Neutral Particles in Light of
the Majorana-Ahuwalia Ideas*

Valeri V. Dvoeglazov†
Escuela de Física, Universidad Autónoma de Zacatecas
Antonio Dovalí Jaime s/n, Zacatecas 98000, ZAC., México
Internet address: VALERI@CANTERA.REDUAZ.MX
(February 28, 1995)

Abstract

The first part of this article (Sections I and II) presents oneself an overview of theory and phenomenology of truly neutral particles based on the papers of Majorana, Racah, Furry, McLennan and Case. The recent development of the construct, undertaken by Ahluwalia [Mod. Phys. Lett. A9 (1994) 439; Acta Phys. Polon. B25 (1994) 1267; Preprints LANL LA-UR-94-1252, LA-UR-94-3118], could be relevant for explanation of the present experimental situation in neutrino physics and astrophysics.

In Section III the new fundamental wave equations for self/anti-self conjugate type-II spinors, proposed by Ahluwalia, are re-casted to covariant form. The connection with the Foldy-Nigam-Bargmann-Wightman-Wigner (FNBWW) type quantum field theory is found. The possible applications to the problem of neutrino oscillations are discussed.

PACS numbers: 03.65.Pm, 11.30.Er, 14.60.Pq, 14.60.St
I. INTRODUCTION

Neutrino physics and astrophysics brought many “black spots” coming from experiment at the cloudless sky of the Standard Model. E. g., Professor Robertson noted in this connection [1]: “The solar neutrino results yield fairly strong and consistent indications that neutrino oscillations [2] are occurring.” Though “other evidence for new physics is less consistent and convincing”, the solar neutrino problem, ref. [1] (and in addition: the “negative mass squared” problem, e. g., ref. [1-4], the atmospheric neutrino anomaly [5], the possibility of neutrinoless double $\beta$- decay [6],7], the “spin crisis” in QCD [11,12], the tentative experimental evidence for a tensor coupling in the $\pi^- \rightarrow e^- + \bar{\nu}_e + \gamma$ decay [13], as well as the dark matter problem, e. g.,[14]) seems to me to provide sufficient reasons for searches of the models beyond the framework of the Standard Model. At the same time, the present experimental situation does not provide clear hints for theoreticians, what principles should be used for explanation of the mentioned phenomena and for construction of the “ultimate” theory. Thus, the Nature leaves us with many “degrees of freedom” of working out the hypotheses which seem for the first sight to be very “exotic” [15-19], if not “crazy” [20,21].

In this essay I continue the study of $j = 1/2$ and $j = 1$ neutral particles (the present-of-day knowledge states that neutrino and photon are the only truly neutral particles in the Nature), undertaken in ref. [22]. The crucial point of those papers is based on realizing “the dynamical role played by space-time symmetries for [fundamental] interactions”. The $ab\ initio$ construction of self/anti-self conjugate spinors in the $(j,0) \oplus (0,j)$ representation space and derivation of some physical relevant properties connected with space-time symmetries were presented there. In fact, the articles [22] are the development of the formalism proposed in the old papers [23-28] and they could be applicable for description of neutrino interactions and clarification of the present experimental situation.

II. THEORY AND PHENOMENOLOGY OF NEUTRAL PARTICLES

Kayser [29] writes: “We have become accustomed to thinking of a neutrino $\nu$ and its antineutrino $\bar{\nu}$ as distinct particles. However, it has long been recognized that the apparent distinction between them may be only an illusion. [Such] models, [in which there is no difference between neutrino and its antineutrino], naturally follow from GUT (grand unification theories).” Moreover, from a viewpoint of a lot of models beyond the Standard one it is very natural for neutrino (the spin-1/2 truly neutral particle) to be a massive \( \text{as opposed to the Glashow-Salam-Weinberg electroweak}\)

1There are opposite opinions on the solar neutrino problems. E. g., in his talk “The steady vanishing of the three solar neutrino problems” at the 27th Int. Conf. on High Energy Physics (1994) Prof. D. R. O. Morrison denies their existence at all: “The evidence for any solar neutrino problem is “not compelling”.

2Thanks to the two-component neutrino theory proposed by Landau, Lee, Yang and Salam [30].

3Surprisingly, the six of the present upper bounds on $m_{\nu_e}^2$ are negative [31]. E. g., the LANL result is $-147 \pm 68 \pm 41 \text{eV}^2$, the LLNL one, $-130 \pm 20 \pm 15 \text{eV}^2$. The most recent measurement (Troitsk, 1994) involves a new kind of systematics and gives $-18 \pm 6 \text{eV}^2$. 
model).

For the moment I take a liberty to present a little of history. In 1937 Majorana has given a derivation of a symmetrical theory of the electron and the positron \[23\]. The essential ingredient of that theory was the reformulation of the variational principle, based on the use of non-commutative variables. This led him to separation of the Dirac equation “into two distinct groups one of which acts on the real part and the other, on the imaginary part of [the spinor wave function], $\Psi = U + iV$ ”. He noted: “... the part of this formalism which refers to the $U$ (or to the $V$) may be considered by itself as a theoretical description of some material system, in conformity with the general methods of quantum mechanics... Equations constitute the simplest theoretical representation of a system of neutral particles.” His ideas have been developed in application to the $\beta$ radioactivity by Racah \[24\] and Furry \[25\]. In fact, they have analyzed the Majorana’s projection $\psi \rightarrow \frac{1}{2} \left\{ \psi + S_{1/2}^c \psi \right\}$.

The matrix of charge conjugation is defined as

$$S_{1/2}^c = e^{i\Theta_{1/2}} \begin{pmatrix} 0 & i \Theta_{1/2} \\ -i \Theta_{1/2} & 0 \end{pmatrix} \mathcal{K} \equiv \mathcal{C}_{1/2} \mathcal{K},$$

where $\mathcal{K}$ is the operation of complex conjugation and

$$\left(\Theta_{1/2}\right)_{\sigma, \sigma'} = (-1)^{j+j'} \delta_{\sigma', -\sigma}$$

is the Wigner’s operator ($\Theta_{1/2}J\Theta_{1/2}^{-1} = -J^*$). Racah noted that the symmetric description of a particle and an antiparticle does not always imply that two types of particle are physically indistinguishable. That is clear for the electron and the positron states, which have opposite electric charge, but this statement can also be applied for neutrino: “a neutrino emitted in a $\beta^-$ process may by absorption induce only a $\beta^+$ process, and vice versa”. However, if consider the symmetric Hamiltonian (the sum of $H_F$, the Fermi Hamiltonian, and $H_{KU}$, the Konopinski-Uhlenbeck Hamiltonian, ref. \[33\]), we come to the physical identity between neutrino and antineutrino and, hence, to the Majorana’s formalism for neutral particles, – Racah writes, from what follows the experimental possibility of the neutrinoless double $\beta$ decay discussed below. In the papers \[25\] the Lorentz invariance of the Majorana’s projection \[1\] as well as the persistence in time and the possibility of interaction of the Majorana’s particle with the nonelectric scalar potential $\gamma_0 \Phi$ had been proven.

Furry also noted the non-invariance of the projection under the change of phase (i.e., in fact, with respect to multiplication by a complex constant, what implies the absence of the simple gauge interactions of the Majorana neutral particle as opposed to the Dirac charged particle). Differing from Racah he has claimed that “the results predicted for ... observed processes [$\beta$-radioactivity] are ... identical with those of the ordinary theory. [However], the physical interpretation is quite different [and] an experimental decision between the formulation using neutrinos and antineutrinos...
and that using only neutrinos will ... be ... difficult. His point of view is now widely accepted: as opposed to the Dirac prescription of the charged particle (that has four states which answer for the same momentum but different spin configurations of particle and antiparticle) in the Majorana theory for \( j = 1/2 \) particles there are just two states corresponding to the two projections of the spin, \( i.e. \) there are no “antiparticles” and any necessity of the negative-energy states.

