Strength and rigidity evaluation of corrugated web I-beams using finite element analysis

T L Dmitrieva and Kh Ulambayar
Irkutsk National Research Technical University, 664074, Irkutsk, Lermontov street, 83, Russia
dmitrievat@list.ru

Abstract. Finite element (FE) models of metal I-beams were analyzed to identify their strength and rigidity characteristics. Shell-type finite elements were used to model corrugated web I-beams. Results reliability was confirmed by solving FE analysis task, using LIRA and ANSYS software packages. Comparison with rod model calculations of the same beams was made and numerical and analytical solution achieved. Analysis was made using three models of corrugated web I-beams with different corrugation configurations and identical steel intensity. Strength and rigidity characteristics of I-beams were evaluated.

1. Introduction
Modern strength and rigidity requirements for metal structures, caused by need of material saving and cost reduction, can be fulfilled by using complex form metal constructions with different principles of their cross-section parameters’ change. However, the usage of such constructions leads to certain problems of their stress-strain state (SSS) analysis. Therefore, the most important stage in calculations is the choice of the structures’ design model and their fixing and loading condition. This choice is most often realized in the form of FEM. Usage of correct constructions finite element model (FEM) is important stage of complete structure’s computer modeling and in many respects determines the adequacy of obtained results.

Current task of metal construction mechanics is perfecting methods of calculation and design of rod elements with non-standard cross-sections. One of the most prospective construction solutions among those is the elements’ usage with corrugated metal inserts. Currently, this task is being intensively researched and is of interest to structural designers, as well as computer modeling specialists [1-8]. Economical expediency of corrugated elements is established in [9]. Same source notes main flaws in design of such constructions, which are mainly caused by lack of normative documents and special computation software. This article, however, would not be touching on design issues, concentrating on challenges of SSS evaluation.

Calculation of frames with corrugated element using rod FE model presumes knowledge of their geometric parameters. Tabulated values of these parameters are known for certain corrugation types – triangle and wave types (SIN beams). It can be noted, that the lack of assortments containing flexural and axial rigidity parameters of cross-sections for different corrugation profiles is one of the problems complicating their calculations.

The aims of this article are
- evaluation of spatial FE model usage adequacy with shell-type elements for calculation of corrugated beams;
- evaluation of rigidity values given in RDS RK 5.04-24-2006 «Mix of fabricated I-beam sections of regular type and with corrugated walls»;
- comparison of strength and rigidity parameters of corrugated web I-beams of various corrugation configurations with same flat web I-beam parameters.

2. Adequacy evaluation of finite element model with shell-type elements exemplified by flat web I-beam

In order to verify spatial FE model based on shell-type elements, a test calculation of flat web I-beam was made and further compared with analytical results.

2.1. Analytical calculation

Analytical calculation was made based on generally-known theoretical formulas of structural mechanics [10].

\[
\begin{align*}
    f_{\text{max}} &= \frac{5qL^4}{384EI} \\
    M_{\text{max}} &= \frac{qL^2}{8} \\
    Q_{\text{max}} &= \frac{qL}{2}
\end{align*}
\]  

\textbf{Figure 1.} Calculation scheme and stress diagrams.

2.2. Numerical calculation

Numerical calculations were made using two software “LIRA-SAPR” and “ANSYS 14.5”. Calculations results can be seen on Figures 2-4. Results are summaries in Table 1.
Figure 2. Finite element model Software “LIRA-SAPR”.

Figure 3. Finite element model Software “ANSYS 14.5”.

Figure 4. Contour plot of displacements along the Z-axis of calculation model: (a) Software “LIRA-SAPR”, (b) Software “ANSYS 14.5”.

