Design of surface profile of pairs of friction unit

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Abstract. To control the processes of friction and wear in the working pairs of the friction unit, it is necessary to study the behavior of the materials of the friction unit during contact interaction, accompanied by the combined action of temperature and dynamic loads. Numerous studies are devoted to the study of the influence of the chemical composition on the friction process of friction units, as well as the creation of new materials for brake units. There are practically no works devoted to the influence of nanogeometry of the surfaces of contact pairs of friction units on contact pressure. In this case, from an uneven distribution of contact pressure over the friction surface, strong heating can occur, which results in the formation of burns, thermal spots and foci of microcracks.

1. Introduction

The researchers [2,3,4] indicate the effect of temperature on the tribocontact of various friction units. In literature [6,7], modified friction units based on new polymeric materials used in various types of brake devices are proposed. In the source [5], the author cites the effect of hydrogen on the interaction process of friction pairs. Literary sources [1,8] describe the interaction of electric fields of the surfaces of contact pads of friction units. The analysis of the works [9, 10] on the wear of the working surfaces of friction units shows that the specific contact pressures of the working surfaces of friction pairs are unevenly distributed over the area of the metal element.

From the literature review, we can conclude that the authors paid great attention to temperature, chemical composition as the primary factor that affects the contacting elements of the friction unit. The consequence of this phenomenon is uneven wear of both metallic and non-metallic friction elements. However, the temperature is the result of contact and partial smoothing of the microprotrusions of non-metallic and metallic elements. The influence of the distribution of nanogeometry over the contact surface of the friction unit is not considered in any source. On this basis, we can conclude that this problem is not sufficiently disclosed in modern science.

To solve the formulated problem, it is necessary to formulate a requirement for the designed friction units, which relate to the minimum uneven distribution of pressure over the area of the working surfaces of the brake. The durability of the friction unit parts depends on the uniformity and wear rate of the working surfaces. The main cause of uneven wear is the contact pressure of the friction pairs.

The problem of choosing nanogeometry of the surface of the working surfaces of the friction unit, which ensures uniform wear of the metal friction element, has not yet been solved by the calculation method.

Based on the choice of nanogeometry of the friction surface, it is possible to obtain a uniform law
of the distribution of contact pressure over the elementary area of friction of the friction assembly, which will further reduce wear on working surfaces.

The value of the optimal value of the contact pressure of the elementary area on the friction surface \( p \) is not known in advance and must be determined when solving the optimization problem.

In addition to wear, the uneven distribution of contact pressure over the contact area of the friction unit affects the occurrence of thermal foci on the surfaces of the metal elements of the friction unit (Figure 1). All these factors naturally do not contribute to the effective functioning of such a critical unit as the braking mechanism.

Figure 1. Thermal stains on the surface of the brake disc

2. Materials and methods

To solve the contact problem, it is necessary to create a mathematical model based on the differential equations of thermoelasticity with boundary conditions. The control variables are the data of nanogeometry of the friction surfaces, i.e. external surfaces of metallic and non-metallic elements of the friction unit. The elements of the friction unit are modeled as an isotropic homogeneous body. We place the elementary area of the friction surfaces in the polar coordinate system. The origin is located in the center of the concentric arcs \( L_0 \) and \( L \) with radii \( R_0 \) and \( R \). We assume that the contour of the nonmetallic friction element \( L ' \) is close to circular in which the function \( H (\theta) \) lies in the definition of a further solution to the optimization problem. Imagine the boundary of the contour of a non-metallic friction element in the form:

\[
\rho = R + \varepsilon H(\theta),
\]

where \( \varepsilon \) is a small parameter equal to the ratio of the maximum height of the profile roughness to the average value of the height of the friction surface.

Similarly, the external contour of the elementary area of the metal friction element is presented, in which the function \( H_1 (\theta) \) is determined when solving the optimization problem:

\[
\rho = R_1 + \varepsilon H_1(\theta),
\]

where \( R_1 \) is the outer radius of the elementary area of friction of the metal friction element, m

Before solving the optimization problem, it is necessary to solve the contact problem of pressing a non-metallic element into the working surface of a metal element, considering the functions \( H (\theta) \) and \( H_1 (\theta) \) known, and each function has the form of a segment of the Fourier series.

The contact task problem has the form of a system of equations:


\[
\begin{align*}
  p^{(1)}(\theta, t) &= p_0^1(\theta) + tp_1^1(\theta) + \ldots + tp_n^1(\theta); \\
  p_0^{(1)}(\theta) &= \alpha_{0,0}^1 + \sum_{n=1}^{\infty}(\alpha_{k,0}^1 \cos \theta + \beta_{k,0}^1 \sin \theta) \\
  p_1^{(1)}(\theta) &= \alpha_{0,1}^1 + \sum_{n=1}^{\infty}(\alpha_{k,1}^1 \cos \theta + \beta_{k,1}^1 \sin \theta), \\
  \alpha_n^{(1)} &= \alpha_k^1 + \alpha_{n,1}^1 t + \ldots; \\
  \beta_n^{(1)} &= \beta_k^1 + \beta_{n,1}^1 t + \ldots;
\end{align*}
\]

where \( t \) is the contact time of elementary sites, \( s \).

