Students' Mathematical Thinking Process Involving Equal Signs

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Abstract. The purpose of this research is to know the process of students’ mathematical thinking in solving mathematics problems involving equal sign. The method used in this research is descriptive qualitative using two stages such as testing and interviewing. This research was conducted at Sekolah Dasar Islam Surya Buana Malang and held at 5th grade involved 30 students, after having the test, will be continued the interview. The interviewed was conducting with two students who choosed based on the communication’s ability; so the students be able to explain the responding that has been given. Based on the result of this study most students understood that the equal sign as a symbol to obtain the results of a calculation. This result also shows that there are students who can understand equal sign as equality, however that understanding just limited to the equal sign with closed type, while in open type all students did not understand. It can be concluded that students more understand the equal sign as an operational symbol. This result is suggested to the teachers to introduce the meaning of equal sign to students especially relating to equality.

1. Introduction

Thinking is a required process to find ideas and able to conclude how to solve a problem [1]. Therefore, the active role of educators is needed to deal these problems. [2] states that in a mathematic learning, a teacher should help students in problem solving, not only to ask the answers by the students but also to guide the students develop their thinking process.

Mathematical thinking process is really needed to show students’ mathematical ability, and to improve their ability in finding a problem solving [3], because mathematics is an important science and tool to assist students in strengthening their mathematical thinking process and interpreting their mathematical problems [4]. Consequently, it is necessary to create a learning atmosphere which is able to build the students’ thinking process in solving mathematical problems. According to [5] in mathematics does not only consist of memorizing or looking for important answers. It is also important that students are able to express their ideas through their mathematical thinking process to find answers to their problems. Furthermore, according to [2] the most important thing in mathematics is how students are able to use their thinking process to solve the mathematical problems.

Mathematics is considered a difficult subject. A very important thing in mathematics learning is how students are able to use their thinking process to solve the math problem [6]. Because math is more than a subject learned by the students. It contains a symbol language which asks how the
students are able to understand the contents and meanings of the language [7]. Thus, it is important for students to understand the math problems.

One of the common math problems for students is how to use the equal sign. The equal sign problem is also faced by students of Sekolah Dasar Islam Surya Buana Malang, most students still had the lack of knowledge in solving problems presented in the form of equal sign, it is also stated in the research of [8] showed that most students still have difficulties to solve the math problems in interpreting equal sign, especially relating to equality.

Based on observation conducted by researcher at the Sekolah Dasar Islam Surya Buana Malang seen There are students has a good ability in mathematics, but the problem is when researcher tried to explore their understanding of the equal sign. Evidently most of elementary students have a lack understanding of the meaning of equal sign. Wherein they only understand equal sign as the operational sign and not as relational sign. According to Cobb [9], children are more likely to see equal sign as a symbol to denote the sum of numbers next to the equal sign. Children are used to an operational thinking, they find it difficult to understand and solve the math problem presented in an equality form. Based on this case, [10] state that the lack of students’ understanding of equal sign as the symbol of equality will result in students’ difficulties to solve the math problems presented in the form of equality. This will be a problem for every student.

Students at the school who have good abilities in mathematics also still have less understanding about equal sign presented in the form of equality. This is in accordance to [11], Students who are good at math and also top students still have a poor understanding of equal sign when it is used as an equality sign. According to [12] most students have a better understanding of operational problem solving which has been embedded in children’s mind so that they have difficulties to solve a problem related to equality. This error is occurred due to lack of understanding or learning experience in elementary school [13]. Therefore, teachers should play an active role to help the students to know and understand about equal sign in mathematical calculations.

2. Research Method

The method used in this research was qualitative descriptive analysis method to know the mathematical thinking process of students in solving mathematical problem related to equal sign. Data was obtained through test and interview. Researcher prepared several questions related to equal sign which were given to the students and then continued to an intensive interview. The interviewed was conducting with two students who choosed based on the communication's ability; so the students be able to explain the responding that has been given.

