Pattern Formation in Exciton System near Quantum Degeneracy

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We discuss models of the modulational instability in a cold exciton system in coupled quantum wells. One mechanism involves exciton formation in a photoexcited electron-hole system in the presence of stimulated binding processes which build up near exciton degeneracy. It is shown that such processes may give rise to Turing instability leading to a spatially modulated state. The structure and symmetry of resulting patterns depend on dimensionality and symmetry. In the spatially uniform 2d electron-hole system, the instability leads to a triangular lattice pattern while, at an electron-hole interface, a periodic 1d pattern develops. Wavelength selection mechanism is analyzed, revealing that the transition is abrupt (type I) for the uniform 2d system, and continuous (type II) for the electron-hole interface. Another mechanism that could possibly drive the instability involves long-range attraction of the excitons. We illustrate how such an interaction can result from plasmon wind, derive stability criterion, and discuss likelihood of such a scenario.

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I. INTRODUCTION

Recent experiments observed striking spatial pattern formation in two-dimensional cold exciton systems in photoexcited AlGaAs/GaAs quantum well (QW) structures.\textsuperscript{1,2,3,4,5} The photoluminescence (PL) patterns span macroscopic scales in excess of 100 \( \mu \text{m} \) and include concentric rings as well as bright spots.\textsuperscript{1,2,3,4,5} In addition to that, the electron-hole system exhibits an abrupt transition at ca. 2 K in which the outermost ring ‘fragments’ into regularly spaced beads of high PL intensity.\textsuperscript{1,3} While the gross features of PL have been explained within the framework of a classical transport theory, attributing the internal rings to nonradiative exciton transport and cooling\textsuperscript{1} and the outermost rings and bright spots \textsuperscript{1,2,3,4,5} to macroscopic charge separation\textsuperscript{1,2,3,4,5}, the origin of the instability remains unidentified.

Spatially modulated exciton density is not to be expected in QW system designed so that excitons interact repulsively as electric dipoles and thus do not form droplets.\textsuperscript{5} The macroscopic character of ring fragments, of 10 – 30 \( \mu \text{m} \) in size and containing about 10\(^4\) excitons each, as well as the observed spatial modulation of the exciton density which appears abruptly at a certain temperature, calls for an explanation involving a symmetry-breaking instability of a homogeneous state to a patterned state. Such behavior is reminiscent of the instability predicted by Alan Turing\textsuperscript{2} to occur in a reaction-diffusion system. Such an instability of a reaction-diffusion system, first predicted in 1952 by Alan Turing\textsuperscript{2}, is characterized by an intrinsic wavelength resulting from an interplay of reaction and diffusion processes. The Turing instability is known to occur in certain chemical reactions\textsuperscript{8,9,10}, as well as in biological systems.\textsuperscript{11}

In this work we propose a mechanism, based on the kinetics of exciton formation from optically excited electrons and holes, that can lead to a Turing instability in the exciton system. Interestingly, these kinetic effects build up and become especially strong in the regime near exciton quantum degeneracy, due to stimulated enhancement of the electron-hole binding rate. The transition to a state with a spatially modulated exciton density reveals itself in the spatial PL pattern, and presents a directly observable signature of degeneracy.

Although, in itself, an observation of an instability does not constitute unambiguous evidence for degeneracy, it may complement other manifestations discussed in the literature, such as changes in exciton recombination\textsuperscript{12} and scattering\textsuperscript{13} rates, in the PL spectrum\textsuperscript{14}, absorption\textsuperscript{15} and PL angular distribution\textsuperscript{16}. While linking the observed instability with degeneracy is premature, our main aim here is to present the Turing instability from a broader viewpoint, as a novel and quite general effect of quantum kinetics that can help to identify the regime of Bose-Einstein condensation (BEC).

Besides the kinetic mechanism, we consider another mechanism based on the assumption that the interaction between excitons is of an attractive character at large distances. The origin of such an attractive contribution may be in the force on excitons due to plasmons or phonons generated at exciton recombination. We describe a simple model for the plasmon wind effect, analyze the stability criterion, and consider the possibility that the observed periodic patterns are related with such an attraction.

