The Lyman-\(\alpha\) forest and WMAP year three

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ABSTRACT

A combined analysis of Cosmic Microwave Background (CMB) and Lyman-\(\alpha\) forest data can constrain the matter power spectrum from small scales of about \(1 h^{-1} \text{Mpc}\) all the way to the horizon scale. The long lever arm and complementarity provided by such an analysis has previously led to a significant tightening of the constraints on the shape and the amplitude of the power spectrum of primordial density fluctuations. We present here a combined analysis of the WMAP three year results with Lyman-\(\alpha\) forest data. The amplitude of the matter power spectrum \(\sigma_8\) and the spectral index \(n_s\) inferred from the joint analysis of high and low resolution Lyman-\(\alpha\) forest data as analysed by Viel & Haehnelt (2006) are consistent with the new WMAP results to within 1\(\sigma\). The joint analysis with the mainly low resolution data as analysed by McDonald et al. (2005) suggest a value of \(\sigma_8\) that is \(\sim 2\sigma\) higher than that inferred from the WMAP three year data alone. The joint analysis of the three year WMAP and the Lyman-\(\alpha\) forest data also does not favour a running of the spectral index. The best fit values for a combined analysis of the three year WMAP data, other CMB data, 2dF and the Lyman-\(\alpha\) forest data are \((\sigma_8, n_s) = (0.78 \pm 0.03, 0.96 \pm 0.01)\).

Key words: Cosmology: observations – cosmology: theory - cosmic microwave background, cosmological parameters – quasars: absorption lines

1 INTRODUCTION

Measurements of the matter power spectrum from Lyman-\(\alpha\) forest data extend to smaller scales and probe a redshift range complementary to estimates of the matter power spectrum from Cosmic Microwave Background (CMB), galaxy surveys or weak gravitational lensing observations (e.g. Croft et al. 1998, Gnedin & Hui 1998, McDonald et al. 2000, Hui et al. 2001, Croft et al. 2002, C02, McDonald 2003, Viel et al. 2003, Meiksin & White 2004, Viel et al. 2004, VHS).

The combined analysis of Lyman-\(\alpha\) forest data with the first year of WMAP data (Spergel et al. 2003, WMAP1) suggested that the fluctuation amplitude of the matter power spectrum on small scales was rather high \((\sigma_8 \sim 0.9)\) and that there was no significant deviation of the spectral index of primordial density fluctuations from a Harrison-Zeldovich spectrum \((n_s = 1)\). There was also no evidence for a (large) running of the spectral index, a non-zero neutrino mass or a deviation from a cold dark matter spectrum at small scales (Viel et al. 2004, VHS, Viel et al. 2004, McDonald et al. 2005, M05, Viel et al. 2005, Seljak et al. 2003, Lidz et al. 2006, Beltran et al. 2005, Abazajian 2006).

Viel et al. (2004) found \(\sigma_8 = 0.94 \pm 0.08, n = 0.99 \pm 0.03\) (1\(\sigma\)) and no evidence for a (large) running of the spectral in a combined analysis of a large sample of high resolution spectra quasar (QSO) absorption spectra at \(z \sim 2.5\) (Kim et al. 2004, Croft et al. 2002) and the WMAP1 data. Similar results, with somewhat smaller errors \((\sigma_8 = 0.90 \pm 0.03, n_s = 0.98 \pm 0.02)\) have been subsequently obtained by the SDSS collaboration in a combined analysis of the WMAP1 and other CMB data, SDSS galaxy survey data and SDSS Lyman-\(\alpha\) forest data (Seljak et al. 2003). The Lyman-\(\alpha\) forest data analysed by M05 and Seljak et al. (2003) consists mainly of low-resolution low S/N SDSS spectra with a wide redshift coverage \((2 < z < 4)\) to which they added a small sample of eight high resolution spectra (McDonald et al. 2004). The flux power spectrum was modelled using dark matter simulations, which take into account hydrodynamical effects in an approximate way and were calibrated with a few hydrodynamical simulations. Viel & Haehnelt (2006) found \(\sigma_8 = 0.91 \pm 0.07, n_s = 0.95 \pm 0.04\) for the SDSS Lyman-\(\alpha\) forest data alone using a suite of state-of-the-art full hydrodynamical simulations. Further studies of Lyman-\(\alpha\) forest data by Desjacques & Nusser (2004), Jen et al. (2003) and Zaroubi et al. (2004) also came to similar conclusions.

The WMAP3 data alone argues for significant deviation from a Harrison-Zeldovich spectrum, \(n_s = 0.95 \pm 0.02\), and a smaller value for the fluctuation amplitude on small...
scales $\sigma_8 = 0.74 \pm 0.06$ (Spergel et al. 2006). The WMAP team chose not to update their combined analysis of CMB and Lyman-\(\alpha\) forest in their WMAP3 data release. In this Letter we will present such a joint analysis.

