Gravitational cubic interactions for a massive mixed symmetry gauge field

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Abstract
In a recent paper (Boulanger et al 2011 J. Phys. A: Math. Theor. 44 415403), cubic gravitational interactions for a massless mixed symmetry field in an AdS space have been constructed. In the current paper, we extend these results to the case of massive field. We work in a Fradkin–Vasiliev approach and use a frame-like gauge-invariant description for the massive field which works in the AdS spaces with arbitrary values of the cosmological constant including a flat Minkowski space. In this, the massless limit in the AdS space coincides with the results of Boulanger et al (2011) while we show that it is impossible to switch on a gravitational interaction for the massless field in the dS space.

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1. Introduction

In a recent paper [1], cubic gravitational interactions for a simplest mixed symmetry field (hook), corresponding to the Young Tableau that contains two boxes in the first row and one box in the second row, have been investigated using a number of different approaches, namely

- the direct construction of all possible cubic vertices using the modified 1- and 1/2-order formalism (for similar construction for spin 3 cubic vertices, see [2]);
- a Fradkin–Vasiliev approach [3, 4] applied to the Alkalaev–Shaynkman–Vasiliev (ASV) description of the massless hook in the AdS space [5–7];
- a Fradkin–Vasiliev approach applied to the Stueckelberg description of the hook [8, 9] that differs from the ASV one by the presence of some Stueckelberg fields;
- a cohomological approach [10, 11] applied to the Stueckelberg description.

While the results of different approaches completely agree, it turns out that the most simple and straightforward way to construct interactions is to use the Fradkin–Vasiliev approach that initially was formulated for the investigation of gravitational interactions for massless particles in the AdS space [3, 4] and then was successfully applied to more general interactions (see e.g. [12–14]). Let us briefly recall the main steps of the procedure.
We begin with frame-like gauge-invariant formulation with the known set of fields and gauge transformations. For each field (both physical and auxiliary), we construct a gauge-invariant object that we will generically call curvature.

Then we rewrite the free Lagrangian as an expression quadratic in these gauge-invariant curvatures. In general, such an expression will contain higher derivative terms so we have to adjust coefficients so that all such terms cancel.

Now we add quadratic corrections to free curvatures supplemented with appropriate corrections to gauge transformations so that variations of deformed curvatures are proportional to the free ones.

Lastly, we replace free curvatures in the Lagrangian with the deformed ones and adjust coefficients so that all variations vanish on shell. This in turn means that off shell all variations can be compensated by additional corrections to gauge transformations.

The paper [1], as most of the works on higher spin interactions till now is devoted to the interactions for massless fields which is possible in the AdS space only. At the same time, many researches believe that for massive particles it should be possible to switch on interactions even in a flat Minkowski space. Moreover, it would be good if such theories could be obtained as smooth deformations of the massless ones (a kind of spontaneous symmetry breaking). Our main motivation in this paper is to give one of the first non-trivial examples of such theories.

As is clear from the description of the Fradkin–Vasilliev approach given above, two main ingredients of such an approach are gauge invariance and frame-like formalism [15–17]. But over the last few years, we have seen that there exists a frame-like gauge-invariant description for massive fields for both symmetric [18, 19] and mixed symmetry ones [8, 9, 20, 21]. Moreover, such a description works well both in the flat Minkowski space and in the AdS space with an arbitrary value of the cosmological constant, including all possible massless and partially massless limits. Thus, it seems natural to use the Fradkin–Vasilev approach applied to the frame-like gauge-invariant description for the investigation of possible interactions for massive and/or massless particles. In our recent paper [22], we have shown how such a procedure works in the case of electromagnetic interactions of the massive hook, while the aim of the current paper is to extend the results of [1] to the case of the massive hook.

In the next section, we will give all necessary information on the free hook including free Lagrangian, gauge transformations and gauge-invariant curvatures. Moreover, we will show that using partial gauge fixing, one can obtain a simple description for the massive hook directly related to the ASV description for the massless one. One of the lessons from [1] is that at least for the particular hook case, such partial gauge fixing ‘commutes’ with the switching on an interaction so we may freely use it to simplify calculations without any loss of generality. Then in section 3, we consider the application of the general procedure to the gravitational interactions for the massive hook including the massless limit in the AdS space, while the investigation of the massless case in the dS space which turns out to be special is given in the appendix.

Notations and conventions. We work in the AdS space with $d \geq 4$ dimensions. We will use the notation $e^a_\mu$ for the background (non-dynamical) frame of the AdS space and $D_\mu$ for AdS covariant derivatives normalized so that

$$[D_\mu, D_\nu] e^a_\sigma = -\kappa e^a_\mu e^\mu_\sigma, \quad \kappa = \frac{2\lambda}{(d-1)(d-2)}.$$ 

We use Greek letters for world indices and Latin letters for local ones. Surely, using the frame $e^a_\mu$ and its inverse $e^\mu_a$, one can freely convert world indices into local ones and vice versa and we indeed will use such conversion whenever convenient. But separation of the world and local
indices plays a very important role in a frame-like formalism. In particular, all terms in the Lagrangians can be written as a product of forms, i.e. as expressions completely antisymmetric on world indices and this property greatly simplifies all calculations. For that purpose, we will often use the notations \( \mu^a_{\mu} = e^a_{\nu} e^b_{\rho} - e^a_{\nu} e^\rho_{\mu} \) and so on.

