A Family-nonuniversal $U(1)'$ Model for Excited Beryllium Decays

Beyhan Pulıcê

Department of Physics, ˙Izmir Institute of Technology, TR35430, ˙Izmir, Turkey

Abstract. Excited beryllium has been observed to decay into electron-positron pairs with a 6.8 $\sigma$ anomaly. The process is properly explained by a 17 MeV proto-phobic vector boson. In present work, we consider a family-nonuniversal $U(1)'$ that is populated by the $U(1)'$ gauge boson $Z'$ and a scalar field $S$, charged under $U(1)'$ and singlet under the Standard Model (SM) gauge symmetry. The SM chiral fermion and scalar fields are charged under $U(1)'$ and we provide them to satisfy the anomaly-free conditions. The Cabibbo-Kobayashi-Maskawa (CKM) matrix is reproduced correctly by higher-dimension Yukawa interactions facilitated by $S$. The vector and axial-vector current couplings of the $Z'$ boson to the first generation of fermions do satisfy all the bounds from the various experimental data. The $Z'$ boson can have kinetic mixing with the hypercharge gauge boson and $S$ can directly couple to the SM-like Higgs field. The kinetic mixing of $Z'$ with the hypercharge gauge boson, as we show by a detailed analysis, generates the observed anomalous decays. In this work, we extend the SM with a family-nonuniversal $U(1)'$ with its associated light gauge boson $Z'$ and a singlet scalar $S$. In the model, there are two mixings with the SM: the gauge kinetic mixing of the hypercharge gauge boson and the $Z'$ boson, and the quartic scalar mixing of the SM-like Higgs and the extra scalar. The masses of the gauge bosons are generated dynamically through spontaneous symmetry breaking (SSB) via vacuum expectation values (vev) of the scalar fields.

There is a new matter content to cancel the anomalies. Another recent interpretation makes an extension of the SM with two gauge groups, $U(1)\gamma' \times U(1)\times$, and they add a new matter content to get rid of the $Z-Z'$ mass mixing. In this work, we construct a framework in which various anomalous SM decays can be discussed.

PACS. XX.XX.XX No PACS code given

1 Introduction

The Atomki experiment has recently observed a 6.8 $\sigma$ anomaly [1] (see also [2,4]) in excited $^8$Be nuclear decays, $^8\text{Be} \to ^8\text{Be} e^+e^-$, in both the distributions of the opening angles and the invariant masses of the electron-positron pairs (IPC). The SM predicts the angular correlation between the emitted $e^+e^-$ pairs to drop rapidly with the separation angle. However, the experiment observed a bump with a high significance at a large angle of $\approx 140^\circ$ which is consistent with creation and subsequent decay of a new particle with an invariant mass of $m_{e^+e^-} = 16.7\pm 0.35\text{(stat)} \pm 0.5\text{(sys)}\text{ MeV}$. In [5], they observed a peak in $e^-e^+$ angular correlations at $115^\circ$ with $7.2\ $\sigma$ in 21.01\text{ MeV} 0^- \to 0^+$ transition of $^4\text{He}$ and it is described with a light particle with a mass of $m_{ee}^2 = 16.84\pm 0.16\text{(stat)} \pm 0.20\text{(sys)}\text{ MeV}$. It is likely the same particle with the one that is observed in [1].

In recent interpretations of the experiment [2,4], possible particle physics interpretations of the $^8$Be anomalous decays are examined and concluded that a proto-phobic, spin-1 boson with a mass of $\approx 17\text{ MeV}$ fit the anomaly. They determine the bounds on the vector current couplings of the new gauge boson to the first generation of the SM fermions via combination of the relevant experimental data. They propose two particle physics models, $U(1)_B$ and $U(1)_{B-L}$ models, that are not initially anomaly-free therefore they add a new matter content to cancel the anomalies. Another recent interpretation [5] makes an extension of the SM with two gauge groups, $U(1)\gamma' \times U(1)\times$, and they add a new matter content to get rid of the $Z-Z'$ mass mixing. In [4], they present a $U(1)'$ extended 2-Higgs doublet model for $^8$Be anomalous decays. In [4], a pseudoscalar and in [1] an axial vector candidates are presented. The extension of the minimal supersymmetric standard model (MSSM) by an extra $U(1)'$ is discussed in [4] with $U(1)'$ charges of the fields to be family-dependent and satisfy the anomaly-free conditions.

In this work, we extend the SM with a family-nonuniversal $U(1)'$ with its associated light gauge boson $Z'$ and a singlet scalar $S$. In the model, there are two mixings with the SM: the gauge kinetic mixing of the hypercharge gauge boson and the $Z'$ boson, and the quartic scalar mixing of the SM-like Higgs and the extra scalar. The masses of the gauge bosons are generated dynamically through spontaneous symmetry breaking (SSB) via vacuum expectation values (vev) of the scalar fields.

Our first intention in this work is to construct the framework of an anomaly-free, family-nonuniversal $U(1)'$ model that fits the Atomki signal with a minimal field content. The model we present is able to explain the Atomki signal with a proto-phobic gauge boson with a mass of $\approx 17\text{ MeV}$. We find the couplings of the $Z'$ boson to the first generation of the SM fermions via the family-nonuniversal charges of the chiral fields that satisfy the
anomaly-free conditions. We show that with these couplings we are able to explain the Atomki signal.

The paper is organized as follows. In Sec.2 we construct the framework of the family-nonuniversal $U(1)'$ model. We summarize the experimental bounds in Sec.3. We give the vector and axial-vector current couplings of the $Z'$ boson to the first generation of the SM fermions in Sec.4. We show that the CKM matrix is properly obtained in the model in Sec.5. In Sec.6, we consider the LHC bound on the invisible decays of the SM Higgs. We summarize the model and discuss future prospects in Sec.7.

