Quark Fragmentation Functions in a Diquark Model for $\Lambda$ Production

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Abstract

Using a simple quark-diquark model, we extract a set of unpolarized and polarized fragmentation functions for the $\Lambda$ based on the available unpolarized $\Lambda$ production data in $e^+e^-$ annihilation. It is found that there is a strong SU(3) flavor symmetry breaking in the unpolarized $\Lambda$ fragmentation functions and that the polarized $u$ and $d$ quark contributions to the polarized $\Lambda$ fragmentation are positive in medium and large $z$ region. The predictions of $\Lambda$-polarization with the obtained quark fragmentation functions are compatible with the available data on the longitudinally polarized $\Lambda$ produced in $e^+e^-$-annihilation, polarized charged lepton deep inelastic scattering (DIS), and neutrino DIS. The spin asymmetry for the $\Lambda$ production in $p\bar{p}$ collisions is also predicted for a future test in experiments at RHIC-BNL.

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1 Introduction

Investigating the detailed structure of the nucleon is one of the most active research directions of high energy and nuclear physics. In order to enrich our understanding of the flavor and spin structure of the nucleon, it is very important to apply the same mechanisms that produce the quark structure of the nucleon to other octet baryons and to find a new domain where the physics invoked to explain the structure of the nucleon can be clearly checked. However, rather little is known about the structure of other octet baryons. This is primarily due to their short life time. Also one obviously cannot produce a beam of charge-neutral hyperons such as Λ. What one can actually measure in experiments is the quark to Λ fragmentation. The Λ hyperon is of special interest in this respect since its decay is self-analyzing with respect to its spin direction due to the dominant weak decay Λ → pπ− and the particularly large asymmetry of the angular distribution of the decay proton in the Λ rest frame. So polarization measurements are relatively simple to be performed and the polarized fragmentation functions of quarks to the Λ can be measured. Actually, there has been some recent progress in measurements of polarized Λ production. The longitudinal Λ polarization in e+e− annihilation at the Z-pole was observed by several collaborations at CERN [1-3]. Recently, the HERMES Collaboration at DESY reported a result for the longitudinal spin transfer to the Λ in the polarized positron DIS process [4]. Also the E665 Collaboration at FNAL measured the Λ and Λ̄ spin transfers from muon DIS [5], and they observed a very different behaviour for Λ’s and Λ̄’s. Very recently, the measurement of Λ polarization in charged current interactions has been done in NOMAD [6]. The high statistics investigation of polarized Λ production is one of the main future goals of the HERMES Collaboration which will improve their detector for this purpose by adding so called Lambda-wheels. In addition, we also expect to get some knowledge of hadronization mechanism from the experiments at the BNL Relativistic Heavy Ion Collider (RHIC) where a polarized pp collider with high luminosity and with a center of mass (c.m.) energy √s = 500 GeV is now running [7].

