Of Inflation and the Inflaton

R. Brout*
Department of Applied Mathematics, University of Waterloo
Waterloo, Ontario N2T 3G1, Canada

Service de Physique Théorique, Université Libre de Bruxelles
The International Solvay Institutes
B1050 Bruxelles, Belgium
robert.brout@ulb.ac.be

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Abstract

Due to intra-field gravitational interactions, field configurations have a strong negative component to their energy density at the planckian and transplanckian scales, conceivably resulting in a sequestration of the transplanckian field degrees of freedom. Quantum fluctuations then allow these to tunnel into cisplanckian configurations to seed inflation and conventional observed physics: propagating modes of QFT in a geometry which responds to the existence of these new modes through the energy constraint of general relativity, \( H^2 = \rho / 3 \). That this tunnelling results in geometries and field configurations that are homogeneous allows for an estimate of the mass of the inflaton, \( m = O(10^{-6}) \), and the amplitude of the inflaton condensate, \( \langle \phi \rangle = O(10) \), both consistent with phenomenology.

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1 Introduction

This paper is an enquiry into the conceptual foundations of cosmogenesis as it is suggested within the inflationary scenario. In particular, the highly successful implementation of inflation through the dynamics of the inflaton field (≡ \( \phi \)) will be discussed.

It is not without interest to brook the subject as it was conceived in the creation scenario of early civilizations. The first lines of Genesis 1.1., thoughts of the ancient Hebrews, believed to date from 700 B.C. or thereabouts, read:

“And the earth was without form and void and darkness was on the face of the deep.
And God said, Let there be light and there was light...”

Similar notions are found in the clay tablets of the more urbanized Sumerians and Babylonians from a millennium beforehand.

That the universe is a manifestation of form created from formless void is a natural thought. One has to begin somewhere.

However the second part of the proposition has now been seriously called into question.

“Let there be light” implies suddenness. It corresponds to the big bang as it was conceived before inflation, the existence of an initial state of homogeneity on a space-like surface from which the cosmos was born, the inception, unexplained, of the era of adiabatic expansion. It took some years for relativity to penetrate our collective scientific consciousness to abandon this view. The universe, concomitant with its homogeneity in the mean, must result from a cause, the sudden onset of homogeneity rejected in favor of its gradual establishment from prior circumstances. Just as action at a distance must be abandoned, the world, in all its aspects, is to be concerned as a succession of events from some initial event onwards. Short of this, our only other resource would indeed be divine intervention.

We shall follow historical precedent and adopt biblical chaos as the precondition of cosmology, in a sense to be more precisely described. General relativity (GR) suggests that the initial seed of creation was planckian in dimensions. From that seed, a background geometry, including time itself, must emerge. Quantum field theory (QFT) incites us to seek, within the seed, the birth of field configurations which serve as the basis of physics as we practice it, the field theoretical modes. Inflation ensues wherein field configurations, modes of quantum fields, propagate in geometry; and geometry, through the constraints of GR, feeds on the energy of the modes. The requirement of causality has led us that far. Inflation is the child of relativity and through the two modern instruments of physics, GR and QFT, we are called upon to bring it to maturity.

In this, the inflaton phenomenology used to implement inflation, is remarkably successful. Though our intention is to adhere as closely as possible to QFT and GR to explain, or rather to interpret cosmogenesis and the inflaton scenario, inevitably, unconventional notions will appear. For the inflatonic phenomenology unfolds at the planckian scale, and even transplanckian when introducing the primeval fluctuations born during inflation as modes, in order to explain present day observations which reflect them. And physics at these scales we do not have. Nevertheless, we shall hazard hypotheses which are based on accessible physics. This procedure follows past traditions in our science, hypotheses elaborated from what is known, through analogy, to attempt to elucidate the unknown. For each fundamental
advance carries with it its own limitations. To overcome these limitations, such hypotheses, first elaborated, are then incorporated into a new logical structure which transcends the past. At the present time, that limitation is met at the planckian scale.

It is the nature of our enquiry to reject the idea of an elementary field having the peculiar properties requisite to having a successful inflaton. Likewise, we eschew a priori constructs which exist beyond the pale of conventional physics, string theory, supergravity, extra dimensions. The particular hypotheses we adopt are, in great measure, forced upon us by pursuing conventional physics to the limits of its validity.

We shall dwell further on the concept of causality applied to cosmogenesis.

Inflation, hence creation, in GR, is naturally born in a planckian seed from which both the modes of QFT and the classical geometric structure of GR are simultaneously created, together with the geometry. No seed, no geometry. No geometry, no physics as we know it, for physics as we know it requires a geometric background to support the propagation of the modes of QFT, and modes are the “ur-stuff” of physics.

The conclusion is that physics is born as the universe is. At present, our limited intellectual horizons, built, as they are, upon QFT and GR forces upon us a phenomenology of creation and precludes a complete theoretical formulation. The tools necessary come by a rational account of pre-cosmological chaos, unendowed, as it is, with geometry, bereft of the modes of QFT, are simply absent from the arsenal of weapons which appear essential to formulate a satisfactory rational account of creation.

