**The TauSpinner approach for electroweak corrections in LHC Z → ll observables**

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**ABSTRACT**

The LHC enters an era of Standard Model Z-boson couplings measurements to match the LEP legacy precision. The calculations of electroweak (EW) corrections available for the Monte Carlo generators become of relevance. Precise predictions of Z-boson production and decay require classes of QED/EW/QCD corrections to be taken into account, preferably in the manner which allows for their separation from the QCD dynamics of the production process.

For the LEP time calculations genuine weak and line-shape corrections were introduced with electroweak form-factors and *Improved Born Approximation*. The multi-loop vacuum polarization contribution and multi-photon bremsstrahlung were necessary. This formalism was well suited, in particular, for handling the so-called doubly-deconvoluted observables around the Z-pole, i.e. observables for which the initial- and final-state QED real and virtual emissions are treated separately or are integrated over. The approach was convenient for implementation into Monte Carlo programs as well. We attempt now to follow the approach for the LHC pp collisions EW corrections.

Our technical focus is on the EW corrections to doubly-deconvoluted observables of $Z \rightarrow \ell\ell$ process at the LHC, with the per-event weights. For this purpose the TauSpinner package is enriched with the EW sector. The Dizet library, as interfaced to the LEP era KKMC Monte Carlo is used. It calculates lepton pair production complete $O(\alpha)$ weak loop corrections with dominant higher-order terms as well. For the program efficiency and for numerical stability, the KKMC format look-up tables are used. The size of the corrections is evaluated for: the Z-boson line-shape, the outgoing leptons forward-backward asymmetry, the effective leptonic weak mixing angles and finally for the spherical harmonic expansion coefficients of the lepton distribution. Results from simplified calculations of *Effective Born* with modified EW couplings are also presented and compared with the predictions of *Improved Born Approximation* where complete set of EW form-factors is used. Comments on precision limits and on some of the necessary for LHC and FCC projects improvements are given, but the subject is not exhausted.
1 Introduction

A theoretically sound separation of QED/EW effects between the QED emissions and genuine weak effects was essential for the phenomenon of LEP precision physics \[1\]. It was motivated by the structure of the amplitudes for single Z or (to a lesser degree) WW pairs production in $e^+e^-$ collisions, and by the fact that QED bremsstrahlung occurs at a different energy scale than the electroweak processes. Even more importantly, with this approach multi-loop calculations for complete electroweak sector could be avoided. The QED terms could be resumed in exclusive exponentiation scheme implemented in Monte Carlo \[2\]. Note that QED corrections modify cross-section at the peak by as much as 40%. The details of this paradigm are explained in \[3\]. It was obtained as a consequence of massive efforts, we will not recall them here. From the present study perspective, it is important that spin amplitudes are semi-factorized into a Born-like term(s) and functional factors responsible for bremsstrahlung \[4\].

A similar separation can be also achieved for dynamics of production process in $pp$ collisions, which can be isolated from QED/EW corrections. It was explored recently in the case of configurations with high-$p_T$ jets associated with the Drell-Yan production of Z \[5\] or W bosons \[6\] at LHC. The potentially large electroweak Sudakov logarithmic corrections discussed in \[7\] (but not here) represent yet another class of weak effects, separable from those discussed above and throughout this paper. They are very small for lepton pairs with a virtuality close to the Z-boson pole mass and if accompanied by the jet, when virtuality of $\ell\ell j$ system is not much larger than 2 $M_W$. Otherwise the Sudakov corrections have to be revisited.

To assess precisely the size and impact of genuine weak corrections to the Born-like cross section for lepton pair production with a virtuality below threshold for WW pair production, the precision calculations and programs prepared for the LEP era: KKMC Monte Carlo \[8\] and Dizet electroweak (EW) library, were adapted to provide pre-tabulated EW corrections to be used by LHC specific programs like TauSpinner package \[9\]. To our time KKMC Monte Carlo use Dizet version 6.21 \[10\] \[11\]. We restrict ourselves to that version as a reference. The TauSpinner package was initially created as a tool to correct with per-event weight longitudinal spin effects in the generated event samples including $\tau$ decays. Implemented there algorithms turned out to be of more general usage. It provides reweighting technique to modify each event hard process production and decay matrix elements which were used for Monte Carlo generation. The most recent summary on its algorithms and their applications is given in \[12\]. The possibility to introduce one-loop electroweak corrections from SANC library \[13\] in case of Drell-Yan production of the Z-boson became available in TauSpinner since \[14\]. This implementation allowed to introduce per-event complete spin weight also with the help of pre-tabulation prepared for EW corrections. However no higher loop contributions were available.

That implementation of EW corrections is now enhanced. The TauSpinner package and algorithms are adapted to allow EW corrections from Dizet library directly into spin amplitudes and weight calculations for the Drell-Yan Z-boson production process. In \[5\] \[6\] we have shown that separating EW and QCD higher order corrections is possible and the Born-level spin amplitudes, if calculated in the adapted Mustraal frame \[4\], provide very good implementation for the EW LO sector even in case of NLO QCD description of the Drell-Yan processes. Beyond LO EW corrections can be thus implemented as well. The EW corrections are introduced as form-factor corrections to Standard Model couplings and propagators entering Born-level spin amplitudes. This approach was very successful in analyses of LEP precision physics and we use the same strategy for the LHC precision physics around the Z-boson pole. It has found already its applications in experimental analysis. A preliminary measurement of effective leptonic weak mixing angle, recently published by ATLAS Collaboration \[15\], is the first one using this approach to EW corrections in measurements of Z-boson properties at LHC.

This paper is organized as follows. In Section 2 we collect main formulae of the formalism, in particular we recall definition of the Improved Born Approximation. In Section 3 we present numerical results for the electroweak form factors. Some details on commonly used EW schemes are discussed in Section 4 which also recall definition of the Effective Born. In Section 5 we comment on the issues of using Born approximation in $pp$ collisions and in Section 6 we give more explanation why the Born approximation of the EW sector is still valid in the presence of NLO QCD matrix elements. In Section 7 we define concept of EW weight which can be applied to introduce EW corrections into already existing samples, generated with Monte Carlo programs with EW LO hard process matrix elements only. In Section 8 we discuss in numerical detail EW corrections to different observables of interest for precision measurements: Z-boson line-shape, lepton forward-backward asymmetry and lepton angular spherical harmonic expansion coefficients. In this Section we include also discussion of effective weak mixing angle in case of $pp$ collision. For results presented in Section 8 we use QCD NLO Powheg+M1NLO $Z+j$ sample of Monte Carlo \[16\] events, generated for $pp$ collision with $\sqrt{s}=8$ TeV and EW LO implementation in matrix elements. Section 9 summarizes the paper.
In Appendix A details on the technical implementation of EW weight and how it can be calculated with help of the TauSpinner framework are given. In Appendix B formulae which have been implemented to allow variation of the weak mixing angle parameter of the Born spin amplitudes are discussed. Illustrative numerical results are provided, but detailed discussion is left for the forthcoming work. In Appendix C initialization details of the Dizet, used for EW form-factors calculation, are given.

2 Improved Born Approximation

At LEP times, to match higher order QED effects with the loop corrections of electroweak sector, the concept of electroweak form factors was introduced [3]. This arrangement was very beneficial and enabled common treatment of one loop electroweak effects with not only higher order QED corrections including bremsstrahlung, but also to incorporate higher order loops into $Z$ and photon propagators, see e.g. documentation of KKMC Monte Carlo [2] or Dizet [11]. Such description has its limitations for the LHC applications, but for the processes of the Drell-Yan type with a moderate virtuality of produced lepton pairs is expected to be useful, even in the case when high $p_T$ jets are present. For the LEP applications [11], the EW form factors were used together with multi-photon bremsstrahlung amplitudes, but for the purpose of this paper we discuss their use for parton level Born processes only (no QED ISR/FSR).

The discussed here is so called the Improved Born Approximation (IBA) [11]. It absorbs some or all of higher order EW corrections into redefinition of couplings and propagators of the Born spin amplitude. This allows for straightforward calculation of doubly deconvoluted observables like various cross-sections and asymmetries.

The initial/final QCD and QED corrections form separately gauge invariant subsets of diagrams [11]. The QED subset consists of QED-vertices, $\gamma\gamma$ and $\gamma Z$ boxes and bremsstrahlung diagrams. Fermionic self-energies have to be also taken into account. The subset corresponding to the initial/final QCD corrections can be constructed also. All the remaining corrections contribute to the IBA: purely EW loop and boxes and internal QCD corrections (line-shape corrections). They can be split into two more gauge-invariant subsets, giving rise to two improved (or dressed) amplitudes: (i) improved $\gamma$ exchange amplitude with running QED coupling where only fermion loops contribute and (ii) improved $Z$-boson exchange amplitude with four, in general complex, EW form factors: $\rho_{\ell f}$, $\chi_{\ell}$, $\chi_f$, $\chi_{\ell f}$. Components of those corrections are as following:

- Corrections to photon propagator, where fermion loops contribute dominantly the so called vacuum-polarization corrections.
- Corrections to $Z$-boson propagator and couplings, called EW form-factors.
- Contribution from the purely weak boxes, the $WW$ and $ZZ$ diagrams. They are negligible at the $Z$-peak (suppressed by the factor $(s-M_Z^2)/s$), but very important at higher energies. They enter as corrections to form-factors and introduce dependence on $\cos \theta$ of scattering angle.
- Mixed $O(\alpha\alpha_s)$ corrections which originate from gluon insertions to the fermionic components of bosonic self-energies. They enter as corrections to all form-factors.

Below, to define notation we present the formula of the Born spin amplitude $\mathcal{A}^{\text{Born}}$. We recall conventions from [11]. Let us start with defining the lowest order coupling constants (without EW corrections) of the $Z$ boson to fermions: $s_W^2 = 1 - M_W^2/M_Z^2 = \sin^2 \theta_W$ defines weak Weinberg angle in the on-mass-shell scheme and $T_{3,\ell f}$ third component of the isospin. The vector $v_\ell, v_f$ and axial $a_\ell, a_f$ couplings for leptons and quarks are defined with the formulae below:\footnote{The inclusion of QED initial and final state bremsstrahlung, does not lead to complications of principle, but would obscure presentation. Necessary extensions are rather straightforward, thanks to properties of QED matrix elements, presented for the first time in [4].}

$$
\begin{align*}
v_\ell &= (2 \cdot T_3^\ell - 4 \cdot q_\ell \cdot s_W^2) / \Delta, \\
v_f &= (2 \cdot T_3^f - 4 \cdot q_f \cdot s_W^2) / \Delta, \\
a_\ell &= (2 \cdot T_3^\ell) / \Delta, \\
a_f &= (2 \cdot T_3^f) / \Delta.
\end{align*}
$$

\footnote{We will use “$\ell$” for lepton, and “$f$” for quarks.}
where
\[ \Delta = \sqrt{16 \cdot s_W^2 \cdot (1 - s_W^2)}. \] (2)

