Magneto-optical Feshbach resonance: controlling cold collision with quantum interference

Bimalendu Deb

Department of Materials Science, Indian Association for the Cultivation of Science, Jadavpur, Kolkata 700032, India
E-mail: msbd@iacs.res.in

Received 23 December 2009, in final form 17 February 2010
Published 6 April 2010
Online at stacks.iop.org/JPhysB/43/085208

Abstract
We propose a method of controlling two-atom interaction using both magnetic and laser fields. We analyse the role of quantum interference between magnetic and optical Feshbach resonances in controlling cold collision. In particular, we demonstrate that this method allows us to suppress inelastic and enhance elastic scattering cross sections. Quantum interference is shown to modify significantly the threshold behaviour and resonant interaction of ultracold atoms. Furthermore, we show that it is possible to manipulate not only the spherically symmetric s-wave interaction but also the anisotropic higher partial-wave interactions which are particularly important for high-temperature superfluid or superconducting phases of matter. (Some figures in this article are in colour only in the electronic version)

1. Introduction
Two-particle interaction is a key to describing interacting many-particle systems at the microscopic level. The means of manipulating this interaction enables us to explore the physics of such systems with controllable interaction. In solid state systems, the scope of externally controlling inter-particle interactions is limited due to crystalline structures. By contrast, ultracold atomic gases offer a unique opportunity since their interatomic s-wave interaction is widely tunable by a magnetic Feshbach resonance (MFR) [1]. New insight into the exotic phases of interacting electrons in solids can be gained from the experiments involving ultracold atoms with tunable interactions. Atom–atom interactions can also be manipulated by an optical Feshbach resonance (OFR) [2], albeit with limited efficiency. Over the last decade, the MFR [3, 4] has been extensively used to study interacting Bose [5–8] and Fermi gases [9–11] of atoms. Electric fields [12, 13] can also be used to alter interatomic interaction.

The MFR relies on the interplay of Zeeman effects and hyperfine interactions while the OFR is based on photoassociation (PA) [14–16] of two colliding ground-state atoms into an excited molecular state. The OFR has been demonstrated in recent experiments [17–19]. Recently, PA spectroscopy in the presence of an MFR has attracted a lot of attention both experimentally [20–23] and theoretically [24–28]. Junker et al [20] have observed asymmetric profile in the PA spectrum under the influence of an MFR. This spectral asymmetry results from Fano-type quantum interference [29] in continuum-bound transitions [26]. The use of quantum interference to control Feshbach resonance had been suggested earlier by Harris [30]. Of late, quantum interference has been observed in two-photon PA [31–33] and coherent atom–molecule conversion [34]. It has also been shown that Fano’s theory [29] can account for the PA spectrum [35, 36] even in the absence of any MFR.

Here we demonstrate theoretically a new method of altering two-atom interaction. Let us consider that a laser field is tuned near a PA transition of two atoms which are simultaneously influenced by a magnetic field-induced Feshbach resonance. There are two competing resonance processes occurring in this system. One is the MFR attempting to associate the two ground-state atoms into a quasi-bound state embedded in the ground continuum. The other one is the PA resonance tending to bind the two atoms into an excited molecular state.

PA transitions can occur in two competing pathways which originate from the perturbed and unperturbed continuum states. The Fano-type quantum interference between these two pathways can be used to control atom–atom interaction. This quantum control of two-body...
interaction due to applied magnetic and optical fields is what we call the ‘magneto-optical Feshbach resonance’ (MOFR). In a strong-coupling regime of PA transitions, the s-wave scattering state gets coupled to higher partial-wave states [37, 38] via two-photon continuum-bound dipole coupling. Since the s-wave scattering amplitude is largely enhanced due to the applied magnetic field, amplitudes of the higher partial waves coupled to the s-wave will also be largely modified. By resorting to a model calculation, we present explicit analytical expressions for phase shifts, elastic and inelastic scattering rates which manifestly show the significant effects of quantum interference in controlling collision. Resonant interaction arises in many physical situations [39–41]. It is therefore important to devise coherent control of resonant interaction.

