Order-Optimal Joint Transmission and Identification in Massive Multi-User MIMO via Group Testing

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Abstract—The number of wireless devices continues to grow, and more antennas per device are being added, increasing the challenge of efficient resource allocation, especially for uplink streams. To address this, we propose a novel massive multiple-user multiple-input-multiple-output (MU-MIMO) scheme based on Group Testing (GT) with non-cooperative self-scheduling users, reducing the required overhead and complexity. Specifically, we show that out of a population of \( N \) devices with \( M \) messages each, it is possible for the base station (BS) to jointly identify and decode up to \( K \) devices, unknown in advance, simultaneously and without any scheduling or channel state information. The BS efficiently decodes the transmissions with vanishing error probability using only \( O(K \log N M) \) antennas, which implies order-optimal sum-rate.

I. INTRODUCTION

Multiple-input-multiple-output (MIMO) systems, which incorporate multiple antennas at the transmitter and receiver, have become ubiquitous due to their ability to boost channel robustness and capacity. Several techniques have been suggested over the years which utilize MIMO to selectively and adaptively optimize the available channel. MIMO can improve performance in single and multi-user (MU) communications. In both cases, there is a tradeoff between the channel state information (CSI) available at the transmitter and receiver, which involves overhead, and the achievable gains. Typical wireless communication systems (e.g., cellular and WLAN) rely on channel reports collected by the base station (BS) or access point (AP). Yet to reduce the enormous overhead in collecting CSI from all users at all times, they compromise on the accuracy of these reports and collect CSI only occasionally and only from a selected set of users each time.

Scheduling which user(s) should transmit next is highly important in communication systems and MIMO in particular. All the more so in MU-MIMO communication, in which the specific set of users chosen for transmission highly influences the performance. Accordingly, numerous studies have focused on user selection (e.g., [1], [2]). Again, user selection involves a tradeoff between overhead and performance. On the one hand, collecting CSI from all users can enable selecting the optimal group of users for transmission, yet it involves a lot of overhead. On the other hand, collecting CSI from only a small set of users reduces the overhead involved, yet compromises performance. Note that the overhead encompasses both the CSI gathering from the users and the BS scheduling announcement. For example, the state-of-the-art WiFi technology, the 802.11ax standard, supports MU-MIMO both for the upstream (users to AP) and for the downstream (AP to users), allowing simultaneous transmission from a small group of users (up to eight users). The standard defines that channel estimation (channel report collection) will be performed from the preselected users before each MU transmission opportunity (TXOP) (e.g., [3, Chapters 3.3.4-3.3.6]). In addition to the overhead involved in collecting CSIs, the AP needs to track which users are backlogged, which in the case of upstream traffic, can incur additional overhead.

In this study, we suggest an upstream self-scheduling mechanism for MU-MIMO, in which the users schedule themselves for transmission. The proposed mechanism eliminates overhead as it requires no CSI collection by the receiver, and there is no need for tracking the current backlogged users. It relies on combining two unrelated techniques, Index Modulation (e.g., [4, Chapter 1.2]), and Group Testing (GT) (e.g., [5], [6]). In this study, Index Modulation refers to the setup in which the transmitter activates a subset of receive antennas. The specific set of activated antennas determines the modulated symbol. Note that the demodulation is quite simple, as it requires a single energy detection for each receive antenna, similar to On-Off Keying [7].

The GT concept tackles the problem of finding \( K \) ill patients (or other defective entities) out of a large population of \( N \) patients. To minimize the number of tests required to identify the \( K \) ill patients, rather than examine each sample individually (\( N \) tests), the samples from a group of patients are mixed and examined together as a single sample. The group of patients participating in each test is determined \textit{a priori} in the form of a test matrix (non-adaptive GT). It has been shown that the number of group tests required for vanishing error probability is in the order of \( K \log N \) (see, for example, [8], [9]). Handling scenarios in which the samples may be contaminated have also been addressed (e.g., [10]–[14]).

