Single-Layer Phase Screen With Pointing Errors for Free Space Optical Communication

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This work was supported in part by the National Natural Science Foundation of China under Grant 61501097, in part by the Chinese Fundamental Research Funds for the Central Universities under Grant ZYGX2019J080, and in part by the Science and Technology Department of Sichuan Province under Grant 2020ZYD016.

ABSTRACT Single-layer phase screen is a classical, simple and accurate approach for simulating the optical field under turbulence, and it is generally used in free space optical communication (FSOC). However, conventional single-layer phase screen studies have rarely considered the pointing errors from FSOC, which prevents the theoretical results for mobile FSOC from being verified using simulated optical fields. This work establishes a simulation method for the single-layer phase screen with pointing errors. Statistics on simulated optical field with pointing errors indicated that the previous theoretical results for the long-term average irradiance and irradiance fluctuation probability density function (PDF) for mobile FSOC can be modified to better fit the simulation results. In addition, the received power fluctuation PDF, average bit-error-rate (BER), and the influence of non-zero mean pointing errors are investigated through the simulated optical field with pointing errors. The proposed simulation method provides a powerful approach for understanding and evaluating the comprehensive influences of atmospheric turbulence and pointing errors on the FSOC.

INDEX TERMS Free space optical communication, atmosphere turbulence, phase screen, pointing errors.

I. INTRODUCTION

Free space optical communication (FSOC) has drawn great attention because of its high bandwidth, high speed, and low power consumption [1]. FSOC links have been successfully applied on various mobile platforms, such as satellites, aircraft, and ships [2]. However, the performance of FSOC is inevitably decayed under atmospheric turbulence [3], [4].

To simulate and evaluate the complicated impact of atmospheric turbulence on laser propagation, phase screen simulations have been developed and widely applied in FSOC link research [5]–[22]. A phase screen can be generated through two main kinds of methods: the power spectrum inversion method and the Zernike polynomial method [7]. The Zernike polynomial method is directly related to turbulence-induced wave-front distortion [8], which is generally applied in adaptive-optics [9] and [10] and FSOC with wave-front distortion correction [11], [12]. The power spectrum inversion method is related to turbulence-induced optical-field irradiance disturbance and wave-front distortion [13]–[16], and it is generally used in FSOC research [17]–[22].

The single-layer phase screen is a classical, simple and accurate approach for simulating the optical field under turbulence, which is generated by the power spectrum inversion method. L. C. Andrews et al. theoretically showed that atmospheric turbulence propagation under weak fluctuations can be equivalent to that of a single-layer phase screen placed at 0.36 times the propagation distance [5], [6]. In this case, the simulation results for both the long-term average irradiance and irradiance fluctuation probability density function (PDF) show excellent agreement with the theoretical results for weak fluctuations. Because the long-term average irradiance and irradiance fluctuation PDF directly determine the atmospheric FSOC performance (e.g., average signal-to-noise ratio, bit error rate, outage capacity and availability) [6], [23], the single-layer phase screen connects the simulated turbulence distorted optical field and the FSOC link performance.

However, conventional single-layer phase screen studies have rarely considered the pointing errors from FSOC, which
errors can be established in the following steps. First, simulating turbulence-distorted optical fields along different can be placed at 0.36 times the transmission distance for can be ignored. Then, the location of single-layer phase screen distance variations along different propagation directions can be expressed as follows \[
\begin{align*}
\theta_x &= \frac{\text{random pointing error angle to the front of the phase screen.}}{
\theta_x \pm \sigma_x 
}
\end{align*}
\]
\[
\theta_y = \pm \sigma_y 
\end{align*}
\]
Therefore, the single-layer phase screen with pointing errors can be established in the following steps. First, the pointing errors are simulated according to their distribution function, with the optical field traveling \(0.36L\) (\(L\) is the whole transmission distance) in the free space at a random pointing error angle to the front of the phase screen. Second, the optical field is transformed into a turbulence-distorted optical field by phase screen’s modulation. The phase screen can be regarded as a turbulent layer without thickness. Finally, the turbulence-distorted optical field travels \(0.64L\) in free space with the same angle as the first step to obtain the final optical field at the transmitter (see Fig. 1).

