Model independent analysis of dark matter points to a particle mass at the keV scale

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ABSTRACT

We present a model independent analysis of dark matter (DM) both decoupling ultrarelativistic (UR) and non-relativistic (NR) based on the DM phase-space density \( D = \rho_{DM}/\sigma_{DM}^3 \). We derive explicit formulas for the DM particle mass \( m \) and for the number of ultrarelativistic degrees of freedom \( g_d \) at decoupling. We find that for DM particles decoupling UR both at local thermal equilibrium (LTE) and out of LTE, \( m \) turns to be at the keV scale. For example, for DM Majorana fermions decoupling at LTE the mass results \( m \simeq 0.85 \) keV. For DM particles decoupling NR, \( \sqrt{mT_d} \) results in the keV scale (\( T_d \) is the decoupling temperature) and the \( m \) value is consistent with the keV scale. In all cases, DM turns to be cold DM (CDM). Also, lower and upper bounds on the DM annihilation cross-section for NR decoupling are derived. We evaluate the free-streaming (Jeans') wavelength and Jeans' mass: they result independent of the type of DM except for the DM self-gravity dynamics. The free-streaming wavelength today results in the kpc range. These results are based on our theoretical analysis, astronomical observations of dwarf spheroidal satellite galaxies in the Milky Way and \( N \)-body numerical simulations. We analyze and discuss the results on \( D \) from analytic approximate formulas both for linear fluctuations and the (non-linear) spherical model and from \( N \)-body simulations results. We obtain in this way upper bounds for the DM particle mass which all result below the 100 keV range.

Key words: dark matter – keV mass scale.

1 THE DARK MATTER PARTICLE MASS

Although dark matter was noticed seventy-five years ago (Zwicky 1933; Oort 1940) its nature is not yet known.

Dark matter (DM) must be non-relativistic by the time of structure formation \( (z < 30) \) in order to reproduce the observed small structure at \( \sim 2 - 3 \) kpc.

DM particles can decouple being ultrarelativistic (UR) at \( T_d > m \) or being non-relativistic (NR) at \( T_d < m \) where \( m \) is the mass of the dark matter particles and \( T_d \) the decoupling temperature. We consider in this paper particles that decouple at or out of local thermal equilibrium (LTE).

The DM distribution function \( F_d \) freezes out at decoupling. Therefore, for all times after decoupling \( F_d \) coincides with its expression at decoupling. \( F_d \) is a function of \( T_d, m \) and the comoving momentum of the dark matter particles \( p_c \).

Knowing the distribution function \( F_d(p_c) \), we can compute physical magnitudes as the DM velocity fluctuations and the DM energy density. For the relevant times \( t \) during structure formation, when the DM particles are non-relativistic, we have

\[
\langle \vec{V}^2 \rangle(t) = \langle \frac{p_{ph}^2}{m^2} \rangle(t) = \int \frac{d^3 p_{ph}}{(2\pi)^3} \frac{p_{ph}^2}{m^2} F_d[a(t) p_{ph}] \int \frac{d^3 p_{ph}}{(2\pi)^3} F_d[a(t) p_{ph}] \tag{1}
\]

where we use the physical momentum of the dark matter particles \( p_{ph}(t) \equiv p_c/a(t) \) as integration variable. The scale factor \( a(t) \) is normalized as usual,

\[
a(t) = \frac{1}{1 + z(t)} \quad \text{, \quad } a(\text{today}) = 1 \tag{2}
\]

namely, the physical momentum \( p_{ph}(t) \) coincides today with the comoving momentum \( p_c \).
We can relate the covariant decoupling temperature \( T_d \), the effective number of UR degrees of freedom at decoupling \( g_d \), and the photon temperature today \( T_\gamma \) by using entropy conservation (Kolb & Turner 1990; B"orner 2003; Yao 2006):

\[
T_d = \left( \frac{2}{g_d} \right)^{1/4} T_\gamma, \quad \text{where} \quad T_\gamma = 0.2348 \text{ meV},
\]

and 1 meV = \( 10^{-3} \) eV.

The DM energy density can be written as

\[
\rho_{\text{DM}}(t) = \frac{m g}{2 \pi^2 a(t)^3} I_2 \equiv m n(t),
\]

where

\[
I_2 = \int_0^{\infty} y^2 F_2(y) \, dy.
\]

\( n(t) \) is the number of DM particles per unit volume and we used as integration variable

\[
y = \frac{p_{ph}(t)}{a(t)^3} \Rightarrow \frac{\rho_c}{T_d}.
\]

From eq. (5) at \( t = 0 \) and from the value observed today for \( \rho_{\text{DM}} \) (Komatsu et al. 2003; Yao 2006),

\[
\rho_{\text{DM}} = \Omega_{\text{DM}} \rho_c = 0.228 \rho_c,
\]

where \( \rho_c \) is the critical density.

Using as integration variable \( y \) [eq. (6)], eq. (11) for the velocity fluctuations, yields

\[
\langle V^2 \rangle(t) = \left[ \frac{T_d}{m a(t)} \right]^{2/3} I_2,
\]

where

\[
I_4 = \int_0^{\infty} y^4 F_4(y) \, dy.
\]

Expressing \( T_d \) in terms of the CMB temperature today according to eq. (3) gives for the one-dimensional velocity dispersion,

\[
\sigma_{\text{DM}}(z) = \sqrt{\frac{3}{2}} \langle V^2 \rangle(z) = \frac{2^{1/4}}{3} \frac{1 + z}{g_d} \frac{T_\gamma}{m} \frac{\sqrt{I_1}}{\sqrt{I_2}} = (10)
\]

\[
= 0.05124 \frac{1 + z}{g_d} \frac{\text{keV}}{m} \left( \frac{I_1}{I_2} \right)^{1/2} \frac{\text{km}}{s}.\]

It is very useful to consider the phase-space density invariant under the universe expansion (Bolotovsky, de Vega & Sanchez 2008; Hogan & Dalcanton 2009; Madsen 2001).

\[
D(t) \equiv \frac{n(t)}{(p_{ph}^2(t))^{3/2}} \rho_{\text{DM}}(t) \rho_c (3/3 m^3 \sigma_{\text{DM}}^2(t))^{1/2}.
\]

where we consider the relevant times \( t \) during structure formation when the DM particles are non-relativistic. \( D(t) \) is a constant in absence of self-gravity. In the non-relativistic regime \( D(t) \) can only decrease by collisionless phase mixing or self-gravity dynamics (Lynden-Bell 1967; Tremaine et al. 1984).

We derive a useful expression for the phase-space density \( D \) from eqs. (5), (10) and (12) with the result

\[
D = \frac{g}{2 \pi} \frac{I_2^2}{I_4},
\]

Observing dwarf spheroidal satellite galaxies in the Milky Way (dSphs) yields for the phase-space density today (Wyse & Gilmore 2007) (Gilmore et al. 2007):

\[
\frac{\rho_s}{\sigma_s^2} \sim 5 \times 10^{-3} \text{ keV/cm}^3 \text{(km/s)}^2 = (0.18 \text{ keV})^4.
\]

The precision of these results is about a factor 10.

After the radiation dominated era the phase-space density reduces by a factor that we call \( Z \)

\[
D(0) = \frac{1}{Z} D(z \sim 3200),
\]

Recall that \( D(z) = \frac{\rho_{\text{DM}}}{(3 \sqrt{3} m^4 \sigma_{\text{DM}}^3)} \) [according to eq. (12)] is independent of \( z \) for \( z \gtrsim 3200 \) since density fluctuations were \( \lesssim 10^{-3} \) before the matter dominated era (Dodelson 2003).

