Supermassive black holes: connecting the growth to the cosmic star formation rate

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ABSTRACT
We present a model connecting the cosmic star formation rate (CSFR) to the growth of supermassive black holes. Considering that the evolution of the massive black hole is dominated by accretion (Soltan’s argument) and that the accretion process can be described by a probabilistic function directly regulated by the CSFR, we obtain the evolution of the black hole mass density. Then, using the quasar luminosity function, we determine both the functional form of the radiative efficiency and the evolution of the quasar duty cycle as functions of the redshift. We analyse four different CSFRs showing that the quasar duty cycle, $\delta(z)$, peaks at $z \sim 8.5$–11 and so within the window associated with the reionization of the Universe. In particular, $\delta_{\text{max}} \sim 0.09$–0.22 depending on the CSFR. The mean radiative efficiency, $\bar{\eta}(z)$, peaks at $z \sim 0.1$–1.3 with $\bar{\eta}_{\text{max}} \sim 0.10$–0.46 depending on the specific CSFR used. Our results also show that it is not necessary for a supercritical Eddington accretion regime to produce the growth of the black hole seeds. The present scenario is consistent with the formation of black hole seeds $\sim 10^3 M_\odot$ at $z \sim 20$.

Key words: black hole physics – galaxies: active – galaxies: evolution – galaxies: nuclei – quasars: general.

1 INTRODUCTION
There is strong evidence that nearly all galaxies contain supermassive black holes (SMBHs) in their centres and that the evolutions of the SMBH and its host galaxy are connected (see e.g. Ferrarese & Merrit 2000). In particular, the masses of the black holes range from $\sim 10^6 M_\odot$ for galaxies with small bulges up to $\sim 10^9 M_\odot$ for galaxies in cores of groups and clusters of galaxies (see e.g. Margonriian et al. 1998). These SMBHs are present not only in the local Universe, in the form of the low-luminosity active galactic nuclei (AGNs), but also in the early stages of galaxy formation as can be seen from quasars discovered beyond $z > 6$ (see e.g. Fan et al. 2003). Furthermore, accretion on to massive black holes is generally accepted as a way to power strong emission as observed in the cases of AGNs and quasars.

On the other hand, it is reasonable to consider that the growth of SMBHs can be regulated, in some way, by the cosmic star formation rate (CSFR; see e.g. Franceschini et al. 1999; Haiman, Ciotti & Ostriker 2004; Heckman et al. 2004; Merloni, Rudnick & Di Matteo 2004; Mahamood, Devriendt & Silk 2005; Wang et al. 2006). This is reinforced by the fact that the CSFR can be directly connected to the masses of dark matter haloes (see e.g. Conroy & Wechsler 2009; Pereira & Miranda 2010). The dark haloes are the natural nursery for the birth and growth of the massive black holes. In principle, measuring the masses and accretion rate of the black holes that drive the AGNs and quasars could help us understand the evolution of these sources, their connection with the CSFR and the contribution of mini-quasars to the reionization of the Universe. Here, we present a formalism permitting us to confront several CSFRs with the quasar luminosity function (QLF). This formalism could also be used for a better estimate of the CSFR up to redshift $\sim 7$ using the QLF as observational data to be fitted by the ‘theoretical’ CSFRs. Furthermore, this work could contribute to the study of the feedback processes associated with both star formation at higher redshifts and growth of SMBHs. As main results, we derive the functional form of the radiative efficiency associated with the accretion process of these black holes and the evolution of the quasar duty cycle. Throughout this Letter, we consider the standard cosmological model ($\Lambda$CDM) with $\Omega_b = 0.04$, $\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$ and $h = 0.73$.

