A model for the $B \rightarrow X_s + \gamma$ decay in the chromomagnetic background

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Abstract

We calculate the shape of the photon spectrum for $B \rightarrow X_s + \gamma$ decay in the presence of a background chromomagnetic field for the case of SU(2) gauge group. The effect of the external field resembles the Zeeman effect - the parton model peak is split into two slightly asymmetric peaks, located around the kinematic endpoint. We investigate the analytic properties of the spectrum and calculate the total decay rate. This decay may serve as a model for the investigation of gluon condensate effects and their influence on the shape of the spectrum.

Recently, there has been considerable interest in the photon energy spectrum in the decay $B \rightarrow X_s + \gamma$, inspired by the first experimental measurement of this decay by the CLEO collaboration [1]. Considerable theoretical progress has been made by applying Wilson’s operator product expansion
(OPE) and the heavy quark effective theory (HQET) to the study of the endpoint region of the photon spectrum in this decay. The inclusion of higher order matrix elements from the OPE smears the parton model spectrum over a region of the order of $\Delta y \approx \Lambda/m_b$, where $y = 2E_\gamma/m_b$ ($E_\gamma$ is the $\gamma$-energy and $\Lambda$ is the QCD parameter). The detailed shape of this nonperturbative spectrum is, however, unknown, because it depends on the expectation values of higher order in $1/m_b$ matrix elements. The radiative corrections give additional uncertainty to the shape of the spectrum, because the higher order terms in the perturbative expansion are important. Some attempts have been made to include the higher order perturbative terms by resumming the “renormalon chains”. However, the terms of a very high order in the “renormalon chain” come from large distances and have to be treated separately, as a condensate.

In this paper we consider the $B \to X_s + \gamma$ decay in the external chromomagnetic background for the case of an SU(2) gauge group. The possibility that a confining ground state in QCD develops a nonzero color background field was discussed a long time ago. The decay in the background field may serve as a model for the investigation of gluon condensate effects. This model allows the nonperturbative analytical calculation of the photon spectrum and the detailed investigation of its properties. The spectrum depends on one parameter, $p = B^2/m_b^2$, where $B \sim \Lambda$ is the magnitude of the external field. We show that the spectrum is nonvanishing in the region $1 - \sqrt{p}/2 \leq y \leq 1 + \sqrt{p}/2$. The shape of the spectrum resembles the shape of spectrum in the case of the quantum mechanical Zeeman effect - the initial $\delta$-function peak is split into two smeared peaks, located slightly
asymmetry around the kinematic endpoint $y = 1$. We also obtain the analytic expression for the total decay rate and calculate several moments of the spectrum. First, we review very briefly the formalism for the theoretical treatment of radiative B decay and then calculate the photon spectrum in the presence of the background field.

The effective Hamiltonian for $B \to X_s + \gamma$ decay may be written as

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} |V_{tb}V_{ts}^*| c_7^{\text{eff}}(m_b) O_7(m_b),$$

(1)

where $O_7 = e/16\pi^2 \bar{s} \sigma^{\mu\nu}(m_b(1 + \gamma_5)/2 + (1 - \gamma_5)/2) b F_{\mu\nu}$ and $c_7^{\text{eff}}(m_b)$ is the Wilson coefficient at the scale $\mu \sim m_b$ (for the recent review see [6]). In this paper we assume for simplicity $m_s = 0$.

The decay rate can be written in terms of the imaginary part of a correlator of two local currents [7]

$$T(v \cdot k) = -i \int d^4x e^{i(m_b-v-k)x} \langle B | T\{J^\mu(x), J^\mu_\mu(0)\} | B \rangle$$

(2)

where the current has the form

$$J^\mu(x) = \bar{b}_v(x) [\gamma^\mu, \hat{k}] m_b(1 - \gamma_5/2) \cdot s(x)$$

and $\bar{b}_v(x) = e^{im_bv-x} b(x)$, $k$ is the photon momentum.

