A comprehensive study of vector leptoquark on the $B$-meson and Muon g-2 anomalies

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ABSTRACT: Recently reported anomalies in various $B$ meson decays and also in the anomalous magnetic moment of muon ($g - 2$) motivate us to consider a particular extension of the standard model incorporating new interactions in lepton and quark sectors simultaneously. Our minimal choice would be leptoquark. In particular, we take vector leptoquark ($U_1$) and comprehensively study all related observables including $(g - 2)_\mu$, $R_{K^{(*)}}$, $R_{D^{(*)}}$, $B \to (K)\ell\ell'$ where $\ell\ell'$ are various combinations of $\mu$ and $\tau$, and also lepton flavor violation in the $\tau$ decays.
Physics is a data-driven science, and we are keen to modify the standard model (SM) when experimental results deviate from the theoretical predictions. Over the past several years, the $B$-physics experiments BaBar, LHCb, and Belle have reported several anomalous results in the $b \rightarrow s \ell \ell$ and $b \rightarrow c \ell \nu$ processes. In particular, the lepton flavor universality (LFU), which is one of the approximate symmetries in the SM, seems to be broken beyond the expected range according to the observables of $R_D, R_{D^*}, R_K$, and $R_{K^*}$, which measured the ratios of different lepton flavors

$$R_{D^(*)} \equiv \frac{B(B \rightarrow D^{(*)} \ell \nu)}{B(B \rightarrow D^{(*)} \ell \nu)} \quad R_{K^(*)} \equiv \frac{B(B \rightarrow K^{(*)} \mu^+ \mu^-)}{B(B \rightarrow K^{(*)} e^+ e^-)}.$$ (1.1)

The precise measurement of those quantities would test the basic structure of the SM since LFU is only violated by the lepton masses in the SM.

The world averaged experimental values based on measurements from BaBar [1], Belle [2–4], and LHCb [5, 6] are

$$R_D = 0.340 \pm 0.027 \pm 0.013 \quad R_{D^*} = 0.295 \pm 0.011 \pm 0.008,$$ (1.2)

and the combined discrepancy to SM prediction is at the $3.1\sigma$ level [7, 8].

The most precise measurement to date of the $R_K$ has been performed by LHCb [9]

$$R_K = 0.846_{-0.041}^{+0.044}, \quad q^2 \subseteq [1.1, 6.0] \text{ GeV}^2,$$ (1.3)
which has 3.1σ deviation from the SM expectation. For the $R_{K^*}$, LHCb Run-1 provides \[10\]

\[
R_{K^*} = \begin{cases} 
0.66_{-0.07}^{+0.11} \pm 0.03, & q^2 \subseteq [0.045, 1.1] \text{ GeV}^2, \\
0.69_{-0.07}^{+0.11} \pm 0.05, & q^2 \subseteq [1.1, 6.0] \text{ GeV}^2. 
\end{cases} 
\tag{1.4}
\]

Combining both $q^2$ bins, it has 2.5σ tension with the SM. On the other side, the $R_{K^*}$ and $R_K$ measurements from Belle \[11, 12\]

\[
R_{K^*} = \begin{cases} 
0.90_{-0.21}^{+0.27} \pm 0.10, & q^2 \subseteq [0.1, 8.0] \text{ GeV}^2, \\
1.18_{-0.32}^{+0.52} \pm 0.10, & q^2 \subseteq [15, 19] \text{ GeV}^2, 
\end{cases} 
\quad R_K = \begin{cases} 
0.98_{-0.23}^{+0.27} \pm 0.06, & q^2 \subseteq [1.0, 6.0] \text{ GeV}^2, \\
1.11_{-0.26}^{+0.29} \pm 0.07, & 14.18 \text{ GeV} < q^2, 
\end{cases} 
\tag{1.5}
\]

are still compatible with SM predictions within their large uncertainties. In the near future, Belle II is expected to significantly improve the uncertainties \[13\].

Recently, the Muon $g-2$ experiment at Fermilab reported the value, $a^\text{FNAL}_\mu = (116592040 \pm 54) \times 10^{-11}$ \[14\], or,

\[
\Delta a^\text{FNAL}_\mu = a^\text{FNAL}_\mu - a^\text{SM}_\mu = (230 \pm 69) \times 10^{-11}, 
\tag{1.6}
\]

which is a 3.3σ discrepancy. Since the value is compatible with the long standing value from BNL \[15, 16\], the significance is strengthened to 4.2σ level:

\[
\Delta a^\text{BNL+FNAL}_\mu = a^\text{exp}_\mu - a^\text{SM}_\mu = (251 \pm 59) \times 10^{-11}. 
\tag{1.7}
\]

Finding a common origin of the $B$-meson and $(g - 2)_\mu$ anomalies is non-trivial, but it is appealing from the theoretical point of view.\footnote{See \[17–23\] for some of our earlier attempts to account $(g - 2)$ and also various anomalies and possible experimental probes.}