## 2 THE DATA SETS

### 2.1 WMAP

The WMAP\(^1\) satellite has mapped the entire sky in five frequency bands between 23 and 94 GHz with polarization sensitive radiometers. The temperature power spectrum has been measured over a large range of scales ($l < 1000$) to an unprecedented accuracy (Hinshaw et al. 2006). We will use the temperature and polarization (Page et al. 2006) power spectra and maps as used by the WMAP likelihood codes (Verde et al. 2003; Spergel et al. 2006) as implemented in the code COSMOMC\(^2\) (Lewis & Bridle 2002).

### 2.2 Lyman-\(\alpha\) forest data sets

We will investigate two different Lyman-\(\alpha\) forest data sets. The sample of high resolution QSO absorption spectra used by VHS and Viel et al. (2004), consisting of the LUQAS sample (27 high resolution QSOs) (Kim et al. 2004) and the (reanalysed) sample of C02 (30 high resolution and 23 low resolution spectra), and the SDSS Lyman-\(\alpha\) forest data as presented by McDonald et al. (2006). The SDSS Lyman-\(\alpha\) forest data set consists of 3035 QSO spectra with low resolution ($R \sim 2000$) and low S/N ($\sim 10$ per pixel) spanning a wide range of redshifts ($z = 2.2 - 4.2$), while the LUQAS and the C02 samples contain mainly high resolution ($R \sim 45000$), high signal-to-noise (> 50 per pixel) QSO spectra with median redshifts of $z = 2.125$ and $z = 2.72$, respectively. Modelling the flux power spectrum of the Lyman-\(\alpha\) forest accurately for given cosmological parameters is not as straightforward as modelling the CMB power spectra and accurate numerical simulations are required. M05 modelled the flux power spectrum using a large number of Hydro Particle Mesh (HPM) simulations (Gnedin & Hu 1998; Viel et al. 2006), calibrated with a few full hydrodynamical simulations. VHS improved instead the effective bias method developed by C02 (see Gnedin & Hamilton 2002 and Zaldarriaga et al. 2003 for a critical assessment of the errors involved), by using a grid of full hydrodynamical simulations run with the Tree-SPH code GADGET-2 (Springel et al. 2001; Springel 2005) to infer the linear matter power spectrum. Viel & Haehnelt (2006) used a Taylor expansion of the flux power spectrum around best fitting values based on full hydrodynamical simulations to model the dependence of the flux power on cosmological and astrophysical parameters in their independent analysis of the SDSS Lyman-\(\alpha\) forest data.

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3 RESULTS

3.1 Incorporating the Lyman-α data into COSMOMC

The linear dark matter power spectra inferred from the two Lyman-α forest data sets have been incorporated into the new public available version of COSMOMC. The VHS power spectrum consists of estimates of the linear dark matter power spectrum at nine values of the wavenumber $k$ at $z = 2.125$ and nine values at $z = 2.72$, in the range $0.003 < k$ (s/km) $< 0.03$. The estimate of the uncertainty of the overall amplitude of the matter power spectrum is 29%. This estimate takes into account possible systematic and statistical errors (see the relevant tables of VHS for a detailed discussion). M05 provided a measurement of slope and amplitude of the matter power spectrum at $z = 3$ at a wavenumber $k = 0.009$ s/km with an estimate of the 1σ error of the amplitude of $\sim 14\%$. M05 have also made available a table that gives the minimum $\chi^2$ for a given cosmological model as a function of the amplitude and slope after marginalization over a wide range of cosmological and “nuisance” parameters. The nuisance parameters characterize a range of astrophysical and noise-related systematic uncertainties. We have furthermore implemented the modelling of the SDSS flux power spectrum with the method of Viel & Haehnelt (2006) based on a Taylor expansion of the flux power spectrum around a best fitting model.

3.2 Constraints on $\sigma_8$ and $n_s$

To make contact with previous analyses we show the marginalized 1σ and 2σ likelihoods in the $\sigma_8$-$n_s$ projection in Figure 1. The coloured contours in the left panel show the constraints for the VHS sample (light cyan), the SDSS sample (dark blue) with the likelihood estimates provided by M05 and the constraints for the WMAP1 data (light green). In all cases we assume the Universe to be flat, no contribution from tensor perturbations, a pure cosmological contribution from tensor perturbations, a pure cosmological and neutrinos with negligible mass. For the analysis of the WMAP1 data we assumed a prior on the Thomson optical depth $0 < \tau < 0.3$.

