Tsallis Agegraphic Dark Energy Model

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Using the nonextensive Tsallis entropy and the holographic hypothesis, we propose a new dark energy (DE) model with time scale as infrared (IR) cutoff. Considering the age of the Universe as well as the conformal time as IR cutoffs, we investigate the cosmological consequences of the proposed DE models and study the evolution of the Universe filled by a pressureless matter and the obtained DE candidates. We find that although these models can describe the late time acceleration and the density, deceleration and the equation of state parameters show satisfactory behavior by themselves, however, these models are classically unstable unless the interaction between the two dark sectors of the Universe is taken into account. In addition, the results of the existence of a mutual interaction between the cosmos sectors are also addressed. We find out that the interacting models are stable at the classical level which is in contrast to the original interacting agegraphic dark energy models which are classically unstable.

I. INTRODUCTION

The cosmological constant $\Lambda$ is the simplest approach to DE puzzle, which is responsible for the current acceleration of the Universe expansion, and fills about $\%70$ of energy content of the cosmos. It may also be described by modifying general relativity (GR) meaning that their evolution is not independent from each other, a result decomposes the total energy-momentum conservation law as

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (1)$$
$$\dot{\rho}_D + 3H(1 + \omega_D)\rho_D = -Q, \quad (2)$$

where $\rho_D$ and $\rho_m$ are the DE and DM energy densities, respectively. $\omega_D = \frac{p_D}{\rho_D}$, where $p_D$ denotes the pressure of DE, is also called the state parameter of DE. Such interaction may solve the coincidence problem, and if $Q < 0$ ($Q > 0$), then there is an energy transfer from DM (DE) to DE (DM). See the review for more details and references about the interacting DE models. Although the $\Lambda$ model is consistent with observations, it suffers some difficulties such as the fine-tuning and cosmic coincidence problems. These difficulties motivate physicists to look for other DE candidates.

Agegraphic dark energy (ADE) is an alternative to the $\Lambda$ model based on the uncertainty relation of quantum mechanics. It was argued that in Minkowskian spacetime, the uncertainty in time $t$ is $\delta t = \beta t_p^{2/3} t^{1/3}$ where $\beta$ is a dimensionless constant of order unity and $t_p$ denotes the reduced Plank time. Due to some difficulties of the original ADE, Wei and Cai proposed a new ADE model, in which the conformal time $\eta$, defined as $\delta t = a \delta \eta$, where $t$ is the cosmic time, is taken into account instead of the universe age. It is worthwhile mentioning that the conformal time $\eta$ satisfies the $d^2s = dt^2 - a^2 dx^2 = a^2(d\eta^2 - dx^2)$ relation in the FRW universe. Since the entropy relation has a crucial role in this approach, each modification to the system entropy may change the ADE model. The ADE models have been investigated widely in the literatures.

It was first pointed out by Gibbs that systems with a long range interaction, such as gravitational systems, do not necessarily obey the Boltzmann-Gibbs (BG) theory, and indeed these systems can violate the extensivity constraint of the Boltzmann-Gibbs entropy. Based on the Gibbs arguments, in 1988 Tsallis introduced a statistical description for the non-extensive systems which leads to a new entropy-area relation for horizons. According to Tsallis, the entropy associated with the black hole is written as $S_B = \gamma A^\theta$, where $\gamma$ is an unknown constant and $\theta$ denotes the non-extensive parameter. Applying this non-extensive entropy relation to the apparent horizon of FRW universe, and using the holographic dark energy hypothesis, a new holographic DE model was proposed with energy density $\rho_D = BL^{2\theta - 4}$, where $B$ is an unknown parameter and $L$ is the IR cutoff. More works in which various non-extensive entropies have been used to study the cosmic evolution can also be found in. Here, we are going to use the nonextensive Tsallis entropy to build two Tsallis ADE (TADE) models by using the age of the Universe and the conformal time as the IR cutoffs, and study their effects on the evolution of the Universe.

