Shedding light on CP violation in the charm system via $D \to V\gamma$ decays

Gino Isidori$^{1,2}$ and Jernej F. Kamenik$^{3,4}$

$^1$CERN, Theory Division, CH-1211 Geneva 23, Switzerland
$^2$INFN, Laboratori Nazionali di Frascati, Via E. Fermi 40, 00044 Frascati, Italy
$^3$J. Stefan Institute, Jamova 39, P. O. Box 3000, 1001 Ljubljana, Slovenia
$^4$Department of Physics, University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia

(Dated: May 1, 2014)

Recent evidence for direct CP violation in non-leptonic charm decays cannot be easily accommodated within the Standard Model (SM). On the other hand, it fits well in new physics models generating CP violating $\Delta C = 1$ chromomagnetic dipole operators. We show that in these frameworks sizable direct CP asymmetries in radiative $D \to P^+ P^- \gamma$ decays $(P = \pi, K)$, with $M_{P^0}$ close to the $\rho$ or the $\phi$ peak, can be expected. Enhanced matrix elements of the electromagnetic dipole operator can partly compensate the long distance dominance in these decays, leading to CP asymmetries of the order of several percent. If observed at this level, these would provide a clean signal of physics beyond the SM and of new dynamics associated to dipole operators. We briefly comment on related CP violating observables accessible via time dependent $D(\bar{D}) \to P^+ P^- \gamma$ studies and angular decay product distributions in rare semileptonic $D$ decays.

I. INTRODUCTION

A significant evidence for direct CP violation in $D \to P^+ P^-$ decays $(P = \pi, K)$ has recently been reported by the LHCb [1] and by the CDF [2] collaborations. Both experiments find a non-vanishing value for $\Delta a_{CP} \equiv a_{K^+ K^-} - a_{\pi^+ \pi^-}$, where

$$a_f \equiv \frac{\Gamma(D^0 \to f) - \Gamma(\bar{D}^0 \to f)}{\Gamma(D^0 \to f) + \Gamma(\bar{D}^0 \to f)} .$$

Combining these recent results with older measurements leads to the following world average [2]

$$\Delta a_{CP}^{\text{exp}} = -(0.67 \pm 0.16)\% ,$$

that differs from zero by about 4$\sigma$.

The theoretical interpretation of this result is puzzling. The value in Eq. (2) exceeds by a factor 5–10 what is naturally expected in the Standard Model (SM) (see e.g. Ref. [3], and the more recent analyses in Ref. [4, 5]). However, we cannot exclude that such result has a SM explanation due to the non-perturbative enhancement of penguin-type hadronic matrix elements [6–8]. On the other hand, this value can naturally be accommodated in well-motivated extensions of the SM. In particular, it fits well in models generating at short distances a sizable CP violating phase for the effective $\Delta C = 1$ chromomagnetic operators [3, 4, 9, 10].

Given this situation, it is important to identify possible future experimental tests able to distinguish standard vs. non-standard explanations of $\Delta a_{CP}$. An interesting strategy that makes use of CP asymmetries in various hadronic $D$ decays (necessarily including neutral mesons) has recently been proposed in Ref. [11]. However, this strategy is effective in isolating possible non-standard contributions to $\Delta a_{CP}$ only if they are generated by effective operators with a $\Delta I = 3/2$ isospin structure. This is not the case for the well-motivated scenario with a new CP violating phase in the $\Delta C = 1$ chromomagnetic operator. As we point out here, in the latter case an efficient strategy is obtained by measuring CP asymmetries in radiative $D$ decays.

