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Optimizing Power and Thermal Efficiency of an Irreversible Variable-Temperature Heat Reservoir Lenoir Cycle

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Abstract: Applying finite-time thermodynamics theory, an irreversible steady flow Lenoir cycle model with variable-temperature heat reservoirs is established, the expressions of power \( P \) and efficiency \( \eta \) are derived. By numerical calculations, the characteristic relationships among \( P \) and \( \eta \) and the heat conductance distribution \( (L_u) \) of the heat exchangers, as well as the thermal capacity rate matching \( (1/wf HCC) \) between working fluid and heat source are studied. The results show that when the heat conductances of the hot- and cold-side heat exchangers \( (H_U, L_U) \) are constants, \( P, \eta \) is a certain “point”, with the increase of heat reservoir inlet temperature ratio \( (\tau) \), \( U_H, U_L \), and the irreversible expansion efficiency \( (\eta_i) \), \( P \) and \( \eta \) increase. When \( u_k \) can be optimized, \( P \) and \( \eta \) versus \( u_L \) characteristics are parabolic-like ones, there are optimal values of heat conductance distributions \( (u_{L_{(opt)})}, u_{U_{(opt)})} \) to make the cycle reach the maximum power and efficiency points \( (P_{max}, \eta_{max}) \). As \( C_{u_{(opt)}}/C_{i_{(opt)}} \) increases, \( P_{max} - C_{u_{(opt)}}/C_{i_{(opt)}} \) shows a parabolic-like curve, that is, there is an optimal value of \( C_{u_{(opt)}}/C_{i_{(opt)}} \) to make the cycle reach double-maximum power point \( \left( (P_{max})_{max} \right) \); as \( C_{u_{(opt)}}/C_{i_{(opt)}} \), \( U_H \), and \( \eta \) increase, \( (P_{max})_{max} \) and \( C_{u_{(opt)}}/C_{i_{(opt)}} \) increase; with the increase in \( \tau \), \( (P_{max})_{max} \) increases, and \( C_{u_{(opt)}}/C_{i_{(opt)}} \) is unchanged.

Keywords: finite-time thermodynamics; irreversible steady-flow Lenoir cycle; cycle power; thermal efficiency; heat conductance distribution; thermal capacity rate matching

1. Introduction

As a further extension of traditional irreversible process thermodynamics, finite-time thermodynamics (FTT) [1–11] has been applied to analyze and optimize performances of actual thermodynamic cycles, and great progress has been made. FTT has been applied in micro- and nano-cycles [12–15], thermoelectric devices [16,17], thermionic devices [18,19], gas turbine cycles [20–22], internal combustion cycles [23,24], cogeneration plants [25,26], thermoradiative cell [27], chemical devices [28,29], and economics [30,31].

According to the nature of the cycle, the researched heat engine (HEG) cycles include steady flow cycles [32–37] and reciprocating cycles [38–48]. For the steady flow HEG cycle, considering the temperature change of the heat reservoir (HR) can make the cycle closer to the actual working state of the HEG, therefore, some scholars have studied the steady flow cycles with variable temperature HR [49–53].

The Lenoir cycle (LC) model [54] was proposed by Lenoir in 1860. From the perspective of the cycle process, the LC lacks a compression process. It looks like a triangle in the cycle \( T-s \) diagram. It is a typical atmosphere pressure compression HEG cycle, the
compression process required by the HEG during operation is realized by atmosphere pressure and it can be used in aerospace, ships, vehicles, and power plants in engineering practice. Georgiou [55] first used classical thermodynamics to study the steady flow Le-noir cycle (SFLC) and compared its performance with that of a steady flow Carnot cycle.

Compared to the classical thermodynamics, the finite time process of the finite rate heat exchange (HEX) between the system and the environment and the finite size device are considered in the FTT [1–11,56–59], therefore, the result obtained is closer to the actual HEG performance.

Considering the heat transfer loss, Shen et al. [60] established an endoreversible SFLC model with constant-temperature HRs by applying FTT theory, analyzed the influences of HR temperature ratio and total heat conductance (HTC) on the power output ($P$) and efficiency ($\eta$) characteristics, and obtained the maximum $P$ and maximum $\eta$ and the corresponding optimal HTC distributions. Based on the NSGA-II algorithm, Ahmadi et al. [61] optimized the ecological performance coefficient and thermoeconomic performance of the endoreversible SFLC with constant-temperature HRs. Based on the Ref. [60], Wang et al. [62] further considered the internal irreversibility loss, established the irreversible SFLC model and optimized its $P$ and $\eta$ performance.

