1. INTRODUCTION

Accretion, disks, and jets are ubiquitous in astrophysics (see, e.g., Blandford & Rees 1992). A consensus has been reached that an extra needed ingredient to obtain outflow from inflow is the presence of strong magnetic fields that thread a disk conventionally assumed to be rotating at Keplerian speeds about a central gravitating object, taken in this paper to be a newly born star. Differences come in ascribing the origin of the magnetic fields to the disk itself or to the central star (Königl & Pudritz 2000; Shu et al. 2000).

Disk winds have been extensively studied, both analytically via the assumptions of self-similarity in two-dimensional (2D) space for axisymmetric, time-independent flows (e.g., Blandford & Payne 1982; Contopoulos & Lovelace 1994) or by taking advantage of arbitrary variations of the gas pressure (e.g., Tsiganos & Trussoni 1991) or by studying the asymptotic properties of the collimation (Heyvaerts & Norman 1997); and numerically by finite-element methods attacking the axisymmetric, time-independent, Grad-Shafranov equation (GSE; e.g., in the relativistic regime by Camenzind 1987) or by finite-difference treatments of the time-dependent equations of ideal magnetohydrodynamics (MHD) in 2D and 3D (e.g., Uchida & Shibata 1985; Pudritz et al. 2006). For a review of these types of calculations, see Ferreira (2004).

The most highly developed semianalytic theory for the second viewpoint is called X-wind theory, in which fast jets arising in young stellar objects (YSOs) owe their existence to the interaction of the accretion disk with the magnetosphere of the central star. The interaction of accretion disks with strongly magnetized central stars has also been studied numerically (e.g., Goodson et al. 1999; Long et al. 2005; Ustyugova et al. 2006). Although both funnel flows and X-like winds have been found, they have yet to appear simultaneously in numerical simulations, probably because the numerical calculations have not yet proceeded to steady state where the condition of disk-locking applies (Shu et al. 1994a). Pure X-wind theory assumes for simplicity that the disk itself is unmagnetized; in fact, all that is needed for the theory to work is for open field lines to be concentrated in a narrow annulus near the inner edge of an accretion disk.

Recently, Bacciotti et al. (2002) and Coffey et al. (2004) identified jet rotation in four T Tau systems, DG Tau, TH 28, RW Aur, and LkHα 321, of an amount too large to be compatible with X-winds, but consistent with launching from disks at radii of 0.5−2 AU. Later, Cabrit et al. (2006) showed from millimeter-wave radio measurements that the disk rotation in RW Aur is actually in the opposite sense to that deduced for the jet from optical lines. Moreover, Pety et al. (2006) find that HH 30, which is observed nearly edge-on and therefore should have had the clearest signature for jet rotation, showed no evidence for outflow rotation at millimeter wavelengths, a conclusion reinforced by optical and ultraviolet observations of the HH 30 jet by Coffey et al. (2007). While the positive results remain for the three other systems, the case of HH 30, where longitudinal velocities occur in the direction transverse to the line of sight, suggests that the slight line asymmetries in the other cases may be more associated with unequal jumps in the velocity of shocked, high-speed jets than to the rotation of collimated outflows.

In contrast, no one has proposed any explanation other than X-winds for the correlated inflow-outflow signatures seen in SU Aur by Giampapa et al. (1993) and Johns & Basri (1995). Apart from SZ 68 (Johns-Krull & Hatzes 1997), we are unaware of any other T Tau star that shows a tilted-dipole magnetic field geometry, and it could be that the dipole component is small on the surface of most T Tau stars (Johns-Krull 2007). Fortunately, S. Mohanty & F. H. Shu (2008, in preparation) show that while funnel flows are sensitive to the detailed assumptions made concerning multipole structure on the surfaces of the central stars, the properties of the X-wind depend mostly on the amount of trapped flux in the
X-region (see also the observational evidence relating to this point collected by Johns-Krull & Gafford 2002).

Recent calculations show that YSOs are unlikely to lose enough magnetic flux in the process of gravitational collapse to make the level of magnetization ignorable in the resultant circumstellar accretion disks (Galli et al. 2006; Shu et al. 2006, 2007). Indeed, the disks are sufficiently magnetized in many cases that, in quasi-steady state, they rotate at sub-Keplerian rates until the inner disk edge is reached. Thus, there are open questions of how much of the trapped flux near the inner edge is to be attributed to the central star versus the disk and how such disks reacquire near-Keplerian rates of rotation at their inner edges. We ignore these complications in the present study of the X-wind phenomenon, but we note that the methods introduced here are easily modified to attack the more complex problem when the accretion disk interacting with a stellar magnetosphere is itself strongly magnetized.

The original X-wind model supposed the outflow to occur from the equator of a magnetized star spun to breakup by the presence of an accretion disk that abutted its surface (Shu et al. 1988). Later, in order to accommodate the slow rotators, such as the classical T Tauri stars which are only rotating at one-tenth of breakup (Vogel & Kuhi 1981; Bouvier 1990; Edwards et al. 1993), Shu et al. (1994a) generalized the X-wind picture to include the case of relatively low accretion when the magnetosphere of the star would truncate the accretion disk at an inner edge before the disk reached the stellar surface (typically a circle of radius 0.2 on the scale of Fig. 1, where the disk’s inner edge is taken to be at $\varpi = 1$). In a quasi-steady state where most of the mass of the central star is built up by disk accretion, the magnetic coupling between the star and the disk regulates the star to corotate at the Keplerian frequency at the truncation radius. For a protostar with magnetic dipole moment $\mu$, mass $M$, and mass accretion rate $\dot{M}$, Ostriker & Shu (1995) estimate this radius to be

$$R_X = \Phi_{dz}^{-4/7} \left( \frac{\mu^4}{GM^2} \right)^{1/7},$$

where $\Phi_{dz}$ is an order unity dimensionless number that parameterizes the amount of stellar magnetic flux that is trapped in the disk. Inside this radius, matter is channeled to the star via a funnel flow. The excess angular momentum of the accreting material is deposited in the magnetic field in the form of Maxwell torque and then transported back to the disk. The gain of angular momentum and approximate field freezing would try to move the footpoint of the funnel-flow field lines outward.

Exterior to the truncation radius $R_X$, the equatorial inward drift in the accretion disk creates an angle between the stellar magnetic field lines and the disk normal. If approximate field freezing holds as the accretion proceeds, a fraction of the field lines will develop an angle larger than 30°, when matter frozen to this flux tube becomes unstable to magnetocentrifugal fling (Blandford & Payne 1982). These field lines are thus responsible for driving a MHD wind from the disk. Since the wind removes angular momentum from the disk, the footpoints of those field lines in the disk will try to migrate inward. The radially inward press of the footpoints of the wind field lines and the radially outward press of the footpoints of the funnel-flow field lines create a magnetic X-configuration that distinguishes the model from similar variants in the literature (the historical choice of the name from the X-point of the equivalent gravitational potential in the corotating frame is common to many models). In quasi-steady state where radial advection into the X is balanced by the resistive diffusion of field lines out of the X, Shu et al. (1994b) estimated that enough stellar flux could be trapped in a small X-region near the inner disk edge to have large dynamical effects, namely, the truncation of the disk by a funnel flow out of the disk plane accompanied by an X-wind that carries away most of the excess angular momentum transported into the X-region.

Apart from the original numerical estimates, there are reasons to suppose that if turbulent resistivity is the source of the diffusion across magnetic field lines (Shu et al. 2007), then the fractional size of the X-region in units of $R_X$ is given by the ratio of sound speed at the surface of the disk where the X-wind is launched to the local Keplerian speed at $R_X$. For the inner disk of a classical T Tauri star, the thermal sound speed is $a \approx 5 \text{ km s}^{-1}$, while the Keplerian speed at $R_X$ is $v_K \sim 100 \text{ km s}^{-1}$ (Najita et al. 2007); thus, the ratio $\epsilon$ is a small number $\sim 0.05$. In an asymptotic analysis where $\epsilon$ is taken to $0$, the X-wind tied to the trapped field lines in the X-region would emerge from virtually a single point in the meridional plane with a fanlike geometry. Seen by an observer rotating at the Keplerian angular frequency of $R_X$, gas flows along streamlines that coincide with field lines if field freezing is assumed, and both patterns of streamlines and field lines remain stationary in the corotating frame.

Viewed in this fashion, the overall problem can be broken into smaller pieces and tackled separately. Using a formulation with a precedent in the work of Lovelace et al. (1986), Shu et al. (1988,
1994b) wrote down the mathematical equations that describe a steady state axisymmetric flow in the corotating frame (Grad & Rubin 1958; Shafranov 1966). By the method of matched asymptotic expansions, Shu et al. demonstrated the existence of an inner solution in the X-region where the flow makes a sonic transition and an outer solution where the sound speed is formally taken to zero and the X-region shrinks down to a point. Najita & Shu (1994) computed numerically the portion of the X-wind in which the fluid velocity is sub-Alfvénic and the governing equation is elliptic. Ostriker & Shu (1995) solved the problem of the funnel flow and the field configuration in the dead zone (in which the field lines do not depart sufficiently from disk normal to load any matter) in an approximation that treated the accretion flow onto the star as highly sub-Alfvénic. Shu et al. (1995) constructed asymptotic solutions that describe the logarithmically slow, far-wind collimation into jets. The free boundaries between various parts of the problem (funnel flow, dead zone, and X-wind) were determined by pressure balance on either side.

