Physics reach of $\beta$-beams and $\nu$-factories: the problem of degeneracies

S. Rigolin$^{a}$

$^a$Theoretical Physics Department and I.F.T, Universidad Autonoma de Madrid, Cantoblanco, Spain

We discuss the physics reach of $\beta$-Beams and $\nu$-Factories from a theoretical perspective, having as a guideline the problem of degeneracies. The presence of degenerate solutions in the measure of the neutrino oscillation parameters $\theta_{13}$ and $\delta$ is, in fact, the main problem that have to be addressed in planning future neutrino oscillation experiments. If degeneracies are not (at least partially) solved, it will be almost impossible to perform, at any future facility, precise measurements of $\theta_{13}$ and/or $\delta$. After a pedagogical introduction on why degenerate solutions arise and how we can get rid of them, we analyze the physics reach of current $\beta$-beam and $\nu$-factory configurations. The physics reach of the "standard" $\beta$-Beam is severely affected by degeneracies while a better result can be obtained by higher-$\gamma$ setups. At the $\nu$-Factory the combination of Golden and Silver channels can solve the eightfold degeneracy down to $\sin^2 \theta_{13} \leq 10^{-3}$.

1. Introduction

The atmospheric and solar sector of the PMNS leptonic mixing matrix have been measured with quite good resolution by SK, SNO and KamLand. These experiments measure two angles, $\theta_{12}$ and $\theta_{23}$, and two mass differences, $\Delta m^2_{12}$ and $\Delta m^2_{23}$. The present bound on $\theta_{13}$, $\sin^2 \theta_{13} \leq 0.04$, is extracted from the negative results of CHOOZ and from three-family analysis of atmospheric and solar data. The PMNS phase $\delta$ is totally unbounded as no experiment is sensitive, up to now, to the leptonic CP violation. The main goal of next neutrino experiments will be to measure these two, still unknown, parameters. In this talk we analyze the physics reach of $\beta$-Beam and $\nu$-Factory following a somehow theoretical perspective: the problem of degeneracies in $(\theta_{13}, \delta)$ measure. The best channel for measuring $(\theta_{13}, \delta)$ is the $\nu_e \rightarrow \nu_\mu$ appearance channel \cite{1} (and/or its T and CP conjugate ones). Unfortunately, this measure is severely affected by the presence of an eightfold degeneracy \cite{2,3}.

2. The eightfold degeneracy

It was originally pointed out in Ref. \cite{2} that the appearance probability $P_{\alpha\beta}$ for neutrinos with a fixed Baseline/Energy (L/E) ratio and input parameters $(\bar{\theta}_{13}, \bar{\delta})$ has no unique solution. Indeed, the equation

\[ P_{\alpha\beta}(\bar{\theta}_{13}, \bar{\delta}) = P_{\alpha\beta}(\theta_{13}, \delta) \]  

has a continuous number of solutions. The locus of the $(\theta_{13}, \delta)$ plane satisfying this equation is called equiprobability curve (see Fig. 1). Consider now the equiprobability curves for neutrinos (+) and antineutrinos (−) with the same L/E (and the same input parameters). The following system of equations

\[ P_{\alpha\beta}(\bar{\theta}_{13}, \bar{\delta}) = P_{\alpha\beta}(\theta_{13}, \delta) \]  

has two intersections (see Fig. 2): the input pair $(\bar{\theta}_{13}, \bar{\delta})$ and a second, L/E dependent, point. This

\[ \text{approaches can be used to get rid of them: combination of different experiments and/or combination of different oscillation channels. In section 3 and 4 we describe shortly the physics reach of } \beta \text{-Beam and } \nu \text{-Factory respectively$^2$.} \]

$^2$The eightfold degeneracy in the $(\theta_{13}, \delta)$ measure has been comprehensively studied in literature. All the technical details and a complete set of bibliographic references can be found in \cite{4,5}. 

$^*$The author acknowledges the financial support of MCYT through project FPA2003-04597 and of the European Union through the networking activity BENE.
second intersection introduces an ambiguity in
the measurement of the physical values of \(\theta_{13}\)
and \(\delta\): the so-called intrinsic clone solution \[2\].
Knowing the two probabilities of eq. \[2\] is conse-
quently not enough for solving the intrinsic de-
genicity. One needs to add more informations.

Two ways are viable: i) using independent ex-
periments (i.e different L/E) and/or ii) using in-
dependent oscillation channels. In case i) one
can think to observe the same neutrino oscilla-
tion channel (i.e. the golden \(\nu_e \rightarrow \nu_\mu\) oscilla-
tion) using neutrino (antineutrino) beams with dif-
f erent L/E. In Fig. 1 one can see that experiments
with different L/E present clone solutions in dif-
ferent regions of the \((\theta_{13}, \delta)\) parameter space. If
the clones are well separated one can solve the
degeneracy. Another possibility, case ii), is to fix
L/E and to use contemporaneously two different
oscillation channels (like for example \(\nu_e \rightarrow \nu_\mu\)
and \(\nu_e \rightarrow \nu_\tau\)). In Fig. 2 one can see how the intrinsic
clones for the two channels appear, generally, in
different locations and so the intrinsic degeneracy
can be solved.

