Bending and breathing modes of the Galactic disc

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ABSTRACT
We explore the hypothesis that a passing satellite or dark matter subhalo has excited coherent oscillations of the Milky Way’s stellar disc in the direction perpendicular to the Galactic mid-plane. This work is motivated by recent observations of spatially dependent bulk vertical motions within ~2 kpc of the Sun. A satellite can transfer a fraction of its orbital energy to the disc stars as it plunges through the Galactic mid-plane, thereby heating and thickening the disc. Bulk motions arise during the early stages of such an event when the disc is still in an unrelaxed state. We present simple toy-model calculations and simulations of disc–satellite interactions, which show that the response of the disc depends on the relative velocity of the satellite. When the component of the satellite’s velocity perpendicular to the disc is small compared with that of the stars, the perturbation is pre-dominantly a bending mode. Conversely, breathing and higher order modes are excited when the vertical velocity of the satellite is larger than that of the stars. We argue that the compression and rarefaction motions seen in three different surveys are in fact breathing-mode perturbations of the Galactic disc.

Key words: Galaxy: kinematics and dynamics – solar neighbourhood – Galaxy: structure.

1 INTRODUCTION
Recently, three independent surveys of stellar kinematics within ~2 kpc of the Sun detected spatially dependent bulk motions in the direction perpendicular to the Galactic plane (Widrow et al. 2012; Carlin et al. 2013; Williams et al. 2013). Widrow et al. (2012) found that the bulk motions of stars, when plotted as a function of position z relative to the Galactic mid-plane, have characteristics of a breathing-mode perturbation with a velocity gradient of ~3–5 km s⁻¹ kpc⁻¹. This result was based on a sample of 11k main-sequence stars from the Sloan Extension for Galactic Understanding and Exploration (SEGUE) survey, which focused on intermediate latitudes above and below the Galactic mid-plane and Galactic longitudes in the range 100° < l < 180° (Yanny & Gardner 2009). Williams et al. (2013) used a sample of 72k red-clump stars from the Radial Velocity Experiment (RAVE) survey (Steinmetz et al. 2006) to map out bulk motions as a function of Galactocentric radius and z and found evidence for compressional motion outside the solar circle and rarefaction inside with peak vertical bulk velocities of ±15 km s⁻¹. Carlin et al. (2013) found similar features in their analysis of 400k F-type stars with proper motions from the PPMXL catalogue (Roederer, Demleitner & Schilbach 2010) and spectroscopic radial velocities from the LAMOST/LEGGUE survey (Cui et al. 2012; Zhao et al. 2012). As stressed by Carlin et al. (2013), the three surveys look in different parts of the extended solar neighbourhood and consider different spatial projections of the data.

Widrow et al. (2012) also found a north–south asymmetry in the number counts when plotted against z. The number count asymmetry was confirmed by Yanny & Gardner (2013) who carried out a careful analysis of the uncertainties and potential systematic effects. The number counts show a 10 per cent (north – south)/((north + south) deficit at |z| ≈ 400 pc and an excess of about the same magnitude at |z| ≈ 800 pc.

It is possible that the north–south asymmetries in number counts and bulk vertical motions are the result of stellar debris from a tidally disrupted satellite galaxy that is mixing in with the disc stars. Widrow et al. (2012) considered an alternative hypothesis in which the north–south asymmetries arise from coherent oscillations of the disc itself, which were excited by a passing satellite or dark matter subhalo. Eventually, the oscillations die away due to phase mixing and Landau damping. The disc settles into a new equilibrium state, albeit one with a higher velocity dispersion. Thus, the bulk motions seen in the data may indicate an early phase of a disc heating event. The aim of this paper is to explore this hypothesis in more detail through toy-model calculations andnumerical simulations.

A similar question was posed by Minchev et al. (2009) in the context of the stellar velocity distribution in the solar neighbourhood. Detailed analyses of Hipparcos data (Creele, Creze & Bienaymé 1998; Dehnen 1998; Creele, Crézé & Bienaymé 1999; Nordström et al. 2004) have revealed rich substructure in the local velocity distribution. There are classical moving groups, which are thought to be the stellar streams from dissolved star clusters (see Eggen 1996 and references therein). Velocity-space features may arise from

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dynamical effects of the bar (Dehnen 2000) or spiral structure (De Simone, Wu & Tremaine 2004; Quillen & Minchev 2005; Chakrabarty 2007) or they may be due to streams of stars that were tidally stripped from accreted satellite galaxies (Navarro, Helmi & Freeman 2004; Helmi et al. 2006). Minchev et al. (2009) consider a different scenario in which the ‘energy kick’ from a passing satellite leaves ripples in the (disc plane) velocity distribution of stars. Their conjecture is that velocity-space substructure in the solar neighbourhood is a manifestation of these ripples.

Satellites, dark matter subhaloes, and globular clusters, for that matter, have long been recognized as possible culprits of disc heating and thickening. In general, a massive object that passes through the disc will transfer a fraction of its orbital energy to the disc stars (Lacey & Ostriker 1985; Toth & Ostriker 1992; Sellwood, Nelson & Tremaine 1998). Satellite interactions can also cause the disc to spread out radially and develop warps and flares (Quinn & Goodman 1986; Quinn, Hernquist & Fullagar 1993; Walker, Mihos & Hernquist 1996; Velazquez & White 1999).

Satellite encounters can excite various modes in the disc such as bending modes and breathing modes (Toomre 1966; Araki 1985; Mathur 1990; Weinberg 1991). It is the latter that corresponds most closely to the velocity perturbations seen in the SEGUE, RAVE, and LAMOST surveys. We will show that a bending-mode perturbation arises when the satellite’s vertical velocity is less than that of the disc stars while breathing and higher order modes are excited when the vertical velocity of the satellite exceeds that of the stars.

