Energetics govern ocean circulation on icy ocean worlds.

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ABSTRACT

Globally ice-covered oceans have been found on multiple moons in the solar system and may also have been a feature of Earth’s past. However, relatively little is understood about the dynamics of these ice-covered oceans, which affect not only the physical environment but also any potential life and its detectability. A number of studies have simulated the circulation of icy-world oceans, but have come to widely different conclusions. To better understand and narrow down these diverging results, we discuss energetic constraints for the circulation on ice-covered oceans, focusing in particular on Snowball Earth, Europa, and Enceladus. Energy input that can drive ocean circulation on ice-covered bodies can be associated with heat and salt fluxes at the boundaries as well as ocean tides and librations. We show that heating from the solid core balanced by heat loss through the ice sheet can drive an ocean circulation, but the resulting flows would be relatively weak and strongly affected by rotation. Salt fluxes associated with freezing and melting at the ice sheet boundary are unlikely to energetically drive a circulation, although they can shape the large-scale circulation when combined with turbulent mixing. Ocean tides and librations may provide an energy source for such turbulence, but their strength remains highly uncertain for the icy moons, which poses a major obstacle to predicting the ocean dynamics of icy worlds and remains as an important topic for future research.

1. INTRODUCTION

Globally ice-covered oceans have been found on multiple moons in the solar system (Carr et al. 1998; Kivelson et al. 2000; Thomas et al. 2016; Nimmo & Pappalardo 2016) and spark our curiosity in part due to their potential to provide hospitable environments for life (Des Marais et al. 2008; Waite et al. 2017; Postberg et al. 2018; Hendrix et al. 2019). Earth’s oceans may also have been covered by a global ice sheet during the so-called “Snowball Earth” events, and indeed eukaryotic life not only appears to have survived through these episodes, but may have evolved significantly during them (e.g. Hoffman et al. 2017). However, relatively little is known about these oceans beyond their existence, and due to our inability to directly observe them at present, we heavily rely on models to decipher their mysteries.

Although their potential biology may represent the holy grail for research on the icy moon oceans, it is natural to start with the somewhat more tractable problem of inferring the physical and chemical environment. In this study we specifically focus on the ocean circulation and mixing processes, which control the transport of heat and chemical tracers, including those that may affect life and our ability to observe its signatures [e.g. via material ejected in plumes].

Ocean circulation on icy moons can broadly speaking be driven by heat and salt fluxes, tidal forcing, or magnetic forces (e.g. Soderlund et al. 2020, and references therein). We here focus primarily on “buoyancy driven” flow, i.e. flows associated with temperature and salinity gradients, although we also include a discussion of the role of tides and librations in driving vertical mixing, which in turn affects the buoyancy field and associated flow (e.g. Wunsch & Ferrari 2004). Following most of the previous work on buoyancy-driven flows, we will neglect magnetic forces, although they may be significant on Jupiter’s moons (Gissinger & Petitdemange 2019).

A number of studies have simulated the buoyancy-driven dynamics of ice covered oceans both in the context of Snowball Earth (e.g. Ashkenazy et al. 2013; Ashkenazy & Tziperman 2016; Jansen 2016) and icy moons (e.g. Soderlund et al. 2014; Soderlund 2019; Ashkenazy & Tziperman 2021; Kang et al. 2020; Zeng & Jansen 2021), and have come...
to widely different conclusions, in particular with regards to the characteristic current speeds in these oceans. The Snowball Earth simulations of Ashkenazy et al. (2013), Ashkenazy & Tziperman (2016) and Jansen (2016) broadly consistently show small Rossby-number turbulent flows dominated by eddies and jets with characteristic velocities on the order of 1 cm/s. For Europa, Soderlund et al. (2014) find moderate Rossby-number convective turbulence, with characteristic velocity scales on the order of 1 m/s, whereas Ashkenazy & Tziperman (2016) instead find largely geostrophic (i.e. low Rossby-number) turbulence and jets with characteristic velocities on the order of 1 cm/s. In a parameter regime deemed applicable to Enceladus, Soderlund (2019) finds characteristic flow velocities on the order of 0.1 m/s, while Kang et al. (2020) and Zeng & Jansen (2021) find velocities on the order of 0.1 mm/s. The flow dynamics and associated kinetic energy levels hence vary widely across these studies, with variations across different studies being larger than variations between different oceans.

To shed some light on these discrepancies and to establish what insights can be gained from first principles (i.e. without running numerical simulations whose results are sensitive to many parameters and implicit but often unstated assumptions), we here consider energetic constraints for the circulation of a globally ice-covered ocean. For simplicity we limit ourselves to an ocean in a statistical equilibrium state (as also assumed in all studies discussed in the previous paragraph). Specifically, we assume that the net global ocean warming or cooling is small compared to the heat fluxes through the lower and upper ocean boundaries, and similarly that the net global mean freezing or melting rate is small compared to the regional rates. Non-equilibrium effects could be important (e.g. Hussmann & Spohn 2004; Nakajima et al. 2019), but would vastly widen the range of possible solutions. Our philosophy is that the better-constrained equilibrium problem should serve as a null-hypothesis, which will be rejected if and only if evidence contradicts the assumptions or predictions of equilibrium ocean dynamics.

2. ENERGETICS OF THE SEAWATER BOUSSINESQ EQUATIONS.

The weak compressibility of water allows us to employ the seawater Boussinesq approximation, with which the dynamical equations reduce to (e.g. Young 2010)

\[ \frac{Dv}{Dt} + 2\Omega \times v + \nabla p = b\hat{k} + \mathcal{F} \]

(1)

\[ \nabla \cdot v = 0 \]

(2)

where \( v \) is the velocity, \( \Omega \) is the planetary rotation, \( p \) is the pressure anomaly relative to a hydrostatic reference state with constant density, \( \rho_0 \), \( b = g(\rho_0 - \rho)/\rho \) is buoyancy\(^3\), \( \hat{k} = g^{-1}\nabla \phi \) is the normal vector in the direction of gravitational acceleration, and \( \mathcal{F} = \mathcal{T} - D \) is an acceleration due to tidal and frictional forces, respectively (where we define the frictional force with a negative sign for illustrative purposes, as it dominantly acts to decelerate the flow).

Multiplying Eq. (1) by \( v \), and using \( \nabla \cdot v = 0 \) yields an equation for the kinetic energy as

\[ \frac{D}{Dt} |v|^2 + \nabla \cdot (vp) = wb + v \cdot T - v \cdot D \]

(3)

where \( w \) is the velocity normal to the geopotential surfaces (in practice usually well approximated by the radial velocity).

Integrating over the entire ocean volume and assuming an equilibrium state and no-normal-flow boundary conditions\(^4\), we find a balance between the conversion from potential to kinetic energy, \( wb \), KE generation by tidal forcing, \( v \cdot T \), and frictional dissipation, \( v \cdot D \):

\[ \int wb dV + \int v \cdot T dV = \int v \cdot D dV \]

(4)

We hence have two potential sources of kinetic energy: 1) the vertical buoyancy flux, which converts potential to kinetic energy and is directly related to the vertical heat and salt flux, and 2) tidal forcing. This paper will focus

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\(^1\) Results are reported in non-dimensionalized units as \(|U|/(2\Omega D)| \sim 0.5\). Re-dimensionalizing with the rotation rate \( \Omega \sim 2 \times 10^{-5} \text{s}^{-1} \) and ocean depth \( D \sim 1 \times 10^5 \text{m} \) gives \(|U| \sim 2 \text{ m/s} \).

\(^2\) Results are reported in non-dimensionalized units as \(|U|/(2\Omega D)| \sim 0.1\). Re-dimensionalizing with the rotation rate \( \Omega \sim 5 \times 10^{-5} \text{s}^{-1} \) and ocean depth \( D \sim 1 \times 10^4 \text{m} \) gives \(|U| \sim 0.1 \text{ m/s} \).

