Heavy-quark transport coefficients in a hot viscous quark-gluon plasma medium

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Abstract. The heavy-quark (HQ) transport coefficients have been estimated for a viscous quark-gluon plasma medium, utilizing a recently proposed quasi-particle description based on a realistic QGP equation of state (EoS). Interactions entering through the equation of state significantly suppress the temperature dependence of the drag coefficient of QGP as compared to that of an ideal relativistic system of quarks and gluons. Inclusion of shear and bulk viscosities through the corrections to the thermal phase space factors of the bath particles alters the magnitude of the drag coefficient, and the enhancement is significant at lower temperatures. In the competition between the effects of the EoS and dissipative corrections through phase space factors; the former eventually dictate how the drag coefficient would behave as a function of temperature, and how much quantitatively digress from the ideal case. The observations suggest significant impact of both the realistic equation of state, and the viscosities, on the HQs transport at RHIC and the LHC collision energies.

Keywords: Heavy quark transport; Quasi-particle model; Effective fugacity; Drag and diffusion; Equation of state; Viscous QGP

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1. Introduction

Experimental heavy-ion collision (HIC) program at Relativistic Heavy Ion Collider (RHIC) indicates the production of a liquid like state of the matter where the properties of the system is governed by quarks and gluons. Such a state of matter is called as quark-gluon plasma (QGP). Two of the most striking finding of the RHIC which lead to the imposition of strongly interacting liquid-like behavior are viz., the observed large elliptic flow and the phenomenon of jet quenching [1]. Preliminary results from LHC [2, 3, 4] confirm these observations. The strongly coupled picture of the QGP is found to be consistent with the lattice simulations of the hot QCD (Quantum Chromodynamics) equation of state (EoS) [5, 6], which predict a strongly interacting behavior even at temperatures, $T \sim 2T_c$, where $T_c$ is the temperature for the quark-hadron transition. Therefore, it calls for the adaption of a realistic equation of states for the QGP (such as obtained form lattice QCD) in the theoretical and phenomenological investigations regarding its bulk and transport properties. We shall closely follow this point of view in the present analysis.

Hadrons containing HQs ($c$, $\bar{c}$, $b$, or $\bar{b}$) are of great interest in investigating the properties of the QGP, since their momentum spectrum get significantly modified while traveling through QGP. This fact has been reflected in the particle spectra at RHIC, and LHC energies. Further, HQ thermalization time is larger than gluons and light quarks, and they do not constitute the bulk of the QGP. Since their formation occurs in the early stages of the collisions, they can travel through the thermalized QGP medium, and can retain the information about the interaction with them very efficiently. For instance, it is pertinent to ask whether a single $c\bar{c}$ can stay together long enough to form a bound state (say $J/\psi$) at the hadronization state. To address this, one requires to study the dynamics of the HQs propagating through the QGP.

One can envisage the HQ transport in the QGP medium as follows. The non-equilibrated HQs travel in the equilibrated QGP medium, therefore, the problem can be studied within the framework of Langevin dynamics [7]. This is to say that the HQs undertake a random motion in the heat bath of the equilibrated QGP. Recall, that the QGP goes through a hydrodynamic evolution process before it reaches the hadronization and subsequently the hadrons freeze-out. The pertinent question to ask is, whether a HQs maintain equilibrium during this entire process of evolution or not. It has been observed [8] within the ambit of Langevin dynamics and pQCD (perturbative QCD) that they may not achieve the equilibrium for the RHIC and LHC collision conditions.

The nuclear modification factor of the HQ, $R_{AA}$ encode the effects of the medium through the modified momentum distribution due to their interaction with the bath particles. It has been observed that their energy loss in the QGP due to gluon radiation is insufficient to describe the medium modification of the spectrum [9, 10, 11]. Therefore, one has to look at the collisions since they have different fluctuation spectrum than radiation, and might contribute significantly as indicated earlier [12, 13]. The collisional effects can be captured well in the HQ drag and diffusion coefficients which have been
calculated within weak coupling QCD by several authors. The formalism, and details are offered in Sec. 2.1.

