Study of conversion decays $\phi \to \eta e^+e^-$, $\eta \to e^+e^-\gamma$ and $\eta \to \pi^+\pi^-e^+e^-$ at CMD-2

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Abstract

Using 15.1 pb$^{-1}$ of data collected by CMD-2 in the $\phi$-meson energy range, the branching ratios of the following conversion decays have been measured:

$$B(\phi \to \eta e^+e^-) = (1.14 \pm 0.10 \pm 0.06) \cdot 10^{-4},$$

$$B(\eta \to e^+e^-\gamma) = (7.10 \pm 0.64 \pm 0.46) \cdot 10^{-3},$$

$$B(\eta \to \pi^+\pi^-e^+e^-) = (3.7^{+2.5}_{-1.8} \pm 0.3) \cdot 10^{-4}.$$
The upper limits for the following rare conversion decays have been obtained at the 90% confidence level:

\[ B(\phi \to \eta \mu^+ \mu^-) < 9.4 \cdot 10^{-6}, \]
\[ B(\eta \to e^+e^-e^+e^-) < 6.9 \cdot 10^{-5}. \]

### 1 Introduction

Conversion decays of low mass vector mesons \( \rho, \omega, \phi \to \pi^0(\eta) + e^+e^-(\mu^+\mu^-) \) as well as Dalitz decays \( \pi^0(\eta) \to e^+e^-\gamma, \eta \to \mu^+\mu^-\gamma \) have been extensively discussed \[1\] as one of the main background sources for experiments in which the yield of dileptons was measured in heavy ion collisions \[2, 3\].

By studying spectra of the invariant mass of a lepton pair \( M_{inv}(l^+l^-) \) in such decays one can determine a so called transition form factor as a function of momentum transfer as well as the corresponding branching ratio and test predictions of various theoretical models, from standard vector meson dominance (VMD) to calculations on the lattice \[1, 4, 5, 6\].

The experimental information on such decays is rather scarce \[7\]. While for the \( \omega \) meson both possible conversion decays into \( \pi^0 \) were observed, \( \omega \to \pi^0\mu^+\mu^- \) and \( \omega \to \pi^0e^+e^- \) \[8\], only a few events of the decay \( \phi \to \eta e^+e^- \) were previously detected \[10\]. The Dalitz decay of the \( \pi^0 \) was well studied before \[7\], but the situation with the Dalitz decay \( \eta \to e^+e^-\gamma \) is much worse: only one experiment reported a measurement of its branching ratio \[11\].

A large data sample of the \( \phi \) mesons collected by SND and CMD-2 detectors at the VEPP-2M electron-positron collider dramatically changed the situation: both groups reliably measured the branching ratio of the decays \( \phi \to \eta e^+e^- \) and \( \eta \to e^+e^-\gamma \) \[12, 13\]. CMD-2 has recently published results of the first observation of the \( \phi \to \pi^0e^+e^- \) decay \[14\].

This work is devoted to the determination of the branching ratio for the conversion decays \( \phi \to \eta e^+e^-, \eta \to e^+e^-\gamma \) and \( \eta \to \pi^+\pi^-e^+e^- \) using the complete data sample available at CMD-2. The upper limits for the branching ratios of the conversion decays \( \phi \to \eta \mu^+\mu^- \) and \( \eta \to e^+e^-e^+e^- \) were also obtained.

### 2 Experiment

The general purpose detector CMD-2 installed at the VEPP-2M \( e^+e^- \) collider \[15\] has been described in detail elsewhere \[16\].

It consists of a cylindrical drift chamber (DC) and double-layer multiwire proportional Z-chamber, both also used for the trigger, and both inside a
thin (0.38 $X_0$) superconducting solenoid with a field of 1T. The momentum resolution of the DC is equal to $\sigma_p/p = (\sqrt{90 \cdot (p(\text{GeV}))^2 + 7}) \%$. The accuracy in the measurement of polar and azimuthal angles is $\sigma_\theta = 1.5 \cdot 10^{-2}$ and $\sigma_\phi = 7 \cdot 10^{-3}$ radians respectively.

The barrel calorimeter with a thickness of $8.1X_0$ is placed outside the solenoid and consists of 892 CsI crystals. The energy resolution for photons is about 9% in the energy range from 50 to 600 MeV. The angular resolution is of the order of 0.02 radians.

The end-cap calorimeter placed inside the solenoid consists of 680 BGO crystals. The thickness of the calorimeter for normally incident particles is equal to $13.4X_0$. The energy and angular resolution varies from 8% to 4% and from 0.03 to 0.02 radians respectively for the photon energy in the range 100 to 700 MeV. Both barrel and end-cap calorimeters cover a solid angle of $0.92 \times 4\pi$ steradians.

