Black-hole solutions correspondence between conformal and massive theories of gravity

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Abstract – In this work, a correspondence between black-hole solutions of conformal and massive theories of gravity is found. It is seen that this correspondence imposes some constraints on parameters of these theories. What is more, a relation between the mass of black holes and the parameters of massive gravity is found. Indeed, the acceptable ranges of massive gravity parameters ($c_1$ and $c_2$) are found. It is shown that by considering the positive mass of black holes, some ranges of $c_1$ and $c_2$ are acceptable.

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Introduction. – The general relativity (GR) is a successful theory for explaining the effects of gravity within the solar system such as the precession of the perihelion of Mercury and the gravitational bending of light around the Sun. Nevertheless, there are some puzzles left on scales beyond the distances of the solar system. For example, the observational evidence of galactic rotation curves is not consistent with the predictions of GR: the unknown dark matter. In addition, GR cannot describe the accelerated expansion of the Universe. In order to provide the energy source to explain this acceleration, the concept of dark energy is introduced. According to the mentioned reasons, the modification of GR is necessary. One of the modified theories of gravity is the conformal (Weyl) gravity, whose action is defined by the square of the Weyl tensor [1–6]. This theory is invariant under a conformal transformation of the metric tensor as

$$g_{\mu\nu} \rightarrow g_{\mu\nu}' = \Omega^2 g_{\mu\nu},$$

where $\Omega = \Omega(x)$ is a nonsingular function of spacetime coordinates.

It is notable that the solutions of the GR equation are a subset of the solutions of conformal gravity. It was shown that conformal gravity with a Neumann boundary condition can select the GR solution out of conformal gravity [4,7]. According to the fact that the conformal gravity possesses more solutions than GR, and also that it can explain dark-matter and dark-energy scenarios [8–11], this motivates us to consider the conformal gravity theory in this work.

Strictly speaking, conformal theory of gravity has some interesting properties, e.g., i) it is useful for constructing super-gravity theories [12,13]. ii) It can be considered as a possible UV completion of GR theory [11,14,15]. iii) It can solve the problem of spacetime singularities [16,17]. iv) It was shown that the observational data (X-ray data) of astrophysical black holes is compatible with this theory [18]. v) It also arises from twist-ener-string theory with both gauge-singlet open strings and closed strings [19]. vi) This theory can explain the dark side of the Universe [8–11]. vii) It can appear as a counterterm in AdS/CFT calculations [20,21].

Another modification of GR is represented by massive gravity theories. From the perspective of modern particle physics, graviton is a massless spin-2 particle in GR. In order to endow mass to the graviton, massive gravity was introduced by Fierz and Pauli in 1939 [22]. It is notable that this theory included massive graviton in flat space (linear level) where in nonflat background (nonlinear level) it encountered the Boulware-Deser ghost [23]. Fortunately, a version of the ghost-free massive theory was proposed by de Rham, Gabadadze and Tolley which is known as dGRT massive gravity [24–27]. In ref. [28], the mass-radius ratio bounds for compact objects have been studied in this gravity. Also, Yamazaki et al. [29], discussed the boundary conditions for the relativistic stars and obtained...
the mass-radius relation of stars in this gravity. Black-hole solutions and their thermodynamical quantities in the dRGT theory were investigated in refs. [30–40].

It is notable that, modification of reference metric related to the physical metric of spacetime leads to the possibility of introduction of different classes of dRGT-like massive gravity [27]. One of the massive theories of gravity is proposed by Vegh [41], which is applied in gauge/gravity duality. In other words, this theory is similar to the dRGT massive gravity with a difference in its reference metric which is a singular one. Graviton in Vegh’s massive gravity may behave like a lattice and exhibits a Drude peak [41]. In addition, it was shown that for an arbitrary singular metric, this theory of massive gravity is ghost free and stable [42].

Generally, massive gravity has some interesting properties, e.g., i) similarly to conformal gravity, this theory of gravity would provide a solution for the cosmological constant problem [43,44] and also can explain the self-acceleration of the Universe without adding the cosmological constant to the field equation [45]. In other words, one of the massive gravity terms plays the role of cosmological constant [46,47]; ii) it results into extra polarization for gravitational waves, and affects the propagation’s speed of gravitational waves [48], and also produces the gravitational waves during inflation [49,50]; iii) the existence of super-Chandrasekhar white dwarfs with mass greater than 1.45 times the mass of the Sun [51], and also of massive neutron stars with mass greater than three times that of the Sun [52]; iv) the topological black holes in this theory can exhibit van der Waals behavior [53], and heat engines [54]; v) the existence of a black-hole remnant in massive gravity with a possible candidate for dark matter [55].

