Poynting-Robertson effect and capture of grains in exterior resonances with planets

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Abstract. Spherical meteoroids orbiting the Sun experience orbital decay due to the Poynting-Robertson effect and can become trapped in commensurability resonances with a planet. Detail conditions for trapping in the planar circular restricted three-body problem with action of solar electromagnetic radiation on spherical grain are presented. The results are given in terms of the radiation pressure factor $\beta$.

In general, the greater value of $\beta$, the shorter capture time in a resonance. Captures of particles may be even several times longer than it is presented in literature. Analytical formula for maximum capture times is given.

Analytical theory for minimal capture eccentricities is presented. The obtained analytical results differ from those presented in the literature. However, the known analytical results are not very consistent with our detail numerical computational results for $\beta$-values 0.01 and 0.05 and for the planet Earth. Minimum eccentricities for captures into resonances are also smaller, even in order of magnitude, than the published values.

The presented results enable comparison with calculations for nonspherical particles.

Key words. meteoroids, electromagnetic radiation, celestial mechanics

1. Introduction

Physics of resonances with Solar System planets is discussed mainly during the last three decades. The orbital evolution of meteoroids near resonances with planets was also investigated. Besides gravitational forces the effect of solar electromagnetic radiation in the form of the Poynting-Robertson (P-R) effect is usually taken into account (Robertson 1937, Klačka 2004, 2008a, 2008b), see e. g., Jackson and Zook (1992), Weidenschilling and Jackson (1993), Marzari and Vanzani (1994), Šidlichovský and Nesvorný (1994), Liou and Zook (1995), Liou et al. (1995).

The aim of our contribution is detail analytical reconsideration of the influence of the P-R effect on a meteoroid which can become trapped in commensurability resonances with a planet. As a motivation
we can introduce that Weidenschilling and Jackson (1993) yields for theoretical minimal value of eccentricity at the beginning of capture the value 0.3506 for dust particle with $\beta = 0.01$ in resonance 2:1 with the Earth, while our simulations show that real values may be even less than 0.1. Detail conditions for trapping in the planar circular restricted three-body problem with action of solar electromagnetic radiation on a spherical grain (P-R effect holds) are presented. We concentrate on real values of orbital elements – width of semimajor axis in resonance, minimal values of eccentricities at the beginning of capture, and, capture time. We use for calculating osculating orbital elements simultaneously solar gravity and dominant part of solar radiation pressure-term (central acceleration have form $-GM_\odot (1 - \beta) e_R/r^2$). Results presented in this contribution may serve for comparison of the results obtained for spherical and nonspherical dust particles (meteoroids), since P-R effect holds only if a special condition is fulfilled (see Eq. (9) in Klačka 2008a; compare Eq. (73) in Klačka 2008b) and real nonspherical particles do not fulfill the condition.

2. Model – Equation of motion

All our numerical simulations are based on equation of motion written in the form (see Eq. (72) in Klačka 2008b for more details)

$$\frac{dv}{dt} = -\frac{GM_\odot}{r^2} e_R - GM_P \left\{ \frac{r - r_P}{|r - r_P|^3} + \frac{r_P}{|r_P|^3} \right\} + \beta \frac{GM_\odot}{r^2} \left\{ \left( 1 - \frac{v \cdot e_R}{c} \right) e_R - \frac{v}{c} \right\},$$

$$e_R \equiv r/|r| ; \quad \beta = 7.682 \times 10^{-4} Q_{\text{pr}}' A' \frac{m}{m [\text{kg}]},$$

where $r$ is position vector of the particle (with respect to the Sun) and $r_P$ is position vector of the planet which moves in circular orbit around the Sun; $m_P$ is mass of the planet, $M_\odot$ is mass of the Sun, $m$ is mass of the particle/meteoroid, $A' = \pi a'^2$, $a'$ is radius of the spherical particle, $Q_{\text{pr}}'$ is dimensionless efficiency factor (see, e. g., van de Hulst 1981). $\beta = 5.761 \times 10^{-5} Q_{\text{pr}}'/ (\rho [\text{g/cm}^3] s[\text{cm}])$, for homogeneous spherical particle in the Solar System, $\rho$ is mass density and $s$ is radius of the sphere. Planar motion will be considered.