## 2 Family-nonuniversal $U(1)'$ Model

In this section, we present the framework of the family-nonuniversal $U(1)'$ model. We extend the SM gauge symmetry, $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$, by an extra $U(1)'$ symmetry

$$G_{SM} \times U(1)' \quad (1)$$

The $U(1)'$ quantum number assignment to chiral fermion and scalar fields is given in Tab.1.

|  | $SU(3)_c$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)'$ |
|---|---|---|---|---|
| $Q_i$ | 3 | 2 | 1/6 | $Q_i$ |
| $u_{R_i}$ | 3 | 1 | 2/3 | $Q_{u_{R_i}}$ |
| $d_{R_i}$ | 3 | 1 | $-1/3$ | $Q_{d_{R_i}}$ |
| $L_i$ | 1 | 2 | $-1/2$ | $Q_{L_i}$ |
| $e_{R_i}$ | 1 | 1 | $-1$ | $Q_{e_{R_i}}$ |
| $H$ | 1 | 2 | 1/2 | $Q_{H}$ |
| $S$ | 1 | 1 | 0 | $Q_{S}$ |

Table 1: The gauge quantum numbers of the fields in the family-nonuniversal $U(1)'$ model for $i = 1, 2, 3$ which refers to the three generations of matter.

### 2.1 Mixing of Higgs Bosons

The Lagrangian of the scalars in the family-nonuniversal $U(1)'$ model is given by

$$\mathcal{L}_{Higgs} = \mathcal{L}_{Higgs}^{SM} + \mathcal{L}_{Higgs}^{mix}; \quad (2)$$

$$\mathcal{L}_{Higgs}^{SM} = |D_{\mu}H|^2 + \mu_s^2 |H|^4 - \lambda_s |H|^4, \quad (3)$$

$$\mathcal{L}_{Higgs}^{S} = |D_{\mu}\hat{S}|^2 + \mu_s^2 |\hat{S}|^2 - \lambda_s |\hat{S}|^4, \quad (4)$$

$$\mathcal{L}_{mix}^{mix} = -\kappa |\hat{H}|^2 |\hat{S}|^2 \quad (5)$$

where the last equation contains a mixing term with a scalar mixing parameter $\kappa$. The hatted fields are used since we will use the fields without hat in the mass-basis.

We parametrize the SM-like Higgs $H$ and the extra scalar $\hat{S}$, respectively as

$$\hat{H} = \frac{1}{\sqrt{2}} \left( \phi_1 + i \phi_2 \right) v + \hat{h} + i \phi_3,$$
$$\hat{S} = \frac{1}{\sqrt{2}} \left( v_s + \hat{s} + i \phi_s \right) \quad (6)$$

where $\phi_1, \phi_2, \phi_3$ and $\phi_s$ are the Goldstone bosons; $v$ and $v_s$ are vevs of the scalar fields that are real and positive.

The scalar potential is bounded from below provided that

$$\lambda > 0, \quad \lambda_s > 0 \quad \text{and} \quad 4\lambda \lambda_s - \kappa^2 > 0. \quad (7)$$

For both nonvanishing values of vevs, the minimum of the potential occurs at

$$\frac{v^2}{2} = 2\lambda \mu^2 - \kappa \mu_s^2 \quad (8)$$
$$\frac{v_s^2}{2} = 2\mu_s^2 - \kappa \mu^2 \quad (9)$$

These solutions are physical for $v^2 > 0$ and $v_s^2 > 0$ which leads to $\lambda s \mu^2 > \kappa \mu_s^2$ and $\lambda_s \mu^2 > \kappa \mu^2$ if Eq.(7) is satisfied. One can realize that for both nonvanishing vevs there are solutions for

- $\mu^2, \mu_s^2 > 0$ for both signs of $\kappa$.
- $(\mu^2 > 0, \mu_s^2 < 0)$ or $(\mu^2 < 0, \mu_s^2 > 0)$ for only $\kappa < 0$.
- There are not any solutions for $\mu^2, \mu_s^2 < 0$.

The scalar mass Lagrangian is given by

$$\mathcal{L}_{massscalar} = -V_{scalar} = \frac{1}{2} \left( \hat{h} \frac{\cos \alpha}{\sqrt{\lambda^2 - \lambda_s v_s^2}} \right) \left( \frac{\sin \alpha}{\sqrt{\lambda^2 - \lambda_s v_s^2}} \right) \left( \frac{\hat{h}}{\hat{s}} \right). \quad (10)$$

We go to the mass basis, $(h, s)$, via transformation

$$\left( \frac{\hat{h}}{\hat{s}} \right) = \left( \cos \alpha \quad \sin \alpha \right) \left( \frac{h}{s} \right) \quad (11)$$

where the mixing angle is given by

$$\tan 2\alpha = -\frac{\kappa \mu_s^2}{\lambda^2 - \lambda_s v_s^2}. \quad (12)$$

The masses of the SM-like Higgs $h$ and the extra scalar $s$ are given by

$$m_h^2 = \lambda v^2 + \lambda_s v_s^2 \pm \sqrt{(\lambda v^2 - \lambda_s v_s^2)^2 + \kappa^2 v^2 v_s^2} \quad (13)$$

where $\lambda v^2 > \lambda_s v_s^2$. In the limit of no scalar mixing, $\kappa \to 0$, the masses of the scalars in Eq.(13) reduce to

$$m_{H,0}^2 = 2\lambda v^2, \quad m_{S,0}^2 = 2\lambda_s v_s^2. \quad (14)$$
2.2 Mixing of Gauge Bosons

The $U(1)'$ symmetry couples to the SM hypercharge symmetry $U(1)_Y$ through the kinetic mixing which leads to the most general gauge Lagrangian of $U(1)_Y \times U(1)'$

\[
\mathcal{L}_{\text{gauge}} = \mathcal{L}_{\text{gauge}}^{\text{SM}} + \mathcal{L}_{\text{gauge}}' + \mathcal{L}_{\text{gauge}}^{\text{mix}},
\]

(15)

\[
\mathcal{L}_{\text{gauge}}^{\text{SM}} = -\frac{1}{4} \hat{B}_{\mu\nu} \hat{B}^{\mu\nu},
\]

(16)

\[
\mathcal{L}_{\text{gauge}}' = -\frac{1}{4} \hat{Z}_{\mu\nu} \hat{Z}^{\mu\nu},
\]

(17)

\[
\mathcal{L}_{\text{gauge}}^{\text{mix}} = -\frac{1}{2} \sin \chi \hat{B}_{\mu\nu} \hat{Z}^{\mu\nu}
\]

(18)

where $\hat{B}_{\mu\nu}$ and $\hat{Z}_{\mu\nu}$ are the field strength tensors of $U(1)_Y$ and $U(1)'$, respectively. The last equation contains a mixing term with a gauge kinetic mixing parameter $\chi$.

