Abstract—The implementation of the process $\gamma \gamma \rightarrow ZZ$ at the one-loop level within SANC system multi-channel approach is considered. The derived one-loop scalar form factors can be used for any cross channel after an appropriate permutation of their arguments — Mandelstam variables $s, t, u$. To check of the correctness of the results we observe the independence of the scalar form factors on the gauge parameters and the validity of Ward identity (external photon transversality). We present the complete analytical results for the covariant and tensor structures and helicity amplitudes for this process. We make an extensive comparison of our analytical and numerical results with those existing in the literature.

DOI: 10.1134/S1547477117060061

1. INTRODUCTION

Physics with $\gamma \gamma$ collider [1–4] and with $e^+e^-$ linear collider [5–7] always demonstrated the great interest to establish the effects from transversal and longitudinal polarization. We begin to create the theoretical support for the $\gamma \gamma$ colliders: first of all it is SANC modules for processes $\gamma \gamma \rightarrow \gamma \gamma (\gamma Z, ZZ)$ at the one-loop level [8, 9]. Second step will be MC generator with these modules taking into account the polarization effects for $\gamma \gamma$ and $e^+e^-$ physics.

In this article we describe some results obtained with SANC (Support of Analytic and Numerical calculations for experiments at Colliders) — a network system for semi-automatic calculations for processes of elementary particle interactions at the one-loop precision level, see [10]. The corresponding FORTRAN modules [11] for processes at the one loop level can be downloaded by request.

All calculations at the one-loop precision level are realized in the spirit of the book [12] in the $R_\xi$ gauge and all the results are reduced up to the scalar Passarino–Veltman functions: $A_0, B_0, C_0, D_0$ [13]. These two distinctive features allow one to perform several checks: e.g. to test gauge invariance by observing the cancellation of gauge parameter dependence, to test symmetry properties and validity of the Ward identities, all at the level of analytical expressions. The SANC system is based on FORM [14] applications. These applications had to be modularized as procedures in a most universal way so as to be used as building blocks for the computation of more complex processes. We used a covariant method for calculating helicity amplitudes (HA) presented in [15]. The numerical computations are done in FORTRAN.

We have implemented in the framework of the SANC system processes such as

$$\gamma(p_1, \lambda_1) + \gamma(p_2, \lambda_2) \rightarrow Z(p_3, \lambda_3) + Z(p_4, \lambda_4)$$

(1)

($\lambda_i, (i = 1, 2, 3, 4)$ are the helicities of the external particles) in the Standard Model (SM) at the one-loop level of accuracy in $R_\xi$-gauge through fermion loops and corresponding precomputation blocks.

The computations of these processes take into account non-zero masses of loop-fermions. Our previous evaluation for processes $\gamma + \gamma \rightarrow \gamma + \gamma$ and $\gamma + \gamma \rightarrow \gamma + Z, Z \rightarrow \gamma \gamma \gamma$ was presented in [8] and [9]. The additional precomputation modules used to calculate massive fermion-box diagrams are briefly described.

We discuss the covariant and tensor structures and present them in a compact form. The helicity amplitude approach and their expressions are given.

First of all we calculate $b b b b \rightarrow 0$ in the multi-channel approach and get the main object form factors (FF) for the annihilation to vacuum. In the second step we produce the FF of the real channel 1 by a permutation of the arguments $s, t$ and $u$.

The 4-momenta of incoming bosons are denoted by $p_1$ and $p_2$, of the outgoing ones by $p_3$ and $p_4$.

The 4-momenta conservation reads

$$p_1 + p_2 - p_3 - p_4 = 0.$$
The Mandelstam variables are\(^2\)

\[
s = (p_1 + p_2)^2, \quad t = -(p_1 - p_3)^2, \quad u = -(p_1 - p_4)^2.
\]

Whenever possible, we compare the results with those existing in the literature.

To check our numerical and analytical results we compare them with other independent calculations [16].

