Comparison of Coding Methods for a Genetic Algorithm in Multi-objective Optimization of an Indeterminate Structure

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Abstract. Two different coding algorithms are used to find optimal support locations for an indeterminate structure using a Genetic Algorithm. The objective is to equalize the load distribution among the supports while maximizing the enclosed polygonal area by the supports to increase stability of the structure. In the first approach, called the continuous method, the candidate surface for the support locations was used as a continuum of space. In the second approach, called the discretized method, the solution space is broken into rectangular grids and the index number of the nodal points are used as genes in coding the problem. The average value of the objective function in the discretized method is found to be 0.0147 compared to the average value of 0.405 in the continuous method. The discretized method also outperformed the continuous method in each component of the multi-objective problem and showed faster convergence towards the optima.

Introduction

Genetic algorithms (GAs) are a metaheuristics approach for solving various optimization problems including multi-objective optimization scenarios. In this paper, GAs have been used to solve for the support locations of a multi-objective structural problem using two different coding algorithms, namely, “Continuous Method” and “Discretized method”. A rectangular shaped indeterminate structure has been considered as the test case as a myriad of examples are found in real life including decorative over hung light structures. The objective function includes minimization of the tension difference in the supports and maximization of the area generated by the support locations to increase stability. The continuous method is a simple genetic algorithm which operates on the real coordinate of the points over the continuous space. The key concept for the discretized method is to divide the whole solution space in smaller rectangular grids. Each node point is designated by two index numbers along the axes. Discrete nodal points are considered to be candidate support locations. GA operates on the integer indices of the nodes rather than the real coordinates. This discretized method can be applicable for a class of structural design optimization problems where a discrete solution space is required, for example, design of a shaft. A normalized objective function has been developed to handle multiple components of the objective function. Two different coding methods along with a normalized objective function have been used to solve the test case and the obtained results have been compared.

Background

Genetic Algorithms, one of the popular optimization techniques, are stochastic global search procedures which impersonate the “Natural Theory of Evolution” developed by Charles Darwin [1]. After being introduced by Holland [2], GAs have been successfully applied in various fields of science, engineering and management.

In real engineering challenges, most of the optimization problems have multiple objectives which are often conflicting. There are two basic approaches to handle multi-objective optimization problems. In the first approach, a weighted sum of all the objectives is used as the objective function. The inherent problem in this approach is to determine the accurate weightage values of the
various components. Another approach is to determine an entire Pareto optimal solution set comprising sets of solutions having no dominance over each other. This approach is preferred as decision makers are given a set of optimal solutions allowing them to choose by trading-off various parameters. Being a popular metaheuristic approach, GA is well suited for multi-objective optimization problems. Jones et al. [3] mentioned that 70% of all metaheuristics approaches use evolutionary algorithms as their fundamental basis. Various regions of the solution space are being searched simultaneously resulting in a diverse set of solutions. In many situations, prioritization, weightage or scaling are not required in a multi-objective GA which makes it more useful for searching non-convex, discontinuous and multi-modal solutions spaces. Schaffer [4] proposed the first multi-objective GA, named Vector evaluated GA (VEGA), with a limitation of having the search direction parallel to the axes of objective space. Following the work of Schaffer, a good number of multi-objective GAs have been developed including Multi-objective Genetic Algorithm (MOGA) [5], Niched Pareto Genetic Algorithm (NPGA) [6], Non-dominated Sorting Genetic Algorithm (NSGA) [7], etc. A complete list of these popular multi-objective GA approaches with their advantages and disadvantages has been discussed in Konak et al. [8]

GAs have gained popularity in the field of structural design optimization because of their proficiency in searching for a global optimal solution. Though GAs are competent with optimization problems having continuous and discrete variables [9, 10], in most cases simple GAs have been used to solve structural design optimization problems having discrete design space [11, 12, 13]. Modification of simple GAs has been performed to improve the reliability of the performance of continuous GA where incremental design variables along with a Novel Genetic Algorithm were used [14]. The performance of this method was tested for several optimization problems including structural design problems but none of the cases were multi-objective.

