We extend this line of research by developing a Bayesian im-

A

where

we refer to as

learned Bayesian reconstruction with gener-

ative models (L-BRGM), entails joint optimization over the

style-code and the input latent code, and enhances the expres-
sive power of a pre-trained StyleGAN2 generator by allowing

the style-codes to be different for different generator layers.

Considering the inverse problems of image inpainting and

super-resolution, we demonstrate that the proposed approach

is competitive with, and sometimes superior to, state-of-the-art

GAN-based image reconstruction methods.

1. INTRODUCTION

Ill-posed inverse problems are encountered routinely in vari-

ous imaging applications, wherein one seeks to estimate an

unknown image \( x \in \mathbb{R}^n \) from its degraded and noisy mea-

surement \( y \in \mathbb{R}^m \). Variational reconstruction circumvents

ill-posedness by incorporating prior knowledge about \( x \) via a

regularizer \( \psi : \mathbb{R}^n \to \mathbb{R} \), and computes an estimate \( \hat{x} \) as

\[
\hat{x} \in \arg \min_x L(y, Ax) + \lambda \psi(x),
\]

where \( A \) denotes the (typically ill-conditioned) measurement

operator and \( L \) measures data-fidelity. The role of \( \psi \) is to

penalize undesirable solutions. Some popular choices for \( \psi \)

include the Tikhonov regularization, total-variation (TV), and

more recently, sparsity-promoting regularizers that encourage

the image to be sparse in a suitable (analytical or learned) basis

\[ [1] \). Notably, for a Gibbs-type prior \( p(x) \propto \exp (-\lambda \psi(x)) \),

\[ [1] \) can be interpreted as the Bayesian maximum a-posteriori

probability (MAP) estimate of the unknown image \( x \).

The success of deep generative modeling in recent years

serves as a major inspiration for learning the regularizer based

on a dataset of training images \([2][6] \). Such data-driven reg-

ularizers have been shown to significantly outperform their

hand-crafted variants on a wide array of imaging inverse prob-

lems. Within the realm of data-driven regularizers, a particu-

larly promising approach has been to seek a reconstruction

\( \hat{x} \) that lies in the range of a pre-trained generator \( G : Z \to \mathbb{R}^n \),

where \( Z \) is a latent space whose dimension is significantly

smaller than \( n \). This amounts to reformulating (1) as

\[
\hat{x} = G(\hat{z}), \text{ where } \hat{z} \in \arg \min_z L(y, AG(z)) .
\]

Training such a generator \( G \) generally tends to suffer from

mode collapse, in which case \( \text{range}(G) \) might fail to represent

the image manifold in its entirety and there could be target

images lying outside \( \text{range}(G) \). In such scenarios, solving

(2) fails to recover the image of interest, and one needs to

allow for some flexibility for the target image to lie close to

\( \text{range}(G) \) through soft constraints. Such methods typically

require computing a latent code for a given image, which

calls for inverting \( G \), and are therefore referred to as GAN-
inversion \([7] \). Methods for inverting a GAN can be learning-
or optimization-based, and we adopt the latter in our work. In

particular, we extend the recently proposed Bayesian recon-

struction through generative models (BRGM) approach \([8] \)

by introducing a data-driven modeling of the regularizer on the

latent space. Since our approach employs learned networks for

prior modeling, we refer to it as learned-BRGM (L-BRGM).

2. THE PROPOSED L-BRGM METHOD

To construct the latent prior, L-BRGM relies on a pre-trained

StyleGAN2, which is a recent state-of-the-art GAN architec-
ture developed by Karras et al. \([9] \). StyleGAN2 is an improved

variant of its predecessor StyleGAN \([10] \), named after its ar-

chitecture’s inspiration from the style transfer literature. In

StyleGAN, the input latent space \( Z \) is warped into an inter-

mediate disentangled feature space \( \mathcal{W} \) (also called the style

space) via a eight-layer fully connected mapping network \( M \).
The StyleGAN generator $G$, consisting of adaptive instance normalization (AdaIN) and convolution blocks, subsequently produces photo-realistic images from these disentangled features. Randomness is introduced in the generated samples by feeding noise into the generator. StyleGAN2 enhances the sample quality by redesigning the generator architecture and by introducing a novel path-length regularizer (PLR). The PLR seeks to ensure that the generator approximately preserves length, i.e., a fixed change in $w \in W$ causes a fixed magnitude change in the sampled image. The effect of PLR is to promote orthogonality in the Jacobian matrix $J_G(w) = \frac{\partial G(w)}{\partial w}$ at any $w$. As remarked in [9], the PLR makes it easier to invert $G$, motivating the usage of StyleGAN2 in our approach.

