Accretion of Dark Energy onto Higher Dimensional Charged BTZ Black Hole

Ujjal Debnath*

Department of Mathematics,
Indian Institute of Engineering Science and Technology,
Shibpur, Howrah-711 103, India.

(Dated: September 17, 2014)

In this work, we have studied the accretion of \((n+2)\)-dimensional charged BTZ black hole (BH). The critical point and square speed of sound have been obtained. The mass of the BTZ BH has been calculated and we have observed that the mass of the BTZ BH is related with square root of the energy density of dark energy which accretes onto BH in our accelerating FRW universe. We have assumed modified Chaplygin gas (MCG) as a candidate of dark energy which accretes onto BH and we have found the expression of BTZ BH mass. Since in our solution of MCG, this model generates only quintessence dark energy (not phantom) and so BTZ BH mass increases during the whole evolution of the accelerating universe. Next we have assumed 5 kinds of parametrizations of well known dark energy models. These models generate both quintessence and phantom scenarios i.e., phantom crossing models. So if these dark energies accrete onto the BTZ BH, then in quintessence stage, BH mass increases upto a certain value (finite value) and then decreases to a certain finite value for phantom stage during whole evolution of the universe. We have shown these results graphically.

PACS numbers: 04.70.Bw, 04.70.Dy, 98.80.Cq

I. INTRODUCTION

In recent years, the type Ia Supernovae and Cosmic Microwave Background (CMB) observations suggest that our universe is currently in the phase of accelerated expansion. This acceleration is caused by some unknown matter which has the property that positive energy density and negative pressure satisfying \( \rho + 3p < 0 \) is known as “dark energy” (DE) \[\text{3-6}\]. The simplest candidate of dark energy is the cosmological constant which is characterized by the equation of state \( p = w \rho \) with \( w = -1 \). Many other theoretical models have been proposed to explain the accelerated expansion of the universe. Another candidate of dark energy is quintessence satisfying \(-1 < w < -1/3\) \[\text{5-8}\].

* ujjaldebnath@gmail.com, ujjal@iucaa.ernet.in
When \( w < -1 \), it is known as phantom energy. Distinct data on supernovas showed that the presence of phantom energy with \(-1.2 < w < -1\) in the Universe is highly likely \([7]\). Several models for the explanation of dark energy were suggested. These usually include k-essence, dilaton, DBI-essence, Hessence, tachyon, Chaplygin gas, etc \([8-16]\).

In Newtonian theory, the problem of accretion of matter onto the compact object was formulated by Bondi \([17]\). The equations of motion for steady-state spherical symmetric flow of matter into or out of a condensed object (e.g. neutron stars, ‘black holes’, etc.) are discussed by Michel \([18]\) and also obtained analytic relativistic accretion solution onto the static Schwarzschild black hole. The accretion of phantom energy onto a static Schwarzschild black hole was first proposed by Babichev et al \([19, 20]\) and established that black hole mass will gradually decrease due to strong negative pressure of phantom energy and finally all the masses tend to zero near the big rip where it will disappear. Jamil \([21]\) has investigated the accretion of phantom like variable modified Chaplygin gas onto Schwarzschild black hole and also showed that mass of the black hole will decreases for dark energy accretion and otherwise will increases. Also the accretion of dark energy onto the more general Kerr-Newman black hole was studied by Madrid et al \([22]\) and Bhadra et al \([23]\). Till now, several authors \([24-38]\) have discussed the accretion of several candidates of dark energy onto black holes.

Recently, there has been a growing interest to study the black hole (BH) solution in (2+1)-dimensions. These BH solutions have all the typical properties that can be found in (3+1) or higher dimensions, such as horizons, Hawking temperature and thermodynamics. The discovery and investigation of the (2+1)-dimensional BTZ (Banados-Teitelboim-Zanelli) black holes \([39-41]\) organizes one of the great advances in gravity. Jamil and Akbar \([42]\) have investigated the thermodynamics of phantom energy accreting onto a BTZ BH. Abhas \([43]\) investigated the phantom energy accretion onto 3D black hole formulated in Einstein-Power-Maxwell theory. The accretion of phantom energy onto Einstein-Maxwell-Gauss-Bonnet black holes was studied in \([44]\). They showed that the evolution of the black hole mass was independent of its mass and depends only on the energy density and pressure of the phantom energy. Interest in the BTZ black hole has recently heightened with the discovery that the thermodynamics of higher dimensional black holes \([45, 46]\). Also, non-static charged BTZ like black holes in \((n + 1)\)-dimensions have been considered by Ghosh et al \([47]\), which in the static limit, for \( n = 2 \), reduces to (2+1)-dimensional BTZ black hole solutions. John et al \([48]\) examined the steady-state spherically symmetric accretion of relativistic fluids, with a polytropic equation of state, onto a higher dimensional Schwarzschild black hole. Also charged BTZ-like black holes in higher
dimensions have been studied by Hendi [49]. There are also charged, rotating, regular extensions of the BTZ black hole solutions [50–57] available in the literature by employing nonlinear Born-Infeld electrodynamics to eliminate the inner singularity [58].

