Universal Isgur-Wise form factors from QCD sum rules in HQET

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We review the role of the universal Isgur-Wise functions parameterizing the $B$ meson semileptonic matrix elements to excited charm states in the infinite heavy quark mass limit. We also discuss the determination of such form factors by QCD sum rules in the framework of the Heavy Quark Effective Theory.

1. UNIVERSAL FORM FACTORS

The heavy quark flavor and spin symmetry of QCD, exactly valid in the infinite $m_Q$ limit, has important consequences on the spectrum, the static properties and both the strong and weak decay matrix elements of the hadrons containing a single heavy quark $Q$.\textsuperscript{1,3} Notably, in such heavy quark effective theory (HQET), for $m_Q \to \infty$ the states can be classified in degenerate doublets by the total angular momentum $J$ and by the angular momentum of the light degrees of freedom (quarks and gluons) $s_Q = J - s_Q$, and the semileptonic matrix elements between members of the doublets identified by $s_Q$ and $s_F$ can be expressed in terms of “universal” form factors which are functions of $y = v \cdot v'$ (with $v$ and $v'$ the initial and final hadron four-velocities). This is a well known result for $B \to D^{(*)}\ell\nu$: the pseudoscalar $B$, $D$ and vector $B^*$, $D^*$ mesons have $s_F^P = \frac{3}{2}$, and the semileptonic matrix elements can be written in terms of a single universal form factor, the Isgur-Wise function $\xi(y)$. The heavy flavour symmetry requires that such form factor is normalized to unity at the zero recoil point $y = 1$.

In case of transitions between members belonging to different HQET multiplets, additional form factors are introduced. An important example involves the four meson states corresponding to the orbital angular momentum $L = 1$ ($P$ waves in the constituent quark model), which can be classified in the doublets: $J^P = (0^+_{1/2}, 1^+_{1/2})$ and $J^P = (1^+_{3/2}, 2^+_{3/2})$, according to the values $s_F^P = \frac{1}{2}$ and $s_F^P = \frac{3}{2}$, respectively. Denoting the corresponding charm mesons by $(D_0, D^*_1)$ and $(D_1, D^*_2)$, the weak matrix elements $< (D_0, D^*_1) | V - A | B >$ and $< (D_1, D^*_2) | V - A | B >$ can be written in terms of two independent functions, $\tau_{1/2}$ and $\tau_{3/2}$, respectively,\textsuperscript{4}, that allow the complete description of the physical processes at the leading order in $1/m_Q$. However, contrary to the case of the Isgur-Wise function $\xi$, one cannot invoke symmetry arguments to predict the normalization at $y = 1$ of $\tau_{1/2}(y)$ and $\tau_{3/2}(y)$.

From the phenomenological point of view, the $B \to D^{**}$ semileptonic transitions ($D^{**}$ being the generic $L = 1$ charmed state) are interesting, since in principle these decay modes may account for a sizeable fraction of the inclusive semileptonic $B$-decay rate. In any case they represent a well-defined set of corrections to the theoretical prediction that, in the limit $m_Q \to \infty$ and under the condition $(m_b - m_c)/(m_b + m_c) \to 0$, the total semileptonic $B \to X_c$ decay rate should be saturated by the $B \to D$ and $B \to D^*$ modes.\textsuperscript{5} Another point of interest, the values of the $B \to D^{**}$ form factors at $y = 1$ provide a lower bound to the slope $\rho^2$ of the function $\xi$:\textsuperscript{6}

$$\rho^2 \geq \frac{1}{4} + |\tau_{1/2}(1)|^2 + 2|\tau_{3/2}(1)|^2 .$$ (1)

They also appear in the “dipole” sum rule:\textsuperscript{7}

$$\frac{\Lambda}{2} = (\Lambda^+ - \Lambda)|\tau_{1/2}(1)|^2 + 2(\Lambda^T - \Lambda)|\tau_{3/2}(1)|^2 + \ldots$$ (2)

where $\Lambda = M_B - m_b$, $\Lambda^+ = M_{D_0} - m_c$, $\Lambda^T = \ldots$.

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$M_{D_1} - m_c$ are heavy meson “binding energies”.

The investigation of the semileptonic $B$ transitions to excited charm states is an important issue for the theoretical analysis of the $D^{**}$ production in nonleptonic $B$ decays \cite{6}, as well as for CP physics at $B$ factories \cite{3}. As for $\Lambda_{QCD}/m_Q$ effects in such processes, they are discussed in \cite{10}.

