Topological solid phase in a quantum dimer model

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We present an example for the phase transition between a topological non-trivial solid phase and a trivial solid phase in the quantum dimer model (QDM) on triangular lattice. Such a transition is beyond the Landau’s paradigm of phase transition. We have characterized such a transition with the topological entanglement entropy (TEE) of the system, which is found to change from $\gamma = \ln 2$ in the topological solid phase to zero in the trivial solid phase, through a pronounced peak around the transition point. We also calculated the correlation function of the vison excitation in the QDM and find that the vison condensate develops right at the topological transition point. These results imply that the topological order and the related fractionalized excitation can coexist with conventional symmetry breaking order.

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Topological order is a novel form of order that goes beyond the Landau’s paradigm of phase transition. The topological order manifests itself in both the topological degeneracy for system defined on multiply connected manifold and the topological contribution to the entanglement entropy. The concept of topological order plays an important role in the study of fractional quantum Hall systems, the spin liquid phase in quantum magnets and the pseudogap phase in the high-$T_c$ superconductors. A direct physical consequence of the topological order is the emergence of fractionalized excitation in the spectrum of the system.

The concept of topological order is best illustrated by the quantum dimer model (QDM), whose degree of freedom is the dimer occupying the nearest neighboring bonds of the lattice. The dimer in the QDM can be viewed as a simplified version of the spin singlet pair in the resonating valence bond theory of quantum spin liquid and should satisfy the constraint that each site of the lattice should be occupied by one and only one dimer. On multiply connected manifold, the Hilbert space of the QDM factorizes into distinct topological sectors that can not be connected by any local Hamiltonian term at finite order. For example, on a two dimensional torus, we can classify the dimer configurations into four topological sectors according to the parity of the number of dimers that are intersected by two large loops wrapping around the two holes of the torus. The $Z_2$ topological degeneracy of the QDM defined on the torus results from the fact that different topological sectors are indistinguishable by local Hamiltonian.

The local order parameter related to the conventional symmetry breaking order can in principle be used to distinguish dimer configuration in the different topological sectors of the QDM. It is thus unclear if the topological order in the QDM is robust or not when one introduce symmetry breaking order in the QDM. For example, the absence of the $Z_2$ topological order can be easily seen when the dimers in the QDM froze into a static valence bond solid dimer configuration of either the staggered or columnar type, which is a simple product state. Spinon excitation is obviously confined in such a classical background. More generally, it is widely believed that the topological order and the related fractionalized excitation should be found in system without any conventional symmetry breaking order. However, a fully symmetric state is usually very difficult to realize in the strict sense. On the other hand, spectral signature that is akin to the continuum of fractionalized excitation has been reported in more than one quantum magnetic systems with long-range magnetic order. It is thus crucial to know if the conventional symmetry breaking order can coexist with topological order.

The purpose of this paper is to provide a concrete example for the coexistence of the topological order and the conventional symmetry breaking order in a QDM. The existence of the topological order is here proved by the non-trivial topological entanglement entropy of the system. We also show that the transition between the topological solid phase and the trivial solid phase is driven by the condensation of the vison excitation.

The quantum state used to demonstrate the coexistence of topological order and conventional symmetry breaking order in the QDM is given by the following generalized Rokhsar-Kivelson (R-K) wave function

$$|\Psi\rangle = \sum_C \psi(C)|C\rangle.$$  \hspace{1cm} (1)

Here $|C\rangle$ denotes a general dimer covering of the triangular lattice. The amplitude $\psi(C)$ is given by $\psi(C) = n^\eta$, in which $n$ is the number of dimers that occupy the thick bonds illustrated in Figure 1. $|\Psi\rangle$ reduces to the standard R-K wave function of the triangular QDM when $\eta = 1$, which is a well known example of $Z_2$ topological liquid. When $\eta > 1$, the lattice symmetry is broken explicitly to establish a columnar valence bond solid order. It is not clear if the topological degeneracy is robust or not in this situation.
Here $C_1$ and $C_2$ denote dimer configuration in the system and the replica respectively. $S(C_1,C_2)$ is 1 if the swapped configuration satisfy the dimer constraint and is otherwise zero.

