Is there a Higgs? Where is it? Is supersymmetry there? Where is it? By discussing these questions, we call attention to the ‘LEP paradox’, which is how we see the naturalness problem of the Fermi scale after a decade of electroweak precision measurements, mostly done at LEP.

Is it wise to spend time in reviewing a subject, which can be summarized in one sentence: neither the Higgs nor supersymmetry have been found so far? Admittedly the question makes sense. For sure these topics are crucial to the central problem of particle physics: the ElectroWeak symmetry breaking. Our main motivation here is, however, more specific. We want to bring the focus on what we like to call the ‘LEP paradox’. By this we mean the way several years of (mostly) LEP results [1] make us see the old and well known naturalness problem of the Fermi scale.

The questions we address, in logical order, are:

1. Is there a Higgs?
2. If yes, where is it?
3. Is there supersymmetry?
4. If yes, where is it?

All of these questions, as well as their answers, have to do with the ‘LEP paradox’ that was just mentioned.

1 Is there a Higgs?

Any decent theory of the EW interactions must contain the Goldstone bosons, two charged and one neutral, that provide the longitudinal degrees of freedom for the W and Z bosons. On top of them, the Standard Model has a neutral Higgs boson. Without the Higgs and without specifying what replaces it, one deals with a gauge Lagrangian with SU(2)_L ⊗ U(1) non-linearly realized in the Goldstone boson sector. This is in formal analogy with the chiral SU(3)_L ⊗ SU(3)_R of strong interactions in the pseudoscalar octet sector.

The predictive power of such a non-linear Lagrangian is reduced with respect to the SM. In practice, at present, the comparison can be made by considering 2 “(g − 2)-like” quantities, $\epsilon_1$ and $\epsilon_3$ [2], which include the EW radiative correction effects more sensitive to the Higgs sector. The experimentally determined $\epsilon_1$ and $\epsilon_3$ [1], mostly by $\Gamma_Z$, $M_W/M_Z$ and $\sin^2\theta_W$, are shown in fig. 1 and compared with the SM prediction. All the radiative corrections not included in $\epsilon_1$ and $\epsilon_3$, less sensitive to the Higgs mass, are fixed at their SM values. The agreement with the SM for a Higgs mass below the triviality bound of about 600 GeV is remarkable and constitutes indirect evidence for the existence of the Higgs boson.

With a non linear Lagrangian, neither $\epsilon_1$ and $\epsilon_3$ can be computed. Some believe, however, that suitable models of EW symmetry breaking may exist where both $\epsilon_1$ and $\epsilon_3$ deviate from the SM values for $m_h = (100 \div 200)$ GeV by less than $(1 \div 2)10^{-3}$, having therefore a chance of also reproducing the data without an explicit Higgs boson in the spectrum [3]. In the case where a reliable estimate can be made, technicolour models with QCD-like dynamics, this is known not to happen [4].

2 Where is the Higgs?

If one accepts the existence of a Higgs boson, the SM Lagrangian $L_{SM}$ becomes an unavoidable effective low energy approximation of any sensible theory. A deviation from it could occur for the need of describing new degrees of freedom with mass comparable or lower than the Fermi scale. Barring this possibility, the predictions of the SM — hence the indirect determination of $m_h$ from the EW Precision Tests — could only be al-
tered by the presence of operators $O_i^{(4+p)}$ of dimension $4 + p \geq 5$ weighted by inverse powers of a cut-off scale, $\Lambda$, associated with some kind of new physics. The lower limits that the same EWPT set on the operators and with $\Lambda$ parameters. We take one operator at a time with the dimensionless coefficients $c_i = +1$ or $c_i = -1$ and different values of the Higgs mass. The blanks in the columns with $m_h = 300$ or 800 GeV are there because no fit is possible, at 95% C.L., for whatever value of $\Lambda$. A fit is possible, however, for $m_h = (300 \div 500)$ GeV with suitable operators and with $\Lambda$ in a defined range \cite{10}, as shown in fig. 2.

