1-D/2-D/3-D Hopfield Associative Memories

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Abstract. In this research paper storage as well as retrieval of 1-D/2-D/3-D information using Hopfield type Associative Memories (AMs) is discussed. Various Artificial Neural Networks(ANN) architectures are proposed. Also, implementational issues associated with those Associative Memories are discussed. Cascade connection of AM and Convolutional Neural Network is proposed for noise immunity.

Keywords: Associative Memories, Hopfield Network, Ceiling Neuron

1. Introduction

In an effort to the model, biological memory, Hopfield proposed an Associative Memory (the so-called Discrete Time Hopfield Neural Network(DTHNN)). The proposed Associative Memory (AM) is based on a vector of \{+1,-1\} as the state of the dynamical system. Thus, in such an Artificial Neural Network(ANN), only one dimensional information can be stored.

In [8], the author proposed the design of Multi-Dimensional Neural Networks. A natural question that arises is how to design an associative memory which can store two/three dimensional information. As a first attempt to answer the question, researchers attempted designing associative memory with \{+1,-1\} arrays as the state of the system. Further some researchers studied associative memories with multi-state neurons, whose state vector has more than two elements [11], [9]. These efforts have many applications for content based Image Retrieval and other problems.

In this research paper we propose storage/retrieval of 1-D/2-D/3-D information from 1-D/2-D/3-D queries based on the ideas of associative memory.

This research paper is organized as follows. In Section-II, review of related research literature is presented. In Section-III, an architecture based on Parallel Hopfield Neural Network is proposed. In Section-IV, we proposed new architectures by stacking of parallel Hopfield Associative Memories for conversion of lower dimension to higher dimensions. We also discussed on Ceiling Neuron based Associative Memory. In Section-V, a novel deep learning architecture is proposed. In Section-VI, applications are discussed. In Section-VII, results of all the proposed architectures are discussed and implemented on Black & White Images. The research paper concludes in Section-VIII.
2. Review of Related Literature
In research literature on Associated Memories, many researchers realized that Hopfield Associated Memory is very restricted from the point of view of state space. Aizenberg et al proposed multi-state neuron based associative memories. In most of these efforts, the state of the neural network (dynamical System) was a vector. In [8], the author proposed the design of multi-dimensional associative memories. In fact convergence theorem of multi-dimensional Hopfield neural network was proved [3]. 2-D as well as 3-D associative memories find many applications to store images as well as video data [2]. In this research paper, we attempt simple models of 2-D or 3-D associative memories in the spirit of Hopfield’s effort.

3. Parallel Hopfield Neural Network
In this section, we propose a variation of DTHNN. The architecture proposed in this section is motivated by the concept of CEILING NEURON proposed in [4]. In this model, there are multiple thresholds at each neuron instead of a single one. We utilize such an idea to arrive at the following nonlinear dynamical system which acts as a two dimensional associative memory.

3.1. Architecture-1 :
\[ \tilde{V}(n + 1) = \text{Sign}\{\bar{W}\tilde{V}(n) - \tilde{T}\} \]  
(1)

with, \( \tilde{V}(0) \) as the initial state matrix. In (1), \( \{\tilde{V}(n) : for \ n \geq 0\} \) is a \{+1,-1\} valued state matrix and \( \tilde{T} \) is a matrix of thresholds (motivated by the idea in [4]). It should be noted that (1) corresponds to fully parallel mode of operation of 2-D AM. It readily follows that serial mode of operation of such an associative memory corresponds to updating just one component of \( \tilde{V}(n + 1) \). We ensure that the diagonal elements of weight matrix, \( W \) are all non-negative.