In this paper we wish to address the X-wind part of the overall problem. In order for the X-wind to accelerate from rest to supersonic speeds, it must smoothly pass three surfaces on which the fluid velocity is equal to the slow MHD, Alfvén, and fast MHD velocity, respectively (Heinemann & Olbert 1978; Sakurai 1985). These critical surfaces manifest themselves as singularities in the governing equation (see Weber & Davis 1967) and thus need to be handled analytically. In an axisymmetric problem, if the shapes of the streamlines are known in the meridional plane, the conserved quantities of the problem (mass-to-flux loading, angular momentum flux including that carried in the Maxwell stress, and Bernoulli’s integral along a streamline) suffice to give a completely analytic solution, including the locations and conditions required to cross the critical surfaces smoothly. Unfortunately, the streamline distribution in the meridional plane is not known a priori but must be obtained, in principle, from a solution of the GSE. The spatial location of the critical surfaces are part of the overall solution of the GSE; indeed, they characterize the regions where this partial differential equation (PDE) is elliptic or hyperbolic. The mixed character of the GSE makes a direct numerical attack extremely difficult when self-similarity does not apply, perhaps the hardest problem in the mathematical theory of nonlinear PDEs of second order (see Garabedian 1986). The current work sidesteps the mathematical solution of the GSE as a nonlinear PDE of second order and approaches it instead as a much more amenable problem in variational calculus.

The rest of this paper is organized as follows. In § 2 we review the basic formulation in terms of a stream function and an Alfvén discriminant that yields the PDEs—the so-called Grad-Shafranov and Bernoulli equations—that govern a steady, axisymmetric, X-wind flow. In § 3 we write down an action whose variations with respect to the stream function $\psi(w,z)$ and the Alfvén discriminant $\mathcal{A}(w,z)$ yield, respectively, the GSE and the Bernoulli equation (BE) when the action reaches an extremum. We also perform a transformation where we replace the vertical coordinate $z$ in cylindrical coordinates $(w, \varphi, z)$ by $\psi$. In § 4 we show how to incorporate boundary conditions into the problem, as well as how to take advantage of the fact that analytic forms are known for the solutions in the near-neighborhood of the X-point and in the asymptotic regime far from the X-point (Shu et al. 1994b, 1995). In § 5 we outline a practical implementation of the principle of extremal action, making use of only variations of $\psi$—or, more precisely, of $w(\varphi, \psi)$ in our actual working space—as the substitute to attacking the GSE, while we solve BE directly for reasons that are expounded on in this section. In § 6 we present numerical results for three specific cases of mass loading onto wind flux tubes, finding good agreement with previous approximate solutions obtained by Shang (1998) that have been used for many different astrophysical applications (e.g., Shang et al. 1998, 2002, 2004). In § 7 we summarize the recipes needed to convert the numerical solutions of § 6 into practical dimensional models. We then offer our conclusions and suggestions for needed future research.

2. BASIC EQUATIONS

From the fundamental parameters of the problem, we may construct units of length, time, and density as $R_X$, $\Omega_X^{-1}$, and $M_w / 4\pi R_X^3 \Omega_X$, respectively. By assuming axisymmetry and stationarity in a frame that is rotating with angular velocity $\Omega_X$, we may write down the dimensionless governing equations in the above units,

$$\nabla \cdot (\rho u) = 0,$$

$$\nabla \left( \frac{1}{2} |u|^2 \right) + (2e_z + \nabla \times u) \times u = -\frac{\epsilon}{\rho} \nabla \rho - \nabla V_{\text{eff}} + \frac{1}{\rho} (\nabla \times B) \times B,$$

$$B \times u = 0,$$

$$\nabla \cdot B = 0,$$

where $\epsilon \equiv a/R_X^2 \Omega_X$ is the sound speed measured in units of Keplerian velocity at the X-point and is assumed to be a small parameter of the problem. The effective potential in the corotating frame is defined as

$$V_{\text{eff}} = -\frac{1}{\sqrt{\omega^2 + z^2}} - \frac{1}{2} \omega^2 + \frac{3}{2}.$$  

Here we have added a constant term to the effective potential so that its numerical value is zero at the X-point.

2.1. Constants of Motion

The continuity equation (2a) is satisfied identically if we define the poloidal velocity through a stream function (Shu et al. 1988, 1994a),

$$\rho u_\varphi \equiv \frac{1}{\omega} \frac{\partial \psi}{\partial \varphi}, \quad \rho u_z \equiv -\frac{1}{\omega} \frac{\partial \psi}{\partial \omega}.$$  

For steady state axisymmetric flow in the corotating frame, the field freezing condition from equation (2c) demands that the magnetic field and the velocity are related by (see, e.g., Mestel 1968)

$$B = \beta \rho u.$$  

With this identification, the continuity equation (2a) and the absence of magnetic monopoles (eq. [2d]) imply $u \cdot \nabla \beta = 0$. In terms of the stream function, this means $\beta$ is constant along each streamline, or $\beta = \beta(\psi)$.

The Euler equation describes momentum and energy balance in three spatial dimensions. If we take the component along the fluid velocity by taking the inner product of equation (2b) with $u$, we obtain the BE along streamlines

$$u \cdot \nabla H = 0,$$

where $H \equiv \frac{1}{2} |u|^2 + e^2 \ln \rho + V_{\text{eff}}$.  

In other words, $H = H(\psi)$, and the energy per unit mass of an isothermal gas, including its specific enthalpy, is conserved along a streamline in the corotating frame where the flow occurs parallel...
to $B$. Similarly, if we take the toroidal component of the Euler equation (2b), we obtain a third conserved quantity along streamlines, the angular momentum of the gas allowing for that part carried away by the Maxwell torque of the field,

$$J \equiv \omega^2 + \omega(1 - \beta^2 \rho)u_e = J(\psi). \quad (7)$$

As we see below, the determination of the conserved quantities, $H(\psi)$ and $J(\psi)$, is achieved by demanding that the X-wind crosses the slow MHD and fast MHD surfaces smoothly. The loading of mass onto flux, which is governed by $\beta(\psi)$, is freely specifiable within certain limits to be detailed below.

The last component of the Euler equation describes momentum balance in the direction perpendicular to the poloidal field lines. It is the famous GSE (Heinemann & Olbert 1978; Sakurai 1985),

$$\nabla \cdot (A\nabla \psi) + \frac{1}{A} \left( \frac{J}{\omega^2} - 1 \right) \frac{J'}{\omega^2} + \frac{\beta^2 V_{\text{eff}} + \epsilon^2 \ln[\epsilon^2 h/((\beta^2 - \omega^2)A)]}{(\beta^2 - \omega^2)A^2} - \frac{\epsilon^2 h/h}{\beta^2 - \omega^2} = 0, \quad (8)$$

where we have rescaled Bernoulli’s function as

$$H \equiv -\epsilon^2 \ln(\epsilon^2 h), \quad (9)$$

so that $h$ remains an order unity quantity in our calculation. Here $A$ is the Alfvén discriminant defined by

$$A \equiv \frac{M_\alpha^2 - 1}{\omega^2 \rho}, \quad (10)$$

where

$$M_\alpha^2 \equiv \frac{\rho u_e^2}{B^2} = \frac{1}{\beta^2 \rho} \quad (11)$$

is the Alfvén Mach number. Hence $A$ is positive when the total velocity is less than the Alfvén speed and negative when the total velocity is larger than the Alfvén speed. From the form of the GSE (8), we see that the conserved angular momentum flux $J$ is not freely specifiable. It is determined by the condition of smooth Alfvén transition. In order for the solution to remain continuous and differentiable, one must impose

$$J = \omega^2 \text{ whenever } A = 0. \quad (12)$$

The elimination of $\rho$ in the equations in favor of $A$ is based on numerical considerations, since $\rho$ will in general vary by many orders of magnitude, while $A$ only varies moderately. As we argued in § 1, the sound speed $\epsilon$ is likely to be small. In terms of these variables, the BE takes the form

$$|\nabla \psi|^2 + \frac{1}{A^2} \left( \frac{J}{\omega^2} - 1 \right)^2 + \frac{2\omega^2 V_{\text{eff}} + \epsilon^2 \ln[\epsilon^2 h/((\beta^2 - \omega^2)A)]}{(\beta^2 - \omega^2)A^2} = 0. \quad (13)$$

2.2. The Cold Limit

With $A$ implicitly defined in the BE (13), the GSE is a PDE of mixed type, which demands different numerical methods in different regions (see Heinemann & Olbert 1978 and the Appendix). There are three relevant signal speeds (which we term sonic, slow, and fast in the Appendix) involved in an MHD flow (see Jackson 1975; Shu 1992). The loci where the poloidal fluid speed equals those signal speeds separate the flow into four regions. As the poloidal velocity exceeds the sonic speed, the governing GSE changes from elliptic to hyperbolic. A wise strategy might start with the search of appropriate boundary conditions in the disk where $u_e^* = 0$ and at the sonic surface (whose location is still undetermined), followed by a standard scheme (e.g., relaxation) to obtain the interior solution. Beyond the sonic surface, the GSE becomes hyperbolic. The boundary condition on the sonic surface now serves as the initial condition, which we use to integrate forward along characteristics toward the slow surface. We then follow similar procedures to obtain solutions from the slow surface to the fast surface, and beyond.