We use osculating orbital elements which considers solar gravity and dominant part of solar radiation pressure-term, simultaneously: central acceleration $-GM_\odot (1 - \beta) e_R/r^2$ is used in calculations of orbital elements and the corresponding quantities will be denoted with a subscript $\beta$. 

$$\frac{dv}{dt} = -\frac{GM_\odot}{r^2} e_R - GM_P \left\{ \frac{r - r_P}{|r - r_P|^3} + \frac{r_P}{|r_P|^3} \right\} + \beta \frac{GM_\odot}{r^2} \left\{ \left( 1 - \frac{v \cdot e_R}{c} \right) e_R - \frac{v}{c} \right\},$$

$$e_R \equiv r/|r| ; \quad \beta = 7.682 \times 10^{-4} Q_{\text{pr}}' A' \frac{m}{m [\text{kg}]},$$
3. Perturbation equations of celestial mechanics

Rewriting Eq. (1) into the form

\[
\frac{dv}{dt} = -\frac{GM_\odot}{r^2} (1 - \beta) e_R - Gm_p \left\{ \frac{r - r_p}{|r - r_p|^3} + \frac{r_p}{|r_p|^3} \right\}
- \beta \frac{GM_\odot}{r^2} \left( \frac{v \cdot e_R}{c} e_R + \frac{v}{c} \right),
\]

we can immediately write for perturbation acceleration to Keplerian motion

\[
F_\beta = (F_\beta)_G + (F_\beta)_{PR},
(F_\beta)_G = - Gm_p \left\{ \frac{r - r_p}{|r - r_p|^3} + \frac{r_p}{|r_p|^3} \right\},
(F_\beta)_{PR} = - \beta \frac{GM_\odot}{r^2} \left( \frac{v \cdot e_R}{c} e_R + \frac{v}{c} \right).
\]

Perturbation equations of celestial mechanics yield for osculating orbital elements ($a_\beta$ – semimajor axis; $e_\beta$ – eccentricity; $i_\beta$ – inclination (of the orbital plane to the reference frame); $\Omega_\beta$ – longitude of the ascending node; $\omega_\beta$ – argument of pericenter; $\Theta_\beta$ is the position angle of the particle on the orbit, when measured from the ascending node in the direction of the particle’s motion, $\Theta_\beta = \omega_\beta + f_\beta$):

\[
\begin{align*}
\frac{da_\beta}{dt} &= \frac{2 a_\beta}{1 - e_\beta^2} \sqrt{\frac{p_\beta}{\mu (1 - \beta)}} \left\{ F_{\beta R} e_\beta \sin f_\beta + F_{\beta T} (1 + e_\beta \cos f_\beta) \right\}, \\
\frac{de_\beta}{dt} &= \sqrt{\frac{p_\beta}{\mu (1 - \beta)}} \left\{ F_{\beta R} \sin f_\beta + F_{\beta T} \left[ \cos f_\beta + \frac{e_\beta + \cos f_\beta}{1 + e_\beta \cos f_\beta} \right] \right\}, \\
\frac{di_\beta}{dt} &= \sqrt{\frac{r}{\mu (1 - \beta) p_\beta}} F_{\beta N} \cos \Theta_\beta, \\
\frac{d\Omega_\beta}{dt} &= \sqrt{\frac{r}{\mu (1 - \beta) p_\beta}} F_{\beta N} \frac{\sin \Theta_\beta}{\sin i_\beta}, \\
\frac{d\omega_\beta}{dt} &= -\frac{1}{e_\beta} \sqrt{\frac{p_\beta}{\mu (1 - \beta)}} \left\{ F_{\beta R} \cos f_\beta - F_{\beta T} \frac{2 + e_\beta \cos f_\beta}{1 + e_\beta \cos f_\beta} \sin f_\beta \right\} \\
&- \sqrt{\frac{r}{\mu (1 - \beta) p_\beta}} F_{\beta N} \frac{\sin \Theta_\beta}{\sin i_\beta} \cos i_\beta, \\
\frac{d\Theta_\beta}{dt} &= \frac{\sqrt{\mu (1 - \beta) p_\beta}}{r^2} - \frac{r}{\sqrt{\mu (1 - \beta) p_\beta}} F_{\beta N} \frac{\sin \Theta_\beta}{\sin i_\beta} \cos i_\beta,
\end{align*}
\]

where $\mu \equiv GM_\odot$ and $r = p_\beta/(1 + e_\beta \cos f_\beta)$; $F_{\beta R}$, $F_{\beta T}$ and $F_{\beta N}$ are radial, transversal and normal components of perturbation acceleration.
3.1. P-R effect and perturbation equations of celestial mechanics