The experiment was performed in the $\phi$ meson energy range (985-1060 MeV). The integrated luminosity collected during the runs of 1993 (PHI93), 1996 (PHI96) and 1998 (PHI98) was 1.5, 2.1 and 11.5 pb$^{-1}$ respectively, so that our analysis is based on the data sample corresponding to 15.1 pb$^{-1}$.

3 Data analysis of decay $\phi \rightarrow \eta e^+e^-$

3.1 General approach

Three different decay modes of the $\eta$ have been used for this analysis: $\eta \rightarrow 2\gamma, 3\pi^0, \pi^+\pi^-\pi^0$. In the first and third cases event selection was performed using kinematic reconstruction based on energy-momentum conservation. For each decay mode of the $\eta$ the number of detected events due to the decay $\phi \rightarrow \eta e^+e^-$ is given by:

$$N_{\phi \rightarrow \eta e^+e^-} = N_{\phi} \cdot B(\phi \rightarrow \eta e^+e^-) \cdot B(\eta \rightarrow f) \cdot \varepsilon_{\phi \rightarrow \eta e^+e^-},$$  

(1)

where $N_{\phi}$ is the total number of the produced $\phi$ mesons, $B(\phi \rightarrow \eta e^+e^-)$ is the branching ratio of the decay under study, $B(\eta \rightarrow f)$ is the product of the branching ratios for all intermediate decays involved in the decay chain leading to some particular final state $f$ used for selection and $\varepsilon_{\phi \rightarrow \eta e^+e^-}$ is the corresponding detection efficiency.

One of the most important backgrounds to the studied process comes from events of the decay $\phi \rightarrow \eta \gamma$ followed by the $\gamma$ conversion in the material in front of DC (hereafter referred to as “conversion”). Since the DC resolution is not sufficient to separate events with the conversion in the material from
those due to conversion decays at the interaction point, the contribution of this background was calculated from the simulation:

\[ N_{\phi \rightarrow \eta \gamma} = N_{\phi} \cdot B(\phi \rightarrow \eta \gamma) \cdot B(\eta \rightarrow f) \cdot \varepsilon_{\phi \rightarrow \eta \gamma}, \quad (2) \]

where \( \varepsilon_{\phi \rightarrow \eta \gamma} \) is the detection efficiency of the \( \phi \rightarrow \eta \gamma \) decay with the \( \gamma \) conversion in the material.

The total number of observed events is a sum of the two contributions above:

\[ N_{\text{exp}, \phi \rightarrow \eta e^+ e^-} = N_{\phi \rightarrow \eta e^+ e^-} + N_{\phi \rightarrow \eta \gamma}. \quad (3) \]

The following expression for the branching ratio of the decay \( \phi \rightarrow \eta e^+ e^- \) can be obtained from (1), (2) and (3):

\[ B(\phi \rightarrow \eta e^+ e^-) = \frac{N_{\text{exp}, \phi \rightarrow \eta e^+ e^-}}{N_{\phi} \cdot B(\eta \rightarrow f) \cdot \varepsilon_{\phi \rightarrow \eta \gamma} - B(\phi \rightarrow \eta \gamma) \cdot \left( \frac{\varepsilon_{\phi \rightarrow \eta \gamma}}{\varepsilon_{\phi \rightarrow \eta e^+ e^-}} \right)}. \quad (4) \]

Detection efficiencies were determined from Monte Carlo simulation [17]. To take into account effects not properly reproduced by the simulation, the detection efficiencies were if necessary corrected using the information obtained from “clean” experimental events. For example, for all conversion decays into an \( e^+ e^- \) pair the angle between the electron and positron is small. Therefore, to determine the reconstruction efficiency for events with a small opening angle between the tracks, a sample of experimental events of the process \( \phi \rightarrow \pi^+ \pi^- \pi^0, \pi^0 \rightarrow e^+ e^- \gamma \) was used.

For the analysis of the decay mode \( \eta \rightarrow 3\pi^0 \) in which \( e/\pi \) separation described in detail elsewhere [14, 18] was applied, the corresponding correction to the detection efficiency was obtained from the sample of experimental events of the processes \( e^+ e^- \rightarrow e^+ e^- \gamma \) and \( \phi \rightarrow \pi^+ \pi^- \pi^0, \pi^0 \rightarrow 2\gamma \). In this procedure the distributions of the parameter \( E_{\text{CsI}}/p \) were studied for both particle types \( (e/\pi) \) and signs \( (+/-) \), where \( E_{\text{CsI}}/p \) is the ratio of the energy deposition in the CsI calorimeter and the momentum of the particle with a track matching the cluster in CsI. For each event the probability \( W_{e^+ e^-} \) for two tracks to be an \( e^+ e^- \) pair is determined from the probability density functions for the parameter \( E_{\text{CsI}}/p \) as a function of particle momentum.

