The role of heat loss at the fuel–shell interface during the fast ignition of cylindrical DT targets

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Abstract. The problem of fast heating by a proton beam of a precompressed DT-fuel target in the form of a cylinder surrounded by a gold shell is considered. Three models of the heat transfer through the fuel–shell interface (zero heat flux, a conventional model with the condition of temperature continuity and a model with a temperature jump at the interface) are compared with each other. It is shown that the presence of a magnetic field suppressing heat fluxes through the interface do not have a noticeable effect on the threshold energy of the fast ignition.

1. Introduction
High-gain targets for the inertial confinement fusion are meant for the ignition of a small part of the fuel with the subsequent propagation of the thermonuclear burn wave on its main part. The concept of fast ignition [1, 2] is based on using two drivers. The first driver compresses the target up to a necessary value of density, while the second driver provides for fast rise of temperature in a volume, which is sufficient for the ignition.

Cylindrical shell targets with precompressed DT fuel irradiated from the target end by heavy ion beams are considered in papers [3–5]. In the present work, we consider the close problem of the fast ignition by a proton beam with the proton energy 1 MeV having the stopping range about 10 times less than the heavy ion beams and, nevertheless, sufficient for the ignition.

As is known, a sufficiently strong magnetic field along the boundary between the shell and the fuel suppresses heat transfer through the interface and, thereby, reduces the threshold ignition energy [6, 7]. In this paper, we study the question of how significant the presence of a magnetic field is when the target is fast ignited.

2. Problem statement
The problem of heating the precompressed DT fuel, which has the form of a cylinder surrounded by a gold shell, is considered. At the initial instant of time, both the fuel and the shell are motionless and have constant in space values of thermodynamic functions. The fuel is heated from the end of the cylinder by a beam of protons. Thus, we assume that a target configuration close to the ideal one under consideration can be created in the process of target compression.
Figure 1. Scheme of the target and boundary conditions: “Soft” denotes the boundaries with the soft conditions; “Free”—the free boundary.

2.1. Shell density
Two cases of DT fuel compressed isentropically from the normal state (at the normal pressure $p_a = 0.1$ MPa, temperature 4 K and density $\rho_a \approx 0.22$ g/cm$^3$) up to density ratios $\delta_{a0} = \rho_0 / \rho_a = 100$ and 500 are considered. For the equation of state (EOS) of DT mixture from [8] based on the wide-range semiempirical EOS model [9], the calculation gives the initial pressure $p_0 \approx 20$ and 400 TPa at $\delta_{a0} = 100$ and 500 respectively.

The initial density of the shell $\rho_{sh0}$ is obtained from the normal state of gold ($p_a = 0.1$ MPa, $\rho_{sh0} \approx 19$ g/cm$^3$) by isentropic compression up to the pressure $p_0$. For EOS of gold from [10] that ensures a transition to the Fermi gas behavior at strong compression, the calculation gives $\rho_{sh0} \approx 100$ and 300 g/cm$^3$ when $\delta_{a0} = 100$ and 500 respectively.

As the EOS of gold in the problem under consideration, we choose the Thomas–Fermi model with quantum and exchange corrections [11, 12]. In the case of $\delta_{a0} = 500$, the initial values of pressure and density coincide with the values given above while the values of temperature and specific internal energy are determined by the EOS [11, 12]. In the case of $\delta_{a0} = 100$, the given above values of $\rho_{sh0} \approx 100$ g/cm$^3$ and $p_0 \approx 20$ TPa are incompatible with the EOS [11,12]. To be able to use the EOS [11,12], it is necessary to slightly reduce the value of $\rho_{sh0}$ to about 95 g/cm$^3$.

2.2. Boundary conditions
Initial configuration of the fuel and the shell in cylindrical coordinates $z$, $r$ is shown in figure 1. The fuel has the form of a cylinder of radius $R$. In front of the cylinder, there is an additional domain intended for calculating the fuel emitted from the target. Initially, this domain is filled by a DT gas of a low density and the pressure of 0.1 MPa. The shell has the form of a cylindrical layer with a thickness of $\Delta R$ along $r$.

Figure 1 also displays the types of conditions at different boundaries. On the boundaries of the additional domain except for the symmetry axis, the equality of zero to the spatial derivatives of thermodynamic functions and velocity (soft conditions) are set. The soft boundary conditions are also set on a rectilinear boundary AB that coincides initially with the end boundary of the shell. The outer lateral boundary of the shell is the free boundary with the pressure of 0.1 MPa. The self-radiation of plasma is not taken into account in the additional domain in order to avoid unphysical solutions of the diffusion equation in the optically thin case. The boundary condition of the absence of external radiation for the diffusion equation is set on the line BC. In the additional domain, we also exclude the heating of a rarefied DT mixture by the proton beam.
The boundaries AB and BC lie on a fixed straight line, but the points B and A can move along the line in accordance with the motion of the fuel–shell interface and the lateral boundary of the shell.

