Efficiency of the Wireless Power Transfer System with Planar Coils in the Periodic and Aperiodic Systems

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Abstract: This article presents the results of the proposed numerical and analytical analysis of the Wireless Power Transfer System (WPT). The system consists of a transmitting surface and a receiving surface, where each of them is composed of planar spiral coils. Two WPT systems were analysed (periodic and aperiodic) considering two types of coils (circular and square). In the aperiodic system, the adjacent coils were wound in the opposite direction. The influence of the type of coils, the winding direction, the number of turns, and the distance between the coils on the efficiency of the WPT system was compared. In periodic models, higher efficiency was obtained with circular rather than square coils. The results obtained with both proposed methods were consistent, which confirmed the correctness of the adopted assumptions. In aperiodic models, for a smaller radius of the coil, the efficiency of the system was higher in the square coil models than in the circular coil models. On the other hand, with a larger radius of the coil, the efficiency of the system was comparable regardless of the coil type. When comparing both systems (periodic and aperiodic), for both circular and square coils, aperiodic models show higher efficiency values (the difference is even 57%). The proposed system can be used for simultaneous charging of many sensors (located in, e.g., walls, floors).

Keywords: wireless power transfer; wireless charging; circuit modeling; numerical analysis; planar coils

1. Introduction

The traditional charging of devices with cables and wires turned out to be very lossy (approx. 30%), difficult to install, and subject to frequent breakdowns [1]. The energy transmission from the source to the load should primarily concern active energy, which over time is directly related to the voltage at the load terminals as well as the current resulting from the voltage and the load response. In most systems, to obtain the flow of active energy, it is necessary to properly compensate the reactive power and balance the load [2].

The number of portable devices continues to grow, but wired chargers mean the devices are not fully portable. For the indicated reasons, alternative solutions related to energy transmission and distribution were sought. Such a solution is the wireless power transfer (WPT) technology described in [1,3,4]. The WPT system can effectively transmit power from the source to the device by electromagnetic induction. The advantage of this system is the mobility of the devices and the lack of radiation. It is important to also pay attention to health aspects because the operation of WPT systems with the use of inductive/capacitive coupling concerns the generation of strong electromagnetic fields [5]. Although these magnetic fields can cause disease in humans, and one should respect their accepted limitations [6,7].

There are many studies and experiments on electric vehicles (EVs) aimed at reducing fuel and energy consumption [8,9]. Also in the medical field, WPT is studied and even used when charging or powering medical devices (e.g., pacemakers, endoscopy) [10,11].
WPT system is also considered in beacon systems [12] and intelligent buildings containing sensors placed inside walls [13].

Renewable energy resources are used in many applications, e.g., charging of electric vehicles, islanded power generation, or heating systems. Various concepts have been introduced in the literature to ensure a more economical, safe, and convenient transfer of power from renewable energy sources to the load. For this reason, research has begun on the use of WPT systems for renewable energy resources, including for charging electric vehicles in a solar photovoltaic garage or building integrated photovoltaics (BIPV). For example, in [14] an adaptive virtual impedance controller was developed to increase the efficiency of the WPT system at the variable load provided by a photovoltaic generator. The described controller uses an original method of adaptive evaluation of the coupling coefficient and the resonant frequency of the WPT system coils as a function of the distance between the coils.

WPT multi-coil systems operate at high frequencies ($f \geq 1$ MHz) [15,16]. At higher frequencies, mainly one transmitter and many receivers are considered [17]. Usually, an ideal WPT system is expected to transmit power efficiently, regardless of the relative positions of the transmitter and receiver. However, in practice, when the distance between the receiver and transmitter does not correspond to the optimal transmission area, the efficiency is quite low [18]. However, relatively little research has been done on WPT systems with multiple receivers [19,20]. The authors in [21] proposed a method of designing circuit parameters based on different receivers, but without controlling the power assigned to each receiver.

In the literature, there are solutions for the WPT system using e.g., a pair of coils [22,23] and even a coil system [15,24–27]. With large misalignment or the distance between the transmitting-receiving coils, the use of intermediate resonators/coils can increase the efficiency of these systems, as well as enable power transmission over longer distances [28–30]. For low frequencies ($f < 1$ MHz) these resonators can be arranged in a linear resonator array (planar array) [28,29,31] or in a domino configuration (domino array) [32,33], where the energy transfer in the space between the transmitter and the receiver is supported by the use of several resonators. The downside to this approach is that it requires more space than any other transmission system. The WPT system presented in [29] consisted of the array of identical resonators (cells) arranged in a plane along a line, creating the planar structure. Two adjacent resonators were spaced at the same distance and magnetically coupled with a mutual inductance. In this approach, the authors omitted the coupling between nonadjacent resonators, which was also done in [34].

A detailed analysis was carried out for the series configuration of resonators [29,35,36], while the series-parallel topology of planar coils, acting as a group of energy transmitters and receivers, is still not fully developed yet. For this reason, we decided to analyse multi-transmitter and multi-receiver systems using planar coils.

In [30] the authors proposed a model of the periodic circuit. The method of analysis of the low-power WPT periodic system with the use of field and circuit models was also presented. The article supplemented the mathematical formulations with the analysis of selected geometric variants of the arrangement of circular coils. In these formulations the authors used their own approach by numerical integrations and rectangle rule formula, because in the literature no analytical solution was found. The analysis concerned: transmitter current, receiver current, and power transfer efficiency. The comparison of the results from the numerical and circuit analysis showed that the difference was 12.44%. Additionally, an analysis of the horizontal misalignment in the discussed periodic WPT system was performed.

According to the authors, since the difference between the two methods was large [37], presents a modified analytical approach. As a result of modifying the equations, similar results were obtained, where the error was a maximum of 2%. In this approach, due to the current flow through the “skin” of the conductor at high frequencies of the magnetic field, the effective cross-section of the wire was also taken into account. To compare the
performance of WPT systems with different coils, it was necessary to determine the source power, the load power, and efficiency. The analysis was divided into two separate cases: (i) operation with the maximum efficiency in determining the optimal load, (ii) operation with the maximum transmitted power in determining the optimal load. The results are presented for circular planar coils in an infinite periodic system and separately for a system with nine coils.

In order to compare the efficiency of the system depending on the type of coils used in the periodic WPT system, a multivariate analysis was performed in [38] for the system with circular or square coils. For this reason, it was necessary to modify the formulas proposed in [37] in order to be able to calculate the periodic WPT system with flat square coils. It was also necessary to create new numerical models taking into account the geometry of the square coil. The estimation of the maximum efficiency and maximum power for both proposed WPT systems at low frequencies is described.

The article presents a wireless power transmission system with periodically arranged planar coils. The authors propose and study a numerical and analytical model that can be used to analyse the power transmission conditions in the presented WPT systems. Both solutions reduce the size and complexity of numerical and analytical models. The proposed single-cell analysis with periodic boundary conditions does not require a three-dimensional model with many coils. The advantage of this solution is a large reduction in the degrees of freedom. The simplified model is an alternative to the large matrix formula [39,40].

The proposed numerical and analytical models allow the assessment of the influence of the coil structure on power transmission. Adjustment of geometric parameters makes it possible to obtain high efficiency. The necessary lumped parameters in the proposed analytical model are calculated by using analytical equations. The numerical results in the frequency domain of the presented WPT systems are compared with the results obtained from the analytical analysis. The convergence of the results confirmed the correctness of both methods and the adopted conditions of the analysis.