In all cases the number of the \( \phi \) mesons was determined from a process with a similar final state, so that some of systematic uncertainties cancel. It was specially checked that the difference in the energy dependence of the process under study and the normalization one produced negligible effect on the final results.


3.2 Selection of $\phi \to \eta e^+e^-$ by $\eta \to \gamma\gamma$ mode

In this process the final state contains two charged particles and two photons, $B(\eta \to f) = B(\eta \to \gamma\gamma)$.

The following selection criteria were used for the decay $\phi \to \eta e^+e^-$:

- the angle between two tracks $\Delta \psi < 0.5$;
- the number of photons taken for the kinematic reconstruction $N_{KREC}^\gamma = 2$, if the number of photons in an event $N_\gamma > 2$, the combination with the best $\chi^2$ was chosen;
- the energy of each photon $50 < E_\gamma < 490$ MeV.

Figure 1 shows the distribution of the invariant mass $M_{inv}(\gamma\gamma)$ for the selected events of the PHI98 run. A clear signal is observed at the $\eta$ meson mass which can be fit with a Gaussian, its variance taken from simulation. The number of events in the peak is $167 \pm 18$ and the expected number of “conversion” events is $23 \pm 2$. The detection efficiencies from simulation are $\varepsilon_{\phi\to\eta e^+e^-} = 22.9\%$ and $\varepsilon_{\phi\to\gamma\gamma} = 3.1 \cdot 10^{-4}$.

The selection criteria for the the normalization process $\phi \to \eta\gamma$, $\eta \to \pi^+\pi^-\gamma$ were described elsewhere [14]. The number of detected events for it is $1858 \pm 58$ and the detection efficiency is $20.8\%$. The branching ratio of the decay $\phi \to \eta e^+e^-$ is $(1.10 \pm 0.15) \cdot 10^{-4}$.

![Image of Figure 1: Invariant mass $M_{inv}(\gamma\gamma)$ for $\phi \to \eta e^+e^-$, $\eta \to \gamma\gamma$](image)

Experimental data of the PHI93 and PHI96 runs were processed similarly. The total detected number of the $\phi \to \eta e^+e^-$, $\eta \to \gamma\gamma$ candidates is $214 \pm 20$ with an expected “conversion” background of $31 \pm 2$ events.
The main sources of systematic uncertainties and their contributions to the error of the branching ratio are listed below:

- A limited sample of simulated events used to determine detection efficiencies — 2.6%.
- Statistical errors of the parameters in the correction for a small opening angle — 3.4%.
- Dependence on the transition form factor model — 4.5%.
- The shape of the invariant mass distributions used in the fits — 2.4%.
- The branching ratios of intermediate decays, mainly $B(\phi \to \eta \gamma)$ and $B(\eta \to \pi^+\pi^-\gamma)$ — 3.8%.
- Inaccurate knowledge of the material thickness for the calculations of the "conversion" — 1.3%.

Two first sources of the error are of statistical nature and since their values were obtained for each run separately, they are run-to-run uncorrelated. Their total contribution in each run was quadratically added to the statistical error and the values of the branching ratio in different runs were averaged.

The average value of the branching ratio of the $\phi \to \eta e^+e^-$ decay is $(1.13 \pm 0.14 \pm 0.07) \cdot 10^{-4}$ where the first error is statistical including the uncorrelated systematic errors described above and the second one is a systematic correlated error common for all runs.

### 3.3 Selection of $\phi \to \eta e^+e^-$ by $\eta \to 3\pi^0$ mode

In this case the final state contains two charged particles and 6 photons and $B(\eta \to f) = B(\eta \to 3\pi^0) \cdot (B(\pi^0 \to \gamma\gamma))^3$.

The following selection criteria were used for the decay $\phi \to \eta e^+e^-$:

- $\Delta \psi < 0.5$;
- $N_\gamma \geq 4$;
- $E^\gamma_{\text{max}} < 490$ MeV.

