Interaction of an electromagnetic $E$-wave with a thin conducting film between two dielectric media in the case of an anisotropic isoenergetic surface and impurity scattering

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Abstract. The coefficients of reflection, transmission and absorption are calculated in the framework of the kinetic approach, when an electromagnetic $E$-wave interacts with a thin conducting film located between two dielectric media. To account for the surface scattering of charge carriers is used a model of mirror-diffuse boundary conditions, assuming that the specularity coefficients of the upper and lower surfaces of the film differ from each other. The electromagnetic wave falls on the upper surface of the film at an arbitrary angle. The case of an anisotropic isoenergetic surface of a conductor having the form of a three-axis ellipsoid, one of the main axes of which is parallel to the magnetic field strength of the wave, and the other is perpendicular to the film surfaces, is considered. The impurity scattering of electrons (holes) is dominated in the volume of the conductor. The dependence of the absorption coefficient on the parameters of the isoenergetic surface of the conductor is analyzed.

1. Introduction

Much attention has been devoted to the optical properties of thin conducting films, in particular the coefficients of absorption, reflection and transmission electromagnetic wave [1-4]. The coefficients of reflection, transmission and absorption in the interaction of a metal film with an $H$-wave were calculated in [1], and with an $E$-wave in [2]. The film is located in a vacuum, and the specularity coefficients of the film have equal values in these works. The calculation method in [1,2] uses the impedance approach that was used in the article of Kliever-Fuchs [3]. In articles [4] and [5], which are a continuation of works [1] and [2], for the $H$- and $E$-waves, respectively, different specularity coefficients of the film surface and the presence of different media between which the film is located are considered.

Thin films are widely used in various fields. The phenomenon of interference in thin films underlies the illumination of optics [6]. Accurate calculation of optical parameters allows increasing the efficiency and energy efficiency of multilayer solar cells [7].

We study the effect of anisotropic isoenergetic surface and impurity scattering on the reflection, transmission and absorption coefficients of a thin conducting film located between two dielectric media, in the case of arbitrary incidence angles of an electromagnetic $E$-wave and different
coefficients of mirror image of the film boundaries in this paper. Similar calculations for the electromagnetic $H$-wave were made in the work [8].

2. Formulation of the problem

We consider a thin conducting film of thickness $a$ located between two different media. The upper surface of the film has a specularity coefficient $q_1$ when reflecting conduction electrons (holes) from it and borders on a medium having a dielectric constant $\varepsilon_1$. The lower surface of the film has a specularity coefficient $q_2$ when reflecting charge carriers from it and borders on a substrate with a dielectric constant $\varepsilon_2$. We neglect quantum dimensional effects. A flat monochromatic $E$-wave (the magnetic field vector of the wave is parallel to the film surface) falls on the film boundary at an angle $\theta$. We assume that the thickness of the layer is much less than the length of the incident electromagnetic wave and the depth of the skin layer, this allows us to ignore the skin effect and consider the component of the electric field parallel to the surface as homogeneous. The layer thickness can be comparable to or less than the free path of charge carriers in a macroscopic sample. We assume that the frequency of the wave is much lower than the frequency of the plasma resonance in the conducting film. The media are non-magnetic, and the values $\varepsilon_1$ and $\varepsilon_2$ are real. We take a Cartesian coordinate system, the beginning of which is located on the upper surface of the film, the $Y$ axis is parallel to the magnetic field strength of the wave, the $X$ axis is perpendicular to the plane of the film and directed deep into the film (Figure 1).

![Figure 1](image_url)

**Figure 1.** Scheme of the interaction of an electromagnetic wave with a thin conducting film located between two dielectric media.

