The Identification of the Concept Understanding on Limit of Functions Based on Students’ Learning Styles in Mathematics Department at FMIPA UNM

A W Saputra¹, Ilham Minggi², Djadir³

Mathematics Education Department, Universitas Negeri Makassar, Makassar, Indonesia
Email: ¹arwinwahyus26@gmail.com; ²ilham.minggi@unm.ac.id; ³djadir.math@unm.ac.id

Abstract. This paper is a qualitative research with a descriptive approach. The aim of this paper is to identify the understanding on limit of functions concept based on students’ learning styles in Mathematics Department at FMIPA UNM. The subjects of the study were 6 students in Mathematics Department at FMIPA UNM consisted of 1 student at each category with a visual, auditory, kinesthetic, visual-auditory, auditory-kinesthetic, and visual-kinesthetic learning styles. The data collections were conducted through the data analysis of written test results on mathematical concept understanding and interviews. The indicators of understanding on limit of functions concept in this study among others are understanding the concept of epsilon-delta, understanding the absolute value concept as distance, understanding the implication statement concept, understanding to interpreting the limit of functions definition using graph, and understanding to proving the limit of functions in accordance with the formal definition. The results showed that subjects who were dominant in the two categories of learning styles were generally able to understand the limit of functions concept relationally, while subjects who were dominant in one learning style category were generally able to understand the limit of functions concept instrumentally.

Keywords. Mathematical Concept Understanding, Limit of Functions, and Learning Styles.

1. Introduction

The mathematical concept understanding is important in mathematics learning because in addition to being one of the objectives of mathematics learning, the concept understanding can also help students not only memorize the formulas, but can understand correctly what means in mathematics learning [1].

The success of a mathematical concept understanding is not easy thing because it is done individually. Each student has different abilities for understanding the mathematical concepts. However, a mathematical concept understanding must be pursued. For this reason, instructors are required to be a professional in planning, implementing and evaluating the learning processes that are oriented towards the success of mathematical concept understanding.

The limit of functions is a mathematical concept that requires special attention to be understood by students, especially for mathematics students. The reasons why this concept is important to be understood by mathematics students because this concept is the basic concept that builds several othermathematical concepts, for example the concept of derivatives and integrals. In other words, students must be smart in this concept first before learning the next concepts such as derivatives and integrals concepts. This is important because the definitions of derivatives and integrals using
the limit of functions definition for constructing the concept.

The importance of understanding the concept of limit function was also stated by several experts' opinions, including [2] arguing that the notion of limits of functions is an important part of calculus. It is the foundation of many other important concepts. He also arguing that “if students do not understand what limits are about, how can they understand concepts as, for example, derivatives and integrals?” Understanding limit concept is crucial for calculus students since it establishes a ground for development of the concepts of continuity, derivative, and integral. Although importance of limit understanding has been recognized, introduction of this concept, because of its complexity, causes serious difficulties [3],[4]. Salas and Hille in [5] also states that limit is a basic idea in calculus. Without proper grasp of the limit concept, a very important branch of mathematics known as analysis would also not exist. This is illustrating that the importance of the limit of function concept, especially for mathematics students.

Related to the understanding of limit of functions, the theory of concept understanding that used in this paper is the theoretical of understanding by Skemp. [6] expressed the opinion that "To understand something means to assimilate it into an appropriate schema " in his famous article entitled "Relational Understanding and Instrumental Understanding", describing two types of understanding are: (1) Instrumental understanding, defined as "rules without reasons" or in other words a person’s ability to use mathematical procedures to solving a problem without knowing why the procedure is used. In this case a person only understand the sequence of construction or the algorithm. (2) Relational Understanding, defined as " knowing what to do and why " or in other words the ability to use a rule with a full awareness of why using these rules. According to Skemp, at this stage a person not only knows and memorizes something but also knows how and why it happened.

This paper is limited to the aspects of instrumental understanding and relational understanding because in generally many people are able to name something correctly but are unable to explain why it is true. Students' understanding of the concept of limit functions is said to be instrumental if students are able to write and implement the concept of limit functions but have not been able to explain precisely why. Meanwhile, students’ understanding is said to be relational if they are able to write and implement the the concept of limit functions and they are able to explain precisely why.

