Black strings induced by the Casadio-Fabbri-Mazzacurati Braneworld black holes solutions

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Abstract. This paper aims to evince the bulk metric near the brane and in particular the black string warped horizon associated to the Casadio-Fabbri-Mazzacurati braneworld black holes, and their physical consequences. The black string can be shown to be more unstable to large-scale perturbations than the Schwarzschild standard black strings, namely the Gregory-Laflamme instability. The two solutions of Einstein equations proposed by Casadio, Fabbri, and Mazzacurati, regarding black hole metrics presenting a post-Newtonian parameter measured on the brane are analyzed, in particular when suitable values for the post-Newtonian parameter precluding Hawking radiation on the brane are chosen. The associated black string profile is studied. Further details can be found in [1].

1. Introduction

Extra-dimensional spacetimes are scenarios for extensions of the general relativity, wherein Einstein’s field equations can present solutions as black holes in higher dimensions. Furthermore, the hierarchy solution is obtained by leading gravity to leak into extra dimensions [2]. A particularly useful approach to deal with the hierarchy is provided an effective 5D reduction of the Hořava-Witten theory [3, 4, 5, 6].

Motivated by the study of gravity on 5D braneworld scenarios and some applications, in particular the black holes/black strings horizons variations induced by braneworld effects, [7, 8, 9], further aspects concerning generalized black strings and their warped horizons are here reviewed. The Casadio-Fabbri-Mazzacurati metrics on the brane, namely the type I and type II black hole solutions [10, 11] are studied as generators the bulk metric, inducing an associated black string warped horizon [1].

This article is organized as follows: in Sec. 2 the Einstein field equations on the brane are revisited. For a static spherical metric on the brane, the propagating effect of 5D gravity is evinced from the Taylor expansion along the extra dimension of the brane metric, providing the black string warped horizon profile. Such expansion can provide the bulk metric near the brane merely from the metric on the brane. In Sec. 3, the type I and type II Casadio-Fabbri-Mazzacurati black string solutions and their respective warped horizons are obtained, analyzed and depicted. We analyze such solutions in the particular case where the associated post-Newtonian parameter
2. Black string warped horizon and the bulk metric near the brane

Hereupon the notation in [12, 13] is adopted, where \{\theta_\mu\} typify a tetrad on a brane embedded in a bulk. A frame \(\theta^A = dx^A\) (\(A = 0, 1, 2, 3, 4\)) in the bulk is represented in local coordinates. In the brane defined by \(y = 0\), [hereon \(y\) denotes the associated Gaussian coordinate] the 1-form \(dy\) is orthogonal to the brane. The metric

\[ g_{\mu\nu}(x^\alpha, y)\, dx^\mu\, dx^\nu + dy^2 \]

endows the bulk. The fine tuning is provided by \(\Lambda_4 = \frac{\kappa_4^2}{2} \left( \frac{1}{5} \kappa_5^2 \lambda^2 + \Lambda_5 \right)\) and the 5D coupling constants are related by \(\kappa_5^2 = \frac{1}{5} \lambda \kappa_5^4\). Here \(\Lambda_4\) denotes the effective brane cosmological constant, and \(\lambda\) is the brane tension. The constant \(\kappa_5 = 8\pi G_5\), where \(G_5\) is the 5D Newton gravitational constant, denotes the 5D gravitational coupling, related to the 4D gravitational constant \(G\) by \(G_5 = G\ell_{\text{Planck}}^4\). The junction condition provides the extrinsic curvature tensor by [14, 15, 12]

\[ K_{\mu\nu} = -\frac{1}{2} \kappa_5^2 \left( T_{\mu\nu} + \frac{1}{3} (\lambda - T) g_{\mu\nu} \right), \]

where \(T = T_{\mu\nu}\) is the trace of the energy-momentum tensor. The trace-free and symmetric components of the 5D Weyl tensor are respectively denoted by \(B_{\mu\nu\alpha\beta} = g_{\mu\rho} g_{\nu\sigma} C_{\rho\sigma\alpha\beta} n^\mu n^\nu\) and \(\varepsilon_{\mu\nu} = C_{\mu\nu\rho\sigma} n^\rho n^\sigma\).

