Research on the stiffness of Grillage Method in Virtual Cross Beam based on Fuzzy recognition

Lixiang Sun
School of Civil Engineering, Lanzhou Jiaotong Univ., 88 West Anning Rd., Lanzhou, Gansu Province 730070, China
*Corresponding author’s e-mail: 121211235@qq.com

Abstract. The stiffness of virtual cross beam is an important parameter which affects the calculation results when using beam grillage method to analyze the stress of bridge structure. In order to obtain a reasonable value of the stiffness about the virtual cross beam, it is necessary to make comparative analysis. On the basis of the comprehensive comparative analysis of the stress and deflection of the control section in hollow slab bridge and simply supported beam bridge with twin-box single cross section, the value range of the virtual cross beam stiffness is obtained. Based on the theory of fuzzy mathematics, a fuzzy recognition model for the stiffness of the virtual beam was proposed. By calculating the relative membership degree vector of the stiffness in each virtual beam, the length and width of the cross section of the virtual beam is obtained to be 0.7 times height of the beam. In order to further verify the reasonable of the value, the stress and deflection values of the control section have been compared. These values are taken from the single-box and double-chamber plexiglass box girder model and highway single-box three-chamber box beam. The compared results show that the solution of the grillage model with the stiffness of the virtual beam in this paper is in good agreement with shell solutions and solid solutions.

1. Introduction
The grillage method presented by Lightfoot and Sawko [1,2] is used for analysis of the structural discrete into several cross beams, and by analyzing the equivalent grillage model to obtain the mechanical behavior of the original structure. Grillage method can realize rapid modeling by means of computer. As long as the parameter setting is reasonable, the analysis results can meet the requirements of engineering design [3,4]. It can be used for the analysis of beam-bridge, slab-bridge, and beam-slab structures, especially suitable for the analysis of curved bridges and skew bridges. According to the preliminary analysis of multi-plate hollow slab, small box girder and special-shaped slab bridge, grillage method is one of the most widely used analytical methods [5-10]. The grillage method is used to set up a numerical model, and it is necessary to set a certain distance of a virtual cross beam with stiffness and no weight at the position where the main girder does not have a transverse horizontal partition, and the stiffness and the spacing of the virtual cross beam are the most important factors for determining the calculation accuracy of the grillage. Hambly [3] just gives the value requirements for the distance between virtual cross beams: multi-plate hollow slab, T-beam and small box girder can obtain good precision by calculating span from 1/8 to 1/4; the spacing of box girder structure shall be 1/4 of the distance between longitudinal bending and reverse bending points. Therefore, for simply-supported beams, no matter what the superstructure is, the spacing of virtual cross beams is about 1/8 of the calculated span. However, the research on the value of the stiffness of
the virtual cross beam is less, and the existing research has not given the specific value [7, 11], resulting in different analysis results for the same structure. Therefore, it is necessary to study the specific value of the stiffness of the virtual cross beam.

The simply-supported beam has advantages as simple structure, definite mechanics, and convenient maintenance. It is widely used in the railway and highway bridge in China. In order to simplify the mechanical analysis, most of the simply-supported beams need to be analyzed by the grillage method. In this study, the numerical models of simply-supported box girder and multi-plate hollow slab bridge are established by using the grillage method. The influence of stiffness of the virtual cross beam on the stress and deformation of the control section in the structure is investigated. The value range of the virtual cross beam stiffness is obtained by comparing with the numerical solution of the solid element. The specific value of the virtual cross beam stiffness is obtained by using the fuzzy identification model. The virtual cross beam stiffness value obtained by the grillage method is validated by the measured value (or shell and plate value) and the value of solid element calculated results.