The charmed $2^{+}_{1/2}$ state, $D^*_1(2460)$, has been observed with $m_{D^*_1} = 2458.9 \pm 2.0$ MeV, $\Gamma_{D^*_1} = 23\pm 5$ MeV and $m_{D^*_2} = 2459\pm 4$ MeV, $\Gamma_{D^*_2} = 25^{+8}_{-7}$ MeV for the neutral and charged states, respectively. The HQET state $1^+_{1/2}$ can be identified with $D_1(2420)$, with $m_{D_1} = 2422.2 \pm 1.8$ MeV and $\Gamma_{D_1} = 18.9^{+6.6}_{-5.5}$ MeV (a $1^+_{1/2}$ component is contained in this physical state, due to the mixing allowed for the finite value of the charm quark mass). There is also some experimental evidence of beauty $s^P_2 = \frac{3}{2}^+$ states \cite{11}.

Both the states $2^{+}_{3/2}$ and $1^{+}_{1/2}$ decay to hadrons by $d$-wave suppressed transitions. On the other hand, the $s^P_2 = \frac{1}{2}^+$ doublet ($D_0$, $D_1^*$) has not been observed yet. The strong decays of such states occur through $s$-wave transitions, with expected larger widths than in the case of the doublet $\frac{3}{2}^+$. Theoretical analyses of the coupling constants governing the two-body hadronic transitions can be carried out by QCD sum rules, in analogy with the determinations of the $B^* B\pi$, $D^* D\pi$ couplings \cite{4}, obtaining $\Gamma(\bar{D}_0^0 \rightarrow D^+\pi^-) \approx 180$ MeV and $\Gamma(\bar{D}_1^0 \rightarrow D^+\pi^-) \approx 165$ MeV, and the mixing angle $\alpha$ between $D_1^*$ and $D_1 \simeq 16^\circ$ \cite{12}.

### 2. FORM FACTOR $\tau_{1/2}$ FROM QCD SUM RULES IN HQET

The matrix elements of the semileptonic $B \rightarrow D_0 \ell \bar{\nu}$ and $B \rightarrow D_1^* \ell \bar{\nu}$ transitions can be parameterized in terms of six form factors:

$$< D_0(v') | \bar{c} \gamma_{\mu} \gamma_5 b | B(v) > = \frac{g_+(y)(v + v')_{\mu} + g_-(y)(v - v')_{\mu}}{\sqrt{m_{D_0} m_{D_0}}} ,$$

$$< D_1^*(v', \epsilon) | \bar{c} \gamma_{\mu} (1 - \gamma_5) b | B(v) > = \frac{g_{V_1}(y) \epsilon_{\mu} + \epsilon \cdot v [g_{V_2}(y) v_{\mu} + g_{V_3}(y) v'_{\mu}] - i g_A(y) \epsilon_{\mu \alpha \beta \gamma} \epsilon^{\alpha \beta \gamma} v^\mu v'^\nu}{\sqrt{m_{D_1^*} m_{D_1^*}}} .$$

The relation of $g_i(y)$ in \cite{5} to $\tau_{1/2}(y)$ in \cite{5}, in the limit $m_Q \rightarrow \infty$, involves short-distance coefficients which depend on the heavy quark masses $m_b, m_c$, on $y$ and on a mass-scale $\mu$, and connect the QCD vector and axial vector currents to the HQET currents. At the next-to-leading logarithmic approximation in $\alpha_s$ such relations are:

$$g_+ + g_- = -2 \left( C_1^5 + (y - 1) C_6^5 \right) \tau_{1/2}$$

$$g_+ - g_- = 2 \left( C_1^5 - (y - 1) C_6^5 \right) \tau_{1/2}$$

$$g_{V_1} = 2(y - 1) C_1 \tau_{1/2}$$

$$g_{V_2} = -2 C_2 \tau_{1/2}$$

$$g_{V_3} = -2 \left( C_1 + C_3 \right) \tau_{1/2}$$

$$g_A = -2 C_1^5 \tau_{1/2} .$$

The Wilson coefficients $C_i$, depending on a scale $\mu$, are reported, e.g., in \cite{13}. Since the form factor $\tau_{1/2}$ is defined by the matrix elements of weak currents in the effective theory, it depends on $\mu$; it is possible to remove the scale-dependence by compensating it by the $\mu$-dependence of the Wilson coefficients, defining $\tau_{1/2}^{ren}$.

The universal functions can be estimated by non-perturbative approaches; a genuinely field theoretical method is represented by HQET QCD sum rules, which allow to relate hadronic observables to QCD parameters via the Operator Product Expansion (OPE) of suitable Green functions. The expansion involves $\alpha_s$ corrections in the coefficients of the OPE, as well as non-perturbative quark and gluon vacuum condensates.