![FIGURE 1. Schematic of the establishment of the single-layer phase screen with boresight pointing errors.](image)

As shown in Fig. 1, the optical field at the transmitter is a TEM\(_{00}\) Gaussian beam [6]:
\[
U_0(x, y, 0) = \exp \left( -\frac{x^2 + y^2}{W_0^2} \right)
\]
where \(x\) and \(y\) represent the coordinate value along the horizontal and vertical direction, respectively, and \(W_0\) represents the beam radius at the transmitter.

The pointing errors from the tracking mechanism can be generally described by the Gaussian distribution in both the horizontal and vertical axes [23]–[25]. The PDF of the horizontal pointing errors angle \(\theta_x\) is given as follows [25]:
\[
f(\theta_x) = \frac{1}{\sqrt{2\pi} \sigma_x} \exp \left( -\frac{\theta_x^2}{2\sigma_x^2} \right)
\]
where \(\sigma_x\) is the standard variance of \(\theta_x\). The PDF of the vertical pointing errors angle \(\theta_y\) is given as follows [25]:
\[
f(\theta_y) = \frac{1}{\sqrt{2\pi} \sigma_y} \exp \left( -\frac{\theta_y^2}{2\sigma_y^2} \right)
\]
where \(\sigma_y\) is the standard variance of \(\theta_y\).

Based on Eq. (2) and Eq. (3), a series of horizontal and vertical pointing errors angles \(\theta_{xj}\) and \(\theta_{yj}\) are generated, respectively. In addition, \(j\) represents the serial number from 1 to \(M\), with \(M\) denoting the number of the simulation. The phase perturbations generated by the phase screen and pointing errors in each simulation group are random.

Then, the \(j\)-th optical field \(U_{ij}(x, y, 0.36L)\) in the front of phase screen at \(0.36L\) can be expressed as follows:
\[
U_{ij}(x, y, 0.36L) = \mathcal{F}^{-1} \left\{ \mathcal{F}[U_0(x, y, 0)] \times H_1 \times T_{ij} \right\}
\]

\[
U_0(x, y, 0) = \exp \left( -\frac{x^2 + y^2}{W_0^2} \right)
\]
\[
f(\theta_x) = \frac{1}{\sqrt{2\pi} \sigma_x} \exp \left( -\frac{\theta_x^2}{2\sigma_x^2} \right)
\]
\[
f(\theta_y) = \frac{1}{\sqrt{2\pi} \sigma_y} \exp \left( -\frac{\theta_y^2}{2\sigma_y^2} \right)
\]

\[
U_{ij}(x, y, 0.36L) = \mathcal{F}^{-1} \left\{ \mathcal{F}[U_0(x, y, 0)] \times H_1 \times T_{ij} \right\}
\]
where $L$ represents the propagation distance; $F^{-1}$ denotes an inverse Fourier transform, and $F$ denotes a Fourier transform. The Fourier transform and inverse Fourier transform are used to achieve the optical field diffraction and the atmospheric turbulence modulation [37]. $H_1 = \exp[-i0.36L/(k_x^2 + k_y^2)]$ denotes the transmission of the optical field at $0.36L$ in free space [37], $k_x$ and $k_y$ denote the spatial wavenumbers, $k = 2\pi/\lambda$ is the optical wavenumber, and $\lambda$ is the wavelength. $T_{ij} = \exp[-i2\pi(0.36\theta_i/\lambda_f + 0.36\theta_j/\lambda_f)]$ is the $j$-th Fourier shift factor, which represents the pointing errors angles induced random displacement of optical field at $0.36L$.

When the optical field $U_1(x, y, 0.36L)$ passes through the single-layer phase screen, the turbulence-distorted optical field $U_2(x, y, 0.36L)$, modulated by the phase screen at $0.36L$, can be expressed as follows [6], [37]:

$$ U_2(x, y, 0.36L) = U_1(x, y, 0.36L) \times \exp[i\phi(x, y)] $$  \hspace{1cm} (5)

where $\phi(x, y)$ denotes the $j$-th phase perturbations caused by atmospheric turbulence:

$$ \phi(x, y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} c(k_x, k_y) \sqrt{\Psi_\phi(k_x, k_y)} \times \exp[i(k_x x + k_y y)] \Delta k_x \Delta k_y $$  \hspace{1cm} (6)

where $c(k_x, k_y)$ corresponds to the complex Gaussian distribution statistics with mean zero and unit variance. $\Psi_\phi(k_x, k_y)$ is the power spectrum function and complied with the Kolmogorov spectrum, $\Psi_\phi(k_x, k_y) = 2\pi k^2 \times 0.033L^2C_2^2k^{-11/3}$. $\Delta k_x$ and $\Delta k_y$ are the sampling intervals, and $C_2$ is the atmospheric refractive index structure constant. The Rytov approximation is applied in the power spectrum inversion method. Therefore, the phase screen simulation should be valid under weak fluctuations [6], [37], where the Rytov variance ($\sigma_R^2 = 1.23C_2^2L^2/\lambda f^{11/6}$) is less than 1.

