Long distance electromagnetic contribution to
$B^{\pm} \to (K^{\pm}, \pi^{\pm})\ell^{+}\ell^{-}$ rare processes

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Abstract. We evaluate a long distance weak annihilation contribution to the decay rates of the $B^{\pm} \to (K^{\pm}, \pi^{\pm})\ell^{+}\ell^{-}$ rare processes. Despite a visible kinematic leptonic universality violation is obtained in the low $q^2$ region, its effect almost disappears in the region of interest for the $R_K$ anomaly. We show that sizeable CP asymmetries are obtained when short and long distance prescriptions are taken into account in different energy intervals.

1. INTRODUCTION

Indirect evidence of new physics (NP) effects can be extracted by studying the most suppressed (rare) processes within the standard theory. The ones involving flavor changing neutral currents (FCNC) have triggered a lot of interest since some deviations from the standard model (SM) predictions have been observed experimentally. Short time ago, the LHCb collaboration reported the measurement of the ratio of muon to electron pairs produced in $B^{\pm} \to K^{\pm}$ decays, $R_K \equiv \frac{B(B^- \to K^-\mu^+\mu^-)}{B(B^- \to K^-e^+e^-)} = 0.745^{+0.090}_{-0.074} \pm 0.036$ [1], observing a deficit with respect to the SM value $R_K = 1.0003 \pm 0.0001$ [2, 3] in the (1,6) GeV$^2$ region for the squared invariant-mass of the lepton pair. Many models have been proposed in the literature to solve the anomaly [4], some of them involving non-universal leptonic interactions. In this work, we calculate a long-distance (LD) weak annihilation (WA) contribution to the $B^{\pm} \to (K^{\pm}, \pi^{\pm})\ell^{+}\ell^{-}$ decays, and study its effect on the branching ratios and the CP asymmetry for each of these rare processes.

2. SHORT DISTANCE APPROACH

In the SM, the $B^- (p_B) \to P^- (p_P)\ell^+(p_+)\ell^-(p_-)$ decay (with $P = \pi$ or $K$) involves contributions of penguin (Figure 1a), as well as $W$ boson box (Figure 1b) diagrams, which are dominated by loops involving the top quark. This short distance picture (SD) is accurately described by the effective amplitude

$$M_{QCDf} = \frac{G_F\alpha}{\sqrt{2}\pi} V_{tb} V^{\dagger}_{td} \xi P(q^2)p_B^\mu \left[ F_V(q^2)\ell\gamma_\mu\ell + F_A(q^2)\ell\gamma_\mu\gamma_5\ell \right],$$

where $q = p_+ + p_-$ is the momentum of the lepton pair, $D$ stands for $d(s)$ in the case of $|\Delta S| = 0(1)$ transitions. The factors $F_V \approx C_9 = 4.214$ and $F_A = C_{10} = -4.312$, correspond to the vector and axial Wilson coefficients at NNLO [5], obtained from the weak...
effective Hamiltonian for the $b \to q \ell^+ \ell^-$ transition. The short-distance WA contributions induced by four quark operators can be absorbed into the effective $F_V$ coefficient by replacing $\xi_P(q^2)F_V \to \xi_P(q^2)F_V + F^{WA}$. $\xi_P(q^2)$ stands for the weak form factor of the specific process, where we will use the expressions obtained from light-cone sum rules (LCSR) [2, 6] regarding the $P_P(q^2)$ Gegenbauer moments as

$$\xi_\pi(q^2) = \frac{0.918}{1 - q^2/(5.32 \text{ GeV})^2} - \frac{0.675}{1 - q^2/(6.18 \text{ GeV})^2} + \mathcal{P}_\pi(q^2),$$

$$\xi_K(q^2) = \frac{0.0541}{1 - q^2/(5.41 \text{ GeV})^2} + \frac{0.2166}{[1 - q^2/(5.41 \text{ GeV})^2]} + \mathcal{P}_K(q^2).$$

The SD WA diagrams have been studied in the QCD factorization (QCDf) framework [7], where the virtual photon exchange between hadrons and the dilepton pair and hard gluon scattering have been analysed [5, 8]. Along this paper we will refer to Eq. (1) indistinctly as the SD or QCDf contribution.

