Transmission Schemes for Four-Way Relaying in Wireless Cellular Systems

Huaping Liu†,*, Petar Popovski*, Elisabeth de Carvalho*
Yuping Zhao†, Fan Sun* and Chan Dai Truyen Thai*

†State Key Laboratory of Advanced Optical Communication Systems and Networks
Peking University, China
*Department of Electronic Systems, Aalborg University, Denmark
Email: {liuhp,yuping.zhao}@pku.edu.cn, {petarp,edc,fs,ttc}@es.aau.dk

Abstract

Two-way relaying in wireless systems has initiated a large research effort during the past few years. While one-way relay with a single data flow introduces loss in spectral efficiency due to its half-duplex operation, two-way relaying based on wireless network coding regains part of this loss by simultaneously processing the two data flows. In a broader perspective, the two-way traffic pattern is rather limited and it is of interest to investigate other traffic patterns where such a simultaneous processing of information flows can bring performance advantage. In this paper we consider a scenario beyond the usual two–way relaying: a four-way relaying, where each of the two Mobile Stations (MSs) has a two-way connection to the same Base Station (BS), while each connection is through a dedicated Relay Station (RS). While both RSs are in the range of the same BS, they are assumed to have antipodal positions within the cell, such that they do not interfere with each other. We introduce and analyze a two-phase transmission scheme to serve the four-way traffic pattern defined in this scenario. Each phase consists of combined broadcast and multiple access. We analyze the achievable rate region of the new schemes for two different operational models for the RS, Decode-and-Forward (DF) and Amplify-and-Forward (AF), respectively. We compare the performance with a state-of-the-art reference scheme, time sharing is used between the two MSs, while each MS is served through a two-way relaying scheme. The results indicate that, when the RS operates in a DF mode, the achievable rate regions are significantly enlarged. On the other hand, for AF relaying, the gains are rather modest. The practical implication of the presented work is a novel insight on how to improve the spatial reuse in wireless cellular networks by coordinating the transmissions of the antipodal relays.
I. INTRODUCTION

Relay-based communication has matured in the past decades, both in terms of knowing the fundamental limits, but also with respect to practical system implementation and utilization. Capacity bounds and various cooperative strategies for relay networks have been studied in [1]. [2] developed Decode-and-Forward (DF) relaying to multiple access relay channels and broadcast relay channels, and generalized Compress-and-Forward (CF) relaying to multiple relays as well. A paradigm shift occurred with the concept of network coding [3], in which the relays process multiple data flows simultaneously and transmit functions of the incoming communication flows, rather than only replicating the incoming flows. This idea can bring profound gains in a wireless setting, notably in a scenario with two-way relaying [4], [5]. An overview of bidirectional relay protocols is given in [6]. The achievable rate regions for the two-way relaying channel under full-duplex assumption were given in [7], but a viable assumption for wireless receivers is that they can operate in a half-duplex manner [8] [9]. Note that for the half-duplex wireless transceivers, one-way relaying suffers from a loss in spectral efficiency as the relay cannot transmit and receive at the same time. Wireless network coding can help to regain part of this loss when two or more data flows are served simultaneously through the same relay. The work [10] has analyzed the achievable rate regions of two-way relaying under half-duplex condition and compared various bidirectional relay protocols. Another related work is [11], which proves the optimal broadcast strategy for bidirectional relaying.

Although the benefits of wireless network coding have largely been confined to the canonical two-way relaying scenario, one can identify the underlying principles and then apply them in more generalized scenarios. Those principles are (1) simultaneous service of multiple flows over the wireless medium and (2) cancellation of interference based on previously gathered information. We have utilized these principles in order to devise transmission schemes in the case when the flows to a relayed and a direct user are simultaneously served [12]. One of the main objectives of this paper is to leverage on these principles and investigate how they can be applied in a new, and very practical scenario. Namely, this involves four-way relaying, in which two Mobile Stations (MSs) have each a two-way connection to the same Base Station (BS), while each of them uses a dedicated Relay Station (RS). This is depicted on Fig. 1. Both RSs are in the range of the same BS, but they are at the “opposite sides” of the cell, i. e.
have antipodal positions, in a sense that they do not interfere with each other. For brevity, let BS→RS1 denote the directional link from BS to RS1 (and similar for the other links). The state-of-the-art conventional scheme for this scenario is multiplexing two independent two-way relaying schemes in time as shown in Fig. 2(a). Another conventional scheme is simultaneously conducting transmissions which do not interfere with each other as seen in Fig. 2(b).

A similar scenario has been considered [13], where DS-CDMA is used to avoid interference, the nodes use BPSK modulation and the relay applies physical layer network coding (denoise-and-forward). Unlike [13], in our work we do not use orthogonal CDMA codes, but take advantage of the antipodal relay deployment in order to coordinate the interference. Furthermore, in our work we are not constrained to a specific modulation type, since we analyze the achievable rate region when the nodes use information-theoretic Gaussian codebooks. A careful look at the scenario reveals that the number of design possibilities is significantly increased. For example, an alternative scheme (Fig. 2(b)) could be the following 4-stage scheme: (1) BS→RS1 and U2→RS2, (2) RS1→U1 and RS2→BS, (3) BS→RS2 and U1→RS1, and (4) RS1→BS and RS2→U2. With similar observation, other transmission schemes can be proposed. We focus on a scheme that consists of only two stages, where each phase consists of combined broadcast and multiple access. In the first phase, the BS broadcasts to RS1 and RS2 using superposition coding, while U1 and U2 are simultaneously carrying out their uplink transmissions. In the second phase, the relays RS1 and RS2 are broadcasting: RS1 to BS and U1, while RS2 to BS and U2. Assuming DF or AF operation at the relay, a reference scheme that represents the state-of-the-art can be defined as follows. Each MS is served through a two-way relaying scheme, while the BS applies time-sharing in order to serve both MSs. The achievable rate region for the considered scenario is four-dimensional. In order to obtain a better insight, we parametrize the two-way traffic associated with each of the MSs by defining downlink-uplink ratio. The results show that, when the RS operates in a DF mode, the achievable rate regions are significantly enlarged. When AF is applied, in most cases the proposed two-phase transmission scheme has larger sum-rate than the reference scheme which has four phases.

