Phenomenological Scaling of Rapidity Dependence for Anisotropic Flows in 25 MeV/nucleon Ca + Ca by Quantum Molecular Dynamics Model

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Anisotropic flows (v1, v2, v3 and v4) of light fragments up till the mass number A = 4 as a function of rapidity have been studied for 25 MeV/nucleon 40Ca + 40Ca at large impact parameters by Quantum Molecular Dynamics model. A phenomenological scaling behavior of rapidity dependent flow parameters vn (n = 1, 2, 3 and 4) has been found as a function of mass number plus a constant term, which may arise from the interplay of collective and random motions. In addition, v4/v22 keeps almost independent of rapidity and remains a rough constant of 1/2 for all light fragments.

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Collective flow is an important observable in heavy ion collisions and it can bring some information on nuclear equation of state and in-medium nucleon-nucleon cross section. Many studies of the properties of the Fourier expansion for the transverse momentum in and perpendicular to the reaction plane. Anisotropic flows generally depend on both particle transverse momentum and rapidity, and for a given rapidity the anisotropic flows at transverse momentum pt (pt = √p_x^2 + p_y^2) can be evaluated according to

\[ v_n(p_t) = \langle \cos(n\phi) \rangle, \]

where \( \langle \cdots \rangle \) denotes average over the azimuthal distribution of particles with transverse momentum \( p_t \). The anisotropic flows \( v_n \) can further be expressed in term of single-particle averages:

\[ v_1 = \langle \cos\phi \rangle = \frac{p_x}{p_t}, \]

\[ v_2 = \langle \cos(2\phi) \rangle = \frac{p_x^2 - p_y^2}{p_t^2}, \]

\[ v_3 = \langle \cos(3\phi) \rangle = \frac{p_x^3 - 3p_x p_y^2}{p_t^3}, \]

\[ v_4 = \langle \cos(4\phi) \rangle = \frac{p_x^4 - 6p_x^2 p_y^2 + p_y^4}{p_t^4}, \]

where \( p_x \) and \( p_y \) are, respectively, projections of particle transverse momentum in and perpendicular to the reaction plane.

The intermediate energy heavy-ion collision dynamics is complex since both mean field and nucleon-nucleon collisions play the competition roles. Furthermore, the isospin dependent role should be also incorporated for asymmetric reaction systems. Isospin dependent Quantum Molecular Dynamics model (IDQMD) has been affiliated with isospin degrees of freedom with mean field and nucleon-nucleon collision. The IDQMD model can explicitly represent the many body state of the system and principally contain correlation effects to all orders and all fluctuations, and can describe...
the time evolution of the colliding system well. When the spatial distance ($\Delta r$) is closer than 3.5 fm and the momentum difference ($\Delta p$) is smaller than 300 MeV/c between two nucleons, two nucleons can coalesce into a cluster \[22\]. With this simple coalescence mechanism which has been extensively applied in transport theory, different size clusters can be recognized.

In the model the nuclear mean-field potential is parameterized as

$$ U(\rho, \tau_z) = \alpha \left( \frac{\rho}{\rho_0} \right) + \beta \left( \frac{\rho}{\rho_0} \right)^\gamma + \frac{1}{2} (1 - \tau_z) V_c $$

$$ + C_{sym} \left( \frac{\rho_n - \rho_p}{\rho_0} \right) \tau_z + U^{Yuk} \tag{7} $$

where $\rho_0$ is the normal nuclear matter density (0.16 fm$^{-3}$), $\rho_n$, $\rho_p$ and $\rho$ are the neutron, proton and total densities, respectively. $\tau_z$ is the $z$-th component of the isospin degree of freedom, which equals 1 or -1 for neutrons or protons, respectively. The coefficients $\alpha$, $\beta$ and $\gamma$ are parameters for nuclear equation of state. $C_{sym}$ is the symmetry energy strength due to the density difference of neutrons and protons in nuclear medium, which is important for asymmetric nuclear matter (here $C_{sym} = 32$ MeV is used), but it is trivial for the symmetric system studied in the present work. $V_c$ is the Coulomb potential and $U^{Yuk}$ is Yukawa (surface) potential. In this work, we take $\alpha = 124$ MeV, $\beta = 70.5$ MeV and $\gamma = 2$ which corresponds to the so-called hard EOS with an incompressibility of $K = 380$ MeV.