The paper is organized as follows. In Section 2 we briefly describe the precomputation strategy (see also, [10]). In Section 3 we discuss the tensor structures of covariant amplitudes for these processes and present their compact form. The idea of form factors is described in Section 4. In Section 5 we consider the sets of the corresponding helicity amplitudes. In the conclusions, Section 3, we present the comparison of our results with those existing in the literature.

2. PRECOMPUTATION STRATEGY

In this section we briefly describe the modules relevant for \(bb \rightarrow bb\), \((b-\) is \(A\) or \(Z\)). The contributions of fermionic loop boxes form a gauge-invariant and UV-finite subset.

In SANC, the idea of precomputation becomes vitally important for boxes [10]. The calculation of some boxes for some particular processes takes so much time that an external user should refrain from repeating the precomputation. Furthermore, the richness of boxes requires a classification. Depending on the type of external lines \((f-\)fermion or \(b-\)boson), we distinguish three large classes of boxes: \(ffff\), \(ffbb\) and \(bbbb\).

The precomputation file \(bbbb\) Box, i.e. \(AAAA\) Box, \(AAAZ\) Box, \(AAZZ\) Box contains the sequence of procedures to calculate the covariant amplitude. At this step we suppose that all momenta are incoming (denoted by \(p_i\)) and photons are not on-mass-shell. Therefore, these results can be used for other processes which need these parts as building blocks.

When we implement the processes \(bb \rightarrow bb\), we use this building block several times by replacing the incoming momenta \(p_i\) by the corresponding kinematical momenta \(k_i\), calculate FFs by the module \(bb \rightarrow bb\) (FF), then helicity amplitudes by the modules \(bb \rightarrow bb\) (HA) and finally the differential and total process cross section by the module \(bb \rightarrow bb\) (XS).

3. COVARIANT AMPLITUDE

The covariant one-loop amplitude (CA) corresponds to the result of the straightforward standard calculation of all diagrams contributing to a given process at the one-loop (1-loop) level. The CA is being represented in a certain basis, made of strings of Dirac matrices and/or external momenta (structures), contracted with polarization vectors of vector bosons, \(e(k)\), if any.

A CA can be written in an explicit form with the aid of scalar FFs.

\[
\mathcal{A}_{\gamma \gamma \rightarrow ZZ} = 4e^4 \sum_{i=1}^{20} \left[ \mathcal{F}_{\gamma i}^{{\text{bos}}} (s,t,u) + \mathcal{F}_{\gamma i}^{{\text{fer}}} (s,t,u) \right] T_i^{\alpha \beta \mu \nu}.
\]

All masses, kinematical factors and coupling constant and other parameter dependences are included into these FFs \(\mathcal{F}_{\gamma i}\), but tensor structure with Lorenz indexes made of strings of Dirac matrices are given by basis. The number of form factors is equal to the number of independent structures.

The covariant amplitude for the channel \(bb \rightarrow bb\), i.e. \(\gamma \gamma \rightarrow ZZ\) can be obtained from annihilation to the vacuum with the following permutations of the 4-momenta:

\[
p_1 \rightarrow p_1, \quad p_2 \rightarrow p_2, \quad p_1 \rightarrow -p_3, \quad p_4 \rightarrow -p_4.
\]

To describe the tensor structures of the CA for \(AA \rightarrow ZZ\) we introduce the following five auxiliary tensorial strings:

\[
\tau^i = p^i + \frac{1}{2}s \delta^i, \quad \tau^j = p^j - \frac{1}{2}(M_Z^2 - t) \delta^i, \quad \tau^k = p^k - \frac{1}{2}(M_Z^2 - u) \delta^i, \quad \tau^l = p^l + \frac{M_Z^2 - t}{s} p^i, \quad \tau^m = p^m + \frac{M_Z^2 - u}{s} p^i.
\]

