Study of controlled motion bionic mini robot

E N Politov, A N Rukavitsyn
Southwest State University, 94, 50 Let OctiabriaAv., Kursk, 305040, Russia
E-mail: politovyevgeny@rambler.ru

Abstract. The article describes a dynamic model of a bionic mini-robot capable of moving on a rough surface or separately from it. Differential equations describing the robot’s motion in the phases of flight and movement on the support surface were obtained. The debalance’s angular velocity was used as the controlled parameter. The results of numerical modelling of the equations of motion supported theoretical conclusions on the character of the dependence of height and length of a jump on the frequency of rotation. It was simultaneously established that the shape of the trajectory of the center of mass depends on the controlled parameter.

1. Introduction
Mobile bionic devices that move separately from the surface at the expense of periodic motion of the internal masses are presented in [1-7]. The difference between such apparatus and robots with spring drives is the fact that the kinetic energy of the mobile masses, which is converted to the robot’s kinetic energy, is used as the source of motion instead of the spring’s potential energy. This allows us to control motion at the expense of one parameter – debalance’s angular velocity.

Methods of calculating the systems using the kinetic energy of the internal masses for motion are not inadequately developed. Therefore, the aim of this work is the design of a mathematical model of motion of the new type of robot with one rotating mass and the study of the main principles of controlled motion[8-10].

2. Materials and methods
The considered jumping mini-robot is a mechanical system consisting of two rigid bodies, one of which is a cylindrical casing of mass m1 that is periodically in contact with the surface and the second – a debalance of mass m2 that rotates uniformly with respect to the casing (Figure 1).

On the diagram point O1 – the casing’s center of mass; the debalance’s axis of rotation passes through point O2 and point C – the robot’s center of mass. The control parameter in this given system is the debalance’s rotation frequency, ω and the subordinate parameter – height, length of jump and the average speed of the robot’s motion. The mass ratio of the system’s elements, λ=m2/(m1+m2) is a variable parameter. Let us look at motion in a fixed coordinate system Oxyz. Two coordinate systems: O1x1y1z1 whose axes are principal and central axes of inertia and O2x2y2z2 whose axis passes the point modelling the debalance are rigidly bound to the robot’s links. The authors assume that all the points of the system move in parallel vertical planes. This is possible if the debalance’s relative rotation coincides with the casing’s symmetry plane. Then, by choosing abscissa axis, it is sufficient to consider projections of the robot in plane Oxy.

The debalance’s relative motion is defined and occurs with a constant angular speed. The angle of rotation can be presented as φ2=ωt.
Figure 1. An analytical diagram of the motion of a jumping mini-robot

Let us introduce radiuses –vectors \( \vec{r}_1, \vec{r}_2, \vec{r}_C \) that define the coordinates of the centers of mass of the casing and the debalance in the absolute coordinate system, as well as the coordinates of the center of mass of the entire system, \( C \). It is obvious that

\[
\vec{r}_2 = \vec{r}_1 + \vec{r}_{21},
\]

where \( \vec{r}_{21} \) – debalance’s radius vector relative the casing (Figure 2).

From the definition of the center of mass it is possible to obtain the following expression using the given vector equality:

\[
\vec{r}_C = \vec{r}_1 + \frac{m_2}{m_1 + m_2} \vec{r}_{21}.
\]

Let us determine the vector, \( \vec{r}_{21} \). From the diagram in Figure 2a it is possible to see that \( \vec{r}_{21} \) is the sum of vectors \( O_1O_2 \) and \( \vec{r}_{22} \), which coordinates are conveniently given in the relative coordinate systems: \( \vec{O_2O_1} = (O_1O_2, 0,0)^T \) – in the body’s system, \( \vec{r}_{22} = (l,0,0)^T \) – in the debalance’s system. Here and further, the index “T” before a row matrix denotes transposition. The distance \( O_1O_2 \) and debalance’s length \( l \) are given.