Were there no such homogeneous structuralism of space, brought about by the hubble expansion, then there would be no modes of QFT, hence no positive energy to drive the hubble expansion which is brought about by the GR energy constraint \[ H^2 = \rho / 3; \] \( H \) = hubble expansion parameter, \( \rho \) = vacuum energy density]. The conclusion is that physics is born when the universe is, at least physics as we know it.

Nevertheless, the conjectured nature of the planckian seed of creation is in great measure the substance of this paper. These conjectures will lead to a formulation of the inflaton, the nature of its substance in terms of fields, and its mass, which is more intelligible than the more familiar phenomenology as expressed, for example, in [1]. The conjectural character is based on analogy with known physics, for example, the role of quantum tunnelling out of chaos. As such it is still phenomenology.

The hope is that, in the future, there will emerge a coherent mathematical scheme, a rational account of our world and the transplanckian pre-world from which our world has sprung. Call it quantum gravity, a discipline which will be called upon to describe chaotic configurations without the support of geometry, but which lays the foundations of geometry on the macroscopic scale (and possibly even at a cisplanckian scale only slightly greater than planckian). Loop quantum gravity affords some hope, but unfortunately there has not yet been forthcoming a large scale geometry that would emerge from the embryonic notions that form the canonical formulation of LQG.

Thus we must abide by our ignorance and live with a phenomenological, yet suggestive, account of creation from “formless void”. This paper is to be read as a suggestive interpretation of a highly successful phenomenology.

To appreciate the unfamiliar and enigmatic issues that arise in inflatonary phenomenology we first introduce the subject with a brief synopsis.

The inflaton is a scalar field, \( \phi \), whose dynamics is governed by a covariantized Klein-Gordon (KG) equation, the metric for which is an expanding flat homogeneous space. The
hubble expansion parameter, \( H(\phi, \dot{\phi}) \), is self-consistently determined in virtue of the GR energy constraint, \( H^2/3 = m^2(\langle \phi \rangle)^2/2 + (\dot{\phi})^2/2 \). We set \( m_{pl} = 1 \), and also treat other irrelevant constants as equalling 1. The averaging, indicated by \( \langle \rangle \), is over the homogeneous expanding patch. If \( \phi \) is not homogeneous, it rapidly becomes so, in virtue of the expansion, for a wide class of initial conditions. The phenomenology is most often presented with initial conditions \( \dot{\phi} = 0, \langle \phi \rangle \neq 0 \). Once more, if these conditions are not met initially, there is an attractor solution of the KG equation which rapidly brings about the “slow roll” condition \( \langle \dot{\phi} \rangle \ll m \langle \phi \rangle \), hence a quasi stationary hubble parameter whereupon the homogeneous patch expands quasi-exponentially.

The KG equation has an in-built friction term, which results from the expansion itself, so \( \langle \dot{\phi} \rangle \Rightarrow 0 \) after a few \( e \)-folds of the scale factor, \( a \). For the homogeneous patch to expand sufficiently to account for the homogeneity of space within our present horizons, determined by the adiabatic expansion subsequent to inflation, one requires initially \( \langle \phi \rangle = O(10) \), a numerical result that follows from the use of the mass, \( m \) of \( \phi \) determined observationally to be \( O(10^{-6}) \).

The condition \( (\langle \phi \rangle/m) \gg 1 \), required for the slow roll, is amply fulfilled and that is the sine qua non for inflation to work. [Often the term \( m^2\phi^2 \) is replaced by a more general function \( V(\phi) \), but the quadratic approximation for \( V(\phi) \) seems to suffice.]

During the period called “reheating”, as \( \langle \phi \rangle \rightarrow 0 \), the energy stocked in \( \langle \phi \rangle \), principally in the potential energy \( m^2(\langle \phi \rangle)^2 \), gets converted into quanta through a presumed coupling of \( \phi \) to the conventional fields of QFT. The mechanism of coupling and the resulting dynamics is less well understood than the rest. But one postulates that, in the end, there is thermalization of the resulting quanta and the adiabatic expansion ensues. Thus inflation prior to reheating, and reheating itself, is the phenomenology that replaces what was the big bang prior to the invention of inflation. It is not only causal, but it has physical consequences which are very interesting and observationally confirmed, to wit: the formation of primeval fluctuations behind the so-called inflationary horizon at \( H^{-1} \). Upon following the evolution of these fluctuations throughout cosmological history, one shows that their traces survive to this day. They account both for the large scale distribution of galaxies as well as the characteristically scale-dependent temperature fluctuations of the cosmic microwave background (CMB). The analysis is quite general and its success is a triumph of no small accomplishment.

From this synopsis, the principal conclusion is that inflation is a consequence of the dynamics of the gravitation/field complex which unfolds at the planckian level. Moreover, nothing prevents extrapolation to its beginning wherein the scale factor is of planckian dimensions. Hence inflation is an invitation to new physics.

