A study on M-joins of super mean labeling of some digital numbers

S. Sriram\(^1\)\(^*\) and K. Thirusangu\(^2\)

Abstract
Let G be a graph and define an injective function \( f : V(G) \to \{1, 2, 3, \ldots, p\} \). Now for suppose for each edge we assign the labels such that \( E(G) \to \{1, 2, 3, \ldots, p+q\} \) the induced edge \( f^* \) is \( f^*(e) = \frac{f(u)+f(v)}{2} \) if \( f(u) + f(v) \) is even and \( f^*(e) = \frac{f(u)+f(v)+1}{2} \) if \( f(u) + f(v) \) is odd then \( f \) is called super mean labelling of graph. In this paper we wish to study the super mean labelling of some of digital numbers 2,5,7 and also intend to study on the M-joins of Super mean labelling of digital numbers 2,5,7 and extend our study to prove some characteristics property of M-Joins of Super mean labelling of digital numbers 2,5,7.

Keywords
Super Mean Labeling, Digital numbers, Digital labelling of graphs, Joins of Super Mean Labeling of Digital numbers.

AMS Subject Classification
05C78.

1. Introduction
We consider a finite simple graph with set of vertices and edges for our discussion and follow the notation and basic concepts\(^2\). In the discussion of labelling of graphs by different labelling techniques we are motivated and found that there is a quite significance with the Super Mean Labeling of graphs with that of the technological improvements related to transmitting computer signals. Hence we consider here the understanding of Super Mean labelling of graphs towards the digital numbers. Super Mean labelling is introduced by P.Jeyanthi, R.Ponraj and D.Ramya \(^3\) and further contribution made in various papers\(^4\)\(^5\)\(^6\). We consider the literary survey based on graphs given by J.A.Gallian \(^1\). Digital numbers are the one which in remote sensing systems assign pixel in the form of binary integer. We here consider the digital numbers 2,5,7 and tried to label them and prove that they are Super Mean labelling graphs. In continuation we study on the joins of the digital numbers 2,5,7 and extent the same towards M-Joins of the digital numbers 2,5,7.

2. Preliminaries

**Definition 2.1.** We define an injective function \( f : V(G) \to \{1, 2, 3, \ldots, p\} \). Now for suppose for each edge we assign the labels such that \( E(G) \to \{1, 2, 3, \ldots, p+q\} \) the induced edge \( f^* \) is \( f^*(e) = \frac{f(u)+f(v)}{2} \) if \( f(u) + f(v) \) is even and \( f^*(e) = \frac{f(u)+f(v)+1}{2} \) if \( f(u) + f(v) \) is odd then \( f \) is called super mean labelling of graph.

**Definition 2.2.** The digital number 2 is defined as a graph consisting of 6 vertices and 5 edges with the following structure

---

*Corresponding author: 1 sanksriram@gmail.com; 2kthirusangu@gmail.com

©2020 MJM.
Definition 2.3. The digital number 5 is defined as a graph consisting of 6 vertices and 5 edges with the following structure.

Fig. Digital Number 5

Note 2.4. The digital number 2 and 5 are mirror images as being seen from the above structure.

Definition 2.5. The digital number 7 is defined as a graph consisting of 3 vertices and 2 edges with the following structure.

Fig. Digital Number 7

In this paper we wish to prove that the digital numbers 2, 5, 7 are super mean labelling graphs and we construct 1-join of the digital numbers 2, 5, 7 and prove that they are super mean labelling and we extend to M-joins of the digital numbers 2, 5, 7 and prove that they are super mean labelling graphs.

3. Main Results

Theorem 3.1. The digital number 2 is super mean labeling graph.

Proof. Consider the digital number 2 which has 6 vertices and 5 edges. Let the vertex set be $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and the edge set be $E = \{e_1, e_2, e_3, e_4, e_5\}$ corresponding to the vertices $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6\}$. Now we label the vertices as follows $f(v_i) = 2i - 1$ for $1 \leq i \leq 6$ and hence on obtaining the induced edge labelling we have $f^*(e_i) = \frac{f(v_i) + f(v_{i+1})}{1} = \frac{2i - 1 + 2i}{2} = i$, for $1 \leq i \leq 5$. We find that the induced edge labelling follows super mean labelling of graph and hence the digital number 2 is super mean labelling graph. Hence the proof.

Theorem 3.2. The digital number 5 is super mean labeling graph.

