The goal of the ultra-relativistic heavy ion collisions is to create and study quark gluon plasma (QGP). The pre-requisite for the formation of QGP is to create a hot and dense system of nuclear matter. Experimental results from the Relativistic Heavy Ion Collider (RHIC) have indicated the formation of such a state of matter in the early stage \( 1 \). Among others, one of the promising signals of QGP formation is the enhanced production of strange and anti-strange particles \( 2 \). As a result, the production of particles with hidden (e.g., \( \phi \)) as well as open strangeness have become a field of intense theoretical \( 3,4,5,6 \) and experimental \( 7,8,9,10 \) activities.

In the present work we study the production of \( \phi \) meson at RHIC. The \( \phi \) meson is composed of a strange (s) and anti-strange (\( \bar{s} \)) quark and its interaction with nuclear matter is suppressed according to Okubo-Zweig-Izuka (OZI) rule. In QGP s and \( \bar{s} \) quarks are produced mainly by gluon fusion and annihilation of light (u and d) quarks and anti-quarks. These \( s\bar{s} \) will form \( \phi \) through the hadronization process. This process is not OZI suppressed. Therefore, we expect excess \( \phi \) if QGP is formed in the initial state of heavy ion collisions as compared to hadronic initial state. The production of \( \phi \) mesons from sources other than plasma is expected to be small, so they will not overshadow the plasma signal. The other advantage of \( \phi \) is that after its production during the hadronization of the QGP, it suffers less re-scattering in the hadronic matter, thereby retaining information of the thermodynamic state of the matter during its hadronization stage. Therefore, study of \( \phi \) spectra will be very useful to determine the transition temperature \( (T_c) \) for quark-hadron phase transition. However, it may be mentioned that the kaons which come from decay of \( \phi \) mesons will undergo re-scattering, and the \( \phi \)'s experimentally reconstructed from these kaons may carry some re-scattering effect. Still reliable information on \( T_c \) can be extracted by measuring \( \phi \) productions in high energy heavy ion collisions \( 11 \).

Hence we shall show the usefulness of \( \phi \) as a probe for co-existence phase by studying its production at \( \sqrt{s_{NN}} = 200 \text{ GeV} \). First we demonstrate from the experimental data that \( \phi \) mesons freeze out at a temperature close to \( T_c \), the value of which is obtained from lattice QCD simulations \( 12 \). We argue, using the yield of \( \phi \) mesons at mid-rapidity \( (\frac{dN}{dy}) \) that the enhanced production is possible if the system has passed through a mixed phase of quarks, gluons and hadrons and the temperature during the hadronization is \( > 160 \text{ MeV} \). Finally, attempt will be made to characterize the mixed phase by putting bounds on the the effective statistical degeneracy.

Relativistic hydrodynamics with \( (3+1) \) dimensional expansion \( 13 \) and boost invariance along the longitudinal direction \( 14 \) has been used to study the evolution of the matter formed in heavy ion collisions. We take the equation of state from lattice QCD simulations \( 12 \). The initial velocity and energy density profiles are taken as \( v(r, \tau_i) = 0 \) and \( c(r, \tau_i) = c_0/(1 + \exp((r - R)/\delta)) \) respectively. Here \( \tau_i \) is the thermalization time, \( R \) is nuclear radius. Results will be shown for two sets of values of \( \tau_i = 0.15 \) and 0.2 fm/c. Such values of \( \tau_i \) have recently been used to reproduce the thermal photon spectra measured by PHENIX collaboration \( 17 \) in \( \text{Au + Au} \) collisions at \( \sqrt{s_{NN}} = 200 \text{ GeV} \). The corresponding values for the initial temperatures are \( 590 \) and \( 480 \text{ MeV} \) respectively. Results of transverse momentum spectra of \( \phi \) mesons are shown in figs. \( 11 \). It appears that the results for \( T_i = 590 \text{ MeV} \) describes the data quite well for the entire range of transverse momentum \( (p_T) \) under consideration. However, for \( p_T > 2 \text{ GeV} \) (or \( m_T - m_\phi \sim 1.2 \text{ GeV} \), where \( m_T = \sqrt{p_T^2 + m_\phi^2} \) is the transverse mass and \( m_\phi \) is the mass of the \( \phi \) meson) the application of hydrodynamics may be questionable and contributions from perturbative QCD may become important. In fact, at low \( p_T \) domain the contribution of thermal \( s\bar{s} \) for \( \phi \) formation is dominant \( 18 \). In that case, the description for

