Dynamical properties of heavy-ion collisions from the Photon-Photon intensity correlations

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Abstract

We consider here the bremsstrahlung emission of photons at low and intermediate energies \( E_{\text{lab}} \leq 1000\,\text{MeV/u} \) of the projectile and derive expressions more general than previous results obtained by Neuhauser which were limited to the case of isotropic systems. We find that the two-photon correlation function strongly depends not only on the space-time properties of the collision region but also on the dynamics of the proton-neutron scattering process in nuclear matter. As a consequence of polarisation correlations it turns out that for a purely chaotic source the intercept of the correlation function of photons can reach the value 3 (as compared with the maximum value 2 for isotropic systems). Furthermore even for “hard” photons (\( E_{\gamma} > 25\,\text{MeV} \)) the maximum of the correlation function can reach the value of 2 in contrast with the value of 1.5 derived by Neuhauser for this case. The formulae obtained in this paper which include also the possible presence of a coherent component can be used as a basis for a systematic analysis of photon intensity-interferometry experiments.

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1 Introduction

The Hanbury-Brown and Twiss intensity interferometry \[1\] plays at present an important role in particle and nuclear physics in a wide range of c.m. energies, because this method can provide information about the space-time properties of the sources. From hadron interferometry one obtains information mainly about the properties of the fireball just near the freeze-out. Moreover, in this case we are faced with some dynamical problems like the final state interaction and contribution from resonances which mask the real space-time size of the collision region. In contrast, photons and leptons have a large mean free path and leave the collision region just after their production. Additionally we can assume that photons have no final state interaction (light-light scattering can be neglected). Therefore direct photons (i.e. photons not originating from $\pi^0$ decay) and dileptons are good probes for the earliest stages of the reaction. Another advantage of electro-magnetic probes is that their emission is controlled by QED rather than by strong interaction dynamics for which no comparable theory exists yet.

In this paper we consider mostly the emission of “hard” photons (their definition will be given below) in heavy ion reactions in the energy range up to 1 Gev/u. We will show that the photon-photon correlation function can provide there not only space-time information but also some dynamical information about the proton-neutron scattering in nuclear matter. It is known \[2, 3\] that at intermediate energies hard photons are produced mainly through bremsstrahlung and $\pi^0$ decay. Since the photon pairs from the $\pi^0$ decay can be eliminated to a great extent by measuring the photon pair invariant mass, we concentrate ourselves on bremsstrahlung photons.

2 Bremsstrahlung photons and their correlations

The photon spectrum is usually described by a superposition of two contributions, both parametrized by exponentials \[2, 3\] and referring to different photon energies respectively: the low energy part is assumed to be due to thermal photons and the slope is just given by the temperature. The second (high energy) part can be considered \[2\] as due to a mixture of bremsstrahlung (see Chapter \[3\]) and photons originating from decays (mostly $\pi_0 \to \gamma\gamma$). Measurements of the angular distribution of radiation, the photon-source velocity \[2, 8\] as well as the impact-parameter dependence of the photon multiplicity \[4\] suggest that the direct hard photons mainly originate from bremsstrahlung in independent proton-neutron collisions\[8\].

As was shown in \[3\] the non-relativistic recoilless bremsstrahlung formula for the current

\[1\] The photon emission from proton-proton collisions is of quadrupole form and therefore highly suppressed as compared with the dipole radiation from proton-neutron collisions.
in a $p-n$ collision:

$$j^\lambda(k) = \frac{i e}{m k^0} \mathbf{p} \cdot \bm{\epsilon}_\lambda(k)$$

(1)

(2)

$\mathbf{p} = \mathbf{p}_i - \mathbf{p}_f$ is the difference between the initial and final momentum of the proton, $\bm{\epsilon}_\lambda(k)$ is the vector of linear polarisation and $k$ the 4-momentum) works well even in the relativistic case due to the fact that relativistic and recoil corrections to some extent compensate each other. If we have $N$ such sources we can write down the transition current as follows:

$$J^\lambda(k) = \sum_{n=1}^{N} e^{i k y_n} j^\lambda_n(k)$$

The index $n$ here labels the independent $p-n$ collisions taking place at different space-time points $y_n$. Formulas (1), (2) are examples of radiation by classical currents, where the influence of the emitted photons on the $p-n$ collision process is negligible.

