Reynolds-number scaling of a vorticity-annihilating boundary layer

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To mimic the unsteady vortex–wall interaction of animal propulsion in a canonical test case, a vorticity-annihilating boundary layer was examined through the spin-down of a vortex from solid-body rotation. A cylindrical, water-filled tank was rapidly stopped, and the decay of the vortex from solid-body rotation was observed by means of planar and stereo particle image velocimetry. High Reynolds-number ($Re$) measurements were achieved by combining a large-scale facility (diameter, $D = 13$ m) with a novel approach to reduce end-wall effects. The influence of the boundary-layer formation at the tank’s bottom wall was minimised by introducing a saturated salt-water layer. The experimental efforts have allowed us to assess the $Re$ dependency of the laminar–turbulent transition of the vorticity-annihilating side-wall boundary layer at scales similar to large cetaceans. The scaling of the transition mechanism and its onset time were found to agree with predictions from linear stability analysis. Furthermore, the growth rate of the curved turbulent boundary layer was also in good agreement with an empirical scaling formulated in the literature for much smaller $Re$. Eventually, the scaling of vorticity annihilation was addressed. The earlier onset of transition at high $Re$ compensates for the reduced effects of viscosity, leading to similar vorticity annihilation rates during the early stages of the spin-down for a wide $Re$ range.

Key words: swimming/flying, vortex breakdown, transition to turbulence

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1. Introduction

Vortex–wall interactions are ubiquitous in fluid dynamics. Examples include swirling flows to enhance heat transfer (Kitoh 1991; Mitrofanova 2003), tank draining (Park & Sohn 2011), centrifugal separators (Hoffmann, Stein & Bradshaw 2003), and the interaction of tip vortices with the ground during take-off or landing of aircrafts (Kopp 1994). One of the most prominent examples of vortex–wall interaction is the formation of leading-edge vortices (LEVs; figure 1(a), Eldredge & Jones 2019). While already adapted in engineering solutions (such as helicopter flight (Ham & Garelick 1968), low-inertia rotor design (El Makdah et al. 2019), and micro-aerial vehicles (Pitt Ford & Babinsky 2013)), the aerodynamic principle is often associated with biological propulsion. An example of an LEV is shown in figure 1(a), where the acceleration of propulsors (such as wings or flippers) initiate the separation of a shear layer that in turn rolls up into a vortex. Subsequently, the vortex on the suction side interacts with the propulsor and thereby forms a unique boundary layer. The vorticity inside the vortex interacts with the opposite-signed vorticity at the wall and thereby initiates the vorticity-annihilating process. Wojcik & Buchholz (2014), Eslam Panah, Akkala & Buchholz (2015) and Akkala & Buchholz (2017) showed that this vorticity annihilation in the suction-side boundary layer has a significant contribution to the circulation budget of the LEV as it annihilates up to 50% of the vorticity fed by the separated shear layer. However, how the vorticity diffuses and/or convects from the near-wall region into the vortex and how the vorticity-annihilation process ensues remains to be fully understood. Buchner, Honnery & Soria (2017) observed centrifugal instabilities in the suction-side boundary layer, suggesting a complex and three-dimensional vorticity-annihilation process for medium Reynolds numbers $Re = O(10^4)$. However, in nature, vortices are formed over a wide range of scales ranging from $Re \approx 10^2$ (small insects) to $Re \approx 10^7$ (large cetaceans) (Gazzola, Argentina & Mahadevan 2014), leading to vorticity-annihilating boundary layers at significantly different $Re$. The present investigation, therefore, addresses the unsteady evolution of such a vorticity-annihilating boundary layer to understand its sensitivity to scale, i.e. $Re$.

In the current study, the interaction of a vortex with a wall is abstracted to the canonical flow of a decaying vortex from solid-body rotation (SBR), which in the following is referred to as the spin-down process; see figure 1(b). The vortex in SBR (angular velocity $\Omega$) is placed inside an impulsively stopped cylinder of radius $R$ with a cylindrical coordinate system ($r, \phi, z$) placed at its centre. For a fluid of viscosity $\nu$ the temporal evolution of the spin-down process from the initial conditions,

$$u_\phi(r) = \Omega r, \quad u_r = u_z = 0,$$

depends on the Reynolds number

$$Re = \frac{\Omega R^2}{\nu}$$

and a dimensionless time

$$\theta = \frac{\nu t}{R^2}.$$ 

Alternatively, the external dimensionless time $\Omega t = \theta Re$ can be utilised, where $\Omega t = 2\pi$ represents a full revolution of the SBR. For a limited range of $Re < 2.8 \times 10^4$, Kaiser et al. (2020) identified five successive stages of the spin-down process. Figure 1(b) shows the evolution of vorticity during the early stages, while figure 1(c) presents the spatially averaged azimuthal velocity profile $\langle u_\phi \rangle = \langle u_\phi \rangle_{\phi,z}(r,t)$. After the formation of

924 A11-2
Scaling of a vorticity-annihilating boundary layer

Figure 1. (a) Sketch of an LEV on a bird’s wing with an arrow pointing to the position of the vorticity-annihilating boundary layer. (b) Vorticity evolution (colour coded) and (c) the azimuthal velocity profile \( \langle u_\phi \rangle \) in stages I to III of the spin-down: initial condition (i.c.); laminar stage (I); instabilities and transition to turbulence (II); and sustained turbulence with intact vortex core and a region of constant angular momentum (III).

A laminar boundary layer (stage I) a primary centrifugal instability mechanism occurs and leads to the formation of streamwise laminar Taylor rolls that subsequently break down owing to secondary instabilities (stage II), initiating the transition to turbulence. During the turbulent stage III, the vortex core is still intact and in SBR. However, between the vortex core (blue in Figure 1c) and a turbulent near-wall region (red), a turbulent region is established (purple) that – in spatial average – is vorticity-free (\( \langle \omega_z \rangle = 0 \)). In this vorticity-free region the angular momentum \( l(t) = \langle u_\phi \rangle r \) is spatially constant and only a function of time \( t \), see Figure 1c). Eventually the vortex core breaks down (stage IV) and the flow relaminarises (stage V). Kaiser et al. (2020) define the boundary-layer thickness \( \delta_{99} \) as the distance from the cylinder wall, where \( \langle u_\phi \rangle \) deviates 1% from the initial condition; see Figure 1c). The magnitude of \( \delta_{99} \) grows continuously during the spin-down until the vortex core breaks down and the complete cylinder contains a turbulent flow (\( \delta_{99}/R = 1 \)).

