Transfer of an arbitrary photon state along a cavity array without initialization

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Abstract
We propose a quantum state transfer (QST) scheme that transfers any single-mode photon state along a one-dimensional coupled-cavity array (CCA). By building a map from QST in a CCA to that in a spin-$\frac{1}{2}$ chain, we show that many previous results of QST schemes for the spin chain system find similar applications in the CCA system. Furthermore, high fidelity QST along a long CCA can be achieved for arbitrary initial states. Using numerical simulations we provide a visual presentation of the result: at some time $\tau$ the CCA system gets high fidelity QST under different initial conditions. Finally we discuss possible experimental realizations of our QST scheme.

Introduction
Recently the technology of producing high quality cavities has been improving remarkably [1–4]. The toroidal micro-cavity supporting whispering-gallery modes circulating around the outer circumference of a toroid can reach an ultra high Q-factor greater than $10^8$ [1]. Photonic crystals represent an appealing alternative owing to their mature technology [2–4]. Cavities with Q-factor $10^6$ have been realized using photonic crystals [4]. In addition, many quantum computation schemes based on optical photons have been proposed for the dirigibility of light [5], where cavity QED systems are used as phase shifters, beamsplitters are used as the Hadmard gates [6], and optical fibers are used as the corresponding communication proposals for photons. Here a qubit is encoded on the location of a single photon between two space modes or its polarization [7, 8]. All-optical apparatuses have also been proposed to realize the all-optical communication of information [9–11]. For the scheme of quantum computation based on photonic qubits, the photonic channel is natural and reasonable.

In this paper we propose a QST scheme that transfers any single-mode photon state along a one-dimensional coupled-cavity array (CCA). The coupled cavities can be realized using photonic crystals [2–4], toroidal microresonators [1] or could be integrated in an optical fiber with a periodic index of fabrication as shown in figure 1(a). With the development of technology of the control of photons in a cavity [12–16], the writing in and reading out of photon states could be realized.

It is worth mentioning that for the scheme of quantum computation based on solid qubits, such as quantum dots [17], superconducting qubits [18], and the N-V center [19], spin chains are proposed as the quantum channel [20, 21]. It is shown that the high fidelity of state transfer could be achieved through a long unmodulated spin chain. QST along an unmodulated spin chain can be perfect only when the length of the spin chain is less than 4. For a chain of any length perfect QST can be achieved by modulating the coupling strengths between adjacent spins [22–25]. Other schemes are also discussed, such as only tuning the coupling strengths to get high fidelity QST [26–40], QST without initialization [41, 42], optimizing basis [43, 44], using special external potential [45–47] or measurement [48] and generalizing to high spin QST [49–52]. A number-theoretic relation between QST and the length of a one-dimension spin chain is found in [53].

We will show that all the previous results obtained in the spin chain system mentioned above find similar applications in our scheme, and the initialization step is not needed. Notice that the Hilbert space dimension of a single mode is infinite, and our scheme can be used to transfer the quantum states in any finite dimensional...
Hilbert space with an appropriate mapping of quantum states. So our proposal provides a possibility to realize a more easily controlled and arbitrary dimension quantum state transfer scheme than those using spin chains.

This article is organized as follows. First we propose the QST scheme, where the Hamiltonian of the system is given. Next we analyse the fidelity of QST in our scheme and give the condition of perfect QST. Then we solve the dynamic problem about fidelity. After that we simulate the QST using our scheme in three cases: uniform coupling CCA, perfect modulated CCA and CCA with coupling strengths in the ballistic regime. Finally we discuss the possible experimental realization of our scheme.

Scheme and analysis

The system of our scheme is a CCA as depicted in figure 1(b). Every cavity has the same cavity mode with frequency $\omega$. Photons can hop between adjacent cavities due to the overlap of the light mode [54]. The Hamiltonian is given by

$$H = \omega \sum_{n=1}^{N} \hat{a}_{n}^{\dagger} \hat{a}_{n} + \sum_{n=1}^{N-1} J_n \left( \hat{a}_{n}^{\dagger} \hat{a}_{n+1}^{\dagger} + \hat{a}_{n} \hat{a}_{n+1} \right),$$

where $\omega$ is the frequency of the cavity mode, $J_n$ is the coupling strength between the $n$th cavity and the $(n+1)$th cavity, which can be adjusted by changing the thickness of the mirror between the two cavities.

