Nilsson parameters $\kappa$ and $\mu$ in the relativistic mean field models

A. Sulaksono, T. Mart
Departemen Fisika, FMIPA, Universitas Indonesia, Depok 16424, Indonesia

C. Bahri
Departernen Fisika, FMIPA, Universitas Indonesia, Depok 16424, Indonesia and Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803, USA

PACS numbers: 21.10.Pc, 21.60.-n, 21.60.Cs

I. INTRODUCTION

The finite-range (FR) (see Refs. 1, 2, 3, 4 for a review) and point-coupling (PC) (see Refs. 5, 6, 7, 8, 9 for a review) types of relativistic mean-field (RMF) models have been successful to describe the bulk properties as well as the deformation in a wide mass spectrum of nuclei. The inter-connection of the models and their relations to non-relativistic models, like the Skyrme Hartree-Fock (SHF) ones, have been established [10]. The role of the exchange in RMF-FR [8, 9, 10, 11, 12] and RMF-PC [14] models for finite nuclei have also been explored.

A significant attention has been paid to explore the role of the spin-orbit potential ($V_{L-S}$) in RMF models for various problems and applications that are connected to single particle spectra predictions (see for examples Refs. 2, 11, 16, 17, 18, 20, 21). The standard parameter set of the Nilsson model has been also quite successful in reproducing single particle spectra of stable nuclei [22]. New parameter sets were proposed to improve the predictability of the model for neutron rich [23] and proton rich [24] nuclei. Single particle levels of this parameter sets are compared in Refs. 23, 24 with those obtained by SHF and RMF models. We also note that the origin of the prolate dominance shapes over the oblate ones can be explained in the framework of a Nilsson model as an effect of the strong interference between spin-orbit and orbit-orbit terms of the Nilsson potential [25]. So far, however, except for the pseudo-spin symmetry study in finite nuclei [26], there have been no other investigations of the role of the orbit-orbit potential ($V_{LL}$) which is directly derived from RMF models.

Odd nuclei and single particle spectra in an RMF-FR model using different level of approximations (spherical and deformed) were computed and compared in Ref. 27. In Ref. 12, single particle splitting energies between spin-orbit partners along some isotonic chain (O, Ca, Sn) are also examined in the framework of RMF, SHF and relativistic Hartree-Fock models. Furthermore, there is another method to study spin-orbit potential by exploring high-spin data [28]. With this method one can avoid the scarce and uncertain data available on spin-orbit splittings and their isotopic as well as isotonic dependences. Here we quote from Ref. 28, for example, that the most recent experimental data evaluations [29] give $\Delta \epsilon_{d_{3/2} - d_{5/2}} \approx 6$ MeV for $^{40}$Ca and $\Delta \epsilon_{d_{3/2} - d_{5/2}} \approx 5$ MeV for $^{48}$Ca, while older works give $\Delta \epsilon_{d_{3/2} - d_{5/2}} \approx 6.8$ MeV [30], 7.3 MeV [31], 7.7 MeV [32] for $^{40}$Ca and $\Delta \epsilon_{d_{3/2} - d_{5/2}} \approx 5.3$ MeV for $^{48}$Ca. More detailed information on single particle levels can be found in Ref. 33. Since the method is based on a direct comparison of the excitation energies of terminating states, the correlations beyond mean-field can be strongly suppressed. Nevertheless, this method is still constrained by the limited knowledge on the time-odd component of the nonrelativistic mean-field [25].

In this paper, we will revisit and study the single particle spectra (SPS) of $^{208}$Pb, $^{132}$Sn and $^{40}$Ca in the RMF models in order to understand the origin of their predictive powers for spin-orbit splitting and the reason that the relative placements of the states with different orbital angular momenta $l$ are not well reproduced [27]. Afterward, we will try to find the connection between their SPS predictions with their effective masses ($M^*$) through their $V_{L-S}$ and $V_{LL}$ potentials in $^{208}$Pb. Spherically symmetric calculations are used due to the robustness of the spectral differences against polarization effects [27].

