Primordial Density Fluctuations in Phase Coupling Gravity

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Abstract

In this paper we study the evolution of primordial density perturbations in the framework of Phase Coupling Gravity, proposed by Bekenstein. We show that in the very early universe, these perturbations grow with an exponential-like behaviour.

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1 Introduction

Dark matter: a must.

The issue of the material content of the Universe is one of the most actual and controversial problems in Modern Cosmology. Nucleosynthesis puts a very stringent bound on the amount of the barionic component \( \Omega_B \approx 0.016h^{-1} \). On the other hand, the flatness problem requires \( \Omega = 1 \), otherwise an extreme fine tuning of \( \Omega(t) \) would be called for in the early universe \[3\]. For this and other reasons, nowadays a putative non-barionic dark matter component permeating the Universe is tacitly assumed.

Now at a quite different scale, spiral galaxies are known to have flat rotation curves, meaning that the graph of the velocity squared of test particles (stars, HII clouds,...) displays a plateau at about 5kpc away from galactic center extending as far as many tens of kiloparsecs. This results is at odds with Newtonian prediction that at large distances the graph should fall off as \( 1/r \) – unless we are witnessing here another manifestation of the same putative unseen cosmological material. The manner dark matter clumps in the galaxy is evaluated by recalling that a flat rotation curve must be generated by a logarithmic Newtonian potential. Inserting this potential into Poisson’s equation gives a density profile of the dark matter component that falls off as \( 1/r^2 \). Adding up this component to the visible part accounts for \( \Omega_{\text{Halo}} \approx 0.1 \). Similar dynamical methods applied to the motion of clusters due to their gravitational field at the scale of 20 Mpc, yields \( \Omega_{20} \approx 0.2 \). Clearly this is a long way away from \( \Omega = 1 \) required from cosmological considerations. Therefore, consistency of the dark matter scenario requires a smoothly distributed component at scales larger than 20 Mpc \( \Omega_{\text{smooth}} \approx 0.8 \) \[3\].

A must?

The raison d’être of postulating a smoothly distributed and undetectable dark matter component is just to yield the right bookkeeping for \( \Omega = 1 \). If this were not enough, a pitfall awaits the dark matter scenario at the galactic scale. Tully and Fisher \[4\] discovered the empirical law bearing their name that relates the luminosity \( L \) of a spiral galaxy to the velocity at the plateau:

\[
V = 220\text{km/s}(L/L_*)^{1/4},
\]

where \( L_* \) is a constant corresponding to the typical luminosity of a galaxy \[2\]. Now, sources that contribute mostly to the luminosity in the frequency
bandwidth where the law is stated correspond to white dwarfs which, in turn, are mainly located in the galactic disk. This brings about a conundrum because if dark matter in the halo is to be blamed for the flat rotation curves, then a very fine tuning between disk and halo parameters would have to be called for, which is hard to explain and even harder to implement. As a matter of fact, adjusting halo and disk parameters yields unavoidably to a “bump” in all rotation curves just before the plateau is reached, which is seldom observed. The dark matter scenario becomes more intricate when one comes to the question of its very nature (massive neutrinos, WIMPS,...), because all the candidates are of very hard direct detection.

A radically different approach would be to say that there is no considerable amount of dark matter permeating the Universe, what we are rather witnessing in the spiral galaxies is the breakdown of General Relativity (at a given scale). Since galactic dynamics involves weak gravitational fields and non-relativistic motion, this clearly entails a modification of Newton’s law too. Indeed, this was the step taken by Milgrom’s who put forward an explicit modification of the Newtonian dynamics that takes place when the Newtonian acceleration is of the order of $a_0 = 2 \times 10^{-8}\text{cm/s}^2$ or smaller. He introduces a distinction between the Newtonian gravitational field $\vec{g}_N$ and the actual acceleration a test particle is subjected to, $\vec{g}$. In his proposal Modified Newtonian Dynamics (MOND, for brief) these accelerations are related through:

$$\mu\left(\frac{g}{a_0}\right)\vec{g} = \vec{g}_N,$$

where $\mu$ is a function satisfying,

$$\begin{cases} 
\mu(x) \to x & \text{if } x \ll 1 \\
\mu(x) \to 1 & \text{if } x \gg 1 \text{ (Newtonian limit)} 
\end{cases}$$

MOND can be shown to

- reproduce the flat rotation curves.
- be consistent with Tully-Fisher’s law.
- satisfy the weak equivalence principle but not the strong one.