The event by event $e/\pi$-separation described previously was employed, so that $W_{e^+e^-}$ was determined for each event. Figure 2 shows the distribution of the probability $W_{e^+e^-}$ versus the missing mass of tracks $M_{\text{mis}}(e^+e^-)$ for the PHI98 data after these cuts. Events with an $e^+e^-$-pair have $W_{e^+e^-} \sim 1$, while
for those with pions $W_{e/π} \sim 0$. Events of the process $\phi \to \eta e^+e^-$, $\eta \to 3\pi^0$ (region 1 in Fig. 2) are separated from events of the process $\phi \to K_S K_L$, $K_S \to \pi^+\pi^-\gamma$ (region 2 in Fig. 2) and from events of the process $\phi \to K_S K_L$, $K_S \to \pi^0\pi^0$, $\pi^0 \to e^+e^-\gamma$ (region 3 in Fig. 2). It is clear that the following additional cut can efficiently separate events with pions from those with electrons:

- $W_{e/π} > 0.5$.

![Figure 2](image1.png) **Figure 2**: Distribution of the probability $W_{e/π}$ versus the missing mass $M_{mis}(e^+e^-)$ for $\phi \to \eta e^+e^-$, $\eta \to 3\pi^0$.

![Figure 3](image2.png) **Figure 3**: Missing mass $M_{mis}(e^+e^-)$ for $\phi \to \eta e^+e^-$, $\eta \to 3\pi^0$.

Figure 3 shows the distribution of the missing mass $M_{mis}(e^+e^-)$ for the PH198 data after all cuts. The left peak contains the events of the decay under study while the right peak (at $\sim 800$ MeV) corresponds to $\phi \to K_S K_L$, $K_S \to \pi^0\pi^0$, $\pi^0 \to e^+e^-\gamma$ events. The corrected detection efficiencies are $\varepsilon_{\phi\to\eta e^+e^-} = 19.3\%$ and $\varepsilon_{\phi\to\eta\gamma} = 3.9 \cdot 10^{-4}$.

From the fit the number of events in the left peak is $131 \pm 12$, while the expected number of “conversion” events is $23 \pm 12$. Normalizing to the same process $\phi \to \eta\gamma$, $\eta \to \pi^+\pi^-\gamma$ as in subsection 3.2, one obtains the branching ratio of the decay $\phi \to \eta e^+e^-$ ($1.23 \pm 0.16 \cdot 10^{-4}$).

The total number of candidates for the process $\phi \to \eta e^+e^-$, $\eta \to 3\pi^0$ is $158 \pm 13$ events with an expected “conversion” background of $28 \pm 2$ events.

In addition to the sources of systematic uncertainties described in the previous subsection, there is a systematic uncertainty arising from $e/\pi$ separation. Since this procedure is based on the independent data sample and is therefore run-to-run uncorrelated, the corresponding systematic uncertainty
equal to 1.8% can be added to the statistical error of the branching ratio in each run.

The average value of the branching ratio of the $\phi \rightarrow \eta e^+e^-$ decay is 
$$(1.21 \pm 0.14 \pm 0.09) \cdot 10^{-4}.$$ 

3.4 Selection of $\phi \rightarrow \eta e^+e^-$ by $\eta \rightarrow \pi^+\pi^-\pi^0$ mode

For this process the final state contains four charged particles and two photons, $B(\eta \rightarrow f) = B(\eta \rightarrow \pi^+\pi^-\pi^0) \cdot B(\pi^0 \rightarrow \gamma\gamma)$.

Two tracks with opposite charges and a smaller opening angle were assumed to be an $e^+e^-$ pair, while two others — correspondingly a pion pair.

The selection criteria for the normalization process $\phi \rightarrow \pi^+\pi^-\pi^0$, $\pi^0 \rightarrow e^+e^−\gamma$ are:

1. $\Delta \psi(e^+e^-) < 0.3$;
2. $0.5 < \Delta \psi(\pi^+\pi^-) < 2.5$, to suppress $\phi \rightarrow K_SK_L$;
3. the number of photons $N^K_{REC} = 1$;
4. the photon energy $50 < E_\gamma < 250$ MeV.

Figure 4 shows the distribution of the invariant mass $M_{inv}(e^+e^-\gamma)$ for all the data after the cuts. The detection efficiencies from simulation are $\varepsilon_{\pi^0 \rightarrow e^+e^-\gamma} = 4.8\%$ and $\varepsilon_{\pi^0 \rightarrow \gamma\gamma} = 9.4 \cdot 10^{-5}$. The number of events in the peak for this process is $1745 \pm 43$.

