Linking the exotic structure of $^{17}$C to its unbound mirror $^{17}$Na

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Abstract

The structure of $^{17}$C is used to define a nuclear interaction that, when used in a multichannel algebraic scattering theory for the $n+^{16}$C system, gives a credible definition of the (compound) excitation spectra. When couplings to the low-lying collective excitations of the $^{16}$C-core are taken into account, both sub-threshold and resonant states about the $n+^{16}$C threshold are found. Adding Coulomb potentials to that nuclear interaction, the method is used for the mirror system of $p+^{16}$Ne to specify the low-excitation spectrum of the particle unstable $^{17}$Na. We compare the results with those of a microscopic cluster model. A spectrum of low excitation resonant states in $^{17}$Na is found with some differences to that given by the microscopic-cluster model. The calculated resonance half-widths (for proton emission) range from $\sim$ 2 to $\sim$ 672 keV.

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I. INTRODUCTION

The spectra of radioactive nuclei at or just beyond a drip line are most intriguing. To date, details of their spectra are poorly known at best. Few if any excited states have been identified. Likewise the spin-parities of many of the known states have not been or are uncertainly assigned. Nowadays, opportunities exist to investigate spectra of such exotic systems using isotope separator on-line facilities with which production of radioactive ion beams having energies typically 0.1 A to 10 A MeV is possible. Reactions using these beams with higher incident energy, particularly from hydrogen targets, can be, and have been, used to study the structure of the radioactive ions as well [1, 2]. However, it is the low energy domain that interests us as we wish to consider structures of compound systems formed by amalgamation of the beam ion and a nucleon.

With light mass systems having charge number $\pi$ and neutron number $\nu$, there is often the possibility to link the structures of mirror systems. Usually there is a reasonably well known spectrum of a nucleus $(A+1)ZX_{\nu=N+1}$, which we treat as a compound of a neutron $(n)$ with $(A+1)ZX_{\nu=N}$ to define a chosen nuclear model interaction. With that model, assuming charge invariance of the nuclear force and adjusting for Coulomb effects, the spectrum of the mirror, $(A+1)Q_{\nu=Z}$ may be predicted. This has been done [3], for example, for the mass-7 isobars with multichannel algebraic scattering (MCAS) [4] evaluations of the spectra of the compound systems; $^7$Li (as $n+\ ^6$Li), $^7$Be (as $p+\ ^6$Li), $^7$He (as $n+\ ^6$He), and $^7$B (as $p+\ ^6$Be). Also, the approach predicted a spectrum for the (particle unstable) nucleus $^{15}$F when treated as a compound of $p+\ ^{14}$O [5]. In that study, the nuclear interaction was set by an analysis of the mirror system $^{15}$C treated as $n+\ ^{14}$O. It was found that key requirements for obtaining resonance states of $^{15}$F were, a) the Coulomb barrier, which is essential in recreating the resonance aspect of the observed spin $1/2^+$ ground state, b) coupling of the extra core proton to distinct states of $^{14}$O and c) consideration of the Pauli principle within a coupled-channel collective model prescription. The latter two features ensured a credible sequence of spin-parity values, a very good fit to the high-quality elastic scattering cross sections known at that time [6, 7], and prediction of a set of narrow resonances only a few MeV above the (two) known ones. Subsequently, narrow resonances in the region of that excitation energy were observed [8, 9, 10]. (Note: There is an error in Ref. [8] relating to citation of the results of our earlier study [5]. In their Table I, references 6 and 7 have been reversed in both the table caption and the header row. A similar error has also been made in Table I in Ref. [9] involving their references 16 and 17.)

