Bound on local minimum-error discrimination of bipartite quantum states

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We consider the optimal discrimination of bipartite quantum states and provide an upper bound for the maximum success probability of optimal local discrimination. We also provide a necessary and sufficient condition for a measurement to realize the upper bound. We further establish a necessary and sufficient condition for this upper bound to be saturated. Finally, we illustrate our results using an example.

Quantum state discrimination is one of the fundamental tasks in quantum information processing [1−3]. In discriminating orthogonal quantum states, there is always a measurement of perfect discrimination. On the other hand, non-orthogonal quantum states cannot be perfectly discriminated by means of any measurement. For this reason, there has been a huge amount of research effort focused on finding good state-discriminating strategies [4].

In discriminating multiparty quantum states, it is known that some optimal state discrimination cannot be realized only by local operations and classical communication (LOCC) [5−8]. To characterize the limitation of LOCC discrimination, many studies have been contributed to optimal local discrimination of multiparty quantum states [9−15]. Nevertheless, due to the difficulty of mathematical characterization for LOCC, it is still a hard task to realize optimal local discrimination.

One efficient way to handle this difficulty is to investigate possible upper bounds for the maximum success probability of optimal local discrimination. Moreover, establishing good conditions on measurements realizing such upper bounds is also important for a better understanding of optimal local discrimination.

Here, we consider bipartite quantum state discrimination and provide an upper bound for the maximum success probability of optimal local discrimination. We also provide a necessary and sufficient condition for a measurement to realize the upper bound. Moreover, we establish a necessary and sufficient condition for this upper bound to be saturated; it is equal to the maximum success probability of optimal local discrimination. Finally, we illustrate our results using an example.

In bipartite quantum systems, a state is represented by a density operator \( \rho \), that is, a Hermitian operator having positive semidefinite \( \rho \geq 0 \) and unit trace \( \text{Tr} \rho = 1 \), acting on a bipartite complex Hilbert space \( \mathcal{H} = \mathbb{C}^{d_1} \otimes \mathbb{C}^{d_2} \). A measurement is described by a positive operator-valued measure (POVM) \( \{ M_i \}_i \) that is a set of Hermitian operators \( M_i \) on \( \mathcal{H} \) satisfying positive semidefiniteness \( M_i \geq 0 \) for all \( i \) and the completeness relation \( \sum_i M_i = 1 \), where \( 1 \) is the identity operator on \( \mathcal{H} \). When a measurement \( \{ M_i \}_i \) is performed for input state \( \rho \), the probability of obtaining measurement outcome with respect to \( M_j \) is \( \text{Tr}(\rho M_j) \).

**Definition 1.** A Hermitian operator \( E \) on \( \mathcal{H} \) is called positive partial transpose (PPT) if its partial transposition, denoted \( E^{\text{PT}} \), is positive semidefinite [16−18]. Similarly, we say that a set of Hermitian operators \( \{ E_i \}_i \) is PPT if \( E_i \) is PPT for all \( i \).

A LOCC measurement is a measurement that can be realized by LOCC. We note that every LOCC measurement is a PPT measurement [19].

Throughout this paper, we only consider the situation of discriminating \( n \) bipartite quantum states \( \rho_1, \ldots, \rho_n \) in which the state \( \rho_i \) is prepared with the probability \( \eta_i \). We denote this situation as an ensemble \( \mathcal{E} = \{ \eta_i, \rho_i \}^n_{i=1} \).

