Viscous and resistive accretion flows with radially self-similar outflows

Kazem Faghei* and Azam Mollatayefeh*

School of Physics, Damghan University, Damghan, Iran

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ABSTRACT
The existence of outflows in accretion flows has been confirmed by observations and by magnetohydrodynamics simulations. In this paper, we study the outflows of advection-dominated accretion flows (ADAFs) in the presence of resistivity and a toroidal magnetic field. The mechanism of energy dissipation in the flow is assumed to be the viscosity and the magnetic diffusivity as a result of turbulence in the accretion flow. It is also assumed that the magnetic diffusivity and the kinematic viscosity are not constant and that they vary by position, and the $\alpha$-prescription is used for these. The influence of outflows emanating from an accretion disc is considered as a sink for mass, angular momentum and energy. The self-similar method is used to solve the integrated equations that govern the behaviour of the accretion flow in the presence of outflows. The solutions represent the disc that rotates faster and becomes cooler for stronger outflows. Moreover, by adding magnetic diffusivity, the surface density and rotational velocity decrease, while the radial velocity and temperature increase. A study of the present model with the magnitude of a magnetic field implies that the disc rotates and accretes faster and becomes hotter, while the surface density decreases. The thickness of the disc increases when adding a magnetic field or resistivity, while the disc becomes thinner for more mass and energy losses resulting from the outflows.

Key words: accretion, accretion discs – MHD – ISM: jets and outflows.

1 INTRODUCTION
Accretion discs are important physical objects in astrophysics. The standard model of a thin accretion disc is widely regarded as a successful model for explaining the observational features in active galactic nuclei (AGNs) and X-ray binaries (Shakura & Sunyaev 1973). However, the standard disc model cannot be used to explain the spectral energy distributions (SEDs) of many sources, such as Sgr A$^\ast$. To understand such systems, the model of advection-dominated accretion flow (ADAF) is introduced (Ichimaru 1977; Narayan & Yi 1994). In this accretion flow, the energy released as a result of dissipation processes is retained in the fluid rather than being radiated away. The models of ADAF are in an intermediate position between the model of the spherically symmetric accretion flow of a non-rotating fluid (Bondi 1952) and the classical accretion theory of a cool, thin disc (e.g. Pringle 1981).

The observational evidence of accretion flows implies that the outflow is an important property in these systems. For example, a comparison of the accretion luminosities of neutron stars and white dwarfs in quiescence with similar binary companions implies that the accretion rate on to white dwarfs is larger by three orders of magnitude than that on the surface of a neutron star (Loeb, Narayan & Raymond 2001). This indicates significant outflows in accretion flows. Moreover, outflows seem to be common in the nuclei of galaxies. Marrone et al. (2006) have suggested that the accretion rate of Sgr A$^\ast$ at small radii, much smaller than the Bondi radius, must be low, below $10^{-7} \dot{M}_\odot \text{ yr}^{-1}$. However, Baganoff et al. (2003) have estimated that the hot plasma surrounding Sgr A$^\ast$ should supply the accretion rate of $10^{-6} \dot{M}_\odot \text{ yr}^{-1}$ at the Bondi radius. This significant difference between the inner and outer mass accretion rates indicates mass loss from the accretion flow as a result of the outflow (Kawabata & Mineshige 2009). Thus, several authors have studied accretion discs in the presence of outflows (Knegge 1999; Fukue 2002, 2004; Shadmehri 2008; Xie & Yuan 2008; Bu, Yuan & Xie 2009; Kawabata & Mineshige 2009; Li & Cao 2009; Abbassi, Ghanbari & Ghasemnezhad 2010).

