On the determination of the parity of the $\Theta^+$

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December 30, 2021

Abstract

We critically examine the possibility of determining the parity of the $\Theta^+(1540)$ from the reactions $NN \rightarrow \Theta^+Y \ (Y = \Lambda, \Sigma)$ recently discussed in the literature. Specifically, we study the energy dependence of those observables that were suggested to be the most promising ones to unravel the parity of the $\Theta^+$, namely the spin correlation coefficient $A_{xx}$, and the spin transfer coefficient $D_{xx}$. We show that the energy dependence of $\sigma_0(1 + A_{xx})$, corresponding to the spin-triplet production cross section, guarantees unambiguous information on the parity of the $\Theta^+$. Here, $\sigma_0$ denotes the unpolarized cross section. Also, the possibility to determine the parity of the $\Theta^+$ through the energy dependence of $\sigma_0D_{xx}$ is discussed.
1 Introduction

By now there is increasing an experimental evidence for the existence of an exotic baryon state, the $\Theta^+(1540)$ pentaquark [1]. Its parity, however, is not yet identified. This quantum number is a decisive quantity regarding the substructure of the observed strangeness $S = +1$ resonance [2]. In recent months several proposals were put forward to determine its parity, in both hadronically [3] [4] [6] [7] [8] [9] [10] and electromagnetically [11] [12] induced reactions. In these reactions, both the model-dependent and model-independent analyses reveal that certain spin observables are directly related and/or very sensitive to the parity of the $\Theta^+$. Indeed, based on reflection symmetry in the scattering plane, it has been shown in Ref. [12] that it is straightforward to identify the spin observables which are directly related to the parity of the $\Theta^+$. Although the considerations in that work have been confined to the photoproduction reaction, the method discussed there for the parity determination is quite general and can be easily applied to other reactions induced either by hadronic or electromagnetic probes. However, the method requires the polarization of the $\Theta^+$ to be measured, a requirement that poses an enormous experimental challenge.

Amongst the existing proposals, the one by A. Thomas et al. [4] to measure $\vec{p}\vec{p} \rightarrow \Theta^+\Sigma^+$ seems to be most appealing, for it points to a model-independent determination of the parity without the need to determine the $\Theta^+$ polarization. Moreover, the first observation of the $\Theta^+$ in a nucleon–nucleon ($NN$) collision [5] suggests that the production cross section is in the order of 0.4 $\mu b$ so that concrete experimental investigations on the $\Theta^+$ parity in this reaction seem to be indeed feasible. The idea by A. Thomas et al. exploits the fact that in the $NN$ system the Pauli principle links spin and parity: In order to obey Fermi statistics the quantum numbers characterizing the $NN$ system have to fulfill

$$(-)^{L+S+T} = -1,$$

where $L$, $S$ and $T$ are the orbital angular momentum and the total spin and isospin, respectively. Thus—for a given isospin—the total parity $\pi$ of the system is closely linked to its spin, since $\pi = (-)^L$. If the final state is in an $s$–wave we have, in addition, $\pi = \pi(\Theta^+)$, which allows to determine the parity of the exotic baryon through a manipulation of the initial spin state. In Ref. [6] this proposal was worked out in detail and, in particular, the spin correlation parameter $A_{xx}$, was identified as the crucial observable and its energy dependence was discussed based on general arguments. The findings of Ref. [6] were shortly afterwards supported by a particular model calculation [7].

A very important observation was made in Refs. [9], namely, that if the $\Theta^+$ is an isoscalar then the channel $pn \rightarrow \Theta^+\Lambda$ offers independent additional information. As should be clear from Eq. (11), the role of the positive and negative parities is interchanged when switching from isospin $T = 1$ to $T = 0$. In addition, other observables and the case of arbitrary spin of the $\Theta^+$ were discussed in Refs. [10]. In particular, it was argued that the spin transfer coefficient $D_{xx}$ plays a special role amongst all possible polarization observables that can be measured in $NN$ induced reactions, for it might allow a determination of the parity of the $\Theta^+$ with currently available experimental facilities. This is because the hyperons have a self-analyzing decay that allows to determine the hyperon polarization solely from the angular pattern of the decay particles. So far these considerations have been restricted to threshold kinematics only.
In this paper we re-examine the suggested methods for a parity determination in $NN$ collisions. Furthermore, we provide detailed information for an explicit experimental determination of the parity of the $\Theta^+$. Specifically, we shall

- show that the energy dependence of $\sigma_0(1 + A_{xx})$, where $\sigma_0$ denotes the unpolarized cross section, leads to an unambiguous determination of the parity of the $\Theta^+$;
- discuss the possibilities to determine the parity from a measurement of the spin transfer coefficient $D_{xx}$.

