DEUTERON P-WAVE IN ELASTIC BACKWARD PROTON-DEUTERON SCATTERING

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Abstract
The elastic backward proton-deuteron scattering is analyzed within a covariant approach based on the invariant expansion of the reaction amplitude. The relativistic invariant equations for all the polarization observables are presented. Within the impulse approximation the relation of the tensor analyzing power $T_{20}$ and the polarization transfer $\kappa_0$ to $P$-wave components of the deuteron wave function is found. The comparison of the theoretical calculations with experimental data is presented. An experimental verification of the reaction mechanism is suggested by constructing some combinations of different observables.

Key-words: elastic backward proton-deuteron scattering, invariant amplitudes, helicity amplitudes, polarization observables, “Magic Circle”, deuteron wave function, small components, reaction mechanism, one-nucleon exchange.

1 Introduction

As known, the study of polarization phenomena in hadron and hadron-nucleus collisions gives more detailed information about dynamics of their interaction and the structure of colliding particles. Among the simplest reactions with hadron probes are processes of forward or backward scattering of protons off the deuteron. In particular the tensor analyzing power $T_{20}$ by backward $pD$ elastic scattering has been measured in Saclay yet fifteen years ago [1]. These interesting data yet can’t be understood theoretically especially at the kinetic energy of protons emitted backward $T_p > 0.6$ GeV. The intensive experimental study of the elastic and inelastic $pD$ reaction has been continued in Dubna and Saclay (see for instance [2, 3]) and is also planed to be investigated in the nearest future at COSY [4]. All these data can’t be described within the impulse approximation by using the usual deuteron wave function having only $S$- and $D$-waves as it is shown in [3].

In this paper we concentrate our attention on the study of the contribution of a possible $P$-wave component in the deuteron wave function (DWF) by using helicity amplitudes formalism to all the polarization observables and in particular such as the tensor analyzing power $T_{20}$ and deuteron-proton polarization transfer $\kappa_0$. This contribution is investigated within the impulse approximation. We suggest an experimental test of the reaction mechanism by measuring some combinations of the polarization characteristics.
2 General formalism

*Invariant expansion of \( pD \rightarrow Dp \) backward reaction amplitude*

In general the amplitude of reaction \( pD \rightarrow Dp \) can be written in the following form (see Fig.1):

\[
M_{\beta_f\sigma_i}(s,t,u) = \bar{u}_{\sigma_f}(p_f) \, Q^{\mu\nu}(s,t,u) \, u_{\sigma_i}(p_i) \, \xi^{*(\beta_f)}(D_f) \, \xi^{(\beta_i)}(D_i),
\] (1)

where \( u_{\sigma_i}(p_i) \equiv u_i \) and \( \bar{u}_{\sigma_f}(p_f) \equiv \bar{u}_f \) are the spinors of the initial and final nucleons with spin projections \( \sigma_i \) and \( \sigma_f \) respectively; \( \xi_\mu(D) \) is the polarization vectors of deuterons; \( s, t, u \) are invariant Mandelstam’s variables:

\[
s = (D_i + p_i)^2,
\]

\[
t = (D_i - D_f)^2,
\]

\[
u = (D_i - p_f)^2 = \bar{s}.
\]

For the backward \( pD \rightarrow Dp \) scattering the amplitude (1) depends only on the one kinematical variable which is chosen usually as \( s \), e.g., square of the initial energy in the c.m.s. The amplitude \( Q^{\mu\nu} \) for this process contains four amplitudes and can be written in the form:

\[
Q^{\mu\nu}(s) = Q_0(s) \, (-g_{\mu\nu} + q_\mu q_\nu) + Q_1(s)q_\mu q_\nu + Q_2(s)q_\mu \gamma_\nu + iQ_3(s)\gamma_5 q_\mu \rho \gamma^\rho q^\sigma,
\] (2)

where we introduce the unit 4-vector \( q = Q/\sqrt{Q^2}, \ Q = (D_i + D_f)/2 \).