The GT concept was adapted to communication terminology (e.g., [15]). The users and their messages are analogous to the population of \( N \) patients. The \( K \) ill patients are the self-scheduled users who actually send messages whose identity is unknown. The test matrix is akin to a binary codebook. The tests conducted are typically energy detection results on different system resources such as timeslots, frequency bands, or, as in the suggested scheme, excited antennas. Over the years, several studies have suggested GT-originated codes to devise communication protocols capable of simultaneous decoding of many messages using a simple decoding algorithm.
For example, in [16], Robin and Erkip proposed an energy-efficient sensor discovery in power-constrained clustered networks. In [17], Inan et al. have leveraged the constant weight constructions suggested in [18], [19] to solve and analyze an asynchronous GT variant, in which each user’s transmissions are received with arbitrary slot offsets. Cohen et al. proposed a GT-based communication protocol in [20], based on the binning ideas from [21]. Robin and Erkip have analyzed a protocol similar to [20] in [22], albeit for activity discovery rather than reliably conveying messages, where they assumed a slow-fading Rayleigh channel where each party has only one antenna, and the channel remains constant during the transmission procedure. The main idea is to reduce the continuous signal and noise models into discrete binary models using energy detection, where the energy at a timeslot is compared to some threshold, followed by Noisy CoMa for decoding ([12]). All these works assumed transmissions over separate timeslots or frequency bands, which replaced the original GT test tubes of the human patients. However, the extension to the spatial dimension, where test tubes are replaced by antennas, is challenging. Unlike different timeslots or frequency bands in which there is an inherent separation between the resources just as with the GT test tubes, beamforming to a specific set of antennas results in energy leakage to all other antennas, which depends on the channels between the transmit and receive antennas and can result in an unintended antenna receiving power that exceeds the energy received by the aimed antenna, all the more so when $K$ users are transmitting simultaneously.

Accordingly, adjusting existing schemes for MU-MIMO requires careful design to prevent self-interference.

The suggested scheme relies on a codebook generated using methods from GT. To tackle self-interference, users leverage their knowledge of the channels which requires minimal overhead as can be accomplished by a short single training sequence sent by the receiver to all users simultaneously. Yet, rather than beamforming to the desired antennas, the transmitting users null their transmitted signals’ energy at the unintended antennas. Still, since the transmitter utilizes the multi-antenna degree of freedom to null-steer energy to the un-stimulated antennas, the energy received by an un-nulled (stimulated) antenna is random, depends on the channels to this antenna, and may be depleted. Accordingly, the receiver needs to devise an energy detection mechanism (converting channel output to a binary vector) to estimate which antenna is targeted (not nulled) by at least one user. The binary vector is treated as the result vector of GT and serves as the input of a decoding algorithm that returns the sent messages (consequently, the identities of the transmitting users as well).

We devise an optimal energy threshold which maximizes the system sum-rate. The suggested scheme requires no scheduling overhead (headers, control messages, CSI collection at the receiver, etc.) and has remarkably low complexity. We thoroughly analyze the suggested scheme’s error probability and antenna scaling laws. We show that the number of required antennas is logarithmic in the total number of users in the system and is linear with the number of self-scheduled users.

Moreover, we show that our scheme is order-optimal in the number of users or the number of messages per user.

II. SYSTEM MODEL

A. Notation

Matrices appear in bold (e.g., $H$) and vectors are underlined (e.g., $\mathbf{x}$). We use subscripts for user indices (e.g., $H_k$). Components of a vector or matrix are specified with subscripts after square brackets (e.g., $[y]_m$ is $y$’s $m^{th}$ component, $[H_k]_{i,j}$ is $H_k$’s component in the $i^{th}$ row and $j^{th}$ column). For matrix columns, we put a single subscript followed by a colon after square brackets. E.g., $[H_k]_{:,j}$ is $H_k$’s $j^{th}$ column. Similarly, for the $j^{th}$ row we write $[H_k]_{i,:}$ and $(\cdot)^T$ denotes the transpose operation. Unless explicitly specified, all logarithms in this manuscript are in base 2; $\ln(\cdot)$ is the natural logarithm.