The temperature, $T$ and chemical potential, $\mu_B$ dependence of the drag and diffusion coefficient enter through the thermal distributions of gluons and light quarks. In the present case, we ignore the $\mu_B$ dependence in view of the fact that the QGP produced at RHIC and LHC energies at the mid-rapidity region has negligibly small net baryon density. Therefore, one has to implement the realistic QGP EoS in terms of appropriate form of the thermal distribution functions. Lattice QCD EoS (LEoS) may be a good choice for the description of the QGP. Another important aspect is the viscosities (both shear and bulk) of the QGP. One also has to include the viscous modifications in the distribution functions by maintaining the near equilibrium picture so that the mathematical formalism of HQ transport remain intact. This is possible since the QGP posses small shear and bulk viscosities for temperature, $T > T_c$ ($T_c$ is the quark-hadron transition). Both these aspect have been included in the present work while studying the temperature dependence of HQ drag and diffusion coefficients in the QGP. Discussion regarding the former is offered in Sec. 2.2, and the latter in 2.3. Our results suggest that both of these modifications have significant impact on the temperature dependence of HQ transport coefficients. Interestingly, the inclusion of interaction through the EoS modulate the transport coefficients quite significantly as compared to those obtained by employing an ideal EoS (assuming the QGP as a relativistic gas of non-interacting quarks and gluons).

The paper is organized as follows. In section 2.1, formalism of HQ drag and diffusion has been presented. Section 2.2 deals with the quasi-particle description of hot QCD and modeling of thermal distribution functions of gluons, and quarks which constitute hot QGP medium. Section 2.3 deals with the viscous modifications to the thermal distribution functions. In Section 3, results and discussions have been presented for the drag and diffusion coefficients with the LEoS which may be treated a realistic EoS for the QGP. Section 4 is devoted to summary and conclusions.

2. Heavy-quark drag and diffusion in the viscous QGP

The special role of HQs to characterize QGP stem from the fact that they are produced very early in the collisions and remain extant throughout the evolution and hence can witness the entire evolution of the system. Moreover, HQs do not constitute the bulk part of the system because their masses is considerably larger than the temperature attainable in HIC at RHIC and LHC.

2.1. Formalism of HQ drag and diffusion

In the present work, we consider the elastic interaction experienced by HQs while propagating through the QGP. For the process, $c(p) + l(q) \rightarrow c(p') + l(q')$ ($l$ stands for light quark, anti-quark and gluon), the drag, $\gamma$ can be calculated by using the following
Heavy-quark transport coefficients in a hot viscous quark-gluon plasma medium

expression [14, 15]:
\[ \gamma = p_i A_i/p^2 \]  

(1)

where \( A_i \) is given by
\[
A_i = \frac{1}{2E_p} \int \frac{d^3q}{(2\pi)^3E_q} \int \frac{d^3p'}{(2\pi)^3E_{p'}} \int \frac{d^3q'}{(2\pi)^3E_{q'}} \\
\frac{1}{g_Q} \sum |M|^2 (2\pi)^4 \delta^4(p + q - p' - q') \\
f(q)(1 \pm f(q'))[(p - p')_i] \equiv \langle \langle (p - p')_i \rangle \rangle 
\]

