TWO BODY $B$ DECAYS, FACTORIZATION AND $\Lambda_{QCD}/m_b$ CORRECTIONS

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Abstract. By using the recent experimental measurements of $B \to \pi\pi$ and $B \to K\pi$ branching ratios, we find that the amplitudes computed at the leading order of the $\Lambda_{QCD}/m_b$ expansion disagree with the observed $B\!R$s, even taking into account the uncertainties of the input parameters. Beyond the leading order, Charming and GIM penguins allow to reconcile the theoretical predictions with the data. Because of these large effects, we conclude, however, that it is not possible, with the present theoretical and experimental accuracy, to determine the CP violation angle $\gamma$ from these decays.

INTRODUCTION

The advent of the $B$ factories, Babar [1] and Belle [2], opens new perspectives for very precise measurements of non-leptonic $B$-decays [3] and calls for a significant improvement of the theoretical predictions. In this respect, important progress has been recently achieved by systematic studies of factorization in $B$ decays [4, 5]. These studies confirmed the physical idea [6] that factorization holds in heavy hadron decays, $m_Q \gg \Lambda_{QCD}$, for the leading terms of the $\Lambda_{QCD}/m_Q$ expansion. They leave open, however, the central question of whether i) the leading terms predict with sufficient accuracy the relevant $B$-meson decay rates or ii) the power-suppressed corrections, which cannot be evaluated without some model assumption, are phenomenologically important. This problem has been addressed in a series of papers [7]–[13]. In particular, the main conclusion of ref. [12] is that non-perturbative $\Lambda_{QCD}/m_b$ corrections from the leading operators of the effective weak Hamiltonian, conventionally called Charming (or GIM) penguins, are very important in cases where the factorized amplitudes are either colour or Cabibbo suppressed. One of the consequences of this observation is that factorization at the leading order in $\Lambda_{QCD}/m_b$ is unable to reproduce the observed $B \to \pi\pi$ and $B \to K\pi$ BRs even taking into account the uncertainties of the input parameters.

The results of ref. [12] seem to be at variance with the conclusions of ref. [13] where it is stated that, in the QCD factorization approach, “an acceptable fit to the branching
fractions is obtained even if we impose $\gamma < 90^\circ$ as implied by the standard constraints on the unitarity triangle \(^2\).

Given the contradictory conclusions of the different studies, it is very important to clarify the situation by comparing input parameters, methods of analysis and results. In order to help the debate on this subject, we discuss in the following:

- The theoretical framework of factorization and the calculation of the amplitudes at the leading order of the $\Lambda_{QCD}/m_b$ expansion;
- A comparison of the leading terms, including their uncertainties, with the measured $B \to \pi\pi$ and $B \to K\pi$ branching ratios;
- Results for the $B \to \pi\pi$ and $B \to K\pi$ branching ratios including charming (and GIM) penguins;
- A comparison of the results obtained with charming and GIM penguins \(^1\) with the model of the $\Lambda_{QCD}/m_b$ corrections adopted in \(^1\).

**FACTORIZATION**

In this section we recall the basic ingredients necessary to compute the relevant amplitudes either within factorization or including the corrections which arise at higher order in the $\Lambda_{QCD}/m_b$ expansion.

The physical amplitudes for $B \to K\pi$ and $B \to \pi\pi$ can be conveniently written in terms of RG-invariant parameters built using the Wick contractions of the effective Hamiltonian \(^1\). In the heavy quark limit, following the approach of ref. \(^5\), it is possible to compute these RG invariant parameters using factorization. The formalism of ref. \(^5\) has been developed so that it is also possible to include the perturbative corrections to order $\alpha_s$, i.e. at the next-to-leading order in perturbation theory \(^3\).

For the sake of discussion, it is instructive to start from the explicit expression of the $B_d \to K^+\pi^-$ amplitude. In terms of the parameters defined in \(^1\), this amplitude reads

$$
\mathcal{A}(B_d \to K^+\pi^-) = -V_{ts}V_{tb}^\ast \left( E_1(s,u,u;B_d,K^+,\pi^-) - P_{1}^{\text{GIM}}(s,u;B_d,K^+,\pi^-) \right) + V_{ts}V_{tb}^\ast P_1(s,u;B_d,K^+,\pi^-). \tag{1}
$$