Our basis of CA is given by:

\[
T_1^{\alpha \beta \mu \nu} = t_1^\alpha p_1^\mu p_1^\nu, \quad T_2^{\alpha \beta \mu \nu} = t_1^\beta t_1^\gamma t_1^\mu t_1^\nu, \quad T_3^{\alpha \beta \mu \nu} = t_2^\gamma t_3^\mu t_3^\nu t_3^\alpha, \quad T_4^{\alpha \beta \mu \nu} = t_1^\alpha t_1^\beta t_1^\mu t_1^\nu, \quad T_5^{\alpha \beta \mu \nu} = t_3^\gamma t_3^\mu t_3^\nu t_3^\alpha,
\]

\[
T_6^{\alpha \beta \mu \nu} = t_2^\gamma t_2^\mu t_2^\nu t_2^\alpha, \quad T_7^{\alpha \beta \mu \nu} = \frac{1}{2} p^\mu \left(2p^\nu t_2^\beta + s \delta^\alpha t_4^\beta\right), \quad T_8^{\alpha \beta \mu \nu} = t_4^\alpha p^\mu t_4^\nu t_4^\beta, \quad T_9^{\alpha \beta \mu \nu} = t_5^\nu t_5^\mu t_5^\alpha t_5^\beta, \quad T_{10}^{\alpha \beta \mu \nu} = t_1^\alpha t_1^\beta t_1^\mu t_1^\nu, \quad T_{11}^{\alpha \beta \mu \nu} = t_3^\alpha t_3^\beta t_3^\mu t_3^\nu, \quad T_{12}^{\alpha \beta \mu \nu} = t_5^\alpha t_5^\beta t_5^\mu t_5^\nu, \quad T_{13}^{\alpha \beta \mu \nu} = t_1^\alpha t_1^\beta t_1^\mu t_1^\nu, \quad T_{14}^{\alpha \beta \mu \nu} = t_3^\alpha t_3^\beta t_3^\mu t_3^\nu, \quad T_{15}^{\alpha \beta \mu \nu} = \delta_{\alpha \beta} p^\mu p^\nu - \delta_{\mu \nu} p^\alpha p^\beta + \delta_{\alpha \nu} t_1^\alpha t_1^\beta,
\]

\[
T_{16}^{\alpha \beta \mu \nu} = \left(\delta_{\alpha \nu} p^\mu - \delta_{\alpha \mu} p^\nu\right) t_4^\beta, \quad T_{17}^{\alpha \beta \mu \nu} = \left(\delta_{\beta \nu} p^\mu - \delta_{\beta \mu} p^\nu\right) t_4^\alpha.
\]

\(^2\) Note, that in SANC we use the Pauli metric.
4. FORM FACTORS

In the multi-channel approach we calculate \( bbbb \rightarrow 0 \), and get the main object formfactors (FF) for the annihilation into the vacuum. The form factors \( F \) are presented as combinations of scalar PV functions \( A_0, B_0, C_0, D_0 \) [13], and depend on invariants \( s, t, u \), and also on fermion and boson masses. They do not contain ultraviolet poles. These one-loop scalar form factors can be used for any cross channel after an appropriate permutation of their arguments \( s, t, u \).

Explicit expressions for the boson and fermion parts of the form factors are not shown in this article because they are very cumbersome. A complete answer for \( \gamma \gamma \rightarrow \gamma \gamma \) can be found in the package which is downloadable from the homepage of the computer system SANC. For massless loop fermions the FFs are rather compact for \( \gamma \gamma \rightarrow \gamma \gamma \), see [8]. Note that the expression for the amplitude of boson diagrams are similar to those for the fermion diagrams except for the explicit representation of form factors.

5. HELICITY AMPLITUDES

In SANC we use the helicity amplitude approach. In the expression for CA, as one can see in subsection 3, one has tensor structures and a set of scalar FFs. To calculate an observable quantity, such as cross section, one needs to take the square of the amplitude, calculate products of Dirac spinors and contract Lorenz indices with polarization vectors. In the standard approach of taking amplitude square one gets squares for each diagram and their interferences. This leads to a large number of terms.