(2)

g_Q being the statistical degeneracy of the HQ propagating through the medium. The expression for \( \gamma \) indicates that the drag coefficient is the measure of the thermal average of the momentum transfer, \( p - p' \) due to interaction weighted by the square of the invariant amplitude, \( |M|^2 \). The factor \( f(q) \) denotes the thermal phase space for the particle in the medium. We shall see in the subsequent subsection that \( f(q) \) will involve three types of thermal phase space distribution corresponding to the gluons (\( g \)), light-quarks (\( q \equiv \text{up and down} \)) and the strange quarks (\( s \)) and their anti-quarks. Therefore, \( f(q) \) jointly denote these three sectors as,
\[
f(q) \equiv \{f_g, f_q, f_s\}, 
\]

in the absence of any dissipative effects. It may be recalled here that for zero baryonic chemical potential the thermal phase space for quarks and anti-quarks are same. In the presence of dissipation, we denote it as,
\[
f(q) \equiv \{f_{gg}, f_{qq}, f_{ss}\}. 
\]

(4)

We shall discuss these distributions in the subsequent subsections.

Similar to drag, the diffusion coefficient, \( B_0 \) can be defined as:
\[
B_0 = \frac{1}{4} \left[ \langle \langle p'^2 \rangle \rangle - \frac{\langle \langle (p.p')^2 \rangle \rangle}{p^2} \right] 
\]

(5)

With an appropriate choice of \( \mathcal{F}(p') \) both the drag and diffusion co-efficients can be evaluated from the following expression:
\[
\langle \langle \mathcal{F}(p) \rangle \rangle = \frac{1}{512\pi^4} E_Q \int_0^\infty q^2 dq d(\cos \chi) \frac{f(q)}{E_q} \\
\frac{w^{1/2}(s, m_Q^2, m_p^2)}{\sqrt{s}} \int_1^{-1} d(\cos \theta_{c.m.}) \\
\frac{1}{g_Q} \sum |M|^2 \int_0^{2\pi} d\phi_{c.m.} \mathcal{F}(p') 
\]

(6)

where \( s \) is the Mandelstam variable, \( E_p = \sqrt{p^2 + m_Q^2} \) is the energy of the HQ, \( E_q \) is the energy of the bath particle, \( \chi \) is the polar angle of \( \vec{q} \) and \( w(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ac \), is the triangular function. Note that the Bose enhancement and Pauli suppression of phase space is neglected in Eq. 6.
2.2. The quasi-particle description of hot QCD

Now we discuss the quasi-particle nature of the hot QCD medium which is employed recently in [16]. This description has been developed in the context of the recent (2+1)-flavor lattice QCD EoS [17] at physical quark masses. There are more recent lattice results with the improved actions and refined lattices [6], for which we need to re-look the model with specific set of lattice data specially to define the effective gluonic degrees of freedom. This is beyond the scope of the present analysis. Henceforth, we will stick to the set of lattice data utilized in the model described in [16].

The model initiates with an ansatz that the LEoS can be interpreted in terms of non-interacting quasi-partons having gluon and quark effective fugacities $z_g$ and $z_q$ respectively which encode all the interaction effects. In this approach, the hot QCD medium is divided into two sectors, viz., the effective gluonic sector, and the matter sectors (light and strange quark). The former refers to the contribution of gluonic action to the pressure which also involves contributions from the internal fermion lines. On the other hand, latter involve interactions among quarks, anti-quarks, as well as their interactions with gluons. The ansatz can be translated to the form of the equilibrium distribution functions, $f_{eq} = \{f_g, f_q, f_s\}$ (this notation will be useful later while writing the transport equation in both the sector in compact notations) as follows,

\[
\begin{align*}
    f_{g/q} &= \frac{z_{g/q} \exp[-\beta E_q]}{1 \mp z_{g/q} \exp[-\beta E_q]}, \\
    f_s &= \frac{z_q \exp[-\beta E_q]}{1 + z_q \exp[-\beta E_q]},
\end{align*}
\]

where $E_q = \sqrt{q^2 + m^2}$ is the energy of the particle in the thermal bath moving with momentum, $q = |q|$. Light quarks and gluons are taken as massless ($m = 0$), for strange quark $m = m_s (\sim 0.1$ GeV) and $\beta = T^{-1}$ denotes inverse of the bath temperature.

