Archetypal Factorization and Gluon Poles in semi-exclusive reactions

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ABSTRACT: Within the archetypal factorization procedure, we study the gluon pole contributions manifesting in the nucleon-lepton hard processes of hadron production. We prove the dominant role of gluon pole contributions in the processes of such kind. We analyse the different sources of complexity associated with the gluon pole functions. As a practical application, we derive a new single spin asymmetry related directly to the gluon poles which can be studied experimentally.
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1 Introduction

The hadron production in nucleon-lepton processes attracts still much attentions not only of the experimental collaborations (for example, at JLab) which investigate the composite (spin) structure of hadrons. From the theoretical viewpoint, this kind of processes is interested because it opens the window for the studies of the transverse momentum dependent functions, in particular, appearing in the frame of semi-inclusive kinematics, see Fig. 1. In this kinematics, the hadron tensor involves both the distribution and fragmentation functions which are linked by the loop integrations over the transverse components of momenta (this is a general feature of TMD-factorizations). Based on the classical formulation of the factorization theorem [1–4], the different non-perturbative functions (such as distribution functions and fragmentation functions) bound by the dimensionful transverse momentum integration cannot be considered as totally independent ones. Nevertheless, there are different approaches where the mentioned non-factorized effects can be diminished by the corresponding summation taking into account the transverse momenta of partons (see, for example, [5] and the references therein). Moreover, in the framework of semi-inclusive kinematics, the photon and produced hadron transverse momenta are related each other with the necessary condition $\mathbf{q}_\perp = \frac{\mathbf{P}_h \perp}{z} \sim \mathbf{k}_\perp$, where $\mathbf{k}_\perp$ denotes the primordial transverse momentum of quark.

On the other hand, the attempts to use the approaches inspired by TMD-factorization for description of the existing experimental data from JLab are encountered the difficulties at rather a large value of $\mathbf{q}_\perp$ (sometimes, it is named as the “$q_T$ crisis”).

However, the hadron production in nucleon-lepton collision can be also considered using the semi-exclusive kinematics where the hadron tensor contains two distribution amplitudes instead of one fragmentation function, see Fig. 1. In the present paper, alternatively to the semi-inclusive description, we propose an approach based on the well-defined and archetypal factorization procedure (see for example [6–10]) applied to the semi-exclusive mode of the hadron production processes. Within this approach, the hadron tensor is given by the corresponding convolution of the distribution function and the distribution amplitudes if a value of intermediate quark fraction is large, $x \to 1$. Another feature of the used approach is that the photon and produced hadron transverse momenta are not tied in contrast to the approaches based on TMD-factorization. In this context, we calculate a new single spin asymmetry which is associated with the gluon pole contribution for a wide region of produced hadron transverse momentum. This asymmetry can be also a object of experimental measurements at a large value of $\mathbf{P}_h \perp$ using the existing data of JLab.

In the paper, we also give a proof that, in the limits of $x \to 1$ and $m_q \to 0$, the leading order hadron tensor is actually suppressed in comparison with the gluon pole contribution to the hadron tensor where the gluon radiation has been taken into account. Based on this our principle statement, we concentrate on the manifestation of gluon poles in the lepton-hadron collision of hadron production.

As shown in [7–11], the use of contour gauge conception gives the strong mathematical evidences for the nature of functional complexity in the treatment of gluon poles. From the viewpoint of practical uses, the complexity of the corresponding distribution functions related to the gluon poles is very important because it leads to the different kinds of asymmetries. In addition to the contour gauge conception, we investigate the different sources of functional complexity which are associated with the gluon pole effects.
Within the factorization procedure we adhere in our paper, we introduce a new kind of the transverse momentum dependent distribution functions which leads to the different new observables. With the help of these new functions and the gluon pole contributions it becomes to be possible to study the hadron spin structure in a different light using the experimental data extracted from the measurements of processes with a large value of transverse momentum of produced hadron.

The paper is organized as follows. The kinematics is introduced and the Sudakov decompositions are presented in Sec. 2. In Sec. 3, we obtain the leading order hadron tensor and the hadron tensor which involves the gluon pole contribution. Also, we describe the main stages of our factorization procedure focusing on its important features. In Sec. 4, we formulate the principal statement on the dominant role of gluon pole contributions to the corresponding hadron tensor in comparison with the hadron tensor without the gluon pole contribution. Sec. 5 is devoted to the derivation of the gauge-invariant hadron tensor with gluon pole contributions and to the introduction of the new transverse momentum dependent functions. We also analyse the different sources of complexity for the function involving the gluon pole. The new single spin asymmetry is presented and discussed in Sec. 6. Sec. 7 is reserved for a summary and outlook. Some technical issues are presented in appendices. Appendix A contains the discussion on the different choices of the frame systems. The alternative representation of functions with the on-shell fields in correlators is presented in Appendix B. The off-shell extension of the fields in the corresponding correlators is discussed in Appendix C. In Appendix D, the extension of standard Lorentz parametrization is presented and Appendix E is devoted to the method of covariant integrations.

2 Kinematics

To begin with, we specify our kinematics. We study the hadron (pion, ρ-meson) production in the hard process which can be considered, from the kinematical point of view, as the intermediate one between the semi-inclusive and semi-exclusive reactions. Namely, we deal with

\[ \ell(l_1) + N^{(\uparrow \downarrow)}(P_2) \rightarrow \ell(l_2) + h(P_h^\uparrow) + \bar{q}(K) + X(P_X) \]  

or

\[ \gamma^*(q) + N^{(\uparrow \downarrow)}(P_2) \rightarrow h(P_h^\uparrow) + \bar{q}(K) + X(P_X). \]  

In Eqns.(2.1) and (2.2), \( h \) denotes \( \pi \) or \( \rho \) triplet of mesons (in general, any hadron) and the virtual photon produced by the lepton \( (l_1 = l_2 + q) \) has a large mass squared \( (q^2 = -Q^2) \) while the photon transverse momenta are small.

The Sudakov decompositions take the standard forms (for the sake of shortness, we omit the four-dimension indices) [7–10]

\[ P_2 \approx \frac{\hat{Q}}{y_B \sqrt{2}} n + P_{2 \perp}, \quad P_{h}^\uparrow \approx \frac{\hat{Q}}{x_B \sqrt{2}} n^* + P_{h \perp}^\uparrow, \]  

\[ n^* = (1/\sqrt{2}, 0_T, 1/\sqrt{2}), \quad n = (1/\sqrt{2}, 0_T, -1/\sqrt{2}) \]

\[ S \approx \frac{\lambda}{M_2} P_2 + S_\perp \]
for the hadron momenta and spin vector;
\[ q = \frac{\hat{Q}}{\sqrt{2}} n^* + \frac{\hat{Q}}{\sqrt{2}} n + q_\perp, \quad q_\perp \ll \hat{Q} = \sqrt{-Q^2}, \tag{2.5} \]
for the photon momentum. The hadron momenta \( P_h^1 \) and \( P_2 \) have the plus and minus dominant light-cone components, respectively, see Fig. 1.

The differential cross-section takes therefore the following form
\[ d\sigma = \frac{d^3\mathbf{P}_X^2}{(2\pi)^3 2\Omega_0} \frac{d^3\mathbf{P}_1^h}{(2\pi)^3 2E_1^h} \mathcal{L}_{\mu\nu} \mathcal{W}_{\mu\nu}, \tag{2.6} \]
where \( \mathcal{L}_{\mu\nu} \) and \( \mathcal{W}_{\mu\nu} \) denote the lepton and hadron tensors, respectively.

### 3 The hadron production in semi-exclusive processes

The process under our discussion resembles the standard Drell-Yan process, see Fig. 1, where one of the initial hadron states is being replaced by the final hadron state. Moreover, all unobserved hadron states \( |P_X\rangle \) can be divided into two (after the applied factorization procedure, independent) sets, i.e. \( |P_X\rangle = |P_X^1\rangle \oplus |P_X^2\rangle \). The set given by \( |P_X^1\rangle \) is associated with the upper part of the hadron tensor, while the other set given by \( |P_X^2, P_X^1\rangle = |P_X^1, K\rangle + \ldots \) (see Fig. 1) is forming the lower part of hadron tensor. In this context, we mainly follow the stages of our analysis that have been used in a series of papers [7–10]. So, by definition, the hadron tensor of (2.1) is defined to be given by the following expression
\[ \mathcal{W}_{\mu\nu} = \int (dP_S) \delta^{(4)}(q + P_2 - P_1^h - K - P_X^2) \times \langle P_2|J_\mu(0)|P_1^h, K, P_X^2\rangle \langle P_1^h, K, P_X^2|J_\nu(0)|P_2\rangle, \tag{3.1} \]
where the matrix elements are written in the Heisenberg representation and the phase space measure is defined by
\[ (dP_S) = \frac{d^3\mathbf{K}}{(2\pi)^3 2\Omega_0} \frac{d^3\mathbf{P}_X^2}{(2\pi)^3 2E_2}. \tag{3.2} \]
Notice that from the viewpoint of factorization theorem the hadron tensor of (3.1) is not yet in a factorized form. To apply the factorization procedure, the hadron momenta \( P_2 \) and \( P_1^h \) should be associated with the light-cone minus and plus directions, respectively, which are well-separated. In other words, there are no the strong interferences between the hadron dominant directions.

In addition, it is important to stress that due to the used factorization procedure the above-mentioned two sets of states, \( |P_2 X_2⟩ \) and \( |P_1^h, P_X⟩ \Rightarrow |P_1^h, K⟩ \), are independent ones because the light-cone plus direction, dominated in the lower blob, and light-cone minus direction, dominated in the upper blob, are separated well.

With these, the Fock states corresponding to the initial and final hadrons can be permuted resulting in

\[
\mathcal{W}_{\mu\nu} \sim \left\{ (0|\bar{\psi}(0)|P_1^h, K) \gamma_\nu \langle P_2 |\psi(0) |P_X⟩ \right\} \times \\
\left\{ \langle P_X |\bar{\psi}(0)|P_2⟩ \gamma_\mu \langle P_1^h, K|\psi(0)|0⟩ \right\} 
\]

for the matrix elements in (3.1). Here, as mentioned, the unobserved hadrons with \( P_X \) are fully associated with the upper part of the process, see Fig. 1 (the left panel), and they are independent from the detected hadron with \( P_1^h \).

In what follows, since the states with \( P_X \) have been traded for the (jet) quark with \( K \) and the only-remained unobserved states are the states with \( P_X \), we simplify the notations by the tacit replacement \( P_X \equiv P_X \) unless it leads to some misunderstanding.

### 3.1 The hadron tensor at leading order

In the paper, our main interest is associated with the gluon pole generated by the radiative corrections. Nevertheless, we begin our consideration with the detail analysis of leading order hadron tensor.

Making used the Fourier transform for the delta-function and the translation transforms in Eqn. (3.1), the leading order hadron tensor \( \mathcal{W}^{(0)}_{\mu\nu} \) can readily be written down as

\[
\mathcal{W}^{(0)}_{\mu\nu} = \int (d^4 \xi) e^{i q \xi} \text{tr} \left[ \gamma_\nu \Phi(\xi) \gamma_\mu \bar{\Phi}(\xi) \right],
\]

(3.4)

where the hidden spinor indices related to the functions \( \Phi \) and \( \bar{\Phi} \) are open, and

\[
\Phi(\xi) = \sum_X \int \frac{d^3 \bar{P}_X}{(2\pi)^3 2E_X} \langle P_2 |\bar{\psi}(\xi)|P_X⟩ \langle P_X |\psi(0)|P_2⟩,
\]

(3.5)

\[
\bar{\Phi}(\xi) = \int \frac{d^3 K}{(2\pi)^3 2K_0} \langle 0|\bar{\psi}(\xi)|P_1^h, K⟩ \langle P_1^h, K|\psi(0)|0⟩.
\]

(3.6)

In (3.4), the functions \( \Phi(\xi) \) and \( \bar{\Phi}(\xi) \) can be moved apart to different positions in \( x \)-space by the integration with the corresponding delta-functions, \( \delta \)

\[
\mathcal{W}^{(0)}_{\mu\nu} = \int (d^4 \xi) e^{i q \xi} \int (d^4 \eta_1)(d^4 \eta_2) \times \\
\delta^{(4)}(\eta_1 - \xi) \delta^{(4)}(\eta_2 - \xi) \text{tr} \left[ \gamma_\nu \Phi(\eta_1) \gamma_\mu \bar{\Phi}(\eta_2) \right].
\]

(3.7)
Afterwards, a series of Fourier transforms gives the following representation for the hadron tensor through the momentum loop integrations

\[ \mathcal{W}^{(0)}_{\mu\nu} = \int (d^4k_1)(d^4k_2) \delta^{(4)}(k_1 + k_2 - q) \times \text{tr} \left[ \gamma_\nu \Phi(k_2) \gamma_\mu \Phi(k_1) \right] , \]  
(3.8)

and

\[ \Phi(k_2) = \int (d^4\eta_2) e^{ik_2 \eta_2} \times \sum_X \int \frac{d^3\vec{P}_X}{(2\pi)^3 2E_X} \langle P_2 | \psi(\eta_2) | P_X \rangle \langle P_X | \bar{\psi}(0) | P_2 \rangle , \]  
(3.9)

\[ \bar{\Phi}(k_1) = \int (d^4\eta_1) e^{ik_1 \eta_1} \times \int \frac{d^3\vec{K}}{(2\pi)^3 2K_0} \langle 0 | \bar{\psi}(\eta_1) | P_{1h, K} \rangle \langle K, P_{1h} | \psi(0) | 0 \rangle . \]  
(3.10)

These functions can be expressed in terms of the corresponding distribution functions after the use of Lorentz decompositions.