The Einstein brane field equations can be expressed as

\[ G_{\mu\nu} = -\varepsilon_{\mu\nu} - \frac{1}{2} \Lambda_5 g_{\mu\nu} + \frac{1}{4} \kappa_5^2 \left( \frac{1}{2} g_{\mu\nu} \left( T^2 - T_{\alpha\beta} T^{\alpha\beta} \right) + TT_{\mu\nu} - T_{\mu\alpha} T^{\alpha}_{\;\nu} \right). \]

The Weyl tensor electric term \(\varepsilon_{\mu\nu}\) carries an imprint of high-energy effects sourcing Kaluza-Klein (KK) modes.

Considering vacuum on the brane, where \(T_{\mu\nu} = 0\) outside a black hole, the field equations \(G_{\mu\nu} = -\varepsilon_{\mu\nu} - \frac{1}{2} \Lambda_5 g_{\mu\nu}\) and \(R = 0 = \varepsilon_{\mu\nu}\) hold for braneworlds with \(Z_2\)-symmetry. The vacuum field equations in the brane are \(\varepsilon_{\mu\nu} = -R_{\mu\nu}\), where the bulk cosmological constant is comprised into the warp factor. Although a Taylor expansion of the metric was used to probe properties of a black hole on the brane in, e. g., [16, 12], in order to enhance the range of our analysis throughout this paper a more complete approach to analyze braneworld corrections in the black string profile can be accomplished, based on [9].

A Taylor expansion of the metric along the extra dimension allows us to analyze the black string more deeply. The effective field equations are complemented by other ones, obtained from the 5D Einstein and Bianchi equations in Refs. [13, 12]. Hereupon, since we are concerned with the Taylor expansion of the metric along the extra dimension up to the fourth order, besides the effective field equation \(\mathcal{L} K_{\mu\nu} = K_{\mu\alpha} K^{\alpha}_{\;\nu} - \varepsilon_{\mu\nu} - \frac{1}{6} \Lambda_5 g_{\mu\nu}\), the effective equations are considered:

\[ \mathcal{L} \varepsilon_{\mu\nu} = \nabla^\alpha B_{\alpha(\mu\nu)} + \left( K_{\mu\alpha} K_{\nu\beta} - K_{\alpha\beta} K_{\mu\nu} \right) K^{\alpha\beta} + K^{\alpha\beta} R_{\mu\alpha\nu\beta} - K \varepsilon_{\mu\nu} + \frac{1}{6} \Lambda_5 \left( K_{\mu\nu} - g_{\mu\nu} K \right) + 3K^{\alpha}_{\left(\nu\mu\varepsilon_{\mu\nu}\right)} \alpha, \]

\[ \mathcal{L} B_{\mu\nu\alpha} = K^{\alpha\beta} B_{\mu\nu\beta} - 2 \nabla_{\mu} \varepsilon_{\nu\alpha} - 2 B_{\alpha\beta\mu} K_{\nu}^{\beta}. \]

These expressions are used to compute the terms in the Taylor expansion of the metric, along the extra dimension, providing the black string profile and further physical consequences as well.
By denoting $K = K_\mu^\nu$ and $g_{\mu\nu}(x,0) = g_{\mu\nu}$, the Taylor expansion is given [9, 1], for a vacuum in the brane, by:

$$g_{\mu\nu}(x,y) = g_{\mu\nu} - \frac{1}{3} \kappa_5^2 \lambda g_{\mu\nu} |y| + \left[ \frac{1}{6} \left( \frac{1}{3} \kappa_5^2 \lambda^2 - \Lambda_5 \right) g_{\mu\nu} - \mathcal{E}_{\mu\nu} \right] y^2$$

$$- \frac{1}{6} \left( \frac{193}{36} \lambda^3 \kappa_5^8 + \frac{5}{3} \Lambda_5 \kappa_5^2 \lambda \right) g_{\mu\nu} + \kappa_5^2 R_{\mu\nu} \right] \frac{|y|^3}{3!} +$$

$$+ \left[ \frac{1}{6} \Lambda_5 \left( R - \frac{1}{3} \Lambda_5 - \frac{1}{18} \lambda^2 \kappa_5^4 + \frac{7}{324} \lambda^4 \kappa_5^8 \right) g_{\mu\nu} + \left( R - \Lambda_5 + \frac{19}{36} \Lambda_5 \right) \mathcal{E}_{\mu\nu}$$

$$+ \frac{1}{6} \left( \frac{37}{36} \lambda^2 \kappa_5^4 - \Lambda_5 \right) R_{\mu\nu} + \mathcal{E}^{\alpha\beta} R_{\mu\alpha\nu\beta} \right] \frac{y^4}{4!} + \cdots \quad (2)$$

This expression is shown to be prominently relevant for our subsequent analysis.

