Response to Fackerell’s Article

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Abstract

E. D. Fackerell claims: 1) that Alley and Yılmaz treatment of parallel slabs in general relativity is wrong because the Yılmaz metric used is not a solution of the field equations of general relativity; 2) he also claims that the correct treatment of the parallel slab problem in general relativity must be based on the so-called Taub metric. We show below that both of Fackerell’s claims are false. His first claim is based on his failure to distinguish the matter-free regions and the regions with matter. His second claim is based on his failure to recognize that for the Taub metric the left-hand side, hence also the right-hand side of the field equations, are identically zero everywhere. Thus no material systems can be treated via the Taub metric.

1 Introduction

In an article titled “Remarks on the Yılmaz and Alley Papers” E.D. Fackerell [1] claims that the Yılmaz metric, used by H. Yılmaz [2] and C.O. Alley [3] for the treatment of parallel slabs in general relativity, is not a solution to the field equations of general relativity. His train of thought in making this statement seems to be as follows:

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The Yılmaz metric used in the said treatments is of the form

\[ ds^2 = e^{-2\phi} dt^2 - e^{2\phi} \left[ e^{2\epsilon\phi} (dx^2 + dy^2) + e^{4\epsilon\phi} dz^2 \right] \]  \quad (1)

\[ \phi = az + \frac{1}{2} \sigma z^2 + C \]  \quad (2)

where \( \epsilon = \pm 1 \) for general relativity. \( \epsilon = 0 \) for the Yılmaz theory. But Fackerell, based on some calculations of his, seems to conclude that the field equations will everywhere require \(-g_{zz}\) to be

\[ -g_{zz} = e^{2(1 + 2\epsilon)\phi} + 2 \ln|\partial_z \phi| + K. \]  \quad (3)

Therefore, he says, the Yılmaz metric will be valid if and only if \( \partial_z \phi = \text{Constant} \) so that one can set \( 2 \ln|\partial_z \phi| + K = 0 \) to regain the Yılmaz form. Since \( \partial_z \phi \) is not a constant in the Yılmaz field \( \phi \) in Eq. (2) above, the metric used in the treatment, he claims, cannot be a solution to the field equations of general relativity!

However, a simple calculation, which can be done by hand, shows that the Yılmaz metrics for \( \epsilon = \pm 1 \) satisfy the field equations of general relativity everywhere exactly. If Fackerell did this calculation he would have seen that it is indeed true. Therefore the question is not whether Yılmaz and Alley used wrong metrics in their works but rather how and where precisely Fackerell himself made a fundamental error in his calculations.

## 2 Fackerell’s Fundamental Error

To find this out, we shall first verify that Yılmaz’ metrics are indeed solutions of the required field equations. Fackerell’s error will be clearly seen in the course of this exercise. The general relativity equations to satisfy with \( \tau^\nu_\mu \) as a matter tensor are

\[ \tfrac{1}{2} G^\nu_\mu = \tau^\nu_\mu. \]  \quad (4)

There are two distinct cases with two sub-cases each: [Note 1]
A) : $\sigma = 0, \, \epsilon = \pm 1$ (Matter – free regions)

B) : $\sigma \neq 0, \, \epsilon = \pm 1$ (Regions with matter)

where $\sigma$ corresponds to the ordinary Laplacian of $\phi$. The Yilmaz metric is of the form

$$ds^2 = e^{-2\phi} dt^2 - e^{\mu}(dx^2 + dy^2) - e^\eta dz^2. \quad (5)$$

where $\phi$, $\mu$ and $\eta$ are functions only of the coordinate $z$. Using the shorthand $(') = \partial_z \phi$, the general relativity equations (4) for metric (5), after multiplying by $8e^\eta$, are:

$$\mu' (3\mu' - 2\eta') + 4\mu'' = 8e^\eta \tau_0^0 \quad (6)$$

$$4\phi'^2 + (\mu' - 2\phi') (\mu' - \eta') + 2(\mu'' - 2\phi'') = 8e^\eta \tau_1^1 = 8e^\eta \tau_2^2 \quad (7)$$

$$\mu' (\mu' - 4\phi') = 8e^\eta \tau_3^3. \quad (8)$$

Let us first consider the matter-free case (A):

