Deduction of gear system proportion in any leaf rose line drawing

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Abstract. Based on the planetary gear mechanism which realizes the three-leaf rose line drawing method, the ratio of the number of teeth of the sun gear, the planetary gear, the internal gear, and the length of the tracing rod are deduced. The calculation formulas of the number of teeth and the length of the tracing rod for drawing rose lines of any leaf are obtained. SolidWorks is used for modeling and dynamic simulation. The simulation results show that the conclusion is correct.

1. Introduction
The rose line comes from the European chart. In medieval nautical maps, there were no longitudes and latitudes, but only some intersecting straight lines radiated from the center in an orderly way.

To generate rose lines, we can use point by point generation algorithm [1], or we can use computational simulation [2]. You can also draw the rose line of three leaves by designing the planetary gear mechanism [3] (figure 1, figure 2).

Any leaf rose line can be generated by changing the gear ratio of the planetary gear mechanism that generates the three-leaf rose line.

![Figure 1. The initial position of a planetary gear mechanism.](image1)

![Figure 2. The planetary gear mechanism turns an angle.](image2)
When the coefficient of $\theta$ is odd, the number of leaves generating rose line is the same as the coefficient. When the coefficient of $\theta$ is even, the number of leaves generating rose line is twice of the coefficient.

The module of sun gear 1 is $m$, the number of teeth is $z_1$, the diameter of graduation circle is $m z_1$; the module of planet gear 2 is $m$, the number of teeth is $z_2$, the diameter of graduation circle is $m z_2$; the module of internal gear 3 is $m$, the number of teeth is $z_3$, and the diameter of graduation circle is $m z_3$. As shown in the figure, the tracing rod 4 is fixedly connected with the planet wheel 2 and rotates around the axis $O_1$ of the planet wheel 2. The length from the rotation center to the top of the tracing rod is $(m z_1 + m z_2)/2$.

### 2. Calculation of speed ratio of sun wheel 1, planet wheel 2 and planet carrier

If the rotation speed of sun wheel 1 is $\omega_1 = \omega$, then the rotation speed ratio of sun wheel 1 and planet carrier is:

$$ i_{1H} = \frac{\omega_1}{\omega_H} = 1 - i^H_{13} = 1 - \left(-\frac{z_3}{z_1}\right) = \frac{z_1 + z_3}{z_1} \quad (1) $$

The planet carrier speed is:

$$ \omega_H = \frac{\omega_1}{i_{1H}} = \frac{\omega_1}{\frac{z_1 + z_3}{z_1}} = \frac{z_1}{z_1 + z_3} \omega \quad (2) $$

Speed ratio of sun wheel 1 and planet wheel 2:

$$ i_{12}^H = \frac{\omega_1 - \omega_H}{\omega_2 - \omega_H} = -\frac{z_2}{z_1} \quad (3) $$

The speed of planetary gear 2 is:

$$ \omega_2 = -\frac{z_1(z_3 - z_2)}{z_2(z_1 + z_3)} \omega \quad (4) $$

The speed ratio of sun wheel 1, planet carrier and planet wheel 2 is:

$$ \omega_1 : \omega_H : \omega_2 = 1 : \frac{z_1}{z_1 + z_3} : -\frac{z_1(z_3 - z_2)}{z_2(z_1 + z_3)} \quad (5) $$

### 3. Derivation of polar coordinate trace line of point A

The initial position is shown in Figure 1. If the rotation speed of sun wheel 1 is $\omega$, the rotation speed of planet wheel 2 is $\frac{z_1(z_3 - z_2)}{z_2(z_1 + z_3)} \omega$, and the rotation speed of planet carrier is $\frac{z_1}{z_1 + z_3} \omega$. When the sun wheel turns $\omega t$ clockwise, the planet wheel 2 turns $\frac{z_1(z_3 - z_2)}{z_2(z_1 + z_3)} \omega t$, and the planet carrier turns $\frac{z_1}{z_1 + z_3} \omega t$, as shown in Figure 2.

Let the length of $O_1 A$ be $l$, and then $\Delta O_1 A$ is isosceles triangle,

$$ \rho = OA = 2 l \sin \frac{\omega_1 O_1 A}{2} = 2 l \sin \frac{\varphi_1 + \varphi_2}{2} = 2 l \sin \frac{\frac{z_1}{z_1 + z_3} \omega t + \frac{z_1(z_3 - z_2)}{z_2(z_1 + z_3)} \omega t}{2} = 2 l \sin \frac{z_1 z_3}{2 z_2(z_1 + z_3)} \omega t \quad (6) $$

$$ \theta = \frac{\pi}{2} - \frac{z_1}{z_1 + z_3} \omega t - \alpha O_1 O A = \frac{\pi}{2} - \frac{z_1}{z_1 + z_3} \omega t - \frac{\pi - \frac{z_1}{z_2(z_1 + z_3)} \omega t}{2} = \frac{z_1 z_3 - 2 z_2 z_3}{2 z_2(z_1 + z_3)} \omega t \quad (7) $$

### 4. Deduction of tooth number of n-leaf rose line corresponding gear

#### 4.1. Deduction of tooth number of 3-leaf rose line corresponding gear

The three leaf rose line is defined as:

$$ \rho = a \sin 3 \theta \quad (8) $$

Let $2l = a$, take formula (7) into (6), Then formula (6) becomes

$$ \rho = a \sin \frac{z_1 z_3}{z_2 z_3} \theta \quad (9) $$
Let
\[
\frac{z_1 z_3}{z_1 z_3 - 2 z_1 z_2} = 3
\]  
(10)
Formula (9) is the formula of three leaf rose line. At this time, the corresponding gear train can draw three leaf rose line.