\(^3\) Notice that Young (2010) defines \( b \) using a constant reference gravity \( g_0 \) and writes the first term on the R.H.S. of Eq. (1) as \( b\nabla Z \), where \( Z \) is \( \phi/g_0 \), with \( \phi \) the geopotential (\( g \equiv |\nabla \phi| \)), such that \( \nabla Z = g/g_0 k \). We here absorb the factor \( g/g_0 \) into the definition of buoyancy, assuming that \( g \) is itself only a function of \( x \).

\(^4\) Notice that the no-normal flow boundary condition here amounts to neglecting any possible kinetic energy injected by jets emanating from the sea floor or ice shell (e.g. Kite & Rubin 2016). The effect of geothermal vents or freezing and melting at the ice-ocean interface on buoyant plumes, however, is included via the heat and salt flux boundary conditions.
primarily on buoyancy-driven circulations, that is we assume a balance between the first term on the LHS of Eq. (4) and the dissipation on the RHS. The potential role of tides in modulating the buoyancy-driven circulation will also be discussed.

Importantly, Eq. (4) highlights that buoyancy forcing can drive a circulation if and only if it is distributed such that the global mean buoyancy flux required to balance the forcing is directed upwards.

2.1. Buoyancy forcing

The upward buoyancy flux is related to the upward flux of heat or compositional buoyancy. In this manuscript we assume a water ocean with dissolved salts, such that compositional buoyancy effects are encapsulated by the salinity. In general we thus allow buoyancy to be some function of potential temperature, salinity, and geopotential height (e.g. Young 2010), i.e.

\[ b = \tilde{b}(\Theta, S, z), \]  
(5)

where \( \Theta \) is potential temperature\(^5\), \( S \) is salinity, and \( z \) is depth relative to some reference geopotential height level.

Temperature and salinity evolve according to

\[ \frac{D\Theta}{Dt} = \kappa_T \nabla^2 \Theta \]  
(6)

\[ \frac{DS}{Dt} = \kappa_S \nabla^2 S \]  
(7)

with the boundary conditions

\[ -\kappa_T \partial_z \Theta(z_{bot}) = \frac{1}{\rho c_p} Q_{bot}, \quad -\kappa_T \partial_z \Theta(z_{top}) = \frac{1}{\rho c_p} Q_{top}, \]  
(8)

\[ -\kappa_S \partial_z S(z_{bot}) = S_{bot}, \quad -\kappa_S \partial_z S(z_{top}) = S_{top}, \]  
(9)

where \( \kappa_T \) and \( \kappa_S \) are the molecular diffusivities for heat and salt, respectively, and \( Q_{bot/top} \) and \( S_{bot/top} \) denote the heat and salt fluxes through the bottom and top boundaries, respectively. For simplicity we here do not include significant heat sources in the interior, although such sources could be added.

If the equation of state is nonlinear, the conservation equations for \( \Theta \) and \( S \) cannot readily be translated into a conservation equation for buoyancy. The most general way to account for nonlinearities in the equation of state, which is discussed in Appendix A is to introduce the dynamic enthalpy, which is related to the potential energy. A significantly simpler (and more intuitive) result can be obtained if we assume that horizontal variations in the thermal and haline expansion coefficients are small, such that we can approximate

\[ b \approx \langle b \rangle + \langle \alpha \rangle (\Theta - \langle \Theta \rangle) + \langle \beta \rangle (S - \langle S \rangle) \]  
(10)

where \( \alpha \equiv g^{-1} \partial_b b \) and \( \beta \equiv g^{-1} \partial_S b \) are the thermal and haline expansion coefficients and \( \langle \cdot \rangle \) denotes a horizontal average at any given depth. We can then directly relate the total upward buoyancy flux (which represents the source of kinetic energy in a buoyancy-driven flow) to the upward heat and salt fluxes:

\[ B \equiv \int wbdV \approx \int \left[ g\langle \alpha \rangle \int w\Theta dA \right] dz + \int \left[ g\langle \beta \rangle \int wS dA \right] dz \equiv B^\Theta + B^S \]  
(11)

where we used that \( \int \int wdA = 0 \) due to volume conservation, and we defined \( B^\Theta \) and \( B^S \) as the total net upward buoyancy fluxes at each level associated with heat and salt fluxes, respectively.

The total upward heat and salt fluxes can be related to the sources and sinks of heat and salt via their conservation equations (see also sketches in Figs. 1 and 2). Specifically, the area-integrated vertical advective fluxes of heat and salt are constrained by the conservation of heat and salt above or below a respective level. In a statistically steady state, where the mean rate of change of \( \Theta \) and \( S \) at any given depth is approximately zero, Eq. (6) can be integrated

\(^5\) Notice that temperature is conserved following adiabatic motion in a Boussinesq fluid and hence temperature and potential temperature are formally identical. However, \( \theta \) should be interpreted as potential temperature when comparing to observed fluids.
Figure 1. Sketch of the vertical heat flux through the ocean, which provides the energy source for thermally driven flows. In this manuscript we assume that the ocean and ice sheet are in equilibrium, and that heat sources in the interior of the ocean are negligible, such that the total heat flux from the core (\( \int Q_{\text{bot}} dA \)) is equal to the net heat flux from the ocean to the ice sheet (\( \int Q_{\text{top}} dA \)). Notice that the vertical heat flux in the ocean (\( \mathcal{F}^Q(z) \)) can also be constrained from the potentially more observable heat flux emanating from the planetary body’s surface (\( \int Q_{\text{surf}} dA \)), which is given by the sum of the core heating (\( \int Q_{\text{bot}} dA \)) and the tidal energy dissipation in the ice layer (\( \int Q_{\text{ice}} dA \)). Specifically, the maximum vertical heat flux in the ocean is limited to \( \mathcal{F}^Q \leq \int Q_{\text{surf}} dA \).

over the volume above or below some depth \( z \) to yield:

\[
\int (w\Theta - \kappa_T \partial_z \Theta) dA = \frac{1}{\rho c_p} \left[ \int Q_{\text{bot}} \mathcal{H}(z - z_{\text{bot}}) dA - \int Q_{\text{top}} \mathcal{H}(z - z_{\text{top}}) dA \right] \\
= \frac{1}{\rho c_p} \left[ \int Q_{\text{top}} \mathcal{H}(z_{\text{top}} - z) dA - \int Q_{\text{bot}} \mathcal{H}(z_{\text{bot}} - z) dA \right] \\
\equiv \frac{1}{\rho c_p} \mathcal{F}^Q 
\]  

(12)

where \( \mathcal{H} \) is the Heaviside function and we defined \( \mathcal{F}^Q \) as the the vertical heat and salt flux across any depth \( z \) needed to balance the net heat and salt fluxes through the boundaries above and below the respective level. Similarly, Eq. (7) can be integrated to obtain

\[
\int (wS - \kappa_S \partial_z S) dA = \int S_{\text{bot}} \mathcal{H}(z - z_{\text{bot}}) dA - \int S_{\text{top}} \mathcal{H}(z - z_{\text{top}}) dA \\
= \int S_{\text{top}} \mathcal{H}(z_{\text{top}} - z) dA - \int S_{\text{bot}} \mathcal{H}(z_{\text{bot}} - z) dA \\
\equiv \mathcal{F}^S
\]  

(13)

Notice that interior sources of heat or salt could be included by modifying the definition of \( \mathcal{F}^Q \) and \( \mathcal{F}^S \), which generally need to balance any net sources or sinks above or below the respective level.

These definitions allow us to express the KE generation associated with heat and salt flux forcing as:

\[
\mathcal{B}^\Theta \approx \int g(\alpha) \frac{\mathcal{F}^Q}{\rho c_p} dz + \int g\alpha \kappa_T \partial_z \theta dV, 
\]  

(14)

and

\[
\mathcal{B}^S = \int g(\beta) \mathcal{F}^S dz + \int g\beta \kappa_S \partial_z S dV, 
\]  

(15)

The first term on the RHS of equations (14) and (15) can be interpreted as the potential energy source associated with heat and salt forcing at the lower and upper boundaries, while the second term is the potential energy source or sink.
Figure 2. Sketch of the vertical salt flux through the ocean. If melting occurs under thinner ice and freezing occurs under thicker ice, as sketched here, the salt flux required in the ocean to balance brine rejection from freezing and melting ($\mathcal{F}^S$) is upward, which implies a downward buoyancy flux that converts kinetic to potential energy. In this scenario salt fluxes therefore do not energetically drive a circulation. Instead, kinetic energy from an alternative source is required to maintain a circulation that can flux salt upwards.

due to diffusion (which provides a source of mechanical energy whenever the stratification is statically stable, and a sink when it is unstable).

