Multiple giant resonances in nuclei: their excitation and decay

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The excitation of multiphonon giant resonances with heavy ions is discussed. The conventional theory, based on the use of the virtual photon number method in conjunction with the harmonic model is presented and its shortcomings are discussed. The recently developed model that invoke the Brink-Axel mechanism as an important contribution to the cross-section is discussed and compared to the conventional, harmonic model. The decay properties of these multiple giant resonances are also discussed within the same coherent + fluctuation model in conjunction with the hybrid decay model. It is demonstrated that the Brink-Axel mechanism enhances the direct decay of the states, as data seem to require. Comparison of our model with other recent theoretical works is presented.

1. INTRODUCTION

The study of the double giant dipole resonance in nuclei has received a considerable amount of attention over the last 15 years [1]. Both the pion double charge exchange and relativistic heavy ion Coulomb excitation reactions have been used to probe this large amplitude collective motion in many fermion systems. The quest for the similar double plasmon resonance in metallic clusters is underway [2]. Plans are also in progress to search for the triple giant dipole resonance (TGDR) in nuclei [3]. It is clearly of importance to supply theoretical estimates of the cross-section as well as the different decay branching ratios of these exotic collective modes. This is the purpose of the present paper. We use the recently developed coherent plus incoherent excitation theory of Ref. [4] in conjunction with the hybrid decay model of Dias-Hussein-Adhikari (DHA) of Ref. [5].

The existing models for the calculation of the excitation cross-section of DGDR can be grouped into four categories: a microscopic structure model in conjunction with second order Coulomb excitation perturbation theory [6], a macroscopic, oscillator model in the Weisäcker-Williams approximation [7] coupled channels Coulomb excitation theory [8] and finally the recently developed average plus fluctuation model [4,9]. In this latter model the average cross-section is calculated according to the theory developed in [11], where the simple, double, etc. giant resonances are considered as doorway states belonging to the
spectrum of a damped harmonic oscillator. The recent work of Gu and Weidenmüller [12], based on random matrix theory, lends full support to our model [4,9].

In this review we describe our work based on the Brink-Axel mechanism. In section 2 we describe the harmonic oscillator model and the multi-step reaction theory. In section 3 we consider the static and dynamic effects associated with the damping of the multiphonon resonances. In section 4 we present the “Absorption-Fluctuation Theorem (AFT)”. In section 5 the Brink-Axel mechanism is invoked to implement the AFT. In section 6 the evolution equation of the density matrix is presented and discussed. Application of the theory to the excitation of the DGDR and TGDR in several systems is presented in section 7. In section 8 the decay of these resonances is discussed within the hybrid model of DHA [5]. Finally, in section 9 concluding remarks are given.

2. MULTISTEP REACTION THEORY

In this section we describe the semiclassical excitation of a harmonic oscillator, which represents the spectrum of the giant resonances. The spectrum of an harmonic oscillator is given by,

\[ E_n = E_0 + n\hbar\omega, \]  

where \( E_0 \) is the zero point energy, \( n \) is the number of phonons and \( \omega \) is the frequency.

We now allow the harmonic oscillator to be excited by an external pulse \( V(t) \) (a passing fast ion). The excitation, for zero width, is described by the Poisson formula for the probabilities, vis.

\[ P_n = \frac{\left[P_1^{(0)}\right]^n}{n!} P_0, \]  

Above, \( n \) is the number of phonons in the excited state, \( P_1^{(0)} \) is the first order perturbation probability of exciting a one phonon state,

\[ P_1^{(0)} = \left| \frac{1}{i\hbar} \int_{-\infty}^{\infty} V(t) e^{i\omega t} dt \right|^2, \]  

and \( P_0 \) is the probability for the oscillator to remain in the ground state,

\[ P_0 = \exp\left[ -P_1^{(0)} \right]. \]  

Thus for the double giant resonance (DGR) excitation one has,

\[ P_2 = \frac{1}{2} P_1^{(0)} P_1^{(0)} \exp\left[ -P_1^{(0)} \right]. \]  

For the triple giant resonance (TGR), one has,
\[ P_3 = \frac{1}{6} P_1^{(0)} P_1^{(0)} P_1^{(0)} \exp \left[ -P_1^{(0)} \right], \]  

or

\[ P_3 = \frac{1}{3} P_1^{(0)} P_2. \]  

In all of the above, the cross section \( d\sigma_n / d\Omega \) is calculated using the semi-classical formula

\[ \frac{d\sigma_n}{d\Omega} = \frac{d\sigma_{Ruth}}{d\Omega} P_n, \]  

and the integrated cross section is simply

\[ \sigma_n = \int \frac{d\sigma_{Ruth}}{d\Omega} P_n(b) \, d\Omega = 2\pi \int_0^\infty b db P_n(b) \]  

3. EFFECT OF DAMPING

Giant resonances are damped because of their coupling to the complicated fine structure states and to the open decay channels. By far the former type of damping is the dominant one. Thus the harmonic oscillator model must be damped by including an appropriate damping width. The excitation energy of the n-phonon state of the damped oscillator is given by

\[ E_n = nE_1 \]  

and \( E_1 \) is the excitation energy of the single giant resonance. From the Boson nature of the phonons, one would expect

\[ \Gamma_n = n \Gamma_1. \]  

Damping contains escape (to open channels) plus spreading (to complex chaotic states). According to sum rule arguments [12] the widths of multiple giant resonances should behave as \( \Gamma_n = \sqrt{n} \Gamma_1 \). The experimental width is situated between the two limits. This fact points to the conclusion that the experimentally determined width is an effective one which may embody some reaction dynamic effects [9].