We diagonalize the field strength terms via a $GL(2, R)$ transformation

\[
\begin{pmatrix}
\hat{Z}_{\mu}' \\
\hat{B}_{\mu}'
\end{pmatrix} =
\begin{pmatrix}
\sqrt{1 - \sin^2 \chi} & 0 \\
\sin \chi & 1
\end{pmatrix}
\begin{pmatrix}
\hat{Z}_{\mu} \\
\hat{B}_{\mu}
\end{pmatrix}
\]

(19)

The mass eigenstates of the neutral gauge bosons are obtained via the transformation

\[
\begin{pmatrix}
\hat{B}_{\mu} \\
W^3_{\mu} \\
\hat{Z}_{\mu}'
\end{pmatrix} =
\begin{pmatrix}
\cos \theta_W & -\sin \theta_W \cos \varphi & \sin \theta_W \sin \varphi \\
\sin \theta_W & \cos \theta_W \cos \varphi & -\cos \theta_W \sin \varphi \\
0 & \sin \varphi & \cos \varphi
\end{pmatrix}
\begin{pmatrix}
A_{\mu} \\
\hat{Z}_{\mu} \\
\hat{Z}_{\mu}'
\end{pmatrix}
\]

(24)

where $\theta_W$ is the Weinberg angle and and $\varphi$ is the gauge mixing angle which is given by

\[
\tan 2\varphi = \frac{2(g'\eta + 2e\tilde{g}Q_H)\sqrt{g^2 + \eta^2}}{(g'\eta + 2e\tilde{g}Q_H)^2 + 4\eta^2Q_H^2g^2 - g^2 - \eta^2}.
\]

(25)

The masses of the physical gauge bosons read as

\[
\mathcal{D}_\mu = \partial_\mu + igT^i W^i_\mu + ig' Q_Y \hat{B}_\mu + i(e\tilde{g}Q' + \eta g)\hat{Z}_\mu' = 0,
\]

(20)

where $T^i = \frac{1}{2}\sigma^i$ is the third component of isospin in which $\sigma^i$ are the Pauli spin matrices with $i = 1, 2, 3$; $W^i_\mu$ is the $SU(2)_L$ gauge field; $g$ and $g'$ are the $SU(2)_L$ and $U(1)_Y$ gauge couplings, respectively. In Eq. (20), we have introduced

\[
\tilde{g} \equiv \frac{g}{\cos \chi}, \quad \eta \equiv -\tan \chi
\]

(21)

where $\tilde{g}$ is the normalized $U(1)'$ gauge coupling

\[
\tilde{g} \equiv \frac{g_{U(1)'}}{e}.
\]

(22)

The mass squared matrix of the gauge bosons in the $(\hat{B}_\mu, \hat{Z}_\mu')$ gauge-basis is given by

\[
\mathcal{L}_{\text{gauge}}^{\text{mass}} = \frac{1}{2} \begin{pmatrix}
\hat{B}_\mu \\
W^3_{\mu} \\
\hat{Z}_{\mu}'
\end{pmatrix} \begin{pmatrix}
The masses of the physical gauge bosons read as
the gauge mixing angle in Eq. (25) vanishes identically. This ensures zero mixing between the $Z$ and the $Z'$ bosons so that the $Z'$ mass is set by the vev $v_s$ of the extra scalar

$$M_{Z'}^2 = e^2 g_s^2 Q_3^2 v_s^2. \tag{29}$$

The condition in Eq. (28) can be relaxed. We know that the mixing of the $Z$ and the $Z'$ can be at most at the level of the $Z^0$ mass

$$\frac{1}{2} g' v_s^2 \left( \frac{g' v_s}{2} + e g Q_H \right) \lesssim M_{Z'}^2 \tag{30}$$

which gives

$$\left( \frac{g' v_s}{2} + e g Q_H \right) \lesssim 10^{-8} \tag{31}$$

for a $Z'$ mass of $M_{Z'} = 17$ MeV which implies $\tan 2 \varphi \lesssim 10^{-8}$. The current limit on the $Z - Z'$ mixing angle from the LEP data is about $|\varphi| = 10^{-3} - 10^{-4}$. It is thus clear that the $Z - Z'$ mixing angle in our family-nonniversal $U(1)'$ model is well below the limit from the electroweak precision data.

### 2.3 Leptons and Quarks

The kinetic Lagrangian of the fermions is given by

$$\mathcal{L}_{\text{fermionic}} = i \bar{Q}_i \gamma^\mu \partial_\mu Q_i + i \bar{u}_R i \gamma^\mu \partial_\mu u_R + i \bar{d}_R i \gamma^\mu \partial_\mu d_R$$

$$+ i \bar{\nu}_L \gamma^\mu \partial_\mu \nu_L + i \bar{e}_R i \gamma^\mu \partial_\mu e_R \tag{32}$$

where $i = 1, 2, 3$ is the family index, $Q_i$ is for the left-handed quark doublets and $(u_R, d_R)$ are for the right-handed quark singlets

$$Q = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}, \quad (u_R, d_R), \tag{33}$$

and $L$ is for the left-handed lepton doublet and $e_R$ is for the right-handed lepton singlet

$$L = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix}, \quad e_R. \tag{34}$$