The process of the QST along the CCA is as follows. First, the state we want to transfer is encoded on the photons in the first cavity (cavity $A$). After some time $\tau$, the state is transferred to the other end of the array (cavity $B$). The Gauss curve denotes the transmitted photon state. The wavy lines in the cavities represent some arbitrary states of the single-mode photon. (c) Coupled cavities realized using an array of toroidal microresonators coupled by evanescent-field between the high-Q whispering-gallery modes. The dark disks are the cavities formed from fused SiO$_2$ and the light-colored rectangle represents the silicon chip.

![Figure 1. Our quantum communication protocol. (a) Coupled cavities realized in an optical fiber with a periodic index of refraction. The dark columns with a concave surface represent the parts with a high index of refraction. (b) Initially the state to be transferred is prepared in the first cavity (cavity $A$). After some time $\tau$, the state is transferred to the other end of the array (cavity $B$). The Gauss curve denotes the transmitted photon state. The wavy lines in the cavities represent some arbitrary states of the single-mode photon. (c) Coupled cavities realized using an array of toroidal microresonators coupled by evanescent-field between the high-Q whispering-gallery modes. The dark disks are the cavities formed from fused SiO$_2$ and the light-colored rectangle represents the silicon chip.](image-url)
When $\hat{a}_N(t) = \hat{a}_i$, the fidelity of the system at time $\tau$ is

$$F(\tau) = \text{Tr}\left( U^\dagger(\tau) \left| \phi_N \right\rangle \left\langle \phi_N \right| U(\tau) \left| \phi_i \right\rangle \left\langle \phi_i \right| \otimes \rho_{2...N} \right)$$

$$= \text{Tr}\left( \left( a_N^\dagger(\tau) \right| 0 \right) \left( 0 | f^\dagger(a_N^\dagger(\tau)) \right) \left| \phi_i \right\rangle \left\langle \phi_i \right| \otimes \rho_{2...N} \right)$$

$$= \text{Tr}\left( \left( a_i^\dagger \right| 0 \right) \left( 0 | f^\dagger(a_i^\dagger) \right) \left| \phi_i \right\rangle \left\langle \phi_i \right| \otimes \rho_{2...N} \right)$$

$$= \text{Tr}\left( \left| \phi_i \right\rangle \left\langle \phi_i \right| \otimes \rho_{2...N} \right) = 1.$$ 

In other words, to get a perfect photon state transfer means to get a time $\tau$ that $\hat{a}_N(\tau) = \hat{a}_i$. It can be easily verified by noting that the expected value of any operator in the Hamiltonian $N$-th cavity at time $\tau$ is equal to that of the operator in the first cavity at initial state, e.g., $\left\langle \hat{a}_N^\dagger \hat{a}_N(\tau) \right\rangle = \left\langle \hat{a}_i^\dagger \hat{a}_i(0) \right\rangle$.

Now we analyze the dynamics of $\hat{a}_N(t)$, which satisfies the Heisenberg equation

$$\frac{d\hat{a}_N(t)}{dt} = i\left[ H, \hat{a}_N(t) \right].$$

First we note that the set $\{|a_n^\dagger| n = 1, 2, 3, \ldots, N\}$, is closed under the action $[H, \cdot]$. So $\hat{a}_N(t)$ can be expanded as

$$\hat{a}_N(t) = \sum_{n=1}^{N} \alpha_n(t) \hat{a}_{N+1-n}.$$ 

Now we come to the solution of $\hat{a}_N(t)$, which is determined from the Heisenberg equation for $\hat{a}_N$:

$$\frac{dA}{dt} = i(G + i\omega)A,$$

where $A = [\alpha_1^2, \alpha_2^2, \ldots, \alpha_N]^T$ with $T$ being the transpose operation, $G$ is a tri-diagonal matrix

$$G = \begin{bmatrix}
0 & J_{N-1} & 0 & 0 & \cdots \\
J_{N-1} & 0 & J_{N-2} & 0 & \cdots \\
0 & J_{N-2} & 0 & J_{N-3} & \cdots \\
0 & 0 & J_{N-3} & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{bmatrix}.$$ 