We use the notation $\nu_g = 2(N_c^2 - 1)$ for gluonic degrees of freedom, $\nu_q = 2 \times 2 \times N_c \times 2$ for light quarks, $\nu_s = 2 \times 2 \times N_c \times 1$ for the strange quark for SU($N_c$). Note that we are working at zero baryonic chemical potential, therefore quark and antiquark distribution function are the same. This fact has been taken care of by counting the antiquark degrees of freedom in $\nu_q$ and $\nu_s$. Here, we are dealing with SU(3), so $N_c = 3$. Since the model is valid in the deconfined phase of QCD (beyond $T_c$), therefore, the mass of the light quarks can be neglected as compared to the temperature of the bath.

Clearly, the determination of $f_g$, $f_q$, and $f_s$ require the temperature dependence of the free parameters $z_g$ and $z_q$ respectively. To do this, we rewrite lattice QCD pressure symbolically as,

\[ P = P_g + P_{qg}, \]

where $P$ is full (2+1)-flavor lattice QCD pressure, $P_g$ is the contribution from the gluonic action alone and $P_{qg}$ is the remaining part obtained after subtracting $P_g$ from $P$. The
fraction $P_g$, is utilized to define effective gluonic (quasi-gluons) degrees of freedom, and $P_{qg} \equiv P - P_g$ to effective quark-antiquark degrees of freedom through an effective Grand canonical partition function $Z = Z_g \times Z_{qg}$ via the relation $P\beta V = \ln(Z)$ as follows,

$$P_g\beta V = \frac{V\nu_g}{8\pi^3} \int d^3\vec{q} \ln \left(1 - z_g \exp[-\beta|\vec{q}|] \right)$$
$$P_{qg}\beta V = \frac{V}{8\pi^3} \int d^3\vec{q} \left\{ \nu_q \ln \left(1 + z_q \exp[-\beta|\vec{q}|] \right) + \nu_s \ln \left(1 + z_q \exp[-\beta\sqrt{\vec{q}^2 + m^2}] \right) \right\}. \tag{9}$$

One can read-off $Z_g$ and $Z_{qg}$ which are their standard definitions for bosons and fermions. We know the left hand side of these equation from lattice QCD, and by calculating roots of these equations at each temperature, we can determine $z_g$ and $z_q$ as a function of temperature. This exercise has been performed numerically.

It is worth emphasizing that the effective fugacity is not merely a temperature dependent parameter which encodes the hot QCD medium effects. Its physical significance is reflected through the modified dispersion relation both in the gluonic and quark sector obtained from the thermodynamic relation of energy density and partition function, $\epsilon = -\partial_\beta \ln(Z)$. One thus find that the effective fugacities modify the single quasi-parton energy as follows,

$$\omega_g = |\vec{q}| + T^2 \partial_T \ln(z_g)$$
$$\omega_q = |\vec{q}| + T^2 \partial_T \ln(z_q)$$
$$\omega_s = \sqrt{|\vec{q}|^2 + m^2} + T^2 \partial_T \ln(z_q), \tag{10}$$

and this lead to the new energy dispersions for gluons ($\omega_g$), light-quark antiquarks ($\omega_q$), and strange quark-antiquarks, ($\omega_s$). These dispersion relations can be explicated as follows. The single quasi-parton energy not only depends upon its momentum but also gets contribution from the collective excitations of the quasi-partons. The second term is like the gap in the energy-spectrum due to the presence of quasi-particle excitations. This makes the model more in the spirit of the Landau’s theory of Fermi-liquids. For a detailed discussions of the interpretation and physical significance of $z_g$, and $z_q$, we refer the reader to [16]. There are other quasi-particle descriptions in the literature, those could be characterized as, effective mass models [18, 19], effective mass models with gluon condensate [20], and effective models with Polyakov loop [21]. Our model is fundamentally distinct from all these models. Another crucial point is regarding the definition of the energy momentum tensor, $T^{\mu\nu}$. As described in [22], in the presence of non-trivial temperature dependent energy dispersion (as in all these quasi-particle models), we need to modify the definition of the $T^{\mu\nu}$ so that the trace anomaly effects in QCD can be accommodated in the definition. The modified $T^{\mu\nu}$ for the effective mass models is obtained in [22], and for the current model in [23].
2.3. Viscous QGP: modification to thermal distributions