In the helicity amplitude approach we also derive tensor structures and FFs. But the next step is a projection to the helicity basis and as a result one gets a set of non-interfering amplitudes, since all of them are characterized by different sets of helicity quantum numbers. In this approach we can separate the calculations of Dirac spinors and the contractions of Lorenz indices from calculations of FFs. We can do this before taking the amplitude squares. So, proceeding in this way, we get a profit on calculation time (a smaller number of terms due to zero interference) and also a clearer step-by-step control.

In the SANC system helicity amplitudes are the result of an application of the procedure TRACEHelicity.prc. A description of the main SANC procedures is given in [8].

In this section we collect the analytical expressions of the HAs. The total number of the HA is 36.

We have verified the analytical zero between the cross sections of [16] and SANC.

\[
\gamma(p_1, \lambda_1) + \gamma(p_2, \lambda_2) \rightarrow Z(p_3, \lambda_3) + Z(p_4, \lambda_4)
\]

\( \lambda_i, (i = 1, 2, 3, 4) \) are the helicities of the external particles.

The relationship between HA obey parity and Bose symmetries:

\[
\mathcal{H}_{\lambda_1, \lambda_2, \lambda_3, \lambda_4}(s, t, u, \lambda) = \mathcal{H}_{-\lambda_1, -\lambda_2, -\lambda_3, \lambda_4}(s, t, u, \lambda),
\]

\[
\mathcal{H}_{\lambda_1, \lambda_2, \lambda_3, \lambda_4}(s, t, u, \lambda) = \mathcal{H}_{\lambda_1, \lambda_2, \lambda_3, \lambda_4}(s, t, u, \lambda).
\]

Finally eight independent HA remain due to parity transformation and corresponding rotation about the \( y \)-axis:

\[
\mathcal{H}_{+, +, +, +}(s, t, u, \lambda) = \mathcal{H}_{+, +, +, -}(s, t, u, -\lambda)\mathcal{H}_{+, +, -, +}(s, t, u, \lambda)
\]

\[
= -\mathcal{H}_{+, +, +, -}(s, t, u, -\lambda).
\]

There are 10 sets of HA for this process. Inside each set the HA are equal to each other or replace the sign of \( \lambda \), or change the sign of \( \cos \theta_\gamma \):

\[
\mathcal{H}_{+, +, +, +}, \mathcal{H}_{+, +, +, -}, \mathcal{H}_{+, +, -, +}, \mathcal{H}_{+, +, -, -}, \mathcal{H}_{+, -, +, +}, \mathcal{H}_{+, -, +, -}, \mathcal{H}_{+, -, -, +}, \mathcal{H}_{+, -, -, -}, \mathcal{H}_{+, 0, +, +}, \mathcal{H}_{+, 0, +, -}, \mathcal{H}_{+, 0, -, +}, \mathcal{H}_{+, 0, -, -}, \mathcal{H}_{+, -1, +, +}, \mathcal{H}_{+, -1, +, -}, \mathcal{H}_{+, -1, -, +}, \mathcal{H}_{+, -1, -, -}, \mathcal{H}_{-, +, +, +}, \mathcal{H}_{-, +, +, -}, \mathcal{H}_{-, +, -, +}, \mathcal{H}_{-, +, -, -}, \mathcal{H}_{-, -, +, +}, \mathcal{H}_{-, -, +, -}, \mathcal{H}_{-, -, -, +}, \mathcal{H}_{-, -, -, -}, \mathcal{H}_{-, 0, +, +}, \mathcal{H}_{-, 0, +, -}, \mathcal{H}_{-, 0, -, +}, \mathcal{H}_{-, 0, -, -}, \mathcal{H}_{-, -1, +, +}, \mathcal{H}_{-, -1, +, -}, \mathcal{H}_{-, -1, -, +}, \mathcal{H}_{-, -1, -, -}, \mathcal{H}_{-, -2, +, +}, \mathcal{H}_{-, -2, +, -}, \mathcal{H}_{-, -2, -, +}, \mathcal{H}_{-, -2, -, -}.
\]