The range of values of \( Z \) (which is necessarily \( Z > 1 \)) is analyzed in detail in sec. 2.3 below.

We can express the phase-space density today from eqs. (12) and (13) as

\[
D(0) = \frac{1}{3 \sqrt{3} m^4} \frac{\rho_s}{\sigma_s^2},
\]

Therefore, eqs. (12), (15) and (16) yield,

\[
\frac{\rho_s}{\sigma_s^2} = \frac{1}{Z} \frac{\rho_{\text{DM}}}{\sigma_{\text{DM}}^3}(z \sim 3200),
\]

where \( \rho_{\text{DM}}/\sigma_{\text{DM}}^3(\sim 3200) \) follows from eqs. (12) and (14).

\[
\frac{\rho_{\text{DM}}}{\sigma_{\text{DM}}^3}(z \sim 3200) = 3 \sqrt{3} m^4 \frac{g}{I_4} \frac{I_2^3}{I_4^2}.
\]

We can express \( m \) from eqs. (16)–(18) in terms of \( D \) and observable quantities as

\[
m^4 = \frac{Z}{3 \sqrt{3}} \frac{\rho_s}{\sigma_s^2} = \frac{2 \pi^2}{3} \frac{Z}{\sqrt{3}} \frac{\rho_{\text{DM}}}{\sigma_{\text{DM}}^3} I_2^2,
\]

\[
m = 0.2504 \left( \frac{Z}{g} \right)^{1/4} I_2^2.
\]

Combining this with eq. (5) for \( m \) we obtain the number of ultrarelativistic degrees of freedom at decoupling as

\[
g_d = \frac{2^{3/4}}{\pi^2} \frac{1}{\Omega_{\text{DM}}} \frac{\rho_c}{\sigma_s^2} \left( \frac{T_d}{m} \right)^{1/2} [I_2 I_4]^{1/2},
\]

\[
= 35.96 \frac{Z}{g} \frac{g}{[I_2 I_4]^{1/2}}.
\]

If we assume that dark matter today is a self-gravitating gas in thermal equilibrium described by an isothermal sphere solution of the Lane-Emden equation, the relevant quantity
characterizing the dynamics is the dimensionless variable (de Vega & Sánchez 2002; Destri & de Vega 2007)
\[ \eta = \frac{G m^2 N}{L T} = \frac{2}{3} G L^2 \frac{\rho_s}{\sigma_s^2}, \] (22)

which is bound to be \( \eta \lesssim 1.6 \) to prevent the gravitational collapse of the gas (de Vega & Sánchez 2002; Destri & de Vega 2007). Here \( V = L^2 \) stands for the volume occupied by the gas, \( N \) for the number of particles, \( G \), for Newton’s constant and \( T = \frac{1}{2} m \sigma^2 \) is the gas temperature. (The length \( L \) is similar to the so-called King radius (Binney & Tremaine 1987). Notice however that the King radius follows from the singular isothermal sphere solution while \( L \) is the characteristic size of a stable isothermal sphere solution (de Vega & Sánchez 2002; Destri & de Vega 2007).)

The compilation of recent photometric and kinematic data from ten Milky Way dSphs satellites (Wyse & Gilmore 2007; Gilmore et al. 2007) yields values for the one dimensional velocity dispersion \( \sigma_s \) and \( L \) in the ranges

\[ 0.5 \, \text{kpc} \leq L \leq 1.8 \, \text{kpc}, \quad 6.6 \, \text{km/s} \leq \sigma_s \leq 11.1 \, \text{km/s}. \] (23)

Combining eqs. (12), (17) and (22) yields the explicit expression for the DM particle mass,
\[ m^4 \sim \frac{1}{2 \sqrt{3} G} \frac{Z}{L^2 \sigma_s} = \frac{4 \pi}{\sqrt{3}} \frac{M_{pl} \eta}{L^2} \frac{Z}{L^2 \sigma_s} = 0.5279 \times 10^{-4} \frac{\eta Z}{\rho_s} \frac{10 \, \text{km/s}}{\sigma_s} \left( \frac{\text{kpc}}{L} \right)^2 \left( \text{keV} \right)^4 . \] (24)

This formula provides an expression for the DM particle mass independent of eq. (19). We shall see below that eqs. (19) and (24) yield similar results.

We investigate in the subsequent sections the cases where DM particles decoupled UR or NR both at LTE and out of LTE. We compute there \( m \) and \( \rho_s \) explicitly in the different cases according to the general formulas eqs. (20), (21) and (24).

1.1 Jeans’ (free-streaming) wavelength and Jeans’ mass

It is very important to evaluate the Jeans’ length and Jeans’ mass in the present context (Börner 2003; Gilbert 1968; Bond & Szalay 1983). The Jeans’ length is analogous to the free-streaming wavelength. The free-streaming wavenumber is the largest wavenumber exhibiting gravitational instability and characterizes the scale of suppression of the DM transfer function \( T(k) \) (Bojanovskiy, de Vega & Sánchez 2008).

The physical free-streaming wavelength can be expressed as (Börner 2003; Bojanovskiy, de Vega & Sánchez 2008)
\[ \lambda_{fs}(t) = \lambda_J(t) = \frac{2 \pi}{k_{fs}(t)} \] (25)

where \( k_{fs}(t) = k_J(t) \) is the physical free-streaming wavenumber given by
\[ k_{fs}(t) = \frac{4 \pi G \rho_{DM}(t)}{(V^2)(t)} = \frac{3}{2} \left[ 1 + z(t) \right] \frac{H^2_{0} \Omega_{DM}}{(V^2)(0)} . \] (26)

where we used that \( \rho_{DM}(t) = \rho_{DM}(0) (1 + z)^3 \) and eq. (7).

We obtain the primordial DM dispersion velocity \( \sigma_{DM} \) from eqs. (4), (7) and (17).
\[ \frac{1}{3} (V^2)(0) = \sigma_{DM} = \left( 3 \frac{M_{pl} H_{0}^2 \Omega_{DM}}{Z \rho_s} \right)^{\frac{3}{2}} \] (27)

This expression is valid for any kind of DM particles. Inserting eq. (27) into eq. (26) yields for the physical free-streaming wavelength
\[ \lambda_{fs}(z) = \frac{2 \sqrt{3} \pi}{\Omega_{DM}^{\frac{3}{2}}} \left( \frac{3 M_{pl} H_{0}^2 \Omega_{DM}}{Z \rho_s} \right)^{\frac{3}{2}} \frac{1}{\sqrt{1 + z}} = \frac{16.3}{Z^2} \frac{1}{\sqrt{1 + z}} \, \text{kpc} . \] (28)

where we used 1 keV = 1.563738 \( 10^{29} \) (kpc)\(^{-1} \).

Notice that \( \lambda_{fs} \) and therefore \( \lambda_J \) turn to be independent of the nature of the DM particle except for the factor \( Z \).

The approximated analytic evaluations in sec. 2 together with the results of N-body simulations (Peirani et al. 2006; Hoffman et al. 2007; Lapi & Cavaliere 2004; Romano-Diaz et al. 2004; Vass et al. 2009) indicate that for dSphs \( Z \) is in the range
\[ 1 < Z < 10000 . \]

Therefore, \( 1 < Z^2 < 21.5 \) and the free-streaming wavelength results in the range
\[ 0.757 \frac{1}{\sqrt{1 + z}} \, \text{kpc} < \lambda_{fs}(z) < 16.3 \frac{1}{\sqrt{1 + z}} \, \text{kpc} . \]

These values at \( z = 0 \) are consistent with the N-body simulations reported in Gao & Theuns (2007) and are of the order of the small DM structures observed today (Wyse & Gilmore 2007; Gilmore et al. 2007).

