Asymmetric balance in symmetry breaking

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Spontaneous symmetry breaking is central to our understanding of physics and explains many natural phenomena, from cosmic scales to sub-atomic particles. It can also be exploited for applications, but engineered systems naturally present some deviations from perfect symmetry. Surprisingly, the impact of such asymmetries has barely been studied. Here, we experimentally study spontaneous symmetry breaking in presence of two controllable asymmetries. We remarkably find that the characteristic features of spontaneous symmetry breaking, while dramatically destroyed by one asymmetry, can be entirely restored when a second asymmetry is introduced. In essence, asymmetries balance each other. Our study illustrates aspects of the universal unfolding of the pitchfork bifurcation. It also has practical implications, in particular showing that symmetry-breaking-based sensors can potentially exhibit divergent sensitivity even in presence of imperfections. Our experimental implementation built around an optical fibre ring additionally constitutes the first observation of the polarization symmetry breaking of passive nonlinear resonators.

Spontaneous symmetry breaking (SSB) is a concept of fundamental importance. It is central to the standard model of particle physics, underpins phenomena as diverse as ferromagnetism and superconductivity, and plays a key role in convection cells and fluid mechanics, morphogenesis, embryo development and more generally self-organization. SSB can also be exploited for many applications. Physical systems however often exhibit deviations from perfect symmetry because of naturally occurring imperfections. Surprisingly, the impact of asymmetries on SSB-related dynamics has barely been considered in experiments, and even then restricted to situations with only one asymmetry parameter. Here, we report for the first time an experimental study of a system that exhibits spontaneous mirror-symmetry breaking and that presents two controllable asymmetry parameters. While the characteristic dynamics of SSB — random, spontaneous selection between two mirror-like states with opposite symmetries — is destroyed by one asymmetry, we remarkably observe that a second asymmetry can restore them. In essence, the two asymmetries can balance each other. Interestingly, this is conceptually related to the design principle of asymmetric balance, by which a design or an art composition can be asymmetric, and yet, still look balanced.

Spontaneous symmetry breaking is underlain by the fundamental pitchfork bifurcation. For a system with left/right or mirror symmetry, that bifurcation describes how a symmetric state transitions to two equivalent, stable, mirror-like asymmetric states (see e.g. panel b of Fig. 1). The pitchfork is however a structurally fragile, degenerate bifurcation: in the presence of small asymmetries, one of the asymmetric state dominates while the other cannot be spontaneously excited. It turns out that only two parameters are needed to describe all the possible topologies of the perturbed pitchfork — its so-called universal unfolding. This argument has been used to reduce the number of parameters in the search of simplified models of complex problems, such as limb coordination — a feature found in movements of a huge range of animals — or the emergence of the ubiquitous homochirality of biological molecules. It has also guided recent engineering research in buckling-resistant structures and led to the discovery that optimal designs with imperfect symmetry only emerge when considering two asymmetry parameters. We note that evolution, which is inherently guided by optimization, has produced countless designs with near but not perfect symmetry, including the functional neural wiring of the brain. Clearly, considering two asymmetry parameters instead of one in the study of SSB can have dramatic and intriguing consequences. To the best of our knowledge, our work is the first to experimentally address how two asymmetries can balance each other.

Our experiment is based on a nonlinear optical fibre ring resonator that presents two distinct orthogonal polarization

