Aspects of warm-flat directions

Tomohiro Matsuda

Laboratory of Physics, Saitama Institute of Technology,
Fusaiji, Okabe-machi, Saitama 369-0293, Japan

Abstract

Considering the mechanism of dissipative slow-roll that has been used in warm inflation scenario, we show that dissipation may alter usual cosmological scenarios associated with SUSY-flat directions. We mainly consider SUSY-flat directions that have strong interactions with non-flat directions and may cause strong dissipation both in thermal and non-thermal backgrounds. An example is the Affleck-Dine mechanism in which dissipation may create significant (both qualitative and quantitative) discrepancies between the conventional scenario and the dissipative one. We also discuss several mechanisms of generating curvature perturbations in which the dissipative field, which is distinguished from the inflaton field, can be used as the source of cosmological perturbations. Considering the Morikawa-Sasaki dissipative coefficient, the damping caused by the dissipation may be significant for many MSSM flat directions even if the dissipation is far from thermal equilibrium.
1 Introduction

Inflation has been considered as an important cosmological event in the very early Universe. In most models of inflation, a scalar field (inflaton) rolls slowly during inflation and the vacuum energy associated with the inflaton potential is responsible for accelerating the expansion of the Universe. Since the primeval radiation rapidly red-shifts during inflation, the Universe during inflation would be cold. Reheating after inflation is thus required to recover the hot Universe. However, if there are interactions between inflaton and other fields, the radiation background during inflation may not be completely red-shifted away and the Universe may be warm during inflation. The warm scenario is based on the idea that the dissipation sources radiation and raises the temperature during inflation. The scenario with the dissipating inflaton field is known as warm inflation. If the inflaton is dissipative, the equation of motion is given by

$$\ddot{\phi} + 3H(1 + r)\dot{\phi} + V(\phi, T_\phi) = 0,$$

(1.1)

where the subscript denotes the derivative with respect to the field $\phi$. The strength of the damping is measured by the rate $r$ given by the dissipative coefficient $\Upsilon$ and the Hubble parameter $H$;

$$r \equiv \frac{\Upsilon}{3H}.$$  

(1.2)

Here the effective potential of the field $\phi$ may depend on the radiation temperature $T$, and thus the potential is expressed as $V(\phi, T)$. The dissipative coefficient $\Upsilon$ of the inflaton, which is related to the microscopic physics of the interactions, leads to the energy conservation equation;

$$\dot{\rho}_R + 4H\rho_R = \Upsilon \dot{\phi}^2,$$  

(1.3)

where $\rho_R$ is the energy density of the radiation and the right hand side ($\Upsilon \dot{\phi}^2$) represents the source of the radiation. Scenario with $r > 1$ is called “strongly dissipating warm inflation” and the one with $r < 1$ is called “weakly dissipating warm inflation”. A very confusing situation would be that inflaton is slow-rolling because of the strong dissipation, while the radiation redshifts away because the thermalization process cannot catch up with

---

2 The Hawking temperature is intrinsic to de Sitter space. Non-thermal background is considered when the Hawking temperature is not significant for the dissipation mechanism[1][2].
the rapid red-shifting. Although inflation is not “warm” in this peculiar situation, the scenario is sometimes classified into strongly dissipating warm inflation. According to this classification, “warm direction” in this paper involves strongly dissipating field that evolves in a cold ($T \simeq 0$ and $\rho_R \simeq 0$) Universe.

If we consider the dissipative field equation, the effective slow-roll parameters should be different from the conventional ones. They are given by

$$\epsilon_w \equiv \frac{\epsilon}{(1 + r)^2},$$
$$\eta_w \equiv \frac{\eta}{(1 + r)^2},$$

where the usual slow-roll parameters ($\epsilon$ and $\eta$) are defined by

$$\epsilon \equiv \frac{M_p^2}{2} \left( \frac{V_{/\phi}}{V} \right)^2,$$
$$\eta \equiv \frac{M_p^2 V_{/\phi \phi}}{V}.$$  \tag{1.5}

Here the subscript denotes the derivative with respect to the dissipative field $\phi$.

To quantify the dissipative coefficient, it would be useful to list some past results in different models. The dissipative coefficient may depend on the temperature $T$ and the expectation value of the inflaton field. Considering interactions given by the superpotential

$$W = g_1 \Phi X^2 + g_2 XY^2,$$  \tag{1.6}

where $g_1$ and $g_2$ are coupling constants, and $\Phi, X, Y$ are superfields whose scalar components are given by $\phi, x, y$. Fermionic partners of the scalar components $\phi, x, y$ are $\psi_\phi, \psi_x, \psi_y$. During inflation, when $\phi$ is large, the field $y$ and its fermionic partner $\psi_y$ are massless in the global supersymmetric limit. The mediating field is $x$, which obtains large mass $m_x \sim g_1 \phi$ from the interaction. In this model the dissipation is caused by the excitation of the mediating field $x$ that decays into massless fermions. At high temperature ($m_x \ll T$) the dissipative coefficient is given by $\Upsilon \propto (g_1^2/g_2^2)T$, while at low temperature ($m_x \gg T$) the coefficient is given by $\Upsilon \propto g_1^2(T^3/\phi^2)$. In addition to the dissipation in thermal background, there may be significant dissipation in non-thermal background. In fact, considering the basic mechanism of black-hole radiation in the time-dependent background, it is straightforward to find that the similar mechanism leads to the coherent
excitation of the mediating field, which appears in the zero-temperature Universe and contributes dissipation due to the time-dependent mass caused by $\dot{\phi} \neq 0$. Dissipation in the non-thermal background was studied by Morikawa and Sasaki\cite{1}, and later it was found that in the presence of the mediating field the dissipation gives $\Upsilon \propto g_1^2 g_2^2 \phi$ for $T = 0\cite{1}$. In this paper we consider $\Upsilon \simeq C_T T^3/\phi^2$ for the dissipation after reheating (low $T$, in the region where Morikawa-Sasaki coefficient is negligible) and $\Upsilon \propto \phi$ for the dissipation during inflation (zero $T$, when Morikawa-Sasaki coefficient is significant).