Surfaces of the equal energy are ellipsoids in the impulse space near the energy extremum of the conductors [9]. We assume that the surface constant energy is a three-axis ellipsoid whose main axes coincide with the coordinate axes in the impulse space $(p_x, p_y, p_z)$; the extreme point is located in the center of the Brillouin zone $(p_0 = 0)$; the energy is counted from the bottom of the conduction zone for electrons or the ceiling of the valence zone for holes $(\varepsilon(p_0) = 0)$, so the energy of charge carriers in the crystal will have the form [9]:

$$
\varepsilon(p_x, p_y, p_z) = \frac{p_x^2}{2m_1} + \frac{p_y^2}{2m_2} + \frac{p_z^2}{2m_3},
$$

(1)

where $m_1, m_2, m_3$ are the effective masses of the quasiparticle along the main axes of the constant energy ellipsoid or along the axes $p_x, p_y$ and $p_z$ respectively.
The direction of the vectors $\mathbf{j}$ and $\mathbf{E}$, generally speaking, do not coincide in an anisotropic sample (single crystal) and the linear relationship between them is expressed by formulas of the form [10]:

$$j_i = \sigma_{ik} E_k,$$

where the values $\sigma_{ik}$ make up a symmetric tensor of the second rank (the conduction tensor), and summation is assumed by repeating indexes. If the axes of the energy ellipsoid coincide with the axes $p_x, p_y$ and $p_z$, then the non-diagonal components of the tensor $\sigma_{ik}$ are zero.

We assume that the volume relaxation time $\tau$ is inversely proportional to the velocity of the charge carrier (for example, in the case of impurity scattering [9]):

$$\tau(v) = \frac{\lambda}{v} = \frac{\lambda}{\sqrt{v_x^2 + v_y^2 + v_z^2}},$$

where $\lambda = \text{const}$ is the length of the free path, which does not depend on the speed of electrons (holes).

Expressions for the reflection coefficients $R$, passage $T$ and absorption $A$ of an electromagnetic $E$-wave when it interacts with a thin conducting film located between two media with permittivity $\varepsilon_1$ and $\varepsilon_2$, in the case of different mirror coefficients $q_1$ and $q_2$ of the film surfaces and arbitrary incidence angles of the electromagnetic wave $\theta$, were obtained in [5]:

$$R = \left| \frac{\beta_{12} \sqrt{\varepsilon_{12}} - \sin^2 \theta (\beta + p^{(1)} p^{(2)}) + \cos \theta (\beta - p^{(1)} p^{(2)})}{\beta_{12} \sqrt{\varepsilon_{12}} - \sin^2 \theta (1 + \beta) + \cos \theta (1 - \beta)} \right|^2,$$

$$T = \cos \theta \beta_{12} \Re \sqrt{\varepsilon_{12}} - \sin^2 \theta \left| \frac{p^{(2)} - p^{(1)}}{\beta_{12} \sqrt{\varepsilon_{12}} - \sin^2 \theta (1 + \beta) + \cos \theta (1 - \beta)} \right|^2,$$

$$A = 1 - T - R,$$

$$\beta_{12} = \frac{\beta_1}{\beta_2}, \quad \varepsilon_{12} = \frac{\varepsilon_2}{\varepsilon_1}, \quad \beta = \frac{1}{2} (p^{(1)} + p^{(2)}),$$

$$p^{(1)} = \frac{\sqrt{\varepsilon_1} Z^{(1)} \cos \theta - 1}{\sqrt{\varepsilon_1} Z^{(1)} \cos \theta + 1}, \quad p^{(2)} = \frac{\sqrt{\varepsilon_2} Z^{(2)} \cos \theta - 1}{\sqrt{\varepsilon_2} Z^{(2)} \cos \theta + 1},$$

$$\beta_1 = 1 - \frac{\sin^2 \theta}{\varepsilon_1}, \quad \beta_2 = 1 - \frac{\varepsilon_2 \sin^2 \theta}{\varepsilon_2},$$

where $c$ is speed of the light.

$Z^{(1)}$ and $Z^{(2)}$ are impedances on the lower and upper surfaces of the film, respectively. $Z^{(1)}$ corresponds to a symmetric magnetic field configuration (case 1): $H_y(0) = H_y(a), E_x(0) = E_x(a), E_z(0) = -E_z(a)$; a $Z^{(2)}$ corresponds to a antisymmetric magnetic field configuration (case 1): $H_y(0) = -H_y(a), E_x(0) = -E_x(a), E_z(0) = E_z(a)$.

