BEAMSTRahlung INDUCed MINIJET BACKGROUNDS AT FUTURE COLLIDERS AND A MODEL FOR $\sigma_{\gamma\gamma}^{inel}$

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The phenomenon of beamstrahlung can give rise to potentially dangerous hadronic backgrounds due to minijet production at the future linear colliders as well as at the $\gamma\gamma$ colliders that are under consideration. In this talk I will review briefly the current estimates of these backgrounds and predictions of the eikonalised minijet model for $\sigma_{\gamma\gamma}^{inel}$. I end by pointing out issues that need to be studied in more detail to firm up our estimates of these backgrounds.

1 Introduction

The phenomenon of beamstrahlung at high bunch densities is one of the most interesting quantum aspect of the beam physics (QABP). I want to begin this discussion by pointing out the particle physics facts which make it clear that in the quest of ever rising energies, the phenomenon of beamstrahlung is unavoidable in the energy range ($300 \leq \sqrt{s} \leq 2000$ GeV) of the next generation $e^+e^-$ colliders (NLC’s). I then want to summarise briefly how photons develop ‘strong’ interactions at high energies and how these combined with beamstrahlung photons can give rise to a new class of hadronic backgrounds at the NLC’s. Since these backgrounds are caused by the ‘hadronic’ interactions of the photon, they are relevant (perhaps even more so) for the $\gamma\gamma$ colliders that are being planned using backscattered laser photons. I will then comment on the current estimates of these backgrounds clearly pointing out the sources of uncertainties. I will then present a newer, convenient parametrisation of the $\gamma\gamma$ minijet cross-sections incorporating the recent information on the photon structure function as well as on $\sigma_{\gamma\gamma}^{inel}$. I will also present results of a new calculation of $\sigma_{\gamma\gamma}^{inel}$ in an eikonalised minijet model and will end by pointing out the improvements necessary in the estimates of the minijet induced backgrounds.

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\footnote{On leave of absence from University of Bombay, Mumbai, India.}
2 Particle Physics and beamstrahlung at NLC’s

The 'clean' environment of the $e^+e^-$ colliders has played an essential role in the development of particle physics. A quick comparison of the cross-sections of physically interesting processes such as $e^+e^- \rightarrow f \bar{f}$, $(e^+e^- \rightarrow W^+W^-)$, at the current $e^+e^-$ colliders like LEP-1 (LEP 200) ($\sim 1000(30)$ pb) to the ones expected at the NLC’s ($\sim 1 - 10$ pb), shows that the NLC’s will have to operate at luminosities at least 3-4 orders of magnitude higher than those at LEP. This has to be coupled with the fact that due to the higher energies these colliders will have to be linear colliders. If we recall that at a storage ring collider like LEP, a given bunch circulates $\sim 10^8$ times, it is clear that the bunch densities required at the NLC’s will have to be very high indeed. It can be seen that for almost all the designs under consideration there will be beamstrahlung, the spectra depending on the machine designs. A very convenient parametrisation for the calculation of the beamstrahlung spectra, for small values of the beamstrahlung parameter $\Upsilon$ has been given by Chen. Thus along with each $e^+e^-$ collision there will also be an underlying $\gamma\gamma$ collision due to the beamstrahlung photons.

If the construction of $\gamma\gamma$ colliders using backscattered laser beams becomes a reality, which might be possible as indicated by recent experiments from SLAC, the luminosity and the energy of the $\gamma\gamma$ collisions will be close to those of the parent $e^+e^-$ machine. Just like the parent NLC’s the $\gamma\gamma$ colliders may also suffer from the minijet induced backgrounds due to the photon structure.

3 Minijet induced backgrounds due to hadronic structure of photon

3.1 Hadronic structure of photons and ‘resolved’ contributions to jet cross-section in $2\gamma$ processes.

Before beginning to discuss the size of the backgrounds caused by the combination of the hadronic structure of the photon and the phenomenon of beamstrahlung at the NLC’s, let us briefly recapitulate what is known experimentally about $\gamma\gamma$ interactions at present. All the current information comes from the study of jet/hadron production in the two photon processes

$$e^+e^- \rightarrow e^+e^-\gamma\gamma \rightarrow \text{hadrons} + e^+e^-$$  \hspace{1cm} (1)$$