2 SMBHs AND THE CSFR
We consider that black holes grow by accreting matter (Soltan’s argument; see e.g. Soltan 1982; Wang et al. 2008) and that the accretion process can be described by a probabilistic function directly regulated by the CSFR. This can be described in the following way:

$$\rho_{\text{BH}}(z) = \rho_{\text{BH}}^0 \frac{\rho_{\text{BH}}(z)}{\rho_{\text{BH}}(z = 0)},$$

where $\rho_{\text{BH}}$ is the mass density of the black holes and $\rho_{\text{BH}}^0$ is the mass density of the black holes at $z = 0$. The function $\rho_{\text{BH}}(z)$ is a probabilistic function directly regulated by the CSFR.
where $\rho_{BH}^{\text{init}}$ represents the black hole mass density in our local Universe. The function $\rho_{BH}(z)$ makes the connection between the CSFR and the black hole mass density at redshift $z$. In particular, we consider

$$\rho_{BH}(z) = \frac{\dot{\rho}_*(z)}{(1 + z')} P(t_d) \frac{d\Omega_{\text{BH}}}{dz} dz'.\quad (2)$$

In equation (2), $\dot{\rho}_*(z)$ represents the CSFR at redshift $z$, $P(t_d)$ is the probability per unit of time of the black hole growing up by accreting matter from the environment and the $(1 + z')$ term in the denominator considers the time dilatation due to the cosmic expansion. The time delay $t_d$ makes the connection between the redshift $z_1$ at which the accretion disc forms around the black hole and the redshift $z$ at which the material is incorporated into the black hole. Thus, we have

$$t_d = \int_{z'}^z \frac{9.78 \, h^{-1} \, \text{Gyr}}{(1 + z') \sqrt{\Omega_{\Lambda} + \Omega_m (1 + z')}} dz'.\quad (3)$$

We consider that $P(t_d) = B t_d^n$, and this function is normalized as $\int_{t_{\text{min}}}^{t_{\text{max}}} P(t_d) dt_d = 1$, where $t_{\text{min}}$ is the minimum time required for the accretion process to take place and $t_{\text{max}}$ is the age of the universe. We assume $t_{\text{max}} = t_i = 4.2 \times 10^9$ yr, where $t_i$ is the Salpeter time. It is worth stressing that $P(t_d) \propto t_d^n$ has been used by different authors in several astrophysical contexts (see e.g. Regimbau & de Freitas Pacheco 2006; Regimbau & Hughes 2009, and references therein).

In this Letter, we consider the CSFR, $\dot{\rho}_*(z)$, derived by Perreira & Miranda (2010, hereafter PM) who obtained this function from the hierarchical scenario for structure formation using a Press–Schechter-like formalism; Springel & Hernquist (2003, hereafter SH) who obtained the CSFR from hydrodynamic simulations; Hopkins & Beacom (2006, hereafter HB); and Fardal et al. (2007, hereafter F) who derived the CSFR using the available observational data. In Fig. 1, we summarize all these different CSFRs.

In Fig. 2 can be seen the black hole mass density as a function of the redshift. The results are presented for four different CSFRs, as explained above, and using the formulation contained in equations (1) and (2). Observe that for the CSFR derived by PM, $\rho_{BH}/\rho_{BH}^{\text{init}}$ peaks at $z = 1.1$ ($1.5$) for $n = -1.0$ ($n = -1.5$). Considering $n = -1.0$, note that PM CSFR produces higher values for $\rho_{BH}(z)$ at $z > 4$ than those produced by others’ CSFRs. We also plot in Fig. 2 $\rho_{BH}(z)$ derived from the standard model (SM). In this case, $\rho_{BH}(z)$ is obtained from the integration of the QLF, $\Phi_b(L_b, z)$, in the following way (see e.g. Small & Blandford 1992; Yu & Tremaine 2002; Merloni, Rudnick & Di Matteo 2004; Wang, Chen & Zhang 2006; Shankar 2009; Shankar, Weinberg & Miralda-Escude 2009):

$$\rho_{BH}(z) = \int_{L_{\text{min}}}^{L_{\text{max}}} \frac{d\Omega_{\text{BH}}}{dz} \int_{t_i}^{\infty} \frac{1 - \eta}{c^2 \eta} L_b \Phi_b(L_b, z) dL_b,$$\quad (4)

where $L_b$ is the bolometric luminosity taken from Hopkins, Richards & Hernquist (2007), $\eta$ is the radiative efficiency and $c$ is the speed of light. Note that in order to solve equation (4), some consideration about $\eta$ value should be made. In general, $\eta$ is assumed constant with typical value $\sim 0.1$. However, we will show in the next section that $\eta$ must be a function of time if the growth of the SMBHs is regulated by the CSFR.