We consider that in the case under consideration, the wavelength of incident radiation significantly exceeds the thickness of the film, and the frequency of the external field is less than the plasma frequency in the conducting film, the impedances $Z^{(1)}$ and $Z^{(2)}$ will have the following form:

$$Z^{(1)} = 0, \quad Z^{(2)} = \frac{c}{2 \pi a \sigma_{zz}},$$

(9)
\[ \bar{\sigma}_{zz} = \frac{1}{a} \int_{0}^{a} \sigma_{zz}(x) dx, \quad j_z(x) = \sigma_{zz}(x) E_z. \] (10)

To calculate the reflection coefficients \( R \), passage \( T \), and absorption \( A \) (3-10), it is necessary to calculate the average specific electrical conductivity \( \bar{\sigma}_{zz} \) over the film thickness.

Since there are no restrictions on the ratio between the thickness of the conducting film \( a \) and the free path of the charge carriers in the macroscopic sample, the calculation of the electrical conductivity \( \sigma_{zz} \) of the film is carried out in the framework of kinetic theory.

3. Calculation of the average film thickness specific electrical conductivity

We use the kinetic equation in the approximation of the relaxation time \( \tau \) and in the linear external field approximation to find the electrical conductivity [11]:

\[ \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + e \mathbf{E} \frac{\partial f}{\partial \mathbf{p}} = -\frac{f_1}{\tau}. \] (11)

Here \( \mathbf{E} \) is the electric field strength of the wave acting on the charge carriers in the film; \( \mathbf{p} \) and \( e \) are the impulse and the charge of the electron (hole), respectively; \( f = f_0 + f_1 \) is nonequilibrium charge carrier distribution function; \( f_0 \) is equilibrium Fermi-Dirac distribution function; \( f_1 \) is non-equilibrium correction that occurs under the influence of the external electric field of the wave (the magnetic field of the wave is ignored).

The velocity of \( \mathbf{v} \) electrons (holes) in the conductor is equal to [9]:

\[ v_i = \frac{\partial \varepsilon}{\partial p_i}, \] (12)

where \( i = x, y, z \).

The electric field transverse to the surface away from the plasma resonance is effectively shielded, so the influence of the \( x \)-components of the electric field \( E_x \) on the charge carriers is neglected, in a conductor.

We will use the model of mirror-diffuse reflection of charge carriers from the upper and lower boundaries of the film as boundary conditions for the distribution function \( f \) [12]:

\[ \begin{align*}
  f_1^+(0, v_x, v_y, v_z) &= q_1 f_1^-(0, -v_x, v_y, v_z) \\
  f_1^-(a, -v_x, v_y, v_z) &= q_2 f_1^+(a, v_x, v_y, v_z)
\end{align*} \] (13)

where \( f_1^+ \) and \( f_1^- \) are small deviations from the equilibrium Fermi-Dirac distribution function of electrons (holes) with positive and negative velocity projections on the \( x \)-axis), respectively.

The current density \( \mathbf{j} \) is determined using a non-equilibrium function \( f \) from the kinetic equation (5) as follows [11]

\[ \mathbf{j} = e n \langle \mathbf{v} \rangle = e \int \mathbf{v} \ f \ \frac{2 d^3 p}{h^3} = 2e \ \frac{m_1 m_2 m_3}{h^3} \int \mathbf{v} f_1 \ d^3 v, \] (14)

where \( h \) is Planck’s constant. The \( n \) concentration is defined as [12]

\[ n = \int \ f \ \frac{2 d^3 p}{h^3} = 2 \ \frac{m_1 m_2 m_3}{h^3} \int f_0 \ d^3 v. \] (15)
Figure 2. The dependence of the absorption coefficient $A$ on the dimensionless effective mass $k_{m1}$ for the degenerate case ($e^{-\mu} \gg 1$) (a) and for the non-degenerate case ($e^{-\mu} \gg 1$) (b). Solid curves $k_{m3} = 0.1$, dotted curves $k_{m3} = 1$, point curves $k_{m3} = 10$. Curves 1 - $q_1 = q_2 = 0$, curves 2 - $q_1 = 0.5$, $q_2 = 0.6$, curves 3 - $q_1 = q_2 = 1$.