We choose NL-Z, NL-Z2, NL-VT1 (RMF-FR) 2, 12, 34 and PC-F1 (RMF-PC) 3 because they nearly have the same procedure to adjust their coupling constants, hence the prediction bias due to the different fitting procedure can be minimized.

II. SPS PREDICTIONS OF RMF MODELS

In this section we will study the $^{208}$Pb, $^{132}$Sn and $^{40}$Ca SPS predictions of RMF models. The experimental single particle data are taken from [33]. The $^{208}$Pb neutron $(2f_{7/2} \text{ and } 1h_{9/2})$ and proton $(2d_{5/2} \text{ and } 1g_{7/2})$ data as well as the $^{40}$Ca neutron $(1d_{5/2})$ and proton $(1d_{5/2})$ data are obtained by averaging over the spectroscopic...
The deviations show up significantly in the 1\textit{h}_{9/2} states, with experimental data. For the proton spectrum, it occurs between 1\textit{h}_{9/2} and 2\textit{f}_{7/2} states as well as between 2\textit{d}_{5/2} and 2\textit{g}_{7/2} states.

The trends of the spin-orbit splitting (see the lower-left panel of Fig. 1) and the relative position of SPS (see the lower-right panel of Fig. 1) of NL-Z are similar to the case of NL-Z2 but different from those of NL-VT1 and PC-F1. For proton, except for the splitting in 3\textit{p} states of PC-F1, all parameter sets have only 15% deviation from their experimental values. Nevertheless, since the positions of 3\textit{p} states are quite far from the Fermi surface, we can say that all parameter sets give a good prediction of the proton spin-orbit splitting. In the case of neutron, only the splitting of 2\textit{f} states deviates by less than 10% from the experimental value. NL-VT1 has four gaps with deviations in splitting less than 20%. Unfortunately, it has a gap (1\textit{i} states) with a more than 20% deviation and the gap is larger than the corresponding experimental data. The position of these states is above the Fermi surface. NL-Z2 and NL-Z have more or less 20% deviation in the splitting of 3\textit{p} states (the position of these states is around Fermi surface) and for PC-F1 the deviation of that splitting is larger than 20%. A quite large deviation appears in the splitting of 3\textit{d} and

$E_l = \frac{1}{2(l+1)} \sum_{j=l-1/2}^{l+1/2} (2j+1)E_j$, \hspace{1cm} \alpha_{lk} = E_l - E_k$, (1)

where $E_l$ is the average energy of the spin-orbit partner with angular momentum $l$. The relative position between $k$ and $l$ levels can be determined from the difference between $E_l$ and $E_k$ ($\alpha_{lk}$).

Figure 1 shows the SPS for $^{208}$Pb. In the top panels, it can be seen that all models have a similar trend in placing and ordering the proton and neutron single particle energies. The gap between an occupied and an unoccupied levels is relatively well reproduced for proton but quite poor in the case of neutron. Compared with experimental data, the SPS lines do not coincide. The deviations show up significantly in the 1\textit{g}_{7/2}, 3\textit{p}_{3/2} and 3\textit{p}_{1/2} proton states and in the 1\textit{h}_{9/2} neutron state. For the neutron spectrum, similar to Ref. 22, the ordering is reversed between 1\textit{i}_{13/2}, 3\textit{p}_{3/2} and 2\textit{f}_{5/2} states. There are quite significant discrepancies in the spacing between 1\textit{i}_{11/2} and 3\textit{d}_{5/2} states, as well as between 1\textit{h}_{9/2} and 2\textit{f}_{7/2} states, with experimental data. For the neutron spectrum, it occurs between 1\textit{h}_{9/2} and 2\textit{f}_{7/2} states as well as between 2\textit{d}_{5/2} and 2\textit{g}_{7/2} states.
2g states (the positions of both spin-orbit partners are above Fermi surface). It seems that all parameter sets are unable to give good predictions in the neutron spin-orbit splitting. The SPS relative position of proton has a better prediction than that of neutron. Proton has two os which have deviation less than 30% and one α above the Fermi level (α_{2f}). We also note that α_{2f} has 60% deviation. Neutron has three os with deviations from experimental data between 30% and 50% and it has even one α (α_{3f}) that deviates by about 100% from experimental data. The positions of those states (3p and 2f) are around the Fermi surface.