Important reformulations of the Majorana’s work have been undertaken by McLennan and Case in 1957, ref. [27,28]. Let me reproduce the main points of the Case’s paper[6]. By using Majorana ansatz[7]

\[
\psi_L = C_{[1/2]}^{-1} \psi_R^* , \tag{4}
\]

where \( \psi_{R,L} = \frac{1}{2}(1 \pm \gamma_5)\psi \), the Dirac equation was shown to be re-written to

\[
\eta^\mu \partial_\mu \phi + \kappa \phi^* = 0 , \tag{5}
\]

and its complex conjugated

\[
\eta^{\mu*} \partial_\mu \phi^* + \kappa \phi = 0 . \tag{6}
\]

Here \( \eta^\mu = C_{[1/2]}^0 \gamma_\mu = C_{[1/2]} (1 - \gamma_5) \gamma^\mu / 2, \ \phi = \psi_R \) and \( \kappa \) is mass of the particle in the notation of ref. [28]. The matrices \( \eta^\mu \) satisfy the anticommutation relation:

\[
\eta^{\mu*} \eta^\nu + \eta^{\nu*} \eta^\mu = 2g^{\mu\nu} . \tag{7}
\]

The signature was chosen to be \((-1,+1,+1,+1)\). The corresponding Hamiltonian equations are

\[
i \frac{\partial \phi}{\partial t} = \frac{1}{i} \sigma \cdot \nabla \phi + \kappa (A\phi^*) , \tag{8}
\]

\[
i \frac{\partial (A\phi^*)}{\partial t} = -\frac{1}{i} \sigma \cdot \nabla (A\phi^*) + \kappa \phi . \tag{9}
\]

5Of course, in the case of massless states this assertion does not cause any opposite opinions. Also, in ref. [32] the equivalence of description of the neutrino in terms of Majorana spinors and Weil spinors was claimed, but let us not forget that their arguments implied zero neutrino mass. I would like to mention the very detailed pedagogical introduction of ref. [34] to the Majorana theory, which has included a discussion of mass eigenstates of the neutrino. Nevertheless, in the case of massive neutrinos more explanations are required to the problem of the equivalence of the two descriptions and to the question of the number of independent states. See also the footnote # 22 in ref. [25a], ref. [22,35] and the discussion below.”

6The papers of Serpe [26] and McLennan [27] are concerned with the massless neutrino and could be accounted as the particular cases. Let us not forget that we don’t have a strong theoretical principle that forbids mass of neutrino.

7Let us note that the definitions of K. M. Case and D. V. Ahluwalia differ by the overall phase factor.
with \( \eta^\mu = -iA\sigma^\mu \). The matrix \( A \) can be chosen \( \sigma_2 \) in the conventional representation (see [28, p.308]). The law of association for the proper Lorentz transformation is usual, \( \Lambda = \exp \left( \frac{i}{\hbar} v \sigma \cdot q \right) \) with velocity \( v \) in the direction \( q \). However, for spatial reflections one has to impose

\[
\phi' (x') = \Lambda \phi^* (x), \quad \text{or} \quad \phi^* (x) = \Lambda^{-1} \phi' (x'). \tag{10}
\]

This form ensures that \( \Lambda = i\rho A \), where \( \rho \) is a real number with the absolute value unity. By using similar arguments for time reflections one has \( \phi' (x') = \Lambda \phi^* (x) \) where \( \Lambda = \mu A \), with \( \mu \) being real (and its absolute value being equal to the unit). However, the McLennan-Case consideration does not exhaust all possible Majorana-like construct. For instance, the possibility of the anti-self conjugate construct, \( i.e., \)

\[
\phi^* (x) = \Lambda^{-1} \phi' (x').
\]

(11)

has been realised much later [34]. From a physical point of view this corresponds to the two neutrino with opposite \( CP \) quantum numbers, \( e.g., [36] \).

Recently, the theory of neutral Majorana-like particles has been developed substantially in the papers of Ahluwalia [22]. Particularly, the generalization to higher-spin particles has been proposed. The formalism is based on the type-II bispinors (another Majorana-like construct which could be important for description of higher spin particles) introduced by him. The fundamentally new wave equation has been proposed there. We are going to discuss it in the next Section.

The type-II \((j, 0) \oplus (0, j)\) bispinors are defined in the following way:

\[
\lambda(p^\mu) \equiv \begin{pmatrix} \zeta_{\lambda} \Theta_{[j]} \phi^*_L (p^\mu) \\ \phi^*_L (p^\mu) \end{pmatrix}, \quad \rho(p^\mu) \equiv \begin{pmatrix} \phi_R (p^\mu) \\ \zeta_{\rho} \Theta_{[j]}^* \phi^*_R (p^\mu) \end{pmatrix}. \tag{12}
\]

\( \zeta_{\lambda} \) and \( \zeta_{\rho} \) are phase factors that are fixed by the conditions of self/anti-self conjugacy:

\[
S^c_{[1/2]} \lambda(p^\mu) = \pm \lambda(p^\mu), \quad S^c_{[1/2]} \rho(p^\mu) = \pm \rho(p^\mu), \tag{13}
\]

for a \( j = 1/2 \) case; and

\[
[\Gamma^5 S^c_{[1]}] \lambda(p^\mu) = \pm \lambda(p^\mu), \quad [\Gamma^5 S^c_{[1]}] \rho(p^\mu) = \pm \rho(p^\mu), \tag{14}
\]

for a \( j = 1 \) case. The spin-1 counterpart of the equation (12) is

\[
S^c_{[1]} = e^{i\Phi_{[1]}} \begin{pmatrix} 0 & \Theta_{[1]} \\ -\Theta_{[1]} & 0 \end{pmatrix} \mathcal{K} \equiv C_{[1]} \mathcal{K}. \tag{15}
\]

The phase factors are determined as \( \zeta_{\lambda}^S = \zeta_{\rho}^S = +i \), for the self charge conjugate \( j = 1/2 \) spinors, \( \lambda^S(p^\mu) \) and \( \rho^S(p^\mu) \); and \( \zeta_{\lambda}^A = \zeta_{\rho}^A = -i \), for the anti-self charge conjugate \( j = 1/2 \) spinors, \( \lambda^A(p^\mu) \)

\( ^8 \)The self/anti-self conjugate type-II spinors were shown in ref. [22] not to exist for bosons. This fact is related with the FNBWW-type construct and it follows from the analysis of ref. [18a]. However, \( [\Gamma^5 S^c] \) self/anti-self conjugate type-II spinors have been introduced there.
and \(\rho^A(p^\mu)\). The equations (14) determine \(\zeta^S\) = \(\zeta^S_\rho\) = +1 for the self \([\Gamma^5 S_{[1]}]\)-conjugate \(j = 1\) spinors; and \(\zeta^A\) = \(\zeta^A_\rho\) = −1 for the anti-self \([\Gamma^5 S_{[1]}]\)-conjugate \(j = 1\) spinors. The remarkable property of the self/anti-self conjugate spinors, which seems not to be realised before an appearance of the papers [22], is: they cannot be in the definite helicity eigenstates. In fact, let the 2-spinors \(\phi^h_{L,R}(p^\mu)\) be an eigenstate of the helicity operator

\[
J \cdot \hat{p} \phi^h_{L,R}(p^\mu) = h \phi^h_{L,R}(p^\mu)
\]

(16)

then, by using the Wigner-identity (see formula above, Eq. (3)), we convince ourselves

\[
J \cdot \hat{p} \Theta_{[j]} [\phi^h_{L,R}(p^\mu)]^* = -h \Theta_{[j]} [\phi^h_{L,R}(p^\mu)]^*
\]

(17)

Thus, if \(\phi^h_{L,R}(p^\mu)\) are eigenvectors of \(J \cdot \hat{p}\), then \(\Theta_{[j]} [\phi^h_{L,R}(p^\mu)]^*\) are eigenvectors of \(J \cdot \hat{p}\) with opposite eigenvalues to those associated with \(\phi^h_{L,R}(p^\mu)\), ref. [22c,d]. The unusual properties of the type-II spinors under space (time) reflections have also been noted in ref. [22]. They are not eigenspinors of the parity operator, see formulas (36a,b) and (37a,b) in the fourth paper.

The key test for a Majorana neutrino is the neutrinoless double-beta decay. An antineutrino emitted in the beta decay of one neutron is supposed to interact with another neutron and to cause it to transform into a proton and an electron. So in the final state there are two protons, two electrons and no neutrinos, \((A,Z) \rightarrow (A,Z + 2) + 2e^-\). The conservation of lepton number is violated. Such a possibility, originally proposed by Racah [22], did not yet observe in experiment in spite of the fact that the available phase space for this process is larger than for the two-neutrino double \(\beta\) decay [25]. The experimental bound for a halftime of neutrinoless \(\beta\) decay is \(T_{1/2} > 2 \times 10^{24}\) years (the enriched isotope \(^{76}\)Ge was used, ref. [9]). The failure of its observation was explained by the statement that apart from the non-conservation of lepton number the Racah processes is inhibited by helicity. In order to complete the second step of the Racah process, the antineutrino has to flip its helicity and turns itself into a neutrino. Rosen has shown [10] that such a flip may be induced only by a Majorana mass term. “... Even if right-handed currents provide the phenomenological mechanism for no-neutrino decay, the fundamental mechanism underlying the process must be [a presence of] neutrino mass [term].” In the case of neutral particles the electric charge conservation (superselection rules) no longer forbids transitions between particle and antiparticle \(\nu_eL \leftrightarrow \bar{\nu}_eR\) or \(\bar{\nu}_eL \leftrightarrow \nu_eR\). It is these oscillations that provide the ground for the Racah process. For the first time a theoretical model of neutrino oscillations has been proposed by Pontecorvo in 1957, ref. [2]10, see also [37], by using the analogy with oscillations in the \(K^0 - \bar{K}^0\) spinless meson system [38]. This old idea had eventually been gone of the use. But it has found a new life in the idea of oscillations between different flavours [3,4,39,40] in connection with the discovery of muon and \(\tau\)-lepton neutrinos.