The average electrical conductivity (10) over the thickness of the film, taking into account (1, 2, 11-15), is equal to

$$\overline{\sigma_{zz}} = \sigma_0 \Sigma(x_0, y_0, q_1, q_2, k_{m1}, k_{m3}, u_\mu),$$  \hspace{1cm} (16)

$$m_0 = \sqrt{m_1 m_2 m_3}, \quad k_{m1} = \frac{m_1}{m_0}, \quad k_{m2} = \frac{m_2}{m_0}, \quad k_{m3} = \frac{m_3}{m_0}, \quad \Omega = \frac{z_0}{v_1} v_1, \quad (17)$$

$$x_0 = \frac{a}{\lambda}, \quad y_0 = \frac{\omega a}{v_1}, \quad z_0 = \frac{va}{v_1} = x_0 \bar{v} - i y_0, \quad u_\mu = \frac{\mu}{k_B T}, \quad u = \frac{\varepsilon}{k_B T}, \quad (18)$$

$$\Sigma = \frac{e^{-\mu_\mu x_0}}{k_{m3} \pi^{3/2} F_{1/2}(u_\mu)} \int_0^\infty \int_0 \int_0 d\phi d\bar{d}du \frac{u^3 e^{u \sin^2(\bar{\varphi})}}{z_0 (1 + e^{u - u_\mu})^2} \times \left\{2 - \frac{1}{\Omega} \left[1 - q_1 q_2 \exp(-2\Omega) \right] \right\}.$$  \hspace{1cm} (19)

Here $\sigma_0$ is static electrical conductivity; $k_{m1}, k_{m2}, k_{m3}$ are dimensionless effective masses; $x_0$ is dimensionless inverse mean free path length; $y_0$ is dimensionless frequency of the electric field; $\lambda$ is free path length of electrons (holes); $v_1$ is the characteristic speed [13], $\mu$ is chemical potential; $k_B$ is Boltzmann constant; $T$ is temperature. Thus, expressions (3-9, 16-19) completely determine the reflection, passage, and absorption coefficients.
4. Analysis of results
We consider the behavior of the absorption coefficient $A$ as a function of the dimensionless effective mass of charge carriers $k_m$ (figure 2) for a specific case. We suppose that an electromagnetic wave falls from a vacuum ($\varepsilon_1 = 1$) on a thin conducting film with a thickness of $a = 10$ nm, located on a glass substrate ($\varepsilon_2 = 5$), at an angle of $\theta = 20^\circ$. The frequency of the electromagnetic wave lies in the far infrared range: $\tilde{f} = 1$ THz. We consider the case when the free path length for the degenerate case is $\lambda = 50$ nm, and for the non-degenerate case it is $\lambda = 500$ nm; the specific static electrical conductivity of a massive sample is $\sigma_0 = 10^{17}$ s$^{-1}$.

5. Conclusion
An analytical expression is obtained for the specific electrical conductivity of a conducting film in the case of an anisotropic dispersion law. The difference in the specularity coefficients of the film surfaces is taken into account. The coefficients of reflection, transmission and absorption, depending on the electrical conductivity of the film, are calculated. The absorption coefficient of the effective mass along the ellipsoid axis perpendicular to the surfaces and directed deep into the film is analyzed. It is shown that the dependence of the absorption coefficient on the effective mass has a maximum, which is shifted towards higher values of the effective mass when the specularity coefficients of the film surfaces increase. The absorption coefficient ceases to depend on the specularity coefficients at high values of effective mass.

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