Measuring the CP Angle $\beta$ in Hadronic $b \to s$ Penguin Decays

David London$^a$ and Amarjit Soni$^b$

Abstract: Asymmetric $e^+e^-$ colliders running on the $\Upsilon(4S)$ ($B$ factories) will much more readily measure CP-violating asymmetries in the decays of $B_d^0$ and $B^\pm$ mesons than in the decays of $B_s^0$ mesons. As such, they will seemingly not be able to probe new phases in $B_s^0$-$\overline{B_s^0}$ mixing, i.e. in $b \to s$ transitions. However, by measuring the CP angle $\beta$ via $b \to s$ hadronic penguin decays such as $\overline{B_d} \to \eta'K_S$ and $\overline{B_d} \to \phi K_S$, and comparing its value to that obtained in $\overline{B_d} \to \Psi K_S$, it is possible to detect the presence of new physics in the $b \to s$ flavour-changing neutral current. Recent CLEO results are encouraging in this regard. They suggest that the branching ratio of the $b \to s$ penguin decay $B_d^0 \to \eta' K_S$ is anomalously large, about $4 \times 10^{-5}$, which will make it much easier to search for new physics in $b \to s$ transitions.
To date, the only experimental evidence for CP violation in the weak interactions comes from the kaon system, where CP violation in $K^0\overline{K}$ mixing has been observed. According to the standard model (SM), this CP violation is due to a complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Within the Wolfenstein parametrization of the CKM matrix [1], the only elements which have non-negligible phases are $V_{td}$ and $V_{ub}$:

$$V_{CKM} = \begin{pmatrix}
1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix},$$

where $\lambda = 0.22$ is the Cabibbo angle. These two complex matrix elements are conventionally parametrized as $V_{td} \equiv |V_{td}| \exp(-i\beta)$ and $V_{ub} \equiv |V_{ub}| \exp(-i\gamma)$. The phase information in the CKM matrix can be elegantly displayed using the well-known unitarity triangle (Fig. 1), which is due to the orthogonality of the first and third columns of the CKM matrix.

In the coming years, this explanation of CP violation will be tested at B factories with the BABAR, BELLE and CLEO detectors. The three angles of the unitarity triangle, $\alpha$, $\beta$ and $\gamma$ can be extracted through the measurements of CP-violating asymmetries in $B$ decays to hadronic final states [2]. For example, the CP asymmetries in $\overline{B}_d \to \pi^+\pi^-$ and $\overline{B}_d \to \Psi K_S$ probe $\alpha$ and $\beta$, respectively, and the angle $\gamma$ can be extracted from the CP asymmetry in $B^\pm \to DK^\mp$ [3]. In all cases, the CP phases can be obtained with virtually no hadronic uncertainty. (For the extraction of $\alpha$, since penguins are unlikely to be negligible, an isospin analysis (for the 2$\pi$ final state [4]) and/or a Dalitz plot analysis (3$\pi$ final state [5]) will probably be necessary.) Of course, there are many other decays which can be used to obtain the CP angles, but those mentioned above are the ones which are most often discussed – they have become the “standard” decay modes.

One alternative way of measuring $\gamma$ is via the CP asymmetry in $\overline{B}_s \to D^{\pm}_s K^{\mp}$ [6]. However, since in all probability the B factories are not going to be able to measure CP asymmetries in $B^0_d$ decays, this method will not be available. This will be an important point in the following discussion.

If there is new physics, there are basically three ways in which it can show up in measurements of CP asymmetries [7]:

- The relation $\alpha + \beta + \gamma = \pi$ is violated.
- Although $\alpha + \beta + \gamma = \pi$, one finds values for the CP phases which are outside of the SM predictions.
- The CP angles measured are consistent with the SM predictions, and add up to 180°, but are inconsistent with the measurements of the sides of the unitarity triangle.

2
Figure 1: The unitarity triangle. The angles $\alpha$, $\beta$ and $\gamma$ can be measured via CP violation in the $B$ system.

The principal way in which physics beyond the SM affects the CP asymmetries is via new contributions to $B^0 - \overline{B^0}$ mixing. There are, in fact, many models of new physics which can significantly affect this mixing \cite{4}. However, because $B$ factories will not measure CP asymmetries involving $B^0_s$ mesons, the measurements will be sensitive only to new physics in $B^0_s - \overline{B^0_s}$ mixing, i.e. in the $b \to d$ flavor-changing neutral current (FCNC). This has the consequence that if, as is expected, $\alpha$ and $\beta$ are measured using CP asymmetries involving $B^0_d$ decays, and $\gamma$ is obtained via $B^\pm \to DK^\pm$, the $B$ factories will automatically find that $\alpha + \beta + \gamma = \pi$ \cite{8}. This is because any new-physics effects in $B^0_d - \overline{B^0_d}$ mixing cancel when $\alpha$ and $\beta$ are added, and there are no mixing effects in the measurement of $\gamma$. If, on the other hand, $\gamma$ were measured using $B^0_s$ decays, it would be possible to find $\alpha + \beta + \gamma \neq \pi$ if there were new phases in $B^0_s - \overline{B^0_s}$ mixing, i.e. in the $b \to s$ FCNC.