Using a simple, parametric approach that is sufficiently general to be applicable to many types of dynamical disc–wind models, Knigge (1999) derived the radial distribution of the dissipation rate and effective temperature across a Keplerian, steady-state, mass-losing accretion disc. Fukue (2002) has examined a hydrodynamical wind, which emanates from an accretion disc and is driven by thermal and radiation pressure, under a one-dimensional approximation along supposed streamlines. In another study, Fukue (2004) studied a supercritical accretion regime, where the mass accretion rate was regulated just at the critical rate with the help of wind mass loss. He derived a critical radius outside of which the disc is in a radiation-pressure-dominated standard state. Inside this critical radius, the disc is in a critical state, where the excess mass is...
expelled by wind, and the accretion rate is kept just at the critical rate at any radius inside the critical radius. Shadmehri (2008) has studied the effects of thermal conduction and outflows on ADAFs. He found that in comparison to accretion flows without winds, the disc rotates faster and becomes cooler because of the angular momentum and energy flux, which are taken away by the winds. Based on a one-and-a-half-dimensional description of accretion flow, Xie & Yuan (2008) considered the interchange of mass, radial and azimuthal momentum, and the energy between the outflow and inflow. Bu et al. (2009) have presented self-similar solutions for ADAFs with outflows and ordered magnetic fields. They have assumed that the magnetic field has a strong toroidal component and a vertical component, in addition to a stochastic component. They found that the dynamical properties of ADAFs can be changed significantly in the presence of ordered magnetic fields and outflows. Abbassi et al. (2010) examined the effects of a hydrodynamical wind on ADAFs in the presence of a toroidal magnetic field under a self-similar treatment. Their results imply that, in the presence of the wind, the disc temperature decreases because of energy flux, which is taken away by winds, and the accretion velocity is enhanced.

The importance of magnetic diffusivity has been studied in several accreting systems, such as protostellar discs (Stone et al. 2000; Fleming & Stone 2003), discs in dwarf nova systems (Gammie & Menou 1998), the discs around black holes (Kudoh & Kaburaki 1996) and the Galactic Centre (Melia & Kowalenko 2001; Kaburaki et al. 2010). Moreover, two- and three-dimensional magnetohydrodynamics (MHD) simulations have shown that resistive dissipation is one of the crucial processes that determine the saturation amplitude of the magnetorotational instability (MRI). Thus, the linear growth rate of MRI can be reduced significantly as a result of the suppression by ohmic dissipation (Sano, Inutsuka & Miyama 1998; Fleming, Stone & Hawley 2000; Masada & Sano 2008). Moreover, from a comparison of ideal and resistive MHD simulations, it seems that the magnetic diffusivity might play an important role in astrophysical outflows (Fendt & Ćemeljčić 2002; Ćemeljčić et al. 2008).

As mentioned, semi-analytical studies of ADAFs with outflows are typically related to systems without magnetic diffusivity. However, non-ideal MHD simulations imply that the resistivity plays an important role for outflows (e.g. Fendt & Ćemeljčić 2002; Ćemeljčić et al. 2008). Akizuki & Fukue (2006) have proposed a self-similar solution for ADAFs with a highly ionized gas. They have assumed that the plasma resistivity is zero, and only viscosity is a result of the turbulence and dissipation in the disc. Also, they ignored the effects of outflow/wind in their model. In this paper, we want to explore the influences of resistivity on magnetized ADAFs with outflows. Thus, we adopt the solutions presented by Knigge (1999), Akizuki & Fukue (2006) and Shadmehri (2008). The paper is organized as follows. In Section 2, we define the basic equations for constructing a model for an ADAF in the presence of a toroidal magnetic field, resistivity and outflows. In Section 3, we use self-similar method to solve the equations that govern the behaviour of the accreting gas. We give the results of the model in Sections 4 and 5, and we provide a summary of the model in Section 6.

2 BASIC EQUATIONS

We consider the non-ideal MHD of steady, axisymmetric, viscous, accreting and rotating flow in the presence of a purely toroidal magnetic field. We use cylindrical coordinates \((r, \phi, z)\) centred on the accreting object. We ignore the general-relativistic effects and we use Newtonian gravity. Under these assumptions, the continuity equation is

\[
\frac{\partial}{\partial r}(r \Sigma v_r) + \frac{1}{2\pi} \frac{\partial M_w}{\partial r} = 0. 
\] (1)

