Research on Location Selection of Pre-warehouses Based on Extended CFLP Model

Cong Li\textsuperscript{1,a}, Zhenlin Wei\textsuperscript{2,b,*}

\textsuperscript{1} School of Traffic and Transportation, Beijing Jiaotong University, Beijing, China  
\textsuperscript{2} Institute of Systems Engineering and Control, Beijing Jiaotong University, Beijing, China

\textsuperscript{a} 19120835@bjtu.edu.cn, \textsuperscript{*,b} Corresponding author e-mail: zhlwei@bjtu.edu.cn

Abstract. The warehousing logistics mode of “pre-warehouse plus terminal distribution” is a decentralized resource allocation scheme around the logistics industry at the end of distribution, supporting the development of New Retail, a data-driven extensive retail form centered on consumer experience. Firstly, taking the stand-alone pre-warehouse as the research object, this paper analyzed the distribution characteristics of new retail fresh goods, the consumption structure of demand and the functional characteristics of the pre-warehouse. Secondly, the extended CFLP (Capacitated Facility Location Problem) model was established with processing fee, freshness loss, time penalty and warehouse service capacity. In addition, the one-to-one distribution relationship between demand point and storage facilities in the traditional CFLP model was changed to many-to-many in order to improve the overall facility utilization. Finally, an improved coding genetic algorithm was adopted to solve the nonlinear mixed 0-1 programming model. Therefore, location, matching and distribution volume were showed at the same time. The results of the example demonstrated that the model gave priority to meet the timeliness and fully utilized the facility capacity.

Keywords: logistics engineering, facility location, CFLP, genetic algorithm.

1. Introduction
New retail format attaches great importance to consumer experience, and the corresponding level of logistics services required has been unprecedentedly improved. "Minute level" distribution is the development trend of the logistics industry and will become a new type of service in the logistics industry [1]. Therefore the “pre-warehouse + terminal distribution” is well adapted to the diversified changes in the terminal distribution center because it provides delivery service from the warehousing facility closest to the consumer. And it has realized the transformation from a centralized supply chain to a distributed supply chain and the transition from "customers looking for goods" to "goods looking for customers" [2].

The pre-warehouse is mainly in the form of integrated storefront and warehouse and independent warehouse. Both of them attaches to consumer clusters to form a minute-level distribution network. The general independent warehouse-style pre-warehouse model is shown in Fig. 1.
Figure 1. Warehousing logistics mode for pre-warehouse.

For the operating entity, the front warehouse has achieved a balance between the storage cost of commodities with different sales expectations and the turnover efficiency. Most urban warehouses contain long-tail, low-frequency, and out-of-season products, while the pre-warehouses contain hot-selling, high-frequency, and seasonal products, especially fresh products. The rapid delivery of high-frequency products is achieved under the premise of ensuring the diversity of commodities. For consumers, the pre-warehouse is backed by a superior logistics system that integrates the service level and category breadth of large stores with the convenience of small stores, which means that it meets consumers' expectations of finding a balance between time cost and product preferences.

2. Solution Ideas
In addition to the three principles of minimum cost, dynamics, and consideration of the distribution of demand points in general site selection [3], the location pre-warehouses under new retail also need to have the highest time satisfaction principle. Under the pre-warehouse mode, the goods arrive at the pre-warehouses from the urban warehouse in advance everyday thus there is no overtime during this period of transportation. But there is a loss of freshness in transportation of truck-load. Besides, the pre-warehouse undertakes a lot of distribution business. the costs of picking, processing, packaging and distribution need to be considered. Last, pre-warehouses are set up near communities or business districts where land resources are tight. The effective area of a pre-warehouse is generally below 400 m², meaning the service capacity is limited.

In summary, this article regarded the site-selection of pre-warehouse as a multi-warehouse location problem considering time cost, processing cost, freshness loss and service capability.