The Yukawa Lagrangian is

$$\mathcal{L}_{Yukawa} = -Y_u \bar{Q} \tilde{H} u_R - Y_d \bar{Q} \tilde{H} d_R - Y_e \bar{L} \tilde{H} e_R + \text{h.c.} \tag{35}$$

where $(Y_u, Y_d, Y_e)$ are the Yukawa matrices and $\tilde{H} = i \sigma_2 H^*$. The gauge invariance conditions from the diagonal elements of the Yukawa interactions in Eq. (35) are given by

$$Q_{u_{Li}} = Q_{Q_e} + Q_H$$

$$Q_{d_{Li}} = Q_{Q_e} - Q_H$$

$$Q_{e_{Li}} = Q_{L_e} - Q_H \tag{36}$$

It is clear that the conditions in Eq. (36) involve only the diagonal elements of the Yukawa interactions. Actually, they are general enough to cover also conditions coming from off-diagonal Yukawa entries. One will realize in Sec. 4 that the $U(1)'$ charges give rise to a specific mass matrix structure. The first two families of the up and down-type quarks have the same $U(1)'$ charges while the third family has a different charge, which implies that $(M_{u_{13}}, (M_{u_{23}}, (M_{u_{33}}, (M_{d_{13}}, (M_{d_{23}}, (M_{d_{33}})$ all vanish. These zeroes leave no Yukawa interactions between the first two families and the third family of the up and down-type quarks. There can arise thus no non-trivial gauge invariance conditions in these sectors. The general Yukawa interactions between the first two families are trivial in that their $U(1)'$ charges are universal. Moreover, leptons have family-universal $U(1)'$ charges. It therefore is clear that Eq. (36) covers all cases.

### 3 Constraints from Experiments

It is argued that the new boson is likely a vector boson $[6, 7]$ that couples to the SM fermion currents as

$$\mathcal{L} \supset i Z'_\mu J^\mu = i Z'_\mu \sum_{i=u,d,e,\nu_e,...} \varepsilon_i \varepsilon_i J^\mu_i, \quad J^\mu_i = f_i \gamma^\mu f_i \tag{37}$$

where $\varepsilon^\nu$ is the vector current couplings of the $Z'$ with superscript 'u' referring to 'vector'. It is showed that the vector current couplings of the $Z'$ to the SM fermions are constrained by several experimental data $[6, 7]$. The Atomki signal $[1]$, the neutral pion decay, $H^0 \rightarrow X\gamma$, by NA48/2 experiment $[14, 15]$, the SLAC E141 experiment $[16, 17]$, constraint via the electron anomalous magnetic dipole moment $(g - 2)_e$ $[18]$ and the $\bar{\nu}_e - e$ scattering by TEXONO $[20]$ put constraints on the vector current couplings of the $Z'$ to the first generation of the SM fermions

$$|\varepsilon^e_v| \lesssim 1.2 \times 10^{-3}$$

$$|\varepsilon^e_n| = (2 - 10) \times 10^{-3}$$

$$|\varepsilon^e_p| = (0.2 - 1.4) \times 10^{-3}$$

$$\sqrt{\varepsilon^e_v \varepsilon^e_n} \lesssim 7 \times 10^{-5}. \tag{38}$$

The constraints on the couplings of the $Z'$ from the neutral pion decay $[6, 7]$ require it to be proto-phobic.
i.e., it has a suppressed coupling to the proton compared with the neutron

\[-0.067 < \frac{\varepsilon_{p}^v}{\varepsilon_{n}^v} < 0.078 \]  

where the nucleon couplings are explicitly given by

\[\varepsilon_{p}^v = 2\varepsilon_{u}^v + \varepsilon_{d}^v, \quad \varepsilon_{n}^v = \varepsilon_{u}^v + 2\varepsilon_{d}^v.\]

### 4 Z' Couplings

In this section, we find the vector and axial-vector current couplings of the Z' that are able to explain the Atomki anomaly. First, we show the vector and axial-vector current couplings of the Z' to the first generation of the fermions in terms of the model parameters including the U(1)' charges of the related chiral fermions in Tab. 2.

In Tab. 2, we have introduced

\[\epsilon \equiv -\frac{1}{2} \left( \cot \theta_w + \tan \theta_w \right) \sin \varphi + \frac{\cos \varphi}{\cos \theta_w} \eta, \quad \delta \equiv \tan \theta_w \sin \varphi + \frac{\cos \varphi}{\cos \theta_w} \eta.\]

The SM chiral fermion and scalar fields are charged under U(1)'. We determine the couplings by providing that the charges satisfy the anomaly-free conditions and the gauge invariance conditions. In order to avoid gauge and gravitational anomalies, the U(1)' charges of the chiral fields must satisfy

\[U(1)' - SU(3) - SU(2) : \quad 0 = \sum_i (2Q_{Q_i} - Q_{u_{R_i}} - Q_{d_{R_i}}),\]

\[U(1)' - SU(2) - SU(2) : \quad 0 = \sum_i (3Q_{Q_i} + Q_{L_i}),\]

\[U(1)' - U(1)_Y - U(1)_Y : \quad 0 = \sum_i (\frac{1}{6}Q_{Q_i} - \frac{1}{3}Q_{d_{R_i}} - \frac{4}{3}Q_{u_{R_i}} + \frac{1}{2}Q_{L_i} - Q_{e_{R_i}}),\]

\[U(1)' - \text{graviton} - \text{graviton} : \quad 0 = \sum_i (6Q_{Q_i} - 3Q_{u_{R_i}} - 3Q_{d_{R_i}} + 2Q_{L_i} - Q_{e_{R_i}}),\]

\[U(1)' - U(1)' - U(1)' : \quad 0 = \sum_i (Q^2_{Q_i} + Q^2_{d_{R_i}} - 2Q^2_{u_{R_i}} - Q^2_{L_i} + Q^2_{e_{R_i}}),\]

\[U(1)' - U(1)' - U(1)' : \quad 0 = \sum_i (6Q^3_{Q_i} - 3Q^3_{d_{R_i}} - 3Q^3_{u_{R_i}} + 2Q^3_{L_i} - Q^3_{e_{R_i}}).\]

There are 16 charges and 6 anomaly free conditions with additional conditions from Yukawa interactions such that as we show in Tab. 3, one could express 12 charges in terms of 4 free charges

\[Q_H, \quad Q_{Q_2}, \quad Q_{Q_3}, \quad \text{and} \quad Q_{L_3}.\]

We parametrize the vector current coupling of the Z' boson to the proton as

\[\varepsilon_{p}^v = 2\varepsilon_{u}^v + \varepsilon_{d}^v = \delta', \quad |\delta'| \lesssim 10^{-3}.\]

Then, by Eq. (44), we get

\[\delta = \delta' - \frac{1}{2} \epsilon + \cos \varphi \tilde{g} \left( 3Q_{Q_2} + 3Q_{Q_3} - \frac{7}{2} Q_H \right).\]

which together with the charge solutions in Tab. 3 lead to the couplings in Tab. 3 with a vanishing gauge mixing limit from now on.