In a previous work, a very efficient ratio trick is proposed to evaluate the above trace with the Monte Carlo method [21]. More specifically, $\text{Tr}\rho_s^2$ can be represented as the product over a series of ratios as follows

$$\text{Tr}\rho_s^2 = \prod_{i=1,m} \frac{\sum_{C_1,C_2} S(C_1,C_2) (r_i-1)^{N_{err}}}{\sum_{C_1,C_2} (r_i)^{N_{err}}},$$

in which $r_i \in [0,1]$ is a series of real numbers satisfying $r_0 = 0$, $r_m = 1$ and $r_i < r_j$ for $i < j$. $N_{err}$ is the number of times that the dimer constraint is violated in the swapped configuration [21]. If the difference between the successive $r_i$ is chosen to be sufficiently small, then each ratio in the product can be simulated efficiently with Monte Carlo sampling. The essence of the ratio trick is to find a smooth interpolation between the distribution in the numerator of Eq.(2) and that in the denominator of Eq. (2), which are realized when $r_0 = 0$ and $r_m = 1$.

To extract the value of TEE from $S_2$, we adopt the Kitaev-Preskill subtraction scheme and divide the subsystem into three smaller parts, A, B and C, as is illustrated in Figure 2. Both A and B contain $n_a \times n_a$ sites and C contains $n_a \times 2n_a$ sites. Then we have

$$-\gamma = S_A^2 + S_B^2 + S_C^2 - S_A^2 S_B^2 C^2 - S_A^2 S_B^2 C^2 + S_A^2 S_B^2 C^2.$$

This algorithm has been used to demonstrate the ln2 TEE for the triangular QDM [20, 22].

We have adopted the above algorithm to study the topological property of the generalized R-K state $|\Psi\rangle$. The system is defined on a $L \times L$ torus. Here we set $L = 24$. The subsystem size considered varies from $n_a = 3$ to $n_a = 6$. We have introduced 10 intervals for $r_i$ in the ratio trick. For each ratio in the product, we have performed Monte Carlo simulation with up to $5 \times 10^7$ independent samples.

The result of TEE as a function of $\eta$ is plotted in Figure 3. The most prominent feature in the plot is the broad peak in $\gamma(\eta)$ around $\eta^2 \approx 2.75$. For $\eta < \eta_c$, $\gamma(\eta)$ converges to ln 2, the value expected for a $Z_2$ topological ordered phase. For $\eta \gg \eta_c$, $\gamma(\eta)$ converges to zero, as expected for a trivial solid phase. The broad peak in $\gamma(\eta)$ is thus the signature of phase transition between a topological non-trivial solid phase and a trivial solid phase.

In principle, both $L$ and $n_a$ should be extrapolated to infinity to extract the value of $\gamma(\eta)$. For the fixed torus size of $L = 24$, we find that $\gamma(\eta)$ converges gradually to ln2 when $\eta < \eta_c$ as we increase the subsystem size $n_a$, provided that $2n_a < L/2$ [23]. We thus believe that the $Z_2$ topological order is robust in the whole region of $1 < \eta < \eta_c$. For the same reason, we believe that the system
The larger subsystem size is needed to extract the intrinsic boundary length of the subsystem. At the same time, the value of $\text{Tr}p_s^2$ for larger $L$ and the width of the peak in $\gamma(\eta)$ is topological trivial when $\eta > \eta_c$. We thus expect that the width of the peak in $\gamma(\eta)$ should shrink to zero when both $L$ and $n_a$ are extrapolated to infinity. Calculation for larger $L$ and $n_a$ is definitely needed to confirm such an expectation. However, the calculation of TEE for large $L$ and $n_a$ becomes prohibitively expensive as a result of critical behavior of the vison correlation seems to indicate a continuous transition between the topological solid phase and the trivial solid phase. A more thorough study is definitely needed to learn more accurately the critical behavior of this phase transition.

We note that the generalized R-K state proposed in this work has the same symmetry as the columnar VBS phase of the triangular QDM\cite{8,15}. It is however not clear if the columnar VBS phase realized by the standard triangular QDM is topological trivial or not. Subtle modification on the standard QDM may be necessary to stabilize the topological solid phase proposed in this paper. This is very similar to the situation of supersolid which, although allowed by physical principle, is highly non-trivial to find model realization.

In conclusion, through the calculation of the topological entanglement entropy of a generalized R-K state of the triangular QDM, we established a concrete example for the coexistence of the $Z_2$ topological order and conventional symmetry breaking order. Our result implies that the fractionalized excitation is not necessarily confined in the background of conventional symmetry breaking order. We believe that such coexistence is
FIG. 4: The correlation function between two visons at the largest distance on a $36 \times 36$ torus in the $\vec{a}_1$ direction of the triangular lattice. $\vec{a}_1$ is the translation vector in direction of the thick bond in Figure 1.

not limited to the lattice symmetry breaking studied in this work, but is relevant for general Landau symmetry breaking order. In particular, we expect that the topological order can coexist with the magnetic order. This may be relevant for the recent experimental observation of spinon-continuum-like spectral feature in several magnetic ordered quantum magnets [10–12].

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