*Helicity amplitudes*

To calculate the observables, differential cross sections and polarization characteristics, it would be very helpful to construct the helicity amplitudes \( M^{\lambda_i\lambda_f}_{\mu_i\mu_f} \) of the considered process \( pD \rightarrow Dp \), where we introduced initial (final) proton helicities \( \mu_{i,f} = \pm 1/2 \) and the initial (final) deuteron helicities \( \lambda_{i,f} = \pm 1,0 \). The number of independent helicity amplitudes is the same as the one for corresponding amplitudes incoming to \( Q^{\mu\nu}(s) \) (2) and equal to four. They can be chosen as the following

\[
\Phi_1 = M^{\pm\mp}_{\pm\mp} = -M^{\mp\pm}_{\mp\pm} \ ; \ \Phi_2 = M^{00}_{0+} = -M^{00}_{0-} ;
\]

\[
\Phi_4 = M^{+0}_{++} = -M^{0+}_{++} = M^{0-}_{--} = -M^{--}_{-} ,
\] (3)

and related to the corresponding relativistic invariants \( Q_i \) (2):

\[
\Phi_1 = \frac{\varepsilon}{m} Q_0 \pm Q_3 ; \quad (4)
\]

\[
\Phi_2 = -\frac{\varepsilon}{m} Q_0 - \frac{p^2}{M^2} \left( \frac{\varepsilon}{m}[Q_0 - Q_1] - 2Q_2 \right) ; \quad (5)
\]

\[
\Phi_4 = -\sqrt{2} \frac{p^2}{Mm} Q_2 - \sqrt{2} \frac{\varepsilon \varepsilon D}{Mm} Q_3 ; \quad (6)
\]

or to the corresponding Pauli’s amplitudes \( g_i \):

\[
\Phi_1 = g_1 \mp g_4 \ ; \ \Phi_2 = -g_2 \ ; \ \Phi_4 = \sqrt{2} g_3 . \quad (7)
\]
Polarization observables

Having the helicity amplitudes given by Eq.(3) one may define various polarization characteristics for the discussed process. Applying the notations used in Refs. [6, 7] we define the set of all the possible polarization observables as the following:

\[
(\alpha; \mu | \beta; \nu) = \frac{Tr [\sigma_\alpha O_\mu M^+ \sigma_\beta O_\nu M]}{Tr [M^+ M]},
\]

with a normalization \((0; 0|0; 0) = 1\). The subscripts \(\alpha\) and \(\mu\) (\(\beta\) and \(\nu\)) refer to the polarization characteristics of the initial (final) proton and deuteron respectively; \(\sigma_\alpha\) is the Pauli matrix, and \(O_\mu\) stands for a set of \(3 \times 3\) operators defining the deuteron polarization.

The quantity \(\Sigma = Tr [M^+ M]\)

\[
\Sigma = \sum_{\lambda \mu, \lambda i} |M^\lambda_{\mu} M^i_{\nu}|^2 = 2[|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2 + 2|\Phi_4|^2]
\]

is proportional to the unpolarized differential cross section.

As mentioned in Ref.[8], one of the goals of the future experiments is a direct reconstruction of the complex amplitudes (3). Since the process is described by using four complex amplitudes, one needs to measure at least seven independent observables. At the present time the tensor analyzing power \(T_{20}\) and polarization transfer \(\kappa_0\) measurements was done only. In terms of the helicity amplitudes this observables can be written as the following:

\[
(0; NN|0; 0) = - [2|\Phi_1|^2 + 2|\Phi_3|^2 - |\Phi_4|^2]^2 \cdot \Sigma^{-1} = A_{yy} = -T_{20}/\sqrt{2};
\]

\[
(0; N|N; 0) = 2\sqrt{2} \text{Re} [(\Phi_3 - \Phi_2) \Phi_4^*] \cdot \Sigma^{-1} = (4/3) \kappa_t = (2/3) \kappa_0.
\]