From equation (10)
\[
z_2 : z_3 = 1: 3
\]  
(11)
The number of teeth of gear 1:
\[
z_1 = \left(\frac{z_2}{2} - z_2\right) \times 2
\]  
(12)
Length of tracing rod:
\[
l = \frac{m(z_3 - z_2)}{2}
\]  
(13)
Where l is the length of the tracing rod and m is the module of gear.
The length of the planet carrier is the same as that of the tracing rod.

If the modulus is 3, and the number of teeth of gear 2 \( z_2 = 20 \), then from formula (11), the number of teeth of gear 3 \( z_3 = 60 \), from formula (12), the number of teeth of gear 1 \( z_1 = 20 \), and from formula (13), the length of tracing rod is \( l = 60 \text{mm} \).

4.2. Derivation of the number of teeth corresponding to the n-leaf rose line

By substituting different \( n \) values for 3 on the right side of equation (10) and calculating by 3-leaf rose line, we can get different numbers of gear teeth and planet carrier length. Some calculation structures are shown in Table 1.

| Coefficient value | Leaves of rose line | \( z_3 : z_2 \) | \( z_3 \) | \( z_2 \) | \( z_1 \) | Tracing rod (mm) | Planet carrier (mm) | Module (mm) |
|-------------------|--------------------|----------------|-------|-------|-------|------------------|---------------------|-----------|
| 2                 | 4                  | 4:1            | 80    | 20    | 40    | 30               | 30                  | 3         |
| 3                 | 3                  | 3:1            | 60    | 20    | 20    | 60               | 60                  | 3         |
| 4                 | 8                  | 8:3            | 80    | 30    | 20    | 90               | 90                  | 3         |
| 5                 | 5                  | 5:2            | 100   | 40    | 20    | 90               | 90                  | 3         |
| 6                 | 12                 | 12:5           | 120   | 50    | 20    | 150              | 150                 | 3         |
| 50                | 100                | 100:49         | 1000  | 490   | 20    | 1470             | 1470                | 3         |

5. Modeling and trajectory test of the planetary gear system

5.1. Modeling of gear, planet carrier and tracing rod

According to the parameters of the three-leaf rose line, the planetary gear system is modeled.

Gear 3 is an internal gear with 60 teeth, 3 mm module, and 50 mm tooth thickness. It is named CYLINDER_INNER_GEAR.

Gear 2 is an external gear with 20 teeth, 3 mm module, and 40 mm tooth thickness. It is named CYLINDER_GEAR.

Gear 1 is a sun gear with 20 teeth, 3 mm module, and 40 mm tooth thickness. It is named SOLAR_GEAR.

The length of the planet carrier is 60mm, named PLANET_CARRIER. The length of the tracing rod is 60mm, named TRACER. As shown in figure 3.
5.2. Assembly
In SolidWorks, create an assembly named ROSE_CURVE with a subtype of design.
1) Assemble the inner gear in the default way; 2) Then assemble the sun gear in the form of pin, which coincides the central axis of the sun gear with the central axis of the inner gear, And the end faces of the two gears are overlapped; 3) Assemble the planet carrier in the form of pin, which overlaps the central axis of the planet carrier and the central axis of the internal gear, and aligns the end faces of the planet carrier and the internal gear; 4) Assemble the planet gear in the form of pin, which overlaps the axis of the planet gear and the planetary axis of the planet carrier.

5.3. Simulation adding motor and gear pair
Enter SolidWorks's motion module, mesh the teeth of the sun wheel and the planet wheel through cam pair, and then mesh the teeth of the planet wheel and the internal gear through cam pair. Then delete the two cam constraints.

First, create the gear pair connection of the sun wheel and the planet wheel, and then create the gear pair connection of the planet wheel and the internal gear. Through the connection of the two gear pairs, the kinematic relationship of the whole planetary gear train is determined.

5.4. Trace the top of the tracing bar
Set the servo motor to drive the sun wheel to rotate, and carry out the motion analysis. After that, create the trace curve of the endpoint of the tracing rod, and then trace the trace of the 3-leaf rose line.

4.5 change parameters to generate new gear train and new rose line
As shown in Table 1, a new gear system can be generated by changing the parameters of the planetary gear mechanism, including the number of teeth of the internal gear, the planetary gear, the sun gear, the length of the mapping rod, and the planet carrier.

At the same time, it is necessary to change the corresponding gear diameter of two pairs of gear pairs and increase the movement time of the servo motor to generate a new multi-leaf rose line trajectory. As shown in figure 4 and figure 5.

6. Conclusion
Through strict formula derivation, the formula of the parameters of the components of the planetary gear system which can trace the rose line is obtained, and through SolidWorks parametric modeling, assembly, and motion simulation, the expected rose line track is traced. Prove that the derivation is completely correct.

References
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