Search for a promising tetraquark candidate \(X(uds\bar{s})\) in \(pn \to \Lambda\Lambda X\)

Xiao-Hai Liu\(^1\) and Qiang Zhao\(^{1,2,3}\)

1) Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, P.R. China
2) Department of Physics, University of Surrey, Guildford, GU2 7XH, United Kingdom and
3) Theoretical Physics Center for Science Facilities, Chinese Academy of Sciences, Beijing 100049, P.R. China

(Dated: October 3, 2008)

We propose to search for a tetraquark candidate \(X(uds\bar{s})\) in \(pn \to \Lambda\Lambda X(uds\bar{s}) \to \Lambda\Lambda K^+K^0\) or \(\Lambda\Lambda K^+K^*\). The existence of tetraquark state \(X(uds\bar{s})\) with \(J^P = 1^-\) or \(1^+\) was predicted in the literature based on specific diquark effective degrees of freedom inside hadrons. In order to understand the underlying dynamics for exotic hadrons, a search for the tetraquark \(X(uds\bar{s})\) is strongly recommended. The proposed reaction involves two \(\Lambda\) production, of which the narrow widths make it a great advantage in the analysis of the final state missing mass spectrum. We make an estimate of the production rate of \(X(uds\bar{s})\) in an effective Lagrangian theory and find that for \(J^P = 1^-\) the sample events of \(\sim 2200\) \(nb^{-1}\) will be able to identify \(X(uds\bar{s})\) with five standard deviations at a width of 10 MeV to \(K^+K^0\) near threshold. For \(J^P = 1^+\) with a width of 20 MeV to \(KK^*\), the sample events of \(\sim 130\) \(nb^{-1}\) will be needed. Large production cross sections are expected in a kinematic region beyond the threshold. We emphasize the advantage of low background in this transition channel, and in the meantime caution the large uncertainties in the present estimate due to lack of knowledge about the \(X(uds\bar{s})\) state. Implications for its heavy-flavored partners \(qq\bar{c}\bar{c}\) and \(qq\bar{b}\bar{b}\) are briefly discussed.

PACS numbers: 13.25.-k, 13.75.Ev

I. INTRODUCTION

One puzzling question in low-energy QCD is the apparent absence of “exotic states” in hadron spectrum. Here, “exotic states” are referred to baryons beyond \(qqq\) or mesons beyond \(q\bar{q}\) which are allowed by QCD symmetry but different from conventional constituent quark model classifications. So far, although there are some candidates for such exotic states, none of those has been indisputably established in experiment [1].

For those exotic states, if they have quantum numbers that cannot be constructed by conventional \(qqq\) or \(q\bar{q}\), determination of their exotic quantum numbers is a direct way of confirming their existence. However, for those with conventional quantum numbers, evidence for their existence can be contaminated by conventional quark model configurations due to possible configuration mixings. Their determination hence is a challenge for both experiment and theory. In 2003, experimental evidence for a pentaquark \(\Theta^+ (uudds)\) was reported by LEPS Collaboration at SPring-8 [2], which immediately initiated tremendous interests and activities in both experiment and theory. With a flavor configuration of \(uudds\), i.e. strangeness \(S = +1\), confirmation of such a state would be a direct evidence for the existence of exotic states.

Although the final confirmation of this pentaquark state still needs further experimental studies, it sparked many novel ideals in the understanding of quark-quark interactions and effective degrees of freedom within hadrons, among which the diquark effective degrees of freedom have attracted a lot of attention [3, 4, 5]. We shall not review those progresses here since there have been a lot of detailed discussions in the literature. We only point out that as a follow-up of the pentaquark scenario it will give rise to specific properties of effective diquark degrees of freedom, hence will predict existence of other multiquark exotics such as tetraquark states due to the attractive forces between the diquark clusters [6]. It is interesting to realize that such diquark effective degrees of freedom may lead to a formation of broad tetraquarks even though a narrow pentaquark presumably does not exist. Studying their possible
manifestations in experiment forms our motivation of this work. In particular, we shall identify a unique channel for a tetraquark candidate production, and investigate possibilities of establishing or eliminating it in future experimental measurements.