Taking into account some similar results of conformal gravity and massive theory, one may be motivated to find a possible fundamental relation between them. In this paper we want to show that there is a correspondence between the black-hole solutions in conformal gravity and those of Vegh’s massive gravity. In other words, we will show that the exact solutions of the metric function in conformal gravity can be the same, term by term, as the corresponding solutions of Vegh’s massive gravity in a special case. We also use this correspondence to obtain some constraints on the massive parameters.

**Black-hole solutions in conformal gravity.** – The 4-dimensional action of conformal gravity is given by a square of the Weyl tensor in the following form:  
\[ I_C = -\alpha \int d^4x \sqrt{-g} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}, \]

where \(\alpha\) is a dimensionless coupling constant and plays an important role in critical gravity [56–58]. Hereafter, we set \(\alpha = 1\) without loss of generality.

Varying the action (2), with respect to the metric tensor \(g_{\mu\nu}\), leads to the following equation:
\[ (2\nabla^\mu \nabla_\mu + R^\sigma_\sigma) C_{\mu\nu\rho\sigma} = 0. \]  

Now we consider the field eq. (3), and a 4-dimensional general spacetime in the following form:
\[ ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 h_{ij}dx_i dx_j, \quad i, j = 1, 2, \]

in which \(h_{ij}dx_i dx_j\) is a 2-dimension line-element for an Euclidian space with constant curvature \(2k\) and volume \(V_2\), as
\[ h_{ij}dx_i dx_j = \begin{cases} \cdots & k = 0, \\ \cdots & k = 1, \\ \cdots & k = -1. \end{cases} \]
The solution of conformal gravity has been extracted in refs. [59,60] as
\[ f(r) = s_0 + s_1 r + \frac{s_2}{r} - \frac{\Lambda}{3r^2}, \]

where \(s_0, s_1, s_2\) and \(\Lambda\) are four constants. Due to the fact that the nonzero components of the field equation are, at least, third-order differential equations, one expects three integration constants. In order to extract the solution (6), we have to restrict three of these constants \((s_0, s_1, s_2)\) in the following constraint, as (see ref. [60], for more details)
\[ s_0^2 = 3s_1 s_2 + k^2. \]

Considering \(s_1 = 0\), this solution reduces to a Schwarzschild \((A)dS\) spacetime. Another interesting constant of the solution (6), is called \(\Lambda\). This constant plays the role of cosmological constant. It is notable that \(\Lambda\) appears as an integral constant and has not been inserted in the action by hand. Indeed, we can extract the cosmological constant by solving the field equation of conformal gravity (3), by using the mentioned metric (4). In other words, conformal gravity would solve the dark-matter problem and explains the current acceleration of our Universe.

Now, we briefly examine the geometrical structure of this solution. Our aim is to establish the possibility of having a black-hole solution. For this goal, we first look for the essential singularity(ies) by investigating two scalar curvatures: Ricci and Kretschmann scalars. Considering the metric (4), with (6), the Ricci scalar is extracted as
\[ R = 4\Lambda - \frac{6s_1}{r} + \frac{2(1 - s_0)}{r^2}. \]

Evidently, the Ricci scalar diverges at the origin (\(\lim_{r \to 0} R \to \infty\)). On the other hand, the Kretschmann scalar is given by
\[ K = \frac{8\Lambda^2}{3} - \frac{8s_1}{r} + \frac{8(1 - s_0)}{r^2} - \frac{8s_1 (1 - s_0)}{r^3} + \frac{4(1 - s_0)}{r^4} - \frac{8s_2 (1 - s_0)}{r^5} + \frac{12s_2^2}{r^6}. \]

One can show that this scalar has the following behavior:
\[ \lim_{r \to 0} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \to \infty, \]
\[ \lim_{r \to \infty} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \to \frac{8\Lambda^2}{3}. \]
which can confirm that: i) there is a curvature singularity at \( r = 0 \); ii) the asymptotic behavior of this solution is \((A)dS\), since the Kretschmann scalar is 8\(\Lambda\). Indeed, it is possible to find more than two horizons for black holes in conformal gravity. The thermodynamic properties of this solution have been studied in the literature [62–64].