In past studies the dissipative slow-roll has been considered for warm inflation. However, a natural expectation is that similar phenomenon (dissipative slow-roll) may be observed in the evolution of other scalar fields if there are many scalar fields in the theory. Based on this simple idea, we address the generic situation where many scalar fields in the early Universe dissipate their energy during or after inflation. Thermalization condition of the decay products would be a serious problem if we discuss warm background during inflation; however it is not important in just solving the slow-roll problem using the dissipative motion in non-thermal background. Thermalization of the decay products during inflation (which is required for the warm background) would enhance the dissipative coefficient, but we modestly put an assumption that the thermalization is not significant during inflation\cite{3}. Some implications beyond this assumption will be discussed in this paper for the enhancement of the field fluctuations for $T > H$ during inflation.

The most important topic in this paper is the dissipative motion of a scalar field that is not identified with the inflaton field. For the scalar field, we consider SUSY-flat direction. First note that supersymmetric gauge theories appear with gauge invariant polynomials along which the classical scalar potential in the global supersymmetric limit vanishes. These polynomials are related to the so-called flat directions of the supersymmetric theory. Flatness of the direction is not exact if supersymmetry is broken by soft terms, or non-perturbative corrections lift the potential. Within the minimal supersymmetric standard model (MSSM), the classical scalar potential is given by the sum of the F and D term contributions. Although the MSSM flat directions are usually parameterized by gauge

\footnote{Analytic calculations of dissipation are usually performed in the region $\dot{\phi}/\phi < H < \Gamma_d$, where $\Gamma_d$ is the decay rate of the mediating field. Dissipation beyond the above condition (e.g., preheating) may be important, but it occurs when the slow-roll conditions are violated\cite{4,5}.}
invariant monomials of superfields, each scalar component in the monomial has charges and breaks gauge symmetries by its vacuum expectation value (VEV). A generic argument of the MSSM flat directions is obviously beyond the scope of this paper, however a simple example would be useful to gain some insight into the dynamics related to the MSSM flat directions. We thus consider the \( H_u L \equiv \epsilon_{\alpha\beta} H_u^\alpha L^\beta \) flat direction, which is a polynomial (see the right hand side) given by the superfield of a Higgs field \( (H_u) \) and the superfield of a lepton \( (L) \) in the MSSM action. The F-terms are obtained from the MSSM-superpotential

\[
W = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e} + \mu_H H_u H_d,
\]

where the last term is the \( \mu \)-term. The \( \mu \)-term is usually disregarded in the calculation of the flat directions because \( \mu_H \) is of the order of the soft mass. The flat direction associated with \( H_u L \) is

\[
H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \quad L = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix},
\]

where \( \phi \) is a complex scalar field that parameterizes the flat direction. Note that the flat direction interacts with other scalar fields, gauge bosons and fermions. Note also that \( \phi \neq 0 \) gives mass to other directions through Yukawa couplings in the superpotential, as well as to gauge bosons. The neutrino mass is not explicit in the MSSM action, but it would be natural to expect that neutrinos are effectively massless in the early Universe. Besides neutrinos, there would be fermions to which mediating field (scalar fields or gauge bosons that obtain mass from the direction \( \phi \neq 0 \)) can decay.\(^4\)

A very confusing property of the flat directions would be that they are not flat in the early Universe. The flat directions, which are flat in the global supersymmetric limit, will obtain \( O(H) \) mass from the supergravity corrections. This is the famous \( \eta \)-problem that appears in supersymmetric inflationary models. It would be possible to forbid such corrections introducing symmetry, but our modest assumption is that flat directions are not flat due to the supergravity corrections.

Any field in the early Universe has two distinctive types of evolution. Non-dissipating evolution has been studied by many authors for many applications. Significant results

\(^4\) A more interesting case was discussed in ref.\(^{29}\), where the gravitational decay into gravitinos is efficient for the dissipative slow-roll in a supersymmetric hybrid inflation model.
have been obtained for the Affleck-Dine mechanism [6, 7, 8] and the formation of Q-balls [9, 10] using non-dissipating directions. Unfortunately, dissipative slow-roll has not been studied for such models. In addition to the Affleck-Dine mechanism and Q-balls, dissipative motion may be important for models of generating curvature perturbations, even if the dissipative field is not the inflaton field. In fact, the creation of the curvature perturbations may be due to entropy perturbations of a light field that is distinguished from the inflaton. Such mechanism can be associated with fluctuations of the phase transition [11, 12, 13, 14] or to fluctuations in the ratio of the energy densities [15, 16, 17]. The curvature perturbations may be generated from fluctuations of the inflaton velocity caused by entropy perturbations, which in terms of the $\delta N$ formalism [18, 19] explains the creation of the curvature perturbations at the “bend” of the trajectory. The effect of warm entropy field has not been studied for the evolution of the curvature perturbations, except for a multi-field model discussed in Ref. [22]. Taking into account the dissipative motion of these scalar fields would alter the cosmological consequences of the scenarios. Since cataloging these applications is not appropriate for the purpose of this paper, we first explore the typical situation of the Affleck-Dine mechanism using the dissipative MSSM-flat direction, and then discuss briefly other possible applications of the warm scenario.

2 Warm directions

2.1 Dissipative AD field for the AD baryogenesis

Evolution of a flat direction depends on the shape of the potential. We consider the typical form of the potential:

$$V(\phi) = V_I + \frac{c_s}{2} m_s^2 |\phi|^2 + \frac{c_H}{2} H^2 |\phi|^2 + \left( \frac{\lambda_s m_s + \lambda_n H}{n M_p^{n-3}} \phi^n + h.c. \right) + |\lambda_n|^2 \frac{|\phi|^{2n-2}}{M_p^{2n-6}},$$

(2.1)

where $m_s$ denotes the supersymmetry-breaking mass and $c$ and $\lambda$ with subscripts are dimensionless parameters. As we are considering conventional flat directions, $n$ is an

---

5The adiabatic inflaton field is defined for multi-field inflation so that it points the direction along the inflaton motion. Entropy fields are defined to point the directions transverse to the adiabatic field.
integer greater than 4. The first term $V_I$ denotes the inflaton potential. The last three terms originate from the typical supergravity action.