2. The model

As a simple model, we consider three-channel time-independent scattering of two homonuclear alkali atoms in the presence of a magnetic and a PA laser field. Here, channel implies asymptotic hyperfine or electronic states of the two atoms. There are two ground hyperfine channels of which one is energetically open (labelled as channel 1) and the other one is closed (channel 2) in the separated atom limit. Channel 3 belongs to an excited molecular state which asymptotically corresponds to two separated atoms with one ground and the other excited atom. We assume that the collision energy is close to the binding energy of a quasi-bound state supported by the ground closed channel. It is further assumed that the rotational energy spacing of the excited molecular levels is much larger than PA laser linewidth so that the PA laser can effectively drives transitions to a single ro-vibrational level (v, J) of the excited molecule, where v stands for vibrational and J for rotational quantum numbers. The angular state of the two atoms in the molecular frame of reference can be written as |JΩM⟩ = iℓ! 2jπM D;j,j(0), where Ω is the projection of the electronic angular momentum along the internuclear axis and M is the z-component of J in the space-fixed coordinate (laboratory) frame. D;j,j(0) is the rotational matrix element with ĵ representing the Euler angles for transformation from the body-fixed to space-fixed frame. In our model, we assume that the PA laser is tuned near the resonance of the J = 1 level of the excited molecule.

The energy-normalized dressed state of these three interacting states with the energy eigenvalue E can be written as

\[ \Psi_E = \sum_{M} \frac{\phi_{vJM}(r)}{r} |e⟩|JΩM⟩ + \frac{χ(r)}{r} |g_{1}⟩|000⟩ + \int dE′ \frac{\beta_{E'}}{r} \sum_{M} \frac{\Psi_{E'JM}(r)}{r} |g_{1}⟩|00m_{l}⟩ \]

where \( \phi_{vJM}(r) \) is the radial part of the excited molecular state, \( χ(r) \) is the bound state in the closed channel and \( \Psi_{E'JM}(r) \) represents the energy-normalized scattering state of the partial wave \( m_{l} \) with \( m_{l} \) being the projection of \( J \) along the space-fixed z-axis. \( |g_{1}⟩ \) and \( |e⟩ \) denote the internal electronic states of the \( n^th \) ground and excited molecular channels, respectively. Here \( E' = h^2 k^2/(2μ) \) is the collision energy, where \( k \) and \( μ \) are the relative momentum and reduced mass of the two atoms, respectively. \( β_{E'} \) denotes the density states of the unperturbed continuum. Note that \( φ_{vJM}(r) \) and \( χ(r) \) are the perturbed bound states. In the limit \( r \to \infty \), we have

\[ rΨ_E \to \int dE' β_{E'} \sum_{M} \langle g_{1}|Ψ_{E'JM}(r)⟩|00m_{l}⟩, \]

and thus the scattering properties in the MOFR are determined by the asymptotic behaviour of \( Ψ_{E'JM} \).

From the time-independent Schrödinger equation, under the Born–Oppenheimer approximation, we obtain the following coupled differential equations:

\[ \hat{h}_{J} + V_{1}(r) − h\delta_{1} − E − ihγJ/2)\hat{φ}_{vJM} = −\sum_{M} Λ_{JM} ψ_{EJM} + Λ_{0JM} χ, \]

\[ \hat{h}_{0} + V_{2}(r) − E)χ = −\sum_{M} Λ_{JM} \phi_{vJM} − V_{12}Ψ_{E00}, \]

\[ \hat{h}_{1} + V_{1}(r) − E)Ψ_{EJM} = −\sum_{M} Λ_{JM} \phi_{vJM} − δ_{01}V_{12}χ, \]

where \( Ψ_{EJM} = \int dE' β_{E'} \Psi_{E'JM} \), \( h\delta_{1} \) is the free-molecule energy, \( χ \) is the excited electronic state, \( Ψ_{E00} \) is the ground state, \( ψ_{EJM} \) are the partial-wave states, \( V_{12} \) is the scattering potential, and \( \Lambda_{JM} = −(JMΩ)[\hat{D}_{J}, \hat{E}_{PA}](0,0) \), where \( \hat{D}_{J} \) is the transition dipole moment between the excited and the ground states.