### 3.2 The archetypal (original) factorization procedure

The factorization theorem is the main tool of the hadron tensor calculation. Generally speaking, the QCD factorization theorem, as one of asymptotic methods, allows only the estimation of the amplitudes or hadron tensors instead of exact calculations due to the quark confinement problem.

It is worth to remind that the original factorization theorem states the following: if the kinematics has a large parameter, the short (hard) and long (soft) distance dynamics can be independently separated out during the asymptotical estimation procedure applied to given amplitudes (or hadron tensors). The final result of such estimation or factorization has to be presented in the form of the mathematical convolution of the hard and soft parts, which are independent each other. The mathematical convolution implies that the products of hard and soft parts are integrated out over the dimensionless parton fractions. [1–4].

To illustrate this typical factorization procedure, we consider the arbitrary DY-like hadron tensor which involves two non-perturbative blobs, it reads (here, for the sake of brevity, we omit all possible Lorentz indices)

\[ W = \int (d^4k_1)(d^4k_2) E(k_1, k_2, q) \Phi_1(k_1) \Phi_2(k_2) , \]  
(3.11)

where

\[ E(k_1, k_2, q) = \delta^{(4)}(k_1 + k_2 - q) \delta^{(4)}(k_1, k_2, q) \]

\[ \Phi_1(k_1) \xrightarrow{\mathcal{F}} \langle \psi(z_1) \Gamma_1 \psi(0) \rangle , \]

\[ \bar{\Phi}_2(k_2) \xrightarrow{\mathcal{F}} \langle \bar{\psi}(0) \Gamma_2 \bar{\psi}(z_2) \rangle \]  
(3.12)

and \( \mathcal{F} \) denotes the corresponding Fourier transforms. In Eqn. (3.12), \( \delta^{(4)}(k_1, k_2, q) \) indicates the product of different propagators appearing after the use of Wick’s theorem, while the \( \delta \)-function reflects the corresponding momentum conservation at the subprocess level, see Eqns. (3.4)-(3.8).
We emphasize that the representation of hadron tensor given by Eqn. (3.11) is exact, before
the factorization procedure has been applied. One of the features pointing out that we deal with the
non-factorized representation is the presence of dimensionful loop integrations binding the product
of propagators with the non-perturbative functions. The dimensionful loop integrations do not
allow to treat the product of propagators, which pretends to form the hard part, as independent of
the non-perturbative (soft) functions.

We begin with choosing the dominant directions dictated by the given process kinematics. In
the example of Eqn. (3.11), we have two dominant directions: one of them is associated with the
plus light-cone direction, the other to the minus light-cone direction. The important condition for
factorization is that the dominant directions must be separated rather well in order to be independent
ones.

In the next step, we have to introduce the definitions of the dimensionless parton fractions with
the help of the following replacement:

\[ d^4k_i \Rightarrow d^4k_i \int_{-1}^{1} dx_i \delta(x_i - k_i^+/P_i^+) \]  

(3.13)
depending on the chosen dominant direction. The physical spectral properties of fractions are
determined by the \( \alpha \)-representation where the estimation procedure should be established.

Further, we expand the product of propagators (together with the \( \delta \)-function, see Eqn. (3.16))
around the chosen dominant directions. As a result, we obtain that

\[
W^{(0)} = \int (dx_1)(dx_2)E(x_1P_1^+,x_2P_2^-;q) \\
\times \left\{ \int (d^4k_1) \delta(x_1 - k_1^+/P_1^+) \Phi_1(k_1) \right\} \\
\times \left\{ \int (d^4k_2) \delta(x_2 - k_2^-/P_2^-) \bar{\Phi}_2(k_2) \right\}
\]  

(3.14)
if we neglect the \( k_i^\perp \)-terms in the expansion; and

\[
W^{(i,j)} = \int (dx_1)(dx_2) \sum_{i,j} E^{(i,j)}(x_1P_1^+,x_2P_2^-;q) \\
\times \left\{ \int (d^4k_1) \delta(x_1 - k_1^+/P_1^+) \prod_{f=1}^{i} k_{1f}^\perp \Phi_1(k_1) \right\} \\
\times \left\{ \int (d^4k_2) \delta(x_2 - k_2^-/P_2^-) \prod_{f=1}^{j} k_{2f}^\perp \bar{\Phi}_2(k_2) \right\}
\]  

(3.15)
if the transverse momentum terms are essential in the expansion. Notice that in Eqn. (3.15) the
\( k_i^\perp \)-dependence appears in the form of the corresponding integral moments only. Moreover, the
parts of \( W^{(0)} \) and \( W^{(i,j)} \) with \( E \)-function and \( \Phi \)-functions are not related by the integration over \( k_i^\perp \).

Last but not least, in order to achieve the results of Eqns. (3.14) and (3.15), in the DY-like pro-
cesses the four-dimensional \( \delta \)-function with the argument describing the momentum conservation
at the parton level should be treated as the “hard” parts of hadron tensor. This statement has been
proven with the help of the factorization links introduced in [12]. In particular, the decomposition
of delta function around the hadron dominant directions takes the form of

\[ \delta^{(4)}(k_1 + k_2 - q) = \delta^{(4)}(x_1 P_1 + x_2 P_2 - q) + \frac{\partial \delta^{(4)}(k_1 + k_2 - q)}{\partial k_i^\rho} \bigg|_{k_i = x_i P_i} k_i^\rho + \ldots \] (3.16)

Notice that for our purposes it is enough to be limited by the first term of decomposition in the r.h.s. of (3.16). The other important feature of \( \delta^{(4)}(\langle \text{mom.\ conserv.} \rangle) \) is that the momentum conservation law is valid for any system chosen for the kinematical reason. In this context, we may consider this delta function as a kind of the Lorentz invariant despite the covariant form of momentum conservation.

Thus, once the factorization procedure applied, the leading order hadron tensor takes the factorized form

\[ \mathcal{W}^{(0)}_{\mu\nu} = \delta^{(2)}(q_{\perp}) \int (dx)(dy) \delta(x P_1^+ - q^+) \delta(y P_2^- - q^-) \times \text{tr}[\gamma_\mu \Phi(y) \gamma_\nu \Phi(x)], \] (3.17)

where, as stressed above, the mathematical convolution is given by the integration over dimensionless fractions \( x \) and \( y \). In other words, in our case, the parts of \( \mathcal{W}^{(0)}_{\mu\nu} \) with the delta function and \( \Phi - \) and \( \Phi - \) functions are not linked by the integration over the dimensionful \( k^\perp \).

There are, however, the alternative approaches which are based on the formalism with the nonzero photon transverse momentum and without the \( \delta \)-function expansion, see [5]. These approaches result in the leading order hadron tensor with the \( k^\perp \)-dependent non-perturbative functions which has the following form

\[ \tilde{W}^{(0)} = \int (dx_1)(dx_2) \delta(x_1 P_1^+, x_2 P_2^-, q) \times \left\{ \int (d^2 k_1^+)(d^2 k_2^+) \delta^{(2)}(k_1^+ + k_2^+ - q^+) \right. \\
\times \left. \left( \int (dk_1^+ dk_1^-) \delta(x_1 - k_1^+ / P_1^+) \Phi_1(k_1) \right) \times \left( \int (dk_2^- dk_2^+) \delta(x_2 - k_2^- / P_2^-) \Phi_2(k_2) \right) \right\} e^{-S(k_1^+ / A, k_2^+ / A)} - (\text{the other sources of } k_{\perp}). \] (3.18)

Here, generally speaking, the functions \( \Phi(k_1) \) and \( \Phi(k_2) \) cannot be considered as the independent of each other because the two-dimensional integrations with the \( \delta \)-function relate them. This would lead to the factorization breaking effects. However, the additional Sudakov-like exponentials of \( e^{-S(k_1^+ / A, k_2^+ / A)} \)-form, generated by the corresponding summation of gluon radiations, should minimize the mentioned non-factorized effects. This way of proceeding is really complicated and it demands a very careful analysis which is beyond of our paper.
3.3 The leading order function \( \Phi(k_1) \)

Let us now dwell on the function \( \Phi(k_1) \) of (3.10), see Fig. 2, the right panel. This function contains the integration over the intermediate on-shell (anti)quark Fock state

\[
\mathcal{J}_K = \int \frac{d^3\vec{K}}{(2\pi)^3 2K_0} |K\rangle \langle K| =
\int \frac{d^3\vec{K}}{(2\pi)^3 2K_0} d^+_\lambda(K)|0\rangle \langle 0|d_\lambda(K)
\]

which can also be presented as

\[
\mathcal{J}_K = \frac{E}{m_q} \int (d^4P) \delta(P^2) \int (d^4K) \delta(K^2) \delta^{(3)}(\vec{P} - \vec{K}) \times
\left[ d^+_\lambda(K) v_\lambda(K) [d_{\lambda'}(P) \bar{v}_{\lambda'}(P)] \right]_{K_0 = P_0 = E},
\]

where \( E \) and \( m_q \) denote the energy and mass of the intermediate (anti)quark, respectively.

In (3.20), \( m_q \) is a result of the normalization condition

\[
v_\lambda(K) \bar{v}_{\lambda'}(K) = 2m_q \delta_{\lambda \lambda'},
\]

and the integral unit given by

\[
1 = \int (d^4\vec{P}) \delta^{(3)}(\vec{P} - \vec{K}) = 2E \int (d^4P) \delta(P^2) \delta^{(3)}(\vec{P} - \vec{K})
\]

has been used. Also, the imposed condition \( K_0 = P_0 = E \), which is shown in Eqn. (3.20), can be written in the equivalent form as \( E \delta(P_0 - K_0) \). All these tricks lead to the following representation

\[
\mathcal{J}_K = \frac{E^2}{m_q} \int (d^4P) \int (d^4K) \delta^{(4)}(P - K) \times
\left\{ \int (d^4x) e^{-iKx} \psi_{on-sh.}(x) \right\} \left\{ \int (d^4y) e^{iP_y} \bar{\psi}_{on-sh.}(y) \right\},
\]

Here, \( \psi_{on-sh.} \) denotes the on-shell particle (similary, for \( \bar{\psi}_{on-sh.} \), \( i.e. \)

\[
\int (d^4x) e^{-iKx} \psi_{on-sh.}(x) = \delta(K^2)[d^+_\lambda(K) v_\lambda(K)].
\]

Hence, the function \( \Phi(k_1) \) of Eqn. (3.10) takes the form of

\[
\Phi(k_1) = \frac{E^2}{m_q} \Phi_0^{(2)}(P^h_1 - k_1) \Phi_0^{(1)}(k_1)
\]

where

\[
\Phi_0^{(1)}(k_1) = \int (d^4\xi_1) e^{ik_1 \xi_1} \langle 0|\psi(\xi_1) \psi_{on-sh.}(0)|P^h_1\rangle
\]
\[ \Phi^{(2)}(P^h_1 - k_1) = \int (d^4 \eta_1) e^{-i(P^h_1 - k_1)\eta_1} (P^h_1 | \Psi_{\text{on-sh.}}(\eta_1) \psi(0) \rangle. \] (3.27)

For the sake of completeness, it is instructive to give the representation of \( \Phi(k_1) \) in Eqn. (3.25) if the factorization procedure has been applied. It reads (here, \( \bar{x}_1 = 1 - x_1 \))

\[ \Phi(x_1) = \int (d^4 k_1) \delta(x_1 - k_1^+/P^h_1^+) \Phi(k_1) = \]
\[ P^h_1^+ \int (dk^-_1)(dk^+_1) \]
\[ \times \frac{E^2}{m_q} \Phi^{(2)}(\bar{x}_1, k^-_1, \bar{k}^+_1) \Phi^{(1)}(x_1, k^-_1, \bar{k}^+_1). \] (3.28)

It is worth to notice that there is an alternative way to present the functions like \( \Phi(k_1) \) involving the on-shell fields in correlators, see App. B for details.

