A plane stress yield surface using Bézier curve interpolation in two directions

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Abstract. Current advanced yield criteria, like BBC2005, Yld2000 and Yld2004, offer significant flexibility and are proven to be convex. The number of coefficients of the function is fixed, which limits the number of experimental results that can be fitted independently. A common approach is to define an error function. The optimal anisotropy coefficients are established by minimizing the error function. The Vegter yield criterion is very flexible with respect to experimental results. It is in fact defined as a function that smoothly connects experimental results, while ensuring a continuous function for the derivative. Fitting is therefore a non-issue; the experimental results are the coefficients for the yield criterion. Second order Bézier curves are used for the interpolation between experimental results. Extreme values for these results can cause concavity in the definition of the yield locus. Although rarely an issue, convexity needs to be validated as part of the implementation of the yield criterion. Recently we have seen some cases of concavity that could be attributed to the cosine interpolation in the original Vegter yield criterion definition. Cosine interpolation is used between experimental results at different angles to the rolling direction. An improved method is proposed that uses Bézier interpolation not only between biaxial stress states in one orientation, but also between stress states with a different material orientation. The benefits are greater flexibility in defining experimental results and a coherent approach towards a 3D extension. The benefits of this new yield criterion will be demonstrated in a direct comparison with Yld2000 and Yld2004. The comparison will include the definition of the yield locus, showing actual cases, where in particular biaxial stress states like plane strain and shear are difficult to fit accurately. The comparison will also include improved accuracy in forming simulations.

1. Introduction
Advanced yield criteria in present FE-modeling require complex parameter identification and are not sufficiently flexible to accommodate the data obtained from material testing. Therefore, the Vegter yield criterion has been developed to obtain a high accuracy combined with a simple mathematical description and a large flexibility. This plane stress yield criterion is constructed by interpolation in two directions being being a Bézier interpolation between different stress states and a cosine interpolation over different material orientations. For this model, no complex parameter determination is required. The drawback of this model is that by its large flexibility yield surfaces could be concave in some areas. This concavity does not occur in regular cases but at extreme values of measured input. A way to overcome such problems is replacing the cosine interpolation for the input over different material orientations by a second Bézier interpolation.
2. Basic input data for construction of the Vegter yield surface

The basic idea of the Vegter criterion is constructing a plane stress yield surface directly based on the measured information according to Table 1 in multiple (typically 3) material orientations. In this table known yield criterion information is marked with the symbol √, and additional required information is marked with the symbol ☒. The original Vegter criterion is considered as a starting point (Figure 1).

For the new Bézier interpolation in two directions all stress states and strain vectors of the reference points in at least 3 orientations are required. It is assumed principal stress state coincide with the orientation angle of the tests. As a consequence, in plane shears exist in tests orientations at other angles to the rolling directions than 0º and 90º. These shears are not measured and are estimated as indicated in Table 1.

Table 1. Measurements required for a yield criterion using double Bézier interpolation

| Measurement                        | √σ₁ | √σ₂ | Strain vector (δε₁/δε₁) | **Shear strain vector (δε₂/δε₁) |
|------------------------------------|-----|-----|-------------------------|--------------------------------|
| Equi-biaxial stress test (compressive test) (BI) | ☒   | ☒   | √σ₁                     | 0                              |
| Plane strain test (PS)             | ☒   | ☒   | 0                       | ☒                              |
| Uniaxial tensile test (UN)         | ☒   | ☒   | 0                       | -1                             |
| Shear test: pure shear stress (SH) | ☒   | ☒   | -1                      | ☒                              |
| (simple shear measured)            |     |     |                         |                                |

*) It is implicitly assumed that principal directions of stress coincide with the reference directions.

**) The shear strain vector in the reference directions is generated by using cosine interpolation of the reference stresses over the angle to the rolling direction as in the original Vegter yield criterion. By using the normality rule a shear strain vector is obtained at the principal direction of stress. Due to orthogonality ε₁ and ε₂ at 0º and 90º to the rolling direction are principal strains.

