New higher-derivative couplings in 4D $\mathcal{N} = 2$ supergravity

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Abstract

Using the off-shell formulation for general $\mathcal{N} = 2$ supergravity-matter systems developed in arXiv:0905.0063, we propose a construction to generate a restricted chiral superfield from any real weight-zero projective multiplet $\mathcal{L}$. One can choose $\mathcal{L}$ to be composed of tensor multiplets, $\mathcal{O}(2n)$ multiplets, with $n = 2, 3, \ldots$, and polar hypermultiplets. In conjunction with the standard procedure to induce a $\mathcal{N} = 2$ linear multiplet from any chiral weight-one scalar, we obtain a powerful mechanism to generate higher-derivative couplings in supergravity. In the case that $\mathcal{L}$ is a homogeneous function of $n$ tensor multiplets of degree zero, we show that our construction is equivalent to that developed by de Wit and Saueressig in arXiv:hep-th/0606148 using the component superconformal tensor calculus. We also work out nontrivial examples with $\mathcal{L}$ composed of $\mathcal{O}(2n)$ and tensor multiplets.
## 1 Introduction

Recently, an off-shell formulation for general $\mathcal{N} = 2$ supergravity-matter couplings in four space-time dimensions has been constructed \cite{1, 2, 3}. It is a curved-space extension of the superconformal projective multiplets and their couplings \cite{4} in 4D $\mathcal{N} = 2$ projective superspace \cite{5, 6, 7}. In the present paper, we will demonstrate that the methods developed in \cite{1, 2, 3} and closely related works \cite{8, 9} allow us to generate
new off-shell higher-derivative couplings in $\mathcal{N} = 2$ supergravity, a problem which has been of some interest recently \[10, 11\].

The construction we pursue in this paper is based on a duality between two basic off-shell representations of $\mathcal{N} = 2$ supersymmetry – the vector multiplet \[12\] and the tensor multiplet \[13\]. The vector multiplet can be described in curved superspace by its covariantly chiral field strength $W$ subject to the Bianchi identity, \[12, 14\]

$$\bar{D}^{\hat{a}} W = 0, \quad \Sigma^{ij} := \frac{1}{4} \left( D^{\alpha(i} D^{j)} + 4 S^{ij} \right) W = \frac{1}{4} \left( \bar{D}_{\hat{a}}(i \bar{D}^{j})^{\hat{a}} + 4 \bar{S}^{ij} \right) \bar{W}, \quad (1.1)$$

where $S^{ij}$ and $\bar{S}^{ij}$ are special dimension-1 components of the torsion. The superfield $\Sigma^{ij}$ is real, $\bar{\Sigma}_{ij} := (\Sigma^{ij})^*$ $= \varepsilon_{ik} \varepsilon_{jl} \Sigma^{kl}$, and obeys the constraints

$$D^{(i} \Sigma^{jk)} = \bar{D}_{(i} \bar{\Sigma}^{jk)} = 0. \quad (1.2)$$

These constraints are characteristic of the $\mathcal{N} = 2$ linear multiplet \[15, 16\].

There are several ways to realize $W$ as a gauge invariant field strength. One possibility is to introduce a curved-superspace extension of Mezincescu’s prepotential \[19\] (see also \[20\]), $V_{ij} = V_{ji}$, which is an unconstrained real SU(2) triplet. The expression for $W$ in terms of $V_{ij}$ is

$$W = \frac{1}{4} \bar{\Delta} \left( D^{ij} + 4 S^{ij} \right) V_{ij}, \quad (1.3)$$

where $\bar{\Delta}$ is the chiral projection operator \[B.4\]. Within the projective-superspace approach of \[11, 2, 3\], the constraints on $W$ can be solved in terms of a real weight-zero tropical prepotential $V(v^i)$. The solution \[8\] is

$$W = \frac{1}{8\pi} \oint v^i dv_i \left( (D^-)^2 + 4 S^{--} \right) V(v), \quad (1.4)$$

$$D^a v = \bar{D}_{\hat{a}} V = 0, \quad V(c v^i) = V(v^i), \quad \bar{V} = V.$$

Here $v^i \in \mathbb{C}^2 \setminus \{0\}$ denote homogeneous coordinates for $\mathbb{C}P^1$, and the contour integral is carried out around the origin in $\mathbb{C}P^1$; see Appendix C for our isotwistor notation

\[^1\]Such a superfield is often called reduced chiral.

\[^2\]Our curved-superspace conventions follow Ref. \[3\]. In particular, we use Howe’s superspace realization \[14\] (see also \[18\]) of $\mathcal{N} = 2$ conformal supergravity in which the structure group is $\text{SL}(2, \mathbb{C}) \times \text{U}(2)$. The relevant information about Howe’s formulation is collected in Appendix A. In what follows, we will use the notation: $D^{ij} := D^{\alpha(i} D^{j)}$ and $\bar{D}^{ij} := \bar{D}_{\hat{a}}(i \bar{D}^{j})^{\hat{a}}$. It should be noted that Howe’s realization of $\mathcal{N} = 2$ conformal supergravity \[14\] is a simple extension of Grimm’s formulation \[17\] with the structure group $\text{SL}(2, \mathbb{C}) \times \text{SU}(2)$. The precise relationship between these two formulations is spelled out in \[3\].
and conventions, including the definition of the covariant derivatives $D_\alpha^\pm$ and $\bar{D}_\dot{\alpha}^\pm$. We discuss the relations between these two formulations in Appendix E.

In the rigid supersymmetric case, the representation (1.4) can be derived from a more general result in harmonic superspace \[21, 22\]

$$W = \frac{1}{4} \int du (\bar{D}^-)^2 V^{++},$$

(1.5)

which is given in terms of an analytic prepotential $V^{++}(u_i^+, u_j^-)$ for the vector multiplet \[23\]. Such a derivation makes use of the singular reduction procedure introduced in \[24\]. This has been carried out explicitly in five space-time dimensions \[25\]. Unfortunately, a curved-superspace generalization of (1.5) has not yet been found.

The tensor (or linear) multiplet can be described in curved superspace by its gauge invariant field strength $G^{ij}$ which is defined to be a real SU(2) triplet (that is, $G^{ij} = G^{ji}$ and $\bar{G}_{ij} := (G^{ij})^*$ $= G_{ij}$) subject to the covariant constraints \[15, 16\]

$$D_\alpha^{ij} G^{jk} = \bar{D}_{\dot{\alpha}}^{ij} G^{jk} = 0 .$$

(1.6)

These constraints are solved in terms of a chiral prepotential $\Psi$ \[20, 23, 27, 28\] via

$$G^{ij} = \frac{1}{4} \left( D^{ij} + 4 S^{ij} \right) \Psi + \frac{1}{4} \left( \bar{D}^{ij} + 4 \bar{S}^{ij} \right) \bar{\Psi} , \quad \bar{D}^i \Psi = 0 ,$$

(1.7)

which is invariant under shifts $\Psi \rightarrow \Psi + i \Lambda$, with $\Lambda$ a reduced chiral superfield. Associated with $G^{ij}$ is the real $O(2)$ projective multiplet $G^{++}(v) := G^{ij} v_i v_j$ \[11, 24, 28\]. The constraints (1.6) are equivalent to

$$D_\alpha^+ G^{++} = \bar{D}_{\dot{\alpha}}^+ G^{++} = 0 , \quad D_\alpha^+ := v_i D_\alpha^i , \quad D_{\dot{\alpha}}^+ := v_i \bar{D}_{\dot{\alpha}}^i .$$

(1.8)

The above properties of vector and tensor multiplets are related and complementary. In fact, they can be used to generate linear multiplets from reduced chiral ones and vice versa. Consider a system of $n_V$ Abelian vector multiplets described by covariantly chiral field strengths $W_I$, $I = 1, \ldots, n_V$. Let $F(W_I)$ be a holomorphic homogeneous function of degree one, $F(c W_I) = c F(W_I)$. Then, we can define a composite linear multiplet

$$G^{ij} := \frac{1}{4} \left( D^{ij} + 4 S^{ij} \right) F(W_I) + \frac{1}{4} \left( \bar{D}^{ij} + 4 \bar{S}^{ij} \right) \bar{F}(\bar{W}_I) .$$

(1.9)

Using the algebra of the covariant derivatives, one can check that $G^{ij}$ indeed obeys the constraints (1.6) and that the construction is invariant under shifts of the form

$$F(W_I) \rightarrow F(W_I) + i r_J W_J ,$$

(1.10)
for real constants $r_J$. Eq. (1.9) is a standard construction to generate composite linear multiplets. Of course, the construction (1.9) is a trivial application of (1.7).

Conversely, we may take a system of $n_T$ tensor multiplets described by their field strengths $G^{ij}_A$, with $A = 1, \ldots, n_T$, and let $G^{++}_A := v_i v_j G^{ij}_A$ be the corresponding covariant $O(2)$ multiplets. Let $L(G^{++}_A)$ be a real homogeneous function of degree zero, $L(c G^{++}_A) = L(G^{++}_A)$, and thus $L(G^{++}_A)$ is a covariant real weight-zero projective multiplet. Then, we can generate a composite reduced chiral multiplet defined by

$$W = \frac{1}{8\pi} \oint_C v^i d v_i \left( (\bar{D}^-)^2 + 4 \bar{S}^{--} \right) L(G^{++}_A) . \tag{1.11}$$

The integration in (1.11) is carried over a closed contour $C$ in $\mathbb{C}^2 \setminus \{0\}$. Similarly to eq. (1.4), the right-hand side of (1.11) involves a constant isospinor $u_i$ chosen to obey the constraint $(v, u) := v^i u_i \neq 0$, but otherwise is completely arbitrary. Using the constraints

$$\mathcal{D}_a^+ L(G^{++}) = \bar{\mathcal{D}}_a^+ L(G^{++}) = 0 , \tag{1.12}$$

one can check that the right-hand side of (1.11) is invariant under arbitrary projective transformations of the form:

$$(u^i, v^i) \rightarrow (u^i, v^i) R , \quad R = \begin{pmatrix} a & 0 \\ b & c \end{pmatrix} \in \text{GL}(2, \mathbb{C}) , \tag{1.13}$$

and the same property holds for the right-hand side of (1.4). This invariance guarantees that (1.4) and (1.11) are independent of the isospinor $u_i$. Of course, the reduced chiral construction (1.11) is a simple application of (1.4).

The relation (1.11) is an example of applying our new construction to generate composite reduced chiral multiplets. This construction will be derived in the main body of the paper. A rigid supersymmetric version of (1.11) was described for a special case by Siegel twenty-five years ago [29]. More recently, it has been rediscovered and fully elaborated by de Wit and Saueressig [30] in a component approach using the superconformal tensor calculus to couple it to conformal supergravity, but without a contour-integral representation.

This paper is organized as follows. In section 2 our main construction, eq. (2.11), is derived. As a simple application, in section 3 we consider the improved tensor multiplet. We also analyse implications of fixing the super-Weyl gauge freedom with a tensor multiplet. Section 4 is devoted to a general system of self-interacting tensor multiplet. We demonstrate that our superfield construction makes it possible to
re-derive, in a simple way, the key results of [30] obtained originally within the superconformal tensor calculus. In section 5 we show how to construct reduced chiral multiplets out of $O(2n)$ multiplets. In section 6 we discuss the new higher-derivative couplings and some other implications of our approach. The main body of the paper is accompanied by five technical appendices. In appendix A we give a summary of the superspace geometry for $\mathcal{N} = 2$ conformal supergravity introduced originally in [14] and elaborated in [3]. Appendix B describes the properties of the $\mathcal{N} = 2$ chiral projection operator. Appendix C contains our isotwistor notation and conventions for projective superspace. In Appendix D we describe manifestly $\text{SU}(2)$-covariant techniques to evaluate contour integrals in $\mathbb{C}P^1$. Finally, Appendix E is devoted to two prepotential formulations for the $\mathcal{N} = 2$ vector multiplet.