The Jeans’ mass is given by
\[ M_J(t) = \frac{4}{3} \pi \lambda_J^3(t) \rho_{DM}(t) . \] (29)

and provides the smallest unstable mass by gravitational collapse (Kolb & Turner 1990; Börner 2003). Inserting here eq. (19) for the DM density and eq. (25) for \( \lambda_J(t) = \lambda_{fs}(t) \) yields
\[ M_J(z) = 192 \sqrt{2} \pi^4 \sqrt{\Omega_{DM} M_{pl} H_{0}^2 \rho_s^3 \Omega_{DM}} (1 + z)^{\frac{3}{2}} = \frac{0.4464}{Z} 10^7 M_\odot (1 + z)^{\frac{3}{2}} . \] (30)

Taking into account the \( Z \)-values range yields
\[ 0.4464 10^3 M_\odot < M_J(z) (1 + z)^{\frac{3}{2}} < 0.4464 10^7 M_\odot . \]

This gives masses of the order of galactic masses \( \sim 10^{11} M_\odot \) by the beginning of the MD era \( z \sim 3200 \). In addition, the comoving free-streaming wavelength scale by \( z \sim 3200 \)
\[ 3200 \times \lambda_{fs}(z \sim 3200) \sim 100 \, \text{kpc} , \]
turns to be of the order of the galaxy sizes today.

2 THE PHASE-SPACE DENSITY \( \mathcal{Z} \) FROM ANALYTIC APPROXIMATION METHODS AND FROM N-BODY SIMULATIONS

We analytically derive here formulas for the reduction factor \( Z \) defined by eq. (15) in the linear approximation and in the
spherical model. The results obtained (see Table I) are in fact upper bounds for $Z$. We then analyze the results on $D$ from $N$-body simulations.

### 2.1 Linear perturbations

The simplest calculation of $D$ follows by considering linear perturbations around the homogeneous distribution $\rho_{DM}(z)$ as

$$\rho = \rho_{DM}(z) \left[ 1 + \delta(z, k) \right].$$

We have $\rho_{DM}(z) = \rho_{DM}(0) \left( 1 + z \right)^{3}$ and in a matter dominated universe

$$\delta(z, k) \sim \delta(t) \frac{1 + z}{1 + z_i}.$$  \hspace{1cm} (32)

The peculiar velocity in the MD universe behaves as (Dodelson 2003),

$$v \sim a H \delta \sim 1/\sqrt{1 + z}.$$ \hspace{1cm} (33)

We can thus relate the phase-space density at redshift $z$ $D(z) \sim \rho/v^3$ eq. (12) with the phase-space density at redshift $z_i$ as,

$$D(z) \sim D(z_i) \left( \frac{1 + z}{1 + z_i} \right)^{\frac{2}{3}}.$$ \hspace{1cm} (34)

Since the linear approximation is valid for $|\delta|^2 \ll 1$, we find from eq. (32) that eq. (34) applies in the redshift range ($z_i, z$) where

$$1 + z \gg (1 + z_i) \delta(t).$$ \hspace{1cm} (35)

We can apply eq. (34) to relate $D(z)$ at equilibrium ($z = z_i \simeq 3200$) and $D(z)$ at the beginning of structure formation $z \sim 30$, since $\delta(t) \sim 10^{-3}$ as a scale average of the density fluctuations at the end of the RD dominated era (Dodelson 2003) and eq. (35) is satisfied. We thus obtain from eq. (34) in the linear approximation:

$$D(z \simeq 3200)$$

$$D(z \simeq 30) \sim 1.3 \times 10^3.$$ \hspace{1cm} (36)

Notice that eq. (35) does not hold for $z \sim 0$. Therefore, in order to evaluate $Z$, we should combine the linear approximation result eq. (36) for $30 \lesssim z \lesssim 3200$ with the results of $N$-body simulations for $0 \lesssim z \lesssim 30$. This is done in sec. 2.3 to obtain upper bounds for $Z$.

### 2.2 The spherical model

Let us now consider the spherical model where particles only move in the radial direction where $D(z)$ is invariant under the universe expansion except for the self-gravity dynamics that diminishes $D(z)$ in its evolution (Lynden-Bell 1967; Tremaine et al. 1986). Numerical simulations show that $D(z)$ decreases sharply during phases of violent mergers followed by quiescent phases (Peirani et al. 2019). We can thus relate the phase-space density at redshift $z$ $D(z) \sim \rho/v^3$ eq. (12) with the phase-space density at redshift $z_i$ as,

$$D(z) \sim D(z_i) \left( \frac{1 + z}{1 + z_i} \right)^{\frac{2}{3}}.$$ \hspace{1cm} (34)

The spherical shell reaches its maximum radius of expansion $R_m = R_i/\delta_i$ at $\theta = \pi$ and then it turns around and collapses to a point at $\theta = 2 \pi$. However, well before that, the approximation that matter only moves radially and that random particle velocities are small will break down. Actually, the DM relaxes to a virialized configuration where the velocity and the virial radius follow from the virial theorem (Padmanabhan 1999)

$$v^2 = \frac{6 G M}{5 R_m}, \quad R_v = \frac{1}{2} R_m.$$  \hspace{1cm} (40)

We can now compute the initial phase-space density at $z_i$ and the phase-space density at virialization. We get at $z_i$ from eqs. (12) and (39)

$$D_i = \frac{\rho_{DM}(0)}{3 \sqrt{3} m^4} (1 + z_i)^3 \left( \frac{3 t_i}{2 R_i} \right)^3.$$ \hspace{1cm} (41)

and at virialization for $\theta = 2 \pi$ from eqs. (12) and (40)

$$D_v = \frac{\rho_{DM}(0)}{3 \sqrt{3} m^4} (1 + z_i)^3 \left( \frac{t_i}{R_i} \right)^3 \frac{32}{9 \pi^2} \left( \frac{15}{4} \delta_i \right)^{\frac{2}{3}}.$$ \hspace{1cm} (42)

Therefore, the $Z$ factor in the spherical model takes the value

$$Z = \frac{D_v}{D_i} = \frac{9}{32} \left( \frac{3}{5 \delta_i} \right)^{\frac{2}{3}} = \frac{1.2909}{\delta_i^{\frac{2}{3}}}.$$ \hspace{1cm} (43)

Setting $\delta_i \sim 10^{-3}$ as a scale average of the density fluctuations at the end of the RD dominated era (Dodelson 2003) yields

$$Z \sim 4.08 \times 10^3.$$ \hspace{1cm} (43)

The spherical model approximates the evolution as a purely radial expansion followed by a radial collapse. Since no transverse motion is allowed neither mergers, the spherical model result for $Z$ eq. (43) is actually an upper bound on $Z$. The phase-space density $D(z)$ is invariant under the universe expansion except for the self-gravity dynamics that diminishes $D(z)$ in its evolution (Lynden-Bell 1967; Tremaine et al. 1986). Numerical simulations show that $D(z)$ decreases sharply during phases of violent mergers followed by quiescent phases (Peirani et al. 2019).
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| Approximation used     | Upper limit on $Z$     | Upper limit on $m$ 0.5 $Z^+$ keV |
|------------------------|------------------------|----------------------------------|
| Linear fluctuations    | $\sim 1.3 \times 10^{11}$ | 96 keV                           |
| Spherical Model        | $\sim 1.29 \times \delta^4_i \simeq 4.1 \times 10^4$ | 7.1 keV                           |

Table 1. Upper bounds for the $Z$-factor [defined by eq.(19)] and for the mass of the DM particle obtained for two different approximation methods. Notice that only the spherical model takes into account non-linear self-gravity effects. The mass $m$ mildly depends on $Z$ through the power $1/4$. In any case $m$ results in the keV range.

