Toroidal, compression, and vortical dipole strengths in $^{144-154}$Sm: Skyrme-RPA exploration of deformation effect

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(Dated: May 30, 2013)

A comparative analysis of toroidal, compressional and vortical dipole strengths in the spherical $^{144}$Sm and the deformed $^{154}$Sm is performed within the random-phase-approximation using a set of different Skyrme forces. Isoscalar ($T=0$), isovector ($T=1$), and electromagnetic excitation channels are considered. The role of the nuclear convection $j_{con}$ and magnetization $j_{mag}$ currents is inspected. It is shown that the deformation leads to an appreciable redistribution of the strengths and causes a spectacular deformation splitting (exceeding 5 MeV) of the isoscalar compressional mode. In $^{154}$Sm, the $\mu=0$ and $\mu=1$ branches of the mode form well separated resonances. When stepping from $^{144}$Sm to $^{154}$Sm, we observe an increase of the toroidal, compression and vortical contributions in the low-energy region (often called pygmy resonance). The strength in this region seems to be an overlap of various excitation modes. The energy centroids of the strengths depend significantly on the isoscalar effective mass $m_0$. Skyrme forces with a large $m_0$ (typically $m_0/m \approx 0.8-1$) seem to be more suitable for description of experimental data for the isoscalar giant dipole resonance.

PACS numbers: 24.30.Cz, 21.60.Jz, 13.40.-f, 27.80.+w

I. INTRODUCTION

Possible evidence of a vortical flow in nuclei was suggested by the analysis of the structure of the isoscalar giant dipole resonance (ISGDR) observed in $(\alpha,\alpha')$ scattering experiments [1-3] (see Ref. [6] for a review and an exhaustive list of references). In fact, the prevailing conclusion from this analysis was that the high-energy peak of the ISGDR is produced by compressional vibrations [6-8], whereas the low-energy bump should be attributed to vortical nuclear flow associated with a toroidal dipole mode [9, 12]. The toroidal moments emerge as second order terms of multipole expansions of electric currents in physical systems [13, 14]. In nuclei, the toroidal mode (TM) was predicted within a hydrodynamical model [13].

On the other hand, it was argued that a strong mixing between compressional and vortical vibrations in the isoscalar E1 states should be expected [12]. Moreover, it is not yet completely settled how the vorticity relates to the toroidal and compressional modes. This depends on the way the vorticity is defined. In hydrodynamical (HD) models, where it is characterized by a non-vanishing curl of the velocity field [10], the vorticity is solely associated to the TM while the compressional mode (CM) is irrotational. An alternative definition, more linked to nuclear observables, was proposed by Ravenhall and Wambach (RW) [17]. It adopts, as a measure of the vorticity, the multipole component $\langle f_{\lambda m}(\hat{r})i \rangle$ of the transition density current $\langle f_{\lambda m}(\hat{r})i \rangle$. The motivation for this choice is that this component is not constrained by the continuity equation. With this measure of the vorticity, the TM and CM describe mixed flows of both vortical and irrotational nature. In the recent study [18], the vortical operator of RW type was derived and related in a simple manner to the CM and TM operators. Then the vortical, toroidal and compression E1 strengths in $^{208}$Pb were compared and thoroughly scrutinized within a separable random-phase-approximation (SRPA) [19, 20] using the SLy6 Skyrme force [21]. Later a similar study was performed for the isotopes $^{100,124,132}$Sn [22].

Following the above SRPA exploration, the TM falls into the energy region of so-called pygmy dipole resonance (PDR) [6] supposed to be induced in neutron rich nuclei by a relative translational oscillation of the neutron skin against the residual N=Z core. So an interplay of TM and PDR may be expected. As shown in our recent study [23] within the full (non-separable) RPA [24], the PDR energy region indeed embraces various modes with a strong TM fraction. Moreover, the vortical flow dominates in the nuclear interior while the irrotational motion (relevant for E1 transitions in the long-wave approximation) prevails at the nuclear surface. This point deserves a further investigation. In particular, a comparative analysis of experimental data from $(\alpha,\alpha'\gamma)$ (relevant for both TM and PDR in $T=0$ channel), $(\gamma,\gamma')$ observations (e.g. in Sn isotopes [23] and N=82 isotones.
In this study, we continue the exploration of the toroidal, compression, and vortical (RW) E1 strengths in various mass regions using SRPA. While the previous analysis concerned $^{208}$Pb and Sn isotopes and $^{144}$Sm to axially deformed $^{154}$Sm. Thereby we look particularly at the influence of the nuclear deformation. Like in Ref. [22], we use a representative set of Skyrme forces (SLy6 [21], SkT6 [27], SVbas [28], SkM* [29], and SkI3 [30]) covering a wide range of the isoscalar effective mass, $m_0/m = 1 - 0.58$. The isoscalar ($T=0$), isovector ($T=1$), and pure proton ($elm$) channels of E1 excitations are considered. Like in Refs. [18, 22], the relative contributions and pure proton (elm) channels of E1 excitations are discussed. The HD reaction flows and their operators are discussed. The HD SRPA method is given.

The SRPA method used in the present calculations has been already successfully applied to description of various kinds of nuclear excitations (electric [18, 51, 54] and magnetic [53, 57] giant resonances, E1 strength near the particle thresholds [34, 58], and TM/CW/RW modes [18, 22]) in both spherical and deformed nuclei and was shown as an efficient and reliable theoretical tool.

