Glueball molecules

Alexey A. Petrov

Department of Physics and Astronomy, Wayne State University, Detroit, MI 48201, USA
Theoretical Physics Department, Fermilab, P.O. Box 500, Batavia, IL 60510, USA
Excellence Cluster ORIGINS, Technische Universität München, Boltzmannstr. 2, D-85748 Garching, Germany
(Dated: April 26, 2022)

Experimental searches for pure glueball states have proven challenging and so far yielded no results. This is believed to occur because glueballs mix with the ordinary \( q\bar{q} \) states with the same quantum numbers. We will discuss an alternative mechanism, the formation of the glueball-meson molecular states. We will argue that the wave functions of already observed excited meson states may contain a significant part due to such molecular states. We discuss the phenomenology of glueball molecules and comment on a possible charmless component of the \( XYZ \) states.

INTRODUCTION

The existence of glueballs, strongly interacting states supposedly made of the pure glue degrees of freedom, is expected in Quantum Chromodynamics (QCD). This follows from the non-abelian nature of the theory, where the force carriers are also charged under the color gauge group. Various studies of the properties of glueballs have been performed both in lattice QCD and in other, more model-dependent approaches. Despite various predictions for their properties, no candidates for the pure glueball states have been experimentally observed, even in such glue-rich environments as nuclear collisions. Comprehensive reviews of the current state of theoretical calculations of glueball properties and their experimental searches can be found in 1,3.

Spectroscopic studies of glueballs 1 suggest that possible quantum numbers of the pure glue states, such as \( 0^{++}, 0^{-+}, 1^{--} \), etc., match with those of the ordinary mesons, i.e., states built out of \( q\bar{q} \) pairs. This fact alone implies quantum mechanical mixing of such states.

Phenomenological studies of the spectrum of states above one GeV reveal that no resonance that has been experimentally discovered so far could be unambiguously classified as a pure glueball state. A consensus appears to be reached that the admixture of pure glue and \( q\bar{q} \) states is not tiny, i.e., glueball can only appear as part of the wavefunction of ordinary meson states above 1 GeV. Several studies of such mixing have been performed 4,8.

In this paper, we propose another possibility. We will prove that the glueballs could also manifest themselves differently as glueball-meson molecular states. Depending on the coupling of the glueball and meson states, such molecular states may or may not exhibit a universal behavior predicted for the \( X(3872) \) or deuteron states. The molecular nature makes these states distinct from the hybrid meson states.

For simplicity, here we shall concentrate on the lightest glueball state \( G \), the state with the quantum numbers of a scalar, \( 0^{++} \). This is probably the most-studied state with various predictions for its mass and mixing patterns with other meson scalar states, all pointing that this is the lightest glueball state with a mass between 1 and 2 GeV. To determine the properties of the molecular states, we shall employ the so-called extended SU(2) Linear Sigma Model (eLSM) 3,6. In this model, the standard SU(2) linear sigma model is complemented by the scalar dilaton field \( G \), representing the glueball’s interpolating field. The parameters of the dilaton-pion interactions are fixed by fitting the decay widths of the scalar fields, whose masses and mixing parameters are fitted to the masses of \( f_0(1710) \), \( f_0(1500) \), and \( f_0(1370) \) states. The resulting lowest-energy molecular state will have quantum numbers of the pseudoscalar, \( 0^{-+} \), so we will denote it as \( P \) in what follows.

Has such a state been observed? We will argue that a light quark state \( \pi(1800) \) could have a significant \( P \) molecular component, which could explain its somewhat narrow width, which is rather unusual for such a massive light-quark \( q\bar{q} \) state 4. We will discuss the decay patterns of glueball molecular states. Finally, we will comment on the possible implications of glueball molecules with other quantum numbers.

SIGMA MODEL AND A MOLECULAR STATE

To describe pion-glueball interactions, we need to specify a framework in which the description of the system would be done. This is especially important for the description of the glueball states. As it turns out, a symmetry-inspired description of the scalar \( 0^{++} \) glueball field is possible and follows from the definition of the QCD trace anomaly. The pure glue sector of classical QCD is invariant under a dilatation symmetry, which is broken by quantum corrections. This results in the nonzero value of the trace of the energy-momentum tensor \( T_{\mu\nu} \). The vacuum expectation value of this trace is proportional to the value of the gluon condensate,

\[
\langle T_{\mu\nu} \rangle = -\frac{11N_c}{48} \left( \frac{\alpha_s}{\pi} G^a G^a_{\mu\nu} \right) = -\frac{11N_c}{48} C^4,
\]

(1)
where $C^4 = (300 - 600 \text{ MeV})^4$. An effective field theory can be built for the scalar field $G$, describing the trace anomaly, where $G$ can be used as an interpolating field for the scalar glueball.

