Appendix to "Structural equation modeling reveals determinants of fitness in a cooperatively breeding bird".

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A1 ADDITIONAL INFORMATION ABOUT THE VARIABLES CONSIDERED

| Variable name                      | LV | ID | Type | Levels | Categories            |
|------------------------------------|----|----|------|--------|-----------------------|
| number of offspring intra-group    | R  | 1  | ord  | 3      | 0, 1, 2+              |
| number of offspring extra-group    | R  | 2  | ord  | 3      | 0, 1, 2+              |
| number of offspring only one known parent | R  | 3  | ord  | 3      | 0, 1, 2+              |
| group size                         | SE | 4  | ord  | 3      | 0, 1, 2+              |
| number of helpers                  | SE | 5  | ord  | 3      | 0, 1, 2+              |
| territory size                     | TQ | 6  | ord  | 4      | ≤ 1500, (1500, 2500], (2500, 3500], > 3500 m² |
| insectXvegetation                  | TQ | 7  | ord  | 5      | ≤ 3, (3, 6], (6, 11], (11, 20], > 20 insect dm² |
| age, age²                          | fc |    | cont | -      | -                     |

Table A1: List of variables included in the models. For each observed variable, we report the corresponding latent variable (LV) or if the observed variable is treated as a fixed covariate (fc). R stands for reproductive potential, SE for social environment and TQ for territory quality. The ID is a sequential number ID useful to interpret the JAGS scripts. The type denotes if a variable is ordinal (ord) or continuous (cont). When applicable, we report also the number of categories (levels) and the thresholds between categories. Open intervals are denoted with parentheses, while closed intervals are denoted with square brackets.

A2 CALCULATION OF THRESHOLDS FOR ORDINAL DATA

The ordinal variable $z_{\text{Q},o}^{\text{Q},\text{o}}$ was transformed into its continuous underlying variable $z'_{\text{Q},o}^{\text{Q},\text{o}} \sim N[0, \sigma^2]$ (Gelman et al. 2014; Lee 2007; Thanoon and Adnan 2015). To do so, we specified appropriate threshold values that relate $z_{\text{Q},o}^{\text{Q},\text{o}}$ to $z'_{\text{Q},o}^{\text{Q},\text{o}}$ Fig. A1. For example, given an ordinal variable $z$ including $a$ categories, then

$$z = a \quad \text{if} \quad \alpha_{a-1} < x' \leq \alpha_a$$  \hspace{1cm} (1)

where $z \in \{1, 2, \cdots, a\}$. The vector $\alpha = \{\alpha_0, \alpha_1, \cdots, \alpha_a\}$ includes the thresholds and $\alpha_0 = -\infty$, $\alpha_a = \infty$. To avoid identification problems, the values of $\alpha$ are set to constants and estimated through MCMC samples. To make sure the underlying continuous variables in the females and males have the same scale and are comparable we set identical values of $\alpha$ in both samples (Song et al. 2011).
Figure A1: Graphical representation of a hypothetical ordinal categorical variable \( z \) with three categories. Panel (a) represents the proportions of the three categories in the variable. Panel (b) represents the underlying continuous variable, \( z' \sim N[0, 1] \). The values of \( \alpha_0, \ldots, \alpha_3 \) are the appropriate thresholds that relate \( z \) to \( z' \). From a practical perspective, the sampler substitutes each data point within the categorical variable with a random value between the corresponding thresholds. For example, a data point equal to category two is substituted with a random value within -0.7 and zero \((-0.7, 0]\).
A3 STRUCTURAL EQUATIONS OF THE MODELS

The two-sample three-level structural model for the reproductive success of females and males is given for each observation \(o\) of individual \(i\) nested in a social group \(s\) by

\[
y_{\varphi io} = \Lambda_{\varphi 1io}(b_{\varphi 1io}\zeta_{\varphi 1io} + (I - \Pi_{\varphi 1})^{-1}\Gamma_{\varphi 1}F_{1}(\xi_{\varphi 1io} + \delta_{\varphi 1io}) + \Lambda_{\varphi 1i\xi}\xi_{\varphi 1io} + \epsilon_{\varphi 1io} + \eta_{\varphi})
\]

\[
\mu_{\varphi} + \Lambda_{\varphi 2\eta}((I - \Pi_{\varphi 2})^{-1}\Gamma_{\varphi 2}F_{2}(\xi_{\varphi 2i}) + \delta_{\varphi 2i}) + \Lambda_{\varphi 2i\xi}\xi_{\varphi 2i} + \epsilon_{\varphi 2i} + \eta_{\varphi} + \mu_{s} + \Lambda_{s\eta}(\Gamma_{s}F_{2}(\xi_{s}) + \delta_{s}) + \Lambda_{s\xi}F_{2}(\xi_{s}) + \epsilon_{s}
\]

and

\[
y_{\sigma io} = \Lambda_{\sigma 1io}(b_{\sigma 1io}\zeta_{\sigma 1io} + (I - \Pi_{\sigma 1})^{-1}\Gamma_{\sigma 1}F_{1}(\xi_{\sigma 1io} + \delta_{\sigma 1io}) + \Lambda_{\sigma 1i\xi}\xi_{\sigma 1io} + \epsilon_{\sigma 1io} + \eta_{\sigma})
\]

\[
\mu_{\sigma} + \Lambda_{\sigma 2\eta}((I - \Pi_{\sigma 2})^{-1}\Gamma_{\sigma 2}F_{2}(\xi_{\sigma 2i}) + \delta_{\sigma 2i}) + \Lambda_{\sigma 2i\xi}\xi_{\sigma 2i} + \epsilon_{\sigma 2i} + \eta_{\sigma} + \mu_{s} + \Lambda_{s\eta}(\Gamma_{s}F_{2}(\xi_{s}) + \delta_{s}) + \Lambda_{s\xi}F_{2}(\xi_{s}) + \epsilon_{s}
\]