If vertical variations in $g\alpha$ and $g\beta$ are also small, thermal forcing at the boundaries provides a source of energy that can drive a circulation if, and only if,

$$\alpha \int \mathcal{F}^Q dz > 0,$$

i.e. for $\alpha > 0$ heating, on average, needs to occur at a greater depth than cooling, while for $\alpha < 0$ cooling needs to occur at greater depth than heating. Similarly salt fluxes can drive a circulation only if

$$\beta \int \mathcal{F}^S dV > 0.$$

Since generally $\beta < 0$, this requires that salt needs to be removed at greater depth and added at shallower depth. Large vertical variations in $g\alpha$ and $g\beta$ can be accounted for using the generalized relation in Eqs. (14) and (15), which allow us to compute the mechanical energy input associated with any given heat and salt flux boundary conditions more generally. When horizontal variations in the thermal and haline expansion coefficient are non-negligible, the amount of energy that can be converted to kinetic energy depends on the specifics of the circulation, as elaborated in Appendix A.

2.2. Energy dissipation and flow properties

In equilibrium the total sources and sinks of mechanical energy have to be in balance. The energy sources thereby provide a constraint on the energy dissipation, which in turn provides some constraint on the flow. Relating the energy dissipation rate to the kinetic energy of the flow itself is not straightforward, as it depends on the characteristics of the flow field, but we can make some progress by considering specific flow regimes and estimating the range of parameters and scales over which the flow regimes are expected to hold. In the following we first discuss the relationship between the kinetic energy and the dissipation rate for a turbulent flow that is largely unaffected by rotation, as well as the conditions under which the assumption that rotation is negligible breaks down. We then derive an alternative relationship between the kinetic energy and the dissipation rate in the opposite limit of so-called geostrophic turbulence, where rotation is a leading-order effect and flows are largely geostrophically balanced (i.e. the leading order momentum balance is between the pressure gradient and Coriolis acceleration).

2.2.1. Isotrophic turbulence and the role of rotation

We first assume a fully turbulent flow unconstrained by the influence of rotation (i.e. high Reynolds and Rossby numbers) and we assume that, in this limit, the dissipation rate is independent of the value of molecular viscosity$^6$. Specifically, we start by considering Kolmogorov’s theory for the kinetic energy spectrum in the inertial cascade of isotropic turbulence, which suggests that

$$E(k) = Kc^{2/3}k^{-5/3}$$

$^6$ Notice that it is unclear in how far this holds true in the vicinity of boundaries. We here assume that most dissipation occurs in the interior in this fully turbulent regime, but we will return to the issue of boundary-layer dissipation below.
where $E(k)$ is the kinetic energy spectrum as a function of wavenumber, $k$, $\epsilon$ is the turbulent spectral KE flux, which in turn is equal to the dissipation rate, and $K \approx 1.5$ is the Kolmogorov constant (e.g. Vallis 2006). Integrating over the energy inertial range yields

$$E_t \approx 2k_0^{-2/3}\epsilon^{2/3}$$

(19)

where $k_0$ is the "injection scale" (where the KE spectrum flattens out), and $E_t$ is the total turbulent kinetic energy in the inertial range.

We can use Eq. (19) to estimate the kinetic energy of turbulent flows up to the largest scales of an isotropic turbulent energy inertial range. Isotropy may be broken by the geometry (e.g. the vicinity to a boundary), or by the effects of stratification or rotation. We will here consider in particular the importance of rotation, which can break isotropy and fundamentally change the nature of the turbulent flow throughout the watercolumn.

The role of rotation can be characterized by the Rossby number, which, using Eq. (19) and $L \equiv 2\pi/k_0$, can be estimated as

$$Ro_t = \sqrt{\frac{\epsilon}{\Omega L}} \approx \frac{\epsilon^{1/3}}{\Omega L^{2/3}}.$$  

(20)

Equation (20) suggests a maximum length scale for turbulent flows unaffected by rotation (c.f. Fernando et al. 1991; Jones & Marshall 1993; Maxworthy & Narimousa 1994; Bire et al. 2022):

$$L_{rot} \approx \frac{\epsilon^{1/2}}{\Omega^{3/2}}.$$  

(21)

Once rotation becomes important, Maxworthy & Narimousa (1994) and others find that the convective Rossby number of a rotating plume scales as

$$Ro_{rc} = \frac{\epsilon^{1/2}}{\Omega_{1/2}H} = Ro_t^{3/2},$$  

(22)

where $H$ is the depth of the convecting fluid (and we used Eq. (20) with $L = H$ in the second equality). Eq. (22) is likely to be a better predictor of the convective flow Rossby number when ocean-depth convection is strongly affected by rotation (i.e. $L_{rot} < H$). However, both give the same prediction for the length scale at which $Ro_t \approx 1$ and hence rotation becomes important, which indeed also follows directly from dimensional analysis if we postulate that this scale shall depend only on $\epsilon$ and $\Omega$.

2.2.2. Boundary layer dissipation in geostrophic dynamics

When rotation becomes of dominant importance it is likely that much of the energy becomes trapped in geostrophically balanced vorticies and large-scale mean flows, which result from the up-scale kinetic energy cascade associated with quasi-balanced turbulent motions (e.g. Vallis 2006). The lack of a forward energy cascade means that dissipation is likely to be limited mostly to turbulent boundary layers near the seafloor and the ice-ocean interface.

We can estimate the energy dissipation per unit area in a turbulent boundary layer as (e.g. Jansen 2016, and references therein):

$$\int \epsilon dz = c_D|U_g|^3$$

(23)

where the integral on the L.H.S. is over the depth of the turbulent boundary layer, $c_D$ is the turbulent drag coefficient, and $U_g$ is the characteristic near-surface geostrophic velocity\(^7\). The value of $c_D$ depends on the surface roughness, with $c_D \approx 0.0025$ an empirical average value that is commonly used for the drag coefficient for Earth’s seafloor (e.g. Egbert et al. 2004; Sen et al. 2008). The skin drag coefficient under smooth ice can be estimated to be around $c_D \approx 0.002$, although rough morphology can significantly increase this value (e.g. Brenner et al. 2021). Lacking information about the ice roughness, and noting the order-of-magnitude nature of our estimates, we will here use $c_D \approx 0.0025$ at both the sea floor and under the ice sheet at the top. In addition to surface roughness, the drag coefficient may depend on other flow properties, but experience from Ocean and Atmospheric modelling has shown that drag coefficients of this order produce reasonable results for a wide range of flows (e.g. Smagorinsky et al. 1965; Egbert et al. 2004; Sen et al.

\(^7\) Notice that Eq. (23) also follows from the Kolmogorov scaling in Eq. (19) if we take $k_0$ to scale as the inverse of the bottom boundary layer depth, in which case $c_D \approx E_t^{1/2}/|U|^3$, i.e. it is related to the ratio of the boundary layer turbulent kinetic energy to the kinetic energy of the large-scale geostrophic flow.
2008; Chen et al. 2018; Adcroft et al. 2019), which instills confidence that a similar coefficient can be used to obtain useful order-of-magnitude estimates for icy moon oceans.

Averaging the energy dissipation in the top and bottom boundary layers over the depth of the whole water column gives an average energy dissipation rate per unit volume

\[ \epsilon \approx \frac{2^{5/2} c_D}{H} E_g^{3/2}, \]  

(24)

where \( H \) is the depth of the ocean and \( E_g = 1/2 |U_g|^2 \) is the characteristic KE of the geostrophic flow. We can solve Eq. (24) to get a crude estimate for the characteristic KE in a flow that is dominated by large-scale balanced dynamics, where KE dissipation occurs primarily in turbulent boundary layers:

\[ E_g \approx \frac{H^{2/3}}{2^{5/3} c_D^{2/3}} \epsilon^{2/3}. \]  

(25)

In general, balanced dynamics may co-exist with intermediate and high Rossby number convective turbulence and/or tidal waves, in which case interior energy dissipation may be important and boundary layer dissipation may be enhanced due to the tidal flows. In this case, Eq. (25) is expected to overestimate the KE. It is also important to note that numerical simulations of planetary circulation typically use very high artificial viscosities for numerical stability, which can be a large, albeit probably unphysical, source of dissipation for balanced KE (e.g. Jansen 2016).