### 3 The Mean Radiative Efficiency and the Quasar Duty Cycle

The bolometric luminosity of a black hole with accretion rate $\dot{M}_a$ is given by $L = \dot{\eta} M_a c^2$ (with $\dot{\eta}$ the mean radiative efficiency). Considering that $\dot{M}_a$ is related to the mass variation of the black hole by $\dot{M}_a = M_{BH}/(1.0 - \dot{\eta})$, and that $\dot{\eta}$ is a function only of the redshift $z$, we obtain

$$U = c^3 \frac{\dot{\eta} c^2}{(1 - \dot{\eta})} \dot{\rho}_{BH},$$\quad (5)

with $U$ being the luminosity density. Deriving the equation (1) in $z$ and using the result in (5) produces

$$U(z) = \int_{L_{\text{min}}}^{L_{\text{max}}} L_b \Phi_b(L_b, z) dL_b.$$\quad (7)

where $L_{\text{min}}$ is the lower limit of the QLF. Here, we take $L_{\text{min}}$ and $\Phi_b(L_b, z)$ from Hopkins et al. (2007). Then, defining

$$f(z) = \frac{\dot{\eta}}{1 - \dot{\eta}},$$\quad (8)

Figure 2. The comoving massive black hole mass density. We present $\rho_{BH}(z)/\rho_{BH}^{\text{init}}(z)$ using the four CSFRs presented above. The results are shown for two values of the $n$-exponent of the probability function ($n = -1.0$ and $-1.5$). We also present for comparison $\rho_{BH}$ derived from the SM.

Figure 1. CSFR derived by Perreira & Miranda (2010, PM); Springel & Hernquist (2003, SH); Hopkins & Beacom (2006, HB) and Fardal et al. (2007, F). The observational points were taken from Hopkins (2004, 2007).
we can write
\[
U = c^2 f(z) \rho_{\text{BH}}. \tag{9}
\]

Now, we define a functional \( f'(z, n_i) \) which will be used to map \( f(z) \) given by equation (8). In particular, \( n_i \) is a vector of parameters and so
\[
f'(z, b_1, b_2, t_q) = c_0 \left[ \left( \frac{t_q(z)}{t_q} \right)^{b_1} + \left( \frac{t_q(z)}{t_q(z)} \right)^{b_2} \right]^{-1}, \tag{10}
\]
where \( c_0 \) is a normalization constant which gives \( \bar{\eta}(z = 0) = \bar{\eta}_0 \) (with \( \bar{\eta}_0 = 0.1 \); see e.g. Hopkins et al. 2007), \( t_q \) can be understood as a characteristic time-scale, \( b_1 (i = 1, 2) \) are dimensionless constants and \( t_q(z) \) is the age of the Universe at redshift \( z \).

The parametric form of equation (10) is widely used in the literature. For example, equation (12) of Hopkins & Hernquist (2009) and equation (13) of Hopkins, Narayan & Hernquist (2006) are similar to equation (10) presented here. Furthermore, Wang et al. (2006) used the same parametric form to determine the mass function of SMBHs. Note that, we are using equation (7) as a source of information to obtain \( \bar{\eta}(z) \). Thus, using equation (10) in equation (9), it is possible to write
\[
\eta_i(n_i, U_i, z_i) = \| U(z_i) - c^2 f'(z_i, n_i) \rho_{\text{BH}}(z_i) \|. \tag{11}
\]
The function that will be minimized is
\[
J(n_i) = \sum_{i=0}^{N-1} \eta_i^2(n_i, U_i, z_i). \tag{12}
\]

See that \( \bar{\eta} = f(z)/(1.0 + f(z)) \) and \( f(z) = f'(z, n_i) \), where \( n_i \) gives the best fit from equation (12). In Table 1, we present the best-fitting parameters which permit us to derive the function \( \bar{\eta}(z) \).