where, with regard to the female (♀) sample, \(y_{\varphi io}\) is the vector of observed variables; \(\Lambda_{\varphi 2, \varphi 1i}\) and \(\Lambda_{s}\) are the matrix of factor loadings at the within-individual, between-individual, and social group level respectively and can be defined as \(\Lambda_{\varphi 2} = (\Lambda_{\varphi 2\eta, \varphi 2\xi}, \Lambda_{\varphi 1i} = (\Lambda_{\varphi 1i\eta, \varphi 1i\xi})\) and \(\Lambda_{s} = (\Lambda_{s\eta, \Lambda_{s\xi}} where the subscript \(\eta\) refers to the endogenous latent variables; and the subscript \(\xi\) refers to the exogenous latent variables; \(b_{\varphi}\) is the vector of regression coefficients; \(\zeta_{\varphi}\) is the matrix of fixed covariates; \(\Gamma_{\varphi 1, \varphi 2}, \Pi_{\varphi 1, \varphi 2}\) are matrices of unknown parameters; the values of \(\Gamma_{s}\) were set to be independent of \(F_{2}(\xi_{s})\) because the social-group level was merely introduced to correct for pseudoreplication and we wanted to avoid overfitting the model; \(\xi_{\varphi 2i, \varphi 1io}\) and \(\xi_{s}\) are matrices of exogenous latent variables; \(F_{2}(\xi_{\varphi 2i}) = \) and \(F_{1}(\xi_{\varphi 1io})\) are vector-values functions transforming the values of \(\xi_{\varphi 2i}\) and \(\xi_{\varphi 1io}\) respectively; \(\mu_{\varphi}\) and \(\mu_{s}\) are vectors of intercepts; \(\delta_{\varphi 2i} \sim N[0, \Psi_{\varphi 2i}], \delta_{\varphi 1i\delta} \sim N[0, \Psi_{\varphi 1i\delta}]\) and \(\delta_{s} \sim N[0, \Psi_{s}\xi, \xi]\) are vectors of measurement errors where \(\Psi_{\varphi 2i}, \Psi_{\varphi 1i\delta}\) and \(\Psi_{s}\xi, \xi\) are diagonal matrices; \(\epsilon_{\varphi 2i} \sim N[0, \Psi_{\varphi 2i}], \epsilon_{\varphi 1i\delta} \sim N[0, \Psi_{\varphi 1i\delta}]\) and \(\epsilon_{s} \sim N[0, \Psi_{s}\xi, \xi]\) are vectors of measurement errors where \(\Psi_{\varphi 2i}, \Psi_{\varphi 1i\delta}, \Psi_{s}\xi, \xi\) are diagonal matrices; and \(I\) is the identity matrix.

The measurement errors within-, between-individual and between-groups are assumed to be independent. All the vectors (bold-face lowercase letters) and matrices (bold-face uppercase letters) for the male sample are indicated by the same letters of the female sample but include the \(\sigma\) subscript. To solve the variance-covariance matrix of the model above defined it is necessary to impose some constraints on the parameters (Lee and Song 2012; Song et al. 2012).
Cubaynes et al. 2012). For example, some values of $\Lambda_{2,\sigma}$, and $\Lambda_{4,\sigma}$ are set to 1 (see Fig. 1). This practice does not impact the interpretation of the model but allows to include variables with a different unit of measurement as an expression of the same latent variable (Cubaynes et al. 2012).

Equations for each model considered are reported below:

- **$M1, M2, M3$:**
  \[
  \begin{align*}
  \eta_{Q1io,1} &= \left[ b_{1i,1} \ b_{1i,2} \right] \times \begin{bmatrix} \zeta_{Q1io,1} \\ \zeta_{Q1io,2} \end{bmatrix} + \left[ \gamma_{Q1i,11} \ \gamma_{Q1i,12} \right] \times \begin{bmatrix} \xi_{Q1io,1} \\ \xi_{Q1io,2} \end{bmatrix} + \left[ \delta_{Q1io,1} \right] \\
  \eta_{Q2i,1} &= \left[ \gamma_{Q2i,11} \ \gamma_{Q2i,12} \right] \times \begin{bmatrix} \xi_{Q2i,1} \\ \xi_{Q2i,2} \end{bmatrix} + \left[ \delta_{Q2i,1} \right] \\
  \eta_{\sigma_{1io},1} &= \left[ b_{1i,1} \ b_{1i,2} \right] \times \begin{bmatrix} \zeta_{\sigma_{1io},1} \\ \zeta_{\sigma_{1io},2} \end{bmatrix} + \left[ \gamma_{\sigma_{1i,11}} \ \gamma_{\sigma_{1i,12}} \right] \times \begin{bmatrix} \xi_{\sigma_{1io},1} \\ \xi_{\sigma_{1io},2} \end{bmatrix} + \left[ \delta_{\sigma_{1io},1} \right] \\
  \eta_{\sigma_{2i,1}} &= \left[ \gamma_{\sigma_{2i,11}} \ \gamma_{\sigma_{2i,12}} \right] \times \begin{bmatrix} \xi_{\sigma_{2i,1}} \\ \xi_{\sigma_{2i,2}} \end{bmatrix} + \left[ \delta_{\sigma_{2i,1}} \right] \\
  \eta_{s,1} &= \left[ \delta_{s,1} \right]
  \end{align*}
  \]

- **$M4$:**
  \[
  \begin{align*}
  \eta_{Q1io,1} &= \left[ b_{1i,1} \ b_{1i,2} \right] \times \begin{bmatrix} \zeta_{Q1io,1} \\ \zeta_{Q1io,2} \end{bmatrix} + \left[ \gamma_{1i,11} \ \gamma_{1i,12} \right] \times \begin{bmatrix} \xi_{Q1io,1} \\ \xi_{Q1io,2} \end{bmatrix} + \left[ \delta_{Q1io,1} \right] \\
  \eta_{Q2i,1} &= \left[ \gamma_{2i,11} \ \gamma_{2i,12} \right] \times \begin{bmatrix} \xi_{Q2i,1} \\ \xi_{Q2i,2} \end{bmatrix} + \left[ \delta_{Q2i,1} \right] \\
  \eta_{\sigma_{1io},1} &= \left[ b_{1i,1} \ b_{1i,2} \right] \times \begin{bmatrix} \zeta_{\sigma_{1io},1} \\ \zeta_{\sigma_{1io},2} \end{bmatrix} + \left[ \gamma_{1i,11} \ \gamma_{1i,12} \right] \times \begin{bmatrix} \xi_{\sigma_{1io},1} \\ \xi_{\sigma_{1io},2} \end{bmatrix} + \left[ \delta_{\sigma_{1io},1} \right] \\
  \eta_{\sigma_{2i,1}} &= \left[ \gamma_{2i,11} \ \gamma_{2i,12} \right] \times \begin{bmatrix} \xi_{\sigma_{2i,1}} \\ \xi_{\sigma_{2i,2}} \end{bmatrix} + \left[ \delta_{\sigma_{2i,1}} \right] \\
  \eta_{s,1} &= \left[ \delta_{s,1} \right]
  \end{align*}
  \]
• M5: 
\[ \eta_{Q11o,1} = [b_{i1,1} b_{i1,2}] \times \left[ \zeta_{Q11o,1} \zeta_{Q11o,2} \right] + [\gamma_{Q11i,12} \gamma_{Q11i,12}] \times \left[ \xi_{Q11o,1} \xi_{Q11o,2} \right] + [\delta_{Q11o,1}] \]
\[ \eta_{Q21i,1} = [\gamma_{Q21i,12} \gamma_{Q21i,12}] \times \left[ \zeta_{Q21i,1} \zeta_{Q21i,2} \right] + [\delta_{Q21i,1}] \]
\[ \eta_{Q311o,1} = [b_{i1,1} b_{i1,2}] \times \left[ \zeta_{Q311o,1} \zeta_{Q311o,2} \right] + [\gamma_{Q31i,12} \gamma_{Q31i,12}] \times \left[ \xi_{Q31o,1} \xi_{Q31o,2} \right] + [\delta_{Q31o,1}] \]
\[ \eta_{Q321i} = [\gamma_{Q321i} \gamma_{Q321i}] \times \left[ \xi_{Q321i} \xi_{Q321i} \right] + [\delta_{Q321i}] \]
\[ \eta_{s,1} = [\delta_{s,1}] \]