The effects of curved streamlines, in combination with the oppositely signed vorticity layers, influences all stages starting from the centrifugal instability (Rayleigh 1917; Euteneuer 1972) over the turbulent statistics (Meroney & Bradshaw 1975) to the decay of anisotropic turbulence (Ostilla-Mónico et al. 2017). For increasing \( Re \), however, transition to turbulence occurs earlier (Kim & Choi 2006), and therefore the ratio of the boundary-layer thickness \( \delta_{99} \) to the integral length scale \( R \), at which transition occurs, decreases (\( \delta_{99}/R \ll 1 \)). In the related Taylor–Couette problem, the impact of vanishing curvature effects \( (d/R \ll 1, \text{ where } d \text{ is the gap between inner and outer cylinder}) \) was extensively studied. The flow approaches either the linearly stable plane-Couette problem that usually undergoes a nonlinear instability (Faisst & Eckhardt 2000) or the rotating plane-Couette flow (Nagata 1990; Nagata, Song & Wall 2021). For the present spin-down problem, small values of \( \delta_{99}/R \ll 1 \) lead to a flow that approaches the acceleration of a flat plate relative to a fluid. This so-called Stokes’ first problem is linearly unstable and destabilises owing to Tollmien–Schlichting (TS)-like waves (Luchini & Bottaro 2001). Comparing the stability analyses for centrifugal instability and TS waves, respectively, the critical times \( \theta_c \), at which transition is expected, can be retrieved (Luchini & Bottaro 2001; Kim & Choi 2006). As discussed in Appendix A, the critical times scale with \( \rho \xi \propto Re^{-4/3} \) in the case of centrifugal instability and with \( \theta_{TS} \propto Re^{-2} \) in the case of a TS instability. This is in agreement with the intuition that the TS instability eventually dominates at very large \( Re \). As it yet remains unclear whether the centrifugal instability
dominates over the entire Re range relevant for animal propulsion, the present study aims to close this gap in the literature.

Furthermore, in stage III and at moderate Re, the region of constant angular momentum \( \langle \omega_z \rangle = 0 \) can be explained in terms of curved streamlines (Rayleigh 1917). As streamline curvature is expected to be small during the onset of turbulence at high Re, the present study also explores whether and when a region of constant angular momentum is established in the investigated Re range.

To assess how a very high Re influences vorticity annihilation for both the transition-to-turbulence (stage II) and the sustained-turbulence stage (stage III), spin-down experiments up to stage III of the spin-down process were performed for \( Re \leq 4 \times 10^6 \). The experiments explored in the current study increase the Re range significantly in comparison with prior direct numerical simulations (DNS) in Kaiser et al. (2020) (\( Re \leq 2.8 \times 10^4 \)) and experimental investigations by Euteneuer (1972), and Mathis & Neitzel (1985) (\( Re \leq 2.5 \times 10^4 \)). This large advancement in Re becomes possible through the unique combination of a large-scale facility with a novel approach to reduce end-wall effects.

2. Experimental methods

To systematically expand the Re range from data available in the literature (\( Re \leq 2.8 \times 10^4 \)) up to very high Re (\( Re \leq 4 \times 10^6 \)), two experimental campaigns with complementary length scales were performed; see figure 2. A small-scale experiment (SSE; \( 2R = 0.49 \) m, \( 2.8 \times 10^4 \leq Re \leq 5.6 \times 10^5 \)) at the Karlsruhe Institute of Technology and a large-scale experiment (LSE; \( 2R = 13.00 \) m and \( 5.2 \times 10^5 < Re < 4 \times 10^6 \)) at the CORIOLIS II platform in Grenoble were conducted. In both campaigns the cylindrical containers were filled with water (height \( H \), kinematic viscosity \( \nu \), density \( \rho_w \)). Duck & Foster (2001) provide a comprehensive review of the effects of end-wall boundary layers on the flow in accelerated cylinders. As in the present study only the vorticity annihilation in the side-wall boundary layer is explored, end-wall effects are not of interest and should be minimised. To reduce end-wall effects, the experiments were performed in an open-surface configuration. Furthermore, in some runs (both SSE and LSE) a saturated salt-water layer (\( \rho_s = 1.2 \) g cm\(^{-3} \)) of height \( H_s \) was placed under the measurement volume at the bottom of the respective cylindrical container. Appendix B provides a detailed description of the experimental design of the salt-water layer approach and a comparison with results without this additional layer. It is shown that the salt-water layer successfully mitigates end-wall effects as long as the interface between the salt-water layer and the fresh-water layer remains stable.

The spin-down process was captured by means of planar and stereoscopic particle image velocimetry (PIV). Therefore, a horizontal laser light sheet was introduced for both campaigns (SSE and LSE) at height \( H_m = 0.5H \) and the water was seeded with polyamide 12 particles. Further details of the respective set-ups and experimental parameters are outlined in the following.

2.1. Small-scale experiments

As depicted in figure 2(a) an acrylic glass cylinder with an inner diameter of \( 2R = 0.49 \) m was filled with distilled water (\( \rho_w = 997 \) kg m\(^{-3} \)) and mounted to an aluminium framework via ball bearings that allow co-axial rotation of the cylinder. The cylinder was propelled by a closed-loop controlled step motor that allowed for fast deceleration without an additional braking device. Spin-down experiments in the
Scaling of a vorticity-annihilating boundary layer

Figure 2. Set-up of the SSE (a) and the LSE (b). Stereo PIV system focused on the near-wall region (red), planar PIV system covering the long-term boundary-layer evolution (blue), and planar PIV to assess the coherence of the vortex core (orange). Filling height \( H \), axial position of the measurement plane \( H_m \) and height of the saturated salt-water layer \( H_s \).

range of \( 0.47 \text{s}^{-1} \leq \Omega = 2\pi f \leq 9.42 \text{s}^{-1} \) were performed, which corresponds to the aforementioned \( Re \) range of \( 2.8 \times 10^4 < Re < 5.6 \times 10^5 \). For all angular velocities \( (\Omega) \) tested, the cylinder decelerated to rest within less than one revolution.

A low-speed planar PIV system consisting of a double-frame CCD camera (PCO Pixelfly, 1392 px x 1040 px, lens: Nikon AF Nikkor 50 mm f/1.4D) and a dual-pulsed Nd:YAG laser (Quantel Evergreen 70) was utilised to provide double-frame images at 5 Hz, leading to a delay of \( \Delta t = 0.2 \text{s} \) between two consecutive velocity fields. The field of view (FOV) in the image plane resulted in 110 mm x 82 mm with a resolution of 12.7 px mm\(^{-1} \). The pulse distance was adapted to the initial condition (i.e. \( \Omega \)) of the respective experiments to ensure a maximum displacement of 15 px. Tests at different aspect ratios \((A)\) \( 0.5 \leq A = H/R \leq 2.0 \) were performed to examine the influence of \( A \) on the end-wall effects.