A critical aspect of the sum rule calculations in HQET is represented by the size of non-leading terms, such as the $\alpha_s$ corrections. For example, the predictions for the leptonic constants of $q\bar{q}$ pseudoscalar mesons are affected by considerably large next-to-leading corrections in $\alpha_s$ \cite{3}. Conversely, for the Isgur-Wise function $\xi(y)$ the next-to-leading order $\alpha_s$ corrections turn out to be small and well under control \cite{14}.

In what follows we present a recent determination of $\tau_{1/2}(y)$ in the frameworks of HQET QCD sum rules at the next-to-leading order in the strong coupling $\alpha_s$ \cite{3}. The method is based
on the three-point correlator

$$\Pi(\omega, \omega', y) = i^2 \int dx \, dz e^{ik'x - k'z} < 0|T[J_s^\prime(x), \tilde{A}_\mu(0), J_5^\prime(y)]|0 > \tag{5}$$

with \(\tilde{A}_\mu = \bar{h}^\mu_Q \gamma_\mu \gamma_5 h^5_Q\) the \(b \to c\) weak axial current in HQET, \(J_s^\prime = \bar{q}_h \gamma_5 h_5^5\) and \(J_5^\prime = \bar{q}_s \gamma_5 h_5^5\) local currents interpolating \(D_0\) and \(B\), \(h^5_Q\) being HQET quark fields. Here, \(\omega = 2v \cdot k\) and \(\omega' = 2v' \cdot k'\), and \(k, k'\) are residual momenta in the expansion of the heavy meson momenta: \(P = m_Q v + k, P' = m_Q v' + k'\). The residual momenta remain finite in the heavy quark limit.

Using the analyticity of \(\Pi\) in \(\omega\) and \(\omega'\) at fixed \(y\), we can represent \(\Pi\) in terms of physical hadronic states, giving poles at positive values of \(\omega\) and \(\omega'\) and a continuum. The lowest-lying contribution, determined by the \(B, D_0\) pole, introduces the form factor of interest:

$$\Pi_{pole}(\omega, \omega', y) = \frac{-2\tau_{1/2}(y, \mu) F(\mu) F^+(\mu)}{(2\lambda - \omega - i\epsilon)(2\lambda^+ - \omega' - i\epsilon)}. \tag{6}$$

\(F(\mu), F^+(\mu)\) are HQET couplings of the pseudoscalar and scalar interpolating currents to their respective \(0^-\) and \(0^+\) bound states:

$$< 0|J_5^\prime|B(v) > = F(\mu) \tag{7}$$

$$< 0|J_5^\prime|B_0(v) > = F^+(\mu). \tag{8}$$

Notice that \(F(\mu)\) is related to the familiar \(B\)-meson leptonic decay constant \(f_B\). The parameters \(\Lambda\) and \(\Lambda^+\) identify the position of the poles in \(\omega\) and \(\omega'\). As noted previously, \(\mu\) independent quantities \(F^+\) and \(F\) can be obtained by suitable perturbative factors.

The contribution of higher states to \(\Pi\) in \(\Pi\) is taken into account by invoking the quark-hadron duality ansatz, and is modeled by a perturbative QCD continuum.

The correlator \(\Pi\), at large negative \(\omega, \omega'\), can also be expressed in QCD by using the short-distance operator product expansion, in terms of perturbative and nonperturbative contributions:

$$\Pi = \Pi^{pert} + \Pi^{np}. \tag{9}$$

In \(\Pi^{np}\) represents the series of power corrections in the "small" variables \(\frac{1}{\Lambda}\) and \(\frac{1}{\Lambda^+}\). These corrections are determined by quark and gluon vacuum condensates, which account for the features of the nonperturbative strong interactions.

The QCD sum rule for \(\tau_{1/2}\) is obtained by imposing that the QCD and the hadronic representations numerically match in a suitable range of Euclidean values of \(\omega\) and \(\omega'\).

To "optimize" the sum rule, a double Borel transform in the variables \(\omega\) and \(\omega'\) is applied. This allows to eliminate subtraction terms, improve the convergence of the nonperturbative series and enhance the role of the lowest-lying meson states.

The same method can be applied to determine \(F^+, \Lambda^+\) (fig.1) and \(F, \Lambda\), considering two-point correlators. The result is \(\Lambda^+ = 1.0 \pm 0.1\) GeV.