II. SHORT-DISTANCE EFFECTIVE HAMILTONIAN

The first key ingredient of our strategy is the strong link between the $\Delta C = 1$ chromomagnetic operator,

$$Q_8 = \frac{m_c}{4\pi^2} \bar{u}_L q_\mu T^a g_s G^{\mu\nu}_a e_{CR} ,$$

and the $\Delta C = 1$ electromagnetic-dipole operator,

$$Q_7 = \frac{m_c}{4\pi^2} \bar{u}_L \sigma_{\mu\nu} Q_u e F^{\mu\nu}_{CR} .$$

In most explicit new-physics models the short-distance Wilson coefficients of these two operators $(C_7,8)$ are expected to be similar. Moreover, even assuming that only a non-vanishing $C_8$ is generated at some high scale, the mixing of the two operators under the QCD renormalization group (RG) implies $C_{7,8}$ of comparable size at the charm scale. The same is true for the pair of operators with opposite chirality $Q_{7,8}^{\prime}$, obtained from $Q_{7,8}$ with the replacement $L \leftrightarrow R$.

To quantify the size of these coefficients, we normalize the effective Hamiltonian describing the $\Delta C = 1$ new-physics contributions as

$$H_{\Delta C=1}^{\text{eff-NP}} = \frac{G_F}{\sqrt{2}} \sum_i C_i Q_i + \text{h.c.} ,$$

The complete list of potentially relevant operators can be found in Ref. [4]; however, for the purpose of our analysis we can restrict our attention only to $Q_{7,8}$ and $Q_{7,8}^{\prime}$. Assuming the initial conditions of these operators are generated at some scale $M > m_c$, taking into account
the RG evolution of the operators at the leading log level (assuming only SM degrees of freedom below the scale $M$), leads to [12]

$$C_7^{(0)}(m_c) = \eta \left[ \eta C_7^{(0)}(M) + 8(\eta - 1) C_8^{(0)}(M) \right],$$  \hspace{1cm} (6)

$$C_8^{(0)}(m_c) = \eta C_8^{(0)}(M),$$  \hspace{1cm} (7)

where

$$\eta = \left[ \frac{\alpha_s(M)}{\alpha_s(m_b)} \right]^{\frac{7}{2}} \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{\frac{7}{2}},$$  \hspace{1cm} (8)

and

$$\tilde{\eta} = \left[ \frac{\alpha_s(M)}{\alpha_s(m_b)} \right]^{11/2} \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{11/2}.$$  \hspace{1cm} (9)

Following the analysis in Ref. [9], the new-physics contribution to $\Delta a_{CP}$ induced by $Q_8$ can be written as

$$|\Delta a_{CP}| \approx -1.8 |\text{Im}[C_8^{NP}(m_c)]|,$$  \hspace{1cm} (10)

where the numerical value assumes strong phases and is affected by $O(1)$ uncertainties due to the theoretical error on $(PP|Q_8|D)$. Assuming this contribution saturates the experimental value of $\Delta a_{CP}$ leads to $|\text{Im}[C_8^{NP}(m_c)]| \approx 0.4 \times 10^{-2}$. If we further assume that the initial scale $M$ is around 1 TeV, and that at this scale $|C_7^{NP}(M)| \ll |C_8^{NP}(M)|$, the RG evolution implies

$$|\text{Im}[C_7^{NP}(m_c)]| \approx |\text{Im}[C_8^{NP}(m_c)]| \approx 0.4 \times 10^{-2}.$$  \hspace{1cm} (11)

This is for instance what happens in supersymmetry, where the gluino-mediated amplitude proportional to $(\delta_{LR}^D)_{12}$ leads to the initial condition

$$C_7^{\text{SUSY}}(m_{\text{SUSY}}) = \frac{4}{15} C_8^{\text{SUSY}}(m_{\text{SUSY}}).$$  \hspace{1cm} (12)

Taking into account the $O(1)$ uncertainties in the determination of $|\text{Im}[C_8^{NP}(m_c)]|$, and the additional uncertainties in the initial conditions of $C_8^{NP}(M)$, we consider the following range for $|\text{Im}[C_8^{NP}]|$ at the charm scale

$$|\text{Im}[C_7^{NP}(m_c)]| = (0.2 - 0.8) \times 10^{-2}.$$  \hspace{1cm} (13)

The same range holds for $|\text{Im}[C_7^2]|$, if the leading contribution to $\Delta a_{CP}$ is generated by $Q_8$, rather than $Q_8$. At low energies $C_7$ receives contributions also from the mixing with the SM four-fermion operators. However, to a good accuracy these contributions are CP conserving. The leading effect is the two-loop mixing between $C_7$ and $C_{1,2}^{CS}$ [13]. According to the analysis in Ref. [13], integrating out also light quark loops one obtains

$$|C_7^{SM-eff}(m_c)| = (0.5 \pm 0.1) \times 10^{-2},$$  \hspace{1cm} (14)

with an $O(1)$ strong phase and a negligible CP-violating phase (more than two orders of magnitude smaller).

