Comparison of the focused optical vortices produced by high-aperture phase conventional and spiral zone plates

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Abstract. Using the FDTD simulation, sharp focusing of a linearly polarized Gaussian beam with an embedded topological charge \( m = 3 \) by a phase zone plate and focusing of a Gaussian beam by a phase spiral zone plate with topological charge \( m = 3 \) were studied. The obtained results showed that proposed elements formed different patterns of intensity at a focal plane. The spiral zone plate forms a focal spot with three petals. At a distance of 13.5 \( \mu \text{m} \) from the focus, the lobe structure of the intensity (and energy flux) is replaced by an annular distribution.

1. Introduction

Vortex laser beams are often used in different areas, for example, in optical communications [1, 2], in nanomanipulation [3, 4], in quantum computing [5], cryptography [6], and imaging systems [7]. For the formation of optical vortices (OV), various optical elements can be used, such as axicons [8], holograms [9], liquid crystals [10], phase helical plates (PHP) [11], spatial light modulators [12], etc. However, these methods are quite resource-intensive. Another way for implementing a topological charge (TC) is binary spiral zone plates (SZP), which can be phase or amplitude [13, 14]. Both of them can form vortex fields with specified characteristics. At the same time, their manufacture is a simpler task than, for example, fabrication of phase spiral plates [15].

This paper presents a study of optical beams transformation when they are focused by a conventional zone plate (ZP) and SZP made of silica glass. The study aims to assess the possibility of using SZP for the simultaneous formation and focusing of an optical vortex.

2. Simulation of ZP

Firstly, we consider the binary phase ZP with 13 rings. Formula (1) was used for the calculation of binary relief [16]:

\[
\rho_m = \sqrt{m \lambda f + m^2 \lambda^2 / 4}
\]

where \( \rho_m \) is the radius of the zone with number \( m \), \( \lambda \) is the incident wavelength, \( f \) is the focal length. The parameter values were as follows: \( \lambda = 0.532 \ \mu\text{m}, f = \lambda, M = 27 \) (total number of zones which gives 13 rings).
The formula (1) allows calculating the radii of the binary relief zones. Thus, the width of the corresponding ring of the binary ZP can be calculated by subtracting adjacent zone radii. The radii of ZP’s zones are presented in Table 1. The height of the ZP’s relief is equal to \( \lambda = 0.532 \mu m \).

**Table 1. Radii of ZP’s zones.**

| m   | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| r_m, \( \mu m \) | 0.595 | 0.921 | 1.219 | 1.505 | 1.784 | 2.060 | 2.334 | 2.606 | 2.877 |
| m   | 10  | 11  | 12  | 13  | 14  | 15  | 16  | 17  | 18  |
| r_m, \( \mu m \) | 3.147 | 3.417 | 3.686 | 3.954 | 4.223 | 4.491 | 4.758 | 5.026 | 5.293 |
| m   | 19  | 20  | 21  | 22  | 23  | 24  | 25  | 26  | 27  |
| r_m, \( \mu m \) | 5.561 | 5.828 | 6.095 | 6.362 | 6.629 | 6.896 | 7.162 | 7.429 | 7.696 |

The linearly polarized Gaussian vortex beam with a wavelength of \( \lambda = 0.532 \mu m \), waist of \( \omega = 8 \mu m \), and topological charge \( m = 3 \) was proposed as incident light for ZP:

\[
\begin{align*}
E_r(r, \theta) &= \exp\left(-\frac{r^2}{\omega^2}\right) \exp(im\theta); \\
E_r(r, \theta) &= 0.
\end{align*}
\]

The FDTD-method implemented in the FullWAVE software package was used for modelling. The following parameters of the grid were used: step along transverse coordinates was 15 nm, a step along longitudinal coordinate was 7 nm, the pseudo-time step \( c\Delta t \) (\( c \) is a speed of light in the vacuum, \( \Delta t \) is a time step) was 5 nm. The averaging of the calculated fields for 10 periods was carried out while analysis of simulation results.

Simulation results at the focal plane are presented below in Fig. 1-3. Fig. 1a shows the intensity distribution in the focal plane at a distance of 0.532 \( \mu m \) from the relief. Fig. 1b–c additionally shows cross-sections of the intensity and of the longitudinal component of the Pointing vector.

**Figure 1.** 3D intensity distribution in the focal plane on the distance of 0.532 \( \mu m \) from the relief of ZP (a), the cross-section of the intensity (b) and the longitudinal component of the Pointing vector \( S_z \) (c) in the focal plane along X (dotted black line) and Y (green line) for ZP.

For more detail, we present distributions of the electric field vector components in Fig. 2 and distributions of the Poynting vector \( S \) components in Fig. 3.

**Figure 2.** 3D distributions of the electric field vector components in the focal plane at a distance of 0.532 \( \mu m \) from the relief of ZP.
It can be seen from Fig. 1 that the focal spot has form of a ring. However, two local intensity maxima appear on the vertical axis at the focus of the ZP. Fig. 2 shows that after focusing formed field has all components of the electric field [17]. Fig. 3a-b show presence of the transverse components of the Poynting vector. They determine the longitudinal component of the orbital angular momentum (OAM) vector and show the magnitude and direction of the energy flux in the transverse plane. A microparticle captured in such a light beam will rotate either along a circular path or around its center of mass. Fig. 3c presents that the axial energy flux at the focus for the ZP has the form of a ring.