II. KINETIC INSTABILITY

A. Model

Following Ref.\textsuperscript{3}, here we consider a transport theory formulated in terms of electron, hole, and exciton densi-
ties \( n_{e,h,x} \) obeying a system of coupled nonlinear diffusion equations:

\[
\begin{align*}
(e) & \quad \partial_t n_e = D_e \nabla^2 n_e - u n_e n_h + J_e \\
(h) & \quad \partial_t n_h = D_h \nabla^2 n_h - u n_e n_h + J_h \\
(x) & \quad \partial_t n_x = D_x \nabla^2 n_x + \omega n_e n_h - \gamma n_x.
\end{align*}
\] (1)

The nonlinear couplings account for exciton formation from free electron and hole binding. Here, no attempt has been made to describe the detailed and complicated density and temperature dependence of the physical parameters entering the model, nor to account for nonequilibrium exciton energy distribution and cooling due to phonon emission, \( \text{phon} \). Instead, we adopt a more phenomenological approach and assume that, as a result of stimulated electron-hole binding processes mediated by phonons. These processes enhance the binding rate by a factor \( 1 + N_{E}\text{ph} \), where \( N_{E}\text{ph} \) denotes the occupation of exciton states. In thermal equilibrium, and at low temperatures, one can ignore the reverse processes of exciton dissociation: The binding energy, carried away by phonons, is much larger than \( k_B T \).

For a degenerate exciton gas with \( N_{E}\text{ph} > 1 \), the dominant process involves scattering into the ground state and the stimulated enhancement factor is expressed as

\[
f = e^u, \quad u \equiv \frac{n_x}{n_0(T)}; \quad n_0(T) = \frac{2g m_e k_B^2 T}{\hbar^2}.
\] (2)

Here \( m_x \approx 0.21m_0 \) represents the exciton mass, and \( g \) denotes the degeneracy (in the indirect exciton system, the exchange interaction is extremely weak, and \( g = 4 \)). Equivalently, when reparameterized through its dependence on temperature, \( u \equiv T_0/T \) where \( T_0 = (\pi \hbar^2/2g m_e k_B) n_x \) is the degeneracy temperature. At \( T \sim T_0 \) (equivalently \( n_x \sim n_0 \)), there is a crossover from classical to quantum Bose-Einstein statistics, and the stimulated enhancement factor \( f \) increases sharply.

Qualitatively, the stimulated transition mechanism for hydrodynamic instability can be understood as follows: A local fluctuation in the exciton density leads to an increase in the stimulated electron-hole binding rate. The associated depletion of the local carrier concentration causes neighboring carriers to stream towards the point of fluctuation presenting a mechanism of positive feedback. The wavelength, determined by the most unstable harmonic of the density, characterizes the length scale of spatial modulation in the nonuniform state.

Before turning to the analysis of instability, it is useful to discuss intrinsic constraints on the dynamics due to electric charge and particle number conservation. These are obtained by considering the linear combinations \( (e) - (h), (e) + (h) + 2(x) \) of the transport equations. In both cases, the nonlinear term drops out and one obtains linear equations

\[
\begin{align*}
\tilde{L}_e n_e - \tilde{L}_h n_h &= J_e - J_h \\
\tilde{L}_e n_e + \tilde{L}_h n_h + 2\tilde{L}_x n_x + 2\gamma n_x &= J_e + J_h
\end{align*}
\] (3, 4)

with \( \tilde{L}_{e(h,x)} = \partial_t - D_{e(h,x)} \nabla^2 \). Note that, since the origin of the relations is rooted in conservation laws, they are robust and insensitive to the exact form of the electron-hole binding term.

### B. The two-dimensional problem

The simplest to analyze is instability of a steady state with spatially uniform carrier sources and density distribution, which can be achieved in the presence of spatially extended photoexcitation, described by constant \( J_e(r) = J_h(r) \equiv J \) in Eqs. (1). In a uniform steady state, ignoring the dependence of the radiative recombination rate \( \gamma \) on exciton density, we have

\[
\tilde{n}_x = J/\gamma, \quad \tilde{n}_{e,h} = (J/w(\tilde{n}_x))^{1/2}.
\] (5)