Methodology

Problem Formulation

Two different coding approaches are applied to find the optimum support layout for an overhanging, rectangular structure commonly found in architectural applications such as hanging light fixtures. The objective of the problem is to optimize the support layout so that the load is equally distributed while increasing the stability of the structure. The design variables are the locations of the supports on the candidate surface of the structure with respect to a 2-D Cartesian coordinate system defined on the surface with its origin at the center of gravity. A solution vector \( \mathbf{X} \), of the problem is thus \( \mathbf{X} \in \mathbb{R}^{2n} \), where \( n \) is the number of supports. Let \( s \subseteq \mathbb{R}^{2} \), is the collection of all points on the candidate surface. The solution vector is comprised of \( n \) ordered pairs \((a_i, b_i)\)\( \in s \), where \( a_i \) is the abscissa and \( b_i \) is the ordinate of the \( i \)th support. Let, \( \varphi = \{1,2,3,4\ldots n\} \), and \( R_i \) is the reaction force at the \( i \)th support then the optimization problem can be formulated as,

\[
\begin{align*}
\text{minimize } z &= f(X) + g(X) \\
\text{subject to, } &\mathbf{X} \in \mathbb{R}^{2n} | \cup_{i \in \varphi} (a_i, b_i) \in s \\
\text{where } f(X) &= \max \{ R_i \ \forall \ i \in \varphi \} - \min \{ R_i \ \forall \ i \in \varphi \} \\
g(X) &= - \Delta_{\varnothing \in \Omega} = - \text{maximum polygonal area formed by any subsets of supports } | \Omega \subseteq \varphi
\end{align*}
\]

Forming No-Preference, Normalized Objective Function

Different units, huge numerical differences, and disparate requirements of the functions \( f \) and \( g \) make it quite difficult to assign preference weightage to the components of the objective function. Therefore a no-preference strategy [15] defining global criteria by semi-norm mapping [16] of the functions is chosen. Further, the sensitivity to units [17] is addressed by applying a heuristic based normalization technique reducing both \( f \) and \( g \) into a dimensionless quantity.
Table 1. Differences between continuous method and discretized method.

| Criteria         | Continuous Method                                      | Discretized Method                                   |
|------------------|--------------------------------------------------------|------------------------------------------------------|
| **Solution Space** | ![Diagram](image1.png)                                  | ![Diagram](image2.png)                                |
|                  | $(-L/2, W/2)$                                           | $(-L/2, W/2)$                                        |
|                  | $(-L/2, -W/2)$                                          | $(-L/2, -W/2)$                                      |
| Solution space   | $((a_i, b_i))$                                          | $((l_i, w_i))$                                       |
|                  |                                                        | $I = \{l_0, l_1, ..., l_{n-1}, l_n\}$                |
|                  |                                                        | $W = \{w_0, w_1, ..., w_{n-1}, w_n\}$                |
|                  |                                                        |                                                        |
| Coded Chromosome | $X = \{a_1, a_2, ..., a_m, b_1, b_2, ..., b_m\}$, which comprises real-valued genes of abscissa and ordinates of the support locations | $X = \{l_1, l_2, ..., l_{m}, w_1, w_2, ..., w_{m}\}$, which comprises integer genes of indices of nodal points, where each integer is converted to 8-bit binary. |
| Initial Population | Random generation of real valued numbers within solution space, $s$, is performed. | Randomly numbers are generated within $s$, converted to integers and then to 8-bit binary numbers. |
| Mapping and Fitness | Generated populations are evaluated using Eq. 2. | Genotypes, $X$, generated in previous step are mapped into phenotypes, $X^\prime$, where, $X^\prime = \{a_1, a_2, ..., a_m, b_1, b_2, ..., b_m\}$. Then, phenotypes are evaluated using Eq. 2. |
| Selection         | Stochastic acceptance with elitist strategy.           | Stochastic acceptance with elitist strategy.         |
| Crossover         | Generate $m \in (0,1)$ using uniform distribution.    | Single point crossover, $Child_1 = [H_i + L_i]$ and $Child_2 = [H_j + L_j]$. Here $X_i$ and $X_j$ are split into $H_i$ and $L_i$, and $H_j$ and $L_j$ respectively. |
|                  | $Child_1 = (m \times X_i) + ((1 - m) \times X_j)$     |                                                      |
|                  | $Child_2 = ((1 - m) \times X_i) + (m \times X_j)$     |                                                      |
| Mutation          | Generate $p \in (0,1)$ using uniform distribution.    | Generate a random number to select a bit to be mutated and the bit value is altered. |
|                  | Then, $X_{muted} = X \pm p \in s$.                    |                                                      |

The semi-norm mapping of the two components in the objective function is achieved by $\|f_{best}\|$ and $\|g + g_{best}\|$. It should be noted that in Eq. 1, $g$ is defined as the negative of the maximum polygonal area formed by any subset of the current layout.