Much work has already been done to relate the flavor and spin structure of the Λ
to various fragmentation processes [8-25]. Explicit calculations have been performed for the quark fragmentation functions in a quark-diquark model [11]. The quark distributions inside the Λ have also been studied in the MIT bag model and novel features in cases similar to the nucleon one have been suggested [12]. One of the most interesting observations is related to the polarization of quarks inside the Λ. In the naive quark model, the Λ spin is exclusively provided by the strange (s) quark, and the u and d quarks are unpolarized. Based on novel results concerning the proton spin structure from DIS experiments and SU(3) symmetry in the baryon octet, it was found that the u and d quarks of the Λ should be negatively polarized [9]. However, based on the light cone SU(6) quark diquark spectator model and the perturbative QCD (pQCD) counting rules analysis, it was found that the u and d quarks should be positively polarized at large x, even though their net spin contributions to the Λ might be zero or negative [13]. In order to convert the spin structure of the Λ into predictions for future experiments, Florian, Stratmann and Vogelsang [14] made a QCD analysis of the polarized Λ fragmentation function within three different scenarios. Scenario 1 corresponds to the SU(6) symmetric non-relativistic quark model, according to which the u and d quark contributions to the spin of the Λ are zero; Scenario 2, based on a SU(3) flavor symmetry analysis and on the first moment of g_1, predicts that the u and d quarks of the Λ are negatively polarized. Scenario 3 is built on the assumption that all light quarks are positively polarized in the Λ and contribute equally to the Λ polarization. It is very interesting that the best agreement with data was obtained within scenario 3, i.e. for strong flavour symmetry violation. In the above analyses, the unpolarized Λ fragmentation functions were usually parametrized [14] with the assumption of SU(3) flavor symmetry and the polarized Λ fragmentation functions were proposed [14, 16, 20] based on simple ansatz such as \( \Delta D^\Lambda_q(z) = C_q(z)D^\Lambda_q(z) \) with assumed coefficients \( C_q(z) \), or Monte Carlo event generators without a clear physics motivation. In fact there is a real need for more realistic predictions for the spin structure of the Λ for future experiments. The unpolarized Λ fragmentation functions have been relatively well determined by means of the unpolarized Λ production in \( e^+e^- \) annihilation. The main purpose of this work is to extract the Λ fragmentation functions from the available experimental data with
a clear physics motivation. We find that a simple quark-diquark model can provide a relationship between the polarized and unpolarized fragmentation functions with a clear physics picture. In addition, the model can be used to consider the SU(3) symmetry breaking in the fragmentation functions. We plan to propose a set of quark fragmentation functions for the Λ based on a quark-diquark model. The unpolarized Λ fragmentation functions are optimized by a fit to the unpolarized cross section of the produced Λ in $e^+e^-$ annihilation. Then we relate the polarized Λ fragmentation functions to the unpolarized ones at the initial scale within the framework of the diquark model. Finally, we check the obtained fragmentation functions by means of the available measurement results on the Λ polarization.

The paper is organized as follows. In Sec. 2, we briefly describe the quark to Λ fragmentation functions based on a simple quark-diquark model. In Sec. 3, a fit to the unpolarized cross section of the produced Λ in $e^+e^-$ annihilation is done in order to optimize the unpolarized fragmentation functions for the Λ. In Sec. 4, we calculate the Λ polarization in $e^+e^-$-annihilation at the Z pole, the longitudinal spin transfer to the Λ in polarized charged lepton DIS and the Λ ($\bar{\Lambda}$) polarizations in neutrino (antineutrino) DIS. We also predict the spin asymmetry for the Λ production in $p\bar{p}$ collisions for a future test in experiments. Finally, we present a discussion and summary of our new knowledge together with our conclusions in Sec. 5.

2 Input quark fragmentation functions in a quark-diquark model

HERMES plans for extensive Λ production experiments after the HERA shut-down. It is very timely as more reliable unpolarized and polarized Λ fragmentation functions with a clear physical motivation are necessary to serve as basis for the analysis of the new HERMES data. We find that the diquark model given in Ref. [11] has a clear physics picture. In this model, the fragmentation of a quark into the Λ is modeled with a quark-diquark-Λ vertex. The SU(6) spin-isospin structure of the Λ can be re-expressed in terms of quark and diquark states. The SU(3) symmetry breaking can
be reflected by considering the mass difference between a scalar diquark and vector
diquark states. In addition, we will show that the polarized quark to Λ fragmentation
functions can be related to the unpolarized fragmentation functions by means of some
definitive ratios given by the model.

Within the framework of the diquark model [11], the unpolarized valence quark
to Λ fragmentation functions can be expressed as

\begin{equation}
D_{u_s}(z) = D_{d_s}(z) = \frac{1}{12} a_s(\Lambda)(z) + \frac{1}{4} a_V(\Lambda)(z),
\end{equation}

\begin{equation}
D_{s_s}(z) = \frac{1}{3} a_s(\Lambda)(z),
\end{equation}

where \(a_D(\Lambda)(z)\) \((D = S \text{ or } V)\) is the probability of finding a quark \(q\) splitting into \(\Lambda\)
with longitudinal momentum fraction \(z\) and emitting a scalar \((S)\) or axial vector \((V)\)
antidiquark. The form factors for scalar and axial vector diquark are customarily
taken as the same form

\begin{equation}
\phi(k^2) = N \frac{k^2 - m_q^2}{(k^2 - \Lambda_0^2)^2}
\end{equation}

