Performance Investigations of the Uniformly and Non-Uniformly Excited Linear Arrays

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Abstract. Uniform linear arrays include the arrangement of all array elements in one dimension with equal amplitude excitations. Such types of arrays usually have good directivity, narrow beam-width, and they suffer from high sidelobe levels that may cause interference and degrade the overall system performance. The problem of high side lobe levels may overcome by using non-uniformly excited arrays instead of its uniform counterpart. In this paper, the Dolph and Taylor excited arrays are adopted as a non-uniformly excited array. The performance in terms of half power beam width (HPBW), Peak sidelobe level (SLL), directivity (D), and first null-to-null beam width of the uniformly and the two non-uniformly excited arrays are investigated and compared. Simulation results show that the non-uniformly excited arrays can significantly reduce the peak SLL at the cost of lower directivity and wider HPBW. Thus, it is advised to use the non-uniformly excited arrays in the environments that borne high interference pollution.

1. Introduction

Many practical applications of the antenna arrays require high demand on the required radiation characteristics that cannot be met by a single radiant antenna; thus, multiple elements can be used to fulfill such requirements. An antenna array is a collection of two or more space-separated antennas arranged in a specific structure. Generally, an antenna array can increase the overall gain, increase diversity, cancel interference from a set of directions and steer the main beam in a particular direction [1]. By appropriately exciting each individual element with certain amplitudes and phases, arrays can produce the desired radiation characteristics. They are referred to as linear antenna arrays when the antenna components are positioned in a straight line [2]. An array of identical elements of the same magnitude, equal spaced, and each with a constant progressive phase is known as a uniformly excited array, and if its main beam is found to be perpendicular to the array axis, it’s considered to be a broadside array. The authors in [3] studied the spacing and phase progression between the array elements. Arrays with equal spacing between their elements and unequal amplitudes are called non-uniformly excited arrays such as Dolph-Chebyshev and Taylor antenna arrays. The authors in [4] introduced a backfire antenna pattern with non-linear excitation (like Taylor) and uniform space between array elements. The authors also show that the directivity and the peak SLL of the designed antenna where the directivity as a function of antenna length was plotted. In [5], the authors studied a non-uniform antenna array for which the elements are equally spaced with unequal amplitude excitation using Chebyshev excitation method. They also investigated the effect of number of array elements, separation distance, and the amplitude excitations of the array elements on the radiation characteristics. The separation distance between array elements were allowed to vary from 0.1λ to 2λ in steps of 0.02λ for a number of elements up to 10 elements. Kailash et al in,[6], offered a comparative analysis of the effects of different spacing and the number of the array elements in terms of array factor, directivity and half-power beam-width (HPBW) for the Broadside case. They found that the directivity and the radiation characteristics can be improved proportionally with the increased number of the array elements. In [7-10], different methods were proposed to design Dolph
and Taylor arrays, while in [11-18] other methods were used to synthesis an optimum radiation patterns with desired constraints.

2. Principles of the Uniform and Non-Uniform Arrays

2.1 Uniform Array

In this type of arrays, all the antenna elements are fed with equal amplitude and phase. Generally, there are two main types of such arrays namely: Broadside array and End-Fire array. Broadside array designed to radiate its radiation pattern that is perpendicular to the direction of the axis of an array. The maximum radiation direction is always perpendicular to the line or plane of the array according to the elements on a line or a plane. The following equations of the Array factor (AF), half power beam width (HPBW), first null beam width (FNBW) and the side lobe level (SLL) for the broadside uniform linear array are:

$$AF_n = \frac{1}{N} \sin \left( \frac{N \Psi}{2} \right)$$  \hspace{1cm} (1)

Where $N$ is the number of array elements, $\Psi = kd \cos \theta + \beta$, $k=\frac{2\pi}{\lambda}$, $d$ is the separation distance between array elements, $\beta$ is the phase excitation difference (for broadside antenna, $\beta=0$).

$$FNBW = 2 \left[ \frac{\pi}{2} - \cos^{-1}\left( \frac{\lambda}{Nd} \right) \right]$$ \hspace{1cm} (2)

$$HPBW = 2 \left[ \frac{\pi}{2} - \cos^{-1}\left( \frac{1.391 \lambda}{\pi d} \right) \right]$$ \hspace{1cm} (3)

$$SLL = 20 \log_{10} \left( \frac{2}{3\pi} \right)$$ \hspace{1cm} (4)

2.2 Non-Uniform Array

In non-uniform arrays the elements are fed with different amplitudes and/or phases. Some common and widely used Types of non-uniform linear amplitude arrays include Dolph-Chebyshev array and Taylor array [4].

