Shear bands in granular flow through a mixing-length model

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Abstract – We discuss the advantages and results of using a mixing-length, compressible model to account for the shear banding behaviour in granular flow. We formulate a general approach based on two functions of the solid fraction to be determined. Studying the vertical chute flow, we show that the shear band thickness is always independent of flow rate in the quasistatic limit, for Coulomb wall boundary conditions. The effect of bin width is addressed using the functions developed by Pouliquen and coworkers, predicting a linear dependence of shear band thickness on channel width, while the literature reports contrasting data. We also discuss the influence of wall roughness on shear bands. Through a Coulomb wall friction criterion we show that our model correctly predicts the effect of increasing wall roughness on the thickness of shear bands. Then a simple mixing-length approach to steady granular flows can be useful and representative of a number of original features of granular flow.

Introduction. – Granular materials are ubiquitous in everyday’s life, but a satisfying comprehension of their complex dynamics has not yet been achieved, despite the efforts of condensed matter science [1]. From the latin poet Lucretius to nowadays, we have experienced a perspective reversal; while, in his De Rerum Natura, he used seeds as an example to explain the “flowability” of liquids, today we try and use hydrodynamic analogies (e.g. [2–5]) to explain the flow of granular materials. However, towards those poppy seeds our various theories remain almost a divinatory exercise.

Many of the theoretical approaches appeared in last decade literature neglect the compressibility of granular materials, assuming it as an incompressible fluid with \( \rho \approx \text{const} \). From a phenomenological point of view, we believe that dilatancy is a requirement for shearing a granular material, in other words, the material has to dilate in order to shear. Accordingly, we expect that neglect of dilatancy loses an important part of the granular flow physics.

In this work we formulate a simple model explicitly involving the solid fraction influence on the flow properties of granular materials. The model aims at being a generalization of the one developed by the GDR MIDI [6–9] based on the dimensionless parameter \( I \), considered as the ratio between shearing time and rearranging time due to pressure. In the GDR’s formulation, the solid fraction is derived from \( I \) as being linearly depending on it. Here we reverse the formulation assuming the solid fraction \( \phi \) to be the critical variable, instead of \( I \), to re-establish the physical relevance of the dilation of the medium to determine the flow features. Giving the model an appropriate account of the solid fraction can become important for those geometries (like silos) in which \( \phi \) varies significantly (more than 10%) all over the flow section. In silos flow can be seen to originate from fluidization due to the injection of voids from the exit hole, where the solid fraction is quite different from its value in the core.

In this perspective, we illustrate the \( \phi \)-based model derivation and its application to the vertical chute arrangement, to verify the constitutive relations proposed. We specifically used the model to predict the shear bands extension. This issue was considered [9] as a weakness of the mixing-length approach; predicted shear bands width and its dependence on geometrical and flow parameters apparently do not match some experimental data. In the following we show that a simple mixing-length approach to steady granular flows can appropriately predict the shear band thickness.

The model. – The model outlined here is formulated for 2D, steady granular flows. The relevant equations are
momentum balance with its two components, and the equation of continuity.

As a fundamental assumption, we consider the flow structure to be solvable with the steady, compressible Navier-Stokes (N-S) equations, with a solid fraction, pressure and shear-dependent viscosity. In addition, we need a constitutive equation for \( \eta \), and we saturate the degree of freedom introduced by \( \phi \) with a sort of Equation of State (EqS) that involve pressure.

Accordingly, equations are

\[
\nabla \cdot \rho \vec{u} = 0
\]

\[
\nabla \cdot (\rho \mu \vec{u}) = -\frac{\partial p}{\partial x} + 2 \frac{\partial}{\partial x} \eta \left( \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \eta \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \rho g_x,
\]

\[
\nabla \cdot (\rho v \vec{u}) = -\frac{\partial p}{\partial y} + 2 \frac{\partial}{\partial y} \eta \left( \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial x} \eta \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \rho g_y,
\]

where \( \rho \) is the local density of the compressible pseudo-homogeneous medium, which is, having neglected the interstitial fluid, and taken \( \rho_p \) as material density,

\[
\rho = \rho_p \phi.
\]

Constitutive relations. We assume a rheological law based on dimensional analysis, like Prandtl’s approach to turbulent flows, as discussed by Ertas and Halsey [10]. Because of the eminent precursor, we will call it “mixing-length approach”.