This subsection follows considerations presented in Klačka (2004 – Sec. 6.1), or, in Klačka (1992). On the basis of Eq. (3), we can immediately write for components of perturbation acceleration to Keplerian motion:

\[
\begin{align*}
(F_{\beta R})_{PR} &= -2 \beta \left( \frac{\mu}{r^2} \right) \frac{v_{\beta R}}{c}, \\
(F_{\beta T})_{PR} &= -\beta \left( \frac{\mu}{r^2} \right) \frac{v_{\beta T}}{c}, \\
(F_{\beta N})_{PR} &= 0,
\end{align*}
\]

where \(\mu \equiv GM_{\odot}, r = p_{\beta}/(1 + e_{\beta} \cos f_{\beta})\) and two-body problem yields

\[
\begin{align*}
v_{\beta R} &= \sqrt{\mu \left(1 - \beta\right)/p_{\beta} e_{\beta} \sin f_{\beta}}, \\
v_{\beta T} &= \sqrt{\mu \left(1 - \beta\right)/p_{\beta} (1 + e_{\beta} \cos f_{\beta})}.
\end{align*}
\]

Inserting Eqs. (5) – (6) into Eq. (4), one easily obtains

\[
\begin{align*}
\left( \frac{da_{\beta}}{dt} \right)_{PR} &= -\beta \left( \frac{\mu}{r^2} \right) \frac{2a_{\beta}}{c} \left( 1 + e_{\beta}^2 + 2e_{\beta} \cos f_{\beta} + e_{\beta}^2 \sin^2 f_{\beta} \right) / \left( 1 - e_{\beta}^2 \right), \\
\left( \frac{de_{\beta}}{dt} \right)_{PR} &= -\beta \left( \frac{\mu}{r^2} \right) \frac{1}{c} \left( 2e_{\beta} + e_{\beta} \sin^2 f_{\beta} + 2 \cos f_{\beta} \right), \\
\left( \frac{di_{\beta}}{dt} \right)_{PR} &= 0, \\
\left( \frac{d\Omega_{\beta}}{dt} \right)_{PR} &= 0, \\
\left( \frac{d\omega_{\beta}}{dt} \right)_{PR} &= -\beta \left( \frac{\mu}{r^2} \right) \frac{1}{c} \frac{1}{e_{\beta}} \left( 2 - e_{\beta} \cos f_{\beta} \right) \sin f_{\beta}, \\
\left( \frac{d\Theta_{\beta}}{dt} \right)_{PR} &= \sqrt{\mu \left(1 - \beta\right)p_{\beta}} / r^2.
\end{align*}
\]

3.2. Gravity of a planet and perturbation equations of celestial mechanics

As for gravitational acceleration generated by a planet \((F_{\beta})_G\) presented in Eq. (3), we can write (compare Eqs. (44) in Klačka 2004 for the case \(i_P = \Omega_P = \omega_P = 0\) and Eqs. (57) in Klačka 2004)

\[
\begin{align*}
r_P &= r_P \, e_{PR}, \quad r = r \, e_R, \\
e_{PR} &= (\cos \Theta_P, \sin \Theta_P, 0), \\
e_R &= (\cos \Theta_\beta, \sin \Theta_\beta, 0), \\
e_T &= (- \sin \Theta_\beta, \cos \Theta_\beta, 0), \\
e_{PR} &= \cos (\Theta_P - \Theta_\beta) \, e_R + \sin (\Theta_P - \Theta_\beta) \, e_T, \\
\Theta_P &= n_P(t - t_0) + \Theta_{P0}, \quad n_P = \sqrt{G (M_{\odot} + m_P)} \, r_P^{-3/2}. \quad (8)
\end{align*}
\]
Inserting Eqs. (8) into \((F_\beta)_G\) presented in Eq. (3), we obtain
\[
(F_\beta)_G = Gm_P \left( X_I - X_{II} \right),
\]
\[
X_I = \left[ r_P \cos (\Theta_P - \Theta_\beta) - r \right] e_R + r_P \sin (\Theta_P - \Theta_\beta) e_T,
\]
\[
X_{II} = \frac{1}{r_P} \left[ \cos (\Theta_P - \Theta_\beta) e_R + \sin (\Theta_P - \Theta_\beta) e_T \right].
\]
In order to obtain \((da_\beta/dt)_G\), \((dc_\beta/dt)_G\), \(\ldots\), \((d\Theta_\beta/dt)_G\), it is sufficient to put Eqs. (9) into Eqs. (4).