2.2. Large-scale experiments

The CORIOLIS II platform was filled up to \( H = 1 \text{ m} \) with a weak saline solution \((\rho_w = 1004 \text{ kg m}^{-3})\) resulting in \( A = 2/13 \). The slightly higher density of the saline solution compared to the distilled water used in the SSE decreased the density difference between water and polyamide 12 particles \((\rho_p = 1014 \text{ kg m}^{-3})\) and thereby minimised settling of the particles during the extended spin-up times to SBR of 3 h before the start of the spin-downs. Experiments at various revolution times of the initial SBR within the range \( 120 \text{s} \leq 1/f \leq 470 \text{s} \) were performed, resulting in a \( Re \) range of \( 0.5 \times 10^6 \leq Re \leq 4.0 \times 10^6 \) for the LSE. Figure 2(b) provides an overview of the three PIV setups that were operated simultaneously. All three PIV set-ups utilised the same horizontal laser light sheet produced by a continuous 25 W Spectra Physics Millennia laser. A stereo set-up (red) took advantage of the optical access through the same window as the light sheet. Two Phantom Miro M310 cameras (Nikon AF Micro-Nikkor 60 mm f/2.8D) were
used to record the near-wall region with a resolution of 14.5 px mm\(^{-1}\). The long-term evolution of the boundary layer was recorded with a planar, time-resolved three-camera system consisting of two pco.edge 5.5 cameras (lens: Samyang ED AS UMC 35 mm) and a Dalsa Falcon 4M camera (lens: Nikon AF Nikkor 28 mm \(f/2.8D\)). The combined FOV had a radial extent of approximately 75 cm and a resolution of 10 px mm\(^{-1}\). The frame rate of the time-resolved measurements was set to achieve a maximum particle displacement of 30 px between two consecutive images. The frame rates varied within the range of 25–400 Hz (0.0025 s \(\leq \Delta t \leq 0.04\) s) dependent on \(Re\) and the FOV of the planar and the stereo set-up, respectively. Finally, a pco.1200 HS camera (lens: Nikon AF Nikkor 50 mm \(f/1.4D\); orange in figure 2b) was used to record the coherence of the vortex core at an approximate side-wall distance of 1.5 m (\(R_{ls} = 5\) m). A FOV of 25 cm \(\times\) 20 cm resulted in a resolution of approximately 5 px mm\(^{-1}\).

3. Results

The present section discusses the \(Re\) scaling of the vorticity-annihilating boundary layer. In particular, the onset of laminar-to-turbulent transition (§ 3.1) and the turbulent boundary-layer growth (§ 3.2) are investigated. Finally, the findings on the scaling of the transition to turbulence and the scaling of the turbulent boundary-layer growth are combined to examine the \(Re\) scaling of vorticity annihilation (§ 3.3).

3.1. Transition

Two aspects of the transition \(Re\) scaling are observed in the present section. First, it is to be validated if at very high \(Re\) centrifugal instabilities are still the primary instability mechanism yielding transition to turbulence in the boundary layer. Second, the time instance \(\theta_o = \nu \omega_o / R^2\), where \(\langle u_{\phi} \rangle\) first deviates from a stable laminar profile, is captured and compared with analytical, numerical, and experimental studies at smaller \(Re\), as reported in the literature.

As mentioned in § 1, a higher \(Re\) causes an earlier onset of the primary instability and therefore a thinner boundary layer (\(\delta_{99}\)) when the instability occurs. Therefore, diminishing curvature effects (\(\delta_{99}/R \ll 1\)) are expected and the boundary layer asymptotically approaches Stokes’ first problem, which is a boundary layer with TS-like waves rather than Taylor rolls as its primary instability. To identify which instability mechanism dominates at high \(Re\), the vorticity distribution is assessed by means of the highly resolved stereo PIV data. In case the centrifugal instability occurs first, characteristic plumes evolve from the wall that lead to rapidly growing laminar Taylor rolls as sketched in figure 3(a). Dependent on the relative position of the PIV laser sheet to the Taylor rolls, the footprint of the Taylor rolls in the captured data varies. In the centre of the plumes (case i, in figure 3a) slow fluid is transported from the near-wall region into the flow, while at the outer border of the plumes fast fluid is transported towards the wall (case ii). The most unique footprint of the Taylor rolls occurs in the plane where slow fluid engulfs the faster fluid (case iii). The resulting inflection points in the azimuthal velocity profile cause wall-parallel layers of axial vorticity \(\omega_z\) with opposite signs in the PIV data; see figure 3(b). The relative position of the laser sheet and the Taylor rolls varies from experiment to experiment, and even within a single run as the Taylor rolls grow in size and/or are not perfectly axisymmetric. In approximately 90\% of the LSEs the characteristic footprint of laminar Taylor rolls (case iii) was detected in the PIV data during transition. Thus, it can be concluded that the centrifugal instability dominates even for the very weak streamline curvature in the present \(Re\) range.
Scaling of a vorticity-annihilating boundary layer

Figure 3. (a) Sketch of the \(\omega_z\) distribution in the streamwise vortices; (b) footprint of streamwise vortices in the PIV measurements during a spin-down at \(Re = 1 \times 10^6\); and (c) onset of the centrifugal instability. Comparison of SSE and LSE with numerical data (empty markers), prior experiments (filled markers) and stability theory (lines). For \(Re \geq 2 \times 10^6\) (data to the right of the dotted line), the CORIOLIS II platform was not fully at rest at the onset of the instability. The data contained in (c) are also provided in Appendix A.

After the presence of Taylor rolls is confirmed, their onset time \((\theta_o = \nu t_o / R^2)\) can be compared with prior experiments of smaller \(Re\) and predictions from stability analysis. We define the time instant \(\theta_o\) as the time where the mean velocity profile \((\langle u_\phi \rangle)\) first deviates from the analytical stable laminar solution given by Neitzel (1982). The large scales of the LSE facility \((R = 6.5 \text{ m})\), in combination with the time-resolved measurements \((25 - 400\) frames per second, dependent on \(Re\) and PIV set-up), lead to a high temporal resolution in normalised units \((\theta = \nu t / R^2)\) and thereby allow for an accurate estimate of \(\theta_o\). The supplementary movies (movie 1 and movie 2) available at https://doi.org/10.1017/jfm.2021.600, show the process to determine \(\theta_0\) for two experiments at \(Re \approx 1 \times 10^6\) and \(Re \approx 2.6 \times 10^6\), respectively. In movie 1 case ii occurs first, resulting in a temporally decreasing \(\delta_{99}\) before case iii is detected. Eventually, the secondary instability initiates the transition to turbulence. In movie 2 only case iii is observed. Note that owing to the very high \(Re\) in movie 2, the centrifugal instability emerged before the CORIOLIS II platform was completely at rest. Therefore, the azimuthal velocity \((\langle u_\phi \rangle)\) was normalised by \(\Delta \Omega (t) R\), where \(\Delta \Omega (t)\) is the instantaneous change of the platform’s angular velocity and \(\Delta \Omega = 2 \Omega\) once the platform is at rest.

Figure 3(c) shows a comparison between the present results with the onset times of prior SSE, numerical simulations, and stability theory. Kim & Choi (2006) utilised propagation theory to determine the scaling of the critical time \(\theta_c = v t_c / R^2 = 9.4 Re^{-4/3}\) at which the linear instability first grows at a faster rate than the mean flow decays. Comparing critical and onset times, Kim & Choi (2006) observed that \(\theta_o \approx 4 \theta_c\). While the present data exceeds the \(Re\) range of prior measurements by two orders of magnitude, the experimental results still follow the predicted scaling.

Only for \(Re \geq 2 \times 10^6\) (right of the dotted line in figure 3c) do the data deviate from the expected behaviour. The deviation can be explained by the significant deceleration times of the CORIOLIS II platform. Owing to its high mass, the platform cannot be

924 A11-7
stopped instantly. Therefore, even though the platform was decelerated at the fastest possible rate, it was not fully at rest when instabilities occurred for the highest $Re$ experiments.

