Three Neutrino Flavors are Enough

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Abstract

It is shown that it is possible to account for all three experimental indications for neutrino oscillations with just three neutrino flavors. In particular, we suggest that the solar and atmospheric neutrino anomalies are to be explained by the same mass difference and mixing. Possible implications and future tests of the resulting mass-mixing pattern are given.

1 Introduction

Currently there are three pieces of evidence which suggest that neutrinos have non-zero mass differences and mixings. These are: (i) the observations of solar neutrinos, (ii) the anomaly in the $\nu_\mu/\nu_e$ ratio in atmospheric neutrinos at low energies and (iii) the possible $\bar{\nu}_\mu - \bar{\nu}_e$ conversion seen in the LSND experiment. With the conventional interpretation of these effects as being due to neutrino oscillations; the solar neutrino anomaly needs a $\delta m^2 (\nu_e - \nu_x)$ of either about $10^{-5} - 10^{-6} \text{eV}^2$ (MSW) or about $10^{-10} \text{eV}^2$ (long wavelength vacuum oscillations), the atmospheric neutrino anomaly calls for a $\delta m^2 (\nu_e - \nu_\mu)$ or $(\nu_\mu - \nu_\tau)$ of $10^{-2} - 10^{-3} \text{eV}^2$ and the LSND effect needs a $\delta m^2 (\nu_e - \nu_\mu)$ in the neighborhood of $1 - 2 \text{eV}^2$. For these three independent $\delta m^2$'s at least one more neutrino state (beyond the three flavors) is necessary [1].

In this letter we explore the possibility that all the neutrino anomalies may yet be accounted for with just three flavors of neutrinos. We assume only two distinct values of $\delta m^2$'s. One value of $\delta m^2$ is selected to explain
the low energy atmospheric data, while the second value is selected with the LSND effect in mind.

Specifically we choose the following spectrum of $\delta m^2$'s:
\begin{align}
\delta m^2_{31} &\sim \delta m^2_{32} \sim (1 - 2) eV^2 \\
\delta m^2_{21} &\sim 10^{-2} eV^2
\end{align}

We then seek to determine if, with this spectrum of $\delta m^2$'s, an explanation of the LSND, solar, atmospheric neutrino data can be found by appropriate choice of neutrino mixing angles.

We begin by calculating the neutrino survival and transition probabilities. In general, these are given by
\begin{equation}
P_{\alpha\beta} = \left| \sum_i U_{\beta i} \exp(-iE_it)U_{\alpha i}^* \right|^2
\end{equation}

Here the $U_{\alpha i}$ are elements of the matrix $U$ describing the mixing between the flavor eigenstates ($\nu_\alpha$) and the mass eigenstates ($\nu_i$); that is $\nu_\alpha = \sum_i U_{\alpha i}\nu_i$. For now we ignore possible CP violation, then $U$ is real, and Eq (3) may be written as
\begin{equation}
P_{\alpha\beta} = \sum_i (U_{\beta i})^2(U_{\alpha i})^2 + 2 \sum_{i>j} U_{\beta i}U_{\beta j}U_{\alpha i}U_{\alpha j} \times \cos \left( \frac{\delta m^2_{ij}L}{2E} \right)
\end{equation}

where $\delta m^2_{ij} = m_i^2 - m_j^2$ and $L$ is the distance between the neutrino source and detection. We present below an explicit form of the $3 \times 3$ matrix $U$

\begin{equation}
U = \begin{pmatrix}
C_{12}C_{13} & C_{13}S_{12} & S_{13} \\
-S_{23}S_{12} - S_{23}S_{13}C_{12} & C_{23}C_{12} - S_{23}S_{13}S_{12} & S_{23}C_{13} \\
S_{23}S_{12} - C_{23}S_{13}C_{12} & -S_{23}C_{12} - C_{23}S_{13}S_{12} & C_{23}C_{13}
\end{pmatrix}
\end{equation}

where $C_{12} = \cos \theta_{12}$, $S_{12} = \sin \theta_{12}$, etc.. The explicit form of the transition probabilities depends on the spectrum of the $\delta m^2$'s. For the choice of $\delta m^2$'s considered here, all of the oscillating terms in Eq (4) average to zero for the energies and path lengths relevant to both low energy atmospheric and solar neutrinos. Hence, for our model, the form of the transition and survival probabilities relevant to solar and atmospheric neutrinos are:
\begin{equation}
P_{ee} = \sum_i (U_{ei})^4
\end{equation}
\[ P_{\mu \mu} = \sum_i (U_{\mu i})^4 \]  
\[ P_{e \mu} = P_{\mu e} = \sum_i (U_{ei}U_{\mu i})^2 \]

Note that the above expressions are functions of the mixing angles only, and are independent of the neutrino energy.