Recently, Timofeyuk and Descouvemont [10] used the same philosophy of fixing the nuclear aspect by a two-center, microscopic cluster model (MCM) of $^{17}$C, treated as $n+\ ^{16}$C, to find a spectrum of $^{17}$Na as $p+\ ^{16}$Ne. They predict narrow resonances in the low excitation spectrum of the particle unstable nucleus, $^{17}$Na, with very broad ones above that. Those results were a stimulus to use the MCAS approach as a complementary study. Thus we consider the mirror systems, $^{17}$C ($n+\ ^{16}$C) and $^{17}$Na ($p+\ ^{16}$Ne), especially since the low excitation spectrum of $^{17}$C has been found experimentally [11, 13] and microscopic models for the structure of that nucleus have been proposed [2, 10, 12]. An MCAS analysis of the low excitation spectrum of $^{17}$C has been made before [2]. However, the results of that study came from an overly simple, two-channel, evaluation. Therein, also, we considered a distorted wave approximation (DWA) analysis of inelastic scattering data [13] of 70 A MeV $^{17}$C ions from hydrogen targets. Those DWA evaluations were made using no-core shell model wave functions and the complete set of results allowed us to pose some constraints upon the
structure of $^{17}$C.

However, inadequacies remained due, in part, to simplifications in the previous MCAS evaluations based upon limited available data. Subsequent experiments \[12\] have suggested a number of additional spin-parity assignments. The spin-parities of the three closely spaced sub-threshold states of $^{17}$C are identified and current no-core shell models fail to match them adequately. It has been suggested \[10\] that coupling of a neutron to states at about 4 MeV excitation in $^{16}$C is needed to improve the results. That coupling was not included in our previous study \[2\] and so we present herein results of MCAS evaluations in which coupling to the 4$^+$ state in the mass-16 nuclei is included.

II. MCAS EVALUATIONS OF $^{17}$C AS AN $n+^{16}$C SYSTEM

MCAS calculations of the $n+^{16}$C system were made using three states in $^{16}$C; the 0$^+$ (ground), 2$^+$ (1.766 MeV), and 4$^+$ (4.142 MeV) states. We assume that couplings to the other states (presumed 0$^+_2$, 2$^+_2$, and 3$^+_2$) in the spectrum between 3 and 5 MeV excitation are not strong and that the rotational model for the interactions, as defined previously \[4\], suffices in seeking the spectrum of $^{17}$C. The parameter values required to get the results displayed in Fig. 1 are listed in Table I. The OPP scale values coincide with Pauli blocking of the 1$s$ and 1$p$ orbits while the 1$d$ and 2$s$ values equate to a Pauli hindrance in those orbits as was needed in predicting \[3\] narrow states in the spectrum of $^{15}$F; a nucleus beyond the proton drip line.

The potential parameter values and Pauli blocking/hindrance weights are quite similar to the set used previously \[2\], now with inclusion of a small hexadecapole deformation to link the 4$^+$ state to the ground state in first order and with some Pauli hindrance of the 2$s$ orbit in the connections to the 4$^+$ target state.

The spectrum that results is shown in Fig. 1 and labelled “mcas(0+2+4)”. It is compared with the spectrum found previously from the 2-state MCAS evaluation (“mcas(0+2)”), and with the experimentally-known one that is a combination of states listed in Table 4 of Ref. 12 and in Fig. 5 of Ref. 14. The energies of that tabled spectrum have been adjusted by $-0.729$ MeV; the value of the $n+^{16}$C threshold in $^{17}$C. Some states specified by Raimann et al. \[14\] are displayed by the dash-dot lines. Only positive parity states are known in the

| State in $^{16}$C | OPP $\lambda_{ij}$ |
|------------------|------------------|
| $(1s_{\frac{1}{2}}, 1p_{\frac{3}{2}}, 1p_{\frac{1}{2}})$ | $1d_{\frac{5}{2}}$ | $2s_{\frac{1}{2}}$ |
| 0$^+$ (ground)   | $10^6$           | 2.7             | 0.0            |
| 2$^+$ (1.766)    | $10^6$           | 2.7             | 0.0            |
| 4$^+$ (4.142)    | $10^6$           | 0.0             | 2.0            |
FIG. 1: MCAS results for the spectrum of $^{17}$C compared with experimental data. The energy scale is set with the $n^{+16}$C threshold as zero emphasising that the lowest three states are stable against neutron emission.

low excitation spectrum, and the integers associated with the individual energy levels shown are two times their spin.