3.2. Turbulent boundary-layer growth

After the transition to turbulence, stage III of the vortex decay begins. For small $Re$, the flow during stage III can be partitioned into three regions (see figure 1c): a core in SBR, a turbulent region of, in spatial average, constant angular momentum $I(t) = \langle u_\varphi \rangle(t) r$ (equivalent to $\langle \omega_z \rangle = 0$), and a turbulent shear layer near the wall. As a result, the region of constant angular momentum represents the marginal case of the Rayleigh criterion (Rayleigh 1917). As such, the region of constant angular momentum only exists owing to the curved streamlines of the mean flow. Furthermore, Kaiser et al. (2020) observed that $\delta_{99}$ grows at the same rate ($\sim \sqrt{vt}$) during stage III ($\delta_{99}^{III}$) as for the laminar stage (stage I, $\delta_{99}^I$); however, with different coefficients, $a_{turb}(Re)$ and $a_{lam}$, respectively,

$$
\delta_{99}^I = a_{lam}\sqrt{vt}, \quad \delta_{99}^{III} = a_{turb}(Re)\sqrt{vt},
$$

(3.1a,b)

where $a_{lam} \approx 3.68$ for all $Re$. For the limited range $3 \times 10^3 < Re < 2.8 \times 10^4$, Kaiser et al. (2020) suggested the empirical scaling law

$$
a_{turb} = 0.1 Re^{1/3} a_{lam}.
$$

(3.2)

The present section expands on whether (a) the region of constant angular momentum, and (b) the scaling given by (3.2) prevail for high $Re$.

To identify if the region of constant angular momentum also exists for large $Re$, a robust estimate of the mean azimuthal velocity profile ($\langle u_\varphi \rangle = \langle u_\varphi \rangle_{\varphi, \tau} (r, t)$) is needed. Figure 4 shows PIV data for five time instances throughout the laminar, transitional and turbulent stages for $Re = 2.6 \times 10^6$. To obtain a good approximation of the mean velocity profile ($\langle u_\varphi \rangle$), the data are averaged along the $\varphi$-direction over the azimuthal extent of the PIV–FOV ($\langle u_\varphi \rangle_{\varphi}$, red lines). During the laminar stage, the azimuthal velocity profile only depends on $r$ and thus $u_\varphi = \langle u_\varphi \rangle = \langle u_\varphi \rangle_{\tau}$ for $t < t_0$ (see inlay in figure 4). However, as only a small subset of the flow is captured via the PIV measurements, the spatial average over the FOV does not suffice to provide a converged estimate of the mean velocity once transition to turbulence begins ($\langle u_\varphi \rangle_{\varphi} \neq \langle u_\varphi \rangle_{\tau}$ for $t > t_0$). To further smooth the data during the transitional and the turbulent stage, a moving time average ($\langle u_\varphi \rangle_{\varphi, \tau}$, black lines) over a small window $t_{avg}$ is calculated as follows:

$$
\langle u_\varphi \rangle_{\varphi, \tau} (r, t) = \frac{1}{(\varphi_{\text{max}} - \varphi_{\text{min}})t_{avg}} \int_{\varphi = \varphi_{\text{min}}}^{\varphi_{\text{max}}} \int_{t^{\text{avg}}/2}^{t + t_{avg}/2} u_\varphi(r, \varphi, t^*) \, dr \, d\varphi, \quad (3.3)
$$

where $\varphi_{\text{min}}$ and $\varphi_{\text{max}}$ are the limits of the FOV. For the LSE, $t_{avg}$ was set to 2.5% of the revolution time ($t_{avg} = 0.025/f$). In the case of the SSE, the data from three subsequent time steps of the low-speed PIV system (5 Hz) were averaged ($t_{avg} = 0.6$ s).

After the slow boundary-layer growth during stage I ($\Omega t = 1.1$ in figure 4) the boundary layer grows rapidly once the flow destabilises (stage II). Eventually, once $\delta_{99}/R = O(10^{-1})$, a region of constant angular momentum is established ($\Omega t = 8.7$ in figure 4). Thus, independent of $Re$, the ratio $\delta_{99}/R$ is large enough that curvature effects cause the region of constant angular momentum and thereby indicate the onset of stage III. As such, the same consecutive stages of vortex decay (stages I–III) are expected for all $Re$. 

924 A11-8
Scaling of a vorticity-annihilating boundary layer

Figure 4. Fitting techniques for a robust approximation of $\delta_{99}$ are exemplarily shown for an experiment at $Re \approx 2.6 \times 10^6$ with a salt-water layer at five arbitrary time instances $\Omega t \in \{1.1, 2.2, 4.3, 8.7, 23.9\}$ corresponding to $\theta = vt/R^2 \in \{4.1, 8.2, 16.4, 32.9, 90.0\} \times 10^{-7}$. The applied fitting functions are: analytical solution of the laminar profile during laminar stage (also shown in inlay); linear fit during transition to turbulence; and a potential vortex during the turbulent stage. The three-camera-FOV of the planar-PIV measurements is shown highlighted in light green.

In the following, the empirical scaling laws for $\delta_{99}(t)$ ((3.1a,b), (3.2)) are tested for the present high $Re$. As $\delta_{99}$ is susceptible to measurement noise, further post-processing steps are applied in order to calculate an accurate estimate of $\delta_{99}$. Figure 4 presents the fitting functions (blue lines) that are applied to $\langle u_\varphi \rangle_{\varphi,t}$ (black lines). For the laminar stage, the highly resolved stereo PIV data are fitted with the analytical solution of Neitzel (1982) as shown in the inlay of figure 4 (see also movie 1 and movie 2). During the transition to turbulence (stage II), $\delta_{99}$ quickly exceeds the FOV of the stereo-PIV system, which only recorded the near-wall region (see figure 2b). Therefore, the data of the planar three-camera system are utilised. In stage II, a linear fit is applied in the boundary layer (plus-shaped markers). For stage III, $l(t)/R$ is selected as the fitting function (triangular markers). For all stages, the intersection of the fit with the SBR (yellow markers in figure 4) is used to obtain an estimate for $\delta_{99}$. Note that this approach provides the possibility to estimate $\delta_{99}$ even beyond the FOV of the PIV measurements as exemplarily shown for $\langle u_\varphi \rangle_{\varphi}(r, \Omega t = 23.9)$ in figure 4.

Figure 5(a) shows a comparison of the evolution of $\delta_{99}$ in the SSE and the LSE with the numerical data of Kaiser et al. (2020). The low-speed PIV system of the SSE only allowed us to track the complete transition process for the smaller $Re$; for larger $Re$, the normalised temporal resolution ($\Delta \theta = v \Delta t/R^2$) of the SSE is insufficient. In contrast, time-resolved measurements and a large radius ($R$) during the LSE allowed tracking of $\delta_{99}$ during the laminar and the transitional stage despite the high $Re$. Owing to the finite spin-down times of the large-scale facility used for the LSE, $\delta_{99}$ grows slightly slower during the laminar stage, as compared to an impulsively stopped cylinder (dotted black vs solid orange and blue lines in figure 5a).