According to the system of equations of the contact problem, the pressure of microroughness linearly depends on the coefficients of the Fourier series \( \alpha_{k,0}^1 \) and \( \beta_{k,0}^1 \) of the function \( H(\theta) \) and the coefficients \( \alpha_{k,1}^1 \) and \( \beta_{k,1}^1 \) of the function \( H(\theta) \).

For the elementary contact pad of the friction unit, the contact pressure is represented as:

\[
p(\theta, t) = f(\theta, t, \alpha_{k,0}^1, \beta_{k,0}^1, \alpha_{k,1}^1, \beta_{k,1}^1), \quad (k = 1, 2, \ldots, m); \quad (4)
\]

Contact time is considered a free parameter. Coefficients from the Fourier series \( \alpha_{k,0}^1, \beta_{k,0}^1, \alpha_{k,1}^1, \beta_{k,1}^1 \) are constant and must be determined. First it is necessary to split the segment \([\theta_1, \theta_2]\) of the variable \( \theta \) into \( M \) parts, where \( M > 4m + 2 \).

\[
p(\theta_i, t) = f(\theta_i, t, \alpha_{k,0}^1, \beta_{k,0}^1, \alpha_{k,1}^1, \beta_{k,1}^1), \quad (i = 1, 2, \ldots, M); \quad (5)
\]

\[
\theta_i = \theta_1 + \Delta \theta_i; \quad \Delta \theta_i = \frac{\theta_2 - \theta_1}{M};
\]

It is necessary to find the values of unknown parameters, which will optimize the function of contact pressure. Deviations of the contact pressure of the elementary friction pad from the desired constant \( \bar{p} \) are calculated by the formula:

\[
\varepsilon_i = f(\theta_i, t, \alpha_{k,0}^1, \beta_{k,0}^1, \alpha_{k,1}^1, \beta_{k,1}^1) - \bar{p}, \quad (i = 1, 2, \ldots, M); \quad (6)
\]

The basic principle of the least squares method states that the values of the contact pressure of the elementary friction site of the friction unit will be most likely, provided that the sum of the squared deviations \( \varepsilon_i \) is minimal:

\[
U = \sum_{i=1}^{M} [f(\theta_i, t, \alpha_{k,0}^1, \beta_{k,0}^1, \alpha_{k,1}^1, \beta_{k,1}^1) - \bar{p}]^2 \to \min; \quad (7)
\]

At any time, the coefficients \( \alpha_{k,0}^1, \beta_{k,0}^1, \alpha_{k,1}^1, \beta_{k,1}^1 \) \( k=1,2,\ldots,m \) and \( \alpha_{k,0}^1, \beta_{k,0}^1, \alpha_{k,1}^1, \beta_{k,1}^1 \) \( k=1,2,\ldots,m \) and the quantity the optimal values of the contact pressure of the elementary area on the friction surface \( \bar{p} \) are considered as independent variables and, equating the partial derivatives of the left side with respect to these variables to zero, we obtain exactly \( 4m + 3 \) equations with \( 4m + 3 \) unknowns. Since the function \( f(\theta_i, t, \alpha_{k,0}^1, \beta_{k,0}^1, \alpha_{k,1}^1, \beta_{k,1}^1) \) is linear with respect to unknown parameters, then the compilation and solution of the system is simplified.

The contact pressure at the elementary friction site of the friction unit is determined by the formula:

\[
p(\theta, t) = p^{(0)}(\theta, t) + \varepsilon p^{(1)}(\theta, t); \quad (8)
\]

The function \( p^{(0)}(\theta, t) \) does not depend on the desired coefficients, and the function \( p^{(1)}(\theta, t) \), on the contrary, linearly depends on the coefficients \( \alpha_{k,0}^1, \beta_{k,0}^1, \alpha_{k,1}^1, \beta_{k,1}^1 \). The dependence of the function \( p^{(1)}(\theta, t) \) on the coefficients can be represented in the following form:

We substitute the equation of contact pressure at the elementary friction site of the friction unit (9) into the dependence (7):
\[ U = \sum_{i=1}^{M} \left\{ \varepsilon[x_1 f_1(\theta_i) + x_2 f_2(\theta_i) + \ldots + x_{4m+2} f_{4m+2}(\theta_i)] + p^{(0)}(\theta_i, t) - \bar{p} \right\}^2 \rightarrow \text{min} \] (10)