2006; Hoffman et al. 2007; Lapi & Cavaliere 2009; Romano-Diaz et al. 2006, 2007; Vass et al. 2009). $\Delta(z)$ decreases at these violent phases by a factor of the order $\gtrsim 1$. (See fig. 3 in Peirani et al. 2006, fig. 1 in Hoffman et al. 2007, fig. 6 in Lapi & Cavaliere 2009; and fig. 5 in Vass et al. 2009). These sharp decreases of $\Delta$ are in agreement with the linear approximation of section 2.1 as we show below.

A succession of several violent phases happens during the structure formation stage ($z \lesssim 30$). Their cumulated effect together with the evolution of $\Delta$ for $3200 \gtrsim z \gtrsim 30$ produces a range of values of the $Z$ factor which we can conservatively estimate on the basis of the $N$-body simulations results Peirani et al. 2006; Hoffman et al. 2007; Lapi & Cavaliere 2009; Romano-Diaz et al. 2006, 2007; Vass et al. 2009; and the approximation results eq.(39) and (43). This gives a range of values $1 \lesssim Z \lesssim 10000$ for dSphs.

Indeed, more accurate analysis of $N$ body simulations should narrow this range for $Z$ which depends on the type and size of the galaxy considered.

The dSphs observations, for which the best observational data are available, take mostly into account the cores of the structures since the dSphs have been stripped of their external halos. Hence, the observed values of $\rho_\star/\sigma^2_0$ may be higher than the space-averaged value represented by the right hand side of eqs.(18) and (17). Higher values for $\rho_\star/\sigma^2_0$ correspond to lower values for $Z$.

The approximate formula eq.(43) indicates a sharp decrease of the phase-space density with the redshift. This sharp decreasing is in qualitative agreement with the simulations in the violent phases Peirani et al. 2006; Hoffman et al. 2007; Lapi & Cavaliere 2009; Romano-Diaz et al. 2006, 2007; Vass et al. 2009). This gives a range of values $1 \lesssim Z \lesssim 10000$ for dSphs.

In summary, the linear approximation suggests a reduction of $\Delta$ at each violent phase by a factor $\gtrsim 1$ while such approximation is valid. Successive violent phases can reduce $\Delta$ by a factor up to $\sim 10$ in the range $0 \leq z \leq 30$ as shown in the simulations Peirani et al. 2006; Hoffman et al. 2007; Lapi & Cavaliere 2009; Romano-Diaz et al. 2006, 2007; Vass et al. 2009).

Combining the approximate decrease of $\Delta(z)$ given by eq.(39) with an upper bound of a decrease by a factor $\sim 100$ for the interval $0 \lesssim z \lesssim 30$ yields in the linear approximation the upper bound

$$Z < 1.3 \times 10^{11},$$

and we have eq.(43) for $Z$ in the spherical model. The fact that $Z$ in the spherical model turns to be several orders of magnitude below the $Z$ value in the linear approximation arises from the fact that the spherical model does include non-linear effects and it is therefore somehow more reliable than the linear approximation.

The range $1 \lesssim Z \lesssim 10000$ for dSphs from $N$-body simulations corresponds to realistic initial conditions in the simulations.

The evolutions in the two approximations considered (see Table I) are simple spatially isotropic expansions, followed by a collapse in the case of the spherical model. There is no possibility of non-radial motion neither of mergers in these approximations contrary to the case in $N$-body simulations. For such reasons, the $Z$-values in Table I are upper bounds to the true values of $Z$ in galaxies. The largest bound on $Z$ yield DM particle masses below $\sim 100$ keV. Moreover, the more reliable spherical model yields 7.1 keV as upper bound for the DM particle mass.

In summary, with realistic initial conditions $\Delta$ will not decrease more than $\lesssim 10000$ and it is therefore fair to assume that $Z < 10000$ for dSphs.

2.4 Synthetic discussion on the evaluation of $Z$ and its upper bounds.

The DM particle mass scale is set by the phase-space density for dSphs eq.(14). Those galaxies are particularly dense and exhibit larger values for $\rho_\star/\sigma^2_0$ than spiral galaxies. Since the primordial phase-space density is an universal quantity only depending on cosmological parameters, the $Z$-factor must be galaxy dependent, larger for spiral galaxies than for dSphs.

We want to stress that the values of the relevant quantities $m$ and $g_d$ are mildly affected by the uncertainty of $Z$ through the factor $Z^4$ [see eqs.(20)–(21)].

Eqs.(43) provides an extreme high estimate for the decrease of $\Delta$ and hence an extreme high estimate for $Z$. The $N$-body simulations show that the violent decrease of $\Delta$ is restricted to a factor of order one at each violent phase Peirani et al. 2006; Hoffman et al. 2007; Lapi & Cavaliere 2009; Romano-Diaz et al. 2006, 2007; Vass et al. 2009).

3 DM PARTICLES DECOUPLING BEING ULTRARELATIVISTIC

3.1 Decoupling at Local Thermal Equilibrium (LTE)

If the dark matter particles of mass $m$ decoupled at a temperature $T_d \gg m$ their freeze-out distribution function

...
only depends on
\[
\frac{p_c}{T_d} = \frac{p_{ph}(t)}{T_d(t)}, \quad \text{where} \quad T_d(t) \equiv \frac{T_d}{a(t)}.
\]
That is, the distribution function for dark matter particles that decoupled in thermal equilibrium takes the form
\[
F_d^{\text{'eq'}[p_c]} = \exp\left[\frac{\sqrt{m^2 + p^2}}{T_d} \right],
\]
where \(F_d^{\text{'eq'}}\) is a Bose-Einstein or Fermi-Dirac distribution function:
\[
F_d^{\text{'eq'}[p_c]} = \frac{1}{\exp\left[\frac{\sqrt{m^2 + p^2}}{T_d} \right] + 1}.
\]
Notice that for eq.(45) in this regime:
\[
\frac{\sqrt{m^2 + p^2}}{T_d} \geq \frac{y}{1 + y} \left(\frac{m^2}{T_d^2}\right).
\]
where \(y\) is defined by eq.(40) and we can use as distribution functions
\[
F_d^{\text{'eq'}[y]} = \frac{1}{\exp\left[\frac{y}{1 + y}\right] + 1}.
\]
Using eqs.(8) and (45), we find then for Fermions and for Bosons decoupling at LTE
\[
m = g_d \left\{ \begin{array}{ll}
3.874 \text{ eV} & \text{Fermions} \\
2.906 \text{ eV} & \text{Bosons}
\end{array} \right.
\]
We see that for DM that decoupled at the Fermi scale: \(T_d \sim 100 \text{ GeV}\) and \(g_d \sim 100\), \(m\) results in the keV scale as already remarked in Bond & Szalay (1983); Pagels & Primack (1982); Bond, Szalay & Turner (1983). DM particles may decouple earlier with \(T_d > 100 \text{ GeV}\) but \(g_d\) is always in the hundreds even in grand unified theories where \(T_d\) can reach the GUT energy scale. Therefore, eq.(47) strongly suggests that the mass of the DM particles which decoupled UR in LTE is in the keV scale.

