Economics of limiting cumulative CO$_2$ emissions

Ashwin K Seshadri

Divecha Centre for Climate Change, Indian Institute of Science, Bangalore 560012, India. email: ashwin@fastmail.fm

Abstract

Global warming from carbon dioxide (CO$_2$) is known to depend on cumulative CO$_2$ emissions. We introduce a model of global expenditures on limiting cumulative CO$_2$ emissions, taking into account effects of decarbonization and rising global income and making an approximation to the marginal abatement costs (MAC) of CO$_2$. Discounted mitigation expenditures are shown to be a convex function of cumulative CO$_2$ emissions. We also consider minimum-expenditure solutions for meeting cumulative emissions goals, using a regularized variational method yielding an initial value problem in the integrated decarbonization rate. A quasi-stationary solution to this problem can be obtained for a special case, yielding decarbonization rate that is proportional to annual CO$_2$ emissions. Minimum-expenditure trajectories in scenarios where CO$_2$ emissions decrease must begin with rapid decarbonization at rate decreasing with time. Due to the shape of global MAC the fraction of global income spent on CO$_2$ mitigation ("burden") generally increases with time, as cheaper avenues for mitigation are exhausted. Therefore failure to rapidly decarbonize early on reduces expenditures by a small fraction (on the order of 0.01 %) of income in the present, but leads to much higher burden to future generations (on the order of 1 % of income).

1 Introduction

Global warming from carbon-dioxide (CO$_2$) is related to its cumulative emissions, i.e. emissions integrated across time, and independent of emissions pathway (Allen et al. (2009); Matthews et al.)
Mitigation of global warming requires large-scale and expensive efforts to decarbonize the world economy (Manne and Richels (1993); Grubb (1993); Weyant (1993); Nordhaus (1993a)). Estimated costs of mitigation vary across different studies and depend on the assumptions that are made (Grubb (1993); Rogelj et al. (2015)). The Stern Review of Climate Change (Stern (2007)) estimated an average cost of reducing anthropogenic emissions of climate forcers to a 550 ppm CO\(_2\) equivalent stabilization level of about 1 percent of Global Gross Domestic Product (GGDP), which is much smaller than damage costs of unmitigated global warming (Stern (2007)). In a recent study Rogelj et al. (2015) illustrate using the integrated assessment model MESSAGE that for scenarios limiting global warming to less than 2 degrees C the cost of mitigation ranges from a small fraction of 1% to a few % of GGDP, and an important factor governing the costs is baseline energy demand (Rogelj et al. (2015)).

This paper introduces an analytical model for global expenditures on reducing CO\(_2\) emissions from economic activity ("decarbonization"). We represent the marginal abatement cost (MAC) of reducing CO\(_2\) emissions as a function of emissions intensity of GGDP. If increase in GGDP leads to relative increase in CO\(_2\) emissions by a smaller factor, because global income elasticity of CO\(_2\) emissions is less than 1, there will occur exogenous reductions in emissions intensity as the global economy grows; this will happen even in the absence of any deliberate mitigation effort, in the "business as usual" scenario. Exogenous reductions in emissions intensity may be viewed as being the result of technological improvements with time, in models where the production function is explicitly present (Nordhaus (1993b)). Our model does not include the production function, but instead uses the income elasticity of global CO\(_2\) emissions as a constant parameter. This permits a simple treatment of the time-dependence of the MAC curve, even in the presence of exogenous reductions in emissions intensity, but our discussion is limited by the assumption of constant elasticity.

We integrate the resulting MAC curve to consider global expenditures on mitigation. Our model considers expenditures for reducing emissions intensity (i.e. decarbonization) as well as those involved in scaling up decarbonization activity as the global economy expands. This can help understand factors behind the scale of economic effort involved in global decarbonization. Because of the centrality of cumulative CO\(_2\) emissions to the global warming problem (Stocker et al. (2009); Stocker et al. (2013); Seshadri (2017)).
(2013); Friedlingstein et al. (2014); Raupach et al. (2014); Rozenberg et al. (2015); Peters (2016); Pfeiffer et al. (2016)), we also consider how to minimize costs of limiting cumulative emissions over time. This is examined through a constrained variational problem (van Brunt (2004)), minimizing a functional describing discounted total mitigation expenditures while satisfying a constraint on cumulative emissions across a specified time-horizon. The policy variable is the decarbonization rate, the latter being the rate with which emissions intensity is reduced. Minimizing the functional leads to the familiar Euler-Lagrange equation (van Brunt (2004)). For our model of CO₂ mitigation expenditures, the variational problem is degenerate, and the Euler-Lagrange equation is algebraic, whereas we seek an initial condition problem in the integrated decarbonization rate. This is an ill-posed problem (Tikhonov and Arsenin (1977)), because a solution satisfying the initial condition on the integrated decarbonization rate does not exist for the algebraic Euler-Lagrange equation that is obtained. One approach for dealing with ill-posed problems is through regularization (Tikhonov and Arsenin (1977)), by adding an additional term to the quantity being minimized in order to render it soluble. In our case we include an additional contribution to the functional being minimized for rendering an initial value problem in the Euler-Lagrange equation.

The aforementioned approach contrasts with the optimal control problem of choosing an emissions pathway to maximize discounted utility, taking into account costs of mitigation as well as damage costs of global warming, which underlies integrated assessment models of global warming such as DICE (Nordhaus (1993a,b)). In such approaches the optimal mitigation pathway is such that the effect on utility of marginal increment to current consumption from increasing emissions is balanced by the present value of the diminution in future consumption (Nordhaus (1993a)). Without considering the production function or climate damages in the present paper we cannot model the above features, and instead assume that cumulative emissions goals are specified exogenously, following contemporary discussions in climate policy (Meinshausen et al. (2009); Peters (2016)).

Section 2 introduces the models of global CO₂ emissions and mitigation expenditures, and the results derived from them. Section 3 considers the expenditure-minimizing pathways of decarbonization, subject to a cumulative emissions constraint. The control variable is the integrated decarbonization rate. As mentioned previously, the original problem is degenerate, and must be

See, for e.g., (van Brunt (2004)) for a general discussion of such cases.
regularized to give rise to an initial value problem in the integrated decarbonization rate. Section 4 presents numerical illustrations of the main results of the paper.

2 Models

2.1 CO₂ emissions under business as usual

We describe global CO₂ emissions \( m(t) \) as the product of GGDP, denoted by \( g(t) \), and global emissions intensity \( \mu(t) \); with present values \( m_0, g_0 \) and \( \mu_0 \). In the absence of deliberate mitigation, under business as usual growth in GGDP leads to increase in global CO₂ emissions that is governed by the constant-elasticity model \( \Delta m/m = \theta \Delta g/g \), where \( \theta \) is the global income elasticity of global CO₂ emissions. Considering small time-interval \( \Delta t \) the emissions intensity at \( t + \Delta t \) has formula:

\[
\frac{m + \Delta m}{g + \Delta g} = \left(1 + (\theta - 1) \frac{\Delta g}{g}\right) \mu
\]

The change \( \Delta \mu \) in emissions intensity is the above ratio minus \( \mu(t) \), so its rate of change during interval \( \Delta t \) is

\[
\frac{\Delta \mu}{\Delta t} = -(1 - \theta) r \mu
\]

where \( r = (\Delta g/\Delta t)/g \) is growth rate of GGDP during this period. Generally \( \theta \) is smaller than one, leading to exogenous decrease of emissions intensity at rate \( \sigma = (1 - \theta) r \). This effect is independent of any deliberate mitigation effort, occurring under business as usual. It is larger for higher growth rates of GGDP if \( \theta < 1 \). For example, GGDP growth at constant rate \( r = 4.0\% \) (which is close to the historical mean during 1950-2014) and \( \theta = 0.75 \) yields constant exogenous decarbonization rate of 1% per year. However we caution that income elasticity of energy demand varies by country and is smaller for developed nations (Webster et al. (2008)). Therefore it is liable to change with time as countries develop, and our assumption of constant elasticity is only an idealization.

Absence of mitigation can also adversely impact economic growth. Quoting Stern (2016), "So the business-as-usual baseline, against which costs of action are measured, conveys a profoundly misleading message to policymakers that there is an alternative option in which fossil fuels are consumed in ever greater quantities without any negative consequences to growth itself."

We write \( \frac{m + \Delta m}{g + \Delta g} \) as \( \mu \frac{1 + \Delta g}{1 + \frac{\Delta g}{g}} \), expand the denominator by its Taylor series in \( \frac{\Delta g}{g} \) and approximate to first-degree in \( \frac{\Delta g}{g} \), obtaining equation (1).
Smaller GGDP growth rates would lead to smaller exogenous decarbonization rates. If the rate of
growth of GGDP were only 1.2 % then $\sigma = 0.3$ % per year. This paper considers only scenarios
with constant growth rate of GGDP, so that with constant $r$ and $\theta$ and in the absence of deliberate
reductions, the emissions intensity at time $t$ would be $\mu(t) = \mu_0 e^{-\sigma t}$, where $t = 0$ denotes the
present. Of course, if elasticity $\theta = 1$ then there is no exogenous decarbonization in our model, and
decarbonization occurs only through deliberate mitigation.

2.2 Marginal abatement cost of mitigation

Increase in GGDP leads to increases in emissions as described above and corresponding expansion of
mitigation possibilities, thereby stretching horizontally the marginal abatement cost (MAC) curve
as the global economy grows. In the absence of deliberate reductions, this effect is governed by
exogenous decarbonization rate $\sigma$ so the MAC curve expressed in terms of emissions intensity is

$$C(\mu(t)) = \frac{\alpha}{\left(\frac{\mu(t)}{\mu_0 e^{-\sigma t}}\right)^\nu}$$

where $C(\mu(t))$ is cost of reducing emissions intensity by one unit, for each unit of GGDP and
$\nu > 0$ is a constant parameter. In case there is no deliberate mitigation so that $\mu(t)$ is equal to
$\mu_0 e^{-\sigma t}$ for all times then the MAC in our model remains constant at $\alpha$, and thereby this model
neglects effects of learning. As deliberate mitigation proceeds thereby reducing ratio $\mu(t)/\mu_0 e^{-\sigma t}$
the marginal cost increases. The MAC $C(\mu(t))$ is in units of billion $$/ (Gton CO_2 year^{-1})$. The
graph represented by $C(\mu(t))$ has the same scale as MAC curves described in $$/ ton CO_2.