B. Model

We assume a time-slotted network of $N$ users transmitting to a single BS. The time slot duration is adjusted to encompass an energy detection for a single pulse (bit) transmission. We assume that the BS and all the users are slot-wise synchronized. Even though there is no schedule and each user determines whether to transmit in a timeslot randomly regardless of other transmitters, we assume that at each time slot, a set of up to $K$ users transmit, their identities are unknown a priori.

Each transmitter has $M_t$ antennas, and the receiver has $M_r$ antennas. We assume $M_t = M_r$, and a Massive MIMO setting, s.t. $1 \ll M_t, M_r$. Each user sends $\log M$ bits per TXOP. We denote the set of user $k$’s $M$ messages by $W_k$. Without loss of generality, we denote the message sent by user $k$ by $w_k$. There is no a priori knowledge about the distribution of which message is sent, or which user is transmitting.

The channel matrix of the $k^{th}$ user is denoted by $H_k \in \mathbb{C}^{M_r \times M_t}$. We assume a scatter-rich urban environment with no line of sight, so each entry in $H_k$ is a zero-mean Complex Gaussian Random Variable (CGRV) (e.g., [23, Chapter 10.1]). For convenience, we shall assume each component of $H_k$ has a unit variance, that is, $[H_k]_{i,j} \sim \mathcal{CN}(0,1)$ for all $i,j,k$.

Since $H_k$ is known to the user $k$, its encoder can utilize it to determine the transmitted symbol from its antennas, e.g., beamforming. We denote user $k$’s transmitted symbol by a complex vector $\mathbf{z}_k \in \mathbb{C}^{M_t \times 1}$ which is a function of the transmitted message $w_k \in W_k$ and $H_k$. The transmission is bounded by power level, $P$, i.e., $\|\mathbf{z}_k\|^2 \leq P$.

The BS has no CSI or any other information regarding the transmitting users’ channel nor their identities. We assume a zero-mean White Complex Gaussian Additive Noise, $\mathbf{n} \in \mathbb{C}^{M_r \times 1}$, where $[n]_i \sim \mathcal{CN}(0,N_0)$ for all $i$. Finally, the BS obtains

$$y = \sum_{k=1}^{N} \delta_k H_k \mathbf{z}_k + \mathbf{n}.$$  

(1)

where $\delta_k \in \{0,1\}$ indicates whether user $k$ is active or not.
The decoder uses $y$ to obtain both the messages sent and the identity of the $K$ users. We say that the system is message-user reliable if the decoder obtains exact correct $K$ messages.

III. A MIMO-GT-BASED TRANSMISSION SCHEME

In this section, we describe our suggested scheme in depth. The scheme is comprised of three parts: codebook generation, transmission scheme, and receiver algorithm. Next, we elaborate on each part. Some of the parameters mentioned will be determined later.

Codebook- We utilize a typical random codebook similar to the one suggested in other studies, e.g., [5]. Specifically, we generate $NM$ binary $M_r$-lengthed codewords, $c_j$ for $1 \leq j \leq NM$, and distribute $M$ codewords to each user. E.g., user $k$ obtains codewords indexed by $(k-1)M + 1$ to $kM$. Each bit in these codewords is generated using i.i.d. Bernoulli distribution with parameter $p$, which will be determined later.

Transmission- To transmit a codeword, each transmitter beamforms its signals to null the energy at all antennas whose indices correspond to zeros in the desired codeword. Specifically, to transmit the codeword, $c_j \in \{0, 1\}^{M_r \times 1}$, the user follows the following procedure: Collect all rows of $H_i$ whose index corresponds to a zero in $c_j$. That is, define $Z^j \triangleq \{ l : [c_j]_l = 0 \}$, and construct $H_{Z^j} \triangleq \{ [H_i]_l \}_{l \in Z^j} \in C^{|Z^j| \times M_i}$. Next, pick $Z^j$, an arbitrary vector satisfying the power constraint from $H_{Z^j}$’s nullspace. We call this technique “Randomized Zero-Forcing” (RZF) beamforming. An example of the RZF algorithm can be found in [24].

