Kinematic censorship as a constraint on allowed scenarios of high energy particle collisions

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Abstract. In recent years, it was found that the energy $E_{c.m.}$ in the centre of mass frame of two colliding particles can be unbounded near black holes. If collision occurs exactly on the horizon, $E_{c.m.}$ is formally infinite. However, in any physically reasonable situation this is impossible. We collect different scenarios of such a kind and show why in every act of collision $E_{c.m.}$ is indeed finite (although it can be as large as one likes). The factors preventing infinite energy are diverse: the necessity of infinite proper time, infinite tidal forces, potential barrier, etc. This prompts us to formulate a general principle according to which the limits in which $E_{c.m.}$ becomes infinite are never achieved. We call this the kinematic censorship (KC). Although by itself the validity of KC is quite natural, its application allows one to forbid scenarios of collisions predicting infinite $E_{c.m.}$ without going into details. The KC is valid even in the test particle approximation, so explanation of why $E_{c.m.}$ cannot be infinite, does not require references (common in literature) to the non-linear regime, backreaction, etc. The KC remains valid not only for free moving particles but also if particles experience the action of a finite force. For an individual particle, we consider a light-like continuous limit of a time-like trajectory in which the effective mass turns into zero. We show that it cannot be accelerated to an infinite energy during a finite proper time under the action of such a force. As an example, we consider dynamics of a scalar particle interacting with a background scalar field.

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1. Introduction

Some of fundamental principles in physics have a form of prohibition. Say, impossibility to reach the absolute zero of temperature constitutes the third law of thermodynamics. A similar statement in black hole physics implies that one cannot convert a nonextremal black hole into the extremal one during a finite number of steps. The principle of cosmic censorship states that one cannot see a singularity from the outside. Investigations on particle collisions revealed one more such a principle that remained shadowed until recently. This can be called "kinematic censorship" (KC): the energy in the centre of mass frame of any colliding particles cannot be infinite. In usual laboratory physics this looks quite trivial. Indeed, in, say, flat space-time any energy gained or released in any process is always finite. And, starting with the total finite energy, one cannot obtain something infinite due to the energy conservation. The situation radically changed after findings made in [1, 2]. It was shown there that if two particles collide in the extremal Kerr background, under certain conditions the energy $E_{c.m.}$ in the centre of mass becomes unbounded when a point of collision approaches the horizon $r_+$:

$$\lim_{r \to r_+} E_{c.m.}(r) \to \infty. \quad (1)$$

This was found for head-on collisions in [1] and for particles moving in the same direction towards the horizon, provided one of particles is fine-tuned, in [2]. In the latter case it is called the BSW effect. Later on, other versions of this process were found in different contexts. In doing so, the input was fi-
finite (particles with finite masses and energies) but the output is formally infinite in some limiting situations. The fact that each time (i) something prevents an infinite $E_{c.m.}$ and (ii) the nature of this "something" is completely different depending on the concrete scenario, suggests that there exists a rather general principle that unifies all so different particular cases. We want to point out the following subtlety. The results on unbounded $E_{c.m.}$ were obtained in the test-field approximation when self-gravitation and backreaction were neglected. As for sufficiently large $E_{c.m.}$, this can be no longer true, there is temptation to ascribe formally infinite $E_{c.m.}$ to the test-field approximation with the expectations (confirmed in some simple models, e.g., see [3]) that in the non-linear regime $E_{c.m.}$ will become finite. However, it turns out that even in the test-particle approximation infinite $E_{c.m.}$ are forbidden, although arbitrarily large but finite $E_{c.m.}$ are possible. Thus the limit under discussion turns out to be unreachable in each case.

It is worth noting that in [4] several objections were pushed forward against the BSW effect, in particular — impossibility to reach the extremal state and backreaction. Meanwhile, it was explained [5] that for nonextremal black holes the BSW effect does exist, although in a somewhat different setting. It was also shown in [6] for extremal black holes and in [7] for nonextremal ones that account of a finite force does not spoil the BSW effect. Therefore, the reasons why infinite $E_{c.m.}$ are, nonetheless, unreachable, should have more fundamental nature and explanations should be done just within the test-particle approximation, without attempts to ascribe them to some factor neglected in this approximation.

