Light-Cone Sum Rules for the Nucleon Form Factors

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Abstract

We argue that soft non-factorisable terms give a significant contribution to the baryon form factors at intermediate momentum transfers and set up a framework for the calculation of such terms in the light-cone sum rule approach. Among them, contributions of three-quark states with different helicity structure compared to the leading twist prove to be the most important. The leading-order sum rules are derived and confronted with the experimental data.

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1 Introduction

Electromagnetic form factors of the nucleon present a classical observable that characterizes the nucleon’s spatial charge and current distributions. The earliest investigations of the proton form factor [1] established the dominance of the one-photon exchange process in the electron-proton scattering. The matrix element of the vector current taken between nucleon states is conventionally written in terms of the Dirac and Pauli form factors $F_1(Q^2)$ and $F_2(Q^2)$, respectively:

$$
\langle P-q| j_{\mu}^{em}(0) |P \rangle = \bar{N}(P-q) \left[ \gamma_{\mu} F_1(Q^2) - i\frac{\sigma_{\mu\nu} q^\nu}{2M} F_2(Q^2) \right] N(P) ,
$$

where $P_{\mu}$ is the nucleon four-momentum in the initial state, $M$ is the nucleon mass, $P^2 = (P-q)^2 = M^2$, $q_{\mu}$ is the (outgoing) photon momentum, $Q^2 = -q^2$, and $N(P)$ is the nucleon spinor. The values of Dirac and Pauli form factors at $Q^2 = 0$ define the electric charge and the anomalous magnetic moment of the nucleon:

$$
F_1^p(0) = 1, \quad F_1^n(0) = 0, \quad F_2^p(0) = \kappa_p = 1.79, \quad F_2^n(0) = \kappa_n = -1.91 .
$$

Hereafter, ‘$p$’ and ‘$n$’ stand for the proton and the neutron, respectively. From the experimental point of view it is more convenient to work with the electric $G_E(Q^2)$ and magnetic $G_M(Q^2)$ Sachs form factors defined as

$$
G_M(Q^2) = F_1(Q^2) + F_2(Q^2), \quad G_E(Q^2) = F_1(Q^2) + \frac{q^2}{4M^2} F_2(Q^2) ,
$$

$$
G_M^p(0) = \mu_p = 2.79 , \quad G_M^n(0) = \mu_n = -1.91 .
$$

In a special frame of reference, the Breit-frame, $G_E(Q^2)$ corresponds to the distribution of electric charge and $G_M(Q^2)$ to the magnetic current distribution. In the same frame $G_M(Q^2)$ stands for the helicity conserving amplitude, while $G_E(Q^2)$ corresponds to a helicity-flip. In the infinite momentum frame $F_1(Q^2)$ and $F_2(Q^2)$ are helicity conserving and helicity violating, respectively.

It is known that the experimental data for $G_M(Q^2)$ at values of $Q^2$ up to 5 GeV$^2$ are very well described by the famous dipole formula:

$$
\frac{1}{\mu_p} G_M^p(Q^2) \sim \frac{1}{\mu_n} G_M^n(Q^2) \sim \frac{1}{(1 + Q^2/\mu_0^2)^2} = G_D(Q^2) ; \quad \mu_0^2 \sim 0.71 \text{ GeV}^2 .
$$

For the electric form factor a dipole behavior is observed for $Q^2$ below 1 GeV$^2$. For larger momentum transfers the experimental situation was unclear until recently, because of SLAC [2] data contradicting the older DESY results [3]. Both of these measurements were based on the traditional Rosenbluth separation of the cross section. Very recently, the Jefferson Lab Hall A Collaboration determined the ratio $G_E^p(Q^2)/G_M^p(Q^2)$ from a simultaneous measurement of longitudinal and perpendicular polarization components of the recoil nucleon [4, 5]. A systematic deviation from the dipole behavior for the electric form factor was observed confirming the tendency seen in earlier measurements at DESY.
The theoretical calculation of the form factors from the underlying field theory presents a classical problem of the physics of strong interactions. For very large $Q^2$, the hard gluon exchange contribution proves to be dominant. The corresponding formalism was developed in [6], presenting one of the highlights of perturbative QCD. This approach introduces the concept of hadron distribution amplitudes as fundamental nonperturbative functions describing the hadron structure in rare parton configurations with a minimum number of Fock constituents (and at small transverse separations). The Dirac form factor $F_1(Q^2)$ exhibits the leading asymptotic $1/Q^4$ scaling behaviour and can be written in a factorized form as a convolution of two nucleon distribution amplitudes of leading twist and a calculable hard part. On the other hand, the Pauli form factor $F_2(Q^2)$ turns out to be additionally suppressed by an extra power of $1/Q^2$. It is, therefore, of higher twist and is not accessible within the standard approach.

In practice the calculations of nucleon form factors for realistic values of $Q^2$ using the standard hard-scattering picture appear to be not convincing. For the asymptotic form of the leading twist three-quark distribution amplitude the proton form factor turns out to be zero to leading order [7], while the neutron form factor compared to the data is small and of opposite sign. A remedy that has been suggested in [8] is that nucleon distribution amplitudes at intermediate momentum transfers may deviate strongly from their asymptotic shape. Chernyak and Zhitnitsky have shown that it is indeed possible to get a satisfactory description of the magnetic form factors using very asymmetric distribution amplitudes in which a large fraction of the nucleon momentum is carried by one valence quark. One drawback of this suggestion is, however, that even at large values of $Q^2 \sim 10 \text{ GeV}^2$ the “hard” amplitude is dominated by small gluon virtualities [9], casting doubt on the consistency of the perturbative approach. Attempts have been made to increase the region of applicability of perturbative QCD by resumming Sudakov-type logarithmic corrections to all orders [10]. Unfortunately, the Sudakov suppression of large transverse separations is most likely not strong enough to suppress nonperturbative effects, see e.g. [11] for a detailed discussion.

Another point of view that is becoming increasingly popular in recent years is that the onset of the perturbative QCD regime in exclusive reactions is postponed until very large momentum transfers and nonperturbative so-called “soft” or “end-point” contributions to exclusive reactions play a dominant role at present energies. In particular, it proves to be possible to get a good description of the existing data by the “soft” contribution alone, modeled by an overlap of nonperturbative wave functions, see [12]. A weak point of this approach is a possibility of double counting, with hard rescattering contributions “hidden” in model-dependent hadron wave functions.

In this paper we develop an approach to the calculation of baryon form factors based on light-cone sum rules (LCSR) [13]. Although the LCSR predictions do involve a certain model dependence and the leading-order sum rules may be not very accurate, this technique offers an important advantage of being fully consistent with QCD perturbation theory. LCSR reveal that the distinction between “hard” and “soft” contributions appears to be scale- and scheme-dependent [14]. It was demonstrated for the case of the pion [14] that the contribution of hard rescattering is correctly reproduced in the LCSR approach as a part of the $O(\alpha_s)$ correction. In recent years there have been numerous applications of LCSRs to mesons, see [15] for a review. Baryon form factors, however, were never considered. One reason for this
is that the LCSR calculations require certain knowledge about the distribution amplitudes of higher twists and for baryons these were not available until recently \[16\]. Another reason is that, as we will see, the LCSR formalism for baryons appears to be considerably more cumbersome.

Apart from resolving several technical issues, our main finding in this work is that the soft contribution to the nucleon form factors is dominated by valence quark configurations with different helicity structure compared to the leading-twist amplitude. Large contributions of the distribution amplitudes with “wrong” helicity are important for the electric form factor and in a more general context can explain why helicity selection rules in perturbative QCD appear to be badly broken in hard exclusive processes at present energies. The sum rules in the present paper are derived to leading order in the QCD coupling. Comparing the results with the available data we conclude that nucleon distribution amplitudes that deviate significantly from their asymptotic shape are disfavored. With the asymptotic distribution amplitudes, the accuracy of the sum rules proves to be of order 50% in the range $1 < Q^2 < 10$ GeV$^2$ both for the proton and the neutron. We believe that the accuracy can be improved significantly by the calculation of $O(\alpha_s)$ corrections to the sum rules and especially if lattice data on the moments of higher-twist distribution amplitudes become available.