![Figure 1. Combination of Bézier interpolation and cosine interpolation of the original Vegter yield criterion.](image)

The original Vegter yield criterion [1] is formulated in principal stress space. The angular interpolation using Fourier series is applied on this principal stress information of the reference points. The Bézier interpolation at arbitrary angle is applied on these angular interpolation results of the reference points. The new double Bézier yield criterion is defined in the deviatoric stress space instead of the σ₁-σ₂-2θ coordinates of the original Vegter yield criterion.
3. Description of the Vegter yield criterion using Bézier interpolation in two directions in two steps in the deviatoric stress space.

The interpolation is made in the deviatoric stress space in which all the normal stress components are treated equivalently. Compared to the usual $\sigma_x - \sigma_y - \tau_{xy}$ space under plane stress conditions:

- Capturing of the complete isotropic case including the von Mises criterion could be made in a straightforward way
- A future extension to a complete 3D stress interpolation can be made easier than using the plane stress state as the starting point.

\[ sd_1 = \sigma_x \cdot \cos(2 \cdot \theta) \]
\[ sd_2 = \sigma_y \cdot \cos(2 \cdot \theta) \]

\[ s_{xy} = sd_1 \cdot \sin(2 \cdot \theta) \]

**Figure 2.** Handling of coordinates in the deviatoric plane to a 3D-plot of all plane stress components of the deviatoric stress

By projection of the principal plane stress components $\sigma_1$ and $\sigma_2$ to the deviatoric plane in the $(\sigma_1, \sigma_2, \sigma_3)$ stress space, principal deviatoric stress coordinates $sd_1$ and $sd_2$ are obtained. In the deviatoric plane, the pure shear axis and the equibiaxal stress axis are defined as x-axis and y-axis respectively. By this projection method, the principal deviatoric stress coordinates can be expressed in the principal plane stress components:

\[
\begin{bmatrix}
sd_1 \\
\frac{1}{\sqrt{6}} \\
\frac{1}{\sqrt{6}} \\
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} \\
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} \\
\end{bmatrix}
\begin{bmatrix}
\frac{1}{\sqrt{6}} \\
\frac{1}{\sqrt{6}} \\
\end{bmatrix}
\begin{bmatrix}
sd_1 \\
\frac{1}{\sqrt{6}} \\
\frac{1}{\sqrt{6}} \\
\end{bmatrix}
\]

(1)

The projected principal coordinates $sd_1$ and $sd_2$ of equation (1) are preferred to the deviatoric stresses which principal components $s_1$, $s_2$ and $s_3$ are presented in Figure 2a. These coordinates are the principal components of the plane stress deviatoric coordinates $sd_x$, $sd_y$ and $s_{xy}$ which will be used as an orthogonal coordinate system in the proposed Bézier surface interpolation method in three dimensions.

According to Figure 2b, the deviatoric shear stress coordinate, $s_{xy}$, scales one to one with the $sd_1$ coordinate. Using this frame work, the von Mises criterion is graphically represented as a sphere with a radius $\sqrt{2/3} \cdot \sigma_{eq}$ ($\sigma_{eq}$ is the equivalent stress). The principal deviatoric coordinates $sd_1$ and $sd_2$ can be expressed in the general deviatoric plane stress coordinates $sd_x$, $sd_y$ and $s_{xy}$ in the following way:
The deviatoric components \( s_d \) and \( s_d \) can be expressed in the normal plane stress components \( \sigma_x \) and \( \sigma_y \) using the same projection method as proposed for the principal components according to equation (1). Inclusion of the scaling of the deviatoric shear stress coordinate \( s_{xy} \) with the shear stress \( \tau_{xy} \) delivers the following relationship between the deviatoric plane stress components \( s_d, s_d \) and \( s_{xy} \) and the plane stress components \( \sigma_x, \sigma_y, \tau_{xy} \):

\[
\begin{pmatrix}
    s_d \\
    s_d \\
    s_{xy}
\end{pmatrix} = \begin{pmatrix}
    \frac{1}{\sqrt{2}} & 0 & 0 \\
    0 & \frac{1}{\sqrt{2}} & 0 \\
    0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
    \sigma_x \\
    \sigma_y \\
    \tau_{xy}
\end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix}
    \frac{1}{\sqrt{2}} & 0 & 0 \\
    0 & \frac{1}{\sqrt{2}} & 0 \\
    0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
    s_d \\
    s_d \\
    s_{xy}
\end{pmatrix}
\]

