Abstract

The field of reinforcement learning can be split into model-based and model-free methods. Here, we unify these approaches by casting model-free policy optimisation as amortised variational inference, and model-based planning as iterative variational inference, within a ‘control as hybrid inference’ (CHI) framework. We present an implementation of CHI which naturally mediates the balance between iterative and amortised inference. Using a didactic experiment, we demonstrate that the proposed algorithm operates in a model-based manner at the onset of learning, before converging to a model-free algorithm once sufficient data have been collected. We verify the scalability of our algorithm on a continuous control benchmark, demonstrating that it outperforms strong model-free and model-based baselines. CHI thus provides a principled framework for harnessing the sample efficiency of model-based RL and the asymptotic performance of model-free RL (Wang & Ba, 2019).

In this work, we show that the control as inference framework (Levine, 2018; Rawlik et al., 2013; Ziebart, 2010; Abdolmaleki et al., 2018) provides a principled methodology for combining model-based and model-free RL. This framework casts decision making as probabilistic inference, enabling researchers to derive principled (Bayesian) objectives and draw upon a wide array of approximate inference techniques. While the framework encompasses many different methods, they all share the goal of inferring a posterior distribution over actions, given a probabilistic model that is conditioned on observing ‘optimal’ trajectories.

Since computing the posterior distribution over actions is generally analytically intractable, variational methods are often used to implement approximate inference (Beal et al., 2003). Here, we highlight a distinction between amortised and iterative approaches to variational inference (Kim et al., 2018), and show that, in the context of control as inference, amortised inference naturally corresponds to model-free policy optimisation, whereas iterative inference naturally corresponds to model-based planning.

Leveraging these insights, we propose control as hybrid inference (CHI), a framework for combining amortised and iterative inference in the context of control. This framework proposes two inference algorithms – one amortised and one iterative – which work collaboratively to recover an (approximate) posterior over sequences of actions. To combine these processes, we implement an algorithm in which amortised inference sets the initial conditions for a subsequent phase of iterative inference, leading to a natural and adaptive interaction between the two inference approaches.

We utilise a didactic experiment to investigate the interaction between amortised and iterative inference over the course of learning and explore how this is affected when environmental contingencies change. We find that iterative inference dominates action selection when amortised predictions are uncertain, such as at the onset of learning, and that amortised inference determines action selection when sufficient data have been collected. We demonstrate the scalability of our algorithm using a high-dimensional control benchmark and demonstrate that it outperforms strong model-based and model-free baselines, both in terms of sample efficiency and...
asymptotic performance. These results suggest that CHI could provide a principled framework for combining the sample efficiency of model-based planning with the asymptotic performance of model-free policy optimisation.

2. Background

We consider a finite-horizon Markov decision process (MDP) defined by a tuple \( \{S, A, p_{\text{env}}, r\} \), where \( s \in S \) denotes states, \( a \in A \) denotes actions, \( p_{\text{env}}(s_{t+1}|s_t, a_t) \) is the environment’s dynamics and \( r(s_t, a_t) \) is the reward function. Traditionally, RL problems look to identify the policy \( p_\theta(a|s_t) \) which maximizes the expected sum of reward \( \mathbb{E}_{p_\theta(r)} \left[ \sum_{t=1}^T r(s_t, a_t) \right] \), where \( \theta \) are the policies parameters, \( \tau \) denotes a trajectory \( \tau = \{(s_t, a_t)\}_{t=1}^T \), and \( p_\theta(\tau) \) denotes the probability of trajectories under a policy, \( p_\theta(\tau) = p(s_1) \prod_{t=1}^T p_\theta(a_t|s_t) p_{\text{env}}(s_{t+1}|s_t, a_t) \).