The fully massive case of the analytical expressions of the helicity amplitudes \( \mathcal{H}_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} \) has the following form:
\[ \mathcal{H}_{+++} = \frac{1}{32} \sin^{-} c^{+} \left\{ 2s \left( -\mathcal{F}_7 + \mathcal{F}_8 + \mathcal{F}_9 - \mathcal{F}_{10} \right) - k_3 \left( \mathcal{F}_{11} - \mathcal{F}_{12} - \mathcal{F}_{13} - \mathcal{F}_{14} + \mathcal{F}_{15} + \mathcal{F}_{16} \right) - \frac{\sqrt{s}}{2s} \left( c^{-} \mathcal{F}_{17} + c^{+} \mathcal{F}_{20} \right) + \frac{k_2}{2s} \left( \mathcal{F}_{18} + \mathcal{F}_{19} \right) \right\}, \]

\[ \mathcal{H}_{++-} = \frac{1}{64} c^{-} c^{+} \left\{ 8 \frac{s}{s} \mathcal{F}_6 + 2s \left[ 4 \left( \mathcal{F}_1 + \mathcal{F}_2 \right) - 4 \frac{\sqrt{s}}{s} \left( \mathcal{F}_3 + \mathcal{F}_4 \right) - k_2 \mathcal{F}_{12} - k_2 \mathcal{F}_{13} + c^{+} \mathcal{F}_{14} \right] + c^{-} \mathcal{F}_{11} + \mathcal{F}_{15} - \mathcal{F}_{16} - \frac{k_2}{2s} \mathcal{F}_{17} - \frac{k_1}{2s} \mathcal{F}_{20} + k_2 \mathcal{F}_{18} + k_1 \mathcal{F}_{19} \right\}, \]

\[ \mathcal{H}_{+-+} = \frac{1}{32} (c^{-})^2 s \left[ 4 \left( \mathcal{F}_1 + \mathcal{F}_2 \right) + \left( \frac{\sqrt{s}}{s} c^{+} - u + t - s \right) \left[ \mathcal{F}_{12} + \mathcal{F}_{13} + \frac{1}{2} c^{+} \frac{\sqrt{s}}{s} \left( \mathcal{F}_{17} + \mathcal{F}_{20} \right) \right] + \sqrt{s} \mathcal{F}_{11} - \mathcal{F}_{14} - \mathcal{F}_{16} = \mathcal{F}_{17} + \mathcal{F}_{19} + 2M^2 \left( \mathcal{F}_{18} + \mathcal{F}_{19} \right) \right\}, \]

\[ \mathcal{H}_{---} = \frac{1}{64} \frac{1}{M^2} \left[ 4s \mathcal{F}_1 + \mathcal{F}_2 \right] + 2 \lambda \left[ 2(\mathcal{F}_3 + \mathcal{F}_4) + \frac{1}{s^2} k_3 \mathcal{F}_0 \right] - s \left[ \sqrt{s} \mathcal{F}_{11} \left( \mathcal{F}_1 + \mathcal{F}_3 \right) + k_2 \mathcal{F}_{12} + 4s \mathcal{F}_2 \mathcal{F}_3 + \sqrt{s} \mathcal{F}_{14} - \mathcal{F}_{16} \right] + 2M^2 \left( \sqrt{s} \mathcal{F}_{18} + \mathcal{F}_{19} - \sqrt{s} \mathcal{F}_{20} \right) \right], \]