Here, \(v_r\) is the radial velocity, \(\Sigma = 2\rho H\) is the surface density of the disc (where \(\rho\) and \(H\) are the mid-plane density and half-thickness of the disc, respectively) and \(M_w\) is the mass-loss rate by the outflow/wind. The half-thickness of the disc is given by \(H = c_s \sqrt{1 + \Pi/\Omega_1^2}\), where \(\Omega_1\) is the Keplerian angular velocity, \(c_s\) is the sound speed and \(\Pi\) is defined in equation (9). It will be reduced to its traditional form of \(H = c_s/\Omega_1\) in the absence of the toroidal component of the magnetic field \(B_{\phi}\) (equation 8). The sound speed is defined as \(c_s = (p_{\text{gas}}/\rho)^{1/2}\), where \(p_{\text{gas}}\) is the gas pressure. The cumulative mass-loss rate from the disc can be written as

\[
M_w(r) = \int_{r_i}^{r} 4\pi r' \dot{m}_w(r') \, dr'.
\] (2)

where \(r_i\) denotes the radius at the inner edge of the disc and \(\dot{m}_w(r)\) is the mass-loss rate per unit area from each disc face. Because the mass accretion rate is \(\dot{M} = -2\pi r \Sigma v_r\), from equations (1) and (2), we can write

\[
\frac{\partial \dot{M}}{\partial r} = \frac{\partial M_w}{\partial r}.
\] (3)

Equation (3) implies that the mass accretion rate varies by radius as a result of the outflow. Thus, we exploit a power-law dependence for the mass accretion rate as follows (e.g. Blandford & Begelman 1999):

\[
\dot{M}(r) = \dot{M}(R) \left(\frac{R}{r}\right)^s.
\] (4)

Here, \(R\) is the radius at the outer edge of the disc, \(\dot{M}(R)\) is the mass accretion rate at \(R\) and \(s\) is a free parameter. For a disc without outflow/wind, \(s = 0\), and in the presence of the outflow/wind, \(s > 0\) (e.g. Fukue 2004). The observed broad-band spectra of Sgr A* and soft X-ray transients can also be fitted by ADAF models with moderate outflows, \(s \sim 0.3–0.4\), if the direct heating of electrons in ADAFs is efficient (Quataert & Narayan 1999; Yuan, Quataert & Narayan 2003). Equations (1)–(4) imply that

\[
\dot{m}_w(r) = \frac{\dot{M}(R)}{4\pi R^2} \left(\frac{r}{R}\right)^{s-2}.
\] (5)

The radial equation of momentum is

\[
v_r \frac{d}{dr} \left(r^2 \Omega^2 \right) = r \left(\Omega^2 - \Omega^2_{\phi}\right) - \frac{1}{\Sigma} \frac{d}{dr} \left(\Sigma c_s^2\right) - \frac{c_s^2}{r} - \frac{1}{2\Sigma} \frac{d}{dr} \left(\Sigma c_\phi^2\right).
\] (6)

where \(\Omega\) is the angular velocity of the flow and \(c_\phi\) is the Alfvén speed, which is defined as \(c_\phi^2 = B_{\phi}^2/(4\pi \rho) = 2p_{\text{mag}}/\rho\), where \(p_{\text{mag}}\) is the magnetic pressure.

Taking the outflow/wind into consideration, the angular momentum transfer equation can be written as (e.g. Knigge 1999)

\[
\Sigma v_r \frac{d}{dr} \left(r^2 \Omega\right) = \frac{1}{r} \frac{d}{dr} \left(r^2 \Omega \Sigma v_r \right) - \frac{\left(\Omega^2 - \Omega^2_{\phi}\right) - (lr^2 \Omega) \frac{d M_w}{2\pi r^2} dr}{d r}.
\] (7)

The two terms on the right-hand side of equation (7) describe the effects of viscous torques due to shear \((\nu\), which is the effective kinematic viscosity\) and the angular momentum sink provided by the outflow. Here, we assume that matter that outflows at radius \(r\) on the disc carries away specific angular momentum \((lr^2 \Omega)\). Thus, \(l = 0\) corresponds to a non-rotating disc wind and \(l = 1\) to outflowing material that carries away the specific angular momentum it had at the point of outflow (Knigge 1999).
The hydrostatic balance in the vertical direction is integrated to
\[
\frac{GM}{r^3} H^2 = c_i^2 \left[ 1 + \frac{1}{2} \left( \frac{c_A}{c_i} \right)^2 \right] = (1 + \Pi)c_i^2.
\]
(8)