3. The solution

The coupled equations (2)–(4) can be conveniently solved by the method of Green’s function. Let \( φ^{0}_{JM} \) be the excited bound state solution of the homogeneous part of (2) with the binding energy \( E_{vJ} \). Using the Green’s function \( G_{v}(r, r', J) = −\frac{ϕ^{0}_{JM}(r')}{ΔE_{vJ} + iγJ/2} \), where \( ΔE_{vJ} = h\delta_{1} + E - E_{vJ} \), we can write

\[ φ_{vJM}(r) = \int dE' β_{E'} \sum_{M}Λ_{E'JM} + \Lambda_{bb} φ^{0}_{JM}(r) \]

where \( Λ_{E'JM} = \int dE' Ψ_{E'JM}(r')δ_{01}Ψ_{E'00}(r') \) is the free-bound dipole coupling between the unperturbed bound state
\[ \phi_{ij}^{(1)} \] and the perturbed scattering state \( \psi_{EL} \) and \( \Lambda_{bb} = \int dr \Lambda_{bb}^{(2)}(r)\phi_{ij}^{(1)}(r)\chi(r) \] is the bound–bound dipole coupling between \( \phi_{ij}^{(1)} \) and the perturbed bound state \( \chi \). Let \( \chi_0(r) \) be the solution of the homogeneous part of (3) with the binding energy \( E_B \). Writing \( \phi_{ij} \) in the form \( \phi_{ij} = (0) \), one can express \( \Lambda_{bb} \) in terms of \( \Lambda_{bb}^{(1)} \) and \( \Lambda_{bb}^{(0)} \). After some minor algebra, we obtain

\[ \Lambda_{bb}^{(1)} = \frac{E - E_J}{\lambda} \int dE' \beta_{ij}^* (A_E | \Lambda_{bb}^{(0)} |^2 + V_{EL}) \chi_0(r) \tag{6} \]

where \( \Lambda_{bb}^{(0)} \) is the Rabi frequency between the two bound states \( \phi_{ij}^{(1)} \) and \( \chi_0 \) and \( \Lambda_{bb}^{(1)} \) and \( \Gamma_{bb} \) are given by

\[ \int dE' \beta_{ij}^* (A_E | \Lambda_{bb}^{(0)} |^2 + V_{EL}) \chi_0(r) \]

Note that the right-hand side of (7) involves the laser coupling \( \Lambda_{bb}^{(1)} \) with the perturbed continuum states. Here, \( A_E \) is related to the coefficient of \( \phi_{ij}^{(1)} \) in the energy-normalized dressed state (1) of three interacting states of which two are bound states and one is ground continuum state. Since \( \phi_{ij}^{(1)} \) is unit-normalized, \( A_E \) has the dimension of inverse of square root of energy. Physically, the PA excitation probability for collision energies ranging from \( E' \) to \( E' + \Delta E' \) is given by \( \lambda |A_E|^2 dE' \). Now, substituting (7) into (5) and (6) and then using the resultant form of \( \phi_{ij} \) and \( \chi_0 \) into (4), it is easy to see that the equation of motion for a particular \( \ell \)-wavefunction gets coupled to other \( \ell \)-wavefunctions.

The Green’s function for the homogeneous part of (4) can be written as \( K_{\ell}(r, r') = -\pi \psi_{EL}^{0 \text{reg}}(r) \psi_{EL}^{0 \text{reg}}(r') \), where \( r_{\text{reg}} \) implies either \( r \) or \( r' \) whichever is smaller (greater) than the other. Here, \( \psi_{EL}^{0 \text{reg}}(r) = \psi_{EL} + i \psi_{EL} \) where \( \psi_{EL}^{0 \text{reg}} \) and \( \psi_{EL}^{0 \text{irr}} \) represent regular and irregular scattering wavefunctions, respectively, in the absence of optical and magnetic fields. Asymptotically, \( \psi_{EL}^{0 \text{reg}}(r) \sim j_\ell c_\ell n_\ell - n_\ell c_\ell j_\ell n_\ell \) and \( \psi_{EL}^{0 \text{irr}}(r) \sim -(n_\ell c_\ell j_\ell + j_\ell c_\ell n_\ell) \), where \( j_\ell \) and \( n_\ell \) are the spherical Bessel and Neumann functions for the partial wave \( \ell \) and \( n_\ell \) is the phase shift in the absence of laser and magnetic field couplings. According to Wigner threshold laws, as \( \ell \to 0 \), \( n_\ell \sim k^{2\ell+1} \) for \( \ell \leq (n-3)/2 \); otherwise, \( n_\ell \sim k^{2\ell} \) with \( n \) being the exponent of the inverse power-law potential at large separation. Using \( K_{\ell}(r, r') \), the perturbed wavefunction \( \psi_{EL} \) can be formally expressed in terms of \( V_{EL} \), \( A_E \) and \( \Lambda_{bb} \) and the partial-wave free-bound dipole transition matrix elements \( \Lambda_{bb}^{(1)}_{\ell,\ell',J} = \int dr \phi_{ij}^{(1)}(r) \Lambda_{bb}^{(1)}(r) \psi_{EL}^{0 \text{reg}}(r) \). Next, substituting this into (7) and the expression for \( \psi_{EL} \), we can express \( A_E \) exclusively in terms of couplings between unperturbed states. Explicitly, we have

