FURTHER ANALYSIS OF EXCITATIONS OF QUARKS AT FINITE TEMPERATURE
– MASS EFFECT AND POLE STRUCTURE –

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We calculate the spectral function of the massive quark at finite temperature (T) using a Yukawa model and show that the peak in the negative energy region among the three-peaks found in a previous work for the massless quark is largely suppressed. To explore the underlying mechanism of this behavior, we also investigate the pole structure of the retarded Green function of the quark. We will show the result only for the massless quark. We find the residues of the poles corresponding the three-peaks are all comparable at T ∼ mb. We also show that the multi-peak structure of the quark spectra is well described in the pole approximation which indicates that the quasi-particle picture is valid in this T region.

1. Introduction

The recent analyses of the experiment in the Relativistic Heavy Ion Collider (RHIC) suggest that quark-gluon plasma (QGP) near and above the critical temperature Tc of QCD phase transition is a rather strongly coupled system. Such a picture is consistent with the lattice calculations and model calculations which suggest the existence of hadronic excitations even above Tc. In fact, the possible existence

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of hadronic excitations above $T_c$ itself had been suggested earlier by Hatsuda and Kunihiro. They showed using an effective chiral model that hadronic $\sigma$- and $\pi$-like excitations appear as the soft mode of the chiral transition near but above $T_c$.

Recently Kitazawa et al. showed that the quarks may have a collective nature and show also an anomalous behavior at $T$ near but above $T_c$ in the chiral limit: The quark coupled to those soft modes give rise to a three-peak structure in the spectral function. They also clarified with use of a Yukawa model that the novel structure of the quark spectra is owing to the level mixing between quark (anti-quark) states and anti-quark hole (quark hole) states in the thermally excited anti-quark (quark) distribution via the “resonant scattering” of quarks with the soft modes.

In this report, we further examine their results on the anomalous behavior of the quarks focusing on the pole structure of the collective quark excitations and the possible effects of the finite quark mass. The relevance of our new findings of the collective nature of the quarks to the RHIC experiments will be briefly discussed.

2. Formulation

We start from the following Lagrangian composed of a massive quark field $\psi$ and a massive scalar boson $\phi$:

$$\mathcal{L} = \bar{\psi} \left( i\gamma^\mu \partial_\mu - m_f - g\phi \right) \psi + \frac{1}{2} \left( \partial_\mu \phi \gamma^\mu \phi - m_b^2 \phi \right),$$

where $g$ is the coupling constant, $m_f$ the quark mass and $m_b$ the boson mass.

In this report, we focus on the case of a vanishing external momentum of the quark.

The quark self-energy in the imaginary-time formalism at one-loop order is given by

$$\tilde{\Sigma}(i\omega_n) = -g^2 T \sum_n \int \frac{d^3k}{(2\pi)^3} G_0(k, i\omega_n) D(-k, i\omega_m - i\omega_n),$$

where $G_0(k, i\omega_n) = [i\omega_n\gamma_0 - (k \cdot \gamma + m_f)]^{-1}$ and $D(k, i\nu_n) = [(i\nu_n)^2 - k^2 - m_b^2]^{-1}$ are Matsubara Green functions for the free quark and the scalar boson, respectively, with $\omega_n = (2n + 1)\pi T$ and $\nu = 2n\pi T$ being the respective Matsubara frequencies. One can get the retarded self-energy $\Sigma^R(\omega)$ by carrying out the analytic continuation $i\omega_n \rightarrow \omega + i\eta$ after summing over the Matsubara mode $n$.

The spectral functions of the quark and anti-quark are expressed with the retarded Green function $G^R(\omega)$ as $\rho_{\pm}(\omega) \equiv -(1/\pi) \text{Tr} [\text{Im} G^R(\omega) \gamma_0 \Lambda_{\pm}] \equiv -(1/\pi) \text{Im} G^R_{\pm}(\omega)$ where $\Lambda_{\pm} = (1 \pm \gamma_0)/2$ are the number projection operators at vanishing momentum and $G^R_{\pm}(\omega)$ the quark and anti-quark components of the retarded Green functions.

We analytically continue the retarded Green function of the quark to whole complex energy plane and search poles $z_+$ of the Green function by solving following equation in the lower-half plane:

$$[G^R_+ (z_+)]^{-1} = 0,$$
Further Analysis of Collective Excitations of Quarks at Finite Temperature

The pole $z_+$ can be expressed as

$$z_+ = M - i\Gamma/2,$$

where $M$ is the mass of the excitation and $\Gamma$ the width.

The residue at the pole is given by

$$Z = \lim_{z \to z_+} (z - z_+^*) G^R(z),$$

which represents the strength of the excitation.