The organization of this paper is as follows. In the next section, we address TADE model in which the age of the Universe is used as the IR cutoff and study the evolution of the cosmos, whenever there is no interaction between the two dark sectors. In addition, the results of considering a mutual interaction between the dark sectors of cosmos are also investigated. Considering the conformal time instead of the universe age, we introduce a new ADE...
model and study the cosmic evolution in both interacting and non-interacting FRW universes in the third section. The last section is devoted to a summary and concluding remarks.

II. TSALLIS AGEGRAPHIC DARK ENERGY (TADE) MODEL

Considering the age of the Universe as IR cutoff, which is defined as

\[ T = \int_a^a \frac{dt}{H(a)} , \]  

(4)

where \( a \) and \( H = \dot{a}/a \) are the scale factor and the Hubble parameter, respectively, one can use Eq. (3) to write the energy density of TADE as

\[ \rho_D = B T^{2\delta - 4} , \]  

(5)

recovering the primary ADE model of Cai [16] at the limit of \( \delta = 1 \). The first Friedmann equation of a flat FRW universe filled by a pressureless fluid \( \rho_m \) and TADE (\( \rho_D \)) is written as

\[ H^2 = \frac{1}{3m_p^2} (\rho_m + \rho_D) , \]  

(6)

which can also be rewritten as

\[ \Omega_m + \Omega_D = 1 , \]  

(7)

by defining the fractional energy densities

\[ \Omega_m = \frac{\rho_m}{3m_p^2H^2} , \quad \Omega_D = \frac{\rho_D}{3m_p^2H^2} . \]  

(8)

Finally, we easily get

\[ r = \frac{\Omega_m}{\Omega_D} = -1 + \frac{1}{\Omega_D} , \]  

(9)

for the energy densities ratio. As it is apparent from Eq. (8), using the observational values of \( H \), and the fractional energy densities, one may find primary estimations for the allowed intervals of \( B \) and \( \delta \) satisfying observations. The more certain results are achievable by employing observational outcomes on other cosmic parameters such as the deceleration parameter and etc. In the following, since we are eager to show the power of this model, considering \( \Omega_D^0 = 0.73 \) and \( H(a = 1) = 67 \) as the current values of these parameters [37], we only choose some values of the system parameters such as \( \delta \) and \( B \) producing distinctive behaviors.

A. Noninteracting case (\( Q = 0 \))

Inserting the time derivative of Eq. (4) in the conservation equation (2), one obtains

\[ \omega_D = -1 - \frac{2\delta - 4}{3TH} , \]  

(10)

where

\[ T = \left( \frac{3H^2\Omega_D}{B} \right)^{\frac{1}{2\delta - 4}} . \]  

(11)

FIG. 1: Evolution of \( \omega_D \) versus redshift parameter \( z \) for non-interacting TADE. Here, we have taken \( \Omega_D^0 = 0.73, B = 2.4 \) and \( H(a = 1) = 67 \).

Additionally, by combining the time derivative of Eq. (10) and using Eqs. (11) and (2), we reach

\[ \frac{\dot{H}}{H^2} = -\frac{3}{2} (1 - \Omega_D) + \frac{(\delta - 2)\Omega_D}{TH} , \]  

(12)

which can also lead to

\[ q \equiv -1 - \frac{\dot{H}}{H^2} = -\frac{1}{3} - \frac{3\Omega_D}{2} - \frac{(\delta - 2)\Omega_D}{TH} , \]  

(13)

for the deceleration parameter. It is also a matter of calculation to show

\[ \dot{\Omega}_D = \frac{(2\delta - 4)\Omega_D}{T} + 2\Omega_D H(1 + q) , \]  

(14)

where dot denotes the derivative with respect to the cosmic time. In order to study the effects of perturbations on the classical stability of the model, the squared of the sound speed (\( v_s^2 \)) should be evaluated,

\[ v_s^2 = \frac{dP_D}{d\rho_D} = \frac{\dot{P}_D}{\dot{\rho}_D} = \frac{\rho_D}{\dot{\rho}_D} \dot{\omega}_D + \omega_D , \]  

