On the detection of point sources in CMB maps based on cleaned K-map

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ABSTRACT

We use the Wilkinson Microwave Anisotropy Probe 7-year data (WMAP7) to further probe point source detection technique in the sky maps of the cosmic microwave background (CMB) radiation. The method by Tegmark et al. for foreground reduced maps and the Kolmogorov parameter as the descriptor are adopted for the analysis of WMAP satellite CMB temperature data. Part of the detected points coincide with point sources already revealed by other methods. However, we have also found 2 source candidates for which still no counterparts are known, and identified 7 point sources listed in Planck Early Release Compact Source Catalogue as high reliability sources.

Key words. cosmic microwave background, point sources, non-Gaussianity

1. Introduction

The cosmic microwave background (CMB) maps have been examined for detecting point sources as potential foregrounds, which could be cosmological or Milky Way objects emitting thermally or non-thermally. A catalog is made available on the NASA WMAP team web page. Various methods, including the wavelets and needlets, have been used for the detection of point sources in pixelized sky maps (see, e.g., Scodeller et al. (2012), Batista et al. (2011)). Most of the detected sources in CMB maps coincide with known radio sources, quasars, blazars, although some sources are still unidentified (Wright et al. 2009a,b). For such a purpose the so-called empirical cumulative distribution function \( F_N(x) \) is calculated,

\[
F_N(x) = \begin{cases} 
0, & x < X_1 \\
\frac{k}{N}, & X_1 \leq x, \quad k = 1, 2, \ldots, N - 1 \\
1, & X_N \leq x 
\end{cases}
\]

where \( k \) is the number of elements which obey to the relation \( X_i \leq x \). Having assumed a particular theoretical cumulative distribution function (CDF) \( F(x) \), the parameter \( \lambda_N \) is easily calculated,

\[
\lambda_N = \sqrt{N} \sup_{x} |F_N(x) - F(x)|. \tag{2}
\]

Kolmogorov proved that in the limit \( N \rightarrow \infty \), \( \lambda_N \), which is a random variable, has a cumulative distribution function \( \Phi(\lambda) \) reading as

\[
\Phi(\lambda) = \sum_{j=-\infty}^{+\infty} (-1)^j e^{-2j^2\lambda^2}. \tag{3}
\]

The function \( \Phi(\lambda) \) can be expressed as a particular value of theta functions, since \( \Phi(\lambda) = \vartheta_3(0, e^{-2\lambda^2}) = \vartheta_3(0, -e^{-2\lambda^2}) \) (see Abramowitz & Stegun 1970, 16.27).

Some general worries were expressed by Frommert et al. (2012) for the use of the KSP parameter on the ground that the CMB fluctuations, despite Gaussian to a high degree, are angularly correlated. In a short note Gurzadyan...
about point sources. For more details see Ghahramanyan et al. (2009); Gurzadyan et al. (2011).

A known issue that can impact the KSP calculations is beaming and other instrumental correlation effects. In this paper we use only a single WMAP band CMB map for K-map calculations, therefore the beaming effect of different bands are not relevant. The mean value of the K-map varies very slightly between different bands if the Galactic disk region is not taken into account. Also the beaming effect adds some unexpected Gaussian noise into the CMB data. So it hardly has any effect on the K-map calculations aimed at detecting strong departure from a Gaussian distribution.

3. Cleaned CMB K-map
3.1. WMAP7 W CMB and K-maps

It is well known that the CMB sky has an nearly Gaussian distribution. So, the Gaussian distribution is used in Kolmogorov method to construct a K-map. For every compact region of the CMB map containing 256 neighboring temperature pixels, one corresponding pixel value of the K-map is obtained. For WMAP7 W band CMB map with \( n_{side} = 512 \) resolution parameter (details in Górski et al. (2005)), we obtain a K-map with \( n_{side} = 32 \) parameter. This means that the CMB map has \( n_{map} = 12 n_{side} = 3145728 \) pixels and the K-map \( n_{map} = 12288 \) pixels. This is due to the fact that KSP is a statistical parameter. Then the KSP distribution maps over the whole sky can be obtained for WMAP7 data. It is seen that the Galactic disk region has higher and saturated KSP values, which indicates that it has a non-Gaussian distribution, distinct from the CMB. Also a lot of pixels have a high value of KSP, but most of them, as it will be shown below, are due to instrumental and other types of noise, also of non-Gaussian nature.