Now, it is conceivable that the new physics significantly affects the $b \to s$ FCNC, without affecting the $b \to d$ FCNC appreciably. It would be a shame if this possibility remained untested at the $B$ factories. Is it really impossible to detect any new phases appearing in $B^0_s - \overline{B^0_s}$ mixing?

Fortunately, the answer to this question is no. It is in fact possible to detect new phases in the $b \to s$ FCNC, but in a somewhat different way. One of the alternative possibilities for measuring the angle $\beta$ involves $b \to s$ penguin decays \cite{9}. Such decays, being dominated by virtual $t$-quarks, involve the combination of CKM matrix elements $V_{tb}^*V_{ts}$, which is real in the Wolfenstein parametrization. Thus, in the context of the SM, if one measures the CP asymmetry in $(B_d \to f)$, where $f$ is a CP eigenstate and the decay is dominated by a $b \to s$ hadronic penguin, one is probing the phase of $B^0_d - \overline{B^0_d}$ mixing, which is simply $\beta$. 

\[ \text{Figure 1: The unitarity triangle. The angles } \alpha, \beta \text{ and } \gamma \text{ can be measured via CP violation in the } B \text{ system.} \]
The key point here is that this is the same CKM phase as that probed in $\overline{B}_d \to \Psi K_s$. Therefore, if one finds a discrepancy between the values of $\beta$ as measured in $\overline{B}_d \to \Psi K_s$ and in hadronic $b \to s$ penguins, this clearly points to new physics, with new phases, in the $b \to s$ FCNC. In particular, it indicates that there are new amplitudes contributing to these hadronic $b \to s$ penguin decays $[10]$. This same new physics will, in general, also lead to new phases in $B_s^0 - \overline{B}_s^0$ mixing, and hence to disagreements with the SM predictions for CP asymmetries in $B_s^0$ decays. Thus, by measuring $\beta$ in hadronic $b \to s$ penguins, one is performing essentially the same tests as if one measured CP asymmetries in $B_s^0$ decays.

(Note that this holds even if there is new physics in $B_d^0 - \overline{B}_d^0$ mixing.)

There are several models of new physics which can lead to new phases in the $b \to s$ FCNC $[7,10]$: four generations, non-minimal supersymmetric models such as effective supersymmetry $[11]$, and models with enhanced chromomagnetic dipole operators $[12]$. Models with $Z$-mediated FCNC’s $[13]$ will affect $b \to s$ hadronic penguins only marginally $[7]$. It should also be noted that if the new physics in the $b \to s$ FCNC has the same phase as in the SM (i.e. $\approx 0$), then the SM predictions will be unchanged. That is, there will be no discrepancy in the value of $\beta$ extracted from $\overline{B}_d \to \Psi K_s$ and from hadronic $b \to s$ penguins.

We now turn to an examination of the prospects for measuring $\beta$ via $b \to s$ hadronic penguins. One needs a final state which is a CP eigenstate. The decay mode first considered in this regard was $\overline{B}_d \to \phi K_s$ $[9]$. However, there is another possibility which has generated considerable excitement: $\overline{B}_d \to \eta' K_s$. The reason that this mode is interesting is that the branching ratio for the corresponding charged $B$ decay has recently been found to be anomalously large: $B(B^+ \to \eta' K^+) = (7.8^{+2.7}_{-2.2} \pm 1.0) \times 10^{-5}$ $[14]$. We therefore expect that $B(B_d^0 \to \eta' K_s) \approx 4 \times 10^{-5}$. This large branching ratio greatly improves the usefulness of the decay mode $\overline{B}_d \to \eta' K_s$ for getting at $\beta$.

In fact, there are a large number of final states that can be used: $\phi K_s, \eta' K_s, \pi^0 K_s, \rho^0 K_s, \omega K_s, \eta K_s$, etc. In principle, one can simply add the measured CP asymmetries in all of these modes, including a minus sign if the CP of the final state is negative, to obtain a larger signal.