(15)
FIG. 2: Evolution of $q$ versus redshift parameter $z$ for non-interacting TADE. Here, we have taken $\Omega_0^D = 0.73$, $B = 2.4$ and $H(a = 1) = 67$.

which finally leads to
\[
v_s^2 = \frac{\Omega_D - 3}{2} + \frac{3^{\frac{3}{4}}(H^2\Omega_DB^{-1})^{\frac{1}{4}}(5 - 2\delta + (\delta - 2)\Omega_D)}{3H},
\]

for the non-interacting case. The evolution of the system parameters are plotted in Figs. 1-4. It is apparent that the model is classically unstable ($v_s^2 < 0$). Moreover, it is apparent that there are some values of $\delta$ for which $\Omega_D$ and $q$ can show satisfactory behavior by themselves. The $\delta = 2.6$ case is interesting, because, in addition to $\Omega_D$ and $q$, it leads to suitable behavior for the state parameter ($\omega_D \approx -1$) during the cosmic evolution.

B. Interacting case ($Q \neq 0$)

As mentioned earlier, recent observations indicate that the evolution of DM and DE is not independent, a key to solve the coincidence problem [14]. Here, the $Q = 3b^2H(\rho_D + \rho_m)$ mutual interaction between the dark sectors of cosmos [12] is assumed to get the expressions for deceleration parameter, the equation of state, the evolution of density parameter, and also $v_s^2$ as

\[
q = -\frac{1}{3} - \frac{3\delta - 4}{2} - \frac{3\Omega_D}{(\delta - 2)\Omega_D} - \frac{(\delta - 2)\Omega_D}{TH},
\]

\[
\omega_D = -1 - \frac{b^2}{\Omega_D} - \frac{2\delta - 4}{3TH},
\]

\[
\dot{\Omega}_D = \frac{(2\delta - 4)\Omega_D}{T} + 2\Omega_DH(1 + q),
\]

\[
v_s^2 = \frac{3 + b^2 - \Omega_D}{2} - \frac{3^{\frac{7}{4}}b^2H(\Omega_DB^{-1})^{\frac{1}{4}}(1 + b^2 + \Omega_D)}{6(\delta - 2)\Omega_D}
\]

\[
- \frac{3^{\frac{3}{4}}b^2H(\Omega_DB^{-1})^{\frac{1}{4}}(5 - 2\delta + (\delta - 2)\Omega_D)}{H},
\]

plotted in Figs. 5-9 which show satisfactory behaviors for $\Omega_D$ and $q$. As it is apparent, this case is classically stable, a behavior unlike those of THDE [24] and the non-interacting case, and moreover, $\omega_D$ acted as that of the phantom sources in past.
FIG. 4: Evolution of $v_2^s$ versus redshift parameter $z$ for non-interacting TADE. Here, we have taken $\Omega_0^D = 0.73$, $B = 2.4$ and $H(a = 1) = 67$.

FIG. 5: Evolution of $\omega_D$ versus redshift parameter $z$ for interacting TADE. Here, we have taken $\Omega_0^D = 0.73$, $B = 2.4$, $\delta = 2.6$ and $H(a = 1) = 67$.

III. NEW TSALLIS AGEGRAPHIC DARK ENERGY MODEL (NTADE)

Due to some problems of the original ADE [16], a new ADE was proposed by Wei and Cai [17], in which the conformal time $\eta$ is used as the IR cutoff instead of the age of the Universe. The conformal time is defined as

FIG. 6: Evolution of $q$ versus redshift parameter $z$ for interacting TADE. Here, we have taken $\Omega_0^D = 0.73$, $B = 2.4$, $\delta = 2.6$ and $H(a = 1) = 67$.

FIG. 7: Evolution of $\Omega_D$ versus redshift parameter $z$ for interacting TADE. Here, we have taken $\Omega_0^D = 0.73$, $B = 2.4$, $\delta = 2.6$ and $H(a = 1) = 67$.

