Introduction. While a universal fault-tolerant quantum computer with thousands of qubits remains elusive, noisy intermediate-scale quantum (NISQ) devices with a few qubits are already operational [1–3], albeit with limitations due to a lack of reliable error-correction [4]. This progress has stirred a flurry of research activity to identify problems that can take advantage of this recently developed quantum technology [5]. Utilizing NISQ systems as digitized synthetic platforms to study physics phenomena challenging to investigate otherwise has emerged as a critical frontier [6].

In strongly-correlated electron materials, fractional quantum Hall (FQH) states are widely studied for their exotic topological properties, such as excitations with fractional charge [7, 8] and fractional statistics [9, 10]. Recently, FQH states have come into focus due to their universal geometric features such as Hall viscosity [11–13] and the Girvin-MacDonald-Platzman magnetoroton collective mode [14, 15]. In the long-wavelength limit \( k \to 0 \), the magnetoroton forms a quadrupole degree of freedom that carries angular momentum \( L = 2 \) and can be represented by a quantum metric, \( \tilde{g} \) [16]. For this reason, the \( k \to 0 \) limit of the magnetoroton has been referred to as “FQH graviton” [17, 18], due to its formal similarity with the fluctuating space-time metric in a theory of quantum gravity [19, 20].

The experimental detection of the FQH graviton for \( \nu = 1/3 \) Laughlin state remains an outstanding challenge. While at large momenta, \( k \sim \ell_B^{-1} \), with \( \ell_B = \sqrt{\hbar/eB} \) being the magnetic length, the magnetoroton mode may be probed via inelastic light scattering [21–24], the magnetoroton enters the continuum near \( k \to 0 \) for the \( \nu = 1/3 \) Laughlin state (in contrast to the mode for \( \nu = 7/3 \) [25, 26]). Haldane proposed that quantum-metric fluctuations can be exposed by breaking rotational symmetry [16]. Following up on this idea, recent theoretical works [27, 28] have probed the FQH graviton by quenching the metric of “space”, i.e., by suddenly making the FQH state anisotropic (see also alternative proposals [29–31]). It was found that such geometric quenches induce coherent dynamics of the FQH graviton [27], even though the graviton mode resides at finite energy densities above the FQH ground state. In contrast, near the FQH liquid-nematic phase transition [32, 33], the graviton is expected to emerge as a gapless excitation [34–36].

In this paper, we realize the FQH graviton in a synthetic NISQ system – the IBM open-access digitized quantum processor – and simulate its out-of-equilibrium dynamics. We first map the problem onto a one-dimensional quantum spin chain, corresponding to the FQH state on a thin cylinder. While topological properties of FQH states have been extensively studied in this regime [39–46], we show that this limit remarkably captures some geometric properties of FQH systems, in particular their quench dynamics. As a second step, we implement the quench dynamics on the IBM NISQ device, using two complementary approaches. On the one hand, we used an optimally-compiled, noise-aware Trotterization circuit with error mitigation methods [47–49]. This allowed us to successfully simulate quench dynamics on
we can generally write it as $g = \exp(\hat{Q})$ where $\hat{Q} = Q(2d_\nu d_\nu - d_{a,b})$ is a Landau-de Gennes order parameter and $d = (\cos(\phi/2), \sin(\phi/2))$ is a unit vector [65]. Parameters $Q$ and $\phi$ intuitively represent the stretch and rotation of the metric, respectively. The FQH state is invariant under area-preserving deformations of $g$, illustrated in Fig. 1(a).

Since the Hamiltonian in Eq. (1) is positive semi-definite, it has a unique (unnormalized) ground state with zero energy [44]

$$|\psi_0\rangle = \prod_j \left(1 - \frac{1}{V_{1,0}} e^{\frac{i2\pi c_1}{L^2} \frac{g_{12}}{g_{11}} \hat{S}_j}\right)|\ldots 100100\ldots \rangle,$$

where $\hat{S}_j = c_{j+1}^\dagger c_{j+2} c_{j+3} c_j$ is an operator that “squeezes” two neighbouring electrons while preserving their center-of-mass position [66]. The ground state in the limit $L_2 \rightarrow 0$ is the product state $|\ldots 100100\ldots \rangle$. The off-diagonal squeezing operator is essential for the $1/3$ Laughlin state [39].

In previous works [39, 44–46], the ground state of the model in Eq. (1) and its neutral excitations were studied on isotropic cylinders, $g_{11} = g_{22} = 1$, $g_{12} = 0$. In particular, it was found that the state in Eq. (3) has $\sim 98\%$ overlap with the ground state of the full Hamiltonian in the range of circumferences $5\ell_B \lesssim L_2 \lesssim 7\ell_B$, where $V_{2,1}/V_{1,0} \approx 0.2 \pm 0.3$, justifying the use of the truncated model Eq. (1) in this regime [44]. We have confirmed that the same conclusions continue to hold in the presence of mass anisotropy [67].

As a further justification of the model in Eq. (1), we plot the entanglement entropy $S_{\text{ent}}$ of the Laughlin state in a large system of 100 electrons as a function of the circumference $L_2$ in Fig. 1(b). We see that it is possible to reduce $L_2$ to approximately $5\ell_B$, where the “area law” for entanglement entropy [68, 69] still holds, but long-range electron hopping is strongly suppressed. Below we focus on this regime, where the key aspects of 2D physics are preserved, but the system can be mapped to a 1D spin chain model and thus efficiently simulated on quantum hardware.

**Geometric quench.** We now show that, in addition to the ground state, the effective model in Eq. (1) captures the high-energy excitations that govern the graviton dynamics in the FQH phase. We initially prepare the system in the ground state $|\psi_0\rangle$ in Eq. (3) with isotropic metric $(g_{11} = g_{22} = 1, g_{12} = 0)$. At time $t = 0$, we instantaneously introduce diagonal anisotropy, $g_{11} = 1/g_{22} > 1$, and let the system evolve unitarily, under the dynamics generated by the post-quench anisotropic Hamiltonian. We are interested in the dynamical fluctuations of its quantum metric $\tilde{g}$ as the system is taken out of equilibrium.