Here, we introduce \( \Pi \) as
\[
\Pi = \frac{P_{\text{mag}}}{p_{\text{bar}}} = \frac{1}{2} \left( \frac{c_A}{c_i} \right)^2,
\]
(9)
which is the degree of magnetic pressure to the gas pressure. Because we apply a steady self-similar method to solve the system equation, \( \Pi \) is constant throughout the disc. In fact, \( \Pi \) is a function of position and time (Machida, Nakamura & Matsumoto 2006; Oda et al. 2007; Khesali & Faghei 2008, 2009). Studies of hot accretion flows have found that the typical value of \( \Pi \) is in the range 0.01–1 (De Villiers, Hawley & Krolik 2003; Beckwith, Hawley & Krolik 2008). However, we also consider the magnetically dominated case (\( \Pi > 1 \)). The MHD simulation by Machida et al. (2006) shows that thermal instability grows in an accretion flow, and thus the magnetic pressure exceeds the gas pressure because the disc shrinks in the vertical direction and the toroidal magnetic flux is conserved. This results in a large value of \( \Pi \), and a magnetically dominated accretion flow is formed (Oda et al. 2007).

We assume that both the viscosity and the diffusivity are the result of turbulence in the disc, so it is reasonable to use these parameters in analogy with the \( \alpha \)-prescription of Shakura & Sunyaev (1973) for the turbulence:
\[
v = P_m \eta = \alpha c_i H.
\]
(10)
Here, \( P_m \) is the magnetic Prandtl number of the turbulence, assuming a constant of order of unity, \( \eta \) is the magnetic diffusivity and \( \alpha \) is a free parameter less than unity.

Here, we can write the energy equation, considering the energy balance in the system. We assume that the energy released because of viscous and resistive dissipations can be balanced by the advection cooling and energy loss of the outflow (e.g. Shadmehri 2008). Thus,
\[
\frac{\Sigma v_r}{\gamma - 1} \frac{dc_i^2}{d\ln r} - 2H v_r c_i^2 \frac{d\rho}{d\ln r} = \frac{\alpha \sqrt{1 + \Pi}}{\Omega_k} f c_i^2 \\
\times \left\{ \Sigma r^2 \left[ \frac{d\Omega}{d\ln r} \right]^2 + \frac{H}{2\pi \rho} \left[ \frac{1}{r} \frac{d}{d\ln r} \left( r B_\phi \right) \right]^2 \right\} \\
- \frac{1}{2} \xi \bar{m}_w(r) v_r^2(r),
\]
(11)
where \( f \) is called the advection parameter and is a constant less than unity. The parameter \( f \) measures how much the flow is advection-dominated (Narayan & Yi 1994). The first two terms on the right-hand side of equation (11) represent the energy generated as a result of viscous and resistive dissipation, respectively. The resistive dissipation is derived by \( (4\pi/\gamma \xi c_i)J^2 \), where \( J = (c/4\pi)\nabla \times B \) is the current density. Moreover, the last term on the right-hand side of equation (11) is the energy loss resulting from the wind or outflow. Depending on the energy-loss mechanism, the dimensionless parameter \( \xi \) might change. We consider this to be a free parameter of our model, so that larger \( \xi \) corresponds to more energy extraction from the disc resulting from the outflows (Knigge 1999; Shadmehri 2008).

The creation/escape rate of the magnetic field can be described by dynamo and diffusion. We define the advection rate of the toroidal magnetic field as (Oda et al. 2007)
\[
\Phi = \int v_r B_\phi d\zeta,
\]
(12)
which is used instead of the induction equation. Because we study a steady-state accreting system, the above quantity will be constant in the absence of the dynamo and the diffusion effects. This quantity can vary with radius because of the presence of the dynamo/diffusion effect (Machida et al. 2006; Oda et al. 2007). In the present model, we expect that the magnetic flux advection rate varies with radius because of the presence of resistivity. We consider this property in Section 3.