We define the partial derivatives with respect to each unknown parameter and equate them to zero according to the principle of the least squares method:

\[ \frac{1}{2} \frac{\partial U}{\partial \varepsilon_{x_1}} = \sum_{i=1}^{M} f_1(\theta_i) \left[ \varepsilon[x_1 f_1(\theta_i) + x_2 f_2(\theta_i) + \ldots + x_{4m+2} f_{4m+2}(\theta_i)] + p^{(0)}(\theta_i, t) - \bar{p} \right] = 0; \]
\[ \frac{1}{2} \frac{\partial U}{\partial \varepsilon_{x_2}} = \sum_{i=1}^{M} f_2(\theta_i) \left[ \varepsilon[x_1 f_1(\theta_i) + x_2 f_2(\theta_i) + \ldots + x_{4m+2} f_{4m+2}(\theta_i)] + p^{(0)}(\theta_i, t) - \bar{p} \right] = 0; \]
\[ \frac{1}{2} \frac{\partial U}{\partial \varepsilon_{x_{4m+2}}} = \sum_{i=1}^{M} f_{4m+2}(\theta_i) \left[ \varepsilon[x_1 f_1(\theta_i) + x_2 f_2(\theta_i) + \ldots + x_{4m+2} f_{4m+2}(\theta_i)] + p^{(0)}(\theta_i, t) - \bar{p} \right] = 0; \]
\[ \frac{1}{2} \frac{\partial U}{\partial \varepsilon_{p^{(0)}}} = \sum_{i=1}^{M} (-1) \left[ \varepsilon[x_1 f_1(\theta_i) + x_2 f_2(\theta_i) + \ldots + x_{4m+2} f_{4m+2}(\theta_i)] + p^{(0)}(\theta_i, t) - \bar{p} \right] = 0; \] (11)

The coefficients for unknowns in the system of equations (11) depend on the interaction time of the contacting pads, the specific loads on the friction pairs, and the temperature developed on the contacting surfaces of the friction unit. For design, the solution of greatest interest is the initial interaction of the contacting friction pairs, i.e. \( t = 0 \).

Solving the system of linear equations (11) for a fixed instant of time by the Gauss method, using computer simulation as applied to disk brake mechanisms, we find the desired coefficients of the Fourier series to solve the contact problem.

The results of computer simulation of the coefficients of the contact problem at the beginning of the interaction of a friction pair \( (t = 0) \) are given in Table 1.

To find the contact pressure \( p \), it is necessary to set the contact time of the working platforms \( (t = 15s) \). Simulations of the contact pressure of an elementary site will be considered for two cases. In the first, the polar angle of the contact pad of the friction unit will be 30 degrees, in the second, the polar angle increases to 50 degrees. In addition to the polar angle of interaction of the elementary areas of the contact pair of the friction unit, the law of the technological roughnesses of the area will change.

**Table 1. Calculation results**

| \( \alpha^1_{k,0} \) | \( \beta^1_{k,0} \) | \( \alpha^1_{k,1} \) | \( \beta^1_{k,1} \) | \( \alpha^1_{k,2} \) | \( \beta^1_{k,2} \) | \( \alpha^1_{k,3} \) | \( \beta^1_{k,3} \) |
|---|---|---|---|---|---|---|---|
| 0.379 | 0.411 | 0.323 | 0.231 | 0.128 | 0.455 | 0.279 | 0.195 |
| \( \alpha^{(1)} \) | \( \alpha^{(1)} \) | \( \alpha^{(1)}_{32} \) | \( \beta^{(1)} \) | \( \beta^{(1)} \) | \( \beta^{(1)} \) |
| 0.582 | 0.779 | 0.976 | 0.388 | 0.652 | 0.849 |

The results of modeling the contact pressure of the elementary site of the friction unit versus the value of the polar angle are presented in Figure 2. Curves 1 and 3 correspond to the cosine law of the distribution of technological roughnesses, curves 2 and 4 of the sinusoidal distribution of technological roughness. The knowledge of the contact pressure of the working platform of the friction unit allows one to further find the temperature distribution on the surfaces of the friction unit, to determine the stress-strain state, as well as their wear.
Figure 2. Dependences of the contact pressure of the elementary pad of the contact pair of the friction unit on the value of the polar angle: curves 1 and 2, the angle of the contact pad 50 degrees, curves 3 and 4, the angle of the contact pad 30 degrees.

3. Conclusion
Modeling showed that under the same braking forces and the values of the free parameters of the friction unit parts, the contact pressure of the elementary area depends both on the nano-geometry of the surfaces of the working surfaces of the friction unit and on the roughness class. A higher roughness class helps to reduce the contact pressure of the working surfaces of the friction unit and, accordingly, the temperature on the working surfaces of its elements. This fact is explained by the fact that the energy for deformation of the protrusions of the nano-geometry of the contact surface is less affected, and thus the working surfaces of the contact pads absorb less energy. According to the sinusoidal and cosinosoidal laws of the distribution of technological roughnesses, the maximum contact pressure will be shifted in the direction of rotation of the metal friction element relative to the axis of symmetry of the elementary area. The simulations also confirmed with decreasing girth, contact pressure will increase.

The developed mathematical model of the contact pressure of the working areas of the friction unit allows determining the dependence of the nano-geometrical working surfaces of the contact areas of the friction unit. These studies allow designing friction units taking into account the distribution of contact pressure of the working pairs of the friction unit, which will affect the stress-strain and thermal state of the working pairs of the friction unit.

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