SPH simulation of surge waves generated by aerial and submarine landslides

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Abstract. Since the Indian Ocean Tsunami in 2004, there has been extensive research on tsunami modelling. Tsunami catastrophes were generally generated by earthquake fault plate or mass landslide. This article is focused on surge waves induced by mass landslides. Here we restrict to two dimensional study, in which a solid mass was sliding down over a sloping beach. Our approach is numerical simulation using the Smoothed Particle Hydrodynamics (SPH) method. The SPH method is a Lagrangian meshless method, commonly used to describe complex events. Here, the solid mass was modeled as a solid box with triangular-cross section. Its movement follows analytical solution derived by Watts in [1]. The SPH method was used to simulate surge waves induced by two types of landslides; aerial and submarine. Our results were validated using the experimental data of Heinrich [2]. It was shown that the resulting waves induced by aerial and submarine landslides as well as the solid box movement agree quite well with the experimental data. The Root Mean Square Error (RMSE) of free surface deformation in aerial simulation recorded at time $t = 0.6, 1.0, 1.5$ are $0.02053, 0.02342, 0.02221$, respectively and in submarine simulation recorded at time $t = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0$ are $0.02908, 0.04085, 0.03772, 0.03843, 0.03753, 0.02582$, respectively. Whereas motion of the solid box in submarine simulation has better accuracy than in aerial simulation with RMSE $0.00799$ and $0.03831$, respectively.

1. Introduction
Tsunami is natural phenomena that has potential to create a great damage in coastal areas. In recent years, the phenomena have caused a huge loss of human life and catastrophic damage. An example is the Indian Ocean Tsunami in 2004 that occured in Aceh Province, Sumatera-Indonesia. The tsunami has caused huge economic loss, great destruction to infrastructure and coastal area, and killed around 170,000 to 220,000 people. Further, around 40,000 homes were lost due to the natural disaster [3].

Since the Indian Ocean Tsunami in 2004, there has been extensive research on tsunami modelling. The phenomena attract researchers to get better understand to potential threat of tsunami. Normally, there are two tsunami generation mechanism, i.e. driven by tectonic earthquake fault plate or by mass landslide. The tsunami generated by landslide is the most recent tsunami event occured in Indonesia. The tsunami event is generated by Mount Anak Krakatau eruption and happened on 22 December 2018. According to official website of the national disaster agency of Indonesia, the tsunami at least killed 431 people, injured 7200 people, and severely damaged 1527 homes. This article presents a numerical simulation of a solid mass sliding down over an inclined beach, generating a surge wave. The mass is modeled
as a solid box with triangular-cross section and two types of simulations were conducted; aerial and submarine.

In literature, research on landslide-induced tsunami is extensive, those that involve experimental, analytical, and numerical study. One important experimental work was conducted by Heinrich in 1992 [2]. The experiments of aerial and submarine landslides were carried out in a wave tank with dimension 20 m x 0.55 m x 1.5 m at the Hydraulic National Laboratory of Chatou in France. In the experiment surge waves were generated by a sliding motion of a solid box with triangular-cross section over a plane beach with slopes 15° and 45°. This experimental data are often used as a benchmark test for mass landslides numerical models.

Theoretical studies on submarine landslide was done by Philip Watts [1, 4]. In his thesis in 1997 [1], he derived the equations of motion for a landslide mass. In deriving the equation, several forces considered are added mass, gravitational, buoyancy, dynamic friction, and drag force. Solution of the problem is a natural logarithm of trigonometric function. In this article, we adopt the equations of landslide motion used in our model.

In recent years, there was a growing interests on researches of landslide motions that induce tsunami. Extensive researches in numerical scheme are focused on simulating free surface deformation due to landslide motion. These kind of simulations requires numerical schemes that can handle dry-wet procedure effectively. Basically, there are types of approaches; grid based and mesh less based method. Note that, simulating free surface deformation due to aerial landslide, is not an easy task. Most numerical grid based methods fail to do this.

Tjandra and Pudjaprasetya [5] implement the staggered conservative scheme to explain the submarine landslide numerically. In terms of meshless based method, one of them is Smothed Particle Hydrodynamics (SPH) method that will be used in this article.

Many authors focused their work on solving the problem using SPH method, see for instances [6, 7, 8, 9]. In those references, different ways was used to move the solid box. Qiu [9] used experimental data of Heinrich directly, Ashtiani and Shobeyri [7] used a prescribed piecewise linear velocity, whereas Lin et al. [6] calculate all forces acting on the solid box as the way to move it. In our study, we use analytical solution from Watts as described in [1]. Further, we use standard SPH as our previous works as described in [10, 11]. To validate our numerical results, comparison between numerical results and experimental data of Heinrich [2] is carried out.