One of the aspects that affects the students’ absorption in learning mathematics is learning styles aspect. Learning styles have an important role in the learning process of students. The importance of aspects of learning styles can be a reasoning for mathematics teachers to make good learning in the classroom. In other words, the teaching and learning process can run effectively if the learning strategies that applied in the classroom accommodating the learning styles possessed by students.

Each student has a unique learning style. In other words, no learning styles is better or worse than another. Knowing the appropriate learning styles for themselves is important because every student will find it easier to absorb information. By recognizing a dominant learning style, each student will be smarter to determining a more effective way of learning for themselves. Thus, each student can make the most of their learning abilities so that the learning outcomes obtained can be optimal.

Based on the diversity of learning styles regarding the individual differences to mathematical concept understanding, it is necessary to carry out further studies and analysis to determine the understanding of students' mathematical concepts based on learning styles. In this paper, the author intends to researchs about the learning styles of students who are focused on understanding the concept of limit of functions.

The indicators of understanding the concept of limit functions discussed in this paper including understanding the concept of epsilon-delta, understanding the concept of absolute value as distance, understanding the concept of implication statements, understanding to interpreting the formal definition of limit of functions using graphs, and understanding to proving the truth value of the limit of functions according to the formal definition.
2. **Research Method**

This paper is a qualitative research with a descriptive approach. This study aims to describe the understanding of mathematical concepts in limit of functions topic based on students’ learning styles. This research was conducted on Mathematics Department of FMIPA UNM in the odd semester of the 2019/2020 academic year.

The subjects in this study were students of the Mathematics Department of FMIPA UNM who were given a learning styles questionnaire to grouping the types of students’ learning styles. The subjects in this study consisted of 6 students, such as 1 student each category with a visual, auditory, kinesthetic, visual-auditory, auditory-kinesthetic, and visual-kinesthetic learning styles.

The instruments used in this study were grouped into 4 types namely the researcher himself as the main instrument, a learning style questionnaire to classify the types of students’ learning styles, a written test to identify the concept understanding of the subjects, as well an interview guidelines for triangulation purposes.

The data collection techniques used in this study include: (1) The student learning styles data obtained through the provision of a learning style questionnaire to classify the types of students’ learning styles (2) The data of concept understanding obtained by giving written test to all six subjects were selected. In this case the written test contains a problem with the limit of functions topic. This test is given after the subjects have completed a learning style questionnaire. (3) The Interview was conducted after giving a written test of the concept understanding using semi-structured interview guidelines.

According to [7], there are three activity in qualitative data analysis that occur simultaneously. The activities in data analysis in this study include: (1) *Data Condensation*. At this stage, the researcher compiles the interview transcript based on the recordings between the researcher and the subjects. Then the researcher made coding on each item of the interview transcript. Then these points are condensed by choosing items that only a transcript of the interview according to the research questions. (2) *Data Display*. At this stage, the researcher identifies the subject’s understanding mathematical concepts based on the results of the interview transcripts and the results of written tests that have been carried out in the previous data condensation process. (3) *Conclusion Drawing / Verifications*. At this stage, the researcher compared the results of the written test with the interview transcript that had been condensed previously through a process of methods *triangulation*. The results of the methods *triangulation* is used as a reference for the writer to drawing a conclusion.

3. **Results and Discussion**

The data collection in this study was carried out by giving a learning styles questionnaires, giving a written tests of mathematical concepts understanding, and interviews. The data was initially obtained by classifying the category of students’ learning styles from the results of the learning style questionnaires that given to all students of the Mathematics Science class batch 2017 as many as 30 students. The result of the learning style questionnaires scoring can be seen in the following table.

| No | Type of Learning Styles                  | Number of Subjects |
|----|-----------------------------------------|--------------------|
| 1  | Visual Learning Styles                  | 13 subjects        |
| 2  | Auditory Styles                         | 5 subjects         |
| 3  | Kinesthetic Learning Styles             | 6 subjects         |
| 4  | Visual – Auditory Learning Styles       | 1 subjects         |
| 5  | Auditory – Kinesthetic Learning Styles  | 2 subjects         |
| 6  | Visual – Kinesthetic Learning Styles    | 3 subjects         |
|    | **Total**                               | **30 subjects**    |
Subsequently, six subjects were selected consisting of one student each category with a visual, auditory, kinesthetic, visual-auditory, auditory-kinesthetic, and visual-kinesthetic learning styles. The selection of the six subjects was based on the learning styles score of each student, where the student who had the highest learning styles score in each type of learning style category would be selected as the research subject. The description of the understanding mathematical concepts in the limit of functions topic based on learning styles is as follows:

3.1 Visual Learning Style Category

3.1.1 Understanding the Concept of Epsilon-Delta. The subject describes the meaning of epsilon as a positive number without being able to relate to other mathematical concepts such as the concept of distance. The subject does not understand the meaning of delta because the subject argues that delta is not a number.