The above two points will be examined on general grounds. Furthermore, in order to corroborate our findings, we complement them with a concrete calculation within a meson–exchange model. To be concrete we consider the case of a spin $1/2$–$\Theta^+$ only.

## 2 The ideal observable

In Ref. [6] it was shown that

$$^3\sigma_{\Sigma} = \frac{1}{2} \sigma_0(2 + A_{xx} + A_{yy})$$

projects on spin triplet initial states in the $NN \to \Theta^+Y$ reaction ($Y = \Sigma, \Lambda$). From this, it was concluded [6] that a measurement of $A_{xx}$ in these reactions would be the ideal quantity for the parity determination of the $\Theta^+$. The reason is that, when approaching the production threshold energy, this quantity has to approach the value of -1 in the case of a positive parity pentaquark but necessarily a positive value in the case of a negative parity pentaquark in the reaction channel $pp \to \Theta^+\Sigma^+$. In the $pn \to \Theta^+\Lambda$ channel, this role is interchanged, i.e., $A_{xx}$ approaches -1 in the case of a negative parity but a positive value in the case of a positive parity pentaquark. However, to make these findings of any practical use, it was necessary to provide an educated guess for the behavior of $A_{xx}$ away from the physical threshold, i.e. for excess energies up to 50 MeV, say, for only there an experiment can be performed with reasonable count rates. In Ref. [6] this was done using the so-called ‘naturalness assumption’: it was assumed that an $l$–th partial wave introduces a factor of $(p'/\Lambda)^l$ into the corresponding amplitude, where $p'$ is the relative momentum of the outgoing particles and $\Lambda$ is a scale typical for the production itself. Apart from this factor, the (reduced) partial wave amplitudes were assumed to be of the same magnitude for all partial waves. For later reference the corresponding result for the energy dependence of $A_{xx}$ is shown in Fig. 1 as a function of the excess energy $Q$.

Naturally the question arises how reliable are the scale arguments mentioned above. Final state interactions can introduce additional large scales into the problem. However, as was argued in Ref. [6], there are good reasons to believe that the $\Theta^+–\text{hyperon}$ interaction is weak. It is also straightforward to show that the Coulomb interaction has no influence anymore for excess energies of 10 MeV or higher. However, scale arguments typically indicate only the order of magnitude of particular contributions and thus can well be off by factors of 2–3. In addition, they can not account for possible cancellations amongst different production mechanisms. Unfortunately, these uncertainties can lead to wrong conclusions on the parity of the $\Theta^+$, because they could change significantly the energy dependence of $A_{xx}$ compared to the
one shown in Fig. 1. For illustration of this point in the middle row of Figs. 2 and 3 we show the results for $A_{xx}$ calculated within various models described in detail in the appendix. A comparison with the prediction in Fig. 1 shows that $A_{xx}$ in case of a positive parity pentaquark (dashed curves) in the second and third columns in Fig. 2 would lead to an erroneous assignment of the parity of the pentaquark. The same can be said about the negative parity results for the $pn$ reaction, shown as the solid curve in the third column of Fig. 3. It is thus necessary to look for observables that allow solid conclusions about the parity of the resonance produced irrespective of the relative importance of various amplitudes.

Such observables indeed exist! The key observation is that $^3\sigma_\Sigma$ projects onto the spin triplet initial states and, therefore, its energy dependence will unambiguously point to the parity of the pentaquark. Let us discuss this argument in more detail: we consider first the $pp \to \Sigma^+\Theta^+$ reaction. Odd (even) partial waves in the final state correspond to odd (even) partial waves in the initial state in case of a positive parity pentaquark but to even (odd) partial waves in the initial state for a negative parity pentaquark. From Eq. (1) we find that for this reaction the initial state is in a spin triplet (singlet) state for positive parity but in a spin singlet (triplet) state for negative parity. Thus, the lowest (even) partial wave in the final state corresponds to a spin singlet or triplet initial state for positive or negative parity, respectively. Since the centrifugal barrier leads to a momentum dependence of the $l$–th partial wave amplitude of $p^{l+1}$ we find that the spin triplet cross section in the $pp$ induced reaction should scale as $\sqrt{Q}$ for a negative parity pentaquark and as $Q^{3/2}$ for a positive parity pentaquark.\(^\dagger\) This observation on the energy dependence of the spin triplet cross section holds irrespective of the relative importance of different partial waves. For illustration, in Fig. 2 we show the results of various model calculations (see Appendix for details) for the energy dependence of the total cross section $\sigma_0$, $A_{xx}$ and the spin triplet cross section $^3\sigma_\Sigma$. (Note that the results shown in Figs. 2 and 3 are divided by the factor $\sqrt{Q} \propto p'/q$, i.e. by the two-body phase-space volume.) The results for the positive parity pentaquark are shown as dashed lines whereas those for the negative parity pentaquark are shown as solid lines. In order to exhibit better the difference in