Parameter $\alpha$ describes the present MAC, in billion $$/ (Gton CO_2 year^{-1})$. The present cost of
reducing emissions intensity by $\Delta \mu$ in a year is $\alpha g_0 \Delta \mu$. The factor $g_0 \Delta \mu$ has units of emissions
(Gton CO_2 year^{-1}). For decreasing emissions intensity between time $t$ and $t + \Delta t$ by $\Delta \mu(t)$,
the mitigation expenditure is $C(\mu(t)) g(t) \Delta \mu(t)$. GGDP is in units of trillion $$/ year, and
global emissions intensity in Gtons CO_2 / trillion $\$, so expenditure is in (billion $$/ (Gton CO_2
year^{-1}))*(trillion $$/ year)*(Gton CO_2/ trillion $\$), or billions of dollars.

\footnote{A simple model for exogenous learning would make $\alpha$ a function of time.
2.3 Effect of mitigation at rate $k(t)$

Deliberate reduction of emissions intensity, or mitigation, occurs at rate $k(t)$, with the effect over time-interval $\Delta t$ being $\Delta \mu(t) = -k(t) \mu(t) \Delta t$. In conjunction with exogenous reductions arising during economic expansion in case $\theta < 1$, the rate of change in emissions intensity is $\Delta \mu(t) / \Delta t = -(k(t) + \sigma) \mu(t)$, which is integrated for $\mu(t) = \mu_0 e^{-\int_0^t k(s) ds} e^{-\sigma t}$. We write this in terms of integrated decarbonization rate $K(t) = \int_0^t k(s) ds$, so that $\mu(t) = \mu_0 e^{-K(t)} e^{-\sigma t}$. Decarbonization in the form of this integrated rate is the policy variable in the optimization problem of Section 3.

2.4 Mitigation expenditure

The expenditure on mitigation has two contributions: initial cost associated with reducing emissions intensity, and scaling up mitigation to maintain reduced levels of emissions intensity as GGDP increases.

2.4.1 Reducing emissions intensity

Deliberate reductions in emissions intensity require expenditures at levels described by the MAC curve. Reducing emissions intensity by $\Delta \mu(t)$ between time $t$ and $t+\Delta t$ costs $C(\mu(t)) g(t) |\Delta \mu(t)|$. Only the deliberate reduction in emissions intensity $\Delta \mu(t) = -k(t) \mu(t) \Delta t$ contributes to mitigation expenditures, so this contribution between time $t$ and $t+\Delta t$ is $\alpha (\mu(t) / (\mu_0 e^{-\sigma t}))^{-\nu} g(t) k(t) \mu(t) \Delta t$, with discounted sum across the specified time-period being

$$E_\mu = \alpha \mu_0^\nu \Delta t \sum_t e^{-\delta t} e^{-\nu \sigma t} g(t) k(t) (\mu(t))^{1-\nu}$$  \hspace{1cm} (4)

where $\delta$ is the rate of time-discounting. Substituting for emissions intensity $\mu(t) = \mu_0 e^{-K(t)} e^{-\sigma t}$

$$E_\mu = \alpha \mu_0 \Delta t \sum_t e^{-\delta t} e^{-\sigma t} g(t) \hat{K}(t) e^{(\nu-1)K(t)}$$  \hspace{1cm} (5)
2.4.2 Expansion of mitigation with growth in GGDP

The second contribution to expenditures is due to scaling up of mitigation for maintaining lower levels of emissions intensity as the global economy expands. Between times $t$ and $t + \Delta t$ the activities involved in reducing emissions intensity from $\mu_0 e^{-\sigma t}$, the value it would have in the absence of deliberate reductions, to $\mu(t)$ that it actually has must be expanded proportionally to increase in GGDP during this period. For each unit of GGDP, total cost of reducing emissions intensity from $\mu_0 e^{-\sigma t}$ to $\mu(t)$ at time $t$ is

$$\int_{\mu_0 e^{-\sigma t}}^{\mu(t)} \frac{\alpha}{\left(\mu / (\mu_0 e^{-\sigma t})\right)^\nu} (-d\mu)$$  \hspace{1cm} (6)

Time $t$ is fixed in the above equation so factor $e^{-\nu \sigma t}$ can be taken outside the integral. This contribution increases proportionally with change in GGDP, so expenditure between time $t$ and $t + \Delta t$, on scaling up mitigation when GGDP increases from $g(t)$ to $g(t) + \Delta g(t)$, is

$$\frac{\alpha \mu_0 e^{-\nu \sigma t}}{\nu - 1} \left(\frac{1}{\mu(t)^{\nu - 1}} - \frac{1}{\mu_0^{\nu - 1} e^{-(\nu - 1) \sigma t}}\right) \Delta g(t)$$  \hspace{1cm} (7)

and substituting for $\mu(t)$ as before the total discounted expenditure becomes

$$E_g = \frac{\alpha \mu_0}{\nu - 1} \sum_t e^{-\delta t} e^{-\sigma t} \left(e^{(\nu - 1)K(t)} - 1\right) \Delta g(t)$$  \hspace{1cm} (8)

2.4.3 Total expenditure

The total discounted expenditure $E_\mu + E_g$ in continuous-time is

$$E(t) = \int_0^t e^{-\delta s} P_\mu(s) \, ds + \int_0^t e^{-\delta s} P_g(s) \, ds$$  \hspace{1cm} (9)

where

$$P_\mu(t) = \beta e^{-\sigma t} g(t) \dot{K}(t) e^{(\nu - 1)K(t)}$$  \hspace{1cm} (10)
is annual expenditure from reducing emissions intensity, where $\beta = \alpha \mu_0$, and

$$P_g(t) = \frac{\beta}{\nu - 1} e^{-\sigma t} g(t) \left( e^{(\nu - 1)K(t)} - 1 \right)$$  \hspace{1cm} (11)

is that from expansion of mitigation. Exogenous decarbonization has the effect of decreasing both contributions to future expenditure by $e^{-\sigma t}$. We therefore introduce the parameter $\rho = \sigma + \delta$, combining its effect with that of discounting in time.

In scenarios with constant growth rate of GGDP and constant mitigation rate $k$ the discounted expenditure then becomes

$$E(t) = \beta g_0 k \int_0^t e^{((\nu - 1)k + r - \rho)s} ds + \frac{\beta g_0 r}{\nu - 1} \int_0^t \left( e^{((\nu - 1)k + r - \rho)s} - e^{(r - \rho)s} \right) ds$$  \hspace{1cm} (12)

integrating to

$$E(t) = \beta g_0 k \frac{e^{((\nu - 1)k + r - \rho)t} - 1}{(\nu - 1)k + r - \rho} + \frac{\beta g_0 r}{\nu - 1} \left( \frac{e^{((\nu - 1)k + r - \rho)t} - 1}{(\nu - 1)k + r - \rho} - \frac{e^{(r - \rho)t} - 1}{r - \rho} \right)$$  \hspace{1cm} (13)

In case of large mitigation rates and long time-horizons $\frac{e^{((\nu - 1)k + r - \rho)t} - 1}{(\nu - 1)k + r - \rho} \gg \frac{e^{(r - \rho)t} - 1}{r - \rho}$, so that

$$E(t) \approx \beta g_0 \left( k + \frac{r}{\nu - 1} \right) \frac{e^{((\nu - 1)k + r - \rho)t} - 1}{(\nu - 1)k + r - \rho}$$  \hspace{1cm} (14)

and the ratio of expenditures from expansion and reducing emissions intensity is approximately

$$\frac{1}{\nu - 1} \frac{r}{k}.$$

For short time-horizons such that $((\nu - 1)k + r - \rho)t \ll 1$, the expenditure from reducing emissions intensity increases linearly with time as $E_\mu(t) \cong \alpha \mu_0 kt$, being proportional to the decarbonization rate. The second contribution from expansion is quadratic in time as $E_g(t) \cong \frac{\alpha \mu_0 k}{2} rt^2$, and increases with the decarbonization rate and GGDP growth rate. Their ratio is $E_g(t) / E_\mu(t) = \frac{1}{2} rt$, and initially $rt \ll 1$ so expenditures are governed by costs of reducing emissions intensity, but the second contribution from expansion becomes increasingly important.

---

5Strictly, the model of constant exogenous decarbonization rate $\sigma$ assumes constant GGDP growth rate, so this is the implicit assumption throughout the paper.

6We expand the corresponding exponentials in equation (13) by their Taylor series and consider the leading-order terms.
2.5 Burden of mitigation expenditure as fraction of GGDP

The global economy of the future is expected to be richer than the present, and therefore better poised to manage CO$_2$ mitigation expenditures. However marginal abatement costs increase with time as cheaper mitigation activities are exhausted and more expensive activities must be undertaken for continued decarbonization. Consider "burden", defined as the ratio of mitigation expenditure in a year and corresponding GGDP; the numerator is the integrand in equation (9) without the time-discount factor. The burden is

\[ b(t) = \beta e^{-\sigma t}k(t) e^{(\nu - 1)K(t)} + \frac{\beta r}{\nu - 1} e^{-\sigma t} \left( e^{(\nu - 1)K(t)} - 1 \right) \]  

(15)

using $\hat{g}/g = r$. The two terms above arise from reducing emissions intensity and expansion respectively. Let us consider their respective contributions to $\dot{b}(t)$ for the case of constant decarbonization rate $k$.