The principal physical ingredient which we call upon to explain cosmogenesis and inflation is intra-field gravitational interactions. At short length scales the resulting negative energy density dominates the more familiar zero point energy density. The latter is often cited as the problem of the cosmological constant, which is the problem of how one must tune that component of energy to get finite reasonable physics. Such a consideration, however, neglects to take into account the intra-field gravitational energy. Whereas zero point energy density roughly scales like \( l^{-4} \) at length scale \( l \) (from \( \sum_{k}^{l^{-1}} |k| \) in free field theory), the latter in a perturbative scheme is expected to scale roughly like \( l^{-6} \) (in units where \( l_{pl} = m_{pl}^{-1} = 1 \)). For example a Newtonian estimate is \( \int_{l}^{l^{-1}} d^3k \int_{l}^{l^{-1}} d^3k' \bar{k}||k'||/||k - k'||^2 \).

We suppose that on scales \( l > l_{pl} \), a more precise perturbative treatment will not give a
result qualitatively different.

Guided by common sense as well as the mode description of field configurations, presumably a guide of some reliability at scales \( l > l_{pl} \), we may suppose that the number of field degrees of freedom scales like \( l^{-3} \). Thus, save for special circumstances we surmise that field configurations are characterized by behaviour at small \( l \). But, for \( l < l_{pl} \), gravitational interaction energy within a field configuration is dominant. If so, this would render unnecessary the introduction of a subtraction term in the form of a cosmological constant to counter infinite zero point energy.

Let us now go further along these lines. The energy constraint of GR, applied to homogeneous flat space, reads \( H^2 = \rho/3 \). From the previous paragraph this implies that, save special circumstances, cosmology as we view it (i.e., as an expanding large scale homogeneous space which is the habitat of field configurations) cannot be realized. This is because, as argued, save special circumstances, configurations are dominated by the small \( l \) scale for which gravitational interaction energy dominates, where \( \rho < 0 \).

The problem of cosmogenesis is thus the search for special circumstances wherein field configurations with scale \( l > l_{pl} \) become relevant. We shall argue that this happens when a mechanism that leads to homogeneity sets in, and this happens in virtue of a very particular tunnelling mechanism out of the chaotic small \( l \) configurations to those of larger \( l \). Specifically, let us denote by \( \Lambda_0 \) that momentum scale at which \( \rho(\Lambda_0) = 0 \) so that, for momenta \( \Lambda \), for which \( \Lambda < \Lambda_0 \), we have \( \rho(\Lambda) > 0 \).

Then, the configuration of larger length scale towards which one tunnels is at the momentum scale \( \Lambda \approx \Lambda_0 - m \), where \( m \ll 1 \) in planckian units. In that configuration, there arises approximate local homogeneity, a planckian seed slightly larger than planckian in volume. Within that seed, configurations start to get sorted out to form modes, giving rise to \( \rho > 0 \), hence a local hubble expansion. Inflation sets in and field configurations similar to those within the seed also arise, this time being stimulated by the hubble expansion whose effect continues to extend from the seed to its neighbours. Thus a expands exponentially and a universe is born.

From whence arrive these small length scale degrees of freedom of negative energy in which the energy has small absolute value? Note that in their sequestered state they do not contribute to the hubble expansion until they tunnel out to be converted to positive energy configurations whereupon they then do contribute to the local hubble expansion. They arise as quantum fluctuations, or if one will, as virtual configurations of positive energy. Under normal circumstances these virtual configurations which tunnel out of sequestration would simply reenter into the ensemble of the sequestered configurations giving rise to no net effect. However, gravity in the form of the hubble expansion seizes those fluctuations, so as to stabilize them, converting virtuality to reality. That is the working of the energy constraint \(-H^2 + \rho/3 = 0\).

It remains to model the mechanism of sequestration. We take sequestered configurations to be bound state space-time entities which are localized within the causet sites of Sorkin. This picture is an interpretation of [4] wherein dark energy is given as a vacuum fluctuation. It is adopted here because it has the virtue of Lorentz invariance, and moreover has met with some success in accounting for present day dark energy, albeit it requires refinement to deal with dark energy at an earlier epoch of the adiabatic expansion! One may envision other models of sequestration such as foam on a space-like surface, but it would seem that Lorentz invariance is a good guide. In this respect, see Section 3 where the causet scenario...
is briefly presented.

From this rather naive model, one can start to estimate the parameters of the inflaton scenario (Section 5). The inflaton is taken to be a collective degree of freedom formed from the positive energy degrees of freedom of all elementary fields, which lie in the momentum range \((\Lambda_0 - m, \Lambda_0)\), with \(\rho(\Lambda_0) = 0\). The parameter, \(m\), is the inflaton’s mass. We will estimate from the tunnelling mechanism, as roughly \(O(10^{-5})\), as against the phenomenologically favored value of \(O(10^{-6})\). This is a satisfactory preliminary result which is an encouraging sign. From this value of \(m\), and further argumentation in Section 5, one can estimate the amplitude \(\langle \phi \rangle\) of the inflaton condensate: \(\langle \phi \rangle \approx O(\sqrt{N})\) where \(N\) is the number of species of fields contributing to known (cisplanckian) physics. Thus \(\langle \phi \rangle = O(10)\), which is once more a phenomenologically acceptable estimate, wherein the requirement to have sufficient inflation to accommodate the “size” of our present universe is instrumental. Further arguments are given in Section 5 wherein the causal set discretization of space-time indicates that \(\phi\) is a scalar field governed by a Klein-Gordon evolution operator appropriately coarse-grained. The condensate \(\langle \phi \rangle\) is the average of \(\phi\) over the homogeneous expanding patch.