The paper is organized as follows. In Sec. II, the theoretical background and calculation scheme are presented. In Sec. III, the supplemented characteristics are inspected and numerical results for TM, CM, and RW strengths are discussed. In Sec. IV, the conclusions are drawn. In Appendix A, the vortical, toroidal, and compression flows and their operators are discussed. The HD and RW conceptions of the vorticity on nuclear flow are outlined. Appendix B sketches the derivation of the RW equations. In Appendix C, a basic information about the SRPA method is given.

II. THEORETICAL BACKGROUND AND CALCULATION SCHEME

The main topic of this paper is the influence of nuclear deformation on the toroidal, compressional and vortical dipole strength functions. The corresponding transition operators are [18]

$$
\mathcal{M}(\text{tor}; 1\mu) = -\frac{2}{2\sqrt{3}} \int d^3 r \tilde{j}_{\text{nuc}}(r) \cdot \frac{\sqrt{7}}{5} r^2 \hat{Y}_{120}(\hat{r}) \hat{Y}_{10\mu}(\hat{r}),
$$

$$
\mathcal{M}(\text{com}; 1\mu) = \frac{1}{10} \int d^3 r \tilde{\rho}(\hat{r}) \cdot \left[ r^3 - \delta_{T,0} \left( r^2 \right)_0 \right] Y_{1\mu}(\hat{r}),
$$

where $\tilde{j}_{\text{nuc}}(r)$ and $\tilde{\rho}(\hat{r})$ are operator of nuclear current and nuclear density, respectively. Symbols $Y_{\lambda\mu}(\hat{r})$ and $\hat{Y}_{\lambda\mu}(\hat{r})$ stand for spherical harmonics and vector spherical harmonics, respectively, and $\langle r^2 \rangle_0 = \int d^3 r \rho_0(r) r^2$ is the ground state square radius. The derivation of these operators and their connection to the long-wavelength limit of the standard E1 operator

$$
\mathcal{M}(E 1\mu) = -\int d^3 r \tilde{\rho}(\hat{r}) \cdot \hat{Y}_{1\mu}(\hat{r})
$$

can be found in the Appendix A and Ref. [18]. The toroidal, compressional, and vortical strengths determined by the operators (1), (2), and (3), respectively. The indices $\nu$ stand for the RPA states with the energies $E_\nu$, $|0\rangle$ is the RPA ground state. Further,

$$
\xi_{\Delta}(E - E_\nu) = \frac{1}{2\pi} \frac{\Delta}{(E - E_\nu)^2 + (\frac{\Delta}{2})^2}
$$

is the Lorentz weight with the averaging parameter $\Delta$. The averaging is needed for the convenience of comparison of the results with the experimental data and to simulate roughly the smoothing effects beyond the SRPA (escape widths and coupling to complex configurations). For the broad and poorly known TM,CM, and RW strengths, a constant averaging width $\Delta=1$ MeV is optimal. The strengths are analyzed in $T=0$, $T=1$, and electromagnetic ($elm$) channels characterized by the effective charges $e_{\text{eff}}$ and gyromagnetic ratios $g_{\text{eff}}$

$$
T = 0 : \quad e_{\text{eff}} = e_p = 1, \quad g_{n,p} = \frac{\zeta}{2} (g_n + g_p),
$$

$$
T = 1 : \quad e_{\text{eff}} = -e_p = -1, \quad g_{n,p} = \frac{\zeta}{2} (g_n - g_p),
$$

$$
elm : \quad e_{\text{eff}} = 0, \quad e_{\text{eff}} = 1, \quad g_{n,p} = \zeta g_{n,p},
$$

where $g_n = -3.82$ and $g_p = 5.58$ are free neutron and proton gyromagnetic ratios, $\zeta \approx 0.7$ is the usual quenching factor [39]. See details in Appendix A.

The photoabsorption cross-section is fully determined by the $E1$ transitions and thus reads [40]

$$
\sigma_{\text{phot}}(E) = \frac{16 \pi^3 \alpha_e}{9 \epsilon^2} E S(E1; E),
$$
where \( \alpha_e = 1/137 \) is the fine-structure constant and \( S(E1; E) \) is the strength function calculated with the standard dipole operator

\[
\hat{M}(E1\mu) = \frac{N}{A} \sum_{p=1}^{Z} r_p Y_{1\mu}(\Omega_p) - \frac{Z}{A} \sum_{n=1}^{N} r_n Y_{1\mu}(\Omega_n). \tag{11}
\]

For the photoabsorption cross-section, detailed experimental data are available \(^{11,12}\). So it is worth to compute this observable more accurately. It should be taken into account that i) the escape widths appear above the particle emission thresholds and grow with energy due to the widening of the emission phase space, ii) the collisional widths, induced by the coupling with complex configurations, also increase with the excitation energy. To simulate these trends, one should use in \(^{13}\) an energy-dependent averaging parameter \( \Delta(E) \). This can be done by implementing a double folding scheme \(^{14}\). We calculate first the strength function \(^{14}\) with the operator \(^{11}\) by using a small but fixed value of \( \Delta \). This gives the strength distribution \( S'(E1; E) \) very close to one obtained in RPA but with an equidistant energy grid. This strength is then folded again by using an energy dependent \( \Delta(E) \):

\[
S(E1; E) = \int dE' S'(E1; E') \xi_{\Delta(E')}(E - E'). \tag{12}
\]

In the present study, we use a simple linear dependence

\[
\Delta(E') = \begin{cases} 
\Delta_0 & \text{for } E' \leq E_{\text{th}} \\
\Delta_0 + a(E' - E_{\text{th}}) & \text{for } E' > E_{\text{th}} \end{cases} \tag{13}
\]

where \( E_{\text{th}} \) is the energy of the first emission threshold and \( \Delta_0 = 0.1 \) MeV is a minimal width. The parameter \( a \) is chosen to reproduce the GDR strength distribution.