From this example we learn that the best way
for solving the degeneracies is to add all the pos-
sible available informations: different baselines,
different energy bins (i.e. different L/E) and dif-
f erent channels. Therefore, in planning future ex-
periments one has to understand which combina-
tions of experiments can give the largest set of
really independent informations. The existence
of unresolved degeneracies could, in fact, mani-
fests itself in a complete lost of predictability on
the aforementioned parameters as we will see in
the next sections.

Unfortunately, the appearance of the intrinsic
degeneracy is only a part of the “clone problem”.
As it was pointed out in \[3\], two other sources of ambiguities arise due to the present (and near
future) ignorance of the sign of the atmospheric
mass difference, \(s_{atm} = \text{sign}[\Delta m^2_{23}]\), and of the
\(\theta_{23}\) octant, \(s_{oct} = \text{sign}[\tan(2\theta_{23})]\). These two
discrete variables assume the values \(\pm 1\), depend-
ing on the physical assignments of the \(\Delta m^2_{23}\) sign
\((s_{atm} = 1 \text{ for } m^2_3 > m^2_2 \text{ and } s_{atm} = -1 \text{ for } m^2_3 < m^2_2)\) and of the \(\theta_{23}\)-octant \((s_{oct} = 1 \text{ for } \theta_{23} < \pi/4 \text{ and } s_{oct} = -1 \text{ for } \theta_{23} > \pi/4)\). Conse-

![Figure 1. Correlation of \(\theta_{13}\) and \(\delta\) if only neu-
trinos (or antineutrinos) are measured: infinite de-
genicity.](image1)

![Figure 2. Correlation of \(\theta_{13}\) and \(\delta\) if neutri-
nos (full line) and antineutrinos (dashed line) are
measured: twofold degeneracy.](image2)
subsequently, future experiments will have to measure four unknowns: two continuous variables ($\theta_{13}, \delta$) plus two discrete variables ($s_{atm}, s_{oct}$).

From these considerations it follows that eq. (2) should be more correctly replaced by the following four systems of equations (each for any possible choice of the $s_{atm}$ and $s_{oct}$ signs)\(^3\):

$$
P_{\alpha\beta}^{\pm}(\theta_{13}, \delta; \bar{s}_{atm}, \bar{s}_{oct}) =
\quad P_{\alpha\beta}^{\pm}(\theta_{13}, \delta; s_{atm} = \pm \bar{s}_{atm}; s_{oct} = \pm \bar{s}_{oct}). \quad (3)
$$

Solving the four systems of eq. 3 will result in obtaining the true solution plus additional clones to form an eightfold degeneracy. These eight solutions are respectively: the true solution and its intrinsic clone (when the right $s_{atm}$ and $s_{oct}$ signs are used in eq. 3), the $\Delta m_{23}^2$-sign clones (when $s_{atm} = -\bar{s}_{atm}$ is used), the $\theta_{23}$-octant clones (when $s_{oct} = -\bar{s}_{oct}$ is used) and finally the mixed clones (when simultaneously $s_{atm} = -\bar{s}_{atm}$ and $s_{oct} = -\bar{s}_{oct}$ are used).

\(^3\)For simplicity of notation we express eqs. 1–3 in terms of probabilities. However as noticed in \(^4\) one should use instead the number of (leptonic) measured events.

3. Physics reach of the $\beta$-Beam

In Fig. 5 one can see the dramatic impact that degeneracies can have in the precision measurement of ($\theta_{13}, \delta$) at the “standard” $\beta$-Beam configuration\(^4\) (i.e. $L=130$ km and $\gamma=(60, 100)$ for He and Ne ions respectively): (1) the error in the $\theta_{13}$ measurement is increased by a factor four (two) for large (small) values of $\theta_{13}$; however, the presence of degeneracies has a small impact on the ultimate $\theta_{13}$ sensitivity; (2) the error in the $\delta$ measurement grows in a significant way in presence of the clones, almost spanning half of the parameter space for small values of $\theta_{13}$. These facts are well understood: being the “standard” $\beta$-Beam a (short distance) counting experiment there are not enough independent informations to cancel any of the degeneracies.

