Analysis of Temperature Effect on Saturated Hydraulic Conductivity of the Chinese Loess

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Abstract: Saturated hydraulic conductivity ($K_s$) is a significant basic hydraulic parameter in the field of soil science. However, it is typically used as a constant, and that will lead to some inaccurate results in numerical simulations of water movement. In this study, a laboratory experiment was conducted to investigate the effect of temperature on $K_s$ of Chinese loess. The results indicated that $K_s$ had a special relationship with temperature for this soil: $K_s$ increased in the range of temperatures ($t$) from 7 to 44 °C. A formula in Forest Standard of China LY/T 1218-1999 can well describe the distribution characteristics of the experimental data, but it may not apply if bulk density changes. The study improved the formula by analyzing another set of published experimental data. In the improved formula, the influence of bulk density and temperature on $K_s$ is fully considered. The results show that the improved formula can be applied to the simulation of saturated water flow by accurately describing $K_s$ variation with temperature and bulk density.

Keywords: Chinese loess; saturated hydraulic conductivity; temperature; bulk density

1. Introduction

Saturated hydraulic conductivity ($K_s$) has been widely used in the modeling of flow and transport-related phenomena in the soil [1]. Saturated hydraulic conductivity has a relationship between particle shape and packing density characteristics of soils [2], and it is strongly influenced by the fluid properties (viscosity, chemistry), soil mineral composition, the relative amount of soil fluid (saturation) present in the soil matrix, and temperature [3]. Temperature is known as a very important factor, which has effects on hydraulic conductivity [4,5]. Geothermal exploration in the volcanic area and heating pipeline burst in the city would lead to the intermittent movement of high-temperature water in the soil [6]. Surface water recharging the aquifer has an influence on groundwater flow pattern in the well field [7]. It is necessary to build a mathematical model and simulate for water in the soil to make a series of sensitivity analyses [8]. Whereas $K_s$ is actually variable, it is typically used as a constant in numerical simulations of water movement. The assumption is not consistent with the actual. Therefore, the simulation results in some inaccurate results, especially for the simulation at various temperatures. Therefore, it is necessary to analyze the temperature influence on $K_s$ deeply, and to establish the corresponding model.

At present, some research about temperature and water movement in soil has been reported. Temperature has effects on soil water movement and retention [9]. Hydraulic conductivity has been shown to increase within a temperature range from 20 to 80 °C [10], though the effect of temperature elevation on hydraulic conductivity is not large [11]. The relationship between temperature and $K_s$ has been described by some formulas based on thermodynamics [12] and activation energy [13]. The effects of temperature on hydraulic conductivity can be predicted to some degree from changes in viscosity [14]. Moreover, models that describe the relationship between temperature and $K_s$ have been established or applied. For example, the numerical solutions have been proposed for the drying of...
porous media. In the experiment, it is found that temperature has an effect on diffusivity of water content, but it is not extremely sensitive to temperature [15]. Saturated hydraulic conductivity can be estimated by using the Hagen–Poiseuille equation [16] and Darcy’s law [17]. Chen et al. suggested that pore size plays a key role in the process of changing hydraulic conductivity [18]. Most studies to date have focused on the theory of energy to describe the effect of temperature on Ks values.

The viscosity of water and porosity of soil are also factors on hydraulic conductivity, and they will change with temperature. Viscosity has been found to greatly impact intrinsic permeability values [19]. Soil permeability is known to increase with increased temperature, due to the attendant decrease in water viscosity [20,21]. It is obvious that the value of hydraulic conductivity also depends on soil density [22–25]. The soil density state can be described by the soil porosity and volumetric water content at saturation. However, many parameters, including viscosity and porosity, will change with temperature. The simple study of viscosity, porosity, and other single factors is not enough to reveal the specific process of Ks varying with temperature, due to the complexity of process changes caused by temperature.