Also, energetic considerations impose the creation of a charge neutral universe. Therefore we distinguish matter anti-matter neutrality and charge neutrality. From the above, the scenario which is proposed would result in a universe containing only matter degrees of freedom, but which is charge neutral, as is required by observation.

While we note that fermionic fields tend to contribute negatively to the vacuum energy, we will here postpone a detailed discussion of the implications.

We conclude this introduction with a summary of the issues that will be presented in more detail than is indicated in the introduction.

1. \(\langle \phi \rangle = O(10)\) is a sort of condensation of \(\phi\), that emerges at creation, formed from modes in the momentum interval \((\Lambda_0 - m < |k| < \Lambda_0)\) where \(\rho(\Lambda_0) = 0\). \(\langle \phi \rangle^2\) may be viewed as a superfluid density where the fluid is composed of matter.

2. \(\phi\) obeys a KG equation wherein figures the mass, \(m = O(10^{-5})\), this estimate resulting from a tunnelling mechanism from sequestration into a homogeneous positive energy density configuration. The d’Alembertian in the KG operator arises from a coarse-grained description of propagation on a causet.

3. Primeval fluctuations arise behind the inflationary horizon at \(O(H^{-1})\) through essentially the same mechanism as \(\langle \phi \rangle\) arises, save that it is their wave vector that characterizes them rather than mass. What is important in both cases is the creation of cisplanckian modes, not quanta. This occurs through tunnelling. In the text, comparison is made with the similar situation that occurs in black hole evaporation.

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2 Negative Pressure

There are interesting points in common between field theory and fluid dynamics. These are often fruitful when searching for the dynamics of fields in the cosmological setting and will be called upon throughout this paper.

This section is devoted to the strange concept of negative pressure. It was encountered in the earliest work on inflation \cite{2,3} when it was noticed that the exponential expansion of space, which was postulated to be the causal prelude to the big bang was characterized by a negative pressure when expressing the energy-momentum tensor $T_{\mu\nu}$ of a field in terms of fluid variables, with $p =$ pressure and $\rho =$ mass density. To see this, one begins with $T_{\mu\nu}$ for a field expressed in terms of its 4 velocity, $u_\mu$, and one replaces $u_\mu$ by the field momentum $\partial_\mu \phi$. Identifying the tensors gives the interpretation of the field expression for $T_{\mu\nu}$ in terms of pressure:

$$\rho = \frac{1}{2} \phi^{\mu \nu} \phi_{\mu \nu} + V(\phi) \quad (1)$$

$$p = \frac{1}{2} \phi^{\mu \nu} \phi_{\mu \nu} - V(\phi) \quad (2)$$

It is readily seen that if $V(\phi) > 0$ and dominates the derivative terms, then the space expands exponentially; and $p < 0$.

Does this formal result express some physical content? It does and it is suggestive of how one must proceed to model inflation.

The simple fluid setup to illustrate $dE = -pdV$ is to calculate the work done on a fluid enclosed within an insulated cylinder one end of which is a piston which exerts pressure on the fluid. The pressure is due to forces external to the fluid, taken positive if they are exerted inwards so that $dV < 0$. Then $p > 0$ assures $dE = -pdV > 0$ due to work done on the system. Mutatis mutandis, if $dV > 0$ the systems works on the piston and $dE < 0$, the principle of the steam engine which does work in its expansion phase.

For the cosmological fluid during inflation, one has $dV > 0$ and $p < 0$ so that, under the cosmological expansion, $dE > 0$. Apparently, there are sources of energy within the system which push the system out so that energy within the fluid increases as the volume does. These sources must be rather homogeneously distributed so that there is not too much dilution of these internal energy sources as the system expands. One wants inflation to last for a sufficiently long time to assure the existence of a universe at least as large as ours. Therefore the total number of sources must increase in number. And if the expansion is quasi-stationary (called the slow roll corresponding to a very slow decrease of the density of potential energy), their density should remain quasi constant. Thus, the expansion is exponential.

This roughed out macroscopic sketch of inflation indicates the way towards a more detailed mechanism. One must search for energy sources within field configurations which, like the field itself, are homogeneously distributed on the coarse-grained level.
The phenomenological KG equation containing a mass term, and covariantized, offers a successful rendering of what is required. In virtue of the attractor solution, \( \langle \phi \rangle \) varies little on a time scale \( O(\langle \phi \rangle_{initial}/m) \) maintaining the energy density constant during this period at \( \rho \approx m^2 \langle \phi \rangle_{initial}^2 \) whereupon the system then exits from inflation; i.e., \( \langle \phi \rangle \rightarrow 0 \). After this “reheating” period, the inflationary energy density degenerates into quanta, hence temperature and entropy.

3 The Cis-Trans Dichotomy

The action of a scalar field in flat non-expanding homogeneous space is \( \int d^4 x \partial_\mu \chi \partial^\mu \chi \). Masslessness, in the context of this paper, is an adequate approximation since we shall be interested in dynamics at the planck scale and all known masses are \( \ll 1 \). Flat homogeneous space is used for the sake of simplicity. Its use is vindicated at a later stage by virtue of inflation itself.