A significant simplification can be achieved when the sound speed is negligible, as in the outer problem of the X-wind (see § 4 of Shu et al. 1994b). The governing equations are treated as a power series expansion in $\epsilon$. The leading term in the GSE (8) and the BE (13) are

$$|\nabla \psi|^2 + \frac{1}{A^2} \left( \frac{J}{\omega^2} - 1 \right)^2 + \frac{2\omega^2 V_{\text{eff}}}{(\beta^2 - \omega^2)A^2} = 0. \quad (14a)$$

$$|\nabla \psi|^2 + \frac{1}{A^2} \left( \frac{J}{\omega^2} - 1 \right)^2 + \frac{2\omega^2 V_{\text{eff}}}{(\beta^2 - \omega^2)A^2} = 0. \quad (14b)$$

Note that the lowest order term in $H$ vanishes independent of the form of $h$. In this limit, both the sonic speed and the slow speed reduce to zero, and the first elliptic and hyperbolic parts of the flow shrink down to the X-point. We are thus spared the vicissitudes of this portion of the problem. Once the fluid leaves the X-region (with poloidal velocity greater than the slow speed), it proceeds to the fast surface, where the governing equation becomes hyperbolic.

Najita & Shu (1994) solved the GSE in the sub-Alfvénic region. By introducing a generalized coordinate system, they were able to map the location of the Alfvén surface to a known location and determine the functional form of $\beta(\psi)$ based on the position and shape of the Alfvén surface. Their numerical scheme to find $\beta(\psi)$ by iteration encountered a systematic “drift problem,” however, and an artificial “Alfvén seam” was invented to cope with this difficulty.

In a later treatment by Shang (1998), $\beta(\psi)$ was specified in advance, limited in its functional form by considerations of how the gas exits the X-region, an analysis that we repeat in § 4.2 (see also § 5). The GSE was not solved as a PDE, but rather as an error estimator in a Weber-Davis type of analysis, where $\psi$ as a trial function of spatial location is obtained by interpolating between the known analytic forms in the X-point neighborhood (see § 4.1) and at asymptotic infinity (see § 4.3). The interpolation formula has a number of degrees of freedom, which are adjusted to give “least error” in some sense when the trial solution for $\psi$ is substituted back into the GSE. The rest of the problem, including the constraints of the conserved quantities and smooth passage through the Alfvén and fast surfaces, are performed exactly. She verified the result derived by Goldreich & Julian (1970) that passage through the Alfvén surface is automatic in such a scheme if one has guaranteed it through the fast surface. In fact, § 5.2 demonstrates the falsity of the frequent claim made otherwise in the literature that $J(\psi)$ is set at the Alfvén surface; the claim holds only if one already has a solution such that the wind passes smoothly through the fast surface.
3. VARIATIONAL PRINCIPLE

Based on the above arguments, the X-wind is a fierce mathematical beast, and a direct numerical attack is unlikely to subdue it fully. To construct a global solution of the X-wind that accelerates elements of plasma from the disk to supermagnetosonic speeds, we must resort to a different approach. Consider the following action written down by inspection,

$$ S = \int \left\{ \frac{1}{2} A |\nabla \psi|^2 - \frac{1}{2} A \left( \frac{J}{\varpi^2} - 1 \right) \right\} d^3 x, $$

It is straightforward to demonstrate that variation against \( \psi \) yields the GSE (8), while variation against \( A \) gives the BE (13). The challenges of constructing solutions to a nonlinear PDE of mixed type is now transformed to tuning trial functions of \( \psi \) and \( A \) until a local extremum of the action from equation (15) is reached.

To formulate a scheme that is easy to implement numerically, we consider a change of independent variables from the usual cylindrical coordinates

\[(\varpi, z, \varphi) \to (\varpi, \psi, \varphi).\]

For a given value of \( \psi \), the functional form of \( z(\varpi) \) determines the shape of the given streamline, and \( A(\varpi) \) offers information on the velocity distribution along that streamline. Written in these new coordinates and taking the cold limit as \( \epsilon \to 0 \), the action reads

$$ S = 2 \pi \int \left\{ \frac{1}{2} A \left( \frac{J}{\varpi^2} - 1 \right) \left[ \frac{1}{2} \left( \frac{\partial \varpi}{\partial \psi} \right)^2 + \frac{\partial \varpi}{\partial \psi} \beta \varpi \right] \right\} \varpi d\psi d\varpi. \tag{16} $$

Since \( A \) only enters the action as a constraint rather than a dynamic variable (i.e., its derivative is absent in the action), variation with respect \( A \) yields the BE as before, but now written in a different set of coordinates,

$$ \left( \frac{\partial \varpi}{\partial \psi} \right)^{-2} \left[ 1 + \left( \frac{\partial \varpi}{\partial \psi} \right)^2 \right] + \frac{1}{A^2} \left( \frac{J}{\varpi^2} - 1 \right)^2 + \frac{2 \varpi^2 V_{\text{eff}}}{\left( \beta^2 - \varpi^2 A \right)^2} = 0. \tag{17} $$

Variation with respect to \( z \) gives

$$ \delta S_z = 2 \pi \int \left\{ \frac{1}{2} A \left( \frac{J}{\varpi^2} - 1 \right) \left[ \frac{1}{2} \left( \frac{\partial \varpi}{\partial \psi} \right)^2 + \frac{\partial \varpi}{\partial \psi} \beta \varpi \right] \right\} \varpi d\psi d\varpi. $$

Integrating by parts, we have

$$ \delta S_z = 2 \pi \int \left\{ \frac{1}{2} A \left( \frac{J}{\varpi^2} - 1 \right) \left[ \frac{1}{2} \left( \frac{\partial \varpi}{\partial \psi} \right)^2 + \frac{\partial \varpi}{\partial \psi} \beta \varpi \right] \right\} \varpi d\psi d\varpi. $$

Since we specify the boundary condition \( z = 0 \) on \( \psi = 0 \) and \( z \equiv Z(\varpi) \) on \( \psi = 1 \), where \( Z(\varpi) \) is a known function, we see that \( \delta z \) vanishes on these two boundaries. As we see below (see Shu et al. 1994b, 1995), the solution near the X-point and asymptotically can be constructed analytically. Thus, \( \delta z \) also vanishes when \( \varpi = 1 \) and \( \varpi \to \infty \) in our variational scheme, and both surface terms vanish in the above expression. In order for the action to be stationary against any choice of \( \delta z \), the solution must satisfy the Euler Lagrange equation,

$$ \frac{\partial}{\partial \psi} \left( \varpi \frac{\partial V_{\text{eff}}}{\partial \psi} \right) - \frac{\partial V_{\text{eff}}}{\partial \psi} \varpi + \frac{\partial \varpi}{\partial \psi} = 0. \tag{18} $$

Dividing both sides by \( \varpi \), we can simplify the Euler-Lagrange equation (18) to obtain

$$ \frac{\partial}{\partial \psi} \left( \varpi \frac{\partial V_{\text{eff}}}{\partial \psi} \right) - \frac{\partial V_{\text{eff}}}{\partial \psi} \varpi + \frac{\partial \varpi}{\partial \psi} = 0. \tag{19} $$

We notice that the coefficient of \( \partial A / \partial \psi \) is simply the BE, which vanishes at a local extremum of the action. One may easily check that the other terms yield the conventional GSE, but written in our new coordinates.

4. BOUNDARY CONDITIONS

4.1. X-Point

With the new coordinates, the computational domain is bounded by \( \psi \in [0, 1] \) and \( \varpi \in [1, \infty) \). The X-point in these coordinates is a
singularity given by \( \varpi = 1 \) for all values of \( \psi \). Fortunately, we have analytic solutions there. From this point onward, we work with a scaled Alfvén discriminant

\[
\chi = \frac{A}{\beta \varpi}.
\]

(20)

This function has the advantage of remaining finite even when \( \beta \) diverges. For a given functional form of \( \beta \) (which tells us how matter is loaded on the field lines), the Alfvén discriminant has the series expansion in \( \varpi - 1 \),

\[
\chi_X = 1 + \chi_1(\varpi - 1) + \chi_2(\varpi - 1)^2 + \ldots,
\]

(21)

where a subscript \( X \) reminds us that this series solution is valid near the X-point. In order to match asymptotically onto the outer limit of the inner problem (Shu et al. 1994b), the density near the X-point forms an angle of

\[
\theta = \tan \left( \frac{1}{2} \right),
\]

which has the solution

\[
\theta = \tan \left( \frac{1}{2} \right), \quad \phi = \frac{1}{K} \int_0^\psi \beta d\psi.
\]

(23)

If we assume that the upper boundary of the X-wind (\( \psi = 1 \)) near the X-point forms an angle of \( \theta_X \) with the \( x \)-axis, we have

\[
\tan \theta_X \approx \frac{z}{\varpi - 1} = z_1(\varpi - 1) = \tan \left( \frac{1}{K} \int_0^1 \beta d\psi \right).
\]

Thus, the integration constant is given by

\[
K = \frac{\beta}{\partial \chi}, \quad \beta = \frac{1}{K} \int_0^1 \beta d\psi.
\]

(24)

Substituting back into the BE allows us to solve for \( \chi_2 \),

\[
\chi_2 = 3 - \frac{\sqrt{4 \cos^2 \theta - 1}}{K \beta \cos^2 \theta}.
\]

(25)

