Research Article

Testing an Inflation Model with Nonminimal Derivative Coupling in the Light of Planck 2015 Data

Kourosh Nozari and Narges Rashidi

Department of Physics, Faculty of Basic Sciences, University of Mazandaran, P.O. Box 47416-95447, Babolsar, Iran

Correspondence should be addressed to Kourosh Nozari; knozari@umz.ac.ir

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We study the dynamics of a generalized inflationary model in which both the scalar field and its derivatives are coupled with the gravity. We consider a general form of the nonminimal derivative coupling in order to have a complete treatment of the model. By expanding the action up to the second order in perturbation, we study the spectrum of the primordial modes of the perturbations. Also, by expanding the action up to the third order and considering the three-point correlation functions, the amplitude of the non-Gaussianity of the primordial perturbations is studied in both equilateral and orthogonal configurations. Finally, by adopting some sort of potentials, we compare the model in hand with the Planck 2015 released observational data and obtain some constraints on the model’s parameters space. As an important result, we show that the nonminimal couplings help to make models of chaotic inflation that would otherwise be in tension with Planck data, in better agreement with the data. This model is consistent with observation at weak coupling limit.

1. Introduction

The existence of an inflationary stage in the history of the early universe was proposed to solve some problems of the standard big bang cosmology, such as the flatness, horizon, and relics problems. A successful inflationary paradigm can also provide a causal mechanism for generating the initial density perturbations needed to seed the formation of structures in the universe [1–8]. A simple single field inflationary scenario (in which the early universe was dominated by the potential energy of the scalar field dubbed inflaton) predicts the dominant mode of the primordial density fluctuations to be highly adiabatic, scale invariant, and Gaussian [9]. But recent observational data have detected a level of scale dependence in the primordial perturbations [10, 11]. On the other hand, although there is no direct signal for primordial non-Gaussianity in observation, Planck team has obtained some tight limits on primordial non-Gaussianity [12]. Some inflationary models also predict a level of non-Gaussianity in the perturbation’s mode [9, 13–20]. This is really an important and interesting point, because a large amount of information on the cosmological dynamics deriving the initial inflationary expansion of the universe can be found in the primordial scalar non-Gaussianities [21, 22]. In this regard, the extended inflationary models, which can show the non-Gaussianity and scale dependence of the primordial perturbation, are more favorable.

One successful class of the extended inflationary models is the one with a nonminimal coupling between the scalar field and the curvature scalar $R$. This coupling usually is shown by the Lagrangian term as $f(\phi)R$. There are some motivations behind including an explicit nonminimal coupling in the action. It is necessary for the renormalization term arising in quantum field theory in curved space [23]. Considering the quantum corrections to the scalar field theory, this term would arise at the quantum level [24, 25]. With nonminimal coupling term, it is possible to have gravity as a spontaneous symmetry-breaking effect [26, 27]. NMC term allows having an oscillating universe [28]. Also, with this term, it is possible to solve the graceful exit problem of the old inflation model by slowing the false-vacuum expansion [29, 30]. So, it seems that the presence of nonminimal coupling...
between the scalar field and Ricci scalar is a useful extension of the theory and in this regard many authors have studied such models [17, 31–40].

It has been shown that the presence of NMC makes the inflation model more viable. For instance, the author of [41] has considered an inflation model with nonminimal coupling and noncanonical kinetic terms leading to a chaotic inflation model. It is shown that this chaotic inflation model is favored by the BICEP2 observation. But this model is in tension with recent observational data released by Planck 2015. The authors of [42] have considered an inflation model with a nonminimal coupling between the scalar field and gravity as \((\alpha \phi^2 + \beta \dot{\phi}^4)R\). They compared their model with Planck 2015 data in Einstein frame to explore the viability of the model. It is shown that this model just for \(N = 65\) and with some maximum values of \(n_s\) is consistent with Planck 2015 data. In [43], a nonminimally coupled inflationary model with quadratic potential in Einstein frame has been considered. The authors of this paper have shown that their model for some values of the NMC parameter is consistent with observational data. For other literature dealing with these issues, we refer the reader to [44–53].

Another extension of the inflation theory arises from considering a nonminimal coupling between the derivatives of the scalar field and the curvature, which leads to the interesting cosmological behaviors [54]. This coupling is usually given by the Lagrangian term as \((R_{\mu \nu} - (1/2)g_{\mu \nu}R)\nabla^\mu \phi \nabla^\nu \phi\). Some authors have shown that, during inflation, with a nonminimal coupling between the scalar field and gravity, the unitary bound of the theory is violated [55–57]. However, it has been shown that this model with nonminimal couplings between the kinetic term of the inflaton (derivatives of the scalar field) and the Einstein tensor preserves the unitary bound during inflation [58]. Also, the presence of nonminimal derivative coupling is a powerful tool to increase the friction of an inflaton rolling down its own potential [58]. Some authors have considered the model with this coupling term and have studied the early time accelerating expansion of the universe as well as the late time dynamics [59–61]. As an important extension of the nonminimal models, in addition to the standard coupling between the scalar field and curvature, it is possible also to have coupling between the derivatives of the scalar field and gravitational sector which makes the inflationary model more powerful to be a successful scenario.

In this work, we consider a generalized inflationary model in which both the scalar field and its derivatives are coupled to the curvature. We also consider an extension of the nonminimal derivative term in the Lagrangian as \(\mathcal{F}(\phi)G_{\mu \nu}\nabla^\mu \nabla^\nu (\phi)\). In this term, \(\mathcal{F}\) is a general function of \(\phi\) which, for \(\mathcal{F} \sim (1/2)\phi\), leads to the simple cases introduced earlier in the literature. In Section 2, we present some main equations of this generalized inflationary model. In Section 3, we use the ADM metric and study the linear perturbations of the model. By expanding the action up to the second order in perturbation and considering the 2-point correlation functions, we obtain the amplitude of the scalar perturbation and its spectral index. Also, by considering the tensor part of the perturbed metric, we obtain the tensor perturbation and its spectral index as well. In Section 4, by expanding the action up to the cubic order in perturbation, we explore the nonlinear perturbation in this model. By considering the 3-point correlation functions, we study the non-Gaussian modes of the primordial perturbations. In this section, we obtain the amplitude of the non-Gaussianity in the equilateral and orthogonal configuration and in \(k_1 = k_2 = k_3\) limit. Then, in Section 5, we test our generalized inflationary model in confrontation with the recently released observational data. To this end, we consider two types of nonminimal coupling parameter (\(\xi \phi^2\) and \(\xi \phi^4\)), several types of nonminimal derivative coupling function, and also several types of potential. By these functions, we study the behavior of the perturbative parameters in the background of the Planck 2015 observational data. Our main goal is to see whether this nonminimal model has the potential to make models of chaotic inflation in better agreement with Planck 2015 data or not.

