Quantum measurements between a single spin and a torsional nanomechanical resonator

B D'Urso¹, M V Gurudev Dutt¹, S Dhingra and N M Nusran

Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA
E-mail: dursobr@pitt.edu and gdutt@pitt.edu

New Journal of Physics 13 (2011) 045002 (13pp)
Received 30 September 2010
Published 4 April 2011
Online at http://www.njp.org/
doi:10.1088/1367-2630/13/4/045002

Abstract. While the motions of macroscopic objects must ultimately be governed by quantum mechanics, the distinctive features of quantum mechanics can be hidden or washed out by thermal excitations and coupling to the environment. We propose a system consisting of a graphene nanomechanical oscillator (NMO) coupled with a single spin through a uniform external magnetic field, which could become the building block for a wide range of quantum nanomechanical devices. The choice of graphene as the NMO material is critical for minimizing the moment of inertia of the oscillator. The spin originates from a nitrogen-vacancy (NV) center in a diamond nanocrystal that is positioned on the NMO. This coupling results in quantum non-demolition (QND) measurements of the oscillator and spin states, enabling a bridge between the quantum and classical worlds for a simple readout of the NV center spin and observation of the discrete states of the NMO.

Contents

1. Introduction 2
2. The proposed system 2
3. Measurements 6
   3.1. Measure spin state with the nanomechanical oscillator (NMO) 6
   3.2. Measure NMO number state with spin 7
4. Applications 10

Acknowledgments 11
References 11

¹ Authors to whom any correspondence should be addressed.
1. Introduction

There is much interest in an experimental realization of a harmonic oscillator with behavior that can only be understood with quantum mechanics. Micromechanical and nanomechanical devices utilizing flexing components are attractive because of their simplicity of fabrication, robustness and adaptability. Existing approaches have mostly focused on one of two goals. The first goal is to increase the oscillation frequency to decrease the significance of thermal excitations. Unfortunately, decreasing the resonator size, which tends to increase the resonance frequency, also tends to result in lower quality factor ($Q$) resonators. Fabricated devices span a wide range of oscillation frequency and $Q$ values, from silicon nitride membranes oscillating at $\sim 100$ kHz and $Q > 10^7$ at 4 K [1] to silicon carbide doubly clamped beams with frequencies over 1 GHz and $Q = 500$ [2]. The second goal is to make the oscillator interact with an optical [3]–[5] or microwave [6] cavity field. This adds active ‘cold damping’ to the oscillator, which has a very low effective temperature and can overwhelm the fluctuations associated with the mechanical damping for a particular mode. Thus, it may be possible to reach the ground state at the expense of increased damping.

We will use the spin of a single nitrogen vacancy (NV) center in a diamond nanocrystal as part of our quantum nanomechanical system. The electronic ground state of this defect forms a spin triplet $S = 1$ and is a highly promising system for this purpose because of the remarkably long spin depolarization ($T_1 \sim 4$–100 s at 4 K) [7]–[9] and decoherence times ($T_2 \sim 2$ ms at 300 K) [10]. Further, the defect state can be optically spin polarized and measured by combining confocal microscopy techniques with spin-state-dependent optical fluorescence [11, 12]. The ground state exhibits a zero-field splitting of 2.87 GHz (2.88 GHz at low temperatures), which defines the $\hat{z}$-axis of the electron spin. An additional magnetic field can be applied to split the $m_s = \pm 1$ sublevels, which allows for fast coherently driven spin transitions using microwave fields [13, 14]. Experiments have shown that the electron spin of the NV center is an excellent sensor of the local magnetic environment, with the ability to measure and even manipulate proximal nuclear spins [15]–[17]. This sensitivity was used recently to demonstrate nanoscale magnetic sensing and magnetic imaging with NV centers [18, 19], and we will use it to measure the number state changes in a nanomechanical oscillator (NMO).

2. The proposed system

While active mode cooling may at first appear to be the most direct path to observing quantum behavior in an NMO, it may only serve to initialize the oscillator in a known state. Despite active cooling, the quantum decoherence rate is essentially limited at $\bar{n}\gamma_m$, where $\bar{n} = k_B T/\omega_m$ is the average thermal number state of the oscillator with oscillation frequency $\omega_m$ and damping rate $\gamma_m$. $T$ is the physical temperature of the device, which is unaffected by active optical or microwave cooling.

