Hadronic EDMs induced by the strangeness and constraints on supersymmetric CP phases

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Abstract

We consider hadronic CP violation induced by chromoelectric dipole moments (CEDMs) of light quarks and the QCD theta parameter ($\theta$). We concentrate on the strange quark CEDM especially in this paper. Using effective CP violating nucleon interactions induced by the CEDMs and $\theta$ with the SU(3) chiral Lagrangian technique, we calculate the EDMs for the $^{199}$Hg atom, neutron and deuteron. We also discuss supersymmetric contributions to the electric dipole moments with the mass insertion approximations and give stringent constraints on the CP phases of the flavor changing SUSY breaking terms.
1 Introduction

The origin of CP violation in nature is a very important issue in particle physics since CP violation is indispensable for the baryon asymmetry in universe. In the standard model (SM), there are two CP violating parameters; the phase of the CKM matrix and the QCD theta ($\theta$) parameter. The former induces CP violation in flavor changing processes, such as CP violating $K$ and $B$ meson decay modes whereas the latter induces flavor conserving CP violation, such as neutron electric dipole moment (EDM). The experimental upperbound on the neutron EDM gives a strong constraint on $\theta$, as $|\theta| \lesssim 10^{-10-11}$. On the other hand, the recent measurements of CP asymmetry in the $B$ decay modes at the Babar and Belle experiments confirm that the phase of the CKM matrix is the dominant source of the CP violation in the $K$ and $B$ decays. It is known that the phase of the CKM matrix is not much enough to explain the baryon asymmetry in universe. Therefore, it is very important to search for new CP violating phases.

The minimal supersymmetric SM (MSSM) is one of the most attractive models which describe physics beyond the SM. In the MSSM many new CP phases may be introduced in both the flavor-diagonal and changing soft SUSY breaking terms and these new phases can contribute to the CP violation at low energy experiments. Recently, the Belle collaboration announced that CP asymmetry in $B \rightarrow \phi K_s$ is $-0.96 \pm 0.50$, which is $3.5 \sigma$ deviation from the SM prediction [1]. On the other hand, the Babar result on the CP asymmetry in $B \rightarrow \phi K_s$ is consistent with the SM prediction [2]. Many papers appear to explain the anomaly in SUSY models [3, 4]. One of the most interesting possibilities is that the deviation is a signal of CP violation in right-handed bottom and strange squarks mixing. In particular, in the SUSY SU(5) GUT with right-handed neutrinos, the large right-handed bottom and strange squarks mixing may be induced due to the large neutrino mixing [5]. However, we have pointed out that the CP violation in right-handed squark mixing is strongly constrained by the $^{199}$Hg atomic EDM through the chromoelectric dipole moment (CEDM) of the strange quark [6].

Motivated by the observation that strange quark contribution to $^{199}$Hg atomic EDM may give an important implication to other observables, we revisit various hadronic EDMs induced by the quark CEDMs and the QCD theta term, especially paying an attention
to the strange quark CEMD. There are several theoretical approaches to calculate the hadronic EDMs induced by the quark CEDMs so far. In particular, the hadronic EDMs, including the neutron EDM, are evaluated in detail from the QCD sum rule [7, 8, 9, 10, 11, 12]. In this paper, we take a rather “traditional” approach based on the chiral Lagrangian. While the QCD sum rule is a good tool to dictate the QCD dynamics, it is difficult to incorporate the sea quark dynamics, especially in the neutron EDM. In fact, the sigma term in the chiral perturbation theory suggests that the sea quark dynamics is important in the baryon physics [13]. Thus, we adopt the chiral Lagrangian approach in this paper, and incorporate the strange quarks in it. We derive the CP violating effective Lagrangian with SU(3) chiral Lagrangian in the presence of the QCD theta term and the quark CEDMs for the evaluation.

From this Lagrangian we find that the strange quark CEDM contribution to the $^{199}$Hg EDM is induced by the $\eta^0$-$\pi^0$ mixing originated from the isospin violation. While the strange quark CEDM contribution is evaluated by the eta exchange contribution in the previous paper [15], the recent calculation for the Shiff moment [14] suggests that the contribution is suppressed by $O(10^{-2})$. However, we show that the derived constraint on the strange quark CEDM is comparable to the previous result from the eta exchange contribution. Also, for the neutron EDM, the loop calculation shows that the constraint on the strange quark CEDM is stronger than that from $^{199}$Hg EDM. The improvement of the deuteron EDM is proposed recently [16], and the sensitivity may reach to $d_D \sim (1 - 3) \times 10^{-27} e cm$. We discuss the sensitivity to the CP violating parameters in QCD, including the strange quark CEDM.

We also discuss the SUSY contributions to the hadronic EDMs through the quark CEDMs. We consider the gluino contribution with the mass insertion approximations and show the constraints on the CP phases of the flavor changing SUSY breaking terms, which are relevant to the SUSY flavor physics, from the hadronic EDM experiments.

This paper is organized as follows: In Section 2 the CP violation at quark level up to the dimension five operators is reviewed. In Section 3 we derive the effective nucleon interactions induced by the CP violating interaction at quark level. In Section 4 we estimate various hadronic EDMs with the CP violating nucleon interactions. In Section 5 we consider SUSY contributions to the hadronic EDMs with the mass insertion approxima-
tion and show the constraints on the SUSY parameters. Also we discuss the correlation between the hadronic EDM and the CP asymmetry in $B \rightarrow \phi K_S$. Finally, Section 6 is devoted to conclusion and discussion.

2 Hadronic CP violation in quark level

The CP violation in the strong interaction of the light quarks is dictated by the following effective operators,

$$\mathcal{L}_{\text{CP}} = \bar{\theta} \frac{\alpha_s}{8\pi} G \tilde{G} + \sum_{q=u,d,s} \frac{i}{2} \bar{q} \gamma_5 g_s (G \sigma) \gamma_5 q,$$

(1)

up to the dimension five ones. Here, $G_{\mu\nu}$ is the SU(3) gauge field strength, $\tilde{G}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}$ and $G \sigma = G^{\mu
u}\sigma^{\mu\nu} T^a$. The first term in Eq. (1) is the effective QCD theta term, and the second term is for the quark CEDMs. The $\bar{\theta}$ parameter can be $\mathcal{O}(1)$ generically, however, it is strongly constrained by the neutron EDM experiments. One of the most elegant solution is to introduce the Peccei-Quinn (PQ) symmetry [17], since the axion makes $\bar{\theta}$ vanish dynamically. On the other hand, when the quark CEDMs are non-vanishing, the theta term is induced again even if the PQ symmetry is introduced, as pointed out by Ref. [18]. Thus, in the evaluation of the effect of the quark CEDMs, we need to include the QCD theta term systematically. In this section we review the CP violating effective interactions in the strong interaction of the light quarks up to the dimension five operators and the role of the PQ symmetry.

We start from the case without the PQ symmetry, first. Let us take a basis, in which the QCD theta term is vanishing, by the chiral rotation as

$$\mathcal{L}_{\text{CP}} = - \sum_{q=u,d,s} m_q \bar{q} i \alpha_q \gamma_5 q + \sum_{q=u,d,s} \frac{i}{2} \bar{q} g_s (G \sigma) \gamma_5 q,$$

(2)

where we have used $|\bar{\theta}| \ll 1$. The first term is induced by the axial anomaly and $\alpha_q$’s satisfy the following relations,

$$\bar{\theta} = \sum_{q=u,d,s} \alpha_q.$$

(3)
Furthermore, we also impose for convenience a condition that the CP violating tadpoles for \( \pi^0 \) and \( \eta^0 \) should vanish, \( \langle \pi^0 | L_{\text{CP}} | 0 \rangle = \langle \eta^0 | L_{\text{CP}} | 0 \rangle = 0 \). Using the PCAC relation, these conditions and Eq. (3) determine \( \alpha_q \)'s as

\[
\alpha_u = \frac{m_d}{m_u + m_d} \left( \theta - m_0^2 \frac{d_s}{2m_s} \right) + m_0^2 \frac{d_u - d_d}{2(m_u + m_d)},
\]

(4)

\[
\alpha_d = \frac{m_u}{m_u + m_d} \left( \theta - m_0^2 \frac{d_s}{2m_s} \right) - m_0^2 \frac{d_u - d_d}{2(m_u + m_d)},
\]

(5)

\[
\alpha_s = \frac{m_u m_d}{m_s(m_u + m_d)} \left( \theta - m_0^2 \left( \frac{d_u}{2m_u} + \frac{d_d}{2m_d} \right) \right) + m_0^2 \frac{d_s}{2m_s},
\]

(6)

where \( m_0^2 \) is the ratio of the quark-gluon condensate to the quark one. From the QCD sum rule, it is estimated as [19]

\[
m_0^2 = \frac{\langle 0 | q g_s (G\sigma) q | 0 \rangle}{\langle 0 | q q | 0 \rangle} \simeq 0.8 \text{ GeV}^2.
\]

(7)

Notice that the CEDM of the strange quark gives sizable contributions to \( \alpha_u \) and \( \alpha_d \) when \( \tilde{d}_u / m_u \sim \tilde{d}_d / m_d \sim \tilde{d}_s / m_s \).