Clearly the three known subthreshold ($n^{+16}$C) states are now matched well in energy and spin-parity by the new MCAS results. The other known and uncertain spin-parity states also have matching MCAS partners in proximity of their excitation energies. Additionally, the uncertain states from Raimann et al. [14] seem to have possible matches, and the first low-lying state above threshold of that set we expect to be a $\frac{3}{2}^{+}$ resonance. The (0+2+4) MCAS spectra have a number of aspects in common with that shown in Fig. 1 of Ref. [10]. Besides the three closely spaced and weakly bound sub-threshold states, the MCM study gave a group of states ($\frac{3}{2}^{+}$, $\frac{5}{2}^{+}$, $\frac{7}{2}^{+}$, and $\frac{9}{2}^{+}$) in the region of 2 MeV above the $n^{+16}$C threshold and a second group in the region of 4 MeV above that threshold. There is also a higher excited $\frac{1}{2}^{+}$ state found with both calculations above 6 MeV, notable by being very broad (half-width $\sim$ 5.6 MeV with MCAS). The third $\frac{3}{2}^{+}$ state in the MCAS spectrum (centroid $E_x$ $\sim$ 4.5 MeV) is also very broad (half-width $\sim$ 4 MeV). Our MCAS result shows more states, including two very narrow ones of spin-parity $\frac{13}{2}^{+}$ and $\frac{11}{2}^{+}$ at 3.82 and 4.16 MeV excitation respectively.

Specifics of the states in the $^{17}$C spectrum are listed in Table III. Columns 1 to 3 display
TABLE II: Spectra for $^{17}$C to 6 MeV excitation. Units are MeV. The $n+^{16}$C threshold lies at 0.728 MeV.

| $J^π$ | Ref. [12] $E$ | $\frac{1}{2}Γ$ | Ref. [14] $E$ | $\frac{1}{2}Γ$ | MCAS $E$ | $\frac{1}{2}Γ$ |
|-------|--------------|----------------|--------------|----------------|---------|----------------|
| $\frac{3}{2}^+$ | 0.00 | 0.00 | 0.00 | 0.00 |
| $\frac{1}{2}^+$ | 0.21 | 0.292 | 0.201 | 0.00 |
| $\frac{5}{2}^+$ | 0.31 | 0.295 | 0.390 | 0.00 |
| (3/2+, 7/2+) | 2.06 | 0.50 | (1.18) | 1.196 | 2x10$^{-7}$ |
| (9/2+) | 3.10 | 0.20 | (2.64) | 2.714 | 0.017 |
| (5/2+, 7/2+, 9/2+) | 4.25 | 0.28 | (3.82) | 4.038 | 0.086 |

Values of spin-parity $J^π$, excitation energy (or centroid) $E$, and half-widths $\frac{1}{2}Γ$, of resonances ascertained in a study [12] of three neutron transfer cross sections for $^{12}$C scattering from $^{14}$C. The excitation energies of states shown in Fig. 5 of Ref. [14] are listed in column 4, while the MCAS results are displayed in columns 5, 6, and 7. The three nucleon transfer reaction widths in general do not match those from the MCAS evaluation but the two sets are quite different; the latter being single nucleon removal values.

The resonant states found using MCAS in the region of 3 MeV excitation have widths that agree well with most of the matching ones from the MCM evaluation [10]. Widths quoted in Ref. [10] are $\frac{7}{2}^+$ $E_1$ (10$^{-12}$ MeV), $\frac{9}{2}^+$ $E_1$ (10$^{-6}$ MeV), and $\frac{5}{2}^+$ $E_2$ (0.015 MeV).

We note also that Satou et al. [13] from their measurements of 70A MeV $^{17}$C radioactive ion beam scattering from hydrogen suggest that there are resonances in $^{17}$C at 2.2, 3.05, and 6.13 MeV with spin-parities anticipated to be $\frac{2}{2}^+$, $\frac{9}{2}^+$, and $\frac{5}{2}^+$ respectively. In a recent article [15], results of using a simple shell model approach and considering single-particle widths suggests that the 2.2 MeV resonance is in fact two or three narrow but closely spaced ones.