For an arbitrary classical current $J^\mu(x)$ coupling with the photon field

$$A^\mu(x) = \int \tilde{d}k \left( a_\lambda(k) \epsilon^\mu_\lambda(k) e^{-ikx} + a^+_\lambda(k) \epsilon^\ast_\mu_\lambda(k) e^{ikx} \right) ,$$

(3)

through the conventional interaction lagrangian

$$L_{int} = -J^\mu(x) A^\mu(x)$$

(4)

we can easily find the exact $S$-matrix

$$S = \exp \left[ -\frac{1}{2} \sum_{\lambda=1}^{2} \int \tilde{d}k \left| J^\lambda(k) \right|^2 \right] \exp \left[ -i \int \tilde{d}k \left( J^\lambda(k) a^+_\lambda(k) + J^{+\lambda}(k) a_\lambda(k) \right) \right] :$$

(5)

which contains all the information on the photon production and absorption. We use in (5) the notation:

$$J^\lambda(k) \equiv \epsilon^\mu_\lambda(k) \tilde{J}^\mu(k) \ .$$

(6)

where $\tilde{J}^\mu(k)$ is the Fourier transform of the current $J^\mu(x)$. Before the collision there are no photons, i.e. we have the photon vacuum in the initial state.

The amplitude to produce $n$-photons from the photon vacuum by the classical current $J$ follows directly from (5):

$$<0| \prod_{i=1}^{n} a_\lambda (k_i) |0> = <0|S|0> \prod_{i=1}^{n} \left( -i J^\lambda_i(k_i) \right) .$$

(7)

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$^2$The experimentally found one-photon spectra are of exponential form. This property can be also incorporated into this model, see Chapter 5.
The corresponding cross-section can be written in a condensed and convenient form by means of the generating functional for the exclusive processes. Let us introduce the notation \( q \equiv (k, \lambda) \) for the momentum \( k \) and polarisation \( \lambda \) degrees of freedom of photon. Integration over \( q \) means integration over momentum (with the invariant measure \( d\tilde{k} \)) and summation over polarisation \( \lambda \). We define the exclusive generating functional \( g^{ex}[Z] \) as follows:

\[
g^{ex}[Z] = \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} dq_i Z(q_i) \frac{1}{\sigma^{ex}} \cdot \frac{d^n \sigma^{ex}}{dq_1 \times \ldots \times dq_n}
\]

The conservation of probability gives the normalisation \( g^{ex}[Z = 1] = 1 \). The method of the generating functionals allows one to avoid the complicated combinatorics which usually appears when one consider inclusive processes. The generating functional for the inclusive cross-section can be introduced in a similar way

\[
g^{in}[Z] = \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} dq_i Z(q_i) \frac{1}{\sigma^{in}} \cdot \frac{d^n \sigma^{in}}{dq_1 \times \ldots \times dq_n}
\]

(with the normalisation condition \( g^{in}[Z = 0] = 1 \)) and can be very simple expressed through the exclusive one

\[
g^{in}[Z] = g^{ex}[Z + 1].
\]

In our case (see eq.(7)) the summation over \( n \) can be performed explicitly and we are left with the simple expressions

\[
g^{ex}[Z] = \exp \left[ \sum_{\lambda=1}^{2} \int d\tilde{k} |J^{\lambda}(k)|^2 (Z_{\lambda}(k) - 1) \right]
\]

\[
g^{in}[Z] = g^{ex}[Z + 1] = \exp \left[ \sum_{\lambda=1}^{2} \int d\tilde{k} |J^{\lambda}(k)|^2 Z_{\lambda}(k) \right]
\]

If the classical current obeys some random behaviour which is the present case, as we consider chaotic sources, the generating functional should be subjected to the averaging over the current distribution

\[
G^{in} = \left< g^{in}[Z] \right>.
\]

Taking the variational derivatives of \( G^{in}[Z] \) with respect to \( Z \) one gets the inclusive spectra. For example single and double inclusive spectra read as follows:

\[
\rho^{\lambda}(k) = \frac{\delta G^{in}[Z]}{\delta Z_{\lambda}(k)} \bigg|_{Z=0} = \left< |J^{\lambda}(k)|^2 \right>
\]

\[
\rho^{\lambda_1,\lambda_2}(k) = \frac{\delta^2 G^{in}[Z]}{\delta Z_{\lambda_1}(k_1) \delta Z_{\lambda_2}(k_2)} \bigg|_{Z=0} = \left< |J^{\lambda_1}(k_1)J^{\lambda_2}(k_2)|^2 \right>
\]

An essential ingredient in eqs. \(13\) - \(15\) is the average prescription \( < ... > \).

For the model considered here, where hard bremsstrahlung photons originate mostly from independent proton-neutron collisions at the early stage of the heavy-ion reaction, the averaging prescription \( < ... > \) appears quite naturally and consists of two parts:
1. The points \( y_n \) where the independent \( p - n \) collisions take place (see eq. (2)) are considered to be randomly distributed in the space-time volume of the source (e.g. fireball) with a distribution function \( f(y) \) for each \( p - n \) collision. This kind of averaging is typical for Bose-Einstein correlation studies.