Hereon, the black hole horizon evolution along the extra dimension — the warped horizon [17] — shall be investigated, exploring the component $g_{00}(x,y)$ in (2). Indeed, let us consider any spherically symmetric metric associated to a black hole — in particular the Schwarzschild and the Casadio-Fabbri-Mazzacurati ones here investigated. The black hole solution, namely, the black string solution on the brane, is regarded when $\sqrt{g_{00}(x,0)} = R$, where $R$ denotes the coordinate singularity, usually calculated by the component $g_{rr}^{-1} = 0$ in the metric in the braneworld context. In the Schwarzschild metric $R = R_S = \frac{2GM}{r}$. The coordinate singularities for the Casadio-Fabbri-Mazzacurati metrics are going to be analyzed in what follows, in the black string context as well. Such singularities shall be shown to be also physical singularities, by analyzing their respective 4D and 5D Kretschmann scalars.

3. Casadio-Fabbri-Mazzacurati braneworld solutions

The analysis of the gravitational field equations on the brane is not straightforward, due to the fact that the propagation of gravity into the bulk does not allow a complete presentation of the brane gravitational field equations as a closed form system [13]. The solutions provided by Casadio, Fabbri, and Mazzacurati for the brane black holes metrics [16, 18, 10] take into account the post-Newtonian parameter $\beta$, measured on the brane. The case $\beta = 1$ generates forthwith an exact Schwarzschild solution on the brane, and elicits a black string prototype. Furthermore, it was observed in [10, 11] that $\beta = 1$ holds in solar system scales measurements, and $\beta$ gives information about the vacuum energy of the braneworld [10, 11].

One of the main motivation regarding the Casadio-Fabbri-Mazzacurati setup is that black holes solutions of the Einstein equations on the brane must depart from the Schwarzschild solution. Indeed, it is important to emphasize that, as we shall see for the Casadio-Fabbri-Mazzacurati black string, it might be possible to find out points along the extra dimension for which the Kretschmann scalar $(5)K = (5)R_{\mu\nu\rho\sigma} (5)R_{\mu\nu\rho\sigma}$ diverges, indeed naked singularities along the extra dimension. Since the pure black string configuration is unstable [19], this structure is not physical ab initio. Anyway for $y = 0$ one reproduces the Kretschmann scalars standard 4D behavior. The Casadio-Fabbri-Mazzacurati black string solutions and their respective braneworld corrections are going to be presented and their stability analyzed as well.

Given a general static spherically symmetric

$$g_{\mu\nu}dx^\mu dx^\nu = -B(r)dt^2 + A(r)dr^2 + r^2d\Omega^2, \quad (3)$$

the Casadio-Fabbri-Mazzacurati 4D black hole solution was obtained in [10, 11]. The Schwarzschild 4D metric is obtained when $B(r) = (A(r))^{-1}$ and $B(r) = 1 - \frac{2GM}{r}$. Its unique extension into the bulk is a black string warped horizon, with the central singularity extending
all along the extra dimension, and the bulk horizon singular \[10, 11, 20\]. If the Schwarzschild metric on the brane is demanded with a regular AdS horizon, there is no matter confinement on the brane: in this case matter percolates into the bulk \[22\]. The condition \(B(r) = (A(r))^{-1}\) holds in the 4D case, although the most general solution is the Reissner-Nordström one \[10, 11, 13\], related to the case II analyzed in which follows.

### 3.1. Casadio-Fabbri-Mazzacurati black string: Case I

This case was analyzed by Casadio, Fabbri, and Mazzacurati in \[10, 11\], regarding the 4D black hole solution (3). They obtained a solution of the Einstein’s equations provided by the metric coefficients

\[
\begin{align*}
B(r) &= 1 - \frac{2GM}{c^2r} \\
A(r) &= \frac{1 - \frac{3GM}{2c^2r}}{(1 - \frac{2GM}{c^2r}) (1 - \frac{GM}{2c^2r}(4\beta - 1))}
\end{align*}
\]

with respect to (3). The Casadio-Fabbri-Mazzacurati black string classical horizon, in the brane, is the solution of the algebraic equation \(A(r) = 0\). In order to extract phenomenological information of numerical calculations, we first consider in this Subsection the case where \(\beta = 5/4\). Note that the metric above was also derived as a possible geometry outside a star on the brane \[18\]. The corresponding Hawking temperature is calculated in \[10\].