3 The one-nucleon exchange mechanism (ONE)

Let us consider our reaction within the framework of the impulse approximation, Fig.2. In ONE model the amplitude of the \(pD \rightarrow Dp\) backward reaction has a very simple form [5]:

\[
Q_{\mu \nu}^N = \Gamma_\nu \frac{\vec{n} - m}{m^2 - u} \Gamma_\mu,
\]

where \(\Gamma_\nu (\Gamma_\mu = \gamma_0 \Gamma^+ \gamma_0)\) is a deuteron vertex with one off-shell nucleon and can be written with four form factors parameterization exactly coinciding with the one used, for instance, by Gross [9] or Keister and Tjon [10]. To connect this relativistic invariant formalism with the non-relativistic one we also express the reaction amplitude in the deuteron rest frame in terms of partial deuteron waves, namely in the \(\rho\)-spin classification, the two large components of the DWF \(U =^3 S_{1^+}^+\) and \(W =^3 P_{1^+}^+\), and the small components \(V_s =^1 P_{1^+}^-\) and \(V_t =^3 P_{1^+}^-\) as like as in [9].
By making use of the identities \( n = D_i - p_f, \) \( n^2 = u \leq (M - m)^2, \) after computing the quantities (2), one can find the forms of the helicity amplitudes (4-6) within the ONE model in terms of this positive- and negative-energy wave function \( \Phi \):

\[
\Phi_N^1 = 0; \quad (13)
\]

\[
\Phi_N^2 = -2\pi^2 \left( m^2 - u \right) \left[ \frac{\xi_D}{M} \left( U + \sqrt{2}W \right) - 2\sqrt{3} \frac{p}{m} V_s \right] \left( U + \sqrt{2}W \right) - 6\pi^2 M \xi_D V_s^2; \quad (14)
\]

\[
\Phi_N^3 = 2\pi^2 \left( m^2 - u \right) \left[ \frac{\xi_D}{M} \left( \sqrt{2}U - W \right) - 2\sqrt{3} \frac{p}{m} V_i \right] \left( \sqrt{2}U - W \right) + 6\pi^2 M \xi_D V_i^2; \quad (15)
\]

\[
\Phi_N^4 = 2\pi^2 \left( m^2 - u \right) \left[ \frac{\xi_D}{M} \left( \sqrt{2}U - W \right) \left( U + \sqrt{2}W \right) \right]
- \sqrt{3} \frac{p}{m} \left\{ \left( \sqrt{2}U - W \right) V_s + \left( U + \sqrt{2}W \right) V_i \right\} \left( U + \sqrt{2}W \right) - 6\pi^2 M \xi_D V_s V_i. \quad (16)
\]

where \( P_{\text{lab}} \) is the final proton momentum. Firstly, one can see, that all the \( \Phi_N^i(W) \) amplitudes are real, e.g., all the T-odd polarization correlations are equal to zero within this approximation. For example, \( (N; LS|0; 0) = 0. \) Secondly, within the ONE approximation the helicity amplitude \( \Phi_N^1(W) \) is vanished because the spin-down proton in the incident channel cannot result in the spin-down deuteron in the final channel due to the lack of the spin non-flip of the proton. This leads to \( (0; NN|0; NN) = (0; NN|0; SS). \)

This consequence of the ONE mechanism can be verified experimentally by measuring and combining the different observables given by Eq.(8). For example, one can find the helicity amplitude \( \Phi_1(W) \) from the following combination:

\[
\left| \Phi_1(W) \right|^2 = (1 + T_{20}/\sqrt{2} + 2\kappa_l) \cdot \Sigma/6,
\]

where \( \kappa_l = -(3/4) \left( 0; L|L; 0 \right) = (3/2) \left[ \left| \Phi_1 \right|^2 - \left| \Phi_3 \right|^2 - \left| \Phi_4 \right|^2 \right] \cdot \Sigma^{-1}. \)