\[ A_E = \frac{e^{i\theta}}{\sqrt{2}\Lambda_{bb}^{(0)}} \frac{E - E_J + i\Gamma_{bb}}{\lambda} \sum_{\ell,\ell',J} \Lambda_{bb}^{(1)}_{\ell,\ell',J} \frac{\sqrt{2}\Lambda_{bb}^{(0)}}{\sqrt{2}\Lambda_{bb}^{(1)}_{\ell,\ell',J}} + \frac{\sqrt{2}\Lambda_{bb}^{(0)}}{\sqrt{2}\Lambda_{bb}^{(1)}_{\ell,\ell',J}} + \frac{\sqrt{2}\Lambda_{bb}^{(0)}}{\sqrt{2}\Lambda_{bb}^{(1)}_{\ell,\ell',J}} + \frac{\sqrt{2}\Lambda_{bb}^{(0)}}{\sqrt{2}\Lambda_{bb}^{(1)}_{\ell,\ell',J}} \]
given by $K_{\text{d}} = (\nu_{\text{rel}} \sigma_{\text{d}})$ and $K_{\text{inel}} = (\nu_{\text{rel}} \sigma_{\text{inel}})$ where $(\cdot, \cdot)$ stands for thermal averaging over the relative velocity $\nu_{\text{rel}} = \hbar \kappa / \mu$. Far from MFR ($\epsilon \to \pm \infty$) we have $T_{\text{mf}} \to 0$, $E_{\text{shift}}^{\text{off}} \to 0$ and $\Gamma_{q} \to \Gamma_{0}$. In this limit $T_{\text{d}}$ reduces to the form $T_{\text{d}} = -\Gamma_{0}/[D + i\hbar (\gamma_{J} + \sum \gamma_{J\ell})]$ which is the $T$-matrix element of standard OFR for which both elastic and inelastic scattering rates increase as laser intensity increases [42].

We can define an energy-dependent complex MOFR scattering length by $a_{\text{mf}} = -\tan \eta_{\text{mf}} / k$. In the limit $k \to 0$, we have

$$a_{\text{mf}} \simeq a_{\text{mf}} + q_{f}^{2} h \bar{\Gamma}_{0}/[k(D - E_{\text{shift}}^{q} + i\hbar \gamma_{J})] + i \frac{\gamma_{J}}{k^{2} a_{\text{mf}}}$$

(13)

where $a_{\text{mf}} = -\lim_{k \to 0} \tan \eta_{\text{mf}} / k$ is the MFR scattering length. Since $(k^{2} \bar{\Gamma}_{0} / \gamma_{J})$ tends to be independent of $k$ at ultralow energy, it is possible to have the condition $\Re \{k^{2} \bar{\Gamma}_{0} / \gamma_{J} \} > 0$ satisfied near MFR ($a_{\text{mf}} \to \pm \infty$) and PA resonance ($D \simeq 0$) in the strong-coupling regime ($\Gamma_{0} \gg \gamma_{J}$). Note that $D = \Delta E_{\text{f}} - \sum \gamma_{J\ell} E_{\text{shift}}^{\ell}$ is the PA resonance condition in the absence of MFR. Furthermore, it is to be noted that $E_{\text{shift}}^{q}$ as given by (11) is independent of $k$ in the limits $k \to 0$ and $\epsilon \to 0$ and can greatly exceed the spontaneous linewidth $\gamma_{J}$ in the strong-coupling regime [27]. Under such conditions, we can write

$$a_{\text{mf}} \simeq \left( \frac{D - E_{\text{shift}}^{q}}{k^{2} \bar{\Gamma}_{0} / \gamma_{J}} + 1 \right) + i \left( \frac{\gamma_{J}}{k^{2} a_{\text{mf}}} \right).$$