2. The amplitude of photon production (11) is very sensitive to the momentum transfer of the proton in the proton-neutron collision. That is why we have to take into account also the fluctuations of the initial momentum of the proton: \( p_i = p_0 + \Delta p_F \). Here \( p_0 \) is the momentum of the nucleus as a whole and \( \Delta p_F \) is the Fermi-motion of a nucleon. The final momentum distribution of the proton \( (p_f) \) can be in principle determined by dynamical models of nucleus-nucleus collisions, e.g. BUU (8), etc.

Since in practice we need only the first two moments of this distribution we keep them as a free phenomenological parameters to be determined in the experiment. The comparison of the experimental values with model predictions can serve as an indirect test of the model.

The fundamental quantity in our approach is the two-current correlator in momentum space:

\[
\langle J^{\lambda_1}(k_1)J^{\lambda_2}(k_2) \rangle = \frac{1}{N} \sum_{n,m=1}^{N} \int \prod_{l=1}^{N} d^4 y_l f(y_l) \times \\
\exp(i k_1 y_n - i k_2 y_m) \left\langle j_n^{\lambda_1}(k_1)j_m^{\lambda_2}(-k_2) \right\rangle = \sum_{n=1}^{N} \left[ \tilde{f}(k_1 - k_2) \right. \\
\left. - \tilde{f}(k_1) \tilde{f}(-k_2) \right] + \left\langle J^{\lambda_1}(k_1)J^{\lambda_2}(-k_2) \right\rangle.
\]  

(16)

Here we use the properties \( (J^{\lambda}(k))^* = J^{\lambda}(-k) \), \( j_n^{\lambda_1}(k_1)j_n^{\lambda_2}(-k_2) = j_n^{\lambda_1}(k_1)j_n^{\lambda_2}(-k_2) \) for \( n \neq m \) (that is the hypothesis of independent proton-neutron collisions), and \( \tilde{f}(k) \) denotes the Fourier transform of \( f(y) \) with the normalisation \( \tilde{f}(k = 0) = 1 \).

Usually the function \( \tilde{f}(k) \) has a steep maximum around \( k = 0 \) with a width of the order of the size of the source \( R \). In the region of the Bose-Einstein peak for hard photons in particular for \( 2kR \gg 1 \) we can therefore neglect \( \tilde{f}(k_1)\tilde{f}(-k_2) \) as compared to \( \tilde{f}(k_1 - k_2) \). This approximation strongly simplifies the algebra without influencing significantly the accuracy of the calculations.

3 Chaotic sources

Let us consider the case of a chaotic source for which \( \langle J^{\lambda}_{ch}(k) \rangle = 0 \). The current correlator (16) is now given by the simple formula:

\[
\langle J^{\lambda_1}(k_1)J^{\lambda_2}(-k_2) \rangle = F^{\lambda_1\lambda_2}(k_1, k_2) \equiv \tilde{f}(k_1 - k_2) \sum_{n=1}^{N} j_n^{\lambda_1}(k_1)j_n^{\lambda_2}(-k_2).
\]
\[ f(k_1 - k_2) \frac{e^2/m^2}{k_1^0 k_2^0} \epsilon^j_{\lambda_1}(k_1) \left( \sum_{n=1}^{N} < p^i_n p^j_n > \right) \epsilon^i_{\lambda_2}(k_2). \] (17)

\(< p^i_n p^j_n >\) denotes here the averaging with respect to the distribution of the momentum transfer of the proton in the collision \( n \) and \( \sum_{n=1}^{N} \) goes through all the relevant proton-neutron collisions. As mentioned above the quantity \(< p^i_n p^j_n >\) can be extracted from dynamical models of heavy-ion collisions. But one can reach important and general conclusions without specifying this quantity as follows. We use the axial symmetry around the beam direction. The tensor decomposition of \(< p^i_n p^j_n >\) gives then:

\[ < p^i_n p^j_n > = \frac{\sigma_n}{3} \delta^{ij} + \delta_n \ell^i \ell^j, \] (18)

where \( \ell \) is the unit vector in the beam direction and \( \sigma_n, \delta_n \) are real positive constants. Note that this expression is more general than the corresponding one used in [3] where because of the isotropy assumption \( \delta_n \) was assumed to vanish. This generalization has important consequences to be exhibited below.