In section II, we assume the \((n + 2)\)-dimensional charged BTZ black hole (BH) in presence of dark energy filled universe. The critical point has been obtained. If dark energy accretes onto the BTZ BH, the rate of change of mass of the black hole is expressed in terms of the density and pressure of dark energy and also find the expression of BH mass in terms of density. In our previous work, we have investigated accretions of various types of dark energies (including some kinds of parametrizations of dark energy) onto Morris-Thorne wormhole [59]. Our main motivation of the work is to examine the natures of the mass of the black hole during accelerating expansion of the FRW universe if several kinds of dark energies accrete around the BH. In section III, we have assumed some versions of dark energy like modified Chaplygin gas (MCG) and some kinds of parametrizations of dark energy candidates. The mass of the BTZ BH has been calculated for all types of dark energies and its natures have been analyzed during evolution of the universe. Finally, we give some concluding remarks of the whole work in section IV.

II. ACCRETION PHENOMENA OF HIGHER DIMENSIONAL CHARGED BTZ BLACK HOLE

In recent years there has been increasing interest about black hole solutions whose matter source is power Maxwell invariant, i.e., \((F_{\mu\nu}F^{\mu\nu})^s\) [46, 49], where \(s\) is the power of non-linearity. In the special case \((s = 1)\), it can reduces to linear electromagnetic field. In addition, in \((n + 2)\)-dimensional gravity, for the special choice \(s = (n + 2)/4\), matter source yields a traceless Maxwell’s energy-momentum tensor which leads to conformal invariance, which is the analogues of the four dimensional Reissner-Nordstrom solutions in higher dimensions [49, 60]. Also, it is valuable to find and analyze the effects of exponent \(s\) on the behavior of the new solutions, when \(s = (n + 1)/2\). In this case the solutions are completely different from another cases \((s \neq (n + 1)/2)\).

The \((n + 2)\)-dimensional action in which gravity is coupled to nonlinear electrodynamics field is given by [49]
\[ S = \frac{1}{16\pi} \int d^{n+2}\sqrt{-g} \left[ R + 2\Lambda - (\alpha F)^s + L_m \right] \] (1)

where \( R \) is scalar curvature, \( \Lambda \) refers to the positive cosmological constant which is in general equal to \( \frac{n(n+1)}{2l^2} \) for asymptotically AdS solutions, in which \( l \) is a scale length factor, \( \alpha \) is a constant and \( s = \frac{(n+1)}{2} \) gives BTZ-like solutions. Varying the action (1) with respect to the metric \( g_{\mu\nu} \) and the gauge field \( A_\mu \) (with \( s = \frac{(n+1)}{2} \)) the field equations are obtained as

\[ G_{\mu\nu} - \Lambda g_{\mu\nu} = T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(EM)} \] (2)

Here,

\[ T_{\mu\nu}^{(m)} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} \] (3)

is the energy-momentum tensor for matter. Here \( \rho \) and \( p \) are the energy density and pressure of the matter while \( u^\mu = (u^0, u^1, 0, 0, ..., 0) \) is the velocity vector of the fluid flow satisfying \( u_\mu u^\mu = -1 \). Also \( u^1 = u \) is the radial velocity of the flow. Also

\[ T_{\mu\nu}^{(EM)} = \alpha (\alpha F)^{\frac{n-1}{2}} \left( \frac{1}{2} g_{\mu\nu} F - nF_{\mu\lambda}F^\lambda_\nu \right) \] (4)

is the energy-momentum tensor for electro-magnetic field and

\[ \partial_\mu \left( \sqrt{-g} F^{\mu\nu}(\alpha F)^{\frac{n-1}{2}} \right) = 0 \] (5)

Let us consider static spherically symmetric \((n + 2)\)-dimensional charged BTZ black hole metric given by [49]

\[ ds^2 = -f(r)dt^2 + \frac{1}{f(r)} dr^2 + r^2 \sum_{i=1}^{n} d\phi_i^2 \] (6)