2 Main construction

Within the formulation for $\mathcal{N} = 2$ supergravity-matter systems developed in [3], the matter fields are described in terms of covariant projective multiplets. In addition to the local $\mathcal{N} = 2$ superspace coordinates $z^M = (x^m, \theta^i, \bar{\theta}^\dot{i})$, such a supermultiplet, $Q^{(n)}(z, v)$, depends on auxiliary isotwistor variables $v^i \in \mathbb{C}^2 \setminus \{0\}$, with respect to which $Q^{(n)}$ is holomorphic and homogeneous, $Q^{(n)}(cv) = c^n Q^{(n)}(v)$, on an open domain of $\mathbb{C}^2 \setminus \{0\}$. The integer parameter $n$ is called the weight of $Q^{(n)}$. In other words, such superfields are intrinsically defined in $\mathbb{C}P^1$. The covariant projective supermultiplets are required to be annihilated by half of the supercharges,

$$
\mathcal{D}_\alpha^+ Q^{(n)} = \bar{\mathcal{D}}^\dot{\alpha}_+ Q^{(n)} = 0, \quad \mathcal{D}_\alpha^+ := v_i \mathcal{D}_\alpha^i, \quad \bar{\mathcal{D}}^\dot{\alpha}_+ := v_\dot{i} \bar{\mathcal{D}}^\dot{\alpha}_\dot{i},
$$

(2.1)

with $\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}^i_\alpha, \bar{\mathcal{D}}^\dot{i}_\dot{\alpha})$ the covariant superspace derivatives. The dynamics of supergravity-matter systems are described by locally supersymmetric actions of the form [3]:

$$
S(\mathcal{L}^{++}) = \frac{1}{2\pi} \oint_C v^i dv_i \int d^4x d^4\theta d^4\bar{\theta} E \frac{WW \mathcal{L}^{++}}{(\Sigma^{++})^2}, \quad E^{-1} = \text{Ber}(E_A^M),
$$

(2.2)

with $\Sigma^{++}(v) := \Sigma^{ij} v_i v_j$. Here the Lagrangian $\mathcal{L}^{++}(z, v)$ is a covariant real projective multiplet of weight two. The vector multiplet $W$ is used as a supergravity compensator and $\Sigma^{ij}$ is defined as in (1.1). As shown in [9], the action (2.2) can be written as an integral over the chiral subspace

$$
S(\mathcal{L}^{++}) = \int d^4x d^4\theta E W \mathbb{W},
$$

$$
\mathbb{W} = \frac{1}{8\pi} \oint_C v^i dv_i \left( (\bar{\mathcal{D}}^-)^2 + 4\bar{S}^{--} \right) \mathbb{V}, \quad \mathbb{V} := \mathcal{L}^{++} \Sigma^{++}.
$$

(2.3)
It was also proved in [9] that $W$ is a restricted chiral superfield.

We now give an alternative chiral representation for the action (2.2) in the case that $L^{++}$ has the form

$$L^{++} = G^{++} L,$$  \hspace{1cm} (2.4)

where $G^{++}$ is a tensor multiplet, and $L$ is a real weight-zero projective multiplet,

$$D^+ \alpha L = \bar{D}^+ \dot{\alpha} L = 0, \quad L(c v^i) = L(v^i), \quad \bar{L} = L.$$  \hspace{1cm} (2.5)

Rewriting $G^{++}$ in terms of its chiral potential $\Psi$ (1.7),

$$G^{++}(v) = \frac{1}{4} \left( (D^+)^2 + 4 S^{++} \right) \Psi + \frac{1}{4} \left( (\bar{D}^+)^2 + 4 \bar{S}^{++} \right) \bar{\Psi}, \quad \bar{D}^i \dot{\alpha} \Psi = 0.$$  \hspace{1cm} (2.6)

one may rearrange (2.2) into the form

$$S(G^{++} L) = \frac{1}{2 \pi} \oint_C v^i d v_i \int d^4 x d^4 \theta d^4 \bar{\theta} E \frac{\Psi \bar{W} \bar{L} \Sigma^{++}}{\Sigma^{++}} + c.c.$$  \hspace{1cm} (2.7)

This can be reduced to a chiral integral by acting with the chiral projector $\bar{\Delta}$ defined by eq. (B.4) (see [9] for a detailed derivation)

$$S(G^{++} L) = \frac{1}{2 \pi} \int d^4 x d^4 \theta \bar{E} \bar{\Delta} \oint_C v^i d v_i \frac{\Psi \bar{W} \bar{L} \Sigma^{++}}{\Sigma^{++}} + c.c.$$  \hspace{1cm} (2.8)

Now, it only remains to make use of an important representation discovered in [9]. Specifically, given an arbitrary isotwistor superfield $U^{(-2)}(z, v)$ of weight $-2$ (see [1] for the definition of isotwistor supermultiplets), it was shown in [9] that

$$\bar{\Delta} \oint C v^i d v_i U^{(-2)} = \frac{1}{16} \oint C v^i d v_i \left( (D^-)^2 + 4 S^{-} \right) \left( (\bar{D}^+)^2 + 4 \bar{S}^{++} \right) U^{(-2)}.$$  \hspace{1cm} (2.9)

Applying this representation to (2.7) gives

$$S(G^{++} L) = \int d^4 x d^4 \theta E \bar{W} \bar{W} + c.c.$$  \hspace{1cm} (2.10)

where we have introduced the following composite chiral superfield:

$$\bar{W} = \frac{1}{8 \pi} \oint C v^i d v_i \left( (D^-)^2 + 4 S^{-} \right) \bar{L}, \quad \bar{D}^i \bar{W} = 0.$$  \hspace{1cm} (2.11)

Because $\Psi$ is defined only up to gauge transformations

$$\delta \Psi = i \Lambda, \quad \bar{D}^i \dot{\alpha} \Lambda = \left( D^a (i D^a) + 4 S^{ij} \right) \Lambda - \left( \bar{D}^a (i \bar{D}^a) + 4 \bar{S}^{ij} \right) \bar{\Lambda} = 0,$$  \hspace{1cm} (2.12)

the superfield $\bar{W}$ must indeed be reduced chiral,

$$\left( D^a (i D^a) + 4 S^{ij} \right) \bar{W} = \left( \bar{D}^a (i \bar{D}^a) + 4 \bar{S}^{ij} \right) \bar{W}.$$  \hspace{1cm} (2.13)

We present a more direct proof of this result in Appendix E.
3 The improved tensor multiplet

The improved $\mathcal{N} = 2$ tensor multiplet \cite{31, 32} is a unique theory of the $\mathcal{N} = 2$ tensor multiplet which is superconformal in the rigid supersymmetric case, and super-Weyl invariant in the presence of conformal supergravity. This theory is a natural generalization of the improved $\mathcal{N} = 1$ tensor multiplet \cite{33}. It occurs as a second compensator in the minimal formulation of $\mathcal{N} = 2$ Poincaré supergravity proposed by de Wit, Philippe and Van Proeyen \cite{31}. Within the projective-superspace formulation for $\mathcal{N} = 2$ supergravity \cite{1, 2, 3}, the Lagrangian for the tensor compensator is

$$\mathcal{L}^{++} = G^{++} \ln \left( \frac{G^{++}}{i\Upsilon^+ \tilde{\Upsilon}^+} \right),$$

see \cite{2} for more details. Here $\Upsilon^+$ is a weight-one arctic hypermultiplet, and $\tilde{\Upsilon}^+$ its smile-conjugate. As shown in \cite{2, 9}, the superfields $\Upsilon^+$ and $\tilde{\Upsilon}^+$ are pure gauge degrees of freedom in the sense that they do not contribute to the tensor compensator action. Using the formulation (2.10) and (2.11), the tensor compensator action can be rewritten as a chiral action of the form

$$S = \int d^4x \, d^4\theta \, \mathcal{E} \Psi \mathcal{W} + \text{c.c.},$$

where $\mathcal{W}$ denotes the following reduced chiral scalar

$$\mathcal{W} = \frac{1}{8\pi} \oint_C v^i dv_i \left( (\bar{D}^-)^2 + 4\tilde{S}^{--} \right) \ln \left( \frac{G^{++}}{i\Upsilon^+ \tilde{\Upsilon}^+} \right).$$

(3.3)

It can be seen that the arctic multiplet $\Upsilon^+$ and its conjugate $\tilde{\Upsilon}^+$ do not contribute to the contour integral, and so they will be ignored below. Our goal in this section is to evaluate (3.3) as well as to study the properties of $\mathcal{W}$.

3.1 Evaluation of the contour integral

We begin by evaluating the derivatives in (3.3):

$$\mathcal{W} = \frac{1}{8\pi} \oint_C v^i dv_i \left( \frac{(\bar{D}^-)^2 G^{++}}{G^{++}} - \bar{D}^- G^{++} \bar{D}^- G^{++} \right) + 4\tilde{S}^{--} \ln G^{++}.$$

(3.4)

It follows from first principles that (3.3) is invariant under the projective transformations (1.13) and therefore independent of $u_i$. We wish to show explicitly that all of the $u$-dependence in the integrand (3.4) can be eliminated using the properties of
the tensor multiplet. First, using the constraints (1.6) and the (anti)commutation relations (A.3b), one can show that

\[
\bar{D}_{\dot{a}} G^{++} = \frac{2}{3} \bar{\chi}^{++} \equiv \frac{2}{3} \bar{\chi}^{i} v_{i} ,
\]

\[(D^{-})^{2} G^{++} + 8 \bar{S}^{--} G^{++} - 4 \bar{S}^{--} \partial^{-} G^{++} = \frac{1}{3} \bar{M} ,
\]

where we have defined

\[
\bar{\chi}^{\dot{a}i} := \bar{D}_{\dot{a}} G^{\dot{a}i} ,
\]

\[\bar{M} := (\bar{D}_{jk} + 12 \bar{S}_{jk}) G^{jk} .
\]

Using these formulae, we rewrite \(W\) as

\[ W = \frac{1}{8\pi} \oint_{C} v^{i} dv_{i} \left( \frac{1}{3} \bar{M} G^{++} - \frac{4}{9} (G^{++})^{2} + 4 \partial^{--} \left( \bar{S}^{++} \log G^{++} \right) - 8 \bar{S}^{--} \right) .
\]

The terms involving \(\bar{S}\) explicitly in this expression can be rearranged into

\[ \frac{1}{8\pi} \oint_{C} v^{i} dv_{i} \partial^{--} \left( 4 \bar{S}^{--} \log G^{++} - 8 \bar{S}^{--} \right)
\]

and so they vanish using integration by parts (see Appendix D for technical details). We are left with

\[ W = \frac{1}{8\pi} \oint_{C} v^{i} dv_{i} \left( \frac{1}{3} \bar{M} G^{++} - \frac{4}{9} (G^{++})^{2} \right) .
\]

The contour integral in (3.9) can be evaluated in a manifestly SU(2) covariant way using the technique described in Appendix D. Making use of the result (D.20) leads to

\[ W = -\frac{1}{24G} \bar{M} + \frac{1}{36G^{3}} \bar{\chi}^{i} \bar{\chi}^{j} G_{ij} ,
\]

where

\[ G^{2} := \frac{1}{2} G^{ij} G_{ij} .
\]

This expression (up to normalizations) was discovered originally in [31] using the superconformal tensor calculus. It was later reconstructed in curved superspace [28] with the aid of the results in [31] and [29]. Its contour origin was explored in the globally supersymmetric case by Siegel [29]. Here we have derived it from superfield supergravity and shown its contour integral origin, eq. (3.3), for the first time.
A curious feature of the reduced chiral superfield (3.10) is that it can be rewritten in a more elegant and compact form

\[ \mathcal{W} = -\frac{G}{8} (\mathcal{D}_{ij} + 4S_{ij}) \left( \frac{G^{ij}}{G^2} \right). \]  

(3.12)

Its rigid supersymmetric version appeared in our recent work on \( \mathcal{N} = 2 \) supercurrents [34]. It is a laborious task to check that this does indeed match (3.10) in supergravity by straightforwardly applying the properties of the tensor multiple. We will demonstrate the equivalence of (3.12) to (3.10) via an indirect route, through the use of super-Weyl transformations [14, 3]. Using the representation (3.12), it is easy to show that the super-Weyl transformation law of \( \mathcal{W} \) coincides with that of the vector multiplet field strength.