A promising tetraquark candidate with the valence quark content $ud\bar{s}\bar{s}$ has been broadly studied recently, while some relevant earlier work has been done by Jaffe decades ago. Burns, Close and Dudek suggested that if the diquark degrees of freedom such as the ones proposed for the pentaquark $\Theta^+ (uudd\bar{s})$ exist, there should also exist an isoscalar tetraquark $ud\bar{s}\bar{s}$ with $J^P = 1^-$, mass about 1.6 GeV, and decaying into $K^0 K^+$ with a width around $\mathcal{O}(10 \sim 100)$ MeV. They used a novel configuration to turn a “bad diquark” into a “good diquark”. Karliner and Lipkin thought there would be an isoscalar $ud\bar{s}\bar{s}$ meson with $J^P = 0^+$. As it cannot couple to $K^0 K^+$ or $KK\pi$ considering the generalized Bose statistics and parity conservation, the lowest decay mode would be four-body $KK\pi\pi$ channel which may result in a narrow width and make the state detectable. In contrast, Kanada-En’yo et al. argued that the $ud\bar{s}\bar{s}$ meson with $J^P = 1^+$ may be a stable and low-lying state, and decay into $KK^+$ via $S$-wave.

Motivated by the above studies, we propose to search for this tetraquark candidate $X(ud\bar{s}\bar{s})$ in $pn \rightarrow \Lambda \Lambda \Lambda X(ud\bar{s}\bar{s})$ which is illustrated in Fig. 1. Although there are still large uncertainties with the model predictions, the cross section may be small and the width of $X(ud\bar{s}\bar{s})$ may be broad, the background which contains two narrow $\Lambda$’s, is so clean that it may be possible to extract resonance signals in the missing mass spectrum of two $\Lambda$ final states.

II. THE MODEL

In this section we shall study tetraquarks of $J^P = 1^-$ and $1^+$ as two most possible candidates with effective Lagrangians. In principle, the tetraquark state can be produced via several typical transitions as illustrated in Figs. 1 and 2 which can be labeled by the Mandelstam variables, i.e. $t$, $u$ and $s$-channel. In Fig. 1 ($t$-channel), two meson exchanges are required. In Fig. 2a) ($u$-channel) a meson and a $\Xi$ state will mediate the transition while in Fig. 2b) ($s$-channel) a meson and a pentaquark state $\Theta$ exchange are needed. Since we have no applicable information on the $u$ and $s$-channel transitions, we do not discuss their contributions in this work. We shall focus on the $t$-channel transition and the details of the model are given as follows.

A. Production of $X(ud\bar{s}\bar{s})$ with $J^P = 1^-$

For the process illustrated in Fig. 1 the following effective Lagrangians are adopted for the production of $X(ud\bar{s}\bar{s})$ with $J^P = 1^-$:

$$\mathcal{L}_{KNA} = -ig_{KNA} \bar{N} \gamma_5 \Lambda K + H.c. \quad (1)$$

$$\mathcal{L}_{K^*NA} = -g_{K^*NA} \bar{N} \left( \gamma_\mu \Lambda K^{*\mu} - \frac{K^{*\mu} \Lambda}{2M_N} \sigma_{\mu\nu} \Lambda \partial^{\nu} K^{*\mu} \right) + H.c. \quad (2)$$

$$\mathcal{L}_{K\Xi} = ig_{K\Xi} X^\mu \left( \partial_\mu \bar{K} - \bar{K} \partial_\mu K \right) + H.c. \quad (3)$$

$$\mathcal{L}_{K^*\Xi} = g_{K^*\Xi} \varepsilon^{\alpha\beta\gamma\delta} \partial_\alpha X_\beta \partial_\gamma K_\delta^* \bar{K} + H.c. \quad (4)$$