Here, \( f(r) \) is termed as the lapse function, which is obtained as [49]

\[ f(r) = \frac{r^2}{l^2} - r^{1-n} \left[ M + 2\frac{n+1}{2} Q^{n+1} \ln \left( \frac{r}{l} \right) \right] \] (7)

where \( M \) is the mass and \( Q \) is the charge of the BTZ black hole. Here \( \sqrt{-g} = r^n \). Using \( u_\mu u^\mu = -1 \), we get \( g_{00}u^0 u^0 + g_{11}u^1 u^1 = -1 \) (since \( u^0 \) and \( u^1 \) are the non-zero components of velocity vector), so we can obtain \((u^0)^2 = (\frac{(u^i)^2 + f(r)}{f(r)})\) and since \( u^1 = u \), so we have \( u_0 = g_{00}u^0 = \sqrt{u^2 + f(r)} \).
A proper dark-energy accretion model for BTZ black hole should be obtained by generalizing the Michel’s theory [18]. Such a generalization has been already performed by Babichev et al [19, 20] for the case of dark-energy accretion onto Schwarzschild black holes. We shall follow now the procedure used by Babichev et al [19, 20]. We assume that the in-falling dark energy fluid does not disturb the spherical symmetry of the black hole. The relativistic Bernoulli’s equation after the time component of the energy-momentum conservation law $T_{\mu\nu}^{;\nu} = 0$, we obtain (consider steady state condition and spherically symmetric)

$$\frac{d}{dr} (T_0^1 \sqrt{-g}) = 0$$

(8)

which provides the first integral,

$$(\rho + p)u_0 u^1 \sqrt{-g} = C_1$$

(9)

i.e.,

$$ur^n (\rho + p) \sqrt{u^2 + f(r)} = C_1$$

(10)

where the integration constant $C_1$ has the dimension of the energy density.

Moreover, the second integration of motion is obtained from the projection of the conservation law for energy-momentum tensor onto the fluid four-velocity, $u_{\mu} T_{\nu;\nu}^{\mu} = 0$, which gives

$$u^n \rho_{;\mu} + (\rho + p) u_{\mu}^{;\mu} = 0$$

(11)

which yields

$$ur^n \exp \left[ \int_{\rho_{\infty}}^{\rho_{h}} \frac{d\rho}{\rho + p} \right] = -A$$

(12)

where $A$ is integration constant and the associated minus sign is taken for convenience. Also $\rho_h$ and $\rho_{\infty}$ are the energy densities at the BTZ horizon and at infinity respectively. Combining these two, we obtain,

$$\int (\rho + p) \sqrt{u^2 + f(r)} \exp \left[ - \int_{\rho_{\infty}}^{\rho_{h}} \frac{d\rho}{\rho + p} \right] = C_2$$

(13)

where, $C_2 = -C_1 = \rho_{\infty} + p(\rho_{\infty})$. Further the value of the constant $C_2$ can be evaluated for different dark energy models.
The equation of mass flux $J_{\mu}^\mu = 0$ is given by $\frac{d}{dr} (J^1 \sqrt{g}) = 0$, which integrates to $\rho u^1 \sqrt{g} = A_1$ yields

$$\rho u^\mu = A_1$$  \hspace{1cm} (14)

where, $A_1$ is the integration constant. From (10) and (14), we obtain,

$$\frac{\rho + p}{\rho} \sqrt{u^2 + f(r)} = \frac{C_1}{A_1} = C_3$$  \hspace{1cm} (15)

Let,

$$V^2 = \frac{d \ln(\rho + p)}{d \ln \rho} - 1$$  \hspace{1cm} (16)

So from (14) and (15), we obtain

$$\left[ V^2 - \frac{u^2}{u^2 + f(r)} \right] \frac{du}{u} - \left[ nV^2 - \frac{rf'(r)}{2(u^2 + f(r))} \right] \frac{dr}{r} = 0$$  \hspace{1cm} (17)

It is evident that if one or the other of the bracketed factors in (17) vanishes one has a turn-around point, and the solutions are double-valued in either $r$ or $u$. Only solutions that pass through a critical point correspond to material falling into (or flowing out of) the object with monotonically increasing velocity along the particle trajectory. The critical point of accretion is located at $r = r_c$ which is obtained by taking the both bracketed factors in Eq. (17) to be zero. So at the critical point, we have

$$V^2_c = \frac{u^2_c}{u^2_c + f(r_c)}$$  \hspace{1cm} (18)

and

$$nV^2_c = \frac{r_c f'(r_c)}{2(u^2_c + f(r_c))}$$  \hspace{1cm} (19)

Here, subscript $c$ refers to the critical quantity and $u_c$ is the critical speed of flow at the critical points. From above two expressions, we have

$$u^2_c = \frac{r_c}{2n} f'(r_c)$$  \hspace{1cm} (20)

At the critical point, the sound speed can be determined by

$$c_s^2 = \left. \frac{dp}{d\rho} \right|_{r=r_c} = \frac{C_3 V_c (V^2_c + 1)}{u_c} - 1$$  \hspace{1cm} (21)

We mentioned that the physically acceptable solutions of the above equations are obtained if $u^2_c > 0$ and $V^2_c > 0$ which leads to

$$u^2_c > -f(r_c) \quad \text{and} \quad f'(r_c) > 0$$  \hspace{1cm} (22)
i.e.,
\[ u_c^2 > -\frac{r_c^2}{l^2} + r_c^{1-n} \left[ M + 2^{\frac{n+1}{2}} Q^{n+1} \ln \left( \frac{r_c}{l} \right) \right] \]  \tag{23}

and
\[ 2r_c^{n+1} + l^2 \left[ (n-1)M + 2^{\frac{n+1}{2}} Q^{n+1} \left\{ (n-1) \ln \left( \frac{r_c}{l} \right) - 1 \right\} \right] > 0 \]  \tag{24}