The Lagrangian of the axial-vector current interaction of the Z' boson is given by

\[\mathcal{L} \supset i Z'_{\mu} \sum_{i=u,d,e,\nu} \varepsilon_{i}^a \varepsilon_{f} \gamma^{\mu} \gamma^{5} f_{i}.\]

where \(\varepsilon_{i}^a\) is the axial-vector current coupling with superscript 'a' referring to 'axial-vector'.

We obtain the solutions of the free charges \(Q_{Q_2}, Q_{Q_3}, Q_H\) and \(Q_{L_3}\) as follows.

- In the limit of minimal flavor violation there holds the relation \(\varepsilon_{u}^a = \varepsilon_{d}^a\) by which we obtain the solution

\[Q_{Q_3} = Q_H - 2Q_{Q_2}.\]
Next, we parametrize the vector current coupling of the Z′ boson to the electron vanishes, $e_{d}^{n} = \frac{1}{2} \epsilon + \frac{1}{2} \delta + \cos \varphi \frac{gQ_{L1} + Q_{Q3}}{2}$, and the axial-vector current coupling of the Z′ boson to the proton as $e_{p}^{n} = 2e_{u}^{n} + e_{d}^{n} = \delta'$ with $|\delta'| \lesssim 10^{-3}$. Consideration of other constraints reduce the couplings in this table to the couplings in Table 3.

| $Q_{Q1}$ = $Q_{H} - Q_{Q2} - Q_{Q3}$ | $Q_{Q2}$ = $2Q_{H} - Q_{Q2} - Q_{Q3}$ | $Q_{Q3}$ = $-Q_{Q2} - Q_{Q3}$ |
|---|---|---|
| $Q_{L1}$ = $-Q_{H}$ | $Q_{L2}$ = $-2Q_{H}$ | $Q_{L3}$ = $-Q_{H}$ |
| $Q_{L3}$ = $Q_{L1} - Q_{H}$ | $Q_{L3}$ = $Q_{L1} - Q_{H}$ | $Q_{L3}$ = $Q_{L1} - Q_{H}$ |

Table 2: The Z′ couplings to the first generation of fermions in terms of the model parameters including the $U(1)'$ charges of the related chiral fermions.

Table 3: The $U(1)'$ charge solutions of the chiral SM fermions by the gauge invariance and the anomaly-free conditions.

| $e_{u}^{n} = \frac{1}{2} \epsilon + \frac{1}{2} \delta + \cos \varphi \frac{gQ_{L1} + Q_{Q3}}{2}$ | $e_{d}^{n} = \frac{1}{2} \epsilon + \frac{1}{2} \delta + \cos \varphi \frac{gQ_{L1} - Q_{Q3}}{2}$ | $e_{v}^{n} = \frac{1}{2} \epsilon + \cos \varphi \frac{gQ_{L1}}{2}$ |

Table 4: The Z′ couplings after using the charge solutions in Table 3 and parametrization of the vector current coupling of the Z′ boson to the proton as $e_{p}^{n} = 2e_{u}^{n} + e_{d}^{n} = \delta'$ with $|\delta'| \lesssim 10^{-3}$. Consideration of other constraints reduce the couplings in this table to the couplings in Table 3.

Next, we parametrize the vector current coupling of the Z′ boson to the neutron

$$e_{u}^{n} = e_{u}^{n} + 2e_{d}^{n} = \epsilon'$$

where parameter $\epsilon'$ satisfies

$$|\epsilon'| \approx (2 - 10) \times 10^{-3}.$$ (50)

Then, by Eq. (48) and Eq. (49), we obtain the solutions of $Q_{Q2}$ and $Q_{Q3}$

$$Q_{Q2} = \frac{1}{3g} (\epsilon + \epsilon'),$$

$$Q_{Q3} = \frac{1}{3g} (\epsilon - 2\epsilon').$$ (51)

The axial-vector coupling to the electron vanishes, $e_{a}^{n} = 0$, identically via the zero $Z - Z'$ mixing condition in Eq. (22), as well as the the axial-vector current couplings to the up and down quarks $e_{a}^{u} = e_{a}^{d} = 0$; the vector and axial-vector current couplings to the electron neutrino, $e_{v}^{n} = e_{d}^{n} = 0$ with the following $U(1)'$ charge of the SM-like Higgs boson $Q_{H} = \frac{\epsilon}{g}$. (52)

Using the solution of $Q_{H}$ in (52) we get $\eta \lesssim 10^{-4}$, which well agrees with the bounds.

The axial-vector current coupling of the Z′ boson to the electron is constrained via the neutral pion decay process, $\Pi^{0} \to e^{+}e^{-}$ [21]. The matrix element of this process is proportional to $e_{a}^{n}(e_{u}^{n} - e_{d}^{n})$ [22]. However, in our model the axial-vector current coupling of the Z′ to the electron vanishes, $e_{a}^{n} = 0$, as well as the axial-vector current couplings to the up and down quarks $e_{u}^{n} = e_{d}^{n} = 0$. Therefore this rare process imposes no constraints on the axial-vector current coupling of the Z′. The axial-vector current coupling of the Z′ to the electron is constrained also by the atomic parity violation [23] and the parity-violating Møller scattering [24] which constrain the products $e_{a}^{n}e_{a}^{v}$ and $e_{a}^{n}e_{a}^{e}$, respectively. It is obvious that due to vanishing $e_{a}^{n}$, there arise no constraints from these processes. As a result of these, the vector and axial-vector current couplings of the Z′ to the first generation of the SM fermions take the forms in Table 3.

In Table 3, we present the Z′ couplings to the first generation of the SM fermions that fit the Atomki signal. The couplings of the Z′ are proto-phobic.
The couplings of the fermions that fit the Atomki signal with Table 5: The $\delta_n \approx 10^{-3}$ and $\epsilon_n = \epsilon_u + 2\epsilon_d \equiv \epsilon'$, $|\epsilon'| \approx (2 - 10) \times 10^{-3}$. The couplings of the $Z'$ are proto-phobic, Eq. (39), and satisfy the experimental constraints in Eq. (43).