The paper is organized as follows. Section (II) introduces the notations and definitions about system and channel models. Section (III) includes the analysis of the achievable rate region for two-phase transmission protocols. Section (IV) presents the achievable rate region of the reference transmission protocols which have four phases. Section (VI) shows the figures under
some special conditions to give out an intuitive insight for the achievable rate regions.

II. SYSTEM AND CHANNEL MODELS

Consider a bidirectional cellular network in which a BS intends to exchange information with two mobile stations U1 and U2 with the aid of two relay stations RS1 and RS2, as Fig. 1 shows. All the nodes are half-duplex, such that a node can either transmit or receive at a given time. We assume that the MSs do not have direct links to/from the BS. For simplicity, we use 1, R1, B, R2, 2 as the indices in the formulas to denote the mobile station U1, relay station RS1, base station BS, relay station RS2 and mobile station U2 respectively. The channels are denoted by \( h_{11}(U1-RS1) \), \( h_{12}(RS1-BS) \), \( h_{22}(BS-RS2) \) and \( h_{21}(RS2-U2) \). Each channel \( h_l, l \in \{11, 12, 22, 21\} \), is reciprocal, known at all the nodes. Each MS has a two-way, uplink/downlink traffic to/from the BS. The noise at all receivers \( z_j \sim CN(0, \sigma^2), j \in \{1, R1, B, R2, 2\} \) is independent Additive White Gaussian Noise (AWGN) with zero mean and unit variance. If node \( j \) is transmitting, its transmission power is bounded by \( \bar{P}_j, j \in \{1, R1, B, R2, 2\} \), i.e., \( E\{|x_j|^2\} = P_j \leq \bar{P}_j \). The capacity of a single link is \( C(\gamma) = \log_2(1 + \gamma) \), where \( \gamma \) is the Signal-to-Noise Ratio (SNR).

Fig. 2(a) illustrates a state-of-the-art transmission scheme based on time-division between two different two-way relaying instances. The first phase is the multiple access (MA) from U1 and BS to RS1. The second phase is the broadcast (BC) from RS1 to U1 and BS. Similarly, the third phase is the MA from U2 and BS to RS2, while the fourth phase is the BC from RS2 to U2 and BS. This four-phase four-way relaying scheme will serve as a reference scheme.

In the proposed scheme, the two downlink signals for U1 and U2 are broadcast by the BS using superposition coding. The whole transmission contains only two phases where communications with the two users occur simultaneously. Furthermore, a MA process and a BC process occur simultaneously in each phase, as the Fig. 3 shows. The proposed scheme makes full use of the side information at U1, U2 and BS to cancel the self-interference.

III. ACHIEVABLE RATE REGIONS OF TWO-PHASE FOUR-WAY RELAYING

The signals sent by U1 and U2 are denoted by \( x_1 \) and \( x_2 \), respectively. The BS uses superposition coding and the broadcast signal is

\[
x_B = \sqrt{\alpha} x_{B1} + \sqrt{1 - \alpha} x_{B2}, \quad \alpha \in [0, 1]
\]
where $x_{B1}(x_{B2})$ is the signal intended for $U1(U2)$ and $E\{|x_{B1}|^2\} = E\{|x_{B2}|^2\} = E\{|x_{B}|^2\} = P_{B} \leq \bar{P}_{B}$. Define $R_{d}^i$ as the downlink data rate of $U_i$ and define $R_{u}^i$ as the uplink data rate of $U_i$.

There are one BC process and two MA processes in the first phase: BS broadcasts the data intended to $U1$ and the data intended to $U2$ using superposition coding, while $U1$ and $U2$ transmit their uplink data. A MA occurs at RS1 with BS and $U1$ as transmitters, while another MA occurs at RS2 with BS and $U2$ as transmitters. In the second phase, there are two BC processes and one MA process. Each relay broadcasts a signal that is a function of the signal received in the first phase, while the BS acts as a receiver over a MA channel as the two RSs are transmitting simultaneously. Note that this is not an ordinary MA channel, since BS has a side information (i.e., its own information sent in phase 1) about the transmitted signals from the RSs. Finally, in the second phase, each of the users receives a signal only from the relay and, similar to the usual two-way relaying, removes the self-information and decodes the desired signal. Next, we derive the rate region of the four-way relaying scheme when the relay operates in AF and DF mode, respectively.

A. Amplify-and-forward

At the end of phase 1, $RS_i$ receives
\[ y_{Ri} = h_{i1}x_{i} + h_{i2}x_{B} + z_{Ri}, i = 1,2. \]  
(2)

After reception, RS1 and RS2 amplify the received signals as follow:
\[ x_{Ri} = \beta_{i}y_{Ri}, \beta_{i} = \frac{P_{Ri}}{\sqrt{|h_{i1}|^2P_{i} + |h_{i2}|^2P_{B} + 1}}, i = 1,2 \]  
(3)

Here, $\beta_{i}$ is the amplification factor according to the relay transmission power constraints. $x_{Ri}$ is the signal broadcast by $RS_i$.

At the end of phase 2, $U_i$ receives $y_{i}$. Based on the side information about $x_{i}$, the channels and $\beta_{i}$, $U_i$ can cancel the contribution of $x_{i}$ from $y_{i}$ to get $\bar{y}_{i}$,
\[ \bar{y}_{i} = h_{i1}h_{i2}\beta_{i}(\sqrt{\alpha x_{B1}} + \sqrt{1-\alpha x_{B2}}) + h_{i1}\beta_{i}z_{Ri} + z_{i} , i = 1,2. \]  
(4)

As the BS has the side information about $x_{B}$, it can remove the contribution of $x_{B}$ from the received signal $y_{B}$ to get $\bar{y}_{B}$,
\[ \bar{y}_{B} = h_{11}h_{12}\beta_{1}x_{1} + h_{22}h_{21}\beta_{2}x_{2} + h_{12}\beta_{1}z_{R1} + h_{22}\beta_{2}z_{R2} + z_{B}. \]  
(5)
Equation (4) describes a BC channel where the signals are sent through superposition coding: $x_{Bi}$ is intended to user $i$. Equation (5) describes a MA channel. Using side information, the system becomes equivalent to 2 direct communication channels between the BS and the users. The first channel is a BC channel and the second channel is a MA channel, for which known information theory results can be used.