$^{17}$C has been noted [8, 10] as having a “peculiar” structure which may be connected with the neutron separation energy from the ground state being only 0.728 MeV. That is typical of a halo nucleus. Indeed, the channel-coupling interaction we require to give the spectrum of $^{17}$C reflects that with diffuseness being large. Also, features of this interaction resemble those required [10] in a two-body potential model of this system, viz. “The bound $^{17}$C spectrum cannot be understood in the two-body potential model with deformation and the 2$^+$ excitation of the $^{16}$C core either, if standard sets of potentials are used. An $ℓ$-dependent and nonstandard spin-orbit $n+^{16}$C potential must be used for these purposes.” That is illustrated in Fig. 2 which shows how spin states arise from underlying components of the
FIG. 2: Spectra from MCAS evaluations restricting couplings to single and two channels of the three compared with the full three-channel spectrum. Again the energy scale is set with the $n^{+16}C$ threshold as zero to emphasise the sub-threshold from resonance states in each evaluation.

coupled-channel approach. Calculations have been made for each subdivide of the full three target state coupled-channel problem. The numerals indicate two times the spin values. The first three columns from the left give the states found when each state alone is considered as a single channel problem. Obviously, within the searched energy range (to 6 MeV), the nucleon on the ground state gives only a single state from adding a $d_2$ neutron. The addition of a neutron (probably into a $d_3$ single-particle state) gives the set shown for the single channels of the $2^+$ and $4^+$. The order shown is due to the spin and angle dependent features used in the base interaction. The existence of the second $2^+$ state when a neutron is added to the isolated $4^+$ state may be reflecting addition in a $d_3$ state.

The next three columns are the spectra found when two of the three $^{16}C$ states are allowed in the coupling. The results found coupling the ground and $2^+$ states are very similar to what was published in an earlier paper [2]; differences reflecting changed interaction parameter values. The inclusion of the $4^+$ state, whether in a 2- or the full 3-state coupling study, leads to a richer spectrum and changes the order of states. Of note, only evaluations in which the $2^+$ state is included give a low lying $\frac{3}{2}^+$ state. Moreover, the three-state coupling markedly moves the energy values from that found otherwise.
The key feature is finding the three closely spaced sub-threshold states and in this spin order. Non-trivial changes of the parameters, notably with the deformations, radius and diffuseness to smaller values, cause such packing to be lost. The coupling of the $4^+$ state in $^{16}\text{C}$ with the ground and $2^+$ states causes notable changes to the predicted spectrum; changes that better align with (so far) experimentally defined states in $^{17}\text{C}$, a few of which have been assigned spins. Clearly we predict many other resonance states, but whether they exist or can be found if they do, remains an open question. It is most unlikely that any direct $n+^{16}\text{C}$ experiment will ever be done so one must rely on some surrogate approach, such as $^{16}\text{C}(d,p)$, or some study that identifies neutron emissions from $^{17}\text{C}$.

III. MCAS EVALUATIONS OF $^{17}\text{Na}$ AS A $p+^{16}\text{Ne}$ SYSTEM

$^{17}\text{Na}$ ($p+^{16}\text{Ne}$) is the mirror system to $^{17}\text{C}$ ($n+^{16}\text{C}$) but, to date, none of its states have been identified. Its ground state mass is not listed in the Ame2003 mass table [16]. However, it is expected [17] that the spectrum of this nucleus should have a number of low-excitation resonant states, some of which are to be quite narrow. The MCM study of the structure anticipates that the unbound ground state should lie around 2.4 MeV above the proton-$^{16}\text{Ne}$ threshold; a value notably less than 3.65 MeV expected using the Kelson and Garvey formula [17].

The MCAS calculations of the $p+^{16}\text{Ne}$ system were made adding a Coulomb potential to the nuclear interaction set by the analysis of the $^{17}\text{C}$ spectrum. The Coulomb potentials are those derived from a Woods-Saxon charge distribution having the same parameters (geometry and deformation) as the nuclear interaction. The spectrum of $^{16}\text{Ne}$ known to date is just the ground state ($0^+$), and an excited $2^+$ state at 1.69 MeV. We presume, based upon the spectrum of its mirror, $^{16}\text{C}$, that there will be a $4^+$ state in the vicinity of 4 MeV excitation.