This situation is changed when the PQ symmetry is introduced, since \( \tilde{\sigma} \) is promoted to a dynamical field, axion (a),

\[
\mathcal{L} = a \frac{\alpha_s}{8\pi} G\tilde{G}.
\]

(8)

If the quark CEDMs are vanishing, \( \tilde{\sigma} (\equiv \langle a \rangle) \) is aligned dynamically to the zero at the minimum of the axion potential. However, the axion potential is modified in the presence of the quark CEDMs [18],

\[
V_{\text{eff}}(a) = K_1 a + \frac{1}{2} K a^2,
\]

(9)

where

\[
K = - \lim_{k \to 0} \int d^4 x e^{ikx} \left( \left| T \left[ \frac{\alpha_s}{8\pi} G\tilde{G}(x) \frac{\alpha_s}{8\pi} G\tilde{G}(0) \right] \right| 0 \right),
\]

(10)

\[
K_1 = - \lim_{k \to 0} \int d^4 x e^{ikx} \left( \left| T \left[ \frac{\alpha_s}{8\pi} G\tilde{G}(x) \sum_{q=u,d,s} i \frac{d_q}{2} \bar{q} g_s (G\sigma) \gamma_5 q(0) \right] \right| 0 \right).
\]

(11)

The linear term with respective to \( a \), which is proportional to the quark CEDMs, induces non-vanishing vacuum expectation value for \( a \). The matrix elements, \( K \) and \( K_1 \), are
evaluated using the current algebra [20, 18], and they are given as

\[ K = \frac{m_u m_d}{(m_u + m_d)} \langle 0 | \bar{q} q | 0 \rangle, \quad (12) \]

\[ K_1 = -\frac{m_u m_d}{2(m_u + m_d)} \sum_{q=u,d,s} \frac{\tilde{d}_q}{m_q} \langle 0 | \bar{q} i g_s (G\sigma) \gamma_5 q | 0 \rangle. \quad (13) \]

Thus, the minimum of the axion potential is shifted in the presence of the quark CEDMs and the following \( \theta \) parameter is effectively induced as

\[ \theta_{\text{ind}}(\equiv \langle a \rangle) = -\frac{K_1}{K} = m_0^2 \sum_{q=u,d,s} \frac{\tilde{d}_q}{2m_q}. \quad (14) \]

Plugging Eq. (14) into Eqs. (4-6), it is found that \( \alpha_q \)'s become simpler under the assumption of the PQ symmetry,

\[ \alpha_q = m_0^2 \frac{\tilde{d}_q}{2m_q} \quad (q = u, d, s). \quad (15) \]

The PQ symmetry suppresses the contribution of the strange quark CEDM to \( \alpha_u \) and \( \alpha_d \).

This means that the valence quarks themselves in nucleon do not suffer from the strange quark CEDM.

### 3 CP violating nucleon interactions

In the previous section we have considered the CP violating interactions at the quark level. We need to translate those quark level interactions into hadronic interactions in order to calculate the hadronic EDMs. This is a rather difficult task, and the sizable theoretical uncertainties are expected for evaluation of hadronic matrix elements. Here, we consider an approach based on the chiral Lagrangian technique. It is possible to discuss hadronic CP violation with the effective Lagrangian in a systematical and simple way. In this section we derive the effective CP violating nucleon interactions using the SU(3) chiral Lagrangian in order to incorporate the strange quark contribution in nuclear interactions.

First, let us review the chiral Lagrangian without CP violation. We consider the SU(3) \(_L\times SU(3)\_R\) chiral symmetry and the effective Lagrangian is written in terms of the
meson fields

\[ M = \begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\
\pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & K^0 \\
K^- & K^0 & -2\frac{\eta^0}{\sqrt{6}}
\end{pmatrix}, \tag{16}\]

and the baryon fields

\[ B = \begin{pmatrix}
\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\
\Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\
\Xi^- & \Xi^0 & -2\frac{\Lambda^0}{\sqrt{6}}
\end{pmatrix}. \tag{17}\]

The effective Lagrangian invariant under SU(3)_L × SU(3)_R is given by

\[ \mathcal{L}_0 = \frac{f_\pi^2}{4} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) + \text{Tr} (\bar{B} \gamma_5 (i \phi - m) B) \]

\[ + \frac{i}{2} \text{Tr} (\bar{B} \gamma_\mu (\xi \partial^\mu \xi^\dagger + \xi^\dagger \partial^\mu \xi) B) + \frac{i}{2} \text{Tr} (\bar{B} \gamma_\mu B (\partial^\mu \xi \xi^\dagger + \partial^\mu \xi^\dagger \xi)) \]

\[ + \frac{i}{2} (D + F) \text{Tr} (\bar{B} \gamma_\mu \gamma_5 (\xi \partial^\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi) B) \]

\[ - \frac{i}{2} (D - F) \text{Tr} (\bar{B} \gamma_\mu \gamma_5 B (\partial^\mu \xi \xi^\dagger - \partial^\mu \xi^\dagger \xi)) \]

where

\[ \xi = \exp \left( \frac{i M}{\sqrt{2f_\pi}} \right) \tag{19}\]

with \( U = \xi^2 \). The phenomenological parameters \( D \) and \( F \) are 0.81 and 0.44, respectively. The effective Lagrangian is expanded by the derivatives, and the lowest order terms are important at the low energy. The terms relevant in our paper are summarized in Appendix A.

The pseudoscalars are massless due to the SU(3)_L × SU(3)_R symmetry. However, the SU(3)_L × SU(3)_R symmetry is violated by small quark masses and the pseudoscalars become massive. Also, the isospin breaking effect has an important role in the EDMs of the \(^{199}\text{Hg}\) atom and deuteron as shown later, since it leads to the \( \pi^0-\eta^0 \) mixing,

\[ \begin{pmatrix}
\pi' \\
\eta'
\end{pmatrix} = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
\pi \\
\eta
\end{pmatrix}, \tag{20}\]

where \( \pi' \) and \( \eta' \) are for the mass eigenstates. The mixing angle is given as \( \tan \theta = -\sqrt{3}(m_u - m_d)/(4m_s) \approx 0.01 \) for \( m_u = 5.1 \text{ MeV}, m_d = 9.3 \text{ MeV} \) and \( m_s = 175 \text{ MeV} \).
Now we introduce the CP violating terms to the chiral Lagrangian. We need to evaluate the matrix element $\langle B_a \pi^c | i \mathcal{L}_{\text{CP}} | B_b \rangle$ in order to obtain the CP violating interactions between nucleons and pseudoscalars induced by the CP violating terms in Eq. (2). It is reduced by the PCAC relation,

$$\langle B_a \pi^c | i \mathcal{L}_{\text{CP}} | B_b \rangle = \sum_{q, q' = u, d, s} \frac{i}{\pi} \left[ (\alpha_q m_q + \alpha_{q'} m_{q'}) \langle B_a | q_T c q | B_b \rangle \right. \\
- \left( \frac{d_q}{2} + \frac{d_{q'}}{2} \right) \langle B_a | \overline{q} g_s (G\sigma) T_c q | B_b \rangle],$$

(21)

where $T_c$ is a generator for the flavor SU(3) symmetry.