### 3.2 Super-Weyl gauge fixing with a tensor multiplet

Within Howe’s formulation for \( \mathcal{N} = 2 \) conformal supergravity [14], the super-Weyl transformation law of the spinor covariant derivatives is

\[ \delta \mathcal{D}^i_\alpha = \frac{1}{2} \sigma \mathcal{D}^i_\alpha + 2(\mathcal{D}^i_\sigma)M_{\gamma\alpha} - 2(\mathcal{D}_{ak}\sigma)J^{ki} - \frac{1}{2}(\mathcal{D}_a\sigma)J, \]  

(3.13a)

\[ \delta \bar{\mathcal{D}}^{\dot{\alpha}}_i = \frac{1}{2} \sigma \bar{\mathcal{D}}^{\dot{\alpha}}_i + 2(\bar{\mathcal{D}}^{\dot{\alpha}}_i\sigma)\bar{M}_{\dot{i}\dot{\alpha}} + 2(\bar{\mathcal{D}}^{\dot{\alpha}}_\sigma)\bar{J}^{\dot{k}i} + \frac{1}{2}(\bar{\mathcal{D}}_{\dot{\alpha}}\sigma)\bar{J}, \]  

(3.13b)

where the transformation parameter \( \sigma \) is real unconstrained, and \( M_{\alpha\beta}, J_{ij} \) and \( J \) denote the Lorentz, SU(2)\(_R\) and U(1)\(_R\) generators respectively, see [3] and Appendix A for more details. The super-Weyl transformations of the torsion and the curvature are given in [3]. The field strengths of the vector multiplet \( W \) and the tensor multiplet \( G^{ij} \) transform as primary fields,

\[ \delta_\sigma W = \sigma W, \]  

(3.14a)

\[ \delta_\sigma G^{ij} = 2\sigma G^{ij}. \]  

(3.14b)

The composite reduced chiral superfield (3.12) transforms as the vector multiplet field strength.

In \( \mathcal{N} = 2 \) supergravity, one of the two conformal compensators is usually a vector multiplet \( W \) (see, e.g., [31] for more details), \( W \neq 0 \). Usually, the super-Weyl invariance is fixed by imposing a condition on \( W \). Specifically, one may use the super-Weyl and local U(1)\(_R\) freedom to impose the gauge condition

\[ W = 1. \]  

(3.15)
This amounts to switching off the spinor $U(1)_R$ connections,

$$\Phi^i_\alpha = \Phi^i_\dot{\alpha} = 0 \ ,$$  \hspace{1cm} (3.16)

along with certain restrictions [14, 3] on the torsion, namely

$$G^{i\dot{j}}_{\alpha\dot{\beta}} = 0 \ , \quad S^{i\dot{j}} = \bar{S}^{i\dot{j}} \ ,$$  \hspace{1cm} (3.17)

as well as identifying the torsion $G^{i\dot{j}}_{\alpha\dot{\beta}}$ with the $U(1)_R$ vector connection

$$G_{\alpha\dot{\beta}} = \Phi_{\alpha\dot{\beta}} \ .$$  \hspace{1cm} (3.18)

If the second conformal compensator is a tensor multiplet $G^{ij}$, with $G^2 \neq 0$, we may instead choose this multiplet to fix the super-Weyl gauge freedom. Specifically, we may use the super-Weyl gauge freedom to impose the condition

$$G^2 = \frac{1}{2} G^{ij} G_{ij} = 1 \ .$$  \hspace{1cm} (3.19)

This fixes $G^{ij}$ to be a unit isovector subject to the tensor multiplet constraints (1.6). Actually, these constraints are sufficient to enforce that $G^{ij}$ is covariantly chiral and antichiral. To prove the former, note that (1.6) is equivalent to

$$\mathcal{D}^i_\alpha G_{jk} = \frac{2}{3} \delta^{ij}_k \mathcal{D}^m G_{km} \ .$$  \hspace{1cm} (3.20)

It follows that

$$G_{ij} \mathcal{D}^j_\alpha G^2 = G_{ij} G^{kl} \mathcal{D}^j_\alpha G_{kl} = \frac{2}{3} G^2 \mathcal{D}^j_\alpha G_{ji} \ .$$  \hspace{1cm} (3.21)

When we set $G^2 = 1$, (3.21) must vanish, which in turn implies that (3.20) must vanish as well. Because $G^{ij}$ is real, we may conclude that it must be both chiral and antichiral in this gauge.

These conditions have a number of interesting consequences for the superspace geometry. In accordance with (A.3a), the consistency condition for $G^{ij}$ being chiral reads

$$0 = \{\mathcal{D}^i_\alpha, G^{ij} \} G^{kl} = 2 \epsilon_{\alpha\dot{\beta}} \epsilon^{ij} S^{mn} J_{mn} G^{kl} + 4Y_{\alpha\dot{\beta}} J^{ij} G^{kl} \ .$$  \hspace{1cm} (3.22)

This condition and its conjugate imply

$$Y_{\alpha\dot{\beta}} = 0 \ , \quad \bar{Y}_{\dot{\alpha}\dot{\beta}} = 0 \ ;$$  \hspace{1cm} (3.23)

$$S^{ij} \propto G^{ij} \ , \quad \bar{S}^{i\dot{j}} \propto \bar{G}^{i\dot{j}} \ .$$  \hspace{1cm} (3.24)
Because $Y_{\alpha\beta}$ vanishes, $S^{ij}$ is now completely antichiral

$$D^k_{\alpha}S^{ij} = 0 ,$$

(3.25)
due to the dimension-3/2 Bianchi identities \([A.9]\). We conclude that

$$S^{ij} = -\bar{\phi} G^{ij} , \quad \bar{S}^{ij} = -\phi G^{ij} ,$$

(3.26)

where $\phi$ is some chiral scalar.

Furthermore, in accordance with eq. \([A.3b]\), the consistency condition for $G^{ij}$ being both chiral and antichiral reads

$$0 = \{D^i_{\alpha}, \bar{D}^j_{\beta}\} G^{kl} = -2i\delta^i_j D^i_{\alpha\beta} G^{kl} + 8G^{i\beta}J^j_k G^{kl} - 4i\delta^i_j G^{\alpha\beta} J^{mn} G^{kl} .$$

(3.27)

This implies

$$G_{\alpha\dot{\alpha}} = 0 ,$$

(3.28)
as well as

$$D^i_{\alpha\dot{\alpha}} G^{ij} = 4G_{\alpha\dot{\alpha}}^{k(i} G^{j)}_{k} .$$

(3.29)

This last condition is quite interesting. If we now use the SU(2)$_R$ gauge freedom to fix $G^{ij}$ to a constant unit isovector, we find

$$-2\Phi_{\alpha\dot{\alpha}}^{k(i} G^{j)}_{k} = 4G_{\alpha\dot{\alpha}}^{k(i} G^{j)}_{k}$$

(3.30)

which implies that the vector SU(2)$_R$ connection is $-2G^{ij}_{\alpha\dot{\alpha}}$ up to terms which are proportional to $G^{ij}$.

Thus we have some rather interesting structure\[1\]. Imposing the super-Weyl gauge \((3.19)\) eliminates $G_{\alpha\dot{\alpha}}$ and $Y_{\alpha\beta}$, as well as implying the relation \((3.26)\) for chiral $\phi$. In fact, the field $\phi$ is actually reduced chiral. Making use of the algebra of covariant derivatives as well as the dimension-3/2 Bianchi identities \([A.9]\), one may show that (in this gauge)

$$D^i_{ij} S^{kl} - \bar{D}^i_{ij} S^{kl} = -4S^{ij} S^{kl} + 4S^{ijkl} S^{ij} .$$

(3.31)

Contracting with $G_{kl}/2$ gives

$$D^i_{ij} \phi - \bar{D}^i_{ij} \bar{\phi} = -4S^{ij} \phi + 4\bar{S}^{ij} \bar{\phi}$$

(3.32)

\[1\]An analogue of the super-Weyl gauge \((3.19)\) naturally occurs in $N = 3$ and $N = 4$ supergravity in three dimensions \([35]\). Implications of such a gauge fixing are highly nontrivial in the case of $N = 4$ supergravity.
which shows $\phi$ to indeed be reduced chiral.

This formulation allows us to show that (3.12) is indeed (3.10) and that this is reduced chiral. Equality follows since both expressions have identical super-Weyl transformation properties and reduce in the gauge $G^2 = 1$ to

$$\mathbb{W} = -\frac{1}{2} \bar{S}_{ij} G^{ij} = \phi.$$  \hfill (3.33)

Moreover, since $\phi$ is reduced chiral, both (3.10) and (3.12) must be as well, since $\mathbb{W}$ changes as a primary field under the super-Weyl transformations,

$$\delta_\sigma \mathbb{W} = \sigma \mathbb{W}. \hfill (3.34)$$

### 4 Self-interacting tensor multiplets

The improved tensor multiplet is the unique super-Weyl invariant action available which involves a single tensor multiplet. In the case of several tensor multiplets $G_{A}^{++}$, we can consider a locally supersymmetric and super-Weyl invariant action generated by the Lagrangian

$$\mathcal{L}^{++} = G_{A}^{++} \mathcal{F}^{A}(G_{B}^{++}), \hfill (4.1)$$

where $\mathcal{F}^{A}$ is a homogeneous function of degree zero, $\mathcal{F}^{A}(c G_{B}^{++}) = \mathcal{F}^{A}(G_{B}^{++})$. This Lagrangian is simply a superposition of several terms of the form (2.4). Associated with this Lagrangian are reduced chiral superfields

$$\mathbb{W}^{A} = \frac{1}{8\pi} \int_{C} v^{i} dv_{i} \left( (\bar{D}^{-})^2 + 4\bar{S}^{--} \right) \mathcal{F}^{A}(G_{B}^{++}). \hfill (4.2)$$

Evaluating the derivatives on $\mathcal{F}^{A}$ is a nearly identical procedure to what we considered in the previous section with a single tensor superfield. It is straightforward to find

$$\mathbb{W}^{A} = \frac{1}{8\pi} \mathcal{F}^{A,B} \bar{M}_{B} + \frac{4}{9} \mathcal{F}^{A,B,C}_{ij} \bar{\chi}^{i}_{B} \bar{\chi}^{j}_{C}, \hfill (4.3)$$

where $\bar{\chi}^{i}_{B}$ and $\bar{M}_{B}$ are defined as in (3.6a) and (3.6b), with $G^{ij}$ replaced by $G^{ij}_{B}$, and

$$\mathcal{F}^{A,B}_{ij} = \frac{1}{8\pi} \int_{C} v^{j} dv_{j} \frac{\partial \mathcal{F}^{A}}{\partial G^{++}_{B}} \hfill (4.4)$$

$$\mathcal{F}^{A,B,C}_{ij} = \frac{1}{8\pi} \int_{C} v^{k} dv_{k} v_{i} v_{j} \frac{\partial^2 \mathcal{F}^{A}}{\partial G^{++}_{B} \partial G^{++}_{C}} \hfill (4.5)$$
It is worth noting that the second of these expressions can be written as

$$ F^{A,B,C}_{ij} = \frac{\partial F^{A,B}}{\partial G^{ij}_C}. \quad (4.6) $$

The reduced chiral superfield \((4.3)\) was first constructed a few years ago by de Wit and Saueressig \[30\]. They considered \(F^{A,B}\) as a general function obeying certain consistency conditions and defined \(F^{A,B,C}_{ij}\) via \((4.6)\). Here we will show that their consistency conditions are indeed satisfied by the construction \((4.4)\).

The first set of consistency conditions are

$$ F^{A,B,C}_{ij} = F^{A,C,B}_{ij}, \quad \epsilon^{jk} F^{A,B,C}_{ij} D^{kl} = 0 \quad (4.7) $$

and it is very easy to see that these are satisfied by \((4.5)\). The first of them is trivial, while the second follows from

$$ F^{A,B,C}_{ij} D^{kl} = \frac{1}{8\pi} \int_C v^m d v^m v_i v_j v_k v_l \frac{\partial^3 F^A}{\partial G^{++}_B \partial G^{++}_C \partial G^{++}_D} \quad (4.8) $$

and the property that \(\epsilon^{jk} v_j v_k = 0\).

The second set of conditions, required for the superconformal case, are\(^2\)

$$ F^{A,B,C}_{ik} G^{ki}_C = -F^{A,B}, \quad (4.9) $$

$$ F^{A,B,C}_{k} (G^{C}_{ij}) = 0. \quad (4.10) $$

The first of these follows from the fact that \(F^A\) is a homogeneous function of degree zero and so

$$ \frac{\partial F^A}{\partial G^{++}_C} G^{++}_C = 0. \quad (4.11) $$

Taking a derivative with respect to \(G^{++}_B\), we find

$$ \frac{\partial^2 F^A}{\partial G^{++}_B \partial G^{++}_C} G^{++}_C = -\frac{\partial F^A}{\partial G^{++}_B}. \quad (4.12) $$

Then \((4.9)\) follows since

$$ F^{A,B,C}_{ik} G^{ki}_C = \frac{1}{8\pi} \int_C v^k d v_k G^{++}_C \frac{\partial^2 F^A}{\partial G^{++}_B \partial G^{++}_C} = -F^{A,B}. \quad (4.13) $$

\(^2\)These were originally written in \[30\] as a single equation, but here we will consider them separately.
using \([4.12]\). To prove \((4.10)\) is a little trickier. One may begin by introducing a fixed isotwistor \(u_i\) and writing the condition as

\[
\int_C v^k d v_k u^i u^j v_k G_{C_j}^k = 0.
\]

Then we note that

\[
\int_C v^k d v_k (v, u)^2 G_{C}^{++} \frac{\partial^2 F^{A}}{\partial G_{B}^{++} \partial G_{C}^{++}} = \frac{1}{16} \int_C v^k d v_k \partial^{-} \left( (v, u)^2 \frac{\partial F^{A}}{\partial G_{B}^{++}} \right) \quad (4.14)
\]

The right-hand side is a total contour derivative and so it must vanish, which implies \((4.10)\).