For better numerical accuracy, we carry out the computation to next order. The next term in the series expansion of the GSE is \( O((\varpi - 1)^{-1}) \),

\[
\frac{\partial^2 Q}{\partial \theta^2} + Q = \sin \theta,
\]

where \( Q = z_2 \cos^3 \theta \). Given the boundary condition \( z = 0 \) at \( \psi = 0 \) for all values of \( \chi \), the above equation may be solved to give

\[
z_2 = \frac{1}{2} \sec^2 \phi (q \tan \theta - \phi).
\]

(26)

The integration constant \( q \) can be determined by expanding the upper boundary near the X-point. Substituting into the BE, we can determine the last term without the knowledge of \( J \) as

\[
\chi_3 = \frac{\cos^2 \theta - 2 + \tan \phi (q \tan \theta - \phi)}{2K \beta \cos^2 \phi \sqrt{4 \cos^2 \phi - 1} - 1} + \sqrt{4 \cos^2 \phi - 1} (4 - q \cos^2 \phi) - 4.
\]

(27)

### 4.2. Specifying Mass Loading and Difficulties with the Boundary Layer

Since the X-wind is driven magnetocentrifugally, one would naively expect that it is bounded away from the polar axis (at least in the immediate vicinity of the X-point) by some curve which intersects the disk. In the outer limit of the inner problem (see eq. [3.10d] of Shu et al. 1994b), the gas pressure \( \rho = \epsilon \rho \) takes the form

\[
p = \left[ \frac{\beta}{\partial \phi(0) \beta} \right] \sigma^2 (4 \cos^2 \phi - 1)^{-1/2}, \quad \text{as } \sigma \to \infty,
\]

where \( \tan \phi = \varpi(\varpi - 1) \). As \( \theta \to \pi/3 \) (which is the critical angle for the last matter-carrying streamline), the pressure diverges unless

\[
\beta \propto (4 \cos^2 \phi - 1)^{-1/2}, \quad \text{as } \phi \to 1.
\]

Substitute this functional form into the lowest order equation (23), finite magnetic field and pressure on the last streamline demands

\[
\beta \propto (1 - \psi)^{-1/3}, \quad \text{as } \psi \to 1.
\]

(28)

The divergence of \( \beta \) should not come as a surprise. Recall that the last streamline is defined to be the boundary between the X-wind and the dead zone. To ensure analyticity across this boundary, we must have \( \rho \to 0 \) as \( \psi \to 1 \). Now since neither the magnetic field nor the velocity become singular, we must take \( \beta \to \infty \) on that last streamline, so that the product \( \beta^2 \rho^2 = B^2 / \mu^2 \) remains finite. With this limit in place, we see that the rescaled Alfvén discriminant

\[
\frac{1 - 1/\beta^2 \rho}{\varpi^2} \to 1/\varpi^2
\]

remains positive for all points along the last streamline. In other words, the flow on the last streamline is always sub-Alfvénic, since the Alfvén speed is infinite there. This behavior of the last streamline requires a double limiting procedure if we were to accurately construct the asymptotic solution on that interface. We speculate that this difficulty is an indication that the last streamline needs to be treated as a boundary layer. This speculation is
reinforced by the fact that the last X-wind streamline is the outer bounding surface to a sheet of axisymmetric current defined by opened stellar field lines that reverse poloidal directions as the current sheet is crossed and we find ourselves in the dead zone of the overall X-wind/ funnel-flow configuration (see Fig. 1). Until we actually construct such a boundary layer/current sheet theory, we adopt a simple modification to deal with the problem; we truncate the formal wind solution at some \( \psi_1 < 1 \), below which \( J \) and \( \beta \) remain finite. We then add the part between \( \psi_1 \) and 1 to the dead zone fields of the problem, i.e., treat the last few streamlines as opened vacuum fields and impose the pressure balance condition at \( \psi_1 \).

4.3. Asymptotic Solution

The asymptotic solution at large distances from the X-point was constructed by Shu et al. (1995). In particular, for a wind reaching more or less constant terminal velocity, its density scales roughly as \( \rho \propto \varpi^{-2} \), and the Alfvén discriminant \( \chi \rightarrow -1/\beta^2 \rho \varpi^{-2} \) is a slowly varying function of \( r \). By ignoring all radial derivatives compared to angular derivatives, the GSE and the BE admit solutions of the form

\[
\chi = -1/\beta C, \quad \sin \theta = \text{sech} [C^{-1} I(C, \psi)],
\]

\[
I(C, \psi) = \int_0^{\psi} \frac{\beta d\psi}{\sqrt{2J - 3 - 2C\beta}},
\]

where \( \theta \) is the usual polar angle in spherical coordinates and \( C \) is a “constant of integration” that varies slowly in \( \psi \). For any given (large) value of \( r \), the wind reaches a terminal velocity given by

\[
v_w = (2J - 3 - 2C\beta)^{1/2}.
\]

To determine the constant \( C \), we impose pressure balance between the X-wind and the dead zone. Since the dead zone field lines carry no inertia, they do not develop a toroidal component, and the poloidal field satisfies the vacuum equation (Ostriker & Shu 1995). Asymptotically, we do not expect the field lines to pinch toward the rotational axis, since the hoop stress is vanishingly small (Shu et al. 1995). For simplicity, we assume the boundary layer deviates only slightly from a cylindrical surface at the asymptotic infinity (an assumption which is checked a posteriori for consistency). For any given (large) value of \( r \), we can approximate the boundary locally by \( \varpi = \text{const} \). Then a particular solution is

\[
B = B_{\text{hc}} \hat{\varpi}.
\]

For the hollow-cone region to trap the same amount of net flux as the wind part and have a cross-sectional area of \( \pi \varpi_{\text{bc}}^2 \), we have

\[
B_{\text{bc}} = \frac{2\phi_{\text{hc}} \beta}{\varpi_{\text{bc}}},
\]

where \( \phi_{\text{hc}} \) is a number ranging from 1 to 3 depending on the fraction of closed field lines in the dead zone (compare Fig. 1 of this paper with Fig. 1 of Shu et al. 2001). The maximal case \( \phi_{\text{hc}} = 3 \) has 3 times as many field lines as the minimal case \( \phi_{\text{hc}} = 1 \), but the extra field lines cancel in oppositely directed pairs and contribute no net flux. The overall solution does not depend sensitively on the number \( \phi_{\text{hc}} \), and we take \( \phi_{\text{hc}} \) henceforth to be unity as illustrated in Figure 1 of the current paper.

In contrast, the wind region is dominated by the toroidal field

\[
B_{w, \varphi} = \beta p u_{\varphi} = \frac{J - \varpi^2}{\varpi^2 \chi \beta} \rightarrow -\frac{C}{\varpi}.
\]

The poloidal field \( B_{w, \varphi} = \beta p u_{\varphi} \propto 1/\varpi^2 \) is much weaker in this limit. By equating the magnetic pressures on both sides of the boundary, \( B_{\text{bc}}^2 = B_{w, \varphi}^2 \), we obtain

\[
C = \frac{2\beta}{\varpi_{\text{bc}}} = \frac{2\beta}{r} \cosh [C^{-1} I(C, 1)],
\]

which implicitly defines \( C(r) \). Since \( I \) only depends very weakly on \( C \), this expression shows that \( C \rightarrow 0 \) logarithmically as \( r \rightarrow \infty \). Note that this limiting behavior of \( C \) ensures that \( \varpi_{\text{bc}} \) deviates from a constant only logarithmically slowly, which validates our assumption on the geometry of the boundary layer. Written in our coordinates, the asymptotic geometry of each streamline is given by

\[
z = \varpi \sinh [C^{-1} I(C, \psi)].
\]

In other words, each streamline is approximately radial, with a logarithmic collimation toward the axis.

With \( \rho \rightarrow C/\beta \varpi^2 \) and \( v_w \rightarrow \text{const} \), the poloidal and toroidal Alfvén speeds are given by

\[
v_{A, \varphi}^2 = \frac{B_{w, \varphi}^2}{\rho} \rightarrow C\beta, \quad v_{A, \varphi}^2 = \frac{B_{w, \varphi}^2}{\rho} \rightarrow C\beta.
\]

Thus, the Alfvén speed is dominated by the toroidal component, which decreases logarithmically. This means that the (poloidal) terminal velocity is superfast in the asymptotic regime, and the wind has to make a fast mode transition along each streamline. The above analysis simply reiterates the claims made in the Appendix that the asymptotic behavior of the flow is governed by a hyperbolic differential equation.

5. GLOBAL SOLUTIONS

As a particular example, let us suppose that diffusive mass loading onto field lines in the X-region produces a \( \beta \)-function which has the form

\[
\beta = \frac{2}{3} \beta(1 - \psi)^{-1/3}.
\]

It is easy to verify that \( \int_0^1 \beta \psi d\psi = \bar{\beta} \). To be definite, let us also assume that the upper boundary of the X-wind near the X-point forms the maximum angle \( \bar{\theta}_x = \pi/3 \) with the equatorial plane in order for magneto-centrifugal acceleration to operate. The \( O((\varpi - 1)^2) \) solution from equation (23) takes the form

\[
z_1 = \tan \vartheta = \tan \frac{\pi}{3} \left[ 1 - (1 - \psi)^{2/3} \right].
\]

In fact, we have chosen a very special value for the opening angle. Recall that the formal boundary between the X-wind and the dead zone is characterized by vanishing \( \rho \) with finite magnetic field. That results in \( \beta \rightarrow \infty \) and \( \chi = \varpi^{-2} \). If \( \bar{\theta}_x \) were smaller than \( \pi/3 \), one may check that the boundary condition \( \chi = \varpi^{-2} \) agrees with the series solution of \( \varpi \rightarrow 1 \) to the second order for all values of \( q \). This integration constant is computed by expanding the shape of the last streamline near the X-point. However, when \( \bar{\theta}_x = \pi/3 \), the series solution agrees with the boundary condition only if the quantity \( q \) in equation (26) satisfies

\[
q = \frac{1}{3} \left( \frac{7}{4} + \frac{\pi}{\sqrt{3}} \right).
\]

If \( \bar{\theta}_x > \pi/3 \), the solution becomes discontinuous. This behavior is consistent with our physical intuition. When the flow is cold,
As one eases up the external pressure, \( \varepsilon \) increases. However, even when the external pressure drops to zero, the matter-carrying streamlines are confined to \( \vartheta \leq \pi/3 \), since it is the boundary where centrifugal effects can overcome gravity. At least near the X-point, there is no freedom to choose the shape of the last streamline. Any excursion across this boundary requires additional pressure support from the X-wind, which calls for a warm rather than cold outflow.