Combined with Eqs. (1) - (6), the series turbulence-disturbed optical field with pointing errors at the receiver can be described as follows:

$$ U_j(x, y, L) = F^{-1} \left[ F \left[ U_2(x, y, 0.36L) \right] \times H_2 \times T_{2j} \right] $$  \hspace{1cm} (7)

where $U_j(x, y, L)$ denotes the $j$-th optical field at $L$, $H_2 = \exp[-i0.64L/(k_x^2 + k_y^2)]$ denotes the free space transmission of optical field from $0.36L$ to $L$, and $T_{2j} = \exp[-i2\pi(0.64\theta_i/\lambda_f + 0.64\theta_j/\lambda_f)]$ denotes the $j$-th Fourier shift factor from $0.36L$ to $L$. When $\theta_x = \theta_y = 0$, Eq. (7) represents the result of the classical single-layer phase screen without pointing errors.

Compared with the classical single-layer phase screen, $T_{1j}$ and $T_{2j}$ are newly added Fourier shift factors that represent the pointing error-induced random displacement.

Because the long-term average irradiance with pointing errors and irradiance fluctuation PDF with pointing errors are fundamental for the mobile FSOC performance in atmospheric turbulence [6] and [23], the following work is focused on these two functions, which have seldom been investigated through simulated turbulence-distorted optical fields under pointing errors. The long-term average irradiance and irradiance fluctuation PDF with pointing errors are obtained from the simulated turbulence-distorted optical fields under pointing errors.

The long-term average irradiance with pointing errors can be obtained:

$$ \langle I_p(x, y, L) \rangle = \frac{1}{M} \sum_{j=1}^{M} [I_p(x, y, L)] $$

$$ = \frac{1}{M} \sum_{j=1}^{M} \langle |U_j(x, y, L)|^2 \rangle $$  \hspace{1cm} (8)

when $\theta_x = \theta_y = 0$, Eq. (8) is simplified to the long-term average irradiance without pointing errors. When $\theta_x = \theta_y = 0$, Eq. (8) represents the long-term average irradiance without pointing errors.

The series of irradiance at the receiver on the propagation axis is described as $I_p(0, 0, L)$ and $I_p(0, 0, L) = |U_j(0, 0, L)|^2$. The irradiance fluctuation PDF is calculated by the kernel smoothing method, which is a nonparametric PDF estimation method that performs based on a data histogram and weighted average. The specific steps are as follows. To obtain the kernel estimation of the density function, the estimation interval is divided into $M$ parts, with a partition width of $\Delta I$. Then, the mathematical expression of the irradiance fluctuation PDF with pointing errors can be obtained as follows [38]:

$$ f(I) = \frac{1}{M \cdot \Delta I} \sum_{i=1}^{M} K_S \left[ \frac{2(I - I_j)}{\Delta I} \right], \quad K_S(u) = \begin{cases} 1 & |u| \leq 1 \\ 0 & |u| > 1 \end{cases} $$  \hspace{1cm} (9)

where $\Delta I$ represents the irradiance interval and $M$ is the statistic number of irradiance and $K_S$ is a uniform kernel function, which is associated with the frequency of $I_p(0, 0, L) \in [I_j - 0.5\Delta I, I_j + 0.5\Delta I]$ and can be obtained through the histogram function in MATLAB. When $\theta_x = \theta_y = 0$, Eq. (9) represents the irradiance fluctuation PDF without pointing errors.

III. SIMULATION RESULTS

The simulation parameters are as follows: wavelength $\lambda = 850$ nm, transmitting aperture $W_0 = 0.04$ m, transmission distance $L = 1000$ m, grid points $512 \times 512$, phase screen size $= 0.4$ m, and the simulated turbulence-disturbed optical field $M = 10,000$. $C_2^2$ is set to $5 \times 10^{-15}$m$^{-2/3}$, $1 \times 10^{-14}$m$^{-2/3}$ and $2 \times 10^{-14}$m$^{-2/3}$, and the corresponding Rytov variance is equal to 0.2, 0.4, and 0.8, respectively. The standard variance of pointing errors is set to 5 $\mu$rad, 10 $\mu$rad, and 15 $\mu$rad.