The expression for the decay rate is

$$d\Gamma = \frac{d^3k}{(2\pi)^3} \frac{\alpha G^2_F}{8\pi^3} |V_{tb}V_{ts}^*|^2 |c_7^{\text{eff}}(m_b)|^2 \Im T(v \cdot k).$$

(3)

The leading term in $1/m_b$ expansion corresponds to the parton model result and is given by

$$T(v \cdot k) = -i m_b^2 \int d^4x e^{i(m_b v - k) \cdot x}$$

$$\langle B | \bar{b}_v(x) [\gamma^\mu, \hat{k}] \frac{(1 - \gamma_5)}{2} (iS_F(x)) \frac{(1 + \gamma_5)}{2} [\hat{k}, \gamma^\mu] b_v(0) | B \rangle$$

(4)
where $S_F$ is the final $s$-quark free propagator.

This expression leads to the differential decay rate

$$\frac{d\Gamma}{dy} = \Gamma_0 \delta(1-y),$$

where $y = \frac{2E_\gamma}{m_b}$ and

$$\Gamma_0 = \frac{\alpha m_b^5 G_F^2}{32\pi^4} | V_{tb} V_{ts}^*|^2 | c_7^{\text{eff}}(m_b) |^2$$

is the parton model total decay rate [7].

Now we want to investigate the influence of the external gluon field on the shape of photon spectrum. The external field with field strength of the order of $\Lambda$ gives a nonperturbative example of how the gluon condensate corrections to the final state $s$-quark would affect the spectrum.

We consider the case of the SU(2) gauge group, which allows analytical calculations. We work in an axially symmetric anzatz for the background field. The axially symmetric anzatz may represent the solution of the Yang-Mills equations in the strong coupling limit [10], if we assume that the external source is axially symmetric, what represents, probably, the final event symmetry of the decay. We neglect also any space-dependence of the external field. We have to note the unphysical features of the model, such as the apparent lack of the Lorentz and gauge invariance due to the presence of a constant background field. However, we restore, formally, these symmetries. We take only gauge-independent contribution to the decay rate and integrate it with respect to the possible direction of the magnetic field, restoring the gauge and rotational invariance. To restore formally relativistic invariance, we assume that the value of parameter of the model corresponds to the
Lorentz invariant condensate value.

Thus, we chose the background SU(2) field configuration as

$$A_0 = 0; \quad gA_1 = B\frac{\sigma_1}{2}; \quad gA_2 = B\frac{\sigma_2}{2}; \quad A_3 = 0$$ (6)

where $\sigma_i$ are the $\sigma$-matrices. This configuration corresponds to the chromomagnetic field directed along the third axis in the configuration and coordinate spaces

$$gF_{12} = B^2\frac{\sigma_3}{2}$$ (7)

To take into account the effect of this field on the shape of the spectrum we have to use the $s$-quark propagator in the presence of the external background. We will find the analytic expression for the propagator, so this model may be considered as a model for the summation of the infinite series of the gluon condensates insertions into the final $s$-quark. The expression for the propagator,

$$G(q) = \frac{(\bar{Q})}{Q^2} = \frac{(\bar{Q})}{Q^2 + 1/2ig\sigma_{\mu\nu} F^{\mu\nu}}, \quad \sigma_{\mu\nu} = 1/2[\gamma_{\mu}, \gamma_{\nu}]$$ (8)

(where the $Q_\mu = q_\mu + gA_\mu$ and $q = m_b\cdot v - k$) may be found using the Schwinger proper time formalism [8]. The propagator can be written as

$$G(q) = -i(\hat{q} - 1/2 B (\sigma_\gamma)_\perp) \int_0^\infty ds e^{is(q^2-B^2/2)} e^{isD_0}, \quad \text{where we use the notation} \ (a \cdot b)_\perp = a_1b_1 + a_2b_2 \text{ and the operator} \ D_0 \text{ is given by} \ D_0 = -B(q\sigma)_\perp + B^2/2 (\Sigma_3\sigma_3); \quad \Sigma_3 = i\gamma_1\gamma_2.$$
Using that $D_0^2$ is proportional to the unit operator, the result is

$$G(q) = -i(\dot{q} - 1/2B(\sigma\gamma))_\perp \int_0^\infty ds \, e^{is(q^2 - B^2/2)} \{\cos(s \mid D_0 \mid) + \frac{iD_0}{|D_0|} \sin(s \mid D_0 \mid)\},$$

(10)

where $|D_0| = B\sqrt{q^2_\perp + B^2/4}$.