With this notation, the \( \mathcal{A}_{\text{Born}} \) spin amplitude for the \( q\bar{q} \rightarrow Z/\gamma' \rightarrow \ell^+\ell^- \) can be written as:
\[
\mathcal{A}_{\text{Born}} = \frac{\alpha}{s} \left\{ \left[ i\tau^\mu \gamma_\mu \chi_f(s) \cdot (q_\ell \cdot q_f) \right] + \left[ i\tau^\mu \gamma_\mu \chi_f(s) \cdot (\bar{v}_f \cdot v_\ell) \right] 
+ i\tau^\mu \gamma_\mu \bar{v}_f^\alpha u^\beta \cdot (\bar{v}_f \cdot v_\ell) + i\tau^\mu \gamma_\mu v_\ell^\alpha \bar{u}_f^\beta \cdot (a_\ell \cdot a_f) \right\} \chi_Z(s),
\] (3)

where \( u, v \) denote fermion spinors. The \( Z \)-boson and photon propagators are defined respectively as:
\[
\chi_y(s) = 1, \quad \chi_Z(s) = \frac{G_u \cdot M_Z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha \cdot s - M_Z^2 + i \cdot \Gamma_Z \cdot s/M_Z}. \] (4)

We can redefine vector and axial couplings and introduce EW form-factors \( \rho_{\ell f}(s, t), \chi_f(s, t), \chi_f(s, t), \chi_{\ell f}(s, t) \) as follows:
\[
\begin{align*}
v_\ell &= (2 \cdot T_5^f - 4 \cdot q_\ell \cdot s_W^2 \cdot \chi_f(s, t))/\Delta, \\
v_f &= (2 \cdot T_5^f - 4 \cdot q_f \cdot s_W^2 \cdot \chi_f(s, t))/\Delta, \\
a_\ell &= (2 \cdot T_3^f)/\Delta, \\
a_f &= (2 \cdot T_8^f)/\Delta.
\end{align*} \] (5)

Normalization correction \( Z_{\text{VH}} \) to \( Z \)-boson propagator is defined as
\[ Z_{\text{VH}} = \rho_{\ell f}(s, t). \] (6)

Re-summed vacuum polarization corrections \( \Gamma_{\text{VH}} \) to \( \gamma \) propagator are expressed as
\[ \Gamma_{\text{VH}} = \frac{1}{2 - (1 + \Pi_\gamma(s))}, \] (7)
where \( \Pi_\gamma(s) \) denotes vacuum polarization loop corrections of virtual photon exchange. Both \( \Gamma_{\text{VH}} \) and \( Z_{\text{VH}} \) are multiplicative correction factors. The \( \rho_{\ell f}(s, t) \) can be also absorbed as multiplicative factor into definition of vector and axial couplings.

The EW form-factors \( \rho_{\ell f}(s, t), \chi_f(s, t), \chi_f(s, t), \chi_{\ell f}(s, t) \) depend on two Mandelstam invariants \( (s, t) \) due to contributions of the \( WW \) and \( ZZ \) boxes. The Mandelstam variables satisfy the identity
\[ s + t + u = 0 \quad \text{where} \quad t = -\frac{s}{2}(1 - \cos \theta) \] (8)
and \( \cos \theta \) is the cosine of the scattering angle, i.e. angle between incoming and outgoing fermion directions.

Note, that in this approach the mixed EW and QCD loop corrections, originating from gluon insertions to fermionic components of bosonic self-energies, are included in \( \Gamma_{\text{VH}}, Z_{\text{VH}} \).

One has to take special attention to the angle dependent vector coupling product. The correction breaks factorization, formula (3), of the couplings into ones associated with either \( Z \) boson production or decay. The mixed term has to be introduced:
\[
\begin{align*}
\chi_{\ell f} = \frac{1}{v_\ell \cdot v_f} & \left[ \frac{(2 \cdot T_5^f)(2 \cdot T_5^f) - 4 \cdot q_\ell \cdot s_W^2 \cdot \chi_f(s, t)(2 \cdot T_5^f) - 4 \cdot q_f \cdot s_W^2 \cdot \chi_f(s, t)(2 \cdot T_5^f)}{\Delta^2} 
+ (4 \cdot q_\ell \cdot s_W^2)(4 \cdot q_f \cdot s_W^2) \cdot \chi_{\ell f}(s, t) \right] \chi_Z(s).
\end{align*} \] (9)

Finally, we can write the spin amplitude for Born with EW corrections, \( \mathcal{A}_{\text{Born+EW}} \), as:
\[
\mathcal{A}_{\text{Born+EW}} = \frac{\alpha}{s} \left\{ \left[ i\tau^\mu \gamma_\mu \chi_f(s) \cdot (q_\ell \cdot q_f) \right] \cdot \Gamma_{\text{VH}} \cdot \chi_f(s) + \left[ i\tau^\mu \gamma_\mu \chi_f(s) \cdot (v_\ell \cdot v_f) \cdot \chi_{\ell f} \right] 
+ i\tau^\mu \gamma_\mu v_\ell^\alpha \bar{u}_f^\beta \cdot (a_\ell \cdot a_f) \right\} \chi_Z(s),
\] (10)
The EW form factor corrections: ρ_ℓf, K_ℓf, K_f, K_ℓf can be calculated using the Dizet library. This library invokes also calculation of vacuum polarization corrections to photon propagator Π_{γγ}. For the case of pp collisions we do not introduce QCD corrections to vector and axial coupling of incoming fermions. They are assumed to be included later as a part of the QCD NLO calculations for the initial parton state, including convolution with proton structure functions.

The Improved Born Approximation uses the spin amplitude ω^{Born+EW} of Eq. (11) and 2 → 2 body kinematics to define the differential cross-section with EW corrections for q̅q → Z/γ* → ll process. The presented above formulae very closely follow the approach taken for implementation of EW corrections to KKMC Monte Carlo [2].

3 Electroweak form-factors

For the calculation of EW corrections, we use Dizet library as of the the KKMC Monte Carlo [2] (version of 2010). The interface prepares look-up tables with EW form-factors and vacuum polarization corrections. The pre-tabulation grid granularity and ranges of the centre-of-mass energy of outgoing leptons and lepton scattering angle are adapted to variation of the tabulated functions. Theoretical uncertainties on the predictions for EW form-factors have been estimated in times of LEP precision measurements, in the context of either benchmark results like [17] or specific analyses [3]. The predictions are updated with the known Higgs boson mass and better measurements of the top-quark mass. In existing code of the Dizet library, certain types of the corrections or options of the calculations of different corrections can be switched off/on. In Appendix C, we show in Table 11 almost the complete list of available options for calculations. Now we do not attempt to estimate the size of theoretical uncertainties, delegating it to the followup work in the context of LHC EW Precision WG studies. The other versions of electroweak calculations, like of [13, 18], can and should be studied then as well. Already now the precision requirements [15] are comparable to those of individual LEP measurements.

3.1 Input parameters to Dizet

The Dizet package relies on the so called on-mass-shell (OMS) normalization scheme [19, 20] but modifications are present. The OMS uses the masses of all fundamental particles, both fermions and bosons, the electromagnetic coupling constant α(0) and the strong coupling α_s(M^2_Z). The dependence on the ill-defined masses of the light quarks u, d, c, s and b is solved by dispersion relations, for details see [11]. Another exception is the W-boson mass M_W, which still can be predicted with better theoretical error than experimentally measured values, exploiting the very precise knowledge of the Fermi constant in μ-decay G_μ. For this reason, M_W is usually replaced by G_μ as an input.

The knowledge about the hadronic vacuum polarization is contained in ∆α_{h}^{(5)}(s), which is treated as input. It can be either computed from quark masses or, preferably, fitted to experimental low energy e^+ e^- → hadrons data.

The basic choice implemented in the Dizet library, for calculating EW corrections at the Z-resonance, is to use G_μ and M_Z as input parameters and then calculate M_W.

The M_W is calculated iteratively from the equation

\[
M_W = \frac{M_Z}{\sqrt{2}} \sqrt{1 + \sqrt{1 - \frac{4A_0^2}{M_Z^2(1 - \Delta r)}}}
\]

(12)

where

\[
A_0 = \frac{\pi \alpha(0)}{\sqrt{2} G_\mu}
\]

(13)

The Sirlin’s parameter ∆r [21]

\[
\Delta r = \Delta \alpha(M_Z^2) + \Delta r_{EW}
\]

(14)

is also calculated iteratively, and the definition of ∆r_{EW} involves re-summation and higher order corrections. Since this term implicitly depends on M_W and M_Z iterative procedure is needed again. The re-summation term in formula

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3 Compatibility with this program is also part of the motivation why we leave updates for the Dizet library to the forthcoming work. Dizet 6.21 is also well documented.
Table 1: The Dizet initialization: masses and couplings. The $M_W$ and $s_W^2$ from internal calculations are also shown.

| Parameter | Value | Description |
|-----------|-------|-------------|
| $M_Z$     | 91.1876 GeV | mass of Z boson |
| $M_H$     | 125.0 GeV   | mass of Higgs boson |
| $m_t$     | 173.0 GeV   | mass of top quark |
| $m_b$     | 4.7 GeV     | mass of b quark |
| $1/\alpha(0)$ | 137.0359895(61) | Electromagnetic coupling in Thomson limit |
| $G_\mu$   | $1.166389(22) \cdot 10^{-5}$ GeV$^{-2}$ | Fermi constant in $\mu$-decay |
| $M_W$     | 80.353 GeV  | calculated with formula (12) |
| $s_W^2$   | 0.22351946  | calculated with formula (16) |

(14) is not formally justified by renormalisation group arguments, the correct generalization is to compute higher order corrections, see more discussion in [11].

Note that once the $M_W$ is recalculated from formula (12), the Standard Model relationship between the weak and electromagnetic couplings

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}M_W^2 \sin^2\theta_W}$$

(15)
is not anymore fulfilled, unless the $G_\mu$ is redefined away from the measured value. This is an approach of some EW LO schemes, but not the one used by Dizet. It requires therefore the complete, as defined by formula (5), expression for $\chi_Z(s)$ propagator in spin amplitude Eq. (11).

In the OMS renormalisation scheme the weak mixing angle is defined uniquely through the gauge-boson masses:

$$\sin^2\theta_W = s_W^2 = 1 - \frac{M_W^2}{M_Z^2}.$$  

(16)With this scheme, measuring $\sin^2\theta_W$ will be equivalent to indirect measurement of $M_W^2$ through the relation (16).

In Table 1 we collect numerical values for all parameters used in the presented below evaluations. Note that formally they are not representing EW LO scheme, as the relation (15) is not obeyed. The $M_W$ in (16) is recalculated with (12) but $G_\mu$, $M_Z$ remain unchanged.

### 3.2 The EW form-factors

Real parts of the $\rho_{\ell f}(s,t)$, $\kappa_{\ell f}(s,t)$, $\kappa_f(s,t)$, $\kappa_{\ell f}(s,t)$ EW form-factors are shown in Figure 1 for a few values of $\cos\theta$, the angle between directions of the incoming quark and the outgoing lepton, calculated in the outgoing lepton pair centre-of-mass frame. The Mandelstam variables $(s,t)$ relate to the invariant mass and $\cos\theta$ by Eq. (9).

The box correction $\cos\theta$ dependence is more sizable for the up-quarks. Note, that at the Z-boson peak, Born like couplings are only weakly modified; form-factors are close to 1 and of no numerically significant angular dependence. At lower virtualities corrections are relatively larger because the Z-boson contributions are non resonant and thus smaller. In this phase-space region the Z-boson is dominated by the virtual photon contribution anyway. Above the peak, the $WW$ and later also $ZZ$ boxes contributions become sizable, the dependence on the $\cos\theta$ gradually appears. Those contributions become gradually doubly resonant and sizable.