Based on the convergence theorem of ordinary HNN in the serial mode of operation, 2-D AM converges to a stable state (matrix of +1,-1) and to a cycle of length 2 in the fully parallel mode of operation. A more general model of AM motivated by the idea in [6] is the following one:

\[ \tilde{V}(n + 1) = \text{Sign}\{\bar{W}(n)\tilde{V}(n) - \tilde{T}(n)\} \]  
(2)

In the spirit of Parallel HAM in (1), we can use three/higher-dimensional state tensors. Thus, we have the following general multi-dimensional associative memory.

\[ \tilde{V}(n + 1) = \text{Sign}\{\bar{W} \circ \tilde{V}(n) - \tilde{T}\} \]  
(3)

where \( \{\tilde{V}(n) : for \ n \geq 0\} \) are state tensors and ‘\( \circ \)’ denotes suitable inner product. Also \( \tilde{T} \) is the tensor of thresholds.  

Note: Synthesis of Hopfield Associative memory with desired stable states, when threshold vector is a zero vector was documented in [1] and when the threshold vector is a non-zero vector was documented in [10] naturally generalises to tensor based linear operators with eigen values and eigen tensors. These results naturally apply for architecture-1 above.

4. Stacking of Parallel Hopfield Associative Memories
In the two-dimensional associative memory proposed in the above section, the synaptic weight matrix remains same for updating the state vectors in parallel. An effort to relax this assumption leads to the following architecture.

4.1. Architecture -2
We propose stacking based architecture where the input of ‘M’ HAMs/HNNs is a matrix of \{+1’s,-1’s\} i.e., Let \( \tilde{V}(0) \) be a \{+1,-1\} component matrix. \( \tilde{V}(0) = [V_1(0) \ V_2(0) \ldots \ V_M(0)] \)

Note: In this architecture, a two dimensional \{+1,-1\} matrix is associated with a 2-D matrix of \{+1,-1\}s whose columns correspond to the stable states of associative memories.
4.2. Architecture-3

In this architecture of stack, at each level of stack, we have a different 2-Dimensional associative memory with different initial state matrix that converges to a different stable state matrix. 

**Note:** 3-Dimensional initial state tensor is associated with a 3-Dimensional stable state tensor.

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It is possible to conceive architectures in which a higher dimensional information is associated with a lower dimensional information. We have following three cases:

- 2-Dimensional to 1-Dimensional
- 3-Dimensional to 2-Dimensional
- 3-Dimensional to 1-Dimensional

Such associative memories require higher dimensional stable states to be reduced to lower dimensional stable states.

**CEILING NEURON - BASED ASSOCIATIVE MEMORY : 2-D to 1-D**

Associative Memory

In Ceiling neuron model [4], every neuron has multiple thresholds. Using multiple thresholds, the net contribution (i.e., $\sum_{i=1}^{N} W_i x_i$) is thresholded and the resulting state of the network is a matrix. In Section-5 of [4], we propose an AM where the state is a vector. Thus, in such an AM, 2-D state information is associated with a 1-D stable states. We, thus expect design of AMs where higher dimensional information is associated with lower dimensional information.

**Note-1:** When functions of human memory are understood, it becomes evident that higher dimensional information is "associated" with lower dimensional information (and vice-versa) in an effortless manner. One of our goals in this research paper is to arrive at models of ANN which can achieve these functions.

**Note-2:** Using Parallel, Stacked HNN architectures, 1-D/2-D/3-D information can be stored and retrieved. Thus, a total of Nine architectures[3 x 3] are possible and included three of them.

**Note-3:** With two stages of Stacked/parallel architectures, there are twenty seven (9 X 3) possible associative memories. The effort is to model, biological associative memories.
5. Novel Associative Memories: Deep Learning
In most of the applications 1-D/2-D/3-D data (vectors/matrices/3-D arrays) is corrupted by noise [5]. In the case of 1-D neural networks, the author proposed the concept of “HYBRID Neural Networks” [7]. In that research paper, AM (e.g. HNN) is utilized to filter the noise. The input vectors after filtering are fed to a Multi-Layer Perceptron, which performs classification. Generalizing the idea, we employ 2-D/3-D associative memory to filter noise from images, videos. The filtered input is fed to a Deep Convolutional network for performing classification.

The Block diagram representation of such a Deep neural network is provided below.