\(^\dagger\)The individual production mechanisms will induce an additional weak energy dependence on top of this. We do not expect the production operator to introduce any stronger energy dependence since it is of rather short-ranged nature. In fact, the momentum transfer at threshold is given by $q = \sqrt{(m_\Theta + m_\Sigma)^2/4 - m_N^2} \approx 5 \text{ fm}^{-1}$ which corresponds to a range of $1/q \approx 0.2 \text{ fm}$. 

Figure 1: Sketch of the expected energy dependence of $A_{xx}$ for the two different parities of the $\Theta^+$. In case of the $pp \to \Theta^+\Sigma^+$ $(pn \to \Theta^+\Lambda)$ reaction the hatched area corresponds to negative (positive) parity, whereas the filled area corresponds to positive (negative) parity.
Figure 2: Energy dependence of the total cross section and the angular integrated polarization observable $A_{xx}$ and $^3\sigma_\Sigma$ for the reaction $pp \rightarrow \Sigma^+\Theta^+$. Solid (dashed) lines correspond to a negative (positive) parity $\Theta^+$. Shown are results for three different models for the production operator: the left column shows the results for the model with only kaon exchange, the middle one those for the one with $K^*$ exchange and the right one those for the model including $K^*$ and $K$ exchange. All results for $\sigma_0$ and $^3\sigma_\Sigma$ are normalized to 1 at an excess energy of 20 MeV and are divided by the phase-space volume.

the energy dependence of the observables corresponding to different parities of the pentaquark, the results for $\sigma_0$ and $^3\sigma_\Sigma$ are scaled to 1 at $Q = 20$ MeV for all models. Although the energy dependence of $A_{xx}$ exhibited by two of the models considered in Fig. 2 is significantly stronger than what is expected for natural-size amplitudes (c.f. Fig. 1), the energy dependence of $^3\sigma_\Sigma$ for the two parities always follows the features discussed above and, consequently, allows definite conclusions on the parity of the pentaquark.

Analogously, for the $pn \rightarrow \Lambda\Theta^+$ reaction the spin triplet cross section should scale as $Q^{3/2}$ for odd parity pentaquarks and as $\sqrt{Q}$ for even parity pentaquarks. In Fig. 3 we show the results of the same models used before for the $pn$ channel. Again, we observe that the scaling argument leading to the results of $A_{xx}$ presented in Fig. 2 is violated here for one of the models considered, but the energy dependence of $^3\sigma_\Sigma$ allows definite conclusions on the parity of the pentaquark.

One might still ask what would happen if for some reason the $s$–wave amplitude is strongly
Figure 3: Energy dependence of the total cross section and the angular integrated polarization observable \( A_{xx} \) and \( ^1\sigma_\Sigma \) for the reaction \( pn \rightarrow \Lambda \Theta^+ \). Same description of curves and panels as in Fig. 2. All results for \( \sigma_0 \) and \( ^3\sigma_\Sigma \) are normalized to 1 at an excess energy of 20 MeV and are divided by the phase-space volume.

suppressed or even absent. Even in this case the method of parity determination proposed will work. To be concrete let us look at the channel \( pp \rightarrow \Sigma^+ \Theta^+ \) only. If the pentaquark has positive parity a spin triplet initial state is to be at least in a \( p \)-wave. Thus, \( ^3\sigma_\Sigma \) should follow a \( Q^{3/2} \) behavior independent of the strength of the \( s \)-wave. If the pentaquark has negative parity, spin triplet initial states lead to even partial wave final states. In the absence of an \( s \)-wave this implies a behavior at least as \( Q^{5/2} \) for \( ^3\sigma_\Sigma \). Consequently we should expect the spin triplet cross section to be strongly suppressed in comparison to the total cross section which should show a \( Q^{3/2} \) behavior. Again this difference is decisive.

It should also be stressed that all arguments given above hold even if the \( \Theta^+ \) signal seen in the experiment would be due to an interference of the resonance signal with the background.