Note, even though $g$ and $\tilde{g}$ are related to one another, $\tilde{g}$ is an emergent property of a many-body state and not necessarily equal to $g$. Nevertheless, we can formally parameterize $\tilde{g}$ using the parameters $Q$ and $\phi$, representing the IBM device, overcoming the problem of the large circuit depth. On the other hand, we devised an efficient optimal-control-based [50–52] variational quantum algorithm [53–56], analogous to the Quantum Approximate Optimization Algorithm (QAOA) [57–62], that creates the post-quench state using a hybrid classical-quantum circuit depth and only two variational parameters. Scales favorably with system size, with a linear-depth circuit model and thus efficiently simulated on quantum hardware.

![Fig. 1](image.png)
the stretch and rotation of the emergent metric. In order
to determine the equations of motion for \( \tilde{Q} \) and \( \tilde{\phi} \),
we maximize the overlap between \( |\psi(t)\rangle \) and the family
of trial states in Eq. (3) [70]. When this overlap is close
to unity, we can be confident that we found the optimal
metric parameters \( \tilde{Q} \) and \( \tilde{\phi} \) describing the state \( |\psi(t)\rangle \).

In Fig. 2, we summarize the results of the graviton
dynamics in the model in Eq. (1) when anisotropy is sud-
denly changed from \( Q=0 \) to \( Q\approx 0.18 \) while keeping \( \phi=0 \).
Fig. 2(a) shows the dynamics of \( \tilde{Q} \) and \( \tilde{\phi} \) for different
system sizes \( N \). The dynamics is in excellent agreement
with the bimetric theory in the linear regime [71],

\[
\tilde{Q}(t) = \pm 2A \sin \frac{E_g t}{2}, \quad \tilde{\phi}(t) = \pm \frac{\pi}{2} - \frac{E_g t}{2},
\]

where \( E_g \) is the energy of the graviton mode in units of \( V_{1,0} \).
As can be seen in Fig. 2(a), the numerical data can be
accurately fitted using Eqs. (4). The fit yields the
oscillation frequency \( E_g=1.29 \). Note that this
energy is much higher than the first excited energy of
the quench Hamiltonian. We identify this energy with
the graviton state as evidenced by the sharp peak in the
quadrupole (\( L=2 \)) spectral function \( I_{1,2}(\omega) \) [72]. The later
spectral function is designed to detect the characteristic
\( d \)-wave symmetry of the graviton. Analogous to an
oscillating space-time metric induced by a gravitational
wave, \( I_{1,2}(\omega) \) is the associated transition rate due to the
dynamics of the oscillating mass-tensor [72]. Thus, the
model in Eq. (1) reproduces the graviton oscillation as
described by the bimetric theory.

**Spin chain mapping.** We use the reduced registers
scheme introduced in Ref. [43] to map the model
(1) to a spin chain, see also [67] for further details.
The reduced register is a block of three consecutive orbitals
that encodes whether or not the block is “squeezed” with
respect to the root state \( |100,100,\ldots\rangle \). For each block
of three sites, the state of the reduced register is \( \uparrow \) if
it is squeezed (i.e., 011) or 0 if not (i.e., either 000 or
100). In the root state, none of the blocks are squeezed
and it maps to \( |0,0,0,\ldots\rangle \). If we apply the squeezing
operator to one block of the root state, we obtain, e.g.,
\[ |100,011,000,\ldots\rangle \rightarrow |0,1,0,\ldots\rangle \]. In terms of reduced
registers, squeezing acts as flip of 0 to 1, so it can be
viewed as the Pauli X matrix. However, there is an
important difference in that the Hilbert space is not a
tensor product of reduced registers, since the squeezing
can never generate two neighboring \ldots \|11\ldots\| configurations
of the reduced registers [73, 74]. This type of con-
strained Hilbert space arises e.g., in the Fibonacci anyon
chain [75]. The inverse mapping is constructed as fol-
loows: for any \( \uparrow \) we make a 011 block. A 0 that follows
a \( \uparrow \) (0) gives a 000 (100) block. With this mapping of
states, we can show that the Hamiltonian (1) maps to a
local spin-chain Hamiltonian

\[
\hat{H} = \sum_\ell \left( (V_{1,0} - 3V_{3,0})N_\ell + V_{3,0}N_\ell N_{\ell+2} \right) \\
+ (1 - N_{\ell-1})[\text{Re}(V_{2,1})X_\ell - \text{Im}(V_{2,1})Y_\ell](1 - N_{\ell+1}),
\]

\[
(5)
\]

where we omitted the boundary terms for simplicity
and introduced the occupation number \( N=|0\rangle \langle 1| \), Pauli
\( X=|0\rangle \langle 1|+|1\rangle \langle 0| \), and Pauli \( Y=-i|0\rangle \langle 1|+i|1\rangle \langle 0| \) opera-
tors.

**Quantum simulation.** The standard procedure for
simulating the time evolution \( e^{-i\hat{H}t} \) is to use Trotter
decomposition. Here \( \hat{H} \) is given in equation Eq. (5) with
real \( V_{2,1} \) and it has the form \( \hat{H} = \sum_\ell H_\ell \). We de-
compose the evolution operator into \( k \) Trotter steps as

\[
\hat{U}_\ell = e^{-iH_\ell t/k}
\]

FIG. 2. (a) Dynamics of \( \tilde{Q} \) and \( \tilde{\phi} \) following the geometric
quench in the TC limit, with \( L_2=5.477\ell_0 \) and post-quench
anisotropy \( Q\approx 0.18 \). Data is for system sizes \( N=7,9,15 \)
electrons. (b) Quadrupole spectral function, \( I_{1,2}(\omega) \), shows a sharp
peak at the graviton energy \( E_g\approx 1.29 \), which agrees well with the frequency
of the oscillations in (a).

