Bi-maximal Neutrino Mixing With SO(3) Flavour Symmetry

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We demonstrate that an $SU(2)_L \times U(1)_Y$ model with extended Higgs sector gives rise to bi-maximal neutrino mixing through the incorporation of SO(3) flavour symmetry and discrete symmetry. The neutrino and the charged lepton masses are generated due to higher dimensional terms. The hierarchical structures of neutrinos and charged leptons are obtained due to inclusion of SO(3) flavour symmetry and discrete symmetry. The model can accommodate the vacuum oscillation solution of solar neutrino problem, through reasonable choice of model parameters along with the atmospheric neutrino experimental result.

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Evidence in favour of neutrino oscillation (as well as neutrino mass) has been provided by the Super-Kamiokande (SK) atmospheric neutrino experiment [1] through the measurement of magnitude and angular distribution of the $\nu_\mu$ flux produced in the atmosphere due to cosmic ray interactions. Observed depletion of $\nu_\mu$ flux in earth has been interpreted as the oscillation of $\nu_\mu$ to some other species of neutrino. In a two flavour neutrino oscillation scenario, oscillation between $\nu_\mu - \nu_\tau$, the experimental result leads to maximal mixing between two species $\sin^2 2\theta \sim 0.82$ with a mass-squared difference $\Delta m^2_{atm} \sim (5 \times 10^{-4} - 6 \times 10^{-3}) \text{ eV}^2$. The solar neutrino experimental results [2] are also in concordance with the interpretation of atmospheric neutrino experimental result and the data provide the following values as $\Delta m^2_{\text{e}\mu} \sim (0.8-2) \times 10^{-5} \text{ eV}^2$, $\sin^2 2\theta \sim 1$ (Large angle MSW solution) or $\Delta m^2_{\text{e}\mu} \sim (0.5-6) \times 10^{-10} \text{ eV}^2$, $\sin^2 2\theta \sim 1$ (vacuum oscillation solution). Furthermore, the CHOOZ experimental result [3] gives the value of $\Delta m^2_{\text{eX}} < 10^{-3} \text{ eV}^2$ or $\sin^2 2\theta_{\text{eX}} < 0.2$. In order to reconcile with the solar and atmospheric neutrino experimental results, a distinct pattern of neutrino mixing emerges, namely, bi-maximal neutrino mixing [4], in which $\theta_{12} = \theta_{23} = 45^o$, and if, the CHOOZ experimental result is interpreted in terms of $\nu_e - \nu_\tau$ oscillation, then $\theta_{31} < 13^o$.

In the present work, we demonstrate that an SU(2)$_L \times$ U(1)$_Y$ model with extended Higgs sector coupled with an SO(3) flavour symmetry [5,6] and discrete $Z_3 \times Z_3 \times Z_4$ symmetry, gives rise to nearly bi-maximal neutrino mixing along with the vanishing value of $\theta_{31}$. We have also discussed the situation when the mixing is exactly bi-maximal. Instead of three almost degenerate neutrinos [5,7], we obtain a hierarchical pattern of neutrino masses. The
charged lepton masses are also hierarchical in nature and this is due to the inclusion of SO(3) flavour symmetry, which, when spontaneously broken, gives rise to the desired hierarchy in mass [6]. The discrete $Z_3 \times Z'_3 \times Z_4$ symmetry prohibits unwanted mass terms in the charged lepton and neutrino mass matrices as well as when accompanied with the choice of residual phases of the mass matrices give rise to required mixing pattern. We consider soft discrete symmetry breaking terms in the scalar potential, which are also responsible to obtain non-zero values of the VEV’s of the Higgs fields upon minimization of the scalar potential. The Majorana neutrino masses are obtained due to explicit breaking of lepton number through higher dimensional terms. The leptonic fields ($l_{iL}, E_{iR}, i = 1, 2, 3$ is the generation index) and the Higgs fields ($\chi, \xi, \phi_1, \phi_2, \phi_3, \xi_e, \xi_\mu, h$) have the following representation contents:

