Hencky’s model for elastomer forming process

A A Oleinikov and A I Oleinikov

1 Faculty of informational technologies, mathematics and physics, Amur State University of Humanities and Pedagogy, Komsomolsk-na-Amure, 681000, Russia
2 Research and Technology Center, Central Aerohydrodynamic Institute, Zhukovsky, MR, 140180, Russia
3 Department of Aeromechanics and Flight Engineering, Moscow Institute of Physics and Technology, MR, 140180, Zhukovsky, Russia

E-mail: andriy.oleinikov@mail.ru

Abstract. In the numerical simulation of elastomer forming process, Hencky’s isotropic hyperelastic material model can guarantee relatively accurate prediction of strain range in terms of large deformations. It is shown, that this material model prolongate Hooke’s law from the area of infinitesimal strains to the area of moderate ones. New representation of the fourth-order elasticity tensor for Hencky’s hyperelastic isotropic material is obtained, it possesses both minor symmetries, and the major symmetry. Constitutive relations of considered model is implemented into MSC.Marc code. By calculating and fitting curves, the polyurethane elastomer material constants are selected. Simulation of equipment for elastomer sheet forming are considered.

1. Introduction

The greatest problem of machine building consists in permanent upgrading of the manufacturing methods. The main idea behind the process of equipment development is to replace the die forming, that is based on rigid dies and punches, by flexible dies and punches variable in forming. In the equipment, that is considered, the die and punch are formed as a system of coaxial stem rods with the blank being secured between them. The high points of the rods, contacting to the panel blank are defined by Hencky’s elasticity shaped as hemispheres in unloading state. Such equipment ensures die forming simultaneously for both principal curvatures without development of cylindrical stiffness and allows parts with double curvature to be manufactured within one process pass. The paper deals with simulation of equipment actuator operation of industrial equipment for the pressure treatment of materials, including elastomer forming process.

It is well known, that application of complex material models that are efficient in all the range of elastomers deforming needs accurate experiment definition and huge work of parameters searching for description of experimental curves. It is shown, that the Hencky’s isotropic hyperelastic material model provide good approximation of elastomers strains up to 50 % of scale.

2. Hencky’s isotropic hyperelastic constitutive material model

Currently theoretical fundamentals of the hyperelastic theory are well developed, but there are many open questions yet, like constructing more efficient forms of equations in terms of their usage in the applications. In the present work new approach for formulation of constitutive
relations of Hencky’s isotropic hyperelastic material is suggested. This approach is stated on tensors representations through eigenprojections of right Cauchy–Green strain tensor $C$.

2.1. Basic kinematics

Introduce the velocity vector $v$ and the velocity gradient $l \in T^2$

$$v = \dot{x}, \quad l \equiv \text{grad} v \equiv r_F = \dot{F} \cdot F^{-1}. \quad (1)$$

Introduce the symmetric Eulerian strain rate tensor $d \in T_{sym}^2$ and the Lagrangian counterpart $D$ of the tensor $d$ called the rotated strain rate tensor $D = R \cdot D \cdot R^T$

$$d \equiv \text{sym} l = \frac{1}{2}(l + l^T), \quad D = \frac{1}{2}(\dot{U} \cdot U^{-1} + U^{-1} \cdot \dot{U}), \quad d = R \cdot D \cdot R^T. \quad (2)$$

Let $H \in T^2$ is a Lagrangian tensor. Enter the Lagrangian D-rate of the tensor $H$ [1]:

$$H^D \equiv \dot{H} - \Omega^D \cdot H + H \cdot \Omega^D, \quad (3)$$

where

$$\Omega^D \equiv \Psi_r(U, D) = \sum_{i \neq j=1}^m r(z)U_i \cdot D \cdot U_j. \quad (4)$$

Here $\Psi_r(U, D) \equiv$ skew-symmetric tensor function, where $z \equiv \lambda_i/\lambda_j$, and scalar-valued function $r(z)$ evaluated in this way:

$$r(z) \equiv \begin{cases} 2z - z^{-1} + \frac{1}{\ln z}, & \text{if } z \neq 1, \\ 0, & \text{if } z = 1 \equiv \lim_{z \to 1}(2z - z^{-1} + \frac{1}{\ln z}). \quad (5) \end{cases}$$

Notice, that the D-rate of the tensor $H$ possesses remarkable property

$$E^{(0)D} = D. \quad (6)$$

2.2. Hyperelastic Constitutive Relations

Let consider constitutive relations of the Hencky’s hyperelastic isotropic material model in form

$$\tau = \lambda(\text{tr} E^{(0)})I + 2\mu E^{(0)}, \quad (7)$$

where $\tau$ is Noll’s (Lagrangian) stress tensor, tensor $E^{(0)}$ called the right logarithmic strain tensor (right (Lagrangian) Hencky strain tensor) [2], $\lambda$, $\mu$ — Lame constants. Or [3]

$$\bar{\tau} = C_E : E^{(0)}, \quad (8)$$

where $C_E \in T_{sym}^4$ — Lagrangian tensor:

$$C_E \equiv \lambda C_1 + \mu(C_{II} + C_{III}) = \lambda C_1 + 2\mu S. \quad (9)$$

For usage in applications, we express the second Piola–Kirchhoff stress tensor $S^{(2)}$ through the eigenprojections of the right Cauchy–Green strain tensor $C$, principal stretches $\lambda_i$ express through the eigenvalues $\mu_i$ of the tensor $C$ ($\mu_i = \lambda_i^2 \ (i = 1, \ldots, m)$).
Constitutive relations of the Hencky’s isotropic hyperelastic medium we enter in form

\[ S^{(2)} = \sum_{i=1}^{m} \tau_i / \mu_i C_i. \]  