FIG. 3. Circuit implementation of the trotterized unitary \( U_\ell \)
in the bulk of the spin chain.
\( e^{-iHt} \approx \prod_{\ell} U_{\ell}(t/k)^k \), where \( \delta t = t/k \) and the approximation improves for larger \( k \). In [67] we derive the circuit which implements a Trotterized time evolution of our Hamiltonian and the subcircuit for the bulk \( U_{\ell}(\delta t) \) is shown in Fig. 3. Below we demonstrate this circuit yields good results on current IBM devices with 5 qubits after using noise-aware error mitigation methods and optimized compilations [47–49].

While the trotterization algorithm emulates the actual quantum evolution resulting from FQH quenched Hamiltonian, it has a relatively large number of entangling gates. We can access large systems by a hybrid classical-quantum method that requires classical optimization, using the following variational ansatz for the final post-quench state \( |\psi(t)\rangle \):

\[
|\psi_{\text{var}}(\alpha, \beta)\rangle = \prod_{\ell} e^{-i\alpha N_{\ell} \ell X_\ell} e^{-i\beta (1-N_{\ell-1}) X_\ell} |000 \ldots\rangle, \tag{6}
\]

where on each reduced register \( \ell \), we apply alternating gates \( N_{\ell} \) and \((1-N_{\ell-1})X_\ell\) (on the very first site, due to open boundary condition, we use \(X_1\) instead of \((1-N_{\ell-1})X_\ell\) for \( \ell=1 \)).

The optimal parameters \( \alpha^*, \beta^* \in [0, 2\pi) \) are determined at each time step \( t \) using classical optimization by the dual annealing algorithm to maximize the overlap, \(|\langle \psi_0 |U^\dagger(t)|\psi_{\text{var}}(\alpha, \beta)\rangle|\), with the exact state. Naively, it appears that the classical optimization needs to be performed for each \( t \) and system size. Importantly, however, we find the optimal parameters \( \alpha^*, \beta^* \) to exhibit a simple oscillatory behavior as a function of time, as well as weak dependence on the system size as shown in Fig. 4(a)-(b). The data for system sizes \( N = 7, \ldots, 13 \) almost collapse, allowing a smooth extrapolation to the thermodynamic limit \( (N \to \infty) \), shown as the solid black line. In Fig. 4(c), we have checked using time-evolved block decimation (TEBD) [76] that the extrapolated parameters produce excellent agreement with direct TEBD calculation of \( |\psi(t)\rangle \) for larger systems. Thus, the weak system-size dependence of the variational parameters eliminates the need to directly perform the classical optimization for the actual size of the system, providing access to system sizes for which the classical optimization is infeasible.

Our variational algorithm’s circuit depth scales as the number of qubits \( N \) independent of the evolution time \( t \). As for trotterization, since we have a local lattice model in one dimension with no explicit Hamiltonian time dependence, the total circuit depth is expected to scale as \( N t \) for a fixed error tolerance [77, 78]. Despite higher complexity, trotterization corresponds to the actual unitary operator describing the quantum evolution and does not need any classical optimization or variational ansatz. Both algorithms have good scalability potential to more qubits.

**Results on the IBM Quantum Processor.** In Fig. 5, we present our measurements of the root state fidelity \( |\langle \psi(t) |100100 \ldots \rangle|^2 \), the local density \( \langle n_i \rangle \) and the equal-time density-density correlation function \( C_{ij}(t) = \langle n_i(t)n_j(t) \rangle + \langle n_i(t) \rangle \langle n_j(t) \rangle \). While these quantities are in terms of the original fermionic basis, they are easily extracted from measurements in the reduced basis using the rules discussed above Eq. (5). As shown in Fig. 5, the variational results are in excellent agreement the simulations. Similarly, the error-mitigated Trotter algorithm faithfully generates oscillations with the expected graviton frequency despite deeper circuits and higher execution-time error rates than the variational algorithm, which only induce quantitative shifts.

We note that the noise levels of the IBM devices vary widely. Using qiskit library, we executed error-mitigated circuit for the trotterization algorithm on ibmq_perth processor [79] with readout error, CNOT noise and T2 dephasing time of roughly 1.4%, 1.7% and 109 \( \mu s \) respectively. The variational ansatz was executed on IBM’s ibmq_santiago processor [79] with averaged readout error, CNOT noise and T2 dephasing time of roughly 1.5%, 0.6% and 120 \( \mu s \), respectively. We also performed simulations of our circuits in qiskit for comparison. Using post-selection methods, we improve the
measurements by discarding states that lie outside the physical Hilbert space.

**Conclusions.** We showed that quantum-geometrical features of FQH states can be realized in an effective 1D model that has an efficient quantum-circuit representation. Our 1D model makes efficient use of resources, as each qubit corresponds to three Landau orbitals, reminiscent of holographic quantum simulation [80]. As a proof of principle, utilizing the quantum-circuit mapping, we developed efficient quantum algorithms that allowed us to simulate graviton dynamics on IBM quantum processors. We used state-of-the-art error mitigation to successfully run the deep trotterization circuit, which does not require any classical optimization. We also developed a variational algorithm with a linear circuit depth (independent of the evolution time), which makes use of classical optimization but can be scaled to the thermodynamic limit. We expect these results will motivate further analytical investigations into tractable models of graviton dynamics in condensed matter systems, as well as their realizations on NISQ devices.

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Supplemental Material for “Probing Geometric Excitations of Fractional Quantum Hall States on Quantum Computers”

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In this Supplementary Material, we give a detailed derivation of the FQH Hamiltonian on a thin cylinder and in the presence of mass anisotropy. We briefly review the bimetric theory of FQH states and present additional evidence that the model studied in the main text captures the graviton dynamics. Finally, we discuss the Trotterization approach to simulating quantum dynamics in this model, and demonstrate its inadequacy in the currently available NISQ devices.

FQH HAMILTONIAN AND THE THIN-CYLINDER LIMIT

In this Section, we derive the parent Hamiltonian for the Laughlin ν=1/3 state on a thin cylinder with general mass metric. We pick the Landau gauge, in which the single-electron orbitals are given by

$$\phi_j^c(r) = \frac{1}{\sqrt{\pi^{1/2} L_2}} e^{i\kappa x_j y} e^{-(x-j)^2}, \quad (S1)$$

where $j$ denotes the orbital index within the lowest Landau level (LLL) and $L_2$ is the circumference of the cylinder, with $\kappa = 2\pi / L_2$ (we work in units $\ell_B=1$). The number of available single-electron orbitals is given by the magnetic flux as $N_\phi = (L_1 L_2)/(2\pi)$.