Viscosities usually modifies the equilibrium thermal distributions as follows,
\[ f = f_\text{eq} + \delta f_\eta + \delta f_\zeta. \]  
(11)

where \( f \) jointly denote the viscous modified thermal distribution for gluons, quark and anti-quarks (in the present case gluons, light quarks and strange quark), therefore, \( f \equiv \{f_{gg}, f_{qq}, f_{ss}\} \). For the shear and bulk viscous corrections, \( \delta f_\eta \) and \( \delta f_\zeta \) respectively, we utilize the results of [22] by using only first order terms in the expansion of shear and bulk part of the stress tensor, and choosing the local rest frame of the fluid. Precisely, we have minimally extended the expressions in the present case by substituting the thermal distributions of quarks and gluons as the form obtained in the effective fugacity quasi-particle model. This is solely based on the fact that the quasi-partons in the present case are non-interacting up to their respective effective fugacities that encode all the interactions. This leads to the following expressions,
\[ f_{gg} = f_g + \frac{f_g [1 + f_g]}{2T^3\tau} \left[ \left( \frac{1}{3} - q_z^2 \right) \frac{\eta}{s} + \frac{2|\vec{q}|^2}{5} \frac{\zeta}{s} \right] \]  
(12)
\[ f_{qq} = f_q + \frac{f_q [1 - f_q]}{2T^3\tau} \left[ \left( \frac{1}{3} - q_z^2 \right) \frac{\eta}{s} + \frac{2|\vec{q}|^2}{5} \frac{\zeta}{s} \right] \]  
(13)
\[ f_{ss} = f_s + \frac{f_s [1 - f_s]}{2T^3\tau} \left[ \left( \frac{1}{3} - q_z^2 \right) \frac{\eta}{s} + \frac{2|\vec{q}|^2}{5} \frac{\zeta}{s} \right], \]  
(14)

where \( \eta \) and \( \zeta \) are the shear and bulk viscosities of the QGP, \( s \) is the entropy density and \( \tau \) is the thermalization time of the QGP. We will employ the same temperature independent values of \( \eta/s \) and \( \zeta/s \) while comparing the effects of ideal and non-ideal EoS with viscosities. For simplicity, same constant values are also taken for \( \eta/s \) and \( \zeta/s \) for gluons, and quarks.

There are several interesting reports on the estimation of a small value of \( \eta/s \) for QGP, and the emergence of near perfect fluid picture [24, 25, 26, 27]. It has been realized in the recent past that bulk viscosity of the QGP is significant near the transition temperature, \( T_c \) [23, 28, 29, 30]. This is attributed to the large value of the interaction measure near \( T_c \) [28]. Before we present our results on the effects of \( \eta \) and \( \zeta \) on the phase space distribution it is imperative to mention viscous correction to phase space distribution is based on momentum expansion, which will be valid when the momentum \( (q) \) is small [31]. It should be noted here that the momentum of the thermal particles are integrated out in evaluating the drag and diffusion coefficients of HQ and the thermal phase space factor become small at \( q \sim 3 - 4 \) GeV for the temperature range considered here. In the present work the drag of the HQ are evaluated as a function of its momentum and temperature of the bath.