Let \( \nu(\Lambda) \) denote the number of modes, cut-off at \( \Lambda \), and let \( \rho(\Lambda) \) denote the energy density. Then, in the absence of gravity, one has:

\[
\nu(\Lambda) = \Lambda^3 \tag{3}
\]

\[
\rho(\Lambda) = \Lambda^4 \tag{4}
\]

The absence of an a priori length scale allows one to let \( \Lambda \rightarrow \infty \) with impunity since the absence of gravity implies zero coupling of \( \rho(\Lambda) \) to anything.

All is changed when gravity enters the game, i.e., when one adds a term \( \int \sqrt{g} R d^4 x \) to the action and makes the field action invariant converting it to \( \int \sqrt{g} \partial_\mu \chi \partial^\mu \chi d^4 x \) where \( g_{\mu \nu} \) is used to make \( \partial_\mu \) and \( \partial^\mu \) covariant and contravariant. It is beyond present knowledge to calculate \( \rho(\Lambda) \) as \( \Lambda \rightarrow 1 \), but the Newtonian approximation and more generally its refinement in perturbation theory (in powers of \( m_{pl}^{-2} \)) offer a guide. The Newtonian approximation gives for the interaction energy density

\[
\rho(\Lambda)|_{\text{Newtonian Interaction}} = - \int^\Lambda d^3 k \int^\Lambda d^3 k' |\vec{k}| |\vec{k}'| / |\vec{k} - \vec{k}'|^2
\]

\[
= - \Lambda^6 , \tag{5}
\]

\[
\rho(\Lambda)|_{\text{Newtonian}} = \Lambda^4 - \Lambda^6 . \tag{6}
\]

To cisplanckian scales (i.e., \( \Lambda < 1 \)) a perturbative estimate would be expected to yield the form

\[
\rho(\Lambda)|_{\text{Pert.}} = \alpha(\Lambda) \Lambda^4 - \beta(\Lambda) \Lambda^6 , \tag{7}
\]

where \( \alpha \) and \( \beta \) are slowly varying positive functions. There is no need here to enter into complications of renormalization. But we do specify that the word “perturbative” implies the prescription that all momentum integrals are cut off at scales \( \Lambda = O(1) \). For one of the principal theses of this paper is that perturbative QFT, based as it is on the modes of the fields, is inadequate at planckian and transplanckian scales.

In subsequent sections it shall be argued that straightforward perturbative QFT makes sense only at scales \( \Lambda < \Lambda_p \), where \( \Lambda_p = \Lambda_0 - m \); where \( \rho(\Lambda_0) = 0 \). For brevity, in this
section we ignore the interesting subtleties (which are essential to describe inflation) for \( \Lambda \) near \( \Lambda_0 \) and simply class modes with \( (|\vec{k}| < \Lambda_0) \) as cis. They propagate and the trans degrees of freedom \( (|\vec{k}| > \Lambda_0) \) do not, nor do they contribute to the energy constraint \( (H^2 = \rho/3) \) since \( \rho < 0 \) for \( \Lambda > \Lambda_0 \).

We now formulate the postulate that only the cis sector of field configurations determines the space-time geometric back reaction occasioned by those configurations. Thus, the hubble expansion parameter is determined from the cis configurations. And analogously, the mass, \( M \), of a black hole which varies due to evaporation is determined (directly!) only by the cis sector. The qualification "directly" is further explained below.

In similar fashion we expect dark energy, if it is indeed a vacuum effect as most physicists believe, to be directly influenced by cis configurations.

Our postulate is most reasonable. Consider that \( \rho(\Lambda) \) goes negative for \( \Lambda > \Lambda_0 \) in consequence of the gravitational attraction responsible for the \( \Lambda^6 \) term of Eq.6. For \( \Lambda > \Lambda_0 \), the fact that \( \rho(\Lambda) \) goes negative in the perturbative calculation indicates that the transplanckian degrees of freedom are localized in bound states. It is this fact that vitiates transplanckian perturbation theory.

We postulate that the transplanckian class of field configurations are aggregates of bound states, each aggregate of length scale \( \Lambda_0^{-1} \). This postulate goes back to the foam idea of Wheeler of the mid-60's. It has been given a Lorentz invariant setting in more recent times in terms of Sorkin's causet scenario [4]. Since such configurations are sequestered and do not extend beyond length scale \( O(\Lambda_0^{-1}) \), it is natural to postulate that their influence cannot extend over macroscopic regions of space-time, hence the postulate: only cis configurations influence macroscopic GR.

But that does not mean that the transplanckian configurations are inert. On the contrary, the main content of Section 4 is based on cis-trans communication. Whence, it will be argued, the mass, \( m \), in the KG equation for the cis-class concept \( \phi \), arises in consequence. This is the origin of the indirect influence of the trans class in cis physics.

An alternative way to show up the cis-trans dichotomy is to be found in an important series of papers of Parentani [5] who has shown, by perturbative methods, that modes become overdamped at the planckian scale. Thus they become useless to characterize transplanckian field configurations.