The matrix elements of the scalar operators in the right-handed side in Eq. (21) are represented as

$$\langle B_a | q_T c q | B_b \rangle \equiv X \text{Tr} \left( B_a B_b T_c \right) + Y \text{Tr} \left( B_a T_c B_b \right),$$

(22)

$$\langle B_a | 1 q | B_b \rangle \equiv \delta_{ab} Z,$$

(23)

from consideration of the transformation property under SU(3)$_L \times$SU(3)$_R$. Here, 1 is the $(3 \times 3)$ unit matrix. The phenomenological parameters $X$, $Y$ and $Z$ can be determined by the baryon mass splittings and the nucleon sigma term [13].

$$X = \frac{1}{2m_s} (-2m_\Xi + 3m_\Lambda - m_\Sigma) = -\langle \overline{d}d \rangle + \langle \overline{s}s \rangle,$$

(24)

$$Y = \frac{1}{m_s} (m_\Xi - m_\Sigma) = -\langle \overline{d}d \rangle + \langle \overline{u}u \rangle,$$

(25)

$$Z = \frac{3\sigma}{m_u + m_d} - \frac{3m_\Xi - m_\Lambda}{2m_s} = \langle \overline{u}u \rangle + \langle \overline{d}d \rangle + \langle \overline{s}s \rangle,$$

(26)

where $\langle \overline{q}q \rangle \equiv \langle p|\overline{q}q|p \rangle$ and $\sigma$ represents the nucleon sigma term, which is estimated as $\sigma \simeq 45$ MeV. Numerically they are given by

$$X = -1.35, \quad Y = 0.72, \quad Z = 7.7,$$

(27)

$$\langle \overline{u}u \rangle = 3.5, \quad \langle \overline{d}d \rangle = 2.8, \quad \langle \overline{s}s \rangle = 1.4.$$  

(28)

For the evaluation of $\langle B_a | \overline{q} g_s (G\sigma) T_c q | B_b \rangle$, we use the following relation adopted in Ref. [15], which is inspired by the QCD sum rule,

$$\langle B_a | \overline{q} g_s (G\sigma) q | B_b \rangle \simeq \frac{5}{3} m_0^2 \langle B_a | \overline{q}q | B_b \rangle$$

(29)
for each quark. From Eqs. (22) and (29)

\[
\langle B_a | \bar{q} g_s (G\sigma) T_c q | B_b \rangle \equiv \frac{5}{3} m_0^2 \left( X \text{Tr} \left( \overline{B}_a B_b T_c \right) + Y \text{Tr} \left( \overline{B}_a T_c B_b \right) \right).
\]

(30)

This relation is, of course, approximate one, however, it makes the formula simple as shown now.

From the above consideration, Eq. (21) can be reduced as

\[
\langle B_a \pi^c | i \mathcal{L}_{\overline{CP}} | B_b \rangle = \frac{i}{f_\pi} \langle B_a | \overline{\sigma} \{ T_c, A \} q | B_b \rangle.
\]

(31)

The matrix $A$ in Eq. (31) is a diagonal matrix, $A = \text{diag}(A_u, A_d, A_s)$, with the components

\[
A_q = \alpha_q m_q - \frac{d_q}{2} \frac{5}{3} m_0^2.
\]

(32)

Therefore, the CP violating Lagrangian is obtained in terms of the baryon and meson fields,

\[
\mathcal{L}_{\overline{CP}} = \frac{1}{\sqrt{2} f_\pi} \left( X \text{Tr} \left( \overline{B} B \{ M, A \} \right) + Y \text{Tr} \left( \overline{B} \{ M, A \} B \right) \right)
+ \frac{2}{3} (Z - X - Y) \text{Tr} (AM) \text{Tr} \left( \overline{B} B \right).
\]

(33)

The relevant terms to our discussion in Eq. (33) are summarized in Appendix B.

The matrix elements $A_q$’s are written in terms of the quark CEDMs and the QCD theta parameter. When there is no PQ symmetry, they are given by

\[
A_u = 3.3 \times 10^{-3} \overline{\theta} - 0.53 \overline{d}_u - 0.14 \overline{d}_d - 7.5 \times 10^{-3} \overline{d}_s \text{ GeV}, \quad (q = u, d, s).
\]

(34)

\[
A_d = 3.3 \times 10^{-3} \overline{\theta} - 0.26 \overline{d}_u - 0.41 \overline{d}_d - 7.5 \times 10^{-3} \overline{d}_s \text{ GeV},
\]

(35)

\[
A_s = 3.3 \times 10^{-3} \overline{\theta} - 0.26 \overline{d}_u - 0.14 \overline{d}_d - 0.27 \overline{d}_s \text{ GeV}.
\]

(36)

Here, $\overline{d}_q$’s are written in GeV unit. On the other hand, when the PQ symmetry is introduced, $A_q$’s are written in terms of the quark CEDMs and become rather simple,

\[
A_q = -0.27 \overline{d}_q \text{ GeV}, \quad (q = u, d, s).
\]

(37)
4 Hadronic EDMs

In this section we consider the hadronic CP violation. Among various observables, the EDMs of neutron or atoms are very sensitive to the flavor conserving CP violation. In particular, the experimental upper bound on the EDMs of neutron [21] and $^{199}$Hg atom [22],

$$|d_{\text{Hg}}| < 2.1 \times 10^{-28} e\text{ cm},$$  \hspace{1cm} (38)

$$|d_n| < 6.3 \times 10^{-26} e\text{ cm},$$  \hspace{1cm} (39)

give strong constraints on hadronic CP violation. Also, the improvement of the deuteron EDM is proposed recently, and the sensitivity may reach to $d_D \sim (1 - 3) \times 10^{-27} e\text{ cm}$ [16]. Therefore, we estimate the EDMs of $^{199}$Hg atom, neutron and deuteron with the chiral Lagrangian obtained in the previous section.

4.1 $^{199}$Hg Atomic EDM

The $^{199}$Hg atom is a diamagnetic atom, in which electrons make a close shell. In such atoms, the atomic EDMs are primary sensitive to the CP violation in nucleons and represented by the Shiff moments ($S$). The Shiff moment generates the T odd electrostatic potential $V_{\text{eff}} = 4\pi S (\vec{I} \cdot \vec{\nabla})\delta(\vec{r})$ with the nucleus of spin ($\vec{I}$). The $^{199}$Hg atomic EDM is calculated in terms of the Shiff moment $S$ [23]

$$d_{\text{Hg}} = -2.8 \times 10^{-17} S^{(199}\text{Hg}) e\text{ cm},$$  \hspace{1cm} (40)

where $S$ is in unit of $e\text{ fm}^3$. This implies that the Shiff moment is bounded as [24]

$$|S| < 0.75 \times 10^{-11} e\text{ fm}^3,$$  \hspace{1cm} (41)

from Eq. (38).

The Shiff momentum is induced by the pion and eta exchanges with the CP violating coupling. From the formula derived in the previous section, the CP violating pion interactions are given as

$$\mathcal{L}_{\text{CP}}^{NN\pi} = \frac{1}{2f_\pi} \left( (A_u + A_d) - \frac{\theta}{3\sqrt{3}}(A_u - A_d) \right) \left( \langle \bar{u}u \rangle - \langle \bar{d}d \rangle \right) \bar{N}_\tau N^\alpha \pi^\alpha$$
\[
\frac{1}{2f_\pi} \left( (A_u - A_d) \left( \langle \bar{u}u \rangle + \langle \bar{d}d \rangle \right) + \frac{4\theta}{\sqrt{3}} A_s \langle \bar{s}s \rangle \right) \vec{N} \pi^0
\]
\[
+ \frac{\theta}{6\sqrt{3} f_\pi} (A_u - A_d) \left( \langle \bar{u}u \rangle - \langle \bar{d}d \rangle \right) \left( \vec{N} \tau^a N \pi^a - 3N \tau^3 N \pi^0 \right)
\]
\[
\equiv \vec{g}_{\pi NN} \vec{N} \tau^a N \pi^a + \vec{g}_{\eta NN} \vec{N} \pi^0 + \frac{2}{3} g_{\pi NN} \left( N \pi^a - 3N \pi^0 \right)
\]

where \( N = (p, n)^T \), and \( \vec{g}_{\pi NN} \), \( \vec{g}_{\eta NN} \) and \( \vec{g}_{\eta NN} \) correspond to the isoscalar, isovector and isotensor coupling constants, respectively. Here, we include the \( \pi^0 - \eta^0 \) mixing. The CP violating eta ones are

\[
\mathcal{L}_{\text{CP}}^{NN\eta} = \frac{1}{\sqrt{3} f_\pi} \left( \frac{1}{2} (A_u + A_d) \left( \langle \bar{u}u \rangle + \langle \bar{d}d \rangle \right) - 2A_s \langle \bar{s}s \rangle \right) \vec{N} \eta
\]
\[
+ \frac{1}{2\sqrt{3} f_\pi} (A_u - A_d) \left( \langle \bar{u}u \rangle - \langle \bar{d}d \rangle \right) \vec{N} \tau_3 N \eta
\]
\[
\equiv \vec{g}_{\pi NN} \vec{N} \eta + \vec{g}_{\eta NN} \vec{N} \tau_3 N \eta.
\]

