Visualization of the photoelastic method data in the angular zone of a region

L U Frishter
Moscow State University of Civil Engineering, Yaroslavskoe shosse, 26, Moscow, 129337, Russia

Abstract. The experimental photoelastic method, which is a continuum method, and the method of forced deformations defrosting, allow us to obtain a stress-strain state in the stress concentration zone on the contact surface of composite structures with a surge of forced deformations on models made of optically-responsive material. The experimental solution is obtained on the model with an angular cut of the boundary, the vertex of which is occupied by the rupture of forced (temperature) deformations. The fringe contour obtained on the model by photoelastic method and the method of defrosting in the stress concentration area, in the zone of angular cut of the boundary and forced deformations surge, is characterized by significant gradients of experimental data: the order of fringes (isochromes), isolines parameters, stresses, deformations. Modern methods of visualization of experimental data based on digital shooting expand the capabilities of the photoelastic method. The study purpose is to estimate the capabilities of using photoelastic method and the method of deformations defrosting to obtain the stress-state deformation in the angular zone of a region, which is as close as possible to the stress concentration area during digital shooting and data processing, by the standard methods of stress separation.

1. Introduction
Investigation of the local stress-strain state in the composite area of significant structural nonuniformity is relevant in the process of the calculation and design of structural elements of nuclear power plants, power equipment, hydraulic structures, during a sharp change in the shape of the structural boundary, such as angular or step form. The stress-strain state of composite structures and constructions is characterized by the significant concentration of stresses at the intersection of elements with different options of the border structural design: special lines, points, e.g. reentering angle, etc. A complex stress-strain state occurs in the area of stress concentration based on both the shape of the boundary or “geometric factor”, in particular, the angular cuts of the boundary area, and the finite rupture of the given forced deformations, mechanical properties, which extend to an irregular point of the boundary area.

The experimental photoelastic method, which is a continuum method, and the method of defrosting forced deformations allow us to obtain stress-strain state in the angular zone of stress concentration on the contact surface of composite structures on models made of optically-responsive material [1-13]. Numerical calculation methods in the stress concentration zone based on the sampling of the computational area require additional estimates for stress-strain state in the area of a special point of the elastic body to verify the results of the solution.

The issues of behavior of solutions of the Laplace, Poisson equations, and the elliptic equations for the regions with rough boundaries are reviewed in the studies of V. A. Kondrat’ev [14], V. V. Fufaev,
M. L. Williams, Ya. S. Uflyand, A. I. Kalandia, G. P. Cherepanov [15], D. B. Bodzhi, O. K. Aksent'jan [16], A. Ya. Aleksandrov, K. S. Chobanyan [17], I. T. Denisjuk [18], V. D. Kuliev [19] et al.

In the fundamental work of V. A. Kondrat’ev [14], it is proved that in the vicinity of irregular points of the region boundary the solution of the common elliptical boundary value problem is represented in the form of an asymptotic series and infinitely differentiable function. The components of this series contain the solutions of homogeneous boundary value problems for model regions: wedge or cone. These solutions depend on local characteristics: the size of solid or plane angle and the type of boundary conditions, mechanical characteristics for piecewise homogeneous bodies. The values of the solution expansion coefficients in the vicinity of the special point are unknown and depend on the problem as a whole. Methods of determining the mentioned expansion coefficients are complex and difficult to access during actual determining the stresses of composite structures having a complex shape of the boundary.

In the studies of I. T. Denisjuk, the asymptotics of the elastic solution for the plane composite region with angular points on the lines of the section [18] is given. The advantage of the fundamental research of the boundary value problem singular solution in the V. D. Kuliev’s study [19] is the opportunity of applying its results to researching the problems of residual stresses and crack propagation across the boundary of heterogeneous material compounds [6, 19, 20]. It is shown [6, 20] that the order of stresses singularity depends on the degree of approaching the vertex of the crack to the area of junction of the materials consisting of different modules.

In the M. L. Williams’ studies, which are referred to by many authors, stresses, deformations, Airy stress function near the vertex of the sector with rectilinear sides have a degree form. In the studies of A. I. Kalandia, K. S. Chobanyan, L. A. Bagirov, O. K. Aksent’jan, G. P. Cherepanov, V. P. Netrebko, V. Z. Parton and many others, the solution of the homogeneous boundary value problem in the vicinity of the irregular border point is reviewed in a degree form.

In the study [16, 20], a local curvilinear coordinate system, in which the Lame’s equations are put down, is introduced for researching stress-strain state in the vicinity of an irregular point on a special line of the elastic body boundary. When approaching to an irregular point of the boundary from within the region, the solution of the elastic problem is reduced to the solution of two homogeneous plane problems: planar deformation and out-of-plane deformation or out-of-plane shear.