Since in the third Section I am going to deal with a scheme of neutrino oscillations on the ground of Majorana-like theory with type-II spinors let me reproduce here main points of the well-known

\[9\] Of course, this explanation is appropriate only in the framework of the Standard Model.

\[10\] As mentioned in ref. [10], some rumors of the positive result concerning no-neutrino decay were circulated in the end of the fifties.
flavour mixing scheme and of the common-used consideration of neutrino mass terms.

Schemes of neutrino mixing are usually characterized by the type of the relevant mass term. According to the modern literature it is possible to form the following mass terms in the Lagrangian:

- **Dirac mass term:**

  \[ L^D = - \sum_{\nu', l=e, \mu, \tau} \bar{\nu}_{\nu'} R M_{\nu' i} \nu_L + h.c. \; \]  
  (18)

- **Majorana mass term (left-left):**

  \[ L^M = - \frac{1}{2} \sum_{\nu', l=e, \mu, \tau} (\bar{\nu}_{\nu'} L)^c M_{\nu' i} \nu_L + h.c. \; \]  
  (19)

- **Dirac plus Majorana mass term**

  \[ L^{D+M} = - \frac{1}{2} \sum_{\nu', l=e, \mu, \tau} (\bar{\nu}_{\nu'} L)^c M_{\nu' i} \nu_L - \sum_{\nu', l=e, \mu, \tau} \bar{\nu}_{\nu'} R M^D_{\nu' i} \nu_L - \frac{1}{2} \sum_{\nu', l=e, \mu, \tau} \bar{\nu}_{\nu'} R M^R_{\nu' i} (\nu_L)^c + h.c. \; \]  
  (20)

So, in a general case it is necessary to consider three (six) mass eigenstates that correspond to the diagonalized mass matrix obtained by the unitary transformation with the \(3 \otimes 3\) (or \(6 \otimes 6\) in the case of the \(D+M\) mass term) matrix, e.g., \(\nu_L = \sum_{i=1}^{3} U_{ii} \nu_{iL}\). We will denote the mass eigenstates \(\mid \nu_i \rangle\), \(i = 1, 2, 3\). Thus, one can obtain the diagonalized mass term in the Lagrangian:

\[ L^D = - \sum_{i=1}^{3} m_i \bar{\nu}_i \nu_i \; , \]  
(21)

\[ L^{M(D+M)} = - \frac{1}{2} \sum_{i=1}^{3(6)} m_i \bar{\psi}_i \psi_i \; . \]  
(22)

The most general mass matrix (Dirac and Majorana mass term) can be represented in the following form:

\[ \Psi_{L,R} M \Psi_{L,R} = \]  
(23)

---

11 More extended consideration could be found in [4,41,42].

12 The present experimental data restrict the number of light neutrino species to three (electron, muon and \(\tau\)-lepton neutrino [13]).
In the vacuum mass eigenstates propagate independently, i. e. let assume that they are orthogonal. If a physical state is the linear combination of mass eigenstates which have different masses (for the sake of simplicity let me consider only two species) one has:

\[
\begin{align*}
    |\nu_e(0)\rangle &= \cos\theta_{\nu} |\nu_1\rangle + \sin\theta_{\nu} |\nu_2\rangle \\
    |\nu_\mu(0)\rangle &= -\sin\theta_{\nu} |\nu_1\rangle + \cos\theta_{\nu} |\nu_2\rangle
\end{align*}
\] (24)

the partial content of species in it may vary with time. Let in the instan t of time \( t = 0 \) we have the mixing (24), then at a later time \( t \)

\[
|\nu_e(t)\rangle = \cos\theta_{\nu} e^{-iE_1t} |\nu_1\rangle + \sin\theta_{\nu} e^{-iE_2t} |\nu_2\rangle =
\]

\[
= \left( e^{-iE_1t} \cos^2\theta_{\nu} + e^{-iE_2t} \sin^2\theta_{\nu} \right) |\nu_e(0)\rangle + \sin\theta_{\nu} \cos\theta_{\nu} \left( e^{-iE_1t} - e^{-iE_2t} \right) |\nu_\mu(0)\rangle ,
\]

and

\[
|\nu_\mu(t)\rangle = -\sin\theta_{\nu} e^{-iE_1t} |\nu_1\rangle + \cos\theta_{\nu} e^{-iE_2t} |\nu_2\rangle =
\]

\[
= \sin\theta_{\nu} \cos\theta_{\nu} \left( e^{-iE_2t} - e^{-iE_1t} \right) |\nu_e(0)\rangle + \left( e^{-iE_2t} \cos^2\theta_{\nu} + e^{-iE_1t} \sin^2\theta_{\nu} \right) |\nu_\mu(0)\rangle .
\]

Thus, a electron neutrino produced at \( t = 0 \) has non-zero probability of being a muon neutrino at a later time (and vice versa). The probability is calculated to give

\[
P_{\nu_e\rightarrow\nu_\mu} = | <\nu_\mu(0)|\nu_e(t)>|^2 = |\sin\theta_{\nu} \cos\theta_{\nu} \left( e^{-iE_1t} - e^{-iE_2t} \right) |^2 =
\]

\[
= 2 \sin^2\theta_{\nu} \cos^2\theta_{\nu} \left[ 1 - \cos(E_1 - E_2)t \right] .
\]

(27)

For the sake of completeness let us note that

\[
P_{\nu_e\rightarrow\nu_e} = | <\nu_e(0)|\nu_e(t)>|^2 = 1 - 2 \sin^2\theta_{\nu} \sin^2 \left[ \frac{1}{2} (E_2 - E_1)t \right] .
\]

(28)

Since in the high-velocity limit \( (p >> m) \)

\[
E_1 - E_2 = \sqrt{p^2 + m_1^2} - \sqrt{p^2 + m_2^2} \approx \frac{m_1^2 - m_2^2}{2p} ,
\]

(29)

one obtains

\[
P_{\nu_e\rightarrow\nu_\mu} \approx 2 \sin^2\theta_{\nu} \cos^2\theta_{\nu} \left[ 1 - \cos \left( \frac{m_1^2 - m_2^2}{2p} \right) \frac{c^3}{\hbar} t \right] ,
\]

(30)

where we restored \( c \) and \( \hbar \) in order cosine to be dimensionless. Since the velocity of neutrino is approximately \( (? \approx \frac{c}{12}) \) equal to the light velocity, one has

\[
P_{\nu_e\rightarrow\nu_\mu} \approx 2 \sin^2\theta_{\nu} \cos^2\theta_{\nu} \left[ 1 - \cos \left( \frac{m_1^2 - m_2^2}{2p} \right) \frac{c^2}{\hbar} x \right] =
\]

\[
= 2 \sin^2\theta_{\nu} \cos^2\theta_{\nu} \left[ 1 - \cos \left( \frac{2\pi \frac{x}{l_1}}{\hbar} \right) \right] ,
\]

(31)

where
\[ l_{12} = \frac{4\pi p\hbar}{(m_1^2 - m_2^2)c^2} \quad (32) \]

is the “vacuum oscillation length”. In the case of almost “degenerate” neutrinos \( (m_1^2 - m_2^2) \approx (10^{-2}\text{eV}/c^2)^2 \) the “oscillation length” \( l_{12} \) is of the order of meters. The readers are able to find the numerous literature on the other versions of the oscillations (including three species etc.), see for the recent review \[31\].

The present-of-day experiments have not detected any such oscillations for terrestrially (nuclear reactors, accelerators) created neutrinos. That is usually explained by very small mass differences between eigenstates. On the other hand, the study of solar neutrinos reveals a strong possibility that, before they reach the Earth, the neutrinos undergo a significant oscillation. Besides vacuum oscillations, plasma processes also should be taken into account in the analysis of the solar neutrino flux. However, we aren’t going to discuss here the transmission through matter (the Mikheyev-Smirnov-Wolfenstein effect \[44\]), referring the reader to known reviews \[45\].