Parentani’s calculations, being perturbative, do not indicate the destiny of overdamped modes. The calculation must be followed up non perturbatively, one suggestion for which are the bound state aggregates which we have called upon above to give an effective discretization of space (space-time).

The model of discretization which will serve as a basis for our discussion of inflaton dynamics is the causet construction. It has the advantage of explicit covariance, i.e., propagation by discreet movements (hopping) on the causet network is, on the average, covariantly described. For “on the average” we shall sometimes use the term coarse grained, referring to an average over scales \( O(1) \). The causet scheme is designed for the purpose.

This is not to say that a non-covariant scheme such as foam lain out on space-like surfaces could not equally well serve the purpose, and in fact, even be more suggestive of physical models. This is Parentani’s view. It is for definitiveness and convenience that we adopt causets.

For the non cognoscenti a brief summary of the causet scenario follows. The prescription
is sketchy and mathematically non rigorous. For more details, see [4].

4 Causets

Space-time is here given as a manifold upon which we distribute points. For simplicity we deal with 1 spatial dimension. Generalization to 3 dimensions is straightforward, see [6]. Also, we shall use flat space-time. This suffices to describe the space-time patches we call upon for the description of propagation in the cosmological spaces we envision.

Each point in the manifold is the site of an event; hence it can serve as the tip of a light cone. Signals are sent from within the cone to the tip. The causet prescription is to distribute these points at random in a Poisson distribution. Then under Lorentz transformation not only are the boundaries of the cone invariant, but so are the lengths \((\Delta t^2 - \Delta x^2 = \Delta u \Delta v)\) between events where \(u, v = t \mp x\). That is because Poisson distributions are lain down with equal areas (on average) subtended by the parallelograms drawn with \(\Delta u, \Delta v\) on a side. (The Minkowski version of the distribution of raindrops on a flat pavement is a convenient visualization.) Since under Lorentz transformations, space-time intervals transform as \(\Delta v \rightarrow \Delta v \exp[w]\) and \(\Delta u \rightarrow \Delta u \exp[-w]\) where \(w\) is the rapidity, a coarse grained view of the distribution is conserved.

From these, Sorkin and collaborators have constructed a mean d’Alembertian. It is the inverse of a retarded Green’s function, this latter constructed as a hopping matrix from site to site in the forward direction. The ingenious rules for its construction make it sufficient to go backwards from the tip of the light cone to sites a few units of length in the past. There results, on taking the average of the inverse, a Lorentz invariant d’Alembertian.

This “tour de force” lays the foundation of a field theory possessed of a Lorentz-invariant cutoff. In [6], details on the propagator are further developed and it is hoped that shortly we will have a full-fledged QFT based on this construction. In Section 5 we make use of this knowledge to rationalize the KG equation that governs the dynamics of \(\phi\).

5 Cosmogenesis and the Inflaton

We now assemble the considerations of the previous sections. To start we discuss the formation of the inflaton condensate \(\langle \phi \rangle\), then how one can rationalize the value of its mass.

The basic assumption is that the inflaton field emerges from sequestered degrees of freedom, which, for definiteness we take to be bound state aggregates within causet elements. \(\phi\) is a collective field comprised of the momentum components of all fields contributing to conventional (cisplanckian) QFT. \(\langle \phi \rangle_t\) is the spatial average of \(\phi\) at time \(t\) over the expanding homogeneous patch of space that is described by inflation. The momentum components have \(\vec{k}\) in the interval \((\Lambda_0 - m < |\vec{k}| < \Lambda_0)\) where \(\rho(\Lambda_0) = 0\) defines \(\Lambda_0\) and \(m \ll \Lambda_0\). \(m\) will be identified with the inflaton mass.

First we devote ourselves to the question of how to rationalize \(\langle \phi \rangle_{t_0} = O(10)\), where \(t_0\) is the initial time of inception of inflation, here taken to be the time of cosmogenesis.

Let \(\nu(\Lambda)\) be the number of all modes up to scale \(\Lambda\). For \(t < t_0, \nu(\Lambda) = 0\). There are no modes of QFT, all field configurations being sequestered and they all have negative energy density, \(\rho(\Lambda) < 0\), so there is no expansion. In fact, as emphasized in Section 1, there is no physics, at last in the sense as we conventionally formulate physics.
Now consider a single causet element. Field degrees of freedom tunnel in and out as fluctuations. Tunnelling is important when their configurations have nearly vanishing energy density. We may think of virtual transitions wherein negative energy configurations spend a finite duration of time exterior to the main bulk of the causet element having virtual positive energy. Here we appeal to analogy with familiar quantum physics. That is all we have to go on.

Under normal circumstances these virtual configurations will fall back into their sequestered state leaving no net effect. The exceptional circumstance resulting in a planckian seed of inflation arises when that fluctuation has a sizable homogeneous component of energy over a volume slightly greater than planckian. This will result in a local hubble expansion whose effect will extend in an isotropic manner to the neighboring causet elements. In this way the cooperative process gets set up as originally envisioned in [1] wherein curvature drives positive energy field fluctuations to give permanent effects, and these positive energy effects drive curvature.

