Electron mass renormalization and absorption of hard photons

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Abstract Mass renormalization of the electron in configurations such as metallic hydride surfaces due to electromagnetic field fluctuations leads to mass enhancement of the electron, which is known as the \textit{heavy electron}. The effective mass renormalization has substantial consequences in the theory of electromagnetic field interaction with matter (QED). One of the fascinating effects appears when an external photon interacts with the heavy electron. In this case, the wavelength of the scattered photon from the electron increases and the hard photon turns into a soft photon. In this paper, we present a novel mechanism to show how the heavy electron results in hard photon absorption.

1 Introduction

According to the perturbative field theories, one can expand n-point functions around the coupling constant in low energies. At high orders consisting of loop Feynman diagrams, the physical quantities achieve quantum radiative corrections. So, to calculate any physical quantity, such as mass, two parts contribute, namely the \textit{bare} value that appears in the Lagrangian and quantum corrections $\Delta m$ as

$$m_{\text{physical}} = m_{\text{bare}} + \Delta m.$$  \hspace{1cm} (1)

Because of the undetermined momentum in the loop Feynman diagrams, $m_{\text{physical}}$ in the above equation has no definite value and diverges, called \textit{ultraviolet divergences}. The renormalization program as an instruction controls all infinity in the whole of the theory. The renormalization program not only leads to the absorption of divergences of the amplitudes but also modifies the results of the calculation of physically observable quantities such as the mass and the electric charge.

The electron as a fundamental particle does not exist alone in real processes. Rather, it is coupled with a gauge field (the photon), known as QED theory. Beyond the leading order of QED, as mentioned earlier, electron mass and charge are modified due to quantum corrections. In this manner, in surveying any physical processes, the electrons should be considered as the \textit{renormalized electron}, not the bare one [1,2]. Renormalization procedures for quantities such as mass and electric charge have been very well known for many years. Mass renormalization has vital consequences in physical processes (see for instance [3,4]). In condensed matter physics, mass renormalization is usually interpreted as mass enhancement of the particle; for example, the renormalized electron is considered a \textit{heavy electron}.

The most serious problem with the renormalization procedure is the presence of boundary conditions on the quantum fields. The presence of the boundaries imposes crucial constraints on the propagators and for n-point functions. Here, propagators do not have the standard form (wave functions are not plane waves) and depend completely on the nontrivial properties that break translational symmetry. Therefore, all physical quantities will be modified according to symmetry breaking. Altogether, the renormalization program with nontrivial boundary conditions is complicated. However, attempts have been made in [5–8] for scalar field theory and in [9] for QED theory.

As mentioned earlier, the modification of the electron propagators due to boundary conditions (for example, in a lattice) causes that electron to become heavy. The first image of heavy electrons was captured in [10] using spectroscopic imaging scanning tunneling microscopy (SI-STM) by measuring the wavelength of electrons on the surface of the material. It was shown that under extreme conditions, electrons take large mass. In another experiment, in [11], by visualizing the electron wave in the crystal with STM, it has been shown that moving electrons in some lattices can make them more massive than the free one.
The effective mass renormalization is one of the most interesting areas and very well known in physics; for instance, superconductivity in heavy fermion material [12–16], Hall effect evolution in a heavy electron critical point [17,18], heavy electron systems in Fermi liquid theory [19], non-Fermi liquid behavior in heavy fermion metal [20,21], and magnetic properties of the heavy fermion [22,23]. Recently, some notable works have emerged in this area: nonlocal Kondo effect and quantum critical phase in heavy fermion metals [24], heavy fermion thin films [25], and heavy fermions in Kondo lattice models [26]. However, in [27], the authors have also used the concept of heavy electron to explain the theory of low-energy nuclear reaction (LENR) [28–34]. According to the LENR, the electrons in the metallic hydride surfaces are subjected to the localized condensed electromagnetic fields. In these conditions, the proton can capture the renormalized electrons, where very low-momentum neutrons (along with neutrinos) are produced to induce chains of nuclear reactions in neighboring condensed matter [35,36]. Usually, a proton can capture a charged lepton to produce a neutron and a neutrino.

\[ l^- + p^+ \rightarrow n + \nu_l \]  

(2)

For the above reaction, it is necessary that the following condition on the mass of the lepton is preserved (see please [37,38]),

\[ M_l > 2.53M_e^0, \]  

(3)

where \( M_e^0 \) is the free electron mass. Equation (2) holds for the muon, but not for a free electron. In [27], the authors explain how the modification of the electron mass, yields such chain reactions. Also, the heavy electron may absorb gamma rays from nuclear reactions due to (2).

