Contextual problem in mathematical problem solving: core ability in Realistic Mathematics Education

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Abstract. Problem solving is one of the ability existed in the component of Realistic Mathematics Education. This ability becomes more important for a student to understand, analyse and formulate the daily-life problem into formal mathematics, such as real-life problem, problem-based specific context, and the problem itself. Even though problem solving is an important component that must be mastered by the student in learning mathematics, not every student has a certain skill to understand and solve the problems occurred in daily life. Many students do neither master the basic mathematics concepts nor know the specific criteria in problem solving ability. Regarding the fulfilment, this paper aims to investigate how to optimize problem solving ability using Realistic Mathematics Education as a part of student's learning. The method used in this article is literature review in which the data collected through a study of several theories and the results of relevant scientific research such as mathematics learning, problem solving concept, Realistic Mathematics Education theory and the others. The result shows that the ability of problem solving and Realistic Mathematics Education interconnected each other, which is Realistic Mathematics Education has the important role in optimizing problem solving, especially in solving contextual problem. According to the result, this article contribute to improving teachers and students mathematics proficiency and deepening the learning theories especially the theories of mathematics problem solving to be more skilful and well-experienced in learning mathematics.

1. Introduction
Problem solving is the ability that must be developed by the children since elementary school [1], hence, it becomes the main focus for learning mathematics in the school [2]. Also, the ability to solve mathematics problems in the various contexts especially in daily life is also seen as the main goal in mathematics education [3,4] as well as how students can identify the relationship between mathematics and real-world problems that can be applied in daily life [5]. As stated by programme for International Student Assessment (PISA), the assessment of mathematics student achievement focussed on the ability to solve real-world problems [6]. Such an ability is the need for improving mathematical knowledge and how mathematics can be used to assist the working and daily life [5] in the current and future world.

Solving a mathematics problem is the process to solve mathematics contextually that can increase mathematics skills and the knowledge of mathematics concepts. It involves the student's competency on how to solve and to recognise the mathematics problem [7]. Also, It helps the student to improve
their critical and creative thinking skills [8,9]. Problem solving skill, moreover, becomes the important student’s ability for learning mathematics. However, the students tend to be struggling to solve mathematics problems [10] since mathematics is a difficult subject to understand. They often encounter a few problems related to the well-structured problem which is inconsistent with the problem genuinely [11], however, they need to solve it and associate with the real-world problems. Therefore, it is undeniable that there must be a further literature review to discuss the ability of problem solving knowledge which is later expected to improve on how to think realistically and reasonably, particularly in learning mathematics.

Mathematics is inherent in human activities in everyday life [12] in which problem solving skill is the highest specific achievement of human intelligence that affects human's activity [13]. In the PISA framework assessment, solving a problem means being able to understand, identify, solve, evaluate and justify the problems [6]. In this case, students are required to master a variety of basic abilities such as the ability to read, understand and analyze a problem. Moreover, the sensitivity of the students into the problems that occur in the surrounding environment is needed to hone the ability to solve the mathematical problems. PISA also emphasizes the importance of student capabilities in explaining real-world situations [6], moreover, mathematical problems are closely related to the context of problems that exist in everyday life. As a result, this problem-solving ability encourages the students to formulate and identify non-routine problems that then students have the opportunity to find routine solutions so that students can later develop new strategies and solutions.

Mathematical problems do not always have to use existing problems. In this case, students must grope and formulate their context that they want to solve. While the context has been provided clearly, it is usually more targeting students to solve these problems using certain methods. Therefore, the problem solving process can be linked to realistic and realistic learning, that is, learning that puts forward the power of thinking and reasoning of students to solve a problem and learning that utilizes the conditions that are around students. Realistic Mathematics Education, later we called it RME, is a process of learning mathematics that has the characteristics of realistic learning, namely learning that utilizes situations in the real world [14]. As the optimization of student knowledge obtained from the objects of the surrounding environment gives rise to real mathematics learning for students [15].

In addition, RME is an approach in which mathematics education is understood as human activity [16,17,18] which deals with reality, is close to the world of children, and has relevance to values humanity in society [19]. In RME, students must learn mathematics by developing and applying mathematical concepts and tools in problem situations that exist in everyday life which also make sense to them [19]. In this case, the problem does not have to be a real problem, however, a problem is the context that can be understood and imagined by students in the form of student fantasy, fairy tales, and formal mathematics [20].