**Control as Inference** To reformulate the problem of RL in the language of probability theory, we introduce an auxiliary ‘optimality’ variable \( O \in [0, 1] \), where \( O_t = 1 \) denotes that time step \( t \) was optimal (we drop \( 1 \) for conciseness). We assume agents encode a generative model over trajectories and optimality variables:

\[
p(\tau, O_{1:T}) = p(s_1) \prod_{t=1}^T p(O_t|s_t, a_t) p_\lambda(s_{t+1}|s_t, a_t)p(a_t)
\]

(1)

where \( \lambda \) are parameters of the dynamics model, which may be learned in a model-based context. We assume an uninformative action prior \( p(a_t) = \frac{1}{|A|} \). The optimality likelihood \( p(O_t|s_t, a_t) \) describes the probability that some state-action pair \( (s_t, a_t) \) is optimal and is defined as \( p(O_t|s_t, a_t) = \exp(r(s_t, a_t)) \) (Levine 2018).

The goal of control as inference is to maximise the marginal-likelihood of optimality \( p(O_{1:T}) \). While it is generally intractable to evaluate this quantity directly, it is possible to construct a variational lower bound \( \mathcal{L} \) which can be evaluated and optimised through variational inference (Jordan et al. 1999). To achieve this, we introduce an arbitrary distribution over trajectories \( q(\tau) = q(s_1) \prod_{t=1}^T q(s_{t+1}|s_t, a_t)q_\phi(a_t|s_t) \), which we refer to as an approximate posterior. The variational lower bound \( \mathcal{L} \) is then given by (see Appendix D for a derivation):

\[
\mathcal{L} = -D_{\text{KL}} \left( q(\tau) \parallel p(\tau|O_{1:T}) \right) \leq \log p(O_{1:T})
\]

(2)

Maximising \( \mathcal{L} \) with respect to the parameters of the approximate posterior provides a tractable method for maximising the (log) marginal-likelihood of optimality. We can further simplify Eq. 2 by fixing \( q(s_1) = p(s_1) \) and \( q(s_{t+1}|s_t, a_t) = p_\lambda(s_{t+1}|s_t, a_t) \), giving (see Appendix D for a derivation):

\[
\mathcal{L} = \mathbb{E}_{q(\tau)} \left[ \sum_{t=1}^T r(s_t, a_t) \right] + \mathcal{H} \left[ q_\phi(a_{1:T}|s_{1:T}) \right]
\]

(3)

where \( \mathcal{H}[\cdot] \) is the Shannon entropy. The inclusion of the action entropy term provides several benefits, including a mechanism for offline learning (Nachum et al. 2017), improved exploration and increased algorithmic stability. Empirically, algorithms derived from the control as inference framework often outperform their non-stochastic counterparts (Haarnoja et al. 2018).

**Iterative Inference** Equation 2 demonstrates that control as inference corresponds to a process of variational inference. In the wider literature on variational inference, a key distinction is made between iterative and amortised approaches. Iterative approaches to variational inference directly optimise the parameters of the approximate posterior \( \theta \) in order to maximise \( \mathcal{L} \), a process which is carried out iteratively for each data point. Examples of this approach include stochastic variational inference (Hoffman et al. 2013), variational message passing (Winn & Bishop 2005), belief propagations (Weiss & Freeman 2000) and variational expectation-maximisation (Marino et al. 2018b). Within the control as inference framework, the approximate posterior is over actions and, when considering an approximate posterior over sequences of actions, several model-based planning algorithms can be cast as a process of iterative inference (Okada & Taniguchi 2019, Piché et al. 2018, Williams et al. 2017, Friston et al. 2015, Tschantz et al. 2019).

**Amortised Inference** In contrast, amortised variational inference learns a parameterised function \( f_\phi(\cdot) \) which maps directly from data \( x \) to the parameters of the approximate posterior \( \theta = f_\phi(x) \). Amortised inference models are learned by optimising the parameters \( \phi \) in order to maximise \( \mathcal{L} \), an optimisation that takes place over the available dataset \( D \). Amortised variational inference form the basis of popular methods such as variational autoencoders (Kingma & Welling 2013). In the context of control as inference, the parameterised function corresponds to a policy, and the approximate posterior is again over actions. This approach is closely related the field of maximum-entropy RL (Eysenbach & Levine 2019), which has inspired several influential model-free algorithms (Levine 2018, Haarnoja et al. 2018, Abdolmaleki et al. 2018).