Fig. 3 presents the luminosity density of this work when compared to that obtained from equation (7), which comes from the integration of the QLF (Hopkins et al. 2007). In particular, for \( n = -1.0 \), we see that PM and SH CSFRs produce an excellent agreement with \( U(z) \) up to \( z \approx 6 \). In the case \( n = -1.5 \), we verify that F CSFR has the best agreement with \( U(z) \) at \( z \leq 1 \). On the other hand, from \( z \approx 1 \) up to \( z \approx 6 \), PM and SH, beyond F CSFR, have good agreement with the integrated QLF if we consider the parametric form given by equation (10).

In Fig. 4, we present the mean radiative efficiency \( \bar{\eta}(z) \) as a function of the redshift, while in Table 2, we show the redshift where \( \bar{\eta}(z) \) peaks. See that \( \bar{\eta}_{\text{max}} \) is within the range 0.10–0.46, depending on the specific CSFR. In particular, in these cases, we have \( \bar{\eta}_{\text{max}} \) in the range 0.1–1.3. These results are concordant with accreting black holes which could reach \( \bar{\eta} \approx 0.2 \) for the most massive systems (Narayan 2005).

On the other hand, the Eddington mass can be written as \( \dot{M}_{\text{edd}} = M_{\text{BH}} / t_e \). The bolometric luminosity weighted by the Eddington mass

\[
\langle L \rangle = \langle n \dot{m} \rangle \rho_{\text{BH}} c^2 / t_e. \tag{13}
\]

Using equations (13) and (5), we obtain
\[
\langle \dot{m} \rangle = \frac{t_q \dot{\rho}_{\text{BH}}}{(1 - \bar{\eta}) \rho_{\text{BH}}}. \tag{14}
\]

In Fig. 5, we present the evolution of \( \langle \dot{m} \rangle \) with the redshift. We can see that the accretion processes are more active at higher redshifts.

**Table 1.** Best-fitting parameters of \( \bar{\eta}(z) \).

| \( n \) | CSFR | \( b_1 \) | \( b_2 \) | \( t_q \) (Gyr) |
|---|---|---|---|---|
| -1.5 | PM | 2.85 | 2.39 | 5.74 |
| -1.5 | HB | 1.51 | 1.29 | 13.19 |
| -1.5 | F | 0.82 | 1.47 | 5.99 |
| -1.5 | SH | 2.74 | 3.17 | 4.93 |
| -1.0 | PM | 1.81 | 1.96 | 5.40 |
| -1.0 | HB | 1.91 | 0.74 | 14.81 |
| -1.0 | F | 0.46 | 1.06 | 4.23 |
| -1.0 | SH | 2.37 | 2.72 | 4.72 |

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SMBHs can be derived from the ratio of efficiency associated with the growth of the black holes, $\tau$ have the lifetime of an SMBH (Merloni et al. 2004). In particular, we equation (14). It is possible to find the mean accretion weighted by formation up to the present time. Another point can be derived from ‘well behaved’ during all time associated, from the seed black hole growth process associated with the growth of SMBHs is that the accretion processes never reach a super-Eddington regime.

Figure 5. The mean dimensionless accretion rate for $n = -1.0$ and $-1.5$ as functions of the redshift.

for all CSFRs studied in this Letter. However, our results also show that the accretion processes never reach a super-Eddington regime. The accretion process associated with the growth of SMBHs is ‘well behaved’ during all time associated, from the seed black hole formation up to the present time. Another point can be derived from equation (14). It is possible to find the mean accretion weighted by the lifetime of an SMBH (Merloni et al. 2004). In particular, we have

$$\tau_{\text{DC}}(z) = \int_z^{z_{\text{ini}}} \langle \dot{m} \rangle \frac{dz'}{dz}. \quad (15)$$

On the other hand, the quasar duty cycle, $\delta(z)$, associated with the SMBHs can be derived from the ratio of $\tau_{\text{DC}}(z)$ to the Hubble time. This result is presented in Fig. 6. See that $\delta(z)$ is also defined as the fraction of active black holes to their total number. As pointed by Wang et al. (2006), this parameter is a key to understand how many times and how many black holes are triggered during their lifetimes. In Table 3, we show the redshifts where $\delta(z)$ peaks for each CSFR studied here. Note that, typically, $\delta$ is maximum in the redshift range $8.5–11.0$, and this is very curious because the redshift of reionization is $10.5 \pm 1.2$ (Jarosik et al. 2011).