The stability of the system can be assessed by linearizing Eqs. (1) about the uniform solution (5) with a harmonic modulation \( \delta n_{e,h,x} \propto e^{i\ell \cdot r} \). Using (5) and writing \( \delta \hat{L}_e \delta n_e = \hat{L}_e \delta n_e = -\hat{L}_x \delta n_x - \gamma \delta n_x \), one can express \( \delta \hat{n}_{e,h,x} \) in terms of \( \delta n_e \) and obtain

\[
\hat{L}_e(\lambda, \k) + \gamma \tilde{n}_x \left( 1 + \frac{L_e(\lambda, \k)}{L_h(\lambda, \k)} \right) = \gamma u L_e(\lambda, \k) \hat{L}_x(\lambda, \k) + \gamma
\] (6)

where \( u = d \ln w/d \ln \tilde{n}_x \) is evaluated at the steady state, and \( \hat{L}_e(\lambda, \k) = \lambda + D_{e(h,x)} k^2 \). Solving Eq. (6), one obtains the growth rate dispersion \( \lambda(\k) \). The stability criterion \( \text{Re} \lambda < 0 \) is first violated at the parameter values such that the sign of \( \lambda(\k) \) reverses for some \( \k \). The condition \( \lambda(\k) = 0 \) gives

\[
(\k^2 \ell_x)^2 + r = \frac{u \ell_x^2}{1 + (\k^2 \ell_x^2)},
\] (7)

where \( \ell_x = \sqrt{D_x/\gamma} \) denotes the exciton diffusion length and \( r = D_h(D_e^{-1} + D_h^{-1})\tilde{n}_x/\tilde{n}_e \). Eq. (7) has solutions if

\[
u \equiv \frac{d \ln w}{d \ln \tilde{n}_x} \geq \frac{u_c}{1 + u_c^{1/2}}
\] (8)

as illustrated in Fig. 1. At \( u = u_c \), we obtain the most unstable wavenumber \( \k_c = r^{1/4}/\ell_x \) selected by competition of the stimulated binding and diffusion processes.

The binding rate \( u(\tilde{n}_x) \) builds up near degeneracy due to the growth of stimulated processes, leading to instability at temperatures approaching \( T_{\text{BEC}} \).
interacting Bose gas, the transport coefficients, and thus the constant \( r \), are practically insensitive to the degree of exciton degeneracy at \( T > T_{\text{BEC}} \).

On symmetry grounds, since the triangular lattice pattern is stabilized by quadratic terms, the mean field analysis predicts that the transition to the modulated state in this case is abrupt, of a type I kind. Indeed, the triangular lattice geometry arises in various 2d pattern selection problems, from Bénard convection cells\(^{18} \) to the mixed state of type II superconductors\(^{19} \).

### C. Instability of a 1D electron-hole interface

The application of these ideas to the 1d modulation seen in exciton rings\(^{15} \) was explored recently in Ref.\(^{20} \). The analysis starts with first determining the profile of the uniform distribution. The rings mark the interface between regions populated by electrons and holes at which they bind to form excitons. The steady state is maintained by a constant flux of carriers arriving at the interface. The parameter regime which is both relevant and simple to analyze is that of long exciton lifetime \( \gamma^{-1} \) where the diffusion length \( \ell_d \) exceeds the range of the electron and hole profile overlap. In this case, approximating the source of excitons by a straight line \( c\delta(x) \), where \( c \) is the total carrier flux and \( x \) is the coordinate normal to the interface, the exciton density profile is given by \( (c\ell_d/2D_d)e^{-|x|/\ell_d} \). Accordingly, one can seek the electron and hole profile treating \( w(n_e) \) as constant and restoring its dependence on \( n_h \) later when turning to the instability. The profiles can be inferred from two coupled nonlinear diffusion equations

\[
\partial_t n_e(x) - D_e \partial_x^2 n_e(x) = -w n_e n_h, \tag{10}
\]

with the boundary condition: \( D_e \partial_x n_e \big|_{x=\pm \infty} = \pm c \theta (\pm x) \). From Eq.\(^{8} \) one obtains \( D_e n_e - D_h n_h = cx \) which allows the elimination of \( n_h \). Applying the rescaling \( n_{e(h)}(x) = \ell g_{e(h)}(x) / D_e(h) \), where \( \ell = (D_e D_h / wc)^{1/3} \), one obtains

\[
\partial_x^2 g_e = g_h (g_e - \bar{g}), \hspace{1cm} \bar{g} \equiv x / \ell. \tag{11}
\]

From the rescaling one can infer that the electron and hole profiles overlap in a range of width \( \ell \sim e^{-1/3} \) while

\[
g_e(|x| \gg \ell) = \bar{g} \theta (\bar{x}) + O \left( |\bar{x}|^{-1/4} e^{-2 |\bar{x}|^{3/2}/3} \right),
\]

The solution of this problem, obtained numerically, is displayed in Fig.\(^{4} \).