The above-mentioned were all studies on the SFLC with constant temperature HR. Based on Refs. [60–62], an irreversible SFLC with a variable temperature HR will be established in this paper, and the influence of internal irreversibility, HR inlet temperature ratio, thermal capacity rate (TCR) matching, and total HTC on cycle performance will be studied.

2. Cycle Model and Thermodynamic Performance

Figure 1 shows the $T$-$s$ diagram of an irreversible variable temperature HR SFLC. Process $1 \rightarrow 2$ ($3 \rightarrow 1$) is a constant volume (pressure) endothermic (exothermic) one, and process $2 \rightarrow 3$ is an irreversible expansion one ($2 \rightarrow 3_s$ is the corresponding isentropic one). Assuming the cycle working fluid is an ideal gas, as well as the inlet (outlet) temperature of the hot- and the cold-side fluid are $T_{Hin}$ ($T_{Hout}$) and $T_{Lin}$ ($T_{Lout}$).

Figure 1. Cycle $T$-$s$ diagram.
The irreversible expansion efficiency ($\eta_e$) is defined as [41,44,46,51]:

$$\eta_e = \frac{T_2 - T_i}{T_2 - T_i}$$  \hspace{1cm} (1)

Assuming the heat transfer between the working fluid and HR obeys the law of Newton heat transfer, according to the ideal gas properties and the theory of HEX, the cycle heat absorption and heat release rates are, respectively:

$$Q_{hi} = C_{sf1}(T_2 - T_i) = C_{lima}E_{hi}(T_{lim} - T_i)$$  \hspace{1cm} (2)

$$Q_{li} = C_{sf2}(T_3 - T_i) = C_{limb}E_{li}(T_3 - T_{lim})$$  \hspace{1cm} (3)

where $C_{H}$ ($C_L$) and $C_{sf1}$ ($C_{sf2}$) are heat source and working fluid TCRs ($C_{sf1} = mC_v$, $C_{sf2} = mC_P = kC_{sf1}$), respectively, $\dot{m}$ is the working fluid mass flow rate, $C_v$ ($C_P$) is the constant volume (pressure) specific heat, $k$ is the specific heat ratio. $E_{hi}$ and $E_{li}$ are the effectiveness of hot- and cold-side HEXs, respectively:

$$E_{hi} = \frac{1 - e^{-N_{hi}(1 - C_{lima}/C_{limb})}}{1 - (C_{lima}/C_{limb})e^{-N_{hi}(1 - C_{lima}/C_{limb})}}$$  \hspace{1cm} (4)

$$E_{li} = \frac{1 - e^{-N_{li}(1 - C_{lima}/C_{limb})}}{1 - (C_{lima}/C_{limb})e^{-N_{li}(1 - C_{lima}/C_{limb})}}$$  \hspace{1cm} (5)

where $N_{hi}$ and $N_{li}$ are the heat transfer unit number of the two HEXs, $C_{lima}$ ($C_{limb}$) is the larger (smaller) of $C_H$ and $C_{sf1}$, and $C_{lima}$ ($C_{limb}$) is the larger (smaller) of $C_L$ and $kC_{sf1}$. Their expressions are, respectively:

$$N_{hi} = \frac{U_H}{C_{lima}}$$  \hspace{1cm} (6)

$$N_{li} = \frac{U_L}{C_{limb}}$$  \hspace{1cm} (7)

$$C_{lima} = \max\{C_H, C_{sf1}\}, C_{limb} = \min\{C_H, C_{sf1}\}$$  \hspace{1cm} (8)

$$C_{lima} = \max\{C_L, kC_{sf1}\}, C_{limb} = \min\{C_L, kC_{sf1}\}$$  \hspace{1cm} (9)

According to the second law of thermodynamics, one obtained:

$$\frac{T_2}{T_i} = \left(\frac{T_{lim}}{T_i}\right)^{\eta_e}$$  \hspace{1cm} (10)

From Equations (2) and (3), the expressions of $T_2$ and $T_3$ are, respectively:

$$T_2 = \frac{C_{lima}E_{hi}(T_{lim} - T_i)}{C_{sf1}} + T_i$$  \hspace{1cm} (11)

$$T_3 = \frac{C_{limb}E_{li}T_{lim} - kC_{sf1}T_i}{C_{limb}E_{li} - kC_{sf1}}$$  \hspace{1cm} (12)