Researcher tried to explored the students’ understanding about equal sign and how their thinking process in solving the given problems. Subjects in this research were 30 students of 5th grade of Sekolah Dasar Islam Surya Buana Malang. There were 10 questions consisted of two categories: as a sign to get results and as an equality sign (close equal sign and open equal sign). Those 10 questions are given by Table 1 as the following.

| Question                  | Information                                           |
|---------------------------|-------------------------------------------------------|
| $17 + 28 = \ldots$        | Questions of equal sign as the operational symbol used to get a final result. |
| $43 - 27 = \ldots$        |                                                       |
| $27 + 14 = \ldots + 21$   |                                                       |
| $27 + \ldots = 35 + 21$   | Questions of equality result with a Close Equal Sign type used to equate the numbers on the left and right of equal sign. |
| $36 - 15 = \ldots + 12$   |                                                       |
| $38 - \ldots = 15 - 5$    |                                                       |
| $27 + \ldots = \ldots + 21$ |                                                       |
| $\ldots + 14 = 35 + \ldots$ |                                                       |
| $15 - \ldots = \ldots + 5$ |                                                       |
| $\ldots - 11 = 9 - \ldots$ |                                                       |

Table 1. List of test questions.
3. Research Results and Discussion

3.1. Research result

Research results showed that most students still had difficulties to solve the math problems related to equal sign. Most problems by the students were equal sign as the operational and equality sign. According to Kieran [14], equal sign would be viewed as an equality symbol if the numbers on the left and right of the equal sign would produce the same value. Operational symbol was the symbol used to obtain the result of addition or subtraction of each number [12]. However, in reality, most students still considered equal sign as an operational sign, for example $5 + 3 = \ldots + 1$. Most students put 8 on the equality sum of those numbers, and there was also students who put 9 as the result. This occured due to [15] the students’ assumption that equal sign was a sign to get results.

Related to this finding, researcher conducted interviews with the students to find out their thinking process in understanding and solving the math problems. According to the test results, there were 25 students with poor results and 5 students with good results. From those two type of results, researcher chose one student of each category to be the subjects of this research. Subjects were chosen based on their communication ability to answer and provide a well and fluent explanation. Researcher (R) chose student GR to represent the group with a good result and student PR to represent the group with a poor result.

The following was the interviews conducted by the researcher and the students:

3.1.1. The meaning of equal sign as a result

Earlier researcher showed that most students only viewed the equal sign as the symbol to obtain result of the sum of numbers. According to Byrd (2015), most children interpreted equal sign as a sign used to obtain result. For example, $17 + 28 = \ldots$, from the operational understanding by the students, they would easily answer that the result of that the sum of the numbers was 45. This occured because students had a good understanding of operational sign.

![Figure 1. The answer of GR.](image1)

![Figure 2. The answer of PR.](image2)

According to the answers above as given by Figure 1 and Figure 2, to understand the thinking process of the students in solving the problems would be delivered the interview results in the following section.

R : Are you sure that your answer is correct?
GR : Yes, I’m sure.
PR : Yes.
R : Could you explain how do you get the your answer?
GR : I add the number 17 with 28, then I get the result of 45.
PR : I just add 17 and 18.
R : Could you explain how do you add the numbers?
GR : If we arrange the numbers of $7 + 8$, the result is 15, we store 1. Next, the stored number is added to the number in front of it $(1+1+2)$, the result is 4.
PR : $7 + 8$ is 15, one is stored and then, $1 + 1$ is 2. 2 is added 2 more, it will be 45.
R : How about the question number 2? Is it the same way?
GR : For number 2, I also use the same way but it’s a subtraction. First, I take 1 from 4, so $13 – 7$ is 6. Because 4 is already subtracted, then it’s left 3. 3 minus 2, the result is 1. Then I get the result of 16.
PR : Question number 2. I only need to subtract 13 with 7 and the result is 6. 3 – 2 is 1, so the result is 16.