The SNRs of an equivalent BC channel defined in (4) are given by

$$S_i = \frac{E \{ |h_{i1}h_{i2} \beta_i x_B|^2 \}}{E \{ |h_{i1} \beta_i z_{Ri} + z_i|^2 \}} = \frac{|h_{i1}|^2 |h_{i2}|^2 P_B P_{Ri}}{|h_{i1}|^2 (P_i + P_{Ri}) + |h_{i2}|^2 P_B + 1}, \quad i = 1, 2. \tag{6}$$

The SNRs of an equivalent MA channel defined in (5) are given by

$$S'_i = \frac{E \{ |h_{i1}h_{i2} \beta_i x_i|^2 \}}{E \{ |h_{i1} \beta_i z_{R1} + h_{i2} \beta_i z_{R2} + z_B|^2 \}} = \frac{|h_{i1}|^2 |h_{i2}|^2 P_{R1}}{|h_{i1}|^2 P_i + |h_{i2}|^2 P_{B} + 1} + \frac{|h_{i2}|^2 P_{R2}}{|h_{i1}|^2 P_i + |h_{i2}|^2 P_{B} + 1} + 1, \quad i = 1, 2. \tag{7}$$

**Proposition 1**: The achievable rate region of the two-phase four-way relaying scheme using AF is

when $S_1 > S_2$

$$R^d_1 < \frac{1}{2} C(S_1 \alpha), \quad R^d_2 < \frac{1}{2} C \left( \frac{S_2(1 - \alpha)}{S_2 \alpha + 1} \right), \quad R^u_1 < \frac{1}{2} C \left( S'_1 \right)$$

$$R^u_2 < \frac{1}{2} C \left( S'_2 \right), \quad R^u_1 + R^u_2 < \frac{1}{2} C \left( S'_1 + S'_2 \right) \tag{8}$$

when $S_1 \leq S_2$

$$R^d_1 < \frac{1}{2} C \left( \frac{S_1 \alpha}{S_1(1 - \alpha) + 1} \right), \quad R^d_2 < \frac{1}{2} C \left( S_2(1 - \alpha) \right), \quad R^u_1 < \frac{1}{2} C \left( S'_1 \right)$$

$$R^u_2 < \frac{1}{2} C \left( S'_2 \right), \quad R^u_1 + R^u_2 < \frac{1}{2} C \left( S'_1 + S'_2 \right). \tag{9}$$

**Proof**: The whole system is composed by a BC channel and a MA channel.

1) **Broadcast Channel**: From [14], when $S_1 > S_2$ the optimal decoding consists in decoding $x_{B2}$ at U2 treating $x_{B1}$ as Gaussian noise. At U1, the decoder first decodes $x_{B2}$, then cancels $x_{B2}$ from the received signal $\tilde{y}_1$. Finally, U1 decodes $x_{B1}$. The capacity region of the AWGN BC channel in (4) is

$$R^d_1 < \frac{1}{2} C \left( S_1 \alpha \right), \quad R^d_2 < \frac{1}{2} C \left( \frac{S_2(1 - \alpha)}{S_2 \alpha + 1} \right) \tag{10}$$

here, $R^d_1$ is the rate of $x_{B1}$, $R^d_2$ is the rate of $x_{B2}$. When $S_1 \leq S_2$ the capacity region is similar.

The scaling factors $1/2$ in (10), account for the half-duplex constraint. Furthermore, if RS1 and
RS2 use AF, phase 1 and phase 2 have the same duration. Then U1 and U2 only spend half of the time to receive the signal. BS spends half of the time to transmit. When \( S_1 \leq S_2 \), the capacity region of the AWGN BC channel in (4) is,
\[
R_1^d < \frac{1}{2} C \left( \frac{S_1 \alpha}{S_1 (1 - \alpha) + 1} \right), \quad R_2^d < \frac{1}{2} C (S_2 (1 - \alpha)).
\] (11)

2) Multiple access channel: From [14], the capacity region of AWGN MA channel in (5) is
\[
R_1^u < \frac{1}{2} C (S_1'), \quad R_2^u < \frac{1}{2} C (S_2'), \quad R_1^u + R_2^u < \frac{1}{2} C (S_1' + S_2').
\] (12)

We have scaling factors 1/2 in (12), because U1 and U2 only spend half of the time to transmit the signal. BS spends half of the time to receive the signal.

Combining (10)-(12), we obtain the achievable rate region of two-phase AF scheme as shown in proposition 1.

B. Decode-and-forward

At the end of phase 1, RS\(i\) receives
\[
y_{Ri} = h_{i1} x_i + h_{i2} \sqrt{\alpha_i} x_{B_i} + h_{i2} \sqrt{1 - \alpha_i} x_{B_j} + z_{Ri}.
\] (13)
here \((i, j) \in \{(1, 2), (2, 1)\}\), \(\alpha_1 = \alpha, \alpha_2 = 1 - \alpha\).

The decoder in RS\(i\) decodes the signal \(x_i\) and \(x_{B_i}\), then re-encodes the messages of \(x_i\) and \(x_{B_i}\) into \(x_{Ri}\). RS\(i\) does not need to decode \(x_{B_j}\), as it is intended to U\(j\). In fact, \(x_{B_j}\) should only be decoded if there is a benefit for decoding \(x_i\) and \(x_{B_i}\), otherwise \(x_{B_j}\) should be treated as noise. However, the successful decoding of \(x_{B_j}\) will impose a rate limitation on \(R_j^d\).

At the end of phase 2, Ui receives
\[
y_i = h_{i1} x_{R_i} + z_i.
\] (14)

Meanwhile BS receives,
\[
y_B = h_{12} x_{R_1} + h_{22} x_{R_2} + z_B.
\] (15)

Equations (14) and (15) describe the BC channel from RS\(i\) to Ui and BS which have the side information. Equation (13) describes the MA channel at BS with side information.