Now let us separate the average value of the proton momentum transfer \(< p_n >\):

\[ p_n = \Delta p_n + < p_n > \] (19)

We have then:

\[ < p^i_n p^j_n > = < \Delta p^i_n \Delta p^j_n > + < p^i_n > < p^j_n > = < \Delta p^i_n \Delta p^j_n > + < p >^2 \ell^i \ell^j \] (20)

where the tensor \(< \Delta p^i_n \Delta p^j_n >\) can be represented (due to axial symmetry) again as the sum of the two terms:

\[ < \Delta p^i_n \Delta p^j_n > = \frac{\sigma_n}{3} \delta^{ij} + \xi_n \ell^i \ell^j. \] (21)

In order to find the coefficients \( \sigma_n, \xi_n, \delta_n \) we split the 3-vectors in the transverse and the longitudinal parts:

\[ \Delta p^i_n = \ell \cdot (\ell \cdot \Delta p_n); \quad \Delta p^i_n = \Delta p_n - \Delta p^i_n. \] (22)

The coefficients in (18), (21) read:

\[ \frac{\sigma_n}{3} = \frac{1}{2} < (\Delta p^i_n)^2 >; \quad \xi_n = < (\Delta p^i_n)^2 > - \frac{\sigma_n}{3}; \]

\[ \delta_n = < (\Delta p^i_n)^2 > - \frac{\sigma_n}{3} + < p_n >^2. \] (23)

With the help of (18), (23) one can express the current correlator (17)

\[ F^{\lambda_1 \lambda_2}(k_1, k_2) = f(k_1 - k_2) \frac{N \cdot e^2/m^2}{k_1^0 k_2^0} \epsilon^j_{\lambda_1}(k_1) \left[ \frac{\sigma}{3} \delta^{ij} + \delta \ell^i \ell^j \right] \epsilon^i_{\lambda_2}(k_2). \] (24)
through two parameters $\sigma$ and $\delta$

$$\frac{\sigma}{3} = \frac{1}{N} \sum_{n=1}^{N} \frac{1}{2} < (\Delta p_n^i)^2 >$$  \hspace{1cm} (25)$$

$$\delta = \frac{1}{N} \sum_{n=1}^{N} [ < (\Delta p_n^i)^2 > + < p_n >^2 ] - \frac{\sigma}{3},$$  \hspace{1cm} (26)$$

which absorb the relevant dynamical information about the proton-neutron scattering in the medium. The parameters $\sigma$ and $\delta$ can be extracted from experimental data on photon-photon Bose-Einstein Correlations (cf. below) and/or calculated from dynamical models for heavy-ion collisions.

The single inclusive cross-section for a detector, which does not measure polarizations follows directly from (14), (24)

$$\rho_1(k) = \sum_{\lambda=1}^{2} F_{(k,k)}^{\lambda \lambda} = N \frac{e^2/m^2}{(k_0)^2} \left[ \frac{2}{3} \sigma^2 + \delta^2 \cdot \sin^2 \theta \right]$$  \hspace{1cm} (27)$$

where $\theta$ is the angle between the photon and the beam directions and the polarization sum is calculated using the well-known identity:

$$\sum_{\lambda=1}^{2} \epsilon_{\lambda}^i(k) \epsilon_{\lambda}^j(k) = \delta^{ij} - n^i n^j$$  \hspace{1cm} (28)$$

with $n = k/|k|$.

To calculate the double inclusive cross-section (as well as the higher order inclusive spectra) one has to know higher order current correlators. In our case when all proton-neutron collisions are considered to be independent from each other and the number of the participating protons is sufficiently large $N > 10$, we can apply the central limit theorem and express the higher order current correlators through the first and second ones. Assuming $<J^\lambda(k)> = 0$ (no coherence) the double inclusive cross-section is represented through the sum of products of the two-current correlators

$$\rho_2(k_1, k_2) = \sum_{\lambda_1, \lambda_2=1}^{2} <J^{\lambda_1}(k_1), J^{\lambda_1}(-k_1), J^{\lambda_2}(k_2), J^{\lambda_2}(-k_2) >=$$

$$= \rho_1(k_1) \rho_1(k_2) + \sum_{\lambda_1, \lambda_2=1}^{2} F^{\lambda_1 \lambda_2}(k_1, k_2) F^{\lambda_2 \lambda_1}(k_2, k_1) + F^{\lambda_1 \lambda_2}(k_1, -k_2) F^{\lambda_2 \lambda_1}(-k_2, k_1)$$

$$= \rho_1(k_1) \rho_1(k_2) + \sum_{\lambda_1, \lambda_2=1}^{2} F^{\lambda_1 \lambda_2}(k_1, k_2) F^{\lambda_2 \lambda_1}(k_2, k_1) + (k_2 \leftrightarrow -k_2)$$  \hspace{1cm} (29)$$

