2MASS TWO-COLOR INTERSTELLAR REDDENING LINES: 
THE BAND-WIDTH EFFECT

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Abstract. The band-width effect on interstellar reddening lines in the \(J−H\) 
vs. \(H−K_s\) diagram of the 2MASS survey is investigated using synthetic color 
indices and color excesses based on the Kurucz model atmospheres. At large in-
terstellar reddenings (\(E_{H−K_s} \geq 1.0\)) reddening lines deviate considerably from a 
straight line. The lines can be approximated by a parabolic equation: 
\[E_{J−H} = rE_{H−K_s} + sE_{H−K_s}^2,\]
where the slope coefficient, \(r\), and the curvature coefficient, \(s\), depend slightly on the intrinsic energy distribution of the source. The curva-
ture of the reddening lines is confirmed by the \(J−H\) vs. \(H−K_s\) diagrams plotted 
by Stražys and Langalys (2008) from 2MASS observations.

Key words: ISM: extinction – stars: fundamental parameters – photomet-
ric systems: infrared, 2MASS

1. INTRODUCTION

The color excess in a two-color monochromatic system (defined by the wave-
lengths \(\lambda_1\) and \(\lambda_2\)) is a difference of interstellar extinctions \(A(\lambda_1)\) and \(A(\lambda_2)\) ex-
pressed in stellar magnitudes. The values of the monochromatic extinctions for 
the unit dust mass \(x\) can be taken from the interstellar extinction law, i.e., the 
dependence of \(A\) on \(\lambda\) or \(\lambda^{-1}\). In the monochromatic or narrow-band photometric 
systems the extinction increases linearly with increasing of the dust mass.

In the case of a heterochromatic photometric system the extinctions are defined 
by the equation

\[A_m = -2.5 \log \frac{\int F(\lambda) R_m(\lambda) \tau(\lambda) d\lambda}{\int F(\lambda) \ R_m(\lambda) \ d\lambda}, \tag{1}\]

where \(F(\lambda)\) is the spectral energy distribution function of a star or a model atmo-
sphere, \(R_m(\lambda)\) is the response function of the passband, \(\tau(\lambda)\) is the transmittance 
function of the unit mass of dust and \(x\) is the number of dust masses.

This means that the heterochromatic extinction depends on the spectral energy 
distribution and the amount of interstellar dust. A red star, affected by the same 
cloud of interstellar dust, will exhibit smaller extinction \(A(\lambda)\) than a blue star.
Also, if a dust cloud gives the extinction \( A(\lambda) \), the addition of the second identical cloud will raise the extinction not to up \( 2A(\lambda) \) but to a smaller quantity. The broader the response function, the larger is the dependence of the extinction on the spectral energy distribution and the amount of interstellar reddening. This dependence is known as the band-width effect. The reason for the effect can be understood as the dependence of the effective wavelength on spectral type and interstellar reddening.

Since color excesses are differences of extinctions in two passbands, the dependence of \( A(\lambda_1) \) and \( A(\lambda_2) \) on spectral energy distribution of the star and on its interstellar reddening transfers the band-width effect to color excesses and color-excess ratios. However, in an exceptional case the band-width effect on a color excess can be zero, when the band-width effect in both passbands is the same.

The band-width effect was well known to stellar photometrists long ago; see, e.g., the reviews by one of the authors (Straižys 1977, 1992). However, in some new photometric systems the effect sometimes becomes forgotten. The near-infrared \( J, H, K \) system is one of such examples.

Jones & Hyland (1980) were probably the first who tried to estimate the band-width effect on the form of reddening line in the \( J-H \) vs. \( H-K \) diagram. By synthetic photometry they found some deviation of heavily reddened stars at \( J-H > 3.5 \). A similar effect was also calculated by Nagata et al. (1993). Naoi et al. (2006) found the decline of the reddening line slope in Ophiuchus and Chamaeleon star-forming regions by observations in the SIRIUS \( J, H, K_s \) system, but failed to confirm the effect by synthetic photometry.

One of the authors of the present paper (Straižys 1992) has estimated the band-width effect in the \( UBVRIJHKLM \) system by calculating color excesses and their ratios for black bodies of different temperatures. A clear decline of the ratio \( E_{J-H}/E_{H-K} \) from 2.0 to 1.7 was found when the temperature of the radiation source has decreased from 20000 K to 2000 K.

Recently, during the investigation of the \( E_{J-H}/E_{H-K_s} \) ratios in various Milky Way directions and in star-forming regions (Straižys & Laugalys 2008), we have noted that in most directions heavily reddened stars deviate down from the linear reddening line of red giants. This stimulated the investigation of possible band-width effect for heavily reddened stars in the two-color diagram of the 2MASS system.