3. Model Establishment

3.1. Problem Description
As shown in Fig. 2, the problem can be regarded as selecting some pre-warehouses (at most n pre-warehouses) in certain positions from m alternative pre-warehouses, and these selected warehouses distribute goods to n demand points. The total cost should be minimized under the premise of meeting all requirements. The solution result should include three aspects: (1) the set of selected pre-warehouses, (2) the distribution relationship between each warehouse and the demand point, and (3) the distribution volume of each distribution relationship.
CFLP (Capacitated Facility Location Problem) [4,5] was selected and expanded to adapt to this problem. In addition of some new cost components, the one-to-one relationship between facilities and demand nodes in the traditional CFLP model has been changed to a many-to-many relationship to avoid poor inventory distribution and waste of service capacity.

3.2. Model Assumptions
- The transportation from the Urban warehouse to the pre-warehouses is delivered at one time in advance. While the delivery from the pre-warehouses to the demand nodes is delivered multiple times in proportion to delivery volume.
- One pre-warehouse can supply multiple demand nodes One demand node can be supplied by multiple pre-warehouses.
- The pre-warehouses can be selected only in the range of alternatives.
- The demand at each demand node is known.
- All nodes are regarded as mass points, and the distance between them is a straight line.
- The transportation from the Urban warehouse to pre-warehouse is truck-load, using refrigerated trucks. The delivery from pre-warehouse to demand node is instant, using electric scooter.
- The cost of building, operating warehouse and processing goods is known.
- No reverse logistics such as return or exchange.

3.3. Symbol Description
Model variables: Let \( Z_i \in \{0,1\} \) be the selection result of the \( i \)-th pre-warehouse, and \( Q_{ij} \) be the delivery volume result between the \( i \)-th pre-warehouse and the \( j \)-th demand node. \( x_{ij} \in \{0,1\} \) indicates whether the end distribution between the \( i \)-th pre-warehouse and the \( j \)-th demand node timed out (\( x_{ij}=0 \) means not timeout). \( Y_{ij} \in \{0,1\} \) indicates whether the \( i \)-th pre-warehouse deliver to the \( j \)-th demand node (\( Y_{ij}=0 \) means does not).

Model parameters: \( M=\{1,2,\ldots,m\} \) is the set of alternative pre-warehouses while \( N=\{1,2,\ldots,n\} \) represents the set of demand nodes. \( C_1 \) is the freight rate from Urban warehouse to pre-warehouse while \( C_2 \) is the freight rate from pre-warehouse to demand node? \( D_j \) is the demand volume of the \( j \)-th demand node. \( E_i \) is the receiving volume of the \( i \)-th pre-warehouse. \( F_j \) is the linear coefficient of penalty cost of the \( j \)-th demand node. \( S_i \) is the capacity constrain of the \( i \)-th pre-warehouse? \( A_j \) is the maximum delivery time promised for the \( j \)-th demand node, that is to say time threshold? \( D_{ij} \) is the linear distance between Urban warehouse and the \( i \)-th pre-warehouse while \( d_{ij} \) represents the linear distance between the \( i \)-th pre-warehouse and the \( j \)-th demand node. \( f_i \) is the fixed cost of building and operating the \( i \)-th pre-warehouse while \( h_i \) means the unit processing cost of the \( i \)-th pre-warehouse? \( g \) is the coefficient of penalty cost for freshness loss per unit volume of goods. \( k \) is the average delivery quantity of single terminal distribution. \( t_{ij} \) is the time-consuming from the \( i \)-th pre-warehouse to the \( j \)-th demand node. \( V \) is the delivery speed from the \( i \)-th pre-warehouse to the \( j \)-th demand node. \( \theta \) is the conversion parameter.
of processing workload and distribution quantity. λ is the freshness function based on transportation distance?

