Nonadiabatic ratchet effect in superconducting films with a tilted cosine pinning potential

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Abstract. The influence of an ac current of arbitrary amplitude and frequency on the mixed-state dc-voltage-ac-drive ratchet response of a superconducting film with a dc current-tilted uniaxial cosine pinning potential at finite temperature is theoretically investigated. The results are obtained in the single-vortex approximation, i.e., for non-interacting vortices, within the frame of an exact solution of the appropriate Langevin equation in terms of a matrix continued fraction. The dc voltage ratchet response is discussed as a function of ac current amplitude and frequency for various dc biases, in a wide range of corresponding dimensionless parameters. Our theoretical results are discussed in comparison with recent experimental work on the high-frequency ratchet response in nanostructured superconducting films.

1. Introduction

Within the last decade vortex ratchets, which exploit asymmetric vortex dynamics, have been attracting considerable attention, both theoretically \cite{1, 2} and experimentally \cite{3, 4, 5}. We refer the reader to Refs. \cite{6, 7} for comprehensive reviews. Only recently, B. B. Jin \textit{et al.} \cite{5} have experimentally investigated a very important issue in the vortex ratchet study, namely the frequency dependence of the dc voltage at large amplitudes of the ac driving force in a frequency range between 0.5 MHz and 2 GHz, whereas ac frequencies were always lower than 1 MHz in vortex ratchet measurements up to that work \cite{3, 4}. This has stimulated us to theoretically scrutinize the exactly solvable model of the two-dimensional temperature-dependent nonlinear vortex dynamics in thin-film superconductors with a uniaxial washboard pinning potential (WPP), subjected to an ac drive of any arbitrary current densities and frequencies at various dc biases \cite{8}. The main experimentally achievable values in the problem, namely the dc voltage ratchet response and absorbed ac power have been exactly calculated \cite{9} on the basis of an exact solution of the Langevin equation for non-interacting vortices in terms of a matrix continued fraction, for any intermediate current angles with regard to the WPP channels. Although our theoretical predictions \cite{8, 9} can be examined on superconducting films with a WPP \cite{10, 11, 12}, these lack experimental scrutiny so far, especially at microwave and GHz frequencies. For this reason, in the present work where applicable, we will be referring to the results of B. B. Jin \textit{et al.} \cite{5} as a most closely related experimental work to our new tilted-ratchet results, though that work \cite{5} is dealing with an initially asymmetric ratchet potential.
The aim of the present contribution is to present novel frequency-dependent results for the dc voltage ratchet response [9] as a function of ac current amplitude and frequency as well as dc current inducing a tilt of the WPP, in a wide range of corresponding dimensionless parameters. It will be pointed out which new nonadiabatic ratchet effects in the vortex dynamics appear even within the single-vortex approximation.

2. Formulation of the problem
Our theoretical treatment of the system relies upon the Langevin equation for a vortex moving with velocity \( \mathbf{v} \) in a magnetic field \( \mathbf{B} = nB \) (\( B \equiv |\mathbf{B}| \), \( n = nz \), \( z \) is the unit vector in the \( z \) direction and \( n = \pm 1 \)) which, neglecting the Hall effect, has the form

\[
\eta \mathbf{v} = \mathbf{F}_L + \mathbf{F}_p + \mathbf{F}_{th},
\]

where \( \mathbf{F}_L = n(\Phi_0/c)j \times z \) is the Lorentz force (\( \Phi_0 \) is the magnetic flux quantum, and \( c \) is the speed of light). \( j = j(t) = j^{dc} + j^{ac} \cos \omega t \), where \( j^{dc} \) and \( j^{ac} \) are the dc and ac current density amplitudes and \( \omega \) is the angular frequency. \( \mathbf{F}_p = -\nabla U_p(x) \) is the anisotropic pinning force, \( U_p(x) = (U_p/2)(1 - \cos kx) \) is the periodic washboard pinning potential with \( k = 2\pi/a \) [8], where \( U_p \) is its depth and \( a \) is the period. \( \mathbf{F}_{th} \) is the thermal fluctuation force represented by a Gaussian white noise and \( \eta \) is the vortex viscosity.

