Multiple passages of light through an absorption inhomogeneity in optical imaging of turbid media

M. Xu, W. Cai and R. R. Alfano

Institute for Ultrafast Spectroscopy and Lasers,

New York State Center of Advanced Technology for Ultrafast Photonic Materials and Applications,

and Department of Physics,

The City College and Graduate Center of City University of New York,

New York, NY 10031
Abstract

The multiple passages of light through an absorption inhomogeneity of finite size deep within a turbid medium is analyzed for optical imaging using the “self-energy” diagram. The nonlinear correction becomes more important for an inhomogeneity of a larger size and with greater contrast in absorption with respect to the host background. The nonlinear correction factor agrees well with that from Monte Carlo simulations for CW light. The correction is about 50% – 75% in near infrared for an absorption inhomogeneity with the typical optical properties found in tissues and of size of five times the transport mean free path.

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*Electronic address: minxu@sci.ccny.cuny.edu
The main objective of optical imaging of turbid media is to locate and identify the embedded inhomogeneities by essentially inverting the difference in photon transmittance in time or frequency domains due to the presence of these inhomogeneities. The key quantity involved is the Jacobian which quantifies the influence on the detected signal due to the change of the optical parameters of the medium. The linear perturbation approach is only suitable to calculate the Jacobian for a small and weak absorption inhomogeneity, and not valid when the absorption strength is large. This failure can be attributed to the multiple passages through the abnormal site by the photon. The most important correction is the “self-energy” correction which takes into account the repeated visits made by a photon through the site up to an infinite number of times. The presence of other inhomogeneity “islands” can be ignored because the photon propagator decreases rapidly with the distance between two separate sites.

In this Letter, the nonlinear correction for an absorption inhomogeneity of a large strength due to repeated visits by the photon is modeled by a nonlinear correction factor (NCF) to the linear perturbation approach. NCF as a function of the size and the strength of the inhomogeneity is estimated using the “self-energy” diagram. NCF is obtained based on the cumulant approximation to radiative transfer, and verified by Monte Carlo simulations for CW light. The magnitude of NCF is $0.5 - 1$ for an absorptive inhomogeneity of size up to $5l_t$ ($l_t$ is the mean transport free path of light), and of the typical optical properties of human tissues ($\mu_a l_t / c \sim 0.01 - 0.05$ where $\mu_a$ is the absorption coefficient and $c$ is the speed of light in the medium).

Consider an absorption site centered at $\bar{r}$ and far away from both the source and the detector, the change of the detected light, $\Delta I$, at the detector $r_d$ from a modulated point
source at \( \mathbf{r}_s \) including the multiple passages through the site, is given by

\[
\Delta I = -G(\mathbf{r}_d, \omega|\bar{\mathbf{r}}) V \delta \mu_a(\bar{\mathbf{r}}) \sum_{n=0}^{\infty} \left[ -\bar{N}_{self}(\omega; R) V \delta \mu_a(\bar{\mathbf{r}}) \right]^n G(\bar{\mathbf{r}}, \omega|\mathbf{r}_s) \tag{1}
\]

\[
= -G(\mathbf{r}_d, \omega|\bar{\mathbf{r}}) \frac{V \delta \mu_a(\bar{\mathbf{r}})}{1 + \bar{N}_{self}(\omega; R) V \delta \mu_a(\bar{\mathbf{r}})} G(\bar{\mathbf{r}}, \omega|\mathbf{r}_s)
\]

where \( \delta \mu_a \) is the excess absorption of the absorption site of size \( R \) and volume \( V \), \( \omega \) is the modulation frequency of light, \( G \) is the propagator of photon migration in the background medium, and \( \bar{N}_{self}(\omega; R) \) is the self-propagator which describes the probability that a photon revisits the volume \( V \).

Here \( G(\mathbf{r}_2, \omega|\mathbf{r}_1) \) gives the probability density that a photon leaves the volume at \( \mathbf{r}_1 \) and reenters it at \( \mathbf{r}_2 \). The scattering property of the site is the same as that of the background. In Eq. (1) \( G(\mathbf{r}_d, \omega|\bar{\mathbf{r}}) \) and \( G(\bar{\mathbf{r}}, \omega|\mathbf{r}_s) \) are well modeled by the center-moved diffusion model as long as the separations \( |\mathbf{r}_d - \bar{\mathbf{r}}|, |\mathbf{r}_s - \bar{\mathbf{r}}| \gg l \), however, the diffusion Green’s function can not be used in Eq. (2) to evaluate \( \bar{N}_{self}(\omega; R) \) because the diffusion approximation breaks down when \( \mathbf{r}_1 \) is in the proximity of \( \mathbf{r}_2 \).