\[
l_{iL} \ (1, 2, -1), \ E_R \ (3, 1, -2), \ \chi \ (3, 1, 0), \ \xi \ (3, 1, 0), \\
\phi_1 \ (3, 1, 0), \ \phi_2 \ (3, 1, 0), \ \phi_3 \ (3, 1, 0), \ h \ (1, 2, 1), \\
\xi_e \ (1, 1, 0), \ \xi_\mu \ (1, 1, 0)
\]

where the digits in the parentheses represent SO(3), SU(2)$_L$ and U(1)$_Y$ quantum numbers. The subscript of the $\phi$ Higgs fields denote the direction of development of non-zero VEV, such as $<\phi_1> = (v_1, 0, 0)$ etc. and the subscript below $\xi$ fields denote the respective coupling with the charged leptons.

Regarding Higgs content and the symmetry breaking pattern, the present model is analogous to a supersymmetric model discussed in Ref.[6], where the Higgs scalars are replaced by ‘flavon’ chiral superfields. Regarding the representation content of leptonic fields, three lepton doublets ($l_{1L}, l_{2L}, l_{3L}$) are forming an SO(3) triplet while the right-handed charged leptons ($e, \mu, \tau$) are singlet under SO(3) in Ref.[6] and this is just opposite to our case. We
consider the following discrete $Z_3 \times Z'_3 \times Z_4$ symmetry transformation of the lepton and Higgs fields:

$Z_3 \times Z'_3 \times Z_4$ Symmetry

\[
 l_{1L} \rightarrow i\omega l_{1L}, \ l_{2L} \rightarrow -il_{2L}, \ l_{3L} \rightarrow -il_{3L}, \ E_R \rightarrow i\alpha E_R \\
\chi \rightarrow \alpha \omega \chi, \ \xi \rightarrow i\xi, \ h \rightarrow h, \ \phi_1 \rightarrow \alpha \omega^2 \phi_1, \ \phi_2 \rightarrow \alpha^2 \omega \phi_2, \\
\phi_3 \rightarrow \alpha^2 \phi_3, \ \xi_e \rightarrow \alpha^2 \omega \xi_e, \ \xi_\mu \rightarrow -\alpha^2 \omega \xi_\mu
\]

where $\omega$ and $\alpha$ are the generators of $Z_3$ and $Z'_3$ group, respectively. The most general lepton-Higgs Yukawa interaction in the present model generating Majorana neutrino masses is given by

\[
 L'_Y = \beta_1 \frac{(l_{1L}l_{2L})(\chi\chi)hh}{M_f^3} + \beta_2 \frac{(l_{1L}l_{3L})(\chi\chi)hh}{M_f^3} + \beta_3 \frac{(l_{2L}l_{2L})(\xi\xi)hh}{M_f^3}
 + \beta_4 \frac{(l_{2L}l_{3L})(\xi\xi)hh}{M_f^3} + \beta_5 \frac{(l_{3L}l_{3L})(\xi\xi)hh}{M_f^3}
\]

and the Yukawa interaction which is responsible for generation of charged lepton masses is given by

\[
 L_Y^E = \beta_6 \frac{(e_R\phi_1)l_{2L}h\xi_e^2}{M_f^3} + \beta_7 \frac{(e_R\phi_1)l_{3L}h\xi_e^2}{M_f^3} + \beta_8 \frac{(e_R\phi_2)l_{1L}h\xi_\mu}{M_f^3}
 + \beta_9 \frac{(e_R\phi_3)l_{2L}h}{M_f} + \beta_{10} \frac{(e_R\phi_3)l_{3L}h}{M_f}
\]