where the $e^+,e^-$ are lost along the beam pipe. In this process the photons are ‘almost’ real and the spectrum is ordinary Weiszäcker-Williams (WW) spectrum. The $2\gamma$ physics was studied for the first time at PEP/PETRA with $\sqrt{s} = W_{\gamma\gamma} = 5 - 10$ GeV. Strong interactions of the photon arise due to its
fluctuations into a short lived $\bar{q}q$ pair. At high energies and hence on shorter time scales, this can be computed using perturbative QCD. The photon thus can be looked upon as surrounded by a ‘cloud’ of quarks and gluons. Some part of the interactions of the high energy photons can therefore be computed as though the photon ‘consists’ of these partons. The early theoretical investigations indicated that already at TRISTAN the jet production in $2\gamma$ interactions would be dominated by the ‘resolved’ processes, where the partonic constituents of the photon and not the photon itself, participate in the hard scattering process giving rise to jets.

There exist three different types of contributions to jet production in $\gamma\gamma$ processes where both/one/no photon take part ‘directly’ in the hard process. We call these direct/1-resolved/2-resolved processes respectively. Schematically one can write down the jet production cross-section as follows:

$$\frac{d\sigma}{dp_T}|_{\text{direct}} = \int dz_2 \int dz_1 \int dx \int dx_2 f_{\gamma/e}(z_1) f_{\gamma/e}(-z_2) \sum_{P_1,P_2,P_3,P_4} f_{P_1/\gamma}(x_1) f_{P_2/\gamma}(x_2) \frac{d\sigma}{dp_T}(P_1P_2 \to P_3P_4).$$

(2)

Here $P_i (i = 1, 4)$ denote the partons, $f_{a/b}(x)$ denotes the flux of partons of type ‘a’ in ‘b’ carrying momentum fraction ‘x’ of ‘b’. For the ‘1-res’ (‘direct’) processes one (both) the flux factors $f_{P_i/\gamma}(x_i)$ are to be replaced by $\delta(1-x_i)$, corresponding to the fact that the entire energy of the photon is available for the scattering process giving rise to the jet production. For the case of TRISTAN/LEP1/LEP 200 etc. the photon flux factors $f_{\gamma/e}(z)$ are just the WW flux factors, whereas the beamstrahlung flux will have to be added to these at the NLC’s.

The $2\gamma$ processes in general dominate production of low invariant mass hadron production over the annihilation processes at higher energy, due to the logarithmic enhancement of the WW flux factors. Our investigations indicated that the jet production in $2\gamma$ processes is dominated by ‘resolved’ processes already for TRISTAN energies; i.e.

$$\frac{d\sigma}{dp_T}|_{\text{direct}} < \frac{d\sigma}{dp_T}|_{\text{1-res}} < \frac{d\sigma}{dp_T}|_{\text{2-res}}.$$ 

(3)

The existence of resolved contributions in $2\gamma$ production of jets was clearly demonstrated by the TRISTAN experiments. The dominance of the jet production by the ‘resolved’ processes increases with energy. This is demonstrated in Fig. The figure shows cross-section for production of central jets ($|y| < 2$) integrated above a certain $p_{T_{\text{min}}}$ in $2\gamma$ processes in $e^+e^-$ collisions, as a function of $\sqrt{s}$. Here we notice the following,
If we recall that at these energies $\sigma(e^+e^- \to W^+W^-) \approx 10$ pb, we realise that the cross-section for inclusive production of central jets in $2\gamma$ processes at the NLC’s is very large indeed.

- It is dominated by the ‘resolved’ processes.
- It also has a strong dependence on the assumed parton content of the photon. The figure shows the results for two parametrisations of the photonic parton densities.
- The cross-section increases almost linearly with $\sqrt{s}$. Of course this has no problem with unitarity as this is an inclusive cross-section.

Fig. 1 above shows the expected cross-sections without inclusion of any beam-strahlung photons. Inclusion of the beamstrahlung contribution to the spectra show that the predicted cross-sections vary widely, depending on the machine design even for a given $\sqrt{s}$. From the point of view of the minjet-induced hadronic backgrounds more interesting is the size of the $\sigma_{e^+e^-\gamma\gamma}$ cross-sections for lower values of $p_T$. We show in Fig. 2 the effect of including beamstrahlung contribution, at a fixed $\sqrt{s}$. The contribution without any beamstrahlung, though not shown in this figure is pretty close to the curve labelled T, as for this design (TESLA) the beamstrahlung spectrum is pretty soft. We notice from Fig. 2 the following:

- Inclusion of beamstrahlung increases $\sigma_{e^+e^-\gamma\gamma} \approx \sigma(e^+e^- \to e^+e^-\gamma\gamma \to e^+e^- + \text{jets})|_{p_T > p_T}$ by an order of magnitude or more as can be
Figure 2: $p_T$ integrated jet production cross-section in $2\gamma$ processes as a function of $p_{T_{min}}$ for different machine designs with $\sqrt{s} = 500$ GeV including the beamstrahlung photons.

seen by comparing Fig. 1 and Fig. 2.