The recent evaluation for the Schiff moment of \(^{199}\)Hg [14], in which the many-body treatment is performed, reveals that the core polarization effect reduces the isoscalar and tensor channel contributions while the isovector channel one is comparable to the previous results [25]. Their result for the Schiff momentum is\(^1\)

\[
S^{(199)\text{Hg}} = -4 \times 10^{-4} \times \left( g_{\pi NN}^{(0)} \vec{g}_{\pi NN}^{(0)} - \frac{1}{3} \frac{m_\pi^2}{m_\eta} g_{\eta NN}^{(0)} \vec{g}_{\eta NN}^{(0)} + \frac{m_\pi^2}{m_\eta} g_{\eta NN}^{(0)} \vec{g}_{\eta NN}^{(0)} \right) - 5.5 \times 10^{-2} \times \left( g_{\pi NN}^{(0)} \vec{g}_{\pi NN}^{(0)} - \frac{m_\pi^2}{m_\eta} g_{\eta NN}^{(0)} \vec{g}_{\eta NN}^{(0)} \right) + 9 \times 10^{-3} \times g_{\pi NN}^{(2)} \vec{g}_{\pi NN}^{(2)} [e \text{ fm}^3].
\]

Here, \( g_{\pi NN}^{(0)} \) and \( g_{\eta NN}^{(0)} \) is the CP even pion and eta coupling constants, respectively,\(^2\)

\[
\mathcal{L}_{\text{CP}} = \frac{m_\pi^2}{f_\pi} (D + F) \vec{N} \gamma_5 \tau^a N \pi^a - \frac{m_\pi^2}{\sqrt{3} f_\pi} (D - 3F) \vec{N} \gamma_5 N \eta^0
\]
\[
\equiv g_{\pi NN}^{(0)} \vec{N} \gamma_5 \tau^a N \pi^a + g_{\eta NN}^{(0)} \vec{N} \gamma_5 N \eta^0.
\]

The results in Ref. [25], which neglect the core polarization, correspond to \( S^{(199)\text{Hg}} = -0.086 \times g_{\pi NN}^{(0)} \vec{g}_{\pi NN}^{(0)} + \vec{g}_{\pi NN}^{(1)} - 2\vec{g}_{\pi NN}^{(2)} \) \( e \text{ fm}^3 \) for the pion contribution [14].

\(^1\)While only pion exchange is included in in their calculation, they show that the correction from the finite range of the pion interaction is small. Thus, the above introduction of the eta contribution is justified.

\(^2\)The isovector coupling in the CP even pion interaction is also induced by the \( \eta^0 - \pi^0 \) mixing, but we neglect it, since it does not change our qualitative discussion.
From this evaluation, it is found that the EDM of $^{199}$Hg atom is sensitive to $(A_u - A_d)$ through the isovector channel, and the contribution proportional to $(A_u + A_d)$ is suppressed by $O(10^{-2})$ due to the $\eta^0$-$\pi^0$ mixing or the isoscalar channel. This means that the $^{199}$Hg atomic EDM is insensitive to the QCD theta parameter. Also, notice that $g^{(1)}_{\pi NN}$ depends on $A_s$ through $\pi^0$-$\eta^0$ mixing. While eta itself has the CP violating interaction which depends on $A_s$, the eta exchange contributions are suppressed due to the isoscalar channel or the mass. Thus we concentrate on the isovector channel of the pion exchange from the CP violating pion interaction with non-vanishing $\eta^0$-$\pi^0$ mixing.

The Shiff moment from non-vanishing $g^{(1)}_{\pi NN}$ is represented in terms of $A_q$’s as

$$S(^{199}\text{Hg}) = -(23 \times (A_u - A_d) + 0.13 \times A_s) \ e \ fm^3,$$ (48)

where $A_q(q = u, d, s)$ are given in GeV unit. The current experimental bound on the EDM of $^{199}$Hg atom implies that

$$e|\tilde{d}_u - \tilde{d}_d + 0.0051\tilde{d}_s| < 2.4 \times 10^{-26} e \ cm.$$ (49)

This is almost independent of whether we impose the PQ symmetry or not, since it depends on $A_u - A_d$. The strange quark CEDM is bounded as

$$e|\tilde{d}_s| < 4.7 \times 10^{-24} e \ cm.$$ (50)

It is found that this bound does not have a qualitative difference, especially in the contribution of the strange quark CEDM, from one based on Ref. [25], that is, $e|\tilde{d}_u - \tilde{d}_d + 0.012\tilde{d}_s| < 7 \times 10^{-27} e \ cm$ [15]. In our evaluation the contribution of the strange quark CEDM comes from the $\eta^0$-$\pi^0$ mixing, and it is suppressed by $\sim (m_u - m_d)/(m_s m_\pi^2)$. On the other hand, the eta exchange contribution in the previous evaluation is suppressed by $\sim 1/m_\eta^2$. Thus, it is found that they are comparable.

### 4.2 Neutron EDM

The evaluation of the neutron EDM has a long history. However, the precious calculation is still a very difficult issue. One of the simple estimations is based on the Naive Dimensional Analysis [26]. However, we can know only the order of magnitude of the neutron
EDM in this method. More elaborated calculations are done based on the QCD sum rule [7, 8, 9, 10, 11, 12]. The QCD sum rule is an established technique, and it has a merit for the evaluation of the neutron EDM. In Ref. [11] the QCD sum rule analysis reproduces the ratio of the quark EDM contributions in the neutron EDM, which is expected from the non-relativistic quark model. On the other hand, they argue that the PQ symmetry suppresses the strange quark CEDM contribution in the neutron EDM. However, the suppression is for the valence quarks, not for the sea quarks. They introduce the contribution of the strange quark CEDM only through the vacuum expectation value of $\bar{q}i\gamma_5q$ with $q$ the valence quark under the CP violating background. It should be suppressed under the PQ symmetry as discussed in Section 2. In fact, the sigma term in the chiral perturbation theory suggests that the sea quark dynamics is important in the baryon physics [13]. As shown in Eq. (28), the matrix element of the strange quark is comparable to those of the other light quarks in nucleon.

Thus, we evaluate the neutron EDM from the effective Lagrangian obtained in the previous section by the traditional loop calculation so that we include the strange quark contribution. The neutron EDM ($d_n$) is defined by

$$L_{\text{EDM}} = -\frac{d_n}{2} \bar{n} \sigma_{\mu\nu} \gamma_5 n F^{\mu\nu}. \quad (51)$$

The EDM is induced by the one loop diagrams with the charged mesons in Fig. 1 and they are written as

$$d_n = \frac{e}{4\pi^2 f_\pi^2} \left( A_u x_u^{(n)} + A_d x_d^{(n)} + A_s x_s^{(n)} \right), \quad (52)$$

where

$$x_u^{(n)} = (D + F) \left( \langle \bar{u}u \rangle - \langle \bar{d}d \rangle \right) \log \frac{m_p}{m_\pi} + (D - F) \left( \langle \bar{d}d \rangle - \langle \bar{s}s \rangle \right) \log \frac{m_p}{m_K} = 2.0, \quad (53)$$

$$x_d^{(n)} = (D + F) \left( \langle \bar{u}u \rangle - \langle \bar{d}d \rangle \right) \log \frac{m_p}{m_\pi} = 1.7, \quad (54)$$

$$x_s^{(n)} = (D - F) \left( \langle \bar{d}d \rangle - \langle \bar{s}s \rangle \right) \log \frac{m_p}{m_K} = 0.33. \quad (55)$$

Here, we have considered the most singular terms, which have logarithmic singularities in the chiral limit originated from $\pi$ and $K$ mesons.
Let us consider the constraint from the neutron EDM experiment. When there is no
PQ symmetry, the neutron EDM is given in terms of the quark CEDMs and the QCD
theta parameter,

\[ d_n = (7.7 \times 10^{-16} \theta - 4.6 \times \tilde{d}_u - 3.0 \times \tilde{d}_d + 0.33 \times \tilde{d}_s) \text{ e cm}. \]  

(56)

If we impose the PQ symmetry,

\[ d_n = (1.6 \times \tilde{d}_u + 1.3 \times \tilde{d}_d + 0.26 \times \tilde{d}_s) \text{ e cm}. \]  

(57)

Here, the quark CEDMs in the above equations are written in cm unit. For comparison
to the result of the QCD sum rule in Ref. [11], the contributions from the up and down
CEDMs are slightly larger in our evaluation. It is reasonable since the sea quarks also
contribute to the neutron EDM in addition to the valence quarks.