From Equations (1) and (10)–(12), the expression of $T_i$ can be obtained as:

$$T_i = \frac{[(C_{lima}E_{hi}T_{lim} - C_{sf1}T_i) / (C_{lima}E_{hi} - C_{sf1})] + [(C_{limb} / C_{sf1})E_{hi}(T_{lim} - T_i) + T_i]\eta_e - 1}{1 - \frac{1}{\eta_e}[C_{lima} / C_{sf1}]E_{hi}(T_{lim} - T_i) + T_i] \frac{1}{T_i} \eta_e$$  \hspace{1cm} (13)

From to Equations (2), (3), and (11)–(13), the expressions of $P$ and $\eta$ can be obtained as:
\[ P = Q_H - Q_L = \frac{C_{\text{lin}} E_H (T_{\text{fin}} - T_i) (C_{\text{lin}} E_{i1} - k C_{\text{sfl}}) - k C_{\text{lin}} E_{i1} C_{\text{sfl}} (T_{\text{fin}} - T_i)}{C_{\text{lin}} E_{i1} - k C_{\text{sfl}}} \]  
\[ (14) \]

\[ \eta = \frac{P}{\dot{Q}_H} = \frac{C_{\text{lin}} E_H (T_{\text{fin}} - T_i) (C_{\text{lin}} E_{i1} - k C_{\text{sfl})} - k C_{\text{lin}} E_{i1} C_{\text{sfl}} (T_{\text{fin}} - T_i)}{C_{\text{lin}} E_H (C_{\text{lin}} E_{i1} - k C_{\text{sfl}}) (T_{\text{fin}} - T_i)} \]  
\[ (15) \]

When \( \eta = 1 \), substituting into Equation (13), the expression of \( T_i \) for an endoreversible SFLC with variable temperature HR can be obtained as:

\[ T_i = \frac{(C_{\text{lin}} E_{i1} T_{\text{fin}} - k C_{\text{sfl}} T_i)}{(C_{\text{lin}} E_{i1} - k C_{\text{sfl}}) \{ (C_{\text{lin}} E_{i1} / C_{\text{sfl}}) (T_{\text{fin}} - T_i) + T_i \}^{\frac{1}{2}} T_i^{\frac{1}{2}}} \]  
\[ (16) \]

Combining Equations (4)–(9) and (14)–(16), by the numerical solution, the relationship between the \( P \) and \( \eta \) characteristics of the variable temperature HR endoreversible SFLC can be obtained.

Substituting \( C_H = C_L \rightarrow \infty \) into Equations (4), (5) and (13)–(15) yields the expressions of the effectiveness of the two HEXs, \( \eta \), and \( T_i \) for an irreversible SFLC with constant temperature HR [62]:

\[ E_H = 1 - \exp(-N_H) \]  
\[ (17) \]

\[ E_L = 1 - \exp(-N_L) \]  
\[ (18) \]

\[ T_i = \frac{E_H T_H (\eta - 1) + (T_i - E_i T_i) / (1 - E_i)}{\{ (1 - E_H)(1 - \eta) + [E_H T_H + (1 - E_H) T_i] / T_i^{\frac{1}{2}} \eta \}} \]  
\[ (19) \]

\[ P = \dot{Q}_{H_{in}} - \dot{Q}_{L_{in}} = \dot{m} C_L (T_H - T_i) - \frac{k E_L (T_i - T_i)}{1 - E_L} \]  
\[ (20) \]

\[ \eta = \frac{P}{\dot{Q}_{H_{in}}} = 1 - \frac{k E_L (T_i - T_i)}{E_H (1 - E_i) (T_H - T_i)} \]  
\[ (21) \]

When \( \eta = 1 \) and \( C_H = C_L \rightarrow \infty \), all of the expressions become the results of an endoreversible SFLC with constant temperature HR [60].