The selection criteria for the process under study $\phi \rightarrow \eta e^+e^-$, $\eta \rightarrow \pi^+\pi^-\pi^0$, $\pi^0 \rightarrow \gamma\gamma$ are listed below:

1. $\Delta \psi(e^+e^-) < 0.3$;
2. $0.5 < \Delta \psi(\pi^+\pi^-) < 2.5$;
3. the number of photons $N^K_{REC} = 2$;
4. the photon energy $50 < E_\gamma < 250$ MeV;
5. $M_{inv}(e^+e^-\gamma) > 300$ MeV, to suppress the Dalitz decay of the $\pi^0$;
6. $M_{inv}(\gamma\gamma) < 250$ MeV.

Figure 5 shows the distribution of the missing mass $M_{mis}(e^+e^-)$ for all the data after the cuts. The detection efficiencies are $\varepsilon_{\phi \rightarrow \eta e^+e^-} = 10.4\%$ and $\varepsilon_{\phi \rightarrow \eta\gamma} = 2.3 \cdot 10^{-4}$. The number of $\phi \rightarrow \eta e^+e^-$ events in the peak is $53 \pm 8$
and the expected number of “conversion” events is $11 \pm 1$. The corresponding branching ratio of the decay $\phi \to \eta e^+e^-$ is $(1.04 \pm 0.20 \pm 0.08) \cdot 10^{-4}$.

Table 1 summarizes results of the data processing for the $\phi \to \eta e^+e^-$ decay and shows the number of selected events, the expected number of “conversion” events as well as the obtained branching ratios. It is clear that the values of $B(\phi \to \eta e^+e^-)$ determined from various decay modes of the $\eta$ are consistent within the errors and can be averaged. The averaging procedure took into account that some of the sources of the systematic error like e.g. the branching ratios of the intermediate decays are common for three measurements.

| Mode               | $N_{\phi \to \eta e^+e^-}$ | $N_{\phi \to \eta \gamma}$ | $B(\phi \to \eta e^+e^-)$, $10^{-4}$ |
|--------------------|-----------------------------|-----------------------------|--------------------------------------|
| $\eta \to \gamma\gamma$ | 214 $\pm$ 20               | 31 $\pm$ 2                 | 1.13 $\pm$ 0.14 $\pm$ 0.07          |
| $\eta \to 3\pi^0$      | 158 $\pm$ 13               | 28 $\pm$ 2                 | 1.21 $\pm$ 0.14 $\pm$ 0.09          |
| $\eta \to \pi^+\pi^-\pi^0$ | 53 $\pm$ 8               | 11 $\pm$ 1                 | 1.04 $\pm$ 0.20 $\pm$ 0.08          |
| Total               | 425 $\pm$ 25               | 70 $\pm$ 3                 | 1.14 $\pm$ 0.10 $\pm$ 0.06          |
4 Search for decay $\phi \rightarrow \eta \mu^+ \mu^-$

Events with four tracks and two or more photons were analyzed to search for the decay $\phi \rightarrow \eta \mu^+ \mu^-$, $\eta \rightarrow \pi^+ \pi^- \pi^0$. For conversion decays into a $\mu^+ \mu^-$ pair the angle between muons is not necessarily small.

The analysis employed a kinematic fit taking into account energy-momentum conservation. A pair of tracks with opposite charges and the missing mass closest to the $\eta$ mass was assumed to be a muon pair.

The invariant mass of the photon pair was required to be near the $\pi^0$ mass:

$$|M_{\gamma\gamma} - m_{\pi^0}| < 30 \text{ MeV}.$$  

We restricted the ionization losses of the tracks (in arbitrary units):

$$dE/dx < 2000(1 + 2|\cos \theta|)\left(1 + \frac{(Q + 15Q^2)^2}{p^2}\right),$$

where $Q$ and $p$ are the track charge and momentum (in MeV/c). This requirement suppressed the background from the decay $\phi \rightarrow K^+ K^-$ in which products of kaon nuclear interactions scatter back to the drift chamber and induce two extra tracks or one of the kaons decays via the $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ channel, accompanied by fake photons.

Then a pair of oppositely charged particles with the minimum space angle $\psi_{\text{min}}$ between the tracks was looked for. Assuming it to be an $e^+ e^-$ pair and taking a photon with the smaller energy, the invariant mass $M_{e^+ e^- \gamma}$ was calculated. The requirements $\psi_{\text{min}} > 0.3$ and $M_{e^+ e^- \gamma} > 170$ MeV reduced the background from the reaction $e^+ e^- \rightarrow \omega \pi^0$ with the conversion decay of one of the neutral pions.