For this reason one is cautious about saying that the Higgs is between 100 and 200 GeV, as obtained in a pure SM fit with $\Lambda = \infty$. To fake a light Higgs, however, a coincidence is needed. From table 1, a more likely conclusion seems that $\Lambda$ is indeed bigger than about 5 TeV and the Higgs is light.

In spite of this, an interesting question is the following \cite{4}: suppose that a Higgs heavier than 250 GeV appears light in the EWPT because of a suitable operator in the above list. Would any direct effect of such operator be visible in high energy collisions? Not possible, we think, at the Tevatron and unlikely at the LHC, where it would be definitively easier to discover the Higgs itself.

3 Is supersymmetry there?

The naturalness problem of the Fermi scale, caused by the quadratic divergences in the Higgs mass, is with us since more than 20 years. We think that a Higgs mass in the $(100 \div 200)$ GeV range and, especially, a lower bound on the scale of new physics of about 5 TeV turn the naturalness problem of the Fermi scale into a clear paradox. The loop with a top of 170 GeV gives a contribution to the Higgs mass

$$\delta m_h^2(\text{top}) = \frac{3}{\sqrt{2} \pi^2} G_F m_t^2 k_{\text{max}}^2 = (0.3 k_{\text{max}})^2$$  \(1\)

where $k_{\text{max}}$ is the maximum momentum of the virtual top. The paradox arises if one thinks that 5 TeV is also a lower bound on $k_{\text{max}}$, since in this case $\delta m_h^2(\text{top})$ would exceed $(1.5 \text{ TeV})^2$, about 100 times the indirect value of $m_h^2$. We like to call this the “LEP paradox”, for obvious reasons.

Supersymmetry offers a neat solution to this paradox. A stop loop counteracts the top loop contribution to the Higgs mass, turning $k_{\text{max}}^2$ of eq. (1) into

$$k_{\text{max}}^2 \rightarrow m_t^2 \ln \frac{k_{\text{max}}^2}{m_t^2}$$  \(2\)

In this way a stop mass $m_t$ in the Fermi-scale range keeps the top-stop contribution to $m_h$ under control, while not undoing the success of the SM in passing the EWPT. This is a non trivial constraint for any possible solution of the LEP paradox. The success of supersymmetric grand unification adds significant support to this view \cite{5}.

The contrary arguments to the supersymmetric solution of the LEP paradox are of general character. One argument is that power divergences in field theory are not significant. This looks problematic to us: the top loop is there and something must be done with it. A quadratic divergence explains the $\pi^\pm/\pi^0$ mass splitting $m_{\pi^\pm} - m_{\pi^0} \sim (\alpha_{em}/4\pi)\Lambda_{QCD}^2$ \cite{8}. More relevant may be the observation that the cosmological constant poses another serious unsolved problem, also related to power divergences.

Alternative physical pictures are proposed for solving the hierarchy problem (top-colour \cite{9}, extra dimensions without supersymmetry \cite{10}, ...). As far as we know, they all share a common problem: the lack of calculative techniques and/or of suitable conceptual developments do not allow to address the LEP paradox. Maybe the fundamental scale of these theories is low and the agreement of the EWPT with the SM and a high cut-off is accidental. Alternatively, the separation between the Higgs mass and the scale of these theories may be considerable. In this last case, unfortunately, the related experimental signatures may become elusive.
Table 1: 95% lower bounds on $\Delta\chi^2$ for the individual operators and different values of $m_h$. $\chi^2_{\text{min}}$ is the one in the SM for $m_h > 115$ GeV.