The field operator creating an electron at position $r$ is given by

$$\Psi^c(r) = \sum_j \phi_j^c(r)c_j^\dagger, \quad (S2)$$

where $c_j^\dagger$ is the creation operator which creates an electron at orbital $j$ within LLL (and similarly for the annihilation operator). From the field operator, the density operator in momentum space is given by

$$\hat{\rho}(q) = \int d^2r \Psi^c(r) \Psi(r) e^{iq \cdot r} = e^{-\frac{q^2 + q_y^2}{4}} \sum_j e^{iq_x \kappa y_j} c_j^\dagger c_j \equiv F(q) \hat{\rho}(q), \quad (S3)$$

where, we introduced the LLL form factor $F(q) = e^{-\frac{q^2 + q_y^2}{4}}$ and $\hat{\rho}(q)$ is the projected density operator.

Finally, the FQH Hamiltonian in momentum space takes the form

$$\hat{H} = \frac{1}{N_\phi} \sum_q \bar{\rho}(-q) \bar{V}(q) \bar{\rho}(q), \quad \bar{V}(q) = |F(q)|^2 v(q), \quad (S4)$$

where $v(q)$ is the interaction potential. For the $\nu = 1/3$ Laughlin state, the potential is given by $V_1$ Haldane pseudopotential and it takes the form $1 - |q|^2$ in momentum space, which translates into the Trugman-Kivelson real space potential $\nabla^2 \delta(r)$.

In the previous derivations, we have implicitly assumed that the electron band mass tensor,

$$g = g_m = \begin{pmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{pmatrix}, \quad (S5)$$

is isotropic, i.e., $g_{11} = g_{22} = 1$, $g_{12} = g_{21} = 0$. In a more general case, mass anisotropy modifies the single-electron wave functions in Eq. (S1), which turn modifies the effective interaction matrix elements. In momentum space, the effect of band mass metric is given by:

$$\bar{V}(g_m, q) = (1 - g_{11} q_x^2 - g_{22} q_y^2 - 2g_{12} q_x q_y) \times \exp \left( \frac{g_{11} q_x^2 + g_{22} q_y^2 + 2g_{12} q_x q_y}{2} \right). \quad (S6)$$

To obtain the cylinder Hamiltonian for a general metric, we integrate out $q_x$ in Eq. (S4) and use the above expression for $\bar{V}(g_m, q)$. Dropping the overall multiplicative constant and making use of translation invariance, we obtain

$$\hat{H} = \sum_{l,m,j} V_{l,m} c^\dagger_{l+m} c_{j+m} + \text{h.c.}, \quad (S7)$$

$$V_{l,m} = \kappa^3 m^2 (g_{11} g_{22} - g_{12}^2) e^{-\frac{2 \kappa^2 m^2 + 2m^2 \delta_{m,lm}}{g_{11}^2 g_{22}^2}} \quad (S8)$$

which can be simplified due to the unimodular property of the mass tensor, $g_{11} g_{22} - g_{12}^2 = 1$, due to the magnetic...
flux through the system being fixed \((\det g = 1)\). We will use the following convenient reparametrization of \(g\)

\[
g = \begin{pmatrix}
\cosh Q + \cos \phi \sinh Q & \sin \phi \sinh Q \\
\sin \phi \sinh Q & \cosh Q - \cos \phi \sinh Q
\end{pmatrix}
\]  

(S9)

in terms of real numbers \(Q\) and \(\phi\), which represent the stretch and rotation in the plane. With these simplifications, the final Hamiltonian can be written as

\[
\hat{H} = \sum_{j=0}^{N_a-1} \sum_{k>\lvert m\rvert} V_{k,m} c_{j+m}^{\dagger} c_{j+k} + c_{j+m+k} c_{j},
\]

\[
V_{k,m} \propto (k^2 - m^2) \exp \left(-\kappa^2 \left(k^2 + m^2 - 2i k m g_{12}\right) \right) / 2 g_{11}.
\]  

(S10)

In the thin torus limit, we can truncate the Hamiltonian in Eq. (S10) by dropping long-range scattering processes, which are exponentially suppressed in the small parameter \(\exp(-\kappa^2/2)\), arriving at the Hamiltonian studied in the main text,

\[
\hat{H} = \sum_j V_{1,0} \hat{n}_j \hat{n}_{j+1} + V_{2,0} \hat{n}_j \hat{n}_{j+2} + V_{3,0} \hat{n}_j \hat{n}_{j+3}
\]

\[
+ V_{2,1} c_{j+1}^{\dagger} c_{j+2}^{\dagger} c_{j+3} + \text{h.c.,}
\]  

(S11)

where \(\hat{n}_j \equiv c_j^{\dagger} c_j\) and \(V_{2,1} = 3 e^{-\kappa^2 0.25 \pi^2 g_{11} 12} / 2 g_{11}\). Comparison of the dynamics generated by the Hamiltonian in Eq. (S10) and the truncated Hamiltonian of Eq. (S11) are given in Fig S1. We find that both models give approximately the same dynamics near the thin-cylinder limit, where we expect the truncation to be justified.

The Hamiltonian in Eq. (S11) is positive semi-definite for general \(g_{12}\), so we can write it in the form with

\[
\hat{H} = \sum_j \left( Q_j^{\dagger} Q_j + P_j^{\dagger} P_j \right),
\]  

(S12)

\[
Q_j = \sqrt{V_{1,0}} c_j + 2 c_{j+1} + \sqrt{V_{3,0}} e^{i \frac{2 \pi}{L_2}} g_{12} c_{j+3},
\]  

(S13)

\[
P_j = \sqrt{V_{2,0}} c_j + 2 c_{j+2},
\]  

(S14)

where \(V_{1,0}, V_{2,0}\) and \(V_{3,0}\) are given in Eq. (S10) and \(e^{i \frac{2 \pi}{L_2}} g_{12}\) is the complex phase due to anisotropy parameterized by the previously mentioned tensor \(g\).