In the particular example we are studying here, the second-order coefficient for \( z \) becomes

\[
\frac{1}{2} \sec^2 \vartheta \left[ \left( \frac{7}{12} + \frac{3}{\sqrt{3}} \right) \tan \vartheta - \vartheta \right].
\]

For numerical tractability, we place the boundary layer at \( \psi = 0.99 \). Given the choice of mass loading in equation (34), the \( \beta \)-function is not much larger than unity there.

5.1. Fixing the Free Function \( J(\psi) \)

The BE (14b) is actually a quartic algebraic equation for the Alfvén discriminant once the shape of the streamlines are known. The Alfvén surface here is not a real singularity of the equation; it simply ensures that \( \chi = 0 \) is a solution when \( J = \varpi^2 \). In other words, smooth crossing of the Alfvén surface does not uniquely determine the value of \( J \). To see this, let us define

\[
\mathcal{L} = \varpi \rho u_z = -\frac{1}{\beta^2 \chi} \left( \frac{J}{\varpi^2} - 1 \right).
\]

It is always negative and asymptotes to zero for the wind, since the magnetic field lines form a trailing spiral. The BE can be written as

\[
(\nabla \psi)^2 + \mathcal{L}^2) (\beta^2 \mathcal{L} + J - \varpi^2)^2 + 2 \varpi^2 V_{\text{eff}} \mathcal{L}^2 = 0.
\]

As long as \( J \) is larger than some critical value, there are always real and finite solutions to this equation, which means the Alfvén surface is automatically crossed. On the other hand, the fast point is a real critical point for the BE. A smooth fast mode transition demands the BE to have a double root at the critical point (see Fig. 2). That means not only does the left-hand side of equation (37) need to vanish, its derivative with respect to \( \mathcal{L} \) must vanish as well.

After some algebra, these requirements can be written as

\[
\mathcal{L} = |\nabla \psi|^{2/3} (J - \varpi^2)^{1/3} \beta^{-2/3}.
\]

This expression is simply a statement that at the fast point, the poloidal fluid velocity is equal to the magnetosonic speed, which for \( \varepsilon = 0 \) is equal to the total Alfvén speed. Note that both equations (37) and (38) are automatically satisfied by \( \mathcal{L} = (J - \varpi^2) = 0 \). This solution, however, is unphysical since it has a discontinuity on the Alfvén surface when \( \chi = 0 \). Substituting equation (38) back into the BE (37), we have

\[
\left[ |\nabla \psi|^{2/3} \beta^{4/3} + (J - \varpi^2)^{2/3} \right]^3 + 2 \varpi^2 V_{\text{eff}} = 0.
\]

For a given value of \( J \), the solutions to this equation give the locations where the BE has degenerate roots. If \( J < J_c \), equation (39) has no roots in the super-Alfvén region (\( \varpi^2 > J \)). If \( J > J_c \), then equation (39) has two roots in the super-Alfvén part of the flow. The desired solution is obtained when \( J = J_c \), and there is only one double root occurring at the fast mode transition point (see Fig. 2).

5.2. Interpolation Schemes and Numerical Strategy

Our strategy is then to find interpolations between the X-point solution in §4.1 and the asymptotic solution of §4.3 so that the action from equation (16) is extremized. Since the action involves an integral extending to \( r \to \infty \) and the streamlines are approximately radial, in general the action integral is infinite. In the X-wind problem, however, the assumption of stationarity is an approximation that must fail physically at very large distances from the X-point. If the flow extends all the way to spatial infinity, then steady state cannot be established in finite time. To make the practical aspect of this problem manageable, we opt to truncate the action integral at some finite spatial surface and assume that the solution is identical to the asymptotic solution beyond that point. Then the interpolation requires the intermediate solution to join smoothly onto the asymptotic solution at the boundary. Since the parameter \( C \) that appears in the asymptotic solution is purely a function of \( r \), it is natural to choose the boundary surface at \( r = r_0 < \infty \). Thus, along a given streamline labeled by \( \psi \), the action involves an integral over the range \( \varpi \in [1, \varpi_\infty] \), where

\[
\varpi_\infty = \frac{2 \beta}{C} \cosh[C^{-1}I(C, 1)] \cosh[C^{-1}I(C, \psi)].
\]

Here \( I \) is the integral defined in equation (29), and the asymptotic value of \( z \) is given by

\[
z_{\varpi_\infty}(\varpi, \psi) = \varpi_\infty \sinh[C^{-1}I(C, \psi)].
\]

Since the asymptotic behavior of the streamlines are predominantly radial with a logarithmic collimation toward the pole, we may approximate them by linear functions. There is a large class of basis functions in which \( z(\psi; \varpi) \) can be expanded. To avoid unphysical oscillations introduced by higher order polynomial interpolations, we approximate \( z \) by a cubic spline such that the second derivative \( z_{\varpi \varpi} \) is a continuous piecewise linear function,

\[
z_{\varpi \varpi} = f_i + (\varpi - \varpi_i) \frac{f_{i+1} - f_i}{\varpi_{i+1} - \varpi_i}, \quad \text{for} \ \varpi_i \leq \varpi < \varpi_{i+1},
\]

for some \( i \).
where $i = 0, \ldots, N - 1$, with $\varpi_0 = 1$ and $\varpi_N = \varpi_\infty$. The boundary conditions on $z = x$ read

$$f_0 = 2z_2, \quad f_N = 0. \quad (43)$$

Direct integration yields (Press et al. 1992)

$$z = ay_i + by_i + ci + df_i, \quad (44)$$

$$\varpi_\infty = \frac{y_{i+1} - y_i}{\varpi_{i+1} - \varpi_i} - \frac{3a^2 - 1}{6}(\varpi_{i+1} - \varpi_i) f_i + \frac{3b^2 - 1}{6}(\varpi_{i+1} - \varpi_i) f_{i+1}, \quad (45)$$

where

$$a = \frac{\varpi_{i+1} - \varpi_i}{\varpi_{i+1} - \varpi_i}, \quad b = 1 - a = \frac{\varpi_i - \varpi_{i+1}}{\varpi_{i+1} - \varpi_i},$$

$$c = \frac{1}{6}(a^2 - a)(\varpi_{i+1} - \varpi_i)^2, \quad d = \frac{1}{6}(b^3 - b)(\varpi_{i+1} - \varpi_i)^2,$$

and $y_i = \varpi(z_i)$ for that particular streamline. The $y_i$ are determined by demanding $z = x$ is continuous throughout the domain. Explicitly,

$$\frac{\varpi_i - \varpi_{i-1}}{6} f_{i-1} + \frac{\varpi_{i+1} - \varpi_{i-1}}{6} f_i + \frac{\varpi_{i+1} - \varpi_i}{6} f_{i+1}$$

$$= \frac{y_{i+1} - y_i}{\varpi_{i+1} - \varpi_i} - \frac{y_i - y_{i-1}}{\varpi_i - \varpi_{i-1}} f_i,$$

which is a set of $N - 2$ linear equations for the $N y_i$. The boundary conditions $y_0 = 0$ and $y_N = x_\infty$ close the equations and allow for a unique determination of $y_i$ once $f_i$ are given. Since we have information on the slope of the solution on both boundaries, they impose two further constraints

$$z_1 = \frac{y_i - y_0}{\varpi_i - \varpi_0} - \frac{1}{3}(\varpi_i - \varpi_0) f_0 - \frac{1}{3}(\varpi_i - \varpi_0) f_i, \quad (47)$$

$$z_{\infty, x} = \frac{y_N - y_{N-1}}{\varpi_N - \varpi_{N-1}} + \frac{1}{6}(\varpi_N - \varpi_{N-1}) f_{N-1}$$

$$+ \frac{1}{3}(\varpi_N - \varpi_{N-1}) f_N, \quad (48)$$

To demonstrate the principles, we choose $N = 3$, so that all the $f_i$ are constrained. For a given set of $\varpi_i$, the equations (46), (47), and (48) form a set of four linear equations, which can be solved by standard means. We also define $\varpi_2$ by

$$\frac{\varpi_2 - \varpi_1}{\varpi_2 - \varpi_1} = \frac{\varpi_\infty - \varpi_2}{\varpi_\infty - \varpi_1}, \quad (49)$$

i.e., we demand that the interval between interpolation points increases exponentially. Thus, the shape of each streamline is parameterized by a single variable, $\varpi_1$.