**Black-hole solutions in massive gravity.** – The action of the Einstein gravity by adding massive terms and in the presence the cosmological constant \((\Lambda)\) is given by [41]

\[
I = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[ R - 2\Lambda + m^2 \sum_{i} U_i (g, f) \right]; \quad (12)
\]

in the above action \( R \) and \( m \) are the Ricci scalar and the mass of the graviton, respectively, and \( \Lambda = \frac{\kappa^2}{2} \). Also, \( g \) and \( f \) are a metric tensor and a fixed symmetric tensor, respectively. It is notable that \( c_i \)'s are arbitrary constants whose values can be determined according to observational or theoretical considerations. In addition, \( U_i \)'s are symmetric polynomials of the eigenvalues of the \( d \times d \) matrix \( K_{\mu}^{\nu} = \sqrt{g^{\mu\nu}} f_{\alpha\nu} \). Indeed, \( U_i \)'s are interaction terms which are introduced in the following forms [41]:

\[
\begin{align*}
U_1 &= [K], \\
U_2 &= [K]^2 - [K^2], \\
U_3 &= [K]^3 - 3[K][K^2] + 2[K^3], \\
U_4 &= [K]^4 - 6[K^2][K^2] + 8[K^3][K] + 3[K^2]^2 - 6[K^4], \\
U_5 &= [K]^5 - 10[K]^3[K^2] + 20[K]^2[K^3] - 20[K^2][K^3] + 15[K][K^2]^2 - 30[K][K^4] + 24[K^5],
\end{align*}
\]

\( \ldots \) \( (13) \)

Using the action (12) and varying it with respect to the metric tensor, \( g_{\mu\nu} \), one obtains

\[
G_{\mu\nu} + \Lambda g_{\mu\nu} + m^2 X_{\mu\nu} = 0, \quad (14)
\]

where \( G_{\mu\nu} \) is Einstein’s tensor. In the field equation (14), \( X_{\mu\nu} \) is the massive term which is given by

\[
X_{\mu\nu} = -\frac{c_1}{2}(U_1 g_{\mu\nu} - K_{\mu\nu})
- \frac{c_2}{2}(U_2 g_{\mu\nu} - 2U_1 K_{\mu\nu} + 2K_{\mu\nu})
- \frac{c_3}{2}(U_3 g_{\mu\nu} - 3U_2 K_{\mu\nu} + 6U_1 K_{\mu\nu} - 6K_{\mu\nu})
- \frac{c_4}{2}(U_4 g_{\mu\nu} - 4U_3 K_{\mu\nu} + 12U_2 K_{\mu\nu} - 24U_1 K_{\mu\nu}^3 + 24K_{\mu\nu}^4) + \ldots . \quad (15)
\]

In order to extract the static charged black holes in the context of massive gravity with \((A)dS\) asymptote, we consider the metric (4), and an appropriate reference metric introduced in refs. [41,65,66],

\[
f_{\mu\nu} = \text{diag}(0, 0, C^2 h_{ij}), \quad (16)
\]

where \( C \) is a positive constant. Considering the above reference metric, for 4-dimensional spacetime, \( U_i \)'s are as [65,66]

\[
\begin{align*}
U_1 &= \frac{2C}{r}, \\
U_2 &= \frac{2C^2}{r^2}, \\
U_i &= 0, \quad i > 2. \quad (17)
\end{align*}
\]

Using the metric (4) with the field equation (14), the metric function \( f(r) \) is obtained in the following form [65,66]:

\[
f(r) = k - \frac{M}{r} - \frac{\Lambda}{3} r^2 + m^2 \left( \frac{C c_1}{2} r + C^2 c_2 \right), \quad (18)
\]

where \( M \) is an integration constant related to the total mass of black holes. It is shown that the solutions (18) can be interpreted as black holes with more than two horizons [61,65,66]. The thermodynamic aspects of this solution have been studied in [32,53].