Let us consider the quantum state discrimination of \( \mathcal{E} \) using a measurement \( \{ M_i \}_i \) where the click of \( M_i \) means the detection of \( \rho_i \). The minimum-error discrimination \([20−22]\) of \( \mathcal{E} \) is to achieve the minimum error in correctly guessing the prepared state. Equivalently, the minimum-error discrimination of \( \mathcal{E} \) is to achieve the so-called guessing probability of \( \mathcal{E} = \{ \eta_i, \rho_i \}^n_{i=1} \), defined as

\[
 p_G(\mathcal{E}) = \max_{\text{POVM}} \sum_{i=1}^n \eta_i \text{Tr}(\rho_i M_i),
\]

where the maximum is taken over all possible POVMs. The POVMs providing the optimal success probability \( p_G(\mathcal{E}) \) in Eq. (1) can be verified from the following conditions \([21−24]\):

\[
 \sum_{j=1}^n \eta_j \rho_j M_j - \eta_i \rho_i \geq 0 \quad \forall i = 1, \ldots, n. \tag{2a}
\]

\[
 M_i(\eta_i \rho_i - \eta_j \rho_j) M_j = 0 \quad \forall i, j = 1, \ldots, n. \tag{2b}
\]

Note that Condition (2a) is a necessary and sufficient condition for a measurement \( \{ M_i \}_i \) to realize \( p_G(\mathcal{E}) \), whereas Condition (2b) is a necessary but not sufficient condition for a POVM \( \{ M_i \}_i \) to provide \( p_G(\mathcal{E}) \).

When the available measurements are limited to PPT POVMs, we denote the maximum success probability by

\[
 p_{\text{PPT}}(\mathcal{E}) = \max_{\text{POVM}} \sum_{i=1}^n \eta_i \text{Tr}(\rho_i M_i). \tag{3}
\]
We denote by \( p_L(\mathcal{E}) \) the maximum of success probability that can be obtained by using LOCC measurements; that is,
\[
p_L(\mathcal{E}) = \max_{\text{LOCC}} \sum_{i=1}^{n} \eta_i \text{Tr}(\rho_i M_i).
\]
From the definitions of \( p_G(\mathcal{E}) \) and \( p_{\text{PPT}}(\mathcal{E}) \), \( p_G(\mathcal{E}) \) is obviously an upper bound of \( p_{\text{PPT}}(\mathcal{E}) \). Moreover, \( p_L(\mathcal{E}) \) is a lower bound of \( p_{\text{PPT}}(\mathcal{E}) \) because all LOCC measurements are PPT [19]. Thus, we have
\[
p_L(\mathcal{E}) \leq p_{\text{PPT}}(\mathcal{E}) \leq p_G(\mathcal{E}).
\]
We also note that \( p_L(\mathcal{E}) = p_{\text{PPT}}(\mathcal{E}) \) if and only if there exists a LOCC measurement realizing \( p_{\text{PPT}}(\mathcal{E}) \) since both \( p_{\text{PPT}}(\mathcal{E}) \) and \( p_L(\mathcal{E}) \) have the same objective function for maximization.

For a given ensemble \( \mathcal{E} = \{\eta_i, \rho_i\}_{i=1}^{n} \), let us consider the maximum quantity
\[
q_G(\mathcal{E}) = \max_{\text{PVM}} \sum_{i=1}^{n} \eta_i \text{Tr}(\rho_i^{\text{PPT}} M_i)
\]
over all possible POVMs. The following lemma shows that \( q_G(\mathcal{E}) \) in Eq. (6) is an upper bound of \( p_{\text{PPT}}(\mathcal{E}) \):

**Lemma 1.** For a bipartite quantum state ensemble \( \mathcal{E} = \{\eta_i, \rho_i\}_{i=1}^{n} \),
\[
p_{\text{PPT}}(\mathcal{E}) \leq q_G(\mathcal{E}),
\]
where the equality holds if and only if there exists a PPT measurement realizing \( q_G(\mathcal{E}) \).