About 60000 events have been simulated for the collision system of $^{40}$Ca + $^{40}$Ca at 25 MeV/nucleon with impact parameter from 4 fm to 6 fm. The physics results were extracted at the time of 200 fm/c when the system has been in the freeze-out stage. Fig. 1(a) shows rapidity dependence of the average in-plane transverse momentum per nucleon ($\langle p_x/A \rangle$) around mid-rapidity for light fragments. Here rapidity $y$ is the rapidity of fragment in the center of momentum (c.m.) frame which is normalized to the initial projectile rapidity, i.e., $y = y_{c.m.}/y_{proj}$. It shows that $\langle p_x/A \rangle$ decreases monotonously with the increasing rapidity, and decreases more rapidly for heavier fragments. The negative slope is mainly driven by the attractive mean field in this energy region $[11, 27]$. Fig. 1(b) shows mass dependence of the so-called sideways flow parameter (slope value) $d\langle p_x/A \rangle/dy$, which can be easily extracted via linear fits near mid-rapidity in Fig. 1(a). The flow parameter is negative and its absolute value increases with mass apparently, which seems consistent with coalescence-like cluster production mechanism $[16]$.

In Fig. 2, we show integrated anisotropic flows $v_1$, $v_2$, $v_3$ and $v_4$ as a function of rapidity $y$. The trend of the directed flow $v_1$ shown in Fig. 2(a) is similar to that of $\langle p_x/A \rangle$, i.e., the directed flow decreases monotonously with the increasing rapidity around mid-rapidity. It is positive in target-like rapidity region and negative in projectile-like rapidity region. And it gets to the extremum near $y = \pm 1.5$, and then the absolute value of directed flow goes down with the increasing $|y|$. It also shows at a given rapidity the absolute magnitude of $v_1$ is larger for heavier clusters. Fig. 2(b) presents the rapidity dependence of elliptic flow $v_2$. It shows that the flow is positive and the curve for a given fragment is symmetric with $y = 0$, and descends monotonously when the
The magnitude of collective flow at the same rapidity. The rapidity dependences of $v_1$ and $v_2$ are similar to the results at RHIC energies for charged hadrons \[28\], but the interaction level of the matter in the two different energy region are obviously different. In this low energy heavy ion collisions, the positive elliptic $v_2$ essentially stems from the collective rotational motion \[11, 27\]. Fig. 2(c) and (d) show rapidity dependence of higher order flows of $v_3$ and $v_4$, respectively. The trend of $v_3$ and $v_4$ are similar to that of $v_1$ and $v_2$, respectively, but they show smaller magnitude of flows for a given particle in comparison with $v_1$ and $v_2$, respectively. Nevertheless the magnitude of flows of $v_3$ and $v_4$ are not too small and hence higher order flows should not be neglected in this energy region.