2.1. A Bayesian formulation of L-BRGM

The proposed L-BRGM approach has two key features: (i) formulating the reconstruction problem as a joint optimization over the style-code $w$ and the input latent code $z$, and (ii) using the pre-trained mapping network $M : \mathcal{Z} \rightarrow \mathcal{W}$ to model the distribution of $w$. Both of them emerge naturally from the Bayesian framework explained in the following.

Let $p(y|w, z)$ be the probability density function (p.d.f.) of the measurement $y$ conditioned on the style-code $w$ and the input noise $z$. Given $w$, the distribution of $y$ is independent of $z$, and is determined by the generator $G$ and the measurement operator $A$. Consequently, we have that $p(y|w, z) = p(y|w)$. The posterior distribution of the joint latent code $(w, z)$ conditioned on $y$ can be expressed using Bayes’ rule:

$$p(z, w | y) = \frac{p(y | w, z)p(w, z)}{p(y)} = \frac{p(y | w)p(w, z)}{p(y)}. \quad (3)$$

Further, $p(w, z)$ factors as $p(w, z) = p(w|z)p(z)$, leading to

$$\log p(z, w | y) = \log p(y | w) + \log p(w | z) + \log p(z) - \log p(y).$$

Since $w$ is obtained by applying the mapping network $M$ on $z$, it is natural to model $p(w|z)$ as a Gaussian perturbation around $M(z)$, i.e., $p(w|z) = \mathcal{N}(w; M(z), \lambda_{\text{map}}^{-1} I)$. Subsequently, noting that $p(z) = \mathcal{N}(z; 0, \lambda_{\text{lat}}^{-1} I)$, and assuming the same measurement noise model considered in [8] eq(4) (i.e., Gaussian noise in the pixel space as well as in the VGG-16 embedding space), the MAP estimate of $(w, z)$ reduces to

$$\min_{(z, w)} \lambda_{\text{map}} \|y - A G(w)\|^2_2 + \lambda_{\text{agg}} \|\varphi(y) - \varphi(A G(w))\|^2_2$$

$$+ \lambda_{\text{map}} \|w - M(z)\|^2_2 + \lambda_{\text{lat}} \|z\|^2_2, \quad (4)$$

where $\varphi$ computes the VGG-16 features. Finally, we extend (4) by optimizing over $w^+$, where $w^+ = \{w_i\}_{i=1}^L$, allowing for different style-codes $w_i$ at different layers $i$ of the generator. The resulting optimization reads

$$\min_{(z, w^+)} J(w^+, z),$$

where

$$J(w^+, z) := \lambda_{\text{pix}} \|y - A G(w^+)\|^2_2 + \lambda_{\text{map}} \sum_{i=1}^L \|w_i - M(z)\|^2_2$$

$$+ \lambda_{\text{agg}} \|\varphi(y) - \varphi(A G(w^+))\|^2_2 + \lambda_{\text{lat}} \|z\|^2_2. \quad (5)$$

$J$ is minimized via an iterative optimization algorithm (namely, Adam [11]) starting from an appropriate initialization (c.f. Sec 2.3), and the final reconstructed image is computed as \(\hat{x} = G(w^+)\), where \(\hat{w}^+\) is the optimal style-code.

Since L-BRGM extends the BRGM approach by exploiting the StyleGAN2 mapping network for modeling $p(w)$, it is imperative to compare and contrast the Bayesian probability models for the two methods. As the ground-truth image $x$ can be modeled as $x = G(w)$ for an appropriate $w$, the BRGM approach seeks to maximize $p(x|y) \propto p(y|G(w))p(G(w))$. Subsequently, the image prior in BRGM is rewritten via the change of variable formula as $p(G(w)) = p(w) (\det(J_G(w)))^{-1}$. Finally, making the simplifying assumption that the Jacobian $J_G(w)$ is not dependent on $w$ (i.e., $G(w)$ varies approximately linearly in $w$, which can be justified by the PLR during StyleGAN2 training), one ends up with the task of solving eq(6) posed in [8]. To model $p(w)$ in BRGM, one allows for the flexibility of having different $w_i$’s at different layers of $G$, while encouraging the $w_i$’s to be similar via a cosine similarity prior. Additionally, each $w_i$ is modeled as a Gaussian random vector, with the parameters empirically estimated from a large number of samples fed through $M$.