Let us first consider evolution of the AD field without strong dissipation. Following the conventional argument, we consider potential with $c_H < 0$ and $H \gg m_s$ during inflation. Then the instability near the origin leads to a global minimum at

$$|\phi_0| \sim \left| \frac{c_H H M_p^{n-3}}{\lambda_n} \right|^{1/(n-2)}.$$  \hspace{1cm} (2.2)

Disregarding the dissipation, the field will rapidly settle down to a minimum at $\phi = \phi_0$. The fluctuations of the field are negligible for $c_H \sim 1$, since the amplitude of the fluctuations will quickly fall off. The motion in the $U(1)$ direction, which is associated with rotation of the complex scalar, is determined by the explicit breaking term, which appears as the so-called A-term caused by $\lambda_n H \neq 0$. The A-term leads to a fast motion in the $U(1)$ direction, with a gap in the phase between the $H$-induced vacuum and the $m_s$-induced one. Eventually, the gap leads to rotational motion that is required for the baryogenesis[20]. Assuming that inflaton decays when $H < m_{3/2}$, where $m_{3/2}$ is the gravitino mass, the baryon to entropy ratio subsequent to reheating is given by[20]

$$\frac{n_b}{s} \sim \frac{n_b T_R \rho_\phi}{n_\phi m_\phi \rho_I},$$  \hspace{1cm} (2.3)

where $T_R$ and $\rho_I$ are the reheating temperature and the inflaton mass density, $n_\phi$ and $n_b$ are AD field and baryon number densities. Here the energy densities are estimated at the time of inflaton decay. The energy density of the AD field ($\rho_\phi$) is calculated from the potential $V(\phi)$. Typical value of $n_b/n_\phi$ is $O(0.1)$, which has been obtained in Ref.[20] considering the field equation for the AD field $\phi$ and integrating over the angle parameter. If the AD field is non-dissipative, the “initial” condition of the AD field is given by $\phi = \phi_0$, while for a dissipative field the mechanism that determines the “initial” condition is quite different. The difference that appears in the “initial” condition is the main result of this section.

When we consider the dissipation of the field motion, we must consider two distinctive cases for the AD-field: dissipation of the angular motion and the radial motion. The dissipation of the angular motion has been discussed for the so-called “longevity of the AD field”. In fact, non-perturbative decay of the rotating field has been discussed by
many authors\cite{21}. When the effective “slow-roll” in the radial direction is simply due to the rotational motion, the field starts to roll quickly as soon as the angular velocity decreases through dissipation. This situation is similar to a satellite losing their energy due to the air resistance, and it is in contrast to the dissipative slow-roll scenario. In contrast to the “slow-roll” due to the angular motion, the dissipative slow-roll, which is considered in this paper, is possible even if the rotational motion is strongly dissipative.\footnote{Baryogenesis will be suppressed if the angular motion is highly dissipative\cite{21}. We are not arguing this point.}

In the presence of strong \((r > 1)\) dissipation, the scenario of the Affleck-Dine baryogenesis may be quite different. The AD field may roll slowly in spite of the supergravity corrections, and its fluctuations may become significant. To understand the dissipative slow-roll of the Affleck-Dine field during inflation, we follow the typical treatment of the warm inflationary scenario\cite{2,22}, since the equations describing the AD field are completely the same as the warm inflationary scenario. We note again that the direction associated with the field motion is flat at the tree level but obtains \(O(H)\) mass due to the supersymmetry-breaking and the supergravity corrections. The field motion is dissipative, but the field cannot cause warm inflation by itself. The AD field is distinguished from the inflaton field. We consider evolution of the AD field both during and after inflation. The simplest situation is that the background radiation is negligible \((T \simeq 0\) and \(\rho_R \simeq 0)\) during inflation, but the radiation is not negligible after reheating. Therefore, the dissipative coefficient may be enhanced after reheating. We first consider the simplest situation in which dissipation during inflation occurs in non-thermal background.

For the typical interaction of a supersymmetric flat direction we consider the interaction Lagrangian

\[
\mathcal{L}_I = -\frac{1}{2} g^2 \phi^2 \chi^2 - h \chi \bar{\psi} \psi,
\]

where \(\chi\) is the scalar field (mediating field) that obtains large mass \(m_\chi \sim g \phi\) from the flat direction \(\phi\), and \(\psi\) is a matter that satisfies the condition \(m_\chi > 2 m_\psi\) for the decay \(\chi \rightarrow 2\psi\).

During inflation, when \(T = 0\), calculation of the dissipative slow-roll parameters is straightforward. Far from the origin, terms proportional to \(\phi^n\) dominate the potential. Then, using \(V_I \sim H^2 M_p^2\) and \(m_\psi^2 \ll H^2\) during inflation, the slow-roll parameters are
The dissipative coefficient in the non-thermal background, the dissipative rate during inflation is given by

\[ r \sim 10^{-2} N_\chi N_\psi g^3 h^2 \frac{\phi}{H}, \]  

where \( N_\chi \) and \( N_\psi \) are the effective numbers of the mediating field and the light fermions.

The effective slow-roll conditions give

\[ \epsilon_w < 1 \rightarrow \phi < \phi_\epsilon \equiv M_p \times \left( \frac{10^{-2} N_\chi N_\psi g^3 h^2}{\lambda_n} \right)^{1/(n-2)}, \]
\[ \eta_w < 1 \rightarrow \begin{cases} \phi < \phi_\eta \equiv M_p \left[ \left( \frac{H}{M_p} \right) \frac{10^{-4} N_\chi^2 N_\psi^2 g^6 h^4}{\lambda_n (n-1)} \right]^{1/(n-4)} \left( n > 4 \right), \\ H < M_p \times \frac{10^{-4} N_\chi^2 N_\psi^2 g^6 h^4}{\lambda_n (n-1)} \left( n = 4 \right). \end{cases} \]

where we considered strong dissipation \( r > 1 \). The strong dissipation requires

\[ \phi > H \left( \frac{10^2}{N_\chi N_\psi g^3 h^2} \right). \]

From the above equations, we find that the dissipative slow-roll is common for the AD field if the initial condition is chaotic (\( \phi \sim M_p \)) and the interaction coefficients (\( g \) and \( h \)) are not suppressed.

Our most important result is that the initial amplitude of the AD field is not always determined solely by the scalar potential. For the dissipative direction, the amplitude of the flat direction is determined by (1) the scalar potential, (2) the chaotic initial condition and (3) the interactions of the component scalar fields. Note that only (1) is important for determining the initial condition in the usual scenario.

