The existence of dark energy is one of the most significant cosmological discoveries over the last century. Various models of dark energy have been proposed, such as a small positive cosmological constant, quintessence, k-essence, phantom, holographic dark energy, etc., see [2] for recent reviews with fairly complete lists of references of different dark energy models. However, although fundamental for our understanding of the universe, its nature remains a completely open question nowadays.

Inspired by string/M theory, the idea that our universe is a 3-brane embedded in a higher dimensional spacetime has received a great deal of attention in recent years. Among various braneworld models, the model proposed by Dvali, Gabadadze and Porrati (DGP) [3] is impressive. In the DGP model, the bulk is a flat Minkowski spacetime, but a reduced gravity term appears on the brane without tension. In this model, gravity appears 4-dimensional at short distances but is altered at distance large compared to some freely adjustable crossover scale through the slow evaporation of the graviton off our 4-dimensional brane world universe into an unseen, yet large, fifth dimension. In the DGP model, the gravitational behaviors on the brane are commanded by the competition between the 5-dimensional curvature scalar $(5)R$ in the bulk and the 4-dimensional curvature scalar $R$ on the brane. At short distances the 4-dimensional curvature scalar $R$ dominates and ensures that gravity looks 4-dimensional. At large distances the 5-dimensional curvature scalar $(5)R$ takes over and gravity spreads into extra dimension. The late-time acceleration is driven by the manifestation of the excruciatingly slow leakage of gravity off our four-dimensional world into an extra dimension. This offers an alternative explanation for the current acceleration of the universe [4]. However, just as LCDM model, it is also suffered from coincidence problem. In LCDM model, $\lambda$ is a constant with respect to redshift $z$, while the density of matter evolves as $(1+z)^3$. Hence why do they approximately equal each other now? This is the coincidence problem in LCDM model. To solve the dark energy problem in frame of DGP model, people require $r_c \sim H_0^{-1}$, where $r_c$ is a constant, and $H_0$ is present Hubble parameter. Similarly, we can ask why $r_c$ approximately equals the Hubble radius now.

The coincidence problem in LCDM model. This is the coincidence problem in frame of braneworld scenario a brane is moving in the bulk, and generally speaking, its proper acceleration does not vanish. One may expect that a brane, as an observer in the bulk space, should perceive the Unruh temperature. The Unruh effect of RS II braneworld is investigated in [5].

However, there is a serious problem to deduce the Unruh effect in AdS spacetime. It is found that there are several natural choices of vacua, corresponding to the fact that AdS spacetime is not globally hyperbolic. The specification of the vacuum state then depends on boundary conditions imposed at timelike infinity. A particle detector following a timelike worldline in an AdS spacetime will in some cases detect a flux of particles with a thermal or modified thermal spectrum. Compared to the AdS spacetime the nature choice of the vacuum state of the Minkowski space, as the bulk of DGP model, is unique. We study the Unruh effect for a DGP brane in detail in [10], which shows that the Unruh temperature does not vanish even at the limit $r_c \to \infty$, which means the gravitational effect of the 5th dimension vanishes. Furthermore, the Unruh temperature is always higher than geometric temperature for a dust dominated brane in the whole history of the universe for both of the two branches and for all three types of spatial curvature. So an influx from bulk to brane will appear if brane interacts with bulk based on the most sound principle of thermodynamics.

Although this difference in temperature indicates an energy influx, it offers no hint of the form of the inter-
action term. Alternatively, in this letter, we refer ourselves to considerations based on statistical mechanics to derive the interaction term between Unruh radiation and dark matter confined to the brane. Before proposing our construction, it will be helpful to give a brief review of DGP cosmology and Unruh effect for a DGP model.