As above mentioned, the intermediate quark (see, for example, Eqn. (3.28)) is an on-shell particle, \textit{i.e.}

\[ (k_1 - P^h_1)^2 \equiv (k_1^+ - P^h_1^+)k^-_1 - (\bar{k}^+_1 - \bar{P}^h_1^+)^2 = m_q^2 \]
\[ \implies (k_1^+ - P^h_1^+) = \frac{m_q^2}{k_1^+}, \] (3.29)

where the condition \( |\bar{k}^+_1| \sim |\bar{P}^h_1^+| \ll P^h_1^+ \) has been used. Therefore, using \( k_1^+ = xP^h_1^+ \), Eqn. (3.29) shows that the on-shell quark condition corresponds to the limits of \( x \to 1 \) and \( m_q \to 0 \).

### 3.4 Hadron tensor with the radiative correction before factorization

A straightforward extension of [7–10] gives the sum of standard and non-standard contributions to the hadron tensor with radiative corrections, see Fig. 2, the left panel, and Fig. 3.
Before the factorization procedure applied, we have
\[
\mathcal{Y}^{(\text{stand.})}_{\mu\nu} = \int (d^4k_1) (d^4k_2) \delta^{(4)}(k_1 + k_2 - q) \times \int (d^4\ell) \mathcal{D}_{\alpha\beta}(\ell) \text{tr}[\gamma_5\gamma_\alpha S(k_2 - \ell)\gamma_\mu\gamma_\beta S(k_1)]
\]
\[
\times \Phi^{[\Gamma]}(k_2) \Phi^{[\Gamma]}_1(k_1; \ell) \Phi^{[\Gamma]}(k_1) \delta\big((P^h_1 - k_1)^2\big),
\]
(3.30)
for the standard diagram contribution; while, the non-standard diagram contribution is given the following expression
\[
\mathcal{Y}^{(\text{nonstand.})}_{\mu\nu} = \int (d^4k_1) (d^4k_2) \delta^{(4)}(k_1 + k_2 - q) \times \int (d^4\ell) \mathcal{D}_{\alpha\beta}(\ell) \text{tr}[\gamma_5\gamma_\alpha S(k_1)\gamma_\alpha\Gamma_1\gamma_\beta\Gamma_2]
\]
\[
\times \Phi^{[\Gamma]}(k_2) \Phi^{[\Gamma]}_1(k_1; \ell) \Phi^{[\Gamma]}(k_1) \delta\big((P^h_1 - k_1)^2\big).
\]
(3.31)
The non-perturbative functions of Eqns. (3.30) and (3.31) are defined as
\[
\Phi^{[\Gamma]}(k_2) = \sum_X \int (d^4\eta_2) e^{-ik_2\eta_2} \times \langle P_2, S_\perp | \text{tr}\left[\bar{\psi}(\eta_2)\Gamma\psi(\eta_2)\right] | S_\perp, P_2 \rangle
\]
\[
\equiv - \int (d^4\eta_2) e^{-ik_2\eta_2} \langle P_2, S_\perp | \psi(\eta_2)\Gamma\psi(0) | S_\perp, P_2 \rangle
\]
(3.32)
and
\[
\Phi^{[\Gamma]}_1(k_1; \ell) = \int (d^4\eta_1) e^{-i(P^h_1 - \ell - k_1)\eta_1} \langle P^h_1 | \psi(\eta_1)\Gamma_1 \psi(0) | 0 \rangle,
\]
(3.33)
\[
\Phi^{[\Gamma]}(k_1) = \int (d^4\xi) e^{ik_1\xi} \langle 0 | \psi(\xi) \Gamma_2 \psi(0) | P^h_1 \rangle,
\]
(3.34)
where the correlators contain the off-shell fermion operators, cf. Eqns. (3.26) and (3.27). Moreover, for the sake of practical application, we introduce the dimensionless analog of delta-function in the form of (see appendix B)
\[
\delta\big((P^h_1 - k_1)^2\big) = \lim_{\{m_\psi, k_1\} \to 0} \frac{m_\psi^2}{2\xi_1P^h_1 + \delta\left(k_1 - \frac{\left(\vec{k}_1^\perp - \vec{P}^h_1\right)^2 + m_\psi^2}{2\xi_1P^h_1}\right)},
\]
(3.35)
which ensures that one of fermions in the function \(\Phi^{[\Gamma]}_1(k_1)\) becomes the on-shell one.

3.5 The next-to-leading order function \(\Phi^{(4)}(k_1, \ell)\) with gluon radiation

In the standard and non-standard hadron tensors, see Eqns. (3.30) and (3.31), the next-to-leading function \(\Phi^{(4)}(k_1, \ell)\) (see Fig. 2, the left panel) extends the leading order function \(\Phi(k_1)\) (see Fig. 2, the right panel) and includes the functions \(\Phi^{(4)}(k_1, \ell)\) and \(\Phi^{(4)}_1(k_1)\) of Eqns. (3.33) and (3.34) (here,
we omit the $\Gamma$-structure in each functions). The gluon momentum $\ell$ dependence of $\Phi_{(2)}(k_1; \ell)$ indicates the fact that we deal with the next-to-leading order due to the gluon radiation.

To demonstrate this, we begin with the representation of the leading order function $\Phi(k_1)$ given by Eqn. (3.25). We now take into account the first order of interaction in the correlator and write the following

$$
\bar{\Phi}^{(A)}(k_1) = \frac{E^2}{m_q} \left\{ \int (d^4 \xi) e^{-i(P_1^h - k_1) \xi} \int (d^4 z) \right.
\times \langle P_1^h | \psi(0) \left[ \bar{\psi}(z) \hat{A}(z) | \psi(z) \right] \psi_{\text{on-sh.}}(\xi) | 0 \rangle \left\} 
\times \left\{ \int (d^4 \eta) e^{ik_1 \eta} \langle 0 | \psi \left( \xi \rangle \right) P_1^h \right\}.
$$

(3.36)

Here, the first figure-bracketed correlator with interactions has to be understood as the correlator written in the interaction representation (the time-ordering symbol has been omitted). Using Wick’s theorem and Fourier transformations, we derive that

$$
\bar{\Phi}^{(A)}(k_1) = \frac{E^2}{m_q} \left\{ \int (d^4 \xi) e^{-i(P_1^h - k_1) \xi} \int (d^4 \ell) \right.
\times \langle P_1^h | \psi(0) \bar{S}(k_1 - P_1^h + \ell) \hat{A}(\ell) \psi_{\text{on-sh.}}(z) | 0 \rangle \left\} 
\times \left\{ \int (d^4 \eta) e^{ik_1 \eta} \langle 0 | \psi \left( \xi \rangle \right) P_1^h \right\},
$$

(3.37)

or in the equivalent form it reads

$$
\bar{\Phi}^{(A)}(k_1) = \frac{E^2}{m_q} \left\{ \int (d^4 \ell) \tilde{\Phi}^{(2)}_0 (P_1^h - k_1) \right.
\times \bar{S}(k_1 - P_1^h + \ell) \hat{A}(\ell) \left\} \tilde{\Phi}^{(1)}_0 (k_1).
$$

(3.38)

4 Dominant role of gluon pole contribution

In this section, we give a proof that the next-to-leading order (gluon pole contribution) function $\Phi^{(A)}(k_1, \ell)$ is dominating over the leading order function $\Phi(k_1)$ in the kinematical region of $x_1 \rightarrow 1$ and $m_q \rightarrow 0$. We remind that the limits of $x_1 \rightarrow 1$ and $m_q \rightarrow 0$ correspond to the fact that the intermediate quark is on-shell, see Eqn. (3.29). The result of this section can be considered as a principle finding of our study.

The kinematical and model-independent prefactor $E^2/m_q$ which appears in both the leading and next-to-leading functions plays the crucial role in our analysis. Namely, one can see that $E^2/m_q$ at the leading function $\tilde{\Phi}(k_1)$ of Eqn. (3.25) nullify this contribution in the certain domain. Indeed, using the kinematical constrains inspired by (2.3), we have

$$
\Phi(k_1) \Rightarrow \frac{E^2}{m_q} = \frac{(k_1^+ - P_1^h + k_1^-)^2}{2m_q}
\Rightarrow \frac{(k_1^+ - P_1^h + k_1^-)^2}{2m_q} \mid_{x_1 \rightarrow 1} \lim_{m_q \rightarrow 0} \frac{\varepsilon^2}{\lim_{\delta \rightarrow 0} \delta} \sim \lim_{\varepsilon \rightarrow 0} \varepsilon = [0]^4,
$$

(4.1)
where \( \sim \) means “behaves as”.

On the other hand, the situation is changed drastically if we consider the the next-to-leading function related to the radiative correction. Focusing on the gluon pole contribution to Eqn. (3.38), which exists at \( x_2 = x_1 \), we obtain that

\[
\Phi^{(A)}(k_1) \bigg|_{x_1 \to 1, m_q \to 0} \Rightarrow \\
\frac{(k_1^+ - P_1^{h+} + k_{-})^2}{2m_q} S(k_1 - P_1^h + \ell) \bigg|_{x_1 \to 1, m_q \to 0} = \frac{(k_1^+ - P_1^{h+} + k_{-})^2}{2m_q} \gamma_+ \bigg|_{x_1 \to 1, m_q \to 0}
\]

\[
\sim \lim_{\epsilon \to 0} \frac{\epsilon^2}{\lim_{\delta \to 0} \frac{\delta}{\lim_{\eta \to 0} \eta}} \sim \lim_{\epsilon \to 0} \frac{\epsilon^2}{\epsilon^2} \equiv [1], \tag{4.2}
\]

where the representation given by

\[
k_1^+ - P_1^{h+} + \ell^+ = -\vec{x}_1 P_1^{h+} + (x_2 - x_1) P_1^{h+} \tag{4.3}
\]

has been applied. For the function \( \Phi^{(A)}(k_1) \), we are interested in the combination of \( \Phi^{(2)\tw-2} \otimes \Phi^{(1)\tw-3} \) which results in the \( \gamma^+ \)-term of the propagator in the second line of Eqn. (4.2). Thanks for Eqn. (4.2), one can see that \([0]\) stemming from the combination \( E^2/m_q \) is compensated by \([0]\) from the (anti)quark propagator if we deal with the gluon pole contribution.

It is important to stress that an alternative consideration of function \( \Phi^{(A)}(k_1) \) described in Appendix B leads to the same behaviour of \( \Phi^{(A)}(k_1) \) as in Eqn. (4.2).

Therefore, we can conclude that the leading order hadron is actually suppressed in comparison with the gluon pole contribution to the hadron tensor in the limits of \( x_1 \to 1 \) and \( m_q \to 0 \). Based on that, we are able to concentrate entirely on the contributions associated with the gluon poles.