By using WPT systems, one can also charge multiple devices that are close together. The authors propose a system with a grid of periodically arranged planar coils. In this solution, the coils form two surfaces: transmitting and receiving. This approach increases the density of transmitted power, and synchronous power delivery to multiple devices is possible thanks to a single power source. The proposed system can be used for simultaneous charging of many sensors (located in, e.g., walls, floors). Taking into account high power applications, the proposed WPT solution can be used when charging cars in large parking lots. An example of the application of periodic and aperiodic systems is the extra-low voltage lighting. In accordance with IEC 60364 standard (part 7), separated extra-low voltage (SELV) lightning installation requires a transformer to minimize the risk of electric shock in highly hazardous areas such as bathrooms and industrial spaces [41]. When light emitting diode (LED) matrices are predominantly rectangular in shape and composed of multiple chipsets, such light sources can be powered by IPT, providing both the required galvanic isolation and the ability to independently control each light source. The proposed solution of the WPT system also allows for the simultaneous supply of multiple receivers. Therefore, periodically arranged transmitting coils can transmit energy to several LED chipsets through e.g., wall or ceiling. This application also proves the highest level of insulation between the source and the luminaire.

Two types of coils were analysed: circular and square. Two systems were also considered: the periodic and the aperiodic checkerboard type. The difference between these systems concerned the winding direction of the turns in the adjacent coils. The calculations of the exemplary WPT systems were made in the frequency range from 0.1 MHz to 1 MHz. Using both proposed methods, the authors analysed the impact of the type of coils, the winding direction, and geometric parameters (coil radius, number of turns, and distance between the coils) on the power transfer efficiency. The analysis was multivariate and the influence of the above parameters on the power of the transmitter and receiver was also
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Section 2 presents and describes four analysed models of the WPT system. The authors also described numerical and analytical models with their conditions and formulae. The results (e.g., tables, characteristics) are presented in the third section. This section is divided into five paragraphs. The first (Section 3.1) includes e.g., general results of all variants of the analysed models. For a better and more precise analysis the results are divided into four subsections: (i) comparison of periodic and aperiodic systems for circular coils (Section 3.2); (ii) comparison of periodic and aperiodic systems for square coils (Section 3.3); (iii) comparison of periodic systems for both coils: circular and square (Section 3.4); and (iv) comparison of aperiodic systems for both coils: circular and square (Section 3.5).

2. Models of Wireless Power Transfer System

2.1. Geometry and Description of the Analysed Models of the WPT System

The presented WPT system is composed of pairs of transmitting-receiving coils which are called WPT cells (Figure 1). Two main approaches to the WPT are considered. One is connected to circular coils (Figure 1a) and the other to square coils (Figure 1b). The outer dimensions of each transmitter/receiver cell are $d \times d$, the same radius ($r$), and the number of turns wounded around a dielectric carcass ($n$).

![Figure 1](image_url)

**Figure 1.** The studied WPT system composed of: (a) circular planar coils, (b) square planar coils.

The analysis concerned two variants of the coils. The first one was made of circular coils (Figure 2a), and the second one was made of square coils (Figure 2b). Two types of coils were considered: with a smaller radius ($r = 5 \text{ mm}$) and larger ($r = 20 \text{ mm}$).
Additionally, the influence of the winding direction on the parameters of the system was considered. The system with the transmitting-receiving surface composed of coils wound in the same direction is called the periodic system (Figure 3a,c).

Figure 2. Two types of modelled coils: (a) circular, (b) square.

Figure 3. The studied transmitting/receiving surface composed of: (a) circular coils with the same winding direction, (b) circular coils with the opposite winding direction, (c) square coils with the same winding direction, (d) square coils with the opposite winding direction.
On the other hand, the WPT system with alternately wound turns was called aperiodic (Figure 3b,d). For example, elements marked in yellow contained coils wound the same way. Whereas, cells adjacent to the edge of the exemplary cell G_{x,y} contained reverse-wound coils marked with a blue line, e.g., G_{x+1,y}. The cell G_{x+j,y+k} is an element of the set with identical inductors (j-column number, k-row number in the grid). Each adjacent coil is separated by the distance $d$ taking into account that $d \approx 2r$.

The transmitting surface is powered so that each transmitter is connected in parallel to a sinusoidal voltage source with the effective value $U_t$. The coils forming the receiving surface are connected directly to the load. Each WPT cell is assigned a separate load, which is $Z_l$. The analysed models of the WPT system guarantee an increase in the density of transmitted power in the area between the transmitting and receiving surfaces. In the four proposed variants of the WPT system, it is possible to select the power conditions in each cell depending on the desired requirements. It is also possible to supply multiple independent receivers simultaneously.

The presented four models constituting a solution for the WPT system were solved using the numerical method (Section 2.2) and the proprietary analytical method (Section 2.3). The results of both methods were compared in Section 3.

2.2. Numerical Solution of the WPT System

The analysis of the WPT system can be performed by numerical methods (e.g., FEM, FDTD, FDFD) [38,42–45], by the analytical method [27], or experimentally. Each numerical method requires the preparation of an accurate three-dimensional model and the assignment of different boundary conditions. Among other aspects, if the actual model is not accurately mapped, the results of the analysis will not be reliable. Also, the accuracy of the solution depends on the size of the model described by the number of degrees of freedom ($N_{DOF}$). A compromise must always be found between the computing power of a computer and the number of degrees of freedom. The proposed solution reduces the model to the 2D system, thanks to which the quick and accurate analysis is possible, and at the same time does not affect the quality of the results. In contrast, experimental models require the construction of a multi-segment prototype with appropriate geometry. This allows for the analysis of various electrical parameters (e.g., frequency), but without taking into account the influence of the model geometry, e.g., the number of turns, on the operating parameters. For the initial analysis of the WPT system properties (e.g., power and efficiency), it is enough to use analytical models.

For the numerical analysis, four models of the WPT system, the Finite Element Method (FEM) was used. It is a method of solving differential equations found in physics and technology [45]. Finite difference methods (e.g., FDTD, FDFD) are treated as a special variant of FEM with a peculiar way of presenting the approximating function. Various types of finite elements are used in solving problems using FEM. Their classification can be made on the basis of the type and degree of the approximating polynomial. A complex sets of shape and mediums can be approximated with high accuracy by means of curvilinear elements. The dimensions of the elements may vary by volume. Thanks to this, it is possible to increase or decrease the dimensions of elements in certain zones of the considered volume (the so-called adaptation of the finite element mesh).

This analysis requires considering e.g., the geometry and distribution of the coil turns, the number of WPT cells, and the electrical circuit elements connected to each coil. The turns of the coils are made of thin wire ($w$) with an insulation layer ($i$). The compensating capacitor is modelled as an element with a concentrated capacity ($C$). The capacity of the capacitor is determined on the basis of the parametric analysis of the system for different $C$. On the basis of the series resonance, it is possible to find the compensating capacity ($C$), at the specified frequency [27].

The compensating capacitor is modeled as an element with lumped capacity ($C$), attached to each inductor. A voltage source with an amplitude $U_t$ and a specific frequency $f$ is connected to the transmitting coil, while the load $Z_l$ is connected to the receiving coil. The
parameters of the numerical model, coil windings, and the frequency domain are shown in Table 1.