![Figure 1](https://example.com/figure1.png)  
**Figure 1 | Polarization symmetry breaking.** a. Schematic diagram of a driven nonlinear fibre ring resonator. b. Pitchfork bifurcation diagram showing how the intensities of the two polarization modes part above a certain threshold. The control parameter can be either the driving power or the driving laser frequency. c. Resonance of the system illustrating how in an asymmetric state the stronger mode (large blue dot) can undergo a smaller effective nonlinear shift (black arrow) and be closer to resonance than the weaker mode (small orange dot). This occurs when the driving laser (green line) is red-detuned (left side of the resonance).
modes. The resonator is externally driven by intense laser light (Fig. 1a) so as to excite both of these modes; hereafter $E_+$ and $E_-$ denote their electric field amplitudes inside the resonator. Ideally, when the two polarization modes are equally driven and are degenerate (i.e., the resonator material is isotropic, and the modes have identical resonance frequencies), the system is symmetric with respect to an interchange of the two modes, $E_+ = E_-$. In the simplest case, the stationary intracavity field solution assumes that symmetry, $E_+ = E_-$, and the two modes have the same intensities. Symmetry breaking occurs above a certain threshold, and manifests itself by the parting of the intensities of the two polarization modes, $|E_+|^2 \neq |E_-|^2$ (Fig. 1b): the symmetric solution loses its stability in favour of two mirror-like asymmetric solutions. The instability stems from the cubic (Kerr) nonlinearity of silica optical fibres which couples the two modes through their intensities. Critically, in optical fibres, cross-coupling between polarization modes can be stronger than self-coupling so that the weaker mode can experience a larger nonlinear effect than the stronger mode. In these conditions, the weaker mode is pushed away from resonance while the stronger mode is pulled towards it, reinforcing any initial intensity imbalance (Fig. 1c). This polarization SSB is formally identical to the SSB that occurs in the same system when considering two counter-propagating beams and which was recently studied experimentally.\(^{35}\) In both cases, an imbalance of driving power between the two modes (beams) readily provides a controllable asymmetry parameter. In our experiment, we have also manipulated the wavenumbers of the two driven polarization components as the second asymmetry parameter.

Figure 2 illustrates the experimental setup. The ring resonator is built around a fibre coupler (beam-splitter) that recirculates 90% of the intracavity light, and allows for the injection of the driving field (entering from the right in the figure). Another 1% tap coupler extracts a small fraction of the intracavity field for analysis. The rest of the resonator is made up of “spun” single-mode silica optical fibre, a type of fibre which presents very little polarization anisotropy (birefringence). Due to unavoidable fibre bending and other imperfections, the two polarization modes of the resonator are however slightly linearly coupled. As a consequence, the interactions between the two modes are not only dependent on the modal intensities as described above, but are also phase sensitive. To avoid this complication, we drive the two polarization modes with slightly different carrier frequencies. At the same time, we purposefully introduce some fixed birefringence in the resonator (set through an intracavity polarization controller) to counterbalance the associated difference in wavenumbers, and to realize effective isotropic (or close to isotropic) conditions for the two driven polarization components. The dual carrier driving field is prepared, ahead of injection into the resonator, by splitting the output of a single frequency laser into two components with a polarization beam-splitter (PBS), and frequency-shifting one of these with an acousto-optic modulator (AOM). Through recombination in a second PBS, these two components are given orthogonal polarizations and, using another PC (labelled PC\(_{\text{map}}\)), are mapped onto the two resonator modes (see Methods). This arrangement enables simple and reproducible control of two separate asymmetry parameters. On the one hand, adjusting the RF frequency applied onto the AOM (around 80 MHz) controls the effective isotropy, specifically the difference in wavenumbers $\delta \Delta$ with which the two polarization components...
propagate inside the resonator (see Methods for normalization). On the other hand, controlling the polarization state of the beam ahead of the first PBS changes the ratio of driving power between the two modes, without affecting the total driving power. This is achieved with an electronic polarization modulator, complemented with a manual bias (PC\(\chi\)).

In the following, we express the driving power ratio as \(\tan^2 \chi\), in terms of an effective driving polarization ellipticity angle \(\chi\). An ellipticity angle \(\chi\) of \(45^\circ\) represents perfectly balanced driving. In our experiment, \(\chi\) is measured and monitored (see Methods) before PC\(\chi\), close to the resonator input, and stabilized against any environmental drift through feedback to the polarization modulator. The driving laser frequency can be either scanned or locked at a set detuning \(\Delta\) from resonance using another feedback loop (not shown in Fig. 2) using the method of Ref. 42). Note that changing the detuning \(\Delta\) changes the wavenumbers of the two driven polarization components by the same amount and does not introduce any asymmetry. When both feedback loops are engaged, all the parameters of the resonator are quantifiably controlled and stable. Finally, to reach more easily the power threshold of SSB, the resonator is synchronously driven by flat-top 1 ns long pulses carved into the continuous-wave (cw) beam of our driving laser with an amplitude modulator (AM). Two such pulses are launched per roundtrip, for reasons explained below. Note that the total peak driving power is kept at the same level for all the results discussed below (see Methods).

