On the $\Delta I = 1/2$ Rule in Holographic QCD

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We study the $\Delta I = 1/2$ rule for kaon decays and the $B_K$ parameter for $K^0 - \bar{K}^0$ mixing in a dual 5-dimensional holographic QCD model. We perform, in the chiral limit, computations of the relevant four-point current-correlators, which depend upon self-interactions among the 5D bulk fields. Spontaneous chiral symmetry breaking ($\chi$SB) is realized through boundary conditions on the bulk fields. Numerical results are analyzed in comparison with QCD, chiral perturbation theory ($\chi$PT) and data, finding reasonable agreement with the experimental values of the $g_8$ and $g_{27}$ parameters describing the $\Delta I = 1/2, 3/2$ decay channels.

I. INTRODUCTION

The gauge/gravity duality [1, 2] provides new tools to investigate non-perturbative QCD. Holographic dual models which allow the study of confinement and $\chi$SB have been developed in essentially two different approaches: models based on string theory/supergravity in 10D [4], or phenomenologically inspired holographic dual models in 5D [6, 8]. In the past year, a study of meson masses and decay constants in the 5D models have shown quite good agreement with data, at least for the leading meson excitations. These encouraging results are based on two-point current-current correlators, and in the 5D description the leading contributions to such correlators are obtained from bare 5D propagators. One can also compute $n$-point correlators at tree level; however, the fact that 5D interactions must be taken into account makes an important difference that we shall investigate here. The holographic calculation of four-point correlators is a non-trivial test for 5D dual models of non-perturbative QCD, with, perhaps, the most interesting aspect being the holographic computation of observables which are important in QCD, particularly the celebrated $\Delta I = 1/2$ rule for kaon decays. Since our calculation is the first within this framework, we focus upon the simplest AdS/QCD model.

II. THE $\Delta I = 1/2$ RULE, AND THE $B_K$ MIXING PARAMETER

We start with a review of the relevant facts. Neglecting CP-violation there are two independent $K^0$ decays: $K^0 \rightarrow \pi^+\pi^-$ and $K^0 \rightarrow \pi^0\pi^0$. These two decays are combinations of $\Delta I = 1/2$ and $\Delta I = 3/2$ isospin amplitudes ($A_0$ and $A_2$, respectively). Experimentally $ReA_0/ReA_2 = 22.2$, and the unexpected largeness of this ratio is the $\Delta I = 1/2$ rule. In the chiral limit, and at $O(g^2)$ in the chiral expansion, these two amplitudes are expressed in terms of the $g_8$ and $g_{27}$ parameters (see e.g. [3, 10]). In the following we will also discuss the related but simpler observable, $B_K$, parameterizing $K^0 - \bar{K}^0$ mixing. Performing an OPE, its calculation reduces to that of the hadronic matrix element $\langle \bar{K}^0 | Q_{\Delta S = 2} | K^0 \rangle \sim (\mu) \equiv \frac{1}{2} f_K^2 M_K^2 B_K(\mu)$ of the four-quark left handed current operator $Q_{\Delta S = 2} = 4\eta_{\mu\nu}L^\mu_{sd}L^\nu_{ud}$ with $L^\mu_{q\bar{q}} = \bar{q}_\gamma \gamma^\mu (\frac{1-\eta}{2})q_\gamma$. Similarly, $g_{8,27}$ involve the $K \rightarrow \pi\pi$ matrix elements of two $\Delta S = 1$ operators four-quark left handed current operators (above the charm threshold), $Q_1 = 4\eta_{\mu\nu}L^\mu_{sd}L^\nu_{uu}$ and $Q_2 = 4\eta_{\mu\nu}L^\mu_{sd}\bar{L}^\nu_{ud}$.