Table 1. Parameters of the wire used to form the coils.

| Parameter                              | Symbol | Value     |
|----------------------------------------|--------|-----------|
| diameter of the wire                   | \( w \) | 150 \( \mu \)m |
| thickness of the wire insulation        | \( i \) | 1 \( \mu \)m |
| conductivity of the wire               | \( \sigma \) | 5.6\( \times \)10\( ^7 \) S/m |
| voltage source                         | \( U_l \) | 1 V |
| load impedance                         | \( Z_l \) | 50 \( \Omega \) |
| frequency domain                       | \( f_{\text{min}} \div f_{\text{max}} \) | 100 \( \div \) 1000 kHz |

The numerical analysis of the four WPT models are taken into account as all cells forming the transmitting and receiving surfaces. In the proposed numerical approach, the entire WPT system is simplified to a single cell \( G_{x,y} \) containing a pair of transmitting and receiving coils. The analysed cell \( G_{x,y} \) is filled with air (Figure 4); the infinite array of resonators was modeled using periodic/antiperiodic boundary conditions (PBC). They are applied to the four side surfaces of the cell. At the top and bottom of the model there is a perfectly matched layer (PML) imitating an infinite dielectric background. Due to the fact that four variants are analysed (two periodic and two aperiodic, because the winding direction is variable), there are different boundary conditions on the surfaces perpendicular to the XY axis. In the periodic models, an infinite array of coils was modeled using periodic boundary conditions (PBC) [44,45]. Whereas, in aperiodic models, antiperiodic boundary conditions are applied in order to project an infinite set of WPT cells.

![Diagram](image1.png)

Figure 4. The proposed numerical model of the single WPT cell with: (a) circular coils, (b) square coils.

The model was made in the Comsol Multiphysics 4.3b software, which has built-in functions that enable solving the presented issues. This software is based on the FEM method. Thanks to this method, it was possible to significantly reduce the number of degrees of freedom by using the adaptation of the finite mesh element. This solution allowed to refine the mesh in significant places of the model, i.e., coils or corners, in order to obtain accurate results. We performed an analysis in the frequency domain, taking Magnetic fields physics in conjugation with the fragmentary Equivalent circuit. The coils were modeled using the built-in current sheet approximation of planar inductors (Multi-turn coil boundary condition), while the lumped voltage source (\( U_l \)), capacitors (C), and load (\( Z_l \)) were connected to the coils by internal coupling with a fragmentary Equivalent circuit. The
analysed numerical models with circular coils contained approximately 246.540 degrees of freedom or 232.270 for square coils.

The problem of energy transport in the presented model can be solved using the magnetic vector potential \( \mathbf{A} = [\mathbf{A}_x, \mathbf{A}_y, \mathbf{A}_z] \) in [Wb/m]. The Helmholtz equation which formulates the magnetic field phenomena in the frequency domain was used

\[
\nabla \times \left( \mu_0^{-1} \nabla \times \mathbf{A} \right) - j \omega \sigma \mathbf{A} = \mathbf{J}_e
\]

where \( \omega \) is the angular frequency [rad/s], \( \sigma \) is the electrical conductivity [S/m], \( \mathbf{J}_e \) is the external current density [A/m²]. Periodic boundary conditions on the four external surfaces were defined as magnetic insulation

\[
\mathbf{n}_{surf} \times \mathbf{A} = 0
\]

where \( \mathbf{n}_{surf} \) is a surface normal vector \( \mathbf{n}_{surf} = [1 \ x \ y \ z] \). The current density \( \mathbf{J}_e \) results from the value of the supply voltage \( \mathbf{U}_t \) connected to the transmitting coil. The goal is to find the spatial distribution of the magnetic potential vector \( \mathbf{A}(x,y,z) \) at a specific frequency \( f \).

2.3. Analytical Solution of the WPT System

Modeling and solving the numerical model of the WPT system is a multi-stage task. Even with numerical calculations, a simpler model with a similar scope of analysis is desirable. This will enable easier modeling and shorten the calculation time. As an alternative to the numerical solution, the authors proposed an analytical model combining a two-port network with analytical equations for calculating lumped parameters (Figure 5). In this method, the same approach as in the numerical model was used, i.e., the WPT wide area network analysis was reduced to a single WPT cell. The solution of the analytical model in the frequency domain is possible by any peripheral method. On the other hand, the greatest problem is the determination of the values of the lumped parameters taking into account the influence of the neighboring cells on the equivalent inductances of the transmitting coil \( L_t \) and receiving coil \( L_r \) as well as their mutual inductance \( M_{tr} \) (Figure 5) [27,37]. In [14] was proposed controller, a unique method is employed to adaptively estimate the coefficient of coupling and resonant frequency of the WPTS coils as a function of the distance between the coils. The purpose of this solution is to increase the efficiency of the wireless power transfer of the photovoltaic generator to the load.

![Circuit model of the cell in the WPT system: (a) model with indicated magnetic coupling, (b) two-port network model of the cell with identical Tx and Rx coils.](image)

**Figure 5.** Circuit model of the cell in the WPT system: (a) model with indicated magnetic coupling, (b) two-port network model of the cell with identical Tx and Rx coils.

The length of all windings in a circular coil \( l_{cir} \) is presented by Equation (3), whereas for a square coil \( l_{squ} \) by Equation (4):

\[
l_{cir} = 2\pi n[r - 0.5(n - 1)(w + i)],
\]

\[
l_{squ} = 4n[2r - n(w + i)]
\]
Taking into account that the transmitter and receiver coils are identical, the resistances of the inductor are $R_t = R_r = R_c$. Substituting Equation (3) or Equation (4) into the formula for the resistance of a conductor with a constant cross section, the inductor resistance ($R_c$) can be calculated:

- for circular coil:
  \[
  R_c = \frac{l_{cir}}{\sigma \pi \frac{w^2}{4}} = \frac{2\pi n[r - 0.5(n - 1)(w + i)]}{\sigma \pi \frac{(w^2)}{4}},
  \]

- for square coil:
  \[
  R_c = \frac{l_{squ}}{\sigma \pi \frac{w^2}{4}} = \frac{4n[2r - n(w + i)]}{\sigma \pi \frac{w^2}{4}}
  \]

It is possible to reduce the analysis to a single WPT cell, because in the network each coil has the same electrical parameters and there is a magnetic coupling with its neighbors (Figure 5). The mutual inductances between the coils affect the internal inductance of the coil in $G_{x,y}$:

\[
L_c = L_{self} + \sum_i \sum_j (M_{x+i,y+j}),
\]

where: $L_c$—effective inductance in [H], $M_{x+i,y+j}$—mutual inductance in [H] between coils adjacent in the horizontal plane, for $i \neq 0$ and $j \neq 0$, $L_{self}$—self-inductance of a planar coil in [H]. The self-inductance of a plane coil can be determined from [37,46]:

\[
L_{self} = \frac{\mu_0 c_1 d_{ave}^2 n^2}{2} \ln \left( \frac{c_2}{\nu} \right) + c_3 \nu + c_4 \nu^2
\]

where: $d_{ave}$ is a mean diameter:

\[
d_{ave} = 2r - (w + i)n,
\]

and $\rho$ is a fill coefficient:

\[
\rho = \frac{(w + i)n}{2r - (w + i)n}
\]