In the present work, we do not perform new concrete calculations. Instead, we collect a number of results, already obtained earlier, and make new qualitative generalizations based on them.

The fact that an infinite $E_{c.m.}$ cannot occur in any act of collisions, is more or less obvious in the flat space-time. In a curved space-time, the issue is not so trivial since formally infinite $E_{c.m.}$ was obtained with finite initial energies of colliding particles. And, although from physical grounds the KC is more or less obvious, it would be of interest to give a proof of the KC. However, in the present article we do not pretend for such a proof in an arbitrary curved space-time. Instead, we enumerate several typical situations and trace which conditions precisely prevent collisions with infinite $E_{c.m.}$. The corresponding factors turn out to be rather diverse. We also demonstrate that the KC works as some regulator that imposes constraint on possible scenarios and enables us to reject some of them in advance, even without going into details.

Another separate issue is behavior of a single particle that formally approaches the speed of light locally in a curved background and becomes potential source of difficulties with infinite $E_{c.m.}$. We suggest rather general and rigorous proof that this cannot happen under the action of finite forces. This is illustrated by physically interesting example of a scalar particle interacting with a background scalar field.

We use the systems of units in which the speed of light $c = 1$.

2. Can a massive particle be accelerated until a speed of light?

The question of the possibility or impossibility of infinite $E_{c.m.}$ reduces to the behavior of individual particles and their Killing energy $E$. If $E$ remains finite (equivalently, the velocity of a massive particle is less than a speed of light), infinite $E_{c.m.}$ is impossible. In the flat space-time, it is more or less obvious, that any particle cannot reach the speed of light during a finite amount of the proper time, provided all the components of four-acceleration $a^\mu$ are finite. Meanwhile, even in such a relatively simple case, there are some subtleties. Because of the Lorentz signature, it would seem that although separate components of $a^\mu$ diverge, the scalar $a^2 = a_\mu a^\mu$ can, nonetheless, be finite. However, now we argue that in the situation under discussion this is impossible.

Statement: if a massive particle moving under the action of some force reaches the speed of light during the finite proper time, the absolute value of acceleration $a \to \infty$ in this point.

To the best of our knowledge, the proof of the corresponding statement is absent from literature, so we suggest it below. We would like to stress that the issue under discussion is not exhausted by a simple fact that the square of the four-velocity is equal to $-1$ even in a curved space-time and cannot turn into zero by jump. The problem is that a time-like trajectory can approach the light-like one continuously. The corresponding situation was considered in the monograph [8] (see there Eqs. (23.2.4)–(23.2.6) and corresponding discussion). However, the authors of [8] restricted themselves by a particular example whereas we consider a general approach. Also, in the next Section we consider a particular example how
this is realized in motion of a scalar particle interacting with the background scalar field. We demonstrate that the effective mass vanishes in some point, so although the square of the four-velocity remains equal to $-1$, the square of the four-momentum vanishes.

At first, let us consider the situation in the flat space-time. Let in the Minkowski metric

$$ds^2 = -dt^2 + \eta_{ik}dx^i dx^k$$

(2)
a particle move along the trajectory $x^\mu = x^\mu(\tau)$, where $\tau$ is the proper time. Then, we have

$$a^\mu = \frac{du^\mu}{d\tau},$$

whence along a given trajectory from point 1 to point 2 one obtains

$$u^\mu = \int_1^2 a^\mu d\tau,$$

(4)

where on the trajectory $a^\mu$ is a function of $\tau$ only. If all components of $a$ are finite, $u^\mu$ remains finite as well, so the speed of light cannot be reached.

Now, we assume that as $\tau \rightarrow \tau_0$, $v \rightarrow 1$. Here, the velocity is defined according to $v^2 = v_i v^i$, $v^i = dx^i/dt$. The four-velocity has a standard form

$$u^\mu = \left(\frac{1}{\sqrt{1-v^2}}, \frac{v^i}{\sqrt{1-v^2}}\right) = \gamma(1,v^i),$$

(5)

where $\gamma = 1/\sqrt{1-v^2}$ is the Lorentz factor.