Satellites and subhaloes can also excite spiral structure and bars in stellar discs. (See Sellwood 2013 for a recent review of the stellar dynamics of disc galaxies.) The seminal work of Toomre & Toomre (1972) showed that the tidal interaction between a stellar disc and a companion galaxy of comparable mass can generate grand design spiral structure similar to what is seen in M51. Alternatively, multimassed and flocculent spiral structure can arise from the continual interactions between the disc and a system of satellite galaxies and dark matter subhaloes. Cosmological simulations of structure formation in a Λ cold dark matter (ΛCDM) universe suggest that the haloes of Milky Way-sized galaxies harbour a wealth of substructure in the form of subhaloes (Klypin et al. 1999; Moore et al. 1999; Gao et al. 2004). These results motivated Gauthier, Dubinski & Widrow (2006) and Dubinski et al. (2008) to explore satellite–disc interactions for an M31-like galaxy. For their particular M31 model, when the halo is smooth, the disc remains stable against bar formation for 10 Gyr and relatively weak spiral structure develops, presumably seeded by the shot noise of the N-body realization. Conversely, when 10 per cent of the halo mass initially resides in compact subhaloes, the disc develops prominent spiral features and forms a strong bar. Similar results were found in a series of simulations by Kazantzidis et al. (2008).

More recently, Purcell et al. (2011) considered a model Milky Way disc that was perturbed by a single satellite galaxy. The prototype for their perturber was the Sagittarius dwarf spheroidal galaxy (Ibata, Gilmore & Irwin 1994; Ibata et al. 1997), which is believed to have survived several orbits about the Galaxy. In the Purcell et al. (2011) simulations, spiral structure emerges that is similar to the spiral structure observed in the Milky Way. Gómez et al. (2013) reanalysed these simulations and found that vertical perturbations in the number density of disc particles at roughly the Sun’s position from the Galaxy’s centre were also generated and qualitatively similar to those seen in Widrow et al. (2012) and Yanny & Gardner (2013).

It is not at all surprising that a Sagittarius-like dwarf produces vertical oscillations similar to what is found in the data. The solar neighbourhood is characterized by a circular speed about the Galactic centre of ≈220–230 km s⁻¹ (see, e.g., Bovy et al. 2012b, and references therein) and a stellar surface density of ≈50 M⊙ pc⁻² (see, e.g., Holmberg & Flynn 2004). Stars in the disc have vertical velocities in the range of 10–40 km s⁻¹ (Robin et al. 2003; Bovy et al. 2012b). A satellite with a comparable surface density to that of the solar neighbourhood, with an orbit that is matched to the local standard of rest (LSR), and with a vertical velocity through the mid-plane in resonance with the vertical motions of disc stars will produce the strongest perturbations.

In Section 2, we review earlier discussions of disc heating and thickening and then present a simple toy-model calculation for the excitation of bending and breathing modes. In Section 3, we present results from one-dimensional N-body simulations that support the toy-model calculation and also illustrate the potentially long-lived nature of the oscillations. We provide preliminary results from fully self-consistent 3D N-body simulations of satellite–disc encounters in Section 4. Finally, we summarize our results and give concluding remarks in Section 5.

2 BENDING AND BREATHING MODES

This section focuses on the physics of bending- and breathing-mode perturbations of a stellar disc. We begin with a heuristic discussion that makes contact with the previous work of disc heating and thickening.

2.1 Free-particle approximation

In an early paper on disc heating, Toth & Ostriker (1992) calculated the transfer of energy to a stellar disc from a passing satellite by treating the disc stars as free particles that are scattered in the Keplerian potential of the satellite. The satellite loses energy via dynamical friction (Chandrasekhar 1943) and it is this energy that heats the disc.

Consider a satellite of mass $M_s$ that passes through the disc with speed $v_s$ at an angle $\theta_v$ relative to the disc normal as measured in the LSR. For illustrative purposes, we focus on stars initially in the plane that contains the satellite’s orbit and the disc normal. Following Toth & Ostriker (1992), we ignore the random motion of the stars with respect to the LSR. The impact parameter for a star–satellite scattering event is $b = v_s \cos \theta_v + z \sin \theta_v$, where $(x, z)$ is the position of the star relative to the point at which the satellite passes through the Galactic mid-plane. The change in the vertical component of the star’s velocity is then

$$\Delta v_z(x, z) = \frac{v_s}{v_b} \left[ 1 + \left( \frac{b^2}{b_{\infty}^2} \right)^{-1} \left( \cos \theta_v + \frac{b}{b_{\infty}} \sin \theta_v \right) \right], \quad (1)$$

where $b_{\infty} \equiv GM_s/v_b^2$ is the impact parameter that leads to scattering by 90°. Note that

$$b_{\infty} = 4.3 \text{kpc} \left( \frac{M_s}{10^{10} M_\odot} \right) \left( \frac{v_s}{100 \text{ km s}^{-1}} \right)^{-2}. \quad (2)$$

As a measure of bending- and breathing-mode perturbations, we define

$$\Delta v_{\text{bend}} \equiv \frac{1}{2} (\Delta v_z(x, h) + \Delta v_z(x, -h)) \quad (3)$$

and

$$\Delta v_{\text{breathe}} \equiv \Delta v_z(x, h) - \Delta v_z(x, -h). \quad (4)$$
In the small and large impact parameter limits, we have
\[
\Delta v_{\text{c, bend}} = \begin{cases} 2v_s \cos \theta_s & b \ll b_{90} \\ 2v_s (b_{90}/x) \tan \theta_s & b \gg b_{90} ; b \gg h \end{cases}
\]
and
\[
\Delta v_{\text{c, breathe}} = \begin{cases} 4v_s (h/b_{90}) \sin^2 \theta_s & b \ll b_{90} \\ -4v_s (h/b_{90}/x^2) \tan^2 \theta_s & b \gg b_{90} ; b \gg h. \end{cases}
\]
Thus, both breathing and bending modes are excited by a passing satellite.