It should be noticed that the Lee-Weinberg (Lee & Weinberg 1977; Sato & Kobayashi 1977; Vysotsky, Dolgov & Zeldovich 1979) lower bound as well as the Cowick-McClelland (Cowick & McClelland 1972) upper bound follow from eq.(8) as shown in Bovannovsky, de Vega & Sanchez (2002a).

Computing the integrals in eq.(13) with the distribution functions eq.(46) yields for DM decoupling UR in LTE
\[
\mathcal{D} = g \left\{ \begin{array}{ll}
\frac{1}{\pi^2} \sqrt{\frac{\zeta(3)}{\zeta(5)}} = 1.9625 \times 10^{-3} & \text{Fermions} \\
\frac{1}{\pi^2} \sqrt{\frac{\zeta(3)}{\zeta(5)}} = 3.6569 \times 10^{-3} & \text{Bosons}
\end{array} \right.
\]
where \(\zeta(3) = 1.2020569\ldots\) and \(\zeta(5) = 1.0369278\ldots\).

Inserting the distribution function eq.(46) into eqs.(19) and (24) for \(m\) and \(g_d\), respectively, we obtain
\[
\mathcal{m} = \left(\frac{Z}{g}\right)^\frac{3}{2} \text{ keV} \left\{ \begin{array}{ll}
0.568 & \text{Fermions} \\
0.484 & \text{Bosons}
\end{array} \right.
\]
\[
g_d = \frac{g_d}{\eta^2} Z^\frac{5}{2} \left\{ \begin{array}{ll}
155 & \text{Fermions} \\
180 & \text{Bosons}
\end{array} \right.
\]

A further estimate for the DM mass \(m\) follows by inserting eq.(48) for \(\mathcal{D}\) in eq.(21)
\[
m \sim \left[ \frac{\eta Z}{g} \frac{10 \text{ km/s}}{\sigma_s} \right]^{\frac{1}{2}} \sqrt{\frac{\text{kpc}}{L}} \text{ keV} \left\{ \begin{array}{ll}
0.405 & \text{Fermions} \\
0.347 & \text{Bosons}
\end{array} \right.
\]
Taking into account the observed values for \(\sigma_s\) and \(L\) from eq.(23) and the fact that \(\eta \lesssim 1.6\), \(g \simeq 1 - 4\), \(1 < Z^\frac{5}{2} < 10\), eq.(50) gives again a mass \(m\) in the keV scale as in eq.(49). Both equations (49) and (50) yield a mass larger in 17% for the fermion than for the boson.

We can express the free-streaming wavelength as a function of the DM particle mass from eqs.(28) and (49), with the result,
\[
\lambda_{fs}(z) = \left(\frac{\text{keV}}{m}\right)^\frac{1}{2} \frac{\text{kpc}}{g^\frac{1}{2}} \frac{1}{\sqrt{1 + z}} \left\{ \begin{array}{ll}
7.67 & \text{Fermions} \\
6.19 & \text{Bosons}
\end{array} \right.
\]
We display in fig.1 \(\lambda_{fs}(0)\) in kpc vs. \(m g^\frac{1}{2}\) in keV.

3.2 Decoupling out of LTE
In general, for DM decoupling out of equilibrium, the DM particle distribution function takes the form
\[
F_d(p_c) = F_d \left(\frac{p_c}{T_d} \frac{m}{T_d} \ldots\right)
\]
Typically, thermalization is reached by the mixing of the particle modes and scattering between particles that redistributes the particles in phase space: the larger momentum modes are populated by a cascade whose front moves towards the ultraviolet akin to a direct cascade in turbulence, leaving in its wake a state.
of nearly LTE but with a lower temperature than that of equilibrium (Boyanovsky, Destri & de Vega 2004; Destri & de Vega 2006). Hence, in the case the dark matter particles are not yet at thermodynamical equilibrium at decoupling, their momentum distribution is expected to be peaked at smaller momenta since the ultraviolet cascade is not yet completed (Boyanovsky, Destri & de Vega 2004; Destri & de Vega 2006). The freeze-out of equilibrium distribution function can be then written as

$$F_{\text{out of LTE}}(p_c) = \frac{F_0}{\xi T_d} \left[ \frac{a(t)}{p_{th}(t)} \right] \theta(p_c - p_{\text{c}}),$$

(53)

where $\xi = 1$ at thermal equilibrium and $\xi < 1$ before thermodynamical equilibrium is attained. $F_0 \sim 1$ is a normalization factor and $p_{\text{c}}^0$ cuts the spectrum in the UV region not yet reached by the cascade.

Inserting the out of equilibrium distribution eq. (53) in the expression for the DM particle mass eq. (8) and using eq. (40), we obtain the generalization of eq. (47) for the out of LTE case:

$$m = \frac{g_d}{g} m_{\text{out of LTE}} \left\{ \begin{array}{ll} 3.593 & \text{Fermions} \\ 2.695 & \text{Bosons} \end{array} \right.,$$

(54)

where $s = p_{\text{c}}^0/\xi T_d$. Here we used eq. (40) and

$$F_{\pm}(s) = \int_0^{s} \frac{y^2 dy}{c^0 + 1}, \quad F_+(\infty) = \frac{3}{2} \zeta(3), \quad F_-(\infty) = 2 \zeta(3),$$

(55)

Inserting the out of equilibrium distribution eq. (53) into eqs. (19) and (21) for $m$ and $g_d$, respectively, and using eq. (40), we obtain the estimates

$$m \sim \left( \frac{Z}{g} \right)^{\frac{4}{3}} W_{\pm}(s) \text{ keV} \left\{ \begin{array}{ll} 0.568 & \text{Fermions} \\ 0.484 & \text{Bosons} \end{array} \right..$$

(56)

where

$$W_{\pm}(s) \equiv \left( \frac{G_{\pm}(s) F_{\pm}(\infty)}{G_{\pm}(\infty) F_{\pm}(\infty)} \right)^{\frac{1}{2}}, \quad X_{\pm}(s) = \left( \frac{G_{\pm}(s) F_{\pm}(s)}{G_{\pm}(\infty) F_{\pm}(\infty)} \right)^{\frac{1}{2}}.$$

(58)

Here $F_{\pm}(s)$ is defined by eq. (57) and

$$G_{\pm}(s) \equiv \int_0^{s} \frac{y^2 dy}{c^0 + 1}, \quad G_+(\infty) = \frac{45}{2} \zeta(5), \quad G_-(\infty) = 24 \zeta(5).$$

(59)

For small arguments $s$ we have:

$$W_+(0) = \frac{\sqrt{7}}{5} \left[ \zeta^2(3) \right]^{\frac{1}{3}} = 0.573298 \ldots ,$$

$$W_-(s) = \frac{s^3}{27 \zeta(3)} = 0.475316 \ldots s^3,$$

$$X_+(s) = \frac{s^3}{2025 \zeta(3)} = 0.052992 \ldots s^3,$$

$$X_-(s) = \frac{s^3}{4320 \zeta(3)} = 0.039886 \ldots s^3.$$

As seen in fig. 2

$$W_{\pm}(s) \leq 1 \quad \text{and} \quad X_{\pm}(s) \leq 1 \quad \text{for} \quad s \geq 0.$$

We see from eq. (57) that for relics decoupling out of LTE, $m$ is in the keV range. From eqs. (50)–(54) we see that both $m$ and $g_d$ for relics decoupling out of LTE are smaller than if they would decouple at LTE. In addition, since $X_{\pm}(s)$ vanishes for $s \to 0$, $g_d$ may be much smaller than for decoupling at LTE.