2. The model of surge waves generated by landslides
In the SPH method, the fluid body is considered as a collection of fluid particles. The motion of these particles is governed by the following equations

\[
\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{u},
\]

\[
\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho} \nabla p + \mathbf{F},
\]

\[
\frac{D\mathbf{x}}{Dt} = \mathbf{u},
\]

where \(\mathbf{x}(t)\) denotes the position of particles and \(\mathbf{u}(t)\) the vector velocity. Parameters are: fluid density \(\rho\), pressure \(p\), and forces acting on the fluid is denoted as \(\mathbf{F}\). Moreover, \(\frac{D}{Dt}\) indicates the material derivative.

Several authors used different set of fluid forces, for instance Qiu in [9] accounts for gravitational force only, whereas Ashtiani et al. in [7] adds one more term namely the laminar kinematic viscosity term. Lin et al. as described in [6] considers forces acting on the fluid namely gravitational acceleration, laminar kinematic viscous, and sub-particle scale (SPS) turbulence stress. In this article, we only account for gravitational force, and therefore, in the following descriptions, notation \(\mathbf{F}\) is just replaced by \(g\).
2.1. Numerical approach

In this section we will give a short resume about the SPH method. This method was introduced in [12, 13] to solve astrophysical problems. In recent development, the method is used in many kinds of fluid flow problems including the landslide-induced tsunami problem. Working with SPH has an advantage, one does not need to solve an equation for describing free surface dynamics. Explanation of the method can be found in many references, see for instances [14, 15, 16, 17, 18]. In this article, standard formulation as elaborated in [10, 11] is used. Explanations of the SPH method in the current article are taken from those references.

Using the SPH method, in the fluid domain, we set finite particles with certain distance. Each particle has characteristic of the fluid such as position, velocity, density, etc. Further, a kernel function is used to interpolate any function and its spatial derivative. Here, the cubic spline kernel is chosen

$$W(r, \kappa) = \beta \begin{cases} 1 - \frac{3}{2} R^2 + \frac{3}{4} R^3 & 0 \leq R \leq 1 \\ \frac{1}{4} (2 - R)^3 & 1 \leq R \leq 2 \\ 0 & R \geq 2 \end{cases}$$

where $\beta$ is the normalization parameter for two-dimensional space defined by $10/(7 \pi \kappa^2)$, $R = r/\kappa$ denotes the non-dimensional distance between particles $i$ and $j$, and $r$ is the distance between particles.

By applying the SPH method, discrete form of the Euler equations (1)–(2) can be written as

$$\frac{D\rho_i}{Dt} = \sum_{j=1}^{N} m_j \left( u_i - u_j \right) \cdot \nabla_i W_{ij},$$

$$\frac{Du_i}{Dt} = -\sum_{j=1}^{N} m_j \left( \frac{p_j}{\rho_j^2} + \frac{p_i}{\rho_i^2} + \Pi_{ij} \right) \nabla_i W_{ij} + g,$$

where $p_i$ and $p_j$ are the pressure of particle $i$ and $j$, respectively. The term $\Pi_{ij}$ represents an artificial viscosity added to help achieve numerical stability, and is defined using

$$\Pi_{ij} = \begin{cases} -\alpha \bar{c}_{ij} \mu_{ij} \bar{\rho}_{ij} & u_{ij} \cdot x_{ij} < 0 \\ 0 & \text{otherwise} \end{cases}$$

where

$$\mu_{ij} = \frac{\kappa_{ij} u_{ij} \cdot x_{ij}}{r_{ij}^2 + (0.1 \kappa_{ij})^2}, \quad u_{ij} = u_i - u_j, \quad x_{ij} = x_i - x_j,$$

$$\bar{c}_{ij} = 0.5 (c_i + c_j), \quad \bar{\rho}_{ij} = 0.5 (\rho_i + \rho_j), \quad \kappa_{ij} = 0.5 (\kappa_i + \kappa_j).$$

Notations $c_i$ and $c_j$ represent the speed of sound of particles $i$ and $j$ respectively. Whereas the coefficient $\alpha$ is a non-dimensional constant. Its value is adjustable depends on the problem and need calibration [19]. In this article, the coefficient is chosen carefully depends on the type of landslide.

Further, in the SPH method, the fluid is assumed to be slightly compressible by employing an equation of state used to update pressure of particle. Another way to update the pressure of particle is by solving Poisson equation of pressure directly as described in detail in [6, 7]. In the current article, the following equation of state proposed by Monaghan in [15] is used.

$$p(\rho) = \frac{c_i^2 \rho_0}{\gamma} \left[ \left( \frac{\rho}{\rho_0} \right)^\gamma - 1 \right],$$

(8)
where \( \gamma = 7 \), \( \rho_0 = 1000 \text{ kg m}^{-3} \) is the reference density, and \( c_0 = c(\rho_0) = \sqrt{(\partial p/\partial \rho)|_{\rho_0}} \) represents the speed of sound at the reference density. As for the time integration used to solve equation (3), the leapfrog scheme is used see [16] for more detail.