• The different legends on the figure refer to different machine designs (for details see 3). We can see that the predicted jet cross-sections depend on the machine designs in a striking manner, the dependence of course reflecting the different hardness of the beamstrahlung spectra expected. There is no cut on the rapidities of jets here.

• The value of $\sigma_{e^+e^-\gamma\gamma}^{\text{jet}}$ of course depends very strongly on $p_{T_{min}}$ and on the parton content of the photon. For the parton distributions used in the figure here (DG16), $\sigma_{e^+e^-\gamma\gamma}^{\text{jet}} \sim p_{T_{min}}^{(-3.3)}$.

• The curve labelled Laser is the one expected for a $\gamma\gamma$ collider constructed using the spectra as given in Ref. [8].

• Note that here we have decreased $p_{T_{min}}$ to a very low value ($\geq 1$ GeV). This means we have strictly gone to the limit of the applicability of perturbation theory. We will discuss the implication of this for the minijet induced backgrounds in the next subsection.

3.2 Minijet induced backgrounds at NLC’s

The discussion of Sec. 3.1 shows that the $\sigma_{e^+e^-\gamma\gamma}^{\text{jet}}$ does indeed grow very significantly at the high energy $e^+e^-$ colliders with energy with the inclusion of
beamstrahlung and at high energy $\gamma\gamma$ colliders. The results of Fig. 3 show that the ‘minijet’ cross-sections do indeed rise very fast and are very large at the NLC’s. This indicates that associated with every ‘effective’ bunch crossing there will be underlying events with low $p_T$ hadrons being produced from the $\gamma\gamma$ collision which have nothing to do with the $e^+e^-$ event under study. In our original study we constructed the following measure for the ‘messiness’ of an event. We first defined an effective luminosity per bunch crossing in the following fashion. If $\delta t$ is the time resolution of the detector, $\Delta t$ is the temporal separation between different bunches and $L_b$ is the luminosity per bunch crossing,

$$L_{eff} = L_b, \quad \text{if} \quad \Delta t > \delta t$$

$$= L_b \times \frac{\delta t}{\Delta t}, \quad \text{if} \quad \Delta t < \delta t$$  \quad (4)

The ‘effective’ number of ‘minijet’ events which will then produce an underlying event will be given by multiplying the cross-sections given in Fig. 2 by $L_{eff}$ for different machine designs. In Table 1 taken from Ref. [3], I show the number of ‘minijet’ events expected per effective bunch crossing. If this number is well below 1 then the collider has no significant minijet-induced, beamstrahlung dependent hadronic backgrounds. In Table 1, $\sigma^{Semi-hard}$ is the above mentioned ‘minijet’ cross-section. The value of $p_{Tmin}$ used in this Table is 1.6 GeV. As

| Collider | $\sigma^{Semi-hard}$ (µb) | $\sigma^{soft}$ (µb) | No. of minijet events |
|----------|--------------------------|---------------------|----------------------|
| T        | 0.016                    | 0.041               | 0.004                |
| D-D (nbh)| 0.020                    | 0.051               | 0.021                |
| P-F      | 0.042                    | 0.072               | 0.46                 |
| JLC1     | 0.069                    | 0.12                | 1.1                  |
| $\gamma\gamma$ (500) | 1.9 | 0.25 | $>\sim O(10 \sim 20)$ |
| T(1000)  | 0.057                    | 0.099               | 0.0036               |
| T(2000)  | 0.21                     | 0.13                | 0.013                |
| T(10000) | 0.17                     | 0.27                | 0.043                |