To reject the background from the decay $\phi \rightarrow K_SK_L$, $K_S \rightarrow \pi^+ \pi^-$ and $K_L \rightarrow \pi^+ \pi^- \pi^0$ we excluded events in which at least one pair of tracks satisfies the conditions: $|M_{\pi^+ \pi^-} - m_{K_S}| < 30 \text{ MeV}/c^2$ and $|P_{\pi^+ \pi^-} - P_{K_S}| < 30 \text{ MeV}/c$. Here $M_{\pi^+ \pi^-}$ and $P_{\pi^+ \pi^-}$ are the invariant mass and momentum of the pair calculated under the assumption that particles in this pair are pions.

For the twelve events surviving the above conditions, the distribution of the maximum impact parameter of tracks $d$ was analyzed.

The decay under study contributes to the region of small impact parameters, $d < 0.3$ cm, while the main background reaction $\phi \rightarrow K_SK_L$ has a broad distribution of $d$. We have detected $N_{\eta \mu \mu} = 2$ candidate events with $d < 0.3$ cm. Using the number of $K_SK_L$ events $N_{K_SK_L} = 10$ observed in the region $d > 0.3$ cm and the ratio $N_{K_SK_L}(d < 0.3)/N_{K_SK_L}(d > 0.3) = 0.20 \pm 0.03$ obtained from simulation, the number of background events was estimated to be $N_{bg} = 2.0 \pm 0.6$. This leads to an upper limit for the number of signal events: $N_{\eta \mu \mu} < 3.9$ at 90% CL. From simulation the detection efficiency was $\varepsilon = 9.1\%$. The upper limit for the decay probability is $B(\phi \rightarrow \eta \mu^+ \mu^-) < 9.4 \cdot 10^{-6}$ at 90% CL.
5 Data analysis for $\eta$ conversion decays

5.1 General approach

Conversion decays of the $\eta$-meson were studied using the radiative decay of the $\phi$: $e^+e^- \rightarrow \phi \rightarrow \eta \gamma$.

Similarly to the $\phi \rightarrow \eta e^+e^-$ decay, the number of detected events for the conversion decay $\eta \rightarrow Xe^+e^-$ is given by the following expression:

$$N_{\eta \rightarrow Xe^+e^-} = N_\phi \cdot B(\phi \rightarrow \eta \gamma) \cdot B(\eta \rightarrow Xe^+e^-) \cdot \varepsilon_{\eta \rightarrow Xe^+e^-},$$

(5)

where $B(\eta \rightarrow Xe^+e^-)$ is the branching ratio of the $\eta$ decay under study ($X = \gamma, \pi^+\pi^-$) and $\varepsilon_{\eta \rightarrow Xe^+e^-}$ is the corresponding detection efficiency.

The contribution of the background caused by the conversion in the material was calculated from simulation:

$$N_{\eta \rightarrow X\gamma} = N_\phi \cdot B(\phi \rightarrow \eta \gamma) \cdot B(\eta \rightarrow X\gamma) \cdot \varepsilon_{\eta \rightarrow X\gamma},$$

(6)

where $\varepsilon_{\eta \rightarrow X\gamma}$ is the detection efficiency of the $\eta$ decay with the $\gamma$ conversion in the material.

The total number of observed events is a sum of the two contributions above:

$$N_{\eta \rightarrow Xe^+e^-}^{\exp} = N_{\eta \rightarrow Xe^+e^-} + N_{\eta \rightarrow X\gamma}. \quad (7)$$

The following expression for the branching ratio of the decay $\phi \rightarrow \eta e^+e^-$ can be obtained from (3), (6) and (7):

$$B(\eta \rightarrow Xe^+e^-) = \frac{N_{\eta \rightarrow Xe^+e^-}^{\exp}}{N_\phi \cdot B(\phi \rightarrow \eta \gamma) \cdot \varepsilon_{\eta \rightarrow Xe^+e^-} - B(\eta \rightarrow X\gamma) \cdot \left(\frac{\varepsilon_{\eta \rightarrow X\gamma}}{\varepsilon_{\eta \rightarrow Xe^+e^-}}\right).$$

(8)

5.2 Selection of $\eta \rightarrow e^+e^-\gamma$ decay

The final state contains two charged particles and two photons.

The normalization process used for the decay $\eta \rightarrow e^+e^-\gamma$ was the same as for process $\phi \rightarrow \eta e^+e^-$, $\eta \rightarrow \gamma\gamma$ (see subsection 3.2). The selection criteria are also similar to those for $\phi \rightarrow \eta e^+e^-$, $\eta \rightarrow \gamma\gamma$ and include two additional cuts:

- $M_{\text{inv}}(\gamma\gamma) > 250$ MeV to suppress events of the decay $\phi \rightarrow \pi^+\pi^-\pi^0$;
- $|M_{\text{inv}}(\gamma\gamma) - 547.5| > 30$ MeV to suppress events of the conversion decay $\phi \rightarrow \eta e^+e^-$, $\eta \rightarrow \gamma\gamma$. 