If the contributions in Eqs. (11) and (14) where the dominant contributions to radiative $D$ decays, we could expect $O(1)$ direct CP asymmetries in these modes. As we discuss below, this is not the case due to genuine long-distance contributions that dominate the decay rates.

### III. SHORT- VS. LONG-DISTANCE CONTRIBUTIONS IN $D \to V \gamma$

The second important ingredient of our analysis is the observation that in the Cabibbo-suppressed $D \to V \gamma$ decays, where $V$ is a light vector meson with $u\bar{u}$ valence quarks ($V = \rho, \omega$), $Q_7$ and $Q_8^{(1)}$ have a sizable hadronic matrix element. More explicitly, the short-distance contribution induced by $Q_8^{(1)}$, relative to the total (long-distance) amplitude, is substantially larger with respect to the corresponding relative weight of $Q_8$ in $D \to P^+P^-$ decays.

The decay amplitudes for $D \to V \gamma$ decays can be decomposed as follows

$$\mathcal{A}[D(p) \to V(p,\ell\gamma)(q,\ell)] = -i A_{PC}^V \epsilon_{\mu
u\alpha\beta} q^\mu e^\nu p^\alpha \bar{\ell}^\beta,$$

$$+ A_{PV}^V [(\tilde{\epsilon}^\nu q^\nu)p - (q^\nu \tilde{\epsilon}^\nu)] ,$$  \hspace{1cm} (15)

with the corresponding rates

$$\Gamma(D \to V \gamma) = \frac{m_D^3}{32\pi} \left(1 - \frac{m_V^2}{m_D^2}\right)^3 \left[|A_{PV}|^2 + |A_{PC}|^2\right].$$  \hspace{1cm} (16)

The short-distance contribution induced by $Q_7$ to the effective couplings $A_{PV,PC}^V$ is

$$(A_{PC(PV)})_{1,2}^{s.d.} = \frac{eQ_8G_F m_c}{\sqrt{2}} C_7(m_c) T_V^{(1,2)},$$  \hspace{1cm} (17)

where $T^{(1,2)}_V$ are defined by

$$\langle V(p,\ell)[\bar{u}q\sigma_{\mu\nu}(1 + \gamma_5)c]|D(p)\rangle = -2i\epsilon_{\mu\nu\alpha\beta} \epsilon^\alpha p^\beta T_1^V$$

$$+ T_1^V [(m_D^2 - m_V^2)\epsilon^\mu - (\tilde{\epsilon}^\nu p)(p + \bar{p})^\nu],$$  \hspace{1cm} (18)

and $T_1^V = T_1^V | T_2^V |$ via the identity $\gamma_5 \sigma_{\mu\nu} = \frac{i}{2} \epsilon_{\mu\nu\alpha\beta} \sigma_{\alpha\beta}$. A recent sum-rule estimate finds [14] $T_2^V | T_2^V | \approx 0.70(7)$. We note in passing that at leading order in $\alpha_s$ and in the infinite charm quark mass limit, heavy quark symmetry predicts $T_2^V = V_2^V(0)/(1 - m_V/m_D)$ [15], where $(q^2 \equiv (p - \bar{p})^2)^2$.

$$\langle V(p,\ell)[\bar{u}q\epsilon^\mu c]|D(p)\rangle = 2iV_2^V(q^2) \frac{m_D}{m_V} T_2^V,$$  \hspace{1cm} (19)

