The Generalized Counting Rule and Oscillatory Scaling

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We have studied the energy dependence of the $pp$ elastic scattering data and the pion-photoproduction data at 90° c.m. angle in light of the new generalized counting rule derived for exclusive processes. We show that by including the helicity-nonconserving amplitudes and their interference with the Landshoff amplitude, we are able to reproduce the energy dependence of all the $pp$ elastic cross-section and spin-correlation ($A_{NN}$) data available above the resonance region. The pion-photoproduction data can also be described by this approach, however, data with much finer energy spacing is needed to confirm the oscillations about the scaling behavior. This study strongly suggests an important role for helicity-nonconserving amplitudes related to quark orbital angular momentum and for the interference of these amplitudes with the Landshoff amplitude at GeV energies.

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The transition between perturbative and non-perturbative regimes of Quantum Chromo Dynamics (QCD) is of long-standing interest in nuclear and particle physics. Exclusive processes play a central role in studies trying to map out this transition. The differential cross sections for many exclusive reactions [1] at high energies and large momentum transfers appear to obey dimensional scaling laws [2] (also called quark counting rules). In recent years, the onset of this scaling behavior and Brodsky [8], they could break hadron helicity conservation in exclusive processes [9], and experimental data in similar energy and momentum regions tend not to agree with these helicity conservation selection rules [10]. Although contributions from non-zero parton orbital angular momenta are power suppressed, as shown by Lepage and Brodsky [3], they could break hadron helicity conservation rule [11]. Interestingly recent re-analysis of quark orbital angular momenta seems to contradict the notion of power suppression [12]. Furthermore, Ref [13] argues that non-perturbative processes could still be important in some kinematic regions even at high energies. Thus the transition between the perturbative and non-perturbative regimes remains obscure and makes it essential to understand the exact mechanism governing the early onset of scaling behavior.

Towards this goal, it is important to look closely at claims of agreement between the differential cross section data and the quark counting rule prediction. Deviations from the quark counting rules have been found in exclusive reactions such as elastic proton-proton ($pp$) scattering [14]. In fact, the re-scaled 90° center-of-mass $pp$ elastic scattering data, $s^{10} \frac{d\sigma}{ds}$ show substantial oscillations about the power law behavior. Oscillations are not restricted to the $pp$ elastic scattering channel: they are seen in elastic $\pi p$ fixed angle scattering [14] and hints of oscillation about the $s^{-7}$ scaling have also been reported in the recent data [5] from Jefferson Lab (JLab) on photo-pion production above the resonance region. In addition to violations of the scaling laws, spin correlations in polarized $pp$ elastic scattering also show significant deviations from perturbative QCD (pQCD) expectations [15, 16]. Several sets of arguments have been put forward to account for these deviations from scaling laws and the unexpected spin correlations. Brodsky and de Teramond [17] explain the $pp$ scattering data in terms of the opening up of the charm channel and excitation of $c\bar{c}u\bar{c}u\bar{u}d\bar{d}$ resonant states. Alternatively the deviations are said to be an outcome of the interference between the pQCD (short distance) and the long distance Landshoff amplitude (arising from multiple independent scattering between quark pairs in different hadrons) [18]. Gluonic radiative corrections to the Landshoff amplitude give rise to an energy dependent phase [19] and thus the energy dependent oscillation. Carlson, Chachkhunashvili, and Myhrer [20] have also applied a similar interference concept to explain the $pp$ polarization data. The QCD re-scattering calculation of the deuteron photo-disintegration process by Frankfurt, Miller, Sargsian and Strikman [21] predicts that the additional energy dependence of the differential cross-section, beyond the $s^{10} \propto s^{-11}$ scaling, arises primarily from the $n−p$ scattering in the final state. In this scenario the oscillations may arise due to QCD final state interaction. If these predictions are correct, such oscillatory behavior may be a general feature of high energy exclusive photo-reactions.

Recently, a number of new developments have generated renewed interest in this topic. Zhao and Close [22] have argued that a breakdown in the locality of quark-hadron duality (dubbed as “restricted locality” of quark-hadron duality) results in oscillations around the scaling curves predicted by the counting rule. They explain that the smooth behavior of the scaling laws arise due
to destructive interference between various intermediate resonance states in exclusive processes at high energies. However, at lower energies this cancellation due to destructive interference breaks down locally and gives rise to oscillations about the smooth behavior. On the other hand, Ji et al. [23] have derived a generalized counting rule based on a pQCD inspired model, by systematically enumerating the Fock components of a hadronic light-cone wave function. Their generalized counting rule for hard exclusive processes include parton orbital angular momentum and hadron helicity flip, thus they provide the scaling behavior of the helicity flipping amplitudes. The interference between the different helicity flip and non-flip amplitudes offers a new mechanism to explain the oscillations in the scaling cross-sections.