A. Sharp target state results

Initially we suppose that the three states in $^{16}\text{Ne}$ are like those assumed for $^{16}\text{C}$ namely to have zero widths. Using those states in an MCAS evaluation with a Coulomb interaction added to the nuclear one gave the spectra of $^{17}\text{Na}$ ($p+^{16}\text{Ne}$) as indicated in Fig. 3. The resulting spectrum (labelled MCAS) is compared with that given in Ref. [10] (identified in the figure by the label T+D). The three states of $^{16}\text{Ne}$ used in both studies are shown on the left of this diagram. The spins of the (positive parity) states found for $^{17}\text{Na}$ are indicated by two times their values and are shown for each level found. The excitation spectra depicted for $^{17}\text{Na}$ have the order of the spins interchanged and the size of energy gaps differ from those in $^{17}\text{C}$. The three lowest excited states from each calculation have spin-parities of $\frac{1}{2}^+, \frac{3}{2}^+$, and $\frac{5}{2}^+$. But while the $\frac{3}{2}^+$ state is the first excited state in the MCM spectrum, it is the second excited state the MCAS evaluation. The energy gaps of the three lowest states predicted by each calculation are different.

It may be thought that, the Thomas-Ehrman (TE) shift could make the (MCAS) $\frac{1}{2}^+$ state, in particular, change noticeably in energy. The TE shift is of note when there is a weakly bound s-wave proton coupling to a core state. However, since the dominant attributes in the low lying states of $^{17}\text{Na}$ in the MCAS evaluation are so strongly defined by coupling of a $d_{5/2}$ proton to the $2^+$ and $4^+$ states in $^{16}\text{Ne}$, a large TE shift is unlikely since the centrifugal
FIG. 3: Results from model evaluations of the spectrum of $^{17}$Na. The energy scale is relative to the $p+^{16}$Ne threshold.

barrier helps contain the $d$-wave function.

With both model calculations, the states found are resonances. The resonance centroids and half-widths for proton decay that the models give are listed in Table III. The widths found with the MCM and MCAS model are a mix of narrow and broad while the MCAS results range between 2 and 500 keV. However, it must be remembered that $^{16}$Ne is itself a proton emitter, and so the three target states considered should also be treated as resonances. Doing so with MCAS can have marked effect, in particular upon the evaluated compound nucleus widths [18]. With the exception of the ground state, widths of the states in $^{16}$Ne are not known, and indeed even the existence of a $4^+$ state at $\sim 4$ MeV excitation is not known.

B. Resonant target state results

Since the ground state of $^{17}$Na lies above the 1$p$-, the 2$p$-, and the 3$p$-emission thresholds, break-up into those channels give widths to the (resonant) states of $^{17}$Na. Mukha et al. [9] found the ground state of $^{16}$Ne to have a width of 122 keV while they assigned a value of 200 keV to the first excited $2^+$ state near 1.69 MeV excitation. They did not observe a $4^+$ resonance with centroid $\sim 4$ MeV. They did observe other resonance states with the lowest in excitation centred about 7.6 MeV.

In MCAS calculations made using resonant states for the target, we retained the 3-state model taking the ground state to have a width of 122 keV, the $2^+$ (1.69 MeV) state to have
TABLE III: MCAS energies and half-widths of $^{17}$Na states.