FIG. 8: Evolution of $v_2^s$ versus redshift parameter $z$ for interacting TADE. Here, we have taken $\Omega_0^D = 0.73$, $B = 2.4$, $\delta = 2.6$ and $H(a = 1) = 67$. 
FIG. 9: Evolution of $v_s^2$ versus redshift parameter $z$ for interacting TADE. Here, we have taken $\Omega_D^0 = 0.73$, $B = 2.4$, $b^2 = 0.01$ and $H(a = 1) = 67$.

dt = \text{d}\eta$ leading to $\dot{\eta} = 1/a$ and thus

$$\eta = \int_0^a \frac{\text{d}a}{Ha^2}. \quad (18)$$

In this manner, using Eq.(19), the energy density of NTADE is written as

$$\rho_D = B\eta^2 - 4\delta - 4. \quad (19)$$

A. Non-interacting case

Whenever there is no interaction between the dark sectors of cosmos ($Q = 0$), one can insert Eq.(19) and its time derivative into Eq.(2) to reach

$$\omega_D = -1 - \frac{2\delta - 4}{3\eta H}, \quad (20)$$

where $\eta = \left(\frac{3H^2\Omega_D}{B}\right)^{\frac{1}{2}}$. Differentiating Eq.(8) and using Eqs. (19) and (11), we arrive at

$$q = -\frac{1}{3} \left(\frac{3\Omega_D}{2} - \frac{(\delta - 2)\Omega_D}{\a \eta H}\right), \quad (21)$$

for the deceleration parameter. In addition, it is a matter of calculation to use Eqs. (8) and (19) in order to show that

$$\dot{\Omega_D} = \frac{(2\delta - 4)\Omega_D}{a \eta} + 2\Omega_D H (1 + q). \quad (22)$$

Finally, the squared of the sound speed is find out as

$$v_s^2 = \frac{3\Omega_D - 7}{6} + \frac{6\Omega_D - 2\delta + (\delta - 2)\Omega_D - 4}{3H}. \quad (23)$$

The evolution of the system parameters has been depicted in Figs. 10-13 claiming that although $\Omega_D$, $\omega_D$ and $q$ can show acceptable behavior by themselves during the cosmic evolution, the model is classically unstable ($v_s^2 < 0$).

B. Interacting case

Considering the $Q = 3b^2H(\rho_D + \rho_m)$ mutual interaction between the dark sectors of cosmos [12], it is a matter of calculations to find

$$\omega_D = -1 - \frac{\Omega_D}{3\eta H}, \quad (24)$$

$\Omega_D = \frac{2\delta - 4}{3\eta H}$, and

$$\dot{\Omega_D} = \frac{(2\delta - 4)\Omega_D}{a \eta} + 2\Omega_D H (1 + q).$$

It is apparent that the model is classically stable, and its parameters, including $q$, $\omega_D$ and $\Omega_D$, have satisfactory behaviors. Thus, just the same as the TADE model, the existence of the $Q = 3b^2H(\rho_D + \rho_m)$ interaction between the cosmos sectors make the model stable at the classical level.
FIG. 11: Evolution of $q$ versus redshift parameter $z$ for non-interacting NTADE. Here, we have taken $\Omega_0^D = 0.73$, $B = 2.4$ and $H(a = 1) = 67$.