The intention of RZF is that the receiver will put an energy threshold on each antenna and determine which receive antenna received no energy (i.e., its index corresponds to zeros from all transmitting users) and which at least one transmitting user did not null (relying on the transmitter’s un-nulled antennas to receive some energy). However, in the presence of noise, also the receiver antennas that have been nullled by all users will detect some energy and the energy threshold should distinguish between the antennas that experience only noise and those that receive some of the transmitted power. Accordingly, the RZF can be improved. Specifically, it is possible that rather than a random choice, each transmitter selects a vector from $H_{Z^j}$’s nullspace that maximizes the SNR at the antennas which are not nulled. However, we omit this analysis as analyzing the scheme with the optimized vector can be complex and does not substantially change the qualitative discussion in terms of the order-optimality we wish to accomplish.

Receiver Algorithm- The receiver obtains $y$ according to (1), and compares $|y_i|^2$ to an energy threshold $N_0 \gamma$ for all $i$ (the selection of $\gamma$ will be discussed later). The result of the hard decision comparison, $Y_i$ is a binary vector similar to the infected test tubes in the GT context. $Y$ is sent to a GT-decoding algorithm to obtain the codewords and their senders’ identities.

As previously explained, since there is no control over the energy received by the desired antennas (which can be quite low), and due to noise, a hard decision according to energy threshold may introduce erroneous bits in $Y$. These errors are characterized by crossover probabilities from ‘1’ to ‘0’ (miss-detecting an antenna that at least one user targets) and vice-versa from ‘0’ to ‘1’ (identifying an antenna that no user targets as an antenna that at least one user targets). We denote these probabilities by $q_{10}$ and $q_{01}$, respectively. Consequently, we need to utilize a GT decoder that handles errors. We adapted the Noisy CoMa algorithm ([12]), which obtains the messages from the $Y$.

Basically, Noisy CoMa examines all codewords and discards those that almost surely were not transmitted. Note that in the absence of errors an index of a transmitted codeword $c_j$ of ‘1’ should result in energy detection in the antenna with the same index hence a ’1’ in the same index of $Y$. On the other hand, a transmitted codeword’s index of ‘0’ will not necessarily result in ‘0’ on the same index of $Y$, as it depends on the other transmitters. Accordingly, when Noisy CoMa discards the almost surely not transmitted codewords, it focuses on the codeword indices that are ‘1’, and discards all codewords that the fraction of ‘1’ in the codeword that are ‘0’ in $Y$ exceeds a tunable low threshold. Formally, Noisy CoMa obtains the messages from the $Y$ according to the following criterion:

Noisy CoMa Decision Criterion [22] Fix $\Delta > 0$ ($\Delta$ is tunable and will be discussed later). Denote $\text{supp}(c_j)$ and $\text{supp}(Y)$ as the set of indices where $c_j$ and $Y$ have non-zero components, respectively. Let $T_j \triangleq |\text{supp}(c_j)|$ and $S_j \triangleq |\text{supp}(c_j) \cap \text{supp}(Y)|$. Noisy CoMa declares that $c_j$ has been transmitted if and only if $S_j \geq T_j^\beta (1 - q_{10}(\Delta + 1))$.

IV. SUFFICIENT CONDITIONS

In this section, we examine sufficient requirements for reliable communication of the proposed MIMO-GT scheme. Specifically, we analyze the required number of transmit-receive antennas and the associated energy threshold ($\gamma$) in order to achieve a vanishing error probability. The main result is summarized in the following theorem:

**Theorem 1 (Direct):** Fix $N, K$, and $M$. Let $\delta > 0$. MIMO-GT with error probability less than $(N,M)^{-\delta}$ requires no more than $(1 + \delta)\beta^2 K \ln NM$ receiver antennas for some constant $\beta > 1$. Consequently, MIMO-GT is message-user reliable.

The theorem proof comprises three steps: (i) Computing the per antenna error probability, i.e., the error probability in estimating whether at least one user targets antenna $i$ in the energy detection phase. (ii) Devising sufficient conditions on $M_r$ such that Noisy CoMa’s error probability vanishes. (iii) Minimizing the sufficient conditions, in light of the first step, and showing that the minimizer is unique.

