The New $\Sigma_b$ multiplet, heavy baryon mass predictions, meson-baryon universality and effective supersymmetry in hadron spectrum

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Abstract

The recent measurement by CDF $M(\Sigma_b) - M(\Lambda_b) = 192$ MeV is in striking agreement with our theoretical prediction $M(\Sigma_b) - M(\Lambda_b) = 194$ MeV. In addition, the measured splitting $M(\Sigma_b^*) - M(\Sigma_b) = 21$ MeV agrees well with the predicted splitting of 22 MeV. We point out the connection between these predictions and an effective supersymmetry between mesons and baryons related by replacing a light antiquark by a light diquark. We discuss the theoretical framework behind these predictions and use it to provide additional predictions for the masses of spin-\textsuperscript{1}{\textsubscript{2}} and spin-\textsuperscript{3}{\textsubscript{2}} baryons containing heavy quarks, as well as for magnetic moments of $\Lambda_b$ and $\Lambda_c$.

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I. STRIKING AGREEMENT WITH MESON-BARYON PREDICTIONS

A new challenge demanding explanation from QCD is posed by the remarkable agreement shown in Fig. 1 between the experimental masses 5808 MeV and 5816 MeV of the newly discovered Σ⁺_b and Σ⁻_b and the 5814 MeV quark model prediction [1] from meson masses

\[
\frac{M_{\Sigma_b} - M_{\Lambda_b}}{M_{\Sigma} - M_{\Lambda}} = \frac{(M_\rho - M_\pi) - (M_{B^*} - M_B)}{(M_\rho - M_\pi) - (M_{K^*} - M_K)} = 2.51
\]

This then predicts that the isospin-averaged mass splitting is

\[
M_{\Sigma_b} - M_{\Lambda_b} = 194 \text{ MeV}
\]

and

\[
M(\Sigma_b) = 5814 \text{ MeV},
\]

using the most recent CDF Λ_b mass measurement [2] \(M(\Lambda_b) = 5619.7 \pm 1.2\) (stat.) \(\pm 1.2\) (syst.) MeV.†

CDF obtained the masses of the Σ⁻_b and Σ⁺_b from the decay \(\Sigma_b \rightarrow \Lambda_b + \pi\) by measuring the corresponding mass differences [3,4]

\[
M(\Sigma_b^-) - M(\Lambda_b) = 195.5^{+1.0}_{-0.9} \text{ (stat.)} \pm 0.1 \text{ (syst.) MeV}
\]

(1.2)

\[
M(\Sigma_b^+) - M(\Lambda_b) = 188.0^{+2.0}_{-2.3} \text{ (stat.)} \pm 0.1 \text{ (syst.) MeV}
\]

with isospin-averaged mass difference \(M(\Sigma_b) - M(\Lambda_b) = 192 \text{ MeV}\).

The final values for Σ⁻_b and Σ⁺_b are [3]

\[
M(\Sigma_b^-) = 5816^{+1.0}_{-1.0} \text{ (stat.)} \pm 1.7 \text{ (syst.) MeV}
\]

(1.3)

\[
M(\Sigma_b^+) = 5808^{+2.0}_{-2.3} \text{ (stat.)} \pm 1.7 \text{ (syst.) MeV}
\]

with isospin-averaged mass \(M(\Sigma_b) = 5812 \text{ MeV}\).

There is also the prediction for the spin splittings, good to 5%

\[
M(\Sigma_b^*) - M(\Sigma_b) = \frac{M(B^*) - M(B)}{M(K^*) - M(K)} \cdot [M(\Sigma^*) - M(\Sigma)] = 22 \text{ MeV}
\]

(1.4)

to be compared with 21 MeV from the isospin-average of CDF measurements [3]

\[
M(\Sigma_b^{*-}) = 5837^{+2.1}_{-1.8} \text{ (stat.)} \pm 1.7 \text{ (syst.) MeV}
\]

(1.5)

\[
M(\Sigma_b^{*+}) = 5829^{+1.6}_{-1.8} \text{ (stat.)} \pm 1.7 \text{ (syst.) MeV}
\]

†Ref. [1] used an older value \(M(\Lambda_b) = 5624 \text{ MeV}[5]\), yielding \(M(\Sigma_b) = 5818 \text{ MeV}\).
Fig. 1. Experimental results from CDF for $M(\Sigma_b^+) - M(\Lambda_b)$ and $M(\Sigma_b^-) - M(\Lambda_b)$ compared with the theoretical prediction in Ref. [1].

