Study on flow measurement method of vertical axial flow pump based on the relationship between flow and differential pressure of two points in elbow inlet passage

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Abstract: Under the support of computational fluid dynamics (CFD), a new flow measurement method for vertical axial flow pump with elbow inlet passage was put forward. In order to select suitable turbulence model to predict wall pressure in elbow inlet passage, Standard k-ε, RNG k-ε and Realizable k-ε models were used to simulate the flow field. In comparison with experimental results, the conclusion could be drawn that it is feasible to predict the wall pressure of elbow inlet passage with Realizable k-ε turbulence model. For model pump, based on the data from the model test and the numerical simulation respectively, the equations about discharge of pump and two-point differential pressure were obtained with the least square method and the results indicated that coefficients of the equations were slightly different. Based on the same numerical simulation method as model pump, the differential pressures were obtained by cloud computing with large parallel for prototype pump. In order to improve the prediction accuracy of discharge of prototype pump, a new corrected method was proposed on the condition that the relative errors about two-point differential pressures between numerical simulation and experiment for model pump were the same as those for prototype pump according to Euler similarity criterion. It is concluded that the method improves the accuracy and simplicity of flow measurement for vertical axial flow pump with elbow inlet passage and the approach to predict the flow of pump is feasible.

1. Introduction
The discharge of vertical axial flow pump is the key parameter to determine the efficiency and unit energy consumption in the pumping station, which is related to whether the performance test of pump device can be completed scientifically and accurately. It is very difficult to measure and monitor the discharge of axial flow pump because the length of elbow inlet passage is very short, section of passage is irregular, the flow variation is very complex and streamline curvature of flow is very large in the elbow inlet passage. Measurement of flow rate in the pumping station has been a major research issue in this field. At present, the discharge of vertical axial flow pump is obtained by conversion with the performance curve of prototype pump device base on the differential water level between pump sump and outlet sump [1]. Due to the disadvantage such as the limitation of accuracy of water level measurement, scale effect of similarity conversion and errors in data fitting and interpolation, it is difficult to achieve high accuracy of this method for flow measurement. The field test is also a common method for flow measurement. For instance, Qiu et al[2] et al used five-hole probes and...
acoustic Doppler current profiler (ADCP) to measure the discharges of water pumps. Yan[3] proposed a differential pressure flowmeter with by-pass tube. Zheng et al[4] and Zhou J R et al[5] et al used ultrasonic flowmeter to measure the flow in the large low-lift pumping stations. However, these research results are limited to the flow measurement method itself without considering the effect of additional errors from flow measurement instruments. In other fields, many researchers have also done a lot of research on flow measurement. Kazushi et al[6] proposed a real-time implementation of Kalman filter for unsteady flow measurement in a pipe. Han et al[7] used arc-type conductivity probes (ATCP) and electromagnetic flowmeter (EMF) to measure the flow of oil-in-water emulsions. Werner et al[8] developed a flowmeter based on the differential pressure method to obtain mass flow data in small centrifugal compressor. But those methods may not suitable to measure the discharge of vertical axial flow pump in the elbow inlet passage. However, the traditional method of flow measurement by differential pressure was proposed, which is a relatively simple and accurate method in the pump sump[9]. According to the equation about the discharge of pump and differential pressure, the discharge of pump can be obtained by simply measuring the differential pressure. But in the calibration of the equation, the current-meter method [10] and flow measurement method of salt water concentration [11] are used to measure the discharge of pump where the differential pressure of inlet passage is obtained by pressure equalizer ring, which has many disadvantages such as too much work, too many restriction conditions, too much waste of time, and too poor the measurement accuracy.

In recent years, more and more researchers at home and abroad pay attention to the simulation of pumping stations by numerical means[12, 13]. In the paper, the method of computational fluid dynamics (CFD) is introduced to the traditional method for flow measurement by differential pressure in the pump sump, and a new definition of method for flow measurement is proposed to improve the accuracy and simplicity in the calibration of equation about the discharge of pump and differential pressure. It should be pointed out that the differential pressure in the paper refers to the head of differential pressure between the two designated pressure measurement points inlet passage, rather than the head of average differential pressure between the two sections used in the calculation of the head loss of inlet passage. In the paper, in order to select suitable turbulence model to predict wall pressure in elbow inlet passage, the commercial computing software ANSYS Fluent and the RANS method based on Standard k-ε[14, 15], RNG k-ε[16, 17] and Realizable k-ε[18] turbulence models are used to calculate the flow field in the elbow inlet passage under conditions of the design water level and several flow schemes, and the results are compared with model test data. The Realizable k-ε turbulence model based on Reynolds-averaged Navier–Stokes equations is analyzed to simulate accurately the flow field in the elbow inlet passage, especially for prediction of wall pressure. In order to improve the prediction accuracy of discharge of prototype pump, a new corrected method was proposed. Then the equation about discharge of pump and differential pressure of elbow inlet passage is obtained, which provides a basis for popularizing and applies the flow measurement method of vertical axial flow pump based on the relationship between flow and differential pressure of two point in elbow inlet passage.