In order to test the sensitivity of the modes, we consider a selection of sufficiently different Skyrme parametrizations (SkT6 \(^{27}\), SVbas \(^{28}\), SkM* \(^{29}\), SLy6 \(^{21}\), and SkI3 \(^{30}\)) covering a wide range of isoscalar effective masses: \( m_0/m = 1, 0.9, 0.79, 0.69, \) and 0.58, respectively. All the parametrizations provide a good description of basic ground properties of nuclei.

The configuration space in the calculations covers all particle-hole (two-quasiparticle) states with an excitation energy up to \( E_{\text{cut}} \approx 175 \) MeV. Such a big basis allows to exhaust in the SRPA calculations about 100\% of the isovector Thomas-Reiche-Kuhn (TRK) energy-weighted sum rule (EWSR) \(^{40}\) for the GDR and isoscalar Harakeh’s EWSR for the ISGDR \(^{43}\) (see also an extensive discussion at the end of the next subsection and Appendix C). Besides that, the large basis is crucial to lower the energy position of the spurious E1(T=0) peak towards the correct zero energy. In the present study, it lies in spherical \(^{144}\)Sm at 1.5 – 2.0 MeV, depending on the Skyrme parametrization. This is not so good as in some full RPA calculations for \(^{208}\)Pb, which use a larger configuration space and manage to yield the spurious peak below ~ 0.6 MeV, see e.g. SLy5 \(^{44}\) and SLy6 \(^{23}\) results. However, the present value is sufficient for our aims since the T=0 strengths of interest are situated at \( E > 5 \) MeV and further extension of the configuration space does not significantly affect them. To be on the safe side, the TM, CM, and RW strength functions are given below only in the range \( E > 5 \) MeV.

The calculations exploit a 2D representation in cylindrical coordinates using a mesh size of 0.3 fm and a calculation box of 21 fm. For open-shell isotopes, we use zero-range pairing forces. The Hartree-Fock-Bogoliubov (HFB) equations are implemented at the BCS level \(^{15}\). For closed shell nuclei, this reduces automatically to a Hartree-Fock (HF) treatment.

III. RESULTS AND DISCUSSIONS

A. Supplemented characteristics

In Figures 1 and 2, some basic features (binding energies and photoabsorption cross sections) are presented to demonstrate the accuracy of our approach. In addition to spherical \(^{144}\)Sm and deformed \(^{154}\)Sm, the transitional isotopes \(^{146,150,152}\)Sm are also shown to illustrate the trends with development of the deformation. The treatment of these soft isotopes within RPA is known to be insufficient and has to be amended by the coupling with complex configurations \(^{46,47}\). Nevertheless, we find useful to present these RPA results to outline the trends.

In Figure 1, the binding energies of Sm isotopes

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FIG. 1: (color online) HF/HFB binding energies of Sm isotopes versus the dimensionless parameter of quadrupole deformation \( \beta \), calculated with different Skyrme parametrizations. The binding energies at an equilibrium deformation are to be compared with the experimental values (dashed horizontal lines) obtained from mass measurements \(^{48}\).
For each isotope, we indicate the dimensionless quadrupole deformation $\beta$ of the prolate ground state computed with the force SLy6.

Calculated within the HF/HFB approach are depicted in dependence on the dimensionless parameter of the quadrupole deformation $\beta$. For all Skyrme forces used here, we see the pronounced main minima at $\beta=0$ in the spherical $^{144}\text{Sm}$ and at $\beta=0.33-0.34$ in the prolate deformed $^{154}\text{Sm}$. In the latter case, the internal quadrupole moment is $Q=6.4-6.6$ b. Both computed $\beta$ and $Q$ are in a good agreement with the experimental values $\beta_{\text{exp}}=0.34$ and $Q_{\text{exp}}=6.6$ b [66]. In the transitional $^{148,150,152}\text{Sm}$, the calculations produce two shallow minima with roughly the same depth, corresponding to prolate and oblate shapes, respectively. Despite all the Skyrme parametrizations are fitted to experimental binding energies of selected doubly magic or semi-magic nuclei, in our calculations only the SLy6, SV-bas, and SkI3 forces reproduce the measured binding energy [48] in the semi-magic $^{144}\text{Sm}$. The two older parametrizations SkM* and SkT6 were tuned to doubly-magic nuclei only and do not perform so well for the rather soft $^{144}\text{Sm}$.

In Figure 2, the calculated photoabsorption cross section is inspected. We get a good agreement with experiment [42] for most Skyrme forces in the spherical $^{144}\text{Sm}$ and the well-deformed $^{154}\text{Sm}$. The deformation splitting of the GDR in $^{144}\text{Sm}$ is also reproduced. For the transitional isotopes, the agreement is acceptable as well. In all the nuclei, the main deviation arises at the high-energy wing of the GDR. This is probably an effect of neglecting the complex configurations.

