Localized energy associated with Bianchi-Type VI universe in $f(R)$ theory of gravity

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Abstract. In the present work, focusing on one of the most popular problems in modern gravitation theories, we consider generalized Landau-Lifshitz energy-momentum relation to calculate energy distribution of the Bianchi-Type VI spacetime in $f(R)$ gravity. Additionally, the results are specified by using some well-known $f(R)$-gravity models.

Keywords: Bianchi; energy localization; modified gravity.

PACS Numbers: 04.20.-q; 04.50.-h; 04.90.+e.

1. Introduction

Gravitational energy-momentum localization problem is one of the most popular in gravitation theories and it still remains unsolved. Einstein is known as the first scientist who worked on energy-momentum pseudotensors[1] and different energy momentum prescriptions[2, 3, 4, 5, 6, 7, 8] were put forward after his studies. All energy-momentum formulations except for the Møller prescription[9] were restricted to make computations in cartesian coordinates. In 1990, Virbhadra and his collaborators reopened the energy-momentum localization problem[10, 11, 12, 13, 14, 15, 16] and after those pioneering papers great numbers of work have been prepared[17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29] by considering different energy momentum complexes and spacetime models.

Recently, modified gravitation theories especially $f(R)$ gravity which extends the general theory of relativity have also been taken into account by many scientists to discuss gravitational puzzles again[30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48]. The $f(R)$-gravity should be defined by modifying the Einstein-Hilbert action:

$$S = -\frac{1}{2\kappa} \int \sqrt{-g} f(R) d^4 x + S_m$$

where $\kappa = 8\pi G$, $g$ represents the determinant of the metric tensor, $f(R)$ denotes a general function of Ricci scalar and $S_m$ is the matter part of action[49]. It is known that

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the Ricci curvature scalar is given by

$$R = g^{\mu\nu} R_{\mu\nu},$$

where $R_{\mu\nu}$ is the Ricci tensor related with the Riemann tensor, i.e. $R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}$, given below:

$$R^\lambda_{\mu\nu\sigma} = \partial_\nu \Gamma^\lambda_{\mu\sigma} - \partial_\sigma \Gamma^\lambda_{\mu\nu} + \Gamma^\eta_{\mu\nu} \Gamma^\lambda_{\eta\sigma} - \Gamma^\eta_{\mu\sigma} \Gamma^\lambda_{\eta\nu},$$

and $\Gamma^\lambda_{\mu\sigma}$ is known as the Christoffel symbols:

$$\Gamma^\lambda_{\mu\sigma} = \frac{1}{2} g^{\lambda\beta} \left( \partial_\alpha g_{\mu\beta} + \partial_\mu g_{\sigma\beta} - \partial_\beta g_{\mu\sigma} \right).$$

Now, varying equation (1) with respect to the metric tensor yields the following field equation

$$F(R) R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} - \left[ \nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla_\alpha \nabla^\alpha \right] F(R) = \kappa T_{\mu\nu}.\ (5)$$

Here, it has been defined that $F(R) \equiv \frac{d f(R)}{dR}$ and $\nabla_\mu$ represents the covariant derivative. After construction for the vacuum case, i.e. $T = 0$, the corresponding field equation transforms to the following form

$$F(R) R - 2 f(R) + 3 \nabla_\alpha \nabla^\alpha F(R) = 0.\ (6)$$

It can be easily seen that for any constant curvature scalar equation (6) becomes

$$F(R_0) R_0 - 2 f(R_0) = 0,$$

here we have used that $R = R_0 = constant$. In non-vacuum case, the constant curvature scalar condition is described by

$$F(R_0) R_0 - 2 f(R_0) = \kappa T.\ (8)$$

Making use of the generalized Landau-Lifshitz prescription for the Schwarzschild-de Sitter universe, Multamäki et al. [50] calculated energy distribution for some well known $f(R)$ gravity models including constant curvature scalar. Later, Amir and Naheed [51] considered a spatially homogeneous rotating spacetime solution of $f(R)$ gravity to calculate Landau-Lifshitz energy density. Moreover, using some well-known $f(R)$ theory suggestions, Salti et al. [52] also discussed energy-momentum localization problem for Gödel-Type metrics. These studies motivate us to discuss energy-momentum problem for another background in $f(R)$-gravity and extend those works.