L-BRGM extends this Bayesian formalism by formulating a Bayesian MAP estimation problem over the joint code $(w, z)$ and alleviates the need to hand-craft a suitable prior on $w$ by exploiting the mapping network $M$. L-BRGM also allows for different style-codes $w_i$ for $L = 18$ different layers of $G$. Further, all $w_i$’s are encouraged to be close to $M(z)$, thus ensuring mutual similarity among each pair of $w_i$’s. L-BRGM does not require any Gaussian assumption on the style-codes $w_i$, but instead makes this assumption on the latent $z$, thereby making the framework more realistic (since $z$ is indeed sampled from a Gaussian distribution during training).

2.2. The effect of regularization

It is worth addressing the importance of modeling the distribution $p(w)$ of the style-code, which acts as a regularizer in the reconstruction process. If one optimizes the latent vector $w$ by solely minimizing the data-likelihood term (i.e., a combination of the pixel-wise error and VGG perceptual loss), the reconstructed images tend to have unnatural artifacts. This phenomenon is demonstrated in Fig. [I] for inpainting on the FFHQ dataset consisting of human face images of resolution 1024. The reconstructed image with no regularization on $w$ appears to be smooth around the inpainting boundary, but fails to recover important facial features (such as hair color and texture) far from this boundary. Notably, the L-BRGM reconstruction in Fig. [II] matches the ground-truth more closely than BRGM. This highlights the fact that the regularization term on the style-code can lead to realizing the full capacity of the underlying generative model. While no regularization leads to worse reconstructions, hand-crafting the regularizer also limits the potential of the Bayesian framework considerably.
Fig. 1. Effect of regularization. Without any regularization, the reconstructed image tends to look unnatural.

Fig. 2. Illustration of the effect of early stopping for L-BRGM and BRGM. Further, despite strong quantitative performance, we noted that GFP-GAN struggled to generate a natural-looking image.

2.3. Key implementation details

The reconstruction loss $J(w^+, z)$ arising from the Bayesian MAP formalism is a non-convex objective of the joint latent code $(w^+, z)$. Therefore, it is generally challenging to devise an iterative algorithm that succeeds in recovering the global minimum regardless of the initialization. We employ the Adam algorithm developed by Kingma and Ba [11] in view of its wide applicability for solving high-dimensional non-convex problems while requiring minimal hyper-parameter tuning. Nevertheless, the success of Adam in solving (5) depends to a great extent on careful initialization and the stopping rule, both of which are elucidated in the following.

Multiple initialization: To initialize the latent code $z$ for (5), we draw $N_1$ i.i.d. samples $\{z(i)\}_{i=1}^{N_1}$ from the standard normal distribution, and select the best $z(i)$ that minimizes the perceptual loss: $z_{\text{init}} = \arg\min_{1 \leq i \leq N_1} \|\varphi(y) - \varphi(AG(M(z(i))))\|_2^2$. Computing $z_{\text{init}}$ does not require any expensive gradient computation and only entails forward passes through the generator and the mapping network. Such an initialization approach prevents any bias that might potentially arise from just one initial point and helps the reconstruction algorithm avoid local minima in the loss landscape. We found that $N_1 = 100$ sufficed for our experiments.

Early stopping: We found that for both BRGM and L-BRGM, the iterative scheme for reconstruction can potentially diverge if the iterations are not terminated appropriately. This phenomenon is illustrated in Fig. 2 for the task of image super-resolution. The example in the second row of Fig. 2 illustrates how early stopping might be necessary to prevent divergence of BRGM and L-BRGM from the image manifold, whereas the first row shows a representative example where such an early stopping may not be required. Therefore, we track the learned perceptual image patch similarity (LPIPS) metric [12] between the ground-truth and the reconstructed image over the iterations and report the best LPIPS (i.e., the smallest) distance achieved during the iterative optimization process. Empirically, we found that BRGM also attained the optimal LPIPS distance around the same iteration as L-BRGM. Such early stopping is not applicable to the GFP-GAN approach [13], considered for numerical comparison in Section 3, since GFP-GAN is a learning-based inversion method, as opposed to BRGM and L-BRGM that employ optimization for inversion.