Additional useful insight may be obtained from the analysis of the sum rules. As was mentioned above, the TRK sum rule for the photoabsorption cross section is exhausted in our calculations by about 100%, see details in Appendix C. For the purposes of the present study, it is crucial to check the sum rule for the ISGDR [43]

$$\text{EWSR}_{\text{ISGDR}} = \frac{1}{100} \left[ \frac{3h^2}{8\pi m} A \left( \langle r^4 \rangle_0 - \frac{25}{3} \langle r^2 \rangle_0^2 \right) \right],$$

obtained for the transition operator (2) and $\langle r^4 \rangle_0 = \int d^3r \rho_0(r) r^4$. The results are shown in Table I and they prove that this sum rule is also nicely fulfilled. Thus, extracting both isovector TRK and isoscalar ISGDR E1 sum rules confirms that our configuration space is sufficiently large. Altogether, the above results demonstrate reliability of our approach in description of static and E1 dynamical properties of spherical and deformed nuclei and thus justifies its further application to TM/CW/RW E1 strength functions.

|      | SkI3 | SLy6 | SkM* | SV-bas | SkT6 |
|------|------|------|------|--------|------|
| ISGDR| 24.4 | 24.7 | 25.1 | 24.3   | 24.4 |
| RPA  | 25.0 | 25.4 | 24.8 | 23.1   | 24.3 |
| $^{144}\text{Sm}$ | 34.1 | 33.8 | 34.1 | 33.2   | 33.2 |
| $^{154}\text{Sm}$ | 34.8 | 34.0 | 33.5 | 32.6   | 33.1 |

TABLE I: The ISGDR and RPA EWSR (in $e^2 fm^6$) for CM(T=0) in $^{144,154}\text{Sm}$, computed with different Skyrme forces.

B. TM, CM, and RW strength functions

In Figure 3, the E1(T=0) and E1(T=1) strength functions for the CM in spherical $^{144}\text{Sm}$ and deformed $^{154}\text{Sm}$ are shown for the chosen set of Skyrme forces. The strengths are computed purely within the HF/HFB approach (without the residual interaction) by using the transition operator (2). As seen from the figure, the calculations give two broad CM bumps, a smaller one at low-energy and a larger one at high-energy, which is in accordance to previous theoretical and experimental studies, see review [6]. In $^{154}\text{Sm}$, the strength is larger and more uniform than in $^{144}\text{Sm}$, as expected for a heavier and more deformed nucleus.

Three non-trivial effects are visible in both T=0 and T=1 channels. First, in $^{154}\text{Sm}$ we see a substantial growth of the CM strength in the region $4 \leq E \leq 11$ MeV where the PDR is supposed to appear. The comparison with the constrained case $\beta=0$ (no deformation) for the force SLy6 indicates that this effect should be mainly ascribed to increasing the number of neutrons (from magic 82 to 92). Indeed, unlike $^{144}\text{Sm}$, in $^{154}\text{Sm}$ particular low-energy s.p. transitions $\nu\nu[1h_9/2 \leftrightarrow 2f_7/2]$ become active, which may lead to the growth of the strength. However, following Fig. 4, where the deformation splitting of $\mu=0$ and $\mu=1$ CM(T=0) components for SkT6 and SLy6 forces is demonstrated, the deformation effect is also important. Just due to deformation, the low-energy
On the left, the isoscalar effective masses \( m_0/m \) of the forces are listed. The upper sector is excited solely in the low-lying E1 spectra into an upper and lower energy parts of the region \( 4 \leq E \leq 11 \text{ MeV} \) are dominated by \( \mu=0 \) and \( \mu=1 \) branches, respectively. Moreover, the comparison of T=0 and T=1 strengths allows to state that the PDR region may be also separated into two isospin sectors, a low-energy T=0 and a high-energy T=1 ones. This conjecture finds support in recent experimental analysis \([23, 29]\), which have split the low-lying E1 spectra into an upper and lower energy sectors. The upper sector is excited solely in \( (\gamma, \gamma') \) and, therefore, has T=1 nature. The lower sector, instead, is composed of levels excited in both \( (\gamma, \gamma') \) and \( (\alpha, \alpha'\gamma) \) and thus may be related to T=0 excitations.

Second, Fig. 3 shows that in \( ^{154}\text{Sm} \) a substantial bump appears at 20-30 MeV. This is particularly pronounced in T=0. The comparison to the case \( \mu=0 \) indicates that the bump is caused by the deformation. Fig. 4a) shows that this is just the \( \mu=0 \) branch of the high-energy CM. We also see in Fig. 4a) a huge deformation splitting (\( \sim 10 \) MeV) of the CM(T=0) strength computed with SLy6 force (similar results are obtained for other Skyrme forces). Thus deformation gives for the high-energy CM indeed dramatic effect.

Third, Fig. 3 shows that the CM strength in general and low-energy and high-energy CM bumps in particular are noticeably upshifted with decreasing \( m_0/m \). This places for both isotopes \( ^{144}\text{Sm} \) and \( ^{154}\text{Sm} \) and in both channels, T=0 and T=1. The effect is straightforwardly explained by well known spread of s.p. spectra below the Fermi level with decreasing \( m_0/m \), see examples in \([49]\).

In Figure 5, the RPA CM strength functions are depicted. The residual interaction significantly downshifts (upshifts) the strength in T=0 (T=1) channels. Nevertheless all three effects discussed above for HF/HFB case remain the same. The \( (\alpha, \alpha') \) experiment \([3, 5]\) gives for the ISGDR in \( ^{144}\text{Sm} \) two bumps with the energies and widths E=14.2 MeV, \( \Gamma=4.8 \) MeV and E=25.0 MeV, \( \Gamma=19.9 \) MeV. The narrow low-energy and broad high-energy bumps of the ISGDR are commonly treated as TM \([10, 12]\) and CM \([7, 8]\), respectively. Alternatively,
both bumps may be treated as merely CM branches with the strong CM/TM coupling at a low energy [23]. Following Fig. 5, our calculations for $^{144}$Sm roughly reproduce the experimental data [2,3] for the forces SkT6, SVbas, SkM*, and SLy6. For SkI3 with a low effective mass $m_0/m=0.58$, the agreement for the high-energy CM bump is not so good. It seems that Skyrme forces with a large $m_0/m$ are more preferable for description of CM.