5 Adding \(O(2n)\) multiplets

In this section we construct reduced chiral multiplets out of \(O(2n)\) multiplets.

5.1 The case of a single \(O(2n)\) multiplet

We consider next a more general projective Lagrangian of the form

\[
\mathcal{L}^{++} = \frac{Q^{(2n)}}{(G^{++})^{n-1}} = G^{++} \frac{Q^{(2n)}}{(G^{++})^n},
\]

where \(Q^{(2n)}\) is a real covariant \(O(2n)\) multiplet having the functional form

\[
Q^{(2n)} = Q^{i_1 \cdots i_{2n}} v_{i_1} \cdots v_{i_{2n}}, \quad (Q^{i_1 \cdots i_{2n}})^* = Q_{i_1 \cdots i_{2n}} \quad (5.2)
\]

and obeying the analyticity constraints

\[
\mathcal{D}^{+}_a Q^{(2n)} = \mathcal{D}^{+}_{\dot{a}} Q^{(2n)} = 0. \quad (5.3)
\]

We note that \(Q^{(2)} \equiv Q^{++}\) is a tensor multiplet.

The reduced chiral superfield which we construct from (5.1) is

\[
\mathbb{W}_n = \frac{1}{8\pi} \int_C v^i d v_i \left( (\mathcal{D}^-)^2 + 4 \mathcal{S}^{--} \right) \left( \frac{Q^{(2n)}}{(G^{++})^n} \right). \quad (5.4)
\]

Expanding this out gives

\[
\mathbb{W}_n = \frac{1}{8\pi} \int_C v^i d v_i \left\{ \frac{(\mathcal{D}^-)^2 Q^{(2n)}}{(G^{++})^n} - 2n \mathcal{D}^{+}_a Q^{(2n)} \mathcal{D}^{+}_{\dot{a}} G^{++} \right. \\
- n Q^{(2n)} (\mathcal{D}^-)^2 G^{(2)} \right. \\
+ n(n + 1) Q^{(2n)} \mathcal{D}^{+}_a G^{++} \mathcal{D}^{+}_{\dot{a}} G^{++} + 4 \mathcal{S}^{--} \left( \frac{Q^{(2n)}}{(G^{++})^n} \right) \left(5.5\right)
\]
As before, it turns out that all of the explicit $u$-dependence in this expression can be removed using the analyticity properties. For a general $O(2n)$ multiplet, it is a straightforward exercise to show that

\[ \mathcal{D}_\alpha Q^{(2n)} = \frac{2n}{2n + 1} \eta^{(2n-1)}_\alpha, \]  

(5.6a)

\[ (\mathcal{D}^-)^2 Q^{(2n)} + 8nS^{--}Q^{(2n)} - 4S^{-+} \partial^{--} Q^{(2n)} = \left( \frac{2n - 1}{2n + 1} \right) \mathcal{H}^{(2n-2)}, \]  

(5.6b)

where

\[ \eta^{(2n-1)} := \mathcal{D}_k Q^{k i_1 \cdots i_{2n-1} v_{i_1} \cdots v_{i_{2n-1}}}, \]  

(5.7a)

\[ \mathcal{H}^{(2n-2)} := (\mathcal{D}_{jk} + 4(2n + 1) S_{jk}) Q^{j k i_1 \cdots i_{2n-2} v_{i_1} \cdots v_{i_{2n-2}}}. \]  

(5.7b)

These are, of course, generalizations of equations in subsection 3.1 involving the $O(2)$ multiplet $G^{++}$. We will need those other results, too. Applying these relations, we find

\[ \mathbb{W}_n = \frac{1}{8\pi} \int_C v^i dv_i \left\{ \frac{2n - 1}{2n + 1} \mathcal{H}^{(2n-2)} - \frac{8n^2}{3(2n + 1)} \eta^{(2n-1)} \bar{\chi}^+ \right. 
\]

\[ - \frac{n}{3} \bar{M} Q^{(2n)} (G^{++})^{n+1} + \left. \frac{4n(n + 1)}{9} \frac{Q^{(2n)} \bar{\chi}^+ \bar{\chi}^+}{(G^{++})^{n+2}} \right\}, \]  

(5.8)

where we have again eliminated a total derivative term

\[ -\frac{1}{8\pi} \int_C v^i dv_i \partial^{--} \left( 4S^{-+} \frac{Q^{(2n)}}{(G^{++})^n} \right) = 0. \]  

(5.9)

Now we apply eq. (D.20) to each of the four terms in (5.8). The result is a rather unwieldy expression:

\[ \mathbb{W}_n = -\frac{(2n)!}{2^{2n+1} (n!)^2} \left\{ \frac{n}{2(2n + 1)} \frac{\mathcal{H}^{i_1 \cdots i_{2n-2} G_{(i_1 i_2} \cdots G_{i_{2n-3} i_{2n-2})}}{G^{2n-1}} \right. 
\]

\[ - \frac{2n^2}{3(2n + 1)} \frac{\eta^{i_1 \cdots i_{2n-1} \bar{\chi}^{i_2 n} G_{(i_1 i_2} \cdots G_{i_{2n-1} i_{2n})}}{G^{2n+1}} \right. 
\]

\[ - \frac{n}{12} \frac{\bar{M} Q^{i_1 \cdots i_{2n} G_{(i_1 i_2} \cdots G_{i_{2n-1} i_{2n})}}{G^{2n+1}} \right. 
\]

\[ + \left. \frac{n(2n + 1)}{18} \frac{Q^{i_1 \cdots i_{2n} \bar{\chi}^{i_{2n+1}} \bar{\chi}^{i_{2n+2}} G_{(i_1 i_2} \cdots G_{i_{2n+1} i_{2n+2})}}{G^{2n+3}} \right\}. \]  

(5.10)

However, as with the improved tensor action, there is a simpler, more compact expression which is equivalent to this one:

\[ \mathbb{W}_n = -\frac{(2n)!}{2^{2n+2} (n + 1)! (n - 1)!} G (\mathcal{D}_{ij} + 4 \bar{S}_{ij}) R_n^{ij}, \]  

(5.11)
where
\[ R_{ij}^n = \left( \delta_{ij}^{kl} - \frac{1}{2G_{ij}G_{kl}} \right) Q^{kl i_1 \cdots i_{2n-2}} G_{i_1i_2} \cdots G_{i_{2n-3}i_{2n-2}} G^{-2n}. \] (5.12)

The expression for \( W_n \) has an overall structure quite similar to the improved tensor action result (3.12), except the argument \( R_{ij}^n \) of the derivative is much more complicated.

Thankfully, many of these complications may be easily understood. The factor in parentheses in (5.12) is simply an orthogonal projector on the \( ij \) indices; it ensures in particular that if we choose \( Q^{(2n)}_n = (G^{++})^n \), we get \( W_n = 0 \), which follows trivially from the contour integration. Furthermore, a nontrivial check on the combinatoric factors of (5.11) can be made by considering the replacement \( Q^{(2n)}_n \rightarrow Q^{(2n-2)}_n G^{++} \), under which we ought to find \( W_n \rightarrow W_{n-1} \). (This is obvious by considering the original expression (5.4) for \( W_n \).) This is a straightforward combinatoric exercise, the most difficult step of which is to make the replacement in (5.12) of \( Q_{kl i_1 \cdots i_{2n-2}} \) with
\[ \frac{G^{kl} Q^{i_1 \cdots i_{2n-2}}}{2n^2 - n} + \frac{2n - 2}{2n^2 - n} \left( G^{k(i_{2n-2} \cdots i_{2n-3}) l} + G^{l(i_{2n-2} \cdots i_{2n-3}) k} \right) + \frac{(n-1)(2n-3)}{2n^2 - n} G^{(i_{2n-3} \cdots i_{2n-2}) l} Q^{i_1 \cdots i_{2n-4}} G^{(i_{2n-5} \cdots i_{2n-4}) k} \] (5.13)
The first term vanishes when contracted with the orthogonal projector. The second and third terms, when contracted with \( G_{i_1i_2} \cdots G_{i_{2n-3}i_{2n-2}} G^{-2n} \) sum to
\[ \frac{2(n^2 - 1)}{2n^2 - n} Q^{kl i_1 \cdots i_{2n-4}} G_{i_1i_2} \cdots G_{i_{2n-5}i_{2n-4}} G^{-2(n-1)} \] (5.14)
the numeric prefactor of which is exactly right to convert the expression for \( W_n \) to that of \( W_{n-1} \).

Our \( O(2n) \) multiplet \( Q^{(2n)} \) may be an independent dynamical variable. If \( n > 1 \), it may be chosen instead to be a composite field. For instance, we can choose \( Q^{(2n)} = Q^{(2m)}_1 Q^{(2n-2m)}_2 \), with \( Q^{(2m)}_1 \) and \( Q^{(2n-2m)}_2 \) being \( O(2m) \) and \( O(2n-2m) \) multiplets respectively, \( m = 1, \ldots, n-1 \). Another option is to realize \( Q^{(2n)} \) as a product of \( n \) tensor multiplets \( H^{A++} \),
\[ Q^{(2n)} = H^{1++} \cdots H^{n++}. \] (5.15)

In the case \( n = 2 \) choosing \( Q^{(4)} = (H^{++})^2 \) leads to the Lagrangian
\[ \mathcal{L}^{++} = \frac{(H^{++})^2}{G^{++}} \] (5.16)
which is a curved-superspace version of that proposed in \[36, 37\] to describe the classical universal hypermultiplet \[38\]. Using that result, we have

\[
\frac{1}{8\pi} \oint_C v^i dv_i \left( (\mathcal{D}^-)^2 + 4\mathcal{S}^{--} \right) \left( \frac{H^{++}}{G^{++}} \right)^2 = -\frac{G}{16} (\mathcal{D}_{ij} + 4\mathcal{S}_{ij}) \mathcal{R}^{ij}_2 ,
\]

where

\[
\mathcal{R}^{ij}_2 = \frac{1}{G^4} \left( \nu^{ij}_{kl} - \frac{1}{2G^2} G^{ij} G_{kl} \right) H^{(kl} H^{mn)} G_{mn} .
\]

In the simplest case \(n = 1\), \(Q^{(2)} \equiv H^{++}\) we get

\[
\mathcal{W}_1 = \frac{1}{8\pi} \oint_C v^i dv_i \left( (\mathcal{D}^-)^2 + 4\mathcal{S}^{--} \right) \frac{H^{++}}{G^{++}} = -\frac{G}{16} (\mathcal{D}_{ij} + 4\mathcal{S}_{ij}) \mathcal{R}^{ij}_1 ,
\]

where

\[
\mathcal{R}^{ij}_1 = \frac{1}{G^2} \left( H^{ij} - G^{ij} \frac{G \cdot H}{G^2} \right) , \quad G \cdot H := \frac{1}{2} G^{kl} H_{kl} .
\]

The reduced chiral scalar \(\mathcal{W}_1\) is such that

\[
\int d^4 x d^4 \theta \mathcal{E} \Psi \mathcal{W}_1 + \text{ c.c.} = 0 ,
\]

as a consequence of the identity (see eq. (4.61) in \[9\])

\[
\oint_C v^i dv_i \int d^4 x d^4 \theta d^4 \bar{\theta} E \frac{W \bar{W}}{(\Sigma^{++})^2} H^{++} = 0 .
\]

### 5.2 Generalization to several \(\mathcal{O}(2n)\) multiplets

We consider next a more general projective Lagrangian constructed out of a set of \(\mathcal{O}(2n_A)\) multiplets \(Q^{(2n_A)}_A\) and at least one tensor multiplet \(G^{++}\), specifically

\[
\mathcal{L}^{++} = G^{++} F(Q^{(2n_A)}_A) .
\]

The function \(F\) is required to be a homogeneous function of degree zero in \(v^i\); this implies

\[
F(c^{n_A} Q^{(2n_A)}_A) = F(Q^{(2n_A)}_A) .
\]