We now generalize eq. (48) for the phase-space density $D$ to the out of LTE case eq. (49). Using eqs. (49), (50) and (57) we have

$$D = g \frac{W^4_{\pm}(s)}{X_{\pm}(s)} \left\{ \begin{array}{ll} 1.9625 \times 10^{-3} & \text{Fermions} \\ 3.6569 \times 10^{-3} & \text{Bosons} \end{array} \right..$$

(60)

Inserting now eq. (60) into eq. (24) leads to the out of LTE generalization of the estimate for the DM particle mass $m$ eq. (54)

$$m \sim \left( \frac{Z}{g} \right)^{\frac{4}{3}} \left( \frac{10 \text{ km} \text{ skm}^{-1}}{\sigma_s} \right)^{\frac{1}{3}} W_{\pm}(s) \sqrt{\frac{kpc}{L}} \text{ keV} \left\{ \begin{array}{ll} 0.405 & \text{Fermions} \\ 0.347 & \text{Bosons} \end{array} \right..$$

(61)

Taking into account the observed values for $\sigma_s$ and $L$ from eq. (23) and the fact that $\eta \lesssim 1.6$, $g \simeq 1 - 4$, $1 < Z^4 < 10$, eq. (61) gives again a mass $m$ in the keV scale as in eq. (54). Both equations (50) and (54) yield a mass larger in 17% for the fermion than for the boson.

3.2.1 An instructive example: Sterile neutrinos decoupling out of LTE.

We consider in this subsection a sterile neutrino $\nu$ as DM particle decoupling out of LTE in a specific model where

Figure 2. $X_+(s)$, $X_-(s)$, $W_+(s)$ and $W_-(s)$ as functions of $s$ according to eq. (53).

155 Fermions
180 Bosons
ν is a singlet Majorana fermion \((g = 2)\) with a Majorana mass \(m_\nu\) coupled with a small Yukawa-type coupling \((\sim 10^{-5})\) to a real scalar field \(\chi\) (Chikashige et al. 1981; Gelmini & Roncadelli 1981; Schechter & Valle 1982; Shaposhnikov & Tkachev 2006; McDonald & Sahn 2009). \(\chi\) is more strongly coupled to the particles in the Standard Model plus to three right-handed neutrinos. As a result, all particles (except \(\nu\)) remain in LTE well after \(\nu\) decouples from them.

The distribution function after decoupling of the sterile neutrino \(\nu\) is known for small coupling \(Y\) to be (Boyanovsky 2008).

\[
F_d^\nu(y) = \frac{\tau}{\sqrt{y}} \frac{g_\nu^2(y)}{n^2} \quad \text{where} \quad g_\nu^2(y) = \sum_{n=1}^{\infty} \frac{e^{-ny}}{n^2} \quad (62)
\]

and the coupling \(\tau\) is in the range \(0.035 \lesssim \tau \lesssim 0.35\) (Boyanovsky 2008).

It is interesting to compare the small \((y \to 0)\) and large momenta \((y \to \infty)\) behaviour of this out of equilibrium distribution \(F_d^\nu(y)\) with the Fermi-Dirac equilibrium distribution eq. (62). We find

\[
\frac{F_d^\nu(y)}{F_d^{\text{equil}}(y)} \to \frac{2}{\sqrt{y}} \frac{\tau \zeta(\frac{5}{2})}{\sqrt{y}} \to \infty, \quad \zeta(\frac{5}{2}) = 1.341 \ldots,
\]

\[
\frac{F_d^\nu(y)}{F_d^{\text{equil}}(y)} \to \frac{\tau}{\sqrt{y}} \to 0.
\]

Therefore, \(F_d^\nu(y)\) exhibits an enhancement compared with the Fermi-Dirac equilibrium distribution for small \((y \to 0)\) and a suppression for large momenta \((y \to \infty)\). Qualitatively, the out of equilibrium distribution eq. (62) exhibits the same effect when compared to the equilibrium distribution \(F_d^{\text{equil}}\) as a consequence of the incomplete UV cascade.

We now evaluate the relevant physical quantities inserting \(F_d^\nu(y)\) in the relevant equations of sect. 3.1. We find for \(m_\nu\) from eqs. (8) and (62)

\[
m_\nu = 2.34 \frac{g_d}{\tau} \text{eV}, \quad (63)
\]

which must be compared with the LTE result for fermions eq. (17) with \(g = 2\).

The phase-space density \(D\) from eqs. (13) and (62) takes the value,

\[
D = \frac{6 \tau \zeta(\frac{5}{2})}{35 \pi \zeta(\frac{7}{2})^2} = 5.627 \times 10^{-3} \tau \quad \text{where} \quad \zeta(7) = 1.0083493 \ldots
\]

(64)

This result is to be compared with the LTE result for fermions eq. (18) with \(g = 2\).

Inserting the sterile neutrino distribution function eq. (62) into eqs. (13) and (21), that take into account the decrease of the phase-space density due to the self-gravity dynamics, we obtain the following mass estimates for the \(\nu\) DM particles that decoupled out of LTE,

\[
m_\nu \sim \left(\frac{2}{\tau}\right)^\frac{1}{4} 0.434 \text{keV}, \quad g_d \sim \tau \sqrt{\frac{2}{\pi}} Z \frac{1}{2} 185. \quad (65)
\]

Again, these formulas must be compared with the LTE result for fermions eqs. (19), \(g_d \sim 100\) corresponds to \(T_d \sim 100\) GeV (see Kolb & Turner (1990)) which is the expected value for \(T_d\) in Boyanovsky (2008).

More precisely, for the typical range \(0.035 \lesssim \tau \lesssim 0.35\), from eq. (65) we find

\[
0.56 \text{keV} \lesssim m_\nu \lesssim 1.0 \text{keV}, \quad 15 \lesssim g_d Z \frac{1}{2} \lesssim 84
\]

while for \(g = 2\) fermions decoupling in LTE, the mass turns to be smaller: \(m_\nu \sim 0.48 \text{keV}\) and \(g_d\) larger: \(g_d Z \frac{1}{2} = 184\) [from eq. (29)].

A further estimate for \(m_\nu\), independent of eq. (65), follows by inserting eq. (64) into eq. (21) valid for a self-gravitating gas of DM:

\[
m_\nu \sim 0.3105 \left[ \frac{\eta Z}{\tau} \frac{10 \text{km/s}}{\sigma_s} \right]^{\frac{1}{2}} \frac{\sqrt{L}}{\text{kpc}} \text{ keV}, \quad (66)
\]

which gives for the typical \(\tau\) range,

\[
0.40 \text{keV} \lesssim m_\nu \lesssim 40 \text{keV}, \quad 10^{-5} \lesssim \frac{\eta}{\tau} \frac{10 \text{km/s}}{\sigma_s} \lesssim 0.72 \text{keV}
\]

[Recall that \(0.1 < Z \frac{1}{2} \lesssim 1\).]