3.4. Objective Function

\[ \min W = \sum_{i=1}^{n} w_i \]  
(1)

\[ w_i = \sum_{i=1}^{n} C_i E_i d_i \] 
(2)

\[ w_2 = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} Q_{ij} d_{ij} / k \] 
(3)

\[ w_3 = \sum_{i=1}^{n} Z_i f_i \] 
(4)

\[ w_4 = \sum_{i=1}^{n} Z_i h_i E_i^y \] 
(5)

\[ w_5 = \sum_{i=1}^{n} \lambda Z_i g E_i \] 
(6)

\[ w_6 = \sum_{i=1}^{n} \sum_{j=1}^{n} Z_i F_j (t_{ij} - a_j) x_{ij} y_{ij} Q_{ij} \] 
(7)

Equation (1) is the optimization function. (2) is the transportation cost from the Urban warehouse to the selected pre-warehouses. (3) is the delivery cost from the selected pre-warehouses to the demand nodes. (4) is the fixed cost for building and operating selected pre-warehouses. (5) is processing cost (generally \( \theta = 0.5 \)). (6) is penalty cost of freshness loss during transportation from urban warehouse to pre-warehouses. In (6), \( \lambda = ae^{bd} - a \) is a continuously decreasing function of transport distance (generally \( a, b = (0.03, 0.004) \)) [7]. (7) is the penalty cost when the pre-warehouse delivery time exceeds the time threshold. \( F_j \) is a positive coefficient. The larger the value is, the more sensitive it is to time. And \( a_j \) is also a positive parameter. The greater the value is, the less time the demand node requires or the lower the delivery efficiency promised by the company.

3.5. Constraints

\[ \sum_{j=1}^{n} Q_{ij} = D_j, j = 1,2,\ldots,n \] 
(8)

\[ \sum_{j=1}^{n} Q_{ij} = E_i, i = 1,2,\ldots,m \] 
(9)

\[ \sum_{j=1}^{n} Q_{ij} \leq S_i, i = 1,2,\ldots,m \] 
(10)

\[ 0 < \sum_{i=1}^{n} Z_i \leq n \] 
(11)

\[ Q_{ij} \geq 0, i = 1,2,\ldots,m; j = 1,2,\ldots,n \] 
(12)

\[ x_{ij}, y_{ij}, Z_i \in \{0,1\}, i = 1,2,\ldots,m; j = 1,2,\ldots,n \] 
(13)

Formula (8) indicates that the total distribution volume of the pre-warehouses meets all demands. (9) is that the inflow of the pre-warehouses should be equal to the outflow volume. (10) is the constraint of warehouse service capacity. (11) is the quantity limit of the selected pre-warehouses. (12) is the non-negative constraint of distribution volume. (13) is the integer constraint of decision variables.
4. Empirical Analysis

4.1. Case description

It is planned to build at least one of the five pre-warehouse alternatives in an area of about 30 km² to meet the needs of ten demand nodes. The location and the assumed distribution relationship are shown in Fig. 3, indicating that pre-warehouses No.1, No.3, No.4, No.5 are selected.

![Figure 3. Sketch diagram of demand nodes, pre-warehouses and assumed distribution relationship.](image)

The delivery speed from the pre-warehouse to the demand node \( V \) is 2.25 km/h (not the vehicle speed; the process includes picking, packing, waiting and other activities). The average delivery quantity of single terminal distribution \( k \) is 3.7 orders/time (Measuring goods volume by order quantity). The penalty cost coefficient for the freshness loss per unit volume \( g \) is 45. The freight rate from Urban warehouse to pre-warehouse \( C_1 \) is 0.012 CNY/(km*order) while the freight rate from pre-warehouse to demand node \( C_2 \) is 1 CNY/(km*order). Other data are shown in Tab. 1 to Tab. 3.