**Proof.** Inequality (7) holds because
\[
q_G(\mathcal{E}) = \max_{\sum_{i=1}^{n} M_i = 1, M_i \geq 0} \sum_{i=1}^{n} \eta_i \text{Tr}(\rho_i^{\text{PPT}} M_i) = \max_{\sum_{i=1}^{n} M_i = 1, M_i \geq 0} \sum_{i=1}^{n} \eta_i \text{Tr}(\rho_i M_i),
\]
where the first equality follows from \( \text{Tr}(AB) = \text{Tr}(A^{\text{PT}} B^{\text{PT}}) \) for any two operators \( A \) and \( B \), the second equality holds due to \( 1 = 1^{\text{PT}} \), and the inequality is from the fact that a PPT POVM \( \{M_i\}_{i=1}^{n} \) implies \( M_i \geq 0 \) for all \( i \) along with \( M_i^{\text{PT}} \geq 0 \) for all \( i \) and \( \sum_{i=1}^{n} M_i = 1 \).

If \( p_{\text{PPT}}(\mathcal{E}) = q_G(\mathcal{E}) \), then
\[
q_G(\mathcal{E}) = p_{\text{PPT}}(\mathcal{E}) = \sum_{i=1}^{n} \eta_i \text{Tr}(\rho_i M_i) = \sum_{i=1}^{n} \eta_i \text{Tr}(\rho_i^{\text{PPT}} M_i^{\text{PT}})
\]
for some PPT POVM \( \{M_i\}_{i=1}^{n} \). Since \( \{M_i^{\text{PT}}\}_{i=1}^{n} \) is also a PPT POVM, there exists a PPT measurement giving \( q_G(\mathcal{E}) \). Conversely, if \( \{M_i^{\text{PT}}\}_{i=1}^{n} \) is a PPT POVM providing \( q_G(\mathcal{E}) \), then
\[
p_{\text{PPT}}(\mathcal{E}) \leq q_G(\mathcal{E}) = \sum_{i=1}^{n} \eta_i \text{Tr}(\rho_i^{\text{PPT}} M_i) = \sum_{i=1}^{n} \eta_i \text{Tr}(\rho_i M_i^{\text{PT}}) \leq p_{\text{PPT}}(\mathcal{E})
\]
where the first inequality is from Inequality (8) and the second inequality follows from the fact that \( \{M_i^{\text{PT}}\}_{i=1}^{n} \) is also a PPT POVM. Thus, \( p_{\text{PPT}}(\mathcal{E}) = q_G(\mathcal{E}) \). \( \square \)

**Corollary 1.** For a bipartite quantum state ensemble \( \mathcal{E} = \{\eta_i, \rho_i\}_{i=1}^{n} \),
\[
p_L(\mathcal{E}) = q_G(\mathcal{E})
\]
if and only if there is a POVM \( \{M_i\}_{i=1}^{n} \) giving \( q_G(\mathcal{E}) \) such that \( \{M_i^{\text{PT}}\}_{i=1}^{n} \) is a LOCC measurement.

**Proof.** Suppose that \( \{M_i\}_{i=1}^{n} \) is a POVM giving \( q_G(\mathcal{E}) \) and \( \{M_i^{\text{PT}}\}_{i=1}^{n} \) is a LOCC measurement. Since \( \{M_i^{\text{PT}}\}_{i=1}^{n} \) is a PPT POVM, \( p_{\text{PPT}}(\mathcal{E}) = q_G(\mathcal{E}) \) due to Lemma 1. Also, \( \{M_i^{\text{PT}}\}_{i=1}^{n} \) gives \( p_{\text{PPT}}(\mathcal{E}) \) in Eq. (3) because
\[
\sum_{i=1}^{n} \eta_i \text{Tr}(\rho_i M_i^{\text{PT}}) = \sum_{i=1}^{n} \eta_i \text{Tr}(\rho_i^{\text{PT}} M_i) = q_G(\mathcal{E}) = p_{\text{PPT}}(\mathcal{E}).
\]
The existence of a LOCC measurement giving \( p_{\text{PPT}}(\mathcal{E}) \) implies \( p_L(\mathcal{E}) = p_{\text{PPT}}(\mathcal{E}) \). Thus, Eq. (11) holds. Conversely, if Eq. (11) is satisfied, then
\[
q_G(\mathcal{E}) = p_L(\mathcal{E}) = \sum_{i=1}^{n} \eta_i \text{Tr}(\rho_i M_i) = \sum_{i=1}^{n} \eta_i \text{Tr}(\rho_i^{\text{PT}} M_i^{\text{PT}}),
\]
where \( \{M_i\}_{i=1}^{n} \) is a LOCC measurement realizing \( p_L(\mathcal{E}) \) in Eq. (4). Since every LOCC measurement is PPT, it follows that \( \{M_i^{\text{PT}}\}_{i=1}^{n} \) with \( M_i = M_i^{\text{PT}} \) for all \( i \) is a POVM giving \( q_G(\mathcal{E}) \) such that \( \{M_i^{\text{PT}}\}_{i=1}^{n} \) is a LOCC measurement. \( \square \)

