Magnetic Energy Conversion in Magnetohydrodynamics: Curvature Relaxation and Perpendicular Expansion of Magnetic Fields

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Abstract

The mechanisms and pathways of magnetic energy conversion are an important subject for many laboratory, space, and astrophysical systems. Here, we present a perspective on magnetic energy conversion in magnetohydrodynamics through magnetic field curvature relaxation (CR) and perpendicular expansion (PE) due to magnetic pressure gradients, and quantify their relative importance in two representative cases, namely 3D magnetic reconnection and 3D kink-driven instability in an astrophysical jet. We find that the CR and PE processes have different temporal and spatial evolutions in these systems. The relative importance of the two processes tends to reverse as the system enters the nonlinear stage from the instability growth stage. Overall, the two processes make comparable contributions to magnetic energy conversion, with the PE process somewhat stronger than the CR process. We further explore how these energy conversion terms can be related to particle energization in these systems.

Unified Astronomy Thesaurus concepts: Magnetohydrodynamics (1964); Relativistic jets (1390); Solar magnetic reconnection (1504)

1. Introduction

The energization and heating of plasma is a universal phenomenon in laboratory, space physics, and astrophysics, such as in the solar corona, solar wind, disks around active galactic nuclei, astrophysical jets, etc. Magnetic energy is frequently found to be the free energy source for energizing plasma in these systems, and thus the conversion of magnetic energy is a key element for understanding these plasma systems (e.g., Parker 1979; Colgate et al. 2001; Kronberg et al. 2001). Observations of many magnetically dominated astrophysical environments and systems have demonstrated fast magnetic energy release and strong particle acceleration. Blazar jets powered by supermassive black holes can exhibit ≈minutes variations in TeV emissions (Aharonian et al. 2007) and gamma-ray flares from the Crab Nebula have been observed as well (Abdo et al. 2011).

For example, magnetic reconnection is commonly considered one of the most important magnetic energy conversion processes. The rapid change in magnetic field connectivity converts the magnetic energy stored in the antiparallel magnetic field into plasma energy (e.g., Sweet 1958; Parker 1957, 1963; Priest & Forbes 2007; Birn et al. 2012), and can lead to efficient particle acceleration and heating (e.g., Drake et al. 2006; Li et al. 2019). In addition, it is well established that magnetic reconnection occurring in global-scale current sheets can contribute to the energy release in solar flares and the Earth’s magnetosphere (Dungey 1961). More recently, Ripperda et al. (2020) demonstrate with general-relativistic resistive magnetohydrodynamic (MHD) simulations that magnetic reconnection can generate plasmoids and explain black hole flares. Another important case is kink instability-enabled magnetic energy conversion. In astrophysical jets, the free magnetic energy can be stored in large-scale helical magnetic fields, and is released due to kink instability (e.g., Li et al. 2006; Nakamura et al. 2007; Mizuno et al. 2009; Zhang et al. 2017). A third case that is often studied is magnetized turbulence, in which the injected magnetic energy can be converted to plasma energy as well. Here, the compression effect has been discussed in the context of single-fluid MHD, and it is especially important for plasma heating (Birn et al. 2012; Du et al. 2018). In particular, it was proposed (e.g., Yang et al. 2016; Matthaeus et al. 2020) that electromagnetic energy interconverts with flow kinetic energy via the $j \cdot E$ term, and the “pressure-strain” interaction converts energy between flow kinetic energy and internal energy. The pressure-strain interaction consists of both a compressive part $−p \nabla \cdot \mathbf{u}$ (pressure dilatation; Aluie et al. 2012) and a “Pi-D” term, which is the product of a traceless pressure tensor and the velocity shear tensor, though its robustness may continue to be under scrutiny (Du et al. 2020).

Rapid progress has also been made in the area of plasma kinetic studies of magnetic reconnection and associated particle energization, along with theoretical developments (e.g., Dahlin et al. 2014; Guo et al. 2014; Sironi & Spitkovsky 2014; Zank et al. 2014; Werner et al. 2016; Li et al. 2015, 2017, 2018, 2019; le Roux et al. 2015, 2018; Lazarian et al. 2020). In particular, Li et al. (2017) constructed an electric current from various particle drift motions and evaluated the particle energization due to drifts in kinetic particle-in-cell (PIC) simulations of magnetic reconnection. While curvature drift is found to be the dominant particle acceleration mechanism, other drifts also make some minor contributions. Of particular importance is the gradient (or grad-B) drift, which usually has a decelerating effect and counteracts the curvature drift acceleration. Li et al. (2018) further show that combining the various drift acceleration terms yields an expression for the energization that can be interpreted as the summation of fluid compression, shear, and inertial energization effects. In
addition to the first-principles kinetic studies, the usage of the test particle approach in reconnection and kink configurations (e.g., Kowal et al. 2012; Medina-Torreyón et al. 2021) has also yielded useful understanding of particle energization processes.