3. NUMERICAL RESULTS

We consider two prototypical inverse problems, namely (i) image inpainting and (ii) super-resolution to validate L-BRGM quantitatively and compare it with state-of-the-art GAN-based image reconstruction approaches. For inpainting, we choose BRGM [8] and deep generative inpainting network (DeepGIN) [14] as the competing methods. BRGM is a natural candidate for comparison since L-BRGM is a direct extension of it, while the choice of DeepGIN, which was an entry in the Aim-2020 inpainting challenge [15], was motivated by its applicability to inpainting with extreme masking.
For super-resolution, we choose BRGM and the generative facial prior (GFP) approach [13] that seeks to restore facial images by exploiting the prior encapsulated by a GAN trained on face images (referred to as GFP-GAN) for comparing with L-BRGM. GFP-GAN uses the existing architecture from Real-ESRGAN [16] and is reported to yield state-of-the-art performance for facial image restoration. Contrary to the generators in BRGM and L-BRGM, both GFP-GAN and DeepGIN were trained on images of resolutions lower than 1024×1024. Consequently, we include comparisons with GFP-GAN and DeepGIN for target ground-truth images of resolution 256×256, whereas for the images of resolution 1024×1024, the comparison is restricted to only BRGM. Both L-BRGM and BRGM reconstructions are computed with 2000 iterations. The Adam parameters are chosen as $\eta, \beta_1, \beta_2 = 0.1, 0.96, 0.9999$; whereas the regularization penalties are: $\lambda_{\text{pix}}, \lambda_{\text{vgg}}, \lambda_{\text{map}}, \lambda_{\text{lat}} = 2 \times 10^{-5}, 2 \times 10^{-7}, 30, 0.4$. A set of 99 images (not used during training) are utilized for performance validation. The LPIPS scores are calculated using AlexNet features [17] for a fair evaluation, since the perceptual loss based on VGG-16 embedding is already a part of the reconstruction objective $J$.

For inpainting on images of size 256×256, L-BRGM outperforms both BRGM and DeepGIN in terms of LPIPS (c.f. Table 1). In case of super-resolution, L-BRGM defeats BRGM, but performs slightly worse as compared with GFP-GAN. The superiority of GFP-GAN can presumably be attributed to the fact it is trained with bespoke loss terms that are particularly designed for face images (such as the facial component and identity preserving losses [13]). For inpainting, L-BRGM also emerges as the best performing method in terms of LPIPS on a majority of the test images. The inpainting examples in Fig. 3 demonstrate that L-BRGM is conspicuously superior to its competitors. However, L-BRGM turns out to be slightly inferior in terms of SSIM, but we found that, unlike LPIPS, SSIM does not correlate well with the visual image quality and we therefore report only the LPIPS distances for inpainting and super-resolution on 1024×1024 images (see Table 2). In this case, L-BRGM was found to significantly surpass BRGM for both inpainting masks and for the task of super-resolution. The superiority of L-BRGM over BRGM underscores the need for a better regularizer on the latent space, and highlights the advantages of data-driven modeling of such a regularizer.

### 4. CONCLUSIONS AND OUTLOOK

We proposed a novel optimization-based approach for StyleGAN2 inversion, with potential applications to imaging inverse problems. The proposed L-BRGM algorithm leverages the full expressive power of the StyleGAN2 generator and mapping network for modeling the joint latent- and style-code prior in a Bayesian estimation framework. The proposed approach yields competitive performance with state-of-the-art GAN-based approaches for prototypical imaging inverse problems, such as inpainting and super-resolution. Nevertheless, developing a generic method to outperform the state-of-the-art on all possible target images is an ambitious and rather challenging objective. Therefore, further research is needed to not only produce novel regularization schemes that effectively make use of a pre-trained GAN, but also to learn generative models with more interpretable (and hence regularizable) latent spaces for image reconstruction problems.
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