May 2, 2014 DRAFT
Proposition 2: The achievable rate region of the two-phase four-way relaying scheme using DF is the convex closure of all 4-dimensional rate tuples satisfying \((R_1^u, R_1^d, R_2^u, R_2^d) \in D\).

\[
D^{(1)}_{R_1} = M_{R_1}^2 \cup M_{R_1}^3 \quad (16a)
\]

\[
D^{(1)} = D^{(1)}_{R_1} \cap D^{(1)}_{R_2} \quad (16b)
\]

\[
D = D^{(1)} \cap D^{(2)} \quad (16c)
\]

where \(M_{R_1}^2, M_{R_1}^3\) and \(D^{(2)}\) are defined as follows

\[
M_{R_1}^2 = \{ (R_i^u, R_i^d, R_j^u, R_j^d) \mid (R_i^u, R_i^d) \in \{20\} \} \text{ where } (i, j) \in \{(1, 2), (2, 1)\} \quad (17)
\]

\[
M_{R_1}^3 = \{ (R_i^u, R_i^d, R_j^u, R_j^d) \mid (R_j^u, R_j^d) \in \{21\} \} \text{ where } (i, j) \in \{(1, 2), (2, 1)\} \quad (18)
\]

\[
D^{(2)} = \{ (R_1^u, R_1^d, R_2^u, R_2^d) \mid (R_i^u, R_i^d, R_j^u, R_j^d) \in \{22\} \} \quad (19)
\]

where \(0 \leq \alpha_i, \tau \leq 1\).

\[
\begin{align*}
R_i^u < \tau C \left( \frac{|h_{i1}|^2 P_i}{(1 - \alpha_i)|h_{i2}|^2 P_B + 1} \right), & \quad R_i^d < \tau C \left( \frac{\alpha_i|h_{i2}|^2 P_B}{(1 - \alpha_i)|h_{i2}|^2 P_B + 1} \right) \\
R_i^u + R_i^d < \tau C \left( \frac{|h_{i1}|^2 P_i + \alpha_i|h_{i2}|^2 P_B}{(1 - \alpha_i)|h_{i2}|^2 P_B + 1} \right). & \quad (20a)
\end{align*}
\]

\[
\begin{align*}
R_i^u < \tau C \left( |h_{i1}|^2 P_i \right), & \quad R_i^d < \tau C \left( \alpha_i|h_{i2}|^2 P_B \right), & \quad R_j^d < \tau C \left( (1 - \alpha_i)|h_{i2}|^2 P_B \right) \quad (21a)
\end{align*}
\]

\[
\begin{align*}
R_i^u + R_i^d < \tau C \left( |h_{i1}|^2 P_i + \alpha_i|h_{i2}|^2 P_B \right), & \quad (21b)
\end{align*}
\]

\[
\begin{align*}
R_i^u + R_j^d < \tau C \left( |h_{i1}|^2 P_i + (1 - \alpha_i)|h_{i2}|^2 P_B \right) & \quad (21c)
\end{align*}
\]

\[
\begin{align*}
R_i^d + R_j^d < \tau C \left( |h_{i2}|^2 P_B \right) & \quad (21d)
\end{align*}
\]

\[
\begin{align*}
R_i^u + R_i^d + R_j^d < \tau C \left( |h_{i1}|^2 P_i + |h_{i2}|^2 P_B \right) & \quad (21e)
\end{align*}
\]

\[
\begin{align*}
\begin{align*}
R_1^u < (1 - \tau) C \left( |h_{i1}|^2 P_{R1} \right), & \quad R_2^d < (1 - \tau) C \left( |h_{i2}|^2 P_{R2} \right) \quad (22a)
\end{align*}
\]

\[
\begin{align*}
\begin{align*}
R_1^u < (1 - \tau) C \left( |h_{i1}|^2 P_{R1} \right), & \quad R_2^u < (1 - \tau) C \left( |h_{i2}|^2 P_{R2} \right) \quad (22b)
\end{align*}
\]

\[
\begin{align*}
\begin{align*}
R_1^u + R_2^u < (1 - \tau) C \left( |h_{i2}|^2 P_{R1} + |h_{i2}|^2 P_{R2} \right) \quad (22c)
\end{align*}
\]

Proof: Let us focus on the communication of U1 in phase 1; the other user is treated similarly. In (17) and (18) we set \(i = 1\) and \(j = 2\). U1 does not need to receive \(x_{B2}\) and therefore the relay RS1 will only decode \(x_{B2}\) if it helps to obtain higher rates \((R_1^u, R_1^d)\) for
the data flows supported through the RS. $M^2_{R1}$ is the rate region that is obtained when RS1 receives $x_1$ and $x_{B1}$ over a MA channel, while treating $x_{B2}$ as noise. Note that $M^2_{R1}$ is a four-dimensional region in which no constraints are put on $(R^u_2, R^d_2)$, which means that they can have arbitrary non-negative values. This is made for the sake of consistent notation; the actual upper limits on $(R^u_2, R^d_2)$ will be set by intersecting the regions in (16). If we put the constraint that RS1 should decode $x_{B2}$, then we need to consider a three-user MA channel at RS1 with the signals $x_1, x_{B1}, x_{B2}$. The 4-dimensional rate region is given by (18) where $R^u_2$ is unconstrained. The union $M^2_{R1} \cup M^3_{R1}$ is a four-dimensional region $D^{(1)}_{R1}$. The projection of $D^{(1)}_{R1}$ on the plane $(R^u_1, R^d_1)$ defines all possible rate pairs that can be decoded at the RS1. Clearly, some rate pairs are decodable when $x_{B2}$ is decoded, others when it is treated as noise. Using the similar analysis, we obtain the MA region at RS2 in phase 1 which is $D^{(1)}_{R2}$.