According to the crucial application of the heavy electron, especially in shielding gamma rays, it is substantial to briefly discuss the procedure for the electron mass renormalization and heavy electron. We examine the renormalization of the mass as it affects the kinematics of the electron. In Sect. 3, we investigate the photon scattering from the electron in the presence of an electromagnetic field to prove the absorption of hard photons. It is Compton scattering with the difference that the bare electron is replaced with a renormalized electron. We show that the cross-section of modified Compton scattering is truly different from the usual one. Our results reduce to the standard Compton scattering when the renormalized electron is considered a free electron. In Sect. 4, we present our results.

2 Mass renormalization of the electron: a brief review

In this section, we briefly investigate the renormalization of the electron mass. Our approach is in the framework of renormalized perturbation theory. The Lagrangian for QED is

\[ L_{\text{QED}} = -\frac{1}{4} (F^{\mu\nu})^2 + \bar{\Psi} (i\gamma^\mu - m_0) \Psi - e_0 \bar{\Psi} \gamma_\mu \Psi A^\mu. \]  

(4)

where \( m_0 \) and \( e_0 \) are the bare mass and the bare electric charge, respectively. \( \Psi(x) \) and \( A^\mu(x) \) are fermion and photon fields, respectively, and can be written as

\[ \Psi(x) = \int \frac{d^3p}{(2\pi)^3} \sum_{s=1,2} \frac{1}{\sqrt{2E_p}} \left[ c^\dagger_p \psi^s(x) + d^\dagger_p \phi^s(x) \right] \]  

(5)

\[ A_\mu(x) = \int \frac{d^3p}{(2\pi)^3} \sum_{s=0}^3 \frac{1}{\sqrt{2E_p}} \left[ a^\dagger_p A^s_\mu(x) + a^\dagger_p \bar{A}^s_\mu(x) \right]. \]  

(6)

where, in the first line, \( c^\dagger_p (c_p) \) and \( d^\dagger_p (d_p) \) create (annihilate) a fermion and anti-fermion with momentum \( p \) and spin direction \( s \), respectively. Here, \( \psi^s(x) \) and \( \phi^s(x) \) are the particle and anti-particle solutions of the Dirac equation, respectively. In the second line, \( a^\dagger_p A^s_\mu(x) \) (\( a_p \bar{A}^s_\mu(x) \)) creates (annihilates) a photon with momentum \( p \) and polarization \( e^s_\mu(p) \), and \( \bar{A}^s_\mu(x) \) are the momentum-space solution of the equation \( \partial^\mu A_\mu = 0 \).

According to the above, the electron propagator (Green’s function \( G(\not{p}) \)) takes some corrections and can be written in perturbative series of QED as [41]:

\[ iG(\not{p}) = + + \ldots \]  

(7)

\[ iG(\not{p}) = \frac{i}{\not{p} - m_0} + \frac{i}{\not{p} - m_0} \left( i\Sigma_2(\not{p}) \right) \frac{i}{\not{p} - m_0} + \frac{i}{\not{p} - m_0} \left( i\Sigma_2(\not{p}) \right) \frac{i}{\not{p} - m_0} + \ldots \]  

(8)
where, in Feynman gauge,
\[
i \Sigma_2(\not{p}) = (-ie)^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu \gamma^\nu i(k + m_0) \gamma^\nu \frac{-i}{k^2 - m_0^2 + i\epsilon} \frac{1}{(p-k)^2 + i\epsilon},
\]
\tag{9}

is known as electron self energy. Equation (9) is a part of some geometric series. One can rewrite that
\[
i G(\not{p}) = \frac{i}{\not{p} - m_0 - \Sigma_2(\not{p})}.
\]
\tag{10}