In this perspective the apparent viscosity of the medium is locally formulated as

\[
\eta = \rho_p L^2 |\dot{\gamma}|,
\]

where the unique timescale is \(|\dot{\gamma}|^{-1}\); \( L \) is a characteristic length, that has to be a function of \( d \) and \( \phi \) only, with a generic relation of the form

\[
L^2 = d^2 f(\phi).
\]

The function \( f(\phi) \) is not known so far, but some features of it may be prescribed: it should diverge when \( \phi \to \phi_{\text{max}} \), to limit the material flow (that becomes “jammed”), i.e. \( \eta \to \infty \). To achieve this limit, \( \eta \) should diverge faster than \(|\dot{\gamma}|^{-\frac{1}{2}}\), as can be easily seen from eq. (5). Interestingly, for \( f(\phi) = 1 \), eq. (6) reduces to Bagnold scaling for shear stress (valid for rapid granular flows), providing a further requirement that \( f(\phi \to 0) = 1 \). However, the present work addresses dense flow of granular material, and we are not interested, at the moment, in the liquid-gas-like transition.

We also need a relation between pressure and solid fraction, which is similar to an EqS. Assuming shear rate plays the role of temperature in a gas, and \( \phi \) acts through a geometrical (excluded-volume) function \( h(\phi) \) to be specified, we can use a dimensional analysis to obtain the following EqS:

\[
p = \rho_p h(\phi) (|\dot{\gamma}| d)^2.
\]

To keep pressure finite when shear rate vanishes, \( h(\phi) \) has to diverge when \( \phi \to \phi_{\text{max}} \) (although the value and physical meaning of \( \phi_{\text{max}} \) are still a matter of debate [11]).

When dealing with eq. (7), we must remember that the model is valid only for stationary flows: of course, a static granular packing can be found in a wide range of solid fraction, but we assume that when the system is flowing and is in a stationary state, the only state where the material behaves rigidly is when \( \phi \approx \phi_{\text{max}} \). In the dynamic regime the material explores its phase space to approach a unique solid fraction profile.

Dimensional analysis is broadly used in granular flow modeling attempts, starting from Bagnold’s works [12]. The formulation of Josserand et al. [2,11] uses dimensional analysis with Coulomb friction to develop a constitutive relation for shear stress that is composed by a rate-dependent part and a rate-independent one, and where the isotropic part of the stress tensor is related to the solid fraction by means of entropic considerations. We express normal and shear stresses according to Pouliquen [8,9], with the difference that we explicitly use the solid fraction as the key variable, instead of the dimensionless shear rate. Note that these laws, are very similar to those developed from hydrodynamic analogies [4,5], where granular temperature is used to represent the local mobility of the medium. We prefer our simple closure, based on \( \phi \) and an EqS for it, also because granular temperature is a variable which is difficult to measure and then correctly validate.

Rearranging eqs. (5), (6), we obtain

\[
\eta = p \frac{f(\phi)}{|\dot{\gamma}|} h(\phi) = p G(\phi).
\]

For the sake of simplicity, we replaced the ratio \( f/\eta \) with \( G(\phi) \) and introduce

\[
F(\phi) = [h(\phi)]^{-1/2}
\]

as a simple replacement, provided \( h \) always appears in this form in the following developments of N-S equations. Note that \( G \) must vanish if \( \eta |\dot{\gamma}| = \tau \to 0 \).

It is easy to see that the functions \( F \) and \( G \) correspond, respectively, to the inertial number \( I = \frac{\rho \dot{\gamma}^2}{\sqrt{\rho_p / \rho_g}} \) and to the effective friction coefficient \( \mu^* \) as discussed by the GDR MiDi [6,7], which we generalized to any configuration and dimensions beyond 1D. In simple, quasi-1D geometries, one can interpret \( G \) as the ratio of shear to normal stresses; in this sense \( \mu^* \) was measured from DEM simulations by da Cruz et al. [7] and possible fittings for its dependence on \( I \) were discussed either by da Cruz et al. and by Pouliquen et al. [9]. Our formulation is a generalization of those results in a 2- or 3-dimensional case, where the fundamental role played by solid fraction is well recognized.