(14)

Let us recall that $a_{\text{mf}} = -1/(k \epsilon)$, $-\Gamma_{\text{mf}} / [2k(E' - \bar{E}_{J})]$. where $\bar{E}_{J} = E_{f}^{q} + E_{\text{shift}}^{q}$ and $E' = \hbar^{2} k^{2} / (2m)$. Therefore, in the case of finite $\bar{E}_{J} > E'$, the real part of $a_{\text{mf}} \Re \{a_{\text{mf}} \}$ becomes inversely proportional to energy and hence $\sigma_{\text{q}} \sim 1/k^{4}$ as $k \to 0$. In the case of $\bar{E}_{J} = 0$, $\Re \{a_{\text{mf}} \}$ goes to a constant in the limit $k \to 0$. In both the cases, the imaginary part of $a_{\text{mf}} \Im \{a_{\text{mf}} \}$ becomes independent of $k$ but inversely proportional to laser intensity suggesting that $K_{\text{inel}}$ can be made very small by increasing the laser intensity. On the other hand, for $D \to 0$, (14) indicates that $\Re \{a_{\text{mf}} \}$ becomes independent of laser intensity. Thus, we can infer that the inelastic scattering rate can be suppressed while the elastic rate can be enhanced by using quantum interference in the strong-coupling regime at ultralow temperatures. Very recently, Bauer et al. [23, 43] have experimentally demonstrated the effect of suppression of the inelastic rate in PA due to the influence of a magnetic Feshbach resonance.

The amplitudes of higher partial-wave scattered wavefunctions can also be enhanced by the MOFR. The higher partial waves that can be manipulated are given by the condition $J = L + S + \ell$. In the case of a singlet-to-singlet PA transition for $J = 1$, the maximum partial wave that can be significantly affected is $\ell = 2$ (d-wave), and in the case of a triplet-to-triplet transition it is $\ell = 3$. For $\ell \neq 0$, we have $T_{\ell 0} = \pi A_{E} \exp(i\eta_{J}) \Delta_{0}^{\ell}$. Using (8), in the leading order in dipole coupling at ultralow energy we have

$$T_{\ell 0} \simeq \exp(i\pi \eta_{J}) \left( q_{f} + \epsilon \right) / (\epsilon + i) \pi A_{E} \Delta_{0}^{\ell}. \frac{D - E_{\text{shift}}^{q} + i\hbar \gamma_{J} + \Gamma_{0}}{2}.$$  

(15)

In the limit $\epsilon \to \infty$, $T_{\ell 0}$ reduces to that of the OFR [37] for $\ell \geq 1$.

Figure 1. Subplots (a) and (b) show elastic and inelastic scattering cross sections $\sigma_{\text{el}}$ (solid line) and $\sigma_{\text{inel}}$ (solid-dotted line), respectively, in unit of cm$^{2}$ as a function of the magnetic field $B$ in Gauss (G) for $\Gamma_{0}/\gamma = 0.1$ (a) and $\Gamma_{0}/\gamma = 10.0$ (b) at the collision energy $E = 10 \mu$K and $q_{J} = -6.89$. Subplot (c) displays $\sigma_{\text{el}}$ versus $B$ (solid and dashed lines) and $\sigma_{\text{inel}}$ versus $B$ (dotted and solid-dotted lines) plots for $\Gamma_{0}/\gamma = 10.0$ (solid and solid-dotted lines) and $\Gamma_{0}/\gamma = 0.1$ (dashed and dotted lines) at $E = 100$ nK and $q_{J} = -68.88$. Subplot (d) exhibits the variation of $\Re \{a_{\text{mf}} \}$ (solid line) and $a_{\text{mf}}$ (dashed lines) as a function of $B$ for $\Gamma_{0}/\gamma = 10.0$, $B = 10 \mu$K and $q_{J} = -6.89$. The other fixed parameters for all the subplots are $\Gamma_{0}/\gamma = 16.67 MHz$ and $\gamma = 11.7 MHz$.