The vectors will be determined in the absolute system with the help of the rotation matrix, \( T_{10} \) of coordinate system \( O_1x_1y_1z_1 \) relative \( Oxyz \) and rotation matrix \( T_{21} \) of system \( O_2x_2y_2z_2 \) relative \( O_1x_1y_1z_1 \):

\[
O_1O_2^{(0)} = T_{10}O_1O_2, \quad \vec{r}_{22}^{(0)} = T_{10}\vec{r}_{22}.
\]

The upper index “0” before the vectors (in brackets) indicates the absolute coordinate system. The required radius-vector of a point mass relative the body’s center of mass in the absolute coordinate system is determined by the following equality:

\[
\vec{r}_{21} = T_{10}(O_1O_2 + T_{21}\vec{r}_{22}).
\]

The radius-vector of the center of mass in the absolute coordinate system corresponds to the column matrix \( \vec{r}_1 = (x_1, y_1,0)^T \).

The motion of the considered system is described by vector equations that express theories of dynamics of the conservation of momentum and angular momentum of a system:

\[
\frac{d\vec{Q}}{dt} = \vec{F}, \quad (4) \quad \frac{d\vec{L}_c}{dt} = \vec{M}_c,
\]
where $\dot{Q}$ - momentum of the system; $L_c$ - angular momentum of the system relative the center of mass, $C$; $\mathbf{F}$, $\mathbf{M}_c$ - the principal vector and principal moment of external forces applied to the system.

The momentum of the system by definition is equal to the vector sum: $\dot{Q} = m_1\dot{r}_1 + m_2\dot{r}_2$, in which $\dot{r}_1$, $\dot{r}_2$ - the velocity of the body’s centre of mass and the debalance, while it is derivative with respect to time - $\frac{d\dot{Q}}{dt} = m_1\ddot{r}_1 + m_2\ddot{r}_2$. The acceleration of the body’s centre of mass is represented by the vector, $\ddot{r}_1 = (x_1, y_1, 0)^T$ and that of the debalance, according to (1), by the sum: $\ddot{r}_2 = \ddot{r}_1 + \ddot{r}_{21}$. The debalance’s acceleration for its motion around point $O_1$ is obtained by differentiating (3):

$$\ddot{r}_{21} = T_{10}(O_1O_2 + T_{12}r_{22}) + 2T_{10}T_{21}r_{22} + T_{10}T_{21}r_{22}.$$  

(5)

3. Results and Discussion

By substituting the values of the derivatives of the matrices and vectors in (4) and switching from vector form to scalar it is possible to find differential equations of motion of the center of mass:

$$(m_1 + m_2)x_1 + m_2\phi_1(-\sin \phi_1(O_1O_2 + l \cos \phi_2) - \cos \phi_1 \cdot l \sin \phi_2) =$$

$-m_2\phi_1^2(\cos \phi_1(O_1O_2 + l \cos \phi_2) - \sin \phi_1 \cdot l \sin \phi_2) +$ $+2m_2\phi_1\omega_l(l \sin \phi_2 - \cos \phi_1 \cos \phi_2) + m_2l\omega_l^2(\sin \phi_1 \sin \phi_2 - \cos \phi_1 \cos \phi_2) = 0$;

$$(m_1 + m_2)y_1 + m_2\phi_1(\cos \phi_1(O_1O_2 + l \cos \phi_2) - \sin \phi_1 \cdot l \sin \phi_2) =$$

$+m_2\phi_1^2(-\sin \phi_1(O_1O_2 + l \cos \phi_2) - \cos \phi_1 \cdot l \sin \phi_2) + 2m_2\phi_1\omega_l(-\cos \phi_1 \sin \phi_2 -$ $-\sin \phi_1 \cos \phi_2) + m_2l\omega_l^2(-\sin \phi_1 \cos \phi_2 - \cos \phi_1 \sin \phi_2) = -(m_1 + m_2)g$.

Let us determine the derivatives of the angle of rotation in the obtained equations using the principle of conservation of momentum of a system (5).

