Pinning Control of Spatio Temporal Chaos in Nonlinear Optics

C. Mendoza1, P.L. Ramazza2, J.Martínez-Mardones1 and S. Boccaletti2

1 Institute of Physics, Pontifical Catholic University of Valparaíso, 234-0025 Valparaíso, Chile.
2 CNR- Istituto dei Sistemi Complessi, Via Madonna del Piano 10, 50019 Sesto Fiorentino (FI), Italy.
E-mail: caromendoza@gmail.com

Abstract. We have studied numerically the influence of the number of controllers in the control of a spatial pattern in an optical device. In this article, we focus on the liquid crystal light valve (LCLV) which is known to exhibit spatio-temporal chaotic states in some range of parameters. By applying a correcting term in the intensity proportional to the difference between the light intensity of the target pattern and the chaos state, the system is driven to the target pattern in finite time. In addition, we study the number of pinning points and their positions to reach the control of the pattern.

1. Introduction
The importance of optical devices in today technology is mainly due to the ever growing need of bandwidth in the telecommunication industry [1]. The manipulation and control of light are crucial in the different stages of the transmission of information. The ability to design all-optical circuits will be decisive in the next few years for satisfying the demand of bandwidth.

Many studies search the minimum number of controllers necessary to stabilize a state out of a spatiotemporal chaotic evolution[2, 3, 4] in the context of the one dimensional complex Ginzburg-Landau equation in the proximity of a Hopf bifurcation [5].

The liquid crystal light valve (LCLV) has been widely used both experimentally and theoretically for describing chaotically evolving two dimensional optical patterns. The LCLV system has become also an experimental model of optical device due to its rather easy manipulation in laboratories and its large use in display media [6]. The main reason is the slow time scale that governs the pattern formation inside the LCLV, of about one second. The pioneering works of lasers shining onto a Kerr media by Oppo et al. [7] have been followed by many experimental works on the LCLV [8, 9] which have permitted to reveal the richness of pattern formation in such nonlinear optical device.

The experimental setup on which this theoretical work is based is described in its full details in reference [10] and will not be reported here. Using the standard eikonal approximation of wave optics [11], the equation that describe the evolution of the LCLV is [9]:

\[
\frac{\partial \phi}{\partial t} = -\frac{\phi - \phi_0}{\tau} + D\nabla_\perp^2 \phi + \phi_{sat}(1 - e^{-\frac{\alpha I_0}{\phi_{sat}}}),
\]

(1)
where the light intensity $I$ is given by:

$$I = |Ee^{i\sigma\nabla^2}|^2,$$

(2)

Here $\phi(x, y)$ is the phase of the optical beam at the output of the LCLV, $\phi_0$ is the uniform phase obtained by opening the feedback loop, $\tau$ is the relaxation time of the liquid crystals inside the valve, $\phi_{sat}$ the value of phase saturation, $\alpha$ is a control parameter of the LCLV, $I_0$ is the intensity of the impinging laser beam, $E$ the electric field. The beam is transmitted following the axis of the liquid crystal director, $D$ is the diffusion constant of the phase inside the LCLV and $L$ is the diffractive propagation length.

The equation (1) is of reaction-diffusion type [12] and its simulation leads to a stationary hexagonal pattern for small control parameter $\alpha I_0 = 0.6$ and to spatio-temporal chaos for larger value of the control parameter $\alpha I_0 = 1.7$, as it is illustrated in Fig.1.

This work is based on numerical simulations of Eq.(1). We have used a spectral method with 128x128 Fourier modes in the two space dimensions [14]. The time integration follows a Runge-Kutta scheme of fourth order with a time step of $\Delta t = 0.25$. The parameters used in the simulations, relevant for comparison with the experimental, are: $D = 1.51 \times 10^{-8}$ mm$^2$/s, $\phi_{sat} = 5\pi$ rad, $\sigma = 1$ mm$^{-2}$, $\tau = 1$s, $\phi_0 = 0$, and $L_x = L_y = 2$ mm. The wavelength of the laser light used in the experiment is $\lambda_0 = 633$ nm.

2. Methods and indicators

In order to control the spatio–temporal patterns in the LCLV for a future experimental implementation, the disturbance term is in term of the intensity of the light rather than the phase of the electric field:

$$I_{pert} = I + c(I_t - I)$$

(3)

where $c$ is the control strength and $I_t(x, y)$ is the intensity of the target pattern.

The pinning points are the points where the control perturbation is applied. We used a regular grid, $np = 1$ means that all grid points are pinning points, $np = 2$ corresponds to the situation of leaving a space between the pinning points in each direction, for $np = 3$ two mesh points are left uncontrolled in each directions between pinning points, and so on for larger values of $np$. Fig. 2 summarize the number and location of the pinning points for different values of $np$ in the regular case.
In order to characterize quantitatively the ability of controlling the system, one defines the cross-correlation between the actual pattern and the desired target pattern as follow:

$$\rho = \frac{\langle (I - \langle I \rangle)(I_t - \langle I_t \rangle) \rangle}{\sqrt{\langle (I - \langle I \rangle)^2 \rangle \sqrt{\langle (I_t - \langle I_t \rangle)^2 \rangle}}}$$  \hspace{1cm} (4)$$

where \(\langle \rangle\) denotes a full space–time average.