We have

$$
E_1(s,u,u;B_d,K^+,\pi^-) = a^u_1(K\pi)\langle Q_1^u \rangle_{\text{fact}} + a^u_2(K\pi)\langle Q_2^u \rangle_{\text{fact}} + \tilde{E}_1
$$

$$
P_1(s,u;B_d,K^+,\pi^-) = \sum_{i=3}^{10} a_i^u(K\pi)\langle Q_i \rangle_{\text{fact}} + \tilde{P}_1
$$

$$
P_{1}^{\text{GIM}}(s,u;B_d,K^+,\pi^-) = \sum_{i=3}^{10} (a_i^u(K\pi) - a_i^a(K\pi))\langle Q_i \rangle_{\text{fact}} + \tilde{P}_{1}^{\text{GIM}}, \tag{2}
$$

\(^2\) Indeed the analysis of ref. \(^{14}\) gives $\gamma = (54.8 \pm 6.2)^\circ$ and ref. \(^{15}\) quotes $34^\circ < \gamma < 82^\circ$.

\(^3\) An alternative framework, provided by the approach of ref. \(^4\), will not be discussed here. For a recent discussion see also \(^17\).
where \( \langle Q_i \rangle_{\text{fact}} \) denotes the factorized matrix elements and \( a^f_i \) the parameters introduced in [5]. Eqs. (1) and (2) are exact. Unfortunately, similar equations require the knowledge of several non-perturbative parameters, which at present cannot be extracted from the data. To be predictive, we will then use our physical intuition to reduce their number. In eqs. (1) and (2):

1. The CKM matrix elements can either be taken from other experimental measurements or from a fit to the non-leptonic \( BRs \), assuming that factorization is accurate enough;
2. The coefficients \( a^f_i (M_1 M_2) \) (e.g. \( a^u_1 (K \pi) \)) are renormalization group invariant, as the corresponding factorized matrix elements, and have been computed perturbatively at the NLO in refs. [5, 13];
3. The coefficients \( a^f_6 \) and \( a^f_8 \), which have also been computed to one-loop order, are instead scheme dependent. Their scheme-dependence is cancelled by the hadronic matrix elements of the penguin operators \( Q_6 \) and \( Q_8 \) respectively. Assuming factorization for the chirally enhanced contributions [5], the latter can be expressed in terms of the ratio

\[
r^K_{\chi} (\mu) = \frac{2m_K^2}{m_b(\mu)(m_s(\mu) + m_q(\mu))},
\]

which is formally of \( O(\Lambda_{\text{QCD}}/m_b) \) but numerically important (an analogous parameter \( r^K_\pi (\mu) \) can be defined for \( \pi \pi \) decays). We will discuss the rôle of these terms below.
4. The leading amplitudes \( \langle Q_i \rangle_{\text{fact}} \) are computed in terms of decay constants and semi-leptonic form factors. The form factors can either be taken from theoretical calculations [18, 19] or fitted from the experimental \( BRs \) (the possibility of extracting them from the corresponding \( B \to \pi \) semileptonic form factor at small momentum transfer is at present rather remote).
5. The tilded parameters, namely \( \tilde{P}_1, \tilde{P}^{\text{GIM}}_1 \) and \( \tilde{E}_1 \), are genuine, non-perturbative \( \Lambda_{\text{QCD}}/m_b \) corrections which cannot be computed at present. If we neglect Zweig-suppressed contributions, by \( SU(2) \) symmetry one can show that all the Cabibbo-enhanced \( \Lambda_{\text{QCD}}/m_b \) corrections to \( B \to K \pi \) decays can be reabsorbed in \( \tilde{P}_1 \). Several corrections are contained in \( \tilde{P}_1 \); this parameter includes not only the charming penguin contributions, but also annihilation and penguin contractions of penguin operators. It does not include leading emission amplitudes of penguin operators \( (Q_3 - Q_{10}) \) which have been explicitly evaluated using factorization. Had we included these terms, this contribution would exactly correspond to the parameter \( P_1 \) of ref. [16]. The parameter \( \tilde{P}_1 (P_1) \) encodes automatically not only the effect of the annihilation diagrams considered in [20], but all the other contributions of \( O(\Lambda_{\text{QCD}}/m_b) \) with the same quantum numbers of the charming penguins. In this respect it is the most general parameterization of all the perturbative and non-perturbative contributions of the operators \( Q_5 \) and \( Q_6 (Q_3 \) and \( Q_4) \), including the worrying higher-twist infrared divergent contribution to annihilation discussed in ref. [13, 21]. The parameter \( \tilde{P}_1 \) has the same quantum numbers and physical effects as the original charming penguins proposed in [7], although it has a more general meaning.
6. If one also includes $B \to \pi\pi$ decays we have several other parameters, for example $P_1^{\text{GIM}}$ and $P_3$, in the formalism of ref. [16]. A closer look to $P_3$ shows that this term is due either to Zweig suppressed annihilation diagrams (called CPA and DPA in ref. [7]) or to annihilation diagrams which are colour suppressed with respect to those entering $\tilde{P}_1$. For this reason in ref. [12] $P_3$ was taken to be zero. $\tilde{P}_1$ is equal to the corresponding parameter in $K\pi$ decays if $SU(3)$ symmetry is assumed. In our analysis we have used the same value of $\tilde{P}_1$ for all $K\pi$ and $\pi\pi$ channels. $P_1^{\text{GIM}}$ will be discussed later on.