Applying the model to the vertical chute. – We choose the vertical chute configuration as a standard benchmark for model evaluation. Original flow structures, principally related to the width of shear zones, can be
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found in the chute flow, like in Couette cells. Reference for these configurations is the well known paper by GDR MiDi [6]. Broadly speaking, the material flows in a plug-like fashion in the central part of the chute, while it is sheared near the wall. The extent of shear bands apparently approaches a typical dimension, of order 10 – 15 particle diameters. Predicting shear bands’ thickness is a benchmark for all models applied to the chute and Couette flow [9].

**Vertical chute equations.** A scheme of the chute is given in fig. 1. For the steady vertical chute 2D flow, N-S equations simplify thanks to

\[ v = 0 \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial t} = 0, \]

leading to

\[
\begin{align*}
\frac{\partial}{\partial y} \eta \left( \frac{\partial u}{\partial y} \right) + \rho_p \phi g &= 0, \\
\frac{\partial p}{\partial y} &= 0.
\end{align*}
\]

We also included Janssen effect, assuming pressure does not vary in the vertical direction. Equation (11) additionally states that pressure will not vary horizontally either. The other equation can be integrated. With \( y \) originating from the center of symmetry and directed to the wall, as shown in fig. 1, we have

\[ |\gamma|^2 = -\left( \frac{\partial u}{\partial y} \right), \]

that should be replaced in the \( x \)-momentum balance in combination with eq. (8), to give:

\[ \frac{\partial}{\partial y} \eta \left( \frac{\partial u}{\partial y} \right) + \rho_p \phi g = -p \frac{\partial}{\partial y} [G(\phi)] + \rho_p \phi g = 0, \]

or

\[ G(\phi) = \frac{\rho_g y}{\rho} \int_0^y \phi \, dy; \]

\( G(y = 0) = 0 \) follows from eq. (8) and the symmetry, which requires that the shear stress at the centerline should vanish.

From eq. (14) we expect to identify \( \phi(y) \) provided \( p \) and an invertible form of \( G \) are given. At the same time, \( u(y) \) can be obtained from the EoS:

\[ |\gamma|^2 = \frac{p}{\rho_p} \frac{h(\phi) dy}{d} \Rightarrow \frac{\partial u}{\partial y} = -\frac{\sqrt{p/\rho_p}}{d} F(\phi), \]

or

\[ u(y) = u_{slip} + \frac{\sqrt{p/\rho_p}}{d} \int_y^b F(\phi) \, dy, \]

where \( b \) is the half-width of the channel. So far, the unknown functions \( \phi(y) \) and \( u(y) \) can be formally obtained solving the coupled eqs. (14) and (16), but in practice \( F(\phi) \) and \( G(\phi) \) must be specified, and also the pressure calculated.

The continuity equation can be used in its integral form, correlating local profiles to the total mass flow rate, \( \dot{M} \). Experiments are easily performed with constant flow rate, either controlled by a simple hole in the bottom of the silo, or using a moving plate with fixed velocity. Accordingly:

\[ 2 \rho_p \int_0^b \phi u \, dy = \dot{M} = \text{const.} \]

Developing eq. (17), we can formulate an expression for the slip velocity by using eq. (16):

\[ \int_0^b \phi \left( u_{slip} + \frac{\sqrt{p/\rho_p}}{d} \int_y^b F(\phi) \, dy' \right) \, dy = \frac{\dot{M}}{2 \rho_p} \]

or

\[ u_{slip} = \frac{1}{\int_0^b \phi \, dy} \left[ \frac{\dot{M}}{2 \rho_p} - \frac{\sqrt{p/\rho_p}}{d} \int_0^b \left( \int_y^b F(\phi) \, dy' \right) \, dy \right]. \]

**On boundary conditions.** One of the most critical issue in granular flow simulation is the identification and application of boundary conditions. For the solid fraction, we will assume that in the central zone of the chute the material reaches \( \phi_{max} \) (which could be random close packing, or some other critical value of \( \phi \) that leads \( h(\phi) \) to diverge [11]). The divergence of \( h \) at the center is due to the fact that pressure is constant in the chute, but shear rate has to vanish at \( y = 0 \).