For the Yilmaz metric, where $\mu = 2(1 + \epsilon)\phi$ and $\eta = 2(1 + 2\epsilon)\phi$, all terms quadratic in $\phi'$ cancel. (The algebra is exhibited in [Note 2].) The equations (6) and (7) in this case become:

$$(1 + \epsilon)\phi'' = 0 \quad (9)$$

$$\frac{1}{2}\epsilon \phi'' = 0 \quad (10)$$

with the solution

$$\phi'' = 0 \quad (11)$$

$$\phi = az + C \quad (12)$$

where $a$ and $C$ are constants, hence $\phi' = \text{Constant}$ is automatic. Setting $2\ln |a| + K = 0$ in equation (3) above, as Fackerell himself suggests, we have

$$-g_{zz} = e^{2(1 + 2\epsilon)\phi} \quad (13)$$

as originally stated by Yilmaz.
Let us now consider also the regions with matter, case (B):
The $\tau_{\mu}^{\nu} \neq 0$ equations (6) - (8) are satisfied with the matter tensor of the form

\[ \tau_{\mu}^{\nu} = e^{-\eta} \text{diag}[(1 + \epsilon, \epsilon/2, \epsilon/2, 0)\sigma] \]  

where by directly substituting $\mu = 2(1 + \epsilon)\phi$, $\eta = 2(1 + 2\epsilon)\phi$, all terms quadratic in $\phi'$ again cancel for $\epsilon = \pm 1$ (as shown in [Note 2]) and we have

\[(1 + \epsilon)\phi'' = (1 + \epsilon)\sigma \]  

\[ \frac{1}{2} \epsilon \phi'' = \frac{1}{2} \epsilon \sigma \]  

with the solution

\[ \phi'' = \sigma \]  

\[ \phi = az + \frac{1}{2} \sigma z^2 + C \]  

as stated by Alley and Yılmaz. But now there are no logarithmic terms in the exponential because $\phi''$ terms causing them are taken away by equating them to the matter terms. The result is again the form of equation (13) above

\[-g_{zz} = e^{2(1 + 2\epsilon)\phi} \]

as originally stated by Yılmaz.

The whole discussion can be reduced into the realization that

\[ \phi'' = \begin{cases} 0 & \text{(Outside a slab),} \\ \sigma & \text{(Inside a slab)} \end{cases} \]  

Nothing else is needed, not even Fackerell’s way of removing the logarithmic terms above, since the form given by Yılmaz makes it superfluous. [Note 3]

Thus, whether we like it or not, the Yılmaz metric is a solution of the field equations of general relativity, both in the matter-free regions and in the regions with matter. Then, where did Fackerell make his mistake? In the light of the above discussion we do not have far to look. Since in a matter-free region $\phi$ is linear ($\phi'' = 0$) Fackerell thought this would contradict the quadratic term in equation (18) above

\[ \phi = az + \frac{1}{2} \sigma z^2 + C. \]
But those are different regions with different material contents as indicated above. Linearity in coordinates of a potential in one region does not prevent it from being nonlinear in coordinates in a different region. Thus, contrary to this fact, Fackerell is requiring that the linearity of $\phi$ in the matter-free region be imposed on the region with matter. This is equivalent to requiring that there be no matter terms at all, anywhere. A more elementary calculational error is hardly imaginable.

We should really stop here and go no further with Fackerell’s paper. All his other asides such as his appeals to authority and his prima facie case, etc., are irrelevant since his main tenet is false. Therefore it is useless to go over every conceivable argument. We must however, mention just one additional error, because this one is so hopelessly hidden that the reader, not being able to penetrate, might think he has something.

3 Fackerell’s Second Error: His Treatment of the Parallel Slab Problem in General Relativity

Let us now also examine Fackerell’s own treatment of the parallel slabs. Fackerell claims that the appropriate metric to use for the slab calculations is the Taub metric

$$ds^2 = f^{-1/2}dt^2 - f(dx^2 + dy^2) - f^{-1/2}dz^2$$  

(20)

where

$$f = (1 + kz).$$  

(21)

However, he does not use the slabs as originally defined, and he does not use equations of motion. Instead, he considers infinitely thin sheets. Under some sophisticated looking maneuvers he claims he gets certain results which sound impressive. But it can be shown by a simple calculation that, for the Taub metric the left-hand side (hence also the right-hand side) of the field equations are identically zero. Again, if Fackerell performed this calculation he would have seen that it is true. In any case the proof is so simple that we
shall here reproduce it: First, identify from the Taub metric by comparing with the general metric, equation (5), that $\mu = 4\phi$, $\eta = -2\phi$. Then simply calculate to get,

$$e^{4\phi} = f = 1 + k z$$
$$\phi = \frac{1}{4} \ln(1 + k z)$$
$$\phi' = (k/4)/(1 + k z)$$
$$\phi'' = -(k^2/4)/(1 + k z)^2 = -4\phi'^2.$$  

Finally, substituting into the required field equations (6) - (8) one finds that all expressions on the left-hand side are identically zero. (The steps in this calculation are exhibited in [Note 4].) Therefore the right-hand side is

$$\tau_\mu^\nu = \text{diag} [(0, 0, 0, 0)].$$

This is because, unlike the Yilmaz metrics, the term linear in $\phi''$ (that is, the Laplacian) also disappears and nothing is left to represent matter.