For a given state ensemble \( \mathcal{E} = \{\eta_i, \rho_i\}_{i=1}^{n} \), the following theorem provides a necessary and sufficient condition on a measurement \( \{M_i\}_{i=1}^{n} \) to realize \( q_G(\mathcal{E}) \) in Eq. (6).

**Theorem 1.** For a bipartite quantum state ensemble \( \mathcal{E} = \{\eta_i, \rho_i\}_{i=1}^{n} \), a POVM \( \{M_i\}_{i=1}^{n} \) gives \( q_G(\mathcal{E}) \) if and only if it satisfies
\[
\sum_{j=1}^{n} \eta_j \rho_j^{\text{PT}} M_j - \eta_i \rho_i^{\text{PT}} \geq 0 \quad \forall i = 1, \ldots, n.
\]
Moreover, if a POVM \( \{M_i\}_{i=1}^{n} \) realizes \( q_G(\mathcal{E}) \), then
\[
M_i (\eta_i \rho_i^{\text{PT}} - \eta_j \rho_j^{\text{PT}}) M_j = 0 \quad \forall i, j = 1, \ldots, n.
\]
**Proof.** To prove the sufficiency of the first statement, we suppose that \( \{M_i\}_{i=1}^{n} \) is a POVM satisfying Condition (14). For any POVM \( \{M_i\}_{i=1}^{n} \), we have
\[
\sum_{j=1}^{n} \eta_j \text{Tr}(\rho_j^{\text{PT}} M_j) - \sum_{k=1}^{n} \eta_k \text{Tr}(\rho_k^{\text{PT}} M_k) = \text{Tr} \left[ \sum_{j=1}^{n} \eta_j \rho_j^{\text{PT}} M_j \left( \sum_{i=1}^{n} M_i^{\text{PT}} \right) - \sum_{k=1}^{n} \text{Tr}(\eta_k^{\text{PT}} M_k) \right] = \sum_{i=1}^{n} \text{Tr} \left[ (\sum_{j=1}^{n} \eta_j \rho_j^{\text{PT}} M_j - \eta_i \rho_i^{\text{PT}}) M_i^{\text{PT}} \right] \geq 0.
\]
where the first equality follows from \( \sum_{i=1}^{n} M_i^T = \mathbb{1} \) and the inequality is from \( M_i^T \geq 0 \) for all \( i \) and Condition (14). Thus, the definition of \( q_G(\mathcal{E}) \) leads us to

\[
\sum_{i=1}^{n} \eta_i \text{Tr}(\rho_i^{PT} M_i) = q_G(\mathcal{E}),
\]

which proves the sufficiency of the first statement.