### 2.2 Resonant interaction

The previous calculations ignore the epicyclic motions of stars in the vertical direction. Stars near the Galactic mid-plane, where the gravitational potential \( \psi \) is approximately quadratic in the vertical direction, oscillate about the mid-plane with frequency
\[
\kappa_z \equiv \left( \frac{\partial^2 \psi}{\partial z^2} \right)^{1/2}_{\mid z=0},
\]
Stars whose orbits take them further from the mid-plane oscillate with a frequency \( v < \kappa_z \). Sellwood et al. (1998) calculated the change in vertical energy of a star due to the tidal field from the passing satellite in the impulse approximation (see also Spitzer 1958). They averaged over satellite directions and found
\[
\Delta E_z \approx \frac{4}{3} \left( \frac{G M_s}{b \kappa_z^2} \right)^2 \frac{L(\beta)}{v_s^3} \rho_b (\beta) \equiv \frac{E_i}{\Delta \kappa_z},
\]
where \( \Delta E_z = \Delta v_z^2/2 \), \( E_i \equiv \hbar^2 \kappa_z^2 \) is the characteristic vertical energy for stars in the disc, \( \beta \equiv 2k_s b/v_s \), and the function \( L(\beta) \) is unity for \( \beta \to 0 \) and is exponentially small for \( \beta \to \infty \).

By focusing on the tidal field, Sellwood et al. (1998) pick out the breathing and higher order modes. When the satellite velocity is high, \( \Delta E_z \propto v^{-2} \) as in the free-particle approximation (equation 6 with \( b \gg b_{90} \)). On the other hand, if the time-scale for a satellite to pass through the disc is long as compared to \( v^{-1} \), then the interaction between the satellite and star will be adiabatic and the energy transfer will be exponentially small. The energy transfer peaks when \( v_s \approx \hbar \kappa_z \) (see fig. 1 of Sellwood et al. 1998), that is, when the satellite is in resonance with the stellar orbit.

Note that the above equations ignore the finite size of the satellite in calculating the stellar orbits. The perturbing force due to a real satellite will be softened inside a characteristic scale radius \( a_s \). In general, \( a_s = \eta G M_s / \sigma_z^2 \), where \( \sigma_z \) is the vertical velocity dispersion of the satellite and \( \eta \) is a constant of order unity. Then, \( a/b_{90} = \eta v_z^0 / \sigma_z^2 \). With a softened perturber, \( \Delta v_{\text{c, bend}} \to 0 \) for \( b \to 0 \) whereas for a point mass perturber, \( \Delta v_{\text{c, bend}} \) is non-zero and constant for \( b \to 0 \), as in equation (5).

### 2.3 Energy transfer to disc stars

The change in vertical energy of a star due to a perturbing vertical force \( F_z \) is
\[
\Delta E_z = \int_{-\infty}^{\infty} \mathrm{d}t' F_z(z, t') v_z(t').
\]
In both Toth & Ostriker (1992) and Sellwood et al. (1998), the change in vertical energy of a star is approximated as \( \Delta E \approx \frac{1}{2} \Delta v_z^2 \). In other words, they assume that \( v_z(t) \) in equation (9) is generated entirely by \( F_z \). The function \( L(\beta) \) introduced by Sellwood et al. (1998) is a crude way of accounting for the orbital motion. If the time-scale for the satellite to pass through the disc is long as compared to the orbital period, then the integral on the right-hand side of equation (9) approaches zero.

Our interest here is in the details of the vertical perturbations that are excited by a passing satellite: What modes are excited by a passing satellite and what is their subsequent evolution? In the sections that follow, we address these questions numerically. Here, we discuss the excitation of vertical perturbations under the assumption that the perturbations are small. We can then approximate equation (9) by substituting the unperturbed orbit for \( v_z^0 \) for \( v_z \) inside the integral. This prescription resembles the Born approximation that is used to calculate scattering amplitudes in quantum mechanics.

The isothermal plane (Spitzer 1942; Camm 1950) provides a simple equilibrium model for the vertical phase space structure of a stellar disc. In this model, the distribution function (DF) is
\[
f(z, v_z) = \frac{\rho_0}{(2\pi \sigma_z^2)^{1/2}} e^{-E_z/\sigma_z^2}.
\]
where \( E_z = v_z^2/2 + \psi(z) \) is the vertical energy and \( \sigma_z \) is the velocity dispersion in the vertical direction. The density and potential are given by
\[
\rho(z) = \rho_0 \text{sech}^2 (z/z_0)
\]
and
\[
\psi(z) = 2\sigma_z^2 \ln \cosh (z/z_0),
\]
where \( \rho_0 \) is the density at the mid-plane and \( z_0 = (\sigma_z^2/2\pi G \rho_0)^{1/2} \). Stars with \( E_z \ll \sigma_z^2 \) execute simple harmonic motion about the mid-plane with a maximum excursion of \( z_{\max} \approx z_0 (E_z/\sigma_z^2)^{1/2} \) and a period of \( T \approx 2\pi/\kappa_z \approx 21/\pi \sigma_z/\sigma_z \) that is approximately independent of \( E_z \). On the other hand, for stars with \( E_z \gg \sigma_z^2 \), we have \( z_{\max} \approx z_0 (E_z/2\sigma_z^2) \) and \( T \approx T(E_z) = (z_0/\sigma_z)(8E_z/\sigma_z^2)^{1/2} \). In what follows, we set \( \sigma_z \) and \( G \) to unity and \( z_0 = 21/\pi \) so that \( \rho_0 = 1/4\pi \) and \( \kappa_z = 1 \).

