Signal Analysis and Filtering using one Dimensional Hilbert Transform

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Abstract. In the ground of Digital Signal dispensation, the discrete Hilbert Transform has found more and more important. The representation of a signal as the real part of a complex function in time is very much useful in many areas of signal analysis. In this paper, the demonstration of a significant improvement of a real seismic signal in the form of the complex envelope (amplitude and phase) which may be obtained by using the discrete Hilbert transform techniques. The pulse here considered for the study is similar to the Berlage function that is used to replicate seismic signals. In addition to this, it also presents discussion on the application of Hilbert transform on the spectral factorization which is related to the minimum phase and inverse filters.

1. Introduction
The improvement of the one-dimensional (1-D) Hilbert transform is narrowly patterned after Gold and Rader (1969), Cizek (1970). The 1-D Hilbert transform is often distinct as a quadrature filter which introduces a 90 degree phase shift. In a complex function the factual and unreal parts are related together through the Hilbert transform. In almost every field where Fourier transform techniques are used to represent and analyze physical processes, one finds that readily available are situation anywhere there continue living associations flanked by the actual and unreal[1] part or the enormity and phase of the Fourier transform. These associations are frequently represented by one form or other of the Hilbert transform relations. Hilbert transform relations for the Fourier transform of causal sequences and the isolated Fourier transform of episodic sequence which are "causal" in the intelligence that they are nil in the moment part of every period. It is possible to base[2] the derivation of the Hilbert renovate relations on the notion of causality. Since we are interested in relating the actual and unreal parts of a composite sequence, "causality" will be applied to the Fourier transform of the sequence. In this paper, the demonstration of a significant improvement of a real seismic signal in the form of the
complex envelope (amplitude and phase) which may be obtained by using the discrete Hilbert transform techniques. The pulse here considered for the study is similar to the Berlage function that is worn to imitate seismic signals. In addition to this, it also presents discussion on the application of Hilbert transform on the spectral factorization which is related to the minimum phase and inverse filters.

2. Hilbert Transform in Signal Analysis
Application of the Hilbert transform (1D) in seismic signal analysis has been very useful, especially when we represent the signal as the real part of a complex function in time. In general terms, the compensation of the composite wrapping stem from the natural departure of amplitude in sequence beginning point of view information. In a real signal, these are blended in such a way which can be confusing to visual analysis. In this section we will see how the complex envelope of a seismic wavelet can be calculated. Let us start our discussion by assuming a signal, $p(t)$, which can represent a seismic trace of finite duration that has been digitized for computer processing. Thus, the time erratic $t$ assume just numeral principles, $t = 0, 1, 2, ...N-1$, where the period of $p(t)$ is N samples. We try to represent this signal as the real part of a complex envelope $\hat{p}(t)$ (the cap over each symbol indicating a complex quantity),

$$ p(t) = \text{Re}[\hat{p}(t)] = \text{Re}[|\hat{p}(t)| \exp \{j \text{ph}(t)\}] \quad 0 \leq t \leq N \quad (1) $$

Where $|\hat{p}(t)|$ is the envelope of $p(t)$. some times we call it $E(t)$, the $\text{Ph}(t)$ is the angle function or phase function. it is the polar notation of equation (1) that will be most useful in what follows. If one can visualize the $p(t)$ in the complex plane as a vector with length $|\hat{p}(t)|$ rotating with angle $\text{Ph}(t)$ with respect to the real axis, the analogy with a "time-varying" phasor is clear.

Consequently the definition of instantaneous amplitude and frequency as the length and rotational velocity, respectively, of a vector gives $\hat{p}(t)$.

- instantaneous amplitude = $|\hat{p}(t)| = E(t)$
- instantaneous radian frequency = $\frac{d\text{ph}(t)}{dt}$

(2)

of course, equation (1) is insufficient for calculating $\hat{p}(t)$ from a given $p(t)$, since the imaginary part is completely unspecified. However, the complex envelope can be derived from equation (1) by specifying that it be linear, that is, the scaling and addition of real signals correspond to a similar scaling and addition of their complex envelopes, and that (1) reduces to a phasor representation in the special case where $P(t)$ is a pure sinusoid with a numeral digit of period in time space $t = (0, N)$. It is important for us here to reminder with the intention of the composite wrapper so derived is a complex signal which lends to equation (2), an interpretation which is intuitively meaningful. That is the instantaneous amplitude and frequency agrees with amplitudes measured [9] at signal peaks and frequencies calculated from zero crossings.