Therefore, we propose to measure the total cross section as well as the corresponding (angle integrated) spin-correlation parameter \( A_{xx} \) for \( NN \) induced pentaquark production at two well separated energies, e.g. at excess energies of \( Q = 20 \) and \( 40 \) MeV.

\(^2\)We tuned the parameters of the \( K-K^* \) model such that this case is realized in the right column of Fig. 3.

\(^3\)We were not able to construct a model, where this scenario is realized.
3 The spin transfer coefficient $D_{xx}$

In addition to the spin correlation coefficient $A_{xx}$ also the spin transfer coefficient $D_{xx}$ is discussed in the literature as a useful observable to determine the parity of the pentaquark in $NN$ collisions.

The energy dependence of $D_{xx}$ cannot be studied on as general grounds as that of the spin triplet cross section $\sigma_0^\Sigma$ discussed in the previous section. However, using the methods outlined in, e.g., Refs. [6, 13] we may give the general angular and energy dependence of $D_{xx}$ up to terms of order $(p')^2$. This analysis leads to the following structure for negative parity pentaquarks produced in the $pp \rightarrow \Theta^+\Sigma^+$ reaction,

$$4\sigma_0 D_{xx} = \alpha + \left(\frac{p'}{\Lambda}\right) \beta \cos(\theta) + \left(\frac{p'}{\Lambda}\right)^2 \left[\gamma \cos^2(\theta) + \delta \sin^2(\theta) \sin^2(\phi)\right] + \mathcal{O}\left(\frac{p'^3}{\Lambda^3}\right), \quad (3)$$

where all the coefficients ($\alpha, \beta, ...$) turn out to be independent from each other. On the other hand, for positive parity pentaquarks produced in the $pp$ induced reaction we find

$$4\sigma_0 D_{xx} = \left(\frac{p'}{\Lambda}\right) \beta' \cos(\theta) + \left(\frac{p'}{\Lambda}\right)^2 \left[\epsilon' + \gamma' \cos^2(\theta) + \delta' \sin^2(\theta) \sin^2(\phi)\right] + \mathcal{O}\left(\frac{p'^3}{\Lambda^3}\right), \quad (4)$$

where again all the coefficients ($\beta', \gamma', ...$) turn out to be independent from each other. Eqs. (3) and (4) reflect the different threshold behavior for the two different parities first pointed out...
Figure 5: Results for the energy dependence of $D_{xx}$ and $|D_{xx}|\sigma_0$ in the channel $pn \rightarrow \Lambda\Theta^+$ for the various models. The calculations for the angular distributions were performed for $Q = 40$ MeV. The calculations for the angular distributions were performed for $Q = 40$ MeV. Same description of curves and panels as in Fig. 2.

in Refs. [9]: only for negative parity pentaquarks produced in the $pp$ reaction a non-vanishing threshold value is allowed for $D_{xx}$. However, nothing prevents $\alpha$ from being small and thus it can not be said a priori, whether a conclusion on the parity of the pentaquark can be drawn from a measurement of $D_{xx}$—we come back to this issue below. The relevant expressions for the $pn \rightarrow \Theta^+\Lambda$ channel are identical to Eqs. (3) and (4), except that the roles of the different parities are interchanged.

To simplify the discussion we will now focus on the $pp$ system only. The terms higher order in $p'/\Lambda$ are given here for the first time. For both parities the term linear in $p'$ stems from an interference of $s$– and $p$–waves in the final state. The terms quadratic in $p'$ originate from the interference of various spin triplet initial states and thus stem from $p$–waves interfering with each other in case of an even parity pentaquark and from $s$–waves interfering with $d$–waves in case of an odd parity pentaquark.

As was stressed in the previous paragraph, we should identify observables that allow conclusions on the parity without the need to employ scale arguments for the relative importance of different partial waves. It turns out that $D_{xx}\sigma_0$ is such a quantity. A close look at Eqs. (3) and (4) reveals that ideally one may extract from the experiment $D_{xx}\sigma_0$ at $\phi = 0$ and $\cos(\theta) = 0$ at two different energies, for in this case the production of a negative parity pentaquark can lead to an energy independent non–zero result (up to the energy dependence from phase space
and small corrections due to the energy dependence of the production operator), whereas a positive parity pentaquark can lead to a linear energy dependence (in this case an energy independent result consistent with zero would not be conclusive, for the parameter $\epsilon'$ can also be 0). However, this would require a highly differential measurement. Fortunately, already the angle integrated result for $D_{xx}\sigma_0$ can provide valuable information when one extrapolates linearly from the data to the threshold after dividing by the phase space factor $p'$. Here a finite value is possible only for a positive parity pentaquark whereas a negative parity pentaquark has to lead to a vanishing $D_{xx}\sigma_0$ at threshold. Note, however, that no conclusion on the parity can be drawn if this extrapolation leads to a value consistent with zero (unfortunately we do not have a model calculation to illustrate this point—see also footnote 3).