Then we adopt the following values for the $KNA$ and $K^*NA$ couplings from Refs. [12, 13] which are also adopted by Ref. [14]:

$$g_{K^*NA} = -4.26, \quad \kappa_{K^*NA} = 2.66,$$

$$g_{KNA} = -\frac{1}{\sqrt{3}} (1 + 2F) g_{\pi NN} = -13.24, \quad (5)$$

where $g_{KNA}$ is obtained by using SU(3) flavor symmetry relation.
The coupling constant $g_{KKX}$ is determined by the decay width of $X$ which is predicted by Ref. 3: 
$$
\Gamma(X \rightarrow K^+ K^0) \sim O(10 \sim 100) \text{ MeV}. 
$$
With the effective Lagrangian mentioned previously, we have:
$$
\Gamma(X \rightarrow K^+ K^0) = \frac{g_{KKX}^2 (m_X^2 - 4 m_K^2)^{3/2}}{48 \pi m_X^2},
$$
which leads to
$$
g_{KKX} = 1.4, \text{ when } \Gamma(X \rightarrow K^+ K^0) = 10 \text{ MeV}. 
$$
We also assume that $\Gamma(X \rightarrow KK^*) \sim O(10 \sim 100) \text{ MeV}$, with the similar process, we have:
$$
\Gamma(X \rightarrow KK^*) = \frac{g_{KK^*X}^2}{48 \pi} \frac{[(m_X^2 - (m_K^* + m_K)^2)(m_X^2 - (m_K^* - m_K)^2)]^{1/2}}{2 m_X^2} \times [(m_X^2 + m_{K^*}^2 - m_K^2)^2 - 4 m_K^2 m_{K^*}],
$$
and
$$
g_{KK^*X} = 2.6, \text{ when } \Gamma(X \rightarrow KK^*) = 10 \text{ MeV}. 
$$
For the vertices $KNA$ and $K^*NA$, we introduce the covariant monopole form factor at each interaction vertex as the intermediate meson may be off-shell,
$$
F_{M}(q^2) = \frac{\Lambda^2 - M^2_{ex}}{\Lambda^2 - q^2},
$$
where $\Lambda = \Lambda_{KNA}$, $\Lambda_{K^*NA}$ here, and $M_{ex}$ are the corresponding masses of the exchanged mesons, $m_K$ or $m_{K^*}$. The following commonly used values are adopted [13]:
$$
\Lambda_{KNA} = 1.1 \text{ GeV}, \quad \Lambda_{K^*NA} = 1.0 \text{ GeV}. 
$$
So far there are no available data for the determination of the $KKX$ and $KK^*X$ form factors. We hence introduce a dipole form factor for the meson coupling vertices, i.e.
$$
F_{X}(q^2) = \left(\frac{\Lambda_X^2 - M^2_{ex}}{\Lambda_X^2 - q^2}\right)^2,
$$
where $\Lambda_X$ is the cut-off energies for $KKX$ and $KK^*X$, and $M_{ex}$ denotes the corresponding masses of the exchanged mesons, $m_K$ or $m_{K^*}$, respectively. The $\Lambda_X$-dependence of the total cross section will be illustrated later.

In Fig. 1 some of the kinematic parameters are defined as follows:
$$
q_1 = r_1 - s_1, \quad q_2 = r_2 - s_2 \\
w_1 = r_1 - s_2, \quad w_2 = s_1 - r_2.
$$
The corresponding transition matrix elements are then given as follows:
$$
T_{1f_i} = g_{KKX} g_{KNA}^2 \bar{u}(s_1) \gamma_5 u(r_1) \bar{u}(s_2) \gamma_5 u(r_2) \frac{(q_1 - q_2) \cdot \epsilon_X}{(q_1^2 - m_k^2)(q_2^2 - m_k^2)} F_K(q_1^2) F_K(q_2^2) F_{KKX}(q_2^2) \\
- g_{KKX} g_{KNA}^2 \bar{u}(s_2) \gamma_5 u(r_1) \bar{u}(s_1) \gamma_5 u(r_2) \frac{(w_1 - w_2) \cdot \epsilon_X}{(w_1^2 - m_k^2)(w_2^2 - m_k^2)} F_K(w_1^2) F_K(w_2^2) F_{KKX}(w_2^2),
$$
$$
T_{2f} = \Gamma_{KNA} (s_1, r_1) \Gamma_{K^*NA}^\mu (s_2, r_2) \Gamma_{KK^*X}^{\delta \beta} (t_0, q_2) \frac{i P_{\mu \delta} (q_2) \epsilon_\beta}{(q_1^2 - m_k^2)(q_2^2 - m_k^*)} F_K(q_1^2) F_{K^*}(q_2^2) F_{KK^*X}(q_2^2) \\
- \Gamma_{KNA} (s_2, r_1) \Gamma_{K^*NA}^\mu (s_1, r_2) \Gamma_{KK^*X}^{\delta \beta} (t_0, w_2) \frac{i P_{\mu \delta} (w_2) \epsilon_\beta}{(w_1^2 - m_k^2)(w_2^2 - m_k^*)} F_K(w_1^2) F_{K^*}(w_2^2) F_{KK^*X}(w_2^2),
$$
(15)
where