4.2. Numerical results

To illustrate further the analytical results discussed above, we present selective numerical results. As a model system, we consider $^7$Li atoms with the PA transition $^3 \Sigma_{u}^{+} \to ^3 \Sigma_{u}^{+}$. The parameter $\epsilon$ is related [44] to the magnetic field $B$, the resonance width $\Delta$ and the background scattering length $\eta_{B}$ by $\epsilon \sim -(B - \eta_{B}) / (k_{0} \eta_{B} \Delta)$, where $\eta_{B}$ is the resonance magnetic field. We use the realistic parameters taken or estimated from earlier experimental results [45, 46]. These parameters are the spontaneous linewidth $\gamma_{J} = 11.7 MHz$, $\Delta = -192.3 Gauss$ (G) and $\eta_{B} = -24.5 a_{0}$ ($a_{0}$ is Bohr radius). We take $B_{0} = 730.5 G$. From the reported Fano profile of the PA spectrum [20], we extract $q_{J} = -6.89$ at $E = 10 \mu$K. Using low-energy behaviour $q_{J} \sim 1/k$, we extrapolate $q_{J}$ at other collision energies. The Feshbach resonance linewidth $\Gamma_{mf}$ is taken to be 16.66 MHz for $E = 10 \mu$K. In all our numerical plots we set $D = 0$.

In figures 1(a)–(c), $\sigma_{00}$ as a function of $B$ is compared with $\sigma_{\text{inel}}$. We note that compared to weak-coupling results of figure 1(a), the strong-coupling result $\sigma_{00}$ of figure 1(b) largely exceeds $\sigma_{\text{inel}}$ in almost the entire range of $B$. Because of the interference between the two resonances, two closely spaced maxima appear near $B_{0}$ in figure 1(b). Even in figure 1(a), there is a prominent maximum at and near which $\sigma_{00}$ exceeds $\sigma_{\text{inel}}$. The reason for such a feature is that, as can be inferred from (14), for a given collision energy and $D = 0$, $\Re \{a_{\text{mf}} \}$ becomes independent of laser intensity as $\epsilon \to 0$ while $\Im \{a_{\text{mf}} \}$ goes to zero in the strong-coupling regime. Figure 1(c) shows that at much lower energy ($E = 100 nK$) inelastic scattering rates...
are further suppressed while elastic ones are enhanced in both weak- and strong-coupling regimes. Figure 1(d) illustrates how the MFR is split into a double resonance owing to Fano interference. This explains the appearance of two peaks near $B_0$. The minimum at $B = 710$ G arises due to a Fano minimum at which the PA transition amplitude vanishes.

We show the partial p- and d-wave scattering amplitudes in Figure 2 in the strong coupling regime. Typically, the higher partial-wave stimulated linewidths $\Gamma_{J=\pm 1}$ and $\Gamma_{J=0}$ are smaller than $\Gamma_{J=\pm 2}$ by one and four orders of magnitudes, respectively [37]. Comparing Figure 2 with Figure 1(b), we note that p- and d-wave scattering cross sections show a maximum near $B_0$ at which $\sigma_{n=0}$ is of the same order of magnitude as $\sigma_0$ while $\Gamma_{J=\pm 2}$ is three orders of magnitude smaller than that of $\sigma_0$. The minimum near $B \approx 730$ G can be attributed to the quantum interference-induced anomalously large positive shift as shown in the inset of Figure 2.