Throughout the analysis we mainly use the HEALPIX program package for calculation of the spherical harmonic coefficients and to reconstruct the maps Górski et al. (2005). For some tedious manipulating procedures with the spherical harmonic coefficients \( a_{lm} \) (see Eq. (6) below) we use the GLESP program package Doroshkevich et al. (2005). For maps with low resolution (in our case for K-map, \( n_{side} = 32, n_{map} = 12288 \) the \( a_{lm} \) calculation and map reconstruction the use of HEALPIX program package led to accuracy problems. To test whether the calculation error is small or not for a low resolution map, we construct a unity HEALPIX map with \( n_{side} = 32 \), and another one with \( n_{side} = 512 \), and run them through the \( a_{lm} \) calculation and the map reconstruction procedures. The result, multiplied by 1000, is given in Fig. 2. For both cases this procedure adds some non-isotopic noise which has almost zero mean \( \langle e \rangle \) and a very small standard deviation \( \sigma \). We obtain \( \langle e_{512} \rangle = 10^{-6}, \langle e_{32} \rangle = 2 \cdot 10^{-4}, \sigma_{512} = 2.3 \cdot 10^{-4}, \sigma_{32} = 3.71 \cdot 10^{-3} \). Although some pixels around the poles have more error, the fraction of these pixels on the sky for \( n_{side} = 32 \) is less than 0.5%. So for \( n_{side} = 32 \), the error and standard deviation are sufficiently small, which enables one to calculate the \( a_{lm} \) and to construct the cleaned map for KSP using Tegmark et al. method.

![Fig. 1. WMAP7 Cosmic microwave background temperature (top) and Kolmogorov maps (bottom).](image-url)
3.2. Modified Tegmark et al. method

Tegmark et al. method (Tegmark et al. 2003) is commonly used for obtaining a CMB foreground-reduced map from the original WMAP CMB maps. In this method every map from different bands is weighted with weights \( w_i^l \). Here \( l \) is the multipole number and \( i \) the band index. It differs from the interlinear combination (ILC) method of weighting different bands suggested by the WMAP team (Jarosik et al. 2011) by dependence of weights on the multipole numbers. These weights are simply calculated from the cross-correlation matrix between the bands.

We use Tegmark et al. method (Tegmark et al. 2003; Saha et al. 2006, 2008; Gurzadyan et al. 2009b) to develop a cleaned CMB K-map for eight WMAP bands: Q1, Q2, V1, V2, W1, W2, W3, W4. This method, based on the power spectrum comparison, assigns a weight \( w_i^l \) for each map \( i \) and multipole \( l \). In order not to distort the original map, \( w_i^l \) should obey the relation

\[
\sum w_i^l = 1.
\]

But a priori \( w_i^l \) could be any real numbers, including negative ones. But KSP must belong to the interval \( 0 \leq \Phi \leq 1 \), so the use of negative weights to construct the cleaned \( a_{lm} \) and then to reconstruct the map could result in negative values of \( \Phi \) for some pixels. To avoid this problem, we normalize the weights by the following formula:

\[
w_i^l = \frac{w_i^l - w_{\text{min}}^l}{1 - n w_{\text{min}}^l},
\]

where \( w_{\text{min}}^l \) is the minimal value of the original weights for fixed \( l \) multipole. It is easy to see that for the new weights \( \tilde{w}_i^l \) the relation \( \sum_i \tilde{w}_i^l = 1 \) holds. This is the unique linear relationship satisfying \( \tilde{w}_i^l \geq 0 \) and minimally modifying the original \( w_i^l \).

In the Kolmogorov method, if a sequence of random numbers \( T_n \) obeys a theoretical distribution function \( F(x) \) then \( N \) realizations of this sequence gives \( \Phi_N, \lambda_N \). The remarkable point of the method is that, \( \Phi_N \) has a uniform distribution and the mean value \( \langle \Phi \rangle = 0 \). Therefore for a CMB K-map the value 0.5 is a natural threshold for distinguishing non-Gaussian areas from others, since Gaussian temperature pixels cannot have \( \Phi > 0.5 \). Further,
the cleaned K-map mean value and sigma are respectively $\langle \Phi \rangle = 0.22$, $\sigma_\Phi = 0.14$, so pixels with $\Phi > 0.5$ mostly exceed the 3-$\sigma$ region (see Fig. 11). In Fig. 3 one can see the Galactic disk, the Large Magellanic Cloud (LMC) and other possible point sources. There are in total 398 non-Gaussian pixels among 12'288 (about 3.2%). Only 27 among them are outside the Galactic region ($|b| < 20$) and the Large Magellanic Cloud ($l = 280.4136, b = 32.9310$). So, these 27 regions are point source candidates. Indeed, most of them are found in the catalogs of Gold et al. (2011), Lanz et al. (2011) and Ade et al. (2011). It is interesting that seven of the point sources are also listed in the Planck Early Release Compact Source Catalogue as high reliability point sources (Ade et al. 2011). Two possible point sources remain unidentified in any of the known catalogs.

Another interesting feature of our cleaned K-map is that its distribution differs from the uncleaned maps in a minor way. The restrictions from the weighting scheme Eq. 4 keep the mean value almost unchanged. This means that structures in the cleaned map are preserved during the cleaning procedure, but for a lot of regions the KSP value is decreased.