• M6: 
\[ \eta_{Q11o,1} = [b_{i1,1} b_{i1,2}] \times \left[ \zeta_{Q11o,1} \zeta_{Q11o,2} \right] + [\gamma_{11i,12} \gamma_{11i,12}] \times \left[ \xi_{Q11o,1} \xi_{Q11o,2} \right] + [\delta_{Q11o,1}] \]
\[ \eta_{Q21i,1} = [\gamma_{21i,12} \gamma_{21i,12}] \times \left[ \xi_{Q21i,1} \xi_{Q21i,2} \right] + [\delta_{Q21i,1}] \]
\[ \eta_{Q311o,1} = [b_{i1,1} b_{i1,2}] \times \left[ \zeta_{Q311o,1} \zeta_{Q311o,2} \right] + [\gamma_{11i,12} \gamma_{11i,12}] \times \left[ \xi_{Q31o,1} \xi_{Q31o,2} \right] + [\delta_{Q31o,1}] \]
\[ \eta_{Q321i} = [\gamma_{21i,12} \gamma_{21i,12}] \times \left[ \xi_{Q321i} \xi_{Q321i} \right] + [\delta_{Q321i}] \]
\[ \eta_{s,1} = [\delta_{s,1}] \]
$M_7: \begin{bmatrix} \eta_{Q_11o,1} \\ \eta_{Q_11o,2} \end{bmatrix} = \begin{bmatrix} b_{1i,1} & b_{1i,2} \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} \zeta_{Q_11o,1} \\ \zeta_{Q_11o,2} \end{bmatrix} + \begin{bmatrix} 0 & \pi_{Q_11,12} \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} \eta_{Q_11o,1} \\ \eta_{Q_11o,2} \end{bmatrix} + \begin{bmatrix} 0 \\ \gamma_{Q11o,2} \end{bmatrix} \times \begin{bmatrix} \xi_{Q11o,1} \end{bmatrix} + \begin{bmatrix} \delta_{Q11o,1} \\ \delta_{Q11o,2} \end{bmatrix}$

$M_8: \begin{bmatrix} \eta_{Q_11o,1} \\ \eta_{Q_11o,2} \end{bmatrix} = \begin{bmatrix} b_{1i,1} & b_{1i,2} \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} \zeta_{Q_11o,1} \\ \zeta_{Q_11o,2} \end{bmatrix} + \begin{bmatrix} 0 & \pi_{Q_11,12} \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} \eta_{Q_11o,1} \\ \eta_{Q_11o,2} \end{bmatrix} + \begin{bmatrix} 0 \\ \gamma_{Q11o,2} \end{bmatrix} \times \begin{bmatrix} \xi_{Q11o,1} \end{bmatrix} + \begin{bmatrix} \delta_{Q11o,1} \\ \delta_{Q11o,2} \end{bmatrix}$
where

- $\eta_{Q1,1}, \eta_{Q2,1}$ = female reproductive potential within- and between-individual,
• $\xi_{\text{fi},1}, \xi_{\text{fi},2} = \text{female social environment within- and between-individual},$

• $\xi_{\text{fi},2}, \xi_{\text{fi},2} = \text{female territory quality within- and between-individual},$

• $\eta_{\text{si},1} = \text{reproductive potential between social group}.$

• note that in $M_7$ to $M_{10}$: $\eta_{\text{fi},2}, \eta_{\text{fi},2} = \text{indicate the female social environment within- and between-individual, while } \xi_{\text{fi},1}, \xi_{\text{fi},1} = \text{indicate the female territory quality within- and between-individual},$

• Similar definitions apply to the male sample (replace the $\varphi$ symbol with the $\sigma^*$ symbol).
\[
\begin{align*}
\begin{bmatrix}
\rho_{1,1} \\
\rho_{2,1} \\
\rho_{3,1} \\
\rho_{4,1}
\end{bmatrix} & = 
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\rho_{1,1} \\
\rho_{2,1} \\
\rho_{3,1} \\
\rho_{4,1}
\end{bmatrix} + 
\begin{bmatrix}
\lambda_{2,1-2} \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\eta_{1,1} \\
\eta_{2,1} \\
\eta_{3,1} \\
\eta_{4,1}
\end{bmatrix} + 
\begin{bmatrix}
\lambda_{2,1-3} \\
\lambda_{2,1-2} \\
\lambda_{2,1-3} \\
\lambda_{2,1-3}
\end{bmatrix}
\begin{bmatrix}
\zeta_{1,1} \\
\zeta_{2,1} \\
\zeta_{3,1} \\
\zeta_{4,1}
\end{bmatrix} \\
\begin{bmatrix}
\rho_{1,1} \\
\rho_{2,1} \\
\rho_{3,1} \\
\rho_{4,1}
\end{bmatrix} & = 
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\rho_{1,1} \\
\rho_{2,1} \\
\rho_{3,1} \\
\rho_{4,1}
\end{bmatrix} + 
\begin{bmatrix}
\lambda_{2,1-2} \\
\lambda_{2,1-2} \\
\lambda_{2,1-3} \\
\lambda_{2,1-3}
\end{bmatrix}
\begin{bmatrix}
\eta_{1,1} \\
\eta_{2,1} \\
\eta_{3,1} \\
\eta_{4,1}
\end{bmatrix} + 
\begin{bmatrix}
\lambda_{2,1-3} \\
\lambda_{2,1-2} \\
\lambda_{2,1-3} \\
\lambda_{2,1-3}
\end{bmatrix}
\begin{bmatrix}
\zeta_{1,1} \\
\zeta_{2,1} \\
\zeta_{3,1} \\
\zeta_{4,1}
\end{bmatrix}.
\end{align*}
\]
The measurement equations of the models $M_2$ to $M_6$ can be easily derived from those of $M_1$ by setting the relevant parameters to be invariant (see Table A2).
Figure A2: Graphical representation of the alternative path diagram in $M_7, M_8, M_9$, and $M_{10}$. Similar to Fig. 1, circles represent the latent variables ($R =$ reproduction potential, $SE =$ social environment, and $TQ =$ territory quality), while rectangles represent fixed covariates and observed variables. To highlight the relationship between observed and latent variables, we colored variables related to the social environment in pink, to territory quality in green, and to reproduction in yellow. Fixed covariates (age, age$^2$) are colored in grey. The path represents both the female and male samples. This path describes an indirect effect of TQ on R. In fact, TQ is directly causing variations in the SE, which is then affecting R (TQ $\rightarrow$ SE $\rightarrow$ R). These relationships are present in models $M_7$ to $M_{10}$, but some of these lines are kept invariant between females and males in different models. For example, the relationships between TQ $\rightarrow$ SE and SE $\rightarrow$ R differ between females and males in models $M_7$, while are invariant in models $M_{10}$. The thickness of the lines is proportional to the relative importance of the corresponding parameter estimate in $M_{10}$, where all the lines are invariant between females and males. The number 1 above the lines indicates that the corresponding observed variable was set as the reference level for the latent variable. The plus and minus signs at the top of each line indicate if the parameter is positive or negative.
### A5 SUMMARY OF THE CONSTRAINTS APPLIED TO PARAMETERS