In the space of the SU(2) gauge group, the propagator may be written as

$$G(q) = C_0 + C_1\sigma_1 + C_2\sigma_2 + C_3\sigma_3,$$

(11)

where $C_i$ are the functions of the moment.

We have to plug this propagator into the expression (4) for the $T$ operator and take the heavy quark matrix elements. Because the result has to be gauge invariant, we assume that only the $C_0$ term (which is gauge independent) gives contribution to $T$. Moreover, we will integrate the imaginary part of $T$ with respect to the possible direction of the magnetic field, restoring, therefore, the rotational invariance.

The expression for the $C_0$ is

$$C_0 = \frac{\dot{q}(q^2 - B^2/2) - B^2/2(q\gamma)_\perp}{(q^2 - B^2/2)^2 - B^2(q^2_\perp + B^2/4)}.$$

(12)

The final expression for $T$ has the following form

$$T = -4m_b^4 y^2 \frac{(1 - y - p/2 + p/4 y \sin^2\Theta)}{(1 - p/4)(y - y_1)(y - y_2) + p/4 y^2 \cos^2\Theta}$$

(13)

where we introduce the parameter $p = B^2/m_b^2$, $\Theta$ is the angle between the space direction of the magnetic field and the photon 3-momentum $k$. Here

$$y_{1,2} = 1 + \frac{(-p \pm 2\sqrt{p})}{(4 - p)}.$$
The imaginary part of $T$ is nonzero for $y_1 \leq y \leq y_2$. For small $p$ it corresponds to a cut (the branch singularity), located at

$$1 - \frac{\sqrt{p}}{2} \leq y \leq 1 + \frac{\sqrt{p}}{2}$$

(14)

We take the imaginary part of $T$ and integrate it with respect to the angle $\Theta$. The regularization prescription $q^2 \to q^2 + i\epsilon$ (where $q = m_b v - k$) is used. Also we use the prescription

$$\text{Im} \frac{1}{x \pm i\epsilon} = \mp \pi \delta(x).$$

The result for the differential decay rate is

$$\frac{d\Gamma}{dy} = \begin{cases} \frac{\Gamma_0}{\sqrt{p}} \left( \frac{y^2(1 - y - p/2 + p/4 y)}{\sqrt{(1 - p/4)(y - y_1)(y_2 - y)}} - y \sqrt{(1 - p/4)(y - y_1)(y_2 - y)} \right) & \text{for } y_1 \leq y \leq \frac{y_1 + y_2}{2} \\ \frac{\Gamma_0}{\sqrt{p}} \left( - \frac{y^2(1 - y - p/2 + p/4 y)}{\sqrt{(1 - p/4)(y - y_1)(y_2 - y)}} + y \sqrt{(1 - p/4)(y - y_1)(y_2 - y)} \right) & \text{for } \frac{y_1 + y_2}{2} \leq y \leq y_2 \end{cases}$$

(15)

The shape of the spectrum is shown in Fig.(1). The spectrum has two peaks, located at $y_{1,2} \approx 1 \mp \frac{\sqrt{p}}{2}$, smeared over the region between this points. Thus, the effect from the chromomagnetic field resembles the Zeeman effect - the parton model single peak, located at the kinematic endpoint $y = 1$, is split into two peaks. The right peak corresponds to the final $s$-quark with the polarization directed along the magnetic field, while the left peak corresponds to the $s$-quark with the opposite polarization. The spectrum is negative in the region $(1 - p/2) \lesssim y \lesssim (1 - p/4)$ and has a discontinuity at $y \approx 1 - p/4$. 