This construction is a generalization of that presented in Section \[4\] which involved only \(\mathcal{O}(2)\) multiplets.\(^3\)

\(^3\)In Section \[4\] an index \(A\) was placed on the function \(F\) and the tensor multiplet \(G^{++}\) in the corresponding construction \([11]\) to match the notation in \([30]\). Here we leave such an index off and consider only a single function \(F\) for simplicity.
The reduced chiral superfield which we construct from (5.23) is
\[ W = \frac{1}{8\pi} \oint_{C} v^{i} dv_{i} \left( (\mathcal{D}^{-})^{2} + 4\mathcal{S}^{-} \right) \mathcal{F}(Q_{A}^{(2n_{A})}) . \] (5.25)

Expanding this out and applying (5.7a) and (5.7b) gives
\[ W = \frac{2n_{B} - 1}{2n_{B} + 1} F^{A}_{i_{1} \cdots i_{2n_{A}}-2} \hat{H}_{A}^{i_{1} \cdots i_{2n_{A}}-2} + \left( \frac{2n_{A}}{2n_{A} + 1} \right) \left( \frac{2n_{B}}{2n_{B} + 1} \right) F^{AB}_{i_{1} \cdots i_{2(n_{A}+n_{B}-1)}} \eta_{A}^{i_{1} \cdots i_{2n_{A}}-1} \eta_{B}^{i_{1} \cdots i_{2n_{A}}-2(n_{A}+n_{B}-1)} \] (5.26)

where
\[ F^{A}_{i_{1} \cdots i_{2n_{A}}-2} := \frac{1}{8\pi} \oint_{C} v^{k} dv_{k} \frac{\partial F}{\partial Q_{A}^{(2n_{A})}} v_{i_{1}} \cdots v_{i_{2n_{A}}-2} \] (5.27)
\[ F^{AB}_{i_{1} \cdots i_{2(n_{A}+n_{B}-1)}} := \frac{1}{8\pi} \oint_{C} v^{k} dv_{k} \frac{\partial^{2} F}{\partial Q_{A}^{(2n_{A})} \partial Q_{B}^{(2n_{B})}} v_{i_{1}} \cdots v_{i_{2(n_{A}+n_{B}-1)}} \] (5.28)

are both totally symmetric in their isospin indices, and \( \eta_{A} \) and \( \hat{H}_{A} \) are as defined in (5.7a) and (5.7b) respectively, with \( Q \) replaced by \( Q_{A} \). It is worth noting that the second of these expressions can be written
\[ F^{AB}_{i_{1} \cdots i_{2n_{A}}-2 j_{1} \cdots j_{2n_{B}}} = \frac{\partial F^{A}_{i_{1} \cdots i_{2n_{A}}-2}}{\partial Q_{B}^{j_{1} \cdots j_{2n_{B}}}} . \] (5.29)

6 Discussion

In this paper we have proposed a construction to generate reduced chiral superfields from covariant projective multiplets, including tensor multiplets, \( \mathcal{O}(2n) \) multiplets, etc. It is given by the relation (2.11). In conjunction with the standard construction to derive \( \mathcal{N} = 2 \) linear multiplets from vector ones, eq. (1.9), we are now able to generate nontrivial higher derivative couplings. For simplicity, we illustrate the idea by considering models with vector and tensor multiplets.

We can start from a system of tensor multiplets \( G_{A}^{++} \), where \( A = 1, \ldots, n \), and introduce a function \( \mathcal{F}_{\text{tensor}}(G_{A}^{++}) \) which is homogeneous of degree zero,
\[ G_{A}^{++} \frac{\partial}{\partial G_{A}^{++}} \mathcal{F}_{\text{tensor}} = 0 . \] (6.1)

Then the following superfield
\[ W = \frac{1}{8\pi} \oint_{C} v^{i} dv_{i} \left( (\mathcal{D}^{-})^{2} + 4\mathcal{S}^{-} \right) \mathcal{F}_{\text{tensor}}(G_{A}^{++}) \] (6.2)

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is reduced chiral, in accordance with the consideration in section 2.

As a next step, we consider two types of reduced chiral superfields $W_I$ and $\mathbb{W}_j$. Here $W_I$ is the field strength of a physical vector multiplet, while $\mathbb{W}_j$ is a composite field strength of the form $[6.2]$. We introduce a function $\mathcal{F}_{\text{chiral}}(W_I, \mathbb{W}_j)$ which is homogeneous of degree one

$$
\left( W_I \frac{\partial}{\partial W_I} + \mathbb{W}_j \frac{\partial}{\partial \mathbb{W}_j} \right) \mathcal{F}_{\text{chiral}} = \mathcal{F}_{\text{chiral}} \quad (6.3)
$$

Then the superfield

$$
\mathbb{G}^{++} = \frac{1}{4} \left( (D^+)^2 + 4S^{++} \right) \mathcal{F}_{\text{chiral}}(W_I, \mathbb{W}_j) + \text{c.c.} \quad (6.4)
$$
is an $\mathcal{O}(2)$ multiplet.

Now, the two procedures described can be repeated. We can consider two types of $\mathcal{O}(2)$ multiplets, tensor multiplets $G^{++}_A$ and composite ones $\mathbb{G}^{++}_B$ by the rule (6.4). We next pick a function of these multiplets, $\mathcal{F}_{\mathcal{O}(2)}(G^{++}_A, \mathbb{G}^{++}_B)$, which is homogeneous of degree zero,

$$
\left( G^{++}_A \frac{\partial}{\partial G^{++}_A} + \mathbb{G}^{++}_B \frac{\partial}{\partial \mathbb{G}^{++}_B} \right) \mathcal{F}_{\mathcal{O}(2)} = 0 \quad (6.5)
$$

Using this function, we generate the following reduced chiral superfield

$$
\mathbb{W} = \frac{1}{8\pi} \oint_C v^i dv_i \left( (\bar{D}^-)^2 + 4\bar{S}^{--} \right) \mathcal{F}_{\mathcal{O}(2)}(G^{++}_A, \mathbb{G}^{++}_B) \quad (6.6)
$$

Next, we can make use of this reduced scalar to derive new linear multiplets, and so on and so forth.

Each of the two constructions employed adds two spinor derivatives (or one vector derivative). This differs from a more traditional way (see, e.g., [11] and references therein) to generate higher derivative structures using the chiral projection operator $\bar{\Delta}$, eq. (6.4). We recall that given a scalar, isoscalar and $U(1)_R$-neutral superfield $U(z)$, which is inert under the super-Weyl transformations, its descendant $\bar{\Delta}U$ has the properties

$$
\bar{\Delta}U = -4\bar{\Delta}U, \quad \bar{D}_i^\alpha \bar{\Delta}U = 0, \quad \delta_\sigma \bar{\Delta}U = 2\sigma \bar{\Delta}U \quad (6.7)
$$

where $\delta_\sigma \bar{\Delta}U$ denotes the super-Weyl variation of $\bar{\Delta}U$. Given a vector multiplet $W$ such that $W$ is nowhere vanishing, we can define the chiral scalar $W^{-2}\bar{\Delta}U$ which is neutral under the local $U(1)_R$ and the super-Weyl transformations. The latter superfield can be used to construct an antichiral superfield of the form $W^{-2}\bar{\Delta}(W^{-2}\bar{\Delta}U)$,
which is neutral under the local $U(1)_R$ and the super-Weyl transformations, and so on and so forth.

Using these chiral operators, one may construct higher derivative actions involving chiral Lagrangians. However, it is usually possible to convert the chiral Lagrangian, which involves an integral over the chiral subspace, into an integral over the whole superspace by eliminating one of the chiral projection operators. Schematically, if $\mathcal{L}_c = \Phi \bar{\Delta} U$ for some chiral superfield $\Phi$ and a well-defined local and gauge-invariant operator $U$, then

$$\int d^4x \, d^4\theta \, \Phi \bar{\Delta} U = \int d^4x \, d^4\theta \, d^4\bar{\theta} \, E \, \Phi \, U . \quad (6.8)$$

Thus, higher derivative actions of this type are invariably most naturally written as integrals over the entire superspace and are not intrinsically chiral. This has important ramifications for perturbative calculations, where non-renormalization theorems place strong restrictions on intrinsic chiral Lagrangians.

The constructions we are considering are interesting partly because they are higher derivative terms which cannot be written as full superspace integrals, at least not without introducing prepotentials. As an example, let us choose the Lagrangian $\mathcal{L}^{++}$ in (2.2) as $\mathcal{L}^{++} = \mathcal{G}^{++} \mathcal{F}_{(2)}(G_A^{++}, G_B^{++})$, where $\mathcal{G}^{++}$ is given by eq. (6.4). Upon integration by parts we get

$$I = \frac{1}{2\pi} \oint_C v^i dv_i \int d^4x \, d^4\theta \, d^4\bar{\theta} \, E \frac{\bar{W}}{\Sigma^{++}} \Omega , \quad (6.9)$$

where we have defined the composite Lagrangian

$$\Omega(W_I, \mathbb{W}_J, G_A^{++}, G_B^{++}) := \mathcal{F}_{\text{chiral}}(W_I, \mathbb{W}_J) \mathcal{F}_{(2)}(G_A^{++}, G_B^{++}) . \quad (6.10)$$

The specific feature of this Lagrangian is that it obeys the single constraint

$$\bar{D}^+_a \Omega = 0 . \quad (6.11)$$

Although we have written (6.9) as an integral over the full superspace, this is really the locally supersymmetric generalization of the globally supersymmetric action

$$I_{\text{rigid}} = -\frac{1}{8\pi} \oint_C v^i dv_i \int d^4x \, (\bar{D}^-)^2 D^4 \Omega , \quad \bar{D}^+_a \Omega = 0 , \quad (6.12)$$

---

4This is a generalization of the construction of rigid superconformal invariants containing $F^n$, with $F$ the electromagnetic field strength.

5It is important to assume that $U$ is a well-defined local and gauge-invariant operator and not, for example, a prepotential; else, any chiral action may be rewritten as an integral over the full superspace in this way.

6This way of writing Lagrangians over subspaces in terms of the full superspace is familiar from
where the spinor derivatives may be understood as arising from an integration over six Grassmann coordinates. For a large class of such Lagrangians, this action cannot be rewritten as an integral over the whole superspace of eight Grassmann coordinates without the introduction of prepotentials. As with $\mathcal{N} = 1$ theories, this has implications for non-renormalization theorems.

We should point out that special holomorphic three-derivative contributions to $\mathcal{N} = 2$ supersymmetric Yang-Mills effective actions, which are given as an integral over $3/4$ of superspace, have been discussed in the literature [43].

The results of this paper allow us to obtain a simple form for the projective-superspace action [2] of the minimal formulation for $\mathcal{N} = 2$ Poincaré supergravity with vector and tensor compensators [31]. Using the techniques developed, the gauge-invariant supergravity action can be written as

$$S_{\text{SUGRA}} = \frac{1}{\kappa^2} \int d^4 x \, d^4 \theta \, \mathcal{E} \left\{ \Psi \mathcal{W} - \frac{1}{4} W^2 + m \Psi W \right\} + \text{c.c.}$$

$$= \frac{1}{\kappa^2} \int d^4 x \, d^4 \theta \, \mathcal{E} \left\{ \Psi \mathcal{W} - \frac{1}{4} W^2 \right\} + \text{c.c.} + \frac{m}{\kappa^2} \int d^4 x \, d^4 \theta \, d^4 \bar{\theta} \, E G^{ij} V_{ij} ,$$

(6.13)

where $\kappa$ is the gravitational constant, $m$ the cosmological constant, $\mathcal{W}$ is given by eq. (3.12), and $V_{ij}$ is the Mezincescu prepotential (E.7). Within the projective-superspace approach of [1, 2, 3], this action is equivalently given by (2.2) with the following Lagrangian

$$\kappa^2 \mathcal{L}^{++}_{\text{SUGRA}} = G^{++} \ln \frac{G^{++}}{\mathcal{T}^+ + \bar{\mathcal{T}}^+} - \frac{1}{2} V \Sigma^{++} + m V G^{++} ,$$

(6.14)

with $V$ the tropical prepotential for the vector multiplet, and $\mathcal{T}^+$ a weight-one arctic multiplet (both $\mathcal{T}^+$ and its smile-conjugate $\bar{\mathcal{T}}^+$ are pure gauge degrees of freedom). The first term in the right-hand side of (6.14) is (modulo sign) the locally supersymmetric version of the projective-superspace Lagrangian for the improved tensor multiplet constructed in [5]. The fact that the vector and the tensor multiplets are compensators means that their field strengths $W$ and $G^{ij}$ should possess non-vanishing expectation values, that is $W \neq 0$ and $G \equiv \sqrt{\frac{1}{2} G^{ij} G_{ij}} \neq 0$. These

$\mathcal{N} = 1$ superspace, where there are two ways to write chiral actions, either as an integral over the chiral subspace [40], $\int d^4 x \, d^2 \theta \, \mathcal{E} \, \mathcal{L}_c$, or as an integral over the full superspace [41, 42], $\int d^4 x \, d^2 \theta \, d^2 \bar{\theta} \, \mathcal{E} \bar{\mathcal{L}}_c$. Eq. (6.9) is analogous to the second of these forms, and so is the projective action (2.2). The locally supersymmetric version of (6.12), analogous to the first form, has not yet been written down within the approach of [1, 2, 3].
conditions are consistent with the equations of motion for the gravitational superfield (see [34] for a recent discussion)

\[ G - W \bar{W} = 0. \]  
(6.15)

The equations of motion for the compensators are

\[ \Sigma^{++} - mG^{++} = 0, \]  
(6.16a)

\[ \mathbb{W} + mW = 0. \]  
(6.16b)

A remarkable feature of the supergravity action (6.13) is that its reduction to component fields can readily be carried out using the technique developed in [44].