and the polarization sum

$$\sum_{\lambda_1 \lambda_2=1}^{2} F^{\lambda_1 \lambda_2}(k_1, k_2) F^{\lambda_2 \lambda_1}(k_2, k_1) = |\tilde{f}(k_1 - k_2)|^2 \cdot N^2 \frac{e^4/m^4}{(k^0_1 k^0_2)^2} \times$$  \hspace{1cm} (30)$$
\[
\left\{ \frac{\sigma^2}{9} (1 + \cos^2 \psi) + \delta^2 \sin^2 \theta_1 \sin^2 \theta_2 + \frac{2}{3} \sigma \delta [1 - \cos^2 \theta_1 - \cos^2 \theta_2 + \cos \psi \cos \theta_1 \cos \theta_2] \right\}
\]
is performed with the help of (24), (28).

The general expression for the second order correlation function is defined by

\[
C_2(k_1, k_2) = \frac{\rho_2(k_1, k_2)}{\rho_1(k_1) \rho_1(k_2)}
\]

and has a complicated angular dependence, but the two limiting cases 1) \( \sigma \gg \delta \) and 2) \( \sigma \ll \delta \) lead to very simple expressions:

1. For the case \( \sigma \gg \delta \)

\[
C_2(k_1, k_2|\sigma \neq 0, \delta = 0) = 1 + \frac{1}{4} (1 + \cos^2 \psi) \left[ |\tilde{f}(k_1 - k_2)|^2 + |\tilde{f}(k_1 + k_2)|^2 \right]
\]

which is the result derived in [5] and which gives for the intercept

\[
C_2(k, k) = \frac{3}{2} + \frac{1}{2} |\tilde{f}(2k)|^2 .
\]

2. The opposite case \( \sigma \ll \delta \) leads to another formula:

\[
C_2(k_1, k_2|\sigma = 0, \delta \neq 0) = 1 + |\tilde{f}(k_1 - k_2)|^2 + |\tilde{f}(k_1 + k_2)|^2 ;
\]

with the intercept exceeding 2:

\[
C_2(k, k) = 2 + |\tilde{f}(2k)|^2 .
\]

For hard photons \( |\tilde{f}(2k)|^2 \ll 1 \) (cf. [5]) and one can neglect this contribution to the intercept while for soft photons \( \tilde{f}(2k) \) is non-negligible and in the limit \( k = 0, \tilde{f}(0) = 1 \) so that \( C_2(k, k) = 3 \). The real situation (when both \( \sigma \) and \( \delta \) contribute) is between (32) and (34) and exhibits a more complicated angular behaviour than the considered above limiting cases [6]. For instance the general expression for the intercept which follows directly from (27), (29), (30)

\[
C_2(k, k) = \int \frac{d\Omega \rho_2(k, k)}{d\Omega \rho_1^2(k)} = 1 + \frac{1}{2} \left[ 1 + |\tilde{f}(2k)|^2 \right] \left[ 1 + \frac{1.2 \delta^2}{\sigma (\sigma + 2 \delta) + 1.2 \delta^2} \right]
\]
depends on both parameters \( \sigma \) and \( \delta \) and varies in the range between 3/2 and 3. The solid angle integration over all possible orientations of the photon momentum \( \mathbf{k} \) corresponds to a 4\( \pi \) detector.

The function \( f(x) \) and its Fourier transform \( \tilde{f}(k) \) reflects the space-time properties of the photon source. It depends in principle on three constants: the time duration \( R_0 \), the longitudinal radius \( R_l \) and the transverse radius \( R_t \). One can propose a concrete parametrization for the source geometry \( \tilde{f}(k) \) and then fit the inclusive data using (31). However, because of insufficient statistics one has usually to limit oneself to a smaller number of parameters.
In particular the choice of two parameters $T = R_0$ and $R = R_t = R_t$ is good enough for
the present state of the art. With this assumption we can perform analytically the angular
integration over $\theta_1$ and $\theta_2$ in (27) and (29), (30) keeping constant the angle between the two
photons $\mathbf{n}_1 \cdot \mathbf{n}_2 = \cos \psi = \text{const}$. The corresponding integration
\[
\int d\mu = 2\pi \int_0^\pi \sin \theta_1 d\theta_1 \int_0^{2\pi} d\varphi
\]
extends over the rotations of $\mathbf{n}_2$ around $\mathbf{n}_1$ ($d\varphi$) and over all orientations of $\mathbf{n}_1$ around the
beam direction. After some algebra we are left with the general expression for the Bose-
Einstein correlation function
\[
C_2(k_1, k_2) \equiv \frac{\int d\mu \rho_2(k_1, k_2)}{\int d\mu \rho_1(k_1) \rho_1(k_2)} = 1 +
\]
\[
\frac{1}{4} \left\{ \left( |\tilde{f}(k_1 - k_2)|^2 + |\tilde{f}(k_1 + k_2)|^2 \right) \cdot \left[ 1 + \cos^2 \psi + \frac{0.3\delta^2(3 + \cos^2 \psi)(3 - \cos^2 \psi)}{\sigma(\sigma + 2\delta) + 0.3\delta^2(3 + \cos^2 \psi)} \right] \right\}
\]
which has a pronounced dependence not only on the space-time characteristics $R_0$, $R$ but
also on the angle $\psi$ and the “dynamical” constants $\sigma$, $\delta$. The limiting cases (32) and (34)
as well the intercept formula (36) can be rederived directly from (38).