2.2. The solid box treatment

Motion of the solid box is elaborated in detail in [1, 4]. Here, we resume it for clarity. Let \( S(t) \) denotes the position of solid box as a function of time \( t \). The motion of the solid box is governed by the Newton second law (equation (9)) in which forces are added mass, bouyancy, drag, friction, and gravitational force.

\[
(m_s + C_m m_w) \frac{d^2 S}{dt^2} = (m_s - m_w) (\sin \theta - C_n \cos \theta) g - 0.5 C_d \rho_w w l \cos \theta \sin \theta \left( \frac{dS}{dt} \right)^2,
\]

where \( m_s \) denotes mass of the solid box, \( C_m \) the initial position of the submerged solid box for given incline angle \( \theta \), \( m_w \) mass of water, \( C_d \) drag force coefficient, \( \rho_w \) density of water, \( w \) and \( l \) width and length of the triangular-cross section and \( t \) denotes time.

Derivation as well as solution of the equation (9) are described in detail in [1]. According to the reference, solution of the equation (9) is given as follows

\[
S(t) = S_0 \ln \left[ \cosh \left( \frac{t}{t_0} \right) \right]
\]

where \( S_0 = u_t^2/a_0 \), \( t_0 = u_t/a_0 \), and

\[
a_0 = \frac{(m_s - m_w) (\sin \theta - C_n \cos \theta) g}{(m_s + C_m m_w)},
\]

\[
u_t = \sqrt{\frac{2 (m_s - m_w) (\sin \theta - C_n \cos \theta) g}{C_d \rho_w \cos \theta \sin \theta}}.
\]

Whereas velocity and acceleration of the solid box are given as follows

\[
u(t) = \frac{S_0}{t_0} \tanh \left( \frac{t}{t_0} \right),
\]

\[
a(t) = \frac{S_0}{t_0^2} \left[ \frac{1}{\cosh \left( \frac{t}{t_0} \right)} \right]^2.
\]

In this current article, we treat the solid box as a collection of particles with solid label and its movement follows the equation (10). Let \( x_0 \) denotes initial position of the solid particles, thus movement of the solid particles can be written as

\[
x(t) = x_0 + S(t).
\]

3. Numerical simulation

In this article, we conducted a simulation of surge waves generated by aerial and submarine landslides. Further, we compare the numerical results with experimental data of Heinrich [2]. Note that all parameters in the numerical computation use SI unit, we write the parameters without unit.

Heinrich as described in [2] conducted an experiment in a wave tank with dimensions 20 m x 0.55 m x 1.5 m in the Hydraulic National Laboratory of Chatou in France. On the far right of
Figure 1. Sketch of fluid domain and notations used in our numerical simulation to mimic the experiment of Heinrich [2].

In the experiment, there was a shoreline with a mild slope \( \tan(15^\circ) \), see Figure 1. In the experiment, water depth is denoted as \( h_0 \), see Figure 1. The equilibrium situation is disturbed by the motion of a solid box which is sliding down along 45° sloping bottom. In the experiment, the movement of the solid box is stopped after it hits a rubber buffer at the bottom, see Figure 1.

Depends on the initial positions of the solid box, two numerical experiments were distinguished as submarine and aerial, see Figure 1. In the experiment of submarine landslide, the water depth is 1 m and the box with 140 kg mass is placed 1 cm below surface of water. Whereas for the aerial landslide, the water depth is 0.4 m and the box with 105 kg mass is placed just above still water level. Parameters used in simulations can be seen in Table 1.

| Name of Parameter                              | Symbol | value (aerial) | value (submarine) |
|------------------------------------------------|--------|----------------|-------------------|
| Density of water                               | \( \rho_w \) | 1000           | 1000              |
| Density of solid box                           | \( \rho_s \) | 1528           | 2037              |
| Speed of sound                                 | \( c_0 \) | 39.6182        | 62.6418           |
| Artificial viscosity coefficient               | \( \alpha \) | 0.15           | \( 10^{-6} \)     |
| Distance between particle (at the beginning)   | \( r_i \) | 0.02           | 0.02              |
| Time step                                      | \( \Delta t \) | \( 10^{-4} \) | \( 10^{-4} \)     |
| Characteristic length scale                    | \( S_0 \) | 5.39972        | 0.15618           |
| Characteristic time scale                      | \( t_0 \) | 1.50158        | 0.256785          |

3.1. Aerial Landslide

Here, we mimicked Heinrich experimental landslide of aerial type; water depth is set 0.4 m and the solid box is set just above the water surface, see Figure 1. In our SPH numerical experiment, we used 6321 number of fluid particles, and 351 number of solid particles. In our simulation, position of the solid particles are set constant after the lowest corner of the of solid particle has reached the bottom.