For the components of $a^\mu$ we have $a^0 = \dot{\gamma}, a^i = d(\gamma v^i)/d\tau$. Here, dot denotes $d/d\tau$. After some algebra, one can obtain

$$a^2 = \frac{\dot{\gamma}^2}{\gamma^2} + \gamma^2 \frac{dv^i}{d\tau} \frac{dv^k}{d\tau} \eta_{ik}.$$\hspace{1cm} (6)

Now, let us consider a concrete law of approaching to the speed of light near the value $\tau = \tau_0$:

$$v^i = (v^i)_0 - \beta^i(\tau_0 - \tau)^n + \ldots,$$

(7)

where $\beta^i$ are some constants. Then,

$$v^2 \approx 1 - 2\alpha(\tau_0 - \tau)^n,$$

(8)

where

$$\alpha = \beta_i (v^i)_0.$$\hspace{1cm} (9)

Then, direct evaluation of different terms in (6) gives us

$$a^2 \approx \frac{n^2}{4(\tau - \tau_0)^2} \rightarrow \infty.$$\hspace{1cm} (10)

Thus, not only separate components of the four-acceleration diverge but also the scalar product does so.

In the curved space-time, the situation looks quite similar from the physical point of view. However, it would seem that rigorous proof becomes much more complicated. In particular, this happens because of the appearance of the Christoffel symbols $\Gamma^\rho_{\mu\nu}$ in the expression for $a^\mu$. Fortunately, this is not so. The key point here is the existence of coordinate frame in which $\Gamma^\rho_{\mu\nu} = 0$ along a given line. This was shown by Fermi [9]. See also textbook [10], Ch. VII.91. Then, (3) and (4) apply with the same result.

Thus if the interval $\tau_1 \leq \tau \leq \tau_2$ is finite, $u^\mu$ remains finite for any finite $a$. Otherwise, a particle can indeed reach the speed of light but by expense that $a$ diverges in a corresponding point. One can pass from the coordinate frame under discussion to any other one by a finite Lorentz boost, so a particle’s speed is less than the speed of light in a new frame as well.

The context under discussion can be considered as a rather unexpected practical application of Fermi’s finding.

To be more precise, we must make a reservation. There exists a situation in which a particle can indeed reach a speed of light during a finite proper time with a finite or even zero acceleration but this is due to incompleteness and/or singular character of the frame. For example, this happens if a particle falls in the Schwarzschild black hole (see, say, eq. (102.7) in [11]). However, this velocity is measured by a static observer who becomes singular on the horizon, so this is a consequence of impossibility of using such a frame on the horizon and beyond. Also, the boost between the corresponding frame and any regular frame becomes singular as well.

In the next Section, we will consider a concrete physical example in which such a situation occurs.

### 3. Explicit example: vanishing of the effective mass

In this Section, we consider a concrete physical example. We deal with a scalar particle interacting with some background field $\psi$. Let the action have a simple form

$$S = -\int (m - q\psi) d\tau,$$\hspace{1cm} (11)

where $\psi$ is the background scalar field, $q$ being the scalar charge of a particle, $m$ is its mass. For such a
system, the exact spherically symmetric static solution describing a black hole was found \cite{12}–\cite{15}. Its metric formally coincides with that of the extremal Reissner-Nordström black hole:

\[
\text{ds}^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\omega^2,
\]

where \( f = (1 - \frac{r_+}{r})^2 \), \( r_+ \) is the horizon radius. Let us consider pure radial motion. Then, one can obtain easily that

\[
m_* \dot{t} = \frac{E}{f}, \tag{13}
\]

the effective “mass” \( m_* = m - q\psi \), the radial momentum

\[
P = \sqrt{E^2 - m_*^2 f}, \tag{15}
\]

the energy \( E = \text{const.} \).

We choose \( q > 0 \). Let us denote as \( r_0 \) the point in which \( \psi = m/q \), so \( m_*(r_0) = 0 \). It is implied that \( r_0 > r_+ \).