Assuming an Friedmann-Robertson-Walker (FRW) metric on a DGP brane, one can derive the Friedmann equation on the brane \[4\],

\[
H^2 + \frac{k}{a^2} = \frac{\rho}{3\mu^2} + \frac{2\epsilon}{r_c} + \frac{2\epsilon}{r_c} \left( \frac{\rho}{3\mu^2} + \frac{1}{r_c^2} \right)^{1/2},
\]

where \(\rho\) denotes matter energy density on the brane, \(r_c = \kappa^2\mu^2\), denotes the cross radius of DGP brane, \(\kappa^2\) is the 5-dimensional gravitational constant, \(\mu\) is the 4-dimensional reduced Planck mass, \(\epsilon = \pm 1\), represents the two branches of DGP model, \(H\) is Hubble parameter, \(k\) is spatial curvature, and \(a\) is the scale factor of the brane. By using the junction condition across the brane, one derives the acceleration of brane \[11\],

\[
A = \frac{\kappa^2}{6}(2\rho_e + 3p_e).
\]

Here \(\rho_e\) and \(p_e\) are the effective energy density and pressure of the brane, respectively. We see that, generally speaking, \(A\) does not vanish. So the brane should perceive Unruh-type radiation in the bulk. In \[10\] we consider a detector coupling to a massless scalar field \(\phi\), which satisfies 5 dimensional Klein-Gordon equation,

\[
\Box^{(5)}\phi = 0.
\]

By using Green function method, we derive the temperature of the Unruh radiation \[10\] is similar to a particle detector,

\[
T = \frac{|A|}{2\pi}.
\]

Here, if \(A\) is a negative number, which only means that the direction of the acceleration is contrary to our reference direction. The corresponding Unruh temperature is the same as the case of the positive acceleration \(-A\). So we just take the absolute value of \(A\).

To investigate the microscopic mechanism of interaction between brane and bulk, one should decompose the 5-dimensional modes, which satisfies \[13\], into modes along the brane and modes transverse (not necessarily vertical) to the brane. Following \[12\], we call them type I modes and type II modes, respectively. In a pseudo-Euclidean coordinates system, type I mode reads,

\[
\phi^{(I)} = e^{-ik_\mu x^\mu},
\]

where \(k_\mu\) denotes the momentum of the mode in different directions, and \(\mu = 0, 1, 2, 3\). Type II modes reads,

\[
\phi^{(II)} = e^{-ik_\mu x^\mu - ik_y y},
\]

where \(y\) is the coordinate of the extra dimension, and we require \(k_y \neq 0\). An arbitrary 5 dimensional mode must be either type I or type II. It was shown in \[12\] that Type II modes of the bulk gravitational fluctuations field for which the brane is effectively a mirror. A completely similar argument proves that the brane also behave as a perfect mirror for the type II scalar fluctuations. Hence only type I mode can exchange energy with matter confined to the brane. We show a schematic plan of type I and type II modes in Fig. 1.

By a standard process of canonical quantization, type I modes become photons on the brane. Here photons mean quanta without mass, which do not necessarily satisfy Maxwell equations. In fact we do not even need to study its commutativity in detail. Whether they obey Fermi-Dirac statistics or Bose-Einstein statistics the number density of the photons gas \(n_\gamma\) is proportional to the cubic of its temperature,

\[
n_\gamma \propto T_\gamma^3,
\]

where \(T_\gamma\) is just the temperature of the Unruh radiation \[4\]. The particles confined to the brane are immerged in the thermal bath of these photons. Therefore, based on statistical mechanics the reaction rate \(\Gamma\) between dark matter particles and the type I photons is proportional to the number densities of photons and dark matter particles, and the scattering cross section \(\Gamma \propto n_{dm} n_\gamma \sigma\). Here, \(n_{dm}\) denotes the number density of dark matter particle, and \(\sigma\) represents the scattering cross section. In a low energy region, the internal freedoms can not be excited. This letter concentrates on the late time universe, hence the cross section is effectively constant. Therefore, the reaction rate can be written as an equation by inserting a constant \(b\),

\[
\Gamma = b|A|^3,
\]

where we have used \[11\], \[14\] and \(n_{dm} \propto \rho\). Here, following the previous works on interacting models \[5\], as a good approximation we assume there is only pressureless
dark matter on the brane, which interacts with the Unruh photons. Therefore, the continuity equation of the brane becomes,

\[ \dot{\rho} + 3H\rho = \Gamma. \]  

(7)

As we explain before, the DGP braneworld model also suffers from coincidence problem, which says why the three RHS term of (4) are at the same scale today. To overcome this hurdle, we consider a limiting DGP model, that is, \( r_e \gg H_0^{-1} \). In such a model, the coincidence model is evaded, and at the same time the gravitational effect of the 5th dimension is no longer responsible for the present acceleration. It is shown in [10] that the Unruh effect does not vanish even when \( r_e \rightarrow \infty \). We shall prove that under the situation \( r_e \gg H_0^{-1} \) the interaction between dark matter on the brane and Unruh radiation can drive the observed acceleration of the universe.