For the moment, many physicists don’t consider seriously the Pontecorvo’s original idea. “Since the helicity of a free particle is conserved, in vacuum the oscillations \( \nu_L \leftrightarrow \bar{\nu}_R \) cannot occur... For the above reasons it was generally supposed that Pontecorvo’s original oscillations are just the oscillations of active neutrinos into sterile states\[13\], whereas the true neutrino-antineutrino oscillations were considered impossible”, – claimed Akhmedov et al.\[14\] in ref. \[16\]. Nevertheless, the same authors realised that under certain conditions particle-antiparticle oscillations can occur and revisited the original idea on the ground of introduction of magnetic (or electric) dipole moment of neutrino with an addition of neutrino of other specie. The similar conclusion has been reached in \[41, p.378\] where was said that “traversal of the solar magnetic field may flip the neutrino spin.” However, the estimated order of the transition magnetic moment is \( \mu_\nu \sim 10^{-11} - 10^{-10} \mu_B \). “[Nevertheless], resonant effects in a full treatment may well enhance the spin-flip to a level where it is important.”

...It seems to me the history of the Majorana theory (as well as of neutrino physics itself) is very dramatic: one can see from the above that many outstanding physicists were not able to find the common answers on the experimental consequences of this description...

Next, in the following Section we shall work with spin-1 fields in the Weinberg formulation. Therefore, it is useful to repeat the key points of this particular model presented in the papers \[44\] [45][46][47][48][49][18][50][21]. The pioneer study of the \((j,0) \oplus (0,j)\) representation space for description of higher spin particles has been undertaken in ref. \[17\]. This way of a consideration is on an equal footing with the Dirac’s way of description of spin-1/2 particles and, in fact, has its origin from the Wigner’s classic work \[51\]. In the Weinberg theory a \(2(2j + 1)\) bispinor is constructed from left- and right-spinors \( \phi_R \) and \( \phi_L \), with they transforming according to the \((j,0) \oplus (0,j)\) representation of the Lorentz group. Without reference to any wave equation it can be shown that

\[ (j, 0) : \quad \phi_R(p^\mu) = \Lambda_R(p^\mu \leftarrow \bar{p}^\mu) \phi_R(\bar{p}^\mu) = \exp (+J \cdot \varphi) \phi_R(\bar{p}^\mu) \quad , \]
\[ (0, j) : \quad \phi_L(p^\mu) = \Lambda_L(p^\mu \leftarrow \bar{p}^\mu) \phi_L(\bar{p}^\mu) = \exp (-J \cdot \varphi) \phi_L(\bar{p}^\mu) \quad , \]

\[13\] E. g., \( \nu_L \leftrightarrow \bar{\nu}_L \).

\[14\] Cf. the thoughts of refs. \[46\] and \[10, p.4,5\] on neutrino-antineutrino oscillations.
where $\Lambda_L$ and $\Lambda_R$ are the Lorentz boost matrices for left- and right-$j$-dimensional spinors from the rest system $\hat{p}^\mu$; $\varphi$ are the Lorentz boost parameters; the operator $\mathbf{J}$ is presented by the angular momentum matrices. The Weinberg equation contains solutions with tachyonic dispersion relations. In 1971 Tucker and Hammer [49] have shown that it is possible to reformulate the $2(2j + 1)$ theory and to obtain the spin-$j$ equations which possess the correct physical dispersion. Positive- and negative-energy spinors coincide in their construct. However, introduction of electromagnetic gauge interaction in their equation for $j = 1$ mesons appears to be difficult. The resulting theory is not renormalizable for all $j \geq 1$. Another reformulation has been recently (1993) proposed. Based on the analysis of the transformation properties left- and right-spinors and a choice of appropriate rest spinors (spinorial basis), Ahluwalia et al. [18] have noted that it is possible to construct the Dirac-like theory in $(j, 0) \oplus (0, j)$ space for arbitrary spin $j$. The remarkable feature of this construct is the fact that boson and its antiboson have opposite relative intrinsic parities. Such a type of theory has been named as the Foldy-Nigam-Bargmann-Wightman-Wigner (FNBWW) quantum field theory. Finally, in my recent works [21] another Weinberg-Tucker-Hammer equation ("Weinberg double") with a correct physical dispersion has been given. These equations turn out to be equivalent to the equations for the antisymmetric tensor $F_{\mu\nu}$ and its dual, which could be deduced from the Proca theory. The field consideration of the Weinberg doubles partly clarified contradictions with the Weinberg theorem, occurred in the earlier works [53,50]. The contradictions were caused by the application of the generalized Lorentz condition (formulas (18) of ref. [53a]) to physical quantum states what resulted in equating the eigenvalues of the Pauli-Lyuban’sky operator to zero. The propagators for the Weinberg-Tucker-Hammer construct have also been obtained, ref. [21c].

However, these new constructs deal with the Dirac-type spinors (type-I spinors) and they are applicable mainly to the charge particles. Many questions related to neutral particles have left unsolved in ref. [18,21].

15Let us note that the massless first-order “Weinberg” equations for any spin have proven in ref. [20, Table 2] to possess another kinematical acausalities. Apart from the correct physical dispersion $E = \pm p$ there is a wrong dispersion relation $E = 0$ in the case of $j = 1$ (in the case of higher spins one has even more acausal solutions). This fact doubts their application for all processes (including quantumelectrodynamic processes). Nevertheless, the massless limits of the modified $2j$-order Weinberg equations ($\varphi_{u,v} = \pm 1$ for bosons)

$$\left[\gamma^{\mu_1\mu_2...\mu_{2j}}\partial_{\mu_1} \partial_{\mu_2} ... \partial_{\mu_{2j}} + \varphi_{u,v} m^{2j}\right] \Psi(x) = 0,$$

turn out to be well-defined and has no any kinematical acausality [20]. The $\gamma$-matrices are the covariantly defined $2(2j + 1) \otimes 2(2j + 1)$-matrices. See also refs. [21,50,52] for discussion on the connection of the Weinberg formulation with the antisymmetric tensor field description and for attempts of explanation of the origins and the consequences of incorrect dispersion relations.

16See also the old works of Sankaranarayanan and Good [48].

17The Weinberg theorem says that for massless particles $B - A = helicity$, if field transforms on the $(A, B)$ Lorentz group representation.
III. NEW FUNDAMENTAL EQUATION PROPOSED BY AHLUWALIA AND RELEVANT PHYSICAL CONSEQUENCES

The general wave equation for any spin in the instant-front formulation of QFT is given in [22c,d]\(^{18}\)

\[
\left( \zeta_\lambda \exp (-J \cdot \varphi) \Theta_{[j]} \Xi_{[j]} \exp (-J \cdot \varphi) \right) \lambda(p^\mu) = 0. \quad (36)
\]

The particular cases \((j = 1/2 \text{ and } j = 1)\) are also given there (Eqs. (31) and (32), respectively). The \(\lambda^S(p^\mu)\) appear to be the positive energy solutions with \(E = +\sqrt{m^2+p^2}\), the \(\lambda^A(p^\mu)\), negative energy solutions with \(E = -\sqrt{m^2+p^2}\) for both spin-1/2 and spin-1 cases. However, to re-write these equations to a covariant form is a difficult task. For instance, an attempt of the author of the formalism [22] to put the equation in the form

\[
(\lambda_{\mu
u} p_\mu p_\nu + m\lambda^\mu p_\mu - 2m^2 \mathbb{I}) \lambda(p^\mu) = 0
\]

was in a certain sense misleading. He noted himself: “it turns out that matrices \(\lambda_{\mu\nu}\) and \(\lambda^\mu\) do not transform as Poincaré tensors.” Below I try to explain in what way the equations for \(\lambda(p^\mu)\) and \(\rho(p^\mu)\) spinors are re-written to a covariant form.

The crucial point of derivation of the equation (36) is the generalized Ryder-Burgard relation for type-II spinors\(^{19}\)

\[
[\phi^h_L(\bar{p}^\mu)]^* = \Xi_{[j]} \phi^h_L(\bar{p}^\mu),
\]

where

\[
\Xi_{[1/2]} = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}, \quad \Xi_{[1]} = \begin{pmatrix} e^{i\phi} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i2\phi} \end{pmatrix},
\]

\(h\) is the helicity, \(\phi\) is the azimuthal angle associated with \(p\). In this framework \((j = 1/2 \text{ case})\) the best, what can be done, is to re-write Eq. (36) to the form:

\[
\left( \frac{i\zeta_\lambda}{\sin \theta} \gamma_5 [\gamma \times \bar{p}]_3 + \mathbb{I} \right) \lambda(p^\mu) = 0,
\]

\((\theta\) is the polar angle associated with \(p\)) by using the following identities:

\[
\Theta_{[1/2]} \Xi_{[1/2]} = \Xi_{[1/2]}^{-1} \Theta_{[1/2]} = i\sigma_1 \sin \phi - i\sigma_2 \cos \phi = i \frac{[\sigma \times p]_3}{\sqrt{p_1^2 + p_2^2}},
\]

\(^{18}\)See the corresponding equation in the light-front formulation in ref. [22a].