And finally, the following relation between amplitudes:

\[
\Delta_N^N = \Phi_N^2 \Phi_N^3 + (\Phi_N^4)^2 = -12\pi^4 \frac{M^3 E_{\text{lab}}^2}{m^2} \frac{(2E_{\text{lab}} - M)}{m^2} \left[ \left( \sqrt{2}U - W \right) V_s - \left( U + \sqrt{2}W \right) V_i \right]^2 \quad (17)
\]

has a purely P-wave dependence. We have for a “Magic Circle” in the \( \kappa_0-T_{20} \) plane the following equation:

\[
\left( \frac{\kappa_0^N}{2} \right)^2 + \frac{(T_{20}^N + 1/(2\sqrt{2}))^2}{9/8} = 1 - \left( \frac{\Delta_N^N}{\Sigma_N^N} \right)^2. \quad (18)
\]

In terms of positive-energy waves \( U \) and \( W \) only the helicity amplitudes have a well-known non-relativistic form. For this simple case there is the following relation: \( \Delta_N = 0. \) And the Lorenz boost effects do not contribute to the polarization observables.

4 Results and Discussions

Let us present the calculation results for the deuteron tensor analyzing power \( T_{20} \), the polarization transfer \( \kappa_0 \) and their link given by Eq.(18) obtained within the relativistic
impulse approximation. In Fig's. (3,4) $T_{20}$ and $\kappa_0$ for different kinds of the DWF are presented. It can be seen from these figures the inclusion of the $P$-wave to the DWF according to \cite{9} changes the form of $T_{20}$ at $P_{lab} > 0.2$ GeV/c. The shape of these observables is changed towards the experimental data by increasing the probability of $P$-wave $P_V$ in the DWF. The form of the polarization transfer $\kappa_0$ is closed to the experimental data at $P_V = 0.4\% - 0.5\%$. Although the description of the experimental data about $T_{20}$ and $\kappa_0$ isn’t satisfactory even by inclusion of the $P$-wave to the DWF nevertheless the $P$-wave contribution improves the description of data and shows a big sensitivity of the polarization observables presented in Fig's. (3,4) to this effect.

The link between $T_{20}$ and $\kappa_0$ given by Eq.(18) is presented in Fig.5. The big sensitivity of this relation to the contribution of the $P$-wave probability $P_V$ is also seen from this figure. There isn’t also a satisfactory description of the experimental data nevertheless the shape of the ”Magic Circle” which is right for the conventional DWF is deformed towards the experimental data.

In principle, there is some analogy between the effects of the deuteron $P$-wave and secondary interactions contributing to the discussed observables for elastic and inelastic backward $pD$ reactions \cite{13,14} and \cite{15}. The contribution of secondary interactions, in particular the triangle graphs with a pion in intermediate state, results in an improvement of the description of discussed experimental data on observables for the deuteron stripping reaction $Dp \rightarrow pX$ \cite{15}.

The consequence of the ONE mechanism can be verified experimentally by measuring and combining the different observables given by Eq.(8). At least, one can find experimentally the kinematical region where $\Phi^N_1(W) = 0$ and the ”Magic Circle” Eq.(18) can be applicable to find some information about the $P$-wave contribution to the DWF.

## 5 Conclusions

The performed analysis has shown the following. The discussed polarization observables $T_{20}$ and $\kappa_0$ are very sensitive to a possible contribution of $P$-wave to the relativistic DWF. There is some analogy between the inclusion of $P$-wave to the DWF and effect of the secondary interactions which are some corrections to the ONE graph. One can propose a verification of the reaction mechanism for the elastic backward $pD$ scattering from the measuring of the polarization observable like as $(0; SN|0; SN)$, which have to be equal zero within the relativistic ONE approximation as it is seen from Eq.(13). Any way, combining the another polarization observables which are more available for the measurement one can find experimentally whether the helicity amplitude $\Phi^N_1(W)$ is equal zero or not at some kinematical region. Therefore one can verify experimentally the validity of the relativistic invariant impulse approximation. At least, one can find some
kinematical region where it is valid more less and extract some information about the 
P-wave contribution to the DWF.

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Figure 1: Elastic backward proton-deuteron amplitude.

Figure 2: The one-nucleon exchange diagram.

Figure 3: Tensor analyzing power $T_{20}$. 
Figure 4: Polarization transfer coefficient $\kappa_0$.

Figure 5: “Magic Circle” in the $\kappa_0$–$T_{20}$ plane.