In this regard, the model with nonminimal coupling between the scalar field and its derivative to gravity is a subset of Horndeski theory which satisfies the necessary conditions for being a good theoretical inflationary model. Also, this model is observationally viable, since this model with several inflationary potentials is consistent with Planck 2015 observational data. As we can see from Planck team’s released paper, a simple inflationary model, with potential corresponding to \(\phi^2, \phi^3,\) and \(\phi^{2+3}\), is not consistent with observation. In [53], it has been shown that a nonminimal model (a model with nonminimal coupling between the scalar field and gravity) is viable with potentials they have considered. However, their model has been studied in Einstein frame. We will show that adding a nonminimal coupling between the gravity and derivatives of the scalar field leads to a viable inflationary model even in Jordan frame.

The main properties of the cosmological perturbations are described by the tensor-to-scalar ratio and the scalar spectral index. In this regard, several observational teams are trying to find some constraints on these perturbative parameters. WMAP team has found the constraints \(r < 0.13\) and \(n_s = 0.9636 \pm 0.0084\) from the combined WMAP9 + eCMB + BAO + H_0 data [62]. The constraints obtained by the joint Planck 2013 + WMAP9 + BAO data are as \(r < 0.12\) and \(n_s = 0.9643 \pm 0.0059\) [63]. Then, in 2014, BICEP found a surprising result. The joint Planck 2013 + WMAP + BICEP2 + BAO data has set the constraints \(r = 0.2096^{+0.043}_{-0.068}\) and \(n_s = 0.9653 \pm 0.0129\) on the perturbative parameters [64, 65]. This large value of the tensor-to-scalar ratio, which shows the large amplitude of the gravitational wave modes generated during inflation, was a really challenging result. But Planck collaboration has performed a new study on the polarized dust emission in our galaxy and has shown that the part of the sky observed by BICEP2 is contaminated significantly by galactic-dust emission. So, the polarized dust emission might contribute a significant part of the B-mode detected by BICEP2. The Planck collaboration released the constraints \(r < 0.099\) and \(0.9652 \pm 0.0047\) from Planck TT, TE, and EE + lowP + WP data (note that Planck TT, TE, and EE + lowP
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2. The Setup

The four-dimensional action for a cosmological model where a scalar field is nonminimally coupled to the Ricci scalar, in the presence of a nonminimal derivative coupling between the scalar field and gravity, is given by the following expression:

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} R + f(\phi) R - \frac{\alpha}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] + \mathcal{F}(\phi) G_{\mu\nu} \nabla^\mu \nabla^\nu(\phi), \]

where \( R \) is the Ricci scalar, \( \phi \) is a scalar field (inflaton) with the potential \( V(\phi) \), and \( f(\phi) \) and \( \mathcal{F}(\phi) \) are general functions of the scalar field. Also, in the nonminimal derivative term of action (1), \( G_{\mu\nu} = R_{\mu\nu} - (1/2) g_{\mu\nu} R \) is the Einstein tensor. Note that, in some papers, there is a coefficient \( \kappa^2/2 \) in front of the nonminimal derivative term, where the constant parameter \( \kappa^2 \) has length-squared dimension. In this paper, we absorb this constant parameter in function \( \mathcal{F}(\phi) \).

By assuming the FRW metric, variation of action (1) with respect to the metric leads to the Friedmann equation of the model as

\[ H^2 \left( 1 + 2\kappa^2 f \right) = \frac{\kappa^2}{3} \left( \phi^2 \left( \frac{1}{2} - 9H^2 \mathcal{F}' \right) - 6H f' \phi + V(\phi) \right), \]

where a dot refers to a time derivative of the parameter and a prime denotes a derivative with respect to the scalar field. By varying action (1) with respect to the scalar field, we get the following equation of motion:

\[ \ddot{\phi} - 1 + 6\mathcal{F}' H^2 + \left( 12\mathcal{F}' H + 18 \mathcal{F}'^2 H^2 - 3H \right) \phi + 3\mathcal{F}'' H^2 \phi^2 + 6 f' R - V' = 0. \]

Now, by defining the slow-roll parameters as \( \epsilon \equiv -\dot{H}/H^2 \) and \( \eta = -(1/H)(\dot{H}/H) \), we find these parameters in our setup as follows:

\[ \epsilon = \frac{A}{1 + 2\kappa^2 f + \kappa^2 \mathcal{F}' \phi^2}, \]

\[ \eta = 2\epsilon - \frac{1}{H \dot{\phi}} \left( 1 + 2\kappa^2 f + \kappa^2 \mathcal{F}' \phi^2 \right) \]

where parameter \( A \) is defined as

\[ A \equiv \frac{\kappa^2 \phi^2 - \kappa^2 f' \phi - 3\kappa^2 \mathcal{F}' \phi^2 + \frac{\kappa^2}{2} f'' \phi^2 + \frac{\kappa^2}{2} \mathcal{F}'' \phi^4}{H}, \]

\[ + \frac{\phi}{H \phi} \left( \frac{\kappa^2 f' \phi + \kappa^2 \mathcal{F}' \phi^2 + \kappa^2 \mathcal{F}'' \phi}{H} \right). \]

During the inflationary era, the evolution of the Hubble parameter is so slow, so that in this era the conditions \( \epsilon \ll 1 \) and \( \eta \ll 1 \) are satisfied. As one of these two slow-varying parameters reaches unity, the inflation phase terminates.

The number of e-folds during inflation which is defined as

\[ N = \int_{t_{in}}^{t_{end}} H \, dt \]

in our setup and within the slow-roll limit \( \phi \ll |3H\dot{\phi}| \) and \( \phi^2 \ll V(\phi) \) takes the following form:

\[ N = \int_{0}^{\phi_{in}} \frac{3H^2 \left( 4\mathcal{F}' H + 6\mathcal{F}'^2 H^2 - 1 \right)}{V' - 6f'R} \, d\phi, \]

where \( \phi_{in} \) shows the value of the inflaton at the horizon crossing of the universe scale and \( \phi \) denotes the value of the field when the universe exits the inflationary phase. In the next section, we study the linear perturbation of the model which is the key test of any inflationary model. In this regard, we calculate the spectrum of perturbations produced due to quantum fluctuations of the fields about their homogeneous background values.

