Wavelet transform analysis of composite signal under multi-source excitation

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Abstract. In practical engineering application, the response of mechanical structure may be the superposition effect of vibration source interaction, while the traditional detection method only considers the main vibration source, ignoring other vibration sources will cause analysis error. Therefore, it is of practical engineering significance to study the detection method of compound excitation in mechanical testing. This paper aims to present an extraction method of composite modulation signal under multi-source excitation by wavelet transform. The modulated signal is analyzed by short-time Fourier transform and wavelet transform. The time-frequency analysis results show that the resolution of wavelet can be automatically adjusted at high frequency, and the effect of wavelet is better at high frequency. The key contribution is to establish a method for extracting multi-source excitation signals and to provide certain technical support for the modulated signal under multi-source excitation.

1. Introduction
In practical engineering application, the response of mechanical structure may be the superposition effect of vibration source interaction, while the traditional detection method only considers the main vibration source, ignoring other vibration sources will cause analysis error. Therefore, it is of practical engineering significance to study the detection method of compound excitation in mechanical testing.

Multi-source excitation signals are complex, so it is necessary to adopt effective signal processing methods in order to extract the state features of mechanical structures. When both amplitude modulation and frequency modulation exist in the detection signal, a single frequency domain analysis cannot effectively separate these modulation features. The composite excitation method is applied to monitor the structural vibration response characteristics by applying different excitation parameters, and it is gradually referenced by the scholars at home and abroad in the field of engineering structure monitoring. The Missouri-Rolla University, Boeing and the Air Force Research Institute jointly proposed a hybrid fault detection technology based on ultrasonic and eddy current. Some scholars have tried compound detection technology research: Zhang Qinghua studied the mechanism and application of combined ultrasonic and eddy current nondestructive testing[1]. Klepka studied the application of nonlinear vibration-acoustic modulation technique in damage detection of composite laminates[2]. Ming Hong studied damage characterization by modeling nonlinearities of ultrasonic waves[3]. Among various signal analysis methods, time-frequency analysis can capture signal features in time domain and frequency domain to improve the extraction accuracy of arrival time. At present, some scholars have introduced the time-frequency method and wavelet transform into the analysis of modulated signals. These studies show that the required modes can be separated by multi-scale analysis of scattered signals using wavelet transform. Lin used Hilbert transform to demodulate the structural modal analysis signal and evaluate the damage [4]. Zumpano et al. used the average power
as the damage index to perform time-frequency analysis on the modulated signal[5]. Courtney et al. used bispectrum transform to process acoustic modulation signal, and discussed the influence of damage degree on modulation effect[6]. Generally speaking, the traditional signal processing methods can no longer meet the feature analysis of multi-source excitation detection signals. At present, time-frequency analysis, wavelet transform, Hilbert transform and other analysis methods have been constantly tried and updated.

Therefore, combining with the development trend of detection technology, this paper aims to present an extraction method of composite modulation signal under multi-source excitation by wavelet transform and the key contribution is to establish a method for extracting multi-source excitation signals and to provide certain technical support for the modulated signal. This paper is structured as follows. Section 2 reviews the basic principle of wavelet transform. Section 3 is devoted to the characteristics of composite modulation signal under multi-source excitation based on wavelet transform by simulation. Different wavelets are selected and the results show that the larger the bandwidth parameter and center frequency of wavelets, the better the time-frequency aggregation reflected on the time-frequency graph. At the same time, the modulated signal is analyzed by short-time Fourier transform and wavelet transform. The time-frequency analysis results show that the resolution of wavelet can be automatically adjusted at high frequency, and the effect of wavelet is better at high frequency. Section 4 summarizes the full text.

2. Principle of wavelet transform

Short time Fourier transform is local stationary. A long stochastic process can be regarded as the superposition of a series of short-term stationary signals by adding window function in time domain. Once the window is selected, it cannot be changed. Although the windowed Fourier transform can solve the problem of time-domain localization of the transform function, the size and shape of the window are fixed, and the window is not adaptive, so it can not be optimized for specific problems. In practice, for high-frequency signals, the sampling frequency is high because the waveform is relatively narrow in order to give a better accuracy, on the other hand for low-frequency signals, the time period is longer to give complete signal information because the waveform is relatively wide.

Wavelet transform just overcomes these problems:

1) Wavelet transform is a group of windows, and this group of windows have orthogonal relationship, so the frequency domain does not overlap.

2) This set of windows is from small to large in time scale, so that we can effectively intercept the spectrum characteristics of time domain signals of different scales.

The main steps of multi-source signal extraction using continuous wavelet transform are as follows: Firstly, the scale coefficient matrix is obtained by continuous wavelet transform and transformed into frequency matrix. Then, the corresponding instantaneous amplitude and frequency are obtained by using the frequency matrix.