The factorized contribution for these matrix elements is well known; the unfactorized contribution is more controversial. In the chiral limit the leading $N_c$ unfactorized contributions reduce to the calculation of integrals in $Q^2$ (with $Q$ the Euclidean momentum flowing between the left handed currents) of two distinct four-point current correlators [11, 12]. For $B_K \equiv B_K(\mu) \cdot C_{\Delta S = 2}(\mu)$,

$$B_K(\mu) = \frac{3}{4} - \frac{3}{4} \frac{1}{32\pi^2 F^2_\pi} \int dQ^2 W_{LRLR}(Q^2(\mu)), \quad (1)$$

$$W_{LRLR}(Q^2) = -\frac{Q^2}{3F^2_\pi} \eta_{\alpha\beta} \eta_{\mu \nu} \int \frac{dQ^2}{4F^2_\pi} W^{\mu\nu\alpha\beta}_{LRLR}(q),$$

$$W^{\mu\nu\alpha\beta}_{LRLR}(q) = \lim_{k \to 0} i^3 \int d^4x d^4y d^4z \ e^{i q(x-y)} \langle 0| T\{L^\mu_{sd}(x)R^\nu_{ds}(y)L^\alpha_{ua}(0)R^\beta_{us}(z)\} |0\rangle_{\text{conn}}, \quad (2)$$

while $g_8$ and $g_{27}$ are given by

$$g_8(\mu) = z_1(\mu) \left( \frac{4B_K(\mu)}{5} - 1 \right) + \quad (3)$$

$$z_2(\mu) \left( 1 - \frac{8B_K(\mu)}{15} \right) - \int dQ^2 W_{LLRR}(Q^2) \mu^2 F^2_\pi, \quad (4)$$

In the above, $C_{\Delta S = 2}(\mu)$ and $z_{1,2}(\mu)$ are the Wilson coefficients of $Q_{\Delta S = 2}$ and $Q_{1,2}$, $R^\mu_{q\bar{q}} = \bar{q}_\gamma \gamma^\mu (\frac{1+\eta}{2})q_\gamma$, and

$$W_{LLRR}(Q^2) = -\frac{Q^2}{3F^2_\pi} \eta_{\alpha\beta} \eta_{\mu \nu} \int \frac{dQ^2}{4F^2_\pi} W^{\mu\nu\alpha\beta}_{LLRR}(q),$$

$$W^{\mu\nu\alpha\beta}_{LLRR}(q) = \lim_{k \to 0} i^3 \int d^4x d^4y d^4z \ e^{i q(x-y)} \langle 0| T\{L^\mu_{ua}(x)L^\nu_{ud}(0)R^\alpha_{ds}(y)R^\beta_{us}(z)\} |0\rangle_{\text{conn}}. \quad (5)$$
THREE HOLOGRAPHIC QCD MODEL

To compute the four-point correlators through the gauge/gravity correspondence, we consider the \( L^\mu_{M,N} \) and \( R^a_{M,N} \) 4D quark currents. By virtue of the AdS/CFT correspondence, these operators couple to the boundary values of 5D gauge fields \( L^\mu(x^\nu,z) \) and \( R^a(x^\nu,z) \), and the 5D gauge fields are massless. The 5D action is a \( SU(3)_L \times SU(3)_R \) Yang-Mills theory

\[
\frac{M_5}{4} \int dz \, d^4x \, \sqrt{g} \left( L^a_{M,N} L^{a,MN} + R^a_{M,N} R^{a,MN} \right),
\]

where \( M_5 \) is an undetermined mass scale, and \( M = (\mu, 5) \). The fifth coordinate \( z \) runs from \( L_0 \) to \( L_1 \), which are the positions of the UV and IR branes respectively. It is possible to set \( L_0 = 0 \) as in Refs. \[4, 5\] as our results are smooth as \( L \to 0 \). We use the AdS\(_5\) metric \( ds^2 = a(z)^2(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \), where \( a(z)^2 = (L/z)^2 \) and \( \eta_{\mu\nu} \) is given by \( (+, -, -, -) \). The field strengths are \( F_{M,N} = \partial_M L_N - \partial_N L_M - i[\lambda_M, L_N] \), where \( \lambda_M = L^a_{\mu\nu} T^a \), and similarly for \( R_{M,N} \). We normalise \( tr(T^a \Gamma) = \delta^{ab} \), and define the vector and axial gauge bosons as \( V_M = (L_M + R_M)/\sqrt{2} \) and \( A_M = (L_M - R_M)/\sqrt{2} \), respectively.