Note that, following Fig. 4, the residual interaction considerably decreases the deformation splitting. As compared to the HFB case, it is reduced for the high-energy CM($T=0$) bump from $\sim 10$ MeV (HFB) to $\sim 5$ MeV (RPA). Nevertheless, the deformation splitting remains very large. Such strong splitting is known only for the isovector GDR.

In Fig. 6, the RPA TM strength in $^{144,154}$Sm is de-
FIG. 7: (color online) The same as in Fig. 3 but for the RPA vortical (RW) strength function.

FIG. 8: (color online) RPA toroidal (upper), RW vortical (middle), and compression (bottom) strength functions calculated in T=0 (left), T=1 (middle), and electromagnetic (right) channels with SLy6 Skyrme force in spherical $^{144}$Sm. For TM and RW, the contributions from total (black/bold line), convection (blue/thin line), and magnetization (red/dotted line) currents are depicted.

In contrast to the CM, the TM is concentrated at low energy while the high-energy strength is rather weak. Following our analysis, this is because of the instructive (destructive) summation of $\lambda l=10$ and 12 terms in the TM operator (11) in the low-energy (high-energy) regions. For the CM operator (18), may be decomposed in similar manner (18), we have the opposite result.

The TM(T=0) strength in Fig. 6 demonstrates the same three effects discussed above for CM: i) pumping the strength to the PDR region due to neutron excess/deformation impact, ii) appearance of an apprecia-
the QRPA plus phonon-coupling model \cite{50, 51} and the tensions are already available. Among them, there are like an octupole-quadrupole excitation. Several RPA core excitations coupled to collective surface vibrations, may couple to two-phonon states describing collective-

In accordance to the previous calculations for Pb \cite{18}, the maximal nuclear vorticity is expected \cite{18}.

In Fig. 7, a similar behavior is observed for the vortical RW strength generated by the operator \cite{33}. The main difference from TM is that low-energy and high-energy RW bumps in T=0 channel are of a comparable strength. However, before performing more detailed investigations with including multiphonon states, a first exploration with mere RPA exploration is desirable and this is just what we have presented here.

IV. CONCLUSION

The E1 compression, toroidal, and vortical strengths in even Sm isotopes (from spherical \textsuperscript{144}Sm to deformed \textsuperscript{154}Sm) were investigated within the self-consistent separable random-phase approximation (SRPA) approach \cite{13, 20} with Skyrme forces. A representative set of different Skyrme parametrizations (SkT6 \cite{27}, SVbas \cite{28}, SkM* \cite{29}, SLy6 \cite{21}, and SkI3 \cite{30}) was used. Three reaction channels were inspected: isoscalar (T=0), isovector (T=1) and electromagnetic (with the proton part of the transition operator). Particular attention was paid to effects of the quadrupole deformation.

As a first step, the accuracy of the scheme was checked with respect to binding energies, photoabsorp-
Votruba - Blokhintsev (Czech Republic - BLTP JINR) grants. W.K. and P.-G.R. are grateful for the BMBF support under contracts 05P12ODDU and support from the F+E program of the Gesellschaft für Schwerionenforschung (GSI). The support of the research plan MSM 0021620859 (Ministry of Education of the Czech Republic) and the Czech Science Foundation project No P203-13-07117S to this work are also appreciated.

Appendix A: Vorticity conceptions and the relevant transition operators

1. Velocity and nuclear current fields

In hydrodynamics (HD) [16], the vortical motion of a fluid is characterized by a non-vanishing value of the curl of the velocity field

$$\nabla \times \vec{v}_{\text{HD}}(\vec{r}) \neq 0$$

(A1)

where \(\vec{v}(\vec{r})\) is defined as a time derivative of the displacement field, see e.g. [12]. Instead, a quantum-mechanical treatment of the nuclear vorticity should involve the actual nuclear physical observables, namely the nuclear current defined by the operator

$$\hat{j}_{\text{nucl}}(\vec{r}) = \sum_{\nu \nu'} \langle \nu | \hat{j}(\vec{r}) | \nu' \rangle a_\nu^+ a_{\nu'},$$

(A2)

where \(a_\nu^+, a_{\nu'}\) are single particle (s.p.) creation and annihilation operators, respectively. The nuclear current consists of a convection and a magnetization parts:

$$\hat{j}_{\text{nucl}} = \hat{j}_{\text{con}} + \hat{j}_{\text{mag}}.$$  

(A3)

Thus the current transition density (CTD) reads

$$\langle \nu | \hat{j}_{\text{nucl}}(\vec{r}) | \nu' \rangle = \langle \nu | \hat{j}_{\text{con}}(\vec{r}) | \nu' \rangle + \langle \nu | \hat{j}_{\text{mag}}(\vec{r}) | \nu' \rangle.$$  

(A4)

The convection CTD is given by

$$\langle \nu | \hat{j}_{\text{con}}(\vec{r}) | \nu' \rangle = \frac{i e_{q \nu}^\text{eff} \hbar}{2m} \times \left[ (\nabla \Psi_\nu(\vec{r}))^+ \Psi_{\nu'}(\vec{r}) - \Psi_\nu(\vec{r}) \nabla \Psi_{\nu'}(\vec{r}) \right],$$