2. Formatting the title, authors and affiliations

2.1. Formatting the title

Taking the 32m double chamber box girder of railway in reference [11] as the research object, the elastic modulus (E) is $3.45 \times 10^4$ MPa, and the Poisson ratio is 0.2. For the convenience, the virtual cross-section of grillage method is specified to be square, and 11 groups of sections are selected, in which the lower limit of section is the minimum plate thickness and the upper limit is infinity (100000cm in this paper), and the remaining cross-section dimensions are based on beam height, just from 0.1 times beam height to 1.0 times beam height (same below). The analysis shows that in the beam-grillage model, the effect of the virtual cross beam stiffness on the stress and deformation of the span in the simply-supported beam and the cross-section of L/4 is completely consistent, and the deformation law of the longitudinal members of the box beam is the same as that of each longitudinal member of the box beam. Therefore, this paper only shows the influence on the stress and deformation of the mid-span section (same as below). The parameter $\lambda$ represents the ratio of the calculated results of the beam-grillage corresponding to the stiffness of each virtual cross beam to the theoretical value of the entity element in figure 1. The end cross beam is defined according to the actual size, the minimum size of the virtual cross beam is 20 cm, and the longitudinal spacing is 1/8 of the calculated span. There are two types of loads are applied, the one is concentrated load ($P=400kN$, equivalent to single-row concentrated force for double-track railway) and the other is full-span uniformly distributed load ($q=172.24kN/m$, equivalent to calculated train load for this calculated span). The solid element numerical model of the simply supported beam is established by using ANSYS solid45 element as a comparison. The influence curve of the virtual cross beam stiffness on the stress and deformation of the mid-span section of the structure is shown in Figure 1.

![Figure 1. Influence of virtual beam stiffness on mid span stress and deformation of double chamber box girder.](image-url)
It can be seen that the inflection point of the curve on the stress and deformation is at 0.4h, and then tends to be gentle, where h is the height of the beam. At this time, the calculated value of the beam-grillage is close to the value of the solid element. The length and width dimension of the virtual cross beam is about 0.8h×0.8h, and the beam-grillage value is closest to the value of the solid element.

2.2. Formatting author affiliations
Taking a 21m hinged joint hollow slab bridge of a highway as the research object, the concrete grade is C50, and 10 longitudinal members are connected with each other by hinged joints. The size of the middle section of the side slab and the middle slab is shown in Figure 2.

The grillage method is used to establish the numerical model, and the hinge joint is simulated by releasing beam end constraint. The vehicle load grade is first-class highway. According to the two-lane load, the deformation and stress of L/4-span and mid-span section are analyzed. Because of the large number of hollow slabs, the side plates and mid-plates are chosen as the research objects, and the influence curves of virtual cross beam stiffness on the stress and deformation of hollow slab edges and mid-span sections are shown as shown in figure 3.

As can be seen from figure 3, the inflection point of the side plate curve appears at 0.5h, while the inflection point of the influence curve on the stress and deformation of the medium plate appears at 0.6h and 0.5h. There is no significant difference up to 1.0h between the beam stress and deflection of the plate and the value of the solid element. If the stiffness of virtual cross beam is infinitely large, the result of calculation will be distorted.
3. Formatting the text

3.1. The general idea of fuzzy model recognition

First, the evaluation index and the scheme to be selected are put forward, and their membership functions are calculated. Then the non-normalized weight vectors of each index are determined according to the scale of judgment matrix. Finally, the optimal ranking is obtained by calculating the relative optimal degree of each scheme.

There are two types of indexes, first, the standard eigenvalues of indexes from grade 1 to grade c decrease and second the standard eigenvalues of indexes of grade c increase from grade 1 to grade c.

The relative membership of class (1) index to A is

\[ r_{ij} = \begin{cases} 
0, & x_{ij} \leq y_{ic} \\
\frac{x_{ij} - y_{ij}}{y_{il} - y_{ij}}, & y_{il} > x_{ij} > y_{ic} \\
1, & x_{ij} \geq y_{il} 
\end{cases} \]

(1)

The relative membership degree of type (2) index to A is

\[ r_{ij} = \begin{cases} 
0, & x_{ij} \geq y_{ic} \\
\frac{x_{ij} - y_{ij}}{y_{il} - y_{ij}}, & y_{il} < x_{ij} < y_{ic} \\
1, & x_{ij} \leq y_{il} 
\end{cases} \]

(2)

3.2. Establishment of Fuzzy recognition Model

To set up a model of fuzzy pattern recognition is to transform the eigenvalue matrix of the index.

\[ X = \begin{pmatrix} 
  x_{i1} & x_{i2} & \cdots & x_{in} \\
  x_{i1} & x_{i2} & \cdots & x_{in} \\
  \vdots & \vdots & \ddots & \vdots \\
  x_{i1} & x_{i2} & \cdots & x_{in} 
\end{pmatrix} \]

(3)

In the equation: \( x_{ij} \) is the eigenvalue of sample j index i, \( i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \).