\[ \Gamma_{K^0 N A} (s_1, r_1) = -i g_{K^0 N A} \bar{u} (s_1) \gamma_5 u (r_1) \]  
\[ \Gamma_{K^* N A} (s_2, r_2) = g_{K^* N A} \bar{u} (s_2) \left( \gamma^\mu - \frac{i k_{K^* N A} \sigma^\mu \nu q_{2\nu}}{2M_N} \right) u (r_2) \]  
\[ \Gamma_{KK^* X}^{\beta\delta} (t_0, q_2) = g_{KK^* X} e^{\alpha\beta\gamma t_0} q_{2\gamma} \]  
\[ P_{\mu\delta} (q_2) = g_{\mu\delta} - \frac{q_{2\mu} q_{2\delta}}{m_{K^*}^2} \]  

The transition matrix element \( T_{3f_1} \) for Fig. 1(c) can be obtained by making a momentum substitution in \( T_{2f_1} \), i.e. \( r_1 \rightarrow r_2 \) and \( s_1 \rightarrow s_2 \).

### B. Production of \( X(u d \bar{s} s) \) with \( J^P = 1^+ \)

As studied in [8], axial vector meson \( ud \bar{s} s \) (\( J^P = 1^+ \)) is a good tetraquark candidate. Within a fluid-tube model, this state might appear around 1.4 GeV with a width of \( 4(20 \sim 80) \) MeV. We apply these quantities as an input to give an estimation of the cross section. The process is similar to the previous section except for the effective Lagrangian for the coupling of \( KK^*X \):

\[ \mathcal{L}_{KK^* X} = g_{KK^* X} (\partial^\alpha K^{*\beta} \partial^\gamma \bar{K} X_{\beta} - \partial^\alpha K^{*\beta} \partial^\gamma \bar{K} X_{\alpha}) + H.c. \]  

The meson \( X(u d \bar{s} s) \) with \( J^P = 1^+ \) can be produced via \( KK^* \)-exchange but not \( KK \)-exchange in this process. Thus we only need to consider Fig. 1(b) and (c). The following is the transition amplitude:

\[ T_{2f_1} = \Gamma_{K^0 N A} (s_1, r_1) \Gamma_{K^* N A}^{\mu} (s_2, r_2) \Gamma_{KK^* X}^{\beta\delta} (t_0, q_2) \frac{i P_{\mu\delta} (q_2) \epsilon_\beta}{(q_1^2 - m_{K^*}^2)(q_2^2 - m_{K^*}^2)} F_K (q_1^2) F_{K^*} (q_2^2) F_{KK^* X} (q_2^2) \]

\[ -\Gamma_{K^0 N A} (s_2, r_1) \Gamma_{K^* N A}^{\mu} (s_1, r_2) \Gamma_{KK^* X}^{\beta\delta} (t_0, w_2) \frac{i P_{\mu\delta} (w_2) \epsilon_\beta}{(w_1^2 - m_{K^*}^2)(w_2^2 - m_{K^*}^2)} F_K (w_1^2) F_{K^*} (w_2^2) F_{KK^* X} (w_2^2), \]