3.1.2 Understanding the Absolute Value Concept as Distance. The subject describes the concept of absolute value based only on what the subject seen without being able to relate it to other mathematical concepts. The subject also does not thoroughly understand the concept of absolute value inequality, either containing the epsilon symbol or containing the delta symbol.

3.1.3 Understanding the Implication Statement Concept. The subject describes the implication statement on the written test based on what the subject seen, but the subject is unable to relate it to other mathematical principles to construct a concept of formal definition of the limit of functions such as the quantity of epsilon-delta, the meaning of absolute value, or the meaning of absolute value inequality concept.

3.1.4 Understanding to Interpreting the limit of functions definition using graph. The subject interprets the graph of the limit of functions by relating the concept of the epsilon-delta quantity to the axis coordinate, although the answer is not entirely correct. The subject is unable to understand the graphs depicted and unable to relating to other mathematical concepts such as inequality of absolute values and implication statements to interpreting the existing graphs.

3.1.5 Understanding to proving limit of linear functions with the formal definition. The subject proves the statement of the limit value statement of a linear function is true by relating to the existential quantor meaning. However, the subject was not able to comprehend thoroughly the principles of selecting delta quantity based on inaccuracy containing the epsilon that the subject had previously completed.

3.1.6 Understanding to proving limit of rational quadratic functions with the formal definition. The subject proves that the limit value statement of the rational quadratic function is true by making algebraic manipulations to determine the delta quantity in the existential quantor. However, some steps still not completely precise and the subject is often confused when the writer tries to confirm based on what the subject has written on the answer sheet.

3.2 Auditory Learning Style Category

3.2.1 Understanding the Concept of Epsilon-Delta. The subject is confused when the writer tries to ask several questions that are related to each other in this section.

3.2.2 Understanding the Absolute Value Concept as Distance. The subject does not thoroughly understand the concept of absolute value, both the concepts of \(0 < |x - c| < \delta\) and the concepts of \(|f(x) - L| < s\), and the subject unable to relate it with other mathematical concepts.

3.2.3 Understanding the Implication Statement Concept. The subject describes the meaning of the statement \(0 < |x - c| < \delta \Rightarrow |f(x) - L| < s\) based only on what the subject sees, namely an implication statement. The subject is unable to describe the meaning of these implications and is unable to relate to other mathematical concepts.

3.2.4 Understanding to Interpreting the limit of functions definition using graph. The subject did not fully understand how to interpret the limit of functions graph because no answers that
the subject wrote on the answer sheet, and was unable to express any opinion during the 
interview process.
3.2.5 Understanding to proving limit of linear functions with the formal definition. The subject 
does not understand how to prove the limit value statement of a linear function.
3.2.6 Understanding to proving limit of rational quadratic functions with the formal definition. 
The subject does not understand how to prove the limit value statement of the rational 
quadratic function.

3.3 Kinesthetic Learning Style Category
3.3.1 Understanding the Concept of Epsilon-Delta. The subject describes the meaning of the 
epsilon and delta symbol by relating to the meaning of quantor sentence and can describe 
this symbol by associating with other mathematical objects, namely the concept of distance. 
However, the subject is doubt to suggesting several reasons when the writer gave several 
related questions.
3.3.2 Understanding the Absolute Value Concept as Distance. The subject is only able to describe 
the meaning of absolute value of $0 < |x - c| < \delta$ based on what they see. Meanwhile, the 
subject did not understand at all the meaning of the inequality $|f(x) - L| < s$ because no answer on the answer sheet.
3.3.3 Understanding the Implication Statement Concept. The subject describes the statement $0 < |x - c| < \delta \Rightarrow |f(x) - L| < s$ not only based on what the subject seen, but 
can relate to the hyphen of the epsilon-delta and can explain the objects around the point of 
interest with certain conditions.
3.3.4 Understanding to Interpreting the limit of functions definition using graph. The subject is 
unable to understand how to interpret the graph of the limit of functions. Likewise, the 
answers written by the subject and the opinions expressed by the subject during the 
talk did not match with the expected answers.
3.3.5 Understanding to proving limit of linear functions with the formal definition. The subject 
proves that the limit value statement of a linear function is true by referring to the meaning 
of existential quantors. The subject also understands the results obtained from algebraic 
manipulation. However, the subject did not fully understand the delta quantity selection and 
to make a conclusions.
3.3.6 Understanding to proving limit of rational quadratic functions with the formal definition. 
The subject is not able to solve the algebraic manipulation on the existing quadratic rational 
function. The subject is also unable to choose the quantity in the existential quantor and 
unable to confirm and draw conclusions from the existing problems.