\(^{19}\)In ref. [18] the relation \(\phi_R(\bar{p}^\mu) = \pm \phi_L(\bar{p}^\mu)\) for type-I spinors (in fact, for the Dirac bispinor) has been named as the Ryder-Burgard relation, see also [54, p.44]. Through this paper I also use this name, but I understand that this relation could be found in earlier papers and books, see, e. g., the discussion surrounding equations (25,26) of Ch.5, ref. [55]. It can be deduced also from Eq. (22a) of ref. [56].
\[ [\sigma \times p] (\sigma \cdot p) = -(\sigma \cdot p) [\sigma \times p] = i\sigma p^2 - ip (\sigma \cdot p) \]

and

\[ \exp(\pm \sigma \cdot \varphi/2) = \cosh \frac{\varphi}{2} \pm (\sigma \varphi) \sinh \frac{\varphi}{2} \quad \varphi = \hat{p} = \frac{p}{|p|}. \]

However, the obtained equation can’t be considered as a dynamical equation (energy operator does not present there). In fact, Eq. (33) is only a reformulation of the condition of self/anti-self conjugacy.

Let us undertake another attempt. From the analysis of the rest spinors (formulas 22a-22b and 23a-23c of ref. [22d]) one can conclude that another form of the generalized Ryder-Burgard relation is possible. Namely, the form connecting 2-spinors of the opposite helicity is:

\[ [\phi^h_L (\hat{p}^\mu)]^\ast = (-1)^{1/2-h} e^{-i(\theta_1 + \theta_2)} \Theta_{[1/2]} \phi_{L}^{-h} (\hat{p}^\mu) \]

for a \( j = 1/2 \) case; and

\[ [\phi^h_R (\hat{p}^\mu)]^\ast = (-1)^{1-h} e^{-i\delta} \Theta_{[1]} \phi_{L}^{-h} (\hat{p}^\mu) \]

for a \( j = 1 \) case (\( \delta = \delta_1 + \delta_3 \) for \( h = \pm 1 \) and \( \delta = 2\delta_2 \), for \( h = 0 \)). Provided that the overall phase factors of the rest spinors are chosen to be \( \theta_1 + \theta_2 = 0 \) (or \( 2\pi \)) in a spin-1/2 case and \( \delta_1 + \delta_3 = 0 = \delta_2 \), in a spin-1 case, the Ryder-Burgard relation is written

\[ [\phi^h_L (\hat{p}^\mu)]^\ast = (-1)^{-h} \Theta_{[1]} \phi_{L}^{-h} (\hat{p}^\mu) \]

This choice is convenient for calculations. The same relations exist for right-handed spinors \( \phi^h_R (\hat{p}^\mu) \) in both a \( j = 1/2 \) case and a \( j = 1 \) case.

By using (45) and following to the procedure of deriving the wave equation developed in ref. [22] one can obtain for a \( j = 1/2 \) case (\( \hat{p} = \gamma^\mu p_\mu \)):

\[ \left[ \frac{i}{m} \gamma_5 \hat{p} - 1 \right] \Psi^{(S)}_{+1/2} (p^\mu) = 0 \quad \left[ \frac{i}{m} \gamma_5 \hat{p} + 1 \right] \Psi^{(A)}_{+1/2} (p^\mu) = 0 \]

\[ \left[ \frac{i}{m} \gamma_5 \hat{p} + 1 \right] \Psi^{(S)}_{-1/2} (p^\mu) = 0 \quad \left[ \frac{i}{m} \gamma_5 \hat{p} - 1 \right] \Psi^{(A)}_{-1/2} (p^\mu) = 0 . \]

Here we defined new spinors:

\[ \Psi^{(S)}_{+1/2} (p^\mu) = \left( \begin{array}{c} \Theta_{1/2} \phi_{L}^{-1/2} (p^\mu) \\ \phi_{L}^{1/2} (p^\mu) \end{array} \right) \quad \text{or} \quad \Psi^{(S)}_{+1/2} (p^\mu) = -i \left( i \Theta_{1/2} \phi_{L}^{1/2} (p^\mu) \right)^\ast , \]

\[ \Psi^{(S)}_{-1/2} (p^\mu) = \left( \begin{array}{c} \Theta_{1/2} \phi_{L}^{1/2} (p^\mu) \\ \phi_{L}^{-1/2} (p^\mu) \end{array} \right) \quad \text{or} \quad \Psi^{(S)}_{-1/2} (p^\mu) = i \left( i \Theta_{1/2} \phi_{L}^{-1/2} (p^\mu) \right)^\ast , \]

\[ \Psi^{(A)}_{+1/2} (p^\mu) = \left( \begin{array}{c} -i \Theta_{1/2} \phi_{L}^{-1/2} (p^\mu) \\ \phi_{L}^{1/2} (p^\mu) \end{array} \right) \quad \text{or} \quad \Psi^{(A)}_{+1/2} (p^\mu) = i \left( i \Theta_{1/2} \phi_{L}^{1/2} (p^\mu) \right)^\ast , \]

\[ \Psi^{(A)}_{-1/2} (p^\mu) = \left( \begin{array}{c} -i \Theta_{1/2} \phi_{L}^{1/2} (p^\mu) \\ \phi_{L}^{-1/2} (p^\mu) \end{array} \right) \quad \text{or} \quad \Psi^{(A)}_{-1/2} (p^\mu) = -i \left( i \Theta_{1/2} \phi_{L}^{-1/2} (p^\mu) \right)^\ast . \]
As opposed to $\lambda(p^\mu)$ and $\rho(p^\mu)$ these spinor functions are the eigenfunctions of the helicity operator of the $(1/2, 0) \oplus (0, 1/2)$ representation space, but they are not self/anti-self conjugate spinors.

The equations similar to (61,62) can also be obtained by the procedure described in footnote #1 of ref. [22d] with type-I spinors ($\Psi = \text{column}(\phi_R(p^\mu) \ \phi_L(p^\mu))$) if imply that the Ryder-Burgard relation has the form

$$\phi_R(\hat{p}^\mu) = \pm i \phi_L(\hat{p}^\mu) \quad .$$

The equations of kind (61,62) have been discussed in the old literature, ref. [57]. Their relevance to the problem of describing the neutrino has been noted in the cited paper. The properties of this bispinors respective to the parity ($\gamma_0$) operation are the following (cf. with formulas (36a,b) in ref. [22d]):

\begin{align*}
\gamma_0 \Psi^{(S)}_{+1/2}(p^\mu) &= -i \{ \Psi^{(A)}_{-1/2}(p^\mu) \}^c \
\gamma_0 \Psi^{(S)}_{-1/2}(p^\mu) &= +i \{ \Psi^{(A)}_{+1/2}(p^\mu) \}^c \
\gamma_0 \Psi^{(A)}_{+1/2}(p^\mu) &= -i \{ \Psi^{(S)}_{-1/2}(p^\mu) \}^c \
\gamma_0 \Psi^{(A)}_{-1/2}(p^\mu) &= +i \{ \Psi^{(S)}_{+1/2}(p^\mu) \}^c
\end{align*}

By using the formulas relating $\Psi$, Eq. (51), with self/anti-self conjugate spinors it is easy to find corresponding equations for spinors $\lambda(p^\mu)$ and $\rho(p^\mu)$. In the case of spin-1/2 field we obtain

\begin{align*}
\hat{p} \lambda^S_\uparrow(p^\mu) + im \lambda^S_\downarrow(p^\mu) &= 0 \quad , \quad \hat{p} \rho^S_\uparrow(p^\mu) - im \rho^S_\downarrow(p^\mu) = 0 \
\hat{p} \lambda^S_\downarrow(p^\mu) - im \lambda^S_\uparrow(p^\mu) &= 0 \quad , \quad \hat{p} \rho^S_\downarrow(p^\mu) + im \rho^S_\uparrow(p^\mu) = 0 \
\hat{p} \lambda^A_\uparrow(p^\mu) - im \lambda^A_\downarrow(p^\mu) &= 0 \quad , \quad \hat{p} \rho^A_\uparrow(p^\mu) + im \rho^A_\downarrow(p^\mu) = 0 \
\hat{p} \lambda^A_\downarrow(p^\mu) + im \lambda^A_\uparrow(p^\mu) &= 0 \quad , \quad \hat{p} \rho^A_\downarrow(p^\mu) - im \rho^A_\uparrow(p^\mu) = 0
\end{align*}

(provided that $m \neq 0$). The indices $\uparrow$ or $\downarrow$ should be referred to the chiral helicity introduced in [22c,p.10]. If imply similarly to [22d] that $\lambda^S_\uparrow(p^\mu)$ (and $\rho^A_\uparrow(p^\mu)$) are the positive-energy solutions and $\lambda^A_\uparrow(p^\mu)$ (and $\rho^S_\uparrow(p^\mu)$) are the negative-energy solutions, the equations (57-60) in the coordinate space can be written

\begin{align*}
\partial_\mu \gamma^\mu \lambda_\eta(x) + \varphi_{\uparrow\downarrow} m \lambda_{-\eta}(x) &= 0 \
\partial_\mu \gamma^\mu \rho_\eta(x) + \varphi_{\uparrow\downarrow} m \rho_{-\eta}(x) &= 0 \quad ,
\end{align*}

where $\varphi_{\uparrow\downarrow} = \pm 1$ with the sign is “+” if $\eta = \uparrow$ and the sign is “–” provided that $\eta = \downarrow$. These equations (61) and (62) are very similar to the Dirac equation, however, the sign at the mass term can be opposite and spinors enter in the equations with opposite chiral helicities. The Dirac equation with the opposite sign at mass term had been considered (in different aspects) in refs. [58,60]. Eqs. (61,62) should be compared with the new form of the Weinberg equation for $j = 1$ spinors in a coordinate representation, ref. [58].