Figure 2 shows the SPS for 132Sn. In the top panels, it can be seen that all models have similar trend in levels placing and ordering. Their SPS lines do not coincide with experimental data. For the neutron spectrum, the reversed ordering between 2d_{3/2} with 3s_{1/2} and 2h_{11/2} states occurs. A significant deviation in spacing with experimental data occurs between 2d_{5/2} and 1g_{7/2} states of proton. The bottom panels show that all parameter sets predict acceptable spin-orbit splittings for proton but not for neutron. The relative position between 2d and 2f levels for neutron (α_{2f}) deviates by almost 40% from experimental data. The trends of the spin-orbit splitting (see the lower-left panel of Fig. 2) and the relative position of the SPS (see the lower-right panel of Fig. 2) of NL-Z are similar to those of NL-Z2 but different from those of NL-VT1 and PC-F1.

Figure 3 shows the SPS for 40Ca. In the upper-left panel (proton) it is shown that NL-VT1 reproduces experimental values of every single particle energy. PC-F1 has a too narrow spacing between 1d_{3/2} and 2s_{1/2} states. The reversed ordering of 2f_{5/2} and 2p_{1/2} states occurs in the case of NL-Z and NL-Z2. In the upper-right panel (neutron) it can be seen that each model does not really have a similar trend in levels placing and ordering. Similar to the case of proton, here PC-F1 has a too narrow spacing between 1d_{3/2} and 2s_{1/2} states. The lower panels show that, in contrast to 208Pb and 132Sn, 40Ca has a better spin-orbit splitting for neutron rather than for proton. The relative positions between 1d and 1f levels for neutron and proton (α_{2f}) deviate by less than 20% from experimental data.

These results confirm the findings of Ref. [27] that the relative placement and ordering of the states in RMF models are not well reproduced. In addition, we note that RMF-FR (NL-Z, NL-Z2 and NL-VT1) has a better prediction than RMF-PC (PC-F1) and the presence of the tensor terms (NL-VT1) enhances the improvements in placing and ordering of single particle states, particularly in lighter nuclei (e.g., 40Ca). However, these improvements are still not adequate to overcome the problem.

### III. NILSSON PARAMETERS κ AND µ OF RMF MODELS

It has been known that all models presented here have $M^*/m \approx 0.6$. Unlike the non-self-consistent calculations (models using Wood-Saxon or Nilsson potentials), where the SPS has a direct connection with the potential parameters, the connection is not so obvious in the RMF models because it is hidden by the self consistency condition. Therefore, it is natural to translate $M^*/m$ of RMF models into $V_c$, $V_{LL}$ and $V_{L-S}$ by taking a non-relativistic limit, where they resemble a Wood-Saxon or Nilsson potential. The interpretation of the results of this section will be given in the next section by varying $M^*/m$ in one model and studying its SPS prediction for 208Pb.

The Hamiltonian of the RMF model in spherical systems is

$$H = \alpha \cdot (\vec{p} + i\gamma_0 \vec{T}) + \gamma_0 (m + S) + V_0,$$

where $H\Psi_k^\pm = \epsilon_k^\pm \Psi_k^\pm$ is fulfilled. Using the general convention for $\Psi_k^+$, i.e., $\Psi_k^+ = \left( \frac{g_k \chi_{k^+}}{i \int f \chi_{k^-}} \right)$, the positive energy equation for the upper component becomes

