Hadronic production of baryons, containing two heavy quarks
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Abstract

In the framework of QCD perturbation theory, total and differential cross sections of the Ξ°_{bc}, Ξ_{bc}^{(*)} and Ξ^{(*)}_{cc} baryons production in gluon collisions are calculated in the leading order over \( \alpha_s \) for the doubly heavy (bc) and (cc) diquarks. At both small and large transverse momenta of the baryons, a use of the mechanism of the heavy quark fragmentation into the heavy diquark is shown to underestimate the cross section values in comparison with the exact numerical calculations of complete set of diagrams. The expected in Tevatron experiments yield of baryons with two heavy quarks is evaluated as \((1.3 \pm 0.3) \cdot 10^5 bcq\)-baryons and \((1.6 \pm 0.3) \cdot 10^4 ccq\)-baryons at \( p_T > 5 \) GeV and \(|y| < 1 \) of the baryon momentum and rapidity cuts, with account for the antiparticle yields.

1 Introduction

One of new directions in the heavy quark physics is related with a description of baryons with two heavy quarks \([1]\). In a large heavy quark mass limit, a typical size of a \( Q_1Q_2 \)-diquark configuration is much less than the characteristic radius of light hadrons \( r_{Q_1Q_2} \ll \Lambda^{-1} \). For light quarks, the heavy-heavy diquark object looks like a heavy antiquark at low virtualities, so that one can relate the \( Q_1Q_2q \)-baryon characteristics with the properties of a meson with a single heavy quark \([2]\). The form factors of the doubly heavy baryons are straightforwardly related with those of heavy light mesons \([3]\), described by the Isgur-Wise functions, up to the accuracy of a probability for the transition of the doubly heavy diquark to another one. The latter point of view to the \( Q_1Q_2q \) baryon production \([4]\).

First, the process under consideration can be represented as the process of a hard production of the \( Q_1Q_2 \) diquark, that further has a hadronization into the \( Q_1Q_2q \) baryon. The hadronic production of the diquark with a mixed flavor is calculated under the analogy with the \( Q_1Q_2 \) heavy quarkonium production, on the basis of computations of the fourth \( \alpha_s \)-order diagrams in the QCD perturbation theory. Second, the nonperturbative soft part of the matrix element describes the quark binding in the \( Q_1Q_2 \) diquark and it is given by the diquark wave function at the origin.

At present, the basis for the current estimates of the \( Q_1Q_2q \) baryon production cross section is the consideration of the \( Q_1Q_2 \) diquark production cross sections at large transverse momenta via the fragmentation mechanism \([4]\). The fragmentation function of the heavy quark \( Q_1 \rightarrow (Q_1Q_2) + Q_2 \) is taken in the same form as the function of fragmentation into the heavy quarkonium \( Q_1Q_2 \) \([5, 6]\) with the same quantum numbers over the Lorentz...
group. This approach is quite exclusive and valid in the high energy \( e^+e^- \)-annihilation, where the fragmentational mechanism does practically dominate [6], but the situation is more complex for the hadronic production [7]. The complication is caused by that a number of the leading order diagrams, describing the hadronic production of doubly heavy diquark, is much greater than the number of diagrams in the \( e^+e^- \)-interaction. Moreover, the former diagrams do not allow one completely to interpret them in terms of the fragmentational mechanism. The fragmentation is not exactly defined in the hadronic production. As was shown [7], the diagrams of the recombinational type give a dominant contribution even at quite large transverse momenta. So, a simple formula, corresponding to the convolution of the differential cross section of the heavy quark production with the function of the heavy quark fragmentation into the \( Q_1\bar{Q}_2 \) quarkonium or \( Q_1Q_2 \) diquark, gives only a cross section evaluation in the order of magnitude. To obtain an exact result is necessary to take into account all 36 diagrams of the fourth order over \( \alpha_s \).

In what follows, we present the results of exact calculations for the \( bcq \) and \( ccq \) baryons yields in the framework of the computations for the heavy quark pair production in the fourth order diagrams of QCD. The difference between the given approach and the fragmentational one is connected with that the contribution of the complete set of diagrams is calculated with no neglect of a part of them. For the states with a various quantum numbers, the performed calculations result in the yield ratios, different from those of in the approximate approach of the fragmentation.