Figure 3 shows the energy dependence of elastic and inelastic scattering cross sections at three different values of $B$ in both the strong- (main figure) and weak-coupling (upper inset) regimes. The main figure and the upper inset clearly show that when $B = 730$ G which is close to $B_0$, the elastic part of the scattering cross section largely exceeds the inelastic part in the low-energy regime. We note that the elastic scattering cross section $\sigma_0$ (solid curve) at $E = 10$ nK and $B = 730$ G exceeds the inelastic scattering cross section $\sigma_{inel}$ (dashed curve) by two orders of magnitudes. In contrast, this does not happen if $B$ is tuned far away from $B_0$. For instance, when $B = 700$ G and $E = 10$ nK, $\sigma_0$ (plus solid curve) is smaller than $\sigma_{inel}$ (plus curve) by two orders of magnitude. The effect of laser intensity on the scattering cross sections at low energy can be understood by comparing the main figure with the upper inset of Figure 3. The stimulated linewidth ($\Gamma_{J=0}$) in the strong-coupling regime (main figure) is taken to be 20 times larger than that in the weak-coupling regime (upper inset). In other words, PA laser intensity for the strong-coupling case is taken to be 20 times larger compared to the weak-coupling case. Let us now compare the plots of the main figure with the corresponding plots of the upper inset: when $B$ is tuned close to $B_0$ or MFR, the elastic scattering cross section $\sigma_0$ (solid curve) for strong- (main figure) as well as weak-coupling (upper inset) regime tends to be equal as the energy $E$ decreases. At $E = 10$ nK, we find $\sigma_0 \approx 1.7 \times 10^{-10} \text{ cm}^2$ in both the regimes. In contrast, when $B = 700$ G which is away from MFR, $\sigma_0$ (plus solid curves) at $E = 10$ nK for weak- and strong-coupling regimes are $1.7 \times 10^{-11} \text{ cm}^2$ and $9.9 \times 10^{-11} \text{ cm}^2$, respectively. Thus in conformity with our previous analysis, by comparing the plots in the main and in the upper inset of Figure 3, we can infer that when $B$ is tuned near $B_0$, the elastic cross section at low energy becomes independent of laser intensity. The minimum at $B \approx B_0$ in the $\sigma_0$ versus $E$ plots of Figure 3 can be attributed to the large positive shift $E_q^\text{shift}$ as depicted in the lower inset of this figure.

5. Conclusions and outlook

Quantum interference is shown to change threshold and resonance behaviour significantly. This may in turn change the character of near-zero energy dimer states. Therefore, the crossover physics between the Bardeen–Cooper–Schrieffer (BCS) state of atoms and the Bose–Einstein condensate (BEC) of such dimers is likely to be affected by the MOFR. Although the MFR can most efficiently tune the s-wave scattering length, there does not exist any standard method of tuning higher partial-wave interatomic interaction. The MOFR will be particularly useful for tuning higher partial-wave interaction. The MFR is not applicable for atoms having
no spin magnetic moment and so is the MOFR. However, the underlying principle of the MOFR can also be applicable to such atoms provided a quasi-bound state embedded in the ground continuum is tunable by nonmagnetic means.

Appendix

We discuss how to derive (8). Using $K_\ell$ we first convert (4) (with the index $M = m_\ell$ being suppressed) into an integral equation of the form

$$\Psi_{E\ell}(r) = \exp(i\eta)\Psi_{E\ell}^0 + \int dr'K_{\ell}(r, r')\times[A_{\ell,j}(r')\phi_{\ell,j}(r') + V_{12}(r')\chi(r')\delta_{0,0}].$$

(A.1)

Substituting $\phi_{\ell,j} = \int dE'\beta_{E'}A_{E'}\phi_{E\ell}^0$ and (6) into (A.1), we get

$$\Psi_{E\ell} = e^{i\eta}\Psi_{E\ell}^0 + \frac{A_{0}\beta_{E}V_{E}\delta_{0}}{E - E_{x}}\times\int dr'K_{0}(r, r')V_{12}(r')\chi^0(r') + A_{E}\int dr'K_{\ell}(r, r')A_{\ell,j}(r')\phi_{\ell,j}(r').$$

(A.2)

Putting the above equation for $\ell = 0$ ($\Psi_{E0}$) into the equation $V_{E} = \int dr\Psi_{E0}^*(r)V_{12}(r)\chi^0(r)$ and after a minor algebra we obtain

$$V_{E} = \frac{(E - E_{x})(e^{i\eta}V_{E}^0 + A_{E}(V_{\text{eff}} - i\pi A_{0}V_{E}^0))}{E - (E_{x} + E_{\text{shift}} + i\Gamma_{f}/2)} + \frac{A_{E}A_{0}^{\beta_{E}}(E_{\text{shift}}^{\beta_{E}} - i\Gamma_{f}^{\beta_{E}})/2}{E - (E_{x} + E_{\text{shift}} + i\Gamma_{m}/2)}.$$  

(A.3)

After having substituted (A.3) into (A.2), we are left with the only unknown parameter $A_{E}$. Now, substituting (A.2) and (A.3) into (7) and using $\epsilon = (E - (E_{x} + E_{\text{shift}}))/(\Gamma_{m}/2)$ and the parameter $q_{f}$ defined by (9), we obtain (8). Thus, (12) is finally expressed in terms of all the known or unperturbed parameters.

