A “Gaussian” Approach to Computing Supersymmetric Effective Actions

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Abstract

For nonsupersymmetric theories, the one-loop effective action can be computed via zeta function regularization in terms of the functional trace of the heat kernel associated with the operator which appears in the quadratic part of the action. A method is developed for computing this functional trace by exploiting its similarity to a Gaussian integral. The procedure is extended to superspace, where it is used to compute the low energy effective action obtained by integrating out massive scalar supermultiplets in the presence of a supersymmetric Yang-Mills background.

1 Introduction

Low energy effective actions are important in physical situations in which one is interested in phenomena at an energy scale which is small compared to the masses of some of the fundamental degrees of freedom in the theory. Although these heavy degrees of freedom can only occur as virtual states at the energy scale of interest, they still produce observable effects, which are summarized in a low energy effective action for the “light” degrees of freedom. The low energy effective action is determined by “integrating out” the heavy degrees of freedom.

Recently there has been renewed interest in perturbative computations of effective actions in supersymmetric Yang-Mills theories [1–4]. This interest was stimulated by the work of Seiberg and Witten [7], who deduced the full

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nonperturbative low energy effective action for an N=2 supersymmetric Yang-Mills theory. The perturbative computations have focused on the corrections to the low energy Kähler potential for the scalar supermultiplets in the N=1 superfield formulation of N=2 supersymmetric Yang-Mills theory. The effective Kähler potential was first considered in superfield form in the work of Buchbinder et al [5,6].

In this paper, we introduce a new technique for the perturbative calculation of low energy effective actions, and then illustrate it in superspace by determining the low energy effective action obtained by integrating out massive N=1 scalar multiplets in the presence of an N=1 supersymmetric Yang-Mills background. This is a case not treated fully in the earlier calculations quoted above. The approach is from the point of view of heat kernels and zeta function regularization, as opposed to the supergraph calculations in [2–4]. Buchbinder et al [5,6] have also developed functional techniques for computing low energy effective actions in superspace, but our approach differs significantly from theirs. Also, although zeta function regularization is only really useful in the computation of one-loop effective actions, there are nonrenormalization theorems in N=2 supersymmetric Yang-Mills theories which ensure the absence of higher loop corrections to the effective action, so this is not a disadvantage if one is ultimately interested in these theories.

The plan of the paper is as follows. We begin with a brief summary of an approach to the computation of nonsupersymmetric low-energy effective actions which relies on the similarity of the functional trace of the heat kernel to a Gaussian integral. The technique is then applied in superspace to determine the low energy effective action obtained by integrating out massive N=1 scalar multiplets coupled to an N=1 supersymmetric Yang-Mills background. The paper ends with a short discussion.

2 The Nonsupersymmetric Case

An efficient way to generate the one-loop effective action is via zeta function regularization. We consider for definiteness a massive scalar field in the presence of a Yang-Mills background with Euclidean action

\[ S[A, \phi] = \frac{1}{2} \int d^4x \, \phi^\dagger (-D_\alpha D_\alpha + m^2) \phi, \]

where \(D_\alpha\) is the covariant derivative in the representation \(R\) of the gauge group to which the scalar fields belong, with \([D_\alpha, D_\beta] = F_{\alpha\beta}\). The one loop effective action for the Yang-Mills fields obtained by “integrating out” the scalar fields
\[ \Gamma[A] = -\ln \int [d\phi] e^{-S[A,\phi]} = \ln \det(-D_aD_a + m^2). \]

Using zeta function regularization, this is just \( -\zeta'(0) \), where the zeta function is defined by
\[
\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty ds \, t^{s-1} e^{-m^2t/\mu^2} Tr \tilde{K}(t/\mu^2).
\]

In this expression, \( Tr \) is a trace over gauge indices and \( \tilde{K}(t) \) is the functional trace of the heat kernel for the operator \( D_aD_a \),
\[
\tilde{K}(t) = \int d^4x \lim_{x \to x'} e^{tD_aD_a} \delta^{(4)}(x, x') \equiv \int d^4x K(t).
\]

Since \( t \) in \( K(t) \) has dimensions of inverse mass squared, the usual parameter \( \mu \) with dimensions of mass (representing the renormalization point) has been introduced into the zeta function to make \( t \) dimensionless.

If we introduce a plane wave basis for the delta function, \( K(t) \) takes the form
\[
K(t) = \lim_{x \to x'} \int \frac{d^4k}{(2\pi)^4} e^{ik.(x-x')} \left( e^{-ik.(x-x')} e^{tD_aD_a} e^{ik.(x-x')} \right)
\]
\[
= \int \frac{d^4k}{(2\pi)^4} e^{X_aX_a} tX_aX_a
\]

where \( X_a = D_a + ik_a \). It then follows that \( K(t) \) satisfies the differential equation
\[
\frac{dK(t)}{dt} = K_{aa}(t), \tag{1}
\]

where the tensors \( K_{a_1 \cdots a_n}(t) \) are defined by
\[
K_{a_1 \cdots a_n}(t) = \int \frac{d^4k}{(2\pi)^4} X_{a_1} \cdots X_{a_n} e^{tX.X}.
\]

To solve the differential equation, it is necessary to obtain an expression for \( K_{ab}(t) \) in terms of \( K(t) \). The approach we take is to use the identity
\[
0 = \int \frac{d^4k}{(2\pi)^4} \frac{\partial}{\partial k_{a_1}} \left( X_{a_1} \cdots X_{a_{m-1}} e^{tX.X} \right)
\]

to study the properties of these tensors (the boundary term in the integral vanishes because of the \( e^{-k^2} \) factor in the integrand). This is the same method that can be used to determine the moments \( \int \frac{d^4k}{(2\pi)^4} k_{a_1} \cdots k_{a_n} e^{-k^2} \) of an ordinary Gaussian in terms of the Gaussian itself.