3. Linear Perturbation

In this section, we study the linear perturbation arising from the quantum behavior of both the scalar field \( \phi \) and the space-time metric, \( g_{\mu\nu} \), around the homogeneous background solution. To this end, we should expand action (1) up to the second order in small fluctuations. In this regard, it is convenient to work in ADM metric formalism [66]:

\[ ds^2 = -N^2 dt^2 + h_{ij} \left( dx^i + N^i dt \right) \left( dx^j + N^j dt \right), \]

where \( N \) is the lapse function and \( N^i \) is the shift vector. To obtain a general perturbed form of metric (8), we should expand the shift and lapse functions as \( N^i = B^i + \delta^i_j \partial_j B + v^i \) and \( N = 1 + \Phi \), respectively. \( \Phi \) and \( B \) are 3-scalar and \( v \) is a vector satisfying the condition \( \partial_j v_j = 0 \) [67, 68]. Note that it is sufficient to compute \( N \) or \( N^i \) up to the first order. This is because the second-order perturbation is multiplied by a factor which is vanishing using the first-order solution. Since the third-order term is multiplied by a constraint equation at the zeroth-order obeying the equations of motion, the contribution of this term also vanishes [9, 15, 69]. \( h_{ij} \) should be written as \( h_{ij} = \alpha^2 [(1 - 2\Psi)\delta_{ij} + 2\mathcal{F}_{ij}] \), where \( \Psi \) is...
the spatial curvature perturbation and $\mathcal{F}_{ij}$ is a spatial shear 3-tensor which is symmetric and traceless. So, perturbed metric (8) takes the following form:

$$ds^2 = -(1 + 2\Phi) dt^2 + 2a(t) B_2 dt dx^i + a^2(t) \left[(1 - 2\Psi) \delta_{ij} + 2\mathcal{F}_{ij}\right] dx^i dx^j. \quad (9)$$

In order to study the scalar perturbation of the theory, we work within the uniform-field gauge with $\delta \Phi = 0$ and also neglect the gravitational wave ($\mathcal{F}_{ij} = 0$). So, by considering the scalar part of the perturbations at the linear level, we rewrite perturbed metric (9) as $[67, 68, 70]$ for the scalar part at the linear level, we neglect the gravitational wave

$$ds^2 = -(1 + 2\Phi) dt^2 + 2a(t) B_2 dt dx^i + a^2(t) (1 - 2\Psi) dx^i dx^j. \quad (10)$$

Replacing the metric (10) in action (1) and expanding the action up to the second order in perturbations give

$$S_2 = \int dtd^3x a^3 \left[ -\frac{1}{2} \left( \kappa^2 + f + \phi^2 \mathcal{F}' \right) \Psi^2 - \frac{2}{a^2} \left( \kappa^2 + f + \phi^2 \mathcal{F}' \right) \Phi \partial^2 \Psi \right. \left. + \frac{1}{a^2} \left( \kappa^2 + f + \phi^2 \mathcal{F}' \right) \Psi \right. \left. - \left( \frac{2}{a} \right)^2 \left( \kappa^2 + f + \phi^2 \mathcal{F}' \right) \Phi \partial^2 B \right]. \quad (11)$$

By substituting (12) into action (11) and integrating by parts, we reach

$$S_2 = \int dtd^3x a^3 \left[ N^2 - \frac{c_s^2}{a^2} (\partial \Psi)^2 \right], \quad (14)$$

where the parameters $N$ and $c_s$ (which is known as the sound velocity) are defined as

$$N = -\frac{4}{a^2} \left( \kappa^2 + f + \phi^2 \mathcal{F}' \right)^2 \left( \frac{9\kappa^2 H^2 (1 + \kappa^2 f) - \frac{3}{2} \phi^2 + 18H\phi f' + 54H^2 \phi^2 \mathcal{F}'}{(2\kappa^2 H (1 + \kappa^2 f) + 2\phi f' + 6\phi^2 \mathcal{F}')^2} \right) + 3 \left( \kappa^2 + f + \phi^2 \mathcal{F}' \right)^2, \quad (15)$$

$$c_s^2 = \frac{3}{a^2} \left[ 2 \left( \kappa^2 + f + \phi^2 \mathcal{F}' \right) \left( \kappa^2 + f + \phi^2 \mathcal{F}' \right) \frac{d}{dt} \left( \kappa^2 + f + \phi^2 \mathcal{F}' \right) - 2 \left( \kappa^2 + f + \phi^2 \mathcal{F}' \right) \left( \kappa^2 + f + \phi^2 \mathcal{F}' \right) \frac{d}{dt} \right] - 2 \left( \kappa^2 + f + \phi^2 \mathcal{F}' \right)^2, \quad (16)$$

$$\left[ \left( \frac{d}{dt} \right)^2 \left( \frac{\kappa^2 + f + \phi^2 \mathcal{F}'}{d\mathcal{F}'} \right) \right]^{-1},$$
respectively. To see more details of driving this type of equations, one can refer to [15, 16, 22, 71].

Now, we calculate the quantum perturbations of $\Psi$. To this end, by variation of action (14), we find the equation of motion of $\Psi$ as

$$
\ddot{\Psi} + \left( 3H + \frac{\mathcal{W}'}{\mathcal{W}} \right) \dot{\Psi} + c_s^2 k^2 \Psi = 0.
$$

(17)

To solve this equation, it is convenient to rewrite it in terms of the conformal time. By introducing the new variable as $\psi = \Psi z$, where $z = a \sqrt{2 \mathcal{W}}$, (17) becomes

$$
\ddot{\psi} + c_s^2 k^2 \left( - \frac{z''}{z} \right) \psi = 0,
$$

(18)

where a prime refers to derivative with respect to the conformal time. By considering the slow variation of $\mathcal{W}$, we have $z''/z = 2 / r^2$ in the de Sitter background. In this regard, the solution of (18) is given by the following expression:

$$
\psi = C_1 (1 + i c_s r) e^{i c_s r} + C_2 (1 - i c_s r) e^{-i c_s r}.
$$

(19)

By choosing the Bunch-Davis vacuum characterized by the mode function $\psi = \exp (-i c_s r) / \sqrt{2 c_s k}$ in the asymptotic past, when the $k$-mode is far inside the horizon ($k \tau \rightarrow -\infty$), we obtain

$$
C_1 = \frac{i a H}{\sqrt{2 c_s^3 k}},
$$

$$
C_2 = 0.
$$

(20)

In this way, the solution of (17), up to the lowest order of the slow-rolloff approximation, is given by the following expression:

$$
\Psi = \frac{i H e^{-k r}}{2 c_s^3 k \sqrt{2 c_s k \mathcal{W}}} (1 + i c_s k r).
$$

(21)

There are two other treatments to obtain the constants $C_1$ and $C_2$, which have been mentioned in [72]. To study the power spectrum of the curvature perturbation, we should compute the two-point correlation function in our setup. We find the two-point correlation function, sometimes after the horizon crossing, by obtaining the vacuum expectation value of $\Psi$ at $\tau = 0$ (corresponding to the end of inflation):

$$
\langle 0 | \Psi (0, k) \Psi (0, k') | 0 \rangle
$$

$$
= (2\pi)^3 \delta^3 (k_1 + k_2) \frac{2 \pi^2}{k^3} \mathcal{A}_s,
$$

(22)

where $\mathcal{A}_s$ is called the power spectrum and is given by

$$
\mathcal{A}_s = \frac{H^2}{8 \pi^2 \mathcal{W} c_s^3}.
$$

(23)

The scalar spectral index of the perturbations at the Hubble crossing is defined as

$$
n_s - 1 = \frac{d \ln \mathcal{A}_s}{d \ln k} \bigg|_{k, \tau = a H},
$$

(24)

which in our setup takes the following form:

$$
n_s - 1 = -2e - \frac{5 k^2 f' \phi}{2 H (1 + k^2 f)}
$$

$$
- \frac{1}{H} \frac{d \ln \left( e + 5 k^2 f' \phi / 4 H (1 + k^2 f) \right)}{d t}
$$

$$
- \frac{1}{H} \frac{d \ln c_s}{d t}.
$$

(25)

Any deviation of $n_s$ from the unity shows that the evolution of perturbations is scale dependent.