The mass of the strange quarks does not play any prominent role at high temperature \( (T \geq T_c) \) because \( m/T < 1 \). In fact, the mass effects are negligibly small (of the order of \( (m/T)^2 \)). Therefore, the strange quark mass can be ignored which makes the numerical calculations simpler.
Let us set the notation here to distinguish the results obtained for the cases with ideal EoS and LEoS as well as for zero and non-zero values of $\eta$ and $\zeta$. The quantity $\gamma_{Id}$ stands for the drag coefficient for ideal case, $\gamma_{eos}$ denotes that for the LEoS, and $\gamma_{\eta}$ ($\gamma_{\zeta}$) stands for drag coefficients when the effect of $\eta$ ($\zeta$) is included through the phase space of the bath particles. $\gamma_{\eta+\zeta}$ denotes the drag coefficient while both $\eta$ and $\zeta$ are taken into account with ideal QGP EoS and as $\gamma_{\eta+\zeta}^{eos}$ in the case of the realistic QGP EoS. Similarly, the quantity $D_{\eta+\zeta}^{eos}$ denotes the HQ diffusion coefficient while LEoS and both $\eta$ and $\zeta$ are taken into account, the notation $D_{Id}$ denotes same for ideal EoS without viscous effects.

Figure 1. Variation of the ratio $\gamma_{\eta}/\gamma_{Id}$ as a function of momentum with different values of $\eta/s$ at a given temperature are shown (upper panel). Same quantity at a different temperature is depicted in the lower panel.
3. Results and discussions

In this section, we display the momentum and temperature dependence of the HQ drag coefficient employing the realistic EoS for the QGP and the dissipative environment mainly induced by the shear and bulk viscosities. Our focus is mainly on the impact of the effects of the viscosity and the realistic EoS on the HQ transport coefficients. Our aim is served by the ratio of drag coefficients evaluated with ideal (excluding dissipation) and LEoS (including viscous effects). These effects are demonstrated through the results displayed in the Figs. 1-9.

![Graph 1](image1.png)

**Figure 1.** The variation of the ratio $\gamma_{\eta}/\gamma_{Id}$ as a function of temperature with different values of $\eta/s$ at a given thermalization time, $\tau$ are shown (upper panel). Same quantity with the variation of $\tau$ for fixed $\eta/s$ is shown in the lower panel.

In Fig 1 (upper panel), the effect of shear viscosity on drag coefficient has been depicted as a function of momentum at $T = 0.25$ GeV through the ratio of the drag coefficients with and without shear viscous effects with ideal EoS. The drag coefficient for
Heavy-quark transport coefficients in a hot viscous quark-gluon plasma medium

Figure 3. The variation of the ratio $\gamma_c/\gamma_{Id}$ as a function of momentum with different values of $\zeta/s$ at a given temperature are shown (upper panel). Same quantity at a different temperature is shown in the lower panel.

zero shear viscosity is higher compared to the case of nonzero shear viscosity in the low momentum range. A reduction in the ratio of the drag coefficients for low $p$ is obtained because of the decrease in the phase space factor of the particles in the thermal bath due to the possible negative contribution from the shear viscous corrections as indicated in Eqs. 13-14. The situation get reversed at higher momenta ($p > 2$ GeV). In Fig 1 (lower panel) the same quantity has been plotted at a higher temperature ($T = 0.4$ GeV). The nature of the variation of the ratio remains similar. However, the magnitude of the ratio reduces with increasing temperature, indicating the modulation of relative drag coefficient induced by $\eta/s$.

In Fig 2 (upper panel), the effect of shear viscosity on drag coefficient has been depicted as a function of temperature ($> T_c$) for momentum, $p = 1.5$ GeV. The drag coefficient for zero shear viscosity is higher compared to the case of nonzero shear
viscosity in the entire temperature range considered here. This is because of the fact that we have performed the calculation at low momentum. The same quantity is displayed in Fig. 2 (lower panel), as a function of temperature for different values of thermalization time at a given value of $\eta/s$. It is observed that effects of shear viscosity get milder with the increase in thermalization time (as expected from the terms appearing as the coefficients of $\eta/s$).