3. SCALING LAWS FOR THE KINETIC ENERGY OF THERMALLY-DRIVEN FLOWS ON SNOWBALL EARTH, ENCELADUS AND EUROPA

We can derive estimates for the kinetic energy of buoyancy-driven flows by assuming a statistically steady state where the source of KE equals the sink. If the primary source of energy is given by thermal forcing, Eq. (4) together with Eq. (11) and (14) provide a scaling relation for the mean kinetic energy dissipation per unit volume:

\[ \epsilon \equiv \frac{1}{V} \int \mathbf{v} \cdot \mathbf{D} \sim \frac{R^3}{V} \approx \frac{g \alpha F Q H}{\rho_c p} \approx \frac{Q \alpha g}{\rho_c p}, \]  

(26)

where \( Q \) is the average heat flux through the sea floor per unit area, and we for now neglect variations in the thermal expansion coefficient, \( \alpha \), as well as radial variations in the gravity and the surface area throughout the depth of the ocean. (The latter assumption leads to \( O(10\%) \) errors for Europa and Enceladus, which is not of concern here. The former may lead to larger errors if the ocean is relatively fresh, and we will return to this issue later.) We also ignored the energetic effect of molecular diffusion (the second term in Eq. (14)), which is likely to be small\(^8\).

Eq. (26) provides a key constraint for the energetics of thermally-driven flows. It predicts that the energy dissipation rate per unit volume increases linearly with the bottom buoyancy flux, which in turn is given by the heat flux multiplied by the thermal expansivity and the gravity. The results are shown in Fig. 3 for the parameter range occupied by the icy moons and Snowball Earth. In the following, we use this result together with the results from sections 2.1 and 2.2 to obtain order-of-magnitude estimates for thermally-driven ocean flows on Snowball Earth, Europa, and Enceladus.

To estimate the kinetic energy dissipation rate for thermally-driven flows in the ocean’s of Snowball Earth, Europa, and Enceladus, we assume an average heating rate at the ocean floor of \( Q \approx 0.1 \text{ W/m}^2 \) for Earth (Jansen 2016) and \( Q \approx 0.03 \text{ W/m}^2 \) for Europa and Enceladus (Ruiz 2005; Hemingway et al. 2018; Choblet et al. 2017), \( g \approx 10 \text{ m/s}^2 \) for Earth, \( g \approx 1 \text{ m/s}^2 \) for Europa and \( g \approx 0.1 \text{ m/s}^2 \) for Enceladus. The thermal expansion coefficient is uncertain, in particular for Enceladus, as it depends on pressure and ocean salinity. Indeed for salinities below about 20 g/kg the thermal expansion coefficient at the ice-ocean interface on Enceladus is expected to be negative (Zeng & Jansen 2021; Kang et al. 2021). For now we will assume \( \alpha \approx 2 \times 10^{-4} \text{ K}^{-1} \) for Earth and Europa and \( \alpha \approx 1 \times 10^{-4} \text{ K}^{-1} \) for Enceladus, although we note that this is likely to be an upper-end estimate, and we will return to a low salinity-scenario for Enceladus below. (Assumed parameters are also summarized in table 1.) Using further that \( \rho_c p \approx 4 \times 10^6 \text{ J m}^{-3} \text{ K}^{-1} \) we find

\(^8\) Notice that his formally amounts to assuming a large Nusselt number - i.e. we assume that the advective heat flux is large compared to the diffusive heat flux.
Figure 3. Energy dissipation rate per unit volume for thermally-driven flows predicted by the scaling in Eq. (26) as a function of the bottom heat flux \(Q\) times \(\alpha/(\rho c_p)\) and the gravitational acceleration \(g\). The white stars mark the estimates for Snowball Earth (Ea), Europa (Eu) and Enceladus (En), assuming the parameters given in Table 1. The colorbar is logarithmic with the contour interval a factor of \(\sqrt{10}\).

\[
\epsilon \approx 5 \times 10^{-11} \text{m}^2\text{s}^{-3} \quad \text{for Snowball Earth} \tag{27}
\]
\[
\epsilon \approx 2 \times 10^{-12} \text{m}^2\text{s}^{-3} \quad \text{for Europa} \tag{28}
\]
\[
\epsilon \approx 8 \times 10^{-14} \text{m}^2\text{s}^{-3} \quad \text{for Enceladus}. \tag{29}
\]

The energy dissipation rate is thus largest for Snowball Earth and smallest for Enceladus, with differences mostly driven by the differences in the gravity and further amplified by differences in the estimated vertical heat flux and thermal expansivity. For comparison, the kinetic energy dissipation rate in Earth’s deep ocean today is around 2 TW (Wunsch & Ferrari 2004), which divided by the mass of the ocean amounts to about \(1.5 \times 10^{-9} \text{m}^2\text{s}^{-3}\). This dissipation is balanced by energy input primarily from winds and tides, which provide significantly more energy to the present-day ocean circulation than geothermal heating.

3.1. The role of rotation

Given the energy dissipation rate, \(\epsilon\), and the rotation rate of the planetary body, \(\Omega\), we can use Eq. (21) to estimate a critical scale \(L_{\text{rot}}\) above which we may expect turbulent motions to become strongly affected by rotation (Fig. 4). Using the energy dissipation rates estimated above and \(\Omega \approx 8 \times 10^{-5} \text{s}^{-1}\) for Snowball Earth, \(5 \times 10^{-5} \text{s}^{-1}\) for Enceladus, and \(2 \times 10^{-5} \text{s}^{-1}\) for Europa, we find the scale at which convection is expected to become strongly affected by rotation from Eq. (21) to be \(L_{\text{rot}} \approx 10 \text{m}\) for Snowball Earth and Europa and \(L_{\text{rot}} \approx 1 \text{m}\) for Enceladus. Uncertainties in many of our assumed parameters are substantial (and not easily quantified), such that these results should only be viewed as indicative of the expected order of magnitude. Importantly, however, it is clear that \(L_{\text{rot}}\) is much smaller than the ocean depth in all three cases and we therefore expect deep convection and any other potential large-scale flow to be strongly affected by rotation. It is also worth noting that this length scale is significantly smaller than the smallest length scales that can be resolved in global-scale numerical simulations of these oceans, indicating that “large-eddy-simulations” that can explicitly resolve part of the isotropic turbulent inertial range are not feasible with realistic parameters (c.f. Bire et al. (2022)).

COMPARISON TO EXISTING SIMULATION RESULTS

The dominant importance of rotation in the oceans of Snowball Earth, Europa, and Enceladus is qualitatively consistent with the Snowball Earth simulations of Ashkenazy et al. (2013) and Jansen (2016), the Europa simulations of Ashkenazy & Tziperman (2021) and the Enceladus simulations of Kang et al. (2020) and Zeng & Jansen (2021). However, the result appears to be at odds with simulation results for Europa’s ocean by Soderlund et al. (2014), which
show strong turbulent convection that is only relatively weakly affected by rotation. The apparent contradiction can readily be understood by noting that the vertical heat flux in the simulations of Soderlund et al. (2014) (which use fixed temperature boundary conditions) amounts to about $10^5 \text{Wm}^{-2}$—in stark contrast to the heat flux of $0.03 \text{Wm}^{-2}$ assumed here. Increasing the vertical heat flux on Europa to $10^5 \text{Wm}^{-2}$ would yield $L_{\text{rot}} \approx 25 \text{km}$, which in turn would make significant turbulent convection that is only relatively weakly affected by rotation plausible. Indeed, using Eq. (20) with $L \approx H \approx 100 \text{km}$ (appropriate for Europa’s ocean) and an energy input consistent with a heat flux of $10^5 \text{Wm}^{-2}$ (which gives $\epsilon \approx 5 \times 10^{-6} \text{m}^2\text{s}^{-3}$) we can estimate a Rossby number for full-depth turbulent convection of $R_o \approx 0.4$, which is broadly consistent with the Rossby numbers of convective motions found in the simulations of Soderlund et al. (2014). However, using the more realistic estimate of $\epsilon \approx 2 \times 10^{-12} \text{m}^2\text{s}^{-3}$, Eq. (20) gives $R_o \approx 3 \times 10^{-3}$, again confirming that rotation is expected to be very important. Using the Rossby number for rotating convection in Eq. (22) further reduces our estimated convective Rossby number to $R_{o,c} \approx 1 \times 10^{-4}$.