For testing if the number of nucleon scaling works for rapidity dependent flows, we plot the flows per nucleon as a function of rapidity which is shown in Fig. 3. It seems that the curves for different fragments do not stay together as we imagined except for $v_1/A$. One reason is that the effect of random motion which is independent of mass may weaken the mass scaling in rapidity space. For details, if we compare the magnitude of collective flows of other fragments with that of proton or neutron, it shows the flow magnitude of fragments with mass number of 2, 3, and 4 is smaller than proton or neutron for $v_1$ and $v_2$, but larger than proton or neutron for $v_3$ and $v_4$. However, if we use a mass number dependent function $C(A) = \frac{3}{2}(A + \frac{3}{2})$ instead of $A$ for $v_1$ and $v_2$, and use $C(A) = 3(A - \frac{3}{2})$ for $v_3$ and $v_4$, and plot $v_n/C(A)$ versus $y$ again which is shown in Fig. 4, we can now see the curves for different fragments coincide with each other excellently except for slight deviation in large rapidity region, i.e. in the spectators region. The coefficient $C(A)$ may be qualitatively understood by two parts: the constant part reflects the effect of random motion which is independent of mass, and the part including mass number $A$ reflects the collective motion which increases linearly with mass \[18, 29\]. From the mass dependent coefficient of $C(A)$, there are two classes: one for $v_1$ and $v_2$, and another for $v_3$ and $v_4$. The larger coefficient for $v_3$ and $v_4$ may indicate that the contribution of collective motion to higher order flows $v_3$ and $v_4$ is larger than that to $v_1$ and $v_2$.

The RHIC experimental data demonstrated a scaling relationship between 2nd flow $v_2$ and $n$-th flow $v_n$ for hadrons, namely $v_n(p_t) \sim p_t^{n/2}(p_t)$ and $v_4/v_2^2 \sim 1.2$ in the STAR data \[30\]. And we have already shown the scaling of $v_4/v_2^2$ as a function of $p_t$ in intermediate energy heavy ion collisions \[10, 20, 21\], i.e., the ratio of $v_4/v_2^2$ is independent of transverse momentum and shows a constant value of 0.5 for different light fragments. Similarly, we plot $p_t$-integrated $v_4/v_2^2$ as a function of rapidity which is shown in Fig. 5. The figure shows that the ratios of $v_4/v_2^2$ for different fragments up to $A = 4$ roughly keep a constant of 0.5 in the studied rapidity region except for some fluctuations at large rapidities. Therefore the ratio of $v_4/v_2^2$ is almost independent of transverse momentum and rapidity and its value is around 0.5.
frame of relativistic fluid dynamics which works in RHIC energies, the ratio value of 0.5 reflects that the collision system reaches to thermal equilibrium and its subsequent evolution follows the laws of ideal fluid dynamics. A coincident value of 0.5 as relativistic fluid dynamics in the present intermediate energy may also indicate some kinds of thermal equilibrium has been also reached.

To summarize, phenomenological behaviors of anisotropic flows as a function of rapidity for light fragments up till mass number 4 have been investigated by the simulation of 25 MeV/nucleon $^{40}$Ca + $^{40}$Ca collision in peripheral collisions in the framework of the Quantum Molecular Dynamics model. It was shown that $v_1$ and $v_3$ of light fragments decrease monotonously with rapidity from positive value near target-like rapidity to negative value near projectile-like rapidity, while $v_2$ and $v_4$ are positive and show Gaussian-like shape with a peak around $y_{c.m.} = 0$. When we plot anisotropic flows per nucleon ($v_1/A$, $v_2/A$, $v_3/A$ and $v_4/A$) versus rapidity for light fragments, the curves of different particles do not stay together perfectly except for $v_1/A$. But when we plot $v_n/A$ ($n = 1, 2, 3, 4$) versus rapidity where $C(A)$ is a linear function of the mass number $A$ plus an additional constant term, the curves of different particles collapse onto the same curve. That indicates that the fragment flows may arise from the interplay of collective (a term proportional to mass number in $C(A)$) and random thermal motion (a constant term in $C(A)$) of nucleons. $C(A)$ shows a classification between $v_{1,2}$ and $v_{3,4}$, this might reflect that the different role of collective motion on the different anisotropies in momentum space. Additionally, it was found that the ratio of $v_4/v_2$ are almost independent of rapidity and the value is about 0.5, which implies a thermalization scenario of the ideal fluid-like dynamics could be applied even in such low energy heavy ion collisions. Further theoretical investigations and experimental checks are awaiting.

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