The action integral and the asymptotic solution can be treated as solutions to a set of simultaneous “ordinary” differential equations

$$\frac{dS}{d\varphi} = \int_{1}^{\varpi}(\varpi) L \varpi d\varpi, \quad \frac{dL}{d\varphi} = \frac{\beta}{\sqrt{2J(\varphi) - 3 - 2C\beta(\varphi)}}, \quad (50)$$

subject to the boundary conditions

$$S(0) = 0, \quad I(0) = 0.$$

Here $L$ represents the Lagrangian appearing in the action from equation (16). For each value of $\varphi$, to compute the right-hand side of equation (50), one needs the values of $\varpi(\varphi)$, $I(\varphi)$, and $J(\psi)$, where $\varphi = 0.99$ is the label of the boundary layer discussed in § 4.2. Ideally, one would like to specify the shape of the last streamline by fixing the values of $\varpi(\varphi)$ and $J(\psi)$ as boundary conditions and varying the function $\varpi_1(\psi)$ in a constrained manner to achieve a local extremum of the action. In practice, we find it more convenient to implement a scheme where only $\varpi_1(\psi)$ is given and $I(\psi)$ is determined as an eigenvalue. This approach allows more freedom in the parameter search for the desired $\varpi_1(\psi)$. With each streamline fully parameterized, one can proceed to determine the necessary value of $J(\psi)$ that allows a smooth fast mode transition according to the procedure outlined in § 5.1.

Once $J(\psi)$ and $I(\psi)$ are both known, we can easily solve the BE (14b) as an algebraic equation along each streamline for $L$ using standard techniques such as Laguerre’s method (see Press et al. 1992). In particular, note that we do not actually use the extremal property of the action principle with respect to $A$ to attack the BE, but effect direct solutions of it instead. Increased numerical accuracy constitutes only one reason for a mixed procedure, where we do find the extremal action through variations of $\psi$, or equivalently, through variations of $\varphi(\varpi, \psi)$, as a substitute for solving the GSE. There is a yet more practical reason. It turns out the the correct solution sits on a saddle, where the extremal action is minimized by variations of $\psi$ but maximized by variations of $A$. This combination makes a numerical search for the extremal action extremely difficult to execute in practice, perhaps even impossible, if the search is carried out in the double-function space of allowable $\psi$ and $A$.

One further obstacle to overcome is that the action integral from equation (16) is logarithmically divergent at the X-point. Recall that the Jacobian of the coordinate transformation vanishes at the X-point, since it maps the entire axis of $\varpi = 1$ onto a single point. A series expansion of the Lagrangian using the series solution of § 4.1 shows that it diverges as

$$L_s = \frac{K\beta}{2(\varpi - 1)}. \quad (51)$$

Fortunately, this term does not enter into the variation scheme, and we may safely remove it as a counter term from the Lagrangian, as is the standard practice in quantum field theory.

Finally, the function $\varpi_1(\psi)$ is modeled by a Hyman filtered spline (Hyman 1983) interpolating over evenly spaced control points $\psi_i \in [0, 1)$. The values of $\varpi_1$ at these control points, $\varpi_1(\psi_i)$, are the parameters we can adjust in our variation scheme. We restrict the parameter space to that which satisfies the condition that the streamlines do not cross and that each streamline is monotonic. We then adopt a genetic algorithm to search for a set of $\varpi_1(\psi)$ that gives a local extremum of the action from equation (16).

6. NUMERICAL RESULTS

We compute the streamlines for three cases of average mass loading corresponding to $\beta = 1, 2, 3$. In each case, we place the outer boundary of the computational domain at a constant radius so that it intersects the last streamline at $\varpi_\infty(\varphi) = 20$ (which yields $C = 0.1\beta$). After a multidimensional search, we locate the desired set of control points that extremize the action. They are tabulated in Table 1, and the function $\varpi_1(\psi)$ is interpolated between these points as described in § 5.

For each converged solution, we can numerically integrate the asymptotic equation to evaluate $I(\psi)$. For practical purposes, we
present here an interpolation formula that is a seventh-degree polynomial in \( \beta^{-1} \), and the coefficients are tabulated in Table 2. Once \( I(\psi) \) is known, one may determine the outer boundary of the computational domain in accordance with the asymptotic condition from equation (29). With the combination of \( \varpi_i(\psi) \) and \( I(\psi) \), we are able to reconstruct the streamlines with the spline interpolation scheme, and they are depicted in Figure 3. The location of the Alfvén surface determines the value of \( J(\psi) \), as a function of \( \psi \), which ultimately allows us to compute the angular momentum being transported as well as the terminal velocity along each streamline. For convenience, we also present an interpolation formula for \( J(\psi) \) as a polynomial in \( \beta \), with the coefficients tabulated in Table 3.

The solid lines in Figure 3 show the logarithmically spaced contours of constant density. It is evident that even though the dotted streamlines become asymptotically radial and only collimate logarithmically slowly, the density becomes cylindrically stratified very quickly, giving the X-wind the illusion of a jetlike appearance (Shang et al. 1998, 2002).

Detailed comparisons of the results obtained here with those given by Shang (1998) show some differences, but the main impression is how remarkably well the solutions obtained by the two very different methods for the same mass-to-flux loading \( \beta(\psi) \) agree with one another. Shang (1998) had a similar experience in comparing her approximate, but analytic, solutions for the sub-Alfvénic region to the exact, but numerical, solutions obtained by Najita & Shu (1994).

We attribute the fortunate circumstance to the following causes. If one is somehow given the geometric shape of the streamlines (or, equivalently, the field lines in the meridional plane), then the Weber-Davis procedure used by Shang, which includes an exact solution of BE, would give an exact solution of the two-dimensional flow problem, provided one takes care to cross each of the critical points properly. In realistic circumstances, the geometric shape of streamlines in the meridional plane is not given a priori, but is to be found from the GSE (or, equivalently, from minimizing the action by variations of the stream function \( \psi \)). However, if one has analytic solutions to the GSE (or from the work of Shu et al. 1994b, 1995) near and far from the X-point, then there are only so many ways that one can adjust the function \( z(\varpi, \psi) \) for values of \( \psi \) from 0 to 1 and of \( \varpi \) close to 1 (or dimensionally, \( R_X \)) to \( \varpi \gg 1 \) (or \( R_Y \)) that will connect the streamlines near the X-point (a fan) smoothly to those appropriate at asymptotic infinity (radial outflow). The procedures used by Shang (1998) and those used here to make such adjustments differ, but the global solution is relatively insensitive to these details as long as one correctly gets the conserved quantities: mass-to-flux loading \( \beta(\psi) \), angular momentum distribution \( J(\psi) \), and Bernoulli's constant \( H(\psi) = 0 \).

### 7. DISCUSSION AND CONCLUSIONS

#### 7.1. Recipe for Use of Results

For the convenience of the reader, we summarize the recipes needed to convert the results of \( \S \) 6 into numerical X-wind models for astronomical and meteoritical applications. Begin with the equation that describes the dimensionless locus of a streamline for given \( \psi \) with numerical value between 0 and 1,

\[
z = z(\varpi, \psi),
\]

where the functional form of \( z(\varpi, \psi) \) is computed numerically by the technique described in \( \S \) 5.2.

The reconstruction of streamline shapes, i.e., the function \( z(\varpi, \psi) \), is performed over three radial intervals whose endpoints are \( \varpi_0 \equiv 1 \), \( \varpi_1(\psi) > 1 \), \( \varpi_2(\psi) > \varpi_1(\psi) \), and \( \varpi_3(\psi) \equiv \varpi_\infty(\psi) > \varpi_2(\psi) \) that give a geometrically increasing separation,

\[
\frac{\varpi_2 - \varpi_1}{\varpi_1 - 1} = \frac{\varpi_\infty - \varpi_2}{\varpi_2 - \varpi_1},
\]

where \( \varpi_\infty(\psi) \) is given by equation (40),

\[
\varpi_\infty = \frac{2 \beta}{C} \cosh[C^{-1}I(C,1)]
\]

For practical computations, we choose \( C = 0.1 \beta \) so that \( \varpi_\infty = 20 \) on the \( \psi = 1 \) streamline. The asymptotic integral \( I(C, \psi) \) in equation (29) can be approximated by a seventh-degree polynomial in \( \beta^{-1} \),

\[
I(\psi) = I_0 + I_1 \beta^{-1}(\psi) + \ldots + I_7 \beta^{-7}(\psi),
\]

where the coefficients \( I_0, I_1, \ldots, I_7 \) are given in Table 2 for the three values of \( \beta = 1, 2, 3 \). The function \( \varpi_1(\psi) \) represents the first nontrivial abscissa of the spline beyond the X-point for each value of \( \psi \) and is tabulated in Table 1 for \( \psi_i = 0.0, 0.2, 0.4, 0.6, 0.8, 0.99 \). For intermediate values, we interpolate \( \varpi_1 \) by a piecewise cubic polynomial,

\[
\varpi_1 = h_0(\psi_i) + h_1(\psi)(\psi - \psi_i) + h_2(\psi)(\psi - \psi_i)^2 + h_3(\psi)(\psi - \psi_i)^3
\]

for \( \psi_i \leq \psi \leq \psi_{i+1} \). In Table 4 we list the values of \( h_i(\psi) \) for each case of \( \beta \). To get \( \varpi_2(\psi) \) for any value of \( \psi \), one should use