This matrix element enters semileptonic $D \to V$ decays and thus $V_2^V(0)$ can be accessed experimentally. Unfortunately, for the interesting $D \to (\rho, \omega)V\ell\nu$ transitions, no such analyses are available at present. On the other hand, in the heavy charm quark limit, $T_2^V$ can be related to hadronic matrix elements entering radiative $B \to V$ transitions $- T_B^{(V)}(B)$. Starting from the (quenched) Lattice QCD estimate $T_2^{(B)}(0) = 0.20(4)$ [16], and running it in both the perturbative matching scale (from $\mu_b = 4.6$ GeV to $\mu_c = 1.4$ GeV) as well as the heavy quark mass scaling (including leading power corrections) [16, 17], we obtain $T_2^{(D)} \approx 0.7(2)$. Using instead existing sum-rule estimates
of $T_{(D)}^\rho$ [18] typically leads to $\mathcal{O}(20\%)$ larger values. Consequently we employ the value of $T_{(D)}^\omega$ with a conservative uncertainty estimate of

$$T_{(D)}^\rho \approx T_{(D)}^\omega \approx 0.7 \pm 0.2 ,$$

which leads to

$$\left| (A_{\text{PC},PV}^\rho)^{\text{d}} \right| \approx \frac{0.6(2) \times 10^{-9}}{m_D} \left| C_7(m_c) \right| \left| C_7(m_c) \right| \frac{0.4 \times 10^{-2}}{0.4 \times 10^{-2}} .$$

The contribution induced by $Q_F^\rho$ is obtained with the replacement $C_7 \to \pm C_7$ in $A_{\text{PC},PV}^\rho$. The only $D^0 \to V^0\gamma$ modes the CP violating asymmetries can reach 10% (see Sec. V). The observed rates satisfy to a good accuracy the relation $B(D \to K^{*}\gamma)/B(D \to K^{*}\rho^0) = B(D \to \phi\gamma)/B(D \to \phi\rho^0)$, generally expected by vector meson dominance (VMD). The three Cabibbo-suppressed $D^0 \to V^0\gamma$ modes, $V^0 = \rho^0, \omega, \phi$, are expected to have similar rates.\footnote{According to explicit VMD predictions [20] $B(D \to \rho \gamma)$ and $B(D \to \omega \gamma)$ are very similar, possibly a factor $\sim 2$ smaller than $B(D^0 \to \phi \gamma)$. In the following we assume $B(D \to (\rho, \omega)\gamma) \geq 10^{-5}$.} We can thus estimate the typical size of their long-distance amplitudes $|A_{\text{PC}}^{V}(l)\,^{\text{d}}| \approx \left| (A_{\text{PC},PV}^\rho)^{\text{d}} \right|$ as follows

$$\left| (A_{\text{PC},PV}^\rho)^{\text{d}} \right| = \left[ \frac{32\pi}{m_D^3} \left( 1 - m_T^2 \right) \frac{\Gamma(D \to V\gamma)}{2} \right]^{1/2} \approx 5.8(4) \times 10^{-8} \quad \text{for} \quad V = \phi .$$

In the limit where the strong phases of the amplitudes have a mild energy dependence, and assuming we can neglect the weak phase of the direct CP asymmetry, defined in Eq. (1), can be decomposed as

$$|a_{V\gamma}| = 2 \zeta_{\text{weak}} \sin(\Delta \phi_{\text{strong}}) ,$$

where

$$\zeta_{\text{weak}} = \frac{\text{Im}(A_{\text{PC},PV}^{V})^{\text{d}}}{|A_{\text{PC},PV}^{V}(l)^{\text{d}}|} .$$

As a result, according to Eqs. (21) and (22), in the $\rho$ and $\omega$ modes the CP violating asymmetries can reach 10% for maximal strong phases:

$$|a_{(\rho, \omega)\gamma}|_{\text{max}} = 0.04(1) \frac{\text{Im}[C_7(m_c)]}{0.4 \times 10^{-2}} \times \left[ \frac{10^{-5}}{B(D \to (\rho, \omega)\gamma)} \right]^{1/2} \lesssim 10\% .$$

The case of the $\phi$ resonance, or better the $|K^+K^-\gamma|$ final state with $M_{KK}$ close to the $\phi$ peak, is more involved since the hadronic matrix element (18) vanishes, in the large $m_s$ limit, if $V$ is a pure $s \bar{s}$ state. However, as we discuss in more detail in the next section, a non-negligible CP asymmetry can be expected also in this case for two main reasons: 1) the matrix element in (18) is not identically zero even for $V = \phi$, both because $O(\Lambda_{QCD}/m_c)$ corrections and because of the tiny $u \bar{u}$ component of $\phi$; 2) non-resonant contributions due to (off-shell) $\rho$ and $\omega$ exchange can also contribute to the $|K^+K^-\gamma|$ final state.