$$\begin{align*}
\left[ \frac{\partial^2}{r^2} + \frac{2}{r} \frac{\partial}{r} - \frac{\vec{L}^2}{r^2} - \frac{(\partial_r \Delta)}{(2m + \epsilon_k^+ - \Delta)} - 2T_r \right] \frac{\vec{\sigma} \cdot \vec{L}}{r} + \frac{(\partial_r \Delta)}{(2m + \epsilon_k^+ - \Delta)} \frac{\partial_r}{r} + \left( \frac{2T_r}{r} - T^2_r + \partial_r T_r + \frac{(T_r \partial_r \Delta)}{(2m + \epsilon_k^+ - \Delta)} \right) + (\epsilon_k^+ - \Sigma) (2m + \epsilon_k^-) & g_k = 0, \quad (3)
\end{align*}$$

with $\Sigma = S + V_0$, $\Delta = V_0 - S$, and $\epsilon_k^+ = \epsilon_k^+ - m$, while $S$, $V_0$ and $\vec{T}$ indicate the scalar, time-component of the vector and tensor potentials, respectively. The non-relativistic form of Eq. (4) can be derived. The Darwin term $\frac{(\partial_r \Delta)}{(2m + \epsilon_k^+ - \Delta)} \frac{\partial_r}{r}$ in Eq. (4) can be absorbed by transforming the $g_k$ wave function into the new one, $G_k^+$. This leads to a Schrödinger form, i.e.,

$$\begin{align*}
\left( \frac{\vec{p}^2}{2m} + \frac{\vec{L}^2}{2mr^2} + V_c (r, \epsilon^+^+) + V_{L-S}(r, \epsilon^+^+) \frac{\vec{\sigma} \cdot \vec{L}}{r} \right) G_k^+ & = \epsilon^{NR} G_k^+, \quad (4)
\end{align*}$$

where $\epsilon^{NR}$ is the non-relativistic energy.
where $\epsilon^{NR} = \epsilon^{+}\left(1 + \frac{\epsilon^{+}}{2m}\right)$ \cite{16}. In finite nuclei, the second term in $\epsilon^{NR}$ is smaller than one, and

$$V_{L-S}(r, \epsilon^{+}) = \frac{1}{2m} \left[ \frac{(\partial_{r} \Delta)}{(2m + \epsilon^{+} - \Delta)} - 2T_{r} \right], \quad (5)$$

In heavy nuclei, the mean-field central potential $V_c$ and the spin-orbit potential $V_{L-S}$ are closer to the non-relativistic results obtained by using the Wood-Saxon potential \cite{35}. Unlike the Wood-Saxon results, the RMF model has a strong energy-dependent $V_c$ and a weak energy-dependent $V_{L-S}$ (Fig. 4). In $^{208}$Pb, $V_n$ is deeper than $V_p$ due to the fact that more neutrons are present rather than protons. $V_{L-S}$ of RMF model (NL-VT1) is deeper than the Wood-Saxon prediction \cite{35}. The tensor term gives only a minor additional contribution in spin-orbit potential near the Fermi surface. There is no significant difference between neutron and proton $V_{L-S}$ of $^{208}$Pb in the RMF model (NL-VT1).

The study of non-relativistic potentials of RMF models has been done in many places with different intentions and different methods to obtain $V_c$ and $V_{L-S}$ \cite{2, 12, 15, 17, 18, 19, 20}. However, a non-relativistic form of the RMF model like the Nilsson one has not yet been explored, especially in analyzing the SPS. The advantage of using the Nilsson model is that $V_{LL}$ could be employed to