The presentation is organized as follows. In Section 2 we introduce the necessary notation and explain basic ideas and techniques of the LCSR approach on the example of the leading-twist contribution to the sum rule. Section 3 contains the derivation of sum rules including higher twist corrections, which is our main result. The numerical analysis of the LCSRs is carried out in Section 4, together with a summary and discussion. The paper has two Appendices devoted to technical aspects of the calculation: In Appendix A we collect the necessary expressions for the conformal expansions of nucleon distribution amplitudes and in Appendix B a derivation of nucleon mass corrections to the sum rules is given.

2 Getting Started: Leading Twist

The method of light-cone sum rules \[13\] combines the standard technique of QCD sum rules \[17\] with the specific light-cone kinematics of hard exclusive processes. As the most apparent distinction, the short-distance Wilson operator product expansion in contributions of vacuum condensates of increasing dimension is replaced by the light-cone expansion in terms of distribution amplitudes of increasing twist. Our calculation is similar to the calculation of the pion form factor in Ref. \[14\].

Throughout this work we consider the following correlation function

$$T_\nu(P,q) = i \int d^4x \ e^{iq \cdot x} \langle 0 | T \left\{ \eta(0)j^\em_{\nu}(x) \right\} | P \rangle$$

which includes the electromagnetic current

$$j^\em_{\nu} = e_u \bar{u} \gamma_\nu u + e_d \bar{d} \gamma_\nu d$$

and an interpolating nucleon (proton) field \[8\]

$$\eta_{CZ}(0) = \varepsilon^{ijk} \left[ u^i(0)C \gamma_5 u^j(0) \right] \gamma_5 \not{d}^k(0),$$
\[ \langle 0 | \eta_{CZ} | P \rangle = f_N (P \cdot z) \not z N(P). \]  

(2.3)

Here \( z \) is a light-cone vector, \( z^2 = 0 \), and the coupling \( f_N \) determines the normalization of the leading twist proton distribution amplitude \([4]\). This choice is convenient for our purposes as it leads to the same hierarchy of the contributions of different twists as in the perturbative approach \([3]\), see below. From the definition in Eq. (1.1) the contribution of the nucleon intermediate state in the correlation function Eq. (2.1) is readily derived to be

\[ z^\nu T_\nu(P, q) = f_N \left( P' \cdot z \right) \not z N(P) \]  

(2.4)

where

\[ P' = P - q \]  

(2.5)

and the dots stand for higher resonances and the continuum. In order to simplify the Lorentz structure we have contracted the correlation function with \( z^\nu \) to get rid of contributions \( \sim z^\nu \) that give subdominant contributions on the light-cone.

On the other hand, at large Euclidean momenta \( P'^2 \) and \( q^2 = -Q^2 \) the correlation function can be calculated in perturbation theory. The leading order contribution is obtained from the diagram shown in Fig. 1. A simple calculation yields

\[ z^\nu T_\nu(P, q) = \frac{f_N}{M^2 - P'^2} (P' \cdot z) \left\{ 2F_1(Q^2) (P' \cdot z) - F_2(Q^2)(q \cdot z) \right\} \not z \]  

\[ + F_2(Q^2) \left[ (P' \cdot z) + \frac{1}{2} (q \cdot z) \right] \frac{\not q}{M} N(P) + \ldots \]  

(2.6)

where

\[ P' = P - q \]  

(2.7)
Each distribution amplitude $V_1, A_1$ and $T_1$ can be represented as

$$F(a_k p \cdot z) = \int \mathcal{D}x \, e^{-ipz} \sum_j x_j a_j F(x_i).$$

(2.8)

The integration runs over the longitudinal momentum fractions $x_1, x_2, x_3$ carried by the quarks inside the nucleon with $\sum_i x_i = 1$ and the integration measure is defined as

$$\int \mathcal{D}x = \int_0^1 dx_1 dx_2 dx_3 \, \delta(x_1 + x_2 + x_3 - 1).$$

(2.9)

The normalization is fixed by

$$\int \mathcal{D}x \, V_1(x_1, x_2, x_3) = f_N,$$

cf. Eq. (2.3). With these definitions, we find for the contribution in Eq. (2.6)

$$z_\nu T_\nu = - \left[ e_d \int \mathcal{D}x \frac{x_3 V_1(x_i)}{(q - x_3 P)^2} + 2 e_u \int \mathcal{D}x \frac{x_2 V_1(x_i)}{(q - x_2 P)^2} \right] 2(p \cdot z)^2 g N(P) + \ldots$$

(2.11)

where the ellipses stand for contributions that are nonleading in the infinite momentum frame kinematics $P \to \infty, q \sim \text{const}, z \sim 1/P$. Note that in this case only $V_1$ contributes and there is no Lorentz structure $\sim (P \cdot z)^2 / g \not z$ that would give rise to the Pauli form factor $F_2$.

The common idea of QCD sum rules is to match the dispersion representation in Eq. (2.4) with the QCD calculation at certain “not so large” Euclidian values of the momentum $P^2$ flowing through the nucleon interpolation current. To this end we can rewrite the perturbative result in Eq. (2.11) in the form of a dispersion relation with a certain spectral density

$$z_\nu T_\nu(P, q) = \int_0^{s_0} ds \, \frac{\rho(s, Q^2)}{s - P^2} 2(P \cdot z)^2 g N(P) + \ldots.$$  

(2.12)

Restricting the region of integration to the mass region below the Roper resonance, $s_0 \sim (1.5 \text{ GeV})^2$, one eliminates contributions other than the nucleon [17]. If terms of $O(M^2/Q^2)$ are neglected — which is consistent with twist-3 accuracy — such a representation is obtained easily by the substitution $s = (1 - x_3)Q^2/x_3$ or $s = (1 - x_2)Q^2/x_2$ for the contribution of the $d$-quark and the $u$-quark, respectively. The upper bound in the dispersion integral then translates into a lower bound in the integral over the corresponding momentum fraction: $x_2 > Q^2/(s_0 + Q^2)$ or $x_3 > Q^2/(s_0 + Q^2)$. Finally we follow the usual QCD sum rules procedure to use a Borel transformation to convert the power suppression of higher mass contributions into an exponential suppression

$$\frac{1}{-(q - xP)^2} = \frac{1}{x(s - P^2)} \rightarrow \frac{1}{x} \exp \left\{ -\frac{s}{M_B^2} \right\}. $$

(2.13)
As the result, a new variable — the Borel parameter $M_B$ — enters instead of $P^2$. Equating the Borel transformed versions of Eq. (2.4) and Eq. (2.11) we finally arrive at the sum rules

$$ F_{1}^{tw-3}(Q^2) = \frac{1}{f_N} \int_0^{s_0} dx \, V_1(x_i) \exp \left( -\frac{\bar{x}_3 Q^2 - x_3^2 M_B^2}{x_3 M_B^2} \right) \Theta \left( x_3 - \frac{Q^2}{Q^2 + s_0} \right) + 2 e_u \int_0^{s_0} dx \, V_1(x_i) \exp \left( -\frac{\bar{x}_2 Q^2 - x_2^2 M_B^2}{x_2 M_B^2} \right) \Theta \left( x_2 - \frac{Q^2}{Q^2 + s_0} \right) , $$

$$ F_2^{tw-3}(Q^2) = 0 . \tag{2.14} $$

where the superscript ‘tw-3’ reminds of twist-three accuracy.

The $\Theta$-functions in Eq. (2.14) originate from the restriction of the spectral density to the duality region $s < s_0$ and confine the integration region to values of $x_3 \to 1$ and $x_2 \to 1$ at $Q^2 \to \infty$ for the contributions of $d$ and $u$ quarks, respectively. It follows that in our approximation the form factor is dominated by parton configurations where the scattered quark carries almost all the momentum of the nucleon. This is precisely the soft, or Feynman mechanism the form factor is dominated by parton configurations where the scattered quark carries almost all the momentum of the nucleon. This is precisely the soft, or Feynman mechanism, and the standard wisdom tells that this contribution has to be subleading at very large momentum transfers. Indeed, using the asymptotic wave function $V_1(x_i) = 120 f_N x_1 x_2 x_3$ and expanding in $1/Q^2$ we find that the sum rule result in the limit $Q^2 \to \infty$ behaves as

$$ F_1^{tw-3}(Q^2) = 20 (e_d + 2 e_u) \frac{1}{Q^8} \int_0^{s_0} ds \, s^3 e^{-\frac{s+M_B^2}{M_B^2}} , \tag{2.15} $$

i.e. it is suppressed by two additional powers of $1/Q^2$ compared with the expected asymptotic behavior. This observation is in agreement with an analysis of the soft contribution in the framework of the Drell-Yan description of $F_1$ \cite{18}. The soft overlap contribution of two leading twist light-cone wave functions computed in \cite{19} asymptotically vanishes as $1/Q^8$. We will find, however, that this strong suppression does not hold for contributions of wave functions with different helicity structure.

