NONLINEAR ELASTOPLASTIC DEFORMATION OF A HOLLOW CYLINDRICAL SHELL OF FINITE LENGTH UNDER INFLUENCE OF INTERNAL PULSE LOADING

Abstract: The process of nonlinear elastoplastic deformation of a hollow cylindrical shell of finite length under the influence of an internal axisymmetric load is investigated. Shell calculations were carried out for elastically and elastoplastic deformable models on the basis of the Kirchhoff-Love theory. A comparative evaluation of the methodology for the numerical solution of the problem is given. The influence of physico-mechanical and structural characteristics, as well as the parameters of the acting load on the deformed state of the shell, is analyzed.

Key words: cylindrical shell, Kirchhoff-Love theory, elastoplastic deformation, numerical solution.

Introduction. Among all types of hydroelastic shells used by human, the greatest interest is represented by cylindrical shells, in which simplicity, compactness and high processability are successfully combined. Cylindrical shells, as structural elements, find wide application in the creation of machines and structures (pipelines, chemical equipment, protective structures, underwater vehicles and structures). In various emergency situations, they are subjected to shock-wave or pulse loadings. On the other hand, in the production environment, it is necessary to create a detail of the desired shape with a press or explosion. With such external or internal influences in the
structural elements, as a rule, elastoplastic deformation processes develop. Therefore, the development of modern machinery and technology requires the creation of new high-strength materials and optimize the size of parts, while maintaining the standard size, the ideal smoothness of the surface, and the high strength of the products. The process of creating such objects from production requires constant replacement or improvement of forging and stamping equipment, presses with large subgrades and installations forming shock-wave or pulse loadings. At the same time, we can be helped by an intensively developing new technology, in particular, related to press, explosion or hydraulic shock [2-7, 9, 11-12]. The mastering of such equipment requires in some cases ideal conditions, huge investment and time costs. In such cases, numerical modeling of elastoplastic deformation of shell elements of structures will help us [4, 5, 12, 13]. This rather complicated problem of hydroelastic-plasticity consists in determining the hydrodynamic forces that arise on the surfaces of the interaction of a structure with a liquid, as well as the investigation of the dynamic characteristics (frequencies, vibration modes) of thin-walled bodies bordering a liquid or containing a liquid in itself [3-7].

To date, quite a lot of results have been obtained in this area, but they mainly refer to idealized objects [3, 6, 7, 9, 11]. This is due to the fact that the problems of interaction of non-stationary pressure waves with deformed bodies are one of the most complicated problems of mechanics. Rapid changing of the process parameters in time, the presence of wave fronts moving in time, and cavitation phenomena, the appearance of plastic zones in the material of the barrier, as well as reflected and radiated waves-all this makes the investigation much more difficult and necessitates a number of simplifying assumptions and hypotheses.

Numerous theoretical and experimental studies [2-6] indicate that in the shell structures, with intensive pulse loads, transient processes arise with low transverse strength. For example, in [1] the influence of the reinforcement structure on the character of elastic and elastoplastic deformation of layered glass-plastic cylindrical shells of finite length, loaded with a single pulse of internal pressure, is numerically analyzed. In order to investigate emergency situations, such as the leakage of petroleum products, the planar problem of elastoplastic deformation of a pipeline immersed in a fluid, with and without allowance for the external medium, under the action of impulse and shock load, has been solved numerically in [8].

Therefore, in the long term, the study of dynamic response and strength under pulse loading of such structural elements is actually. Below we investigate the process of elastoplastic deformation of a hollow cylindrical shell of finite length under the influence of an internal axisymmetric pulse loading.

**Formulation of the problem.**

The problem of numerical investigation of the process of nonlinear elastic-plastic deformation of a hollow cylindrical shell of finite length under influence of an internal pulse loading applied to the inner surface is considered. It is assumed that the shell edges are free and the beginning of the deformation process corresponds to the instant of time \( t = 0 \). The equations of the Kirchhoff-Love theory are used as the basic equations. The necessary equations for the nonlinear theory of thin shells can be obtained if the shear strains are neglected in the nonlinear equations of motion of the hollow shell [2]. In addition, in the case of the cylindrical shell, the motion is axisymmetric, and therefore the terms containing the circular mixing and the derivatives with respect to the angular coordinate in these equations are zero. Then in the notation of [2,4] we have

\[
\frac{\partial N_x}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2},
\]

\[
\frac{\partial^2 M_{xx}}{\partial x^2} + \frac{1}{R} N_y + \frac{\partial}{\partial x} \left( N_y \frac{\partial w}{\partial x} \right) = q - \rho \frac{\partial^2 w}{\partial t^2},
\]

where \( y, x \) are circumferential and axial coordinates; \( u, w \) are longitudinal and radial displacements; \( N_x, N_y \) are longitudinal and circumferential forces; \( M_{xx} \) is the axial moment; \( q \) is loading; \( t \) is time; \( \rho, R, h \) are density, radius and thickness of the shell, respectively.