Instead of damping the NMO into the quantum ground state, we propose to introduce quantum behavior into the NMO by coupling it to a well-behaved quantum system: the spin of an NV center in a diamond nanocrystal. The challenge is then to carefully engineer the coupling between the NMO and the spin such that the interaction allows each system to make repeated measurements of the other without leading to uncontrolled changes in the quantum state despite thermal excitation of the NMO. Fortunately, there is a class of measurements that meets our requirements, known as quantum non-demolition (QND) measurements [20, 21]. The essential
requirement for a QND measurement is that the coupling Hamiltonian between the two systems should commute with the QND observables. One system, e.g. the NV (NMO), is designated as the quantum system to be measured in QND fashion and the other, i.e. the NMO (NV), serves as a probe that is then read out classically. In addition, it is often experimentally helpful, although not essential, if (i) it is possible to prepare the probe system in a well-defined quantum state and delay readout, thus reducing or controlling ‘back-action’ on the quantum system, and (ii) the environment or extraneous Hamiltonian also commutes with the QND observables. In reality, the environment Hamiltonian does not perfectly commute with the QND observable, and causes it to change or jump. Naturally, the time for a measurement should be faster than the so-called jump time. However, the nonlinear coupling typically required for QND measurements often makes the coupling too weak to detect or utilize due to the above constraints. Coupling an NV center and an NMO has been proposed elsewhere, but with a complex measurement scheme utilizing dressed states [22]. Single electronic spins have also been detected, but with very weak coupling to a cantilever by magnetic resonance force microscopy [23].

We present a particularly simple geometry that strongly couples an NMO to an electronic spin in a solid-state system and results in a QND measurement. The geometry, as shown in figure 1, begins with an NMO in the form of a doubly clamped beam or ribbon. We plan to utilize the lowest torsion mode of the ribbon, where the twisting occurs around the axis running the length of the ribbon (the \( \hat{x} \)-axis). A diamond nanocrystal containing an NV center that provides the spin is mechanically attached to the center of the ribbon. Finally, the twisting motion of the ribbon is coupled to the spin with a uniform external magnetic field oriented perpendicular to the plane of the ribbon and substrate. Experimentally, the direction of the external magnetic field will be tuned to match the NV center \( \hat{z} \)-axis. For maximum NMO–NV coupling, the NV center \( \hat{z} \)-axis should be perpendicular to the NMO torsional axis, but the coupling is not dependent on the angle of the NV center axis around the NMO torsional axis. For convenience, we analyze the system assuming that the NV center \( \hat{z} \)-axis is perpendicular to the substrate when the NMO is not twisted.
We analyze the system starting with the quantum mechanical Hamiltonian for the system, with terms for the NMO and NV centers,

\[ H = H_{\text{NMO}} + H_{\text{NV}}. \]  

We use the standard harmonic oscillator Hamiltonian for the NMO contribution,

\[ H_{\text{NMO}} = \hbar \omega_m \left( \hat{n} + \frac{1}{2} \right). \]  

The frequency \( \omega_m \) is the classical oscillation frequency, which can be calculated from the dispersion relation of torsional waves on a graphene ribbon [24], giving

\[ \omega_m = \frac{\pi}{L} \sqrt{\frac{2 Y t^3}{3 \rho w^2 (1 + \nu)}}, \]  

where \( \nu \) is the Poisson ratio, \( Y \) is the Young’s modulus, \( L \) is the length, \( w \) is the width, \( t \) is the thickness and \( \rho \) is the mass density of the NMO ribbon material. We use the usual Hamiltonian for the NV center [11, 25],

\[ H_{\text{NV}} = \hbar \Delta S_z^2 + g \mu_B \left( \vec{S} \cdot \vec{B}_{\text{ext}} \right), \]  

where \( \Delta \) is the zero-field splitting which defines the \( \hat{z} \)-axis of the NV, \( g \approx 2 \) is the electron \( g \)-factor in the NV center, \( \mu_B \) is the Bohr magneton and \( \vec{S} \) is the electronic spin of the NV center. \( \vec{B}_{\text{ext}} \) is the external magnetic field, which results in coupling between the NMO and NV centers.

Although the applied magnetic field remains fixed in a certain direction, \( \vec{S} \cdot \vec{B}_{\text{ext}} \) in equation (4) depends on the NMO angle \( \theta \) since the angle of the NV \( \hat{z} \)-axis relative to \( \vec{B}_{\text{ext}} \) depends on \( \theta \). For an external field that is in the \( \hat{z} \)-direction when \( \theta = 0 \), we have

\[ H_{\text{NV}} = \hbar \Delta S_z^2 + g \mu_B B_{\text{ext}} \left( S_y \sin \theta + S_z \cos \theta \right). \]  

Assuming that changes in \( \theta \) due to the NMO are slow compared with the relevant NV center timescales, we can treat the changes in the magnetic field as adiabatic for the NV center spin. Then, we can analyze the NMO–NV interaction by finding the eigenvalues of \( H_{\text{NV}} \) with \( \theta \) as a parameter. Some results of these calculations are shown in figure 2, where \( B_{\text{zfs}} = \hbar \Delta / g \mu_B \sim 0.1025 \text{ T} \) is the magnitude of the effective internal magnetic field that results in the zero-field splitting \( \Delta \). For QND measurements, we seek a large \( \theta^2 \) dependence of the energy eigenvalues. We further analyze the sharp nonlinear dependence of the energy gap between the \( \mid 0 \rangle \) and \( \mid -1 \rangle \) spin states around \( \theta = 0 \), which is the result of an avoided level crossing between these states when \( B_{\text{ext}} \) approaches \( B_{\text{zfs}} \).