There are two comments to be made here. First, although $\phi K_s$ is pure $b \to s$ penguin, the other final states get some contributions from the quark-level $\bar{b} \to \bar{u} u s\bar{s}$ tree diagram. Since the tree diagram has a different weak phase than that of the penguin, this can spoil the cleanliness of the method for extracting $\beta$. In order to see how important the effect is, one has to estimate the ratio of the tree ($T$) and penguin ($P$) amplitudes.

Let us first consider $\overline{B}_d \to \eta' K_s$. We write

$$\frac{T_{\eta' K_s}}{P_{\eta' K_s}} = \frac{T_{\eta' K_s}}{T_{\pi^+ \pi^-}} \frac{T_{\pi^+ \pi^-}}{P_{\eta' K_s}}. \tag{2}$$

The first piece on the right-hand side (RHS) can be estimated by a simple comparison
of the Feynman diagrams. The tree contribution to the decay \( B_d^0 \to \pi^+\pi^- \) is color-allowed and is controlled by the CKM matrix elements \( V_{ub}^*V_{ud} \). Compared to this, the analogous contribution to \( B_d^0 \to \eta'K_s \) is suppressed by three factors: (i) it is colour suppressed, (ii) it has the CKM matrix elements \( V_{ub}^*V_{us} \), which is a factor \( \lambda \) smaller, and (iii) one has to include the normalizations \( u\bar{u} \sim \eta'/\sqrt{3} \) and \( d\bar{s} \sim K_s/\sqrt{2} \). There may be additional \( SU(3) \)-breaking effects involving form factors or decay constants, but these are unknown and are probably of \( O(1) \). The second piece on the RHS can be obtained from the measured branching ratios, assuming that the decays \( B_d^0 \to \pi^+\pi^- \) and \( B_d^0 \to \eta'K_s \) are dominated by the tree and penguin contributions, respectively. The upper limit on \( B_d^0 \to \pi^+\pi^- \) is \( 1.5 \times 10^5 \) \( [14] \). We thus obtain

\[
T_{\eta'K_s}/P_{\eta'K_s} < \frac{\lambda}{\sqrt{6}} \frac{a_2}{a_1} \sqrt{\frac{1.5}{4}} = 0.018 ,
\]

where we have conservatively taken the color-suppression factor to be \( a_2/a_1 = 1/3 \). (For \( B \) decays into heavier final states, the suppression factor has been found to be even smaller: \( a_2/a_1 = 0.2 \) \( [15] \).) Thus the tree contribution to \( B_d^0 \to \eta'K_s \) is negligible, so that the CP asymmetry in this mode does indeed measure \( \beta \) to a very good approximation.

The ratio of \( T/P \) for the other decay modes can be calculated similarly. Isospin implies that the amplitude for \( B_d^0 \to \pi^0K_s \) is half that of \( B_d^0 \to \pi^-K^+ \). From the measured branching ratio of \( B(B_d^0 \to \pi^-K^+) = (1.5^{+0.5}_{-0.4}(0.2)) \times 10^{-5} \) \( [14] \) one therefore obtains \( B(B_d^0 \to \pi^0K_s) \simeq 4 \times 10^{-6} \). The remaining final states are not really related to \( \pi^-K^+ \), but one can still estimate their branching ratios by comparing their penguin contributions to that of \( \pi^-K^+ \). In this case, one finds that \( B(B_d^0 \to \rho^0K_s) \simeq B(B_d^0 \to \omega K_s) \simeq 4 \times 10^{-6} \) and \( B(B_d^0 \to \eta K_s) \simeq 1 \times 10^{-6} \). In all cases, we then obtain

\[
\frac{T}{P} \lesssim 0.04 .
\]

Thus, these modes are also pure \( b \to s \) penguin to a good approximation.

There is one detail which is worth mentioning. Due to \( SU(3) \) breaking, the physical \( \eta \) and \( \eta' \) are in fact linear combinations of the \( \eta \) and \( \eta' \) states used above. However, since in all cases the tree contribution is very small, the inclusion of \( \eta-\eta' \) mixing does not affect the analysis.

To sum up this point, CP asymmetries in \( b \to s \) penguins do indeed measure the CP angle \( \beta \). The tree contributions to these decays are quite small, at most a few percent. It is therefore possible to add up the measured CP asymmetries in all these modes to obtain a larger signal. If the value of \( \beta \) extracted in this way differs by more than about 10% from that found in \( \Psi K_s \), then it is a clear signal of new physics, with new phases, in the \( b \to s \) FCNC. If the difference is less than about 10%, it could in principle be due to the tree contamination. However, this can be checked by using only the final states \( \phi K_s \) and \( \eta'K_s \) (to a very good approximation).
The second comment concerns the standard way of extracting $\beta$ via the CP asymmetry in \((B_d^\pm) \rightarrow \Psi K_s\). Given that one can also obtain $\beta$ through $b \rightarrow s$ penguins, how do these two methods compare with one another?