From the current experimental bound of the neutron EDM in Eq. (39), we obtain the
following bounds for the the theta parameter and the quark CEDMs for a case that the
PQ symmetry does not exist,

\[ |\theta| < 8.2 \times 10^{-11}, \]

\[ e |\tilde{d}_u| < 1.4 \times 10^{-26} \text{ e cm}, \]

\[ e |\tilde{d}_d| < 2.1 \times 10^{-26} \text{ e cm}, \]

\[ e |\tilde{d}_s| < 1.9 \times 10^{-25} \text{ e cm}. \]  

(58)

If the PQ symmetry works,

\[ e |\tilde{d}_u| < 3.9 \times 10^{-26} \text{ e cm}, \]

\[ e |\tilde{d}_d| < 4.8 \times 10^{-26} \text{ e cm}, \]

\[ e |\tilde{d}_s| < 2.4 \times 10^{-25} \text{ e cm}. \]  

(59)

Here, we assume no accidental cancellation among the various contributions.

Notice that the constraint on the strange quark CEDM is one order of magnitude
stronger than that from the $^{199}$Hg atomic EDM, though the prediction for the neutron
EDM may be expected to have more theoretical uncertainty. While the experimental
bound on the neutron EDM is weaker than that of the $^{199}$Hg atomic EDM, the strange
quark constraint is not suppressed by the $\eta^0$ mass or the isospin violation.
4.3 Deuteron EDM

The new measurement for the deuteron EDM is proposed in Ref. [16], and it is argued that one or two orders of magnitude improvement may be achieved relative to the current bounds on the QCD theta parameter and the nuclear force. It may give a stringent constraint on the new physics or discover the signal. Also, the nuclear dynamics in deuteron is rather transparent and the theoretical uncertainty is expected to be small. The deuteron EDM is given by

\[
d_D = (d_n + d_p) + d_N^N, \tag{60}
\]

where \(d_p\) is the proton EDM and the second term comes from the CP violating nuclear force.

In Ref. [27] they show that the chiral logarithms in the sum of the proton and neutron EDM exactly cancel in the chiral perturbation theory, and they adopt the QCD sum rule for it. However, the cancellation in the chiral perturbation theory does not come from some symmetry. The chiral logarithms in the neutron and proton EDMs come from loop diagrams of charged mesons in which photons are attached to the changed mesons. Thus, it is obvious that the chiral logarithms are canceled in the SU(2) chiral perturbation theory. However, it is not true if we introduce the strange quark and the corresponding mesons.

In the SU(3) chiral perturbation theory, the sum of the proton and neutron EDMs is

\[
(d_p + d_n) = \frac{e}{4\pi f^2} \left( A_u x_u^{(n+p)} + A_d x_d^{(n+p)} + A_s x_s^{(n+p)} \right), \tag{61}
\]

where

\[
x_u^{(n+p)} = -\frac{1}{3} \left( D(\langle \bar{u}u \rangle - 5\langle \bar{d}d \rangle + 4\langle \bar{s}s \rangle) + 3F(\langle \bar{u}u \rangle + \langle \bar{d}d \rangle - 2\langle \bar{s}s \rangle) \right) \log \frac{m_p}{m_K} \tag{62}
\]

\[
x_d^{(n+p)} = \frac{1}{3} D(3\langle \bar{u}u \rangle - 10\langle \bar{d}d \rangle + 3\langle \bar{s}s \rangle) + F(\langle \bar{u}u \rangle - \langle \bar{s}s \rangle) \tag{63}
\]

\[
x_s^{(n+p)} = -\frac{1}{3} \left( D(\langle \bar{u}u \rangle - 5\langle \bar{d}d \rangle + 4\langle \bar{s}s \rangle) + 3F(\langle \bar{u}u \rangle + \langle \bar{d}d \rangle - 2\langle \bar{s}s \rangle) \right) \log \frac{m_p}{m_K} \tag{64}
\]
and then $x^{(n+p)}_u = 6.4$, $x^{(n+p)}_d = -2.7$, and $x^{(n+p)}_s = -0.40$. The logarithmic terms in the proton EDM come from the $K^+\Lambda^0$ loop in addition to the $K^+\Sigma^-$ and $\pi^+\nu$ loops. We also include constant terms which are not suppressed by proton mass. The strangeness has an important role in the deuteron EDM. When the PQ symmetry does not exist,

$$(d_p + d_n) = (6.4 \times 10^{-10}\theta - 7.7 \times \tilde{d}_u + 0.73 \times \tilde{d}_d + 0.23 \times \tilde{d}_s) \ e\ cm. \quad (65)$$

If it does,

$$(d_p + d_n) = (-5.1 \times \tilde{d}_u + 2.1 \times \tilde{d}_d + 0.32 \times \tilde{d}_s) \ e\ cm. \quad (66)$$

In Ref. [28] it is shown that $d^{NN}_D$ comes from the isovector coupling of pion $g^{(1)}_{\pi NN}$, given in Eq. (42), as

$$d^{NN}_D = -2.1 \times 10^{-15} \times g^{(0)}_{\pi NN}g^{(1)}_{\pi NN} \ e\ cm. \quad (67)$$

It implies

$$d^{NN}_D = (-11 \times \tilde{d}_u + 11 \times \tilde{d}_d - 0.063 \times \tilde{d}_s) \ e\ cm. \quad (68)$$

From the above evaluation, the measurement of the deuteron EDM will be a stringent test for the SM. If they establish the sensitivity of $d_D \sim (1 - 3) \times 10^{-27} e\ cm$, they can probe $ed_u \sim ed_d \sim 10^{-28} e\ cm$ from the nuclear force as shown in Ref. [27], and $\theta \sim 10^{-11} e\ cm$ from the nucleon EDMs. Also, for the strange quark CEDM, they may reach to $ed_s \sim 10^{-26} e\ cm$ from the nucleon EDMs.

5 SUSY contributions

5.1 Hadronic EDMs

Let us consider the SUSY contributions to the hadronic EDMs. In the MSSM many CP violating parameters can be introduced in the soft SUSY breaking terms. If there are SUSY CP phases, the quark CEDMs are induced through one loop diagrams. The bounds on the SUSY contributions to the hadronic EDMs can be estimated from the results in Section 4.
In our previous paper [6], we have considered the gluino contribution to the strange quark CEDM induced by the flavor mixing in the MSSM. While the EDMs are flavor conserving phenomena, the stringent bounds for the EDMs give constraints on even the flavor mixing, and they have an impact on the flavor physics, such as $B$ meson decay. Thus, we consider the gluino contributions including the flavor violation inside the Feynman graph in order to demonstrate the constraints on the SUSY CP phases from the our calculations.\footnote{In Ref. [29], it is pointed out that the chargino diagrams can also give large contribution to $^{199}\text{Hg}$ EDM. A comprehensive analysis of the SUSY contributions to the hadronic EDMs will be discussed elsewhere.}