3. Numerical Examples and Discussions

3.1. Cycle Performance Analysis When the HTC of Hot- and Cold-Side HEXs Is Constant

Determining the relevant parameters according to the Refs. [53,60–62]:
\[ C_s = 0.7165 \text{kJ} / (\text{kg} \cdot \text{K}), \quad \dot{m} = 1.1165 \text{kg} / \text{s}, \quad T_i = 300 \text{K}, \quad C_H = C_L = 1.2, \quad k = 1.4, \quad \text{and} \quad \eta_s = 0.92. \]

Because the LC lacks an adiabatic compression process, it is a three-branch cycle, missing constraints on cycle pressure ratio, and the basic optimization relationship between \( P \) and \( \eta \) cannot be obtained. When \( U_H \) and \( U_L \) are given, it can be seen from Equations (4)–(7) and (13)–(15), when the corresponding effectiveness of HEXs and HR inlet temperature are given, the cycle \( P \) and \( \eta \) can be obtained as a certain point. Figure 2 shows the \( P-\eta \) characteristic of the cycle. It can be seen that when \( E_{i1} \) and \( E_{i1} \) take 0.8 and 0.9, as well as the HR inlet temperature ratio \( \tau = T_{\text{fin}} / T_{\text{lin}} \) are 3.25 and 3.75, respectively, the corresponding \( P-\eta \) show a “point” change. Parameters \( U_H, U_L, \tau, \) and \( \eta_s \) have significant effects on \( P \) and \( \eta \). When \( U_H, U_L, \tau, \) and \( \eta_s \) increase, \( P \) and \( \eta \) increase. When \( \eta_s \) changes from 0.75 to 1, \( P \) and \( \eta \) increase by about 639.3 and 632.2%, respectively.
Figure 2. The characteristic of $P - \eta$ (a) and the influence of $\eta_e$ on $P - \eta$ characteristics when $U_H$ and $U_L$ are given (b).

3.2. Cycle Performance Optimization When the HTC Distributions of the Two HEXs Can Be Optimized

Assuming that the sum of the HTCs of the two HEXs are a constant value:

$$U_L + U_H = U_T$$ (22)

$$U_H = (1 - u_L)U_T, \quad U_L = u_L U_T$$ (23)
where \( u_t = \frac{U_t}{U_f} \) and \( 0 < u_t < 1 \). Combining Equations (4)–(7), (22) and (23), \( E_{H_1} \) and \( E_{L_1} \) expressions are, respectively:

\[
E_{H_1} = \frac{1-e^{-\left(1-u_t\left(U_t/C_{\text{lim}_{\text{max}}}-1\right)\right)/\left(C_{\text{lim}_{\text{max}}}/C_{\text{lim}_{\text{max}}}-1\right)}}{1-(C_{\text{lim}_{\text{min}}}/C_{\text{lim}_{\text{max}}})e^{-\left(1-u_t\left(U_t/C_{\text{lim}_{\text{max}}}-1\right)\right)/\left(C_{\text{lim}_{\text{max}}}/C_{\text{lim}_{\text{max}}}-1\right)}}
\]

\[
E_{L_1} = \frac{1-e^{-\left(1-u_t\left(U_t/C_{\text{lim}_{\text{max}}}-1\right)\right)/\left(C_{\text{lim}_{\text{max}}}/C_{\text{lim}_{\text{max}}}-1\right)}}{1-(C_{\text{lim}_{\text{min}}}/C_{\text{lim}_{\text{max}}})e^{-\left(1-u_t\left(U_t/C_{\text{lim}_{\text{max}}}-1\right)\right)/\left(C_{\text{lim}_{\text{max}}}/C_{\text{lim}_{\text{max}}}-1\right)}}
\]

Combining Equations (13)–(15) and (24)–(25), the relationships between \( P \) and \( \eta \) versus \( u_t \) of the irreversible SFLC with variable temperature HR can be obtained.

Figures 3 and 4 show the \( P \) and \( \eta \) versus \( u_t \) characteristics when \( u_t \) can be optimized. The two figures show that the characteristics of \( P-u_t \) and \( \eta-u_t \) are parabolic-like ones, that is, there are maximum \( P \) \( (P_{\text{max}}) \) and maximum \( \eta \) \( (\eta_{\text{max}}) \) as well as the corresponding optimal HTC distributions \( u_{t_{\text{opt}}} \) and \( u_{t_{\text{opt}}} \).

**Figure 3.** The curve of \( P-u_t \).
Figures 5–8 show the influences of $\tau$, $U_T$, and $\eta_v$ on $P_{\text{max}}$, $\eta_{\text{max}}$, $u_{T,(\text{opt})}$, and $u_{T,(\text{opt})}$. It can be seen from Figures 5 and 6, when $\tau$ is fixed and as $U_T$ increases, $P_{\text{max}}$ and $\eta_{\text{max}}$ increase, $u_{T,(\text{opt})}$ and $u_{T,(\text{opt})}$ first increase and then decrease; when $U_T$ is fixed and as $\tau$ increases, $P_{\text{max}}$, $\eta_{\text{max}}$, $u_{T,(\text{opt})}$, and $u_{T,(\text{opt})}$ increase; according to Figures 7 and 8, with the increases of $\eta_v$, $P_{\text{max}}$, and $\eta_{\text{max}}$ increase, $u_{T,(\text{opt})}$ and $u_{T,(\text{opt})}$ decrease.