We start by characterizing our system in symmetric conditions: the driving ellipticity \(\chi\) is maintained at \(45^\circ\) by the feedback loop while \(\delta \Delta\) is set to zero. To this end, we scan the driving laser frequency across the cavity resonance while resolving the two polarization modes (Fig. 3 blue and orange curves respectively). The modes are separated at the cavity output by a PBS and their intensities measured with slow photodiodes that do not resolve the individual driving pulses. In Figs 3a and 3b, we present histograms of each mode intensity accumulated over many scans. Remarkably, while the two modes have equal intensities near the base of the resonance (in line with the symmetric conditions), the peak of the resonance exhibits a high degree of variability. We observe that high intensity in one mode always correlates with low intensity in the other mode, as is made evident by the two individual scans shown in Figs 3c-d: symmetry is markedly broken. Here we have also plotted the total output intensity (black curves) measured at the resonator output by a third photodiode. The total intensity does not display any sign of the underlying pitchfork instability, thus highlighting that the SSB studied here is a purely polarization phenomenon. To the best of our knowledge, this is the first observation of a polarization SSB in passive driven resonators. Additionally, a comparison between Figs 3 and 4 highlights the very high degree of mirror symmetry in our system, with the two sets of curves very nearly matching each other. The high variability in Figs 3a-b can be interpreted as due to different subparts of the two pulses circling the resonator spontaneously breaking their symmetry one way or the other, randomly. This leads to averaged intensities spanning the entire range of values between those observed when pulses switch as a whole (panels c and d correspond to that latter case) even though the system has only two stationary solutions that are mirror of each other.

Departing from symmetric conditions through a change in \(\chi\) or \(\delta \Delta\) leads to the disappearance of the behaviour reported in Fig. 3. The system then always favours the same mode: resonance scans look either like the one presented in Fig. 3c, or the one in Fig. 3d, depending on the direction of the change. A secondary state, an almost mirror image of the first one, is never excited spontaneously although it might be present in the system. In order to probe its existence under asymmetric conditions, we apply strong perturbations to the two driving pulses through the polarization modulator (see top left of Fig. 2). The two pulses driving the resonator are subject to opposite perturbations to maximize the chance that one of the pulses will switch to the other solution, irrespective of which solution is initially spontaneously excited. Before and after applying the perturbations, measurements of the output pulse peak intensity levels for each of the polarization modes are taken with fast photodiodes connected to a high sampling-rate.
The results are summarized in Fig. 4, where we plot the modal intensities of the identified stationary solutions (squares). Panel e has been obtained with $\delta\Delta \approx 0$. The two solutions identified for $\chi = 45^\circ$ are exact mirror image of each other, as expected from perfect symmetry conditions (see Fig. 3). As $\chi$ is varied around that point, we can observe that both solutions continue to exist, even though their degeneracy is lifted. Theoretical predictions (smooth curves) agree very well with the measurements. Note that some solutions predicted theoretically are not observed in the experiment because they are only metastable in our pulse driving conditions. The range of co-existence is highlighted as a yellow band, and is reasonably wide, covering almost 10$^\circ$ of ellipticity change. Outside that band, however, the driving asymmetry becomes too strong, and only one solution remains: the symmetry breaking instability effectively disappears. In Fig. 4e, we have introduced some asymmetry between the wavenumbers. Interestingly, we observe that the coexistence region seems to simply shift to a different range of values of $\chi$. In particular, there exists a value of $\chi \neq 45^\circ$ where the two solutions are again mirror images of each other (where the blue and orange curves intersect). The two asymmetries, in $\chi$ and $\delta\Delta$, are now balancing each other. This asymmetric balance is very robust: in Fig. 4c, $\delta\Delta$ is large enough for the coexistence region not to even overlap with the balanced driving condition at $\chi = 45^\circ$. Eventually, when too much asymmetry is present (panel d), the coexistence region disappears: symmetry breaking is well and truly destroyed and cannot be restored through a balance of asymmetries. Fig. 4e highlights how all the experimental data in Figs 4a–d (plus some extra measurements) fit together. We must stress that the theoretical fits shown in Fig. 4 have all been obtained for the same parameters values and with the values of $\delta\Delta$ directly measured experimentally. This makes the overall agreement all the more remarkable.