Although Eqs.\(^{10} \) are nonlinear, their diffusive character does not straightforwardly admit a spatial instability: A fluctuation in the position of the interface initiates an increased electron-hole flux which, in time, restores the uniform distribution. However, if one restores the dependence of the binding rate on exciton density, the same mechanism of positive feedback which characterized the instability in the uniform system becomes active.

To explore the instability, one may again expand linearly in fluctuations around the spatially uniform solution, \( g_e(x, y) = g_0(x) + \delta g_e(x) e^{iy} \) (similarly \( g_h \) and \( g_0 \)), where \( y \) is the coordinate along the interface and \( g_0(x) \)

\[
\begin{align*}
k^{(j)} &= k_s (\cos(\frac{2\pi}{3} j + \theta), \sin(\frac{2\pi}{3} j + \theta)), \tag{9}
\end{align*}
\]

\( j = 1, 2, 3 \), with the phase parameter \( \theta \) describing the degeneracy with respect to 2d rotations. This leads to a density distribution \( \delta n \propto \sum_j e^{ik \cdot r} \) with maxima arranged in a triangular lattice.

FIG. 1: Graphical solution of Eq.\(^{4} \) that selects the most unstable wavelength. Inset: The 3-fold symmetric star of wavevectors describing the modulation near the instability threshold and the corresponding triangular pattern of exciton density variation.

To what extent are these results insensitive to the origin of the nonlinearity in the binding rate? If enhanced by intraband Auger processes, which transfer the binding energy released in exciton formation to other excitons, one expects the binding rate \( w \) to scale linearly with low exciton density, viz. \( w(n_e) = w_0 (1 + n_e/n_0) \), where \( n_0 \) denotes some constant involving a ratio of the two-body and three-body cross-section of the electron and hole in the presence of excitons. Crucially, in this case, the left hand side of Eq.\(^{4} \) is bounded by unity, while the right hand side is in excess. Therefore, at least over the parameter range considered here, one can infer that a simple linear scaling of the binding rate with density does not lead to instability. Indeed, the instability may be used to discriminate against certain mechanisms in the kinetics of exciton formation.

Turning to the discussion of the spatial pattern resulting from the instability, we note that the wavevector selection determines its modulus, but not direction. At threshold \( u = u_c \) all modes with \( |k| = k_s \) become unstable simultaneously. The resulting 2d density distribution can be found by considering the effect of mixing different harmonics due to higher order terms in \( \delta n_{e,x} \) about the uniform state. Since these equations contain quadratic terms, the favored combination of harmonics is a 3-fold symmetric star

\[
k^{(j)} = k_s (\cos(\frac{2\pi}{3} j + \theta), \sin(\frac{2\pi}{3} j + \theta)), \tag{9}
\]
FIG. 2: Numerical solution of the (normalized) bound state wavefunction $\psi(x)$ representing fluctuation in the electron density, together with the effective potential $U_{\text{eff}} = \bar{g}_e + \bar{g}_h - a_0 \bar{g}_e \bar{g}_h$ of the Schrödinger-like equation (12).

denotes the uniform profile obtained from Eq. (11). With $\ell_x \gg \ell$, the exciton density remains roughly uniform over the electron-hole interface. Denoting this value by $\bar{n}_x(0)$, in the vicinity of the interface, one may again develop the linear expansion $w[n_x] \approx w[\bar{n}_x(0)] (1 + u \delta n_x / \bar{n}_x(0))$ where, as before, $u = d \ln w / d \ln n_x$. Noting that Eq. (9) enforces the steady state condition $\delta g_e = \delta \bar{g}_h$, a linearization of Eqs. (11) obtains the Schrödinger-like equation

$$[-\delta^2_x + (\ell k)^2 + \bar{g}_e + \bar{g}_h] \delta g_e + \frac{u}{\bar{g}_e(0)} \bar{g}_e \delta g_x = 0,$$  

(12)

where the particle density Fourier components $\delta g_{z(x),h}(q) = \int e^{-iqx} \delta g_{z(x),h}(x) dx$ obey the relation

$$\delta g_z(q) = -\delta g_e(q) \left(1 - \frac{1}{Q^2 + (q \ell_x)^2}\right),$$  