It is convenient to employ the extended linear sigma model, whose parameters have already been fitted to describe scalar-glueball mixing and glueball decays [3, 6]. As we shall see, no new interaction terms are needed to describe the molecular $\mathcal{P}$ states. Additional details on the choice of the Lagrangian and the parameter-fitting procedure can be found in [3, 6].

The effective Lagrangian of the eLSM is

$$\mathcal{L} = \mathcal{L}_{\text{LSM}} + \mathcal{L}_{\text{dilaton}} + \mathcal{L}_{\text{int}},$$

where the first term, $\mathcal{L}_{\text{LSM}}$, corresponds to the Lagrangian of the ordinary SU(2) linear sigma model (a generalization to the SU(3) LSM is straightforward [3]),

$$\mathcal{L}_{\text{LSM}} = \text{Tr} \left[ (\partial^a \Phi)^4 (\partial_a \Phi) \right] - \lambda_1 \left( \text{Tr} \left[ (\Phi^4) \right] \right)^2$$

$$- \lambda_2 \text{Tr} \left[ (\Phi^2)^2 + \text{Tr} \left[ H (\Phi^4) \right] \right] + c (\text{det}(\Phi^4) + \text{det}(\Phi)),$$

where $\Phi$ contains both scalar $\sigma$ and $a_0$ and pseudoscalar $\eta_N$ and $\pi_i$ fields,

$$\Phi = (\sigma + i \eta_N t^0 + (a_0 + i \vec{\pi}) \cdot \vec{t}),$$

where $t^0$ and $\vec{t}$ are the generators of U(2). Note that $\eta_N = (\vec{u}u + \vec{d}d)/\sqrt{2}$ only contains non-strange degrees of freedom. The last two terms in Eq. (3) represent the explicit breaking of chiral symmetry $\text{Tr} \left[ H (\Phi^4) \right] \sim h \sigma$, where $h \sim m_\pi^2$ [3, 6], and a contribution due to the chiral anomaly. Those terms also explicitly break dilaton symmetry.

The effective Lagrangian for the dilaton field can be written as [10, 11]

$$\mathcal{L}_{\text{dilaton}} = \frac{1}{2} \left( \partial_\mu G \right)^2 - \frac{m_G^2}{4 \Lambda^2} \left[ G^4 \log \left( \frac{G}{\Lambda} \right) - \frac{1}{4} G^4 + \frac{1}{2} \left( \partial_\mu G \right)^2 - V(G) \right].$$

The minimum of the dilaton potential $V(G)$ is at $\tilde{G} = G_0 = \Lambda$. Expanding around the minimum of the potential $\tilde{G} \rightarrow \Lambda + G$ we obtain the effective Lagrangian for the field glueball field $G$ with the mass parameter $m_G \sim 1.6$ GeV obtained from the lattice computations. It can be related to the trace of the energy-momentum tensor, implying that [3, 6]

$$\Lambda^2 = \frac{11 C^4}{4 m_G^2}. \tag{6}$$

The glueball-pion interaction piece $\mathcal{L}_{\text{int}}$ can be written as

$$\mathcal{L}_{\text{int}} = -m_\sigma^2 \text{Tr} \left[ \left( \frac{G}{\Lambda} \right)^2 (\Phi^4) \right]. \tag{7}$$

This Lagrangian contains both glueball-pion and glueball-sigma interaction terms, which are relevant for computing the properties of the molecular state $\mathcal{P}$. The $\pi \pi G^2$ interaction term can be directly read off Eq. (7), while the $\sigma$-exchange interaction can be obtained from the following procedure. Setting all fields but $\sigma$ and $G$ in Eq. (4) to zero and shifting the remaining fields by their vacuum expectation values (VEVs) $\tilde{G} \rightarrow G + G_0$ and $\sigma \rightarrow \sigma + \langle \sigma \rangle$ we obtain the $\sigma G^2$ interaction term,

$$\mathcal{L}_{\sigma G} = - \frac{m_\sigma^2}{4 \Lambda^2} \sigma G^2 \sigma + ... \tag{8}$$

The ellipses in Eq. (8) represent terms that are not relevant for building a molecular state. However, they contain terms responsible for mixing glueballs and $q\bar{q}$ states in this model. The terms generating the $\pi \pi \sigma$ vertex can also be obtained from Eq. (3) by shifting $\sigma \rightarrow \sigma + \langle \sigma \rangle$, as in the usual LSM.

