On the kinematic modelling of an accurate tracking system to be implemented into solar-thermal collectors

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Abstract. One of the possibilities for increasing the energy performance of a parabolic trough-type solar thermal collector is by its diurnal orientation. The paper presents the kinematics of a tracking system with linkages, having as a starting point a patent proposal elaborated by the authors. The tracking system consists of a monoaxial mechanism, driven by three linear actuators, designed for the accurate orientation on large angular strokes of concentrated solar thermal collectors. The tracking system has two linear actuators in a triangular arrangement, articulated at the fixed ends, with the role of achieving the orientation accuracy and favourable transmission angles, and a third actuator aimed to allow a fine increase of the angular stroke of the collector under controlled conditions. The first steps in the kinematic modelling are to determine the orientation stroke of the collector and the transmission angles required in the static modelling by considering the third actuator as fixed and the strokes of the other two actuators as independent of each other. The aim of this modelling is to allow the subsequent optimization of an orientation program for a parabolic trough solar collector that takes into account the geometrical, kinematic, static and operating requirements. The simulation results allow drawing conclusions that are useful to the designers and developers in the field for dimensioning the collector tracking system under correct functioning conditions.

1. Introduction

The increase of living standard has been followed by an increase in the energy requirement at global level. Greenhouse gases, pollution and global warming have led more and more states to adopt targets for obtaining energy by using renewable energy sources, reducing pollution, developing and using systems that have low energy consumption.

According to REN21 (Renewable Energy Policy Network for the 21st Century) report, the largest energy consumer at the end of 2016 is the heating & cooling sector which consumes 51% of the total energy, the second consumer being transportation, which represents 32%, while power represents 17% of total energy consumption [1].

The concentrated solar-thermal systems allow the use of the obtained thermal agent both for heating during the cold or transient periods (spring and autumn), as well as for cooling. The thermal energy obtained at the end of 2018 by using concentrated solar systems was of 16.6 GWh, the increase being of 3.8% compared to 2017 [1].

The main concentrated solar systems are the parabolic trough-type collectors, with central tower and the dish-type parabolic collectors. These systems use only direct solar radiation, which leads to the
use of tracking systems aiming to change the position of the reflecting surface.

Two types of tracking systems are used for the orientation of the concentrated collectors: passive tracking systems that do not have motors and use shape memory materials, and active tracking systems, which are composed of motors and control systems. Orientation can be performed on one direction or on two directions [2].

The parabolic trough collectors are the most used concentrated systems due to their orientation simplicity, the low cost and the use in applications with large surfaces. These systems are equipped with monoaxial tracking systems [3].

The efficiency of this type of systems can be diminished by the following factors: the accuracy of orientation and / or the quality of the reflecting surface. The greatest influence in diminishing the system efficiency is given by the orientation accuracy. The technical literature states that the orientation accuracy is achieved with angles of 0.2° for cloudy days and 0.05° for clear days, the largest amount of solar radiation being obtained in the time interval 11-13 [2, 5].

The tracking system can contain gears, cam mechanisms, chain or cable transmissions and linkages [3,6,7,8]. In order to obtain large angular strokes, two or three linear actuators in series and / or in parallel connection can be used in the linkage based tracking systems [6,8,9].

The paper presents the kinematic modelling of a monoaxial tracking system with linkages, driven by three linear actuators, which is used for the orientation of parabolic trough collectors in order to reduce the incidence angle of solar radiation. The modelling allows the optimization of the tracking system with which to obtain an efficient orientation program.

The numerical simulation results enable drawing conclusions that are useful to the designers and developers in the field for dimensioning the collector tracking system under imposed functioning conditions.

2. Problem formulation

According to Figs. 1 and 2, the tracking system has two linear actuators arranged in a triangle, articulated at the fixed ends (joints A and B, Fig. 2), with the role of achieving the orientation accuracy of the solar collector with large angular strokes, concomitant with favourable transmission angles, and a third actuator used for fine adjustments of the angular displacement and maximization of the collector orientation stroke under controlled conditions.

![Figure 1. Representative positions (morning-East, noon and evening-West) for the collector diurnal stroke.](image)

The first step in the kinematic modelling is to consider that the third actuator is inactive (blocked) and that the transmission angle $\gamma = 90^\circ$ (see Fig. 2). According to Fig. 3 and under these premises, the angular displacement of the solar-thermal collector ($\psi$) and the angles between the three linear
actuators ($\alpha_1$, $\alpha_2$, $\alpha_3$) can be determined from the geometry of the tracking system ($a$, $b$, $l$) (see Fig. 3) as functions of the active actuators displacements ($a_1$ and $a_2$).