| Model | Constraints on Parameters |
|-------|---------------------------|
| M1    | $\zeta^f = \zeta^m$      |
| M2    | $\Lambda^{q_1} = \Lambda^{q_2}, \Lambda^{\mu_1} = \Lambda^{\mu_2}, \text{ and } \zeta^q = \zeta^p$ |
| M3    | $\Lambda^{q_1} = \Lambda^{q_2}, \Lambda^{\mu_1} = \Lambda^{\mu_2}, \text{ and } \zeta^q = \zeta^p$ |
| M4    | $\Lambda^{q_2} = \Lambda^{q_2}, \Lambda^{q_1} = \Lambda^{q_2}, \mu^q = \mu^p, \Gamma^q_1 = \Gamma^p_1, \Gamma^q_2 = \Gamma^p_2, \text{ and } \zeta^q = \zeta^p$ |
| M5    | $\Lambda^{q_2} = \Lambda^{q_1}, \Lambda^{q_1} = \Lambda^{q_2}, \mu^q = \mu^p, \Gamma^q_1 = \Gamma^p_1, \Gamma^q_2 = \Gamma^p_2, \text{ and } \zeta^q = \zeta^p$ |
| M6    | $\Lambda^{q_1} = \Lambda^{q_1}, \Lambda^{q_2} = \Lambda^{q_2}, \mu^q = \mu^p, \Gamma^q_1 = \Gamma^p_1, \Gamma^q_2 = \Gamma^p_2, \text{ and } \zeta^q = \zeta^p$ |
| M7    | $\Lambda^{q_2} = \Lambda^{q_2}, \Lambda^{q_2} = \Lambda^{q_2}, \mu^q = \mu^p, \Gamma^q_1 = \Gamma^p_1, \Gamma^q_2 = \Gamma^p_2, \text{ and } \zeta^q = \zeta^p$ |
| M8    | $\Lambda^{q_2} = \Lambda^{q_2}, \Lambda^{q_2} = \Lambda^{q_2}, \mu^q = \mu^p, \Pi^{q_1} = \Pi^{p_1}, \Pi^{q_2} = \Pi^{p_2}, \text{ and } \zeta^q = \zeta^p$ |
| M9    | $\Lambda^{q_2} = \Lambda^{q_2}, \Lambda^{q_2} = \Lambda^{q_2}, \mu^q = \mu^p, \Pi^{q_1} = \Pi^{p_1}, \Pi^{q_2} = \Pi^{p_2}, \text{ and } \zeta^q = \zeta^p$ |
| M10   | $\Lambda^{q_1} = \Lambda^{q_1}, \Lambda^{q_2} = \Lambda^{q_2}, \mu^q = \mu^p, \Gamma^q_1 = \Gamma^p_1, \Gamma^q_2 = \Gamma^p_2, \text{ and } \zeta^q = \zeta^p$ |

Table A2: Summary of the constraints applied to parameters in the SEMs.

### A6 SOFTWARE

Models were implemented in JAGS 4.3.0 and R 4.0.0 (R Core Team 2020) with the use of the packages rjags 4-10 (Plummer 2016) and doMc 1.3.7 (Revolution Analytics and Weston 2015). For each model, we ran five MCMCs of 500,000 samples with a thin of 50, a burn-in of 300,000 and an adaptation phase of 1,000,000. As recommended by Lee and Song (2012) we choose parameter expanded normal priors or normal priors for all the regression parameters and inverse gamma priors for the precision parameters. The specification of prior distributions of the parameters differs among the models because invariant parameters among females and males share a single prior distribution, while not invariant parameters have independent prior specifications (Lee and Song 2012). Actual values of prior distributions can be found within the code to run the models on Gitlab.
### Table A3: Threshold estimates for the ordinal variables analyzed in the SEMs.

For each variable, we report the threshold id, the mean estimate, the lower 95% high posterior density interval (HPDI), and the upper 95% HPDI. The number of threshold values equals the number of levels of the categorical variable minus one and varies between variables because each variable has a different number of levels. The number of offspring intra-group, extra-group and with only one known parent, the group size, and the number of helpers only had three levels of categories, while the insect per vegetation had five levels and the territory size had four levels.

| Variable                                      | Threshold | Mean  | lower 95% HPDI | upper 95% HPDI |
|-----------------------------------------------|-----------|-------|----------------|----------------|
| number of offspring intra-group               | \( \alpha_1 \) | 9.9   | 9.46           | 10.33          |
| number of offspring intra-group               | \( \alpha_2 \) | 22.13 | 21.21          | 23.09          |
| number of offspring extra-group               | \( \alpha_1 \) | 12.35 | 11.86          | 12.82          |
| number of offspring extra-group               | \( \alpha_2 \) | 21.7  | 20.8           | 22.58          |
| number of offspring only one known parent      | \( \alpha_1 \) | 17.03 | 16.43          | 17.67          |
| number of offspring only one known parent      | \( \alpha_2 \) | 24.46 | 23.28          | 25.64          |
| group size                                    | \( \alpha_1 \) | 1.53  | 1.17           | 1.9            |
| group size                                    | \( \alpha_2 \) | 11.53 | 11.07          | 11.99          |
| number of helpers                             | \( \alpha_1 \) | 8.64  | 8.23           | 9.05           |
| number of helpers                             | \( \alpha_2 \) | 19.01 | 18.29          | 19.72          |
| insectXvegetation                             | \( \alpha_1 \) | -13.42| -14.02         | -12.85         |
| insectXvegetation                             | \( \alpha_2 \) | -4.98 | -5.42          | -4.55          |
| insectXvegetation                             | \( \alpha_3 \) | 1.92  | 1.51           | 2.36           |
| insectXvegetation                             | \( \alpha_4 \) | 7.89  | 7.41           | 8.35           |
| territory size                                | \( \alpha_1 \) | -9.01 | -9.44          | -8.58          |
| territory size                                | \( \alpha_2 \) | 6.32  | 5.91           | 6.7            |
| territory size                                | \( \alpha_3 \) | 14.85 | 14.3           | 15.41          |
| M | $L_{\nu=0.5}$ | $\Delta L_{0.5}$ | M | $L_{\nu=0.8}$ | $\Delta L_{0.8}$ |
|---|---|---|---|---|---|
| M6 | 21767.88 | - | M6 | 25838.84 | - |
| M3 | 21791.88 | 24.00 | M3 | 25918.76 | 34.92 |
| M1 | 21793.53 | 25.65 | M5 | 25920.47 | 36.63 |
| M4 | 21798.95 | 31.07 | M4 | 25921.55 | 37.71 |
| M5 | 21801.38 | 33.50 | M2 | 25999.46 | 115.62 |
| M2 | 21803.85 | 35.97 | M1 | 26007.18 | 123.34 |
| M8 | 22127.84 | 359.96 | M9 | 26553.72 | 669.88 |
| M9 | 22271.19 | 503.31 | M7 | 26570.66 | 686.82 |
| M7 | 2285.01 | 517.13 | M10 | 26583.66 | 699.82 |
| M10 | 22297.72 | 529.84 | M8 | 26953.67 | 1069.83 |