In early attempts \[16, 24–27\], the $U_1 = (3, 1)_{2/3}$ singlet vector Leptoquark, in general couples to both left-handed (LH) and right-handed (RH) SM fermions, as single-mediator accounts for all the low-energy data. Its simultaneously explanations of the $R_{K^*}$ and $R_{D^*}$ anomalies only requires the LH couplings between second and third-generation quarks and leptons. However, the LH couplings cannot produce large enough muon magnetic moment for $(g - 2)_\mu$ anomaly. In this work, we further extend non-zero RH couplings of $U_1$, such that it substantially enhances the contribution to $(g - 2)_\mu$. We explored the plausible parameter space and search the common solution for both $B$-meson and $(g - 2)_\mu$ anomalies.\footnote{(Note added) When we are finishing our paper, similar idea has been considered in \[28\]. Unfortunately, however, they have missed some relevant constraints from low-energy experiments. In particular, the experimental $B_s \to \mu \mu$ data conflicts with their preferred parameter region.}
2 Vector Leptoquark $U_1 = (3, 1)_{2/3}$

We focus on the $U_1 = (3, 1)_{2/3}$ weak singlet vector leptoquark because it provides a simultaneous explanations for $R_{K^{(*)}}$ and $R_{D^{(*)}}$ anomalies \[16, 24-27\]. The general Lagrangian includes the $U_1$ couplings to both LH and RH fermion under the SM gauge symmetry. However, to explain $R_{K^{(*)}}$ and $R_{D^{(*)}}$ anomalies, the most relevant interactions are the left-handed couplings to the 2nd and 3rd generations of leptons and quarks

$$
\mathcal{L} \supset U_{1\mu} \sum_{i,j=1,2,3} \left( x_{ij}^L (\bar{d}_L^n \gamma^i e^j_L) + \left( V_{CKM} x_{L}^{U_{PMNS}} \right)_{ij} (\bar{u}_L^i \gamma^i_\mu e^j_R) + x_{ij}^R (\bar{d}_R^n \gamma^i_\mu e^j_R) \right) + h.c.
$$

(2.1)

Here we only include the real parts of CKM and PMNS matrices \[16\] as

$$
V_{CKM}^{xL} U_{PMNS} =
\begin{pmatrix}
0.974 & 0.224 & 0.009 \\
0.225 & 0.974 & 0.041 \\
0.004 & 0.042 & 0.999
\end{pmatrix}
\begin{pmatrix}
x_{L}^1 & x_{L}^{12} & x_{L}^{13} \\
x_{L}^{21} & x_{L}^{22} & x_{L}^{23} \\
x_{L}^{31} & x_{L}^{32} & x_{L}^{33}
\end{pmatrix}
\begin{pmatrix}
0.821 & 0.551 & -0.150 \\
-0.307 & 0.600 & 0.739 \\
0.481 & -0.580 & 0.657
\end{pmatrix}.
$$

The couplings $x_{ij}^L$ to the first generation leptons and quarks are set to zero in order to avoid the constraints from $\mu - e$ conversion on nuclei, and atomic parity violation on $B(K \rightarrow \pi \nu \bar{\nu})$.

The LH couplings contribute to the Wilson coefficients of the effective Lagrangian \[24\]

$$
C_9^{\mu \nu} = -C_{10}^{\mu \nu} = -\frac{\pi v_{EW}^2}{V_{tb} V_{ts}^{*} \alpha_{EM}} \frac{x_{L}^{22} (x_{L}^{32})^*}{m_{U_1}^2},
$$

(2.2)

where $v_{EW} = 246$ GeV is the Higgs’ vacuum expectation value, and $\alpha_{EM}$ is the fine structure constant. The fit to $R_K, R_{K^{*}}$ and $B(B_s \rightarrow \mu \mu)$ data prefers the parameter window \[24\]

$$
C_9^{\mu \nu} = -C_{10}^{\mu \nu} \subseteq [-0.85, -0.50] \Rightarrow -\frac{\pi v_{EW}^2}{V_{tb} V_{ts}^{*} \alpha_{EM}} \frac{x_{L}^{22} (x_{L}^{32})^*}{m_{U_1}^2} \subseteq [0.83, 1.41] \times 10^{-3}$ TeV$^{-2}.
$$

(2.3)

Here, we turn off the RH couplings $x_{R}$, since they contribute to the Wilson coefficients $(C_S)' = (C_D)'$ and are disfavored by the $b \rightarrow s$ data.

