Current distribution in narrow translation-invariant quantum-Hall-systems with lateral density modulation

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Abstract
A previously developed self-consistent screening and magneto-transport theory for laterally confined, translation-invariant quantum-Hall-systems is applied to two-dimensional electron systems created by a donor sheet with a lateral density modulation. The previous calculations, assuming a homogeneous donor charge density, could explain experimental results on the spatial distribution of an applied source-drain-current, and the resulting Hall potential, only for the ‘edge-dominated’ low-magnetic-field part of a quantum-Hall-plateau, where the current flows through incompressible stripes near the edges. For the high-magnetic-field regime of the plateau, they predicted current flow only in a narrow stripe in the center of the sample, whereas the experiments found current in a wide region of its bulk. Assuming a suitably modulated donor charge density, we can avoid this discrepancy, and we obtain a strong dependence of the distribution of the applied current on magnetic field, lattice temperature, and the current-strength.

1. Introduction

The question, where an applied source-drain-current (ASDC) flows under the conditions of the integer quantum Hall effect (IQHE) through a laterally confined two-dimensional electron system (2DES), is important for the understanding of the underlying physics. Unfortunately, so far there exist no experiments, which can directly measure the local distribution of the ASDC in macroscopic samples.

In microscopically narrow samples (width \( W \gtrsim 10 \mu m \)), on the other hand, scanning-force-microscopy experiments allow to measure the spatial distribution of the ASDC from the resulting Hall potential across the sample under the conditions of the IQHE \([1–4]\) and its breakdown \([5]\). The early experiments showed that, for magnetic field values \( B \) in the quantum Hall plateau (QHP) corresponding to the integer Landau-level filling factor \( \nu = k \), the ASDC flows dissipationless through so-called ‘incompressible stripes’ (ISs), which had been predicted by Chklovskii et al \([6, 7]\) as a consequence of the zero-temperature screening properties of a 2DES in a strong perpendicular magnetic field. Later selfconsistent thermal-equilibrium calculations \([8, 9]\) confirmed that such ISs should exist at finite, sufficiently low temperatures, and that they should be able to carry an ASDC nearly dissipationless \([10-12]\).

To describe quantum Hall states with finite ASDC, the current-carrying non-equilibrium state was approximated by a local-equilibrium state with position-dependent electrochemical potential \( \mu^*(r) \) and conductivity tensor \( \sigma(r) \), which determine the density \( j(r) = \sigma(r) E(r) \) of the ASDC by a local version of Ohm’s law, with the gradient of the electrochemical potential defining the driving electric field, \( E(r) = \nabla \mu^*(r) / e \) \([10–12]\). These calculations revealed a strongly nonlinear response behavior under the conditions of the IQHE, with a current-induced asymmetry between the ISs near opposite sample edges. These asymmetries were nicely confirmed by experiments \([3, 13]\) using unidirectional currents of different strengths. For sufficiently strong current, a breakdown of the IQHE was observed, which occurred as a continuous increase of the voltage drop inside the sample with increasing strength of the applied bias voltage, i.e. of the ASDC \([5]\). To allow for a
current-induced breakdown of the IQHE, the effect of Joule heating was included into the selfconsistent theory of screening and magneto-transport [14].

With this modification of the theory, one finds under the conditions of the IQHE and its breakdown a nice qualitative agreement of the results of experiments [5] and of calculations [14], if one considers the low-\(B\) part of a QHP. In this magnetic field regime the transport is ‘edge-dominated’ [5], since the current flows through well defined ISs not very far from the edges of the Hall bar. This experimental situation can be simulated sufficiently well by the simple model of a relatively narrow (width 3 \(\mu\)m), translation-invariant Hall bar with a constant density of positive background charges, defining a confinement potential, which leads to an electron density profile decreasing monotonically from the center towards the edges of the bar. As \(B\) is increased towards the high-\(B\) part of the QHP, in this model the ISs become wider and move towards the center of the sample, so that the ASDC flows only through a narrow region in the middle of the Hall bar [13]. In the experiment, on the other hand, the high-\(B\) part of the QHP is ‘bulk-dominated’, since the ASDC is distributed over a wide region of the bulk of the Hall bar [5]. This is probably due to the larger sample width (\(\gtrsim 10 \mu\)m) and spatial fluctuations of the confinement potential, created by inhomogeneities of the donor distribution [5], which may lead to a wide region of fluctuating electron density in the bulk of the sample, with incompressible regions far from its center, which carry the dissipationless current. It seems interesting to investigate a situation, in which the ASDC can flow through several of such incompressible regions, and to learn, how the current distribution depends on parameters like lattice temperature, applied magnetic field and the strength of the ASDC, which is expected to lead to strongly nonlinear feedback effects.