5 Gauge-invariant hadron tensor with the radiative correction in the factorized form

We are in a position to discuss the final expression for the hadron tensor. We remind that the concrete \( \Gamma \) matrix projections (or the Fierz projections) in the corresponding functions depend on the considered case. For example, in order to describe the pion or rho meson production, the corresponding functions receive the \( \gamma \)-structure as (see (3.30) and (3.33), (3.34))

\[
\Gamma \otimes \Phi^{[\Gamma]} \Rightarrow \gamma^+ \otimes \Phi^{[\gamma^+]} \otimes \sigma^+ \otimes \Phi^{[\sigma^+]} \tag{5.1}
\]

for the leading twist two nucleon distribution functions;

\[
\Gamma_1 \otimes \Phi^{[\Gamma_1]} \Rightarrow \gamma^- (\gamma_8) \otimes \Phi^{[\gamma^- (\gamma_8)]} \tag{5.2}
\]

\[
\Gamma_2 \otimes \Phi^{[\Gamma_2]} \Rightarrow \gamma^0 (\gamma_8) \otimes \Phi^{[\gamma^0 (\gamma_8)]} + \sigma^- (\gamma_8) \otimes \Phi^{[\sigma^- (\gamma_8)]} \tag{5.3}
\]

for the pion and rho meson distribution amplitudes which correspond to the twist two (5.2) and the twist three (5.3).
While the factorization procedure applied, we deal with the gauge invariant hadron tensor that takes the form of

\[ W_{\mu\nu} = \delta^{(2)}(\vec{q}_\perp) \int (dx_1) (dy) \delta(x_1 P^+_1 - q^+) \delta(y P^-_2 - q^-) \]

\[ \times F(y) \int (dx_2) \tilde{B}(x_1, x_2) \frac{T^\nu}{P^\mu_1 P^\mu_2} \left[ \frac{P^\mu_1}{y} - \frac{P^\mu_2}{x_1} \right], \]

(5.4)

where

\[ F(y) = \left( \frac{f_T(y)}{h_1(y)} \right), \quad T^\nu = \left( \frac{\epsilon^{\mu_1 + S_\perp} P^\nu_1 V^\nu_\perp}{\epsilon^\nu S_\perp P^\mu_1} \right) \]

(5.5)

with \( V^\nu_\perp = \{ P^\perp_1, e^\perp_1 \}^\nu \). The distribution function \( F(y) \) represents the \( k^\perp \)-integrated function [5].

The structure function (or distribution function) \( \tilde{B}(x_1, x_2) \) related to gluon pole is defined as [7]

\[ \tilde{B}(x_1, x_2) = \frac{1}{2} \frac{\vec{k}_1^+ \otimes \vec{k}_1^+ \otimes \vec{k}_2^+ \otimes \vec{k}_2^+}{x_2 - x_1 - i\epsilon}, \]

(5.6)

where the transverse momentum integration is given by

\[ \tilde{B}(x_1, x_2) = \frac{1}{2} \frac{\vec{k}_1^+ \otimes \vec{k}_1^+ \otimes \vec{k}_2^+ \otimes \vec{k}_2^+}{x_2 - x_1 - i\epsilon} \times \]

\[ \int (d\ell^- d^2 \ell_\perp) \frac{\tilde{B}^{\mu w-3}(\ell_2; \vec{k}_1^+; \ell_2^- \vec{\ell}_\perp)}{\ell^- - \ell_2^- - P^+_1 (2x_2 P^+_1) + i\text{sign}(x_2) \epsilon}, \]

(5.7)

Figure 3. The hadron production in the semi-exclusive process: the standard diagram contribution (the left panel) and the non-standard diagram contribution (the right panel) to the gauge invariant hadron tensor. The dominant quark and gluon momenta \( k_1 \) and \( \ell \) lie along the plus direction while the dominant antiquark momentum \( k_2 \) — along the minus direction.
5.1 Hadron tensor with the radiative correction: the case of \( k \)-dependent nucleon distribution functions

The gauge invariant hadron tensor of (5.4) can be readily extended to the case with the essential \( k \)-dependence of the nucleon distribution function. As the first stage, we consider the nucleon matrix element of the quark vector projection given by (see (5.1) and Appendix D)

\[
\Phi^{\gamma\mu}_\alpha(y, k_\perp^2) = \epsilon^{-\pm k_\perp^2} \left( f_{(1)}(y, \vec{k}_2^\perp) S_{\alpha}^\perp + f_{(2)}(y, \vec{k}_2^\perp) s_{\alpha}^\perp \right),
\]

where \( S_{\alpha}^\perp \) and \( s_{\alpha}^\perp \) stand for the nucleon and quark spin covariant vectors, respectively. Notice that the function \( f_{(1)}(y, \vec{k}_2^\perp) \) is nothing but the well-known function \( f_{\perp T}(y, \vec{k}_2^\perp) \), while the \( k \)-dependent function \( f_{(2)}(y, \vec{k}_2^\perp) \) is a new function which has been first introduced in [14].

It is important to notice that the quark spin cannot directly be observed, but we can judge on the quark spin presence implicitly. As shown in [14], despite the angle \( \phi \), our new function \( \Phi^{\gamma\mu}_\alpha(y, k_\perp^2) \) (see Subsec. 5.2 for details) gives the kinematical constraints on this angle relating the quark spin angle to the corresponding hadron angle. Namely, the orthogonality condition required by the covariant integration leads to the (anti)collinearity of \( P_2^\perp \) and \( s^\perp \). As a result, we deal with \( \phi_P = \phi_0 \pm n\pi \) that relates the hadron momentum with the quark spin vector. In other words, this constraint can be treated as one of conditions for the existence of our new function.

Hence, for the gauge invariant hadron tensor we have the following expression

\[
\gamma^{\mu\nu}_\perp(\vec{q}_\perp) = \delta^{(2)}(\vec{q}_\perp) \int (dx_1)(dy) \delta(x_1 P_1^{h^+} - q^+) \times \delta(y P_2^- - q^-) \left\{ \int (d^2 \vec{k}_2^\perp) F_{(1),(2)}^\alpha(y, \vec{k}_2^\perp) k_2^\perp \right\} \times \int (dx_2) B(x_1, x_2) V^{\mu\nu}_\perp \frac{P_1^\mu}{P_1^\perp} \frac{P_2^\mu}{P_2} \frac{y}{x_1},
\]

where

\[
F_{(1),(2)}^\alpha(y, \vec{k}_2^\perp) = \left( f_{(1)}(y, \vec{k}_2^\perp) \epsilon^{P_1^\perp + \alpha s^\perp} + f_{(2)}(y, \vec{k}_2^\perp) \epsilon^{P_2^\perp + \alpha s^\perp} \right),
\]

with the functions \( f_{(1)}(y, \vec{k}_2^\perp) \) and \( f_{(2)}(y, \vec{k}_2^\perp) \) corresponding to the nucleon polarized and nucleon unpolarized cases. Then, we focus on the quark axial-vector projection of the nucleon correlator which reads

\[
\Phi^{\gamma\gamma}_\perp(y, k_\perp^2) = \epsilon^{-\pm k_\perp^2} f_{(3)}(y, \vec{k}_2^\perp)
\]

and defines the other new \( k \)-dependent function [14].

The covariant (invariant) integration is one of our essential tool (see, for example, [17]). It needs to define the set of exterior Lorentz tensors which are, in our case, \( (P_2^\perp, S^\perp, s^\perp) \). Appendix E summarizes the main stages of this technique.
Notice that the covariant integration of \( f_{(3)}(y, \vec{k}_2^+) k_2^+ \) over \( d^2 \vec{k}_2^+ \) does not contain the vector \( P_2^+ \). Instead, the transverse \( k_\perp \)-dependence should be realized by the quark spin covariant vector only. Indeed, the Lorentz covariant integration gives us the following

\[
\int (d^2 \vec{k}_2^+) f_{(3)}(y, \vec{k}_2^+) k_2^+ \, e^{as^+ - +} A[f_{(3)}] = \delta^{(2)}(\hat{q} \perp) \int (dx_1) (dy) \delta(x_1 P_1^{h_1} - q^+) \delta(y P_2^--q^-) \times \int (dx_2) B(x_1, x_2) |P_2^+| \cos \phi_{PS} e^{\nu_1 + P_1^{h_1} \perp} \times \frac{1}{P_1^{h_1} P_2} \frac{P_{h_1}^\mu}{y} - \frac{P_{h_2}^\mu}{x_1} A[f_{(3)}],
\]

where \( |P_2^+| \cos \phi_{PS} = (s^+ P_2^+ \perp) \) provided \( |s^\perp| = 1 \).

To conclude this subsection, one remark is in order. Having restored the corresponding Wilson line in the nucleon correlators, we may conclude that the \( k_\perp \)-dependent parton functions possess the non-trivial properties under the time-reversal transforms because these transforms convert the future-pointed WL to the past-pointed WL [13].

### 5.2 Kinematical constraints for extractions of \( k_\perp \)-dependent \( f_{(i)} \)-functions

The covariant (invariant) integrations of (5.8) and (5.11) give the constraint conditions imposed on the corresponding angles in order to single out the contribution of given \( f_{(i)} \)-functions, see Appendix E.

Let us consider the following integration, which appears in the hadron tensor after the factorization theorem used,

\[
\mathcal{J}_1 = \int (d^2 \vec{k}_2^+) f_{(1)}(y, \vec{k}_2^+) \, e^{-S_\perp k_\perp^+},
\]

where \( e^{-S_\perp k_\perp^+} = \vec{S}_\perp \wedge \vec{k}_2^+ \). Hence, we have (we omit the unit vector \( \mathbf{n} \) defining the direction of vector product)

\[
\mathcal{J}_1 = |\vec{S}_\perp| \int (d^2 \vec{k}_2^+) f_{(1)}(y, \vec{k}_2^+) |\vec{k}_2^+| \sin(\phi_2 - \phi_{k_\perp}),
\]

where, and in what follows, the angle \( \phi_2 \) is defined in \((\hat{x}, \hat{y})\)-plane as the angle between the vector \( A \) and \( \hat{x} \)-axis with \( A = (S^\perp, s^+, P_2^+, k_\perp^+) \). On the other hand, the integration \( \mathcal{J}_1 \) can be presented with the help of the covariant integration as (see Appendix E)

\[
\mathcal{J}_1 = \int (d^2 \vec{k}_2^+) f_{(1)}(y, \vec{k}_2^+)
\]

\[
\times \left\{ e^{-S_\perp P_2^+} \frac{(k_2^+ \cdot P_2^+)}{(P_2^+)^2} + e^{-S_\perp \alpha_2} e^{as_2 - +} \frac{e^{k_\perp^+ s_\perp^+}}{s_\perp^2} \right\}.
\]
Comparing the representations with and without the covariant integrations, we obtain that

\[
\int (d^2 \vec{k}_\perp) f_1(y, \vec{k}_\perp) \left[ \phi \right] \left\{ \sin(\phi_S - \phi_k) - \sin(\phi_S - \phi_p) \right\} \cos(\phi_p - \phi_k) - \sin(\phi_S - \phi_k) \cos(\phi_p - \phi_S) \right\} = 0, \tag{5.18}
\]

where all angles are determined in the two-dimensional Euclidian space forming the perpendicular plane, \( i.e. \forall \phi \in \mathbb{R}^2 \). The equation of (5.18) has a solution given by \( \phi_p = \phi_s - \pi n \). On the other hand, this condition can be considered as a necessity condition for the covariant integration.

In the similar manner, we can analyse the other \( f_{(i)} \)-functions. Finally, we can fix all the kinematical restrictions on the angles which allow to extract the corresponding \( f_{(i)} \)-function contributions to the hadron tensor. We have

\[
\begin{align*}
&f_1(y, \vec{k}_\perp) \text{ extracted at } \{ \phi_p = \phi_s - \pi n \}; \\
&f_2(y, \vec{k}_\perp) \text{ extracted at } \{ \phi_p = \phi_s + \pi, \phi_S = \phi_s - \frac{\pi}{2} \}; \\
&f_3(y, \vec{k}_\perp) \text{ extracted at } \{ \phi_{(s(e))} = \phi_k, \phi_{(s(e))} = \phi_p \}.
\end{align*}
\tag{5.19}
\]

The remaining angles, which are not shown in (5.19), should be considered as free angles.

On the other hand, the kinematical constraints of Eqn. (5.19) give a possibility to express the quark spin angle \( \phi_s \) through the hadron angles \( \phi_p \) and \( \phi_S \) which are available in experiments.

### 5.3 On the complexity of \( \tilde{B}(x_1, x_2) \)-function

The complexity of \( \tilde{B}(x_1, x_2) \)-function given by the representation of (5.6) together with (5.7) is very important property which influences on the observables. We concentrate on the analysis of complexity and we determine the possible reasons of its origin.