For small NMO oscillation angle \( \theta \), we can expand the eigenvalues as

\[ \lambda_i(\theta) = \hbar \omega_i + \frac{1}{2} \kappa_i \theta^2 + O(\theta^4), \]  

where \( i = -1, 0 \) or \(+1\) is a label, which is equivalent to the NV center \( m_s \) when \( \theta = 0 \). In the \( \theta \)-independent term in \( \lambda_i(\theta) \), \( \omega_i \) is simply the result of combining the zero-field splitting with the static external field at \( \theta = 0 \),

\[ \omega_{\pm 1} = \Delta \left( \pm B_{\text{ext}} / B_{\text{zfs}} \right), \]  

\[ \omega_0 = 0. \]
Figure 2. Plots of the variation in spin state energies with NMO angle $\theta$ with two different external field magnitudes. (a) Variation over full range of angles. (b) Smaller range near $\theta = 0$ showing the $\theta^2$ dependence of the energy gap.

The $\theta^2$ coefficient in $\lambda_i(\theta)$, $\kappa_i$, is an NV-state-dependent effective spring constant associated with the NMO–NV interaction. For the three NV spin states we consider, it is

$$\kappa_{\pm 1} = -\hbar \Delta \frac{B_{\text{ext}}}{B_{\text{ext}} \pm B_{\text{zfs}}}.$$  \hspace{1cm} (9)

$$\kappa_0 = \hbar \Delta \frac{2B_{\text{ext}}^2}{B_{\text{ext}}^2 - B_{\text{zfs}}^2}.$$ \hspace{1cm} (10)

Since the effective spring constant depends on the NV state, there is a state-dependent frequency shift in the NMO oscillation frequency. Treating the NMO, which actually has a distributed moment of inertia and torsional spring constant, as a localized torsion pendulum located at the center of the diamond NV center with effective moment of inertia $I_{\text{eff}}$ (equal to half of the total moment of inertia), we calculate the frequency shift $\delta_i$ due to the small change in spring constant $\kappa_i$ to be

$$\delta_i \approx \frac{\kappa_i}{2I_{\text{eff}}\omega_m}.$$ \hspace{1cm} (11)

So, we find the frequency shifts to be

$$\delta_{\pm 1} = -\frac{\hbar \Delta}{2I_{\text{eff}}\omega_m} \frac{B_{\text{ext}}}{B_{\text{ext}} \pm B_{\text{zfs}}}.$$ \hspace{1cm} (12)

$$\delta_0 = \frac{\hbar \Delta}{2I_{\text{eff}}\omega_m} \frac{2B_{\text{ext}}^2}{B_{\text{ext}}^2 - B_{\text{zfs}}^2}.$$ \hspace{1cm} (13)

We can simplify the system Hamiltonian by expanding $\theta^2$ in the interaction term of $H_{\text{NV}}$ in creation and annihilation operators and dropping rotating terms, giving

$$H = \hbar \omega_m \left( \hat{n} + \frac{1}{2} \right) + \sum_{i=-1}^{+1} \hbar \omega_i |i\rangle \langle i| + \sum_{i=-1}^{+1} \hbar \delta_i \left( \hat{n} + \frac{1}{2} \right) |i\rangle \langle i|. \hspace{1cm} (14)$$
The sum involving $\delta_i$ represents the measurement (interaction), and it commutes with the rest of the Hamiltonian as is needed for a QND measurement. Remarkably, this Hamiltonian is nearly identical to that of a single electron in a Penning trap, which has a proven record of precision measurements [26]. Experimentally, the effect of $\delta_i$ on the NMO frequency may be detectable only when the NV spin state changes. The observable shifts, considering only transitions that can be driven directly, are then

$$\delta_{0\pm} = \delta_{\pm 1} - \delta_0 = \frac{\hbar \Delta}{I_{\text{eff}} \omega_m} \frac{B_{\text{ext}} (3B_{\text{ext}} \mp B_{\text{zfs}})}{B^2_{\text{zfs}} - B^2_{\text{ext}}}. \quad (15)$$

In the strong interaction limit where $B_{\text{ext}}$ approaches $B_{\text{zfs}}$, we obtain

$$\delta_{0-} \approx 2\delta_{0+} \approx \frac{\hbar \Delta}{I_{\text{eff}} \omega_m} \frac{1}{1 - B_{\text{ext}}/B_{\text{zfs}}}. \quad (16)$$

The result of the interaction is that a change in the spin state (e.g. from $|0\rangle$ to $|-1\rangle$) appears as a change of the NMO frequency $\Delta \omega_m = \delta_{0-}$. Similarly, a change in the NMO number state (e.g. by $\Delta n = 1$) results in a change in the spin transition frequency $\Delta (\omega_{-1} - \omega_0) = \delta_{0-}$. Note that the measurement is QND in both directions.