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We suppose that this negative “gap” and discontinuity are spurious and related to the presence of the constant background field in this model. For a gauge-dependent background field $C_i \ (i = 1, 2, 3)$ terms in propagator (11) give additional contribution to the $T$ operator (4) and may cure these defects. Moreover, the motion of the $b$-quark inside the $B$-meson [11] and the radiative corrections must smear the spectrum and “raise” it in this region.

We calculate the total decay rate in this model. The result is

$$\Gamma = \Gamma_0 \frac{8 (16 - 16p + 7p^2 - p^3)}{(4 - p)^{7/2}} = \Gamma_0 (1 - \frac{p}{8} + \frac{7}{128}p^2 + O(p^3)) \quad (16)$$

We note that the contribution from the right peak to the total decay rate is slightly larger than the contribution from the left peak; the difference is of the order of $O(\sqrt{p})$. This asymmetry arises because the $s$-quark polarization...
along the magnetic field is more preferable energetically than the opposite polarization. We suppose that asymmetry of the spectrum with respect to the kinematic endpoint \( y = 1 \) is a general feature of the spectrum, related to the presence of \( \sigma_{\mu\nu} F^{\mu\nu} \) term in propagator (8).

We calculate also the moments of the photon energy spectrum in this model. We define the moments as

\[
\bar{a}_k = \int (y - s)^k \frac{1}{\Gamma} \frac{d\Gamma}{dy} dy,
\]

where we normalize the differential decay rate on the total decay rate (16) and define the moments around the reference point \( y = s \approx 1 + p/6 \). For such a choice

\[
\bar{a}_0 = 1, \quad \bar{a}_1 = 0
\]

The next two moments are

\[
\bar{a}_2 = p/6, \quad \bar{a}_3 = -\frac{2p^2}{15}.
\]

(17)

Even for the case of SU(2) gauge group it is helpful to make some estimate for the value of the parameter \( p \). Restoring formally the relativistic invariance, we assume that the value of parameter \( p^2 \) corresponds to the condensate value

\[
p^2 \sim \frac{\langle 0 | \alpha Tr F_{\mu\nu} F^{\mu\nu} | 0 \rangle}{m_b^4}.
\]

Because the condensate value is of the order of \( \Lambda^4/m_b^4 \), the value of the parameter \( p \) is \( p \approx \Lambda^2/m_b^2 \). Thus, according to the equation (16), the expansion around \( p = 0 \) resembles the \( 1/m_b \) expansion (11). The estimate for the moment \( a_2 = \bar{a}_2 m_b^2/\Lambda^2 \) is \( a_2 \sim 0.2 \) and \( a_3 = \bar{a}_3 m_b^3/\Lambda^3 \sim -0.01 \). The recent
estimates for these moments are $a_2 \sim 0.7$, $a_3 \sim -0.3$ (see [12] for the review), so the value of second moment and the sign of the third moment are consistent with these estimates.

We want to note that the absence of $\sqrt{p} \approx \Lambda/m_b$ term in the total decay rate ([16]) is in agreement with Luke’s theorem [13], and the size of the smeared region of the spectrum $\sqrt{p} \approx \Lambda/m_b$ (what may be seen from (14)) is consistent with the estimates in other models [11].

In summary, we have calculated the shape of the photon spectrum in the chromomagnetic SU(2) field. This model may be considered as a model for the summation of the infinite series of the gluon condensates. The effect of the external field resembles the Zeeman effect. We investigated the analytical properties of the spectrum and calculated the total decay rate. In a more rigorous model we would take into consideration the fluctuation of the magnitude and the direction of the background field. We expect that these fluctuations lead to overlapping of the peaks in Fig. (1). Corrections due to motion of the $b$-quark inside the B-meson [11] and radiative corrections should smear the spectrum further, leading to the broad peak, observed in the CLEO experiment [1].

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