For linear equation of state \( p = w\rho \), we obtain \( c_s^2 = w \) and \( V_c^2 = 0 \) and from (18), we obtain \( u_c = 0 \). From (20), we see that the critical point occurs at the point \( r_c \) where \( r_c \) can be found from the equation
\[ r_c^{n+1} = l^2 \left[ M + 2^{\frac{n+1}{2}} Q^{n+1} \ln \left( \frac{r_c}{l} \right) \right] \]  \tag{25}

The rate of change of mass \( \dot{M} \) of the BTZ black hole is computed by integrating the flux of the dark energy over the \( n \)-dimensional volume of the black hole and given by \[ 48 \]
\[ \dot{M} = -\frac{2\pi^{\frac{n+1}{2}}}{\Gamma\left(\frac{n+1}{2}\right)} r^n T_{10} \]  \tag{26}

Using equations (12) and (13), the above equation can be written as
\[ \dot{M} = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma\left(\frac{n+1}{2}\right)} A(\rho_\infty + p(\rho_\infty)) \]  \tag{27}

If we neglect the cosmological evolution of \( \rho_\infty \) then from (26) we obtain the mass of the black hole as
\[ M = M_0 + \frac{2\pi^{\frac{n+1}{2}}}{\Gamma\left(\frac{n+1}{2}\right)} A(\rho_\infty + p(\rho_\infty))(t - t_0) \]  \tag{28}

where \( M_0 \) is the initial mass corresponding to the initial time \( t_0 \). The result (26) is also valid for any equation of state \( p = p(\rho) \), thus we can write
\[ \dot{M} = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma\left(\frac{n+1}{2}\right)} A(\rho + p) \]  \tag{29}

We see that the rate for the BTZ black hole exotic mass due to accretion of dark energy becomes exactly the positive to the similar rate in the case of a Schwarzschild black hole, asymptotically. Since the BTZ black hole is static, so the mass of the black hole depends on \( r \) only. When some fluid accretes outside black hole, the mass function \( M \) of the black hole is considered as a dynamical mass function and hence it should be a function of time also. So \( \dot{M} \) is time dependent and the increasing or decreasing of the black hole mass \( M \) sensitively depends on the nature of the fluid which accretes upon the black hole. If \( \rho + p < 0 \) i.e., for phantom dark energy accretion, the mass of the black hole decreases but if \( \rho + p > 0 \) i.e., for quintessence dark energy accretion, the mass of the black hole increases.
III. DARK ENERGY ACCRETES UPON BTZ BLACK HOLE

In the following, we shall assume different types of dark energy models such as modified Chaplygin gas and some parameterizations of dark energy models. The natures of mass function of black hole will be analyzed for present and future stages of expansion of the universe when the dark energies are accreting upon BTZ black hole.

A. Modified Chaplygin Gas

We consider the background spacetime is spatially flat represented by the homogeneous and isotropic FRW model of the universe which is given by

$$ds^2 = -dt^2 + a^2(t) \left[ dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]$$

(30)

where \(a(t)\) is the scale factor. The Einstein’s equations for FRW universe are (choosing \(8\pi G = c = 1\))

$$H^2 = \frac{1}{3} \rho \ ,$$

(31)

$$\dot{H} = -\frac{1}{2} (p + \rho)$$

(32)

Conservation equation is given by

$$\dot{\rho} + 3H(\rho + p) = 0$$

(33)

where \(H = \frac{\dot{a}}{a}\) is the Hubble parameter. Now assume the modified Chaplygin gas (MCG) \([61]\) as dark energy model, whose EoS is \(p = w\rho - \frac{B}{\rho^\alpha}, \ (B > 0, \ 0 \leq \alpha < 1)\). For MCG model, we obtain the solution of \(\rho\) as

$$\rho = \left[ \frac{B}{1+w} + \frac{C}{a^3(1+w)(1+\alpha)} \right]^{\frac{1}{1+\alpha}}$$

(34)

where \(C > 0\) is an arbitrary integration constant. From above, we can obtain the present value of the energy density \(\rho_0 = \left[ \frac{B}{1+w} + C \right]^{\frac{1}{1+\alpha}}\). For MCG model, we obtain \(c_s^2 = w + \frac{\alpha B}{\rho^{\alpha+1}}\) and \(V^2_c = \frac{(\alpha+1)B}{(1+w)\rho^{\alpha+1} + B}\).