After inflation and reheating, the dissipation may be enhanced by the thermal background. We thus consider the dissipative coefficient after reheating:

\[ \Upsilon = C_T \frac{T^3}{\phi^2}, \]  

See appendix.
which leads to strong dissipative region \( (r > 1) \) given by
\[
\phi < \sqrt{C_T M_p T},
\]
(2.11)
where \( H \sim T^2 / M_p \) is assumed. Interestingly, the result shows that even if the Morikawa-Sasaki coefficient is not significant (i.e. when the “initial” condition before reheating is given by \( \phi = \phi_0 \)), AD field that starts oscillation from \( \phi = \phi_0 \) can be trapped at \( \phi \sim \sqrt{C_T M_p T} \) due to the frictional force caused by the strong dissipation. Note that rotational oscillation of the AD field is important for the Affleck-Dine baryogenesis, which suggests that the oscillation does not hit the origin \( (|\phi| = 0) \) but there is a minimum of \( |\phi| \) for the oscillation. Our result shows that the minimum must satisfy the condition \( |\phi|_{\text{min}} > \sqrt{C_T M_p T} \) during the rotational oscillation, which seems very severe for the conventional scenario, in which the initial amplitude is given by \( \phi_0 \sim \left( c_H H M_p^{n-3} / \lambda_n \right)^{1/(n-2)} \). In fact, \( |\phi_0| \gg |\phi|_{\text{min}} \) gives significant condition for the potential with \( n < 6 \).

If the Morikawa-Sasaki coefficient is large \( (r \gg 1) \) during inflation, the field rolls slowly. The dissipation may lead to the creation of isocurvature perturbations caused by the AD field. In fact, since the AD field satisfies the dissipative slow-roll conditions during inflation, the perturbations of the AD field can exit the horizon. To understand the significance of the perturbations, it will be important to consider (temporarily) “warm” \( (T > H) \) background during inflation. Following the standard argument of warm inflation, the root-mean square fluctuation amplitude of the light field \( \delta \phi \) after the freeze out is obtained to be
\[
\delta \phi_{\Upsilon > H} \sim (\Upsilon H)^{1/4} T^{1/2} \sim r^{1/4} T^{1/2} H \quad (\Upsilon > H),
\]
(2.12)
\[
\delta \phi_{\Upsilon < H} \sim (HT)^{1/2} \sim r^{1/2} T^{1/2} H \quad (\Upsilon < H)
\]
(2.13)
where \( r_T \) denotes the ratio defined by \( r_T \equiv T / H \). The fluctuation is enhanced for \( r_T > 1 \) and \( r > 1 \). Note that in the standard scenario of the Affleck-Dine mechanism the fluctuation is negligible \( (\delta \phi = 0) \) due to the supergravity corrections that lead to \( \eta \sim 1 \) during

\small
\footnote{Our modest assumption for the dissipative motion after reheating is that the thermalization of the dissipative decay product is not significant and it does not become a significant source of radiation after reheating. The evolution of the temperature \( T \) and the radiation energy density \( \rho_R \) is thus given by the conventional cosmological equations.}

\small
\footnote{Following previous studies of the warm inflation scenario\cite{[1][2][3]}, strong dissipation would be robust for SUSY flat directions.}
inflation. The discrepancy is important for the study of the isocurvature perturbations related to the Affleck-Dine baryogenesis\cite{8, 17}.

2.2 Warm directions for curvature perturbations

In the previous section we showed that the baryon number asymmetry of the Universe can be associated with the dissipative field. Next we consider models of generating curvature perturbations in which the seed of the cosmological fluctuation is associated with the dissipative field that is distinguished from the inflaton field.

2.2.1 Curvatons

Assume that there are many components in the Universe, such as vacuum energy, matter, oscillating field or radiation. If the ratio of these components in the energy density is not homogeneous in space, and their scaling rules are distinguishable, density perturbations are generated during cosmological evolution. Assume that there is an inhomogeneity related to the density of the field $\varphi$, and the inhomogeneity is caused by the cosmological fluctuation of the field $\delta \varphi \neq 0$. Then, one can define the ratio $r_\varphi \equiv \rho_\varphi / \rho_{\text{total}}$ and the inhomogeneity $\delta r_\varphi \equiv \delta \rho_\varphi / \rho_{\text{total}}$. Note that initially ($t = 0$) the total energy density is homogeneous ($\delta \rho_{\text{total}}(0) \equiv 0$). We are considering isocurvature perturbations at $t = 0$.

The perturbation caused by the fluctuation of the field $\varphi$ ($\delta \rho_\varphi / \rho_{\text{total}} \sim r_\varphi \times (\delta r_\varphi / r_\varphi)$) may be suppressed when $r_\varphi \ll 1$. However, if $\rho_\varphi$ scales milder than other components of the Universe, the ratio $r_\varphi$ may grow and may finally reach $r_\varphi \sim 1$ after a time period.

The typical example is curvatons, whose ratio grows after inflation and finally dominates the density of the Universe before decay. Usually the curvaton scales as a matter during curvaton oscillation, but in some extended models the curvatons can be associated with topological defects\cite{17} or hybrid states\cite{27} that do not scale as a matter. In this section we discuss whether the warm scenario can accommodate the usual curvaton scenario.

Assume that the curvaton $\varphi$ is identified with the warm-flat direction, then the amplitude of the fluctuation $\delta \varphi$ will be enhanced if the warm background is significant ($r_T \gg 1$). This causes enhancement of the perturbations, as in the case of the Affleck-Dine field. If the dissipation is strong for the curvatons, the slow-roll period will be extended because of
the large $r$ in the field equation. This enhances the “initial” ratio of the curvaton density at the beginning of the curvaton oscillation, since the energy density of the curvaton is a constant before oscillation. These effects enhance both $r_\varphi$ and $\delta r_\varphi$ at the beginning of the oscillation helping curvatons generate the curvature perturbations. However, in contrast to the situation before the oscillation, the evolution during the curvaton oscillation may cause a serious problem. For the strongly dissipating curvatons we have to consider a field $\varphi$ with interactions between mediating field, where the dissipation is significant only when the mediating field decays into light matter. Since the typical interaction required for the strong dissipation is nothing but the interaction required for instant preheating followed by instant reheating\cite{25}, the first oscillation will lead to efficient production of the mediating field near the origin and subsequently the instant decay\cite{10}. As a result, strongly dissipating curvatons cannot oscillate for a long time. The curvaton density cannot grow during the oscillation. Since the oscillation of the dissipative curvaton cannot last long, the ratio $r_\varphi$ associated with the warm curvaton cannot grow during oscillation\cite{11}. To understand the situation, we consider an optimistic case in which strong dissipation ($r \gg 1$) delays the curvaton oscillation until $H_{osc} \simeq r^{-1}m_\varphi$. The ratio at the beginning of the oscillation is given by

$$r_\varphi \simeq \frac{m_\varphi^2 \varphi^2}{H_{osc}^2 m_p^2} \simeq r^2 \frac{\varphi^2}{M_p^2},$$  \hspace{1cm} (2.14)

which suggests that $r_\varphi \sim 1$ can be reached before the oscillation. The curvaton decays before cosmological scales start to enter the horizon. The curvature perturbation is given by

$$\zeta = r_\varphi \zeta_\sigma \equiv r_\varphi \frac{1}{3} \frac{\delta \rho_\varphi}{\rho_\varphi}.$$  \hspace{1cm} (2.15)