- $^{132}$Sn, PROTON
- $^{132}$Sn, NEUTRON

FIG. 2: Same as Fig. 1 but for $^{132}$Sn.
analyze the relation between states of different \( l \) and their orderings. Reference [26] calculates \( V_{LS} \) and \( V_{LL} \) from several RMF models by using some different approximations to study the origin of the pseudo-spin symmetry. We note that the maximum value of \( \epsilon^+ \) has the same order of magnitude with the difference value between the scalar and the time components of vector potentials (\( \Sigma \)), e.g., in \( {}^{208}\text{Pb} \) neutron, the corresponding value is around 50 MeV. On the other hand, \( \Delta \) is a summation of the scalar and the time components of the vector potential and the corresponding maximum value for \( {}^{208}\text{Pb} \) neutron is more or less 400 MeV. It is also known that the nucleon mass is around 1000 MeV [2]. Therefore to obtain a “Nilsson form” we can use the assumption that \( \epsilon^+ \approx \Delta < m \), so that \( \epsilon^+ = \epsilon^+ \left(1 + \frac{\Sigma}{2m}\right) \approx \epsilon^+ \) and \( (2m + \epsilon^+ - \Delta)^{-1} \approx (2m - \Delta)^{-1} \). After that, Eq. (4) can be written as

\[
\left[ \frac{p_r^2}{2m} + \frac{L^2}{2mr^2} + V_{c}^{\text{eff}}(r) + V_{LL}^{\text{eff}}(r) + V_{L-S}^{\text{eff}}(r) s \cdot L \right] \approx \epsilon^+ \epsilon^+_k ,
\]

(7)

As another consequence, the energy-dependent \( V_c \) transforms into energy-independent \( V^+ \) plus a small nonlocal term (\( p_r \) dependent). Equation (7) can be considered as the Nilsson form of the RMF model. Both potentials can be compared with their partners from the Nilsson model through Nilsson parameters \( \kappa \) and \( \mu \) [36].

The dominant parts of \( V_{c}^{\text{eff}} \) and \( V_{L-S}^{\text{eff}} \) are taken merely to compare the predictions among the presented models (parameter sets). They have similar \( V_{c}^{\text{eff}} \) and \( V_{L-S}^{\text{eff}} \)
predictions not only for proton but also for neutron, especially near the Fermi surface. Small differences appear in the region around the center of nuclei and a small $V_{c}^{\text{eff}}$ deviation also appears in the unoccupied region of PC-F1. Thus, the forms of $V_{c}^{\text{eff}}$ and $rV_{L-S}^{\text{eff}}$ are essentially almost model-independent (see Fig. 5).

In the Fermi surface, all parameter sets have similar $\kappa$ and $\mu$ for proton and neutron. On the contrary, Nilsson model has different $\mu$ for proton and neutron (see Fig. 6). Compared with the Nilsson model (shaded regions) RMF ones have larger $\kappa$ but smaller $\mu$. The differences among all parameter sets (models) in $\kappa$ only appear in the region close to the center of nuclei. It means that $\kappa$ and $\mu$ of RMF models can be considered as model (parameter set) independent. Unlike in the Nilsson model, where $\kappa$ and $\mu$ are independent from the position $\langle r \rangle$, in RMF models both quantities depend on $\langle r \rangle$. The spatial dependence of $V_{L-S}^{\text{eff}}$ originates mainly from the energy-dependent potential $V_{c}$. It should be noted that the non-relativistic model, like SHF, does not have such dependence. Therefore, we can consider this spatial dependence as a genuine feature of self-consistent RMF models.

**IV. INTERPRETATION**

We prepared two variations of parameter sets to properly interpret the results. First, we varied the scalar coupling constant $g_{s}$ of NL-Z until we obtained the desired $M^{*}/m$, while the four parameter sets were kept constant. Second, we fit the four parameter sets with $M^{*}/m$ varied. The parameter sets in the second procedure were obtained by fitting the four parameter sets into the same observable that were used in obtaining the NL-Z param-
TABLE I: Numerical values of coupling constants used in the parameter sets. Except for the NL-Z parameterization, these values are adjusted with respect to $M^*/m$.