- As one can realize, our model is proto-phobic in both vector and axial-vector current interactions. The axial-vector current coupling to up and down quarks vanish identically via the zero $Z - Z'$ mixing condition in Eq. (28) so the $Z'$ has purely vector current interactions with up and down quarks.
- The vector current coupling to the electron does not vanish as it should not for the IPC and it is able to take value satisfying the experimental constraints. The axial-vector current coupling to the electron vanishes identically via the zero $Z - Z'$ mixing condition in Eq. (28).
- The experimental constraints require the vector current coupling to the electron neutrino to be significantly below the vector current coupling to the neutrino. The vector and axial-vector current couplings to the electron neutrino vanish identically with zero $Z - Z'$ mixing condition in Eq. (28) and this obviously satisfies the experimental data.
- In order to have universal charges in the lepton sector, we assume

$$Q_{L3} = -Q_H.$$ (53)

As a result of these, the first two families of the quarks have the same $U(1)'$ charges which is different from the third family charge and the leptons have universal $U(1)'$ charges, as we show in Tab. 6.

5 CKM Matrix

There are several texture-specific quark mass matrices in the literature [25, 31]. The goal has always been avoiding the large number of parameters in these mass matrices. Some elements of these matrices are assumed to be zero and they are generally referred to as ‘texture zero matrices’. These kind of matrices provide a viable framework to obtain the flavor mixing matrix, the CKM matrix, which is compatible with the current data [32].

For definiteness, we focus here on the texture-specific quark mass matrices in [33, 34]

$$M_{u,d} = \begin{pmatrix} x \times 0 \\ x \times x \\ 0 \times x \end{pmatrix}$$ (54)

which are known to reproduce the CKM matrix. The viability of these mass matrices are analyzed in [35] by showing the compatibility with the CKM matrix. In our model the Higgs field leads to $(M_{u,d})_{13} = 0$, $(M_{u,d})_{31} = 0$ and $(M_{u,d})_{23} = 0$, $(M_{u,d})_{32} = 0$. In order to match to Eq. (54), we need to induce matrix elements $(M_{u,d})_{23} \neq 0$ and $(M_{u,d})_{32} \neq 0$. One way to do this is by higher-dimensional operators [36, 39].

As a minimal approach that fits to our $U(1)'$ set up, we introduce the Yukawa interactions

$$\mathcal{L} \supset \lambda_{u,d}^{23} \left( \frac{S}{\Lambda} \right)^{23} Q_2 \tilde{H}_R + \lambda_{u,d}^{23} \left( \frac{S^*}{\Lambda} \right)^{23} \delta S^3 Q_2 \tilde{H}_R + h.c.$$ (55)

where $\lambda_{u,d}^{23}$ is the Yukawa coupling, $\Lambda$ is the mass scale for flavor physics, $\delta_{u,d}^{23}$ and $\delta S^3$ are parameters that will be determined below. From Eq. (55), we get the gauge invariance conditions

$$-Q_{Q_2} = Q_H + Q_{u,d} + \delta_{u,d}^{23} Q_S = 0,$$ (56)

$$-Q_{Q_2} + Q_H + Q_{d,u} + \delta_{u,d}^{23} Q_S = 0$$

which lead to

$$\delta_{u,d}^{23} = \delta_{d,u}^{23} = \frac{\epsilon'}{Q s g}.$$ (57)

after using the solutions of the charges in Tab. 6. This method of generating the hierarchy can be extended to the other Yukawa entries (in terms of their 33 entries or few other entries) [36, 39].

The parameters $\delta_{u,d}^{23}$ and $\delta S^3$ are positive integers so that we adopt $Q_S = \frac{1}{g}$ to obtain $\delta_{u,d}^{23} = \delta S^3 = 1$. This solution of $Q_S$ leads to $v_S \approx O(10)$ GeV for a 17 MeV $Z'$ boson. The charge of the extra scalar $\tilde{S}$ is $Q_S \approx O(10^{-2})$ for the coupling $g \approx O(10^{-1})$. If we use the optimized values of the matrix elements of $(M_{u,d})_{23}$ from [35], we find that $\delta_{u,d}^{23} \approx 2$ for $\Lambda \approx O(10)$ GeV and $\lambda_{u,d}^{23} = 1$. The solutions via Eq. (55) are not necessarily specific to the texture in Eq. (54). One can consider different textures and generate the same CKM structure by modifications or extensions of Eq. (55).

In the present model in the interaction basis the couplings of the $Z'$ to the SM quarks are diagonal.
are put into the couplings in Tab.(4).

Table 6: The $U(1)'$ charges of the chiral SM fermions. One obtains the $Z'$ couplings in Tab.(5) if these charge solutions are put into the couplings in Tab.(4).

but nonuniversal. This nonuniversality gives rise to flavor changing neutral currents (FCNCs). From $B^0 - ar{B}^0$ mixing there arise stringent constraints for these FCNCs.

\[ |\epsilon^{L(R)}| \lesssim 10^{-6} \]  

(58)

where $\epsilon^{L(R)}$ are the chiral couplings of the $Z'$ to the $s\gamma^\mu b$ current.