The Langevin equation (1) has been solved in Ref. [8] in terms of a matrix continued fraction allowing one to exactly calculate the main quantities of physical interest in our problem, namely (i) the time-independent (but frequency-dependent) dc electrical field response and (ii) the stationary ac response on the frequency \( \omega \), independent on the initial conditions. Both these are determined by the appropriate components of the average electric field induced by the moving vortex system [9] (see Eqs. (2-5) therein).

3. Main ratchet results
We now proceed to the presentation of theoretical results for the time-independent dimensionless dc electric field \( E^d \), scaled in units of \( \rho_f j_c \), as a function of its dimensionless external driving parameters, i.e., dc bias \( \xi^d = |j^{dc}|/j_c \), amplitude \( \xi^a = |j^{ac}|/j_c \), and frequency of the ac input \( \Omega = \omega \bar{\tau} \) with \( \bar{\tau} = 2\eta/U_p k^2 \) being the relaxation time, \( \rho_f \equiv B\Phi_0/\eta c^2 \) being the flux-flow resistivity, and \( j_c \equiv cU_p k/2\Phi_0 \) [8]. For simplicity, below we put emphasis on the case when both the currents flow along the WPP channels provoking the vortex movement perpendicular to them. As a result, next we consider only the dc ratchet response component along the WPP channels, as the only reason to provide the reader with most intuitive figure data.

The single vortex approximation used in Ref. [8] supposes the WPP period, \( a \), to be larger than the effective magnetic field penetration depth, \( \lambda \), and the temperature low enough to prevent smearing of singularities in the ratchet responses. To accomplish this, the figure data are calculated for the dimensionless inverse temperature \( g = U_p/2T = 100 \) [8] representing a reasonable value, experimentally achievable, e.g., for thin Nb films either furnished with nanofabricated PPP landscapes [10, 11, 12], or grown on faceted sapphire substrates [13] where \( U_p \approx 1000 \div 5000 \) K and \( T \approx 8 \) K.

Entering the discussion of the obtain results, it should be noted that one of the main questions in the study of the function \( E^d(\xi^a, \xi^d, \Omega) \) relies upon the determination of the frequency and dc bias dependences of the ac amplitude threshold value, \( \xi^a(\Omega, \xi^d) \) [5, 9], which can be considered as an ac critical current magnitude for the dc ratchet response \( E^d \) such that \( E^d = 0 \) for \( \xi^a < \xi^a \). To accomplish this, we begin the graphical analysis with the ac amplitude dependence of the dc ratchet response considering specific features in \( E^d(\xi^a, \Omega, \xi^d) \) at intermediate frequencies, \( \Omega = 1 \), as depicted in Fig. 1.a.
Figure 1. a) The ratchet voltage $E^d$ versus $\xi^a$ in the nonadiabatic regime for a set of biases $\xi^d$, as indicated. Thresholds and phase-locking features in the curves can be more steeper if calculated at lower temperatures, i.e., at $g > 100$ (see Ref. [9] for details). b) The frequency dependence of $\xi^a_c$ for the dc-tilted cosine pinning potential for a set of biases $\xi^d$, as indicated. The navy and orange (online) dashed lines represent rough separations between low ($\Omega \ll 1$), intermediate ($\Omega \sim 1$), and high frequency ($\Omega \gg 1$) regimes. The curves behave qualitatively similar to those obtained experimentally in Ref. [5] on superconducting Pb films with a non-tilted ratchet pinning potential.