In Fig. 3 (upper panel), the effect of the bulk viscosity on drag coefficient (relative to the ideal case) has been displayed as a function of momentum for $T = 0.25$ GeV. It is observed that the drag coefficient get enhanced in the presence of bulk viscosity. This enhancement in the ratio of the drag coefficients for low $p$ may arise from the increase in the phase space factor of the particles in the thermal bath due to the positive bulk viscous corrections as indicated in Eqs. 13-14. That is, since the term appearing as the

Figure 4. Upper panel: The variation of the ratio $\gamma/\gamma_{id}$ as a function of temperature with different values of $\zeta/s$ at a given value of thermalization time $\tau$ are depicted. Lower panel: same quantity has been plotted for fixed $\zeta$ but with varying $\tau$. 
coefficient of $\zeta/s$ is always positive definite, the presence of bulk viscosity will enhance the phase space density and hence the drag coefficients. If we increase the $\zeta/s$, drag coefficient is also enhanced. In Fig 3 (lower panel), the same quantity has been plotted for $T = 0.4$ GeV. It is observed that the ratio reduces with increase in temperature.

In Fig 4 (upper panel), the effect of the bulk viscosity on drag coefficient (relative to the ideal case) has been displayed as a function of temperature for $p = 1.5$ GeV. The effects of thermalization time is displayed in Fig 4 (lower panel). It is found that the effect of bulk viscosity becomes weaker at larger thermalization time.

Considering both the shear and the bulk viscosities, we compute the drag coefficient $(\gamma_{\eta+\zeta})$ and study its variation with the momentum for temperatures $T = 0.25$, 0.40 GeV. The results are depicted in the upper and lower panels of Fig. 5. We observe that the ratio $\gamma_{\eta+\zeta}/\gamma_{Id}$ get enhanced with increase in momentum. The increase is faster at low

Figure 5. The variation of the ratio $\gamma_{\eta+\zeta}/\gamma_{Id}$ as a function of momentum with different values of $\eta/s$ and $\zeta/s$ at a given temperature are shown (upper panel). Same quantity is plotted at a different temperature in the lower panel.
Heavy-quark transport coefficients in a hot viscous quark-gluon plasma medium

0.2 0.3 0.4 0.5 0.6 0.7 0.8
T (GeV)

0.8 1 1.2 1.4 1.6 1.8

\gamma_{\eta+\zeta}

\eta/s=0.08, \zeta/s=0.02, \tau=0.3
\eta/s=0.08, \zeta/s=0.01, \tau=0.3
\eta/s=0.16, \zeta/s=0.01, \tau=0.3

Figure 6. $\gamma_{\eta+\zeta}$ stands for drag coefficients when the effects of both the shear and bulk viscosities are included through the thermal distribution of the bath particles in evaluating the drag coefficient. The variation of the ratio of the drag coefficients as a function of temperature with different values of $\zeta/s, \eta/s$ and $\tau$ are shown.

0.2 0.3 0.4 0.5 0.6 0.7 0.8
T (GeV)

0.2 0.4 0.6 0.8

$\gamma_{\eta+\zeta}/\gamma_{Id}$

Figure 7. $\gamma_{eos}$ stands for drag coefficients when effect of EoS is considered in evaluating the drag coefficient. The variation of the ratio of the drag coefficients as a function of temperature are shown.

momentum, slower at higher momentum. The magnitude of the quantity $\gamma_{\eta+\zeta}/\gamma_{Id}$ is lower at higher value of $T$ (0.4 GeV) through out the range of the momentum considered here. Interestingly, the low momentum domain is dominated by shear viscous effects, whereas high momentum part is dominated by bulk viscous effects.