Table 1: Estimate of minijet-induced ‘messiness’ for different designs of NLC’s
seen from Fig. 2, $\sigma_{e^+e^-\gamma\gamma}^{\text{jet}}$ depends strongly on the value of $p_{T \text{min}}$. Hence that is indeed a major source of uncertainty of the minijet-induced messiness. We will comment on it a little later. $\sigma_{e^+e^-\gamma\gamma}^{\text{jet}}$ calculated above can basically be used as a figure of merit for different machines. Table 1 shows that by choosing the machine design judiciously and reducing beamstrahlung, it is possible to reduce the minijet-induced hadronic background to an acceptable level for $e^+e^-$ colliders up to a $\sqrt{s}$ of $\sim 1000$ GeV. However, for a ‘Laser’ collider even for $\sqrt{s} = 500$ GeV, the expected number of ‘minijet’ events is unacceptably large. Also please note that in calculating the last column we have not included the contribution from $\sigma_{\text{soft}}^{\gamma\gamma\to\text{hadrons}}$ as most of the hadrons produced in the soft $\gamma\gamma$ reactions will be lost along the beam pipe. This contribution is essentially computed using a constant $\gamma\gamma\to\text{hadrons}$ cross-section and using $W_{\gamma\gamma} > 5$ GeV.

Here a comparison with the results obtained by Chen and collaborators is in order. They, very correctly, stressed that while it is clear that $\sigma_{e^+e^-\gamma\gamma}^{\text{jet}}$ grows in the manner we noticed, the following three questions are also relevant in assessing this background: 1) How much of the increase of $\sigma(\gamma\gamma\to\text{jets})$ is reflected in the increase in $\sigma_{\gamma\gamma}^{\text{inclus}}$, 2) How many of the hadrons produced by these minijet interactions reach the detectors; i.e. the issue of the scalar $E_T$ distribution and 3) How best to determine the value of $p_{T \text{min}}$. At the end of their analysis they also conclude that it is possible to design $e^+e^-$ colliders up to $\sqrt{s} \sim 1$ TeV which are free from minijet backgrounds. It is worth pointing out here that the ‘minijet crisis’ here has vanished not because the estimates of backgrounds have been lowered but the beam designs have been modified. Recently (see talks at these conferences) people have started thinking about yet higher energy $e^+e^-$ colliders, with $\sqrt{s} \leq 5$ TeV. I will comment about the estimates of the minijet backgrounds at these in the last section. We do however, disagree with Chen et al, in our conclusion for $\gamma\gamma$ colliders. The two analyses did differ greatly in the choice of $p_{T \text{min}}$ and we will comment on it in the next section.

It is true that for $\gamma\gamma$ colliders effects of multiple parton interactions and hence eikonalisation becomes more important than in the case of $e^+e^-$ colliders. Since the analysis of Ref. [4], full MC generators for $\gamma\gamma$ events at TRISTAN/LEP, including the ‘resolved’ contributions have become available and for a better assessment of the ‘minijet’ background, perhaps a newer MC analysis is needed. Since the size of these backgrounds depend crucially on the parton densities in photon, it is important to also include the updated parametrisations that are available now. A determination of $p_{T \text{min}}$ using the latest data on $\sigma_{\gamma\gamma}^{\text{inclus}}$ is also necessary. In the analysis of Ref. [4] the $\sigma(\gamma\gamma\to\text{jets})$ was eikonalised. In the eikonalisation it was assumed that the multiple parton
interaction causing it are completely independent of each other. The existence of ‘pedestal’ effect shows that that is perhaps not completely correct.

In general the issue of how much of the rise of $\sigma(\gamma\gamma \rightarrow \text{jets})$ is reflected in $\sigma_{\gamma\gamma}^{\text{inel}}$ is an important one. While I discuss that in the next section, here I give a newer parametrisation of the ‘minijet’ cross-sections in $\gamma\gamma$ collisions which can be used in estimating the hadronic backgrounds at the NLC’s by folding it with appropriate beamstrahlung spectra. This supercedes the corresponding parametrisation that was given in Eq. 3.