Extending this guideline into the relativistic domain clearly entails framing a new covariant theory. It would be a wise step to take General Relativity as a building block for such a theory because precision tests in the solar system seem to confirm General Relativity to a very high extent. MOND suggests one to demand such a theory to comply to the weak equivalence
principle but not to the strong one. Furthermore, stability considerations requires a positive energy flux. A further imposition is that causality must not be violated at any rate. A first candidate, AQUAL (Aquadratic Lagrangian Theory) \cite{6} was proposed in the early eighties but was soon shown not to be a viable candidate because it was plagued with superluminal propagation. The most promising candidate nowadays was proposed by Bekenstein and Milgrom \cite{6} an was baptized as Phase Coupled Gravity (PCG, for brief). In this theory, in addition to the metric tensor, $g_{\mu\nu}$ the gravitational interactions are mediated by a complex scalar $\chi$ field. The corresponding action is

$$S_{\chi} = -\frac{1}{2} \int \left( g^{\alpha\beta} \chi_{,\alpha} \chi_{,\beta} + V(\chi^* \chi) \right) \sqrt{-g} d^4 x, \quad (4)$$

where $V(x)$ represents the scalar field self-interaction. One expresses this action in a more convenient form by decomposing $\chi$ in terms of its amplitude $q$ and phase $\psi$:

$$S_{q,\phi} = -\frac{1}{2} \int \left( q_{,\alpha} q^{,\alpha} + q^2 \phi_{,\alpha} \phi^{,\alpha} + V(q^2) \right) \sqrt{-g} d^4 x. \quad \text{(5)}$$

PCG is defined via the composition of this action with Einstein-Hilbert’s and the matter action which is defined through the replacement rule $L_m \rightarrow e^\phi L_m$. Put into words, matter couples only to the phase of the complex scalar field. Clearly, predictions depend upon the choice of the potential $V(q^2)$. Minimal PCG ($V(x) = 0$) and the sextic potential were show to lead to instabilities \cite{7}.

There are two alternative and equivalent representations of a scalar tensor theory. The first, written in i. Einstein’s frame ($g_{\mu\nu}^*$) where the scalar field interacts directly with matter and test particles do not follow geodetic lines; ii. the physical frame in which the scalar field is absorbed by the metric tensor via a conformal transformation

$$g_{\mu\nu} = g_{\mu\nu}^* e^{-\eta \phi}, \quad \text{(6)}$$

where $\eta$ is some parameter. In the physical frame the PCG action takes the form:

$$S_f = \frac{1}{16\pi G_0} \int \sqrt{-g} d^4 x e^\eta \left[ R - q_{,\alpha} q^{,\alpha} - (q^2 - \frac{3}{2} \eta^2) \phi_{,\alpha} \phi^{,\alpha} - e^{\eta \phi} V(q^2) \right] + S_m. \quad \text{(7)}$$

Here, $R$ is the scalar curvature and $G_0$, Newton’s constant. Inspection of this equation reveals that PCG corresponds to a Brans-Dicke theory with
variable $\omega_{BD} = q^2 - 3\eta/2$. In order to grapple with cosmological issues, a definite choice of the potential is needed. As we said, minimal PCG was discarded from stability grounds and the next simple candidate is a quadratic potential $V(q^2) = Aq^2 + B$. In order to reproduce the observed (flat) rotation curves in a flat Universe, Sanders obtained as the best fit for these parameters $A = 4.0 \times 10^4$, $B = 6.7$ and $\eta = 10^{-7}$. In this paper we shall study the very early Universe in the framework of this particular model and, in particular, study the evolution of primordial density fluctuations.