where

\[ \Gamma_{KK^* X}^{\beta\delta} = g_{KK^* X} (q_1 \cdot q_2 g^{\beta\delta} - q_2^\beta q_1^\delta), \]  

and the other terms are the same as the previous section. The coupling constant \( g_{KK^* X} \) is determined by the width:

\[ \Gamma (X \rightarrow KK^*) = g_{KK^* X}^2 \frac{24\pi m_X^3}{|p_{K^*}|} \left[ 2 \frac{1}{m_X^2} \right], \]

which leads to

\[ g_{KK^* X} = 7.5, \quad \text{when } \Gamma (X \rightarrow KK^*) = 20 \text{ MeV}. \]  

It should be cautioned that the adopted couplings still bare large uncertainties due to our limited knowledge on the tetraquark states. This will consequently bring uncertainties to the estimated production cross sections. However, we emphasize that our strategy here is to single out the \( pn \rightarrow \Lambda K^+K^0 \) and \( \Lambda\Lambda K^* \) channel which are advantageous for detecting the tetraquark states in experiment. Our calculation is to provide a reasonable estimate of the feasibility for future experiments.
C. Numerical Results

Experimental data for $pn \to \Lambda K^+ K^0$ and $\Lambda \Lambda K K^*$ are not available so far. Interestingly, there are a few experiments in 1980's on the inclusive reaction $pp \to 2 \Lambda + \text{anything}$ and exclusive reaction $pp \to 2 \Lambda + 2 K^+$ motivated by the search for dibaryon state formed by two $\Lambda$. The total cross section of the inclusive reaction is about $19 \pm 10 \mu b$ with the beam energy at $E_n = 32.1$ GeV ($W = 7.87$ GeV), and that of the exclusive reaction has an upper limit about $460 \text{nb}$ at $E_n = 7.8$ GeV ($W = 4.05$ GeV). These data still possess large uncertainties and need to be improved. But at this moment, they can serve as a guidance for constraining our parameter space, and give an estimate of the cross sections for $X$ tetraquark productions.

Although the values for coupling constants can be determined indirectly by other experimental data such as strangeness productions and theoretical model prediction for $X \to K^+ K^0$, we still lack information about the choice of the cut-off energies in the form factors. Therefore, a thorough investigation of the parameter space is necessary. We first fix the parameters $\Lambda_{KNA}, \Lambda_{K^*NA}$ as in Eq. (11) and set $\Lambda_{KKX} = \Lambda_{K^*K^*X} \equiv \Lambda_X = 1.2 \text{GeV}$, which is a commonly adopted value for meson-meson interaction form factors. This allows us to obtain the total cross sections for different $X$ states. We then examine the model sensitivities to $\Lambda_X$ by extracting the $\Lambda_X$ dependence of the total cross sections at a given energy.

In Fig. 3 the energy dependence of the total cross section for $pn \to \Lambda \Lambda X$ is presented for $X$ with $J^P = 1^-$. It shows that the contribution is dominated by $KK$-exchange, while $K K^*$-exchange is small. The total cross section also exhibits enhancement above threshold at $W \simeq 7.0 \text{GeV}$, and then dies out with the increasing $W$. The peak value is about $0.115 \mu b$, which is much smaller than the cross section for $pp \to \Lambda \Lambda + \text{anything}$. This value is also below the upper limit of the exclusive $pp \to 2 \Lambda + 2 K^+$ cross section near threshold $18$. As shown by the solid curve in Fig. 3, the cross section near threshold at $E_n = 7.8$ GeV ($W = 4.05$ GeV) is about $2.3 \text{nb}$. This value can be seen more clearly in Fig. 4(a) at $\Lambda_X = 1.2 \text{GeV}$.