The spectrum of the curvature perturbation is given by

$$P_{\zeta}^{1/2} = 2r_\varphi \frac{\delta \varphi_I}{\varphi_I},$$  \hspace{1cm} (2.16)

where the subscript $I$ means that quantities are to be estimated during inflation. In the above equation we assumed that the curvaton potential is quadratic: $V(\sigma) \simeq m_\varphi^2 \varphi^2$. In

\footnote{We will discuss this issue later in sec.2.2.3 for the inhomogeneous preheating scenario.}

\footnote{Heavy curvaton accompanied by phase transition\cite{16} may avoid this problem. Efficient production of stable Q-balls\cite{17} may also avoid this problem\cite{26}. These exceptional cases are not excluded but should be discussed elsewhere.}
order to generate the curvature perturbations of the Universe, the curvaton fluctuation must (at least) satisfy the bound given by

\[ r \varphi \frac{\delta \varphi}{\varphi} \simeq r \varphi \left[ r^{1/4} r_T^{3/2} \frac{H}{\varphi} \right] \sim 10^{-5}, \tag{2.17} \]

where \( T \) represents the temperature when the fluctuation exits horizon. Combining Eq. (2.14) and (2.17), and assuming that \( \varphi \) is constant until oscillation begins, we find

\[ \varphi \sim M_p \frac{M_p \times 10^{-5}}{H_I} r^{-9/4} r_T^{-1/2} \tag{2.18} \]

which suggests that warm curvatons may accommodate MSSM flat directions even if it decays fast at the first oscillation. The result is in contrast with the usual MSSM-curvaton model[26].

### 2.2.2 Inhomogeneous phase transition

In this section we consider an inhomogeneous phase transition. The \( i \)-th cosmological period (we call this period a “phase” of the Universe) can be characterized by the scaling property of the dominant component of the energy density \( \rho_{dom} \propto a^{-n_i} \). At the boundary of the “phases” in the time-direction, a gap \( \Delta n_i \equiv n_{i+1} - n_i \) appears. The local delay of the “phase transition” is a natural source of density perturbations[11]. There are at least three distinctive mechanisms of this kind:

1. “Generation of curvature perturbations at the end of inflation[12, 13, 14]” occurs for a change in \( n_i \) from \( n_i = 0 \) to \( n_{i+1} = 3 \) (inflaton decays into matter) or to \( n_{i+1} = 4 \) (inflaton immediately decays into radiation).

2. “Inhomogeneous reheating” occurs for a change in \( n_i \) from \( n_i = 3 \) to \( n_{i+1} = 4 \) (matter decays into radiation).

3. “Inhomogeneous phase transition” is possible for the conventional phase transition in the early Universe[11].

The phase boundary is inhomogeneous if the field value (\( \varphi \)) that determines the boundary is inhomogeneous (\( \delta \varphi \neq 0 \)). For instance, consider the potential for hybrid inflation given by

\[ V = V_0 + \frac{1}{2} m_\varphi^2 \varphi^2 - \frac{1}{2} m_\chi^2 \chi^2 + \frac{1}{4} \lambda \chi^4 + \frac{1}{2} \lambda_\varphi \varphi^2 \chi^2 + \frac{1}{2} \lambda_\varphi \varphi^2 \chi^2 + V_2(\varphi), \tag{2.19} \]
where $\phi$ and $\chi$ are the inflaton and the waterfall field. This is the same as the original hybrid model except for the additional light field $\varphi$. Here $\varphi$ is distinguished from the inflaton field. Assuming that $\varphi$ is light during inflation, inflation ends when $\chi$ is destabilized at $\phi = \phi_c$, which corresponds to

$$\lambda_\phi \phi_c^2 + \lambda_\varphi \varphi^2 = m_\chi^2. \quad (2.20)$$

In this case the fluctuation $\delta \varphi$ associated with the additional light field $\varphi$ causes inhomogeneous end of hybrid inflation and leads to $\delta N \neq 0$. This is the basic mechanisms for generating curvature perturbations at the end of inflation. The mechanisms 1. 2. 3. will work with warm scenario in which $\varphi$ (the field that sources cosmological perturbation) is identified with the warm direction. $\varphi$ may be a moduli in the string theory.

Compared with the usual (non-dissipative) scenario, the most significant discrepancy will appear in the slow-roll condition and the amplitude of the field fluctuations. Namely, if the dissipation is strong for the field ($r \gg 1$), the $O(H^2)$ mass-term correction from the supergravity interactions does not cause the $\eta$ problem for the field $\varphi$. This is the first significant advantage of the warm scenario. Moreover, if the warm background is significant for the field, the amplitude of the field fluctuations ($\delta \varphi$) will be enhanced. This is the second advantage of the warm scenario. As a consequence, the curvature perturbations caused by the warm direction may depend on the dissipative coefficient $\Upsilon$ (i.e. it depends not only on the scalar potential but also on the interactions) and the temperature.

As far as one is considering the usual non-dissipative scenario, directions that are not protected from the $O(H^2)$ supergravity corrections are not considered as the source of the cosmological perturbations. We claim that what is important in solving the $\eta$ problem is not the conventional mass protection but the interactions of the field that cause dissipative motion. In fact, there can be many flat directions that are not protected from $O(H)$ corrections but can roll slowly due to the strong dissipation caused by the interactions. Moreover, considering warm background during inflation, the amplitudes of the field fluctuations may not be uniform but characterized by their interactions. Therefore, the cosmological perturbations sourced by these warm directions are potentially very important for analyzing the interactions of the underlying theory. In contrast to the usual
non-dissipative scenario, the cosmological perturbations in the warm scenario depend not only on the potential but also on the interactions of the field.