It is well known that the form factor is strongly sensitive to the shape of the nucleon distribution amplitude. The general QCD description of distribution amplitudes is based on the conformal expansion \cite{6, 7, 22}. There exist several concrete models that take into account the first few conformal partial waves. In Fig. 2 we show the twist-3 LCSR prediction (Eq. (2.14)) for the magnetic form factor $G_M^{p,n}$ (to this accuracy $G_M = F_1$) normalized to the phenomenological dipole parametrization Eq. (1.4), for three different choices of the distribution amplitude

$$ V_1^{asy}(x_i, \mu \simeq 1\mbox{GeV}) = 120 x_1 x_2 x_3 f_N , $$

$$ V_1(x_i, \mu \simeq 1\mbox{GeV}) = 120 x_1 x_2 x_3 f_N \left[ 1 + \tilde{f}_3^+(\mu)(1 - x_3) \right] , $$

$$ V_1^{COZ}(x_i, \mu \simeq 1\mbox{GeV}) = 120 x_1 x_2 x_3 f_N \left[ 1 + \tilde{f}_3^+(\mu)(1 - x_3) + \tilde{f}_3^{d1}(3 - 21 x_3 + 28 x_3^2 + \tilde{f}_3^{d2}(5(x_1^2 + x_2^2) - 3(1 - x_3^2))) \right] . \tag{2.16} $$

\footnote{Throughout this paper we give explicit expressions for the proton. Neutron form factors are obtained by the obvious substitution $e_u \leftrightarrow e_d$.}
Figure 2: Twist-3 approximation to the soft contribution to the magnetic form factor of the proton (left) and the neutron (right), see Eq. (2.14). Plotted is the ratio $G^p/n/(\mu p/n G_D)$ where $G_D$ is the dipole fit Eq. (1.4). The three curves correspond to different models of the leading twist distribution amplitudes: The full line corresponds to $V^{asy}_1$, the dashed line to $V_1$, and the dashed-dotted line to $V^{COZ}_1$, respectively, see Eq. (2.16). Sum rule parameters are taken to be $M^2_B = 2$ GeV$^2$ and $s_0 = (1.5$ GeV$)^2$. The data points are taken from [20, 21].

where

$$f_N(\mu = 1$GeV$) = (5.3 \pm 0.5) \times 10^{-3}$GeV$^2,$$
$$\tilde{\phi}_3^+(\mu = 1$GeV$) = 1.1 \pm 0.3,$$
$$\tilde{\phi}_3^{d_1}(\mu = 1$GeV$) = 0.615,$$
$$\tilde{\phi}_3^{d_2}(\mu = 1$GeV$) = 3.68. \tag{2.17}$$

The first expression in Eq. (2.14) defines the asymptotic distribution amplitude in the limit $Q^2 \to \infty$ and corresponds to the contribution of the lowest conformal spin ("S-wave") [4]. The second and the third expressions correspond to taking into account the next-to-lowest ("P-wave") and also the next-to-next-lowest ("D-wave") conformal spin, respectively [4]. The parameters in Eq. (2.17) are taken from the last reference in [8]. They were obtained using QCD sum rules for the lowest moments of the distribution amplitudes and their accuracy is subject of debate. Note that the dependence on $f_N$ in fact cancels out of the sum rule in Eq. (2.14). The numerical impact of the scale dependence of the remaining parameters is negligible compared to the intrinsic uncertainty of their estimates, so that we do not take this effect into account.

From Fig. 2 it is seen that the results vary by factor 2–3 depending on the choice of the distribution amplitude. The “P-wave” approximation in the second line in Eq. (2.14) seems to fit best while for the asymptotic distribution amplitude the neutron form factor turns out to be zero. One has to keep in mind, however, that in this calculation $F_2(Q^2)$ is zero both for proton and neutron and for arbitrary choice of the distribution amplitude. Hence we have, e.g. $G_M(Q^2) = G_E(Q^2)$ identically, which is clearly not a satisfactory approximation in a few GeV region. We conclude that the leading twist approximation to the LCSRs is not

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1We have rewritten the original expressions given in [8] using the conformal basis defined in the work [22]. In this basis, all terms are mutually orthogonal with respect to the conformal weight function $V_1^{asy}(x_i)$. [22]
sufficient for the quantitative analysis and in the next section we proceed to the complete
treatment, including higher-twist effects.

3 Beyond the Leading Twist

Apart from radiative corrections, the correlation function in Eq. (2.1) receives further contributions of higher-twist distribution amplitudes. These contributions arise in two ways. First, one has to remember that the hard quark propagator $\sim \not{x} / x^4$ in Eq. (2.6) receives corrections in the background color field [23], which are proportional to the gluon field strength tensor and give rise to four-particle (and five-particle) nucleon distribution amplitudes corresponding to parton states with a gluon or quark-antiquark pair in addition to the three valence quarks. Such corrections are usually not expected to play any significant role (see e.g. [19]), and in this work we do not take them into account. Second, one has to improve on the treatment of the matrix elements of the three-quark operators in Eq. (2.6) taking into account contributions of other Lorentz structures and less singular contributions on the light-cone, beyond the leading-twist approximation in Eq. (2.7).

We only need to retain vector-like Dirac structures since it is easy to see (cf. Eq. (2.6)) that all others do not contribute to the sum rule we are considering. To this accuracy, the complete decomposition of the three-quark operator reads [16]:

$$4 \langle 0 | \varepsilon^{ijk} u_\alpha(a_1 x) u_\beta(a_2 x) d_\gamma(a_3 x) | P \rangle =$$

$$= \left( V_1 + \frac{x^2 M^2}{4 V_1^M} \right) (PC)_{\alpha\beta} (\gamma_5 N) \gamma + V_2 M (PC)_{\alpha\beta} (\not{x} \gamma_5 N) \gamma + V_3 M (\gamma_\mu C)_{\alpha\beta} (\gamma^\mu \gamma_5 N) \gamma + V_4 M^2 (\not{x} C)_{\alpha\beta} (\not{x} \gamma_5 N) \gamma + V_5 M^2 (\gamma_\mu C)_{\alpha\beta} (i\sigma^{\mu\nu} x \gamma_5 N) \gamma + V_6 M^3 (\not{x} C)_{\alpha\beta} (\not{x} \gamma_5 N) \gamma .$$

The expansion in Eq. (3.1) has to be viewed as an operator product expansion to the leading order in the strong coupling. Each of the functions $V_i$ depends on the deviation from the light-cone at most logarithmically, and we have also included the $O(x^2)$ correction to the leading-twist-3 structure, denoted by $V_1^M$. The invariant functions $V_1(x_i), \ldots, V_6(x_i)$ can be expressed readily in terms of higher-twist nucleon distribution amplitudes $V_1(x_i), \ldots, V_6(x_i)$ introduced in Ref. [16]:

$$V_1 = V_1 , \quad 2 p \cdot x V_2 = V_1 - V_2 - V_3 ,$$

$$2 V_3 = V_3 , \quad 4 p \cdot x V_4 = -2 V_1 + V_3 + V_4 + 2 V_5 ,$$

$$4 p \cdot x V_5 = V_4 - V_3 , \quad (2 p \cdot x)^2 V_6 = -V_1 + V_2 + V_3 + V_4 + V_5 - V_6 .$$

In difference to the “calligraphic” functions $V_1(x_i), \ldots, V_6(x_i)$ each of the distribution amplitudes $V_1(x_i), \ldots, V_6(x_i)$ has definite twist, see Table [1] and corresponds to the matrix element of a (renormalized) three-quark operator with exactly light-like separations $x^2 \to 0$, see Table 2 and Appendix C in Ref. [16] for the details. The higher-twist distribution amplitudes $V_2(x_i), \ldots, V_6(x_i)$ correspond to “wrong” components of the quark spinors and have
Table 1: Twist classification of the distribution amplitudes in Eq. (3.2).

| twist-3 | twist-4 | twist-5 | twist-6 |
|---------|---------|---------|---------|
| $V_1$   | $V_2, V_3$ | $V_4, V_5$ | $V_6$   |

different helicity structure compared to the leading twist amplitude. For baryons these “bad” components cannot all be traded for gluons as in the case of mesons \cite{24}. They are not all independent, but related to each other by the exact QCD equations of motion. As the result, to the leading conformal spin accuracy the five functions $V_2(x_i), \ldots, V_6(x_i)$ involve only one single nonperturbative higher twist parameter. In the calculations presented below we use the conformal expansions of higher twist distribution amplitudes to the next-to-leading order (include “P-wave”). This accuracy is consistent with neglecting multiparton components with extra gluons (quark-antiquark pairs) that are of yet higher spin. Explicit expressions for the distribution amplitudes are collected in Appendix A.