2. Material and methods

2.1. Governing equation

The flow in the elbow inlet passage can be considered as a steady incompressible flow. The governing equations can be expressed as continuous equations and momentum equations[19]:

Continuous equations:

$$\frac{\partial u}{\partial x} = 0 \tag{1}$$

Momentum equations:
\[
\frac{\partial}{\partial x_j} (\rho u_i u_j) = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_i} \left( \mu \frac{\partial u_i}{\partial x_j} \right) - \frac{\partial}{\partial x_j} (\rho u_i u_j) \quad (2)
\]

The modeled transport equations for \(k\) and \(\varepsilon\) in the Realizable \(k-\varepsilon\) model [20] are:

\[
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho u_i k)}{\partial x_i} = \frac{\partial}{\partial x_i} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + G_k - \rho \varepsilon \quad (3)
\]

\[
\frac{\partial (\rho \varepsilon)}{\partial t} + \frac{\partial (\rho u_i \varepsilon)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + \rho C_t E \varepsilon - \rho C_2 \frac{\varepsilon^2}{k + \sqrt{\varepsilon}} \quad (4)
\]

2.2. Computational domain and mesh

In the paper, a calculation model of a pumping station is established, which includes pump sump, elbow inlet passage, division pier, gate slot, hub and outlet pipe, as shown in Figure 1(a), and the diameter \(d\) of suction pipe is 312 mm. In the numerical simulation, the inlet of the calculation domain is extended by 17.5\(d\) and the outlet of suction pipe is extended by 5\(d\) to reduce the influence of the calculated boundary on the calculation results and avoid additional calculation distortion.

The grid is not only an indirect expression of geometric model, but also an important carrier of numerical calculation and analysis, the quality of which directly affects the accuracy and efficiency of calculation and the reliability of calculation results. The grid generation of computational domain is done with hexahedral meshes by the mesh division software ANSYS ICEM to ensure the accuracy of the numerical simulation.(Figure1.b) Considering the complexity of the flow in the elbow inlet passage with different radius of curvature, the meshes are encrypted to predict the water flow in the region exactly, where the minimum mesh length is \(\Delta l=2.00\) mm with \(\Delta l /d=1/44\). Because the complexity of flow is relatively small in other regions of the computational model, the meshes are relatively sparse. The adjacent grid scale ratio is 1.05 in order to ensure that the change of grid scale does not result in numerical dispersion which affects the calculation accuracy.

![Figure 1. Geometric parameters and calculation grid of the model.](image)

![Figure 2. The variation trend of the equation coefficients with the different grid quantities.](image)
points were given in the elbow inlet passage, and the corresponding differential pressures were collected under condition of different discharges. The least square method is used to linearly fit the discharges of pump and the differential pressures. Figure 2 shows that the variation trend of the equation coefficients with the different grid quantities. As can be seen from Figure 2, the calculation error of the equation coefficients is large when the number of grids is small. With the increase of number of grids, the equation coefficients tend to be stable. The computational results of the numbers of grids which are 0.88 million, 1.21 million and 2.11 million are very close. In order to ensure the rational use of computing resources, the mesh size and encryption method of computational case with 0.88 million are selected as the grid generation scheme in the paper.

2.3. Boundary conditions and solution method
Numerical simulation is conducted with software ANSYS Fluent by using finite volume method to discrete the governing equations, in which diffusion term is solved by central-differencing, and convection term is solved with second-order up wind scheme, and pressure and velocity coupling is used with SIMPLEC algorithm.

The inlet velocity boundary of where the size and direction are given. For free outflow of the outlet boundary, the transport variable gradient of outlet section is zero. The free surface is assumed to be a stress-free boundary. Here the solid wall boundaries are treated by the standard wall functions. The advantage of the standard wall functions is that it combines the physical quantities on the wall with the unknown quantities in the turbulent core region, not necessary to solve the flow in the wall region, and the nodal parameter values of adjacent control volume can be obtained directly.

3. Results and discussion

3.1. Selection and Verification of Turbulence Model

Figure 3. The plane velocity vector distribution near the bottom of the inlet passage (y=0.05m).

Figure 4. The pressure distribution near the bottom of the inlet passage (y=0.05m).