## 2 Calculation technique

The calculation technique, applied in the present paper, is analogous to that of the hadronic production of \( B_c \) mesons [6]. The only difference is due to the binding of two heavy quarks (\( Q_1 \) and \( Q_2 \)) in contrast to the binding of a heavy quark with a heavy antiquark.

We suppose that the binding energy in the diquark is much less than the masses of quarks, composing the diquark, and hence, the quarks are on the mass shells. Therefore, the quark four-momenta are related with the diquark momentum \( P \) in the following way

\[
p_1 = \frac{m_1}{M} P, \quad p_2 = \frac{m_2}{M} P,
\]

where \( M = m_1 + m_2 \) is the diquark mass, \( m_{1,2} \) are the quark masses.

In the given approach, the diquark production can be described by the 36 leading order Feynman diagrams, corresponding to the production of four free quarks in the way of combining of two quarks into the colour antitriplet diquark with the given quantum numbers over the Lorentz group. The latter procedure is performed by means of the projection operators

\[
\frac{1}{\sqrt{2}} \{ \bar{u}_1(+)\bar{u}_2(-) - \bar{u}_1(-)\bar{u}_2(+) \}
\]

for the scalar state of diquark (the corresponding baryon is denoted as \( \Xi'_{12}(J = 1/2) \)),

\[
\frac{1}{\sqrt{2}} \{ \bar{u}_1(+)\bar{u}_2(+) \},
\]

\[
\frac{1}{\sqrt{2}} \{ \bar{u}_1(+)\bar{u}_2(-) + \bar{u}_1(-)\bar{u}_2(+) \},
\]

\[
\bar{u}_1(-)\bar{u}_2(-)
\]

(3)
for the vector state of diquark (the baryons are denoted as $\Xi_{12}(J = 1/2)$ and $\Xi^*_{12}(J = 3/2)$).

To produce the quarks, composing the diquark, in the $\bar{3}_c$-state, one has to introduce the colour wave function as $\varepsilon_{ijk}/\sqrt{2}$ into the diquark production vertex, so that $i = 1, 2, 3$ is the colour index of the first quark, $j$ is that of the second one, and $k$ is the colour index of the diquark.

The diquark production amplitude $A$ is expressed via the amplitude $A'$ for the free quark production in kinematics (4) as

$$A = \frac{\sqrt{2M}}{\sqrt{2m_1}\sqrt{2m_2}4\pi} \sum_{h_1,h_2,i,j} P(h_1, h_2) A'(h_1, h_2, i, j) \frac{\varepsilon_{ijk}}{\sqrt{2}},$$

where $h_1, h_2$ are the spiralities of corresponding quarks, $i, j$ are its colour indices, $R(0)$ is the diquark radial wave function at the origin, and the $P(h_1, h_2)$ operators have the following explicit form ($H = h_1 + h_2$)

$$P(h_1, h_2) = \frac{1}{\sqrt{2}} (-1)^ {h_1 - \frac{1}{2}} \delta_{H0}$$

for the scalar state, and

$$P(h_1, h_2) = |H| + \frac{1}{2} \delta_{H0}$$

for the vector one.

In the numerical calculations, giving results, which will be discussed in the next section, we suppose the following values of parameters

$$\alpha_s = 0.2,$$
$$m_b = 4.9 \text{ GeV},$$
$$m_c = 1.7 \text{ GeV},$$
$$R_{bc(1S)}(0) = 0.714 \text{ GeV}^{3/2},$$
$$R_{cc(1S)}(0) = 0.263 \text{ GeV}^{3/2},$$

where the $R_{bc}(0)$ value has been calculated by means of a numerical solution of the Schrödinger equation with the Martin potential [8], multiplied by the 1/2 factor, caused by the colour antitriplet state of quarks instead of singlet one, and the $R_{cc}(0)$ value is taken from ref.[4] for the sake of convenience of the result comparison.

To calculate the production cross section of the diquarks, composed of two $c$-quarks, one has to account for their identity. One can easily find, that the antisymmetrization over the identical fermions leads to the scalar diquark production amplitude, equal to zero, and it results in that the amplitude of the vector $cc$-diquark production can be obtained by the substitution of equal masses in the amplitude of the vector $bc$-diquark production with the account for the 1/2 factor, following from the identity of quarks and antiquarks.