\( \phi \) production at RHIC: characterization of co-existence phase

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We extract the effective degrees of freedom that characterize the co-existing phase of quark gluon plasma and hadrons. Experimental data on \( \phi \) at mid-rapidity is used to set a lower bound to the critical temperature of quark hadron phase transition. The production and evolution of strangeness have been studied by using Boltzmann equation. The results have been contrasted with the experimental data obtained by STAR collaboration at RHIC for \( \text{Au + Au} \) collisions at \( \sqrt{s_{NN}} = 200 \text{ GeV} \). Our study reveals that the \( \phi \) mesons will form through the hadronization processes. Still reliable information on \( T_c \) can be extracted by measuring \( \phi \) productions in high energy heavy ion collisions.

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$T_i = 480$ MeV is better suited. From the inverse slope (for $0.45 \leq p_T$ (GeV)$\leq 2$) of the data we get the effective temperature as $257 \pm 8$ MeV. We find that the average radial velocity $<v_r> \sim 0.43$ at the freeze-out surface. Therefore, using the relation $T_{eff} = T_{true} + (1/2)m_\phi <v_r>^2$, we get the ‘true’ freeze-out temperature $T_{true} \sim 162 \pm 8$ MeV (error bars correspond to the experimental error bars [8]). It should be noted that the value of $T_{true}$ is close to $T_c$ predicted by lattice QCD simulation. The value of the hadronization temperature in statistical hadronization model [19] was found as $T_c = 155 \pm 8$ MeV (see also [20]). The similar value of hadronization temperature was obtained [21] from the analysis of experimental data on heavier hadrons.

Now we turn to the yield of $\phi$ mesons at mid rapidity. The experimental value of $dN_\phi/dy$ for 0-5% central collisions is $7.7 \pm 0.30$ [8]. It is more convenient to deal with the ratio, $dN_\phi/dN_c$, where $dN/dy$ is the total particle multiplicity at mid-rapidity. This ratio has the advantage in the sense that some of the uncertainties (e.g. volume) are removed. From thermal model calculations it can be shown that at the critical temperature $T_c$,

$$\frac{dN_\phi}{dN_c} = 0.7 \frac{1}{g_{eff}(T_c)} \int_{M_\phi/T_c}^\infty \frac{x^2 dx}{e^x - 1},$$  \hspace{1cm} (1)$$

where, $m_\phi=1.02$ GeV, and $g_{eff}$ is the effective statistical degeneracy. Eq. (1) indicates that the ratio $dN_\phi/dN_c$ depends on $T_c$ and the statistical degeneracy where the $\phi$ mesons freeze out. Taking $T_c$ and $g_{eff}$ from lattice QCD simulations [12] as 170 MeV and 18 respectively, we get $dN_\phi/dN_c = 4.77 \times 10^{-3}$ from Eq. (1). The experimental value of $dN_\phi/dN_c$ is $7.7 \times 10^{-3}$. This indicates that there is a clear over production of $\phi$ mesons. The data can be expressed in terms of the calculated value of $dN_\phi/dy |_{eq}$ by a simple relation:

$$\gamma_\phi = \frac{dN_\phi/dy |_{eq}}{dN/dy |_{eq}}$$  \hspace{1cm} (2)$$

$dN_\phi/dy |_{eq}$ is the value of $\phi$ meson multiplicity expected from thermal model at $T_c$ and $\gamma_\phi$ is the enhancement factor (a value of $\gamma_\phi = 1$ ($<1$) indicates equilibrium (under production). The experimental data requires the value of $\gamma_\phi$ to be $\sim 1.3$. This indicates that there seems to be a over saturation of strangeness production at the temperature where $\phi$ freezes out.