The holographic dual of this 5D theory with three quark flavours and a global \( SU(3)_L \times SU(3)_R \) chiral symmetry \[4, 5\]. We work in the chiral limit and include the effect of the spontaneous \( \chi_\text{SB} \) via the IR boundary condition on the axial vector fields \[4, 5\]. This is clearly a rather severe limit, yet the calculational simplification it affords it worthwhile. The effects of spontaneous and explicit \( \chi_\text{SB} \) could be treated more precisely by the inclusion of a bulk scalar in the bi-fundamental representation of the gauge group \[4, 5\].

A. Propagators and 5D Interactions

By the holographic correspondence, the UV boundary values of \( V_\mu \) and \( A_\mu \) act as classical sources coupled to the 4D vector and axial global symmetry currents. Moreover, the 5D action evaluated on the solutions of the equations of motion (EOM) of the bulk fields defines the generating functional for current-current correlators in the 4D theory. We therefore need to solve the EOM with given UV boundary values of the fields, and substitute back into the action. The EOM are non-linear, so we must solve them iteratively. This is the origin of Witten diagram construction for calculating the \( n \)-point Greens functions of operators in the dual theory \[4\]. The iterative solution of the EOM requires the bulk-to-bulk and bulk-to-boundary propagators for the gauge fields in the 5D theory.

We work in the unitary gauge in both the \( SU(3)_V \) and \( SU(3)_A \) sectors. \( V_5 \) is eliminated completely in this gauge if we impose null Dirichlet boundary conditions for it on both branes. We impose warped Neumann boundary conditions on the \( A_5 \) field (i.e. \( \partial_z (a A_5) = 0 \) on both branes). In this gauge \( A_5 \) is a physical field because it has a zero mode which cannot be gauged away. The \( V_\mu \) and \( A_\mu \) propagators have both transverse and longitudinal parts in this gauge. Following Ref. \[12\] we have

\[
\langle V_\mu V^\nu \rangle = -iG_0^V(z, z') P^\mu_\nu - iG_0^V(z, z') P^\mu_\nu, \quad (7)
\]

where \( P^\mu_\nu \) = \( \eta^\mu_\nu - p^\mu p^\nu \) and \( P^\mu_\nu \) = \( p^\mu p^\nu \). The \( A_\mu \) propagator has a similar form, and both solve

\[
\left( \partial_z^2 - \frac{1}{z} \partial_z + p^2 \right) G^V_{A}(z, z') = \frac{z \delta(z - z')}{M_5 L} \quad (8)
\]

The boundary conditions on \( G_\mu^V(z, z') \) are Dirichlet on the UV brane, \( G_\mu^V(z, z')|_{z = L_0} = 0 \), and Neumann on the IR brane, \( \partial_z G_\mu^V(z, z')|_{z = L_1} = 0 \). We impose Dirichlet boundary conditions on both branes in the axial sector \( G^A_\mu(z, z')|_{z = L_0, L_1} = 0 \), to encode \( \chi_\text{SB} \). The bulk-to-boundary propagator is defined as:

\[
\langle V_\mu V^\nu \rangle \bigg|_{\text{B}} = -\frac{M_5 L}{z} \partial_z \langle V_\mu V^\nu \rangle \bigg|_{z = L_0}, \quad (9)
\]

(with a similar equation for \( A_\mu \)), where

\[
\langle V_\mu V^\nu \rangle \bigg|_{\text{B}} = -iK_\mu^V(z') P^\mu_\nu - iK_0^V(z') P^\mu_\nu, \quad (10)
\]

\[
K^V_{A}(z') = -\frac{M_5 L}{z} \partial_z G^V_{A}(z, z')|_{z = L_0}, \quad (11)
\]

and similarly for \( K^A_0(z') \).