It is possible to show that the representation of the solution of elastic problem in the vicinity of an irregular point on a special boundary line in the form of two homogeneous plane problems is correct if: a) the given forced deformations, volumetric forces are continuous along the area of elastic body; b) the given forced deformations, volumetric forces are piecewise continuous functions, and the surge of values of the forced deformations, and volumetric forces along the inner surface of the contact areas reaches a special line of the body boundary; c) the contact surface of the areas \( V_1 \) and \( V_2 \) of elastic composite body \( V \) having different mechanical characteristics \( E_i, v_i, i = 1, 2 \), respectively, reaches a special line of the body’s boundary. In the case of c), the solution of the elastic problem in the vicinity of the boundary irregular point is reduced to two plane problems for the composite body.

In the studies [11, 15], the similarity theory is applied for researching the solution of the elastic problem in the displacements in the vicinity of an irregular point on a special line of the region boundary.

We review the small vicinity of an irregular point on a special line: the line of rupture, for example, of boundary conditions or the first-order derivatives of the region surface function. In the small vicinity of an irregular surface boundary point, a similarity group is used: \( x_i = tx; \quad y_i = ty; \quad z_i = z; \quad t > 0 \), where \( t \) is a group parameter [15]. If we put down the Lame’s equation in the small vicinity of a point on a special line, \( t \to +\infty \), the solution of the elastic problem is reduced to the solution of two homogeneous plane problems: plane and antiplane deformation.

The study [15] introduces the concept of canonical singular problem, which characterizes the peculiarity of stress-strain state in the vicinity of the irregular border point, for which the following two theorems are given.
Any canonical singular problem corresponds to the transcendental equation, each root of which corresponds to a homogeneous solution, the number of any real constants in this solution is equal to the multiplicity of the root. In the infinitely small vicinity of a special point, the solution of the correct boundary value problem of the elasticity theory behaves as an asymptotically maximal in absolute value eigenfunction of the corresponding canonical singular problem.

Analytical solutions of elasticity problems in the zone of irregular point, lines of the region boundary are characterized by singularity of the solution based on the idealization of the boundary form.

Theoretical and experimental researches of stress concentration based on the shape of the boundary are reflected in studies of G. Naber, R. Peterson, N. G. Savin, V. I. Tulchy, B. N. Ushakov, I. P. Fomin, N. A. Makhutov, V. V. Vasilyev, V. P. Netrebko and many others.

Freezing of deformations in blanks and the subsequent annealing – defrosting of compound polymeric model allow to obtain the photoelastic method of stress-strain state in the model corresponding to the required one [1-7]. The advantage of the freezing method is that there is no need to reproduce the temperature field in the model, the standard equipment is used to conduct the experiment and the stress-strain state of structures with a complex shape of boundaries (geometric concentration) is simulated.

The method of defrosting is practically multiple-purpose and is recommended for solving the plane problem of the elasticity theory, as well as in some cases of the volumetric problem, which are characterized by the presence of restrictions for temperature exposures. For a particular case when temperature deformations in one of the directions do not cause stresses, many problems of engineering practice are solved: research of the thermal stress state of structural elements of nuclear reactors, power equipment, in welded and threaded joints, structures with concentration zones, in problems of mechanical engineering, hydraulic engineering, etc. [1-7].

Simulation of problems with forced deformations in a standard form not fulfilling the compatibility conditions is reviewed in studies of G. S. Vardanjan, N. I. Prigorovskiy, S. E. Bugayenko, M. N. Dveres, B. N. Evstratov, I. A. Razumovskij, V. N. Bronov, V. N. Savostyanov, L. U. Frishter et al. The disadvantage of defrosting method regarding the impossibility to simulate volumetric deformations on models because of incompressibility of material in a high elastic state is overcome in works of S. E. Bugayenko, N. I. Prigorovskiy, M. N. Dveres, B. N. Evstratov, V. N. Savostyanov, D. I. Omelchenko et al.

The fringe contour obtained on the model by photoelastic method and the method of defrosting in the stress concentration area, is characterized by significant gradients of experimental data: the order of fringes (isochromes), isolines parameters, stresses, deformations. Modern methods of visualization of experimental data based on digital shooting expand the capabilities of the photoelastic method. Therefore, there is a need to estimate the opportunities of obtaining stressed state by photoelastic method and method of deformations defrosting in the region which is as close as possible to the area of stress concentration during digital shooting and data processing, by standard methods of stress separation.

The purpose of the study is to analyze the opportunities of obtaining stress state in the area of angular cut of the boundary under the action of rupturing forced (temperature) deformations by means of defrosting method with the use of visualization of experimental data during digital shooting and processing, and standard methods of stress separation developed in the photoelastic method.

2. Materials and Methods

The experimental solution by defrosting the deformations [1, 7] of an elastic problem with forced deformations for a plane area is reviewed by the example of a 180 mm long and 24 mm wide composite beam model.