- \((B_d^\pm) \rightarrow \Psi K_s\): The branching ratio is $4 \times 10^{-4}$. Assuming that the $K_s$ is detected in the $\pi^+\pi^-$ mode (67% branching ratio), and that the $\Psi$ is detected via its decay to $\mu^+\mu^-$ or $e^+e^-$ (12% b.r.), the product branching ratio for \((B_d^\pm) \rightarrow \Psi K_s\) is $\sim 32 \times 10^{-6}$.

- \((B_d^\pm) \rightarrow \eta' K_s\): The branching ratio for this process has been found to be about $4 \times 10^{-5}$. The $\eta'$ has two important modes through which it may be detected: $\eta' \rightarrow \eta\pi\pi$ (followed by $\eta \rightarrow \gamma\gamma$) with a product b.r. of 17%, and $\eta' \rightarrow \rho^0\gamma$ (followed by $\rho^0 \rightarrow \pi^+\pi^-$) with a product b.r. of about 30%. Thus the total efficiency can approach 47%. Although CLEO’s detection efficiency for the $\eta'$ is at present only about 5% (where so far only the $\eta\pi\pi$ mode has been used), it is clearly important to improve upon this at the $B$-factory detectors. For the purpose of our discussion we will assume a combined efficiency of 20%, yielding an effective b.r. of $5 \times 10^{-6}$, which includes the b.r. for $K_s \rightarrow \pi^+\pi^-$.

- \((B_d^\pm) \rightarrow \phi K_s\): The branching ratio for this decay has not yet been measured, but we can estimate its value from measured quantities. The decay $B_d^0 \rightarrow \pi^-K^+$, which has a branching ratio of $1.5 \times 10^{-5}$, should be in the same ballpark as \((B_d^\pm) \rightarrow \phi K_s\). Taking into account that $d\bar{s} \sim K_s/\sqrt{2}$, we therefore estimate its branching ratio to be in the range of $(7 - 15) \times 10^{-6}$. (This is slightly larger than theoretical estimates which also include electroweak penguin contributions [10].) Assuming that the $\phi$ is detected through its $K^+K^-$ decay mode (49% b.r.), the product branching ratio for this decay is $\sim (2 - 5) \times 10^{-6}$.

The remaining modes — $\pi^0 K_s$, $\rho^0 K_s$, $\omega K_s$, $\eta K_s$ — can be analyzed similarly, using the decays $\pi^0 \rightarrow \gamma\gamma$ (99% b.r.), $\rho^0 \rightarrow \pi^+\pi^-$ (100% b.r.), $\omega \rightarrow \pi^+\pi^-\pi^0$ and $\pi^+\pi^-$ (91% b.r.), and $\eta \rightarrow \gamma\gamma$ (39% b.r.). In this case one needs estimates of the branching ratios and of the detection efficiencies for the neutral mesons, which at this point are unknown. A reasonable educated guess is that all of these product branching ratios are in the range $\sim (2 - 5) \times 10^{-6}$. Thus, when all the $b \rightarrow s$ penguin decay modes are added, they could have a combined yield of up to $3 \times 10^{-5}$. This approach is therefore quite promising: it may end up requiring just about the same number of $B$’s as compared to $\Psi K_s$, or perhaps a factor of two or three more.

To conclude, our observations are quite simple. $B$ factories are going to measure CP-violating asymmetries using $B_d^0$ and $B^\pm$ decays. Although there are a variety of ways of detecting the presence of physics beyond the SM, such measurements are sensitive only to new physics in $B_d^0-B_d^\pm$ mixing, i.e. in the $b \rightarrow d$ FCNC. However, it is conceivable that
the new physics enters only in the $b \to s$ FCNC, leaving the $b \to d$ FCNC unaffected. Since $B$ factories are not going to be able to measure CP asymmetries in $B^0_s$ decays, it appears that this possibility will remain untested.

In fact, there is a way of probing new physics in the $b \to s$ FCNC. In the SM, CP asymmetries in $\bar{B}_d \to \Psi K_s$ and $b \to s$ hadronic penguins both measure the CP angle $\beta$. If there is a difference between the measurements of $\beta$ obtained in these two ways, this directly indicates the presence of new physics, with new phases, in the $b \to s$ FCNC. In general, this same new physics will affect $B^0_s - \bar{B}^0_s$ mixing, and will show up in CP asymmetries involving $B^0_s$ decays.