The paper is organized as follows. In the second section, we give a brief information about the Landau-Lifshitz distribution in $f(R)$ gravity for the Bianchi-VI type spacetime. Next, in the third section, we calculate energy density considering Landau-Lifshitz distribution for some specific $f(R)$ models. Finally, we devote the last section to discussions.
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2. Generalized Landau-Lifshitz Prescription in Bianchi-Type VI Spacetime

The generalized Landau-Lifshitz distribution is given by

\[ \tau^{\mu\nu} = F(R_0)\tau_{LL}^{\mu\nu} + \frac{1}{6\kappa}[F(R_0)R_0 - f(R_0)]\frac{\partial}{\partial x^\gamma}(g^{\mu\nu}x^\gamma - g^{\nu\gamma}x^\mu), \quad (9) \]

where \( \tau_{LL}^{\mu\nu} \) is the Landau-Lifshitz energy-momentum of general relativity and defined by

\[ \tau_{LL}^{\mu\nu} = (-g)(T^{\mu\nu} + t_{LL}^{\mu\nu}) \quad (10) \]

with

\[ t_{LL}^{\mu\nu} = \frac{1}{2\kappa} \left[ (2\Gamma_{\alpha\beta}^\gamma \Gamma_{\gamma\delta}^\delta - \Gamma_{\alpha\gamma}^\gamma \Gamma_{\beta\gamma}^\delta - \Gamma_{\alpha\gamma}^\delta \Gamma_{\beta\gamma}^\gamma)(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\alpha\beta}) \right. \]
\[ + g^{\mu\alpha}g^{\nu\gamma}(\Gamma_{\alpha\delta}^\nu \Gamma_{\delta\gamma}^\beta + \Gamma_{\beta\gamma}^\nu \Gamma_{\alpha\delta}^\delta - \Gamma_{\alpha\gamma}^\nu \Gamma_{\beta\delta}^\delta - \Gamma_{\alpha\beta}^\nu \Gamma_{\gamma\delta}^\delta) \]
\[ + g^{\mu\alpha}g^{\nu\beta}(\Gamma_{\alpha\delta}^\nu \Gamma_{\gamma\delta}^\beta + \Gamma_{\beta\gamma}^\nu \Gamma_{\alpha\delta}^\delta - \Gamma_{\alpha\gamma}^\nu \Gamma_{\beta\delta}^\delta - \Gamma_{\alpha\beta}^\nu \Gamma_{\gamma\delta}^\delta) \]
\[ + g^{\mu\beta}g^{\nu\gamma}(\Gamma_{\alpha\gamma}^\mu \Gamma_{\beta\delta}^\nu + \Gamma_{\beta\gamma}^\mu \Gamma_{\alpha\delta}^\nu - \Gamma_{\alpha\gamma}^\mu \Gamma_{\beta\delta}^\delta - \Gamma_{\alpha\beta}^\nu \Gamma_{\gamma\delta}^\nu). \quad (11) \]

Consider 00-component of equation (9) gives energy density associated with the universe and it can be written as given below [50]:

\[ \tau^{00} = F(R_0)t_{00}^{00} + \frac{1}{6\kappa}[F(R_0)R_0 - f(R_0)](\frac{\partial}{\partial x^i}g^{00}x^i + 3g^{00}). \quad (12) \]

In the canonical cartesian coordinates, the homogenous Bianchi-Type VI spacetime is defined by the following line-element [53]:

\[ ds^2 = dt^2 - dx^2 - e^{2(A-1)x}dy^2 - e^{2(A+1)x}dz^2, \quad (13) \]

where \( A \) is a constant with \( 0 \leq A \leq 1 \). The metric tensor \( g_{\mu\nu} \), its form \( g^{\mu\nu} \) and \( \sqrt{-g} \) for the Bianchi-Type VI model can be written, respectively, as:

\[ g_{\mu\nu} = (1, -1, -e^{2(A-1)x}, -e^{2(A+1)x}), \quad (14) \]
\[ g^{\mu\nu} = (1, -1, -e^{2(1-A)x}, -e^{-2(A+1)x}), \quad (15) \]
\[ \sqrt{-g} = e^{2Ax}. \quad (16) \]