In one area of the beam $\Omega_2$, temperature deformations $\alpha T \delta_0$ were created, and the other area $\Omega_1$ was not loaded. Temperature deformation surge on the contact surface of areas expands to an irregular point t. O(0.0) of the boundary – the vertex of the region cut. The experimental solution is analyzed in the vicinity of the vertex of the beams’ boundary cut, the
ends of which have different span angles: a) squared one $2\alpha=180^0$; b) ‘cut off’ squared end $\alpha+\beta=180^0$, $\alpha=105^0$, $\beta=75^0$; b) end with symmetrical cut out angle $2\alpha=260^0$.

Using detailed experimental data obtained during digital shooting, it is possible to fragment the beam end area in such a way that standard methods of stress separation of the photoelastic method are applicable [1-4, 6]: graphic and shear stresses difference method in the region closest to the stress concentration area.

Selecting different fragments of the beam area, pictures of the isostaths as a whole in the area of the beam end and pictures of the isostaths in the area adjacent to the stress concentration area are constructed.

For example, Figure 1 shows the catalogue of experimental data for the selected fragment of the area $\Omega_2$ of the span beam’s end $2\alpha=260^0$. 

![Diagram](image-url)
Figure 1. Catalogue of experimental data for the selected fragment of the beam end with span $2\alpha = 260^\circ$

For the selected fragment of the beam end area, experimental data are shown in Figure 2: a) fringe contour; b) combined contour of fringes and isoclines. According to Figure 2, the isostaths shown in Figure 3 are graphically constructed for the selected fragment of the region.

According to the experimental data of Figure 2, stress diagrams are constructed by the method of a difference of shearing stresses in several vertical sections of the beam, which are close enough to the area of stress concentration. Auxiliary sections I, II, III, IV are parallel to the axis $OX$ and are situated from this axis at the distance of $y_i/h$, equal, respectively, to: I $0,02$; II $0,05$; III $0,08$; IV $0,11$, $h = 24$ mm. The shear stresses difference method is used to obtain stresses in midsections I-II, II-III, III-IV, which are situated from the axis $OX$ at the distance equal, respectively, to: $y_{I-II} = 0,035h = 0,84$ mm, $y_{II-III} = 0,065h = 1,6$ mm, $y_{III-IV} = 0,1h = 2,4$ mm, $h = 24$ mm. Figure 4 shows stress diagrams in midsections of the beam end.

Figure 2. Input data for a fragment of the beam end area with a span of $2\alpha = 260^\circ$: a) isochromes contour; b) combined isochromes and isoclines contour $\theta$ through $5^\circ$
Figure 3. Graphical method of stress separation in the beam end area with a span of $2\alpha = 260^\circ$: a) isochromes and isoclines contour indicating the directions of the main stresses; b) combined isostaths, isochromes and isoclines contour

Figure 4. Shear stresses difference method for the area of beam end with a span of $2\alpha = 260^\circ$: a) stresses $\sigma_1$, $\sigma_2$, $\sigma_x$, $\sigma_y$ in midsections I-II, II-III, III-IV of the beam end; b) pure shear line

The result of the stress separation shows that in all reviewed beams sections with different boundary cut angles, there are points where $\sigma_1 = -\sigma_2$, and the platforms inclined at an angle $45^\circ$ to the main ones are in pure shear conditions: $\tau_{max} = \sigma_1$; $\sigma_j = \frac{\sigma_1 + \sigma_2}{2} = 0$. 
Points at which the “pure shear” platforms can be seen are located at the vertexes of the sharp corners of the isochromes and their vicinity. The result of the stress separation shows that the line connecting the vertexes of the sharp corners of the isochromes, expanding to the vertex of the cut t. O(0.0) at the boundary of the area is a “pure shear” line, at each point of which there are pure shear platforms.

3. Results
On the plane models of the photoelastic method and the method of deformations defrosting, the stress state is obtained in the area of the angular cut of the boundary, in the vertex of which the rupture of the forced (temperature) deformations expands. The obtained results of stress separation show that modern capabilities of visualization of experimental data during digital shooting and data processing, application of standard methods of stress separation allow to obtain a stress state in the vicinity adjacent to the area of stress concentration, which expands the capabilities of photoelastic method for the analysis of experimental solutions. The singular solution to the problem of elasticity (stress concentration) theory, in which fringes are not readable with any increase in the region’s fragment, requires the development of a method to extrapolate the confident data of the experiment to the stress concentration area.

4. Discussion
The research of the stress-strain state of the structure in the area of angular cut of the boundary, in the vertex of which the rupture of forced (temperature) deformations expands, has multiple approaches. The analytical solution in the area of irregular point of the boundary – the vertex of the corner cut – is characterized by the singularity of the solution based on the idealization of the boundary shape.