2. Kinematics

The frame-like gauge-invariant description [8, 9] requires four pairs of physical and auxiliary fields: \((\Omega_{\mu}^{abc}, \Phi_{\mu}^a), (\Omega_{\mu}^{ab}, f_{\mu}^a), (C_{\muab}, \Phi_{\mu}^c)\) and \((B_{ab}, B_{\mu})\) where the fields \(\Phi_{\mu}^c, f_{\mu}^a, C_{\muab}\) and \(B_{\mu}\) are physical ones, while \(\Omega_{\mu}^{abc}, \Omega_{\mu}^{ab}, C_{\muab}\) and \(B_{ab}\) are the auxiliary ones. The free Lagrangian describing the massive particle in the AdS space has the form

\[
\mathcal{L}_0 = -\frac{3}{4} \mu^a_{\rho} \Omega_{\mu}^{abc} \xi_{\rho} + \frac{1}{2} \mu^a_{\rho} \Omega_{\mu}^{abc} D_\rho \Phi_\mu^a + \frac{3}{4} \mu^a_{\rho} \Omega_{\mu}^{abc} \Phi_\mu^d + \frac{1}{2} \mu^a_{\rho} \Omega_{\mu}^{abc} \Phi_\mu^c \Phi_\mu^b
\]

Here \(8m_1^2 - 24m_2^2 = -3(d - 3)\kappa\), \(\bar{m}_{1,2} = \sqrt{(d - 2)} m_{1,2}\).

The Lagrangian is invariant under the following set of gauge transformations:

\[
\delta_0 \Phi_{\mu}^a = D_\mu \zeta^a - \eta_{\mu}^a + \frac{2m_1}{3(d - 3)} e^a_{[\mu} \xi_{\nu]} + \frac{4m_2}{(d - 2)} e^a_{\mu} \Lambda
\]

\[
\delta_0 \Omega_{\mu}^{abc} = D_\mu \eta_{\mu}^{abc} + \frac{4m_1}{3(d - 3)} e^a_{\mu} \eta^{bc}
\]

\[
\delta_0 f_{\mu}^a = D_\mu \xi^a + \eta_{\mu}^a + 4m_1 \xi^a + \frac{4m_2}{(d - 2)} e^a_{\mu} \Lambda
\]

\[
\delta_0 C_{\muab} = D_\mu \eta_{\muab} - 2m_1 \eta_{\muab}
\]

\[
\delta_0 B_{ab} = D_\mu \Lambda + 2m_2 \xi_{ab} + 4\bar{m}_1 \xi_{ab}
\]

As the relation on the parameters \(m_{1,2}\) clearly shows for non-zero values of the cosmological constant \(\kappa\), it is not possible to set both \(m_1\) and \(m_2\) equal to zero simultaneously. In the AdS space \((\kappa < 0)\), one can set \(m_2 = 0\). In this, the whole system decomposes into two disconnected subsystems. One of them with the Lagrangian and gauge transformations

\[
\mathcal{L}_0 = -\frac{3}{4} \mu^a_{\rho} \Omega_{\mu}^{abc} \xi_{\rho} + \frac{1}{2} \mu^a_{\rho} \Omega_{\mu}^{abc} D_\rho \Phi_\mu^a - \frac{1}{2} \mu^a_{\rho} \Omega_{\mu}^{abc} D_\rho f_{\mu}^a - m_1 \left[ \frac{\mu^a_{\rho} \Omega_{\mu}^{abc} f_{\mu}^c}{abc} + \frac{\mu^a_{\rho} \Omega_{\mu}^{abc} \Phi_\mu^c}{abc} \right]
\]
corresponds to the massless representation of the AdS group (which differs from that of the Poincare group [23–25]), while the other one just gives the gauge-invariant description of the massive antisymmetric second rank tensor. In turn, in the dS space one can set

$$
\delta_0 \Phi_{\mu \nu} = D_{[\mu} \zeta_{\nu]} + \eta_{\mu \nu} + \frac{2m_1}{3(d-3)} \epsilon^{\mu \nu \alpha \beta} \eta_{\alpha \beta}
$$

$$
\delta_0 \Omega_{\alpha \beta}^{abc} = D_\alpha \eta^{abc} + \frac{4m_1}{3(d-3)} \epsilon^{\alpha \beta \gamma \delta} \eta_{\gamma \delta}
$$

$$
\delta_0 f_\mu^a = D_\mu \xi^a + \frac{4m_1 z_\mu^a}{3(d-3)} + \frac{2m_1}{3(d-3)} \epsilon^{\mu \nu \alpha \beta} \eta_{\alpha \beta}
$$

$$
\delta_0 \Omega_{\mu \nu}^{ab} = D_\mu \eta^{ab} - 2m_1 \eta_{\mu \nu}
$$

(4)

In our recent paper [22], we have investigated electromagnetic interactions for the same mixed symmetry fields (both physical and auxiliary):

$$
\mathcal{L}_0 = -\frac{1}{4} \eta_{\mu \nu} \Omega_{\alpha \beta}^{abcd} \Omega^{\alpha \beta}_{\nu \mu} + \frac{1}{3} \eta_{\mu \nu} \Omega_{\alpha \beta}^{abcd} \Omega^{\alpha \beta}_{\nu \mu} D_{\alpha \beta} \Phi_{\mu \nu} - \frac{1}{8} \eta_{\mu \nu} \Omega_{\alpha \beta}^{abcd} \Omega^{\alpha \beta}_{\nu \mu} + \frac{1}{3} \eta_{\mu \nu} \Omega_{\alpha \beta}^{abcd} \Omega^{\alpha \beta}_{\nu \mu}
$$

$$
+ m_2 \left[ \frac{\epsilon_{\mu \nu \alpha \beta}}{\Omega^{\mu \nu}_{\alpha \beta}} + \frac{\epsilon_{\mu \nu \alpha \beta}}{\Omega^{\mu \nu}_{\alpha \beta}} + \frac{\epsilon_{\mu \nu \alpha \beta}}{\Omega^{\mu \nu}_{\alpha \beta}} \right]
$$

(5)

$$
\delta_0 \Phi_{\mu \nu} = D_{[\mu} \zeta_{\nu]} + \eta_{\mu \nu} + \frac{4m_2}{(d-3)} \epsilon^{\mu \nu \alpha \beta} \eta_{\alpha \beta}
$$

$$
\delta_0 \Omega_{\mu \nu}^{ab} = D_\mu \eta^{ab}
$$

$$
\delta_0 C_{\mu \nu} = D_{[\mu} \zeta_{\nu]} - 2m_2 \zeta_{\mu \nu}
$$

$$
\delta_1 C_{\mu \nu} = 6m_2 \eta^{ab}
$$

(6)

corresponds to the massless representation of dS group, while the other one describes a so-called partially massless spin-2 particle.