2.2.3 Inhomogeneous number density from preheating

Preheating uses typical interaction given by

\[ \mathcal{L}_{int} \simeq \frac{g_{PR}^2}{2} \phi^2 \varphi^2 \]  \hspace{1cm} (2.21)

and oscillation of the field \( \phi \). Then the interaction leads to efficient production of \( \varphi \) near \( \phi = 0 \). Assuming so-called instant preheating scenario, the number density of \( \varphi \) created by preheating is given by

\[ n_{\varphi} = \frac{\left( g_{PR} |\dot{\phi}(t_*)| \right)^{3/2}}{8\pi^3} \exp \left[ -\frac{\pi m_{\varphi}^2}{g_{PR} |\dot{\phi}(t_*)|} \right], \]  \hspace{1cm} (2.22)

where \( \phi \) is the oscillating field and \( t_* \) denotes the time when \( \phi \) reaches its minimum at \( \phi = 0 \). The mass \( (m_{\varphi}) \) of the preheat field \( \varphi \) is assumed to be very small near the enhanced symmetric point (ESP) at \( \phi = 0 \). In order to create inhomogeneities in the number density of the preheat field \( \varphi \), the mass of the preheat field must be inhomogeneous in space. Typically, a light field \( \phi_l \) is introduced in addition to the oscillating field \( \phi \), with the interaction \( \sim g_{PR}^2 \phi^2 (\phi^2 + \phi_l^2) \). Then, the mass of the preheat field is given by

\[ m_{\varphi}^2 \simeq g_{PR}^2 (\phi^2 + \phi_l^2), \]  \hspace{1cm} (2.23)

which is inhomogeneous for \( \delta \phi_l \neq 0 \) at the ESP(\( \phi = 0 \)).

The light field \( \phi_l \) can be identified with dissipative direction, if it has interactions with mediating fields \( \chi_i \) that decay into light particles. The significant dissipation of the field \( \phi_l \) does not cause serious problem to the original inhomogeneous preheating model. As a consequence, our warm-flat scenario accommodates inhomogeneous preheating, solving the \( \eta \)-problem for the field \( \phi_l \) and enhancing the amplitude of the perturbations \( \delta \phi_l \) when inflation is warm.

In the above discussion we showed that dissipative direction \( \phi_l \) accommodates inhomogeneous preheating scenario. However, if the instant decay after preheating is assumed for the \( \phi \)-field oscillation, which is the case in the first model of inhomogeneous preheating proposed in Ref.\[5\], the dissipative effect may not be negligible for the motion of
the oscillating field \( \phi \). In fact, if the instant decay is assumed for the preheat field \( \varphi \), \( \varphi \) can play the role of the mediating field. Then, significant dissipation is expected before preheating, which typically leads to the delay of the \( \phi \) oscillation. To show explicitly the dissipative effect that may be significant for \( \phi \), we consider the potential with mass hierarchy \((m_l \ll m)\):
\[
V(\phi, \phi_l) = \frac{1}{2}m_l^2 \phi_l^2 + \frac{1}{2}m^2 \phi^2,
\]
(2.24)
and the dissipation based on the zero-temperature approximation. The interaction with the preheat field is given by
\[
\mathcal{L}_{\text{int}} \simeq \frac{g^2 P_R}{2} \phi^2 \varphi^2,
\]
(2.25)
where the preheat field \( \varphi \) causes dissipation. The decay into light fermions, which is needed for the preheating scenario followed by the instant decay, is induced by the term
\[
\mathcal{L}_{\psi \varphi} = h \phi \bar{\psi} \psi.
\]
(2.26)
Obviously, for \( g \sim h \sim O(1) \), the dissipative process \( \phi \to \varphi \to \psi \) is efficient for the model. Typically, the dissipative coefficient in the zero-temperature approximation is given by
\[
\Gamma \simeq 10^{-2} g_{PR}^3 h^2 m_{\psi}.
\]
(2.27)
Then the effective slow-roll conditions for the field \( \phi \) are given by
\[
\epsilon_w \simeq M_p^2 \left( \frac{m^2 \phi}{V} \right)^2 \frac{1}{(1 + r)^2} \sim \left( \frac{m}{10^{-2} \times \phi g_{PR}^3 h^2} \right)^2 < 1
\]
(2.28)
\[
\eta_w \simeq \frac{m^2}{H^2 (1 + r)^2} \sim \frac{m^2}{10^{-4} \times \phi^2 g_{PR}^6 h^4} < 1,
\]
(2.29)
which lead to simple condition given by \( \phi > 10^2 \times m/g_{PR}^3 h^2 \), where the motion of the field \( \phi \) is slow because of the dissipation caused by the preheat field. Therefore, the requirement for instant preheating followed by instant reheating leads to a new condition for the model, which affects the initial condition of the oscillation. Namely, the amplitude of the \( \phi \)-field oscillation is not given by the usual condition \( \phi \simeq M_p \), but by the new condition \( \phi \simeq \text{Min}[10^2 \times m/g_{PR}^3 h^2, M_p] \) that is obtained considering the dissipative effect caused by the preheat field \( \varphi \).
3 Conclusions and discussions

In this paper we considered cosmological scenarios associated with the dissipative scalar fields that are not identified with the inflaton field. The strong dissipation \((r > 1)\) solves the \(\eta\)-problem associated with the supergravity interactions. Field fluctuations of the dissipative field may be enhanced by the warm background during inflation. We investigated the possibility that these effects may alter the usual cosmological scenarios associated with SUSY-flat directions. For an obvious example we first considered the Affleck-Dine mechanism, then briefly discussed several mechanisms of generating curvature perturbations in which the dissipative field sources the cosmological perturbations. Our point is that in generic situations there are many flat directions that are not protected from supergravity corrections but can roll slowly due to the strong dissipation caused by interactions with non-flat directions. The dissipative field is not the inflaton field, but can source cosmological perturbations. Since the cosmological perturbations created by the dissipative field are characterized by their interactions, they are potentially very important for analyzing the interactions of the underlying theory using cosmological observations.

4 Acknowledgment

We wish to thank K.Shima for encouragement, and our colleagues at Tokyo University for their kind hospitality.