The success of the relation (1.1) goes back to the question first raised by Andrei Sakharov: “The $\Lambda$ and $\Sigma$ are made of the same quarks; why are their masses different?” One can now pose the same question for all baryon pairs denoted by $\Lambda_f$ and $\Sigma_f$, consisting of a $u$ and a $d$ quark and a third quark $q_f$ of another flavor $f$ which can be $s$, $c$ or $b$. The spins are coupled differently and the hyperfine interaction between a $u$ quark and a $d$ quark is stronger than the hyperfine interaction between one $u$ or $d$ quark and $q_f$ quark. We also note that the hyperfine interaction in mesons between a $u\bar{d}$ pair is stronger than the hyperfine interaction between a $u$ or $d$ and an antiquark $\bar{q}_f$ of flavor $f$.

The lhs of the relation (1.1) takes the ratio of the $\Sigma_b - \Lambda_b$ mass difference which measures the hyperfine interaction difference between a $ud$ pair and a $ub$ or $db$ pair to the $\Sigma - \Lambda$ mass difference which measures the hyperfine interaction difference between a $ud$ pair and a $us$ or $ds$ pair. The rhs of the relation (1.1) takes the ratio of a combination of meson mass differences which measures the hyperfine interaction difference $u\bar{d}$ pair and a $u\bar{b}$ or $d\bar{b}$ pair. The relation (1.1) is based on the assumption that the $qq$ and $q\bar{q}$ interactions have the same flavor dependence. This automatically follows from the assumption [10] that both hyperfine
interactions are inversely proportional to the products of the same quark masses. But all that is needed here is the weaker assumption of same flavor dependence \[6\],

\[
\frac{V_{\text{hyp}}(q_i\bar{q}_j)}{V_{\text{hyp}}(q_i\bar{q}_k)} = \frac{V_{\text{hyp}}(q_i\bar{q}_j)}{V_{\text{hyp}}(q_i\bar{q}_k)}
\]

(1.6)

The original derivation \[1\] assumed that hyperfine interactions were inversely proportional to the products of quark masses, reflecting the fact that the spin-spin interaction is linear in color-magnetic moments of the quarks, which in turn are inversely proportional to quark masses,

\[
\left(1 - \frac{m_u}{m_c}\right) = M_{\Sigma^0} - M_{\Lambda^0} = 2.16 = \left(1 - \frac{m_u}{m_s}\right) = \frac{(M_{\rho} - M_\pi) - (M_{D^*} - M_D)}{(M_{\rho} - M_\pi) - (M_{K^*} - M_K)} = 2.10
\]

(1.7)

The simplicity of eq.(1.7) is somewhat misleading, because it hides the fact that the strength of the color hyperfine interaction also depends on the hadron wavefunction at the origin, which is model-dependent \[11\]. We avoid this difficulty by using the weaker assumption eq. (1.6) which is model-independent and unlike quark masses related to directly measurable observables.

Extending the relation (1.1) to any two different flavors and rearranging the two sides to create baryon-meson ratios gives \[1\]

\[
\frac{M_{\Sigma^0} - M_{\Lambda^0}}{(M_\rho - M_\pi) - (M_{B^*} - M_B)} = \frac{M_{\Sigma^0} - M_{\Lambda^0}}{(M_\rho - M_\pi) - (M_{D^*} - M_D)} = \frac{M_{\Sigma^0} - M_{\Lambda^0}}{(M_\rho - M_\pi) - (M_{K^*} - M_K)}
\]

\[
0.32 \approx 0.33 \approx 0.325
\]

(1.8)

The baryon-meson ratios are seen to be independent of the flavor \(f\).

The challenge is to understand how and under what assumptions one can derive from QCD the very simple model of hadronic structure at low energies which leads to such accurate predictions.

We shall present here many results relating meson and baryon masses which have been obtained without any explicit model for the hyperfine interaction beyond their flavor dependence. They relate experimental masses of mesons and baryons containing quarks of five different flavors \(u, d, s, c, b\) with no free parameters. It is difficult to believe that these relations are accidental when they relate so many experimentally observed masses of mesons and baryons. This suggests that any model for hadron spectroscopy which treats mesons and baryons differently or does not yield agreement with data for all five flavors is missing essential physics.