In our recent paper [22], we have investigated electromagnetic interactions for the same massive mixed symmetry field. We have shown that it is impossible to take a limit \( m_1 \to 0 \) without switching off minimal \( \epsilon / m \) interactions, while nothing prevents one from taking a limit \( m_2 \to 0 \). It turns out that the situation with gravitational interactions is the same. Namely, in a very recent paper [1], cubic gravitational interactions for the case \( m_2 = 0 \) were constructed, while in the appendix of the current paper, we consider the case \( m_1 = 0 \) and show that it is impossible to switch on the gravitational interaction. Thus, in the rest of the paper we will always assume that \( m_1 \neq 0 \).

Let us return to the general massive case. Having at our disposal an explicit form of the gauge transformations, we can construct gauge-invariant objects (curvatures) for all eight fields (both physical and auxiliary):

$$
R^{\alpha \beta}_{\mu \nu} = D_{[\mu} \Omega_{\nu]}^{\alpha \beta} + \frac{4m_1}{3(d-3)} \epsilon^{\mu \nu \alpha \beta} \eta_{\alpha \beta} + \frac{4m_2}{3(d-3)} \epsilon^{\mu \nu \alpha \beta} \eta_{\alpha \beta}
$$

$$
T_{\mu \nu \alpha} = D_{[\mu} \Phi_{\nu \alpha]} + \frac{2m_1}{3(d-3)} \epsilon^{\mu \nu \alpha \beta} \eta_{\beta} + \frac{4m_2}{(d-3)} \epsilon^{\mu \nu \alpha \beta} \eta_{\beta}
$$

$$
F^{\mu \nu}_{\alpha \beta} = D_{[\mu} \Omega_{\nu]}^{\alpha \beta} + 2m_1 \Omega_{[\mu \nu]}^{\alpha \beta} + \frac{2m_2}{(d-2)} \epsilon^{\mu \nu \alpha \beta} \eta_{\beta}
$$

$$
T_{\mu \nu} = D_{[\mu} \Phi_{\nu]} + \frac{4m_1}{3(d-3)} \epsilon^{\mu \nu \alpha \beta} \eta_{\beta} + \frac{4m_2}{(d-2)} \epsilon^{\mu \nu \alpha \beta} \eta_{\beta}
$$

$$
C_{\mu \nu} = D_\mu \Phi_{\nu} + \frac{4m_1}{3} \epsilon^{\mu \nu \alpha \beta} \eta_{\beta} + \frac{2m_2}{(d-2)} \epsilon^{\mu \nu \alpha \beta} \eta_{\beta}
$$

$$
C_{\mu \nu \alpha} = D_{[\mu} \Phi_{\nu \alpha]} + C_{\mu \nu \alpha} + 2m_2 \Phi_{[\mu \nu \alpha]}
$$

$$
B_{\mu \nu} = D_{[\mu} \Phi_{\nu]} + 4 \frac{m_2}{3} \Omega_{[\mu \nu]}^{\alpha \beta} + \frac{4m_1}{3} \epsilon^{\mu \nu \alpha \beta} \eta_{\beta}
$$

$$
B_{\mu \nu} = D_{[\mu} B_{\nu]} + 2m_2 f_{[\mu \nu]} - 4 \frac{m_1}{3} C_{\mu \nu}
$$

(7)
Now let us partially gauge fix such a description by settling $f_{\mu}{}^a = 0$ and $B_{\mu} = 0$. At the same time, we solve the corresponding algebraic equations $T_{\mu
u}{}^a = 0$ and $B_{\mu
u} = 0$:

$$\Phi_{\mu\nu}{}^a = -\frac{1}{4m_1}\Omega_{[\mu,v]}{}^a, \quad C_{\mu\nu} = -\frac{1}{4m_1}B_{\mu\nu}. $$

Then (after some field rescaling) we obtain the following simple Lagrangian for the remaining four fields:

$$\mathcal{L}_0 = -\frac{3}{4}\{\nu\}_{ab}\Omega_{\mu}{}^{acd}\Omega_{\nu}{}^{bcd} + \frac{3}{8}\{\nu\}_{abc}\Omega_{\mu}{}^{abcd}\Omega_{\nu}{}^{cd} - \frac{1}{6}\Omega_{\mu}{}^{abcd}D_{\alpha}\Omega_{\nu}{}^{\alpha cd} - \frac{1}{4}\epsilon^{a}{}_{cde}D_{\mu}B_{\nu}{}^{cde}$$

$$- m_2\epsilon^a{}_{\mu} \left[ \frac{3}{2}\Omega_{\mu}{}^{abc}B_{\nu}{}^{bc} + C^{abc}\Omega_{\nu}{}^{bc} \right] - \frac{m_1^2}{2}\{\nu\}_{\mu}{}^{\alpha}\Omega_{\nu}{}^{\alpha bc} - \frac{m_1^2}{4}B_{\mu\nu}{}^{2} \cdot \tag{8} \label{8}$$

This Lagrangian is invariant under the following remaining gauge transformations:

$$\delta\Omega_{\mu}{}^{abc} = D_{\mu}\eta^{abc} + \frac{4m_2}{3(d-3)}\epsilon_{\mu}{}^{[a}\eta^{bc]}, \quad \delta\Omega_{\mu}{}^{ab} = D_{\mu}\eta^{ab} - \frac{2m_2}{d-3}\epsilon_{\mu}{}^{[a}\Omega_{[v]}{}^{b]}$$

$$\delta C^{abc} = 6m_2\eta^{abc}, \quad \delta B_{\mu\nu} = -4m_2\eta^{ab}. \tag{9} \label{9}$$

Note that such a description turns out to be closely related to the Alkalaev–Shaynkman–Vasiliev description for mixed symmetry fields \[5\]( see also \[6, 7\]). Indeed the gauge fields $\Omega_{\mu}{}^{abc}$ and $\Omega_{\mu}{}^{ab}$ (up to different normalization) correspond to the ASV description of the (partially) massless hook in AdS, while the zero forms $C^{abc}$ and $B^{ab}$ play the roles of Stueckelberg fields making them massive.