Other important parameters in an inflationary model are amplitude of the tensor perturbation and its spectral index. To study the tensor perturbation, we use the tensor part of perturbed metric (9). We can write $\mathcal{T}_{ij}$, in terms of the two polarization tensors, as follows:

$$
\mathcal{T}_{ij} = \mathcal{T}_{+} e^{+}_{ij} + \mathcal{T}_{x} e^{x}_{ij},
$$

(26)

where $e^{(+)\times}_{ij}$ are symmetric, traceless, and transverse. The normalization condition imposes the constraints

$$
e^{(+)\times}_{ij} (k) e^{(+)\times}_{ij} (-k) = 0,
$$

(27)

and the reality condition gives

$$
e^{(+)\times}_{ij} (-k) = \left( e^{(+)\times}_{ij} (k) \right)^*.
$$

(28)

Now, we can write the second-order action for the tensor mode (gravitational waves) as follows:

$$
S_T = \int dt d^3 x \mathcal{W} \left[ \mathcal{T}_{+}^2 - \frac{c_T^2}{a^2} \left( \partial \mathcal{T}_{+} \right)^2 + \mathcal{T}_{x}^2
$$

$$
- \frac{c_T^2}{a^2} \left( \partial \mathcal{T}_{x} \right)^2 \right],
$$

(29)

where $\mathcal{W}$ and $c_T^2$ are defined with the following expressions, respectively:

$$
\mathcal{W} = \frac{1}{4 k^2} \left( 1 + kf \right) \left( 1 + \frac{k^2 \mathcal{F}'}{1 + kf} \right),
$$

(30)

$$
c_T^2 = \frac{\kappa^2 - f - \phi^2 \mathcal{F}'^2}{\kappa^2 + f + \phi^2 \mathcal{F}'^2}.
$$

(31)

By following the strategy as applied for the scalar mode case, we can find the amplitude of the tensor perturbations as follows:

$$
\mathcal{A}_T = \frac{H^2}{2 \pi^2 \mathcal{W} c_T^3},
$$

(32)

which, by using the definition of the tensor spectral index

$$
n_T = \frac{d \ln \mathcal{A}_T}{d \ln k},
$$

(33)
leads to the following expression for $n_T$:

$$n_T = -2\epsilon - \frac{\kappa^2 f' \phi}{2H(1 + \kappa^2 f)}.$$  \hfill (34)

The ratio between the amplitudes of the tensor and scalar perturbations (tensor-to-scalar ratio) is another important inflationary parameter which in our setup is given by

$$r = \frac{a_T}{a_S} = 16c_s^3 \left( \epsilon + \frac{5\kappa^2 f' \phi}{4H(1 + \kappa^2 f)} \right).$$  \hfill (35)

### 4. Nonlinear Perturbations and Non-Gaussianity

Non-Gaussianity of the primordial density perturbations is another important aspect of an inflationary model. To explore the non-Gaussianity of the density perturbations, one has to study the nonlinear perturbation theory. The two-point correlation function of the scalar perturbations gives no information about the non-Gaussianity of the model. The non-Gaussian property shows itself up in the three-point correlation function. This is because, for a Gaussian perturbation, all odd $n$-point correlators vanish and the higher even $n$-point correlation functions can be expressed in terms of sum of products of the two-point functions. To calculate the three-point correlation function, we should expand action (1) up to the third order in perturbation, all odd $n$-point fluctuations and nonlinearities in the evolution. By expanding action (1) up to the third order in perturbation, we obtain a complicated expression which can be seen in the Appendix. We use (12) to eliminate the perturbation parameter $\Phi$ in the expanded action. Then, by introducing the new parameter $\mathcal{X}$ as

$$B = \frac{2}{\kappa^2 H(1 + \kappa^2 f)} \Psi \left( \epsilon + \frac{5\kappa^2 f' \phi}{4H(1 + \kappa^2 f)} \right) - \frac{a^3 \mathcal{X} f'}{\kappa^2 + f + \phi^2 f'},$$  \hfill (36)

we can rewrite the third-order action, up to the leading order, as follows:

$$S_3 = \int dt d^3x \left[ \left( \frac{3a^3}{\kappa^2} \left( \frac{1 + \kappa^2 f}{c_s^2} \right) \left( \frac{1}{c_s} - 1 \right) \right) \Psi \Psi + \left[ \frac{\alpha}{\kappa^2} \left( \frac{1 + \kappa^2 f}{c_s^2} \right) \right] \left( \frac{1}{c_s^2} - 1 \right) \right] \cdot \left( \epsilon + \frac{5\kappa^2 f' \phi}{4H(1 + \kappa^2 f)} \right) \Psi \Psi + \left[ \frac{\alpha}{\kappa^2} \left( \frac{1 + \kappa^2 f}{c_s^2 H} \right) \right] \left( \frac{1}{c_s} - 1 \right) \Psi (\partial \Psi)^2 + \left[ \frac{\alpha^3}{\kappa^2} \left( \frac{1 + \kappa^2 f}{c_s^2 H} \right) \right] \left( \frac{1}{c_s^2} - 1 \right) \Psi (\partial \Psi)^2 \right. $$

$$+ \left[ \frac{\alpha^3}{\kappa^2} \left( \frac{1 + \kappa^2 f}{c_s^2 H} \right) \right] \left( \frac{1}{c_s} - 1 \right) \Psi (\partial \Psi)^2 \right. $$

Now, we are in the position to calculate the three-point correlation function by using the interacting picture. In this picture, we obtain the vacuum expectation value of the curvature perturbation $\Psi$ for the three-point operator in the conformal time interval between the beginning and end of the inflation as [9, 22, 71]

$$\langle \Psi(k_1) \Psi(k_2) \Psi(k_3) \rangle = -i \int_{\tau_1}^{\tau_2} d\tau a \langle 0 |$$

$$\left[ \Psi(\tau, k_1) \Psi(\tau, k_2) \Psi(\tau, k_3) \right] \left[ H_{\text{int}}(\tau) \right] | 0 \rangle, \tag{38}$$

where the interaction Hamiltonian, $H_{\text{int}}$, is equal to the Lagrangian of the third-order action. To solve the integral of (38), we can approximate the coefficients in the brackets of the Lagrangian (37) to be constants since these coefficients would vary slower than the scale factor. By solving the integral, we find the three-point correlation function of the curvature perturbation in the Fourier space as

$$\langle \Psi(k_1) \Psi(k_2) \Psi(k_3) \rangle = (2\pi)^3 \delta^3(k_1 + k_2 + k_3) \mathcal{B}_\Psi(k_1, k_2, k_3). \tag{39}$$