with a normalization constant \(N\) and a mass parameter \(\Lambda_0\). \(\Lambda_0 = 500\) MeV will
be adopted in our numerical calculations. In (3), \(m_q\) and \(k\) are the mass and the
momentum of the fragmenting quark \(q\), respectively. According to Ref. [11], \(a_D^{(q)}(z)\)
can be expressed in the quark-diquark model as

\begin{equation}
a_D^{(q)}(z) = \frac{N^2 z^2 (1-z)^3 [2(M_\Lambda + m_q z)^2 + R^2(z)]}{64 \pi^2 R^6(z)}
\end{equation}

with

\begin{equation}
R(z) = \sqrt{zm_D^2 - z(1-z)\Lambda_0^2 + (1-z)M_\Lambda^2},
\end{equation}

where \(M_\Lambda\) and \(m_D\) \((D = S \text{ or } V)\) are the mass of the \(\Lambda\) and a diquark, respectively.
In consideration of the mass difference \(M_\Lambda - M_p = 176\) MeV, we choose the diquark
mass \(m_S = 900\) MeV and \(m_V = 1100\) MeV for non-strange diquark states, \(m_S =\)
(900 + 176) MeV and \( m_V = (1100 + 176) \) MeV for diquark states \((qs)\) with \( q = u, d\). The quark masses are taken as \( m_u = m_d = 350 \) MeV and \( m_s = (350 + 176) \) MeV.

Similarly, the polarized quark to \( \Lambda \) fragmentation functions can be written as

\[
\Delta D^\Lambda_{uu}(z) = \Delta D^\Lambda_{dd}(z) = \frac{1}{12} \tilde{a}^{(u)}_S(z) - \frac{1}{12} \tilde{a}^{(u)}_V(z),
\]

and

\[
\Delta D^\Lambda_{ss}(z) = \frac{1}{3} \tilde{a}^{(s)}_S(z),
\]

with

\[
\tilde{a}^{(q)}_D(z) = \frac{N^2 z^2 (1 - z)^3}{64 \pi^2} \left[ \frac{2(M_\Lambda + m_q z)^2 - R^2(z)}{R^6(z)} \right],
\]

for \( D = S \) or \( V \). What we are interested is not the magnitude of the fragmentation functions but the flavor and spin structure of them which are given by the diquark model. In order to extract the flavor and spin structure information, we introduce the flavor structure ratios

\[
F^{(u/s)}_S(z) = \frac{a^{(u)}_S(z)}{a^{(s)}_S(z)},
\]

\[
F^{(u/s)}_M(z) = \frac{a^{(u)}_V(z)}{a^{(s)}_S(z)},
\]

and the spin structure ratio

\[
W^{(q)}_D(z) = \frac{\tilde{a}^{(q)}_D(z)}{\tilde{a}^{(q)}_D(z)},
\]

with \( D = S \) or \( V \). Then we can use the fragmentation function \( D^\Lambda_{uu}(z) \) to express all other unpolarized and polarized fragmentation functions as follows

\[
D^\Lambda_{uu}(z) = \left[ \frac{3}{4} F^{(u/s)}_M(z) + \frac{1}{4} F^{(u/s)}_S(z) \right] D^\Lambda_{ss}(z),
\]

\[
\Delta D^\Lambda_{uu}(z) = \frac{1}{4} \left[ W^{(u)}_S (z) F^{(u/s)}_S(z) - W^{(u)}_V (z) F^{(u/s)}_M(z) \right] D^\Lambda_{ss}(z),
\]

and
\[ \Delta D_{s_v}^A(z) = W_s^{(s)}(z)D_{s_v}^A(z). \] (14)