Let us dwell on the procedure of finding angular momentum. The choice of using the system’s center of mass is not accidental, because the acceleration of the body’s centre of mass will be absent in the final equations of rotation. The robot’s centre of mass is a stationary point. Let us connect it with coordinate system $C\xi\eta\zeta$, whose axes perform translational motion, i.e. the axes at all times remain parallel to the axes of the fixed system. In this system one will determine the quantities in the principle of conservation of angular momentum (5). Let us take note that angular characteristics of the robot’s
motion in the system of the centre of mass and the fixed system coincide.

The angular momentum is the vector sum:

\[ \mathbf{L}_c = \mathbf{L}_{c1} + \mathbf{L}_{c2} \]  

(6)

The angular momentum of the body is made up of the angular momentum of its rotation around its own centre of mass and its momentum (as a material point whose mass equals that of the body) relative the common centre of mass:

\[ \mathbf{L}_{c1} = \mathbf{L}_{0i} + \mathbf{L}_{c1}' = J \dot{\omega} + \mathbf{r}_i \times \mathbf{m}_i \dot{\mathbf{r}}_i \]  

(7)

where \( \mathbf{L}_{0i} \) – angular momentum of the body relative its centre of mass, \( \mathbf{O}_i \), \( \mathbf{\omega} = (0; 0; \phi) \) – vector of the robot’s angular velocity, \( J \) – inertia tensor of the body in the system of axes parallel to the axes of the fixed system.

It is known that the inertia tensor has the simplest form in the system of principal axes of inertia i.e., in system \( O_{x' x' y' z'} \):

\[ \mathbf{J}_1 = \begin{pmatrix} I_{x' x'} & 0 & 0 \\ 0 & I_{y' y'} & 0 \\ 0 & 0 & I_{z' z'} \end{pmatrix} \]

where the elements of the main diagonal – axial moments of inertia of the body are given. From here using the rotation matrix \( T_{0s} \), one can go to the system axis parallel to the axes of the fixed system, in which the inertia tensor is given as follows:

\[ J = T_{10}^T J_1 T_{10}^T \]  

(8)

The angular momentum of the body is made up of the angular momentum of its rotation around its own centre of mass and its momentum (as a material point whose mass equals that of the body) relative the common centre of mass:

\[ \mathbf{L}_{c2} = m_s \mathbf{r}_{sc} \times \dot{\mathbf{r}}_s \]  

(9)

Combining (6)-(9) and taking into account the representations of the velocities of the body’s points and the definition of the system’s centre of mass, one can write the angular momentum of the system as:

\[ \mathbf{L}_c = T_{10} J_s T_{10}^T \dot{\mathbf{\omega}}_1 + \sum_{i=1}^{2} \mathbf{r}_{ic} \times m_i \dot{\mathbf{r}}_{ic} \]  

(10)

From (1) and (2), let us write the radius-vectors of the system’s point relative the centre of mass as follows:

\[ \mathbf{r}_{ic} = -\frac{m_2}{m_1 + m_2} \mathbf{r}_{21}, \quad \mathbf{r}_{2c} = \frac{m_1}{m_1 + m_2} \mathbf{r}_{21} \]  

(11)

After differentiating (10) with respect to time and taking into consideration equation (11), one obtains the expression:

\[ \frac{d\mathbf{L}_c}{dt} = T_{10} J_s T_{10}^T \dot{\mathbf{\omega}}_1 + \frac{m_1 m_2}{m_1 + m_2} \mathbf{r}_{21} \times \dot{\mathbf{r}}_{21} \]  

(12)

Taking into account the fact that the principal moment of external gravitational forces relative the center of mass equals zero and substituting them in the process of transformation of the coordinates of the vectors in (5) and taking the scalar form, one obtains the required differential equation of rotation:

\[ \sum m_i (O_{1i} \dot{O}_1 - 2 O_{1i} O_2 \cos \varphi_{21} + \dot{t}^2)] \varphi_i = \frac{m_1 m_2}{m_1 + m_2} O_1 O_2 l \sin \varphi_{21} \dot{\varphi} (2 \dot{\varphi} + \varphi) \]  