### 3.2. Quasi-balanced flows

At scales much larger than $L_{\text{rot}}$, we expect the motion to be strongly affected by rotation, leading to quasi-balanced flows that do not undergo a forward energy cascade. Energy dissipation then may be limited mostly to turbulent boundary layers. In this case, Eq (25) may provide a useful estimate for the total KE and thus typical flow speeds as a function of the energy dissipation rate, $\epsilon$, and the depth of the ocean, $H$ (Fig. 5). Assuming that $H = 3 \text{km}$, $100 \text{km}$, and $30 \text{km}$ for Snowball Earth, Europa and Enceladus, respectively, we find

\begin{align*}
E_g &\approx 5 \times 10^{-4} \text{m}^2\text{s}^{-2} \quad \text{for Snowball Earth} \\
E_g &\approx 5 \times 10^{-4} \text{m}^2\text{s}^{-2} \quad \text{for Europa} \\
E_g &\approx 3 \times 10^{-5} \text{m}^2\text{s}^{-2} \quad \text{for Enceladus.}
\end{align*}

These results suggest potential balanced flow velocities of up to a few cm/s for Snowball Earth and Europa and a few mm/s for Enceladus. These estimates should likely be viewed as an upper bound as we here assumed that all energy input goes into balanced motions and we ignored any additional potential routes to dissipation (e.g. via energy transfer to internal waves) that would reduce the expected kinetic energy level.

### COMPARISON TO EXISTING SIMULATION RESULTS

Flow velocities of a couple of cm/s for Snowball Earth are broadly consistent with the GCM simulations of Ashkenazy et al. (2013) and Jansen (2016), both of which reveal flow patterns dominated by geostrophic turbulence and mean
Figure 5. Estimated geostrophic flow speeds \( U_g = \sqrt{2E_g} \) as a function of the energy dissipation rate \( \epsilon \) and ocean depth \( H \) as predicted by the scaling in Eq. (25) assuming that most dissipation happens in turbulent boundary layers near the sea floor and ice-ocean interface with a drag coefficient \( C_d = 0.025 \). The white stars mark the estimates for Snowball Earth (Ea), Europa (Eu) and Enceladus (En), assuming the parameters given in table 1. The colorbar is logarithmic with the contour interval a factor of \( 10^{1/4} \).

flows. A flow velocity of a couple of cm/s for Europa is also broadly consistent with the simulation results of Ashkenazy & Tziperman (2021).

The Enceladus Ocean simulations with sea-floor heating of Kang et al. (2020) and the high-salinity simulations of Zeng & Jansen (2021) have flow velocities on the order of 0.1 mm/s, which is at least an order of magnitude weaker than our estimate for the maximum KE. Some discrepancy may be expected due to the parameterized boundary-layer drag. Instead of the quadratic boundary layer drag assumed in Eq. (24), the simulations of Kang et al. (2020) and Zeng & Jansen (2021) apply a relatively strong linear drag parameterization. With a linear drag parameterization the average dissipation rate due to drag at the boundaries is

\[
\epsilon \approx 2n_b \gamma HE_g \tag{33}
\]

where \( \gamma \) is the linear drag coefficient, and \( n_b = 1 \) in the setup of Zeng & Jansen (2021) where frictional drag with \( \gamma = 1 \times 10^{-4} \) m/s is applied only at the lower boundary, while \( n_b = 2 \) in the setup of Kang et al. (2020) where frictional drag with \( \gamma = 2 \times 10^{-3} \) m/s is applied at both the upper and lower boundaries. Replacing Eq. (24) by Eq. (33) in our estimate of the maximum geostrophic kinetic energy for Enceladus yields \( E_g \approx 10^{-5} \) m²s⁻² with the frictional drag of Zeng & Jansen (2021) and \( E_g \approx 3 \times 10^{-7} \) m²s⁻² with the frictional drag of Kang et al. (2020). This amounts to characteristic velocities of around 4 mm/s and somewhat less than 1 mm/s respectively, which is closer to but still significantly larger than the velocities observed in the simulations. Strong dissipation of kinetic energy associated with the (numerically necessary) large interior viscosity, as well as reduction of the conversion of potential to kinetic energy due the parameterized vertical eddy diffusivity (i.e. the second term on the R.H.S. of Eq. 14), are likely to contribute to reduced flow speeds in the simulations of Kang et al. (2020) and Zeng & Jansen (2021). Notice also that the choice of a relatively strong linear drag is practical in numerical simulations (as it allows the model to equilibrate faster and improves numerical stability), but is not justified on a physical basis. We therefore conclude that the simulations of Kang et al. (2020) and Zeng & Jansen (2021) likely underestimate the strength of the buoyancy-driven circulation.

3.3. Thermally-driven flows in a low-salinity, low-pressure ocean

If salinity and pressure are relatively low, the thermal expansion coefficient near the freezing point is negative. This scenario was first suggested for Europa by Melosh et al. (2004), although due to the modest pressures that are required for a negative thermal expansion coefficient, it appears most likely to be relevant for Enceladus, and was considered in the low-salinity Enceladus ocean simulations of Zeng & Jansen (2021) and Kang et al. (2021). To estimate the energetics of such an ocean we assume a stably stratified layer below the ice sheet and above some depth \( z_{\text{strat}} \), as
Of course, the energy input and hence typical flow speeds would be even smaller than estimated above. For illustrative purposes, we assume that the sea floor and ice sheet are flat (i.e. both boundaries follow a geopotential height surface), such that $F^\Omega$ is vertically constant and equal to the total bottom heat flux. Assuming again that horizontal variations in the thermal expansion coefficient are small, Eq. (14) gives the KE generation as

$$B^\Theta = \int g\langle \alpha \rangle \left[ \frac{F^\Omega}{\rho c_p} + A\kappa T \partial_z \langle \Theta \rangle \right] dz$$

In the stratified layer, $\langle \alpha \rangle < 0$, such that the first term in the integral in (34) amounts to an energy sink. One plausible solution is then that the vertical heat flux through the stratified layer is balanced by molecular diffusion (i.e. the second term in the integral in 34), which amounts to a conversion from internal to potential energy. In this case the stratified layer would only be a few tens of meters thick (Zeng & Jansen 2021), and most of the ocean would be convective with $\Theta > \Theta_c$ and hence $\alpha > 0$. The dynamics in the convective layer would follow scaling laws analogous to those presented above, although the very small thermal expansion coefficient at temperatures only marginally warmer than $\Theta_c$ implies that the energy input and hence typical flow speeds would be even smaller than estimated above.

It is theoretically possible that kinetic energy is transferred from the convective layer into the stratified layer, which could allow for a turbulent circulation throughout the depth of the ocean as long as $\int g\langle \alpha \rangle dz > 0$ (i.e. the net mechanical energy input is positive), which in turn is possible if the stratified layer is sufficiently shallow. More plausibly, tides or librations may provide a source of kinetic energy that can balance the negative $B^\Theta$ associated with turbulent vertical heat transport in the stratified layer.

