Analysis of the angles of contact in a two-roll module

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Abstract. The study is devoted to the analysis of the angles of contact in a two-roll module. An asymmetric two-roll module is considered, in which the rolls are inclined to the right relative to the vertical line, they have unequal diameters and elastic coatings made of materials of different stiffness and friction coefficients; the material layer is fed down inclined relative to the centerline. Analytical expressions for the angles of contact are obtained, which are the main quantities that determine the boundary conditions for the problems of contact interaction in a two-roll module. It was stated that the sum of the contact angles does not depend on the inclination of the material layer feeding to the centerline and on the inclination of the upper roll relative to the vertical line.

1. Introduction
Roller machines are widely used in many industries, in the processing of various materials. The main working body of roller machines is a two-roll module, which consists of a roll pair and processed material. The main thing in the improvement of technological processes performed in roller machines is the study and solution of the problem of contact interaction in two-roller modules. The theory of contact interaction should predict the shape of the contact area and the patterns of its growth with increasing load, and the magnitude and distribution of normal and tangential forces transmitted through the contact surfaces [1-5].

The problems of contact interaction of a two-roll module mainly depend on the magnitude of the contact angles, since they determine the boundary conditions for these problems. The contact angles in two-roll modules are investigated and analyzed to further develop the theory of contact interaction.

In many machines, two-roll modules are asymmetric. Often, several types of asymmetry are realized simultaneously, for example, two types of geometric asymmetry (different diameters and inclinations of the material layer relative to the horizontal line), kinematic asymmetry (one roller is driven, the other roller is free).

At present, there are many publications [6] devoted to the theoretical analysis of contact angles in a symmetric two-roll module. There are studies [7-11], in which one or several types of asymmetry in two-roll modules are investigated. However, at present, there is no comprehensive analysis of contact angles that considers all possible types of asymmetry in two-roll modules.
2. Theoretical study of the problem

Consider an asymmetric two-roll module (figure 1), in which the rollers are inclined to the right at an angle $\beta$ relative to the vertical line, have unequal diameters ($R_1 \neq R_2$) and elastic coatings made of materials with different stiffness and friction coefficients ($f_1 \neq f_2$), both rollers are driven. The layer of material has a uniform thickness $\delta_1$ and is fed inclined down at an angle $\gamma_1$ to the axis $O_1y'$ (the centerline), the distance between the rollers is $h_1$.

The contact angles are determined by the geometrical conditions in the two-roll module. The geometrical conditions are studied in three positions of the front end of the material layer: the first position is the time of contact with the rollers; the second position is the time of touching the centerline; the third position is the time of exit from the contact zone. Analysis of the first position makes it possible to find the angles of the initial contact of the rollers, the analysis of the second position determines the angles of the initial contact of the steady-state process, and the analysis of the third position determines the exit angles.

Let us analyze the contact angles in the first position. Let the layer of material conveyed to the rollers touch them in cross section $A_1A_2$ (figure 1). From figure 1, it follows that

$$R_1 + h_1 + R_2 - O_1B_1 - B_1B_2 - B_2O_2 = 0, \quad A_1B_1 = A_1D_1 + A_2B_2. \tag{1}$$

From right-angled triangles $\Delta A_1B_1O_1$, $\Delta A_2B_2O_2$ and $\Delta A_1D_1A_2$ we determine:

$$O_1B_1 = R_1 \cos(\alpha_1 + \beta_1), \quad A_1B_1 = R_1 \sin(\alpha_1 + \beta_1); \quad O_2B_2 = R_2 \cos(\alpha_2 - \beta_1),$$

$$A_2B_2 = R_2 \sin(\alpha_2 - \beta_1); \quad B_1B_2 = A_2D_1 = \delta_1 \cos \gamma_1, \quad A_1D_1 = \delta_1 \sin \gamma_1.$$

With these expressions, from equation (1) we have

$$R_1 - R_1 \cos(\alpha_1 + \beta_1) - R_2 - R_2 \cos(\alpha_2 - \beta_1) + \delta_1(1 - \cos \gamma_1) + h_1 - \delta_1 = 0,$$

$$R_2 \sin(\alpha_2 - \beta_1) = R_1 \sin(\alpha_1 + \beta_1) - \delta_1 \sin \gamma_1.$$