It is possible to draw conclusions from the simulation results. The energy flux in the focus of a high-aperture ZP illuminated by an optical vortex with linear polarization and topological charge $m = 3$ has the form of a ring (Fig 3c) as in the case of sharp focusing of an optical vortex with linear polarization by an ideal spherical lens [18].

3. Simulation of SZP
In this section, we consider the binary phase SZP. The transmittance function of SZP can be calculated by equation (3):

$$T(r, \theta) = \exp\left[im\theta + ik\left(\sqrt{f^2 + r^2} - f\right)\right]$$

(3)

where $r$ and $\theta$ are polar coordinates, $k$ is wave number, $f$ is focal length. The parameter values were as follows: $m = 3$, $\lambda = 0.532 \mu m$, $f = \lambda$, $r \in [-4, 4]$.

The linearly polarized Gaussian beam with a wavelength of $\lambda = 0.532 \mu m$ and waist of $\omega = 4 \mu m$ is proposed as incident light for SZP. The spatial distribution of the incident field with the necessary polarization was calculated in MATLAB by formula (2) for $m = 0$. As in a previous section, simulation results at the focal plane are presented in Fig. 4-6.

Figure 4. 3D intensity distribution in the focal plane on the distance of 0.532 $\mu m$ from the relief of SZP (a), the cross-section of the intensity (b) and the longitudinal component of the Pointing vector $S_z$ (c) in the focal plane along X (dotted black line) and Y (green line) for SZP.
Figure 5. 3D distributions of the electric field vector components in the focal plane at a distance of 0.532 μm from the relief of SZP.

Figure 6. 3D distributions of the Poynting vector components in a focal plane at a distance of 0.532 μm from the relief of SZP.

It can be seen from Fig. 4a and Fig. 6c that both the intensity distribution and the axial energy flux at the focus of the SZP have three local maxima located at the corners of the regular triangle. It should be noted that the shape of the electric strength vector components of the formed by the spiral and conventional zone plates fields is practically the same.

It is possible to draw conclusions from the simulation results. The SZP "behaves" differently from ZP while focusing a Gaussian beam with linear polarization. Due to the three relief “arms” emerge from the centre of the SZP the energy flux in a sharp focus also has three "petals" (Fig. 6c). In the focus of the SZP, the intensity distribution also acquires a multi-lobed structure.

4. Simulation in far-field

Since different distributions in the near-field were obtained for ZP and SZP, the fields in the far zone were calculated for the SZP. Fig. 7 shows diffraction pattern in the XZ and YZ planes. Fig. 8-10 show intensity distribution at different distance from the SZP's relief. The distribution of the electric field vector components and the Poynting vector components for each intensity cut is also presented right after their snapshots (Fig. 8-10).

Figure 7. 3D intensity distribution in the XZ (a) and YZ (b) planes.

Figure 8. 3D intensity distribution (a), 3D distributions of the electric field vector components(b-d), 3D distributions of the Poynting vector components (e-g) at the distance of 5 μm from the relief of SZP in the XY plane.
Figure 9. 3D intensity distribution (a), 3D distributions of the electric field vector components (b-d), 3D distributions of the Poynting vector components (e-g) at the distance of 14 µm from the relief of SZP in the XY plane.

Figure 10. 3D intensity distribution (a), 3D distributions of the electric field vector components (b-d), 3D distributions of the Poynting vector components (e-g) at the distance of 23 µm from the relief of SZP in the XY plane.

From Fig. 8-10 we can see that the multi-lobe structure of the intensity and energy flux, is not preserved, and at a certain distance from the SZP, it disappears. The three-lobe intensity pattern is retained from the focus (z = 0.5 µm) to a distance of z = 5 µm (Fig. 8). Moreover, it can be seen from a comparison of the intensity at the focus in Fig. 4a (or the flux in Fig. 6c) and the intensity at a distance of z = 5 µm (Fig. 8a), that the intensity pattern rotates. While further propagation of light from the SZP the contribution to the light field will be made not by the central “arms”, but by the periodic rings of the SZP, which, like the spiral axicon [19], should form a ring. A such an intensity ring is formed at a distance of z = 14 µm (Fig. 9). Since some part of the Gaussian beam passed outside the SZP aperture, being refracted at the SZP edges, the Gaussian beam formed a focus (light spot) at a distance of z = 23 µm (Fig. 10). Note that the intensity distributions in Fig. 8a, 9a, 10a are almost the same as the distribution of the axial energy flux at these distances (Fig. 8g, 9g, 10g). This confirms the well-known identity between intensity and energy flux for paraxial light fields [20].

5. Conclusion
Using the FDTD simulation, vortices focusing by ZP and SZP is investigated. The linearly polarized Gaussian beam with an embedded optical vortex was chosen as incident light for ZP, while SZP was illuminated by an ordinary Gaussian beam with the same polarization. The field formed in the focus by ZP and SZP were compared. The intensity distribution at the focus of a high-aperture ZP illuminated by an optical vortex with linear polarization and topological charge \( m = 3 \) has the form of two local maxima lying on the vertical axis (the direction of the initial polarization along the horizontal axis). Thus, when focusing, the binary ZP "behaves" almost like an ideal spherical lens. The SZP "behaves" differently when focusing a Gaussian beam with linear polarization. Due to the multi-beam structure of the SZP near the center, the energy flux and intensity distribution in a sharp focus have three "petals". However,
at a distance of 13.5 µm from the focus, the lobe structure of the intensity (and energy flux) is replaced by an annular distribution.

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