**The correspondence between black-hole solutions of conformal and massive gravities.** – Here, we want to show that the black-hole solutions in conformal and massive theories of gravity are the same. In other
words, we indicate that these theories of gravity describe the same black holes. For this goal, we compare these solutions. Our analysis shows that these solutions are the same provided we impose some constraints on parameters of conformal and massive gravities in the following forms:

\[ s_0 = k + m^2C^2c_2, \]
\[ s_1 = \frac{m^2C^2c_1}{2}, \]
\[ s_2 = -M. \]  

(19)

Adjusting the parameters of conformal and massive gravities in the above equation, black-hole solutions in these theories are the same. Here, we can ask why these solutions are the same. Although the answer to this question is not trivial, some similar properties help us to deepen our insight for these theories. For example, i) both of these theories can explain the dark matter, ii) both of them nicely address the cosmological constant problem and explain the self-acceleration of the Universe without introducing the cosmological constant, iii) these gravities consider the quantum nature of black holes, iv) both of the black holes have multi-horizons.

**Physical limitations based on this correspondence.** Another interesting result of this correspondence is related to the mass of black holes in massive gravity. Indeed, we can extract a relation for the mass of black holes which depends on the parameters of massive gravity. Considering the adjustment (19) and putting it into eq. (7), we obtain a relation for the mass of black holes vs. the parameters of massive gravity and the topological factor \( k \) as

\[ M = \frac{-2C^2c_2}{Sc_1} \left( 2k + m^2C^2c_2 \right). \]  

(20)

The above equation imposes some constraints on the mass of black holes. In other words, in order to have a positive mass of black holes, we obtain some conditions reported in table 1. According to the reported conditions of table 1, we obtain the acceptable ranges for \( c_1, c_2 \) and \( k \).

On the other hand, in order to study white dwarfs and neutron stars with spherical symmetric assumption, we have to consider \( k = 1 \). In addition, our studies on white dwarfs [51] and neutron stars [52] with spherical symmetry \( (k = 1) \) in massive gravity showed that the sign of \( c_2 \) should be negative, but the sign of \( c_1 \) can be both positive and negative. In order to explain compact objects such as neutron stars and white dwarfs with spherical symmetry in massive gravity, the acceptable ranges for the \( c_1 \) and \( c_2 \) are the I and IV cases in table 1.

As one can see, the parameters of the conformal gravity in eq. (19), depend on the massive graviton and parameters of massive gravity. Therefore, we may think that gravitons in the conformal gravity can behave like a massive particle.

**Closing remarks.** In this paper, in order to find a correspondence between black-hole solutions in conformal and massive theories of gravity, we have analyzed their metric functions in four dimensions. Comparing these black holes, we have found a correspondence between them which imposed some constraints on the parameters of these theories. Indeed, we have shown that these black holes are the same, when we impose some conditions between the parameters of conformal and massive gravities as i) \( s_0 = k + m^2C^2c_2 \), ii) \( s_1 = \frac{m^2C^2c_1}{2} \), and iii) \( s_2 = -M \).

On the other hand, in order to have a positive mass of black holes in massive gravity, we have obtained some conditions originated from the correspondence between black holes in conformal and massive gravities. For more investigations, we have compared the reported constraints of white dwarfs and neutron stars in massive gravity with these conditions. We have found seven physical ranges for studying black holes in massive gravity with different topologies (cases I to VII in table 1), and two physical ranges for evaluating the compact objects such as white dwarfs and neutron stars in massive gravity with spherical symmetric assumption. In other words, the suitable ranges were: i) \( c_1 > 0 \) and \( \frac{-2}{C^2c_1} < c_2 < 0 \), and ii) \( c_1 < 0 \) and \( c_2 < \frac{-2}{m^2C^2} \) (cases I and IV in table 1).

As future works, one can study the possible relation between the mentioned theories with electrically charged and higher-dimensional modifications. More fundamentally, it is interesting to look for a possible relation between the action and the field equation of conformal gravity and massive theory. In addition, regardless of a massless spin-2 graviton which is a standard mode of gravity, one can find that there are additional degrees of freedom related to massive or conformal theories of gravity. It is shown that in conformal gravity there is an additional spin-2 mode which can be a tachyonic or a massive ghost [67,68]. Besides, as regards massive gravity it is proven that different classes of massive gravity may have additional degrees of freedom [27]. It will be interesting to investigate the degrees of freedom of the mentioned theory and look for a possible relation between them.

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