Lepton Masses protect the Primordial Baryogenesis from Sphaleron Erasure

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ABSTRACT

We present a revision of the analysis of sphaleron baryon-number violating processes in the standard model including lepton-mass effects. We find the surprising result that a GUT-scale matter-asymmetry can survive the $B$ and $L$ violating sphaleron interactions even though $(B - L)$ is conserved and equals zero for all temperatures. We extend the analysis to cover the minimal supersymmetric standard model (MSSM). As a new result we present the MSSM analysis below the temperature of electroweak breaking and for $B - L = 0$ at all temperatures\footnote{This is a very concise summary of work performed in collaboration with G.G. Ross Ref.\[3\]. The only new result is given in Eq.\[1\]. Throughout we shall use the notation of\[4\].}

It has been observed that the requirement of a cosmological baryon-asymmetry leads to powerful constraints on the initial nature of the baryon asymmetry. The basic point is that sphaleron induced processes are thought to be in thermal equilibrium above the electroweak breaking scale leading to the possible erasure of any pre-existing baryon-number excess. They conserve $(B - L)$ and so, it is normally argued, any baryon-number excess produced at an early epoch by a $(B - L)$ conserving interaction will be erased. Thus $(\frac{1}{3}B - L_i) = 0$ channel. It should also be clear that $B - L = 0$ at all temperatures does not necessarily imply $(\frac{1}{3}B - L_i) = 0$, and thus a baryonasymmetry can in principle survive. We shall see in the chemical potential analysis how this happens.

The net number density $n_k$ of a given particle species is approximately given by

$$n_k \approx \frac{g_k}{\pi^2} T^3 \left( \frac{\mu_k}{T} \right) \int_{m_k/T}^{\infty} y \sqrt{y^2 - m_k^2/T^2} \frac{e^y}{(1 + e^y)^2} dy = \frac{g_k}{\pi^2} T^3 \left( \frac{\mu_k}{T} \right) F_\pm \left( \frac{m_k}{T} \right),$$

proportional to $\frac{g_k}{\pi^2} T^3 (\mu_k/T)$. We have assumed $\mu/T \ll 1$. We denote the mass dependent proportionality factor

$$F_\pm \left( \frac{m_k}{T} \right) = F_\pm (0) \alpha \frac{m_k}{T}.$$\footnote{$T_{sphal}$ is the lowest temperature where the sphalerons are still in equilibrium.}

The introduction of $\alpha \not= 1$ for non-zero masses is the decisive new point in our analysis.

At sufficiently high temperatures and densities the chemical potentials of the photon, gluon and $Z^0$ vanish. Therefore the chemical potentials of particles and antiparticles are equal and opposite and different coloured quarks have equal chemical potentials. The off-diagonal Yukawa interactions guarantee that the chemical potential of all up-like quarks and all down-like quarks are respectively equal. We are thus left with $3N + 7$ chemical potentials. It is convenient to introduce the following notation ($\mu_i = \mu_{\nu_i}$)

$$\alpha_i = \frac{\alpha(m_{\nu_i}/T),}{\Delta_i = N - \sum_i \alpha_i,}{\mu = \sum_i \alpha_i \mu_\nu_i,}\quad \Delta \mu = \mu - \bar{\mu},\quad \Delta_u = N - \sum_i \alpha (m_{u_i}/T_{sphal}),\quad \Delta_d = N - \sum_i \alpha (m_{d_i}/T_{sphal}),\quad \alpha_+ = \alpha (m_{\phi}/T),\quad \alpha_0 = \alpha (m_{\varphi}/T),\quad \alpha_{W} = \alpha (T_{sphal}/T).$$

The gauge interactions are in thermal equilibrium well below $T = M_W$. The Sphaleron interactions violate baryon-number $B_i$ and lepton-number $L_i$ but conserve $\frac{1}{3}B - L_i$, $B_i - B_j$, of which five are linearly independent. They are in thermal equilibrium for all temperatures $T > T_{sphal}$.