(13)

with $Q^2 \equiv (k \ell_x)^2 + 1$. Now, since the product $\bar{g}_e \bar{g}_h$ is strongly peaked around the interface, the typical contribution from the last term in (12) arises from Fourier elements $q \ell \sim 1$. Then, with $\ell_x \gg \ell$, the second contribution to $\delta g_z(q)$ can be treated as a small perturbation on the first and, to leading order, neglected, i.e. $\delta g_x \approx -\delta g_e$. In this approximation, the most unstable mode occurs at $k = 0$. Qualitatively, an increase in $u$ will trigger an instability of the $k = 0$ mode at a critical value $u_c$ when the linear equation first admits a non-zero solution for $\delta g_e$. At the critical point, the corresponding fluctuation in the electron density then acquires the profile of the (normalized) zero energy eigenstate $\psi(x)$. Numerically one finds that the critical point for the instability occurs when $u_c / \bar{g}_e(0) = (2/\ell_x) u_c \equiv a_0 \approx 6.516$, while the corresponding solution $\psi(x)$ is shown in Fig. 2.

While the approximation above identifies an instability, a perturbative analysis of the $k$-dependent corrections implied by (13) reveals that the most unstable mode is spatially modulated. To the leading order of perturbation theory, an estimate of the shift of $u_c$ obtains

$$\frac{\delta u_c}{u_c} = \frac{\ell}{a_0 a_1} \left(\frac{(k \ell)^2}{Q^2} + \frac{a_0 a_2(Q)}{\ell_x^2 Q}\right),$$

where $a_1 = \int_{-\infty}^{\infty} dx \bar{g}_e(x) \bar{g}_h(x) \psi^2(x) \approx 0.254 \ell$, and

$$a_2(Q) = \frac{1}{2} \int_{-\infty}^{\infty} dxdx' \bar{g}_e(x) \bar{g}_h(x) \psi(x)e^{-Q|x-x'|/\ell_x} \psi(x').$$

With $\ell_x \gg \ell$, it will follow that $Q \ell \ll \ell_x$, and the latter takes the constant value $a_2 \approx 0.4612$ independent of $k$. Finally, minimizing $\delta u_c$ with respect to $k$, one finds that the instability occurs with a wavevector

$$k_x \ell_x \simeq (a_0 a_2 \ell_x / \ell^2)^{1/3}$$  

(14)

implying a shift of $u_c$ by $\delta u_c / u_c \sim (\ell / \ell_x)^{4/3}$. As a result, one can infer that the spatial modulation wavelength $\lambda_c \sim \ell_x^{2/3} \ell_x^{4/3}$ is typically larger than the electron-hole overlap $\ell$, but smaller than $\ell_x$. Finally, an expansion of the nonlinear equations to higher order in fluctuations shows that, below the transition (i.e. for $u > u_c$), the amplitude of the Fourier harmonic $k_c$ grows as $(u - u_c)^{1/2}$.

Once the instability is strongly developed (or when $\ell_x \ll \ell$) the above linear stability analysis fails to produce quantitatively accurate results. Instead, one can perform numerical search for a steady state of the full nonlinear problem. Such an analysis, reported in Ref. 20, indeed demonstrates a modulational instability developing in a soft way, with a square root dependence of the modulation amplitude on the deviation from critical point (see Fig. 3 in Ref. 20). The characteristic instability wavelength obtained numerically is consistent with that predicted by Eq. (14).

III. LONG-RANGE ATTRACTION MECHANISM

Here we discuss the hydrodynamical forces between excitons that arise due to polarization charge relaxation triggered by exciton recombination. For the indirect excitons in 2d coupled quantum wells we find a long-range attraction that falls off very slowly as a function of distance. The strength of this interaction is controlled by the specifics of the model, such as carrier mobility in the wells and adjacent doped regions. Even if week, this interaction may overcome the repulsive electric dipole interaction at a certain length scale, thereby making the uniform state unstable with respect to density modulation formation.

The idea of the ‘plasmon wind’ effect is as follows. Each exciton polarizes charges in the doped regions above and adjacent doped regions. Even if week, this interaction at a certain length scale, thereby making the uniform state unstable with respect to density modulation formation.
spreading in 2d. The spreading is controlled by charge continuity and electro/hydrodynamics of the doped region. Thus an exciton cloud in which excitons disappear at a constant rate will represent a steady source of 2d plasmons spreading radially away from it. The polarization charge in this situation is described by a 2d Poisson equation with a source proportional to exciton density and, therefore, the force on other excitons far away will be of a long-range 1/r form. The sign of this force is attraction, since the polarization charge dipole is opposite to that of exciton dipole.