A. LONG-TERM AVERAGE IRRADIANCE

The expression for the long-term average irradiance without pointing errors is deduced as follows [6]:

$$ \langle I(x, y, L) \rangle = \frac{W_0}{W_{LT}} \exp \left[ -\frac{2(x^2 + y^2)}{W_{LT}^2} \right] $$  \hspace{1cm} (10)
where $W_{LT}$ is the long-term beam radius, with $W_{LT} = W \sqrt{1 + 1.33\sigma_R^2 A^{5/6}}$, and $\Lambda = 2L/kW_0^2$.

The expression for long-term average irradiance with pointing errors is fitted from the satellite-ground FSOC experiment [24]:

$$\langle I(x, y, L) \rangle = \frac{W_0^2}{W_E^2} \exp \left[ -\frac{2(x^2 + y^2)}{W_E^2} \right]$$  \hspace{1cm} (11)

where $W_E^2 = W_{LT}^2 + (8 \ln 2)\sigma_R^2 L^2$ and $\sigma_E$ represents the standard variance of the pointing errors and is equal to $1.87 \mu\text{rad}$ [24].

Profiles of the long-term average irradiance under different pointing errors and Rytov variances are shown in Fig. 2. The lines represent the theoretical results, the dots represent the phase screen simulation results, and the red color represents the theoretical and simulation results without pointing errors. The theoretical results for long-term average irradiance without pointing errors are associated with Eq. (10). The blue, green, and purple color represents the theoretical and simulation results with pointing errors of 5 $\mu\text{rad}$, 10 $\mu\text{rad}$, and 15 $\mu\text{rad}$, respectively. The theoretical results for long-term average irradiance with pointing errors are associated with Eq. (11). The simulation results for long-term average irradiance with pointing errors are associated with Eq. (8).

![FIGURE 2. Profiles of the long-term average irradiance under different pointing errors and Rytov variances.](image)

The correlation coefficients between Eq. (8) and phase screen simulation without pointing errors are all 0.999 with increases in the Rytov variance from 0.2 to 0.8, which means that the single-layer phase screen can reach a high accuracy simulation for the long-term average irradiance without pointing errors.

The correlation coefficients between Eq. (11) and the phase screen simulation are all 0.985 at 5 $\mu\text{rad}$ with increasing Rytov variance, 0.972 at 10 $\mu\text{rad}$ with increasing Rytov variance, and 0.961 at 15 $\mu\text{rad}$ with increasing Rytov variance. The correlation coefficients between Eq. (10) and phase screen simulations decrease as the pointing errors increase.

Eq. (11) is fit under the pointing errors of 1.87 $\mu\text{rad}$; thus, we infer that the broadened coefficient $(8\ln 2 \approx 5.54)$ before the pointing errors in Eq. (11) could be modified to fit the simulation results at different pointing errors. Then, the long-term average irradiance with pointing errors is modified as follows:

$$\langle I'(x, y, L) \rangle = \frac{W_0^2}{W_S^2} \exp \left[ -\frac{2(x^2 + y^2)}{W_S^2} \right]$$  \hspace{1cm} (12)

where $W_S^2 = W_{LT}^2 + q\sigma_R^2 L^2$ and $q$ is a constant that is fit based on the phase screen simulation results.

Based on the phase screen simulation results from Fig. 2, the $q$ in Eq. (12) is fit as 4. In this case, the correlation coefficients between Eq. (12) and phase screen simulation are all improved to 0.999 under different pointing errors and Rytov variances (see Fig. 3).

![FIGURE 3. Profiles of the modified long-term average irradiance with pointing errors.](image)

We find that the broadened coefficient can be modified to 4 in the long-term average irradiance with pointing errors.

