Spin dependent structure function $g_1$ at small $x$ and small $Q^2$  

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The SMC has recently published the optimal set of results for the virtual photon-proton and virtual photon-deuteron cross section asymmetries $A_{p1}^p(x, Q^2)$ and $A_{d1}^d(x, Q^2)$, obtained from spin dependent inclusive muon–proton and muon–deuteron scattering at 100 and 190 GeV. The data covered the kinematic interval of $0.0008 < x < 0.7$ and $0.2 < Q^2 < 100$ GeV$^2$. Events with lower values of $x$ were not measured to avoid a contamination with muon scattering off atomic electrons at $x = 0.000545$. Recently the SMC has obtained results for the $A_{p1}^p$ and $A_{d1}^d$ at very small $x$ and $Q^2$, acquired from the 190 GeV $\mu$-p and $\mu$-d scattering, Fig.1. The data were collected with a dedicated "low $x$ trigger" in which both a minimal energy deposit in the hadronic part of the calorimeter and a scattered muon were demanded. This together with off-line selections removed all but $(5\pm1)\% \mu e$ events and considerably reduced the radiative background. Measured asymmetries are very small, cf.Fig.1, and special care has to be taken to well control the systematic errors, in particular those due to uncertainties of the spin averaged structure functions $R$ and $F_2$ at low $x$ and low $Q^2$. The new data complement the optimal set of the SMC results in the region of $0.01 < Q^2 < 0.2$ GeV$^2$ and $0.00006 < x < 0.0008$ and include the lowest values of $x$ ever measured for the spin dependent inelastic scattering.

Below we shall review the predictions concerning the $g_1(x, Q^2)$ valid in the region of the new measurements. To obtain predictions for the asymmetry $A_{p1}^p$ rather than for $g_1^p$, $A_1 \approx 2x(1+R)g_1/F_2$, models for $F_2^p$ and $R$, valid at low $x$ and $Q^2$, like e.g. these of ref. [3] may be used.

The $g_1$ function is expected to be a finite function of $W^2$ in the limit $Q^2 \rightarrow 0$ for fixed $W^2$, free from any kinematical singularities or zeros at $Q^2 = 0$, similarly to the structure function $F_1$.

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The new SMC data include the kinematic region where the four momentum transfer is smaller than any other energy scale, \( W^2 \) or \( 2M\nu \), \( Q^2 \ll W^2 \) and \( W^2 \) is high, \( W^2 \gtrsim 100 \text{ GeV}^2 \). Thus one should expect that the Regge model should be applicable there. This model gives the \( x \) (or \( W^2 \)) dependence of \( g_1 \) at fixed \( Q^2 \). However \( W^2 \) changes very little in the kinematic range of the data: from about 100 GeV\(^2\) at \( x = 0.1 \) to about 220 GeV\(^2\) at \( x = 0.0001 \), contrary to a quite strong change of \( Q^2 \) (from about 20 GeV\(^2\) to about 0.01 GeV\(^2\) respectively). This means that the new SMC measurements cannot test the Regge behaviour of \( g_1 \) through the \( x \) dependence of the latter. For completeness we shall only list the Regge predictions. According to the Regge model 

\[ g_1(x, Q^2) \sim x^{-\alpha} \text{ for } x \to 0 \text{ (for fixed } Q^2 \text{)}, \]

where \( \alpha \) denotes the intercept of the Regge pole trajectory corresponding to axial vector mesons. It is expected that \( \alpha \sim 0 \) for both \( I = 0 \) and \( I = 1 \) trajectories, [4]. This behaviour of the \( g_1 \) should go smoothly to the \( W^2 \alpha \) dependence for \( Q^2 \to 0 \). Other considerations related to the Regge theory predict \( g_1^p \sim \ln x \), [5], while the model based on exchange of two nonperturbative gluons gives \( g_1^p \sim 2 \ln(1/x) - 1 \), [6]. A perverse behaviour, \( g_1 \sim 1/(x\ln x) \), recalled in [5], is not valid for \( g_1 \), [7].