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Figure 6 shows the distribution of the invariant mass $M_{inv}(e^+e^-\gamma)$ for the selected events of the PHI98 run.

A clear signal is observed at the $\eta$ meson mass which can be fit with a Gaussian, its variance taken from simulation. The number of events in the peak is $303 \pm 21$ and the expected number of “conversion” events is $38 \pm 3$. The detection efficiencies are $\epsilon_{\eta\rightarrow e^+e^-\gamma} = 19.7\%$ and $\epsilon_{\eta\rightarrow \gamma\gamma} = 5.1 \cdot 10^{-4}$.

The total detected number of the $\phi \rightarrow \eta\gamma$, $\eta\rightarrow e^+e^-\gamma$ candidates from all experimental runs is $374 \pm 24$ with an expected background of $51 \pm 3$ events. The systematic errors are the same as for the process $\phi \rightarrow \eta e^+, \eta \rightarrow \gamma\gamma$. The average value of the branching ratio of the $\eta\rightarrow e^+e^-\gamma$ decay is $(7.10 \pm 0.64 \pm 0.46) \cdot 10^{-3}$.

![Figure 6: Invariant mass $M_{inv}(e^+e^-\gamma)$ for $\phi \rightarrow \eta\gamma$, $\eta\rightarrow e^+e^-\gamma$](image)

![Figure 7: Invariant mass $M_{inv}(\pi^+\pi^+e^+\gamma)$ for $\phi \rightarrow \eta\gamma$, $\eta \rightarrow \pi^+\pi^-e^-$](image)

### 5.3 Selection of $\eta \rightarrow \pi^+\pi^- e^+e^-$ decay

The final state contains four charged particles and one photon.

The normalization process used for the decay $\eta \rightarrow \pi^+\pi^- e^+e^-$ was the same as for the process $\phi \rightarrow \eta e^+, \eta \rightarrow \pi^+\pi^-\pi^0$ (see Section 3.4). The selection criteria were:

- $\Delta\psi(e^+e^-) < 0.5$;
- the number of photons $N_{\gamma}^{KREC} = 1$;
- $M_{inv}(e^+e^-) > 200$ MeV to suppress the Dalitz decay of the $\pi^0$;
• $M_{mis}(e^+e^-) > 600$ MeV to suppress events of the decay $\phi \rightarrow \eta e^+e^-$. 

Figure 7 shows the distribution of the invariant mass $M_{inv}(\pi^+\pi^-e^+e^-)$ for all the data after applying these cuts. The detection efficiencies are $\varepsilon_{\eta \rightarrow \pi^+\pi^-e^+e^-} = 5.04\%$ and $\varepsilon_{\eta \rightarrow \pi^+\pi^-\gamma} = 4.73 \cdot 10^{-5}$. Four events were detected with the expected “conversion” background of 0.4. The background from events of the process $\phi \rightarrow \eta \gamma$, $\eta \rightarrow \pi^+\pi^-\pi^0$, $\pi^0 \rightarrow e^+e^-\gamma$ with one lost photon was estimated from simulation to be less than 0.1 event. The systematic errors are similar to these for the process $\phi \rightarrow \eta e^+e^-$. The branching ratio for the decay $\eta \rightarrow \pi^+\pi^-e^+e^-$ is $(3.7^{+2.5}_{-1.8} \pm 0.3) \cdot 10^{-4}$.

6 Search for decay $\eta \rightarrow e^+e^-e^+e^-$

Selection criteria similar to those for the decay $\eta \rightarrow e^+e^-\pi^+\pi^-$ were used for a search for the decay $\eta \rightarrow e^+e^-e^+e^-:

• $\Delta \psi(e^+e^-)_{1,2} < 0.5$;
• the number of photons $N^K_{\gamma REC} = 1$;
• $M_{inv}(e^+e^-\gamma) > 200$ MeV to suppress the Dalitz decay of the $\pi^0$.

After applying all these cuts no events survive with the invariant mass $M_{inv}(e^+e^-e^+e^-)$ near the $\eta$ mass. The detection efficiencies are $\varepsilon_{\eta \rightarrow e^+e^-e^+e^-} = 17.5\%$ and $\varepsilon_{\eta \rightarrow e^+e^-\gamma} = 9.4 \cdot 10^{-5}$. The normalization process is the same as in the previous case. The corresponding upper limit for the branching ratio of the decay $\eta \rightarrow e^+e^-e^+e^-$ is $6.9 \cdot 10^{-5}$ at the 90% confidence level.