From Eqns. (5.6) and (5.7), one can immediately see the first source of complexity which is due to the gluon pole at \( x_2 = x_1 \) regularized by the complex prescription in the frame of contour gauge conception \([7, 11]\).

Then, we discuss the momentum dependence of nonperturbative function related to the correlator. The exact form of momentum dependence is dictated by the detailization of interactions in the correlator. Indeed, as an illustrative example, let us consider the simplest function \( \Phi^{[\Gamma]}(k) \) given by (the symbol of time-ordering is not shown)

\[
\Phi^{[\Gamma]}(k) = \int (d^4 z) e^{+ikz} \langle \psi(0) \Gamma \psi(z) S \rangle. \tag{5.20}
\]

Here, we use the interaction representation with the \( S \)-matrix for the correlator. If we neglect the interactions in the correlator, \( i.e. \ S \rightarrow 1 \), we can see that all \( k \)-dependence is accumulated in the exponential:

\[
\Phi^{[\Gamma]}(k) \bigg|_{S=1} = \int (d^4 z) e^{+ikz} \langle \psi(0) \Gamma \psi(z) \rangle. \tag{5.21}
\]

Having integrated the function \( \Phi^{[\Gamma]}(k) \) over \( k^- \) and \( \vec{k}_\perp \), we obtain that the coordinate \( z \) of \( \psi \)-function lies on the light-cone minus direction:

\[
\int (dk^-)(d^2 \vec{k}_\perp) \Phi^{[\Gamma]}(k^+, k^-, \vec{k}_\perp) \bigg|_{S=1} = \int (d^4 z) e^{+ik^+z - \delta(z^+)} \delta(\vec{z}_\perp) \langle \psi(0) \Gamma \psi(z) \rangle. \tag{5.22}
\]
However, the momentum dependence becomes more involved if we take into account, say, the second order of interactions. Namely, replacing $S$ on $S^{(2)}$ in Eqn. (5.20), we have the following expression

$$\Phi^{[\gamma]}(k) \bigg|_{S=S^{(2)}} = \int (d^4\xi) e^{i\ell \cdot \xi} \langle S(0) | S(\xi) \rangle = \left( \int (d^4\xi) \psi(0) \Gamma^r(z) \left[ \int (d^4\xi) \psi(\xi) \hat{A}_r(\xi) \psi(\xi) \right]^2 \right)$$

$$+ \int (d^4\xi) e^{i\ell \cdot \xi} \langle S(0) | S(k) \int (d^4\ell) \gamma_\alpha S(k - \ell) \gamma_\beta \hat{D}_{\alpha\beta}(\ell) \rangle \psi(\xi)$$

+(the other contributions from Wick’s theorem).

One can see that in Eqn. (5.23) the $k$-dependence of $\Phi^{[\gamma]}(k)$ is determined by the exponential and propagators, cf. Eqn. (5.21). Thus, neglecting the corresponding interaction, we simplify the momentum dependence of nonperturbative functions.

In this connection, if we neglect the interaction in the channel defined by the light-cone minus direction, using Eqns. (3.33) and (5.7) we can get that

$$\int (d\ell^-) e^{i\ell \cdot \eta^-} = \frac{e^{i\ell \cdot \eta^-}}{\ell^- - \bar{\ell}^-/(2x_{21}P_{1}^{\perp}) + i\bar{\ell}^-} = -2\pi i \theta(-\eta^-_1) \theta(\eta^-_1) e^{i\eta^-_1 \bar{\ell}^-/(2x_{21}P_{1}^{\perp}) - i\bar{\ell}^-},$$

where $|\bar{\ell}^-| \ll P_{1}^{\perp}$ has been always assumed. That is, in this case the Cauchy theorem use provides the complex $i$ accompanying the corresponding residue. Hence, the $\tilde{B}$-function takes the form of

$$\tilde{B}_{(apr.)}(x_1, x_2) \equiv \tilde{B}(x_1, x_2) = \frac{i}{x_2 - x_1 - i\epsilon} \int (d^2\tilde{k}^{\perp}) \Phi^{[\gamma]}(1) \Phi^{[\gamma]}(2)(x_1; \tilde{k}^{\perp}) + \tilde{B}(x_1, x_2) \hat{\Phi}^{[\gamma]}(2) \tilde{B}(x_2; \hat{\Phi}^{[\gamma]}(2)(x_2; \tilde{k}^{\perp} - \tilde{k}^{\perp} + \bar{\ell}^{\perp})).$$

So, we have demonstrated that the origin of complexity of $\tilde{B}$ is being entirely determined by the interaction order. Within our approximation, the function $\Phi^{[\gamma]}(2)$ contributing to $\tilde{B}$ has to be treated as a function with the correlator involving the non-interacting quarks.

### 6 Spin Asymmetry Parameters

In this section, we concentrate on the differential cross sections and the several spin asymmetry parameters which are available for the experimental studies.

First, we want to mention that, in the most general case, the cross section takes the following standard form (for the sake of shortness the normalization factors have been absorbed by the definition of integration measure)

$$\int d\sigma = \int (d\hat{P}) \delta^{(4)}(\langle \text{mom.conserv.} \rangle) |\mathcal{M}|^2,$$

where the integrated phase space given by $(d\hat{P})$ expressed through the function $\lambda(s, t, u)$ is the Lorentz invariant object as well as the process amplitude $\mathcal{M}$. Therefore, the integrated cross section is a frame-independent function of Lorentz scalars. Moreover, the contributions of vector components to the given scalars which appear in the integrated phase space and in the amplitude
can be different ones. Namely, let $s$ be a scalar that parametrizes the phase space and is equal to $(a \cdot b)_{\text{frame-1}}$ defined by components in the given frame. The same value of $s$ which is now an argument of the amplitude can be presented by different product $(a \cdot b)_{\text{frame-2}}$ defined in another frame. It means that the integrated phase space and the given amplitude as the functions of scalars can be calculated in the different frames depending on our practical choice. However, the angular dependence of differential cross-sections under the experimental studies is a frame-dependent subject.

Further, every of spin asymmetries is given by the hadron tensor that contracted with the lepton tensor. In our case, we define the lepton tensor as

$$L_{\mu \nu} = \bar{u}(l_2) \gamma^\nu u(l_1) - \bar{u}(l_2) \gamma^\mu u(l_1),$$

where the first term corresponds to the unpolarized leptons while the second term appears if the initial lepton is longitudinally polarized.

For the present study, the lepton center-of-mass system as known as the CS-frame is the most convenient frame for both the considered factorization procedure and the analysis of spin asymmetries related to the gluon pole contributions. Indeed, within the CS-frame, (a) the initial and final hadron dominant momenta are separated well ensuring the needed conditions for factorization; (b) the hadrons possess the nonzero transverse momentum components which are necessary to construct the observables associated with the gluon poles. However, from the experimental point of view, the best choice for the investigation of several observables with the essential angular dependence is provided by the the initial hadron rest frame. For example, for the JLab experiments the hadron target at the rest has being used.

In this connection, we can first calculate the contraction of lepton and hadron tensors in terms of Lorentz invariant scalar products. Afterwards, all scalar products which are appeared in according to the considered case are expressed through the frame-dependent components which match the preferable phase space system. We refer the reader to Appendix A for all details of the frame choice and of the transitions between the frames.

### 6.1 Differential cross sections

Let us now go over to the discussion on the differential cross section. At the begining, we consider the phase space, we have the following

$$d^3 P_S = \frac{d^3 P_1}{(2\pi)^3 2E_1^h} \frac{d^3 P_2}{(2\pi)^3 2E_2^h} \frac{d^3 q}{(2\pi)^3 2E_q} \int (d^4 q) \delta \left(q^2 - 2(l_1 q)\right).$$

Hence, the differential cross-section reads

$$d\sigma = \frac{d^3 P_1^h}{(2\pi)^3 2E_1^h} \int (d^4 q) \delta \left(q^2 - 2(l_1 q)\right) \times L_{\mu \nu} T_{\mu \nu}^{CI}(P_1, P_2)(x_B, y_B) \delta^{(2)}(\mathbf{q}_\perp),$$

where

$$L_{\mu \nu} = \bar{u}(l_2) \gamma^\nu u(l_1) - \bar{u}(l_2) \gamma^\mu u(l_1),$$

and

$$T_{\mu \nu}^{CI}(P_1, P_2) = T_{\mu \nu}^{CI}(x_B, y_B) \delta^{(2)}(\mathbf{q}_\perp).$$
where the gauge invariant hadron tensor, see (5.4), has been presented in the form of

$$W_{\mu\nu} = T_{\mu\nu}^{GI}(P^h_1, P_2)\mathcal{F}(x_B, y_B)\delta^{(2)}(\vec{q}_\perp)$$  \hspace{1cm} (6.5)

with the gauge invariant algebraic tensor given by

$$T_{\mu\nu}^{GI}(P^h_1, P_2) = V_{\nu}^{\perp}(P^h_1) \cdot P_2 \left[ \frac{P_{1\mu}}{y} - \frac{P_{2\mu}}{x_1} \right]$$  \hspace{1cm} (6.6)

and

$$\mathcal{F}(x_B, y_B) = \int (dx_1) (dy) \delta(x_1 P^h_1 \cdot q^+ - y P^+_2 - q^-) \times \left\{ \int (d^2\vec{k}_2^\perp) F(y; \vec{k}_2^\perp) w(\vec{k}_2^\perp) \right\} \int (dx_2) B(x_1, x_2).$$  \hspace{1cm} (6.7)

The functions $F(y; \vec{k}_2^\perp)$ and $w(\vec{k}_2^\perp)$ denote the corresponding distribution function and the weight function.

We now assume that the contraction of (6.2) and (6.5) is being expressed in terms of Lorentz scalars. The two-dimension delta function $\delta^{(2)}(\vec{q}_\perp)$ of Eqns. (5.9) and/or (6.5) is appeared thanks for the decomposition of (3.16).

Introducing the photon rapidity as

$$2 y_\gamma = \ln \frac{q^+}{q^-},$$  \hspace{1cm} (6.8)

the differential cross section takes the form of

$$d\sigma = \frac{d^3\vec{P}^h_1}{(2\pi)^3 2E_h} \int dy_\gamma \mathcal{F}(x_B, y_B) \left| \left. T_{\mu\nu}^{GI}(P^h_1, P_2) \right|_{\vec{q}_\perp = 0} \right|$$  \hspace{1cm} (6.9)

or

$$(2\pi)^3 d\sigma = \frac{d\tilde{z}}{\tilde{z}} d^2\vec{P}^h_1 \int dy_\gamma \mathcal{F}(x_B, y_B) \left| \left. T_{\mu\nu}^{GI}(P^h_1, P_2) \right|_{\vec{q}_\perp = 0} \right|$$  \hspace{1cm} (6.10)

where

$$\frac{d(P^h_1)_3}{E^h_1} \approx \frac{d(P^h_2)_3}{(P^h_2)_3} \approx \frac{dP^h_+}{P^h_+} \approx \frac{d\tilde{z}}{\tilde{z}},$$

$$\tilde{z} = \frac{P_2 \cdot P^h_1}{P_2 \cdot q} \bigg|_{CS} = \frac{P^{h+}_1}{q^+}. \hspace{1cm} (6.11)$$

Following [5], we can also introduce the other fracture parameter as

$$\tilde{y} = \frac{P_2 \cdot q}{P_2 \cdot l_1 \bigg|_{CS}} = \frac{q^+}{l^+_1}, \hspace{1cm} (6.12)$$

and we can readily derive that

$$1 - \tilde{y} = e^{2(y_\gamma - y_l)}$$  \hspace{1cm} (6.13)

with the lepton rapidity $y_l$. Hence, if $y_\gamma \to 0$ (see Eqn. (6.8)) then

$$1 - \tilde{y} = e^{-2y_l} \Rightarrow d\tilde{y} = 2e^{-2y_l} dy_l. \hspace{1cm} (6.14)$$

For the region of small lepton rapidity, we get that $d\tilde{y} \approx 2dy_l$. The photon momentum integration in the phase space is very crucial to complete the factorization procedure for the process under our consideration.
6.2 The case of unpolarized leptons and hadrons

Since every single spin asymmetry has to be normalized by the unpolarized cross section, we begin with a consideration of the case where both the lepton and hadron tensors correspond to the unpolarized particles. In Section 4, it has been proven that if we focus on the particular kinematics where $x \to 1$ and the intermediate (anti)quark is massless one, the leading order hadron tensor is suppressed in comparison with the next-to-leading hadron tensor related to the gluon pole contributions.