(A5)

where \(\Psi_\nu(\vec{r})\) are s.p. wave functions and \(e_{q \nu}^\text{eff}\) is a proton (q=p) or neutron (q=n) effective charge. The magnetization CTD has the form

$$\langle \nu | \hat{j}_{\text{mag}}(\vec{r}) | \nu' \rangle = \frac{e \hbar}{2m} g_{q \nu}^\text{eff} \Psi_\nu(\vec{r}) (\nabla \times \vec{\sigma}) \Psi_{\nu'}(\vec{r}),$$

(A6)

where \(g_{q \nu}^\text{eff}\) is the effective gyromagnetic ratio.

As shown below, the nuclear vorticity may be defined either following a semi-classical HD approach, see e.g. [12, 16], or a quantum-mechanical RW route [17].

Acknowledgments

The work was partly supported by the GSI-F+E-2010-12, Heisenberg-Landau (Germany - BLTP JINR), and
2. HD treatment of the vorticity

In this approach, the spin part of the nuclear current is usually neglected. Then one may define the velocity operator \( \hat{\mathbf{v}}_{\text{HD}} \) through the nuclear current operator as

\[
j_{\text{nuc}}(\hat{r}) = \rho_0(\hat{r}) \hat{\mathbf{v}}_{\text{HD}}(\hat{r}),
\]

where \( \rho_0(\hat{r}) \) is the density in the ground state. After taking the curl of the current \( (A7) \), we obtain the quantity

\[
\rho_0(\hat{r}) \nabla \times \hat{\mathbf{v}}_{\text{HD}}(\hat{r}) = \nabla \times j_{\text{nuc}}(\hat{r}) - \nabla \rho_0(\hat{r}) \times \hat{\mathbf{v}}_{\text{HD}}(\hat{r}) = \nabla \times j_{\text{nuc}}(\hat{r}) - \nabla \rho_0(\hat{r}) \times j_{\text{nuc}}(\hat{r}) / \rho_0(\hat{r})
\]

which may be adopted to measure the nuclear vorticity. This definition, however, does not take into account that the charge and current densities are not independent but related by the continuity equation.

3. RW treatment of the vorticity

In order to eliminate the drawback of the hydrodynamical definition, the RW prescription \([17]\) starts with decomposing the nuclear current into an irrotational and a vortical components

\[
j_{\text{nuc}}(\hat{r}) = j_{\text{irrot}}(\hat{r}) + j_{\text{vort}}(\hat{r})
\]

It is further assumed that the quantity

\[
\hat{w}(\hat{r}) = \nabla \times j_{\text{vort}}(\hat{r}) = \nabla \times j_{\text{nuc}}(\hat{r}) - \nabla \times j_{\text{irrot}}(\hat{r})
\]

is a measure of a vortical flow. This motion sets in once \( \langle f | \hat{w}(\hat{r}) | i \rangle \neq 0 \) and vice versa.

One proceeds with a multipole decomposition in terms of spherical harmonics \( Y_{\lambda\mu}(\hat{r}) \) or vector spherical \( \tilde{Y}_{\lambda\mu}(\hat{r}) \) harmonics obtaining

\[
\langle f | \hat{w}_{\text{nuc}}(\hat{r}) | i \rangle = \sum_{\lambda \mu} b^{(fi)}_{\lambda\mu} \omega_{\lambda}(r) \tilde{Y}_{\lambda\mu}(\hat{r}),
\]

where

\[
b^{(fi)}_{\lambda\mu} = \frac{(j_{\text{irrot}}(\hat{r}) \times \hat{\mathbf{v}}_{\text{HD}}(\hat{r})) / \rho_0(\hat{r})}{\sqrt{2j_f + 1}}
\]

and

\[
u_{\lambda\lambda}^{(fi)}(r) = \sqrt{\frac{2\lambda + 1}{\lambda}} \left( \frac{d}{dr} + \frac{\lambda + 2}{r} \right) j_{\lambda\lambda+1}(r),
\]

which follows from applying the continuity equation.

Using the the multipole decomposition, the second term in \( (A10) \) can be written as

\[
\left[ \nabla \times j_{\text{irrot}}(\hat{r}) \right]_{\lambda} = \frac{1}{\lambda} \left[ (\nabla \hat{\rho}(\hat{r})) \times \hat{\mathbf{r}} \right]_{\lambda}.
\]

This quantity resembles the second term in the hydrodynamic equation \( (A8) \). It is therefore legitimate to adopt the matrix element \( (A11) \) as a measure of vorticity \( [17] \).