To investigate such questions, we will modify our previous calculations for narrow translation-invariant Hall bars [14] so that, for certain regimes of the applied magnetic field, more than two ISs with the same integer value of the local filling factors are possible. Of course inhomogeneities of the donor distribution and the presence of source and drain contacts will destroy the translation invariance, and a quantitative agreement of results of such calculations with the experimental results cannot be expected. Another oversimplification of our model is the neglect of spin-splitting of the Landau levels, and of the exchange interaction, which leads to a magnetic–field-dependent enhancement of the Zeeman g-factor [15] and, in simple approximations, to a reduction of screening effects. A careful and demanding many-body perturbation theory, a so-called ’screened Hartree–Fock approximation’, is necessary to obtain reasonable results for the g-factor [15] and for the screening [16]. Since these screening results are qualitatively similar to those neglecting the exchange interaction [16] and since we can expect only qualitatively correct results due to our simplifying symmetry assumptions, we neglect spin-splitting and exchange interaction, which reduces the necessary computer time by orders of magnitude.

2. Model and results

2.1. Equilibrium and linear response

In the following we use the model developed in previous work [14], neglect the extent of the 2DES in \(z\)-direction and the distances between donor layer, 2DES and top gates, so that all charges are in the plane \(z = 0\). We assume translation invariance in \(y\)-direction and lateral confinement of donor density \(n_D(x)\) and electron density \(n_0(x)\) to the interval \(|x| < d\) by the in-plane metal gates at \(|x| > d\). The relation between the charge densities and the potentials, created by these densities, are given by equations (1)–(3) of [14]. Whereas in [14] a constant density of donor charges was assumed, \(n_0(x) \equiv n_D = \text{const}\), which leads to the potential \(V_D(x, 0) = -E_D \left[ 1 - (x/d)^2 \right]^{1/2}\), with \(E_D = 2\pi e^2 n_D d / \kappa\), we now use

\[
n_D(x) \equiv n_D(x; w) = n_D \left[ 1 - w \cos(k_D \pi x / d) \right]
\]  

(1)

with an integer \(k_D = 0\), so that the average donor density is still \(n_D\), and for \(w = 0\) one gets again the constant \(n_0(x; 0) = n_D\) and the previous results. For \(w = 0\) we have to calculate \(V_D(x, 0)\) numerically. Figure 1 shows results for \(B = 0\), \(T = 0\), \(n_D = 4 \cdot 10^{11} \text{ cm}^{-2}\), \(k_D = 1\) and the two cases \(w = 0\) (black lines) and \(w = 0.1\) (red lines), under the conditions \(n_D(x; w) \equiv 0\) for \(|x| > 0.9d\). In the inset, the horizontal broken line labeled \(n_D(B)\) indicates the electron density which, in a perpendicular magnetic field of strength \(B\), corresponds to filling factor \(\nu = 2\). The intersections of these lines with the curve \(n_D(x; w)\) fix the positions, at which ISs with local filling factor \(\nu(x) = 2\) may develop at sufficiently low temperatures. For both cases, \(w = 0\) and \(w = 0.1\), the electron density has the symmetry \(n_D(-x; w) = n_D(x; w)\). Since \(n_D(x; 0)\) decreases monotonically from the center \(x = 0\) towards the edges, for each \(B\)-value at most two ISs with filling factor \(\nu = 2\) are possible. In contrast, \(n_D(x; 0.1)\) has a minimum in the center and in the interval 6.45 \(T \leq B \leq 6.95\) \(T\) up to four ISs are possible, which do not move towards the center of the bar for \(B\)-values near the high-\(B\) edge of the QHP. Of course, the model given in equation (1) can describe situations, which allow for more ISs. Some results for \(k_D = 5\) and \(w = -0.1\), where the model allows up to ten ISs with filling factor \(\nu = 2\) [17], have been discussed in [3].
The values given in [14] for the longitudinal resistance are plotted as functions of the magnetic consequence, the ISs at these (length reasonable results near the low-temperatures are shown and, for comparison, the $T_{L} = 2.5$ K result for the unmodulated case is included$^1$. To get reasonable results near the low-$B$ edge of the QHP, a spatial average of the conductivity tensor over the magnetic length ($\lambda = \ell_B$) is performed, as described in equation (A6) of [14].

A rather well defined ($\nu = 2$)-QHP is seen near $B = 6.6$ T and the indication of a ($\nu = 4$)-QHP near $B = 3.3$ T. Well quantized resistance values $R_{long} \ll 10^{-4} R_{LL}$ are only obtained for $T \leq 2.5$ K in the ($\nu = 2$)-QHP. The sharp cusps in the logarithmic plot of $R_{long}$ versus $B$ (lowest panel of figure 2) occur at the $B$-values, at which the number of possible ISs changes (see figure 1).

Apparently the ($\nu = 2$)-QHP of the system with modulation ($w = 0.1$) is smaller than that of the system without modulation ($w = 0$). The high-$B$ edge of the plateau for $w = 0.1$ occurs at a smaller $B$-value than for $w = 0$, since the maximum electron density for $w = 0.1$ is smaller than for $w = 0$. The low-$B$ edge of the plateau occurs at somewhat higher $B$-values than for $w = 0$, because for these $B$-values filling factor $\nu = 2$ occurs at densities $n_d(B)$, at which according to figure 1 the slope of $n_d(x; 0.1)$ is steeper than that of $n_d(x; 0.0)$. As a consequence, the ISs at these $B$-values are narrower for $w = 0.1$ than for $w = 0$ [6], and are destroyed already at lower $B$-values.