### 3 SELF-SIMILAR SOLUTIONS

We seek self-similar solutions in the following forms (e.g. Akizuki & Fukue 2006; Shadmehri 2008):
\[
\Sigma(r) = c_0 \Sigma_0 \left( \frac{r}{R} \right)^{s/(1/2)};
\]
(13)
\[
v_r(r) = -c_1 \sqrt{\frac{GM}{R}} \left( \frac{r}{R} \right)^{-(1/2)};
\]
(14)
\[
\Omega(r) = c_2 \sqrt{\frac{GM}{R^3}} \left( \frac{r}{R} \right)^ {-(3/2)};
\]
(15)
\[
c_i^2(r) = c_3 \left( \frac{GM}{R} \right) \left( \frac{r}{R} \right)^{-1};
\]
(16)
\[
c_\alpha^2(r) = \frac{B_\alpha^2}{4\pi \rho} = 2\Pi c_3 \left( \frac{GM}{R} \right) \left( \frac{r}{R} \right)^{-1}.
\]
(17)
Here, \( \Sigma_0 \) and \( R \) are exploited to write the equations in the non-dimensional forms. Substituting the above solutions into the continuity, radial momentum, angular momentum, hydrostatic and energy equations (equations 1, 6–8 and 11), we can obtain the following relations:
\[
c_0 c_1 = \bar{m};
\]
(18)
\[-\frac{1}{2} c_i^2 + \left( s + \frac{1}{2} \right) \Pi + s - \frac{3}{2} c_1 - c_i^2 + 1 = 0;
\]
(19)
\[-\frac{1}{2} c_0 \left[ c_1 - 3\alpha c_3 \left( s + \frac{1}{2} \right) \sqrt{1 + \Pi} \right] + \bar{m} \bar{s} m_0 = 0;
\]
(20)
\[
\frac{H}{r} = \sqrt{(1 + \Pi)c_3};
\]
(21)
\[
\alpha \Pi f (\gamma - 1) \left( s - \frac{1}{2} \right) c_0 c_3^2 + P_m c_0 c_3
\]
\[
\times \left\{ \frac{9}{2} \alpha f (\gamma - 1) c_2 - \frac{2c_1}{\sqrt{1 + \Pi}} \left[ \left( s - \frac{3}{2} \right) \gamma - \left( s - \frac{5}{2} \right) \right] \right\} \\
- \frac{2\bar{s} m_0 \xi P_m (\gamma - 1)}{\sqrt{1 + \Pi}} = 0;
\]
(22)
Here, \( \bar{m} \) is the non-dimensional mass accretion rate, defined as
\[
\bar{m} = \frac{\dot{M}(R)}{2\pi \Sigma_0 \sqrt{GM} R}.
\]
(23)
Using equations (18)–(22), we obtain a quadratic equation for the coefficient of \( c_3 \):

\[
-\frac{81}{16} \alpha^2 (1 + \Pi) \left[ \frac{1 + 2s}{2^{s-1}} - 1 \right] \, \alpha^2 + \left\{ \frac{3}{(\gamma - 1)f} \left[ \frac{1 + 2s}{2^{s-1}} - 1 \right] \right. \\
\left. \times \left[ \left( s - \frac{3}{2} \right) \gamma - \left( s - \frac{5}{2} \right) \right] + \frac{9}{2} \left[ \left( s + \frac{1}{2} \right) \Pi + \left( s - \frac{3}{2} \right) \right] \right. \\
\left. + \frac{\Pi}{P_m} \left( s - \frac{1}{2} \right)^2 \left[ \alpha + \frac{9}{2} \left[ \frac{1 + 2s}{3f} \left( \frac{1 + 2s}{2^{s-1}} - 1 \right) \right] \right] \right\} = 0. \tag{24}
\]

The rest of the coefficients are

\[
c_0 = -\frac{2}{3} \alpha \sqrt{1 + \Pi} \left( \frac{2^{s-1} - 1}{1 + 2s} \right) c_3^{-1}, \tag{25}
\]

\[
c_1 = -\frac{3}{2} \alpha \sqrt{1 + \Pi} \left( \frac{1 + 2s}{2^{s-1}} - 1 \right) c_3, \tag{26}
\]

and

\[
c_2 = 1 - \frac{9}{8} \alpha^2 (1 + \Pi) \left( \frac{1 + 2s}{2^{s-1}} - 1 \right)^2 c_3 \\
+ \left\{ \left[ s + \frac{1}{2} \right] \Pi + s - \frac{3}{2} \right\} c_3. \tag{27}
\]

Without mass outflows, resistivity and a toroidal magnetic field (i.e. \( s = l = \xi = 0, P_m = \infty \) and \( \Pi = 0 \)), the above relations reduce to the standard ADAF solutions (Narayan & Yi 1994). Moreover, in the absence of wind and resistivity but with a toroidal magnetic field, the above relations reduce to the result of Akizuki & Fukue (2006). However, the present model includes a toroidal magnetic field, outflows and resistivity.

