Application and analysis of interacting multimode Kalman filter in location algorithm

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Abstract—Aiming at the non-line-of-sight (NLOS) problem that affects the accuracy of wireless location in cellular networks, the IMM method is introduced. The method based on interactive multimode Kalman filter (IMMKF) is studied and analyzed in detail. A more complex second-order and fourth-order Markov transfer model is established to describe the wireless communication channel. The positioning performance and the factors that affect the performance of IMMKF filter are analyzed by simulation. The simulation results show that IMMKF can adaptively switch between models with the change of channel propagation environment, suppress non line of sight error, and show good positioning performance and filtering performance.

1. INTRODUCTION
The location system of cellular network generally uses the time of arrival (TOA), time difference of arrival (TDOA), angle of arrival (AOA) and signal strength (RSS) received by three or more base stations to estimate the location. NLOS error is the decisive factor affecting the accuracy of this kind of algorithm. How to effectively identify and suppress NLOS error have become an important issue in the study of improving the accuracy of mobile positioning methods based on TDOA and TOA [1][2].

At present, the commonly used NLOS error elimination method is to directly process the measured value to suppress NLOS error. The classical methods include Wylie method [2], offset Kalman filter method [3], etc., but these methods have estimation lag and cannot meet the real-time positioning needs. In this paper, the IMM method is applied to the location of cellular network. The IMMKF method is used to estimate the distance between the mobile station and the base station, and then the location is calculated according to the geometric principle. The performance of the location algorithm based on the second-order channel and the fourth-order channel model is verified by simulation, and various factors affecting the location performance of the IMMKF are analyzed.

2. IMMKF INTRODUCTION
2.1 State Space Model
Assuming the distance measurement \( r_k(n) \) obtained by TOA from the mobile station to the base station \( k \) at time \( n \) [2]

\[
r_k(n) = d_k(n) + n_k(n) + NLOS_k(n)
\]

In the above formula, \( d_k(n) \) is the real distance value, \( n_k(n) \) is the distance offset caused by standard measurement noise, which is the Gaussian random variable obeying the distribution...
\( N(0, \sigma^2) \), \( \text{NLOS}_k(n) \) is the distance offset caused by NLOS propagation, which is the main error source obeying the distribution \( N(m_{\text{NLOS}}, \sigma_{\text{NLOS}}^2) \). Since the two kinds of errors are independent, formula (1) can also be expressed as [4]:

\[
 r_k(n) = d_k(n) + m_{\text{NLOS}} + b_k(n)w_k(n)
\]  

(2)

In IMMKF filtering process, the state vector of base station \( k \) is defined as [5][6]:

\[
 x_k(n) = [d_k(n), \quad \dot{d}_k(n)]^T
\]  

(3)

\( d_k(n) \) represents the moving speed of the mobile station relative to the base station \( k \), and the dynamic state equation is:

\[
 x_k(n+1) = Fx_k(n) + C\nu_k(n)
\]  

(4)

\[
 r_k(n) = Gx_k(n) + m_{\text{NLOS}} + b_k(n)w_k(n)
\]  

(5)

among them

\[
 F = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} T_s/2 \\ T_s \end{bmatrix}, \quad G = [1, 0]
\]

\( T_s \) is the sampling period and \( \nu_k(n) \) is the driving noise of the motion process, which depends on the acceleration of the mobile station.

2.2 IMMKF Estimates the Distance from the Mobile Station to the base station

The working principle of IMMKF includes four steps: input interaction, Kalman filtering, model probability updating and state fusion [7][8].

1) Input interaction

The covariance of input state variable and estimation error at time \( n \) are expressed as \( \hat{x}_{k,0,j}(n−1/n−1) \) and \( P_{k,0,j}(n−1/n−1) \), \( i \) and \( j \) are the number of channel models.

\[
 \mu_{k,i,j}(n−1/n−1) = p_{ij}\mu_{k,i}(n−1)/\bar{\sigma}_{k,j}
\]  

(6)

\[
 \bar{\sigma}_{k,j} = \sum_p p_{ij}\mu_{k,i}(n−1)
\]  

(7)

\[
 \hat{x}_{k,0,j}(n−1/n−1) = \sum_j \hat{x}_{k,j}(n−1/n−1)\mu_{k,d,j}(n−1/n−1)
\]  

(8)

\[
 P_{k,0,j}(n−1/n−1) = \sum_i[P_{i,j}(n−1/n−1) + \Delta_i\times\Delta_i^T]\mu_{d,j}(n−1/n−1)
\]  