3.3. Testing the method

Here we represent a simulated Gaussian CMB map without any correlation and a K-map for it. We have not made any assumptions about the theoretical cumulative distribution function (i.e., mean and standard deviation of a Gaussian distribution), although it is fixed during the map generation. So we calculate these parameters for every region on the sky. This approach generates some false non-Gaussian regions with high values of the KSP due to varying these two parameters. As one can see, the K-map for the generated Gaussian map is blue almost everywhere, i.e., $\Phi$ is very small, but due to the numerical effect mentioned above, about 900 of 12288 pixels (about 7%) of it are above the reasonable threshold of 0.5. For different simulated maps this point appears at random positions on the maps. Very rare pixels have a KSP value higher than 0.9. A similar situation appears in the real WMAP7 CMB maps. We can not assume any proper Gaussian distribution parameters for any region on the sky. Some remarkable non-Gaussian regions (such as the Galactic disk) can be seen in regions with high values of the KSP, but they are mixed with other regions affected by numerical inaccuracies. Hence, our problem is to distinguish false and real non-Gaussian regions (such as the Galactic disk) can be seen in regions with high values of the KSP, but for a lot of regions the KSP value is decreased.

4. CMB and K-map power spectra

Any full sky map can be represented via a series of Legendre spherical functions $Y_{lm}(\theta, \varphi)$ (Hivon et al. 2002), where

$$T(\theta, \varphi) = \sum_{l,m} a_{lm} Y_{lm}(\theta, \varphi),$$

$$a_{lm} = \int T(\theta, \varphi) Y_{lm}^*(\theta, \varphi) \sin \theta \, d\theta \, d\varphi.$$  

As easily shown, the coefficients $C_l$ of the Legendre polynomials $P_l(\cos \theta)$ in the two point correlation function $C(\theta)$ of the power spectrum are related to the $a_{lm}$ by

$$C_l = \langle a_{lm}^* a_{lm} \rangle,$$

$$C(\theta) = \frac{1}{4\pi} \sum_{l,m} (2l + 1) C_l P_l(\cos \theta).$$  

To see if any correlation exists between the CMB temperature and the KSP parameter map, we degrade the resolution of the CMB temperature map to $n_{side} = 32$ and also normalize it to the same $0 \leq T \leq 1$ interval as $\Phi$. So, instead of the original temperature we use a dimensionless parameter $T$, to prevent any discrepancy between temperature and Kolmogorov maps. For power spectrum and cross power spectrum calculations we use WMAP7 (Jarosik et al. 2011) eight different bands $T$ and the Kolmogorov
maps (Q1, Q2, V1, V2, W1, W2, W3, W4). Then, the foreground reduced map is obtained using Tegmark et al. (2003) weighting technique for triplets of different bands for the calculation of the power spectra both for the temperature and the KSP correlation functions, via standard technique as shown in Hinshaw et al. (2003) and Gurzadyan et al. (2009a). The lines in Fig. 10 are the mean value of the $C_l$ for different cross power spectra between different bands. The error bars are obtained as the variance square root of the different cross power spectra.

The results for the foreground cleaned maps are shown in Fig. 10. It can be seen that the $C_l$ have a different behavior and the main striking fact is that they are crossed at $l \approx 25$.

Fig. 8. Simulated Gaussian K-map points above 0.9 value.

Fig. 9. CMB K-map points above 0.9 value.

Fig. 10. CMB temperature and K-map correlation function power spectra.

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Fig. 11. Histograms for WMAP7 W band and cleaned K-maps.

5. Cross power spectra of CMB temperature and K-maps

For cross power spectra calculation we use the common technique described in Hinshaw et al. (2003) via taking the cross-correlation power spectrum coefficients for different type of $a_{lm}$, as

$$\tilde{C}_l = \langle a_{lm}^T a_{lm}^\Phi \rangle.$$  \hspace{1cm} (8)

Here we again use the foreground reduced maps (see Tegmark et al. 2003) for the calculation of the cross power spectra between $T$ and $\Phi$, since the original ones are very noisy, while the aim is to get the smallest possible error bars in the final power spectrum. The power spectrum estimation is done without taking into account the Galactic disk plane region, i.e., we use the window function which is zero for the region $\theta = \pm 20$ for both $T$ and $\Phi$, and unity elsewhere. This cutting method influences mostly the even $l$ inducing some unreasonable peaks (for example at $l = 2, 12, 22, \ldots$) around the window function power spectrum peaks. One could use Peebles (1973) method to reduce this effect on even $l$ values and adjust odd $l$ values but then one would have to calculate the power spectrum at least up to $l = 250$. The low resolution map of $\Phi$ allows the accurate estimation of the power spectrum up to $l = 97$ which makes impossible the use of this method. Therefore we have to use only odd $l$ values.