2 PCG Early Universe.

Sanders [8] obtained the evolution of FRW models in the framework of PCG solving numerically the differential equations for $a(t)$, $q(t)$ and $\phi(t)$. He obtained that at the very early universe (equation of state $p = -\rho$) the last two quantities are nearly constant. Inspired by his results, we took the ansatz $q(t) = \text{const.}$ and solved the flat model equations. Consistently, we obtained a large and slowly varying field $\phi(t)$. Then we studied the fate of primordial density perturbations in the early Universe. In contrast to the standard inflationary scenario, density perturbations are shown to grow during PCG inflation.

Variations of the action (7) with respect to the metric and both scalar fields yield the following equations:

- **metric variations**

$$8\pi G_0 T^{\alpha\beta} = (R^{\alpha\beta} - \frac{1}{2} R g^{\alpha\beta}) + \frac{1}{2} q_{\mu} q^{\mu} g^{\alpha\beta} + (q^2 - 3/2\eta^2) (\frac{1}{2} \phi^{\mu} \phi^{\mu} g^{\alpha\beta} - \phi^{\alpha} \phi^{\beta}) - q^{\alpha} q^{\beta} + \frac{1}{2} e^{\eta \phi} V(q^2) g^{\alpha\beta} + \eta (\eta \phi^{\mu} \phi^{\mu} + \phi^{\mu}_{\mu}) g^{\alpha\beta} - \eta (\eta \phi^{\beta} \phi^{\alpha} + \phi^{\alpha\beta})$$

(8)

where $T^{\alpha\beta}$ is the energy momentum tensor of matter.

- **$\phi$ variations**

$$\eta R - \eta q_{\alpha} q^{\alpha} + \eta (q^2 - \frac{3\eta^2}{2}) \phi_{\alpha} \phi^{\alpha} - 2V(q^2) \eta e^{\eta \phi} + 4qq_{\alpha} \phi^{\beta} g^{\alpha\beta} + 2(q^2 - \frac{3\eta^2}{2}) \Box \phi = 0$$

(9)

- **$q$ variations**

$$e^{-\eta \phi} [e^{\eta \phi} q^{\alpha}]_{;\alpha} - q \phi^{\alpha} \phi_{;\alpha} - V'(q^2) q e^{\eta \phi} = 0.$$  

(10)
A handy equation follows after combining eq. (9) with the trace of eq. (8)

\[ 4\pi G_0 T = \left( q^2 \phi,_{\alpha} \phi,^{\alpha} + \frac{2qq_{\alpha,\alpha} \phi,^{\alpha}}{\eta} + \frac{q^2 \Box \phi}{\eta} \right) e^{\eta \phi} \]  \hspace{1cm} (11) 

where \( T \) is the trace of the energy-momentum tensor.

For a homogeneous and isotropic model the cosmological equations are enormously simplified. Indeed the 00 component of eq. (8) reduces to

\[ \frac{8\pi G_0}{3} \rho e^{-\eta \phi} = \left( \frac{\dot{a}}{a} + \frac{\eta \ddot{\phi}}{2} \right)^2 - \frac{q^2}{6} e^{\eta \phi} V(q^2) - \frac{1}{6} q^2 \dot{\phi}^2, \]  \hspace{1cm} (12) 

Similarly, eqs. (9) and (10), collapse respectively to

\[ \eta R + \eta q^2 - \eta (q^2 - \frac{3}{2} \eta^2) \dot{\phi}^2 - 2V(q^2) \eta e^{\eta \phi} - 4q \dot{\phi} \dot{q} - 2(q^2 - \frac{3}{2} \eta^2) \dot{\phi} = 0 \]  \hspace{1cm} (13) 

and

\[ \ddot{q} + \frac{3}{a} \dddot{q} + \eta \dot{q} \dddot{\phi} - q \dot{\phi}^2 = -q V'(q^2) e^{\eta \phi}. \]  \hspace{1cm} (14) 

Finally, eq. (11) goes into:

\[ 4\pi G_0 T = e^{\eta \phi} (-q^2 \dot{\phi}^2 - \frac{2}{\eta} q \dot{q} \dot{\phi} - \frac{q^2 \ddot{\phi}}{\eta} - \frac{3}{\eta} q^2 \frac{\dddot{a}}{a} \frac{\dot{\phi}}{\phi}) \]  \hspace{1cm} (15) 