Connecting the microscopic particle motion and acceleration processes with the macroscopic magnetic energy conversion processes remains a long-standing challenge. Beresnyak & Li (2016) discussed the conversion of magnetic energy in MHD and its relation to first-order Fermi particle acceleration. Their analysis suggests that the energy transfer from magnetic field to kinetic motions is inherently related to the curvature drift acceleration. Beresnyak & Li (2016) also discuss the distinction between different types of turbulence. Specifically, energization in decaying magnetic turbulence and magnetic reconnection-driven turbulence are compared against each other using incompressible MHD simulations. While both systems see a conversion of magnetic energy into kinetic energy, the magnetic reconnection case is found to have stronger energization. This is likely due to the larger free magnetic energy that is available in the reconnection case. In terms of the curvature drift, one may argue that the magnetic field possesses “good” curvature that helps the release of magnetic energy and the particle acceleration during reconnection. However, as we will show in this paper, the conclusions made by Beresnyak & Li (2016) are contingent on the incompressible nature of the flow, and need to be revisited for a general compressible plasma.

In this paper, we will present detailed analyses of the magnetic energy conversion processes in two typical configurations, reconnection and kink, using 3D MHD simulations. We show that two dominant processes can be identified in regulating the magnetic energy conversion. We will present the results from the theoretical analysis in Section 2, and the numerical analysis in Section 3. A summary and conclusions are given in Section 4.

2. Energy Conversion in MHD

2.1. Two Processes in Magnetic Energy Conversion

The ideal MHD energy equations, including magnetic energy and plasma energy, are

\[
\frac{\partial}{\partial t}\left(\frac{B^2}{8\pi}\right) + \nabla \cdot \left[ -\frac{(u \times B) \times B}{4\pi} \right] = -j \cdot E;
\]

\[
\frac{\partial}{\partial t}\left(\frac{1}{2} \rho u^2 + \frac{p}{\gamma - 1}\right) + \nabla \cdot \left[ \left(\frac{1}{2} \rho u^2 + \frac{\gamma p}{\gamma - 1}\right) u \right] = j \cdot E.
\]

We use the centimeter-gram-second (CGS) Gaussian unit system throughout the paper. As usual, the quantities in the equations are defined as magnetic field \( B \), electric field \( E \), current density \( j \), plasma mass density \( \rho \), flow velocity \( u \), thermal pressure \( p \), and adiabatic index \( \gamma \). The work done by the electric field \( j \cdot E \) converts the magnetic energy to plasma energy, and thus contributes to the energization of the plasma.

It can be expanded as

\[
j \cdot E = -(u \times B) \cdot \nabla \times B + \frac{1}{4\pi} u \cdot (B \cdot \nabla B) - u \cdot \nabla\left(\frac{B^2}{8\pi}\right).
\]

We further expand the first term on the right-hand side of Equation (3) as

\[
\frac{1}{4\pi} u \cdot (B \cdot \nabla B) = \frac{1}{4\pi} (u \cdot B) (b \cdot \nabla B) + \frac{B^2}{4\pi} u \cdot \kappa,
\]

where the magnetic field curvature is defined as \( \kappa = b \cdot \nabla b \), and \( b = B/B \) is the unit vector tangent to the magnetic field. Combining these terms, we can recast Equation (3) as

\[
j \cdot E = \frac{B^2}{4\pi} u \cdot \kappa - u \cdot \nabla\left(\frac{B^2}{8\pi}\right).
\]

Here, we denote the gradients parallel and perpendicular to the magnetic field as \( \nabla_\parallel = b (b \cdot \nabla) \) and \( \nabla_\perp = \nabla - \nabla_\parallel \). The equation shows that the total magnetic energy transfer in Equation (1) can be decomposed into two parts. Physically, the first term corresponds to the relaxation of high magnetic field line tension—curvature relaxation (CR) when the flow velocity is along the curvature of the magnetic field, which releases magnetic energy. The second term corresponds to the perpendicular expansion (PE) of the magnetic energy, which