A typical example of the non-uniformly excited arrays is the Dolph-Chebyshev array. The Dolph-Chebyshev method is a compromise between the uniform and binomial methods, as if no side lobes are produced by the Dolph-Chebyshev method; it decreases to the binomial model [5][6]. Dolph-Chebyshev produced a pattern for a uniformly spaced array that has a minimum possible beam width for a given maximum sidelobe level. The pattern of this array is obtained from the Chebyshev polynomial, and it has $2N$ nulls for an array of $2N + 1$ elements. The level or height of all the sidelobes is equal. In some cases of the Dolph distributions, the element amplitude excitations of the last end element on each side of the array may gradually increase in their amplitudes with compared to the previous value to achieve low side lobe array pattern [7].

To design the Dolph-Chebyshev array, the following equations are used:

$$f = 1 + 0.636 \left( \frac{2}{R_0} - \cosh( (\cosh^{-1} R_0)^2 - \pi^2)^{1/2} \right)$$ \hspace{1cm} (7)

where $f$ represents the broadening factor in the main beam of the Dolph excited array with compared to that of the uniformly excited array.

$$D_0 = \frac{2R_0^2}{1+(R_0^2-1)^2/4}$$ \hspace{1cm} (8)

Where $D_0$ is the maximum directivity in the Dolph excited array.

$$R_0 \text{ (voltage ratio)} = 10^{\frac{R_0 \text{ dB}}{20}}$$ \hspace{1cm} (9)
Where $R_0$ is the major side lobe voltage ratio and $L$ is the length of the array. The HPBW of the Dolph-Chebyshev array can be calculated by firstly compute the value of the HPBW for the uniformly excited array under the same number of the array elements and spacing between them. After that we may find the value of HPBW of the Dolph-Chebyshev array by multiplying the HPBW of the uniform array by the beam broadening factor ($f$) as follows:

$$\text{HPBW}_{\text{uniform}} = \cos^{-1}\left[ \cos \theta_0 - 0.443 \frac{\lambda}{L+d} \right] - \cos^{-1}\left[ \cos \theta_0 + 0.443 \frac{\lambda}{L+d} \right]$$

(10)

$$\text{HPBW}_{\text{Dolph}} = \text{HPBW}_{\text{uniform}} \times f$$

(11)

Another type of the non-uniform arrays is the Taylor array, which is similar to the Dolph-Chebyshev array, but it is more applicable to the continuous distributions. An array pattern that is optimal compromise between the beam width and the side lobe level is the one that produced by the Taylor design [8]. The minor lobes are maintained at an equal and specific level in an ideal design. Since the minor lobes are of equal ripple and stretch to infinity, an infinite power is implied. The technique as applied by Taylor, however, realistically leads to a pattern whose first few minor ones are monotonically decaying. Practically, a small monotonically decay exhibits even the degree of the nearest minor lobes. This decay is a function of the $u$-space over which these minor lobes must be located. The rate of decay of the closest minor lobes increases as this space decreases. For a very large space of $u$ (over which the closest minor lobes are required to have an equal ripple), the rate of decay is negligible [9].

To design the Taylor array, the following equation are usually used:

$$AF_{\text{Taylor}} = \frac{\sin u \prod_{n=1}^{n=2} \left[ 1 - \left( \frac{\lambda}{\lambda l \cos \theta_n} \right)^2 \right]}{u \prod_{n=1}^{n=2} \left[ 1 - \left( \frac{\lambda}{\lambda l \cos \theta_n} \right)^2 \right]}$$

(12)

Where $u = \frac{1}{\lambda} \cos \theta$, and $u_n = \frac{1}{\lambda} \cos \theta_n$

The value of the HPBW of such type of array is computed by

$$\Theta_0 = 2 \sin^{-1} \left[ \frac{\lambda \sigma}{\pi} \left( \cosh^{-1} R_0 \right)^2 - \left( \cosh^{-1} \frac{R_0}{\sqrt{2}} \right)^2 \right]^{1/2}$$

(13)

Where

$$\sigma = \frac{n}{\left( A^2 + \left( \frac{n-2}{2} \right)^2 \right)^{1/2}}$$

(14)

$$A = \frac{1}{\pi} \cosh^{-1}(R_0)$$

(15)

$$\text{SLL} = -20 \log (\cosh(\pi A))$$

(16)