![Diagram of tracking mechanism](image)

**Figure 2.** The tracking mechanism in 2 representative positions: reference (noon-interrupted line) and current (continuous line), with the highlighting of the angular stroke ($\Delta \psi$) of the solar-thermal collector.

As an example, the case study considers the angular stroke $\Delta \psi = 114^\circ$ corresponding to the winter solstice between 7:30 a.m. ($\psi = 57^\circ$) and 4:30 p.m. ($\psi = -57^\circ$) [8].

3. Kinematic Modelling of the Tracking System

The kinematic modelling presented in this paper aims to further prepare the mathematical model of the control program for the solar-thermal collector orientation. The following steps are considered in the kinematic modelling:

3.1. Notations

The following notations are used in the kinematic modelling (see Fig. 3):

- $ED = ED_0 = ED_1 = ED_2 = a$ the length of the rocker;
- $DC = D_0C_0 = D_1C_1 = D_2C_2 = b$ the length of the blocked actuator / connecting rod;
- $OA = OB = OC_0 = l$ half of the distance between the joints with the base of the mobile actuators;
- $OE = OC_0 + C_0E$ the maximum distance between the rocker joint and the base;
- $AC = a_1$; $BC = a_2$ the current length of the actuator;
- $AC_0 = a_1^0 = a_0^0$; $BC_0 = a_2^0 = a_0$ the length of the actuator in the reference position;
- $\angle EDC = \gamma$ the angle between the connecting rod and the rocker;
- $\angle DCA = \alpha_1$; $\angle DCB = \alpha_2$ the angles between the connecting rod and the actuators;
- $\angle ACB = \alpha_2$ the angle between the two mobile actuators;
- $\psi$ the angular displacement of the solar collector;
- $\Delta \psi = \psi_{\text{max}} - \psi_{\text{min}}$ the angular stroke of the solar collector.
3.2. The angles between actuators

The angles involved in the orientation algorithm of an accurate solar-thermal collector (which will also take into account the forces in the actuators) are illustrated in Fig. 3.

![Figure 3](image)

**Figure 3.** The tracking mechanism in the reference position (interrupted line) and current position (continuous line), with the collector angular displacement (ψ) and kinematic angles (α1, α2, α3) highlighted

According to Fig.3, the coordinates of joint C(xC, yC)) are firstly establish based on the correlations given by the triangles ΔACQ and ΔBCQ:

\[
CQ^2 = AC^2 - AQ^2 = BC^2 - BQ^2
\]

\[
y_C^2 = a_1^2 - (l + x_C)^2 = a_2^2 - (l - x_C)^2
\]

Rel. (2) allows determining the coordinates of joint C(xC, yC):

\[
\begin{align*}
x_C &= \frac{a_1^2 - a_2^2}{4l} \\
y_C &= \frac{(4la_1)^2 - (4l^2 + a_1^2 - a_2^2)^{3/2}}{4l}
\end{align*}
\]

The segment EC0, further denoted by c, is obtained from the triangle ΔED0C0 (rel. 4, 5),

\[
EC_0 = [ED_0^2 + D_0C_0^2]^{3/2}
\]

\[
c = [a^2 + b^2]^{3/2}
\]

In the current position (Fig. 3), the variable segment EC and the angle γ can be obtained from ΔEPC and ΔEDC (rel. 8):

\[
EC^2 = PC^2 + EP^2 = ED^2 + DC^2 - 2ED \cdot DC \cos(\angle EDC)
\]
The angle $\alpha_1$ between the connecting rod (the inactive actuator) and the actuator $a_1$ is determined from relations (10), (14) and (16), and it is modelled by rel. (17):

$$\alpha_1 = \langle DCA \rangle = \langle ECA \rangle - \langle ECD \rangle$$

$$AE^2 = EC^2 + AC^2 - 2EC \cdot AC \cos(\langle ECA \rangle) = AO^2 + OE^2$$

$$\langle ECA \rangle = \arccos \frac{EC^2 + AC^2 - AE^2}{2EC \cdot AC}$$

$$AE = \left[ l^2 + (l + c)^2 \right]^{\frac{1}{2}}$$

$$\langle ECA \rangle = \arccos \frac{x_c^2 + (l + c - y_c)^2 + a_1^2 - l^2 - (l + c)^2}{2a_1 \left( x_c^2 + (l + c - y_c)^2 \right)^{\frac{1}{2}}}$$