To explore further the regime of asymmetric balance, we performed additional resonance scan measurements with parameters close to those where we find mirror-like solutions. For $\delta\Delta = 0.63$ (corresponding to Fig. 4c), this is illustrated in Fig. 5 using the same format as Fig. 4a. Remarkably, the histograms of Figs 5a,b reveal, for a critical value of driving ellipticity $\chi \approx 53.5^\circ$, and despite the strong asymmetries, the system presents again a high degree of variability similar to that observed under symmetric conditions. The two individual scans plotted in Figs 5c,d highlight that the observed variability stems from the random selection of one of two solutions of opposite symmetries, i.e. in which a different polarization mode dominates. The fact that the “orange” mode is driven more strongly than the “blue” mode (the driving ellipticity corresponds here to a factor of about 1.8 difference in driving intensity) is evident in Fig. 5 yet it does not preclude a “blue” dominant solution to be spontaneously excited (Fig. 5b). Similar to the symmetric case, and perhaps more remarkably, we again observe no sign of the instability in the total intensity (black curves in Figs 5c,d). These results show that the concept of asymmetric balance introduced above goes much further than just restoring solutions that are mirror of each other. A proper SSB, and a pitchfork-like dynamics, truly exists when asymmetries are balanced. We note that this behaviour agrees with what would be expected from the two-parameter unfolding of the pitchfork bifurcation. Moreover, these findings are not specific to the parameters of Fig. 5; we have observed them for all other values of $\delta\Delta$ in the range
Figure 5 | Nonlinear resonances in conditions of asymmetric balance. Same as Fig. 3 but for δΔ = 0.63 (same value as in Fig. 4) and χ = 53.5°. These measurements illustrate that a SSB-like response can be found under asymmetric conditions when asymmetries in χ and δΔ are critically balanced. In particular, a “blue” dominated solution can still be spontaneously excited, even though the “orange” mode is driven stronger. Grey lines in c,d highlight how the two solutions observed for Δ = 5.45 are close mirror of each other, corresponding to the crossing point in Fig. 4.

where mirror-like solutions can be restored. Note that the critical value of χ found in Fig. 5 (53.5°) is slightly different with that observed to give mirror-like solutions in Fig. 4 (51°), but this is consistent with the dependence of the critical point on the cavity detuning Δ and matches numerical predictions.

In summary, we have studied experimentally a system presenting a SSB instability in presence of two asymmetry parameters. By systematically tracking the different stationary solutions of the system with controlled and quantified asymmetries, we have observed that, while the SSB dynamics is destroyed by one asymmetry, it can actually be restored by a second properly balanced asymmetry. To the best of our knowledge, this is the first experimental realization of a full SSB recovering through a controlled balance of two asymmetries. Given the importance and ubiquity of SSB in the physical sciences, our work is relevant to numerous fields. In particular, it could be extended to other multimode systems, and it shows that applications of SSB in sensing based on real, necessarily imperfect, physical platforms, can potentially still exhibit divergent sensitivity.

Measurement of the driving ellipticity. The driving ellipticity χ is measured in our experiment by monitoring the powers of the

METHODS

Model and notations. The evolution of our system over time t can be described by a set of coupled cw (dispersion-less) Lugiato-Lefever equations for the complex scalar amplitudes of the electric fields of the two polarization modes, E+ and E−:

\[
\frac{\partial E_+}{\partial t} = \left[ -1 + i(\Delta_+ + \Delta) \right] E_+ + \sqrt{\chi} \cos \chi \\
\frac{\partial E_-}{\partial t} = \left[ -1 + i(\Delta_+ + \Delta) \right] E_- + \sqrt{\chi} \sin \chi
\]  

(1)

(2)