From reducing emissions intensity this is

\[ \dot{b}(t) = \beta k ((\nu - 1)k - \sigma) e^{-\sigma t} e^{(\nu - 1)kt} \]  

(16)

so its sign depends on the sign of $(\nu - 1)k - \sigma$. If the MAC rises steeply enough that $\nu > 1$ then $\dot{b}(t) > 0$ if the decarbonization rate is large enough compared to the exogenous rate $\sigma$ so that $(\nu - 1)k > \sigma$. Increasing burden to future generations can result from a sharply rising MAC curve not being compensated adequately by exogenous reductions in emissions intensity. In case $0 < \nu < 1$ then the burden from decarbonization decreases with time. 

From expansion the contribution to $\dot{b}$ is simplified to

\[ \dot{b}(t) = \frac{\beta r}{\nu - 1} \left\{ ((\nu - 1)k - \sigma) e^{-\sigma t} e^{(\nu - 1)kt} - \sigma e^{-\sigma t} \right\} \]  

(17)

and generally the last term is small so that if \((\nu - 1)k - \sigma) / (\nu - 1) > 0\) this contribution to burden increases with time. Only if \(1 < \nu < 1 + \sigma/k\) does this contribution decrease with time, \(^7\)

\(^7\)In the approach \(\nu \to 1\) the contribution to decarbonization is approximately $b(t) = \beta e^{-\sigma t}k(t)$, so this generally decreases with time unless decarbonization rate is increasing.
and for all other conditions it increases. If $\sigma/k \ll 1$ the burden from expansion is almost certain to increase with time because of the shape of the MAC curve.

In summary, both decarbonization and expansion are likely to impose increasing burdens on future generations if the MAC curve rises steeply enough that

$$\nu > 1 + \frac{(1 - \theta)r}{k}$$

(18)

since increasing MAC would not be compensated by effects of exogenous reductions in emissions intensity.

In the future if $e^{(\nu-1)K(t)} \gg 1$ in equation (15) the burden simplifies to

$$b(t) = \beta e^{-\sigma t} e^{(\nu-1)K(t)} \left( k(t) + \frac{r}{\nu-1} \right)$$

(19)

so for approximately constant values of integrated decarbonization $K(t)$ the burden from reducing emissions intensity is proportional to decarbonization rate $k(t)$.

### 2.6 Convexity of relation between mitigation costs and cumulative emissions

Here we examine the approximate relation between discounted mitigation expenditures and cumulative CO$_2$ emissions. Such a relation of course depends on the function $K(t)$, and this subsection considers only scenarios with constant decarbonization rate so $K(t) = kt$. We also fix the GGDP growth rate and autonomous decarbonization rate, so the graph between expenditures and cumulative emissions reflects only differences in the decarbonization rate.

Cumulative CO$_2$ emissions between the present at time $t = 0$ and the time-horizon at $t = T$ is $M(T) = \int_0^T m(t) \, dt$. With $m(t) = \mu_0 g(t) e^{-K(t)} e^{-\sigma t}$, and in case of constant decarbonization rate $k$ this becomes $M(T) = m_0 \left( 1 - e^{-\chi T} \right) / \chi$, with $\chi = k + \sigma - r$. The slope of the graph between discounted expenditure $E(T)$ and cumulative emissions $M(T)$ equals derivative $\partial E / \partial M$, which is written as $\partial E / \partial M = (\partial E / \partial k) (\partial k / \partial \chi) (\partial \chi / \partial M)$. The value of $\partial M / \partial \chi = \frac{m_0}{\chi} \left( (1 + \chi T) e^{-\chi T} - 1 \right)$.

We examine separately the very different cases with short and long time-horizons.

For short time-horizons the leading order terms are $\partial E / \partial k \cong \alpha m_0 T$, and $\partial M / \partial \chi \cong -m_0 T^2$. Then,
since $\partial k/\partial \chi = 1$ with constant $r$ and $\sigma$, we obtain $\partial E/\partial M \approx -\alpha/T$. This slope is constant for fixed time-horizon $T$. For $T = 1$ the slope of the graph is $-\alpha$, which also represents the increase in expenditure (in billion $\$s$) during one year from the present time that is needed for decreasing emissions by $1 \text{ Gton year}^{-1}$. This much should be obvious from the model of the MAC curve, but there is also an analogous linear relation for short time-horizons of a few years.

For long time-horizons $T$ for which $e^{-\chi T} \ll 1$, $M \equiv m_0/\chi$, and eliminating $k$ and substituting into equation (14) one obtains the relation between expenditure and cumulative emissions $E \equiv \beta g_0 \frac{e^{(r-\rho)T} - 1}{r-\rho} \left( \frac{m_0}{\rho} + \frac{\kappa\nu}{\nu-1} - \sigma \right)$, where $E$ is a convex function of $M$ and has increasing slope for smaller $M$. For $\nu > 1$ the expenditure in equation (14) depends more strongly on decarbonization rate $k$ due to the exponential term in $k$, so the effect is even larger. Convexity of the graph between $E$ and $M$ arises from two effects in the case of constant decarbonization rates: $\partial M/\partial \chi$ for the case of large $T$ behaves as $-m_0/\chi^2$, so larger $\chi$ in case of more rapid decarbonization has progressively smaller effect on limiting cumulative emissions; secondly, increasing decarbonization rate increases $E$ nonlinearly for long time-horizons, due to the exponential term in $k$. These effects are more general and therefore not limited to scenarios with constant decarbonization rates.

In summary, for short time-horizons the graph of mitigation expenditures versus cumulative emissions is linear, whereas for longer time-horizons it is convex. CO$_2$ is long-lived and policies to limit cumulative emissions should take into account sufficiently long time-horizons. Under these conditions the marginal cost, measured in terms of additional discounted expenditures needed to achieve more stringent mitigation goals, is increasing. Parameters used in the paper are listed in Table 1.

---

8The slope of the graph between expenditure and cumulative emissions becomes smaller as time-horizon $T$ is increased because sensitivity of expenditure to decarbonization rate grows linearly with $T$, whereas that of cumulative emissions has a quadratic relation with $T$. The benefits of higher decarbonization rates are sensitive to $T$, because decarbonization takes time to influence cumulative emissions. Therefore for time horizons of a few years, decarbonization appears more attractive while considering the longer periods.
| Symbol | Description | Units |
|--------|-------------|-------|
| \(m(t)\) | CO\(_2\) emissions | Gton CO\(_2\) year\(^{-1}\) |
| \(M_0\) | cumulative CO\(_2\) emissions goal | PgC |
| \(\mu(t)\) | CO\(_2\) emissions intensity | Gton CO\(_2\) (trillion \(\$\))\(^{-1}\) |
| \(g(t)\) | Global gross domestic product (GGDP) | trillion \(\$\) year\(^{-1}\) |
| \(r\) | annual GGDP growth rate | year\(^{-1}\) |
| \(k(t)\) | decarbonization rate | year\(^{-1}\) |
| \(\theta\) | income elasticity of CO\(_2\) emissions | dimensionless |
| \(\sigma\) | exogenous decarbonization rate | year\(^{-1}\) |
| \(\alpha\) | coefficient for MAC curve | billion \(\$\) / (Gton CO\(_2\) year\(^{-1}\)) |
| \(\nu\) | exponent in MAC curve | dimensionless |
| \(E_\mu(t), E_g(t), E(t)\) | discounted expenditures until year \(t\) | billion \(\$\) |
| \(P_\mu(t), P_g(t), P(t)\) | expenditure in year \(t\) | billion \(\$\) year\(^{-1}\) |
| \(b(t)\) | expenditure / GGDP in year \(t\) | dimensionless |
| \(K(t)\) | integrated decarbonization rate | dimensionless |
| \(\delta\) | time-discount rate | year\(^{-1}\) |
| \(\rho\) | \(\sigma + \delta\) | year\(^{-1}\) |
| \(\chi\) | \(k + \sigma - r\) | year\(^{-1}\) |
| \(\beta\) | \(\alpha \mu_0\) | year |
3 Minimum-expenditure pathways for reducing emissions intensity of CO₂

This section examines quasi-stationary pathways of decarbonization that minimize mitigation expenditure subject to constraint on cumulative CO₂ emissions. Such pathways need not correspond to constant decarbonization rate. There is a constraint on the integrated decarbonization rate, which is \( K(0) = 0 \) at the present time, and we therefore seek the initial value problem in \( K(t) \) whose solution minimizes mitigation expenditure.

Cumulative CO₂ emissions is written as \( M(T) = \int_0^T m(t, K, \dot{K}) \, dt \), where \( m(t, K, \dot{K}) = \mu_0 g(t) e^{-K(t)} e^{-\sigma t} \) is emissions.\(^9\) We wish to find \( K(t) \) that minimizes discounted mitigation expenditure \( E(T) = \int_0^T f(t, K, \dot{K}) \, dt \), where \( f(t, K, \dot{K}) = \beta e^{-\delta t} e^{-\sigma t} g(t) \dot{K}(t) e^{(\nu-1)K(t)} + \frac{\beta}{\nu} e^{-\delta t} e^{-\sigma t} g(t) \left( e^{(\nu-1)K(t)} - 1 \right) \).

Consider choosing stationary pathway of integrated decarbonization \( K(t) \) in order to minimize \( E(T) \) subject to cumulative emissions constraint

\[
\int_0^T m(t, K, \dot{K}) \, dt = M_0
\]

where \( M_0 \) is the cumulative emissions goal. For such a pathway, the derivative of functional \( I(K, \dot{K}) = \int_0^T f(t, K, \dot{K}) \, dt + \lambda_1 \left\{ \int_0^T m(t, K, \dot{K}) \, dt - M_0 \right\} \) must be stationary with respect to \( K(t) \) for arbitrary perturbations \( \delta K \) satisfying the initial condition. This yields the familiar Euler-Lagrange (E-L) equation derived in Appendix 1

\[
\frac{\partial f}{\partial K} + \lambda_1 \frac{\partial m}{\partial K} = \frac{d}{dt} \left( \frac{\partial f}{\partial \dot{K}} + \lambda_1 \frac{\partial m}{\partial \dot{K}} \right)
\]

simplifying to \( e^{\nu K(t)} = \frac{\lambda_1 \mu_0}{(\delta + \sigma) \beta} e^{\delta t} \), where \( \lambda_1 \) can be eliminated using the constraint on cumulative emissions. In addition the solution must satisfy a "natural boundary condition" \( \frac{\partial f}{\partial K}(T) + \frac{\partial m}{\partial K}(T) = 0 \) arising from the fact that the value of \( K(T) \) is not fixed by the specification of our problem (Appendix 1). However such satisfaction is not possible, and we seek a solution that is only stationary with respect to perturbations that leave intact not only \( K(0) \), which follows from initial condition \( K(0) = 0 \), but also \( K(T) \). Fixing \( K(T) \) is an artificial constraint on our problem that has

\(^9\)Emissions do not depend explicitly on \( \dot{K} \), but we include this argument for consistency with the formulation of the rest of the functional that we seek to minimize.
been introduced for tractability. In this sense our solution is "quasi-stationary", i.e. only relative to a restricted type of perturbations which preserves both endpoints, although the second endpoint is not constrained by a final condition on \( K(t) \).