![Figure 1. Top panel: time history of energization terms in Equation (5) for the magnetic reconnection simulation, including the curvature-related term \((B^2/4\pi) u \cdot \kappa\), the gradient-related term \(-u \cdot \nabla_\perp(B^2/8\pi)\), and the total energization \(j \cdot E\). All these terms are integrated over the entire simulation box. Bottom panel: evolution of different energy components as reconnection proceeds, including the \(z\)-component of the magnetic field, total magnetic field, total kinetic energy, and internal energy. All the energies are normalized to the initial magnetic energy.](image)
can drive flows in the opposite direction of the magnetic energy gradient, releasing magnetic energy.

A complementary view is expressed via plasma energization in several previous studies (e.g., Birn et al. 2012; Du et al. 2018), where Equation (2) can be rewritten as the plasma flow energy and thermal energy evolution,

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho u^2 \right) + \nabla \cdot \left( \frac{1}{2} \rho u^2 u \right) = j \cdot E - u \cdot \nabla p \label{eq:flow_energy_thermal}
\]

(6)

\[
\frac{\partial p}{\partial t} + 2 \gamma - 1 \nabla \cdot \left( \frac{\gamma p}{\gamma - 1} u \right) = u \cdot \nabla p \label{eq:thermal_energy}
\]

(7)

These equations suggest that the work done by the electric field \( j \cdot E \) contributes strictly to the bulk acceleration in ideal MHD, and that heating is only facilitated by compression \( u \cdot \nabla p \).

### 2.2. The Roles of Fluid Compression and Shear

Another way to express \( j \cdot E \) is by using the perpendicular velocity \( u_\perp \) in place of the total velocity \( u \), since the parallel velocity does not contribute to the electric field. It can be shown that the curvature drift term is related to the flow shear as

\[
\frac{B^2}{4\pi} u \cdot \kappa = - \frac{B^2}{4\pi} bb : \nabla u_\perp, 
\]

and that the total \( j \cdot E \) is decomposed into the sum of the perpendicular expansion and shear,

\[
j \cdot E = -u_\perp \cdot \nabla \left( \frac{B^2}{8\pi} \right) - \frac{B^2}{4\pi} bb : \nabla u_\perp, \label{eq:j_E_decomposition}
\]

(8)

or

\[
j \cdot E = \nabla \cdot \left( \frac{B^2}{8\pi} u_\perp \right) - \frac{B^2}{8\pi} \nabla \cdot u_\perp + \frac{B^2}{4\pi} bb : \sigma, \label{eq:j_E_decomposition2}
\]

(9)

where \( \sigma_{ij} = (1/2)(\partial \mu_{ij} + \partial \mu_{ji} - 2\nabla \cdot u_\perp \delta_{ij}/3) \) is the shear tensor (\( \delta_{ij} \) is the Kroneker delta). Equation (9) is very reminiscent of the results obtained by Li et al. (2018), who show from the Vlasov equation that in the limit of a gyrotropic (aka Chew-Goldberger-Low (CGL)) pressure (Chew et al. 1956; Hunana et al. 2019), the plasma energization in the perpendicular direction \( j_\perp \cdot E_\perp \) can be expressed as the sum of a compression term, a shear term, and an inertial term (see Equation (9) in Li et al. (2018), where \( j_\perp \) is the perpendicular component of the current density with respect to the local magnetic field, and such a current is produced by various particle motions and drifts). Similarly, in our Equation (9), the first term on the right does not contribute to magnetic energy conversion, while the second and third terms correspond to magnetic energy conversion via flow expansion/compression and flow shear, respectively.
The fact that Equations (5) and (9) are mathematically equivalent suggests that both the CR and PE processes are mixed together in the shear term in Equation (9). In this paper, we opt to use Equation (5) as our primary approach for analyzing magnetic conversion processes.

3. MHD Simulations

In this section, we demonstrate how magnetic energy conversion occurs in different types of systems, namely magnetic reconnection and kink-unstable jet. We have performed a set of 3D ideal MHD simulations, using the Athena++ code (Stone et al. 2008, 2020) and the LA-COMPASS code to study these systems. We focus on the temporal and spatial evolutions of the main terms described in Equation (5).