To make quantitative progress, let $\nu(\Lambda(x,t))$ be the number of modes which issue from the causet site at $x, t$ in a manner that is approximately homogeneous and isotropic and which is localized around $x, t$. Then the momentum components contributing to $\nu(\Lambda(x,t))$ will be planckian in character (since causet elements are planckian distributed). Since $\nu(\Lambda(x,t))$ is composed of cisplanckian field configurations, the corresponding energy carried by them is positive as well as $[\Lambda_0 - \Lambda(x,t)] > 0$. [We recall $\rho(\Lambda_0) = 0$ and $\Lambda(x,t) < \Lambda_0$ to make $\rho(\Lambda(x,t)) > 0$.] It then makes sense to define the positive energy quantity $\phi^2$ through

$$m\phi^2 = \nu(\Lambda_0) - \nu(\Lambda(x,t)),$$

and the spatial (homogeneous) average of $\phi$

$$\langle \phi \rangle^2_t = [\nu(\Lambda_0) - \langle \nu(\Lambda(x,t)) \rangle] / m.$$

where $\langle \nu(\Lambda(x,t)) \rangle$ is the spatial mean of the number of modes contained on the cloud of cisplanckian field configurations centered about $t$. The quantity $m$ is a mass parameter and $\langle \Lambda(x,t) \rangle = \Lambda_0 - m$ with $m \ll \Lambda_0$.

In Eq. 8 one may interpret $\nu(\Lambda(x,t))$ as a fluid density, that fluid being composed of the modes of QFT. $\nu(\Lambda)$ is the number of modes of all species having $|\vec{k}| < \Lambda$ (= $\Lambda^3$ for free field theory). Then to good approximation

$$\langle \phi \rangle^2_0 \approx \frac{d\nu}{d\Lambda}|_{\Lambda_0} \approx \frac{\nu(\Lambda_0)}{\Lambda_0}.$$

$\langle \phi \rangle_t$ is to be interpreted as the inflation condensate wave function. It is the analog of a superfluid wave function at absolute zero since the fluid carries no entropy. The factorization of superfluid density into a square is the analog of the Penrose-Onsager construction.

That $\langle \phi \rangle^2_0$ is indeed the homogeneous part of the phenomenological inflaton is seen by multiplying Eq. 8 and integrating over $d^3x$ in the volume about the site $x, t$. The conventional potential energy $\int m^2 \langle \phi \rangle^2_0 d^3x$ that is stocked in that volume is then checked out to be equal to $m \int [\nu(\Lambda_0) - \nu(\Lambda(x,t))] d^3x$. This latter is the number of modes that have leaked out into the volume multiplied by the energy, $m$, carried by each of those modes.

The value of $\langle \phi \rangle_t$ can be estimated from Eq. 10. Taking $\Lambda_0 = \mathcal{O}(1)$ one has

$$\langle \phi \rangle_t = \mathcal{O}(\sqrt{N}).$$
At \( t_0 \), one requires phenomenologically \( \langle \phi \rangle_{t_0} = O(10) \). Since \( N = O(10^2) \) the result conforms to the phenomenological requisite.

The next step is to estimate \( m \). The *sine qua non* for inflation to take place is the homogeneous isotropic production of energy per tunnelling event. Were energy produced helter skelter from chaos, without giving rise to a homogeneous distribution of cisplanckian field energy, we would not be able to avail ourselves of the GR energy constraint and induce a global hubble expansion. Rather such a random event would result in an uneventful fluctuation, i.e., \( \langle \phi \rangle = 0, H = 0 \).

We now interpret \( m \) in terms of the product of the energy involved in each tunnelling event and the amplitude, \( A \), for a tunnelling to occur. For the former we take the energy \( O(1) \), since that is the only scale present. For the latter, in order for a tunnelling event to set up the cascade of successive events which take place from neighbor to neighbor, the tunnelled wave function must have extension out to the distance of neighbour also \( O(1) \), in virtue of the causet construction. Thus without any further additions, if such a tunnelling event were responsible for inflationary cooperative process, we would have \( m = 1e^{-1} \) where the exponent comes from \( e^{-\Delta E d} \) with \( \Delta E = \text{planck energy and } d = \text{mean interval between causet elements} \). But, as mentioned above, to set up the tunnelling events necessary for inflation, the tunnelled configuration should approximate to an isotropic homogeneous distribution. Therefore tunnelling out of a given causet site should be a synchronized act of tunnelling from a given site to neighbors. If there are \( Z \) in number then one obtains \( m = O(e^{-Z}) \). In three dimensions \( Z = O(10) \) whence \( m = O(10^{-5}) \) as against the phenomenological value \( O(10^{-6}) \). Thus this rough estimate once more conforms to phenomenology in order of magnitude.

It is well to emphasize that we are still working within a phenomenological framework making use of analogies borrowed from conventional physics. The hope is that these encouraging estimates render accepted phenomenology somewhat more intelligible, but even more that they will direct further research towards a complete theoretical structure.

