Self-Duality of Super D3-brane Action on $AdS_5 \times S^5$ Background

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Abstract

We show that the super D3-brane action on $AdS_5 \times S^5$ background recently constructed by Metsaev and Tseytlin is exactly invariant under the combination of the electric-magnetic duality transformation of the worldvolume gauge field and the $SO(2)$ rotation of $N = 2$ spinor coordinates. The action is shown to satisfy the Gaillard-Zumino duality condition, which is a necessary and sufficient condition for a action to be self-dual. Our proof needs no gauge fixing for the $\kappa$-symmetry.
1 Introduction

Recently much attention is concentrated on the connection between string theory and M-theory on \( AdS_{p+2} \times S^{D-p-2} \) and extended superconformal theories in \( p + 1 \) spacetime dimensions \([1, 2, 3]\). \( AdS_5 \times S^5 \) describes a maximally supersymmetric vacuum (besides flat space) of type IIB supergravity and is the near horizon geometry of the D3-brane solution. \( AdS_7 \times S^4 \) and \( AdS_5 \times S^5 \) are maximally supersymmetric vacua of 11-dimensional supergravity and are the near horizon geometries of the M2- and M5-branes, respectively.

The actions of super \( p \)-branes in these near horizon background are described by a Dirac-Born-Infeld (DBI) type term and a Wess-Zumino term, and are invariant under local diffeomorphisms of the worldvolume, the \( \kappa \)-symmetry and the global isometries of the background.

Recently the super \( p \)-brane actions on \( AdS_{p+2} \times S^{D-p-2} \) background geometries have been constructed for the superstring \([4, 5, 6, 7, 8]\) and the D3-brane \([9]\) on the \( AdS_5 \times S^5 \) background, for the M2-brane on the \( AdS_4 \times S^7 \) solution \([10, 11]\) and M5-brane on the \( AdS_7 \times S^4 \) solution \([12]\).

On the other hand electric-magnetic duality relations among various super \( p \)-brane effective actions on the flat background have been found in semiclassical approximation \([13, 14]\) and for some cases on quantum level \([15, 16, 17]\). It will be of great interest to investigate whether these duality relations hold valid for super \( p \)-brane actions on \( AdS \) background. In some recent literatures the quantum equivalence of super D-string and F-string on \( AdS_5 \times S^5 \) and the semiclassical self-duality of super D3-brane on \( AdS_5 \times S^5 \) have already been discussed \([18, 19]\). However in these works proofs of the self-duality relied on a specific gauge choice for the \( \kappa \)-symmetry.

The purpose of the present paper is to show that the super D3-brane action on \( AdS_5 \times S^5 \) constructed by Metsaev and Tseytlin \([9]\) is exactly self-dual. The action is shown to satisfy the Gaillard-Zumino(GZ) duality condition \([20, 21, 22]\). Igarashi et.al. have shown that the GZ-condition is actually necessary and sufficient condition for the theory to be self-dual \([23]\) and they have shown that the super D3-brane action on the flat background indeed satisfies the GZ-duality condition \([17]\).

This paper is organized as follows. In the next section a brief review of the super D3-brane action on \( AdS_5 \times S^5 \) constructed by Metsaev and Tseytlin \([9]\) is given. In section 3 the super D3-brane action on \( AdS_5 \times S^5 \) is shown to satisfy the GZ duality condition, thereby establishing its exact self-duality without resort to any semiclassical approximation.

2 Super D3-brane Action on \( AdS_5 \times S^5 \)

In this section, for later use, we briefly review the D3-brane action constructed by Metsaev and Tseytlin \([9]\).