As discussed in § 3.1, the onset of transition scales with $Re$, which leads to small values of $\delta_{99}/R$ at the onset of the centrifugal instability. Once the centrifugal instabilities set in, the boundary layer grows rapidly during stage II for all $Re$ until the region of constant angular momentum is established and stage III (fully turbulent boundary layer with intact vortex core) begins.
As previously observed for the lower Re DNS data (Kaiser et al. 2020), $\delta_{99}$ grows again proportionally to $\sqrt{v_t}$ during stage III for all Re, with a Re-dependent proportionality factor $a_{\text{turb}}(Re)$. Figure 5(b) shows a comparison between the present experiments and the empirical scaling law given in (3.2) and shows good agreement for the largely extended Re range. Therefore, the flow properties of this particular growth rate of the turbulent boundary layer, combined with the existence of a turbulent-flow region with constant angular momentum, appears characteristic of the spin-down over a wide range of Re.

From a turbulence-modelling point of view, the observed similarity between turbulent and laminar boundary-layer growth rate suggests the use of an eddy-viscosity approach. Introducing an effective eddy viscosity $\overline{\nu_t}$, which is constant across the boundary layer and constant over time during stage III, yields

$$\delta_{99}^{\text{III}} = a_{\text{lam}} \sqrt{(v + \overline{v_t})t} = a_{\text{lam}} \sqrt{1 + \overline{v_t} / v \sqrt{v_t} \overline{a_{\text{turb}}}}. \quad (3.4)$$

The Reynolds-number dependency of $a_{\text{turb}}$ is thus fully contained in $\overline{v_t}$. The overbar for $\overline{v_t}$ is chosen to indicate the surprising fact that the present data suggest the presence of an effective eddy viscosity that remains constant in a statistically unsteady turbulent flow.

While the experimental data do not provide the possibility of a deeper analysis, the lower-Re DNS data of Kaiser et al. (2020) do provide some additional insight, which is summarised in Appendix C. It is found that the observed Re scaling of $a_{\text{turb}} / a_{\text{lam}}$ can be recovered if one assumes a near-wall plateau of the turbulent Reynolds number, located underneath the region of constant angular momentum, to be the driving force behind $\overline{v_t}$.

3.3. Scaling of vorticity annihilation

After scaling laws for the onset of transition (§ 3.1) and the turbulent stage (§ 3.2) are established, the Re scaling of vorticity annihilation can be discussed. According to

924 A11-10
Scaling of a vorticity-annihilating boundary layer

Figure 6. Vortex core circulation ($\Gamma_c$) for various $Re$. The plotted curves correspond to $Re \in [0.3, 0.6, 1.2, 2.8] \times 10^4$ (DNS, Kaiser et al. 2020), $Re \in [2.8, 5.6] \times 10^4$ (SSE, with salt-water layer, $A = 0.5$), $Re \in [2.8, 5.6] \times 10^5$ (SSE, no salt-water layer, $A = 2.0$), and $Re \in [2.0, 2.7] \times 10^6$ (LSE, salt-water layer, $A = 2/13$). (a) Time normalised in viscous units ($\theta = vt/R^2$); and (b) time normalised in outer units ($\Omega t$).

Morton (1984) vorticity is only introduced at a wall, when the wall is either accelerated or a streamwise pressure gradient exists. For the spin-down, no more vorticity is introduced when the cylinder is at rest, as no streamwise pressure gradient exists. Assuming an instantaneous deceleration, at $t = 0$ all negative vorticity is in an infinitesimally small layer at the wall. As such, the cylinder circulation, written

$$\Gamma = \int_A \omega_z \, dA = 2\pi \int_{r=0}^R \langle \omega_z \rangle \, r \, dr = 0,$$

(3.5)

is zero after the cylinder walls are impulsively stopped. During spin-down, the negative vorticity propagates into the core until eventually $\langle \omega_z \rangle = 0$ for all $r$ when the flow is at rest. This implies that there is a temporally decreasing amount of circulation $\Gamma_c$ in the core of the cylinder:

$$\Gamma_c = 2\pi \int_{r=0}^{R-\delta_{99}} \langle \omega_z \rangle \, r \, dr.$$

(3.6)

The core circulation can further be approximated to be

$$\Gamma_c \approx 2\Omega \pi (R - \delta_{99})^2$$

(3.7)

as $\langle \omega_z \rangle = 2\Omega$ is constant in the vortex core while the vortex core is still intact (stages I–III). Combining (3.7) with (3.2) and (3.1a,b) allows us to reconstruct the decay of $\Gamma_c$ during stages I and III for a given $Re$.

In the following, the temporal evolution of vorticity annihilation is captured by comparing the evolution of $\Gamma_c$ for various $Re$. Figure 6 presents the decay of the core circulation for $3 \times 10^3 \leq Re \leq 2.7 \times 10^6$. As $\Gamma_c$ can be approximated as a function of $\delta_{99}$ (see (3.7)), the stages of the boundary-layer growth shown in figure 5 are also visible in the evolution of $\Gamma_c$. During the laminar stage, $\Gamma_c$ decays slowly, before the transition to turbulence increases the decay rate. Eventually, once stage III is entered, the decay rate of $\Gamma_c$ decreases again. In viscous units ($\theta$), the earlier onset of transition leads to a faster decay of $\Gamma_c$ with increasing $Re$ (figure 6a).
Considering vortex–wall interactions on accelerated propulsors (wings or flippers, see § 1), the outer time scale $\Omega t$ provides a better comparison of the $Re$ scaling of vorticity annihilation. Observations in nature show that similar propulsors perform similar kinematics over a wide range of $Re$ (Taylor, Nudds & Thomas 2003; Gazzola et al. 2014). As such, a relevant metric for comparing the $Re$ scaling would be how much vorticity is annihilated on the time scale of the kinematics, e.g. after a full revolution of the vortex on the propulsor ($\Omega t = 2\pi$). Figure 6(b) compares the decay of $\Gamma_c$ on an outer time scale ($\Omega t$). In this scaling, a smaller $Re$ leads to a faster decay of $\Gamma_c$ owing to more pronounced viscous effects during the laminar stage. However, this is (in part) compensated by earlier transition for high $Re$. As such, around a full revolution of the SBR ($\Omega t = 2\pi$, highlighted in figure 6b) multiple lines intersect, suggesting similar $\Gamma_c$ despite significantly different $Re$. The similar amount of annihilated vorticity (similar $\Gamma_c$) after one core revolution ($\Omega t = 2\pi$) could explain why similar kinematics are observed during vortex formation in animal locomotion for a wide range of $Re$. Typically, the formed vortices are shed from the respective propulsor after each stroke. Therefore, the vorticity-annihilating boundary prevails only for a finite time before the stroke ends. Note, however, that if the boundary-layer evolution takes place over an extended period of time ($\Omega t > 20$), for instance by stabilising a LEV on a propulsor, a $Re$ scaling of the vorticity-annihilation process is likely to become more apparent, as shown in figure 6(b).

