LHCb strategies for $\gamma$ from $B \rightarrow DK$

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One of the most promising ways to determine the angle $\gamma$ of the CKM unitarity triangle is through measurement of the tree level processes $B \rightarrow DK$. The LHCb collaboration has studied the potential of these decays employing the Atwood-Dunietz-Soni (ADS) and Dalitz methods, making use of a large sample of simulated data. For each method the expected sensitivities to the angle $\gamma$ are presented in this report.

1. INTRODUCTION

This report is arranged as follows. In Section 1 we present a physics motivation, followed by a general discussion of $B \rightarrow DK$ decays \textsuperscript{1}. Section 2 gives a brief description of the LHCb detector, the simulation and the event selection techniques. An introduction to the ADS method to extract the CKM angle $\gamma$ from $B \rightarrow DK$ decays and its application and expected performance in LHCb is presented in Section 3. Section 4 describes the use of the Dalitz method to extract $\gamma$ from $B \rightarrow DK$ decays and the expected LHCb sensitivities. We conclude in Section 5.

1.1. Motivation

In B hadron decays, tree level processes are generally dominated by Standard Model contributions while new physics mainly affects loop diagrams. Any difference between the CKM unitarity triangle as measured in tree level processes compared to loop level processes would indicate new physics in the flavour sector. Currently the unitarity triangle as determined by the tree level measurements of $\gamma$ and the CKM matrix element $|V_{ub}|$ is consistent with the triangle determined by measurements of the loop dominated parameters $\epsilon_K$, $\Delta m_d$, $\Delta m_s$ and sin2$\beta$ \textsuperscript{1}. However, the present measurements of $\gamma$ from $B \rightarrow DK$ at B factories have large errors: $\gamma = (92 \pm 41 \pm 11 \pm 12)\circ$ (BaBar) \textsuperscript{2}, $\gamma = (53^{+15}_{-18} \pm 3 \pm 9)\circ$ (Belle) \textsuperscript{3}. The precision on the angle $\gamma$ from $B \rightarrow DK$ has to be improved to at least 5$\circ$ \textsuperscript{1} to match the precision of its indirect estimate from a global fit to CKM parameters excluding direct measurements of $\gamma$.

In LHCb, the tree level decays $B_{s}^{0} \rightarrow D_{s}^{\pm}K^{\mp}$ can also be used to determine the angle $\gamma$ in a theoretically clean way. This measurement is expected to have a sensitivity of 14$\circ$ with 2 fb$^{-1}$ of data. We will see below that the $B \rightarrow DK$ decays have a greater statistical sensitivity to the angle $\gamma$ with an equivalent amount of data.

1.2. Features of $B \rightarrow DK$ decays

The CKM favoured process $B \rightarrow \bar{D}^{0}K$ and disfavoured process $B \rightarrow D^{0}K$ can be described with three parameters: a weak phase difference $\gamma = arg(-(V_{ub}V_{ab}^{*})/(V_{cd}V_{cb}^{*}))$, a strong phase difference $\delta_B$ and the ratio of magnitudes between the disfavoured and favoured amplitudes, defined as $r_B$. If the $\bar{D}^{0}$ and $D^{0}$ decay to a common final state, then the interference between the two amplitudes via $\bar{D}^{0}$ and $D^{0}$ allows the extraction the angle $\gamma$ with several methods, as illustrated below.

It is expected that $r_B$ is small in the $B^{\pm} \rightarrow DK^{\pm}$ case and has a value of about 0.1 \textsuperscript{4} due to colour suppression in the CKM disfavoured amplitude. For $B^{0} \rightarrow DK^{*0}$, both amplitudes are colour suppressed, therefore $r_B$ is expected to be larger \textsuperscript{4}.

\textsuperscript{1}In this report $D$ represents a $D^0$ or $\bar{D}^0$, $B$ represents a $B^{\pm}$, $B^0$ or $\bar{B}^0$ and $K$ represents a $K^{\pm}$, $K^{*0}$ or $K^{*0}$.
2. THE LHCb DETECTOR

The LHCb detector is a single arm spectrometer dedicated to the study of CP violation in B meson decays at the Large Hadron Collider, which will start operation at CERN in 2007. The detector and its expected performance is described in detail in [1]. Here we only emphasize that LHCb has a 94% tracking efficiency for tracks with momentum above 10 GeV/c, a 93% $K^{\pm}$ identification efficiency and a corresponding probability of 4.7% for a $\pi^{\pm}$ to be misidentified as a $K^{\pm}$ for the momentum range 2 – 100 GeV/c.