A Calculation of the dissipative coefficient at zero temperature

A key mechanism of dissipative motion, which is generic in realistic flat direction involves the scalar flaton \(\phi\) exciting a heavy bosonic field (mediating field) \(\chi\) which decays to light fermions \(\psi_d\). This mechanism is expressed by the typical interaction Lagrangian, which is given by

\[
\mathcal{L}_{int} = -\frac{1}{2}g^2\phi^2\chi^2 - h\chi\bar{\psi}\psi, \tag{A.1}
\]
where $\psi_d$ are light fermions to which the mediating field $\chi$ can decay (i.e. $m_\chi > 2m_\psi$ at the decay). For typical MSSM directions, the mediating fields $\chi$ are the heavy gauge bosons, Higgses or scalar supersymmetric partners that obtain large mass from $\phi$. The light fermions are neutrinos (which is massless in MSSM), fermion supersymmetric partners of flat directions or any other particles with the mass satisfying $m_\chi > 2m_\psi$.

A significant example\[1\] leading to a non-trivial flaton dissipation is for $m_\chi > 2m_\psi$ and $m_\chi > m_\phi$, where $\chi \rightarrow 2\psi$ is possible but the direct decay “$\phi \rightarrow$other” is not allowed. Nevertheless, it is not difficult to imagine the physics underlying the dissipation of the field $\phi$. It is useful to look into these processes involving $\phi$ that have an imaginary term associated to dissipation. The imaginary term is interpreted in terms of an effective theory for $\phi$, which appears after integrating over other fields. First integrating $\psi$ that only couples to $\chi$, the $\chi$ propagator is dressed, which has a real part (a shift in the mass) and an imaginary part (rate of the decay). Then integrating over the dressed $\chi$, the relevant contributions to the dissipative mechanism appear from the decay of $\chi$\[12\]. For the decay rate

$$\Gamma_\chi \simeq \frac{N_\psi}{8\pi} h^2 m_\chi \simeq \frac{N_\psi}{8\pi} h^2 g \phi,$$

(A.2)

where $N_\psi$ is the number of the light fermions, the dissipative coefficient is given by\[1\]

$$\Gamma \sim N_\chi \sqrt{2} g^3 N_\psi h^2 \phi \frac{1}{8^3 \pi^2},$$

(A.3)

where $N_\chi$ is the number of the mediating fields\[13\].

References

[1] M. Morikawa and M. Sasaki, “Entropy Production In The Inflationary Universe,” Prog. Theor. Phys. 72, 782 (1984). A. Berera and R. O. Ramos, “Dynamics of interacting scalar fields in expanding space-time,” Phys. Rev. D 71, 023513 (2005) [arXiv:hep-ph/0406339]. M. Bastero-Gil and A. Berera, “Sneutrino warm-inflation in the minimal supersymmetric model,” Phys. Rev. D 72, 103526 (2005)

\[12\] The original calculation given by Morikawa and Sasaki\[1\] is different from the above description. Their calculation is based on the Bogoliubov transformation with the time-dependent background $\dot{\phi} \neq 0$ and the quartic interaction $\propto \phi^4$. The mediating field is not considered in the original calculation.

\[13\] See ref.\[1\] for details of the calculation.
[2] A. Berera, “Warm Inflation,” Phys. Rev. Lett. 75, 3218 (1995) [arXiv:astro-ph/9509049]. A. Berera, “The warm inflationary universe,” Contemp. Phys. 47, 33 (2006) [arXiv:0809.4198 [hep-ph]].

[3] I. G. Moss and C. Xiong, “Dissipation coefficients for supersymmetric inflatonary models,” arXiv:hep-ph/0603266.

[4] L. Kofman, A. D. Linde and A. A. Starobinsky, Towards the theory of reheating after inflation, Phys.Rev.D56(1997)3258-3295,1997 [hep-ph/9704452].

[5] E. W. Kolb, A. Riotto and A. Vallinotto, “Curvature perturbations from broken symmetries,” Phys. Rev. D 71, 043513 (2005) [arXiv:astro-ph/0410546]; T. Matsuda, “Generating the curvature perturbation with instant preheating,” JCAP 0703, 003 (2007) [arXiv:hep-th/0610232]; T. Matsuda, “Generating curvature perturbations with MSSM flat directions,” JCAP 0706, 029 (2007) [arXiv:hep-ph/0701024].

[6] I. Affleck and M. Dine, “A New Mechanism For Baryogenesis,” Nucl. Phys. B 249, 361 (1985).

[7] M. Postma and A. Mazumdar, “Resonant decay of flat directions: Applications to curvaton scenarios, Affleck-Dine baryogenesis, and leptogenesis from a sneutrino condensate,” JCAP 0401, 005 (2004) [arXiv:hep-ph/0304246]; M. Kawasaki and K. Nakayama, “Late-time Affleck-Dine baryogenesis after thermal inflation,” Phys. Rev. D 74, 123508 (2006) [arXiv:hep-ph/0608335]; T. Matsuda, “Affleck-Dine baryogenesis after thermal brane inflation,” Phys. Rev. D 65, 103501 (2002) [arXiv:hep-ph/0202209].

[8] K. Koyama and J. Soda, “Baryon isocurvature perturbation in the Affleck-Dine baryogenesis,” Phys. Rev. Lett. 82, 2632 (1999) [arXiv:astro-ph/9810006]; K. Enqvist and J. McDonald, “Inflationary Affleck-Dine scalar dynamics and isocurvature perturbations,” Phys. Rev. D 62, 043502 (2000) [arXiv:hep-ph/9912478]; S. Kasuya, M. Kawasaki and F. Takahashi, “Isocurvature fluctuations in Affleck-Dine mechanism...
and constraints on inflation models,” JCAP 0810, 017 (2008) [arXiv:0805.4245 [hep-ph]].

[9] A. Kusenko and M. E. Shaposhnikov, “Supersymmetric Q-balls as dark matter,” Phys. Lett. B 418, 46 (1998) [arXiv:hep-ph/9709492].

[10] S. Kasuya and M. Kawasaki, “Q-ball formation through Affleck-Dine mechanism,” Phys. Rev. D 61, 041301 (2000) [arXiv:hep-ph/9909509]; S. Kasuya and M. Kawasaki, “Q-ball formation: Obstacle to Affleck-Dine baryogenesis in the gauge-mediated SUSY breaking?,” Phys. Rev. D 64, 123515 (2001) [arXiv:hep-ph/0106119]; T. Matsuda, “Q-ball inflation,” Phys. Rev. D 68, 127302 (2003) [arXiv:hep-ph/0309339].

[11] T. Matsuda, “Cosmological perturbations from an inhomogeneous phase transition,” Class. Quant. Grav. 26, 145011 (2009) [arXiv:0902.4283 [hep-ph]].