Table A4: Model ranking of the ten competing models based on the $L_{\nu=0.5}$ and $L_{\nu=0.8}$ measure. The $\Delta L_{0.5}$ and $\Delta L_{0.8}$ represent the difference between each model and the top ranking model considering, respectively, the $L_{\nu=0.5}$ and $L_{\nu=0.8}$ measure.
| Variable                          | Model | Variable                          | Model |
|----------------------------------|-------|-----------------------------------|-------|
| social environment               | M3b   | number offspring extra−group      | M6    |
|                                  | M4b   |                                   | M3    |
|                                  | M6b   |                                   |       |
| territory quality                | M3b   | number offspring one known parent | M6    |
|                                  | M4b   |                                   | M3    |
|                                  | M6b   |                                   |       |
|                                   |       | number helpers                    | M6    |
|                                   |       |                                   | M3    |
|                                   |       | insect X vegetation               | M6    |
|                                   |       |                                   | M3    |

(a) Slopes of latent variables between-individual, $\Gamma_2$

| Variable                          | Model | Variable                          | Model |
|-----------------------------------|-------|-----------------------------------|-------|
| number offspring intra−group      | M6    | number offspring intra−group      | M6    |
|                                   | M3    |                                   | M3    |
| number offspring extra−group      | M6    | number offspring extra−group      | M6    |
|                                   | M3    |                                   | M3    |
| number offspring one known parent | M6    | number offspring one known parent | M6    |
|                                   | M3    |                                   | M3    |
| group size                        | M6    | group size                        | M6    |
|                                   | M3    |                                   | M3    |
| number helpers                    | M6    | number helpers                    | M6    |
|                                   | M3    |                                   | M3    |
| territory size                    | M6    | territory size                    | M6    |
|                                   | M3    |                                   | M3    |
| insect X vegetation               | M6    | insect X vegetation               | M6    |
|                                   | M3    |                                   | M3    |

(b) Slopes of observed variables between-individual, $\Lambda_{2\xi}$

(c) Intercepts of observed variables within-individual, $\mu$

(d) Intercepts of observed variables between-pair, $\mu_s$
| Variable                  | Model | Model |
|--------------------------|-------|-------|
| number offspring intra−group | M6    | M6    |
| number offspring extra−group | M6    | M6    |
| number offspring one known parent | M6    | M6    |
| group size              | M6    | M6    |
| number helpers           | M6    | M6    |
| territory size           | M6    | M6    |
| insect X vegetation      | M6    | M6    |

(e) Slopes of observed variables between-pair, $\Lambda_s$

| Variable                  | Model | Model |
|--------------------------|-------|-------|
| number offspring intra−group | M6    | M6    |
| number offspring extra−group | M6    | M6    |
| number offspring one known parent | M6    | M6    |
| group size              | M6    | M6    |
| number helpers           | M6    | M6    |
| territory size           | M6    | M6    |
| insect X vegetation      | M6    | M6    |

(f) Errors of observed variables between-pair, $\epsilon_s$

| Variable                  | Model | Model |
|--------------------------|-------|-------|
| number offspring intra−group | M6    | M6    |
| number offspring extra−group | M6    | M6    |
| number offspring one known parent | M6    | M6    |
| group size              | M6    | M6    |
| number helpers           | M6    | M6    |
| territory size           | M6    | M6    |
| insect X vegetation      | M6    | M6    |

(g) Errors observed variables within-individual female, $\epsilon_{Q_{110}}$

(h) Errors observed variables within-individual male, $\epsilon_{Q_{110}}$
Figure A3: Mean and 95% high-posterior density interval (HPDI) for the parameter estimates that are not reported in the main text for the best two models: $M_6$ (filled circles), and $M_3$ (diamonds). Pink symbols represent parameters of the sample of females, blue symbols of males, and black symbols of both sexes. Panel a represents the effect of the latent variables at the between-individual level, while Panel b represents the effect of the factor-loadings of the observed-variables between-individual. The values of $\mu = \mu_2 = \mu_\delta$ (Panel A5(c)), and $\mu_s$ (Panel A5(d)) represent the intercepts of the equations and should be interpreted as the value of the latent variable when the observed variables correspond to their mean. The production of extra-pair offspring highly drives variations in reproduction between social groups (Panel e). Variations in the number of helpers drive variations in the social environment between social groups (Panel e), while variations in the territory quality between social groups are mainly driven by insect availability rather than territory size (Panel e). Errors related to the estimation of the observed variables between social groups, of the observed variables within-individual female and male, and of the latent variables are reported in Panels f, g, h, and i respectively. Smaller mean error estimates indicate higher accuracy in the identification of that variable, smaller 95% HPDI indicate higher precision. Panel j shows the legend of the figure.
Data on Seychelles warblers were collected on Cousin Islands (29 km$^2$, 4°20′S, 55°40′E), Republic of Seychelles between 1995 and 2016. The population of Cousin island is constituted by circa 320 adult individuals and is at carrying capacity (Komdeur et al. 2016; Komdeur and Pels 2005). The population was enclosed into the island and migration/emigration to/from other islands was negligible (Komdeur et al. 2016). Monitoring was intense and almost all breeding attempts were recorded every year during the month of June and September, and often during January-March, which corresponds to the minor breeding season (Richardson et al. 2002).

Birds were caught using mist nets and marked with a metal ring from the British Trust for Ornithology and an individual color ring combination. At capture a small blood sample of circa 25 µl was collected to determine the sex and assign parentage (Hadfield et al. 2006; Komdeur and Pels 2005). The age of the individuals was estimated based on a mixture of behavioral and morphological characteristics (Richardson and Burke 2003). Most birds were captured in their first year of age.

Thanks to molecular parentage assignment we were able to determine the identity of the parents (Edwards et al. 2016; Hadfield et al. 2006; Richardson et al. 2001; Sparks et al. 2020). The combination of molecular parentage assignment with behavioral observations and the identification of the individuals residing in a territory allowed us to identify if paternity was intra- or extra-group. However, for some offspring we were able to only assign the identity of one parent. The uncertainties in the determination of the mother or father arise because the population is highly inbred (i.e., close inbreeding occurred in circa 5% of matings, Richardson et al. 2004). Mothers were especially difficult to assign because the dominant breeding female and cobreeding helpers can be closely related (Richardson and Burke 2003).