4. RW vortical operator

The vortical multipole operator was introduced in \([18]\). One starts with the standard \( E\lambda \mu \) multipole operator \([?]\)

\[
\hat{M}(E\lambda\mu, k) = (2\lambda + 1)!! \sqrt{\frac{\lambda}{\lambda + 1}} \int d^3r \left[ j_\lambda(k \hat{r}) \tilde{Y}_{\lambda\mu}(\hat{r}) \right] \cdot \left[ \nabla \times \hat{\mathbf{v}}_{\text{nuc}}(\hat{r}) \right].
\]

After expanding the spherical Bessel functions \( j_\lambda(k \hat{r}) \) in powers of \( (kr) \) and using the continuity equation, one obtains \([11]\)

\[
\hat{M}(E\lambda\mu; k) = \hat{M}(E\lambda\mu) + k \hat{M}(\text{tor}; \lambda\mu) + \ldots,
\]

where \( \hat{M}(E\lambda\mu) \) and \( \hat{M}(\text{tor}; \lambda\mu) \) are, respectively, the long-wavelength limits of the standard and toroidal \( E\lambda \mu \) multipole operators \([11, 18]\)

\[
\hat{M}(E\lambda\mu) = -\int d^3r \hat{\rho}(\hat{r}) r^\lambda \tilde{Y}_{\lambda\mu}(\hat{r}),
\]

\[
\hat{M}(\text{tor}; \lambda\mu) = -\frac{i}{2c} \sqrt{\frac{\lambda}{\lambda + 1}} \int d^3r \hat{\mathbf{v}}_{\text{nuc}}(\hat{r}) \cdot \left[ \nabla \times \hat{\mathbf{v}}_{\text{nuc}}(\hat{r}) \right](A10)
\]

\[
\cdot \left[ \nabla \times \hat{\mathbf{v}}_{\text{nuc}}(\hat{r}) \right].
\]

The minus sign in \( \hat{M}(E\lambda\mu) \) is appropriate for excitations processes \( (E_f > E_i) \).

According to the RW concept of vorticity \([17]\), the vortical multipole operator \( \hat{M}(\text{vor}; \lambda\mu; k) \) is obtained from \( (A15) \) by the substitution

\[
\left[ \nabla \times j_{\text{nuc}}(\hat{r}) \right] \rightarrow \hat{w}(\hat{r}) = \nabla \times j_{\text{nuc}}(\hat{r}) - \nabla \times j_{\text{irrot}}(\hat{r}).
\]

Using \( (A14) \), one gets in the long wavelength limit

\[
\hat{M}(\text{vor}; \lambda\mu; k) = k \hat{M}(\text{vor}; \lambda\mu) + \ldots
\]

\[= k \left[ \hat{M}(\text{tor}; \lambda\mu) + \hat{M}(\text{com}; \lambda\mu) \right] + \ldots \]

The toroidal term \( \hat{M}(\text{tor}; \lambda\mu) \) is given by \( (A17) \) and the compressional operator is

\[
\hat{M}(\text{com}; \lambda\mu) = -k \hat{M}(\text{com}; \lambda\mu),
\]

where \( \hat{M}(\text{com}; \lambda\mu) \) is the familiar compression multipole operator \([11, 12]\)

\[
\hat{M}(\text{com}; \lambda\mu) = \frac{1}{2 (2\lambda + 3)} \int d^3r \hat{\rho}(\hat{r}) r^{\lambda+2} Y_{\lambda\mu}(\hat{r}).
\]

From the above equations it follows \([18]\) that the vorticity multipole operator has the form

\[
\hat{M}(\text{vor}; \lambda\mu) = \hat{M}(\text{tor}; \lambda\mu) + \hat{M}(\text{com}; \lambda\mu)
\]

\[
= -\frac{i}{c} \sqrt{\frac{2\lambda + 1}{\lambda + 1}} \int d^3r \hat{\mathbf{v}}_{\text{nuc}}(\hat{r}) \cdot \hat{\mathbf{r}}^{\lambda+1} \tilde{Y}_{\lambda\lambda+1,\mu}(\hat{r}).
\]
This definition shows that the vortical operator $M(\text{vor}; \lambda \mu; k)$ is a first order correction to the leading $E\lambda$ operator.

For the dipole transitions, the operators are to be accordingly modified in order to remove the spurious contribution induced by the center of mass motion. Following [18], only the toroidal (1) and compressional (2) operators are actually modified, while the vorticity operator (3) remains unchanged.

Appendix B: SRPA approach

The SRPA Hamiltonian is self-consistently derived [19, 20] from the functional [62]

\[ \mathcal{E} = \mathcal{E}_{\text{kin}} + \mathcal{E}_{\text{Sk}} + \mathcal{E}_{\text{pair}} + \mathcal{E}_{\text{Coul}} \quad (B1) \]

involving kinetic-energy, Skyrme, pairing, and Coulomb terms. The Skyrme functional $\mathcal{E}_{\text{Sk}}(\rho, \sigma, \tau, j, j', s, T)$ [63–65] depends on the time-even nucleon ($\rho$), kinetic-energy ($\sigma$), and spin-orbit ($j$) densities as well as on the time-odd current ($j'$), spin ($s$), and vector kinetic-energy ($T$) densities.

The Hamiltonian is composed of the HFB mean field and residual potential [19, 20]

\[ \hat{H} = \hat{h}_{\text{HFB}} + \hat{V}_{\text{res}}, \quad (B2) \]

The HFB mean field is given by

\[ \hat{h}_{\text{HFB}} = \int d^3 \tau \sum_{\alpha_+} \left[ \frac{\delta \mathcal{E}}{\delta \hat{j}_{\alpha_+}(\tau)} \right] \hat{\delta j}_{\alpha_+}, \quad (B3) \]

where $J_{\alpha_+}$ ($J_{\alpha_-}$) are time even (odd) densities and $\hat{J}_{\alpha_\pm}$ are the corresponding operators, with $\alpha_+$ ($\alpha_-$) denoting the densities.