4 Partially coherent sources

In general, photon sources are not totally chaotic but may contain also a coherent compo-
nent $< J^\lambda(k) > \equiv I^\lambda(k) \neq 0$ so that the total current $J^\lambda = J^\lambda_{\text{ch}} + I^\lambda$ leads to a partially
coherent field. There are several mechanisms which are responsible for coherence: collective
deacceleration of the initial nuclei, collective flow, coherent radiation from nuclear fragments,
etc. The possibility to investigate such collective phenomena is an interesting and important
part of heavy-ion physics. In this chapter we discuss phenomenologically the influence of
the coherent part of the electric current on the photon correlation function. The single and
double inclusive cross-sections (see (27) and (29) ) read:
\[
\rho_1(k) = \sum_{\lambda=1}^{2} \left| F^{\lambda}(k; k) + |I^\lambda(k)|^2 \right|^2 ,
\]
\[
\rho_2(k_1, k_2) = \sum_{\lambda_1, \lambda_2=1}^{2} < J^{\lambda_1}(k_1)J^{\lambda_1}(-k_1)J^{\lambda_2}(k_2)J^{\lambda_2}(-k_2) > = \rho_1(k_1)\rho_1(k_2) +
\]
\[
+ \sum_{\lambda_1, \lambda_2=1}^{2} \left\{ |F^{\lambda_1\lambda_2}(k_1, k_2)|^2 + 2\Re[I^{\lambda_1}(k_1)F^{\lambda_1\lambda_2}(k_1, k_2)I^{\lambda_2}(k_2)] + (k_2 \leftrightarrow -k_2) \right\} .
\]
As before we assume axial symmetry around the beam direction and parametrise $I^\lambda(k)$ as
follows:
\[
I^\lambda(k) = \frac{ie}{mk_0} \sqrt{N} S(k_0) \hat{\ell} \cdot \epsilon^\lambda(k) .
\]
We assume also that on the scale important for the Bose-Einstein study (\(\psi\) explicitly the integrations over all the angles except for dependence of the photon source is still arbitrary. It is then possible again to perform the function correlation function in the compact form chaotic case there is now one more function \(S(k_0)\) entering in \(\rho_1(k)\) and \(\rho_2(k_1, k_2)\).

Using (24), (28) and (41) we get
\[
\sum_{\lambda=1}^{2} |I^\lambda(k)|^2 = N \left(\frac{e/m}{k^0} \right)^2 |S(k_0)|^2 \sin^2 \theta , \tag{42}
\]
\[
\sum_{\lambda_1, \lambda_2=1}^{2} I^{\lambda_1}(k_1)F^{\lambda_1\lambda_2}(k_1, k_2)I^{\lambda_2}(k_2) = \tilde{f}(k_1 - k_2)N^2 \left(\frac{e^2/m^2}{k_1^0 k_2^0} \right)^2 S^*(k_1^0)S(k_2^0) \times
\]
\[
\left[ \frac{\sigma}{3}(1 - \cos^2 \theta_1 - \cos^2 \theta_2 + \cos \psi \cos \theta_1 \cos \theta_1) + \delta \sin^2 \theta_1 \sin^2 \theta_2 \right] \tag{43}
\]
which together with \(F^{\lambda_1}(k, k)\) and \(F^{\lambda_1\lambda_2}(k_1, k_2)F^{\lambda_2\lambda_1}(k_2, k_1)\) (see (27),(30)) determine completely the single and double inclusive spectra (39,40). As compared to the completely chaotic case there is now one more function \(S(k_0)\) entering in \(\rho_1(k)\) and \(\rho_2(k_1, k_2)\).