Moreover, pQCD calculations of the nucleon formfactors including quark orbital angular momentum [24, 25] and those computed from light-front hadron dynamics [26] both seem to explain the $\frac{1}{Q^4}$ fall-off of the proton form-factor ratio, $G_E^p(Q^2)/G_M^p(Q^2)$, measured recently at JLab in polarization transfer experiments [27].

In this letter we examine the role of the helicity flipping amplitudes in the oscillatory scaling behavior of $pp$ scattering and charged photo-pion production from nucleons and the oscillations in the spin correlations observed in polarized $pp$ scattering. We have used the generalized counting rule of Ji et al. [23] to obtain the scaling behavior of the helicity flipping amplitudes.

It is well known that $pp$ scattering can be described by five independent helicity amplitudes [28]. According to the dimensionless as well as the generalized counting rules the three helicity-conserving amplitudes, $M(+, +; +, +), M(+, -; +, -)$ and $M(-, +; +, -)$, have an energy dependence of $\sim 1/s^4$. On the other hand, the simple constituent quark interchange models [26] assume the two helicity flipping (nonconserving) amplitudes, $M(+, +; +, -)(NC1)$ and $M(-, -; +, +) (NC2)$ to be zero. Later analysis by Lepage and Brodsky [8] have shown these amplitudes to be non-zero but power suppressed. The new generalized counting rule predicts their energy dependence to be $\sim 1/s^{4.5}$ and $\sim 1/s^5$ respectively [29]. Thus the generalized counting rule which includes the helicity flipping amplitudes and the interference between them, gives rise to additional energy dependence beyond the $s^{-10}$ scaling predicted by dimensional scaling.

In addition to these short distance amplitudes, Landshoff [30] has shown that there can be contributions from three successive on-shell quark-quark scattering. Although each scattering process is itself a short distance process, different independent scatterings can be far apart, limited only by the hadron size. The Landshoff amplitude also carries energy dependent phase arising from gluonic radiative corrections which are calculable in pQCD [14] and has a known energy dependence, similar to the renormalization-group evolution: $\phi(s) = \frac{\pi}{\ln s/\Lambda_{QCD}^2}$. This effect is believed to be analogous to the Coulomb-nuclear interference that is observed in low-energy charged-particle scattering. It has been shown that this energy dependence of the phase occurs at medium energies [31] and becomes independent of energy at asymptotically high energies [31, 32]. In Ref. [13], Ralston and Pire have used the helicity-conserving amplitudes, the Landshoff amplitude with an energy dependent phase and the interference between them to reproduce the oscillations in the $pp$ scattering data at 90° c.m. angle (a similar method was used by Carlson et. al [21] to describe oscillation in the cross-section as well as the spin-correlation). They write the two amplitudes as $M = M_S + e^{i\phi(s) + i\delta} M_L$, where $M_S \sim 1/s^4$ represents the three helicity-conserving short distance amplitudes, $M_L \sim 1/s^3.5$ is the Landshoff amplitude and $\phi(s)$ is the energy dependent phase, $\delta$ is an arbitrary energy independent phase. By fitting to the existing $pp$ scattering data at 90° c.m. angle, they find that the ratio of $M_L$ to $M_S$ is 1:0.04 for an energy dependent phase given by $\phi(s) = \frac{\pi}{\ln s/\Lambda_{QCD}^2}$, where $\Lambda_{QCD} = 100$ MeV. It has been argued that the asymptotic leading limit used to calculate this energy dependence phase of the Landshoff amplitude is not entirely valid [33] and thus the Landshoff term is better parametrized as,

$$M_L = b_j s^{-3.5} e^{ic_j[i\ln(s/\Lambda_{QCD})] + id_j} \frac{[\log(s)]^{d_j}}{[\log(s)]^{d_j}} ,$$

(1)

where $b_j, c_j, d_j$ and the energy independent phase $d_j$ are now parameters which are not exactly calculable. Fig. [11] shows the fit of Ref. [13] compared to the world data, and Fig. [14] is a fit using the more general parametrization of the Landshoff described above. Both these fits deviate drastically from the data at $s < 10$ GeV$^2$ and are not sensitive to the different parameterizations of the Landshoff amplitude. Since the Landshoff amplitude is expected to be significant only at high energies, it is not unreasonable that the above formalism does not describe the data at low energies.