According to the interview results above, students had a good operational understanding in solving the problem. The results also showed that the students not only directly gave the results of the addition or subtraction, but the students were also able to use good math steps to solve the problem.

3.1.2. The meaning of equal sign as an equality
Most students still had difficulties to solve the math problem on the equal sign as an equality, both in open equal sign and open equal sign. This was occurred because students were more likely understanding more about equal sign as the symbol to gain result. According to Kieran [16], students were more likely to view equal sign as a process to gain result rather than as an equality relation between the numbers. For example, in solving the problem of $7 + 6 = ... + 8$. Most students assumed that the value to be placed on the dots was 13 because they counted that the value was the sum of $7 + 6$. There were also students who gave 21 as the result because they assumed that the answer was $7 + 6 + 8$. As well as in the open equal sign such as $45 + ... = ... + 25$. In this section, students were asked to use numbers which might resulted in equality of the equal sign. For example, students were able to use number 5 on the left and 25 on the right side, also other numbers which might give an equivalence value.

However, in general, students were difficult to solve and there were students who used operational result number to obtain final result. For example, students would use numeral 10 to be added with numeral 45 and obtained the result of 55 that would be placed in right part of equal sign. Thus, the statement of student in giving answer for the question was lack of make sense [17].

In this part, equality is viewed in 3 angles as follow:

a. Equality with close equal sign type in left side

| Figure 3. The answer of GR. | Figure 4. The answer of PR. |
|-----------------------------|-----------------------------|

GR student in solving the problem considered that the numerals in left side of equal part was added, so the result would the same as number of numerals in right side. For example, from the question on Figure 3 above, students would think that $27 + 14 = 41$, so student would use numeral if it was added with 21, it would result the same value.

Whereas from Figure 4 we can see that PR students considered that numerals in left side of equal sign $27 + 14 = ... + 21$, if it was added it would obtain the result that it would be used in right side, so that the numeral would be added to the next numeral, 21. It would give expected result. Students’ understanding and reason in solving problem can be shown through the result of interview as follow,

R : Are you sure that your result is right?
GR : Yes, I’m sure
PR : Yes
R : Why?
GR : Because $27 + 14$ is the same as $20+21$ that is 41
PR : Because 27 is added 14 is equal 41 and it is added by 21 is equal 62.
R : Can you explain how did you get the result?
GR : Because $27 + 14$ is equal 41, then $41 - 21$ is equal 20 then $20 + 21$ is equal 41
PR : Because I used the way I told before in number 1, so I just added it.
R : Can numeral 20 be replaced by numeral 4?
GR : It can’t be
R : Why?
GR : Because, the result will be not the same and 41 is from the result of 27 + 14
R : So how did you get 62 as the result?
PR : Because I have obtained 41 as the result and I add with 21.
R : Why did you do so?
PR : Because here, the is added sign (while pointing at +)
R : How is about the question number 7? Is it in the same way to solve?
GR : It is
PR : Yes, it is

Based on the result of the interview above, it showed that GR student considered that numeral in right side would result the same as left side. If the point was fulfilled numeral 20, so that numeral in both sides would result the same, 41. However, PR student in solving used numeral 41 as the result of numeral addition of right side of equal sign.

b. Equality with close equal sign type in right side

Figure 5. The answer of GR.

Figure 6. The answer of PR.

GR student in solving this problem considered that numerals in right side of equal sign if it was added so the result would the same as numbers of numerals in left side. For example, from the problem on Figure 5 above, student would think that 35 + 21 = 56, so student will result numeral that if it was added 27, it would result the same number. Whereas from Figure 6 we obtain that PR student considered that numerals in right side of equal sign 27 + ... = 35 + 21, if it was added, it would obtain result that would be used in left side, whereas in solving this question, student considered numerals that would use dots if it was added with 27 was equal 35 and numeral in right side if it was added. It would result certain value. 56. Understanding and reason of students can be seen in interview result as follow,

R : How was about the question number 4?
GR : This question is the same as question number 3,
PR : It is the same as number 3
R : Why?
GR : Because 35 + 21 is the same as 27 + 29 is equal 56
PR: Because 27 added 8 is equal 35 and it is added 21 and it is equal 56.