In particular, applying it in the case \( m = 2 \),
\[ 0 = i\delta_{ab} K(t) + \int \frac{d^4 k}{(2\pi)^4} X_a \frac{\partial}{\partial k^b} e^{t X \cdot X} \]
\[ = i\delta_{ab} K(t) + 2it \int \frac{d^4 k}{(2\pi)^4} X_a \left( \int_0^1 ds e^{-st X \cdot X} X_b e^{st X \cdot X} \right) e^{t X \cdot X}. \quad (2) \]

Because \( \int_0^1 ds e^{-st X \cdot X} X_b e^{st X \cdot X} = X_b + \cdots \), this identity yields an expression for \( K_{ab}(t) \) in terms of \( K(t) \) which can then be used to solve the linear differential equation (1).

The quantity
\[ \int_0^1 ds e^{-st X \cdot X} X_b e^{st X \cdot X} = \sum_{n=0}^{\infty} \frac{t^n}{(n+1)!} \text{ad}^{(n)}(X \cdot X)(X_b) \quad (3) \]

cannot be evaluated exactly and so must be approximated in some way. In the case of a computation of the low energy effective action, one is interested in the piece of the effective action which contains no covariant derivatives of the field strength of the Yang-Mills background. This is the covariant generalization of the low energy effective action for the \( U(1) \) case, where a constant field strength is equivalent to the long wavelength limit for the background electromagnetic field. A covariantly constant background corresponds to setting \( (D_a F_{bc}) = [X_a, [X_b, X_c]] \) to zero, so that only the commutators \( [X_a, X_b] = F_{ab} \) need be retained in evaluating (3). In this case, it is easy to show that \( \text{ad}^{(n)}(X \cdot X)(X_b) = (-2)^n (F^n)_{bc} X_c \), so that
\[ \int_0^1 ds e^{-st X \cdot X} X_b e^{st X \cdot X} = B_{bc}(t) X_c \]

where
\[ B_{bc}(t) = \left[ \frac{e^{-2tF} - 1}{-2tF} \right]_{bc}. \]

Note that the power series expansion for the matrix \( B_{bc}(t) \) begins with \( \delta_{bc} \) and only involves positive powers of \( F_{ab} \). As a result, the inverse matrix \( B_{bc}^{-1}(t) = \left[ \frac{-2tF}{e^{-2tF} - 1} \right]_{bc} \) exists and does not require the inverse of \( F_{ab} \) to be determined. Inserting this result into (2), and using the fact that \( B_{bc}(t) \) commutes with \( X_a \) to the order that we are working, one finds
\[ 0 = \delta_{ab} K(t) + 2t B_{bc}(t) K_{ac}(t). \]

It follows that
\[ K_{ab} = \left[ \frac{F}{e^{-2tF} - 1} \right]_{ba}. \]
Thus (1) becomes
\[
\frac{dK(t)}{dt} = \text{tr} \left[ \frac{F}{e^{-2tF} - 1} \right] K(t),
\]
where the trace \( \text{tr} \) is over spacetime indices and \textit{not} gauge indices; the kernel is still a matrix with respect to its gauge indices. Noting that
\[
\text{tr} \left[ \frac{F e^{2tF}}{1 - e^{2tF}} \right] = -\frac{1}{2} \text{tr} \left[ C^{-1} (1 - e^{2tF})^{-1} \frac{d}{dt} (1 - e^{2tF}) C \right],
\]
with \( C \) a matrix independent of \( t \), and using the boundary condition that \( K(t) \) reduces to the ordinary Gaussian \( \int \frac{d^4k}{(2\pi)^4} e^{-k^2} \) in the limit \( F_{ab} \to 0 \), one finds the standard result \([9–12]\) for the functional trace of the heat kernel:
\[
K(t) = \frac{1}{4\pi^2} \text{det} \left[ \frac{1 - e^{2tF}}{F} \right]^{-\frac{1}{2}} = \frac{1}{16\pi^2 t^2} \text{det} \left[ \frac{tF}{\sinh tF} \right]^{-\frac{1}{2}}.
\]
(4)

This technique is readily generalises to quantum fields of different spin, or to the inclusion of a potential for the scalar fields. For later use, we note that for a Dirac spinor in a Yang-Mills background, the kernel for the Laplace-type operator given by the square of the Dirac operator is \([9,10]\)
\[
K_{\Sigma/2}(t) = \text{tr} \left( e^{-t\Sigma_{ab} F_{ab}} \right) K(t),
\]
(5)
where \( \Sigma_{ab} = \frac{1}{4} [\gamma_a, \gamma_b] \), the trace \( \text{tr} \) is over the spinor indices on the gamma matrices and \( K(t) \) is the spin zero kernel (4).

The above considerations were motivated to a certain extent by the work of Avramidi [8], who computed
\[
\int \frac{d^4k}{(2\pi)^4} \sqrt{\gamma} e^{-t(\gamma_{ab} k_a k_b - 2k_a D_a)},
\]
where \( \gamma_{ab} \) is a metric in \( k \) space, and \( \gamma = \text{det} \gamma_{ab} \). The result is of the form
\[
f(t, \gamma, F) e^{g_{ab}(t) D_a D_b},
\]
where \( g_{ab} \) is a functional of \( \gamma_{ab} \) and \( F_{ab} \). By choosing \( \gamma_{ab} \) so that \( g_{ab} = \delta_{ab} \), it is possible to obtain an expression for \( e^{tD_a D_a} \) which can be used to compute the heat kernel and hence its functional trace. It is not immediately clear how this could be applied to superspace calculations.

Another common approach to computing effective actions is to determine the Green’s function for the quantum fields in the presence of the background; this is then used to compute the functional trace of the heat kernel. In the above approach, we compute the functional trace of the heat kernel directly without the need for this intermediate step. This is an advantage when it comes to computing effective actions in supersymmetric theories, as there are many different Green’s functions. In [5], where the effective Kähler potential for the Wess-Zumino model is computed, Buchbinder et al expressed all the Green’s functions in terms of a single one, thereby providing some simplification. In
the next section, we show that it is possible to extend the techniques developed above to calculations of effective actions for supersymmetric theories in superspace. To illustrate this, we compute the supersymmetric analogue of the results (4) and (5).