3. LONG DISTANCE WA CONTRIBUTION

The long distance WA diagrams that contribute to $B^-(p_B) \to P^-(p_P)$ transitions are depicted in (Figure 1c) and (Figure 1d). The decay amplitude for these diagrams can be written as

$$\mathcal{M}_{LD,WA} = \sqrt{2}G_F(4\pi\alpha)V_{ub}V^*_{ud}f_Bf_P \frac{1}{q^2(m_B^2 - m_P^2)} \times \left[m_B^2 \left(F_P(q^2) - 1\right) - m_P^2 \left(F_B(q^2) - 1\right)\right] p_B^\mu \tilde{\ell} \gamma_\mu \ell,$$

where $F(q^2)$ denotes the electromagnetic form factor (FF) of the corresponding meson and $f_M$ the meson decay constant. At first sight, it can be seen that the $D = s$ mode is CKM-suppressed compared to $D = d$, and that the second term between squared brackets, corresponding to photon emission from the $B$ meson, is suppressed by the ratio of masses of the light meson to the heavy. In addition, we have found that the FF contribution of the $B$ meson is suppressed.
itself due to the reduced charged radius of the heavy meson, such that its final contribution is negligible. We have analysed other kinds of structure dependent contributions, but these vanish because of the gauge invariance and/or are suppressed by $1/N_c$ [9]. The vector nature of the LD amplitude in Eq. (2) makes plausible its incorporation in a re-definition of the coefficients coming from operator $O_9$, namely

$$\xi_P(q^2) F_V \rightarrow \xi_P(q^2) F_V + \kappa_P m_B^2 \left[ \frac{F_P(q^2) - 1}{q^2} \right],$$

(3)

where

$$\kappa_P = -8\pi^2 V_{ub} V_{ud}^* f_B f_P \frac{m_B^2 - m_P^2}{m_B^2 - m_P^2}.$$ 

(4)

To complete our analysis [10], we will use the electromagnetic form factors (FF) of the light pseudoscalar mesons [11] obtained from Resonance Chiral Theory (RχT) [12], which are saturated by the lowest-lying light flavor resonances, $\rho$ and $\omega$ for $P = \pi$, and the extremely narrow $\phi(1020)$ meson for $P = K$ [13]. For the sake of completeness, we will also use the fit to the experimental data provided by the BaBar collaboration to estimate the uncertainties owing to changes in the FF [13]. In order to avoid double counting of WA contributions, we will consider in the interval from threshold to 2 GeV$^2$ the RχT approach since we expect that chiral Lagrangians give an adequate description for low $q^2$ values, while QCDf will be more appropriate for describing the higher $q^2$ region. The matching of $C_{9f}^{eff}$ obtained with both descriptions at this intermediate energy scale is considered in detail in Ref. [10].

4. Results
The comparison between both approaches used for the FF can be found in Fig. 2, where good agreement was found in the low lepton invariant mass region, with minor differences above 1.5 GeV; these differences are taken into account in the final quoted errors. Figure 3 shows the spectra of the lepton pair, normalized to the $B^-$ meson decay width, for low values of the lepton-pair energies. A marked violation of lepton universality is observed in this limited region of the $q^2$ range owing to kinematical effects.

In Table 1 we show separately the contribution of QCDf and LD prescriptions for the WA diagrams for different ranges of $q^2$. We have restricted our analysis to regions below 8 GeV$^2$ (above this value the charm effects are expected to dominate). We can observe that the LD term contributes at the same level of the SD part when integrated branching ratios are computed from
Figure 3. Normalized spectrum of the lepton-pair for the LD WA contribution, $P = \pi$ (left) and $P = K$ (right), in the region going from threshold up to 1.5 GeV$^2$.

the threshold, $q^2 \sim 0.05$ GeV$^2$. In the region above 1 GeV$^2$ the QCDf contribution dominates, these confirms the feasibility of NP physics searches in the $1 \leq q^2 \leq 6$ GeV$^2$ interval in the kaon mode, and $1 \leq q^2 \leq 8$ GeV$^2$ for the pion mode, where the hadronic pollution can be well under control.