(3)

The Bézier interpolation is defined in the \((s_d, s_d, s_{xy})\) deviatoric stress space, normalized through a division by the equivalent stress, \(\sigma_{eq}\):

\[
X = \begin{pmatrix}
    s_{dx} \\
    s_{dy} \\
    s_{xy}
\end{pmatrix} = \begin{pmatrix}
    \frac{1}{\sigma_{eq}} \\
    1 \\
    1
\end{pmatrix} \begin{pmatrix}
    s_d \\
    s_d \\
    s_{xy}
\end{pmatrix} = \begin{pmatrix}
    \frac{1}{\phi + \sigma_f} \\
    \phi + \sigma_f
\end{pmatrix} \begin{pmatrix}
    s_d \\
    s_d \\
    s_{xy}
\end{pmatrix}
\]

(4)

Where, \(\sigma_f\) is the reference yield stress, which is based on the stress strain curve at uni-axial tension in the rolling direction. This reference is arbitrary and can be any state (it can be a mean yield stress over the directions to RD or the equi-biaxial stress, these two are less common). The yield function \(\phi\) with, \(\phi < \theta\) in the elastic regime and \(\phi = \theta\) during plastic deformation.

The first step is a Bézier interpolation of the stress states in the plane of principal stress of the reference directions. The hinge points are forced in the same principal stress space and are calculated from the two strain vectors in the reference points.

**Figure 3.** Explanation of the double Bézier method.

The method is an extended mathematical formulation. Here a brief description is explained with the help of illustrations in Figures 3 and 4:
a) Firstly, the Bézier interpolation between stress states $i$ and $k$ (typically the 4 states in Table 1) at the individual material orientations $j$ is realised with a Bézier parameter $\mu_\sigma$ and a weight factor $w_\sigma$ for the hinge point at this orientation and stress states

$$X_j(\mu_\sigma) = \frac{(1 - \mu_\sigma)^2 \cdot X_{ij} + \{2 \cdot w_\sigma \cdot (1 - \mu_\sigma) \cdot \mu_\sigma\} \cdot X_{ijk} + \mu_\sigma^2 \cdot X_{kj}}{1 + 2 \cdot (w_\sigma - 1) \cdot (1 - \mu_\sigma) \cdot \mu_\sigma}$$

\[i = 1 \rightarrow 3 \land j = 1 \rightarrow m \land k = i + 1\]  

b) Shear strains at other material orientations than $0^\circ$ and $90^\circ$ are estimated. For the Bézier interpolation between orientations the strain vectors at these angles ($\theta_i$ and $\theta_k$) are needed. This information is not available from step a) at other angles than $0^\circ$ and $90^\circ$ and at other points than the reference points, because only principal stress components are interpolated. Only the strain components in the direction of the principal stress components can be calculated using the normality rule. Avoiding the need of hinge points for the strains, the shears $s\varepsilon_{12,j}$ in the plane of the principal stress are estimated by a Bézier interpolation with a weight factor $w_\sigma = 0$:

$$s\varepsilon_{12,j}(\mu_\sigma) = \frac{(1 - \mu_\sigma)^2 \cdot s\varepsilon_{12,y} + \mu_\sigma^2 \cdot s\varepsilon_{12,kj}}{1 - 2 \cdot (1 - \mu_\sigma) \cdot \mu_\sigma}$$

\[i = 1 \rightarrow 3 \land j = 1 \rightarrow m \land k = i + 1\]  

c) Defining straight sections with a varying angle to the equi-biaxial stress direction. Non-straight sections possibly lead to undesired concavity of the yield surface. The sections start from a pure shear tangent at the pure shear point which gradually change to sections perpendicular to the equi-biaxial strain vector (Figure 4a). These sections are not perpendicular to the $fsd'_y$-axis. As a consequence the angle in this section is smaller than the accompanying angle to the rolling direction.