$\mathcal{B}_\Psi$ in (40) is given by (23). Also, the parameter $\mathcal{B}_\Psi$ is defined as

$$\mathcal{B}_\Psi = \frac{3}{4} \left( 1 - \frac{1}{c_s^2} \right) \delta_1 + \frac{1}{4} \left( 1 - \frac{1}{c_s^2} \right) \delta_2 + \frac{3}{2} \left( \frac{1}{c_s^2} - 1 \right) \delta_3, \tag{41}$$

where

$$\delta_1 = \frac{2}{K} \sum_{i \neq j} k_i^2 k_j^2 - \frac{1}{K^2} \sum_{i \neq j} k_i^2 k_j^2,$$

$$\delta_2 = \frac{1}{2} \sum_{i \neq j} k_i^2 + \frac{2}{K} \sum_{i \neq j} k_i^2 k_j^2 - \frac{1}{K^2} \sum_{i \neq j} k_i^2 k_j^2,$$

$$\delta_3 = \frac{(k_i k_j k_3)^2}{K^3}, \tag{42}$$

As it can be seen from (40), a three-point correlator depends on the three momenta $k_1, k_2,$ and $k_3$. To satisfy the translation
invariance, these momenta should form a closed triangle, meaning that the sum of these momenta should be zero \((k_1 + k_2 + k_3 = 0)\). Also, satisfying the rotational invariance makes the shape of the triangle important \([21, 73–76]\). Depending on the values of momenta, there are several shapes and each shape has a maximal signal in a special configuration of triangle. One of these shapes is a local shape \([77–80]\) which has a maximal signal in the squeezed limit \((k_1 \ll k_2)\). Another shape which is corresponding to the equilateral triangle \([81]\) has a signal at \(k_1 = k_2 = k_3\). A linear combination of the equilateral and orthogonal templates, which is orthogonal \([82]\) to equilateral one, gives a shape corresponding to folded triangle \([83]\) with a pick in \(k_1 = 2k_2 = 2k_3\) limit. We mention that the orthogonal configuration has a signal with a positive peak at the equilateral configuration and a negative peak at the folded configuration.

In order to measure the amplitude of the non-Gaussianity, we define the dimensionless parameter \(f_{NL}\) called “nonlinearity parameter,” as follows:

\[
f_{NL} = \frac{10}{3} \frac{\varepsilon_y}{\sum_{i=1}^3 k_i^3}.
\]  

(43)

In this paper, we study the non-Gaussianity in the equilateral and orthogonal configurations. To this end, we should find \(\varepsilon_y\) in these configurations. In this regard, we should estimate the correlation between two different shapes. So, following \([84–86]\), we define the following quantity:

\[
\mathcal{C} \left( \hat{B}_y, \hat{B}_y' \right) = \frac{\mathcal{F} \left( \hat{B}_y, \hat{B}_y' \right)}{\sqrt{\mathcal{F} \left( \hat{B}_y, \hat{B}_y \right) \mathcal{F} \left( \hat{B}_y', \hat{B}_y' \right)}},
\]  

(44)

where \(\hat{B}_y = B_y / \Delta_y\) and

\[
\mathcal{F} \left( \hat{B}_y, \hat{B}_y' \right) = \int dk_1 dk_2 dk_3 \hat{B}_y (k_1, k_2, k_3) \hat{B}_y' (k_1, k_2, k_3) \omega
\]  

(45)

with \(\omega = (k_1 k_2 k_3)/(k_1 + k_2 + k_3)^3\). The region of integration is \(0 < k_1 < \infty, 0 \leq k_2/k_1 < 1\), and \(1 - k_2/k_1 \leq k_3/k_1 \leq 1\). We note that two shapes satisfying the condition \(|\mathcal{C} \left( \hat{B}_y, \hat{B}_y' \right)| = 0\) are almost orthogonal. Now, we introduce a shape \(\tilde{\delta}^{\text{equi}}\) as \([20, 85]\)

\[
\tilde{\delta}^{\text{equi}} = -\frac{12}{13} (3\delta_1 - \delta_2).
\]  

(46)

By using (40)–(42) and (44)–(46), we can show that the following shape is orthogonal to (46):

\[
\tilde{\delta}^{\text{ortho}} = \frac{12}{14 - 13\beta} (\beta (3\delta_1 - \delta_2) + 3\delta_1 - \delta_2).
\]  

(47)

Note that, to normalize the quantities \(\tilde{\delta}^{\text{equi}}\) and \(\tilde{\delta}^{\text{ortho}}\), we have considered that in the equilateral configuration limit (corresponding to \(k_1 = k_2 = k_3\)) two shapes are equal and we have \(\tilde{\delta}^{\text{equi}} = \tilde{\delta}^{\text{ortho}} = k^3\) (for more details, see \([85]\)).

In this regard, and by solving the condition \(|\mathcal{C} \left( \hat{B}_y, \hat{B}_y' \right)| = 0\), we have \(\beta = 1.1967996\). Now, bispectrum (41) can be expressed in terms of the equilateral and orthogonal basis as

\[
\mathcal{C}_y = \mathcal{C}_1 \tilde{\delta}^{\text{equi}} + \mathcal{C}_2 \tilde{\delta}^{\text{ortho}},
\]  

(48)

where the parameters \(\mathcal{C}_1\) and \(\mathcal{C}_2\) are given by the following expressions:

\[
\mathcal{C}_1 = \frac{12}{13} \frac{1}{24} \left( \frac{1}{c_i} - 1 \right) (2 + 3\beta)
\]  

(49)

By using (43)–(49), we can obtain the amplitude of the non-Gaussianity in the equilateral and orthogonal configuration as follows:

\[
f_{NL}^{\text{equi}} = \frac{130}{36} \sum_{i=1}^3 k_i^3 \left( \frac{1}{24} \left( \frac{1}{c_i} - 1 \right) (2 + 3\beta) \right) \tilde{\delta}^{\text{equi}},
\]  

(50)

\[
f_{NL}^{\text{ortho}} = \frac{140 - 130\beta}{36} \sum_{i=1}^3 k_i^3 \left( \frac{1}{8} \left( \frac{1}{c_i} - 1 \right) \right) \tilde{\delta}^{\text{ortho}}.
\]  

(51)

Since the equilateral shape has a maximal signal at \(k_1 = k_2 = k_3\) limit and also an orthogonal configuration has a positive peak at this limit, we rewrite (50) in this limit as

\[
f_{NL}^{\text{equi}} = \frac{325}{18} \left( \frac{1}{24} \left( \frac{1}{c_i} - 1 \right) (2 + 3\beta) \right),
\]  

(52)

\[
f_{NL}^{\text{ortho}} = \frac{10}{9} \left( \frac{65}{4} \beta + \frac{7}{6} \right) \left( \frac{1}{8} \left( \frac{1}{c_i} - 1 \right) \right).
\]  

(53)

So far, we have obtained the main equations of our setup. In the following section, we examine our model by using the recently released observational data by Planck collaboration and find some constraints on the model’s parameters space.