Proof. Consider the digital number 5 which has 6 vertices and 5 edges. By the construction of digital same labelling technique as we labelled for the vertices of digital number 2 and we further find that the induced edge labelling is super mean labelling graph. Hence the proof.

Theorem 3.3. The digital number 7 is super mean labeling graph.

Proof. Consider the digital number 7 which has 3 vertices and 2 edges. Let the vertex set be $V = \{v_1, v_2, v_3\}$ and the edge set be $E = \{e_1, e_2\}$ corresponding to the vertices $\{v_1v_2, v_2v_3\}$. Now we label the vertices as follows $f(v_i) = 2i - 1$ for $1 \leq i \leq 3$ and hence on obtaining the induced edge labelling we have $f^*(e_i) = \frac{f(v_i) + f(v_{i+1})}{1} = \frac{2i - 1 + 2i}{2} = i$, for $1 \leq i \leq 2$. We find that the induced edge labelling follows super mean labelling of graph and hence the digital number 7 is super mean labelling graph. Hence the proof.

Note 3.4. We call the digital number graph 2,5,7 as basic graph.

Now we construct for the digital number 2,5,7 graph by inserting one vertex between every two vertices in already constructed digital number 2,5,7 graph (basic graph). This modified digital graph 2,5,7 will have some vertices and edges in them. The digital number 2 after inserting one vertex between every two vertices is given as below. Now labelling the vertices as given in the above theorems for 2 and 7 with the number of vertices 9 and 5 and the edges 8 and 4 respectively with the same labelling technique except that increasing the number of vertices from 6 to 9 for digital number 2 and number of vertices from 3 to 5 for digital number 7 and the edges will follow to be a super mean labelling graph. The result for the digital number 5 is same as the theorem proved earlier. The following table illustrates the total number of vertices and edges in a digital number 2,5,7 graph.

To summarise the procedure to find the number of vertices and edges on inserting vertices in between every two vertices of the digital number 2,5,7 graph we prove the following result.

Theorem 3.5. (a) The total number of vertices and edges of digital number 2 and 5 on inserting vertices in between every two vertices of basic digital number 2 and 5 graph is $n(V) = n(V_{bg}) + 5I$, where $I$ stands for the number of vertices inserted and $n(V_{bg})$ the number of vertices in basic graph. The number of edges $n(E) = n(V) - 1$

(b) The total number of vertices and edges of digital number 7 on inserting vertices in between every two vertices of basic...
A study on M-joins of super mean labeling of some digital numbers — 681/685

Digital Number 2  Digital Number 5  Digital Number 7

| Number of vertices - Basic graph | 6 | 6 | 3 |
|----------------------------------|---|---|---|
| Number of edges - Basic graph    | 5 | 5 | 2 |
| Number of vertices by inserting one vertex in between every two vertices of basic graph | 11 | 11 | 5 |
| Number of edges by inserting one vertex in between every two vertices of basic graph | 10 | 10 | 4 |

Table 1. Table to illustrate digital number after insertion of vertices in between every two vertices.

digital number 7 graph is \( n(V) = n(V_{bg}) + 2I \), where \( I \) stands for the number of vertices inserted. The number of edges \( n(E) = n(V) - 1 \).

**Proof.** The theorem can be proved by applying principle of mathematical induction on the number of vertices inserted \( I \). We find from the above table that the result (a) and (b) is true for first positive integer \( I = 1 \). Let us assume that the result is true for \( I = m - 1 \), where \( m \) is some positive integer which is the number of vertices inserted. Now we need to prove that the result is true for \( I = m \) which is obvious as on combining the result of \( I = 1 \) and \( I = m - 1 \) we can prove the result (a) and (b). Hence the proof by principle of mathematical induction.

**Definition 3.6.** We define 1-join of digital number 2 graph by joining the digital number 2 graph with a similar digital number 2 graph by an edge. Similarly we define 2-join of digital number 2 graph as follows

\[
\begin{align*}
\text{Number of vertices} & \quad \text{Number of edges} \\
\text{Basic graph} & \quad \text{Basic graph} \\
\text{by inserting} & \quad \text{by inserting} \\
\text{one vertex in} & \quad \text{one vertex in} \\
\text{between every} & \quad \text{between every} \\
\text{two vertices of} & \quad \text{two vertices of} \\
\text{basic graph} & \quad \text{basic graph} \\
\end{align*}
\]

\( f(v_i) = 2i - 1 \) for \( 1 \leq i \leq 6 \)

\( f(v_i^1) = f(v_i) + 12 \) for \( 1 \leq i \leq 6 \). Then for the induced edge labeling of 1-join of digital number 2 graph is super mean labeling graph.