(9)

\[
 \Delta_i = \hat{x}_{k,i}(n−1/n−1) - \hat{x}_{k,0,j}(n−1/n−1)
\]  

(10)

2) Kalman filtering

\[
 \hat{x}_{k,j}(n/n−1) = F\hat{x}_{k,0,j}(n−1/n−1)
\]  

(11)

\[
 P_{k,j}(n/n−1) = FP_{k,0,j}(n−1/n−1)F^T + \sigma_{e,k,j}^2CC^T
\]  

(12)

\[
 e_{k,j}(n) = r_{k,j}(n) - G\hat{x}_{k,j}(n/n−1)
\]  

(13)

\[
 K_{k,j}(n) = P_{k,j}(n/n−1)G^TS_{k,j}(n)^{-1}
\]  

(14)

\[
 \hat{x}_{k,j}(n/n) = \hat{x}_{k,j}(n/n−1) + K_{k,j}(n)e_{k,j}(n)
\]  

(15)

\[
 S_{k,j}(n) = GP_{k,j}(n/n−1)G^T + b_{k,j}^2
\]  

(16)

\[
 P_{k,j}(n/n) = [I - K_{k,j}(n)G]P_{k,j}(n/n−1)
\]  

(17)

3) Model probability update

\[
 \Lambda_{k,j}(n) = N(e_{k,j}(n)|0, S_{k,j}(n))
\]  

(18)

\[
 \mu_{k,j}(n) = \sum_j \Lambda_{k,j}(n)\bar{e}_{k,j}
\]  

(19)

4) State fusion
\[
\hat{x}_k(n/n) = \sum_{j} \hat{x}_{k,j}(n/n) \mu_{k,j}(n) \\
P_k(n/n) = \sum_{j} [P_{k,j}(n/n) + \{x_{k,j}(n/n) - \hat{x}_k(n/n)\}] \\
\times \{x_{k,j}(n/n) - \hat{x}_k(n/n)\} \times \mu_{k,j}(n) \\
\hat{d}_k(n) = G\hat{x}_k(n/n)
\]

(20)

3. LOCATION PERFORMANCE AND ANALYSIS OF SECOND-ORDER IMMKF

3.1 Second-order channel system model
The change of LOS/NLOS in the propagation channel between the mobile station and the base station can be regarded as a second-order Markov transfer model [9]. As shown in Fig.1, assuming that the channel is in LOS state at the current time, the probability of the channel still in LOS state at the next time is \(p_{11}\), and the probability of LOS/NLOS transfer is \(p_{12}\).

![Second-order Markov transfer model](image)

Figure 1. Second-order Markov transfer model

\[m_{k,NLOS} = \begin{cases} 0 & \text{LOS} \\ m_{NLOS} & \text{NLOS} \end{cases}\]

\[b_k(n) = \begin{cases} \sigma & \text{LOS} \\ \sigma \sqrt{\sigma^2 + \sigma_{NLOS}^2} & \text{NLOS} \end{cases}\]

3.2 Location Performance and Analysis
1) Simulation environment
Suppose that the mobile station moves at a speed of 70 km/h and the motion track is shown in Fig.2. the total length of the sample is 1000, the sampling interval \(T\) is 0.2 s, the standard measurement noise obeys the Gaussian distribution with the mean value of 0 and the standard deviation of 150 m; the error caused by NLOS propagation obeys the Gaussian distribution with the mean value of 513 m and the standard deviation of 409 m [3]. It is assumed that the LOS/NLOS transfer of the propagation channel between the mobile station and the base station is once every 200 samples, and the base stations (BS1 (0, 0), BS2 (0, 5000), BS3 (4330, 2500)) can simultaneously detect the transmission signal from the mobile station. The process driven noise depends on the acceleration of the mobile station and it is assumed to be a Gaussian random variable with mean value of 0 and variance of 1. The initial probability of each model is set to 0.5, and the second-order Markov transfer matrix of channel state between mobile station and base station is as follows:

\[
\begin{bmatrix}
p_{11} & p_{12} \\
p_{21} & p_{22}
\end{bmatrix} = \begin{bmatrix} 0.995 & 0.005 \\
0.005 & 0.995 \end{bmatrix}
\]
2) Performance analysis

The accuracy of location algorithm based on ranging mainly depends on whether the distance between mobile station and base station can be estimated accurately. Fig.3 shows the filtering results of Kalman and second-order IMMKF when the channel state between mobile station and base station is fixed and transferred respectively. It is not difficult to see that when the channel state is fixed to NLOS, Kalman filter cannot accurately estimates the distance between the two. This is because, when the state equation can correctly reflect the actual situation of the channel, the distance estimation is the estimation in the sense of minimum variance. When the channel is NLOS propagation, Kalman cannot adaptively adjust the state equation to match the actual situation of the channel.