Numerical methods of calculation in the stress concentration zone based on the sampling of the calculated area require additional estimates for stress-strain state in the area of a special point of the elastic body to verify the results of the solution. The photoelastic method and the method of deformations “defrosting” allow to obtain a stress state in the area as much as possible approached to a zone of pressure concentration during digital shooting and data processing, standard ways of stress separation. The advantage of the freezing method is that there is no need to reproduce the temperature field in the model, the standard equipment is used to conduct the experiment, and the stress-strain state of structures with a complex shape of boundaries (geometric concentration) is simulated.

The defrosting method is practically multiple-purpose and is recommended for solving the plane problem of the elasticity theory. In the region of singular solution of the elasticity problem (the vertexes of the angular cut of the boundary), the fringe contours are not readable at any increase in the fragment of the region. Noting the advantages of this or that approach to the research of the stress-strain state in the zone of structural ununiformity under the action of rupturing forced deformations, it is reasonable to use a comprehensive approach: experimental, analytical or numerical method to estimate the data obtained.

5. Conclusions
Modern opportunities of visualization of experimental data during digital shooting and data processing, application of standard methods of stress separation allow to obtain a stress state in the vicinity adjacent to the area of stress concentration, which expands the possibilities of photoelastic method for the analysis of experimental solutions.

References
[1] Hesin G L et al 1975 The photoelasticity method vol 3 (Moscow: Stroyizdat) p 311
[2] Frocht M M 1948 Photoelasticity vol 2 ed N I Prigorovskiy (Moscow – Leningrad: GITTL) p 432
[3] Durelli A and Riley W 1970 Introduction to photoelasticity ed N I Prigorovskiy (Moscow: Mir)
[4] Aleksandrov A Ja and Ahmetzjanov M H 1974 Polarization-optical methods for deformable solid mechanics (Moscow: Nauka) p 576
[5] Kobayashi A 1990 Experimental Mechanics vol 1 (Moscow: Mir) p 615, vol 2 (Moscow: Mir) p 551
[6] Razumovskij I A 2007 Interference-optical methods of deformable solid mechanics (Moscow: Publisher MGTU named after N Je Bauman) p 240
[7] Kosheленко A S and Poznjak G G 2004 Theoretical foundations and practice of photomechanics in mechanical engineering (Moscow: Granica) p 296
[8] Pestrenin V M, Pestrenina I V and Landik L V 2013 Vestnik TGU. Math. Mech. 4 80–87
[9] Albaut G N, Kharinova N V, Sadovnichij V P, Semenova J I and Fedin S A 2011 Nonlinear problems of mechanics of destruction Vestnik of Lobach. Univ. of Nizhn. Novgor. 4 1344–48
[10] Netrebko V P 2003 Photelasticity method-based study of stress intensity factors near inclined cracks in orthotropic plates (Moscow: Mechanical Engineering Research Institute of the Russian Academy of Sciences) pp 69–77
[11] Vardanjan G S and Frishter L U 2007 Int. J. Comput. Civil Struct. Eng. 3 75–81
[12] Frishter L U 2017 Photoelasticity-based study of stress-strain state in the area of the plain domain boundary cut-out area vertex ed V Murgul and Z Popovic Int. Scient. Conf. Energ. Manag. of Municip. Transp. Facil. and Transp. EMMFT 2017 Advances in Intelligent Systems and Computing vol 692 (Cham: Springer) p 836–844
[13] Albaut G N, Tabanjuhova M V and Harinova N V 2004 Determination of the first stress intensity factor in elements with angled cut-out (Experimental Mechanics and Calculation of Structures (Kostinsky Read.)) (Moscow: Moscow State University Press) pp 166–175
[14] Kondra'tev V A 1967 Boundary value problems for elliptic equations in domains with conical or corner points (Transactions of the Moscow Mathematical Society vol 16) (Moscow: MSU) pp 209–292
[15] Cherepanov G P 1974 Mechanics of brittle failure (Moscow: Nauka) p 640
[16] Aksent'yan O K 1967 Features of the stress-strain state of the plate in the neighborhood of the edge (Applied Mathematics and Mechanics vol 31 issue 1) pp 178-186
[17] Chobanjan K S and Gevorkjan S H 1971 The behavior of the stress field near the angular point of the separation line in the problem of plane deformation of a composite elastic body (Bull. of the Acad. of Scien. of the Armen. SSR. Mech. Issue XXIV, No. 5) pp 16–24
[18] Denisjuk I T 1995 Stress state close to a singular line of the interface boundary (Bull. of the Rus. Acad. of Scien., Solid mech., No. 5) pp 64–70
[19] Kuliev V D 2005 Singular boundary value problems (Moscow: Nauka) p 719
[20] Parton V Z and Perlin P I 1981 Methods of the mathematical theory of elasticity (Moscow: Nauka) p 688