**Theorem 3.7.** 1-Join of digital number 2 graph is super mean labeling of graph

**Proof.** Consider the 1-join of digital number 2 graph which has the vertex set \( V = \{v_1,v_2,v_3,v_4,v_5,v_6, v_1^1,v_2^1,v_3^1,v_4^1,v_5^1,v_6^1\} \). The edge set

\[
E = \{e_1,e_2,e_3,e_4,e_5,e_1^1\} \cup \{e_2^1,e_3^1,e_4^1,e_5^1,e^1\}
\]

associated with the vertices namely \( \{v_1v_2,v_2v_3,v_3v_4,v_4v_5, v_5v_6\} \cup \{v_1v_2^1,v_2^1v_3^1,v_3^1v_4^1,v_4^1v_5^1,v_5^1v_6^1,v_6^1v_1^1\} \) where the edge \( e^1 \) is associated with the vertices \( v_6v_1^1 \) which joins one digital number 2 graph with another digital number 2 graph.

Now we label the vertices of 1-join of digital number 2 graph as follows

\[
f(v_i) = 2i - 1 \quad \text{for} \quad 1 \leq i \leq 6 \\
f(v_i^1) = f(v_i) + 12 \quad \text{for} \quad 1 \leq i \leq 6
\]

The theorem can be proved by applying principle of mathematical induction.

Algorithm 3.9. Algorithm to label the vertices of M-Join of digital number 2 graph

**Step-1:** Start the Algorithm

**Step-2:** Input the value of \( M \), the number of Join of digital number 2 graph to construct

**Step-3:** Label the vertices of basic digital number 2 graph with \( f(v_1) = 2i - 1 \) for \( 1 \leq i \leq 6 \)

**Step-4:** To Label the joins of digital number 2 graph

For \( i = 1 \) to \( i = M \)

\[
f(v_i^M) = f(v_i) + 12M \quad \text{for} \quad 1 \leq i \leq 6
\]

**Step-5:** Stop the Algorithm.

**Theorem 3.10.** M-Join of digital number 2 graph is super mean labeling

**Proof.** Consider the M-Join of digital number 2 graph and label the vertices as specified in the above algorithm. We then obtain the induced edge labelling of the M-Join of digital number 2 graph as follows

\[
f^*(e_i) = \frac{f(v_i) + f(v_{i+1})}{2} \quad \text{for} \quad 1 \leq i \leq 5
\]

\[
f^*(e_i^M) = \frac{f(v_i^M) + f(v_{i+1}^M)}{2} \quad \text{for} \quad 1 \leq i \leq 5,
\]

where \( M \) is the number of joins and \( M = 1,2 \ldots \)

\[
f^*(e^1) = \frac{f(v_6) + f(v_1^1)}{2}
\]

For \( M \geq 2 \) we have

\[
f^*(e^M) = \frac{f(v_{M+1}^M) + f(v_1^M)}{2}
\]

Hence the induced edge labelling is super mean labeling and hence M-Join of digital number 2 graph is super mean labeling graph. Hence the proof.

**Theorem 3.11.** M-Join of digital number 5 graph is super mean labeling graph

**Proof.** We have understood from our earlier discussion that digital number 2 graph and digital number 5 graph are mirror image to each other. Hence we can consider the same procedure adopted to label the vertices of M-Join of digital number 2 graph for M-Join of digital number 5 graph. Hence M-Join of digital number 5 graph is super mean labeling graph. Hence the proof.
Theorem 3.12. 1-Join of digital number 7 graph is super mean labeling of graph

Proof. Consider the 1-Join of digital number 7 graph which has the vertex set $V = \{v_1, v_2, v_3, v_1', v_2', v_3'\}$. The edge set $E = \{e_1, e_2, e_1', e_2', e_1^2, e_1^3\}$ associated with the vertices namely $\{v_1, v_2, v_3, v_1', v_2', v_3', v_1^2, v_2^2, v_3^2, v_1^3, v_2^3, v_3^3\}$ where the edge $e_1^3$ is associated with the vertices $v_3, v_3'$ which joins one digital number 2 graph with another digital number 2 graph.