\[ \mathcal{H}_{++0} = \frac{1}{32} \sin \frac{\sqrt{s}}{\sqrt{2} M^2} \left[ 2k_1 \left[ 2 \left( \mathcal{F}_1 + \mathcal{F}_2 \right) + \sqrt{s} \left( \mathcal{F}_3 - \mathcal{F}_4 \right) \right] + 2s \left[ k_1 \left( -\mathcal{F}_7 + \mathcal{F}_9 \right) + k \left( \mathcal{F}_8 - \mathcal{F}_{10} \right) \right] + c^{-} k_2 \mathcal{F}_9 - k_2 \left( \mathcal{F}_{11} - \mathcal{F}_{13} \right) - k \mathcal{F}_{16} \right] + \frac{1}{2s} \left[ \sqrt{s} \left( \mathcal{F}_{17} + c^{+} k \mathcal{F}_{20} \right) + k_2 \left( \mathcal{F}_{18} + c^{-} \mathcal{F}_{19} \right) \right] \right], \]

\[ \mathcal{H}_{++00} = \frac{1}{32} \left\{ -2 \left( s^2 c^{+} + k^2 \right) \mathcal{F}_1 - 2 \left( s^2 c^{-} + k^2 \right) \mathcal{F}_2 \right\} \]

\[ - 2c^{-} c^{+} \lambda (\mathcal{F}_3 + \mathcal{F}_4) - 4k_1 \left[ \mathcal{F}_5 - \frac{\mathcal{F}_6}{s} \right] -s \left[ k \mathcal{F}_7 + k^2 \mathcal{F}_8 - k_1 \mathcal{F}_9 + \left( s^2 \cos^2 \frac{\sqrt{s}}{\sqrt{2} M^2} + \lambda \right) \mathcal{F}_{10} \right] + \frac{\cos^2 \frac{\sqrt{s}}{\sqrt{2} M^2}}{2} k_1 k_2 \left[ \mathcal{F}_{11} - \mathcal{F}_{15} + k \mathcal{F}_{16} \right] \right] + \frac{1}{2s} \left[ \sqrt{s} \left( k_1 \mathcal{F}_{17} + k_2 \mathcal{F}_{18} + \mathcal{F}_{19} - \sqrt{s} \mathcal{F}_{20} \right) \right], \]

\[ \mathcal{H}_{+-0} = \frac{1}{32} \sin \frac{\sqrt{s}}{\sqrt{2} M^2} \left[ 4s \left( \mathcal{F}_1 + \mathcal{F}_2 \right) - 2 \frac{\sqrt{s}}{s} \left( k_1 \mathcal{F}_3 + k_2 \mathcal{F}_4 \right) \right] + s \left[ c^{-} \mathcal{F}_{11} - k_2 \mathcal{F}_{12} - 4M^2 \mathcal{F}_{13} + c^{-} \mathcal{F}_{15} - \mathcal{F}_{16} \right] \right. \]

\[ - \sqrt{s} \mathcal{F}_{14} - 2M^2 \left[ c^{-} \mathcal{F}_{17} - \mathcal{F}_{20} - k_2 \mathcal{F}_{18} - k_1 \mathcal{F}_{19} \right] \right], \]

\[ \mathcal{H}_{+-00} = \frac{1}{32} \left\{ 4k_1 \left( \mathcal{F}_1 - \mathcal{F}_2 \right) + \frac{2 \sqrt{s}}{s} \left( k_1 \mathcal{F}_3 - \mathcal{F}_4 \right) \right\} + 2s \left[ k \left( \mathcal{F}_7 - \mathcal{F}_8 \right) - k_1 \left( \mathcal{F}_9 - \mathcal{F}_{10} \right) - \frac{c^{-}}{2} k_2 \mathcal{F}_{12} + 2c^{-} M^2 \mathcal{F}_{13} \right] \right. \]

\[ + k k \mathcal{F}_{14} - 2M^2 \left[ 2s \cos^2 \frac{\sqrt{s}}{\sqrt{2} M^2} \left( \mathcal{F}_{11} - \mathcal{F}_{13} + \mathcal{F}_{15} \right) \right. \]