It is useful to introduce the orbital phase angle \( \theta \), which is related to time by the expression \( d\theta = 2\pi \mathrm{d}t/T(E_z) \). In Fig. 1, we follow the vertical motions of two stars that have the same \( E_z \), but whose phase angles differ by \( \pi \). A satellite with speed greater than the maximum speed of the stars passes through the disc at \( t = 0 \). The upper two panels show the vertical position and velocity, respectively. The third panel shows the tidal force of the satellite on the star, that is, the force of the satellite on the star minus the force of the satellite on a star at \( z = 0 \). Note that we have softened the force law with \( a_s = 1 \). The fourth panel shows the change in vertical energy of the star as
\[
\delta E_z = \int_{-\infty}^{\infty} \mathrm{d}t' \left( \frac{F_z(z, t') - F_z(0, t')} {v_z(t')} \right) v_z(t').
\]
Evidently, both stars gain energy. Likewise, two stars whose phase angles differ from these two stars by \( \pm \pi/2 \) will lose energy. The pre-dominant perturbation in this case is a breathing mode.

Fig. 2 follows two stars that interact with a satellite whose speed is less than their maximum speed. Once again, the two stars differ in phase angle by \( \pi \). However, in this case, one star gains energy while the other loses energy. The perturbation in this case is pre-dominantly a bending mode. To further study the nature of the perturbation, we consider the Fourier transform of \( \delta E_z \) as
\[
\delta E_z(E_z, \theta) = \sum_m e^{2m\pi} \delta E_m(E_z)
\]
Figure 1. Interaction of two stars with a passing satellite. The left- and right-hand columns correspond to two stars that differ by $\pi$ in their orbital angle $\theta$. The speed of the satellite is greater than the maximum speed of the stars. The top two rows show the phase space coordinates $z$ and $v$ as a function of time for the star (solid curves) and satellite (dashed curves). The third panels show the tidal forces acting on the stars while the bottom panels show the change in energy $\delta E_z$.

Figure 2. Same as Fig. 1 but for the case where the satellite speed is less than the maximum speed of the stars.

and the corresponding inverse transform as

$$\delta E_m(E_z) = \frac{1}{2\pi} \int_0^{2\pi} d\theta \ e^{-i m \theta} \delta E(E_z, \theta). \quad (15)$$

The $m = 1$ term corresponds to a bending-mode perturbation while the $m = 2$ term corresponds to the breathing mode. In Fig. 3, we show the fractional change in energy, $\delta E_z/E_z$, as a function of $E_z$ and $\theta$ for $v_s = 1.75$. We see that for $E_z \lesssim 5$, where the characteristic orbital speed of the stars is less than the speed of the satellite, $\delta E_z$ has a pattern characteristic of a breathing mode (i.e. strong $m = 2$ pattern in $\delta E_z/E_z$ as a function of $\theta$ at fixed energy). For $E_z \gtrsim 5$, the pattern for $\delta E_z$ is that of a bending mode ($m = 1$ pattern).

For stars of energy $E_z$, the power in the $m$th mode of the perturbation is

$$P_m(E_z) = |\delta E_m(E_z)|^2 \quad (16)$$

where, by Parseval’s theorem, the total power is

$$P(E_z) = \sum_m P_m(E_z) = \frac{1}{2\pi} \int_0^{2\pi} d\theta \ (\delta E(E_z, \theta))^2. \quad (17)$$

In Fig. 4, we plot the normalized power $P_m/P$ as a function of $E_z$ for various choices of $m$ and $v_s$. For our choice of constants, a star with $E_z = 1$ has a maximum orbital speed of $v_{\text{max}} = 1$. We see that for a slow moving satellite ($v_s = 0.5$), the perturbation is almost
is the effective thickness of the for the Kuijken & Gilmore (1989) model. They chose the following parametric form for the potential:

\[ \psi(z) = K \left( z^2 + D^2 \right)^{1/2} - D + H z^2. \] (18)

The first term on the right-hand side is meant to account for the disc’s contribution to the potential where D is the effective thickness of the disc and K is proportional to the disc’s surface density. The second term is meant to account for the halo where H is proportional to the effective halo density in the solar neighbourhood. Kuijken & Gilmore (1989) fixed D to be 180 pc and imposed the constraint

\[ H = 2\pi G \left( 0.015 - 9.4 \times 10^{-5} \left( \frac{K}{M_\odot pc^{-2}} \right) \pm 0.002 \right) M_\odot pc^{-3} \] (19)

to be consistent with observations of the Galactic rotation curve. They found

\[ K = 2\pi G \left( 46 \pm 9 \right) M_\odot pc^{-2} \] (20)

and therefore

\[ H = 2\pi G \left( 0.011 \pm 0.003 \right) M_\odot pc^{-3}. \] (21)

The Besançon model (Robin et al. 2003), which is one of the most widely cited models of the Milky Way, provides a self-consistent description of the Galactic potential and stellar populations for the Milky Way based on star counts and the rotation curve. The model is presented as a set of density laws for the (multicomponent) thin disc, the thick disc, the bulge, the stellar halo, and the dark halo. From these results, it is straightforward to extract the vertical potential in the solar neighbourhood.

Apart from the physical interpretation ascribed by Kuijken & Gilmore (1989), equation (18) provides a convenient parametric form for the vertical potential in part because the inverse \( z = \frac{1}{\psi} \) is analytic. With the parameters given in Table 1, equation (18) provides an excellent fit to \( \psi(z) \) for the Besançon model in the range \( 0 < z < 2 \).

Widrow et al. (2008) presented a set of 25 disc–bulge–halo models for the Milky Way designed to fit observational data for the rotation curve and local kinematics. Each model is represented by self-consistent expressions for the DF and density of the three components as well as the total gravitational potential. The models are characterized by their susceptibility to bar and spiral instabilities. Here, we consider the most stable model (Toomre parameter \( Q = 2 \), global stability parameter \( K = 4.5 \)) and again fit the vertical potential at \( R = R_{\odot} \) to equation (18).

