Unusual penetration effect in ferromagnetics. Negative refraction under tangential wave incidence.

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Abstract. In this paper we study behaviour of a refracted wave within a ferrite medium and a reflected wave in vacuum if an incident wave propagates along the interface in an isotropic medium. It is shown analytically that a plane harmonic wave can transmit from an isotropic medium to anisotropic medium in this exotic case. It is shown also that wave behaviour in this case is analogous to wave behavior in a metamaterial. The wavenumbers of the refracted waves and the Poyting vectors are found. The reflection and transmission coefficients are investigated also.

1. Introduction
Nowadays there are so many publications in the area of anisotropic material electromagnetic that one may think that all aspects of the electromagnetic theory for anisotropic structures and media have been extensively studied. However, we show in this paper, that some simple specific cases have not been considered deep enough in the literature despite the fact that in these cases interesting exotic phenomena can be observed.

The electromagnetic plane wave propagation in vacuum along an interface with an anisotropic medium has been studied in detail in [1-3]. It has been shown analytically for the first time in [1] that a bulk wave can be excited within a gyrotropic medium under tangential wave propagation. This phenomenon was identified as the “penetration effect”.

Exotic behavior of electromagnetic waves in the vicinity of anisotropic media has been reported in [4-6]. In this case, authors have considered an oblique wave incidence on a metallic film. Extraordinary transmission has been described as being generated by surface plasmons, which are coherent oscillations of a metal’s conducting electrons at the metal surface. Moreover, looking through the history of anisotropic media we find that unusual patterns of reflection and refraction by anisotropic structures have been studied in the middle of the last century [7-9]. For example, it has been shown in [7] that a reflected wave can be directed toward an interface but not from it. Unusual refraction from anisotropic media with negative dispersion have also been described in [8]. Other phenomena such as the decreasing of the refraction angle while increasing the incidence angle, the absence of reflected and refracted waves while an incident wave is presented have been discussed in [9].

In this paper we investigate the dependence of penetration effect parameters (an incident wave propagate parallel to an interface) on the anisotropy axis orientation when this axis is in the plane normal to the incidence plane.
2. **Statement of the problem**

It is assumed that a plane incident wave propagates along y-axis in an isotropic medium. The anisotropic medium is described as

\[
\mu = \begin{pmatrix}
\mu_{xx} & j\mu_{xy} & 0 \\
-j\mu_{xy} & \mu_{xx} & 0 \\
0 & 0 & \mu_{zz}
\end{pmatrix}
\]  

(1)

with \( \mu_{xy} \to 0 \). On other words we consider a nonresonance region of a ferromagnetic.

The geometry of the problem is shown in Fig.1. Here \( x'y'z' \) is the coordinate system connected with the permeability dyadic (1), where \( \mu_{xx}, \mu_{xy}, \mu_{zz} \) are the components of permittivity in the \( x'y'z' \)-system; \( xyz \) is the coordinate system connected with the normal to the interface, \( y \) is always coincide with \( y' \). The \( x' \)- and \( z' \)-axes are orientated in the plane normal to the incidence (the \( xOz \)-plane) under an arbitrary angle \( \beta \) (Fig.1). Our main aim is investigation of behavior of reflected and refracted waves.

3. **Analytical investigation**

Taking into account the continuity conditions for the tangential components of the wavenumbers we can write the dispersion relation

\[
a_4 k_y^4 + a_2 k_z^2 + a_0 = 0
\]

(2)

were

\[
a_4 = -\omega^2 \varepsilon_1 (\mu_{11} \sin^2 \beta + \mu_{33} \cos^2 \beta)
\]

\[
a_2 = \omega^4 \varepsilon_1^2 \mu_{11}(\mu_{33} + \mu_{33} \cos^2 \beta + \mu_{11} \sin^2 \beta) - \omega^2 \varepsilon_1 k_y^2 (\mu_{11} + \mu_{11} \sin^2 \beta + \mu_{33} \cos^2 \beta)
\]

\[
a_0 = -\omega^6 \varepsilon_1^2 \mu_{11} \mu_{33} + \omega^4 \varepsilon_1^2 k_z^2 (\mu_{11} + \mu_{33}) - \omega^2 \varepsilon_1 \mu_{11} k_y^2
\]

(3)