For the mentioned model using equation (1) with $k_B = 5$ and $w = \pm 0.1$ one obtains wider QHPs than for $w = 0$, mainly because the resulting electron density $n_d(x; w)$ has larger maxima for $w = \pm 0.1$ than for $w = 0$ [17].

2.2. Temperature dependence of current density beyond linear response

The strongest effect of the modulation of the donor-density is expected in the $B$-regime, in which more than two ISs with the same local filling factor may develop at low temperatures (see figure 1). Typical examples are shown in figure 3 for $B = 6.7$ T and two values of the ASDC. To obtain reasonable accuracy and convergence of the iteration at low temperatures, the interval $2d = 3$ $\mu$m was divided into $N = 1000$ equal parts for $I = 0.2$ $\mu$A. For $I = 1.2$ $\mu$A, $N = 1500$ subintervals were used, but convergence was obtained only down to $T_L = 1.7$ K. Collision broadening and Joule heating are treated as in [14], with the corresponding parameters $\gamma = 0.1$ and $p = 0.5$. The uppermost panels of figure 3 show the density of the ASDC at relatively high temperatures on a

$^1$The values given in [14] for the longitudinal resistance are too small by a factor 2.
linear scale. At the highest lattice temperature ($T_L = 50$ K, black lines) the Hall field $E_H(x) = \partial_x \mu^e(x)/e$ is nearly constant, so that the normalized Hall potential $[\mu^e(x) - \mu^d(-d)]/\mu^e(d) - \mu^d(-d)$, shown in the lowest panels, increases nearly linearly across the sample, while the current density $j_y(x)$ shows a similar position-dependence as the electron density $n_{el}(x)$ (see figure 1).

With decreasing temperature the Landau quantization becomes noticeable and the longitudinal resistivity $\rho_L(x)$ becomes small at positions with $\nu(x) \approx 1$ (Shubnikov–de Haas effect), so that $j_y(x)$ exhibits there maxima, which are already much larger than the minima for $T_L = 10$ K (blue lines in figure 3). The upper panels of figure 3 show, that for $T_L \gtrsim 20$ K the response is essentially linear, whereas for $T_L \lesssim 10$ K nonlinear feedback

**Figure 2.** Linear response result for $B$-dependence of Hall resistance $R_{Hall} = [\mu^e(d) - \mu^d(-d)]/\epsilon f$ (upper panel) and longitudinal resistance $R_{Long} = 2d E_H^2/I$ in linear (central) and logarithmic scale (lower panel) with modulation of the background charge density as in figure 1. Results are shown for three temperatures for the modulated case ($w = 0.1$), and for $T = 1.5$ K for the unmodulated case ($w = 0$).

**Figure 3.** Current density $j_y(x)$, filling factor $\nu(x)$, and normalized Hall potential for $B = 6.7$ T, two values of the ASDC, $I = 0.2 \mu$A (left panels) and $I = 1.2 \mu$A (right panels), and the indicated values of the lattice temperature $T_L$. The model parameters are $k_D = 1$, $w = 0.1$ (see equation (1)).
effects occur, which become stronger with decreasing temperature, and lead to strongly nonlinear transport properties.

With further decreasing $T_L$ the difference between the maxima and the minima of $j_y(x)$ increases further, to more than six orders of magnitude for $T_L \lesssim 2$ K. To demonstrate this, figure 3 shows $j_y(x)$ for $T_L \leq 10$ K on a logarithmic scale. The reason for this strong spatial variation of the density of the ASDC is the occurrence of incompressible stripes with filling factor $\nu(x) \approx 2$ and exponentially small $\rho_e(x)$ values in their centers. Then our selfconsistent calculation yields a current density, which with decreasing $T_L$ is more and more confined to these ISs, where the current flows essentially dissipation-less. This is an example of the general thermodynamic principle of minimum entropy production in stationary non-equilibrium states.

Since without ASDC electron density and conductivity tensor have reflexion symmetry, $n_{el}(-x; w) = n_{el}(x; w)$ and $\delta(-x) = \delta(x)$, in the linear response limit filling factor and current density have the same symmetry, $\nu(-x) = \nu(x)$ and $j_y(-x) = j_y(x)$, respectively. This symmetry is, however, destroyed already by a rather small ASDC, and the resulting asymmetry of electron and current density increases rapidly with increasing ASDC and with decreasing lattice temperature. Figure 4 demonstrates how the nonlinear feedback of the ASDC on the confining potential and the ISs works. As follows from figure 1, for $B = 6.7$ T ISs may develop at sufficiently low temperatures near $x = \pm 0.3d$ and $x = \pm 0.3d$, and the ISs near $x = \pm 0.3d$ are expected to be wider than those near $x = \pm 0.7d$, since near $x = \pm 0.3d$ the slope of the electron density profile $n_{el}(x)$ is not so steep as near $x = \pm 0.7d$ [6]. The externally ASDC is chosen in such a manner, that its direction agrees with the direction of the intrinsic current, carried by the eigenstates centered near $x = -0.3d$ and $x = 0.7d$, and is opposite to the intrinsic current near $x = -0.7d$ and $x = 0.3d$. So the most favorable region for the current transport at low temperatures is the region near $x = -0.3d$, where a wide IS with intrinsic current in the direction of the ASDC develops at low $T_L$ and high ASDC.