There is no closed form Poisson-type solution for the excitation probabilities of the damped harmonic oscillator above. However, one can solve the problem numerically, and as expected the DGR cross section is reduced at high energies. At low energies there is a slight increase in the cross section as a function of \( \Gamma_1 \), contrary to the claim of ref [14]. The reason is that in the absence of the width the number of virtual photons for low bombarding energies at the position of the resonance is very small, resulting in a small cross section.

We should emphasize that the Poisson formula (or harmonic formula), Eq. (3), can still be used in a microscopic calculation where all fine structure states (that give rise to \( \Gamma_1 \)) are included. This fact was nicely demonstrated in Ref. [10] using a schematic
microscopic picture containing a simple harmonic oscillator, representing the giant resonance spectrum, coupled to a “bath” of many simple harmonic oscillators. The excitation probabilities were evaluated in closed form and were found to be

\[ P_n = \sum_{\nu} P_n^{(\nu)} = \exp \left( -\sum_{\nu'} |\alpha_{\nu'}|^2 \right) \sum_{\nu} \frac{|\alpha_\nu|^2}{n!}, \]  

(11)

where \( |\alpha_\nu|^2 \) is the many-harmonic oscillator generalization of \( P_1^{(0)} \) of Eq.(4), Ref. [10].

Short of the microscopic picture, one “suggests” the validity of the harmonic, \( P_2 = \frac{1}{2} P_1^{(0)} P_1^{(0)} \exp \left[ -P_1^{(0)} \right] \), form for the DGR excitation probability within the damped oscillator model. This is the widely used folding model [15] whose results are employed in the analysis of the data. It is referred to in the literature erroneously as the “harmonic” model. It should be called the “damped harmonic” model.

The data analyses show [1] that the folding model does not fully account for the data (to within 20%) and in the exceptional case of \( Xe + Pb \) at the GSI energies (640 MeV/A) the discrepancy is more than 80%. The reason can be easily the contribution of dynamical effects that accompany the damping, the Brink-Axel effect to be discussed below, as well as higher order anharmonic effects of the type discussed in [16]. We should mention that the dynamic effects become quite significant at lower energies, and it would be important to fully test it with new data taken at, say, 100 MeV/A.

The data also seem to suggest that the direct decay of multiphonon states is enhanced in comparison with the harmonic picture of the independent decay of the multiphonons found in the state.

The above two features found in the data clearly call for the development of a consistent reaction theory that accounts for both the excitation and the decay of multi-phonon states.
in heavy-ion reactions. One would rely on the idea of calculating an average amplitude (damped oscillator) that supplies the coherent, harmonic (folding) contribution to the cross section. The remaining piece of the amplitude, whose energy average is zero, supplies the statistical, fluctuation contribution to the cross section (the Brink-Axel contribution), just like in low energy neutron scattering from nuclei where one has the optical potential supplying the average amplitude and thus the “direct” cross section and the statistical amplitude that gives rise to the fluctuation cross section calculated from the Hauser-Feshbach theory.

4. ABSORPTION-FLUCTUATION THEOREM

In Nuclear Reaction Theory one uses average amplitudes to calculate optical or coherent cross section. Unitarity dictates that one must take into account a fluctuation or statistical cross section and add it incoherently to the optical one to obtain the average cross section to be compared with the data. There are three types of fluctuations.

- Initial state fluctuations: these occur in low energy nuclear reactions that involve the formation of the compound nucleus, \( N + A \rightarrow CN \rightarrow N' + A' \), the S-matrix is given by: \( S = \bar{S} + V G_{CN} V \): with \( \bar{G}_{CN} = 0 \), where the upper bar implies energy average. The resulting cross section contains an optical model piece plus a fluctuation, Hauser-Feshbach one.

- Final state fluctuations: There are cases that involve fluctuations in the final state of the reaction. This happens when the final channel contains a particle in an unbound state (continuum). An example of this type of fluctuations is the reaction \( A (p, \gamma) \) at \( E_p \sim 20 \text{MeV} \). One gets a direct-semidirect cross-section and a direct-compound (fluctuation) one. The latter was found to be dominant [17].