The solutions are given by:

\[
G^V_{0}(z, z')|_{z < z'} = \frac{\pi z z'}{2M_5 L(AD - BC)} \int dz_{1} [J_1(pz_{1}) + BJ_1(pz_{1})][CJ_1(pz_{1}) + DJ_1(pz_{1})],
\]

\[
G^A_{0}(z, z')|_{z < z'} = \frac{1}{2M_5 L} \left( \frac{z'^2 - L_0^2}{L_0^2 - L_1^2} \right), \quad (13)
\]

\[
G^V_{0}(z, z')|_{z > z'} = \frac{1}{2M_5 L} \left( \frac{z^2 - L_0^2}{L_0^2 - L_1^2} \right), \quad (13)
\]

\[
(z > z' \text{ again swaps } z \text{ with } z').
\]

For the vector sector the coefficients are \( A = -J_1(pL_0), C = -J_0(pL_0), B = J_1(pL_0) \) and \( D = J_0(pL_1) \); for \( A_\mu \) we obtain \( A = -J_1(pL_0), C = -J_0(pL_0), B = J_1(pL_0) \) and \( D = J_0(pL_1) \). In the zero momentum limit:

\[
G^V_{0}(z, z')|_{z < z'} = \frac{1}{2M_5 L} \left( \frac{z'^2 - L_0^2}{L_0^2 - L_1^2} \right), \quad (14)
\]

\[
G^A_{0}(z, z')|_{z < z'} = \frac{1}{2M_5 L} \left( \frac{z^2 - L_0^2}{L_0^2 - L_1^2} \right), \quad (14)
\]

\[
(z > z' \text{ again swaps } z \text{ with } z').
\]

For the axial sector

\[
K^V_{A}(z') = -\frac{z'}{L_0} \frac{CJ_1(pz_{1}) + DJ_1(pz_{1})}{AD - BC}, \quad (15)
\]

while \( K^V_{0}(z') = 1 \), and \( K^A_{0}(z') = z'^2 - L_0^2 \). The physical condition of 4D current conservation eliminates the
The interactions are the three-gauge boson and four-gauge boson vertices derived from the 5D action Eq. (4).

B. Four-point current-current correlators

In order to calculate the four-point current-current correlators we have to solve tree level four-point Witten diagrams which contain either one four-gauge field vertex or two three-gauge field vertices. Note that certain 5d Witten diagrams do not contribute to the four-point functions 22 and 23 because they do not preserve the flavor structure of the 4d theory.

We find that the group theory structure is a common factor in all diagrams, so denoting by Σ the sum of all diagrams with group factors amputated we have $\Sigma = \Sigma_X + \Sigma_A + \Sigma_Y$, with

$$\Sigma_X = 3i M_5 L \int \frac{dz}{z} \left( [K_0^V V^2 + K_0^A 2^2] [K_0^2 + K_p^A 2^2] - 4 K_0^V K_0^A K_p^V K_p^A \right)$$

$$\Sigma_A = -\frac{3i}{2} (M_5 L)^2 \int \frac{dz}{z} \frac{dz'}{z'} G_p^0(z, z') A'(z, z') \quad (19)$$

where $A'(z, z')$ is a function whose explicit form is known. As for the Y-diagrams, the integrations are more involved, but the sum can be written as $\Sigma_Y = Y_1 + Y_2 + Y_3 + Y_4 - 2Y_5 - 2Y_6$, where

$$Y_i = -3i \left( \frac{M_5 L}{\sqrt{2}} \right)^2 p^2 \int \frac{dz}{z} \frac{dz'}{z'} K_p^{0i}(z, z') K_p^{0i}(z') K_p^{di}(z) K_p^{di}(z') \quad (20)$$

with $\{abcde\} = \{VVVVD\}, \{VAVAD\}, \{AAAVV\}, \{AVAVA\}, \{AAVVV\}, \{AVVVV\}$ for $i = 1, ..., 6$. Performing the Wick rotation to Euclidean space, we obtain:

$$W_{LLRR}(Q^2) = \frac{i Q^2}{6\pi^2} \Sigma(p = iQ) = -2W_{LLRR}(Q^2). \quad (21)$$

Thus we find that the ratio of the two correlators is independent of the integration in $z$-space, and is solely due to their different $SU(3) \times SU(3)$ symmetry factors. The integrals can be done analytically with finite values for $L_1$ and $L_0$, but our observables have a smooth $L_0 \rightarrow 0$ limit, and much simpler expressions arise if we set $L_0 = 0$ giving:

$$\Sigma(Q) = 3i M_5 L \left[ \frac{32}{Q^6 L_0^5} - \frac{16}{5Q^4 L_0^3} - \frac{69}{60 L_0^2} + \frac{2}{5Q^2 L_0^2} \right]$$

$$+ \frac{69}{60 L_0^2} - \frac{3}{15Q^2 L_0^2} + \frac{8}{5Q^4 L_0^3}$$

$$+ \frac{32}{Q^6 L_0^5} - \frac{64}{Q^6 L_0^5} + \frac{1}{QL_0^3 L_0^3} \right) \quad (22)$$

where $I_{0,1} \equiv I_{0,1}(QL_1)$ are modified Bessel functions of zeroth and first order respectively.