Comparison between simulation results and experimental data can be seen in Figure 2. At the beginning, all particles (both solid and fluid) are set all in the equilibrium positions. At
Figure 2. Surge waves generated by aerial landslide at time $t = 0$ (a), $t = 0.6$ (b), $t = 1$ (c), and $t = 1.5$ (d), compared with experimental data (red dotted line) of Heinrich [2].

![Figure 2](image)

In time $t > 0$, the solid particles start to slide down over the sloping bottom until the outer of the lowest corner particles reach the bottom. The solid movement will push the water particles and generate waves. Figure 2 presents snapshots of solid and fluid particles together with Heinrich experimental data (red dotted line). It is shown that due to landslide motion, fluid particles were deformed, generating a wave at the surface. Moreover, it is shown in Figure 2 the numerical surge waves are in good agreement with the experimental data of Heinrich [2].

3.2. Submarine Landslide

In this simulation, we also reconstruct the Heinrich experiment; water depth is set 1 m and the solid box is set 1 cm below the water surface, see Figure 1. Further, in our SPH numerical experiment, we used 15816 number of fluid particles, and 351 number of solid particles.

Comparison of between simulation results and experimental data can be seen in Figure 3. Figure 3 describes snapshots of solid and fluid particles together with experimental data (red dotted line) of Heinrich. Like in aerial landslide, a wave at the surface was generated due to the landslide motion. In general, it is shown in Figure 3, the numerical surge waves are in good agreement with the experimental data of Heinrich [2]. Moreover, we compare vertical displacement of the solid box obtained from numerical results with experimental data of Heinrich [2], which are given in Figure 4. It is shown that in both cases; aerial and submarine landslide, they are in good agreement with the experimental data.

In order to see quantitative comparison between our numerical results and the experimental data of Heinrich, we compute the Root Mean Square Error (RMSE),

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{N}}$$

where $y$ and $\hat{y}$ are free surface from experimental data and SPH simulation, respectively. Results of the comparison are given in the Table 2. It is shown that generally for both aerial and submarine simulation, each RMSE in the table is small enough. As qualitatively seen in Figure 4, motion of the solid box in submarine simulation has better accuracy than in aerial simulation with RMSE 0.00799 and 0.03831, respectively.
Figure 3. Surge waves generated by submarine landslide at time $t = 0.5$ (a), $t = 1.0$ (b), $t = 1.5$ (c), $t = 2.0$ (d), $t = 2.5$ (e), and $t = 3.0$ (f), compared with the experimental data (red dotted line) of Heinrich [2].

Figure 4. Vertical displacement time history of the solid box for aerial landslide (a) and submarine landslide (b), in the SPH simulation (solid blue line) and Heinrich experiment [2] (red dotted line).

Table 2. RMSE of aerial and submarine simulation

| Time | Aerial RMSE | Submarine RMSE |
|------|-------------|----------------|
| 0.6  | 0.02053     | 0.02908        |
| 1.0  | 0.02342     | 0.04085        |
| 1.5  | 0.02221     | 0.03772        |
| 2.0  |             | 0.03843        |
| 2.5  |             | 0.03753        |
| 3.0  |             | 0.02582        |
4. Conclusions
The smoothed particle hydrodynamic method has been implemented to simulate surge waves generated by aerial and submarine landslides. The landslides are modeled as a solid box with triangular-cross section sliding down over an inclined beach. Movement of the solid box follows analytical solution derived by Watts in [1]. In addition, the comparison of our SPH results is given to show our numerical results are reasonable. Here, the results of free surface deformation and the solid box movement for both aerial and submarine landslides are in good agreement with experimental data of Heinrich [2]. The RMSE of free surface deformation in aerial simulation recorded at time $t = 0.6, 1.0, 1.5$ are $0.02053, 0.02342, 0.02221$, respectively and in submarine simulation recorded at time $t = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0$ are $0.02908, 0.04085, 0.03772, 0.03843, 0.03753, 0.02582$, respectively. Further, the RMSE data in Table 2 show that free surface deformation in the aerial simulation has better accuracy than the submarine simulation. Whereas motion of the solid box in submarine simulation has better accuracy than in aerial simulation with RMSE $0.00799$ and $0.03831$, respectively.

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