ABSTRACT

We develop a new method of modeling microlensing events based on a Monte Carlo simulation that incorporates both a Galactic model and the constraints imposed by the observed characteristics of the event. The method provides an unbiased way to analyze the event especially when parameters are poorly constrained by the observed lightcurve. We apply this method to search for planetary companions of the lens in OGLE-2003-BLG-423, whose maximum magnification $A_{\text{max}} = 256 \pm 43$ (or $A_{\text{max}} = 400 \pm 115$ from the lightcurve data alone) is the highest among single-lens events ever recorded. The method permits us, for the first time, to place constraints directly in the planet-mass/projected-physical-separation plane rather than in the mass-ratio/Einstein-radius plane as was done previously. For example, Jupiter-mass companions of main-sequence stars at 2.5 AU are excluded with 80% efficiency.

Subject headings: Galaxy: bulge — gravitational lensing — planetary systems — stars: low-mass, brown dwarfs

1. INTRODUCTION

High-magnification microlensing events are exceptionally sensitive to the presence of planetary companions to the lens. As the projected separation of the source and the (parent-star) lens decreases, the size of the images increases, thus enhancing the probability that the planet will pass close enough to the lens to generate a noticeable deviation in the lightcurve (Gould & Loeb 1993). And if the source gets sufficiently close to the lens, the lightcurve can be perturbed by the central caustic associated with the parent star itself (Griest & Safzadegh 1998). Groups that monitor microlensing events to search for planets are well aware of this enhanced sensitivity and so devote special attention to these events.

It is therefore somewhat surprising that the vast majority of events monitored by these groups in the past have not been particularly high-magnification, and the vast majority of the observations of the few that did reach high magnification were actually performed well away from the peak, when the event had far less sensitivity to planets. In particular, of the 43 events monitored by the PLANET (Probing Lensing Anomalies NETwork) collaboration (Albrow et al. 2001, Gaudi et al. 2002) over 5 years, most of the sensitivity to planets came from just 5 or 6 events, and most of that from the near-peak regions of these events. Gaudi & Han (2002) used the relatively sparse OGLE data to put limits on planetary systems, although Gaudi & Han (2004) have argued that planets could not be reliably detected from such data alone.

The main reason for this apparent discrepancy was simply a shortage of microlensing alerts. Hence, at any given time, there just were no high-magnification events in progress, or at least none near their peak. The available telescope time then had to be applied to less favorable events. In addition, when devising their observational strategy, PLANET considered that they would have to characterize the events they were monitoring entirely with their own data. Such characterization is absolutely essential to evaluating the sensitivity of each event to planets, and it requires a very large number of observations on the wings and at baseline when the event has very little sensitivity to planets.

With the commencement of the Optical Gravitational Lens Experiment’s OGLE-III project (Udalski et al. 2002), the situation is radically changed. Using its dedicated 1.3 m telescope, large-format ($35 \times 35$') camera, generally excellent seeing, and ambitious observing strategy, OGLE-III is alerting microlensing events toward the Galactic bulge at a rate of 500 per season. Since microlensing events are uniformly distributed in impact parameter $u_0$, and since peak magnification scales $A_{\text{max}} \sim u_0^{-1}$ (for $u_0 \ll 1$), this implies that there are dozens of events with $A_{\text{max}} \gtrsim 10$ each year, and a handful

*BASED IN PART ON OBSERVATIONS OBTAINED WITH THE 1.3 M WARSAW TELESCOPE AT THE LAS CAMANAS OBSERVATORY OF THE CARNEGIE INSTITUTION OF WASHINGTON.

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with $A_{\text{max}} \gtrsim 100$. Moreover, OGLE-III photometry is publicly available (literally hours after it is taken), so there is generally no need for microlensing followup groups to monitor the wings or baseline in order to characterize the event. That is, followup observing time can be concentrated on the highly sensitive peaks of the high-magnification events. In the 2003 season, such peaks were occurring almost continuously. OGLE-III is therefore generating substantial new opportunities for microlensing planet search groups such as PLANET (Albrow et al. 1998), the Microlensing Planet Search (MPS, Rhie et al. 1999, 2000), and the Microlensing Follow-Up Network (μFUN, Yoo et al. 2004).

However, the OGLE-III approach also generates substantial challenges. In order to monitor a very large area during the 2002 and 2003 seasons, OGLE-III returned to each field only of order 50 times over the roughly 9 month season. During the long nights around 21 June, when the bulge transits near midnight, the cadence was relatively high, once every two or three nights. But at the edges of bulge season, the rate of return dropped as low as once per week or less. Hence, some extremely high magnification events appeared quite ordinary as of their last observation before peak, and then could be recognized for what they were only very close to (or past) their peak. Indeed, it is quite possible that their true nature as high magnification events could not be recognized at all from the OGLE-III lightcurve alone. Thus, without additional work, the riches generated by OGLE-III could easily pass by unnoticed. Beginning in the 2004 season, OGLE adjusted its strategy to concentrate on a reduced number of fields that have relatively higher expected event rates. Hence, it is expected that the above-mentioned problems will be mitigated in future seasons.

Here we develop a new method of modeling microlensing events that incorporates both a Galactic model via a Monte Carlo simulation and the constraints imposed by the observed characteristics of the event. We apply this method to the extreme microlensing event (EME) OGLE-2003-BLG-423, which at $A_{\text{max}} \sim 250$, proves to have the highest magnification ever recorded among single-lens events. As such, the event also has the greatest potential sensitivity to planetary companions of the lens, with substantial probability of detecting even Neptune mass planets, whose event timescale would typically be only about 6 hours. This enhanced sensitivity poses special challenges to the analysis because both the form and amplitude of the impact of such small planets on the lightcurve will depend on the relative size of the source compared to the Einstein ring. If this relative size were known, it would be straightforward to calculate its effect. However, since the lightcurve is consistent with a point source, our information on the source size is limited.