In summary the results for the sterile neutrino decoupling out of LTE in the model of Chikashige et al. (1981); Gelmini & Roncadelli (1981); Schechter & Valle (1982); Shaposhnikov & Tkachev (2006); Boyanovsky (2008) are qualitatively similar to those for fermions decoupling at LTE.

### 4 DM PARTICLES DECOUPLING BEING NON-RELATIVISTIC

Particles decoupling non-relativistic at a temperature \(T_d \ll m\) are described by a freeze-out Maxwell-Boltzmann distribution function depending on

\[
\frac{p_c^2}{T_d} = a^2(t) \frac{p^2_{\text{equ}}(t)}{T_d} = T_d y^2.
\]

That is,

\[
F_d^{\text{equil}}(p_c) = 2 \frac{\tau}{\sqrt{2}} \frac{\pi}{45} \frac{g_d}{m} Y_\infty \left(\frac{T_d}{m}\right)^{-\frac{1}{2}} e^{-\frac{p^2}{2m T_d}} =
\]

\[
= 2 \frac{\tau}{\sqrt{2}} \frac{\pi}{45} \frac{g_d}{m} Y_\infty \left(\frac{T_d}{m}\right)^{-\frac{1}{2}} e^{-\frac{a^2(t) p^2_{\text{equ}}(t)}{2m T_d}} =
\]

\[
= 2 \frac{\tau}{\sqrt{2}} \frac{\pi}{45} \frac{g_d}{m} Y_\infty e^{-\frac{p^2}{2m T_d}} \frac{a^2(t) p^2_{\text{equ}}(t)}{2m T_d}, \quad (67)
\]

where \(g_d\) is the effective number of ultrarelativistic degrees of freedom at decoupling, \(Y(t) = n(t)/s(t)\), \(n(t)\) is the number of DM particles per unit volume, \(s(t)\) their entropy per unit volume, \(x \equiv m/T_d\) and \(Y_\infty\) follows from the late time limit of the Boltzmann equation (Kolb & Turner 1990; Börner 2003).

For particles that decoupled NR we obtain inserting eq. (64) into the general formula for the DM particle mass eq. (8),

\[
m = 4 \frac{\Omega_{DM} p_c}{g T_d^3 Y_\infty} = 0.748 \frac{g T_d^3 Y_\infty}{\Omega_{DM} p_c} = 0.748 \text{ eV}.
\]

Solving the Boltzmann equation gives for \(Y_\infty\) (Kolb & Turner 1990; Börner 2003),

\[
Y_\infty = \frac{45}{4 \sqrt{2} \pi} \frac{g}{x} e^{-x}.
\]
Notice that $x \gtrsim 1$ since the DM particles decoupled NR. $Y_\infty$ can also be expressed in terms of $\sigma_0$ (the thermally averaged total annihilation cross-section times the velocity which appears in the Boltzmann equation) as (Kolb & Turner 1990; Börner 2003),

$$Y_\infty = \frac{1}{\pi} \sqrt{\frac{45}{8} \frac{x}{g_\Delta}} \frac{m_\sigma M_{Pl}}{m}$$  \hspace{1cm} (70)

(We assume for simplicity S-wave annihilation). It follows from this relation and eq. (68) that

$$\sigma_0 = 0.414 \times 10^{-9} \frac{g x}{\sqrt{g_\Delta}} \text{GeV}^2$$  \hspace{1cm} (71)

The transcendental equations (68) and (69) fix the values of $m$ and $x$. They can be combined as

$$\frac{x^2}{x} = 193.5 \frac{g^2}{g_\Delta} \text{keV}.$$  \hspace{1cm} (72)

This equation has solutions for $x > 1$ provided

$$\frac{m}{\text{keV}} > \frac{c}{193.5} \frac{g^2}{g_\Delta} = 0.014 \frac{g_\Delta}{g}.$$  \hspace{1cm} (73)

For $x = m/T_d > 1$ we have the analytic solution of eq. (72)

$$\frac{m}{T_d} = x \approx \log \left( 193.5 \frac{g^2}{g_\Delta} \frac{m}{\text{keV}} \right) = 5.265 + \log \left( \frac{g^2}{g_\Delta} \frac{m}{\text{keV}} \right).$$  \hspace{1cm} (74)

We obtain the mass of the DM particle inserting the non-relativistic distribution function eq. (67) into the general formula eq. (19) for $m^*$ with the result

$$m^* T_d = 45 \frac{2}{\pi^2} \frac{1}{g_\Delta} \frac{Z}{\rho_0} \frac{\rho_0}{\sigma_0}.$$  \hspace{1cm} (75)

Combining eq. (68) for $Y_\infty$ with eq. (75) we obtain for the product $m T_d$

$$\sqrt{m T_d} = 1.47 \frac{Z}{g_\Delta} \text{keV} \text{ NR Maxwell – Boltzmann}.$$  \hspace{1cm} (76)

Typical wimps are assumed to have $m = 100 \text{ GeV}$ and $T_d = 5 \text{ GeV}$ (Particle Data Group 2001). Such value for $T_d$ implies $g_\Delta \simeq 80$ (Kolb & Turner 1990). Eq. (74) thus requires for such heavy wimps $Z \sim 10^{23}$ well above the upper bounds derived in sec. 2 (see Table 1). Therefore, wimps in the 100 GeV scale are strongly disfavoured.

We find from eqs. (13) and (17) the phase-space density for DM decoupling NR in LTE

$$D = \frac{g}{135} \frac{2 \pi^2}{\sqrt{3}} \frac{g_\Delta}{m} Y_\infty \left( \frac{T_d}{m} \right)^{\frac{3}{2}} = 8.4418 \times 10^{-2} g_\Delta g \frac{\text{cm}^2}{\text{g}} Y_\infty x^{-\frac{3}{2}}.$$  \hspace{1cm} (77)

We obtain using here the value for $Y_\infty$ from eq. (68)

$$D = 0.6315 \times 10^{-4} \frac{\text{cm}^2}{\text{g}} g_\Delta \frac{\text{keV}}{m^2} T_d \frac{3}{2}.$$  \hspace{1cm} (76)

Inserting this expression for $D$ into the general estimate for the DM mass eq. (21) yields

$$\sqrt{m T_d} \sim 0.94 \left( \frac{\eta Z}{g_\Delta} \right)^{\frac{1}{2}} \left( \frac{1 \text{ keV}}{L} \right)^{\frac{1}{2}} \left( \frac{10 \text{ km/s}}{\sigma} \right)^{\frac{1}{2}} \text{ keV}.$$  \hspace{1cm} (77)

As in eq. (74) but independently from it, we reach a result for $\sqrt{m T_d}$ in the keV scale assuming the DM is a self-gravitating gas in thermal equilibrium.

### 4.1 Allowed ranges for $m$, $T_d$ and the annihilation cross section $\sigma_0$.

We derive here individual bounds on $m$, $T_d$ and $\sigma_0$ for DM particles decoupling NR.