We are now ready to discuss factorization for the leading terms of the $\Lambda_{\text{QCD}}/m_b$ expansion. Factorization is the theory of non leptonic decays which is obtained in the limit $m_b \to \infty$. Thus it consists in neglecting all terms of $O(\Lambda_{\text{QCD}}/m_b)$ ($\tilde{E}_1 = 0$, $\tilde{P}_1 = 0$, $r_{K,\pi}^{\chi} = 0$, etc.). At lowest order in perturbation theory, called also naïve factorization, the $a_i^{\tilde{f}}$ are simple combinations of the Wilson coefficients and do not depend on the hadron wave functions. The inconvenience of naïve factorization is that physical amplitudes still have a marked dependence on the renormalization scale because, contrary to the Wilson coefficients, the factorized matrix elements are scale independent.

The scale dependence is reduced by working at $O(\alpha_s)$, both for the Wilson coefficients and the matrix elements. In refs. [5] it has been shown that, at this order, all the dangerous infrared divergences can be reabsorbed in the definition of the hadronic wave functions. For the leading terms in the $\Lambda_{\text{QCD}}/m_b$ expansion there are strong arguments to support the idea that this will remain true at all orders in $\alpha_s$, see also ref. [26]. Thus, in the limit $m_b \to \infty$, it is likely that factorization is preserved by strong interactions. At $O(\alpha_s)$ or higher, the coefficients $a_i^{\tilde{f}}$ depend on the specific detail of the hadron wave-fuctions. For this reason, the uncertainties relative to the wave functions, as the residual renormalization scale dependence, must be taken into account in the evaluation of the uncertainties for the theoretical predictions. The approximation in which we neglect all the $\Lambda_{\text{QCD}}/m_b$ corrections, but include the perturbative corrections to the leading contribution, is called QCD factorization or simply factorization. Factorization implies an important consequence: predictions of non-leptonic decay rates are model independent to the extent that the few relevant hadronic parameters, namely the kaon and pion decay constants, $f_{K,\pi}$, the semileptonic form factors, $f_{K,\pi}(0)$, and the hadronic wave functions are known.

In cases like $B \to K\pi$ decays, where the factorized amplitudes are Cabibbo suppressed, the corrections of $O(\Lambda_{\text{QCD}}/m_b)$, which unfortunately are model dependent, become important. At lowest order in $\alpha_s$, the chirally enhanced terms proportional to $r_K^{\chi}$ ($r_{K}^{\pi}$) are computable by assuming that factorization can be applied beyond the leading order. A substantial difficulty arises, however, at $O(\alpha_s \Lambda_{\text{QCD}}/m_b)$. Although the chirally enhanced corrections from $Q_{6,8}$ are infrared finite, other contributions of the same order from different operators are infrared divergent, signaling that they belong to the class of the non-perturbative contributions which appear beyond factorization. These cannot be predicted using the same hadronic quantities of the factorized amplitudes. For this reason, any phenomenological analysis which aims at including in a coherent way the terms of $O(\alpha_s \Lambda_{\text{QCD}}/m_b)$ is forced to introduce extra model-dependent non-perturbative parameters besides $f_{K,\pi}$, $f_{K,\pi}(0)$ and the hadronic wave functions. This implies that, at
$O(\Lambda_{QCD}/m_b)$, model dependence is unavoidable (even in the subsector of the chirally enhanced contributions) and it is present in both the analyses of ref. [12] and ref. [13], which we will compare below.