In the mathematics curriculum, problem solving skills become the main goal of learning mathematics which is closely related to the capabilities of students in solving contextual problems that are around them. Whereas RME is a learning approach that focuses on realistic learning which can help students to optimize mathematics learning by using a realistic learning approach, namely mathematics learning using real-world contexts. Thus, problem solving and RME have a relationship in learning mathematics. There has been a lot of research and literature review on problem solving and RME which both have positive impacts related to mathematics learning. However, it is still rare to study literature on enhancing mathematical problem solving using RME as a learning approach. Furthermore, it is a need to conduct an in-depth literature study on optimizing the ability to solve mathematical problems using Realistic Mathematics Education.

2. Method
The method used in this article is a literature review that aims to inform and give a wide-ranging overview theoretically related to the design of mathematical problem solving as well as a contextual problem as a core problem in Realistic Mathematics Education (RME). The data was collected and
analyzed in the form of literature in which this was obtained from the results of several relevant research and scientific journal related to this topic.

3. Problem solving in mathematics education

3.1 Problem Solving Concept

Mathematical problem solving has been seen as an essential subject for teaching and learning mathematics [21]. The core goals of problem solving in mathematics learning were identified in three categories such as training students to develop their ability in solving the problem through creative thinking, learning a standard techniques in mathematical modelling and providing the basic skill to stimulate the skills of critical thinking and analytical reasoning [22]. On the other hand, some knowledge is also needed in problem solving like the knowledge of content, the pedagogy knowledge of problem solving, and affective factor [23].

| Table 1. The knowledge in solving problem |
|------------------------------------------|
| Knowledge | Categories |
| Content | • The ability to solving problems |
|          | • Various problems in mathematics |
|          | • Problem posing |
| Pedagogy of problem solving | • Students as problem solver |
|          | • The practice of teaching |
| Affective factor | • Attitude and self-efficacy |

To construct the ability to solve mathematical problems, the students need several problem solving skills. Because solving the problem needs various activities, procedures in problem solving need to be considered. Jonassen [24] said that understanding problems, ways of representing and individual differences become crucial factors in the problem solving process. The nature of the problem variation is different from one individual to another based on the learner perspective. Problems can be routine and non-routine [25], originating from externals [26] and real-life problems [27]. According to the variety of abovementioned problems, I briefly mentioned that problems include the processes of abstracting, structuring and presenting.

Besides, the Principles and Standards for School Mathematics [28] set standards in problem solving namely building new mathematical knowledge through problem solving, solving problems that arise in mathematics and other contexts, applying and adopting various appropriate strategies to solve problems, monitoring and reflecting on the process of solving mathematical problems. The steps that must be taken to teach students mathematical problem solving are learning from problems, contributing to student learning, and guiding to solve the problem as well as reflecting the results [28]. In this case, several levels will be achieved in solving problems [13,21,29] that it is shown in table 2. Problem solving skill is divided into three parts; problem variations, representation, and individual differences [20]. Firstly, problem variations include complexity and abstractness. Secondly, context, social and historical is the categories for representation. To construct the representation, the learners categorize the existing problems by mapping the schema and use the procedure to solve them. Representation relies on the student's cognitive skills like memorizing, thinking, understanding and the others in which these enable the student to implement the further stage of problem solving. Thirdly, individual differences cover cognitive styles, individual knowledge, motivation, and self-confidence.

According to the mentioned phases, I briefly describe that problem solving process consists several abilities namely understanding, organizing, and confirmation and every phase includes mathematics skills and cognitive abilities [10] such as the ability to focus, memorize, and recall.
3.2 Problem Solving in Realistic Mathematics Education

According to Realistic mathematics education theory, the real world is the source or starting point for the development of mathematical concepts [30]. In this case, the issues raised by RME are contextual to provide opportunities for students to develop informal problem solving abilities, where students can develop strategies to solve specific problem contexts and use them to develop mathematical concepts [31]. In line with Sugiman and Kusumah [32] revealed that RME can improve students' (mathematical) problem solving abilities. In this case, using the RME approach can bridge students to understand real mathematical problems towards formal mathematics. The advantages of RME in that RME gives freedom to students to solve a mathematical problem by using many methods which means that students do not have to have a single solution [33]. Furthermore, the steps to solve mathematical problems are provided with the learning steps at RME. The problem solving steps have been mentioned in table 2, while the steps in learning mathematics using RME according to [34] can be synchronized with the steps of RME in the following table 3.