Now we label the vertices of 1-join of digital number 2 graph as follows
$$f(v_i) = 2i - 1 \text{ for } 1 \leq i \leq 3$$
$$f(v_i') = f(v_i) + 6 \text{ for } 1 \leq i \leq 3.$$ 

Then for the induced edge labeling of 1-join of digital number 2 graph is given by
$$f^*(e_i) = \frac{f(v_i) + f(v_{i+1})}{2} \text{ for } 1 \leq i \leq 2$$
$$f^*(e_i') = \frac{f(v_i') + f(v_{i+1}')}{2} \text{ for } 1 \leq i \leq 2$$
$$f^*(e_1^3) = \frac{f(v_3) + f(v_3')}{2}$$

We find that the induced edge labelling of 1-Join of digital number 7 graph is super mean labelling graph. Hence the proof.

Algorithm 3.13. Algorithm to label the vertices of M-Join of digital number 7 graph

Step-1: Start the Algorithm

Step-2: Consider the label assigned for digital number 7 as follows $f(v_i) = 2i - 1 \text{ for } 1 \leq i \leq 3$

Step-3: Now to label the vertices of M-Joins of digital number 7

For $i = 1$ to $i = M$ 
$$f(v_i^M) = f(v_i) + 6M$$
for $1 \leq i \leq 3$

Step-5: Stop the Algorithm.

Theorem 3.14. M-Join of digital number 7 graph is super mean labeling

Proof. Consider the M-Join of digital number 7 graph and label the vertices as specified in the above algorithm. We then obtain the induced edge labelling of the M-Join of digital number 7 graph as follows
$$f^*(e_i) = \frac{f(v_i) + f(v_{i+1})}{2} \text{ for } 1 \leq i \leq 2$$
$$f^*(e_i^M) = \frac{f(v_i^M) + f(v_{i+1}^M)}{2} \text{ for } 1 \leq i \leq 2,$$ where M is the number of joins of $M = 1, 2..$

For $M \geq 2$ we have $f^*(e_i^M) = \frac{f(v_3^M) + f(v_3')}{}{2}$$

Hence the induced edge labeling is super mean labeling and hence M-Join of digital number 7 graph is super mean labeling graph. Hence the proof.

Now we construct M-Join of digital number 2 graph, M-Join of digital number 5 graph, M-Join of digital number 7 graph by inserting vertices in between every two vertices of basic digital number 2 graph, basic digital number 5 graph, basic digital number 7 graph respectively.

Observation 3.15. We find from Table.1 given above that on inserting one vertex between every vertex of basic digital number 2 graph we have the total number of vertices increases from 6 to 11 and the number of edges increases from 5 to 10. Similarly the basic digital number 5 graph also will have 11 vertices and 10 edges as like basic digital number 2 graph. We find that the number of vertices of basic digital number 7 graph increases from 3 to 5 and the number of edges increases from 2 to 4.

We now initiate our task to prove that the 1-Join of digital number 2 graph, 1-Join of digital number 5 graph, 1-Join of digital number 7 graph, M-Join of digital number 2 graph, M-Join of digital number 5 graph and M-Join of digital number 7 graph are super mean labelling.

Observation 3.16. We prove basic digital 2 graph by inserting vertices in between every vertices is super mean labelling by considering the labelling techniques adopted in Theorem.3.1 and altering the values of $i$ as $1 \leq i \leq 12$ we can prove 1-Join of digital number 2 is super mean labelling graph. In a similar understanding we can prove that 1-Join of digital number 5 graph is super mean labelling graph. We can prove 1-Join of digital number 7 is super mean labelling graph by altering the values of $i$ as $1 \leq i \leq 5$.

Observation 3.17. The same algorithm and theorem can be used to prove M-Join of digital number 2 graph, M-Join of digital number 5 graph and M-Join of digital number 7 graph are super mean labelling by altering the values of $i$ by $1 \leq i \leq 12$ and $1 \leq i \leq 5$ respectively.
All the results holds good for the structure we have obtained. As we know that the tree is acyclic connected graph. We have

(i) G is acyclic and connected graph with

P

(ii) G has unique uv path

(iii) G is minimally connected

(iv) G has n vertices and

n

− 1 edges

Hence we can conclude that M-Joins of digital 2 graph, M-Joins of digital 5 graph, M-Joins of digital 7 graph preserves tree structure.