The vertical potential and force for the three models are shown in Fig. 5. Fig. 6 shows the mean speed and maximum excursion in \( z \) for the three models considered here as well as the DF for the Besançon model. Consider, for example, stars that have \( z_{\text{max}} = 1 \) kpc. These stars have a mean speed of about \( 25–30 \text{ km s}^{-1} \) with the thick and thin discs making similar contributions to the total stellar DF.

### 3 EVOLUTION OF VERTICAL OSCILLATIONS

The mode decomposition scheme described above provides a useful starting point for understanding the evolution of perturbations in a stellar disc. Bending, breathing, and higher order modes are collective excitations (Araki 1985; Mathur 1990; Weinberg 1991; Sellwood et al. 1998), which can lose energy via Landau damping and dynamical friction with the halo. Phase mixing also affects the evolution of the modes and may lead to an effective damping of observables, which invariably involves a projection of the coarse-grained DF. Finally, the interaction region of the disc for a satellite encounter will be sheared by differential rotation. In what follows, we provide a brief discussion of these effects and then present one-dimensional numerical simulations of a satellite that interacts with a plane-symmetric disc.
3.1 Theoretical considerations

It is a well-known result that perturbations in a homogeneous self-gravitating fluid with density \( \rho_0 \) and sound speed \( c_s \) grow via the Jeans instability if the wavelength of the perturbation is greater than the Jeans length \( \lambda_J \equiv (\pi c_s^2/G \rho_0) \). Perturbations whose wavelength \( \lambda \) is less than \( \lambda_J \) oscillate as sound waves with period \( T = 2\pi\lambda/c_s \) (see e.g. Binney & Tremaine 2008). In a homogeneous collisionless system with Maxwellian DF, the Jeans length is the same as for a fluid except that the sound speed is replaced by the velocity dispersion. However, perturbations with \( \lambda < \lambda_J \) are strongly damped by a process known as Landau damping, which was first discussed in the context of plasma waves (Landau 1946). (For an excellent pedagogical discussion, see Stix 1992.) Particles can draw energy from or feed energy to the collective mode. In the case of a spatially homogeneous Maxwellian DF, the net effect of the stars is to damp the wave on a time-scale comparable to the period of the oscillations (Lynden-Bell 1962).

As stressed in Binney & Tremaine (2008) and elsewhere, Landau-damped waves are not true modes but rather the response of the system to a perturbation. True modes of the system would satisfy the collisionless Boltzmann and Poisson equations with a harmonic time-dependence for all times.

In the case when the unperturbed system is spatially homogeneous, one must resort to the Jeans swindle, which side-steps the fact that the unperturbed potential is ill-defined. For the isothermal plane, the existence of a self-consistent equilibrium solution (equations 10–12) allows one to carry out a perturbation analysis without resorting to the Jeans swindle (Araki 1985).

In principle, wave-like perturbations of the isothermal plane will also experience Landau damping, which occurs when there are particles in the distribution in resonance with the wave, i.e. whenever the frequency of the wave matches \( n\nu \), where \( \nu \) is the orbital frequency of a star and \( n \) is an integer. The orbital frequencies \( \nu \) of stars in the isothermal plane range from 0 to \( \nu_{\max} = \kappa_z \). Thus, the spectrum of orbits and their harmonics consist of semi-infinite overlapping segments \( \{0, n\nu_{\max}\} \), where \( n \) is a positive integer. For a mode of frequency \( \nu \), there will be a set of particles with \( n > \text{int}(\nu/\nu_{\max}) \) that are in resonance with, and therefore capable of Landau damping, the mode.

Mathur (1990) and Weinberg (1991) showed that true modes, that is, modes that do not suffer Landau damping, exist for the truncated isothermal plane. In that model, the DF is given by

\[
\begin{align*}
\mathcal{f}(z, v_z) &= \begin{cases} 
\int_0^{\nu_{\max}} \left( E_z + \frac{W}{\sigma_z^2} \right)^{-\frac{3}{2}} \left( 1 - \frac{E_z}{\sigma_z^2} \right)^{\frac{1}{2}} 
\int_0^{\nu_{\max}} \left( E_z + \frac{W}{\sigma_z^2} \right)^{-\frac{3}{2}} \left( 1 - \frac{E_z}{\sigma_z^2} \right)^{\frac{1}{2}} \end{cases} & 0 < E_z < W \\
0 & \text{otherwise}
\end{align*}
\]

Orbital frequencies range from \( \nu_{\min} \) to \( \nu_{\max} \), where \( \nu_{\min} \) is the orbital frequency of a star with energy \( E_z = W \) and \( \nu_{\max} = \kappa_z \) is the frequency for a star with \( E_z \to 0 \), that is, a star whose orbit stays near the mid-plane. The frequency spectrum for orbits has a gap from 0 to \( \nu_{\min} \), called the principal gap (Mathur 1990; Weinberg 1991). If \( 2\nu_{\min} > \nu_{\max} \), then there will be a second gap between \( \nu_{\max} \) and \( 2\nu_{\min} \), and so on. Weinberg (1991) showed that a mode exists for \( \nu = 0 \). This mode corresponds to a displacement of the system as a whole and corresponds to a bending mode. Once the system is embedded in an external potential, the frequency of the mode shifts to a non-zero value.

Weinberg (1991) analysed the linearized collisionless Boltzmann and Poisson equations and showed that modes also exist in the higher frequency gaps and typically lie near the upper frequency end of the gap. Thus, for the first gap beyond the principal one, the frequency of the mode is less than but close to \( 2\nu_{\min} \). That is, the coherent oscillations are near the 2:1 resonance with particles at the upper edge of the energy distribution.