4. Concluding remarks

The interaction of vortices with solid boundaries leads to the formation of a unique boundary layer, where layers of oppositely signed vorticity interact. Two complementary experimental campaigns are presented that explore the $Re$ scaling of an unsteady vorticity-annihilating boundary layer. The interaction of the vorticity from the vortex with the vorticity introduced at the wall is investigated in the canonical flow of a decaying SBR (Taylor, Nudds & Thomas 2003; Gazzola et al. 2014). As such, a relevant metric for comparing the $Re$ scaling would be how much vorticity is annihilated on the time scale of the kinematics, e.g. after a full revolution of the vortex on the propulsor ($\Omega t = 2\pi$). Figure 6(b) compares the decay of $\Gamma_c$ on an outer time scale ($\Omega t$). In this scaling, a smaller $Re$ leads to a faster decay of $\Gamma_c$ owing to more pronounced viscous effects during the laminar stage. However, this is (in part) compensated by earlier transition for high $Re$. As such, around a full revolution of the SBR ($\Omega t = 2\pi$, highlighted in figure 6b) multiple lines intersect, suggesting similar $\Gamma_c$ despite significantly different $Re$. The similar amount of annihilated vorticity (similar $\Gamma_c$) after one core revolution ($\Omega t = 2\pi$) could explain why similar kinematics are observed during vortex formation in animal locomotion for a wide range of $Re$. Typically, the formed vortices are shed from the respective propulsor after each stroke. Therefore, the vorticity-annihilating boundary prevails only for a finite time before the stroke ends. Note, however, that if the boundary-layer evolution takes place over an extended period of time ($\Omega t > 20$), for instance by stabilising a LEV on a propulsor, a $Re$ scaling of the vorticity-annihilation process is likely to become more apparent, as shown in figure 6(b).

4. Concluding remarks

The interaction of vortices with solid boundaries leads to the formation of a unique boundary layer, where layers of oppositely signed vorticity interact. Two complementary experimental campaigns are presented that explore the $Re$ scaling of an unsteady vorticity-annihilating boundary layer. The interaction of the vorticity from the vortex with the vorticity introduced at the wall is investigated in the canonical flow of a decaying SBR (Taylor, Nudds & Thomas 2003; Gazzola et al. 2014). As such, a relevant metric for comparing the $Re$ scaling would be how much vorticity is annihilated on the time scale of the kinematics, e.g. after a full revolution of the vortex on the propulsor ($\Omega t = 2\pi$). Figure 6(b) compares the decay of $\Gamma_c$ on an outer time scale ($\Omega t$). In this scaling, a smaller $Re$ leads to a faster decay of $\Gamma_c$ owing to more pronounced viscous effects during the laminar stage. However, this is (in part) compensated by earlier transition for high $Re$. As such, around a full revolution of the SBR ($\Omega t = 2\pi$, highlighted in figure 6b) multiple lines intersect, suggesting similar $\Gamma_c$ despite significantly different $Re$. The similar amount of annihilated vorticity (similar $\Gamma_c$) after one core revolution ($\Omega t = 2\pi$) could explain why similar kinematics are observed during vortex formation in animal locomotion for a wide range of $Re$. Typically, the formed vortices are shed from the respective propulsor after each stroke. Therefore, the vorticity-annihilating boundary prevails only for a finite time before the stroke ends. Note, however, that if the boundary-layer evolution takes place over an extended period of time ($\Omega t > 20$), for instance by stabilising a LEV on a propulsor, a $Re$ scaling of the vorticity-annihilation process is likely to become more apparent, as shown in figure 6(b).
Scaling of a vorticity-annihilating boundary layer
to applications of unsteady vortex formation such as LEVs. The centrifugal instabilities
during LEV formation reported by Buchner et al. (2017) could only be shown at medium
$Re = O(10^4)$ and it yet remained unclear if at higher $Re$ the centrifugal instability
mechanism would prevail and/or have a significant impact on the flow. In the present study,
we find that Taylor rolls can also be expected during vortex formation at very high $Re$.
Furthermore, the effect of the centrifugal instability on the vorticity annihilation process
is significant throughout all $Re$. During the transition to turbulence (triggered by the
centrifugal instability), the boundary layer grows rapidly until $\delta_{99}/R = O(10^{-1})$. Thereby,
significant amounts of vorticity are annihilated and eventually a region of constant angular
momentum ($\langle \omega_z \rangle = 0$) is formed.

The empirical scaling law for the boundary-layer growth rate (3.2) allows
us to estimate how quickly the vorticity in the boundary layer cross-annihilates. To quantify the temporal
evolution of vorticity annihilation, the decay of the remaining circulation in the vortex core
($\Gamma_c \approx 2\Omega\pi(R - \delta_{99})^2$; (3.7)) is monitored on both a viscous as well as on an outer time
scale for a wide range of $Re$. For higher $Re$ the effects of viscosity are smaller and thus
the viscous diffusion of oppositely signed vorticity from the laminar boundary layer to the
fluid core in SBR is significantly slower. However, the earlier onset of transition ($\theta_o$) and
the rapid vorticity annihilation taking place during the transition to turbulence partially
compensate for the reduced effects of viscosity. As such, the amount of annihilated
vorticity after one revolution of the vortex core is similar for a wide range of $Re$ (see § 3.3).
This similarity in the vorticity-annihilation rate during the early stages could partially
explain the similar propulsor kinematics observed in nature across a wide range of scales
(Taylor et al. 2003; Gazzola et al. 2014). The vortices formed in nature often detach from
the propulsor after a few revolutions (end of the stroke), leading to a similar contribution
of vorticity annihilation to the circulation budget of the LEV, independent of $Re$. Future
work could address the $Re$-scaling effects of permanently stabilised LEVs as for example
investigated by Lentink & Dickinson (2009). It is hypothesised that once the dynamics of
the laminar-to-turbulent transition are excluded, $Re$ will have a more significant influence
on the circulation budget of the LEV.

**Supplementary movies.** Supplementary movies are available at https://doi.org/10.1017/jfm.2021.600.

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Appendix A. Centrifugal instability and TS waves

A.1. Critical time $\theta_c$

Before the onset of the instability the boundary layer grows as $\delta_99 = a_{lam} \sqrt{\nu t}$ (Kaiser et al. 2020). At a critical time $\theta_c = v_t / R^2$ a linear instability occurs. To estimate if, for a given $Re$, either the centrifugal instability ($\theta^{CI}_c$, Kim & Choi 2006) or TS waves ($\theta^{TS}_c$, Luchini & Bottaro 2001) occur first, the present section compares $Re$ scaling of the respective problem based on linear instability analysis. The similarity to the Taylor–Couette problem suggests a Taylor number for the spin-down flow that for small $\delta_99$ simplifies to

$$ Ta = \frac{\Omega (R - \delta_99)^{1/2} \delta_99^{3/2}}{\nu} \approx Re \left( \frac{\delta_99}{R} \right)^{3/2} = Re \left( \frac{a_{lam} \sqrt{vt}}{R} \right)^{3/2}, \quad (A1) $$

where $Re = \Omega R^2 / \nu$. Assuming the onset of transition at a critical time $t^{CI}_c$ and a critical Taylor number $Ta_c$, (A1) leads to

$$ \theta^{CI}_c = \frac{v_t Ta}{R^2} = \frac{Ta_4^{4/3}}{a_{lam}^2} Re^{-4/3} = C_1 Re^{-4/3}. \quad (A2) $$

Kim & Choi (2006) found the same scaling with a stability analysis. Based on the growth rate of the kinetic energy of the disturbance, Kim & Choi (2006) defined $\theta^{IC}_c$ and estimated $C_1 = 9.4$.