To obviate this problem new $\beta$-Beam setups have been recently proposed by \(^1\)\(^1\)\(^2\). In these setups the use of energy bins helps in reducing the number of degeneracies and improves the sensitivity to $\theta_{13}$ and $\delta$. However, to have a significant

\(^4\)For a comprehensive description of old and new $\beta$-Beam setups look at \(^1\)\(^1\)\(^1\)\(^1\)\(^1\)\(^2\).
improvement over the “standard” setup, neutrino energies of $\mathcal{O}(1$ GeV) (i.e. $\gamma \approx 150-300$) and baselines of 300-700 km are needed.

In Fig. 5 the 3σ sensitivity reach to $\theta_{13}$ is shown. Notice that the inclusion of all degeneracies worsen the sensitivity to $\theta_{13}$ for $\delta=0$ of almost and order of magnitude, compared to the only-intrinsic case usually presented in literature.

In Fig. 6 the CP violation exclusion plot at 99% CL for different $\beta$-Beam setups it is shown. One can easily notice in this plot that the sensitivity to CP violation is increased at higher-$\gamma$ (and baseline) configurations. This work, however, assumes the same neutrino flux at any of the $\gamma$s considered. While this hypothesis seems reasonable (within a factor 2), a careful analysis of ions fluxes at different $\gamma$s should be performed.

Besides the appearance channel, the $\nu_e \rightarrow \nu_e$ disappearance channel is available at the $\beta$-Beam. Unfortunately, as it was shown in Fig. 5, this channel does not provide any additional informations once realistic systematic errors are taken into account.

4. Physics reach of the $\nu$-Factory

No much work has been devoted in the last two years in improving the analysis for the $\nu$-Factory. Our understanding of the $\nu$-Factory physics reach is practically still the one presented in ref. [13] (and graphically summarized in Fig. 8) with the addition of the $\nu_e \rightarrow \nu_\tau$ “silver” appearance channel ref. [14]. The combination of the “golden” channel (with a 3000 km baseline and, eventually, a second detector placed at 7000 km) and the “silver” channel could solve practically all the degeneracies down to $\theta_{13} \approx 1^\circ - 2^\circ$. Notice however, that all the golden channel studies strongly dependend on the old-fashion detector configuration presented in [15]. The appearance of the “degeneracies problem” force us to consider a complete re-analysis of the iron detector characteristics. The inclusion of the first energy bins (let’s say between 0-10 GeV) becomes mandatary for solving clones solutions and so improv-

5Originally the detector was designed for reaching the highest sensitivity to $\theta_{13}$ because at the time the detector was designed the SMA solution was not excluded yet.
ing $\nu$-Factory physics reach. In fact the oscillation maximum for a NF with a baseline of 3000 km is around 5 GeV. In the old detector analysis tight background cuts reduce almost to 0 the efficiency for this bin, in such a way that only the above-the-peak energy informations is left.

In our opinion, the full understanding of the $\nu$-Factory physics reach should have to take advantage of its large availability of oscillation channels. Besides the accurate studies of the “golden” and “silver” appearance channels, a deeper analysis of the $\nu_\mu$, $\nu_\mu$ disappearance channels and of the $\nu_\mu \rightarrow \nu_\tau$ appearance one should be considered. Of course this effort could require a (considerable) increase of detectors cost. But it should be considered if the major effort of building the $\nu$-Factory is agreed to be necessary for the full understanding of neutrino oscillation parameters.

REFERENCES

1. A. Cervera et al., Nucl. Phys. B 579 (2000) 17 [Erratum-ibid. B 593 (2001) 731].
2. J. Burguet-Castell et al., Nucl. Phys. B 608 (2001);
3. G. L. Fogli and E. Lisi, Phys. Rev. D 54 (1996) 3667; H. Minakata and H. Nunokawa, JHEP 0110 (2001) 001; V. Barger et al. Phys. Rev. D 65 (2002) 073023.
4. A. Donini et al., JHEP 0406, 011 (2004).
5. A. Donini et al., Nucl. Phys B710 (2005) 402; A. Donini et al., hep-ph/0506100.
6. P. Zucchelli, Phys. Lett. B 532 (2002) 166.
7. J. J. Gomez-Cadenas et al. hep-ph/0105297.
8. A. Donini et al., Phys.Lett. B 621 (2005) 276.
9. M. Apollonio et al., arXiv:hep-ph/0210192.
10. See for example P. Huber et al. Nucl. Phys. B 645 (2002) 3, and references therein.
11. J. Burguet-Castell et al., hep-ph/0503021.
12. See M. Mezzetto these proceedings and P. Huber et al., arXiv:hep-ph/0506237.
13. J. Burguet-Castell et al. Nucl. Phys. B 646 (2002) 301; D. Autiero et al., Eur. Phys. J. C 33, 243 (2004); P. Huber et al., JHEP 0505, 020 (2005).
14. A. Donini et al. Nucl.Phys.B 646, 321 (2002).
15. A. Cervera, F. Dydak and J. Gomez Cadenas, Nucl. Instrum. Meth. A 451, 123 (2000).