If the multiplet of conformal supergravity is considered as a curved superspace background, the action (6.13) describes (modulo sign) a massive vector multiplet or a massive tensor multiplet [2]. The rigid superspace limit of (6.13) was introduced for the first time by Lindström and Roček using \( \mathcal{N} = 1 \) superfields [32]. Their construction was immediately generalized to \( \mathcal{N} = 2 \) superspace [27] as a simple extension (\( \mathcal{D}_{ij}G^{ij} \rightarrow \mathbb{W} \)) of the massive \( \mathcal{N} = 2 \) tensor multiplet model proposed earlier by Howe, Stelle and Townsend [20]. More variant models for massive \( \mathcal{N} = 2 \) tensor multiplets can be found in [45].

The super-Weyl gauge freedom of (6.13) can used to fix \( G = 1 \). The geometric implications of such a gauge fixing have been spelled out in subsection 3.2. Since \( W \neq 0 \), the local U(1)\(_R\) freedom allows us to impose the gauge \( W - \bar{W} = 0 \).

The supergravity theory with Lagrangian (6.14) possesses a dual formulation described solely in terms of a chiral scalar \( \Psi \) and its conjugate \( \bar{\Psi} \) [2]. Using the techniques developed in the present paper, the dual formulation can be written as

\[ S_{\text{SUGRA}} = \frac{1}{\kappa^2} \int d^4 x d^4 \theta \mathcal{E} \left\{ \Psi \mathbb{W} + \frac{1}{4} \mu (\mu + ie) \Psi^2 \right\} + \text{c.c.}, \]  
(6.17)

where \( m^2 = \mu^2 + e^2 \), with \( \mu \neq 0 \). Here \( \Psi \) and its conjugate \( \bar{\Psi} \) are the only conformal compensators. The action is super-Weyl invariant, with \( \Psi \) transforming as

\[ \delta_\sigma \Psi = \sigma \Psi, \]  
(6.18)

in spite of the presence of the mass term. Unlike common wisdom, we see that \( \mathcal{N} = 2 \) Poincaré supergravity can be realized without a compensating vector multiplet.\footnote{The vector multiplet has been eaten up by the tensor multiplet which is now massive. The vector compensator acts as a Stückelberg field to give mass to the tensor multiplet. This is an example of the phenomenon observed originally in [46] and studied in detail in [17, 18, 19, 20, 21, 22, 23]}. 

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The complex mass parameter in (6.17) can be interpreted to have both electric and magnetic contributions which are associated with the two possible mass terms $B \wedge ^* B$ and $B \wedge B$ for the component two-form $B$ (see, e.g., [49] for a pedagogical discussion).

The action (6.17) leads to the following equation of motion for $\Psi$

$$\mathbb{W} + \frac{1}{2} \mu (\mu + i e) \Psi = 0 .$$

(6.19)

Provided eq. (6.19) holds, the equation of motion for the gravitational superfield is

$$G - \mu^2 \Psi \bar{\Psi} = 0 ,$$

(6.20)

compare with (6.16b). The latter implies that $\Psi$ is nowhere vanishing on-shell. We therefore are allowed to impose the super-Weyl gauge $G = 1$ and fix the local $U(1)_R$ symmetry as $\Psi - \bar{\Psi} = 0$.

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A Geometry of conformal supergravity

This section is taken essentially verbatim from [3].

We give a summary of the superspace geometry for $N = 2$ conformal supergravity which was originally introduced in [14], as a generalization of [17], and later elaborated in [3]. A curved four-dimensional $N = 2$ superspace $\mathcal{M}^{48}$ is parametrized by local coordinates $z^M = (x^m, \theta^\mu_i, \bar{\theta}^\dot{\mu}_{\dot{i}})$, where $m = 0, 1, \cdots, 3$, $\mu = 1, 2$, $\dot{\mu} = 1, 2$ and $i = 1, 2$. The Grassmann variables $\theta^\mu_i$ and $\bar{\theta}^\dot{\mu}_{\dot{i}}$ are related to each other by complex conjugation: $\bar{\theta}^\dot{\mu}_{\dot{i}} = \bar{\theta}^{\mu_i}$. The structure group is $\text{SL}(2, \mathbb{C}) \times \text{SU}(2)_R \times U(1)_R$, with $M_{ab} = -M_{ba}$, $J_{ij} = J_{ji}$ and $\mathbb{J}$ be the corresponding Lorentz, $\text{SU}(2)_R$ and $U(1)_R$ generators. The covariant derivatives $\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}^i_\alpha, \bar{\mathcal{D}}^{\dot{i}}_{\dot{\alpha}}) \equiv (\mathcal{D}_a, \mathcal{D}^i_\alpha, \bar{\mathcal{D}}^{\dot{i}}_{\dot{\alpha}})$ have the form

$$\mathcal{D}_A = E_A + \frac{1}{2} \Omega_A^{bc} M_{bc} + \Phi_A^{kl} J_{kl} + i \Phi_A \mathbb{J}$$

$$= E_A + \Omega_A^{\beta\gamma} M_{\beta\gamma} + \Omega_A^{\dot{\beta}\dot{\gamma}} \bar{M}_{\dot{\beta}\dot{\gamma}} + \Phi_A^{kl} J_{kl} + i \Phi_A \mathbb{J} .$$

(A.1)

Here $E_A = E_A^M \partial_M$ is the supervielbein, with $\partial_M = \partial/\partial z^M$, $\Omega_A^{bc}$ is the Lorentz connection, $\Phi_A^{kl}$ and $\Phi_A$ are the $\text{SU}(2)_R$ and $U(1)_R$ connections, respectively.
The Lorentz generators with vector indices \((M_{ab})\) and spinor indices \((M_{\alpha\beta} = M_{\beta\alpha} \quad \text{and} \quad \bar{M}_{\dot{a}\dot{b}} = \bar{M}_{\dot{b}\dot{a}})\) are related to each other by the standard rule:

\[
M_{ab} = (\sigma_{ab})^{\alpha\beta} M_{\alpha\beta} - (\bar{\sigma}_{ab})^{\dot{a}\dot{b}} \bar{M}_{\dot{a}\dot{b}}, \quad M_{\alpha\beta} = \frac{1}{2} (\sigma_{ab})_{\alpha\beta} M_{ab}, \quad \bar{M}_{\dot{a}\dot{b}} = -\frac{1}{2} (\bar{\sigma}_{ab})_{\dot{a}\dot{b}} M_{ab}.
\]

The generators of the structure group act on the spinor covariant derivatives as follows:\(^8\)

\[
\begin{align*}
[M_{\alpha\beta}, D^i_j] &= \varepsilon_{\gamma(\alpha} D^i_{\beta)} , \\
[\bar{M}_{\dot{a}\dot{b}}, \bar{D}^i_{\dot{\gamma}}] &= \varepsilon_{\dot{\gamma}(\dot{a}} \bar{D}^i_{\dot{b})} , \\
[J_{kl}, D^i_j] &= -\delta^i_j \delta^k_l , \\
[J_{kl}, \bar{D}^{i\dot{\gamma}}] &= -\varepsilon_{i(k} \bar{D}^{i\dot{\gamma}}_{l)} , \\
[J, D^i_j] &= D^i_j , \\
[J, \bar{D}^{i\dot{\gamma}}] &= -\bar{D}^{i\dot{\gamma}} ,
\end{align*}
\]

(A.2)

Our notation and conventions correspond to \[53\].

The covariant derivatives obey the algebra

\[
\begin{align*}
\{D^i_j, D^\beta_\beta\} &= 4 S^{ij} M_{\alpha\beta} + 2 \varepsilon^{ij} \varepsilon_{\alpha\beta} Y^{\gamma\delta} M_{\gamma\delta} + 2 \varepsilon^{ij} \varepsilon_{\alpha\beta} \bar{W}^{\gamma\delta} \bar{M}_{\gamma\delta} \\
&\quad + 2 \varepsilon_{\alpha\beta} \varepsilon^{ij} S^{kl} J_{kl} + 4 Y_{\alpha\beta} J^{ij} , \quad \text{(A.3a)} \\
\{D^i_j, \bar{D}^{\dot{\beta}}\} &= -2 i \delta^j_\beta (\sigma^\gamma)_{\alpha} \delta^i_\dot{\beta} D^c \gamma + 4 \left( \bar{\delta}^j_\dot{\beta} \bar{G}^{\dot{\gamma}\dot{\delta}} + i G^{\dot{\gamma}\dot{\beta} j} \right) M_{\alpha\delta} + 4 \left( \delta^j_\beta G^{\dot{\gamma}\dot{\beta} j} + i G^{\dot{\gamma}\dot{\beta} j} \right) \bar{M}_{\beta\dot{\delta}} \\
&\quad + 8 G^\beta_\dot{\beta} J^i j - 4 i \delta^i_\beta G^{\dot{\gamma}\dot{\beta} k} J_{kl} - 2 \left( \delta^j_\beta G^j_\dot{\beta} + i G^j_\dot{\beta} j \right) \bar{J}^\beta , \quad \text{(A.3b)} \\
[D^i_j, \bar{D}^{\dot{\beta}}] &= -i (\bar{\sigma}_a)^{\alpha\gamma} \left( \delta^i_j G^\beta_\dot{\alpha} + i \bar{G}^{\dot{\beta}a} \right) D^\gamma_{\alpha} \\
&\quad + \frac{1}{2} \left( (\sigma_a)^{\beta\gamma} S^j k - \varepsilon^{jk} (\sigma_a)^{\beta\gamma} \bar{W}_{\dot{\delta} \dot{\gamma}} - \varepsilon^{jk} (\sigma_a)^{\gamma \delta} Y_{\alpha\beta} \right) \bar{D}^\gamma_k \\
&\quad + \frac{1}{2} R^{j, k} \bar{M}_{cd} + R^{j, k} \bar{J}_{kl} + i R^{j, k} \bar{J} \quad \text{(A.3c)}
\end{align*}
\]

Here the dimension-1 components of the torsion obey the symmetry properties

\[
S^{ij} = S^{ji} , \quad Y_{\alpha\beta} = Y_{\beta\alpha} , \quad W_{\alpha\beta} = W_{\beta\alpha} , \quad G_{\alpha\dot{\alpha}}^{\dot{i}j} = G_{\alpha\dot{\alpha}}^{\dot{j}i} \quad \text{(A.4)}
\]

and the reality conditions

\[
\overline{S^{ij}} = \bar{S}_{ij} , \quad \overline{W_{\alpha\beta}} = \bar{W}_{\dot{a}\dot{b}} , \quad \overline{Y_{\alpha\beta}} = \bar{Y}_{\dot{a}\dot{b}} , \quad \overline{G_{\beta\dot{\alpha}}} = G_{\alpha\beta} , \quad \overline{G_{\beta\dot{\alpha}}^{\dot{i}j}} = G_{\alpha\beta}^{\dot{j}i} . \quad \text{(A.5)}
\]

The \(U(1)_R\) charges of the complex fields are:

\[
\begin{align*}
\mathbb{J} S^{ij} &= 2 S^{ij} , \quad \mathbb{J} Y_{\alpha\beta} = 2 Y_{\alpha\beta} , \quad \mathbb{J} W_{\alpha\beta} = -2 W_{\alpha\beta} , \quad \mathbb{J} W &= -2 W . \quad \text{(A.6)}
\end{align*}
\]