### 5. Observational Constraints

In this section, we are going to find some observational constraints on the parameters space of the model in hand. To this end, we first define the extension of the nonminimal derivative term as \(f(\phi) \sim (1/2n)\phi^n\), which for \(n = 1\) leads to the usual nonminimal derivative case (as mentioned in Section 1). We set the values \(1, 2, 3,\) and \(4\) for \(n\). Another general function in this setup is \(f(\phi) \sim \xi \phi^k\), for \(k = 2\) and \(f(\phi) \sim \xi \phi^k\). The last general function is the potential term in the action. We adopt various types of potential to explore the linear potential as \(V(\phi) \sim \phi\) \([87]\), the quadratic potential as \(V(\phi) \sim \phi^2\), the cubic potential as \(V(\phi) \sim \phi^3\), the quartic potential as \(V(\phi) \sim \phi^4\), the axion monodromy’s motivated potential as \(V(\phi) \sim \phi^{2/3}\) \([88]\), and the exponential
Figure 1: Tensor-to-scalar ratio versus the scalar spectral index for an inflationary model in which both the inflaton and its derivative are nonminimally coupled to gravity, in the background of the Planck 2015 TT, TE, and EE + lowP data (a) and the amplitude of the orthogonal configuration of the non-Gaussianity versus the amplitude of the equilateral configuration for an inflationary model in which both inflaton and its derivative are nonminimally coupled to gravity, in the background of Planck 2015 TT, TEE, TTE, and EET data (b). Figure is plotted with $n = 1$, $f(\phi) \sim \xi \phi^2$, and various types of the potential, for $N = 60$. Note that the parameter $\xi$ varies along the curves.

| $V$  | $r - n_s$ | $r - n_s$ | $f_{\text{ortho}} - f_{\text{equi}}$ | $f_{\text{ortho}} - f_{\text{equi}}$ |
|------|-----------|-----------|-----------------------------------|-----------------------------------|
| $\phi$ | $\xi < 0.091$ | $\xi < 0.0912$ | $\xi < 0.09$ | $0.015 < \xi < 0.0895$ |
| $\phi^2$ | $\xi < 0.088$ | $\xi < 0.09$ | $0.01 < \xi < 0.089$ | $0.021 < \xi < 0.082$ |
| $\phi^3$ | $0.012 < \xi < 0.09$ | $\xi < 0.08$ | $0.011 < \xi < 0.093$ | $0.01 < \xi < 0.098$ |
| $\phi^4$ | $0.014 < \xi < 0.10$ | $0.001 < \xi < 0.093$ | $0.013 < \xi < 0.10$ | $0.01 < \xi < 0.098$ |
| $\phi^{2/3}$ | $\xi < 0.081$ | $\xi < 0.082$ | $0.015 < \xi < 0.073$ | $0.01 < \xi < 0.109$ |
| $e^{-\phi}$ | $0.016 < \xi < 0.108$ | $0.078 < \xi < 0.080$ | $0.094 < \xi < 0.103$ | $0.017 < \xi < 0.097$ |

Table 1: The ranges of $\xi$ in which the values of the inflationary parameters $r$ and $n_s$ and also $f_{\text{ortho}}$ and $f_{\text{equi}}$, with $n = 1$, are compatible with the 95% CL of the Planck 2015 dataset.

potential as $V(\phi) \sim e^{-\alpha \phi}$. With this specification of the general functions, we can analyze the model numerically and find some constraints on the nonminimal coupling parameter $\xi$. For this purpose, we solve the integral of (7) by adopting the mentioned functions of the scalar field. By solving this equation, we find the value of the inflaton at the horizon crossing in terms of the e-folds number. By substituting $\phi_{\text{hc}}$ into (25), (35), and (51), we can find the scalar spectral index, tensor-to-scalar ratio, and the amplitude of the orthogonal and equilateral configuration of the non-Gaussianity in terms of the e-folds number. Then, we can analyze these parameters numerically to see the observational viability of this setup in confrontation with recently released data. Plotting the figures and analyzing the model, we have rescaled all constant parameters to unity with the choice $G = c = 1$. In this regard, we study the behavior of the tensor-to-scalar ratio versus the scalar spectral index and the orthogonal configuration versus the equilateral configuration. The observational parameters are defined at $k_0 = 0.002Mpc^{-1}$ where subscript 0 refers to the value of $k$ when it left the Hubble radius during inflation. The results are shown in Figures 1–8. We have found that an inflationary model with a nonminimal coupling between the scalar filed and Ricci scalar and also a nonminimal derivative coupling, in some ranges of nonminimal coupling parameter $\xi$, is consistent with Planck 2015 dataset. In fact, these nonminimal couplings help to make models of chaotic inflation that would otherwise be in tension with Planck data, in better agreement with the data. The ranges of nonminimal coupling are shown in Tables 1–4. Note that some other authors have considered inflationary model with nonminimal coupling.
Figure 2: Tensor-to-scalar ratio versus the scalar spectral index for an inflationary model in which both inflaton and its derivative are nonminimally coupled to gravity, in the background of the Planck 2015 TT, TE, and EE + lowP data (a) and the amplitude of the orthogonal configuration of non-Gaussianity versus the equilateral configuration for an inflationary model in which both inflaton and its derivative are nonminimally coupled to gravity, in the background of Planck 2015 TTT, EEE, TTE, and EET data (b). Figure is plotted with $n = 1$, $f(\phi) \sim \xi \phi^{4}$, and various types of potential, for $N = 60$.

Figure 3: Tensor-to-scalar ratio versus the scalar spectral index for an inflationary model in which both inflaton and its derivative are nonminimally coupled to gravity, in the background of Planck 2015 TT, TE, and EE + lowP data (a) and the amplitude of the orthogonal configuration versus the equilateral configuration of non-Gaussianity for an inflationary model in which both inflaton and its derivative are nonminimally coupled to gravity, in the background of Planck 2015 TTT, EEE, TTE, and EET data (b). Figure is plotted with $n = 2$, $f(\phi) \sim \xi \phi^{2}$, and various types of potential, for $N = 60$. 
Figure 4: Tensor-to-scalar ratio versus the scalar spectral index for an inflationary model in which both inflaton and its derivative are nonminimally coupled to gravity, in the background of Planck 2015 TT, TE, and EE + lowP data (a) and the amplitude of the orthogonal configuration versus the amplitude of the equilateral configuration of non-Gaussianity for an inflationary model in which both inflaton and its derivative are nonminimally coupled to gravity, in the background of Planck 2015 TTT, EEE, TTE, and EET data (b). Figure is plotted with $n = 2$, $f(\phi) \sim \xi \phi^4$, and various types of potential, for $N = 60$.