One constraint on velocity can be formulated as an integral condition, by fixing the flow rate as done above. In addition, we must speculate on the interaction between the granular assembly as a continuum and the walls. The simplest view used in the literature is assuming a layer of particles glued at the walls, for which we can use a no-slip boundary condition. This assumption is attracting for its simplicity but we believe it requires caution in its application. We are skeptic that the continuum averaged interaction between nearest particles is the same in the bulk and in the layer of particles facing the glued
ones. In this perspective, experimental investigation and critical theoretical speculations have to be done. A viable alternative to the no-slip assumption is the Coulomb criterion at the wall:

$$\tau_w = \sigma_w \tan \delta,$$

where $\delta$ is a characteristic wall friction angle. In case of particle artificially fixed at the wall, this means assuming them as a wall, with a specific roughness measurable by its own $\delta$. Combining Coulomb’s law with eq. (8) we obtain a condition on the solid fraction,

$$\tau_w = \eta_w |\dot{\gamma}|_w = \frac{p}{|\dot{\gamma}|_w} G(\phi_w) |\dot{\gamma}|_w = p G(\phi_w),$$

which leads to

$$G(\phi_w) = \tan \delta,$$

given that $\sigma_w = p$. Very important, with Coulomb’s criterion the slip velocity is not zero, and has to be determined from eq. (19) using the flow rate. In addition eq. (22) allows us to calculate the pressure (normal stress), provided that

$$G_w = G(y = b) = \frac{\rho_p g}{\tan \delta} \int_0^b \phi dy,$$

which gives

$$p = \frac{\rho_p g}{\tan \delta} \int_0^b \phi dy = \frac{\rho_p \phi_{ave} g b}{\tan \delta},$$

where the average solid fraction, defined by $\int_0^b \phi dy = b \phi_{ave}$, has been introduced.

Combining eqs. (24), (19), and (16), the velocity profile can be explicitly written as

$$u(y) = \frac{1}{\phi_{ave} b} \left[ -\frac{\sqrt{\phi_{ave} g b}}{d \sqrt{\tan \delta}} \int_0^b \phi dy \left( \int_y^b F'(\phi)dy' + \frac{M}{2 \rho_p} \right) \right. + \frac{\sqrt{\phi_{ave} g b}}{d \sqrt{\tan \delta}} \int_y^b F'(\phi)dy.\right]$$

The result of eq. (24) together with eq. (14) leads to

$$G(\phi) = \frac{\tan \delta}{b \phi_{ave}} \int_0^y \phi dy,$$

stating that in the approximation of $\phi \approx \phi_{ave}$

$$G(\phi) \approx \frac{y}{b} \tan \delta, \quad (27)$$

or $G$ is a linear function of $y$, which provides a consistence criterion for the identification of the unknown function $G$.

Interestingly, our model, based also on Coulomb wall criterion, predicts the invariance of the velocity profiles with flow rate in the quasistatic limit (where $\phi \approx \phi_{ave}$ is valid). In other words, the scaled velocity profile:

$$\tilde{u} = \frac{u(y) - u_{slip}}{u_{max} - u_{slip}} = \frac{\int_y^b F'(\phi(y))dy}{\int_0^b F'(\phi(y))dy}, \quad (28)$$

does not depend on the flow rate, that influences only the slip tangential velocity. In this limit, the solid fraction profile also does not depend on the flow rate, as predicted by eq. (27), but only on the bin width and the wall friction angle. We underline that the result is independent of the particular formulation of the functions $F$ and $G$. Also with Pouliquen’s formulation for the effective friction coefficient (which can be seen as a particular choice for $F$ and $G$), but with a Coulomb slip criterion, the independence of shear bands from flow rate is obtained. This is indeed a result supporting the mixing-length approach. We think that the shear bands independence from flow rate could be related to the peculiar behavior of granular matter, which is able to develop internal stresses, supported by walls, to sustain itself; in this perspective, taking into account stresses in the formulation of boundary conditions would be necessary. However, far from the quasistatic limit, or in conditions where $\phi \approx \phi_{ave}$ is more acceptable, the flow rate can significantly affect $\phi(y)$ and $u(y)$ because of the close coupling of the two equations.