Assuming that the angles $\alpha_1$, $\alpha_2$, $\beta_1$ and $\gamma_1$ are small, we can write these equations in a simplified form:
\[ R_1(\alpha_1 + \beta_1)^2 + R_2(\alpha_2 - \beta_1)^2 + \delta_1\gamma_1^2 + 2(h_1 - \delta_1) = 0, \]  
(2)

\[ \alpha_2 - \beta_1 = \frac{R_1}{R_2}(\alpha_1 + \beta_1) - \frac{\delta_1}{R_2}\gamma_1. \]  
(3)

After substitution \((\alpha_2 - \beta_1)\) from equation (3) and simple transformations, equation (2) takes the form

\[ R_1(R_1 + R_2)(\alpha_1 + \beta_1)^2 - 2R_2\delta_1\gamma_1(\alpha_1 + \beta_1) + \delta_1^2\gamma_1^2 + R_2\delta_2\gamma_1^2 + 2R_2(h_1 - \delta_1) = 0. \]

Solving this quadratic equation, we determine

\[ \alpha_1 + \beta_1 = \frac{\delta_1\gamma_1}{R_1 + R_2} + \left( \frac{2R_2(\delta_1 - h_1)}{R_1(R_1 + R_2)} - \frac{R_2\delta_1(\delta_1 + R_1 + R_2)\gamma_1^2}{R_1(R_1 + R_2)^2} \right)^{1/2}. \]

Calculations using this formula indicate that the value of the second term under the radical is small compared to the first term. On this basis, the second term can be ignored, giving the formula for determining the contact angle \(\alpha_1\) a simpler form

\[ \alpha_2 = \left( \frac{2R_1(\delta_1 - h_1)}{R_1(R_1 + R_2)} \right)^{1/2} + \frac{(R_1 + R_2)\beta_1 - \delta_1\gamma_1}{R_1 + R_2}. \]  
(4)

Taking into account expression (4) from equation (3), we find expressions for determining the angle \(\alpha_2\):

\[ \alpha_1 = \left( \frac{2R_2(\delta_1 - h_1)}{R_1(R_1 + R_2)} \right)^{1/2} - \frac{(R_1 + R_2)\beta_1 - \delta_1\gamma_1}{R_1 + R_2}. \]  
(5)
Summing up expressions (4) and (5), after transformations, we find the sum of the angles $\alpha_1$ and $\alpha_2$:

$$\alpha_1 + \alpha_2 = \left( \frac{2(R_1 + R_2)(\delta_1 - h_1)}{R_1 R_2} \right)^{\frac{1}{2}}.$$

(6)

Next, we investigate how the contact angles change under the forces acting on the material layer. For this purpose, we consider the schemes of forces acting on the material layer in section $A_1A_2$ (figure 1). To do this, we derive the force balance equations of the material layer at the time of contact with the rollers: (7)

$$\begin{aligned}
\sum X' &= -N_{1x'} - N_{2x'} + T_{1x'} + T_{2x'} = 0, \\
\sum Y' &= N_{1y'} - N_{2y'} + T_{1y'} - T_{2y'} = 0.
\end{aligned}$$

(7)

From the diagram of forces in figure 1 we determine

$$N_{1x'} = N_1 \sin(\alpha_1 + \beta_1), \quad T_{1x'} = T_1 \cos(\alpha_1 + \beta_1), \quad N_{1y'} = N_1 \cos(\alpha_1 + \beta_1), \quad T_{1y'} = T_1 \sin(\alpha_1 + \beta_1),$$

$$N_{2x'} = N_2 \sin(\alpha_2 - \beta_1), \quad T_{2x'} = T_2 \cos(\alpha_2 - \beta_1), \quad N_{2y'} = N_2 \cos(\alpha_2 - \beta_1), \quad T_{2y'} = T_2 \sin(\alpha_2 - \beta_1).$$