That some kind of meson-baryon or light antiquark-diquark symmetry or effective broken supersymmetry describes a number of relations between meson and baryon masses has been noted \[8\]. The new successful relations (1.8) fit into this effective supersymmetry picture.
We now develop a formal description of this effective supersymmetry [8] and obtain new relations between masses of mesons and baryons. These relations do not have a simple description in traditional QCD treatments which treat meson and baryon structures very differently.

II. THE LS TRANSFORMATION - A NEW MESON-BARYON SUPERSYMMETRY?

A. The prediction for the newly discovered Σ_b baryons

That meson and baryon masses must be related because they are made of the same quarks was first pointed out by Sakharov and Zeldovich [6] in a paper that was completely ignored until the same work was independently rediscovered [7].

We go beyond the simple quark model to find clues to the nonperturbative dynamics of QCD. We search for the minimum set of assumptions needed to derive old and new successful relations between mesons and baryons. This supersymmetry transformation goes beyond the simple constituent quark model. It assumes only a valence quark of flavor \( i \) with a model independent structure bound to “light quark brown muck color antitriplet” of model-independent structure carrying the quantum numbers of a light antiquark or a light diquark. Since it assumes no model for the valence quark, nor the brown muck antitriplet coupled to the valence quark, it holds also for the quark-parton model in which the valence is carried by a current quark and the rest of the hadron is a complicated mixture of quarks and antiquarks.

This light quark supersymmetry transformation, denoted here by \( T^S_{LS} \), connects a meson denoted by \( |\mathcal{M}(\bar{q}Q_i)\rangle \) and a baryon denoted by \( |\mathcal{B}([qq_S]Q_i)\rangle \) both containing the same valence quark of some fixed flavor \( Q_i, i = (u, s, c, b) \) and a light color-antitriplet “brown muck” state with the flavor and baryon quantum numbers respectively of an antiquark \( \bar{q} \) (\( u \) or \( d \)) and two light quarks coupled to a diquark of spin \( S \).

\[
T^S_{LS}|\mathcal{M}(\bar{q}Q_i)\rangle \equiv |\mathcal{B}([qq_S]Q_i)\rangle \tag{2.1}
\]

The mass difference between the meson and baryon related by this \( T^S_{LS} \) transformation has been shown [8] to be independent of the quark flavor \( i \) for all four flavors (\( u, s, c, b \)) when the contribution of the hyperfine interaction energies is removed. For the two cases of spin-zero [8] \( S = 0 \) and spin-one \( S = 1 \) diquarks,

\[
M(N) - \tilde{M}(\rho) = M(\Lambda) - \tilde{M}(K^*) = M(\Lambda_c) - \tilde{M}(D^*) = M(\Lambda_b) - \tilde{M}(B^*) \approx 323 \text{ MeV} \approx 321 \text{ MeV} \approx 312 \text{ MeV} \approx 310 \text{ MeV} \tag{2.2}
\]

\[
\tilde{M}(\Delta) - \tilde{M}(\rho) = \tilde{M}(\Sigma) - \tilde{M}(K^*) = \tilde{M}(\Sigma_c) - \tilde{M}(D^*) = \tilde{M}(\Sigma_b) - \tilde{M}(B^*) \approx 517.56 \text{ MeV} \approx 526.43 \text{ MeV} \approx 523.95 \text{ MeV} \approx 512.45 \text{ MeV} \tag{2.3}
\]

where

\[
\tilde{M}(V_i) \equiv \frac{3M_{V_i} + M_{P_i}}{4}; \tag{2.4}
\]
are the weighted averages of vector and pseudoscalar meson masses, denoted respectively by $M_{V_i}$ and $M_{P_i}$, which cancel their hyperfine contribution, and

$$
\tilde{M}(\Sigma_i) \equiv \frac{2M_{\Sigma^*_i} + M_{\Sigma_i}}{3}; \quad \tilde{M}(\Delta) \equiv \frac{2M_{\Delta} + M_N}{3}
$$

are the analogous weighted averages of baryon masses which cancel the hyperfine contribution between the diquark and the additional quark.

