transition, the thermodynamics of the MSSM can be described by a three-dimensional two Higgs doublet effective theory. This effective theory has a phase where CP is spontaneously violated. We study spontaneous CP violation with non-perturbative lattice simulations, and analyse whether one could end up in this phase for any physical MSSM parameter values.

1. INTRODUCTION

The physics problem we address is CP violation around the finite temperature MSSM electroweak phase transition. CP violation is one of the requirements for baryogenesis, and it is thus of interest to ask whether it would be available to a sufficient extent in the MSSM, where the phase transition can be strong enough to account for the non-equilibrium constraint\cite{1,2}.

In particular, we are here interested in the phenomenon of spontaneous CP violation. Spontaneous CP violation can in principle take place in any two Higgs doublet model\cite{3}, but for the physical MSSM parameters it cannot be realized at $T = 0$\cite{4}. However, it has been suggested that it might be more easily realized at finite temperatures\cite{5}, or even only in the phase boundary between the symmetric and broken phases\cite{6,7}.

As described in\cite{8}, this questions can be studied with 3d effective fields theories and non-perturbative lattice simulations. More specifically, we report here on preliminary results concerning the following two questions:

1. Is there a homogeneous thermodynamical phase in the MSSM for some parameter values and temperatures, where CP is spontaneously violated?
2. Could CP also be violated at the phase boundary between the usual symmetric and CP conserving broken phases?

2. EFFECTIVE THEORY

Let $O_a = H^*_a H_a$, $a = 1, 2$, and $M = H^*_1 \tilde{H}_2$, $\tilde{H}_2 = i \sigma_2 H^*_2$. The theory we consider here is

$$L_{3d} = \frac{1}{4} F_{ij}^a F_{ij}^a + \sum_{a=1}^2 \left[ (D_i H_a)^\dagger (D_i H_a) + m_a^2 O_a + \lambda_a^2 O_a^2 \right] + \lambda_4 M^\dagger M + \lambda_5 M^2 + (\lambda_6 O_1 + \lambda_7 O_2) M + \text{H.c.}$$  \hspace{1cm} (1)

This theory is an effective theory for finite $T$ MSSM if the right-handed stop mass is not smaller than the top mass. The opposite case leading to a strong transition, necessitates a dynamical SU(3) stop triplet field, but we defer that discussion to a future publication — the results do not differ qualitatively from the present ones.

We have also neglected the dynamical effects of the hypercharge U(1) subgroup.

As described in\cite{8}, this theory can be parametrized as

$$H_1 = \frac{v_1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad H_2 = \frac{v_2}{\sqrt{2}} \begin{pmatrix} \cos \theta e^{i \phi} \\ \sin \theta \end{pmatrix}.$$  \hspace{1cm} (2)

It turns out that in the case relevant to us, $\lambda_4 - 2 \lambda_5 < 0$, the angle $\theta$ settles dynamically to zero, so that we do not consider it any more.

For real parameters, this theory is even under both of the discrete symmetries C, P. The C symmetry, $H_a \rightarrow H_a^*$, corresponds to $\phi \rightarrow -\phi$. While parity is not spontaneously broken in this theory, the C symmetry can be, thus violating also CP\cite{8}. CP violation is signalled by $|\cos \phi| < 1$. 

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3. TREE-LEVEL CRITERIA

The region of the parameter space where there is CP violation can be found by minimizing the effective potential of the theory in Eq. (1) with respect to $\phi$. We do this first at the tree-level.

First of all, it is easy to understand that to get $|\cos \phi| < 1$, one needs

$$\lambda_4 - 2\lambda_5 < 0, \quad \lambda_5 > 0.$$  \hspace{1cm} (3)

The first condition is automatically satisfied in, say, the MSSM where $\lambda_4 \approx -g^2/2$. The second condition can also be satisfied: $\lambda_5 > 0$ (as well as $\lambda_6, \lambda_7 \neq 0$) can be generated radiatively. The $\lambda_5$ generated is typically very small, though, $\lambda_5 \sim 0.01$. In fact, the stability of the 3d theory itself sets an upper bound: $\lambda_5 > \sim 0.03$ would make the theory unbounded from below.

