CASPIAN SEA TIDAL MODELING USING COASTAL TIDE GAUGE DATA

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ABSTRACT
The purpose of this article is to model tidal conditions in the Caspian Sea using data from coastal tide gauges of Anzali, Noshahr and Neka Ports. Harmonic Analysis method was used to identify and examine 40 tidal components. The results illustrate that the annual (Sa) and semi-annual Solar (Ssa) components on all of the ports listed have the highest range in comparison with the other components which are respectively 16 cm, 18 cm and 15 cm for annual components and 2.8 cm, 5.4 cm and 3.7 cm for semi-annual components.

Keywords: Caspian Sea, harmonic analysis, tidal modelling, coastal tide gauges.

1. INTRODUCTION
Knowing the causes of water level fluctuations of the seas has been one of the major challenges in all scientific fields and attracts the attention of many researchers. Oceanic effects, aerologic effects, tides, climate change and vertical movement of the earth’s shell can be noted as five factors that contribute to the impacts of climate change on the water level fluctuations (KARABİL S. 2011).

Tidal effects due to their significant impacts on sea water level are of great importance and researchers have always been looking forward to modelling them (PSMSL, 2011).

In this study, the effectiveness of tidal components by the use of coastal tide gauge’s observations is surveyed. Harmonic analysis method was used to determine tidal amplitude and phase of 40 components.

1.1 Study area and relevant data
Caspian Sea is surrounded by five countries, including Iran, Russia, Azerbaijan, Turkmenistan and Kazakhstan. The Sea is the largest remaining section of the old Tethys Sea breakdown that was spread from the Arctic to Indian Ocean through the first to third geological period. In third geological period appearing of Caucasus and Asian mountains leads to this big Sea dividing so rise of European continent and construction of Iranian plateau were the main reasons of creation of Caspian Sea. Having the length of about 1030 Km and the width of 435 to 196 Km, the Caspian Sea locates between the 47° 57´–36° 33´ circuits and 46° 43´–54° 53´ hour circles (Arpe and Leroy, 2007).
2. TIDES

Tides are the regular ebbing and flowing movement of the sea happening as the result of attractions of celestial near-earth bodies such as the sun and the moon. Tidal acceleration of celestial objects such as the moon at one point is the difference between the gravity acceleration of that celestial body and the mass centre of the Earth at that point. Visco elastic earth changes due to tidal forces are roughly one-third of the surface international water. The vector field of these forces can be replaced by a scalar field named potential of tides. In each point of the earth this potential can be computed by (Paul Melchior, 1966):

\[
U(R_e, \lambda, \phi) = \frac{GM}{R} \sum_{i=2}^{\infty} \left( \frac{R_e}{R} \right)^i \cos i \theta
\]

Where \( G \) is the universal constant of gravity, \( M \) is the mass of absorbing body (Moon or Sun), \( R_e \) is the average radius of the Earth, \( R \) is the geocentric distance and \( \theta \) is the geological distance of a point with \((R_e, \lambda, \phi)\) coordinates. Main course of this relation is achieved when \( i=2 \). However, in some cases when \( i=3 \) is also used about the moon. Thus, the main indicator term of tidal potential describes as follow:

\[
U_2(R_e, \lambda, \phi) = \frac{3GMR_e^2}{4R^3} \left( \cos 2 \theta + \frac{1}{3} \right)
\]

And regarding to spherical trigonometry relations:

\[
U_2(R_e, \lambda, \phi) = \sum_{i=0}^{\infty} U_{2m}(R_e, \lambda, \theta)
\]
It can be seen clearly from this relation that the first parameter of the relation relates to the half-daily affects and the second and third parameters respectively show the effects of daily and long period of tides. This relationship suggests that the tidal potential is a function of the absorvent body (Moon and Sun). It should be noted that except the gravitational forces mentioned above, other factors may be effective in producing and intensifying the tides (Doodson, 1922).

In general, the sea water level fluctuations can be obtained as the result of the interaction of following dynamic processes: tides, changes in atmospheric pressure, the dynamic effects of ocean circulation, wind effects, the effects of temperature, water salinity changes, the effects of river discharge into the oceans and melting of polar ices.