Similarly, using the lightcurve data alone the impact parameter $u_0$ is measured only to about 30%. If $u_0$ were known much more precisely (as it often is for events with relatively bright sources), then we would be able to specify with equal precision where in relation to the Einstein ring a planet could be and still avoid detection. With our less perfect knowledge of $u_0$ however, we must be satisfied with a more probabilistic statement about these locations.

Both of these challenges are likely to be generic to the analysis of EMEs. Because such events occur with low probability, their sources are most likely to be the relatively common main-sequence stars that normally lie unnoticed in ground-based bulge images, but which can briefly leap to prominence in an EME. Since these main-sequence stars are faint and hence small, they will most often avoid finite-source effects even in EMEs. Their faintness also induces large photometric errors in the wings of the lightcurve, the region that must be well-measured to accurately determine $u_0$. For similar reasons, these two challenges are likely to be key issues in future, even more aggressive, microlensing experiments that aim to detect Earth-mass planets either by space-based (Bennett & Rhie 2002) or ground-based (Gaudi, Han & Gould 2004) observations.

In our analysis, we will take as our starting point the method pioneered by Gaudi & Sackett (2000) and Albrow et al. (2000), which was then applied to a much larger sample by Gaudi et al. (2002). However, we improve upon this method in several respects. First, we fix the impact parameter $u_0$ at a series of different values consistent with the event data and evaluate the sensitivity to companions at each $u_0$. To find the net sensitivity, we must weight each of these outcomes by the relative probability that the actual event had that particular $u_0$. Second, we determine these relative probabilities not just from the fit to the lightcurve data, but by incorporating the results of a Monte Carlo simulation of events toward the actual line of sight. For each trial $u_0$, we weight the simulated events by how well they reproduce both the observed characteristics of the lightcurve and the probability that the source has the luminosity inferred from the lightcurve combined with the Monte Carlo event parameters, as determined from the Hipparcos luminosity distribution at the observed color of the source. This method not only allows us to more accurately estimate the planetary sensitivity, it also permits us to characterize this sensitivity as a function of planet mass and planet-star separation, since each simulated event has a definite lens mass (drawn from the adopted mass function) and definite lens and source distances (and so definite Einstein radius). In contrast, the original approach of Albrow et al. (2000) yielded sensitivities in terms of two lightcurve-fit parameters, the planet-star mass ratio and the separation in units of the Einstein radius.

This method would also permit a similarly rigorous statistical treatment of finite source effects, since each simulated event has a definite ratio of source size to Einstein radius. However, based on the Monte Carlo, we show that in the case of OGLE-2003-BLG-423, finite-source effects are negligible.

2. DATA

OGLE-2003-BLG-423 was alerted by the Early Warning System (EWS, Udalski 2003) on UT 7:38 14 Sept 2003, almost exactly 24 hours before the peak on HJD = HJD–2450000 = 2897.8070, and less than 5 hours after the triggering observation by the OGLE-III observatory in Las Campanas, Chile. While the automated alert did not itself call any more attention to this event than the other three that were alerted simultaneously, the OGLE web site immediately affixed a “!” to this event, indicating that it was of special interest. Moreover, from the data available at the web site one could see that the event was already 3 mag above baseline and rising rapidly. See Albrow (2004) for a Bayesian approach to determine whether ascending microlensing events are likely to achieve high magnification.

Immediately following the alert, μFUN decided to focus its observations heavily upon this event. Because the event was triggered relatively late in the season when the bulge is already west of the meridian at twilight, the time per night that it could be observed from any one site was restricted: roughly

\[^{10}\text{http://www.astrouw.edu.pl/~ogle/ogle3/ews/ews.html}\]
the Einstein radius. Combining this with the Einstein crossing time $t_E = 97$ days derived from the OGLE data yielded an estimate of $A_{\max} \simeq 1/\mu_0 \sim 400$, which would be the highest magnification single-lens event ever recorded. Recognizing the importance of this event, OGLE and $\mu$FUN worked together to develop an observation plan that would allow us to characterize it as well as possible. Our principal concern was that if OGLE returned to its regular cycle of observations and $\mu$FUN stopped observing the event altogether (as both would normally do several days after the peak), then the OGLE and $\mu$FUN observations might barely overlap in time, meaning that the two photometry systems could not be rigidly linked into a single lightcurve. To resolve this problem, we agreed to each observe the event several times for the next few nights (weather permitting) and to both regularly continue observing it until it got too close to the Sun.

There are a total of 278 $I$ band images including 150 from OGLE, 78 from $\mu$FUN Chile, and 50 from $\mu$FUN Israel. In addition there are 7 $V$ band images from $\mu$FUN Chile, all taken near peak for the purpose of determining the color of the source. Finally, since $\mu$FUN Chile observations are carried out with an optical/infrared camera, all $V$ and $I$ images from this location are automatically accompanied by $H$ band images. However, even at peak, the event was too faint in $H$ for these observations to be useful. For each data set, the errors were rescaled to make $\chi^2$ per degree of freedom for the best-fit point-source/point-lens (PSPL) model equal to unity. We then eliminated the largest outlier and repeated the process until there were no $3 \sigma$ outliers. This resulted in the elimination of 1 OGLE point, 1 $\mu$FUN Chile $I$ point and 1 $\mu$FUN $V$ point. In the neighborhood of each of these four outliers, there are other data points that agree with the PSPL model, showing that the outliers are indeed caused by systematic errors rather than revealing unmodeled structure in the lightcurve. The final rescaling factors were 1.13 and 0.82 for OGLE and $\mu$FUN Chile $I$, respectively. The other two observatory/filter combinations did not require renormalization. The descriptions of the instruments, observing protocol, and reduction procedures are identical to those given in Yoo et al. (2004). The photometry is carried out using the DoPHOT-based PLANET pipeline.