In the above Lagrangian, we consider all the couplings $\beta_1,...,\beta_{10}$ are complex and are given by $\beta_i = |\beta_i|e^{i\delta_i}$ ($i = 1,...,10$). For our analysis, we consider $|\beta_i| = 1$. Among ten phases, it is possible to absorb any five of them by redefining lepton fields and among the residual five phases, we set $\delta_2 = \delta_4 = \delta_7 = \pi$ and rest of them equal to 0. Although the dynamical origin of such
choice of phases is not clear (also not prohibited) in the present framework, however, from the model building point of view such choice plays important role to obtain viable phenomenological scenario. We also consider all the VEV’s are real. The present model contains a large mass scale \( M_f \), and for our analysis we set \( M_f \sim M_{\text{GUT}} \). We also consider that the flavour symmetry group \( \text{SO}(3) \) is broken below the GUT scale, but much above the electroweak scale corresponds to \( \langle h \rangle \). On the otherway, the scale of \( \text{SO}(3) \) symmetry breaking VEV’s, \( \langle \chi \rangle \) and \( \langle \xi \rangle \) are constrained by the solar and atmospheric neutrino experimental results, and the VEV’s of \( \langle \phi_1 \rangle \), \( \langle \phi_2 \rangle \), \( \langle \phi_3 \rangle \), \( \langle \xi_e \rangle \) and \( \langle \xi_{\mu} \rangle \) determine the masses of the charged leptons. The Higgs fields \( \xi_e, \xi_{\mu} \) are singlet under the gauge symmetry and their VEV’s in principle can take values above the \( \text{SO}(3) \) symmetry breaking scale.

In order to avoid any zero values of the VEV’s of the Higgs fields upon minimization of the scalar potential, we have to consider discrete symmetry breaking terms. Without going into the details of the scalar potential, this feature can be realized in the following way. In general, the scalar potential can be written as (keeping upto dim=4 terms)

\[
V = Ay^4 + By^3 + Cy^2 + Dy + E
\]  

(5)

where ‘\( y \)’ is the VEV of any Higgs field and A, B, C, D, E are generic couplings of the terms contained in the scalar potential. Minimizing the scalar potential w.r.t. ‘\( y \)’, we obtain

\[
V' = A'y^3 + B'y^2 + C'y + D
\]  

(6)
Eqn.(6) reflects the fact that as long as $D \neq 0$, and $A'$ or $B'$ or $C'$ is not equal to zero, we will get non-zero solutions for '$y'$. Thus, in order to obtain $y \neq 0$ solution, it is necessary to retain the terms with generic coefficients $D$ and $A'$ or $B'$ or $C'$. In the present model, both the discrete symmetry breaking terms soft and hard, correspond to the term with coefficient $D$. Discarding hard symmetry breaking terms, we retain soft discrete symmetry breaking terms, and, hence, none of the VEV is zero upon minimization of the scalar potential.

Let us look at the charged lepton sector. Substituting the VEV's of the Higgs fields appeared in Eqn.(4), we obtain the charged lepton mass matrix given by

$$M_E = \begin{pmatrix} 0 & d & -d \\ e & 0 & 0 \\ 0 & f & f \end{pmatrix}$$

(7)

where $d = \frac{\langle \phi_1 \rangle < h > < \xi_e >^2}{M_f}$, $e = \frac{\langle \phi_2 \rangle < h > < \xi_\mu >}{M_f}$ and $f = \frac{\langle \phi_3 \rangle < h >}{M_f}$. The hierarchy between the $d$, $e$ and $f$ parameters, $d < e < f$ is manifested due to the large mass scale $M_f$. Diagonalizing $M_E M_E^T$, we obtain the following eigenvalues and mixing angles as

$$m_{E_1} = \sqrt{2}d$$

$$m_{E_2} = e$$

$$m_{E_3} = \sqrt{2}f$$

(8)

and $\theta_{E_{12}} = \theta_{E_{23}} = \theta_{E_{31}} = 0$. It is to be mentioned that the zero values of $\theta_{E_{12}}$, $\theta_{E_{23}}$ is assured due to discrete symmetry invariance however, the zero value of $\theta_{E_{31}}$ is obtained due to our choice of the value of $\delta_7$. The hierarchy in the charged lepton masses arises due to the hierarchy already manifested in
the mass matrix given in Eqn. (7). The three eigenvalues of the matrix $M_E$ can be fitted with the masses of the three charged leptons and the vanishing value $\theta_{31}^E$ is also not in conflict with the experimental value given by CHOOZ experiment as mentioned earlier.