4. Exterior mean motion resonances and \(da_\beta/dt\)

We will consider only terms proportional to \(e_\beta^0\) and \(e_\beta^1\), i.e., we will neglect \(e_\beta^2\) and higher orders in \(e_\beta\). First of Eqs. (4) can be written in the form
\[
\frac{da_\beta}{dt} = 2 \frac{a_\beta}{\mu P} \sqrt{\frac{a_\beta}{(1 - \beta)}} \left\{ F_{\beta R} e_\beta \sin f_\beta + F_{\beta T} (1 + e_\beta \cos f_\beta) \right\}. 
\]
(10)

On the basis of Eqs. (9) we write, then \((\mu P = Gm_P)\):
\[
(F_\beta R)_G = \mu P \left\{ \frac{r_P \cos (\Theta_P - \Theta_\beta) - a_\beta}{X(0)} - \frac{\cos (\Theta_P - \Theta_\beta)}{r_P^2} \right\},
\]
\[
(F_\beta T)_G = \mu P \left\{ \frac{r_P \sin (\Theta_P - \Theta_\beta)}{X(e_\beta)} - \frac{\sin (\Theta_P - \Theta_\beta)}{r_P^2} \right\},
\]
\[
X(e_\beta) = \left[ r_P^2 + a_\beta^2 (1 - 2e_\beta \cos f_\beta) - 2a_\beta r_P (1 - e_\beta \cos f_\beta) \cos (\Theta_P - \Theta_\beta) \right]^{3/2},
\]
(11)

for radial and transversal components of gravitational perturbation acceleration.

A particle is in exterior mean motion resonance with a planet when the ratio of their mean motions is approximately the ratio of two small natural numbers: \(n_\beta/n_P = j/(j + q)\) corresponds to \(q\)-order exterior resonance. For the first-order resonances we have \(n_\beta/n_P = j/(j + 1)\), i.e., any first-order resonance may be defined by the ratio \(T/T_p = (j + 1)/j\).

As for commensurability resonances with a planet moving in the circular orbit, the third Kepler’s law \([a_\beta^3 n_\beta^2 = \mu(1 - \beta), a_\beta^3 n_P^2 = \mu(1 + m_P/M_\odot)]\) yields
\[
a_\beta a_P = (1 - \beta)^{1/3} \left( \frac{T}{T_p} \right)^{2/3} \left( 1 + \frac{m_P}{M_\odot} \right)^{-1/3},
\]
(12)
for the semimajor axis and period of revolution around the Sun of the particle and the planet (subscript \(P\)). Defining any resonance by the ratio \(T/T_p\), Eq. (12) yields immediately the ratio \(a_\beta/a_P\).

On the basis of Eqs. (11) we can write for the important resonant terms
\[
RES = \left\{ (F_\beta R)_G e_\beta \sin f_\beta + (F_\beta T)_G (1 + e_\beta \cos f_\beta) \right\},
\]
\[
RES = \mu P \left\{ \frac{r_P}{X(0)} e_\beta \sin (\Theta_P - \omega_\beta) - \frac{a_\beta}{X(0)} e_\beta \sin f_\beta + \frac{r_P}{X(e_\beta)} \sin (\Theta_P - \Theta_\beta) \right\}. 
\]
(13)
Using the relation

$$X(e^\beta) = X(0) \left\{ 1 - 3 e^\beta \cos f^\beta \frac{1 - \varepsilon \cos (\Theta_P - \Theta_\beta)}{1 - 2 \varepsilon \cos (\Theta_P - \Theta_\beta) + \varepsilon^2} \right\} ,$$

$$\varepsilon \equiv r_P/a_\beta ,$$

(14)