### 3.3 Running $\alpha(s)$

Fermionic loop insertion of the photon propagator, i.e. vacuum polarization corrections, are summed together as a multiplicative factor, Eq. (8), for the photon exchange in Eq. (11). But it can be interpreted as $\alpha(s)$ the running QED coupling:

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(5)(s) - \Delta\alpha_e(s) - \Delta\alpha_e(s) - \Delta\alpha\alpha\alpha(5)(s)}.$$  

(17)
Figure 1: Real parts of the $\rho_{\text{up}}$, $\kappa_{s}$, $\kappa_{\text{up}}$ and $\kappa_{\text{down}}$ EW form factors for $q\bar{q} \rightarrow Z \rightarrow ee$ process, as a function of $\sqrt{s}$ and for the few values of $\cos\theta$. For the up-type quark flavour left side plots are prepared and for the down-type right side plots. Note that $\kappa_{s}$ depends on the flavour of incoming quarks.
The hadronic contribution at $M_Z$ is a significant correction: $\Delta \alpha_s^{(5)}(M_Z^2) = 0.0280398$. It is calculated in the five flavour scheme with use of dispersion relation and input from low energy experiments. We will continue to use LEP times parametrization, while the most recent measured $\Delta \alpha_s^{(5)}(M_Z^2) = 0.02753 \pm 0.00009$ [22]. The changed value, modify predicted form-factors, in particular the effective lepton mixing angle $\sin^2 \theta_{eff}^\ell(\bar{M}_Z^2) = \Re, \mathcal{K}_{\bar{W}}$ is shifted by almost $20 \cdot 10^5$, closer to the measured at LEP value. This is not included in the numerical results presented as we consistently remain with the defaults used in KKMC.

The leptonic loop contribution $\Delta \alpha_s(s)$ is calculated analytically up to the 3-loops, and is a comparably significant correction, $\Delta \alpha_s(M_Z^2) = 0.0314976$. The other contributions are very small. Fig. 2 shows the vacuum polarization corrections to the $\chi_f(s)$ propagator, directly representing the ratio $\alpha(s)/\alpha(0)$ of Eq. (17).

### 4 EW input schemes and Effective Born

Formally, at the lowest EW order, only three parameters can be set, other are calculated using Standard Model constraints, following structure of $SU(2) \times U(1)$ group. One of such constraint is formula (15). The most common, following report [23], choices at hadron colliders are: $G_\mu$ scheme ($G_\mu, M_Z, M_W$) and $\alpha(0)$ scheme ($\alpha(0), M_Z, M_W$). There exists by now family of different modifications of $G_\mu$ scheme, see discussion in [23], and they are considered as preferred schemes for hadron collider physics.

The Monte Carlo generators usually allow user to define set of input parameters ($\alpha, M_Z, M_W$), ($\alpha, M_Z, G_\mu$) or ($\alpha, M_Z, s_\bar{W}$). However, within this flexibility, formally multiplicative factor in the $Z$-boson propagator $\chi_Z(s)$, see formula (5), is always kept to be equal to 1. The

$$ G_\mu \cdot M_Z^2 \cdot \Delta^2 = 1; \text{ and } \Delta^2 = 16 \cdot e_W^2 \cdot s_\bar{W}^2. $$

This term is quite often absent in the programs code. So, with the choice of primary parameters, the others are adjusted to match the constraint Eq. (18), regardless if they fall outside their measurement uncertainty window or not.

Let us recall, that the calculations of EW corrections available in Dizet library work with a variant convention of the $\alpha(0)$ scheme. It is defined by the input parameters ($\alpha(0), G_\mu, M_Z$). Then $M_W$ is calculated iteratively from formula (12). This formally brings it beyond EW LO scheme. The numerical value of $s_\bar{W}$ calculated from (16) does not fulfill the EW LO relation (15) anymore.

At this point we would like to introduce two options for the Effective Born spin amplitudes parametrisation, which work well for the EW corrections near the $Z$-pole. We denote them respectively as LEP and LEP with improved norm.:

- The LEP parametrisation use formula (11) for spin amplitude but with $\alpha(s) = \alpha(M_Z^2) = 1 ./ 128.8667, \ s_\bar{W} = \sin^2 \theta_{eff}^\ell(\bar{M}_Z^2) = 0.23152$, i.e. as measured at the $Z$-pole and reported in [24]. All form factors are set to 1.0.
Table 2: The EW parameters used for: (i) MC events generation, (ii) the EW LO $\alpha(0)$ scheme, (iii) effective Born spin amplitude around the Z-pole and (iv) effective Born with improved normalization. In each case parameters are chosen that the SM relation, formula (18), is obeyed. The $G_\mu = 1.166389 \cdot 10^{-5}$ GeV$^{-2}$, $M_Z = 91.1876$ GeV and $\mathcal{K}_f, \mathcal{K}_e, \mathcal{K}_f = 1$ are taken.

| EW LO MC generator | EW LO $\alpha(0)$ scheme | Effective Born LEP | Effective Born LEP with improved norm. |
|--------------------|--------------------------|-------------------|----------------------------------------|
| $\alpha = 1/128.8886$ | $\alpha = 1/137.3599$ | $\alpha = 1/128.8867$ | $\alpha = 1/128.8667$ |
| $s_W^2 = 0.23113$ | $s_W^2 = 0.21215$ | $s_W^2 = 0.23152$ | $s_W^2 = 0.23152$ |
| $\rho_{\ell f} = 1.0$ | $\rho_{\ell f} = 1.0$ | $\rho_{\ell f} = 1.0$ | $\rho_{\ell f} = 1.005$ |

- The LEP with improved norm. parametrisation also use formula (11) for spin amplitude with parameters are set as for LEP parametrisation. All form-factors are set to 1, except $\rho_{\ell f} = 1.005$. This corresponds to the measured $\rho(M_Z^2) = 1.005$ and reported in [24].

Table 2 collects initialization constants of EW schemes relevant for our discussion. We specify parameters which enter formula (11) used for calculating Born spin amplitudes for: (i) MC events generation, (ii) the EW LO $\alpha(0)$ scheme, (iii) effective Born (LEP) parametrisation and (iv) effective Born (LEP with improved norm.). In each case parameters are chosen that the SM relation, formula (18), is obeyed.

In the Improved Born Approximation complete $O(\alpha)$ EW corrections, supplemented by selected higher order terms, are handled thanks to s-, t-dependent form-factors, which multiply couplings and propagators of the usual Born expressions. The Effective Born absorbs bulk of EW corrections into redefinition of a few fixed parameters (i.e. couplings) instead.

In the following, we will systematically compare predictions obtained with the EW corrections and those calculated with LEP or LEP with improved norm. approximations. As we will see, effective Born with LEP with improved norm. works well around Z-pole both for the line-shape and forward-backward asymmetry.

5 Born kinematic approximation and $pp$ scattering

The solution how to define Born-like kinematics in case of $pp$ scattering is available in the algorithms of TauSpinner package [12]. It assumes that hard-process history sometimes encoded in the so-called history entries of the generated event is not known. In particular the flavour and momenta of the incoming partons have to be therefore reconstructed, from the kinematics of final states, reaction center of mass energy and with the help of probabilities calculated from parton level cross-sections and PDFs. We recall briefly principles and explain optimization.

5.1 Average over incoming partons flavour

Parton level Born cross-section $\sigma^q\bar{q}_{\text{Born}}(\hat{s}, \cos \theta)$ has to be convoluted with the structure functions, and summed over all possible flavours of incoming partons and all possible helicity states of outgoing leptons. The lowest order formula is given below

$$d\sigma_{\text{Born}}(x_1, x_2, \hat{s}, \cos \theta) = \sum_{q_f, \bar{q}_f} [f^{\hat{q}_f}(x_1, \ldots) f^{\bar{q}_f}(x_2, \ldots) d\sigma^q\bar{q}_{\text{Born}}(\hat{s}, \cos \theta) + f^{q_f}(x_1, \ldots) f^{\bar{q}_f}(x_2, \ldots) d\sigma^{q\bar{q}}_{\text{Born}}(\hat{s}, -\cos \theta)],$$

where $x_1, x_2$ denote fractions of incoming protons momenta carried by the corresponding parton, $\hat{s} = x_1 x_2 s$ and $f/\bar{f}$ denotes parton (quark/anti-quark) density functions. We assume that kinematics is reconstructed from four-momenta of the outgoing leptons. The sign in front of $\cos \theta$, the cosine of the scattering angle is negative if the

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4The EW LO initialization is consistent with PDG value $\sin^2 \theta_W^{\text{LO}} = 0.23113$, but commonly used $G_\mu$ scheme, ($G_\mu = 1.1663787 \cdot 10^{-5}$ GeV$^{-2}$, $M_Z = 91.1876$ GeV, $M_W = 80.385$ GeV) correspond to $\sin^2 \theta_W = 0.228972$.

5Valid for the ultra-relativistic leptons.
parton which carry $x_1$ of the first incoming proton (the one which follows the $z$-axis direction) is a quark rather than anti-quark. The two possibilities are taken into account by the two terms of (19). The formula is used for calculating differential cross-section $d\sigma_{\text{Born}}(x_1, x_2, \delta, \cos\theta)$ of each analyzed event, regardless if its kinematics and flavours of incoming partons may be available from the event history entries. The formula can be used to a good approximation in case of NLO QCD spin amplitudes. The kinematics of outgoing leptons is used to construct effective kinematics of the Drell-Yan production process and decay, without need for information on the history of the hard-process itself. It can be constructed, as we will see later, for events where initial state of Feynman diagrams were quark-gluon or gluon-gluon parton pair (as stored in the history event entries) too.

5.2 Effective beams kinematics

The $x_1, x_2$ are calculated from kinematics of outgoing leptons, following formulae of [25]

$$x_{1,2} = \frac{1}{2} \left( \pm \frac{p_z}{4E} + \sqrt{\left(\frac{p_z}{4E}\right)^2 + 4 \left(\frac{m_{\ell\ell}^2}{4E^2}\right)^2} \right), \quad (20)$$

where $E$ denotes energy of the proton beam and $p_z$ denotes $z$-axis momenta of outgoing lepton pairs in the laboratory frame and $m_{\ell\ell}$ lepton pair virtuality.

5.3 Definition of the polar angle

For the cos $\theta$, in case of $q\bar{q} \rightarrow Z \rightarrow \ell\ell$ process, weighted average of the outgoing leptons angles with respect to the beams directions [26] denoted as cos $\theta^*$ can be used. Extension to $pp$ collisions, requires the choice if the $z$-axis is parallel to the quark or to the anti-quark direction. For the further calculation, boost of all four-momenta (also of incoming beams) into the rest frame of the lepton pair need to be performed. The cos $\theta^*$ is the calculated from

$$\cos \theta_1 = \frac{\tau_1(1) b_z(1) + \tau_1(1) b_y(1) + \tau_1(1) b_z(1)}{|\tau_1(1)||\bar{b}(1)|}, \quad \cos \theta_2 = \frac{\tau_2(2) b_z(2) + \tau_2(2) b_y(2) + \tau_2(2) b_z(2)}{|\tau_2(2)||\bar{b}(2)|}, \quad (21)$$

as follows:

$$\cos \theta^* = \frac{\cos \theta_1 \sin \theta_2 + \cos \theta_2 \sin \theta_1}{\sin \theta_1 + \sin \theta_2} \quad (22)$$

where $\tau_1(1), \tau_2(2)$ denote 3-vectors of outgoing leptons and $\bar{b}(1), \bar{b}(2)$ denote 3-vectors of incoming beams four-momenta.