The quark-diquark description of the fragmentation function should be reasonable in large \( z \) region where the valence quark contributions dominate. In small \( z \) region, the sea contribution is difficult to be included in the framework of the diquark model itself. In order to optimize the shape of fragmentation functions, we adopt simple functional forms

\[ D_{s_v}^A(z) = N_s z^{\alpha_s}(1 - z)^{\beta_s}. \] (15)

and

\[ D_{q_s}^A(z) = D_{q}^A(z) = N z^{\alpha_s}(1 - z)^{\beta_s}. \] (16)

to parametrize fragmentation functions of the valence quark \( D_{s_v}^A \), sea quark \( D_{q_s}^A(z) \) and antiquark \( D_q^A(z) \) for \( q = u, d, s \). We assume that \( D_{g}^A \) and \( \Delta D_{g_s}^A \), \( \Delta D_{q_s}^A \), and \( \Delta D_q^A \) at the initial scale are zero and they only appear due to the QCD evolution. Hence, the input unpolarized and polarized \( q \to \Lambda \) fragmentation functions can be written as

\[ D_q^A(z) = D_{q_v}^A(z) + D_{q_s}^A(z), \] (17)

\[ \Delta D_q^A(z) = \Delta D_{q_v}^A(z). \] (18)

3 A fit to unpolarized cross section of produced \( \Lambda \) in \( e^+ e^- \) annihilation

For a fit to the experimental data, the fragmentation functions have to be evolved to the scale of the experiments. We used the evolution package of Ref. [26] suitable modified for the evolution of fragmentation functions in leading order, taking the input scale \( Q_0^2 = M_{\Lambda}^2 \) and the QCD scale parameter \( \Lambda_{QCD} = 0.3 \) GeV.
In order to express the inclusive cross section and polarization for the Λ production in $e^+e^-$ annihilation, we introduce the following quantities

\[
\hat{A}_q = 2\chi_2(v_e^2 + a_e^2)v_qa_q - 2e_q\chi_1a_qv_e, \tag{19}
\]

\[
\hat{C}_q = e_q^2 - 2\chi_1v_ev_q + \chi_2(a_e^2 + v_e^2)(a_q^2 + v_q^2), \tag{20}
\]

with

\[
\chi_1 = \frac{1}{16\sin^2\theta_W\cos^2\theta_W \left( s - M_Z^2 \right)} \frac{s(s - M_Z^2)}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2}, \tag{21}
\]

\[
\chi_2 = \frac{1}{256\sin^4\theta_W\cos^4\theta_W \left( s - M_Z^2 \right)^2 + M_Z^2\Gamma_Z^2} s^2, \tag{22}
\]

\[
a_e = -1 \tag{23}
\]

\[
v_e = -1 + 4\sin^2\theta_W \tag{24}
\]

\[
a_q = 2T_{3q}, \tag{25}
\]

\[
v_q = 2T_{3q} - 4e_q\sin^2\theta_W, \tag{26}
\]

where $T_{3q} = 1/2$ for $u$, while $T_{3q} = -1/2$ for $d$, $s$ quarks, $N_c = 3$ is the color number, $e_q$ is the charge of the quark in units of the proton charge, $s$ is the total c.m. energy squared, $\theta$ is the angle between the outgoing quark and the incoming electron, $\theta_W$ is the Weinberg angle, and $M_Z$ and $\Gamma_Z$ are the mass and width of $Z^0$.

In the quark-parton model, the differential cross section for the semi-inclusive $\Lambda$ production process $e^+e^- \rightarrow \Lambda + X$ can be expressed to leading order

\[
\frac{1}{\sigma_{tot}} \frac{d\sigma}{dx_E} = \sum_q \frac{\hat{C}_q}{\sum_q \hat{C}_q} \left[ D_q^\Lambda(x_E, Q^2) + D_q^\Lambda(x_E, Q^2) \right] \tag{27}
\]

where $x_E = 2E_\Lambda/\sqrt{s}$ with $E_\Lambda$ being the energy of the produced $\Lambda$ in the $e^+e^-$ c.m. frame, and $\sigma_{tot}$ is the total cross section for the process.