The off-diagonal quark interactions violate all $B_i$, but preserve the total baryon-number $B$ and are all in thermal equilibrium for $T > T_{sphal}$. Thus, in the SM the symmetries are reduced to the three conserved quantum numbers

$$\frac{1}{3}B - L_i.$$ \footnote{Provided there is a component produced in a $(\frac{1}{3}B - L_i) = 0$ channel. It should also be clear that $B - L = 0$ at all temperatures does not necessarily imply $(\frac{1}{3}B - L_i) = 0$, and thus a baryonasymmetry can in principle survive. We shall see in the chemical potential analysis how this happens.}
In the massless limit $\Delta \mu = \Delta t = \Delta b = \Delta d = 0$, $\alpha(0) = 1$. At $T_{sphal} = M_W$ and for $m_{top} = 150 GeV$: $\Delta t, \Delta d \ll \Delta u \approx 0.38$. We thus neglect both $\Delta t$ and $\Delta d$. The electroweak interactions lead to the following $4 + 2N = 10$ equilibrium relations among the chemical potentials

\[
\begin{align*}
\mu_W &= \mu - \mu_0 \\
\mu_i &= \mu_L + \mu_i \\
\mu_R &= \mu_0 + \mu_L \\
\mu_{dR} &= -\mu_0 + \mu_W + \mu_{aL} \\
\mu_{dL} &= \mu_{aL} + \mu_W
\end{align*}
\]

independent of the mass corrections $\alpha$ and internal degrees of freedom $g_k$. However, the net value of a quantum number depends on the net number density and thus depends on the product of $\mu$, $g_k$ and most importantly $\alpha$. Using the equations (6), we can express all of the chemical potentials in terms of 6, which we chose to be $\mu_W, \mu_0, \mu_{aL}$, and $\mu_i$. However, $\mu_i$ only appears in the combinations $\mu$ and $\Delta \mu$ leaving only 5 independent chemical potentials.

We can now express the total electric charge $Q$ and the third component of weak isospin $Q_3$, as well as $B, L$ in terms of these 5 chemical potentials, e.g.

\[
Q = 2(N - 2\Delta_u)\mu_{aL} - 2(2N + 2\alpha_W + b\alpha_-)\mu_W - 2(\mu - \Delta \mu) + 2(2N - \Delta u + b\alpha_-)\mu_0,
\]

\[
Q_3 = 3\Delta_u\mu_{aL} - (2N + 4\alpha_W + b\alpha_-)\mu_W + b(\alpha_- - \alpha_0)\mu_0 + \frac{1}{2}\Delta \mu,
\]

\[
B = (4N - 2\Delta_u)\mu_{aL} + 2N\mu_W - \Delta_u\mu_0,
\]

\[
L = 3\mu - 2\Delta \mu + 2N\mu_W - N\mu_0.
\]

We have for $T > T_{sphal}$ one further equation due to the Sphaleron interactions

\[
N(3\mu_{aL} + 2\mu_W) + \mu = 0.
\]

At all temperatures $U(1)_Q$ is a good symmetry and therefore we must have $Q = const. \approx 0$. Above $T_C$, we also have $Q_3 = 0$. For $T < T_C$, $Q_3 \neq 0$ but $\phi > 0$ which implies $\mu_0 = 0$. Thus for $T > T_{sphal}$, we have 3 equations beyond those of Eq. (6), for the five unknowns: $\mu_{aL}, \mu_W, \mu_0, \mu_i$, and $\Delta \mu$. Hence, we can write $B$ and $L$ in terms of $\mu_{aL}$ and $\Delta \mu$.

(1) $T \gtrsim T_C$ Above the scale for electroweak breaking the quarks, leptons and the W-boson are massless, $\Delta_u = \Delta_d = \Delta_t = 0$, $\alpha_W = \alpha_t = 1$, and $\Delta \mu = 0$. We can thus write $B$ and $L$ in terms of $\mu_{aL}$ only

\[
B = 4N\mu_{aL}, \quad L = -\frac{14N^2 + 9N\alpha_-}{2N + b\alpha_-}\mu_{aL}.
\]

Above $T_C$ the mass corrections do not modify the value of $B$ and only very mildly modify $L$. From Eq. (11) and the observational value for $n_B/s$ we obtain $\mu_{aL} \approx 10^{-11}$. Also

\[
B + L = -\frac{6N^2 + 5N\alpha_-}{2N + b\alpha_-}\mu_{aL}
\]

\[
= -\frac{6N + 5\alpha_-}{2N + 13\alpha_-}(B - L),
\]

Thus a non-zero value for $B - L$ implies a non-zero value for $B + L$, even though the $B + L$ violating Sphaleron interactions are in thermal equilibrium.