Figure 5. The variation trend of differential pressure with the variation of pump flow.
Figure 3 shows the plane velocity vector distribution near the bottom of the inlet passage \((y=0.05\, \text{m})\) when the discharge of pump \(Q=0.221\, \text{m}^3/\text{s}\) in the numerical calculation with the turbulence models including RNG \(k-\varepsilon\) (graph a), Standard \(k-\varepsilon\) (graph b) and Realizable \(k-\varepsilon\) (graph c). As can be seen from the figure 3, the velocity vectors calculated by different turbulence models are very similar and have wall vortices at the elbow inlet passage, but the size of the wall vortices is different. The range of the wall vortices calculated by the Standard \(k-\varepsilon\) turbulence model is smaller than that calculated by the turbulence models RNG \(k-\varepsilon\) and Realizable \(k-\varepsilon\). Figure 4 shows that the pressure distribution near the bottom of the inlet passage \((y=0.05\, \text{m})\) at the discharge of pump \(Q=0.221\, \text{m}^3/\text{s}\) in the numerical calculation with the turbulence models such as RNG \(k-\varepsilon\) (graph d), Standard \(k-\varepsilon\) (graph e) and Realizable \(k-\varepsilon\) (graph f). It can be seen from the figure 4 that the pressure distribution calculated by the turbulence model Realizable \(k-\varepsilon\) is more distinguishable and symmetrical, which effectively reflects the influence of the gate slot to the flow distribution. However, the pressure distribution simulated by RNG \(k-\varepsilon\) turbulence model is asymmetrical along both sides of the pier. While the results simulated by Standard \(k-\varepsilon\) turbulence model are symmetrical, the ability of capturing wall vortices is poor. In order to more accurately compare and analyze the predictive ability of different turbulence models for wall pressure of elbow inlet passage, a physical model experiment about elbow inlet passage of vertical axial flow pump was carried out. The experimental model follows the pressure similarity criterion so that the discharges of pump were \(0.221\, \text{m}^3/\text{s}\), \(0.350\, \text{m}^3/\text{s}\), \(0.173\, \text{m}^3/\text{s}\), \(0.340\, \text{m}^3/\text{s}\) and \(0.330\, \text{m}^3/\text{s}\), respectively. At the same time, the numerical simulation of the model test is carried out. Figure 5 shows that the variation trend of differential pressure with the variation of pump flow in the elbow inlet passage, which obtained by model test and calculated by numerical calculation with Realizable \(k-\varepsilon\), RNG \(k-\varepsilon\) and Standard \(k-\varepsilon\) respectively. It can be seen from the figure 5 that the differential pressures of the inlet passage increase with the increase of the pump capacities, and the numerical results calculated by the Realizable \(k-\varepsilon\) turbulence model are the closest to the result of model test. Therefore, the RANS method based on Realizable \(k-\varepsilon\) turbulence model will be used in the paper.

### 3.2. Study on the relationship between pump flow and differential pressure in prototype

Table 1 shows that the differential pressures of two pressure measurement points on the elbow inlet passage under the condition of different discharges in the model test. The least square method is used to fit the functional relationship between discharge of pump \(Q_{\text{mt}}\) and the corresponding differential pressure \(\Delta h_{\text{mt}}\) of elbow inlet passage, and the equation is obtained.

\[
Q_{\text{mt}} = 0.583 \sqrt{\Delta h_{\text{mt}}} \quad (5)
\]

Similarly, Table 1 gives that the differential pressures of two pressure measurement points on the elbow inlet passage under the condition of different discharges in the numerical calculation of the model. The least square method is also used to fit the data, and the equation about the discharges \(Q_{\text{mc}}\) and the differential pressures \(\Delta h_{\text{mc}}\) is obtained.

\[
Q_{\text{mc}} = 0.542 \sqrt{\Delta h_{\text{mc}}} \quad (6)
\]

In order to predict the functional relationship between discharges of pump and differential pressures of elbow inlet passage in practical engineering, CFD method is used to simulate the flow field of elbow inlet passage of vertical axial pump prototype. In the numerical calculation of the prototype, the numerical method same as that in the numerical calculation of the model, including turbulence model, boundary condition, discrete format and size of grid. The grid size of model is applied to the mesh generation of the prototype so that the grids number of prototype is very large (more than 90 million), which causes computational difficulties for computers or workstations. So the method of cloud computing with large parallel (Shenzhen Cloud Computing Center) was applied to the numerical calculation of the prototype, and the results are shown in Table 2.
 Generally, when the equation (5) and equation (6) are identical, the equation about the discharge of pump and differential pressure in the prototype can be directly fitted and applying to practical engineering. But in comparison between equation (5) and equation (6), there is a certain amount of error about equation coefficients. So the differential pressures of elbow inlet passage in prototype calculated by numerical method should be corrected.