References

[1] Tiesinga E, Verhaar B J and Stoof H T 1993 Phys. Rev. A 47 4114
[2] Fedichev P O, Kagan Y, Shlyapnikov G V and Walraven J T M 1996 Phys. Rev. Lett. 77 2913
[3] Kohler T, Goral K and Julienne P S 2006 Rev. Mod. Phys. 78 1311
[4] Chin C, Grimm R, Julienne P S and Tiesinga E 2008 arXiv:0812.1496
[5] Inouye S et al 1998 Nature 392 151
[6] Courtelle Ph et al 1998 Phys. Rev. Lett. 81 69
[7] Roberts J L et al 1998 Phys. Rev. Lett. 81 5109
[8] Timmermans E, Tommasini P, Hussein M and Kerman A 1999 Phys. Rev. Lett. 83 199–230
[9] O’Hara et al 2002 Science 298 2179
[10] Greiner M, Regal C A and Jin D S 2003 Nature 426 537
[11] Zwierlein M W et al 2005 Nature 435 1047
[12] Marinescu M and You L 1998 Phys. Rev. Lett. 81 4596
[13] Kemps R V 2006 Phys. Rev. Lett. 96 123202
[14] Thorsheim H R, Weiner J and Julienne P S 1987 Phys. Rev. Lett. 58 2420
[15] Jones K M, Tiesinga E, Lett P D and Julienne P S 2006 Rev. Mod. Phys. 78 483
[16] Weiner J, Bagnato V S and Zilio S 1999 Rev. Mod. Phys. 71 1
[17] Fatemi F K, Jones K M and Lett P D 2002 Phys. Rev. Lett. 85 4462
[18] Theis M et al 2004 Phys. Rev. Lett. 93 123001
[19] Enomoto K, Kasa K, Kitagawa M and Takahashi Y 2008 Phys. Rev. Lett. 101 203201
[20] Junker M et al 2008 Phys. Rev. Lett. 101 060406
[21] Winkler K et al 2007 Phys. Rev. Lett. 98 043201
[22] Ni K K et al 2008 Science 322 231
[23] Bauer D M et al 2009 Nat. Phys. 5 339
[24] Mackie M et al 2008 Phys. Rev. Lett. 101 040401
[25] Pellegrini P, Gacesa M and Cote R 2008 Phys. Rev. Lett. 101 055201
[26] Deb B and Agarwal G S 2009 J. Phys. B: At. Mol. Opt. Phys. 42 215203
[27] Deb B and Rakshit A 2009 J. Phys. B: At. Mol. Opt. Phys. 42 195202
[28] Kuznetsova E et al 2009 New J. Phys. 11 055028
[29] Fano U 1961 Phys. Rev. 124 1866
[30] Harris S E 2002 Phys. Rev. A 66 010701
[31] Moal S et al 2006 Phys. Rev. Lett. 96 023203
[32] Wynar R et al 2000 Science 287 1016
[33] Winkler K et al 2005 Phys. Rev. Lett. 95 063202
[34] Dunke R et al 2005 Phys. Rev. A 72 041801
[35] Bohn J L and Julienne P S 1996 Phys. Rev. A 54 R4637
[36] Bohn J L and Julienne P S 1999 Phys. Rev. A 60 414
[37] Deb B and Hazra J 2009 Phys. Rev. Lett. 103 023201
[38] Hazra J and Deb B 2010 Phys. Rev. A 81 022711
[39] Holland M, Kokkelmans S J J M F, Chiofolo M L and Walser K 2001 Phys. Rev. Lett. 87 120406
[40] Chen Q, Stajic J, Tan S and Levin K 2005 Phys. Rep. 412 1–88
[41] Yu G, Li Y, Motoyama M and Greiner M 2009 Phys. Rev. Lett. 5 873
[42] Bohn J L and Julienne P S 1997 Phys. Rev. A 56 1486
[43] Baur D M et al 2009 Phys. Rev. A 79 062713
[44] Moerdijk A J, Verhaar B J and Axelsen A 1995 Phys. Rev. A 51 4852
[45] Prodan I D, Pichler M, Junker M, Hulet R G and Bohn J L 2003 Phys. Rev. Lett. 91 080402
[46] I. Abraham E R, McAlexander W I, Sackett C A and Hulet R G 1995 Phys. Rev. Lett. 74 1315