2.1. Data simulation and event selection

Monte Carlo simulation data produced with Pythia and Geant4 are used to study the trigger, the reconstruction and event selection, which in turn allows the physics performance to be assessed.

We use event samples consisting of 260 million minimum bias event for trigger studies, 140 million inclusive $b\bar{b}$ events and dedicated signal events for selection and background studies.

Sensitivities on physics parameters are obtained using fast simulation. These are based on efficiencies, resolutions and background levels obtained from the full simulation.

In our studies we use the following information to discriminate between signal and background events: charged particle identification information based on the Ring Imaging Cherenkov detectors; invariant masses; impact parameters; transverse momenta; $\chi^2$ of decay vertices of the $B$, $D$, $K^*$ and $K_S^0$ particles; the opening angle between the momentum direction of a $B$ and its flight direction; the event topology itself.

All selection cuts are optimized to reject most background events from a large sample of inclusive $b\bar{b}$ events while retaining a reasonable signal efficiency. The ratio of background to signal, $B/S$, is assessed using a separate inclusive $b\bar{b}$ sample.

3. ADS METHOD AND SENSITIVITY

3.1. Description of the method

Atwood, Dunietz and Soni (ADS) [7] suggested a method of determining $\gamma$ based on the reconstruction of non-CP eigenstates for decays common to both $D^0$ and $\bar{D}^0$. An example is the hadronic final state $K\pi$, which may arise from the Cabibbo favoured (CF) decay $D^0 \to K^+\pi^-$ or the doubly-Cabibbo suppressed (DCS) decay $D^0 \to K^+\pi^-$. The relation between the CF and the DCS decay is described by a magnitude ratio $r_{D^0}$ and a strong phase difference $\delta_{D^0}$

$$r_{D^0} e^{i\delta_{D^0}} \equiv \frac{A(D^0 \to K^-\pi^+)}{A(D^0 \to K^-\pi^+)} = \frac{A(D^0 \to K^+\pi^-)}{A(D^0 \to K^+\pi^-)}.$$ (1)

Therefore there are four possible $B$ decays, whose decay rates can be written as follows

$$\Gamma(B^- \to (K^-\pi^+)_D K^-) \propto 1 + (r_{D^0}K_{\pi}^\gamma)^2 + 2r_{D^0}K_{\pi}^\gamma \cos(\delta_{D^0} - \delta_{D^0}^K - \gamma),$$ (2)

$$\Gamma(B^- \to (K^+\pi^-)_D K^-) \propto r_{B}^2 + (r_{D^0}K_{\pi}^\gamma)^2 + 2r_{B}K_{\pi}^\gamma \cos(\delta_{B^+} + \delta_{D^0}^K + \gamma),$$ (3)

$$\Gamma(B^+ \to (K^+\pi^-)_D K^+) \propto 1 + (r_{D^0}K_{\pi}^\gamma)^2 + 2r_{B}K_{\pi}^\gamma \cos(\delta_{B^+} - \delta_{D^0}^K - \gamma),$$ (4)

$$\Gamma(B^+ \to (K^-\pi^+)_D K^+) \propto r_{B}^2 + (r_{D^0}K_{\pi}^\gamma)^2 + 2r_{B}K_{\pi}^\gamma \cos(\delta_{B^+} + \delta_{D^0}^K + \gamma),$$ (5)

where the constant of proportionality is the same in each expression. The relative rates of the four processes yield three observables which depend on five parameters $\gamma$, $r_B$, $r_{D^0}$, $\delta_B$ and $\delta_{D^0}$, of which $r_{D^0}K_{\pi}^\gamma$ is already well known. It is necessary to use different $D$ decays in order to determine all parameters.

Similarly, the four-body decay $D^0 \to K\pi\pi\pi$ provides three new observables which depend on $\gamma$, $r_B$, $\delta_B$ and two new parameters $r_{D^0}$ and $\delta_{D^0}$. Further information can be added by including $D$ decays to CP-eigenstates, such as $K^+K^-$ and $\pi^+\pi^-$. Each provides one more observable without introducing any new parameters:

$$\Gamma(B^- \to (h^+h^-)_D K^-) \propto 1 + (r_{B})^2 + 2r_{B} \cos(\delta_{B}-\gamma),$$ (6)

$$\Gamma(B^+ \to (h^+h^-)_D K^+) \propto 1 + (r_{B})^2 + 2r_{B} \cos(\delta_{B}+\gamma).$$ (7)
3.2. Performance with charged $B$ mesons

Results from the B factors favour a small value of $r_B$ for charged $B$ decays [30]. We set $r_B = 0.077$ for this study. The following assumptions are made: $r_D^{K\pi} = r_D^{K^3\pi} = 0.06$, $-25^\circ < \delta_D^{K\pi} < 25^\circ$ and $-180^\circ < \delta_D^{K^3\pi} < 180^\circ$.