(2) $T_{sphal} \approx T \approx T_C$ Setting $\mu_0 = 0$ and using Eqs. (10) we obtain

\[
B = \left(4N - 2\Delta_u + \frac{4N(2N - \Delta_u)}{2\alpha_W + b\alpha_-}\right)\mu_{aL},
\]

\[
+ \frac{2N}{2\alpha_W + b\alpha_-}\Delta \mu,
\]

\[
L = -\left(9N + 8N(2N - \Delta_u)\right)\mu_{aL},
\]

\[
-2\left(1 + \frac{2N}{2\alpha_W + b\alpha_-}\right)\Delta \mu.
\]

In the massless limit Eqs. (1) agree with [4]. We find a non-zero value for $B + L$, which is not washed out by the sphalerons. The mass corrections due to the top quark are about 5%, the correction due to the Higgs and $W$-mass is about a factor of two. The more important mass effect is that $B + L$ is not proportional to $B - L$ and therefore $B - L = 0$ does not imply $B = B + L = 0$. This is contrary to previous results and is important, since it was assumed that due to the sphalerons one either requires early $B - L \neq 0$ baryogenesis or late baryogenesis.

(3) $B - L = 0$ In models where $B - L$ is conserved at all temperatures one must impose the additional constraint $B - L = 0$. For $T \approx T_C$, this additional equation for the chemical potentials immediately gives $B = L = B + L = 0$, as well as $\mu_{aL} = \mu_\mu = \mu_\mu = 0$. However, we do not necessarily have $\mu_i = 0$.

If the initial conditions are such that $L_1 = 0$, and if the only lepton-number violating interactions are sphaleron processes, then the $\mu_i$ will be equal and the vanishing of $\mu$ implies the vanishing of $\mu_i$. But if there is a pre-existing asymmetry in a given $L_i$ channel, then $L_i - L_j$ conservation means that the vanishing of $L$ can come about only through a cancellation between different, non-zero, $\mu_i$. The non-zero $\mu_i$ imply a non-zero $\Delta \mu$ which will regenerate the lepton- and baryon-asymmetry, similar to models discussed in [8]. This follows because, imposing $B - L = 0$, we can express $\mu_{aL}$ by $\Delta \mu$ and

\[
B = \left[-\frac{4N - 2\Delta_u + \frac{4N(2N - \Delta_u)}{2\alpha_W + b\alpha_-}}{13N - 2\Delta_u}(\alpha_W + \frac{b\alpha_-}{2\alpha_-}) + 6N(2N - \Delta_u)\right].
\]
as well as $B = L = \frac{1}{2}(B + L)$. In this case $\Delta \mu \neq 0$, since $\mu_i \neq 0$ and the baryon number reappears due to the sphaleron interactions converting $L_i$ number excess into $B$ excess. We find the surprising result, that even when $B - L = 0$ for all temperatures we can have non-vanishing $B, L$ and $B + L$.

In the MSSM there are no further chemical potentials because; they are all related to their SUSY partners'. This is because at the energies we consider the neutralino (Majorana fermions!) have vanishing chemical potential. For example the reaction $\tilde{e}^- \leftrightarrow e^- + \tilde{\chi}^0$ in thermal equilibrium implies $\mu_{\tilde{e}} = \mu_e$. Therefore the SM results, which depended on the number of independent chemical potentials, remain qualitatively the same. The main difference is that now three linear combinations of the neutrino chemical potentials appear:

$$\mu, \Delta \mu, \Delta \tilde{\mu} = \mu - \sum \tilde{\alpha}_i \mu_i,$$

where

$$\tilde{\alpha}_i = \alpha(m_{\tilde{e}i})$$

denotes the mass corrections due to the sleptons now instead of the leptons and we have assumed $\tilde{m}_{\mu i} = m_{\tilde{e}i}$. Due to the larger sfermion masses these can be substantially larger. Taking $\Delta \tilde{\mu} \gg \Delta \mu$ we obtain in the case $B - L = 0$

$$B = -2\frac{19 + 63\tilde{\alpha} + 50\tilde{\alpha}^2}{584 + 925\tilde{\alpha} + 216\tilde{\alpha}^2} \Delta \tilde{\mu}. \quad (18)$$

And for $\tilde{\alpha} = 1$, $B = -1.8 \Delta \tilde{\mu}$. Finally we get $L = B$ and $B + L = 2B$.

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