The expression (2),(3) gives us the normal components of the wavevector in an anisotropic media. In this case there are two independent waves: a right-polarized wave and a left-polarized wave. The electromagnetic field components of these waves can be obtained directly from Maxwell equations as

\[
E_x = \frac{\omega \mu_{11} k_y}{k_z^2 - \omega^2 \varepsilon_1 \mu_{11}} H_z
\]

\[
E_y = \frac{k_z \mu_{11}(\omega^2 \varepsilon_1 \mu_{11} - k_z^2)}{\omega \varepsilon_1 \tan \beta} + \frac{\omega^2 \varepsilon_1 \mu_{11}(k_z^2 - k_x^2) - k_x^4}{\omega \varepsilon_1 k_y(\omega^2 \varepsilon_1 \mu_{11} - k_z^2) \tan \beta} H_z
\]

\[
E_z = \frac{\omega^2 \varepsilon_1 \mu_{11}(k_x^2 - k_z^2) - k_z^4}{\omega \varepsilon_1 k_y(\omega^2 \varepsilon_1 \mu_{11} - k_z^2) \tan \beta} H_z + \frac{\omega^2 \varepsilon_1 \mu_{11}[\mu_{33}(k_x^2 + k_z^2) + k_x^2 \mu_{11}] - k_x^2 \mu_{11} (k_x^2 + k_z^2) - \omega^4 \varepsilon_1^2 \mu_{11} \mu_{33}}{\omega \varepsilon_1 k_y(\omega^2 \varepsilon_1 \mu_{11} - k_z^2) \sin \beta} H_z
\]

(4)
\[ H_x = \left[ \frac{\omega^2 \varepsilon_1 \mu_{11} - \mu_{11} \left( k_x^2 + k_z^2 \right)}{(\mu_{33} - \mu_{11})(\omega^2 \varepsilon_1 \mu_{11} - k_z^2) \sin \beta \cos \beta} + (\tan \beta)^{-1} \right] H_z \]

\[ H_y = \frac{k_y k_z}{\omega^2 \varepsilon_1 \mu_{11} - k_x^2} H_z \]

The components of the Poyting vector are written by using the classical microwave theory

\[ S_x = \frac{c}{8\pi} (E_x H_z^* - E_z H_x^*) \]
\[ S_y = \frac{c}{8\pi} (E_y H_z^* - E_z H_y^*) \]
\[ S_z = \frac{c}{8\pi} (E_z H_y^* - E_y H_z^*) \] (5)

It is obviously that the z-components of the wavevectors and the Poyting vectors are nonzero (see (2) and (5)). Thus we can say about bulk wave propagation within an anisotropic material in the considered exotic case. Moreover the x-component of the Poyting vector is nonzero. Thus the Poyting vector is not lie in the incidence plane in contrast to the cases described in [3]. Therefore it is possible to obtain very exotic deflection of an optical beam. These properties can be used for example in switching elements of communication systems and photonic computers.

4. Numerical results

In Fig.2a the dependence of the wavenumber on the incidence angle is presented for \( \mu_{xx} = 10.6, \mu_{xy} = 0, \mu_{zz} = 0.99 \). The components of the wavenumber in Fig.2a and the Poyting vector are shown in Fig.2b. As we can see the positive z-component of the Poyting vector corresponds to the negative z-component of the wavevector. Analogously negative z-component of the Poyting vector corresponds to the positive z-component of the wavevector. Therefore the medium can be considered as a metamaterial.

\[ \text{Fig.2. The dependences of the } z \text{-component of the wavenumber and the Poyting vector components on the inclination angle } \beta \text{ of the anisotropy axis} \]

It is interesting that the resulting reflected wave is elliptically polarized and this wave can be represented as a superposition of the right- and left-polarized waves with the different reflection coefficients. The dependences of the reflection coefficients for the right- and left-polarization waves are presented in Fig.3.
5. Conclusions

The exotic penetration effect for a magnetic medium is studied analytically. It is shown that in this exotic case the anisotropic medium in this case is analogous to a metamaterial. The dependence of the wavenumbers, the field components, and the Poyting vector components on an inclination angle when the anisotropy axis is orientated in the plane normal to the incidence plane are investigated. It is shown that left- and right-polarized waves are existed within the anisotropic medium. Note that the Poyting vectors of these waves are not lying in the incidence plane in contrast to the case when the anisotropy axis is in the incidence plane. Moreover the reflected wave is elliptically polarized in contrast to the case described in [2,3]. Note that the presented results cannot be applied if $\beta = 0$.

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