**Table 1.** Comparative results of calculations.

| №  | Calculation methods                      | $f_{\text{max}}$,mm | $\sigma_{\text{max}}$,MPa | $\tau_{xy,\text{max}}$,MPa |
|----|------------------------------------------|----------------------|----------------------------|-----------------------------|
| 1  | Analytical solution                      | 8.2                  | 110                        | 41                          |
| 2  | Solution in «LIRA-SAPR» for element FE41 | 8.15(0.6%)           | 99.83(10.2%)               | 43(4.9%)                    |
| 3  | Solution in «ANSYS 14.5» for element SHELL181 | 8.27(0.8%)           | 104.13(5.6%)               | 39.7(3.3%)                  |

As it can be seen in Table 1, observational error for stress values is an order higher than for bending deflection values, which correlates with task-given geometrical and physical dependencies. Difference in numerical results is notable and drifts both higher and lower in comparison to analytically found numbers, which is caused by the character of finite elemental net.
3. Evaluation of rigidity parameters of I-beam 40/50BGS1.

Let’s compare results of rigidity calculations for flexural strain \((EI)\) and for axial tensile and compressive stresses \((EA)\) between analytical and numerical methods. Corrugated web I-beam 40/50BGS1 will be used as object of study.

Stiffness values for this beam type are given in RDS RK 5.04-24-2006 «Mix of fabricated I-beam sections of regular type and with corrugated walls meeting strength properties of rolling sections» [11]. This normative document was developed in Republic of Kazakhstan under the general editorship of Maksimov Y.S. and Ostrikov G.M. and was put in use in 2007.

### Table 2. Assortment of BGS-series corrugated web plate girders.

| Cross section | Cross-section specifications, cm | Corrugation specification, \(L_c,\text{cm}\) | Corrugation specification, \(t_n,\text{cm}\) | Area, \(cm^2\) | Mass per 1m, kg | \(I_x, cm^4\) | \(W_x, cm^3\) | \(S_x, cm^3\) | \(t_x, cm\) |
|---------------|---------------------------------|---------------------------------|---------------------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 40/50 BGS1    | 2 3 4 5 6 7 8 9 10             | 50 21 0.25 1 48 10            | 3 54                            | 42.8           | 25210.5        | 1029           | 514.5          | 24.5 |

**Figure 5.** Schematics of corrugated web I-beam.

3.1. Analytical calculation

**Figure 6.** Calculations schemes and the results of calculation.
3.2. Numerical calculation

![Figure 7](image)

**Figure 7.** Contour plot of displacements along the Z-axis of calculation model in Software «LIRA-SAPR».

![Figure 8](image)

**Figure 8.** Contour plot of displacements along the X-axis of calculation model in Software LIRA-SAPR».

**Table 3.** Comparative results of calculations.

| № | Calculation methods | \( f_{\text{max}} \), mm | \( \Delta \), mm |
|---|---------------------|-------------------------|-----------------|
| 1 | Analytical solution | 15.1                     | 1.5             |
| 2 | Numerical solution in software «LIRA-SAPR» for FE41, FE42 | 16.3 | 1.78 |
| 3 | Calculation error, \( \varepsilon \) | 8% | 18.7% |

4. Numerical study of SSS for corrugated web I-beams

Let’s perform a numerical analysis of three I-beams types with different corrugation shape, which are shown on Figure 9 (MODEL 1-3). Results of calculation for flat web I-beam, examined in point 1 (MODEL 4 on Figure 9), will be used as comparison. FE model for calculations of I-beam with trapezoidal corrugation (MODEL 1 on Figure 9) is shown on Figure 11.

Results of other calculations are summarized in table 4. Shell-type finite elements FE41, FE42 from software package «LIRA-SAPR» and FE “SHELL181” from software package «ANSYS 14.5» were chosen for modeling.

![Figure 9](image)

**Figure 9.** Isometric projection of calculation models.
Based on displacement values for each model, their following rigidity characteristics were identified: moment of inertia \( I \), moment of resistance \( W \) and cross-section area \( A \) (see table 4).

\[
I = \frac{5qL^4}{384Ef_{\text{max}}},
\]

\[
W = \frac{I}{H/2},
\]

\[
A = \frac{PL}{E\Delta},
\]

(4) (5) (6)

where \( \Delta \) is amount of X-axis displacement for beams (M1-M4), which was found for corrugated web I-beam by solving an axial compression problem by numerical method.