The significance of this force is that, while being weak compared to the direct exciton repulsion, it falls off slowly and comes to dominate the physics at sufficiently large distances. Being attractive, it favors exciton clumping on large length scales. The situation can be understood by analogy with gravitation forces which are very weak, but long range, and thus are irrelevant on small length scales, while controlling the structures forming on the astronomical length scales.

To estimate the effect of dynamical screening and plasmon wind on the interaction between indirect excitons, we consider a simplified model. The excitons will be treated as electric dipoles positioned in the xy plane and oriented along the z axis. The doped regions will be described as 2d conducting planes with conductivities $\sigma_{1,2}$ at a distance $a_{1,2}$ from the $xy$ plane and parallel to it.

The dynamical screening problem for indirect excitons, described by dipoles $d$ with density $n_x(r, t)$, is formulated in terms of the charge and current densities in the doped regions:

$$\partial_t \rho_{1,2} + \nabla j_{1,2} = s_{1,2}, \quad j_{1,2} = -\sigma_{1,2} \nabla \varphi_{1,2}$$  \hspace{1cm} (15)

where $s_1 = -\lambda(\rho_1 - \rho_2)/2$, $s_2 = -s_1$ describes leakage through the structure. The potentials $\varphi_{1,2}$ are related to the charge density in the two planes as

$$\varphi_i = \sum_j U_{ij}(k) \rho_j + \varphi^{(i)}_i$$  \hspace{1cm} (16)

with $\varphi^{(i)}_i$ the exciton dipole potential in the planes $z = a_1, -a_2$. Using the Coulomb interaction 2d Fourier components

$$\frac{2\pi}{k} e^{-ka} = \int \frac{e^{-ikr}}{(r^2 + a^2)^{1/2}} d^2r$$  \hspace{1cm} (17)

we obtain

$$U_{1\to j}(k) = 2\pi/k, \quad U_{i\to j}(k) = (2\pi/k)e^{-k(a_1 + a_2)}$$  \hspace{1cm} (18)

Similarly, $\varphi^{(0)}_i = 2\pi d e^{-ka_1}$, $\varphi^{(2)}_i = -2\pi d e^{-ka_2}$ is found for the potential in the planes $z = a_1, -a_2$ of a dipole $d$ situated at the origin.

To determine the force on a remote exciton, $f = -\nabla U_{\text{eff}}$, $U_{\text{eff}} = -dE_z$, we compute the electric field $z$ component $E_z(k) = -2\pi(e^{-ka_1} \rho_1 - e^{-ka_2} \rho_2)$ using $\rho_{1,2}$ obtained from Eqs. (15, 16). In the simplest symmetric situation, when $\sigma_{1,2} = \sigma$ and $a_{1,2} = a$, we obtain

$$U_{\text{eff}}(k, \omega) = 2\pi d^2 \left( \frac{2k^3 U_{12}}{2k^2 U_{11} + (\lambda - i\omega)/\sigma} + k \right) n(k, \omega)$$  \hspace{1cm} (19)

where $U_{11} = U_{11} - U_{12}$ and $n(k, \omega)$ is exciton density. The second term corresponds to the direct exciton dipole interaction, while the first term describes the effect of polarization charge.

In order to extract the static interaction, as well as the part that depends on the exciton recombination, we expand Eq. (19) to the first order in $\omega$ and obtain

$$U_{\text{eff}}(k, \omega) = (D_1(k) + i\omega D_2(k)) n_x(k, \omega)$$  \hspace{1cm} (20)

with

$$D_1(k) = 2\pi d^2 \left( \frac{2k^3 U_{12}}{2k^2 U_{11} + (\lambda + i\omega)/\sigma} + k \right)$$  \hspace{1cm} (21)

$$D_2(k) = 2\pi d^2 \frac{2k^3 U_{12}}{\sigma(2k^2 U_{11} - U_{12} + \lambda/\sigma)^2}$$  \hspace{1cm} (22)