The coefficients: $c_1, c_2, c_3, c_4$ depend on the geometry of the coil [46]. The values of the coefficients for a circular coil are as follows: $c_1 = 1$, $c_2 = 2.5$, $c_3 = 0$, $c_4 = 0.2$. For a square coil, these are: $c_1 = 1.46$, $c_2 = 1.9$, $c_3 = 0.18$, $c_4 = 0.13$. From Equation (7), the inductance of the considered coil in the segment $G_{x,y}$:

\[
L_c = L_{self} - M_{pe},
\]

where: $M_{pe}$ is the sum of mutual inductances in the periodic grid. If loads $Z_l = \infty$ and there is no capacitor in series with the transmitter coils, $M_{pe}$ is presented by [37]:

\[
M_{pe} = \frac{|U_{tr,\infty} - R_c|}{2\pi f} - L_{self},
\]

where:

$|U_{tr,\infty}| - \text{RMS value of the source current in [A]},$

$|I_{tr,\infty}| - \text{source current in [A]},$

$\Psi - \text{phase angle between source voltage and current in [rad]}.\]

Taking into account the equivalent circuit (Figure 5b), instead of calculating $M_{tr,x+i,y+j}$, the mutual inductance ($M_{tr}$) between the transmitter and the receiver can be described by the formula [37]:

\[
M_{tr} = \frac{|U_{tr,\infty}|}{2\pi f L_{tr,\infty}},
\]

where:
$U_{r,\infty} = |U_{r,\infty}| e^{j\theta}$, voltage induced in the receiving coil in [V],
$|U_{r,\infty}|$ - RMS value of the voltage induced in [V],
$\Theta$ - phase angle between source voltage and induced voltage in [rad].

The compensating capacity (C) at a given frequency is presented by the Equation (14):

$$C(f) = \frac{1}{4\pi^2 f^2 L_c} = \frac{1}{4\pi^2 f^2 (L_{\text{self}} - M_{pe})}$$ (14)

The compensating capacity can be calculated after previously determining the self-inductance ($L_{\text{self}}$) and the mutual inductance of adjacent coils ($M_{pe}$).

3. The Results Obtained with Both Methods (Analytical and Numerical)

3.1. The Analysed Variants of the WPT System

Four variants of the WPT system were analysed (Figure 3):
- periodic model with circular coils (marked on the characteristics by: P cir);
- aperiodic model with circular coils (AP cir);
- periodic model with square coils (P sqr);
- aperiodic model with circular coils (AP sqr).

For each variant, the influence was analysed: changes in the coil radius ($r$), the number of turns ($n$) and the distance between the transmitting and receiving surface ($h$) on e.g., system efficiency (Table 2).

Table 2. Variants of geometric parameters taken for the analysis of two types of coils.

| $r$ (mm) | $n$  | $h$ (mm) |
|---------|------|---------|
|         | 15   | 2.5     | 5.0     |
|         | 20   | 2.5     | 5.0     |
|         | 25   | 2.5     | 5.0     |
|         | 40   | 10.0    | 20.0    |
| 20      | 50   | 10.0    | 20.0    |
|         | 60   | 10.0    | 20.0    |

The analysis of many of the models considered was based on the following calculation: transmitter power, receiver power, and power transfer efficiency. A wide range of frequencies (100 ÷ 1000 kHz) is also considered.

Because the passive load ($Z_l$) was considered, its active power is calculated from the equation:

$$P_o = Z_l |I_r|^2,$$ (15)

The power of the transmitter is represented by:

$$P_z = U_t I_t,$$ (16)

where:
- $I_r$ - the current flowing through the receiving coil in [A],
- $I_t$ - the current flowing through the transmitting coil in [A].

The power transfer efficiency is represented by:

$$\eta = \frac{P_o}{P_z} \times 100\%$$ (17)

The results presented in Figures 6–29 and Tables 3–13 apply to both proposed methods (analytical and numerical). The results obtained by the analytical method are marked with dots in the figures. On the other hand, the results obtained by the numerical method are
marked with a line. Lumped parameters of the electrical circuit from Figure 5b for circular and square coils were presented in Tables 3 and 4.

Table 3. The values of the compensating capacity at $f_{\text{max}} = 1$ MHz, self, and mutual inductance for small circular and square coils ($r = 5$ mm).

| $n$ | $L_{\text{self}}$ ($\mu$H) | $M_{\text{tr}}$ (nH) | $C$ at $f_{\text{max}}$ (nF) |
|-----|-----------------|-----------------|-----------------|
|     |                 | $h = 0.5 \ r$   | $h = r$         | $h = 0.5 \ r$ | $h = r$ | Periodic | Aperiodic |
| 15  | 2.36            | 373             | 86              | 784           | 286     | 13.26     | 10.72     |
| 20  | 3.14            | 547             | 127             | 1062          | 384     | 9.77      | 8.06      |
| 25  | 3.64            | 659             | 152             | 1227          | 439     | 8.30      | 6.95      |
|     |                 |                 |                 |               |         |           |           |
| 15  | 3.09            | 334             | 72              | 1009          | 373     | 12.28     | 8.20      |
| 20  | 4.05            | 519             | 116             | 1365          | 505     | 9.03      | 6.25      |
| 25  | 4.64            | 645             | 145             | 1575          | 581     | 7.66      | 5.46      |

The $L_{\text{self}}$ values are higher for a square coil than for a circular one. This inductance increases as the number of turns increases. The $M_{\text{tr}}$ values are higher for a circular coil than for a square one for the periodic model and lower for the aperiodic model. This inductance also increases as the number of turns increases. The values of the compensating capacity are higher for a circular coil than for a square one. This capacity decreases as the number of turns increases.

Table 4. The values of the compensating capacity at $f_{\text{max}} = 1$ MHz, self, and mutual inductance for small circular and square coils ($r = 20$ mm).

| $n$ | $L_{\text{self}}$ ($\mu$H) | $M_{\text{tr}}$ (nH) | $C$ at $f_{\text{max}}$ (pF) |
|-----|-----------------|-----------------|-----------------|
|     |                 | $h = 0.5 \ r$   | $h = r$         | $h = 0.5 \ r$ | $h = r$ | Periodic | Aperiodic |
| 40  | 91              | 12.00           | 2.72            | 10.46         | 348     | 280       |
| 50  | 122             | 17.85           | 4.09            | 14.50         | 258     | 208       |
| 60  | 151             | 23.89           | 5.51            | 18.30         | 207     | 167       |
|     |                 |                 |                 |               |         |           |           |
| 40  | 120             | 9.52            | 1.93            | 13.29         | 329     | 211       |
| 50  | 160             | 15.14           | 3.17            | 18.65         | 242     | 158       |
| 60  | 198             | 21.36           | 6.60            | 23.84         | 192     | 128       |

The $L_{\text{self}}$ values are higher for a square coil than for a circular one. This inductance increases as the number of turns increases. The $M_{\text{tr}}$ values are higher for a circular coil than for a square one for the periodic model and lower for the aperiodic model. This inductance also increases as the number of turns increases. The values of the compensating capacity are higher for a circular coil than for a square one. This capacity decreases as the number of turns increases.

Tables 5 and 6 compare the values of power transfer efficiency ($\eta$) at $f_{\text{max}} = 1$ MHz depending on the type of coils, the number of turns, and the distance between the transmitter and receiver. Whereas Tables 7 and 8 summarize the efficiency values ($\eta$) at the frequency $f_{\text{max}}/2 = 500$ kHz.