$$ED^2 = EC^2 + DC^2 - 2EC \cdot DC \cos(\langle ECD \rangle)$$

$$\langle ECD \rangle = \arccos \frac{EC^2 + DC^2 - ED^2}{2EC \cdot DC} = \arccos \frac{x_c^2 + (l + c - y_c)^2 + b^2 - a^2}{2b \left( x_c^2 + (l + c - y_c)^2 \right)^{\frac{1}{2}}}$$

$$\alpha_1 = \arccos \frac{x_c^2 + (l + c - y_c)^2 + a_1^2 - l^2 - (l + c)^2}{2a_1 \left( x_c^2 + (l + c - y_c)^2 \right)^{\frac{1}{2}}} - \arccos \frac{x_c^2 + (l + c - y_c)^2 + b^2 - a^2}{2b \left( x_c^2 + (l + c - y_c)^2 \right)^{\frac{1}{2}}}$$

The angle $\alpha_2$ between the two mobile actuators $a_1$ and $a_2$ can be obtained based on rel. (20):

$$AB^2 = AC^2 + CB^2 - 2AC \cdot CB \cos(\langle ACB \rangle)$$

$$\langle ACB \rangle = \arccos \frac{AC^2 + CB^2 - AB^2}{2AC \cdot CB}$$

$$\alpha_2 = \arccos \frac{a_1^2 + a_2^2 - 4l^2}{2a_1 a_2}.$$ 

The angle $\alpha_3$ between the connecting rod and the actuator $a_2$ is given by the sum of the previous angles, from rel. (17) and (20):

$$\langle DCB \rangle = \langle DCA \rangle + \langle ACB \rangle$$

$$\alpha_3 = \alpha_1 + \alpha_2$$

3.3. Collector displacement and stroke

The angular displacement of the solar-thermal collector ($\psi$) can be determined based on Fig. 4.
According to Fig. 4, the angular displacement of the solar collector is obtained from the following relation:

$$\psi = m - n + q$$  \hspace{1cm} (23)

where: $q$ can be established from $\Delta ED_0 C_0$ with rel. (24), $m$ from $\Delta EPC$ with rel. (26), and $n$ from $\Delta EDC$ with rel. (28), combined with rel. (8) and (16):

$$\langle D_0 EC_0 \rangle = \arcsin \frac{D_0 C_0}{EC_0}$$  \hspace{1cm} (24)

$$q = \arcsin \frac{b}{c}$$  \hspace{1cm} (25)

$$\langle PEC \rangle = \arctg \frac{PC}{EP}$$  \hspace{1cm} (26)

$$m = \arctg \frac{x_c}{l + c - y_c}$$  \hspace{1cm} (27)

$$\langle DEC \rangle = 180^\circ - (\langle EDC \rangle + \langle ECD \rangle)$$  \hspace{1cm} (28)

$$n = 180^\circ - \left( \gamma + \arcsin \frac{a \sin \gamma}{\sqrt{x_c^2 + (l + c - y_c)^2}} \right)$$  \hspace{1cm} (29)

The angular displacement $\psi$ is obtained by replacing rel. (25), (27) and (29) into rel. (23):

$$\psi = \arcsin \frac{b}{c} \left[ 180^\circ - \left( \gamma + \arcsin \frac{a \sin \gamma}{\sqrt{x_c^2 + (l + c - y_c)^2}} \right) \right] + \arctg \frac{x_c}{l + c - y_c}.$$

The solar collector stroke can be determined as the difference between the extreme displacements:

$$\Delta \psi = \psi_{\text{max}} - \psi_{\text{min}}$$  \hspace{1cm} (31)

### 4. Numerical Simulations and Discussions

The numerical simulations presented below start from the following premises (Fig. 3):

1) It is required the angular displacement of the solar collector for an angular stroke $\Delta \psi=114^\circ$, corresponding to the winter solstice between 7.30 a.m. ($\psi=57^\circ$) and 4.30 p.m. ($\psi=-57^\circ$);
2) The angle between the connecting rod and the rocker is imposed: \( \gamma = 90^\circ \); 
3) The following dimensions are known: the rocker length \( a = DE = 300 \text{ mm} \), the connecting rod length \( b = DC = 100 \text{ mm} \), the distance between the actuators joints with the base \( 2l = AB = 600 \text{ mm} \), the distance between the revolute joint and the base \( OE = 616.22 \text{ mm} \); the lengths of the mobile actuators in the reference position \( a_0 = AC_0 = a_1(\psi = 0^\circ) = BC_0 = a_2(\psi = 0^\circ) = 424.26 \text{ mm} \).