### Table 4. Research results for 4 types of finite element models.

| Models | Mass per 1m, kg | \( f_{\text{max}} \) mm | \( \sigma_{\text{max}} \) MPa | \( \tau_{\text{xy, max}} \) MPa | \( l \), m \(^4\) | \( W \), m \(^3\) | \( A \), m \(^2\) |
|--------|----------------|-----------------|----------------|----------------|-------------|-------------|-------------|
| M1     | 65             | 11.8            | 130            | 93             | 0.000322    | 0.00129     | 0.0078      |
| M2     | 67             | 12              | 127            | 90             | 0.000316    | 0.00127     | 0.0078      |
| M3     | 68             | 12.2            | 138            | 96             | 0.000311    | 0.00125     | 0.0078      |
| M4     | 66             | 9.9             | 114            | 69             | 0.000383    | 0.00153     | 0.0083      |

Software «LIRA-SAPR»

| M1     | 65             | 11.9            | 140            | 105            | 0.000322    | 0.00128     | 0.0078      |
| M2     | 67             | 12.1            | 133            | 86             | 0.000311    | 0.00124     | 0.0078      |
| M3     | 68             | 12.4            | 139            | 97             | 0.000322    | 0.00126     | 0.0078      |
| M4     | 66             | 10              | 119            | 69.4           | 0.00038     | 0.00152     | 0.0083      |

Software «ANSYS 14.5»
As it can be seen from table, rigidity parameters calculated by software packages «LIRA-SAPR» and «ANSYS 14.5» are almost identical.

5. Conclusions

Based on made calculations following results were received:

- Flexural rigidity of corrugated web I-beam is inferior to typical flat web I-beam (with average $I_{\text{corrugate}}/I_{\text{flat}}$ ratio of 0.84);
- Axial compression rigidity of corrugated web I-beam is inferior to flat web I-beam (with average $A_{\text{corrugate}}/A_{\text{flat}}$ ratio of 0.94);
- Average calculated values of normal vector stresses are $\sigma_{\text{corrugate}}^{\text{max}} / \sigma_{\text{flat}}^{\text{max}} 1.16$;
- Average calculated values of tangent vector stresses are $\tau_{\text{corrugate}}^{\text{max}} / \tau_{\text{flat}}^{\text{max}} 1.35$.

Thus, it follows that with their weight being equal, corrugated web I-beam strength and rigidity parameters are inferior to that of a flat web I-beam. However, this articles research requires several additions. For example, these calculations neither take into account general and local stability tests, nor investigate the effect of torsion deformations. Additionally, more optimized corrugation configurations could exist which would result in better rigidity and strength parameters. Research of such configurations is this article authors’ further task.

References

[1] Lukin A O, Alpatov V Yu and Chernyshev D D 2016 Vestnik SGASU. Town Planning and Architecture 2 23 4–9
[2] Martynyuk A Ya, Nilov A A and Laznyuk M V 2008 Metal Constructions 4 14 237–44
[3] Ahmed S Elamary, Amr B Saddek and Mamdooh Alwetaishi 2017 In. J. of Applied Engineering Research 4 12 470–81
[4] Fatimah De’nan, Nor Salwani Hashim 2012 Int. J. Energy Engineering 1 2 1–4
[5] Mr Akshay M Bankar, Pallavi K Pasnur 2017 Int. J. Sci. Technol. Eng. 3 4 1–3
[6] R Rajkumar, R Aravindh, B Gokula Krishnan and B Mukul Anand 2017 Advances in Computational Sciences and Technology 9 10 2745–64
[7] Revathi N, Satheshkumar G K and Dr Arunkumar G Int. J. Res. Innovation Eng. Technol 12 2 48–53
[8] Wang Z Y, Wang Q Y and Jiang R J 2015 Int. J. Civil Engineering 4 13 419–31
[9] Bryancev A A, Absimetov V E and Lalin V V 2017 Construction of Unique Buildings and Structures 3 54 93–104
[10] Ikrin V A 2004 Mechanics of Materials with Elements of Theory of Linear Elasticity (Moscow: ASV Press) p 424
[11] Maksimov Yu S and Ostrikov G M 2007 Mix of Fabricated I-Beam Sections of Regular Type and With Corrugated Walls Meeting Strength Properties of Rolling Sections, RDS RK 5.04-24-2006 (Astana: KAZGOR Press) p 22