The ground state of Hamiltonian of Eq. (S12) is given by the following expression given in the main text:

\[
|\psi_0\rangle = \mathcal{N} \prod_j \left(1 - \sqrt{V_{3,0}} e^{i \frac{2 \pi}{L_2} g_{12}} S_j \right) |\ldots 100100\ldots\rangle,
\]  

(S15)

where \(S_j = c_{j+1}^{\dagger} + 2 c_{j+2} + 2 c_{j+3} c_j\) is the ‘squeezing’ operator mentioned in the main text, and \(\mathcal{N}\) is a normalization constant. It is evident that \(Q_j |\psi_0\rangle = P_j |\psi_0\rangle = 0\) for all sites \(j\), and \(|\psi_0\rangle\) is the ground state with zero energy. The geometric dependence enters our wave function in Eq. (S15) through the complex phase.

### GRAVITON EXCITATION NEAR THE THIN-CYLINDER LIMIT

The Hamiltonian in Eq. (S10) has several symmetries, most importantly due to the specific form of \(V_{k,m}\), it conserves the center-of-mass position of the electrons, \(K \equiv \sum_j \hat{n}_j\). This is because each \(V_{k,m}\) term destroys two particles, initially separated by \(|k-m|\) orbitals, and creates two particles at a distance \(k+m\) orbitals (and vice versa). This means that, in the Landau gauge, the total momentum along the circumference is conserved. Thus, we can simultaneously label our energy states as momentum eigenstates denoted by quantum number \(K\) (in the units of \(2\pi/L_2\)). It is convenient to label the orbitals \(j = 0, \pm 1, \ldots, \pm N_a/2\) such that \(\sum_j j \hat{n}_j |\ldots 100100\ldots\rangle = 0\), i.e. the root state lies in the zero momentum sector.

Before presenting the energy spectra of the models in Eqs. (S10)-(S11), we explain the construction of their bases in Fock space. In the case of the model in Eq. (S10), we work in the full Fock basis corresponding to \(N\) spinless fermions residing in \(N_0 = 2\) orbitals (note that the last two are always unoccupied sites). However, when considering the truncated Hamiltonian in Eq. (S11), the effective Hilbert space is much smaller: it consists of all configurations obtained by applying all possible squeezes to the root state. For example, the squeezed basis for \(N=4\) electrons is \(|100100100\rangle, |011000100100\rangle, |100011000100\rangle, |01100011000100\rangle\) and \(|0110000110000\rangle\). In general, the number of states in the squeezed basis is

![Fig. S1. Comparison of the geometric quench dynamics between the full Hamiltonian, Eq. (S10) and the truncated Hamiltonian, (S11), for cylinder circumference \(L_2=6.245\) and \(N=6\) electrons. The quench is driven by changing \(Q=0 \rightarrow Q=0.26\). Fit against the bimetric theory prediction in Eq. (S19) below.](image)
given by the Fibonacci number. Hence, the Hilbert space dimension still grows exponentially with system size, but it is much smaller than the full Fock basis. This is due to the special structure in the Hamiltonian (S11) which causes it to fracture into many dynamically-disconnected sectors [1].

![Spectrum](image)

**FIG. S2.** Spectrum of the Hamiltonian in Eq. (S11) with energies labelled by momentum $K$. Blue squares: spectrum obtained by restricting the Hamiltonian to the squeezed basis. Red dots: spectrum in the full Fock basis. The green line corresponds to the graviton energy.

The energy spectrum $E_K$ of the quench Hamiltonian is plotted in Fig. S2. We notice that the dominant state ($E_K \approx 1.29$) contributing to the dynamics is much higher than the first excited state in $K=0$ momentum sector, for both the squeezed basis and the full Fock basis. Note that the excited states with momenta $K \neq 0$ do not contribute to the dynamics since our quench preserves translation symmetry (hence, $K$ is conserved). In the main text, we identified $E_g \approx 1.29$ as the frequency of the emergent graviton mode which lies in the zero momentum sector. We have observed minor size and edge effects i.e. $E_g \approx 1.26$ for $N = 5$, 1.29 for $N = 7 - 9$ and $E_g \approx 1.3$ for $N = 15$.

**BREAKDOWN OF THE BIMETRIC THEORY IN THE STRICT 1D LIMIT**

For the observation of graviton oscillations in the main text, it was crucial that the FQH system is not in the strict 1D limit, $L_2 \to 0$. We require a sufficiently large $L_2$, typically $L_2 \gtrsim 5\ell_B$, such that the system can accommodate the essential correlations in neutral excitations underpinning the graviton mode. Taking $L_2 \to 0$ destroys these correlations, the graviton oscillations disappear and the bimetric theory description breaks down, as we illustrate in Fig. S3.

![Dynamics](image)

**FIG. S3.** Breakdown of graviton oscillations in the strict 1D limit. Plots show the deviation of microscopic dynamics of $\tilde{Q}(t)$ and $\tilde{\phi}(t)$ from the bimetric theory, for two values of the cylinder circumference. (a) The circumference is $3.15\ell_B$ and the dynamics still roughly follows the bimetric theory prediction, albeit with visible deviations. (b) The circumference is reduced to $2.75\ell_B$ and there is no agreement with the bimetric theory. For this value of the circumference, we are in the strict 1D Tao-Thouless limit where the dynamics is trivial.