The initial condition is $A(0) = [1, 0, \ldots, 0]^T$. To solve the differential equations we can apply the Laplace transformation on both sides of the equation as done in [55]. Note that multiplying both sides of Equation (5) by $i$, we can rewrite it as $i\frac{dA}{dt} = \hat{H}_{\text{new}} A$, which has the same form as the Schrödinger equation with $\hat{H}_{\text{new}} = -(G + i\omega)$. The new Hamiltonian $\hat{H}_{\text{new}}$ is in an $N$-dimensional Hilbert space, which is much more tractable than the original Hamiltonian $H$ that is in the $D^N$-dimensional Hilbert space. $A$ is the wave function of the new Hamiltonian, and we denote it as $A(A)$. In other words, the operator $a_N^\dagger(t)$ is represented as a vector $|A\rangle$ in the Hilbert space of the new Hamiltonian. It is worth mentioning that the explanation for the lesser Hilbert space is the element number of the set, which contains $A^\dagger$ and is closed under the operator $[H, \cdot]$, is only $N$ rather than the excitation number conservation. This can be seen clearly in the XY Hamiltonian with the coupling strength that cannot conserve the excitation number [55].

In the uniform condition, $J_0 = 1$, the eigenvalues and eigenstates of the new Hamiltonian $\hat{H}_{\text{new}}$ are

$$E_n = -2 \cos \frac{\pi n}{N+1} - i\omega,$$

$$|\psi_n\rangle = \frac{1}{\sqrt{N+1}} \sin \frac{\pi n}{N+1}.$$ 

We consider the question: what is the state of the new system at the given time $\tau$? The Hamiltonian of the system is $\hat{H}_{\text{new}}$ and the initial state is $|A_0\rangle = [1, 0, \ldots, 0]^T$. Using the Schrödinger equation we know

$$|A(t)\rangle = \sum_n \exp(-iE_nt)|\psi_n\rangle \sqrt{\frac{2}{N+1}} \sin \frac{\pi n}{N+1}.$$ 

The last element of the state is

$$\alpha_N(t) = \sum_n (-1)^{n-1} \frac{2}{N+1} \exp(-iE_nt) \sin \frac{\pi n}{N+1}.$$ 

From [53, 56] we know that if and only if the number of length is $N = p \cdot 2^m - 1$, where $p$ is a prime, or $N = 2p - 1$ (for convenience we call it the pretty good length condition), is a time $\tau$ that

$$\exp(-iE_n\tau) \approx (-1)^{n-1}e^{\gamma},$$

where $\gamma = 1$ if $N \equiv 1 \mod 4$, $\gamma = -1$ if $N \equiv 3 \mod 4$, $\gamma = \pm i$ if $N$ is even, and $E_n = E_n + i\omega$. 


So we have $\alpha_N(\tau) \approx \gamma e^{i\omega\tau}$. From $|\gamma e^{i\omega\tau}| = 1$, and the normalization of the state we get that $\alpha_1(\tau) \approx \alpha_2(\tau) \approx \cdots \approx \alpha_{N-1}(\tau) \approx 0$. So at the time $\tau$ we have

$$a_N^\dagger(\tau) \approx \gamma e^{i\omega\tau}a_1.$$

As for the phase $e^{i\omega\tau}$, we can adjust the cavity mode to a proper value $\omega$ that makes $e^{i\omega\tau} = 1$. So we get the conclusion that pretty good state transfer (PGST) is obtained at time $\tau$ if the length of the cavities satisfies the pretty good length condition and the cavity mode is $\omega$. Note that when the frequency $\omega$ is not equal to $\omega$, the phase $e^{i\omega\tau}$ can also be canceled by a proper time period of free evolution of the last cavity after switching off its coupling with the other cavities at time $\tau$.

Compared with the PGST in spin chains we do not need the initialization of the cavities or the single excitation condition. In XY spin chain systems QST is proportional to the parity of the initial state \cite{55}, while in the CCA system if we can achieve perfect QST at time $\tau$, the initial state of cavities $2 - N$ has nothing to do with QST at the perfect time. The reason is that in XY spin chain systems the operators in the other part of system by $X_i$ $a_i$ $N$ while in the CCA system $\hat{a}_i^\dagger$ is standalone in the Heisenberg equation related with the operators of the $N$th site.