The invariant function $V_1^M$ corresponding to the corrections of order $\mathcal{O}(x^2)$ to the leading-twist Lorentz structure in Eq. (3.1) is twist-5 and in general is a complicated function of distribution amplitudes. In the present context there is an important simplification that two of the quarks in Eq. (2.6) always appear to be at the same space-time point, cf. Fig. 1. In this specific configuration, $V_1^M$ can be obtained using the technique of Ref. \cite{23} that has been designed for the studies of mesonic operators. The corresponding calculation is carried out in Appendix B and the expressions for $V_1^M$ in the above mentioned limits present one of the new results of this paper. Substituting the full expansion Eq. (3.1) in Eq. (2.6) we obtain the result:

$$z_\nu T^{\nu} = 2(P \cdot z)^2 \not N (P) \left\{ e_d \left[ - \int Dx \frac{x_3 V_1(x_i)}{(q - x_3 P)^2} \right] + M^2 \int_0^1 dx_3 x_3^2 \frac{2 \tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_5 - \frac{2}{x_3} V_1^{M(d)}}{(q - x_3 P)^4} (x_3) \right. \right.$$

$$+ 2M^4 \int_0^1 dx_3 x_3^3 \frac{\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_5 + \tilde{V}_6}{(q - x_3 P)^6} (x_3) \left. \right] + 2e_u \left[ x_3 \leftrightarrow x_2 \right] \right\}$$

$$+ \frac{(P \cdot z)^2}{M} \not q N (P) \left\{ e_d \left[ M^2 \int_0^1 dx_3 2x_3 \frac{V_2 + \tilde{V}_3 - \tilde{V}_1}{(q - x_3 P)^4} (x_3) \right. \right.$$

$$- 4M^4 \int_0^1 dx_3 x_3^2 \frac{\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_5 + \tilde{V}_6}{(q - x_3 P)^6} (x_3) \left. \right] + 2e_u \left[ x_3 \leftrightarrow x_2 \right] \right\} + \ldots .$$

We see that at this stage a contribution to $F_2(Q^2)$ arises. The functions $V_1^{M(d)}$ and $V_1^{M(u)}$ (the second one appears in the u-quark contribution) are given in Eq. (3.13) and Eq. (3.16), respectively.
while the distribution amplitudes with a ‘tilde’ are defined as

\[
\tilde{V}(x_3) = \int_1^{x_3} dx_3' \int_0^{1-x_3'} dx_1 V(x_1, 1 - x_1 - x_3', x_3'),
\]

\[
\tilde{V}(x_2) = \int_1^{x_2} dx_2' \int_0^{1-x_2'} dx_1 V(x_1, x_2', 1 - x_1 - x_2'),
\]

\[
\tilde{V}(x_3) = \int_1^{x_3} dx_3' \int_1^{x_3'} dx_3'' \int_0^{1-x_3''} dx_1 V(x_1, 1 - x_1 - x_3'' - x_3'''),
\]

\[
\tilde{V}(x_2) = \int_1^{x_2} dx_2' \int_1^{x_2'} dx_2'' \int_0^{1-x_2''} dx_1 V(x_1, x_2'', 1 - x_1 - x_2''),
\]

(3.4)

and result from partial integration in \(x_2\) or \(x_3\), respectively: The integration by parts in \(x_2\) or \(x_3\) is done in order to eliminate the \(1/P \cdot x\) factors that appear in Eq. (3.1) when the distribution amplitudes Eq. (3.2) are inserted. After this, the \(\int d^4x\) integration becomes trivial. The surface terms sum up to zero.

The Borel transformation and the continuum subtraction are performed by using the following substitution rules:

\[
\int dx \frac{\varrho(x)}{(q - xP)^4} = \int_0^1 \frac{dx}{x^2} \frac{\varrho(x)}{s - P^2} = \frac{1}{M_B^2} \int_0^1 \frac{dx}{x^2} \varrho(x) \exp \left( -\frac{\bar{x}Q^2}{xM_B^2} - \frac{\bar{x}M^2}{M_B^2} \right) + \frac{\varrho(x_0)}{Q^2 + \bar{x}_0^2 M^2},
\]

(3.5)

\[
\int dx \frac{\varrho(x)}{(q - xP)^6} = -\int_0^1 \frac{dx}{x^3} \frac{\varrho(x)}{s - P^2} = -\frac{1}{2M_B^2} \int_0^1 \frac{dx}{x^3} \varrho(x) \exp \left( -\frac{\bar{x}Q^2}{xM_B^2} - \frac{\bar{x}M^2}{M_B^2} \right) - \frac{1}{2} \frac{\varrho(x_0)}{Q^2 + \bar{x}_0^2 M^2} \frac{1}{M_B^2} \frac{\varrho(x_0)}{Q^2 + \bar{x}_0^2 M^2} M_B^2
\]

\[
+ \frac{1}{2} \frac{\bar{x}_0^2}{Q^2 + \bar{x}_0^2 M^2} \left[ \frac{d}{dx_0} \frac{\varrho(x_0)}{Q^2 + \bar{x}_0^2 M^2} \right] e^{-\bar{x}_0^2/2M_B^2} \left( 1 + \frac{Q^2}{M_B^2} \right) e^{-\bar{x}_0^2/2M_B^2}
\]

(3.6)

where in difference to Eq. (2.14) we have to keep the nucleon mass in the kinematical relation \(s = \frac{1}{x} Q^2 + (1 - x) M^2\) and \(x_0\) is the solution of the corresponding quadratic equation for \(s = s_0\):

\[
x_0 = \frac{\left[ \sqrt{(Q^2 + s_0 - M^2)^2 + 4M^2Q^2} - (Q^2 + s_0 - M^2) \right]}{(2M^2)}.
\]

(3.7)

The contributions \(\sim e^{-s_0/M_B^2}\) in Eq. (3.5) and Eq. (3.6) correspond to the “surface terms” arising from successive partial integrations to reduce the power in the denominators \((q -
$xP)^{2n} = (s - P^2)^{2n}(-x)^{2n}$ with $n > 1$ to the usual dispersion representation with the denominator $\sim (s - P^2)$. Without continuum subtraction, i.e. in the limit $s_0 \to \infty$ these terms vanish.

Collecting everything together, we get the final expressions:

$$F_1^p(Q^2) = \frac{e_d}{f_N} \left\{ \int_{x_3}^{1} \frac{dx_3}{x_3} \exp \left( -\frac{x_3Q^2}{x_3M_B^2} + \frac{x_3M^2}{M_B^2} \right) \left[ \left( \int_0^{1-x_3} dx_1 V_1(x_1, 1-x_1-x_3, x_3) \right) \right] \right. $$

$$+ \frac{M^2}{M_B^2} \left[ 2\tilde{V}_1 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_5 - \frac{2}{x_3} V_1^{M(d)} \right] (x_3) $$

$$+ \frac{M^4}{M_B^2} \left( \tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_6 \right) (x_3) $$

$$+ \frac{M^2(x_3^0)^2}{Q^2 + (x_3^0)^2M^2} e^{-(s_0-M^2)/M_B^2} \left[ \left( 2\tilde{V}_1 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_5 - \frac{2}{x_3} V_1^{M(d)} \right) (x_3) \right. $$

$$+ \frac{M^2}{M_B^2} \left( \tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_6 \right) (x_3) $$

$$- M^2 \frac{d}{dx_3} \left( \frac{(x_3^0)^2}{Q^2 + (x_3^0)^2M^2} (\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_6) (x_3) \right) \right] \left\{ x_3 \leftrightarrow x_2, V_1^{M(d)} \to V_1^{M(u)} \right\},$$