Although \( K(t) \) increases in time in the presence of a non-zero discount rate, the absence of a term in \( \dot{K} \) in the E-L equation precludes imposing initial condition \( K(0) = 0 \). The E-L equation is algebraic in our optimization problem because integrand \( f + \lambda \frac{\partial f}{\partial \dot{K}} \) depends linearly on \( \dot{K} \), leading to a degenerate case \( \text{[van Brunt (2004)]} \) as shown in Appendix 1.

In order to introduce a term in \( \dot{K} \) in the E-L equation, we seek a second integral constraint involving a different function \( h(t, K, \dot{K}) \) that obeys

\[
\frac{d}{dt} \left( \frac{\partial h}{\partial \dot{K}} \right) = \dot{K} e^{-\gamma t} \tag{22}
\]

with \( \gamma > 0 \), for reasons that will become evident. Then \( \frac{\partial h}{\partial \dot{K}} = \int_0^t \dot{K}(s) e^{-\gamma s} ds \) and integrating by parts

\[
\frac{\partial h}{\partial \dot{K}} = e^{-\gamma t} K(t) + \gamma \int_0^t K(s) e^{-\gamma s} ds \tag{23}
\]

using initial condition \( K(0) = 0 \). Furthermore, choosing \( \gamma \) large so that the first term can be neglected we obtain

\[
h(t, K, \dot{K}) = \gamma \dot{K}(t) \int_0^t K(s) e^{-\gamma s} ds + h_1(K(t), t) \tag{24}
\]

We seek only one such function parameterized by \( \gamma \) satisfying equation (22), so make the simplest choice and set \( h_1(K(t), t) \) to zero. Then \( h(t, K, \dot{K}) = \gamma \dot{K}(t) \int_0^t K(s) e^{-\gamma s} ds \), and we choose \( \gamma \) large so that \( \int_0^T h(t, K, \dot{K}) dt \) can be made small\(^{10}\).

We therefore impose a further equality constraint on our original problem

\[
\int_0^T h(t, K, \dot{K}) dt = \varepsilon \tag{25}
\]

\(^{10}\)To see how this is possible, consider the example of constant decarbonization rate where \( K(t) = \zeta t \) with \( \zeta > 0 \), so that \( h(t) = \gamma \zeta^2 \int_0^t s e^{-\gamma s} ds \). Integrating by parts this becomes \( -\zeta^2 t e^{-\gamma t} + \frac{\zeta^2}{\gamma} \left( 1 - e^{-\gamma t} \right) \), which can be made small by choosing \( \gamma \) to be sufficiently large. The terms involved in defining \( h(t) \) are positive so \( h(t) > 0 \) and furthermore in this example \( h(t) < \zeta^2 / \gamma \), so \( \int_0^T h(t) dt < \zeta^2 T / \gamma \), which can be made small by choosing \( \gamma \) sufficiently large.
with \( \varepsilon \ll 1 \) and the new functional to be minimized contains an additional term \( \lambda_2 \left\{ \int_0^T h \left( t, K, \dot{K} \right) dt - \varepsilon \right\} \), so the modified E-L equation becomes

\[
\frac{\partial f}{\partial K} + \lambda_1 \frac{\partial m}{\partial K} + \lambda_2 \frac{\partial h}{\partial K} = \frac{d}{dt} \left( \frac{\partial f}{\partial \dot{K}} + \lambda_1 \frac{\partial m}{\partial \dot{K}} + \lambda_2 \frac{\partial h}{\partial \dot{K}} \right) \tag{26}
\]

Here too there is an analogous natural boundary condition, but we cannot satisfy it and only seek a solution that is stationary with respect to the restricted class of perturbations discussed above. Recall that \( \frac{d}{dt} \left( \frac{\partial h}{\partial \dot{K}} \right) = \dot{K} e^{-\gamma t} \). Furthermore, \( \frac{\partial h}{\partial K} = \gamma \dot{K} \left( t \right) \int_0^t e^{-\gamma t} dt = \dot{K} \left( 1 - e^{-\gamma t} \right) \). Then the E-L equation is

\[
-\lambda_1 \mu_0 g \left( t \right) e^{-\sigma t} e^{-K(t)} + \lambda_2 \dot{K} \left( t \right) \left( 1 - e^{-\gamma t} \right) = -\left( \delta + \sigma \right) \beta \dot{K} \left( t \right) e^{(\nu - 1)K(t)} + \lambda_2 \dot{K} \left( t \right) e^{-\gamma t} \tag{27}
\]

We simplify by neglecting \( e^{-\gamma t} \) compared to 1 because \( \gamma t \gg 1 \), so the evolution equation for \( K \) becomes

\[
\dot{K} \left( t \right) = \frac{\lambda_1 \mu_0}{\lambda_2} e^{-\sigma t} g \left( t \right) e^{-K(t)} - \frac{\left( \delta + \sigma \right) \beta}{\lambda_2} e^{-(\delta + \sigma) t} g \left( t \right) e^{(\nu - 1)K(t)} \tag{28}
\]

with initial condition \( K \left( 0 \right) = 0 \). Multiplying by \( e^{K(t)} \) on both sides and defining \( x \left( t \right) = e^{K(t)} \) the evolution equation for \( x \left( t \right) \) is

\[
\dot{x} \left( t \right) = \frac{\lambda_1 \mu_0}{\lambda_2} e^{-\sigma t} g \left( t \right) - \frac{\left( \delta + \sigma \right) \beta}{\lambda_2} e^{-(\delta + \sigma) t} g \left( t \right) \left( x \left( t \right) \right)^\nu \tag{29}
\]

with \( x \left( 0 \right) = 1 \). The above equation is not of a standard type that can be solved exactly, and we resort to approximate solutions (Appendix 2). Integrated decarbonization rate \( K \left( t \right) \) must increase to be economically relevant, but with an initial value problem in \( K \left( t \right) \) we face the risk that it actually decreases in the solution to the above equation. In case of either \( \sigma > 0 \) or \( \delta > 0 \) this possibility is realized and, as shown in Appendix 2 for \( \sigma > 0 \), \( K \left( t \right) \) is a decreasing function. We cannot avoid this by regularizing the E-L equation for a 2-point boundary value problem. The value of \( K \left( 0 \right) \) is known but not \( K \left( T \right) \), and the latter is not uniquely determined by the constraint on cumulative emissions.\[11\]

\[11\]The cumulative emissions constraint of equation (20) discretized in time corresponds to a single equation in several unknown values of \( K \) at the various time-steps, so this does not yield a unique constraint on \( K \left( T \right) \).
Therefore we can obtain a meaningful solution to the E-L equation only for the special case of \( \sigma = 0 \), involving unit income elasticity of emissions, and in the absence of time-discounting. For this special case, \( x(t) = 1 + \frac{\lambda_1 \mu_0}{\lambda_2} G(t) \), where \( G(t) = \int_0^t g(s) \, ds \) is integrated GGDP, and hence the quasi-stationary solution is

\[
K(t) = \ln \left( 1 + \frac{\lambda_1 \mu_0}{\lambda_2} G(t) \right)
\]  (30)

which is increasing. Lagrange multipliers \( \lambda_1 \) and \( \lambda_2 \) are estimated by substituting equation (30) into integral constraints provided by equations (20) and (25). The E-L equation has reduced our infinite-dimensional problem of choosing function \( K(t) \) to the finite-dimensional one of estimating \( \lambda_1 \) and \( \lambda_2 \) satisfying these constraints.

For the above solution, being quasi-stationary only if \( \sigma = 0 \), we have \( e^{-K(t)} = 1 / \left( 1 + \frac{\lambda_1 \mu_0}{\lambda_2} G(t) \right) \); and therefore in this case cumulative emissions at time \( t \) is \( M(t) = \frac{\lambda_1}{\lambda_1} \ln \left( 1 + \frac{\lambda_1 \mu_0}{\lambda_2} G(t) \right) \), or \( M(t) = \frac{\lambda_1}{\lambda_2} K(t) \) in the quasi-stationary solution.

Thus the quasi-stationary solution, stationary relative to a restricted class of perturbations in \( K \), exists for the special case of \( \sigma = 0 \) and \( \delta = 0 \), and has cumulative emissions graph proportional to the graph of \( K(t) \), and the emissions graph \( m(t) \) proportional that of decarbonization rate \( k(t) \), with \( k(t) = \frac{\lambda_1}{\lambda_2} m(t) \). Scenarios requiring decreasing emissions, so that cumulative emissions eventually becomes approximately constant, have \( K(t) \) increasing at diminishing rate in this solution; decarbonization rate \( k(t) \) is initially large and decreases with time.