| Parameter | NL-Z | P-0.67 | P-0.70 | P-0.75 | P-0.80 |
|-----------|------|--------|--------|--------|--------|
| $g_s$     | 10.06| 8.91   | 8.45   | 7.58   | 7.22   |
| $g_V$     | 12.91| 11.02  | 10.26  | 8.73   | 7.94   |
| $g_R$     | 9.69 | 9.69   | 9.69   | 9.69   | 9.69   |
| $b_2$     | -13.51| -13.44| -13.41| -13.06| -13.49|
| $b_3$     | -40.22| -29.74| -24.48| -3.69 | 30.07  |
| $m_S$     | 488.67| 488.67| 488.67| 488.67| 488.67|
| $m_V$     | 780  | 780    | 780    | 780    | 780    |
| $m_R$     | 763  | 763    | 763    | 763    | 763    |

For the unfitted parameter sets (see the top right panel of Fig. 7), the depth of $V_{\text{eff}}$ and its trend around Fermi surface drastically change. On the other hand, for the fitted parameter sets (see top left panel of Fig. 7), the depth and the trend around Fermi surface do not significantly change when $M^*/m$ is varied. It means that the nuclear observable requires a cancellation between scalar and vector potentials. The scalar and vector potentials tend to weaken if $M^*/m$ becomes larger than 0.6, but the $V_{L-S}^{\text{eff}}$ is sensitive to the variation of $M^*/m$ (see the lower panels of Fig. 7). $V_{L-S}^{\text{eff}}$ decreases when $M^*/m$ increases. The weakening of $V_{L-S}^{\text{eff}}$ is due to the smallness of the spin-orbit splitting (see the upper-left panel of Fig. 7). Fitting the parameters to the nuclear observable does not help in this case. Therefore, only appropriate values of scalar and vector potentials can yield a correct $V_{c}^{\text{eff}}$ and $V_{L-S}^{\text{eff}}$ simultaneously. These potentials correspond to an $M^*/m$ of about 0.6.

As can be seen in Fig. 7, the Nilsson parameter $\kappa$ depends on $M^*/m$. The value of $\kappa$ at Fermi surface decreases when $M^*/m$ increases and the major effects are found for the fitted parameter sets. For these parameter sets, if $M^* = 0.77 m$, it coincides with the prediction from the Nilsson model (shaded area). On the other hand, $\mu$ from fitted parameter sets is almost independent from the $M^*/m$ variation, while $\mu$ of the unfitted parameter

sets, on the contrary, depends on $M^*/m$. It means that for the RMF models, a constant value of $\mu$ is the requirement for the correct nuclear bulk properties. The value of $\mu$ of the RMF models around Fermi surface is smaller than that of the Nilsson model. In the RMF models, $\kappa$ and $\mu$ depend on each other. As $M^*/m$ is getting larger, $\kappa$ is getting smaller and the spin-orbit potential is getting smaller in order to keep $\mu$ constant.

The variation of the neutron and proton SPS with re-

TABLE II: Nuclear matter properties predicted by NL-Z, P-067, P-0.70, P-0.75, and P-0.80 parameterizations.

| Parameter | NL-Z | P-0.67 | P-0.70 | P-0.75 | P-0.80 |
|-----------|------|--------|--------|--------|--------|
| $E/A$ (MeV) | -16.18| -16.30| -16.38| -16.34| -15.85 |
| $\rho_{0m}$ (fm$^{-3}$) | 0.15 | 0.16 | 0.16 | 0.17 | 0.16 |
| $M^*/m$ | 0.58 | 0.67 | 0.70 | 0.75 | 0.80 |
| $a_4$ (MeV) | 41.7 | 41.1 | 42.0 | 42.8 | 39.7 |

FIG. 7: Same as in Fig. 6, but using the P-067, P-0.70, P-0.75, P-0.80, II-0.65, II-0.70, II-0.75 and II-0.77 parameter sets. The fitting procedure is the same as in the case of NL-Z. II-0.65, II-0.70, II-0.75 and II-0.77 are unfitted parameter sets (obtained only by adjusting the value of coupling constant $g_s$).