One can incorporate the same chiral helicity states in equations by using the identities (48a,b) of ref. [22d].

\begin{align*}
\rho^S_\uparrow(p^\mu) &= -i \lambda^A_\uparrow(p^\mu) \quad , \quad \rho^A_\downarrow(p^\mu) = +i \lambda^A_\uparrow(p^\mu) \
\rho^A_\uparrow(p^\mu) &= +i \lambda^S_\downarrow(p^\mu) \quad , \quad \rho^S_\downarrow(p^\mu) = -i \lambda^S_\downarrow(p^\mu)
\end{align*}
Thus, one can come to
\[
\hat{p}\lambda^{S}_{\uparrow\downarrow}(p^\mu) + m\rho^{S}_{\uparrow\downarrow}(p^\mu) = 0, \quad \hat{p}\lambda^{A}_{\uparrow\downarrow}(p^\mu) + m\rho^{S}_{\uparrow\downarrow}(p^\mu) = 0, \quad (65)
\]
\[
\hat{p}\rho^{S}_{\uparrow\downarrow}(p^\mu) + m\lambda^{A}_{\uparrow\downarrow}(p^\mu) = 0, \quad \hat{p}\rho^{S}_{\uparrow\downarrow}(p^\mu) + m\lambda^{S}_{\uparrow\downarrow}(p^\mu) = 0. \quad (66)
\]

It is also useful to note the connection of type-II spinors $\lambda(p^\mu)$ and $\rho(p^\mu)$ with the type-I Dirac bispinor $\psi^{D}(p^\mu)$ and its charge conjugate $(\psi^{D}(p^\mu))^c$:
\[
\lambda^{S}(p^\mu) = \frac{1 - \gamma_5}{2} \psi^{D}(p^\mu) + \frac{1 + \gamma_5}{2} (\psi^{D}(p^\mu))^c, \quad (67)
\]
\[
\lambda^{A}(p^\mu) = \frac{1 - \gamma_5}{2} \psi^{D}(p^\mu) - \frac{1 + \gamma_5}{2} (\psi^{D}(p^\mu))^c, \quad (68)
\]
\[
\rho^{S}(p^\mu) = \frac{1 + \gamma_5}{2} \psi^{D}(p^\mu) + \frac{1 - \gamma_5}{2} (\psi^{D}(p^\mu))^c, \quad (69)
\]
\[
\rho^{A}(p^\mu) = \frac{1 + \gamma_5}{2} \psi^{D}(p^\mu) - \frac{1 - \gamma_5}{2} (\psi^{D}(p^\mu))^c. \quad (70)
\]

The equations (65-66) could then be re-written to the form with type-I spinors:
\[
(\hat{p} + m) \psi^{D}_{\pm 1/2}(p^\mu) + (\hat{p} + m) \gamma_5 (\psi^{D}_{\mp 1/2}(p^\mu))^c = 0, \quad (71)
\]
\[
(\hat{p} - m) \gamma_5 \psi^{D}_{\pm 1/2}(p^\mu) + (\hat{p} - m) (\psi^{D}_{\mp 1/2}(p^\mu))^c = 0, \quad (72)
\]
\[
(\hat{p} + m) \psi^{D}_{\mp 1/2}(p^\mu) + (\hat{p} + m) \gamma_5 (\psi^{D}_{\pm 1/2}(p^\mu))^c = 0, \quad (73)
\]
\[
(\hat{p} - m) \gamma_5 \psi^{D}_{\mp 1/2}(p^\mu) + (\hat{p} - m) (\psi^{D}_{\pm 1/2}(p^\mu))^c = 0. \quad (74)
\]

So, we can consider the $(\psi^{D}_{h}(p^\mu))^c$ (or $\gamma_5 \psi^{D}_{h}(p^\mu)$, or their sum) as the positive-energy solutions of the Dirac equation and $\psi^{D}_{h}(p^\mu)$ (or $\gamma_5 (\psi^{D}_{h}(p^\mu))^c$, or their sum) as the negative-energy solutions. The field operator can be defined
\[
\Psi = \int \frac{d^3p}{(2\pi)^3 2p_0} \sum_{h} \left[ (\psi^{D}_{h}(p^\mu))^c a_h \exp(-ip \cdot x) + \psi^{D}_{h}(p^\mu) b_h \exp(ip \cdot x) \right]. \quad (75)
\]

The similar formulation has been developed by Nigam and Foldy, ref. [21].

Let us note a interesting feature. We can obtain the another interpretation (namely, $\psi^{D}(p^\mu)$ corresponds to the positive-energy solutions and $(\psi^{D}(p^\mu))^c$, to the negative ones) if choose other overall phase factors in the definitions of the rest-spinors $\phi_L(\tilde{p}^\mu)$ and $\phi_R(\tilde{p}^\mu)$, formulas (22) of ref. [22d]. The signs at the mass term depend on the form of the generalized Ryder-Burgard relation; if $\theta_1 + \theta_2 = \pi$ the signs would be opposite. One can obtain the generalized equations (71-74) for an arbitrary choice of the phase factor. For $\lambda^{S}(p^\mu)$ spinors they are following:
\[
\begin{cases}
  i\hat{p}\lambda^{S}_{\uparrow}(p^\mu) - mT \lambda^{S}_{\downarrow}(p^\mu) = 0 \\
  i\hat{p}\lambda^{S}_{\downarrow}(p^\mu) + mT \lambda^{S}_{\uparrow}(p^\mu) = 0,
\end{cases} \quad (76)
\]
where
\[
T = \begin{pmatrix}
  e^{i(\theta_1 + \theta_2)} & 0 \\
  0 & e^{-i(\theta_1 + \theta_2)}
\end{pmatrix} \quad (77)
\]
and $m \neq 0$. In the case $\theta_1 + \theta_2 = \pm \frac{\pi}{2}$ we also have the correct physical dispersion, $p_0^2 - \mathbf{p}^2 = m^2$, for $\lambda(p^\mu)$ spinors.
Next, one can see from (61,62) that neither $\lambda^{S,A}(x)$ nor $\rho^{S,A}(x)$ are the eigenfunctions of the Hamiltonian operator (we have different chiral helicities in the “Dirac” equations). They are not in mass eigenstates. However $\psi^D$ and $(\psi^D)^c$ are in mass and helicity eigenstates. In ref. [61] it was shown that even without a resort to a plane-wave expansion, if the eigenvector $|\phi>$ has the eigenvalue “−1” of the normalized Hamiltonian $\hat{H}/|E|$ in the Hilbert space, then $|\phi^c>$ has the eigenvalue “+1”. This analysis is in accordance with the Feynman-Stückelberg interpretation of “antiparticle” as the particle moving backward in time [62], which seems to be deeper with respect to the Dirac’s hole concept, because the former permits us to describe bosons on the equal footing with fermions [18]. Thus, one can come to the conclusion that matrix elements, e. g.,

\[
\begin{align*}
\langle \lambda_A - \eta | \lambda_S - \eta \rangle (\mu_0) &> \sim \sin^2 \left( \frac{Et}{\hbar} \right) , \quad \langle \lambda_S - \eta | \lambda_S - \eta \rangle (\mu_0) > \sim \cos^2 \left( \frac{Et}{\hbar} \right), \\
\langle \lambda_A - \eta | \lambda_A - \eta \rangle (\mu_0) &> \sim \sin^2 \left( \frac{Et}{\hbar} \right) , \quad \langle \lambda_S - \eta | \lambda_A - \eta \rangle (\mu_0) > \sim \cos^2 \left( \frac{Et}{\hbar} \right) .
\end{align*}
\]

(78, 79)

We are ready to put the question forward: can the high-energy neutrino described by the field (Eq. (47) of ref. [22d])

\[
\nu^{ML} \equiv \int \frac{d^3p}{(2\pi)^3} \sum_\eta \left[ \lambda^S_\eta (p^\mu) a_\eta (p^\mu) \text{exp}(-ip \cdot x) + \lambda^A_\eta (p^\mu) a^\dagger_\eta (p^\mu) \text{exp}(ip \cdot x) \right]
\]

(80)

“oscillate” from the state of one chiral helicity to another chiral helicity with the oscillation length of the order of the de Broglie wavelength, $\lambda = h/p$?