| state | $E(\text{MeV})$ | $\frac{1}{2}\Gamma(\text{MeV})$ | $E(\text{MeV})$ | $\frac{1}{2}\Gamma(\text{MeV})$ | $\frac{1}{2}|\Gamma|_2(\text{MeV})$ |
|-------|----------------|-------------------------------|----------------|-------------------------------|-------------------------------|
| $\frac{3}{2}^+$ | 2.40 | 1.360 | 1.03 | 0.005 | 0.110 |
| $\frac{5}{2}^+$ | 2.57 | 0.025 | 2.26 | 0.004 | 0.231 |
| $\frac{7}{2}^+$ | 2.97 | 0.144 | 1.13 | 0.002 | 0.092 |
| $\frac{9}{2}^+$ | 4.35 | 0.025 | 3.98 | 0.036 | 0.242 |
| $\frac{11}{2}^+$ | 4.38 | 1.593 | 4.45 | 0.208 | 0.129 |
| $\frac{13}{2}^+$ | 5.41 | 0.210 | 3.36 | 0.011 | 0.158 |
| $\frac{15}{2}^+$ | 5.27 | 1.383 | 4.28 | 0.103 | 0.249 |
| $\frac{17}{2}^+$ | — | — | 6.04 | 0.186 | 0.242 |
| $\frac{19}{2}^+$ | — | — | 6.64 | 0.084 | 0.600 |
| $\frac{21}{2}^+$ | — | — | 6.81 | 0.457 | 0.184 |

* The widths are the summed widths in Table II of Ref. [10].

A width of 200 keV, and assumed the existence of a $4^+$ state analogous to that in $^{16}$C centred at 4 MeV and with a width of 1 MeV. As with recent MCAS studies of other systems [19, 20], the results of this calculation made only small changes to the centroid energies of the states in $^{17}$Na found using zero widths for the three $^{16}$Ne states. But it had a serious effect upon the half-widths of the compound nucleus. The results obtained when the target states are resonances are listed in column identified as $\frac{1}{2}|\Gamma|_2$ in Table III. Comparison with the values found using zero target-state widths (given in the adjacent column) show the changes. With the exception of the $\frac{5}{2}|_2$ and of the $\frac{7}{2}|_2$ which have smaller half-widths, all other resonances increase in half-widths; some even more than 100-fold. The maximum value in the set is 672 keV whereas values of up to 1.6 MeV were found in the MCM evaluation [10].

It is essential that new experimental information be provided for more detailed investigations of model structures for these exotic nuclei. Nonetheless, a most important theoretical aspect is channel coupling; the effects of which we discuss next.

IV. EFFECTS OF CHANNEL COUPLING IN THE MCAS APPROACH

In Fig. 4 we show MCAS spectra found with (on the right) and without (in the center) deformations of the nuclear interactions. The spectrum on the left is that when both the deformation parameters and the spin-spin strength, $V_{Is}$, are set to zero. As usual, the spins of the states considered are indicated by two times their values and matching pairs (of spin values) are indicated by the dashed lines linking the no deformation to the full results. Of course the deformations result in states that are admixtures of those of the same spin-parity shown in the center spectrum. Setting the spin-spin interaction to zero and having no deformation shows that these states are the result of coupling a $1d_{3/2}$ proton to the three selected states for $^{16}$Ne. Such coupling allows twelve states as a basis, namely the set $\frac{1}{2}^+$, two $\frac{3}{2}^+$, three $\frac{5}{2}^+$, two $\frac{7}{2}^+$, two $\frac{9}{2}^+$, an $\frac{11}{2}^+$, and a $\frac{13}{2}^+$. However, with larger (quadrupole)
FIG. 4: MCAS spectra for $^{17}$Na found with no deformation and $V_{ls} = 0$ (left), with no deformation (center), and the full result (right).

deformation, extra states built largely upon the coupling of $2s_{\frac{1}{2}}$ and/or $1d_{\frac{3}{2}}$ proton states to the three $^{16}$Ne ones can be found in the spectrum within that excitation energy range. Clearly the deformation of the nuclear interaction has a very large effect on the resultant spectrum.