In general SUSY models, both left-handed and right-handed squarks have flavor mixings. In this case, the CEDM of the $i$-th light quark $q_i$ is generated by a diagram in Fig. 2(a), and it can be enhanced by $m_{q_j}/m_{q_i}$ when $j > i$, ($i, j = 1, 2, 3$). Using the mass insertion technique, it is given by

$$
\tilde{d}_{qi} = c \frac{\alpha_s m_{\tilde{g}}}{4\pi m_{\tilde{q}}^2} \left( -\frac{1}{3} N_1(x) - 3N_2(x) \right) \text{Im} \left[ J_{ij}^{(q)} \right],
$$

where $m_{\tilde{g}}$ and $m_{\tilde{q}}$ are the gluino and averaged squark masses and $c$ is the QCD correction, $c \sim 0.9$. The functions $N_i$ are given as

$$
N_1(x) = \frac{3 + 44x - 36x^2 - 12x^3 + x^4 + 12x(2 + 3x) \log x}{6(x - 1)^6},
$$

$$
N_2(x) = \frac{-10 + 9x - 18x^2 - x^3 + 3(1 + 6x + 3x^2) \log x}{3(x - 1)^6}.
$$

The flavor violation in the squark mass terms contributes to the quark EDMs via combinations of $J_{ij}^{(q)}$,

$$
J_{ij}^{(q)} \equiv (\delta_{LL}^{(q)})_{ij} (\delta_{LR}^{(q)})_{jj} (\delta_{RR}^{(q)})_{ji}.
$$

The mass insertion parameters, $(\delta_{LL}^{(q)})_{ij}$, $(\delta_{RR}^{(q)})_{ij}$ and $(\delta_{LR}^{(q)})_{ij}$, are defined as

$$
(\delta_{LL}^{(q)})_{ij} = \left( \frac{m_{\tilde{q}L}^2}{m_{\tilde{q}}^2} \right)_{ij}, \quad (\delta_{RR}^{(q)})_{ij} = \left( \frac{m_{\tilde{q}R}^2}{m_{\tilde{q}}^2} \right)_{ij},
$$

for $q = u, d$, and

$$
(\delta_{LR}^{(d)})_{ij} = \delta_{ij} \frac{m_{d_j} (A_{ij}^{(d)} - \mu \tan \beta)}{m_d^2}, \quad (\delta_{LR}^{(u)})_{ij} = \delta_{ij} \frac{m_{d_j} (A_{ij}^{(u)} - \mu \cot \beta)}{m_u^2}.
$$
Here, \((m_{\tilde{q}_{L(R)}}^2)\) is the left-handed (right-handed) squark mass matrix. Here, we assume that the flavor-conserving SUSY breaking terms do not have the CP phases since they are already strongly constrained, and we keep terms induced by the off-diagonal terms in the squark mass matrices.

In the typical SUSY models, the left-handed squark mixings are governed by the CKM matrix, and even if the universal scalar mass hypothesis is imposed at high energy scale, the radiative correction induces the off-diagonal terms as

\[
(\delta^{(d)}_{LL})_{12} = O(\lambda^5) \simeq 3 \times 10^{-4}, \quad (\delta^{(d)}_{LL})_{23} = O(\lambda^2) \simeq 4 \times 10^{-2},
\]

\[
(\delta^{(d)}_{LL})_{13} = O(\lambda^3) \simeq 8 \times 10^{-3},
\]

\((75)\)

for \(\lambda \sim 0.2\). On the other hand, the right-handed squark mixing is rather model dependent. In the SU(5) SUSY GUT with right-handed neutrinos, large right-handed squark mixings can be induced by the neutrino Yukawa couplings [5]. They are constrained from the \(K^0-\bar{K}^0, D^0-\bar{D}^0, B^0-\bar{B}^0\) mixings experiments [30];

\[
\begin{align*}
|\delta^{(d)}_{RR})_{12}| & \lesssim 4 \times 10^{-2}, \\
|\delta^{(u)}_{LL})_{12}| & \lesssim 1 \times 10^{-1}, \\
|\delta^{(d)}_{RR})_{13}| & \lesssim 1 \times 10^{-1},
\end{align*}
\]

\((76)\)

for \(m_{\tilde{q}} = m_{\tilde{g}} = 500 \text{ GeV}\).

The quark CEDMs are estimated from Eq. (69) as

\[
ed_{\tilde{q}_u} = -7.9 \times 10^{-28} \sin \theta^{(2)}_u \text{ e cm} \times
\]

\[
\left( \frac{m_{\tilde{q}}}{500 \text{ GeV}} \right)^{-3} \left( \frac{(\delta^{(u)}_{LL})_{12}}{3 \times 10^{-4}} \right) \left( \frac{(\delta^{(u)}_{RR})_{21}}{0.1} \right) \left( \frac{A_c - \mu \cot \beta}{500 \text{ GeV}} \right),
\]

\[
ed_{\tilde{q}_d} = -4.0 \times 10^{-28} \sin \theta^{(2)}_d \text{ e cm} \times
\]

\[
\left( \frac{m_{\tilde{q}}}{500 \text{ GeV}} \right)^{-3} \left( \frac{(\delta^{(d)}_{LL})_{12}}{3 \times 10^{-4}} \right) \left( \frac{(\delta^{(d)}_{RR})_{21}}{4 \times 10^{-2}} \right) \left( \frac{\mu \tan \beta}{5000 \text{ GeV}} \right),
\]

\((77)\)
where we take $m_q = m_{	ilde{q}}$ and $\theta_{q_i}^{(j)}$ is the phase of the SUSY parameters, i.e. $\theta_{q_i}^{(j)} = \text{arg}[J_{ij}^{(q)}]$. Here, we take the experimental bounds for mass insertion parameters of the right-handed squarks in Eq. (76) while the left-handed ones are given by Eq. (75). We have neglected the $A$-terms in $\tilde{d}_d$ and $\tilde{d}_s$ since they are subdominant.

When one of the quark CEDMs saturates the hadronic EDM bounds, we can obtain the following bounds on the SUSY CP phases from the $^{199}$Hg atomic (neutron) EDM experiments,\(^4\)

$$
\left| \sin \theta_u^{(2)} \right| < 30 \times (47) \left( \frac{\delta_{LL}^{(u)}_{12}}{3 \times 10^{-4}} \right)^{-1} \left( \frac{\delta_{RR}^{(u)}_{21}}{0.1} \right)^{-1} \left( \frac{A_t - \mu \cot \beta}{500 \text{GeV}} \right)^{-1},
$$

$$
\left| \sin \theta_u^{(3)} \right| < 4.2 \times (6.5) \times 10^{-3} \left( \frac{\delta_{LL}^{(u)}_{13}}{8 \times 10^{-3}} \right)^{-1} \left( \frac{\delta_{RR}^{(u)}_{31}}{0.1} \right)^{-1} \left( \frac{A_t - \mu \cot \beta}{500 \text{GeV}} \right)^{-1},
$$

$$
\left| \sin \theta_d^{(2)} \right| < 60 \times (113) \left( \frac{\delta_{LL}^{(d)}_{12}}{3 \times 10^{-4}} \right)^{-1} \left( \frac{\delta_{RR}^{(d)}_{21}}{4 \times 10^{-2}} \right)^{-1} \left( \frac{\mu \tan \beta}{5000 \text{GeV}} \right)^{-1},
$$

$$
\left| \sin \theta_d^{(3)} \right| < 2.5 \times (4.7) \times 10^{-2} \left( \frac{\delta_{LL}^{(d)}_{13}}{8 \times 10^{-3}} \right)^{-1} \left( \frac{\delta_{RR}^{(d)}_{31}}{0.1} \right)^{-1} \left( \frac{\mu \tan \beta}{5000 \text{GeV}} \right)^{-1},
$$

$$
\left| \sin \theta_s^{(3)} \right| < 0.98 \times (0.048) \left( \frac{\delta_{LL}^{(s)}_{23}}{0.04} \right)^{-1} \left( \frac{\delta_{RR}^{(s)}_{32}}{0.1} \right)^{-1} \left( \frac{\mu \tan \beta}{5000 \text{GeV}} \right)^{-1},
$$

where we take $m_q = m_{\tilde{q}} = 500 \text{GeV}$. The above bounds on $\theta_u^{(2)}$ and $\theta_d^{(2)}$ are looser than than the stringent $K^0 - \bar{K}^0$ constraint. On the other hand, the CP phases related to the 1-3 and 2-3 mixing angles are constrained by the hadronic EDMs significantly. These bounds are expected to be improved furthermore by the deuteron EDM measurements.