\[ - c^{-} \mathcal{F}_{17} - \mathcal{F}_{20} - k_1 \left( \mathcal{F}_{18} - \mathcal{F}_{19} \right) \right\} \].
SANC: THE PROCESS $\gamma \gamma \rightarrow ZZ$

$$\mathcal{H}_{++0} = \frac{1}{32} \sin \theta_{\gamma} \frac{\sqrt{s}}{2M_Z} e^{-\left[4s(F_1 + F_2) - 2\sqrt{s}[k_1 \overline{F}_3 - k_2 \overline{F}_4] \right]}$$
$$- \sqrt{\lambda} k_{11} (\overline{F}_{11} + \overline{F}_{15}) - k_{12} \overline{F}_{12} - s k_{13} \overline{F}_{13} - e^+ s \sqrt{\lambda} \overline{F}_{14}$$
$$+ \frac{1}{\lambda_{33}} \left[ 2s \overline{F}_{16} - k_{1} \overline{F}_{20} + \sqrt{\lambda} k_{12} \overline{F}_{17} + k_{3} (k_{2} \overline{F}_{18} + k_{4} \overline{F}_{19}) \right],$$

where
$$u = \cos \theta_{\gamma} \sqrt{\lambda} + t, \quad e^\pm = 1 \pm \cos \theta_{\gamma}$$
$$k_1 = s - \sqrt{\lambda}, \quad k_2 = s + \sqrt{\lambda}, \quad k_3 = s^2 + \lambda,$$
$$k_{11} = sc^+ - k_1, \quad k_{12} = sc^+ - k_2, \quad \lambda = s(s - 4M_Z^2).$$

The definitions of cross-sections $\sigma_y$ were given in [17]:

$$\frac{d\sigma_0}{d \cos \theta^*} = \left( \frac{\beta_Z}{64\pi s} \right) \sum_{\lambda,\lambda_4} \left[ |\mathcal{H}_{++\lambda,\lambda_4}|^2 + |\mathcal{H}_{-\lambda,\lambda_4}|^2 \right]$$

$$\frac{d\sigma_{22}}{d \cos \theta^*} = \left( \frac{\beta_Z}{64\pi s} \right) \sum_{\lambda,\lambda_4} \left[ |\mathcal{H}_{++\lambda,\lambda_4}|^2 - |\mathcal{H}_{-\lambda,\lambda_4}|^2 \right]$$

$$\frac{d\sigma_{33}}{d \cos \theta^*} = \left( \frac{\beta_Z}{64\pi s} \right) \sum_{\lambda,\lambda_4} Re \left[ \mathcal{H}_{++\lambda,\lambda_4} \mathcal{H}^*_{-\lambda,\lambda_4} \right],$$

where
$$\beta_Z = 1 - \frac{M_Z^2}{s}.$$

6. CONCLUSIONS

This paper is devoted to the description of implementing the complete one-loop electroweak calculations for the process $\gamma \gamma \rightarrow ZZ$ into the SANC framework. We presented analytical expressions for the Covariant Amplitude and for the Helicity Amplitudes. To be assured of the correctness of our analytical results, we checked the independence of the form factors on gauge parameters (all calculations were done in $R_\xi$ gauge), the validity of Ward identities for covariant amplitudes and, finally, the SANC results for this processes were compared with other independent calculations [16], [17] (see Figs. 1–4). For all of the contributions good agreement was obtained with the results, given in the literature.

We begin to develop MC SANC generator at the one-loop level taking into account the polarization for future linear $e^+ e^-$ colliders—ILC and CLIC. This
study for Bhabha process we are going to present in the near future. To fill the generator we have library for the complete one-loop electroweak modules.

The full version MC SANC generator will contain the processes $\gamma\gamma \rightarrow \gamma\gamma(\gamma Z, ZZ)$ and some part of the library of the necessary complete one-loop modules we present here.