In these equations, the incoherent coupling between the two modes is determined by the cross-coupling coefficient B. Other terms on the right-hand side represent, respectively, losses, the Kerr effect (self-phase modulation), the detuning of the driving frequency with respect to resonance, and the driving strength, with X representing the total driving power. Because of residual birefringence in our fibre resonator, the resonances of the two polarization mode families are normally observed for different driving laser frequencies, which correspond to using different detunings in the equations above, Δ+ ≠ Δ−. The difference in detuning, Δ = Δ+ − Δ−, is tuned in our experiment by shifting the carrier frequency of the − mode with the AOM as explained in the main text. Symmetry is obtained with χ = 45° and δΔ = 0. The normalization is the same as that used in Ref. 46 (see also section below regarding normalization of wavenumbers).

Extra experimental details. Our fibre ring resonator has a total length of about 10.5 m, corresponding to a free spectral range of 19.76 MHz (±20 kHz) and a round trip time τp of 50.60 ns (±50 ps). The measured finesse is 24.1 (±0.1), amounting to total losses of 26% per roundtrip. The associated photon lifetime and resonance linewidth are, respectively, about 4.9 ns and 820 kHz. The driving laser is a Koheras AdjustiK E15 laser, with a linewidth <1 kHz. Apart from the couplers, the resonator is made of a spun fibre, that exhibits normal group-velocity dispersion (−40 ps/nm/km) at the 1550 nm driving wavelength. The normal dispersion has been selected to avoid modulation instabilities. Driving pulses are 1.04 ns long, and separated by 24.5 ns. The separation is chosen large enough to minimize unwanted ripples in the AM driving electronics, while at the same time avoiding the acoustic echo generated by the leading pulse and that would affect the shape of the trailing pulse for separations in the 20–22 ns range. Calibration of the normalized driving power X was obtained by observing the nonlinear shift of the resonance as a function of driving power. All the experimental and numerical results presented here have been obtained with X = 10.8, which corresponds to about 110 mW of total averaged power entering the resonator. The nonlinear cross-coupling coefficient B was obtained from the ratio of the nonlinear shift of the cavity resonance peak for balanced driving condition (χ = 45°) to that observed when only one mode was driven (χ = 0°). That ratio is (1 + B)/2, and is independent of driving power. Two separate measurements gave values of B of 1.55 and 1.6. That value was subsequently refined to B = 1.57 by fitting the data shown in Figs 4d–d to theoretical curves. The fit was found to depend very sensitively on the value of B.
two polarization components of a small fraction of the driving field with two carefully calibrated photodetectors. First, we have made sure that the photodetectors are operated strictly in a linear regime. Second, we have measured a calibration factor that corrects for the small difference in responsivities between the two diodes, so that we get the same reading when they are illuminated by the same intensity. Finally, before each set of measurements, we carefully measure the zero level of both diodes. tan^2 χ is then obtained as the ratio of the two photodiodes readings after zeroing and responsitivity correction. This leads to the value of χ with an uncertainty that we estimate at least less than 0.5°.

**Adjustments of all the polarization controllers.** The setup incorporates a number of polarization controllers (PCs) which must all be set in well defined positions for the experiment to run as intended. We describe here how they can all be adjusted one by one unequivocally. We first adjust the intracavity PC (PC_{intrac}) such that the two families of orthogonally polarized resonances of the cavity (observed on the total output intensity) are well separated — to avoid linear mode coupling. In our case, the separation was set at about 45 % of the FSR (the precise value is not critical). Next, we turn off the AOM, which suppresses one of the polarization components of the driving beam, and we adjust PC_{monitor} so that one of the photodiode used to monitor the driving ellipticity reads zero. With the AOM still off, we then adjust PC_{map} so as to excite a single family of cavity resonances, carefully cancelling any trace of the other (orthogonally polarized) family through observation of the total output intensity. In that configuration, the beams that go through the two paths of the Mach-Zehnder interferometer each drive one corresponding polarization mode of the resonator. Finally, PC_{out} at the cavity output is adjusted by observing the output signals behind the PBS that follows using fast photodiodes, with both modes driven (AOM is on) and the cavity detuning locking loop engaged. Here we use the fact that the two orthogonally polarized modes that we are trying to separate are also frequency-shifted, by about 80 MHz. Oscillations resulting from the beating of the two modes can thus be observed with fast photodiodes whenever the two modes are not properly separated by the PBS. We thus simply adjust PC_{out} so as to cancel that beating on both photodiodes. This guarantees that the corresponding photodiodes are only sensitive to one particular cavity polarization mode. Finally, PC_y is used to bias the driving ellipticity, one of our asymmetry parameters. Note that changing the polarization state before the Mach-Zehnder interferometer (through PC_y or through the polarization modulator) has no effect on the total driving power, as the variable attenuator placed in the interferometer is set so as to introduce the same losses as the AOM in the other arm. This is adjusted by monitoring the absence of any modulation of the total power behind the recombining PBS while applying a sinusoidal modulation onto the polarization modulator.