It is possible to generate a direct CP asymmetry because the LD and SD WA contributions to the amplitude of the $B^\pm \rightarrow P^\pm \ell^+ \ell^-$ decays have different weak and strong phases. In this way, a (partially integrated over a finite $q^2$ range) CP asymmetry,

$$A_{CP}(P) = \frac{\Gamma(B^+ \rightarrow P^+ \ell^+ \ell^-) - \Gamma(B^- \rightarrow P^- \ell^+ \ell^-)}{\Gamma(B^+ \rightarrow P^+ \ell^+ \ell^-) + \Gamma(B^- \rightarrow P^- \ell^+ \ell^-)}, \quad (5)$$

is obtained from the interference of the diagrams shown in Figure 1. By inserting the amplitudes of LD WA and QCDf contributions in the above expression, and considering the matching of the LD and QCDf descriptions of the WA contributions described in the previous section, Eq. (5) leads to:

$$A_{CP}(P) = \begin{cases} 
(16.1 \pm 1.9)\%, & \text{for } P = \pi, \ 0.05 \leq q^2 \leq 8 \text{ GeV}^2, \\
(7.8 \pm 2.9)\%, & \text{for } P = \pi, \ 1 \leq q^2 \leq 8 \text{ GeV}^2, \\
(-1.0 \pm 0.3)\%, & \text{for } P = K, \ 1 \leq q^2 \leq 6 \text{ GeV}^2.
\end{cases} \quad (6)$$

In the $\pi$ meson case, we have found that the CP asymmetry for the first range is widely dominated by the LD WA contribution, while within the second range is dominated by QCDf effects. For kaons, the CP asymmetry is basically obtained from LD WA contributions (which are almost 70% within this range). The uncertainties in our results stem from the systematic error owing to parametrizations of the light meson electromagnetic form factors. For $P = \pi$, the CP asymmetries have been computed in [14, 15] considering the QCDf approach. These results and ours agree within the experimental error with the recent determination made by the LHCb collaboration for the integrated CP asymmetry, namely $A_{CP} = 0.11 \pm 0.12 \pm 0.01$ [16].

5. Concluding Remarks
We have calculated the effects of the WA diagrams in the LD regime to the $B^{\pm} \rightarrow (K^{\pm}, \pi^{\pm}) \ell^+ \ell^-$ rare processes and shown their effect over the CP asymmetries. Despite a visible kinematic leptonic universality violation is obtained in the low $q^2$ region, the effect of LD WA contributions almost disappears in the region of interest for the $R_K$ anomaly. This confirms the feasibility of the $1 \leq q^2 \leq 6$ GeV$^2$ range for NP searches in the $P = K$ case, and suggests to take
Table 1. Branching ratios of $B^- \to P^- \ell^+ \ell^-$ decays for $P = \pi$ (left hand side) and $P = K$ (right hand side) integrated in different $q^2$ ranges.

| $B^- \to \pi^- \ell^+ \ell^-$ | $B^- \to K^- \ell^+ \ell^-$ |
|-----------------------------|-----------------------------|
| 0.05 \leq q^2 \leq 8 \text{ GeV}^2 | 1 \leq q^2 \leq 8 \text{ GeV}^2 | 1 \leq q^2 \leq 6 \text{ GeV}^2 |
| LD | (9.06 \pm 0.15) \cdot 10^{-9} | (4.74 \pm 0.05) \cdot 10^{-10} | (1.70 \pm 0.21) \cdot 10^{-9} |
| interf. | (-2.57 \pm 0.13) \cdot 10^{-9} | (-2.8^{+2}_{-1.1}) \cdot 10^{-10} | (-6 \pm 2) \cdot 10^{-11} |
| QCDf | (9.57^{+1.45}_{-1.05}) \cdot 10^{-9} | (8.43^{+1.31}_{-1.41}) \cdot 10^{-9} | (1.90^{+0.69}_{-0.41}) \times 10^{-7} |
| Total | (1.61^{+0.07}_{-0.11}) \cdot 10^{-8} | (8.69^{+0.41}_{-0.87}) \cdot 10^{-9} | (1.92^{+0.41}_{-0.41}) \times 10^{-7} |

$1 \leq q^2 \leq 8 \text{ GeV}^2$ for NP searches when $P = \pi$. Similarly, we have obtained that measurable CP asymmetries are generated. Our results for the CP asymmetry in the $P = \pi$ case agree with the recent experimental measurements.

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