To prove the second statement along with the necessity of the first statement, we assume that \( \{M_i\}_{i=1}^{n} \) is a POVM providing \( q_G(\mathcal{E}) \). We first show the positive semidefiniteness of the following Hermitian operators

\[
H_i = \frac{1}{2} \sum_{j=1}^{n} \left( \eta_j \rho_j^{PT} M_j + \eta_j M_j \rho_j^{PT} \right) - \eta_i \rho_i^{PT}, \quad i = 1, \ldots, n.
\]

To show it, we first suppose \( \langle v \vert H_i \vert v \rangle < 0 \) for some unit vector \( \vert v \rangle \) and lead to a contradiction. For \( 0 < \epsilon < 1 \), let us consider the following POVM \( \{M_i^{(\epsilon)}\}_{i=1}^{n} \),

\[
M_i^{(\epsilon)} = \left( (1 - \epsilon) \vert v \rangle \langle v \vert + \epsilon \vert v \rangle \langle v \vert \delta_{i1}, \quad i = 1, \ldots, n,
\]

where \( \delta_{ij} \) is the Kronecker delta.

From a straightforward calculation, we can easily see that

\[
\sum_{j=1}^{n} \eta_j \text{Tr}(\rho_j^{PT} M_j^{(\epsilon)}) - \sum_{j=1}^{n} \eta_i \text{Tr}(\rho_i^{PT} M_i) = -2\epsilon \langle v \vert H_i \vert v \rangle + \epsilon^2 \left[ \sum_{j=1}^{n} \eta_j \langle v \rangle \langle v \vert M_j \rangle \langle v \rangle - \eta_i \langle v \rangle \langle v \vert \right].
\]

For given real numbers \( a \) and \( b \) with \( a > 0 \), we note that there is \( \epsilon \in (0, 1) \) such that \( a\epsilon + H_i^2 > 0 \). Thus, the right-hand side of Eq. (20) is positive for some \( \epsilon \in (0, 1) \). This contradicts the assumption that \( \{M_i\}_{i=1}^{n} \) realizes \( q_G(\mathcal{E}) \), therefore \( H_i \geq 0 \). Since the choice of \( H_i \) with a negative eigenvalue can be arbitrary, \( H_i \geq 0 \) for all \( i \).

Now, we show the satisfaction of Conditions (14) and (15). It is straightforward to verify that

\[
2 \sum_{i=1}^{n} H_i M_i = \sum_{i=1}^{n} \eta_i M_i \rho_i^{PT} - \sum_{j=1}^{n} \eta_j \rho_j^{PT} M_j.
\]

Since the right-hand side of Eq. (21) is traceless, we have

\[
\sum_{i=1}^{n} \text{Tr}(H_i M_i) = 0,
\]

which implies

\[
H_i M_i = M_i H_i = 0 \quad \forall i = 1, \ldots, n
\]

due to the positive semidefiniteness of \( H_i \) and \( M_i \) for all \( i \). Equations (21) and (23) lead us to

\[
\sum_{i=1}^{n} \eta_i M_i \rho_i^{PT} = \sum_{i=1}^{n} \eta_i \rho_i^{PT} M_i
\]

Applying Eq. (24) to Eq. (18), we can show that

\[
\sum_{j=1}^{n} \eta_j \rho_j^{PT} M_j - \eta_i \rho_i^{PT} = H_i \geq 0 \quad \forall i = 1, \ldots, n,
\]

therefore Condition (14) holds. That is, the necessity of the first statement is true.

Moreover, Eq. (23) leads us to

\[
M_i (\eta_i \rho_i^{PT} - \eta_j \rho_j^{PT}) M_j = M_i H_j M_j - M_i H_i M_j = 0 \quad \forall i, j = 1, \ldots, n,
\]

this is, Condition (15) is satisfied. Therefore, the second statement is also true.

From Corollary 1 and Theorem 1, we have the following corollary.

**Corollary 2.** For a bipartite quantum state ensemble \( \mathcal{E} = \{\eta_i, \rho_i\}_{i=1}^{n} \),

\[
pl(\mathcal{E}) = q_G(\mathcal{E})
\]

if and only if there is a POVM \( \{M_i\}_{i=1}^{n} \) satisfying Condition (14) such that \( \{M_i^{PT}\}_{i=1}^{n} \) is a LOCC measurement.