As the interference between the Landshoff and the short distance amplitudes fail to describe the data at low energies, it is possible that the helicity flip amplitudes and their interference may play an important role at these energies. The helicity flip amplitudes arising from the parton orbital angular momentum are non-negligible when the parton transverse momentum can not be ne-
forms for the phase $\phi$ where $\phi$ be written as; combined as one amplitude and the two helicity flipping
activities to them. The three helicity-conserving amplitudes
above, were employed in the fits to examine their sensi-
dence of the phase in the Landshoff amplitude, described
\[23\]. The two different forms for the energy depen-
dal.\[23\] predicts a much faster fall-off with energy for the helicity
amplitudes according to the generalized counting rule of Ji
\[23\], this fit had two parameters; the overall
normalization $A_1$ and the arbitrary phase $\delta$. (b) The same
data fitted with the new more general parametrization of the
Landshoff amplitude, this fit includes the 3 additional pa-
rameters $b_1, c_1$ and $d_1$ mentioned in Eq. 1. The data are from
Ref. \[12\]

We have neglected the helicity flipping Landshoff contri-
glected compared with the typical momentum scale in
the exclusive processes at relatively low energies. Thus
one would expect the helicity flip amplitudes to be a sig-
nificant contribution to the cross-section at low energies.
Moreover, the generalized counting rule of Ji et al. \[23\]
predicts a much faster fall-off with energy for the helicity
flip amplitudes as expected. We have refitted the world
data by including the two helicity-nonconserving ampli-
tudes according to the generalized counting rule of Ji et
al. \[23\]. The two different forms for the energy depen-
dence of the phase in the Landshoff amplitude, described
above, were employed in the fits to examine their sensi-
tivity to them. The three helicity-conserving amplitudes
combined as one amplitude and the two helicity flipping
amplitudes, along with the Landshoff contributions, can
be written as;

$$
M_{HC} = s^{-4}(a_1 + b_1 s^{0.5} e^{i\phi_1(s)})
$$

$$
M_{NC1} = s^{-4}(a_2 s^{-0.5} + b_2 s^{0.5} e^{i\phi_2(s)})
$$

$$
M_{NC2} = s^{-4}(a_3 s^{-1} + b_3 s^{0.5} e^{i\phi_3(s)}),
$$

where $\phi_j(s)$ is the energy dependent phase. Two different
forms for the phase $\phi_j(s)$ were used in our fits; $\phi_j(s) = \frac{\pi}{0.06} \ln(n(s/\Lambda_{QCD}^2)) + \delta_j$ and $\phi_j(s) = c_j \frac{\ln(n(s/\Lambda_{QCD}^2)) + \delta_j}{(\log(s))^{d_j}}$.

We have neglected the helicity flipping Landshoff contrib-
utions. The scaled cross-section is then given by,

$$
R = s^{10} \frac{d\sigma}{dt} \propto |M_{HC}|^2 + 4|M_{NC1}|^2 + M_{NC2}|^2,
$$

The factor of four associated with the $NC1$ helicity flip-
ing amplitude arises because of the two possible config-
urations of this single spin flip amplitude \[23\].

Fig 2 shows the results of our fit and also shows the
explicit contributions from the $s^{-11}$ and $s^{-12}$ term for
this approach. The value of $\Lambda_{QCD}$ was fixed at 100 MeV
for all fits. This new fit is in much better agreement with
the data. The helicity flip amplitudes (mostly the term
$\sim s^{-4.5}$) are significant at low energies and seem to help
in describing the data at low energies. It is interesting
to note that among the helicity flip amplitudes the one
with the lower angular momentum dominates. These are
very promising results and should be examined for other
reactions.

As mentioned earlier the $A_{NN}$ spin-correlation in
polarized $pp$ elastic scattering also shows large devia-
tions \[16\] from the expectations of pQCD (assuming
hadron helicity is conserved). In terms of the helicity amplitudes \( A_{NN} \) is given by \( \text{[20]} \):

\[
RA_{NN} = 2\text{Re}[M^*(++;++)(---;++)] \\
+ 2\text{Re}[M^*(---;++)(-++;++)] \\
+ 4|M(---;++)|^2,
\]

(4)

where \( R \) has been defined in Eq. \( \text{[8]} \). At \( \theta_{cm} = 90^\circ \) the ratio of the three helicity non-flip amplitudes is \( 2:1:1 \) \( \text{[20]} \). Taking this into account we have fit the \( A_{NN} \) data by including the helicity flipping amplitudes. Fig. \( \text{[3]} \) shows the results for the case where the helicity flip amplitude is neglected and only the interference between short distance amplitude and the Landshoff amplitude is used (in this case the expression for \( A_{NN} \) simplifies to \( RA_{NN} = 2\text{Re}[M^*(---;++)(-++;++)] \)). These results are similar to those obtained by Carlson et. al \( \text{[20]} \) and they described the \( A_{NN} \) data at high energies but fail to describe the low energy data using this idea of interference between short distance and Landshoff terms. Fig. \( \text{[3]} \) shows the results of our fit when the helicity flipping amplitudes are included. It is clear that this method is a better fit to a large fraction of the data which includes some low energy data. This suggests that even in case of the spin correlation \( A_{NN} \) in polarized \( pp \) elastic scattering the helicity flip amplitudes play an important role at low energies (\( s < 10 \text{ GeV}^2 \)).