In Fig. S3 we contrast the dynamics of $\tilde{Q}(t)$ and $\tilde{\phi}(t)$ for two values of the cylinder circumference, $L_2 = 3.15\ell_B$ (a) and $L_2 = 2.75\ell_B$ (b). While in the first case the dynamics still largely follows the bimetric theory prediction (albeit with visible deviations), in the second case the dynamics no longer conforms to the bimetric theory. In the second case, we are in the regime of the 1D Tao-Thouless limit, where the dynamics is trivial due to the initial state being close a product state and an eigenstate of the post-quench Hamiltonian. Thus, we conclude that finite $L_2$ is necessary to observe the graviton oscillation.
GEOMETRIC QUENCH IN THE BIMETRIC THEORY

In FQH systems, “geometry” appears in three distinct guises. First, the form factor $F_m(q) = \exp(-g_{ab}q_0q_0\ell^2_b/4)$ is a function of the band mass tensor $g_m$, where we use the Einstein summation convention. As discussed in Eq. (S9), the symmetric, unimodular tensor $g_m$ can be conveniently parametrized by $Q$ and $\phi$. Second, the interaction potential $V_q$, in general depends on another rank-2 tensor $g_i$, characterizing the underlying solid state material ($g_i$ originates from the dielectric tensor of the material for the Coulomb interaction). Both $g_m$ and $g_i$ are set by extrinsic experimental conditions. For simplicity, in this paper we study the model Hamiltonian for the Laughlin state, where we take $g_m = g_i$.

In the presence of extrinsic tensors $g_m$ and $g_i$, a FQH state develops its own, intrinsic geometric degree of freedom. This intrinsic degree of freedom defines the shape of particle-flux composite droplets in the FQH state and originate from the dielectric tensor of the solid state material (originates from the dielectric tensor of the material for the Coulomb interaction). Both $g_m$ and $g_i$ are modified by extrinsic tensors, modifying the ambient metric, $g^\text{amb} = g^\text{amb} + g^\text{ex}$. Quadrupolar anisotropy is introduced by modifying the ambient metric, $g^\text{amb} = g^\text{amb} + g^\text{ex}$, where $e_i^A$ are the vielbeins that describe the ambient geometry (we assume ambient space is flat, i.e., $e_i^A = \delta_i^A$) and $m_{AB}$ is a unimodular matrix, assumed to be of the form diag $[e_i^A, e^{-A}]$, where $A$ is the effective anisotropy.

In a homogeneous magnetic field, the bimetric Lagrangian is [6]

$$L = \frac{\nu^c}{2\pi\ell^2_B} \hat{\omega}_0 - \frac{m}{2} \left( \frac{1}{2} \hat{e}_i^m \gamma^m_{ij} - \gamma \right)^2,$$

where $\hat{\omega}_0 = \frac{1}{2} e_{\alpha\beta} \hat{E}_{\alpha}^\text{amb} \partial_0 \hat{e}_\beta^i$ is the temporal component of the (dynamic) Levi-Civita spin connection. The phenomenological coefficient $m$ sets the energy scale which determines the gap of the spin-2 mode, $E_g = 2\Omega(1 - \gamma)$, where $\Omega = (m/\nu)(2\pi\ell^2_B/\nu)$, and the quantized coefficient $\gamma$ is determined by the “shift” [9]. Parameter $\gamma$ is used to tune the theory close to the nematic phase transition in the gapped phase $\gamma < 1$, where the GMP description is exact.

In order to compute the dynamics of $g$, we parametrize it in terms of $Q$ and $\phi$ as in Eq. (S9). Both $Q$ and $\phi$ are functions of time but not space, since we consider global (homogeneous) quench. The equations for $\phi(t)$ and $Q(t)$ have been shown to take the form [5]

$$\hat{Q} = -2\Omega \left( \sinh A \cosh Q \cos \phi - \cosh A \sinh Q \right)$$

$$Q(t) = \frac{\pm 2A \sin \left( \frac{E_g t}{2} \right)}{\pi}$$

which was used for fitting the dynamics data in the main text. Note that there is only one linearly independent solution because the system is invariant under $Q \rightarrow -Q$ and $\phi \rightarrow \phi + \pi$. Thus, we can focus on the $Q \geq 0$ part and consider $\phi \mod 2\pi$. By inspection of $\dot{Q}(t)$, we see the solution alternates between two branches, which doubles the frequency from $E_g/2$ to $E_g$. The overall prefactor (written as $2A$) is expected to be proportional to the anisotropy of the post-quench Hamiltonian.

Ref. [5] established a good quantitative agreement between the numerically exact quench dynamics and the above prediction of the bimetric theory. The agreement is observed in the regime where the ground state before and after the quench remains in the Laughlin phase. Intuitively, the agreement is due to the fact that the quench involves an exponentially vanishing fraction of eigenstates that contribute to the dynamics, which a posteriori justifies the fundamental assumption of the bimetric theory that assumes a single spin-2 degree of freedom.

DERIVATION OF THE SPIN-CHAIN HAMILTONIAN

The Hamiltonian in Eq. (S11) can be mapped onto the spin model in Eq. (S22) in terms of the reduced registers.
Each reduced register corresponds to a block of three consecutive sites, where the reduced register $\ell$ contains three fermionic sites $j = 3\ell - 3, 3\ell - 2, 3\ell - 1$ as shown below:

\[
\begin{array}{cccc}
\bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet \\
\cdots
\end{array}
\]

To derive the mapping, note that it is sufficient to restrict to the subspace of states obtained by repeated action of the Hamiltonian on the root state, i.e., $H|100100\ldots, H^2|100100\ldots, \text{etc.}$, or, equivalently, states obtained by repeated action of the squeezing operators $\tilde{S}_j = c_j\dagger c_j + 2c_j^2 + 3c_j$ on the root state. We refer to this subspace as the squeezing subspace. Within this subspace, the terms $\hat{n}_j\hat{n}_{j+2}$ can be dropped as none of the states generated by the application of $\tilde{S}_j$ on $|100100\ldots\rangle$ (or $011, 001, 011, 000, 100$) has exactly two $1$s and two nearest neighbor occupied pairs. Therefore,

\[
\sum_{\ell=1}^{N-3} N_\ell + \sum_{\ell=1}^{N-3} N_\ell N_{\ell+2} + N_1 + N_{N-1}. 
\]

Finally, we map the squeezing (off-diagonal) term. Unless $j$ is the first site of a reduced block, the operator $V_{2,1}c_j^\dagger c_j + 2c_j^2 c_j + V_{2,1}c_j^\dagger c_j + 2c_j$ annihilates the state. Thus we focus on an individual term corresponding to block $\ell$.