Based on the interview above, it showed that GR student considered that numeral in left side would obtain the same result as right side, if dots were fulfilled numeral 29, so that numeral in both sided would result the same, 56. But PR student in solving used numeral 8 that if it was added numeral 27, it would obtain 35 as the result.

c. Equality with open equal sign type
Based on the answers from students on Figure 7, it showed that GR student in answering questions though that if numeral that would be used in dots left side of equal sign was the result of minus of 27 with 21, and the numeral would be added 27 so that it would obtain expected result. Whereas from Figure 8 we can conclude that PR student considered that numeral that would be placed in dots was the result of add from numeral 27 and 21. To find out the thinking process of students in solving problem, it can be seen from interview result as follow,

R : How about question number 5, do you understand?
GR : No
PR : No
R : Is it difficult?
GR : Yes
PR : Yes
R : But you have answered, can you explain about numeral you have been written for your answer?
GR : In first dots I used numeral 6, because 27 - 21 is equal 6. Second dots is because 27 + 6 is equal 33
PR : In dots I write 48, because 27 + 21 is equal 48
R : Are you sure with your answer?
GR : No
PR : No

GR as well as PR showed that they did not have good understanding in solving the problem related to open equal sign and they more considered that the question could only be answered in operational form. For example GR student in initial equal sign fulfilled numeral 6 that was minus result from existed numeral, then the numeral would be added with numeral in left side so it obtained 35 as the result. Whereas PR student fulfilled numeral 48 in both sides because he considered that numeral that would be used was the result of addition from existed numeral.

4. Discussion
Based on the result of the research above, it showed that equal sign related to operational used to obtain the best understanding for student in the level of basic education. It is because most students have understood, known and used equal sign as operational sign [18]. Beside, according to [19] equal sign tends to be understood by students as symbol to reveal calculation result, in solving this operational problem, student argued that the result of right side was the final result from addition as well as minus in each numeral in left side. Students also could understand and explain well about steps used in solving the question. It is because they consider that given equal sign requires them to finish operationally [20].

On the other hand, equality problem with close sign type in left side of right side (PR) still had difficulty. Students face difficulty in solve this kit of problem because they have lack understanding about given problem [18]. Other than that, this mistake can occur as the given problem is not understood by students [20], because the understanding about equal sign was still very minimal. It was because students more considered equal sign as symbol connects that would be used to obtain expected result from addition or minus of two or more numeral in equal sign.
 Whereas student of GR could explain that numerals in left and right equal sign would be the same if the numerals had the same big numerous. Whereas the numerals would be considered equal and balance and at the same time equal sign would be considered as equal symbol or equality between both numerals/value. According to [19], students who are able to finish equal sign question/problem in equality form are those who can consider equal sign as relational sign.

However, on the other side, students had not been able to understand the problem of equal sign as equality in open equal sign. It is because most students have still had difficulty in understanding meaning in equal sign [20]. So it makes them doing mistake in answering given [19]. It occurred because in solving the problem, student more understood and answered as operational sign to obtain final result.

5. Conclusion and Suggestion

Based on the result of the research and the discussion above, it can be concluded that in general, students understand more about equal sign as the sign used to obtain or give expected result. It showed students still had the lack of knowledge about equal sign, so that they haven’t not been able to categorize an differentiate equal sign as operational sign and equality, especially equality in open equal sign type.

Therefore, it is suggested to mathematics teachers to recognize deeper about the meaning of equal sign and the way to calculate to students so that they do not have difficulties in solving mathematics problem related to equal sign and equality

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