The local effective Lagrangian relevant for the scales $\mu < m_\pi$ can be obtained by matching the $\pi G$ scattering amplitude and integrating out the $\sigma$ field,

$$\mathcal{L}_{\pi G} = -\lambda \sigma G^2, \tag{9}$$

where the effective coupling $\lambda$ is given by

$$\lambda = \frac{m_\sigma^2}{2 \Lambda^2} \left[ 1 - \frac{\langle \sigma \rangle^2}{m_\sigma^2} (2 \lambda_1 + \lambda_2) \right]. \tag{10}$$

The numerical values of the parameters in Eq. (10) can be obtained from [6]. To compute the properties of the molecular states $\mathcal{P}$, we will use the formalism employed in describing the properties of deuterium [14] or $X(3872)$ state [16]. In particular, we will consider non-perturbative scattering amplitude for the scattering $\pi G \rightarrow \pi G$. The pole of this amplitude, if exists, corresponds to the bound state in the $\pi G$ channel, i.e., the molecular $\mathcal{P}$ state. The transition amplitude can be obtained from the Lippmann-Schwinger equation (see Fig. 1),

$$i T_{\pi G} = -i \lambda + \int \frac{d^4 q}{(2\pi)^4} (i T_{\pi G}) G_{\pi G} (-i \lambda) \tag{11}$$

where $G_{\pi G}$ is given by

$$G_{\pi G} = - \frac{1}{4 m_G m_\pi} \frac{1}{E^2 - q^2/2m_\pi + i\epsilon} \times \frac{1}{E^2 - q_0^2 - q^2/2m_\pi + i\epsilon}. \tag{12}$$

Here $E$ is the energy of the mesons in the center-of-mass frame. The solution of Eq. (11) is

$$T_{\pi G} = \frac{\lambda}{1 + i \tilde{A}}, \tag{13}$$

$\tilde{A}$ is the (divergent) integral given by

$$\tilde{A} = - \frac{i}{2 m_G m_\pi} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - 2m_G E - i\epsilon}. \tag{14}$$
where $\mu_{\pi G}$ is the reduced mass of the $\pi G$ system, and we have used the residue theorem to evaluate the integral over $q_0$.

The integral in Eq. (14) diverges, so its divergence needs to be removed by renormalizing the coupling $\lambda$. Following S. Weinberg [14, 15], we choose to define the renormalized $\lambda_R$ in the MS subtraction scheme in dimensional regularization [16]. Computing the integral of Eq. (14) in $d - 1$ dimensions yields

$$A \to \tilde{A} = \frac{iq}{8\pi m_\pi m_G},$$

(15)

where $E = p^2/2\mu_{\pi G}$. This implies that the scattering amplitude $T_{\pi G}$ which solves the Lippmann-Schwinger equation of Eq. (11) is

$$T_{\pi G} = \frac{\lambda_R}{1 + \lambda_R \frac{\mu_{\pi G}}{8\pi m_\pi m_G} q^2},$$

(16)

The scattering amplitude of Eq. (16) has a pole, which corresponds to the bound state with the energy

$$E_b = \frac{32\pi^2 m_\pi^2 m_G^2}{\lambda_R^2 \mu_{\pi G}}.$$  

(17)

Assuming a nonrelativistic bound state implies that the binding energy is small. The mass of the molecular state $\mathcal{P}$ would then be

$$m_\mathcal{P} = m_G + m_\pi - E_b.$$  

(18)

Since the bulk of theoretical predictions points to the glueball mass values of $m_G \sim 1.6 - 1.7$ GeV [1, 3], it is natural to assume that the mass of the molecular state should be around $m_\mathcal{P} \approx 1.8$ GeV. Interestingly, the state $\pi(1800)$ has the correct quantum numbers and could be seen as a candidate for the glueball molecular state.

**PHENOMENOLOGY**

In the previous section, we predicted the existence of the pseudoscalar glueball molecular state $\mathcal{P}$ by identifying a pole in a $\pi G$ scattering amplitude. Yet, both $\pi$ and $G$ are not asymptotically stable states. While the small width of the $\pi$, which only decays via weak interactions, can be neglected, little is known about glueballs’ widths. With the glueball decaying strongly, one can legitimately ask if the molecular bound state has enough time to form at all. In general, one can argue that in the large $N_c$ limit, the glueball widths scale like $1/N_c^2$, while the $q\bar{q}$ meson widths scale only as $1/N_c$ [12]. This gives us confidence that the glueball molecular states can be formed. It must be noted that the scalar glueball considered in this paper is uniquely linked to trace anomaly, so its width could be computed [3, 6].