Using equations (29), (31) and (33), we have

$$\dot{M} = -\frac{2\pi \frac{\alpha+1}{2} A}{\sqrt{3} \Gamma(\frac{n+1}{2})} \frac{\dot{\rho}}{\sqrt{\rho}}$$

(35)

which integrates to yield

$$M = M_0 - \frac{4\pi \frac{\alpha+1}{2} A}{\sqrt{3} \Gamma(\frac{n+1}{2})} (\sqrt{\rho} - \sqrt{\rho_0})$$

(36)
where, \( M_0 \) is the present values of the BTZ black hole mass. In the late stage of the universe i.e., \( a \) is very large \((z \rightarrow -1)\), the mass of the black hole will be
\[
M = M_0 + \frac{2\pi \frac{n+1}{2} A}{\sqrt{3} \Gamma(\frac{n+1}{2})} \sqrt{\rho_0} \tag{37}
\]

If we put the solution \( \rho \) from equation (34) in equation (36), the mass of black hole \( M \) can be expressed in terms of scale factor \( a \) and then use the formula of redshift \( z = \frac{1}{a} - 1 \), \( M \) will be in terms of redshift \( z \), i.e.,
\[
M = M_0 - \frac{4\pi \frac{n+1}{2} A}{\sqrt{3} \Gamma(\frac{n+1}{2})} \left\{ \left[ \frac{B}{1 + w} + C(1 + z)^3(1+w)(1+a) \right]^{\frac{1}{n+1}} - \left[ \frac{B}{1 + w} + C \right]^{\frac{1}{n+1}} \right\} \tag{38}
\]

Now \( M \) vs \( z \) is drawn in figure 1. Since our solution for MCG model generates only quintessence, so from the figure, we see that the mass \( M \) of the BTZ BH always increases with \( z \) decreases. So we conclude that the mass of the BTZ BH increases if the MCG accretes onto the BTZ BH.

**B. Some Parameterizations of dark energy Models**

In astrophysical sense, the dark energy is popular to have a redshift parametrization (i.e., taking the redshift \( z \) as the variable parameter of the EoS only) of the EoS as \( p(z) = w(z)\rho(z) \). The EoS parameter \( w \) is currently constrained by the distance measurements of the type Ia supernova observation with the range of EoS as \(-1.38 < w < -0.82 \) and WMAP3 observation to constraint on the EOS \( w = -0.97^{+0.07}_{-0.09} \) for the DE, in a flat universe \([63]\). We consider following three models of well known parametrizations (Models I, II, III). We shall also assumed other two parametrizations (Models IV, V). Since the following models generate both quintessence \((w(z) > -1)\) and phantom \((w(z) < -1)\) dark energies for some suitable choices of the parameters.

- **Model I (Linear):** The “Linear” parametrization is given by the EoS \( w(z) = w_0 + w_1 z \). For Linear parametrization and using equation (33), we get the solution as
\[
\rho = \rho_0(1 + z)^3(1+w_0-w_1) e^{3w_1 z} \tag{39}
\]
where, \( \rho_0 \) is the present value of the energy density. The above model generates phantom energy if \( w(z) < -1 \) i.e., \( z < -\frac{1+w_0}{w_1} \) provided \( w_1 > 0 \) and \( w_1 - w_0 > 1 \). Using equation (2), the mass of the black hole is obtained as
\[
M = M_0 - \frac{4\pi \frac{n+1}{2} A \sqrt{\rho_0}}{\sqrt{3} \Gamma(\frac{n+1}{2})} \left[ (1 + z)^{\frac{1}{2}}(1+w_0-w_1) e^{\frac{3}{2} w_1 z} - 1 \right] \tag{40}
\]
Fig. 1 shows the variation of BTZ BH mass $M$ against redshift $z$ for MCG. Figs. 2-6 show the variations of BTZ BH mass $M$ against redshift $z$ for Models I-V respectively.
Since this model is the phantom crossing model, so if this dark energy accretes onto BTZ BH, for quintessence era, BH mass increases up to a certain limit and after that for phantom era, the mass of the BH decreases. We have shown this scenario in figure 2. From the figure, we see that BTZ BH mass $M$ increases for redshift $z$ decreases up to certain stage of $z$ (ΛCDM stage) and then $M$ decreases (phantom era) as universe expands.

- **Model II (CPL):** “CPL” parametrization \([65, 66]\) is given by the EoS $w(z) = w_0 + w_1 \frac{z}{1+z}$. In this case, the solution becomes

$$
\rho = \rho_0 (1 + z)^{3(1+w_0+w_1)} e^{-\frac{3w_1 z}{1+z}}
$$

The above model generates phantom energy if $w(z) < -1$ i.e. $z < -\frac{1+w_0}{1+w_1}$ provided $w_1 > -1$ and $w_1 - w_0 > 0$. The mass of the black hole is obtained as

$$
M = M_0 - \frac{4\pi^{\frac{n+1}{2}} A \sqrt{\rho_0}}{\sqrt{3} \Gamma(\frac{n+1}{2})} \left[ (1 + z)^{\frac{3}{2}(1+w_0+w_1)} e^{-\frac{w_1 z}{2(1+z)}} - 1 \right]
$$

This model is also the phantom crossing model. We have drawn $M$ vs $z$ in figure 3. From the figure, we observe that BTZ BH mass $M$ increases for redshift $z$ decreases up to certain stage of $z$ (ΛCDM stage) and then $M$ decreases (phantom era) as universe expands.