In Fig. 5 are shown the changes in the spectrum that occur as the quadrupole (bottom segment) and hexadecapole (top segment) deformations are varied separately. The energies are values relative to the $p^+^{16}$Ne threshold. The set of states at $\sim 8$ MeV excitation are hardly influenced by the hexadecapole deformation, but, albeit small, that deformation causes some changes to the lower excitation states. The three lowest excitation states, of spins $\frac{1}{2}^+$, $\frac{5}{2}^+$ and $\frac{3}{2}^+$ in sequence, become more bound with increasing $\beta_4$, and by about the same amount. Likewise the second band of resonant states show an interesting variation with the conjectured $\frac{3}{2}^+$ and $\frac{5}{2}^+$ states becoming less bound with increasing hexadecapole deformation, while the $\frac{7}{2}^+$ state is little changed but the $\frac{9}{2}^+$ state becomes noticeably more bound.

The variation in the spectrum caused by changes to the size of quadrupole deformation is shown in the bottom segment of the top panel of this figure. With increasing quadrupole deformation, the states of the spectrum spread with most, but not all, decreasing in excitation energy. The influence on the lowest three (resonant) states is most marked for changes
FIG. 5: Variations of the spectrum of $^{17}$Na upon changing the quadrupole (bottom) and hexadecapole (top) deformations. The $\frac{1}{2}^+$ state is depicted by the stars while $\frac{3}{2}^+, \frac{5}{2}^+, \frac{7}{2}^+$ and $\frac{9}{2}^+$ states are identified by the filled squares, diamonds, up-triangles, and down-triangles respectively. The high spin states, $\frac{11}{2}^+$ and $\frac{13}{2}^+$, are portrayed by the left- and right-triangles respectively.

in $\beta_2$ in the range 0.3 to 0.4, with the $\frac{1}{2}^+$ state eventually becoming the ground state in the MCAS model evaluations.

V. CONCLUSIONS

We have used the MCAS approach to suggest a low-excitation spectrum for the exotic nucleus $^{17}$Na treated as a $p+^{16}$Ne system. The mirror isospin concept was used to define the nuclear $p+^{16}$Ne interaction as that which gave a reasonable low excitation spectrum
for $^{17}\text{C}$ when treated as a $n+^{16}\text{C}$ coupled-channel problem. Three states, a $0^+$ (ground) a first excited $2^+$ ($\sim 1.7 \text{ MeV}$), and a $4^+$ ($\sim 4 \text{ MeV}$), were taken as the target states in the coupled-channel studies. An interaction was found that gave sub-threshold and low excitation resonances for $^{17}\text{C}$ with centroid energies in quite reasonable agreement with the limited set of known values. The $^{17}\text{C}$ spectrum contains both sharp and narrow resonances. By adding Coulomb interactions for the interaction between a proton and the individual states of $^{16}\text{Ne}$, the spectrum of the nucleon emissive $^{17}\text{Na}$ also contained narrow and broad resonances. Those MCAS results had some similarities with, but also a number of variations from, the spectrum of $^{17}\text{Na}$ defined by the MCM of the $p+^{16}\text{Ne}$ system \cite{10}. Both evaluations gave a resonance state spectrum for that nucleus which is known to lie outside of the proton drip line; spectra that had both narrow and broad resonances. The MCAS result placed the resonant ground state of $^{17}\text{Na}$ an MeV or so lower than that of the MCM study and reversed the spins of the first and second excited states. A larger energy ($\sim 3 \text{ MeV}$) is expected from an older estimate \cite{17} though the actual mass excesses of relevant nuclei that calculation were not well fixed. Nonetheless, our results confirm the general findings of the MCM study \cite{10}; notably that channel coupling effects are crucial in ascertaining both the centroid energies as well as a range of widths from narrow to broad in low-excitation spectra. Such channel couplings were also important in predicting the character of another nucleus that lies just beyond the proton drip line, namely $^{15}\text{F}$ \cite{5}. In detail, our results and those of the MCM study do not match well with the effects of using actual resonance states to describe $^{16}\text{Ne}$ being very significant as to what half-widths are found for the resonances in $^{17}\text{Na}$.

Of course, just what couplings are important, how many channels are relevant, what are the optimal nuclear and Coulomb interactions, and what microscopy underlies the spectrum, await improved experiments of high quality from which more details of the nuclei, $^{16,17}\text{C}$, $^{16}\text{Ne}$, and $^{17}\text{Na}$, can be ascertained.

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