Different model-dependent assumptions were made in the two approaches:

1. In ref. [12], $\tilde{E}_1$ and $\tilde{P}_3$ were neglected and $SU(2)$ symmetry was assumed for $\tilde{P}_1$ ($SU(3)$ when it was used for $B \to \pi\pi$ decays). The same approximations were made for $\tilde{P}_1^{\text{GIM}}$. The complex parameter $\tilde{P}_1$ ($\tilde{P}_1^{\text{GIM}}$) was then fitted to reproduce the experimental $BR$s.

2. In ref. [13] the effects of the chirally enhanced $\Lambda_{QCD}/m_b$ corrections were either computed perturbatively or encoded in two complex phenomenological parameters called $X_H$ and $X_A$. An uncertainty of 100 % to the “default” values (e.g. $X_H = 2.4$) was assigned to these parameters in order to determine allowed bands for the predicted $BR$s. The bands include all other sources of uncertainties.

We now compare the leading amplitudes to the experimental results in order to test factorization. Before using predictions based on factorization to test the Standard Model and look for signals of new physics, it is crucial to check how large are the errors induced by our ignorance of the $O(\Lambda_{QCD}/m_b)$ corrections which we are unable to compute. Our position, indeed, is that we have more confidence in the SM rather than in factorization. In order to test factorization, we ought to use all the information that we have from other measurements. Thus, for example, whereas the size of the error on $|V_{cb}|$ and $|V_{ub}|$ can be debated, there is no question that these experimental inputs must be included in any analysis that aims at testing (or using) factorization. We also stress that the value of $|V_{ub}|$ is not expected to be affected by the presence of new physics beyond the SM.

The CP parameter $\gamma$ does in general change if there is physics beyond the SM. It remains an interesting exercise, however, to verify whether, by taking the value of $\gamma$ from the Unitarity Triangle Analysis (UTA) in the SM, the predicted $BR$s are in agreement with the data. If, by using $\gamma$ from the UTA, one is unable to reproduce the experimental $B \to \pi\pi$ and $B \to K\pi$ $BR$s, this implies that “either there is new physics or $\Lambda_{QCD}/m_b$ corrections are important” [12].

In our analysis we have used the likelihood method which has been described in all details in ref. [14]. Without entering in the “ideological” controversy about frequentistic and bayesian methods, we only note here that in [14] it has been shown that, at 95 % C.L., the Bayesian analysis give the same results as the frequentistic Babar Scanning method (and its variations) when the same inputs are used. Thus we will present our contour plots, corresponding to fig. 17 of ref. [13], both with factorization plus chirally enhanced contributions and with the non-factorizable charming (and GIM) penguin corrections. Besides this, we will also give tables with the relevant $BR$s, both in the factorization approximation with chirally enhanced terms and with the charming (and GIM) penguin corrections included.

We end this section with some remarks. In our approach, we have first checked that, within factorization and the SM, it is impossible to fit the experimental $BR$s. The $\Lambda_{QCD}/m_b$ terms, that we are then forced to include in order to reproduce the experimental results, are non-perturbative quantities, infrared divergent in perturbation theory, on which we do not have any knowledge a priori. For this reason we decided to
fit them on the data. The experimental numbers are nicely reproduced and the corrections to factorization are well consistent with the expected size (i.e. $\tilde{P}_1$ is of $O(\Lambda_{QCD}/m_b)$ with respect to the leading contributions).

In ref. [13] the subleading power corrections are varied in predefined intervals and the change in the predicted $BR$s is interpreted as uncertainty on the factorized amplitudes. In our opinion the uncertainties on factorization are only those coming from the CKM matrix elements or the form factors, etc. The $\Lambda_{QCD}/m_b$ terms instead are really contributions beyond factorization: if they are necessary to reproduce the data then it is not possible to make model independent predictions. This would remain true even if we knew without any uncertainty the hadronic parameters entering at the lowest order of the $\Lambda_{QCD}/m_b$ expansion.