The residual interaction is a sum of separable pieces

\[ \hat{V}_{\text{res}} = \frac{1}{2} \sum_{k,k'=1}^{K} (\kappa_{kk'} \hat{X}_k \hat{X}_{k'} + \eta_{kk'} \hat{Y}_k \hat{Y}_{k'}) \quad (B4) \]

with one-body fields

\[ \hat{X}_k = i \int d^3 r d^3 r' \sum_{\alpha_+} \frac{\delta^2 \mathcal{E}}{\delta \hat{j}_{\alpha_+}(\hat{r}) \delta \hat{j}_{\alpha_+}(\hat{r}')} \langle \hat{P}_k, \hat{J}_{\alpha_+} \rangle \hat{J}_{\alpha_+}, \quad (B5) \]

\[ \hat{Y}_k = i \int d^3 r d^3 r' \sum_{\alpha_-} \frac{\delta^2 \mathcal{E}}{\delta \hat{j}_{\alpha_-}(\hat{r}) \delta \hat{j}_{\alpha_-}(\hat{r}')} \langle \hat{Q}_k, \hat{J}_{\alpha_-} \rangle \hat{J}_{\alpha_-}, \quad (B6) \]

and strength constants

\[ \kappa_{kk'}^{-1} = -i \langle \hat{P}_k, \hat{X}_{k'} \rangle, \quad \eta_{kk'}^{-1} = -i \langle \hat{Q}_k, \hat{Y}_{k'} \rangle. \quad (B6) \]

The fields $\hat{X}_k$ and $\hat{Y}_k$ are time-even and time-odd, respectively. The corresponding generators are time-odd $\hat{P}_k = i [\hat{H}, \hat{Q}_k]$ and time-even $\hat{Q}_k$. The two-body potential includes all possible separable terms allowed by the functional. In practice, the number of fields is determined by the number of the generators $Q_k$ or $P_k$ chosen as input. Such a choice is crucial for generating a minimal set of one-body fields appropriate for the excitation mode to be studied and, therefore, render feasible the numerical implementation. An optimal set of generators were determined for $E1$ [19, 20] and $M1$ [35–36] modes. The set of generators needed for the modes under investigation here was discussed in the Appendix D of Ref. [18]. The Skyrme separable RPA may also be derived by an alternative way [54, 55].

The self-consistent separable form [B4] of the residual interaction enables us to transform the RPA equations of motion into a system of homogeneous algebraic equations in the forward $c_{ij}^{(vq-)}$ and backward $c_{ij}^{(vq+)}$ amplitudes of the RPA phonon creation operators [19]

\[ \tilde{c}_{ij}^{\nu} = \sum_{\nu = n, p} \sum_{ij \in q} \left( c_{ij}^{(vq-)} \alpha_i^+ \alpha_j^+ - c_{ij}^{(vq+)} \alpha_j \alpha_i \right), \quad (B7) \]

where $\alpha_i^+$ and $\alpha_i$ are quasiparticle creation and annihilation operators, respectively. The rank of the matrix of the system of algebraic equations is $4K$, where $K$ is the number of separable terms in [B4]. Usually $K = 3 - 5$ is sufficient to describe a given mode [18, 55]. So low rank

![Figure 10: (color online) The HF/HFB (blue curve with triangles) and RPA (red curve with circles) EWSR in $^{144–154}$Sm isotopes, calculated with different Skyrme forces. For the comparison, the TRK values computed with the bare nucleon mass $m$ (pink horizontal line) and isovector effective mass $m_1$ (green curve with rectangles) are depicted. The values of $m_1/m$ are given in the lower panel.](image-url)
allows very efficient and fast solution of the SRPA equations. They yield one-phonon states \( |\nu\rangle = C_\nu^{+} (\text{RPA}) \) of energy \( E_\nu \), with \(|\text{RPA}\rangle\) being the phonon vacuum.

**Appendix C: Thomas-Reiche-Kuhn sum rule**

In Fig. 10, the integral HFB and RPA energy-weighted strengths \[ \langle \text{HFB} \rangle \] are compared to the TRK sum rule for the Skyrme forces \[ \langle \text{TRK} \rangle \].

\[
\text{EWSR}_{\text{TRK}} = \frac{9}{8 \pi m_1} \frac{N Z}{A}. \quad (C1)
\]

Unlike the common case \[ \langle \text{TRK} \rangle \], this expression includes not the bare nucleon mass \( m \) but the isovector effective mass \( m_1 \) so as to take into account the contribution of the velocity-dependent terms (VDT) of the Skyrme forces, see an extensive discussion in Refs. \[ \text{33, 67} \]. It is seen that, for all Skyrme forces, the RPA strengths follow closely the TRK values. The agreement takes place both for \( m_1/m < 1 \) (SkM*, SVbas, SkI3, SLY6 with active VDT) and \( m_1/m = 1 \) (SKT6 with zero VDT). This implies that \( m_1 \) successfully incorporates almost all VDT contributions to the EWSR, ii) our configuration space is sufficiently large. For most of the forces, the HFB and RPA values overestimate by 20-30% the TRK values evaluated with the bare nucleon mass \( m \). This indicates an important role of the VDT. Besides, the VDT causes a substantial difference between HFB and RPA results.

Note that unlike the TRK sum rule \[ \langle C1 \rangle \], the isoscalar EWSR for the ISGDR \[ \text{44, 14} \] uses the bare mass \( m \). Following \[ \text{67} \], the VDT contribute to both mean field and residual interaction but, for \( T=0 \) RPA calculations, these contributions almost compensate each other. Thus we may use the bare mass \( m \). Instead, for \( T=1 \) RPA calculations considered above, the compensation does not take place, which results in using \( m_1 \) in \[ \langle C1 \rangle \].
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