Tab. 1 shows the expected signal and background yields in 2 fb$^{-1}$ of data. Of the 17.7 $k$ background events in the favoured $B \to (K\pi)_D K$ modes, 17.0 $k$ are from the decay $B \to D\pi$, which has a 13 times larger branching ratio, with a $\pi$ misidentified as a $K$, and 0.7 $k$ are combinatorial background events. In the suppressed $B \to (K\pi)_D K$ modes, the combinatorial background dominates.

Our simulation study indicates that the signal yields and background level are very similar for the $D \to K3\pi$ modes. Therefore we use the same yields and $B/S$ as in $D \to K\pi$.

A large number of fast samples are generated based on the yields and background levels given above to estimate the statistical precision of $\gamma$. As shown in Tab. 2 a precision of $5^\circ - 15^\circ$ for $\gamma$ is achievable for 2 fb$^{-1}$ of data, depending on the parameter values of $\delta_D^{K\pi}$ and $\delta_D^{K^3\pi}$ and with $r_B = 0.077$. Better precision can be achieved for larger $r_B$ values.

LHCb is also investigating the feasibility to include the decay $B^\pm \to D^{*0}K^\pm$ in the ADS analysis. An attractive feature of the $D^{*0}$ meson is that it can decay to two final states $D^{0}\pi^0$ and $D^{0}\gamma$. These have opposite CP eigenvalues, which lead to a difference of $\pi$ in their strong phases. This is a useful constraint if the two decays can be distinguished experimentally. However, these decays are difficult to fully reconstruct in LHCb because the detection efficiency of a soft photon in the electromagnetic calorimeter is very low while the background is enormous. Alternative approaches which employ a partial reconstruction or which make use of the event topology to reconstruct the momentum of the $\pi^0/\gamma$ are under study.

3.3. Performance with neutral $B$ mesons

The same method can also be applied to the decay $B^0 \to DK^{*0}$ with $D \to K\pi, KK$ or $\pi\pi$. Assuming $r_B = 0.4, 55^\circ < \gamma < 105^\circ$ and $-20^\circ < \delta_B < 20^\circ$, the expected signal yields in 2 fb$^{-1}$ of data and the background-to-signal ratio $B/S$ are given in Tab. 3. The corresponding statistical precision of the angle $\gamma$ is expected to be $7^\circ - 10^\circ$.

4. DALITZ METHOD AND SENSITIVITY

4.1. Three-body $D$ decay

This method for determining $\gamma$ was proposed in [10]. It makes use of the decay $B^\pm \to DK^\pm$ followed by a multibody $D$ decay into a CP eigenstate. Here we explain the basic idea using $D \to K_S^0\pi^+\pi^-$ as an example. The Dalitz phase space of this decay $D \to K_S^0\pi^+\pi^-$ can be fully parameterized with two effective masses $m_+^2 \equiv m^2(K_S^0\pi^+)$ and $m_-^2 \equiv m^2(K_S^0\pi^-)$. The $D^0$ and $D^0$ decay amplitudes can be written as functions $f(m_+^2, m_-^2)$ and $f(m_+^2, m_-^2)$.

The total Dalitz decay amplitudes, defined as $A^- \equiv A(B^- \to (K_S^0\pi^+\pi^-)DK^-)$ and $A^+ \equiv A(B^+ \to (K_S^0\pi^+\pi^-)DK^+)$, are sums of contributions via $D^0$ and $D^0$:

$$A^- = f(m_+^2, m_-^2) + r_B e^{i(-\gamma+\delta_B)} f(m_+^2, m_-^2), \quad (8)$$
$$A^+ = f(m_+^2, m_-^2) + r_B e^{i(\gamma+\delta_B)} f(m_+^2, m_-^2). \quad (9)$$

In the isobar model [11] $f(m_+^2, m_-^2)$ is a coherent sum of contributions of different resonances:

$$f(m_+^2, m_-^2) = \sum_{j=1}^N a_j e^{i\alpha_j} A_j(m_+^2, m_-^2) + b e^{i\beta}, \quad (10)$$

where $a_j, \alpha_j, b$ and $\beta$ are model parameters which have been measured well at the B factories [23].