In the present model the chiral couplings in the down quark sector are given by

\[
g_{dL} \equiv \text{diag}(g^1_{dL}, g^1_{dL}, g^3_{dL}), \tag{59}
g_{dR} \equiv \text{diag}(g^1_{dR}, g^2_{dR}, g^3_{dR}) \tag{60}
\]

where $g^1_{dL} = g^2_{dL} = \epsilon' / 3$, $g^3_{dL} = g^3_{dR} = -2 \epsilon' / 3$. If we introduce the CKM matrix, in the quark mass eigenstate basis the chiral couplings become

\[
\epsilon^{L}_{ab} \equiv (V_{CKM} g_{dL} V_{CKM}^\dagger)_{23}, \tag{61}
\]

\[
\epsilon^{R}_{ab} \equiv (V_{CKM} g_{dR} V_{CKM}^\dagger)_{23}. \tag{62}
\]

Then one obtains the following condition from both of the chiral couplings above

\[ |\epsilon'| = 2 \times 10^{-3}. \tag{63} \]

6 LHC bound

In our family-nonuniversal $U(1)'$ model, the SM-like Higgs boson is charged under $U(1)'$ which leads to decay of $(h \to Z' Z')$ that should be sufficiently small such that the branching fraction of the SM-like Higgs to the $Z'$ boson pairs has to be $BR(h \to Z' Z') \lesssim 10\%$. The decay rate of this process is given by

\[
\Gamma(h \to Z' Z') = \frac{3}{32 \pi m_h} \epsilon^2 \left( 1 - \frac{4 M^2_{Z'}}{m^2_h} \right)^{1/2} \left( 1 - \frac{m^2_h}{3 M^2_{Z'}} + \frac{m^4_h}{12 M^2_{Z'}} \right) \tag{64}
\]

where we have introduced

\[
\xi = \frac{4}{3} \left[ \cos \alpha \sin^2 \theta_W m_{Z'} v^2 \sin \frac{M^2_{Z'}}{v^2} - \sin \alpha \frac{M^2_{Z'}}{v^2} \right] - \frac{\cos \alpha}{2 \cos \theta_W} \left( g' - \frac{e}{2 \cos \theta_W} \right) \eta^2. \tag{65}
\]

In Fig.(1), we show the region where the partial decay width $\Gamma(h \to Z' Z')$ is less than 10% of the SM Higgs total decay width

\[
BR(h \to Z' Z') = \frac{\Gamma(h \to Z' Z')}{\Gamma^{SM}_{\text{total}}(h) + \Gamma(h \to Z' Z') \lesssim 0.10 \tag{66}
\]

where $\Gamma^{SM}_{\text{total}}(h) = 4.07 \times 10^{-3}$ GeV.

Fig. 1: We show the region where the partial decay width $\Gamma(h \to Z' Z')$ is less than 10% of the SM Higgs total decay width $BR(h \to Z' Z') \lesssim 10\%$. The Higgs mixing angle is $\sin \alpha \sim O(10^{-3})$ for $m_h = 125.09$ GeV and $\eta = 10^{-4}$. The vertical red line is for the $Z'$ boson mass $M_{Z'}$ determined via the experimental data. The scalar mixing angle is $\sin \alpha \sim O(10^{-3})$ and accordingly the scalar mixing parameter is $\kappa \sim O(10^{-3})$ required for $BR(h \to Z' Z') \lesssim 10\%$ for the SM Higgs boson mass of $m_h = 125.09$ GeV and $\eta = 10^{-4}$. The scalar mixing remains at the same order for different values of the kinetic mixing $\eta = 10^{-5}, 10^{-6}$. The decay process of $(h \to ZZ')$ would also be relevant however, the $(hZZ')$ vertex factor, which is given by

\[
Q_{Q_1} = Q_{Q_2} = \frac{1}{3}(\epsilon + \epsilon') \quad Q_{u_1} = Q_{u_2} = \frac{1}{3}(4\epsilon + \epsilon') \quad Q_{d_1} = Q_{d_2} = \frac{1}{3}(-2\epsilon + \epsilon')
\]

\[
Q_{Q_3} = \frac{1}{3}(\epsilon - 2\epsilon') \quad Q_{u_3} = \frac{1}{3}(2\epsilon - \epsilon') \quad Q_{d_3} = -\frac{1}{3}(\epsilon + \epsilon')
\]

\[
Q_{L_1} = Q_{L_2} = Q_{L_3} = -\frac{1}{3} \quad Q_{e_1} = Q_{e_2} = Q_{e_3} = -\frac{1}{3}
\]
\[ h ZZ' = -\frac{\cos \alpha}{\sin 2\theta_W} e^\nu \left( \frac{g^f \eta}{2} + e \bar{g} Q_H \right) \]

is proportional to the left-hand side of the zero \( Z - Z' \) mixing condition in Eq. (28). Therefore this vertex is zero and there arise no constraints from this decay.

### 7 Summary and Outlook

In this work, we construct the framework of a family-nonuniversal \( U(1)' \) model, which is a minimal, anomaly-free extension of the SM that is able to explain the 6.8 \( \sigma \) anomaly in \( ^6\text{Be} \) nuclear decays at the Atomki pair spectrometer experiment.

One possible interpretation of the Atomki signal is a spin-1, proto-phobic gauge boson with a mass of \( \approx 17 \text{ MeV} \). We present a family-nonuniversal \( U(1)' \) model with its associated \( Z' \) boson with a mass of \( \approx 17 \text{ MeV} \) which fulfills all the experimental constraints on its vector and axial-vector current couplings to the first generation of fermions that are necessary to explain the \( ^6\text{Be} \) anomalous decays.

The previously proposed models have a large new content of fields. However, we have a minimal field content with the \( Z' \) boson and the extra scalar. Our family-nonuniversal \( U(1)' \) model is an anomaly-free extension of the SM with a minimum field content that can explain the observed beryllium anomaly.

The CKM matrix is reproduced correctly by higher-dimensional Yukawa interactions facilitated by \( S \). The model provides new couplings to probe new physics at low energies. It may provide framework for anomalous SM decays and forms a framework in which various low-energy phenomena can be addressed. The low-energy phenomena such as \( s^s \to ff \) and \( Z'Z' \to ff \) can be relevant for phenomenological (dark matter) purposes.

### Acknowledgments

The author thanks Durmuş Demir for suggestions and discussions on the problem.