Significant simplifications can be achieved again assuming \(R_t = R_{t} = R\). The time dependence of the photon source is still arbitrary. It is then possible again to perform explicitly the integrations over all the angles except for \(\psi\) and write the photon-photon correlation function in the compact form
\[
C_2(k_1, k_2) = \frac{\int d\mu \rho_2(k_1, k_2)}{\int d\mu \rho_1(k_1) \rho_1(k_2)} = \tag{44}
\]
\[
= 1 \ + \ \{ \lambda^2(k_0)A f(k_1 - k_2)^2 + 2\lambda(k_0)(1 - \lambda(k_0))B Re \tilde{f}(k_1 - k_2) + (k_2 \leftrightarrow -k_2) \}
\]
where we have introduced a new set of parameters:
\[
\lambda(k_0) = \frac{\sigma + \delta}{\sigma + \delta + |S(k_0)|^2} , \quad x = \frac{\sigma}{\sigma + \delta} , \tag{45}
\]
\[
A = \frac{1}{4} \left( 1 + \frac{1 + \cos^2 \psi + (1 - x)^2(13 + \cos^2 \psi)/5}{1 + (1 - x \lambda)^2(3 \cos^2 \psi - 1)/10} \right) ,
\]
\[
B = \frac{1}{4} \left( 1 + \frac{1 + \cos^2 \psi + (1 - x)(13 + \cos^2 \psi)/5}{1 + (1 - x \lambda)^2(3 \cos^2 \psi - 1)/10} \right) .
\]

Here \(\lambda(k_0)\) is of the chaoticity parameter which is the ratio of the number of chaotically produced photons with energy \(k_0\) and their total number \(\lambda(k_0) =< n(k_0) >_{ch} / < n(k_0) >\). We assume also that on the scale important for the Bose-Einstein study (\(|k_1 - k_2| \sim 1/R\)) the function \(\lambda(k_0)\) does not vary significantly: \(|(k_1^0 - k_0^0)d\lambda(k^0)/dk^0| \ll (\lambda(k_0^0) + \lambda(k_0^0))/2\).

The other parameter which influences strongly the behaviour of the correlation function \(C_2(k_1, k_2)\) is the isotropy parameter \(x\) (see (13)). Both these parameters \(\lambda\) and \(x\) can be extracted from the analysis of the angular distribution of the radiation and the value of the intercept \(C_2(k, k)\):
\[
\rho_1(k) / < n(k_0) > = \frac{1}{4\pi}[x \lambda(k_0) + \frac{3}{2}(1 - x \lambda(k_0)) \sin^2 \theta] , \tag{46}
\]
\[
C_2(k, k) = 1 + \frac{(1 + |\tilde{f}(2k)|^2)/2}{5 + (1 - x \lambda)^2} \left\{ \lambda^2(k_0)[5 + 7(1 - x)^2] + 2\lambda(1 - \lambda)[5 + 7(1 - x)] \right\} . \tag{47}
\]
(for the case of hard photons $|\tilde{f}(2k)|^2 \ll 1$ and one can neglect this contribution in (17)). After this has been done the only unknown function in (14) is the space-time distribution function of the source $f(x)$ (or $\tilde{f}(k)$) which can now be obtained by fitting the experimental data. This function has the physical meaning of the space-time distribution of the radiating region and reflects the geometry of the early stages of the collision.

We would like to stress once again that both the parameters $\lambda$ and $x$ strongly influence $C_2(k_1,k_2)$ and the knowledge of only the one of them (e.g. $\lambda$) is not enough to determine in a unique way the two-photon correlation function. For instance, one can check that all the $\lambda, x$ pairs, for which $\lambda = (\sqrt{2} - 1)/(\sqrt{2} - x)$ and $0 \leq x \leq 1$, lead to the same intercept $C_2(k,k) = 3/2$.

5 Exponential fall of the bremsstrahlung amplitude

The experimental observations [2, 3] show that the one-particle inclusive spectrum of bremsstrahlung photons has exponential form suggesting that the underlying proton current reads:

$$j^\lambda(k) = ie^{mk_0} \mathbf{p} \cdot \epsilon^\lambda(k) \cdot \exp[-k_0/(2E_0)]$$

rather than (1). The single and double inclusive cross-sections calculated with (48) instead of (1) can be obtained from the previous results multiplying them by the corresponding powers of $\exp[-|k_1^2|/(2E_0)]$ and $\exp[-|k_0^2|/(2E_0)]$.

The homogeneous functions like the two-photon correlation function (31) and the angular distribution of radiation (46) remain unchanged and all the conclusions about the Bose-Einstein correlations obtained above hold.

In the following we shall derive the non-relativistic classical trajectory of a proton which leads to the mentioned above current (48).