Thus, one arrives at a system of differential equations of motion of the robot:
\[
\begin{align*}
\dot{\phi} &= \frac{A \cos \omega t}{B + A \cos \omega t} \left( \phi + \frac{1}{2} \omega \right) \\
x_1 - D\ddot{\phi}(O_1O_2 \sin \phi + \sin(\phi + \omega t)) &= D(\phi + \omega t)^2 \cos(\phi + \omega t) + D\phi^2 O_1O_2 \cos \phi, \\
y_1 + D\ddot{\phi}(O_1O_2 \cos \phi + \cos(\phi + \omega t)) &= D(\phi + \omega t)^2 \sin(\phi + \omega t) + D\phi^2 O_1O_2 \sin \phi - mg,
\end{align*}
\]

where the constant coefficients are expressed in terms of the system’s parameters:

\[ A = \frac{2m_1m_2}{m_1 + m_2} O_1O_2, B = I_x \omega^2 + \frac{m_1m_2}{m_1 + m_2}, D = \frac{m_2}{m_1 + m_2}. \]

The system of three differential equations (12) describes the motion of the robot in flight i.e. if the body’s center of symmetry coincides with the centre of mass and the following condition holds: \( y_1 > R \), where \( R \) – the radius of the body, then the robot’s motion in flight is described by the system of three equations (12). For a complete description of the jump-like motion it is necessary to supplement system (12) with the robot’s equations of motion on the supporting surface and determine the variation of motion parameters during landing and separation. The analytical condition for motion on a flat supporting surface is the equality \( y_1 = R \). Let us notice that the equations of motion on the surface can be obtained with the help of the already stated theorems (4) and (5), taking into account the fact that the normal reaction \( N \), dry friction \( F_f \), and moment of rolling resistance \( M_r \) are added to the forces acting on the system (Fig.1). Assuming the absence of the rolling, the number of equations is reduced to two:

\[ \begin{align*}
(m_1 + m_2)x_1 &= m_2 \omega^2 \cos \omega t + F_f, \\
0 &= m_2 \omega^2 \sin \omega t + N - (m_1 + m_2)g.
\end{align*} \]

Dry friction is determined in accordance with the analytical model:

\[ F_f = \begin{cases} -f_0 N \text{sign}(x), & x \neq 0, \\ -F_0, & x = 0, |F_0| \leq f_0 N, \\ -f_0 N \text{sign}(F_0), & x = 0, |F_0| > f_0 N, \end{cases} \]

where \( F_0 \) – the projection of all forces applied to the robot besides dry friction; \( f_0 \) – coefficient of dry friction; \( N \) – normal reaction of the surface; \( x \) – the robot’s velocity along \( Ox \).

At the point of landing the velocity of the point of contact changes direction i.e. the system experiences a collision. In this study the authors restrict themselves to the case when the kinetic energy is lost on impact with the exception of the energy of self-rotation of the debalance. This means that the velocity of the contact point and the body’s angular velocity attain zero values as a result of landing and these will be used as initial values during the study of the next phase of motion. One will now study the influence of the control parameter on the characteristics of movement of the robot. Let us refer to the resulting system of differential equations of motion of the robot in the air. The first equation – the equation of rotation – as a first order equation with respect to velocity has an obvious analytical solution, which under zero initial conditions is given as follows:

\[ \phi = \frac{\omega}{2} \left( \frac{A + B}{B + A \cos \omega t} - 1 \right). \]

Thus, it is clear that the time variation of the rotation speed is represented by a periodic bounded by a continuous function whose frequency coincides with the frequency of rotation of the debalance. Furthermore, the expression (14) does not change its sign. For the lowest value of the denominator of the fraction in brackets one will have:

\[ \min \{ B + A \cos \omega t | t \in R \} = B - A = I_x \omega^2 + \frac{m_1m_2}{m_1 + m_2}(O_1O_2 - l)^2 > 0, \]

i.e. the denominator is positive. But since \( A + B > B + A \cos \omega t > 0 \), the sign of the angular velocity
obviously will coincide with that of the debalance’s angular velocity. In a particular case, if the coefficient $A$ in system (12) is zero, the angular velocity remains constant. This case is possible for the studied robot if the centers $O_1$ and $O_2$ coincide. This means that after separating from the surface with zero angular velocity, the body does not rotate in subsequent flight and it can be considered as a material point of mass $M=m_1+m_2$, moving under the action of the force $F=m_2\omega^2l$, which vector rotates uniformly with angular velocity $\omega$.