Our discussions of the next section will show that for $\gamma\gamma$ interactions $p_{T\text{min}} = 2$ GeV is a choice consistent with the data on $\sigma_{\gamma\gamma}^{\text{inel}}$ in the context of an eikonalised minijet model. The cross-section

$$\sigma_{\text{minijet}} \equiv \sigma(\gamma\gamma \rightarrow \text{jets}) \bigg|_{p_{T\text{min}}} = \int_{p_{T\text{min}}} d\sigma(\gamma\gamma \rightarrow \text{jets})$$

(5)

for the two parametrisations GRV densities is given (in nb)

$$\sigma_{\text{minijet}} = \begin{bmatrix} \frac{222}{\text{GeV}} \left( \frac{2 \text{ GeV}}{p_{T\text{min}}} \right)^2 - 161 \left( \frac{2 \text{ GeV}}{p_{T\text{min}}} \right) + 36.6 \bigg( \frac{\sqrt{s}}{50} \bigg)^{1.23} \\ 77.6 \left( \frac{2 \text{ GeV}}{p_{T\text{min}}} \right)^2 - 45.9 \left( \frac{2 \text{ GeV}}{p_{T\text{min}}} \right) + 9.5 \bigg( \frac{\sqrt{s}}{50} \bigg)^{1.17} \end{bmatrix}$$

(6)

(7)

by Eqs. 6 and 7 respectively. Here $\sqrt{s}$ is in GeV. Since the dependence on $p_{T\text{min}}$ of $\sigma(\gamma\gamma \rightarrow \text{jets})$ is extremely strong, it is essential to fix that well. One way of fixing the correct choice of $p_{T\text{min}}$ is to use the eikonalised minijet model to describe the observed features of the $2\gamma$ reactions at TRISTAN/LEP. To that end in the next section I present a new calculation of $\sigma_{\gamma\gamma}^{\text{inel}}$ in the eikonalised minijet model.

I end this section by commenting on the new theoretical issues that will have to be taken in to account in extending these calculations to the higher energy ($\sqrt{s} \leq 3−5$ TeV) $e^+e^−$ and $\gamma\gamma$ colliders. At these energies the $x_\gamma$ values at which photonic parton densities will be sampled will be small ($\sim 10^{-3}$) and hence saturation effects might have to be taken into account. At present, no complete theoretical discussion of the subject is available.

4 An eikonalised minijet model for $\sigma_{\gamma\gamma}^{\text{inel}}$

As is well known, even though the ‘minijet’ cross-sections rise very fast with energy (as shown by Eqs. 6 and 7) the $\sigma_{\gamma\gamma}^{\text{inel}}$ certainly do not rise that fast. The ‘minijet’ cross-section has to be eikonalised so that unitarity is not violated.
In general for photon induced processes, the inelastic cross-section obtained by eikonalisation (and hence unitarisation) of the minijet cross-section is given by

\[ \sigma_{ab}^{inel} = P_{ab}^{had} \int d^2\vec{b}[1 - e^{n(b,s)}] \]  

with the average number of collisions at a given impact parameter \( \vec{b} \) given by

\[ n(b,s) = A_{ab}(b)(\sigma_{ab}^{soft} + \frac{1}{P_{ab}^{had}}\sigma_{ab}^{jet}) \]  

where \( P_{ab}^{had} \) is the probability that the colliding particles \( a, b \) are both in a hadronic state, \( A_{ab}(b) \) describes the transverse overlap of the partons in the two projectiles normalised to 1, \( \sigma_{ab}^{soft} \) is the non-perturbative part of the cross-section while \( \sigma_{ab}^{jet} \) is the hard part of the cross-section (of order \( \alpha \) or \( \alpha^2 \) for \( \gamma p \) and \( \gamma \gamma \) respectively). Notice that, in the above definitions, \( \sigma_{soft} \) is a cross-section of hadronic size since the factor \( P_{had}^{ab} \) has already been factored out.

Letting \( P_{\gamma p}^{had} = P_{\gamma}^{had} \equiv P_{had} \) and \( P_{\gamma\gamma}^{had} \approx (P_{\gamma}^{had})^2 \)

The overlap function \( A_{ab}(b) \) is

\[ A_{ab}(b) = \frac{1}{(2\pi)^2} \int d^2\vec{q} F_a(q) F_b(q) e^{i\vec{q} \cdot \vec{b}} \]  

where \( F \) is the Fourier transform of the b-distribution of partons in the colliding particles. Normally, \( A_{ab} \) is obtained using for \( F \) the electromagnetic form factors of the colliding hadrons. In general, for photons people normally used the form factor for a pion. We take a slightly different approach and calculate the ‘b- distribution’ of the partons by taking the Fourier transform of the transverse momentum distribution of the partons, which in the case of the photons is expected to be, at least for the perturbative part,

\[ f(k_T) = \frac{C}{(k_T^2 + k_{T,0}^2)} \]  