Hence, the differential cross section is given by the following contractions of lepton and hadron tensors:

$$d\sigma^{\text{unpol}} \sim \mathcal{L}^{\mu\nu}_\mu \left\{ \mathcal{W}^{(2)}_{\mu\nu}(\Re \tilde{B}) + \mathcal{W}^{(3)}_{\mu\nu}(3m\tilde{B}) \right\},$$  \hspace{1cm} (6.15)

where the hadron tensor is presented by (5.9) and (5.14), while the lepton tensor is taken from (6.2).

In Eqn. (6.15), the tensor contractions give the following expressions

$$\mathcal{L}^{\mu\nu}_\mu \mathcal{W}^{(2)}_{\mu\nu}(\Re \tilde{B}) =$$

$$\int (dx_1)(dy)\delta(x_1 P_1^{h,+} - q^+)\delta(y P_2^+ - q^-)$$

$$\times \left\{ \int (d^2\vec{k}_2^+) f_2(y, \vec{k}_2^+) e^{P_2^+ + k_2^+ s^+} \right\}$$

$$\times \int (dx_2)\Re \tilde{B}(x_1, x_2) \frac{(\hat{\gamma} \cdot P_1^{h,+})}{P_1^+ \cdot P_2} \sin 2\theta \cos \varphi,$$  \hspace{1cm} (6.16)

and

$$\mathcal{L}^{\mu\nu}_\mu \mathcal{W}^{(3)}_{\mu\nu}(3m\tilde{B}) =$$

$$\int (dx_1)(dy)\delta(x_1 P_1^{h,+} - q^+)\delta(y P_2^+ - q^-)$$

$$\times \left\{ \int (d^2\vec{k}_2^+) f_3(y, \vec{k}_2^+) e^{P_2^+ + k_2^+ s^+} \right\}$$

$$\times \int (dx_2)3m\tilde{B}(x_1, x_2) \frac{\hat{\gamma} \wedge P_1^{h,+}}{m_N P_1^+ \cdot P_2} \sin 2\theta \sin \varphi.$$  \hspace{1cm} (6.17)

We remind once more that the angular dependences have been fixed in the CS-frame, see Appendix A.

6.3 Single spin asymmetry

The single spin asymmetry can be determined for our process with unpolarized leptons provided the (target) initial hadron has a transverse polarization. The practical reason for the experimental study of this asymmetry can be formulated as the following: the given asymmetry directly probes the gluon pole, i.e. if the asymmetry is nonzero, the gluon pole exists. Indeed, the single spin asymmetry is usually related to the imaginary part of the gluon pole which is encoded in the distribution function $\tilde{B}(x_1, x_2)$ (see, the prefactor $1/(x_2 - x_1 - i\varepsilon)$ of Eqn. (5.6)). Notice that, within our approximation described in subsection 5.3, the single spin asymmetry is given by $\Re \tilde{B} \sim \delta(x_2 - x_1)$. 

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Thus, we have
\[ \mathcal{A}_{UT} \sim \mathcal{L}_{\mu \nu} \left\{ \mathcal{W}_{\mu \nu} (\Re \tilde{B}) + \mathcal{W}^{(1)}_{\mu \nu} (\Re \tilde{B}), \right\} \tag{6.18} \]
where the hadron tensor is determined by Eqns. (5.4) and (5.9). The expression for this contraction reads
\[
\begin{align*}
\mathcal{L}_{\mu \nu} \left\{ \mathcal{W}_{\mu \nu} (\Re \tilde{B}) + \mathcal{W}^{(1)}_{\mu \nu} (\Re \tilde{B}), \right\} = \\
\int (dx_1) (dy) \delta (x_1 P^h_1 + q^+) \delta (y P_2 - q^-) \int (dx_2) \\
\times \Re \tilde{B}(x_1, x_2) \left[ \frac{\hat{x} \cdot P^h_1}{P^h_1 \cdot P_2} \sin 2\theta \cos \phi \\
+ \int (d^2 \vec{k}_2) f_1 (y, \vec{k}_2) e^{P^h_{1 \perp} + k_2^\perp} \right] \\
+ m_N h_1 (y) \hat{y} \wedge S_{\perp} \sin 2\theta \sin \phi \right].
\end{align*}
\tag{6.19}\]

6.4 Double spin asymmetry

It is important to show that the double spin asymmetry, associated with the longitudinally polarized initial lepton and the transverse polarized initial hadron, does not exist within our approach. Indeed, the double spin asymmetry is related to the following contraction between the lepton and hadron tensors
\[
\begin{align*}
\mathcal{A}_{LT} \sim \mathcal{L}_{\mu \nu} \left\{ \mathcal{W}_{\mu \nu} (3m \Re \tilde{B}) + \mathcal{W}^{(1)}_{\mu \nu} (3m \Re \tilde{B}), \right\} \\
\sim 2 \lambda_l \epsilon^{\nu l q} \left[ \frac{P^h_{1 \mu}}{y} - \frac{P^h_{2 \mu}}{x_1} \right] \left( p^h_{1 \nu} \epsilon_{1 \nu}^{P^h_{1 \perp} + S_{\perp} \perp} \oplus m_N \epsilon_{\perp}^{V S_{\perp} P^h_{1 \perp}} \right) \\
\sim \epsilon^{\nu l q} \left( p^h_{1 \nu} \perp \epsilon_{\perp}^{V S_{\perp} P^h_{1 \perp}} \right) = (\hat{z} \cdot q) \mathbf{I}_{\perp} \wedge \mathbf{T}_{\perp} = 0
\end{align*}
\tag{6.20}\]
due to the gauge invariant combination \((\hat{z} \cdot q) = 0\).

7 Conclusions

In the paper, we have used the approach which is ground on the archetypal factorization procedure established in a series of seminal papers [1–4]. This approach has been applied to the semi-exclusive mode of the hadron production processes where the hadron tensor is given by the corresponding convolution of the distribution function and the distribution amplitudes provided a large value of intermediate quark fraction, \(x \rightarrow 1\). We have single out the fact that, in our approach, the transverse momenta of photon and produced hadron are not tied by any conditions in contrast to the TMD-factorization approaches. This is definitely a preponderance of our approach which gives a possibility to obtain a new single spin asymmetry associated with the gluon pole contribution in a wide region of the transverse momentum of produced hadron. We have suggested to consider this kind of asymmetry as a object of experimental measurements at a large value of \(\hat{P}_{h \perp}^\ast\) using the existing data of JLab.
In the paper, we have demonstrated that the leading order hadron tensor is suppressed by the model-independent kinematical factor \((\bar{x}_1 P_h^{+})^2/m_q\), see Eqn. (4.1), for \(x_1 \to 1\) and \(m_q \to 0\) in comparison with the gluon pole contribution to the hadron tensor associated with the gluon radiations. Based on this finding, we have investigated the manifestation of gluon poles in the lepton-hadron collision of hadron production at large \(x\), where the produced hadron can be described by the corresponding distribution amplitude.

In the frame of the contour gauge conception, it is known that the twist three distribution function [11] which arises from the quark-gluon correlators have to be treated as a complex function due to the gluon poles. In this context, we have studied the different sources of functional complexity which are associated with the gluon pole effects.

We have also introduced the new transverse momentum dependent distribution functions, see Eqns. (5.8) and (5.11), which should extend our knowledge on the hadron spin structure [14]. We have demonstrated that these new functions together with the gluon pole contributions have formed the new observables which should be accessible in the experimental studies based on the recent data of JLab.

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**A On the frame choice**

In this Appendix, we present the Lorentz transforms which are tying the CS-frame and the initial hadron rest frame. The boost for the initial hadron can be performed by two steps: (a) the Lorentz rotation in \(((P_2)_0, (P_2)_1)\)-plane and, then, (b) the Lorentz rotation in \(((P'_2)_0, (P'_2)_3)\)-plane. As a result, we have

\[
P_{2\mu}\bigg|_{RS} = B_{\mu\nu} P_{2\nu}\bigg|_{CS}
\]

where

\[
B_{00} = \cosh \theta_2 \cosh \theta_1, \quad B_{01} = \cosh \theta_2 \sinh \theta_1,
B_{02} = 0, \quad B_{03} = \sinh \theta_2;
B_{10} = \sinh \theta_1, \quad B_{11} = \cosh \theta_1, \quad B_{12} = 0, \quad B_{13} = 0;
B_{20} = 0, \quad B_{21} = 0, \quad B_{22} = 1, \quad B_{23} = 0;
B_{30} = \sinh \theta_2 \cosh \theta_1, \quad B_{31} = \sinh \theta_2 \sinh \theta_1,
B_{32} = 0, \quad B_{33} = \cosh \theta_2;
\]

with

\[
2\theta_1 = \ln \frac{(P_2)_0 - (P_2)_1}{(P_2)_0 + (P_2)_1}, \quad 2\theta_2 = \ln \frac{(P'_2)_0 - (P'_2)_3}{(P'_2)_0 + (P'_2)_3}
\]
and

\[ (P'_2)_0 = (P_2)_0 \cosh \theta_1 + (P_2)_1 \sinh \theta_1. \quad (A.4) \]

From Eqn. (A.3), we can readily obtain that

\[ (P_2)_0 \neq \pm (P_2)_1, \quad (P'_2)_0 \neq (P_2)_3, \quad P_2^2 = M_2^2 \neq 0. \quad (A.5) \]

Having used the hadron momentum representations in CS-frame given by (cf. (2.3))

\[
P^h_1 = \left( E^h_1, |\vec{P}^h_1| \sin \alpha, 0, |\vec{P}^h_1| \cos \alpha \right)
\]

\[
P_2 = \left( E_2, |\vec{P}_2| \sin \alpha, 0, -|\vec{P}_2| \cos \alpha \right), \quad (A.6)
\]

we derive that

\[
2\theta_1 = \ln \frac{1 - \beta_2 \sin \alpha}{1 + \beta_2 \sin \alpha},
\]

\[
2\theta_2 = \ln \frac{\sqrt{1 - \beta_2^2 \sin^2 \alpha + \beta_2 \cos \alpha}}{\sqrt{1 - \beta_2^2 \sin^2 \alpha - \beta_2 \cos \alpha}}, \quad (A.7)
\]

where

\[
\beta_2 = |\vec{P}_2|/E_2, \quad P_{2,\perp}^2 \approx M_2^2 \ll \tilde{Q}^2. \quad (A.8)
\]

Hence, we have

\[
cosh \theta_1 = \frac{1}{\sqrt{1 - \beta_2^2 \sin^2 \alpha}}, \quad \sinh \theta_1 = -\frac{\beta_2 \sin \alpha}{\sqrt{1 - \beta_2^2 \sin^2 \alpha}},
\]

\[
cosh \theta_2 = \frac{1 - \beta_2^2 \sin^2 \alpha}{1 - \beta_2^2 \sin^2 \alpha}, \quad \sinh \theta_2 = \frac{\beta_2 \cos \alpha}{\sqrt{1 - \beta_2^2}}. \quad (A.9)
\]

So, we adhere the system where the factorization of hadron tensor is more simple from the theoretical point of view. As mentioned, this system can be traced from the CS-frame (the lepton center-of-mass system). Then, we make a transformation to the initial hadron rest system which is more suitable for the considered experiment.