For very high $Re$ and with vanishing curvature effects, the spin-down flow approaches Stokes’ first problem. A boundary-layer Reynolds number $Re_\delta$ can be approximated to

$$ Re_\delta = \frac{U \delta_99}{\nu} = \frac{\Omega (R - \delta_99) \delta_99}{\nu} \approx Re \left( \frac{\delta_99}{R} \right) = Re \left( \frac{a_{lam} \sqrt{vt}}{R} \right). \quad (A3) $$

Equation (A3) provides the relation of a critical Reynolds number $Re^{c}_\delta$ and the critical time

$$ \theta^{TS}_c = \frac{v_t TS}{R^2} = (Re^{c}_\delta / a_{lam})^2 Re^{-2} = C_2 Re^{-2}. \quad (A4) $$

For Stokes’ first problem Luchini & Bottaro (2001) defined $\theta^{TS}_c$ as the instance in time where disturbances of a critical wavelength first get amplified, leading to $C_2 \approx 1485.68^2$.

It is apparent that $\theta^{TS}_c \propto Re^{-2}$, while $\theta^{CI}_c \propto Re^{-4/3}$. As such, for very high $Re$ the TS waves should occur before the centrifugal instabilities. Note, however, that a quantitative comparison of the critical times based on (A2) and (A4) cannot be performed as Luchini & Bottaro (2001) and Kim & Choi (2006) use deviating definitions of $\theta_c$. Furthermore, the remaining curvature of the spin-down problem, even at very high $Re$, might influence $\theta^{TS}_c$ and was neglected for the scaling estimates performed in this section.

A.2. Onset time $\theta_0$

The centrifugal instability dominates in the preset $Re$ range and scales according to the predictions of Kim & Choi (2006). To allow for the validation of future numerical studies, the data of figure 3(c) are listed in table 1. As $Re$ not only depends on $\Omega$ and $R$ but also on $\nu$, the temperature changes of the water in between different runs were monitored and explain the slight variations of $Re$ for LSE with the same initial conditions (i.e. $\Omega$, $R$).
Scaling of a vorticity-annihilating boundary layer

Table 1. Onset of the centrifugal instability for the SSE and the LSE. The table contains the data of figure 3.

| Set-up | Set-up | Re(×10^5) | θ_0(×10^{-6}) | Re(×10^5) | θ_0(×10^{-6}) |
|--------|--------|------------|----------------|------------|----------------|
| SSE    | SSE    | 0.28       | 98.58          | LSE        | 20.30          |
| SSE    | SSE    | 0.28       | 52.69          | LSE        | 19.68          |
| SSE    | SSE    | 0.00       | 81.58          | LSE        | 19.68          |
| SSE    | SSE    | 0.56       | 57.78          | LSE        | 19.68          |
| SSE    | SSE    | 0.56       | 49.29          | LSE        | 19.84          |
| SSE    | SSE    | 0.56       | 39.09          | LSE        | 19.84          |
| SSE    | SSE    | 0.90       | 29.57          | LSE        | 26.57          |
| SSE    | SSE    | 0.90       | 24.47          | LSE        | 26.24          |
| LSE    | LSE    | 5.24       | 1.282          | LSE        | 26.45          |
| LSE    | LSE    | 5.15       | 1.531          | LSE        | 39.86          |
| LSE    | LSE    | 10.12      | 0.634          | LSE        | 39.36          |
| LSE    | LSE    | 20.97      | 0.449          | —          | —              |

Appendix B. End-wall effects and their reduction

Duck & Foster (2001) provide a comprehensive review of the flow in angular-accelerated (enclosed) cylinders of height H with walls at the top and bottom. The end-wall boundary layer significantly influence all stages of the flow and thereby modifies the behaviour of the side-wall boundary layer. As such, end-wall effects are minimised in the present study. While in numerical investigations end walls can easily be removed by choosing periodic boundary conditions in the axial direction, a more elaborate solution was used for the present experimental implementations of the decaying SBR. Prior small-scale experimental campaigns by Euteneuer (1972), Mathis & Neitzel (1985) and Burin et al. (2006) successfully minimised the end-wall influence through either large-aspect-ratio cylinders (A = H/R) or segmented end walls. However, both aforementioned methods come with high cost, when applied to large scales (large R). An alternative approach was suggested by Pedlosky (1967), who used two layers of immiscible fluids with different densities. If the upper layer is confined by a free surface, linear theory predicts a significantly reduced contribution of end-wall effects on the upper layer. The experimental results of O’Donnell & Linden (1992) on accelerating cylinders validated the theoretical predictions. As expensive and/or hazardous fluids cannot be used for LSEs, a saturated salt-water solution (density ρ_s) was chosen as the high-density fluid and fresh water was used for the upper layer. As the two fluids were then miscible, the stability of the interface, as well as the reduction of end-wall effects, were recorded in the SSE and the LSE via a GoPro camera focused on the interface between the two layers.

B.1. Interface stability of the saline-solution-water interface

O’Donnell & Linden (1992) stated that if the interface between two stratified fluids interacts with the end wall during the spin-down process, the interface destabilises. To avoid end-wall-interface interaction, the minimum required height of the salt-water layer H_s^{min} was estimated analytically. The conservative (while non-physical) assumption of an instantaneous deceleration to rest of the salt-water layer during the spin-down process, while the top layer remains in SBR, leads to

\[ H_s^{min} = \frac{\rho_w}{\rho_s - \rho_w} \frac{\Omega^2 R^2}{4g}. \]  

(B1)
Table 2. Interface stability between the salt-water layer and the fresh-water layer. Reynolds number (Re), Interface Froude number ($Fr_s$), minimum salt-water level ($H_{smin}$) and the height of the salt-water level ($H_s$).

| Type | $f$ (Hz) | $\Omega$ (1 s$^{-1}$) | $Re$ | $Fr_s$ | $H_{smin}$ (cm) | $H_s$ (cm) | stable |
|------|---------|-----------------|------|--------|----------------|---------|--------|
| SSE  | 0.075   | 0.47            | $2.8 \times 10^4$ | 0.20   | 0.17          | 4.00    | yes    |
| SSE  | 0.150   | 0.94            | $5.6 \times 10^4$ | 0.82   | 0.68          | 4.00    | yes    |
| SSE  | 0.200   | 1.26            | $7.5 \times 10^4$ | 1.45   | 1.21          | 4.00    | yes    |
| SSE  | 0.250   | 1.57            | $9.4 \times 10^4$ | 2.27   | 1.89          | 4.00    | no     |
| LSE  | 1/120   | $\pi/60$        | $2.0 \times 10^6$ | 0.89   | 1.77          | 8.00    | yes    |
| LSE  | 1/90    | $\pi/45$        | $2.7 \times 10^6$ | 1.57   | 3.15          | 8.00    | yes    |
| LSE  | 1/60    | $\pi/30$        | $4.0 \times 10^6$ | 3.54   | 7.08          | 8.00    | no     |

For all experiments the height of the salt-water layer was chosen such that $H_s > H_{smin}$.