3. POINT-LENS MODELS

The signature of a planetary companion will usually be a brief excursion from an otherwise "normal" point-lens magnification lightcurve. Indeed, as outlined by Gould & Loeb (1992), it is often possible to estimate the planet's properties from the gross characteristics of this deviation. The first step to searching for planets is therefore to fit the lightcurve to a point-lens model (Albrow et al. 2000). However, planetary deviations can be strongly affected by the finite size of the source, even if the rest of the lightcurve is perfectly consistent with a point source, which can lead to degeneracies in the interpretation of the deviation (Gaudi & Gould 1997), or even to a complete failure to detect the deviation. Hence, we begin by presenting the best-fit PSPL model, and then investigate to what extent finite-source effects can be detected or constrained within the context of point-lens models.

3.1. Point-Source Point-Lens Model

We fit the data to PSPL models, defined by three lensing geometry parameters ($t_0$, $u_0$, and $t_E$) as well as a source flux $F_s$ and a blended-light flux $F_b$ for each observatory-filter com-

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11 http://www.astronomy.ohio-state.edu/~microfun
Third, the microlensing model could be in error, either statistically or systematically. We make a more detailed investigation on this offset in §4.4.

3.2. Color-Magnitude Diagram

The first step to understanding the impact of these measurements and their errors is to place the source on an instrumental color-magnitude diagram (CMD). In Figure 2, we have translated the instrumental CMD to place the clump at \([V-I]_0, I_0 = (1.00, 14.32)\), which is the dereddened color and absolute magnitude of the Hipparcos (ESA 1997) clump when placed at the Galactocentric distance, \(R_0 = 8\) kpc (Yoo et al. 2004). The source position (as determined from the model fit) is shown as a triangle. Since the source is substantially fainter than any of the CMD stars, we also plot the Hipparcos lower main sequence stars placed at \(R_0 = 8\) kpc are represented as red points. The source (blue triangle with 1\(\sigma\) errorbar) is significantly fainter than the Hipparcos stars.

There are several notable features of this fit. First, the impact parameter is extremely small, \(u_0 = 0.00250 \pm 0.00072\) implying that the maximum magnification is \(A_{\text{max}} = 400 \pm 115\). Second, the source is extremely faint, \(I_s = 22.0 \pm 0.3\). The OGLE photometry is not rigorously calibrated, but is believed to be accurate to a few tenths. Finally, the errors are quite large, roughly 30\% for each of \(t_{\text{eff}}\), \(u_0\), and \(F_s\). In fact, these errors are extremely correlated: appropriate combinations of these parameters, \(t_{\text{eff}} \equiv u_0 t_{\text{E}}\) and \(F_s \equiv F_s / u_0\), have much smaller errors,

\[
t_{\text{eff}} = 0.2429 \pm 0.0037 \text{ days}, \quad I_{\text{min}} = 15.459 \pm 0.018.
\]

3.3. Finite-Source Effects

We now explore a set of models that are constrained to hold \(u_0\) and \(z_0\) at a fixed grid of values. Here, \(z_0 \equiv u_0 / \rho_s\) and \(\rho_s = \theta_s / \theta_E\) is the angular size of the source \(\theta_s\) in units of the angular Einstein radius \(\theta_E\). We take account of limb darkening.

### Table 1. OGLE-2003-BLG-423 Parameters

| Lightcurve Alone | Lightcurve & Monte Carlo Simulation |
|------------------|-----------------------------------|
| \(t_0\) (days)  | \(u_0\) | \(t_0\) (days) | \(A_{\text{max}}\) | \(I_s\) | \(t_{\text{base}}\) | \(t_0\) (days) | \(u_0\) | \(t_0\) (days) | \(A_{\text{max}}\) | \(I_s\) | \(t_{\text{base}}\) |
|-----------------|---------|----------------|-----------------|--------|--------|-----------------|---------|----------------|-----------------|--------|--------|
| Value           | 2897.8070 | 0.00250        | 97.4            | 400    | 22.0   | 20.21           | 2897.8070 | 0.00391       | 62.1            | 256    | 21.47  | 20.21           |
| Error           | 0.0030   | 0.00072        | 27.9            | 115    | 0.3    | 0.03            | 0.0030   | 0.00066       | 10.5            | 43     | 0.08   | 0.03            |

### Table 2. OGLE-2003-BLG-423 Fluxes Parameters (Lightcurve Alone)

| OGLE-1 | \(\mu\)FUN-I Chile | \(\mu\)FUN-I Israel | \(\mu\)FUN-V Chile |
|-------|---------------------|---------------------|---------------------|
| \(f_s\) | 0.02596             | 0.02596             | 0.02596             |
| \(\sigma_{f_s}\) | 0.00754             | 0.00745             | 0.00745             |
| \(f_b\) | 0.10411             | 0.02396             | -0.23491            |
| \(\sigma_{f_b}\) | 0.00607             | 0.00484             | 0.05733             |