The $B^\pm$ decay rates are given by

$$\Gamma^-(m_+^2, m_-^2) = |f(m_+^2, m_-^2)|^2 + r_B^2 f(m_+^2, m_-^2)^2$$
$$+ 2 r_B Re[f^*(m_-^2, m_+^2) f(m_+^2, m_-^2) e^{i(-\gamma+\delta_B)}], \quad (11)$$
$$\Gamma^+(m_+^2, m_-^2) = |f(m_+^2, m_-^2)|^2 + r_B^2 f(m_+^2, m_-^2)^2$$
$$+ 2 r_B Re[f^*(m_-^2, m_+^2) f(m_-^2, m_+^2) e^{i(\gamma+\delta_B)}]. \quad (12)$$

We can see that the interference terms are sensitive to $\gamma$, which can be determined by measuring $\Gamma^-(m_+^2, m_-^2)$ and $\Gamma^+(m_+^2, m_-^2)$ across the Dalitz phase space.
Table 1
Expected signal yields $S$, number of background events $B$ and the ratio $B/S$ in 2 fb$^{-1}$ of data for ADS decay modes of $B^\pm$ corresponding to $\delta_B = 130^\circ$ and $\delta^{K\pi}_D = 0^\circ$. The uncertainty on the background estimates is around 60% for the rarest modes.

| decay mode                  | $S$  | $B$  | $B/S$ |
|-----------------------------|------|------|-------|
| $B^+ \to (K^+\pi^-)DK^+$   | 28 $k$ | 17.7 $k$ | 0.6   |
| $B^- \to (K^-\pi^+)DK^-$   | 28 $k$ | 17.7 $k$ | 0.6   |
| $B^+ \to (K^-\pi^+)DK^+$   | 530  | 770  | 1.5   |
| $B^- \to (K^+\pi^-)DK^-$   | 180  | 770  | 4.3   |
| $B^+ \to (K^+K^-/\pi^+\pi^-)DK^+$ | 4.3 $k$ | 4.3 $k$ | 1.0 |
| $B^- \to (K^+K^-/\pi^+\pi^-)DK^-$ | 3.3 $k$ | 3.3 $k$ | 1.0 |

Table 2
The statistical error of $\gamma$ for different values of $\delta^{K\pi}_D$ and $\delta^{K_{3\pi}}_D$ for $r_B = 0.077$. Numbers with * are RMS values quoted for non-Gaussian distribution of fit results due to close lying ambiguous solutions. These will disappear as the signal yields increase.

| $\delta^{K\pi}_D$ | $-25^\circ$ | $-16.6^\circ$ | $-8.3^\circ$ | $0^\circ$ | $8.3^\circ$ | $16.6^\circ$ | $25^\circ$ |
|-------------------|-------------|---------------|-------------|----------|-----------|-------------|----------|
| $\delta^{K_{3\pi}}_D = -180^\circ$ | 8.6$^o$ | 7.5$^o$ | 6.5$^o$ | 6.8$^o$ | 7.2$^o$ | 7.3$^o$ | 6.0$^o$ |
| $\delta^{K_{3\pi}}_D = -120^\circ$ | 6.0$^o$ | 6.3$^o$ | 6.3$^o$ | 6.4$^o$ | 6.2$^o$ | 6.2$^o$ | 4.7$^o$ |
| $\delta^{K_{3\pi}}_D = -60^\circ$ | 8.0$^o$ | 7.9$^o$ | 8.1$^o$ | 7.8$^o$ | 7.4$^o$ | 6.7$^o$ | 6.2$^o$ |
| $\delta^{K_{3\pi}}_D = 0^\circ$ | 10.3$^o*$ | 11.1$^o*$ | 12.0$^o*$ | 11.5$^o*$ | 12.1$^o*$ | 13.1$^o*$ | 13.0$^o*$ |
| $\delta^{K_{3\pi}}_D = 60^\circ$ | 9.1$^o*$ | 10.6$^o*$ | 11.2$^o*$ | 12.9$^o*$ | 13.4$^o*$ | 15.0$^o*$ | 15.2$^o*$ |
| $\delta^{K_{3\pi}}_D = 120^\circ$ | 11.6$^o*$ | 11.3$^o*$ | 11.8$^o*$ | 11.0$^o*$ | 10.9$^o*$ | 11.1$^o*$ | 10.9$^o*$ |
| $\delta^{K_{3\pi}}_D = 180^\circ$ | 8.5$^o$ | 7.4$^o$ | 6.5$^o$ | 6.8$^o$ | 7.1$^o$ | 7.3$^o$ | 6.5$^o$ |