Considering these expressions, we rewrite system (7) in the following form

$$\begin{aligned}
[N_1 \sin(\alpha_1 + \beta_1) - T_1 \cos(\alpha_1 + \beta_1)] - (N_2 \sin(\alpha_2 - \beta_1) - T_2 \cos(\alpha_2 - \beta_1)) &= 0, \\
[N_1 \cos(\alpha_1 + \beta_1) + T_1 \sin(\alpha_1 + \beta_1)] + (N_2 \cos(\alpha_2 - \beta_1) + T_2 \sin(\alpha_2 - \beta_1)) &= 0.
\end{aligned}$$

(8)

We divide the first equation of this system by the second equation and transform system (8), expressing the friction forces $T_1$ and $T_2$ through normal forces $N_1$ and $N_2$ according to Amonton’s law of friction:

$$\frac{\sin(\alpha_1 + \beta_1) - f_1 \cos(\alpha_1 + \beta_1)}{\cos(\alpha_1 + \beta_1) + f_1 \sin(\alpha_1 + \beta_1)} = \frac{\sin(\alpha_2 - \beta_1) - f_2 \cos(\alpha_2 - \beta_1)}{\cos(\alpha_2 - \beta_1) + f_2 \sin(\alpha_2 - \beta_1)},$$

where $f_1$, $f_2$ – are the coefficients of friction of the rollers over the material layer at points $A_1$ and $A_2$, respectively.

After a series of transformations, we determine $tg(\alpha_1 + \alpha_2) = \frac{f_1 + f_2}{1 - f_1 f_2}$. Then, considering $f_1 = tg v_1$ and $f_2 = tg v_2$ (where $v_1$ and $v_2$ – are the friction angles at points $A_1$ and $A_2$), respectively, we have:

$$\alpha_1 + \alpha_2 = v_1 + v_2.$$

(9)

Thus, at the point of time when the material layer touches the rollers, the sum of the contact angles is equal to the sum of the friction angles, regardless of the inclination of the upper roller of the material layer.

Taking into account equation (9), expression (6) has the form:

$$v_1 + v_2 = \left( \frac{2(R_1 + R_2)(\delta_1 - h_1)}{R_1 R_2} \right)^{\frac{1}{2}}.$$

(10)
\[ \alpha_1 = \frac{R_2(v_1 + v_2) - (R_1 + R_2)\beta_1 + \delta_1\gamma_1}{R_1 + R_2}, \quad \alpha_2 = \frac{R_1(v_1 + v_2) + (R_1 + R_2)\beta_1 - \delta_1\gamma_1}{R_1 + R_2}. \] (11)

Now let us analyze the geometrical conditions in the second position of the front end of the material layer. Let the front end, brought up to the rollers of the material layer, touch them in section \(A_1A_2\) and pass to section \(C_1C_2\) lying on the centerline (figure 2). During this time, the passage of the layer of material between the rollers is accompanied by the rise of the upper roller to a height \(\Delta = h_2 - h_1\), and section \(A_1A_2\) changes to section \(B_1B_2\).

At that, the layer of material and the elastic coatings of the rollers deform. Deformation occurs until the vertical component of the elastic force of the material is equal to the pressure of the upper roller. Let (at the time of equality of the indicated forces) the two-roll module have the following parameters: the angles of the initial contact of the steady-state process - \(\varphi_{11}\) and \(\varphi_{21}\), the angle of inclination of the upper roller relative to the vertical line - \(\beta_2\), the distance between the rollers - \(h_2\).

We assume that

\[ h_2 = kh_1, \quad k > 1. \] (12)

By analogy with formulas (4), (5) and (6), considering equation (12), we determine \(\varphi_{11}, \varphi_{21}\), and their sum:

\[ \varphi_{11} = \left(\frac{2R_2(\delta_1 - kh_1)}{R_1(R_1 + R_2)}\right)^{1/2} - \frac{(R_1 + R_2)\beta_2 - \delta_1\gamma_1}{R_1 + R_2}, \quad \varphi_{21} = \left(\frac{2R_1(\delta_1 - kh_1)}{R_1(R_1 + R_2)}\right)^{1/2} + \frac{(R_1 + R_2)\beta_2 - \delta_1\gamma_1}{R_1 + R_2}, \] (13)

\[ \varphi_{11} + \varphi_{21} = \frac{2(R_1 + R_2)(\delta_1 - kh_1)}{R_1R_2}. \] (14)

According to figure 2 for the considered two-roll module we have:

\[ \varphi_{11} + \varphi_{21} = v_{11} + v_{21}, \] (15)

where \(v_{11}, \ v_{21}\) - are the angles of friction at points \(B_1\) and, \(B_2\) respectively.