Transform to the index relative membership matrix which meets the requirements of the design code

\[ X = \begin{pmatrix} 
  r_{i1} & r_{i2} & \cdots & r_{in} \\
  r_{i1} & r_{i2} & \cdots & r_{in} \\
  \vdots & \vdots & \ddots & \vdots \\
  r_{i1} & r_{i2} & \cdots & r_{in} 
\end{pmatrix} \]

(4)

Matrix R is the basis of fuzzy analysis and design. The simplified form of fuzzy pattern recognition model is as follows.

\[ r_{ij} = \begin{cases} 
0, & h < a_j, \text{or}, h > b_j \\
\frac{w_{ij}(r_{ij} - s_{ij})}{\left[ \sum w_{ij}(r_{ij} - s_{ij}) \right]^2}, & y_{il} < x_{ij} < y_{ic} \\
1, & d_{ij} = 0 
\end{cases} \]

(5)

Because there are only two levels, in the fuzzy pattern recognition model (5), \( a_j = 1, b_j = 2, j = 1, 2, 3, \ldots, n \). The relative membership degree model of sample j to grade 1 is as follows.
\[ U_j = \frac{1}{1 + \left( \frac{1}{\sum_{i=1}^{m} w_i (r_{ij} - 1)^p} \right)} \cdot \sum_{i=1}^{m} w_i = 1 \]

The relative membership of the sample \( j \) to the level 2 is \( 1 - u_j \). When \( p = 1 \), Equation (4) becomes:

\[ U_j = \frac{1}{1 + \left( \frac{1}{\sum_{i=1}^{m} w_i r_{ij}} \right)} \]

When \( p = 2 \), the formula (5) becomes:

\[ U_j = \frac{1}{1 + \left( \frac{1}{\sum_{i=1}^{m} w_i r_{ij}^2} \right)} \]

3.3. The general idea of fuzzy model recognition

When determining the specific value of the stiffness of the virtual cross beam, four evaluation indexes are considered. The first is to ensure the convergence of the calculated results of the stress of the control section, and the second is to ensure the convergence of the calculated results of the deformation of the control section, and the third is to ensure that the difference between the calculated results of stress and the value of entity unit (or test value) is less than 5%, the forth to ensure that the difference between the calculated result of displacement and the value of entity unit (or test value) is less than 5%.

According to the above calculation, the cross-section size of the virtual cross beam is considered in seven schemes, as is shown in Table 1.

| Project number | Length and Width Section Size of Virtual Cross Beam |
|----------------|-----------------------------------------------------|
| 1              | 0.60h×0.60h                                         |
| 2              | 0.65h×0.65h                                         |
| 3              | 0.70h×0.70h                                         |
| 4              | 0.75h×0.75h                                         |
| 5              | 0.80h×0.80h                                         |
| 6              | 0.85h×0.85h                                         |
| 7              | 0.90h×0.90h                                         |

In order to converge the results of stress calculation, the lower limit of evaluation class 1 should be 0.6h, that is, \( y_{12} = 0.6 \), and the upper limit should be 1.0 h, that is, \( y_{11} = 1.0 \). The base formula 3, the relative membership vector of class 1 should be as follows.

\( r_1 = (0.0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75) \)

In order to converge the results of deformation calculation, \( y_{12} = 0.5 \), \( y_{11} = 0.91 \), and the relative membership vector of calculation class 2 is as follows.

\( r_2 = (0.25, 0.375, 0.5, 0.625, 0.75, 0.875, 1.0) \)

In the same way, the \( y_{12} \) of the evaluation class 3 is 0.63, \( y_{11} = 2.65 \), and the relative membership degree vector of the class 3 according to the formula (5) is.