The simulations of Zeng & Jansen (2021) use a relatively large “eddy diffusivity”, which is much larger than the molecular diffusivity. The vertical heat flux through the stratified layer is balanced by this “eddy diffusion”, such that from the perspective of the model’s energetics, the two terms in the integral in (34) balance in the stratified layer. The simulations of Kang et al. (2021) use an even larger eddy diffusivity, such that the entire depth of the ocean becomes stratified with the upward heat flux and associated downward buoyancy flux accomplished by “eddy diffusion”. However, in both studies the model’s diffusivities, which are much larger than the molecular value, need to be interpreted as representing turbulent mixing. If these turbulent mixing rates are real, the associated vertical heat flux represents turbulent advection, which amounts to a negative $B^\Theta$ - that is a conversion of turbulent kinetic energy to large-scale potential energy. An additional implied energy source is therefore needed to generate the turbulent kinetic energy. A possible source are tides and/or librations, to which we will return below.

4. SALINITY FORCING

A number of studies have recently argued for the importance of salt forcing in driving a circulation on Europa and Enceladus (Zhu et al. 2017; Kang et al. 2020; Ashkenazy & Tziperman 2021; Lobo et al. 2021). Since $\beta$, as defined in Eq. (10), is always negative, salinity forcing provides a source of mechanical energy if, and only if, salt is added (e.g. by freezing) at a shallower depth and removed (or freshwater is added by melting) at a greater depth (Eqs. 13 and 15). At steady state, however, freezing and melting need to balance the convergence of ice flow, which tends to flow from regions of thick ice to regions of thin ice. We therefore expect melting to occur where the ice is thin, and hence the ice-ocean interface is shallow, while freezing occurs where the ice is thick and hence the ice-ocean interface is deep (c.f. Fig. 2). In this case the required upward salt flux reduces the kinetic energy of the ocean and hence cannot, energetically speaking, drive a circulation (although it can still affect the circulation by shaping the horizontal buoyancy gradients).
As for virtually all numerical simulations, the models of Zhu et al. (2017), Kang et al. (2020), Ashkenazy & Tziperman (2021) and Lobo et al. (2021) employ vertical diffusivities that are orders of magnitude larger than the molecular value. In this case vertical diffusion acting against a statically stable stratification can drive a substantial circulation, via the second term in Eq. (15). Indeed, salinity forcing will often tend to increase the stratification at least in some parts of the ocean, which will lead to an increase in the energy input by the prescribed vertical diffusion. However, as in the low-salinity Enceladus simulations of Zeng & Jansen (2021) discussed above, the large vertical diffusivities need to be interpreted as representing mixing by unresolved small-scale turbulence. In reality, this turbulent mixing requires a source of kinetic energy, and the mechanistic basis for this energy source must be established in order for the results of the simulations to be sound. In Earth’s ocean today the source of energy for small-scale turbulent kinetic energy ultimately comes from winds and tides (e.g. Wunsch & Ferrari 2004). Surface wind stress is absent in globally ice covered oceans, which leaves tides as the most obvious source of turbulent kinetic energy in an ocean where buoyancy forcing is primarily due to salt fluxes at the ice-ocean interface.

5. TIDES AND TURBULENT MIXING

5.1. Theory

Tidal forcing generates kinetic energy in the ocean (via the second term on the LHS of Eq. 4). The tidal perturbation potential is typically approximately vertically constant throughout the depth of the ocean, thus generating barotropic (i.e. vertically constant) tides. In Earth’s ocean these barotropic tidal waves are largely linear until they encounter shallow shelves, where the tidal amplitude increases and most of the tidal energy dissipation is assumed to occur (Wunsch & Ferrari 2004). The icy moons do not have shelf seas similar to Earth’s ocean, although tidal dissipation may nevertheless be significantly enhanced in specific regions (e.g. Hay & Matsuyama 2019). Linear tides in the deep ocean do not contribute significantly to the transport of heat, salt and other properties in the ocean, such that global models of Earth’s ocean circulation can reproduce realistic large-scale circulations and tracer distributions without explicitly accounting for tides. However, it is possible that tidal waves on some icy moons become sufficiently non-linear to lead to substantial rectified mean flows (e.g. Huthnance 1981; Brink 2011). Since the current manuscript is focussed on buoyancy-driven circulation, we will not further consider the possible role of rectified tidal flows here, but note that this remains a potentially important topic for further research.

Despite weak direct interactions with the large-scale flow, ocean tides can have an important effect on the large-scale circulation in the presence of a statically stable stratification (e.g. driven by freshwater fluxes at the ice-ocean interface). Barotropic tides interacting with a rough sea-floor topography in the presence of a statically stable stratification can transfer their energy into baroclinic tides, i.e. internal waves that propagate both horizontally and vertically and can be associated with substantial vertical shears. These shears in turn can lead to instabilities and eventually the generation of turbulence within the water column. This pathway is believed to be one of the main drivers of 3D turbulence in Earth’s deep ocean (Vic et al. 2019). Although most of the the turbulent kinetic energy is dissipated into heat, some fraction, usually denoted by the "mixing efficiency", $\Gamma$, is converted to potential energy via vertical mixing of the stratified water column, i.e.:

$$B_t = \int \langle \omega^{t'} \rangle dbdV = - \int \Gamma \epsilon_t dV$$

(35)

where $-B_t$ is the potential energy generation due to tidally-driven turbulent mixing, $\langle \omega^{t'} \rangle_t$ is the vertical buoyancy flux associated with tidally-driven turbulence, $\epsilon_t$ is the tidally generated turbulent kinetic energy dissipation rate per unit mass, and $\Gamma \lesssim 0.2$ is the mixing efficiency (e.g. Peltier & Caulfield 2003).

The potential energy generated by turbulent mixing in a stratified ocean can then drive a large-scale circulation that converts the potential energy back to kinetic energy via a positive conversion $\bar{\omega} \bar{b}$ (where the overbars denote the large-scale circulation). If turbulent mixing is the dominant source of potential energy (i.e. when buoyancy gain and buoyancy loss from external forcing occur at approximately the same depth) the conversion $\bar{\omega} \bar{b}$ is, on average, approximately equal and opposite to the downward buoyancy flux associated with tidally-driven turbulence:

$$\mathcal{B} = \int wbdV = \int \bar{\omega} \bar{b} dV + \int \langle \omega^{t'} \rangle_t dbdV \equiv \mathcal{B} + B_t \approx 0.$$  

(36)

where $\mathcal{B}$ is the kinetic energy generated (and ultimately dissipated) by the mean flow. The energy cycle can then be summarized as follows. Tides generate turbulent kinetic energy, a fraction of which is converted into large-scale
potential energy via downward turbulent buoyancy flux (with the remainder being dissipated into heat). This potential energy can then be converted back to kinetic energy (and ultimately be dissipated) by the large-scale circulation. This mechanism requires a statically stable stratification such that turbulent mixing transports buoyancy downwards. In the absence of a statically stable stratification, we still expect tidally-driven turbulence to contribute to the mixing of properties, but we do not expect it to contribute to driving a large-scale circulation.

In addition to tides, librations and injection of fluid through fissures in the ice or solid core may provide sources of kinetic energy. Liberations are expected to play a similar role to tides, and as for tides, their effect on the large-scale circulation is expected to depend on their ability to generate turbulence in the ocean interior and on the presence of a statically stable stratification. Injection of water trough fissures in the boundaries (which is not formally included in the global kinetic energy-budget in Eq. (4), where we assumed no-normal-flow boundary conditions) is likely to drive mechanical turbulence primarily in the vicinity of the boundaries, but strong jets emanating from the ice shell, as proposed by Kite & Rubin (2016) may play an important role in the dynamics of the upper ocean. A full investigation of the effects of such jets is beyond the scope of this study, but remains as an interesting subject for future research.