Since in the ISs the Hall field $E_H(x) = \partial_x \mu^e(x) / e$ is proportional to the current density $j_y(x)$, the increase of $\mu^e(x)$ across an IS is a measure for the fraction of the ASDC carried by this stripe, and the current distribution is easily read off from the lowest panels of figure 3, which show the normalized change of $\mu^e(x)$ across the sample. Whereas for not too low temperatures ($T_L \gtrsim 4$ K) similar amounts of currents flow through all the stripes with $\nu(x) \approx 2$, these changes at very low temperatures ($T_L \lesssim 2$ K). At these low temperatures nearly all the imposed current flows through the IS near $x = -d/3$, so that nearly the total increase of $\mu^e(x)$ happens across this IS (see lowest panels of figure 3). How this situation is reached with decreasing $T_L$ and at which temperature the system shows the IQHE, depends strongly on the strength of the ASDC.

For small ASDC, as for $I = 0.2 \mu$A in figures 3 and 4, we observe with decreasing $T_L$ a continuous transition from the resistive high-temperature state to an IQH state at $T_L \gtrsim 4$ K. Whereas for $T_L \gtrsim 4$ K a considerable part of the ASDC flows through other parts of the sample, for $T_L \lesssim 2$ K nearly all the ASDC flows through the IS near $x = -0.3d$. For high ASDC, as for $I = 1.2 \mu$A in figures 3 and 4, Joule heating is more important and the resistive high-temperature state prevails at lattice temperatures down to $T_L \sim 4$ K, and changes at lower $T_L$.

**Figure 4.** Upper panels: effective confinement potentials (solid lines) and electrochemical potentials (dashed) for three lattice temperatures, and the lowest two Landau bands (thin violet lines) for $T_L = 2$ K; medium panels: corresponding filling factors; lower panels: difference $\Delta T_E$ between the local electron temperature and the lattice temperature. Model parameters are as in figure 3.
suddenly to an IQH state, in which nearly all the ASDC flows through the IS near $x = -0.3d$. This IS is now much broader than the others and than that for $I = 0.2 \, \mu A$.

The strong influence of the strength of the ASDC on the temperature dependence of Hall and longitudinal resistance of the considered model situation is demonstrated in figures 5(a) and (b). Whereas for $I < 1 \, \mu A$ a continuous transition from the resistive high-temperature state to the IQH state is observed, for $I > 1 \, \mu A$ this transition happens abruptly at a critical temperature $T_{cr}(I)$. The transition temperature $T_{cr}(I)$ decreases with increasing $I$, whereas the magnitude of the discontinuity of $R_{\text{long}}$ increases. If we start with such a low-temperature state at $T_e < T_{cr}(I)$ and increase the temperature while keeping $I$ fixed, our iteration procedure yields low-resistance states up to a critical temperature $T_{cr}^f(I) > T_{cr}(I)$, where a sudden transition to the dissipative high-temperature state takes place. Such a hysteretic behavior was also found and discussed for the model with homogeneous donor charge, which allows only two ISs with the same filling factor [14]. It is also obtained in figures 5(c) and (d), which consider our present modulated model at $B = 6.4 \, T$, where only two ISs with $\nu(x) = 2$ are possible. Since these ISs are relatively narrow, for $T_L \geq 1.5 \, K$ quantum Hall states exist only for relatively low ASDC, $I \leq 0.25 \, \mu A$.

The sudden transition from a dissipative high-temperature state to a nearly dissipation-less low-$T_L$ state explains also the results for the ASDC of $I = 1.2 \, \mu A$, shown in the right panels of figure 3. For $T_L \geq 2.1 \, K$ the system is in a dissipative state, where Joule heating dominates and the profiles of electron- and current-density depend only weakly on temperature. Below this temperature the system is in a nearly dissipation-free state, where nearly all the ASDC flows through the broad IS centered around $x = -d/3$. (The lowest temperature, at which this calculation converged, was $T_L = 1.7 \, K$.)

Figure 5 demonstrates the well known fact (see e.g. [14]), that there is no strict rule to decide whether a state at finite temperature shows the IQHE or not, since at finite temperature (and for finite collision broadening of the Landau density of states) there exist no exact quantization $R_{\text{Hall}} = h/(2e^2)$, $R_{\text{long}} = 0$. For the bulk-dominated situation at $B = 6.7 \, T$, which allows for more than two ISs, we may require as criterion for the IQHE, that, at $T_L = 1.5 \, K$, $R_{\text{long}}$ (and $R_{\text{Hall}} = h/(2e^2)$), which is of a similar order of magnitude) has to be less than $10^{-8}h/e^2 \approx 2.58 \times 10^{-4} \, \Omega$. This condition is satisfied in figure 5(b) for $I \leq 1.5 \, \mu A$. But for small currents, $I \lesssim 0.2 \, \mu A$, the state near $T_L = 2.5 \, K$ shows already similar quantization effects, and a reasonable criterion for the IQHE at $T_L = 2.5 \, K$ may be $R_{\text{long}} \lesssim 10^{-4}h/e^2$. Such a weaker criterion makes also sense for the edge-dominated situation (e.g. Figure 5(d)), where only two ISs near the edges are possible.