For the general case that $f_i$s are not uniform, $a_N(t)$ is provided in \cite{55} as

$$\alpha_N(t) = \begin{cases} \frac{\sum_{i=1}^{M} \sin \left( q_i t \right) \prod_{j \neq i} \left( q_j^2 - q_i^2 \right)}{\det A_N^N} & \text{for } N = 2m, \\ \frac{\sum_{i=1}^{M} \cos \left( s_j t \right) \prod_{j \neq i} \left( s_j^2 - s_i^2 \right)}{\det A_N^N} & \text{for } N = 2m + 1, \end{cases}$$

where $A = p - G$ with $p$ as Laplace complex argument, $A_N^N$ is the matrix $A$ whose $N$th column vector is replaced by $A(0)$. $q$ and $s$ are roots of det $A_N^N$. $m$ is an integer. When $\alpha_N(\tau) = 1$ perfect QST is obtained.

### Numerical simulation

Now we numerically simulate the QST in the unmodulated CCA and show its result in figure 2. The system we simulate has length $N = 5$ with coupling strength $J_0 = 1$. Here we choose units such that $\hbar = 1$. It shows that at time $\tau = 21.8$, which requires $\omega/k = \frac{2\pi}{4}$, $k = 0, 1, 2, \cdots$, we get a good fidelity $F(\tau) = 0.9999$, for any initial state of the chain $2 - N$. The perfect QST is obtained from the first line (solid line) when the cavity modes are $1000$ and the sent state being the coherent state $|\alpha\rangle = e^{-|\alpha|^2/2} e^{i\alpha\sigma_z} |0\rangle$, $\alpha = 1$. The initial state of cavities $2 - N$ for the other two lines (dashed and dashdotted lines) are $|1000\rangle$, $|1100\rangle/\sqrt{2}$ and the sent states are $\frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)$ and $\frac{1}{\sqrt{14}}(|00\rangle + |21\rangle + |32\rangle)$, respectively. The $\omega$ we choose is $\omega(1) = 0.288$.

Note that other conclusions of QST in the spin chain system are also applicable in the CCA system. As we know, the spin chains with modulated coupling strength have perfect QST when the parity of the initial state...
(except the first spin) is 1. The modulated coupling strengths are \( J_n = J^{(k)}_n = \sqrt{n(N-n)} \) for even \( n \) and \( J_n = J^{(k)}_n = \sqrt{(n+2k)(N-n+2k)} \) for odd \( n \), where \( k \in \{0, 1, 2, \ldots\} \) [22, 23]. For the case \( k=0 \), the matrix \( G \) is identical to the representation of the Hamiltonian \( \hat{H} \) of a fictitious spin \( S = \frac{1}{2}(N-1) \) particle: \( \hat{H} = 2S_x \), where \( S_x \) is angular momentum in \( x \) direction [22].

Now we consider the case that the coupling strengths are the perfect modulated ones with \( k=0 \). So the new Hamiltonian is \( \hat{H}_{\text{new}} = -(2S_x + \hbar \omega) \). \( \alpha_N(t) \) can be written directly as

\[
\alpha_N(t) = [i \sin(t)]^{N-1} e^{\frac{\hbar \omega t}{2}}.
\]

So at time \( t = \frac{\pi}{2} \), \( |\alpha_N(t)\rangle = 1 \). The required frequency is \( \omega = 4k + 1 - N \), \( k = 0, 1, 2, \ldots \). In figure 3 we demonstrate the fidelity \( F(t) \) versus \( t \) for the modulated CCA system with length \( N=8 \). The sent state is \( \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle + |2\rangle) \), and the initial states of cavity \( 2-N \) are thermal states \( e^{\frac{\hbar \omega t}{2}} \) with \( \beta = 0.5, 1, 10, 20 \) and \( \omega = 17, 9, 5, 1 \) respectively. It shows that at time \( t = \frac{\pi}{2} \) fidelity \( F(t) \) of QST is high and the transmission time is \( t \sim N \) [57, 58].