IMMKF is composed of two Kalman filters of LOS and NLOS model. The Kalman filter state equation of NLOS model considers NLOS propagation error and matches NLOS channel state. Effective fusion between different models can overcome the limitation of model mismatch caused by single state equation. Therefore, IMMKF can accurately estimates the distance between the mobile station and the base station when the channel state is fixed or transferred. When the location algorithm based on ranging is adopted, 67% of the positioning errors are 47.64m and 95% of positioning error is 73.54m. The positioning accuracy meets the requirements of FCC. But when the single-mode Kalman filter is used for ranging, 67% of the positioning errors is 422.77m and 95% of positioning errors is 490.21m.

4. Location Performance and Analysis of Fourth-order IMMKF

4.1 Fourth-order Channel System Model

Figure 2. Real motion track of mobile station

Figure 3. Kalman and IMMKF filter comparison
In order to be closer to the real communication environment, the communication environment is designed to include LOS environment, rural environment, suburban environment and urban environment according to the cost259 channel model. The change of propagation channel between mobile station and base station is regarded as a fourth-order Markov transfer model, as shown in Fig.4. $m_{\text{NLOS}}$ and $b_k(n)$ in (2) are defined as follows:

$$m_{\text{NLOS}} = \begin{bmatrix} 0 & m_{\text{NLOS, Rural}} & m_{\text{NLOS, Suburban}} & m_{\text{NLOS, Urban}} \end{bmatrix}$$

$$b_k(n) = \begin{bmatrix} \sqrt{\sigma^2 + \sigma^2_{\text{NLOS, Rural}}} \\ \sqrt{\sigma^2 + \sigma^2_{\text{NLOS, Suburban}}} \\ \sqrt{\sigma^2 + \sigma^2_{\text{NLOS, Urban}}} \end{bmatrix}$$

4.2 Location Performance and Analysis

1) Simulation environment

It is assumed that the standard measurement noise obeys the distribution of $N(0, 150)$, the NLOS error in rural obeys the distribution of $N(313, 309)$, the NLOS error of suburban obeys the distribution of $N(413, 209)$, and the NLOS error of urban obeys the distribution of $N(513, 409)$ [3]. The initial probability of each model is set to 0.25. Other parameters are the same as Section III. Markov transfer matrix of channel state between mobile station and base station is as follows:

$$H = \begin{bmatrix} 0.94 & 0.02 & 0.02 & 0.02 \\ 0.02 & 0.94 & 0.02 & 0.02 \\ 0.02 & 0.02 & 0.94 & 0.02 \\ 0.02 & 0.02 & 0.02 & 0.94 \end{bmatrix}$$

2) Performance analysis

Fig.5~fig.7 are the filtering results of Kalman filter, offset Kalman filter and fourth-order IMMKF respectively. When the channel state is constantly changing, the traditional Kalman filter cannot adaptively adjust the state equation to match the actual channel situation, so it is difficult to estimate the distance from the mobile station to the base station, as
Figure 5. Kalman filtering results

Figure 6. Offset Kalman filtering results

Figure 7. IMMKF filtering results
TABLE 1 POSITIONING RESULTS BASED ON THREE KALMAN RANGING AND POSITIONING ALGORITHMS

| Positioning method | parameter | 67% error (m) | 95% error (m) |
|--------------------|-----------|---------------|---------------|
| Kalman             |           | 206.09        | 334.29        |
| Offset Kalman      |           | 104.08        | 214.55        |
| IMMKF              |           | 47.64         | 83.54         |
| IMMKF H₂           |           | 81.59         | 114.44        |

shown in Fig. 5. The offset Kalman in Fig. 6 contains NLOS suppression technology, which can accurately estimate the distance between mobile station and base station, but cannot meet the need of real-time positioning. The fourth-order IMMKF estimation results in Fig. 7 are closer to the actual distance. This is because IMMKF is composed of four Kalman filters under LOS, rural, suburban and urban models. Through effective fusion between models, it can overcome the limitations of model mismatch caused by single state equation model and improve the accuracy of estimation. The positioning results based on the above three Kalman ranging methods are shown in Tab. 1.