3. Control as Hybrid Inference

In this section, we introduce the control as hybrid inference (CHI) framework. Like the control as inference framework, CHI proposes that agents infer an approximate posterior over actions, given a generative model that is conditioned on ‘optimality’. However, CHI additionally proposes that inference is achieved via two processes – an amortised process which maps from states to the parameters of an approximate posterior over actions, and an iterative process which directly updates the parameters of the approximate posterior in an iterative manner. To ensure consistency between these
processes, both amortised and iterative inference utilise the same generative model and optimise the same variational objective. By utilising the correspondence between (i) amortised inference and policy optimisation, and (ii) iterative inference and planning, we demonstrate that this perspective allows for a principled combination of model-based and model-free RL.

**Iterative Inference Algorithm** We consider an iterative inference algorithm which optimises an approximate posterior over action sequences of a fixed horizon $H$ extending from the current time step $t$, $q(a_{t:T}; \theta)$, where we have used $T = t + H$ to simplify notation. We consider this distribution to be a time-dependent diagonal Gaussian, $q(a_{t:T}; \theta) = \mathcal{N}(a_{t:T}; \mu_{t:T}, \text{diag } \sigma_{t:T}^2)$, where $\theta = \{\mu_{t:T}, \sigma_{t:T}^2\}$.

At each time step $t$, agent’s observe the state of the environment $s_t$. Iterative inference then proceeds by iteratively updating the parameters of $q(a_{t:T}; \theta)$ in order to maximise $\mathcal{L}$. As demonstrated in (Okada & Taniguchi, 2019), this can be achieved by utilising of mirror descent (Bubeck, 2014; Okada & Taniguchi, 2018), providing the following update rule:

$$q^{(i+1)}(a_{t:T}; \theta) \leftarrow q^{(i)}(a_{t:T}; \theta) \cdot \mathcal{W}(a_{t:T}) \cdot q^{(i)}(a_{t:T}; \theta) \cdot \mathcal{E}^{q^{(i)}(a_{t:T}; \theta)} \mathcal{W}(a_{t:T}) \cdot q^{(i)}(a_{t:T}; \theta)$$

(4)

where $i$ denotes the current iteration and $\mathcal{W}(a_{t:T}) = \mathbb{E}_{q(s_{t:T}|a_{t:T}, s_t)}[r(\tau)]$. After $I$ iterations, the mean of the approximate posterior over action $\mu_{t:T}$ is returned and the first action $\mu_1$ from this sequence is executed. Equation (4) is a Bayesian generalisation of model predictive path integral control (MPPI) (Williams et al., 2017), a popular method for model predictive control (MPC) (Camacho & Alba, 2013).

**Amortised Inference Algorithm** Our amortised inference algorithm infers an approximate posterior over the current action $q_\phi(a_1|s_1; \theta)$. This implies that $q_\phi(a_1|s_1; \theta) = \mathcal{N}(a_1; \mu_1, \text{diag } \sigma_1^2)$, where $\theta = \{\mu_1, \sigma_1^2\}$. Rather than optimising $\theta$ directly, amortised inference employs a parameterised function $f_\phi(s_t)$ which maps directly from $s_t$ to $\theta$. The parameters of this function $\phi$ are then updated in order to maximise the variational bound $\mathcal{L}$. This optimisation takes place in a batched fashion over the available dataset $D = \{(s_t, a_t, r(s_t, a_t), s_{t+1})\}_{t=1}^B$, where $B$ is the size of the dataset, such that the optimisation problem $\argmax_\phi \mathcal{L}(\phi)$ is augmented to $\argmax_\phi \mathbb{E}_D [\mathcal{L}(\phi)]$.

In the current work, we utilise the Soft Actor-Critic (SAC) algorithm (Haarnoja et al., 2018) to optimise $\phi$. Rather than directly differentiating the variational bound $\mathcal{L}$, SAC employs a message passing approach. We refer readers to (Haarnoja et al., 2018) for a description of the SAC algorithm, and (Levine 2018) for a description of its relationship to variational inference.