Further development of inflatonionary physics requires an understanding of how the inflaton propagates. Fortunately, propagation of fields on the causet network has been the subject of recent work which complements previous work of Sorkin and collaborators [6]. We shall not review this work in the present paper, but do hope in the future to return to a detailed consideration of the hopping mechanism in terms of the localized field configurations which have been posited to exist at the causet sites. One should not overlook the possibilities of interesting dynamics involving cis–trans communication as part of hopping, whence possible interesting asymmetric effects. For the nonce, combining the conceptual understanding of \( \phi \), \( m \) and propagation, there exists some element which makes inflaton physics intelligible.

In this paper we shall not touch upon reheating. Once \( \langle \phi \rangle_t = O(1) \), after a few \( e \)-folds, the KG equation predicts that \( \dot{\phi} \) and \( m\phi \) become comparable. Thereupon the slow roll and inflation stop; the universe is made and there is no more significant creation. Interaction of the cis degrees of freedom among themselves, gravitationally and otherwise, cause \( \phi \), a coherent collective degree of freedom, to fall apart. Analogy with fluid dynamics points to the transition between laminar and turbulent flow as a possible theoretical point of departure to seek inspiration. Another possibility is a quantum formalism of the disintegration into myriads of lower energy quanta. For all we know this may be another way to look upon turbulence.

The problem of the generation of primeval fluctuations presents an interesting construct to the creation of inflationary energy. Yet the fundamentals are the same, the latter being
the creation of homogeneous configurations out of sequestrations.

Suppose the seed of cosmogenesis is at length scale $a$, where $a = O(1)$, which then increases to macroscopic size. A fluctuation of comoving wave vector $\vec{k}$ comes into existence when the physical wave vector, $\vec{k}/a$ is $O(\Lambda_0)$ whereupon it emerges from sequestration into the cis world. As $a$ expands, so do the values of $\vec{k}$, so as to give a scale invariant system of observable fluctuations. The birth of fluctuations is thus the analogy of the birth of $\phi$, mode by mode, without condensation for their energy does not participate significantly to the homogeneous Hubble expansion. An account of the density of these fluctuations is the content of [1].

6 Concluding Remarks

The essential message of this paper is the existence of a length scale, $\Lambda_0^{-1}$, below which the energy density at scale $\Lambda^{-1} \ (= \rho(\Lambda))$ becomes negative. The most important consequence is that, save for exceptional circumstances, the field configurations do not expand, owing to the constraint $H^2 = \rho/3$. There are exceptional circumstances which permit field propagation. Namely, this can occur when the field is $O(\sqrt{N})$.

The exceptional circumstances which result in a cosmos through inflation, arise when an ordered cisplanckian field configuration of positive energy density $\rho(\Lambda)$, with $\Lambda < \Lambda_0$, tunnels out of sequestration. The requirement of homogeneity whence the subsequent existence of the modes of QFT, is met by the existence of a mean scalar field, $\langle \phi \rangle$, the inflaton and $\rho(\Lambda) = m^2 \langle \phi \rangle^2$, where $\langle \phi \rangle$ is the mean of $\phi$ over the expanding homogeneous patch and $m$ is the mass of the inflaton.

$\langle \phi \rangle$ arises from a seed, emerging from chaos by tunnelling out of a bound state configuration that is centered on a causet site. If one makes the natural hypothesis that the energy scale of binding is $O(1)$ and the inter-causet length scale is $O(1)$ as well, then the requirement that this seed yield an approximately homogeneous field distribution concomitantly with an approximately homogeneous space surrounding that seed, yields a value for $m$ consistent with the requirements of observation: $m = O(10^{-6})$. This follows from the existence of roughly 12 neighbors about each site. The same interpretation of the $\phi$ field in terms of mode density and mass yields the estimate $\langle \phi \rangle = O(\sqrt{N}) = O(10)$ where $N$ is the number of species of fields contributing to QFT.

The second main subject of this conclusion concerns the parallelism between black hole evaporation, dark energy and cosmic creation.

Degrees of freedom, which are destined to become Hawking evaporated quanta, near to the black hole horizon at $r = 2M$, exhibit a strong blue shift and enter into the transplanckian regime. Therefore, unless otherwise solicited, they will remain sequestered on the causet sites near the horizon. However, as for cosmogenesis they give rise to fluctuations which tunnel out of sequestration. Being cisplanckian at this point they can elicit macroscopic response, in this case the change of mass of the black hole metric. As for the cosmic problem, one effect drives the other, ultimately leading to permanent effects wherein the frequency of asymptotic Schwarzschild space-time is equal to the loss of black-hole mass as expressed in the metric change. It is interesting that the dynamics of both black hole evaporation and cosmic creation of quanta requires such elaborate mechanisms: the Bogoljubov transformation for Kruskal to Schwarzschild modes on the former, inflation and the formation of the inflaton, followed
by reheating in the latter.

A final point concerns dark energy, where hopefully the considerations above can be substantiated in the presence of cosmic inhomogeneous fluctuations as well as more complicated distributions of on mass shell quanta than were dealt with in the literature. If successful, there is hope that all processes concerning creation from “formless” transplanckian void can be united. Whereupon we shall have some guidance for constructing the fundamental theory. The concept of unitarity will have to be modified, at least in the sense of dealing with constantly varying cisplanckian Hilbert spaces.

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