2. CALCULATIONS AND RESULTS

Interstellar extinctions in the passbands of the 2MASS system were calculated by Equation (1) with the functions taken from the following sources. Spectral energy distributions \( F(\lambda) \) were taken for 409 synthetic spectra of solar metallicity and various temperatures and gravities from Kurucz (2001). Response functions of the 2MASS passbands were taken from Cutri et al. (2006) and Skrutskie et al. (2006). The transmittance function of the interstellar dust for a unit mass \( (x = 1) \) is taken from Straižys (1992, Table 3), with some small modification at wavelengths longer than 2.0 \( \mu \)m to adjust the extinction law to the ratio of color excesses \( E_{J-H}/E_{H-K_s} = 1.9 \). In calculations the dust mass \( x \) was varied from 2 to 10; these values correspond to \( A_V = 6.2 \) and 31 mag.

In Table 1 we present the calculated color excesses and their ratios for a selected set of 87 models with different temperatures and gravities and for five values of
Table 1. Ratios of color excesses $E_{J-H}/E_{H-K_s}$, for the Kurucz models with various interstellar extinctions.

| $T_{\text{eff}}$, log $g$ | $x = 2$ | $x = 4$ | $x = 6$ | $x = 8$ | $x = 10$ |
|--------------------------|---------|---------|---------|---------|---------|
| 3500, 1.0 | 1.942 | 1.906 | 1.871 | 1.838 | 1.808 |
| 3500, 2.0 | 1.991 | 1.901 | 1.867 | 1.836 | 1.806 |
| 3500, 3.0 | 1.942 | 1.905 | 1.865 | 1.836 | 1.807 |
| 3500, 4.0 | 1.943 | 1.903 | 1.870 | 1.837 | 1.807 |
| 3500, 5.0 | 1.940 | 1.902 | 1.868 | 1.834 | 1.804 |
| 4000, 1.0 | 1.932 | 1.900 | 1.864 | 1.832 | 1.803 |
| 4000, 2.0 | 1.939 | 1.898 | 1.865 | 1.832 | 1.802 |
| 4000, 3.0 | 1.936 | 1.898 | 1.864 | 1.831 | 1.801 |
| 4000, 4.0 | 1.939 | 1.900 | 1.864 | 1.831 | 1.802 |
| 4000, 5.0 | 1.936 | 1.902 | 1.869 | 1.836 | 1.807 |
| 4500, 1.0 | 1.943 | 1.906 | 1.870 | 1.838 | 1.807 |
| 4500, 2.0 | 1.940 | 1.904 | 1.868 | 1.835 | 1.805 |
| 4500, 3.0 | 1.943 | 1.900 | 1.866 | 1.834 | 1.804 |
| 4500, 4.0 | 1.936 | 1.900 | 1.864 | 1.833 | 1.802 |
| 4500, 5.0 | 1.940 | 1.900 | 1.863 | 1.832 | 1.802 |
| 5000, 1.0 | 1.947 | 1.908 | 1.873 | 1.841 | 1.811 |
| 5000, 2.0 | 1.943 | 1.906 | 1.871 | 1.840 | 1.809 |
| 5000, 3.0 | 1.940 | 1.906 | 1.871 | 1.839 | 1.808 |
| 5000, 4.0 | 1.943 | 1.908 | 1.871 | 1.837 | 1.808 |
| 5000, 5.0 | 1.940 | 1.903 | 1.867 | 1.836 | 1.805 |
| 5500, 1.0 | 1.954 | 1.917 | 1.881 | 1.847 | 1.816 |
| 5500, 2.0 | 1.958 | 1.915 | 1.878 | 1.846 | 1.816 |
| 6000, 1.0 | 1.954 | 1.917 | 1.881 | 1.847 | 1.816 |
| 6000, 2.0 | 1.961 | 1.921 | 1.887 | 1.853 | 1.822 |
| 6000, 3.0 | 1.951 | 1.919 | 1.883 | 1.849 | 1.819 |
| 6000, 4.0 | 1.943 | 1.908 | 1.871 | 1.837 | 1.808 |
| 6000, 5.0 | 1.940 | 1.903 | 1.867 | 1.836 | 1.805 |
| 6500, 1.0 | 1.951 | 1.913 | 1.876 | 1.844 | 1.813 |
| 6500, 2.0 | 1.947 | 1.910 | 1.875 | 1.842 | 1.812 |
| 6500, 3.0 | 1.947 | 1.910 | 1.874 | 1.842 | 1.811 |
| 6500, 4.0 | 1.947 | 1.910 | 1.874 | 1.842 | 1.811 |
| 6500, 5.0 | 1.947 | 1.910 | 1.874 | 1.842 | 1.811 |
| 7000, 1.0 | 1.966 | 1.926 | 1.890 | 1.856 | 1.826 |
| 7000, 2.0 | 1.961 | 1.921 | 1.887 | 1.853 | 1.822 |
| 7000, 3.0 | 1.951 | 1.919 | 1.883 | 1.849 | 1.819 |
| 7000, 4.0 | 1.947 | 1.910 | 1.874 | 1.842 | 1.811 |
| 7000, 5.0 | 1.943 | 1.908 | 1.871 | 1.837 | 1.808 |
| 7500, 1.0 | 1.954 | 1.917 | 1.881 | 1.847 | 1.816 |
| 7500, 2.0 | 1.958 | 1.915 | 1.878 | 1.846 | 1.816 |
| 7500, 3.0 | 1.951 | 1.913 | 1.876 | 1.844 | 1.813 |
| 7500, 4.0 | 1.947 | 1.910 | 1.875 | 1.842 | 1.812 |
| 8000, 1.0 | 1.947 | 1.910 | 1.874 | 1.842 | 1.811 |
| 8000, 2.0 | 1.947 | 1.910 | 1.874 | 1.842 | 1.811 |
| 8000, 3.0 | 1.947 | 1.910 | 1.874 | 1.842 | 1.811 |
| 8000, 4.0 | 1.947 | 1.910 | 1.874 | 1.842 | 1.811 |
| 8000, 5.0 | 1.947 | 1.910 | 1.874 | 1.842 | 1.811 |