| Operators | $m_h = 115$ GeV | $m_h = 300$ GeV | $m_h = 800$ GeV |
|-----------|-----------------|-----------------|-----------------|
| $O_{WB}$  | $\frac{(H^1\tau^a H)W_{\mu\nu}B_{\mu\nu}}{\tau^a}$ | 9.7  | 7.5  | 7.5  |
| $O_H$     | $|H^1D_H|^2$    | 4.6  | 3.4  | 2.8  |
| $O_{LL}$  | $\frac{(L\gamma^a L)}{\tau^a}$ | 7.9  | 6.1  | 6.1  |
| $O_{HL}$  | $i(H^1 D_{\mu} H)(L\gamma^a L)$ | 8.4  | 8.8  | 8.8  |
| $O'_{HQ}$ | $i(H^1 D_{\mu} H)(\gamma^a Q)$ | 6.6  | 6.8  | 6.8  |
| $O_{HE}$  | $i(H^1 D_{\mu} H)(E\gamma^a E)$ | 7.3  | 9.2  | 9.2  |
| $O_{HU}$  | $i(H^1 D_{\mu} H)(U\gamma^a U)$ | 5.8  | 3.4  | 3.4  |
| $O_{HD}$  | $i(H^1 D_{\mu} H)(D\gamma^a D)$ | 2.4  | 3.3  | 3.3  |

Note how this simply arises by the replacement (2) into (1) and the identification of $k_{\text{max}}$ with $Q$, the RGE scale at which $m_h$ vanishes. In specific models $Q$ is a function of the various parameters.

As well known, $m_h^2$ can also be computed from the quartic coupling of the Higgs potential. Including the one loop large top corrections, one has (\tan\beta \gtrsim 4)

$$m_h^2 \approx \frac{3}{\sqrt{2}\pi^2} G_F m_t^2 m_t^2 \ln \frac{Q^2}{m_t^2}.$$  \hspace{1cm} (3)

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Eqs (3) and (4) may be viewed as a relation between $Q$ and $m_t$, graphically represented in fig. 3.

As mentioned $Q$ is a model dependent function of the various parameters, ranging from the weak scale to the Planck scale. A random choice of the original parameters leads most often to a point on the prolongation of the left branch of the curve in fig. 3, where $\ln(Q/m_t) \gg 1$. However, given the correlation between stop masses and the other sparticle masses expected in explicit models, experiments have excluded this region, requiring that $Q \sim m_t$.

'Where is supersymmetry?' depends on the interpretation of this fact. If it is due to an accidental fine-tuning, it is no longer unlikely to have sparticles above a TeV due to a slightly more improbable accident. At the same time the explanation of the LEP paradox becomes cloudy.

If instead $Q \sim m_t$ is not accidental, it is important to notice that experiments do not yet require that we live on the right branch in fig. 3, with $Q$ very close to $m_t$. If,
for some reason, $Q \sim m_t$, so that sparticle masses are related to the weak scale by a one loop relation (rather than by the usual tree level relation), sparticles should be around the corner. We have recently conjectured that suitable models may exist where $Q$ is predicted to be close to the minimum in fig. 3, where $m_\tilde{t} \sim 400$ GeV \cite{11}. In fig. 4 we show a sampling of the spectra expected in these models, if we also assume minimal supergravity relations between soft terms.

5 Conclusion

A straight interpretation of the results of the EWPT, mostly performed at LEP in the last decade, gives rise to an apparent paradox. The EWPT indicate both a light Higgs mass $m_h \approx (100 \div 200)$ GeV and a high cut-off, $\Lambda \gtrsim 5$ TeV, with the consequence of a top loop correction to $m_h$ largely exceeding the preferred value of $m_h$ itself. The well known naturalness problem of the Fermi scale has gained a pure ‘low energy’ aspect. At present, supersymmetry at the Fermi scale is the only way we know of to attach this problem.

This way of looking at the data may be too naive. As we said, in EWPT the SM with a light Higgs and a large cut-off can at least be faked by a fortuitous cancellation. In any case the point is not to replace direct searches for supersymmetry or for any other kind of new physics. Rather, we wonder if a better theoretical focus on the LEP paradox might be not without useful consequences. Its solution, we think, is bound to give us some surprise, in a way or another.

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