For any integer \( d \geq 2 \), let us consider the two-qudit state ensemble \( \mathcal{E} = \{\eta_{ij}^{(k)}, \rho_{ij}^{(k)}\}_{i,j,k} \) consisting of \( 2d(d-1) \) states with equal prior probability,

\[
\eta_{ij}^{(k)} = \frac{1}{2d(d-1)}, \quad \rho_{ij}^{(k)} = \lambda \vert \Psi_{ij}^{(k)} \rangle \langle \Psi_{ij}^{(k)} \vert + (1 - \lambda) \sigma,
\]

where \( 0 < \lambda \leq 1 \), \( \sigma \) is an arbitrary two-qudit state, and

\[
\vert \Psi_{ij}^{(1)} \rangle = \frac{1}{\sqrt{2}} (\vert i \rangle \otimes \vert i \rangle + \vert j \rangle \otimes \vert j \rangle),
\]

\[
\vert \Psi_{ij}^{(2)} \rangle = \frac{1}{\sqrt{2}} (\vert i \rangle \otimes \vert i \rangle - \vert j \rangle \otimes \vert j \rangle),
\]

\[
\vert \Psi_{ij}^{(3)} \rangle = \frac{1}{\sqrt{2}} (\vert i \rangle \otimes \vert j \rangle + \vert j \rangle \otimes \vert i \rangle),
\]

\[
\vert \Psi_{ij}^{(4)} \rangle = \frac{1}{\sqrt{2}} (\vert i \rangle \otimes \vert j \rangle - \vert j \rangle \otimes \vert i \rangle).
\]

For a POVM \( \{M_{i,j}^{(k)}\}_{i,j,k} \) with

\[
M_{i,j}^{(1)} = \frac{1}{2d(d-1)} \vert \Psi_{ij}^{(1)} \rangle \langle \Psi_{ij}^{(1)} \vert, \quad M_{i,j}^{(3)} = \vert \Psi_{ij}^{(3)} \rangle \langle \Psi_{ij}^{(3)} \vert,
\]

\[
M_{i,j}^{(2)} = \frac{1}{2d(d-1)} \vert \Psi_{ij}^{(2)} \rangle \langle \Psi_{ij}^{(2)} \vert, \quad M_{i,j}^{(4)} = \vert \Psi_{ij}^{(4)} \rangle \langle \Psi_{ij}^{(4)} \vert,
\]

Condition (2a) holds, that is,

\[
\sum_{i',j',k'} \eta_{i',j'}^{(k')} \rho_{i',j'}^{(k')} M_{i',j'}^{(k')} - \eta_{ij}^{(k)} \rho_{ij}^{(k)} = \frac{\lambda}{2d(d-1)} \left( \mathbb{1} - \left| \Psi_{ij}^{(k)} \rangle \langle \Psi_{ij}^{(k)} \right| \right) \geq 0, \quad \forall i, j, k.
\]

Therefore, the optimal success probability \( p_G(\mathcal{E}) \) in Eq. (1) is

\[
p_G(\mathcal{E}) = \sum_{i,j,k} \eta_{ij}^{(k)} \text{Tr}(\rho_{ij}^{(k)} M_{i,j}^{(k)}) = \frac{1 + \lambda (d^2 - 1)}{2d(d-1)}.
\]
To obtain $q_G(\mathcal{E})$ in Eq. (6), we use a POVM $\{M^{(k)}_{i,j}\}_{i,j,k}$ that consists of

$$M^{(1)}_{i,j} = \frac{1}{d} |i\rangle\langle i| \otimes |j\rangle\langle j|, \quad M^{(2)}_{i,j} = \frac{1}{d} |i\rangle\langle i| \otimes |j\rangle\langle j|, \quad M^{(3)}_{i,j} = |i\rangle\langle i| \otimes |j\rangle\langle j|.$$  \hspace{1cm} (33)