The dark solid (SDSS) and light dashed (VHS) contours show the joint constraints for the Lyman-α forest and WMAP1 data. As pointed out by Viel & Haehnelt (2005), there is remarkable agreement between the two joint analyses with the high resolution absorption spectra as anal-

![Figure 2. 1- and 2σ likelihoods for $\sigma_8$ and $n_s$ marginalized over all other parameters. Left panel: The blue contours show the constraints for SDSS only as analysed by M05 with an HST prior $72 \pm 8$ km/s/Mpc (blue). The empty solid contours are for the SDSS data set as analysed by [Viel & Haehnelt (2006), SDSS-d]. The WMAP3 results are shown in green. Right panel: The combined analysis with WMAP3 using the same colour/line coding as in the left panel.](image-url)
ysed by VHS and Viel et al. (2004) and the larger sample of low resolution SDSS spectra as analysed by M05 and Seljak et al. (2003). The Lyman-α forest data break some of the degeneracies of the WMAP1 data which are responsible for the elongated shape of the error contours. The joint analysis tightens the constraints in the $\sigma_8 - n_s$ plane by a factor $\sim 2$ (VHS) and $\sim 4$ (M05), compared to the constraints from the WMAP1 data alone but offers less help in improving the constraints on $n_s$. As discussed above and summarized in Table 1, in the combined analysis with the WMAP1 data the best fitting value of the spectral index is not significantly different from $n_s = 1$ and $\sigma_8 \sim 0.9$. The significantly smaller error bars of the SDSS data set are due to the much larger sample: the wider range of redshifts covered is responsible for breaking some of the degeneracies intrinsic to the Lyman-α forest data (M05).

In the right panel of Figure 1 we show how the situation has changed with the WMAP3 data. As discussed in Spergel et al. (2006) and Page et al. (2006), the contours for the WMAP3 data have shrunk by a factor $\sim 3$ compared to those of WMAP1 and lie at the bottom part of the region allowed by the latter. This is mainly due to the improved measurement of the optical depth from the large scale polarization (Page et al. 2006; Lewis 2006).

In the case of the high resolution VHS sample the errors are too large to significantly tighten the WMAP3 constraints. The joint analysis of WMAP3 with the SDSS Lyman-α forest data places now even tighter constraints at the bottom end of the range preferred by the SDSS Lyman-α forest data alone, about $2\sigma$ above the best fit value from the WMAP3 only data: $\sigma_8 = 0.86 \pm 0.03$ (see also Lewis 2006). The discrepancy is larger than may be naively inferred from the overlap of the Lyman-α only and WMAP3 only analysis because the data sets prefer different values for some of the other parameters in particular $\Omega_m$. The best fit value for $\Omega_m (\sim 0.3)$ is also $\sim 2\sigma$ higher for the combined analysis than for the WMAP3 data alone. The SDSS Lyman-α forest data as analysed by Viel & Haehnelt (2004) and the new WMAP results appear to be marginally consistent. Possible explanations for the (small) discrepancy, if there exists one, may be somewhat too optimistic errors for one or both of the data sets. As discussed in Page et al. (2006) the polarization measurements of the CMB are very difficult mainly due to foreground polarization. Moreover, modelling the Lyman-α forest data also has a range of not yet fully understood systematic uncertainties.

As discussed extensively by McDonald et al. (2006) the mayor systematic uncertainties are the still not very accurately known thermal state of the IGM, the determination of the effective optical depth, the modelling of the effect of strong absorption line systems (Viel et al. 2004) and UV fluctuations and the remaining deficiencies in our ability to accurately predict the flux power spectrum for a large parameter space. To investigate the last issue issue further we compare in the left panel of Figure 2 the analysis of the SDSS data by M05 with that by Viel & Haehnelt (2004) which is based on a Taylor expansion of the flux power spectrum around a best fitting model (labelled as SDSS-d in Figure 2). The analysis of Viel & Haehnelt (2004) uses more accurate full hydro-simulations instead of the approximate simulations of M05 at the expense of a much less complete sampling of parameter space, especially far from the best-fitting values, where the errors are possibly underestimated. Note that here, both for the forest data and the joint analysis, we assumed an HST prior for the Hubble constant (Freedman et al. 2001) that significantly shrinks the error contours for the Lyman-α forest data compared to that of Figure 1. As discussed by Viel & Haehnelt (2004) there is remarkable agreement between the two analyses (note that the analysis of Viel & Haehnelt (2004) does not use the last three redshift bins nor high resolution QSOs compared to that of M05). In the right panel of Figure 2 we show the constraints for the joint analysis with the WMAP3 data. The SDSS Lyman-α forest data as analysed by Viel & Haehnelt (2004) combined with WMAP3 give a smaller value best fit value of $\sigma_8 = 0.80 \pm 0.04$ which is in agreement with that from the WMAP3 data alone to within $1\sigma$. The joint analysis of Lyman-α forest data and the new WMAP data including a possible running of the spectral index gives $n_{\text{run}} = -0.002 \pm 0.015$ at $k = 0.002$ Mpc$^{-1}$ also in agreement with the estimate by Spergel et al. (2006).