The interactions from Eq.(2.1) also give rise to the effective coefficient \[24\]

$$
g_{V_L} = \frac{v_{EW}^2}{2m_{U_1}^2} \left( x_{L}^{bc} \right)^* \left( x_{L}^{bc} + \frac{V_{cs}}{V_{cb}} x_{L}^{c\ell} + \frac{V_{cd}}{V_{cb}} x_{L}^{d\ell} \right),
$$

(4.4)

and contribute to $b \rightarrow c \ell \nu$. It becomes one solution for the $R_D$ and $R_{D^{*}}$ anomalies, and the $1\sigma$ region for $b \rightarrow c \tau \nu$ requires

$$
g_{V_L} \subseteq [0.09, 0.13] \Rightarrow \frac{(V_{cs} x_{L}^{33} + V_{cb} x_{L}^{33}) (x_{L}^{33})^*}{m_{U_1}^2} \subseteq [0.12, 0.18]$ TeV$^{-2}.
$$

(5.5)
3 Low-energy observables

In this section, we summarize the low-energy observables with the $U_1$ vector leptoquark contributions.

3.1 $(g-2)_\mu$

The previous result of Muon $g-2$ experiment with the BNL E821 is $3.7\sigma$ from the SM. After the FNAL result, the deviation between experiment and SM is [14, 29]

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}.$$  \hspace{1cm} (3.1)

And it has $4.2\sigma$ significance from the SM prediction and this results grow up the motivation for SM extensions for new couplings with leptons. In this paper, we present the single leptoquark which is described in Sec. 2 to explain not only $(g-2)_\mu$ anomaly but also $B$-meson anomalies.

For the large mass of $U_1$ leptoquark, it contributes to the $(g-2)_\mu$ anomaly as

$$\Delta a_\mu = \frac{N_c}{16\pi^2} \sum_i \left[ 2Q_\mu \bar{\kappa}_Y \text{Im}(x_L^2 (x_R^2)^*) \frac{m_d m_\mu}{m_{U_1}^2} \left( \ln \left( \frac{\Lambda_{UV}^2}{M_{U_1}^2} \right) + \frac{5}{2} \right) + 2\text{Re}(x_L^2 (x_R^2)^*) \frac{m_d m_\mu}{m_{U_1}^2} \left( 2Q_d + Q_{U_1} \left( 1 - \kappa_Y \right) \ln \left( \frac{\Lambda_{UV}^2}{M_{U_1}^2} \right) + \frac{1 - 5\kappa_Y}{2} \right) - \left( |x_L^2|^2 + |x_R^2|^2 \right) \frac{m_\mu^2}{m_{U_1}^2} \left( \frac{4}{3} Q_d + Q_{U_1} \left( 1 - \kappa_Y \right) \ln \left( \frac{\Lambda_{UV}^2}{M_{U_1}^2} \right) - \frac{1 + 9\kappa_Y}{6} \right) \right] ,$$  \hspace{1cm} (3.2)

If $\kappa_Y \neq 1$ and $\bar{\kappa}_Y \neq 0$, the dipole moment exhibits logarithmic dependence on the cut-off scale $\Lambda_{UV}$ not far above the leptoquark mass. So, the leptoquark contribution to $(g-2)_\mu$ anomaly becomes

$$\Delta a_\mu = \frac{N_c}{16\pi^2} \sum_i \left[ 2\text{Re}(x_L^2 (x_R^2)^*) \frac{m_d m_\mu}{m_{U_1}^2} \left( 2Q_d + Q_{U_1} \left( 1 - \kappa_Y \right) \ln \left( \frac{\Lambda_{UV}^2}{M_{U_1}^2} \right) \right) - \left( |x_L^2|^2 + |x_R^2|^2 \right) \frac{m_\mu^2}{m_{U_1}^2} \left( \frac{4}{3} Q_d + Q_{U_1} \left( 1 + 9\kappa_Y \right) \ln \left( \frac{\Lambda_{UV}^2}{M_{U_1}^2} \right) \right) \right] ,$$  \hspace{1cm} (3.3)

where we use $\kappa_Y = 1$, $\bar{\kappa}_Y = 0$, $N_C = 3$, $Q_b = -1/3$, and $Q_{U_1} = 2/3$ [7].

3.2 $R_{K^(*)}$, $R_{D^(*)}$

To explain the experimental result of $R_{K^(*)}$, it requires the Wilson coefficients as

$$\Delta C_{9}^{\mu \mu} |^{\text{exp}} = -0.40 \pm 0.12 , \quad \Delta C_{9}^{d^*} |^{\text{exp}} = -0.50 \pm 0.38 ,$$  \hspace{1cm} (3.4)

with correlation $-0.5$ [26, 27, 30] between them. And the ratio of the SM predictions and experimental observation is

$$\frac{R_{D^*}^{\exp}}{R_{D^*}^{\text{SM}}} = 1.14 \pm 0.10 , \quad \frac{R_{D}^{\exp}}{R_{D}^{\text{SM}}} = 1.14 \pm 0.05 ,$$  \hspace{1cm} (3.5)
with correlation $-0.37$ [27]. For the $U_1$ leptoquark, the contribution to Wilson coefficients is [26, 27]