The 4-dimensional rate region that describes phase 1, including the operation at both RS1 and RS2, is denoted by $D^{(1)}$. Note that, for some points in $D^{(1)}_{R1}$, the values of $R^u_2$ or $R^d_2$ can be arbitrarily large; and the same is valid for the values of $R^u_1$ or $R^d_1$ in $D^{(1)}_{R2}$. Nevertheless, due to intersection, $D^{(1)}$ is set of non-negative coordinate points in which each coordinate has an upper bound.

We use $D^{(2)}$ to denote 4-dimensional rate region that is achievable during the second phase. The proof of $D^{(2)}$ is provided in Appendix A. The rate region $D$ is the intersection of the 4-dimensional rate region in phase 1 and phase 2, as indicated by (16c). Note that the region coming from the intersection of different convex regions is also convex. However the the region coming from the union of different convex regions may not be convex, such that $D$ may be non-convex. Therefore, the achievable rate region is the convex closure of $D$.

**Remark 1:** In phase 2 there are two BC processes from RS1 and RS2. The BC destinations are U1, U2 and BS which all have side information. These two BC processes intersect at BS and become a MA process to the BS. A similar scenario is introduced in [11] and [15] which give an optimal BC strategy to two terminals where side information is available. However, [11] and [15] do not deal with a MA channel with side information and, to the best of our knowledge, there is no proved optimal BC strategy for this scenario. Phase 2 of the transmission scheme in this paper is the MA extension of the schemes in [11] and [15].
IV. ACHIEVABLE RATE REGIONS OF FOUR-PHASE FOUR-WAY RELAYING

In this paper, we view the four-phase relaying scheme as the reference scheme. This is depicted on Fig. 2(a), where the four-way transmission is decomposed into two time-multiplexed two-way relaying transmissions.

As will be shown later, the expression of the achievable rate regions of this four-phase relaying scheme are independent from the time-sharing fraction (time-ratio) between the two-way relay processes, for AF, and additionally for DF, the BC and MA phases. Unlike the two-phase relaying scheme, the time-ratio is an implicit variable and the achievable rate region can be obtained in closed-form.

A. Amplify-and-forward

Proposition 3: The achievable rate region of the four-phase four-way relaying scheme using AF is characterized by $l_1 + l_2 < 1$, where $l_1$ and $l_2$ are defined as follow

$$ l_1 = \max \left\{ \frac{2R^u_1}{C \left( \frac{|h_{11}|^2|h_{12}|^2P_1P_{R1}}{|h_{11}|^2P_1 + |h_{12}|^2(P_{R1} + P_B) + 1} \right)}, \frac{2R^d_1}{C \left( \frac{|h_{11}|^2|h_{12}|^2P_BP_{R1}}{|h_{11}|^2P_B + |h_{12}|^2(P_{R1} + P_1) + 1} \right)} \right\} $$

(23a)

$$ l_2 = \max \left\{ \frac{2R^u_2}{C \left( \frac{|h_{21}|^2|h_{22}|^2P_2P_{R2}}{|h_{21}|^2P_2 + |h_{22}|^2(P_{R2} + P_B) + 1} \right)}, \frac{2R^d_2}{C \left( \frac{|h_{21}|^2|h_{22}|^2P_BP_{R2}}{|h_{21}|^2P_B + |h_{22}|^2(P_{R2} + P_2) + 1} \right)} \right\} $$

(23b)

Proof: If the relay stations RS1 and RS2 use AF, then the duration of phase 1 (3) is equal to the duration of phase 2 (4). Assuming that the total duration of all the phases is equal to unity, then phase 1 and phase 2 together occupy the time ratio $\eta$, while phase 3 and phase 4 is $1-\eta$. From [10], we get the constraints for phase 1 and phase 2

$$ R^u_1 < \frac{1}{2} \eta C \left( \frac{|h_{11}|^2|h_{12}|^2P_1P_{R1}}{|h_{11}|^2P_1 + |h_{12}|^2(P_{R1} + P_B) + 1} \right) $$

(26a)

$$ R^d_1 < \frac{1}{2} \eta C \left( \frac{|h_{11}|^2|h_{12}|^2P_BP_{R1}}{|h_{11}|^2P_B + |h_{12}|^2(P_{R1} + P_1) + 1} \right). $$

(26b)

Similarly, the constraints for phase 3 and phase 4 can be obtained as

$$ R^u_2 < \frac{1-\eta}{2} C \left( \frac{|h_{21}|^2|h_{22}|^2P_2P_{R2}}{|h_{21}|^2P_2 + |h_{22}|^2(P_{R2} + P_B) + 1} \right) $$

(27a)

$$ R^d_2 < \frac{1-\eta}{2} C \left( \frac{|h_{21}|^2|h_{22}|^2P_BP_{R2}}{|h_{21}|^2P_B + |h_{22}|^2(P_{R2} + P_2) + 1} \right). $$

(27b)
Using the definition of $l_1$ and $l_2$ in (23a) and (23b), we can write (26) and (27) as

$$l_1 < \eta, \quad l_2 < 1 - \eta.$$  \hspace{1cm} (28)

The inequalities in (28) are equivalent to $l_1 + l_2 < 1$ which is the closed-form expression of the achievable rate region for two-way AF relaying scheme.

B. Decode-and-forward

Proposition 4: The achievable rate region of the four-phase four-way relaying scheme using DF is characterized by $k_1 + k_2 + k_3 + k_4 < 1$.

$$k_1 = \max \left\{ \frac{R_u^1}{C \left( |h_{11}|^2 P_1 \right)}, \frac{R_d^1}{C \left( |h_{12}|^2 P_B \right)}, \frac{R_u^1 + R_d^1}{C \left( |h_{11}|^2 P_1 + |h_{12}|^2 P_B \right)} \right\} \hspace{1cm} (29a)$$

$$k_2 = \max \left\{ \frac{R_d^2}{C \left( |h_{11}|^2 P_{R1} \right)}, \frac{R_u^2}{C \left( |h_{12}|^2 P_{R1} \right)} \right\} \hspace{1cm} (29b)$$

$$k_3 = \max \left\{ \frac{R_u^2}{C \left( |h_{21}|^2 P_2 \right)}, \frac{R_d^2}{C \left( |h_{22}|^2 P_B \right)}, \frac{R_u^2 + R_d^2}{C \left( |h_{21}|^2 P_2 + |h_{22}|^2 P_B \right)} \right\} \hspace{1cm} (29c)$$

$$k_4 = \max \left\{ \frac{R_d^3}{C \left( |h_{21}|^2 P_{R2} \right)}, \frac{R_u^3}{C \left( |h_{22}|^2 P_{R2} \right)} \right\}. \hspace{1cm} (29d)$$