The definition of cosine polar angle, Eq. (22) is a default of TauSpinner algorithms. Alternatively, one can use also polar angles of Mustraal [4] or Collins-Soper [27] frames. We will return later to the frame choice, suitable best when NLO QCD corrections are included in the production process of the generated events.

6 QCD corrections and angular coefficients

For the Drell-Yan production [28], one can factorize QCD and EW components of the fully differential cross-section with the help of early 90’s formalism [29, 30, 31] and describe the $Z \rightarrow \ell\ell$ sub-process with factorized out lepton angular dependence in differential cross-section

$$\frac{d\sigma}{dp_T^2 dY d\Omega} = \frac{1}{16\alpha_s} \frac{d\sigma^\alpha}{dY} = \frac{1}{16\alpha_s} \frac{d\sigma^\alpha}{dY}, \quad (23)$$

where the $g_\alpha(\theta, \phi)$ denotes second order spherical harmonics, multiplied by normalization constants and helicity cross-sections $d\sigma^\alpha$, for each of nine helicity configurations of $q\bar{q} \rightarrow Z/\gamma^* \rightarrow \ell\ell$. The polar and azimuthal ($\theta$ and $\phi$) angles of $d\Omega = d\cos\theta d\phi$ are defined in the $Z$-boson rest-frame. The $p_T, Y$ denote laboratory frame transverse momenta and rapidity of the intermediate $Z$-boson. For the $Z$-boson rest frame $z$-axis there is some flexibility. The most common, so called helicity frame choice, is to take it as $Z$-boson laboratory frame momentum. For the Collins-Soper frame [27] it is defined from directions of the two beams in the $Z$-boson rest frame and is signed with the $Z$-boson $p_z$ laboratory frame sign.
The Eq. (23) with explicit spherical harmonics and coefficients reads

\[
\frac{d\sigma}{dp_T^2 dY d\cos \theta d\phi} = \frac{3}{16\pi} \frac{d\sigma^{U+L}}{dp_T^2 dY} \left[ (1 + \cos^2 \theta) + \frac{1}{2} A_0 (1 - 3 \cos^2 \theta) + A_1 \sin(2\theta) \cos \phi + \frac{1}{2} A_2 \sin^2 \theta \cos(2\theta) + A_3 \sin \theta \cos \phi + A_4 \cos \theta + A_5 \sin^2 \theta \sin(2\phi) + A_6 \sin(2\theta) \sin \phi + A_7 \sin \theta \sin \phi \sin \theta \right],
\]

where \(d\sigma^{U+L}\) denotes the unpolarised differential cross-section (notation used in several papers of the 80’s). The coefficients \(A_i(p_T, Y)\) are related to ratios of intermediate state helicity configurations cross-sections to \(d\sigma^{U+L}\). The first \((1 + \cos^2 \theta)\) term of the polynomial expansion, is because spin 1 of the intermediate boson.

The dynamics of the production process is hidden in the angular coefficients \(A_i(p_T, Y)\). In particular, all the hadronic physics is described implicitly by the angular coefficients and it decouples from the well understood leptonic and intermediate boson physics.

For the present paper discussion, of particular interest are couplings implicitly given with Eq. (24) for \(A_i(p_T, Y)\):

\[
\begin{align*}
\sigma^{U+L} & \sim (v_T^2 + a_T^2)(v_q^2 + a_q^2), \\
A_0, A_1, A_2 & \sim 1, \\
A_3, A_4 & \sim \frac{v_T a_T v_q a_q}{(v_T^2 + a_T^2)(v_q^2 + a_q^2)}, \\
A_5, A_6 & \sim \frac{(v_T^2 + a_T^2)(v_T a_q)}{(v_T^2 + a_T^2)(v_q^2 + a_q^2)}, \\
A_7 & \sim \frac{v_T a_T (v_T^2 + a_T^2)}{(v_T^2 + a_T^2)(v_q^2 + a_q^2)}.
\end{align*}
\]

Thanks to the orthonormality of the formula (24) polynomials, integration over azimuthal angle \(\phi\) reduces Eq. (24) to

\[
\frac{d\sigma}{dp_T^2 dY d\cos \theta} = \frac{3}{8\pi} \frac{d\sigma^{U+L}}{dp_T^2 dY} \left[ (1 + \cos^2 \theta) + \frac{1}{2} A_0 (1 - 3 \cos^2 \theta) + A_4 \cos \theta \right].
\]

Both Eqs. (24) and (26), are valid in any rest frame of the outgoing lepton pairs, however the \(A_i(p_T, Y)\) are frame dependent. The Collins-Soper frame is the most convenient and usual choice for the analyses dedicated to QCD dynamics. In this frame, in the low \(p_T\) limit, the \(A_4\) is the only non-zero formula (26) coefficient. It carries direct information on the EW couplings, as can be concluded from formulae (25). All other coefficients depart from zero with increasing \(p_T\) while at the same time \(A_4\) gradually decreases.

For the EW couplings studies, more convenient is the lepton pair rest frame where production dynamics is absorbed into definition of the frame itself (choice of the \(z\)-axis) and the non-zero \(A_i\) coefficients over full range of \(p_T\) are those sensitive to the EW couplings of the intermediate boson to outgoing leptons and incoming quarks. Such a frame was developed at LEP times for the Mustraal program [4]. Recently, extension of this Mustraal frame, for the case of hadron-hadron collisions was introduced and discussed in [5]. As shown in that paper, both Collins-Soper and Mustraal frames are equivalent in the \(p_T = 0\) limit. Then the only non-zero coefficient \(A_4\) is numerically very close for both frames. With the increasing \(p_T\), in the Mustraal frame \(A_4\) remains as the only sizably non-zero one, while many of \(A_i\) coefficients depart from zero with the Collins-Soper frame.

In the proton-proton collision, the \(A_4\) coefficient to remain non-zero in the \(p_T = 0\) limit is not straightforward. It requires choice from two possible \(z\)-axis orientations. In case of Collins-Soper frame, orientation of the \(z\)-axis is to follow direction of the intermediate \(Z\)-boson in the laboratory frame. In case of Mustraal frame the choice of the sign is made statistically using information of the system of leptons and outgoing accompanying visible jets, for details see [5], alternatively the same choice for the sign of the \(z\)-axis as in the Collins-Soper case can be used.

The shape of \(A_i\) coefficients as a function of laboratory frame \(Z\)-boson transverse momenta \(p_T\), depend on the choice of lepton pairs rest-frame. In Fig. 3 \(A_i\) coefficients of the Collins-Soper and Mustraal frames are shown. The Mustraal frame is designed specifically to preserve in the presence of multiple high \(p_T\) jets, decomposition of the distribution into two Born-like terms. As intended, in this frame only \(A_4\) coefficient is sizably non-zero for large \(p_T\).
Figure 3: The $A_i$ coefficients for $Z \rightarrow e^+e^-$ in invariant mass range $80 < m_{ee} < 100$ GeV. The $Z + j$ production process in $pp$ collisions at 8 TeV centre-of-mass energy, generated with Powheg+MiNLO Monte Carlo generator was chosen. The $A_i$ coefficients are calculated in the Collins-Soper and Mustraal frames with moments method \cite{30}.
7 Concept of the EW weight

The EW corrections enter expression for the $\sigma_{\text{Born}}(s,\cos \theta)$ through the definition of the vector and axial couplings and propagators of photon and Z-boson. They modify normalization of the cross-sections, the line-shape of the Z-boson, polarization of the outgoing leptons and asymmetries.

Given that, to a good approximation we were able to factorize QCD and EW components of the cross-section and we can define per-event weight which specifically corrects for EW effects. Such weight modify events generated with EW LO to the one including the EW corrections as well. This is very much the same idea as already implemented in TauSpinner for introducing corrections for other effects: spin correlations, production process, etc.

The per-event weight $w_{\text{EW}}$ is defined as ratio of the Born-level cross-sections with and without EW corrections

$$w_{\text{EW}} = \frac{d\sigma_{\text{Born}+\text{EW}}(s,\cos \theta)}{d\sigma_{\text{Born}}(s,\cos \theta)}, \quad (27)$$

where $\cos \theta$ can be taken according to $\cos \theta^\ast$, $\cos \theta_{\text{Mistraal}}$ (Mistraal frame) or $\cos \theta_{\text{CS}}$ (Collins-Soper) prescription. The $w_{\text{EW}}$ weight allows for flexible and straightforward implementation of the EW corrections using TauSpinner framework and form-factors calculated e.g. with Dizet library.

The formula for $w_{\text{EW}}$ can be used to reweight from one to another EW LO scheme too. In that case, both the numerator and denominator of Eq. $\ref{(27)}$ will use lowest order $d\sigma_{\text{Born}}$, but calculated in different EW scheme.$^6$

8 EW corrections to doubly-deconvoluted observables

With all components needed for calculating $w_{\text{EW}}$ defined, we will now show selected examples of numerical results for doubly-deconvoluted observables around the Z-pole.

The Powheg+MiNLO Monte Carlo, with NLO QCD and LO EW matrix elements, was used to generate $Z + j$ events with $Z \rightarrow e^+e^-$ decays in $pp$ collisions at 8 TeV. No selection was applied to generated events, except outgoing electrons invariant mass in the $70 < m_{ee} < 150$ GeV range. For events generation, the EW parameters as shown in left-most column of Table 2 were used. The $\alpha$ and $s_W^2$ close to the ones of MSbar discussed in $\ref{24}$ were taken. Note that they do not coincide with the precise LEP experiments measurements at the Z-pole $\ref{11}$. The initialization of Table 2 left most column, is often used as a default for phenomenological studies at LHC.

To quantify the effect of the EW corrections, we reweight generated MC events to EW LO with the scheme used by the Dizet, Table 2 second column, and then introduce gradually EW corrections and form-factors calculated with that library. For each step appropriate numerator of the $w_{\text{EW}}$ is calculated, while for the denominator the EW LO $\sigma_{\text{Born}}$ matrix element is used, parameters as in the left-most column of Table 2. The sequential steps, in which we illustrate effects of EW corrections are given below:

1. Reweight with $w_{\text{EW}}$, from EW LO scheme used for MC events generation to EW LO scheme with $s_W^2 = 0.21215$, Table 2 second column. The $\sigma_{\text{Born}}$ matrix element, Eq. $\ref{3}$, is used for calculating numerator of $w_{\text{EW}}$.

2. As in step (1), but include EW corrections to $M_W$, effectively changing to $s_W^2 = 0.22352$ in calculation of $w_{\text{EW}}$. Relation, formula $\ref{15}$, is not obeyed anymore.

3. As in step (2), but include EW loop corrections to the normalization of Z-boson and $\gamma$ propagators, i.e. QCD/EW corrections to $\alpha(0)$ and $\rho_{s_f}(s)$ form-factor calculated without box corrections. The $\sigma_{\text{Born}+\text{EW}}$ is used for calculating numerator of $w_{\text{EW}}$.