\(^4\)Here we do not consider the quark EDM contributions to the neutron EDM. We find that they are subdominant compared with those of the quark CEDMs in our calculation.
5.2 Hadronic EDMs and CP asymmetry in $B \to \phi K_S$

Let us consider a correlation between $\tilde{d}_s$ and $S_{\phi K_s}$ in the SUSY models. In Ref. [6], we have shown that there is a strong correlation between them when both left-handed and right-handed squarks have flavor mixings. In such a case, the dominant contribution to $S_{\phi K_s}$ is supplied by a diagram with the double mass insertion of $(\delta^{(d)}_{RR})_{32}$ and $(\delta^{(d)}_{RL})_{33}$ (Fig. 2(b)). The contribution of Fig. 2(b) is given as

$$H = - C_8^R \frac{g_s}{8\pi^2} m_b s_R (G \sigma) b_L,$$

where

$$C_8^R = \frac{\pi \alpha_s m_\tilde{g}}{m_{\tilde{q}}^2 m_b} (\delta^{(d)}_{LR})_{33} (\delta^{(d)}_{RR})_{32} \left( - \frac{1}{3} M_1(x) - 3 M_2(x) \right),$$

up to the QCD correction. Here,

$$M_1(x) = \frac{1 + 9x - 9x^2 - x^3 + (6x + 6x^2) \log x}{2(x - 1)^5},$$

$$M_2(x) = - \frac{3 - 3x^2 + (1 + 4x + x^2) \log x}{(x - 1)^5}.$$

In a limit of $x \to 1$, $C_8^R$ is reduced to

$$C_8^R = \frac{7\pi \alpha_s}{60 m_b m_\tilde{q}} (\delta^{(d)}_{LR})_{33} (\delta^{(d)}_{RR})_{32}.$$

Using Eqs. (69) and (89), we find a strong correlation between $\tilde{d}_s$ and $C_8^R$ as

$$\tilde{d}_s = - \frac{m_b}{4\pi^2} \frac{11}{21} \text{Im} \left[ (\delta^{(d)}_{LL})_{23} C_8^R \right]$$

up to the QCD correction. The coefficient $11/21$ in Eq. (90) changes from 1 to $1/3$ for $0 < x < \infty$.

In Fig. 3, we show the correlation between $\tilde{d}_s$ and $S_{\phi K_s}$ assuming a relation $\tilde{d}_s = - \frac{m_b}{(4\pi^2)(11/21)} \text{Im}[(\delta^{(d)}_{LL})_{23} C_8^R]$ up to the QCD correction. Here, we take $(\delta^{(d)}_{LL})_{23} = -0.04$, arg$[C_8^R] = \pi/2$ and $|C_8^R|$ corresponding to $10^{-5} < |(\delta^{(d)}_{RR})_{32}| < 0.5$. The matrix element of chromomagnetic moment in $B \to \phi K_s$ is

$$\langle \phi K_s | \frac{g_s}{8\pi^2} m_b (\bar{s}_i \sigma^{\mu\nu} T_j^a P_R b_j) G^{a}_{\mu\nu} | \bar{B}_d \rangle = \frac{4\alpha_s}{9\pi} (\epsilon_{\phi P B}) f_{\phi} m_{\phi}^2 F_{+}(m_{\phi}^2).$$
and $\kappa = -1.1$ in the heavy-quark effective theory [4]. Since $\kappa$ may suffer from the large hadron uncertainty, we show the results for $\kappa = -1$ and $-2$. From this figure, the deviation of $S_{\phi K_s}$ from the SM prediction due to the gluon penguin contribution should be suppressed when the constraints on $d_s$ from the $^{199}$Hg atomic and the neutron EDMs are applied. Comparing Fig. 2 in Ref. [6], the $^{199}$Hg atomic EDM bound allows a sizable deviation of $S_{\phi K_s}$, especially in cases of $\kappa = -2$ and $|\langle \delta_{LL}^{(d)} \rangle_{23}|$ smaller than 0.04. It comes from the new theoretical estimation of the strange CEDM constraint in Eq. (50) and the numerical error in previous calculation. However, we find that the neutron EDM gives a strong bound on $S_{\phi K_s}$. Moreover, $S_{\phi K_s}$ may be constrained further by the future deuteron EDM measurements. Therefore, the hadronic EDMs give a very important implication to $S_{\phi K_s}$.

6 Conclusion and discussion

We have considered hadronic CP violation induced by chromoelectric dipole moments of light quarks and the QCD theta parameter, especially paying an attention to the strange quark CEDM. First, we have derived the effective CP violating nucleon interactions induced by the CEDMs and the theta parameter using the chiral Lagrangian technique. In order to take into account the strange quark contributions, we have used the SU(3) chiral Lagrangian. Using the effective CP violating Lagrangian, we calculated the EDMs of the $^{199}$Hg atom, neutron and deuteron.

The $^{199}$Hg atomic EDM is sensitive to the CP violating nuclear force induced by the $\pi$ and $\eta^0$ exchange diagrams. Though the contribution of strange quark EDM is evaluated from the eta exchange diagrams in the previous papers, it is found in the new evaluation of the Schiff momentum that the isoscalar channel contribution in the $\pi$ and $\eta^0$ exchange is suppressed. We found that the isospin breaking nucleon, interactions originated from the $\pi^0$-$\eta^0$ mixing, leads to the similar constraint on the strange quark CEDM to the previous one.

We evaluated the strange quark CEDM contribution to the neutron EDM using the standard meson loop calculation. This is originated from the one loop diagram involving the $K$ meson. We have found that the neutron EDM gives a stronger bound on the strange
quark CEDM in this calculation than the current $^{199}$Hg atomic EDM experiment, since the contribution to the neutron EDM is suppressed by the loop factor at most. While this calculation has theoretical uncertainties, it also suggests that the strange quark CEDM should be small.

The new technique for the measurement of the deuteron EDM has a great impact on the strange quark CEDM if it is realized. If they establish the sensitivity of $d_D \sim 10^{-27} \text{e cm}$, we may probe the new physics to the level of $e\tilde{d}_s \sim 10^{-26} \text{e cm}$, which is stronger than the bound from the neutron EDM.

In order to demonstrate an implication of our result on the SUSY models, we calculate the gluino contributions to the quark CEDMs with the flavor violating mass insertions. It is usually considered that the EDMs are sensitive to the flavor diagonal CP phases. However, when both left-handed and right-handed quark mixing exist, the CEDMs can be enhanced by the left-right squark mixings. Since the typical SUSY models have the left-handed squark mixing, the EDMs can give strong constraints on the flavor dependent SUSY phases. These constraints on the SUSY phases can give important implications to other SUSY phenomenology, including the $B$ physics. As an example, we have show that there is a strong correlation between the strange quark CEDM and $S_{\phi K_S}$. The current bounds on the strange quark CEDM from $^{199}$Hg atomic and the neutron EDMs imply that the deviation of $S_{\phi K_S}$ from the SM should be strongly suppressed.

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A CP even terms in chiral Lagrangian

In the chiral Lagrangian the interaction can be expanded by derivatives and the lowest order terms are important at the low energy. Here, we summarize the relevant CP even terms in the chiral Lagrangian. The relevant CP conserving couplings between the nucleons and the pseudoscalars are

\[
i\mathcal{L}_p = g_{\pi^c N u_s} \overline{N} \gamma_5 N u^c
\]

\[
= \frac{m_p}{f_\pi} (D + F) \overline{\nu} \gamma_5 p u^0 - \frac{m_n}{f_\pi} (D + F) \overline{\nu} \gamma_5 n u^0 + \frac{m_p + m_n}{\sqrt{2} f_\pi} (D + F) \overline{\nu} \gamma_5 p \pi^-
\]

\[
- \frac{m_p}{\sqrt{3} f_\pi} (D - 3F) \overline{\nu} \gamma_5 p \eta^0 - \frac{m_n}{\sqrt{3} f_\pi} (D - 3F) \overline{\nu} \gamma_5 n \eta^0
\]

\[
+ \frac{m_\Sigma + m_p}{\sqrt{2} f_\pi} (D - F) \overline{\nu} \gamma_5 p K^0 - \frac{m_\Sigma + m_n}{2 f_\pi} (D - F) \overline{\nu} \gamma_5 n K^0
\]

\[
- \frac{m_\Sigma + m_p}{2 \sqrt{3} f_\pi} (D + 3F) \overline{\nu} \gamma_5 n K^0 + \frac{m_\Sigma + m_p}{2 f_\pi} (D - F) \overline{\nu} \gamma_5 p K^-
\]

\[
- \frac{m_\Sigma + m_p}{2 \sqrt{3} f_\pi} (D + 3F) \overline{\nu} \gamma_5 p K^- + \frac{m_\Sigma + m_n}{\sqrt{2} f_\pi} (D - F) \overline{\nu} \gamma_5 n K^-, \quad (92)
\]

where we have used the equation of motion for the nucleon fields, \( \overline{N} \gamma_\mu \gamma_5 N' \partial^\mu M = -i (m_N + m_{N'}) \overline{N} \gamma_5 N' M. \) The coupling constants are given by numerically,

\[
g_{\pi^p} = 12.6, \ g_{\pi^m} = -12.6, \ g_{\pi^n} = 17.9, \quad (93)
\]

\[
g_{\eta^p} = 2.98, \ g_{\eta^m} = 2.98, \ g_{K^\Sigma n} = 5.98, \quad (94)
\]

for \( D = 0.81 \) and \( F = 0.44. \)