Regardless of the frequency, for the periodic model in each case, higher power transfer efficiency values were obtained for a circular coil than for a square one. For the aperiodic model and the small coil, higher power transfer efficiency values were obtained for a square coil than for a circular one. However, for a large coil, the values are higher for a circular coil than for a square one.

Based on the results presented in Tables 5 and 6, a reduction in efficiency can be observed in the aperiodic model for circular and square coils, at a maximum frequency of 1 MHz. The decrease in efficiency applies to the model with a large coil. The reason for the slight decrease in value is that as the number of turns increases, the resistance of the coil increases.
In order to compare the results obtained with both methods (analytical and numerical), the calculated absolute error is presented in Table 9. The results between the two proposed methods are consistent and do not exceed 0.8%.

To better present and interpret many of the results, the rest of the article has been divided into four sections:
- comparison of periodic and aperiodic systems for circular coils (Section 3.2);
- comparison of periodic and aperiodic systems for square coils (Section 3.3);
- comparison of periodic systems for both coils: circular and square (Section 3.4);
- comparison of aperiodic systems for both coils: circular and square (Section 3.5).

Table 5. The values of the power transfer efficiency at $f_{max} = 1$ MHz for a circular coil (periodic and aperiodic).

| n          | $\eta$ (%) at $f_{max}$           |
|------------|----------------------------------|
|            | Periodic (Circular Coil) | Aperiodic (Circular Coil) |
|            | $h = 0.5\ r$     | $h = r$   | $h = 0.5\ r$ | $h = r$   |
| Small coil |                        |                    |             |            |
| 15 ($r = 5$ mm)       | 22.45 | 1.52 | 55.25 | 14.19 |
| 20 ($r = 5$ mm)       | 33.95 | 2.69 | 65.08 | 19.78 |
| 25 ($r = 5$ mm)       | 39.98 | 3.46 | 68.87 | 22.40 |
| Large coil |                        |                    |             |            |
| 40 ($r = 20$ mm)      | 88.34 | 50.69 | 91.27 | 87.79 |
| 50 ($r = 20$ mm)      | 88.62 | 63.03 | 90.25 | 87.87 |
| 60 ($r = 20$ mm)      | 88.75 | 70.10 | 89.18 | 87.42 |

Table 6. The values of the power transfer efficiency at $f_{max} = 1$ MHz for a square coil (periodic and aperiodic).

| n          | $\eta$ (%) at $f_{max}$           |
|------------|----------------------------------|
|            | Periodic (Square Coil) | Aperiodic (Square Coil) |
|            | $h = 0.5\ r$     | $h = r$   | $h = 0.5\ r$ | $h = r$   |
| Small coil |                        |                    |             |            |
| 15 ($r = 5$ mm)       | 15.49 | 0.85 | 62.28 | 18.58 |
| 20 ($r = 5$ mm)       | 26.84 | 1.78 | 71.20 | 25.75 |
| 25 ($r = 5$ mm)       | 33.58 | 2.50 | 74.49 | 29.07 |
| Large coil |                        |                    |             |            |
| 40 ($r = 20$ mm)      | 83.03 | 29.28 | 89.56 | 86.28 |
| 50 ($r = 20$ mm)      | 84.92 | 45.70 | 88.05 | 86.03 |
| 60 ($r = 20$ mm)      | 84.93 | 57.27 | 86.69 | 85.27 |

Table 7. The values of the power transfer efficiency at 500 kHz for a circular coil (periodic and aperiodic).

| n          | $\eta$ (%) at $(f_{max}/2)$ |
|------------|-------------------------------|
|            | Periodic (Circular Coil) | Aperiodic (Circular Coil) |
|            | $h = 0.5\ r$     | $h = r$   | $h = 0.5\ r$ | $h = r$   |
| Small coil |                        |                    |             |            |
| 15 ($r = 5$ mm)       | 6.76  | 0.38  | 23.72  | 3.97   |
| 20 ($r = 5$ mm)       | 11.42 | 0.69  | 32.06  | 5.82   |
| 25 ($r = 5$ mm)       | 14.34 | 0.89  | 36.00  | 6.75   |
| Large coil |                        |                    |             |            |
| 40 ($r = 20$ mm)      | 78.91 | 21.73 | 89.07  | 77.19  |
| 50 ($r = 20$ mm)      | 83.09 | 33.11 | 88.79  | 80.67  |
| 60 ($r = 20$ mm)      | 84.50 | 42.55 | 88.13  | 82.02  |
Table 8. The values of the power transfer efficiency at 500 kHz for a square coil (periodic and aperiodic).

| \( n \) | Periodic (Square Coil) | Aperiodic (Square Coil) |
|---|---|---|
| | \( h = 0.5 \, r \) | \( h = r \) | \( h = 0.5 \, r \) | \( h = r \) |
| Small coil | | | | |
| 15 (\( r = 5 \, mm \)) | 4.29 | 0.21 | 29.46 | 5.41 |
| 20 (\( r = 5 \, mm \)) | 8.43 | 0.45 | 38.70 | 8.00 |
| 25 (\( r = 5 \, mm \)) | 11.27 | 0.64 | 42.89 | 9.33 |
| Large coil | | | | |
| 40 (\( r = 20 \, mm \)) | 67.21 | 9.69 | 88.05 | 76.67 |
| 50 (\( r = 20 \, mm \)) | 75.95 | 18.68 | 87.12 | 79.76 |
| 60 (\( r = 20 \, mm \)) | 79.34 | 28.32 | 86.04 | 80.72 |

Table 9. The values of the absolute error calculated for power transfer efficiency at all frequencies and cases.

| Number of Turns \((n)\) | \( \delta \) (%) |
|---|---|
| | Circular Coil | Square Coil |
| | | Periodic | Aperiodic | Periodic | Aperiodic |
| \( h = 0.5 \, r \) | \( h = r \) | \( h = 0.5 \, r \) | \( h = r \) | \( h = 0.5 \, r \) | \( h = r \) |
| 15 (\( r = 5 \, mm \)) | 0.05 | 0 | 0.36 | 0.11 | 0.01 | 0 | 0.05 | 0.02 |
| 20 (\( r = 5 \, mm \)) | 0.09 | 0.01 | 0.35 | 0.14 | 0.02 | 0 | 0.05 | 0.02 |
| 25 (\( r = 5 \, mm \)) | 0.13 | 0.01 | 0.32 | 0.14 | 0.02 | 0 | 0.03 | 0.02 |
| 40 (\( r = 20 \, mm \)) | 0.04 | 0.13 | 0.13 | 0.77 | 0.01 | 0.01 | 0.02 | 0.03 |
| 50 (\( r = 20 \, mm \)) | 0.01 | 0.08 | 0.17 | 0.45 | 0 | 0.02 | 0.02 | 0.03 |
| 60 (\( r = 20 \, mm \)) | 0.06 | 0.08 | 0.16 | 0.41 | 0.01 | 0.02 | 0.02 | 0.03 |

3.2. Comparison of Periodic and Aperiodic Systems for Circular Coils

In order to show the difference in the efficiency values for the selected exemplary one frequency \((f = 1 \, MHz)\), the modulus of the difference between the efficiency values for the periodic and aperiodic systems \((\Delta \eta)\) was calculated (Tables 10–13).