We assume, that the produced diquark has the fragmentation into the baryon, practically carrying away a total diquark momentum, with the unit probability. Therefore, discussing the results, we will talk on the differential cross sections of $\Xi'_{bc}$, $\Xi^{(*)}_{bc}$ and $\Xi^{(*)}_{cc}$ baryons production, since we suppose that the latters negligibly small deviate from the differential cross sections of diquarks.
3 Discussion

The total energy dependence of the gluonic production cross sections of $\Xi^{(*)}_{bc}$ (○) and $\Xi^{(*)}_{bc}$ (●) baryons is shown on Fig.1 and in Tab.1. To compare, the predictions of the fragmentational mechanism for $\Xi^{(*)}_{bc}$ (solid line) and $\Xi^{'}_{bc}$ (dashed line) are also presented. One can see from the figure, that the fragmentational production mechanism, assuming the validity of factorization in the cross section at $M^2/s \ll 1$ and at large transverse momenta via the formula

$$
\frac{d\sigma_{gg\rightarrow \Xi^{(*)}_{bc}}} {dz} = \sigma_{gg\rightarrow b\bar{b}} \cdot D_{b\rightarrow \Xi^{(*)}_{bc}}(z),
$$

with $z = 2|\vec{p}|/\sqrt{s}$, does not work at low gluon energies, where it overestimates the cross section, because of the incorrect evaluation of the phase space, and it is not valid also at large energies, where the predictions of the fragmentational mechanism are essentially less that the exact results. So, the fragmentational values underestimate the $\Xi^{(*)}_{bc}$ and $\Xi^{'}_{bc}$ cross sections in 9 and 4 times, respectively, at $\sqrt{s} = 100 \text{ GeV}$. When the fragmentational predictions give the ratio $\sigma_{\Xi^{(*)}_{bc}}/\sigma_{\Xi^{'}_{bc}} \simeq 1.4$, the exact perturbative calculations result in $\sigma_{\Xi^{(*)}_{bc}}/\sigma_{\Xi^{'}_{bc}} \simeq 3.2$ even at $\sqrt{s} = 100 \text{ GeV}$.

The agreement with the fragmentational production at $\sqrt{s} = 100 \text{ GeV}$ is poor even at large transverse momenta of the baryon, as one can see from the distributions over $p_T$ for the $\Xi^{(*)}_{bc}$ and $\Xi^{'}_{bc}$ production, shown on Fig.2 in comparison with the predictions of the fragmentational mechanism. Note, that in contrast to the doubly heavy baryon production, the exact perturbative calculations of the gluonic production of $B_c(B^*_c)$ mesons with $p_T > 35 \text{ GeV}$ at $\sqrt{s} = 100 \text{ GeV}$ agree with the fragmentational predictions. For the baryon production, a visible deviation is observed up to the largest values of $p_T$.

The differential cross section $d\sigma/dp_T$ of the $\Xi^{'}_{bc}(\Xi^{(*)}_{bc})$ production in $p\bar{p}$ interactions at $\sqrt{s} = 1.8 \text{ TeV}$ is presented on Fig.3 in comparison with the fragmentational predictions. In the $B_c$ meson production, the difference between the exact and fragmentational approaches was slightly hidden due to the convolution with the hadron structure functions. In the process of baryon production, this is not the case, and the invalidity of the fragmentational approach explicitly follows from the form of the distribution under consideration.

The calculation results on the $\Xi^{(*)}_{cc}$ production point out that the deviation between the exact perturbative and fragmentational values is also essential as in the $\Xi^{(*)}_{bc}$ production. This disagreement can be noted on Fig.4 and in Tab.2, where the dependence of the $\Xi^{(*)}_{cc}$ production cross section on the energy of the gluon interactions is presented, as well as on Fig.5, where the differential cross section $d\sigma/dp_T$ of the $\Xi^{(*)}_{cc}$ production in the gluon interactions is shown at $\sqrt{s} = 100 \text{ GeV}$ in comparison with the fragmentational mechanism predictions. From the latter figure, one can see that even at $p_T > 40 \text{ GeV}$, the exact result slightly overestimates the fragmentation.