We can Eq. (13) rewrite to the following form

$$RES = \frac{\mu_P e^\beta}{a_\beta^2 \left( 1 - 2 \varepsilon \cos (\Theta_P - \Theta_\beta) + \varepsilon^2 \right)^{5/2}} \left\{ -\sin (\Theta_\beta - \omega_\beta) ight. \\
+ \varepsilon \left[ \frac{7}{2} \sin (\Theta_P - \omega_\beta) - \frac{1}{2} \sin (2\Theta_\beta - \Theta_P - \omega_\beta) \\
+ \varepsilon^2 \left[ \frac{7}{4} \sin (\Theta_\beta - 2\Theta_P + \omega_\beta) + \frac{3}{4} \sin (3\Theta_\beta - 2\Theta_P - \omega_\beta) - 2 \sin (\Theta_\beta - \omega_\beta) \right] \\
+ \varepsilon^3 \sin (\Theta_P - \omega_\beta) \right\} .$$

(15)

In this expression we neglect terms proportional to \(\sin(\Theta_\beta - \Theta_P)\), because they have not large influence when we take into account terms important in first-order mean motion resonance. We use Laplace coefficients \(b_j\) defined by the relation

$$(1 - 2 \varepsilon \cos(\Theta_\beta - \Theta_P) + \varepsilon^2)^{-5/2} = \frac{1}{2} \sum_{\pm \infty} b_j \cos j(\Theta_\beta - \Theta_P).$$

(16)

From Eq. (15) using Eq. (16) we get for the first-order exterior resonance

$$RES = \frac{\mu_P e^\beta}{a_\beta^2} \left[ \frac{1}{2} b_j + \varepsilon \left( \frac{7}{4} b_{j+1} - \frac{1}{4} b_{j-1} \right) + \varepsilon^2 \left( -b_j - \frac{7}{8} b_{j+2} + \frac{3}{8} b_{j-2} \right) \\
+ \varepsilon^3 \frac{7}{2} b_{j+1} \right] \sin[(j + 1)\Theta_\beta - j\Theta_P - \omega_\beta] ,$$

(17)

where we have taken into account only terms with sine argument \((j + 1)\Theta_\beta - j\Theta_P - \omega_\beta = \Gamma\). We define \(C_j\) as

$$C_j \equiv - \frac{1}{j + 1} \left[ -\frac{1}{2} b_j + \varepsilon \left( \frac{7}{4} b_{j+1} - \frac{1}{4} b_{j-1} \right) + \varepsilon^2 \left( -b_j - \frac{7}{8} b_{j+2} + \frac{3}{8} b_{j-2} \right) \\
+ \varepsilon^3 \frac{7}{2} b_{j+1} \right] .$$

(18)

Using Eqs. (10), (13), (17) and (18) we obtain

$$\left( \frac{da_\beta}{dt} \right)_G = - (j + 1) a_\beta e^\beta n_\beta \frac{\mu_P}{\mu (1 - \beta)} C_j \sin \Gamma .$$

(19)

For the first-order exterior mean motion resonance Eq. (12) reduces to

$$\frac{a_\beta}{a_P} = (1 - \beta)^{1/3} \left( \frac{j + 1}{j} \right)^{2/3} \left( 1 + \frac{m_P}{M_\odot} \right)^{-1/3} .$$

(20)
Table 1. Values of $C_j$ for particle with $\beta = 0.01$ and $\beta = 0.05$ in first-order exterior mean motion resonance with the Earth in a circular orbit.

| $j$ | $C_j(\beta = 0.01)$ | $C_j(\beta = 0.05)$ |
|-----|---------------------|---------------------|
| 1   | 1.31819             | 1.35328             |
| 2   | 1.68270             | 1.77804             |
| 3   | 2.05585             | 2.23845             |
| 4   | 2.43605             | 2.73560             |
| 5   | 2.82257             | 3.27183             |
| 6   | 3.21514             | 3.85027             |
| 7   | 3.61366             | 4.47467             |
| 8   | 4.01813             | 5.14933             |
| 9   | 4.42856             | 5.87920             |
| 10  | 4.84502             | 6.66992             |
| 11  | 5.26757             | 7.52797             |
| 12  | 5.69630             | 8.46078             |
| 13  | 6.13128             | 9.47693             |
| 14  | 6.57262             | 10.58637            |

The values of $C_j$ are given in Tab. 1 for particles with $\beta \in \{0.01, 0.05\}$ in the first-order exterior resonance $j \in \{1, 2, 3, ..., 14\}$ with planet Earth in circular orbit.

