THERMAL RATIO OF THE DISORDER DEVIATION AND THE
SPACE-TIME DECONFINED PHASE SIZE

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Abstract

A systematic study of the strongly interacting matter under extreme conditions via
the form of the thermal ratio of the disorder deviation is presented. The evolution
of fermi- and bose- particles (quarks and gluons) is studied in the framework
of multi-particle correlation and distribution functions to predict the size of the
finite-temperature deconfined phase.

Key-words: multi-particle correlation functions.

1 Introduction

The existence of a deconfined phase of gluons and quarks has been predicted by quan-
tum chromodynamics (QCD) [1]. Aspects of QCD at finite density are very important as
well as instructive in the sense of the observation of quark-gluon plasma (QGP), especially
from the heavy vector meson (HVM) suppression, strangeness enhancement, hadron yield
distributions versus temperature and dilepton excess. There is large amount of consist-
cy among the different signatures of deconfined phase transition to a new excited state
of matter. In the case of HVM suppression (e.g., $c\bar{c}$ bound state suppression), the CERN-
SPS experiments clearly show that it’s visible in Pb-Pb collisions starting from a definite
value of the energy density in the right range. Enhanced strange particle production in
Pb-Pb collisions offers another indication for the phase transition: in the deconfined phase
strange quarks are more easily produced. The threshold for the producing strange quarks
is much lower than that for strange hadrons. In addition, the mass of the strange quark
goes down in the case of the restoration of the chiral symmetry. Hence, the observation
of large ratios for strange particles is considered as a remnant of the unconfined phase.
The dilepton abundance, while well explained by ordinary soucers in all other cases, for
Pb-Pb collisions a neat excess is instead observed.

There is a very popular point of view in literature that a deconfinement phase tran-
sition is predicted to occur at the typical energy scale involved, the temperature $T_c \sim$
QCD scale $\Lambda_{QCD} \sim$ the pion mass $m_\pi \sim$ the mass of a strange quark $m_s \sim 140-200$
MeV. Putting all indications above mentioned together, the case of a high density state
of matter onsetting at a critical temperature around 170 MeV is reasonably suitable.

Among the issues related to QGP, we attract attention to the problem of deconfined
phase through the calculation of correlation and distribution functions [2] in the thermal
theory of quantized fields. We consider the semiphenomenological model for the deconfinement existence within the framework of the Langevine-type equation. To do this, we follow the standard theory of quantized fields replacing:
1. the asymptotic field operators and
2. the vacuum expectation values by
1. the thermal field operators and
2. the thermal statistical averages, respectively, in order to formulate correlation and distribution functions of produced particles.

The method of Langevin equation and its extensions to the quantal case have been suggested and considered in papers [3-5] and [6-8], respectively. We propose that real physical processes to happen in the finite-temperature QCD should be replaced by a one-constituent (e.g., gluon or quark) propagation provided by a special kernel operator (in the master evolution equation) to be considered as an input of the model and disturbed by the random force $F$. We assume $F$ to be the external source proposed as both a c-number function and an operator. The master equation is an operator one, so that there appear new additional issues about the commutation relations and the ordering of operators, which do not exist in the classical case [8].

Based on the thermal operator-field technique, we introduce a thermal ratio of the disorder deviation (TRDD), reflecting the degree of deviation, from unity, of the ratio of the two-particle thermal momentum-dependent distribution to two one-particle thermal distribution functions of produced particles, gluons and/or quarks (g/q) in a partly deconfined phase state. We study the four-momentum correlations of identical particles which can be both useful and instructive to infer the shape of the particle emitter-source. We estimate the sensitivity of the TRDD-functions to the size of the emitter. Within these features, the canonical formalism in a stationary state in the thermal equilibrium (SSTE) is formulated, and a closed structural resemblance between the SSTE and standard quantum field theory is revealed.

To clarify the internal structure of the disordering of particles, we use the consistent approach based on the evolution of dynamical variables as well as the extension to different modes provided by virtual transitions.