\( r_3 = (0, 0.010, 0.035, 0.059, 0.084, 0.109, 0.134) \)

The \( y_{12} \) of the evaluation class 4 is 0.55 and the \( y_{11} = 0.92 \). The relative membership vector of the calculated class 4 is as follows.

\( r_4 = (0.135, 0.270, 0.405, 0.541, 0.676, 0.811, 0.945) \)
The relative membership matrix of the available indexes is as follows.

\[
R = \begin{bmatrix}
0.000 & 0.125 & 0.250 & 0.375 & 0.500 & 0.625 & 0.750 \\
0.250 & 0.375 & 0.500 & 0.625 & 0.750 & 0.875 & 1.000 \\
0.000 & 0.010 & 0.035 & 0.059 & 0.084 & 0.109 & 0.134 \\
0.135 & 0.270 & 0.405 & 0.541 & 0.676 & 0.811 & 0.945 \\
\end{bmatrix}\]

(9)

According to the principle of consistent binary comparison matrix, the qualitative scale matrix of consistent binary comparison can be obtained.

\[
F = \begin{bmatrix}
0.5 & 0.0 & 0.0 & 0.0 & 1.0 & 1.0 \\
1.0 & 0.5 & 0.5 & 1.0 & 1.0 & 0.0 \\
1.0 & 0.5 & 0.5 & 0.0 & 0.0 & 0.5 \\
1.0 & 0.0 & 0.0 & 0.5 & 0.5 & 0.0 \\
1.0 & 0.0 & 0.0 & 0.5 & 0.5 & 0.5 \\
1.0 & 1.0 & 1.0 & 1.0 & 0.5 & 0.5 \\
\end{bmatrix}\]

(10)

Determination of relative preference vector calculation and value

According to the ordered binary comparison consistent ordering matrix \(F\), the importance ranking of seven indexes is obtained, and the bivariate comparison judgment of importance degree is given. Indicator 2c is as important as 3c, 6c is as important as 7c, but 6c is between same and the slightly compared with 3c, 6c is slightly more important than 4c, 6c is slightly important to 5c, 4c is between slightly and less slightly compared to 5c. According to the above-mentioned binary comparison, the non-normalized weight vector calculation model using the index or factor [2].

\[
P = \left[ \frac{1-g_{12}}{g_{11}}, \ldots, \frac{1-g_{1n}}{g_{11}} \right]
\]

(11)

Table 2: Judgement matrix scale (part) [2]

| Mood operator       | Same   | Slightly | Somewhat |
|---------------------|--------|----------|----------|
| Quantitative        | 0.500  | 0.550    | 0.600    |
| Scales              | 0.525  | 0.575    | 0.625    |
| Relative            | 1.000  | 0.818    | 0.667    |
| Degree of membership| 0.905  | 0.739    | 0.600    |

According to Table 2, the non-normalized weight vector of the index is: \(W = (0.667, 1, 1, 0.818, 0.739, 0.905, 0.905)\), then the index weight vector is: \(W = (0.111, 0.166, 0.166, 0.135, 0.122, 0.15, 0.15)\).

When the parameter \(P\) equal to 1, the relative dominance vector of the scheme can be obtained by using formula (11): \(u = (0.751, 0.794, 0.845, 0.803, 0.801, 0.772, 0.738)\). When the parameter \(P\) equal to 2, the relative dominance vector of the scheme can be obtained by using formula (12): \(u = (0.742, 0.788, 0.858, 0.811, 0.792, 0.761, 0.725)\).

The optimal order of the two distance parameters was as follows: (3), (4), (5), (2), (6), (1), (7). Therefore, when the grillage method is used to model and analyze, the length and width of the cross section of the virtual cross beam should be \(0.7h \times 0.7h\), and the cross-section stiffness is \(0.2401E/12\), in which the parameter \(E\) is the elastic modulus of the structure.