[12] F. Bernardeau, L. Kofman and J. P. Uzan, “Modulated fluctuations from hybrid inflation,” Phys. Rev. D 70, 083004 (2004) [arXiv:astro-ph/0403315].

[13] D. H. Lyth, “Generating the curvature perturbation at the end of inflation,” JCAP 0511, 006 (2005) [arXiv:astro-ph/0510443].

[14] T. Matsuda, “Elliptic inflation: Generating the curvature perturbation without slow-roll,” JCAP 0609, 003 (2006) [arXiv:hep-ph/0606137].

[15] A. D. Linde and V. F. Mukhanov, “Nongaussian isocurvature perturbations from inflation,” Phys. Rev. D 56, 535 (1997) [arXiv:astro-ph/9610219]; D. H. Lyth and D. Wands, “Generating the curvature perturbation without an inflaton,” Phys. Lett. B 524, 5 (2002) [arXiv:hep-ph/0110002]; T. Moroi and T. Takahashi, “Effects of cosmological moduli fields on cosmic microwave background,” Phys. Lett. B 522, 215 (2001) [Erratum-ibid. B 539, 303 (2002)] [arXiv:hep-ph/0110096].

[16] D. H. Lyth, “Can the curvaton paradigm accommodate a low inflation scale,” Phys. Lett. B 579, 239 (2004) [arXiv:hep-th/0308110]; T. Matsuda, “Curvaton paradigm can accommodate multiple low inflation scales,” Class. Quant. Grav. 21, L11 (2004) [arXiv:hep-ph/0312058]; T. Matsuda, “Hilltop Curvatons,” Phys. Lett. B 659, 783 (2008) [arXiv:0712.2103 [hep-ph]].
[17] T. Matsuda, “Topological curvatons,” Phys. Rev. D 72, 123508 (2005) [arXiv:hep-ph/0509063].

[18] T. Matsuda, “Evolution of curvature perturbation in generalized gravity theories,” Class. Quant. Grav. 26, 145016 (2009) [arXiv:0906.0643 [hep-th]]; T. Matsuda, “Delta-N formalism for the evolution of the curvature perturbations in generalized multi-field inflation,” [arXiv:0906.2525 [hep-th]]; T. Matsuda, “Modulated Inflation,” Phys. Lett. B 665, 338 (2008) [arXiv:0801.2648 [hep-ph]]; T. Matsuda, “Successful D-term inflation with moduli,” Phys. Lett. B 423, 35 (1998) [arXiv:hep-ph/9705448].

[19] C. Gordon, D. Wands, B. A. Bassett and R. Maartens, “Adiabatic and entropy perturbations from inflation,” Phys. Rev. D 63, 023506 (2001) [arXiv:astro-ph/0009131]; D. Wands, K. A. Malik, D. H. Lyth and A. R. Liddle, “A new approach to the evolution of cosmological perturbations on largescales,” Phys. Rev. D 62, 043527 (2000) [arXiv:astro-ph/0003278].

[20] M. Dine, L. Randall and S. D. Thomas, “Baryogenesis From Flat Directions Of The Supersymmetric Standard Model,” Nucl. Phys. B 458, 291 (1996) [arXiv:hep-ph/9507453].

[21] R. Allahverdi and A. Mazumdar, “Longevity of supersymmetric flat directions,” JCAP 0708, 023 (2007) [arXiv:hep-ph/0608296]; A. E. Gumrukcuoglu, K. A. Olive, M. Peloso and M. Sexton, “The nonperturbative decay of SUSY flat directions,” Phys. Rev. D 78, 063512 (2008) [arXiv:0805.0273 [hep-ph]]; A. Basboll, “SUSY Flat Direction Decay - the prospect of particle production and preheating investigated in the unitary gauge,” Phys. Rev. D 78, 023528 (2008) [arXiv:0801.0745 [hep-th]].

[22] T. Matsuda, “Evolution of the curvature perturbations during warm inflation,” JCAP 0906, 002 (2009) [arXiv:0905.0308 [astro-ph.CO]]; M. Bellini, “Fresh inflation with increasing cosmological parameter,” Phys. Rev. D 67, 027303 (2003) [arXiv:gr-qc/0211044]; W. Lee and L. Z. Fang, “Correlated hybrid fluctuations from inflation with thermal dissipation,” Phys. Rev. D 69, 023514 (2004) [arXiv:astro-ph/0310856].
[23] T. Matsuda, “Remote Inflation as hybrid-like sneutrino/MSSM inflation,” arXiv:0905.4328 [hep-ph]; T. Matsuda, “Remote Inflation: Hybrid-like inflation without hybrid-type potential,” JCAP 0907, 003 (2009) [arXiv:0904.2821 [astro-ph.CO]].

[24] I. G. Moss and C. Xiong, “Dissipation coefficients for supersymmetric inflatonary models,” arXiv:hep-ph/0603266.

[25] G. N. Felder, L. Kofman and A. D. Linde, “Instant preheating,” Phys. Rev. D 59, 123523 (1999) [arXiv:hep-ph/9812289].

[26] K. Enqvist, A. Jokinen, S. Kasuya and A. Mazumdar, “MSSM flat direction as a curvaton,” Phys. Rev. D 68, 103507 (2003) [arXiv:hep-ph/0303165]; S. Kasuya, M. Kawasaki and F. Takahashi, “MSSM curvaton in the gauge-mediated SUSY breaking,” Phys. Lett. B 578, 259 (2004) [arXiv:hep-ph/0305134]; R. Allahverdi, K. Enqvist, A. Jokinen and A. Mazumdar, “Identifying the curvaton within MSSM,” JCAP 0610, 007 (2006) [arXiv:hep-ph/0603255].

[27] T. Matsuda, “Hybrid Curvatons from Broken Symmetry,” JHEP 0709, 027 (2007) [arXiv:0708.4098 [hep-ph]]; T. Matsuda, “NO Curvatons or Hybrid Quintessential Inflation,” JCAP 0708, 003 (2007) [arXiv:0707.1948 [hep-ph]].

[28] T. Matsuda, “Cosmological perturbations from inhomogeneous preheating and multi-field trapping,” JHEP 0707, 035 (2007) [arXiv:0707.0543 [hep-th]].

[29] T. Matsuda, “A new perspective on supersymmetric inflation,” JCAP 0911, 022 (2009) [arXiv:0911.2350 [hep-ph]].