4 Numerical Results

4.1 Parameter estimates and scenarios

We estimate the global MAC curve (in 1990 US dollars) for CO\(_2\) by aggregating estimates presented in [Morris et al. (2008)](https://example.com), which in turn are based on the MIT Emissions Prediction and Policy Analysis (EPPA) model ([Paltsev et al. (2005)](https://example.com)). Figure 1a shows results from [Morris et al. (2008)](https://example.com) for the year 2050. Estimation of the model of MAC is illustrated in Figure 1b, and we must know the reference emissions in the business as usual case, corresponding to the effect of \( \mu_0 e^{-\sigma t} \). This is taken
from reference-case emissions in 2050 documented for the EPPA model in [Paltsev et al. (2005)]. Least squares regression yields estimates for $\alpha$ and $\mu$ (Table 2). The exponent $\nu$ is significantly larger than one, and this relation affects the properties examined in Section 2. There is large uncertainty in MACs ([Criqui et al. (1999); Klepper and Peterson (2004); Amann et al. (2009)]) but it appears that this exponent in our model cannot be much smaller than 2, because that would lead to very slow increase of the MAC as decarbonization proceeds. Figure 1c shows the effect of changing $\nu$ in our model. In case $\nu$ is closer to 1 then the MAC even after 50% reductions compared to the reference case would be less than 50 $/tonne in 2005 prices, which is substantially smaller than studies suggest ([Ellerman and Decaux (1998); Klepper and Peterson (2004); Amann et al. (2009))].

For income elasticity of CO$_2$ emissions, we use constant value of $\theta = 0.75$ yielding exogenous decarbonization rate of $\sigma = 1$ % per year in a 4 % GGDP growth scenario that is close to the recent historical value in real dollars ([DeLong (1998)]). As a result the estimate of $\sigma$ corresponds to the 2015 value in the DICE model ([Nordhaus and Sztorc (2013)]). In DICE the autonomous decarbonization rate is decreasing at 0.1% at each time-step of five years ([Nordhaus and Sztorc (2013)]). If were were to consider scenarios with decreasing GGDP growth rate, a similar decrease in exogenous decarbonization rate would occur in our emissions model.

For future GGDP growth estimates, considering that our time-horizon is 100 years from the present, most IAMs show a gradual decrease in GGDP growth reflecting demographic transitions during the present century (e.g. [Paltsev et al. (2005)]). Nevertheless we consider annual growth rates (in real dollars) ranging between 0.012 - 0.036, which is a larger range than in IPCC’s Special Report on Emissions Scenarios (SRES) ([Krakauer (2014)]). Generally there is a tendency even among experts to underestimate uncertainty ([Morgan and Henrion (1990)]), and we included a wide range of long-term growth rates in our calculations.$^{12}$ Estimates of parameters of the model, including present emissions, GGDP, and emissions intensity, are listed in Table 2.

We study cumulative emissions goals during the next 100 years of 300, 600, 900 and 1200 PgC.$^{13}$ Global warming is approximately proportional to cumulative CO$_2$ emissions, with the ratio esti-

$^{12}$The same caution applies to long term MACs but we do not consider that effect here.

$^{13}$Only the cumulative emissions are specified in PgC, because carbon accounting is usually performed in these units. However the economic model is carried out in units of Gton CO$_2$. 

17
mated to vary between 0.8–2.5 K per 1000 PgC (Allen and Stocker (2014)). Assuming a symmetric distribution of this "transient climate response to cumulative carbon emissions" with mean value of 1.65 K per 1000 PgC yields mean global warming contribution of CO₂ of approximately 0.5, 1.0, 1.5, and 2.0 K respectively during the next 100 years. With present global warming of about 1 K, if other contributions remain unchanged, the above cumulative emissions goals correspond roughly to mean forecasts of global warming of 1.5, 2.0, 2.5, and 3.0 K relative to preindustrial conditions. We should emphasize that there are uncertainties in the relation between cumulative emissions and global warming (Meinshausen et al. (2009); Peters (2016)).
Table 2: Parameter estimates for model of expenditures. Monetary units refer to 1990 USD.

| Parameter | Value     | Unit                                      |
|-----------|-----------|-------------------------------------------|
| $\alpha$  | 10.4 ± 2.7| billion $ / (Gton CO$_2$ year$^{-1}$)     |
| $\nu$     | 2.4 ± 0.46| dimensionless                             |
| $\theta$  | 0.75      | dimensionless                             |
| $m_0$     | 36        | Gton CO$_2$ year$^{-1}$                   |
| $\mu_0$   | 0.46      | Gton CO$_2$ (trillion $)^{-1}$            |
| $g_0$     | 77.8      | trillion $ $ year$^{-1}$                  |
| $\beta$   | $4.8 \times 10^{-3}$ | year                                      |
Figure 1: Estimate of MAC (in 1990 USD) and effect of changing exponent \( \nu \) in our model of MAC: 
(a) crosses show global MAC versus quantity of emissions reductions from \cite{Morris et al. (2008)} for the year 2050, after conversion from 2005 to 1990 dollars; (b) results of panel (a) presented in the form of the model in equation (3), where \( \mu_0 e^{-\sigma t} \) is taken to correspond to a 60% increase in 2050 CO\(_2\) emissions compared to the present in the reference scenario, roughly based on results documented for the EPPA integrated assessment model \cite{Paltsev et al. (2005)}. The power law is estimated from a least squares fit using the logarithm of the ordinate, yielding \( \alpha = 10.4 \pm 2.7 \) billion $/(Gton a^{-1}) \) and \( \nu = 2.4 \). In both panels, curves correspond to estimate of equation (3); (c) effect of changing \( \nu \) in the model, keeping \( \alpha \) constant. Values of \( \nu \) much smaller than 2 lead to very slow increase of MAC (see text).

4.2 Expenditure-minimizing decarbonization pathways in case of unit income elasticity of emissions

Figure 2 shows expenditure-minimizing pathways in case global income elasticity of emissions is unity, so there is no exogenous decarbonization. Recall that these are "quasi-stationary", i.e. stationary relative to a restricted type of perturbations in the variable \( K(t) \) that keep the endpoints as fixed. The quasi-stationary pathway of annual CO\(_2\) emissions is approximately invariant of GGDP growth rate. The time-dependence of emissions arises from term \( e^{rt - \int_0^t k(s)ds} \), and larger \( r \) is being compensated by larger \( k \). More rapid economic growth requires faster decarbonization for limiting...
cumulative emissions to the same levels.

As noted in Section 3, the quasi-stationary solution has $k(t) \propto m(t)$ so the decarbonization rate is constant only in scenarios involving constant emissions, and increasing in time in scenarios involving increasing emissions. For scenarios requiring decreasing emissions, the decarbonization rate is initially larger and decreases with time. Large annual decarbonization rates of more than 10 percent are initially involved in the quasi-stationary solution for limiting cumulative emissions to 300 PgC or less.

### 4.3 Mitigation expenditures in the presence of exogenous decarbonization

Henceforth we will only describe cases with income elasticity of emissions $\theta = 0.75$, so exogenous decarbonization occurs at a rate increasing with GGDP growth rate. Figure 3 shows pathways satisfying equation (30) with the cumulative emissions constraint taking into account exogenous decarbonization at rate $\sigma$. Limiting cumulative emissions below 600 PgC requires near-term decarbonization rates of at least a few percent. Emissions pathways are generally not invariant of
GGDP growth rate in the presence of autonomous decarbonization, and larger early emissions in high growth scenarios is compensated by reduced emissions later on. When autonomous decarbonization is present, it is not necessary that decreasing emissions pathways be accompanied by initially high decarbonization rates that subsequently decrease. However even in this case stringent mitigation scenarios generally involve initially larger decarbonization rates.

Despite decreasing decarbonization rates in such scenarios, mitigation expenditures are rising here as in all mitigation scenarios, as Figure 4 shows. Expenditures increase from a few to tens of billion dollars in the present time to a few orders of magnitude more as the MAC rises and GGDP increases. The carbon price needed to achieve these reductions in CO$_2$ is equal to the MAC, with formula $\alpha e^{\nu K(t)}$ following equation (3). Figure 4a shows the carbon price (in constant 1990 US dollars) increasing to 100 $/\text{ton CO}_2$ in a few decades in the 300 PgC scenario, and much more rapidly in case GGDP growth can be expected to be rapid, because decarbonization must occur more quickly. However the average rate of growth of GGDP during the next 100 years will not be known until the end of this period.

In our model of mitigation cost, there are two contributions: from reducing emissions intensity, and from subsequent expansion of mitigation as the global economy grows. Recall from Section 2.4 that the first contribution initially grows linearly in time, whereas the second grows quadratically. The first contribution is initially much larger, but the second contribution grows in importance with time (Figure 4d). Its role increases with time because, as the economy grows, decarbonization efforts have to be scaled up in proportion to the level of decarbonization at that time present the global economy, as measured by $e^{(\nu-1)K(t)} - 1$. The ratio of the two contributions is $P_g(t)/P_\mu(t) = 1/\nu \cdot \frac{r}{K(t)} \left(1 - e^{-(\nu-1)K(t)}\right)$. For large $t$ this approaches $1/\nu / K(t)$. Since $\nu \approx 2$ in our model, this always remains smaller than one in scenarios where the decarbonization rate must be larger than the GGDP growth rate.

One lesson from Figures 2-4 is the importance of assumptions about future global economic growth to estimates of the scale of effort involved in decarbonization. Faster growth requires further decarbonization, entailing not only higher marginal costs, but also these must be scaled up to a larger extent with greater economic expansion. Uncertainty in future economic growth thereby presents substantial uncertainty for estimates of the scale of mitigation required.
Figure 3: Quasi-stationary decarbonization rates for limiting cumulative emissions during the next 100 years to 300, 600, 900 and 1200 PgC in the different colors, where effects on emissions of exogenous decarbonization arising from income elasticity of emissions $\theta = 0.75$ have been included: (a) decarbonization rate $k(t)$; (b) annual CO$_2$ emissions. Solid, dashed, and dotted curves indicate GGDP growth rate $r$ of 0.012, 0.024, and 0.036 respectively.

4.4 Mitigation burden

Figure 5 plots the mitigation expenditure as fraction of GGDP ("burden") for the cases shown in the previous figures. This generally increases with time because the condition of equation (18) is met, and the effect of increasing MAC is not mitigated by exogenous decarbonization. For short times the second contribution from expansion is small and the burden approximates to $b(t) \approx \beta e^{(\nu - 1)K(t) - \sigma t} k(t)$, and with $\beta = 4.8 \times 10^{-3}$ years in the present model this becomes a very small fraction of GGDP even for large decarbonization rates. At the current time there are many low-cost options for decarbonization and the mitigation burden therefore can be very small.