Since $\delta_{0-}$ is approximately a factor of 2 larger than $\delta_{0+}$, we will focus on detecting transitions between the $|0\rangle$ and $|-1\rangle$ spin states. The experimental value of $\delta_{0-}$ will depend strongly on the details of the system geometry. For a graphene ribbon of width $= 200$ nm, length $= 2 \mu$m, we estimate $\omega_m \approx 2\pi \times 2$ MHz using the material constants from [24]. Adding a diamond nanocrystal with diameter $= 50$ nm and an applied external field $B_{\text{ext}} = 0.95B_{\text{zfs}}$, we obtain $\delta_{0-} \approx 2\pi \times 1$ kHz, which should be easily detectable. The interaction can be made even larger as $B_{\text{ext}}$ approaches $B_{\text{zfs}}$, but the adiabatic approximation will break down if $\omega_{-1}$ approaches $\omega_m$. From equation (7), we can see that the two frequencies become equal when $B_{\text{ext}}/B_{\text{zfs}} = 1 - \omega_m/\Delta \approx 0.9993$.

3. Measurements

3.1. Measure spin state with the nanomechanical oscillator (NMO)

We can measure the spin state of the NV center by driving the NMO to a classical state and measuring the oscillation frequency. Since this is a QND measurement, it could be performed continuously, allowing us to observe discrete jumps in the NMO frequency as the NV center spin state changes. For a high-fidelity measurement of the NV spin state, we need to resolve the shift of $\delta_{0-}$ associated with a spin state change quickly compared with the lifetime of the spin state. With the proposed values of $\delta_{0-} \approx 2\pi \times 1$ kHz, we anticipate being able to resolve the shift quickly compared with any of these times. So, this experiment could be successful over a wide range of temperatures, and either thermally or microwave-driven spin transitions should be observable.

This yields a new method of reading the NV center spin state with nearly perfect fidelity in a single shot, with no need for optical readout of a single NV center, which is difficult due to the weak emission of individual centers. However, even though this is a QND measurement, it does have an impact on the NV center; the transition lineshape of the NV center is broadened and shifted in frequency by the effective magnetic field change due to the NMO motion. For precision spectroscopy of the NV center, it may be desirable to allow the NMO to damp to a
thermal state and decrease the coupling between the NMO and NV by tuning $B_{\text{ext}}$ away from $B_{\text{NMO}}$ during part of the measurement sequence. In the next section, we discuss how the changes in the NV transition lineshape can be used to probe the NMO state.

### 3.2. Measure NMO number state with spin

The interaction term of the Hamiltonian shifts the NMO frequency depending on the NV spin state and similarly shifts the NV spin transition frequency depending on the NMO number state. In principle, the NMO number state could be read off continuously by making a continuous measurement of the NV center frequency. In practice, a continuous readout of the NV transition frequency on the required timescale is not practical. Instead, we are left with two options. We can probe the NV center transition frequency slowly with a weak drive, which does not uniquely determine the NMO number state but can still provide information about the NMO state structure. Alternatively, we can use pulsed measurements of the NV spin transition frequency to directly detect quantum jumps in the NMO number state. Because the NV can be prepared in a well-defined quantum state before the interaction and read out afterwards, this reduces or eliminates back-action on the NMO state until the NV is read out and allows us to perform precision spectroscopy of the oscillator.

#### 3.2.1. Weak drive measurements

The NV spin transition frequency depends on the NMO number state, so the NV spin transition lineshape can be shifted and broadened in complex ways, even by an undriven thermal NMO state, where the NMO motion is damped by the natural NMO dissipation and driven by thermal fluctuations. Fortunately, the resulting lineshapes have been calculated in an analogous system, a single electron in a Penning trap [27]. The NMO motion is analogous to the harmonic, damped axial motion of the trapped electron and the NV spin transition is analogous to the electron anomaly or cyclotron transition (considering only the lowest two cyclotron states).

The transition lineshapes depend on the damping and thermal excitation amplitude of the NMO, and can be calculated with a classical Brownian motion [27] or quantum density matrix master equation [28] technique. As shown in figure 3, for large $\gamma_m/\delta_0$, the lineshape is a shifted Lorentzian, but for $\gamma_m/\delta_0 \ll \tilde{n}$, the lineshape takes on an exponential shape created by the Boltzmann distribution of the NMO number state. The exponential shape can be used to measure the effective temperature of the NMO, as with one electron in a Penning trap [27]. In the extreme limit of a weak mechanical damping or strong NMO–NV center coupling $\delta_0 \gg \tilde{n}\gamma_m$, the frequency shift $\delta_0$ of the NV spin transition due to a number state change $\Delta n = 1$ can be resolved before the number state changes with the rate $\tilde{n}\gamma_m$. Then, the lineshape breaks up into a series of discrete lines reflecting the discretized NMO number states, in principle revealing the quantum structure of the NMO. In these calculations, we took $\tilde{n} = 10$ to keep numerical solution of the equations tractable and to make the lineshape structure easily visible in the plots.