3.4 Visual-Auditory Learning Style Category
3.4.1 Understanding the Concept of Epsilon-Delta. The subject describes the meaning of the 
epsilon and delta by relating to the concept of distance obtained from the results of the 
talk even though the subject did not write down the answer on the answer sheet.
3.4.2 Understanding the Absolute Value Concept as Distance. The subject describes the meaning 
of the absolute value is not only based on what seen by the subject but the subject can relate 
it with the other mathematical objects such as a distance, although not entirely correct. The 
subject was also unable to state a logical reason based on the answers that the subject 
written down.
3.4.3 Understanding the Implication Statement Concept. The subject describes a statement $0 < 
|x - c| < \delta \Rightarrow |f(x) - L| < s$ based on what the subject seen that it’s the statement of 
implications. The subject did not write down the answer at all on the answer sheet but 
during the interview process, the subject was able to describe the statement $0 < |x - c| < \delta \Rightarrow 0 <$
\[ |x - c| < \delta \] as an implication statement with its consequent and antecedent concepts.

3.4.4 Understanding to Interpreting the limit of functions definition using graph. The subject is able to interpret the definition of the limit of functions by using a graph but the subject is not understood as a whole. The subject is only able to use the concepts of epsilon-delta on the graph, but the other concepts such as the sentences of quantor, absolute values, and the implication statements cannot be understood by the subject.

3.4.5 Understanding to proving limit of linear functions with the formal definition. The subject is able to prove the statement of the limit value of a linear function is true by relating to the quantor existential meaning. But the subject still did not understand the oneness principles of the quantities of epsilon and delta if the question form was changed. The subject is unable to provide a logical reasons when the writer asks several indicators on the subjects’ answer sheet.

3.4.6 Understanding to proving limit of rational quadratic functions with the formal definition. The subject is able to prove the statement of the limit value of the rational quadratic function is true by relating to the quantor existential meaning. But the subject still did not understand the oneness principles of the quantities of epsilon and delta if the question form was changed. The subject is unable to provide a logical reasons when the writer asks several indicators on the subjects’ answer sheet.

3.5 Auditory-Kinesthetic Learning Style Category

3.5.1 Understanding the Concept of Epsilon-Delta. The subject describes the meaning of the concepts of epsilon and delta by relating to other mathematical concepts, such as the concept of distance and the concept of graph functions intuitively.

3.5.2 Understanding the Absolute Value Concept as Distance. The subject describes the concept of absolute value is not only based on what the subject seen, but the subject is able to relate it with the other mathematical concepts such as the distance and the subject is able to relate it with the intuitive meaning of limit functions.

3.5.3 Understanding the Implication Statement Concept. The subject describes the implication statement on the problem not only based on what the subject seen, but the subject can relate to the meaning of epsilon-delta, and the subject is able to emphasize the objects that around the point of interest with certain conditions.

3.5.4 Understanding to Interpreting the limit of functions definition using graph. The subject is able to interpret the graph of the limit function by relating to the other mathematical concepts, such as the concept of absolute value, the concept of distance, and the concept of implication statements. The subject can explain the existing graph according to the intuitive meaning of limit functions.

3.5.5 Understanding to proving limit of linear functions with the formal definition. The subject proves the statement of the limit value of a linear function is true by relating to the existential quantors meaning. The subject is also able to make algebraic manipulation of the inequality that contains the epsilon to selecting the appropriate of delta value.

3.5.6 Understanding to proving limit of rational quadratic functions with the formal definition. The subject proves the statement of the limit value of the quadratic rational function is true not only based on the existential quantors meaning, but the subject is also able to make algebraic manipulation of the absolute value inequality on the epsilon form. The Subject is also able to confirm and draw conclusions based on the selected delta value.