3. TIDAL HARMONIC ANALYSIS

In this study, the Fourier Harmonic analysis method is used to determine the effect of tidal components. As tidal components frequency, water level in each moment and time were given, the amplitude and phase of needed component were determined.

\[ u(\phi, \lambda, t) = \text{MSL}(\phi, \lambda) + \sum_{i=1}^{n} a_i(\Phi, \lambda) \cos \omega_i t + b_i(\phi, \lambda) \sin \omega_i t \]  
(4)

Where \( u(\phi,\lambda,t) \) is water level in \( t \) moment obtained by tide gauge or satellite altimetry, \( \text{MSL}(\phi, \lambda) \) is sea average level, \( \omega_i=2\pi f_i \) is angular frequency derived from tidal components frequency, \( t \) is the observation time and \( a_i \) and \( b_i \) are the Fourier coefficients we need to determine. Considering equation 4 and proration, the amplitude and the phase are calculated as below:

\[ \begin{pmatrix} u(\phi, \lambda, t_1) \\ u(\phi, \lambda, t_2) \\ \vdots \\ u(\phi, \lambda, t_m) \end{pmatrix}_{m \times 1} = \begin{pmatrix} 1 & \cos \omega_1 t_1 & \sin \omega_1 t_1 & \ldots & \cos \omega_n t_1 & \sin \omega_n t_1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & \cos \omega_1 t_m & \sin \omega_1 t_m & \ldots & \cos \omega_n t_m & \sin \omega_n t_m \end{pmatrix}_{m \times (2n+1)} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \\ b_1 \\ \vdots \\ b_n \end{pmatrix}_{(2n+1) \times 1} \]

It is also known that determined parameters accuracy \( \tilde{x} \) is calculated by variance covariance matrices (Vanicek, 1986)

4. TIDE MODELLING

The following equation is used to analyze and predict the tide (Luick J.L., 2004; Do-Seong Byun and Chang-Woo Cho., 2009):

\[ h(t) = Z_0 + \sum_{n=1}^{N} \left[ f_n(t) H_n \cos(\delta_n t - g_n + V_n(t)) + u_n(t) \right] \]

And \( n = 1, 2, \ldots, N \) tidal components

In the above equation,

\( Z_0 \) is the average sea level, \( N \) is the number of tidal components, \( \delta_n \) is angular frequency or speed (degree per hour), \( V_n(t) \) is astronomical argument, \( f_n(t) \) is Nodal factor, \( u_n(t) \) is Nodal phase, \( H_n \) is the component amplitude and \( g_n \) is the phase lag.

In above relations Nodal corrections \( (f_n(t), u_n(t)) \) for each tidal component and
astronomical argument \( V_n(t_0) \) must be calculated in order to determine the revised amplitude and phass. In this step \( V_n(t_0) \) is computed as below:

\[
V_n(t_0) = i_i \lambda_s(t) + i_i \lambda_h(t) + i_i \lambda_p(t) + i_i \lambda_N(t) + i_i \lambda_P(t) + \Phi_n
\]  

Where \( \left(i_i, i_c, i_d, i_e, i_f\right) \) are the constituent’s Doodson Numbers in which Solar coefficients is preferred to Lunar coefficients and the desired time is the time data starts in zero hour UT and remaining terms are as follows:

- \( \lambda_s(t) \) or simply “s” is the Mean Longitude of Moon
- \( \lambda_h(t) \) or simply “h” is the Mean Longitude of Sun
- \( \lambda_p(t) \) or simply “P” is the Longitude of Lunar Perigee
- \( \lambda_N(t) \) or simply “N” is the Longitude of Lunar Ascending Node
- \( \lambda_P'(t) \) or simply “\( P' \)” or “P1” is the Longitude of Perihelion
- \( \Phi_n \) is stationary phase (a multiple of 90°)

Stationary phase exists in most of Doodson Number’s Tables. Algebraic formula to determine the geographic Astronomical Longitude (the Celestial Sphere) \( \lambda_s(t), \lambda_h(t), \lambda_p(t), \lambda_N(t), \lambda_P'(t) \) or in some references in form of \( (S, h, P, N, P') \) has provided (Cartwright, 1982; Doodson, 1921; Franco, 1988; Schureman, 1941; and Tsak- 2000, IOS, Uk). In this study Task- 2000 method has been used.

Task- 2000 based on zero hour epochs UT, January 1900. This formula is accurate at least from 1800 to 2100 (Bell et al., 1999).

\[
\lambda_s(t) = 277.0247 + 129.38481.IY + 13.17639.DL
\]
\[
\lambda_h(t) = 280.1895 - 0.23872.IY + 0.98565.DL
\]
\[
\lambda_p(t) = 334.3853 + 40.66490.IY + 0.11140.DL
\]
\[
\lambda_N(t) = 25901568 - 19.32818.IY - 0.05295.DL
\]
\[
\lambda_P'(t) = 28102209 + 0.017192.IY
\]

Where

- IY = year- 1900
- DL = IL+IDAY-1

\[
IL = (IY - 1) / 4
\]

IDAY is the number of days from January 1th of that year.