### 2.3. Magnetic field dependence at intermediate temperature

For the Hall bar model considered here, number and position of possible ISs, and as a consequence the spatial distribution of the ASDC, depend of course strongly on the magnetic field. Figure 6 gives an overview over the $B$-dependence of the current distribution in the $B$-regime of the $(\nu = 2)$-QHP at the lattice temperature $T_L = 2.5 \, K$ and for two values of the ASDC.

Near the low-$B$ edge of this QHP, i.e. for $B \lesssim 6.45 \, T$, only two ISs with $\nu = 2$ can exist, since in the center of the sample $\nu(x)$ is larger than two. For weak ASDC, $I \lesssim 0.2 \, \mu A$, this leads to stripes near the edges, which carry
nearly all the ASDC with maxima of $j_y(x)$, which are about three orders of magnitude larger than the weak maximum in the center. With increasing ASDC the part of the current carried by the IS near the right edge ($x \approx 0.75d$), which has the direction of the intrinsic equilibrium current density in this stripe, becomes larger than the part carried by the IS on the opposite side. This is already seen in figure 6 for $B = 6.45 \, \text{T}$ and $I = 0.1 \, \mu\text{A}$. With increasing ASDC the current density starts to leak out of this stripe and transport becomes resistive (according to figure 5 at $B = 6.4 \, \text{T}$ for $I \gtrsim 0.1 \, \mu\text{A}$). For $I = 0.5 \, \mu\text{A}$ the current density peaks near the edges are much smaller and a considerable part of the ASDC flows through the dissipative center region. The $I$-dependence of Hall and longitudinal resistance for these situations is plotted in figure 7, which shows that for $T_L = 2.5 \, \text{K}$ and $B \lesssim 6.45 \, \text{T}$ approximate quantum Hall conditions hold only for very small ASDC.

As $B$ becomes larger than 6.45 T, the electron density minimum in the center of the sample approaches filling factor two. For $B = 6.5 \, \text{T}$ an IS appears in the center, which carries a part of the ASDC. For $I = 0.1 \, \mu\text{A}$ this
central stripe is broader than the two stripes near the edges, but carries a smaller part of the nearly dissipationless current than the latter (see the red lines in the left panels of figure 6 and in figure 7). With increasing ASDC the part of the current carried by the center region increases and leaks out of the stripe, so that at \( I = 0.5 \mu A \) the state is dissipative and shows no pronounced resistance quantization.

With further increasing \( B \), the IS in the center splits into two ISs, since the filling factor in the center of the sample becomes less than two, and the separation of the split ISs increases with \( B \), as is expected from figure 1. In the left of these central stripes the equilibrium Hall current has the same direction as the ASDC, and with increasing \( I \) this stripe widens and carries an increasing part of the imposed current. This is nicely shown by figure 6 for the \( B \)-values 6.55 T and 6.60 T, for which according to figure 7 the resistance values are reasonably well quantized up to ASDCs \( I \leq 0.5 \mu A \). For \( B = 6.55 \) T and small ASDC, \( I = 0.1 \mu A \) (first column of figure 6), the current density \( j_\parallel(x) \) shows a local minimum near \( x = 0 \), indicating the tendency of the central IS to split into two stripes. At stronger ASDC, \( I = 0.5 \mu A \) (second column), the current-induced broadening of the (left) IS counteracts this tendency and the central minimum of \( j_\parallel(x) \) is less pronounced. For slightly smaller \( B \)-field (e.g. \( B = 6.53 \) T), increasing current will be able to remove the splitting completely (see figure 10 below). For \( B = 6.6 \) T the splitting into two central ISs is clearly seen, also for \( I = 0.5 \mu A \).

As \( B \) increases further, all the ISs move towards the maxima of the electron density near \( x = \pm 0.56 d \), and the outer ISs become broader and carry a larger part of the ASDC (see data for \( B = 6.8 \) T). For \( B \approx 6.9 \) T the inner and outer ISs merge and broad ISs with local filling factor two are generated near \( x = \pm 0.56 d \), which represent the maxima of the electron density and carry all the ASDC. A close look on the data shows that the left IS at \( x = -0.56 d \) carries somewhat more current than the right one at \( x = 0.56 d \), and that the difference increases with \( I \). The resistance values at \( B = 6.9 \) T are well quantized for \( I \lesssim 0.5 \mu A \). As \( B \) increases above this value, the ISs at the maxima of the electron density shrink rapidly. We find that at \( T_L = 2.5 \) K no ISs and no resistance quantization exists for \( B \gtrsim 6.92 \) T.