It is obvious that the pole of the propagator is shifted away from \( m_0 \) by \( \Sigma_2(\not{p}) \). In fact, The \( \Sigma_2(\not{p}) \) as a radiative correction modifies not only the propagator but also the bare mass of the electron and results in Renormalized mass \( m_R \):
\[
m_R = m_0 + \Sigma_2(\not{p}).
\]
\tag{11}

Up to now, the renormalization program has been done in momentum space. However, if the translational invariance of the system breaks strongly, then the momentum is no longer a good quantum number. Renormalization in configuration space can be applied for such a situation, such as in problems with a nontrivial BC or a nonzero background that cannot be treated as small perturbations. For example, the kink as a constant background in 1+1 dimensions breaks the translational invariance, or in the Casimir effect, we have nontrivial BC on the walls. Another real example of QED is the Lamb shift in which the Coulomb potential in the hydrogen atom breaks the translational symmetry. We present a systematic treatment, up to order \( \alpha \), for the renormalization of quantum electrodynamics in real space, especially for electron mass.

By replacing \( \Psi(x) = \sqrt{z_2} \Psi_r(x) \) and \( A^\mu(x) = \sqrt{z_3} A_r^\mu(x) \) in (4), we have
\[
L_{QED} = -\frac{1}{4} z_3 (F_{\mu\nu}^r)^2 + z_2 \bar{\Psi}_r (i \not{\partial} - m_0) \Psi_r
- \epsilon_0 z_2 \sqrt{z_3} \bar{\Psi}_r \gamma_\mu \gamma_\nu \bar{\Psi}_r \gamma_\mu, \tag{12}
\]

where \( z_2 \) and \( z_3 \) are the field-strength renormalization for \( \Psi \) and \( A^\mu \), respectively. We define a scaling factor \( z_1 \) as \( \epsilon z_1 = \epsilon_0 z_2 z_3 \sqrt{z_3} \) and split each term of the Lagrangian into two pieces
\[
L_{QED} = -\frac{1}{4} \left( F_{\mu\nu}^r \right)^2 + \bar{\Psi}_r (i \not{\partial} - m) \Psi_r - \epsilon \bar{\Psi}_r \gamma_\mu \Psi_r A_r^\mu
- \frac{1}{4} z_3 \left( F_{\mu\nu}^r \right)^2 + i \delta_2 \bar{\Psi}_r \not{\partial} \Psi_r - (\delta_m + m \delta_2) \bar{\Psi}_r \Psi_r
- \epsilon \delta_1 \bar{\Psi}_r \gamma_\mu \gamma_\nu \bar{\Psi}_r A_r^\mu, \tag{13}
\]

with \( z_3 = 1 + \delta_3, z_2 = 1 + \delta_2, m_0 = m + \delta_m \) and \( z_1 = 1 + \delta_1 \), where \( \delta_1, \delta_2, \delta_3 \) and \( \delta_m \) are counterterms. In the calculations of the quantum radiative corrections physical quantity, say electron mass, one usually encounters counterterms and the electron mass becomes renormalized. Here, \( m \) and \( \epsilon \) are the physical mass and charge of the electron (renormalized quantities) measured at large distances. Now, the Feynman rules from the above Lagrangian for fermion fields are
\[
\begin{array}{r}
\text{1PI} & \rightarrow & -i \Sigma(\not{p})
\end{array}
\tag{16}
\]

Here, “1PI” denotes a one-particle irreducible diagram, which is the sum of any diagram that cannot be split in two by removing a single line. To fix the pole of the fermion propagator at the physical mass \( m \) we need two renormalization conditions:
\[
\begin{align}
\Sigma(\not{p} = m) &= 0 \quad \tag{17} \\
\frac{d\Sigma(\not{p})}{d\not{p} \big|_{\not{p}=m}} &= 0. \quad \tag{18}
\end{align}
\]

Now, using the dimensional regularization we can compute \(-i \Sigma(\not{p})\). Applying the above renormalization conditions, up to leading order in \( \alpha \), the divergent parts of the counterterms is derived as
\[
\begin{align}
\delta_2 &\sim -\frac{e^2}{8\pi^2\epsilon}, \quad \tag{19} \\
\delta_m &\sim -\frac{3me^2}{8\pi^2\epsilon}, \quad \tag{20}
\end{align}
\]

where \( d = 4 - \epsilon \) is the spacetime dimension so that we should take the limit \( \epsilon \to 0 \). These counterterms can remove all UV divergences of the QED theory for a fermion propagator in free space.