3 Massive Scalar Multiplet in Gauge Superfield Background

Here, we will be concerned with the computation of the one-loop low energy effective action for a gauge supermultiplet which results from integrating out massive scalar multiplets coupled to the gauge background. This is the superspace analogue of the nonsupersymmetric theory treated in §2. The superspace action for the scalar supermultiplet $\Phi(x, \theta)$ transforming in some (real) representation $R$ of the gauge group $G$ is

$$S = \int d^4x d^2\theta d^2\bar{\theta} \Phi e^{-V} \Phi + \int d^4x d^2\theta \frac{m}{2} \Phi^2 + \int d^4x d^2\bar{\theta} \frac{m}{2} \bar{\Phi}^2.$$ 

Gauge indices are suppressed, and the superspace conventions of Wess and Bagger [13] have been adopted, except that the metric has been Wick rotated to be Euclidean. Because the quantum superfields $\Phi$ are subject to the chirality constraint $\bar{D}_\alpha \Phi = 0$, the effective action cannot simply be formed as the logarithm of the superdeterminant of the operator appearing in the quadratic part of the action. Rather, it is first necessary to write the action in terms of unconstrained superfields. To this purpose we introduce complex scalar superfields $\Psi$ and solve the constraint by expressing $\Phi = \bar{D}^2 \Psi$; the superfields transform under the action of the gauge group in the same way as $\Phi$. In terms of the unconstrained fields, the action can be expressed in the full superspace as

$$S = 8 \int d^4x d^2\theta d^2\bar{\theta} \left( \Psi, \Psi^\dagger \right) \begin{pmatrix} \frac{1}{16} \bar{D}^2 e^V \bar{D}^2 e^{-V} - \frac{m}{4} \bar{D}^2 e^V \\ -\frac{m}{4} D^2 e^{-V} & \frac{1}{16} D^2 e^{-V} \bar{D}^2 e^V \end{pmatrix} \begin{pmatrix} e^V \Psi^\dagger \\ -e^{-V} \Psi \end{pmatrix}.$$ 

So the effective action is $\Gamma_{\text{eff}}[V] = -\frac{1}{2} \ln \text{sdet} \Delta$, where $\Delta$ is the superspace operator in the unconstrained action above [5]. The appropriate zeta function is thus $\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty dt t^{s-1} \int d^8Z Tr K(t/\mu^2)$, where $Tr$ denotes the trace over gauge indices, $\mu$ is the renormalization point and point

$$K(t) = tr \lim_{Z \to Z'} \exp \left( \frac{1}{16} \bar{D}^2 e^V \bar{D}^2 e^{-V} - \frac{m}{4} \bar{D}^2 e^V \\ -\frac{m}{4} D^2 e^{-V} & \frac{1}{16} D^2 e^{-V} \bar{D}^2 e^V \right) \delta^{(8)}(Z, Z').$$

Here, the trace $tr$ is over the $2 \times 2$ matrices, and $\int d^8Z$ and $\delta^{(8)}(Z, Z')$ denote the integration measure and the delta function on the full superspace. Applying the Baker-Campbell-Hausdorff formula, the terms involving the mass $m$
can be placed in a separate exponential; performing the two dimensional trace
then projects out even powers of $m$, giving

\[ K(t) = \sum_{n=0}^{\infty} \frac{(mt)^{2n}}{(2n)!} \lim_{Z \to Z'} \left( \left( \frac{1}{16} \bar{D}^2 e^V \bar{D}^2 e^{-V} \right)^n e^{\frac{1}{16t} \bar{D}^2 e^V \bar{D}^2 e^{-V}} \delta^{(16)}(Z, Z') \right) + \left( \frac{1}{16} D^2 e^{-V} \bar{D}^2 e^V \right)^n e^{\frac{1}{16t} D^2 e^{-V} \bar{D}^2 e^V} \delta^{(8)}(Z, Z') \]

\[ = \sum_{n=0}^{\infty} \frac{(mt)^{2n}}{(2n)!} \frac{d^n}{dt^n} \lim_{Z \to Z'} \left( e^{\frac{1}{16t} \bar{D}^2 e^V \bar{D}^2 e^{-V}} \delta^{(8)}(Z, Z') \right) + e^{\frac{1}{16} \bar{D}^2 e^V \bar{D}^2 e^{-V}} \delta^{(8)}(Z, Z'). \]

Using $\int d^8 Z = \int d^4x \int d^2\theta (-\frac{1}{4}) \bar{D}^2$ and $\int d^8 Z = \int d^4x \int d^2\bar{\theta} (-\frac{1}{4}) D^2$, factors of $(\frac{-1}{4}) \bar{D}^2$ and $(\frac{-1}{4}) D^2$ can be extracted from the exponentials to act on the full superspace delta function to convert the zeta function to the “chiral” form

\[ \zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty dt \ t^{s-1} \sum_{n=0}^{\infty} \frac{1}{(2n)!} \left( \frac{mt}{\mu^2} \right)^{2n} \mu^{2n} \frac{d^n}{dt^n} \left. \left( \int d^4 x \int d^2 \theta \ K_L(t/\mu^2) \right. \right) \]

\[ + \left. \int d^4 x \int d^2 \bar{\theta} K_R(t/\mu^2) \right) \]

where the chiral kernels are

\[ K_L(t) = \lim_{Z \to Z'} e^{\frac{1}{16t} \bar{D}^2 e^V \bar{D}^2 e^{-V}} \delta^{(4)}(x, x') \delta^{(2)}(\theta, \theta'), \]

\[ K_R(t) = \lim_{Z \to Z'} e^{\frac{1}{16t} D^2 e^{-V} \bar{D}^2 e^V} \delta^{(4)}(x, x') \delta^{(2)}(\bar{\theta}, \bar{\theta}'). \]

The mass dependence in (6) involves derivatives of the chiral kernels. It is convenient to remove these derivatives by repeated integration by parts. The boundary terms at $t = \infty$ vanish as the kernels vanish in this limit (this can be checked explicitly using the result (18) for the kernel); the boundary term at $t = 0$ involves a factor $t^{n+s+1}$ and also vanishes as we are only interested in $\zeta(s)$ for $s$ in a small neighbourhood of $s = 0$. The result can be expressed in the form

\[ \zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty dt \ t^{s-1} \left[ \sum_{n=0}^{\infty} \frac{1}{(2n)!} \left( -\frac{m^2 t}{\mu^2} \right)^n \frac{\Gamma(2n + s)}{\Gamma(n + s)} \right] \]

\[ T r \left( \int d^4 x \int d^2 \theta K_L(t/\mu^2) \right) + \int d^4 x \int d^2 \bar{\theta} K_R(t/\mu^2) \right) \].