Deriving expressions for $F(\phi)$ and $G(\phi)$. Following the work of Pouliquen and coworkers, the functions would take the form

$$\left\{ \begin{array}{l}
G(\phi) = \mu_s + \frac{\mu_2 - \mu_s}{y_0/\phi + 1}, \\
F(\phi) = \tilde{\phi},
\end{array} \right. \quad (29)$$

where $\tilde{\phi}$ is the scaled solid fraction given by

$$\tilde{\phi} = \frac{\phi_{max} - \phi}{\phi_{max} - \phi_{min}}, \quad (30)$$

The authors [9] acknowledged a major difficulty in the application to the vertical chute with no-slip at the walls; shear bands are not finite and of constant width in the quasistatic limit. We have already demonstrated that a Coulomb wall slip criterion can correct this. In the following we will illustrate the results of our mixing-length model including the Coulomb wall slip criterion and $F$ and $G$ functions as in eq. (29), for different chute width and wall roughness.

Before that, note the analytical solution achievable in the quasistatic limit, obtained combining eqs. (27) and (29)

$$\tilde{\phi} = \begin{cases}
\frac{y - \mu_s'}{\mu_2' - y}, & \text{for } y > \mu_s', \\
0, & \text{for } y \leq \mu_s',
\end{cases} \quad (31)$$

where $\mu_s' = \mu_s b / \tan \delta$ and $\mu_2' = \mu_2 b / \tan \delta$. The scaled velocity profile, obtained combining eqs. (28), (29) and (31), is indeed a simple function of $y$:

$$\tilde{u} = \begin{cases}
A (y - b) + B \ln (C - Dy), & \text{for } y > \mu_s', \\
1, & \text{for } y \leq \mu_s',
\end{cases} \quad (32)$$

where $A, B, C, D$ are known constants, depending on model parameters, wall friction angle and channel width.
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Table 1: Parameters of the model [8].

| Parameter | Value     | Unit   |
|-----------|-----------|--------|
| $J_0$     | 0.279     | (adim) |
| $\mu_s$  | $\tan(20.9)$ | (adim) |
| $\mu_w$  | $\tan(32.76)$ | (adim) |
| $\phi_{max}$ | 0.83 | (adim) |
| $\phi_{min}$ | 0.75 | (adim) |

![Graph showing shear bands thickness and chute width](image1)

Fig. 2: First two panels: non-dimensional axial velocity $\bar{u}$ and solid fraction vs. non-dimensional channel width, in chutes with different extension (wall friction angle $\delta = 25\text{ deg}$). Third panel: shear bands thickness vs. chute extension.

In the following we perform an analysis based on these analytical results; far from the quasistatic limit, one can repeat the calculations using eq. (26) instead of eq. (27), in a numerical fashion.

Shear bands thickness and chute width. We already demonstrated that the model predicts that the shear band is independent of flow rate. Here we explore the effect of bin width.

As a measure of shear band width, we choose $\Delta y$ from the wall to the point where $\bar{u} \approx 0.99$. Using the quasistatic assumption, we calculate $\bar{u}$ from eq. (32), for different widths, $b$. Other model parameters must be given and they depend on the specific material chosen for the purpose of illustrating the model, we used the values determined by Jop et al. [8], collected in table 1.

Figure 2 shows that the model predicts a linear correlation between shear band width and channel width. The slope of the linear dependency may change with different materials, but remains linear. It is frequently stated that the thickness of shear bands is expected to be independent of channel extension. However, the literature reports data supporting (e.g., [13]) and contrasting [14] this statement. Our results agree with the experimental results by Nedderman and Lahoakul, but the issue requires further investigation of the vertical chute, in order to discriminate the applicability of a mixing-length model to this configuration. From the solid fraction profiles, we argue that $F(\phi) < 0.125$, that is, we are in the dense regime. Even if we are not in the quasistatic limit, the approximation $\phi \approx \phi_{ave}$ is still valid because the body force varies by less than $2\%$, and so profiles can be calculated by means of eqs. (31) and (32). In fact, it is useful to see how the condition $\phi \approx \phi_{ave}$ can be valid beyond the quasistatic limit, and so the analytical solution above can be used also in the dense regime (i.e., out of its rigorous region of validity).