The relationship between nonzero deformations and displacements in the form of [2]

\[
\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 - \frac{\partial^2 w}{\partial x^2}; \quad \varepsilon_y = -\frac{w}{R}.
\]

The relations between stresses and deformations are thus written as

\[
\sigma_x = \frac{E}{1-\nu^2} \left[ \varepsilon_x + \nu \varepsilon_y - \sum_{\alpha=1}^{n} \left( \Delta_{\alpha} \varepsilon_{x}^{\alpha} + \nu \Delta_{\alpha} \varepsilon_{y}^{\alpha} \right) \right],
\]

\[
\sigma_y = \frac{E}{1-\nu^2} \left[ \varepsilon_y + \nu \varepsilon_x - \sum_{\alpha=1}^{n} \left( \Delta_{\alpha} \varepsilon_{y}^{\alpha} + \nu \Delta_{\alpha} \varepsilon_{x}^{\alpha} \right) \right].
\]

Taking into account (1) and (2) for the forces and the moment, we obtain the refined expressions [4]:

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| 0.564       | 4.102     | 4.260        |
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\[ N_i = \frac{Eh}{1-v^2} \left[ \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \right] - \frac{v}{R} w - \sum_{n=1}^{N} \left( \Delta_n^x e, + v \Delta_n^y e, \right) \]

\[ N_i = \frac{Eh}{1-v^2} \left( \frac{w}{R} + v \left[ \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \right] - \sum_{n=1}^{N} \left( \Delta_n^x e, + v \Delta_n^y e, \right) \right) \]

\[ M_i = D \frac{\partial^2 w}{\partial x^2} + \frac{Eh}{1-v^2} \sum_{n=1}^{N} \left[ \Delta_n^x e, + v \Delta_n^y e, \right], \]

which allow us to write the equations of motion (1) in the form:

\[ \frac{\partial^2 u}{\partial t^2} = \rho \left[ 1-v^2 \right] \frac{\partial^2 u}{\partial t^2} - F_i(w) + F_i^p; \]

\[ \frac{\partial^2 w}{\partial t^2} = \left[ \frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \right] - \frac{1-v^2}{12} \frac{Eh}{R^2} \left( P - \rho h \frac{\partial^2 w}{\partial t^2} \right) - F_2(u,w) + F_2^p, \]

where

\[ F_i(w) = \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial x^2} + \frac{v \partial w}{R} \]

\[ F_i^p = \sum_{n=1}^{N} \frac{\partial}{\partial x} \left( \Delta_n^x e, + v \Delta_n^y e, \right) \]

\[ F_2(u,w) = \frac{v}{R} \left[ \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \right] - \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial w}{\partial x} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial w}{\partial x} \right) \right] + \left[ \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \right] \left( \frac{\partial^2 w}{\partial x^2} \right) \]

\[ F_2^p = \frac{\partial^2 w}{\partial x^2} \sum_{n=1}^{N} \left[ \Delta_n^x e, + v \Delta_n^y e, \right] - \sum_{n=1}^{N} \frac{\partial^2}{\partial x} \left( \Delta_n^x e, + v \Delta_n^y e, \right) - \sum_{n=1}^{N} \left[ \frac{1}{R} \left( \Delta_n^x e, + v \Delta_n^y e, \right) - \frac{\partial w}{\partial x} \frac{\partial}{\partial x} \left( \Delta_n^x e, + v \Delta_n^y e, \right) \right]. \]

The boundary conditions of the problem corresponding to the free boundary of the shell have the form \( N_i = 0; M_i = 0; \frac{\partial M_i}{\partial x} = 0 \). In addition, the symmetry conditions with respect to the point \( x = L/2 \), where \( L \) is the length of the shell, are taken into account. The initial conditions of the problem are considered zero.