## 2 Solar Neutrinos

The four currently operating solar neutrino experiments report the following results:

| Experiment | Results |
|------------|---------|
| Homestake  | 2.56 ± 0.16 ± 0.14 SNU |
| Kamioka    | 2.80 ± 0.19 ± 0.33 ×10^{10} m^{-2}s^{-1} |
| SAGE       | 72 ± 12 ± 7 SNU |
| Gallex     | 70 ± 7 SNU |

Recently measurements of the reaction \( \gamma + ^8B \rightarrow ^7Be + p \) have been made\(^6\). These suggest that the cross-section for the inverse reaction \(^7Be + p \rightarrow ^8B + \gamma\) at energies relevant for the solar core may be somewhat smaller than the value used in the Standard Solar Model (SSM) of Bahcall et al. calculations. Hence it is possible that the flux of \(^8B\) neutrinos is somewhat smaller than the SSM, while the other neutrino fluxes are unaffected. We allow for this possibility by defining \( f_B \) as

\[ f_B = \frac{\Phi_B}{\Phi_B^{BP}} \]  

where \( \Phi_B^{BP} \) is the \(^8B\) neutrino flux predicted in the SSM of Bahcall and Pinsonneault, which incorporates helium diffusion\(^7\). Thus the parameter \( f_B \) describes the deviation of the actual \(^8B\) neutrino flux from the SSM value.

We can now proceed to describe the expected counting rates at the various solar neutrino experiments in terms of \( f_B \) and \( P_{ee} \) the solar \( \nu_e \) survival probability.

\(^1\)Solar models which incorporate helium diffusion yield values for the depth of the convective zone and primordial helium abundance which are in excellent agreement with helioseismological data\(^8\).
With a threshold energy of 7.5 MeV the Kamiokande water Cerenkov detector is sensitive only to $^8B$ neutrinos. The expected flux is given by:

$$R(\text{KII}) = (P_{ee} + \alpha(1 - P_{ee}))f_B \times 5.69 \times 10^{10}\text{m}^{-2}\text{s}^{-1}$$

(10)

Where $5.69 \times 10^{10}\text{m}^{-2}\text{s}^{-1}$ is the SSM prediction and $\alpha$ (approximately 0.16) is the ratio of the $\nu_{\mu(\tau)} - e$ to $\nu_e - e$ scattering cross sections integrated over the $^8B$ neutrino spectrum. Note that we have ignored the possibility of oscillation into sterile flavors and assumed that solar neutrinos not interacting as $\nu_e$’s interact as either $\nu_\mu$’s or $\nu_\tau$’s with probability $1 - P_{ee}$.

The expected counting rate in the Homestake $^{37}\text{Cl}$ experiment is given by

$$R(\text{^{37}Cl}) = (6.2f_B + 1.8) \times P_{ee} \text{ SNU}$$

(11)

where 6.2 SNU’s is the expected contribution from $^8B$ neutrinos and 1.8 SNU’s is the contribution from all other solar neutrino fluxes. Similarly, the expected counting rate in the $^{71}\text{Ga}$ experiments SAGE and Gallex is given by

$$R(\text{^{71}Ga}) = (13.8f_B + 117.6) \times P_{ee} \text{ SNU}$$

(12)

where 13.8 SNU’s is the expected contribution from $^8B$ neutrinos and 117.6 SNU’s is the contribution from all other solar neutrino fluxes.