The soft boundary conditions make it possible to exclude from consideration a substance that falls outside the boundary. Our calculations show that the problem considered above is suitable for study of the fuel ignition.

2.3. Beam and target parameters
The beam consisting of protons with energy 1 MeV is considered. The intensity \( J_b \) of the incident beam is independent of \( r \) at \( r \leq R \) and equals zero at \( r > R \). The function \( J_b(t) \) is determined by two parameters, the maximum intensity \( J_0 \) and the beam lifetime \( \Delta t_{pr} \) that in turn determines the initial time interval \( \Delta t_{pr0} = 0.02\Delta t_{pr} \) during which \( J_b(t) \) increases from 0 to \( J_0 \):

\[
J_b(t) = \begin{cases} 
J_0 t/\Delta t_{pr0}, & t \leq \Delta t_{pr0}; \\
J_0, & \Delta t_{pr0} < t \leq \Delta t_{pr}; \\
0, & t > \Delta t_{pr}.
\end{cases}
\] (1)

At the initial density of the fuel \( \rho_0 = 100\rho_a \approx 22 \text{ g/cm}^3 \), the beam parameters are chosen as \( \Delta t_{pr} = 80 \text{ ps}, J_0 = 10^{19} \text{ W/cm}^2 \). At \( \delta_{a0} = 500 \), the beam and the fuel parameters take values \([13]\) \( \Delta t_{pr} = 0.2\Delta t'_{pr}, J_0 = 5J_0', R = 0.2R' \), where the prime denotes the values at \( \delta_{a0} = 100 \). The shell thickness \( \Delta R = 0.25 \text{ and } 0.05 \text{ mm at } \delta_{a0} = 100 \text{ and 500 respectively.} \)

2.4. Governing equations
As applied to DT mixture, the equations of hydrodynamics have the form

\[
\frac{d\rho}{dt} = -\rho \nabla \cdot u, \quad \quad \text{(2)}
\]

\[
\rho \frac{du}{dt} = -\nabla p, \quad \quad \text{(3)}
\]

\[
\rho \frac{d\varepsilon_e}{dt} = -p_e \nabla \cdot u - \nabla \cdot q_e + Q_{ei} + D_e + W_e + S, \quad \quad \text{(4)}
\]

\[
\rho \frac{d\varepsilon_i}{dt} = -p_i \nabla \cdot u - \nabla \cdot q_i - Q_{ei} + D_i + W_i, \quad \quad \text{(5)}
\]

where \( \rho \) is the density, \( u = (u_x, u_z) \) is the mass velocity, \( d/dt = \partial/\partial t + u \cdot \nabla \) is the Lagrangian derivative with respect to time, \( p_e \) and \( p_i \) are the electron and ion pressure, \( p = p_e + p_i \), \( \varepsilon_e \) and \( \varepsilon_i \) are the electron and ion specific internal energy, \( q_e \) and \( q_i \) are the electron and ion heat flux, \( Q_{ei} \) defines the energy exchange between electrons and ions. The rest terms in (4) and (5) define the heating of electrons and ions by the proton beam (\( D_e \) and \( D_i \)) and by \( \alpha \)-particles (\( W_e \) and \( W_i \)) as well as the energy exchange between electrons and self-radiation of plasma (\( S \)). The radiation pressure and the momentum transfer under deceleration of \( \alpha \)-particles are neglected in the equation of motion (3).

The system of equations (2)–(5) is closed by the EOSs for electrons \( p_e(\rho, T_e) \), \( \varepsilon_e(\rho, T_e) \) and ions \( p_i(\rho, T_i), \varepsilon_i(\rho, T_i) \), where \( T_e \) and \( T_i \) are the electron and ion temperature.

Only the primary fusion reaction between deuteron and triton with \( \alpha \)-particle having energy 3.5 MeV and neutron as the reaction products is taken into account. Neutrons are supposed to be escaped from the fuel without interaction.

Mathematical models for burnout of fuel nuclei and for transfer of \( \alpha \)-particles, protons and self-radiation of plasma are given in \([14]\).

The shell is described by the same equations of two-temperature hydrodynamics (2)–(5), but without taking into account the heating by epithermal particles and radiation. The degree of ionization that enters into the two-temperature equations of state and into some coefficients of
equations (4) and (5) is calculated from the well-known formula for the standard Thomas–Fermi model.