Supervielbein superfields on the \( AdS_5 \times S^5 \) background are explicitly constructed by the coset formalism \([4, 5]\). In the explicit coset parametrization \( G(X^M, \theta) = g(X^\hat{m})e^{\theta Q}, \) where \( X^M = (X^m, \theta) \) and \( Q \) are \( N = 2 \) superspace coordinate and su-
percharge, respectively, the supervielbein $L^A = (L^\hat{a}, L)$ and superconnection $L^{\hat{a}b}$ are explicitly given as follows:

$$L^\hat{a} = e^{\hat{a}} - 2i\bar{\theta}\Gamma^{a} W(\theta) D\theta, L = V(\theta) D\theta,$$

(2.1)

$$L^{ab} = \omega^{ab} + 2\bar{\theta}i\tau^{(2)}\Gamma^{ab}\sigma_\theta W(\theta) D\theta, L^{a'b'} = \omega^{a'b'} + 2\bar{\theta}i\tau^{(2)}\Gamma^{a'b'}\sigma_\theta W(\theta) D\theta,$$

(2.2)

where $a, b (a', b')$ are $AdS_5 (S^5)$ coordinate indices and run $0, 1, 2, 3, 4$ ($5, 6, 7, 8, 9$), and $D\theta, V(\theta)$ and $W(\theta)$ are defined by

$$D\theta = (d + \frac{1}{4}\omega^{\hat{a}b}\Gamma^{\hat{a}b} + \frac{1}{2}\tau^{(2)}e^\hat{a}\sigma_+\Gamma^{\hat{a}})\theta.$$

(2.3)

$$V(\theta) = \frac{\sinh \sqrt{m(\theta)}}{\sqrt{m(\theta)}} = 1 + \frac{1}{3!} m(\theta) + \frac{1}{5!} m(\theta)^2 + \cdots,$$

(2.4)

$$W(\theta) = \frac{\cosh \sqrt{m(\theta)} - 1}{m(\theta)} = \frac{1}{2!} + \frac{1}{4!} m(\theta) + \frac{1}{6!} m(\theta)^2 + \cdots,$$

(2.5)

$$m(\theta) = -\sigma_+ \Gamma^\hat{a} i\tau^{(2)}\theta\Gamma^{\hat{a}} + \frac{1}{2}\Gamma^{ab}\theta\bar{\theta}\tau^{(2)}\Gamma^{ab}\sigma_\theta - \frac{1}{2}\Gamma^{a'b'}\theta\bar{\theta}\tau^{(2)}\Gamma^{a'b'}\sigma_\theta.$$

(2.6)

In the above equations Pauli matrices $\tau^{(i)}$ operate on internal $N = 2$ space and Pauli matrices $\sigma_i$ stand for $32 \times 32$ matrices $1 \times 1 \times \sigma_i$.

The D3-brane action depends on the coset superspace coordinates $X^M = (X^\hat{m}, \theta)$ and $U(1)$ world-volume gauge field strength $F = dA = \frac{1}{2} F_{ij} d\sigma^i \wedge d\sigma^j, F_{ij} = \partial_i A_j - \partial_j A_i$, where $i$ and $j$ run over the worldvolume indices 0, 1, 2, 3. The reparametrization and $\kappa$-symmetry invariant super D3-brane action in the $AdS_5 \times S^5$ background consists of two terms; i.e. the Dirac-Born-Infeld(DBI) term and the Wess-Zumino term:

$$S = S_{DBI} + S_{WZ}.$$

(2.7)

The DBI term is given by

$$S_{DBI} = \int_{M_4} d^4 \sigma \sqrt{-\det(G_{ij} + F_{ij})},$$

(2.8)

where $G_{ij}$ is the supersymmetric induced worldvolume metric defined by

$$G_{ij} = L^\hat{a}_i L^\hat{a}_j = \partial_i X^M L^\hat{a}_M \partial_j X^N L^\hat{a}_N, L^\hat{a} = d\sigma^i L^\hat{a}_i,$$

(2.9)

and two-form superfield $F$ is the supersymmetric extension of the $U(1)$ worldvolume gauge field strength defined by

$$F = dA + \Omega^{(3)},$$

(2.10)
where two-form $\Omega^{(i)}$ is defined by

$$\Omega^{(i)} = 2i \int_0^1 ds \hat{L}_s \tau^{(i)} L_s,$$  

(2.11)

and $L^\hat{a} \equiv L^\hat{a}(x, s\theta), L_s \equiv L(x, s\theta)$ and $\hat{L}_s \equiv L^\hat{a}_s \Gamma^\hat{a}.$