## 2 Distribution functions. Evolution equation.

Let us consider a hypothetical system of the quark-gluon excited local thermal phase in QCD where a canonical operator $a$ and its Hermitian conjugate $a^+$ occur. We formulate the distribution functions (DF) of produced particles (gluons and quarks) in terms of point-to-point equal time temperature-dependent thermal correlation functions (CF) of two operators

$$w(\vec{k}, \vec{k}'; t; T) = \langle a^+(\vec{k}, t) a(\vec{k}', t) \rangle = Tr[a^+(\vec{k}, t) a(\vec{k}', t) e^{-H\beta}] / Tr(e^{-H\beta}).$$

Here, $\langle \ldots \rangle$ means the procedure of thermal statistical averaging; $\vec{k}$ and $t$ are, respectively, momentum and time variables, $e^{-H\beta} / Tr(e^{-H\beta})$ stands for the standard density operator in the equilibrium and the Hamiltonian $H$ is given by the squared form of the annihilation $a_\rho$
and creation $a_p^+$ operators for bose- and fermi- particles, $H = \sum_p \epsilon_p a_p^+ a_p$ (the energy $\epsilon_p$ and operators $a_p$, $a_p^+$ carry some index $p$ [9], where $p_\alpha = 2\pi n_\alpha/L, n_\alpha = 0, \pm 1, \pm 2, ...; V = L^3$ is the volume of the system considered); $\beta$ is the inverse temperature of the environment, $\beta = 1/T$. The standard canonical commutation relation

$$[a(\vec{k}, t), a^+(\vec{k}', t)] = \delta^3(\vec{k} - \vec{k}')$$

at every time $t$ is used as usual for Bose (-) and Fermi (+)-operators.

The probability to find two particles, gluons or quarks, with momenta $\vec{k}$ and $\vec{k}'$ in the same event at the time $t$ normalized to the single spectrum of these particles is:

$$R(\vec{k}, \vec{k}', t) = \frac{W(\vec{k}, \vec{k}', t)}{W(\vec{k}, t) \cdot W(\vec{k}', t)} .$$

Here, the one-particle thermal DF is defined as

$$W(\vec{k}, t) = \langle b^+(\vec{k}, t) b(\vec{k}, t) \rangle ,$$

where

$$b(\vec{k}, t) = a(\vec{k}, t) + \phi(\vec{k}, t)$$

under an assumption of occurrence of the random source-function $\phi(\vec{k}, t)$ being an operator, in general. The two-particle DF $W(\vec{k}, \vec{k}', t)$ looks like

$$W(\vec{k}, \vec{k}', t) = \langle b^+(\vec{k}, t) b^+(\vec{k}', t) b(\vec{k}, t) b(\vec{k}', t) \rangle .$$

The evolution properties of propagating particles in a randomly distributed environment comes from the evolution equations

$$i \partial_t b(\vec{k}, t) + A(\vec{k}, t) = F(\vec{k}, t) + P , \quad (1)$$

$$-i \partial_t b^+(\vec{k}, t) + A^*(\vec{k}, t) = F^+(\vec{k}, t) + P , \quad (2)$$

where both $b$ and $b^+$ are the special mode operators of the quark and gluon fields [10], $P$ and $F(\vec{k}, t)$ stand for the stationary external force and the random one, respectively, both acting from the environment. The only operator $F$ has a zeroth value of the statistical average, $\langle F \rangle = 0$. The interaction of the particles considered with the surrounding ones as well as providing the propagation is given by the operator $A(\vec{k}, t)$ which can be defined as the one closely related to the dissipation force:

$$A(\vec{k}, t) = \int_{-\infty}^{+\infty} K(\vec{k}, t - \tau) b(\vec{k}, \tau) \, d\tau . \quad (3)$$

An interplay of quarks and gluons with surrounding particles is embedded into the interaction complex kernel $K(\vec{k}, t)$, while the real physical transitions are provided by the random source operator $F(\vec{k}, t)$ (see eqns. (1) and (2)). The random evolution field operator $K(\vec{k}, t)$ in (3) stands for the random noise and it is assumed to vary stochastically with a $\delta$-like equal time correlation function [10]

$$\langle K^+(\vec{k}, \tau) K(\vec{k}', \tau) \rangle = 2 (\pi \rho)^{1/2} \kappa \delta(\vec{k} - \vec{k}') ,$$
where both the strength of the noise $\kappa$ and the positive constant $\rho \to \infty$ define the effect of the Gaussian noise on the evolution of quarks and gluons in the thermalized environment.