For the equations of hydrodynamics, we use a Godunov-type second-order-accurate quasimonotonous difference scheme on a moving regular grid consisting only of convex quadrangular cells. The fuel–shell interface is identified as a certain grid line. Some details of the code, together with references to works related to this code, are given in [14].

The left boundary of the computational domain has the coordinate \( z = 0 \). The coordinate \( z \) of the end boundary of the target is selected from the condition \( \rho_0 z \approx 1.3 \text{ g/cm}^2 \). The number of grid intervals along the \( z \)-axis in the basic calculation area (between the origin and the point \( C \), see figure 1) \( M = 240 \). The number of intervals in the perpendicular direction \( M_1 = 40 \) in the fuel and \( M_2 = 60 \) in the shell. Control calculations were carried out on more detailed grids.

2.5. Models of heat transfer through the fuel–shell interface

Three models are considered and compared with each other. The first model (H0) assumes the presence of a magnetic field that suppresses heat fluxes through the interface \( q_e = q_i = 0 \), where \( n \) is the interface normal.

The second model (H1), which is usual for the heat equation, presumes the continuity of the temperature and the heat flux at the interface. The calculation formula for the heat flux \( q \) at the interface obtained from these two conditions, in the case of one spatial variable \( x \), has the form [15]

\[
q = \frac{dT}{dx} \approx \frac{T_2 - T_1}{\Delta x_1 + \Delta x_2}, \quad \Delta x = \frac{\Delta x_1 + \Delta x_2}{\Delta x_1/\kappa_1 + \Delta x_2/\kappa_2},
\]

where \( \Delta x_1 = x_0 - x_1, \Delta x_2 = x_2 - x_0 \), \( x_0 \) is the coordinate of a contact discontinuity, \( x_1 < x_0 \) and \( x_2 > x_0 \) are the grid points closest to \( x_0 \). The temperatures \( T_1, T_2 \) and heat conductivities \( \kappa_1, \kappa_2 \) are determined in the points \( x_1 \) and \( x_2 \) respectively. Formula (6) differs from the usual finite-difference expression for the heat flux only by the formula for the thermal conductivity \( \Delta x \) at the contact discontinuity and can be easily generalized to the two-dimensional case, assuming that \( \Delta x_1 \) and \( \Delta x_2 \) are the distances from the centers of the grid cells, which are located on both sides of a grid interval at the interface grid line, to the middle of the interval.

The third model (H2) allows the temperature breaking at the interface by analogy with the known effect of a temperature jump in a gas at the boundary with a solid wall. Here we use the following formula derived from an asymptotic analysis of the Boltzmann equation [16]:

\[
T_g - T_w = \frac{(m\pi)^{1/2}}{(2k)^{3/2}T_g^{1/2}n_g} \frac{2 - 0.287\alpha_e q^{(n)}}{\alpha_e},
\]

where \( T_g \) and \( T_w \) are the temperatures of the gas and the wall respectively, \( m \) and \( n_g \) are the mass and the density of the gas molecules near the wall, \( k \) is the Boltzmann constant; \( \alpha_e \) is the coefficient of accommodation by internal energy, which for the sake of certainty is set equal to 1; \( q^{(n)} = \frac{\partial T_g}{\partial n} \) is the heat flux, \( \alpha \) is the gas heat conductivity, the derivative is taken along the normal from the wall inside the gas.

With respect to equations (4) and (5), we consider the fuel as a gas, the shell as a wall, obtain two separate conditions for electrons and ions and require the continuity of heat fluxes in the fuel and shell at the interface. In mathematical terms, the conditions obtained are a third-kind boundary condition for the heat equation on an inner boundary of a domain.

3. Numerical results

Figure 2 illustrates a typical flow pattern when the target is ignited. A detonation wave arises. The pressure in this wave near the axis of the symmetry is appreciably higher than that near the
Figure 2. Isobars in the fuel (DT) and in the shell (Au) at $t = 0.2$ ns in the case of $\rho_0 = 100\rho_a$, $R = 0.25$ mm. Digits near curves correspond to the pressure in petapascals ($1 \text{ PP} = 10^6 \text{ GPa}$).

shell. The wave front is nonplanar; a rarefaction wave is adjacent to it. One can see a precursor of the detonation wave near the symmetry axis.

In this paper, as the threshold ignition energy $E_{\text{ig}}$, we consider the beam energy, which is minimal over $R$ and sufficient for ignition at the specified values of other beam parameters $J_0$ and $\Delta t_{pr}$.

A convenient and reliable indicator of the ignition is the total power of the plasma heating by $\alpha$-particles divided by the total power of the plasma heating by the proton beam [17],

$$
Y = \frac{1}{J_0 \pi R^2} \int (W_e + W_i) dV,
$$

as a function of the relative time $\tau = t/\Delta t_{pr}$. Here, $W_e, W_i$ are the terms in the right-hand sides of equations (4) and (5), the integration is taken over the fuel volume.