**Combining Amortised & Iterative Inference** We now consider how the amortised and iterative processes can be combined into a single inference algorithm. In our implementation, amortised inference provides an ‘initial guess’ at the approximate posterior which is then refined by a subsequent phase of iterative inference. Formally, at each time step $t$, the parameters $\theta$ are initialised by the amortised mapping $\theta = f_\phi(s_t)$, and then iteratively updated according to Eq. (4).

An immediate challenge for this approach is that amortised inference considers an approximate posterior over a single action $q_\phi(a_t|s_t; \theta)$, whereas iterative inference considers an approximate posterior over a sequence of actions $q(a_{t:T}; \theta)$. To address this challenge, we adapt the amortised algorithm to predict a sequence of actions $q_\phi(a_{t:T}|s_{t}; \theta)$ by applying the factorisation $q_\phi(a_{t:T}|s_t; \theta) = \prod_{t'=t}^T q_\phi(a_{t'}|s_{t'; \theta})$.

Thus, $f_\phi(\cdot)$ still predicts the parameters of a distribution over current actions $\theta = \{\mu_t, \sigma_t^2\}$. However, this factorisation raises a separate issue, in that it requires knowledge of $s_{t:T}$, which are future states and thus unknown. To overcome this issue, we utilise the learned transition model $p_\lambda(s_{t+1}|s_t, a_t)$ (described in Appendix B) to predict the trajectory of future states $s_{t,T}$. Let $p_\phi(\tau)$ denote the probability of trajectories under the amortised policy $\phi_\theta$.

$$p_\phi(\tau) = p(s_t) \prod_{t'=t}^T p_\lambda(s_{t'+1}|s_{t'}, a_{t'}) q_\phi(a_{t'}|s_{t'; \theta})$$

(5)

where we have assumed $p(s_t) = \delta(s_t)$. We can then recover the desired distribution over actions $q_\phi(a_{t:T}|s_{t}; \theta)$ with parameters $\theta = \{\mu_{t:T}, \sigma_{t:T}^2\}$, which can then be used to specify the parameters of a time-dependent diagonal Gaussian $\theta = \{\mu_{t:T}, \sigma_{t:T}^2\}$. These parameters are then used as the initial distribution for the iterative phase of inference. An overview of the proposed method is provided in Algorithm A and discuss our solution to the data bias issue in Appendix C.

**4. Experiments**

**Didatic experiment** To demonstrate the characteristic dynamics of our algorithm, we utilise a simple 2D point mass environment in which an agent must navigate to a goal (top-right corner), with the additional complexity of traversing through a small hole in a wall (see Appendix D for details). We compare the amortised predictions of $q(a_{t:T}|s_{t:T})$ to the final posterior recovered by iterative inference over the course of learning. These results demonstrate...
Figure 1. (a - c): Amortised predictions of $q_\phi(a, \tau | s_t; \theta)$ are shown in red, where $\bullet$ denote the expected states, shaded areas denote the predicted actions variance at each step, and the expected trajectory recovered by iterative inference is shown in blue. At the onset of learning (a), the amortised predictions are highly uncertain, and thus have little influence on the final approximate posterior. As the amortised model $f_\phi(\cdot)$ learns (b), the certainty of the amortised predictions increase, such that the final posterior remains closer to the initial amortised guess. At convergence, (c), the iterative phase of inference has negligible influence on the final distribution, suggesting convergence to a model-free algorithm. (d) Here, we compare our algorithm to its constituent components – the soft-actor critic (SAC) and an MPC algorithm based on the cross-entropy method (CEM). These results demonstrate that the hybrid model significantly outperforms both of these methods.

strate that when the amortised predictions are uncertain, such as at the start of learning, the posterior inferred by iterative inference is relatively unaffected by the amortised predictions, suggesting the model acts in a primarily model-based manner. Once sufficient data has been collected and the amortised predictions are precise, the iterative phase of inference has a negligible effect on the final distribution, suggesting a gradual convergence from a model-based to a model-free algorithm.