This POVM satisfies Condition (14) because

$$\sum_{i',j',k'} \eta_{i',j'} \rho_{i',j'}^{(k')} \rho_{i',j'}^{(k')} \rho_{i',j'}^{(k')} = \frac{1}{d(d-1)} (1 - \mathbb{1}_{i,j} + 2|\Psi^{(5-k)}_{i,j}\rangle\langle \Psi^{(5-k)}_{i,j}|) \geq 0 \quad \forall i, j, k,$$

where

$$\mathbb{1}_{i,j} = (|i\rangle\langle i| + |j\rangle\langle j|) \otimes (|i\rangle\langle i| + |j\rangle\langle j|).$$  \hspace{1cm} (35)

and the equality is due to

$$|\Psi^{(5-k)}_{i,j}\rangle\langle \Psi^{(5-k)}_{i,j}| = \frac{2 + \lambda(d^2 - 2)}{4d(d-1)}.$$  \hspace{1cm} (36)

Thus, Theorems 1 leads us to

$$q_G(\mathcal{E}) = \sum_{i,j,k} \eta_{i,j} \text{Tr}(\rho_{i,j}^{(k)} M_{i,j}^{(k)}) = \frac{2 + \lambda(d^2 - 2)}{4d(d-1)}. \hspace{1cm} (37)$$

Moreover, the POVM $\{M_{i,j}^{(k)}\}_{i,j,k}$ in Eq. (33) is a LOCC measurement since it can be implemented by performing the same local measurement $\{|\rangle\langle i|\}_{i=1}^{d-1}$ on two subsystems. Thus, Corollary 2 and Eq. (37) lead us to

$$p_L(\mathcal{E}) = q_G(\mathcal{E}) = \frac{2 + \lambda(d^2 - 2)}{4d(d-1)}.$$  \hspace{1cm} (38)

In the case of $d = 2$, Eqs. (32) and (38) coincide with the existing results in Ref. [15].

In this paper, we have considered the situation of discriminating bipartite quantum states, and provided an upper bound $q_G(\mathcal{E})$ for the maximum success probability of optimal local discrimination $p_L(\mathcal{E})$ (Lemma 1). We have further established a necessary and sufficient condition for a measurement to realize $q_G(\mathcal{E})$ (Theorem 1). Moreover, we have provided the equality condition between $q_G(\mathcal{E})$ and $p_L(\mathcal{E})$ (Corollaries 1 and 2). Finally, we have illustrated the effectiveness of our results through an example.

We note that finding $p_G(\mathcal{E})$ or $q_G(\mathcal{E})$ in discriminating separable quantum states can be useful in studying the nonlocal phenomenon of separable quantum states, namely nonlocality without entanglement (NLWE) [5, 6]. For the minimum-error discrimination of a separable state ensemble $\{\eta_i, \rho_i\}_{i=1}^n$, NLWE occurs if the guessing probability $p_G(\mathcal{E})$ cannot be achieved only by LOCC, that is, $p_L(\mathcal{E}) < p_G(\mathcal{E})$. From Lemma 1, $q_G(\mathcal{E}) < p_G(\mathcal{E})$ implies $p_L(\mathcal{E}) < p_G(\mathcal{E})$, therefore the occurrence of NLWE. Moreover, even if $q_G(\mathcal{E}) > p_G(\mathcal{E})$, we can show the NLWE phenomenon in terms of $\{\eta_i, \rho_i^{PT}\}_{i=1}^n$ because the partial transposition of any separable state is another separable state and the roles of $p_G(\mathcal{E})$ and $q_G(\mathcal{E})$ are interchanged for the minimum-error discrimination of $\{\eta_i, \rho_i^{PT}\}_{i=1}^n$.

It is an interesting future work to investigate good conditions of optimal local discrimination in multiparty quantum systems having more than two parties. It is also natural to ask if our results are still valid for other optimal discrimination strategies other than minimum-error discrimination.