In Weinberg’s analysis, the phase space DF is written as the sum of the zeroth-order (equilibrium) solution and a linear perturbation. Fig. 7 shows the phase space perturbation for the breathing model of a \( W = 2\sigma^2 \) model. The perturbation rotates in the clockwise direction with a period given by the orbital period of stars near the energy cut-off, \( \nu_{\max} \), while the pattern is periodic with a frequency of \( 2\nu_{\min} \). The phase shown in Fig. 7 is consistent with a velocity perturbation similar to what is seen in the data. In the south \( (z < 0) \), there are more stars with negative velocity than positive velocity. The situation is the reverse in the north. Fig. 8 shows the bulk...
As the system evolves, the perturbation is washed out as a function of position relative to the Galactic mid-plane (equation 22) with $E = 2\sigma^2$ (see also Weinberg 1991). The system is perturbed by a passing satellite whose velocity is $v_s = 2\sigma$ and whose surface density is equal to one-half the surface density of the system. In Fig. 9, we show the change in the DF as a function of $E_z$ and $\theta$ for three epochs. At the first epoch ($t = 0$), the satellite is crossing the mid-plane. The second and third panels show the change in the DF for $t \approx 3T$ and $t \approx 30T$, respectively, where $T = 2\pi\sigma$ is the orbital period for a star near the mid-plane.

Figure 9. Fractional change in vertical energy, $\delta E_z$, as a function of $E_z$ and $\theta$ due to a satellite with speed $v_s = 0.25$. Top, middle, and bottom panels correspond to $t = 0, 3T,$ and $30T$, respectively, where $T = 2\pi\sigma$ is the orbital period for a star near the mid-plane.

We have carried out a new simulation where the initial conditions are that of the truncated isothermal plane (equation 22) with $W = 2\sigma^2$ (see also Weinberg 1991). The system is perturbed by a passing satellite whose velocity is $v_s = 2\sigma$ and whose surface density is equal to one-half the surface density of the system. In Fig. 9, we show the change in the DF as a function of $E_z$ and $\theta$ for three epochs. At the first epoch ($t = 0$), the satellite is crossing the mid-plane. The second and third panels show the change in the DF for $t \approx 3T$ and $t \approx 30T$, respectively, where $T = 2\pi\sigma$ is the orbital period for stars with higher vertical energies and also that those high-energy stars are most easily excited by a passing satellite.

As noted above, in the absence of an external potential, the bending mode for this one-dimensional model corresponds to a displacement of the system in position and velocity. In an actual stellar disc, there is a restoring force to a local vertical displacement due to the gravitational field from the rest of the disc. One can model this effect in the one-dimensional model by adding an external potential. The mode then oscillates but, as discussed in Weinberg (1991), its structure is qualitatively similar to a simple displacement.

Observationally, a bending mode manifests itself as a shift in the position and velocity of the mid-plane across the disc. Indeed, mid-plane displacements of the order of $10^{-100}$ pc have been found in the analysis of CO observations by Nakanishi & Sofue (2006). On the other hand, if we focus on the vertical structure of the disc in the solar neighbourhood, then a bending mode affects our determination of the Sun’s vertical position and velocity but is otherwise unobservable.

3.2 Simulations in one dimension

Widrow et al. (2012) presented one-dimensional $N$-body simulations to illustrate the evolution of a perturbed isothermal plane. In these simulations, the DF is sampled by a set of particles’ each of which represents an infinite plane with surface density $\Sigma_\rho$. The force acting on the $i$th particle is

$$F_i = 2\pi G \Sigma_\rho \left(N_i^R - N_i^L\right),$$  \hspace{1cm} (23)$$

where $N_i^R$ ($N_i^L$) is the number of particles to its right (left). The equilibrium model was the Spitzer solution while the perturbation was chosen to correspond to the velocity and number density perturbations seen in the data. Waves appear to reflect off the low-density regions above and below the mid-plane. The perturbation also appears to decay from the inside out. This result is consistent with the result discussed above (see also Weinberg 1991), namely that coherent excitations of the system are mainly a phenomena of stars with higher vertical energies and also that those high-energy stars are most easily excited by a passing satellite.

We have carried out a new simulation where the initial conditions are that of the truncated isothermal plane (equation 22) with $W = 2\sigma^2$ (see also Weinberg 1991). The system is perturbed by a passing satellite whose velocity is $v_s = 2\sigma$ and whose surface density is equal to one-half the surface density of the system. In Fig. 9, we show the change in the DF as a function of $E_z$ and $\theta$ for three epochs. At the first epoch ($t = 0$), the satellite is crossing the mid-plane. The second and third panels show the change in the DF for $t \approx 3T$ and $t \approx 30T$, respectively, where $T = 2\pi\sigma$ is the orbital period for stars with $E_z \lesssim 0.5$. As the system evolves, the perturbation is washed out for $E_z \lesssim 1.2$, presumably by a combination of phase mixing and Landau damping. However, the perturbation persists mainly for stars with energies near the energy cut-off. This result is jibe with the fact that the true modes of the system are most pronounced near the energy cut-off. (See fig. 4 from Weinberg 1991 and our Fig. 7.)

4 N-BODY SIMULATIONS

4.1 Single satellite perturbations

As discussed above, the coupling between a satellite and the vertical modes of the Galactic disc in the solar neighbourhood is governed by the relative in-plane motions of the satellite and the LSR, the match between the satellite’s vertical velocity and the vertical epicyclic motions of solar neighbourhood stars, and the mass and concentration of the satellite. Gómez et al. (2013) used numerical simulations to show that the Sagittarius dwarf galaxy could have induced wave-like perturbations as it plunged through the stellar disc. The initial Milky Way model is from Widrow et al. (2008). Gómez et al. (2013) considered two models for the Sagittarius progenitor, a light model, with virial mass $M_{\text{vir}} = 3 \times 10^{10} M_\odot$, an NFW (Navarro, Frenk & White 1996) scalelength $r_s = 4.9$ kpc, and a concentration
parameter $c = 16.3$ and a heavy model with $M_{\text{vir}} = 10^{11} \text{M}_\odot$, $r_i = 6.5 \text{kpc}$, and $c = 18$. The surface densities for these models are 220 and $380 \text{M}_\odot \text{pc}^{-2}$. These values are larger than the disc surface density in the solar neighbourhood. However, the satellites suffer mass-loss due to tidal stripping before the pass through the disc. In any case, it is not surprising that the disc is perturbed by a passing satellite with these parameters and indeed, Gómez et al. (2013) find that there are regions in the disc characteristic of the solar neighbourhood (that is, $\sim 8 \text{kpc}$ from the Galaxy’s centre) where the vertical number density profile of disc stars has wave-like perturbations qualitatively similar to what is seen in the data (Widrow et al. 2012; Yanny & Gardner 2013).