Taking as the background solution \( q_0 = \text{const} \), it follows from eq. (14) that:

\[ \dot{\phi}_0^2 = A e^{\eta \phi_0}. \]  \hspace{1cm} (16) 

Integration is trivial,

\[ \phi_0 = \frac{-2}{\eta} \ln \left( \frac{-\eta \sqrt{A t}}{2} + K \right) \]  \hspace{1cm} (17) 

where \( K \) is an integration constant. As anticipated, \( \phi(t) \) is a slowly varying function in the early Universe. Having obtained the evolution of the background fields in the early Universe, we pursue the analysis of the behavior of perturbations:

\[
\begin{align*}
\rho &= \rho_0 + \rho_1 \\
p &= p_0 + p_1 \\
\phi &= \phi_0 + \phi_1 \\
q &= q_0 + q_1 \\
g_{\alpha\beta} &= a^2 \delta_{\alpha\beta} + h_{\alpha\beta}
\end{align*}
\]  \hspace{1cm} (18) 

The evolution of those perturbations are obtained through the linearization of eq’s. (8), (10) and (11). After some tedious but straightforward algebra we obtained the

- metric perturbations (00 component)

\[ 8\pi G_0 (\rho_1 - \rho_0 \eta \phi_1) = e^{\eta \phi_0} [R_{00}^* + \frac{1}{2} R^* - \dot{\phi}_0 \dot{q}_1 - (q_0^2 - \frac{2}{3} \eta^2) (\dot{\phi}_0 \dot{\phi}_1) - q_0 q_1 \phi_0^2 - A q_0 q_1 e^{\eta \phi_0} - \eta \phi_1 (A q_0^2 + B) - \eta \Box q_0 - \eta \ddot{\phi}_1] \]  

(19)

where \( R^* \) e \( R_{00}^* \) are the perturbations of the scalar curvature the Ricci’s tensor oo component.

- \( q \)-perturbations

\[ -\eta \dot{\phi}_0 \dot{q}_1 - \eta \phi_1 \dot{q}_0 + q_1 \phi_0^2 + 2 \dot{\phi}_0 \phi_1 q_0 + A e^{\eta \phi_0} (q_1 + \eta q_0 \phi_1) = 0 \]  

(20)

- \( \phi \) perturbations

\[ 4\pi G_0 e^{-\eta \phi} (T_1 - \eta \phi T_0) = -2 q_0^2 \dot{\phi}_0 \dot{\phi}_1 - 2 q_0 q_1 \phi_0^2 - \frac{2}{\eta} q_0 (\dot{q}_0 \dot{\phi}_1 + q_1 \dot{\phi}_0) - \frac{2}{\eta} q_1 \dot{\phi}_0 + \frac{1}{\eta} q_0^2 \Box \phi_1 + \frac{2}{\eta} q_0 q_1 \Box \phi_0 \]  

(21)

Assuming a period of inflationary evolution where \( a = e^{\Lambda t} \) (and, correspondingly, an equation of state \( p = -\rho \)) and metric perturbations of the form \( h_{ij} = 1/3 h \delta_{ij} \), it follows from the perturbations of the \( \alpha \) component of eq.(8) that:

\[ \dot{h} - 2h \Lambda = -3e^{2\Lambda t} (\dot{\phi}_1 - \Lambda \phi_1), \]  

(22)

where \( h = h_{11} + h_{22} + h_{33} \).