3.1. Magnetic Reconnection

The magnetic reconnection simulation is initialized with force-free current sheets with the following magnetic field configuration,

\[ B_z = B_0 \tanh \left( \frac{d}{\pi L} \sin \left( \frac{\pi z}{d} \right) \right) ; \]

\[ B_y = B_0 \sqrt{1 + \left( \frac{B_x}{B_0} \right)^2 - \tanh^2 \left( \frac{d}{\pi L} \sin \left( \frac{\pi z}{d} \right) \right) } ; \]

\[ B_z = 0. \]

Here, \( B_0 \) is the strength of the upstream reconnecting magnetic field, \( B_y \) is the guide field strength, \( L \) is the half-thickness of the current sheets, and \( d \) is the distance between two adjacent current sheets. We use a box size of \( 2\pi \) in all three directions for our simulation, resolved by \( 512^3 \) cells. The boundary conditions are periodic in all directions. We choose the parameters \( B_0 = 1 \), \( B_y = 0 \), \( L = 0.05 \), and \( d = \pi \) so that there are two current sheets initially. The initial density and temperature profiles are both uniform. The simulation is normalized such that the initial Alfvén speed \( V_A = 1 \) and the
initial plasma $\beta = 0.2$ (the ratio between the thermal pressure and magnetic pressure). An adiabatic equation of state is used with an adiabatic index $\gamma = 5/3$. The characteristic Alfvén time will be $\tau_A = 2\pi$. The current sheets are perturbed initially by random Alfvénic fluctuations that propagate in the $xy$-plane. We introduce 100 random sine waves with transverse velocities and magnetic fluctuations $\delta V_x$ and $\delta B_x$ that are correlated in an Alfvénic fashion ($\delta V_x = \pm \delta B_x$). The total perturbation rms amplitude is $\sim 8\%$, and the cross-helicity is close to zero. The perturbation is restricted to long-wavelength fluctuations, with the wavelength equal to or longer than a third of the box size.

We note that although we use ideal MHD without explicit resistivity and viscosity in the simulation, magnetic reconnection can still occur due to numerical diffusivity at the grid scale. In a high-Lundquist number plasma that is typical of space and astrophysical environments, the energy conversion is dominated by ideal MHD processes, and resistive heating can generally be neglected at large scales. For magnetic reconnection, the nonideal physics is important in the diffusion region where the magnetic field lines break, but the energy conversion process for the entire current sheet system is not sensitive to the nonideal physics. We do caution that, due to the limited spatial resolution, our simulation corresponds to a moderate Lundquist number regime where the plasmoid instability becomes dominant (Loureiro et al. 2007; Bhattacharjee et al. 2009; Huang & Bhattacharjee 2010; Yang et al. 2020).

The top panel of Figure 1 plots the energy conversion terms as in Equation (5). The evolution of the energy in the $z$-component of the magnetic field normalized to the initial magnetic energy is shown in the bottom panel, which also includes the evolution of the total magnetic energy, kinetic energy, and internal energy. Based on our simulation, we find that the evolution may be separated into two stages. During the first stage, the evolution is characterized by a linear instability, which occurs at time $t \lesssim 4$, as indicated by the initial exponential growth of energy in the $B_z$ magnetic field. For the second stage, at a later time, the growth rate slows down, though the magnetic energy is still being released. As illustrated by the top panel of Figure 1, the CR term appears to be dominant in energy conversion at the beginning of the first stage ($t \lesssim 2$). At later times, however, the PE term becomes more important. The overall conversion of the magnetic field is mostly contributed by the PE term at the end of the simulation.

Next, we look into the spatial distribution of the energy conversion terms. Figure 2 displays the spatial distribution of the CR and PE terms and their sum in the $x-z$ plane. Examples from the early ($t = 1.0$) and late ($t = 16.0$) stages are shown in the figure. We find that during the first stage, the strong magnetic energy conversion due to the relaxation of magnetic field curvature occurs near the reconnection $X$-line and the ends of the magnetic islands formed in the two main current sheets. It is interesting to note that there is an approximate spatial anticorrelation between the CR and PE terms, although the CR term is stronger overall. During the second stage, both conversion terms appear to be turbulent, as there are small positive or negative patches mixed in the reconnection and (3D) flux rope regions. The amplitude of the PE term becomes
much stronger and the global summation also indicates its dominance.