The results of a chi-squared analysis are shown in Fig. 1. We find that there is a solution at the 90% C.L. when the electron neutrino survival probability is in the range $0.4 < P_{ee} < 0.55$ and the $^8B$ neutrino flux is in the range $0.55 < f_B < 0.8$. This is consistent with the variation in $^8B$ neutrino flux found in an analysis of solar models.[8]

The results in Fig. 1 can be interpreted in terms of the neutrino mixing matrix $U$. From Eq( 6) the $\nu_ee$ survival probability $P_{ee}$ is a function of $\theta_{12}$ and $\theta_{13}$ only. Each allowed value of $f_B$ in Fig. 1 corresponds to an allowed range of $P_{ee}$ and hence to an allowed range of $\theta_{12}$ and $\theta_{13}$. In Fig. 2 we present a plot of the allowed values (90% C.L.) of $\sin(\theta_{12})$ and $\sin(\theta_{13})$ for $f_B = 0.8$ and $f_B = 0.65$. At $f_B = 0.8$, $P_{ee}$ is required to be $\sim 0.43$, this can be realized only in three flavor mixing. Hence, as shown in Fig. 2, $S_{12}$ and $S_{13}$ must

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2It was first suggested by Acker et al.[9] that solar neutrinos might be accounted for by the same $\delta m^2$ and mixing as atmospheric neutrinos and hence should show an energy independent suppression. A search for an energy independent fit with varying $f_B$ was made in Ref. [10].
both be nonzero. At $f_B = 0.65$, $P_{ee}$ can be greater than 0.5, which can be accomplished in effective two flavor mixing. As shown in the figure, there are allowed regions with $\sin(\theta_{12})$ or $\sin(\theta_{13})$ equal to zero, corresponding to pure $\nu_e - \nu_\tau$ or $\nu_e - \nu_\mu$ mixing respectively.

3 LSND

The Liquid Scintillation Neutrino Detector (LSND) experiment at Los Alamos reports to have observed the possible appearance of $\nu_e$ in an initial beam of $\nu_\mu$’s [11]. These results have been interpreted as evidence of neutrino oscillations and the preferred range of $\delta m^2$ and $\sin^2(2\theta)$ in a two flavor mixing scenario given. For definiteness, we choose $\sin^2(2\theta) \sim 1.2 \times 10^{-3}$ and $\delta m^2 \sim 2\ eV^2$, which lie in this range [12].

In the range of $L/E$ covered by the LSND set-up, $\delta m^2_{31} L/4E \sim 0$ and $\delta m^2_{32} L/4E \sim \delta m^2_{32} L/4E$. Then the $\nu_\mu \to \nu_e$ conversion probability is given by

$$P_{\mu e} = 4U_{e3}^2 U_{\mu 3}^2 \sin^2(\delta m^2_{31} L/4E)$$

Thus the three-flavor interpretation of the LSND result is obtained by letting

$$\sin^2(2\theta_{LSND}) \to 4|U_{\mu 3}|^2 |U_{e 3}|^2$$

This may be expressed as a constraint on the three flavor mixing angles. Using

$$|U_{e 3}|^2 = \sin^2(\theta_{13})$$
$$|U_{\mu 3}|^2 = \cos^2(\theta_{13}) \sin^2(\theta_{23})$$

We obtain

$$\sin^2(\theta_{23}) = \frac{\sin^2(2\theta_{LSND})}{4 \sin^2(\theta_{13}) \cos^2(\theta_{13})}$$

Choosing $\delta m^2_{31} \sim \delta m^2_{32}$ to be near $2\ eV^2$, the LSND results then give $\sin^2(2\theta_{LSND}) \sim 1.2 \times 10^{-3}$, and we have

$$\sin^2(\theta_{23})^2 = \frac{1.2 \times 10^{-3}}{4 \sin^2(\theta_{13}) \cos^2(\theta_{13})}$$
In order to determine the range of validity of this result, constraints from reactor and accelerator experiments must be taken into consideration. For this range of $\delta m^2$, reactor experiments give the bound [13]:

$$|U_{e3}|^2 \leq 0.02 \quad (18)$$

and accelerator experiments give the bound [14]:

$$|U_{\mu 3}|^2 \leq 0.018. \quad (19)$$

As $|U_{\mu 3}|$ is related to $|U_{e3}|$ through Eq (17) these two upper limits can be combined to form bounds on the allowed values of $|U_{e3}|$ and $|U_{\mu 3}|$. We have:

$$0.129 \leq |U_{e3}| \leq 0.141 \quad (20)$$

and

$$0.123 \leq |U_{\mu 3}| \leq 0.134 \quad (21)$$

Hence the requirement that the LSND results be consistent with existing bounds on neutrino mixing leads to rather stringent limits on the allowed values of $|U_{e3}|$ and $|U_{\mu 3}|$. We will find these constraints particularly useful when interpreting the atmospheric neutrino data.