It is interesting to consider the phenomenology of the glueball molecular states. One complicating factor, however, exists. As we pointed out, quantum mechanics requires that the states of different nature but the same quantum numbers mix. If the molecular state mixes with $q\bar{q}$ or other states, all predictions for the decay patterns will depend on the value of the mixing angle(s). We can make predictions for the ratios of various two-body decays assuming that a given physical state is dominated by its glueball-molecular component. If so, the decay patterns of the state would be driven by the decay patterns of the glueball component. In particular, postulating that $\mathcal{P}$ is the dominant component of the $\pi(1800)$ wave-function, we can determine the ratios of its two-body decays into the $f_0$ states.

Since the $0^{++}$ glueball’s quantum numbers coincide with those of the $f_0$ states, they should contain various admixtures of the glue component. The studies of glueball-meson mixing were performed by several authors by studying decays and the production of those states. The results of these studies could be summarized in the following matrix equation,

$$F = M \cdot Q,$$

(19)

where $F = \{|f_0(1370)\rangle, |f_0(1500)\rangle, |f_0(1710)\rangle\}$ and $Q = \{|\eta_N\rangle, |s\bar{s}\rangle, |G\rangle\}$ are the columns for the meson and quark/glueball states, respectively. Note that $\eta_N$ was defined in the previous section. There are several studies that extract the mixing matrix $M$. One study [3, 6] finds that

$$M_1 = \begin{pmatrix}
    0.78 & 0.51 & -0.36 \\
    -0.54 & 0.84 & -0.03 \\
    0.32 & 0.18 & 0.93
\end{pmatrix}.$$  

Alternatively, [3, 6] give a result for the mixing matrix

![FIG. 1: Transition amplitude for the $\pi G$ scattering written in the form of a Lippmann-Schwinger equation.](image-url)
M that is somewhat similar to the one given in Eq. (20),
\[
M_2 = \begin{pmatrix}
0.79 & -0.54 & 0.29 \\
0.49 & 0.84 & 0.22 \\
-0.37 & 0.023 & 0.93
\end{pmatrix}.
\]  

It is interesting to note that both studies agree that the only acceptable scenario is that \( f_0(1710) \) state is mostly gluonic, which we used to justify our assumption that the \( \pi(1800) \) state is primarily a glueball molecule state.

If glueball-molecular component \( P \) dominates, the decay amplitude \( A(\pi(1800) \rightarrow \pi f_0) \) can be written as
\[
A(\pi(1800) \rightarrow \pi f_0) = \langle f_0|G|\pi(1800)\rangle \langle \pi G|H|\pi(1800)\rangle,
\]  

where \( \langle f_0|G \rangle \) parameterizes the glue component of the \( f_0 \) state for different \( f_0 \) states, which can be obtained by inverting the relation in Eq. (19),
\[
\langle G | = \langle f_0(1370)|G|f_0(1370)\rangle + \langle f_0(1500)|G|f_0(1500)\rangle + \langle f_0(1710)|G|f_0(1710)\rangle.
\]  

Then, the ratio of branching ratios for the decays into \( \pi f_0(1500) \) and \( \pi f_0(1370) \) final states would be completely determined by the glue components of the two \( f_0 \) states, and the difference in phase space available in each decay,
\[
\frac{B(\pi(1800) \rightarrow \pi f_0(1500))}{B(\pi(1800) \rightarrow \pi f_0(1370))} = \frac{\langle f_0(1500)|G|f_0(1500)\rangle^2}{\langle f_0(1370)|G|f_0(1370)\rangle^2} r_p, 
\]  

where \( r_p = p_{f_0(1500)}/p_{f_0(1500)} \) is the ratio of the phase space factors for the two-body decay, which is given by the three-momenta of the final state particles,
\[
p_{f_0} = \sqrt{\left(M^2 - (m_\pi + m_{f_0})^2\right) \left(M^2 - (m_\pi - m_{f_0})^2\right)}.
\]

Numerically, the ratio in Eq. (24) differs slightly depending on whether the matrix from Eq. (20) or (21) is used to obtain Eq. (23),
\[
\frac{B(\pi(1800) \rightarrow \pi f_0(1500))}{B(\pi(1800) \rightarrow \pi f_0(1370))} = (4 \div 7) \times 10^{-3},
\]

where the first result corresponds to inverting \( M_1 \), while the second - to inverting \( M_2 \). The trend, however, is similar. That is, assuming that the glueball molecule dominates the \( \pi(1800) \) wave function, the branching ratio of \( B(\pi(1800) \rightarrow \pi f_0(1500)) \) is suppressed compared to \( B(\pi(1800) \rightarrow \pi f_0(1370)) \).