Comparing Eq. (1) to the standard linear perturbation approach, the nonlinear multiple passage effect of an absorption site is represented by a nonlinear correction factor (NCF)

\[
NCF = \left[ 1 + \bar{N}_{self}(\omega; R) V \delta \mu_a(\bar{\mathbf{r}}) \right]^{-1}. \tag{3}
\]

This factor serves as a universal measure of the nonlinear multiple passage effect as long as the absorption site is far away from both the source and the detector and its size is much smaller than its distance to both the source and the detector. This correction is more significant when NCF is further away from unity.

The photon propagator \( N(\mathbf{r}_2, t|\mathbf{r}_1, \mathbf{s}) \), the probability that a photon propagates from position \( \mathbf{r}_1 \) with propagation direction \( \mathbf{s} \) to position \( \mathbf{r}_2 \) in time \( t \), for any separation between
\(\mathbf{r}_1\) and \(\mathbf{r}_2\), was recently derived in a form of the cumulant approximation to radiative transfer.

In the case of interest where the absorption site is deep inside the medium, the photon distribution is isotropic. The photon propagator is simplified to \(N_{\text{eff}}(r, t) \equiv N_{\text{eff}}(|\mathbf{r}_2 - \mathbf{r}_1|, t)\), obtained by averaging \(N(\mathbf{r}_2, t; |\mathbf{r}_1, \mathbf{s}|)\) over the propagation direction \(\mathbf{s}\) of light over the \(4\pi\) solid angle. In the frequency domain, this effective propagator is approximately given by

\[
N_{\text{eff}}(r, \omega) \simeq \begin{cases} 
\frac{1}{4\pi r^3 c} \exp \left( -\frac{1}{3} \kappa^2 l_t r \right) + \frac{\exp(-\kappa l_t)}{4\pi D r c} \sinh(\kappa r) & r < l_t \\
\frac{\exp(-\kappa r)}{4\pi D r c} \sinh(\kappa l_t) & r \geq l_t
\end{cases}
\]

where \(D \equiv l_t c/3\) and \(\kappa \equiv [3(\mu_a - i\omega)/l_t c]^{1/2}\) whose sign is chosen with a nonnegative real part. The two terms in \(N_{\text{eff}}\) when \(r < l_t\) represent ballistic and diffusion contributions respectively. The ballistic term does not depend on scattering because the photon distribution involved is already isotropic. Only diffusion contributes to \(N_{\text{eff}}\) when \(r > l_t\). The self propagator for an absorption sphere deep inside the medium is given by:

\[
\tilde{N}_{\text{self}}(\omega; R) = \frac{1}{V^2} \int_V \int_V N_{\text{eff}}(|\mathbf{r}_2 - \mathbf{r}_1|, \omega) d^3\mathbf{r}_2 d^3\mathbf{r}_1
= \frac{1}{V} \int_0^{2R} N_{\text{eff}}(r, \omega) \gamma_0(r) 4\pi r^2 dr
\]

where \(\gamma_0(r) = 1 - \frac{3r}{4R} + \frac{1}{16} \left( \frac{r}{R} \right)^3\) is the characteristic function for a uniform sphere. An absorption site of an arbitrary shape can be treated in the same fashion. The exact self propagator must be computed by a numerical quadrature. A good approximation of \(\tilde{N}_{\text{self}}(\omega; R)\) is

\[
\tilde{N}_{\text{self}}(\omega; R) = \frac{l_t}{V c} \left\{ \begin{array}{ll}
\left( \frac{3}{4} \xi + \xi^3 \right) - \xi^3 \kappa l_t + O(\kappa^2) & \xi \leq 1/2 \\
\left( \frac{9}{5} \xi^2 + \frac{1}{2} - \frac{3}{4} \xi^{-1} + \frac{3}{16} \xi^{-3} \right) - \xi^3 \kappa l_t + O(\kappa^2) & \xi > 1/2
\end{array} \right.
\]

using Eq. (4) where \(\xi \equiv R/l_t\) when \(|\kappa| R \ll 1\). This approximation compares favorably to the exact \(\tilde{N}_{\text{self}}(\omega = 0; R)\) obtained by numerical quadrature for continuous wave light propagating inside a nonabsorbing medium. The exact and approximate versions of the
dimensionless self-propagator $\bar{N}_{self} V l_t^{-1} c$ are plotted as solid and dashed lines, respectively, in Fig. (1a). The dimensionless self-propagator $\bar{N}_{self} V l_t^{-1} c$ depends solely on two dimensionless quantities $\kappa l_t$ of the background and $R/l_t$ of the absorbing sphere.

It is worthwhile to point out that the self-propagator in time $\bar{N}_{self}(t; R)$, the inverse Fourier transform of (5), includes the contribution from the ballistic motion of the photon when the photon passes through the site. This ballistic contribution manifests itself as the linear decay of $N_{self}(t; R)V$ in the form of $\gamma_0(ct)$ near the origin of time, followed by a transition to diffusion [Fig. (1b)].