(3.8)

and

$$F_2^p(Q^2) = \frac{e_d}{f_N} \int_{x_3}^{1} \frac{dx_3}{x_3} \exp \left( -\frac{x_3Q^2}{x_3M_B^2} + \frac{x_3M^2}{M_B^2} \right) \left[ 2\frac{M^2}{M_B^2} \left( \tilde{V}_2 + \tilde{V}_3 - \tilde{V}_1 \right) (x_3) \right. $$

$$- 2\frac{M^4}{M_B^2} \left( \tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_6 \right) (x_3) \right] $$

$$+ \frac{2M^2x_3^0}{Q^2 + (x_3^0)^2M^2} \left[ \left( \tilde{V}_2 + \tilde{V}_3 - \tilde{V}_1 \right) (x_3) - \frac{M^2}{M_B^2} \left( \tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_6 \right) (x_3) \right. $$

$$\left. + M^2 \frac{d}{dx_3} \left( \frac{x_3^0}{Q^2 + (x_3^0)^2M^2} \left( \tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_6 \right) (x_3) \right) \right] e^{-(s_0-M^2)/M_B^2} $$

$$+ \frac{2e_u}{f_N} \left\{ x_3 \leftrightarrow x_2, V_1^{M(d)} \to V_1^{M(u)} \right\}. \quad (3.9)$$

The sum rules in Eq. (3.8) and Eq. (3.9) present the main result of this paper.

In the limit $Q^2 \to \infty$ the sum rules can be simplified. Employing asymptotic wave functions we get the asymptotic behavior

$$F_1^p(Q^2) = \left[ \frac{37}{3} + 2\frac{\lambda_1}{f_N} \right] e_d + \left[ \frac{37}{3} - 2\frac{\lambda_1}{f_N} \right] e_u \left\{ \frac{M^2}{M_B^2} \right\}$$

11
\[ F_2^B(Q^2) = -2 \left[ \left( \frac{1}{3} \left( 10 - 11 \frac{M^2}{M_B^2} \right) - 2 \frac{\lambda_1}{f_N} \left( 1 + \frac{4}{15} \frac{M^2}{M_B^2} \right) \right) c_d \right. \\
+ \left. 2 \left( 5 - \frac{8}{9} \frac{M^2}{M_B^2} + \frac{\lambda_1}{f_N} \left( 1 + \frac{2}{15} \frac{M^2}{M_B^2} \right) \right) c_u \right] \frac{M^2}{M_B^2} \]
\[ \times \frac{1}{Q^8} \left( s_0^2 M^2 e^{-s_0 - M^2}/M_B^2} + \int_0^{s_0} ds^2 e^{-(s-M^2)/M_B^2} \right) \]
\[ \times \frac{1}{Q^6} \left( s_0^2 M^2 e^{-(s_0-M^2)/M_B^2} + \int_0^{s_0} ds^2 e^{-(s-M^2)/M_B^2} \right) \]  
(3.10)

Note that the contribution \( \sim 1/Q^6 \) to \( F_1(Q^2) \) arises (cf. Eq. (2.13)), which has two sources: The constants \( \sim 37/3 \) originate from the nucleon mass corrections and the terms \( \sim \lambda_1/f_N \) correspond to the contributions of higher-twist three-quark operators. Numerically \( \lambda_1/f_N \sim -5 \), see the next Section, so that both contributions appear to be of the same order. To avoid misunderstanding note that to leading order in \( \alpha_s \) we are dealing with the soft, or Feynman contribution to the form factors only. The true asymptotic behavior \( F_1 \sim 1/Q^4 \) can be reproduced in the LCSR approach as a part of the \( O(\alpha_s^2) \) correction, a calculation which goes far beyond the tasks of this work, see Ref. [14] for the detailed discussion for the pion.

4 Numerical Results and Discussion

For the numerical evaluation of the form factors we used the standard value of the continuum threshold \( s_0 = 2.25 \) GeV\(^2 \) and varied the Borel parameter in the range \( 1.5 < M_B^2 < 2.5 \) GeV\(^2 \). The results appear to be rather stable so that in the Figures given below we take \( M_B^2 = 2 \) GeV\(^2 \) as our standard choice. The solid curves marked ‘asy’ in Fig. 3 and Fig. 4 are obtained using the set of asymptotic distribution amplitudes corresponding to the contributions of operators with the lowest conformal spin. As already discussed in Sec. 2, the dependence on the normalization \( f_N \) of the leading-twist amplitude actually drops out of the sum rules so that to this accuracy the form factors depend on a single nonperturbative parameter - the ratio of matrix elements of the twist-4 and twist-3 operators

\[ \frac{\lambda_1}{f_N} = -5.1 \pm 1.7 \]  
(4.1)

The constant \( f_N \) is defined in Eq. (2.3) while \( \lambda_1 \) is the familiar nucleon coupling to the so-called Ioffe current [27]:

\[ \eta_I(0) = \varepsilon^{ijk} \left[ u_i(0) C \gamma_\mu u_j(0) \right] \gamma_5 \gamma_\mu d_k(0) \] 
\[ \langle 0 \mid \eta_I \mid P \rangle = \lambda_1 M N(P) \]  
(4.2)

The number in Eq. (4.1) is the QCD sum rule estimate at the scale 1 GeV (see [16] and references therein). Notice that the Ioffe coupling \( \lambda_1 \) is large compared to \( f_N \). Hence the
Figure 3: The LCSR prediction for the soft contribution to the magnetic form factor of the proton (left) and the neutron (right). Plotted is the ratio $G_M^{p/n}/(\mu_p/G_D)$ where $G_D$ is the dipole fit Eq. (1.4). The solid curves marked ‘ASY’ are obtained using the set of asymptotic distribution amplitudes corresponding to the contributions of operators with the lowest conformal spin. The curves marked SR show the results including the next-to-leading terms in the conformal expansion with parameters estimated using QCD sum rules. The dashed curves are obtained using asymptotic distribution amplitudes and a reduced relative normalization of the higher-twist contributions within the error range, see text. The data points are taken from [20, 21].

Figure 4: The LCSR prediction for the electric form factor. Plotted is the ratio $\mu_p G_E^p/G_M^p$ for the proton (left) and $(G_E^n/G_D)^2$ for the neutron (right), where $G_D$ is the dipole formula Eq. (1.4). The identification of the curves is the same as in Fig. 3. The data points are taken from [4, 5, 21].

contribution of three-quark states with a different helicity structure compared to leading twist is numerically most important for the form factors in a few GeV range.

Beyond the leading order in the conformal expansion the parameters become more numerous and less known. The curves in Fig. 3 and Fig. 4 marked ‘SR’ are obtained including the contributions of the next-to-the leading spin (“P-wave”) and using QCD sum rule estimates [8, 16] for the four additional parameters that enter the sum rules to this accuracy. For the leading twist this approximation corresponds to the expression in the second line in Eq. (2.16), and $\tilde{\phi}_3^+$ in Eq. (2.17) is the first of the additional parameters. Among the remaining three parameters one is of twist-3 and two are of twist-4, see Appendix A for the details. It is seen that with large “P-wave” contributions the description of the form factors becomes much worse compared to the asymptotic distributions. This is not unexpected, since from the experience of calculations of the pion form factor it is known that QCD sum
rules based on the local operator product expansion tend to overestimate the higher spin corrections considerably, see e.g. [26]. On this evidence, we conclude that large corrections to the asymptotic distribution amplitudes of higher twists are unlikely.

The agreement with the experimental data generally becomes better if the ratio \( \lambda_1/f_N \) is chosen to be 30% lower compared to the estimate in Eq. (4.1), which corresponds to the bottom of the given error range. The corresponding results are shown in Fig. 3 and Fig. 4 by the dashed curves. At the same time, we have checked that the agreement cannot be improved substantially by adding “P-wave” contributions to the distribution amplitudes with their normalizations taken as free parameters. This, again, can be considered as an argument against large corrections to the asymptotic distributions. On the other hand, we believe that quantitative conclusions can only be made after the calculation of radiative \( O(\alpha_s) \) corrections to the sum rules, which goes beyond the tasks of this paper.

To summarize, in this work we have set up a framework for the calculation of baryon electromagnetic form factors in the light-cone sum rule approach. The light-cone sum rules are derived to leading order in the strong coupling and confronted with the experimental data. We argue that soft non-factorisable terms give a significant contribution to the baryon form factors at intermediate momentum transfers. Among them, contributions of three-quark states with different helicity structure compared to the leading twist prove to be the most important. A quantitative analysis of baryon form factors requires the calculation of radiative corrections to the sum rule, which is a challenging but doable task. The approach can easily be generalized to the study of transition form factors like \( \Delta \to N\gamma \) and to weak decays of heavy baryons.