Proof: When the two-way DF relaying scheme is used in the four-way relay cellular networks, the time duration of the 4 phases can be different from each other. We use $\tau_1, \tau_2, \tau_3, \tau_4$ to denote the time ratios spent by phase 1, 2, 3, 4 respectively. Notice that, $\tau_1 + \tau_2 + \tau_3 + \tau_4 = 1$ must be satisfied. From [14], we get the constraints on the rates for the MA process in phase 1,

$$R_u^1 < \tau_1 C \left( |h_{11}|^2 P_1 \right), \quad R_d^1 < \tau_1 C \left( |h_{12}|^2 P_B \right), \quad R_u^1 + R_d^1 < \tau_1 C \left( |h_{11}|^2 P_1 + |h_{12}|^2 P_B \right) \hspace{1cm} (30)$$

The second phase is the BC process with side information to U1 and BS as U1 and BS both know what they sent in phase 1. From [11], the constraints on the rates for the broadcast process from RS1 can be obtained as

$$R_d^1 < \tau_2 C \left( |h_{11}|^2 P_{R1} \right), \quad R_u^1 < \tau_2 C \left( |h_{12}|^2 P_{R1} \right). \hspace{1cm} (31)$$

Similarly, we can obtain the constraints for the MA and BC process at RS2 in phase 3 and phase 4,
\[
\begin{aligned}
R_u^2 < \tau_3 C (|h_{21}|^2 P_2), \quad R_d^2 < \tau_3 C (|h_{22}|^2 P_B) \\
R_u^2 + R_d^2 < \tau_3 C (|h_{21}|^2 P_2 + |h_{22}|^2 P_B) \\
R_d^2 < \tau_4 C (|h_{21}|^2 P_{R_2}), \quad R_u^2 < \tau_4 C (|h_{22}|^2 P_{R_2}) .
\end{aligned}
\] (32a, 32b, 32c)

Applying (29) into (30), (31) and (32), we get

\[
k_i < \tau_i, i = 1, 2, 3, 4.
\] (33)

Incorporating \(\tau_1 + \tau_2 + \tau_3 + \tau_4 = 1\) into (33), we obtain \(k_1 + k_2 + k_3 + k_4 < 1\) which is the closed-form expression of the achievable rate region for two-way DF relaying scheme.

V. Achievable rates with fixed Downlink-Uplink rate ratio

The achievable rate regions described in section III and IV have four dimensions. In order to get a better insight into the achievable rates, we impose a ratio between the uplink and downlink rates. It is practical to fix the downlink-uplink rate ratio for a given type of application e.g. gaming and calls have ratio of 1:1, web browsing has a ratio of about 5:1 [16]. We assume that, for the \(i\)-th user, the downlink rate demand \(R_d^i\) is related to the uplink rate demand \(R_u^i\), as \(R_d^i = \theta_i R_u^i\), see [17]. Applying the downlink-uplink rate ratio, the dimension of the achievable rate region degrades to 2. In this section, for simplicity, we plot the achievable rate regions of the rate pair \((R_u^1, R_u^2)\) and assume each node has equal transmission power which is \(P_j = 10, j \in \{1, R1, B, R2, 2\}\).

Notice that, the achievable rate regions of the four-phase relaying scheme are closed-form and independent from the time ratios as proposition 3 and 4 show. However, the achievable rate region of two-phase AF relaying scheme is related to the superposition ratio as proposition 1 shows. And the achievable rate region of two-phase DF relaying scheme is related to the time ratio and superposition ratio as proposition 2 shows. Then we need to optimize \(\tau\) and \(\alpha\) to get the envelope of the achievable rate regions for two-phase scheme.

In the optimization, we fix the rate \(R_u^1\) of user 1 to a set value \(r_1\) and find the maximal rate \(R_u^2\) of user 2. The parameter \(r_1\) describes the feasible rate values for user 1, \(r_1 \in [0, R_u^1_{\text{max}}]\) where \(R_u^1_{\text{max}}\) is determine via another optimization. The achievable rate regions of the two-phase scheme can be obtained by a two-step optimization. Here we only present the optimization method for DF case, the AF case follows the similar way. The first step is to find \(R_u^1_{\text{max}} = \max \{R_u^1\}\), as
shown below:

\[
\max_{0 \leq \tau \leq 1, 0 \leq \alpha \leq 1} \quad R_1^u, \quad \text{s.t.} \quad (16) \quad \text{and} \quad R_2^u = 0, \quad \text{given} \quad \theta_1, \theta_2. \tag{34}
\]

Then, finding the achievable rate region is equivalently to solve the following optimization problem for each feasible rate point \( r_1 \in [0, R_1^u_{\max}] \):

\[
\max_{0 \leq \tau \leq 1, 0 \leq \alpha \leq 1} \quad R_2^u, \quad \text{s.t.} \quad (16) \quad \text{and} \quad R_1^u = r_1, \quad \text{given} \quad \theta_1, \theta_2. \tag{35}
\]

The formulas (34) and (35) are two conventional constrained nonlinear optimization problems for which a numerical solution for the achievable rate region can be easily found.

VI. NUMERICAL RESULTS

The achievable rate regions of rate pair \((R_1^u, R_2^u)\) are presented in Fig. 4-Fig. 9. S2 stands for two-phase relaying scheme, S4 stands for four-phase relaying scheme. Here we only deal with six typical cases. The six typical cases are divided into two groups of three cases, where the groups have symmetric and asymmetric downlink-uplink ratio, respectively.