In Figs. 4 and 5 we show results for $D_{xx}\sigma_0$, corrected for the phase space behavior, based on the same models used in the previous paragraph. As can be seen in all cases the extrapolation to threshold allows for a discrimination of the different parities.

It should be stressed, however, that the experimental uncertainty on $D_{xx}\sigma_0$ will be large if the value of $D_{xx}$ is small. For example, a small $D_{xx}$ for the $pn \rightarrow \Theta^+\Lambda$ channel resulted from the considered $K-K^*$ model (right column of Fig. 5). We can make this statement somewhat more quantitative. It is well known that the statistical fluctuation in an observable, in this case $D_{xx}\sigma_0 = [\sigma(++)+\sigma(--)]-[\sigma(+-)+\sigma(-+)]$, scales as $\sqrt{N}$, where $N$ denotes the count rate for the individual spin cross sections. On the other hand, the signal itself is given by $D_{xx}N$ counts. Thus we find for the relative uncertainty $\delta$ of $D_{xx}\sigma_0$

$$\delta \simeq \frac{1}{D_{xx}\sqrt{N}}.$$ 

To reduce the uncertainty in $D_{xx}\sigma_0$ to 10 % ($\delta = 0.1$) we find a required count rate for the polarized cross section of $N \simeq 100/D^2_{xx}$, where we assumed 100 % polarization in the beam as well as 100 % acceptance in the final state. More realistic numbers will increase our count rate estimate. This formula shows that if $D_{xx}$ is 0.5 or larger, one needs at least 400 events in the polarized cross section. However, if $D_{xx}$ turns out to be only of the order of 0.1 in the energy range accessible to the experiment (as it is predicted by some of the considered models) then one needs $N \geq 10000$.

To conclude, if it is possible to measure $D_{xx}$ with reasonable accuracy at two different and sufficiently separated energies, it could be possible to extract the parity of the pentaquark. If an extrapolation from the measured values of $D_{xx}\sigma_0$ to the threshold leads to a non–vanishing value in the $pp \rightarrow \Theta^+\Sigma^+$ channel, the parity of the pentaquark must be negative, whereas if it leads to a non–vanishing value in the $pn$ induced reaction, the parity must be positive. It should be stressed, however, that here significantly higher statistics is necessary compared to the measurement of $A_{xx}$ for there the energy dependence of $\sigma_0(1+A_{xx})$ was the quantity of interest whereas here an extrapolation to the threshold is necessary and for both parities the term linear in energy might be sizable.

4 Summary

We examined critically the possibility to determine the parity of the $\Theta^+(1540)$ from the reactions $NN \rightarrow \Theta^+Y$, where $Y$ denotes the $\Lambda$ or $\Sigma$ hyperon, recently discussed in the literature.
Specifically, we studied the energy dependences of the observables that have been suggested in the literature to be the most promising ones to unravel the parity of the $\Theta^+$, namely $A_{xx}$ and $D_{xx}$. The validity and/or breakdown of general scale arguments were critically examined, showing that peculiarities of the so far unknown production mechanism of the $\Theta^+$ could make conclusions based on such scale arguments rather unreliable.

On the other hand, we showed that the energy dependence of $\sigma_0(1 + A_{xx})$, corresponding to the spin-triplet production cross section, guarantees unambiguous information on the parity of the $\Theta^+$, since it does not rely on any assumptions such as scale arguments.

In addition, we have demonstrated that the spin transfer coefficient $D_{xx}$ could, under certain conditions, also be used to determine the parity of the pentaquark. A non–vanishing value of $\sigma_0 D_{xx}$, when extrapolated linearly from the measured values to the threshold, will tell the parity. However, this measurement requires in any case very high statistics. In addition, only a non–vanishing value at the threshold would be conclusive. A threshold value consistent with zero is in accordance with both parities.

Finally, it should be clear that the presented results are applicable for the parity determination of any narrow spin–1/2 baryon resonance in $NN$ collisions, for they are based solely on general considerations.