$x$ to show the significance of the band-width effect. For the model with $T_{\text{eff}} = 35 000$ K, which corresponds to the spectral class O8, the ratio of color excesses is 1.99 at $x = 2$ and 1.85 at $x = 10$. For the model with $T_{\text{eff}} = 4500$ K and log $g = 2.5$, which corresponds to red clump giants (K2III), the ratio is 1.95 for $x = 2$ and 1.81 for $x = 10$.

In Figure 1 we plot the reddening line of red clump giants on the $J-H$ vs. $H-K_s$ diagram in a 1° diameter area in the direction of $t = 330°$, $b = 0°$ (Norma) taken
Fig. 1. Synthetic reddening line for the Kurucz model $T_{\text{eff}} = 4500$ K, log $g = 4.0$ (white circles) plotted on the observational 2MASS $J-H$ vs. $H-K_s$ diagram in the direction with the Galactic coordinates $\ell = 330^\circ, b = 0^\circ$.

from Stražys & Laugalys (2008). The theoretical line fits the observed points very well. In other Milky Way areas investigated by Stražys & Laugalys (2008) the correspondence is not so good since the observed reddening lines exhibit a slightly larger slope. In most of the star-forming regions investigated in that paper, the observed reddening lines end at lower values of color indices and are not suitable for verification of the reddening line curvature.
The reddening line can be expressed by a parabolic equation

\[ E_{J-H}/E_{H-K_s} = r - sE_{H-K_s}. \]  

(2)

The coefficient \( r \) due to the band-width effect shows the usual dependence on the temperature (or on spectral class), decreasing from 2.03 for O-stars down to 1.96 for M-stars. The coefficient \( s \) is almost constant, its average value is \(-0.12\).

We also calculated effective wavelengths of the \( J \), \( H \) and \( K_s \) passbands for various temperatures and gravities defined by the following equation:

\[ \lambda_{\text{eff}} = \frac{\int F(\lambda) R_m(\lambda) \tau_x(\lambda) \lambda d\lambda}{\int F(\lambda) R_m(\lambda) \tau_x(\lambda) d\lambda}. \]  

(3)

The results for five selected models of different temperatures are listed in Table 2. The largest change of the effective wavelengths both with the temperature and interstellar reddening is observed for the \( J \) passband: 0.01–0.02 \( \mu m \) between \( T_{\text{eff}} = 3500 \) K and 35000 K and 0.04–0.06 \( \mu m \) between \( x = 0 \) and 10. For the \( H \) passband the corresponding variations are 0.01 \( \mu m \) and 0.02 \( \mu m \). For the \( K_s \) passband these variations are 0.002 \( \mu m \) and 0.018 \( \mu m \).

The variations of \( \lambda_{\text{eff}} \) for the three passbands help to understand why the reddening line in the \( J-H \) vs. \( H-K_s \) diagram at large redenings is curved down: with increasing reddening the shift of \( \lambda_{\text{eff}} \) for \( J \) is much larger than for \( H \) and this leads to decrease of the base-line of the \( J-H \) color. As a consequence, the increase of \( J-H \) is slowed down in comparison with the dust mass \( x \). At the same time, the difference of \( \lambda_{\text{eff}} \) variation between the \( H \) and \( K_s \) passbands is much smaller, and the values of \( H-K_s \) color remain almost proportional to \( x \) with increasing reddening.