Furthermore, it is hypothesised that the interface stability during spin-down depends on the ratio between the kinetic energy that is dissipated during spin-down ($\rho_s \Omega^2 R^2$) and the stabilising effects of the potential energy $(\rho_s - \rho_w)gH_s$, where $H_s$ is the height of the salt-water layer. The ratio between the destabilising and the stabilising effects yields the interface Froude number

$$Fr_s = \frac{\rho_s \Omega^2 R^2}{(\rho_s - \rho_w)gH_s},$$

where small values of $Fr_s$ imply a stable interface. The critical Froude number $Fr_s^c$ at which the interface destabilises was determined in a series of SSE, before the LSE were designed. Table 2 provides an overview of the experiments that were performed. Despite the large differences in $Re$, the interface destabilised at similar $Fr_s^c$ for both the SSE and the LSE.

### B.2. Reduction of end-wall effects

The impact of the salt-water layer on the influence of end-wall effects was tested via PIV measurements.

Figure 7(a) shows the influence of $A$ and the salt-water layer on the evolution of the azimuthal velocity profile for the SSE at $Re = 5.6 \times 10^4$. For $A = 0.5$, and without the salt-water layer, the vortex core decays rapidly. A larger aspect ratio ($A = 2.0$) reduces the end-wall influence partially. However, introducing a salt-water layer proves to be even more efficient as the vortex core remains in SBR. For $Re = 2.8 \times 10^4$ the experimental data show good agreement with the numerical results from Kaiser et al. (2020), as presented in figure 7(b) and thereby further validates the salt-water-layer approach.

For the LSE the core flow in SBR is captured by a single camera positioned at $R_{hs} = 5$ m. The relative velocity of the fluid towards the platform $(u_{\psi}^{rel})$ is evaluated and spatially averaged over the complete FOV ($\langle u_{\psi} \rangle_{\psi,r}(t)$). Figure 8 presents $\langle u_{\psi} \rangle_{\psi,r}(t)$ for various $Re$.

As the CORIOLIS II platform requires a significant spin-down time owing to its large mass, for high $Re$, and without the salt-water layer, the influence from the bottom wall is found to be already present before the platform is completely at rest (see green line in figure 8a). In comparison, figure 8(b) clearly shows that the salt-water layer allows for a stable vortex core in SBR for large $\Omega t < 25$, approximately corresponding to four full revolutions of the vortex core ($\Omega t = 8\pi$).
Scaling of a vorticity-annihilating boundary layer

Figure 7. Influence of (A) and the presence of a salt-water layer on the flow characteristics during spin-down experiments. Five time instances $\Omega t \in \{2, 8, 16, 32, 64\}$ are presented. The SBR and regions of constant angular momentum $l(t)$ are emphasised using dotted and dashed lines, respectively. (a) $Re = 5.6 \times 10^4$ and (b) comparison of experimental results with DNS data from Kaiser et al. (2020) at $Re = 2.8 \times 10^4$.

Figure 8. End-wall effects and their reduction. While $\langle u_{rel} \rangle_r,\phi/\Omega R_{hs} \approx 1$, the core is still in SBR. The ramp of the acceleration/deceleration process varies for different $Re$. The experiments where a saturated salt-water layer was introduced are marked with the tag ‘salt’.

Appendix C. Turbulent boundary-layer growth rate

The present experimental data reveal that the empirical scaling law for the turbulent boundary-layer growth rate suggested by Kaiser et al. (2020) based on DNS data in the Reynolds-number range $3000 \leq Re \leq 28\,000$ is valid up to $Re = 4 \times 10^6$. During stage III of the spin-down $\delta_{99}$ grows proportionally to $\sqrt{\nu t}$, i.e. similar to the laminar boundary layer, as described by (3.1a,b). The $Re$-dependent proportionality constant of the turbulent boundary-layer growth rate is related to the laminar growth rate by (3.2) throughout the entire investigated $Re$-range. This hints at a peculiar property of the turbulent boundary layer during stage III of the spin-down process.

As already noted by Kaiser et al. (2020) one interesting property of the turbulent boundary layer during this stage is the fact that the growth of $\delta_{99}$ occurs simultaneously with a reduction of the friction velocity $u_\tau$ such that the friction Reynolds number $Re_\tau = u_\tau \delta_{99}/\nu$ remains almost constant. In addition, the strong similarity of the turbulent boundary-layer growth rate with the laminar one enables a very simple modelling approach through a constant effective eddy viscosity $\overline{\nu}$ (3.4).

In the following, the DNS data of Kaiser et al. (2020) are used to explore this idea of $\overline{\nu}$ further. In order to gain some insight into the effective viscosity ratio $\overline{\nu}/\nu$ we employ the
turbulent Reynolds number $Re_t = k^2/\nu \epsilon$ (where $k$ and $\epsilon$ represent turbulent kinetic energy and viscous dissipation, respectively), which is related to the local viscosity ratio $\nu_t/\nu$ in the classical $k-\epsilon$ turbulence model through the model constant $C_\mu$ (Pope 2000).

Figure 9(a) shows $Re_t$ through stage III as a function of the wall distance in viscous units up to $\delta_+=Re_\tau$.

It can clearly be seen that for all investigated $Re$ a near-wall plateau-like behaviour of $Re_t$ is present that remains almost constant over time, while far away from the wall (in the vorticity-free region) the peak of $Re_{t,\text{out}}$ increases.

Figure 9(b) presents the temporal evolution of the peak $Re_t$ value within the near-wall plateau, referred to as $Re_{t,nw}$, in stages I-III. After an initial increase during transition, $Re_{t,nw}$ fluctuates around a constant value during stage III.

Interestingly, $Re_{t,nw}$ appears to scale with $Re_{t,nw} \propto Re_t^{2/3}$ (dashed horizontal lines in figure 9), which recovers the observed $Re$ dependency of $a_{turb}(Re)$ under the assumptions that $\overline{\nu_t}/\nu \gg 1$ and $\overline{\nu_t}/\nu \propto Re_{t,nw}$, we can write

$$a_{turb}/a_{lam} \propto \sqrt{\overline{\nu_t}/\nu} \propto \sqrt{Re_{t,nw}} \propto Re_t^{1/3}. \quad \text{(C1)}$$

The temporally invariant wall-normal location of $Re_{t,nw}$ (visualised through the vertical dotted lines in figure 9a) is located within the buffer layer between the viscous sublayer and the region of logarithmic behaviour and thereby outside of the region of constant angular momentum. The outer peak $Re_{t,\text{out}}$ is located in the centre of the constant angular momentum region.

The observation that the identified $Re$ scaling of the turbulent boundary-layer growth can be recovered with $Re_{t,nw}$ hints at a physical relevance of this quantity for the turbulent boundary-layer growth rate under the spin-down conditions of stage III. This issue deserves further attention in future investigations.
Scaling of a vorticity-annihilating boundary layer

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F. Kaiser and others

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