Let us now focus our attention to the neutrino sector of the model. Substituting the VEV’s of $\chi$, $\xi$ and $h$ Higgs fields in Eqn. (3), we get the Majorana neutrino mass matrix as follows:

$$M_{\nu} = \begin{pmatrix} 0 & a & -a \\ a & b & -b \\ -a & -b & b \end{pmatrix}$$  \quad \text{(9)}$$

where $a = \frac{<\chi>^2<h>^2}{M_f^2}$, $b = \frac{<\xi>^2<h>^2}{M_f^2}$. It is to be noted that the absence of $\nu_e\nu_e$ mass term in the above mass matrix (at the tree level) evades the bound on Majorana neutrino mass due to $\beta\beta_{0\nu}$ decay. Diagonalizing the neutrino mass matrix $M_{\nu}$ by an orthogonal transformation, we obtain the following values of the mixing angles, $\theta_{23} = \frac{\pi}{4}$, $\theta_{31} = 0$ and $\tan^2\theta_{12} = \frac{m_{\nu_1}}{m_{\nu_2}}$. The eigenvalues of the above mass matrix comes out as

$$-m_{\nu_1} = b - x$$
$$m_{\nu_2} = b + x$$
$$m_{\nu_3} = 0$$  \quad \text{(10)}$$