Taking into account the metric in (3), the classical standard black hole radius is given by two solutions of Schwarzschild type \(R_S = \frac{2GM}{c^2}\), for the choice \(\beta = 5/4\), providing zero Hawking black hole temperature. The 4D Kretschmann scalar \(K^{(1)} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\) diverges for \(r = 0\) and \(r = \frac{3GM}{2c^2}\). Now, the Gauss equation is well known to relate the 5D and the 4D Riemann curvature tensor as \(K^{(5)} = K_{\mu\nu\rho\sigma} - K^\mu_{\rho K_{\nu\sigma}} + K^\sigma_{\rho K_{\nu\mu}}\). By taking the junction conditions into account, it follows that \(K_{\mu\nu} = -\frac{1}{5} \kappa_5^2 \lambda g_{\mu\nu}\). By inserting it in the Gauss equation, the 5D Kretschmann scalar \(K^{(5)}\) for the Casadio-Fabbri-Mazzacurati type I black string also diverges for \(r = 0\) and \(r = \frac{3GM}{2c^2}\): the terms involving the extrinsic curvature in Gauss equation do not cancel the divergence provided by the 4D Kretschmann scalar, in the computation for \(K^{(5)}\).

Using the same procedure as \[12\], one can use the metric coefficients (4) in Eq.(2) and calculate the black string warped horizon. In the graphics below, we explicit the value for the black string warped horizon, provided by \(\sqrt{g_{\theta\theta}(R_S, y)}\), where \(R_S\) is the Schwarzschild radius. Further, \(\lambda = \Lambda_5 = 1 = \kappa_5\) hereupon \([M_\odot] \) denotes the sun mass:

![Figure 1](image-url)

**Figure 1.** Graphic of the brane effect-corrected black string horizon \(\sqrt{g_{\theta\theta}(R_S, y)}\) in the Casadio-Fabbri-Mazzacurati first solution, along the extra dimension \(y\), for different values of the black hole mass \(M\). For the dash-dotted line \(M = M_\odot\); for the black dashed line: \(M = 10M_\odot\); for the thick black line: \(M = 10^2 M_\odot\); for the black dotted line: \(M = 10^3 M_\odot\); for the thick gray line \(M = 10^4 M_\odot\); for the gray dotted line \(M = 10^5 M_\odot\).
Fig. 1 evinces a very interesting profile for the black string horizon behavior along the extra dimension $y$ in Gaussian coordinates. It indicates a critical mass $M$ (indeed our simulations provide $M \sim 73 M_\odot$) above which the associated black string warped horizon monotonically increases along the extra dimension. The black string is known to be placed in the bulk, in a tubular neighborhood along the axis of symmetry. We show here that at the coordinate singularities $r = 0$ and $r = \frac{3GM}{2c^2}$ there is a physical singularity for the black string at such values, irrespective of the value for $y$. In fact, the Kretschmann scalar $K = \frac{(5)}{R^{\mu\nu\rho\sigma}}(5)R^{\mu\nu\rho\sigma}$ diverges for such values (see [1]). Notwithstanding, the black string warped horizon $\sqrt{g_{\theta\theta}(R_S, y)}$ does not equal to zero, as illustrated at the Fig. 1.

3.2. Casadio-Fabbri-Mazzacurati black string: Case II

An alternative solution of (3) is obtained in [10, 11] where the metric coefficients

$$B(r) = 1 - \frac{2GM}{c^2 r} + \frac{2G^2M^2}{c^2 r^2} (\beta - 1), \quad A(r) = \frac{1 - \frac{3GM}{2c^2 r}}{(1 - \frac{2GM}{c^2 r})(1 - \frac{GM}{2c^2 r} (4\beta - 1))} \quad (5)$$

are considered in (3). In order that the Hawking temperature be zero on the brane, the choice $\beta = 3/2$ is demanded [10]. The classical solution $R$ for the black hole horizon is given by $R = R_S$ and $R = 5R_S/2$, where $R_S$ denotes the Schwarzschild radius. It implies that the black string horizon now corrected by braneworld effects when (5) is substituted in (2), providing the graphic below.