Continued growth of $Y(\tau)$ after termination of the driver at $\tau > 1$ indicates the ignition of the fuel. If, on the contrary, function $Y(\tau)$ begins to decrease with time, the target does not ignite; a normal shock wave arises, the amplitude of which decreases with time due to the rarefaction wave attached to the shock wave.

On a parametric study of ignition, two ways of changing parameters are of interest. The first way consists in changing the fuel radius $R$ with the same values of the remaining parameters. The second one consists in changing the initial density $\rho_0$, which determines the following changes in the remaining parameters, preserving proximity of fuel heating to the isobaric process (see papers [8, 13]):

$$
R \propto \rho_0^{-1}, \quad \Delta t_{pr} \propto \rho_0^{-1}, \quad J_0 \propto \rho_0.
$$

As shown in [17], the value $Y(1)$ depends weakly on $R$ in the first-way case and on $\rho_0$ in the second-way one. This makes it possible to use the function $Y(\tau)$ for a comparative analysis of the burning of the targets with different parameters.

Define the relative ignition delay $\Delta \tau$ by the equation

$$
Y(1 + \Delta \tau) = 2Y(1),
$$
which means that, during the period of time $\Delta \tau \Delta t_{pr}$ after termination of the ignition driver at $t = \Delta t_{pr}$, the total power of plasma heating by $\alpha$-particles is doubled.

Let $R_{ig}$ be the threshold radius of the ignition. Calculations demonstrate that, at $R$ close to $R_{ig}$ and $R > R_{ig}$, the ignition delay is very large. Formally, an additional threshold radius of ignition $R_{\ast} > R_{ig}$ can be defined so that, at $R \geq R_{\ast}$, the relative ignition delay can not exceed some reasonable value. But the following informal definition turns out to be more convenient.

Suppose that, for a certain set of parameters, the value of $R_{\ast}$ with a reasonable ignition delay is found. The corresponding function $Y(\tau)$ is called the reference one. For other parameter values derived from the original those by one of the two ways described above, let us take $R_{\ast}$ as the value of $R$, at which the function $Y(\tau)$ is visually close to the reference one. The absence of a rigorous definition of the closeness of the functions does not lead to a noticeable uncertainty in the choice of $R_{\ast}$, since the function $Y(\tau)$ varies noticeably with $R$ near $R = R_{\ast}$.

The functions $Y(\tau)$ obtained by the manner described above for the both values of $\delta a_0$ under consideration and for different models of heat transfer across the interface as well as for the case of an absolutely rigid heat-insulated shell are shown in figure 3. As the reference curve, we take the one for $\delta a_0 = 500$ with the model $H1$ (continuity of temperature at the interface) and $R = R_{\ast} = 0.059$ mm, which gives a reasonable ignition delay $\Delta \tau \approx 1.3$. Values of $R_{\ast}$ for other value of $\delta a_0$ and other models of heat transfer are chosen so that the corresponding curve is visually close to the reference one.

It can be seen that the model $H1$ gives the same value $R_{\ast}$ as the model $H0$ (zero heat flux across the interface) for the both values of $\delta a_0$ under consideration. The reason, apparently, is the large ionization degree of gold, which reduces the coefficient of electron thermal conductivity of gold compared to DT-mixture and, consequently, the electron heat flux at the interface [see formula (6)].

The model $H2$ (the temperature jump at the interface) gives some higher value of $R_{\ast}$. Note the importance of taking into account the thermal conductivity in the shell, which reduces the temperature jump at the interface and thus the heat flux [see formula (7)].
Table 1. The threshold-on-$R$ ignition energy (kJ) with a short delay at two values of $\delta_{a0}$ for the three models of heat transfer through the interface and for the rigid heat-insulated shell.

| $\delta_{a0}$ | 500  | 100 |
|---------------|------|-----|
| H0            | 87   | 1680|
| H1            | 87   | 1680|
| H2            | 92   | 1790|
| Rigid         | 27   | 450 |

Table 1 presents the values of the ignition energy

$$E_* = 0.99\pi R^2 J_0 \Delta t_{pr},$$

which can be called the threshold-on-$R$ ignition energy with a short ignition delay. The multiplier 0.99 occurs in integration of function (1) over time.

4. Conclusions

The presence of a magnetic field that suppresses heat fluxes through the interface does not have a noticeable effect on the threshold energy of fast ignition. The threshold energy of a target with a gold shell is about 3–4 times higher than in the ideal case of an absolutely rigid heat-insulated shell.

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