**Appendices**

**A Summary of Nucleon Distribution Amplitudes**

To make the paper self-contained we collect here the necessary information on the set of nucleon distribution amplitudes that enter the LCSRs in Eq. (3.8) and Eq. (3.8) for the nucleon form factors. For definiteness, we consider the proton distribution amplitudes. The presentation in this Appendix follows Ref. [16].

The nucleon distribution amplitudes can be classified according to twist in the light-cone quantization approach of [27]. Hereby the quark field operators are decomposed in “good” and “bad” or “plus” and “minus” components, respectively: \( q = q^+ + q^- \). To this end it is useful to introduce two light-like vectors \( p = p_+ + z = z_- \), \( p^2 = z^2 = 0 \), where \( p_\mu = P_\mu + O(M^2) \). The projection operators on the “plus” and “minus” components are \( \Lambda_+ = \not{p}/(2pz) \) and \( \Lambda_- = \not{z}/(2pz) \), respectively. The leading twist-3 amplitude is identified as the one containing three “plus” quark fields while each “minus” component introduces an additional unit of twist. This classification is adapted to the applications in hard reactions: Higher twist amplitudes in general produce power-suppressed corrections to physical cross sections.

The distribution amplitudes \( V_1(x_1), \ldots, V_6(x_1) \) in Eq. (3.2) can be defined [4, 8, 16] as nucleon-to-vacuum transition matrix elements of non-local operators built of quark fields.
with definite helicity

\[ q^{\uparrow(\downarrow)} = \frac{1}{2}(1 \pm \gamma_5)q \]  (A.1)

and separated by light-like distances. In particular, the leading twist-3 distribution amplitude \( V_1 \) corresponding to the \((u^+u^+d^-)\) component of the three-quark operator can be written as

\[
\langle 0 | \varepsilon^{ijk} [ u_i(a_1z) C \not{u}_j(a_2z) ]_{[10]} | \not{z}d_k^\dagger(a_3z) | P \rangle = -\frac{1}{2}pz \not{z}N^\dagger \int \mathcal{D}x \, e^{-ipz \sum x_a i} V_1(x_i). \]  (A.2)

Here and below we use the notation

\[
\left[ u_i(a_1z) C \not{u}_j(a_2z) \right]_{[10]} = \frac{1}{2} \left( u_i^\dagger(a_1z) C \not{u}_j(a_2z) + u_i^\dagger(a_1z) C \not{u}_j(a_2z) \right). \]  (A.3)

There exist two twist-4 distributions:

\[
\langle 0 | \varepsilon^{ijk} [ u_i(a_1z) C \not{u}_j(a_2z) ]_{[10]} | \not{p}d_k^\dagger(a_3z) | P \rangle = -\frac{1}{2}pz \not{p}N^\dagger \int \mathcal{D}x \, e^{-ipz \sum x_a i} V_2(x_i),
\]

\[
\langle 0 | \varepsilon^{ijk} [ u_i(a_1z) C \not{\gamma}_\perp \not{p}u_j(a_2z) ]_{[10]} | \gamma^\perp \not{d}_k^\dagger(a_3z) | P \rangle = -pz \not{z}M \not{z}N^\dagger \int \mathcal{D}x \, e^{-ipz \sum x_a i} V_3(x_i),
\]  (A.4)

corresponding to the \((u^+u^+d^-)\) and \((u^+u^-d^+)\) projections, respectively. Here \( \perp \) stands for the projection transverse to \( z \) and \( p \), e.g. \( \gamma^\perp \gamma^\perp = \gamma^\mu g^\mu_\nu \gamma^\nu \) with \( g^\mu_\nu = g^\mu_\nu - (p_\mu z_\nu + z_\mu p_\nu)/pz \). Similarly, the two possible projections with two “minus” fields: \((u^-u^-d^+)\) and \((u^-u^+d^-)\) give rise to two distribution amplitudes of twist-5

\[
\langle 0 | \varepsilon^{ijk} [ u_i(a_1z) C \not{p}u_j(a_2z) ]_{[10]} | \not{z}d_k^\dagger(a_3z) | P \rangle = -\frac{1}{4}M^2 \not{z}N^\dagger \int \mathcal{D}x \, e^{-ipz \sum x_a i} V_5(x_i),
\]

\[
\langle 0 | \varepsilon^{ijk} [ u_i(a_1z) C \not{\gamma}_\perp \not{p}u_j(a_2z) ]_{[10]} | \gamma^\perp \not{p}d_k^\dagger(a_3z) | P \rangle = -pz \not{z}M \not{p}N^\dagger \int \mathcal{D}x \, e^{-ipz \sum x_a i} V_4(x_i),
\]  (A.5)

respectively. Finally, there exists a single twist-6 three-quark distribution amplitude

\[
\langle 0 | \varepsilon^{ijk} [ u_i(a_1z) C \not{p}u_j(a_2z) ]_{[10]} | \not{p}d_k^\dagger(a_3z) | P \rangle = -\frac{1}{4}M^2 \not{p}N^\dagger \int \mathcal{D}x \, e^{-ipz \sum x_a i} V_6(x_i) \quad \text{ (A.6)}
\]

corresponding to the \((u^-u^-d^-)\) projection.

The distribution amplitudes are scale-dependent and can be expanded in contributions of conformal operators that do not mix with each other under the renormalization (to one-loop accuracy). To the next-to-leading conformal spin accuracy the expansion reads [16]:

\[
V_1(x_1, \mu) = 20x_1x_2x_3 \left[ \phi_3^0(\mu) + \phi_3^+(\mu)(1 - 3x_3) \right],
\]

15
\[ V_2(x_i, \mu) = 24x_1x_2 \left( \phi_0^0(\mu) + \phi_4^+(\mu)(1 - 5x_3) \right), \]
\[ V_3(x_i, \mu) = 12x_3 \left[ \psi_0^0(1 - x_3) + \psi_4^+(1 - x_3)(x_1^2 + x_2^2 - x_3(1 - x_3)) + \psi_5^+(\mu)(1 - x_3 - 10x_1x_2) \right], \]
\[ V_4(x_i, \mu) = 3 \left[ \psi_3^0(1 - x_3) + \psi_5^+(\mu)(2x_1x_2 - x_3(1 - x_3)) + \psi_5^+(\mu)(1 - x_3 - 2(x_1^2 + x_2^2)) \right], \]
\[ V_5(x_i, \mu) = 6x_3 \left[ \phi_0^0(\mu) + \phi_5^+(\mu)(1 - 2x_3) \right], \]
\[ V_6(x_i, \mu) = 2 \left[ \phi_0^0(\mu) + \phi_6^+(\mu)(1 - 3x_3) \right], \quad (A.7) \]
so that it involves 14 parameters. Not all of them, however, are independent: They are related to each other by exact QCD equations of motion. The corresponding analysis was carried out in [10]. It turns out that the conformal expansion coefficients in Eq. (A.4) can all be expressed in terms of 6 independent matrix elements of local operators. To the leading conformal spin accuracy (“S-wave”) only two parameters enter, \( f_N \) and \( \lambda_1 \), which were defined in Eq. (2.3) and Eq. (4.2), respectively. One obtains [10]
\[ \phi_3^0 = \phi_6^0 = f_N, \quad \phi_4^0 = \phi_5^0 = \frac{1}{2} (\lambda_1 + f_N), \quad \psi_4^0 = \psi_5^0 = \frac{1}{2} (f_N - \lambda_1). \quad (A.8) \]
To the “P-wave” accuracy four new parameters enter, corresponding to matrix elements of the local operators similar to Eq. (2.3) \( (V_1^d, A_1^u) \) and Eq. (4.2) \( (f_1^d, f_1^u) \) but with an additional covariant derivative. The parameters \( V_1^d, A_1^u \) are leading twist-3 and, in particular, the coefficient \( \phi_3^+(\mu) \) appearing in the first line in Eq. (A.7) is given by
\[ \tilde{\phi}_3^+ \equiv \phi_3^+/f_N = (7/2)(1 - 3V_1^d). \quad (A.9) \]
The parameter \( A_1^u \) describes the component in the leading twist distribution amplitude that is antisymmetric in \( x_1 \leftrightarrow x_2 \) and does not contribute to the sum rules directly. It does produce, however, a contribution to the symmetric parts of higher twist amplitudes. In addition, there are two new parameters \( f_1^d, f_1^u \) that are genuine twist-4. One obtains for twist-4:
\[ \phi_3^- = \frac{5}{4} \left( \lambda_1(1 - 2f_1^d - 4f_1^u) + f_N(2A_1^u - 1) \right), \]
\[ \phi_4^- = \frac{1}{4} \left( \lambda_1(3 - 10f_1^d) - f_N(10V_1^d - 3) \right), \]
\[ \psi_4^- = -\frac{5}{4} \left( \lambda_1(2 - 7f_1^d + f_1^u) + f_N(A_1^u + 3V_1^d - 2) \right), \]
\[ \psi_5^- = -\frac{1}{4} \left( \lambda_1(-2 + 5f_1^d + 5f_1^u) + f_N(2 + 5A_1^u - 5V_1^d) \right), \quad (A.10) \]
for twist-5:
\[ \phi_5^- = \frac{5}{3} \left( \lambda_1(f_1^d - f_1^u) + f_N(2A_1^u - 1) \right), \]
\[ \phi_5^+ = -\frac{5}{6} \left( \lambda_1(4f_1^d - 1) + f_N(3 + 4V_1^d) \right), \]
Table A: Numerical values for the expansion parameters of the nucleon distribution amplitudes.