In perturbative QCD the structure function \( g_1 \) is controlled at low \( x \) by the double logarithmic \( \ln^2(1/x) \) contributions [8]. It is convenient to discuss the \( \ln^2(1/x) \) resummation using the formalism of the unintegrated (spin dependent) parton distributions \( f_i(x', k^2) \) \( (i = u, d, \bar{u}, \bar{d}, s, g) \) where \( k^2 \) is the transverse momentum squared of the parton \( i \) and \( x' \) the longitudinal momentum fraction of the parent nucleon carried by a parton [8, 9]. The conventional (integrated) distributions \( \Delta p_i(x, Q^2) \) are related in the following way to the unintegrated distributions \( f_i(x', k^2) \):

\[
\Delta p_i(x, Q^2) = \Delta p_i^{(0)}(x) + \int_{k_0^2}^{W^2} \frac{dk^2}{k^2} f_i(x' = x(1 + k^2/Q^2), k^2)
\]

(1)

where \( W^2 = Q^2(1/x - 1) \) and \( \Delta p_i^{(0)}(x) \) denote the nonperturbative part of the of the distributions. The \( g_1^p \) is related in a standard way to the (integrated) quark and anti-quark distributions, i.e.

\[
g_1^p(x, Q^2) = \frac{1}{2} \left[ \frac{4}{9} (\Delta u_v(x, Q^2) + 2\Delta \bar{u}(x, Q^2)) + \frac{1}{9} (\Delta d_v(x, Q^2) + 2\Delta \bar{d}(x, Q^2) + 2\Delta \bar{s}(x, Q^2)) \right]
\]

(2)

where \( \Delta u_v(x, Q^2) = \Delta p_{u_v}(x, Q^2) \) etc. We assume \( \Delta \bar{u} = \Delta \bar{d} \) and number of flavours \( N_F = 3 \). The parameter \( k_0^2 \) is the infrared cut-off \( (k_0^2 \sim 1 \text{ GeV}^2) \).

The sum of \( \ln^2(1/x) \) terms is generated by the corresponding integral equations for the functions \( f_i((x', k^2) [8, 9, 10] \). These equations lead to approximate \( x^{-\lambda} \) behaviour
Figure 1. $A_1^p$ as a function of $x$ measured by SMC. Filled circles mark the preliminary new data extending to very low $x$, [2], triangles – that of ref. [1]. Errors are statistical. Systematic ones are marked as shaded bands. The curve corresponds to the calculations based on eqs. (1,2).

of the $g_1$ with $\lambda \sim 0.3$ and $\lambda \sim 1$ for the non-singlet and singlet parts respectively which is more singular at low $x$ than that generated by the (non-perturbative) Regge pole exchanges. The $\ln^2(1/x)$ effects are presumably not important in the $W^2$ range of the fixed target experiments, cf.Fig.2 in [3], but they significantly affect $g_1$ in the low $x$ region which may be probed at the polarized HERA, [3, 4].

The formalism based on the unintegrated distributions is very suitable for extrapolating $g_1$ to the region of low $Q^2$ at fixed $W^2$ [3]. Since $x(1 + k^2/Q^2) \to k^2/W^2$ for $Q^2 \to 0$ in the integrand in eq. (1) and since $k^2 > k_0^2$, the $g_1(x, Q^2)$ defined by eqs. (1,2) can be smoothly extrapolated to $Q^2 = 0$ provided that $\Delta p_i^{(0)}(x)$ are free from kinematical singularities at $x = 0$, as in parametrisations used in refs. [3, 4] where $\Delta p_i^{(0)}(x) = C_i(1 - x)^{n_i}$. If $\Delta p_i^{(0)}(x)$ contain kinematical singularities at $x = 0$ then one
may replace $\Delta p_i^{(0)}(x)$ with $\Delta p_i^{(0)}(\bar{x})$ where $\bar{x} = x \left(1 + k_0^2/Q^2\right)$ and leave remaining parts of the calculation unchanged. However the (extrapolated) partonic contribution to the low $Q^2$ region may not be the only one there; the VMD part may play non-negligible role as well. Predictions based on equations (1,2) shown as a curve in Fig.1 reproduce a general trend in the data. They are systematically smaller than the measurements. There is thus a room for non-partonic contributions to $g_1^p$.

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