The Wess-Zumino term is given by

$$S_{WZ} = \int_{M_4} \Omega_4 = \int_{M_5} H_5,$$  

(2.12)

here

$$H_5 = \left. d\Omega_4 = d(\Omega_4 + \Omega^{(1)} \wedge F) \right. = \frac{1}{6} L \wedge \hat{L} \wedge \hat{L} i \tau^{(2)} + \mathcal{F} \wedge \hat{L} \tau^{(1)} \wedge L$$

$$+ \frac{1}{30} (\epsilon^{a_1 \cdots a_5} L^{a_1} \wedge \cdots \wedge L^{a_5} + \epsilon^{a'_1 \cdots a'_5} L^{a'_1} \wedge \cdots \wedge L^{a'_5}).$$  

(2.13)

It is convenient to rewrite $\Omega_4$ in the following form;

$$\Omega_4 = C_4 + \Omega^{(1)} \wedge F,$$  

(2.14)

then we obtain a differential equation for $C_4$

$$dC_4 + \Omega^{(1)} \wedge d\Omega^{(3)} = \frac{1}{6} L \wedge \hat{L} \wedge \hat{L} i \tau^{(2)} \wedge L$$

$$+ \frac{1}{30} (\epsilon^{a_1 \cdots a_5} L^{a_1} \wedge \cdots \wedge L^{a_5} + \epsilon^{a'_1 \cdots a'_5} L^{a'_1} \wedge \cdots \wedge L^{a'_5}).$$  

(2.15)

The equation (2.13) or (2.15) can be explicitly integrated by using the Maurer-Cartan equations and the $\theta \to s\theta$ trick and the result is given in [4]. However for our purpose the explicit form of $C_4$ is not necessary.

As it can be seen from (2.1)-(2.6), the super D3-brane action on $AdS_5 \times S^5$ is a complicated function of supercoordinate $\theta$. However to prove the self-duality of the action (2.7) we need no gauge fixing condition to simplify the fermionic dependence of the supervielbeins and superconnections.

### 3 Self-Duality of Super D3-brane Action on $AdS_5 \times S^5$

In this section we show that the super D3-brane action on $AdS_5 \times S^5$ reviewed in the last section satisfies the Gaillard and Zumino duality-condition, thereby establishing its exact self-duality without resort to any semiclassical approximation.

First let us recall what is the GZ duality condition. Given a generic Lagrangian density $\mathcal{L}(F_{\mu\nu}, g_{\mu\nu}, \phi^A) = \sqrt{-g} L(F_{\mu\nu}, g_{\mu\nu}, \phi^A)$ in four dimensional spacetime which
contains a U(1) gauge field strength $F_{\mu \nu}$, gravitational field $g_{\mu \nu}$ and generic matter fields $\Phi^A$, the constructive relation is given by

$$\tilde{K}_{\mu \nu} \equiv \frac{\partial L}{\partial F_{\mu \nu}}, \quad (3.1)$$

where the Hodge dual components for the anti-symmetric tensor $K_{\mu \nu}$ are defined by

$$\tilde{K}_{\mu \nu} \equiv \frac{1}{2} \eta^{\rho \sigma}_{\mu \nu} K_{\rho \sigma}, \quad \tilde{\tilde{K}}_{\mu \nu} = -K_{\mu \nu}, \quad (3.2)$$

where $\eta_{\mu \nu \rho \sigma} = \sqrt{-g} \epsilon_{\mu \nu \rho \sigma}, \epsilon^{0123} = 1$ and the signature of $g_{\mu \nu}$ is $(-,+,+,+)$. If one defines the infinitesimal $SO(2)$ duality transformation by