According to the Lagrangian (13), the perturbation expansion of the full electron propagator up to order \( \alpha \) is
\[
\begin{array}{r}
-\delta m = \begin{array}{c}
\text{1PI} \\
\rightarrow \\
\text{1PI}
\end{array}
\end{array}
\tag{21}
\]

We choose our renormalization condition in such a way that the pole of the first term of the right-hand side (RHS) gives
the physical mass \(m\) at \(x = x_0\). It requires that the sum of remaining diagrams, which we call \(-i\tilde{\Sigma}(x)\), vanishes at this point, namely

\[
-i\tilde{\Sigma}(x)_{x=x_0} = \left(\begin{array}{c}
\int d^3y \frac{\gamma_\mu(y)}{x_0} \left[\begin{array}{c}
\psi(x) + \bar{\psi}(x) \\
\times \left[\begin{array}{c}
\delta_2(x) \psi + \bar{\psi}(x) \\
\times \left[\begin{array}{c}
\delta_2(x) \phi - im \delta_2(x) - i\delta_m(x) \right] \psi(x) \\
\right]
\end{array}
\right]
\end{array}
\right)
\right)_{x=x_0} = 0,
\]

and \(d\left[-i\tilde{\Sigma}(x)\right]_{x=x_0} = 0.\) (22)

We can write \(-i\tilde{\Sigma}\) to order \(\alpha\) as

\[
-i\tilde{\Sigma}(x) = \int d^3y \frac{\gamma_\mu(y)}{x_0} \left[\begin{array}{c}
\delta_2(x) \psi + \bar{\psi}(x) \\
\times \left[\begin{array}{c}
\delta_2(x) \phi - im \delta_2(x) - i\delta_m(x) \right] \psi(x) \\
\right]
\end{array}
\right)_{x=x_0}
\]

Thus, the first condition in Eq. (22) yields

\[
-i\tilde{\Sigma}(x_0) = \left\{\begin{array}{c}
\int d^3y \frac{\gamma_\mu(y)}{x_0} \left[\begin{array}{c}
\delta_2(x) \psi + \bar{\psi}(x) \\
\times \left[\begin{array}{c}
\delta_2(x) \phi - im \delta_2(x) - i\delta_m(x) \right] \psi(x) \\
\right]
\end{array}
\right)
\right\}_{x=x_0}
\]

\[
= 0,
\]

where \(-i\Sigma_2\) is \(O(\alpha)\) electron self-energy diagram. Now, using the Dirac equation \((i\tilde{\phi} - m)\psi = 0\), up to order \(\alpha\) we obtain

\[
\delta_m = \frac{-1}{\psi(x_0)\bar{\psi}(x_0)} \int d^3y \frac{\gamma_\mu(y)}{x_0} \left[\begin{array}{c}
\left.\psi(x) + \bar{\psi}(x) \right|_{x=x_0}
\end{array}
\right).
\]

(26)

\(\delta_m\) corresponds to \(\Sigma_2(\tilde{\phi})\) up to order \(\alpha\), but for the case that translational symmetry may be broken. It is obvious that \(\delta_m\) depends on the boundary conditions of the fields. Thus, quantum correction of the electron mass depends on nontrivial properties that break translational symmetry. It is customary in condensed matter physics that the impact of \(\delta_m(x)\) on the mass would have been considered a heavy electron (see [42–47] for heavy electron theory). For instance, in interaction \(p + e \rightarrow n + v\), ignoring \(v\) mass, we have

\[
\Delta m = (M_n - M_p - M_e) c^2 \approx 0.78 \text{ MeV}.
\]

(27)

\[
m_e^* = m_e + 0.78 \text{ MeV} = 2.53m_e
\]

(28)

This value is especially important in LENR interactions in the hydrogen-metal environment that lead to the production of ultra-low energy neutrons and nuclear transmutation. In [27], it is stressed that the minimum of the mass growth parameter \(\beta = \frac{2m_e}{m_0}\) for reaction \(p + e^+ \rightarrow n + v\) is \(\beta = 2.53\); \(e^+\) denotes heavy electron. In the next section, we will show how heavy electron via Compton scattering leads to absorption of the gamma rays.