**B. IRRADIANCE FLUCTUATION PDF**

The irradiance fluctuation PDF without pointing errors under weak fluctuations has a log-normal distribution [6], [28]:

$$f(I) = \frac{1}{\sqrt{2\pi \sigma_R^2 I}} \exp \left\{ -\frac{[\ln I + 0.5\sigma_R^2]}{2\sigma_R^2} \right\}$$  \hspace{1cm} (13)
The irradiance fluctuation PDF with pointing errors is deduced as follows [27], [28]:

\[
\text{f}_{PE}(I) = \frac{1}{2} I \beta^{-1} \exp \left[\frac{1}{2} \sigma_R^2 \beta (\beta + 1) \right] \times \text{erfc} \left[ \frac{2 \ln I + (2 \beta + 1) \sigma_R^2}{2 \sqrt{2} \sigma_R} \right] \tag{14}
\]

where \( \beta = \frac{W^2}{4 \sigma^2} \), \( W \) is the beam radius at the receiver in the free space, and \( \text{erfc}() \) represents the complementary error function.

The sky-blue histograms represent the phase screen simulation results from Eq. (9). The red lines represent the theoretical results. The theoretical results of the irradiance fluctuation PDF without pointing errors are associated with Eq. (13). Fig. 4(a)-(c) shows the results without pointing errors. The theoretical results of the irradiance fluctuation PDF with pointing errors are associated with Eq. (14). Fig. 4(d)-(l) shows the results with pointing errors of 5 \( \mu \text{rad} \), 10 \( \mu \text{rad} \), and 15 \( \mu \text{rad} \).

The correlation coefficients between Eq. (13) and Eq. (9) are all above 0.996. Therefore, the single-layer phase screen can reach a high accuracy simulation for the irradiance fluctuation PDF without pointing errors.

As shown in Fig. 4, when the Rytov variance equals 0.4, the correlation coefficients between Eq. (14) and Eq. (9) are 0.995, 0.993, and 0.990 under increasing pointing errors. When the Rytov variance equals 0.8, the correlation coefficients between Eq. (14) and Eq. (9) are 0.994, 0.988, and 0.976 under increasing pointing errors.

The value of the correlation coefficient between Eq. (14) and Eq. (9) is sufficiently close to the theoretical prediction in the above comparison. However, we observe a decrease as the Rytov variance and pointing errors increase, which may be associated with the beam radius.

The beam radius in the original theoretical deduction in Eq. (13) is treated as the beam radius in free space [27] and [28], and it is widely used [29]–[34]. Considering the deduction process for the irradiance fluctuation PDF with pointing errors, there are two parts of the irradiance modulation. One is the atmospheric turbulence-induced scintillation described by a log-normal distribution [6]; and the other is the pointing error-induced irradiance variation along the profile of the optical field described by a beta distribution [24], [25].

In the laser atmospheric propagation, the profile of the turbulence distorted optical field should be modeled as the short-term irradiance [6], which is a kind of Gaussian function.

Thus, the beam radius \( W \) in Eq. (14) should be replaced with the short-term radius \( W_{st} \) [6]:

\[
W_{st} = W \sqrt{1 + 1.33 \sigma_R^2 \Lambda^{5/6}} \left[ 1 - 0.66 \left( \frac{\Lambda_0^2}{1 + \Lambda_0^2} \right)^{1/6} \right] \tag{15}
\]

where \( W_{st} \) is the short-term radius, \( \Lambda_0 = 2L/(kW_0^2) \) and \( \Lambda = 2L/(kW^2) \).

Combined with Eq. (14) and Eq. (15), the irradiance fluctuation PDF with pointing errors is modified as follows:

\[
\text{f}'_{PE}(I) = \frac{1}{2} I \beta'^{-1} \exp \left[\frac{1}{2} \sigma_R^2 \beta' (\beta' + 1) \right] \times \text{erfc} \left[ \frac{2 \ln I + (2 \beta' + 1) \sigma_R^2}{2 \sqrt{2} \sigma_R} \right] \tag{16}
\]

where \( \beta' = \frac{W_{st}^2}{4 \sigma^2} \).

Based on the phase screen simulation results from Fig. 4 with a lower correlation coefficient (Fig. 4 (i), (k) and (l)), the beam radius-modified results for the irradiance fluctuation PDF with pointing errors are shown in Fig. 5.

The red lines represent the theoretical results from Eq. (14), the purple lines represent the beam radius-modified results according to Eq. (16), and the sky-blue histograms represent the results of the phase screen simulation from Eq. (9).

The correlation coefficient between Eq. (14) and simulation is improved to 0.996 with a Rytov variance of 0.4 and pointing errors of 15 \( \mu \text{rad} \), to 0.995 with a Rytov variance of 0.8 and pointing errors of 10 \( \mu \text{rad} \) and to 0.995 with a Rytov variance of 0.8 and pointing errors of 15 \( \mu \text{rad} \).