3. The Spectral function of massive quark

In Fig. 1, we plot the $T$-dependence of the spectral functions of the massless and massive quarks.

Fig. 1. The $T$-dependence of the spectral functions of the massless quark (left panel) and the massive quark with $m_f = 0.2m_b$ (right panel). Each red line is the spectral function of the quark with vanishing momentum at each $T$. We approximate extremely narrow peaks by $\delta$-functions. We express the $\delta$-functions by the vertical lines with the heights which is ten-times of the residues at the peaks. The Green points are the heads of the lines for $\delta$-functions.

The spectral function for the massless quark has three-peak structure at $T \sim m_b$. We call the modes corresponding to the peaks in finite $\omega$ region the massive-mode and the mode corresponding to the peak at the origin the massless-mode. At high $T$, the massive-modes in the positive and negative energy region connect to the normal quasi-quark mode and the ‘plasmino’ mode in the hard thermal loop (HTL) approximation, respectively. On the other hand, the massive-mode peak in the negative energy region is suppressed in the case of the massive quark, and accordingly the three-peak structure hardly remain. This behavior is consistent

$^a$The spectral function for the massless quark in a Yukawa model have been already given by Kitazawa et al.
with the interpretation of the multi-peak structure in the quark spectrum from the aspect of the level mixing in the previous work\cite{8,9} for the massless quark; the quark level is shifted above in energy by quark mass, and hence the level mixing in the negative energy region is suppressed.

We now give a suggestion on the quark spectrum near and above the critical temperature of the chiral phase transition. We assign the ratio of parameters from a calculation in the NJL model\cite{7}, because the model which we use have no phase transition. The extracted ratio of the parameters are $T/m_b \sim 0.8$ and $m_f/m_b \sim 0.2$ at $T \sim 1.1T_c$. The spectral function qualitatively differ from both of zero temperature one and high temperature one. The spectrum of the massive quark at $T \sim T_c$ may have a peak near the origin separately from the peak in the positive-energy region.

4. The Pole structure of quark propagator

We investigate the pole structure of the quark propagator in order to elucidate the underlying mechanism of the behavior of the spectral function shown in the preceding section. Although we should show the result not only for the massless quark but also the massive quark, the research in the later case is now in progress. So we only show the result for the massless quark. We focus on the poles which correspond to the peaks in the spectral function of the quark, although it seems that there are some poles which do not correspond to any peak.

A pole corresponding to the massless-mode stays at the origin of the complex plane irrespective of $T$. The poles of the massive-mode move with $T$ as shown in Fig\textsuperscript{2}.

![Figure 2](image-url)  

Fig. 2. The $T$-dependence of the pole corresponding to the massive-mode in Re $z > 0$ region. Because of the vanishing quark mass, the pole structures in Re $z > 0$ region and Re $z < 0$ region are symmetric. As $T$ is raised from $T = 0.2m_b$ to $T = 40m_b$, the pole denoted by the red point moves from far left to far right.
At low $T$, the poles have large widths and these large widths are consistent with the fact that there is no clear peak in the finite energy region of the spectral function as shown in the left panel of Fig. 1. As $T$ is raised, $\Gamma$ decreases while $M$ increases until some $T$ where $\Gamma$ vanishes. After then the pole moves to large width region as $T$ is raised further. We notice that this behavior is consistent with the HTL approximation; the poles in the HTL approximation may have finite imaginary part in the order of $g^2 T$.

We plot the $T$-dependence of the residues at the poles corresponding to the massless-mode and the massive-mode in Fig. 3. The residues of the massless-mode and the massive-mode are comparable with each other at $T \sim m_b$.

In the vicinity of the pole, the spectrum should be approximated by a Breit-Wigner type formula given by

$$\rho_{BW}(\omega) = -\frac{1}{\pi} \text{Im} \frac{Z}{\omega - z_+},$$  \hspace{1cm} (6)

where $z_+$ is the pole and $Z$ in the residue at $z_+$.

We plot the approximated spectral function in Fig. 4. Comparing it with the spectral function without the approximation, one can say that the approximation by a Breit-Wigner type formula is works well, which supports the validity of the quasi-particle picture of the quarks at $T \sim m_b$.

5. Summary

We have calculated the spectral function of the massive quark using a Yukawa model and found that the peak in the negative energy region is suppressed. This
behavior is consistent with the interpretation of the multi-peak structure in the quark spectra given by Kitazawa et al.\cite{8,9}. Employing the results given in the NJL model, it is suggested that the spectral function of the massive quark at $T \sim T_c$ may have a peak near the origin separately from the peak in the positive-energy region. We have also examined the pole structure of the retarded Green function of the massless quark. The result show that each of the all three peaks are equally significant and the quasi-particle picture is quite valid at $T \sim m_b$.

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