Recent CLEO results are encouraging in this regard. In particular, there is evidence that the branching ratio for the decay $B^0_d \to \eta' K_s$ is anomalously large, about $4 \times 10^{-5}$. The CP asymmetry measured in this decay probes the CP angle $\beta$ to a very good approximation. One can also measure $\beta$ via the CP asymmetries in $B^0_d$ decays to $\phi K_s$, $\pi^0 K_s$, $\rho^0 K_s$, $\omega K_s$, $\eta K_s$, etc. The branching ratios for these decays have not yet been measured, but are expected to be $\sim 5 \times 10^{-6}$. In principle it should be possible to add up the measured CP asymmetries in all these modes to obtain a larger signal. The number of $B^*$s required to measure $\beta$ using this method may be about the same, or perhaps only a factor of 2-3 more, as that needed in the conventional mode, $\bar{B}_d \to \Psi K_s$. In any case, it is clear that the branching ratios for $b \to s$ hadronic penguins are sufficiently large that the CP asymmetries in these decays can probably be measured as easily as those used for the extraction of the other CP angles $\alpha$ and $\gamma$. With this in mind, it will be important to maximize the detection efficiency for the $\eta'$.

This method is perhaps the only way of detecting the presence of new phases in the $b \to s$ FCNC without measuring CP asymmetries in $B^0_s$ decays, and therefore should be a high priority at future $B$ factories.

Acknowledgements

We thank Jim Alexander, Bruce Behrens, Tom Browder, Peter Kim and Sheldon Stone for discussions. DL is grateful for the pleasant hospitality of the Brookhaven National Laboratory, where much of this work was done. This research was financially supported by NSERC of Canada and FCAR du Québec and in part by U.S. DOE contract DE-AC-76CH00016.

References

[1] L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.

[2] For reviews, see, for example, Y. Nir and H.R. Quinn in $B$ Decays, edited by S. Stone (World Scientific, Singapore, 1994), p. 520; I. Dunietz, ibid., p. 550; M. Gronau,
[3] M. Gronau and D. Wyler, *Phys. Lett.* **265B** (1991) 172. See also M. Gronau and D. London, *Phys. Lett.* **253B** (1991) 483; I. Dunietz, *Phys. Lett.* **270B** (1991) 75. Improvements to this method have recently been discussed by D. Atwood, I. Dunietz and A. Soni, [hep-ph/9612433](http://arxiv.org/abs/hep-ph/9612433), *Phys. Rev. Lett.* (to appear).

[4] M. Gronau and D. London, *Phys. Rev. Lett.* **65** (1990) 3381.

[5] Isospin: H.J. Lipkin, Y. Nir, H.R. Quinn and A.E. Snyder, *Phys. Rev.* **D44** (1991) 1454; M. Gronau, *Phys. Lett.* **265B** (1991) 389. Dalitz: A.E. Snyder and H.R. Quinn, *Phys. Rev.* **D48** (1993) 2139.

[6] R. Aleksan, I. Dunietz and B. Kayser, *Zeit. Phys.* **C54** (1992) 653.

[7] For a review of new-physics effects in CP asymmetries in the $B$ system, see M. Gronau and D. London, *Phys. Rev.* **D55** (1997) 2845, and references therein.

[8] Y. Nir and D. Silverman, *Nucl. Phys.* **B345** (1990) 301.

[9] D. London and R. Peccei, *Phys. Lett.* **223B** (1989) 257. See also Y. Nir and H.R. Quinn, ref. [2].

[10] Y. Grossman and M.P. Worah, SLAC-PUB-7351, [hep-ph/9612269](http://arxiv.org/abs/hep-ph/9612269), 1996.

[11] A.G. Cohen, D.B. Kaplan and A.E. Nelson, *Phys. Lett.* **388B** (1996) 588.

[12] A. Kagan, *Phys. Rev.* **D51** (1995) 6196.

[13] Y. Nir and D. Silverman, *Phys. Rev.* **D42** (1990) 1477; D. Silverman, *Phys. Rev.* **D45** (1992) 1800; G.C. Branco, T. Morozumi, P.A. Parada and M.N. Rebelo, *Phys. Rev.* **D48** (1993) 1167; V. Barger, M.S. Berger and R.J.N. Phillips, *Phys. Rev.* **D52** (1995) 1663.

[14] Peter Kim, CLEO collaboration, talk given at FCNC 97, Santa Monica, USA (1997).

[15] M.S. Alam et al. (CLEO Collaboration), *Phys. Rev.* **D50** (1994) 43.

[16] R. Fleischer, *Zeit. Phys.* **C62** (1994) 81.