In practice, there are several obstacles that prevent simple, direct observation of all of these lineshapes in detail. First, weak drive spectroscopy of the NV center spin states is impractical because pulsed drive sequences are generally required to realize the needed spin coherence time $T_2$. An appropriate choice of drive pulse sequence could be utilized, potentially without fundamentally altering the character of the lineshape in some cases, but attainable values of the NMO quality factor will likely make it difficult to realize the fully discretized lineshape. With
Figure 3. Calculated response of NV spin transition to a weak drive while coupled to an NMO with $\bar{n} = 10$ with a range of damping rates $\gamma_m$ illustrating the range of potentially accessible lineshapes.

these limitations, pulsed techniques are a more promising approach for probing the quantum structure of the NMO through the NV center.

3.2.2. Pulsed drive measurements. The Hamiltonian equation (5) can be rewritten by expanding the angle variable $\theta$ in terms of creation and annihilation operators, and using an interaction picture that is co-rotating with the mechanical oscillator,

$$H = \sum_i \hbar \omega_i |i\rangle \langle i| + \hbar \sum_i \delta_i |i\rangle \langle i| \left( \hat{a}^\dagger \hat{a} + \hat{a} \hat{a}^\dagger + \hat{a}^2 e^{-i2\omega_m t} + \hat{a}^\dagger 2 e^{i2\omega_m t} \right).$$

(17)

Note that we have kept the terms rotating at $2\omega_m$ in this Hamiltonian, since we do not assume that we are averaging for long times compared with the mechanical oscillation period during the pulsed measurements. We have assumed, however, that the NV transition frequencies are still large compared to the NMO oscillator frequency, thereby enabling us to assume that the spin still adiabatically follows the external magnetic field. This allows us to use the same diagonalization procedure outlined in section 2 to obtain the interaction terms between the NMO and the NV.

Now, let us also introduce a microwave driving field that is nearly in resonance with the $|0\rangle$ to $|−1\rangle$ transition. By transforming to an interaction picture that is co-rotating with the microwave drive, and making the rotating wave approximation, we obtain

$$H = \hbar \epsilon |−1\rangle \langle −1| + \hbar \sum_{i=0,−1} \delta_i |i\rangle \langle i| \left( \hat{n} + \frac{1}{2} + \frac{\hat{a}^2 e^{-i2\omega_m t} + \hat{a}^\dagger 2 e^{i2\omega_m t}}{2} \right).$$

(18)

where $\epsilon$ is the detuning between the microwave drive and the NV spin transition, which is assumed to be zero for the rest of the discussion herein. At this point, we can consider the
effect of the following pulse sequence, \((\pi/2)_{x} - \tau - (\pi/2)_{y} - T_{i} - (\pi/2)_{y} - \tau - (\pi/2)_{x}\), where \(\tau = N\pi/\omega_{m}\) with \(N\) being an integer, \((\Theta)_{x,y}\) denotes microwave pulses with pulse area of \(\Theta\) around the \(x\)- and \(y\)-axes, respectively, and \(T_{i}\) is an idling time, which can be varied in order to detect quantum jumps in the oscillator. It can be shown, assuming that the NV is initially in the \(|0\rangle\) state, that the probability of remaining in the \(|0\rangle\) state after the sequence is

\[
p(0) = \frac{1}{2} + \frac{\sin(\alpha + \beta)F_{\text{dephase}}(\tau) + \sin(\beta - \alpha)F_{\text{coh}}(\tau)}{4}.
\]  

(19)

Here, \(\alpha = (\delta_{0-\tau})n\), and \(\beta = (\delta_{0-\tau})(n + \Delta n)\) represent the phase accumulated by the NV during each period \(\tau\) due to the number state of the oscillator. We further assumed that \(T_{i}^{NV} \gg T_{i} \gg T_{2}^{*}\). The functions \(F_{\text{dephase}}(\tau)\), \(F_{\text{coh}}(\tau)\) will depend on the details of the environmental noise, but represent the contributions from dephasing and decoherence, respectively. For instance, when the source of noise is from a slowly varying environment with correlation times \(\tau_{c} \gg \tau\) and a Lorentzian power spectral density, it can be shown that \(F_{\text{dephase}}(\tau) = \exp(-(2\tau/T_{2}^{*})^{2})\) and \(F_{\text{coh}}(\tau) = \exp(-(2\tau/T_{2}^{*})^{3})\) [25, 29]. There are three useful attributes of this sequence: (i) changes in the effective magnetic field on the NV due to a quantum jump of the NMO during the time \(T_{i}\) cause changes in the population of the \(|0\rangle\) state, (ii) the effects of the rotating terms are washed out since the period during which the NV is sensitive to them is matched to the semi-period of the oscillator, (iii) the time \(T_{i}\) can be made long as the information about the NMO in the first part of the sequence is encoded into population, not phase. The time \(T_{i}\) does not explicitly appear in the expression for the signal because all spin coherence is assumed to be destroyed during this time. Note that when \(T_{i} = 0\), the sequence looks like a spin–echo sequence. Hence, the decohering effects of the slowly varying environment on the NV center will be refocused during the sequence, and this is made clear from the expression for the fluorescence where the quantum jump sensitive term only decays with the \(T_{2}\) time of the spin. Assuming that \(\tau \gg T_{2}^{*}\), which is likely given the typical NMO frequencies, we can neglect the second term and write \(p(0) = \frac{1}{2} + \frac{\sin(\beta - \alpha)F_{\text{coh}}(\tau)}{4} \approx \frac{1}{2} + \frac{(\delta_{0-\tau}\Delta n)F_{\text{coh}}(\tau)}{4}\).