Here, we present preliminary results from simulations of satellite–disc interactions with a particular focus on vertical velocity perturbations. We choose the most stable of the Galactic models presented in Widrow et al. (2008) as our model for the parent galaxy. In Fig. 10, we present results for a satellite that passes through the mid-plane of the Milky Way on a prograde orbit at a Galactocentric radius of 8 kpc with a vertical speed of roughly 60 km s$^{-1}$. The satellite has a mass of $M_s = 4 \times 10^9 \text{M}_\odot$ and is truncated at a radius of $\sim 0.9 \text{kpc}$. The first column of panels shows the disc as the satellite is passing through the mid-plane while the second column shows the disc some 250 Myr later. The top panels show a logarithmic map of the surface density across the disc. The strong wake generated by the satellite is clearly visible in the upper-left panel. Over time, the disc develops prominent flocculent spiral structure.

To quantify the bending and breathing modes, we model the bulk vertical velocity across the disc plane as

$$\hat{v}_z(x, y, z) = A(x, y) + B(x, y).$$

(24)

That is, for disc stars within a two-dimensional cylinder centred on the point $(x, y)$ in the disc plane, we fit a $\hat{v}_z$ to a linear function in $z$.

The coefficient $B$ is a measure of the bending-mode strength and is shown in the two middle panels of Fig. 10 while $A$ is a measure of the breathing-mode strength and is shown in the bottom two panels. We see that the satellite excites both modes as it passes through the disc. These perturbations are sheared by the differential rotation of the disc and continue to oscillate and reverberate for many hundreds of Myr. Both modes have a pattern across the disc that is qualitatively similar to the spiral structure seen in the density map.

### 4.2 System of satellites

Gauthier et al. (2006) followed the evolution of a disc–bulge–halo galaxy in which the halo comprises a smooth component with an NFW profile (Navarro et al. 1996) and a system of 100 subhaloes. For the parent galaxy, they used the self-consistent equilibrium model of M31 from Widrow & Dubinski (2005) labelled M31a. This model provides a good match to the observed rotation curve, surface brightness profile, and velocity dispersion profile and is stable against the formation of a bar for 10 Gyr so long as the halo mass is assumed to be smoothly distributed. The disc has a mass of $M_D = 7.8 \times 10^{10} \text{M}_\odot$ and an exponential scalelength of $R_D = 5.6 \text{kpc}$. The circular speed curve reaches a peak value of 260 km s$^{-1}$ at a radius of $\sim 10 \text{kpc}$. The complete list of model parameters can be found in table 2 of Widrow & Dubinski (2005) or table 1 of Gauthier et al. (2006). In principle, the model could be ‘rescaled’ to make better contact with Milky Way observations. Nevertheless, the qualitative features of the simulations should be applicable to the Galaxy. A proper Milky Way version of this numerical experiment will be presented in a forthcoming publication.

The 100 subhaloes in the Gauthier et al. (2006) simulation range in mass from $8.7 \times 10^7$ to $1.2 \times 10^{10} \text{M}_\odot$, with a number density mass function given by $dN/dM \propto M^{-1.1}$ (Gao et al. 2004). Initially, each subhalo is modelled as a spherically symmetric truncated NFW system where the truncation radius $r_i$ is given by its tidal radius, as determined by the Jacobi condition, at 50 kpc:

$$\bar{\rho}_s(r_i) = \frac{3}{2} \bar{\rho}_s(r = 50 \text{kpc}).$$

(25)

Here, $\bar{\rho}_s(r)$ is the mean (sub)halo density inside radius $r$. The radius 50 kpc is the mean apocentre for the initial system of subhaloes. Each subhalo is characterized by its mass $M_s$, scale radius $r_s$, and concentration $c = r_i/r_s$. In Fig. 11, we plot $M_s$ and $r_s$ as a function of the average surface density within a projected radius $R = r_s$, $\Sigma(R = r_s)$ for individual subhaloes (see table 2 of Gauthier et al. 2006). Note that $\Sigma(R \lesssim r_s/10)$ is approximately constant and a factor of $\gtrsim 10$ larger than $\Sigma(R = r_s)$.

One of the most striking results from the Gauthier et al. (2006) simulation is the formation of a strong bar at about 5 Gyr (see also Kazantzidis et al. 2008). In Fig. 12, we show the surface density and breathing-mode strength at 2.5 and 10 Gyr. The length of the bar is $\sim 20 \text{kpc}$ as can be seen in the upper-right panel of Fig. 12 and fig. 5 of Gauthier et al. (2006). Prominent spiral structure also develops and indeed, appears to be a precursor to the formation of the bar. Though there is some disc heating during the first 4 Gyr the most significant heating and thickening occurs after the bar forms (see...
Figure 11. Mass and NFW scale radius as a function of average surface density within a projected surface density \( r_s \) for the subhaloes in the Gauthier et al. (2006) simulation.

Dubinski et al. 2008 for a further discussion). By contrast, no bar and only weak spiral structure develops in the control experiment, which assumes a smooth halo. Evidently, satellites and subhaloes provoke spiral structure and bar formation (see also Purcell et al. 2011).