- **Model III (JBP):** The “JBP” parametrization \([67]\) is given by the EoS $w(z) = w_0 + w_1 \frac{z}{(1+z)^2}$. The solution is

$$
\rho = \rho_0 (1 + z)^{3(1+w_0)} e^{-\frac{3w_1 z^2}{2(1+z)^2}}
$$

The above model generates phantom energy if $w(z) < -1$ i.e. $z < -1 + \frac{\sqrt{4(1+w_0)w_1+w_1^2}}{2(1+w_0)}$ provided $w_0 > -1$ and $w_1 < -4(1+w_0)$. The mass of the black hole is obtained as

$$
M = M_0 - \frac{4\pi^{\frac{n+1}{2}} A \sqrt{\rho_0}}{\sqrt{3} \Gamma(\frac{n+1}{2})} \left[ (1 + z)^{\frac{3}{2}(1+w_0)} e^{\frac{3w_1 z^2}{4(1+z)^2}} - 1 \right]
$$

This model is also the phantom crossing model. From figure 4, we see that BTZ BH mass $M$ increases for redshift $z$ decreases up to certain stage of $z$ (ΛCDM stage) and then $M$ decreases (phantom era) as universe expands.

- **Model IV:** Another type of parametrization is considered as in the form of EoS $w(z) = -1 + \frac{A_1 (1+z) + 2A_2 (1+z)^2}{3[A_0 + A_1 (1+z) + A_2 (1+z)^2]}$, where $A_0, A_1$ and $A_2$ are constants \([68, 69]\). This ansatz is exactly the
cosmological constant \( w = -1 \) for \( A_1 = A_2 = 0 \) and DE models with \( w = -2/3 \) for \( A_0 = A_2 = 0 \) and \( w = -1/3 \) for \( A_0 = A_1 = 0 \). In this case, we get the solution

\[
\rho = \frac{\rho_0 [A_0 + A_1 (1 + z) + A_2 (1 + z)^2]}{A_0 + A_1 + A_2}
\]

(45)

The above model generates phantom energy if \( w(z) < -1 \) i.e., \( z < -1 - \frac{A_2}{A_1} \) provided \( A_0 < 0, A_1 > 0, A_2 < 0 \) and \( A_0 + A_1 + A_2 < 0 \). For this condition, \( \rho \) is still positive. The mass of the black hole is obtained as

\[
M = M_0 - \frac{4\pi^{n+1} A \sqrt{\rho_0}}{\sqrt{3} \Gamma \left( \frac{n+1}{2} \right)} \left[ \left\{ A_0 + A_1 (1 + z) + A_2 (1 + z)^2 \right\}^{\frac{1}{2}} - 1 \right]
\]

(46)

This model is also the phantom crossing model. From figure 5, we see that BTZ BH mass \( M \) increases for redshift \( z \) decreases upto certain stage of \( z \) and then \( M \) decreases (phantom era) as universe expands.

**Model V:** Other type of parametrization is assumed to be \( w(z) = w_0 + w_1 \log(1 + z) \). The solution is obtained as

\[
\rho = \rho_0 (1 + z)^{3(1 + w_0)} e^{\frac{3}{2} w_1 \log(1 + z)^2}
\]

(47)

The above model generates phantom energy if \( w(z) < -1 \) i.e., \( z < -1 + e^{-\frac{w_0}{w_1}} \) provided \( w_1 > 0 \). The mass of the black hole is obtained as

\[
M = M_0 - \frac{4\pi^{n+1} A \sqrt{\rho_0}}{\sqrt{3} \Gamma \left( \frac{n+1}{2} \right)} \left[ (1 + z)^{\frac{3}{2}(1 + w_0)} e^{\frac{3}{2} w_1 \log(1 + z)^2} - 1 \right]
\]

(48)

This model is also the phantom crossing model. From figure 6, we see that BTZ BH mass \( M \) increases for redshift \( z \) decreases upto certain stage of \( z \) and then \( M \) again decreases (phantom era) as universe expands.