### 5. FORMING THE OBSERVATIONS TIME SERIES OF COASTAL TIDE GAUGE

The location of tide gauge stations which is formed from their time series data were as follows:

| Station name | Latitude | Longitude | Time spam               |
|--------------|----------|-----------|-------------------------|
| Anzali       | °37.478  | °49.4623  | 21/3/2005-20/3/2014     |
| Noshahr      | °36.6584 | °51.5047  | 21/3/2006-21/3/2014     |
| Neka         | °36.8502 | °53.3656  | 1/1/2000-31/8/2012      |
Figure-2. Time series together with the tidal model and residual of Anzali tide gauge station.

Figure-3. Amplitude and frequency of 40 tidal components of Anzali tidal gauge station.

Figure-4. Time series together with the tidal model and residual of Noshahr tide gauge station.

Figure-5. Amplitude and frequency of 40 tidal components of Noshahr tidal gauge station.

Figure-6. Time series together with the tidal model and residual of Neka tide gauge station.

Figure-7. Amplitude and frequency of 40 tidal components of Neka tidal gauge station.
Table 2. The Amplitude and phase of 40 tidal components for Anzali tide gauge station, obtained from observations and modelling in meter and grade respectively.

| Constituent | Amp-obs (m) | Phase-obs (deg) | Amp-mod (m) | Phase-mod (deg) |
|-------------|-------------|-----------------|-------------|-----------------|
| K2          | 0.003015    | 295.7626        | 0.00208     | 286.43          |
| L2          | 0.001672    | 271.2841        | 0.00171     | 271.46          |
| M2          | 0.008682    | 269.2956        | 0.0084      | 269.245         |
| N2          | 0.003698    | 158.6862        | 0.00374     | 159.22          |
| Ma2         | 0.003133    | 296.7903        | 0.00314     | 296.79          |
| R2          | 0.017878    | 73.2279         | 0.01812     | 78.22           |
| S2          | 0.012064    | 67.1411         | 0.01206     | 67.141          |
| T2          | 0.005672    | 144.9542        | 0.0065      | 142.954         |
| J1          | 0.001707    | 121.5101        | 0.0017      | 121.8201        |
| K1          | 0.022738    | 11.43623        | 0.0233      | 11.3            |
| M1          | 0.007332    | 67.24207        | 0.0073      | 68.45           |
| O1          | 0.006945    | 165.1889        | 0.00693     | 165.1923        |
| P1          | 0.00704     | 217.9986        | 0.00725     | 218.43          |
| Q1          | 0.008204    | 232.6201        | 0.008325    | 232.56          |
| S1          | 0.008428    | 83.31833        | 0.008267    | 84.673          |
| M3          | 0.0072      | 93.20983        | 0.00687     | 93.65           |
| S3          | 0.002717    | 119.0538        | 0.00281     | 119.487         |
| M4          | 0.005868    | 233.6994        | 0.00589     | 233.996         |
| S4          | 0.006028    | 85.39538        | 0.006518    | 85.7            |
| M5          | 0.015845    | 21.3593         | 0.01522     | 21.487          |
| M6          | 0.004693    | 239.5097        | 0.00467     | 240.43          |
| S6          | 0.004883    | 193.316         | 0.0049      | 193.76          |
| M8          | 0.008275    | 265.0774        | 0.00865     | 265.9           |
| S8          | 0.003187    | 159.8985        | 0.00376     | 158.34          |
| Mf          | 0.006459    | 109.5878        | 0.006217    | 107.789         |
| Mn          | 0.007524    | 134.4543        | 0.00776     | 136.89          |
| Msf         | 0.000792    | 77.13807        | 0.000734    | 76.54           |
| Oo1         | 0.006236    | 355.2012        | 0.006236    | 355.34          |
| Ssa         | 0.028563    | 55.42608        | 0.02862     | 53.56           |
| Ms4         | 0.070979    | 15.67987        | 0.0704      | 15.456          |
| Mn4         | 0.018667    | 186.2534        | 0.01889     | 186.789         |
| Mk3         | 0.02533     | 282.7036        | 0.0255      | 282.408         |
| Sa          | 0.163234    | 212.3174        | 0.163198    | 212.456         |
| Mo3         | 0.007534    | 219.1039        | 0.007423    | 218.56          |
| No3         | 0.026181    | 8.642835        | 0.026789    | 9.43            |
| 2N2         | 0.006548    | 46.04756        | 0.006437    | 47.13           |
| So3         | 0.028834    | 137.9187        | 0.02889     | 136.67          |
| Sk3         | 0.006522    | 44.34284        | 0.00657     | 44.389          |
| S01         | 0.001092    | 68.53536        | 0.001034    | 68.576          |
| Mk4         | 0.022139    | 331.8166        | 0.0234      | 331.834         |
Table-3. The Amplitude and phase of 40 tidal components for Noshahr tide gauge station, obtained from observations and modelling in meter and grade, respectively.