IV. RESULTS AND DISCUSSION

The three input parameters of our model are $M_5 L$, $L_0$ and $L_1$. From the expressions in Section II.A one can easily compute the boundary–to–boundary propagator in the vector sector, and match its large Euclidean momentum behaviour to that known from perturbative QCD deriving the relation 22, 23

$$M_5 L = \frac{N_c}{12\pi^2} \quad (23)$$

One is then left with the two dimensionful parameters, $1/L_0$ giving the UV cutoff scale, and $1/L_1$ setting the
IR scale. As mentioned before, one problem of the simple 5D holographic model presented is that it does not correctly reproduce the spectrum of meson resonances in the regime above approximately 1500 MeV. In addition, in computing the physically relevant observables $g_8$ and $g_{27}$ one must in practice choose a scale $\mu$ at which to evaluate the Wilson coefficients $z_1(\mu)$ and $z_2(\mu)$. One simple choice we make in the following is to take $1/L_0 = 1500$ MeV, imposing this as an upper hard cut-off when we evaluate the $Q^2$ integrals in Eqs. (11) and (13), and identifying this hard cut-off with the short distance renormalisation scale $\mu$. Although this matching procedure is quite crude, Refs. [17] have shown that such a procedure captures the dominant contributions. In a later publication we investigate in some detail other more sophisticated matching procedures.  

with $\chi$PT and perturbative QCD calculations of $[15]$ and $[16, 17]$ of the BS and $\Delta I = 1/2$ rule observables, our model, if valid, presents the interesting possibility of calculating the intermediate $Q^2$ contribution around 0.1-2 GeV$^2$ directly, rather than fitting it from an interpolation between the $\chi$PT low $Q^2$ regime $[18]$ and of the OPE high $Q^2$ regime $[16, 17]$. One immediate check on our calculation follows from the fact that the most general form for the Greens function $W_{LRLR}(Q^2)$ in the large $N_c$ limit can be written in terms of the masses, $M_i$, and residues of the meson resonances $[16, 17]$

$$\sum_{i=1}^{\infty} \left( \frac{\alpha_i}{(Q^2 + M_i^2)} + \frac{\beta_i}{(Q^2 + M_i^2)^2} + \frac{\gamma_i}{(Q^2 + M_i^2)^3} \right),$$

and similarly for $W_{LLRR}$ with residues $\alpha'_i$, $\beta'_i$ and $\gamma'_i$. We have explicitly checked that our results for $W_{LRLR}(Q^2)$ and $W_{LLRR}(Q^2)$ agree with this general form.

We now turn to the comparison of our results with the experimental data. Given Eq. (23) and our choice of $L_0 = 1/1500$ MeV$^{-1}$, the one remaining free parameter of our model is $L_1$. The data to which this must be fitted are, most importantly, the pion decay constant, $F_\pi$, the rho vector-meson mass, $m_\rho$, the $a_1$ axial-vector meson mass $m_{a_1}$, and the $g_8$ and $g_{27}$ (or $B_K$) parameters. In the low-$Q^2$ regime chiral perturbation theory calculations $[16, 17]$ give the behaviour of the correlation functions

$$W_{LRLR}(Q^2) = 6 - 24(2l_1 + 5l_2 + l_3 + l_0) \frac{Q^2}{F_\pi^2} + ...$$

$$W_{LLRR}(Q^2) = - \frac{3}{8} - (\frac{15}{2} l_3 + \frac{3}{2} l_0) \frac{Q^2}{F_\pi^2} + ...$$