FIG. 8: Same as in Fig. 6 but using the P-067, P-0.70, P-0.75, and P-0.80 fitted parameter sets. The fitting procedure is the same as in the case of NL-Z. II-0.65, II-0.70, II-0.75 and II-0.77 are unfitted parameter sets (obtained only by adjusting the value of coupling constant $g_s$).
spect to the variation of $M^*/m$ is shown in Fig. 9. Some states are shifted up and some other are shifted down as $M^*/m$ increases. This combination does not improve the SPS relative position. The reason is mainly due to the spin-orbit splitting which is getting narrower when $M^*/m$ increases. In the relative position between two different spin-orbit partners ($\alpha$) a couple of states are closer to experimental data but the majority of states deviate more when $M^*/m$ increases. The value of $\alpha_{dg}$ is negative for P-070 because there is an exchange of ordering in 3d states (the value of the gap is negative). Except for $\alpha_{dg}$, all $\alpha$s change quite drastically. This effect depends on $M^*/m$, but the pattern of changing is not the same for every level. It seems that the relative position ($\alpha$) can be reproduced if $M^*/m$ depends on the states. This dependence can be generated only if we take into account the exchange and/or other correlations (effects) beyond mean-fields. It would be interesting to see whether or not the exchange effect can remedy this problem. Future calculation should address this question.

### V. CONCLUSION

The $^{208}\text{Pb}$, $^{132}\text{Sn}$ and $^{40}\text{Ca}$ SPS of RMF models have been revisited and studied. Qualitatively, all RMF models presented here have a similar trend in SPS. Quantitatively, however, they still have differences due to the models. From the comparison with new experimental data, it is shown that the position of the state is not only poorly reproduced, but some level positions in neutron spectra are reversed.

The non-relativistic limit of the RMF model has been derived in which the potentials resemble a Wood-Saxon and Nilsson forms. The energy-dependent potentials $V_{c}$ and $V_{LS}$ (in Wood-Saxon type) of RMF models can be transformed into energy-independent potentials $V_{c}^{eff}$ and $V_{LS}^{eff}$ (in Nilsson type) but with an additional angular momentum-dependent potential $V_{LL}^{eff}$. These potentials are used to analyze the $^{208}\text{Pb}$ SPS predictions from several RMF models. We found that, first, the behavior of $\kappa$ and $\mu$ of RMF models is different from that of the Nilsson model. Second, due to the inter-dependence of parameters $\kappa$ and $\mu$ in RMF models, the acceptable parameter sets ($M^*/m \approx 0.6$) at the Fermi surface need a relatively large $V_{LL}^{eff}$ in order to maintain a correct spin-orbit splitting.

Since the effect of tensor terms in the RMF model is too small in $V_{LS}$ of heavy nuclei ($^{208}\text{Pb}$), the effect is marginal to give correct level spacings and placement ordering. The suspicion that a relatively small $V_{LL}^{eff}$ (large $M^*/m$) yields a relatively better placement of states is found to be wrong. It is shown in the $^{208}\text{Pb}$ case that when $V_{LL}^{eff}$ is decreasing only two placements of the states...
are getting better, whereas the rest are getting worse. Therefore, it seems that the problem of RMF models in reproducing experimental data on the relative placement of the states is originated in the independence of $M^*$ from $l$ (state).