Thus we are in the Bermuda triangle: i) without the $O(r_\chi)$ and $O(\alpha_s r_\chi)$ terms, that is within QCD factorization, we cannot reproduce the data; ii) the inclusion of the computable subset of $O(\alpha_s r_\chi)$ terms only is inconsistent since there is no reason to exclude the other non-perturbative non-computable contributions of the same order. In any case, we will show that, by using all the available experimental information, also this case is very difficult to reconcile with the data; iii) the complete set of $O(\alpha_s r_\chi)$ corrections leads us beyond factorization and the results are model dependent. Indeed there is no proof that the one-loop finite chirally enhanced terms remain infrared finite at higher orders in $\alpha_s$. Moreover, if the corrections of $O(\alpha_s r_\chi \sim \alpha_s \Lambda_{QCD}/m_b)$ are phenomenologically important, it is difficult to understand why, at the same level of numerical accuracy, other non-chirally enhanced non-perturbative $\Lambda_{QCD}/m_b$ terms should not also be taken into account.

The sad conclusion is that the very nice theory of QCD factorization developed in [5] is insufficient to fit the data because power corrections, which are model dependent, are important in $B \to K\pi$ and $B \to \pi\pi$ decays. Finally, model dependence does not implies that we are unable to make any prediction. If the assumptions made in our approach are reasonable $^5$, by fitting $\tilde{P}_1$ to the data and with the increasing experimental precision we may hope to extract also the value of $\gamma$ or to constrain $\sin 2\alpha$.

\section*{RESULTS}

In this section we present our analysis and a detailed comparison with the results of ref. [13]. In order to obtain our results we used the likelihood method as in [12]. The input parameters, given in table 1, are also the same but for a few differences:

- We added the $O(\alpha_s)$ corrections to the coefficients of the penguin and electropenguin operators computed in [13], which appeared after the completion of [12]. In this respect the criticisms of ref. [27] do not apply to the present analysis. We will see below that, even including these new ingredients, the main physics conclusions of ref. [12] are confirmed.

\footnote{Note that some $BR$s are dominated by the $\Lambda_{QCD}/m_b$ corrections.}

\footnote{For example that we may neglect $\tilde{E}_1$.}
• As for the matrix elements of $Q_{6,8}$, we include the “computable” factorizable chirally enhanced terms in the definition of $\tilde{P}_1$, which in this way contains all the possible Cabibbo enhanced, model-dependent corrections of $O(\Lambda_{QCD}/m_b)$. In practice this corresponds to fit $\tilde{P}_1$ as in ref. [12] with $f_{K,\pi} = 0$. There is no substantial difference between the old choice and the new one, since the contribution of the chirally enhanced terms and of $\tilde{P}_1$ have exactly the same quantum numbers.

• Although this is a minor source of uncertainty we also allow a variation of the renormalization scale and of the parameters of the hadronic wave functions.

We have also verified that by using the inputs of ref. [13] we obtain essentially the same results, and hence arrive to the same physics conclusions.

In this section we present:

1. A brief discussion of the results obtained in QCD factorization, namely with all the terms of $O(\Lambda_{QCD}/m_b)$ set to zero.
2. The results obtained in our approach by fitting $\tilde{P}_1$ and using $\gamma$ as determined from the Unitary Triangle Analysis in ref. [14]

$$\gamma = (54.8 \pm 6.2)^\circ.$$  (4)

On the basis of this study we predict the $B-\bar{B}$ asymmetries of the $BR$s, such as

$$\mathcal{A}(B_d \to \pi^+ \pi^-) = \frac{BR(B_d^0 \to \pi^+ \pi^-) - BR(B_d^0 \to \pi^+ \pi^-)}{BR(B_d^0 \to \pi^+ \pi^-) + BR(B_d^0 \to \pi^+ \pi^-)};$$  (5)

3. The results obtained by letting $\gamma$ free and a comparison of our results with those of ref. [13].

**Results in QCD factorization**

In this subsection we compare the model independent results obtained with QCD factorization, namely including only the terms which survive when $m_b \to \infty$, with the experimental data. In this case there is still a residual model dependence due to our ignorance of the semileptonic form factors $f_{K,\pi}(0)$ and, at order $\alpha_s$, to the ignorance of the hadron wave functions. We vary the semileptonic form factors with flat p.d.f. in the intervals given in table 1 and the parameters of the hadron wave functions in the intervals given in table 2 of ref. [13]. We also vary the renormalization scale between $m_b/2$ and $2m_b$, see table 1 of [13]. The data show a generalized disagreement with the QCD factorization predictions. In particular the allowed region in the $\rho-\eta$ plane and the value of $\gamma$ do not have any overlap with the corresponding ones from the unitarity triangle analysis [14]. This remains true even if we double the uncertainty on $|V_{ub}|$. We conclude that QCD factorization cannot be reconciled with data.
### Table 1