Next, the nonvanishing component of Christoffel symbols are calculated as

\[ \Gamma^1_{22} = (A - 1)e^{2(A-1)x}, \]
\[ \Gamma^1_{33} = - (A + 1)e^{2(A+1)x}, \]
\[ \Gamma^2_{12} = \Gamma_2^{21} = (A - 1), \]
\[ \Gamma^3_{13} = \Gamma_3^{31} = (A + 1). \quad (17) \]

Using the above results, the surviving components of Ricci tensor become

\[ R_{11} = - 2(A^2 + 1), \]
\[ R_{22} = 2A(1 - A)e^{2(A-1)x}, \]
\[ R_{33} = - 2A(1 + A)e^{2(A+1)x}. \quad (18) \]

Additionally, the constant value of Ricci scalar is

\[ R = R_0 = 6A^2 + 2. \quad (19) \]
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Making use of above calculations, the non-vanishing components of \( t^\mu_\nu^{LL} \) are found as
\[
\begin{align*}
t^{00}_{LL} &= \frac{1}{\kappa} (1 - 5A^2), \\
t^{11}_{LL} &= \frac{1}{\kappa} (1 - A^2), \\
t^{22}_{LL} &= \frac{1}{\kappa} \left[ \frac{(1 + A)^2}{e^{2(A-1)x}} \right], \\
t^{22}_{LL} &= \frac{1}{\kappa} \left[ \frac{(A - 1)^2}{e^{2(A+1)x}} \right].
\end{align*}
\] (20)

Also, the non-zero components of \( \tau^\mu_\nu^{LL} \) are calculated as:
\[
\begin{align*}
\tau^{00}_{LL} &= -\frac{8A^2e^{4Ax}}{\kappa}, \\
\tau^{22}_{LL} &= \frac{2}{\kappa} (A + 1)^2 e^{2(A+1)x}, \\
\tau^{33}_{LL} &= \frac{2}{\kappa} (A - 1)^2 e^{2(A-1)x},
\end{align*}
\] (21)

Consequently, in the \( f(R) \)-gravity, one can easily write down the generalized form of Landau-Lifshitz energy distribution as given below
\[
\tau^{00} = -\frac{1}{2\kappa} \left\{ [R_0 F(R_0) - f(R_0)] - 16A^2 e^{4Ax} F(R_0) \right\},
\] (22)
and we also have
\[
\begin{align*}
\tau^{0i} &= \frac{1}{6\kappa} [f(R_0) - R_0 F(R_0)], \quad (i = 1, 2, 3), \\
\tau^{11} &= \frac{2}{3\kappa} [f(R_0) - R_0 F(R_0)], \\
\tau^{22} &= \frac{e^{2(1-A)x}}{3\kappa} \left\{ [2 + (1 - A)x] f(R_0) \\
&\quad + \left[ 6(A + 1)^2 e^{4Ax} + (Ax - x - 2)R_0 \right] F(R_0) \right\}, \\
\tau^{33} &= \frac{e^{-2(1+A)x}}{3\kappa} \left\{ [2 - (1 + A)x] f(R_0) \\
&\quad + \left[ 6(A - 1)^2 e^{4Ax} + (Ax + x - 2)R_0 \right] F(R_0) \right\}.
\end{align*}
\] (23)

3. Energy in specific \( f(R) \) Models

There are many suggested models in the \( f(R) \) theory of gravity\[54\]. In this section of the study, we mainly consider five different well-known models to calculate energy momentum distribution associated with Bianchi-Type VI spacetime exactly.