We also note the striking constancy of the difference between eqs. (2.3) and (2.2), which gives the variation of the spin splitting of the nonstrange diquark in baryons with different companion quarks,

$$
\tilde{M}(\Delta) - M(N) = \tilde{M}(\Sigma) - M(\Lambda) = \tilde{M}(\Sigma_c) - M(\Lambda_c) = \tilde{M}(\Sigma_b) - M(\Lambda_b)
$$

195 MeV $\approx$ 205 MeV $\approx$ 212 MeV $\approx$ 202 MeV

(2.6)

The ratio of the hyperfine splittings of mesons and baryons related by $T_{LS}^1$ is also independent of the quark flavor $i$ for all four flavors ($u, s, c, b$),

$$
\frac{M_\rho - M_\pi}{M_\Delta - M_N} = \frac{M_{K^*} - M_K}{M_{\Sigma^*} - M_\Sigma} = \frac{M_{D^*} - M_D}{M_{\Sigma^*_c} - M_{\Sigma_c}} = \frac{M_{B^*} - M_B}{M_{\Sigma^*_b} - M_{\Sigma_b}}
$$

2.17 $\pm$ 0.01 = 2.08 $\pm$ 0.01 = 2.18 $\pm$ 0.01 = 2.15 $\pm$ 0.20

(2.7)

That masses of boson and fermion states related by this transformation (2.1) satisfy simple relations like (1.8), (2.2), (2.3) and (2.7) remains a challenge for QCD, perhaps indicating some boson-fermion or antiquark-diquark effective supersymmetry.

The meson and baryon ratios (1.7) agree to $\pm$3%. Eq. (1.7) is based on exactly the same logic as the prediction (1.1) which is accurate to 1%. The meson and baryon ratios in eq. (2.7) differ over a range of 5%. These discrepancies at the level of several per cent presumably arise from effects that are not included in our simple model. Two such effects are:

1. Neglect of electromagnetic contributions to hyperfine interactions. These can produce small violations of the relation (1.6) for the case where the two quarks $q_j$ and $q_k$ have different electric charges.

2. Neglect of differences between the wave functions of spin-1/2 and spin 3/2 baryons,

The $\Sigma_b$ prediction (1.1) is particularly insensitive to these effects since it involves only spin-1/2 baryons and relates only hyperfine interactions of $b$ and $s$ quarks which have the same electric charge and the same ratio of the strong to electromagnetic hyperfine interactions.

The relations (2.2) and (2.3) do not assume any strengths for hyperfine interactions, only that their contributions are canceled by suitable spin averaging. They are therefore also insensitive to the electromagnetic contributions to the hyperfine interactions.

The relation (1.7) relates hyperfine interactions of $c$ and $s$ quarks which have different electric charges and different ratios of the strong to electromagnetic hyperfine interactions. This difference can easily account for discrepancies of several per cent in experimental predictions.
B. Prediction for $\Xi_b$ and $\Xi'_b$ baryons

We can now extend this supersymmetry to apply to the case where the brown muck carries one unit of strangeness and has the flavor and baryon quantum numbers respectively of a strange antiquark $\bar{s}$ or a $u$ or $d$ quark pair coupled to spin $S$.

$$M(\Xi_c) - \tilde{M}(D^*_s) = M(\Xi_b) - \tilde{M}(B^*_s)$$

(2.8)

$$\tilde{M}(\Xi'_c) - \tilde{M}(D^*_s) = \tilde{M}(\Xi'_b) - \tilde{M}(B^*_s)$$

(2.9)

These predict

$$M(\Xi_b) \approx M(\Xi_c) - \tilde{M}(D^*_s) + \tilde{M}(B^*_s) = \tilde{M}(B^*_s) + 394 \text{ MeV} = 5795 \text{ MeV}$$

$$\tilde{M}(\Xi'_b) \approx \tilde{M}(\Xi'_c) - \tilde{M}(D^*_s) + \tilde{M}(B^*_s) = \tilde{M}(B^*_s) + 545 \text{ MeV} = 5950 \text{ MeV}$$

(2.10)

Closely related work focusing on estimating the $\Xi_b$ mass appeared recently in Ref. [16], with the prediction $M(\Xi_b) = 5795 \pm 5 \text{ MeV}$. The recent CDF value $M(\Xi_b) = 5792.9 \pm 2.4 \pm 1.7 \text{ MeV}$ [17,19], announced after Ref. [16] appeared, is in surprising agreement with this prediction and with the result (2.10). These results are also consistent with the D0 value $M(\Xi_b) = 5774 \pm 11 \pm 15 \text{ MeV}$ [18].