$$\Delta C_{9}^{\mu u} = \Delta C_{10}^{\mu u} = -\frac{4\pi^2}{\mu_{W}^2} \frac{x_{L}^{33}(x_{L}^{33})^*}{V_{ts}^*V_{tb}} \frac{m_{b}^2}{m_{U_1}^2} , \tag{3.6}$$

$$\Delta C_{9}^U \simeq \frac{1}{V_{tb}V_{ts}^*} \frac{2v^2}{3m_{U_1}^2} x_{L}^{33}(x_{L}^{33})^* \ln \left( \frac{m_{b}^2}{m_{U_1}^2} \right) . \tag{3.7}$$

Similarly, the $U_1$ contribution to $R_D$ and $R_{D^*}$ are [27]

$$\frac{R_D}{R_{D}^{\text{SM}}} \simeq \left[ 1 + \frac{v^2}{m_{U_1}^2} \text{Re} \left\{ \left( x_{L}^{33} - 1.5\eta_S(x_{R}^{33})^* \right) \frac{(V_{cb} x_{L}^{33} + V_{cs} x_{L}^{23} + V_{cd} x_{L}^{13})}{V_{cb}} \right\} \right], \tag{3.8}$$

$$\frac{R_{D^*}}{R_{D^*}^{\text{SM}}} \simeq \left[ 1 + \frac{v^2}{m_{U_1}^2} \text{Re} \left\{ \left( x_{L}^{33} - 1.4\eta_S(x_{R}^{33})^* \right) \frac{(V_{cb} x_{L}^{33} + V_{cs} x_{L}^{23} + V_{cd} x_{L}^{13})}{V_{cb}} \right\} \right],$$

where $\eta_S \simeq 1.8$ accounts for the running of the scalar operator from $m_{U_1} = 4 \text{ TeV}$ to $m_b$.

### 3.3 $B_c \rightarrow \tau \nu, B^\pm \rightarrow \tau \nu$

The bound from observation [31, 32] and SM prediction for $B_c \rightarrow \tau \nu$ are [7]

$$B(B_c \rightarrow \tau \nu)_{\text{exp}} \leq 0.10$$
$$B(B_c \rightarrow \tau \nu)_{\text{SM}} = (2.21 \pm 0.09) \times 10^{-2} \tag{3.9}$$

The proportion of experimental value and SM prediction for $B^\pm \rightarrow \tau \nu$ is [7]

$$\frac{B(B^\pm \rightarrow \tau \nu)_{\text{exp}}}{B(B^\pm \rightarrow \tau \nu)_{\text{SM}}} = 1.30 \pm 0.29 . \tag{3.10}$$

And the $U_1$ leptoquark contributions to each of observables are [7]

$$\frac{B(B_c \rightarrow \tau \nu)_{\text{SM}}}{B(B_c \rightarrow \tau \nu)} = 1 - \frac{(V_{cd} x_{L}^{13} + V_{cs} x_{L}^{23} + V_{cb} x_{L}^{33})}{V_{cb}} \frac{v^2}{m_{U_1}^2} \left( \frac{(x_{L}^{33})^*}{2} + \frac{(x_{R}^{33})^*}{m_{U_1}^2} \right)^2 , \tag{3.11}$$

$$\frac{B(B^\pm \rightarrow \tau \nu)_{\text{SM}}}{B(B^\pm \rightarrow \tau \nu)} = 1 - \frac{(V_{ud} x_{L}^{13} + V_{us} x_{L}^{23} + V_{ub} x_{L}^{33})}{V_{ub}} \frac{v^2}{m_{U_1}^2} \left( \frac{(x_{L}^{33})^*}{2} + \frac{(x_{R}^{33})^*}{m_{U_1}^2} \right)^2 . \tag{3.12}$$

### 3.4 $B_s \rightarrow \tau^+ \tau^-, B_s \rightarrow \mu^+ \mu^-, B_s \rightarrow \tau^- \mu^+$ and $B \rightarrow K \tau^+ \tau^-$

The experimental value from LHCb is [33] and SM prediction is [34]

$$B(B_s \rightarrow \tau^+ \tau^-)_{\text{exp}} = (0.0 \pm 3.4) \times 10^{-3} , \tag{3.13}$$
$$B(B_s \rightarrow \tau^+ \tau^-)_{\text{SM}} = (7.73 \pm 0.49) \times 10^{-7} . \tag{3.14}$$
And the related contribution for $U_1$ leptoquark is [7]

\[
\frac{\mathcal{B}(B_s \rightarrow \tau^+ \tau^-)}{\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)}_{\text{SM}} = \frac{16\pi^4}{e^4} \left( \frac{v_{\text{EW}}}{m_{B_s}^4} \right) \left( \frac{m_{B_s}^2}{m^2_{\mu}} \right) \left( \frac{(x_L^{32})^* x_R^{22}}{V_{ts} V_{tb}} \right)^2 \left( 1 - \frac{4m_{\tau}^2}{m_{B_s}^2} \right)
\]

\[
+ \left| 1 + \frac{4\pi^2}{e^2 C_{10}^{SM}} \frac{v_{\text{EW}}^2}{m_{U_1}^2} \left( \frac{(x_L^{32})^* x_L^{22} + (x_R^{32})^* x_R^{22}}{V_{ts} V_{tb}} \right) \right|^2.
\]