| Constituent | Amp-obs (m) | Phase-obs (deg) | Amp-mod (m) | Phase-mod (deg) |
|-------------|-------------|-----------------|-------------|-----------------|
| K2          | 0.001265    | 205.3896        | 0.001276    | 204.9           |
| L2          | 0.000134    | 104.9257        | 0.000144    | 103.89          |
| M2          | 0.010247    | 275.8484        | 0.01028     | 276.896         |
| N2          | 0.002429    | 110.3829        | 0.002534    | 110.789         |
| Ma2         | 0.002005    | 73.94501        | 0.002007    | 74.289          |
| R2          | 0.00083     | 248.9937        | 0.00076     | 249.3397        |
| S2          | 0.000134    | 10.64485        | 0.000144    | 10.5689         |
| T2          | 0.00104     | 252.2587        | 0.001067    | 252.678         |
| J1          | 0.000575    | 38.258          | 0.000589    | 38.378          |
| K1          | 0.003133    | 206.5065        | 0.003145    | 206.45          |
| M1          | 0.000804    | 197.7024        | 0.000809    | 197.745         |
| O1          | 0.001833    | 74.5144         | 0.001845    | 74.78           |
| P1          | 0.001664    | 171.5155        | 0.001643    | 172.504         |
| Q1          | 0.000887    | 255.0884        | 0.000867    | 255.56          |
| S1          | 0.002105    | 352.2459        | 0.002175    | 352.211         |
| M3          | 0.000044    | 284.4275        | 0.000245    | 284.545         |
| S3          | 0.001738    | 106.7813        | 0.000678    | 106.3413        |
| M4          | 0.000773    | 269.5824        | 0.000989    | 269.6724        |
| S4          | 0.000389    | 334             | 0.000567    | 333.89          |
| M5          | 0.000592    | 308.7407        | 0.000345    | 308.437         |
| M6          | 0.000816    | 107.9719        | 0.00424     | 107.901         |
| S6          | 0.001758    | 297.3552        | 0.000987    | 296.9552        |
| M8          | 0.001783    | 300.9042        | 0.000345    | 300.405         |
| S8          | 0.035218    | 128             | 0.03789     | 128.2362        |
| Mf          | 0.000565    | 63.45231        | 0.000679    | 64.82           |
| Mm          | 0.000585    | 157.1591        | 0.000567    | 156.2271        |
| Msf         | 0.000698    | 171.7511        | 0.000778    | 171.842         |
| Oo1         | 0.001602    | 126.1866        | 0.00189     | 125.731         |
| Ssa         | 0.054341    | 86.44686        | 0.05345     | 86.461          |
| Ms4         | 0.000389    | 142.9945        | 0.00927     | 143.18          |
| Mn4         | 0.000375    | 12.73982        | 0.000678    | 13.7            |
| Mk3         | 0.000346    | 67.02355        | 0.00345     | 66.425          |
| Sa          | 0.188683    | 208.5804        | 0.18907     | 208.2541        |
| Mo3         | 0.000228    | 52.28991        | 0.000789    | 51.75           |
| No3         | 0.000178    | 216.8859        | 0.000845    | 217.323         |
| 2N2         | 0.000519    | 37.64077        | 0.000612    | 37.772          |
| So3         | 0.000414    | 105.4683        | 0.000432    | 105.94          |
| Sk3         | 0.000306    | 334.2423        | 0.000351    | 334.58          |
| S01         | 0.000225    | 195.3219        | 0.000376    | 194.218         |
| Mk4         | 0.000424    | 74.62624        | 0.000387    | 74.947          |
Table-4. The Amplitude and phase of 40 tidal components for Neka tide gauge station, obtained from observations and modelling in meter and grade, respectively.