**CONCLUSIONS**

We proposed the existence of a new family of hadronic states, the glueball molecules. The existence of such states was justified by showing that the \( \pi G \) scattering amplitude contains the pole, whose position identifies the mass of the state.

Based on the hypothesis that \( \pi(1800) \) state contains a dominant \( P \) glueball molecular component, we predicted the ratio of decay branching ratios to \( \pi f_0(1500) \) and \( \pi f_0(1370) \) states. We should note that the main decay channels of a glueball molecular state should be driven by the decays of its glueball component. In particular, we expect the ratios of decay widths into \( 3\pi, K K \pi, \) and \( 2\eta \pi \) final states to mainly follow the quark counting rules for the glueball decays [1].

There are some intriguing implications of this proposal. For example, it provides additional insights into the puzzle of heavy \( XYZ \) states, especially those that decay into charmed mesons. A possible (if small) mixture of a vector \( 1^- \) glueball state \( G_V \) with a vector \( c \bar{c} \) state \( [17] \) could provide a glueball molecular component \( \pi G \) of the \( X(3872) \) state containing no charmed quarks at all. This follows if the mass of the vector glueball is \( m_{G_V} \approx 3.8 \) GeV, as suggested by some lattice QCD studies [3]. Such molecular states would also be different from the hybrid meson states, which can also be described in the framework of effective field theory [18]. Other implications would include the existence of the glueball molecular states with heavy (charm or bottom) quarks. We will address these implications in a forthcoming publication.

**Acknowledgments**

We would like to thank Eric Swanson for his valuable comments. This research was supported in part by the U.S. Department of Energy under contract DE-SC0007983 and by the Excellence Cluster ORIGINS, which is funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany’s Excellence Strategy - EXC-2094 - 390783311. It was also supported by the Visiting Scholars Award Program of the Universities Research Association. Fermilab is managed and operated by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy.

[1] W. Ochs, J. Phys. G 40, 043001 (2013)
[2] E. Klempt and A. Zaitsev, Phys. Rept. 454, 1-202 (2007)
[3] H. X. Chen, W. Chen, X. Liu, Y. R. Liu and S. L. Zhu, arXiv:2204.02649 [hep-ph].
[4] E. Klempt and A. V. Sarantsev, Phys. Lett. B 826, 136906 (2022)
[5] S. Janowski, D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 84, 054007 (2011)
[6] D. Parganlija, P. Kovač, G. Wolf, F. Giacosa and D. H. Rischke, Phys. Rev. D 87, no.1, 014011 (2013)
[7] H. Y. Cheng, C. K. Chua and K. F. Liu, Phys. Rev. D 92, no.9, 094006 (2015)
[8] J. M. Frère and J. Heeck, Phys. Rev. D 92, no.11, 114035 (2015)
[9] P. Eugenio, talk at APS 2016 April meeting.
[10] A. Salomone, J. Schechter and T. Tudron, Phys. Rev. D 23, 1143 (1981)
[11] A. A. Migdal and M. A. Shifman, Phys. Lett. B 114, 445-449 (1982)
[12] S. Nussinov, R. Shrock, Phys. Rev. D 80, 054003 (2009)
[13] F. Giacosa, Acta Phys. Polon. Supp. 10, 1021 (2017)

doi:10.5506/APhysPolBSupp.10.1021 [arXiv:1708.03755 [hep-ph]].
[14] S. Weinberg, Phys. Lett. B 251, 288-292 (1990)
[15] S. Weinberg, Nucl. Phys. B 363, 3-18 (1991)
[16] M. T. Alfiky, F. Gabbiani and A. A. Petrov, Phys. Lett. B 640, 238 (2006)
[17] M. Suzuki, Phys. Rev. D 65, 097507 (2002)
[18] G. Chiladze, A. F. Falk and A. A. Petrov, Phys. Rev. D 58, 034013 (1998)