The sensitivity of the total cross section to the cut-off energy $\Lambda_X$ is examined in Fig. 4. The range of $\Lambda_X = 1.0 \sim 1.5 \text{GeV}$ is a commonly accepted one in the literature. As shown by the solid curve, the cross section varies between 0.05 and $0.3 \mu b$ in terms of $\Lambda_X$, which suggests some sensitivities to the cut-off energies. The $\Lambda_X$-dependences of the exclusive $KK$ and $KK^*$ cross sections are also presented by the dashed and dotted curves, respectively, and a relatively sensitive behavior of the $KK^*$ exchange is found. However, since contributions from this transition are rather small, the overall behavior of the cross sections is dominated by the $KK$ exchange. Although the sensitivity to the cut-off energy brings some uncertainties to the estimate, it does not prevent us from drawing some preliminary conclusions on the production rate for $X$.

Figure 5 shows the results for $X$ production with $J^P = 1^+$. The underlying transition is via the $KK^*$ exchanges and the cross section increases with the energies. A fast rise appears near threshold, and then the cross section becomes rather flat. This behavior is different from the case of $J^P = 1^-$, where an obvious enhancement appears above threshold. At $W < 10 \text{GeV}$, the total cross section is less than $0.21 \mu b$. Then, it slowly increases to $0.25 \mu b$ at $W = 20 \text{GeV}$. Nevertheless, the cross sections for $1^+$ are relatively larger than those for $1^-$ over a wide range of $W$. The different behavior of the near threshold cross sections between $1^-$ and $1^+$ makes it an interesting place for looking for the $X$ state in experiment.

The $\Lambda_X$-dependence of the total cross section is also investigated and the results are displayed in Fig. 6. The cross section turns to be more sensitive to $\Lambda_X$ than the case of $J^P = 1^-$. This will increase the uncertainties of the model predictions. However, in terms of gaining a rough idea about the production rate of $X$ and providing a guidance of the future experimental plan, the range of the uncertainties can still be regarded as acceptable.

Taking the experimental data for $pp \to 2 \Lambda + 2 K^+$ as a guidance, we can estimate the number of sample events for establishing the $X(udss)$. It is reasonable to assume that the cross section from the background contributions is $\sigma_{bg} = 460 \text{nb}$ ($W = 4.05 \text{GeV}$), while the signal cross section is $\sigma_X \simeq 2.3 \text{nb}$ ($J^P = 1^-$) or $\sigma_X \simeq 9.5 \text{nb}$ ($J^P = 1^+$). With a luminosity of $L$ and event-collecting time $t$, we require the tetraquark signal of five standard deviations ($5\sigma$):

$$\frac{N_X}{\sqrt{N_{bg}}} > 5,$$

(25)
where \( N_X \) and \( N_{bkg} \) are the sample events for \( X(ud\bar{s}s) \) and the background, respectively, and they are given by

\[
N_X = L \times t \times \sigma_X , \quad (26)
\]
\[
N_{bkg} = L \times t \times \sigma_{bkg} . \quad (27)
\]

This leads to:

\[
N_{bkg} > 1.0 \times 10^6 \text{ or } N_X > 5.0 \times 10^3 \text{ or } L \times t > 2174 \text{ nb}^{-1}, \text{ for } J^P = 1^- , \quad (28)
\]

and

\[
N_{bkg} > 5.8 \times 10^4 \text{ or } N_X > 1.2 \times 10^3 \text{ or } L \times t > 127 \text{ nb}^{-1}, \text{ for } J^P = 1^+ . \quad (29)
\]

Note that the above estimations are in the near-threshold region with \( W = 4.05 \text{ GeV} \) instead of the kinematics with the largest cross sections, e.g. \( W \simeq 7 \text{ GeV} \). The reason is that we would like to compare our predictions with the only relevant experimental data from Ref. [18], of which the inclusive cross section provide a rough check for the self-consistency of our model. It would be more interesting to look at the energy region of \( W = 7 \text{ GeV} \) where the cross sections are predicted to have a maximum for \( J^P = 1^- \) and be sizeable for \( J^P = 1^+ \). Given the experimental availability in the future, larger cross sections at \( W = 7 \text{ GeV} \) will make the detection of \( X \) much easier there.