![Block diagram of novel associative memories.](image)

The associative memories discussed above, reach stable state given an input. In the fig(3), the stable states constitute the input CNN. Specifically, the associative memory can be synthesized with ‘N’ desired stable states each belonging to a class in the classification problem. The CNN can be trained to perform the desired classification problem. We have experimented with the architecture in Fig.4 and achieved good classification performance. The spurious stable states are eliminated by properly training the CNN architecture.

**Note:** The stable states can be one/two/three dimensional corresponding to audio/text, image, video related data. The synthesis approach proposed in [10] for programming desired stable states can easily be generalized to 2-D/3-D associative memories.

In [7], the author also proposed an architecture in which the noisy output patterns of a Multi-Layer Perceptron (MLP) are filtered using an associative memory.

6. Applications
It is well known that the DTHNN (based on convergence theorem) was successfully utilized to store 1-D patterns (which are specified using (+1’s, -1’s). Generalizing the idea, we innovated the three architectures specified above. These architectures are an effort to model the memory mechanisms in homo-Sapiens. For instance, given the speech input of a person, the memory can correlated with the face image of the person i.e., the biological neural network stores the correspondence between speech signal and the associated image.

In general, human brain is capable of retrieving 1-D/2-D/3-D information, given the 1-D/
2-D/3-D information/signal as the input through the process of association. Hence, out of Nine possible architectures, we provided three architectures in the above sections.

Further, as we can expect, human brains can be expected to have various associative memory units connected to one another using certain “Network Topology”. Such, interacting associative memories are potentially capable of storing heterogeneous types (speech/image/video signals) of information.

7. Results of Implementation

We are currently investigating dedicated purpose hardware to implement 1-D /2-D /3-D associative memories, which can potentially ”Speed up” retrieval of stored 1-D/2-D/3-D information. We now provide some numerical results on data. We also provide some results related to implementation based on black and white images. The results and implementation of all the architectures mentioned in Section-III and Section-IV are provided below:

7.1. Architecture-1
We have taken 3x3 symmetric weight matrix, one state matrix having elements of +1,-1’s and one threshold matrix whose values are in the range of 0 to 1. We implemented AM in parallel mode of operation. Here we have taken, $W = \begin{bmatrix} 0 & -3 & -2 \\ -3 & 0 & -4 \\ -2 & -4 & 0 \end{bmatrix}$. As a result, it converges to a cycle of length 2.

7.2. Architecture-2
We have taken two HAMs having same or different symmetric 3x3 weight matrices and a state matrix of two different 3x3 state vectors are $\tilde{V}(0) = [\tilde{V}_1(0) \tilde{V}_2(0)]$. We have taken, $W_1 = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix}$ and $W_2 = \begin{bmatrix} 0 & 4 & 5 \\ 4 & 0 & 6 \\ 5 & 6 & 0 \end{bmatrix}$. Such an AM converged with cycle of length 2 in parallel mode of operation. Then we stack all the final state vectors, which leads to a matrix of 2-D.

7.3. Architecture-3
We have taken two HAMs having same or different symmetric 3x3 weight matrices and a state matrix of two different 3x3 state vectors as $\tilde{V}_1(0)$ and $\tilde{V}_2(0)$.

We have taken, $W_1 = \begin{bmatrix} 0 & 4 & 5 \\ 4 & 0 & 6 \\ 5 & 6 & 0 \end{bmatrix}$ and $W_2 = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix}$. Such AM converged with cycle of length 2 in parallel mode of operation. Then we stack all the final state matrices, which leads to 3-D. Finally it is shown that 3-D input converges to 3-D output.

7.4. Implementation on Black and White Images:
Here we have taken a black & white image having size of 5x5. Then we converted it into a 5x5 matrix of elements \{+1,-1\}. Then it is given as input to HAM. It converges in parallel mode with cycle of length 2. Then we fix the weights.
8. Conclusions

In this research paper, the concepts of parallel, stacked associative memories are discussed. Various novel architectures of Associative Memories for storage/retrieval of 1-D/2-D/3-D information are discussed. Artificial Neural Network architecture based on cascading of AM’s, CNN’s are proposed. It is expected that these architectures will be of practical utility. We are actively investigating on remaining six architectures.

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