Figure 5: Tensor-to-scalar ratio versus the scalar spectral index for an inflationary model in which both inflaton and its derivative are nonminimally coupled to gravity, in the background of Planck 2015 TT, TE, and EE + lowP data (a) and the amplitude of the orthogonal configuration versus the amplitude of the equilateral configuration of non-Gaussianity for an inflationary model in which both inflaton and its derivative are nonminimally coupled to gravity, in the background of Planck 2015 TTT, EEE, TTE, and EET data (b). Figure is plotted with $n = 3$, $f(\phi) \sim \xi \phi^2$, and various types of potential, for $N = 60$. 
Figure 6: Tensor-to-scalar ratio versus the scalar spectral index for an inflationary model in which both inflaton and its derivative are nonminimally coupled to gravity, in the background of Planck 2015 TT, TE, and EE + lowP data (a) and the amplitude of the orthogonal configuration versus the amplitude of the equilateral configuration of non-Gaussianity for an inflationary model in which both inflaton and its derivative are nonminimally coupled to gravity, in the background of Planck 2015 TTT, EEE, TTE, and EET data (b). Figure is plotted with $n = 3$, $f(\phi) \sim \phi^n$, and various types of potential, for $N = 60$.

Figure 7: Tensor-to-scalar ratio versus the scalar spectral index for an inflationary model in which both inflaton and its derivative are nonminimally coupled to gravity, in the background of Planck 2015 TT, TE, and EE + lowP data (a) and the amplitude of the orthogonal configuration of non-Gaussianity versus the amplitude of the equilateral configuration for an inflationary model in which both inflaton and its derivative are nonminimally coupled to gravity, in the background of Planck 2015 TTT, EEE, TTE, and EET data (b). Figure is plotted with $n = 4$, $f(\phi) \sim \phi^n$, and various types of potential, for $N = 60$. 
term and studied the model numerically. We can compare the result of our setting with their results here. For example, it is shown in [41] that the presence of nonminimal coupling and noncanonical kinetic terms lead to a chaotic inflation model, which is favored by the BICEP2 observation but is in tension with recent observational data. Here, we have shown that the presence of nonminimal coupling between the scalar field and Ricci scalar and also a general nonminimal derivative coupling between the scalar field and gravity make the model observationally more viable. This model with these couplings is consistent with Planck 2015 TT, TE, and EE + lowP data. The authors of [42] have considered a nonminimal coupling between a scalar field and gravity as \((\alpha \phi^2 + \beta \phi^4) R\) in Jordan frame. They perform a frame transformation to Einstein frame and compare their model with Planck 2015 data. Their model just for \(N = 65\) and with some maximum values of \(n_s\) is consistent with observational data and for \(N = 50\) and 60 is observationally disfavored. In our setup, we have considered nonminimal coupling functions \(\phi^2\) and \(\phi^4\) separately. Our numerical analysis shows that the presence of NMC and NMDC makes the model observationally viable for both \(N = 50\) and \(N = 60\) in the Jordan frame (note that although we do not present here the results with \(N = 50\), we have analyzed the model with this value of number of e-folds and have found the viability of the model). In [43], a nonminimally coupled inflationary model with quadratic potential in Einstein frame is considered in the background of Planck 2015 observational data. Their numerical analysis shows that this model for some value of NMC parameter gives \(n_s = 0.96\). Interestingly, our setup also, in some values of \(\xi\), gives \(n_s = 0.96\). Nevertheless, we should emphasize that, for some other choices for model parameters, this would be not the case necessarily.

Our numerical analysis shows that this nonminimal model with small values of the NMC parameter is consistent with observational data and so this model in Jordan frame in weak coupling limit has cosmological viability. In week coupling limit, our setup is applicable to several inflationary potentials. In this limit and in the ranges obtained by numerical analysis, the scalar spectral index is red tilted. Another point is that, in our setup, in some ranges of NMC parameter, it is possible to have large non-Gaussianity, as can be seen in figures. Note that, to obtain the constraints on \(\xi\), we have focused on those ranges of the NMC parameter that lead to the values of the observable quantities allowed by the Planck 2015 observational data. For instance, we have focused on the values of \(\xi\) leading to \(0.95 < n_s < 0.97\) allowed by Planck 2015 TT, TE, and EE + lowP data. In the same way, we focused on the values of the NMC parameter leading to \(-147 < f_{\text{equi}}^\text{NL} < 143\) allowed by Planck 2015 TT, EEE, TTE, and EET data. Also, (25), (35), and (51) show that the scalar spectral index, tensor-to-scalar ratio, and the amplitudes of non-Gaussianity have nonlinear dependence on the NMC parameter \(\xi\). The function \(f(\phi)\) has an explicit nonminimal
Table 2: The ranges of $\xi$ in which the values of the inflationary parameters $r$ and $n_s$ and also $f_{NL}^{\text{ortho}}$ and $f_{NL}^{\text{equi}}$, with $n = 2$, are compatible with the 95% CL of the Planck 2015 dataset.

| $\nu$ | $r - n_s$ | $f(\phi) \sim \xi \phi^2$ | $f(\phi) \sim \xi \phi^4$ | $f_{NL}^{\text{ortho}} - f_{NL}^{\text{equi}}$ |
|-------|-----------|----------------|----------------|-----------------
| $\phi$ | $0.021 < \xi < 0.090$ | $0.019 < \xi < 0.09$ | $0.020 < \xi < 0.088$ | $0.021 < \xi < 0.097$ |
| $\phi^2$ | $0.01 < \xi < 0.055, 0.06 < \xi < 0.114$ | $0.017 < \xi < 0.089$ | $0.014 < \xi < 0.109$ | $0.018 < \xi < 0.098$ |
| $\phi^3$ | $0.022 < \xi < 0.081$ | $0.009 < \xi < 0.086$ | $0.011 < \xi < 0.107$ | $0.007 < \xi < 0.095$ |
| $\phi^4$ | $0.010 < \xi < 0.09$ | $0.011 < \xi < 0.098$ | $0.0108 < \xi < 0.098$ | $0.011 < \xi < 0.098$ |
| $\phi^{2/3}$ | $0.021 < \xi < 0.076$ | $0.018 < \xi < 0.084$ | $0.020 < \xi < 0.086$ | $0.011 < \xi < 0.084$ |
| $\epsilon^\nu$ | $0.086 < \xi < 0.01$ | $0.028 < \xi < 0.098$ | $0.01 < \xi < 0.104$ | $0.01 < \xi < 0.104$ |

Table 3: The ranges of $\xi$ in which the values of the inflationary parameters $r$ and $n_s$ and also $f_{NL}^{\text{ortho}}$ and $f_{NL}^{\text{equi}}$, with $n = 3$, are compatible with the 95% CL of the Planck 2015 dataset.