Thus, the number and position of relevant ISs, which carry the ASDC, is rather different from the unmodulated case, where, as \( B \) increases from the low to the high edge of the QHP, two edge-near ISs move towards the center and merge to a single IS as \( B \) approaches the high edge. Here at a certain \( B \)-value, in addition to the edge-near ISs, a central IS appears, which splits with increasing \( B \), so that four ISs result, which near the high-\( B \) edge of the QHP merge into two ISs at considerable distances from the center.

2.4. Transition edge-bulk-dominated regime under low temperature and high current
As we have seen in figure 3, the current-induced asymmetry of the current distribution is strongly temperature-dependent and increases with decreasing lattice temperature. This \( T_L \)-dependence modifies also the \( B \)-dependence of the current density considerably. To demonstrate this effect, we show in figure 8 the analog to figure 6 for the lower lattice temperature \( T_L = 1.5 \) K. Now the current-induced asymmetry is so strong, that for \( B = 6.45 \) T and \( B = 6.5 \) T nearly all the current flows through the right IS near \( x = 0.75 d \). This holds even for the stronger current \( I = 0.5 \mu A \) (second column of figure 8), which broadens this IS, but cannot destroy the IQHE (see also figure 9). Due to the low temperature, the current-density peak in the center is so small, that it
practically does not contribute to the increase of the Hall potential, even for $B = 6.5\, \text{T}$, where the central minimum of $\nu(x)$ is only slightly larger than 2.

For $B \gtrsim 6.51\, \text{T}$ the central density minimum produces an incompressible structure with $\nu(x) = 2$, which changes its shape and its possibility to carry current drastically with the strength of the ASDC. This structure results from the splitting of the central ISs at low ASDC and the broadening of one of these stripes with increasing ASDC, and will be discussed in some detail below. With further increasing $B$, the splitting of the central ISs survives at higher ASDC, as is indicated by the results for $B = 6.55\, \text{T}$ and clearly seen for $B = 6.6\, \text{T}$. These results and those for $B = 6.8\, \text{T}$ are similar to the corresponding results for $T_L = 2.5\, \text{K}$, although the asymmetries are much stronger and nearly all the ASDC flows through the left of the central ISs. This asymmetry is reduced as $B$ approaches $6.9\, \text{T}$, and for $B = 6.9\, \text{T}$ one finds again two IS centered near $x = \pm 0.56\, \mu\text{A}$, which represent the maxima of the electron density and carry all the ASDC nearly dissipation-less. The IS at $x = -0.56\, \mu\text{A}$ carries now only slightly more current than that at $x = 0.56\, \mu\text{A}$, and the width of the IS and of the $j(x)$-peaks increases slightly with $I$. Near $B = 6.92\, \text{T}$ the high-$B$ edge of the QHP is reached.

Figure 9 contains some additional information. At $T_L = 1.5\, \text{K}$ edge-near ISs exist for $B \gtrsim 6\, \text{T}$. With increasing ASDC, $R_{\text{long}}(I)$ first decreases and then increases, for $B \gtrsim 6.1\, \text{T}$ at a critical $I_{\text{cr}}(B)$ discontinuously by several orders of magnitude. At this $I_{\text{cr}}(B)$, which increases with increasing $B$, also the Hall resistance leaves abruptly the quantized value. In the edge-dominated regime $6.3\, \text{T} \lesssim B \lesssim 6.5\, \text{T}$ our iteration procedure did not converge up to $I_{\text{cr}}(B)$. For $B \gtrsim 6.52\, \text{T}$, where ISs are expected in the center of the sample, we obtained convergence up to $I \approx 1\, \mu\text{A}$ with an interesting structure of $R_{\text{long}}(I)$ near those $I$-values at which the partial current transfer from the center to the right sample edge starts (see figure 10 below). In the bulk-dominated regime, $6.6\, \text{T} \lesssim B \lesssim 6.8\, \text{T}$, where four ISs exist and carry the current dissipation-less, we obtained convergence and well quantized resistances up to $I = 1\, \mu\text{A}$. For $B \gtrsim 6.9\, \text{T}$, where only two ISs exist, at which the electron density assumes its maximum value, we obtain good resistance quantization at small $I$ but an increase of $R_{\text{long}}(I)$ with increasing $I$. For $B \gtrsim 6.92\, \text{T}$ no acceptable quantization is obtained.