**Measurement and normalization of wavenumbers and cavity detunings.** The definitions below apply equally to both polarization modes of the resonator (+ and −) but to simplify the notations we start by focusing on the + mode. Assuming light driven in that polarization component propagates with a wavenumber \( \beta_+ \), that wave accumulates over one roundtrip in the resonator a linear phase shift \( \beta_+ L \) (with respect to the driving field), where \( L \) is the resonator length. We define the corresponding phase detuning \( \Delta \alpha = 2 \pi n_0 - \beta_+ L \) as the distance (in phase) to the closest resonance (of index \( m \)). A positive value of phase detuning corresponds to a driving beam red shifted with respect to the corresponding linear resonance. Normalized detuning is defined as \( \Delta_\alpha = \Delta \alpha / \alpha \), where \( \alpha \) represents the resonator losses, specifically half the percentage of power lost per roundtrip. With that notation, the resonator finesse is simply given by \( \Phi = \pi / \alpha \). The normalized difference in wavenumbers is then defined as \( \delta \alpha = \Delta_\alpha - \Delta_\alpha \).

To calibrate the measurement of \( \delta \alpha \), we first observe for which AOM frequency the linear resonances of the two polarization modes overlap and have maximal total peak intensity. This stage is performed when the resonator is operated purely in the linear regime, and at that point \( \delta \Delta = 0 \). From that starting position, any change \( \Delta f \) in the frequency applied to the AOM corresponds to a change of \( \delta \Delta \) of \( 2 \pi (\Delta f / \text{FSR}) / \alpha = 2 \Phi (\Delta f / \text{FSR}) \). With the uncertainties quoted above for \( \Phi \) and FSR, this is obtained to within 0.5 %. A similar procedure is used to measure the normalized detuning parameter \( \Delta_\alpha \) (corresponding to the mode that is not frequency shifted, i.e. to the blue curves in all our figures); see Ref. [22].

**Measurements of Figure 4.** For each set of parameters (\( \chi \) and \( \Delta_\alpha \)), the intensities of the two output polarization components are measured with 10 GHz-bandwidth photodiodes and acquired with a 40 Gsamples/s oscilloscope. The acquired traces resolve individual output pulses over 8000 subsequent cavity round-trips. The trigger is setup so that the polarization perturbation, applied for about 100 roundtrips, appears in the middle of the acquisition. The instantaneous intensity levels of the recorded pulses are extracted from the traces, and built into histograms over 1000s of pulses, separately for the two pulses driving the resonator, and before and after the perturbation. Care is taken to avoid any transients following the perturbation, and pulses that are only partly switched. The process is repeated for the two polarization components. The levels corresponding to the maxima of the histograms are what is plotted in Fig. 4. The error level is indicated by the size of the markers.

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**AUTHOR CONTRIBUTIONS**

B.G. and J.F. built the preliminary experimental setup, which B.G. and S.C. subsequently improved. B.G. and S.C. had the original idea and interpreted the results. B.G. and S.C. had the original idea and S.C. subsequently improved. B.G. and J.F. built the preliminary experimental setup, which B.G. and S.C. had the original idea and interpreted the results.

**ADDITIONAL INFORMATION**

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**COMPETING FINANCIAL INTERESTS**

The authors declare no competing financial interests.