The following algorithm was used in the numerical simulations: based on rel. (23) and on the variation \( \psi = \psi(t) \) for the winter solstice (between the hours 7.30 a.m. and 4.30 p.m.), the correlation \( \psi = \psi(a_1, a_2, t) \), where \( t \) - time is obtained from premise 1); a correlation \( \gamma = \gamma(a_1, a_2) = 90^\circ \) is obtained from premise 2) and based on rel. (8); from the previous correlations of premises 1) and 2), the following lengths are further determined: \( a_1 = a_1(t) \) and \( a_2 = a_2(t) \); the angles \( \alpha_1(t) \), \( \alpha_2(t) \) and \( \alpha_3(t) \) can be obtained based on relations (17), (20), (22) and the previous lengths.

The variations of the linear displacements \( a_1, a_2 \) of the active actuators related to the actuators length \( a_0 \) in the reference position and the variation of the angular displacement \( \psi \) of the solar-thermal collector are presented in Fig. 5, based on the results of the numerical simulations; the related strokes are highlighted in Tab. 1. Similarly, the variations of the angles between the actuators (required in the subsequent static modelling) are illustrated in Fig. 6 and Tab. 2.

![Figure 5](image)

**Figure 5.** Variations of the length ratios \( a_1/a_0 \) and \( a_2/a_0 \) and of the angular displacement \( \psi \) as functions of the solar hour.

The following conclusions can be highlighted from Fig. 5 and Tab. 1: the variations of the linear displacements of the two active actuators are symmetrical (see also Fig. 3); the two mobile actuators can be identical and they can achieve the required stroke (of \( 114^\circ \)) for a relatively small linear stroke (of 350 mm).

| Solar hour [h] | \( a_1 \) [mm] | \( a_2 \) [mm] | \( \psi \) [deg] |
|----------------|----------------|----------------|----------------|
| 7:30           | 445.2434       | 718.6097       | 56.9737        |
| 10:00          | 369.7078       | 563.525        | 28.4669        |
| 12:00          | 424.2641       | 424.2641       | 0              |
| 14:00          | 563.525        | 369.7078       | -28.4669       |
| 16:30          | 718.6097       | 445.2434       | -56.9736       |

**Table 1.** Displacements and strokes for the active actuators and solar collector.

Stroke \( \Delta a_1 = 350 \) \( \Delta a_2 = 350 \) \( \Delta \psi = 114 \)
Figure 6. Variations of the angles corresponding to the actuators of the tracking system as functions of the solar hour.

Table 2. Angles and strokes of the tracking system

| Solar hour [h] | γ [deg] | α₁ [deg] | α₂ [deg] | α₃ [deg] |
|----------------|---------|-----------|-----------|-----------|
| 7:30           | 90      | 160.9171  | 56.34414  | 217.2612  |
| 10:00          | 90      | 113.0891  | 76.92775  | 190.0168  |
| 12:00          | 90      | 63.43495  | 90        | 153.4349  |
| 14:00          | 90      | 26.85308  | 76.92775  | 103.7808  |
| 16:30          | 90      | -0.3913   | 56.34414  | 55.95284  |

Δα₁ = 161.3  Δα₂ = 33.6  Δα₃ = 161.3

According to Fig. 6 and Tab.2, the premise 2) requires a variation of approx. 160° for the angles between the connecting rod and the actuators, α₁ and α₃, and of approx. 35° for the angle between the two mobile actuators.

5. Conclusions
The implementation of the considered tracking system allows a fine control of the angular displacement of the solar-thermal collector, with an accurate adjustment of the orientation steps throughout the orientation stroke, avoiding blocking in the conditions of higher transmission angles.

The use of the third actuator (adjacent to the rocker) offers additionally the possibility of increasing the maximum orientation stroke, while maintaining the advantage regarding the transmission angles.

The proposed kinematic modelling can be extended under specific requirements, both for the design of the tracking mechanism and for designing optimal control systems for the solar-thermal collector orientation.

6. References
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