The results showed that by using a circular planar coil system, better efficiency can be obtained with the aperiodic model than with the periodic one (Table 10). Using the aperiodic system with a large circular coil \((n = 40)\) at a distance \(h = r = 20 \, mm\), the efficiency is 37% higher than for the periodic system under the same conditions.

Table 10. The difference between the efficiency of the system in the periodic model and the aperiodic model with the use of circular coils.

| \( n \) | \( \Delta \eta \) (%) at \( f_{\text{max}} \) |
|---|---|---|
| | \( h = 0.5 \, r \) | \( h = r \) |
| for small coil \((r = 5 \, mm)\) | | |
| 15 | 33% | 13% |
| 20 | 31% | 17% |
| 25 | 29% | 19% |
| for large coil \((r = 20 \, mm)\) | | |
| 40 | 3% | 37% |
| 50 | 2% | 25% |
| 60 | 0.4% | 17% |

3.2.1. Models with a Small Circular Coil \((r = 5 \, mm)\)

Figures 6–8 show the characteristics of transmitter power, receiver power and efficiency at the distance \(h = r/2 = 2.5 \, mm\) and \(h = r = 5 \, mm\) for periodic and aperiodic models with
small circular coils. The characteristics depend on the number of turns and the frequency. For the aperiodic model with a circular coil, with increasing frequency, the transmitter power $P_x$ decreases faster than for the periodic model (Figures 6a, 7a and 8a). The power $P_x$ increases with increasing frequency until the efficiency of the system reaches 50%, then the receiver power $P_o$ begins to decrease. The power $P_o$ for the aperiodic model is higher than for the periodic one (Figures 6b, 7b and 8b).

In each of the analysed cases, the efficiency of the system increases with the increase in the number of turns (Figure 6c, Figure 7c, and Figure 8c). Regardless of the distance between the coils, the highest efficiency of the system was obtained for the aperiodic model. As the frequency increases, the difference in efficiency between the aperiodic and periodic models increases. The highest difference occurs at the frequency $f = 1$ MHz, the number of turns $n = 15$ and reaches 33% (Figure 6c).

From the analytical and numerical analysis, with the same distance between the circular coils ($h = 2.5$ mm), both in the periodic and aperiodic system, the values of the
receiver power are practically equal at \( f = 1000 \text{ kHz} \) (Figure 7b). At \( h = 2.5 \text{ mm} \) for an aperiodic system, as the frequency increases, the receiver power increases until the efficiency of the system reaches 50\%, then the receiver power begins to decrease. On the other hand, for the periodic system, the receiver power increases in the entire range of the analysed frequencies, because the efficiency of the system has not yet reached 50\%.

It is worth noting that in Figure 8b the receiver power values for the same models, i.e., a system with circular coils (periodic and aperiodic) are almost equal, but at a lower frequency (approx. 900 kHz). On this basis, it can be concluded that in this model, the efficiency of the periodic system increases faster as the number of turns increases.

### 3.2.2. Models with a Large Circular Coil (\( r = 20 \text{ mm} \))

Figures 9–11 show the characteristics of: transmitter power, receiver power and efficiency at the distance \( h = r/2 = 10 \text{ mm} \) and \( h = r = 20 \text{ mm} \) for periodic and aperiodic models with large circular coils.
The transmitter power $P_z$ rapidly decreases with increasing frequency, except for the periodic system with a higher distance between the coils ($h = 20$ mm), in which the decrease is smaller (Figures 9a, 10a and 11a). All characteristics of the power $P_o$ are very similar to each other for two models: periodic at $h = 10$ mm and aperiodic at $h = 20$ mm (Figures 9b, 10b and 11b). The highest values of the receiver power $P_o$, at the frequency $f = 1$ MHz, occur for periodic models at $h = 20$ mm.

As for the receiver power, the efficiency values are comparable for the two models, i.e., periodic at $h = 10$ mm and aperiodic at $h = 20$ mm (Figures 9b, 10c and 11c). At the frequency $f = 1$ MHz, the efficiency of the analysed models reaches about 90%, with the exception of the periodic model at $h = 20$ mm, where the efficiency reaches only 70% at $n = 60$ (Figure 11c).

### 3.3. Comparison of Periodic and Aperiodic Systems for Square Coils

Similarly to models with a circular coil, the efficiency of the system is higher for the aperiodic model with a square coil than for the periodic one (Table 11). For a small coil (aperiodic model) at $h = 2.5$ mm, about 45% higher system efficiency can be obtained than for the periodic model. For a large coil ($n = 40$) at $h = 20$ mm, the efficiency is as much as 57% higher than for the periodic model under the same conditions.

| $n$  | $\Delta \eta$ (%) at $f_{\text{max}}$ |
|------|-----------------------------------|
|      | $h = 0.5 r$                       | $h = r$                       |
| 15   | 47%                               | 18%                          |
| 20   | 44%                               | 24%                          |
| 25   | 41%                               | 27%                          |
| 40   | 7%                                | 57%                          |
| 50   | 3%                                | 40%                          |
| 60   | 2%                                | 28%                          |

**Table 11.** The difference between the efficiency of the system in the periodic model and the aperiodic model with the use of square coils.

#### 3.3.1. Models with a Small Square Coil ($r = 5$ mm)

Figures 12–14 show the characteristics of transmitter power, receiver power and efficiency at the distance $h = r/2 = 2.5$ mm and $h = r = 5$ mm for periodic and aperiodic models with small square coils. All characteristics depend on the number of turns and frequency. The transmitter power $P_z$, in the entire range of the analysed frequencies, has almost identical values for the two models: periodic at $h = 2.5$ mm and aperiodic at $h = 5$ mm (Figure 12a, Figure 13a, and Figure 14a). For models with a circular coil, the receiver power $P_o$ increases with increasing frequency. However, when the efficiency of the system reaches 50%, the power $P_o$ decreases (Figure 12b, Figure 13b, and Figure 14b). The highest value of the receiver power is 0.52 W for the aperiodic model with $n = 15$ and $h = 2.5$ mm (Figure 12b).
Regardless of the frequency, the efficiency of the system is comparable for two models: periodic at $h = 2.5$ mm and aperiodic at $h = 5$ mm (Figure 12c, Figure 13c, and Figure 14c). Regardless of the number of turns, the highest efficiency of the system is for the aperiodic model at $h = 2.5$ mm, e.g., for the number of turns $n = 15$, at two distances ($h = 2.5$ mm and $h = 5$ mm): (a) transmitter power, (b) receiver power, (c) power transfer efficiency.

Figure 12. Results for square coils with $r = 5$ mm and the number of turns $n = 15$ at two distances ($h = 2.5$ mm and $h = 5$ mm): (a) transmitter power, (b) receiver power, (c) power transfer efficiency.

Figure 13. Results for square coils with $r = 5$ mm and the number of turns $n = 20$ at two distances ($h = 2.5$ mm and $h = 5$ mm): (a) transmitter power, (b) receiver power, (c) power transfer efficiency.

Figure 14. Results for square coils with $r = 5$ mm and the number of turns $n = 25$ at two distances ($h = 2.5$ mm and $h = 5$ mm): (a) transmitter power, (b) receiver power, (c) power transfer efficiency.