Assuming that the signal contains two frequency components, corresponding to \( f_H \) and \( f_L \) respectively, the high frequency carrier signal \( x_H(t) \) and low frequency modulation signal \( x_L(t) \) are as follows:

\[
x_H(t) = A_H \cos(2\pi f_H \cdot t), \quad x_L(t) = A_L \cos(2\pi f_L \cdot t)
\]

(1)

In case of frequency modulation, the modulation coefficient \( m_f \) is:

\[
m_f = \frac{A_L}{f_L}
\]

(2)

Then the frequency modulation result \( y \) can be expressed as follows:

\[
y(t) = A_H \cdot \cos(2\pi f_H \cdot t + m_f \sin(2\pi f_L \cdot t))
\]

(3)

The expansion is in the form of Fourier series:
\[ y(t) = A_H \sum_{n=-\infty}^{\infty} S_n(m_f) \cos[2\pi(f_H + nf_L) \cdot t] \] (4)

Where Bessel functions of the first order \( S_n(m_f) \) is expressed as follows:

\[ S_n(m_f) = \sum_{m=0}^{\infty} (-1)^m \frac{(m)^2}{2m!(m+n)!} \] (5)

The following equation is satisfied:

\[ S_{-n}(m_f) = (-1)^n S_n(m_f) \] (6)

The Fourier transform is as follows:

\[
Y(f) = A_H S_0(m_f) \delta(f - f_H) + A_H \sum_{n=1}^{\infty} \left\{ S_n(m_f) [\delta(f - f_H - nf_L) t + (-1)^n \delta(f - f_H + nf_L)] \right\}
\] (7)

It can be seen from equation (7) that the modulated signal is composed of carrier frequency component and side frequency component. Among them, the side frequency components are symmetrically distributed on both sides of the carrier frequency, and the amplitude of the corresponding frequency conversion components on both sides is equal.

Generally speaking, there are numerous frequency conversion components, but the amplitude of high-order side frequency components decreases with the increase. Therefore, in the actual analysis process, only the lower side frequency components in the previous frequency band are considered.

In addition, the amplitude of the frequency conversion component is related to the frequency modulation coefficient. When the frequency modulation coefficient changes, the amplitude of the frequency conversion component also changes.

3. Response simulation under multi-source excitation

According to the above steps of wavelet transform, different wavelets are selected to analyze the modulation signal. The comparison of analysis results shows that the Morlet wavelet is better. Moreover, the larger the bandwidth parameter and the center frequency of Morlet wavelet, the better the time-frequency aggregation reflected in the time-frequency diagram. Figure 1 shows wavelet analysis of modulated signal based on the time-frequency analysis. It can be seen from figure 1 that the wavelet effect is good at high frequency, because the resolution of wavelet can be adjusted automatically at high frequency, the resolution is high.
Here it is assumed that the multi-source excitation signal contains two frequency components, $f_H = 23kHz$ and $f_L = 4kHz$. In order to simplify the signal calculation, the parameters of signal amplitude and modulation coefficient are set as follows: $A_H = 1$, $m_f = 1$. The modulation result can be derived from Formula 3. When frequency modulation occurs, it can be assumed that the frequency modulation result is expressed as follows:

$$y(t) = \sin(2\pi \cdot f_H \cdot t + \sin(2\pi \cdot f_L \cdot t))$$  \hspace{1cm} (8)

Fourier transform is used to get the corresponding time-domain waveform and spectrum as shown in Figure 2, where the overall envelope of the signal does not change, but the density of the signal changes.

In addition, it can be seen from the spectrum diagram of Fig. 2 that multiple side frequency components appear in the spectrum, which are distributed on both sides of the carrier frequency.

Fig. 3 is the analysis of energy distribution characteristics of modulated signal with two frequency components by using continuous wavelet transform. The main energy bands of the signal are concentrated in the vicinity of two carriers, which are strip-shaped and evenly distributed along the time axis.
4. Conclusion
This work focuses on wavelet transform of composite modulation signal under multi-source excitation, differenting from the conventional approaches. In this paper, a detection system by wavelet transform of composite technology has been given. First, the principle of wavelet transform is analyzed. Then the wavelet transform of the composite signal by simulation are studied. Different wavelets are selected for comparative analysis, and the results show that the larger the bandwidth parameter and center frequency of wavelets, the better the time-frequency aggregation reflected on the time-frequency graph. At the same time, the modulated signal is analyzed by short-time Fourier transform and wavelet transform. The time-frequency analysis results show that the resolution of wavelet can be automatically adjusted at high frequency, and the effect of wavelet is better at high frequency.

Acknowledgment
This paper was supported by the scientific research “Research on mechanical structure diagnosis method based on multi-vibration excitation”(No.2020YK042) of Wuhan polytechnic.

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Figure 3  Composite signal waveform and spectrum with two components