![Figure 4](image-url)

**Figure 4.** Defining sections at a certain value of the parameter $fsd'_y$ for the angular interpolation

d) A second Bézier interpolation is made over the angular direction (Figure 4b) at these sections by forcing reference points $X_i(\mu_\sigma)$ and $X_k(\mu_\sigma)$ and a calculated $X_{jk}(\mu_\sigma)$ hinge points is added to these sections. A parameter $fsd'_y(\mu_\sigma)$ is defined for keeping track of
these sections. Bézier interpolation is realised with a Bézier parameter \( \mu \) and weight \( w \) for the hinge point:

\[
X \{fsd', y(\mu), \mu\} = \left[ (1 - \mu)^2 \cdot X_j \{fsd', y(\mu)\} \right] + \frac{2 \cdot w \cdot (1 - \mu) \cdot \mu \cdot X_j \{fsd', y(\mu)\} + \mu^2 \cdot X_j \{fsd', y(\mu)\}}{1 + 2 \cdot (w - 1) \cdot (1 - \mu) \cdot \mu} \]

\( \wedge \ j = 1 \rightarrow m \)

4. The isotropic case, reproducing the von Mises criterion

Using identical values for the input at the individual material orientations does not automatically lead to the isotropic case. The plot of the angular section in Figure 4b for the isotropic case is described by a circle. Adjusting the weight factor, \( w_{\theta} \) depending on the increment of the material orientation angle, \( \Delta \theta_{\mathrm{ref}} \), is required for other orientations.

\[
w_{\theta} = \sin \left( 2 \cdot \Delta \theta_{\mathrm{ref}} \right) \cdot \left[ 2 \cdot \left( 1 - \cos \left( 2 \cdot \Delta \theta_{\mathrm{ref}} \right) \right) \right]^{-1/2}
\]

(8)

This relation is obtained after an extensive derivation. Using data at every 45º results in a value of the weight factor of \( 1/\sqrt{2} \) (0.7071). The same reasoning can be used for the increment of the Lode angle in the deviatoric plane. Replacing the \( 2 \cdot \Delta \theta_{\mathrm{ref}} \) in equation (8) by \( \Delta \alpha_{\mathrm{Lode}, \mathrm{ref}} \) results in a complete match with the von Mises for the weight factor \( w_{\sigma} \) between stress states.

\[
w_{\sigma} = \sin \left( \Delta \alpha_{\mathrm{Lode}, \mathrm{ref}} \right) \cdot \left[ 2 \cdot \left( 1 - \cos \left( \Delta \alpha_{\mathrm{Lode}, \mathrm{ref}} \right) \right) \right]^{-1/2}
\]

(9)

The Vegter-criterion, using four plane stress measurement results (Table 1) at every orientation, has data at every 30º Lode angle increment resulting in a value for the weight factor \( w_{\sigma} \) of 0.9659.

5. Comparison of yield criteria on a highly anisotropic case

The example chosen is an anisotropic packaging steel and is non convex using the usual Vegter yield criterion using cosine interpolation at every 45º. The input data in Table 2 show large differences in r-values and plane strain points. Also, the equi-biaxial strain vector deviates strongly from the isotropic case.

| \( \theta \) | \( f_{\mathrm{un}} \) | R | \( f_{\mathrm{ps}} \) | \( \alpha_{\mathrm{ps}} \) | \( f_{\mathrm{sh}} \) | \( f_{\bi} \) | \( \rho_{\bi} \) |
|---|---|---|---|---|---|---|---|
| 0º | 1.0000 | 0.8352 | 1.1194 | 0.7486 | 0.5788 | 1.0910 | 0.5533 |
| 45º | 1.0301 | 1.2449 | 1.2086 | 0.5638 | 0.5695 | 1.0910 | 1.0000 |
| 90º | 1.0826 | 1.5097 | 1.3024 | 0.5054 | 0.5788 | 1.0910 | 1.8073 |