**IV. DISCUSSIONS**

In this work, we have studied the accretion of \((n + 2)\)-dimensional charged BTZ black hole (BH). A proper dark-energy accretion model for black holes have been obtained by generalizing the Michel theory [18] to the case of black holes. Such a generalization has been already performed by Babichev et al [19, 20] for the case of dark-energy accretion onto Schwarzschild black holes. We have followed the procedure used by Babichev et al [19, 20], adapting it to the case of \((n + 2)\)-dimensional charged BTZ black hole. The critical point and square speed of sound have been obtained. Astrophysically,
mass of the black hole is a dynamical quantity, so the nature of the mass function is important in our black hole model for different dark energy filled universe. We see that the rate for the BTZ black hole exotic mass due to accretion of dark energy becomes exactly the positive to the similar rate in the case of a Schwarzschild black hole, asymptotically. Since the BTZ black hole is static, so the mass of the black hole depends on \( r \) only. When some fluid accretes outside black hole, the mass function \( M \) of the black hole is considered as a dynamical mass function and hence it should be a function of time also. So \( \dot{M} \) is time dependent and the increasing or decreasing of the black hole mass \( M \) sensitively depends on the nature of the fluid which accretes upon the black hole. The sign of time derivative of black hole mass depends on the signs of \((\rho + p)\). If \( \rho + p < 0 \) i.e., for phantom dark energy accretion, the mass of the black hole decreases but if \( \rho + p > 0 \) i.e., for quintessence dark energy accretion, the mass of the black hole increases. The mass of the BTZ BH has been calculated and we have observed that the mass of the BTZ BH is related with square root of the energy density of dark energy which accretes onto BH in our accelerating FRW universe.

We have assumed modified Chaplygin gas (MCG) as a candidate of dark energy which accretes onto BTZ BH. Since in our solution of MCG, this model generates only quintessence dark energy (not phantom) and so BTZ BH mass increases during the whole evolution of the accelerating universe, which is shown in figure 1 also. Next we have assumed 5 kinds of parametrizations (Models I-V) of well known dark energy models (some of them are Linear, CPL, JBP models). These models generate both quintessence and phantom scenarios (phantom crossing models) for some restrictions of the parameters. So if these dark energies accrete onto the BTZ black hole, then for quintessence stage, black hole mass increases up to a certain value (finite value) and then decreases to finite value for phantom stage during whole evolution of the universe. That means, if the 5 kinds of DE accrete onto BTZ black hole, the mass of the black hole increases up to a certain finite value and then decreases in the late stage of the evolution of the universe. We also shown these results graphically clearly. We have drawn the mass of the BTZ black hole for dark energy models I-V in figures 2-6 respectively. Figures 2-6 show the mass of the BTZ black hole first increases to finite value and then decreases to a finite value also.