### Table 1

| \( \beta \) | \( \varpi_0(0.0) \) | \( \varpi_0(0.2) \) | \( \varpi_0(0.4) \) | \( \varpi_0(0.6) \) | \( \varpi_0(0.8) \) | \( \varpi_0(1.0) \) |
|-----------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1.0       | 29.687         | 29.817         | 23.281         | 18.809         | 8.310          | 6.000          |
| 2.0       | 28.281         | 29.165         | 24.644         | 17.774         | 11.150         | 6.000          |
| 3.0       | 28.384         | 28.985         | 23.990         | 19.965         | 10.139         | 6.000          |

**Note.**—The last value \( \varpi_0(1.0) \) is fixed as a boundary condition.

### Table 2

| \( \beta \) | \( I_0 \) | \( I_1 \) | \( I_2 \) | \( I_3 \) | \( I_4 \) | \( I_5 \) | \( I_6 \) | \( I_7 \) |
|-----------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1.0       | 0.732   | -0.446  | 0.886   | -0.511  | -1.781  | 2.648   | -1.397  | 0.263   |
| 2.0       | 0.842   | 0.413   | -2.594  | 2.699   | -11.768 | 24.849  | -22.870 | 7.602   |
| 3.0       | 1.164   | -2.915  | 30.674  | -194.996| 585.524 | -998.405| 930.236 | -347.764|

**Note.**—The interpolated function agrees with the numerical values to within 0.5%.
Fig. 3.— Solutions for $\tilde{\beta} = 1, 2, 3$. The dotted curves represent the streamlines labeled by constant $\psi$, and the solid curves are isodensity contours separated by logarithmic intervals. The dashed curves are the location of the Alfvén surface in each case, and the dash-dotted curves mark the fast surface where the GSE becomes hyperbolic.
equation (53) after first computing $\omega_1(\psi)$ and $\omega_\infty(\psi)$ at the desired value of $\psi$.

The shape of each streamline given by $\psi = \text{const}$ in the three radial intervals whose endpoints are $\omega_0(\psi) = 1$, $\omega_1(\psi)$, $\omega_2(\psi)$, and $\omega_3(\psi) = \omega_\infty(\psi)$ is then described by a piecewise cubic polynomial, whose form, suppressing the implicit dependence on $\psi$, is given by equation (44),

$$z(\bar{\psi}) = y_1a + y_2b + \frac{(\bar{\omega}_{i+1} - \bar{\omega}_i)^2}{6} \left[f_1(a^3 - a) + f_2(b^3 - b)\right],$$

(57)

where

$$a \equiv \frac{\omega_{i+1} - \omega_i}{\omega_{i+1} - \omega_i}, \quad b \equiv \frac{\omega - \omega_i}{\omega_{i+1} - \omega_i}.$$  

(58)

The coefficients $y_1, y_2, f_1,$ and $f_2$ are listed in Table 5 for discrete values of $\psi = 0.0, 0.2, 0.4, 0.6, 0.8, 0.99$ in the three cases $\bar{\psi} = 1, 2, 3$. A Hyman limited spline may be used to compute the streamlines for other values of $\psi$.

The partial derivatives of $\psi$ with $x$ or $z$ are now given by the usual rules of multivariate calculus,

$$\frac{\partial \psi}{\partial \bar{x}} = -\left(\frac{\partial z}{\partial \bar{x}}\right)_z, \quad \frac{\partial \psi}{\partial \bar{z}} = \frac{1}{\left(\frac{\partial z}{\partial \bar{z}}\right)_z}. \quad (59)$$

Table 3 gives $J(\psi)$ as a seventh-order polynomial in $\beta(\psi)$,

$$J(\psi) = J_0 + J_1\beta(\psi) + \ldots + J_7\beta^7(\psi),$$

(60)

where $\beta(\psi)$ is itself given by

$$\beta(\psi) = \frac{2}{3} \bar{\beta}^3(1 - \psi)^{-1/3},$$

(61)

with $\bar{\beta} = 1, 2, 3$ in the three chosen model cases. The coefficients tabulated in Table 3 give a $J(\psi)$ that guarantees that equation (14b),

$$|\nabla \psi|^2 + \frac{1}{A^2} \left(\frac{J}{\psi^2} - 1\right)^2 + \frac{2\omega V_{\text{eff}}}{(\beta^2 - \omega^2A)^2} = 0,$$

(62)

has one real root for $A$ in the computational domain when $V_{\text{eff}}$ is given by equation (3),

$$V_{\text{eff}} = -\frac{1}{\sqrt{\omega^2 + z^2}} - \frac{1}{2} \omega^2 + \frac{3}{2} \rho^2.$$  

(63)

By solving equation (62) as a fourth-order polynomial, we may obtain the relevant value for the Alfvén discriminant $A$. Then the density can be computed through equation (10),

$$\rho = (\beta^2 - \omega^2A)^{-1}.$$  

(64)

Note that this equation produces $\rho = \beta^2$ at the Alfvénic transition $A = 0$.

With the density in place, we may obtain the two components of dimensionless poloidal velocity from the definition in equation (4) of $\psi$,

$$u_x = \frac{1}{\omega \rho} \frac{\partial \psi}{\partial \bar{x}}, \quad u_z = -\frac{1}{\omega \rho} \frac{\partial \psi}{\partial \bar{z}}.$$  

(65)

The toroidal velocity in the corotating frame is given by equation (7),

$$u_\varphi = \frac{J(\psi) - \omega^2}{\omega(1 - \beta^2 \rho)}.$$  

(66)

Note that $J(\psi) = \omega^2$, where $\beta^2 \rho = 1$ keeps the toroidal velocity $u_\varphi$ well behaved across the Alfvén surface, which is not one of the critical surfaces of the overall problem.

The vector magnetic field may now be obtained from equation (5),

$$B = \beta \rho u,$$

(67)

whereas the azimuthal velocity in the inertial frame is given by

$$v_\varphi = u_\varphi + \omega \varphi,$$

(68)

with the term $\omega \varphi$ from the frame rotation being canceled at large $\varphi$ where $u_\varphi \to -\omega \varphi$ because $\rho$ vanishes as $1/\omega^2$ at large distances from the rotation axis. Finally, to convert the computed quantities to their dimensional counterparts, we must multiply velocities, densities, and magnetic fields by $R_A X, M_\omega/4\pi R_A^2 X$, and $(\Omega_X M_\mu/R_A)^{1/2}$, respectively.

For interpolations or extrapolations in $\bar{\beta}$, we recommend computation first of the dimensionless density, velocity, and magnetic
fields for the three cases $\beta = 1, 2, 3$ and then direct interpolations or extrapolations of those fields. Other techniques starting farther back in the process run the danger of obtaining complex roots of $A$ (i.e., complex values of $\rho$) from the solution of the quartic equation (62) because of slight inaccuracies in computing the numerical coefficients.

7.2. Summary

In this paper we have presented a technique by which solutions to the so-called Grad-Shafranov equation for X-wind flow can be solved, not by attacking the partial differential equation (PDE) directly, but by choosing trial functions that minimize an appropriate action integral. While this method has been applied before in problems of plasma confinement in the fusion community, we believe that the example given here is its first application in astrophysics for the notorious case when magnetohydrodynamical flows cross critical surfaces that change the character of the underlying PDE.

Many empirical arguments suggest that funnel flows and X-winds do underlie the accretion hot spots, jets, and winds of YSOs, although a dipolar field geometry near the star (see Fig. 1) may be an oversimplification (Ardila et al. 2002; Unruh et al. 2004; Johns-Krull 2007). Fortunately, although the fractional areal coverage of hot spots depends on the detailed multipole structure of the surfaces of actual young stars, the general validity of X-wind theory depends only on the level of trapped flux in the X-region and is insensitive to the magnetic geometry on the star as long as the fields are strong (S. Mohanty & F. H. Shu 2008, in preparation). The trapped flux in the X-wind models of this paper is computed as

$$2\pi\beta \left( \frac{GM_* M_*}{\Omega_X} \right)^{1/2}$$

and should be compared with the magnetic flux (area times mean field) in hot spots on one hemisphere’s surface of the star impacted by the corresponding funnel flow. (Both fluxes are 1/3 of the total trapped flux in the X-region and equal the net flux of the dead region.) For T Tauri stars, the comparison is pretty good (see, e.g., Johns-Krull & Gafford 2002).

Apart from relative simplicity, the semianalytical solutions summarized in § 7.1 have many other advantages. For example, the solutions hold over a formally infinite dynamic range, showing the asymptotic, logarithmically slow collimation into jets missing in many numerical simulations. These properties make the models of this paper especially suitable for a wide variety of astronomical and meteorite applications, such as detailed comparisons with observations, trajectories of solids entrained in the wind, and interactions with neighboring circumstellar or interstellar matter. A needed generalization for future research is the inclusion of the effects of the intrinsic magnetization of the surrounding accretion disk.