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The sum in square brackets is the generalized hypergeometric function
\[ 2F_2 \left[ \frac{s}{2} + \frac{1}{2}, \frac{s}{2}; \frac{1}{2}, s; -\frac{m^2 t}{\mu^2} \right]. \]

It will be seen later that the chiral kernels \( \int d^4x \int d^2\theta K_L(t) + \int d^4x \int d^2\bar{\theta} K_R(t) \) have a power series expansion in \( t \) of the form \( \sum_{n=0}^{\infty} V_n t^n \), where \( V_n \) represents an effective vertex for the interaction of \( n + 2 \) particles. So, with a rescaling of \( t \),
\[ \zeta(s) = \sum_{n=0}^{\infty} \frac{V_n}{m^2} \left( \frac{m^2}{\mu^2} \right)^{-s} \frac{1}{\Gamma(s)} \int_0^\infty dt \, t^{n+s-1} 2F_2 \left[ \frac{s}{2} + \frac{1}{2}, \frac{s}{2}; \frac{1}{2}, s; -t \right]. \]

Although we have not been able to do the integral (which is the Mellin transform of a generalized hypergeometric function) explicitly, it is possible to deduce the form of the zeta function for \( s \) near zero using the fact that
\[ 2F_2 \left[ \frac{s}{2} + \frac{1}{2}, \frac{s}{2}, \frac{1}{2}, s; -t \right] \rightarrow \frac{1}{2} e^{-t} \] as \( s \rightarrow 0 \). Thus for \( n \neq 0 \), the integral is regular in the limit \( s \rightarrow 0 \), giving \( \frac{1}{2} \Gamma(n) \). This means that in calculating \( \zeta'(0) \), the derivative must act on \( \frac{1}{\Gamma(s)} = s + O(s^2) \) to eliminate the zero at \( s = 0 \). For \( n = 0 \), the integral behaves like \( \frac{1}{2} \Gamma(s) \) for small \( s \), and so the derivative acts on \( (m^2/\mu^2)^{-s} \). The result is
\[ \zeta'(0) = \sum_{n=1}^{\infty} \frac{1}{2} \Gamma(n) \frac{V_n}{m^{2n}} - \frac{1}{2} V_0 \ln \frac{m^2}{\mu^2}. \tag{8} \]

Alternatively, if Schwinger proper time regularization is used, it is only necessary to know the hypergeometric at \( s = 0 \); in this case there is a divergence in the \( n = 0 \) contribution to the effective action which must be removed by hand, but the \( n \geq 1 \) contributions are as above.

It therefore remains to evaluate the chiral kernels; we do this for \( K_L(t) \), as the calculation for \( K_R(t) \) is identical except that left chiral quantities are replaced by right chiral quantities. For the rest of the paper, \( K_L(t) \) will simply be denoted \( K(t) \). Acting on left chiral superfields, the operator \( \frac{i}{16} \bar{D}^2 e^V D^2 e^{-V} \) is equivalent to the Laplace-type operator \( D_a D_a + W^a D_a + \frac{1}{2}(D^a W_a) \), where the \( D \) denote gauge covariant superspace derivatives in the left chiral basis: \( D_a = e^V D_a e^{-V}, D_{\dot{a}} = D_{\dot{a}}, \{ D_a, D_{\dot{a}} \} = -2i(\sigma_a)_{\alpha\beta} D_\alpha, \) and \( W_\alpha = -\frac{1}{4}[D_\alpha, \{ D_{\dot{a}}, D_{\dot{a}} \}] \). Thus the left chiral kernel contains a Laplace-type operator, as required for a well-defined heat kernel:
\[ K(t) = \lim_{Z \rightarrow Z'} e^{t(D_a D_a + W^a D_a + \frac{1}{2}(D^a W_a))} \delta^{(4)}(x, x') \delta^{(2)}(\theta, \theta'). \]

The left chiral delta function has the representation
\[ \frac{1}{4} \delta^{(4)}(x, x') \delta^{(2)}(\theta, \theta') = \int \frac{d^4k}{(2\pi)^4} e^{ik_a(x_a - x'_a - i\theta\sigma_a\bar{\theta} + i\theta'\sigma_a\bar{\theta})} \int d^2\epsilon \, e^{ie^\alpha(\theta - \theta')}, \]
where $\epsilon_\alpha$ is a Grassmann parameter which is the supersymmetric partner of $k_a$.

The delta function in $x$ is a function of the supertranslation invariant interval $x_a - x'_a - i\theta\sigma_\alpha \bar{\theta}' + i\theta'\sigma_\alpha \bar{\theta}$ on superspace.\footnote{Note that $\bar{D}_\dot{a}(x_a - x'_a - i\theta\sigma_\alpha \bar{\theta}' + i\theta'\sigma_\alpha \bar{\theta}) = -i(\theta - \theta')^\alpha (\sigma_a)_{\alpha \dot{a}}$, so that although the superspace invariant interval is not itself left chiral, the fact that $(\theta - \theta')^3 = 0$ means that the full delta function on left chiral superspace is annihilated by $\bar{D}_\dot{a}$, as required.}

Moving the exponential to the left through the differential operators, the coincidence limit of the left chiral kernel has the expression

$$K(t) = 4 \int \frac{d^4 k}{(2\pi)^4} \int d^2 \epsilon \ e^{t(X_a X_a + W^\alpha X_\alpha + \frac{1}{2}(\mathcal{D}^\alpha W_\alpha))},$$

(9)

where

$$X_a = \mathcal{D}_a + ik_a, \quad X_\alpha = \mathcal{D}_\alpha + i\epsilon_\alpha.$$  