4. As in step (3), but include EW corrections to Z-boson vector couplings: $\mathcal{K}_f, \mathcal{K}_e, \mathcal{K}_{f_f}$, calculated without box corrections. The $\sigma_{\text{Born}+\text{EW}}$ is used for calculating numerator of $w_{\text{EW}}$.

5. As in step (4), but $\rho_{s_f}, \mathcal{K}_f, \mathcal{K}_e, \mathcal{K}_{f_f}$ form-factors include box corrections. The $\sigma_{\text{Born}+\text{EW}}$ is used for calculating numerator of $w_{\text{EW}}$.

$^6$ In this way, fixed width description can be replaced with the $s$ dependent one.

$^7$ The MC sample is generated with fixed width propagator. We remain with this convention. This could also be changed with the help of $w_{\text{EW}}$. 

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After step (1) the predictions are according to EW LO and QCD NLO, but with different EW scheme than used originally for events generation. Then steps (2)-(5) introduce EW corrections. Step (3) effectively changes back $\alpha$ to be close to $\alpha(M_Z^2)$, while steps (4)-(5) effectively shift back value of $s_W$ to be close to the one used for events generation. Given the fact that EW LO scheme used for generating events has parameters already close to measured at the Z-pole, we expect the total EW corrections to the generated sample to be roughly at percent level.

In the following, we will also estimate how precise it would be to use effective Born approximation with LEP or LEP with improved norm. parametrisations instead of complete EW corrections. To obtain those predictions, reweight similar to step (1) listed above is needed, but in the numerator of $w_{\text{EW}}^{\text{Born}}$ the $\sigma_{\text{Born}}$ parametrisations as specified in the right two columns of Table 2 are used. For LEP with improved norm. the $\rho_{\epsilon,f} = 1.005$ has to be included.

The important flexibility of proposed approach is that $w_{\text{EW}}^{\text{Born}}$ can be calculated using $d\sigma_{\text{Born}}$ in different frames: $\cos \theta^*$, Mustraal or Collins-Soper. For some observables, frame choice used for $w_{\text{EW}}^{\text{Born}}$ calculation is not relevant at all and the simplest $\cos \theta^*$ frame can be used. We show later an example, where only Mustraal frame for the $w_{\text{EW}}^{\text{Born}}$ calculation leads to correct results of the reweighting procedure.

### 8.1 The Z-boson line-shape

In the EW LO, the Z-boson line-shape, assuming that the constraint [15] holds, depends predominantly on two parameters $M_Z$ and $\Gamma_Z$. The effect on the line-shape from EW loop corrections are due to corrections to the propagators: vacuum polarization corrections (running $\alpha$) and $\rho$ form-factor, which change relative contributions of the $Z$ and $\gamma$, and the Z-boson vector to axial coupling ratio (sin$^2 \theta_{eff}$) too. The above, affects not only shape but normalization of the cross-section too. In the formulae we do not introduce running $Z$ normalization, which remain fixed.

In Fig. 4 (top-left) distributions of generated and EW corrected line-shapes are shown. With the logarithmic scale difference is barely visible. With the following plots of the same Figure we study details. The ratios of the line-shape distributions with gradually introduced EW corrections are shown. For the reference (weight denominators) of the following three plots the schemes: (i) EW LO $\alpha(0)$, (ii) effective Born (LEP) and (iii) effective Born (LEP with improved norm.) are used. At the Z-pole, complete EW corrections contribute about 0.1% with respect to the one of effective Born (LEP with improved norm.). Use of events generated with EW LO matrix element but of different parametrisations significantly reduce the numerical size of missing EW corrections.

Table 3 details numerically EW corrections to the normalization (ratios of the cross-section) integrated in the range $80 < m_{ee} < 100$ GeV and $89 < m_{ee} < 93$ GeV. Results from EW weight with $\cos \theta^*$ definition of the scattering angle are shown. Total EW correction factor is about 0.965 for cross-section normalization and EW LO $\alpha(0)$, while total correction to the effective Born (LEP with improved norm.) is of about 1.001. In Table 4 results with $w_{\text{EW}}^{\text{Born}}$ calculated with different frames are compared. If Mustraal or Collins-Soper frame are used instead of $\cos \theta^*$ for weight calculations, the differences are at most at the 5-th significant digit.

### 8.2 The $A_{FB}$ distribution

The standard way defined forward-backward asymmetry for $pp$ collisions reads

$$A_{FB} = \frac{\sigma(\cos \theta > 0) - \sigma(\cos \theta < 0)}{\sigma(\cos \theta > 0) + \sigma(\cos \theta < 0)}, \quad (28)$$

where $\cos \theta$ of the Collins-Soper frame is used.

The EW corrections change overall normalization and the shape of $A_{FB}$, particularly around the Z-pole too. In Fig. 5 (top-left), the $A_{FB}$ is generated (EW LO) and EW corrected is shown as a function of $m_{ee}$. In the following plots of this Figure, we study details. The $\Delta A_{FB} = A_{FB}^\epsilon - A_{FB}^f$ difference, with gradually introduced EW corrections is shown. For the reference the following choices: (i) EW LO $\alpha(0)$, (ii) effective Born (LEP) and (iii) effective Born (LEP with improved norm.) were taken.

Complete EW corrections to predictions of EW LO $\alpha(0)$ scheme for $A_{FB}$ integrated around Z-pole are about $\Delta A_{FB} = -0.03534$.

The EW correction to $A_{FB}$ of effective Born (LEP with improved norm.), is only $\Delta A_{FB} = -0.00005$. We observe that effective Born (LEP improved norm.) reproduces EW loop corrections precision better than $\Delta A_{FB} = -0.0001$ in the full presented mass range. The remaining box corrections contribute around $m_{ee} = 150$ GeV $\Delta A_{FB} = -0.002$. 

Figure 4: Top-left: line-shape distribution as generated with Powheg+MinLO (blue triangles) and after reweighting introducing all EW corrections (red triangles). The two choices are barely distinguishable. Ratios of the line-shapes with gradually introduced EW corrections are shown in consecutive plots, where as a reference (black dashed line) respectively: (i) EW LO $\alpha(0)$ scheme (top-right), (ii) effective Born (LEP) (bottom-left) and (iii) effective Born (LEP with improved norm.) (bottom-right) was used.
Table 3: EW corrections for cross-sections integrated over the specified mass windows. The EW weight is calculated with \( \cos \theta^* \) scattering angle.

| Corrections to cross-section | \( 89 < m_{ee} < 93 \text{ GeV} \) | \( 80 < m_{ee} < 100 \text{ GeV} \) |
|-----------------------------|-------------------------------|-------------------------------|
| \( \sigma(\text{EW corr. to } m_W) / \sigma(\text{EW LO } \alpha(0)) \) | 0.97114 | 0.97162 |
| \( \sigma(\text{EW corr. to } \chi(Z), \chi(\gamma)) / \sigma(\text{EW LO } \alpha(0)) \) | 0.98246 | 0.98346 |
| \( \sigma(\text{EW/QCD FF no boxes}) / \sigma(\text{EW LO } \alpha(0)) \) | 0.96469 | 0.96602 |
| \( \sigma(\text{EW/QCD FF with boxes}) / \sigma(\text{EW LO } \alpha(0)) \) | 0.96473 | 0.96607 |
| \( \sigma(\text{LEP}) / \sigma(\text{EW/QCD FF with boxes}) \) | 1.01102 | 1.01093 |
| \( \sigma(\text{LEP with improved norm.}) / \sigma(\text{EW/QCD FF with boxes}) \) | 1.00100 | 1.00098 |

Table 4: EW corrections for cross-sections integrated over the mass window around \( Z \)-pole; \( 89 < m_{ee} < 93 \text{ GeV} \). The EW weight is calculated with \( \cos \theta^*, \cos \theta^{\text{Mustraal}} \) or \( \cos \theta^{\text{CS}} \) scattering angle definitions.

| Corrections to cross-section ( \( 89 < m_{ee} < 93 \text{ GeV} \)) | \( w^{\text{EW}}(\cos \theta^*) \) | \( w^{\text{EW}}(\cos \theta^{\text{Mustraal}}) \) | \( w^{\text{EW}}(\cos \theta^{\text{CS}}) \) |
|-------------------------------------------------------------|---------------------------------|---------------------------------|---------------------------------|
| \( \sigma(\text{EW corr. to } m_W) / \sigma(\text{EW LO } \alpha(0)) \) | 0.97114 | 0.97115 | 0.97114 |
| \( \sigma(\text{EW corr. to } \chi(Z), \chi(\gamma)) / \sigma(\text{EW LO } \alpha(0)) \) | 0.98246 | 0.98247 | 0.98246 |
| \( \sigma(\text{EW/QCD FF no boxes}) / \sigma(\text{EW LO } \alpha(0)) \) | 0.96469 | 0.96471 | 0.96470 |
| \( \sigma(\text{EW/QCD FF with boxes}) / \sigma(\text{EW LO } \alpha(0)) \) | 0.96473 | 0.96475 | 0.96474 |
| \( \sigma(\text{LEP}) / \sigma(\text{EW/QCD FF with boxes}) \) | 1.01102 | 1.0103 | 1.01102 |
| \( \sigma(\text{LEP with improved norm.}) / \sigma(\text{EW/QCD FF with boxes}) \) | 1.00100 | 1.00102 | 1.00100 |

Table 5 details numerically EW corrections, for \( A_{FB} \) predictions integrated over the \( 80 < m_{ee} < 100 \text{ GeV} \) and \( 89 < m_{ee} < 93 \text{ GeV} \) range. For calculating EW weight \( \cos \theta^* \) definition of the scattering angle are shown, but for asymmetry definition \( \cos \theta^{\text{CS}} \) was used. In Table 6 results obtained with \( w^{\text{EW}} \) calculated in different frames are compared. When \( \text{Mustraal} \) or \( \text{Collins-Soper} \) frame is used instead of \( \cos \theta^* \), the differences are at most at the 5-th significant digit.

### 8.3 Effective weak mixing angles

The forward-backward asymmetry \( A_{FB} \) at the \( Z \)-pole can be used as an observable for effective weak mixing Weinberg angles, taking into account its dependence on the invariant mass of lepton pairs.

We extend standard LEP times definition of effective weak mixing angles at the \( Z \)-pole, to more suitable for \( pp \) collision at LHC

\[
\sin^2 \theta^\ell_{eff}(s,t) = \text{Re}(\mathcal{K}^f(s,t)) s_W^2 + I^f_2(s,t),
\]

for the off \( Z \)-pole regions. The flavour dependent effective weak mixing angles, calculated using: Eq. (29), EW form-factors of Dizet library, and \( s_W^2 = 0.22352 \) are shown on Fig. 6 as a function of the invariant mass of outgoing lepton pair and for \( \cos \theta = 0.5 \). The imaginary part of \( I^f_2(s,t) \) is about \( 10^{-4} \) only. In Table 8 we display effective weak missing angles averaged over specified mass windows.

The effective \( \sin^2 \theta^\ell_{eff} \) on the \( Z \)-pole, printed by Dizet is shown in Table 7. It is numerically slightly different than of Table 8 which is averaged over close to \( Z \)-pole mass window. Note, that the observed very good agreement at the \( Z \)-pole between \( A_{FB} \) predictions of effective Born with (LEP) or (LEP with improved norm.) parametrisations and fully EW corrected is not reflected on predictions on per flavour effective weak Weinberg angle.