In a limiting DGP model, substitute the Friedmann equation (11) into (7), we derive

\[ \frac{\ddot{a}}{a} = \frac{1}{6\mu} \left(-\rho + \sqrt{3}\rho_b A^3\right). \]

(8)

The acceleration of the brane in the bulk \( A \) does not directly depend on energy density of the matter on the brane \( \rho \), but through the effective density \( \rho_e \) and effective pressure \( p_e \) in (9). The full 5 dimensional field equation and the junction condition across the brane yield (12).

\[ \rho_e = \rho - 3\mu^2(H^2 + k/a^2), \]

(9)

\[ p_e = p + \mu^2 \left(\frac{2\ddot{a}}{a} + H^2 + k/a^2\right). \]

(10)

Associate the above equations (2) and (8), we derive

\[ A^{-2} = K\sqrt{\rho}, \]

(11)

where \( K = \kappa^2\mu\sqrt{3}/6 \). Interestingly, we see that the acceleration of the brane \( A \) inversely correlates to the energy density of the brane, which is completely different from the first sight at equation (2). Therefore, we expect the effect of the bulk Unruh radiation is negligible in the early time for a dust dominated limiting DGP braneworld. Only in some low energy region the bulk Unruh radiation becomes important.

Before presenting the exact cosmic solution, we study some qualitative side of the set Friedmann equation (4) and continuity equation (7). We consider a spatially flat universe, in which the stagnation point dwells at \( \delta H = 0 \), or equivalently \( \delta \rho = 0 \). From the continuity equation (7), we derive the energy density \( \rho_s \) at the stagnation point, \( \rho_s = \delta \rho K^{-3/2}/\sqrt{3} \). To investigate the stability of the cosmic fluid in the neighbourhood of the stagnation point, impose a perturbation to the continuity equation,

\[ \left(\delta \rho\right)_{\delta \rho = \rho_s} = \delta \rho \left(\frac{1}{4}\delta \rho^{3/4}K^{-3/2} - \frac{3\sqrt{3}}{2\mu}\rho_s^{1/2}\right). \]

(12)

At the stagnation point, \( \rho = \rho_s \), hence

\[ \left(\delta \rho\right)_{\rho = \rho_s} = -\frac{5\sqrt{3}}{4\mu}\rho_s^{1/2}\delta \rho, \]

(13)

which means it is a stable point. Now we consider the deceleration parameter, which is the most significant parameters from the viewpoint of observations. Here the deceleration parameter reads,

\[ q = \frac{1}{2} \left(1 - \sqrt{3}\mu_b K^{-3/2}\rho_s^{-5/4}\right). \]

(14)

This equation clearly shows that in the early universe \( q \rightarrow 1/2 \), hence the universe behaves as dust dominated, and with the decreasing of energy density the deceleration parameter becomes smaller. Finally at the stagnation point the deceleration parameter ceases at \( q = \frac{1}{2} \left(1 - \sqrt{3}\mu_b K^{-3/2}\rho_s^{-5/4}\right) = -1 \), which implies that the universe enters a de Sitter phase.

Though for arbitrary spatial curvature analytical solution does not exist, we find an exact solution for a spatially flat universe driven by bulk Unruh radiation. For a spatially flat universe, associate Friedmann equation (11) with the continuity equation (7) we obtain,

\[ \rho = 3^{-4/5} \left(\frac{c}{a^{15}} + \frac{4c^{3/4}L}{a^{15/2}} + \frac{6c^{1/2}L^2}{a^{15/2}} + \frac{4c^{3/4}L^3}{a^{15/4}} + L^4\right)^{1/5}, \]

(15)

where \( c \) is an integration constant, and \( L \) is defined by \( L = \sqrt{3}\mu_b K^{-3/2} \). Clearly, from this exact solution, the universe behaves as dust dominated in a high energy region (a small enough), and becomes de Sitter universe in a low energy region (a large enough), which is exactly the same as we concluded before from behaviors of the deceleration parameter in the history of the universe.