(10)

Note that there is also a shift $-k_a (\sigma_a)_{\alpha \dot{a}} (\bar{\theta} - \bar{\theta}')\dot{\alpha}$ in $\mathcal{D}_\alpha$; however, this vanishes in the coincidence limit as there are no $\mathcal{D}_\dot{a}$ operators present to annihilate the $\bar{\theta} - \bar{\theta}'$. Also note that the integrand in (9) contains an explicit factor $e^{-k^2}$ necessary for the convergence of the $k$ integral. To compute the kernel, we will solve the differential equation

$$\frac{dK(t)}{dt} = K_{aa}(t) + W^\alpha K_\alpha(t) + \frac{1}{2}(\mathcal{D}^\alpha W_\alpha)K(t),$$

(11)

where

$$K_{A_1 A_2 \ldots A_n}(t) = 4 \int \frac{d^4 k}{(2\pi)^4} \int d^2 \epsilon \ X_{A_1} X_{A_2} \cdots X_{A_n} e^{t\Delta},$$

and $X_A$ can represent either a bosonic operator $X_a$ or a fermionic operator $X_\alpha$; the abbreviation $\Delta = X_a X_a + W^\alpha X_\alpha + \frac{1}{2}(\mathcal{D}^\alpha W_\alpha)$ has also been introduced.

The aim is to use identities similar to those used in §2 to express $K_{aa}(t)$ and $W^\alpha K_\alpha(t)$ in terms of $K(t)$. In superspace, there are two kinds of identities involving the vanishing the integral of a total derivative which can be employed, one involving derivatives with respect to the bosonic variables $k_a$ and the other involving derivatives with respect to the fermionic variables $\epsilon_\alpha$:

$$0 = \int \frac{d^4 k}{(2\pi)^4} \int d^2 \epsilon \ \partial_{k^b} \left( X_{A_1} X_{A_2} \cdots X_{A_n} e^{t\Delta} \right);$$

$$0 = \int \frac{d^4 k}{(2\pi)^4} \int d^2 \epsilon \ \partial_{\epsilon^\beta} \left( X_{A_1} X_{A_2} \cdots X_{A_n} e^{t\Delta} \right).$$

(12)
The expression for $K_{ab}(t)$ will arise from the use of the first identity with $X_{A_1} X_{A_2} \cdots X_{A_n}$ replaced by $X_a$, as to the action of the derivative on the exponential pulls down an operator which to leading order is $2itX_a$. Similarly, an expression for $W_\beta K_\alpha(t)$ can be obtained from the second identity with $X_{A_1} X_{A_2} \cdots X_{A_n}$ replaced by $X_\alpha$, as the action of the derivative on the exponential pulls down a factor which to leading order is $-itW_\beta$.

In both cases, the factors $2itX_a$ and $-itW_\beta$ will be accompanied by additional terms containing commutators of $\Delta$ with these factors. As was the case in §2, these will form an infinite series which cannot be summed in general. It is necessary to truncate to a given order in commutators and anticommutators of $X_a$ and $X_\alpha$, corresponding to a particular order in the (super)derivative expansion of the effective action. To lowest order, the effective action contains the superfields $W_\alpha$, but no derivatives of them. Since $W_\alpha$ can be expressed as a double (anti)commutator of spinor derivatives $D_\alpha$ and $\bar{D}_{\dot{\alpha}}$, this corresponds to truncation to at most two (anti)commutators of spinor derivatives (with $D_a$ counting as a first order anticommutator via $\{D_a, D_{\dot{a}}\} = -2i(\sigma_a)_{a\dot{a}} D_{\dot{a}}$). The left chiral kernel is trivial to compute in this approximation, because $[X_a, X_b]$, $[X_a, X_\alpha]$, $[X_a, W_\alpha]$ and $[X_\alpha, W_\beta]$ all involve at least three (anti)commutators of spinor derivatives and therefore must be set to zero. The only potential anticommutator at this order is $\{X_\alpha, X_\beta\}$, but this vanishes due to the torsion and curvature constraints imposed in supersymmetric Yang-Mills theory [13]. The kernel thus reduces in this lowest order approximation to

$$K(t) = 4 \int \frac{d^4k}{(2\pi)^4} \int d^2\epsilon \ e^{-tk^2} e^{itW^\alpha\epsilon_\alpha} = \frac{1}{16\pi^2} W^\alpha W_\alpha.$$  \hspace{1cm} (13)

Note that this does not even reproduce the the nonsupersymmetric low energy effective actions considered in §2; it contains only the leading term, quadratic in the Yang-Mills field strength. To reproduce the supersymmetric analogue of these results, it is necessary to go to the next order in the superspace derivative expansion, namely to consider up to three (anti)commutators of spinor derivatives. This is the truncation which will be made here.

Using the curvature and torsion constraints for supersymmetric Yang-Mills theory [13], and letting $\bar{M}_{ab} = (D\bar{\sigma}_{ab}\bar{W})$, $M_{ab} = (D\sigma_{ab}W)$, and $N_{\alpha\beta} = (D_\alpha W_\beta)$, the nonvanishing commutators in this truncation are:

$$[X_a, X_b] = -\frac{1}{2}(M_{ab} - M_{ba}), \quad [X_a, X_\alpha] = i(\sigma_a)_{a\dot{a}} \bar{W}_{\dot{\alpha}}, \quad \{X_\alpha, W_\beta\} = N_{\alpha\beta}.$$  

In particular, note that $[X_\alpha, W_\alpha]$ must be set to zero at this order.