**Algorithm of solution.**

To solve the problem, we use the finite-difference method in the form of an implicit scheme.

We construct the difference equations for the system of equations of motion of the shell (4). The implicit scheme is obtained with the help of the following representations of \( u \) and \( w \) at the \( m \)-th step in time:

\[ \bar{u}^m = \mu^{m+1} + \bar{\epsilon}^m; \quad \bar{w}^m = \nu^{m+1} + \bar{\epsilon}^{m+1}. \]

Here \( \gamma + \xi = 1 \).

Substitute (5) into the left-hand sides of equations (4). For the \( m \)-th step in time, we obtain

\[ \frac{\partial^2 \bar{u}^m}{\partial x^2} = \frac{1-v^2}{E} \frac{\partial^2 \bar{u}^m}{\partial t^2} - F_i(w) + F_i^p; \]

\[ \frac{\partial^2 \bar{w}^m}{\partial t^2} = \left[ \frac{\partial^2 \bar{w}^m}{\partial x^2} + \frac{1}{2} \left( \frac{\partial \bar{w}^m}{\partial x} \right)^2 \right] - \frac{1-v^2}{12} \frac{Eh}{R^2} \left( P - \rho h \frac{\partial^2 \bar{w}^m}{\partial t^2} \right) - F_2(u,w) + F_2^p; \]

here \( F_1(w), F_1^p, F_2(u,w), F_2^p \) are determined by formulas (5).

Using the central differences and taking into account the boundary conditions, we obtain the algebraic equations corresponding to (7):

\[ B_i u_i^{m+1} - B_i u_i^{m+1} + B_i u_i^{m+1} = Q_i^n; \]

\[ A_i w_i^{m+1} + A_i w_i^{m+1} + A_i w_i^{m+1} + A_i w_i^{m+1} = Q_i^n, \]

where following notation is adopted for the convenience of writing:

\[ B_i = \frac{\gamma E T^2}{h^4} \left( 1-v^2 \right) \quad B_2 = 1 + 2B_1 \quad B_3 = B_1; \]

\[ A_i = \frac{\gamma h^2}{2h_i^4} \quad A_2 = -4A_1 \quad A_3 = A_2 \quad A_4 = A_1; \]

\[ A_i = 6A_i + \frac{\gamma}{R^2} + \frac{\rho(1-v^2)}{R} \]

\[ Q_1^m = -2u_i^m + u_i^{m+1} + \frac{\gamma E T^2}{\rho(1-v^2)} \left[ -\xi \frac{\partial^2 u_i^{m+1}}{\partial x^2} - F_1(w_i^m) + F_1^p \right]; \]

\[ Q_2^m = -\xi \frac{h^2}{12} \frac{\partial^2 w_i^{m+1}}{\partial x^2} - \xi \frac{E T^2}{R} \left( 2w_i^m + w_i^{m+1} \right) - \frac{1-v^2}{E} \left( -2w_i^m + w_i^{m+1} \right) + \frac{1-v^2}{E} \left( P - F_2(u_i^m, w_i^m) + F_2^p \right). \]

The difference expressions for \( Q_i^m, F_i^p \) are easily obtained by replacing the derivatives of \( u_i^{m+1}, w_i^{m+1}, u_i^m, w_i^m \) by the corresponding difference ratios.
The right-hand sides of equations (8) are nonlinearly dependent on $u^m, w^m, u^{m-1}, w^{m-1}$. Here, for brevity, not all the derivatives entering $Q_i^m, Q_2^m$ are written in the difference form, in particular, the terms $F_1(w^m), F_2(u^m, w^m)$ are obtained from (5) by replacing the derivatives of $u, w$ by the corresponding central differences.

In the finite differences, the boundary conditions $N_x = 0, M_z = 0; \frac{\partial M}{\partial x} = 0$ for the shell at the point $x = 0$ were written as:

$$u^m_{x=0} = 1 + \frac{4v}{3R}w_1^m + \frac{8u_3^m - 2u_2^m - 3u_1^m}{3} - 4h^2 \sum_{n=1}^{N} (\Delta^e_n \varepsilon_x + \nu \Delta^e_n \varepsilon_y);$$

$$w^m_{x=0} = -w_1^{m-1} + 4w_2^m - 2w_3^m + 24h^2 \sum_{n=1}^{N} (\Delta^e_n \varepsilon_x + \nu \Delta^e_n \varepsilon_y);$$