[1] Perlmutter, S. J. et al, 1998, Nature 391, 51.
[2] Riess, A. G. et al.[Supernova Search Team Collaboration], 1998, Astron. J. 116, 1009.
[3] Briddle, S. et al, 2003, Science 299, 1532.
[4] Spergel, D. N. et al, 2003, Astrophys. J. Suppl. 148, 175.
[5] P. J. E. Peebles and B. Ratra, Astrophys. J. 325 L17 (1988).
[6] R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. 80 1582 (1998).
[7] U. Alam, V. Sahni, T. D. Saini, and A. A. Starobinsky, Mon.Not.Roy.Astron.Soc. 354, 275 (2004).
[8] C. Armendariz - Picon, V. F. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. 85 4438 (2000).
[9] M. Gasperini et al, Phys. Rev. D 65, 023508 (2002).
[10] B. Gumjudpai and J. Ward, Phys. Rev. D 80 023528 (2009).
[11] J. Martin and M. Yamaguchi, Phys. Rev. D 77 123508 (2008).
[12] H. Wei, R.G. Cai and D.F. Zeng, Class. Quantum Grav. 22 3189 (2005).
[13] A. Sen, JHEP 0207 065 (2002).
[14] R. R. Caldwell, Phys. Lett. B 545 23 (2002).
[15] A. Y. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. B 511 265 (2001).
[16] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15 1753 (2006).
[17] H. Bondi, Mon. Not. Roy. Astron. Soc. 112, 195 (1952).
[18] F. C. Michel, Astrophys. Space Sci. 15, 153 (1972).
[19] E. Babichev et al, 2004 Phys. Rev. Lett. 93, 021102.
[20] E. Babichev, V. Dokuchaev, Y. Eroshenko, J.Exp.Theor.Phys. 100 (2005) 528-538.
[21] M. Jamil, Eur.Phys.J.C62:609,2009.
[22] J. A. J. Madrid and P. F. González-Díaz, Grav. Cosmol. 14, 213 (2008).
[23] J. Bhadra and U. Debnath, Eur. Phys. J. C. 72, 1912 (2012).
[24] S. Chakraborty, N. Mazumder and R. Biswas, Europhys. Lett. 91, 40007 (2010).
[25] A. S. Majumdar, D. Gangopadhyay and L. P. Singh, arXiv:0709.3193v2 [gr-qc].
[26] B. Nayaka and M. Jamil, arXiv:1107.2025v1 [gr-qc].
[27] D. Dwivedee, B. Nayak, M. Jamil and L. P. Singh, arXiv:1110.6350v1 [gr-qc].
[28] J.A.S. Lima, D. C. Guariento and J.E. Horvath, Phys. Lett. B 693, 218 (2010).
[29] M. Sharif and G. Abbas, Chinese Phys. Lett. 28, 090402 (2011).
[30] M. Sharif and G. Abbas, Chinese Phys. Lett. 29, 010401 (2012).
[31] C. Y. Sun, Phys. Rev. D 78, 064060 (2008).
[32] S. W. Kim and Y. Kang, Int. J. Mod. Phys. Conf. Ser. 12, 320 (2012).
[33] P. Martín-Moruno et al, arXiv:0803.2005v1 [gr-qc].
[34] M. Sharif and G. Abbas, arXiv:1106.2415v1 [gr-qc].
[35] M. G. Rodrigues and A. E. Bernardiniz, arXiv:1208.1572v1 [gr-qc].
[36] G. Abbas, arXiv:1303.6945v1 [gr-qc].
[37] G. Abbas, arXiv:1309.0807v1 [gr-qc].
[38] P. Martín-Moruno et al, arXiv:astro-ph/0603761.
[39] M. Banados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. 69 (1992) 1849.
[40] M. Banados, M. Henneaux, C. Teitelboim and J. Zanelli, Phys. Rev. D 48 (1993) 1506.
[41] R. Emparan, G. T. Horowitz and R. C. Myers, JHEP 0001 (2000) 021.
[42] Jamil M and Akbar M, Gen. Relativ. Gravit. 43, 1061 (2011).
[43] G. Abhas, arXiv: 1309.0807 [gr-qc].
[44] M. Jamil and I. Hussain, Int. J. Theor. Phys. 50, 465 (2011).
[45] S. P. Kim, S. K. Kim, K. S. Soh and J. H. Yee, Phys. Rev. D 55 (1997) 2159.
[46] M. Hassaine and C. Martinez, Class. Quantum Gravit. 25 (2008) 195023.
[47] S. G. Ghosh, arXiv:1109.3263v2 [gr-qc].
[48] A. J. John, S. G. Ghosh, S. D. Maharaj, Phys. Rev. D 88, 104005 (2013).
[49] S. H. Hendi, Eur.Phys.J.C71:1551,2011.
[50] W. Shuang et al, Chin.Phys.Lett. 23 (2006) 1096-1098.
[51] D. Birmingham, I. Sachs, S. Sen, Int.J.Mod.Phys. D10 (2001) 833-858.
[52] S. K. Chakrabarti, P. R. Giri, K. S. Gupta, Eur. Phys. J C 60:169-173, 2009.
[53] E. A. L. Rubio, Turk. J. Phys. 32, 1 (2008).
[54] A. Larrañaga, Commun.Theor.Phys.50:1341-1344,2008.
[55] M. R. Setare, M. Jamil, Phys. Lett. B 681 (2009) 469.
[56] M. Akbar, A. A. Siddiqui, Physics Letters B 656 (2007) 217-220.
[57] M. Akbar, H. Quevedo, K. Saifullah, A. Sanchez, S. Taj, Phys.Rev.D83:084031,2011.
[58] S. H. Mazharimosavi, M. Halilsoy and T. Tahamtan, Phys. Lett. A 376, 893 (2012).
[59] U. DebnathEur. Phys. J. C, 74, (2014) 2869 (1-8).
[60] M. Hassaine and C. Martinez, Phys. Rev. D 75 (2007) 027502.
[61] U. Debnath, A. Banerjee and S. Chakraborty, Class. Quant. Grav., 21, 5609 (2004).
[62] Melchiorri, A., Mersini, L., Trodden, M. : Phys. Rev. D 68 043509(2003).
[63] Seljak, U., Slosar, A., McDonald, P.: JCAP 0610 014 (2006).
[64] Cooray, A. R., Huterer, D.: Astrophys. J. 513 L95(1999).
[65] Chevallier, M., Polarski, D.: Int. J. Mod. Phys. D 10 213(2001).
[66] Linder, E. V.: Phys. Rev. Lett. 90 091301(2003).
[67] Jassal, H. K., Bagla, J. S., Padmanabhan, T.: MNRAS 356 L11(2005).
[68] U. Alam, V. Sahni, T. D. Saini and A. A. Starobinski, Mon. Not. R. Astron. Soc. 354 275 (2004).
[69] U. Alam, V. Sahni and A. A. Starobinski, JCAP 0406 008 (2004).
[70] G. Efstathiou, Mon. Not. R. Astron. Soc. 310 842 (1999).
[71] R. Silva, J. S. Alcaniz and J. A. S. Lima, Int. J. Mod. Phys. D 16 469 (2007).