The differential cross section $d\sigma/dp_T$ of the $\Xi^{(*)}_{cc}$ production in $p\bar{p}$ interactions at the energy $\sqrt{s} = 1.8 \text{ TeV}$ is shown on Fig.6 in comparison with the fragmentational estimate. One can see, that at the reasonable values of $p_T$, the fragmentational result is approximately 3 times less than the exact perturbative one.
At the chosen values of parameters and with the account for the cuts over the transverse momentum and rapidity of the baryons ($p_T > 5$ GeV and $|y| < 1$), the production cross section of the $1S$-wave $bcq$-baryons and its antiparticles is evaluated as $\sigma_{bcq} \simeq 1$ nb, and the total cross section of the $1S$-wave $ccq$-baryon production with the account for antiparticles is equal to $\sigma_{ccq} \simeq 0.13$ nb. After the expected end of Run Ib at Tevatron with the integral luminosity $100 \div 150$ pb$^{-1}$, one has the yields of $1.0 \div 1.5 \cdot 10^5$ of $bcq$-baryons and $1.3 \div 1.9 \cdot 10^4$ of $ccq$-baryons.

Thus, we have shown, that the calculations in the leading order approximation of the QCD perturbation theory for the gluonic production of the doubly heavy diquarks, having the hadronization into the baryons, lead to the essential discrepancy in the differential as well as total cross sections of the baryon production in comparison with the predictions of the mechanism of the heavy quark fragmentation into the diquark.

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Table 1: The dependence of the gluonic production cross sections for the $\Xi_{bc}'$ and $\Xi_{bc}^{(*)}$ baryons on the total energy. (The error on last digits stands in brackets.)

| $\sqrt{s}$, GeV | $\sigma_{\Xi_{bc}'}$, pb | $\sigma_{\Xi_{bc}^{(*)}}$, pb |
|-----------------|-------------------------|-------------------------|
| 15.             | 0.4395(9)               | 1.091(3)                |
| 20.             | 1.487(3)                | 5.96(2)                 |
| 30.             | 2.06(1)                 | 8.47(4)                 |
| 40.             | 2.043(18)               | 7.95(6)                 |
| 60.             | 1.63(3)                 | 5.87(9)                 |
| 80.             | 1.25(4)                 | 4.32(11)                |
| 100.            | 0.98(5)                 | 3.31(13)                |

Table 2: The dependence of the gluonic production cross sections for the $\Xi_{cc}^{(*)}$ baryons on the total energy. (The error on last digits stands in brackets.)

| $\sqrt{s}$, GeV | $\sigma_{\Xi_{cc}^{(*)}}$, pb |
|-----------------|-------------------------------|
| 15.             | 3.18(2)                       |
| 20.             | 3.26(3)                       |
| 40.             | 1.97(4)                       |
| 60.             | 1.23(5)                       |
| 80.             | 0.85(7)                       |
| 100.            | 0.68(5)                       |
Figure captions

Fig. 1. The gluonic production cross sections of $\Xi_{bc}^{(s)}$ (○) and $\Xi_{bc}^{(*)}$ (●) in comparison with the predictions of the fragmentational mechanism for $\Xi_{bc}^{(')}$ (dashed line) and $\Xi_{bc}^{(*)}$ (solid line).

Fig. 2. The distributions over the transverse momentum in the gluonic production of $\Xi_{bc}^{(')}$ ($\Xi_{bc}^{(*)}$) in comparison with the fragmentation result at the interaction energy 100 GeV. Here and in what follows, the solid and dashed lines correspond to the $\Xi_{bc}^{(')}$ and $\Xi_{bc}^{(*)}$ production, respectively, and the exact perturbative and fragmentational results are shown as histograms and smooth curves, respectively.

Fig. 3. The differential cross section $d\sigma/dp_T$ of the $\Xi_{bc}^{(')}$ ($\Xi_{bc}^{(*)}$) production in $p\bar{p}$ collisions versus the transverse momentum of the $\Xi_{bc}^{(')}$ ($\Xi_{bc}^{(*)}$)-baryon at the hadron interaction energy 1.8 TeV, in comparison with the fragmentation result.

Fig. 4. The gluonic production cross section of $\Xi_{cc}^{(*)}$ (●) in comparison with the predictions of the fragmentational mechanism (solid line).

Fig. 5. The distribution over the transverse momentum in the gluonic production of $\Xi_{cc}^{(*)}$ in comparison with the fragmentation result at the interaction energy 100 GeV.

Fig. 6. The differential cross section $d\sigma/dp_T$ of the $\Xi_{cc}^{(*)}$ production in $p\bar{p}$ collisions versus the transverse momentum of the $\Xi_{cc}^{(*)}$-baryon at the hadron interaction energy 1.8 TeV, in comparison with the fragmentation result.

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