III. SUMMARY

In this work we studied the production of possible tetraquark candidates \( X(ud\bar{s}s) \) in \( pn \rightarrow \Lambda \Lambda X \). Difficulty in the search for such an exotic state lies on the generally-large background in its productions, while as here we propose that \( pn \rightarrow \Lambda \Lambda X(ud\bar{s}s) \rightarrow \Lambda \Lambda K^+K^0 \text{ and } \Lambda \Lambda KK^* \) are rather clean and ideal for looking for its signals. The numerical results for the cross sections were presented with a reasonable consideration of the parameter spaces. For \( X(ud\bar{s}s) \) of \( J^P = 1^- \), an obvious enhancement were observed above threshold, while for \( J^P = 1^+ \), no enhancement was seen. In both cases, typical values of hundreds of \( nb \) were found for the total cross section above the threshold region.

We adopted the experimental upper limit of \( 460 \text{ nb} \) at \( E_n = 7.8 \text{ GeV} \ (W = 4.05 \text{ GeV}) \) [18], and estimate the signal-background rate. It shows that in order to observe the signal of \( X(ud\bar{s}s) \) at the standard deviations of \( 5\sigma \), the sample events of the background must be larger than \( 1.0 \times 10^6 \) for \( J^P = 1^- \) (or \( 5.8 \times 10^4 \) for \( J^P = 1^+ \)), or the product of luminosity and experiment time should be larger than about \( 2174 \text{ nb}^{-1} \) for \( J^P = 1^- \) (or \( 127 \text{ nb}^{-1} \) for \( J^P = 1^+ \)). It should be noted that the estimates were made near threshold where the cross sections are not the maximum. This is due to the consideration of adopting the only available experimental information of Ref. [18] to estimate the signal-background rate. To search for the signal of \( X(ud\bar{s}s) \), the ideal kinematic region should be around \( W \simeq 7 \text{ GeV} \), where an enhancement of the cross section was predicted for \( J^P = 1^- \) and the cross section also turned to be sizeable for \( J^P = 1^+ \). Meanwhile, we caution that although the theoretical study of the tetraquark properties is more essentially based on the diquark degrees of freedom, our knowledge on the pentaquark should also have influence on the estimate of the tetraquark decay widths. This will bring model-dependence to the theoretical predictions. Taking into account the form factors and couplings, the uncertainties of the results can be as large as two orders of magnitude.

The similar experimental scheme could also be used to search for other exotic states such as \( X(qq\bar{c}c) \) in the process \( NN \rightarrow \Lambda_c\Lambda_cX \) or \( X(qq\bar{b}b) \) in the process \( NN \rightarrow \Lambda_b\Lambda_bX \) and so on. Since we lack experimental constraints on the \( DNA_c \) (\( BNA_b \)) and \( D^*NA_c \) (\( B^*NA_b \)) couplings, we cannot make quantitative estimations for those tetraquark productions. But we stress the advantages of such a reaction process for the search for tetraquark species of \( X(qqQQ) \). Experimental search for such exotics in hadron collider should be able to provide deeper insights into the properties of strong QCD dynamics.

Acknowledgement

We thanks B.S. Zou for very useful discussions and comments on this work. This work is supported, in part, by the U.K. EPSRC (Grant No. GR/S99433/01), National Natural Science Foundation of China
(Grant No.10675131 and 10491306), and Chinese Academy of Sciences (KJCX3-SYW-N2).