5.2. Application to Snowball Earth, Europa, and Enceladus

We begin by estimating the potential role of tidally-driven turbulence in the Snowball Earth ocean. Although little is known specifically about tidal energy dissipation during Snowball Earth periods, average tidal dissipation since the Precambrian was likely somewhat smaller but of similar order of magnitude as the present-day value (Williams 2000; Green et al. 2017). The direct effect of the Snowball Earth ice sheet on tides was likely small (Wunsch 2016). For scaling purposes we therefore here assume a tidal energy input on the order of $10^{12}$ W, which is comparable to the present day value of around $3.5 \times 10^{12}$ W (Wunsch & Ferrari 2004). Assuming a mean ocean depth of about 2 km, an ocean area of $3.5 \times 10^{14}$ m$^2$ and density $\rho_0 \approx 1000$ kg m$^{-3}$, the average tidal energy dissipation per unit mass is around $\epsilon_t \approx 10^{-9} \text{m}^2 \text{s}^{-3}$, which is a little over an order of magnitude larger than the estimated energy input due to thermal forcing (cf. Table 1). If a significant fraction of the dissipated tidal energy is dissipated in the stratified ocean interior and is relatively efficiently converted to potential energy (i.e. $\Gamma \gtrsim 10\%$) it thus may play a significant role in driving a large-scale ocean circulation. Specifically, one may envision a scenario where freezing and melting at the ice-ocean interface combined with tidally-driven turbulent mixing, which pushes the light fresh water into the ocean interior, can drive a significantly stronger large-scale ocean circulation then the thermal forcing alone. This scenario appears to be relevant for the interpretation of simulation results by Ashkenazy et al. (2013) and Jansen (2016), although the turbulent vertical mixing in these simulations is parameterized with prescribed vertical diffusivities (i.e. $\kappa = -\kappa \partial_z \vec{b}$) and is thus not constrained by tidal energy dissipation. The present analysis nevertheless suggests that the vertical mixing may be justifiable on energetic grounds by the expected tidally-driven turbulence, although it would require that a significant fraction of the tidal energy is dissipated in the stratified ocean interior (as opposed to being highly localized in boundary layers where the stratification is vanishingly small) and is relatively efficiently converted to potential energy.

Tidal energy dissipation on Europa has been estimated from modeling to be on the order of $10^9$-$10^{11}$ W (Chen et al. 2014; Matsuyama et al. 2018; Hay & Matsuyama 2019). Assuming, as before, an ocean depth on the order of $10^2$ m, an area of around $3 \times 10^{13}$ m$^2$, and a density $\rho_0 \approx 1000$ kg m$^{-3}$, we estimate a tidal energy dissipation per unit volume of around $\epsilon_t \approx 3 \times 10^{-13} - 3 \times 10^{-11}$ m$^2$ s$^{-3}$, which is between about an order of magnitude smaller and about an order of magnitude larger than the energy input by buoyancy forcing (cf. Table 1). Depending on the estimate for tidal energy dissipation, and the fraction of the dissipated tidal energy that is converted to potential energy, tidally-driven turbulent mixing thus may or may not play a significant role in driving a large-scale ocean circulation on Europa. Specifically, freezing and melting at the ice-ocean interface combined with tidally-driven turbulent mixing may drive a significantly stronger large-scale ocean circulation then the thermal forcing alone, if, and only if, ocean tidal dissipation is near the upper end of these estimates and a large fraction of that energy contributes to vertical mixing against a stable stratification (i.e. $\Gamma \gtrsim 10\%$). A significant salt-driven circulation was found in the simulations of Ashkenazy & Tziperman (2021), although turbulent vertical mixing is parameterized with a prescribed Earth-like vertical diffusivity

9 For steady planar turbulent jets in an unstratified fluid, the peak mean flow speed has been found to decay with distance from the orifice as $U_{\text{max}} \approx 2.5 \sqrt{d/\tau}$, where $d$ is the width of the slot from which the jet emanates and $\tau$ is the distance (e.g. Gutmark & Wygnanski 1976). Turbulent flow speeds are about 20 – 25% of this value. Kite & Rubin (2016) suggest that Enceladus’ “tiger stripes” are associated with $O(1 \text{ m})$ wide slots in which tidal forces generate $O(1 \text{ m/s})$ jets. Assuming a neutral stratification, the results for planar turbulent jets would then suggest peak mean flows of the order of 10 cm/s and associated turbulent velocities on the order of a few cm/s to persist to a depth of a few hundred meters into the ocean. The relatively weak depth dependence ($U_{\text{max}} \propto x^{-1/2}$) moreover indicates that weaker but significant flows may be penetrating much deeper, although results for steady jets are likely to become less applicable to the oscillatory jets proposed by Kite & Rubin (2016) at greater depth. Stratification may further limit the penetration depth of turbulent jets.
in the simulations and is not constrained by tidal energy dissipation. Better estimates of tidal energy dissipation in Europa's ocean are needed to determine whether a significant turbulent vertical diffusivity can be justified for stratified regions of Europa's ocean.

For Enceladus' Ocean, energy dissipation associated with the eccentricity and obliquity tides has been estimated to be relatively small, with a total dissipation between $10^{-4}$ and $10^{-8}$ W suggested by Matsuyama et al. (2018), and Hay & Matsuyama (2019). Tyler (2020) has argued that much larger dissipation rates are possible, accounting for virtually all of the observed $O(10^{10})$ W heat flux, although the calculations generally predict the total dissipation in the ice-ocean system, and it is likely that most of the dissipation in the suggested scenarios would indeed occur in the ice sheet (c.f. Beuthe 2016). Assuming a most likely range of $10^{-4}$ W for ocean tidal dissipation on Enceladus, an average ocean depth of around $3 \times 10^4$ m and an area of around $5 \times 10^{11}$ m$^2$, we find an average tidal energy dissipation of around $\epsilon_t \approx 7 \times 10^{-19} - 7 \times 10^{-16}$ m$^2$ s$^{-3}$, which is at least two orders of magnitude smaller than the estimated mechanical energy input via thermal forcing (cf. Table 1). Mixing driven by the eccentricity and obliquity tides thus is not likely to provide a significant source of mechanical energy for the circulation on Enceladus' ocean.

However, Enceladus ocean may be significantly affected by librations of the ice shell, which is decoupled from the solid interior (Thomas et al. 2016). It is not clear whether librations lead to relatively modest dissipation confined to the ocean-ice boundary layer, thus not contributing significantly to interior mixing, or whether the excitation of internal waves or elliptical instability (an instability that can break up elliptical streamlines) can lead to relatively strong mixing throughout the depth of the ocean (Lemasquerier et al. 2017; Wilson & Kerswell 2018; Rekier et al. 2019; Soderlund et al. 2020). At the extreme end, Wilson & Kerswell (2018) argue that it is possible that all of the $O(10^{10} W)$ of heating on Enceladus may be a result of librational dissipation, which, divided by the mass of Enceladus' ocean would give $\epsilon_t \approx 10^{-9}$ m$^2$ s$^{-3}$—many orders of magnitude larger than the thermal energy input. In this case it is likely that libration-driven turbulence would play the dominant role in ocean mixing. Narrowing down the large uncertainty in the energy dissipation rate associated with libration-driven turbulence will be key to constraining the turbulent diffusivity, which is an essential parameter in numerical simulations of Enceladus' ocean (Kang et al. 2021; Zeng & Jansen 2021).

6. CONCLUSIONS

Consideration of the sources and dissipation rates of kinetic energy provides useful constraints for the circulation of ice-covered oceans. In general, kinetic energy can be generated by tides or by conversion from potential energy, which in turn can be generated by heat and salt flux forcing.

A commonly assumed forcing consists of heating from the solid core balanced by heat loss through the ice sheet, which acts as a source of potential energy and can drive an ocean circulation, as long as the thermal expansivity of the water is positive. In the oceans of Snowball Earth, Europa, and Enceladus, the associated energy input, however, is orders of magnitude smaller than the wind energy input that dominantly drives the circulation in Earth’s present-day ocean. We predict that the resulting thermally-driven flows have flow speeds of at most a few cm/s and will be strongly affected by rotation (i.e. small Rossby numbers). Numerical simulations of thermally-driven flows on icy moons, which typically use artificially large viscous dissipation and sometimes artificially large thermal forcing may misrepresent both energy sources and sinks by multiple orders of magnitude, which can lead to widely different and unrealistic levels of kinetic energy.

Salt fluxes associated with freezing and melting at the ice sheet boundary only provides a source of potential energy if freezing occurs under thinner ice than melting, which is unlikely in equilibrium where freezing and melting needs to be in balance with ice flow. In the more likely scenario where melting on average occurs at a shallower depth than freezing, the salt flux forcing acts as a sink of potential energy. Turbulent vertical mixing, which can push the lighter fresh water into the interior, is then needed to drive a significant circulation. Such turbulent mixing, however, requires an alternative energy source.