Four yield loci descriptions are compared:
- The original Vegter yield criterion indicated as Vegter (original) [1].
- The double Bézier yield criterion indicated as Vegter (2Bez) using the isotropic weight factors of equations (8) and (9).
- The Yld2000 yield criterion based on uniaxial and equi-biaxial stress and strain data [2]. The exponent \( a \) is derived based on the average value of the plane strain points over 0º, 45º and 90º. Nine parameters are optimized in total using nine data points.
- The Yld2004 with 18 parameters [3], using the same input data as the Vegter yield criterion, but without the imposed second component of the plane strain point by the
parameter $\alpha$. Here the exponent $a$ is also optimized and two starting values are used during optimizing the parameters, resulting into somewhat different results. Under plane stress conditions, fifteen parameters (including the exponent $a$) are optimized using thirteen data points. Two additional relations between the parameters must be imposed independently from material type as a consequence of volume invariance during plastic deformation (i.e. no influence of the hydrostatic part of the stress) [4].

The following results are compared:
- Plane stress yield loci plots in principal stress space at $0^\circ/90^\circ$ and $45^\circ/135^\circ$ for comparing the capability to accommodate the input data using the different descriptions.
- Angular plots among the equi-biaxial stress direction for demonstration of convexity (according to Figure 4b) in critical sections.

5.1. Yield loci plots at $0^\circ/90^\circ$ and $45^\circ/135^\circ$

![Figure 5. Enlarged view of yield loci plots of the first quadrant in the plane strain area](image)

![Figure 6. Enlarged view of yield loci plots in the fourth quadrant in the pure shear area](image)

According to the expectations, all descriptions cover the uni-axial and equi-biaxial stress and strain data. The Vegter yield criterion and the double Bézier yield criterion, 2Bez, match all data points by definition and have the same results in these reference angles. In Figures 5a/b both yield loci plots and the plane strain points according to all different descriptions are shown. Two versions of the Yld2004
are optimized using two different starting values for the exponent being \( a = 6 \) or \( a = 8 \) (in the plots indicated as \( > 6 \) and \( > 8 \)). Using these different starting values will result in different sets of optimized parameters including the exponent \( a \) as indicated in the legend of the plots. It is difficult to get one unique description using the Yld2004 yield criterion for this purpose. It is developed to accommodate uni-axial data and R-values at 7 orientations with 15º interval from 0º to 90º [3]. It is unusual to fit the exponent \( a \) of Yld2004. It shows that it is possible to get a good match with the data but with different results.

Moreover, there are larger deviations in the pure shear points (Figure 6a/b). The 0º/90º-results still obey the measured pure shear stress i.e. half the difference between the principal stresses \( \sigma_1 \) and \( \sigma_2 \). The individual principal components differ from one another and also from those of the two Vegter criteria. It is difficult matching the results of the pure shear stress at 45º exactly using Yld2004 as shown in Figure 6b, but the differences are still small. Using Yld2000 (or its equivalent BBC2005 [5]) results in the largest difference with measured plane strain data and pure shear data as expected. But the results are stable and insensitive to the starting values.

5.2. Angular plots around the equi-biaxial axis

In Figure 7 sections around the equi-biaxial axis close to the equi-biaxial point i.e. \((\sigma_1 + \sigma_2)/2 = 0.9945 \cdot \sigma_{bi}\) are shown. These sections are perpendicular to the equi-biaxial strain vector for the Vegter (original) and the Vegter (2Bez) yield criterion. The sections for the YLD2004 and Yld2000 are somewhat different, but these yield criteria are all convex in this critical area and give identical results. Obtaining comparable results, projected coordinates in the equi-biaxial stress direction are used, being \((\sigma_1 - \sigma_2)/\sqrt{2} \cdot \sigma_1\) and \( \tau_{xy} \cdot \sqrt{2}/\sigma_1\). In this area close to the equi-biaxial stress point the Vegter yield surface is concave due to the cosine interpolation of the reference points. The double Bézier yield criterion is completely convex demonstrating its power obtaining a better convexity. However, it is recommended for both the Vegter criterion and the 2Bez-criterion to test for convexity. The preferred method for this test is calculating the eigenvalues of the Hessian matrix on scanned points of the yield surface [2].