We perform the leading order fit since the analysis in Ref. [14] shows that the leading order fit can arrive at the same fitting quality as the next-to-leading order fit. By fitting the inclusive unpolarized $\Lambda$ production data in $e^+e^-$ annihilation, the
optimal parameters in Eqs. (13)-(16) are obtained as $N_s = 2.5$, $\alpha_s = 0.7$, $\beta_s = 3.0$, $\overline{N} = 0.4$, $\overline{\alpha} = -0.51$, and $\overline{\beta} = 7.4$. In Fig. 1, we show our fit results compared with the experimental data [26-31]. In order to show the flavor and spin structure of the $\Lambda$ fragmentation functions, the ratios of $D^{\Lambda}_u(z)/D^{\Lambda}_s(z)$ and $\Delta D^{\Lambda}_s(z)/D^{\Lambda}_s(z)$ ($\Delta D^{\Lambda}_u(z)/D^{\Lambda}_u(z)$) at $Q^2 = 4$ GeV$^2$ are presented in Fig. 2 and 3, respectively. From Fig. 2, we can see that there is a strong flavor symmetry violation in the unpolarized $\Lambda$ fragmentation functions, especially at large $z$. 

Figure 1: The comparison of our results for the $x_E$ dependence of the inclusive $\Lambda$ production cross section $(1/\sigma_{tot}) d\sigma/dx_E$ in $e^+e^-$ annihilation and the experimental data [26-31].
4 Prediction of spin observables in various $\Lambda$ production processes

There has been available data on polarized $\Lambda$ fragmentation functions in $e^+e^-$ annihilation at the Z-pole and also in lepton DIS. We can check our fragmentation based on these experimental data. First, the polarized $s$ quark fragmentation function can be checked in $e^+e^-$ annihilation at the Z-pole since the $\Lambda$ polarization of this process is dominated by the $s$ quark fragmentation function. Second, the spin transfer in $\Lambda$ electroproduction is dominated by the spin transfer from the $u$ quark to the $\Lambda$ due to the charge factor for the $u$ quark. Moreover, due to isospin symmetry the $u$
and $d$ quark spin transfers to the $\Lambda$ are expected to be equal. We can check the $u$ and $d$ quark fragmentation functions by means of the $\Lambda$ production in the polarized charged lepton DIS process. Third, the very recent NOMAD data on the $\Lambda$ polarization in the neutrino DIS process, which has small errors, can help us to draw a clear distinction between different predictions. Finally, the spin transfer in the reaction $p\bar{p} \rightarrow \bar{\Lambda}X$ at RHIC BNL should be also a good tool to discriminate between various sets of polarized fragmentation functions compatible with the LEP data. In order to check the obtained quark fragmentation functions, we apply them to predict the spin observables in the above various $\Lambda$ production processes.

4.1 $\Lambda$ polarization in $e^+e^-$-annihilation

One interesting feature of quark-antiquark ($q\bar{q}$) production in $e^+e^-$-annihilation near the $Z$-pole is that the produced quarks (antiquarks) are polarized due to the interference between the vector and axial vector couplings in the standard model of electroweak interactions, even though the initial $e^+$ and $e^-$ beams are unpolarized. There have been measurements of the $\Lambda$-polarization near the $Z$-pole [1-3]. Theoretically, the $\Lambda$-polarization can be expressed as

$$P_\Lambda = -\frac{\sum_q \hat{A}_q [\Delta D^\Lambda_q(z) - \Delta D^\Lambda_{\bar{q}}(z)]}{\sum_q \hat{C}_q [D^\Lambda_q(z) + D^\Lambda_{\bar{q}}(z)]}.$$  \hspace{1cm} (28)

where $\hat{A}_q$ and $\hat{C}_q$ ($q = u, d$ and $s$) are given in (19) and (20), respectively. Our theoretical prediction for the $\Lambda$ polarization at the $Z$-pole is shown in Fig. 4 together with the experimental data. From Fig. 4, we can see that the $\Lambda$ polarization, which is mainly shaped by the polarized $s$ quark to $\Lambda$ fragmentation (see Fig. 3), is consistent with the experimental data.
Figure 4: The comparison of the experimental data [1-3] for the longitudinal Λ-polarization $P_\Lambda$ in $e^+e^-$-annihilation at the $Z$-pole with our theoretical prediction.