The formal solution of $[1]$ in the operator form in $S(\mathcal{F}_4)$ $(k^\mu = (\omega = k^0, k^j))$ is

$$\tilde{b}(k_\mu) = \tilde{a}(k_\mu) + \tilde{\phi}(k_\mu),$$

where the operator $\tilde{a}(k_\mu)$ is expressed via the Fourier transformed operator $\tilde{F}(k_\mu)$ and the Fourier transformed kernel function $\tilde{K}(k_\mu)$ (coming from $[3]$) as

$$\tilde{a}(k_\mu) = \tilde{F}(k_\mu) \cdot [\tilde{K}(k_\mu) - \omega]^{-1},$$

while the function $\tilde{\phi}(k_\mu) \sim P \cdot [\tilde{K}(k_\mu) - \omega]^{-1}$. The random force operator $F(\vec{k}, t)$ can be expanded by using the Fourier integral

$$F(\vec{k}, t) = \int_{-\infty}^{+\infty} \frac{dw}{2\pi} \psi(k_\mu) \ \hat{c}(k_\mu) \ e^{-i\omega t},$$

where the form $\psi(k_\mu) \cdot \hat{c}(k_\mu)$ is just the Fourier operator $\tilde{F}(k_\mu) = \psi(k_\mu) \cdot \hat{c}(k_\mu)$ and the canonical operator $\hat{c}(k_\mu)$ obeys the commutation relation

$$\left[\hat{c}(k_\mu), \hat{c}^+(k_\mu')\right] = \delta^4(k_\mu - k_\mu').$$

The function $\psi(k_\mu)$ in $[4]$ is determined by the condition $[10]$

$$\int_{-\infty}^{+\infty} \frac{dw}{2\pi} \left[ \frac{\psi(k_\mu)}{\tilde{K}(k_\mu) - \omega} \right]^2 = 1.$$

### 3 TRDD and the space-time size.

The enhanced probability for emission of two identical particles is given by the ratio $R$ of DF in $S(\mathcal{F}_4)$ as follows:

$$R_{bf}(k_\mu, k'_\mu; T) = \frac{\tilde{W}(k_\mu, k'_\mu; T)}{W(k_\mu) \cdot W(k'_\mu)},$$

where $\tilde{W}(k_\mu, k'_\mu; T) = \langle \tilde{b}^+(k_\mu) \tilde{b}^+(k'_\mu) \tilde{b}(k_\mu) \tilde{b}(k'_\mu) \rangle$ and $\tilde{W}(k_\mu) = \langle \tilde{b}^+(k_\mu) \tilde{b}(k_\mu) \rangle$. Using the Fourier solution of equation $[1]$ in $S(\mathcal{F}_4)$, one can get R-ratios for DF obeying to Bose- and Fermi-particles

$$R_b(k_\mu, k'_\mu; T) = 1 + D_b(k_\mu, k'_\mu; T)$$

and

$$R_f(k_\mu, k'_\mu; T) = R_b(k_\mu, k'_\mu; T) - 2 \frac{\Xi(k_\mu) \cdot \Xi(k'_\mu)}{W(k_\mu) \cdot W(k'_\mu)},$$

where

$$D_b(k_\mu, k'_\mu) = \frac{\Xi(k_\mu, k'_\mu)[\Xi(k'_\mu, k_\mu) + \tilde{\phi}^+(k'_\mu)\tilde{\phi}(k_\mu)] + \Xi(k_\mu, k_\mu)\tilde{\phi}^+(k_\mu)\tilde{\phi}(k'_\mu)}{W(k_\mu) \cdot W(k'_\mu)}.$$
and the two-particle CF $\Xi(k_\mu, k'_\mu)$ looks like

$\Xi(k_\mu, k'_\mu) = \langle \hat{a}^+(k_\mu) \, \hat{a}(k'_\mu) \rangle$

$\frac{\psi^*(k_\mu) \cdot \psi(k'_\mu)}{[K^*(k_\mu) - \omega] \cdot [K(k'_\mu) - \omega]} \cdot \langle \hat{c}^+(k_\mu) \, \hat{c}(k'_\mu) \rangle$ .