As mentioned above, for $B \gtrsim 6.52\, \text{T}$ the spatial distribution of the ASDC depends in a peculiar way on its strength. Figure 10 demonstrates this for $B = 6.53\, \text{T}$ and $T = 1.5\, \text{K}$. For small ASDC, i.e. in the linear response limit (black lines in figure 10), nearly all current is carried by two well separated ISs centered at $x = \pm 0.096\, \mu\text{A}$, whereas the current carried by the narrow, edge-near ISs centered at $x = \pm 0.73\, \mu\text{A}$ by orders of magnitude smaller. For small ASDC ($I = 0.1\, \mu\text{A}$, red lines) the IS near $x = 0.1\, \mu\text{A}$ moves toward the center of the sample and carries a smaller part of the ASDC, whereas the IS near $x = -0.1\, \mu\text{A}$ becomes broader and carries a larger part of the current, with a strongly asymmetric density profile $j(x)$. For $I \gtrsim 0.2\, \mu\text{A}$ the central ISs merge to a single broad IS, which for $0.2\, \mu\text{A} < I \lesssim 0.4\, \mu\text{A}$ carries nearly all the current, with an asymmetric current density, which is largest near the left edge of the stripe and decreases towards its right edge. At $I \gtrsim 0.4\, \mu\text{A}$ the situation changes and the fraction of the current carried by the asymmetric central IS decreases with increasing $I$, while the remaining current is taken over by the edge-near IS at $x \sim 0.74\, \mu\text{A}$. The width of this stripe increases with

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Figure 9. Dependence of Hall resistance (upper panel) and longitudinal resistance (lower panel) on the ASDC for the indicated $B$-values at $T_L = 1.5\, \text{K}$. The model is the same as in figures 6-8.
increasing \( I \), so that the increasing amount of current flows dissipation-less within this stripe, at least for the values \( I \leq 1.0 \ \mu A \), for which our calculation converged. The width of the IS near \( x = -0.74 \ d \) turned out to be independent of the ASDC and its contribution to the total current, being largest for \( I \approx 0.4 \ \mu A \), remained extremely small. Calculation of longitudinal and Hall resistances (see figure 9) shows, that for \( T = 1.5 \ \text{K} \) and \( B = 6.53 \ \text{T} \) the IQHE is well developed and that \( R_{\text{long}}(I) \) exhibits a peculiar structure in the interval \( 0.3 \ \mu A \leq I \leq 0.5 \ \mu A \), with a maximum at \( I \approx 0.43 \ \mu A \), where the partial current transfer from the central IS to the edge-near IS starts. For \( B = 6.52 \ \text{T} \) we obtained very similar results, but the structure in \( R_{\text{long}}(I) \), and the related partial current transfer, occurred in the interval \( 0.1 \ \mu A \leq I \leq 0.3 \ \mu A \), with a maximum at \( I \approx 0.24 \ \mu A \). Calculations for \( B = 6.535 \ \text{T} \) indicated such a structure at \( I \geq 0.4 \ \mu A \) with a maximum near \( I = 0.53 \ \mu A \), but did not converge for larger \( I \)-values, probably because of the sharp structures, which occur in the position-dependence of the current density \( j_{y}(x) \).

3. Summary and discussion

The modulation of the donor-charge-density and the resulting modulation of the electron density \( n_{el}(x) \) opens the possibility that, in certain \( B \)-intervals, more than two ISs with filling factor \( \nu(x) = 2 \) may develop at low temperatures. Explicit results have been presented for a donor density modulated according to equation (1) with \( k_{D} = 1 \) and \( w = 0.1 \), which leads to up to four ISs. Calculations for \( k_{D} = 5 \) and for \( w = \pm 0.1 \), where up to ten ISs are observed, have also been performed [17] and compared with experiments [5]. For weak ASDC all these ISs contribute to the current transport, so that the current density extends over a broad part of the sample. This is the ‘bulk-dominated’ regime observed in experiments [5]. The distribution of the ASDC over the existing ISs changes, however, with increasing strength of the current [5] as is explicitly demonstrated in figures 6, 8 and 10.

The actual distribution of the ASDC over the sample in an IQHE state depends on several parameters. It depends, of course, on \( B \), which determines number and position of the ISs, which can carry the current. But it also depends strongly on temperature and the strength of the ASDC, which determine the amount of current carried by the individual stripes. The strength of the temperature dependence will also be influenced by the collision broadening of the Landau levels, which was not changed in this paper. Figure 6 shows, for the relatively high lattice temperature \( T_{L} = 2.5 \ \text{K} \), how with increasing \( B \) the current distribution changes from edge-dominated over center-dominated (when ISs occur near the relative minimum of \( n_{el}(x) \) in the center) to a bulk-dominated case determined by four ISs, which move at the high-\( B \)-edge of the QHP towards the positions of the density maxima. It also shows, that the strength of the total current can modify the fraction of current carried by the individual ISs.
Figure 7 shows, similar to the experiments [5], a continuous breakdown of the IQHE with increasing current. If we argue that the breakdown happens if \( R_{\text{long}}(I) > R_{\text{long}}(I_\text{cr}) = 10^{-4}\Omega/\text{cm}^2 \), then \( I_\text{cr} \) increases with \( B \) for \( B \lesssim 6.8 \text{T} \) and decreases with increasing \( B \) for \( B \gtrsim 6.8 \text{T} \). This and the fact, that in the breakdown regime for \( B \lesssim 6.8 \text{T} \) the slope \( dR_{\text{long}}(I)/dI \) decreases with increasing \( B \), is similar to the experimental finding in [5]. However, in figure 7 these slopes also decrease with increasing \( B \) for \( B \gtrsim 6.8 \text{T} \), which is not in agreement with the experiment [5], which shows in the high-\( B \) part of the (\( \nu = 2 \))-plateau a rather abrupt breakdown of the IQHE with large slopes \( dR_{\text{long}}(I)/dI \) in the breakdown regime (see figures 2 and 3 of [5]). This indicates that the present model is not realistic near the maxima of \( n_\text{el}(x) \).