REFERENCES

1. I. F. Ginzburg, G. L. Kotkin, V. G. Serbo, and V. I. Telnov, “Production of high-energy colliding gamma gamma and gamma e beams with a high luminosity at Vlepp accelerators,” JETP Lett. 34, 491–495 (1981).

2. I. F. Ginzburg, G. L. Kotkin, V. G. Serbo, and V. I. Telnov, “Colliding gamma e and gamma gamma beams based on the single pass accelerators (of Vlepp type),” Nucl. Instrum. Methods Phys. Res. 205, 47–68 (1983).

3. I. F. Ginzburg, G. L. Kotkin, S. L. Panfil, V. G. Serbo, and V. I. Telnov, “Colliding gamma e and gamma gamma beams based on the single pass e+e- accelerators.

4. Polarization effects. Monochromatization improvement,” Nucl. Instrum. Methods Phys. Res. A 219, 5–24 (1984).

5. V. I. Telnov, “Photon collider Higgs factories,” JINST 9, C09020 (2014); arXiv:1409.5563 [physics.acc-ph].

6. E. Accomando et al. (CLIC Physics Working Group Collab.), “Physics at the CLIC multi-TeV linear collider,” in Proceedings of the 11th International Conference on Hadron spectroscopy (Hadron 2005), Rio de Janeiro, Brazil, August 21–26, 2005; arXiv:hep-ph/0412251 (2004).

7. G. Moortgat-Pick et al., “The role of polarized positrons and electrons in revealing fundamental interactions at the linear collider,” Phys. Rep. 460, 131–243 (2008); arXiv:hep-ph/0507011.

8. A. Arbey et al., “Physics at the e+e- linear collider,” Eur. Phys. J. C 75, 371 (2015); arXiv:1504.01726.

9. D. Yu. Bardin, L. V. Kalinovskaya, and E. D. Uglov, “Z$^0$ processes in SANC,” Phys. At. Nucl. 76, 1339–1344 (2013); arXiv:1212.3105 [hep-ph].

10. A. Andonov, A. Arbuzov, D. Bardin, S. Bondarenko, P. Christova, L. Kalinovskaya, G. Nanava, and W. von Schlippe, “SANCscope - v.1.00,” Comput. Phys. Commun. 174, 481–517 (2006); Comput. Phys. Commun. 177, 623(E) (2007); arXiv:hep-ph/0411186.

11. A. Andonov, A. Arbuzov, D. Bardin, S. Bondarenko, P. Christova, L. Kalinovskaya, V. Kolesnikov, and R. Sadykov, “Standard SANC modules,” Comput. Phys. Commun. 181, 305–312 (2010); arXiv:0812.4207 [physics.comp-ph].

12. D. Y. Bardin and G. Passarino, The Standard Model in the Making: Precision Study of the Electroweak Interactions (Clarendon, Oxford, UK, 1999).

13. G. Passarino and M. Veltman, “One loop corrections for e+e- annihilation into $\mu^+\mu^-$ in the Weinberg model,” Nucl. Phys. B 160, 151 (1979).

14. J. A. M. Vermaseren, “New features of FORM,” arXiv:math-ph/0010025 (2000).

15. R. Vega and J. Wudka, “A covariant method for calculating helicity amplitudes,” Phys. Rev. D 53, 5286–5292 (1996); Phys. Rev. D 56, 6037(E) (1997); arXiv:hep-ph/9511318.

16. T. Diakonidis, G. J. Gounaris, and J. Layssac, “A FORTRAN code for $\gamma\gamma \rightarrow ZZ$ in SM and MSSM,” Eur. Phys. J. C 50, 47–52 (2007); arXiv:hep-ph/0610085.

17. G. J. Gounaris, J. Layssac, P. Porfyriadis, and F. M. Renard, “The $\gamma\gamma \rightarrow ZZ$ process and the search for virtual SUSY effects at a gamma gamma collider,” Eur. Phys. J. C 13, 79–97 (2000); arXiv:hep-ph/9909243.