4.2 Spin transfer to $\Lambda$ in polarized charged lepton DIS

For a longitudinally polarized charged lepton beam and an unpolarized target, the Λ polarization along its own momentum axis is given in the quark parton model by [15]

$$P_\Lambda(x, y, z) = P_B D(y) A_\Lambda(x, z),$$

(29)

where $P_B$ is the polarization of the charged lepton beam, which is of the order of 0.7 or so [4, 5]. $D(y)$, whose explicit expression is

$$D(y) = \frac{1 - (1 - y)^2}{1 + (1 - y)^2},$$

(30)

is commonly referred to as the longitudinal depolarization factor of the virtual photon with respect to the parent lepton, and

$$A_\Lambda(x, z) = \sum_q e_q^2 [q^N(x, Q^2) \Delta D_q^\Lambda(z, Q^2) + (q \rightarrow \bar{q})]$$

$$\sum_q e_q^2 [q^N(x, Q^2) D_q^\Lambda(z, Q^2) + (q \rightarrow \bar{q})],$$

(31)

is the longitudinal spin transfer to the Λ. Here $y = \nu/E$ is the fraction of the incident lepton’s energy that is transferred to the hadronic system by the virtual photon. In Eq. (31), $q^N(x, Q^2)$, the quark distribution of the proton, is adopted as the CTEQ5 set 1 parametrization form [33] in our numerical calculations. As shown in Fig. 4, our prediction is compatible with the available experimental data in medium $z$ region,
Figure 5: The z-dependence of the Λ spin transfer in electron or positron (muon) DIS. Note that for HERMES data the Λ polarization is measured along the virtual-photon momentum, whereas for E665 it is measured along the virtual-photon spin. The averaged value of the Bjorken variable is chosen as $x = 0.1$ (corresponding to the HERMES averaged value) and the calculated result is not sensitive to a different choice of $x$ in the small $x$ region (for example, $x = 0.005$ corresponding to the E665 averaged value). $Q^2 = 4 \text{ GeV}^2$ is used and the $Q^2$ dependence of the result is very weak.

which suggests that the $u$ and $d$ quark to the Λ fragmentation functions are likely positive polarized in medium and large $z$ region. However, the experimental data in small $z$ region, whose precision is still poor, can not be explained at the moment.

4.3 Λ polarization in neutrino/antineutrino DIS

The scattering of a neutrino beam on a hadronic target provides a source of polarized quarks with specific flavor structure, and this particular property makes the neutrino (antineutrino) process an ideal laboratory to study the flavor-dependence of quark to hadron fragmentation functions, especially in the polarized case. We find that the Λ polarization in the neutrino (anti-neutrino) DIS process can also be used to check the $u$ and $d$ quark contributions to the polarized Λ fragmentation.

The longitudinal polarizations of the Λ in its momentum direction, for the Λ in the current fragmentation region can be expressed as,

$$P_\nu^\Lambda(x, y, z) = -\frac{[d(x) + \bar{\omega}s(x)]\Delta D_0^\Lambda(z) - (1 - y)^2\pi(x)[\Delta D_2^\Lambda(z) + \bar{\omega}\Delta D_2^\Lambda(z)]}{[d(x) + \bar{\omega}s(x)]D_u^\Lambda(z) + (1 - y)^2\pi(x)[D_d^\Lambda(z) + \bar{\omega}D_d^\Lambda(z)]}, \quad (32)$$
Figure 6: The predictions of $z$-dependence for the $\Lambda$ and $\bar{\Lambda}$ polarizations in the neutrino (antineutrino) DIS process. We adopt the CTEQ5 set 1 quark distributions ~[33] for the target proton at $Q^2 = 4$ GeV$^2$ with the Bjorken variable $x$ integrated over $0.02 \rightarrow 0.4$ and $y$ integrated over $0 \rightarrow 1$.

\[
P^{\Lambda}_{\tau\pi}(x, y, z) = \frac{(1 - y)^2u(x)[\Delta D^\Lambda_d(z) + \varpi \Delta D^\Lambda_s(z)] - [\overline{d}(x) + \varpi \overline{s}(x)]\Delta D^\Lambda_{\tau\pi}(z)}{(1 - y)^2u(x)[\overline{D}^\Lambda_d(z) + \varpi \overline{D}^\Lambda_s(z)] + [\overline{d}(x) + \varpi \overline{s}(x)]\overline{D}^\Lambda_{\tau\pi}(z)},\]  