For \(\rho \approx 0\) the two fields are linearly uncorrelated, \(\rho = 1\) marks complete correlation, and \(\rho = -1\) indicates that the fields are negatively correlated. \(\rho\) is related to a global spatial synchronization of the pattern with respect to the target pattern. In the Fourier plane the energy of the target pattern has six marked modes. The indicator \(S\) compares the energy of the target pattern \((e_t)\) with the energy contained in the actual pattern \((e)\) at the same location in the Fourier plane:

$$S = e/e_t$$  \hspace{1cm} (5)$$

\(S = 1\) implies that the maximum of energy of the current pattern is located in the same six points of the target pattern.

3. Results for pinning control

In all the results presented in the following, Eq.(1) is integrated from \(t = 0\) until \(t = 1,000\) with \(\alpha I_0 = 1.7\).

Fig. 3a shows the hexagonal target pattern. This pattern has been calculated from Eq.(1) and stabilized with a Fourier filter. If not stabilized, this hexagonal pattern would be unstable for \(\alpha I_0 = 1.7\) (see Fig.1). The control method used consists of modifying the intensity according the expression Eq.(3). The two parameters of our analysis are \(c\) (the coupling strength) and \(np\) (the pinning points). Fig. 3b shows a snapshot of the system while only imperfect stabilization is reached for \(c = 0.6\) and \(np = 4\), the numerical values of the indicators are \(\rho = 0.61\) and \(S = 0.95\).

In Fig 3c by increasing the coupling strength \(c\) and the number of pinning points, the control of the target pattern improves. Eventually, for \(c = 1\) and \(np = 2\), the numerical values of the indicators are \(\rho = 0.8\) and \(S = 0.99\).

The surfaces of Fig.4 have been obtained averaging over a time \(t = 1,000\) after a long transitory has elapsed (i.e. \(t = 750\)) with \(\alpha I_0 = 1.7\), and varying \(0 \leq c \leq 2\) and \(1 \leq np \leq 10\). The values of \(S\) are displayed in Fig. 4 (a), and as expected a reduction in the number of pinning points deteriorates the control process.

The variation along parameter \(c\) indicates that the control process is optimum in the vicinity of \(c = 1\). If \(c > 1\), one observes a phenomenon of overshoot: the corrective term is exceedingly large and the control is deteriorated. The same kind of behavior is observed for the indicator \(\rho\) displayed in Fig. 4 (b), only \(c \simeq 1\) and \(np = 1\) lead to \(\rho \approx 1\). The results presented here
Figure 3. Influence of parameters $c$ and $np$ in the control. a) Hexagonal target pattern, b) Output pattern at $c = 0.6$ and $np = 4$. c) Controlled pattern at $c = 1$ and $np = 2$

Figure 4. Influence of $c$ and $np$ on the a) indicator of ”spatial synchronization” $S$, b) correlation $\rho$

for the stabilization of the LCLV are reminiscent of the results obtained for the control and synchronization of the complex Ginzburg-Landau equation [3, 4]. We found that the efficiency of the control is proportional to the product of the density of pinning points by the coupling strength. We called this phenomenon integral behavior of the control [3].

4. Conclusions
In this paper, we have shown through numerical simulations the possibility of driving the actual pattern of an optical system to a target pattern which is an chaotic solution of the system. In particular, we have shown that the coupling strength and the pinning points are important to get a robust stabilization of the target pattern. This does not exclude the possibility that an optimal positioning function exists (not necessarily regular) able to enhance the control process, we leave this possibility for further investigations in this direction.

Acknowledgments
Work partly supported by PBCT project PSD-06, Dr. S.B. acknowledges the Yeshaya Horowitz Association through the Center for Complexity Science.
References

[1] Ana Cárdenas, Mario García-Molina, Salvador Sales, and José Capmany, J. Lightwave Technol. **22**, 2460, (2004).
[2] Lutz Junge and Ulrich Parlitz, Phys. Rev. **E61**, 3736 (2000).
[3] Jean Bragard and Stefano Boccaletti, Phys. Rev. **E62**, 6346 (2000).
[4] Stefano Boccaletti and Jean Bragard, "Controlling spatio-temporal chaos in the scenario of the one-dimensional Complex Ginzburg-Landau equation", Proc. of Royal Soc. London (series A), in press.
[5] Igor Aranson and Lorenz Kramer, Rev. Mod. Phys., **74**, 99 (2002).
[6] Fortunato Tito Arecchi, Stefano Boccaletti and Pier Luigi Ramazza, Phys. Reports **318**, 1 (1999).
[7] Gian–Luca Oppo, Giampaolo D’Alessandro and William Firth, Phys. Rev. **A44**, 471 (1991).
[8] Pier Luigi Ramazza, Stefano Boccaletti, Antonio Giaquinta, Enrico Pampaloni, Samuel Soria and Fortunato Tito Arecchi, Phys. Rev. **A54**, 3472 (1996).
[9] Pier Luigi Ramazza, Stefano Boccaletti and Fortunato Tito Arecchi, Opt. Comm. **136**, 267 (1997).
[10] Luc Pastur, Louis Gostiaux, Umberto Bortolozzo, Stefano Boccaletti and Pier Luigi Ramazza, Phys. Rev. Lett. **93**, 3902 (2004).
[11] Max Born and Emil Wolf, "Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light", Cambridge University Press (7th), 1999.
[12] Ann De Wit, Advances Chem. Phys. **109**, 435 (1999).
[13] Michael Cross and Pierre Hohenberg, Rev. Mod. Phys. **65**, 851 (1993).
[14] William Press, Brian Flannery, Saul Teukolsky, William Vettering, "Numerical Recipes in C: the art of Scientific Computing", Cambridge University Press (2nd), 1992.