To further illustrate the time evolution of the energy conversion terms, the probability density functions (PDFs) of the curvature magnitude $\kappa$ and the cosine of the angle between the velocity and curvature vector $\cos \theta_{\kappa u}$ are shown in Figure 3. The PDFs are calculated by computational cell-based quantities. The velocity distribution does not change much throughout the simulation, and is not shown here. The curvature distribution extends to large curvature values, and develops power law-like distributions in the low- and high-curvature ranges, similar to those reported by Yang et al. (2019) and Bandyopadhyay et al. (2020) in turbulence simulations and observations. The PDFs of $\cos \theta_{\kappa u}$ show that at the early stage, the angle $\theta_{\kappa u}$ concentrates near $0^\circ$ and $180^\circ$ due to the reconnection inflow and outflow. The correlation between the velocity and curvature directions degrades over time, as the distribution becomes more random and tends toward zero. This is probably the reason why the magnetic energy conversion due to curvature relaxation becomes less dominant later in the simulation.

Similarly, we can also inspect the PDFs of the perpendicular gradient $\nabla_\perp (B^2/2)$ and the angle it makes with the velocity $\theta_{\kappa u \perp}$, as shown in Figure 4. The correlation between the velocity and perpendicular gradient vectors also degrades over time. Unlike the curvature distribution, however, the PDF of $|\nabla_\perp (B^2/2)|$ develops a “bump,” starting at high values $\sim 0.5$, which leads to the dominance of the perpendicular expansion process. This bump comes from the turbulent patches, as shown in the lower right panel of Figure 2.
Figure 9. Time history of the energization terms and the energy components similar to Figure 1, but for the kink simulation. The top panel plots the CR term $B^2 \mathbf{u} \cdot \kappa$, the PE term $-\mathbf{u} \cdot \nabla (B^2/2)$, and the total energization. The bottom panel plots the energy in $B_z$ component, total magnetic field, total kinetic energy, and internal energy. The total magnetic energy and the internal energy are plotted in linear scale, and the energy in $B_z$ and total kinetic energy are plotted in log scale.

Figure 5 shows the PDFs of the two conversion terms in Equation (5) at two different time frames. At the earlier time ($t = 1$), the distributions of both terms are clearly skewed, suggesting systematic effects of the positive velocity–curvature correlation and perpendicular fluid compression. In contrast, at the later time ($t = 16$), both PDFs appear to be more symmetric about zero, due to the more turbulent nature of the system. The insets show the cumulative partial moments $PM(x) = \int_{-\infty}^{x} f(x')dx'$, with $f(x')$ being the PDF of $B^2 \mathbf{u} \cdot \kappa$ or $-\mathbf{u} \cdot \nabla (B^2/2)$. Although most of the cells make nearly zero contribution to the energization, as indicated by the sharp peaks in the PDFs, the partial moments illustrate that cells with $-\mathbf{u} \cdot \nabla (B^2/2) > 0.5$ contribute significantly to the total magnetic energy conversion. These analyses strongly suggest that the overall magnetic energy conversion in 3D reconnection is actually dominated by the development of turbulent patches inside the flux ropes undergoing perpendicular expansion owing to magnetic pressure gradients. These patches could be consequences of collisions between reconnection outflows as well as secondary instabilities, as discussed in Huang & Bhattacharjee (2016), Kowal et al. (2017), and Yang et al. (2020).

To further investigate how the CR and PE terms behave under different conditions, we have studied a 3D reconnection set-up with a guide field $B_z = 0.2$ using Equation (10). This is the same set-up as given in a 3D PIC simulation presented by Li et al. (2019). The PIC simulation has a similar initial magnetic field configuration to our MHD simulation, though with only one current sheet, and the box size is $150d_i \times 75d_i \times 62.5d_i$, where $d_i$ is the ion inertial length. Figure 6 shows the evolution of both the CR and PE terms, though interestingly the curvature relaxation term appears to be slightly more important overall. This is different from the case with the guide field $B_z = 0$. We note that the normalization here is different from the MHD simulation. The velocity here is normalized to the speed of light, and the length to the electron inertial length $d_e$. The electron plasma frequency $\omega_{pe}$ is set equal to the electron cyclotron frequency $\Omega_{ce}$, which is defined by the magnetic field strength in the reconnection ($x$–$z$) plane. The ion-to-electron mass ratio is set to 25 so that the Alfvén speed is $V_A \approx d\Omega_{ci} = 0.2c$. We have verified that including the same guide field in the 3D MHD reconnection simulation also enhances the relative importance of the curvature relaxation term. The physical reason may be that a guide field component can suppress the expansion of the magnetic field line perpendicular to the upstream magnetic field. Figure 7 shows the history of the energization terms for two runs with $\beta = 0.5$. The format is the same as Figure 1. The case without the guide field is shown in the left panel, and the case with the guide field $B_z = 0.2B_0$ is shown in the right panel. The general evolutions of these cases are similar to the $\beta = 0.2$ run. Figure 8 plots the fractions of the CR and PE contributions to the total energization (cumulative in time), which illustrate more clearly that the inclusion of a guide field enhances the relative importance of the curvature relaxation term.