3. Results and Discussions

In this section, the radiation patterns and some other characteristics will be investigated to verify the principles of such arrays with highlight of main advantages and disadvantages. After completing the design of the aforementioned arrays in MATLAB software, The code implemented for the uniform arrays that have the following specifications; The number of the considered array elements, $N$ is equal to 10, the separation distances between the elements is $d=0.5\lambda$, the progressive phase $\beta=0$, and the amplitude excitations of the uniformly excited arrays is made to be ones for all array elements and for non-uniformly excited arrays (Taylor and Dolph-Chebyshev) they made varied according to the Dolph
or Taylor distributions. The other specifications are assumed to be same as in the uniform arrays. For non-uniform arrays, the peak sidelobe level is made to be equal to -26 dB. The following results are obtained as shown in figures from Figure 1 to Figure 6 as well as the numerical values listed in Table 1.

The results of the uniformly excited array are shown in Figures 1 and 2 where the radiation pattern and the corresponding amplitude and phase excitations are plotted. It can be seen that the sidelobe level of this array is relatively high about -13.23 dB which is undesirable and may cause many problems in practice. Other performance measures are listed in Table 1. The results of the Dolph excited array are shown in Figures 3 and 4. It can be seen that the sidelobe level was greatly reduced up to -26 dB at the cost of wider half power beam width and lower directivity. As it is clear, the Dolph pattern has equal SLL.

The results of applying the Taylor excited array are shown in Figures 5 and 6. It can be seen that almost the same performance as the Dolph was noticed. Also, it can be seen that the amplitude excitation of the Taylor array has a large difference between the center and the edges of array which makes the implementation issue of the feeding network more complex and expensive.

Finally, Table 2 compares the performances of various designed array that were presented in [14], [15], [17], and [18].

**Table 1:** comparison between uniform array and non-uniform array (Dolph-Chebyshev and Taylor).

|                | Uniform array | Chebyshev array | Taylor array |
|----------------|---------------|-----------------|--------------|
| FNBW (deg.)    | 23.2          | 32.1            | 32.1         |
| HPBW (deg.)    | 10.19         | 10.9            | 10.9         |
| Directivity (dB)| 10            | 9.189           | 9.08         |
| SLL (dB)       | -13.46        | -26             | -26          |
Figure 1. Radiation pattern of the uniformly excited array.

Figure 2. Amplitude excitation (blue) and the phase excitation (red) of the uniform array.

Figure 3. Radiation pattern of the Dolph excited array.

Figure 4. Amplitude excitation (blue) and the phase excitation (red) of the Dolph array.
Table II: Comparison between various designed arrays (Uniform, partially element optimized [14] and [15], End element location [17], Fully element optimized [18]).

|                  | Uniform array | Designed array in [14] and [15] | Designed array in [17] | Designed array in [18] |
|------------------|---------------|----------------------------------|-------------------------|-------------------------|
| N=10, β=0, d=0.5 |               |                                  |                         |                         |
| FNBW (deg.)      | 23.2          | 33.4                             | 32.21                   | 35.0                    |
| HPBW (deg.)      | 10.19         | 10.4                             | 10.25                   | 11.2                    |
| Directivity (dB) | 10            | 9.6                              | 9.8                     | 8.28                    |
| SLL (dB)         | -13.46        | -25                               | -18                     | -40                     |

4. Conclusions

It has been shown from the presented results that the non-uniformly excited Dolph and Taylor arrays give lower sidelobe level with compared to that of the uniformly excited array. This reduction in the sidelobe level is at the cost of wider beam width and lower directivity. Thus, the designer needs to carefully compromise between the required sidelobe level and the desired half power beam width and the directivity. More specifically, the results of the uniformly excited array show that the sidelobe level is at -13.24 dB, directivity was $D = 10$ dB, and the HPBW was 10.19 degree, while for non-uniformly Dolph excited array the sidelobe level was -26 dB, D=9.08, and HPBW= 10.9 degree. For Taylor array the SLL=-26 dB, D=9.189, and HPBW=10.9 degree.

From these results, we may conclude that the uniform array, generally, gives higher directivity and sharper beam width, which is very desirable in the long-distance communication systems. For applications that require a robust performance against interfering signals we need to use, Dolph or...
Taylor arrays or other arrays which they have lowest sidelobes to minimize the effect of interfering signals. This study can be further extended by using an optimization (or adaptive) algorithm [19-20] to optimize the array performance. This left as a future work.

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