![Figure 3](image)

**FIG. 3:** (a) The fit to \( A_{NN} \) from polarized \( pp \) scattering data at \( \theta_{cm} = 90^\circ \) with the helicity non-flip and Landshoff amplitudes only. (b) Fit to the same data when the helicity flip amplitudes are included. The data are from Ref. \( \text{[12] [16]} \). The solid line is the fit and the dashed line is the expectation assuming hadron helicity conservation.

Recently some precision data on pion-photoproduction from nucleons above the resonance region has become available from JLab \( \text{[5]} \). These data show hints of oscillation about the \( s^{-7} \) scaling predicted by the quark counting rule. In pion-photoproduction from nucleons the helicity non-flip amplitudes has an energy dependence of \( s^{-2.5} \), and there is just one helicity flip amplitude which according to the generalized counting rule has an energy dependence of \( s^{-3} \) \( \text{[23]} \). There are no leading order Landshoff terms in pion-photoproduction since the initial state has a single hadron. However, the Landshoff process can contribute at sub-leading order \( \text{[34]} \) (i.e. \( s^{-3} \) instead of \( s^{-2} \)). In principle, the fluctuation of a photon into a \( q\bar{q} \) in the initial state can contribute an independent scattering amplitude at sub-leading order. But, experimentally it has been shown that vector-meson dominance diffractive mechanism is suppressed in vector meson photoproduction at large values of \( t \) \( \text{[32]} \). On the other hand such independent scattering amplitude can contribute in the final state if more than one hadron exist in the final state, as is the case in nucleon photo-pion production reactions. Thus an unambiguous confirmation of such an oscillatory behavior in exclusive photoreactions with hadrons in the final state at large \( t \) may provide a signature of QCD final state interaction.

We have fit the pion-photoproduction data at \( \theta_{cm} = 90^\circ \) including the helicity flip amplitude and the Landshoff amplitude at sub leading order with an energy dependent phase. The Landshoff amplitude was parametrized according to the ansatz given in Ref. \( \text{[33]} \). The amplitudes for \( \gamma p \rightarrow \pi^+ n \) and \( \gamma n \rightarrow \pi^- p \) and the respective Landshoff contribution to each amplitude can be written as;

\[
M_{HC} = s^{-2.5}(a_1 + b_1 s^{-0.5}e^{ic_1\phi(s)+i\delta_1}) \left(\log(s)^d_1\right) \\
M_{NC1} = s^{-2.5}(a_2 s^{-0.5} + b_2 s^{-0.5}e^{ic_2\phi(s)+i\delta_2}) \left(\log(s)^d_2\right),
\]

(5)

and the scaled cross-section is given by;

\[
s^2 \frac{d\sigma}{ds} \propto |M_{HC}|^2 + |M_{NC1}|^2,\text{ where } \phi(s) = lnln(s/A^2).
\]

As seen in Fig \( \text{[4]} \) the existing data can be fit quite well with this form. However, the data are rather coarse distributed in energy and so these results are not a conclusive evidence for oscillations in pion-photoproduction. This underscores the point that a fine scan of energies above the resonance region is urgently needed. This is exactly the issue that will be addressed in the JLab experiment E02010 \( \text{[33]} \) in the near future.

We have shown that the generalized counting rule of Ji et al. \( \text{[28]} \) along with the Landshoff terms and associated interferences does a better job of describing the oscillations about the quark counting rule, in the \( pp \) elastic scattering data at \( \theta_{cm} = 90^\circ \). This is specially true in the low energy region (\( s < 10 \text{ GeV}^2 \)). The contributions from helicity flipping amplitudes which are related to quark orbital angular momentum, seem to play an important role at these low energies, which is reasonable given that the quark orbital angular momentum is non-negligible compared to the momentum scale of the
scattering process. Similarly the spin-correlation $A_{NN}$ in polarized $pp$ elastic scattering data can be better described by including the helicity flipping amplitude along with the Landshoff amplitude and their interference. The photo-pion production data from nucleons at large angles can also be described similarly; however, because of the coarse energy spacing of the data, the results are not as illustrative. This points to the urgent need for more data on pion-photoproduction above the resonance region with finer energy spacing. We expect that our experiment at JLab which is approved for running will help address this need in the near future.

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