Characterization of transiting exoplanets by way of differential photometry

Michael Cowley$^{1,2}$ and Stephen Hughes$^1$

$^1$ Department of Physics and Astronomy, Faculty of Science, Macquarie University, Sydney, NSW 2109, Australia
$^2$ School of Chemistry, Physics and Mechanical Engineering, Science and Engineering Faculty, Queensland University of Technology, Gardens Point Campus, Brisbane, Queensland 4001, Australia

E-mail: michael.cowley@qut.edu.au and sw.hughes@qut.edu.au

Abstract
This paper describes a simple activity for plotting and characterizing the light curve from an exoplanet transit event by way of differential photometry analysis. Using free digital imaging software, participants analyse a series of telescope images with the goal of calculating various exoplanet parameters, including size, orbital radius and habitability. The activity has been designed for a high-school or undergraduate university level and introduces fundamental concepts in astrophysics and an understanding of the basis for exoplanetary science, the transit method and digital photometry.

Introduction
Since the discovery of the first planet outside our Solar System in 1992 [1], the study and characterization of exoplanets has become one of the most dynamic fields of research in astrophysics. Knowledge in the area has grown exponentially and has contributed to an improved understanding of planetary formation and evolution. Most exoplanet discoveries have resulted from observing the stars, as direct observation is hindered by the small amount of light an exoplanet reflects. One such indirect technique, known as the transit method, has successfully yielded more than one hundred discoveries in the last decade. The transit method involves detection of the slight dip in brightness of a host star when an exoplanet passes in front of it, as viewed by an observer (see figure 1).

In this paper, we outline a simple activity that allows students to analyse a series of telescope images, plot their own transit light curve and calculate a number of exoplanetary system parameters.
characteristics, including its size, orbit distance and likelihood for habitability. This is achieved by way of differential photometry, which is an image processing technique used to compare the relative change in brightness between a target and reference stars in an image. The exercise requires access to a computer with spreadsheeting software, Internet access and the ability to download and install free digital imaging software. It can easily be completed within 3 h in a high-school or undergraduate university classroom setting by the students themselves.

Practical activity
Thanks to a number of observatories sharing data publicly, time-series images of numerous host stars are readily available to download from a number of sources. The activity described in this paper involves the detection, analysis and characterization of the exoplanet WASP-2b, which was originally discovered by the SuperWASP project in 2006 by means of the transit method [2]. The data, comprised of ten time-sequenced images of WASP-2, were sourced from the Las Cumbres Observatory Global Telescope Network [3].

Image analysis
To analyse the time-sequenced images, we used the freely available AstroImageJ by the University of Louisville (see figure 2). AstroImageJ is a modified version of the image processing software, ImageJ [4]. AstroImageJ comes with a detailed manual, but we suggest that educators provide students with a basic overview for the activity.

As mentioned above, this activity involves measuring the brightness of stars to look for any changes that may be consistent with a transit event. Unfortunately, the brightness of each star will appear to fluctuate from frame to frame. When images are photographed with a long exposure, fluctuations are averaged and the final image will result in fuzzy stars. Known as astronomical seeing, this phenomenon only impacts ground-based observations and is a result of the turbulent mixing in the Earth’s atmosphere. When measuring the total brightness of individual stars, two different areas of light are considered—light from the star (smeared due to astronomical seeing) and background light (such as moonlight). The aperture photometry tool in AstroImageJ allows for each of these areas to be assigned as a circular annulus, as shown in figure 3. This tool adds all light that has been smeared by the seeing quality and subtracts all light from the background.

For measurements on the WASP-2b images, it is recommended that the radius of the inner aperture be set to 18, the inner background radius to 40 and the outer background radius

1 www.astro.louisville.edu/software/astroimagej/
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Figure 4. Sample light curve of WASP-2b.

Light curve analysis

The dip in brightness observed during an exoplanet transit event is approximately related to the radii of the planet and the star by

$$\Delta F \approx \left( \frac{R_p}{R_*} \right)^2,$$  

where $F$ is the flux measured from the star, $\Delta F$ is the observed change in flux during transit, $R_p$ is the planet radius and $R_*$ is the stellar radius [5]. Represented visually as a light curve, we can geometrically analyse the results to help to determine a number of characteristics of the exoplanet system. However, a good understanding of the host star is first required to accomplish this, and can be achieved by way of stellar classification.

Stellar classification

If one were to plot the luminosity (intrinsic brightness) versus the colour (surface temperature) of many stars, the resulting scatter plot would show some interesting sequences. Known as the Hertzsprung–Russell diagram, this graph provides a visual representation of stellar populations and evolution (see figure 5). The most distinctive band of stars on the graph is known as the main sequence. It can be seen that the luminosity of main-sequence stars increases with mass and is given by the mass–luminosity relation

$$\frac{L_*}{L_\odot} = \left( \frac{M_*}{M_\odot} \right)^4,$$  

where $L_*$ and $M_*$ are the luminosity and mass of the star, and $L_\odot$ and $M_\odot$ are the luminosity and mass of the Sun. By conducting spectral observations, astronomers can determine the stellar luminosity, and by using known values of the Sun, the mass of the observed star can be determined.

Previous observations of WASP-2 have yielded a luminosity of $1.9 \times 10^{26} \pm 0.2$ W [2]. Using this figure with equation (2), its mass was found to be $0.84 \pm 0.10 M_\odot$.