Put the exact solution (15) into (1), we obtain an analytical expression of cosmic time \( t \) as function of the scale factor \( a \) by using a hypergeometric function,

\[ t + c_1 = \frac{2(1 + a^{15/4}c^{-1/4}L)F(2/5, 2/5, 7/5, -a^{15/4}c^{-1/4}L)}{3^{3/5} \left(\frac{c}{a^{15}} + \frac{4c^{3/4}L}{a^{15/2}} + \frac{6c^{1/2}L^2}{a^{15/2}} + \frac{4c^{3/4}L^3}{a^{15/4}} + L^4\right)^{1/10}}, \]

(16)

where \( c_1 \) is an integration constant, and \( F \) denotes Gauss hypergeometric function. The requirement \( \lim_{a \to 0} t = 0 \) yields \( c_1 = 0 \). Thus, we complete a cosmic solution of the limiting DGP braneworld model, where the universe is self-accelerated through the bulk Unruh radiation perceived by the brane.

Now we give a summary and some discussions. The acceleration of the universe is one of the most amazing discoveries in last century and its nature becomes one of the most profound problems in theoretical physics. In this letter we put forward a new answer to the nature of the cosmological acceleration.

DGP braneworld model seems a hopeful candidate to explain the cosmological acceleration. But, as we pointed
out before, it also suffers from coincidenece problem. To evade this problem, we consider the limiting DGP model, that is, $r_c > H_0^{-1}$, which means the gravitational effect of the 5th dimension is neglectable. Hence, the cosmological acceleration does not happen in original DGP model under this situation if there is only dust on the brane.

We find the cosmos can accelerate through interacting the bulk Unruh radiation in a limiting DGP model, which is implied by our previous work, even the brane is dust dominated. Different from previous works on the brane-bulk interaction, we find the interaction form through careful studies on the microscopic mechanism of interaction between brane and bulk. It is shown that the interaction term can be settled up to a constant factor $b$. Base on these constructions we find the acceleration of the brane $A$ is inversely correlates to the energy density of the brane for a dust dominated limiting DGP brane. Therefore, the Unruh effect is not important in the early universe in our model. Finally we derive an exact solution for a spatially flat model. This solution shows clearly that the universe behaves as dust dominated at early time and enters an de Sitter phase at late time, which is consistent with observations. We also show the de Sitter phase is stable.

In this letter only massless Unruh mode is considered. Although it is the most important mode in low energy region, the massive mode also deserves to study further. Using the 5th dimension is neglectable. Hence, the Unruh effect is not important in the early universe in our model. Finally we derive an exact solution for a spatially flat model. This solution shows clearly that the universe behaves as dust dominated at early time and enters an de Sitter phase at late time, which is consistent with observations. We also show the de Sitter phase is stable.

The parameter $b$ in (9) is critical to our model, which encloses all the undetermined information of interaction between dark matter and Unruh radiation. Here we first present a preliminary estimation of its value from the deceleration parameter $q$. From various observations the present value of deceleration parameter $q \sim -0.5$, therefore,

$$\frac{1}{36v^2} \frac{1}{3^{1/4}} \frac{1}{bH_0^2} (r_c H_0)^{-3/2} \left( \frac{\rho}{3\mu^2 H_0^2} \right)^{-5/4} \sim 1, \quad (17)$$

where $\frac{\rho}{\mu^2 H_0^2} = 1$ in our model. We see that a larger cross radius $r_c$ needs a smaller coupling constant $b$. And we have set $r_c H_0 \gg 1$ in previous constructions. Hence the dimensionless coupling constant $bH_0^2$ is a tiny number, which eludes our laboratory experiments even if the dark matter particles were found. Alternatively, we shall turn to astronomical observations to fit our model and hence to determine the value of $b$ in future work, which is also helpful to constraint the cross radius $r_c$ in DGP model.

**Acknowledgments.** This work was supported by the National Natural Science Foundation of China, under Grant No. 10533010, by Program for New Century Excellent Talents in University (NCET) and SRF for ROCS, SEM of China.

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