where $x = \sqrt{b^2 + 2a^2}$. It is to be noted that although the sign of $m_{\nu_1}$ can be made positive by setting appropriate values of residual phases, however, we will see that this can also be achieved due to phenomenological choices of model parameters. In the limit $b \to 0$, $\theta_{12} \to \frac{\pi}{4}$, the two eigenvalues $m_{\nu_1}$ and $m_{\nu_2}$ become degenerate and we can achieve the exact bi-maximal neutrino
mixing. In this situation, although we obtain the exact bi-maximal neutrino mixing however, the obtained eigenvalues $m_{\nu_1} = m_{\nu_2}$ and $m_{\nu_3} = 0$, can be fitted with either the solar or the atmospheric neutrino experimental result. Removal of degeneracy between the two eigenvalues require further higher order corrections \footnote{Almost degenerate neutrinos with bi-maximal mixing can also be achieved by setting $\delta_2 = 0$ (instead of $\pi$) in Eqn.(3) and in this case we obtain $m_{\nu_1} = -\sqrt{2}a$, $m_{\nu_2} = \sqrt{2}a$ and $m_{\nu_3} = 2b$ with $\theta^\nu_{23} = \theta^\nu_{12} = -\frac{\pi}{4}$ and $\theta^\nu_{31} = 0$.}. For oue analysis, we set the value of $\Delta m^2_{21} = \Delta m^2_{sol}$ which in turn sets the value of $\theta_{12}$. The value of $x$ depends on the hierarchical relation between $a$ and $b$ parameters which is manifested from the values of $\Delta m^2_{21} = 4bx$ and $\Delta m^2_{23} = (b + x)^2$. Now, if, $b^2 > 2a^2$, then the value of $x$ comes out as $x \sim b$ and, hence, $m_{\nu_1} = m_{\nu_3} = 0$ and $m_{\nu_2} = 2b$. In this situation also both the mass squared differences are parametrized in terms of a single parameter $b$, and, hence, in this case it is not possible to accommodate both the results of solar and atmospheric neutrino experiments. The same scenario appears for $a = b$ case, and, hence, for a phenomenologically viable model, we have to consider the third option $2a^2 > b^2$ and in this case $m_{\nu_1}$ is also become positive. In this situation, we obtain, $\Delta m^2_{21} = 4\sqrt{2}ab$, $\Delta m^2_{23} = (a\sqrt{2} + b)^2 \sim 2a^2$. For a typical value of $\Delta m^2_{23} \sim 4 \times 10^{-3}$ eV$^2$ which can explain the atmospheric neutrino deficits, we obtain $2a^2 \sim 4 \times 10^{-3}$ eV$^2$ which in turn gives rise to the value of $< \chi > \sim 10^{11}$ GeV for $M_f \sim 10^{12}$ GeV and $< h > \sim 100$ GeV. Using the same values of $M_f$ and $< h >$ parameters, we set the solar neutrino experimental result by setting the value of parameter $b$. For a typical value of $\Delta m^2_{21} \sim 4 \times 10^{-10}$ eV$^2$ which can explain the solar neutrino deficits due to vacuum oscillation, the value of $b^2$ comes out as $b^2 \sim 0.25 \times 10^{-17}$ eV$^2$ which leads to the value of $< \xi > \sim 10^7$ GeV. The mixing angle
θ_{12} in this case comes out as \( \tan \theta_{12} = \frac{\sqrt{2} - b}{a \sqrt{2} + b} \) and since \( a > b \), \( \theta_{12} \to 45^\circ \), and, hence, there is no conflict to satisfy the value of \( \theta_{12} \) well within the allowed range of the experimental value. For large angle MSW solution, a typical value of \( \Delta m_{21}^2 \sim 10^{-5}\text{eV}^2 \) gives rise to \( b^2 \sim 10^{-9}\text{eV}^2 \) and \( \xi \sim 2 \times 10^9 \text{GeV} \), however, in this situation, since \( b \sim a \) we obtain \( \theta_{12} \sim \frac{\pi}{2} \), and, hence, large angle MSW solution is unlikely in the present model.

In summary, we demonstrate that an \( SU(2)_L \times U(1)_Y \) model with an extended Higgs sector, SO(3) flavour symmetry and discrete \( Z_3 \times Z'_3 \times Z_4 \) symmetry, gives rise to nearly bi-maximal neutrino mixing \( \theta_{23} = \frac{\pi}{4} \), \( \theta_{12} \sim \frac{\pi}{4} \) and \( \theta_{31} = 0 \) consistent with the present solar and atmospheric neutrino experimental results. Neutrino masses are generated due to explicit lepton number violating higher dimensional terms (dim=7) and the charged lepton masses are generated due to dim=5,6,7 terms. The hierarchical structure of charged lepton masses is obtained due to the inclusion of SO(3) flavour symmetry. The flavour diagonal structure of charged lepton mass matrix is also obtained due to our choice of residual phases in the charged lepton mass matrix. Due to the same choice of the residual phases in the neutrino mass matrix, three non-degenerate neutrino masses are obtained with the value of the two mixing angles as \( \theta_{23} = \frac{\pi}{4} \) and \( \theta_{31} = 0 \) respecting the atmospheric and CHOOZ experimental results, respectively. The value of \( \theta_{12} \) depends on the masses of the neutrinos and in the exact limit of bi-maximal mixing (\( \theta_{12} = \frac{\pi}{4} \)), we obtain two degenerate neutrino masses. The discrete symmetry prohibits \( \nu_e \nu_e \) mass term in the neutrino mass matrix at the tree level so as to evade the bound on the Majorana neutrino mass from \( \beta \beta_{0\nu} \) decay in the present model. The vacuum neutrino oscillation solution can be achieved in
the present model for a reasonable choice of model parameters along with
the atmospheric neutrino experimental result whereas the large angle MSW
solution is unlikely in the present model.

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