**Figure 4.** The curve of $\eta_u$.

**Figure 5.** The characteristics of $P_{\text{max}}$ and $u_{T,(\text{opt})}$ about $\tau$. 
Figure 6. The characteristics of $\eta_{\text{max}} - U_T$ and $u_{L,(opt)} - U_T$ about $\tau$.

Figure 7. The characteristics of $P_{\text{max}} - \eta_e$ and $u_{L,(opt)} - \eta_e$. 
3.3. TCR Matching Optimization

Setting $C_H=1.2$, $\tau = 3.25$, $U_T=5KW/K$, and $C_L/C_H=1$, and taking $P$ as the objective function and $u_L$ as the optimization variable, the influences of HR TCR ratio ($C_L/C_H$), $U_T$ and $\tau$ on the characteristics of $P_{\text{max}}-C_{\text{opt}}/C_H$ were studied.

Figures 9–12 show the influences of the $C_L/C_H$, $U_T$, $\tau$ and $\eta$ on the characteristics of $P_{\text{max}}-C_{\text{opt}}/C_H$. It can be seen that with the increases of $C_{\text{opt}}/C_H$, $P_{\text{max}}-C_{\text{opt}}/C_H$ shows a parabolic-like change that first increases and then decreases, that is, there is an optimal $C_{\text{opt}}/C_H ((C_{\text{opt}}/C_H)_{\text{opt}})$ which makes the cycle reach double-maximum power point ($\left(P_{\text{max}}\right)_{\text{max}}$).
Figure 9. The characteristics of $P_{\text{max}} \cdot C_{\text{orf}} / C_H$ about $C_L / C_H$.

Figure 10. The characteristics of $P_{\text{max}} \cdot C_{\text{orf}} / C_H$ about $U_T$.
Figure 11. The characteristics of $P_{\text{max}} - C_{\text{nf1}} / C_H$ about $\tau$.

Figure 12. The characteristics of $P_{\text{max}} - C_{\text{nf1}} / C_H$ about $\eta_e$.

Figure 9 shows the influence of $C_L / C_H$ on the characteristics of $P_{\text{max}} - C_{\text{nf1}} / C_H$. As can be seen, with the increase of $C_L / C_H$, $(P_{\text{max}})_{\text{max}}$ and $(C_{\text{nf1}} / C_H)_{\text{opt}}$ increase. When $C_L / C_H$ takes 1, 2, 3, and 5, $(P_{\text{max}})_{\text{max}}$ is 33.78, 38.52, 40.11, and 41.37 kW, and $(C_{\text{nf1}} / C_H)_{\text{opt}}$ is 0.42, 0.49, 0.52, and 0.54, respectively. $C_L / C_H$ increases from 1 to 5, $(P_{\text{max}})_{\text{max}}$ increases by about 22.5%, $(C_{\text{nf1}} / C_H)_{\text{opt}}$ increases by about 28.6%. 
Figure 10 shows the influence of $U_T$ on the characteristics of $P_{\text{max}} - C_{\eta_1}/C_H$. When $U_T$ increases, $(P_{\text{max}})_{\text{max}}$ and $(C_{\eta_1}/C_H)_{\text{opt}}$ increase. When $U_T$ takes 2.5, 5, 7.5, and 10 kW/K, $(P_{\text{max}})_{\text{max}}$ are 20.84, 33.78, 42.44, and 48.56 kW, and $(C_{\eta_1}/C_H)_{\text{opt}}$ is 0.27, 0.42, 0.50, and 0.56, respectively. When $U_T$ increases from 2.5 to 10 kW/K, $(P_{\text{max}})_{\text{max}}$ and $(C_{\eta_1}/C_H)_{\text{opt}}$ increase by about 133.01% and 107.4%, respectively.