Individuals live in groups and defend small territories with defined boundaries. The estimation of territory quality was derived from the territory size and food availability for each group. To calculate the surface of each territory (m$^2$), we draw the territory boundaries on a digital map in the ArcGIS 10.2 (ESRI) environment. To calculate the food abundance relative to the vegetation cover, we first scored the foliage coverage at 20 random points in each territory once a year. More specifically, at each random point, the foliage coverage of the tree species was scored visually at the following height bands: 0 – to 0.75 – m, 0.75 – to 2 – m and at two m intervals after that (Van de Crommenacker et al. 2011). Additionally, we calculated insect availability by counting the number of insects present under 50 leaves of the five most abundant tree species
present in 14 different regions across the island (Brouwer et al. 2009). These regions were chosen with similar exposition to wind-driven salt spray (Brouwer et al. 2009; Komdeur 1992), and insect counts taken in the center of each area were assumed to be representative for all the territories within that region (Komdeur 1992). Given the foliage cover per tree species with broad-leaves $c_x$ and the mean monthly insect count $i_x$ for each tree species $x$ per unit leaf area we calculated the food availability as $\sum (c_x i_x) \text{ (dm}^2)$ (Komdeur 1992).

Each year, between June and September, all the residents in a territory older than six months were counted. The identity and social status of all these individuals was recorded through behavioral observations. When a dominant female started breeding, we performed at least 60 minutes of observations of the nest to determine which individuals visited the nest. If subordinate individuals were incubating the egg(s) and/or feeding the chick(s) they were assigned the status of helpers (Richardson et al. 2002). Some female helpers can lay an egg in the nest (co-breeding; Richardson et al. 2002), but their reproductive success was not analyzed here. There were 29 males that occupied two or more territories simultaneously. In those cases we took the average insectXvegetation, territory size, and we sum up the number of helpers and group size.
A9 COMPARISON WITH PREVIOUS STUDIES

Some of the results presented in the main text contradict previous studies on Seychelles warblers. We showed that territory quality did not impact the reproductive output of females and male breeders and that the number of helpers and group size had a positive effect on reproduction. However, Komdeur (1992) and Brouwer et al. (2009) showed a significant effect of territory quality and territory size on reproduction, respectively. Additionally, Brouwer et al. (2009) showed a positive effect of group size on reproduction only up to a group of five individuals. These discrepancies could be explained by either biological or methodological differences between the three studies. The biological differences could be linked to the different timelines of the three studies (see Table A4 which shows the little overlap between the timelines of the three studies). The studies also use different definitions of variables. For example, Komdeur (1992) and Brouwer et al. (2009) analyzed the number of fledglings that survived to at least three months of age as a proxy for reproduction, while here we analyzed the number of offspring that reached at least 12 months/one year of age. The definition of territory quality also differs: Komdeur (1992) defines territory quality with three categories (low, medium, high quality); Brouwer et al. (2009) calculated the territory size over the number of individuals in the territory; and here, we analyzed the simultaneous effects of food availability and territory size within a latent variable. The main methodological differences are the implementation of generalized linear models (GLM) by Komdeur (1992) and generalized mixed models (GLMM) by Brouwer et al. (2009) versus SEM in this study. There are also other technical differences worth mentioning. The GLM/GLMM in a frequentist framework analyzed the reproductive output of the breeding pair and did not impute missing data. In contrast, with Bayesian SEM, we analyzed the reproductive performance of females and males and imputed missing data points.

To tease apart if biological or methodological differences could explain the discrepancies in the conclusions, we applied the statistical models presented in Brouwer et al. (2009) and Komdeur (1992) to our dataset collected between 1995 and 2016. We ran the GLMs/GLMMs in R (R Core Team 2020) with the glmmTMB package (v. 1.1.2 Bolker 2016) and a log link for count data. Competing models were ranked from the lowest to the highest Akaike information criterion (AIC) values, where a lower AIC indicates a better model fit to the data (Akaike 1987). We present the results of these additional analyses in the following subsections. We predict that the discrepancy between our results with previous studies will be due to:

• biological differences if the implementation of GLM/GLMM on the long-term dataset pro-
vides comparable conclusions with parts of the SEM analysis; or

- methodological differences if results of the GLM/GLMM on the long-term dataset provide comparable conclusions with previous research.

We analyzed both the number of fledglings produced and the number of offspring that reached 12+ months of age (both calculated from the pedigree) with separate GLMs/GLMMs. We speculate that potential discrepancies between the results of the GLM/GLMM on the number of fledglings versus the number of offsprings of 12+ months of age could indicate biological differences in the effect of territory quality and the social environment in the survival of offsprings up to 12+ months of age.

Please note that in the main text, we also refer to a study by Brouwer et al. (2006) who focused on survival. We do not present a formal comparison here as our study focuses on reproduction only.
A9.1 Comparison with Komdeur 1992

Komdeur (1992) showed that yearly reproductive success (the number of fledglings of at least three months of age produced) increased with territory quality and when helpers were present to aid the breeding pair (we copy here the original Table 1 from the paper, now Fig. A5). Following the results from Komdeur (1992), we applied a GLM to our dataset. We ran separate models for the number of fledglings and the number of offsprings of 12+ months. The explanatory variables considered were territory quality (low/medium/high quality where low corresponds to values of territory quality less or equal to the 33rd quantile, medium to values above the 33rd quantile and less or equal to the 66th quantile, and high to values above the 66th quantile), helper(s) presence, and their interaction. Results are presented in Table A5-A6). The model with the lowest AIC value was a model with a single predictive variable, helper presence, both for the number of fledglings and the number of offsprings of 12+ months (Table A5-a and -b). Therefore with the data from 1995-2016 and according to GLMs, there was no effect of territory quality on reproductive output. The effect of helpers’ presence was positive both when analyzing the number of fledglings and the number of offspring of 12+ months of age (see Table A6-a1 and -b1, minimal models). For consistency and to easily compare the results with Table 1 (now Fig. A5) from Komdeur (1992) we also report results of the full model with the interaction between helper(s) presence and territory quality (see Table A6-a2 and -b2, full models). The parameter estimates of the two full models indicate that territory quality did not significantly affect the reproductive output of the breeding pair. Overall, these results meet our first prediction and
show that the discrepancy between the SEM analysis and Komdeur (1992) is due to biological differences.
| Table 1: Effects of territory quality on the Seychelles warbler on Cousin Island (1986–1990) |
|---|---|---|---|---|
| Territory quality ‡ | Low | Medium | High | Statistic ‡ |
| Yearly reproductive success per unaided pair § | 0.19 ± 0.12 | 0.51 ± 0.19 | 0.85 ± 0.21 | $H = 101.04^*$ |
| (pair plus helpers) | (286) | (38) | (28) | |
| Yearly reproductive success per group (V) § | 0.22 ± 0.13 | 0.85 ± 0.21 | 1.27 ± 0.36 | $H = 161.35^*$ |
| Mean group size | 2.4 ± 0.21 | 2.9 ± 0.91 | 3.7 ± 0.93 | $H = 110.12^*$ |
| First-year survival (s) || 0.30 | 0.67 | 0.86 | $X^2 = 28.85^*$ |
| (103) | (23) | (22) | |
| Annual adult survival (s) || 0.76 | 0.88 | 0.91 | $X^2 = 7.82^{**}$ |
| (156) | (64) | (48) | |
| Mean adult life expectancy (x) | 2.7 | 5.4 | 7.4 |