The burden after a long time is $b(t) \approx \beta e^{(\nu - 1)K(t) - \sigma t} \left(k(t) + \frac{r}{\nu - 1}\right)$. The exponent $(\nu - 1) K(t) - \sigma t$ increases substantially as mitigation proceeds (Figure 5a) and the burden approaches 1% of GGDP or more in scenarios with stringent mitigation goals, especially in the presence of rapid GGDP growth (Figure 5b). The latter is generally consistent with results of Rogelj et al. (2015) showing the large influence of baseline energy demand on discounted mitigation costs.

One effect is that delaying decarbonization can impose much larger burdens on future generations if they seek to meet the same stringent cumulative emissions goal. Figure 6 considers emissions scenarios leading to cumulative emissions of 300 PgC. We compare, in the presence of exogenous
Figure 4: Carbon price and CO₂ mitigation expenditures for the scenarios in Figure 3. Colors indicate cumulative emissions goals of 300, 600, 900 and 1200 PgC during the next 100 years. Solid, dashed, and dotted curves indicate GGDP growth rate $r$ of 0.012, 0.024, and 0.036 respectively. Prices and expenditures are in constant 1990 USD: (a) carbon price, which is equal to the marginal abatement cost; (b) total expenditure in each year $P(t)$; (c) expenditure from reducing emissions intensity $P_{\mu}(t)$; (d) ratio $P_g(t)/P_{\mu}(t)$ of expenditures from expansion and reducing emissions intensity. Keeping cumulative emissions below 600 PgC typically requires carbon price of 50 USD (in 1990 dollars) or higher within the next two decades. Carbon price and expenditures are much higher in case of higher GGDP growth. Mitigation expenditures are increasing even as decarbonization rate is decreasing, as the MAC rises. At the present time expenditures are dominated by those on reducing emissions intensity, but the contribution from expansion plays an increasing role.
Figure 5: Mitigation expenditure as a fraction of GGDP ("burden") in the scenarios of Figure 3. Colors indicate cumulative emissions goals of 300, 600, 900 and 1200 PgC for the next 100 years. Solid, dashed, and dotted curves indicate GGDP growth rate $r$ of 0.012, 0.024, and 0.036 respectively: (a) exponent $(\nu - 1) \int K(t) - \sigma t$ appearing in the equation; (b) mitigation burden.

decarbonization, the aforementioned quasi-stationary scenario with one having constant decarbonization rate. The former serves as archetype for rapid and early mitigation. A small fraction ($\leq 0.1\%$) of GGDP is saved in the present time by choosing the constant mitigation rate scenario, but this would entail much higher mitigation burdens in the future. Towards the end of our time-horizon, higher expenditures amount to about 1% of GGDP, and much more in scenarios involving moderate to high growth. Present savings are much more than offset by future increases in mitigation expenditure, because future emissions must be correspondingly smaller in case of failure to reduce emissions substantially in the present. A delay in mitigation requires higher rates of decarbonization in the future under conditions of much higher MAC, where the factor $e^{(\nu - 1) \int K(t) - \sigma t}$ is much larger. The larger future decarbonization rates occurring in the scenario with late mitigation are therefore amplified by a much greater amount than present savings.

4.5 Costs of mitigation

Let us consider the overall costs of mitigation, by evaluating $f = \int_0^T e^{-\delta s} P(s) ds / \int_0^T e^{-\delta s} g(s) ds$, the ratio between discounted expenditures and GGDP for the quasi-stationary solution. Figure 7 shows that this is a convex function of cumulative emissions goal, as Section 2.6 argued. The scale is logarithmic, and mitigation cost increases rapidly for stringent cumulative emissions goals as
Figure 6: Effect of delay in decarbonization on future burden of meeting the same cumulative emissions target of 300 PgC: (a) quasi-stationary emissions pathway with early mitigation and a pathway with constant decarbonization rate; (b) mitigation burden. Solid, dashed, and dotted curves indicate GGDP growth rate of 0.012, 0.024, and 0.036 respectively. The mitigation burden increases with time, and increases more rapidly in scenarios where mitigation is postponed. A small fraction (≤ 0.1 %) of GGDP is saved in the present time by choosing the constant mitigation rate scenario, but leads to much higher mitigation burden in the future (several percent of GGDP).

more rapid mitigation has diminishing effects on cumulative emissions and furthermore expenditure rises. Still the costs are limited to a small fraction of discounted GGDP, although this elides how the burden of mitigation is distributed across time (Section 4.4). Background assumptions about economic growth play an important role, and the ratio in Figure 7 is smaller with higher discount rates since the mitigation burden is increasing with time. Mitigation cost can vary by more than an order of magnitude depending on the assumptions of the model and the mitigation goal, but the results of our idealized model seem to reproduce qualitatively the results from an integrated assessment model (MESSAGE) discussed by Rogelj et al. (2015). The graph shows an approximately linear relationship on a logarithmic scale, so the relation between costs $f$ and the cumulative emission goal $M_0$ roughly obeys power law $f(M_0) = f_1 (M_0/M_{01})^{-n}$, with $M_{01} = 1000$ PgC being a reference goal and leading to cost $f_1$.

Figure 8a shows that higher economic growth increases the reference cost $f_1$ substantially, and this is more sensitive to assumptions about long-term economic growth than to the exponent in the MAC curve. The power $n$ describes sensitivity $\frac{\Delta f/f}{\Delta M_0/M_0}$ of relative changes in $f$ to relative changes in the cumulative emissions goal. Figure 8b shows higher sensitivity to the cumulative emissions goal in case the MAC curve rises sharply and GGDP rises slowly. Rapid economic growth would make meeting cumulative emissions goals more expensive regardless of how stringent they might
be, whereas sharply rising MAC makes cost more sensitive to cumulative emissions. For median values the power in the above model $n \approx 2.2$, so halving the future cumulative emissions goal (for example from a 2 C scenario to a 1.5 C scenario; recall that present warming is about 1.0 C) would increase cost by a factor of about 4.6.

## 5 Conclusions

We have estimated the scale of expenditures involved in reducing cumulative CO$_2$ emissions by reducing emissions intensity of GGDP ("decarbonization"). Our model assumes constant global income elasticity of CO$_2$ emissions, which leads to exogenous decarbonization, independent of mitigation policy, occurring in the business as usual scenario. This occurs at rate increasing with the GGDP growth rate; and in case of constant GGDP growth rate, the effects are analogous to discounting in time. We also neglect decreases in marginal abatement costs with time, but expect that real costs would have to decrease by a large degree to undermine our main conclusions. Mitigation expenditure has two contributions, arising from reducing emissions intensity and expanding effort as the global economy grows. The first contribution is initially dominant for the scenarios we have...
Figure 8: Parameters of power law between cumulative emissions goal and mitigation cost, as a function of GGDP growth rate and exponent of MAC curve. Mitigation cost is measured by $f$, the discounted expenditure as fraction of discounted GGDP, and we fit the model: $f (M_0) = f_1 (M_0/M_{01})^{-n}$, with $f_1$ being the cost of limiting $M_0$ to a reference value of $M_{01} = 1000$ PgC: (a) logarithm of reference cost $f_1$, which is larger with higher growth rate and the MAC exponent, but is more sensitive to the former; (b) exponent $n$ is higher with larger MAC exponent and smaller economic growth. Mitigation cost is more sensitive to cumulative emissions goal if MAC rises sharply and economic growth is slow. Mitigation cost can vary more than an order of magnitude depending on assumptions about the MAC and economic growth.

Examined but the second contribution plays a growing role.

The MAC is estimated to rise steeply enough that the CO$_2$ mitigation burden, defined as expenditure as a fraction of GGDP, is expected to increase following equation (18). Future generations would have to expend larger fractions of GGDP on decarbonization, even in scenarios where the decarbonization rate is decreasing, as cheaper mitigation options become exhausted. Rapid global economic growth increases the mitigation burden on future generations, and despite being wealthier in scenarios with higher-growth they would have to spend larger fractions of GGDP as it becomes necessary to ascend the MAC curve more rapidly.

Discounted mitigation expenditures are convex functions of cumulative CO$_2$ emissions. Over long time-horizons, larger rates of decarbonization have diminishing benefits for reducing cumulative CO$_2$ emissions, whereas mitigation expenditures increase more rapidly. Global warming is approximately proportional to cumulative emissions (Matthews et al. (2009); MacDougall and Friedlingstein (2015); MacDougall (2016); Tokarska et al. (2016)), and the cost of emissions reduction is a convex function of the level of global warming and expenditures increase more steeply if the policy goal involves a smaller degree of global warming. Nonconvexities could still arise in climate change
economics due to non-convex damages from, for example, threshold effects (Fisher and Hanemann (1993); Lempert et al. (1996); Keller et al. (2004)).

We measure overall costs by the discounted mitigation expenditures as a fraction of discounted GGDP, and there is an approximate power law relation between costs and cumulative emissions below 1000 PgC, with exponent substantially larger than one. This exponent depends mainly on the exponent in the MAC. Implications are best understood through an example. Consider two alternate mitigation trajectories where future cumulative emissions differ by a factor of two, for example with global warming of 2.0 K versus 1.5 K where contributions from other forcers are assumed to remain unchanged from present-day values. With the exponent in the power law being generally larger than 2, the more stringent trajectory would involve CO$_2$ mitigation costs that are atleast 4 times larger. However such an account neglects the benefits of early investments in low-carbon technologies and knowledge spillovers to other sectors (Aghion et al. (2014); Dechezleprêtre et al. (2014)).