3 Absorption of gamma rays: revisited Compton scattering

In this section, we survey the photon scattering of the electron surrounded by electromagnetic fields produced by the oscillating protons near them. In this case, the electron does not obey the Dirac equation, while it is considered as a renormalized electron that we have referred to. We will show that these renormalized electrons can absorb hard photons produced in nuclear reactions, changing them to the soft ones. Compton scattering is the scattering of a photon after an interaction with a charged particle (preferably an electron here).

However, we model such photon scattering from the surface renormalized electron in a metal, noting that X-ray photon energy (of order 17 keV) is much higher than the binding energy in the atom, which seems logical.

Two Feynman diagrams contribute to the M-matrix for Compton scattering as follows (for more details, see [48]):

\[
iM = \gamma + \gamma
\]

(29)

\[
iM = -ie^2 \frac{\epsilon_\mu(q')}{\epsilon_\nu(q)} \bar{u}(p') \left(\frac{\gamma^\mu(p' + q + m_0)\gamma^\nu}{(p + q)^2 - m_0^2} \right) u(p),
\]

(30)

where \(p(p')\) and \(q(q')\) represent internal (external) momenta of the electron and photon, respectively, and \(\epsilon_\mu^*(\epsilon_\nu)\) shows the polarization of the output (input) photon. Doing some trace technologies, summing over all spins and polarizations, when the smoke clears, one can derive (see appendix for details)

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To turn Eq. (31) into a cross-section, both a picture of the 

\[ \sigma \left( \omega, \theta \right) | \mathcal{M}_\mu |^2 = \sigma \left( \omega, \theta \right) \rho \left( \omega, \theta \right) \] 

together, we take, 

one can expect the existence of gamma rays. However, this 

\[ \frac{d\sigma}{d\cos \theta} = \frac{1}{8\pi} \left( \frac{\omega'}{\omega R} \right)^2 \left( \frac{1}{2} \sum_{\text{spins}} |M|^2 \right). \] 

Replacing Eq. (32) in Eq. (33) with \( m_r = m_0 \), our result 
is compatible with \textit{Klein–Nishina formula}, originally first 
derived in 1929 [49]. 

\[ \frac{d\sigma}{d\cos \theta} = \frac{\pi \alpha^2}{m^2} \left( \frac{\omega'}{\omega} \right)^2 \left( \frac{\omega'}{\omega} + \frac{\omega'}{\omega - \sin^2 \theta} \right). \]

Defining \( S = \frac{d\sigma}{d\cos \theta} \), one can calculate the energy distribu-

Please note that in the above equation, \( m_0 \) (an electron which 
obeys the Dirac equation) differs from \( m_R \) (heavy electron). 
The momentum of the electron subjected to the electromag-
netic field \( P_\mu = p_\mu - eA_\mu \), where \( P_\mu \) is the momentum 
of an electron interacting with gauge vector field \( A_\mu \). Thus, we 
would have concluded that \( \left( p_\mu - eA_\mu \right) \left( p_\mu - eA^\mu \right) = m_R^2 \).

To turn Eq. (31) into a cross-section, both a picture of the 
kinematics and the reference frame of Compton scattering 
must be considered. Choosing the lab frame in which 
the electron is initially at rest, and \( q = (\omega, \omega'), p = (m_R, 0) \), 
\( q' = (\omega', \omega' \sin \theta, 0, \omega' \cos \theta) \) and \( p' = (E', P') \), altogether, 
we take, 

\[ \frac{d\sigma}{d\cos \theta} = \frac{1}{8\pi} \left( \frac{\omega'}{\omega R} \right)^2 \left( \frac{1}{2} \sum_{\text{spins}} |M|^2 \right). \] 

In Fig. 1, we show the accordance of the Klein–Nishina for-
mula (standard Compton scattering) and our results when 
\( m_r = m_0 \).