As to the equations of motion, we shall then have

$$\sigma d^2 x_\mu/ds^2 = \frac{1}{2} \partial_\mu g_{\alpha\beta} \tau^{\alpha\beta} = 0.$$  

This is clearly the answer to the problem of motion with the Taub metric. But as we have just said, Fackerell does not use the equations of motion of the theory, and he does not use the definition of the slab as originally intended. Instead, he introduces infinitely thin sheets. This is presumably to get around the difficulty that he cannot introduce a matter density $\tau_\mu^\nu$. The truth is that as long as he uses the Taub metric he cannot introduce any matter at all. Under these circumstances how can the “junction conditions” etc., be of any help? Can there be a magic in the junction conditions so as to create something out of absolutely nothing? [Note 5]

Yilmaz and Alley do not use such devices. When they say slab, they mean slab, when they say equations of motion, they mean equations of motion. They do not use junction conditions to supplant the equations of motion nor do they stray from the boundary conditions of the standard field theory. As Fackerell himself points out, the “junction conditions” he uses are not necessarily equivalent to the boundary conditions of the standard field theory. Fackerell’s deductions from the Taub metric are clearly false.
4 Discussion

We have been informed that during a recent discussion on the internet newsgroup sci.physics.relativity, a respected member of the group stated:

“Alley and Yilmaz’s claims along these lines have been demolished, thoroughly and in detail, in a paper by Edward Fackerell, ...[ref. [1] of this paper]..., Fackerell shows that Yilmaz and Alley made a careless mistake in their calculation, and he gives the correct calculation, for both flat plates and spherical shells.”

We cannot expect everyone to dig into the truth of the matter but we must urge Fackerell to honorably retract his position. This sort of thing is not doing any service to science – nor any justice to the scientific truth.

Acknowledgment

We wish to thank Kirk Burrows for carrying out the detailed calculations associated with our treatment of the two parallel slabs problem [4]. His expert knowledge of the symbolic calculational capabilities of Mathematica has been invaluable in our continuing investigations of the theories of Yilmaz and of Einstein.

References

[1] Fackerell, E.D., “Remarks on the Yilmaz and Alley Papers,” in Proceedings of the First Australasian Conference on General Relativity and Gravitation, edited by D.L. Wiltshire, University of Adelaide, 1996, pp. 117-131. [http://www.physics.adelaide.edu.au/ASGRG/ACGRG17]

[2] Yilmaz, H., “Did the Apple Fall?,” in Frontiers of Fundamental Physics, edited by M. Barone and F. Selleri, Plenum Press, New York, 1994. pp. 116-124.
Alley, C.O., “Investigations with Lasers, Atomic Clocks and Computer Calculations of Curved Spacetime and of the Differences Between the Gravitation Theories of Yılmaz and of Einstein,” in *Frontiers of Fundamental Physics*, edited by M. Barone and F. Selleri, Plenum Press, New York, 1994, pp. 125-136.

———, “The Yılmaz Theory of Gravity and its Compatibility with Quantum Theory,” in *Fundamental Problems in Quantum Theory*, edited by D. Greenberger and A. Zeilinger, Ann. NY Acad. Sci., vol. 755, 1995, pp. 464-475.

The second paper includes a treatment of the two slab problem which is similar to that in the first paper, but adds the point that the one dimensional problem of two parallel slabs is realizable experimentally as two circular disks, with the smaller disk surrounded by a “guard ring” in the shape of a circular annulus, similar to the configuration often used for electrical capacitors.

(Two printer’s errors in the second paper should be corrected: on page 472 in Eq. (22), the exponent in the last term in the metric “2φ(1 + ϵ)” should read “2φ(1 + 2ϵ)”); on page 470, following Eq.(15), “in equation 21 below” should read “in equation (15)”.

Burrows, R. K., “A Brief Study of the Metric Theory of Hüseyin Yılmaz using Symbolic Computer Calculations,” Master’s Thesis, Department of Physics, University of Maryland at College Park, 26 August 1994.

Notes

[Note 1] Fackerell treats the cases $\epsilon = +1$ and $\epsilon = -1$ separately from the start. This gets confusing and repetitive. We treat them uniformly with $\epsilon$ as a parameter and only at the end set $\epsilon = +1$ and $\epsilon = -1$.