Algorithm 3.19. To label the vertices of 1-join of digital number 2 graph by inserting one vertex in between every vertex of the basic 1-join of digital number 2 graph

Step : 1 Start the Algorithm

Step : 2. We know that basic 1-Join of digital number 2 graph consists of 11 vertices and 10 edges. Now by inserting vertex in between every vertices of basic 1-Join of digital number 2 graph we find the number of vertices of basic digital number 2 graph increases by 5 and the number of edges of basic digital number 2 graph increases by 5 and total vertices of the 1-Join of digital number 2 graph increases by 10 vertices and 10 edges.

Step : 3. Label the vertices \( f(v_i) = 2i - 1 \) for \( 1 \leq i \leq 11 \)

Label the vertices \( f(v'_i) = 2i - 1 \) for \( 12 \leq i \leq 22 \)

Step : 4. Stop the Algorithm

Theorem 3.20. 1-Join of digital number 2 graph on inserting one vertex in between every vertex of the 1-Join of digital number 2 graph is super mean labelling graph.

Proof. Consider the vertices of 1-Join of digital number graph on inserting one vertex in between every vertex of 1-Join of digital number 2 graph and follow the labelling procedure given in the Algorithm to label the vertices. Now the induced edge labelling is as follows

\[
\begin{align*}
  f^*(e_i) &= \frac{f(v_i) + f(v_{i+1})}{2} & \text{for } 1 \leq i \leq 10 \\
  f^*(e'_i) &= \frac{f(v'_i) + f(v'_{i+1})}{2} & \text{for } 11 \leq i \leq 21 \\
  f^*(e^1) &= \frac{f(v_{10}) + f(v'_1)}{2}
\end{align*}
\]

Hence on obtaining the induced edge labelling we conclude that the 1-Join of digital number 2 with a one vertex inserted between every two vertices is super mean labelling of graph. Hence the proof.

Theorem 3.21. 1-Join of digital number 5 graph on inserting one vertex in between every vertex of the 1-Join of digital number 5 graph is super mean labelling graph.

Proof. We know that 1-Join of digital number 5 graph is a mirror image of 1-Join of digital number 5 graph. The proof is analogous to above theorem.

Theorem 3.22. 1-Join of digital number 7 graph on inserting one vertex in between every vertex of the 1-Join of digital number 7 graph is super mean labelling graph.

Proof. Consider the 1-Join of digital number 7 graph which has the vertex set \( V = \{v_1, v_2, v_3, v_4, v_5, v_1^*, v_2^*, v_3^*, v_4^*, v_5^*\} \). The number of vertices being 6 and the edge set \( E = \{e_1, e_2, e_1^*, e_2^*, e_1^\} \) associated with the vertices set namely \( \{v_1v_2, v_2v_3, v_1^*v_2^*, v_1v_3^*\} \), where the edge \( e_1^\) is associated with the vertices \( \{v_1v_1^*\} \) which joins one digital number 7 graph with another digital number 7 graph. Now inserting vertices in between every two vertices we have the vertex set \( V = \{v_1, v_2, v_3, v_4, v_5, v_1^*, v_2^*, v_3^*, v_4^*, v_5^*\} \).
and the edge set is $E = \{e_1, e_2, e_3, e_4e_1, e_5, e_6, e_7\}$. Now we labelling the vertices as follows we have

$$f(v_i) = 2i - 1 \text{ for } 1 \leq i \leq 5$$

$$f(v_1) = f(v_i) + 10 \text{ for } 1 \leq i \leq 5$$

Then the induced edge labeling of 1-join of digital number 7 graph is given by

$$f(e_i) = f(v_i) + f(v_{i+1}) \text{ for } 1 \leq i \leq 4$$

$$f(e_1) = f(v_1) + f(v_2)$$

$$f^*(e_1) = f(v_1) + f(v_2)$$

we find that the induced edge labeling of 1-join of digital number 7 graph by inserting one vertex between every two vertices is super mean labeling graph. Hence the proof.

**Algorithm 3.23.** To label M-Join of digital number 2 graph by inserting vertices every two vertices of the graph.

**Step 1:** Start the Algorithm

**Step 2:** Let us label M-Join of digital number 2 graph by inserting vertices for every two vertices of the graph.

Let the vertices of basic digital number 2 graph with

$$f(v_i) = 2i - 1 \text{ for } 1 \leq i \leq 6 + 5N$$

where $N$ stands for the number of vertices inserting between every two vertices of basic digital number 2 graph.