### 3.2. Kinked Jets

Another common astrophysical system that may lead to strong magnetic energy conversion is kink instability in astrophysical jets. Here, we present a relativistic MHD simulation of a kink-unstable jet following the setups in Mizuno et al. (2009) and Zhang et al. (2017). The magnetic field has the form of

$$B_z = \frac{B_0}{1 + (r/r_0)^2}; \quad B_0 = \frac{B_0}{1 + (r/r_0)^2} \left( \frac{r}{r_0} \right),$$

where we use $B_0 = 2$ and $r_0 = 1$; $r$ is the radial distance to the central axis. The simulation box size is 40 in the $x$ and $y$ directions, with 640 cells each, and 64 in the $z$ direction, with 1024 cells. Periodic boundaries are used in the $z$ direction, and outflow boundaries in the $x$ and $y$ directions. The system is perturbed by random velocity fluctuations of 0.01 $c$ ($c = 1$ in the simulation). The simulation is run until the time of 350. The initial density $\rho$ and pressure $p$ are uniform ($\rho = 1$ and $p = 0.01$), and the plasma $\beta$ at the central axis is $5 \times 10^{-3}$. The magnetization parameter $\sigma = E_{em}/h$ is about 2 at the central axis, where $E_{em}$ is the electromagnetic energy density and $h = \rho c^2 + \gamma p/(\gamma - 1)$ is the specific enthalpy, with $\gamma = 4/3$ the adiabatic index.

We apply the same analysis as in the reconnection simulation. Similar to Figure 1, we plot the time history of the magnetic energy conversion terms in the top panel of Figure 9. The bottom panel of Figure 9 plots the evolution of the energy in the radial component of the magnetic field developed from the kink instability. The total magnetic energy, kinetic energy, and internal energy are also plotted in the figure. To avoid the effects of the open boundaries, we restrict the calculation to the region $|x| \leq 10$ and $|y| \leq 10$. At the later time $t \geq 200$ (which is five light-crossing times in the transverse direction), the total magnetic energy stops decreasing, while the total magnetic energy conversion $\mathbf{j} \cdot \mathbf{E}$ is positive, which is likely a boundary effect. Nevertheless, similar to the reconnection case, the system undergoes a rapid instability in the...
beginning, as illustrated by the exponential increase in the $E_{Br}$ energy before $t \sim 100$. At the later stage of the evolution, the instability saturates and the system enters a more turbulent state. The top panel of Figure 9 shows that the total energization ($j \cdot E$) is nearly zero in the earlier stage, and maintains a finite positive value at the later stage. It can also be seen that the CR and PE terms are anticorrelated very well with each other, and tend to cancel each other out. This can be understood as both the curvature and gradient vectors of the magnetic field point toward the central axis in a cylindrical kink configuration at zeroth order, so that the two terms in Equation (5) have opposite signs. And it can indeed be verified that $\kappa = \nabla_{\perp} (B^2/2)/B^2$ for the initial kink magnetic field configuration. The finite energization in the later stage is mostly contributed by the CR term, though part of its contribution is canceled out by the PE term. As the kink instability becomes further developed, the $j \cdot E$ term shows that the net magnetic energy conversion becomes appreciable after $t \sim 80$, when the PE term becomes the main contributor in the magnetic energy conversion, though part of its contribution is canceled out by the CR term. At the later, more turbulent stage, the roles of the two terms are reversed, and the net magnetic energy conversion rate is reduced.

4. Summary and Conclusions

In this paper, the conversion of magnetic energy is discussed in the MHD framework. Both theoretical analysis and numerical simulations are presented, illustrating different mechanisms for energy transfer. To summarize, our work yields the following conclusions:

1. In the ideal MHD description, the total energy transfer from magnetic energy to plasma energy $j \cdot E$ can be separated into two dominant processes. Physically, the first part represents the relaxation of magnetic field line tension when the plasma flow velocity is aligned with the magnetic field curvature (the CR term); and the second part represents the situation where the perpendicular magnetic pressure gradients are antialigned with the plasma flow (the PE term).