Now we ask, at what stage of the evolution of the system formed in heavy-ion collisions gives the required over production of strangeness? We adopt the momentum integrated Boltzmann equation to study the evolution of the strangeness density. This has also been used extensively to investigate the density evolution in the early universe [22]. The initial state is considered as a thermalized state of gluons and light quarks with initial temperature, $T_i \sim 480$ MeV and thermalization time, $\tau_i \sim 0.2$ fm/c. The reactions considered for the strange quark productions
In QGP are: \( q \bar{q} \rightarrow s \bar{s} \) and \( gg \rightarrow s \bar{s} \). For the production of strange hadrons the following reactions are considered: \( \pi^+ + \pi^- \rightarrow K^+ + K^-, \pi^+ + \pi^0 \rightarrow K^+ + K^0 \) and \( \pi^0 + \pi^0 \rightarrow K^+ + K^- \) in the meson-meson sector and \( \pi^+ + n \rightarrow \Lambda + K^+, \pi^0 + p \rightarrow \Lambda + K^+, \pi^+ + p \rightarrow \Sigma^+ + K^+, \pi^- + p \rightarrow \Sigma^- + K^+ \) and \( \pi^0 + n \rightarrow \Sigma^- + K^+ \) for the meson-baryon sector.

The evolution of \( s \) in QGP phase with proper time \( \tau \) is given by

\[
\frac{dr_s}{d\tau} = \frac{R_i(T)}{n_s^{eq}} [1 - r_s r_s] \tag{3}
\]

where \( R_i(T) \) is the production rate of particle \( i \) at temperature \( T \).

In the above equation the last term stands for the hadronization of \( s \) quarks to \( K^+ \). \( \alpha \) is a parameter which indicates the fraction of \( s \) quarks hadronizing to \( K^- \). The value of \( \alpha = 0.5 \) if we consider \( K^+ \), \( K^0 \), \( K^- \) and \( K^0 \) formation in the mixed phase. \( r_i \) denotes the ratio of non-equilibrium \( n_i \) to equilibrium \( n_i^{eq} \) density of the particle \( i \). Similar equation can be written for \( r_k \) in the hadronic phase. The evolution of other particles e.g. \( K^- \) can be treated similarly with appropriate cross sections \( \Sigma \) as inputs and solving the coupled set of equations. The initial value of \( s \) and \( \bar{s} \) quarks are taken close to their equilibrium value. However, a small change in the initial values of \( r_s \) do not change the final results drastically. Even with lower initial values of \( s \) and \( \bar{s} \) the system reaches equilibrium very fast due to their production in the high temperature heat bath. In Eq. \( 4 \) and \( 1 - f \) are the fractions of hadrons and QGP respectively in the mixed phase. \( R_i(T) \) is the production rate of particle \( i \) at temperature \( T \).

The cooling of the heat bath is governed by the following equation \( 26 \):

\[
\frac{dT}{d\tau} = -\frac{c_s T}{\tau} \frac{b(r_s + r_\bar{s})}{\alpha(a + b(r_s + r_\bar{s}))} \tag{5}
\]

where \( \alpha = (1 + c_s^2)/c_s^2 \), \( a = 8\pi^2/(45c_s^2) \) and \( b = 7\pi^2n_F/(120c_s^2) \), \( c_s \) is the velocity of sound, \( n_F \) is the number of flavours and \( r_s = dr_s/d\tau \). It may be noted that the Bjorken’s cooling law can be recovered from Eq. \( 5 \) when the second term vanishes. The value of net baryon number at the central rapidity has been assumed to be zero.