Continuous control As a proof of principle, we demonstrate our algorithm can scale to complex tasks by evaluating performance on the challenging Half-Cheetah task (see Appendix D for details). We compare the CHI algorithm to the model-free SAC and a model-based planning algorithm which utilises the cross-entropy method for trajectory optimisation. These results demonstrate that CHI outperforms both baselines in terms of sample efficiency and asymptotic performance. Note that the performance of MPC is lower than what has been reported in previous literature. We believe this is due to the fact that we utilised fewer parameters relative to prior work. These results suggest that a hybrid approach can help stabilize planning algorithms, enabling comparable performance with reduced computational overhead. Indeed, there is no difference between the MPC algorithm and the iterative component of the CHI algorithm, thus establishing the benefit of a hybrid approach.

5. Conclusion

In this work, we have introduced control as hybrid inference (CHI), a framework for combining model-free policy optimisation and model-based planning in a probabilistic setting, and provided proof-of-principle demonstrations that CHI retains the sample efficiency of model-based RL and the asymptotic performance of model-free RL. We finish by highlighting several additional benefits afforded by the CHI framework. First, initialising a model-based planning algorithm with an ‘initial guess’ significantly reduces the search space. Moreover, by employing amortised inference schemes that utilise a value function, it should be possible to estimate the value of actions beyond the planning horizon. Furthermore, the fact that the certainty of amortised predictions increases over the course of learning suggests the possibility of terminating iterative inference once a suitable threshold (in terms of the standard deviation) has been reached, which would decrease the computational cost of model-based planning. We also expect that the relative influence of the two algorithms will be adaptively modulated in the face of changing environmental contingencies, as confirmed in preliminary experiments. Finally, the CHI framework provides a formal model of the hypothesis that model-free and model-based mechanisms coexist and compete in the brain according to their relative uncertainty (Niv et al., 2006; Daw et al., 2005), as well as explaining habituation, or the gradual transition from goal-directed to habitual action after sufficient experience. (Gläscher et al., 2010).

While we have proposed one implementation of CHI based on initialisation, several alternatives exist. For instance, the amortised component could be incorporated as an action prior in the graphical model. Moreover, while we have implemented CHI using particular algorithms, these could be replaced by a wide range of state-of-the-art RL algorithms. This is possible due to the observation that, under a control as inference perspective, model-based planning and model-free policy optimisation generally correspond to iterative and amortised inference, respectively (Millidge et al., in press).
Control as Hybrid Inference

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A. Full Algorithm

Algorithm 1 Inferring actions via CHI

Input: Planning horizon $H$ — Optimisation iterations $I$ — Number of samples $K$ — Current state $s_t$ — Transition distribution $p_\lambda(s_{t+1}|s_t, a_t)$ — Amortisation function $f_\phi(\cdot)$

Amortised Inference:

$\Phi_\theta(\tau) = \delta(s_t) \prod_{i=1}^{T} p_\lambda(s_{t+i}|s_t, a_t) q_\phi(a_t|s_t; \theta)$

Extract $\theta^{(i)} = \left\{ \mu_{i:T}, \sigma_{i:T}^2 \right\}$ from $\Phi_\theta(\tau)$

Initialise $q(a_t; T; \theta)$ with parameters $\theta^{(1)}$

Iterative Inference:

for optimisation iteration $i = 1...I$ do

Sample $K$ action sequences $\{(a_{i:T})_k \sim q(a_{i:T}; \theta)\}_{k=1}^K$

Initialise particle weights $W^{(i)} := \{w_k^{(i)}\}_{k=1}^K$

for action sequence $k = 1...K$ do

$w_k^{(i+1)} \propto W^{(i)}(a_{i:T})_k \cdot q^{(i)}(a_{i:T})_k | \theta$

$\theta^{(i+1)} \leftarrow \arg\max_{\theta} \log E_{W^{(i+1)}} \left[ log p_\lambda(s_{t+1}|s_t, a_t) \right]$ 

end

end

Extract $\mu_{i:T}$ from $q(a_{i:T}; \theta)$

return $\mu_i$

B. Model Details

The proposed CHI algorithm requires a model of the transition dynamics $p_\lambda(s_{t+1}|s_t, a_t)$. This model appears in the iterative inference algorithm, where it is used to evaluate the expected trajectory of states $s_{1:T}$, given some sampled action sequence $(a_{1:T})_k$. The model also appears in the amortised inference algorithm (Eq. 5), where it is again used to calculate the expected trajectory of states under an amortised policy $q_\phi(a_{1:T}; s_{1:T}; \theta)$.