- The first model\[55, 56\] is described in a polynomial form:
\[
f_{1st}(R) = R + \xi R^2,
\] (24)

where \( \xi \) denotes a positive real number.
- The second model\[57\] is given by
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$$f_{2nd}(R) = R - \frac{\epsilon^4}{R}, \quad (25)$$
where $\epsilon$ is a constant parameter. This model is known also as the dark energy model of $f(R)$-gravity.

- The next model\textsuperscript{58} is defined as
  $$f_{3th}(R) = R - pR^{-1} - qR^2, \quad (26)$$
with $p$, $q$ are constant.

- Another one is given by the following definition\textsuperscript{59}:
  $$f_{4th}(R) = R - p \ln \left( \frac{|R|}{\sigma} \right) + (-1)^{n-1}qR^n. \quad (27)$$
Here $n$ represents an integer and $p$, $q$, $\sigma$ are constant parameters.

- The final model is known as the chameleon model and it is given by\textsuperscript{57}
  $$f_{5th}(R) = R - (1 - m)\lambda^2 \left( \frac{R}{\lambda^2} \right)^m - 2\Lambda, \quad (28)$$
where $\Lambda$ denotes the cosmological constant, $m$ shows an integer and $\lambda$ is a constant parameter.

For suitable choices of above constants, all of the $f(R)$ models mentioned above can define the general relativity exactly. Now, considering the above $f(R)$ gravity models and equation (22), one can obtain the following energy densities:

$$\tau_{00}^{1st} = \frac{1}{\kappa} \left\{ 2(3A^2 + 1)^2 \xi - 8A^2 e^{4Ax} \left[ 4\xi(3A^2 + 1) + 1 \right] \right\}, \quad (29)$$

$$\tau_{00}^{2nd} = \frac{1}{2\kappa(3A^2 + 1)^2} \left\{ \epsilon^4 + A^2 \left[ 3\epsilon^4 - 16e^{4Ax}(3A^2 + 1)^2 + 4\epsilon^4 e^{4Ax} \right] \right\}, \quad (30)$$

$$\tau_{00}^{3th} = \frac{1}{2\kappa(3A^2 + 1)^2} \left\{ p(1 + 3A^2 - 4A^2 e^{4Ax}) \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. 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4. Concluding Remarks

Considering Bianchi-Type VI spacetime representation and some popular models of $f(R)$ gravity with constant Ricci curvature scalar, we have mainly evaluated the Landau-Lifshitz energy distribution. All of the calculations have been performed in cartesian coordinates. We have found that the energy distribution of Bianchi-Type VI model in $f(R)$ gravity as below:

$$\tau^{00} = -\frac{1}{2\kappa} \left\{ [R_0 F(R_0) - f(R_0)] - 16A^2 e^{4Ax} F(R_0) \right\}. \quad (34)$$

Assuming $A_{\text{min}} = 0$, one can see that the energy momentum distributions transform into the following forms:

$$\tau^{00}_{\text{1st}(A=0)} = \frac{2\xi}{\kappa}, \quad (35)$$

$$\tau^{00}_{\text{2nd}(A=0)} = \frac{\epsilon^4}{2\kappa}, \quad (36)$$

$$\tau^{00}_{\text{3rd}(A=0)} = \frac{p - 4q}{2\kappa}, \quad (37)$$

$$\tau^{00}_{\text{4th}(A=0)} = \frac{1}{2\kappa} \left[ (-2)^n q(1 - n) - p + p \ln \left( \frac{2}{\sigma} \right) \right], \quad (38)$$

$$\tau^{00}_{\text{5th}(A=0)} = \frac{1}{2\kappa} \left[ 2\Lambda + 2^m (1 - m)^2 \lambda^2 (1 - m) \right]. \quad (39)$$

It is seen that all energy distributions are constant. Therefore, it can be generalized that

$$\tau^{00}_{\text{All}(A=0)} = \text{constant} \quad (40)$$

On the other hand, in case of $A = A_{\text{max}} = 1$, the energy momentum distributions for all models do not have constant values as expected. Moreover, when we take $f(R) = R_0$ in equation (22) it can be concluded that

$$\tau^{00}_{GR} = -\frac{8}{\kappa} A^2 e^{4Ax}. \quad (41)$$

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