Shear bands thickness and wall roughness. Wall roughness can influence the extension of the shear bands, according to our model. Adopting the Coulomb criterion at the wall yields a simple expression of wall roughness, related to the wall friction angle $\delta$, while using a no-slip condition makes it impossible to account for roughness within a continuum approach. Also in view of a real scale application of a mixing-length model, wall friction must be correctly accounted for, and the wall friction angle is a widespread approach. Furthermore, real-world applications aim at perfect wall slip, but are often in an intermediate situation, where wall roughness plays a role.

In the quasi-static regime, Kishida and Uesugi [15] performed experiments in shear cells probing that a linear correlation exists between a normalized wall roughness and the wall friction coefficient $\mu_w$: in the case when wall roughness is built by gluing particles at the wall of the same material of the bulk, we would have

$$\mu_w = m \chi + \mu_p, \quad (33)$$

where $\chi$ is the ratio between wall and bulk particle diameters, $\mu_p$ the lower value of $\mu$, $m$ a coefficient of order 1. However, they showed that $\mu_w$ follows a linear behavior around $\chi = 0.1$ and has already saturated for $\chi = 1$; according to the paper, our values of $\chi$ would belong to a region of constant $\mu_w$. For the inertial regime, a more recent work by Goujon et al. [16] on the role of roughness in flows down inclined planes showed that friction reaches a maximum (in 3-D) for $\chi = 2$, and they related the behaviour at higher roughness to the fact that holes are filled by bulk particles, thus reducing friction. For our 2-D case, using the simple model by Goujon, it turns out that the value of $\chi$ at which friction reaches a maximum is 4. Thus in the range 0–4 friction is an increasing function of the relative roughness $\chi$; however, the behavior depends on the regime of flow.

Results according to eq. (32) are given in fig. 3, showing that the model predicts a dependence of shearing regions on wall roughness. However, the effect of $\delta$ is approaching an asymptote. At small values slip occurs, whereas larger friction reduces its influence on the shear-138 bands. From the solid fraction profiles, the assumption $\phi \approx \phi_{ave}$ can be considered again as valid, because $\phi$ varies by less than $10\%$. The enlargement of shear zones with increasing wall roughness is supported also by the DEM.
The mixing-length approach can capture the effect of between DEM and our mixing-length, continuous model. The above analysis proves that even a simplified mixing-length approach to steady granular flows can be useful and representative of a certain number of features, once the proper boundary conditions are used.

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results of Prochnow [17] (where $\chi$ is, respectively, 0.5, 1 and 4). Figure 4 illustrates qualitatively the comparison between DEM and our mixing-length, continuous model. The mixing-length approach can capture the effect of increasing wall roughness predicted by DEM calculations by means of different sizes of particles fixed at the wall.

Conclusions. – In this work we formulated and applied a mixing-length, compressible continuum model, aiming at discussing the shear band particular behaviour in granular flow. A general approach based on two functions of the solid fraction to be specified has been given. Application to the vertical chute problem demonstrates that for suitable wall boundary conditions (Coulomb friction), the shear band thickness is independent of flow rate, in the quasistatic limit, whatever the form of the generic functions. The relevance of these boundary conditions in real applications was discussed, and a critical evaluation of experimental practice was given.

Further, we addressed the role of bin width, using expression for the $G(\phi)$ and $F(\phi)$ functions based on reformulation of literature results [6] to highlight the variable solid fraction. Results do not agree with the frequently assumed independence of shear band thickness from channel width, but agree with literature experimental data [14]. We expect that additional experimental investigations might elucidate this controversy. Taking advantage of the inclusion of the wall friction criterion instead of a no-slip boundary condition, we correctly predicted an asymptotic increase of shear bands extension with larger wall roughness.

The above analysis proves that even a simplified mixing-length approach to steady granular flows can be useful and representative of a certain number of features, once the proper boundary conditions are used.

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