(33)

where the terms with the factor $\varpi = \sin^2 \theta_c / \cos^2 \theta_c$ ($\theta_c$ is the Cabibbo angle) represent Cabibbo suppressed contributions. There are similar formulae as (32)-(33) for the $\bar{\Lambda}$ whose quark fragmentation functions can be obtained according to the matter and antimatter symmetry, i.e. $D^\Lambda_{q\pi}(z) = D^\bar{\Lambda}_{q\pi}(z)$ and similarly for $\Delta D^\Lambda_{q\pi}(z)$.

The NOMAD data ~[6] on the $\Lambda$ polarization in the neutrino DIS process, which has much smaller errors than the data on the longitudinal spin transfer to the $\Lambda$ in polarized charged lepton DIS, allows to have a further check on the $\Lambda$ fragmentation functions. In Fig. 6, we present our predictions of $z$-dependence for the $\Lambda$ and $\bar{\Lambda}$ polarizations in the neutrino (antineutrino) DIS process and find that our prediction of the $\Lambda$ polarization in neutrino DIS is consistent with the very recent NOMAD data ~[6].
4.4 Spin asymmetry for $\Lambda$ production in $pp\bar{p}$ collisions

In order to enrich the spin knowledge of the nucleon and investigate the hadronization mechanism, many spin physics programs will be undertaken at RHIC-BNL [3, 24]. Theoretically, it has been recently noticed that the $\Lambda$ polarization in polarized proton-proton collisions is also very sensitive to the property of $\Lambda$ fragmentation [17, 20, 21].

In leading order perturbative QCD, the rapidity differential polarized cross section can be schematically written in a factorized form as

$$\frac{d \Delta \sigma_{pp\bar{p} \rightarrow \Lambda X}}{dy} = \int_{p_T^{\text{min}}} dp_T \sum_{f_a f_b \rightarrow c X'} dx_a dx_b dz_c f_a^A(x_a, \mu^2) \Delta f_b^B(x_b, \mu^2) \Delta D_c^A(z_c, \mu^2) \frac{d\Delta \hat{\sigma}}{dy}$$

(34)

where, $f_a^A(x_a, \mu^2)$ and $\Delta f_b^B(x_b, \mu^2)$ are unpolarized and polarized distribution functions of partons $a$ and $b$ in protons $A$ and $B$, respectively, at the scale $\mu^2 = p_T^2$. $\Delta D_c^A(z_c, \mu^2)$ is the polarized fragmentation function of parton $c$ into $\Lambda$ with the momentum fraction $z_c$ of parton $c$. $y$ and $p_T$ are the rapidity and transverse momentum of the produced $\Lambda$. $\frac{d\Delta \hat{\sigma}}{dy}$ is the rapidity differential polarization cross section for the sub-process $a + b \rightarrow c + d$. The directly observable quantity is the spin asymmetry defined by

$$A_{pp}^\Lambda = \frac{\frac{d \Delta \sigma_{pp\bar{p} \rightarrow \Lambda X}}{dy}}{\frac{d\sigma_{pp\bar{p} \rightarrow \Lambda X}}{dy}}$$

(35)

where the unpolarized cross section $d\sigma_{pp\bar{p} \rightarrow \Lambda X}/dy$ is obtained by an expression similar to the one in (34), with all $\Delta$’s removed.

The result of the spin asymmetry in Fig. 7 is obtained by adopting the LO set of unpolarized parton distributions of Ref. [35] and polarized parton distributions of LO GRSV ”standard” scenario [31]. The total c.m. energy $\sqrt{s} = 500$ GeV and the lower cutoff of the transverse momentum $p_T^{\text{min}} = 13$ GeV are taken. By comparing the result in Fig. 7 and the spin structure of the fragmentation function shown in Fig. 3, we can find that the spin asymmetry for the $\Lambda$ production is sensitive to $\Delta D_u^A/\Delta D_u^A$. Therefore, the rapidity dependence of the spin asymmetry for the $\Lambda$ production in $pp\bar{p}$ collisions can provide a good tool to discriminate between various sets of polarized $u$ and $d$ quark fragmentation functions compatible with the LEP data.
Figure 7: The predictions of the spin asymmetry as a function of rapidity for the Λ production in $p\bar{p}$ collisions.