The time evolutions of the ratios of non-equilibrium to equilibrium number density \( (r_i) \) for \( T_c = 150, 170 \) and \( 195 \) MeV are shown in Fig. 2, 3 and 4 respectively. It should be mentioned here that the corresponding \( g_{eff} \) for the above values of temperature are taken from lattice QCD. \( 12 \). We observe a clear over saturation in number density of kaons at the end of the mixed phase (Fig. 5). For \( T_c = 170 - 195 \) MeV the value of \( r_K \) is 1.4 - 1.5.
at the end of the mixed phase, before it reaches the equilibrium value of 1 in the hadronic phase. It may be noted that there is a small difference between \( r_K^+ \) and \( r_K^- \) in the hadronic phase because of their different production cross sections. For the value of \( T_c = 150 \) MeV, we observe that \( r_K^+ \sim 1 \), for \( \alpha = 0.5 \). For smaller values of \( \alpha \), \( r_K^+ \) will be even smaller.

Considering that we need a \( \gamma_\phi > 1 \) to explain the experimental data (as explained before) our calculations set a lower bound on the value of \( T_c \), it must be \( > 160 \) MeV. For a hadronic initial state with temperature of 250 MeV the value of \( r_K \) remains \( \leq 1 \). This indicates that the enhanced production of \( \phi \) mesons in heavy-ion collisions at \( \sqrt{s_{NN}} = 200 \) GeV can be explained if the system passes through a mixed phase during it’s evolution with a \( T_c > 160 \) MeV. At this value of \( T_c \), the enhancement factor, \( \gamma_\phi \sim r_K^+ \times r_K^- \sim \gamma_\phi^2 \sim 1.24 \). Considering the fact that the experimental data indicate \( \gamma_\phi \sim 1.4 \pm 0.1 \), we argue that \( T_c > 160 \) MeV.

The life time of the mixed phase crucially depends on the effective statistical degeneracy of QGP and hadronic phases. \( g_{eff} \sim 9.26 \) and 31 for \( T = 150, 170 \) and 195 MeV respectively. Larger difference in the statistical degeneracy gives rise to larger discontinuity in the entropy density which consequently generates more latent heat. In the mixed phase the cooling due to expansion is compensated by the liberation of latent heat. Hence if larger latent is generated then the system will spend more time in the mixed phase i.e. the life time of the mixed phase will be larger. This is clearly reflected in the results shown in Fig. 2-4.

We characterize the mixed phase in terms of effective statistical degeneracy \( g_{eff} \). This is justified because, (i) \( \phi \) mesons freeze out at \( T_c \) and (ii) its production can only be explained by the existence of mixed phase. For fixed \( g_{eff} \) we calculate \( \frac{dN_\phi}{dy}/\frac{dN}{dy} \) from Eq. 1. This ratio is then multiplied by \( \gamma_\phi \) and compared with the corresponding value from experimental data (lies within the two thick horizontal lines in Fig. 3). The results for \( g_{eff} = 18, 24, \) and 30 are shown in Fig. 5. We find that \( g_{eff} \sim 18 \) may be considered as a lower bound.

In summary, we have studied the production of \( \phi \) mesons in Au+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV. The transverse momentum spectra is well explained by hydrodynamical model with the equation of state taken from lattice QCD. It is observed that \( \phi \) mesons freeze out around the transition temperature predicted by lattice QCD. The over production of \( \phi \) meson at mid-rapidity can only be explained if QGP is formed in the initial state and passes through a co-existence phase of QGP and hadrons, i.e. over saturation in strangeness indicates the formation of co-existing phase of QGP and hadrons (indications of which has already been pointed out at SPS energies [27, 28]). The analysis of the experimental data reveals a lower bound on the transition temperature and the effective statistical degeneracy. The extracted value of \( T_c \sim 162 \pm 8 \) and the lower bound on \( g_{eff} \sim 18 \). The study of \( K/\pi \) in the similar framework is under progress [29].

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