Studies of resistive and magnetized ADAFs (e.g. Faghei 2011) imply that the solution for a set of the input parameters reaches a non-rotating limit at a specific value of \( \Pi \), which we call \( \Pi_c \). Assuming \( c_3 = 0 \) and \( s = l = \xi = 0 \) (no wind case) in equations (18)–(22), \( \Pi_c \) can be written as

\[
\Pi_c = \frac{18 P_m}{f} \left( \frac{5/3 - \gamma}{\gamma - 1} \right). \tag{28}
\]

For typical values of the adiabatic index and advection parameter in ADAFs, \( \gamma = 4/3 \) and \( f = 1 \), we can write \( \Pi_c = 18 P_m \). We cannot extend the solutions to larger values of \( \Pi_c \), because the right-hand side of equation (27) becomes negative, and a negative \( c_2 \) is clearly unphysical. Moreover, \( \Pi_c = 18 P_m \) implies that the flow can be magnetically dominated for \( P_m > 1/18 \sim 0.056 \). This means that the flow can have a strong magnetic field in the presence of resistivity. This property agrees with the resistive MHD simulations of Machida et al. (2006).

Now, we can investigate the radial dependence of the magnetic flux advection rate (equation 12). The self-similar solution for this quantity implies that

\[
\Phi(r) = \Phi_{\text{out}} \left( \frac{r}{R} \right)^{(s-3)/2}, \tag{29}
\]

where \( \Phi_{\text{out}} \) is the magnetic flux advection rate at the outer edge of the disc, \( R \). Because \( s < 3/2 \), the magnetic flux increases when approaching the central object, and the stronger wind/outflow reduces this increase. The radial dependence of \( \Phi \) is qualitatively consistent with the results of Machida et al. (2006) and Oda et al. (2007).

4 RESULTS

Now we can investigate the behaviour of the solutions in the presence of outflows and resistivity. The effects of outflows and resistivity are studied using \( s, l, \xi \) and \( P_m^{-1} \). Here, the inverse of the Prandtl number specifies the resistivity of the fluid. Thus, \( \eta \propto a/\Pi_m \), and \( s = 0.1 \) in all of the figures. The behaviours of the physical variables as a function of \( P_m^{-1} \) are shown in Figs 1 and 2. The solutions in Figs 1 and 2 represent the radial inflow speed, \( c_1 \), and the sound speed, \( c_3 \), both of which increase with the magnitude of resistivity.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Physical quantities of the flow as a function of \( P_m^{-1} \) for \( \gamma = 4/3 \), \( s = 0.1 \), \( l = \xi = 1 \). The solid, dotted, dashed and dot-dashed lines represent \( s = 0, 0.01, 0.02 \) and 0.03.

![Figure 2](https://example.com/figure2.png)

**Figure 2.** Same as Fig. 1, but \( s = 0.1 \), and the solid, dotted, dashed and dot-dashed lines represent \( \xi = 0, 0.1, 0.2 \) and 0.3.
These properties are qualitatively consistent the results of Faghei (2011). The density profiles, \( c_0 \), show that density decreases by adding resistivity. This can be a result of the temperature increase of the flow. The rotational velocity \( c_2 \) decreases with the magnitude of resistivity. Thus, the viscous torque increases with the temperature (\( \nu \propto c_2^2 \propto T \)). These properties are also consistent with the results of Faghei (2011).

In Fig. 1, we have also shown the effect of \( s \) on the physical variables. The value of \( s \) measures the strength of outflow, and a larger \( s \) denotes a stronger outflow. Fig. 1 shows that for non-zero \( s \), the surface density is lower than the standard ADAF solution, and for stronger outflows this reduction of surface density is more evident. We can see that an ADAF with wind rotates more quickly than those without winds and leads to enhanced accretion velocity. The solution shows that the temperature decreases for stronger outflows. However, the outflow acts as a cooling agent. These properties are in agreement with the results of Shadmehri (2008) and Abbassi et al. (2010).

In Fig. 2, the effect of energy loss as a result of outflows is shown by \( \xi \). As with the magnitude of \( \xi \), the more energy can be carried by outflows. Because of this energy loss, we expect the temperature to decrease when adding \( \xi \). The temperature profiles confirm this. Because the turbulent viscosity is proportional to temperature (\( \nu \propto T \)), the efficiency of the angular momentum transport decreases with \( \xi \). Decreasing the viscous torque increases the rotational velocity and decreases the radial infall velocity. These properties are consistent with the results of previous works (e.g. Shadmehri 2008).