5 Discussion and summary

The available experimental data on the Λ polarization supports our prediction that the $u$ and $d$ quark contributions to the polarized Λ fragmentation are positive in medium and large $z$ region. However, this can not be regarded as in contradiction with the result in Ref. [9]. First, in Ref. [9] the $u$ and $d$ quark contributions to the spin content of the Λ were discussed. And here, we talk about the polarized $u$ and $d$ quark fragmentation to the Λ. There has not been yet a definitive relationship between the fragmentation function and the corresponding quark distribution function. Second, the result in Ref. [9] was obtained with the assumption of SU(3) symmetry in the baryon octet. Recently, it has been noticed that the effect of the SU(3) symmetry breaking in hyperon semileptonic decay (HSD) should be significant [37, 38]. The effect has been estimated by a chiral quark soliton model [37] and the large $N_c$ QCD [38]. The consistent results were obtained separately by different approaches. The effect of the SU(3) symmetry breaking in HSD on the spin content of the Λ has been considered in the chiral quark soliton model [39]. It was found that the integrated polarized quark densities for the Λ hyperon should be $\Delta U = \Delta D = -0.03 \pm 0.14$ and $\Delta S = 0.74 \pm 0.17$ in the chiral limit case. When the strange quark mass correction is added, $\Delta U = \Delta D = -0.02 \pm 0.17$ and $\Delta S = 1.21 \pm 0.54$. Therefore, there are
various possibilities for the polarization of the $u$ and $d$ quarks in the $\Lambda$, i.e. the $u$ and $d$ quark contributions to the spin of the $\Lambda$ might be zero, negative or positive. With a statistical model [10], we find that the available experimental data on the $\Lambda$ polarization also suggests that the $u$ and $d$ quarks are positively polarized. Therefore, the situation in which the $u$ and $d$ quarks are positively polarized in the $\Lambda$ seems to be model independent.

We would like to mention that our present knowledge on the $\Lambda$ fragmentation functions is still poor and there are many unknowns to be explored before we can arrive at some definitive conclusion on the quark spin structure of the $\Lambda$. Despite the experimental uncertainties, it seems that the experimental data on the longitudinal spin transfer to the $\Lambda$ in the polarized charged lepton DIS process shows a strong dependence on $z$, especially in low $z$ region. At the moment, it seems to be difficult to understand such a rather strong $z$ dependence in low $z$ region with the available models [14,16,20,22], although the models can provide predictions which are compatible with the data in medium $z$ range. Further studies in order to improve our predictions in small $z$ region are in progress. On the other hand, experimentally, the high statistics measurements on the $\Lambda$ polarization have been regarded as a strongly emphasized project in the COMPASS experiment [40]. Many efforts, both theoretically and experimentally, are being made in order to reduce the uncertainties in the spin structure of the $\Lambda$ since the subject is crucial important for enriching the knowledge of hadron structure and hadronization mechanism.

In summary, we proposed a set of $q \rightarrow \Lambda$ fragmentation functions whose flavor and spin structure are provided by the quark-diquark model. Using the diquark model, we related the polarized fragmentation functions to the unpolarized ones at the initial scale. We optimized the fragmentation functions by fitting the unpolarized cross section for the $\Lambda$ produced in $e^+e^-$ annihilation. It is found that the SU(3) symmetry breaking in the unpolarized $\Lambda$ fragmentation functions is significant, especially in large $z$ region. As a check of the obtained fragmentation functions, our predictions for $\Lambda$ polarization in $e^+e^-$-annihilation and the neutrino DIS are consistent with the experimental data. The prediction on the spin transfer to the $\Lambda$ in polarized charged lepton DIS is compatible with the data in medium $z$ region. For a future test in
experiments at BNL RHIC, we predicted the spin asymmetry for the $\Lambda$ production in $pp\bar{p}$ collisions. It is found that our prediction for the spin asymmetry is very similar to that of scenario 3 in Ref. [14].

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