Effective Born (LEP) and (LEP with improved norm,) are parametrised with \( s_W^2 = 0.23152 \), while Dizet library predicts leptonic effective weak mixing angle \( \sin^2 \theta^\ell_{eff}(M^2_Z) = 0.23176 \). Why then such a good agreement on \( \Delta A_{FB} \) as seen on Fig. 5 bottom plots? Certainly this requires further attention.
Table 5: The difference $\Delta A_{FB}$ in forward-backward asymmetry calculated in the specified mass window. The $\cos \theta^C_S$ is used to define forward and backward hemispheres. The EW weight is calculated from $\theta^*$ definition of the scattering angle.

| Corrections to $A_{FB}$ | $89 < m_{ee} < 93$ GeV | $80 < m_{ee} < 100$ GeV |
|-------------------------|------------------------|-------------------------|
| $A_{FB}(\text{EW corr. } m_W) - A_{FB}(\text{EW LO } \alpha(0))$ | -0.02097 | -0.02103 |
| $A_{FB}(\text{EW corr. prop. } \chi(Z), \chi(\gamma)) - A_{FB}(\text{EW LO } \alpha(0))$ | -0.02066 | -0.02098 |
| $A_{FB}(\text{EW/QCD FF no boxes}) - A_{FB}(\text{EW LO } \alpha(0))$ | -0.03535 | -0.03569 |
| $A_{FB}(\text{EW/QCD FF with boxes}) - A_{FB}(\text{EW LO } \alpha(0))$ | -0.03534 | -0.03567 |
| $A_{FB}(\text{LEP}) - A_{FB}(\text{EW/QCD FF with boxes})$ | -0.00006 | -0.00001 |
| $A_{FB}(\text{LEP with improved norm.}) - A_{FB}(\text{EW/QCD FF with boxes})$ | -0.00005 | -0.00002 |

Table 6: The difference $\Delta A_{FB}$ in forward-backward asymmetry around Z-pole, $m_{ee} = 89 - 93$ GeV. The $\cos \theta^C_S$ is used to define forward and backward hemispheres. The EW weight is calculated respectively from $\cos \theta^*$, $\cos \theta^{M"ustraal}$ or $\cos \theta^C_S$.

| Corrections to $A_{FB} (89 < m_{ee} < 93$ GeV$)$ | $w^{EW}_{\cos \theta^*}$ | $w^{EW}_{\cos \theta^{M"ustraal}}$ | $w^{EW}_{\cos \theta^C_S}$ |
|-------------------------------------------------|------------------------|------------------------|------------------------|
| $A_{FB}(\text{EW/QCD corr. to } m_W) - A_{FB}(\text{EW LO } \alpha(0))$ | -0.02097 | -0.02112 | -0.02101 |
| $A_{FB}(\text{EW/QCD corr. to } \chi(Z), \chi(\gamma)) - A_{FB}(\text{EW LO } \alpha(0))$ | -0.02066 | -0.02081 | -0.02070 |
| $A_{FB}(\text{EW/QCD FF no boxes}) - A_{FB}(\text{EW LO } \alpha(0))$ | -0.03535 | -0.03560 | -0.03542 |
| $A_{FB}(\text{EW/QCD FF with boxes}) - A_{FB}(\text{EW LO } \alpha(0))$ | -0.03534 | -0.03559 | -0.03541 |
| $A_{FB}(\text{LEP}) - A_{FB}(\text{EW/QCD FF with boxes})$ | -0.00006 | -0.00005 | -0.00006 |
| $A_{FB}(\text{LEP with improved norm.}) - A_{FB}(\text{EW/QCD FF with boxes})$ | -0.00005 | -0.00005 | -0.00005 |
Figure 5: Top-left: the $A_{FB}$ as generated with Powheg+MiNLO (blue triangles) and after reweighting introducing all EW corrections (red triangles). The two choices are barely distinguishable. The differences $\Delta A_{FB} = A_{FB} - A_{FB}^\text{eff}$, due to gradually introduced EW corrections are shown in consecutive plots, where as a reference (black dashed line) respectively: (i) EW LO $\alpha(0)$ scheme (top-right), (ii) effective Born (LEP) (bottom-left) and (iii) effective Born (LEP with improved norm.) (bottom-right) was used.

Figure 6: Effective weak mixing angles $\sin^2 \theta_{e\!f}^{\text{eff}}(s,t)$ as a function of $m_{ee}$ and $\cos \theta = 0$, without (left) and with (right) box corrections. The $\mathcal{K}_t(s,t)$ form-factor calculated using Dizet library and on-mass-shell $s_W^2 = 0.22352$ were used. Only the real part is shown, imaginary part of $I^{\text{eff}}_t(s,t)$ is about $10^{-4}$ only.
**Table 7:** The Dizet printout: effective weak mixing angles and $\alpha(M_Z^2)$.

| Parameter                  | Value     | Description                        |
|----------------------------|-----------|------------------------------------|
| $\alpha(M_Z^2)$            | 0.00775995| Calculated with formula (17)       |
| $1/\alpha(M_Z^2)$          | 128.86674 |                                    |
| $ZPAR(6) - ZPAR(8)$         | 0.23176   | $\sin^2\theta_{\ell \text{ eff}}^\ell(M_Z^2)$ ($\ell = e, \mu, \tau$) |
| $ZPAR(9)$                  | 0.23165   | $\sin^2\theta_{\text{up-quark eff}}(M_Z^2)$ |
| $ZPAR(10)$                 | 0.23152   | $\sin^2\theta_{\text{down-quark eff}}(M_Z^2)$ |

**Table 8:** The effective weak mixing angles $\sin^2\theta_{\ell \text{ eff}}^\ell$, for different mass windows with/without box corrections. The form-factor corrections are averaged with realistic line-shape and $\cos\theta$ distribution.

| Parameter                  | $\sin^2\theta_{\ell \text{ eff}}^\ell$ | $\sin^2\theta_{\text{up-quark eff}}$ | $\sin^2\theta_{\text{down-quark eff}}$ |
|----------------------------|----------------------------------------|--------------------------------------|--------------------------------------|
| EW loops without box corrections |                                        |                                      |                                      |
| $80 < m_{ee} < 100 \text{ GeV}$ | 0.23171                                | 0.23171                              | 0.23146                              |
| $78 < m_{ee} < 82 \text{ GeV}$  | 0.23179                                | 0.23172                              | 0.23159                              |
| $89 < m_{ee} < 93 \text{ GeV}$  | 0.23170                                | 0.23169                              | 0.23147                              |
| $108 < m_{ee} < 112 \text{ GeV}$| 0.23168                                | 0.23175                              | 0.23137                              |
| EW loops with box corrections |                                        |                                      |                                      |
| $80 < m_{ee} < 100 \text{ GeV}$ | 0.23171                                | 0.23171                              | 0.23146                              |
| $78 < m_{ee} < 82 \text{ GeV}$  | 0.23136                                | 0.23167                              | 0.23158                              |
| $89 < m_{ee} < 93 \text{ GeV}$  | 0.23168                                | 0.23169                              | 0.23147                              |
| $108 < m_{ee} < 112 \text{ GeV}$| 0.23246                                | 0.23174                              | 0.23130                              |
Figure 7: Top-left: the $A_4$ as function of $m_{ee}$. Overlayed are generated and EW corrected $A_4$ predictions. The results are barely distinguishable. The differences $\Delta A_4 = A_4 - A_4^{ref}$ due to gradually introduced EW corrections are shown in consecutive plots, where as a reference $A_4^{ref}$ (black dashed line) respectively (i) EW LO $\alpha(0)$ scheme (top-right), (ii) effective Born (LEP) (bottom-left) and (iii) effective Born (LEP with improved norm.) (bottom-right) was used.

8.4 The $A_4$, $A_3$ angular coefficients

To complete discussion on doubly-deconvoluted observables, we will turn the attention back to angular coefficients $A_4$ and $A_3$ (proportional to product of vector and axial couplings) and to EW corrections. The coefficients are calculated from the event sample with the well known moments methods [30], in the Collins-Soper frame. The EW weight $wt^{EW}$ is used to introduce EW corrections and is calculated with the help of $\cos\theta^*$, $\cos\theta^{Mustraal}$ or $\cos\theta^{CS}$ angles.

Similarly as for $A_{FB}$, the EW corrections change overall normalization and the shape of $A_4$ as a function of $m_{ee}$, particularly around the $Z$-pole. In Fig. 7 (top-right), the $A_4$ for generated sample (EW LO) and EW corrected as a function of $m_{ee}$ are shown. In the following plots of the figure details are studied. The differences $\Delta A_4 = A_4 - A_4^{ref}$ with gradually introduced EW corrections are shown. For the reference the following choices: (i) EW LO $\alpha(0)$, (ii) effective Born (LEP) and (iii) effective Born (LEP with improved norm.) are taken. Conclusions are very similar as previous Subsection discussion for $\Delta A_{FB}$. Note that $\Delta A_4$ and $\Delta A_{FB}$ scale approximately with the relation $A_4 = 8/3A_{FB}$.

The analogous set of plots, Fig. 8 is prepared for $A_3$ coefficients. In this case, only the Mustraal frame turned out to be adequate for $wt^{EW}$ calculation. Both the $\cos\theta^*$ and $\cos\theta^{CS}$ were unable to fully capture the effects of EW corrections.

The results for $\Delta A_4$ and $\Delta A_3$ are collected; in Table 9 for $A_3$, for $A_4$, multiplied by $\frac{8}{3}$ entries of Table 8 are good enough. The mass $80 < m_{ee} < 100$ GeV and $p_T^{ee} < 30$ GeV windows are chosen. The estimation for $\Delta A_4$ differ little if $\cos\theta^*$, $\cos\theta^{CS}$ or $\cos\theta^{Mustraal}$ is used for calculations of EW corrections. The $\Delta A_3$ is non-zero only if the $\cos\theta^{Mustraal}$ is used in $wt^{EW}$ calculation.
Figure 8: Top-left: the $A_3$ as function of $m_{ee}$. Overlayed are generated and EW corrected $A_3$ predictions. The results are barely distinguishable. The differences $\Delta A_3 = A_3 - A_3^{ref}$ due to gradually introduced EW corrections are shown in consecutive plots, where as a reference $A_3^{ref}$ (black dashed line) respectively (i) EW LO $\alpha(0)$ scheme (top-right), (ii) effective Born (LEP) (bottom-left) and (iii) effective Born (LEP with improved norm.) (bottom-right) was used. In this case, the EW weight is calculated with $\cos \theta^{Mustraal}$.