Rather than treat $\lambda$ as a point estimate, we consider it to a random variable which must be inferred based on the available dataset $D$. Here, we utilise an ensemble approach to approximating $p(\lambda|D)$ (Chua et al., 2018; Kurutach et al., 2018). This approach approximates $p(\lambda|D)$ as a set of particles $p(\lambda|D) \approx \frac{1}{K} \sum_{k=1}^{K} \delta(\lambda - \lambda_k)$, where $E$ is the number of networks in the ensemble and $\delta$ is the Dirac delta function. Each particle $\lambda_k$ is optimised to maximise $\log p(\lambda_k|D) \times \log p(D|\lambda_k)p(\lambda_k)$, and where a uniform prior over $\lambda_k$ is assumed. The model is updated after each episode, iterating over the available dataset for 10 epochs in batches of 50. An ensemble of 5 is used, where each member of the ensemble is a 3 layer neural network with 350 nodes, which is trained to predict a Gaussian distribution over the change in state, as opposed to directly predicting the next state.

C. Parameters

We utilise a relatively small number of parameters relative to previous work. We consider a planning horizon $H$ of 7, 500 samples for the iterative inference procedure. During training, action noise $\epsilon \sim \mathcal{N}(0, 0.3)$ is added to actions to promote exploration. We use a hidden size of 256 for the value and Q-networks used in SAC. We do not use an adaptive $\alpha$ for SAC and instead set it to a constant value of 0.2.

D. Experiment Details

D.1. Didactic Experiment

For the didactic experiment, we use a simple 2D point mass environment, where the agent must simply navigate to a goal. The agent can control its $x$ and $y$ velocity, $a = (\Delta x, \Delta y)$, with a maximum magnitude of $||a|| = 0.05$. The environment is a grid of shape $[0, 1]^2$ that contains a wall with a small opening, making a direct path to the goal impossible. The agent
Figure 2. Graphical model for control as inference.

starts at \((x = 0, y = 0)\) and the goal is at \(g = (x = 1, y = 1.0)\). The reward function is \(r(s_t, a_t) = 1 - ||s_t - g||^2\) which rewards the agent for navigating towards the goal.

D.2. Continuous Control Benchmark

We utilise the Half Cheetah environment \((S \subseteq \mathbb{R}^1, A \subseteq \mathbb{R}^6)\) which describes a running planar biped. We consider a running task, where a reward of \(v - 0.1||a||^2\) is received, where \(v\) is the agent’s velocity and \(a\) are the agent’s actions. Each episode consists of 100 steps, where an action repeat of 4 is used.

E. Evidence Lower Bound Derivation

\[
\log p(O_{1:T}) = \log \int p(\tau | O_{1:T}) ds_{1:T} da_{1:T} \\
= \log \int p(\tau | O_{1:T}) q(\tau) ds_{1:T} da_{1:T} \\
= \log \mathbb{E}_{q(\tau)} \frac{p(\tau | O_{1:T})}{q(\tau)} \\
\geq \mathbb{E}_{q(\tau)} \left[ \log p(\tau | O_{1:T}) - \log q(\tau) \right] 
\]