Starting from a thermal state of the NMO, the NV will measure magnetic field over an integer number \((N)\) of half-periods of the nanomechanical motion, then wait a length of time \(T_{i}\), then again measure over \(N\) half-periods, taking the difference between the initial and final magnetic field measurements with this choice of microwave pulses. The phase accumulated by the NV is dependent only on the number state of the oscillator, while the conjugate operator, i.e. the phase of the oscillator, is made completely indeterminate after the measurement. The magnetic field difference \(\Delta B\) is converted into a probability of a spin transition, which can be read out either through the NMO frequency or through the NV fluorescence. We now consider some of the technical limitations and figures of merit for the observation of quantum jumps.

A histogram of measured magnetic field changes should show peaks at discrete values corresponding to integer changes in the NMO number state. The above expression allows us to derive a minimum detectable number state change \((\Delta n)_{\text{min}} = \sqrt{\frac{2\tau T_{i}}{\delta_{0-\tau} CF_{\text{coh}}(\tau)}}\), where \(C\) is due to finite measurement efficiency of the NV spin state. Typical measurement efficiency using optical readout results in \(C \sim 0.05\); however, by using nearby nuclear spins as a quantum memory, improvements in readout efficiency have already been demonstrated [30, 31]. It may be possible to reach \(C \sim 1\), for instance by using the NMO as described earlier to perform readout. Since we only detect changes in the NMO state during the sequence, the NMO state does not need to be preserved in the times between pulse sequences or during the classical
readout of the NV spin state. Other approaches to improvements in collection efficiency include coupling the NV to photonic crystal slab waveguides, or silica microspheres that can be positioned independently using scanning probe techniques [32, 33]. Finally, the spin coherence times of NV centers in type Ib diamond nanocrystals have been shown, after proper surface cleaning, to be \( T_2 \approx 2 \text{ ms} \) as observed in the bulk case, where \( T_2 \sim 2 \text{ ms} \) was observed as noted earlier. Conservatively, we assume only that \( T_2 \sim 5 \mu \text{s} \) can be obtained. Using \( N = 1, \omega_m = 2\pi \times 2 \text{ MHz}, \delta_{0-} = 2\pi \times 1 \text{ kHz}, T_i \sim 20\tau \), we obtain \( \langle \Delta n \rangle_{\min} \sim 6–0.3 \) for \( C \sim 0.05–1 \), respectively, when averaging for \( \sim 30 \text{ s} \).

This measurement puts more stringent requirements on the NMO. In particular, the NMO should not change state excessively, while the NV makes the initial or final magnetic field measurement (\( \sim 1 \text{ NMO period} \)). Thus, the minimum \( Q \) for fully resolving the discrete structure of the number states is \( Q_{\min} \sim 2\pi \bar{n} \). For the 2 MHz oscillator described earlier at temperature \( T = 2 \text{ K} \), we require \( Q \sim 125 000 \), which is likely to be challenging in graphene. Alternative approaches include cooling to \( T \sim 100 \text{ mK} \) with a dilution refrigerator and/or increasing \( \omega_m \) to \( \sim 10 \text{ MHz} \), decreasing the requirement to \( Q \sim 1500 \). An attractive option in the long term is to add an additional restoring force from optical, magnetic or electrostatic forces to increase the oscillation frequency without adding damping [35].

Classical readout of graphene nanomechanical resonators can be performed optically [36] or electrically [37], with measured \( Q \) factors up to 14 000 at low temperature [37]. Some modifications of these techniques will be required for reading the torsional mode coupled to an NV center, and the \( Q \) of the torsional mode may not be equal to that of the bending modes. For most of the measurements we propose, the details of the classical readout technique are not critical because the result of an NMO–NV interaction can be stored in the NV center spin state and read out through the NMO at a later time (limited by the NV center \( T_1 \)). Furthermore, any heating of the NMO by the classical readout can be allowed to dissipate before the next measurement is carried out.