At large distances, taking the small $k$ limit of the expressions (15), $l^{-1} \ll k \ll a^{-1}$, we obtain

$$D_2(k) = \alpha k^2, \quad D_1(k) = \beta$$  \hspace{1cm} (23)

where $\alpha = d^2/(8\sigma a^2)$, $\beta = \pi d^2/a$, and $l = (8\pi a/\lambda)^{1/2}$ is the characteristic length of plasmon decay due to vertical leakage across the quantum wells. The term $D_2$, which has a long-range form $-\frac{\alpha}{\lambda} \ln(|r - r'|/\ell)$ in position representation, describes the plasmon wind effect. The term $D_1$ describes the static inter-exciton interaction screened by charges in the doped regions. Altogether, we obtain an expression

$$U_{\text{eff}}(r) = \frac{\pi d^2}{a} n_x(r) - \frac{\alpha \gamma}{2\pi} \ln \left( \frac{|r - r'|}{\ell} \right) n_x(r') d^2r'$$  \hspace{1cm} (24)

valid at the length scales from $a$ to $\ell$. In passing from Eq. (21) to Eq. (24), we replaced $\dot{n}_x$ by $-\gamma n_x$, with $\gamma$ the recombination rate. (We note that a similar plasmon shakeup accompanies exciton formation by electron and hole binding, leading to qualitatively similar inter-exciton attraction.)

To see how the long-range attractive part of $U_{\text{eff}}$ generates instability, we consider exciton dynamics

$$\partial_t n_x + \nabla j_x = -\nabla U_{\text{eff}}$$  \hspace{1cm} (25)

$$j_x = -D \nabla n_x - \mu_x n_x \nabla U_{\text{eff}}$$  \hspace{1cm} (26)

where $\mu_x$ is exciton mobility. In a simple, although somewhat artificial situation when the excitons are generated uniformly over the area, the homogeneous state is characterized by constant density $\bar{n}_x = J/\gamma$. Linearizing Eq. (26) about this state, with $\delta n_x \propto e^{\omega t} e^{i k \cdot r}$, and setting $\omega \leq 0$, we obtain the instability criterion:

$$(D + \mu_x \bar{n}_x \beta) k^2 + \gamma \leq \frac{\alpha \gamma \mu_x \bar{n}_x k^2}{k^2 + \ell^{-2}}$$  \hspace{1cm} (27)
This condition is simplest to analyze in the absence of leakage, $\ell = \infty$. Then for instability one must have
\[ \alpha \mu_x n_x \geq 1 \]  
(28)
Replacing conductivity in $\alpha = d^2/(8\sigma a^2)$ by $e^2 \mu d$, where $\mu_d$ is the mobility and $n_d$ is the carrier density in the doped region, we obtain the criterion
\[ d^2 \mu_x n_x \geq 8e^2 a^2 \mu_d n_d \]  
(29)
In a coupled quantum well structure with well separation $w$ the dipole moment is given by $d^2 = e^2 w^2/\epsilon_{GaAs}$ where $\epsilon_{GaAs}$ is the dielectric constant.

The criterion (29) shows that the instability can be reached by increasing exciton number $n_x$ over a threshold value $n_{x,c} = 8\epsilon_{GaAs}(a/w)^2(\mu_d/\mu_x)n_d$. In the geometry (10), the ratio $a/w \approx 10$. With the factor $8\epsilon_{GaAs} \approx 10^2$, even for the in-plane mobility $\mu_x$ orders of magnitude higher than the mobility in the doped region, the critical density $n_{x,c}$ is quite high.

While this difficulty might be overcome by employing a stronger long-range force, e.g. due to phonons, a more serious issue with this mechanism arises from the consideration of temperature dependence. In the experiment (14), the instability occurs at temperature below few Kelvin. To explain this, the above model would have to account for strong temperature dependence of transport properties such as that taking place in the low conductivity regime of electron hopping transport. In contrast, the kinetic effects discussed above lead in a natural way to a low temperature phase transition which occurs as a precursor of exciton degeneracy without any specific assumptions about the nature of transport.

IV. CONCLUSION

In summary, we considered a scenario when the realization of quantum degeneracy in a cold electron/hole-exciton system is signalled by the development of a spatial density modulation. Although our discussion is motivated by the photoexcited CQW system in which electrons and holes are spatially separated, the mechanism is quite general applying also to geometries where the electron and hole sources $J_{e,h}$ have a spatially independent profile. The instability mechanism appears to depend sensitively on there being a strongly nonlinear dependence of the electron-hole binding rate on the exciton density pointing to the importance of stimulated scattering. Although on a qualitative level this model is consistent with experimental observations of a low temperature transition, before considering it as the only explanation more experiments are needed in order to provide a quantitative assessment of the instability mechanism.

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