F. Evidence Lower Bound Simplification

Following the main text, we define \(q(s_1) = p(s_1)\) and \(q(s_{t+1} | s_t, a_t) = p_\lambda(s_{t+1} | s_t, a_t)\), allowing us to simplify Equation 6 as follows:

\[
\mathbb{E}_{q(\tau)} \left[ \log p(\tau | O_{1:T}) - \log q(\tau) \right] \\
= \mathbb{E}_{q(\tau)} \left[ \log p(s_1) + \log p(O_{1:T} | \tau) + \log p_\lambda(s_{2:T} | s_{1:T}, a_{1:T}) \\
- \log p(s_1) - \log q_\theta(a_{1:T} | s_{1:T}) - \log p_\lambda(s_{2:T} | s_{1:T}, a_{1:T}) \right] \\
= \mathbb{E}_{q(\tau)} \left[ \log p(O_{1:T} | \tau) - \log q_\theta(a_{1:T} | s_{1:T}) \right] 
\]

where the last line is derived from the fact that the terms \(p_\lambda(s_{t+1} | s_t, a_t)\) and \(p(s_1)\) appear on both the numerator and denominator.

G. The Data Bias Issue

Training the amortised algorithm from data generated by the CHI algorithm poses a significant issue, which we here refer to as the data bias issue. Because iterative planning algorithms consider hundreds of potential actions before selecting one to be executed, they have an inherent bias to avoiding sub-optimal actions. The resulting dataset is thus highly biased towards good actions, meaning that information about sub-optimal actions does not get propagated to the amortised algorithm. This
poses an issue – even for off-policy algorithms such as SAC. To overcome this, we train the amortised algorithm on the
counterfactual rollouts generated by the iterative planning algorithm. This procedure generates a huge amount of (suboptimal
and optimal) data that would otherwise be discarded. Empirically, we found this method to be crucial for successful learning.

H. Related Work

Combining model-based and model-free RL A number of methodologies exist for combining model-free and model-
based RL (Li, 2020; Che et al., 2018). Previous work has considered using a learned model to generate additional data
for training a model-free policy (Gu et al., 2016; Sutton, 1990; 1991). In (Chebotar et al., 2017), the authors consider
linear-Gaussian controllers as policies and derive both model-based and model-free updates. In (Farshidian et al., 2014), the
authors consider a similar initialisation approach to our own, but use a model-based algorithm to initialize a model-free
algorithm. This is in contrast to our approach, where the model-free policy initializes the model-based planning algorithm.
The initialisation method used in the current paper mirrors the use of policy networks to generate proposals for the Monte-
Carlo tree search in AlphaGo (Silver et al., 2016; 2017). Several papers look to use the learned model to initialize a
model-free policy (Nagabandi et al., 2018).

Combining Amortised & Iterative Inference The idea of combining amortised and iterative inference has been explored
previously the context of unsupervised learning. Such approaches look to retain the computational efficiency of amortised
inference models while incorporating the more powerful capabilities of iterative inference. The semi-amortised variational
autoencoder was introduced in (Kim et al., 2018), which also employs amortised inference to initialize a set of variational
parameters, which are then refined using iterative inference. The authors demonstrate that this approach helps overcome
the ‘posterior collapse’ phenomenon, which describes when the latent code of the auto-encoder is ignored and presents a
common issue when training variational autoencoders. An iterative amortised inference algorithm was proposed by (Marino
et al., 2018b), where posterior estimates provided by amortised inference are iteratively refined by repeatedly encoding
gradients. It was demonstrated that this approach helps overcome the amortisation gap (Krishnan et al., 2017; Cremer
et al., 2018), which describes the tendency for amortised inference models to not reach fully optimised posterior estimates,
likely due to the significant restriction of optimising a direct (and generally feed-forward) mapping from data to posterior
parameters. This iterative amortised inference model was later applied to variational filtering (Marino et al., 2018a). In
(Satorras et al., 2019), the authors propose a hybrid inference scheme for combing generative and discriminative models,
which is applied to a Kalman Filter, demonstrating an improved accuracy relative to the constituent inference systems. The
biological plausibility of hybrid inference schemes has been explored in the context of perception (Marino, 2019), utilising
the predictive coding framework from cognitive neuroscience (Rao & Ballard, 1999; Friston, 2005; Walsh et al., 2020). A
hybrid inference approach which iteratively refines amortised predictions has also been explored in (Hjelm et al., 2016;
Krishnan et al., 2017; Shu et al., 2019).