We first consider the calculation of $W^\alpha K_\alpha(t)$ in terms of $K(t)$. The relevant identity is
\[ 0 = 4 \int \frac{d^4 k}{(2\pi)^4} \int d^2 \epsilon \frac{\partial}{\partial \epsilon_\beta} (X_\alpha e^{t\Delta}) \]
\[ = i\delta_\alpha^\beta K(t) - 4 \int \frac{d^4 k}{(2\pi)^4} \int d^2 \epsilon X_\alpha \frac{\partial}{\partial \epsilon_\beta} e^{t\Delta}. \]

The derivative of the exponential is computed using
\[
\frac{\partial}{\partial \epsilon_\beta} e^{t\Delta} = \left( \int_0^1 ds \ e^{s t \Delta} (-itW^\beta) \ e^{-s t \Delta} \right) e^{t\Delta} = -it \sum_{n=0}^{\infty} \frac{t^n}{(n + 1)!} a_q(n)(\Delta)(W^\beta)e^{t\Delta}. \]

The commutators in the series are easily evaluated to the required order, and we obtain
\[
\frac{\partial}{\partial \epsilon_\beta} e^{t\Delta} = -it \sum_{n=0}^{\infty} \frac{t^n}{(n + 1)!} W^\gamma (N^n)^\gamma_\beta = -iW^\gamma \left( \frac{e^{tN} - 1}{N} \right)_\gamma^\beta. \tag{14} \]

Note that the power series expansion begins at order \(N^0\) and does not involve any negative powers of the matrix \(N\); thus the inverse matrix \(\left( \frac{N}{e^{tN} - 1} \right)_\gamma^\beta\) exists and does not require the inversion of \(N\). Substituting this result and being careful to include the anticommutator which arises from moving \(W^\gamma\) through \(X_\alpha\), we obtain
\[ 0 = \delta_\alpha^\beta K(t) + N_\alpha^\gamma \left( \frac{e^{tN} - 1}{N} \right)_\gamma^\beta K(t) - W^\gamma \left( \frac{e^{tN} - 1}{N} \right)_\gamma^\beta K_\alpha(t). \]

Multiplying by \(\left( \frac{N}{e^{tN} - 1} \right)_\beta^\rho\) and contracting indices appropriately yields a result of the desired form:
\[ W^\alpha K_\alpha(t) = tr \left( \frac{N}{e^{tN} - 1} \right) K(t) + tr(N) K(t). \tag{15} \]

Note that the trace here is over the spinor indices, \(tr(N) = N_{\alpha}^\alpha\); there is not a trace over gauge indices, as the expression is still a matrix with respect to gauge indices.

The first identity in (12) is used to compute \(K_{aa}(t)\) in the same manner as in the bosonic case in §2. To the order in which we are working, one finds
\[
\frac{\partial}{\partial k_b} e^{t\Delta} = 2itB_{bc}(t)X_c + 2itA_b(t), \]
where
\[
B_{bc}(t) = \left( \frac{e^{-t(M - M)} - 1}{-t(M - M)} \right)_{bc}. \]
\[ A_b(t) = \frac{i}{t} \left[ \frac{e^{-t(M-M)} - 1}{(M-M)(M-\frac{1}{2}tr(N)1)} - \frac{e^{-t(M-M)} - 1}{(M-\frac{1}{2}tr(N)1)(M-\frac{1}{2}tr(N)1)} \right]_{bc} (W^\alpha_\sigma \sigma_\alpha \bar{W}^\alpha_\bar{\sigma}). \]

Thus the first identity in (12) with \( X_{A_1}X_{A_2} \cdots X_{A_n} \) replaced by \( X_a \) yields
\[
0 = i\delta_{ab}K(t) + 2itB_{bc}(t)K_{ac}(t) + 2itA_b(t)K_a(t).
\]

The matrix \( B_{bc}(t) \) has a power series expansion in positive powers of \( M \) and \( M \) which begins at order 1, and so it is invertible, yielding
\[
K_{ab}(t) = -\frac{1}{2t}(B^{-1})_{ba}(t)K(t) - (B^{-1})_{bc}(t)A_c(t)K_a(t).
\]

The right hand side involves \( K_a(t) \); this is evaluated by using the first identity in identity (12) with \( X_{A_1}X_{A_2} \cdots X_{A_n} \) replaced by 1. The result is
\[
K_a(t) = -(B^{-1})_{ab}(t)A_b(t)K(t).
\] (16)

At this point an important simplification can be achieved by noting that the kernel in the order that we currently working must reduce to the “zero’th order” result \( K(t) = \frac{1}{16\pi^2}W^2 \) as \( M, \bar{M} \) and \( N \to 0 \), i.e. when the higher order commutators we have allowed at this order vanish. So the kernel must be of the form \( K(t) = F[M, \bar{M}, N] \frac{1}{16\pi^2}W^2 \) with \( F[M, \bar{M}, N] \to 1 \) as \( M, \bar{M}, N \to 0 \). On the other hand, to the order that we are working, \( \{W_\alpha, W_\beta\} \) can be taken to be zero, since it involves a five commutators of spinor derivatives. As a result, expressions involving more than two \( W \)’s vanish at the order we are working because one index must be repeated and \((W_a)^2 = 0\). Therefore, using (16), \( K_a(t) \) vanishes because \( A_b(t) \) involves one factor of \( W \) and \( K(t) \) involves two. Thus the expression for \( K_{ab}(t) \) becomes simply:
\[
K_{ab}(t) = -\frac{1}{2t}(B^{-1})_{ba}(t)K(t).
\] (17)

Substituting (15) and (17) into (11), the differential equation for the left chiral kernel is
\[
\frac{dK(t)}{dt} = \frac{1}{2}tr \left( -\frac{(M-M)}{e^{-t(M-M)} - 1} \right) K(t) + tr \left( \frac{N}{e^{tN} - 1} \right) K(t) + \frac{1}{2}tr(N)K(t).
\]

\[ ^4 \text{In a nonabelian gauge theory, it is not true in general that } \{W_\alpha, W_\beta\} = 0; \text{ rather, } \\
\{W_\alpha, W_\beta\} = W_\alpha W_\beta^\dagger f_{\alpha\beta}T_c, \text{ where } T_a \text{ are generators of the gauge group satisfying } \\
[T_a, T_b] = f_{\alpha\beta}^c T_c. \text{ Also note that using the general result } \epsilon_{\alpha\beta} = \frac{i}{2}\epsilon_{\alpha\beta}W^2 + \\
\frac{1}{2}\{W_\alpha, W_\beta\}, \text{ it follows that to the order we are working, } W_\alpha W_\beta = \frac{i}{2}\epsilon_{\alpha\beta}W^2. \]
Noting the similarity of the first and second terms on the right-hand side of the equation with the differential equation in §2, the solution is easily seen to be