4 Atmospheric Neutrinos

Experimentally measured atmospheric neutrino fluxes are often described in terms of an (observed to predicted) ‘ratio of ratios’ $R$, where

$$R = \frac{(\nu_{\mu}/\nu_e)_{\text{observed}}}{(\nu_{\mu}/\nu_e)_{\text{MonteCarlo}}} \quad (22)$$

The final results [3, 15] from Kamiokande for the low energy atmospheric neutrino $\nu_{\mu}/\nu_e$ ratio place $R$ at $0.62 \pm 0.06 \pm 0.06$. The results from IMB [16] are in excellent agreement with these results. Results from non-water-Cerenkov detectors are somewhat varying: Soudan [17] finds an $R$ of $0.72 \pm 0.19 \pm 0.05 \pm 0.07$ whereas the results from Nusex [18] and Frejus [19] are consistent with an $R$ of unity, although with smaller statistics than the two large water-Cerenkov detectors.
It has been known for some time that this low energy atmospheric neutrino anomaly can be explained by neutrino oscillations. For $\delta m^2$ in the range $(4 \times 10^{-3} - 2 \times 10^{-2}) eV^2$ it has been shown [20] that this anomaly can be explained by $\nu_\mu - \nu_\tau$ oscillations for

$$0.6 \leq \sin^2(2\theta_{\mu\tau}) \leq 1.0$$

or by $\nu_\mu - \nu_e$ oscillations for

$$0.5 \leq \sin^2(2\theta_{\mu e}) \leq 1.0.$$  \hfill (23)

Expressing these bounds in terms of the $U_{ai}$ we have:

$$0.3 \leq (P_{\mu\tau} = \sum_i (U_{\mu i}U_{\tau i})^2) \leq 0.5$$

for $\nu_\mu - \nu_\tau$ oscillations and

$$0.25 \leq (P_{\mu e} = \sum_i (U_{\mu i}U_{ei})^2) \leq 0.5$$

for $\nu_\mu - \nu_e$ oscillations.

In the narrow range of $|U_{e3}|$ values permitted by LSND, reactor and accelerator data (Eq 20), $P_{\mu\tau}$ is less then 0.05 and thus inconsistent with Eq (25); while $P_{\mu e}$ can take on values up to 0.48. Hence, in this region, the atmospheric neutrino anomaly must be explained almost exclusively by $\nu_\mu - \nu_e$ mixing.

With $\theta_{23}$ constrained in terms of $\theta_{13}$ by Eq (17) and $\sin(\theta_{13})$ bound by Eq (20), we find that $0.25 \leq P_{\mu e} \leq 0.5$ if $\sin(\theta_{12})$ is in the range:

$$0.38 \leq \sin(\theta_{12}) \leq 0.92$$  \hfill (27)

Thus we can express this explanation of the low energy atmospheric neutrino anomaly consistent with the LSND, reactor, accelerator data as the region of the $\sin(\theta_{12}) - \sin(\theta_{13})$ plane bounded by Eq (17) and Eq (20).

5 A Combined Solution to the Solar, Atmospheric and LSND Data

It is now a straightforward matter to identify simultaneous solutions to the Solar, Atmospheric and LSND neutrino data. As the energy independent
solution to the solar neutrino problem, shown in Fig. 2, and the combined LSND and atmospheric neutrino solution are both expressed as regions of the $\sin(\theta_{12}) - \sin(\theta_{13})$ plane, any intersection between the allowed regions represents the desired solution.

Fig. 3 presents a plot of the intersecting regions of the Solar neutrino and Atmospheric-LSND solutions. Fig. 3a assumes the $^8B$ solar neutrino flux, $f_B$, is at 80% of its SSM value, Fig. 3b assumes $f_B$ is at 70% of its SSM value and Fig. 3c assumes $f_B$ is at 65% of its SSM value. There is no intersection in Fig. 3a and narrow region of overlap in Fig. 3b broadening somewhat in Fig. 3c as the $^8B$ neutrino suppression is allowed to increase to 0.65.