After partial gauge fixing we have four gauge-invariant objects:

$$\mathcal{R}_{\mu\nu}{}^{abc} = D_{[\mu}\Omega_{\nu]}{}^{abc} + \frac{4m_2}{3(d-3)}\epsilon_{[\mu}{}^{[a}C^{bc]}, \quad \delta\mathcal{R}_{\mu\nu}{}^{abc} = D_{\mu}\eta^{abc} + \frac{4m_1^2}{3(d-3)}\epsilon_{\mu}{}^{[a}\Omega_{[v]}{}^{b]}$$

$$\mathcal{F}_{\mu\nu}{}^{ab} = D_{[\mu}\Omega_{\nu]}{}^{ab} + 2\Omega_{(\mu,\nu)}{}^{ab} - \frac{2m_2}{(d-3)}\epsilon_{[\mu}{}^{[a}\Omega_{[v]}{}^{b]}$$

$$\mathcal{C}_{\mu}{}^{abc} = D_{\mu}C^{abc} - 6m_2\Omega_{\mu}{}^{abc} - \frac{4m_2}{3}\epsilon_{\mu}{}^{[a}\Omega_{[v]}{}^{b]}$$

$$B_{\mu}{}^{ab} = D_{\mu}B_{\nu}{}^{ab} + \frac{4}{3}\epsilon_{\mu}{}^{[a}\Omega_{[v]}{}^{b]} \cdot \tag{10} \label{10}$$

Our next task is to rewrite the free Lagrangian as an expression quadratic in these gauge-invariant curvatures. The most general such Lagrangian looks as follows:

$$\mathcal{L}_0 = \{\nu\}_{abc} [a_{\nu}\mathcal{R}_{\mu\nu}{}^{abc} + b_{\nu}\mathcal{F}_{\mu\nu}{}^{ab} + c_{\nu}\mathcal{C}_{\mu}{}^{abc} + d_{\nu}\mathcal{B}_{\mu}{}^{ab} \} + \{\nu\}_{ab} [a_{\nu}\mathcal{R}_{\mu\nu}{}^{ab} + b_{\nu}\mathcal{F}_{\mu\nu}{}^{ab} + c_{\nu}\mathcal{C}_{\mu}{}^{ab} + d_{\nu}\mathcal{B}_{\mu}{}^{ab} \} + \{\nu\}_{[ab]} [a_{\nu}C_{\mu}{}^{abc}C_{\nu}{}^{bcd} + a_{\nu}\mathcal{B}_{\mu}{}^{bc}\mathcal{B}_{\nu}{}^{bc}]. \tag{11} \label{11}$$

We have usual differential identities for the curvatures:

$$D_{[\mu}\mathcal{R}_{\nu\nu}{}^{abc} = -\frac{4m_2}{3(d-3)}\epsilon_{[\mu}{}^{[a}\mathcal{C}_{\nu]}{}^{bc]} = \frac{4m_1^2}{3(d-3)}\epsilon_{\mu}{}^{[a}\mathcal{F}_{\nu]}{}^{bc]}$$

$$D_{[\mu}\mathcal{F}_{\nu\nu}{}^{ab} = 2\mathcal{R}_{[\mu\nu,\nu]}{}^{ab} + \frac{2m_2}{(d-3)}\epsilon_{\mu}{}^{[a}\mathcal{F}_{\nu]}{}^{bc]}$$

$$D_{[\mu}\mathcal{C}_{\nu}{}^{abc} = -6m_2\mathcal{R}_{\mu\nu}{}^{abc} + \frac{2m_2}{(d-3)}\epsilon_{\mu}{}^{[a}\mathcal{B}_{\nu]}{}^{bc]}$$

$$D_{[\mu}\mathcal{B}_{\nu}{}^{ab} = \frac{4}{3}\mathcal{C}_{[\mu\nu]}{}^{ab} + 4m_2\mathcal{F}_{\mu\nu}{}^{ab}. \tag{12} \label{12}$$

Note that as the solutions of the auxiliary fields $\Omega_{\mu}{}^{abc}$ and $C^{abc}$, we have

$$\mathcal{F}_{[\mu\nu,\nu]}{}^{ab} = 0, \quad \mathcal{B}_{[\mu\nu,\nu]} = 0 \implies \mathcal{R}_{[\mu\nu,\nu]}{}^{ab} = 0. \tag{13} \label{13}$$
Using these differential identities, we can obtain the following four identities for the curvature squares:

\[ I_1 = \left\{ \frac{\mu\nu\alpha\beta}{\gamma\delta\mu\nu} \right\} \frac{1}{D_{\mu}}\left[ R_{\nu\alpha\beta}^{\mu} \right] - 2(d - 4)m_1^2 \frac{R_{\mu\nu}^{ab} R_{\alpha\beta}^{cd}}{(d - 3)} \]

\[ I_2 = I_3 = I_4 = 0. \]