We have also performed an extended combined analysis that includes the further CMB experiments ACBAR (Kuo et al. 2004), CBI (Readhead et al. 2004), VSA (Dickinson et al. 2004), the 2dF galaxy power spectrum (Percival et al. 2001) and the VHS and SDSS-d Lyman-α forest data. In this case we get $(\sigma_8, n_s) = (0.78 \pm 0.03, 0.96 \pm 0.01)$. Further results are listed in Table 2.

4 CONCLUSIONS

We have performed a combined analysis of the WMAP three year results with high and low resolution Lyman-α forest data in order to constrain the shape of the power spectrum of primordial density fluctuations and the amplitude of the matter power spectrum at intermediate scales $\sigma_8$. The main results are as follows.

- The high resolution VHS Lyman-α forest data is consistent to within $1\sigma$ with the three year WMAP results but offers little additional constraining power due to the large error bars. The larger sample of mainly low resolution Lyman-α forest data (SDSS) as analysed by Viel & Haehnelt (2004) is also consistent to within $1\sigma$ with the new WMAP results. However the joint analysis of the SDSS data gives about $2\sigma$ higher $\sigma_8$ and $\Omega_m$ values than those inferred from the new WMAP results alone.

- The best fit values for a combined analysis are $(1\sigma)$:

  $(\sigma_8, n_s) = (0.78 \pm 0.05, 0.96 \pm 0.02)$ and $(\sigma_8, n_s) = (0.86 \pm 0.03, 0.96 \pm 0.02)$ for WMAP combined with high resolution Lyman-α forest data and WMAP combined with low resolution Lyman-α forest data as analysed by M05. The analysis of the SDSS data set as analysed by Viel & Haehnelt (2006) based on full hydrodynamical simulations gives $\sigma_8 = 0.80 \pm 0.04$.

- The joint analysis of Lyman-α forest data and the new WMAP data does not favour a running of the spectral index. The best fitting value is $n_{\text{run}} = -0.002 \pm 0.015$ at $k = 0.002$ Mpc$^{-1}$.

- Adding other CMB data sets, the 2dF galaxy survey and both Lyman-α data sets the constraints on the matter power spectrum become $(\sigma_8, n_s) = (0.78 \pm 0.03. 0.96 \pm 0.01)$. 


Table 2. The marginalized constraints on cosmological parameters from WMAP3 and other data sets. VHS refers to the LUQAS+CROFT sample as analysed by Viel et al. (2002); SDSS refers to the measurement by McDonald et al. (2003); EXT refers to smaller scales CMB data sets: ACBAR (Kuo et al. 2004), CBI (Readhead et al. 2004), VSA (Dickinson et al. 2004) and the 2dF galaxy survey (Percival et al. 2001). SDSS-d refers to the SDSS analysis by Viel & Haehnelt (2006).

| Parameter | WMAP3 | WMAP3+VHS | WMAP3+SDSS | WMAP3+EXT+SDSS-d | WMAP3+SDSS-d (run.) |
|-----------|-------|-----------|------------|------------------|---------------------|
| \( \Omega_{m} h^2 \) | 0.106 ± 0.007 | 0.109 ± 0.008 | 0.120 ± 0.006 | 0.110 ± 0.006 | 0.109 ± 0.006 |
| \( 10^3 \Omega_{b} h^2 \) | 2.222 ± 0.069 | 2.237 ± 0.072 | 2.277 ± 0.065 | 2.226 ± 0.071 | 2.224 ± 0.066 |
| \( \Omega_{m} \) | 0.242 ± 0.032 | 0.257 ± 0.037 | 0.304 ± 0.031 | 0.258 ± 0.029 | 0.253 ± 0.028 |
| \( h \) | 0.729 ± 0.029 | 0.719 ± 0.031 | 0.688 ± 0.025 | 0.719 ± 0.026 | 0.723 ± 0.025 |
| \( \sigma_8 \) | 0.096 ± 0.016 | 0.956 ± 0.017 | 0.964 ± 0.016 | 0.960 ± 0.013 | 0.957 ± 0.014 |
| \( n_s \) | - | - | - | - | - | 0.002 ± 0.015 |

The Lyman-\( \alpha \) forest data appears to be in reasonable agreement with the CMB and other data sets which probe the matter power spectrum at larger scales. The Lyman-\( \alpha \) forest data will thus continue to unfold its special power by a better understanding of the systematic uncertainties, rather than the compilation of larger data sets.

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