We find that the beam radius in the irradiance fluctuation PDF with pointing errors can be modified as the short-term radius in the turbulence.
IV. DISCUSSION

A. COMPARING THE SIMULATED IRRADIANCE FLUCTUATION PDF WITH LOG-NORMAL DISTRIBUTION AND GAMMA-GAMMA DISTRIBUTION

It is widely accepted that the log-normal distribution for the irradiance fluctuation PDF is used for weak fluctuations and the Gamma-Gamma distribution is used for both weak and strong fluctuations [6]. Because the single-layer phase screen is valid under weak fluctuations [6], the simulated irradiance fluctuation PDF is compared with the log-normal distribution and Gamma-Gamma distribution under weak fluctuations.

The Gamma-Gamma distribution is written as follows [6]:

$$f_{GG}(I) = \frac{2(\alpha \beta)^{(\alpha + \beta)/2}}{\Gamma(\alpha) \Gamma(\beta)} I^{(\alpha + \beta)/2 - 1} K_{\alpha - \beta}(2\sqrt{\alpha \beta I})$$

(17)

where $K_v()$ is the second kind modified Bessel functions and $\Gamma()$ is the Gamma-Gamma function. $\alpha$ and $\beta$ are denoted as follows:

$$\begin{align*}
\alpha &= \left\{ \exp\left[ \frac{0.49 \sigma_R^2}{(1 + 1.11 \sigma_R^{12/5})^{7/6}} \right] - 1 \right\}^{-1} \\
\beta &= \left\{ \exp\left[ \frac{0.51 \sigma_R^2}{(1 + 0.69 \sigma_R^{12/5})^{5/6}} \right] - 1 \right\}^{-1}
\end{align*}$$

(18)

where $\sigma_R^2$ is the Rytov variance.

As shown in Fig. 6, the red line denotes the log-normal distribution simulated from Eq. (14), the green line denotes the Gamma-Gamma distribution simulated from Eq. (17) and the sky-blue histograms represent the phase screen simulation results from Eq. (9).

The correlation coefficient between single-layer phase screen simulated irradiance fluctuation PDF and log-normal distribution are all above 0.996 in Fig. 6 (a), (b) and (c), respectively, and the correlation coefficient between the single-layer phase screen simulated irradiance fluctuation PDF and Gamma-Gamma distribution are 0.929, 0.906, 0.854 in Fig. 6 (a), (b) and (c), respectively.

The results show that the log-normal distribution is closer to the single-layer phase screen simulated irradiance fluctuation PDF than the Gamma-Gamma distribution. Few previous studies have investigated these relationships.

The difference between the simulated irradiance fluctuation PDF and various theoretical models (Gamma-Gamma [31], [32], exponential Weibull [33], and M distribution [34]) warrants further investigation.

Furthermore, the phase screen simulation is mainly based on the power spectrum inversion, which is valid for the weak fluctuation. Thus, a new kind of phase screen simulation method for strong fluctuations should be developed.

B. RECEIVED POWER FLUCTUATION PDF

Similar to previous work [27]–[30], we find that the widely used irradiance fluctuation PDF ignores the size of the received aperture and is suitable for long distance propagation. However, the size of the received aperture should be considered for short distance propagation. Then, the received power (irradiance integration on the received aperture) fluctuation PDF should be different from the irradiance fluctuation PDF.

Vetelino et al. [39] and Beason and Gladysz [40], [41] have studied the received power fluctuation PDF. However, neither the theoretical model nor the simulation method have considered the influence of pointing errors. According to the proposed single-layer phase screen simulation method, the received power fluctuation PDF under pointing errors can be obtained.
Based on Eq. (9), the received power on a certain aperture \( D \) can be calculated as follows:

\[
P(D) = \sum_{x=-0.5D}^{0.5D} \sum_{y=-\sqrt{D^2-x^2}}^{\sqrt{D^2-x^2}} [I_p(x, y, L)]
\]  

(19)

Combined with Eq. (19) and by rewriting Eq. (9), the received power fluctuation PDF with pointing errors can be obtained as follows:

\[
f[P(D)] = \frac{1}{M \cdot \Delta P(D)} \sum_{i=1}^{M} K_S \left[ \frac{2[P(D) - P_i(D)]}{\Delta P(D)} \right],
\]

(20)

where \( \Delta P(D) \) represents the power interval, \( M \) is the statistic amount of received power, and \( K_S \) is a uniform kernel function, which is associated with the frequency of \( P(D) \in [P_i(D) - 0.5\Delta P(D), P_i(D) + 0.5\Delta P(D)] \) and can be obtained through the `hist` function in MATLAB.