Chinese loess is commonly distributed in the semi-arid Loess Plateau located in the northern and northwestern parts of China [26]. The underground flow of thermal fluid, such as cooling water leaked from thermal power plants or coking plants, is complicated at different temperatures. Exploring and describing the movement process of water in the soil are conducive to the movement simulation of soil water flow and the optimal management of water resources in a changing environment [27,28]. Therefore, it is necessary to study the relationship between Ks and temperature rigorously. In this study, we aimed to (1) analyze the dynamics of Ks and temperature and (2) improve a formula that describes Ks under different bulk density and temperatures.

2. Materials and Methods

2.1. Materials

Chinese loess samples were taken from an experimental field of Institute for Radiation Protection in Yuci, Shanxi province. Prior to the experiment, soils were air dried and passed through a 2 mm sieve. The soil bulk density (D) was 1.35 g/cm³, and the soil porosity was 0.4924 cm³/cm³. The grain-size distributions of the soil, as measured using the pipette method, are presented in Figure 1; d50 was about 0.075 mm. The samples have a loamy sand texture (78.23% sand, 20.81% silt and 0.96% clay) with low organic matter content (0.392%) [29].

![Figure 1. Particle size distribution of the experimental soil.](image)

**Figure 1.** Particle size distribution of the experimental soil.

2.2. Experimental Methods

The popular device to measure Ks is the head well permeameter such as the Guelph Permeameter [30]. In the study, a new device was designed to measure Ks. Figure 2
shows a schematic diagram of the experimental apparatus used to measure Ks values at various temperatures. The experiments were conducted in a constant temperature tank with a dimension of 0.6 m (length) × 0.4 m (width) × 0.5 m (height). A cutting ring (5 cm ID × 5 cm L) filled with soil was placed in a porous plate to uniformly distribute the water flow above the supporting trestle. The cutting ring outlet was connected to the automatic sampler. Water temperature was monitored by a thermometer and controlled by a heater in a water tank. The heater was set at a predetermined water temperature and held temperature invariability for three hours or more. The tank surface was covered with a plastic sheet to minimize surface evaporation and loss of thermal energy. Water in the tank flowed through the constant temperature water tank, the cutting ring, and then into the automatic sampler. According to the water head difference reading from the meter, the water volume recorded by the automatic sampler, and the time difference, Ks values were calculated by Darcy’s law.

![Figure 2](image-url)

**Figure 2.** A schematic diagram of experimental apparatus used to measure Ks at different temperatures.

The experimental procedure was as follows:

**Step 1:** Filling the cutting ring with soil. Soil samples were hand-loaded into the cutting ring in approximately 1 cm segments each time, with a dry bulk density of 1.375 g/cm³. The bottom of the cutting ring was covered with a 200-mesh nylon wire screen. The soil was then completely saturated by placing the bottom of the soil bed into air-free water.

**Step 2:** Connecting the device. The top of the cutting ring was sealed with a variable diameter connecting pipe. The joints were coated with olefin to eliminate any solution leakage from the pipe during the experiment. The cutting ring was then connected to the other equipment, according to the schema presented in Figure 2. The pinchcock was then fastened, and the sample was then put into the constant temperature water tank slowly.

**Step 3:** Performing experiments. The experiments were conducted under steady-state flow conditions and at various temperatures. The pinchcock was loosened to allow water in the constant temperature water tank through the cutting ring with the soil sample, the variable diameter connected pipe, the overflow water tank, the overflow pipe, and into the automatic sampler. Once steady-state flow was established, that is, when the flux of water no longer changed, we recorded the flux at this time for this temperature, then calculated the Ks value according to Darcy’s law.

3. Results

This experiment surveyed 17 same soil samples; every soil sample was measured at about 20 temperatures (ranging from 7 to 44 °C). It was assumed that there is no change in chemical properties and soil structure during the experiment. According to the outflow
velocity of air-free water at different temperature, $K_s$ values were calculated according to Darcy’s law. Thus, 351 experimental data points were obtained.

Figure 3 shows $K_s$ values at different temperatures. It is obvious that there is a linear increasing relationship between temperature and $K_s$. $K_s$ displayed an increasing tendency over the tested temperature range (7–44 °C), and most of the experimental data are within the 95% confidence interval line. The minimum and the maximum $K_s$ values were 0.158 mm/min and 0.803 mm/min at 9.7 °C and 39.9 °C, respectively. $K_s$ was about 0.300 mm/min at 10 °C and was about 0.680 mm/min at 40 °C. A few individual points deviated from the majority of the values, e.g., the $K_s$ of 0.598 mm/min at 9.8 °C.