4 DISCUSSION

In this Letter, we present a model to obtain the mass density of SMBHs from the CSFR. The key point to do that is to consider Soltan’s argument and that the accretion process can be described by a probabilistic function directly regulated by the CSFR. Our model permits us to determine the function $\rho_{\text{BH}}(z)$, the mean radiative efficiency associated with the growth of the black holes, $\eta(z)$, and the quasar duty cycle $\delta(z)$. In the literature, the common way to obtain the mass density of SMBHs is by integration of the QLF as presented in equation (4) and by considering $\eta$ as a constant. However, here we present a different scenario. In particular, we derive $\rho_{\text{BH}}(z)$ from the CSFR and then we use the quasar luminosity density in order to obtain the mean radiative efficiency as a function of the redshift. From $\eta(z)$, it is straightforward to obtain the dimensionless mass accretion rate and the quasar duty cycle.

If we consider $\rho_{\text{BH}} = (5.9 h^3) \times 10^5 \text{M}_\odot \text{Mpc}^{-3}$ (Graham & Driver 2007; Vika et al. 2009), then our model returns, using the CSFR of PM, $\rho_{\text{BH}} = 4.60 \times 10^3 \text{M}_\odot \text{Mpc}^{-3}$ (with $n = -1.0$). This result is compatible with black hole seeds $\sim 10^3 \text{M}_\odot$ at $z \sim 20$. We also note that the quasar duty cycle has a maximum value close to $z \sim 8.5–11$ and so within the observational uncertainties associated with the redshift of reionization. As the main component of our model is the CSFR, this scenario offers several future possibilities of investigations. One of them is related to the form of the probabilistic function which permits us to determine the growth of SMBHs from the CSFR. In so far as both star formation and growth of SMBHs can be described in the same way, the predictions of different probabilistic functions, $P(t_d)$, can be confronted to different observables at high redshifts. Finally, our results also show that it is not necessary for a supercritical Eddington accretion regime to produce the growth of the black hole seeds.

Table 2. Maximum values for the mean radiative efficiency.

| $n$  | CSFR | $\delta_{\text{max}}$ | $z_{\text{max}}$ |
|------|------|----------------------|------------------|
| $-1.5$ | PM   | 0.38                  | 1.1              |
| $-1.5$ | HB   | 0.10                  | 0.1              |
| $-1.5$ | F    | 0.11                  | 0.6              |
| $-1.5$ | SH   | 0.46                  | 1.2              |
| $-1.0$ | PM   | 0.23                  | 1.1              |
| $-1.0$ | HB   | 0.10                  | 0.3              |
| $-1.0$ | F    | 0.11                  | 0.7              |
| $-1.0$ | SH   | 0.39                  | 1.3              |

Figure 6. The quasar duty cycle derived for $n = -1.0$ and $-1.5$.

Table 3. Maximum values for the quasar duty cycle.

| $n$  | CSFR | $\delta_{\text{max}}$ | $z_{\text{max}}$ |
|------|------|----------------------|------------------|
| $-1.5$ | PM   | 0.13                  | 11.1             |
| $-1.5$ | HB   | 0.15                  | 8.5              |
| $-1.5$ | F    | 0.09                  | 9.9              |
| $-1.0$ | SH   | 0.16                  | 9.4              |
| $-1.0$ | PM   | 0.19                  | 10.9             |
| $-1.0$ | HB   | 0.20                  | 9.0              |
| $-1.0$ | F    | 0.14                  | 9.7              |
| $-1.0$ | SH   | 0.22                  | 9.6              |

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