Neglecting higher orders of eccentricity $e_\beta$, the first of Eqs. (7) yields

$$\left( \frac{da_\beta}{dt} \right)_P = \frac{-2 \beta \mu}{a_\beta c} (1 + 4e_\beta \cos f_\beta) . \quad (21)$$

Eqs. (3) and (4) give

$$\frac{da_\beta}{dt} = \left( \frac{da_\beta}{dt} \right)_G + \left( \frac{da_\beta}{dt} \right)_P . \quad (22)$$

Exterior resonances can produce long-lived trapping: $da_\beta/dt = 0$. On the basis of Eqs. (19), (21) and (22) we can write

$$- (j + 1) a_\beta e_\beta n_\beta \frac{\mu P}{\mu (1 - \beta)} C_j \sin \Gamma = 2 \beta \frac{\mu}{a_\beta c} (1 + 4e_\beta \cos f_\beta) . \quad (23)$$

Relation $n_\beta = n_P j/(j + 1)$ and the last one of Eqs. (8) yield $n_\beta = \sqrt{\mu} (1 + m_P/M_\odot)^{1/2} a_p^{-3/2} j/(j + 1)$. Using this result and also Eq. (20) in Eq. (23),

$$e_\beta = \frac{4 (-\cos f_\beta)}{2} \frac{1}{\mu} \frac{c}{\sqrt{\mu/a_P}} \frac{(1 + m_P/M_\odot)^{-1/6} (j + 1)^{4/3}}{\beta (1 - \beta)^{1/3} j^{1/3} C_j (-\sin \Gamma)} . \quad (24)$$
Eq. (24) immediately yields criterion for resonant trapping:

\[ e_{\beta \text{ min}} = \left\{ 4 + \frac{1}{2} \frac{\mu_P}{\mu} \frac{e}{\sqrt{\mu/a_P}} \frac{(1 + m_P/M_\oplus)^{-1/6}}{\beta (1 - \beta)^{1/3}} \frac{(j + 1)^{4/3}}{j^{1/3}} C_j \right\}^{-1} \] \tag{25}

5. Comparison of analytical and numerical results

We have numerically solved Eq. (1) for various values of \( \beta \) and for various initial conditions. The most important results are presented in Tables 2 and 3 for the case of the first-order exterior resonances with the Earth. Numerical results are compared with analytical results obtained from the theory presented in Secs. 2-4 and, also, with the analytical results presented by Weidenschilling and Jackson (1993). Our theoretical values of semimajor axis for the exterior resonance \((j + 1)/j\) are obtained from Eq. (20). The values are given in columns labeled as \( a_{\beta} \). The values \( a_{\beta \text{ min}} \) and \( a_{\beta \text{ max}} \) are minimal and maximal semimajor axes during the capture in a given resonance. The width of the resonances in semimajor axis can be calculated from the relation \( a_{\beta \text{ max}} - a_{\beta \text{ min}} \). Minimal values of eccentricities for the capture into resonances, found by numerical solution of Eq. (1), are given in the column \( e_{\beta \text{ min num}} \). Minimal capture eccentricities \( e_{\beta \text{ min th. W J}} \), presented in Tab. 2, are taken from Table II in Weidenschilling and Jackson (1993). Maximal capture times for different first-order resonances are also given. The times and the minimum and maximum semimajor axis values are obtained for a given numerical integration of a meteoroid. The extremal values of semimajor axis occur when the maximal capture time exists, for a given resonance. The minimum eccentricity values were found independently. There is no relation between the minimum eccentricity and the set \{maximal capture times, minimum and maximum semimajor axis values\}, as for the presentations in Tables 2 and 3.

General trend seen from Tables 2 and 3 is that the larger value of \( j \) – type of the first-order exterior resonances with the Earth is \((j + 1)/j\), the smaller width of resonances in semimajor axes and the shorter capture times. Moreover, the greater value of \( \beta \), i) the smaller resonant width in semimajor axes, and, ii) the shorter capture time.