The table shows the effect of territory quality on breeding success and mean group size (± standard deviation; numbers in parentheses are territory years), first-year survival (the probability of surviving to age of one year) and annual adult survival (probability of surviving to the next year, starting at age of one year) (numbers in parentheses are bird years based on colour-ringed individuals), and mean adult life expectancy (the length of time after which 50% of the population alive at one year will have died). A pair consists of one male and one female. There was no emigration from the island, so birds that disappeared could be reliably scored as dead. Seychelles warbler pairs occupying high-quality territories fledged significantly more young per year than pairs on lower-quality territories. First year survival and annual adult survival increased significantly with territory quality. Mean adult life expectancy was therefore also significantly correlated with territory quality.

Figure A5: Table 1 copied with permission from Komdeur (1992)
Table A5: Generalized linear models (GLMs) rankings when replicating the analyses from Komdeur (1992). The dependent variables considered were the number of fledglings (3+ months of age) and offsprings surviving to at least 12+ months of age. The preferred model with the lowest Akaike information criterion (AIC) included only helper presence as an explanatory variable in both cases. Column df represents the degrees of freedom of each model. The values of $\Delta$ are the differences in AIC values between the preferred model and each model.

(a) number of fledglings surviving to 3+ months of age

| Predictors                                | df | AIC  | $\Delta$ |
|-------------------------------------------|----|------|---------|
| helper presence                           | 2  | 2766.53 | 0       |
| territory quality and helper presence (without interaction) | 4  | 2768.70 | 2.17    |
| interaction between territory quality and helper presence | 6  | 2771.98 | 5.45    |
| null model                                | 1  | 2796.04 | 29.51   |
| territory quality                         | 3  | 2798.00 | 31.47   |

(b) number of offsprings surviving to 12+ months of age

| Predictors                                | df | AIC  | $\Delta$ |
|-------------------------------------------|----|------|---------|
| helper presence                           | 2  | 3951.42 | 0.00    |
| null model                                | 1  | 3954.51 | 3.10    |
| territory quality and helper presence (without interaction) | 4  | 3955.34 | 3.92    |
| territory quality                         | 3  | 3958.41 | 6.99    |
| interaction between territory quality and helper presence | 6  | 3959.14 | 7.72    |
### MINIMAL MODELS

| Type          | Predictors     | (a1) number of fledglings surviving to 3+ months of age | (b1) number of offsprings surviving to 12+ months of age |
|---------------|----------------|--------------------------------------------------------|----------------------------------------------------------|
|               |                | Log-Mean std. Error CI                                | Log-Mean std. Error CI                                    |
| Intercept     | no helper      | -1.03 0.05 -1.12 – -0.94                              | 0.24 0.02 0.20 – 0.29                                     |
| Contrast      | helper(s)      | 0.48 0.08 0.32 – 0.64                                 | 0.11 0.05 0.02 – 0.21                                     |
| Observations  |                | 1665                                                   | 1665                                                      |

### FULL MODELS

| Type          | Predictors     | (a2) number of fledglings surviving to 3+ months of age | (b2) number of offsprings surviving to 12+ months of age |
|---------------|----------------|--------------------------------------------------------|----------------------------------------------------------|
|               |                | Log-Mean std. Error CI                                | Log-Mean std. Error CI                                    |
| Intercept     | no helper in low TQ | -1.10 0.08 -1.27 – -0.94                              | 0.24 0.04 0.15 – 0.32                                     |
| Contrast      | no helper in medium TQ | 0.13 0.11 -0.10 – 0.35                               | 0.01 0.06 -0.11 – 0.13                                     |
| Contrast      | no helper in high TQ | 0.10 0.12 -0.13 – 0.32                                | 0.00 0.06 -0.11 – 0.12                                     |
| Contrast      | helper(s) in low TQ | 0.48 0.15 0.19 – 0.77                                 | 0.10 0.09 -0.07 – 0.27                                     |
| Contrast      | helper(s) in medium TQ | 0.51 0.15 0.21 – 0.80                               | 0.10 0.09 -0.07 – 0.28                                     |
| Contrast      | helper(s) in high TQ | 0.65 0.14 0.38 – 0.91                                 | 0.15 0.08 -0.02 – 0.31                                     |
| Observations  |                | 1665                                                   | 1665                                                      |

Table A6: Intercepts and contrasts for Poisson models (log link) of the yearly number of offspring that survived to at least three months of age (fledglings, left-hand-side) and at least 12 months of age (right-hand side). Parameters are untransformed (i.e. on the log-scale or log-mean). Explanatory variables considered are the absence or presence of helpers (respectively helpers(s) and no helper) and their interaction with territory quality (respectively low, medium, high TQ). We report the parameter estimates for the minimal models on the top of the table and on the bottom of the table the full models. The column Type indicates if the variable is an intercept or the contrast between a variable and the intercept. The column std. Error represents the standard error, while CI represents the 95% confidence intervals of each parameter. When the confidence intervals did not overlap zero (i.e. there was a significant effect of the corresponding predictor), we highlighted the predictor with the bold font.
A9.2 Comparison with Brouwer et al. 2009

Brouwer et al. (2009) showed that the number of fledglings produced by the breeding pair increased with the relative territory size per bird (or per capita territory size, equal to the ratio between territory size and group size) and with group size up to a group size of five, but decreased afterward (the maximum group size observed was seven). To facilitate comparisons, we copy here Table 2 from Brouwer et al. 2009, now Fig. A6. The GLMMs included year and territory ID (a unique identifier for each territory) as random effects. We replicated the GLMMs for the number of fledglings and offsprings of 12+ months of age with data from 1995-2016. We considered per capita territory size (territory/bird), group size, and group size squared as explanatory variables. Model rankings based on the AIC values showed that the preferred models differ when considering the number of fledglings (Table A7-a) and the number of offspring of 12+ months of age (Table A7-b). When analyzing the number of fledglings produced, the GLMM with relative territory size per bird, group size and group size squared was the preferred model (Table A7-a). Therefore the full model was the preferred model, and the results were similar to those of Brouwer et al. (2009). We observed a positive effect of the relative territory size per bird and a quadratic effect of group size with an initial increase and a decline when there were two or more additional individuals in a group on the number of fledglings produced (Table A8-a1). The GLMM results differ when analysing the number of offspring of 12+ months of age (Table A7-b and A8-b1 and -b2). The minimal model, including only group size, had a lower AIC value and was the preferred model (Table A7-b). We observed a positive linear effect of group size on the reproductive output of the breeding pair (Table A8-b2). For exhaustivity, we also report the full model at the top right of Table A8-b1 confirming that only the parameter estimates for group size had a significant positive effect on the number of offspring of 12+ months of age. The agreement between the results of the GLMM and SEM on the number of offspring of 12+ months of age indicates that different conclusions with Brouwer et al. (2009) were due to biological differences. Since the analysis of the number of fledglings with GLMMs produced similar results with Brouwer et al. (2009) we speculate that the biological differences might be linked with the differential effect of group size and relative territory size on the survival of fledglings to adulthood. Our analyses are preliminary, and further investigations should be conducted to tackle why these differences might occur.
Table 2. Results from analyses examining annual reproductive output per territory (no. fledglings) of Seychelles Warblers on Cousin Island for 8 years between 1986 and 2006 (N = 902 territories).