Consider at first the curves in Fig. 1.a with $\Omega = 1$ which corresponds to the nonadiabatic ratchet response. Several interesting features in the curves should be noted. First, depending on the bias value, we observe qualitatively different behavior for curves with $\xi^d > 1$ and $\xi^d < 1$; namely for $\xi^d = 1.05$ and $\xi^d = 1.2$ the ratchet response is a threshold-free one, whereas a threshold value, $\xi^a_c$, separates the non-dissipative and dissipative states at $\xi^a < 1$. The magnitude of the threshold at subcritical tilts is a decreasing function of $\xi^d$ and, in fact, is equal to $\xi^a_c = 1 - \xi^d$ [8]. The physical reason of the above difference follows from the fact that at $\xi^a = 0$ and $\xi^d > 1$ the vortex is in the running state with a slightly oscillating instantaneous velocity $dx/dt$ and thus, nonzero electric field $E^d$, whereas for $\xi^d < 1$ the vortex is localized in one of the WPP wells. With the increase of the frequency $\Omega$, the threshold values $\xi^a_c(\Omega > 1, \xi^d)$ are larger than those ensuing for $\Omega = 1$ at similar subcritical dc tilts. To illustrate this in detail, we plot the $\xi^a_c(\Omega)$ dependence in Fig. 1.b for a set of biases. All the curves demonstrate qualitatively similar behavior, i.e., a zero plateau at $\xi^a < \xi^a_c$, a linear dependence at large $\xi^a > \xi^a_c$, and a nonlinear transition in between at $\xi^a \gtrsim \xi^a_c$. These segments correspond to the adiabatic, intermediate, and high-frequency modes which are roughly separated by the straight lines $\Omega \simeq 0.1$ and $\Omega \simeq 1$, respectively. It should be noted, that the curves $\xi^a_c(\Omega)$ in Fig. 1.b, calculated in the present work for the dc-tilted cosine pinning potential, are qualitatively similar to those obtained experimentally on superconducting Pb films with a non-tilted ratchet periodic pinning potential (see Ref. [5] and Fig. 5 therein). The transition frequency from the adiabatic to nonadiabatic case has been found at about 1 MHz for that system [5].

Second, a difference in the $E^d(\xi^a, \xi^d, \Omega)$ behavior appears between $0.4 \lesssim \xi^d_{middle} \lesssim 0.7$, which looks like damped oscillating curves, and the curves at $\xi^d \lesssim 0.4$ and $\xi^d \gtrsim 0.7$, which look like curves with phase-locked regions (steps) in $\xi^a$. Whereas at small $\xi^d$ phase-locked regions ensue at $E^d = 0$, at strong biases $\xi^d \gtrsim 0.7$ these flat segments appear at $E^d = 1$. In Ref. [9], this has been investigated in detail and compared with approximate solutions calculated at zero temperature either in terms of the dynamical current-voltage characteristics (CVCs) in the adiabatic mode.
or within an approximate Bessel-function approach at high frequencies.

With the increase of the frequency $\Omega$ at $\xi^a = \text{const}$ and $\xi^d > 1$, the ratchet responses $E^d$ are continuously oscillating curves without thresholds. On the contrary, for $\xi^d < 1$ the responses have thresholds whose magnitudes decrease with increasing $\xi^d$. An interesting property of the dependence $E^d(\xi^a, \xi^d, \Omega)$ in Fig. 1.a is the possibility for $E^d$ to decrease periodically (sometimes down to zero) with increase of the driving amplitude $\xi^a$. Such a behavior of $E^d$ is in contrast to the behavior of the usual dc-driven CVCs, even though in the presence of $\xi^a$, as these are always increasing functions of $\xi^d$.

4. Conclusion

In this work we considered an exactly solvable two-dimensional model structure for the study of the frequency-dependent ratchet effect in superconducting film with a symmetric planar pinning potential, tilted by a dc bias, also known as a tilted ratchet. We have theoretically examined the strongly nonlinear nonadiabatic tilted ratchet behavior of the two-dimensional vortex system of a superconductor as a function of the (ac+dc) transport current density $j$, the frequency $\omega$, and the temperature $T$. In particular, we can describe (i) the critical ac current dependence in a wide frequency range covering the transition from adiabatic to nonadiabatic, with both, a frequency-independent plateau at low frequencies, a direct dependence at high frequencies and a nonlinear transition in between, (ii) the appearance of phase-locking regions in the dependence of $E^d$ on $\xi^a$, and (iii) a weakening of the ratchet effect at extremely high frequencies.

As far as the model described here refers to the tilted-potential ratchet, an exact theoretical description of the rocking-ratchet response in superconducting films with an asymmetric WPP is also possible and will be reported elsewhere [14]. We would like to stress, that our exactly solvable single-vortex model explicitly shows that many important and interesting nonlinear ratchet effects, which can be observed in particular at high frequencies, follow even from such a simple model for one vortex in periodic ratchet WPP. Though experimental verification of the predictions of both the models can be performed, for instance, on Nb thin films with nano-fabricated WPP landscapes [10, 11], the first portion of ratchet data still remains to be seen.

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6. References

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