3.6 Visual-Kinesthetic Learning Style Category

3.6.1 Understanding the Concept of Epsilon-Delta. The Subject describes the meaning of epsilon and delta by relating to an existential quantors. The subject is also able to describe the intuitive meaning of limit of functions. In addition, the subject is also able to interpret the epsilon and delta symbols by linking to other mathematical objects, such as the concept of distance.

3.6.2 Understanding the Absolute Value Concept as Distance. The subject is not fully understand
the meaning of the absolute value inequality on the problem as evidenced by the subjects’ answer sheet and the subject is doubt to answer a questions related to the indicators of concept understanding.

3.6.3 Understanding the Implication Statement Concept. The subject describes the implication statement of $0 < |x - c| < \delta \Rightarrow 0 < |x - c| < \delta$ not only based on what the subject seen, but the subject also understands the basic concept of the implication but not fully understood, and the subject can also relating to the intuitive meaning of limit of functions.

3.6.4 Understanding to Interpreting the limit of functions definition using graph. The subject interprets the graph of the limit of functions by relating to the inequality principle which contains the delta symbol. However, the subject is unable to relate the graph to the other mathematical principles such as inequality that containing epsilon-delta symbol, the absolute value concepts, and the implication statements.

3.6.5 Understanding to proving limit of linear functions with the formal definition. The subject proves the statement of limit value of a linear function is true by relating to the existential quantors. The subject is able to understand that the results obtained from the algebraic manipulations on existential quantor quantity is singular.

3.6.6 Understanding to proving limit of rational quadratic functions with the formal definition. The subject is able to make algebraic manipulations on rational forms to obtain a linear forms of functions and the subject is able to state the reasons to make algebraic manipulations correctly as well.

The following is a table of indicators for concept understanding of limit functions based on students’ learning styles on Mathematics Department of FMIPA UNM based on the Skemp theory which is shown in the following table:

| Table 2. Concept Understanding of Subjects by Skemp |
|-----------------------------------------------|
| Indicators of Concept Understanding | V | A | K | VA | AK | VK |
|-----------------------------------------------|
| The meaning of s and $\delta$ | N | N | I | R | R | R |
| The meaning of absolute value as distance | I | N | I | I | R | I |
| The meaning of implication statement | I | I | I | I | R | I |
| The formal definition of limit using graph | N | N | N | I | R | N |
| Proving the limit of linear functions | I | N | I | I | R | R |
| Proving the limit of rational quadratic functions | I | N | N | I | R | R |

Informations :
I : Instrumental
Understanding R : Relational Understanding N
: Not Understanding

4. Conclusions
4.1 Visual Learning Style Category
The subject with a visual learning style is generally able to understand the concept of limit of functions instrumentally on indicators of understanding the meaning of absolute value, understanding the statement of implications, and prove the truth value of limit of functions.

4.2 Auditory Learning Style Category
The subject with an auditory learning style is generally unable to understand the concept of limit of functions. This is marked by the inability of the subject to solve a questions on the six indicators of understanding and not being able to provide a logical reasons during the interview process.
4.3 Kinesthetic Learning Style Category
The subject with a kinesthetic learning style is generally able to understand the concept of limit of functions instrumentally on four indicators of understanding, such as understanding the meaning of epsilon-delta, the meaning of absolute value, the meaning of the implication statement, and proving the limit value of linear function.

4.4 Visual-Auditory Learning Style Category
The subject with a visual-auditory learning style understands the meaning of epsilon-delta relationally. Meanwhile, the subject is generally able to understand the concept of limit of functions instrumentally. There are at least five indicators that the subject can understand instrumentally.

4.5 Auditory-Kinesthetic Learning Style Category
The subject with an auditory-kinesthetic learning style is generally able to understand the concept of limit of functions relationally. In this case, the subject is not just understand with the concept of limit function at any given indicators, but the subject also knows the reason to solve any indicators understanding of the concept of limit function.

4.6 Visual-Kinesthetic Learning Style Category
The subject with a visual-kinesthetic learning style is generally able to understand a problems related to the meaning of epsilon and delta and understanding to proving the truth value of linear and quadratic functions relationally. In this case, the subject is not only able to solve the problems, but the subject is also capable to expressing the reason of any answers on the indicators. Meanwhile, the subject understands the meaning of absolute value and the implication statement instrumentally which means that the subject is only able to solve the problem without providing a logical reason for the answer that has been completed.

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