\[
W_\ell = V_{2,1}c_j^\dagger c_j + 2c_j^2 c_j + V_{2,1}c_j^\dagger c_j + 2c_j,
\]

where $j = 3\ell - 3$ is the first site of the block $\ell$. We notice that if either of the neighboring blocks of an unsqueezed block $\ell$ are squeezed, $W_\ell$ annihilates a state:

\[
\begin{align*}
W_\ell|...0000\ldots\rangle &= W_\ell|...000011, 00, 000\ldots\rangle = 0, \\
W_\ell|...000000\ldots\rangle &= W_\ell|...0000000, 00, 01, 00\ldots\rangle = 0, \\
W_\ell|...000010\ldots\rangle &= W_\ell|...000010, 00, 01, 00\ldots\rangle = 0.
\end{align*}
\]

We can implement this by introducing a factor $1 - N_{\ell-1}(1 - N_{\ell+1})$. Blocks $\ell = 1$ and $\ell = N-1$ are special because they have only one neighboring block. If block $\ell$ is already squeezed it cannot have a squeezed neighbor, in which case

\[
\begin{align*}
W_\ell|...000010\ldots\rangle &= V_{2,1}|...100, 00, 00, 000\ldots\rangle = V_{2,1}|...000000\ldots\rangle, \\
W_\ell|...000000\ldots\rangle &= V_{2,1}|...100, 00, 00, 000\ldots\rangle = V_{2,1}|...000000\ldots\rangle.
\end{align*}
\]

Therefore if the neighbors are $0$ the action of $W_\ell$ in the $|\Psi, \Psi\rangle$ basis is given by the matrix

\[
W_\ell = \begin{pmatrix} 0 & V_{2,1} \\ V_{2,1}^\star & 0 \end{pmatrix} = \text{Re}(V_{2,1})X - \text{Im}(V_{2,1})Y.
\]

We then represent the full squeezing term for open boundary condition as

\[
\begin{align*}
\sum_{j=0}^{N-4} & V_{2,1}c_j^\dagger c_j + 2c_j^2 c_j + V_{2,1}c_j^\dagger c_j + 2c_j + \text{Re}(V_{2,1})X_\ell - \text{Im}(V_{2,1})Y_\ell \\
& \rightarrow \sum_{\ell=2}^{N-2} (1 - N_{\ell-1}) |\text{Re}(V_{2,1})X_\ell - \text{Im}(V_{2,1})Y_\ell| (1 - N_{\ell+1}).
\end{align*}
\]

**DETAILS OF TROTTERIZATION CIRCUIT**

In the main text, we wrote the quench Hamiltonian as

\[
\hat{H} = \sum_{\ell} H_\ell,
\]

giving rise to a unitary operator $U(t) = [\prod_\ell U_\ell(\delta t)]^k$. Here we explicitly write the $U_\ell$ operators, ignoring the
commutations of the terms in the exponent in the limit of small $\delta t$. For $\ell = 0$, we can write
\[
U_0(\delta t) \approx e^{-i\frac{V_{1,0} - 2V_{3,0}}{2}(I + Z_0)\delta t}e^{-iV_{2,1}X_0(1-N_1)\delta t},
\]
where $I$ is the identity. Ignoring a global phase, the first terms is a $z$ rotation and the second term is a controlled $x$ rotation, resulting in the circuit element of Fig. S4. Similarly the unitary at the right boundary can be written as
\[
U_{N-1}(\delta t) \approx e^{-i\frac{V_{1,0} - 2V_{3,0}}{2}(I + Z_{N-1})\delta t}e^{-iV_{2,1}(1-N_{N-2})X_{N-1}\delta t},
\]
giving the circuit element shown in Fig. S5.

\[
|\psi_{\text{var}}(\alpha, \beta)\rangle = \prod_{\ell \neq 0} e^{-i\alpha N_{\ell}} e^{-i\beta (1-N_{\ell-1}) X_{\ell}} |000\ldots\rangle, \tag{S24}
\]

FIG. S4. Circuit implementation of the Trotterized unitary $U_0$, at the boundary at the left of the chain.

In the bulk of the spin chain, the unitary operator up a global phase has the following structure
\[
U_{\ell}(\delta t) \approx e^{-i\frac{V_{1,0}}{4} Z_{\ell-1} \delta t} e^{-i\frac{V_{1,0} - 3V_{3,0}}{2} Z_{\ell} \delta t} e^{-i\frac{V_{3,0}}{4} Z_{\ell+1} \delta t} 
\times e^{-i\frac{V_{2,1}}{4} Z_{\ell-1} Z_{\ell+1} \delta t} e^{-iV_{2,1}(1-N_{\ell-1})X_{\ell}(1-N_{\ell+1})\delta t}.
\]
Noting that the operator $e^{-i\frac{V_{2,1}}{4} Z_{\ell-1} Z_{\ell+1} \delta t}$ can be implemented by a $z$ rotation acting on qubit $\ell + 1$ sandwiched between two CNOT gates with qubit $\ell - 1$ as the control bit, we find the circuit element presented in the main text for implementing $U_{\ell}(\delta t)$ in the bulk. The unitary constructed above should then act on the prequench ground state. We use the circuit of Ref. 10 to create the Laughlin $\nu = 1/3$ fractional quantum Hall state as the initial state.

\[
U(t) = |\psi(t)\rangle \langle \psi(t)|
\]

\[
|\psi_{\text{var}}(\alpha, \beta)\rangle \succ |\psi_{\text{var}}(\alpha^*, \beta^*)\rangle
\]

VARIATIONAL QUANTUM ALGORITHM.

In the geometric quench, the initial state is the ground state $|\psi_0\rangle$ with the identity metric ($g_{12} = 0$). Ref. 10 found an efficient algorithm to generate $|\psi_0\rangle$ with a linear-depth circuit. Working in the reduced space, we can use only stage 1 of the circuit in Ref. 10 identifying qubits $1+3n$ with the reduced registers. The state after the quench is generated by the unitary evolution operator $U(t) = \exp(-it\hat{H})$, where $\hat{H}$ is the anisotropic post-quench Hamiltonian. Thus, combining an algorithm that generates $U(t)$ with the the ground-state preparation algorithm yields the post-quench state $|\psi(t)\rangle = U(t)|\psi_0\rangle$.