$$\delta F_{\mu \nu} = \lambda K_{\mu \nu}, \quad \delta K_{\mu \nu} = -\lambda F_{\mu \nu}, \quad \delta \Phi^A = \xi^A(\Phi), \quad (3.3)$$

then the consistency of the constructive relation (3.1) and the invariance of the field equations under this $SO(2)$ duality transformation require the following condition:

$$\frac{\lambda}{4} (F_{\mu \nu} \tilde{F}^{\mu \nu} + K_{\mu \nu} \tilde{K}^{\mu \nu}) + \delta \Phi \mathcal{L} = 0, \quad (3.4)$$

and the invariance of the energy-momentum tensor requires $\delta g_{\mu \nu} = 0$. We call the condition (3.4) the GZ self-duality condition [20, 21].

It has been known that the $SO(2)$ duality is lifted to the $SL(2, R)$ duality by introducing a dilaton $\phi$ and an axion $\chi$ [22]. Moreover it has also been shown that the GZ condition (3.4) is actually the necessary and sufficient condition for the action to be invariant under the duality transformation (3.3) [23]. Therefore if one can show an action to satisfy the condition (3.4) under the transformation (3.3) with suitable transformation rule for matter fields, then one establishes the exact self-duality of the theory described by this action without resort to any semiclassical approximation.

Now let us show that the super D3-brane action on $AdS_5 \times S^5$ reviewed in the preceding section satisfies the GZ duality condition under the following $SO(2)$ duality transformation:

$$\delta F_{ij} = \lambda K_{ij}, \quad \delta K_{ij} = -\lambda F_{ij}, \quad \delta \theta = -\lambda \frac{i\tau^{(2)}}{2} \theta, \quad \delta \bar{\theta} = \bar{\lambda} \frac{i\tau^{(2)}}{2} \bar{\theta}, \quad \delta X = 0, \quad (3.5)$$

The $N = 2$ spinor coordinates transform as an $SO(2)$ doublet.

Let us first examine how various quantities in the action (2.7) transform under the above $SO(2)$ duality rotation. The matrix $m(\theta)$ defined in (2.6) transforms as an adjoint representation of $SO(2)$, and therefore matrices $V(\theta)$ and $W(\theta)$ obey the same transformation properties as $m(\theta)$;

$$\delta m(\theta) = -\lambda \frac{i}{2} \tau^{(2)}, \quad \delta V(\theta) = -\lambda \frac{i}{2} \tau^{(2)}, \quad \delta W(\theta) = -\lambda \frac{i}{2} \tau^{(2)}. \quad (3.6)$$
From these transformation properties, we can easily show that the supervielbeins transform as follows:

\[
\delta L^s = 0, \delta L = -\lambda \frac{i \tau^{(2)}}{2} L, \delta \bar{L} = \lambda L \frac{i \tau^{(2)}}{2},
\]

namely the \( N = 2 \) spinor components of the supervielbeins \( L \) and \( \bar{L} \) transform as \( SO(2) \) doublets.

From (3.5) and (3.7), we find \( \Omega^{(1,3)} = 2i \int_0^1 d\bar{s} \bar{L} \tau^{(1,3)} L \) defined in (2.11) transform as an \( SO(2) \) doublet and \( \Omega^{(2)} \) is an \( SO(2) \) singlet:

\[
\delta \Omega^{(1)} = \lambda \Omega^{(3)}, \delta \Omega^{(3)} = -\lambda \Omega^{(1)}, \delta \Omega^{(2)} = 0.
\]

From (3.8) and (2.15) the transformation of \( C_4 \) is easily obtained. Noticing the right hand-side of (2.15) is invariant under \( SO(2) \) rotation, the \( SO(2) \) transformation of (2.15) yields

\[
d \delta C_4 = -\lambda \frac{1}{2} d(\Omega^{(1)} \wedge \Omega^{(1)} + \Omega^{(3)} \wedge \Omega^{(3)}).
\]

Then we obtain the transformation rule for \( C_4 \) up to exact forms;

\[
\delta C_4 = -\lambda \frac{1}{2} (\Omega^{(1)} \wedge \Omega^{(1)} + \Omega^{(3)} \wedge \Omega^{(3)})
\]