We are already in position of solving the equations for the perturbations. Specializing for the case where the background fields are exactly given by eq (17) and \( q_0 = \text{const} \) and further defining a new time variable \( \tau = \frac{\alpha t}{\sqrt{\Lambda}} + K \)

we obtain from eq. (21) in the vanishing wave number limit:

- the evolution of \( \phi_1 \)

\[ \phi_1'' + (\chi_1 - \frac{5}{\tau}) \phi_1' + \phi_1 (\chi_2 + \frac{\chi_4}{\tau^2} + \frac{\chi_5 q_1}{\tau} + \frac{q_1 (\chi_6 + \chi_7)}{\tau^2}) = 0, \]  

(23)

from eq.(21)
the evolution of $q_1$

$$q_1'' + q_1'(\alpha - \frac{2}{\tau}) + \frac{\chi_8 \phi_1'}{\tau} + \frac{\chi_8 \phi_1}{\tau^2} = 0,$$  \hspace{1cm} (24)

and, finally from eq. (19)

• the evolution for $\rho_1$

$$8\pi G_0 (\rho_1 - \eta \rho_0 \phi_1) = \left\{ \phi_1 \left[ -\left(q_0^2 - \frac{3}{2}\eta q_1^2\right) - \frac{2\sqrt{T}}{2K - \eta \sqrt{A t}} - 3\eta^2 \frac{\sqrt{T}}{2K - \eta \sqrt{A t}} \right] 
+ \phi_1 \left[ 3\Lambda^2 \eta - 2\eta \frac{Aq_0^2 + B}{(2K - \eta \sqrt{A t})^2} + 3\eta^2 \Lambda \frac{\sqrt{T}}{2K - \eta \sqrt{A t}} \right] 
+ q_1 \left[ \frac{-8\rho_0 A}{(-\eta \sqrt{A t} + 2K)^2} \right] \right\} \frac{4}{(2K - \eta \sqrt{A t})^2}. \hspace{1cm} (25)$$

In the above equations, primes represent derivatives with respect to $\tau$ and the $\chi$'s are constants displayed in the table beneath:

| Cosmological Parameters |
|--------------------------|
| $\chi_1$                 | $-\frac{6\Lambda}{\eta}$ |
| $\chi_2$                 | $-\frac{6\Lambda^2 \eta^2}{\tau^2}$ |
| $\chi_3$                 | $3\eta \Lambda \frac{\ell}{2} - 3\eta^2 \Lambda \frac{\ell}{2}$ |
| $\chi_4$                 | $\frac{\ell}{Aq_0^2} (Aq_0^2 + B)$ |
| $\chi_5$                 | $\frac{\ell}{q_0}$ |
| $\chi_6$                 | $\frac{b\Lambda}{q_0}$ |
| $\chi_7$                 | $\frac{b\Lambda}{q_0}$ |
| $\chi_8$                 | $\frac{4q_0}{\ell}$ |
| $\alpha$                 | $-\frac{6\Lambda}{\eta}$ |

Integrating eq. (24), it follows that

$$\phi_1 = \frac{1}{\chi_8 \tau} \left\{ q_1' \tau^2 + \alpha \tau^2 q_1 - 4q_1 \tau + \psi \right\}, \hspace{1cm} (27)$$

where $\psi = \int q_1 (4 - 2\alpha \tau) d\tau$.