Figs 3 and 4 show the physical variables as a function of parameter \( \Pi \) and several values of mass and energy losses. By adding \( \Pi \), which indicates the role of the magnetic field on the dynamics of accretion discs, we see that the sound speed becomes larger, while the surface density decreases. Moreover, Fig. 3 shows that the radial and rotational velocities increase with the magnitude of \( \Pi \). An increase in the radial velocity is a result of the magnetic tension term dominating the magnetic pressure term in the radial momentum equation, which assists the radial velocity of accretion flows. Moreover, with an increase in the rotational velocity, the disc should rotate faster than in the case without a magnetic field, which results in magnetic tension. These properties are qualitatively consistent with the results of previous works on magnetized ADAFs (e.g. Akizuki & Fukue 2006; Khesali & Faghei 2009; Abbassi et al. 2010). Figs 3 and 4 imply that the mass and energy losses resulting from wind give the same results as Figs 1 and 2. However, the effects of mass and energy losses on the physical variables do not change for low and high values of the magnetic field.

Figs 5 and 6 show that the disc thickness increases with the magnitude of the resistivity or the magnetic field. Because the temperature increases by adding resistivity or the magnetic field, from equation (21) we can see that the disc thickness increases with temperature. In Figs 5 and 6, the disc thickness is also shown for several values of \( s \) and \( \xi \). The profiles of disc thickness imply that the disc becomes thinner for stronger mass or energy losses resulting from outflows. Thus, \( s \) and \( \xi \) reduce the temperature of the flow.
5 BERNOULLI PARAMETER

Here, we exploit the Bernoulli parameter ($Be$) in order to consider whether the effects of resistivity can generate/enhance outflows in magnetized ADAFs. This parameter measures the likelihood that outflow or wind can originate spontaneously (Narayan & Yi 1994). An adiabatic flow has a constant $Be$ along streamlines. If $Be$ is positive for any accreting gas, then this gas can potentially reach infinity with a net positive kinetic energy. The Bernoulli parameter, defined as the sum of the kinetic energy, the enthalpy and the potential energy of the accretion flow, is

$$Be = \frac{1}{2}(v^2 + r^2\Omega^2) + \frac{\gamma}{\gamma - 1}c_s^2 - r^2\Omega_k^2. \quad (30)$$

Narayan & Yi (1994) have shown that the Bernoulli parameter is positive in height-integrated advection-dominated flows. They have suggested this might explain the frequent occurrence of outflows and wind in many accretion systems. Using the self-similar transformations of equations (13)–(17), the Bernoulli parameter can be written as

$$b = \frac{Be}{\kappa_{K}} = \frac{1}{2}(c_1^2 + c_2^2) + \frac{\gamma}{\gamma - 1}c_1 - 1. \quad (31)$$

where $b$ is the normalized Bernoulli parameter. Fig. 7 shows the behaviour of this parameter as a function of $P_m^{-1}$ for different values of the magnetic field. The $Be$ profiles show that it is positive and increases with the magnitude of resistivity or magnetic field. This could be because of the increase of the flow temperature when adding the resistivity or magnetic field. Moreover, the $Be$ profiles show that the magnetic field is more important in high resistivity. Thus, the outflows can be enhanced in resistive and magnetized ADAFs.

6 SUMMARY AND DISCUSSION

Mass loss appears to be a common phenomenon among astrophysical accretion disc systems. The observational evidence of ADAFs implies that outflow is important in such systems. Moreover, the non-ideal MHD simulation results show that resistivity can play an important role in the outflows of accretion discs.

In this paper, we have investigated the structure of a magnetized ADAF in the presence of resistivity and outflow. We have assumed that the magnetic field has a purely toroidal component. The emanating outflow affects the equations of continuity, angular momentum and energy, and it can therefore act as a sink for mass, angular momentum and energy. We have adopted the solutions presented by Kniege (1999), Akizuki & Fukue (2006) and Shadmehri (2008). Thus, we have assumed that angular momentum transport is a result of viscous turbulence and that the $\alpha$-prescription is used for the kinematic coefficient of viscosity. We have also assumed that the flow does not have a good cooling efficiency, so a fraction of energy accretes along with the matter on to the central object. To solve the equations that govern the structural behaviour of a magnetized ADAF with outflow, we have used a steady self-similar solution.