Both experiments are also consistent with a prediction in Ref. [20], $M(\Xi_b) = M(\Lambda_b) + (182.7 \pm 5.0) \text{ MeV} = (5802.4 \pm 5.3) \text{ MeV}$. But we have used both meson and baryon masses as input and only considered hadrons containing strange quarks, while Ref. [20], has used input only from baryon masses but also included nonstrange baryons. Any differences in these predictions can pinpoint the relative importance of deviations from assumed strangeness dependence and from assumed meson-baryon supersymmetry.

That the value of $m_b - m_c$ obtained from $B$ and $D$ mesons depends upon the flavor of the spectator quark was noted in Ref. [21]. Table I of Ref. [21] shows that the value of the effective quark mass difference $m_b - m_c$ obtained from experimental hadron masses is the same for mesons and baryons not containing strange quarks but different when obtained from $B_s$ and $D_s$ mesons. Some reasons for this difference were noted and the issue requires further investigation.

The values 394 MeV and 545 MeV in eqs. (2.8) and (2.9) can be considered as an effective mass difference between $(us)$ diquarks with respectively spin zero and spin one and a strange antiquark. This gives $545 - 394 \approx 151 \text{ MeV}$ for the hyperfine splitting of a $(us)$ diquark. They can be compared with the corresponding values 312 MeV and 520 MeV for the effective mass difference between $(ud)$ diquarks with respectively spin zero and spin one and a nonstrange antiquark. This gives $520 - 312 \approx 208 \text{ MeV}$ for the hyperfine splitting of a $(ud)$ diquark and $(208/151) \approx 1.4$ for the ratio of the two hyperfine splittings. This is in reasonable agreement with values for the strangeness dependence of hyperfine splittings obtained from other data.

The hyperfine interaction between the heavy quark, $c$ or $b$ acts differently on the spins of the $u$ and $s$ quarks in the strange diquark. The small mixing produced between the baryon states containing $(us)$ diquarks with spin zero and spin one [12–15] has now been shown to be negligible [16].
C. Extending the supersymmetry to doubly strange diquarks

We can now extend the effective supersymmetry to the case of hadrons related by changing a strange antiquark $\bar{s}$ to a doubly strange $ss$ diquark coupled to spin $S = 1$.

$$\frac{M(\Xi^*) - M(\Xi)}{M(K^*) - M(K)} = \frac{M(\Omega_c^*) - M(\Omega_c)}{M(D_s^*) - M(D_s)} = 0.54 \approx 0.50$$ (2.11)

There is a remarkably successful relation for the spin-averaged doubly stranged and charmed-doubly-strange baryons

$$\tilde{M}(\Xi) - \tilde{M}(K^*) = \tilde{M}(\Omega_c) - \tilde{M}(D_s^*)$$

$$668.8 \text{ MeV} \approx 669.2 \text{ MeV}$$ (2.12)

These relations have been recently extended to $b$-flavored hadrons [22].

III. MAGNETIC MOMENTS OF HEAVY QUARK BARYONS

In $\Lambda$, $\Lambda_c$ and $\Lambda_b$ baryons the light quarks are coupled to spin zero. Therefore the magnetic moments of these baryons are determined by the magnetic moments of the $s$, $c$ and $b$ quarks, respectively. The latter are proportional to the chromomagnetic moments which determine the hyperfine splitting in baryon spectra. We can use this fact to predict the $\Lambda_c$ and $\Lambda_b$ baryon magnetic moments by relating them to the hyperfine splittings in the same way as given in the original DGG [10] prediction of the $\Lambda$ magnetic moment,

$$\mu_\Lambda = -\frac{\mu_p}{3} \cdot \frac{M_{\Sigma^*} - M_{\Sigma}}{M_\Delta - M_N} = -0.61 \text{ n.m.} \quad \text{(EXP = -0.61 n.m.)}$$ (3.1)

We obtain

$$\mu_{\Lambda_c} = -2 \mu_\Lambda \cdot \frac{M_{\Sigma_c^*} - M_{\Sigma_c}}{M_{\Sigma^*} - M_{\Sigma}} = 0.43 \text{ n.m.}$$ (3.2)

$$\mu_{\Lambda_b} = \mu_\Lambda \cdot \frac{M_{\Sigma_b^*} - M_{\Sigma_b}}{M_{\Sigma^*} - M_{\Sigma}} = -0.067 \text{ n.m.}$$ (3.3)
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