Putting together this result with eq. (23), yields the differential equation

$$q_1''' \tau + q_1'' \left[ -7 + \tau (\alpha + \chi_1) \right] 
+ q_1' \left[ \frac{1}{\tau} (19 + \chi_4 + \chi_8 \chi_5) + (\chi_3 - 3\chi_1) + \tau (\alpha \chi_1 - 5\alpha + \chi_2) \right] 
q_1 \left[ \frac{1}{\tau^2} (-28 - 4\chi_4 + \chi_7 \chi_8) + \frac{1}{\tau} (7\alpha + 4\chi_1 + \alpha \chi_4 - 4\chi_3 + \chi_8 \chi_6) + (-\alpha \chi_1 + \alpha \chi_3 - 4\chi_2) \right] 
+ \psi \left[ \frac{1}{\tau^2} (7 + \chi_4) + \frac{1}{\tau} (\chi_3 - \chi_1) + \frac{1}{\tau} (\chi_2) \right] = 0. \hspace{1cm} (28)
In the late time limit $|\tau| >> 1$, this hyper colossal equation boils down to
\begin{equation}
q_1''' + q_1''(\chi_1 + \alpha) + q_1'(\alpha\chi_1 + \chi_2) + q_1\alpha\chi_2 = 0.
\end{equation}
(29)
Inserting $q_1 = e^{\frac{6\Lambda\tau}{\eta}}$ into this equation an algebraic equation for $Q$ follows
\begin{equation}
Q(Q - 1)^2 + \frac{\eta^4}{6q_0^2}(1 - Q) = 0,
\end{equation}
(30)
whose solutions for $Q$ are
\begin{equation}
Q = \left\{ \begin{array}{l}
\frac{1}{2} \\
\frac{1}{2} \pm \frac{1}{2} \sqrt{1 + \frac{4\eta^4}{6q_0^2}}
\end{array} \right.
\end{equation}
(31)
Assuming $\eta^4/q_0^2 << 1$, the negative root is approximately :
\begin{equation}
Q = -\frac{\eta^4}{6q_0^2}
\end{equation}
(32)
This result allows us to obtain $\phi_1$, through eq. (24)
\begin{equation}
\phi_1 = \left( \frac{1}{\chi_8} \frac{-\eta^3\Lambda}{q_0^2}\tau + \alpha\tau - 4 \right) + 2\frac{\alpha q_0^2}{\eta^3\Lambda} e^{-\frac{\eta^3\Lambda\tau}{q_0^2}}
\end{equation}
(33)
Our concern in here is the asymptotic behavior of $\rho_1$. This can be obtained combining the the late time regime limit of eq. (25) with eqs. (24) and (33)
\begin{equation}
8\pi G_0 \rho_1 \approx e^{P\tau} \frac{\eta(P + \alpha)}{\chi_8 A} \left\{ \eta P q_0^2 \frac{\eta^2}{2} + 3\Lambda^2 + 8\pi G_0 \rho_0 \right\},
\end{equation}
(34)
where $P = -\eta^3\Lambda q_0^{-2}$. This exponential-like form of density perturbations should be contrasted with the general relativistic prediction that they remain strictly constant during inflation [3].

3 Concluding Remarks

In the standard cosmological scenario density perturbations are frozen during the radiation dominated era, and are allowed to grow only after the decoupling between matter and radiation has taken place [3]. This might leave a very tight time-schedule for the contrast density to grow from $10^{-5}$ at $z \approx 1400$ to
unity at a red-shift of order 1. Likewise, in PCG the density perturbations grown during inflation are frozen during the radiation dominated era. During this era and onwards the PCG cosmological evolution is very much similar to that one of the standard model [8]. Consequently, many predictions of PCG cosmology are similar to those of the standard model. One of the main differences lies in that the exponential-like grow of $\rho_1$ accumulated during inflation will be carried over to the time of decoupling when the perturbations are finally allowed to grow. Therefore, PCG predicts a much larger density contrast, which could alleviate the problem of the tight time-schedule for the contrast density to enter into the non-linear regime.

One of the crucial checks of the standard cosmological scenario is the abundance of light elements. How does PCG predictions of light elements compare to the standard model? As discussed by Sanders [8], the version of PCG with a quadratic potential leads to a somewhat faster expansion of the Universe during nucleosynthesis (of about 6%), causing an apparent overproduction of primordial Helium (earlier freeze-out of neutrons). Nevertheless, the same increase in the expansion rate leads to a reduction in the abundance of neutrons and the two effects would nearly compensate. Thus, the Helium abundance would remain insensitive to the faster expansion and would be within the present observational limits. Unfortunately, the same is not true for the heavier elements, the abundance of heavier nuclei like $H^2$ and $Li^7$ would be changed drastically by a factor from 10 to a 100 [8]. Furthermore, it must be mentioned that the present version of PCG is burdened by other problems, and seems to be in disagreement with precision experiments in the solar system, in particular the precession of the of perihelion of Mercury [10]. Other versions of the theory should be explored in order to come to grips with the observational data.

The bottom line of this paper is to show that viable theories of gravity can be constructed to explain many of the cosmological paradoxes. Furthermore, in contrast to the dark matter scenario, where unseen matter can be placed here and there at will to justify the discrepancies between predictions and observations and with no further consequences, these theories produce many definite predictions which can be checked against the observational data. This fact does turn these theories very attractive.
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