The Hilbert transformed signal is variously known as the quadrature signal or the allied signal, while the complex envelope is in addition identified as the analyti gesture belonging to $P(t)$ since the real part of $p(t)$ is given by equation (1). The envelope $E(t)$ and phase $\text{Ph}(t)$ of a function, $p(t)$, be capable of be obtained by means of the distinct Hilbert renovate as

$$ E(t) = \sqrt{p^2(t) + p_h^2(t)} = \sqrt{p_c p_c^*} \quad (3) $$

and $\text{ph}(t) = \arctan \left[ \frac{p_h}{p_c} \right] \quad (4)$

where $p_h$ - Hilbert transform of $p(t)$, $p_c^*$-Conjugate of $p_c$ and $p_c$ - Complex discrete function of $p(t)$
3. Berlage Function

Let us look first at an artificial pulse whose characteristics can control. The pulse shall study is a form of the Berlage function frequently worn to replicate seismic signals. This throb is shown in the Figure.1. In this example the sine factor makes the pulse oscillate with 0.5 sec between zero-crossings, while the $t^2$ and $\exp$ factor give rise to a defined onset at $t = 0$ and an exponentially [10] decaying tail. The complex representation for the given pulse is found by replacing the sine term by a complex exponential as is done with phasors. That is,

$$p(t) = \text{Re} \ t^2 \exp(2-2t) \exp [2\pi t - \frac{\pi}{2}]$$ \hspace{1cm} (5)

![Figure 1. An Real pulse p(t)](image1)

![Figure 2. Hilbert transform of Real pulse p(t)](image2)

while this is a good representation and provides instantaneous amplitude and angle functions directly, it is not very useful in studying seismic records. A seismic [11-14] record can never be specified mathematically in a form such as Berlage function, and because of the irregularities in a signal, the simple replacement of a sinusoid by a complex exponential function which led to equation (5) is often inappropriate for seismic signals. On the other hand, a method that can calculate the complex envelope of a given recorded signal is an acceptable approach.

The result of this work is demonstrated in the Figure.1. and Figure.2., detailed comparison of the computed values shows that the complex envelope is equivalent to equation (5) to very good approximation. That is,

$$\hat{p}(t) = t^2 \exp(2-2t) \exp [j(2\pi t - \frac{\pi}{2})]$$ \hspace{1cm} (6)
where  $|\hat{p}(t)|=t^2 \exp(2-2t)$ and $p_h(t)=2\pi t-\frac{\pi}{2}$.

The errors are most noticeable in the angle function before $t=0$ sec and beyond $t=5$ sec. They take place at period while the given throb is extremely feeble or not defined, since the angle of a extremely little composite vector is vague in a sensible brains. It can be shown that, the errors in equation (6) are due to spectral truncation effects beyond computer round off error which [15-17] we need not consider in further detail here because they are not important for real seismic signals. These relationships between the envelope and instantaneous amplitude and frequency however be important in and the original pulse what follows.

4. Simulation of Artificial Signal

Since the Berlage function is a simple pulse which has the most properties of seismic signals such as oscillatory nature and the definite onset, the coda of the first onset can be realistically[18] simulated by adding attenuated copies of the basic pulse, if necessary delayed in the wake of the initial onset. That is, if $t_n$ and $a_n$ represent the delay and scale factor of the $n$th pulse added into the coda, the synthetic signal has the form

$$s(t)=p(t)+\sum a_n P(t-t_n)$$

(7)

Where $t_n$ is the positive delay to preserve the initial onset of $s(t)$ at $t=0$. The equation (7) has a very attractive physical interpretation. If the initial throb $P(t)$ represent the major entrance[19] at the soundtrack position, then the delayed pulses may represent minor wavelets arriving subsequently. These may be reverberations from in homogeneities beneath the station or pulses from multiple paths in the layered transmission medium. The positive and negative values of '$a_n$'correspond to positive and negative reflections of the main wavelet or the primary event [20]. From the standpoint of physical interpretation of equation (7), the real justification lies in how realistically it can simulates real seismic records. The Figure.3. shows a synthetic signal constructed in this way.