**Antenna-wise Error Probability** - As previously explained, a hard decision that relies on energy detection introduces the error probabilities $q_{10}$ and $q_{01}$. These crossover probabilities are given in the following two lemmas.

**Lemma 2**: For any $\gamma$, the crossover probability from ’0’ to ’1’ is $q_{01} = e^{-\gamma}$. 

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Lemma 3: Let $\rho \triangleq \frac{E}{N_0}$. For any $\gamma$, the crossover probability from '1' to '0' is

$$q_{10} = \sum_{j=1}^{K} \binom{K}{j} p^j (1-p)^{K-j} \left(1 - \exp \left\{ -\gamma j \rho + 1 \right\} \right).$$

The computations can be found in [24].

Decoding Error Probability- In order to determine the sufficient conditions on the number of required antennas we bound the decoding error probabilities. We shall follow the footsteps of [12, Section V.B], which analyzed the Noisy CoMa algorithm for a Binary Symmetric Channel, with the following adaptations: errors are non-symmetric, and each user has $\mathcal{M}$ codewords. The analysis should be modified accordingly.

We have to consider two types of errors; Mis-detection, $p_{MD}$, where Noisy CoMa fails to find at least one of the transmitted codewords, and false-detection, $p_{FD}$, where Noisy CoMa declares at least one codeword that was not transmitted. $p_{FD}$ also encompasses the event that identical codewords were created by the random code generator. The following two lemmas present the sufficient conditions on $p_{MD}$ and $p_{FD}$:

Lemma 4: Fix some $\delta > 0$. Set $M_r \geq (1 + \delta) \beta_1 K \ln N \mathcal{M}$. $p_{MD} \leq (N \mathcal{M})^{-\delta}$ if

$$\beta_1 \geq \gamma K (1 - \exp \left\{ -2 q_{10} \Delta^2 \right\})^{-1}.$$

Lemma 5: Fix some $\delta > 0$. Set $M_r \geq (1 + \delta) \beta_2 K \ln N \mathcal{M}$ and $\Delta < \frac{p_0}{q_{10}} - 1$. $p_{FD} \leq (N \mathcal{M})^{-\delta}$ if

$$\beta_2 \geq \gamma K (1 - \exp \left\{ -2 p_0 - q_{10} (\Delta + 1) \right\})^{-1}$$

where $p_0$ denotes the probability that the energy detected by a specific antenna was below the threshold, i.e., for any antenna $i$, $p_0 = P(\triangleleft Y_i = 0) = (1 - (1-p)^K) q_{10} + (1-p)^K (1 - q_{10})$. The proofs can be found in [24].

We note that if less than $K$ users transmit and $M_r \geq (1 + \delta) \beta K \ln N \mathcal{M}$, then both lemmas 4 and 5 are still valid, and $\max\{p_{MD}, p_{FD}\} \leq (N \mathcal{M})^{-\delta}$. If a system has a minuscule number of additional time slots, frequency bands to compensate for the difference. E.g., if our solution requires $L = l \cdot M_r$ antennas, use $l$ time slots. In each time slot, save the channel output to obtain $\{y_l^1, y_l^2, \ldots, y_l^T\}$. Therefore, the number of required antennas is a function of $\beta^*$, we briefly discuss how $\beta^*$ scales with $\rho$ and $K$. First, we shall show that $\beta^*$ converges to some constant when $K \to \infty$.

Proposition 7: Let $\rho = \frac{q_{10}}{\gamma}$, where $\alpha > 0$ is some constant. If $1 \leq \gamma \leq \max\{1, \rho\}$, then $\beta^* \leq \frac{b c_{\max}(1, \rho) (\rho + 1)^2}{(1 - \frac{1}{\gamma}) q_{10}}$.

Proof Sketch: First, bound $q_{10}$ from above by its largest addend. Next, bound $\left(1 - \frac{1}{\gamma}\right)^{2K}$ from below by substituting $K = 2$. Finally, apply Taylor series expansion. The formal proof is attached in [24].

Proposition 7 bounds $\beta^*$ from above by a constant with respect to $K$, and $\beta^*$ is monotonously decreasing in $K$. Therefore, $\beta^*$ converges to some constant when $K \to \infty$.