The optimization problem examined here seeks the pathway for global decarbonization that minimizes discounted mitigation expenditures, while meeting an exogenous constraint on cumulative CO$_2$ emissions. Solving it required us to "regularize" the variational problem because the original problem led to an algebraic Euler-Lagrange equation whose solution did not satisfy the initial condition on the integral of decarbonization rate. Regularization yields a soluble Euler-Lagrange equation in the form of an initial value problem in the integrated decarbonization rate. However the solution carries economic meaning only for the special case of absence of exogenous decarbonization and time-discounting. The general problem of minimizing discounted mitigation expenditures in the presence of exogenous decarbonization is important, but beyond our present scope. Furthermore the solution is only "quasi-stationary", with respect to a restricted class of perturbations leaving intact the integrated decarbonization rate at the end of the time-horizon as well.

For the soluble case noted above, the optimal solution has decarbonization rate proportional to emissions. An attraction of this solution is that it does not depend on the parameters of the MAC model. However, policy cannot be chosen without making long-term economic growth forecasts, illustrating another difficulty of choosing climate policy under uncertainty (Roughgarden and Schneider (1999); Pindyck (2013)). The difficulty of forecasting economic growth makes policy-choice in a cumulative
CO$_2$ framework uncertain. For example the levels of price or quantity instruments are likely to determine the position on the MAC reached, but their efficacy cannot be determined except ex-post once average growth rate of GGDP during the long time-horizon of interest becomes known.

The discounted mitigation expenditure is not only a function of cumulative emissions, but depends also on the decarbonization pathway. Expenditures are higher in scenarios requiring larger decarbonization rates in the future during periods of much higher MAC. We illustrate using the quasi-stationary solution for limiting cumulative emissions to 300 PgC over the next hundred years. Although decarbonization starts rapidly at an initially higher rate, the cost of mitigation amounts to a small burden at the present time (< 0.1% of GGDP), but rising to 1% or more at the end of the 100-year period. For trajectories meeting this cumulative emissions constraint, those with lower mitigation burdens in the present involve much higher burdens in the future, and small savings in the present translate to much larger future costs. These results are consistent with the work of Vogt-Schilb and Hallegatte (2014) who suggest that near-term policy should take into consideration longer-term targets as well, otherwise costs of meeting the latter would be higher.

Cumulative carbon accounting appears to make higher mitigation burdens to future generations inevitable, but present choices can mitigate some of this. While future expenditures can be discounted at a goods-discounting rate based on the opportunity cost of capital (Nordhaus (1993b)), discounting of future burdens can only be based on a positive value of the pure rate of time-preference that reflects lower weights being ceded to welfare of future generations.

**Acknowledgments**

This work has been supported by Divecha Centre for Climate Change, Indian Institute of Science. Thanks to several colleagues for helpful suggestions.

---

14 Although not being quasi-stationary in the presence of exogenous decarbonization, it provides an archetype of a pathway involving early mitigation.
Appendix 1: Derivation of quasi-stationary solution and degenerate case

For stationarity of \( I(K, \dot{K}) = \int_{0}^{T} f(t, K, \dot{K}) \, dt + \lambda_{1} \left\{ \int_{0}^{T} m(t, K, \dot{K}) \, dt - M_{0} \right\} \), we require the first-variation \( \delta I \) in the integral due to small changes \( \delta K \) and \( \delta \dot{K} \) to vanish. The variation \( \delta I = I(K + \delta K, \dot{K} + \delta \dot{K}) - I(K, \dot{K}) \) is

\[
\delta I = \int_{0}^{T} \left\{ \frac{\partial f}{\partial K} \delta K + \frac{\partial f}{\partial \dot{K}} \delta \dot{K} + \lambda_{1} \left( \frac{\partial m}{\partial K} \delta K + \frac{\partial m}{\partial \dot{K}} \delta \dot{K} \right) \right\} \, dt = 0 \tag{31}
\]

and, integrating by parts \( \int_{0}^{T} \frac{\partial f}{\partial \dot{K}} \delta \dot{K} \, dt = \frac{\partial f}{\partial \dot{K}}(T) \delta \dot{K}(T) - \int_{0}^{T} \frac{d}{dt} \left( \frac{\partial f}{\partial \dot{K}} \right) \delta \dot{K} \, dt \), using \( \delta \dot{K}(0) = 0 \) and there is a corresponding equation involving \( m(t, K, \dot{K}) \). This yields

\[
\delta I = \left\{ \frac{\partial f}{\partial \dot{K}}(T) + \frac{\partial m}{\partial \dot{K}}(T) \right\} \delta \dot{K}(T) + \int_{0}^{T} \left\{ \frac{\partial f}{\partial K} - \frac{d}{dt} \left( \frac{\partial f}{\partial \dot{K}} \right) + \lambda_{1} \left( \frac{\partial m}{\partial K} - \frac{d}{dt} \left( \frac{\partial m}{\partial \dot{K}} \right) \right) \right\} \delta \dot{K} \, dt = 0 \tag{32}
\]

for arbitrary changes \( \delta \dot{K} \). A subset of arbitrary changes \( \delta \dot{K} \) involves those for which \( \delta \dot{K}(T) = 0 \) and for this \( \frac{\partial f}{\partial \dot{K}} - \frac{d}{dt} \left( \frac{\partial f}{\partial \dot{K}} \right) + \lambda_{1} \left( \frac{\partial m}{\partial K} - \frac{d}{dt} \left( \frac{\partial m}{\partial \dot{K}} \right) \right) \) must vanish. This yields the Euler-Lagrange (E-L) equation \( (21) \), which must also be satisfied when one of the endpoints, in this case at \( t = T \), is not fixed by the specification of the problem. In addition the problem must satisfy the *natural boundary condition* \( \frac{\partial f}{\partial \dot{K}}(T) + \frac{\partial m}{\partial \dot{K}}(T) = 0 \) in order to be stationary for arbitrary changes \( \delta \dot{K} \).

Readers may refer to van Brunt (2004) for general discussion.

However, is not possible to find a solution to our problem that satisfies the aforementioned natural boundary condition. Therefore we consider only changes involving \( \delta \dot{K}(T) = 0 \), so that for these changes equation \( (32) \) is equivalent to the E-L equation. The meaning of this is that, once the E-L equation with initial condition \( K(0) = 0 \) is solved for \( K(t) \), this solution is only stationary with respect to changes that preserve both \( K(0) \) and \( K(T) \). In this sense the solution is *quasi-stationary*, i.e. only relative to a restricted set of changes \( \delta \dot{K} \).

Degeneracy arises in the following manner. In our problem, \( f(t, K, \dot{K}) \) has form \( \dot{K} f_{1}(t, K) + f_{2}(t, K) \) so that \( \frac{d}{dt} \left( \frac{\partial f}{\partial \dot{K}}(t) \right) = \dot{K} \frac{\partial f_{1}}{\partial K} + \frac{\partial f_{1}}{\partial \dot{K}} \) and \( \frac{\partial f}{\partial \dot{K}} = \dot{K} \left( \frac{\partial f_{1}}{\partial K} + \frac{\partial f_{1}}{\partial \dot{K}} \right) \), so that \( \frac{d}{dt} \left( \frac{\partial f}{\partial \dot{K}} \right) = \frac{\partial f_{2}}{\partial \dot{K}} - \frac{\partial f_{1}}{\partial \dot{K}} \), without any terms involving \( \dot{K} \). The term \( \frac{\partial m}{\partial \dot{K}} - \frac{d}{dt} \left( \frac{\partial m}{\partial \dot{K}} \right) \) does not produce any terms involving \( \dot{K} \) since \( m = m(K,t) \), and we are left with an algebraic E-L equation. Such degenerate cases
arise when dependence on \( \dot{K} \) is linear (van Brunt (2004)). To generate an initial value problem in \( K \), regularization is necessary. Therefore we introduce another contribution to the functional, involving \( h(t, K, \dot{K}) \), in Section 3.

Appendix 2: Solution to Euler-Lagrange equation for \( \sigma > 0 \)

Consider equation (29) for \( \delta = 0 \) but \( \sigma > 0 \)

\[
\dot{x}(t) = \frac{\lambda_1 \mu_0}{\lambda_2} n(t) - \frac{\sigma \beta}{\lambda_2} n(t) (x(t))^\nu
\]  

(33)

where \( n(t) = e^{-\sigma t} g(t) \), and expanding \( x(t) \approx x_0(t) + \sigma x_1(t) \) in small parameter \( \sigma \ll 1 \) and substituting in equation (33)

\[
\dot{x}_0(t) + \sigma \dot{x}_1(t) \approx \frac{\lambda_1 \mu_0}{\lambda_2} n(t) - \frac{\sigma \beta}{\lambda_2} n(t) (x_0(t))^\nu \left(1 + \sigma \frac{x_1(t)}{x_0(t)}\right)^\nu
\]

(34)

with terms constant in \( \sigma \) equating to \( \dot{x}_0(t) = \frac{\lambda_1 \mu_0}{\lambda_2} n(t) \) with \( x_0(0) = 1 \) and those of first-degree in \( \sigma \) yielding

\[
\dot{x}_1(t) = -\frac{\beta}{\lambda_2} n(t) (x_0(t))^\nu
\]

(35)

with \( x_1(0) = 0 \). The \( 0^{\text{th}} \)-degree equation is integrated for \( x_0(t) = 1 + \frac{\lambda_1 \mu_0}{\lambda_2} N(t) \), where \( N(t) = \int_0^t n(s) \, ds \).

Then \( x_1(t) = -\int_0^t \frac{\beta}{\lambda_2} n(s) \left(1 + \frac{\lambda_1 \mu_0}{\lambda_2} N(s)\right)^\nu \, ds \), which simplifies to \( x_1(t) = -\frac{\beta}{(\nu+1)\lambda_1 \mu_0} \left(1 + \frac{\lambda_1 \mu_0}{\lambda_2} N(t)\right)^{\nu+1} - 1 \).