![Figure 3. An artificial pulse and its Hilbert transform](image)
5. Inverse Filters

If we assume Y is the known output of a filter B and X is an unknown input, then we have a difficulty with the intention of one often have by a transducer plotter arrangement. Intended for instance, the production of a seismometer is extensively procession beginning which the seismologist might wish to conclude the disarticulation speed or increase of velocity of the position [21]. To unfasten the filter manoeuvre of the riddle B(Z), we will try to come across a different pass through a filter A(Z) where B(Z)A(Z)=1, that earnings A(Z) is inverse of B(Z). Lets take an example, if B(Z)=1-Z/2, then, by Taylors power series formula, we have

$$A(z) = \frac{1}{1-(z/2)} = 1 + \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{8} + \frac{z^4}{16} + \ldots$$

(8)

We can represent this polynomial in a computer [22] since the filter coefficients will drop off rapidly in magnitude. But if B(Z)=1-2Z, then

$$A(z) = \frac{1}{1-2z} = 1 + 2z + 4z^2 + 8z^3 + 16z^4 + \ldots$$

(9)

Here the coefficients of the series increases without bound which in fact will produce a serious problem. A additional arithmetical explanation of the condition of relationships [23, 24] grades beginning solve for the zeros of B(Z). This is equivalent to discover a value of Z for which B(z)=1-0.5z. we find z=1/2. the universal container intended for wavelets with composite coefficients is so as to, if the answer value Z of B(Z)=0, slander within the component circle in the composite flat, next 1/B(Z) will contain coefficients which gust up; and if the cause slander exterior the unit ring, subsequently the opposite 1/B(Z) will be delimited. The minimum-phase function has further applications in geophysics such as the feedback filtering or inverse filters, Homopophic filtering or Homomorphic deconvolution, zero-lag inverse filter, Minimum entropy decnvolution, etc. It determine the smallest amount amount of spreading in gelatinous wave dissemination which is implicit by causality. The Homomorphic system is a nonlinear system which obeys a generalized principle of superposition, which is used to disconnect the wavelet and the desire answer of the transmission path from a seismic record. If the wavelet or the desire reaction of the communication canal is minimum-phase, a seismic documentation is time and again represent as the complexity of a wavelet with the desire reaction of the communication lane. Since the wavelet or the desire reaction of the communication lane obeys the minimum-phase assumption, the seismic record will be a minimum phase, in which case, the Homomorphic filter works nicely, but a problem will arise when the seismic record is a mixed-phase i.e both of the wavelet and the desire reaction of the communication are not minimum phase, in which the Homomorphic system fails. To solve this problem we have to change the seismic record which is mixed phase to minimum phase and this may be achieved by application of Hilbert transform.

6. Minimum Phase

The minimum-phase function has further applications in geophysics such as the feedback filtering or inverse filters, Homomorphic filtering or Homomorphic deconvolution, Zero-lag inverse filter, Minimum entropy deconvolution, etc. It determine the smallest amount amount of spreading in glutinous wave dissemination which is implied by causality. In this section we will discuss one of these uses which is related to seismic exploration. The Homomorphic system is a nonlinear system which obeys a generalized principle of superposition (Oppenheim, 1975), that is used to disconnect the wavelet (seismic source) and the desire reaction of the transmission path from a seismic record. If the wavelet or the desire reaction of the communication channel is minimum-phase, a seismic documentation is often represent as the convolution of a wavelet with the desire reaction of the communication lane. Since the wavelet or the desire reaction of the communication lane obeys the minimum-phase assumption, the seismic record will be a minimum phase, in which case, the Homomorphic filter works nicely but a problem will arise when the seismic record is a mixed-phase i.e both of the wavelet and the desire reaction of the communication are not minimum phase, in which the Homomorphic system fails. To solve this problem we have to change the seismic record which is mixed phase to minimum phase and this may be achieved by application of Hilbert transform.
this problem we have to change the seismic record which is mixed-phase to minimum-phase and this may be achieved by application of Hilbert transform or the other appropriate methods. Figure 4. an example of a mixed-phase and its corresponding minimum-phase function obtained using Hilbert transform technique.

Figure 4. Mixed phase and minimum phase by Hilbert Transform

7. Conclusion
Hilbert Transform can be defined in different ways, and through different mathematical relations, but essentially it is a quadrature filter which introduces a ninety degree phase shift. The complex function components such as real and imaginary parts of real function that are even and odd respectively that are related through the Hilbert transform relationship. The one dimensional has many applications in finding the complex envelope of real seismic signal can give more insight in to the composition of a time series signal than is apparent in the original form. One of the major advantages becomes obvious when the amplitude envelope and phase can be separated from the complex envelope. The information obtained here is very much needed in seismic signal analysis and deserve further attention.

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