Table 9: The $\Delta A_3$ shift of the $A_3$, due to EW corrections, averaged over $p_T^Z < 30$ GeV and $80 < m_{ee} < 100$ GeV ranges. The $\cos \theta^{CS}$ is used for angular polynomials but for the EW weight calculation $\cos \theta^*$, $\cos \theta^{Mustraal}$ or $\cos \theta^{CS}$ are respectively used.

| Corrections to $A_3$ ($p_T^Z < 30$ GeV) | $wT^{EW} (\cos \theta^*)$ | $wT^{EW} (\cos \theta^{Mustraal})$ | $wT^{EW} (\cos \theta^{CS})$ |
|----------------------------------------|--------------------------|-------------------------------|--------------------------|
| $A_3(EW/QCD$ corr. to $m_W) - A_3(EW LO \alpha(0))$ | -0.00060 | -0.00321 | -0.00060 |
| $A_3(EW/QCD$ corr. to $\chi(Z), \chi(\gamma)) - A_3(EW LO \alpha(0))$ | -0.00061 | -0.00322 | -0.00061 |
| $A_3(EW/QCD$ FF no boxes) - $A_3(EW LO \alpha(0))$ | -0.00103 | -0.00546 | -0.00102 |
| $A_3(EW/QCD$ FF with boxes) - $A_3(EW LO \alpha(0))$ | -0.00103 | -0.00545 | -0.00102 |
| $A_3(LEP) - A_3(EW/QCD$ FF with boxes) | 0.00000 | 0.00000 | 0.00000 |
| $A_3(LEP with improved norm.) - A_3(EW/QCD$ FF with boxes) | 0.00000 | 0.00000 | 0.00000 |
9 Summary

In this paper we have shown how the EW corrections for double-deconvoluted observables at LHC can be evaluated using Improved Born Approximation. We have exploited wealth of the LEP era results encapsulated in the Dizet library developed at that time.

We started with formalism used for calculating EW corrections to doubly-deconvoluted observables, following largely discussions available in Dizet documentation. In the following, we introduced notion of the effective Born and explained how, Monte Carlo events generated at NLO QCD can be transformed to reduced kinematics for lowest order spin amplitudes $q\bar{q} \to Z/\gamma^{*} \to \ell\ell$ calculation. This could be achieved thanks to properties of spin amplitudes discussed in [5, 6]. Finally, we explained how per event weight $w_{EW}$ can be used to attribute EW corrections to already generated events. We also re-visited notion of Effective Born with LEP (or with LEP of improved norm.) parametrisations and evaluated how well it works for observables of the paper. Approach for treating EW corrections for Drell-Yan process in pp collisions has been implemented in the Tauola/TauSpinner package [25, 9] to be available from the forthcoming release.

Once formalism explained, numerical results of EW corrections to the Z-boson line-shape, forward-backward asymmetries, lepton angular coefficients were presented. Results were obtained using Dizet library for calculating EW form-factors and Tauola/TauSpinner package for calculating respective EW weights of Improved Born Approximation or Effective Born with LEP (or with LEP improved norm.) parametrisations. The choice of the version of EW library was dictated by the compatibility with KKMC Monte Carlo [2], the program widely used at the LEP times. It relies on a published version of Dizet, thus suits well the purposes of a reference point. Also, omitted effects are rather small. In future, the algorithm of TauSpinner can be useful to quantify the differences among distinct implementations of the electroweak sector.

The updates to Dizet version 6.42 [13, 18] and to other, sometimes unpublished electroweak codes is left however for the future work. One should stress, necessity of numerical discussion and updates due to the photonic vacuum polarization, e.g. as provided in refs. [32, 33] and absent in the last published (or public) version of Dizet 6.42. This is required already at LHC precision of Z-boson couplings measurements.

Finally let us mention that presented implementation of EW corrections as per-even weight, was already found useful for preparation of presentations and discussions during recent workshops, see e.g. Ref. [34].

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References

[1] SLD Electroweak Group, DELPHI, ALEPH, SLD, SLD Heavy Flavour Group, OPAL, LEP Electroweak Working Group, L3 Collaboration, S. Schael et al., Phys. Rept. 427 (2006) 257–454, hep-ex/0509008.

[2] S. Jadach, B. F. L. Ward, and Z. Was, Phys. Rev. D88 (2013), no. 11 114022, 1307.4037.

[3] G. Altarelli, R. Kleiss, and C. Verzegnassi, eds., Z physics at LEP-1. Proceedings, Workshop, Geneva, Switzerland, September 4-5, 1989. vol. 1: Standard Physics, 1989.

[4] F. A. Berends, R. Kleiss, and S. Jadach, Comput. Phys. Commun. 29 (1983) 185–200.

[5] E. Richter-Was and Z. Was, Eur. Phys. J. C76 (2016), no. 8 473, 1605.05450.

[6] E. Richter-Was and Z. Was, Eur. Phys. J. C77 (2017), no. 2 111, 1609.02536.

[7] J. H. Kuhn, A. Kulesza, S. Pozzorini, and M. Schulze, Nucl. Phys. B727 (2005) 368–394, hep-ph/0507178.

[8] S. Jadach, B. Ward, and Z. Was, Comput.Phys.Commun. 130 (2000) 260–325, hep-ph/9912214.

[9] Z. Czyczula, T. Przedzinski, and Z. Was, Eur.Phys.J. C72 (2012) 1988, 1201.0117.
[10] D. Yu. Bardin, M. S. Bilenky, T. Riemann, M. Sachwitz, and H. Vogt, *Comput. Phys. Commun.* **59** (1990) 303–312.

[11] D. Yu. Bardin, P. Christova, M. Jack, L. Kalinovskaya, A. Olchevski, S. Riemann, and T. Riemann, *Comput. Phys. Commun.* **133** (2001) 229–395, [hep-ph/9908433](https://arxiv.org/abs/hep-ph/9908433).

[12] T. Przedzinski, E. Richter-Was, and Z. Was, [1802.05459](https://arxiv.org/abs/1802.05459).

[13] A. Andonov, A. Arbuzov, D. Bardin, S. Bondarenko, P. Christova, L. Kalinovskaya, V. Kolesnikov, and R. Sadykov, *Comput. Phys. Commun.* **181** (2010) 305–312, [0812.4207](https://arxiv.org/abs/0812.4207).

[14] T. Przedzinski, E. Richter-Was, and Z. Was, *Eur. Phys. J.* **C74** (2014), no. 11 3177, [1406.1647](https://arxiv.org/abs/1406.1647).

[15] ATLAS Collaboration, *Measurement of the effective leptonic weak mixing angle using electron and muon pairs from Z-boson decay in the ATLAS experiment at $\sqrt{s} = 8$ TeV*, ATLAS-CONF-2018-037.

[16] S. Alioli, P. Nason, C. Oleari and E. Re, *JHEP* **1006**, 043 (2010) doi:10.1007/JHEP06(2010)043 [arXiv:1002.2581 [hep-ph]].

[17] D. Yu. Bardin, M. Grunewald, and G. Passarino, [hep-ph/9902452](https://arxiv.org/abs/hep-ph/9902452).

[18] A. Akhundov, A. Arbuzov, S. Riemann, and T. Riemann, *Phys. Part. Nucl.* **45** (2014), no. 3 529–549, [1302.1395](https://arxiv.org/abs/1302.1395).

[19] D. Yu. Bardin, P. K. Khristova, and O. M. Fedorenko, *Nucl. Phys.* **B175** (1980) 435–461.

[20] D. Yu. Bardin, P. K. Khristova, and O. M. Fedorenko, *Nucl. Phys.* **B197** (1982) 1–44.

[21] A. Sirlin, *Phys. Rev.* **D22** (1980) 971–981.

[22] M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, *Eur. Phys. J.* **C77** (2017) 827.

[23] S. Alioli et al., *Eur. Phys. J.* **C77** (2017), no. 5 280, [1606.02330](https://arxiv.org/abs/1606.02330).

[24] Particle Data Group Collaboration, C. Patrignani et al., *Chin. Phys.* **C40** (2016), no. 10 100001.

[25] N. Davidson, G. Nanava, T. Przedzinski, E. Richter-Was, and Z. Was, *Comput.Phys.Commun.* **183** (2012) 821–843, [1002.0543](https://arxiv.org/abs/1002.0543).

[26] Z. Was and S. Jadach, *Phys. Rev.* **D41** (1990) 1425.

[27] J. C. Collins and D. E. Soper, *Phys. Rev.* **D16** (1977) 2219.

[28] S. D. Drell and T. M. Yan, *Phys. Rev. Lett.* **25** (1970) 316.

[29] E. Mirkes, *Nucl. Phys.* **B387** (1992) 3–85.

[30] E. Mirkes and J. Ohnemus, *Phys. Rev.* **D51** (1995) 4891–4904, [hep-ph/9412289](https://arxiv.org/abs/hep-ph/9412289).

[31] E. Mirkes and J. Ohnemus, *Phys.Rev.* **D50** (1994) 5692–5703, [hep-ph/9406381](https://arxiv.org/abs/hep-ph/9406381).

[32] H. Burkhardt and B. Pietrzyk, *Phys. Rev. D* **72**, 057501 (2005) doi:10.1103/PhysRevD.72.057501 [hep-ph/0506323].

[33] F. Jegerlehner, [arXiv:1711.06089](https://arxiv.org/abs/1711.06089) [hep-ph].

[34] Presentations at LHC EW WG meetings by E. Richter-Was 2018; https://indico.cern.ch/event/779259/ , https://indico.cern.ch/event/775325/.

[35] See in Development version of [http://tauolapp.web.cern.ch/tauolapp/](http://tauolapp.web.cern.ch/tauolapp/) sub-directory TAUOLA/TauSpinner/examples/Dizet-example.

[36] M. Awramik, M. Czakon, A. Freitas and G. Weiglein, *Phys. Rev. Lett.* **93** (2004) 201805 doi:10.1103/PhysRevLett.93.201805 [hep-ph/0407317].

[37] B. F. L. Ward, S. Jadach, Z. Was and S. A. Yost, “A Precision Event Generator for EW Corrections in Hadron Scattering: ‘3\$\chi$$\chi$MC-hh,” [arXiv:1811.09509](https://arxiv.org/abs/1811.09509) [hep-ph].

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22
A Comment on technical details of TauSpinner EW effects implementation

Although the framework of Tauola/TauSpinner package has been used for numerical results presented in this paper, the code is not yet available with the public release but only in the private distribution and only partly in development version of the code distribution. It updates itself daily from our work repository. Tests and some of the code development need to be completed. Once we achieve confidence the official stable version of the code will become public. Let us nonetheless list main points of the implementation which was already used to obtain numerical results:

- Pre-tabulated EW corrections: form-factors, vacuum polarization corrections in form of 2D root histograms or alternatively ASCII files of the KMC project were used to assure modularity and to enable graphic tests.

- Functions to calculate $\cos\theta^*$, $\sin\theta^{M\nu\tau}$, $\cos\theta^{CS}$ from kinematics of outgoing final state (leptons and partons/jets) used for numerical results are already now in part available in TAUOLA/TauSpinner/examples/Dizet-example directory. The README file of that directory is gradually filled with technical details.

- Routine to initialize parameters of the Born distribution is provided, SUBROUTINE INITWK of TAUOLA/src/tauolaFortranInterfaces/tauola_extras.f has been copied and extended. It is available under the name INITWKSWDELT, with the following input:

  - $G_\mu$, $\alpha$, $M_Z$, $s$,
  - EW form-factors and vacuum polarization corrections,
  - $s_W^2$ and parameters for couplings variations $\delta_{2W}$, $\delta_V$. see Section for details.

- To calculate $d\sigma_{\text{Born}}$ and the $wt^{EW}$ the t_bornew function with flexible options for EW scheme and $\delta_{2W}$, $\delta_V$, is prepared. It is used by TauSpinner library function double default_nonSM_born(int ID, double $S$, double cost, int H1, int H2, int key) now.

- The complete documentation is premature, but some comments on the software used to obtain numerical results are in place.

B How to vary $s_W^2$ beyond the EW LO schemes.