The nonlinear correction factor is obtained by plugging (5) or (6) into (3). In particular, we have

$$NCF \simeq \begin{cases} 
\left[ 1 + \frac{9}{10\pi} q \left( \xi^{-2} + \frac{4}{3} \right) \right]^{-1} & \xi \leq 1/2 \\
\left[ 1 + \frac{9}{10\pi} q \left( \xi^{-1} + \frac{5}{12} \xi^{-3} - \frac{5}{32} \xi^{-4} + \frac{1}{128} \xi^{-6} \right) \right]^{-1} & \xi > 1/2 
\end{cases}$$

(7)

where $q \equiv V \delta \mu_a(\bar{r})/l_t^2 c$ is the dimensionless strength of the absorber when $|\kappa| R \ll 1$. The effectiveness of an absorber of a fixed strength depends on its size. For an absorber of a fixed $q > 0$, the effectiveness of absorbing light is diminished (NCF decreases) when its size is reduced. This can be understood from the fact that the photon spends less time per volume inside the absorber of a smaller dimension because of the ballistic motion of the photon after each scattering event. The photon leaves a small site ($R < l_t$) in an almost straight line. The diffusion behavior for an individual photon is only observed after a large number of scattering and on a scale larger than $l_t$.

Fig. (2) shows plots of NCF versus absorber size for typical absorbers of excess absorption $\delta \mu_a l_t/c$ equal 0.01 and 0.05, respectively. The nonlinear correction factor generally decreases with the size of the absorber whose excess absorption is fixed. With the increase of the background absorption and the modulation frequency, the nonlinear correction becomes less
accentuated. The phase delay is larger for higher modulation frequencies and less background absorption.

Monte Carlo simulations\textsuperscript{10} are performed for CW light propagating in a uniform nonabsorbing and isotropic scattering slab. The thickness of the slab is $L = 80l_t$. A spherical absorber of radius $R$ is located at the center $(0, 0, L/2)$ of the slab. The excess absorption of the absorber is $\delta\mu_{alt}/c = 0.01$. The absorber has the same scattering property as the background. The details of the MC computation has been provided in a previous publication\textsuperscript{11}. The correlated sampling method is used in each simulation to reduce variance.\textsuperscript{12} A single simulation is used to compute the change in light transmittance due to the presence of the absorption site and the corresponding nonlinear correction factor. Fig. (3a) shows the nonlinear correction factors obtained from numerical quadrature, the approximate form Eq. (7), and Monte Carlo simulations, respectively. The agreement between our theoretical NCF and that from Monte Carlo simulations is excellent. The slight difference between them at large radii is accounted for by the fact that the sphere can no longer be regarded as small compared to the dimensions of the slab. The probability of a photon revisiting a large sphere is overestimated by Eq. (5) for the sphere located at the center of the slab.\textsuperscript{15}

Fig. (3b) shows the percentage change of the CW transmittance estimated from the experimental data given in Fig. (9) of Ref. (5). The relevant parameters of the experiment are summarized in the inset. The theoretical predictions from the linear perturbation approach with and without the nonlinear correction are also shown in Fig. (3b) assuming a collimated point source and a point detector in a cofocal setup. The agreement between our theoretical prediction with nonlinear correction and the experimental result is good.

The typical value of the absorption coefficient of human tissues in the near infrared indicates that $\mu_{alt}/c \sim 0.01 - 0.05$.\textsuperscript{13,14} This fact should put our results on NCF in this
range [Figs. 2 and 3] into perspective. Our analysis reveals that the nonlinear correction for absorption inhomogeneities due to multiple passages of light in optical imaging of human tissues becomes important when the size of the inhomogeneity is much larger than the mean transport free path of light. This factor is smaller (further away from unity) for higher excess absorption and weaker background absorption. The value of NCF decreases from \( \sim 0.75 \) to \( \sim 0.5 \) for an absorption site of radius 5\( l_t \) with excess absorption \( \delta \mu_a l_t/c \) increasing from 0.01 to 0.05. This corresponds to 75\% and 50\% underestimations of the excess absorption coefficient in these cases if the standard linear perturbation approach is applied naively.

In conclusion, the nonlinear correction is analyzed for the effect of the multiple passages through an absorption inhomogeneity of finite size deep inside a turbid medium in optical imaging using the “self-energy” diagram. The nonlinear correction factor is verified by Monte Carlo simulations for CW light and supported by experimental results. The nonlinear correction becomes more important for an inhomogeneity of a larger size and with greater contrast in absorption with respect to the background. The standard linear perturbation approach in optical imaging should be augmented to include this nonlinear correction.

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15. Consider the facts: (1) the probability of a photon revisiting the sphere decreases when the position from which the photon leaves the sphere is further away from the center of the sphere, and (2) the photon density inside the sphere is higher in regions closer to its surface for the sphere located at the center of the slab. The arithmetic mean taken in Eq. (5) hence overestimates the revisiting probability.
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