Thus, we have three independent identities so if we require that the Lagrangian quadratic in curvatures correctly reproduce a free Lagrangian for the massive hook given above, we would expect that we obtain a solution with three arbitrary parameters. This turns out to be the case. Note however that one has to be careful using this freedom because as our previous experience shows switching on an interaction tends to partially resolve this ambiguity. We will use the following simple choice for the free Lagrangian:

\[ \mathcal{L}_0 = \left\{ \frac{\mu\nu\alpha\beta}{\gamma\delta\mu\nu} \right\} \left[ a_1 R_{\mu\nu}^{ab} R_{\alpha\beta}^{cd} + a_2 F_{\mu\nu}^{ab} F_{\alpha\beta}^{cd} \right] + \left\{ \frac{\mu\nu}{\alpha\beta} \right\} \left[ a_3 C_{\mu}^{\alpha\beta \mu} C_{\nu}^{\alpha\beta \nu} + a_4 B_{\mu}^{\alpha\beta \mu} B_{\nu}^{\alpha\beta \nu} \right] \]

\[ a_1 = -\frac{9}{512m_1^2}, \quad a_2 = -\frac{3}{256(d - 3)}, \quad a_3 = -\frac{(d - 4)}{16m_1^2(d - 3)}, \quad a_4 = -\frac{3}{32(d - 3)}. \]

Later on we will see that such a choice is compatible with the possibility of switching on an interaction.

3. Cubic gravitational interactions

For the gravitational field, we will use the notations \( h_{\mu}^{\alpha} \) and \( \omega_{\mu}^{\alpha\beta} \). Gauge transformations for the free massless field in the AdS space have the form

\[ \delta_0 h_{\mu}^{\alpha} = D_{\mu} \chi^{\alpha} + \chi_{\mu}^{\alpha}, \quad \delta_0 \omega_{\mu}^{\alpha\beta} = D_{\mu} \chi^{\alpha\beta} + \kappa e_{\mu}^{[a} \chi^{b]} \].

\[ (15) \]
Correspondingly, we have two gauge-invariant objects (linearized curvature and torsion),

\[ R_{\mu\nu}^{ab} = D_{[\mu}a_{\nu]}^{ab} + \kappa e^{[a}_{\mu}h_{\nu]}^{b]}, \]

\[ T_{\mu\nu}^{a} = D_{[\mu}h_{\nu]}^{a} - a_{\mu\nu}^{[a}. \]  

(16)

For non-zero values of the cosmological constant, the free Lagrangian can be written as follows:

\[ \mathcal{L}_0 = - \frac{1}{32\kappa (d-3)} \left\{ \frac{\mu\nu\rho\sigma}{abcd} \right\} R_{\mu\nu}^{ab} R_{\rho\sigma}^{cd}. \]  

(17)

According to the general procedure, our first task is to find deformations for all gauge-invariant curvatures supplemented with appropriate corrections to gauge transformations such that these variations of these deformed curvatures are proportional to the free ones.

**Deformations for hook curvatures.** Let us consider deformations for hook curvatures corresponding to minimal gravitational interactions:

\[ \Delta R_{\mu\nu}^{abc} = \omega_{[\mu}^{[a}e^{bc]}_{\nu]}^{d]- \frac{4m_2}{3(d-3)} [h_{[\mu}^{[a}C_{\nu]}^{bc]} + e_{[\mu}^{[a}C^{bc]}d_{\nu]}^{d]}}} - \frac{4m_1^2}{3(d-3)} h_{[\mu}^{[a}\Omega_{\nu]}^{bc]} \]

\[ \Delta F_{\mu\nu}^{ab} = -\omega_{[\mu}^{[a}e^{bc]}_{\nu]}^{d] - 2\xi [h_{[\mu}^{[a}B_{\nu]}^{bc]} + \frac{2m_2}{3(d-3)} [h_{[\mu}^{[a}B_{\nu]}^{bc]} - e_{[\mu}^{[a}B^{bc]}h_{\nu]}^{c]}}} \]

\[ \Delta C_{\mu}^{abc} = \omega_{[\mu}^{[a}e^{bc]d} + \frac{2m_1^2}{(d-3)} h_{[\mu}^{[a}e^{bc]}h_{\nu]}^{c]}} \]

(18)

Similarly, the appropriate corrections to gauge transformations turn out to be

\[ \delta \Omega_{\mu}^{abc} = -\chi^{d[a}e_{\mu}^{bc]}d^{d]} + \frac{4m_2}{3(d-3)} [C_{\mu}^{[ab}x^{c]} + e_{[\mu}^{[a}C^{bc]}d_{\nu]}^{d]} + \frac{4m_1^2}{3(d-3)} \Omega_{\mu}^{[ab}x^{c]} \]

\[ + \omega_{[\mu}^{[a}\eta^{bc]}d^{d]} - \frac{4m_1^2}{3(d-3)} h_{[\mu}^{[a}\eta^{bc]} \]

\[ \delta \Omega_{\mu}^{ab} = \chi^{d[a}e_{\mu}^{bc]} + \frac{2m_2}{(d-3)} [B_{\mu}^{[ab}x^{c]} + e_{[\mu}^{[a}B^{bc]}x^{c]} - e_{[\mu}^{[a}B^{bc]}h_{\nu]}^{c]}} - \omega_{[\mu}^{[a}\eta^{bc]} + 2\eta^{ab}h_{\nu]}^{c]} \]

\[ \delta C^{abc} = -\chi^{d[a}e^{bc]}d^{d]} - \frac{2m_1^2}{(d-3)} B^{[ab}x^{c]}, \quad \delta B^{ab} = \chi^{[a}B^{bc]} + \frac{4}{3} C^{abc}x^{c}. \]

(19)