We note that the choice of $\Delta$ and $p$, and the antenna-wise error probabilities which are associated with threshold $\gamma$ are crucial to the number of required antennas. For example, taking $\gamma$ too low results in small $q_{10}$, high $\beta_1$ and consequently high $M_r$. We further note that both decoding error probability terms follow the exact same pattern with the only difference on the $\beta$, i.e., in both, the number of required antennas are bounded from below by $M_r \geq (1 + \delta) \beta K \ln N \mathcal{M}$ $i \in \{1, 2\}$. Accordingly, to reduce the number of antennas required for vanishing error probability we need to minimize $\max\{\beta_1, \beta_2\}$:

$$\min_{\Delta, \gamma, \rho} \max\{\beta_1, \beta_2\}$$

s.t. $0 < \Delta < \frac{p_0}{q_{10}} - 1$; $0 < p \leq \frac{1}{2}$; $0 < \gamma$

We note that solving optimization problem (2) determines both the threshold $\gamma$ and the random code generating parameter $p$. The optimization problem can be transformed to the following optimization problem, which is easier to be solved.

Lemma 6: The optimization problem in (2) is equivalent to, and has the same solution as the following optimization problem:

$$\min_{\rho, \gamma \in \mathbb{R}} \{\gamma K (1 - \exp\{-\frac{1}{2} (1-p)^{2K} (1 - q_{10} - q_{10})^2\})\}^{-1}$$

s.t. $0 \leq p \leq \frac{1}{2}$; $0 \leq \gamma$

which has a unique solution.

The proof is technical and appears in [24].

Proof sketch: The proof comprises four main steps: (i) Convert the minimax problem into a minimization problem by eliminating the dependency on $\Delta$. This is done by noting that $\beta_1$ and $\beta_2$ have opposing trends in $\Delta$, hence $\Delta^* = \min\{\frac{1}{2} \frac{(1 - q_{10} - q_{10})^2}{(1-p)^{2K} (1 - q_{10} - q_{10})^2}\}$.

(ii) Show that $p^* \leq \frac{q_{10}}{\gamma}$ and is unique. (iii) Prove that for any $p$, there exists a unique $\gamma$ that minimizes the objective function. Therefore, $\gamma^*$ is unique. (iv) Show that $\gamma^* = O(\ln(K \rho))$ and $\gamma^* \geq \ln(\rho + 1)$.

The optimization problem in lemma 6 is solvable by numerical means which produce the optimal $\gamma^*$ and $p^*$ and consequently $\Delta^*$.

Antenna Scaling Laws- Since the number of required antennas is a function of $\beta^*$, we briefly discuss how $\beta^*$ scales with $\rho$ and $K$. First, we shall show that $\beta^*$ converges to some constant when $K \to \infty$.

Proposition 8: If $\rho \to 0$ then $\beta^* \to \infty$.

Proof Sketch: When $\rho \to 0$, $q_{10}$ tends to $1 - q_{10}$ which implies $\beta^* \to \infty$ for any $\gamma$ and $\gamma^* = O(\ln \rho)$ so $\beta^*$ converges when $\rho \to \infty$ for any $p$ as the crossover probabilities vanish. The proofs can be found in [24].

V. PERFORMANCE

In this section, we evaluate MIMO-GT’s performance in terms of rates and complexity.
Rates- each user has $\mathcal{M}$ codewords, hence sends $\log \mathcal{M}$ bits per transmission. The identification of each transmitting user incurs additional $\log N$ bits. We thus obtain a sum rate of

$$R = K(\log \mathcal{M} + \log N) = K \log N \mathcal{M} = \frac{M_r}{(1 + \delta)^2 \ln 2}$$

bits per channel use. The right-hand side follows Theorem 1 and taking the lowest possible $\beta^*$, maximizing the sum-rate.

Complexity- in MIMO-GT, the coding is straightforward as each transmitted word is mapped to a unique codeword where the user identity is embedded in the codeword. Since each user schedules itself, there is no need for any scheduling mechanisms and their associated complications, i.e., users do not exchange messages to cooperate, there is no need for CSI collection at the receiver, and as far as complexity is concerned, the receiver does not perform any scheduling algorithm, and consequently does not announce scheduling results. Accordingly, the complexity in MIMO-GT is associated with the decoding process.

