Survival analysis of the Swift Optical Transient data

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Summary. — In a systematic search of the OTs at GRBs the Swift satellite determined only an upper limit of the apparent brightness in a significant fraction of cases. Combining these upper limits with the really measured OT brightness we obtained a sample well suited to survival analysis. Performing a Kaplan-Meier product limit estimation we obtained an unbiased cumulative distribution of the V visual brightness. The \( \log_{10}(N(V)) \) logarithmic cumulative distribution can be well fitted with a linear function of \( V \) in the form of \( \log_{10}(N(V)) = 0.234 V + \text{const} \). We studied the dependence of \( V \) on the gamma ray properties of the bursts. We tested the dependence on the fluence, \( T_{90} \) duration and peak flux. We found a dependence on the peak flux on the 99.7% significance level.

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1. – Introduction

Studying the optical transients (OT) of GRBs is one of the major tasks of the Swift satellite. However, a significant fraction of GRBs does not show detectable OT fluences. It means in the number of cases one measured only an upper limit instead of a real event. Treating only those bursts exceeding the limit of detection with a high certainty one never can be sure how representative is the sample obtained in this way for the whole population of the GRBs detected by the BAT. Survival analysis offers a solution to overcome this difficulty.

2. – Statistics with censored data

If one knows a lower/upper bound for the measured data instead of concrete values they are considered as censored. In mathematical terms let we have two independent
stochastic variables: \([X, Y]\). If \(X > Y\) then \(X\) is detected, otherwise \(Y\) which is an upper bound for \(X\) in this way. A sample representing these observations consists of a mixture of real and censored (upper bound) data. In our case \(X\) represents the brightness of an OT and \(Y\) an upper bound for the non real (censored) detection. Kaplan and Meier \[2\] showed it is possible to estimate the true distribution of a sample even in the case of censorship (for the astronomical context see Feigelson and Nelson \[1\]).

3. – Mathematical formulation

According to Kaplan and Meier the following estimate gives the survival function:

\[
S(x) = 1 - F(x) = \prod_{i=1}^{n} P_i
\]

where \(F(x)\) is is the probability distribution function and

\[
P_i = \left[ 1 - \frac{1}{n - i + 1} \right] \delta_i
\]

\(n\) means the number of data, \(\delta\) equals 1 at non censored data and 0 in the censored case. It means the estimated survival function jumps at real data and remains constant in the case of censoring.

4. – Survival analysis with OT data

Until preparing this work Swift observed 144 events and detected OT in 31 cases. 73 bursts have only upper limits and the rest has no optical data. Before making the survival
analysis the effect of the foreground extinction has to be removed. We used the data of Schlegel et al. [3]. Performing the analysis we derived the true cumulative distribution of the V visual magnitudes which can be compared with that of the measured optical events, as displayed in Figure 1.

The logarithmic cumulative distribution of the V magnitudes obtained from the survival analysis can be well fitted with the following linear relationship:

$$\log_{10} N(V) = 0.234 V + \text{const}$$

It is worth mentioning that a homogeneous spatial distribution in Euclidean space would resulted in a coefficient of 0.6 instead of 0.234, as obtained. This result gives an important constraint on estimating the spatial distribution of OT events.

The BAT on the Swift satellite measured the fluence, peak flux and $T_{90}$ duration in the 15-150 keV energy range. We computed the Pearson linear correlation between the extinction-corrected V visual magnitude of the OTs and the logarithms of quantities mentioned. We obtained significant correlation only with the logarithmic peak flux at the 99.7% level (see Fig. 2).

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REFERENCES
[1] Feigelson E.D. and Nelson P.I., ApJ, 293 (1985) 192
[2] Kaplan E.L. and Meier P., J. Am. Stat. Ass, 53 (1958) 457
[3] Schlegel D.J., Finkbeiner D.P. and Davis M., ApJ, 500 (1998) 525