Considering equation (15), expression (14) has the following form:

\[ v_{11} + v_{21} = \left(\frac{2(R_1 + R_2)(\delta_1 - kh_1)}{R_1R_2}\right)^{1/2}. \] (16)

Then we obtain

\[ \varphi_{11} = \frac{R_2(v_{11} + v_{21}) - (R_1 + R_2)\beta_2 + \delta_1\gamma_1}{R_1 + R_2}, \quad \varphi_{21} = \frac{R_1(v_{11} + v_{21}) + (R_1 + R_2)\beta_2 - \delta_1\gamma_1}{R_1 + R_2}. \] (17)

Analyzing the calculation results of the contact angle of the lower roller \(\varphi_{11}\), according to formula (13), the following aspects were revealed:

- the angle \(\varphi_{11}\) increases with increasing \(R_2, \delta_1\) and \(\gamma_1\). The nature of \(\varphi_{11}\) increase with increase in \(R_2\) does not depend on \(R_1\). The angle \(\varphi_{11}\) increases faster at greater values of \(\delta_1\). The angle \(\varphi_{11}\) increases linearly with increasing \(\gamma_1\).
the angle $\varphi_{1}$ decreases with increasing $R_{1}$ and $h_{1}$. The nature of the decrease in $\varphi_{1}$ with increase in $R_{1}$ does not depend on $R_{2}$.

We now proceed to analyze the geometrical conditions at the third position of the front end of the material layer. When passing from the second position to the third position, the front end of the material layer travels from section $C_{1}C_{2}$ to section $D_{1}D_{2}$, and leaves the contact zone of the rollers. During this time, the deformation of the rollers and the layer of material restores.

Let the section $D_{1}D_{2}$ be determined by the following parameters: $\varphi_{12}, \varphi_{22}$ - contact angles at the end of a steady state process; $\delta_{2}$ - finite thickness of the material layer; $\gamma_{2}$ - the angle of inclination of the material layer relative to the centerline; $\beta_{2}$ - the angle of inclination of the upper roller relative to the vertical line (figure 3).

Figure 3 shows that the following equalities must hold:

$$R_{1} - R_{1} \cos(\varphi_{12} - \beta_{2}) + R_{2} - R_{2} \cos(\varphi_{22} + \beta_{2}) + \delta_{2}(1 - \cos \gamma_{2}) + h_{2} - \delta_{2} = 0,$$

$$R_{2} \sin(\varphi_{22} + \beta_{2}) = R_{1} \sin(\varphi_{12} - \beta_{2}) - \delta_{2} \sin \gamma_{2}. $$

Making transformations and simplifications similar to (2) - (4), we obtain a quadratic equation, solving which, we have formulas for determining the exit angles $\varphi_{12}, \varphi_{22}$ and their sum:

$$\varphi_{12} = \left(\frac{2R_{2}(\delta_{2} - kh_{1})}{R_{1}(R_{1} + R_{2})}\right)^{1/2} + \frac{(R_{1} + R_{2})\beta_{2} + \delta_{2}\gamma_{2}}{R_{1} + R_{2}},$$

$$\varphi_{22} = \left(\frac{2R_{1}(\delta_{2} - h_{1})}{R_{2}(R_{1} + R_{2})}\right)^{1/2} - \frac{(R_{1} + R_{2})\beta_{2} + \delta_{2}\gamma_{2}}{R_{1} + R_{2}},$$

$$(18)$$

$$\varphi_{12} + \varphi_{22} = \left(\frac{2(R_{1} + R_{2})(\delta_{2} - h_{1})}{R_{1}R_{2}}\right)^{1/2},$$

$$(19)$$

Assume that

$$\delta_{2} \sin \gamma_{2} = m_{1}\delta_{1} \sin \gamma_{1},$$

$$\delta_{2} \cos \gamma_{2} = m_{2}\delta_{1} \cos \gamma_{1},$$

$$(20)$$

where $m_{1}, m_{2}$ - are the proportionality coefficients.