Enter constitutive relations of the Hencky’s material in velocity:

\[ \dot{\tau} = C^E : \dot{E}^{(0)} \iff \dot{\tau}^D = C^E : D, \]  

where \( \dot{\tau}^D \) identify the objective Lagrangian corotational D-rate of the tensor \( \dot{\tau} \) (see (3)) [1, 4]. From (11) we derive:

\[ S^{(2)} = C : \dot{E}^{(2)}, \]  

where

\[ C \equiv \sum_{i,j=1}^{m} \frac{\lambda}{\mu_i \mu_j} C_i \otimes C_j + \sum_{i=1}^{m} \frac{2(\mu - \tau_i)}{\mu_i^2} C_i \text{ sym} \otimes C_i + \sum_{i \neq j=1}^{m} \frac{2}{\mu_i - \mu_j} \left( \frac{\tau_i}{\mu_i} - \frac{\tau_j}{\mu_j} \right) C_i \text{ sym} \otimes C_j. \]  

The proof of the fact, that derived elasticity tensor possesses both minor symmetries, and also the major symmetry is very useful in applications. This properties have major theoretical meaning in incremental formulations of hyperelasticity, when formulate variational principles for equations, stated with velocities, and criteria of uniqueness and stability of the solutions for this equations.

3. Application of the finite element method for solving of motion equations of deformable bodies of Hencky’s isotropic hyperelastic material

It is well known, that performing a finite element analysis (FEA) on a hyperelastic material is difficult due to nonlinearity, large deformations, and material instability. In this paper Lagrangian formulation of Hencky’s isotropic hyperelastic material constitutive relations is implemented into FEA software package MSC.Marc, that is utilized for the simulations, based on different hyperelastic material models. This implementation is realized for three-dimensional isoparametric finite elements with usage of Fortran-code of user’s subroutine hypela2.f.

In all testing calculations we used constants of the Hencky’s material, which are close to constants of the real elastomers: \( \nu = 0.45, E = 3.37 \text{ MPa}, \rho = 2000 \text{ kg/m}^3 \). Here \( \nu \) — Poisson’s ratio, \( E \) — Young’s modulus, \( \rho \) — mass density of the rod’s material. Young’s modulus and Poisson’s ratio are associated with Lame constants following way:

\[ \lambda = \frac{E \nu}{(1 + \nu)(1 - 2\nu)}, \quad \mu = \frac{E}{2(1 + \nu)}. \]  

Let consider a simple shear problem. This problem is testing problem to approbation elastic and nonelastic material models in terms of large deformations. Solution of this problem, that is derived with usage of the FEA software package MSC.Marc, gives us good agreement for components of Cauchy stress tensor with exact solution. We consider a cube with edge length 0.1 m, which is undergoing simple shear. Finite element model is represented by one eight-nodal three-dimensional isoparametric finite element. Derived solution for stress tensor component \( s_{12} \) coincide with solution, that is derived in [5, 6]. Equilibrium points, that are derived in numerical solution, with pinpoint accuracy lay on curves of such relations of exact solution. The stress tensor component \( s_{12} \) have maximum point, which have coordinates [5, 6]:

\[ \gamma_m = 3.0177171, \quad s_{12,m} = 4 \mu / \gamma_m = 1.32548684 \mu. \]  

\( \gamma_m \) is the maximum shear strain, \( s_{12,m} \) is the maximum shear stress.
Cauchy stress tensor components subject to parameter $\gamma$ in Cartesian coordinate system:

$$s_{11} = \frac{\mu \gamma \ln \mu_1}{\sqrt{4 + \gamma^2}}, \quad s_{22} = -s_{11}, \quad s_{12} = \frac{2\mu \ln \mu_1}{\sqrt{4 + \gamma^2}}. \quad (16)$$

Component of stress tensor $s_{22}$ gives us important information for researching of Pointing’s effect. Pointing’s effect is that, at free bending of rod with circular cross-section in axial direction, in terms of large strain, length of axial line of rod is changing. Simple shear problem is two-dimensional analogue of three-dimensional problem of bending torsion of elastic rod.

4. Simulation of equipment actuator operation

The computation model corresponds to the specially conducted experiment on forming of a 600 $\times$ 400 $\times$ 10 mm flat sheet blank (V95pch2 material). In this experiment, each lower rod moved uniformly with its own speed, so that desired blank bending was achieved at the final time. Owing to blank forming symmetry, the computation model was built for 1/4 of the plate at each instant of rod travel. Results of using software support and simulation of the WP geometry in forming under elastoplastic conditions are further provided.

The first series of computations ensured simulations of the technical procedure of forming through blank bending with upward camber provided by the downward travel of the upper rods. Fig. 1 (a, b) shows the blank position during its bending at the times $t = 10$, and 20, respectively. The time $t = 10$ corresponds to the maximum forming (anticipatory working profile), and $t = 20$ corresponds to the result obtained after springback. A three-dimensional stress analysis is performed at each time instant by the finite element method in the MSC.Marc package with allowance for the geometrical nonlinearity of the DBM equations relative to the default configuration with elastoplastic material performance corresponding to the V95pch2 alloy at 20$^\circ$C. The shear stress intensity distribution on the upper blank surface at $t = 10$ is shown Fig. 1. Thus, three-dimensional simulations of actuator operation allows simulating the work piece performance and variable tooling geometry, and also reproducing the operation of moving equipment parts.

Figure 1. Positions of the blank during its bending by the upper rods: a) at the times $t = 10$ (anticipatory WP); b) at the time $t = 20$ (unloaded condition).

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