(9)

Using the Kubo-Martin-Schwinger condition ( $\mu$ is the chemical potential)

$\langle a(\vec{k}', t') \, a^+(\vec{k}, t) \rangle = \langle a^+(\vec{k}, t) \, a(\vec{k}', t - i\beta) \rangle \exp(-\beta \mu)$ ,

the thermal statistical averages for the $\hat{c}(k_\mu)$-operator should be presented in the following form:

$\langle \hat{c}^+(k_\mu) \, \hat{c}(k'_\mu) \rangle = \delta^4(k_\mu - k'_\mu) \cdot n(\omega, T)$ ,

$\langle \hat{c}(k_\mu) \, \hat{c}^+(k'_\mu) \rangle = \delta^4(k_\mu - k'_\mu) \cdot [1 \pm n(\omega, T)]$

for Bose (+)- and Fermi (-)-statistics, where $n(\omega, T) = \{ \exp[(\omega - \mu)\beta] \pm 1 \}^{-1}$. Inserting CF (9) into (8) and taking into account that the $\delta^4(k_\mu - k'_\mu)$-function should be changed by the smooth sharp function $\Omega(r) \cdot \exp(-q^2/2)$, one can get the following expression for the $D_b$-function

$D_b(k_\mu, k'_\mu; T) = \lambda(k_\mu, k'_\mu; T) \exp(-q^2/2)$

$\times [n(\bar{\omega}, T)\Omega(r) \exp(-q^2/2) + \tilde{\phi}^*(k'_\mu)\tilde{\phi}(k_\mu) + \tilde{\phi}(k'_\mu)\tilde{\phi}(k_\mu)]$ ,

(10)

where

$\lambda(k_\mu, k'_\mu; T) = \frac{\Omega(r)}{W(k_\mu) \cdot W(k'_\mu)} \cdot n(\bar{\omega}, T) , \quad \bar{\omega} = \frac{1}{2}(\omega + \omega')$ .

The function $\Omega(r) \cdot n(\omega; T) \cdot \exp(-q^2/2)$ describes the space-time size of the QGP fire-ball. Choosing the z-axis along the two-heavy-ion collision axis one can put

$q^2 = (r_0 \cdot Q_0)^2 + (r_z \cdot Q_z)^2 + (r_t \cdot Q_t)^2$ ,

$Q_\mu = (k - k')_\mu, Q_0 = \epsilon_k - \epsilon_{k'}, Q_z = k_z - k_z', Q_t = [(k_x - k'_x)^2 + (k_y - k'_y)^2]^{1/2}$ ,

$\Omega(r) \sim r_0 \cdot r_z \cdot r_t^2$ ,

where $r_0$, $r_z$ and $r_t$ are time-like, longitudinal and transverse "size" components of the QGP fire-ball. Formally, the function $D_b$ (10) is the positive one ranging from 0 to 1. The quantitative information (longitudinal $r_z$ and transverse $r_t$ components of the QGP spherical volume, the temperature $T$ of the environment) could be extracted by fitting the theoretical formula (10) to the measured TRDD function and estimating the errors of the fit parameters. Formula (10) indicates that a chaotic g/q source emanating from the thermalized g/q fireball exists. Hence, the measurement of the space-time evolution of the g/q source would provide information of the g/q emission process and the general reaction mechanism. In formula (10) for the $D_b$-function, the temperature of the environment enters through the two-particle CF $\Xi(k_\mu, k'_\mu; T)$. If $T$ is unstable the $R_{b/f}$-functions (5) will change due to a change of DF $\bar{W}$ which, in fact, can be considered as an effective density of the g/q source. Formula (5) looks like the following expression for the experimental $R$-ratio using a source parametrization:

$R_T(r) = 1 + \lambda_T(r) \cdot \exp(-r_t^2 \cdot Q_t^2/2 - r_z^2 \cdot Q_z^2/2)$ ,
where \( r_t(r_z) \) is the transverse (longitudinal) radius parameter of the source with respect to the beam axis, \( \lambda_T \) stands for the effective intercept parameter (chaoticity parameter) which has a general dependence of the mean momentum of the observed particle pair. Here, the dependence on the source lifetime is omitted. Since \( 0 < \lambda_T < 1 \), one can conclude that the effective function \( \lambda_T \) can be interpreted as a function of the core particles to all particles produced. The chaoticity parameter \( \lambda_T \) is the temperature-dependent and the positive one defined by

\[
\lambda_T(r) = \frac{|\Omega(r) n(\bar{\omega}; T)|^2}{W(k_\mu) \cdot W(k'_\mu)}.
\]