Now we are ready to prove the duality symmetry of the super D3-brane action on \( AdS_5 \times S^5 \). First let us calculate \( \frac{\lambda}{4} (F_{ij} \tilde{F}^{ij} + K_{ij} \tilde{K}^{ij}) \). From the definition (3.1) and the action (2.7) we obtain

\[
\tilde{K}^{ij} = \frac{\partial L}{\partial F_{ij}}
= \frac{\sqrt{-\det G_{ij}}}{\sqrt{-\det(G_{ij} + F_{ij})}} (-F_{ij} + \mathcal{T} \tilde{F}^{ij}) + \tilde{\Omega}^{(1)ij},
\]

where we have used the determinant formula for the four-by-four matrix:

\[
det(G_{ij} + F_{ij}) = det(G_{ij})(1 + \frac{1}{2} F_{ij} F^{ij} - \mathcal{T}^2), \mathcal{T} \equiv \frac{1}{4} F_{ij} \tilde{F}^{ij}.
\]

Taking the Hodge dual of (3.9), we find

\[
K_{ij} = -\frac{1}{2} \eta_{ijkl} \tilde{K}^{kl} = \frac{\sqrt{-\det G_{ij}}}{\sqrt{-\det(G_{ij} + F_{ij})}} (\tilde{F}_{ij} + \mathcal{T} F_{ij}) + \Omega^{(1)}_{ij}.
\]

Then we obtain

\[
\frac{\lambda}{4} (F_{ij} \tilde{F}^{ij} + K_{ij} \tilde{K}^{ij}) = \frac{\lambda}{4} (-2\Omega^{(3)}_{ij} \tilde{F}^{ij} + 2\Omega^{(1)}_{ij} \tilde{K}^{ij} - \Omega^{(1)}_{ij} \tilde{\Omega}^{(1)ij} - \Omega^{(3)}_{ij} \tilde{\Omega}^{(3)ij})
= \frac{\lambda}{4} (-4\Omega^{(3)} F + 4\Omega^{(1)} K - 2\Omega^{(1)} \Omega^{(1)} - 2\Omega^{(3)} \Omega^{(3)}).
\]
In the above and below we omit ∧-symbol in products of forms.

Next let us calculate $\delta_\theta L$. In the language of differential forms,

$$
\delta_\theta L = \frac{\partial L}{\partial F} \delta \Omega^{(3)} + \mathcal{F} \delta \Omega^{(1)} + \delta C_4
$$

$$
= \lambda [ -K \Omega^{(1)} + (F + \Omega^{(3)}) \Omega^{(3)} + \frac{1}{2} (\Omega^{(1)} \Omega^{(1)} - \Omega^{(3)} \Omega^{(3)}) ]
$$

$$
= \lambda [ -K \Omega^{(1)} + F \Omega^{(3)} + \frac{1}{2} (\Omega^{(1)} \Omega^{(1)} + \Omega^{(3)} \Omega^{(3)}) ].
$$

(3.15)

It is clearly seen that the right-hand sides of (3.14) and (3.15) exactly cancel with each other and the GZ-duality condition is indeed satisfied. As we have proved the invariance of the action under the infinitesimal $SO(2)$ duality transformation, the action is also invariant under the finite $SO(2)$ duality transformation.

Before closing this section let us mention the extension from the $SO(2)$ duality to the $SL(2, R)$ duality by introducing a constant dilaton $\phi$ and an axion $\chi$ fields. We follow the same line of arguments given in [17]. According to the general method [24], let us define a new Lagrangian $\hat{L}(G, F, \theta, \phi, \chi)$ from the D3-brane Lagrangian $L(G, F, \theta)$ which obey the $SO(2)$ duality:

$$
\hat{L}(G, F, \theta, \phi, \chi) = L(G, e^{-\phi/2} F, \theta) + \frac{1}{4} \chi F \tilde{F}.
$$