It is assumed that the reactions in the support cannot assume negative values; therefore, the maximum tension difference among the supports cannot exceed the average reaction force. The ideal value, $f_{best} = 0$ indicates that the load on each support is equal. Thus, the reaction force component is normalized by dividing it by the average load on each support, $\langle F \rangle$, where $F$ is the total load on the structure. It is further assumed that when $R_i = \langle F \rangle \forall i \in \varnothing \exists \Delta_{max}$, the upper bound of the enclosed area by any subset of supports and a reasonable value of $\Delta_{max}$ can be heuristically determined. Thus, the scalarized objective function becomes, Therefore, $Z \in (0, 2)$. It should be noted that $Z$ can assume a negative value if $\Delta_{max}$ is grossly underestimated by the heuristics. On the other hand, the worst case scenario of $Z$ may be lower than 2 if $\Delta_{max}$ is overestimated.

$$Z = \frac{f(X)}{\langle F \rangle} + [\Delta_{max} + g(X)] \frac{1}{\Delta_{max}}$$

(2)
Coding and Solution Space

Two different coding approaches, named “Continuous Method” and “Discretized method” are used in this paper. The first approach is based on a real valued GA over a continuous solution space whereas the later approach is developed by discretizing the solution space and using the set of discretized points as the candidate solution space. Moreover, each discrete point is represented by a pair of indices which are coded to 8-bit binary numbers. Table 1 describes the similarities and differences of the two methods.

Test Case

In the present investigation, an 8 meter by 6 meter rectangular shaped indeterminate homogeneous plate made of aluminum which is overhung with three supports has been taken as the test case and shown in Figure 1 with detail dimensions. The cables and the weight of the plate are assumed to be acting in the z- direction. Figure 2 shows the area enclosed by the support layout and a solution.

![Figure 1. Rectangular shaped indeterminate structure.](image)

![Figure 2. (a) Heuristically calculated maximum possible area with equally loaded supports; (b) Location of three supports on the surface.](image)

The generalized structural optimization problem can be formulated by satisfying all implicit and explicit constraints. The equations of static equilibrium supply the implicit constraints which are,

\[ \sum \vec{F} = 0 \quad \text{and} \quad \sum \overrightarrow{M_o} = \Sigma (\vec{r} \times \vec{F}) = 0 \] (3)

Now, assuming the only active force is due to the weight of the structure is \( F \), reactions on the supports can be solved by:

\[ \vec{R} = A^{-1} \vec{F}, \quad \text{where}, \quad A = \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \end{bmatrix}, \quad \vec{R} = [\vec{R}_1 \ \vec{R}_2 \ \vec{R}_3] \quad \text{and} \quad \vec{F} = \begin{bmatrix} F \\ 0 \\ 0 \end{bmatrix} \] (4)

Here, \( x_1, x_2 \) and \( x_3 \) are x coordinates and \( x_4, x_5 \) and \( x_6 \) are y coordinates of the support locations respectively. Now, \( f(X) \) can be solved by Eq. 1 and \( g(X) \) can be found by:

\[ g(X) = -\text{Area of the triangle} = -0.5 \times (x_1(x_5-x_6)+x_2(x_6-x_4)+x_3(x_4-x_5)) \] (5)
Now, the force $F$ due to the self-weight of the plate is computed to be approximately 6642 N. Thus, for normalizing the force component, $f(X)$, it is divided by $\bar{F}$, which is 2242 N in this test case. In order to normalize the area component, it is necessary to use a heuristic to find $\Delta_{\text{max}}$. When the three supports are balanced, all reaction forces are equal to $\bar{F}$, i.e., $R_1 = R_2 = R_3 = \frac{F}{3}$. Now, using these values in Eq. 4, following conditions are derived:

\[
x_1 + x_2 + x_3 = 0
\]  
\[
x_4 + x_5 + x_6 = 0
\]

Figure 2 shows the use of these two conditions to heuristically determine the maximum possible area. The maximum possible area is $\Delta_{\text{PQR}}$ found to be 18 m$^2$.