| Constituent | Amp-obs (m) | Phase-obs (deg) | Amp-mod (m) | Phase-mod (deg) |
|-------------|-------------|-----------------|-------------|-----------------|
| K2          | 0.000609    | 248.292         | 0.00071     | 248.35          |
| L2          | 0.000094    | 210.8618        | 0.000078    | 211.23          |
| M2          | 0.003386    | 211.6024        | 0.00421     | 212.316         |
| N2          | 0.000451    | 33.09812        | 0.000418    | 33.789          |
| Ma2         | 0.000203    | 2.616894        | 0.000206    | 3.269           |
| R2          | 0.00027     | 266.0452        | 0.000261    | 266.169         |
| S2          | 0.001491    | 353.4808        | 0.000078    | 353.724         |
| T2          | 0.000322    | 84.26617        | 0.0000341   | 83.972          |
| J1          | 0.00002     | 222.6129        | 0.000237    | 222.7034        |
| K1          | 0.003984    | 232.586         | 0.00412     | 232.213         |
| M1          | 0.000075    | 18.18968        | 0.000087    | 18.226          |
| O1          | 0.001979    | 22.82131        | 0.002141    | 24.718          |
| P1          | 0.000071    | 199.8688        | 0.000076    | 200.1073        |
| Q1          | 0.000133    | 265.089         | 0.001343    | 265.323         |
| R3          | 0.00014     | 238.5899        | 0.00017     | 238.3041        |
| S3          | 0.00127     | 71.53864        | 0.00143     | 70.946          |
| M4          | 0.00041     | 106.7546        | 0.00052     | 106.9928        |
| S4          | 0.00007     | 53.35676        | 0.000083    | 53.468          |
| M5          | 0.00036     | 289.5994        | 0.00038     | 289.783         |
| M6          | 0.0004     | 274.9504        | 0.00047     | 275.7935        |
| S6          | 0.00042     | 59.32671        | 0.00082     | 58.87           |
| M8          | 0.00071     | 493216          | 0.00082     | 78.465          |
| S8          | 0.00007     | 343.1489        | 0.00007     | 343.1376        |
| Mf          | 0.003968    | 358.5273        | 0.003965    | 357.5186        |
| Mn          | 0.001392    | 37.4896         | 0.001323    | 38.418          |
| Msf         | 0.002203    | 332.5874        | 0.002212    | 333.64          |
| Oo1         | 0.00043     | 172.098         | 0.00043     | 172.487         |
| Ssa         | 0.037471    | 90.08432        | 0.0375      | 91.508          |
| Ms4         | 0.00045     | 225.9218        | 0.0005      | 226.5           |
| Mn4         | 0.000063    | 309.734         | 0.00006     | 310.47          |
| Mk3         | 0.000063    | 168.7787        | 0.00007     | 169.681         |
| Sa          | 0.151559    | 218.163         | 0.1518      | 217.2045        |
| Mo3         | 0.00028     | 42.31016        | 0.00027     | 43.401          |
| N3          | 0.00055     | 308.456         | 0.00054     | 307.731         |
| 2N2         | 0.00032     | 294.6541        | 0.00034     | 295.391         |
| So3         | 0.0005     | 94.33536        | 0.0006      | 95.591          |
| Sk3         | 0.000114    | 73.86118        | 0.000145    | 74.584          |
| S01         | 0.000171    | 47.30911        | 0.000182    | 47.835          |
| Mk4         | 0.00058     | 116.3563        | 0.00072     | 116.2482        |
6. CONCLUSIONS

Tidal Analysis of tide gauge stations illustrates the absence of efficiency in tidal components except for annual and semi-annual components. However, unlike other stations, in Anzali station MS4 components was effective.

The results also indicates that the annual (Sa) and semi-annual Solar (Ssa) components on all of the ports listed have the highest range in comparison with the other components which are respectively 16 cm, 18 cm and 15 cm for annual components and 2.8 cm, 5.4 cm and 3.7 cm for semi-annual components.

Ignoring the modulation of solar perihelion and nodal modulation use can be mentioned as the weak points which were effective in Tidal Harmonic Analysis; 18.6 time series must be exists for resolving all the frequencies. As the other defects of this study we can point out that there are no easy way to indicate the significance of amplitude and phase together with lack of appropriate solution for coastal areas that affects the shape of tidal waves.

The main problem with Caspian Sea northern coastal tide gauge data is improper collecting and compiling of them. Moreover, the sampling interval in Anzali and Noshahr tide gauges are daily and each three hours respectively that can affect the tidal modelling and will decrease the accuracy of the computation.

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