Note added in manuscript.—Lee et al. (2007a, 2007b, hereafter L07a, L07b, respectively) describe SMA measurements of the jets in the class 0 sources HH 212 and HH 211, which are systems lying almost in the plane of the sky. In HH 212, most positions show no measurable level of rotation. Although the authors do present one case where it might be present (see Fig. 11c of L07a), they state that higher resolution studies are needed. HH 211 has a jet velocity in the line-of-sight of $\sim 25 \text{ km s}^{-1}$ that probably deprojects to a terminal velocity $v_t = 200 \text{ km s}^{-1}$ or more. The authors detect possible rotation in only two, perhaps four, SiO emission knots out of 15 at a level of $v_\phi = 1.5 \text{ km s}^{-1}$ about a distance $\varpi = 30 \text{ AU}$ from the jet axis. The implied terminal specific angular momentum $j_\phi = \varpi v_\phi = 45 \text{ AU km s}^{-1}$ is an upper limit, because the method of analysis is purposely biased to finding large consistent gradients (see short solid lines in Fig. 10 of L07b). Moreover, as the authors discuss, variations of the shock speed could also have contributed to the observed line asymmetry. The asymptotic analysis of this paper and Shu et al. (1995) give $v_t = (2J - 3)^{1/2} R_0 \Omega_\phi$ and $j_\phi = J R_0^2 \Omega_\phi$ along any streamline centrifugally launched from a base in the disk at radius $R_0$ rotating at angular velocity $\Omega_\phi$. The result is general and applies to all magnetocentrifugally driven steady flows, not just X-wind theory where $b = X$. For $v_t > 200 \text{ km s}^{-1}$ and $j_\phi < 45 \text{ AU km s}^{-1}$, the analysis yields for $J = 4$: $R_0 \Omega_\phi > 89 \text{ km s}^{-1}$ and $R_0^2 \Omega_\phi < 11 \text{ AU km s}^{-1}$; whereas for $J = 10$: $R_0 \Omega_\phi > 49 \text{ km s}^{-1}$ and $R_0^2 \Omega_\phi < 4.5 \text{ AU km s}^{-1}$. Division of the second number by the first in the first case yields $R_0 < 0.13 \text{ AU}$; in the second case, $R_0 < 0.09 \text{ AU}$. In other words, no wind can be launched at even larger distances, because such outflowing material would have been seen in molecular-line emission to carry specific angular momentum in excess of the maximum allowable by the observations. Whereas these results do not prove X-wind theory, they do indicate that any disk-wind scenario purporting to explain the observations will have to look very similar to X-wind theory. Indeed, the telling cases of HH 30 (see § 1), HH 211, and HH 212 prompt us to predict that all jets systems that lie sufficiently in the plane of the sky will fail to exhibit clear-cut evidence for rotation much in excess for what might be expected for launching from the inner edges of accretion disks at typically $\sim 0.05 \text{ AU}$.
CHARACTER OF GOVERNING EQUATION

The GSE (8) resembles the steady state heat diffusion equation with a variable diffusion coefficient \( \mathcal{A} \). This analogy is actually misleading since we do not know its overall character until we substitute in the implicit dependence of \( \mathcal{A} \) on \( \psi \) by solving the (algebraic) BE and examine the characteristics of the GSE. To do so, let us first differentiate the BE with respect to \( \varpi \) and \( z \),

\[
2(\psi,_{\varpi}\psi,_{\varpi} + \psi,_{z}\psi,_{z}) - \frac{2A_{\varpi}}{A^2} \left( \frac{J}{\varpi^2} - 1 \right)^2 + \frac{2\varpi^4 A_{\varpi}}{(\beta^2 - \varpi^2 A)^2} \left[ 2V_{\text{eff}} + 2\varepsilon^2 \ln \left( \frac{\varepsilon^2 h}{\beta^2 - \varpi A} \right) + \varepsilon^2 \right] + \ldots = 0,
\]

\[
2(\psi,_{\varpi}\psi,_{\varpi} + \psi,_{z}\psi,_{z}) - \frac{2A_{z}}{A^2} \left( \frac{J}{\varpi^2} - 1 \right)^2 + \frac{2\varpi^4 A_{z}}{(\beta^2 - \varpi^2 A)^2} \left[ 2V_{\text{eff}} + 2\varepsilon^2 \ln \left( \frac{\varepsilon^2 h}{\beta^2 - \varpi A} \right) + \varepsilon^2 \right] + \ldots = 0,
\]

where in the above equations, a subscript denotes partial derivative and the ellipsis symbols include terms that are irrelevant in determining the character of the GSE. These equations may be solved for \( A_{\varpi} \) and \( A_{z} \) to give

\[
A_{\varpi} = \frac{1}{p} (\psi,_{\varpi}\psi,_{\varpi} + \psi,_{z}\psi,_{z}) + \ldots, \quad A_{z} = \frac{1}{p} (\psi,_{\varpi}\psi,_{\varpi} + \psi,_{z}\psi,_{z}) + \ldots,
\]

where

\[
P = \frac{\varpi^2}{\beta^2 - \varpi^2 A} |\nabla \psi|^2 + \frac{1}{A^2} \left( \frac{J}{\varpi^2} - 1 \right)^2 \frac{\beta^2}{\beta^2 - \varpi^2 A} - \frac{\varepsilon^2 \varpi^4}{(\beta^2 - \varpi^2 A)^2},
\]

after we eliminate \( V_{\text{eff}} \) in the expression by using the BE (14b). The second derivative terms in the GSE (14a) can now be written in the form

\[
a \psi,_{\varpi\varpi} + 2b \psi,_{\varpi z} + c \psi,_{zz} + \ldots = 0,
\]

where

\[
a = \mathcal{A} + \frac{\psi,_{\varpi}}{p}, \quad b = \frac{\psi,_{z}}{p}, \quad c = \mathcal{A} + \frac{\psi,_{z}}{p}.
\]

The character of the GSE is determined by the quantity \( \Delta = b^2 - ac \) (Garabedian 1986): it is elliptic, parabolic, or hyperbolic if \( \Delta \) is negative, zero, or positive, respectively. We may compute \( \Delta \) for our GSE explicitly,

\[
\Delta = -A^2 \left\{ \frac{|\nabla \psi|^2 + (J \varpi^2 - 1)^2 \mathcal{A}^2 - \varepsilon^2 \mathcal{A} \omega^4 [\beta (\beta^2 - \varpi^2 A)]^2}{\varpi^2 A \beta^2 - |\nabla \psi|^2 + (J \varpi^2 - 1)^2 \mathcal{A}^2 - \varepsilon^2 \mathcal{A} \omega^4 [\beta (\beta^2 - \varpi^2 A)]^2} \right\}.
\]

The interpretation of this expression becomes transparent if we transform back into the physical quantities. After some algebra, we have

\[
\Delta = -A^2 \left[ \frac{u^2 - \varepsilon^2 (1 - M_1^2)}{(1 - M_1^2)} \left( u_p^2 - \varepsilon^2 \right) + u_p^2 \right] = A^2 \left[ \frac{u_p^2 - \varepsilon^2}{u_p^2 - v_p^2} \right],
\]

where \( v_{Ap} \equiv (B_p^2/\rho)^{1/2} \) is the poloidal component of the Alfvén velocity \( v_1 \equiv (B_0^2/\rho)^{1/2} \) with \( B^2 = B_p^2 + B_\varphi^2 \), \( u_p \) is the poloidal fluid velocity, and \( v_\varphi \) is defined by

\[
v_\varphi^2 = \frac{\varepsilon^2 v_{Ap}^2}{v_1^2 + \varepsilon^2}.
\]

In the limit \( v_\varphi^2 \gg \varepsilon^2 \), it reduces to the thermal sound speed. In addition, \( v_{\pm p} \) denote the poloidal component of the fast and slow MHD wave speeds, respectively, and are given by

\[
v_{\pm p}^2 = \frac{1}{2} \left( v_1^2 + \varepsilon^2 \right) \left[ 1 \pm \sqrt{1 - \frac{4\varepsilon^2}{v_1^2 + \varepsilon^2}} \right].
\]
A moment of thought reveals that \( v_p < v_s < v_{sp} \). The significance of equation (A2) is now clear. The governing GSE is elliptic when \( u_g < v_s^p < v_p^p < v_s^p \) or \( u_g < v_s^p < v_p^p \), and it is hyperbolic when \( v_s^p < u_g < v_p^p \) or \( u_g > v_p^p \) (Heinemann & Olbert 1978; Sakurai 1985).

To be definite, we refer to the loci where the poloidal velocity squared equals \( v_s^p, v_p^p, \) and \( v_g^p \) as sonic, slow, and fast surfaces, respectively. Despite the deceiving appearance of the GSE (8), note that it does not change character on the Alfvén surface when \( A = 0 \) (or equivalently when \( M_A = 1 \) and the total fluid velocity in the corotating frame equals the total Alfvén speed); it remains elliptic until the fast surface. The fact that the asymptotic flow is described by a hyperbolic PDE is consistent with our physical intuition. When the fluid speed is supermagnetosonic, no information can be sent upstream into the flow. Thus, the asymptotic behavior of the X-wind is determined by the “initial condition” at the place when the fluid velocity first becomes equal to the fastest signal propagation speed—a defining feature of hyperbolic problems.

In the cold limit, the discriminant \( \Delta \) has the simplification

\[
\Delta = -A^2 \left[ \frac{1}{A^2 \beta^2 - |\nabla \psi|^2 + (J \omega^2 - 1)^2 A^{-2}} \right] = -A^2 \left( \frac{1}{1 - u_g^2/v_A^2} \right). \tag{A4}
\]

This equation explicitly states that the transition to the hyperbolic portion of the solution is done through the fast surface, where the poloidal fluid velocity is equal to the magnetosonic speed, which is the total Alfvén speed \( v_A = B/\sqrt{\rho} \) when \( \epsilon \) is set to zero. The axial symmetry of the assumed problem guarantees that any compressions or rarefactions occur only in the meridional plane, so the relevant speed of signal propagation in the limit \( \epsilon \to 0 \) is the magnetosonic speed relative to the poloidal motion of the fluid.

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