![Figure 1. \(\Lambda^+\) and \(F^+\) vs the Borel parameter \(\tau'\).](image-url)

\(F^+ = 0.7 \pm 0.2\) GeV, and \(\Lambda = 0.5 \pm 0.1\) GeV, \(F = 0.45 \pm 0.05\) GeV. The difference \(\Delta = \Lambda^+ - \Lambda\) corresponds to \(\Delta = m_{D_0} - m_{D_0}\), with \(D\) and \(D_0\) the spin averaged states of the \(1^-\) and \(1^+\) doublets. The central value \(\Delta = 0.5\) GeV enables the prediction \(m_{D_0} \approx 2.45\) GeV, with an uncertainty of about 0.15 GeV. Determinations of \(F^+\) at the order \(\alpha_s = 0\) by QCD sum rules gave the result: \(F^+ = 0.46 \pm 0.06\) GeV \cite{3} and \(F^+ = 0.40 \pm 0.04\) GeV \cite{2} and \(\Lambda^+ = 1.05 \pm 0.5\) GeV.
or \( \bar{A}^+ = 0.90 \pm 0.10 \) GeV \(^{[10]}\) depending on the choice of the interpolating currents, which shows the significant size of the \( \alpha_s \) corrections in this case.

Determinations of \( F^+ \) by relativistic quark models give \( F^+ \simeq 0.6 - 0.7 \) GeV\(^{3/2}\) \(^{[15]}\).

The \( \mathcal{O}(\alpha_s) \) corrections to the perturbative part of the sum rule for \( \tau_{1/2} \) are represented by the two-loop diagrams in fig. 2. The result of their calculation, as well as of \( \mathcal{O}(\alpha_s) \) corrections to the condensate terms, is reported in \(^{[13]}\).

In the numerical analysis of the sum rule for \( \tau_{1/2} \), the \( \alpha_s \) contribution to \( \Pi^{pert} \) turns out to be sizeable. However, it is partially compensated by the analogous corrections in \( F \) and \( F^+ \). This is a remarkable result, not expected a priori since the normalization of \( \tau_{1/2} \) is not fixed by symmetry arguments. The obtained \( \tau_{1/2} \) is depicted in fig. 3, where the shaded region essentially represents the theoretical uncertainty of the calculation. Using the expansion near \( y = 1 \):

\[
\tau_{1/2}^{ren}(y) = \tau_{1/2}(1) \left( 1 - \rho_{1/2}^2 (y-1) + c_{1/2}(y-1)^2 \right) \tag{10}
\]

we get \( \tau_{1/2}(1) = 0.35 \pm 0.08 \), \( \rho_{1/2}^2 = 2.5 \pm 1.0 \) and \( c_{1/2} = 3 \pm 3 \). Neglecting \( \alpha_s \) corrections QCD sum rules would give \( \tau_{1/2}(1) \simeq 0.25 \) \(^{[15]}\).

Other determinations of \( \tau_{1/2}(y) \) have appeared in the literature, based on various versions of the constituent quark model \(^{[13]}\). The results range in a broad interval, \( \tau_{1/2}(1) = 0.06 - 0.40 \) and \( \rho_{1/2}^2 = 0.7 - 1.0 \), and critically depend on the detailed features of the models employed in the numerical calculation.

As for \( \tau_{3/2}(y) \), a QCD sum rule analysis to the order \( \alpha_s = 0 \) gives \( \tau_{3/2}(1) \simeq 0.28 \) and \( \rho_{3/2}^2 \simeq 0.9 \) \(^{[15]}\). Quark models give predictions in the range \( \tau_{3/2}(1) \simeq 0.31 - 0.66 \) and \( \rho_{3/2}^2 \simeq 1.4 - 2.8 \) \(^{[13]}\).

As far as the decays \( B \to (D_0, D^*_1) \ell \nu \) are concerned, using \( V_{cb} = 3.9 \times 10^{-2} \) and \( \tau(B) = 1.56 \) ps, we predict \( B(B \to D_0 \ell \nu) = (5 \pm 3) \times 10^{-4} \) and \( B(B \to D^*_1 \ell \nu) = (7 \pm 5) \times 10^{-4} \). This represents only a very small fraction of the semileptonic \( B \to X_c \) decays. Although the branching ratios are small, one can hope that the semileptonic \( B \) transitions to the \( s_{\ell}^{D} = \frac{1}{2}^+ \) charm doublet will be identified at the \( B \)-factories. At present, the \( s_{\ell}^{D} = \frac{1}{2}^+ \) charm doublet is difficult to be distinguished from the non-resonant background.

In conclusion, we stress that predictions derived within HQET should always be supported by the computation of \( 1/m_Q \) as well as radiative corrections. The role of both depend on the various situations. In the case of \( \tau_{1/2} \) we have obtained that \( \alpha_s \) corrections are under control for \( \tau_{1/2} \), while they considerably affect the leptonic constant \( F^+ \).

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