We also note that in the strong-coupling regime where \( \delta_{0-} \gg \bar{n} \gamma_m, (1/T_2^*) \), it may be possible to achieve a single-shot readout of the oscillator number state. This can be done simply by tuning a microwave \( \pi \)-pulse to the appropriate transition, since each of the number states will be resolved, as shown in figure 3. This results in the conditional flip of the NV spin if and only if the oscillator is in the appropriate number state. Assuming the readout of the NV spin does not perturb the oscillator if done through fluorescence measurement, we thereby obtain a single-shot readout of the oscillator number state.

4. Applications

The simplicity and elegance of the proposed nanomechanical-spin system will allow it to serve as a platform for many future experiments in quantum nanomechanics, as well as a building block for future, more complex nanomechanical systems for sensing or quantum information processing applications. In addition to QND measurements, as outlined above, the coupling scheme easily lends itself to other applications, such as cooling, state transfer between NMO and spin and coupling of spins via the NMO. Cooling of the NMO through the spin requires operation in a different regime from the QND measurements, as energy transfer requires an interaction Hamiltonian of the form \( H_I = \hbar \lambda (\hat{a} + \hat{a}^\dagger) S_z \) combined with optical pumping. This Hamiltonian can be achieved in our system by tuning the NV slightly away from the

New Journal of Physics 13 (2011) 045002 (http://www.njp.org/)
level anticrossing, where the energy of the $|0\rangle$ and $|-1\rangle$ states is linearly dependent on $\theta$. Alternatively, it may be possible to reach a non-adiabatic regime where the splitting between the eigenstates is comparable with $\omega_{\text{nr}}$, enabling cooling by energy exchange.

Furthermore, the NV could be prepared in a superposition state, which could then (through the above QND coupling) put the NMO in a superposition state, even a superposition of macroscopically distinguishable states (a ‘Schrödinger Cat’ state). Since the coupling is bidirectional, the state could be coupled back to the NV center or even to another NV center on the same (or a coupled) NMO. Quantum state transfer could be enabled by tuning the $|-1\rangle$ state of the NV center to be energetically close to the spin $|0\rangle$ state, so that, combined with a weak additional transverse magnetic field, transitions between those NV states could be directly driven by the NMO motion. Coupling multiple NMOs or multiple NV centers on the same NMO could further expand the potential applications of the system.

The analysis presented also applies to other materials in addition to the graphene–NV center system described. Other NMO materials, such as silicon nitride or aluminum oxide, may yield devices with higher quality factors and more robust fabrication processes. However, those benefits may come at the cost of lower coupling strengths $\delta_i$ due to the larger required thickness of those materials and thus higher mass of the resulting devices. Sources of a spin other than an NV center may also be beneficial, particularly if the host material can be more directly integrated with the NMO. The requirements include long $T_1$ and $T_2$ times as well as a large zero-field splitting.

Acknowledgments

We acknowledge helpful discussions with P Rabl and D Chang. This material is based on work supported by a DARPA Young Faculty Award under award no. N66001-10-1-4024 (BD), an NSF Faculty Early Career Development Award under award no. DMR-0847195 (GD) and the Alfred P Sloan Research Fellowship (GD).

References

[1] Zwickl B M, Shanks W E, Jayich A M, Yang C, Bleszynski Jayich A C, Thompson J D and Harris J G E 2008 High quality mechanical and optical properties of commercial silicon nitride membranes Appl. Phys. Lett. 92 103125
[2] Huang X M H, Zorman C A, Mehregany M and Roukes M L 2003 Nanoelectromechanical systems: nanodevice motion at microwave frequencies Nature 421 496
[3] Thompson J D, Zwickl B M, Jayich A M, Marquardt F, Girvin S M and Harris J G E 2008 Strong dispersive coupling of a high-finesse cavity to a micromechanical membrane Nature 452 72
[4] Favero I and Karrai K 2009 Optomechanics of deformable optical cavities Nat. Photonics 3 2009
[5] Pinard M and Dantan A 2008 Quantum limits of photothermal and radiation pressure cooling of a movable mirror New J. Phys. 10 2008
[6] Teufel J D, Regal C A and Lehnert K W 2008 Prospects for cooling nanomechanical motion by coupling to a superconducting microwave resonator New J. Phys. 10 2008
[7] Redman D A, Brown S, Sands R H and Rand S C 1991 Spin dynamics and electronic states of NV centers in diamond by EPR and four-wave-mixing spectroscopy Phys. Rev. Lett. 67 3420
[8] Takahashi S, Hanson R, van Tol J, Sherwin M S and Awschalom D D 2008 Quenching spin decoherence in diamond through spin bath polarization Phys. Rev. Lett. 101 047601
Harrison J, Sellars M J and Manson N B 2006 Measurement of the optically induced spin polarisation of N-V centres in diamond Diam. Relat. Mater. 15 586