Note. — \(f\) is rescaled to be the same as in the OGLE-I photometry.
by parameterizing the surface brightness $S$ by,
\[
\frac{S(\vartheta)}{S_0} = 1 - \Gamma \left[ 1 - \frac{3}{2} (1 - \cos \vartheta) \right],
\]
where $\vartheta$ is the angle between the normal to the stellar surface and the line of sight. Since the source has almost exactly the color of the Sun, we assume solar values for $\Gamma$,
\[
\Gamma_Y = 0.528, \quad \Gamma_I = 0.368.
\]

Figure 3 shows contour plots for the various models plotted as functions of $u_0$ and $z_0$. These contours are essentially independent of $z_0$, for $z_0 \gtrsim 1$, i.e., for models in which the lens does not pass directly over the source. Depending on some of the lightcurve parameters $\Delta \chi^2$ cannot be directly specified. While $u_0$ and $t_0$ can be taken as random variables drawn from uniform distributions, $\theta_E$ is a function of several independent physical quantities, namely the lens mass, the distances to the lens and source, and the transverse velocities of the lens and source. We collectively denote these independent physical parameters as $a^\text{phys}$. Therefore, Bayes’ theorem can be rewritten,
\[
P(a^\text{phys}\mid \Delta) = \exp[-\Delta \chi^2(a^\text{phys})/2]P_{\text{rel}}(a^\text{phys}),
\]
where it is understood that some of the lightcurve parameters are determined by the physical parameters. At the end of the day, one may be more interested in the physical parameters (or some subset of them) than the lightcurve parameters, and so after obtaining the general probability distribution given by equation (7), one may integrate over the remaining “nuisance parameters” to get the probability distribution of a specific physical parameter. Indeed, we will do exactly this when we evaluate planet sensitivities in $\S$6.2.

4.2. Relative Likelihood

To apply this general method to OGLE-2003-BLG-423, we first simplify $\Delta \chi^2(a^c)$. In principle, $\Delta \chi^2$ is a function of all five parameters, $t_0, t_{\text{eff}}, F_r, F_s$, and $F_{\text{max}}$. In practice, $t_0$ is extremely well determined from the data, while the remaining four parameters are all highly correlated. That is, since $t_{\text{eff}} = u_0 \theta_E$ and $F_{\text{max}} = F_r/F_s$ (and so $I_{\text{min}}$ are very well determined from the lightcurve data (see eq. (2)), their product $F_{\text{max}}t_{\text{eff}} = F_r/F_s$ is also well determined. Moreover, since the baseline flux is well determined, $F_r$ and $F_s$ are almost perfectly anti-correlated. Hence, once $\theta_E$ is chosen in a particular Monte Carlo realization, $I_r$ is also fixed to within 0.008 mag and all other parameters are rigidly fixed as well. Therefore, the relative likelihood is,
\[
\exp[-\Delta \chi^2(a^c)/2] = \exp[-\Delta \chi^2(\theta_E)/2],
\]
where $\Delta \chi^2(\theta_E)$ is the $\chi^2$ difference relative to the best-fit PSPL model. Since all of the lightcurve parameters are determined from the physical parameters via the well-constrained lightcurve parameters $t_0, t_{\text{eff}}, F_r$, and $F_s$, Bayes’ theorem can be rewritten in our case,
\[
P(a^\text{phys}\mid \Delta) = \exp[-\Delta \chi^2(t_{\text{eff}}a^\text{phys})/2]P_{\text{rel}}(a^\text{phys}).
\]

4. MODELING THE EVENT

We first outline a new method to analyze microlensing events that incorporates both a Galactic model via a Monte Carlo simulation and the constraints imposed by the observed characteristics of the event. This method is completely general and can be applied to any microlensing event. We then apply the method to OGLE-2003-BLG-423.

4.1. General Formalism

In most scientific experiments, ones seeks to determine the posterior probability $P(a\mid \Delta)$ of a parameter set $a = (a_1, \ldots, a_n)$ given a data set $\Delta$. By Bayes’ theorem,
\[
P(a\mid \Delta) = P_{\text{rel}}(\Delta\mid a)P_{\text{pr}}(a),
\]
where $P_{\text{rel}}(\Delta\mid a)$ is the probability of the data given the model parameters $a$ and $P_{\text{pr}}(a)$ is the prior probability of the parameters. For microlensing events,
\[
P_{\text{rel}}(\Delta\mid a^c) = \exp[-\Delta \chi^2(a^c)/2],
\]
where $a^c$ is the set of parameters describing the lightcurve and $\Delta \chi^2(a^c)$ is the $\chi^2$ difference relative to the best-fit model.

Since the prior $P_{\text{pr}}(a^c)$ is often assumed to be uniform, minimization of $\chi^2(a^c)$ is the usual method to find a best parameter set $a^c$. This procedure is appropriate when the lightcurve tightly constrains the parameters, but in general it is more correct to take account of the priors. However, the priors on some of the lightcurve parameters $a^c$ cannot be directly specified. While $u_0$ and $t_0$ can be taken as random variables drawn from uniform distributions, $\theta_E$ is a function of several independent physical quantities, namely the lens mass, the distances to the lens and source, and the transverse velocities of the lens and source. We collectively denote these independent physical parameters as $a^\text{phys}$. Therefore, Bayes’ theorem can be rewritten,
\[
P(a^\text{phys}\mid \Delta) = \exp[-\Delta \chi^2(a^\text{phys})/2]P_{\text{rel}}(a^\text{phys}, a^\text{phys}),
\]
where it is understood that some of the lightcurve parameters are determined by the physical parameters. At the end of the day, one may be more interested in the physical parameters (or some subset of them) than the lightcurve parameters, and so after obtaining the general probability distribution given by equation (7), one may integrate over the remaining “nuisance parameters” to get the probability distribution of a specific physical parameter. Indeed, we will do exactly this when we evaluate planet sensitivities in $\S$6.2.