We assume that \( \psi \) is a smooth function of \( r \) and we examine the behavior of relevant quantities near the point \( r_0 \) where the effective mass vanishes, \( m_*(r_0) = 0 \):

\[
\psi = \psi(r_0) - C(r - r_0) + \ldots, \tag{16}
\]

where \( C \) is some constant. Then,

\[
m_* \approx qC(r - r_0) \tag{17}
\]

Then,

\[
\dot{r} \approx -\frac{E}{Cq(r - r_0)}, \tag{18}
\]

for \( r \geq r_0, \tau \leq \tau_0 \)

\[
r \approx r_0 + B(\tau_0 - \tau), \tag{19}
\]

\[
B = \sqrt{\frac{2E}{Cq}}. \tag{20}
\]

Thus the proper time to reach \( r_0 \) (say, from \( r > r_0 \)) is finite.

For the velocity \( V \) measured by a static observer, we have \( E = m_* \sqrt{f}/\sqrt{1 - V^2} \), whence

\[
V = \sqrt{1 - \frac{m_*^2}{E^2}}. \tag{21}
\]

Near \( r = r_0 \),

\[
V \approx 1 - \frac{C^2 q^2 f(r_0)}{2E^2} (r - r_0)^2, \tag{22}
\]

so \( V \to 1 \).

Thus a particle reaches the speed of light for a finite proper time but in one point \( r_0 \) only.

One can introduce a kinematic momentum according to \( p^\mu = m_* u^\mu \). It is instructive to note that although \( u_\mu u^\mu = -1 \) in any point, including \( r_0 \), \( p_\mu p^\mu = -m_*^2 \to 0 \) when \( r \to r_0 \). In this sense, we have light-like limit of a time-like particle. In the standard case, for a particle of a given mass, this would lead to unbounded energy. However, in the present case \( E \) remains finite due to the fact that \( m_* \to 0 \) in this limit. As a result, we obtain a quite unusual entity — a trajectory with \( u_\mu u^\mu = -1 \) that corresponds, however, to a massless particle in one point.

We would like to stress that these subtleties can have, in principle, further physical consequences, say, in cosmology, where interaction of a scalar particle with the background field can affects time evolution and conditions of thermodynamic equilibrium \cite{13,17}.

Now, we will show that price paid for such non-trivial behavior is divergences in acceleration. For the action \( \{11\} \), equations of motion in this case give rise to the acceleration (see, e.g. Sec. 2 of \cite{13})

\[
a^\alpha = u^\beta \dot{u}^\alpha = \frac{q}{m_*} [\psi^{\alpha\beta} + u^\beta (\psi_\beta u^\alpha)], \tag{23}
\]

whence

\[
a^\alpha = a_{\mu} u^\mu = \frac{q^2}{m_*^2} h_{\mu\nu} \psi_{\mu} \psi_{\nu}, \tag{24}
\]

\[
h_{\mu\nu} = g_{\mu\nu} + u^\mu u^\nu, \tag{25}
\]

where \( u^\mu \) is the four-velocity.

In general \( h_{\mu\nu} \psi_{\mu} \psi_{\nu} \neq 0 \) and, moreover, this quantity can diverge. This happens in the present example. Then, it is seen that acceleration \( a^2 \to \infty \) when \( r \to r_0 \). In other words, a system as a whole is singular although all geometric characteristics like the Kreschmann scalar are perfectly regular in the point \( r_0 \). It is of interest to study further properties of such “intermediate” systems that combine regular and singular properties. For the example under discussion, one can check that Eq. \{11\} is reproduced exactly.