| BR (channels) | Charming + GIM | chirally enhanced | BBNS | Charming + GIM | chirally enhanced | BBNS |
|---------------|----------------|------------------|------|----------------|------------------|------|
| $K^0\pi^0$    | 8.6 $\pm$ 0.9  | 3.6 $\pm$ 1.5    | 4.1 $\pm$ 1.8 | 9.8 $\pm$ 1.0  | 5.7 $\pm$ 2.3    | 6.2 $\pm$ 2.6 |
| $K^0\pi^+$    | 18.7 $\pm$ 1.6 | 10.2 $\pm$ 4.2  | 10.9 $\pm$ 4.8 | 17.9 $\pm$ 1.4 | 8.2 $\pm$ 3.4    | 9.2 $\pm$ 4.0 |
| $\pi^+\pi^-$  | 4.9 $\pm$ 0.8  | 9.2 $\pm$ 3.8    | 9.2 $\pm$ 3.8  | 3.5 $\pm$ 0.8  | 5.7 $\pm$ 2.2    | 6.5 $\pm$ 2.5 |
| $\rho^0\pi^0$ | 0.6 $\pm$ 0.2  | 0.2 $\pm$ 0.1    | 0.4 $\pm$ 0.3  |                |                  |      |

### Table 2

| BR (channels) | Charming + GIM | chirally enhanced | BBNS |
|---------------|----------------|------------------|------|
| $K^0\pi^0$    | 0.27 $\pm$ 0.08 |                  |      |
| $f_K(0)/f_\pi(0)$ | 1.2 $\pm$ 0.1  |                  |      |

### Table 2

| BR (channels) | Charming + GIM | BBNS |
|---------------|----------------|------|
| $B(\rho^0\pi^0)$ | 10.3 $\pm$ 2.6 |      |
| $B(\rho^0\pi^0)$ | 12.0 $\pm$ 1.7 |      |
| $B(\pi^+\pi^0)$  | 17.4 $\pm$ 2.6 |      |
| $B(\pi^+\pi^0)$  | 17.3 $\pm$ 1.6 |      |
| $B(\pi^+\pi^0)$  | 4.4 $\pm$ 0.9  |      |
| $B(\pi^+\pi^0)$  | 5.3 $\pm$ 1.7  |      |

### Factorization with Charming and GIM penguins

We now discuss the effects of charming and GIM penguins, parameterized by $\tilde{P}_1$ and $\tilde{P}_1^{\text{GIM}}$. $\tilde{P}_1$ is a complex amplitude that we fit on the $B \to K\pi$ BRs. In order to have a reference scale for its size, we introduce a suitable “Bag” parameter, $\hat{B}_1$, by writing

$$\tilde{P}_1 = \frac{G_F}{\sqrt{2}} f_\pi f_\pi(0) g_1 \hat{B}_1,$$

where $G_F$ is the Fermi constant, $g_1$ is a Clebsh-Gordan parameter depending on the final $K\pi$ ($\pi\pi$) channel and $B_1 = |B_1| \exp(i\phi)$. Note that $\hat{B}_1$ differs from the parameter defined in ref. [12] because it now includes all the chirally enhanced $\Lambda_{QCD}/m_b$ corrections, part of which were previously explicitly calculated using factorization. In a similar way we introduce $\hat{B}_1^{\text{GIM}}$.

We fit all the BRs given in Table 1 with GIM and charming penguins included and taking the value of $\gamma$ determined from the UTA, see eq. (4). We find

$$|\hat{B}_1| = 0.13 \pm 0.02, \quad \phi = (188 \pm 82)^\circ,$$

$$|\hat{B}_1^{\text{GIM}}| = 0.17 \pm 0.08, \quad \phi^{\text{GIM}} = (181 \pm 59)^\circ,$$

where the notation is self-explaining. Note that the size of the charming and GIM penguin effects is of the expected magnitude. Note that $|\hat{B}_1^{\text{GIM}}|$ is very poorly determined.
FIGURE 1. $\rho$–$\eta$ contour plots obtained with factorization and infrared-finite chirally enhanced terms using $|V_{ub}|$ (up-left) or letting $\rho$ and $\eta$ free (up-right). We also show a comparison of the p.d.f. of $\gamma$ with the one from the UTA analysis of ref. [14] in the two cases.