\[ K(t) = c_1 e^{t \text{tr}(N)} \det \left( \frac{1 - e^{-tN}}{A_1} \right) \det \left( \frac{1 - e^{t(M-M)}}{A_2} \right)^{-\frac{1}{2}}, \]

where \( c_1, A_1 \) and \( A_2 \) are constants (independent of \( t \)). The latter are determined by the requirement that in the limit \( M, \bar{M}, N \to 0 \), the kernel must be of the form (13), from which it follows that

\[ K(t) = \frac{W^2}{16\pi^2} e^{\frac{t}{2} \text{tr}(N)} \det \left( \frac{1 - e^{-tN}}{N} \right) \det \left( \frac{1 - e^{t(M-M)}}{(M-M)} \right)^{-\frac{1}{2}}. \]  

(18)

The power series expansion of the chiral kernel begins at order \( t^0 \), and so is of the form \( \sum_{n=0}^{\infty} V_n t^n \). This yields the vertices in the effective action for the Yang Mills superfield background via (8). As mentioned in the introduction, this case is not considered in the recent computations of effective actions for supersymmetric Yang-Mills theories by graphical techniques [2,4], or in earlier results of Buchbinder et al [5,6] using functional methods. Pickering and West [3] have computed the piece of the effective action corresponding to the lowest order approximation (13) to the heat kernel, but have not computed the corrections involving \( D_\alpha W_\beta \).

We can perform two checks on the result (18). The first is to go back to the expression (9) and perform the \( \epsilon \) integral explicitly. This is done using

\[ 4 \int d^2 \epsilon = \frac{\partial}{\partial \epsilon_\alpha} \frac{\partial}{\partial \epsilon_\alpha} |_{\epsilon=0} \quad \text{and} \quad (14). \]

One finds

\[
K(t) = \frac{1}{2} W^2 \left[ \left( \frac{e^{tN} - 1}{N} \right) \left( \frac{e^{-t(N-tr(N))} - 1}{N-tr(N)} \right) \right] \\
\int \frac{d^4 k}{(2\pi)^4} \exp \left\{ t(X_\alpha X_\alpha + W^\alpha D_\alpha + \frac{1}{2}(D^\alpha W_\alpha)) \right\}.
\]

Note that since \( \epsilon \) has been set to zero, the integral involves the operator \( W^\alpha D_\alpha \) rather than \( W^\alpha X_\alpha \). The remaining integral can be evaluated by the trick of differentiating with respect to \( t \) and using total \( k \) derivative identities. The result is

\[
K(t) = -\frac{1}{32\pi^2} W^2 \left[ \left( \frac{e^{tN} - 1}{N} \right) \left( \frac{e^{-t(N-tr(N))} - 1}{N-tr(N)} \right) \right] e^{-\frac{1}{2} t \text{tr}N} \\
\det \left( \frac{1 - e^{t(M-M)}}{(M-M)} \right)^{-\frac{1}{2}}.
\]

Although this seems to differ from (18), in fact the two expressions can be
shown to be equivalent. This relies on the fact that $N$ is a $2 \times 2$ matrix, so that 
$$\det N = \frac{1}{2} (tr N)^2 - \frac{1}{2} tr (N^2).$$ Also, 
$$\text{tr} \left( e^{-tN} \right) = e^{-\frac{t}{2} tr(N)} \text{tr} \left[ e^{-t(N - \frac{1}{2} \text{tr}(N)1)} \right],$$
and since $N - \frac{1}{2} \text{tr}(N)1$ is a traceless $2 \times 2$ matrix, only traces of even powers are nonvanishing, with 
$$\text{tr} \left[ (N - \frac{1}{2} \text{tr}(N)1)^{2n} \right] = 2 \left[ \frac{1}{2} \text{tr}(N - \frac{1}{2} \text{tr}(N)1)^2 \right]^n.$$

The second check is to compare the result (18) with component results. In the case where the fermionic components of the supersymmetric Yang-Mills background are set to zero, we expect that the supersymmetric kernel should reduce to the difference of the bosonic kernels in §2 for spin zero and spin half quantum fields in a Yang-Mills background. To see that this is so, consider a supersymmetric theory in the presence of a supersymmetric background. The one loop effective action for the background fields is minus the logarithm of the superdeterminant of the operator appearing in the part of the action quadratic in the quantum fields. If the fermionic components of the background fields are set to zero, then this superdeterminant factorises into a ratio of ordinary determinants, so 
$$\Gamma_{\text{eff}} = \ln \det \Delta_B - \ln \det \Delta_F,$$
where $\Delta_B$ and $\Delta_F$ denote the operators in the pieces of the action quadratic in the bosonic and fermionic quantum fields respectively. This is equivalent to 
$$-\zeta'_B(0) + \zeta'_F(0),$$
where $\zeta_B(s)$ and $\zeta_F(s)$ are the usual zeta functions associated with the operators $\Delta_B$ and $\Delta_F$ respectively. The difference $\zeta_B(s) - \zeta_F(s)$ of the zeta functions is of the form 
$$\frac{1}{\Gamma(s)} \int_0^\infty ds \frac{t^{s-1}}{s} \int d^4x K(t/\mu^2),$$
where 
$$K(t) = \lim_{x \to x'} (e^{t\Delta_B} - e^{t\Delta_F}) \delta^{(4)}(x, x') \equiv K_B(t) - K_F(t).$$
As the latter is difference of functional traces, it is a functional supertrace.