Within the CS-frame, the lepton sector is determined as

\[
2l_{1\mu} = Q L_\mu (\theta, \varphi; \hat{x}, \hat{y}, \hat{z}) + q_\mu,
\]

\[
2l_{2\mu} = Q L_\mu (\theta, \varphi; \hat{x}, \hat{y}, \hat{z}) - q_\mu \quad (A.10)
\]

with \(-q^2 = 2(l_1 l_2) = Q^2\) and

\[
L_\mu (\theta, \varphi; \hat{x}, \hat{y}, \hat{z}) = \\
x_\mu \cos \varphi \sin \theta + \hat{y}_\mu \sin \varphi \sin \theta + \hat{z}_\mu \cos \theta, \quad (A.11)
\]
where the normalized axes are determined as \( \hat{x}_\mu = x_\mu / \sqrt{-x^2} \) and so on. In this system, we have the basis formed by (here, symbol \( \sim \) means “behaves as” and the hat symbol means the corresponding normalization)

\[
\hat{x}_\mu \sim \hat{q}_\mu \sim \hat{e}_\mu = (0, 1, 0, 0), \\
\hat{y}_\mu \sim \hat{e}_\mu^{\pm} \sim \hat{e}_\mu = (0, 0, 1, 0), \\
\hat{z}_\mu \sim \hat{P}_1^\mu - \hat{P}_2^\mu.
\]  
(A.12)

After the boost transforms from CS-frame to the RS-frame defined by (A.2), the scalar product \( l_1 \cdot \hat{z} \) takes the following form

\[
l_1 : \hat{z} \bigg|_{RS} = A(\alpha) + B(\alpha) \cos \varphi \sin \theta + C(\alpha) \cos \theta, \tag{A.13}
\]

where

\[
A(\alpha) = \frac{Q \beta_2 \cos \alpha}{1 - \beta_2}, \quad B(\alpha) = -\frac{Q \beta_2 \sin 2\alpha}{2(1 - \beta_2)}, \\
C(\alpha) = \frac{Q(1 + \beta_2 \cos 2\alpha)}{2(1 - \beta_2)}. \tag{A.14}
\]

### B The alternative representation of functions with the on-shell fields in correlators

Now, we give an alternative representations for the functions which contain the correlators with the on-shell fields. As shown in section 3.3, the leading order function \( \tilde{\Phi}(k_1) \), which stems from the squared amplitude, is presented in the product form of functions \( \Phi^{(1)}_0(k_1) \) and \( \Phi^{(2)}_0(k_1) \). Besides this representation, it is convenient to present the function \( \Phi(k_1) \) in the equivalent form as

\[
\tilde{\Phi}(k_1) = \frac{E^2}{m_q} \Phi^{(2)}(P^h_1 - k_1) \Phi^{(1)}(k_1) \delta \left( \frac{(k_1 - P^h_1)^2}{m_q^2} - 1 \right) \tag{B.1}
\]

where

\[
\Phi^{(1)}(k_1) = \int (d^4 \xi_1) e^{ik_1 \xi_1} \langle 0 | \psi(\xi_1) \psi(0) | P^h_1 \rangle \tag{B.2}
\]

and

\[
\Phi^{(2)}(P^h_1 - k_1) = \int (d^4 \eta_1) e^{-i(P^h_1 - k_1) \eta_1} \langle P^h_1 | \psi(\eta_1) \psi(0) | 0 \rangle. \tag{B.3}
\]

That is, the functions \( \Phi^{(1)}(k_1) \) and \( \Phi^{(2)}(P^h_1 - k_1) \) involve now the correlators with only the off-shell fields and, at the same time, the on-shellness of the intermediate (anti)quark with the momentum \( k_1 - P^h_1 \) is ensured by the delta function.

For the use of the limits given by \( m_q \to 0 \) and \( x_1 \to 1 \), the function \( \tilde{\Phi}(k_1) \) can be written as

\[
\tilde{\Phi}(k_1) = \frac{m_q}{4\bar{x}_1P^h_1} \left[ \bar{x}_1P^h_1 + k_1 \right]^2 \tag{B.4}
\]

\[
\times \delta \left( k_1 - \frac{(k_1^\perp - \bar{P}_1^\perp)^2 + m_q^2}{2\bar{x}_1P^h_1} \right) \Phi^{(2)}(P^h_1 - k_1) \Phi^{(1)}(k_1).
\]
It is important to emphasize that the leading order function \( \Phi(k_1) \) exists only if the intermediate (anti)quark is on-shell. In other words, the function \( \Phi(k_1) \) can be extended beyond the on-shell condition and it is uniquely related to the square or interference of two amplitudes which are given by function \( \Phi^{(1)}_o(k_1) \) and \( \Phi^{(2)}_o(P^h_1 - k_1) \). In this connection, the on-shell delta function in Eqns. (B.1) and (B.4) is mandatory for the representations.

Let us now go over to the alternative representations for the next-to-leading function \( \Phi^{(A)}(k_1, \ell) \). We remind that in section 3.5, the representation for this function is obtained by the direct extension of the leading order function \( \Phi(k_1) \). Analogously to the above-mentioned way, there is also the equivalent and alternative representation of \( \Phi^{(A)}(k_1, \ell) \) which is more convenient to use in the most cases. In order to demonstrate it, we begin with (the spin \( \lambda \)-indices are omitted)

\[
\Phi(k_1) = \int (d^4 \xi) e^{-ik_1 \xi} \int \frac{d^3 K}{(2\pi)^3 2K_0} \times \langle 0 | \bar{\psi}(0) d^+(K)|P^h_1 \rangle \langle P^h_1 | d^-(K) | \psi(\xi) | 0 \rangle^I \tag{B.5}
\]

written in the interaction representation. The next stage is a commutation of \( S \)-matrix with the operator \( d^- (K) \) given the following expression in the first order of interaction

\[
\Phi^{(A)}(k_1) = \int (d^4 \xi) e^{-ik_1 \xi} \int \frac{d^3 K}{(2\pi)^3 2K_0} \int (d^4 \eta) e^{iK \eta} \times \langle 0 | \bar{\psi}(0) d^+(K)|P^h_1 \rangle \langle P^h_1 | \left[ \bar{\psi}(\eta) \hat{A}(\eta) \right] \psi(\xi) | 0 \rangle^I. \tag{B.6}
\]

Having used that

\[
\int \frac{d^3 K}{(2\pi)^3 2K_0} = \int (d^4 K) \delta(K^2 - m_q^2) \tag{B.7}
\]

together with Eqn. (3.24), after some algebra we derive

\[
\Phi^{(A)}(k_1) = \Phi^{(2)}(k_1) \hat{A}(0) \Phi^{(1)}_o(k_1 - P^h) \tag{B.8}
\]

where

\[
\Phi^{(2)}(k_1; \ell) = \int (d^4 z) e^{-ik_1 \xi} \langle P^h_1 | \psi(0) \psi(z) | 0 \rangle, \tag{B.9}
\]

\[
\Phi^{(1)}_o(k_1) = \int (d^4 \xi) e^{-i(k_1 - P^h_1) \xi} \langle 0 | \bar{\psi}(0) \psi_{\text{on-sh.}}(\xi) | P^h_1 \rangle. \tag{B.10}
\]

In Eqn. (B.8) \( A_\alpha(0) \) denotes the gluon field operator in the \( x \)-space (the position space). This gluon operator gives the gluon propagator in the hadron tensor after paring with the other gluon field operator appearing in the radiation correction to the corresponding quark field, see Eqns. (3.30) and (3.31).

In analogy with the leading order function, see Eqn. (B.1), the function \( \Phi^{(A)}(k_1) \) can be presented in the form as

\[
\Phi^{(A)}(k_1) = \Phi^{(2)}(k_1) \hat{A}(0) \Phi^{(1)}_o(k_1 - P^h) \times \delta \left( \frac{(k_1 - P^h_1)^2}{m_q^2} - 1 \right) \tag{B.11}
\]
or

\[
\Phi^{(A)}(k_1) = \Phi^{(2)}(k_1) \hat{A}(0) \Phi^{(1)}(k_1 - P^1_h) \\
\times \frac{m_q^2}{2 \xi_1 P^0_{h^+}} \delta \left( k_1 - \frac{(\vec{k}_1^+ - \vec{P}_{h^+})^2 + m_q^2}{2 \xi_1 P^0_{h^+}} \right).
\]

(B.12)

Notice that in contrast to the leading order function, the next-to-leading function \( \Phi^{(A)}(k_1) \) allow the off-shell extension due to the presence the interaction vertex between \( \Phi^{(2)}(k_1) \) and \( \Phi^{(1)}(k_1 - P^1_h) \).

C On the off-shell extension

In the Appendix, we give a short description of the off-shell extension method. The necessity of the off-shell extension is dictated by the fact that, as a rule, the corresponding parton distributions stem from the factorization procedure applied to the amplitudes involving the non-perturbative correlators with the off-shell fields.

The principal basis of our method is a statement that the permutation relations for fields are the translation invariant subjects for both the on-shell and off-shell particles. We remind that this general statement is forming one of the fundament axioms.

For a pedagogical reason, let us start from the permutation relations written for the on-shell scalar particles. We have

\[
[\phi^+_{\text{on-sh.}}(x), \phi^-_{\text{on-sh.}}(y)] = D^+(x, y).
\]

(C.1)

In QFT, it is imposed that the function \( D^+ \) must depend on the difference \( x - y \) only, i.e.

\[
D^+(x, y) = D^+(x - y).
\]

(C.2)

Since the Fourier transforms are the linear operations, the translation invariance property of the permutation relations, see Eqn. (C.2), is determining the commutation relations for the Fourier images of \( \phi^\pm \). Indeed, having used the Fourier transforms, we have the following

\[
[\tilde{\phi}^+_{\text{on-sh.}}(\vec{k}), \tilde{\phi}^-_{\text{on-sh.}}(\vec{p})] = \\
\int (d^3\vec{k})(d^3\vec{p}) e^{ikx - ipy} [\tilde{\phi}^+_{\text{on-sh.}}(\vec{k}), \tilde{\phi}^-_{\text{on-sh.}}(\vec{p})],
\]

(C.3)

where

\[
\tilde{\phi}^\pm_{\text{on-sh.}}(\vec{k}) \equiv \frac{a^\pm_{\text{on-sh.}}(k)}{2E(\vec{k})} \equiv \frac{a^\pm_{\text{on-sh.}}(\vec{k})}{\sqrt{2E(\vec{k})}},
\]

\[
E(\vec{k}) = \sqrt{\vec{k}^2 + m^2}.
\]

(C.4)

From Eqn. (C.3), one can see that in order to ensure the translation invariance of the permutation relation, we have to demand the commutation relation \([\tilde{\phi}^+_{\text{on-sh.}}(\vec{k}), \tilde{\phi}^-_{\text{on-sh.}}(\vec{p})]\) to be proportional to the three-dimensional delta function, i.e.

\[
[\tilde{\phi}^+_{\text{on-sh.}}(\vec{k}), \tilde{\phi}^-_{\text{on-sh.}}(\vec{p})] = \frac{1}{2E(\vec{k})} \delta^{(3)}(\vec{k} - \vec{p}).
\]

(C.5)
Hence, we obtain the standard Paul-Jordan function given by
\[
[\phi^+_{\text{on-sh.}}(x), \phi^-_{\text{on-sh.}}(y)] = \int \frac{(d^3\tilde{k})}{2E(\tilde{k})} e^{ik(x-y)} \equiv D^+(x-y). \tag{C.6}
\]

Notice that the Fourier transforms of on-shell fields, see Eqn. (C.3), can be presented in the four-dimensional forms as
\[
\phi^\pm_{\text{on-sh.}}(x) = \int (d^4\tilde{k}) e^{\pm ikx} \delta(k^2 - m^2) a^\pm_{\text{on-sh.}}(k). \tag{C.7}
\]

With the help of this representation we can readily make an extension of the considered fields beyond the on-shell surface. To this goal, we introduce the following replacement
\[
\delta(k^2 - m^2) a^\pm_{\text{on-sh.}}(k) \Rightarrow a^\pm_{\text{off-sh.}}(k), \tag{C.8}
\]
where \(a^\pm_{\text{off-sh.}}(k)\) denotes the off-shell field operator. As a consequence, the off-shell commutation relation takes the form of
\[
[a^+_{\text{off-sh.}}(k), a^-_{\text{off-sh.}}(p)] = \frac{1}{4E^2(k)} \delta^{(4)}(k - p) \tag{C.9}
\]
which results in
\[
[\phi^+_{\text{off-sh.}}(x), \phi^-_{\text{off-sh.}}(y)] = \int \frac{(d^4k)}{4E^2(k)} e^{ik(x-y)} \equiv D^+_{\text{off-sh.}}(x-y), \tag{C.10}
\]
where \(k_0 \neq E(\tilde{k})\) anymore.