Comparing (8) and (10) one can identify

\[
\Xi(k_\mu, k'_\mu) = \Omega(r) \cdot n(\bar{\omega}; T) \cdot \exp(-q^2/2).
\]

There is no a satisfactory tool to derive the precise analytical form of the random source function \( \tilde{\phi}(k_\mu) \) in (8), but one can put (see (11) and taking into account \( \tilde{\phi}(k_\mu) \sim P \cdot |\tilde{K}(k_\mu) - \omega|^{-1} \) [11,10]

\[
\tilde{\phi}(k_\mu) = [\alpha \cdot \Xi(k_\mu)]^{1/2}.
\]

where \( \alpha \) is of the order \( O \left( P^2/n(\omega; T) \cdot |\psi(k_\mu)|^2 \right) \). Thus,

\[
D_b(q^2; T) = \frac{\tilde{\lambda}^{1/2}(\bar{\omega}; T)}{(1 + \alpha)(1 + \alpha')} e^{-q^2/2} \left[ \tilde{\lambda}^{1/2}(\bar{\omega}; T) e^{-q^2/2} + 2(\alpha')^{1/2} \right]. \tag{11}
\]

It is easily to see that in the vicinity of \( q^2 \approx 0 \) one can get the full correlation if \( \alpha = \alpha' = 0 \) and \( \tilde{\lambda}(\bar{\omega}; T) = 1 \). Putting \( \alpha = \alpha' \) in (11) we find the formal lower bound on the space-time dimensionless size of the fire-ball for Bose-system:

\[
q_f^2 \geq \ln \frac{\tilde{\lambda}(\bar{\omega}; T)}{[\sqrt{(\alpha + 1)^2 + \alpha^2 - \alpha}]^2}.
\]

In the case of Fermi-particles, the following restriction on \( q_f^2 \) is valid (see (7))

\[
\ln \frac{\tilde{\lambda}(\bar{\omega}; T)}{[\sqrt{2\alpha(\alpha + 1) + 3 - \alpha}]^2} \leq q_f^2 \leq \ln \frac{\tilde{\lambda}(\bar{\omega}; T)}{[\sqrt{\alpha^2 + 2 - \alpha}]^2}.
\]

In fact, the function \( D_b(k_\mu, k'_\mu; T) \) in (11) could not be observed because of some model uncertainties. In the standard consideration, the TRDD-function has to contain a background contribution as well as other physical particles (resonances) which have not been included in the calculation of the \( D_b \)-function. In order to be close to the experimental data, one has to expand the \( D_b \)-function as projected on some well-defined function (in \( S(\mathbb{R}_4) \)) of the relative momentum of two particles produced in heavy-ion collisions \( D_b(k_\mu, k'_\mu; T) \rightarrow D_b(Q^2; T) \). Thus, it will be very instructive to use the polynomial expansion which is suitable to avoid any uncertainties as well as characterize the degree of deviation from the Gaussian distribution, for example. In \( (-\infty, +\infty) \), a complete orthogonal set of functions can be obtained with the help of the Hermite polynomials in the Hilbert space of the square integrable functions with the measure \( d\mu(z) = \exp(-z^2/2)dz \). The function \( D_b \) corresponds to this class if

\[
\int_{-\infty}^{+\infty} dq \exp(-q^2/2) |D_b(q)|^n < \infty, \quad n = 0, 1, 2, \ldots.
\]
The expansion in terms of the Hermite polynomials \( H_n(q) \)

\[
D_b(q) = \lambda \sum_n c_n \cdot H_n(q) \cdot \exp(-q^2/2)
\]

is well suited for the study of possible deviation from both the experimental shape and the exact theoretical form of the TRDD function \( D_b \) \((10)\). The coefficients \( c_n \) in \((12)\) are defined via the integrals over the expanded functions \( D_b \) because of the orthogonality condition

\[
\int_{-\infty}^{+\infty} H_n(x) \cdot H_m(x) \cdot \exp(-x^2/2) \, dx = \delta_{n,m}.
\]

Thus, the observation of the two-particle correlation (both for Bose- and Fermi-symmetrization) enable to extract the properties of the structure of \( q^2 \), i.e. the space-time size of QGP formation.