[1] E. Klempt and A. Zaitsev, Phys. Rept. 454, 1 (2007) [arXiv:0708.4016 [hep-ph]].
[2] T. Nakano et al. [LEPS Collaboration], Phys. Rev. Lett. 91, 012002 (2003) [arXiv:hep-ex/0301020].
[3] R. L. Jaffe, Phys. Rev. D 15, 267 (1977); Phys. Rev. D 15, 281 (1977).
[4] R. L. Jaffe and F. Wilczek, Phys. Rev. Lett. 91, 232003 (2003) [arXiv:hep-ph/0307341].
[5] M. Karliner and H. J. Lipkin, Phys. Lett. B 612, 197 (2005) [arXiv:hep-ph/0411136].
[6] F. E. Close, Int. J. Mod. Phys. A 20, 5156 (2005) [arXiv:hep-ph/0411396].
[7] T. Burns, F. E. Close and J. J. Dudek, Phys. Rev. D 71, 014017 (2005) [arXiv:hep-ph/0411160].
[8] Y. Kanada-En'yo, O. Morimatsu and T. Nishikawa, Phys. Rev. D 71, 094005 (2005) [arXiv:hep-ph/0502042].
[9] J. Vijande, F. Fernandez, A. Valcarce and B. Silvestre-Brac, Eur. Phys. J. A 19, 383 (2004) [arXiv:hep-ph/0310007].
[10] Y. Cui, X. L. Chen, W. Z. Deng and S. L. Zhu, Phys. Rev. D 73, 014018 (2006) [arXiv:hep-ph/0511150].
[11] W. L. Wang, F. Huang, Z. Y. Zhang, Y. W. Yu and F. Liu, J. Phys. G 34, 1771 (2007) [arXiv:0707.0390 [nucl-th]].
[12] T. A. Rijken, V. G. J. Stoks and Y. Yamamoto, Phys. Rev. C 59, 21 (1999) [arXiv:nucl-th/9807082].
[13] V. G. J. Stoks and T. A. Rijken, Phys. Rev. C 59, 3009 (1999) [arXiv:nucl-th/9901028].
[14] Y. Oh and H. Kim, Phys. Rev. C 73, 065202 (2006) [arXiv:hep-ph/0602112].
[15] G. Q. Li, C. H. Lee and G. E. Brown, Nucl. Phys. A 625, 372 (1997) [arXiv:nucl-th/9706057].
[16] W. Liu, C. M. Ko and S. H. Lee, Nucl. Phys. A 728, 457 (2003) [arXiv:nucl-th/0308013].
[17] M. Y. Bogolyubsky et al., Sov. J. Nucl. Phys. 50, 424 (1989) [Yad. Fiz. 50, 683 (1989)].
[18] Yu. D. Aleshin, I. L. Kiselevich, I. A. Melnichenko, V. I. Mikhailichenko, S. Y. Nikitin, V. D. Shkarlet and A. V. Shidlovsky, JETP Lett. 43, 200 (1986) [Pisma Zh. Eksp. Teor. Fiz. 43, 159 (1986)].
[19] W. K. H. Panofsky, Phys. Today 40N1, 110 (1987).
FIG. 1: The tetraquark candidate $X(ud\bar{s}\bar{s})$ production via $t$-channel transitions in $pn \rightarrow \Lambda\Lambda X$.

FIG. 2: The tetraquark candidate $X(ud\bar{s}\bar{s})$ production via (a) $u$-channel and (b) $s$-channel transitions in $pn \rightarrow \Lambda\Lambda X$.

FIG. 3: Energy dependence of total cross sections for $pn \rightarrow \Lambda\Lambda X$ with $J^P = 1^-$ for the $X$. We have assumed $\Gamma(X \rightarrow K^+ K^0) = \Gamma(X \rightarrow K K^*) = 10$ MeV [7]. The cut-off energy is set as $\Lambda_{KKX} = \Lambda_{KK^*X} = \Lambda_X = 1.2$ GeV. The dashed line is the contribution of $KK$-exchange, dotted line is that of $KK^*$-exchange, and solid line is the total contribution.
FIG. 4: $\Lambda_X$-dependence of total cross sections for $pn \rightarrow \Lambda \Lambda X$ with $J^P = 1^-$ at two different energies. The notations are the same as those in Fig. 3.

FIG. 5: Energy dependence of total cross sections for $pn \rightarrow \Lambda \Lambda X$ with $J^P = 1^+$ for the $X$. We have assumed $\Gamma(X \rightarrow KK^*) = 20$ MeV. The cut-off energy is set as $\Lambda_X = 1.2$ GeV.
FIG. 6: $\Lambda_X$-dependence of total cross sections for the $X$ with $J^P = 1^+$ at two different energies.