The simulation parameters are the same as previously indicated, and the received aperture is set as 0.08 m. The simulation results for the received power fluctuation PDF are shown in Fig. 7.

The received power fluctuation PDF is quite different from the irradiance fluctuation PDF. Based on the proposed single-layer phase screen with boresight pointing errors, further theoretical research about received power fluctuation PDF should be performed and the simulation should be verified.

**C. AVERAGE BER UNDER POINTING ERRORS**

Based on the simulated received power and received power fluctuation PDF under pointing errors, the average bit-error-rate under pointing errors can be obtained.

For an off-on-keying (OOK) FSO system, the instantaneous BER is expressed as follows [6]

\[
BER = \frac{1}{2} \text{erfc} \left[ \frac{R \times P(D)}{\sqrt{2} \sigma_N} \right]
\]

(21)

where \( R \) denote the responsivity and \( \sigma_N \) denote the current noise, which is mainly determined by the shot noise. \( P(D) \) is the received power in Eq. (19).

The average BER is the weighted average of the instantaneous BER and received power fluctuation PDF, which can be expressed as follows [40], [42]–[43]:

\[
\langle BER \rangle = \int_0^1 BER \times f[P(D)] \, df \, [P(D)]
\]

(22)

As shown in Fig. 8, the average BER significantly decayed as the pointing errors and atmospheric turbulence increased.

**D. INFLUENCE OF NON-ZERO BORESIGHT POINTING ERRORS**

The above research is based on the zero-mean boresight pointing errors. The simulation method can also investigate the influence of non-zero boresight pointing errors on the average irradiance and irradiance fluctuation PDF.

When non-zero mean boresight pointing errors are involved, Eq. (2) and Eq. (3) can be rewritten as follows:

\[
f_{\text{non-zero}}(\theta_x) = \frac{1}{\sqrt{2\pi\sigma_x}} \exp \left( -\frac{(\theta_x - \psi_x)^2}{2\sigma_x^2} \right)
\]

\[
f_{\text{non-zero}}(\theta_y) = \frac{1}{\sqrt{2\pi\sigma_y}} \exp \left( -\frac{(\theta_y - \psi_y)^2}{2\sigma_y^2} \right)
\]

(23)
where $\varphi_x$ and $\varphi_y$ denote the mean value of the boresight pointing errors in the horizontal and vertical direction, respectively. The simulation parameters are the same as previously indicated. The Rytov variance is set to 0.4, the mean value is set to $\varphi_x = 20\mu$rad and $\varphi_y = 30\mu$rad, and the standard variance is set to $\sigma_x = 20\mu$rad and $\sigma_y = 10\mu$rad.

The vertical view of the average irradiance profile is shown in Fig. 9(a), where $O$ denotes the center of propagation axis and $O'$ denotes the center of the average irradiance profile, which is moved by the non-zero boresight pointing errors. Meanwhile, the beam profile turns into an elliptical shape because of the different standard variance of boresight pointing errors between the horizontal and vertical directions. The irradiance fluctuation PDF can be obtained by Eq. (9) and is shown in Fig. 9(b), where the sky-blue histograms represent the single-layer phase screen simulation results. The proposed phase screen simulation method could be useful for verifying the theoretical research of the average irradiance and irradiance fluctuation PDF with non-zero boresight pointing errors, which has rarely been investigated in previous research.

V. CONCLUSION

In this study, a simulation method for the single-layer phase screen with pointing errors was established for the first time. Although the previous theoretical results under pointing errors showed good agreement with the simulation results, we found that they can be modified to fit the simulation results better by setting the broadened coefficient to four in the long-term average irradiance with pointing errors and adopting the short-term radius in the irradiance fluctuation PDF with pointing errors.

Moreover, the received power fluctuation PDF, average BER, and non-zero mean pointing error influence are investigated through the simulated optical field with pointing errors. The proposed simulation method provides a powerful approach to evaluating the theoretical results for FSOC under atmospheric turbulence.

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