2. We present several ideal MHD simulations for two types of system: magnetic reconnection and kinked jets. We find that, for both 3D reconnection and kinked jets, the CR and PE terms make comparable contributions to the conversion of magnetic energy to plasma energy, with the PE process playing a more important role overall in magnetic energy conversion.

3. For both 3D reconnection and kinked jets, which have an inherent instability (tearing and kink) to start the magnetic energy conversion process, a quasi-turbulent state is developed as the system evolves further into the nonlinear stage. The relative importance of the CR and PE terms appears to reverse in the turbulent stage, compared to the linear stage. For 3D reconnection, the CR term dominates the conversion of magnetic energy to plasma energy in the beginning, while the PE term becomes more important in the later, more turbulent
stage. For kinked jets, the PE term dominates the magnetic energy conversion at the beginning, but the CR term becomes more important later.

4. In the 3D reconnection situation, the relative importance of the CR and PE terms depends on the presence of a guide field, most likely due to the impact of the guide field on the strength of the perpendicular expansion owing to magnetic pressure gradients. For a finite guide field \( B_g = 0.2 \), we find that the CR term is overall the more important term.

These findings are potentially important for understanding particle energization in 3D, over many nonlinear times. The role and importance of the PE term might have been previously underappreciated, particularly given its overall dominance in producing substantially more magnetic energy conversion than the CR term. It will be interesting to explore in greater depth its consequences on particle energization and transport processes in 3D magnetically dominated systems.

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**Appendix**

**Relating Magnetic Energy Conversion to Plasma Energization**

The relationship between the CR+PE processes as described in Equation (5) and the plasma energization processes remains relatively unexplored. It is tempting to seek any correspondence between the magnetic energy conversion terms expressed in Equations (5) and (9) and the particle energization processes, particularly in the context of recent kinetic studies (e.g., Li et al. 2017, 2018). We now discuss some possible connections.

**A.1. Single Particle Drifts and Energization**

Beresnyak & Li (2016) have suggested that a particle’s curvature drift acceleration could be related to the CR term discussed in Equation (5). The curvature drift velocity of a single particle is

\[
\mathbf{v}_c = \frac{eE\mathbf{B}}{qB} (\mathbf{b} \times \kappa),
\]

where \( E_p = m\mathbf{v}_p^2/2 \) is the particle parallel energy. Using the convective electric field, particle acceleration by curvature drift can be obtained by calculating the work done by the electric field on the particle’s curvature drift velocity,

\[
\frac{dE}{dt} \bigg|_{\mathbf{v}_c} = qE \cdot \mathbf{v}_c = 2E\mathbf{u} \cdot \kappa.
\]

This process was emphasized in Beresnyak & Li (2016). Similarly, using the grad-B drift velocity,

\[
\mathbf{v}_g = \frac{E_g B}{qB} \frac{\mathbf{B} \times \nabla \mathbf{B}}{B^2},
\]

we find the particle acceleration by grad-B drift as,

\[
\frac{dE}{dt} \bigg|_{\mathbf{v}_g} = qE \cdot \mathbf{v}_g = \frac{E_g}{B^2} \mathbf{u} \cdot \nabla \left( \frac{B^2}{2} \right).
\]

This grad-B drift acceleration term was not included in Beresnyak & Li (2016), as they primarily focused on the incompressible MHD limit where the curvature drift term dominates. In a general compressible system, the conclusions of Beresnyak & Li (2016) need to be revisited and expanded.

Equations (A2) and (A4) contain the terms \( \mathbf{u} \cdot \kappa \) and \( \mathbf{u} \cdot \nabla \left( \frac{B^2}{2} \right) \), respectively, which are included in the two magnetic energy conversion processes of CR and PE as shown in Equation (5). We have placed these expressions side-by-side for comparison,

\[
\mathbf{j} \cdot \mathbf{E} = \frac{B^2}{4\pi} \mathbf{u} \cdot \kappa - \mathbf{u} \cdot \nabla \left( \frac{B^2}{8\pi} \right).
\]

magnetic energy conversion rate in MHD (complete)

\[
\frac{dE}{dt} \sim 2E\mathbf{u} \cdot \kappa + \frac{4\pi E_g}{B^2} \mathbf{u} \cdot \nabla \left( \frac{B^2}{8\pi} \right),
\]

single particle energy change rate(partial)