### References

1. A. J. Krasznahorkay, M. Csatls, L. Csíge, Z. Gcsí, J. Gulys, M. Hunyadi, I. Kuti, B. M. Nyak, L. Stuhl, J. Timr, T. G. Tornyi, Zs. Vajta, T. J. Ketel, and A. Krasznahorkay, Phys. Rev. Lett. 116, 042501 (2016), [arXiv:1504.01527 [nucl-ex]].
2. A. J. Krasznahorkay et al., EPJWeb Conf.142, 01019 (2017).
3. A. J. Krasznahorkay et al., EPJWeb Conf.137, 08010 (2017).
4. A. J. Krasznahorkay et al. 2018 J. Phys.: Conf. Ser.1056 012028.
5. A. J. Krasznahorkay et al., arXiv:1910.10459 [nucl-ex].
6. J. L. Feng, B. Fornal, I. Galon, S. Gardiner, J. Smolinsky, T. M. P. Tait and P. Tanedo, Phys. Rev. Lett. 117, no. 7, 071803 (2016), [arXiv:1604.07411 [hep-ph]].
7. J. L. Feng, B. Fornal, I. Galon, S. Gardiner, J. Smolinsky, T. M. P. Tait and P. Tanedo, Phys. Rev. D 95, no. 3, 035017 (2017), [arXiv:1608.03591 [hep-ph]].
8. P. H. Gu and X. G. He, Nucl. Phys. B 919, 209 (2017), [arXiv:1606.05171 [hep-ph]].
9. L. Delle Rose, S. Khalil and S. Moretti, arXiv:1704.03436 [hep-ph].
10. U. Ellwanger and S. Moretti, JHEP 1611, 039 (2016), [arXiv:1609.01669 [hep-ph]].
11. J. Kozaczuk, D. E. Morrissey and S. R. Stroberg, Phys. Rev. D 95, no. 11, 115024 (2017) [arXiv:1612.01525 [hep-ph]].
12. D. A. Demir, G. L. Kane and T. T. Wang, Phys. Rev. D 72, 015012 (2005), [hep-ph/0503290].
13. J. Erler, P. Langacker, S. Munir and E. Rojas, JHEP 0908, 017 (2009), [arXiv:0906.2433 [hep-ph]].
14. J. R. Batley et al. [NA48/2 Collaboration], Phys. Lett. B 746, 178 (2015), [arXiv:1504.00607 [hep-ex]].
15. M. Raggi, [NA48/2 Collaboration], Nuovo Cim. C38 no. 4, 132 (2016), [arXiv:1508.01307 [hep-ex]].
16. E. M. Riordan et al., Phys. Rev. Lett. 59, 755, (1987).
17. J. D. Bjorken, R. Essig, P. Schuster and N. Toro, Phys. Rev. D 80, 075018 (2009), [arXiv:0906.0580 [hep-ph]].
18. R. Essig et al., [2013], [arXiv:1311.0029 [hep-ph]].
19. H. Davoudiasl, H. S. Lee and W. J. Marciano, Phys. Rev. D 89, no. 9, 095006 (2014), [arXiv:1402.3620 [hep-ph]].
20. M. Deniz et al. [TEXONO Collaboration], J. Phys. Conf. Ser. 203, 012099 (2010).
21. E. Abouzaid et al. [KTeV Collaboration], Phys. Rev. D 75, 012004 (2007), [hep-ex/0610072].
22. Y. Kahn, M. Schmitt and T. M. P. Tait, Phys. Rev. D 78, 115002 (2008), [arXiv:0712.0007 [hep-ph]].
23. S. G. Porsey, K. Beloy and A. Derevianko, Phys. Rev. Lett. 102, 181601 (2009), [arXiv:0902.0335 [hep-ph]].
24. P. L. Anthony et al. [SLAC E158 Collaboration], Phys. Rev. Lett. 95, 081601 (2005), [hep-ex/0504049].
25. A. Rasin, Phys. Rev. D 58, 096012 (1998), [hep-ph/9802356].
26. G. C. Branco, D. Emmanuel-Costa and R. Gonzalez Felipe, Phys. Lett. B 477, 147 (2000), [hep-ph/9911418].
27. H. Fritzsch and Z. z. Xing, Prog. Part. Nucl. Phys. 45, 1 (2000), [hep-ph/9912358].
28. Z. z. Xing and H. Zhang, J. Phys. G 30, 129 (2004), [hep-ph/0309112].
29. G. C. Branco, D. Emmanuel-Costa, R. Gonzalez Felipe and H. Serodio, Phys. Lett. B 670, 340 (2009), [arXiv:0711.1613 [hep-ph]].
30. M. Gupta and G. Ahuja, Int. J. Mod. Phys. A 26, 2973 (2011), [arXiv:1206.3844 [hep-ph]].
31. M. Gupta and G. Ahuja, Int. J. Mod. Phys. A 27, 1230033 (2012), [arXiv:1302.4823 [hep-ph]].
32. C. Patrignani et al. [Particle Data Group], Chin. Phys. C 40, no. 10, 100001 (2016).
33. H. Fritzsch and Z. Z. Xing, Phys. Lett. B 413, 396 (1997), [hep-ph/9707215].
34. H. Fritzsch and Z. z. Xing, Nucl. Phys. B 556, 49 (1999), [hep-ph/9904286].
35. G. Ahuja, Int. J. Mod. Phys. A 31, no. 18, 1630024 (2016).
36. W. Buchmuller and D. Wyler, Nucl. Phys. B 268, 621 (1986).
37. V. Barger, T. Han, P. Langacker, B. McElrath and P. Zerwas, Phys. Rev. D 67, 115001 (2003), [hep-ph/0301097].
38. B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, JHEP 1010, 085 (2010), [arXiv:1008.4884 [hep-ph]].
39. Z. Murdock, S. Nandi and S. K. Rai, Phys. Lett. B 704, 481 (2011), [arXiv:1010.1559 [hep-ph]].
40. D. Becirevic, O. Sumensari and R. Zukanovich Funchal, Eur. Phys. J. C 76, no. 3, 134 (2016) [arXiv:1602.00881 [hep-ph]].
41. J. Kumar and D. London, Phys. Rev. D 99, no. 7, 073008 (2019) [arXiv:1901.04516 [hep-ph]].
42. D. Curtin et al., Phys. Rev. D 90, no. 7, 075004 (2014) [arXiv:1312.4992 [hep-ph]].
43. H. S. Lee and M. Sher, Phys. Rev. D 87, no. 11, 115009 (2013) [arXiv:1303.6653 [hep-ph]].
44. S. Heinemeyer et al. [LHC Higgs Cross Section Working Group], arXiv:1307.1347 [hep-ph].