Since the effective wavelengths depend on the temperature and reddening, their change should be taken into account when plotting the interstellar extinction law: the values of \( A_\lambda \) determined for early-type or less reddened stars should be plotted at shorter wavelengths than for late-type or heavily reddened stars.

3. CONCLUSIONS

Applying the method of synthetic photometry for the Kurucz models we show that interstellar reddening lines in the 2MASS \( J-H \) vs. \( H-K_s \) diagram due to the band-width effect are of parabolic form with a curvature coefficient of \( s = -0.12 \). The slope of the reddening line at constant reddening also decreases with decreasing temperature, but this effect is much smaller. The theoretical results are confirmed by the observed reddening lines in the inner Galaxy investigated by Straizys & Laugalys (2008).

The knowledge of the band-width effect in the \( J, H, K_s \) system on the slope and curvature of reddening lines, as well as on the effective wavelengths, is important in determining the interstellar extinction law in the infrared range (see, e.g., Fitzpatrick 1999; Fitzpatrick & Massa 2005, 2007; Indebetouw et al. 2005; Flaherty et al. 2007; Román-Zúñiga et al. 2007). If one accepts that the reddening line is straight and solves all stars together, the ignorance of the curvature can lead to a smaller ratio of color excesses. Also, the ignorance of the curvature of reddening lines can lead to wrong classifications of heavily reddened stars from photometric data.
Table 2. Effective wavelengths of the 2MASS passbands $J$, $H$ and $K_s$ in $\mu$m for Kurucz models of five values of temperatures and different values of interstellar dust masses ($x = 0, 2, 4, 6, 8$ and $10$).

| $T_{\text{eff}}$, log $g$ | $J$     | $H$     | $K_s$    | $J$     | $H$     | $K_s$    |
|--------------------------|---------|---------|----------|---------|---------|----------|
|                          | $x = 0$ | $x = 6$ | $x = 2$  | $x = 6$ | $x = 8$ | $x = 10$ | $x = 4$  | $x = 6$ | $x = 8$ | $x = 10$ | $x = 4$  | $x = 6$ | $x = 8$ | $x = 10$ |
| 3500, 4.0                | 1.253   | 1.644   | 2.145    | 1.290   | 1.656   | 2.156    |
| 4500, 4.0                | 1.250   | 1.640   | 2.144    | 1.288   | 1.653   | 2.155    |
| 6000, 4.0                | 1.244   | 1.637   | 2.144    | 1.283   | 1.651   | 2.155    |
| 10000, 4.0               | 1.238   | 1.636   | 2.143    | 1.277   | 1.650   | 2.155    |
| 35000, 4.0               | 1.236   | 1.634   | 2.143    | 1.275   | 1.648   | 2.154    |
|                          | $x = 2$ | $x = 8$ | $x = 4$  | $x = 8$ | $x = 10$| $x = 4$  | $x = 8$ | $x = 10$ | $x = 4$  | $x = 8$ | $x = 10$ | $x = 4$  | $x = 8$ | $x = 10$ |
| 3500, 4.0                | 1.266   | 1.648   | 2.148    | 1.300   | 1.660   | 2.160    |
| 4500, 4.0                | 1.264   | 1.645   | 2.147    | 1.299   | 1.657   | 2.158    |
| 6000, 4.0                | 1.258   | 1.642   | 2.147    | 1.294   | 1.655   | 2.159    |
| 10000, 4.0               | 1.251   | 1.641   | 2.147    | 1.288   | 1.654   | 2.158    |
| 35000, 4.0               | 1.249   | 1.639   | 2.146    | 1.287   | 1.652   | 2.158    |
|                          | $x = 4$ | $x = 10$| $x = 4$  | $x = 10$| $x = 4$  | $x = 10$ | $x = 4$  | $x = 10$ | $x = 4$  | $x = 10$ | $x = 4$  | $x = 10$ | $x = 4$  | $x = 10$ |
| 3500, 4.0                | 1.278   | 1.652   | 2.152    | 1.310   | 1.664   | 2.163    |
| 4500, 4.0                | 1.276   | 1.649   | 2.151    | 1.308   | 1.661   | 2.162    |
| 6000, 4.0                | 1.271   | 1.646   | 2.151    | 1.304   | 1.659   | 2.162    |
| 10000, 4.0               | 1.264   | 1.645   | 2.151    | 1.299   | 1.658   | 2.162    |
| 35000, 4.0               | 1.263   | 1.643   | 2.150    | 1.297   | 1.656   | 2.161    |

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