**IV. THE $D \to K^+K^-\gamma$ CASE**

The decay amplitudes for $D \to P^+P^-\gamma$ decays can be decomposed in full generality as follows

$$A[D(p)] = P^+[p_+P^--(p_-)\gamma(q,\epsilon)] = -iM(s,\nu) \epsilon_{\mu\nu\beta\gamma}^{\rho}\epsilon^{es\nu}(p_+ - p_-)^3 + E(s,\nu) \epsilon_\mu^0[q^\mu(qp_+ - qp_-) - qp(p_+ - p_-)\epsilon^\mu] ,$$

where $s = (p_+ + p_-)^2$ and $\nu = (qp_+ - qp_-)$. In the limit where we consider at most electric and magnetic dipole transitions (or neglecting higher order multipoles), we can neglect the $\nu$ dependence of the form factors. In this approximation, the differential rate as a function of $s = M_{PP}^2$ can be written as

$$\frac{d\Gamma}{ds} = \frac{m_D^3}{32\pi} \left( 1 - \frac{s}{m_D^2} \right)^3 \sqrt{\frac{\Gamma_0(s)}{\pi}} \left[ |M(s)|^2 + |E(s)|^2 \right] ,$$

where $\Gamma_0(s) = \sqrt{(1 - 4m_D^2/s)^{3/2}}/(4\pi)$. If the amplitude is dominated by the exchange of vector resonances we can decompose $M$ and $E$ as follows

$$M(s) = \sum_{V} \frac{g_{PP}^V A_{PV}^V}{s - M_{PV}^2 - i\sqrt{s} \Gamma_{PV}^V} ,$$

$$E(s) = \sum_{V} \frac{g_{PP}^V \epsilon_{\mu} A_{PV}^V}{s - M_{PV}^2 - i\sqrt{s} \Gamma_{PV}^V} ,$$

where $g_{PP}^V$ is the $V \to PP$ coupling, defined such that $\Gamma(V \to PP) = g_{PP}^2 \Gamma_0(M_V^2)$. It is then easy to check that in the limit of a single narrow resonance, integrating over $s$, we recover $\Gamma(D \to PP\gamma) = \Gamma(D \to V\gamma) \times B(V \to PP)$. In order to estimate the maximal direct CP asymmetry in the $D \to K^+K^-\gamma$ case, with $M_{KK}$ close to the $\phi$ peak, we evaluate $M(s)$ and $E(s)$ summing over the three light vector resonances ($V = \rho, \omega, \phi$) with the following assumptions:

- In all cases we use the parametric form in Eq. (22) to estimate the overall magnitude of $A_{\text{PC},PV}^V$, assuming further $B(D \to (\rho, \omega)\gamma) \geq 10^{-5}$.
- For $V = \rho, \omega$ we assume the weak phase of $A_{\text{PC},PV}^V$ is $\zeta_{\text{weak}}$, while for $V = \phi$ we use $r_{\text{weak}}$. Here $r =$
with different weak phases, we find smoothly varying strong phases for the contributions

Under these hypotheses, and assuming maximal and smoothly varying strong phases for the contributions with different weak phases, we find

\[ |a_{K+K^-}|^{\text{max}} \approx 2\% , \quad 2m_K < \sqrt{s} < 1.05 \text{ GeV}, \]
\[ |a_{K+K^-}|^{\text{max}} \approx 6\% , \quad 1.05 \text{ GeV} < \sqrt{s} < 1.20 \text{ GeV}. \]

(30)

In the first bin, close to the \( \phi \) peak, the leading contribution is due to the \( \phi \)-exchange amplitude. The contribution due to the non-resonant amplitudes plays a significant role far enough from the \( \phi \) peak, where the charge asymmetry can become larger. However, it must be stressed that away from the \( \phi \) peak the overall rate of the \( D \to K^+K^-\gamma \) process is significantly reduced.