**Step 3:** To Label the joins of digital number 2 graph

For $i = 1$ to $i = M$

$$f(v_i^M) = f(v_i) + 2M(6 + 5N) \text{ for } 1 \leq i \leq 6 + 5N$$

where $N$ stands for the number of vertices inserting between every two vertices of basic digital number 2 graph.

**Step 4:** Stop the Algorithm

**Theorem 3.24.** M-Join of digital number 2 graph on inserting vertices in between every two vertices of the digital number 2 graph is super mean labelling graph.

**Proof.** Consider the vertices of M-Join of digital number graph on inserting one vertex in between every vertex of M-Join of digital number 2 graph and follow the labelling procedure as

$$f(v_i) = 2i - 1 \text{ for } 1 \leq i \leq 3 + 2N$$

Now the induced edge labelling is as follows

$$f^*(e_i) = f(v_i) + f(v_{i+1})$$

where $N$ is the number of vertices inserted.

$$f^*(e_i^M) = f(v_i) + f(v_{i+1})$$

where $M$ is the number of joins and $M = 1, 2, ..$

$$f^*(e_1) = f(v_1) + f(v_2)$$

For $M \geq 2$ we have $f^*(e_i^M) = f(v_i) + f(v_{i+1})$

Hence the induced edge labelling is super mean labelling and hence M-Join of digital number 7 graph by inserting vertices in between every vertex of digital number graph is super mean labelling graph. Hence the proof.

**Theorem 3.25.** M-Join of digital number 5 graph on inserting vertices in between every two vertices of the digital number 5 graph is super mean labelling graph.

**Proof.** Let us label the vertices of M-Join of digital number 5 graph as given in algorithm 3.15 and we find that the M-Join of digital number 2 graph and M-Join of digital number 5 graph are mirror image and hence the proof is analogous to the above theorem. Hence the proof.

**Theorem 3.26.** M-Join of digital number 7 graph on inserting vertices in between every two vertices of the digital number 7 graph is super mean labelling graph.

**Proof.** Consider the vertices of M-Join of digital number graph on inserting one vertex in between every vertex of M-Join of digital number 7 graph and follow the labelling procedure as

$$f(v_i) = 2i - 1 \text{ for } 1 \leq i \leq 3 + 2N$$

Now the induced edge labelling is as follows

$$f^*(e_i) = f(v_i) + f(v_{i+1})$$

where $N$ is the number of vertices inserted.

$$f^*(e_i^M) = f(v_i) + f(v_{i+1})$$

where $M$ is the number of vertices inserted.

$$f^*(e_1) = f(v_1) + f(v_2)$$

For $M \geq 2$ we have $f^*(e_i^M) = f(v_i) + f(v_{i+1})$

Hence the induced edge labelling is super mean labelling and hence M-Join of digital number 7 graph by inserting vertices in between every vertex of digital number 7 graph is super mean labelling graph. Hence the proof.

**4. Conclusion**

In this paper we have discussed the digital number 2, 5 and 7 graph and have constructed joins for the same and studied on how it can be labelled to prove that it is a super mean labelling graph. We have inserted vertices in between every two vertex of the digital number 2, 5, 7 graph and proved that they are super mean labelling graph. We have obtained some characterisation of these digital number graph and also understand that the graph considered are trees fulfilling all the theorem that we have for trees. We in our future discussion like to consider various other graph which can be proved to be a super mean labelling and also like to prove some of its characteristic property.

**Acknowledgment**

We thank the reviewers for their suggestions and improvements on this paper. We also thank them for their support and guidance in publication of this paper.

**References**

[1] J.A. Gallian, *A Dynamic Survey of Graph Labeling*, 22nd Edition, 2019.

[2] F. Harary, *Graph Theory*, Addison-Wesley, 1972.

[3] D.Ramya, R.Ponraj and P.Jeyanthi, Super mean labelling of graphs, Ars Combin., 112(2013), 65–72.

[4] R. Vasuki, P. ASugirtha and J. Venkateswaria, Super Mean Labeling of Subdivision Graphs, *Kragujevac Journal of Mathematics*, 41(2)(2017), 179–201.
[5] S. Sriram and R. Govindarajan, Harmonic Mean Labeling of Joins of Square of Path Graph, *International Journal of Research in Advent Technology*, 7(3)(2019), 1–10.

[6] S. Sriram, R. Govindarajan and K. Thirusangu, Pell Labeling of Joins of Square of Path Graph, *International Journal of Engineering and Advanced Technology*, 9(3)(2019), 2249–8958.