| Parameter                | $B \pm SE$ | $\chi^2$ | df | $P$  |
|--------------------------|------------|----------|----|------|
| Final model              |            |          |    |      |
| Intercept                | $-2.91 \pm 0.38$ |          |    |      |
| Territory size/bird      | $3.24 \pm 0.81$ | $14.0$   | $1$ | $<0.001$ |
| Group size               | $1.12 \pm 0.20$ | $27.4$   | $1$ | $<0.001$ |
| Group size$^2$           | $-0.09 \pm 0.03$ | $11.7$   | $1$ | $<0.001$ |
| Random effects           |            |          |    |      |
| $\sigma^2_{\text{year}}$| $0.16 \pm 0.15$ |          |    |      |
| $\sigma^2_{\text{territory}}$ | $0.01 \pm 0.01$ |          |    |      |
| Rejected                 |            |          |    |      |
| Population density       | $0.14 \pm 0.20$ | $0.50$   | $1$ | $0.48$ |
| Territory quality        | $0.002 \pm 0.01$ | $0.02$   | $1$ | $0.89$ |
| Total rainfall           | $0.001 \pm 0.001$ | $0.46$   | $1$ | $0.50$ |
| Breeding season rainfall | $0.001 \pm 0.001$ | $0.10$   | $1$ | $0.75$ |

Figure A6: Table 2 copied with permission from Brouwer et al. (2009)
### (a) number of fledglings surviving to 3+ months of age

| Predictors                                      | df | AIC     | ∆    |
|------------------------------------------------|----|---------|------|
| (territory size / bird) + group size + group size | 6  | 3866.74 | 0    |
| group size + group size²                         | 5  | 3872.48 | 5.74 |
| (territory size / bird) + group size             | 5  | 3877.15 | 10.42|
| group size                                       | 4  | 3878.45 | 11.71|
| (territory size / bird)                          | 4  | 3949.96 | 83.22|
| null model                                       | 3  | 3975.02 | 108.29|

### (b) number of offsprings surviving to 12+ months of age

| Predictors                                      | df | AIC     | ∆    |
|------------------------------------------------|----|---------|------|
| group size                                      | 4  | 5401.28 | 0    |
| group size + group size²                        | 5  | 5401.47 | 0.19 |
| (territory size / bird) + group size            | 5  | 5403.27 | 1.99 |
| (territory size / bird) + group size + group size² | 6  | 5403.30 | 2.02 |
| (territory size / bird)                         | 4  | 5412.76 | 11.48|
| null model                                      | 3  | 5417.98 | 16.70|

Table A7: Generalized linear mixed models (GLMMs) rankings when replicating the analyses from Brouwer et al. (2009). The dependent variables considered were the number of fledglings and the number of offspring surviving to at least 12+ months of age. When analyzing the number of fledglings the preferred model with the lowest Akaike information criterion (AIC) was the full model including per capita territory size (territory size/bird), group size, and its square. When analyzing the number of offspring of 12+ months of age the preferred model based on its AIC value contained only group size. The column df represents the degrees of freedom of each model. The values of ∆ are the differences in AIC values between the preferred model and each model.
### FULL MODELS

| Predictors                | (a1) number of fledglings surviving to 3+ months of age | (b1) number of offsprings surviving to 12+ months of age |
|---------------------------|--------------------------------------------------------|--------------------------------------------------------|
|                           | Log-Mean  std. Error  CI                               | Log-Mean  std. Error  CI                               |
| Intercept                 | -0.30  0.24  -0.78 – 0.17                              | 0.46  0.13  0.21 – 0.71                                |
| (territory size/ bird)   | 0.14  0.05  0.04 – 0.24                                | 0.01  0.03  -0.04 – 0.06                               |
| group size                | 1.07  0.21  0.67 – 1.48                                | 0.24  0.12  0.01 – 0.48                                |
| group size²               | -0.25  0.07  -0.39 – -0.11                             | -0.06  0.04  -0.14 – 0.02                              |
| **Random Effects**        |                                                        |                                                        |
| $\sigma^2$                | 1.31  0.56                                           |                                                        |
| $\tau_{00}$ year          | 0.31  0.01                                           |                                                        |
| $\tau_{00}$ territory     | 0.08  0.00                                           |                                                        |
| N year                    | 21  21                                               |                                                        |
| N territory               | 178  178                                             |                                                        |
| Observations              | 2258  2258                                           |                                                        |
| Marginal R²               | 0.068  0.012                                         |                                                        |

### MINIMAL MODELS

| Predictors                | (a2) number of fledglings surviving to 3+ months of age | (b2) number of offsprings surviving to 12+ months of age |
|---------------------------|--------------------------------------------------------|--------------------------------------------------------|
|                           | Log-Mean  std. Error  CI                               | Log-Mean  std. Error  CI                               |
| Intercept                 | Not applicable                                        | 0.28  0.02  0.24 – 0.33                                |
| group size                | Not applicable                                        | 0.08  0.02  0.04 – 0.11                                |
| **Random Effects**        |                                                        |                                                        |
| $\sigma^2$                | Not applicable                                        | 0.56                                                  |
| $\tau_{00}$ year          | Not applicable                                        | 0.01                                                  |
| $\tau_{00}$ territory     | Not applicable                                        | 0.00                                                  |
| N year                    | Not applicable                                        | 21                                                    |
| N territory               | Not applicable                                        | 178                                                   |
| Observations              | Not applicable                                        | 2258                                                  |
| Marginal R²               | Not applicable                                        | 0.011                                                 |

Table A8: Intercepts and slopes for Poisson models of the yearly number of offspring that survived to at least 3 months of age (fledglings, left-hand-side) and 12+ months/one+ year of age (right-hand side) as a function of per capita territory size (territory size/bird), group size and the squared of group size. Parameters are untransformed (i.e. in the log-scale or log-mean). On the top of the table we report the parameter estimates for the full models and on the bottom of the table the minimal model. When analyzing the number of fledglings produced the preferred model was the full model. When considering the number of offspring of 12+ months of age the preferred model was the minimal model. The column std. Error represents the standard error, while CI represents the 95% confidence intervals of each parameter. When the confidence intervals did not overlap zero (i.e. there was a significant effect of the corresponding predictor) we highlighted the predictor with the bold font.
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