(3.16)

Then if one define $\hat{F} = e^{-\phi/2} F$ and $\hat{K}$ by taking the dual of $\frac{\partial L(G, \hat{F}, \theta)}{\partial \hat{F}}$, the background dependence is absorbed in the rescaled variables $(\hat{K}, \hat{F})$. These are related with the background dependent $(K, F)$ by

$$
\begin{pmatrix} K \\ F \end{pmatrix} = V \begin{pmatrix} \hat{K} \\ \hat{F} \end{pmatrix},
$$

(3.17)

$$
V = e^\frac{\phi}{2} \begin{pmatrix} e^{-\phi} & \chi \\ 0 & 1 \end{pmatrix}.
$$

(3.18)

Here $V$ is a non-linear realization of $SL(2, R)/SO(2)$ transforming as

$$
V \rightarrow V' = \Lambda V O(\Lambda)^{-1}
$$

(3.19)

Here $\Lambda$ is a global $SL(2, R)$ matrix

$$
\Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, R), ad - bc = 1,
$$

(3.20)

and $O(\Lambda)$ is an $SO(2)$ transformation

$$
O(\Lambda)^{-1} = \begin{pmatrix} \cos \lambda & \sin \lambda \\ -\sin \lambda & \cos \lambda \end{pmatrix} \in SO(2).
$$

(3.21)
The condition that the form of $V$ (3.17) is unchanged under the transformation (3.18) determines the $SO(2)$ rotation angle $\lambda$ and the transformation rule of the background fields $\phi$ and $\chi$;

$$\tan \lambda = \frac{ce^{-\phi}}{c\chi + d},$$

(3.22)

and

$$\tau \rightarrow \tau' = \frac{a\tau + b}{ct + d}; \tau \equiv \chi + ie^{-\phi}.$$

(3.23)

These results show that if the original Lagrangian $L(G, F, \theta)$ is invariant under the $SO(2)$ duality transformation the extended Lagrangian $\hat{L}(G, F, \theta, \phi, \chi)$ with a dilaton and an axion fields is invariant under the $SL(2, R)$ duality transformation of $(K, F)$ and $\tau \equiv \chi + ie^{-\phi}$ and $SO(2)$ rotation of $N = 2$ spinor with rotation angle $\lambda$ given by (3.21).

4 Discussion

We have shown that the super D3-brane action on $AdS_5 \times S^5$ background is exactly self-dual, by proving that the action satisfies the Gaillard and Zumino duality condition which is a necessary and sufficient condition for the action to be self-dual. It should be stressed that our proof of the self-duality needs not any gauge fixing choice for the $\kappa$-symmetry.

Although the super D3-brane action constructed by Metsaev and Tseytlin for which we have proved the self-duality is based on the explicit coset parametrization $G(X^m, \theta) = g(X)e^{\theta Q}$, since the physical properties such as duality relations among super p-branes should not depend on the choice of the parametrization of manifolds, our result should be valid for any other parametrizations of coset manifold. In a recent paper [25] super p-brane theories based on a different parametrizations of the coset manifolds are proposed. It would be interesting to confirm that the super D3-brane action constructed based on this new coset parametrization is indeed self-dual.

Now the self-duality of the super D3-brane action has been proved for the flat and $AdS$ background. Therefore we expect that the self-duality of super D3-brane and various duality relations among super p-branes will be proved in generic background geometries.

There are several effective actions of super D3-brane which are proposed to be manifestly duality symmetric [26, 27]. It would be interesting to investigate possible relations with the duality symmetry discussed in this paper.

We may think about various p-branes on $AdS_5 \times S^5$ background just as in the flat background. It would be interesting to construct various super p-brane actions on $AdS_5 \times S^5$ and investigate duality relations among them known on the flat background in semiclassical approximation [14].

It has been proposed that D-brane dualities are understandable as canonical transformation [28, 13, 17]. It would be also interesting to formulate dualities among super p
D-branes and M-branes both on flat and AdS backgrounds as canonical transformations.

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