6. The influence of the yield criteria on result of a forming simulation

| \( \theta \) | \( f_{un} \) | \( R \) | \( f_{ps} \) | \( \alpha_{ps} \) | \( f_{sh} \) | \( f_{bi} \) | \( \rho_{bi} \) |
|------------|--------|------|--------|--------|--------|--------|--------|
| 0º         | 1.0000 | 2.2157 | 1.2570 | 0.6051 | 0.5408 | 1.1467 | 0.9507 |
| 45º        | 1.0350 | 1.6750 | 1.2293 | 0.5846 | 0.5588 | 1.1467 | 1.0000 |
| 90º        | 0.9898 | 2.4043 | 1.2669 | 0.6177 | 0.5391 | 1.1467 | 1.0518 |

Figure 7. Angular plots close to the equi-biaxial point for comparison convexity yield surfaces
Simulation of punch stretching over a hemispherical dome on DX54 applying different yield criteria (Table 3, Figure 8) for this material is used as a validation case for the different yield criteria descriptions. This same hemispherical dome is used for the verification of Vegter 2017 [6] using a Nakazima sample with a width of 120 mm, in which low friction is applied (assumed coefficient of friction of 0.01). The simulation results of the FLD-plot are compared with the measured FLD at the critical product height (Figure 9).

![Figure 8. Enlarged view of yield loci plots of the first quadrant in the plane strain area using input data of DX54 in Table 3 at 0º/90º to RD](image)

![Figure 9. Comparison of a simulation of hemispherical dome of DX54 at 90º to RD with a measured FLD-plot using at the critical product height, punch diameter of 100 mm and a blank width of 120 mm.](image)

Three yield criteria are compared in Figure 8.

- The original Vegter yield criterion, in this case convex. The 2Bez-version is not implemented in the FE-code PAMSTAMP 2018
- Yld2000 with an optimized exponent $a$ based on the average plane strain values. The individual plane strain values differ from the input data: 0º/90º values are lower and 45º/135º values are higher.
- Hill48 yield criterion with higher values for both the plane strain and the equi-biaxial values.
The underestimation of the plane strain value at 90° to RD in the Yld2000-criterion is the cause for overestimation of critical strains near the plane strain state of the FLD specimen. The combination of high plane strain values and a high equi-biaxial value shifts the strain distribution more to higher values at the left hand side close to uni-axial values. As a consequence Hill48 underestimates the critical strain near the plane strain state at the Right Hand Side in the FLD-plot of Figure 9.

7. Conclusions

- The Vegter (original) yield criterion and the Vegter (2Bez) yield criterion are interpolation methods and are able to accommodate all data points of the yield surface. Due to this large flexibility these two methods can result in a concave surface in some areas. The convexity is much better preserved by the Bézier method for the interpolation between material orientations than the cosine interpolation of the original Vegter criterion.
- The Yld2000 yield criterion (or its equivalent BBC2005) has by its limited number of free parameters (nine in total) an incomplete match in the plane strain area and pure shear area of the yield surface due to the focus on a complete match for the uni-axial and equi-biaxial stress and strain data. By optimising the exponent α an improved match is realised in the plane strain area. The optimisation results in a unique and reliable set of parameters.
- The yield2004 is able to capture the data in the uni-axial and equi-biaxial points and plane strain points and in less extent the pure shear points. The disadvantage is that it is not guaranteed getting a unique set of parameters (including the exponent α) using different starting values during parameter optimisation.
- By choosing the right weight factors and by using the deviatoric stress space in the double Bézier method, the von Mises criterion can be matched in an exact way.
- Compared to the measured results, the best agreement of the strain distribution in a forming simulation of an FLD-part of DX54-material near plane strain state is obtained by using the Vegter-criterion. Using the Hill48-criterion results into overestimation of formability of this part, and using the Yld2000-criterion leads to underestimation of formability of this part, but agreement with measured results is better in the latter case.

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