The pseudoscalars are massless due to the SU(3)_L × SU(3)_R symmetry. However, the SU(3)_L × SU(3)_R symmetry is violated by small quark masses and the pseudoscalars become massive. When the quark masses \( m = \text{diag}(m_u, m_d, m_s) \) are taken into account, we can introduce the following terms,

\[
\mathcal{L}_1 = v^3 \text{Tr} \left( U^\dagger m + m U \right)
\]

\[
+ a_1 \text{Tr} \left( \overline{B} (\xi^\dagger m \xi^t + \xi m \xi) B \right) + a_2 \text{Tr} \left( \overline{B} B (\xi^\dagger m \xi^t + \xi m \xi) \right)
\]

\[
+ b_1 \text{Tr} \left( \overline{B} \gamma_5 (\xi^\dagger m \xi^t - \xi m \xi) B \right) + b_2 \text{Tr} \left( \overline{B} \gamma_5 B (\xi^\dagger m \xi^t - \xi m \xi) \right)
\]

\[
= -2 v^3 \frac{f_\pi}{f_\pi} \text{Tr} (M^2 m) + 2a_1 \text{Tr} (\overline{B} m B) + 2a_2 \text{Tr} (\overline{B} B m)
\]

\[
- \frac{2ib_1}{\sqrt{2} f_\pi} \text{Tr} (\overline{B} \gamma_5 (M m + m M) B) - \frac{2ib_2}{\sqrt{2} f_\pi} \text{Tr} (\overline{B} \gamma_5 B (M m + m M) ) + \ldots \quad (95)
\]
In the estimation of the hadronic EDMs, the isospin breaking effect is important as shown in text. The isospin symmetry is violated by the quark mass term and the $\pi^0$-$\eta^0$ mixing occur through the mass terms. From Eq. (95), the $\pi^0$-$\eta^0$ mass matrix is given by

$$\mathcal{L}_{\pi-\eta} = \frac{2v^3}{f_\pi^2} (\pi, \eta) \left( \begin{array}{cc} m_u + m_d - \frac{1}{\sqrt{3}} (m_u - m_d) & \frac{1}{\sqrt{3}} (m_u + m_d + 4m_s) \\ \frac{1}{\sqrt{3}} (m_u - m_d) & \frac{1}{3} (m_u + m_d + 4m_s) \end{array} \right) \left( \begin{array}{c} \pi \\ \eta \end{array} \right). \quad (96)$$

The mass eigenstates are defined by

$$\left( \begin{array}{c} \pi' \\ \eta' \end{array} \right) = \left( \begin{array}{cc} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{array} \right) \left( \begin{array}{c} \pi \\ \eta \end{array} \right), \quad (97)$$

where $\tan \theta = -\sqrt{3} (m_u - m_d)/(4m_s) \simeq 0.01$ for $m_u = 5.1$ MeV, $m_d = 9.3$ MeV, $m_s = 175$ MeV.

**B CP-odd terms in chiral Lagrangian**

From the effective Lagrangian in Eq. (33), the CP violating nucleon interaction terms are written as follows. Here, we do not include the $\pi^0$-$\eta^0$ mixing.

$$\mathcal{L}_{\text{CP}} \equiv \bar{u}_c N_a N_b \mathcal{N}^a N_b \pi^c$$

$$= \frac{1}{f_\pi} \left( A_u \langle \bar{u}u \rangle - A_d \langle \bar{d}d \rangle \right) \overline{p} p \pi^0 + \frac{1}{f_\pi} \left( A_u \langle \bar{d}d \rangle - A_d \langle \bar{u}u \rangle \right) \overline{m} m \pi^0$$

$$+ \frac{1}{\sqrt{2} f_\pi} (A_u + A_d) \left( \langle \bar{u}u \rangle - \langle \bar{d}d \rangle \right) \overline{p} p \pi^-$$

$$+ \frac{1}{\sqrt{3} f_\pi} \left( A_u \langle \bar{u}u \rangle + A_d \langle \bar{d}d \rangle - 2A_s \langle \bar{s}s \rangle \right) \overline{p} p \eta^0$$

$$+ \frac{1}{\sqrt{3} f_\pi} \left( A_u \langle \bar{d}d \rangle + A_d \langle \bar{u}u \rangle - 2A_s \langle \bar{s}s \rangle \right) \overline{m} m \eta^0$$

$$- \frac{1}{\sqrt{2} f_\pi} (A_d + A_s) \left( \langle \bar{d}d \rangle - \langle \bar{s}s \rangle \right) \overline{\Sigma}^+ p K^0$$

$$+ \frac{1}{2 f_\pi} (A_d + A_s) \left( \langle \bar{d}d \rangle - \langle \bar{s}s \rangle \right) \overline{\Sigma}^0 n K^0$$

$$+ \frac{1}{2 \sqrt{3} f_\pi} (A_d + A_s) \left( \langle \bar{d}d \rangle + \langle \bar{s}s \rangle - 2 \langle \bar{u}u \rangle \right) \overline{\Lambda}^0 n K^0$$

$$- \frac{1}{2 f_\pi} (A_u + A_s) \left( \langle \bar{d}d \rangle - \langle \bar{s}s \rangle \right) \overline{\Sigma}^0 p K^-$$

$$- \frac{1}{2 f_\pi} (A_u + A_s) \left( \langle \bar{d}d \rangle - \langle \bar{s}s \rangle \right) \overline{\Sigma}^+ n K^-$$

$$- \frac{1}{2 \sqrt{3} f_\pi} (A_u + A_s) \left( \langle \bar{d}d \rangle + \langle \bar{s}s \rangle - 2 \langle \bar{u}u \rangle \right) \overline{\Lambda}^0 p K^0.$$
\[ + \frac{1}{2\sqrt{3}f_{\pi}} (A_{u} + A_{s}) \left( \langle dd \rangle + \langle ss \rangle - 2\langle uu \rangle \right) \bar{\Lambda}pK^{-} \]
\[ - \frac{1}{\sqrt{2}f_{\pi}} (A_{u} + A_{s}) \left( \langle dd \rangle - \langle ss \rangle \right) \bar{\Sigma}nK^{-}. \] (98)
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Figure 1: One loop diagrams for the neutron EDM. The blob represents the CP violating coupling.

Figure 2: (a) The dominant diagram contributing to the CEDMs of light quarks when both the left-handed and right-handed squarks have flavor mixings. (b) The dominant SUSY diagram contributing to the CP asymmetry in $B \to \phi K_s$ when the right-handed squarks have a mixing.
Figure 3: The correlation between $\tilde{d}_s$ and $S_{\phi K_s}$ assuming $\tilde{d}_s = -m_b/(4\pi^2)(11/21)\text{Im}[\langle \delta_{LL}^{(d)} \rangle_{23} C_R^8]$. Here, $(\delta_{LL}^{(d)})_{23} = -0.04$ and arg[$C_R^8$] = $\pi/2$. $\kappa$ comes from the matrix element of chromomagnetic moment in $B \to \phi K_s$. The dashed (dotted) line is the upperbound on $d_s$ from the EDM of $^{199}\text{Hg}$ atom (neutron).