The present model shows that the outflows in accretion flows can improve by the resistivity and magnetic field. These properties are agree with the resistive MHD simulations of Fendt & Čemeljić (2002) and Čemeljić et al. (2008). For example, Fendt & Čemeljić (2002) have shown that resistivity can affect the outflow structure in accretion discs. Moreover, they have found that the outflow velocity increases with the magnitude of the resistivity and the toroidal magnetic field.

In this paper, we have assumed a purely toroidal magnetic field, which produces some limitations in the present model. For example, a purely toroidal magnetic field is not enough to have magnetically driven outflows. Thus, the present model is unsuitable to use for discs with magnetically driven outflows. Moreover, we have considered the model in a height-integrated approach and we have applied it to calculate the Bernoulli parameter. Narayan & Yi (1995) have shown that the Bernoulli parameter in ADAFs varies by latitude. In other words, the Bernoulli parameter in equator is negative and becomes positive for high latitude. Thus, we can investigate latitudinal behaviour of the present model.

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REFERENCES

Abbassi S., Ghanbarchi J., Ghasemnezhad M., 2010, MNRAS, 409, 1113
Akizuki C., Fukue J., 2006, PASJ, 58, 469
Baganoff F. K. et al., 2003, ApJ, 591, 891

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Beckwith K., Hawley J. F., Krolik J. H., 2008, ApJ, 678, 1180
Blandford R. D., Begelman M. C., 1999, MNRAS, 303, L1
Bondi H., 1952, MNRAS, 112, 195
Bu D., Yuan F., Xie F., 2009, MNRAS, 392, 325
Čemeljić M., Gracia J., Vlahakis N., Tsinganos K., 2008, MNRAS, 389, 1022
De Villiers J.-P., Hawley J. F., Krolik J. H., 2003, ApJ, 599, 1238
Faghei K., 2011, JAA, in press (arXiv:1111.7302)
Fendt C., Čemeljić M., 2002, A&A, 395, 1045
Fleming T. P., Stone J. M., 2000, ApJ, 530, 464
Fukue J., 2002, PASJ, 54, 415
Fukue J., 2004, PASJ, 56, 569
Gammie C. F., Menou K., 1998, ApJ, 332, 659
Ichimaru S., 1977, ApJ, 214, 840
Kaburaki O., Nankou T., Tamura N., Wajima K., 2010, PASJ, 62, 1177
Kawabata R., Mineshige S., 2009, ApJ, 691, 1135
Khesali A., Faghei K., 2008, MNRAS, 389, 1218
Khesali A., Faghei K., 2009, MNRAS, 398, 1361
Knigge C., 1999, MNRAS, 309, 499
Kudoh T., Kaburaki O., 1996, ApJ, 460, 199
Li S.-L., Cao X., 2009, MNRAS, 400, 1734
Loeb A., Narayan R., Raymond J. C., 2001, ApJ, 547, L151
Machida M., Nakamura K. E., Matsumoto R., 2006, PASJ, 58, 193
Marrone D. P., Moran J. M., Zhao J.-H., Rao R., 2006, ApJ, 640, 308
Masada Y., Sano T., 2008, ApJ, 689, 1234
Melia F., Kowalenkov V., 2001, MNRAS, 327, 1279
Narayan R., Yi I., 1994, ApJ, 428, L13
Narayan R., Yi I., 1995, ApJ, 452, 710
Oda H., Machida M., Nakamura K. E., Matsumoto R., 2007, PASJ, 59, 457
Pringle J. E., 1981, ARA&A, 19, 137
Quataert E., Narayan R., 1999, ApJ, 520, 298
Sano T., Inutsuka S. I., Miyama S. M., 1998, ApJ, 506, L57
Shadmehri M., 2008, Ap&SS, 317, 201
Shakura N. I., Sunyaev R. A., 1973, A&A, 24, 337
Stone J. M., Gammie C. F., Balbus S. B., Hawley J. F., 2000, in Manning V., Boss A., Russell S., eds, Protostars and Planets IV. Univ. Arizona Press, Tuscon, AZ
Xie F.-G., Yuan F., 2008, ApJ, 681, 499
Yuan F., Quataert E., Narayan R., 2003, ApJ, 598, 301

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