6. Extrapolation of capture times

We found, from numerical integrations of Eq. (1), that maximal capture time for a particle in a given mean motion resonance is proportional to the value of the particle’s \( \beta \) and semimajor axis \( a_{\beta} \) according to

\[ \tau \sim \frac{a_{\beta}^2}{\beta} \] \tag{26}
Table 2. Characteristics for the first-order \((j+1)/j\) resonances with Earth for particle with \(\beta = 0.01\).

| \(j\) | \(a_\beta\) | \(a_\beta_{\text{min}}\) | \(a_\beta_{\text{max}}\) | \(\epsilon_\beta_{\text{min}}\) | \(\epsilon_\beta_{\text{min}}\) | \(\epsilon_\beta_{\text{min}}\) | maximal capture time |
|-------|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-------------------|
|       | [AU]     | [AU]            | [AU]            | th.              | th.              | th. W J         | [10^3 years]     |
| 1     | 1.5821   | 1.5761          | 1.5881          | 0.102            | 0.1107          | 0.3506          | 1492             |
| 2     | 1.306    | 1.3023          | 1.3097          | 0.036            | 0.0784          | 0.0387          | 736              |
| 3     | 1.2074   | 1.2046          | 1.2101          | 0.011            | 0.0564          | 0.0227          | 742              |
| 4     | 1.1565   | 1.1544          | 1.1586          | 0.008            | 0.0418          | 0.0148          | 429              |
| 5     | 1.1255   | 1.1238          | 1.1272          | 0                | 0.032           | 0.0104          | 268              |
| 6     | 1.1045   | 1.1031          | 1.1059          | 0.003            | 0.025           | 0.0077          | 252              |
| 7     | 1.0894   | 1.0881          | 1.0907          | 0.001            | 0.0201          | 0.0059          | 227              |
| 8     | 1.0781   | 1.077           | 1.079           | 0                | 0.0164          | 0.0046          | 362              |
| 9     | 1.0692   | 1.0681          | 1.0701          | 0.001            | 0.0136          | 0.0038          | 167              |
| 10    | 1.062    | 1.061           | 1.0626          | 0                | 0.0114          | 0.0031          | 469              |
| 11    | 1.0562   | 1.0551          | 1.0566          | 0.002            | 0.0097          | 0.0026          | 75               |
| 12    | 1.0513   | 1.0503          | 1.0516          | 0.001            | 0.0084          | 0.0022          | 36               |
| 13    | 1.0471   | 1.0459          | 1.0479          | 0.001            | 0.0073          | 0.0019          | 21               |
| 14    | 1.0436   | 1.0423          | 1.0444          | 0.001            | 0.0064          | 0.0016          | 13               |

This relation is also in accordance with the expected dependence of secular time derivation of eccentricity (Liou and Zook 1997; Klačka et al. 2008 – \(\eta \equiv 0\)) on \(a_P\) and \(\beta\). Using Eq. (12) (with assumption \(m_P \ll M_\odot\)) we obtain from Eq. (26)

\[
\frac{\tau(a_{P1}, \beta_1)}{\tau(a_{P2}, \beta_2)} = \left(\frac{a_{P1}}{a_{P2}}\right)^2 \frac{\beta_2 (1 - \beta_1)^{2/3}}{\beta_1 (1 - \beta_2)^{2/3}}.
\]  

Eq. (27) presents the analytical formula for obtaining maximum capture time \(\tau(a_{P2}, \beta_2)\) for the particle of \(\beta_2\) and planetary orbital radius \(a_{P2}\) for a given type of resonance, if maximum capture time \(\tau(a_{P1}, \beta_1)\) is known for the particle of \(\beta_1\) and planetary orbital radius \(a_{P1}\) for the same type of resonance.

Using Eq. (27) we have calculated the values of capture times in Tab. 4 from values of capture times in Tables 2 and 3. We use the values in Tab. 2 in calculations of the expected values in Tab. 3, and vice versa.
Table 3. Characteristics for the first-order \((j+1)/j\) resonances with Earth for particle with \(\beta = 0.05\).