The decoding in MIMO-GT is done using Noisy CoMa. Noisy CoMa matches all columns in the codebook with the output of the energy detection phase, $Y$. Since there are $N \mathcal{M}$ codewords in total and each is of length $M_r$, the total decoding time of Noisy CoMa is $O(N \mathcal{M} M_r)$ (21).

VI. NUMERICAL RESULTS

In order to get some insight into the MIMO-GT performance, in this section, we provide numerical evaluation results.

As a reference, we compare MIMO-GT attainable rates with the ergodic sum rates analyzed in [25]. We emphasize that in contrast to the model suggested herein, [25] analyzed the capacity gain of a MU-MIMO system in which the transmitting users and their CSIs are known to the receiver. We completely omit the overhead required in acquiring these reports. ([25, Equation (40)]) showed that for the case for $M_r < K M_l$, the ergodic sum-rate with CSIT is (left-hand side expression)

$$C_{\text{Full CSIT}} \approx M_r E[\log(1 + \rho) | \mathbf{H}_i[1,1]^2] + O(\log(K M_r)) \leq M_r (1 + \rho) + O(\log(K M_r)). \quad (4)$$

In order to simplify the result, we utilize Jensen’s Inequality and the fact that for $|\mathbf{H}_{ii}[1,1]|^2 \sim \text{Exp}(1)$ [23, Chapter 3.2.2] (right-hand side expression).

We further compare our scheme to two other MAC schemes. (i) TDMA in which the time slots are distributed between the users a priori, and a single user transmits per slot. Since in the suggested model, at most $K$ users are backlogged per time slot, we assume that a user is backlogged in each slot with probability $\frac{N}{K}$. As with the previous comparison, we assume that the user’s identity and CSI are known to the BS, and the user transmits at the highest rate possible (utilizes all transmit and receive antennas). Hence, the average ergodic rate approximation is $\frac{N}{K} \times$ times the approximation for a single user given in [26, Eq. (6)]

$$C_{\text{RR}} \approx \frac{K}{N} \cdot M_r \log(1 + \rho). \quad (5)$$

(ii) An oracle-aided 160MHz state of the art 802.11ax. The oracle schedules users without passing requests or scheduling information and synchronizes transmissions without using dedicated subcarriers for pilots. Following the standard, the receiver sends a trigger frame followed by a short silent interval (SIFS). Afterward, the users transmit their frames simultaneously. For a fair comparison, we assume that the payload size of each user is $\log N \mathcal{M}$, no identification headers are used (as the identities are known), the data rate is MCS2, and the rest is according to the 802.11ax standard (with one training field and 8us packet extension) [27]. The sum rate also considers the scheduling announcement embodied in the trigger frame (sent in MCS0 rate), allocating the entire bandwidth to the $K$ users.

Figure 1a, which compares the sum rates of the four schemes for various $N$, demonstrates that MIMO-GT achieves a higher sum rate than 802.11ax (normalized by a single carrier’s bandwidth) due to the lack of significant overheads. As expected, TDMA outperforms MIMO-GT for a small number of users, as when the percentage of backlogged users is high, TDMA is highly efficient, however, when the probability of a user being backlogged is getting lower, the performance deteriorates. Naturally, the ergodic capacity is much higher than the other schemes, especially since the overhead in acquiring the identities and the channel state of all users is not taken into account.

Figure 1b compares MIMO-GT’s Spectral Efficiency, $\eta \triangleq \frac{R}{M_r}$, with Shannon’s Power Efficiency Limit (SPEL), evaluated for different $K$. The SPEL is calculated like in [28, Chapter 3.5]. For our system, we have used $E_{\text{PHY}} = K P M_r$. The bold dark line is SPEL, and its dashed counterpart is the ultimate SPEL (USL), $10 \log_{10}(\ln 2)$ dB. The cyan line is $\eta^*$’s limit when $K \to \infty$, calculated regardless of $N$ or $\mathcal{M}$.
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