(3.15)

where $C_{10}^{SM} \approx -4.1$ which we use for a normalization such that the SM value for the Wilson coefficient [35].

The ratio between the SM prediction and experimental value is [7, 26]

\[
\frac{\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)}{\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)_{\text{exp}}} = 0.73^{+0.13}_{-0.10}.
\]

(3.16)

And the following $U_1$ vector leptoquark contribution is written by [7]

\[
\frac{\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)}{\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}}} = \frac{16\pi^4}{e^4 (C_{10}^{SM})^2} \left( \frac{v_{\text{EW}}}{m_{B_s}^4} \right) \left( \frac{m_{B_s}^2}{m^2_{\mu}} \right) \left( \frac{(x_L^{32})^* x_R^{22}}{V_{ts} V_{tb}} \right)^2
\]

\[
+ \left| 1 + \frac{4\pi^2}{e^2 C_{10}^{SM}} \frac{v_{\text{EW}}^2}{m_{U_1}^2} \left( \frac{(x_L^{32})^* x_L^{22} + (x_R^{32})^* x_R^{22}}{V_{ts} V_{tb}} \right) \right|^2.
\]

(3.17)

LHCb search on $B^0 \rightarrow \tau^\pm \mu^\mp$ provides an upper limit

\[
\mathcal{B}(B_s \rightarrow \tau^- \mu^+)_{\text{exp}} < 2.1 \times 10^{-5},
\]

(3.18)
at 95% confidence level [36]. SM prediction of this branching fraction is extremely small as $O\left(10^{-54}\right)$ [37]. The expression of $U_1$ contribution to $B_s \rightarrow \tau^- \mu^+$ is [27]

\[
\mathcal{B}(B_s \rightarrow \tau^- \mu^+) = \frac{1}{\Gamma_{B_s}} \frac{m_{B_s} f_{B_s}^2 G_F^2 m_{\tau}^2}{8\pi m_{B_s}^2} \left( 1 - \frac{m_{\tau}^2}{m_{B_s}^2} \right)^2
\]

\[
\times \left( \frac{v_{\text{EW}}^4}{4m_{U_1}^4} \right) \left| \frac{x_L^{22}(x_L^{33})^*}{m_{s}(m_s + m_b)} x_L^{33}(x_L^{33})^* \right|^2.
\]

(3.19)

where $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$ is the Fermi constant, $f_{B_s} = 0.225$ GeV [38] is the leptonic decay constant of $B_s$, and $\Gamma_{B_s} = 4.34 \times 10^{-13}$ GeV is the total width of $B_s$.

BaBar experiment measured the branching fraction [39]

\[
\mathcal{B}(B^+ \rightarrow K^+ \tau^+ \tau^-)_{\text{exp}} = (1.31 \pm 0.71) \times 10^{-3}.
\]

(3.20)

with an upper limit of $\text{Br}(B^+ \rightarrow K^+ \tau^+ \tau^-) < 2.25 \times 10^{-3}$ at the 90% confidence level. The expression of $U_1$ leptoquark contribution to this process is given by [27]

\[
\mathcal{B}(B \rightarrow K \tau^+ \tau^-) \simeq 1.5 \times 10^{-7} + 10^{-7} \frac{v_{\text{EW}}^2}{2m_{U_1}^4} \left[ 1.4 \text{Re} \left( x_L^{23}(x_L^{33})^* \right) - 3.3 \text{Re} \left( x_R^{23}(x_L^{33})^* \right) \right]
\]

\[
+ \frac{v_{\text{EW}}^4}{4m_{U_1}^4} \left[ 1 + 1.4 \text{Re} \left( x_R^{23}(x_L^{33})^* \right) + 95.0 \left| x_L^{33} \right|^2 \right].
\]

(3.21)

where $v_{\text{EW}} = 246$ GeV is the vacuum expectation value of the Higgs.
3.5 $B^+ \to K^+\tau^+\mu^-$, $B^+ \to K^+\tau^-\mu^+$