In order to be close to an experiment one has to replace \( R_{b,f} \) functions \((5), (7)\) with respect to the cylindrical symmetry angles \( \theta \) and \( \phi \) which are non-observable ones at fixed \( Q_t \):

\[
R_{b,f}(k_\mu, k_\mu'; T) \rightarrow \bar{R}_{b,f}(Q_t; T) = N^{-1} \int dq_t \, dQ_z \, d\theta \, d\phi \, \bar{W}(k_\mu, k_\mu'; T),
\]

where

\[
N = \int dq_t \, dQ_z \, d\theta \, d\phi \, \bar{W}(k_\mu) \, \bar{W}(k_\mu'),
\]

\[
q_t = \frac{1}{\cos \theta + \sin \theta} \left\{ k_x + k_y \mp \frac{1}{2} Q_t \left[ \cos(\theta + \phi) + \sin(\theta + \phi) \right] \right\}.
\]

Then, \( \bar{R}_{b,f}(Q_t; T) = 1 + \bar{D}_{b,f}(Q_t; T) \) with

\[
\bar{D}_b(Q_t; T) = \frac{N^{-1}(T)}{(1 + \alpha)(1 + \alpha')} \exp[-(r_t Q_t)^2] \, F_b(Q_t; T),
\]

\[
D_f(Q_t; T) = \frac{1}{(1 + \alpha)(1 + \alpha')} \left\{ N^{-1}(T) \exp[-(r_t Q_t)^2] \, F_b(Q_t; T) - 2 \right\},
\]

\[
F_b(Q_t; T) = \int dq_t \, dQ_z \, d\theta \, d\phi \, n^2(\bar{\omega}; T) \, e^{-\beta_{0Z}} \left[ 1 + 2\sqrt{\alpha \alpha' \lambda^{-1}(\bar{\omega}; T) \, e^{q_t^2/2}} \right],
\]

\[
\beta_{0Z} \equiv (r_0 Q_0)^2 + (r_z Q_z)^2,
\]

\[
\bar{N}(T) = \int dq_t \, dQ_z \, d\theta \, d\phi \, n(\omega; T) \, n(\omega'; T).
\]

To avoid the trivial result, one should restrict the \( \alpha \)-parameter as

\[
\alpha \geq \lambda^{-1/2}(\bar{\omega}; T) \, e^{q_t^2/2} - \frac{1}{2} \lambda^{1/2}(\bar{\omega}; T) \, e^{-q_t^2/2}.
\]

In conclusion of this section, the experimental data are quite desired to make the analysis of the particle fluctuation via TRDD.
4 Conclusion

1. We investigated the finite temperature DF (of produced identical particles, gluons and quarks) which can be both useful and instructive to infer the shape of the gluon/quark source-emitter. In fact, we have presented the method of extracting the intercept and source parameters from the shape of the TRDD-function.

2. The relations between the CF $\Xi(k_{\mu}, k'_{\mu})$ (9) and the full $R$-functions for Bose (8)- and Fermi (8)-particles at the stage of the freeze-out are obtained. We have shown the sensitivity of the correlation functions to the space-time geometry of the source-emitter (10). The TRDD-function $D_b$ describes the size and shape of the space-time domain where the secondary observed particles are generated.

3. One can conclude that formally, the deconfined phase size scale can be determined by the evolution behavior of the field operators and the critical temperature $T = T_c$ (see formula (10)).

4. Since, the TRDD-function $D_b$ is the positive one and restricted by 1, we expect that the $R$-ratio at too small values of $Q_{\mu}$ starts from the fixed point $R(Q_{\mu} \to 0) = 2 - \epsilon$ ($\epsilon \to +0$) and then falls down (with the Gaussian shape) up to unity over some momentum scale interval of an order of the inverse source size.

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