Similar to equations (9) and (15), we have:

$$\varphi_{12} + \varphi_{22} = \nu_{12} + \nu_{22},$$

$$(21)$$

where $\nu_{12}, \nu_{22}$ - are the angles of friction at points $D_{1}$ and $D_{2}$, respectively.

Considering this formula, from equation (19) we determine

$$\nu_{12} + \nu_{22} = \left(\frac{2(R_{1} + R_{2})(\delta_{2} - h_{1})}{R_{1}R_{2}}\right)^{1/2}.$$ 

$$(22)$$

Taking into account expression (22) and equalities $\delta_{2}\gamma_{2} = m_{1}\delta_{1}\gamma_{1},$ $\beta_{1} = \beta_{2}$ expressions (18) take the following form:

$$\varphi_{12} = \frac{R_{2}(\nu_{12} + \nu_{22}) + (R_{1} + R_{2})\beta_{2} + m_{1}\delta_{1}\gamma_{1}}{R_{1} + R_{2}},$$

$$\varphi_{22} = \frac{R_{1}(\nu_{12} + \nu_{22}) - (R_{1} + R_{2})\beta_{2} - m_{1}\delta_{1}\gamma_{1}}{R_{1} + R_{2}}.$$ 

$$(23)$$
Analysis of calculations of $\varphi_{12}$ and $\varphi_{22}$ according to formulas (23), allowed us to draw the following conclusions:

- and $\varphi_{22}$ increases linearly with an increase in sum $(v_{12} + v_{22})$;
- with increase in $R_2$, the angle $\varphi_{12}$ increases, and the angle $\varphi_{22}$ decreases;
- with increase in $\beta_2$, $\gamma_2$ and $\delta_2$, the angle $\varphi_{12}$ increases linearly, and the angle $\varphi_{22}$ decreases linearly.

The analysis of the geometrical conditions of the two-roll module interaction makes it possible to determine the contact angles of the two-roll module with two driven rollers. They are in the following order:

- according to the formula (10) $- v_1 + v_2$;
- according to the formulas (11) and (12) $- \alpha_1, \alpha_2$ and $h_2$;
- according to the formula (16) $- v_{11} + v_{21}$;
- according to the formulas (18) and (20) $- \varphi_{11}, \varphi_{21}, \delta_2$ and $\delta_2 \gamma_2$;
- according to the formula (22) $- v_{12} + v_{22}$;
- according to the formulas (23) $- \varphi_{12}$ and $\varphi_{22}$.

3. Results
The geometrical conditions in a two-roll module are analyzed in the article; the rollers are inclined to the right relative to the vertical line, both rollers are driven, the diameters of the rollers are not identical, the rollers have elastic coatings made of different materials, the material layer is fed down to the centerline.
Analytical expressions for the contact angles are obtained, which are the main quantities that determine the boundary conditions for problems of contact interaction in a two-roll module.

4. Conclusions
Analysis of the calculation results of the dependencies obtained showed the following:

- with an increase in the radius of the upper roller and the angle of inclination of the material layer relative to the centerline, the contact angle of the lower roller increases, and the contact angle of the upper roller decreases;
- with an increase in the radius of the lower roller and the angle of inclination of the centerline relative to the vertical line, the contact angle of the lower roller decreases, and the contact angle of the upper roller increases;
- the pattern of change in the contact angle of the upper roller and the contact angle of the lower roller depending on the distance between the rollers and the thickness of the material layer is the same;
- the sum of the contact angles does not depend on the inclination of the material layer feeding to the centerline and on the inclination of the upper roller relative to the vertical line. They increase with an increase in the initial thickness of the material layer and with a decrease in the roller radius and the distance between the rollers.

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