4. Black vs. white holes as sources of high energy collisions

Now, we turn to concrete scenarios of high energy collisions. The first observations on such collisions near the horizon made in \cite{11} predict formally unbounded \( E_{\text{c.m.}} \) that becomes infinite when \( r \to r_+ \).
However, the crucial point here consists in that actually the corresponding scenario describes collisions of particles moving in the opposite direction (head-on collision). This is especially clear from Eq. (2.57) of [18], where the issue under discussion was elaborated in detail. In turn, this implies that one of particles moves not towards the horizon but away from it. This condition cannot be realized in the immediate vicinity of a black hole horizon (see for details Sec. IV A in [19]) and, rather, corresponds to a white hole. Collisions of particles near white holes were discussed in [20, 21]. Let particle 1 appear from the inner region and collides with particle 2 that comes from infinity or some finite distance outside. It is essential that collision cannot happen exactly on the horizon itself. Otherwise, any massive particle 2 having a finite energy at infinity would become light-like. Instead, one can take a particle 2 to be massless. However, either an observer 1 emits or absorbs a photon of finite frequency or the frequency of particle 2 must be taken infinite from the very beginning that deprives the scenario of physical meaning [22]. Thus, nature of the white horizon protects the energy $E_{c.m.}$ from being infinite.

5. Extremal black holes

Let us consider the metric

$$ds^2 = -N^2 dt^2 + g_{\phi}(d\phi - \omega dt)^2 + \frac{dr^2}{A} + g_\theta d\theta^2$$  \hspace{1cm} (26)

in which the coefficients do not depend on $t$ and $\phi$. We assume that the function $\eta \equiv \sqrt{A}/N$ is finite on the horizon like in the Kerr-Newman metric. By definition of an extremal black hole, near the horizon $r_+$,

$$N \sim r - r_+.$$  \hspace{1cm} (27)

For equatorial motion, it follows from the geodesic equations that the proper time $\tau$ required for a particle to travel from $r_0$ to $r < r_0$ towards the horizon equals

$$\tau = \int_{r_0}^r \frac{m \eta \, dr}{\sqrt{(E - \omega \lambda)^2 - N^2(m^2 + L^2/\eta)}}.$$  \hspace{1cm} (28)

Here, $E$ is the particle energy, $\lambda$ being its angular momentum, $m$ mass.

The essential feature of the Bañados-Silk-West effect is that one of particles has $E - \omega_H \lambda = 0$, where $\omega_H$ is the value of $\omega$ on the horizon. Such a particle is called critical. For small $N$ we have $E - \omega \lambda = O(N)$. Taking into account [23], we see that $\tau \sim |\ln(r - r_+)|$ diverges [4, 23, 24]. It means that a fine-tuned particle never reaches the horizon, so collision never occurs exactly on the horizon. It can happen close to it. Then, $E_{c.m.}$ is large (even unbounded) but finite in each concrete collision. The same situation happens for collision of radially moving particles in the background of the Reissner-Nordström metric [25].

6. Nonextremal black holes, collisions outside

Now, there are no trajectories with infinite $\tau$. However, another difficulty comes into play. The allowed region of motion is characterized by $V_{\text{eff}} \leq 0$ (see Fig. 1). Here, the effective potential $V_{\text{eff}}$ is defined according to (see [26] for details)

$$\left(\frac{dr}{d\tau}\right)^2 + V_{\text{eff}} = 0.$$  \hspace{1cm} (29)

The critical particle cannot approach the horizon at all because of the potential barrier. One can let parameters of a particle to differ slightly from those of the critical one. Then, such a near-critical particle can approach the horizon and collide there with some another one. In such a case $E_{c.m.} \sim 1/\sqrt{\delta}$ where $\delta$ is the parameter that controls the deviation from the exact critical relationship [5]. Thus, $E_{c.m.}$ can be made as large as one likes but not infinite. And, collision of interest can occur within a narrow strip near the horizon only [5, 23]:

$$0 \leq N \leq N_0,$$  \hspace{1cm} (30)
where $N_0 \sim \delta$. For the Kerr metric (see Fig. 1) $r_+ \leq r \leq r_\delta$. When $\delta \to 0$, the region with high energy outcome degenerates into the point, so collision between two particles becomes impossible.