with $l_i$ the standard chiral-Lagrangian coefficients. In our fit procedure we use the $\chi$PT results, Eq. (25), in the integrals over $Q^2$ below $1/L_1$, and the results of the holographic calculation, Eqs. (21) and (22) in the regime $1/L_1$ to $1/L_0$. In terms of $M_5 L$, $L_1$ and $L_0$ the predictions of the 5D holographic model for $F_\pi$ and $m_\rho$ and $m_{a_1}$ are $[14, 15, 16]$  

$$F_\pi^2 \approx \frac{2M_5 L}{L_1^2 - L_0^2}$$

and in a good approximation in the range of interest

$$m_\rho \approx \frac{2.12 (L_1 - 0.282 L_0)}{L_1} \frac{L_1}{(L_1 - L_0)}$$

$$m_{a_1} \approx \frac{3.38 (L_1 - 0.088 L_0)}{L_1} \frac{L_1}{(L_1 - L_0)}.$$  

(We use a precise interpolation to the exact result for our fits.) A non-trivial success of the 5D holographic QCD model is that in the $L_0 = 0$ limit (to which $1/L_0 = 1500$ MeV is a good approximation) the ratio $m_{a_1}/m_\rho \approx 1.6$ is very well reproduced independently of the values of $M_5 L$ and $L_1$, as already noted in $[4, 6]$.  

Numerically performing the $Q^2$ integrations in Eqs. (11), (16) and (17), as a function of $1/L_1$ and utilizing, as appropriate, the leading order values of the Wilson coefficients $z_1$ and $z_2$ (taken from $[16]$ with $\Lambda_{M_S} = 325$ MeV, and $\mu = 1500$ MeV) we can fit the final parameter $L_1$ to the full set of observables. The $\Delta I = 1/2, 3/2$ data requires $g_{8|obs} = 5.1$ and $g_{27|obs} = 0.29$, although the values to be explained are modified to $g_{8|obs} = 3.3$ and $g_{27|obs} = 0.23$ when one takes into account the enhancement already provided by the calculated $O(p^4)$ chiral corrections $[9]$. Fitting to $g_8$ and $g_{27}$ and the values of $F_\pi$, $m_\rho$ and $m_{a_1}$ leads to the result

$$L_1^{-1} \approx 302 \text{ MeV}$$

which implies (in all cases normalized to the data for ease of understanding)

$$\frac{m_\rho|th}{m_\rho|obs} \approx 1.00, \quad \frac{m_{a_1}|th}{m_{a_1}|obs} \approx 1.04, \quad \frac{F_\pi|th}{F_\pi|obs} \approx 0.80,$$

$$\frac{g_8|th}{g_8|obs} \approx 0.49, \quad \frac{g_{27}|th}{g_{27}|obs} \approx 1.39$$

and a value of $B_K(1500 \text{ MeV}) \approx 0.54$ ($\hat{B}_K \approx 0.76$ using the Wilson coefficient value $C_{\Delta S = 2}(1500 \text{ MeV}) = 1.42$, see e.g. Ref. [17]). Considering the relative crudity of the model and the use of the large-$N_c$ expansion of QCD this is a reasonable fit to the data and we find this result very encouraging. We emphasise that $L_1$ and $L_0$ are the only free parameters in this fit to the five observables of equation Eq. (30). It is noteworthy that in the expected domain of validity of the model (i.e. in the $Q^2$ intermediate region), the 5D interactions induce a large increase of $g_8$ and a suppression for $g_{27}$.  

On the other hand the holographic model predicts the low-$Q^2$ behaviour of $W_{LRLR}$ and $W_{LLRR}$ to be identical in form to $\chi$PT but with disagreeing numerical values

$$W_{LLRR} = - \frac{W_{LRLR}}{2} = - \frac{1}{4} + \frac{M_5 L}{4} \frac{29 Q^2}{F_\pi^2} + ...$$

(31)
It is possible that this failing is due to the simplistic truncation of the AdS space in the IR. Alternatively, the fault might be with our approximations, particularly the treatment in the 5D model of $\chi_{SB}$.

It would be interesting to include a bi-fundamental bulk scalar field of mass-squared $-3/L^2$ associated with the $\bar{q}q$ operator, as in the models [4, 5], and a massless singlet scalar associated with the glueball states generated by the gluon field operator $G_{\mu\nu}G^{\mu\nu}$, to see whether their inclusion leads to an even better fit to the full set of observables.

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