IV. CLOSING REMARKS

Since gravity is a long range interaction, one should use the generalized nonextensive Tsallis entropy for studying its related phenomena [23, 26, 28, 34]. In this paper, inspired by the Tsallis entropy [23] and based on the holographic hypothesis, we proposed a new DE model with time scale as IR cutoff. We consider the age of the Universe and the conformal time as system’s IR cutoffs. The behavior of $q$, $\omega_D$, $\Omega_D$ and $v^2_s$ have been studied during the cosmic evolution. It was observed that in the absence of interaction both of these models are classically unstable. In addition, we address the consequences of the existence of a mutual interaction between the dark sectors of cosmos. We found out that, unlike the original ADE models based on the Bekenstein-Hawking entropy [1], the interacting models introduced here are classically stable. This is an interesting result which confirms that interacting TADE and NTADE models may be useful in explaining the late time DE dominated universe. Our study shows also that the predictions of the models for the cosmic evolution are more sensitive to $\delta < 1$ rather than $\delta > 1$. This sensitivity is obtainable by comparing the corresponding curves, and is affected by the initial conditions used for plotting the curves. Holographic hypothesis is the backbone of the ADE models, a fact claiming that, in addition to the sign of the sound speed square, a full analysis on their stability should also consider their non-local features [38, 39]. The second approach is out of our goal in this paper, and can be considered as a serious issue for the future works.

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[1] K. Y. Kim, H. W. Lee and Y. S. Myung, Physics Letters B 660, 118 (2008).
[2] T. Padmanabham, Phys. Rep. 380, 235 (2003);
V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D 9, 373
FIG. 13: Evolution of $v_s^2$ versus redshift parameter $z$ for non-interacting NTADE. Here, we have taken $\Omega_0^D = 0.73$, $B = 2.4$ and $H(a = 1) = 67$.

FIG. 14: Evolution of $\omega_D$ versus redshift parameter $z$ for interacting NTADE. Here, we have taken $\Omega_0^D = 0.73$, $B = 2.4$, $\delta = 2.6$ and $H(a = 1) = 67$.

FIG. 15: Evolution of $q$ versus redshift parameter $z$ for interacting NTADE. Here, we have taken $\Omega_0^D = 0.73$, $B = 2.4$, $\delta = 2.6$ and $H(a = 1) = 67$.

FIG. 16: Evolution of $\Omega_D$ versus redshift parameter $z$ for interacting NTADE. Here, we have taken $\Omega_0^D = 0.73$, $B = 2.4$, $\delta = 2.6$ and $H(a = 1) = 67$.

FIG. 17: Evolution of $v_s^2$ versus redshift parameter $z$ for interacting NTADE. Here, we have taken $\Omega_0^D = 0.73$, $B = 2.4$, $\delta = 2.6$ and $H(a = 1) = 67$.

(2000); P.J.E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 559 (2003).  
[3] A. G. Riess et al, Astron. J. 116, 1009 (1998).  
[4] S. Perlmutter et al, Astrophys. J. 517 565 (1999).  
[5] P. J. E. Peebles and B. Ratra, Astrophys. J. 325, L17 (1988).  
[6] B. Ratra and P. J. E. Peebles, Phys. Rev. D 37, 3406.
FIG. 18: Evolution of $v_2^s$ versus redshift parameter $z$ for interacting NTADE. Here, we have taken $\Omega_0^D = 0.73$, $B = 2.4$, $b^2 = 0.01$ and $H(a = 1) = 67$.

(1988).

[7] C. Wetterich, Nucl. Phys. B 302, 668 (1988).

[8] A. Sheykhi, B. Wang, N. Riazi, Phys. Rev. D 75, 123513 (2007).

[9] A. Joyce, L. Lombriser and F. Schmidt, Ann. Rev. Nucl. Part. Sci. 66, 95 (2016).

[10] L. Amendola, Phys. Rev. D 62, 043511 (2000); W. Zimdahl and D. Pavon, Phys. Lett. B 521, 133 (2001); L. P. Chimento, A. S. Jakubi, D. Pavon and W. Zimdahl, Phys. Rev. D 67, 083513 (2003); S. del Campo, R. Herrera, G. Olivares and D. Pavon, Phys. Rev. D 74, 023501 (2006).

[11] B. Wang, E. Abdalla, F. Atrio-Barandela and D. Pavon, Rept. Prog. Phys. 79, no. 9, 096901 (2016).

[12] D. Pavon, W. Zimdahl, Phys. Lett. B 628, 206 (2005).

[13] A. Sheykhi, M.S. Movahed, E. Ebrahimi Astrophys. and Space Sci. 339, 93 (2012).