| QCDSR          | $\lambda_1/f_N$ | $V_1^d$     | $A_1^u$ | $f_1^d$ | $f_1^u$ |
|----------------|-----------------|-------------|---------|---------|---------|
| asymptotic     | $-5.1 \pm 1.7$  | $0.23 \pm 0.03$ | $0.38 \pm 0.15$ | $0.6 \pm 0.2$ | $0.22 \pm 0.15$ |

Note that there is a mismatch between the twist classification of the distribution amplitudes that implies counting powers of the large momentum $p_+$, and the twist classification of the local operator matrix elements: E.g. the parameters of the twist-4 distribution amplitudes depend both on the leading twist-3 matrix elements $f_N, V_1^d, A_1^u$ and the twist-4 matrix elements $\lambda_1, f_1^d, f_1^u$. This “propagation” of lower-twist matrix elements to higher-twist distribution amplitudes is well known and usually referred to as Wandzura-Wilczek contribution. Note that the distribution amplitudes of twist-5 and twist-6 are entirely of Wandzura-Wilczek type since there exist no genuine twist-5 and twist-6 operators to this order in the conformal expansion. QCD sum rule estimates for the twist-3 [8] and twist-4 [16] parameters are given in Table A together with their asymptotic values in the $Q^2 \rightarrow \infty$ limit.

**B The $O(x^2)$ Corrections**

In this Appendix we present the calculation of the $O(x^2)$ corrections to the light-cone expansion of the three-quark operator in Eq. (3.1). We consider the vector Lorentz projection as it is the only one that contributes to the sum rules. We also make use of the important simplification that to leading order in the strong coupling only the bi-local operator with either $d$ or $u$ quark shifted from the zero space-time point enters, as can be seen from Eq. (2.7). Consider first the $d$-quark contribution:

$$x^a \langle 0| \varepsilon^{ijk} [u^i C \gamma_\alpha u^j] (0) d^k(x) | P \rangle = -x^a \left[ \left( \mathcal{V}_1 + \frac{x^2 M^2}{4} \mathcal{V}_1^{M(d)} \right) P_\alpha (\gamma_5 N)_\gamma + \mathcal{V}_2 MP_\alpha (\not\gamma_5 N)_\gamma \\
+ \mathcal{V}_3 M (\gamma_\alpha \gamma_5 N)_\gamma + \mathcal{V}_4 M^2 x_\alpha (\gamma_5 N)_\gamma + \mathcal{V}_5 M^3 x_\alpha (\not\gamma_5 N)_\gamma \right].$$  (B.1)
We remind that \( \mathcal{V}_1 \) starts at leading-twist-3, and hence \( \mathcal{V}_1^{M(d)} \) is of twist-5. We do not show the \( O(x^2) \) corrections to the other Lorentz structures since they are yet higher twist and will be omitted in what follows\(^\dagger\). The invariant functions \( \mathcal{V}_1, \ldots, \mathcal{V}_6 \) can easily be related to the distribution amplitudes of definite twist by taking the light-cone limit of Eq. (B.1), \( x^2 \to 0 \), see Eq. (3.2) \(^\dagger\). On the other hand, the calculation of \( \mathcal{V}_1^{M(d)} \) is not that immediate since in the light-cone limit this contribution vanishes. The meaning of the separation of \( \mathcal{V}_1 \) and \( \mathcal{V}_1^{M(d)} \) is most easily understood upon the short distance expansion \( x_\mu \to 0 \). In this way, the nonlocal “string” operator in the l.h.s. of Eq. (B.1) is Taylor-expanded in a series of local operators with three quark fields and the increasing number of (covariant) derivatives acting on the \( d \)-quark. The separation of the leading twist part of each local operator corresponds to the symmetrisation over all Lorentz indices and the subtraction of traces. Without loss of generality, we can consider the matrix element contracted with an additional factor \( x_\alpha \), see Eq. (B.1), so that the symmetrization is achieved. To subtract the traces, we formally write

\[
\langle 0 | \tilde{\epsilon}^{ijk} [u^i C \not{u}^j] (0) d^k_\gamma(x) | P \rangle = \langle 0 | [\tilde{\epsilon}^{ijk} [u^i C \not{u}^j] (0) d^k_\gamma(x)]_{-1} | P \rangle 
+ \frac{x^2}{4} \int_0^1 dt \frac{\partial^2}{\partial x_\alpha \partial x^\alpha} \langle 0 | \tilde{\epsilon}^{ijk} [u^i C \not{u}^j] (0) d^k_\gamma(t x) | P \rangle .
\]

(B.3)

The same result can be obtained by observing \(^\dagger\dagger\) that the leading-twist nonlocal operator has to satisfy the homogenous Laplace equation

\[
\frac{\partial^2}{\partial x_\alpha \partial x^\alpha} \langle 0 | [\tilde{\epsilon}^{ijk} [u^i C \not{u}^j] (0) d^k_\gamma(x)]_{-1} | P \rangle = 0 .
\]

(B.4)

Using QCD equations of motion the second line in Eq. (B.3) can be simplified to

\[
\frac{\partial^2}{\partial x_\alpha \partial x^\alpha} \tilde{\epsilon}^{ijk} [u^i C \not{u}^j] (0) d^k_\gamma(t x) = 2 t \tilde{\epsilon}^{ijk} [u^i C \gamma^\alpha u^j] (0) D_\alpha d^k_\gamma(t x) + \text{gluons}
\]

\[
= 2 t \partial_\alpha \tilde{\epsilon}^{ijk} [u^i C \gamma^\alpha u^j] (0) d^k_\gamma(t x) + \text{gluons} ,
\]

(B.5)

where \( \partial_\alpha \) is a derivative with respect to the overall translation \(^\dagger\dagger\); for the matrix element we can make the substitution \( \partial_\alpha \to -i P_\alpha \). Inserting this result in Eq. (B.3) we finally obtain

\[
\langle 0 | \tilde{\epsilon}^{ijk} [u^i C \not{u}^j] (0) d^k_\gamma(x) | P \rangle = \langle 0 | [\tilde{\epsilon}^{ijk} [u^i C \not{u}^j] (0) d^k_\gamma(x)]_{-1} | P \rangle 
+ \frac{x^2}{4} (-i 2 P_\alpha) \int_0^1 dt \langle 0 | \tilde{\epsilon}^{ijk} [u^i C \gamma^\alpha u^j] (0) d^k_\gamma(t x) | P \rangle + \text{gluons} .
\]

(B.6)

\(^\dagger\)Strictly speaking, since we are not taking into account twist-6 contributions induced by \( O(x^2) \) corrections to \( \mathcal{V}_2 \) and \( \mathcal{V}_3 \), in order to be consistent we have to discard the contribution of \( \mathcal{V}_6 \) altogether. This contribution to the sum rules appears to be numerically neglible, however.
Notice that the r.h.s. only involves (up to corrections with additional gluons) the already
known distribution amplitudes. This equation therefore allows us to determine \( \mathcal{V}^{M(d)} \) —
which appears on the l.h.s. of Eq. (B.6) — up to gluonic corrections.