V. DISCUSSION

In order to establish the significance of these results, two important issues have to be clarified: 1) the size of the CP asymmetries within the SM, 2) the role of the strong phases.

As far as the SM contribution is concerned, we first notice that short-distance contributions generated by the operator \( Q_7 \) are safely negligible: using the result in Ref. [13] we find asymmetries below the 0.1% level. The dominant SM contribution is expected from the leading non-leptonic four-quark operators, for which we can apply the general arguments presented in [4]. The CP asymmetries can be decomposed as

\[ |a_j^{\text{SM}}| \approx 2\xi \text{ Im}(R_j^{\text{SM}}) \approx 0.13\% \times \text{ Im}(R_j^{\text{SM}}), \]

(31)

where \( \xi \equiv |V_{cb}V_{ub}|/|V_{ub}V_{us}| \) and \( R_j^{\text{SM}} \) is a ratio of suppressed over leading hadronic amplitudes, naturally expected to be smaller than 1. This decomposition holds both for the \( f = \pi\pi, KK \) channels discussed in Ref. [4] and for the \( f = V\gamma \) case analyzed here. The SM model explanations of the asymmetry in Eq. (2) require \( R_{\pi\pi, KK}^{\text{SM}} \sim 3 \). While we cannot exclude this possibility from first principles, a further enhancement of one order of magnitude in the \( D \to V\gamma \) mode is beyond any reasonable explanation in QCD. As a result, an observation of \( |a_{V\gamma}| \gtrsim 3\% \) would be a clear signal of physics beyond the SM, and a clean indication of new CP-violating dynamics associated to dipole operators.

Having clarified that large values of \( |a_{V\gamma}| \) would be a clear footprint of non-standard dipole operators, we can ask the question if potential tight limits on \( |a_{V\gamma}| \) could exclude this non-standard framework. Unfortunately, the uncertainty on the strong phases does not allow to draw this conclusion. We recall that the maximal values in Eqs. (25) and (30) can be reached only in the limit of maximal constructive interference (namely of \( \pm\pi/2 \) strong phase difference) of the amplitudes with different weak phases. The calculation of light-quark loop contributions in Ref. [13] does suggest the presence of large strong phases in these amplitudes; however, we cannot exclude destructive interference effects leading to \( |a_{V\gamma}| = \mathcal{O}(0.1\%) \) even in presence of a non-standard CP-violating phase in the dipole operator. In principle, this problem could be overcome via time-dependent studies of \( D(\bar{D}) \to V\gamma \) decays or using photon polarization, accessible via lepton pair conversion in \( D \to V(\pi^+\pi^-) \ell^+\ell^- \); however, these types of measurements are certainly more challenging from the experimental point of view.

VI. CONCLUSIONS

Radiative \( D \to P^+P^-\gamma \) decays, with \( M_{PP} \) close to the \( \rho \) or the \( \phi \) peak (for \( P = \pi \) or \( K \), respectively), could help in shedding light on the origin of CP violation in the charm system. If the experimental result in Eq. (2) is due to non-standard dynamics involving dipole operators, we can expect significantly larger direct CP asymmetries in these radiative modes. As we have shown, evidence of \( |a_{PP\gamma}| \gtrsim 3\% \) would be a clear signal of physics beyond the SM, and a clean indication of new CP-violating dynamics associated to dipole operators.

Acknowledgments

We thank G. Giudice and G. Perez for useful comments and discussions and D. Bečirević for help with hadronic matrix element estimates. GI acknowledges the support of the TU München – Institute for Advanced Study, funded by the German Excellence Initiative, and the EU ERC Advanced Grant FLAVOUR (267104). The work of JFK was supported in part by the Slovenian Research Agency.