In Fig[6], the temperature variation (for $p = 1.5$ GeV) of the drag coefficient relative to the ideal case has been shown in the presence of both $\eta/s$ and $\zeta/s$. It is the interplay of these two dissipative effects that dictates the behavior of the ratio as a function of $T$. 
The ratio of the drag coefficients is plotted against $T$ in Fig. 7 to understand the effects of EoS. The effects of interactions contained in LEOs suppresses the phase space distributions of the bath particle which is seen through the reduction of the ratio of drag coefficients at low temperature in Fig. 7. The ratio increases with $T$ because the effects of interaction at high $T$ becomes weaker. Ideally the ratio will become unity in the Stefan-Boltzmann limit.

In Fig. 8 combined effects of both shear and bulk viscosities along with the EoS have been presented as a function of temperature at given values of $\tau$, $\eta/s$, $\zeta/s$ at two different momentum. The reduction in the drag is significant at the temperature domain relevant of nuclear collisions at RHIC and LHC energies for the low momentum.

Figure 8. The variation of the quantity $\gamma_{\eta+\zeta}/\gamma_{Id}$ as a function of temperature.

Figure 9. The variation of the quantity $D_{\eta+\zeta}/D_{Id}$ as a function of temperature.
case. Therefore, this results will have crucial effects on the nuclear suppression of heavy flavors measured at these energies.

The effects of phase space corrections due to non-zero shear and bulk viscosities and realistic EoS on diffusion coefficient \( D \equiv B_0 \) have been shown explicitly in Fig. 9. It is observed that the ratio of diffusion coefficients, \( D_{\eta+\zeta}/D_{Id} \), is significantly smaller than unity for low \( T(<300 \text{ MeV}) \). The effects of realistic EoS, i.e. the LEoS in the present case, is found to play the most dominant role at low \( T \). At higher \( T \) the effects of LEoS becomes weaker and the effects viscous corrections to phase space factors start playing significant role.

4. SUMMARY AND CONCLUSIONS

The drag and diffusion coefficients of the HQs propagating through a viscous system of quarks and gluons have been evaluated by incorporating the corrections to the phase space factors arising from the non-zero viscosities of the system which restrain the system slightly away from the equilibrium. This corrections seem to be significant for the temperature range expected to be attained in the system produced at HIC at RHIC and LHC collision energies. The effect of realistic EoS on the drag has also been ascertained. The combined effects of viscosities and the EoS reduces the drag significantly at moderate momentums. These results will have crucial consequences on the heavy ion phenomenology at RHIC and LHC energies, which is the matter of future investigations.

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Appendix

In this appendix, we quote the invariant amplitude for the elastic processes used for evaluating the drag and diffusion coefficients of charm quark propagating through QGP.

The invariant amplitude, \( |\mathcal{M}|^2 \) for the process \( gc \rightarrow gc \) is given by:

\[
|\mathcal{M}|^2_{gc \rightarrow gc} = \pi^2 \alpha_s^2 \frac{32(s - M^2)(M^2 - u)}{t^2} + \frac{64}{9} \frac{(s - M^2)(M^2 - u) + 2M^2(s + M^2)}{(s - M^2)^2} + \frac{64}{9} \frac{(s - M^2)(M^2 - u) + 2M^2(s + M^2)}{(s - M^2)^2} + \frac{16}{9} \frac{M^2(4M^2 - t)}{(s - M^2)(M^2 - 4)}
\]
Heavy-quark transport coefficients in a hot viscous quark-gluon plasma medium

\[ +16\frac{(s-M^2)(M^2-u)+M^2(s-u)}{t(s-M^2)} - 16\frac{(s-M^2)(M^2-u)-M^2(s-u)}{t(M^2-u)} \] (15)

Similarly for the process, \( qc \rightarrow qc \), the square of the invariant matrix element, \( |M|_{qc\rightarrow qc}^2 \) is given by:

\[ |M|_{qc\rightarrow qc}^2 = \frac{64\pi^2\alpha_s^2}{9} \left[ \frac{(M^2-u)^2+(s-M^2)^2+2M^2t}{t^2} \right]. \] (16)

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