4. Results validation

In order to further verify the correctness of the stiffness value of the virtual crossbeam, taking the highway 25m single-box three-chamber simply-supported box girder as an example, the elastic modulus of the beam is \(3.45 \times 104\) MPa, the Poisson ratio is 0.2, and the size of the cross-section is shown in Figure 4. According to the highway class-1 two-lane load, ANSYS solid45 entity unit value as a comparison.
When analyzing the rationality of the stiffness value of the virtual cross beam, three longitudinal members are selected as the research object. The outermost side component of the loading lane load is numbered 1, and inwards is the edge 2 longitudinal member and the intermediate longitudinal member. The comparison of the beam stress and deflection of the control section with that of the solid element is shown in Table 3.

As can be seen from Table 4, for highway single-box, three-chamber concrete box girder beams, the difference between grillage stress, deflection and solid element values of all longitudinal members is less than 5%, and when the length and width of the virtual cross-section is 0.7 h × 0.7 h, the difference between the grillage stress, the deflection value and the solid element value is less than 5%. The analytical accuracy of the grillage model is high.

### 5. Conclusion

The concrete value of the virtual beam stiffness of grillage is obtained based on the simple supported hollow slab bridge and single-box multi-chamber box girder and validated by the experimental solution and the solution of the entity element. The following conclusions can be drawn:

1. The stiffness of the virtual cross beam has a great influence on the stress and deformation of the control section by the grillage method. When the length and width of the virtual cross-section are 0.6 times to 0.9 times of the beam height, the calculated results of the grillage are close to that of the ANSYS solution. If the stiffness of the virtual cross beam is infinitely large, it may lead to the distortion of the calculated results.

2. Compared with the ANSYS numerical solution, it is shown that when the length and width of the virtual cross beam section is 0.7 times the height of the beam, the grillage model solution of the control cross section is in good agreement with the measured value and the numerical solution of the plate and shell (solid element).

### References

[1] Hambly. Bridge Deck Behavior [M]. London:1976.

[2] Kang Chun-jie, Dai Gong-lian, Li Miao, i. Research on Design and Mechanical Characteristics of Skew Thin Walled Continuous Steel Box Girder [J]. Journal of Railway Science and Engineering, 2015,12(01):119-126.
[3] D.G. Linzell, J.F. Shura. Erection behavior and grillage model accuracy for a large radius curved bridge [J]. Journal of Constructional Steel Research, 2009(66):342-350.

[4] Huang Zu-wei, Lei Jun-qin, Tang Ji-shun. Application Study on Grillage Method for Multiple-Cells Box Girder with Transverse Slope [J]. Journal of Southwest Jiaotong University, 2018,53(01):56-63.

[5] Nassif, H. H; Liu, M. Analytical Modeling of Bridge-Road-Vehicle Dynamic Interaction System [J]. Journal of vibration and control, 2004(2):215-241.

[6] Jeremy Chang, Andrew H. Buchanan, Rajesh P. Dhakal, Peter J. Moss. Shura. Hollow-core concrete slabs exposed to fire [J]. FIRE AND MATERIALS, 2008(6):321-331.

[7] Zhou, K. Optimization of least-weight grillages by finite element method [J]. Structural and Multidisciplinary Optimization, 2009(05):525-532.

[8] Sabeeh Z. Al-Sarraf, Ammar A. Ali, Rana A. Al-Dujaili. Analysis of Composite Bridge Superstructures Using Modified Grillage Method [J]. Engineering and Technology Journal, 2009(05):942-953.

[9] Iman Mohseni; A. R. Khalim Rashid and A. Akbar Pabarja. Influence of skew abutments on behavior of concrete continuous multi-cell box-girder bridges subjected to traffic loads [J]. International Journal of Physical Sciences, 2012(6):966-973.

[10] Kim, Sung Chan; Ryu, Cheolho; Lee, Jang Hyun; Lee, Kyung Seok. Grillage Method Applied to the Planning of Ship Docking [J]. Journal of Constructional Steel Research, 2016(3):150-157.

[11] Lin Peng-zhen, Sun Li-xiang, i. Research on Shear Lag Effect of Twin-Cell Box Girders [J]. Journal of railway engineering society, 2014(1):59-64.