Taking into account these corrections, we obtain the following transformations of deformed curvatures under the hook’s \(\eta^{ab}\) and \(\eta^{bc}\) transformations:

\[ \delta \hat{R}_{\mu\nu}^{abc} = R_{\mu\nu}^{[ab} \eta^{c]} - \frac{4m_1^2}{3(d-3)} T_{\mu\nu}^{[a} \eta^{bc]}} \]

\[ \delta \hat{F}_{\mu\nu}^{ab} = 2\eta^{ab}T_{\mu\nu}^{c} - R_{\mu\nu}^{[a} \eta^{b]}h^{c]}). \]

(20)

**Deformations of gravitational curvatures.** The most general ansatz for such deformations quadratic in fields looks like (schematically)\(^1\)

\[ \hat{R} \sim R \oplus \Omega_1\Omega_3 \oplus aBB \oplus B \oplus \Omega_2 \oplus CC \]

\[ \hat{T} \sim T \oplus \Omega_1\Omega_2 \oplus B \oplus CC \oplus \Omega_2. \]

\(^1\) In fact we have considered the general case without partial gauge fixing where all eight fields are present. In this, the resulting expressions for deformed curvatures contain the auxiliary fields \(\Omega_1\), \(\Omega_2\), \(C\) and \(B\) only. Thus, at least in this particular case, partial gauge fixing ‘commutes’ with switching on interactions and we may use it to simplify calculations without any loss of generality.
where $\Omega_1$ stands for $\Omega_{a^bc}$ and $\Omega_2$ for $\Omega_{a=bc}$. Due to the presence of the zero forms $C_{a^bc}$ and $B_{ab}$, there exists a possibility of making field redefinitions of the form

$$
\omega_{a^bd} \Rightarrow \omega_{a^bd} + \kappa_1 C_{a^bc} B_{b^c} + \kappa_2 C_{b^c} \omega_{a^c} + \kappa_3 \epsilon_{abc} C_{b^c} B_{d} + \kappa_4 \epsilon_{a^b} C_{d} B_{c} + \kappa_5 \epsilon_{a^b} B_{d} B_{c} B_{e} + \kappa_6 \epsilon_{a^b} B_{d} B_{e} B_{f} + \kappa_7 \epsilon_{a^b} B_{d} B_{e} B_{f}
$$

which we will use to simplify all subsequent expressions. In this, the resulting expressions look like

$$
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$$

Now we have to consider all variations that do not vanish on shell and try to adjust coefficients so that all of them vanish. The transformations for hook curvatures we have to take care of look like

$$
\delta \omega_{a^bd} = -a_0 \eta^{e[a} \Omega_{b^d]} + m_2 \omega_{a^b} \eta^{b^d} B_{d} + 4 m_2 \omega_{a^b} \eta^{b^d} B_{d} + \frac{4 m_2 a_0}{3(d - 3)} \eta^{b^d} + \frac{4 m_2 a_0}{3(d - 3)} \eta^{b^d} B_{d} + \frac{4 m_2 a_0}{3(d - 3)} \eta^{b^d} B_{d} B_{e} B_{f}.
$$

We have to consider all variations that do not vanish on shell and try to adjust coefficients so that all of them vanish. The transformations for hook curvatures we have to take care of look like

$$
\delta \omega_{a^bd} = \omega_{a^bd} + \frac{4 m_2 a_0}{3(d - 3)} \eta^{b^d} B_{d} B_{e} B_{f}.
$$

Gravitational interaction. Now according to the general procedure, we consider the sum of free Lagrangians for the hook and graviton where all curvatures are replaced with the deformed ones:

$$
\mathcal{L}_0 = \{ \mu_{a^b} C_{a^b} \}, \quad \mathcal{L}_0 = \{ \mu_{a^b} C_{a^b} \},
$$

Now we have to consider all variations that do not vanish on shell and try to adjust coefficients so that all of them vanish. The transformations for hook curvatures we have to take care of look like

$$
\delta \hat{R}_{a^b} = R_{a^b} \eta^{b^d} + \frac{4 m_2 a_0}{3(d - 3)} \eta^{b^d} B_{d} B_{e} B_{f}.
$$

while for the deformed Riemann tensor, they are given in (23).

Variations under the $\eta^{a^b}$ transformations give us

$$
- \{ \mu_{a^b} \} [4 a_1 \hat{R}_{a^b} \eta^{b^d} B_{d} B_{e} B_{f} + \frac{4 m_2 a_0}{3(d - 3)} \hat{R}_{a^b} \eta^{b^d} B_{d} B_{e} B_{f}].
$$

Using the on-shell identities $R_{a^b} = 0$ and $R_{a^b} = 0$, one can show that the following identity holds:

$$
\{ \mu_{a^b} \} [2 \hat{R}_{a^b} \eta^{b^d} B_{d} B_{e} B_{f} - \hat{R}_{a^b} \eta^{b^d} B_{d} B_{e} B_{f}] = 0.
$$

Thus, we have to put

$$
a_1 = -\frac{a_0}{16k(d - 3)}.
$$

Note that the choice we make here has to be in agreement with the choice for the parameters in the free Lagrangian. As we will see later on, our choices are indeed consistent.
Similarly, variations under the $\eta^{ab}$ transformations produce

$$\{\mu\nu^a\beta_{abcd}\} [4a_2F_{\mu\nu}^{ab}R_{\alpha\beta}^{ce}\eta^{de} - \frac{m_1^2a_0}{6\kappa(d-3)^2} F_{\mu\nu}^{ae}R_{\alpha\beta}^{be}\eta^{de}].$$

Again using the on-shell identities $F_{[\mu\nu,a]}^a = 0$ and $R_{[\mu\nu,a]}^a = 0$, one can show that the following identity holds:

$$\{\mu\nu^a\beta_{abcd}\} [F_{\mu\nu}^{ab}R_{\alpha\beta}^{ce} + F_{\mu\nu}^{ae}R_{\alpha\beta}^{bc}] \eta^{de} = 0.$$

Thus, we obtain

$$a_2 = -\frac{m_1^2a_0}{24\kappa(d-3)^2} = \frac{2m_1^2a_1}{3(d-3)^2}. \quad (27)$$

Note that the resulting relation for $a_1$ and $a_2$ is in agreement with our choice for the free Lagrangian.