4.2. Relative Likelihood

To apply this general method to OGLE-2003-BLG-423, we first simplify $\Delta \chi^2(a^c)$. In principle, $\Delta \chi^2$ is a function of all five parameters, $t_0, t_{\text{eff}}, F_r, F_s$, and $F_{\text{max}}$. In practice, $t_0$ is extremely well determined from the data, while the remaining four parameters are all highly correlated. That is, since $t_{\text{eff}} = u_0 \theta_E$ and $F_{\text{max}} = F_r/F_s$ (and so $I_{\text{min}}$ are very well determined from the lightcurve data (see eq. (2)), their product $F_{\text{max}}t_{\text{eff}} = F_r/F_s$ is also well determined. Moreover, since the baseline flux is well determined, $F_r$ and $F_s$ are almost perfectly anti-correlated. Hence, once $\theta_E$ is chosen in a particular Monte Carlo realization, $I_r$ is also fixed to within 0.008 mag and all other parameters are rigidly fixed as well. Therefore, the relative likelihood is,
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\exp[-\Delta \chi^2(a^c)/2] = \exp[-\Delta \chi^2(\theta_E)/2],
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where $\Delta \chi^2(\theta_E)$ is the $\chi^2$ difference relative to the best-fit PSPL model. Since all of the lightcurve parameters are determined from the physical parameters via the well-constrained lightcurve parameters $t_0, t_{\text{eff}}, F_r$, and $F_s$, Bayes’ theorem can be rewritten in our case,
\[
P(a^\text{phys}\mid \Delta) = \exp[-\Delta \chi^2(t_{\text{eff}}a^\text{phys})/2]P_{\text{rel}}(a^\text{phys}).
\]
We next impose another condition on the prior that constrains $\mathbf{a}^{\text{phys}}$. Since $t_E$ is fixed by the Monte Carlo, $\mathcal{F}_i$ is also fixed (see §4.4). By comparing this to the position of the clump on the instrumental CMD, one can then determine the reddened flux of the source. Since $D_i$ is fixed by the Monte Carlo, the absolute magnitude of the source for this Monte Carlo realization can also be inferred. The prior probability is then proportional to $N_{\text{Hip}}$, the number of Hipparcos stars with this inferred absolute magnitude (and within 0.02 mag of the measured source $V - I$ color). Hence, if we restrict attention to the $k$-th realization of the Monte Carlo with physical parameters $\mathbf{a}^{\text{phys},k}$, the prior probability is given by,

$$P_{\text{pri}}(\mathbf{a}^{\text{phys},k}) \propto (\Gamma N_{\text{Hip}})k.$$  \hfill (12)

### 4.4. Posterior Probability

Combining equations (9) and (12) implies that the posterior probability that a parameter $a_i$ lies in the interval $a_i \in [a_{i,\text{min}}, a_{i,\text{max}}]$ is proportional to,

$$P(a_i \in [a_{i,\text{min}}, a_{i,\text{max}}]) \propto \sum_k P(\mathbf{a}^{\text{phys},k}),$$  \hfill (13)

where

$$P(\mathbf{a}^{\text{phys},k}) = (\Gamma N_{\text{Hip}})\exp[-\Delta \chi^2(\mathbf{t}_E,k)/2] \times \Theta(a_i(\mathbf{a}^{\text{phys},k}) - a_{i,\text{min}}) \Theta(d_{i,\text{max}} - a_i(\mathbf{a}^{\text{phys},k})).$$  \hfill (14)

$a_i$ is one of the physical parameters $\mathbf{a}^{\text{phys}}$ (or possibly a function of several physical parameters as would be the case for $a_i \in \mathbf{a}^\dagger$), both $(\Gamma N_{\text{Hip}})$ and $d_{E,k}$ are implicit functions of $\mathbf{a}^{\text{phys},k}$, and $\Theta$ is a step function.

Letting $a_i = M$, we can evaluate the posterior probability distribution for the lens mass. Figure 4 shows both the event rate $\Gamma$ and the posterior distribution of microlensing events toward the Galactic bulge as a function of mass. The overall event rate is shown in the upper panel. The thin solid and dotted lines represent events from MS stars and brown dwarfs (BDs), and from stellar remnants, respectively. Note that the number of objects in the mass function steeply decreases as the mass increases ($M > 0.7M_\odot$), and in particular that there are no MS stars of $1M_\odot$ in the Galactic bulge because such stars have already evolved off (Holtzman et al. 1998; Zoccali et al. 2000). However, since the cross-section of the microlensing event is proportional to $M^{1/2}$, remnants contribute of order 20% of the bulge microlensing events (Gould 2000). The posterior distribution for the lens mass is shown in the lower panel. Note that high masses are strongly favored, possibly because of the long timescale $t_E$.