### Table 2. The process of problem solving

| Polya’ Problem Solving | PISA’s Mathematical Literacy | The heuristic of Problem Solving |
|------------------------|-----------------------------|---------------------------------|
| Understanding          | Formulating                 | Interpreting                    |
| Planning               | Employing                   | Thinking of solving problem     |
| Looking back           | Interpreting/evaluating     | Representing                    |

### Table 3. The framework of Problem Solving

| Realistic Mathematics Education | Polya’ Problem Solving |
|---------------------------------|------------------------|
| Understanding contextual problem| Understanding the problem |
| Describing contextual problem   | Thinking a plan         |
| Solving the contextual problem  | Doing research          |
| Forming                         | Thinking back           |

4. Realistic Mathematics Education

4.1 Contextual problem

Realistic Mathematics Education that implemented in Indonesia called PMRI (Pendidikan Matematika Realistik Indonesia). It engages student learning with the use of contextual mathematics problem applied more than a decade [35]. A contextual problem that can be called a real-world problem is the actual problem in everyday-life. Mathematics learning-based real-world problem encourages the students to improve their ability in finding and researching the current problem occurred in their surrounding [36] that further they can enhance the skills of understanding and mathematizing the context. For instance, contextual problems provided in the daily-life phenomena can be seen below.

In the parking lot, there are 30 vehicles consisting of motorbikes and cars. The total number of wheels is 90. If many motors are represented by \( x \) and many cars are expressed as \( y \), the two-variable linear equation system of the above statement is ...

Ana, Wendi, and Arya went together to the clock repair because their handwatches did not run according to normal hours. Within a normal hour, Ana’s handwatch is two minutes slower. The three children compared their clocks, apparently Wendi’s handwatch was two minutes faster in an hour than Ana’s handwatch, while Arya’s clock was 2 minutes faster in one hour than Wendi watch. When departing, their watches show the same time as the normal hours, which is exactly at 10 o’clock. After shopping first, finally, exactly at four hours later they arrive at the repair shop. Show what time was Arya's watch when they arrived at the watch repair?
4.2 Contextual Mathematics Learning Design
Realistic Mathematics Education in mathematics learning that is aimed to develop the meaningful student's knowledge and experience through contextual problem [35]. This combined both of teaching and learning mathematics [23]. In learning mathematics, the students facilitated to solve contextual problems [37] by which contextual problem solving, indeed, gives a positive influence in understanding mathematics [38]. Learning mathematics using Realistic Mathematics Education also called mathematical modelling. The procedures of mathematics modelling [39] applying in this following picture describe the real-world problem, mathematical model, mathematical results, and practical consequences as mathematization procedures needed in mathematics problem solving. Besides, the process of mathematization and modelling procedures include mathematical modelling, working, and application.

![Figure 1. The procedure of Modelling and Mathematization](image)

In the mathematical modelling process, the use of real-world problem, called context-based task, which is available in situation and setting context is important for mathematization process [40]. Thus, the characteristic of mathematics contexts is can not straightforward be simplified into mathematical procedure [41], it needs a more implicit process to solve the complexity of the problems [40].

4.3 The Role of Realistic Mathematics Education to Improve Mathematical Problem Solving
Contextual problem is the main problem in Realistic Mathematics Education. This is related to how the students doing mathematics in formal thinking which student’s formal thinking integrates their previous and new knowledge [42]. Hans Freudenthal noted that mathematics is a human activity [43] and is actual activity in doing mathematics [18], also, student’s formal thinking integrates their previous and new knowledge. Furthermore, problem-based context, the ability of problem solving and Realistic Mathematics Education as a mathematical learning approach interconnected each other in which Realistic Mathematics Education has a pivotal role to increase student ability in solving a mathematics-contextual problem.
5. Conclusion
Each student has a different intelligence and in levels that are not the same. Students can explore, grow, and develop their thinking abilities following the characteristics of the intelligence they have. That is why in solving mathematical problems, students have their respective abilities. The process of solving mathematical problems is a complex learning process so it needs to be continuously developed so that students can be facilitated to be able to improve their problem solving abilities. Therefore we need a learning approach that is close to students and their environment and related to their way of thinking so that later children can explore mathematical knowledge more broadly. Realistic Mathematics Education (RME) is an approach that offers a mathematical realization process which is in line with the problem solving process. Therefore, RME is believed to be able to optimize the ability to solve mathematical problems. In this case, students can get the opportunity to make observations and broader scientific exploration.

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