Let us look at the flight of the point, which occurs under the action of force $F$ with variable direction (Figure 2b). The initial conditions of such motion are determined by the point’s state at the point of separation from the surface, namely zero velocity and a certain non-zero angle of inclination of the force, $\alpha$. In the case of detachment the normal reaction equals 0, i.e.:

$$\sin(\omega t) = \frac{m_1 + m_2}{m_2} \cdot \frac{g}{\omega^2l}. \quad (15)$$

It is obvious that depending on the parameters of the system, equation (15) can have a unique solution, multiple solutions or none. If the values of parameters satisfy the inequality $\frac{m_1 + m_2}{m_2} \cdot \frac{g}{\omega^2l} > 1$, then equation (15) does not have a solution and the point is not separated from the surface. In the second case when the following equality holds $\frac{m_1 + m_2}{m_2} \cdot \frac{g}{\omega^2l} = 1$, equation (15) has a unique solution. However, further ascent of this point does not occur, because its acceleration does not attain the required positive value. In the third case the region of the system’s parameter values is determined by the inequality:

$$\frac{m_1 + m_2}{m_2} \cdot \frac{g}{\omega^2l} < 1 \quad (16)$$

In this case the reaction becomes zero at the given values of the angle of inclination (consequently the angle of rotation of the debalance):

$$\alpha = \omega t = \min\{-\arcsin\left(\frac{m_1 + m_2}{m_2} \cdot \frac{g}{\omega^2l}\right) + \pi; \arcsin\left(\frac{m_1 + m_2}{m_2} \cdot \frac{g}{\omega^2l}\right)\} = \arcsin\left(\frac{m_1 + m_2}{m_2} \cdot \frac{g}{\omega^2l}\right).$$

Based on the definition of inverse trigonometric functions, the separation angle lies in the interval $\alpha \in \left(0; \frac{\pi}{2}\right)$. Further, let us look at the body’s vertical motion neglecting its rotation. If the reference time is associated with the point of separation, then the movement of the body centre of mass is described by the differential equation:

$$\ddot{y} = -g + \frac{m_2}{m_1 + m_2} \cdot \omega^2l \sin(\omega t + \alpha) \quad (17)$$

Let us now show that there is a period of time at the beginning of the motion where the point’s acceleration is positive. Indeed, if one transforms (2.4) into the product:

$$\ddot{y}(t) = \frac{m_1}{m_1 + m_2} \omega^2l (\sin(\omega t + \alpha) - \frac{m_1 + m_2}{m_2} \cdot \frac{g}{\omega^2l}) = 2 \frac{m_2}{m_1 + m_2} \omega^2l \cdot \cos\left(\frac{\omega t}{2}\right) \sin\left(\frac{\omega t}{2} + \alpha\right),$$

then it is obvious that acceleration is greater than zero in the time interval $t \in \left(0; \frac{\pi}{\omega}\right)$, as a result of which the point continues to move up after separation. By differentiating equation (17) twice, taking into account the zero initial conditions, it is possible to see that coordinate $y$ can be presented as the sum:

$$y(t) = y_1(t) + y_2(t),$$

with
where \( y_1(t) = -\frac{g}{2} t^2 + \frac{m_2}{m_1 + m_2} \alpha \cos \alpha \cdot t + \frac{g}{\omega^2} \) – is bounded above by the value

\[
y_{1\text{max}} = \frac{1}{2} \left( -\frac{m_2^2 \alpha^2 t^2}{(m_1 + m_2)^2} + \frac{g}{\omega^2} \right),
\]

which monotonically decreases in the interval

\[ t \in \left( \frac{m_2}{m_1 + m_2} \alpha \cos \alpha; +\infty \right) \]

a quadratic time function, \( y_2(t) = -\frac{m_2}{m_1 + m_2} \sin(\omega t + \alpha) \) – a bounded periodic function.