In the case at hand, if the fermionic components of the background Yang-Mills superfields are set to zero, then, with the convention $[D_a, D_b] = F_{ab}$, $N_{\alpha \beta} \rightarrow F_{ab}(\sigma_{ab})_{\alpha \beta}$, $\text{Tr}(N) \rightarrow 0$, $(M - \bar{M})_{ab} \rightarrow -2F_{ab}$, and $W_\alpha \rightarrow -F_{ab}(\sigma_{ab})_{\alpha \beta} \theta_\beta$. Using the antisymmetry of $F_{ab}$, the supersymmetric kernel (18) becomes 
$$K(t) = -\theta^2 \text{det} \left( 1 - e^{-tF} \right) \frac{1}{4\pi^2} \text{det} \left( \frac{1 - e^{2tF}}{F} \right)^{-\frac{1}{2}}$$
where the first determinant is over $2 \times 2$ undotted spinor indices with $F \equiv F_{ab}(\sigma_{ab})_{\alpha \beta}$, and the second determinant is over vector indices with $F \equiv F_{ab}$. On the other hand, using (4) and (5), the difference of the kernel for a scalar
and for a left-handed spinor\footnote{The normalization of the left handed spinor kernel is changed by a factor of $\frac{1}{2}$ relative to that of the scalar kernel because the kernel (5) is computed for the square of the Dirac operator, whereas the effective action contains the determinant of the Dirac operator and not its square.}

\[
\frac{1}{2} tr \left( 1 - e^{-tF} \right) \frac{1}{4\pi^2} \det \left( \frac{1 - e^{2tF}}{F} \right)^{-\frac{1}{2}},
\]

where again the trace is over $2 \times 2$ spinor indices and the determinant is over vector indices. Using the fact that $F_{ab}(\sigma_{ab})_{\alpha\beta}$ is a traceless $2 \times 2$ matrix, it is relatively easy to show the equivalence of $-\det \left( 1 - e^{-tF} \right)$ and $tr \left( 1 - e^{-tF} \right)$, thus ensuring that the supersymmetric kernel (18) for a purely bosonic background does indeed reduce to a difference of kernels for bosonic and fermionic quantum fields.

### 4 Discussion

In this paper, we have illustrated a new approach to computing effective actions in supersymmetric theories by applying it to the determination of the low energy effective action obtained by integrating out massive scalar supermultiplets in the presence of a supersymmetric Yang-Mills background. The approach relies on computing the functional trace of an appropriate heat kernel; however, it is not necessary to compute Green’s functions for the quantum fields in the presence of the background, as advocated by Buchbinder et al [5,6]. Instead, we make use of the similarity of the functional trace of the heat kernel expressed in momentum space to a Gaussian integral. It is expected that the method will also be applicable in the case of scalar superfield backgrounds.

The mass dependence of the effective action involved the Mellin transform of a generalized hypergeometric function. Although we were unable to evaluate the Mellin transform explicitly, the fact that the hypergeometric function reduced to an exponential in the limit $s \to 0$ was enough to determine the form of the mass dependence of the effective action. We note in passing that in the calculation of Buchbinder et al [5] of the low energy effective Kähler potential for the N=1 Wess-Zumino model, the kernel also involves a generalized hypergeometric function. The functional supertrace of the appropriate heat kernel is [5]

\[
K(t) = \frac{\eta \bar{\eta}}{16\pi^2} \sum_{n=1}^{\infty} \frac{n!}{(2n)!} (-t\eta \bar{\eta})^{n-1}.
\]
Here, $\eta$ is the left chiral superfield $\frac{\partial W}{\partial \Phi}$, where $W(\Phi)$ is the superpotential for the chiral scalar superfields $\Phi$ in the Wess-Zumino model. The sum on the right hand side is actually a generalized hypergeometric function,

$$K(t) = \frac{\eta \bar{\eta}}{32\pi^2} \, _1F_1[1, \frac{3}{2}; -\frac{t\eta \bar{\eta}}{4}].$$

This simplifies considerably the computation of the zeta function

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty dt \, t^{s-1} \int d^8 Z \, K(t/\mu^2).$$

Using the integral representation

$$_1F_1[\alpha, \gamma; z] = \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma - \alpha)} \int_0^1 dx \, e^{zx} x^{\alpha-1} (1-x)^{\gamma-\alpha-1}$$

for the confluent hypergeometric functions,

$$\zeta(s) = \int d^8 Z \, \frac{\eta \bar{\eta}}{64\pi^2 \Gamma(s)} \int_0^1 dx \, (1-x)^{-\frac{1}{2}} \int_0^\infty dt \, t^{s-1} e^{-\frac{t\eta \bar{\eta}}{4\mu^2}}$$

$$= \int d^8 Z \, \frac{\eta \bar{\eta}}{64\pi^2} \left(\frac{4\mu^2}{\eta \bar{\eta}}\right)^s \int_0^1 dx \, \frac{(1-x)^{-\frac{1}{2}}}{x^s}.$$  

The integral over $x$ is just the beta function $B(\frac{1}{2}, 1-s) = \frac{\Gamma(\frac{1}{2})\Gamma(1-s)}{\Gamma(\frac{1}{2}-s)}$, so from this it is easy to show

$$\zeta'(0) = \frac{1}{32\pi^2} \int d^8 Z \, \eta \bar{\eta} \left(2 - \ln \frac{\eta \bar{\eta}}{\mu^2}\right),$$

which is the result obtained by Pickering and West in [3] using supergraph techniques. The Schwinger proper time regularization employed by Buchbinder et al [5] yields an expression for the effective Kähler potential which contains a finite constant in the form of an infinite series, which is not evaluated explicitly. Thus recognizing that the kernel is essentially a hypergeometric function in this case yields considerable simplification in the computation of the effective action if zeta function regularization is employed.

Finally, we make a comment of the form of the effective action (18). The effective action computed using the lowest order approximation (13) to the heat kernel is holomorphic in the sense of Seiberg [14], in that it contains only terms with chiral or antichiral gauge superfields. However, when corrections incorporating $\mathcal{D}_a W_b$ are included as in (18), this is no longer true, and the result contains both $\bar{M}_{ab} = (\bar{D}\bar{\sigma}_{ab} W)$ and $M_{ab} = (D\sigma_{ab} W)$ in the one term.
In calculating the effective action for N=2 supersymmetric Yang-Mills theory in N=1 superfield formulation, the result (18) will thus represents a nonholomorphic correction to the effective action which should be included with the nonholomorphic corrections involving scalar superfields computed in [2–4].

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