It should be noted that the selection of any region of the $\sin(\theta_{12}) - \sin(\theta_{13})$ plane determines the complete set of mixing angles, and hence the neutrino mixing matrix $U$, as $\sin(\theta_{23})$ is fixed by Eq (17). Specifically the intersection region of Fig. 3b corresponds to $\sin(\theta_{12}) \sim 0.707$, $\sin(\theta_{13}) \sim 0.140$ and $\sin(\theta_{23}) \sim 0.125$.

Using Eq (5) we present below the explicit form of the $3 \times 3$ mixing matrix $U$ corresponding to solution region of Fig. 3b:

$$ U = \begin{pmatrix} 0.700 & 0.700 & 0.140 \\ -0.714 & 0.689 & 0.124 \\ -0.010 & -0.187 & 0.982 \end{pmatrix} \quad (28) $$

While for the solution region corresponding to Fig. 3c we find that, in addition to Eq (28) above, the following range of matrix values are allowed:

$$ \begin{pmatrix} 0.630 & 0.764 & 0.140 \\ -0.776 & 0.619 & 0.124 \\ -0.010 & -0.187 & 0.982 \end{pmatrix} \leftrightarrow \begin{pmatrix} 0.764 & 0.630 & 0.140 \\ -0.645 & 0.754 & 0.124 \\ -0.028 & -0.185 & 0.982 \end{pmatrix} \quad (29) $$

6 Implications

(i) Both Super-Kamiokande [21] and SNO [22] should see NO spectrum distortion in either $\nu - e$ scattering or the $\nu_e D$ charged current. The suppression in $\nu - e$ scattering should be in the range 0.38 - 0.40 of SSM at all energies and in $\nu_e D$, charged current suppression should be about $f_B P_{ee} \sim 0.32 - 0.34$. 

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(ii) Borexino [23] should observe the $^7$Be line at a rate of $[P_{ee} + \beta(1 - P_{ee})]$ where $\beta$ is the ratio of $\nu_{\mu(\tau)} - e$ to $\nu_e - e$ scattering cross sections. We thus expect 0.56 to 0.58 of the SSM rate, and an identical suppression should hold for the pep line.

(iii) The atmospheric $\nu_{\mu}/\nu_e$ anomaly should be confirmed by Super-Kamiokande. Zenith angle dependence of multi-GeV neutrinos should confirm the tentative evidence seen in Kamiokande [20]. But most important is our prediction [9] that $\nu_{\mu} - \nu_e$ oscillations should be confirmed by observation of excess high energy e-like upcoming shower events (above and beyond $\nu_{\mu}$ neutral current events).

(iv) Future reactor experiments such as CHOOZ [24] and Palo Verde [25] which will be sensitive to $\delta m^2$ up to $10^{-3} \text{ eV}^2$ should see a $P_{ee} \sim 0.48 - 0.5$.

(v) Long baseline experiments (such as MINOS [26], CERN-LNGS [27] and KEK-PS E362 [28]) which will probe $\delta m^2$ up to $10^{-3} \text{ eV}^2$ should see $\nu_{\mu} - \nu_{\tau}$ conversion with $P_{\mu\tau} = \sum_i (U_{\mu i} U_{\tau i})^2 \sim 0.028 - 0.035$ accompanied by $\nu_{\mu} - \nu_e$ conversion at $P_{\mu e} \sim 0.46 - 0.48$.

(vi) Short baseline experiments such as CHORUS [30], NOMAD [30] and COSMOS [31] will probe $\nu_{\mu} - \nu_{\tau}$ conversion for $\delta m^2 \geq 0.1 \text{ eV}^2$. We predict, at $\delta m^2 = 1 - 2 \text{ eV}^2$, an effective $\sin^2(2\theta)$ of $4(U_{\mu3} U_{\tau3})^2$ which is 0.06.

(vii) Large mixings in some ranges of $\delta m^2$ lead to strong conversion of $\nu_{\mu}$ to $\nu_{\tau}$ due to the MSW effect in supernova, leading to a harder energy spectrum of the emerging $\nu_e$’s. This can lead to potential conflict with observation of neutrinos from SN1987A. For the $\delta m^2$ in our scenario this is not a problem [32].