(A5)

One has to be very cautious in drawing any conclusions from this seeming similarity. For example, from a single particle perspective, the particle energy gain associated with curvature drift \( (dE/dtk > 0) \) has the same sign as the CR process. However, the particle energy gain associated with gradient drift \( (dE/dtk > 0) \) has the opposite sign to the PE process with a decrease in energy. Therefore, high-energy particles may gain a significant amount of energy via a Fermi-like mechanism upon encountering a region with \( \mathbf{u} \cdot \kappa > 0 \) or \( \mathbf{u} \cdot \nabla \mathbf{B} > 0 \). This is different from the conversion of magnetic energy in Equation (5), which does not depend on particle energy.

**A.2. Particle Drift Currents and Energization**

Using the kinetic studies of magnetic reconnection, Li et al. (2018) showed from the Vlasov equation that, in the limit when particles are sufficiently magnetized, the particle energization due to the perpendicular electric field can be expressed as the sum of curvature drift, gradient drift, perpendicular magnetization, and bulk acceleration (the inertial term) in the limit of gyrotropic pressure. The curvature and gradient drift terms are found to be dominant over the others (Li et al. 2017, 2018). Specifically, the energization of particles due to drifts can also be calculated as

\[
\mathbf{j} \cdot \mathbf{E} = p\epsilon \mathbf{b} \times \kappa \frac{\mathbf{b} \times \kappa}{B} \cdot \mathbf{E} = p || \mathbf{u} \cdot \kappa; \quad \mathbf{E} = p || \mathbf{u} \cdot \kappa.
\]

(A6)
The two terms are plotted as the green curve $\mathbf{j}_c \cdot \mathbf{E}$ and the red dashed line for comparison.

$$\mathbf{j}_c \cdot \mathbf{E} = \left( \frac{p_\parallel c B}{B^3} \times \nabla B \right) \cdot \mathbf{E} = \frac{p_\parallel}{B^2} \mathbf{u} \cdot \nabla \left( \frac{B^2}{2} \right), \quad (A7)$$

where $p_\parallel$ and $p_\perp$ represent parallel and perpendicular pressure. Fully kinetic PIC simulations have shown that curvature drift acceleration is found to be the dominant particle acceleration mechanism, while grad-B drift typically has the effect of decelerating particles.

By assuming an isotropic pressure, we could use our 3D MHD reconnection simulation described in Section 3.1 to study the evolution of Equations (A6) and (A7). Figure 11 shows the plasma energy change rates due to curvature and gradient drift currents using Equations (A6) and (A7), assuming an isotropic pressure. The figure shows clearly that the plasma energy gain via curvature drift $\mathbf{j}_c \cdot \mathbf{E}$ remains positive throughout the simulation, while the plasma energy change via the grad-B drift process $\mathbf{j}_g \cdot \mathbf{E}$ is negative for most of the time. The results here are qualitatively similar to those from PIC simulations (Li et al. 2017), indicating that particle acceleration is perhaps more sensitive to the global field structure rather than the detailed kinetic physics near the reconnection X-line. We have also plotted the total $\mathbf{j} \cdot \mathbf{E}$ (red dashed line) as given in Equation (5). Note that it exhibits a difference from the sum of Equations (A6) and (A7) (green curve), indicating that these two processes do not capture the total magnetic energy conversion.

It should be emphasized that the two terms shown in Equations (A6) and (A7) should not be equated to the CR and PE terms in Equation (5). For example, the particle gain via the curvature drift current is proportional to the particle (parallel) pressure, whereas the CR magnetic energy conversion term is completely independent of the particle pressure. Intuitively, however, the two descriptions appear to be linked. For a plasma that is approximately in pressure balance, the particle pressure is anticorrelated with the magnetic pressure, so that particles tend to gain a large amount of energy in the high-curvature region where the particle pressure also tends to be high.

As for the grad-B drift term and the PE process, one needs to keep in mind that there is no direct correlation or correspondence between the particle energy change via grad-B drifts and the magnetic energy change. For example, a decrease in the bulk kinetic flow energy could cause a positive $\mathbf{u} \cdot \nabla \cdot (B^2/2)$, so that both the particle energy gain via grad-B and the magnetic energy increase can occur.

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