Thus, Lagrangian (24) with the deformed curvatures defined in (18) and (21) gives us a correct set of cubic gravitational vertices including standard minimal interactions together with non-minimal higher derivatives ones. There are two particular limits that one can consider here. First of all we may take $m_2 = 0$. In this limit, the Stueckelberg fields $C^{abc}$ and $B^{ab}$ completely decouple and the result (up to different field normalization) completely agrees with the results obtained previously in [1].

Another interesting and important limit is a flat limit $\kappa \to 0$ (i.e. $m_2^2 \to m_1^2/3$). The peculiarity here is related to the fact that for the massless graviton, it is possible to rewrite the Lagrangian as an expression quadratic in curvatures for non-zero values of the cosmological constant only. But from relation (26), we obtain

$$a_0 = \frac{9(d-3)\kappa}{32m_1^2} \quad (28)$$

so that at least in the linear approximation, the contribution from the gravity part of the Lagrangian is non-singular in a flat limit.

4. Conclusion

We have seen that the Fradkin–Vasiliev approach together with the frame-like gauge-invariant formalism for massive fields allows one to effectively investigate possible interactions for massive and/or massless fields. The massive hook (as well as massive spin 2) is one of the simplest examples possible but it is clear that such an approach can be applied to higher spin fields (both symmetric and mixed symmetry ones) as well. One of the questions that deserves further study is the problem of the flat limit for gravitational interactions. The reason is that the Lagrangian for the massless graviton can be written as a square of curvature for the non-zero cosmological constant only though as we have seen in the linear approximation the flat limit is non-singular. Also it would be interesting to understand a striking difference between massless representations in the AdS and dS spaces as far as switching on interactions is concerned.

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**Appendix. Partially massless case in a de Sitter space**

Here we will try to switch on gravitational interactions for (partially) the massless mixed symmetry field in the dS space corresponding to the $m_1 \to 0$ limit.

**Kinematics.** For convenience, we reproduce here gauge transformations for this case,

\[
\delta_0 \Phi_{\mu}^a = D_{[\mu} z_{\nu]}^a + \eta_{\mu}^a + \frac{4m_2}{(d-3)} \epsilon_{[\mu}^a \xi_{\nu]}, \quad \delta_0 \Omega_{\mu}^{abc} = D_{\mu} \eta^{abc}
\]

\[
\delta_0 C_{\mu}^{abc} = D_{[\mu} \xi_{\nu]}^{abc} - 2m_2 \xi_{[\mu\nu]}, \quad \delta_0 C_{\mu}^{abc} = 6m_2 \eta^{abc}.
\]  
\[\text{(A.1)}\]

Here $8m_2^2 = (d - 3)\kappa$. Correspondingly, we have four gauge-invariant objects:

\[
\mathcal{R}_{\mu}^{abc} = D_{[\mu} \Phi_{\nu]}^{abc} + \frac{4m_2}{3(d-3)} \epsilon_{[\mu}^{abc} C_{\nu]}^{\alpha} \] \[\mathcal{T}_{\mu \nu \alpha} = D_{[\mu} \Phi_{\nu \alpha]} - \Omega_{[\mu, \nu \alpha]} + \frac{4m_2}{3} \epsilon_{[\mu}^{\alpha} C_{\nu \alpha]} \]

\[
\mathcal{C}_{\mu}^{abc} = D_{\mu} C_{\nu}^{abc}, \quad \mathcal{C}_{\mu}^{abc} = D_{[\mu} C_{\nu]}^{abc} - 2m_2 \Phi_{[\mu \nu, \alpha]}.
\]  
\[\text{(A.2)}\]

It is not possible to express the free Lagrangian in terms of these gauge-invariant objects.

\[\mathcal{L}_0 = \left\{ \frac{\epsilon_{\alpha \beta}^{\mu \nu}}{\epsilon_{\alpha \beta}^{\mu \nu}} \left[ g_{\mu} \mathcal{R}_{\nu}^{\alpha \beta} \mathcal{R}_{\alpha \beta}^{\alpha \beta} + a_3 \mathcal{R}_{\mu \nu}^{\alpha \beta} \mathcal{R}_{\nu}^{\alpha \beta} d + a_3 \mathcal{T}_{\mu \nu \alpha} \mathcal{C}_{\nu}^{\alpha \beta} \right] \right\} \]  
\[\text{(A.3)}\]

Again there is an ambiguity in the choice of parameters related to differential identities. Indeed we have

\[
D_{[\mu} \mathcal{R}_{\nu]}^{abc} = -\frac{4m_2}{3(d-3)} \epsilon_{[\mu}^{abc} C_{\nu]}^{\alpha}, \quad D_{[\mu} \mathcal{C}_{\nu]}^{abc} = -6m_2 \mathcal{R}_{\mu \nu}^{abc}.
\]  
\[\text{(A.4)}\]

As a result, we obtain

\[
\left\{ \frac{\epsilon_{\alpha \beta}^{\mu \nu}}{\epsilon_{\alpha \beta}^{\mu \nu}} \left\{ D_{[\mu} \mathcal{R}_{\nu]}^{abc} \mathcal{C}_{\beta}^{\alpha \beta} \right\} \right\} = -3m_2 \left\{ \frac{\epsilon_{\alpha \beta}^{\mu \nu}}{\epsilon_{\alpha \beta}^{\mu \nu}} \left\{ \mathcal{R}_{\mu \nu}^{\alpha \beta} \mathcal{R}_{\alpha \beta}^{\alpha \beta} + \frac{32(d-4)}{9(d-3)} \mathcal{R}_{\mu \nu}^{\alpha \beta} \mathcal{R}_{\alpha \beta}^{\alpha \beta} \right\} \mathcal{C}_{\nu}^{\alpha \beta} \right\} = 0.
\]