In Fig. 11 no proper correlation can be seen between CMB WMAP7 $T$ and $\Phi$ maps.

6. Conclusions

Using the Kolmogorov stochasticity parameter for the detection of point sources in CMB maps, we proceeded from the K-maps cleaned via the modified Tegmark et al. method. The novelty is that we applied this cleaning algorithm not to the usual temperature maps, but to the K-maps. Other mask construction schemes are based on WMAP K and K1 band maps and are more affected by instrumental and other types of noises. It appears that for about 85% of the cleaned map, the Kolmogorov function has values in the interval $0.0 < \Phi < 0.2$, which implies that the CMB maps are indeed Gaussian with high precision. The non-Gaussian pixels (with $\Phi > 0.5$) are rather
rare (398), i.e., less than 4%. Of course this result is derived for the cleaned K-map but it implies that different types of noises add additional non-Gaussianities into the CMB maps, which may be analyzed by the methods in [de Oliveira-Costa et al. (2004) and Rocha et al. (2005)]. These pixels, when outside the Galactic disk region \( |b| > 20 \), indicate the positions of point source candidates. While most of them have counterparts in existing catalogs, two of them are still unidentified.

Another type of non-Gaussianity discussed here is the fluctuations in the K-map. KSP is a statistical parameter so if one calculates KSP for fixed numbers of elements, then it must have statistical fluctuations. But for the same reason, the full sky K-map should have a uniform distribution, which is not so (Gurzadyan et al. 2011). So in most cases these fluctuations have a non-Gaussian nature.

Some theoretical models (Takeuchi et al. 2012; Komatsu & Spergel 2001; Salopek & Bond 1990; 1991; Gangui 1994) predict primordial non-Gaussianities from inflation era. This effect is very tiny but some authors tried to discover it through the CMB bi-spectrum. We then used a new method to implement the cross-correlation between the CMB temperature and the K-map. Numerical modeling of such a problem was done in Ghahramanyan et al. (2009). It was shown that KSP is sensitive even to small departures from the theoretical distribution. Certain non-Gaussian perturbations would appear in K-map as KSP perturbations. Also, if one uses a proper theoretical distribution function, no correlation between the temperature and KSP should arise. Therefore in regions outside the Galactic disk certain correlations could appear even in the presence of rather small non-Gaussianities. As one can see from Fig. 10 and Fig. 12, any correlation is rather difficult to see, although the intersection of the correlation functions of \( T \) and \( \Phi \) around \( l = 25 \) might be related to certain symmetries, see e.g. Gurzadyan et al. (2008, 2009a).

### Table 1. Coordinates and ID-s of point sources detected in cleaned K-map.

| l    | b     | KSP value listed in | source ID  |
|------|-------|---------------------|------------|
| 15.00 | 58.92 | 0.50250             | Planck     |
| 43.59 | 27.28 | 0.50681             |            |
| 45.00 | 49.70 | 0.50025             | WMAP7      |
| 63.28 | 41.81 | 0.57482             | GB6 J1625+4134 |
| 67.50 | -34.23| 0.52509             | Lanz et al. GB6 J2148+0657 |
| 72.69 | 70.91 | 0.54416             | WMAP7      |
| 85.78 | -38.68| 0.83258             | GB6 J2253+1608 |
| 97.03 | 24.62 | 0.56636             | GB6 J1841+6718 |
| 99.84 | 38.68 | 0.51425             | J1659+6827 G |
| 99.84 | 24.62 | 0.56640             | WMAP7      |
| 105.47| 42.66 | 0.52249             | GB6 J1927+7357 |
| 106.50| 44.99 | 0.61560             | Planck     |
| 132.19| 66.44 | 0.50769             | Planck     |
| 157.50| -20.74| 0.54468             | Lanz et al. GB6 J0336+3218 |
| 158.91| -22.02| 0.54752             | Planck     |
| 195.47| -32.80| 0.71974             | WMAP7      |
| 210.94| -20.74| 0.70129             | Planck     |
| 213.75| -20.74| 0.62933             | Planck     |
| 238.33| -49.70| 0.54351             | WMAP7      |
| 246.09| 24.62 | 0.51170             | Planck     |
| 251.72| -32.80| 0.59206             | WMAP7      |
| 258.75| -72.39| 0.65437             |            |
| 277.03| -35.69| 0.50510             | WMAP7      |
| 282.27| 73.87  | 0.51290             | GB6 J1230+1223 |
| 283.50| 75.34  | 0.73977             | PMN 3046-3826 |
| 288.53| 64.95  | 0.83234             | WMAP7      |
| 293.77| 57.40  | 0.83074             | WMAP7      |
| 293.77| 57.40  | 0.78074             | WMAP7      |

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