3.3.2. Models with a Large Square Coil ($r = 20$ mm)

Characteristics: transmitter power, receiver power, and efficiency at a distance $h = r/2 = 10$ mm and $h = r = 20$ mm for periodic and aperiodic models with large square coils are shown in Figures 15–17.
Figure 15. Results for square coils with \( r = 20 \) mm and the number of turns \( n = 40 \) at two distances \((h = 10 \text{ mm and } h = 20 \text{ mm})\): (a) transmitter power, (b) receiver power, (c) power transfer efficiency.

Figure 16. Results for square coils with \( r = 20 \) mm and the number of turns \( n = 50 \) at two distances \((h = 10 \text{ mm and } h = 20 \text{ mm})\): (a) transmitter power, (b) receiver power, (c) power transfer efficiency.

Figure 17. Results for square coils with \( r = 20 \) mm and the number of turns \( n = 60 \) at two distances \((h = 10 \text{ mm and } h = 20 \text{ mm})\): (a) transmitter power, (b) receiver power, (c) power transfer efficiency.

As the frequency increases, the transmitter power \( P_t \) rapidly decreases for three cases: periodic at \( h = 10 \) mm, aperiodic at \( h = 10 \) mm and \( h = 20 \) mm (Figure 15a, Figure 16a, and Figure 17a). Characteristics of the receiver power \( P_r \) have similar a shape for the two models: periodic at \( h = 10 \) mm, and aperiodic at \( h = 20 \) mm (Figure 15b, Figure 16b, and Figure 17b). Additionally, for the model with the number of turns \( n = 60 \), the \( P_r \) values are almost identical (Figure 17b). In all cases, at the frequency \( f = 1 \text{ MHz} \), the highest receiver power occurs for the periodic model at \( h = 20 \) mm.

At the frequency \( f = 1 \text{ MHz} \), regardless of the number of turns, the efficiency of the system differs from 83% to 90% for the models: periodic at \( h = 10 \) mm, aperiodic at \( h = 10 \) mm, and \( h = 20 \) mm (Figure 15c, Figure 16c, and Figure 17c). However, for the periodic model at \( h = 20 \) mm, the efficiency of the system does not exceed 60% and is much lower in the entire frequency range.
3.4. Comparison of Periodic Systems for Both Coils: Circular and Square

The results showed that by using a periodic arranged coil system, better efficiency can be obtained with circular coils than with square ones (Table 12). The highest differences concern models with a large coil at a distance \( h = r = 20 \text{ mm} \) (e.g., 13–21%).

Table 12. The difference between the efficiency of the system in the periodic models with circular or square coils.

| \( n \) | \( \Delta \eta \) (%) at \( f_{\text{max}} \) |
|-------|-----------------|
|       | \( h = 0.5 \text{ } r \) | \( h = r \) |
| 15    | 7%              | 0.7% |
| 20    | 7%              | 0.9% |
| 25    | 6%              | 1%   |
| 40    | 5%              | 21%  |
| 50    | 4%              | 17%  |
| 60    | 4%              | 13%  |

3.4.1. Models with a Small Coil \( (r = 5 \text{ mm}) \)

Figures 18–20 show the characteristics of transmitter power, receiver power, and efficiency at two distances \( (h = 2.5 \text{ mm and } 5 \text{ mm}) \) for periodic models with small circular or square coils. All characteristics depend on the number of turns and frequency.

**Figure 18.** Results for the periodic system with circular or square coils with \( r = 5 \text{ mm} \) and the number of turns \( n = 15 \) at two distances \( (h = 2.5 \text{ mm and } 5 \text{ mm}) \): (a) transmitter power, (b) receiver power, (c) power transfer efficiency.

**Figure 19.** Results for the periodic system with circular or square coils with \( r = 5 \text{ mm} \) and the number of turns \( n = 20 \) at two distances \( (h = 2.5 \text{ mm and } 5 \text{ mm}) \): (a) transmitter power, (b) receiver power, (c) power transfer efficiency.
Figure 20. Results for the periodic system with circular or square coils with \( r = 5 \) mm and the number of turns \( n = 25 \) at two distances \((h = 2.5 \) mm and \( h = 5 \) mm): (a) transmitter power, (b) receiver power, (c) power transfer efficiency.

Comparing periodic systems with a circular or square coil, it can be noticed that the values of the transmitter power \( P_t \) rapidly decrease at a small distance between the coils, i.e., \( h = 2.5 \) mm (Figure 18a, Figure 19a, and Figure 20a). On the other hand, with a higher distance between the coils \((h = 5 \) mm), the \( P_t \) values slightly decrease with increasing frequency (by a maximum of 4%).

As the frequency increases, regardless of the number of turns, the power of the receiver \( P_r \) increases (Figure 18b, Figure 19b, and Figure 20b). Regardless of the distance between the coils, the higher values of power \( P_r \) are for systems with a circular coil. The highest difference in receiver power occurs at \( f = 1 \) MHz. The highest efficiency values are for models with a circular coil (Figure 18c, Figure 19c, and Figure 20c). The maximum efficiency of the system is 40% with a circular coil and 34% with a square coil, for \( n = 25 \), \( h = 2.5 \) mm, and \( f = 1 \) MHz (Figure 20c).

3.4.2. Models with a Large Coil \((r = 20 \) mm)

Characteristics: transmitter and receiver power and efficiency at the distance \( h = 10 \) mm and \( h = 20 \) mm for periodic models with large circular or square coils are shown in Figures 21–23.

Figure 21. Results for the periodic system with circular or square coils with \( r = 20 \) mm and the number of turns \( n = 40 \) at two distances \((h = 10 \) mm and \( h = 20 \) mm): (a) transmitter power, (b) receiver power, (c) power transfer efficiency.
Figure 22. Results for the periodic system with circular or square coils with \(r = 20 \text{ mm}\) and the number of turns \(n = 50\) at two distances \((h = 10 \text{ mm}\) and \(h = 20 \text{ mm}\)): (a) transmitter power, (b) receiver power, (c) power transfer efficiency.

Figure 23. Results for the periodic system with circular or square coils with \(r = 20 \text{ mm}\) and the number of turns \(n = 60\) at two distances \((h = 10 \text{ mm}\) and \(h = 20 \text{ mm}\)): (a) transmitter power, (b) receiver power, (c) power transfer efficiency.

The transmitter power \(P_z\), at a distance \(h = 20 \text{ mm}\), has higher values for systems with a circular coil, in the frequency range from 100 kHz to 650 kHz. However, for higher frequencies, the \(P_z\) values are higher for models with a square coil (Figure 21a, Figure 22a, and Figure 23a). In the whole frequency range, at \(h = 20 \text{ mm}\), the higher power \(P_v\) values are for circular coil systems. On the other hand, for smaller distances between the coils \((h = 10 \text{ mm})\), the \(P_v\) values are higher for systems with a circular coil only at lower frequencies, i.e., from 100 kHz to 350 kHz (Figure 21b, Figure 22b, and Figure 23b).

Regardless of the number of turns and the distance between the coils, higher efficiency values of the system are for models with a circular coil (Figures 21c, 22c and 23c). For example, for \(n = 60\) at \(f = 1 \text{ MHz}\), the maximum efficiency of the system is 89% at \(h = 10 \text{ mm}\) and 70% at \(h = 20 \text{ mm}\) (Figure 23c). The efficiency of the system with a square coil is lower by up to 13%.