In 2004, the Piantelli group [50] surveyed the energy 
distribution of scattered photons from an experiment done 
in the Ni-H medium (see the right side of Fig. 2). Noting 
that there are circumstances for LENR in the Ni-H medium,
Fig. 1 Energy distribution of scattered photons $\omega'$: Klein–Nishina formula (left), our result with $m_R = m_0$ (right)

old enhancement of the electron mass for beginning LENRs in metal hydride surfaces and absorption of hard photons of electrons, is $m_{e}^* = 2.53 m_e$. More precisely, opposite to Fig. 1, the peak behavior of Figs. 2 and 3 is due to the renormalization of the electron mass which is 2.53 times the original electron mass dominated by Eq. (31).

However, one can survey photon scattering angles over a range of energies. We observe that the angular distribution of scattered photons for standard Compton scattering is shown on the right side in Fig. 3: photons with energy of 1 keV and 100 keV will scatter over all angles, and photons with energy of 3 MeV and 10 MeV almost exclusively exhibit forward scattering. On the other hand, for modified Compton scattering, this distribution is not isotropic, and forward scattering is seen with photons in some specific angles (left side in Fig. 3), which may also explain why gamma-rays are not observed in some experiments.

Fig. 2 Gamma-ray and heavy electron cross-section: a peak under 1 MeV for various energies of an incident photon compatible with experimental data (left). The effects of gamma radiation in the region below 1 MeV from the result of the Piantelli group [50] (right)

4 Conclusion

Enhancement of the electron mass has substantial consequences in different areas of physics, especially in condensed matter, atomic, and particle physics. When an electron in some special conditions is under some electromagnetic field (EM), EM oscillations lead to heavy electrons. This can happen in the Kondo lattice or when we load protons sufficiently and appropriately on metallic hydride surfaces. In these conditions, the electrons are treated as renormalized particles with excess mass, and their physics is far from the free one. As a fascinating application of the heavy electron, we have shown that hard photons scattered from the heavy electron, according to Compton scattering, are turned into soft gamma photons. Our results are completely coincident with experimental observations: the peaks of Fig. 2 near 1 MeV confirm with experiments [50,51]. Numerous efforts have been made over many years to control and absorb high-energy gamma-
Fig. 3 Angular distribution of scattered photons from heavy electron in Eq. (34) (left). Angular distribution of scattered photons for Klein–Nishina formula (right) over a range of commonly encountered energies.

Fig. 4 Schematic of the basic design of a shield system by heavy electrons, the explanation of different parts is provided in Conclusion section.
\[
\frac{B}{((p+q)^2 - m^2)((p-q)^2 - m^2)}
\]
\[
\frac{C}{((p-q)^2 - m^2)(p+q)^2 - m^2)}
\]
\[
\frac{D}{(p-q)^2 - m^2)}
\]
\[
\text{(37)}
\]

where

\[ A = \text{tr} \left( (p^' + m_0)(\gamma^\mu \gamma^\nu + 2\gamma^\mu p^\nu)(\gamma_\mu p_\nu) \right) \]

\[ B = \text{tr} \left( (p^' + m_0)(\gamma^\mu \gamma^\nu + 2\gamma^\mu p^\nu)(\gamma_\nu q_\mu) \right) \]

A, B, C, and D are complicated traces. Noting that replacing \( q \) with \( p' \), we get \( A = D \), and also, due to the ability to reverse the order of the \( \gamma \) matrices we have \( B = C \), here we only survey the trace terms for \( A \), and the other terms are similar. There are 16 separate trace terms for each numerator. We choose one of them, for instance

\[
tr(2p^'\gamma^\mu \gamma^\nu \gamma_\mu p_\nu) = 2 tr(p^'\gamma^\nu \gamma_\mu p_\nu) = 4 tr(\gamma^\nu \gamma_\mu)
\]

\[ = -16 \left( (p^'p)(p,q) - (p,p')(p,q) + (p',q)(p,p) \right) = -16m^2_R(p',q), \]

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