[Note 2] Substituting the expressions $\mu = 2(1 + \epsilon)\phi$ and $\eta = 2(1 + 2\epsilon)\phi$ into the left-hand sides of equations (6) - (8), one obtains the following:

Left-hand side of Equation (6):

$$\mu'(3\mu' - 2\eta') + 4\mu'',$$
\[2(1 + \epsilon)\phi'[6(1 + \epsilon)\phi' - 4(1 + 2\epsilon)\phi'] + 4[2(1 + \epsilon)\phi''],
\]
\[2(1 + \epsilon)\phi'[2(1 - \epsilon)\phi'] + 8(1 + \epsilon)\phi'';
\]
\[4\phi'^2(1 - \epsilon^2) + 8(1 + \epsilon)\phi''\]

Left-hand side of Equation (7):
\[4\phi'^2 + (\mu' - 2\phi')(\mu' - \eta') + 2(\mu'' - 2\phi''),
\]
\[4\phi'^2 + [2(1 + \epsilon)\phi' - 2\phi'][2(1 + \epsilon)\phi' - 2(1 + 2\epsilon)\phi'] + 2[2(1 + \epsilon)\phi'' - 2\phi''],
\]
\[4\phi'^2 + [2\epsilon\phi'][-2\epsilon\phi'] + 4\phi'',
\]
\[4\phi'^2(1 - \epsilon^2) + 4\epsilon\phi''\]

Left-hand side of Equation (8):
\[\mu'((\mu' - 4\phi'),
\]
\[2(1 + \epsilon)\phi'[2(1 + \epsilon)\phi' - 4\phi'],
\]
\[2(1 + \epsilon)\phi'[-2\phi' + 2\epsilon\phi'],
\]
\[-4\phi'^2(1 - \epsilon^2).\]

The factor \((1 - \epsilon^2)\) appears in the left-hand side of each equation multiplying \(\phi'^2\). To satisfy Equation (4), one must have \((1 - \epsilon^2) = 0\), leading to the values \(\epsilon = \pm 1\) for the general relativity solutions. Equations (9) and (10) in the text are obtained after dividing the left-hand sides above by 8.

[Note 3] The only possible demand Fackerell can have would be, that the matter tensor should be \(\tau_{\nu}^{\mu} = e^{-\eta}\text{diag}[(\sigma, 0, 0, 0)]\) for Newtonian correspondence. Ah \cdots, but that requires \(\epsilon = 0\), which is precisely the Yilmaz Theory! Some general relativists refer to the matter tensor \(\tau_{\nu}^{\mu} = e^{-\eta}\text{diag}[(1 + \epsilon, \epsilon/2, \epsilon/2, 0)\sigma]\), with \(\epsilon = \pm 1\), shown here to be a solution of the general relativity field equations for slabs, as “peculiar matter.” However, the standard non-isotropic Schwarzschild metric solutions of general relativity have matter tensors of exactly this form! [3]

[Note 4] To see that the Taub metric is devoid of physical content, substitute into the left-hand sides of the field equations (6) - (8) the relations which hold in the Taub metric: \(\mu = 4\phi, \eta = -2\phi, \phi'' = -4\phi'^2\), to obtain:
Left-hand side of Equation (6):

\[
\begin{align*}
\mu'(3\mu' - 2\eta') + 4\mu'', \\
4\phi'(12\phi' + 4\phi') + 16\phi'', \\
64\phi'^2 - 64\phi'^2 \equiv 0.
\end{align*}
\]

Left-hand side of Equation (7):

\[
\begin{align*}
4\phi'^2 + (\mu' - 2\phi')(\mu' - \eta') + 2(\mu'' - 2\phi''), \\
4\phi'^2 + (4\phi' - 2\phi')(4\phi' + 2\phi') + 2(4\phi'' - 2\phi''), \\
4\phi'^2 + 12\phi'^2 + 4\phi'', \\
16\phi'^2 - 16\phi'^2 \equiv 0.
\end{align*}
\]

Left-hand side of Equation (8):

\[
\begin{align*}
\mu'(\mu' - 4\phi'), \\
4\phi'(4\phi' - 4\phi') \equiv 0.
\end{align*}
\]

[Note 5] The issue of interactive N-body solutions is discussed quite generally by C.O. Alley, D. Leiter, Y. Mizobuchi and H. Yılmaz, “Energy Crisis in Astrophysics (Black Holes vs. N-Body Metrics),” available at the Los Alamos e-print archive, xxx.lanl.gov, as astro-ph/9906458 (28 June 1999). The absence of interaction between the two slabs is a specific example, admitting a simple analytic solution, of the situation in general relativity.