To ensure that the lower limit of the contribution of normalized area to the objective function is 0, a slightly bigger value of 20, has been chosen. The objective function value is determined using Eq. 2.

### Result & Analysis

The genetic algorithm toolbox of Scilab 5.5.0 has been used where the following control parameters shown in Table 2 were set for optimization:

| Parameter                  | Value | Parameter                  | Value |
|----------------------------|-------|----------------------------|-------|
| Population size            | 100   | Crossover probability      | 0.7   |
| Generation number          | 40    | Mutation probability       | 0.1   |

Table 3 shows the summary statistics and Figure 3 shows the distribution of the optimal objective values obtained from 30 runs by each method.

|                          | Continuous Method | Discretized Method |
|--------------------------|-------------------|--------------------|
| Minimum                  | 0.170             | 0.105              |
| Maximum                  | 0.723             | 0.218              |
| Average                  | 0.405             | 0.147              |
| Std. Dev.                | 0.162             | 0.034              |

![Figure 3. Distribution of objective function values from two methods.](image)
It is clearly evident that the discretized method finds better overall solutions minimizing the combined objective function. Moreover, the standard deviation in Table 3 and the distribution in Figure 3 show that the discretized method consistently outperforms the continuous method.

These two approaches can be compared based on the two components of the objective function separately. Table 4 shows area values and the difference of maximum and minimum reaction values for both approaches and Figure 4 shows the distribution of the objective function components.

Table 4. Area and reaction difference obtained from two methods.

| Parameter               | Method   | Minimum | Average | Maximum | ST. DEV |
|-------------------------|----------|---------|---------|---------|---------|
| Area (m²)               | Continuous | 5.55    | 12.14   | 18.41   | 3.382   |
|                         | Discretized | 15.64   | 17.19   | 18.06   | 0.751   |
| Reaction Difference (N) | Continuous | 0.91    | 27.32   | 244.29  | 42.643  |
|                         | Discretized | 0       | 15.28   | 77.080  | 18.023  |

Figure 4. Distribution of optimal area value and reaction differences by two methods.

It is evident from Table 4 and Figure 4 that the discretized method consistently produces support layouts that enclose a higher area while equalizing the force distribution among the supports. It is observed that for both the components of the objective function, the standard deviations of the discretized method are significantly smaller than that of the continuous method. This implies that the discretized method is more reliable and stable than the continuous one. It should be noted that, out of 30 samples, 27 samples of the discretized method provide an area value bigger than 16 m², whereas 26 samples of the continuous method generate an area value smaller than 16 m². Although, the continuous method balances the reaction forces among three supports almost equally, it is worth mentioning that the discretized method provides exactly equal reactions for 11 times out of 30 samples while the continuous method never did. Also, the narrow ranges for both the area value and the reaction difference values bolster the observation that the discretized method worked better with more consistency than the continuous method for this optimization problem.

In order to compare the rate of progression towards optimality two randomly selected runs from each method were observed. Minimum (best) objective values of each generation have been plotted against the generation number (Figure 5).
Figure 5. Minimum (best) objective value VS. generation number for both methods.

In the discretized method, the solution quickly progresses towards the optima and reaches near-optima within 20 generations. While for the continuous method the convergence towards optima is much slower; the best solution of each generation slowly improves in each generation in a linear fashion.

**Conclusion**

This paper has introduced a coding approach named “Discretized Method”. This method divides the entire continuous solution into smaller grids. Collection of the nodal points is considered as the new solution space. The performance of this method has been compared against the continuous simple GA in a test case where a rectangular indeterminate structure is to be hung by three supports. Similar to most of the real engineering problems, the objective function of this test case represents a multi-objective optimization problem where convergence of GA towards the optimal solution satisfying each component objectives is crucial. A no-preference, normalized objective function is developed by semi-norm mapping of the objectives into the objective function. It has been observed that the discretized method has provided consistently better optimal solutions compared to the continuous method. The proposed method was able to achieve excellent results with a relatively low number of generations. This method of discretizing the solution space may be applicable to many other structural problem without losing precision for any practical purposes.

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