It is not at all surprising that system of subhaloes also excites vertical oscillations in the stellar disc.\(^1\) Bending- and breathing-mode perturbations are found across the disc and throughout the simulation, as can be seen in the middle and lower panels of Fig. 12. Prior to bar formation, there are strong large-scale bending modes across the disc with amplitudes of the order of \( 4 \text{ km s}^{-1} \). The breathing modes have a somewhat smaller amplitude (\( \sim 2 \text{ km s}^{-1} \)) and vary on smaller scales. At later times, after the bar has formed, the bending-mode perturbations have diminished. Moreover, in the inner parts of the Galaxy, the breathing mode mirrors the bar. Thus, while subhaloes may have triggered the formation of the bar, it is the bar that generates and maintains compression and rarefaction motions in the inner Galaxy. Of course, the Sun sits well beyond the region of the bar and it is therefore unlikely that the bar could cause the bulk motions seen in the solar neighbourhood.

5 DISCUSSION AND CONCLUSIONS

The implications of spatially dependent bulk motions perpendicular to the Galactic disc were highlighted by Oort (1932) in his seminal work on the structure of the Galactic disc. Oort’s aim was to determine the potential \( \psi(z) \) from the local stellar density and velocity distribution. He based his analysis on the assumption that the local distribution of stars is in equilibrium. To test the assumption, he computed the mean vertical velocity for stars in four separate bins: \( 100 < \pm z < 200 \) and \( 200 < \pm z < 500 \) pc but did not find evidence for systematic motions, a result that he notes ‘lends some support to the assumption . . . that in the \( z \)-direction the stars are thoroughly mixed’ (Oort 1932). Turning Oort’s argument around, the detection of bulk vertical motions by the SDSS/SEGUE, RAVE, and LAMOST surveys suggests that the local Galactic disc is not in equilibrium in the \( z \)-direction.

In this paper, we considered the hypothesis that the observed bulk vertical motions were generated by a passing satellite or dark matter subhalo. The idea that dark matter, in one form or another, might be responsible for heating and thickening the disc dates back to the 1980s. Lacey & Ostriker (1985) calculated disc heating by a dark halo of supermassive black holes while Carr & Lacey (1987) considered dark matter in the form of \( 10^6 \text{ M}_\odot \) dark clusters. In essence, our hypothesis is that the bulk motions seen in the data represent the early stages of a disc heating event.

Our focus has been to explore the theoretical aspects of disc–satellite interactions. We found that the nature of the perturbations is controlled largely by the satellite’s vertical velocity relative to the disc. In particular, a slow moving (as measured in the LSR) satellite induces a bending-mode perturbation. With a higher vertical velocity, higher order modes, such as the breathing mode, are excited. Thus, if a satellite is indeed responsible for the bulk vertical

\(^1\) See the animation found at the URL http://www.cita.utoronto.ca/~ubinski/Rome2007/. Note, in particular, the switch half-way through to an edge-on view of the disc.
motions seen in the solar neighbourhood, then its vertical velocity through the disc would likely have been \( \geq 50 \) km s\(^{-1}\). Moreover, its surface density would have to be comparable to that of the disc in order to produce an appreciable perturbation. The model satellites considered by Gómez et al. (2013) satisfy these conditions and so it is not surprising that they found vertical perturbations in the disc that were qualitatively similar to what was found in the data.

Single satellite simulations show that after a localized breathing-mode perturbation is produced, it is sheared by the differential rotation of the disc. After several orbital periods of the disc, the perturbation assumes a spiral-like pattern. The situation is more complicated with a population of satellites and it may be difficult to disentangle initial perturbations from the accumulated long-lived perturbations.

Our analysis, and that of Weinberg (1991), suggest that stars on the tail of the energy distribution are most responsive to a breathing-mode perturbation. Though our analyses focused on single-component discs, it may well be that the vertical motions is a property more of the thick-disc stars, than the thin-disc stars. It is well known that the vertical velocity dispersion and scaleheight are anticorrelated with metallicity (see, e.g., Bovy et al. 2012a,b; Minchev, Chiappini & Martig 2013; Minchev et al. 2014 and references therein). The arguments presented in this paper suggest that bulk motions should be more prominent in the low-metallicity/high-\( E_\text{c} \) populations. A related issue is radial migration. Sellwood & Binney (2002) argued that spiral waves can change the angular momenta, and hence Galactocentric radii, of individual stars by \( \sim 50 \) per cent. In analysing the cosmological simulations of disc formation by Martig et al. (2012), Minchev et al. (2013) found that satellite interactions can also drive radial migration. Moreover, radial migration can bring high-dispersion stars from the inner disc to the solar neighbourhood, and, as discussed above, these are the stars most susceptible to a recent satellite interaction.

There are 25 known satellites of the Milky Way. Moreover, in an \( \Lambda \)CDM cosmology, the dark halo of a Milky Way-sized galaxy is expected to harbour many more non-luminous subhaloes (Klypin et al. 1999; Moore et al. 1999). Thus, it is likely that the Galactic disc has been continually perturbed over its lifetime. In principle, observations of bulk motions in the stellar disc could provide a probe of the subhalo distribution. To do so, will require a suite of simulations where the slope and amplitude of the subhalo mass function are varied.

Over the next few years, \textit{Gaia} will provide an unprecedented snapshot of the Galaxy by making astrometric, spectral, and photometric observations of approximately one billion Milky Way stars (see, for example Perryman et al. 2001 and de Bruijne 2012). This data set will yield a more accurate and complete map of bulk motions in the stellar disc. By bringing together these observations, theoretical analysis, and \( N \)-body simulations, we hope to better understand Galactic dynamics, and in particular, interactions between the Milky Way’s disc and its satellites and dark matter subhaloes.

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