| $\nu$ | $r - n_s$ | $f(\phi) \sim \xi \phi^2$ | $f(\phi) \sim \xi \phi^4$ | $f_{NL}^{\text{ortho}} - f_{NL}^{\text{equi}}$ |
|-------|-----------|----------------|----------------|-----------------
| $\phi$ | $0.020 < \xi < 0.089$ | $0.017 < \xi < 0.089$ | $0.018 < \xi < 0.107$ | $0.011 < \xi < 0.089$ |
| $\phi^2$ | $0.015 < \xi < 0.084$ | $0.012 < \xi < 0.090$ | $0.011 < \xi < 0.098$ | $0.012 < \xi < 0.090$ |
| $\phi^3$ | $0.018 < \xi < 0.085$ | $0.008 < \xi < 0.083$ | $0.02 < \xi < 0.079$ | $0.019 < \xi < 0.080$ |
| $\phi^4$ | $0.007 < \xi < 0.108$ | $0.032 < \xi < 0.105$ | $0.029 < \xi < 0.081$ | $0.024 < \xi < 0.071$ |
| $\phi^{2/3}$ | $0.04831 < \xi$ | $0.011 < \xi < 0.082$ | $0.086 < \xi < 0.096$ | $0.007 < \xi < 0.095$ |
| $\epsilon^\nu$ | $0.013 < \xi < 0.088$ | $0.043 < \xi < 0.078$ | $0.025 < \xi < 0.077$ | $0.021 < \xi < 0.081$ |

Table 4: The ranges of $\xi$ in which the values of the inflationary parameters $r$ and $n_s$ and also $f_{NL}^{\text{ortho}}$ and $f_{NL}^{\text{equi}}$, with $n = 4$, are compatible with the 95% CL of the Planck 2015 dataset.

| $\nu$ | $r - n_s$ | $f(\phi) \sim \xi \phi^2$ | $f(\phi) \sim \xi \phi^4$ | $f_{NL}^{\text{ortho}} - f_{NL}^{\text{equi}}$ |
|-------|-----------|----------------|----------------|-----------------
| $\phi$ | $\xi < 0.084$ | $\xi < 0.021, 0.025 < \xi < 0.084$ | $\xi < 0.113$ | $0.016 < \xi < 0.093$ |
| $\phi^2$ | $\xi < 0.083$ | $\xi < 0.04, 0.059 < \xi < 0.112$ | $0.125 < \xi < 0.090$ | $0.012 < \xi < 0.092$ |
| $\phi^3$ | $\xi < 0.082$ | $0.019 < \xi < 0.087$ | $\xi < 0.106$ | $0.007 < \xi < 0.109$ |
| $\phi^4$ | $0.075 < \xi < 0.105$ | $0.019 < \xi < 0.081$ | $0.036 < \xi < 0.079$ | $0.019 < \xi < 0.093$ |
| $\phi^{2/3}$ | $0.043 < \xi < 0.112$ | $\xi < 0.086$ | $0.009 < \xi < 0.099$ | $0.008 < \xi < 0.098$ |
| $\epsilon^\nu$ | $0.012 < \xi < 0.099$ | $\xi < 0.109$ | $0.004 < \xi < 0.104$ | $0.013 < \xi < 0.096$ |

Coupling dependence. In the mentioned equations, there are different powers of the function $f(\phi)$ and therefore different powers of the nonminimal coupling parameter. Because of complexity of equations, we have avoided rewriting the equations in terms of the NMC parameter $\xi$. However, we note that the effects of these nonlinearities in $\xi$ have been included in our analysis and graphs. Since quantities such as the scalar spectral index, tensor-to-scalar ratio, and the amplitudes of non-Gaussianity have nonlinear dependence on the NMC parameter $\xi$, any small change in the values of $\xi$ has considerable effect on the values of these quantities. In fact, there is a nonlinear dependence on model parameters that really demands four figures of accuracy in quantities such as $\xi$ to get two figures of accuracy in observable quantities such as $n_s$. So, we have tried to be more accurate and found the values of $\xi$ with higher precision.

Appendix

The expanded action up to third order is as follows:

$$S_1 = \int dt d^3 x a^3 \left\{ \frac{3H^2}{k^2} + \frac{\phi^2}{2 \sqrt{1 - f(\phi)^2}} - \frac{f(\phi)^2}{(1 - f(\phi)^2)^{3/2}} \right.$$ 
$$+ \frac{f(\phi)^2 \phi^6}{2 (1 - f(\phi)^2)^{3/2}} - 80H^2 \alpha \Phi^3 \right.$$ 
$$+ \left( 144H^3 \alpha - \frac{9H^2}{k^2} + \frac{3\phi^2}{2 \sqrt{1 - f(\phi)^2}} + \frac{3f(\phi)^4}{2 (1 - f(\phi)^2)^{3/2}} \right) \cdot \Psi - \left( \frac{6H}{\kappa^2} - 48H^2 \alpha \right) \Psi - \frac{16\alpha}{a^2} \psi^2 \right\}$$
\[ + \frac{2\kappa^2 H + 16H^2 \dddot{\alpha}}{a^4} \Phi^2 + \left( \frac{18H}{\kappa^4} + 216H^2 \dddot{\alpha} \right) \Psi^2 \]

\[ + \frac{16\kappa^4 \dot{\Psi}^2 \dddot{\Psi}}{a^4} + \frac{(16H\dddot{\alpha} - 2\kappa^2)}{a^4} \dddot{\Psi}^2 \Psi - \frac{2\kappa^2 H + 16H^2 \dddot{\alpha}}{a^4} \]

\[ \cdot \dddot{\Psi} \dddot{B} \]

\[ + \frac{8\kappa^2}{a^4} \left( 3\dddot{\alpha} \dddot{B} \dddot{\alpha} - \dddot{\alpha} B^2 \dddot{B} \right) \Psi \]

\[ + \frac{48H\dddot{\alpha} - 2\kappa^2}{a^2} \dddot{\Psi}^2 B + \left( 3\kappa^2 - 72H\dddot{\alpha} \right) \dddot{\Psi}^2 \Psi \]

\[ + \frac{\kappa^2 - 8\kappa^2}{a^2} \Psi \dddot{\Psi}^2 + (72H\dddot{\alpha} - 9\kappa^2) \dddot{\Psi}^2 \Psi \]

\[ + \frac{2\kappa^2 - 16H\dddot{\alpha}}{a^2} \dddot{\Psi} \dddot{\alpha} \dddot{B} - \frac{8\kappa^2}{a^2} \dddot{\alpha} B^2 \dddot{B} + \frac{2\kappa^2 - 16H\dddot{\alpha}}{a^2} \]

\[ \cdot \dddot{\Psi} \dddot{B} - \frac{2\kappa^2 - 16H\dddot{\alpha}}{a^4} \dddot{\alpha} \dddot{B} \dddot{\alpha} B^2 \Psi \]

\[ + \left( \frac{3/2\kappa^2 - 12H\dddot{\alpha}}{a^4} \right) \dddot{\alpha} \dddot{\alpha} \dddot{B} - \dddot{\alpha} B^2 \dddot{B} \right) \]

\[ \left( \dddot{\alpha} \dddot{B} \dddot{\alpha} B - \dddot{\alpha} B^2 \dddot{B} \right) \right] . \]

(A.1)

### Competing Interests

The authors declare that they have no competing interests.

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