For the mass parameters, there are essentially two conditions to be satisfied. The first is rather easy to understand: denoting $m^2_{12} = m^2_{12} + \frac{1}{2} \lambda_6 v_1^2 + \frac{1}{2} \lambda_7 v_2^2$, we note that there is a non-trivial minimum for $\phi$, if

$$\frac{|M^2_{12}|}{2\lambda_5 v_1 v_2} < 1.$$  \hspace{1cm} (4)

Since $\lambda_5$, as well as $\lambda_6, \lambda_7$, are small, this essentially means that $m^2_{12}$ should be very small compared with $v^2 = v_1^2 + v_2^2$.

The second condition is less obvious, but in fact a stronger one. Indeed, one should take into account that $v_1, v_2$ in Eq. (4) are not free parameters but are determined by the equations of motion. As was argued in [3], this leads at the tree-level to the constraint

$$m_1^2 + m_2^2 \lesssim 2\lambda_5 v^2.$$  \hspace{1cm} (5)

The aim of this study is to see how these conditions get modified by radiative corrections, first at 1-loop level, and then in the full non-perturbative system. We can then compare, e.g., with the values of $m^2_{12}, m_1^2 + m_2^2$ allowed by the MSSM.

4. 1-LOOP RESULTS

Adding to the tree-level potential the dominant 1-loop corrections from the vector bosons, the parameter space leading to CP violation can still be completely mapped out. We show a projection of this region in Fig. 1. In this figure, we have imposed an upper bound $v/T \lesssim 3$, since it is only such values which are realized around the electroweak phase transition, and also since otherwise the finite temperature expansion used in the construction of the effective theory breaks down.

After the constraint on $v/T$, $\lambda_5$ is restricted to small values, $\lambda_5 \lesssim 0.02$. We observe that $M^2_{12}$ is small compared with $v^2$, as required by Eq. (4). The upper bound on $m_1^2 + m_2^2$ given by Eq. (5), on the other hand, gets modified by numerical factors, but not in order of magnitude.

5. COMPARISON WITH LATTICE

Finally, we wish to compare the perturbative results in Fig. 1 with non-perturbative lattice results. In order to do so, we choose a particular way of increasing $m_1^2 + m_2^2$ in a way that we cross the boundary of the CP violating phase. We then wish to see whether larger values are allowed in the full system than in Fig. 1.

The operators we measure on the lattice are $H^\dagger_a H_a$ and $R = \text{Re} H^\dagger_1 H_2, I = \text{Im} H^\dagger_1 H_2$. Here $I$ is the order parameter for CP violation.

Typical results from lattice simulations at small volumes are shown in Fig. 2. We observe that the location of the transition does not change signif-
significantly from the perturbative value $\sim 0.015$ for these parameters. We thus conclude that perturbation theory is relatively reliable, and Fig. 1 is quite a satisfactory approximation for the part of the parameter space leading to CP violation.

6. IMPLICATIONS FOR THE MSSM

We can now compare the region leading to CP violation with that allowed by the finite $T$ MSSM. Consider, in particular, Eqs. (4), (5), or the corresponding projections in Fig. 1.

First of all, it turns out that a small $m_{12}$ as required by Eq. (4) can be obtained more easily at finite $T$ than at $T = 0$. Thus, it seems that spontaneous CP violation could be possible.

Unfortunately, the other constraint, Eq. (5), is stronger and works in the opposite direction. Indeed, at finite $T$ in the MSSM (see, e.g., Fig. 1),

$$m_1^2 + m_2^2 \approx m_A^2 + 0.53 T^2 + ...$$

(6)

where the remainder is positive. Thus, at temperatures below $\sim 100$ GeV relevant for the electroweak phase transition, $(m_1^2 + m_2^2)/T^2 \gtrsim 1.2$ for $m_A \gtrsim 80$ GeV. This does not agree with Eq. (4), nor overlap with the region shown in Fig. 1.

7. OUTLOOK

We have addressed the first of the questions of the Introduction, and found a negative answer: one does not end up in a phase where CP is spontaneously violated in the MSSM. A light stop changes some aspects of the analysis, but the conclusion seems to remain the same.

Let us then consider the second question. Now, the analysis above indicated that spontaneous CP violation is always more likely at large vevs, cf. Eqs. (4), (5). Thus it seems quite unlikely that CP would be violated in the phase boundary, if it is not violated in the broken phase. As we have found that the broken phase of the MSSM is always CP symmetric, the answer to the latter question seems also to be negative. A solution of the perturbative equations of motion within the phase boundary points in the same direction.

This issue, as well as other related problems, can of course still be studied with lattice simulations. Work is in progress (see also [10]).

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