The reason is that only $BR(B^0_d \to \pi^+\pi^-)$ in the fit is sensitive to GIM penguins. Thus, in practice, we are trying to fit two parameters, namely $|\hat{B}_{\text{GIM}}|$ and $\phi_{\text{GIM}}$, to a single $BR$.

The results for the $BR$s can be found in table 2 with the label “Charming+GIM”. They show that the extra charming and GIM parameters radically improve the agreement for the measured $B \to K\pi$ and $B \to \pi\pi$ $BR$s. We do not claim, however, to be able to predict $BR(B_d \to \pi^+\pi^-)$ since many effects of the same order besides charming and GIM contributions, which in this case are not Cabibbo enhanced, were ignored: our results instead show that accurate predictions for $B_d \to \pi\pi$ decays can only be obtained by controlling quantitatively the $O(\Lambda_{QCD}/m_b)$ corrections, which is presently far beyond the theoretical reach. To give a complete information, and for comparison with ref. [13] we also fit the data by letting $\gamma$ free. In this case we obtain $\gamma = (89 \pm 42)^\circ$. At present, the precision of the data and the number of free parameters does not allow a useful determination of $\gamma$.

The large absolute values of $\phi$, and the sizable effects that penguins have on the $BR$s, stimulated us to consider whether we could find observable particle-antiparticle asymmetries as the one defined in eq. (5). We find large effects in $BR(B^+ \to K^+\pi^0)$,
TABLE 3. Absolute values of the rate CP asymmetries for $B \rightarrow K\pi$ and $B \rightarrow \pi\pi$ decays.

| $|\mathcal{A}|$ | Charming + GIM | $|\mathcal{A}|$ | Charming + GIM |
|--------------|----------------|--------------|----------------|
| $K^0\pi^0$  | 0.05 ± 0.03    | $K^+\pi^0$  | 0.16 ± 0.08    |
| $K^0\pi^+$  | 0.02 ± 0.02    | $K^+\pi^-$  | 0.15 ± 0.07    |
| $\pi^+\pi^-$| 0.44 ± 0.21    | $\pi^0\pi^0$| 0.61 ± 0.29    |

$BR(B_d \rightarrow K^+\pi^-)$ and $BR(B_d \rightarrow \pi^+\pi^-)$. As discussed before, for $BR(B_d \rightarrow \pi^+\pi^-)$ our predictions suffer from very large uncertainties due to contributions which cannot be fixed theoretically. For this reason, the values of the asymmetry reported in table 3 are only an indication that a large asymmetry could be observed also in this channel. There is a sign ambiguity in $\mathcal{A} \sim \sin \gamma \sin \phi$. This ambiguity can be solved only by an experimental measurement or, but this is extremely remote, by a theoretical calculation of the relevant amplitudes. For each channel, we give the absolute value of the asymmetry in table 3. Note that within factorization all asymmetries would be unobservably small, since the strong phase is a perturbative effect of $O(\alpha_s)$ [5]. The possibility of observing large asymmetries in these decays opens new perspectives. These points will be the subject of a future study.

Comparison with BBNS

In this section we make a critical comparison with the latest analysis of BBNS [13]. We recall that we added all the perturbative corrections and allowed the variations of the non-perturbative parameters which were implemented by BBNS.

We start by discussing the rôle of chirally enhanced terms that do not suffer from infrared divergences at $O(\alpha_s)$. This is an instructive case since, if not for other infrared-divergent contributions of the same order, the inclusion of only these terms, although not justified, would still allow model independent predictions, in the sense discussed before. Thus the question is whether these chirally enhanced terms alone, part of which have the same effect as $\tilde{P}_1$, can describe the data. Thus we have repeated the analysis in the UTA case with only factorization and chirally enhanced infrared-finite terms with the results for the $BR$s given in table 2 with the label “chirally enhanced”. The combined probability that all the predicted values for the $K\pi$ channels are within two $\sigma$s from the experimental numbers is 6%. If we relax the constraint on $\gamma$ we obtain $\gamma = (127 \pm 20)^o$ with a probability of 0.3% that $\gamma < 80^o$ and of 2% that $\gamma < 90^o$. The corresponding $\rho$–$\eta$ contour plot is given in fig. 1 (upper–left). We conclude that this model (model in the sense of including only a chosen subset of the chirally enhanced terms) is strongly disfavoured by the data. The reader may be surprised of the difference between fig. 1 and fig. 17 of ref. [13] (where the non-perturbative parameters $X_{A,H}$ were however included). The difference is explained by the fact that we used the experimental measurements of $|V_{ub}|$. If, following ref. [13, 27], we let both $\rho$ and $\eta$ free, the $\rho$–$\eta$ contour plot changes and becomes that shown in fig. 1 (upper-right), which is very similar to fig. 17 of ref. [13]. For completeness, we also give in fig. 1 the p.d.f. of $\gamma$ in the two cases, together with the
FIGURE 2. Contour plot in the $\gamma - |V_{ub}|$ plane, see text.