Geometric analysis

One of the easiest parameters to determine upon the discovery of an exoplanet is its orbital period, $P$. The orbital period can be deduced from the timing of two mid-transit events, and is related to the semi-major axis of the orbit and the total mass...
of the two bodies via Kepler’s third law,

$$\frac{a^3}{P^2} = \frac{G (M_* + M_p)}{4\pi^2} ,$$  \hspace{1cm} (3)

where \( M_* \) and \( M_p \) are the stellar and exoplanet masses respectively, \( G \) is the gravitational constant and \( a \) is the semi-major axis. Thanks to multiple observations of WASP-2b, its orbital period is known to be 2.152 226 ± 0.000 004 days \([2]\). Assuming \( M_* \gg M_p \) and a circular orbit, the semi-major axis of WASP-2b was found to be 0.031 ± 0.001 AU.

The orbital period and semi-major axis are also proportional to the transit duration, which in turn is dependent on the impact parameter, \( b \). The impact parameter is the projected difference between the centre of the stellar disc and the centre of the planet disc at mid-transit, as seen by the observer (see figure 6(a)). The impact parameter can be given by

$$b = a \cos i .$$ \hspace{1cm} (4)

From figure 6(b), it can be seen that the distance the planet travels from the centre of the stellar disc to the edge of contact is the length \( l \). By setting the impact parameter, \( b \), equal to \( a \cos i \), this length can be expressed in terms of the inclination, the semi-major axis and the radii of the star and the planet,

$$l = \sqrt{(R_* + R_p)^2 - a^2 \cos^2 i} .$$ \hspace{1cm} (5)

The length the planet travels across the star is \( 2l \), as seen by the observer. Figure 6(c) shows that when the planet moves from point A to point B, it subtends an angle \( \alpha \). Assuming a circular orbit of \( 2\pi \) radians and that the stellar radius is much larger than the exoplanet radius, the transit duration, \( t \), can be given by

$$t = \frac{P}{\pi} \sin^{-1} \left( \frac{\sqrt{(R_* + R_p)^2 - a^2 \cos^2 i}}{a} \right) .$$ \hspace{1cm} (6)

If we assume an edge-on orbit (90° angle of inclination) and \( a \gg R_* \gg R_p \), equation (6) can be rewritten as

$$t \approx \frac{PR_*}{\pi a} .$$ \hspace{1cm} (7)

Inspection of the plotted light curve reveals that the transit took 1.8 ± 0.2 h and caused a fractional dip in brightness of 0.020 ± 0.005. Using these data, the radius of the star was found to be (from equation (7)) 0.73 ± 0.10\( R_\odot \) and the radius of the planet was found to be (from equation (1)) 1.05 ± 0.27\( R_{\text{Jupiter}} \).

\textbf{Habitable zone}

The distance at which an exoplanet orbits its host star is an important factor in determining its ability to sustain liquid water, a precursor for life as we know it. Given that the Earth lies within a region that is not too cold or too hot for liquid water to exist, our Solar System is used as a template when classifying the habitable zone of another system (see figure 7).

The size and location of the habitable zone are directly related to the luminosity of the stars. The extent of our own habitable zone is debated amongst scientists, but popular models place it at a range of 0.94–1.72 AU from the Sun \([8]\). Using
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| Parameter          | Calculated value          | Accepted value          |
|--------------------|---------------------------|-------------------------|
| WASP-2 mass        | $0.84 \pm 0.10$ solar masses | $0.85 \pm 0.10$ solar masses |
| WASP-2 radius      | $0.73 \pm 0.10$ solar radii   | $0.78 \pm 0.06$ solar radii   |
| WASP-2b radius     | $1.05 \pm 0.27$ Jupiter radii | $0.96 \pm 0.3$ Jupiter radii  |
| Orbital distance   | $0.031 \pm 0.001$ AU      | $0.031 \pm 0.001$ AU      |

The activity described in this paper represents a basic approach to plotting a light curve from a transiting exoplanet. As such, the calculated results represent rough approximations, caused by a number of limiting factors, such as imperfections in the images, low number of reference stars in the field of view and oversimplification of the light curve geometry. Despite this, the results compare well with currently accepted values. To help students to learn the value of scientific method, they should produce their own results and compare them with accepted values for WASP-2b, shown above or from the SuperWASP site².

This activity has been performed in a number of workshops for advanced secondary-school students and university undergraduates. The workshops involved a 1 h introductory session on exoplanets including a practical simulation, similar to the demonstrations performed in [9, 10]. Students were then allocated 2 h to individually produce and analyse the transit light curves, and later performed a presentation on their findings to the class. Student reactions were positive, as they seemed not only to have enjoyed the activity, but also to have learned something new.

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Michael Cowley is a PhD Student at Macquarie University and previous graduate of the Queensland University of Technology. He is currently involved in extragalactic research, with a focus on AGN and the co-evolution of supermassive black holes and galaxy formation.

Stephen Hughes has a BSc in physics and physiology from Queen Elizabeth College, University of London, an MSc in radiation physics from University College London and a PhD in physics from Kings College London. Stephen is currently a senior lecturer in physics and astrophysics at Queensland University of Technology, and is interested in developing hands-on activities for physics and astronomy education.