With \( \frac{\lambda_1 \mu_0}{\lambda_2} N(t) \ll 1 \) we can further approximate for \( x_1(t) = -\frac{\beta}{\lambda_2} N(t) \) and using \( \beta = \alpha \mu_0 \)

\[
x(t) \approx 1 + \frac{\mu_0}{\lambda_2} (\lambda_1 - \sigma \alpha) N(t)
\]

(36)

As it turns out, \( \lambda_1 \) is small compared to \( \sigma \alpha \) so that \( \lambda_1 - \sigma \alpha < 0 \) in general, and \( x(t) \) is decreasing in time for \( \sigma > 0 \). Hence the Euler-Lagrange solution for \( \sigma > 0 \) does not correspond to relevant economic optima, and such a situation cannot be solved using the method of regularization described here. Analogously it can be shown that with \( \delta > 0 \) the solution \( x(t) \) and hence \( K(t) \) is decreasing in time. Therefore we solve for the quasi-stationary pathway following equation (30), but with
σ present in the emissions model and affecting how the cumulative emissions goal is met. Once this pathway is estimated, it is applied to our model of mitigation expenditures having in general nonzero σ and δ.

References

Aghion, P., C. Hepburn, A. Teytelboym, and D. Zenghelis (2014), Path dependence, innovation and the economics of climate change, Grantham Research Institute on Climate Change and the Environment.

Allen, M. R., and T. F. Stocker (2014), Impact of delay in reducing carbon dioxide emissions, *Nature Climate Change*, 4, 23–26, doi:http://dx.doi.org/10.1038/nclimate2077.

Allen, M. R., D. J. Frame, C. Huntingford, C. D. Jones, J. A. Lowe, M. Meinshausen, and N. Meinshausen (2009), Warming caused by cumulative carbon emissions towards the trillionth tonne, *Nature*, 458, 1163–1166, doi:http://dx.doi.org/10.1038/nature08019.

Amann, M., P. Rafaj, and N. Höhne (2009), GHG mitigation potentials in Annex I countries: Comparison of model estimates for 2020, *Tech. Rep. IR-09-034*, International Institute for Applied Systems Analysis.

Criqui, P., S. Mima, and L. Viguier (1999), Marginal abatement costs of CO2 emission reductions, geographical flexibility and concrete ceilings: an assessment using the POLES model, *Energy Policy*, 27, 585–601, doi:10.1016/S0301-4215(99)00051-8.

Dechezleprêtre, A., R. Martin, and M. Mohmen (2014), Knowledge spillovers from clean and dirty technologies, CEP Discussion Paper No 1300.

DeLong, J. B. (1998), Estimating World GDP, One Million B.C. - Present, University of California at Berkeley mimeo.

Ellerman, A. D., and A. Decaux (1998), Analysis of Post-Kyoto CO2 Emissions Trading Using Marginal Abatement Curves, *Tech. Rep. 40*, MIT Joint Program on the Science and Policy of Global Change.
Fisher, A., and M. Hanemann (1993), *Assessing Surprises and Nonlinearities in Greenhouse Warming*, chap. Assessing climate risks: valuation of effects, pp. 133–154, Resources for the Future.

Friedlingstein, P., R. M. Andrew, J. Rogelj, G. P. Peters, and J. G. Canadell (2014), Persistent growth of CO2 emissions and implications for reaching climate targets, *Nature Geoscience*, 7, 709–715, doi:10.1038/NGEO2248.

Grubb, M. (1993), The costs of limiting fossil-fuel CO2 emissions: a survey and analysis, *Annual Review of Energy and the Environment*, 18, 397–478, doi:10.1146/annurev.eg.18.110193.002145.

Keller, K., B. M. Bolker, and D. F. Bradford (2004), Uncertain climate thresholds and optimal economic growth, *Journal of Environmental Economics and Management*, 48, 723–741, doi:10.1016/j.jeem.2003.10.003.

Klepper, G., and S. Peterson (2004), Marginal abatement cost curves in general equilibrium: The influence of world energy prices, *Tech. Rep. 136*, Nota di Lavoro, Fondazione Eni Enrico Mattei.

Krakauer, N. Y. (2014), Economic growth assumptions in climate and energy policy, *Sustainability*, 6, 1448–1461, doi:10.3390/su6031448.

Lempert, R. J., M. E. Schlesinger, and S. C. Bankes (1996), When we don’t know the costs or the benefits: Adaptive strategies for abating climate change, *Climatic Change*, 33, 235–274, doi:10.1007/BF00140248.

MacDougall, A. H. (2016), The Transient Response to Cumulative CO2 Emissions: a Review, *Current Climate Change Reports*, 2, 39–47, doi:http://dx.doi.org/10.1007/s40641-015-0030-6.

MacDougall, A. H., and P. Friedlingstein (2015), The origin and limits of the near proportionality between climate warming and cumulative CO2 emissions, *Journal of Climate*, 28, 4217–4230, doi:http://dx.doi.org/10.1175/JCLI-D-14-00036.1.

Manne, A. S., and R. G. Richels (1993), *Buying Greenhouse Insurance: The Economic Costs of Carbon Dioxide Emissions Limits*, The MIT Press.

Matthews, H. D., N. P. Gillett, P. A. Stott, and K. Zickfeld (2009), The proportionality of global
warming to cumulative carbon emissions, *Nature*, 459, 829–832, doi:http://dx.doi.org/10.1038/nature08047.

Meinshausen, M., N. Meinshausen, W. Hare, S. C. B. Raper, K. Frieler, R. Knutti, D. J. Frame, and M. R. Allen (2009), Greenhouse-gas emission targets for limiting global warming to 2°C, *Nature (London, United Kingdom)*, 458, 1158–1162, doi:10.1038/nature08017.

Morgan, M. G., and M. Henrion (1990), *Uncertainty: A Guide to Dealing with Uncertainty in Quantitative Risk and Policy Analysis*, Cambridge University Press.

Morris, J., S. Paltsev, and J. Reilly (2008), Marginal Abatement Costs and Marginal Welfare Costs for Greenhouse Gas Emissions Reductions: Results from the EPPA Model, *Tech. Rep. 164*, MIT Joint Program on the Science and Policy of Global Change.

Nordhaus, W., and P. Sztorc (2013), *DICE 2013R: Introduction and User’s Manual*.

Nordhaus, W. D. (1993a), *Economic Progress and Environmental Concern*, chap. How Much Should We Invest In Preserving Our Current Climate?, pp. 255–299, Springer-Verlag.

Nordhaus, W. D. (1993b), Rolling the 'DICE': An optimal transition path for controlling greenhouse gases, *Resource and Energy Economics*, 15, 27–50, doi:10.1016/0928-7655(93)90017-O.

Paltsev, S., J. M. Reilly, H. D. Jacoby, R. S. Eckaus, J. McFarland, M. Sarofim, M. Asadoorian, and M. Babiker (2005), The MIT Emissions Prediction and Policy Analysis (EPPA) Model: Version 4, *Tech. Rep. 125*, The MIT Joint Program on the Science and Policy of Global Change.

Peters, G. P. (2016), The 'best available science' to inform 1.5°C policy choices, *Nature Climate Change*, 6, 646–649, doi:10.1038/nclimate3000.

Pfeiffer, A., R. Millar, C. Hepburn, and E. Beinhocker (2016), The '2°C capital stock' for electricity generation: Committed cumulative carbon emissions from the electricity generation sector and the transition to a green economy, *Applied Energy*, 179, 1395–1408, doi:10.1016/j.apenergy.2016.02.093.

Pindyck, R. S. (2013), Climate change policy: what do the models tell us?, *Journal of Economic Literature*, 51, 860–872.
Raupach, M. R., S. J. Davis, G. P. Peters, R. M. Andrew, and J. G. Canadell (2014), Sharing a quota on cumulative carbon emissions, *Nature Climate Change*, 4, 873–879, doi:10.1038/NCLIMATE2384.

Rogelj, J., G. Luderer, R. C. Pietzcker, E. Kriegler, and M. Schaeffer (2015), Energy system transformations for limiting end-of-century warming to below 1.5 °C, *Nature Climate Change*, 5, 519–528, doi:10.1038/NCLIMATE2572.

Roughgarden, T., and S. H. Schneider (1999), Climate change policy: quantifying uncertainties for damages and optimal carbon taxes, *Energy Policy*, 27, 415–429, doi:10.1016/S0301-4215(99)00030-0.

Rozenberg, J., S. J. Davis, U. Narloch, and S. Hallegatte (2015), Climate constraints on the carbon intensity of economic growth, *Environmental Research Letters*, 10, 1–9, doi:10.1088/1748-9326/10/9/095006.

Seshadri, A. K. (2017), Origin of path independence between cumulative CO2 emissions and global warming, *Climate Dynamics*, pp. 1–19, doi:10.1007/s00382-016-3519-3.

Stern, N. (2007), *The Economics of Climate Change: Stern Review*, Cambridge University Press.

Stern, N. (2016), Current climate models are grossly misleading, *Nature*, 530, 407–409, doi:10.1038/530407a.

Stocker, T. F., D. Qin, G.-K. Plattner, L. V. Alexander, and S. K. Allen (2013), *Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*, chap. Technical Summary, pp. 33–118, Cambridge University Press.

Tikhonov, A. N., and V. Y. Arsenin (1977), *Solutions of Ill-Posed Problems*, V H Winston and Sons.

Tokarska, K. B., N. P. Gillett, A. J. Weaver, V. K. Arora, and M. Eby (2016), The climate response to five trillion tonnes of carbon, *Nature Climate Change*, pp. 1–6, doi:http://dx.doi.org/10.1038/nclimate3036.
van Brunt, B. (2004), *The Calculus of Variations*, Springer.

Vogt-Schilb, A., and S. Hallegatte (2014), Marginal abatement cost curves and the optimal timing of mitigation measures, *Energy Policy*, 66, 645–653, doi:10.1016/j.enpol.2013.11.045.

Webster, M., S. Paltsev, and J. Reilly (2008), Autonomous efficiency improvement or income elasticity of energy demand: Does it matter?, *Energy Economics*, 30, 2785–2798, doi:10.1016/j.eneco.2008.04.004.

Weyant, J. P. (1993), Costs of reducing global carbon emissions, *The Journal of Economic Perspectives*, 7, 27–46, doi:http://www.jstor.org/stable/2138499.