p.d.f. obtained with the UTA analysis of ref. [14]. When the experimental information on $|V_{ub}|$ is used the two p.d.f. have no overlap (lower-left). In the other case, it is still possible to find values of $\gamma$ compatible with the UTA analysis (lower-right), at the price of a rather low value of $|V_{ub}|$. The situation is well illustrated by fig. 2, where the contour plot in the $\gamma - |V_{ub}|$ plane of the joint p.d.f. from non-leptonic decays is compared with the UTA range for $\gamma$ and the allowed interval for the measured $|V_{ub}|$, at $1 \sigma$. This figure demonstrates that, given the strong correlation between $\gamma$ and $|V_{ub}|$, it is crucial to take into account the experimental knowledge of $|V_{ub}|$. An important theoretical remark is in order at this point. The coefficient $a_6^f$ is enhanced by the $O(\alpha_s)$ corrections and may play the same rôle of charming penguins. It is very scaring that its actual value is strongly affected by the contribution of the chromomagnetic operator computed at tree level [13]. It is very hard to believe that this contribution which, besides all general considerations, is also non-local, will remain infrared-finite in higher orders and can be really evaluated in this way.

We now discuss the effect of $X_{A,H}$ on the final results. The addition of these parameters, in the range proposed in ref. [13], leaves the situation substantially unaltered as shown in fig. 3. This holds true also for the BRs, which are given in table 2 with the label BBNS. The reason is that on the one hand $X_A$ and $X_H$ do not have the same quantum numbers, and hence effects, of charming penguins, on the other the range chosen a priori, on the basis on one-loop perturbation theory, is not large enough to improve the agreement with the measured BRs (it essentially increases the uncertainty on the predictions). Of course by choosing a low value of the strange quark mass and of $|V_{ub}|$ (without using the experimental information coming from its measurements), a large
FIGURE 3. ρ–η contour plots obtained with the BBNS model using |V_{ub}| (up-left) or letting ρ and η free (up-right). We also show a comparison of the p.d.f. of γ with the one from the UTA analysis of ref. [14] in the two cases.

value of f_K(0) and a small value for f_π(0) etc. it is still possible to find some point in parameter space where the χ^2 is good. That this can be used to fit γ and test the SM is hard to believe though.

The situation would be different if X_{A,H} are let free to vary and fitted to the data. We have done this exercise and found that the preferred value of ρ_A is much larger than the values allowed in the interval chosen in [13], whereas ρ_H is not determined by the fit. We conclude that since X_A and X_H are infrared divergent quantities, the value of which cannot be predicted, and since without the inclusion of non-perturbative contributions of O(Λ_{QCD}/m_b) is not possible to reproduce the experimental data, we are bound to use model dependent assumptions in the analysis of non-leptonic B → Kπ and B → ππ decays.
CONCLUSION

We have analyzed the predictions of QCD factorization for $B \to \pi \pi$ and $B \to K \pi$ decays. Even taking into account the uncertainties of the input parameters, we find that QCD factorization is unable to reproduce the observed BRs. The introduction of charming and GIM penguins [7] allows to reconcile the theoretical predictions with the data. Instead of varying the non-perturbative phenomenological parameters in preassigned ranges, we prefer to try to fit them on the data. With the present theoretical and experimental accuracy, we find that it is still not possible to determine the CP violation angle $\gamma$. The situation is expected to improve in the near future with more accurate experimental measurements of the relevant BRs.

Contrary to factorization, we predict large asymmetries for several of the particle–antiparticle BRs, in particular $BR(B^+ \to K^+ \pi^0)$, $BR(B_d \to K^+ \pi^-)$ and, possibly, $BR(B_d \to \pi^+ \pi^-)$. This opens new perspectives for the study of CP violation in $B$ systems.

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