**Acknowledgment**

We thank F. Rathman and A. Sibirtsev for very useful discussions. The work of KN is partly supported by COSY grant No. 41445282.

**A Appendix**

To illustrate the points made in the main section, we perform a concrete calculation within the meson–exchange framework. It should be stressed that this model calculation is intended to provide only a qualitative picture of the features discussed in work. For quantitative predictions...
further and important ingredients need to be incorporated into the model. For example, inclusion of the initial state interaction (ISI) would not only change the magnitude of the total cross sections, but it can also change the relative phase of the amplitudes and thus change the results of the polarization observables (for a recent discussion of the effect of the ISI on inelastic $NN$ scattering we refer to Ref. [14]). Also there might be significantly more complicated reaction mechanisms that contribute to the production.

So far there is practically no information on the preferred production mechanism of the $\Theta^+$. In the only model calculation that has presented results for polarization observables [7], it was assumed that the production reaction is dominated by kaon exchange. However, the apparently rather narrow width of the $\Theta^+(1540)$ [15], which implies a fairly small $KN\Theta^+$ coupling constant, makes it plausible that other production mechanisms should be important if not dominant. One of the obvious candidates is the exchange of the $K^*(892)$ vector meson, whose contribution has been already considered in several model calculations. Vector-meson exchanges yield structures that are similar but of opposite character to those generated by pseudoscalar mesons (kaons) so that cancellations can occur. For example, in the $NN$ system the cancellation between the $\pi$- and $\rho$-exchange contributions leads to a significant reduction of the tensor force; the same feature is also seen for the $K$- and $K^*$ exchange in the hyperon-nucleon case [16]. In the reaction $p\bar{p} \to \Lambda\bar{\Lambda}$, on the other hand, there is a strong cancellation between $K$- and $K^*$ exchange in the spin-spin component, resulting in a strong suppression of the spin-singlet amplitude as observed experimentally [17]. Thus, it is not unreasonable to assume that cancellations can also occur in the $\Theta^+$ production reaction.

Although there might well be more complicated production mechanisms for the $\Theta^+$ of relevance [18], we believe that the model of a single kaon exchange (Fig. 6a)), proposed in Ref. [19], or of a single $K^*$ exchange (Fig. 6b)) or a combination of both, proposed in Ref. [20], is well suited to explore the model dependence of those observables relevant for the parity determination. We use the following Lagrangian densities to calculate the diagrams in Fig. 6:

$$\mathcal{L}_{NKY}^\pm = -g_{NKY} \bar{N}i\gamma^\pm Y K + h.c.,$$
$$\mathcal{L}_{NK*Y}^\pm = -g_{NK*Y} \bar{N}\Gamma^\pm \gamma_\mu Y + \frac{\kappa}{m_Y + m_N} \sigma_{\mu\nu} Y \partial^\nu K^* + h.c.,$$

where $\Gamma^+ = \gamma_5$, $\Gamma^- = 1$ and, $Y = \Theta^+, \Lambda, \vec{\tau} \cdot \vec{\Sigma}$. The superscript $\pm$ refers to the positive (+) or negative (−) parity of the $\Theta^+$. We also introduce a monopole form factor, $F(q^2) = (\Lambda_M^2 - m_M^2)/(\Lambda_M^2 - q^2)$ ($M = K, K^*$), at each vertex, where $q^2$ denotes the squared four–momentum of the exchanged meson $M$ and $m_M$ stands for its mass. The parameter values employed in our calculations are summarized in Table 1. Most of them are in the same range as the values employed/extracted in Refs. [7, 8]. The only exception is the cut–off parameter $\Lambda_{K^*}$. We use $\Lambda_{K^*} = 1.3$ GeV instead of 1 GeV employed in Refs. [11, 8], because the latter choice suppresses the $K^*$ exchange contribution almost completely. In addition, we varied the $K^*$ tensor coupling in order to exhibit various scenarios. The three examples shown in the main text are typical representatives of a large number of models investigated.
Table 1: Parameter values used in this work. All parameters are given in units of $g_{NK\Theta^+}$. For the $K^*$, the entries are $(g_{NK^*Y}, \kappa)$.

| M      | $g_{NM\Theta^+}$ | $g_{NM\Sigma}$ | $g_{NM\Lambda}$ | $\Lambda_M$ (MeV) |
|--------|------------------|----------------|-----------------|------------------|
| $K$    | 1                | 3.54           | -13.26          | 1000             |
| $K^*$  | (0.5, 2)         | (-2.46, -0.5) | (-5.80, -0.5)   | 1300             |

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