3.5. Comparison of Aperiodic Systems for Circular and Square Coils

The results have shown that by using the aperiodic arranged coil system, better efficiency can be obtained with a small coil \((r = 5 \text{ mm})\) with square coils than with circular ones. On the other hand, for a large coil \((r = 20 \text{ mm})\), the efficiency of the system for both types of coils had similar values (Table 13).
Table 13. The difference between the efficiency of the system in the aperiodic models with circular or square coils.

| $n$ | $\Delta \eta$ (%) at $f_{max}$ |
|-----|------------------------------|
|     | $h = 0.5 \, r$ | $h = r$ |
|     | for small coil ($r = 5 \, mm$) | for large coil ($r = 20 \, mm$) |
| 15  | 7%   | 4%   |
| 20  | 6%   | 6%   |
| 25  | 6%   | 7%   |
| 40  | 2%   | 2%   |
| 50  | 2%   | 2%   |
| 60  | 3%   | 2%   |

3.5.1. Models with a Small Coil ($r = 5 \, mm$)

Characteristics: transmitter and receiver power and efficiency at two distances ($h = 5 \, mm$ and $10 \, mm$) for aperiodic models with small circular or square coils are shown in Figures 24–26. When analyzing aperiodic models, regardless of the number of turns and the distance between the coils, higher values of the transmitter power $P_z$ are obtained for models with a circular coil (Figure 24a, Figure 25a, and Figure 26a).

![Figure 24](image1.png)

**Figure 24.** Results for the aperiodic system with circular or square coils with $r = 5 \, mm$ and the number of turns $n = 15$ at two distances ($h = 2.5 \, mm$ and $h = 5 \, mm$): (a) transmitter power, (b) receiver power, (c) power transfer efficiency.

![Figure 25](image2.png)

**Figure 25.** Results for the aperiodic system with circular or square coils with $r = 5 \, mm$ and the number of turns $n = 20$ at two distances ($h = 2.5 \, mm$ and $h = 5 \, mm$): (a) transmitter power, (b) receiver power, (c) power transfer efficiency.
Figure 26. Results for the aperiodic system with circular or square coils with \( r = 5 \) mm and the number of turns \( n = 25 \) at two distances \( (h = 2.5 \) mm and \( h = 5 \) mm): (a) transmitter power, (b) receiver power, (c) power transfer efficiency.

With a larger distance between the coils \( (h = 5 \) mm), the values of the receiver power \( P_o \) for circular and square coils are similar in the entire frequency range (Figure 24b, Figure 25b, and Figure 26b). However, for the smaller distance between the coils \( (h = 2.5 \) mm), the power \( P_o \) is higher for the circular coil. As the frequency increases, these differences become larger and larger.

In contrast to periodic systems, regardless of the number of turns and frequency, the efficiency of the system is higher for models with a square coil. The highest difference in efficiency between the model with a square coil and circular coil is 7%, regardless of the distance between the coils (Figure 24c, Figure 25c, and Figure 26c).

3.5.2. Models with a Large Coil \( (r = 20 \) mm)

Figures 27–29 show the characteristics of power \( (P_z, P_o) \) and efficiency \( \eta \) at two distances \( (h = 10 \) mm, \( 20 \) mm) for aperiodic models with large circular or square coils. All characteristics depend on the number of turns and frequency. The higher transmitter power \( P_z \) is in the model with a circular coil (Figure 27a, Figure 28a, and Figure 29a). As the frequency increases, the power \( P_z \) decreases. Receiver power values \( P_o \) are higher for circular coil models. At \( h = 10 \) mm, the power \( P_o \) decreases with increasing frequency, regardless of the type of coil. At a distance \( h = 20 \) mm, the power \( P_o \) increases until the efficiency of the system reaches 50%, and then it begins to decrease (Figure 27b, Figure 28b, and Figure 29b).

Figure 27. Results for the aperiodic system with circular or square coils with \( r = 20 \) mm and the number of turns \( n = 40 \) at two distances \( (h = 10 \) mm and \( h = 20 \) mm): (a) transmitter power, (b) receiver power, (c) power transfer efficiency.
The efficiency of the system is higher for the model with a square coil, but only at low frequencies (Figure 27c, Figure 28c, and Figure 29c). However, at higher frequencies, the efficiency of the system is higher for a circular coil. At $f = 1 \text{ MHz}$, regardless of the number of turns, the distance between the coils, the type of coil, and the efficiency values are similar and reach about 85–90%.

4. Conclusions

Periodic and aperiodic wireless power transfer systems were studied using analytical and numerical analysis. The authors described the methodology of creating models of the WPT system in both proposed methods. Both solutions reduce the size and complexity of numerical and analytical models.

The proposed equivalent circuit model of the WPT cell is an alternative for complex numerical analysis or experimental research of physical prototypes. The proposed analytical model lets to do fast preliminary calculations of WPT cells with different types and geometries of coils. The proposed numerical and analytical models let to evaluate the influence of the coil structure on power transfer efficiency. By adjusting the geometric parameters, high efficiency can be achieved.

The proposed analytical model can replace the three-dimensional field model when an analysis of periodic and aperiodic systems with many WPT cells is considered. The difference in value between both methods is only 0.8%. Based on the comparison of the results obtained with both methods, it can be concluded that the adopted methodology and formulations are correct.

For a small coil, the maximum efficiency of the system occurs for the aperiodic model and amounts to approx. 75%. The fourfold increase in the radius of the coil increases the efficiency to about 91%. Doubling the distance between the coils reduces the efficiency of
a small coil system up to 10 times. However, for a large coil, the efficiency of the system drops up to three times.

Apart from the geometry of the system, the change in load also affects the efficiency of the WPT system. As the load resistance increases, the efficiency of the system increases until it reaches its maximum value. Whereas a further increase in the load resistance reduces the efficiency of the system. System efficiency is highest with resistive load. At a load other than resistive, the efficiency of the system will be lower due to the increase in impedance.

The results showed that by using a planar coils system, better efficiency can be obtained for aperiodic rather than periodic models:

for circular coils:

1. small coil ($r = 5$ mm):
   - $n = 15$: $33\% (h = 0.5\ r)$, $13\% (h = r)$;
   - $n = 20$: $31\% (h = 0.5\ r)$, $17\% (h = r)$;
   - $n = 25$: $29\% (h = 0.5\ r)$, $19\% (h = r)$;

2. large coil ($r = 20$ mm):
   - $n = 40$: $3\% (h = 0.5\ r)$, $37\% (h = r)$;
   - $n = 50$: $2\% (h = 0.5\ r)$, $25\% (h = r)$;
   - $n = 60$: $0.4\% (h = 0.5\ r)$, $17\% (h = r)$;

and for square coils:

3. small coil ($r = 5$ mm):
   - $n = 15$: $47\% (h = 0.5\ r)$, $18\% (h = r)$;
   - $n = 20$: $44\% (h = 0.5\ r)$, $24\% (h = r)$;
   - $n = 25$: $41\% (h = 0.5\ r)$, $27\% (h = r)$;

4. large coil ($r = 20$ mm):
   - $n = 40$: $7\% (h = 0.5\ r)$, $57\% (h = r)$;
   - $n = 50$: $3\% (h = 0.5\ r)$, $40\% (h = r)$;
   - $n = 60$: $2\% (h = 0.5\ r)$, $28\% (h = r)$.

Thanks to a detailed analysis of many variants, it is possible to economically select the type of coils, and the number of turns etc., in order to obtain high efficiency of the system. The authors plan to undertake further research on WPT focusing on other coil shapes and capacitive loads.

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