Figure 5 shows the distributions of posterior probabilities of various other parameters. The thick and thin solid histograms represent bulge-bulge and disk-bulge events, respectively. The upper left panel shows the distribution of impact parameters (histograms) compared to the distribution derived from the lightcurve data alone (solid curve). Note that the best-fit $u_0$ from the lightcurve alone is somewhat lower than the peak of the posterior distribution (see §5.3). The impact parameter and the apparent magnitude are strongly anti-correlated as is discussed in §5.11 and the lower left panel shows the distribution of source dereddened apparent magnitudes (histograms) compared to the distribution based on the lightcurve data alone (as represented by the solid curve in Fig. 5) and by the position and error bar in Fig. 2. Taking account of the prior probabilities $\Gamma$ of the Galactic model and of the $M_i$ distribution of Hipparcos stars at the observed source.
Fig. 5.—Distributions of Monte Carlo microlensing events toward the OGLE-2003-BLG-423 line of sight. The thick and thin solid lines represent bulge-bulge events and bulge-disk events, respectively. The impact parameter, absolute magnitude, dereddened apparent magnitude, and source-lens relative proper motion are denoted as $u_0$, $M_I$, $I_0$, and $\mu$, respectively, while $z_0 \equiv u_0/\rho_*$, where $\rho_*$ is the ratio of the source size to the Einstein radius. The Gaussian curves in the $u_0$ and $M_I$ panels represent the probability distributions derived from the lightcurve fit alone, i.e., before applying the constraints from the Galactic model. The dotted histograms in the middle panels are the distributions of Hipparcos stars at the color of the source, $(V-I)_0 = 0.73$ (left panel), and lenses in the bulge and disk obtained from a Galactic model alone (right panel), respectively.

The dotted histogram in the middle left panel represents the absolute magnitude of Hipparcos MS stars with the same color as the source. However, the Monte Carlo events favored by the lightcurve are dimmer than the average Hipparcos star at $R_0$ (see the middle right panel). The lower right panel shows the distribution of $z_0$. As we discuss in §5, the Monte Carlo effectively takes account of the selection effects that push toward low proper motion (and hence lower $z_0$). The panel shows, however, that the probability that $z_0$ is small enough to generate significant finite-source effects in a point-lens event is extremely small.

The best-fit lightcurve parameters and their errors are shown in Table 1. Note that parameters are different at the 2$\sigma$ level from those with lightcurve alone, and hence the maximum magnification of the event is $A_{\text{max}} = 256 \pm 43$.

5. INFLUENCE OF SELECTION EFFECTS

As mentioned in §2, the event was alerted only 24 hours before peak. The observation just prior to this triggering observation was on 2892.6, about 4 days previous. The event was not alerted from that observation because up to that point there were only two detections on the subtracted images, whereas the alert threshold is set at three in order to avoid spurious events. Even had the event been alerted, it would have been flagged as having an impact parameter $u_0 = 0.0 \pm 0.2$ and therefore would not have been recognized as a high magnification event. This 4-day cycle time, which was typical for OGLE-III observations in mid-September, introduces significant selection effects in the recovery of EMEs.

The primary effect is to select for long events. For example, if we consider an event with the same source star, same impact parameter, and same magnification on 2892.6, but with $t_E$ shorter by a factor 2/3, then it would not have been discovered until after peak. That is, such an event would also not have triggered an alert on 2892.6, but at the 2896.6 observation, at which point it would have already been 0.5 days past peak. By the time followup observations started, it would have been a day past peak, and so magnified only about 40 times. While still impressive, this would not have garnered either the attention or the intensive observations triggered by OGLE-2003-BLG-423.

Thus, the fact that the observed timescale is long compared to that of typical bulge microlensing events is explained largely by selection. However, this selection effect is already fully accounted for in the Monte Carlo. Consider a Monte
source distance is about 2.5 kpc behind the Galactic center. The pressure toward longer events further selects for more distant sources because their larger $\theta_E$ makes the events longer. However, since the FWHM of the prior distribution is about 3.5 kpc, the adopted distance would not be extremely unlikely in any case.

Another abnormal characteristic is the faintness of the source. Up to a point, the event selection procedure would appear to pick out brighter sources. As mentioned above, a brighter source would exactly compensate in the selection process for a shorter event, and these are more common than longer events. However, fainter sources are more common than brighter ones, and this is a larger effect. Moreover, as the source brightness increases, so does its angular size, and this eventually cuts off the peak brightness due to finite source effects. Based on Figure 5, however, we have concluded that the source is probably nowhere near this threshold, so this limitation on source size does not enter as a significant factor.

Finally, the source appears to be dim for its color. If the Hipparcos distribution is representative of bulge stars of solar color, then this feature would actually be selected against: more luminous stars would be both more numerous and, if lensed by exactly the same lens, more easily recognized before peak. However, it may be that the Hipparcos distribution is not representative of the bulge. For example, the stars in the outer bulge may have significantly lower metallicity than those in the solar neighborhood, and therefore be fainter at fixed color as is true of subdwarfs in the solar neighborhood (e.g., Gould 2004).

We conclude that while a number of the features of this event appear unusual at first sight, most are explained in whole or in part by selection effects.

6. SEARCH FOR PLANETS

6.1. Detection Efficiency

As discussed in § 1, Gaudi & Sackett (2000) and Albrow et al. (2000) have already developed a procedure for searching for planets in microlensing lightcurves, and Gaudi et al. (2003) have applied this to a sample of 43 events. For each event, they considered an ensemble of planetary systems characterized by a planet-star mass ratio $q$, a planet-star separation (in units of the Einstein radius) $d$, and an angle $\alpha$ of the source trajectory relative to the planet-star axis. We will begin by following this procedure, but will introduce several important modifications.