7. Nonextremal black holes, collisions inside

Now, both potential barrier and infinite proper time are not encountered in the problem. Formally, when a particle approaches the inner horizon $r_-$, the c.m. energy $E_{c.m.}$ is unreachable in a separate act of collision. But this would mean formation of the horizon, so KC becomes impossible.

$$\lim_{r \to r_-} E_{c.m.}(r) \to \infty. \quad (31)$$

The example from the present Section is especially instructive since it demonstrates the predictive power of the principle under discussion. Several years ago, an intensive discussion developed in literature concerning the possibility of the analogue of the BSW effect inside a black hole. Firstly, it was claimed in [27] that such an analogue does exist. Later on, the author himself refuted this result [28], independently this was done in [29]. More weak version of this effect revived in [30]. Another examples of the same kind (with predictions of infinite $E_{c.m.}$) were suggested in [31] for the nonextremal Kerr metric and in [32] for the cosmological horizon. These results are incorrect, as is explained in [30]. The reason consists in that relevant trajectories (that otherwise would have given infinite $E_{c.m.}$) do not intersect on the horizon.

Meanwhile, such scenarios can be rejected at once only due to contradiction to our KC. This principle is able to reject the scenario described in [27, 31, 32] even without elucidating these details (which can be found in aforementioned references). Indeed, (i) the result [31] is always formally valid for any two particles, (ii) Eq. (31) implies that an infinite $E_{c.m.}$ is reachable in a separate act of collision. But this is what our principle forbids!

8. Scalar field and infinite acceleration

Let a black hole be surrounded by a scalar field and one of colliding particles is minimally coupled to this field. The corresponding action describing interaction of a scalar particle with the background scalar field is described by Eq. (11). However, now we change the sign of the particle’s scalar charge $q$. Then, there are no divergences for acceleration outside the horizon. But, instead, another phenomenon comes into play. It is connected with the immediate vicinity of the horizon. Let the scalar field diverges near the horizon like $\varphi \sim N^{-\beta}$. If $\beta < 1$, the proper time of traveling to to the horizon is indeed finite [33]. Meanwhile, if one calculates the absolute value of the four-acceleration experiences by a particle under action of the scalar field, $a \sim N^2(\beta - 1) \to \infty$ near the horizon [33]. The gradient of $a$ diverges as well. Any particle having small but nonzero size will be teared to pieces, so collision on the horizon becomes impossible.

9. From black holes to naked singularities

Up to now we considered collisions in the background of black holes that is the main subject of our work since it is this case when KC is nontrivial. Meanwhile, we would like to make a short comment how KC reveals itself for naked singularities. Although this case is much more simple, it is quite instructive.

It was noticed in [34] that naked singularities can serve as accelerators in the two-step scenario. One of particles moves from infinity, reflects from the potential barrier and collides with the second particle falling from infinity in the point where $N$ is very small, i.e. on the verge of forming the horizon (but it does not form). Then, $E_{c.m.}^2 \sim N_c^{-2}$, where $N_c$ is the value of $N$ in the point of collision. By its very meaning, $N_c$ cannot reach the value $N_c = 0$ since this would mean formation of the horizon, so KC holds true.

10. Conclusion

We considered several completely different examples in which it seemed to be "obvious" that $E_{c.m.}$ could be infinite. However, we saw that in each of the examples, some "hidden" factor reveals itself which acts to prevent $E_{c.m.}$ from being infinite. The nature of these factors is quite different and depends strongly on the situation. This indeed points to the validity of some fundamental underlying principle called by us "principle of kinematic censorship". From a more practical point of view, this principle, by itself, cannot suggest some new scenarios of high energy collisions. Rather, it can serve as some constraint that enables us to separate forbidden scenarios from physically possible ones and reveal the
factors (sometimes hidden) that make \( E_{c.m.} \) impossible.

Thus we formulated some new principle, suggested arguments in its favour and checked it on concrete examples. There are two qualitative lessons following from our analysis: (i) there is a crucial difference between infinite and unbounded \( E_{c.m.} \) (finite in each act of collision), (ii) the impossibility of infinite \( E_{c.m.} \) is inherent to any scenario in the test particle approximation. There is no need to refer to some not quite understandable factors which will be taken into account in future investigations of non-linear regimes!

In addition to results concerning the properties of collisions, we gave proof for a curved background that under the action of a finite force, a trajectory of a massive particle cannot become light-like. In other words, a massive particle cannot turn into a massless one. This can be of some interest on its own in other contexts [35].

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