To reconstruct the $B^\pm \to (K_S^0\pi^+\pi^-)_D K^\pm$ events is challenging with the LHCb detector as only 25% of the $K_S^0$ particles decay inside the active region of the vertex detector. The expected signal yield in 2 fb$^{-1}$ of data will vary between 1.5 $k$ and 5 $k$, depending on how many of the $K_S^0$ decays successfully found offline can be reconstructed within the CPU constraints of the High Level Trigger. A full simulation has been performed to estimate the background level. The combinatorial background is expected to contribute less than 3.5 $k$ events and contamination from $B^\pm \to (K_S^0\pi^+\pi^-)_D \pi^\pm$ is expected to be around 1200 events. Our present sensitivity studies for $\gamma$ do not include background, and do not take into account the non-flat acceptance efficiency in the Dalitz space, which is expected as a result of the trigger and offline selection. Under these assumptions, a statistical precision of $\sigma_\gamma \approx 8^o - 16^o$ is achievable in 2 fb$^{-1}$ of data. The actual statistical precision will depend on the final background level and on $r_B$.

LHCb is also investigating the decays $B^\pm \to (K_S^0K^+K^-)_D K^\pm$ and $B^0 \to (K_S^0\pi^+\pi^-)_D K^{\ast 0}$.

4.2. Four-body $D$ decay
The Dalitz method can be extended from three to four-body $D$ decays. In this case five parameters are required to describe the Dalitz phase space. The $D$ decay model has been studied in the FOCUS experiment [12] and our $\gamma$ sensitivity studies are based on their results.

Assuming a branching ratio of $B(B^\pm \to (K^+K^-\pi^+\pi^-)_D K^\pm) = 9.5 \times 10^{-7}$, our full simulation yields 1.7 $k$ events in 2 fb$^{-1}$ of data. Based on this we estimate a statistical precision for $\gamma$ to be $\sigma_\gamma \approx 14^o$, where we have assumed $\gamma = 60^o$, $r_B = 0.08$ and $\delta_B = 130^o$, and have not yet in-
Table 3

Expected signal yields in 2 fb$^{-1}$ of data and the background-to-signal ratio $B/S$ for the ADS modes of neutral $B$ mesons. Upper limits with 90% confidence level are quoted for $B/S$.

| decay mode | $S$ | $B/S$ |
|------------|-----|-------|
| $B^0 \to (K^- \pi^+) D K^{*0} + c.c.$ | 3400 | $<0.3$ |
| $B^0 \to (K^+ \pi^-) D K^{*0} + c.c.$ | 500  | $<1.7$ |
| $B^0 \to (K^+ K^- / \pi^+ \pi^-) D K^{*0} + c.c.$ | 500  | $<1.4$ |

cluded background and detector effects. Results from more recent studies can be found in [13].

We are also studying an amplitude analysis for $B^\pm \to (K^\pm \pi^\mp \pi^\mp \pi^\pm) D K^\pm$. Compared with the ADS analysis discussed in Section 3, the advantage of this method is that it takes into account the variation of the strong phase $\delta K^{3\pi}$ in the Dalitz phase space.

4.3. Systematic errors

The biggest systematic uncertainty in the Dalitz method arise from the model dependence of the $D$ decay. In the present B-factory $B^\pm \to (K^0 S \pi^+ \pi^+ \pi^-) D K^\pm$ analyses this uncertainty is around 10°. This error is expected to reduce significantly through exploitation of the coherently produced $D$ mesons available at CLEO-c [14] and BES [15]. A discussion of how these data may be used in a model independent analysis can be found in [10]. Similar techniques can be used for the four-body decay mode, where LHCb also expects large numbers of flavour-tagged $D$ decays for use in model calibration.

5. CONCLUSIONS

We have shown that LHCb will be able to extract the CKM angle $\gamma$ in several ways with $B \to D K$ decays. The combined result is expected to have a precision of around 5° with 2 fb$^{-1}$ of data. Such a result will make it possible to compare the LHCb measurement of the angle $\gamma$ with the indirect determination from a CKM fit and thereby perform a stringent test of the Standard Model. Together with improvement on the $|V_{ub}|$ measurement at the B factories this will provide a precise reference Unitarity Triangle against which new physics searches can be compared.

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