Ocean tides and librations may provide a key energy source for ocean turbulence. Current estimates indicate that tidally-driven vertical mixing is likely to be important in a Snowball Earth ocean, and could possibly play a significant role on Europa, while librations may provide a key source of turbulent kinetic energy on Enceladus. However, the magnitude and spatial distribution of turbulence generated by tides and librations remains highly uncertain, which represents a major hurdle to better constrain the circulations of icy-world oceans. An improved understanding of ocean tides and librations on icy moons thus remains as an important topic for future research.
A GENERALIZED EXPRESSION FOR ENERGY INPUT FROM BUOYANCY FORCING

To quantify the role of heat and salinity forcing on the vertical buoyancy flux, while fully accounting for nonlinearities in the equation of state, it is useful to introduce the dynamic enthalpy (Young 2010):

\[ h^\dagger(\Theta, S, z) = \int_z^0 b(\Theta, S, z') dz' \]  

(A1)

The dynamic enthalpy, \( h^\dagger \), is closely related to the potential energy, and indeed reduces to the well known expression for the potential energy in the limit of an equation of state with no explicit depth dependence (i.e. \( b = \tilde{b}(\Theta, S) \)) where \( h^\dagger \to b z \).

The evolution equation for \( h^\dagger \) is

\[ \frac{Dh^\dagger}{Dt} = -wb + \dot{\Theta} \partial_\Theta h^\dagger + \dot{S} \partial_S h^\dagger \]  

(A2)

where \( \dot{\Theta} \equiv \frac{D\Theta}{Dt} \) and \( \dot{S} \equiv \frac{DS}{Dt} \) are given by Eqs. (6) and (7).

Integrating globally and assuming the global enthalpy budget to be in equilibrium, we can relate the globally integrated vertical advective buoyancy flux, \( B \), which provides a source of KE, to the thermal and salinity forcing, \( \dot{\Theta} \) and \( \dot{S} \):

\[ B = \int wbdV = \int \left( \dot{\Theta} \partial_\Theta h^\dagger + \dot{S} \partial_S h^\dagger \right) dV \]  

(A3)

Defining

\[ \tilde{g}^\alpha(\Theta, S, z) \equiv \frac{1}{z} \partial_\Theta h^\dagger = \frac{1}{z} \int_z^0 g^\alpha(\Theta, S, z') dz' \]  

(A4)

and

\[ \tilde{g}^\beta(\Theta, S, z) \equiv \frac{1}{z} \partial_S h^\dagger = \frac{1}{z} \int_z^0 g^\beta(\Theta, S, z') dz' \]  

(A5)

we can write Eq. (A3) as

\[ B = - \int \left( \dot{\Theta} \tilde{g}^\alpha z + \dot{S} \tilde{g}^\beta z \right) dV \]

\[ = - \int \kappa \left( \nabla^2 \Theta \tilde{g}^\alpha z + \nabla^2 S \tilde{g}^\beta z \right) dV \]

\[ = - \int \kappa \left( \nabla \cdot (\nabla \Theta \tilde{g}^\alpha z) - \nabla \Theta \cdot \nabla (\tilde{g}^\alpha z) + \nabla \cdot (\nabla S \tilde{g}^\beta z) - \nabla S \cdot \nabla (\tilde{g}^\beta z) \right) dV \]

\[ = \int \frac{Q_{\text{top}}}{\rho c_p} \tilde{g}^\alpha z_{\text{top}} dA - \int \frac{Q_{\text{bot}}}{\rho c_p} \tilde{g}^\alpha z_{\text{bot}} dA \]

\[ + \int S_{\text{top}} \tilde{g}^\beta z_{\text{top}} dA - \int S_{\text{bot}} \tilde{g}^\beta z_{\text{bot}} dA \]

\[ + \int g \kappa (\alpha \partial_z \Theta + \beta \partial_z S) dV \]

\[ + \int z \kappa \left( \nabla_h (\tilde{g}^\alpha) \cdot \nabla_h \Theta + \nabla_h (\tilde{g}^\beta) \cdot \nabla_h S \right) dV \]  

(A6)

Eq. (A6) allows us to relate the globally integrated advective upward buoyancy flux (and hence the associated KE generation) to the heat and salt fluxes through the boundaries (the first four terms on the RHS of Eq. A6), reduced by the diffusive buoyancy flux (the fifth term on the RHS of Eq. A6). The last term in Eq. (A6) captures the effect of buoyancy sources or sinks that arise from horizontal diffusive mixing of \( \Theta \) and \( S \) due to the nonlinearity in the equation of state (the cabling effect).
The potential role of the cabbeling term in Eq. (A6) can be illustrated by considering again the example of section 3.3, which may resemble the conditions on Enceladus, assuming a relatively low salinity for the latter. That is we assume a stratified layer with negative thermal expansion coefficient above a convective deep layer with weakly positive thermal expansion coefficient. For illustrative purposes, let us assume the upper and lower boundaries to be horizontal, bottom and surface heat fluxes to be spatially uniform, and we ignore radial variations in the surface area, such that \( Q_{\text{top}} = Q_{\text{bot}} \equiv Q \), as well as vertical variations in gravity. Since variations in the thermal expansion coefficient are primarily due to differences in the temperature, we moreover neglect the direct depth dependence of \( \tilde{b} \) and assume that variations in salinity are small, such that \( \hat{\varphi} = \alpha(\theta) \). In this case, Eq. (A6) simplifies to

\[
\mathcal{B} = \frac{gQ}{\rho c_p} \int (\alpha(\Theta_{\text{top}})z_{\text{top}} - \alpha(\Theta_{\text{bot}})z_{\text{bot}}) dA + \int g\kappa \alpha(\Theta) \partial_z \Theta dV + \int g z \kappa \partial_\Theta \tilde{\alpha} |\nabla_\Theta|^2 dV.
\]

where we used that \( \nabla_\alpha \cdot \nabla_\Theta = \partial_\Theta \alpha |\nabla_\Theta|^2 \). The last term can now be associated with the diffusive destruction of temperature variance, which leads to a buoyancy source if \( \partial_\Theta \alpha > 0 \) (amounting to a positive curvature in \( b(\theta) \)).

Notice that the magnitude (and potentially the sign) of the first and last term in Eq. (A7) depend of the choice of reference level. E.g. setting \( z_{\text{bot}} = 0 \) and \( z_{\text{top}} = H \) where \( H \) is the depth of the ocean (which we assume to be constant with flat upper and lower boundaries) we find

\[
\mathcal{B} = \frac{gQH}{\rho c_p} \int \alpha(\Theta_{\text{top}}) dA + \int g\kappa \alpha(\Theta) \partial_z \Theta dV + \int_0^H g z \kappa \partial_\Theta \alpha |\nabla_\Theta|^2 dV. \tag{A7}
\]

Defining the reference level instead such that \( z_{\text{top}} = 0 \) and \( z_{\text{bot}} = -H \) we get

\[
\mathcal{B} = \frac{gQH}{\rho c_p} \int \alpha(\Theta_{\text{bot}}) dA + \int g\kappa \alpha(\Theta) \partial_z \Theta dV + \int_{-H}^0 g z \kappa \partial_\Theta \alpha |\nabla_\Theta|^2 dV. \tag{A8}
\]

The first term differs dramatically in the two equations, as \( \alpha(\Theta_{\text{top}}) < 0 \) while \( \alpha(\Theta_{\text{bot}}) > 0 \). This difference is compensated by the last term, which also changes sign, as \( z > 0 \) in (A7) and \( z < 0 \) in (A8). The last term in Eq. (A6) hence can be of leading order importance if variations in the thermal (or haline) expansion coefficient are large. Since this term cannot be predicted based on knowledge of the boundary conditions alone, this most general formulation is hence less immediately useful as a predictor of flow energetics. However, if horizontal temperature variations are relatively small, Eq. (14) remains a useful approximation and allows us to estimate the kinetic energy input without explicit consideration of diffusive variance destruction.

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