The current in momentum space is defined through the proton trajectory as follows:

$$j^\lambda(k) = e_\mu^\lambda(k)\tilde{\epsilon}^\mu(k) = -e \int_{-\infty}^{+\infty} dt \exp\{i[k_0t - \mathbf{k}\mathbf{r}(t)]\} (\epsilon^\lambda \cdot \mathbf{v}(t))$$

In non-relativistic case $|\mathbf{v}(t)| \ll 1$ and as a consequence $k_0t - \mathbf{k}\mathbf{r}(t) \approx k_0t$. Therefore $j^\lambda(k)$ reduces to the time Fourier transform of the velocity. Using the identity

$$\int_{-\infty}^{+\infty} dt e^{i\omega t} \frac{1}{2} \left[ 1 - \frac{2}{\pi} \arctg(2E_0t) \right] = \frac{-i}{\omega - i\epsilon} \exp[-|\omega|/(2E_0)]$$

one finds the trajectory

$$\mathbf{v}(t) = \frac{v_0}{2} \left[ 1 - \frac{2}{\pi} \arctg(2E_0t) \right]$$

leading to the proton current (48). The standard formula (1) corresponds to the special case $E_0 \to +\infty$ when the proton trajectory is described by the step function $\mathbf{v}(t) = v_0 \Theta(-t)$. 
The finite value of $E_0$ reflects more smooth then step-like deacceleration of the proton with the characteristic stopping time $\tau \sim 1/(2E_0)$ and stopping length $l \sim v_0\tau$. For instance, for the projectile energy $45\ MeV/u$ (slop-parameter $E_0 = 18\ MeV$\[3\]) one gets $\tau = 5.6\ fm/c$ and $l = 1.7\ fm$ which seem to be quite reasonable.

### 6 Summary

In this paper the production of photons is analysed in the framework of quasi-classical approximation. Our consideration is valid in the region of low and intermediate colliding energies up to $1000\ MeV/u$. We assume that the hard photons ($E_\gamma \geq 25\ MeV$) are produced in independent proton-neutron collisions\[3\] (see Chapter 2) and the whole system has the axial symmetry with respect to the beam direction. Then we derive the expressions for the single and double inclusive cross-sections and for the two-photon correlation function as well (see Chapters 3,4). We show that the behaviour of the photon-photon correlation function depends not only on the space-time properties of the collision region (function $\tilde{f}(k_1 - k_2)$) but also on dynamics of the proton-neutron scattering in matter (parameters $\sigma, \delta$ and the amount of chaoticity $\lambda$). So far the photon intensity interferometry can be considered as an indirect way to check the dynamical properties of the heavy-ion system as well as to obtain the space-time information about a collision.

It turns out that the maximum value of the correlation function (the intercept $C_2(k,k)$) is very sensitive to the details of the proton-neutron scattering and varies generally speaking in the interval $1 \leq C_2(k,k) \leq 3$ (see (17)). If we deal with so hard photons that $|\tilde{f}(2k)|^2 \ll 1$ the intercept finds itself in more narrow range $1 \leq C_2(k,k) \leq 2$ (which is nevertheless wider then the one obtained in $[5]$ $1 \leq C_2(k,k) \leq 1.5$). The reason why the intercept can exceed the value 1.5 (see $[3]$) is strongly connected with the beam-direction anisotropy specific for heavy-ion collisions.

The photons produced with the close momenta can have the correlations in their polarisations. As far as the photons with the same polarisation obey the Bose-Einstein effect like the scalar bosons ($e.x.\ \pi^0$) and ones with the perpendicular polarisations behave like non-identical particles, these correlations in polarisations affect positively on the Bose-Einstein peak increasing intercept. The presence of coherence leads to decreasing of the correlation effect. In order to study the two-photon correlation function including the influence of the anisotropy and the coherent contribution as well ($[14]$) one needs additional information which can be obtained from the angular distribution of the radiation ($[19]$) and the intercept value ($[17]$) analysed together in order to extract the values $\lambda$ and $x$. After it has been done the only unknown quantity in ($[14]$) is the Fourier transform $\tilde{f}(k)$ of the space-time probability distribution $f(x)$ of the photon source which reflects the geometry of the early stages of the

\[3\] the photon emission from the proton-proton collisions has the quadrupole nature and therefore is highly suppressed with respect to dipole radiation from the proton-neutron collisions.
heavy-ion collision.

We would like to acknowledge fruitful comments by Y.Schutz, I.V.Andreev, G.Röpke, T.Alm, J.Clark.

This work was supported in part by the Gesellschaft für Schwerionenforschung, Darmstadt.

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