Figure 11 shows the influence of $\tau$ on the characteristics of $P_{\text{max}} - C_{\eta_1}/C_H$. When $\tau$ increases, $(P_{\text{max}})_{\text{max}}$ increases and $(C_{\eta_1}/C_H)_{\text{opt}}$ is unchanged. When $\tau$ takes 3.25, 3.5, 3.75, and 4, $(P_{\text{max}})_{\text{max}}$ is 33.78, 40.58, 47.75, and 55.25 kW, respectively, and $(C_{\eta_1}/C_H)_{\text{opt}}$ is 0.42. When $\tau$ increases from 3.25 to 4, $(P_{\text{max}})_{\text{max}}$ increases by about 63.6%.

Figure 12 shows the influence of $\eta_\varepsilon$ on the characteristics of $P_{\text{max}} - C_{\eta_1}/C_H$. When $\eta_\varepsilon$ increases, $(P_{\text{max}})_{\text{max}}$ and $(C_{\eta_1}/C_H)_{\text{opt}}$ increase. When $\eta_\varepsilon$ takes 0.85, 0.9, 0.95, and 1, $(P_{\text{max}})_{\text{max}}$ is 22.06, 30.36, 39.01, and 47.97 kW, and $(C_{\eta_1}/C_H)_{\text{opt}}$ is 0.38, 0.41, 0.43, and 0.46, respectively. When $\eta_\varepsilon$ increases from 0.85 to 1, $(P_{\text{max}})_{\text{max}}$ increases by about 117.5%, and $(C_{\eta_1}/C_H)_{\text{opt}}$ increases by about 21.1%.

4. Conclusions

In this paper, an irreversible SFLC model with variable temperature HR is established by applying FTT theory, the expressions of $P$ and $\eta$ are derived, and the influences of $U_T$, $\tau$, $C_l/C_H$, $\eta_\varepsilon$, and $C_{\eta_1}/C_H$ on $P$ and $\eta$ performances are analyzed. The results show that:

1. When $U_H$ and $U_L$ are constants, $P-\eta$ is a certain “point”, and with the increases in $\tau$, $U_H$, $U_L$, and $\eta_\varepsilon$, $P$ and $\eta$ increase. When $u_\varepsilon$ can be optimized, $P$ and $\eta$ versus $u_\varepsilon$ characteristics are parabolic-like ones, there are $u_{\varepsilon,(\text{opt})}$ and $u_{\eta,(\text{opt})}$ which makes the cycle reach $P_{\text{max}}$ and $\eta_{\text{max}}$.

2. With the increase of $C_{\eta_1}/C_H$, $P_{\text{max}} - C_{\eta_1}/C_H$ show a parabolic-like change, there is an $(C_{\eta_1}/C_H)_{\text{opt}}$, which makes the cycle reach $(P_{\text{max}})_{\text{max}}$. With the increases in $C_l/C_H$, $U_T$, and $\eta_\varepsilon$, $(P_{\text{max}})_{\text{max}}$ and $(C_{\eta_1}/C_H)_{\text{opt}}$ increase. With the increases in $\tau$, $(P_{\text{max}})_{\text{max}}$ increases, and $(C_{\eta_1}/C_H)_{\text{opt}}$ is unchanged.

3. Internal irreversibility and variable temperature HR are two general properties of practical cycles. It is necessary to study their influences on the cycle performance. FTT is a powerful theoretical tool for thermodynamic cycles with those properties.

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Nomenclature

\( C_p \) specific heat at constant pressure (kJ/ (kg K))
\( C_v \) specific heat at constant volume (kJ/ (kg K))
\( E \) effectiveness of heat exchanger
\( k \) specific heat ratio (-)
\( m \) mass flow rate of the working fluid (kg/ s)
\( N \) number of heat transfer units
\( P \) cycle power (kW)
\( \dot{Q} \) quantity of heat transfer rate (kW)
\( T \) temperature (K)
\( U \) heat conductance (kW / K)
\( U_T \) total heat conductance (kW / K)
\( u \) heat conductance distribution

Greek symbols
\( \tau \) heat reservoirs inlet temperature ratio
\( \eta \) cycle thermal efficiency

Subscripts
\( H \) hot-side
\( L \) cold-side
\( \text{max} \) maximum value
\( \text{opt} \) optimal
\( P \) maximum power point
\( \eta \) maximum thermal efficiency point
\( 1-3, 3s \) cycle state points

Abbreviations
FTT finite-time thermodynamic
HEG heat engine
HEX heat exchanger
HR heat reservoirs
HTC heat conductance
LC Lenoir cycle
SFLC steady flow Lenoir cycle
TCR thermal capacity rate

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