When the data rate is symmetric, \( \theta_1 = \theta_2 = 1 \), we focus on the performance under different SNR conditions as Fig. 4-Fig. 9 show. We consider three cases. The first case is when the links have the same SNR, that is \( |h_{11}|^2 = |h_{12}|^2 = |h_{22}|^2 = |h_{21}|^2 = 1 \). The second case is when the links that are direct to the BS have a lower SNR than the links that are direct to the users, that is \( |h_{11}|^2 = |h_{21}|^2 = 1, |h_{12}|^2 = |h_{22}|^2 = 0.1 \). The third case is when the links that are direct to the BS have a higher SNR than the links that are direct to the users, that is \( |h_{11}|^2 = |h_{21}|^2 = 0.1, |h_{12}|^2 = |h_{22}|^2 = 1 \).

When the data rate is asymmetric, \( \theta_1 \neq 1, \theta_2 \neq 1 \), we fix the SNR as \( |h_{11}|^2 = |h_{12}|^2 = |h_{22}|^2 = |h_{21}|^2 = 1 \) and focus on the performance under different downlink-uplink rate ratio configurations as Fig. 7-Fig. 9 show. We again consider three cases. The first case is when the downlink date rate is smaller than the uplink date rate for both U1 and U2, that is \( \theta_1 = \theta_2 = 0.5 \). The second case is when the downlink date rate is larger than the uplink date rate for both U1 and U2, that is \( \theta_1 = \theta_2 = 2 \). The third case is the downlink data rate is larger than the uplink date rate for U1 and the downlink data rate is smaller than the uplink date rate for U2, that is \( \theta_1 = 2, \theta_2 = 0.5 \).

For the same relaying scheme (S2 or S4), the rate regions of DF are always larger AF. The main reason is that, in DF, the noise is removed at the relay stations and not forwarded.
Consider the points in the axes where $R_{u1} = 0$ or $R_{u2} = 0$. Only one user communicates with BS. Therefore the four-way communication degrades to two-way communication. This explains why two-phase DF relaying (S2) and four-phase DF relaying (S4) coincide on the axes. However, there is a MA process to BS in the two-phase DF relaying scheme, that the four-phase DF relaying scheme does not have. Because the MA process can enlarge the sum rate, $\max \{R_{u1} + R_{u2}\}$ of the two-phase DF relaying scheme is larger than the four-phase DF relaying scheme. Note that, the shape of the rate region must be convex. Two-phase DF relaying scheme and four-phase DF relaying scheme have the same axes points of the rate regions. However, two-phase DF relaying scheme has larger $\max \{R_{u1} + R_{u2}\}$ than the four-phase DF relaying scheme. Conclude the above factors, the achievable rate region of two-phase DF relaying scheme is larger than the achievable rate region of four-phase DF relaying scheme.

We now consider AF relaying and focus on the point $R_{u2} = 0$ for instance. For two-phase AF relaying, although BS and U2 has no information to communicate, RS2 also receives the information from BS which intends to RS1. In the second phase, RS2 forwards the information received before, along with the noise. Note that this noise will be present, amplified and forwarded even when there is no communication between BS and U2. This explains why the maximal value of the rate $R_{u1}$ at $R_{u2} = 0$ for two-phase AF relaying is lower than the one for four-phase AF relaying. It should be noted that, in some cases, the values in $(R_{u1}, R_{u2})$ do not correspond to the capacities of the individual links, because $(R_{d1}, R_{d2})$ can limit $(R_{u1}, R_{u2})$ through downlink-uplink ratio. For the above situation, the axes points of two-phase AF relaying and four-phase AF relaying can be the same as Fig. 5, Fig. 8 and Fig. 9 show.

In two-phase AF relaying there are more MA processes activated simultaneously than four-phase AF relaying. And the MA process can enlarge the sum rate. This implies that, in most cases the sum-rate of the two-phase AF relaying scheme can be better than the four-phase AF relaying scheme, as Fig. 6, Fig. 7 and Fig. 9 show.

VII. CONCLUSION

We have described a new multi-way relay scenario of practical relevance in wireless cellular networks, termed four-way relaying, in which each of the two Mobile Stations (MSs) has a two-way connection to the same Base Station (BS), while each connection is through a dedicated Relay Station (RS). One of the main assumptions is that the RSs are antipodal, i. e. deployed
at the “opposite side” of the BS, such that they are not interfering. We have proposed novel communication schemes which leverage on the ideas of wireless network coding, but are designed for the considered four-way scenario. The key idea is to have coordinated transmissions to/from the two RSs. The principle can be applied if the RS uses AF or DF relaying. The proposed communication scheme consists of two phases: broadcast and a multiple access, respectively. We compare the performance with a state-of-the-art reference scheme, time sharing is used between the two MSs, while each MS is served through a two-way relaying scheme. The results indicate that, when the RS operates in a DF mode, the achievable rate regions are significantly enlarged. On the other hand, for AF relaying, the gains are rather modest.

An interesting issue for future work is to consider other types of operation for the relay, such as physical-layer network coding, use of lattices and noisy network coding. As another line of research would be to investigate how the Base Stations and the relays should be deployed, assuming that they can use the proposed four-way relaying method.

APPENDIX A

The expression (22) is an upper bound of the achievable rates of phase 2. Here we prove that this upper bound can be achieved. In other words, $R_i^d$ can achieve the capacity of $h_{i1}$ and $R_i^u$ can achieve the whole MA region in Fig. 10. A and B are the corner points of the MA region corresponding to $(C(|h_{12}|^2 P_{R1}/(|h_{22}|^2 P_{R2} + 1)), C(|h_{22}|^2 P_{R2}))$ and $(C(|h_{12}|^2 P_{R1}), C(|h_{22}|^2 P_{R2}/(|h_{12}|^2 P_{R1} + 1)))$ respectively.

The main idea to prove the achievability of the MA region is using one point-to-point codeword at each RS to achieve the corner points (A and B) of the MA region. Then we apply the time-sharing between the point-to-point codeword achieving corner point A and the point-to-point codeword achieving corner point B, thereby proving that the MA region is achievable. For each RS, the transmission in phase 2 is a BC process with side information at the receivers. On the other hand, the simultaneous transmission of RS1 and RS2 are two components of the MA channel, defined at the BS as a receiver. This makes the two BC transmissions interrelated in terms of the achievable rates. In the following we will prove that MA region corner point A can be achieved and $R_i^d$ can achieve the capacity of $h_{i1}$ by the re-encoded codewords $x_{Ri}^A$ at RS$i$.