7 Discussion

Table 2 compares CMD-2 results for conversion decays to those of other existing measurements as well as to the theoretical predictions [1, 19, 20]. Some scatter of the predictions is caused by different assumptions about the transition form factors as well as the values of the coupling constants used by the authors. The CMD-2 result for $B(\phi \rightarrow \eta e^+e^-)$ supersedes the previous value based on a small part of the total data sample collected by CMD-2 [21, 22] as well as the preliminary result of [13]. It can be seen that all results of this work are consistent with other measurements and are more precise. Their accuracy is comparable to that of the theoretical predictions.
Table 2: Branching ratios of conversion decays

| Branching ratio | $B(\phi \rightarrow \eta e^+ e^-)$, $10^{-4}$ | $B(\eta \rightarrow e^+ e^- \gamma)$, $10^{-3}$ | $B(\eta \rightarrow \pi^+ \pi^- e^+ e^-)$, $10^{-4}$ |
|-----------------|---------------------------------------------|---------------------------------------------|---------------------------------------------|
| Theory          | 1.0 - 1.2                                  | 6.5 - 6.8                                   | 3.6                                         |
| CMD-2 [this work] | 1.14 ± 0.10 ± 0.06                          | 7.10 ± 0.64 ± 0.46                          | 3.7$^{+2.5}_{-1.8}$ ± 0.3                   |
| SND $^{[23]}$   | 1.19 ± 0.19 ± 0.07                          | 5.15 ± 0.62 ± 0.39                          | —                                           |
| Others          | $1.3^{+0.8}_{-0.6}$ $^{[10]}$                  | 4.9 ± 1.1 $^{[11]}$                          | $13^{+12}_{-8}$ $^{[24]}$                   |

aAfter SND results were published in $^{[23]}$, its authors came to the conclusion that the systematic uncertainties for $B(\phi \rightarrow \eta e^+ e^-)$ and $B(\eta \rightarrow e^+ e^- \gamma)$ were underestimated by a factor of 1.5-2 $^{[25]}$.

A search for the decay $\phi \rightarrow \eta \mu^+ \mu^-$ has been performed for the first time. The obtained upper limit $B(\phi \rightarrow \eta \mu^+ \mu^-) < 9.4 \cdot 10^{-6}$ at 90% CL is a factor of 1.4-1.8 higher than the theoretical prediction of $(5.3-6.8) \cdot 10^{-6}$ $^{[1, 19, 20]}$.

Finally, the upper limit for the branching ratio of the decay $\eta \rightarrow e^+ e^- e^+ e^-$ has been obtained for the first time: $B(\eta \rightarrow e^+ e^- e^+ e^-) < 6.9 \cdot 10^{-5}$ at 90% CL. It is slightly above the theoretical estimate of $6.5 \cdot 10^{-5}$ based on the result of $^{[26]}$.

The applied cut on the angle between the tracks $\Delta \Psi < 0.5$ selects events with a rather small $q^2$ so that it is practically impossible to study the momentum transfer dependence of the cross section and therefore transition form factors. Such a study will require a much larger data sample.

8 Conclusions

Using the total data sample of 15.1 pb$^{-1}$ collected by CMD-2 in the c.m. energy range 985-1060 MeV, the following results were obtained for various conversion decays of the $\phi$ and $\eta$ mesons:

$$B(\phi \rightarrow \eta e^+ e^-) = (1.14 \pm 0.10 \pm 0.06) \cdot 10^{-4},$$
$$B(\eta \rightarrow e^+ e^- \gamma) = (7.10 \pm 0.64 \pm 0.46) \cdot 10^{-3},$$
$$B(\eta \rightarrow \pi^+ \pi^- e^+ e^-) = (3.7^{+2.5}_{-1.8} \pm 0.3) \cdot 10^{-4},$$
$$B(\phi \rightarrow \eta \mu^+ \mu^-) < 9.4 \cdot 10^{-6} \text{ at } 90\% \text{ CL},$$
$$B(\eta \rightarrow e^+ e^- e^+ e^-) < 6.9 \cdot 10^{-5} \text{ at } 90\% \text{ CL}. $$
The authors are grateful to the staff of VEPP-2M for the excellent performance of the collider, to all engineers and technicians who participated in the design, commissioning and operation of CMD-2. We acknowledge stimulating discussions with R.A. Eichler and M.S. Zolotorev.

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