[14] V. Salvatelli, N. Said, M. Bruni, A. Melchiorri and D. Wands, Phys. Rev. Lett. 113, 181301 (2014).

[15] Planck Collaboration, Astron. and Astrophys. 594, A13 (2016).

[16] R. G. Cai, Phys. Lett. B 657, 228 (2007).

[17] H. Wei and R. G. Cai, Phys. Lett. B 660, 113 (2008).

[18] H. Wei and R. G. Cai, Eur. Phys. J. C 59, 99 (2009).

[19] H. Wei and R. G. Cai, Phys. Lett. B 663 (2008) 1; J. Cui, et al., arXiv:0902.0716

Y. W. Kim, et al., Mod. Phys. Lett. A 23 (2008) 3049; Y. Zhang, et al. [arXiv:0708.1214]

J. P. Wu, D. Z. Ma, Y. Ling, Phys. Lett. B 663, (2008) 152;
K. Y. Kim, H. W. Lee, Y. S. Myung, Phys.Lett. B 660 (2008) 118;
X. Wu, et al., [arXiv:0708.0349];
J. Zhang, X. Zhang, H. Liu, Eur. Phys. J. C 54 (2008) 303;
I. P. Neupane, Phys. Lett. B 673 (2009) 111.

[20] A. Sheykhi, Phys. Lett. B 680 (2009) 113;
A. Sheykhi, Int. J. Mod. Phys. D 18, No. 13 (2009) 2023;
A. Sheykhi, A. Bagheri, M.M. Yazdanpanah, JCAP 09, 017 (2010);
A. Sheykhi, Phys. Rev. D 81, 023525 (2010) ;
A. Sheykhi, Int. J. Mod. Phys. D, Vol. 19, No. 3 (2010) 305;
A. Sheykhi and M.R. Setare, Int. J. Theor. Phys. 49, 2777 (2010).

[21] J. W. Gibbs, Elementary Principles in Statistical Mechanics (New York, 1902).

[22] C. Tsallis, J. Stat. Phys. 52, 479 (1988).

[23] C. Tsallis, L. J. L. Caro, Eur. Phys. J. C 73 (2013) 2487.

[24] M. Tavayef, A. Sheykhi, K. Bamba and H. Moradpour, Phys. Lett. B. 781, 195 (2018).

[25] M. Abdollahi Zadeh et al. [arXiv:1806.07285].

[26] S. Ghaffari et al. Eur. Phys. J. C 78, 706 (2018).

[27] S. Ghaffari et al. Phys. Dark. Univ. DOI: 10.1016/j.dark.2018.11.007.

[28] N. Saridakis, K. Bamba, R. Myrzakulov, A. Lymparis and E. N. Saridakis, [arXiv:1806.04614].

[29] A. Sheykhi, Phys. Lett. B (in press) [arXiv:arXiv:1806.03996].

[30] E. M. Barboza Jr., R. C. Nunes, E. M. C. Abreu, J. A. Neto, Physica A 436, 301 (2015).

[31] R. C. Nunes, E. M. Barboza, E. M. C. Abreu and J. A. Neto, JCAP 1608, 051 (2016).

[32] H. Moradpour, Int. J. Theo. Phys. 55, 4176 (2016).

[33] A. Sayahian Jahromi et al, Phys. Lett. B 780, 21 (2018).

[34] H. Moradpour et al. [arXiv:1803.02195].

[35] H. Moradpour, A. Bonilla, E. M. C. Abreu, J. A. Neto, Phys. Rev. D 96, 123504 (2017).

[36] N. Komatsu, Eur. Phys. J. C 77, 229 (2017).

[37] M. Roos, Introduction to Cosmology (John Wiley and Sons, UK, 2003).

[38] M. Li, C.S. Lin, Y. Wang, J. Cosmol. Astropart. Phys. 0805, 023 (2008).

[39] P. Huang, Y.C. Huang, Eur. Phys. J. C 73, 2366 (2013).