From BaBar experiment, we obtain upper limits [40]

\begin{align}
B(B^+ \to K^+\tau^+\mu^-) &< 2.8 \times 10^{-5}, \\
B(B^+ \to K^+\tau^-\mu^+) &< 4.5 \times 10^{-5}
\end{align}

at 90% confidence level. The leptoquark contribution is given by [27]

\begin{align}
B(B^+ \to K^+\tau^+\mu^-) &\simeq \frac{v^4_{\text{EW}}}{4m_{U_1}} |x_{L}^2|^2 \left[ 8.3 |x_{L}^{33}|^2 + 155.2 |x_{R}^{33}|^2 - 42.3 \text{Re} \left( (x_{L}^{33} x_{R}^{33})^* \right) \right], \\
B(B^+ \to K^+\tau^-\mu^+) &\simeq 8.3 \frac{v^4_{\text{EW}}}{4m_{U_1}} |x_{L}^{32} (x_{L}^{23})^*|^2.
\end{align}

3.6 $\tau \to \mu\gamma$, $\mu\phi$ and LFU in $\tau$ decay

Due to its sizeable couplings to muon and tau leptons, $U_1$ leptoquark can significantly affect the Lepton-flavor-violation in $\tau$ decays. The experimental upper limits are [41, 42]

\begin{align}
B(\tau \to \mu\gamma) &< 3.0 \times 10^{-8}, \\
B(\tau \to \mu\phi) &< 5.1 \times 10^{-8}
\end{align}

at 90% confidence level. In addition, the LFU in the decay of charged leptons can give stringent bounds on the leptoquark couplings. The experimentally measured values are [41, 43]

\begin{align}
(g_{\tau}/g_\mu)_{\text{exp}} &= 1.0000 \pm 0.0014, \\
(g_{\tau}/g_\mu)_\ell &= 1.0010 \pm 0.0015, \\
(g_{\tau}/g_\mu)_\pi &= 0.9961 \pm 0.0027, \\
(g_{\tau}/g_\mu)_K &= 0.9860 \pm 0.0070
\end{align}

The $U_1$ leptoquark contribution to $B(\tau \to \mu\gamma)$ is [7]

\begin{align}
B(\tau \to \mu\gamma) &\simeq \frac{1}{\Gamma_\tau} \frac{\alpha_{\text{EM}}^2 N_c^2 m_\ell^2 m_\tau^2}{256 \pi^4 m_{U_1}^2} |x_{L}^{33}| x_{L}^{32} (x_{L}^{23})^* |^2 \left[ 2Q_b - Q_{U_1} \left( 1 - \frac{5\kappa_Y}{2} \right) \right]^2,
\end{align}

where we use $\kappa_Y = 1$, $\tilde{\kappa}_Y = 0$, $Q_b = -1/3$, and $Q_{U_1} = 2/3$, $\Gamma_\tau = 2.27 \times 10^{-12}$ GeV. For $B(\tau \to \mu\phi)$, it is given by [27, 44]

\begin{align}
B(\tau \to \mu\phi) &\simeq \frac{1}{16 \pi} \frac{f_\phi^2 G_F^2}{m_\tau^2} \left( 1 - \frac{m_\phi^2}{m_\tau^2} \right)^2 \left( 1 + \frac{2m_\phi^2}{m_\tau^2} \right) \frac{v^4}{4m_{U_1}^2} |x_{L}^{23} (x_{L}^{32})^*|^2,
\end{align}

where $\phi$ is the $s\bar{s}$ vector meson with $f_\phi = 0.225$ GeV, $m_\phi = 1.019$ GeV [44]. For LFU in lepton decays, we use the expression [27, 43]

\begin{align}
\left( \frac{g_{\tau}}{g_\mu} \right)_{\ell,\pi,K} &\simeq 1 - 0.08 \times \frac{(x_{L}^{33})^2 v_{\text{EW}}}{4m_{U_1}^2}.
\end{align}
More specifically, the effective Lagrangian for $\tau$ leptonic decay [43]

$$\mathcal{L}_{\ell \rightarrow \nu \bar{\nu}} = -\frac{4G_F}{\sqrt{2}} \left( \left[ C_{\nu e}^{V,LL} \right]_{\rho \sigma \alpha \beta} (\bar{p}_L^\mu \nu_L^\mu)(\bar{p}_R^\mu \gamma^\mu \bar{p}_R^\beta) + \left[ C_{\nu e}^{V,LR} \right]_{\rho \sigma \alpha \beta} (\bar{p}_L^\mu \gamma^\mu \bar{p}_L^\beta) \right) .$$

(3.34)

Similarly, the effective Lagrangian for $\tau$ hadronic decay [43]

$$\mathcal{L}_{\tau \rightarrow h\nu} = -\frac{4G_F}{\sqrt{2}} \sum_\rho \left\{ \left( \delta_{3\rho} (V_{\text{CKM}})_{ji}^* \left[ C_{\nu e u}^{V,LL} \right]_{\rho 3\beta} \right) (\bar{p}_L^\mu \gamma^\mu \bar{p}_L^\beta) + \left[ C_{\nu e u}^{S,RL} \right]_{\rho 3\beta} \right\} \ .$$