| \(j\) | \(a_\beta\) | \(a_{\beta \, \text{min}}\) | \(a_{\beta \, \text{max}}\) | \(e_{\beta \, \text{min}}^{\text{num.}}\) | \(e_{\beta \, \text{min}}^{\text{th.}}\) | \(\text{maximal capture time}\) |
|-------|-------------|----------------|----------------|----------------|----------------|----------------|
|       | [ AU ]      | [ AU ]         | [ AU ]         | [ 10^3 years ] |               |                |
| 1     | 1.5605      | 1.5555         | 1.5656         | 0.196          | 0.1981         | 216            |
| 2     | 1.2882      | 1.2851         | 1.2911         | 0.096          | 0.1701         | 168            |
| 3     | 1.1909      | 1.1887         | 1.1931         | 0.061          | 0.1422         | 187            |
| 4     | 1.1407      | 1.1389         | 1.1426         | 0.045          | 0.1172         | 80             |
| 5     | 1.1101      | 1.1088         | 1.1114         | 0.028          | 0.096          | 56             |
| 6     | 1.0894      | 1.0879         | 1.091          | 0.018          | 0.0786         | 102            |
| 7     | 1.0746      | 1.0735         | 1.0756         | 0.016          | 0.0645         | 62             |
| 8     | 1.0633      | 1.0626         | 1.0643         | 0.01           | 0.0531         | 23             |
| 9     | 1.0546      | 1.0538         | 1.0552         | 0.007          | 0.044          | 20             |
| 10    | 1.0475      | 1.0466         | 1.0481         | 0.005          | 0.0366         | 13             |
| 11    | 1.0418      | 1.0408         | 1.0424         | 0.005          | 0.0307         | 6              |
| 12    | 1.0369      | 1.036          | 1.0376         | 0.004          | 0.0258         | 3              |
| 13    | 1.0328      | 1.0317         | 1.0334         | 0.005          | 0.0218         | 5              |
| 14    | 1.0293      | 1.028          | 1.0301         | 0              | 0.0185         | 1              |

7. Conclusion

The planar circular restricted three-body problem with action of solar electromagnetic radiation on a spherical grain is considered. This Poynting-Robertson effect is studied numerically and analytically. We have found minimal values of capture eccentricities into the first-order exterior mean motion resonances. Unfortunately, our theory for the minimal values of the capture eccentricities cannot explain the found numerically values. The numerical values \(e_{\beta \, \text{min}}\) are also smaller than the values presented in Weidenschilling and Jackson (1993). Thus, any analytical method explaining minimal values of capture eccentricities does not exist, for the first-order exterior mean motion resonances.

Our numerical results offer several resonant characteristics for \(\beta\)–values 0.01 and 0.05 for the first-order exterior resonances with the Earth; capture times for \(\beta = 0.1\) are in order of magnitude shorter than for \(\beta = 0.05\). We have succeeded in finding greater values of capture times than it is presented in Šidlichovský and Nesvorný (1994). Eq. (27) presents an analytical formula for obtaining maximum capture time for the particle of \(\beta_2\) and planetary orbital radius \(a_{P_2}\) for a given type of resonance, if
Table 4. Extrapolation of capture times presented in Tables 2 and 3. Eq. (27) is used.

| $j$ | extrapolated values in Tab. 2 [10^3 years] | extrapolated values in Tab. 3 [10^3 years] |
|-----|---------------------------------|---------------------------------|
| 1   | 1110                            | 290                             |
| 2   | 863                             | 143                             |
| 3   | 961                             | 144                             |
| 4   | 411                             | 83                              |
| 5   | 288                             | 52                              |
| 6   | 524                             | 49                              |
| 7   | 319                             | 44                              |
| 8   | 118                             | 70                              |
| 9   | 103                             | 32                              |
| 10  | 67                              | 91                              |
| 11  | 31                              | 15                              |
| 12  | 15                              | 7                               |
| 13  | 26                              | 4                               |
| 14  | 5                               | 3                               |

maximum capture time is known for the particle of $\beta_1$ and planetary orbital radius $a_{P1}$ for the same type of resonance.

The presented values can be useful in comparison of resonant evolution for spherical and nonspherical meteoroids. Our results, based on the P-R effect, can be used for spherical particles, only. Real nonspherical particles may exhibit nonzero values for other two components of the radiation pressure efficiency factor vector $Q'_2$ and $Q'_3$ (Klačka 2004), and, moreover, values of $\beta$–parameter are time dependent. Thus, the change of semimajor axis $a_\beta$ in orbital resonance caused by the change of parameter $\beta$ may be relatively large. Since the resonant width is a decreasing function of $\beta$, one should await that resonant capture times for nonspherical particles are smaller than the capture times for spherical grains.

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