$$u^m_{N-1} = 1 + \frac{3}{3} \left( w_{N-1}^m - 3w_2^m + w_1^m + 2w_4^m \right) + 8h^2 \sum_{n=1}^{N} \left\{ (\Delta^e_n \varepsilon_x)_4 + 4(\Delta^e_n \varepsilon_y) - 3(\Delta^e_n \varepsilon_y)_2 \right\};$$

$$w_{N-1}^m = -w_{N-1}^{m-1} + 4w_2^m - 2w_3^m + 24h^2 \sum_{n=1}^{N} (\Delta^e_n \varepsilon_x + \nu \Delta^e_n \varepsilon_y);$$

$$w^m_{N=1} = 1 + \frac{3}{3} \left( w_{N-1}^m - 3w_2^m + w_1^m + 2w_4^m \right) + 8h^2 \sum_{n=1}^{N} \left\{ (\Delta^e_n \varepsilon_x)_4 + 4(\Delta^e_n \varepsilon_y) - 3(\Delta^e_n \varepsilon_y)_2 \right\};$$

The points of the three-point and five-point matrix algorithms are known from classical literature. When solving the system of equations (8), a sweep method is used, which has a matrix of tridiagonal and five-diagonal structures, respectively. Solutions are described as:

$$w_{i,k}^{m+1} = \alpha w_{i+1,k}^{m+1} + \beta w_{i,k}^{m+1} + q_{i,k};$$

$$w_{i,k}^{m+1} = P w_{i+1,k}^{m+1} + Q w_{i,k}^{m+1} + q_{i,k}. \quad (9)$$

The coefficients of these three-point and five-point matrix algorithms are calculated from known recurrence formulas, and the initial values of these coefficients are determined from the boundary conditions. Then, according to (9), we calculated $w_{i,k}^{m+1}$. The necessary values of $w_{N+2,k}^{m+1}, w_{N+3,k}^{m+1}$ were found from the boundary conditions:

**Results of calculations.**

Influences of linearity ($F_1 = 0, F_2 = 0$) and nonlinearity ($F_1 \neq 0, F_2 \neq 0$) of motion equations of the shell, boundary conditions (concerning $N$ and $M$), and also the geometrical sizes (thickness, length and radius of the shell), physic-mechanical parameters (steel, D16AT, CuSi3Mn1) and the loading form (an isosceles triangle, sinusoidal) on the nature of deforming of a shell is studied. Calculations were carried out for the steel shell with free ends. On internal surface of the shell the applied pulse loading by duration $10^{-4}$ s which change in time and coordinate correspond to an isosceles triangle is enclosed. Amplitude of loading $q_0$ varied. The shell was divided by length on 20 parts, with a step of 0.01 m. Shell sizes: thickness $h = 0.001$ m; radius $R = 0.014$ m; long $L = 0.2$ m. The step on time providing stability of calculating process was defined from a condition of spectral stability of diagrams [4, 10], then was specified by numerical experiments.

In fig. 1 plotted results of calculations of the central point displacement of the shell in time calculated on the basis of the linear (curve 1) and nonlinear (curve 2) of the equations of the theory of a shell according to the explicit scheme, and a curve 3 - results of calculation with attraction of the nonlinear equations calculated according to the implicit scheme are shown. Intensity loading equaled 100 MPa. Apparently, the accounting of nonlinearity of the equations of vibrations of the shell according to the implicit scheme gives the reduction values of a vibration amplitude and the increased values of its frequency.

Variation of displacement of central point of the shell in time in case of different values of its thickness, length and radius are also calculated. Reduction of thickness and length of a shell and increase in its radius leads to increase in amplitude of elastic vibrations. For example, in case of reduction of thickness twice (in case of $t = 0.5 \cdot 10^{-4}$ s) amplitude increases about 2.4 times; in case of the accounting of plastic deformations displacement increases of nearly 20 times. Changes of the cross size ($R = 0.014; 0.02; 0.03$ m) lead also to sharp
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change of displacement in two and more times (fig. 2, \( h = 0.5 \times 10^{-3} \) m (a curve 1); \( h = 1.0 \times 10^{-3} \) m (a curve 2); \( h = 2.0 \times 10^{-3} \) m (a curve 3)).