### D On the Lorentz parametrization

In this Appendix, we illustrate schematically the new subtleties of Lorentz parametrization observed and discussed, in details, in [14, 15]. For the pedagogical reason, we focus on the simplest hand-bag type of diagrams which describe the forward Compton scattering amplitude (CSA). The forward Compton amplitude reads
\[
\mathcal{M}_{\mu\nu} = \langle P| a^-_{\nu}(q) S[\bar{\psi}, \psi, A] a^+_{\mu}(q)|P \rangle, \tag{D.1}
\]
where the interaction \(S\)-matrix is determined as
\[
S[\psi, \bar{\psi}, A] = T exp \left\{ i \int (d^4z) \left[ \mathcal{L}_{QCD}(z) + \mathcal{L}_{QED}(z) \right] \right\}.
\]
We stress that in contrast to the photon Fock states the hadron states, see Eqn. (D.1), cannot be expressed through the relevant operators of creation and annihilation. Albeit, the creation and annihilation hadron operators can be introduced with the help of the effective Lagrangian describing the transition of partons into hadrons, which is not our case.
Making used the commutation relations of creation (or annihilation) relevant operators with $S$-matrix together with Wick’s theorem, we can readily derive the hand-bag diagram contribution to the CSA. It reads (within the momentum representation)

$$\omega_{\mu\nu}^{\text{hand-bag}} = \int (d^4 k) \text{tr} \left[ E_{\mu\nu}(k) \Gamma \right] \Phi^{[\Gamma]}(k),$$

where $\Gamma$ implies the corresponding $\gamma$-matrix and (here, $c$ stands for the connected diagram contributions)

$$E_{\mu\nu}(k) = \gamma_\mu S(k + q) \gamma_\nu + \gamma_\nu S(k - q) \gamma_\mu,$$

$$\Phi^{[\Gamma]}(k) = \int (d^4 z) e^{ikz} \langle P, S | T \bar{\psi}(0) \Gamma \psi(z) S | \bar{\psi}, \psi, A \rangle | P, S \rangle_c.$$  \hspace{1cm} (D.4)

For the further discussion, it is instructive to introduce the notations as

$$\langle P, S | T \bar{\psi}(0) \Gamma \psi(z) S | \bar{\psi}, \psi, A \rangle | P, S \rangle = \langle P, S | O^{[\Gamma]}(0, z | \bar{\psi}, \psi, A) | P, S \rangle,$$

$$\Phi^{[\Gamma]}(k) = \mathcal{F} \langle P, S | O^{[\Gamma]}(0, z | \bar{\psi}, \psi, A) | P, S \rangle,$$  \hspace{1cm} (D.5)

where $\mathcal{F}$ denotes the corresponding Fourier transformations with the measure as

$$d\mu(z) = (d^4 z) e^{ikz} \quad \text{or} \quad d\mu(k) = (d^4 k) e^{-ikz}. \hspace{1cm} (D.6)$$

As discussed in [15], $S$-matrix generates the Wilson lines, which ensure the gauge invariance of non-local operators, and the contributions which are not being exponentiated. The latter contributions provide (a) the evolutions of the corresponding functions and (b) the tensor structure of Lorentz parametrization (see Fig. 2 in [15]). In the present paper, we mainly focus on the non-exponentiated contributions of $S$-matrix which do not refer to the Wilson lines.

Notice that the forward CSA we now consider corresponds to the simplest example from the viewpoint of factorization in comparison with the standard Drell-Yan-like processes.

Let us concentrate on the function $\Phi^{[\Gamma]}(k)$ which has to be parametrized based on the Lorentz covariance. The hadron-to-hadron matrix element of different quark-gluon operators depends on the hadron (external) and quark-gluon (internal) vectors. Namely, the hadron Fock states give the covariance. The hadron-to-hadron matrix element of different quark-gluon operators depends on the viewpoint of factorization in comparison with the standard Drell-Yan-like processes.

For the tensor structure of $S$-matrix and (here, $c$ stands for the connected diagram contributions)
However, as explained in [14, 15], the presence of interaction in the corresponding correlator from the very beginning opens the window for the new parametrization functions. Indeed, if \( S \neq I \), the quark-gluon operator, for example \( \phi([\gamma^r](0,z|\bar{\psi},A) \), has more complicated structure depending on the decomposition order over the coupling constant (here we use the notations of [14]):

$$
\phi([\gamma^r](0,z|\bar{\psi},\psi,A) \bigg|_{S \neq I} \xrightarrow{\text{Fi.tr.}} \left[ \bar{u}^{(\downarrow)}(k)\gamma^+ \gamma^s u^{(\uparrow)}(k) \right] \left[ \bar{\sigma}^{(\downarrow)}(k)\sigma^{(\uparrow)}(k) \right] \sim s_\perp, \quad (D.9)
$$

where Fi.tr. denotes the implementation of the corresponding Fierz transformations (see [14] for all details).

The r.h.s. of Eqn. (D.9) shows that, thanks to the interaction, one of the internal (quark) vectors can be associated with the quark spin even if the hadron in the correlator is unpolarized. As a result, due to the interaction in the correlator, the parametrization of the vector projection, \( \langle P,S|\phi([\gamma^r](0,z|\bar{\psi},\psi,A) \big| P,S \rangle \), receives the addition parametrization functions:

$$
\langle P,S|\phi([\gamma^r](0,z|\bar{\psi},\psi,A) \big| P,S \rangle \bigg|_{S \neq I} = (D.10)
$$

Here, we do not show the other possible new functions which can be found in [14].

To conclude this Appendix, we would like to notice that, in physical terms, the \( J_{\perp} \)-dependent function \( j_1^{(1)} \) has been entirely generated by the quark spin alignment. From the mechanical point of view, it resembles the deviation of alike-rotated balls from the straightforward motion.

\section{The covariant (invariant) integrations}

In this appendix, we describe the method of the covariant (invariant) integration which is based on the Lorentz decomposition. We begin with the simplest example which clearly illustrates the method. We consider an arbitrary correlator with one Lorentz vector index which corresponds to the forward hadron matrix element of the quark non-local operator. It reads

$$
\langle P|\mathcal{O}_\alpha(\psi(0),\bar{\psi}(z)) \big| P \rangle \equiv \bar{\mathcal{F}} = \int (d^2k_\perp) F(x,k_\perp;P)k^\perp_\alpha \equiv J^\perp_\alpha, \quad (E.1)
$$

where \( \bar{\mathcal{F}} \) denotes the Fourier transform and defined by the integration measure as

$$
d\mu^-(k) = (dz^-)e^{-ik^+z^+}\bigg|_{k^+ = xP^+} \quad (E.2)
$$

and \( a^\perp = (0,\mathbf{a}_\perp,0) \) for any four-vector \( a \). Here and in what follows, we omit the covariant and contravariant indices unless it leads to misunderstandings. In Eqn. (E.1), the hadron momentum \( P \) represents the exterior Lorentz vector characterizing the correlator \( \langle P|\mathcal{O}_\alpha \big| P \rangle \) and we assume that \( P = (P^+,0^-;\mathbf{P}_\perp) \).

We now perform the Lorentz decomposition of \( J^\perp_\alpha \) over the exterior Lorentz vector, we obtain that

$$
J^\perp_\alpha = P^\perp_\alpha \mathcal{O}_\alpha \equiv P^\perp_\alpha \int (d^2k_\perp) F(x,k_\perp;P) \frac{(k^\perp \cdot P^\perp)}{P^2_\perp}, \quad (E.3)
$$
The covariant spin axial-vector \( S_α \), which can form the Lorentz vector \( i e^{αS_{+-}} \), is not participating in the decomposition of Eqn. (E.3) even if the spin axial-vector characterizes the given correlator. Indeed, if \( S = (λP^+/M, 0, B_⊥, \vec{S}_⊥) \) then \( (P \cdot S) = (P_⊥ \cdot S_⊥) = 0 \) and, hence, \( i e^{P_⊥S_⊥} \neq 0 \). So, the Lorentz vector \( i e^{αS_{+-}} \) is not an orthogonal basis vector for the decomposition.

The Eqns. (E.1) and (E.3) correspond, roughly speaking, to two different vector representations of the single Lorentz vector \( J_α \). Let us focus on the scalar product \( (J_⊥ \cdot B_⊥) \) where an arbitrary exterior vector \( B \) which is not associated with the correlator of Eqn. (E.1). We have the following expression

\[
(J_⊥ \cdot B_⊥) = \int (d^2\vec{k}_⊥) F(x, k_⊥; P) \frac{(k_⊥ \cdot B_⊥)}{P_⊥^2}
\]

(E.4)

which involves \( J_α \) written before (see the first line of Eqn. (E.4)) and after (see the second line of Eqn. (E.4)) implementation of the covariant (invariant) integration.

From Eqn. (E.4), we can immediately derive the necessary condition for the self-consistency of the Lorentz decomposition of Eqn. (E.3). Indeed, Eqn. (E.4) can be equivalently rewritten as

\[
\int (d^2\vec{k}_⊥) F(x, k_⊥; P) \left\{ (k_⊥ \cdot B_⊥) - \left( \frac{P_⊥ \cdot B_⊥}{P_⊥^2} \right) \right\} = 0,
\]

(E.5)

where the notation \( φ_{A\beta} = φ_A - φ_β \) with \( (A, B) \equiv (B_⊥, P_⊥, k_⊥) \) has been used for the corresponding angles, see Fig. 4.

One can readily solve Eqn. (E.5), we have the solution given by \( (φ_{Bk} = φ_{BP} + φ_{Bl}) \)

\[
φ_B = φ_P \pm n\pi, \quad (E.6)
\]

\[
φ_P = φ_k \pm n\pi. \quad (E.7)
\]
These conditions ensure the self-consistency of the covariant integration (see Eqn. (E.1)). Moreover, the (anti)collinearity of \( P_\perp \) and \( B_\perp \), see Eqn. (E.6), is a necessary and enough condition for the mentioned self-consistency, while the condition given by Eqn. (E.7) is not crucial and can be omitted.

We now dwell on the practical example of the covariant (invariant) integration applied to Eqn. (5.8). In this case, we have the following decomposition

\[
K_\perp^\alpha \equiv \int (d^2\vec{k}_\perp) f_{(2)}(x, k_\perp, s_\perp; P) k_\perp^\alpha = P_\perp^\alpha \mathcal{B} + \mathcal{C}^{\alpha_+ -}
\]

(E.8)

where \( s_\perp \) implies the covariant quark spin vector and \( B_\perp = \int (d^2\vec{k}_\perp) f_{(2)}(x, k_\perp, s_\perp; P) \frac{(k_\perp \cdot P_\perp)}{P_\perp^2} \)

(E.9)

\[
\mathcal{C}^{\alpha_+ -} = \int (d^2\vec{k}_\perp) f_{(2)}(x, k_\perp, s_\perp; P) \frac{\varepsilon^{k_\perp, s_\perp_\perp}}{s_\perp^2}.
\]

(E.10)

Notice that \( \varepsilon^{k_\perp, s_\perp_\perp} = 0 \) otherwise we deal with a trivial case of \( C = 0 \) provided \( s_\perp^2 \neq 0 \). The important requirement for the Lorentz decomposition of Eqn. (E.8) is that the vectors \( P_\perp \) and \( \varepsilon^{\alpha_+ -} \) have to be orthogonal ones, i.e. they are forming the orthogonal system. In this context, we have to derive the necessary conditions for the orthogonality.

Consider the scalar product \( (P_\perp \cdot K_\perp^\alpha) \) giving

\[
(P_\perp \cdot K_\perp^\alpha) = P_\perp^2 \mathcal{B} + \mathcal{C}^{\alpha_+ -}
\]

(E.11)

which takes place if and only if

\[
\varepsilon^{\alpha_+ -} \sim \sin \varphi_p = 0 \implies \varphi_p = \varphi_\perp \pm n\pi.
\]

(E.12)

That is, the vectors \( P_\perp \) and \( s_\perp \) have to be collinear (or anti-collinear) ones. On the other hand, one may contract \( K_\perp^\alpha \) in (E.8), with the vector \( \varepsilon^{\alpha_+ -} \). This case also leads to the condition of Eqn. (E.12).

To conclude, the condition of Eqn. (E.12) is the necessary condition for the Lorentz decomposition given by Eqn. (E.8).

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