ACKNOWLEDGMENT

A. S. acknowledges J. A. Maruhn and T. Cornelius for some comments in the early stage of this work.

[1] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys 16, 1 (1986).
[2] P.-G. Reinhard, Rep. Prog. Phys. 52, 439 (1989).
[3] B. D. Serot, Rep. Prog. Phys. 55, 1855 (1992).
[4] P. Ring, Prog. Part. Nucl. Phys 37, 193 (1996).
[5] B. A. Nikolaus, T. Hoch, and D. G. Madland, Phys. Rev. C 46, 1757 (1992).
[6] J. J. Rusnak and R. J. Furnstahl, Nucl. Phys. A 627, 95 (1997).
[7] T. Buervenich, D. G. Madland, J. A. Maruhn, and P.-G Reinhard, Phys. Rev. C 65, 044308 (2002).
[8] A. Bouyssy, J. F. Mathiot, N. V. Giai, and S. Marcos, Phys. Rev. C 36, 380 (1987).
[9] H. F. Boersma and R. Malfliet, Phys. Rev. C 49, 233 (1994); Phys. Rev. C 49, 1495 (1994).
[10] J. K. Zhang, Y. Jin, and D. S. Onley, Phys. Rev. C 48, 2697 (1993).
[11] R. N. Schmid, E. Engel, and R. M. Dreizler, Phys. Rev. C 52, 164 (1995); Phys. Rev. C 52, 2804 (1995); Found. Phys. 27, 257 (1997).
[12] M. Lopez-Quelle, N. Van Giai, S. Marcos, and L. N. Savushkin, Phys. Rev. C 61, 064321 (2000).
[13] A. Sulaksono, T. Buervenich, J. A. Maruhn, P.-G Reinhard, and W. Greiner Ann. Phys. (N.Y.) 308, 354 (2003).
[14] A. Sulaksono, T. Buervenich, J. A. Maruhn, P.-G Reinhard, and W. Greiner Ann. Phys. (N.Y.) 306, 36 (2003).
[15] M. Bender, K. Rutz, P.-G. Reinhard, J. A. Maruhn, and W. Greiner, Phys. Rev. C 60, 34304 (1999).
[16] M. Jaminon, C. Mahaux, and P. Rochus, Phys. Rev. C 22, 2027 (1980).
[17] W. Koepf and P. Ring, Z. Phys. A 339, 81 (1991).
[18] A. Baran, Phys. Rev. C 61, 024316 (2000).
[19] F. J. Furnstahl, J. J. Rusnak, and B. D. Serot, Nucl. Phys. A 632, 607 (1998).
[20] S. Yoshida and H. Sagawa, Nucl. Phys. A 658, 3 (1999).
[21] B. G. Todd, J. Piekarewicz, and P. D. Cottle, Phys. Rev. C 69, 021301 (2004).
[22] T. Bengtsson and I. Ragnarsson, Nucl. Phys. A 436, 14 (1985).
[23] J. Zhang, Y. Sun, M. Guidry, L. L. Riedinger, and G. A. Lalazissis, Phys. Rev. C 58, 2663 (1998).
[24] Y. Sun, J. Zhang, M. Guidry, J. Meng and S. Im, Phys. Rev. C 58, 2663 (1998).
[25] N. Tajima and N. Suzuki, Phys. Rev. C 64, 037301 (2001).
[26] C. Bahri, J. P. Draayer, and S. A. Moszkowski, Phys. Rev. Lett. 68, 2133 (1992).
[27] K. Rutz, M. Bender, P.-G. Reinhard, J. A. Maruhn, and W. Greiner, Nucl. Phys. A 634, 67 (1998).
[28] H. Zdunczuk, W. Satula, and R. A. Wyss, nucl-th/0408018 (2004).
[29] A. Oros, Ph.D. Thesis, University of Köln, 1996.
[30] A. Swift and L. R. B. Elton, Phys. Rev. Lett. 17, 484 (1966).
[31] H. Tyren, S. Kullander, O. Sundberg, R. Ramachandran, P. Isacsson, and T. Berggren, Nucl. Phys. 17, 321 (1966).
[32] L. Ray and P. E. Hodgson, Phys. Rev. C 20, 2403 (1979).
[33] V. I. Isakov, K. I. Erokhina, H. Mach, M. Sanchez-Vega, and B. Fogelberg, Eur. Phys. J. A 14, 29 (2002).
[34] M. Rufa, P.-G. Reinhard, J. A. Maruhn, and W. Greiner, Phys. Rev. C 38, 390 (1988).
[35] A. L. Blokhin, C. Bahri and J. P. Draayer, Phys. Rev. Lett. 74, 4149 (1995).
[36] P. Ring and P. Schuck, The Nuclear Many-Body Problem (Springer-Verlag, Heidelberg, 1980).