Using that $T_d < m$ for DM particles that decoupled NR we obtain from eq. (24) a lower bound for $m$ and an upper bound on $T_d$. Furthermore, taking into account that $T_d > b$ eV where $b > 1$ or $b > 1$ for DM particles that decoupled in the RD era, we obtain an upper bound for $m$. In summary,

$$\left( \frac{\eta Z}{g_\Delta} \right)^{\frac{1}{2}} \frac{1.47 \text{ keV}}{m} < \frac{2.16}{b} \text{ MeV} \left( \frac{Z}{g_\Delta} \right)^{\frac{1}{2}},$$  \hspace{1cm} (78)

$$b \text{ eV} < T_d < \left( \frac{Z}{g_\Delta} \right)^{\frac{1}{2}} 1.47 \text{ keV}.$$  \hspace{1cm} (79)

Recalling that (Kolb & Turner 1990)

$$g_\Delta \simeq 3 \quad \text{ for } 1 \text{ eV} < T_d < 100 \text{ keV},$$  \hspace{1cm} (79)

and that $1 < Z < 10^4$, we see from eqs. (78) that

$$1.02 \text{ keV} < m < \frac{482}{b} \text{ MeV}, \quad T_d < 10.2 \text{ keV}.$$  \hspace{1cm} (79)

Notice that $b$ may be much larger than one with $b < 1470 (Z/g_\Delta)^{\frac{1}{2}} < 21960$ to ensure $T_d < m$ for consistency in eqs. (78).

In addition, lower and upper bounds for the cross-section $\sigma_0$ can be derived. From eqs. (71), (76) and $x > 1$ a lower bound follows,

$$\sigma_0 > 0.239 \times 10^{-9} \text{ GeV}^{-2} \text{ g}.$$  \hspace{1cm} (80)

On the other hand, upper bounds for the total DM self-interaction cross sections $\sigma_T$ have been given by comparing X-ray, optical and lensing observations of the merging of galaxy clusters with N-body simulations in Markevitch et al. (2001); Randall et al. (2002); Bradac et al. (2008) (see also Miralda-Escudé (2002); Hennawi & Ostriker 2002; Abazajian et al. 2002):

$$\frac{\sigma_T}{m} < 0.7 \frac{\text{cm}^2}{\text{g}} \simeq 3200 \text{ GeV}^{-2}.$$  \hspace{1cm} (81)

Since the annihilation cross-section must be smaller than the total cross-section we can write the bound

$$\sigma_0 < 3200 m \text{ GeV}^{-3}.$$  \hspace{1cm} (81)

Using the upper bound eq. (78) for $m$ yields the upper bound for $\sigma_0$

$$\sigma_0 < \frac{3.32 \frac{Z^{\frac{1}{2}}}{b}}{\text{ GeV}^{-2}}.$$  \hspace{1cm} (82)

This result leaves at least five orders of magnitude between the lower bound (eq. (24)) and the upper bound for $\sigma_0$. The DM non-gravitational self-interaction is therefore negligible in this context.

Exotic models where very heavy ($\sim 10$ TeV) DM particles are produced very late and decouple non-relativistically were proposed introducing two new fine tuned parameters: (a) the lifetime of unstable particles (neutrinos) that decay into DM (gravitinos) (b) the mass difference between the two particles which must be small enough to lead to non-relativistic DM (Cembranos et al. 2007; Strigari et al. 2007). It is stated in Cembranos et al. 2007.
and Strigari et al. (2007) that such models may describe the observed phase-space density. It is stated in Borzumati et al. (2008) that it is inherently difficult to fulfill all observational constraints in such models.

5 CONCLUSIONS

Our results are independent of the particle model that will describe the dark matter. We consider both DM particles that decouple being NR and UR and both decoupling at LTE and out of LTE. Our analysis and results refer to the mass of the dark matter particle and the number of ultrarelativistic effective degrees of freedom when the DM particles decoupled. We do not make assumptions about the nature of the DM particle and we only assume that its non-gravitational interactions can be neglected in the present context (which is consistent with structure formation and observations).

In case DM particles explain the formation of galactic center black holes, DM particles must be fermions with keV-scale mass (Munyaneza & Biermann 2006).

The mass for the DM particle in the keV range is much larger than the temperature during the MD era, hence dark matter is cold (CDM).

A possible CDM candidate in the keV scale is a sterile neutrino (Dodelson & Widrow 1994; Shi & Fuller 1994; Abazajian, Fuller & Patel 2001; Abazajian 2006; Munyaneza & Biermann 2006; Kusenko 2007) produced via their mixing and oscillation with an active neutrino species. Other putative CDM candidates in the keV scale are the gravitino (Gorbunov et al. 2008; Steffen 2009), the light neutralino (Profumo 2008) and the majoron (Lattanzi & Valle 2007).

Actually, many more extensions of the Standard Model of Particle Physics can be envisaged to include a DM particle with mass in the keV scale and weakly enough coupled to the Standard Model particles.

Lyman-α forest observations provide indirect lower bounds on the masses of sterile neutrinos (Viel et al. 2005, 2007) while constraints from the diffuse X-ray background yield upper bounds on the mass of a putative sterile neutrino DM particle (Dolgov & Hansen 2002; Watson et al. 2006; Bovarsky et al. 2007; Riemer-Sorensen et al. 2006, 2007; Loewenstein et al. 2008). All these recent constraints are consistent with DM particle masses at the keV scale.

The DAMA/LIBRA collaboration has confirmed the presence of a signal in the keV range (Bernabei et al. 2008a). Whether this signal is due to DM particles in the keV mass scale is still unclear (Bernabei et al. 2008b; Pospelov et al. 2008). On the other hand, the DAMA/LIBRA signals cannot be explained by a hypothetical WIMP particle with mass $\gtrsim O(1)$ GeV since this would be in conflict with previous WIMPS direct detection experiments (Aalseth et al. 2008; Fairbairn & Schwetz 2009; Hooper et al. 2009; Savage et al. 2008; Ahmed et al. 2009).

As discussed in sec. 4 typical wimps with $m = 100$ GeV and $T_f = 5$ GeV (Particle Data Group 2009) would require a huge $Z \sim 10^{23}$, well above the upper bounds displayed in Table 1. Hence, wimps cannot reproduce the observed galaxy properties. In addition, recall that $Z \sim 10^{23}$ produces from eq. (29) an extremely short $\lambda_f$, today

$$\lambda_{f_1}(0) \sim 3.51 \times 10^{-4} \text{ pc} = 72.4 \text{ AU}.$$  

If the flyby anomaly would be explained by DM, a keV scale DM mass is preferred (Adler 2004).

Further evidence for the DM particle mass in the keV scale follows by contrasting the observed value of the constant surface density of galaxies to the theoretical calculation from the linearized Boltzmann-Vlasov equation (de Vega & Sánchez 2009). Independent further evidence for the DM particle mass in the keV scale is given by Tikhonov et al. (2009). [See also Gilmore et. al. (2007)].

In summary, our analysis shows that DM particles decoupling UR in LTE have a mass $m$ in the keV scale with $g_d \gtrsim 150$ as shown in sec. 4. That is, decoupling happens at least at the 100 GeV scale. The values of $m$ and $g_d$ may be smaller for DM decoupling UR out of LTE than for decoupling UR in LTE (see sec. 4.2). For DM particles decoupling NR in LTE ($T_d < m$) we find that $\sqrt{m/T}_d$ is in the keV range. This is consistent with the DM particle mass in the keV range.

Notice that the present uncertainty by one order of magnitude of the observed values of the phase-space density $\rho_s/\sigma_v^2$ only affects the DM particle mass through a power $1/4$ of this uncertainty according to eqs. (19)-(20). Namely, by a factor $10^{\mp 1} \approx 1.8$.

We find that the free streaming wavelength (Jeans' length) is independent of the nature of the DM particle except for the $Z$ factor characterizing the decrease of the phase-space density through self-gravity [sec. 1.1]. The values found for the Jeans’ length and the Jeans’ mass for $m$ in the keV scale are consistent with the observed small structure and with the masses of the galaxies, respectively.

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