\(^8\)The (anti)symmetrization of \(n\) indices is defined to include a factor of \((n!)^{-1}\).
The dimension-3/2 components of the curvature appearing in (A.3c) have the following explicit form:

\[
R^j_{\alpha\beta\gamma} = -i(\sigma_d)_{\beta}^j T^j_{\alpha\delta} + i(\sigma_a)_{\beta}^j T^j_{\alpha\delta} - i(\sigma_c)_{\beta}^j T^j_{\alpha\delta}, \tag{A.7a}
\]

\[
R^j_{\alpha\beta\gamma} = -i\varepsilon^{j(k} \hat{D}_a G_{\beta\gamma}^{\delta k} \hat{T}^\delta \hat{W}_{\alpha\delta} - \frac{i}{3} \varepsilon_{\alpha\beta\gamma} \varepsilon^{j(k} \hat{D}_a S^l) q + \frac{4}{3} \varepsilon^{j(k} \hat{D}_a G_{\beta\gamma}^{\delta k} \hat{T}^\delta \hat{W}_{\alpha\delta} + \frac{2}{3} \varepsilon_{\alpha\beta\gamma} \varepsilon^{j(k} \hat{D}_a G_{\delta\alpha}^l) q, \tag{A.7b}
\]

\[
R^j_{\alpha\beta\gamma} = -D^j_{\beta\gamma} G_{\alpha\delta} + \frac{i}{3} D_{(\alpha k} G_{\beta)\gamma}^{jk} + \frac{i}{2} \varepsilon_{\alpha\beta\gamma} D^j_{k} G_{\gamma\delta}^{jk}. \tag{A.7c}
\]

The right-hand side of (A.7a) involves the dimension-3/2 components of the torsion which are expressed in terms of the dimension-1 tensors as follows:

\[
T^{\alpha\beta\gamma}_k \equiv (\sigma_{ab})^{\alpha\beta} T^{\alpha\beta\gamma}_k = - (\tilde{\sigma}_{ab})^{\alpha\beta} \bar{T}^{\alpha\beta\gamma}_k, \tag{A.8a}
\]

\[
T^{\alpha\beta\gamma}_k = -\frac{1}{4} \bar{D}^k Y_{\alpha\beta} + \frac{i}{3} D^l (\alpha G_{\beta\gamma})^{\delta k} t, \tag{A.8b}
\]

\[
T^{\alpha\beta\gamma}_k = -\frac{1}{4} \bar{D}^k \hat{W}_{\alpha\beta} - \frac{1}{6} \varepsilon_{\gamma}(\alpha \bar{D}^{\delta k} S^{\delta k} - \frac{i}{3} \varepsilon_{\gamma}(\alpha \bar{D}^{\delta q} G_{\delta\beta}^{ij} q. \tag{A.8c}
\]

The dimension-3/2 Bianchi identities are:

\[
\bar{D}^i S^{ijk} = 0, \quad \bar{D}^i S^{ijk} = iD^{i(\alpha G_{\beta\gamma})^{jk}}, \tag{A.9a}
\]

\[
\bar{D}^i \hat{W}_{\beta\gamma} = 0, \tag{A.9b}
\]

\[
\bar{D}^i (\alpha Y_{\beta\gamma}) = 0, \quad \bar{D}^i S_{ij} + \bar{D}^j S_{ij} = 0, \tag{A.9c}
\]

\[
\bar{D}^i G_{\beta\gamma} = 0, \tag{A.9d}
\]

\[
\bar{D}^i G_{\beta\gamma}^{ij} = 0, \quad \bar{D}^i G_{\beta\gamma}^{ij} = -\frac{1}{4} \bar{D}^i Y_{\alpha\beta} + \frac{1}{12} \varepsilon_{\alpha\beta\gamma} \bar{D}^{ij} S^{ij} - \frac{1}{4} \varepsilon_{\alpha\beta\gamma} \bar{D}^{ij} \hat{W}_{\gamma\delta} - \frac{i}{3} \varepsilon_{\alpha\beta\gamma} \bar{D}^{ij} G_{\gamma\delta}^{ij}. \tag{A.9e}
\]

B Chiral projection operator

Actions in \( \mathcal{N} = 2 \) supergravity may be constructed from integrals over the full superspace

\[
\int d^4 x d^4 \theta d^4 \bar{\theta} E \mathcal{L} \tag{B.1}
\]

or integrals over a chiral subspace

\[
\int d^4 x d^4 \theta E \mathcal{L}_c, \quad \bar{D}_c^i \mathcal{L}_c = 0 \tag{B.2}
\]
with $E$ the chiral density. Just as in $\mathcal{N} = 1$ superspace, actions of the former type may be rewritten as the latter using a covariant chiral projection operator $\tilde{\Delta}$ [18],

$$
\int d^4x d^4\theta d^4\bar{\theta} E \mathcal{L} = \int d^4x d^4\theta E \tilde{\Delta} \mathcal{L}.
$$

The covariant chiral projection operator is defined as

$$
\tilde{\Delta} = \frac{1}{96} \left( (\bar{D}^{ij} + 16\bar{S}^{ij})\bar{D}_{ij} - (\bar{D}^{\dot{\alpha}\dot{\beta}} - 16\bar{Y}^{\dot{\alpha}\dot{\beta}})\bar{D}_{\dot{\alpha}\dot{\beta}} \right)
$$

$$
= \frac{1}{96} \left( \bar{D}_{ij}(\bar{D}^{ij} + 16\bar{S}^{ij}) - \bar{D}_{\dot{\alpha}\dot{\beta}}(\bar{D}^{\dot{\alpha}\dot{\beta}} - 16\bar{Y}^{\dot{\alpha}\dot{\beta}}) \right).
$$

Its fundamental property is that $\tilde{\Delta}U$ is covariantly chiral, for any scalar, isoscalar and $U(1)_R$-neutral superfield $U(z)$,

$$
\bar{D}^{\dot{\alpha}} \tilde{\Delta} U = 0.
$$

A detailed derivation of the relation (B.3) can be found in [9].

It follows from the explicit structure of the chiral projection operator that

$$
\int d^4x d^4\theta d^4\bar{\theta} E \Phi = 0,
$$

for any covariantly chiral scalar $\Phi$ of zero $U(1)_R$ charge, $\bar{D}^{\dot{\alpha}} \Phi = 0$.

C Isotwistors and projective superspace

In this paper, our isotwistor notation and conventions differ slightly from those adopted in [1 2 3], but agree with those used in [54].

Associated with any completely symmetric SU(2) tensor $V^i_1\cdots i_n = V^{(i_1\cdots i_n)}$ is a superfield $V^{(n)}$ obeying

$$
V^{(n)} = V^{i_1\cdots i_n} v_{i_1} \cdots v_{i_n},
$$

with the $(n)$ superscript referring to the degree of homogeneity in the isotwistor parameter $v_i$. We often have need to introduce an additional isotwistor $u_i$, which is linearly independent of $v_i$

$$
(v, u) = v^k u_k = \epsilon^{kj} v_j u_k \neq 0
$$

in terms of which we may define new isotwistors $V^{(n-m,m)}$

$$
V^{(n-m,m)} = V^{i_1\cdots i_n} v_{i_1} \cdots v_{i_{n-m}} \frac{u_{i_{n-m+1}}}{(v,u)} \cdots \frac{u_{i_n}}{(v,u)}.
$$
with degree \( n - 2m \). For the cases of \( n = 1 \) and \( n = 2 \), we will use a more compact notation involving + and −:

\[
V^+ = V^i v_i, \quad V^- = V^i \frac{u_i}{(v, u)} \tag{C.4}
\]

\[
V^{++} = V^{ij} u_i v_j, \quad V^{-+} = V^{ij} v_i \frac{u_j}{(v, u)}, \quad V^{--} = V^{ij} \frac{u_i u_j}{(v, u)^2} \tag{C.5}
\]

Because the covariant derivatives of \( \mathcal{N} = 2 \) superspace have isospin indices we may identify

\[
D^+_\alpha = v_i D^i_\alpha, \quad \bar{D}^+_\dot{\alpha} = v_i \bar{D}^i_\dot{\alpha} \tag{C.6}
\]

\[
D^-_\alpha = u_i D^i_\alpha, \quad \bar{D}^-_\dot{\alpha} = u_i \bar{D}^i_\dot{\alpha} \tag{C.7}
\]

It will be useful to introduce derivative operations on the isotwistors themselves,

\[
\partial^{-} = \frac{1}{(v, u)} u^i \frac{\partial}{\partial v^i}, \quad \partial^{++} = (v, u) v_i \frac{\partial}{\partial u_i} \tag{C.8}
\]

which have the properties that

\[
\partial^{++} V^{(n-m,m)} = m V^{(n-m+1,m-1)}, \quad \partial^{-} V^{(n-m,m)} = (n-m) V^{(n-m-1,m+1)} \tag{C.9}
\]

For those knowledgable of harmonic superspace [22], the above definitions will seem familiar. They can be derived from corresponding objects in harmonic superspace by the formal replacements

\[
u^i \rightarrow v_i, \quad u_i \rightarrow \frac{u_i}{(v, u)}. \tag{C.10}
\]

The difference from the harmonic superspace definitions is that the isotwistor \( v^i \) is not normalized, and \( u_i \) is not related to \( v^i \) by complex conjugation.

### D Contour integrals in \( \mathbb{C}P^1 \)

In this paper, we have need to evaluate contour integrals of the general form

\[
C_n = \oint_C v^i dv_i \frac{\Omega^{(2n-2)}}{(G^{++})^n}. \tag{D.1}
\]

In this expression \( \Omega^{(2n-2)} \) is some homogeneous function of degree \( 2n - 2 \) in the variable isotwistor \( v^i \), degree zero in the fixed isotwistor \( u_i \), and obeying the analyticity constraints

\[
D^+_\alpha \Omega^{(2n-2)} = D^+_\alpha \Omega^{(2n-2)} = 0. \tag{D.2}
\]
We further assume that the contour encloses a region where the only singularities are those arising from the \((G^{++})^n\) factor in the denominator.

Under these assumptions, the contour may be evaluated using a trick familiar from twistor theory. We write \(G_{ij} = i\omega^i\bar{\omega}^j\) in terms of isotwistors \(\omega^i\) and \(\bar{\omega}^j\). Then
\[
G^2 = \frac{1}{2}G_{ij}G_{ij} = \frac{1}{4}(\omega^j\bar{\omega}^j)^2 \implies G = \frac{1}{2}\omega^j\bar{\omega}^j
\]  
(D.3)
since the reality of \(G_{ij}\) implies \(\bar{\omega}^j = (\omega^j)^*\). So long as \(G\) is nonzero, \(\omega^j\) and \(\bar{\omega}^j\) are linearly independent isotwistors, in terms of which the contour integral may be written
\[
\mathcal{C}_n = \oint_C v^i dv_i \frac{1}{(i\omega^+\bar{\omega}^+)^n} \Omega^{(2n-2)},
\]  
(D.4)
where
\[
\omega^+ = \omega^i v_i, \quad \bar{\omega}^+ = \bar{\omega}^i v_i.
\]  
(D.5)
The pole in the contour appears when either \(\omega^+\) or \(\bar{\omega}^+\) vanishes – that is, when either \(v^i \propto \omega^i\) or \(\bar{\omega}^i\). Because \(G \neq 0\), these poles are distinct and we can consider their residues separately.

Without loss of generality, we will restrict to the case where the contour encircles \(\omega^j\). The \(n\)th order pole may be converted to a first order pole using the relation
\[
(\partial^{--})^j \frac{1}{\omega^+} \left( \frac{(-1)^j j! (\omega^-)^j}{(\omega^+)^{j+1}} \right)
\]  
(D.6)
where
\[
\partial^{--} \equiv \frac{u^i}{(v, u)} \frac{\partial}{\partial v^i}
\]  
(D.7)
and \(\omega^- \equiv \frac{\omega^i u_i}{(v, u)}\)  
(D.8)
and applying integration by parts. To do this, we first rewrite the contour integral using (D.6):
\[
\mathcal{C}_n = \frac{(-1)^{n-1}}{(n-1)!} \oint_C v^i dv_i \frac{1}{(\omega^-)^{n-1}} \left( (\partial^{--})^{n-1} \frac{1}{\omega^+} \right) \frac{\Omega^{(2n-2)}}{(i\omega^+)^n}.
\]  
(D.9)
Next we would like to flip each of the \(\partial^{--}\) operators off the pole. For this step to be valid, we must check that the total derivative terms actually do vanish. Each of them has the form
\[
\oint_C v^i dv_i \partial^{--} \mathcal{F}
\]  
(D.10)
where \( F = F(v, u) \) is a function of degree zero in the isotwistors \( v^i \) and \( u_i \) separately. Because of the homogeneity property, we may trade \( v^i \) and \( u_i \) for projective coordinates \( \zeta \) and \( \xi \) where

\[
v^i = v_\perp(1, \zeta), \quad u_i = u_\perp(1, \xi)
\]

with \( F \) depending only on \( \zeta \) and \( \xi \), and the contour integral rewritten as

\[
\oint_C v^i dv_i \partial^--F = \oint_C d\zeta \partial_\zeta F = \oint_C dt \dot{\zeta} \partial_\zeta F,
\]

where in the second equality we have parametrized the contour with a real variable \( t \). Because \( u_i \) is fixed, \( \xi \) is independent of \( t \) and the integrand is a total derivative in \( t \), so the contour vanishes.