When decoding, BS first treats $x_{R2}^A$ as noise in order to decode $x_{R1}^A$. Then for U1, RS1 and BS, this is a BC process with side information at the receivers. From [15, Theorem 2], $R_1^d$
can achieve the capacity of $h_{11}$ which is $C(|h_{11}|^2 P_{R1})$ (denote as $C_{11}$) and $R_u^1$ can achieve $C(|h_{12}|^2 P_{R1}/(|h_{22}|^2 P_{R2} + 1))$ (denote as $R_u^1(A)$). After the decoding of $x_{R1}^A$, BS cancels $x_{R1}^A$ from the receiving signal. Then for U2, RS2 and BS, this is a BC process with side information at the receivers. From [15, Theorem 2], $R_u^2$ can achieve $C(|h_{22}|^2 P_{R2})$ (denote as $R_u^2(A)$) and $R_d^2$ can achieve $C(|h_{21}|^2 P_{R2})$ (denote as $C_{21}$). By far, we proved that MA region corner point A can be achieved and $R_d^1$ can achieve the capacity of $h_{i1}$ by the re-encoded codewords $x_{RS}^A$ at RSi. In a similar manner we can prove that the rates corresponding to the corner point B can be achieved.

To summarize, the codeword $x_{RS}^A$ achieves $(R_u^1(A), R_u^2(A), C_{11}, C_{21})$ for rate tuple $(R_u^1, R_u^2, R_d^1, R_d^2)$ and the codeword $x_{RS}^B$ achieves $(R_u^1(B), R_u^2(B), C_{11}, C_{21})$ for rate tuple $(R_u^1, R_u^2, R_d^1, R_d^2)$. In order to achieve the rates on the line AB in Fig. 10, we can apply time-sharing between the codewords $x_{RS}^A$ and $x_{RS}^B$. Since U1, U2 and BS know the exact duration of $x_{RS}^A$ and $x_{RS}^B$ in the time-sharing codeword, the decoding process during $t$ is to decode $x_{RS}^A$, the decoding process during $1-t$ is to decode $x_{RS}^B$. Therefore, $(tR_u^1(A) + \bar{t}R_u^1(B), tR_u^2(A) + \bar{t}R_u^2(B), C_{11}, C_{21})$ is achievable for every $t \in [0, 1], \bar{t} = 1-t$. This proves that the line AB in Fig. 10 is achievable while $R_d^1$ can achieve the single-user capacity of the channel with $h_{i1}$. It should be noted that the time-sharing does not affect the decoding at Ui, since Ui knows the detailed structure of the re-encoded codeword, such that it can apply the appropriate side information upon decoding.

Similarly, by time-sharing between corner points and axes points, the rates on the lines CA and BD in Fig. 10 are also achievable. Then the whole MA region in Fig. 10 is achievable while $R_d^1$ achieves the capacity of $h_{i1}$. Summarizing the above results, we get (22). The scaling factor $1-\tau$ corresponds to the duration of phase 2.

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Figure 1. A cellular network with four-way relaying using a Base Station (BS), two Relay Stations (RSs), denoted RS1 and RS2, and two Mobile Stations (MSs) U1 and U2.

![Diagram of cellular network with four-way relaying](image)

(a) Four stages scheme with 2 two-way relaying sessions

U1 → RS1 → BS → RS2 → U2

1: $h_{11}$ → MA ← $h_{12}$

2: $h_{11}$ ← BC → $h_{12}$

3: $h_{22}$ → MA ← $h_{23}$

4: $h_{22}$ ← BC → $h_{23}$

(b) Alternative four stages scheme

U1 → RS1 → BS → RS2 → U2

1: $h_{11}$ → MA ← $h_{12}$

2: $h_{11}$ ← BC → $h_{12}$

3: $h_{22}$ → MA ← $h_{23}$

4: $h_{22}$ ← BC → $h_{23}$

Figure 2. Transmission schemes for four-way relaying consisting of four phases.

Figure 3. A new transmission scheme for four-way relaying consisting of two phases.

![Diagram of four-way relaying scheme](image)
Figure 4. Achievable rate region for the pair of uplink rates, assuming a downlink-uplink ratio of $\theta_1=\theta_2=1$ and channel values of $|h_{11}|^2 = |h_{12}|^2 = |h_{22}|^2 = |h_{21}|^2 = 1$.

Figure 5. Achievable rate region for the pair of uplink rates, assuming a downlink-uplink ratio of $\theta_1=\theta_2=1$ and channel values of $|h_{11}|^2 = |h_{21}|^2 = 1, |h_{12}|^2 = |h_{22}|^2 = 0.1$.

Figure 6. Achievable rate region for the pair of uplink rates, assuming a downlink-uplink ratio of $\theta_1=\theta_2=1$ and channel values of $|h_{11}|^2 = |h_{12}|^2 = 0.1, |h_{22}|^2 = |h_{21}|^2 = 1$.

Figure 7. Achievable rate region for the pair of uplink rates, assuming a downlink-uplink ratio of $\theta_1=\theta_2=0.5$ and channel values of $|h_{11}|^2 = |h_{12}|^2 = |h_{22}|^2 = |h_{21}|^2 = 1$. 

May 2, 2014 DRAFT
Figure 8. Achievable rate region for the pair of uplink rates, assuming a downlink-uplink ratio of $\theta_1=\theta_2=2$ and channel values of $|h_{11}|^2=|h_{12}|^2=|h_{22}|^2=|h_{21}|^2=1$.

Figure 9. Achievable rate region for the pair of uplink rates, assuming a downlink-uplink ratio of $\theta_1=2, \theta_2=0.5$ and channel values of $|h_{11}|^2=|h_{12}|^2=|h_{22}|^2=|h_{21}|^2=1$.

Figure 10. The capacity region of a multiple access (MA) channel.