The \( k_{T,0} \) has actually been measured by ZEUS to be \( 0.66 \pm 0.22 \) GeV. It turns out that the form of \( A_{\gamma\gamma} \) with this transverse momentum ansatz and that for the pion form factor ansatz, are the same, differing only in the value of the parameter \( k_{T,0} \) which is \( 0.735 \) GeV for the \( \pi \) form factor case. Thus one can assess the effect of changing the ansatz for the \( A_{ab} \) for photons by simply changing the value of \( k_{T,0} \).
For the soft part of the cross-section we use a parametrisation,
\[ \sigma_{\gamma p}^{soft} = \sigma^0 + \frac{A}{\sqrt{s}} + \frac{B}{s} \]  
(13)

we then calculate values for \( \sigma^0, A \) and \( B \) from a best fit to the low energy photoproduction data, starting with the Quark Parton Model (QPM) ansatz \( \sigma^0_{\gamma p} \approx \frac{2}{3} \sigma^0_{pp} \) and the form factor ansatz for the \( A_{\gamma p} \). The best fit value for \( p_{T \text{min}} \) that we get is 2 GeV. It might be possible to improve quality of the fit by using a energy dependent \( P_{\text{had}} \), but it needs to be investigated further. The value of 2 GeV is also comparable to the value 1.6 GeV obtained from a fit to the description of minimum bias events in \( pp/\bar{p}p \) collisions.

For \( \gamma\gamma \) collisions, we repeat the QPM suggestion and propose
\[ \sigma_{\gamma\gamma}^{soft} = \frac{2}{3} \sigma_{\gamma p}^{soft} \]  
(14)

We now apply the criteria and parameter set used in \( \gamma p \) collisions to the case of photon-photon collisions, i.e. \( P_{\gamma\gamma} = 1/204, p_{T \text{min}} = 2 \text{ GeV} \), \( A(b) \) from the transverse momentum ansatz with the value \( k_{T,0} = 0.66 \text{ GeV} \). The results of our calculation are shown Fig. 3. The highest of the two full lines corresponds exactly to the same parameter set used in the photoproduction case and appears to be in good agreement with the preliminary results from the OPAL.
Collaboration, whereas the L3 results, everything else being the same, would favour a higher \( k_{T,0} \) value.

The following can be noticed here from the newer data and model calculation:

- The data on \( \sigma_{\gamma\gamma}^{\text{inel}} \) rise faster with the energy than the predictions of the Regge-Pomeron ideas and the rise is consistent with predictions of the eikonalised minijet models where the parameters are fixed by fits to the photoproduction case.

- These calculations use also more up-to-date photonic parton densities and the parameters of the eikonalised minijet model are related here to measured properties of photon induced reactions.

5 Conclusions

We can summarise the discussion so far as follows:

- The knowledge of parton content of the photon has improved considerably in the last few years.

- A combination of beamstrahlung and ‘strong’ interactions of the photon can induce hadronic backgrounds at the future \( e^+e^- \) and \( \gamma\gamma \) colliders. The cross-section \( \sigma(\gamma\gamma \rightarrow \text{jets})|^{\sqrt{s}}_{pT_{\text{min}}} \) of Eq. 3 is a good measure of the ‘messiness’ that this can cause. From the calculation of total inelastic cross-section in the eikonalised minijet model, for photoproduction as well as \( \gamma\gamma \) collisions, 2 GeV seems to be good value for \( p_{T_{\text{min}}} \). However, due to the large number of other parameters involved, this can not be taken to be a determination of \( p_{T_{\text{min}}} \). A MC analysis of the newer 2\( \gamma \) data might help settle this further. The parametrisations of Eqs. (6,7), can be used conveniently to estimate this ‘figure of merit’ for a given collider by folding in with the photon spectra.

- It seems that for \( \sqrt{s} \leq 2 \) TeV, for the current machine designs, the \( e^+e^- \) colliders can be free of ‘minijet’ induced backgrounds. It would be preferable to repeat the analysis of Ref. [4], with the newer MC for photon induced processes that are available now. An analysis of the energy flow due to the hadrons produced by the interactions of the photons, should use the newer information on the \( \sigma_{\gamma\gamma}^{\text{inel}} \) that has become available now.

- For \( \gamma\gamma \) colliders further analysis is necessary. The better knowledge of photonic parton densities available today as well as the newer information
on $\sigma^{inel}_{\gamma\gamma}$ should be used as an input to the newer analyses. At higher energies the saturation effects of the photon structure function should also be included.

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