First, consider the leading twist contribution. We can write

\[
\langle 0 | \varepsilon^{ijk} [u^i C 
\mathfrak{F} u^j] (0) d^k(x) | P \rangle = - \int \mathcal{D}x \left[ e^{-iP \cdot xx_3} P \cdot x \right]|_t \mathcal{V}_1(\gamma_5 N)_\gamma \\
- \int \mathcal{D}x \left[ e^{-iP \cdot xx_3} (\mathfrak{F} \gamma_5 N) \right]|_t P \cdot x \mathcal{V}_2 M \\
- \int \mathcal{D}x \left[ e^{-iP \cdot xx_3} (\mathfrak{F} \gamma_5 N) \right]|_t \mathcal{V}_3 M + \ldots \quad (B.7)
\]

where [28] \( [e^{-iP \cdot xx_3} (P \cdot x)]_t \) and \([e^{-iP \cdot xx_3} \mathfrak{F}]_t \) are the leading-twist components for the free
fields, defined as the solutions of the corresponding homogeneous Laplace equation. Note
that the factor \( P \cdot x \) in the second line in Eq. (B.7) is not included under the \([\ldots]_t \) bracket
since \( (P \cdot x) \mathcal{V}_2 = 1/(V_1 - V_2 - V_3) \) is a function of momentum fractions only and does not
contain any dependence on the position vector \( x \). The solution can easily be constructed
order by order in the \( (M^2 x^2)^n \) expansion. To the required \( \mathcal{O}(x^2) \) accuracy, the result reads

\[
[e^{-iP \cdot xx_3} (P \cdot x)]_t = (P x) \left[ e^{-iP \cdot xx_3} + \frac{x^2 M^2 x_3^2}{4} \int_0^1 dt e^{-iP \cdot xx_3 t} \right] , \\
[e^{-iP \cdot xx_3} \mathfrak{F}]_t = \mathfrak{F} \left[ e^{-iP \cdot xx_3} + \frac{x^2 M^2 x_3^2}{4} \int_0^1 dt t^2 e^{-iP \cdot xx_3 t} \right] + i P \frac{x^2 x_3^2}{4} \int_0^1 dt e^{-iP \cdot xx_3 t} .
\]

The corresponding contribution to \( \mathcal{V}_1^{M(d)} \) is proportional to the nucleon mass squared and
involves the leading twist distribution amplitude, being an exact analogue of the Nachtmann
power suppressed correction in deep inelastic scattering. The second contribution on the
r.h.s. in Eq. (B.6) is special for the exclusive kinematics since it involves a derivative over the
total translation that vanishes for forward matrix elements. Its explicit form is easily
found by contracting the three-quark matrix element in Eq. (B.1) with \( P_\alpha \) instead of \( x_\alpha \) and
inserting the resulting expression in Eq. (B.6). One gets

\[
\frac{x^2}{4} (-i2P_\alpha) \int_0^1 dt \langle 0 | \varepsilon^{ijk} [u^i \gamma^\alpha u^j] (0) d^k(tx) | P \rangle \\
= \frac{x^2 M^2}{4} \int_0^1 dx \int_0^1 dt t e^{-iP \cdot xx_3 t} (V_1 + V_3)(\gamma_5 N)_\gamma + \ldots \quad (B.9)
\]

where the ellipses stand for other Lorentz structures that do not contribute to \( \mathcal{V}_1^{M(d)} \). Inserting
everything into Eq. (B.3) we arrive at

\[
(P x) \int dx_3 e^{-ix_3 P \cdot x} \mathcal{V}_1^{M(d)}(x_3) = P \cdot x \int \mathcal{D}x x_3^2 \int_0^1 dt e^{-iP \cdot xx_3 t} V_1
\]
In order to solve this equation we expand both sides at short distances and obtain the moments of $V_1^{M(d)}$ with respect to $x_3$ expressed through moments of the distribution amplitudes defined as $V_1^{(d)(n)} = \int dx x_3^n V_1(x_i)$. One finds
\[
\int d x_3 \frac{d^n}{d x_3^n} V_1^{M(d)}(x_3) = -\frac{1}{(n+1)(n+2)} \left[ (-2V_1 + V_3 + V_4 + 2V_5)^{(d)(n+2)} \right] \\
+ \frac{1}{(n+1)(n+3)} \left[ (n+3)V_1^{(d)(n+2)} - \frac{1}{2}(V_1 - V_2)^{(d)(n+2)} - (V_1 + V_3)^{(d)(n+1)} \right],
\]
up to contributions of multiparton distribution amplitudes with extra gluons that have been neglected.

The analysis of the $u$-quark contribution is performed in a similar way. We consider the matrix element
\[
x^\alpha \langle 0 | \epsilon^{ijk} [u^i(0) C \gamma_\alpha u^j(x)] d^k(0) | P \rangle = -x^\alpha \left[ (V_1 + \frac{x^2 M^2}{4} V_1^{M(u)}) P_\alpha (\gamma_5 N)_\gamma \right.

+ V_2 M^2 P_\alpha (\gamma_5 N)_\gamma + V_3 M (\gamma_5 N)_\gamma + V_4 M^2 x_\alpha (\gamma_5 N)_\gamma + V_6 M^3 x_\alpha (\gamma_5 N)_\gamma \right],
\]
and find repeating the same steps that lead to Eq. (B.3):
\[
\langle 0 | \epsilon^{ijk} [u^i(0) C \not{u} u^j(x)] d^k(0) | P \rangle = \langle 0 | \epsilon^{ijk} [u^i(0) C \not{u} u^j(x)] d^k(0) | P \rangle_{lt} \]
\[
+ \frac{x^2}{4} \int_0^1 dt \frac{\partial^2}{\partial x^\alpha \partial x^\alpha} \langle 0 | \epsilon^{ijk} [u^i(0) C \not{u} u^j(t x)] d^k(0) | P \rangle_{lt} \]
\[
= \langle 0 | \epsilon^{ijk} [u^i(0) C \not{u} u^j(x)] d^k(0) | P \rangle_{lt} + \text{gluons},
\]
the only difference being that the term corresponding to a total translation does not arise in this case. For the moments with respect to $x_2$ we get
\[
\int d x_2 x_2^n \frac{d^n}{d x_2^n} V_1^{M(u)}(x_2) = \frac{1}{(n+1)(n+3)} \left[ (n+3)V_1^{(u)(n+2)} - \frac{1}{2}(V_1 - V_2)^{(u)(n+2)} \right]

- \frac{1}{(n+1)(n+2)} \left[ (-2V_1 + V_3 + V_4 + 2V_5)^{(u)(n+2)} \right].
\]

The corresponding expressions in the momentum fraction space are easily obtained by inserting the conformal expansions for $V_1, \ldots, V_6$ and inverting the moment equations. The result reads
\[
V_1^{M(u)}(x_2) = \frac{x_2^2}{24} (\lambda_1 C^u_\lambda + f_N C^u_f),
\]
\[
V_1^{M(d)}(x_3) = \frac{x_3^2}{24} (\lambda_1 C^d_\lambda + f_N C^d_f).
\]
with

\[ C^u_\lambda = -(1 - x_2)^3 \left[ 13 - 20 f_1^d + 3 x_2 + 10 f_1^u (1 - 3 x_2) \right] , \]

\[ C^u_f = (1 - x_2)^3 \left[ 113 + 495 x_2 - 552 x_2^2 + 10 A_1^u (-1 + 3 x_2) + 2 V_1^d (113 - 951 x_2 + 828 x_2^2) \right] , \]

\[ C^d_\lambda = -(1 - x_3) \left[ 11 + 131 x_3 - 169 x_3^2 + 63 x_3^3 - 30 f_1^d (3 + 11 x_3 - 17 x_3^2 + 7 x_3^3) \right] - 12 (3 - 10 f_1^d) \ln [x_3] , \]

\[ C^d_f = -(1 - x_3) \left[ 1441 + 505 x_3 - 3371 x_3^2 + 3405 x_3^3 - 1104 x_3^4 \right. \]

\[ -24 V_1^d (207 - 3 x_3 - 368 x_3^2 + 412 x_3^3 - 138 x_3^4) \left. - 12 (73 - 220 V_1^d) \ln [x_3] \right] . \quad (B.16) \]

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