This allows us to prove that the coordinate is bounded above. The character of the variation of the function \( y_1(t) \) suggests that the point will definitely fall and touch the surface.

The analytical functions obtained in this section for the coordinates are investigated further during numerical modeling of the robot’s motion. Let us consider the behaviour of the system for different values of the controlled parameter, \( \omega \). The authors present some of the results of numerical modeling of the robot’s motion. The following parameter values were used: \( m_1 = 0.05 \) kg, \( m_2 = 0.01 \) kg, \( l = 0.01 \) m, \( O_1 O_2 = 0 \) m. The value of the debalance’s angular velocity was changed to obtain different trajectories. From the inequality (16) it follows that the debalance’s velocity cannot be less than 80 rad/s for the robot to separate from the surface. Figure 3 shows the trajectories of motion of the robot at different values of the debalance’s speed of rotation.

\[ \text{Figure 3. Trajectory of the robot} \]

The following patterns are evident. First, as previously assumed, the direction of movement of the robot is opposite in sign to the direction of rotation of the debalance. Secondly, with an increase in the angular frequency of the debalance the trajectory of motion in the air is complicated and self-intersection points whose number increases with increasing frequency appear on it. Thirdly, along with the frequency increases the maximum height of the robot’s ascent from the surface and the step length, which is the distance between the points of separation and subsequent landing.

4. Conclusion

Thus, in this paper the authors consider a mobile two-mass mechanical system diagram that capable of moving on a hard rough surface or separately from it. The developed mathematical model and differential equations, allow us to describe the movement of the robot in flight phase and in the phase of location on the support surface. The analysis of the equation shows that the height and length of the jumps are monotonically increasing functions of the control frequency of rotation. After the calculation it was established that the shape of the trajectory of the centre of mass depends on the value of the controlled parameter. The dependence of the direction of motion on the direction of rotation of the debalance was revealed and the functions of height and length of jump against the frequency of rotation and the mass ratio of the system were determined.
5. Acknowledgments

The study was performed according to the grant of the Ministry of Education and Science of the Russian Federation (project No. 2017-14-576-0053-6416).

References
[1] Kesner S, Plante J-S, Dubovsky S, Boston P 2007 Advances in Climbing and Walking Robots Proc. of 10th Int. Conf. (Singapore) pp 271-280
[2] Miyazaki M, Hirai S 2008 Advances in Climbing and Walking Robots Proc. of the 11 Int. Conf. (Coimbra, Portugal) pp 373-380
[3] Kovac M, Fuchs M, Guignard A, Zufferey J-C, Floreano D 2008 Robotics and Automation Proc. of the IEEE Int. Conf. (Istanbul, Turkey) pp. 373 - 378
[4] Bolotnik N, Yatsun S, Cherepanov 2006 A Proc. Intern. Conf of mechatronics (Budapest, Hungary) pp 438-441
[5] Jatsun S, Dyshenko V, Yatsun A 2008 Advances in Climbing and Walking Robots Proc. of 11 Int. Conf. on (Coimbra, Portugal) pp893-901
[6] Jatsun S, Dyshenko V, Yatsun A, Malchikov A, 2009 Proc. of the 2 Europ. Conf. on Mechanism Science (Cassin, Italy) pp 263-270
[7] Volkova L, Yatsun S 2013 Int. J of Computer and Systems Sciences 52(4) 637-649
[8] Zhuravlev V2001 Foundations of theoretical mechanics. (Moscow: FizmatlitPress) p 320
[9] Lupehina I, Rukavitsyn A2015 Mechanical Engineering, Automation and Control Systems Proc. of Int. Conf. (Tomsk; Russia) pp. 741-749
[10] Rukavitsyn A, Lupekhina I 2011 Sc. J. Proc.of the Rus. Acad. Scien. (Samara) 13(4) 1013-1017