In the paper, a new method is suggested to correct the numerical results in the cloud computing. Because the method for pressure measurement in the model test is same as that in the prototype and method of numerical calculation in the model is also same as that in the prototype so that the paper assume that the relative errors about two-point differential pressures between numerical simulation and experiment for model pump were the same as those for prototype pump according to Euler similarity criterion (named Equal Relative Error Correction Method), and its equation is as follows.

$$
\frac{\Delta p_{pc} - \Delta p_{mt}}{\Delta p_{mt}} = \frac{\Delta p_{pc} - \Delta p_{pt}}{\Delta p_{pt}}
$$

(7)

Table 1. Differential pressures between two measurement points in the elbow inlet passage model.

| Discharges of model $Q_m$ (m$^3$/s) | Differential pressures of model test $\Delta h_{mt}$ (m) | Numerical simulation |
|-------------------------------------|---------------------------------|-------------------|
|                                     | Pressures of P11 (Pa) | Pressures of P24 (Pa) | Differential pressures of numerical simulation $\Delta h_{mc}$ (m) |
| 0.221                              | 0.141                      | -98.997            | 0.162 |
| 0.350                              | 0.359                      | -256.755           | 0.407 |
| 0.173                              | 0.090                      | -60.117            | 0.100 |
| 0.340                              | 0.339                      | -240.865           | 0.383 |
| 0.330                              | 0.318                      | -226.865           | 0.362 |
| 0.311                              | 0.289                      | -200.873           | 0.322 |
| 0.275                              | 0.222                      | -155.269           | 0.251 |
| 0.299                              | 0.262                      | -185.908           | 0.298 |
| 0.287                              | 0.241                      | -170.272           | 0.275 |
| 0.261                              | 0.200                      | -140.004           | 0.228 |
| 0.247                              | 0.179                      | -124.181           | 0.202 |
| 0.233                              | 0.164                      | -110.981           | 0.182 |
| 0.207                              | 0.125                      | -86.600            | 0.142 |
| 0.198                              | 0.115                      | -78.796            | 0.130 |
| 0.185                              | 0.104                      | -69.001            | 0.114 |

Table 2. Differential pressures between two measurement points in the elbow inlet passage prototype.

| Discharges of prototype $Q_p$ (m$^3$/s) | Pressures of P11(Pa) | Pressures of P24(Pa) | Differential pressures $\Delta h_{pc}$ (m) | Corrected differential pressures $\Delta h_{pt}$ (m) |
|----------------------------------------|----------------------|----------------------|------------------------------------------|--------------------------|
| 22.060                                 | -207.364             | -1817.103            | 0.164                                    | 0.143                     |
| 34.977                                 | -469.759             | -4540.022            | 0.415                                    | 0.366                     |
| 17.290                                 | -114.674             | -1104.337            | 0.101                                    | 0.091                     |
| 33.957                                 | -442.814             | -4296.090            | 0.393                                    | 0.348                     |
| 33.012                                 | -418.852             | -4054.296            | 0.371                                    | 0.326                     |
| 31.079                                 | -367.880             | -3530.751            | 0.322                                    | 0.289                     |
The field measured differential pressures of elbow inlet passage is shown in Table 2, which is obtained by correcting the differential pressures of the prototype in the cloud computing with equation (7). Similarly, the least squares method is used to fit the functional relationship between the discharge of pump $Q_{pt}$ and differential pressure $\Delta h_{pt}$ of elbow inlet passage in the field test, and its equation is as follows.

$$Q_{pt} = 57.927 \sqrt{\Delta h_{pt}}$$ (8)

Then, we only need to install high-precision sensors at two measurement points in the elbow inlet passage and measure the differential pressure, bring them into equation (8) to obtain discharge of the vertical axial flow pump in the engineering.

4. Conclusions
Based on definition of the traditional flow measurement method by pressure-difference in the elbow inlet passage, the two-point differential pressure flow measurement method is proposed by introducing CFD technique, which effectively improves the accuracy of the flow of vertical axial flow pump with elbow inlet passage. The main conclusions of this study are as follows.

1. In the numerical simulation results of the RANS method based on Realizable $k$-$\varepsilon$ model, the wall pressures and its variation trend in the elbow inlet passage agree well with the model test results. The calculation accuracy sufficiently reflects the basic motion characteristics of the flow field in the elbow inlet passage.

2. For the model, the coefficients of two equations from model test and numerical simulation about discharge of pump and two-point differential pressure were slightly different, which is inevitable due to test error and numerical error, but the above is still acceptable quality.

3. For the prototype, the simulation adopts the cloud computing technique, in which the numerical method is same as the numerical calculation for the model, including turbulence model, boundary condition, discrete format and size of grid. Based on the above conditions, Equal Relative Error Correction Method can be used to raise the predictive accuracy of discharge of pump.

4. Although the functional relationship between flow of pump and the differential pressure in the prototype has been predicted, the feasibility of the flow measurement method remains to be tested in practical engineering.

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