Transformations and Enhancement Approaches for the Large Deformation EAS Method

Robin Pfefferkorn\textsuperscript{1,*} and Peter Betsch\textsuperscript{1}
\textsuperscript{1} Institute of Mechanics (IFM), Karlsruhe Institute of Technology (KIT), 76131 Karlsruhe, Germany

The computer simulation of large deformation solid mechanics requires highly efficient finite elements. One widely used method enabling the construction of such elements is the enhanced assumed strain (EAS) method (cf. [1]). Within that framework, many transformations for the enhanced as well as the compatible deformation gradient have been used in previous publications (see e.g. [2]). We propose a new frame-invariant alternative which fulfills the patch test and exhibits superior bending performance. Finally, novel mixed finite element methods based on the combination of mixed finite elements for polyconvexity [3] with the EAS method are presented. It is shown that this combination provides a promising framework for novel finite element methods.

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1 EAS method and transformations

The key idea of the EAS method, which was introduced for nonlinear geometry in [1], is to enhance the deformation gradient

\[ \mathbf{F} (\varphi, \alpha) = \mathbf{F}_p (\varphi) + \tilde{\mathbf{F}} (\varphi, \alpha) \]  

by adding an enhanced (or incompatible) part \( \tilde{\mathbf{F}} \) to the compatible deformation gradient \( \mathbf{F}_p \), where \( \varphi \) are the deformations and \( \alpha \) are the additional DOFs for the enhancement. More details on the EAS method can be found e.g. in Simo and Armero [1].

In the present work we propose a new transformation for the enhanced part of the deformation gradient. We start by interpreting the geometry description in the reference frame \( \mathbf{X}^{h,e} \) and current frame \( \mathbf{x}^{h,e} \) of an element as convective coordinates dependent on the isoparametric coordinates \( \xi = [\xi_1, \xi_2, \xi_3]^T \). This allows to define covariant basis vectors \( \mathbf{g}_1, \mathbf{g}_2 \) and their dual (contravariant) bases \( \mathbf{G}^1, \mathbf{G}^2 \) in a standard manner. In this framework the compatible deformation gradient can be computed by

\[ \mathbf{F}^{h,e} = \sum_{i=1}^{n_{enh}} \mathbf{g}_i \otimes \mathbf{G}^i. \]

Under the assumption that the enhanced deformation gradient \( \tilde{\mathbf{F}} \) has the same mixed co- and contravariant structure and some modifications to pass the patch test we arrive at the formula proposed by Simo et al. [4]. A improved and today standard version of that approach was presented by Glaser and Armero [2]. Our novel approach starts with the assumption that the enhanced deformation gradient is a purely covariant field. The procedure by Glaser and Armero [2] and the novel approach are given by

\[ \tilde{\mathbf{F}}^{h,e} = \mathbf{F}_0 \sum_{l=1}^{n_{enh}} \left( \hat{\mathbf{M}} (\xi) \alpha_l \right) \mathbf{J}_0^{-1}, \quad \mathbf{F}^{h,e} = \mathbf{F}_0 \sum_{l=1}^{n_{enh}} \left( \hat{\mathbf{M}} (\xi) \alpha_l \right) \mathbf{J}_0^{-1}, \]

(2)

respectively. Therein, \( \mathbf{F}_0 \) and \( \mathbf{J}_0 \) denote the evaluation of \( \mathbf{F}^{h,e} \) and \( \mathbf{J}^{h,e} \) (Jacobian) at the element center \( \xi = 0 \), respectively. Note, that \( \mathbf{F}_0 \) in (2) is necessary to ensure frame invariance of the method. This is given if \( \tilde{\mathbf{F}} (\varphi^*, \alpha) = Q \mathbf{F} (\varphi, \alpha) \) holds under a superposed rigid body motion \( \varphi^* = Q \varphi + c \) with rotation matrix \( Q \). This holds for both versions presented in (2).