(3.35)

Then

$$\left( \frac{g_\tau}{g_\mu} \right)_\ell = \frac{\sum_\rho \left( \delta_{3\rho} \delta_{\rho 1} + \left[ C_{\nu e u}^{V,LL} \right]_{\rho 13} \right)^2 + \left[ C_{\nu e u}^{V,LR} \right]_{\rho 13} \right)}{\sum_\rho \left( \delta_{3\rho} \delta_{\rho 1} + \left[ C_{\nu e u}^{V,LL} \right]_{\rho 12} \right)^2 + \left[ C_{\nu e u}^{V,LR} \right]_{\rho 12} \right)}^{1/2} ,$$

$$\left( \frac{g_\tau}{g_\mu} \right)_{\pi} = \frac{\sum_\rho \left( \delta_{3\rho} \delta_{\rho 1} + \left[ C_{\nu e u}^{V,LL} \right]_{\rho 31} \right)^2 + \left[ C_{\nu e u}^{S,RL} \right]_{\rho 31} \right)}{\sum_\rho \left( \delta_{3\rho} \delta_{\rho 1} + \left[ C_{\nu e u}^{V,LL} \right]_{\rho 12} \right)^2 + \left[ C_{\nu e u}^{S,RL} \right]_{\rho 12} \right)}^{1/2} ,$$

$$\left( \frac{g_\tau}{g_\mu} \right)_{K} = \frac{\sum_\rho \left( \delta_{3\rho} \delta_{\rho 1} + \left[ C_{\nu e u}^{V,LL} \right]_{\rho 21} \right)^2 + \left[ C_{\nu e u}^{S,RL} \right]_{\rho 21} \right)}{\sum_\rho \left( \delta_{3\rho} \delta_{\rho 1} + \left[ C_{\nu e u}^{V,LL} \right]_{\rho 12} \right)^2 + \left[ C_{\nu e u}^{S,RL} \right]_{\rho 12} \right)}^{1/2} .$$

(3.36)

For $U_1$ leptoquark, by using the Fierz transformation in Eq.(2.1),

$$[\bar{u}_{1L} \gamma^\mu u_{2L}][\bar{u}_{3L} \gamma_\mu u_{4L}] = -[\bar{u}_{1L} \gamma^\mu u_{4L}][\bar{u}_{3L} \gamma_\mu u_{2L}] ,$$

we replace the Wilson coefficients as

$$\left[ C_{\nu e u}^{V,LL} \right]_{\rho \sigma \alpha \beta} = -\frac{2G_F}{\sqrt{2}} \left[ C_{\nu e u}^{V,LL} \right]_{\rho \sigma \alpha \beta} = \left[ C_{\nu e u}^{V,LR} \right]_{\rho \sigma \alpha \beta} = 0 .$$

(3.37)

4 Parameter scanning

We perform the minimal $\chi^2$ parameter scanning, where the value of $\chi^2$ including all the 17 observables are listed in Table 1, which includes anomalies of $R_K^{(\nu)}$, $R_D^{(\nu)}$, constraints from other $B$-meson decay channels, and constraints from $\tau$ decays. For SM, it gives $\chi^2_{\text{SM}} = 26.0$ and $P$-value=0.074 for the observables in Table 1.
Table 1: The list of observable with experiment and SM prediction w/ \( U_1 \) prediction.