For the case spin-1 the situation differs in some aspects. Direct calculations yield a non-dynamical quadratic (in projections of the linear momentum) equation:

\[
\left[ \zeta_\lambda \frac{\gamma_{11}p^2 + \gamma_{22}P^2_1 - 2\gamma_{12}p_1p_2}{P^2 - p_3^2} + \mathbb{I} \right] \lambda(p^\mu) = 0.
\]

(81)

It can also be written in the form\(^{20}\)

\[
\left( \begin{array}{cc}
-\mathbb{I} & \zeta_\lambda D^{(1,0)}(i \frac{[\sigma \times p]}{\sqrt{P^2 - p_3^2}}) \\
\zeta_\lambda \Theta_{[1]} D^{(0,1)}(i \frac{[\sigma \times p]}{\sqrt{P^2 - p_3^2}}) \Theta_{[1]} & -\mathbb{I}
\end{array} \right) \lambda(p^\mu) = 0,
\]

(82)

that is obtained by using, e. g., the technique of ref. [55]: $D^{(J,0)}(A)$ are the Wigner functions for the $(J,0)$ representation, $D^{(0,J)}(A)$, for the $(0,J)$ representation.

If accept another formulation of the Burgard-Ryder relation [53] one has\(^{21}\)

\(20\) We use the notation in terms of the Barut-Muzinich-Williams matrices here, ref. [6].

\(21\) Again, one can obtain the opposite signs in the equations if imply $\delta_1 + \delta_3 = \pi$ for $\phi_L(\tilde{p}^\mu)$ and, correspondingly, for $\phi_R(\tilde{p}^\mu)$.
\[ \gamma_{\mu\nu}p^\nu \lambda^S_s(p^\mu) - m^2 \lambda^S_s(p^\mu) = 0 , \quad \gamma_{\mu\nu}p^\nu \rho^S_s(p^\mu) - m^2 \rho^S_s(p^\mu) = 0 , \quad \gamma_{\mu\nu}p^\nu \lambda^A_s(p^\mu) + m^2 \lambda^A_s(p^\mu) = 0 , \quad \gamma_{\mu\nu}p^\nu \rho^A_s(p^\mu) + m^2 \rho^A_s(p^\mu) = 0 , \]

There exist the identities analogous to (63,64). For instance, under the choice of the phase factors as \( \delta^1_1 = \delta^3_1 = 0 , \delta^1_2 = \delta^3_2 = 0 \) and \( \delta^2_1 = \delta^2_2 = \pi = 0 \) we have:

\[ \rho^S_s(p^\mu) = +\lambda^S_s(p^\mu) , \quad \rho^S_s(p^\mu) = +\lambda^S_s(p^\mu) , \quad \rho^\downarrow_s(p^\mu) = -\lambda^S_s(p^\mu) , \quad \rho^\downarrow_s(p^\mu) = -\lambda^S_s(p^\mu) . \]

Therefore,

\[ \gamma_{\mu\nu}p^\nu \lambda^S_s(p^\mu) - m^2 \rho^S_s(p^\mu) = 0 , \quad \gamma_{\mu\nu}p^\nu \lambda^A_s(p^\mu) - m^2 \rho^A_s(p^\mu) = 0 , \]

Applying relations between type-II and type-I spinors that look like similar to (67-70) except for \( \rho^S \leftrightarrow \rho^A \) we obtain:

\[ \left( \gamma_{\mu\nu}p^\nu - m^2 \right) \psi^D(p^\mu) + \left( \gamma_{\mu\nu}p^\nu - m^2 \right) \gamma_5(\psi^D(p^\mu))^c = 0 , \]

This tells us that \( \psi^D \) (or \( \gamma_5(\psi^D)^c \)) should be considered as the positive-energy solutions of the modified Weinberg equation (18) and (\( \psi^D)^c \) (or \( \gamma_5\psi^D \)) as the negative-energy ones. The analogs of the equations (61,62) can be written:

\[ \gamma^{\mu\nu} \partial_\mu \partial_\nu \lambda_\eta(x) + \varphi_{S,A} \lambda_\eta(x) = 0 , \quad \gamma^{\mu\nu} \partial_\mu \partial_\nu \rho_\eta(x) + \varphi_{S,A} \rho_\eta(x) = 0 , \]

where \( \varphi_{S,A} = \pm 1 \), the sign is “+” for positive-energy solution \( \lambda^S(p^\mu) \) (or \( \rho^S(p^\mu) \)) and the sign is “−” for negative-energy solutions \( \lambda^A(p^\mu) \) (or \( \rho^A(p^\mu) \)). This refers to the \( \eta = \uparrow \) or \( \eta = \downarrow \). As for \( \eta = \rightarrow \) it is easy to see that the equations (85,88) have the opposite signs at mass terms.

The presence of \( \varphi_{\uparrow\downarrow} \) in a \( j = 1/2 \) case or \( \varphi_{S,A} \) in a \( j = 1 \) case hints that we obtained the examples of the FNBWW-type quantum field theory. The analysis of the field operator (75) in the Fock space reveals that fermion and its antifermion can possess same intrinsic parities [61,22d]. Bosons described by the Eqs. (63,66) are found following to ref. [18] to be able to carry opposite intrinsic parities, depending on the choice of the field operator.

\[Cf. \text{ with the formulas (21a-c) in [22a] and (48,49) in ref. [64]. Thus, the form of these relations depends on the choice of the spinorial basis and it is governed by the covariance of the theory under discrete symmetries.} \]
IV. CONCLUDING REMARKS

In this paper I have presented an overview of the theory of truly neutral particles. The question of applicability of the new constructs in the \((j, 0) \oplus (0, j)\) representation space to neutrino physics has been discussed. The connection of the new models with the theories envisaged long ago by Foldy and Nigam \([61]\), and Bargmann, Wightman and Wigner \([65]\) has been found. The particle properties with respect to the operation of parity, being discussed in the present paper (and in refs. \([18, 22]\)), are unusual. In fact, it was shown that these properties depend on the choice of the field operator. Moreover, it was found that the physical content depends on the choice of the spinorial basis. Research in the framework of other constructs representation space deserves further elaboration.

Unfortunately, the present experimental data don’t yet permit us to make reliable conclusions on the sufficiency of the Standard Model (and to what limits). However, the wide interest in neutrino physics in the theoretician community and the forthcoming experimental facilities, SUPER-KAMIOKANDE, SNO (Sudbury), BOREXINO, ICARUS (CERN-Gran Sasso), HELLAZ, HERON (see, e. g., the proceedings of the recent neutrino conference \([66]\)), leave us with a hope that the puzzles of misterious neutral particles can be resolved in a short time.

Acknowledgements. I greatly appreciate many helpful advises of Prof. D. V. Ahluwalia, Prof. A. F. Pashkov and Prof. Yu. F. Smirnov. Discussions with Profs. I. G. Kaplan and M. Torres on neutrino experiments were very useful. The questions of Prof. M. Moreno and Prof. A. Turbiner helped me to realise the necessity of the study of neutral particles.

I would also like to thank an unknown referee of “Phys. Lett. B”, who found that the conclusion of the papers \([21]\) is “a known result”. His report was in a certain part encouraging and it gave me an additional impulse to speed up the work in the needed direction, what resulted in the writing of the present paper.