Figure 3. Experimental data and fitting curves. The dots represent experimental data, the abscissa refers to water temperature (°C), and the ordinate refers to $K_s$ (m/s). The solid line represents the fitting line. The upper and lower dashed lines represent the 95% confidence interval.

The formula of fitting line is fitted as follows and $R^2 = 0.7385$:

$$K_t = 0.0126 t + 0.1755 \quad (1)$$

The parameters involved in the study are listed in Table 1.

| Parameter   | Significance                        | Unit     |
|-------------|-------------------------------------|----------|
| $K_s$       | Soil saturated hydraulic conductivity | mm/min   |
| $K_t$       | Soil saturated hydraulic conductivity at $t$ °C | mm/min   |
| $K_{10}$    | Soil saturated hydraulic conductivity at 10 °C | mm/min   |
| $t$         | Soil water temperature              | °C       |
| $D$         | Bulk density                        | g/cm$^3$ |

4. Discussion

4.1. Comparison with Forest Standard of China

Chinese loess is important in local research on hydrology, geology, and agriculture. Thus, some guidelines about Chinese loess have been published, such as the Forest Standard of China LY/T 1218-1999 (Determination of forest soil percolation rate). This certification included information and guidelines about the relationship between $K_{10}$ and $K_t$ [31]. The
formulation included in the certification (CNSE) is an empirical formula, and is described as follows:

\[ K_t = K_{10} (0.7 + 0.03 t) \] (2)

Fitting CNSE to the experimental data by the least square method, \( K_{10} = 0.337 \) mm/min was obtained, and \( R^2 = 0.739 \). The fitting lines of CNSE are drawn in Figure 4.

![Figure 4. Experimental data and CNSE fitting line. The solid line represents the fitting line. The dashed line represents the CNSE fitting line.](image)

It can be seen from Figure 4 that the CNSE fitting line is very close to the previous fitting line. The slopes of the two lines are 0.0101 and 0.0126 respectively, so the previous fitting line shows that \( K_s \) changes with temperature slightly larger than the CNSE fitting line. Within the experimental temperature range, the maximum difference of \( K_s \) is 0.04919 mm/min. The relative error is about 16.5% at low temperature and −6.71% at high temperature. Therefore, CNSE as an empirical formula has a satisfactory accuracy.

If \( K_{10} \) is taken as the reference value and the form of CNSE is adopted, Formula (2) can be changed to:

\[ K_t = K_{10} (0.0583 + 0.4180 t) \] (3)

where \( K_{10} = 0.3015 \) mm/min.

4.2. Comparison with Another Study Result

Many studies have shown that there is a positive correlation between temperature and \( K_s \). For example, Ren et al. analyzed the effects of different temperature and soil saturated hydraulic conductivity [32]. The soil samples were also taken from Shanxi Province, China. They described the relationship between temperature and \( K_s \) at four different bulk densities \( D \), as shown in Figure 5. The experimental results also show that \( K_s \) has an increased tendency with the increase of temperature. Figure 5 shows that \( K_s \) increases slowly at a temperature less than 20 °C while it increases rapidly at a temperature larger than 20 °C.
From Figure 5, it is obvious that \( K_s \) is affected by \( D \), while changing with temperature. The fitting line 1 is basically close to the experimental data at \( D = 1.35 \). \( K_s \) increases slowly at a temperature less than 20 °C while it increases rapidly at a temperature larger than 20 °C. \( K_s \) is sensitive to \( D \) because they have the same trend. However, fitting lines deviate significantly from experimental data under other \( D \) (1.39, 1.43 and 1.47). The reason is that \( K_s \) passes through the point of \( K_{10} \) and has a slope of 0.4180 \( K_{10} \) according to CNSE. Therefore, the direction of the fitting lines has been fixed, and the fitting result must deviate from the actual data.