| Observable | Experiment | SM predict | \( U_1 \) predict |
|------------|------------|------------|---------------------|
| \( R_{D(*)} \) | \( \frac{R_{D}^{\text{exp}}}{R_{D}^{\text{SM}}} = 1.14 \pm 0.10, \quad \frac{R_{D}^{\text{exp}}}{R_{D}^{\text{SM}}} = 1.14 \pm 0.05 \) | \( \frac{R_{D}^{\text{exp}}}{R_{D}^{\text{SM}}} = 1.14 \pm 0.05 \) | (3.8) |
| \( \Delta C_9^{\mu \mu} = -\Delta C_{10}^{\mu \mu} (R_{K(*)}) \) | \(-0.40 \pm 0.12\) | \(0\) | (3.6) |
| \( \Delta C_9^U \) | \(-0.50 \pm 0.38\) | \(0\) | (3.7) |
| \( B_s \rightarrow \tau \nu \) | \(\leq 0.10\) | \((2.21 \pm 0.09) \times 10^{-2}\) | (3.11) |
| \( B_s^ \mp \rightarrow \tau \nu \) | \((1.09 \pm 0.24) \times 10^{-4}\) | \((8.8 \pm 0.6) \times 10^{-5}\) | (3.12) |
| \( B_s \rightarrow \tau^+ \tau^- \) | \((0.0 \pm 3.4) \times 10^{-3}\) | \((7.73 \pm 0.49) \times 10^{-7}\) | (3.15) |
| \( B_s \rightarrow \mu^+ \mu^- \) | \(BR(B_s \rightarrow \mu^+ \mu^-)^{\text{exp}} = 0.73^{+0.13}_{-0.10}\) | \(\) | (3.17) |
| \( B_s \rightarrow \tau^- \mu^+ \) | \((0.0 \pm 2.1) \times 10^{-5}\) | \(\) | (3.19) |
| \( B \rightarrow K \tau^+ \tau^- \) | \((1.31 \pm 0.71) \times 10^{-3}\) | \((1.20 \pm 0.12) \times 10^{-7}\) | (3.24) |
| \( B^+ \rightarrow K^+ \tau^+ \mu^- \) | \(\leq 2.8 \times 10^{-5}\) BaBar | \(\) | (3.24) |
| \( B^+ \rightarrow K^+ \tau^- \mu^+ \) | \(\leq 4.5 \times 10^{-5}\) BaBar | \(\) | (3.24) |
| \( \tau \rightarrow \mu \gamma \) | \((0.0 \pm 3.0) \times 10^{-8}\) | \(\) | (3.31) |
| \( \tau \rightarrow \mu \phi \) | \((0.0 \pm 5.1) \times 10^{-8}\) | \(\) | (3.32) |

- **scan-1**: varying \( x_L^{22}, x_L^{23}, x_L^{32}, x_L^{33}, x_R^{33} \) and fixing \( m_{U_1} = 2, 10 \) TeV. 
  Results are in Fig. 1.

- **scan-2**: varying \( x_L^{22}, x_L^{23}, x_L^{32}, x_L^{33}, x_R^{32}, x_R^{33} \), and fixing \( m_{U_1} = 2 \) TeV. 
  Results are in Fig. 2.

For **scan-1**, we choose the minimal set of relevant couplings, \( x_L^{22}, x_L^{23}, x_L^{32}, x_L^{33}, x_R^{33} \), that is enough to explain the B-meson anomalies and satisfies all the low-energy observables. It gives rise to the best fit \( \chi^2_{\text{min}} = 9.18 \) and \( P\)-value=0.759 with \((x_L^{22}, x_L^{23}, x_L^{32}, x_L^{33}) = (5.42 \times 10^{-2}, 0.279, -5.05 \times 10^{-2}, 0.947) \) and \( m_{U_1} = 2 \) TeV. Figure 1 shows the 1\sigma region for **scan-1**. The region in upper-left panel of \((x_L^{22}, x_L^{33})\) plane is mainly determined by the \( R_{D(*)}\), meanwhile the \( R_{K(*)} \) dictates the region on the \((x_L^{22}, x_L^{32})\) plane. There is contribution to \( \Delta a_\mu \) from the LH couplings, as shown in the upper-right panels of Fig.1, however, it is not large enough to explain the recent measurement from Fermilab. This becomes the motivation to turn on the RH coupling in the **scan-2**.

In the **scan-2** (\( \chi^2_{\text{min}} = 9.18 \) and \( P\)-value=0.687), according to Eq.(3.3), we found the most efficient way to enhance \( \Delta a_\mu \) is to use \( x_R^{32} \), where the multiplicity of \((x_R^{32}, x_R^{33})\) has milder mass suppression factor \((m_b m_\mu / m_{U_1})\) than that of pure LH couplings or RH couplings. In the \((x_L^{32}, \Delta a_\mu)\) and \((x_R^{32}, \Delta a_\mu)\) planes in Fig.2, there are tiny parameter regions overlap with
Figure 1: Scan-1: The region satisfies $\Delta \chi^2 \equiv \chi^2 - \chi^2_{\text{min}} \leq 2.3$, and $\chi^2_{\text{min}} = 9.18$.

$(g - 2)_\mu$ 1\(\sigma\) region from Fermilab result, which corresponds to $(x^{32}_L, x^{32}_R) \simeq (\pm 1.2, \mp 0.18)$ with $m_{U_1} = 2$ TeV.
Figure 2: Scan-2: The region satisfies $\Delta \chi^2 \equiv \chi^2 - \chi^2_{\text{min}} \leq 2.3$, and $\chi^2_{\text{min}} = 9.18$. Here we fix $m_{U_1} = 2$ TeV.

5 Summary

The recent observational anomalies lead us to consider a vector leptoquark whose couplings with both left and right chiral fermions are essential. It affects various channels of $B$-meson decays and generate lepton flavor universality breaking. At the same time, the leptoquark can contribute to $(g - 2)_\mu$. We examine all possible experimental constraints and conclude that leptoquark can be an actual explanation of all anomalies.

We expect the experimental measurements will be much more improved in the future and leptoquark will be better tested accordingly.
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