For a given $(d, q)$, we define the detection efficiency $\epsilon(d, q)$ as the probability that a companion planetary system described by $(d, q)$ would have produced a lightcurve deviation inconsistent with the observed OGLE-2003-BLG-423 lightcurve:

$$\epsilon(d, q) = \frac{1}{2\pi} \int_0^{2\pi} d\alpha \Theta \left[ \chi^2(d, q, \alpha) - \chi^2_{\text{PSPL}} - \Delta \chi^2_{\text{thr}} \right],$$

(15)

where $\chi^2(d, q, \alpha)$ is the value of $\chi^2$ evaluated for these three parameters, $\chi^2_{\text{PSPL}}$ its best-fit value for the PSPL model, and $\Theta$ is a step function. The $(d, q)$ sampling is 0.1 in the log, and the angular step size is set to be $\Delta \alpha = \sqrt{\frac{\sigma}{2}}$ in order to avoid missing possible planetary perturbations. We choose a Monte Carlo event that has a $t_E$ that is much shorter than the best fit in Table 1, say $t_E = 20$ days rather than 97 days. This event is assigned a source flux that is lower by a factor $\sim 4.5$ so as to reproduce as well as possible the observed lightcurve. In fact, the resulting model lightcurve reproduces the peak region extremely well: most of the $\chi^2$ difference comes from the post-peak wing, which of course did not enter the selection process. Thus, there is no additional selection discrimination among the Monte Carlo events.

The event appears to have several “abnormal” characteristics relative to typical events as represented in the Monte Carlo, and it is of interest to determine which of these are brought about by, or enhanced by selection. The most likely source distance is about 2.5 kpc behind the Galactic center.

For bulge-bulge lensing, there is a general selection effect driving toward distant sources because these have larger $\theta_E$ and so larger cross sections. See equation (10). However, as shown by the dotted lines in the middle right panel of Figure 5, this effect alone pushes the peak of the distribution back only 1.5 kpc (0.7 kpc for bulge-disk lensing) relative to $R_0$, not 2.5 kpc. The pressure toward longer events further selects for more distant sources because their larger $\theta_E$ makes the events longer. However, since the FWHM of the prior distribution is about 3.5 kpc, the adopted distance would not be extremely unlikely in any case.

Another abnormal characteristic is the faintness of the source. Up to a point, the event selection procedure would appear to pick out brighter sources. As mentioned above, a brighter source would exactly compensate in the selection process for a shorter event, and these are more common than longer events. However, fainter sources are more common than brighter ones, and this is a larger effect. Moreover, as the source brightness increases, so does its angular size, and this eventually cuts off the peak brightness due to finite source effects. Based on Figure 5, however, we have concluded that the source is probably nowhere near this threshold, so this limitation on source size does not enter as a significant factor.

Finally, the source appears to be dim for its color. If the Hipparcos distribution is representative of bulge stars of solar color, then this feature would actually be selected against: more luminous stars would be both more numerous and, if lensed by exactly the same lens, more easily recognized before peak. However, it may be that the Hipparcos distribution is not representative of the bulge. For example, the stars in the outer bulge may have significantly lower metallicity than those in the solar neighborhood, and therefore be fainter at fixed color as is true of subdwarfs in the solar neighborhood (e.g., Gould 2004).

We conclude that while a number of the features of this event appear unusual at first sight, most are explained in whole or in part by selection effects.

For each event, they considered an ensemble of planetary systems characterized by a planet-star mass ratio $q$, a planet-star separation (in units of the Einstein radius) $d$, and an angle $\alpha$ of the source trajectory relative to the planet-star axis. We will begin by following this procedure, but will introduce several important modifications.

For a given $(d, q)$, we define the detection efficiency $\epsilon(d, q)$ as the probability that a companion planetary system described by $(d, q)$ would have produced a lightcurve deviation inconsistent with the observed OGLE-2003-BLG-423 lightcurve:

$$\epsilon(d, q) = \frac{1}{2\pi} \int_0^{2\pi} d\alpha \Theta \left[ \chi^2(d, q, \alpha) - \chi^2_{\text{PSPL}} - \Delta \chi^2_{\text{thr}} \right],$$

(15)

where $\chi^2(d, q, \alpha)$ is the value of $\chi^2$ evaluated for these three parameters, $\chi^2_{\text{PSPL}}$ its best-fit value for the PSPL model, and $\Theta$ is a step function. The $(d, q)$ sampling is 0.1 in the log, and the angular step size is set to be $\Delta \alpha = \sqrt{\frac{\sigma}{2}}$ in order to avoid missing possible planetary perturbations. We choose a

Fig. 6.— Detection efficiency of OGLE-2003-BLG-423 in units of planet-star mass ratio $q$ and separation $d$ (normalized to the Einstein radius). The upper panel shows the detection efficiency contours (25%, 50%, 75%, and 95%) by minimizing $\chi^2$ with respect to $t_E$, $d_E$, and $u_0$ (Albrow et al. 2000), and the lower panel shows the 50% efficiency contours for various fixed $u_0$ ($u_0 = 0.002$ to 0.006). As $u_0$ increases, the efficiency decreases monotonically. For comparison, we present the 50% contours of the former method (solid) and the latter method with $u_0 = 0.002$, 0.004 (dashed) in the inset.
In this incarnation of the procedure, we follow Albrow et al. (2000) and adopt for $\chi^2(d, q, \alpha)$ the minimum value of $\chi^2$ with these three parameters held fixed and allowing all other parameters to vary. The results are shown in the upper panel of Figure 6. The curves represent detection efficiencies of 25%, 50%, 75%, and 95%. For $q = 0.1$, companions with separation $0.2 \lesssim d \lesssim 6$ are completely excluded by the data because they would produce deviations $\Delta \chi^2 \equiv \chi^2(d, q, \alpha) - \chi^2_{\text{PSPL}} > 60$ that are not observed. However, the effect of planetary companions of mass ratio $q = 10^{-3}$ would hardly be discernible.