5. PERFORMANCE ANALYSIS OF IMMKF

The performance of IMMKF filter is restricted by many factors. The following is a detailed analysis.

5.1 Model Accuracy and Model Probability Estimation

Model accuracy and model probability estimation are the main factors that affect the performance of IMMKF filter. High model accuracy means that the error of the selected model in describing a certain state is very small, while in describing other states, the error is large; high accuracy of model probability estimation means that the stage in which the model plays a leading role in describing the channel state and the stage in which the model does not play a leading role can be clearly distinguished from the model probability estimation curve.

In this algorithm, the difference between models depends on the variance of distance offset caused by NLOS propagation. Assuming that the channel at time \( n \) is in LOS state, the LOS model is used to describe the channel with small error, and other models are used to describe the channel with large error, that is, the filtering residual under the LOS model is small. According to formulas (18) - (21), the probability of the LOS model is large, and the state estimation is dominant. The output of IMMKF mainly depends on the distance estimation under the LOS model that matches the channel.

Fig. 8 is the model probability curve when \( \sigma_{\text{NLOS}} \), are 409m and 309m in the simulation of second-order IMMKF filter simulation. It can be seen from Fig. 8 that the change of channel state between mobile station and base station (every 40s) and the model that plays a leading role at each time period are obvious. when \( \sigma_{\text{NLOS}} = 309m \), the difference between two models becomes smaller, and the accuracy of model probability estimation also decreases.

In the simulation of fourth-order IMMKF filtering, the standard deviation of distance migration in the four environments is set as 150 m, 209m, 309m and 409m respectively, and the probability estimation curve of LOS model is shown in Fig. 9. In the period of 0 ~ 40s and 160 ~ 200s, the channel is in LOS state, and the estimated probability of LOS model calculated by formula (18) ~ (21) is 95%.

5.2 Markov Transfer Matrix

Markov transfer matrix is actually equivalent to the transition matrix of model state equation, which will directly affect the accuracy of model error and model probability estimation. When the second-order Markov transfer matrix are

\[
H = \begin{bmatrix}
0.995 & 0.005 \\
0.005 & 0.995
\end{bmatrix},
\]

\[
H_1 = \begin{bmatrix}
0.990 & 0.010 \\
0.010 & 0.990
\end{bmatrix},
\]

\[
H_2 = \begin{bmatrix}
0.985 & 0.015 \\
0.015 & 0.985
\end{bmatrix}.
\]
the curve of positioning error is shown in Fig.10. When the Markov matrix is $H_1$, 67% of the positioning errors is 67.09m and 95% of the positioning errors is 97.56m; When the Markov matrix is $H_2$, 67% of the positioning errors is 81.59m and 95% of the positioning errors is 114.44m. Fig.10 shows that the deviation between the model and the actual channel is getting larger and larger with the change of Markov matrix, which leads to the larger and larger ranging error of IMMKF and the larger and larger final positioning error.

When the fourth-order Markov state transition matrix is

$$
H_2 = \begin{bmatrix}
    p_{11} & p_{12} & p_{13} & p_{14} \\
    p_{21} & p_{22} & p_{23} & p_{24} \\
    p_{31} & p_{32} & p_{33} & p_{34} \\
    p_{41} & p_{42} & p_{43} & p_{44}
\end{bmatrix} = \begin{bmatrix}
    0.82 & 0.06 & 0.06 & 0.06 \\
    0.06 & 0.82 & 0.06 & 0.06 \\
    0.06 & 0.06 & 0.82 & 0.06 \\
    0.06 & 0.06 & 0.06 & 0.82
\end{bmatrix}
$$

the deviation from the actual channel state becomes larger, resulting in larger distance estimation error and larger final positioning error (see Tab.1).

5.3 Model Set
IMMKF adopts the method of model set, which can increase, decrease and change the model in real time according to the actual situation of channels. When the law of channel state changing is clear, the filter can choose the model which can describe the state more accurately to improve the estimation accuracy.
6. CONCLUSION
In the complex communication environment, when the adopted model set is close to the actual channel, IMMKF can suppress NLOS error in measurements through the interaction between models, and the filtering result approximates the real distance between mobile station and base station. It is an effective ranging method in the dynamic environment and the location algorithm based on IMMKF can obtain ideal positioning accuracy.

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