The major advantage of the novel approach is its improved performance in numerical tests. We show this with the well-known Cooks-membrane benchmark and compare three enhanced elements based on the transpose Wilson-modes. H1/E9T is the element introduced in [2] and H1/E9T-\( F_0^{-T} \) its counterpart using the novel transformation. Element HM1/E12T is presented by Simo et al. [4]. The results computed with a nearly incompressible Mooney-Rivlin-type material are shown in Figure 1. The novel element outperforms even HM1/E12T, which has three additional enhanced modes.

2 Novel enhancement approaches

The second part of the present work covers novel finite elements based on the combination of the the EAS-method (see above) with a framework for mixed elements based on polyconvexity (see Bonet et al. [3]). In the framework for polyconvexity the three kinematic measures mapping line, area and volume elements play a vital role. They are given by

\[ \mathbf{F} = \nabla \varphi, \quad \mathbf{H} = \text{cof}(\mathbf{F}) = \frac{1}{2} \mathbf{F}^\top \mathbf{F}, \quad J = \det(\mathbf{F}) = \frac{1}{2} \mathbf{H} : \mathbf{F} \]

(3)

denoting the deformation gradient, its cofactor and its determinant, respectively. The key idea for the novel elements is to enhance these measures separately instead of only enhancing the deformation gradient as in (2). Three classes of novel elements are presented in the following.

* Corresponding author: e-mail robin.pfefferkorn@kit.edu

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2.1 Novel element classes

Selective enhancement is the first new approach. It is based on the introduction of a purely displacement based and enhanced version of the deformation gradient given by

\[ F_\varphi = D\varphi, \quad F_E = D\varphi + \tilde{F}. \]  

(4)

The other kinematic measures are computed in both the enhanced and standard version according to (3). Then, a separate decision can be made for every field if they are used in their enhanced version or not.

Cofactor enhancement is based on enhancing the cofactor with an additional field analogously to the deformation gradient. The three kinematic measures can then e.g. be given by

\[ F = D\varphi + \tilde{F}, \quad H = \frac{1}{2}(D\varphi + \tilde{H}) \otimes (D\varphi + \tilde{H}), \quad J = \frac{1}{3} F : H. \]  

(5)

Note, that other version of enhancing the cofactor are possible and have also been tested.

Determinant enhancement denotes the final class of novel elements. Similar to (5) the determinant is enhanced yielding

\[ F = D\varphi + \tilde{F}, \quad H = \frac{1}{2} F \otimes F, \quad J = \frac{1}{3} F : H + \tilde{J}. \]  

(6)

2.2 General properties

A first important property of all three classes is that they are all based on a variational principle and it can be shown that they are consistent with ordinary continuum mechanics in a continuous setting. Furthermore, all elements fulfill the patch test if simple criteria for the ansatz functions of the enhanced fields are met. Finally, the additional degrees of freedom can be statically condensed on element level.

2.3 Numerical investigations

The novel elements have been tested with various shape functions in benchmark tests and show promising results. They exhibit in contrast to the EAS element based on Wilson-modes [1] no spurious hour-glassing for appropriate choice of shape function. Moreover, they can be constructed completely locking-free and show good coarse mesh accuracy. Results of the nearly incompressible Cooks-membrane test are depicted for selected elements in Fig. 1. We compare one element of each novel class to the standard displacement element H1 and EAS element H1/E9T. The selective enhancement element easMix3D-001 uses only enhancement of the determinant and does not perform too well. However, the other two novel elements enhFH3D-TnTd and enhFJ3D-TaB perform in contrast to that even better than the EAS element. Both of them are based on the transpose Wilson modes for the deformation gradient, which are applied only on the off-diagonal elements for enhFH3D-TnTd. The cofactor of enhFH3D-TnTd is enhanced with diagonal elements of the Wilson-modes and the additional determinant of enhFJ3D-TaB uses bilinear shape functions.

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