![Figure 2](image-url)

**Figure 2.** Graphic of the brane effect-corrected Casadio-Fabbri-Mazzacurati type II black string horizon $\sqrt{g_{\theta\theta}(R_S, y)}$, along the extra dimension $y$, for different values of the black hole mass $GM/c^2$ in the brane. For the black dashed line $M = M_\odot$; for the gray line $M = 10M_\odot$; for the black line: $M = 10^2 M_\odot$; the dash-dotted line: $M = 10^3 M_\odot$; for the dotted line: $M = 10^4 M_\odot$; for the gray dashed line $M = 10^5 M_\odot$; for the thick gray line $M = 10^6 M_\odot$.

The black string horizon profile along the extra dimension is qualitatively similar for all values of $M$ depicted here: the warped horizon always increases monotonically. Furthermore, under a similar analysis accomplished this time for the case II Casadio-Fabbri-Mazzacurati, and by taking into account the Kretschmann scalar [1], we conclude that such expression diverges for $r = \frac{3GM}{2c^2}$, for $r = \frac{5GM}{2c^2}$ and $r = \frac{2GM}{c^2}$. Contrary to the Schwarzschild metric, which presents the black hole horizon as a coordinate singularity — which can circumvented by, e. g., the Kruskal-Szekeres coordinates — and not as a physical singularity, the Kretschmann scalar for the Casadio-Fabbri-Mazzacurati case II metric indicates that each black hole horizon on the brane is a physical singularity, since it diverges for such values. Again, the terms involving the
extrinsic curvature in the Gauss equation above are not able to cancel the divergence induced by the 4D Kretschmann scalar, when one calculates \( K \). Hence, the black string also diverges for such values.

Additionally we depict below the CFM type I black string profile along the extra dimension:

![Figure 3. Casadio-Fabbri-Mazzacurati type I black string profile along the extra dimension, for \( M = 2.3 \times 10^3 M_\odot \).](image)

Fig. 2 indicates that the Casadio-Fabbri-Mazzacurati (case II) black string horizon always increases. Since the bulk has no fixed metric a priori, but it can be calculated from (2) by taking into account the metric on the brane, we can calculate the bulk curvature using the metric coefficients in (2).

Compact sources on the brane, such as stars and black holes have a description complicated and analytic solutions lacks. Solutions on the brane are provided by [16, 18, 10].

Delving into the analysis concerning the figures above, the general different profile between the Casadio-Fabbri-Mazzacurati black string warped horizon and the Schwarzschild horizon is expected. The warped horizon is provided by \( \sqrt{g_{\theta\theta}(R_S, y)} \) in (2). All information about the bulk near the brane can be extracted from the Casadio-Fabbri-Mazzacurati metrics on the brane. We considered terms up to \( y^4 \), since irrespective of the black hole horizon radius, the effective distance \( y \) along the extra dimension equals the compactification radius [12], as well as the effective size of the extra dimension probed by a graviton. Besides, in particular our procedure considers suitable values for the post-Newtonian parameter in order that the Hawking radiation on the brane is zero. The physical black hole radii \( R_S = \sqrt{g_{\theta\theta}(R_S, 0)} \) are now effectively dislocated into the bulk, and given by \( R_{\text{brane}} = \sqrt{g_{\theta\theta}(R_S, y)} \).

4. Conclusion

Any phenomenologically successful theory in which our Universe is viewed as a brane must reproduce the large-scale predictions of general relativity on the brane. It implies that gravitational collapse of matter trapped on the brane provides the Casadio-Fabbri-Mazzacurati solutions on the brane: either a localized black hole or an extended black string solution, possessing a warped horizon. It is possible to intersect this solution with a vacuum domain wall and the induced metric is the ones presented in the analysis in Subsections 3.1 and 3.2. In the case I, our analysis is restricted to the case where \( \beta = 5/4 \) since for this values there is a zero (Hawking) temperature black hole associated [10, 11]. Since we want to extract physical information on the braneworld effects on the variation of luminosity exclusively, we opted for this value for the parameter \( \beta \), in such a way that the graphics, concerning this variation on
the luminosity, take into account exclusively the braneworld effects, since Hawking radiation is shown to be suppressed with $\beta = 5/4$ for this metric. Analogously, for the case II the analysis is accomplished taking into account the value $\beta = 3/2$ in Eq.(5), as already discussed. More details can be found in [1]. It can be also shown that the black strings here analyzed are more unstable than the standard ones, in the Gregory-Laflamme context.

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