Parameters of fitting line 1 are listed in Table 2. From the data of \( K_{10} \) and \( D \), it can also be seen that \( K_{10} \) decreases with the increase of \( D \). In addition, root mean square error (RMSE) changes greatly. The minimum value of RMSE is 0.0095 at \( D = 1.35 \), and the maximum value of RMSE is 0.0226 at \( D = 1.39 \).

### Table 2. Parameters of fitting line 1.

| \( D \) (g/cm\(^3\)) | Fitting Line 1 | \( K_{10} \) (mm/min) | RMSE   |
|-----------------------|----------------|-----------------------|--------|
| 1.35                  | \( K_s = 0.0435 + 0.0031 \cdot t \) | 0.0748 | 0.0095 |
| 1.39                  | \( K_s = 0.0179 + 0.0013 \cdot t \) | 0.0307 | 0.0226 |
| 1.43                  | \( K_s = 0.0088 + 0.0006 \cdot t \) | 0.0151 | 0.0153 |
| 1.47                  | \( K_s = 0.0058 + 0.0004 \cdot t \) | 0.0100 | 0.0160 |

In addition, \( K_s \) shows the decrease tendency with the increase of \( D \) at the same temperature. The fitting line 1 (\( D = 1.35 \)) obtained from Formula (2) is satisfactory. However, the fitting results are quite different from the experimental data at \( D = 1.39, 1.43, \) or 1.47. The different slope of the fitting line should be used under different \( D \).

### 4.3. Formula Modification

Considering the influence of \( D \), the model is modified. The formula is improved based on CNSE, and shown as:

\[
K_t = K_{10} (0.0583 + k(D) \cdot t)
\]  

(4)
where \( k(D) \) is a function of \( D \), which is used to determine the slope of a fitting line. To determine this function, fit the experimental data by Formula (4) and let the fitting line 2 pass through \( K_{10} \). The fitting line 2 is shown as Figure 6.

![Figure 6. Experimental data and fitting line 2 at different \( D \).](image)

From Figure 6, it is obvious that fitting line 2 is in good agreement with the experimental data, and their parameters are listed in Table 3. Comparing Tables 2 and 3, it can be seen that RMSE is significantly reduced. According to the fitting results of Formula (4), the maximum RMSE is only 0.0107, which is far less than the fitting results of Formula (3). Therefore, Formula (4) is more suitable for fitting this experimental data.

| \( D \) (g/cm\(^3\)) | Fitting Line 2 | \( K_{10} \) (mm/min) | \( k(D) \) | RMSE |
|------------------|----------------|------------------|----------|------|
| 1.35             | \( K_s = 0.0448 + 0.0030 t \) | 0.0748            | 0.0401   | 0.0065 |
| 1.39             | \( K_s = -0.0003 + 0.0031 t \) | 0.0307            | 0.1010   | 0.0107 |
| 1.43             | \( K_s = -0.0059 + 0.0021 t \) | 0.0151            | 0.1393   | 0.0024 |
| 1.47             | \( K_s = -0.0100 + 0.0020 t \) | 0.0100            | 0.2008   | 0.0023 |

Table 3 also shows that \( k(D) \) increases with \( D \). As shown in Figure 7, \( k(D) \) and \( D \) show an obvious linear relationship. The fitting formula is \( k(D) = 1.30D - 1.71 \), and \( R^2 = 0.992 \).

Therefore, according to the fitting formula \( k(D) \) and Formula (4), a comprehensive formula is obtained:

\[
K_t = K_{10} (0.0583 + 1.30 D t - 1.71 t) \tag{5}
\]

The improved Formula (5) can be applied to the simulation of saturated water flow under different \( D \) and temperatures.
Therefore, according to the above analysis, the following formula is obtained:

\[ t = K_s - 1.71 \exp(-0.0583D + 1.30) \]

The improved formula (5) can be applied to the simulation of saturated water flow at different bulk densities and temperatures. However, CNSE may be not convenient to apply, if bulk density changes. \( K_s \) decreases with the increase of bulk density. An improved formula (Formula (5)) based on CNSE is proposed to adapt to different bulk densities. The fitting results show that the improved formula is applicable to forecast \( K_s \) at different bulk densities and temperatures.

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