In the uniform XX spin channel, perfect QST can be achieved by tuning down the two end coupling strengths limited to zero for arbitrary length \( N \) [31]. But the optimal time of perfect QST becomes long as end coupling strengths decrease. There is a regime, which is called ballistic regime, in which \( 0 < \text{I}_{\text{end}} < 1 \) (the uniform coupling strength is set to 1), the fidelity of QST is high and the transmission time is \( t \sim N \) [57, 58].

In figure 4 we simulate the QST of a CCA system with length \( N = 8 \) at the ballistic regime, \( J_1 = J_2 = 0.3J \). It shows that at time \( t = 20.7 \) the system at the ballistic regime gets a fidelity larger than 0.99.

### Experimental realization

A remarkable cavity that can be used to realize our proposal is the toroidal micro-cavity which is formed from fused SiO\(_2\) [1]. Such a cavity supports whispering-gallery modes circulating around the outer circumference of the toroid with an evanescent field external to the cavity. So it can be coupled with the tapered optical fiber and other toroidal cavities. The cavity has a very high Q-factor (> 10\(^8\)) for light that is trapped as whispering-gallery modes. In [54] this CCA with atoms in the cavities is proposed to form a strongly interacting many-body system.

Another promising candidate for an experimental realization is photonic crystals [2, 3]. Here we proposed a novel optical fiber with periodic index of fabrication as shown in figure 1(c) as a possible scheme to realize the CCA. This optical fiber can be seen as the integration of coupled cavities. In addition, the technology of preparing the coherent or Fock photon state in the cavity and counting the photon number [12-16] can be used to measure the fidelity of the QST of photon state.

The main imperfection in using the CCA to realize the QST comes from cavity loss. In general, it will decrease the fidelity of the channel. In particular, cavity loss will have more effects on the states with larger photons than on those with smaller photons. For example if the transferred state is \( \alpha |0\rangle + \beta |n\rangle \), the cavity loss will have nothing to do with \( \alpha |0\rangle \) while it will change \( \beta |n\rangle \). The effects of cavity loss for QST in our system need detailed investigation in the future.
One of the inspiring questions is whether it is possible to generalize the proposed protocol for fermionic chains like itinerant electrons in a lattice. Here we consider the Hamiltonian that is the same as Equation (1) with \( a^\dagger \) being the fermionic annihilation (creation) operator. We will get the same dynamic function for the operator \( a^\dagger_N \) in the Heisenberg picture, which implies that one fermion may be transferred from one end to the other. But we cannot encode a state onto one for the fermions, such as electrons, as the initial state of the first node

\[
|\psi_0\rangle = \alpha |0\rangle + \beta |1\rangle = \left( \alpha + \beta a^\dagger \right) |0\rangle,
\]

a coherent superposition state with different numbers of fermions, is almost impossible to be realized in a physical system. In other words, we cannot directly generalize our scheme to the fermionic case.

**Conclusion**

In summary, we propose a QST scheme using a CCA system to transfer any single-mode photon state from one end of the array to the opposite end. Our analysis shows that most of the results of QST schemes for a spin chain system are applicable in our scheme and the transferred state is extended to any single-mode photon state without the initialization of the channel. We numerically simulate the schemes in three cases: uniform coupling CCA, perfect modulated CCA and CCA with coupling strengths in a ballistic regime. In every case we use different initial states and sent states, and the expected results are obtained. Using the technology of producing high quality cavity arrays and precisely preparing and measuring photon state in a cavity, our scheme of QST along a CCA may be realized in the near future.

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**Figure 4.** Numerical simulation of the fidelity \( F(t) \) of QST for \( N = 8 \) as a function of \( t \) in the ballistic regime case \( (J_0 = J_1 = 0.3J, J_n = J_0) \) for different initial states and photon frequency \( \omega \). The initial states are \( \rho_1 = \frac{1}{2} (|0\rangle + |1\rangle + |2\rangle) (|0\rangle + |1\rangle + |2\rangle) \) are thermal states with \( \beta = 20 \) (solid line), \( 1000000 \) (dashed line), \( 1100000 \) (dash-dotted line), respectively. The corresponding photon frequencies are 1.29, 0.379, 0.076.
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