Gaudi et al. (2002) discussed a potential shortcoming of this approach: if (as in the present case) $u_0$ is not well constrained by the data, then it is possible that for the procedure to say that certain planetary configurations are permitted by the data when in fact they are excluded. For example, suppose that the measured impact parameter is $u_0 = 0.00250 \pm 0.00072$ while the actual value is $u_0 = 0.003$. For some value of $\alpha$, the caustic induced by a planet could lie right along the $u_0 = 0.003$ trajectory, but the minimization routine might nevertheless find a path that lay $5 \sigma (\Delta \chi^2 = 25)$ from this value at $u_0 = 0.006$ and so avoided the planetary caustic but with $\Delta \chi^2 < 60$ (see Fig. 6 and the accompanying text in Gaudi et al. 2002).

To counter this shortcoming, we evaluate the sensitivity at each allowed value of $u_0$. Our search of $(d, q, u_0)$ parameter space reveals no planets. The best fit is at $u_0 = 0.002$, $d = 1$, $q = 10^{-3}$, but the $\Delta \chi^2$ is only $-2.9$, far short of our adopted threshold of $\Delta \chi^2 = 60$. We evaluate the efficiency by modifying equation (15) to become

$$
\epsilon(d, q; u_0) = \frac{1}{2\pi} \int_0^{2\pi} d\alpha \Theta \left[ \chi^2(d, q, \alpha; u_0) - \chi^2_{\text{PSPL}}(u_0) - \Delta \chi^2_{\text{thr}} \right],
$$

where $\chi^2(d, q, \alpha; u_0)$ and $\chi^2_{\text{PSPL}}(u_0)$ are now evaluated at fixed $u_0$. Here, $u_0$ is defined as the projected separation of the source from the center of the caustic induced by the planetary companion. This is the appropriate generalization from the point-lens case, in which $u_0$ is the projected separation from the (point-like) caustic at the position of the primarly lens. The lower panel of Figure 6 shows 50% contours of detection efficiency for several values of $u_0$ that are consistent with the Monte Carlo simulation (see Fig. 5). For comparison, we present the 50% contours of the Albrow et al. (2000) method (solid) and of our new method with $u_0 = 0.002$ and $u_0 = 0.004$ (dashed) in the inset. Although the difference is small, we find that the previous method of Albrow et al. (2000) tends to overestimate the detection efficiency.

While it is comforting that the magnitude of this effect is small, its sign is somewhat unsettling. Recall that one motivation for integrating over $u_0$ rather than minimizing with respect to $u_0$ was that under the latter procedure, the trajectory could "avoid" planetary caustics and underestimate the sensitivity. In the present case, however, this effect is outweighed by the fact that the most probable value of $u_0$ is increased by taking into account the Monte Carlo compared to the fit to the lightcurve alone. See Figure 5. As discussed in § 1, sensitivity to planets generally decreases with increasing $u_0$.

Note that in this particular case, it is only necessary to integrate over $u_0$ (and not all lightcurve parameters as originally envisaged by Gaudi et al. 2002) because once $u_0$ is specified, all the other lightcurve parameters are highly constrained (see § 4.2).

One might be concerned about finite-source effects during a planet-caustic crossing. However, we have repeated the calculation including finite-source effects for a variety of $(d, q, \alpha)$ combinations and for various plausible source sizes as determined from Monte Carlo. We find no significant difference in planet detection efficiencies.

6.2. Constraints on Planets

Microlensing events provide only degenerate information on physical properties of the source and lens except in so far as other higher order effects such as finite-source effects and parallax are detected. However, our new method based on Monte Carlo simulations allows us, for the first time, to break the degeneracy and place constraints on planetary companions in the planet-mass/physical-separation plane, rather than scaling these quantities to the stellar mass and Einstein radius as was done previously.

For a given ensemble of Monte Carlo events with posterior...
probabilities $P_k(a^\text{phys})$, the detection efficiency $\epsilon$ can be evaluated as a function of the planet mass $m$ and the planet-star projected physical separation $r_\perp$ by,

$$
\epsilon(r_\perp,m) = \frac{\sum_{k=1}^{N} \epsilon \left( r_\perp / d_l, \theta_E, m / M_k, \mu_0 \right) P(a^\text{phys},k)}{\sum_{k=1}^{N} P(a^\text{phys},k)}
$$

(17)

where $N$ is the number of Monte Carlo events and $\theta_E(M, d_l, d_s)$ is the angular Einstein radius.

Figure 7 shows the resulting detection efficiency with the curves representing contours for $\epsilon = 25\%, 50\%, 75\%$, and $95\%$. The left and middle panels show separate detection efficiencies for the MS+BDs, and the remnant stars, respectively. The total efficiency is shown in the right panel. Since remnant stars are more massive than MS+BDs, at fixed planet-star mass ratio, microlensing events by remnant stars probe planets of higher absolute mass. Hence, microlensing is less efficient as a probe of planets of remnants than of MS+BDs at fixed planetary mass.

Because our Galactic model favors substantially lower blending than is implied by the lightcurve alone, the best fit magnitude is reduced from $A = 400$ to $A = 256$. Nevertheless, OGLE-2003-BLG-423 is the highest magnification single-lens event recorded to date. Despite this honor, the detection efficiency is not quite as good as two previous high magnification events, MACHO-98-BLG-35 ($A_{\text{max}} \sim 100$) and OGLE-1999-BUL-35 ($A_{\text{max}} \sim 125$) [Gaudi et al. 2002] see also [Bond et al. 2002]. This is because our observations do not cover the peak of the lightcurve nearly as densely as was the case in those two events. Peak coverage is key because the perturbations by planets mostly occur during a small time interval, basically the full width at half maximum around the peak of the event [Rattenbury et al. 2002].

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