(viii) The neutrino mass spectrum implied by our scenario is:

\begin{align*}
m_1 &\sim m_0 \\
m_2 &\sim m_0 + \epsilon \\
m_3 &\sim \sqrt{m_2^2 + 2\epsilon} \tag{30}
\end{align*}

where $\delta m^2_{12} \sim (2m_0\epsilon + \epsilon^2) \sim 10^{-2} \text{ eV}^2$. There are two limiting cases of interest assuming that the largest mass is in the $eV$ range. One is the hierarchical limit, in which $m_0$ is negligible. Then $m_1 \ll m_2 \sim 0.1eV$ and $m_3 \sim 1.4eV$. The other is the nearly degenerate limit, in which

\begin{align*}
m_1 &\sim 1eV
\end{align*}
\[ m_2 \sim (1 + \epsilon) eV \]
\[ m_3 \sim 1.73 eV \]

with \( \epsilon \sim \frac{1}{2} (10^{-2}) eV \). Then, the sum of the neutrino masses is \( \sum_i m_i \sim 4 eV \). In this case, the Cosmological density parameter associated with neutrinos \( \Omega_\nu = 0.011 h^{-2} \sum_i m_i = 0.044 h^{-2} \approx 0.2 \) (for \( h \) of about 0.5) and the amount of neutrino dark matter component along with cold dark matter makes for a viable and testable scenario for mixed dark matter [33].

(ix) When the neutrinos are Majorana particles, the effective mass \( < m_{\nu_e} > \) relevant in neutrino-less double \( \beta \)-decay analysis is

\[ < m_{\nu_e} > = \sum_i U_{ei}^2 m_i \]

We find that in the case of the hierarchical spectrum \( < m_{\nu_e} > \sim 0.1 eV \) whereas in the degenerate case \( < m_{\nu_e} > \sim 1 eV \) (this could be somewhat smaller when CP phases are taken into account). It is interesting that these values are in the range of what the double beta decay experiments can probe now and in the near future [34].

(x) When the mixing matrix is allowed to have a CP violating phase, the CP violating neutrino flavor conversion probability differences are given by [35]

\[ \Delta P = P_{\mu\tau} - P_{\mu\mu} = P_{\mu\mu} - P_{\mu e} = -4 J_{CP}' [\sin D_{12} + \sin D_{23} + \sin D_{31}] \]

where

\[ J_{CP}' = Im [U_{\mu 2}^* U_{\tau 2} U_{\mu 3} U_{\tau 3}^*] \]
\[ = |U_{\mu 2}||U_{\tau 2}||U_{\mu 3}||U_{\tau 3}| \sin \phi, \]

and

\[ D_{ij} = \delta m_{ij}^2 L/2E \]

with \( \phi \) being the phase in the mixing matrix. With the matrix of Eq.(23), \( J_{CP}' \leq 0.07 \); and \( [\sin D_{12} + \sin D_{23} + \sin D_{31}] \approx \sin D_{12} \) is given by \( -1 \) for \( L/E = 730 \text{km}/10 \text{GeV} \) (relevant for MINOS) and also for \( L/E = \ldots \)
250km/3GeV (relevant for E362). Hence, \( \Delta P \) can be as large as 0.07. (For these parameters, matter effects are negligible [35].)

We conclude by stressing that our proposal to account for both solar and atmospheric neutrino anomalies by the same mass and mixing can be confirmed or ruled out in the very near future.

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Figure Captions

Figure 1 Contour plot showing the allowed values of the parameters $P_{ee}$ and $f_{Bn}$ at the 90 % (solid line) and 95 % (dashed line) confidence levels in the three flavor mixing solution to the solar neutrino problem.

Figure 2 Contour plot showing the allowed values of $\sin(\theta_{12})$ and $\sin(\theta_{13})$ (90 % confidence level) in the three flavor mixing solution to the solar neutrino problem for fixed values of $f_B$. Solid line; $f_B = 0.8$, dashed line; $f_B = 0.65$.

Figure 3 Combined solution: solar atmospheric and LSND results. Dashed line; bounds from LSND, reactor and accelerator experiments, Hatched region; Atmospheric neutrino anomaly explained by $\nu_\mu - \nu_e$ oscillations, Solid line; allowed region at the 90 % C.L. in the three flavor mixing solution to the solar neutrino problem for: (a) $f_B = 0.8$ (b) $f_B = 0.7$ and (c) $f_B = 0.65$. 

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$\sin(\theta_{13})$

Fig. 3c