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[1] A. Chefles, Quantum state discrimination, Contemporary Physics 41, 401 (2000).
[2] S. M. Barnett and S. Croke, Quantum state discrimination, Adv. Opt. Photon. 1, 238 (2009).
[3] J. A. Bergou, Discrimination of quantum states, J. Mod. Opt. 57, 160 (2010).
[4] J. Bae and L.-C. Kwek, Quantum state discrimination and its applications, J. Phys. A: Math. Theor. 48, 083001 (2015).
[5] A. Peres and W. K. Wootters, Optimal detection of quantum information, Phys. Rev. Lett. 66, 1119 (1991).
[6] C. H. Bennett, D. P. DiVincenzo, C. A. Fuchs, T. Mor, E. Rains, P. W. Shor, J. A. Smolin, and W. K. Wootters, Quantum nonlocality without entanglement, Phys. Rev. A 59, 1070 (1999).
[7] R. Duan, Y. Feng, Z. Ji, and M. Ying, Distinguishing arbitrary multipartite basis unambiguously using local operations and classical communication, Phys. Rev. Lett. 98, 230502 (2007).
[8] E. Chitambar and M.-H. Hsieh, Revisiting the optimal detection of quantum information, Phys. Rev. A 88, 020302(R) (2013).
[9] S. Ghosh, G. Kar, A. Roy, A. Sen(De), and U. Sen, Distinguishability of Bell states, Phys. Rev. Lett. 87, 277902 (2001).
[10] J. Walgate and L. Hardy, Nonlocality, asymmetry, and distinguishing bipartite states, Phys. Rev. Lett. 89, 147901 (2002).
[11] H. Fan, Distinguishability and indistinguishability by local operations and classical communication, Phys. Rev. Lett. 92, 177905 (2004).
[12] R. Duan, Y. Feng, Y. Xin, and M. Ying, Distinguishability of quantum states by separable operations, IEEE Trans. Inf. Theory 55, 1320 (2009).
[13] E. Chitambar, R. Duan, and M.-H. Hsieh, When do local operations and classical communication suffice for two-
qubit state discrimination?, IEEE Trans. Inf. Theory 60, 1549 (2014).

[14] S. Bandyopadhyay, A. Cosentino, N. Johnston, V. Russo, J. Watrous, and N. Yu, Limitations on separable measurements by convex optimization, IEEE Trans. Inf. Theory 61, 3593 (2015).

[15] S. Bandyopadhyay and V. Russo, Entanglement cost of discriminating noisy Bell states by local operations and classical communication, Phys. Rev. A 104, 032429 (2021).

[16] A. Peres, Separability criterion for density matrices, Phys. Rev. Lett. 77, 1413 (1996).

[17] M. Horodecki, P. Horodecki, and R. Horodecki, Separability of mixed states: necessary and sufficient conditions, Phys. Lett. A 223, 1 (1996).

[18] PPT property does not depend on the choice of basis or the subsystem to be transposed. For simplicity, we consider the standard basis and the second subsystem throughout this paper.

[19] E. Chitambar, D. Leung, L. Mančinska, M. Ozols, and A. Winter, Everything you always wanted to know about LOCC (but were afraid to ask), Commun. Math. Phys. 328, 303 (2014).

[20] C. W. Helstrom, Quantum Detection and Estimation Theory (Academic Press, New York, 1976).

[21] A. S. Holevo, Probabilistic and Statistical Aspects of Quantum Theory (North-Holland, 1979).

[22] H. Yuen, R. Kennedy, and M. Lax, Optimum testing of multiple hypotheses in quantum detection theory, IEEE Trans. Inf. Theory 21, 125 (1975).

[23] S. M. Barnett and S. Croke, On the conditions for discrimination between quantum states with minimum error, J. Phys. A: Math. and Theor. 42, 062001 (2009).

[24] J. Bae, Structure of minimum-error quantum state discrimination, New J. Phys. 15, 073037 (2013).