Balasubramanian G et al 2009 Ultralong spin coherence time in isotopically engineered diamond Nat. Mater. 8 383

Wrachtrup J and Jelezko F 2006 Quantum information processing in diamond J. Phys.: Condens. Matter 18 S807

Manson N B, Harrison J P and Sellars M J 2006 Nitrogen-vacancy center in diamond: model of the electronic structure and associated dynamics Phys. Rev. B 74 104303

Jelezko F, Gaebel T, Popa I, Gruber A and Wrachtrup J 2004 Observation of coherent oscillations in a single electron spin Phys. Rev. Lett. 92 076401

Fuchs G D, Dobrovitski V V, Toyli D M, Heremans F J and Awschalom D D 2009 Gigahertz dynamics of a strongly driven single quantum spin Science 326 1520

Childress L, Dutt M V G, Taylor J M, Zibrov A S, Jelezko F, Wrachtrup J, Hemmer P R and Lukin M D 2006 Coherent dynamics of coupled electron and nuclear spin qubits in diamond Science 314 281

Dutt M V G, Childress L, Jiang L, Togan E, Mazé J, Jelezko F, Zibrov A S, Hemmer P R and Lukin M D 2007 Quantum register based on individual electronic and nuclear spin qubits in diamond Science 316 1312

Neumann P, Mizuochi N, Rempp F, Hemmer P, Watanabe H, Yamasaki S, Jacques V, Gaebel T, Jelezko F and Wrachtrup J 2008 Multiparticle entanglement among single spins in diamond Science 320 1326

Maze J R et al 2008 Nanoscale magnetic sensing with an individual electronic spin in diamond Nature 455 644

Balasubramanian G et al 2008 Nanoscale imaging magnetometry with diamond spins under ambient conditions Nature 455 648

Braginsky V B, Vorontsov Y I and Thorne K S 1980 Quantum nondemolition measurements Science 209 547

Caves C M, Thorne K S, Drever R W P, Sandberg V D and Zimmermann M 1980 On the measurement of a weak classical force coupled to a quantum-mechanical oscillator. I. Issues of principle Rev. Mod. Phys. 52 341

Rabl P, Cappellaro P, Dutt M V G, Jiang L, Maze J R and Lukin M D 2009 Strong magnetic coupling between an electronic spin qubit and a mechanical resonator Phys. Rev. B 79 041302

Rugar D, Budakian R, Mamin H J and Chui B W 2004 Single spin detection by magnetic resonance force microscopy Nature 430 329

Muñoz E, Lu J and Yakobson B I 2010 Ballistic thermal conductance of graphene ribbons Nano Lett. 10 1652

Taylor J M, Cappellaro P, Childress L, Jiang L, Budker D, Hemmer P R, Yacoby A, Walsworth R and Lukin M D 2008 High-sensitivity diamond magnetometer with nanoscale resolution Nat. Phys. 4 810

Odom B, Hanneke D, D’Urso B and Gabrielse G 2006 New measurement of the electron magnetic moment using a one-electron quantum cyclotron Phys. Rev. Lett. 97 030801

Brown L S and Gabrielse G 1986 Geonium theory: physics of a single electron or ion in a Penning trap Rev. Mod. Phys. 58 233–11

D’Urso B 2003 Cooling and self-excitation of a one-electron oscillator PhD Thesis Harvard University

Maze J R, Taylor J M and Lukin M D 2008 Electron spin decoherence of single nitrogen-vacancy defects in diamond Phys. Rev. B 78 094303

Jiang L et al 2009 Repetitive readout of a single electronic spin via quantum logic with nuclear spin ancillae Science 326 267

Steiner M, Neumann P, Beck J, Jelezko F and Wrachtrup J 2010 Universal enhancement of the optical readout fidelity of single electron spins Phys. Rev. B 81 035205

Larsson M, Dinyari N and Wang H 2009 Composite optical microcavity of diamond nanopillar and silica microsphere Nano Lett. 9 1447

Englund D, Shields B, Rivoire K, Hatami F, Vuckovic J, Park H and Lukin M D 2010 Deterministic coupling of a single nitrogen vacancy center to a photonic crystal cavity Nano Lett. 10 3922–6
[34] Tisler J et al 2009 Fluorescence and spin properties of defects in single digit nanodiamonds ACS Nano 3 1959
[35] Chang D E, Regal C A, Papp S B, Wilson D J, Ye J, Painter O, Kimble H J and Zoller P 2009 Cavity opto-mechanics using an optically levitated nanosphere Proc. Natl Acad. Sci. USA 107 1005–10
[36] Shivaraman S et al 2009 Free-standing epitaxial graphene Nano Lett. 9 3100–5
[37] Chen C, Rosenblatt S, Bolotin K I, Kalb W, Kim P, Kymissis I, Stormer H L, Heinz T F and Hone J 2009 Performance of monolayer graphene nanomechanical resonators with electrical readout Nat. Nano 4 861–7