The carried-out calculations in case of values of amplitude of loading equal 10; 20; 50; 100 MPa showed that the maximum elastic displacement of the central point of the shell in case of \( t=0.5 \times 10^{-4} \) s are equal respectively 0.1 \( \times 10^{-3} \); 0.2 \( \times 10^{-3} \); 0.46 \( \times 10^{-3} \); 0.95 \( \times 10^{-3} \) m. The accounting of plastic properties of material shows that with increase in loading of displacement grow without restrictions (fig. 3, curve 2), and it is possible to receive real results only in case of \( q_0=10 \) MPa (fig. 3, a curve 1).

![Diagram 1](image1.png)

**Fig. 1.** Deflections of central point of the shell calculated on the basis of the equations: linear (curve 1); nonlinear (curve 2) of the equations according to the explicit scheme and the nonlinear equations according to the implicit scheme (a curve 3).

![Diagram 2](image2.png)

**Fig. 2.** Influence change of thickness of a shell on elastic-plasticity deflections of her central point: \( h = 0.5 \times 10^{-3} \) m (a curve 1); \( h = 1.0 \times 10^{-3} \) m (a curve 2); \( h = 2.0 \times 10^{-3} \) m (a curve 3).

The same behavior of deforming yields results of calculations in case of change of parameter values of material of a shell: the maximum elastic displacement for steel \( 0.88 \times 10^{-3} \) m, D16AT – \( 0.488 \times 10^{-3} \) m, CuSi3Mn1 – \( 1.126 \times 10^{-3} \) m. The accounting of plastic properties of material in case of \( t=10^{-4} \) s with leads to increase in displacement almost twice (fig. 4). Influence of small deviation in mechanical properties of material on the behavior of vibrations of a shell is studied. Calculation for D16AT was carried out in case of \( \nu = 0.3; 0.35 \). The received results differed no more than for 1%.
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**Fig. 3.** Influence of the load amplitude on elastic-plasticity displacements of the central point of the shell in time calculated according to the explicit diagram of calculation of \( q_0 = 10 \text{ MPa} \) (curve 1), \( q_0 = 20 \text{ MPa} \) (curve 2).

Influence of the form of loading in time behavior construction deformation is studied. Influence of pulses of relatives to shock on the shell is considered: change of loading under the law of an isosceles triangle, sinusoidal and exponential. Calculations are shown a tendency to increase in displacements with growth of length and amplitude of the falling pulse (fig. 5).

Parameters of the shell remained, as well as in the previous cases. The module of hardening and yield strength characterizing plastic properties of material made 500 and 400 MPa respectively. Values of stored plasticity deformation it was specified through 10 steps on time. Computation carried out with at amplitude of loading of 100 MPa. Calculations showed that length of the shell and small deviation in mechanical properties of material influence on displacement central point, and increase in radius and thickness were followed by growth of displacements. Comparing of curves of displacements central point of the shell for D16AT (curve 1), by CuSi3Mn1 (curve 2) and steel are given in fig. 6 (curve 3).

**Fig. 4.** Influence of parameters of material of the shell (curve 1-D16AT, curve 2-CuSi3Mn1, curve 3 – steel) on elastic-plasticity deflections of her central point.
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Fig. 5. Influence of loading: under the law of isosceles triangle (curve 1, \( q_0 = 100 \) MPa) sinusoidal (curve 2, \( q_0 = 50 \) MPa) and exponential (curve 3, \( q_0 = 5 \) MPa and curve 4, \( q_0 = 10 \) MPa), on elastic-plasticity deflections of the central point of the shell in time.

Fig. 6. Dependences of elastic-plasticity deflection of central point of the shell on time for various materials: a curve 1 – D16AT; 2 – CuSi3Mn1; 3 – steel.

Plastic properties of materials are defined according to the algorithm of [4]. Calculations have shown that change of a step of accounting of accumulation of plastic properties of material influenced a residual deflection a little. The maximum plastic deflections are 30-40% more than the maximum elastic deflections. The zone of plasticity is localized at the central point of a shell.

**Conclusions.**

The shell as a result of influence of pulse loading gets a barrel form. Results of linear and nonlinear calculations differ rather strongly. The accounting of plastic properties of material significantly influences the behavior of deformation of structure. Therefore, the use of the theory of elastoplastic flow in the analysis of this class of problems seems necessary. This numerical technique of the analysis of elastic-plasticity deformation of shell, and also the received results can be widespread for similar calculations of problems of a hydro-elastic-plasticity.

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