In the discussed EW scheme, the $s_W^2$ is directly available for fits. It is calculated from relation of SM. One possibility to vary, but stay within Standard Model framework is to vary some other constants which impact $s_W^2$. The candidates within Standard Model, which are also inputs to the Dizet library, are $G_\mu$ or $m_t$. From the simple estimates, to allow $\pm 100 \cdot 10^{-5}$ variation of $s_W^2$, those parameter will have to be varied far beyond their experimental ambiguities.

One can extend formulae for $s^{\text{Born+EW}}$ beyond the Standard Model too. Additional $v$-like contribution to $Z$-boson $v_\ell,v_f$ couplings can be introduced with $\delta_{2W}$ or $\delta_V$ and as presented later. Below few details and options on implementation into $s^{\text{Born+EW}}$ amplitudes are given:

- $\textbf{optME = 1}$: introduce unspecified heavy particle coupling to the $Z$-boson, to modify fermions vector couplings

\[
\begin{align*}
\nu_\ell &= (2 \cdot T_3^\ell - 4 \cdot q_\ell \cdot (s_W^2 + \delta_{2W}) \cdot \mathcal{K}_\ell(s,t) ) / \Delta, \\
\nu_f &= (2 \cdot T_3^f - 4 \cdot q_f \cdot (s_W^2 + \delta_{2W}) \cdot \mathcal{K}_f(s,t) ) / \Delta, \\
\nu\nu_{\ell f} &= \frac{1}{\nu_\ell \cdot \nu_f} [(2 \cdot T_3^\ell)(2 \cdot T_3^f) \\
&- 4 \cdot q_\ell \cdot (s_W^2 + \delta_{2W}) \cdot \mathcal{K}_\ell(s,t)(2 \cdot T_3^f) \\
&- 4 \cdot q_f \cdot (s_W^2 + \delta_{2W}) \cdot \mathcal{K}_f(s,t)(2 \cdot T_3^\ell) \\
&+ (4 \cdot q_\ell \cdot s_W^2)(4 \cdot q_f \cdot s_W^2) \cdot \mathcal{K}_{\ell f}(s,t) \\
&+ 2 \cdot (4 \cdot q_\ell)(4 \cdot q_f) \cdot s_W^2 \cdot \delta_{2W} \cdot \mathcal{K}_{\ell f}(s,t)] \frac{1}{\Delta^2}
\end{align*}
\]
Figure 9: The $A_4$ variation due shifts induced with the presented earlier options, as a function of $s_W^2$ (left-hand side) and as a function of $\sin^2 \theta_{eff}$ (right-hand side).

but does not alter

$$\Delta = \sqrt{16 \cdot s_W^2 \cdot (1 - s_W^2)}$$

or any other $\delta^{Born+EW}$ couplings or calculations of the EW form-factors.

- **optME = 2**: break the Standard Model $s_W^2 = 1 - M_W^2/M_Z^2$ relation with the $s_W^2 \to s_W^2 + \delta_{S2W}$ shift, wherever $s_W^2$ is present in $\delta^{Born+EW}$.

- **optME = 3**: similar as optME = 1 but redefine directly fermions vector couplings with $\delta_V$. We keep relative normalization (charge structure) of $\delta_V$ similar to $\delta_{S2W}$, to facilitate comparisons. Then

$$\nu_f = \frac{(2 \cdot T_f^2 - 4 \cdot q_f \cdot (s_W^2 \cdot \mathcal{X}_f(s,t) + \delta_V))}{\Delta},$$

$$\nu_f = \frac{(2 \cdot T_f^2 - 4 \cdot q_f \cdot (s_W^2 \cdot \mathcal{X}_f(s,t) + \delta_V))}{\Delta},$$

$$\nu_V = \frac{1}{\nu_V, \nu_f} \left[ (2 \cdot T_f^2)(2 \cdot T_f^2) \right]$$

$$-4 \cdot q_f \cdot (s_W^2 \cdot \mathcal{X}_f(s,t) + \delta_V)(2 \cdot T_f^2)$$

$$-4 \cdot q_f \cdot (s_W^2 \cdot \mathcal{X}_f(s,t) + \delta_V)(2 \cdot T_f^2)$$

$$+(4 \cdot q_f \cdot s_W^2 + 4 \cdot q_f \cdot s_W^2) \cdot \mathcal{X}_f(s,t)$$

$$+2 \cdot (4 \cdot q_f)(4 \cdot q_f) \cdot s_W^2 \cdot \mathcal{X}_f(s,t) \cdot \delta_V \frac{1}{\Delta^2}.$$  \hspace{1cm} (32)

The $\delta_V$ shift is almost equivalent to $\delta_{S2W}$ shift, but affects couplings in a $(s,t)$ independent manner.

The optME = 1, 2, if form-factors are not recalculated, formally differ by the term proportional to $\delta_{S2W}^2$ and only in the expression for $\nu_V$. Change of input parameters $G_\mu$ or $m_t$ as a source for $s_W^2$ variations corresponds to optME = 2, this imply changes of the couplings and recalculation of form-factors. All these options can be realized with the Tauola/TauSpinner package, development version.

Even though discussion of such numerical results is generally out of scope of the present paper and it can not be now exhausted, let us provide some numerical results to illustrate stability of the method. The variations for $A_4(M_Z)$ are presented in Figure 9 as a function of $s_W^2$ on the left-hand side plot and as a function of $\sin^2 \theta_{eff}$ on the right-hand side plot. It is very reassuring, that all presented variation optME methods lead to the same slope of the $A_4(M_Z)$ as a function of $\sin^2 \theta_{eff}$. The $m_t$ (or $G_\mu$) had to be shifted to move $s_W^2$ by $\pm 100 \cdot 10^{-5}$. Then the form-factors were recalculated with these new values with the impact on $\sin^2 \theta_{eff}$. 

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Table 10: Dizet initialization parameters: masses and couplings.

| Parameter | Value               | Description         |
|-----------|---------------------|---------------------|
| $M_Z$     | 91.1876 GeV         | mass of $Z$ boson   |
| $M_h$     | 125.0 GeV           | mass of Higgs boson |
| $m_t$     | 173.0 GeV           | mass of top quark   |
| $1/\alpha(0)$ | 137.0359895(61) | $\alpha_{QED}(0)$  |
| $G_\mu$   | $1.166389(22) \cdot 10^{-5}$ GeV$^{-2}$ | $G_\mu$ - Fermi constant in $\mu$-decay |

C Initialization of the Dizet library

There is a wealth of initialization constants and options available for Dizet library and EW table writing program as installed in KKMC (directory KKMC/KK-all/dizet). The documentation of that program explains options available for the TauSpinner users as well. Tables [10] and [11] recall available Dizet initialization, Table [12] list calculated by Dizet quantities for the TauSpinner library use. Let us recall now some details from Dizet documentation.

The OMS scheme uses the masses of all fundamental particles, both fermions and bosons, and two coupling constants $\alpha_{QED}(0)$ and $\alpha_{QED}(M_Z^2)$. The ill-defined masses of light quarks are replaced by $\alpha(M_Z^2)$ making use of dispersion relation relating the imaginary part of hadronic vacuum polarization with the total cross-section $\sigma_{\text{tot}}(e^+e^- \to \gamma^* \to \text{hadrons})$.

The knowledge about hadronic vacuum polarization is contained in the quantity, usually noted as $\Delta\alpha_h^{(5)}(M_Z^2)$, which can be fitted to low energy experimental annihilation data and used as one of the input parameters. The effective quark masses are chosen to reproduce $\Delta\alpha_h^{(5)}(M_Z^2)$ at the one-loop level.

In the present work we have relied on the Dizet library, version as installed KKMC Monte Carlo [8] and used at a time of LEP 1 detector simulations. Already for the data analysis and in particular for final fits [1], further effects of minor but non-negligible numerical impact were taken into account. Gradually, effects such as improved top contributions [36] or better photonic vacuum polarization [22], were taken into account. This has to be updated for our version of Dizet library too.

Such upgrade is of importance for the KKMC project because of forthcoming applications for example for the Future Circular Collider or for LHC [37].
Table 11: Dizet initialization flags with not modified comments of KKMC code.

| Internal flag | Default value | Optional values | Description |
|---------------|---------------|-----------------|-------------|
| ibox          | 1             | 0,1             | EW boxes on/off |
| Ihvp          | 1             | 1,2,3           | Jegerlehner/Eidelman, Jegerlehner(1988), Burkhardt et al. =4 the best, Degrassi/Gambino |
| Iam4          | 4             | 0,1,2,3,4       | approx/fast/lep1, exact/Slow!/Bardin, exact/fast/Kniehl =1 W mass recalculated |
| Iqcd          | 3             | 1,2,3           | =1 test only, effective quark masses |
| Imoms         | 1             | 0,1             | =1 W mass recalculated |
| Imsf          | 0             | 0,1             | Remainder terms |
| Irel          | 0             | 1               | for 1.3 DALH5 not input |
| Ilam          | 3             | 1,3 or 0,2,     | =0: Quark masses everywhere; =1 Phys. threshold in the ph.sp. |
| Imask         | 2             | -1,0,1,2,       | Barbieri?? |
| Ical          | 1             | 0,1             | FTJR corrections |
| Iale          | 0             | 0,1             | Expansion of δr; =0 none; =3 fully, unrecommended. |
| Ibarb         | 2             | 0,1,2,3         | Expansion of δα; =0 none; =3 fully, unrecommended. |
| Iftjr         | 1             | 0,1             | Barcieri??? |
| Ihiggs        | 0             | 0,1             | Leading Higgs contribution re-summation |
| Iafmt         | 1             | 0,1             | =0 for old ZF |
| Iewlc         | 1             | 0,1             | =0 for old ZF |
| Iczak         | 1             | 0,1             | Czarnecki/Kuehn corrections |
| Ihig2         | 2             | 0,1,2,3         | Two-loop higgs corrections off/on |
| Iale2         | 3             | 1,2,3           | Two-loop constant corrections in δα |
| Igfer         | 2             | 0,1,2           | QED corrections for fermi constant |
| Iddzz         | 1             | 0,1             | ?? DD-ZZ game, internal flag |

Table 12: Dizet recalculated quantities available for the TauSpinner use.

| Parameter          | Value               | Description                                      |
|--------------------|---------------------|--------------------------------------------------|
| α_{QED}(M_Z^2)     | 0.007759            | calculated from Δα_{QED}^{(5)}(M_Z) by Dizet     |
| 1/α_{QED}(M_Z^2)   | 128.882588          |                                                  |
| α_s(M_Z^2)         | 0.1250              | recalculated by Dizet                           |
| α_s(m_t^2)         | 0.1134              | recalculated by Dizet                           |
| ZPAR(1) = δr       | 0.03694272          | the loop corrections to G_μ                      |
| ZPAR(2) = δr_{rem} | 0.01169749          | the remainder contribution O(α)                |
| ZPAR(3) = sin^2θ_W  | 0.22352             | weak mixing angle defined by weak masses         |
| ZPAR(4) = G_μ      | 1.166370 × 10^{-5}  | muon decay constant                              |
| ZPAR(6) – ZPAR(14) | 0.23176-0.23152      | effective weak mixing angles                     |
| ZPAR(15) = α_s(M_Z^2)| 0.12500            | recalculated by Dizet                           |
| ZPAR(16) – ZPAR(30) |                     | QCD corrections                                  |