| Demand Nodes | Values of Parameters Related to Demand Nodes |     |
|--------------|---------------------------------------------|-----|
|              | \( D_j \) (order) | \( a_j \) (hour) | \( F_j \) |
| \( N_1 \)    | 120                                        | 0.85 | 700   |
| \( N_2 \)    | 170                                        | 0.87 | 700   |
| \( N_3 \)    | 150                                        | 0.89 | 720   |
| \( N_4 \)    | 220                                        | 0.78 | 700   |
| \( N_5 \)    | 270                                        | 0.77 | 800   |
| \( N_6 \)    | 200                                        | 0.83 | 700   |
| \( N_7 \)    | 180                                        | 0.76 | 760   |
| \( N_8 \)    | 170                                        | 0.82 | 720   |
| \( N_9 \)    | 220                                        | 0.85 | 700   |
| \( N_{10} \) | 100                                        | 0.88 | 700   |
4.2. Solving process
For this NP-hard problem, the genetic algorithm widely used in CFLP is suitable. It should be noted that the key decision variables of the model are 0-1 variables $Z_i$ and numerical variables $Q_{ij}$, which are arranged in sequence in each chromosome. According to the case, there are 5 candidate pre-warehouses and 10 demand nodes. Therefore $Z_i$ contains 5 variables, and $Q_{ij}$ contains $5 \times 10 = 50$ variables, which means the number of decision variables of the objective function is 55 in total. Besides, as shown in Tab. 4, there is a logical relationship between them, that is, when $Z_i=0$, there must be $Q_{ij}=0$.

Tab.4 Coding of an Individual in Genetic Algorithm

| Pre-warehouses | $Z_i$ | $Q_{ij}$          |
|----------------|-------|------------------|
| $M_1$          | 0     | 0000000000       |
| $M_2$          | 1     | $Q_{2,1}Q_{2,2}Q_{2,10}$ |
| $M_3$          | 1     | $Q_{3,1}Q_{3,2}Q_{3,10}$ |
| $M_4$          | 0     | 0000000000       |
| $M_5$          | 1     | $Q_{5,1}Q_{5,2}Q_{5,10}$ |
The constraint conditions are transformed into a penalty function, which is added to the original objective function to form a new minimized objective function \( f(x) = W + K \). The penalty function \( K \) is represented as (14).

\[
K = \sum_{i=1}^{n} \max(Q_i - D_i, 0) + \sum_{i=1}^{n} \max(Q_i - E_i, 0) + \sum_{i=1}^{n} \max(Q_i - S_i, 0) + \\
\sum_{i=1}^{m} \max(Z_i - n, 0) + \sum_{i=1}^{m} \max(1 - Z_i, 0)
\]

As formula (15) shows, fitness function \( F(x) \) is formed by taking an appropriate large number \( C_{\text{max}} \) to convert the minimization objective function into a maximization problem. Other operations are no different from the general genetic algorithm.

4.3. Result analysis

Figure 4 showed the selected pre-warehouses, distribution relationship between the selected pre-warehouses and demand nodes and distribution volume. With pre-warehouses No.1, No.3, No.4 were selected, all demand was satisfied and none of the pre-warehouses were overloaded (Total capacity utilization rate is 93.26%). In the case of little difference in warehouse capacity, because the alternative pre-warehouses in the same area were not far apart, the transportation cost from the urban warehouse to the front warehouse had little effect on the site selection results. The time penalty constraint caused the model to give priority to the delivery timeliness, avoiding the high penalty cost caused by overtime. Besides, on the premise that the difference in other aspects was not obvious, using \( D_jF_ja_j^{0.5} \) to simply represent the required delivery intensity of \( j \)-th demand nodes, pre-warehouses near the demand nodes No.4, No.5, No.7 and No.9 should be preferred, which is consistent with the site selection result.

5. Conclusion

There are many differences between the pre-warehouse and traditional warehouse in terms of status and function, which makes the location problem of pre-warehouse a novel and characteristic research field. In this paper, a CFLP-based multi-warehouse location model with freshness loss penalty cost and time penalty cost was constructed, and the model was solved by genetic algorithm encoded by multiple variables. The result met the timeliness requirements of terminal distribution, and conformed to the reality of limited capacity of pre-warehouse, which provided a reference for the location of pre-
warehouse. In the further research, more optimization methods and cases will be applied in this model to better analyze the location of pre-warehouse.

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