**Deformations for gravitational curvatures.** This time, due to the presence of zero form, there is an ambiguity related to possible field redefinitions, namely

\[
h_{\mu}^a \quad \Longrightarrow \quad h_{\mu}^a + \alpha_1 C_{\mu}^{bc} C_{\nu}^{\alpha \beta} + \alpha_2 \epsilon_{\mu}^{\alpha \beta} C_{\nu}^{bcd} C_{\rho}^{\beta}.
\]

Using these redefinitions, deformed curvatures can be cast into the form

\[
\tilde{R}_{\mu \nu}^{ab} = D_{[\mu} \omega_{\nu]}^{ab} + \kappa \epsilon_{[\mu}^a h_{\nu]}^b + 6m_2 a_0 (\Omega_{[\mu}^{abc} C_{\nu]}^{\alpha} - \frac{4}{3(d-3)} C_{\mu}^{abc} C_{\nu]}^{\alpha})
\]

\[
\tilde{T}_{\mu \nu}^{ab} = D_{[\mu} h_{\nu]}^{ab} - a_0 \omega_{[\mu \nu]}^{ab} - a_0 (\Omega_{[\mu \nu]}^{abc} C_{\rho]}^{\alpha}) \] \[\text{(A.5)}\]

In this, appropriate corrections to gauge transformations look as follows:

\[
\delta \omega_{\mu \nu}^{ab} = -6m_2 a_0 \eta^{cd} \Omega_{\mu}^{\alpha \beta} \Omega_{\nu]}^{\alpha \beta}, \quad \delta h_{\mu}^{a} = a_0 \eta^{abc} C_{\mu}^{bc} \] \[\text{(A.6)}\]

Taking into account these corrections, we obtain the following transformations for deformed curvatures:

\[
\delta \tilde{R}_{\mu \nu}^{ab} = -6m_2 a_0 \eta^{cd} \Omega_{\mu}^{\alpha \beta} \Omega_{\nu]}^{\alpha \beta}, \quad \delta \tilde{T}_{\mu \nu}^{ab} = a_0 \eta^{abc} C_{\mu}^{bc}.
\]  
\[\text{(A.7)}\]
Deformations for the hook’s curvatures. In this case, the desired results can be easily obtained by the usual substitutions corresponding to minimal gravitational interactions:

\[ \delta \Omega_{\mu \nu}^{abc} = \frac{1}{3(d-3)} \delta \chi_{\mu}^{a C_{\nu}^{b c d}} + \frac{4m^2}{3(d-3)} \delta \alpha_{\mu}^{a \mu \nu} C_{\nu}^{b c d} - \frac{4m^2}{3(d-3)} \delta h_{\mu}^{a C_{\nu}^{b c d}} \]

\[ \delta \mathcal{T}_{\mu \nu a}^{a} = D_{\mu} \Phi_{\nu a}^{a} - \Omega_{\mu \nu [a} C_{b]}^{a \nu} + \frac{4m^2}{d-3} \delta \mu_{\nu}^{a} C_{\nu}^{b c d} - \delta \alpha_{\mu}^{a \mu \nu} b \nu_{a}^{b} - \Omega_{\mu \nu [a} h_{b]}^{b} \]

\[ \frac{4m^2}{3(d-3)} \delta \eta^{a \mu \nu} C_{\nu}^{b c d} \]

\[ \delta C_{\mu \nu} = D_{\mu} C_{\nu}^{a} - C_{\nu}^{a b} + 2m^2 \Phi_{(\mu \nu a)} + \delta h_{\mu}^{a C_{\nu}^{b c d}} - 2m^2 \Phi_{(\mu \nu a)} h_{\nu a}^{a} \]

Corrections to gauge transformations turn out to be:

\[ \delta \Omega_{\mu \nu}^{abc} = -\chi^{a b} C_{\mu}^{b c d} - \epsilon_{\mu}^{a C_{b c d}} c_{d}^{e} \]

\[ \delta \phi_{\mu \nu} = \chi^{a b} \Phi_{\mu \nu}^{a b} + \Omega_{\mu \nu}^{a b} c_{d}^{e} - \frac{4m^2}{(d-3)} \chi_{\mu}^{a b} C_{\nu}^{b c d} + \eta^{a b} \alpha_{\mu}^{a \mu \nu} b \nu_{a}^{b} - \delta \alpha_{\mu}^{a \mu \nu} c_{d}^{e} \]

Gravitational interactions. Following a general procedure, we consider the sum of the free Lagrangians for the hook and graviton but with all curvatures replaced with the deformed ones:

\[ \mathcal{L}_{0} = \{ \mu \\
\begin{array}{c}
\{ ab \\
\{ cd \\
\end{array}
\}
\]

\[ \mu \nu a b \rho \sigma \alpha \beta \]

\[ \mathcal{L} = \{ \mu \\
\begin{array}{c}
\{ ab \\
\{ cd \\
\end{array}
\}
\]

Now we have to take care of variations that do not vanish on shell,

\[ \delta \mathcal{R}_{\mu \nu}^{a b} = -6m^2 \eta^{a b} \mathcal{R}_{\mu \nu}^{b c d} \]

\[ \delta \mathcal{T}_{\mu \nu a}^{a} = -4m^2 \Phi_{\mu \nu a}^{a} \]

and try to adjust coefficients so that all of them vanish. It is easy to see that this time it is impossible. A crucial point is the \( z_{a}^{a} \) transformations that give

\[ \delta \mathcal{L} \sim \{ a b c d \} C_{\mu}^{a b} R_{a b}^{d e} z_{a}^{d e} \]

and this cannot be compensated even on mass shell!

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