Noting that the operator \( \partial^-- \) annihilates \( \omega^- \), the term generated by integrating by parts is

\[
\mathcal{C}_n = \frac{1}{(n-1)!} \oint_C \frac{v^i dv_i}{\omega^+(\omega^-)^{n-1}} \left( \frac{\Omega(2n-2)}{(1\bar{\omega}^+)^n} \right) .
\]

In evaluating the \( \partial^--'s \) on their argument, we would like to eliminate all terms coming from \( \partial^-- \) hitting the \( \bar{\omega}^+ \) factors in the denominator. This is possible if we choose \( u_i = \bar{\omega}^i \):

\[
\mathcal{C}_n = \frac{1}{(n-1)!} \oint_C \frac{v^i dv_i}{\omega^+} \left( \frac{(\partial^-)^{n-1} \Omega(2n-2)}{(\omega^-)^{n-1}(1\bar{\omega}^+)^n} \right) \bigg|_{u_i=\bar{\omega}_i} .
\]

Having reduced our expression to a first order pole, we now apply the residue theorem. Given a contour integral

\[
\oint_C \frac{v^i dv_i}{\omega^j v_j} F^-(v) ,
\]

with \( F^- \) a homogeneous function of \( v^i \) of degree \(-1\) which is nonsingular at \( v^i \propto \omega^i \), we may rewrite it in terms of the inhomogeneous coordinate \( \zeta = v_\perp/w_\perp \). If we exploit the freedom to choose \( v_\perp = w_\perp \), then

\[
v^i = w_\perp(1, \zeta) .
\]

This leads to

\[
\oint_C \frac{d\zeta}{-\omega^2/\omega^\perp + \zeta} F^-(w_\perp(1, \zeta)) = 2\pi i F^-(\omega^i) .
\]

\(^9\)There is a subtlety in this procedure. \( \partial^-- \) annihilates \( 1/\omega^- \) only if \( u_i \) is chosen to be linearly independent of \( \omega_i \). This is analogous to functions of a single complex variable where \( \bar{\partial}(z - z_0)^{-1} = 0 \) only for \( z \neq z_0 \).
We have assumed the contour to be evaluated in a counterclockwise fashion, but in principle the opposite sign may also arise.

Applying this result to (D.14) gives

\[ C_n = \frac{2\pi i}{(n-1)!} \left. \left( \frac{\partial^{--}(2n-2)}{(\omega^-)^{n-1}(i\bar{\omega}^+)^n} \right) \right|_{\bar{u}_i=\bar{\omega}_i, u^i=\omega^i}. \]

The terms in the denominator may be simplified by noting that

\[ \bar{\omega}^++\omega^-=2G, \quad \omega^- = 1 \]

giving

\[ C_n \equiv \oint_C v^i dv_i \frac{\Omega^{2n-2}}{(G++)^n} = \frac{2\pi i^{n+1}}{(n-1)!(2G)^n} \left. \left( \frac{\partial^{--}(2n-2)}{(\omega^-)^{n-1}(i\bar{\omega}^+)^n} \right) \right|_{\bar{u}_i=\bar{\omega}_i, u^i=\omega^i}. \]

For the cases of interest to us, \( \Omega^{(2j)} \) is of the form \( \Omega^{i_1 \cdots i_{2j}} v_{i_1} \cdots v_{i_{2j}} \). Then \( \partial^{--} \) may be evaluated explicitly to give

\[ (\partial^{--})^j \Omega^{(2j)} = \frac{1}{(v, u)^j} \frac{(2j)!}{j!} \Omega^{i_1 \cdots i_{2j}} v_{(i_1} \cdots v_{i_{j}} u_{i_{j+1}} \cdots u_{i_{2j})} \]

\[ = (-i)^j (2j)! \frac{1}{2j!} \Omega^{i_1 \cdots i_{2j}} G_{i_1 i_2} \cdots G_{i_{2j-1} i_{2j}} G^{-j} \]

where we have taken \( v^i = \omega^i \) and \( u_i = \bar{\omega}_i \). This gives our main result

\[ C_n = -\frac{2\pi}{2^{2n-1}} \frac{(2n-2)!}{(n-1)!(n-1)!} \Omega^{i_1 \cdots i_{2n-2}} G_{i_{1} i_{2}} \cdots G_{i_{2n-3} i_{2n-2}} G^{-(2n-1)}. \]

### E Prepotential formulations for vector multiplet

Within the projective-superspace approach of [1, 2, 3], the constraints on the vector multiplet field strength \( W \) can be solved in terms of a real weight-zero tropical prepotential \( V(v^i) \) as in eq. (1.4). Here we use this construction to introduce a curved-superspace analogue of Mezincescu’s prepotential [19].

First of all, let us show how Mezincescu’s prepotential for the vector multiplet can be introduced within standard superspace. For this a simple generalization of the rigid supersymmetric analysis in [20] can be used. One begins with the first-order action

\[ S = \frac{1}{4} \int d^4x d^4\theta \mathcal{E} \mathcal{W} \mathcal{W} + \frac{1}{4} \int d^4x d^4\bar{\theta} \bar{\mathcal{E}} \bar{\mathcal{W}} \bar{\mathcal{W}} \]

\[ - \frac{1}{8} \int d^4x d^4\theta d^4\bar{\theta} E \left( \mathcal{W}(\mathcal{D}^{ij} + 4S^{ij})V_{ij} \right) - \bar{\mathcal{W}}(\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij})\bar{V}_{ij} \right), \quad (E.1) \]
where $\mathcal{W}$ is a covariantly chiral superfield, and $V^{ij} = V^{ji}$ is an *unconstrained* real SU(2) triplet acting as a Lagrange multiplier. Varying (E.1) with respect to $V_{ij}$ gives $\mathcal{W} = W$, where $W$ obeys the Bianchi identity (1.1). As a result, the second term in (E.1) drops out and we end up with the Maxwell action

$$S = \frac{1}{2} \int d^4x \, d^4\theta \, \mathcal{E} \mathcal{W} W .$$

(E.2)

On the other hand, because the action (E.1) is quadratic in $\mathcal{W}$, we may easily integrate $\mathcal{W}$ out using its equation of motion

$$W = iW_D , \quad W_D := \frac{1}{4} \bar{\Delta}(\mathcal{D}^{ij} + 4\mathcal{S}^{ij})V_{ij} .$$

(E.3)

This leads to the dual action

$$S = \frac{1}{2} \int d^4x \, d^4\theta \, \mathcal{E} W_D W_D .$$

(E.4)

The dual field strength $W_D$ must be both reduced chiral and given by (E.3).

We now show how to construct the Mezincescu prepotential $V_{ij}$ within projective superspace. One begins with the expression for $W$ in terms of a weight-zero tropical prepotential $V(v^i)$, eq. (1.4). The analyticity conditions on $V$ may be solved in terms of an unconstrained isotwistor superfield $\mathcal{U}^{(-4)}$ (see [1] for the definition of isotwistor superfields), which is real under the smile-conjugation, as follows

$$V = \frac{1}{16} \left( (\bar{D}^+)^2 + 4\bar{S}^{++} \right) \left( (D^+)^2 + 4S^{++} \right) \mathcal{U}^{(-4)}$$

$$= \frac{1}{16} \left( (D^+)^2 + 4S^{++} \right) \left( (\bar{D}^+)^2 + 4\bar{S}^{++} \right) \mathcal{U}^{(-4)} .$$

(E.5)

Using this construction, one may write $W$, using the results of [9], as

$$W = \frac{1}{128\pi} \oint_C v^i dv_i \left( (\bar{D}^-)^2 + 4\bar{S}^{--} \right) \left( (D^+)^2 + 4S^{++} \right) \mathcal{U}^{(-4)}$$

$$= \frac{\bar{\Delta}}{8\pi} \oint_C v^i dv_i \left( (\bar{D}^+)^2 + 4\bar{S}^{++} \right) \mathcal{U}^{(-4)}$$

(E.6)

where $\bar{\Delta}$ is the chiral projection operator (B.4). This may subsequently be rewritten

$$W = \frac{\bar{\Delta}}{8\pi} (D^{ij} + 4S^{ij}) \oint_C v^k dv_k v_i v_j \mathcal{U}^{(-4)}$$

$$= \frac{1}{4} \bar{\Delta} (D^{ij} + 4S^{ij}) V_{ij} ,$$

(E.7)

where we have defined the Mezincescu prepotential

$$V_{ij} = \frac{1}{2\pi} \oint_C v^k dv_k v_i v_j \mathcal{U}^{(-4)} .$$

(E.8)
This construction for $W$ is manifestly chiral due to the appearance of the projection operator. To prove the Bianchi identity, one may start with (1.4) and replace the dummy integration variable $v^i \to \hat{v}^i$ and rewrite the expression for $W$ in the form

$$W = \frac{1}{8\pi} \oint_C \hat{v}^i d\hat{v}_i u_i u_j \left( \bar{D}^{ij} + 4\bar{S}^{ij} \right) V(\hat{v}).$$  (E.9)

Since this expression does not depend on the constant isospinor $u_i$, we can choose it as $u_i \propto v_i$, and then the last expression turns into

$$W = \left( (\bar{D}^+)^2 + 4\bar{S}^{++} \right) \oint_C \frac{\hat{v}^i d\hat{v}_i}{8\pi(\hat{v}, v)^2} V(\hat{v}).$$  (E.10)

We now can check the fulfilment of the Bianchi identity:

$$\left( (\bar{D}^+)^2 + 4\bar{S}^{++} \right) W - \left( (\bar{D}^+)^2 + 4\bar{S}^{++} \right) \bar{W}$$

$$= \left[ (\bar{D}^+)^2 + 4\bar{S}^{++}, (\bar{D}^+)^2 + 4\bar{S}^{++} \right] \oint_C \frac{\hat{v}^i d\hat{v}_i}{8\pi(\hat{v}, v)^2} V(\hat{v})$$

$$\equiv \left[ (\bar{D}^+)^2 + 4\bar{S}^{++}, (\bar{D}^+)^2 + 4\bar{S}^{++} \right] \Upsilon.  \quad (E.11)$$

Since $\Upsilon$ is a Lorentz scalar and is neutral under $U(1)_R$, we find

$$\left[ (\bar{D}^{++})^2 + 4\bar{S}^{++}, (\bar{D}^{++})^2 + 4\bar{S}^{++} \right] \Upsilon = 8(\bar{D}^{a+}\bar{S}^{++})\bar{D}^+_a \Upsilon - 8(\bar{D}^{a+}S^{++})\bar{D}^{a+} \Upsilon$$

$$+ 8i(\bar{D}^{a+}G^{++}_{a\dot{a}})\bar{D}^{a+} \Upsilon + 8i(\bar{D}^{a+}G^{++}_{a\dot{a}})\bar{D}^+_a \Upsilon + 4 \left( (\bar{D}^+)^2 S^{++} - (\bar{D}^+)^2 \bar{S}^{++} \right) \Upsilon \quad (E.12)$$

vanishes when we apply some of the constraints [A.9],

$$\bar{D}^+_a \bar{S}^{++} = i\bar{D}^a G^{++}_{a\dot{a}}, \quad \bar{D}^{a+} \bar{S}^{++} = i\bar{D}^{a+} G^{++}_{a\dot{a}}. \quad (E.13)$$

and make use of the algebra of covariant derivatives [A.3] to show

$$(\bar{D}^+)^2 \bar{S}^{++} - (\bar{D}^+)^2 \bar{S}^{++} = i\{\bar{D}^{a+}, \bar{D}^{a+}\}G^{++}_{a\dot{a}} = 0. \quad (E.14)$$

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