A large number of Trotterization steps is needed to obtain accurate results for $U(t)$. Furthermore, terms like $(1-N_{\ell-1})X_{\ell}(1-N_{\ell+1})$, which appear in Eq. (S22), require $R_x$ gates controlled by two other qubits, in turn requiring six CNOT gates for each control gate. In order to overcome these difficulties and scale the algorithm to larger systems, we approximate $|\psi(t)\rangle$ using the ansatz:

\[
|\psi_{\text{var}}(\alpha, \beta)\rangle = \prod_{\ell} e^{-i\alpha_{\ell} N_{\ell}} e^{-i\beta_{\ell}(1-N_{\ell-1}) X_{\ell}} |000\ldots\rangle \tag{S23}
\]

The above ansatz has site-dependent variational parameters $\alpha_{\ell}, \beta_{\ell}$. As we have translational symmetry away from the boundaries, we can further simply this ansatz ans approximate the dynamics by only two variational parameters:

\[
|\psi_{\text{var}}(\alpha, \beta)\rangle = \prod_{\ell} e^{-i\alpha N_{\ell}} e^{-i\beta (1-N_{\ell-1}) X_{\ell}} |000\ldots\rangle \tag{S24}
\]
Indeed, we have numerically found that imposing translation invariance on the variational parameters produces high overlaps (~0.99) with the exact state and is sufficient for capturing any local observable expectation values in the bulk of the system, in particular it faithfully reproduces the quantum metric parameters $\bar{Q}$ and $\bar{\phi}$, as well as the full wave function obtained from exact diagonalization (ED) for $N=9$ electrons.

The ansatz in Eq. (S24) has several key advantages that reduce its implementation cost on NISQ hardware, while at the same time it captures any local observable expectation values in the bulk of the system, in particular it faithfully reproduces the quantum metric dynamics, $\bar{Q}(t)$ and $\bar{\phi}(t)$. Importantly, the optimal parameters $\alpha^*$, $\beta^*$ are found to exhibit a simple oscillatory behavior as a function of time as mentioned in the main text.

One advantage of the ansatz in Eq. (S24) is that it relies only on two-qubit control gates, while ensuring no forbidden states (with neighboring 11 registers) are generated as shown in the Fig. S6. Another advantage is that we can obtain the final state by acting on the trivial root state $|000\ldots\rangle$ rather than the ground state $|\psi_0\rangle$. Since all registers are zero in the root state, if we start from the left end of the chain and apply controlled rotations, the qubit to the right is always zero before the application of the unitary operator, allowing us to use two-qubit control gates instead of three-qubit ones, with each register controlled only by its left neighbor.

**EXTRAPOLATION OF VARIATIONAL PARAMETERS**

The weak system-size dependence of the optimal variational parameters $\alpha^*$ and $\beta^*$ allows us to extrapolate them to the thermodynamic limit, see Fig. 5 of the main text. We use $1/N$ as a small parameter, and assume $\alpha^*$ and $\beta^*$ have a Taylor expansion in powers of $1/N$ for each total time $t$:

\[
\begin{align*}
\alpha^*(N,t) &\approx p_0^\alpha(t) + \frac{p_1^\alpha(t)}{N} + \frac{p_2^\alpha(t)}{N^2} + \frac{p_3^\alpha(t)}{N^3} + \ldots \quad (S25) \\
\beta^*(N,t) &\approx p_0^\beta(t) + \frac{p_1^\beta(t)}{N} + \frac{p_2^\beta(t)}{N^2} + \frac{p_3^\beta(t)}{N^3} + \ldots \quad (S26)
\end{align*}
\]

For large $N$, the higher-order terms in the expansion are unimportant. As seen in Fig. 5 of the main text, there is good agreement between the cubic and quadratic fits, with the largest difference on the order 1% for $\beta^*$. Furthermore, the cubic fits are in excellent agreement with TEBD results for large systems. Therefore, the parameters $p_0^\alpha, p_0^\beta$ obtained from the cubic fit provide a smooth, accurate extrapolation of the optimal variational parameters to the thermodynamic limit, $N \to \infty$.

**ERROR MITIGATION TECHNIQUES**

The Hamiltonian simulation via Trotterisation executed on Noisy Intermediate-Scale Quantum (NISQ) computers suffers from physical errors in addition to the algorithmic. Quantum error mitigation approaches are essential to exploit the quantum advantage of NISQ devices. Noise-aware quantum compilers, which translate the quantum algorithm into physical quantum circuits supported by the underlying quantum hardware, incorporate different error mitigation approaches into the compilation process. We apply several optimization layers to boost the fidelity of the quantum circuits, which can be integrated with the state-or-the-art quantum circuit compilation approaches (e.g. Quantum Information Software Kit (Qiskit [11])). We use Qiskit compiler to generate physical quantum circuits. This process involves optimizing the gate count and the circuit depth. Given the physically mapped and optimized quantum circuit, quantum gates are rescheduled based on the gate commutation rules [12, 13] to reduce the quantum circuit error rates [14]. The objective is to push gates with very high error rates to later layers in the quantum circuit to limit their error impact on the circuit when the variation in the gate error rates is significant. The rescheduling algorithm maintains the circuit depth and therefore does not introduce new decoherene errors. This rescheduling algorithm can also create new optimization opportunities, which can further reduce the gate count. We also exploit the trade-off between the algorithmic errors and the physical errors of the trotterisation through the heuristic quantum search compiler [15]. This compiler constructs a quantum sub-circuit that realizes a unitary matrix approximately by searching for a sequence of quantum gates which form a unitary matrix such that the distance between the approximated and the target unitary matrices is within the accepted threshold. To ensure a reduction
in the CNOT gate count, we iteratively increase the error threshold until we observe an improvement in the circuit gate count. The maximum threshold value that we consider is $10^{-2}$. The approximation can be applied to one or more Trotter steps depending on the injected errors in each approximated subcircuit.

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