[1] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 108 (2012) 111602 [arXiv:1112.0938].
[2] A. Di Canto [CDF Collaboration], talk presented at La Thuile 2012 (March 2012); CDF Public Note 10784; see
also T. Aaltonen et al. [CDF Collaboration], Phys. Rev. D 85 (2012) 012009 [arXiv:1111.5023].

[3] Y. Grossman, A. L. Kagan, Y. Nir, Phys. Rev. D 75, 036008 (2007) [hep-ph/0609178].

[4] G. Isidori, J. F. Kamenik, Z. Ligeti and G. Perez, Phys. Lett. B 711, 46 (2012) [arXiv:1111.4987].

[5] H.-Y. Cheng and C.-W. Chiang, Phys. Rev. D 85 (2012) 034036 [arXiv:1201.0785]; E. Franco, S. Mishima and L. Silvestrini, arXiv:1203.3131; H.-n. Li, C.-D. Lu and F.-S. Yu, arXiv:1203.3120.

[6] M. Golden and B. Grinstein, Phys. Lett. B 222, 501 (1989).

[7] J. Brod, A. L. Kagan and J. Zupan, arXiv:1111.5000;

[8] D. Pirtskhalava and P. Uttayarat, arXiv:1112.5451; B. Bhattacharya, M. Gronau and J. L. Rosner, Phys. Rev. D 85 (2012) 054014 [arXiv:1201.2351]; T. Feldmann, S. Nandi and A. Soni, arXiv:1202.3795; J. Brod, Y. Grossman, A. L. Kagan and J. Zupan, arXiv:1203.6659.

[9] G. F. Giudice, G. Isidori and P. Paradisi, arXiv:1201.6204.

[10] W. Altmannshofer, R. Primulando, C.-T. Yu and F. Yu, JHEP 1204 (2012) 049 [arXiv:1202.2866]; G. Hiller, Y. Hochberg and Y. Nir, arXiv:1204.1046.

[11] Y. Grossman, A. L. Kagan and J. Zupan, arXiv:1204.3557.

[12] A. J. Buras, G. Colangelo, G. Isidori, A. Romanino and L. Silvestrini, Nucl. Phys. B 566 (2000) 3 [hep-ph/9908371].

[13] C. Greub, T. Hurth, M. Misiak and D. Wyler, Phys. Lett. B 382 (1996) 415 [hep-ph/9603417].

[14] Y. -L. Wu, M. Zhong and Y. -B. Zuo, Int. J. Mod. Phys. A 21 (2006) 6125 [hep-ph/0604007].

[15] J. Charles, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Rev. D 60, 014001 (1999) [hep-ph/9812358].

[16] D. Becirevic, V. Lubicz and F. Mescia, Nucl. Phys. B 769, 31 (2007) [hep-ph/0611295].

[17] D. Becirevic, private communication.

[18] P. Ball and R. Zwicky, Phys. Rev. D 71 (2005) 014029 [hep-ph/0412079]; C. -D. Lu, W. Wang and Z. -T. Wei, Phys. Rev. D 76 (2007) 014013 [hep-ph/0701265]; M. A. Ivanov, J. G. Korner, S. G. Kovalenko, P. Santorelli and G. G. Saidullaeva, Phys. Rev. D 85 (2012) 034004 [arXiv:1112.3536].

[19] K. Nakamura et al. [Particle Data Group Collaboration], J. Phys. G G 37, 075021 (2010).

[20] G. Burdman, E. Golowich, J. L. Hewett and S. Pakvasa, Phys. Rev. D 52 (1995) 6383 [hep-ph/9502299]; S. Fajfer, S. Prelovsek and P. Singer, Eur. Phys. J. C 6 (1999) 471 [hep-ph/9801279]; S. Fajfer, S. Prelovsek, P. Singer and D. Wyler, Phys. Lett. B 487 (2000) 81 [hep-ph/0006054].

[21] B. Bhattacharya and J. L. Rosner, Phys. Rev. D 77, 114020 (2008) [arXiv:0803.2385]; Phys. Rev. D 79, 034016 (2009) [arXiv:0812.3167]; Phys. Rev. D81, 014026 (2010) [arXiv:0911.2812].