Comparison of figures 6 and 8 shows, that lowering the lattice temperature may have a strong effect on the current distribution, especially in the edge-dominated regime, which for lower \( T_L \) extends to smaller \( B \)-values (\( 6.0 \text{T} \lesssim B \lesssim 6.5 \text{T} \) for \( T_L = 1.5 \text{ K} \)). For \( B \gtrsim 6.1 \text{T} \) the edge-dominated quantized states show with increasing ASDC an abrupt breakdown at a critical current \( I_\text{cr} \), which increases rapidly with \( B \) (see figure 9). For \( 6.2 \text{T} \gtrsim B \lesssim 6.5 \text{T} \) our iteration procedure did not converge for currents much larger than \( 0.5 \mu\text{A} \), probably because of the sharp spatial structure of the current density \( j_\text{el}(x) \), which becomes very different in different ISs.

Figure 10 shows an example for the big change of the current distribution with increasing ASDC. As discussed in detail at the end of section 2.4, the increasing strength of the ASDC leads to an overlap of the two ISs in the center of the sample, connected with a strong asymmetry of the current density, and finally to a redistribution of current from the center to an IS near one edge. This redistribution leads to the peculiar structure of the \( R_{\text{long}}(I) \) curve for \( B = 6.53 \text{T} \) in figure 9. Similar results were obtained for \( B = 6.52 \text{T} \) and \( B = 6.535 \text{T} \), where the structure occurs at lower and larger \( B \)-values, respectively. To demonstrate these interesting effects, figure 9 shows \( R_{\text{long}} \) on a logarithmic scale. A linear plot, which shows the resistance in both, the normal and the IQHE state, cannot resolve these effects. The existing linear plots of the measured longitudinal voltage as function of the applied bias voltage show structures in the regime of normal dissipative states, but not in the regime of the IQHE.

For \( B \gtrsim 6.9 \text{T} \) the \( R_{\text{long}}(I) \) curves obtained for \( T_L = 1.5 \text{ K} \) are similar to those for \( T_L = 2.5 \text{ K} \), although with values, which are a few orders of magnitude smaller.

Current-induced breakdown of the IQHE is clearly obtained for \( B \)-values close to the low-\( B \) edge of the QHP and very close to its high-\( B \) edge, but also for some \( B \)-values which produce ISs far away from local extrema of the electron density. In other \( B \)-regions, especially when the position of the corresponding ISs is close to a local extremum of \( n_\text{el}(x \mid w) \), our Newton–Raphson iteration procedure did not converge or the IQHE state was stable up to \( I > 1.0 \mu\text{A} \).

As expected, our calculations for stationary non-equilibrium states of translation-invariant Hall systems cannot quantitatively explain the experimental results obtained on narrow Hall bars, but they make many aspects of these results plausible. If the calculations predict a current-induced breakdown of the IQHE, we can expect that the experiment on a similar, long sample will also show such a breakdown. The opposite is, however, not true. If our calculation yields a quantum Hall state up to high ASDC, we cannot be sure, that the experiment behaves similar. Source and drain contacts destroy the translation invariance, and usually produce ‘hot spots’ in the Hall bar close to the contacts, in which the local current density is large and a local breakdown of the IQHE may happen. But also local fluctuations of the electron density somewhere in the sample may lead to narrow regions of the (not translation-invariant) incompressible stripes with large current density, where the spatial distance between adjacent Landau levels becomes small and quasi-elastic-inter-Landau-level scattering [18] becomes possible, which has not been included in the present work. As has been discussed before [19], such a local breakdown will spatially expand and finally extend over the whole sample, even if our calculation for a translation-invariant local-equilibrium system with the same parameter values of \( B, T_L \), and \( I \) predicts a quantum Hall state.

There are, however, several things we can learn from these calculation. First, for the appearance of the IQHE we do not need a constant electron density in the sample or any kind of localization of individual electronic states. The IQHE will occur, if the ASDC flows only through incompressible regions, not necessarily translation invariant, with the same integer value of the filling factor. These regions may be connected or separated by regions with different filling factors, and they may surround islands with different filling factors. If, due to long-range density fluctuations, the shape of these incompressible regions changes along the sample, the current density profile across the sample will also change. At low temperatures and high currents, these changes of the current density profile may be large, similar to the experimental observations documented in figures 5(f) and A4(d)–(f) of [5]. We must also expect, that, due to nonlinear feedback effects, the spatial distribution of the ASDC may change with its strength.

Thus, the models with modulated donor charge density allow to understand qualitatively several experimental results in the bulk-dominated regime, in addition to those in the edge-dominated regime, which are not changed qualitatively by the modulation. In view of the simplifying model assumptions and
approximations of our model calculation, a quantitative agreement of our results with experiments can, of course, not be expected.

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