I am grateful to Zacatecas University for a professorship.
REFERENCES

[1] R. G. H. Robertson, in Proc. Dallas HEP Conference (Am. Inst. Phys., 1993), p. 140

[2] B. M. Pontecorvo, ZhETF 33 (1957) 549; ibid 34 (1958) 247; ibid 53 (1967) 171 [English translation: Sov. Phys. JETP 6 (1958) 429; ibid 7 (1958) 172; ibid 26 (1968) 984]

[3] V. Gribov and B. Pontecorvo, Phys. Lett. 28B (1969) 493

[4] S. M. Bilen’kii and B. Pontecorvo, Usp. Fiz. Nauk 123 (1977) 181 [English translation: Sov. Phys. JETP 6 (1958) 429; ibid 7 (1958) 172; ibid 26 (1968) 984]

[5] D. R. O. Morrison, “The steady vanishing of three solar neutrino problems.” Preprint CERN-PPE/94-125, Aug. 1994

[6] P. Langacker, “Solar neutrinos.” Preprint UPR-0640-T [hep-ph/9411339], Penn. U., Nov. 1994

[7] E. Kh. Akhmedov, “Atmospheric neutrinos.” Preprint FTUV-94-09 [hep-ph/9402207]. Valencia, Oct. 1993; Y. Fukuda et al., Phys. Lett. 335B (1994) 237

[8] E. g., Ch. Weinhammer et al., Phys. Lett. 330B (1993) 210; see also S. Boris et al., Phys. Rev. Lett. 58 (1987) 2019; V. A. Lyubimov, in Proc. Int. Conf. on Neutrino Physics and Astrophysics “Neutrino’88.” (Ed. J. Schneps et al. World Scientific, Singapore, 1989), p. 2, for the experiment in which the opposite result ($m^2 > 0$) has been reached.

[9] A. Balysh et al., Phys. Lett. B283 (1992) 32; ibid B322 (1994) 176

[10] S. P. Rosen, “Double beta decay.” Preprint UTAPHY-HEP-4 [hep-ph/9210202], Arlington, Sept. 1992

[11] J. Ashman et al., Phys. Lett. 206B (1988) 364

[12] A. E. Dorokhov, N. I. Kochev and Yu. A. Zubov, Int. J. Mod. Phys. A8 (1993) 603

[13] V. N. Bolotov et al., Phys. Lett. 243B (1990) 308; see for theoretical models: M. V. Chizhov, Mod. Phys. Lett. A8 (1993) 2573; M. V. Chizhov and L. V. Avdeev, Phys. Lett. 321B (1994) 212

[14] J. Binney and S. Tremaine, “Galactic dynamics.” (Princeton University Press, Princeton, 1987); K. Griest, “The particle- and astro-physics of dark matter.” Preprint UCSD-PTH-94-20 [astro-ph/9411038], San Diego, June 1994

[15] A very exotic idea of non-zero electric charge of neutrino has been proposed for explanation of the solar neutrino problem in the following papers: A. Yu. Ignatiev and G. C. Joshi, Mod. Phys. Lett. A9 (1994) 1479; “Nonzero electric charge of the neutrino and the solar neutrino problem.” Preprint UM-P-94/73, RCHEP-94/21 [hep-ph/9407340], Melbourne, July 1994. They are based on the realization of the old Einstein’s idea [for account see: A. Piccard and E. Kessler, Arch. Sci. Phys. Nat. 7 (1925) 340] of the electric charge dequantization and its dependence on the time [e. g., K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. 63 (1989) 938; A. Yu. Ignatiev and G. C. Joshi, Phys. Rev. D48 (1993) 4481; R. Foot, H. Lew and R. R. Volkas, J. Phys. G19 (1993) 361; ibid 1067; and ref. [21d]].

[16] In the following papers you can find exploration of the possibility of the existence of the mirror matter (as a particular case: mirror photon with electric charge and/or mass) R. Foot, “Experimental signature of a massive mirror photon.” Preprint McGill-94-36-REV [hep-ph/9407331], McGill, July 1994; Mod. Phys. Lett. A9 (1994) 169; S. M. Barr, D. Chang and G. Senjanovic, Phys. Rev. Lett. 67 (1991) 2765. See also A. Giveon and E. Witten, Phys. Lett. B332 (1994) 44.

[17] P. Bandyopadhyay, Phys. Rev. 173 (1968) 1481; Nuovo Cim. 55A (1968) 367. I would like to cite several paragraphs from the first paper: “... In view of the neutrino theory of light, photons are likely to interact weakly also, apart from the usual electromagnetic interactions... This assumed photon-neutrino weak interaction, if it exists, will have important bearing on astrophysics. In fact, this interaction can then be held responsible for the following neutrino-generating processes in stars:
\begin{align*}
(1) \quad \gamma + e^- & \leftrightarrow e^- + \nu + \bar{\nu}, \\
(2) \quad e^- + Z & \leftrightarrow e^- + Z + \nu + \bar{\nu}, \\
(3) \quad e^- + e^+ & \leftrightarrow \nu + \bar{\nu}, \\
(4) \quad \gamma + \gamma & \leftrightarrow \nu + \bar{\nu}, \\
(5) \quad \gamma + \gamma & \leftrightarrow \gamma + \nu + \bar{\nu}, \\
(6) \quad \Gamma & \rightarrow \nu + \bar{\nu} \quad (\Gamma \rightarrow e^- + e^+ \rightarrow \gamma \rightarrow \nu + \bar{\nu}) \quad \text{plasma process).}
\end{align*}

The energy dependence of the cross sections for these processes according to the present theory will be significantly different from that in other theories.”

[18] D. V. Ahluwalia, M. B. Johnson and T. Goldman, Phys. Lett. B316 (1993)102; D. V. Ahluwalia and T. Goldman, Mod. Phys. Lett. A8 (1993)2623. These papers are concerned with a theoretical construct of the Foldy-Niagam-Bargmann-Wightman-Wigner (FNBWW) type quantum field theory; its remarkable features are the facts that a boson can possess the opposite parity from its antiparticle, and a fermion and its antifermion, the same parities.

[19] J. Rembieliński, “Quantization of the tachyonic field.” Preprint KFT UL 2/94 (hep-th/9410079). Lódź, Oct. 1994; “Tachyonic neutrinos?” Preprint KFT UL 5/94 (hep-th/9411230) Lódź, Nov. 1994. The possibility that neutrinos are fermionic tachyons (according to the present experimental data) was considered there. The principle of the “absolute causality” holds for all kind of events.

[20] D. V. Ahluwalia and D. J. Ernst, Mod. Phys. Lett. A7 (1992) 1967

[21] V. V. Dvoeglazov, “Mapping between antisymmetric tensor and Weinberg formulations.” Preprint EFUAZ FT-94-05 (hep-th/9408077), Zacatecas, Aug. 1994; “What particles are described by the Weinberg theory.” Preprint EFUAZ FT-94-06 (hep-th/9408140), Zacatecas, Aug. 1994; “The Weinberg propagators.” Preprint EFUAZ FT-94-07 (hep-th/9408170), Zacatecas, Aug. 1994; “Can the Weinberg-Tucker-Hammer equations describe the electromagnetic field?” Preprint EFUAZ FT-94-09-REV (hep-th/9410174), Zacatecas, Oct. 1994

[22] D. V. Ahluwalia, M. B. Johnson and T. Goldman, Mod. Phys. Lett. A9 (1994) 439; Acta Phys. Polon. B25 (1994) 1267; D. V. Ahluwalia, “Incompatibility of self-charge conjugation with helicity eigenstates and gauge interactions.” Preprint LA-UR-94-1252 (hep-th/9404100), Los Alamos, Apr. 1994; “McLennan-Case construct for neutrino, its generalization, and a fundamentally new wave equation.” Preprint LA-UR-94-3118 (hep-th/9409134), Los Alamos, Sept. 1994

[23] E. Majorana, Nuovo Cim. 14 (1937) 171 [English translation: D. A. Sinclair, Tech. Trans. TT-542, National Research Council of Canada]

[24] G. Racah, Nuovo Cim. 14 (1937) 322

[25] W. H. Furry, Phys. Rev. 54 (1938) 56; ibid 56 (1939) 1184

[26] J. Serpe, Physica 18 (1952) 295

[27] J. A. McLennan, Phys. Rev. 106 (1957) 821

[28] K. M. Case, Phys. Rev. 107 (1957) 307

[29] B. Kayser, Comm. Nucl. Part. Phys. 14 (1985) 69

[30] T. D. Lee and C. N. Yang, Phys. Rev. 105 (1957) 1671; L. Landau, ZhETF 32 (1957) 407 [English translation: Sov. Phys. JETP 5 (1957) 337]; Nucl. Phys. 3 (1957) 127; A. Salam, Nuovo Cim. 5 (1957) 299

[31] G. Gelmini and E. Roulet, “Neutrino masses.” Preprint UCLA/94/TEP/36 (hep-ph/9412278), Los Angeles, Dec. 1994 (to appear in Rep. Prog. Phys.)

[32] G. Ryan and S. Okubo, Nuovo Cim. Suppl. 2 (1964) 234

[33] E. Fermi, Nuovo Cim. 11 (1934) 1; E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. 48 (1935) 7
the Istanbul Summer School of Theoretical Physics, 1962”. Ed. F. Gürsey

[66] Nucl. Phys. B (Proc. Suppl.) 35 (1994) – Proc. of the 16th Int. Conf. “Neutrino’94”. Eilat, Israel, 29 May - 3 June, 1994.