- Intermediate state fluctuations: These fluctuations occur in multistep direct reactions. They involve fluctuation in the excited nucleus (subsystem) in, e.g., a heavy-ion reaction, such as the excitation of the DGDR: \( GS \rightarrow GDR \rightarrow DGDR \). The S-matrix can be represented by \( S = \bar{S} + \Gamma_{\downarrow} G V \), with \( \bar{G} = 0 \). The corresponding cross-section can be written as \( \sigma = \sigma_c + \sigma_{fl} \).

5. THE BRINK-AXEL MECHANISM

Another important feature arising from the damping is the dynamical effect owing to the existence of a mechanism that allows for the excitation of a collective phonon on top of the background of chaotic compound states to which the one phonon state decays in a multistep reaction proceeding sequentially. The Brink–Axel phonons contribute incoherently to the cross section since they correspond to a slower process compared to the sequential excitation of the damped oscillator states. In fact one can calculate the bombarding energies at which theses contribution become significant by dividing the one-phonon decay time, \( \tau_d \), by the collision time, \( \tau_c \), via

\[
\frac{\tau_d}{\tau_c} = \frac{\hbar}{\Gamma_1} \frac{2b}{\gamma v}.
\]

For \(^{208}\text{Pb} \) with \( \Gamma_1 \approx 4 \text{MeV} \), \( b = 15 \text{fm} \) at \( E = 300 \text{MeV}/A \), the above ratio is about 0.6 and increasing as the energy is lowered, indicating the increased competition between
the one phonon decay into the background and continuation of the sequential excitation. Therefore one may envisage the following picture of the reaction dynamic.

Figure 2: Cartoon depiction of the conventional double giant dipole resonance excitation (DGDR) and the alternative ‘hot’ giant dipole excitation (HGDR) discussed here.

6. EVOLUTION EQUATION

The equation that governs the evolution of the density matrix, from which the excitation probabilities are calculated is obtained from the damped oscillator equation. The space used to derive the evolution equation is depicted in Figure 3.

\[
\hbar \frac{\partial \rho_{nm}}{\partial t} = -i \sum_m \left( \tilde{V}_{nm}(t) \rho_{nm} - \rho_{nm} \tilde{V}_{nm}(t) \right) - \frac{(\Gamma_{ns} + \Gamma_{n's})}{2} \rho_{nn'} + \delta_{nn'} \sum_r \Gamma_{ns,mr} \rho_{mm'}
\]

(13)

Figure 3: Schematic representation of the collective/statistical states and their transitions. The vertical arrows represent the two-way coherent excitation/de-excitation of collective phonons. The horizontal arrows represent the one-way statistical decay of the collective phonons. \(\varepsilon\) denotes the excitation energy.

The time evolution equation for the density matrix was derived originally by Ko [18] and in the present context by Carlson et al. [4]
The second line of Eq. (13) contains the incoherent contributions of the statistical loss and gain terms, respectively.

Assuming that the collective excited states are harmonic $n$-phonon giant dipole states, the interaction matrix elements can be written as

$$
\tilde{V}_{nn'}(t) = \left( \exp \left[ i \varepsilon_d t / \hbar \right] \sqrt{n} \delta_{n',n-1} + \exp \left[ -i \varepsilon_d t / \hbar \right] \sqrt{n+1} \delta_{n',n+1} \right) V_{01}(t)
$$

(14)

where $\varepsilon_d$ is the excitation energy of the giant dipole resonance and $V_{01}(t)$ is the semiclassical matrix element coupling the ground state to the giant resonance, which we take to have the simple form

$$
V_{01}(t) = V_0 \frac{(b_{\text{min}}/b)^2}{1 + (\gamma vt/b)^2},
$$

(15)

as given in Ref. [11]. As is done there, we neglect the spin degeneracies and magnetic multiplicities of the giant resonance states and approximate the projectile-target relative motion as a straight line.

In the case of harmonic phonons, the decay widths can be approximated as

$$
\Gamma_{ns} = n \Gamma_1,
$$

(16)

where $\Gamma_1$ is the spreading width of the giant dipole resonance. We have neglected the contribution to the width of the hot statistical background of states since, at the low temperatures involved here, the decay widths of the hot Brink-Axel resonances are very similar to those of the cold ones.

We have performed calculations of multiple giant dipole resonance excitation within the model for the system $^{208}\text{Pb} + ^{208}\text{Pb}$ in the projectile energy range from 100 to 1000 MeV/nucleon. In Fig. 4, we show the differential excitation cross section that we obtain for the system $^{208}\text{Pb} + ^{208}\text{Pb}$ at 640 MeV/nucleon as a function of the excitation energy. This was obtained by summing Breit-Wigner expressions with the appropriate excitation energy and width for each of the total $n$-phonon cross sections. We show only the contributions of the first three giant dipole resonances, as the higher order ones are almost invisible even on our theoretical curve. Only the first and second giant dipole resonances have been observed experimentally.

### 7. DGDR AND TGDR EXCITATION

To be consistent with the discussion of the AFT of section 4, it seems appropriate to write the cross-section for the excitation of the DGDR and the TGDR as, respectively,

$$
\sigma^{(2)} = \sigma_c^{(2)} + \sigma_{fi}^{(2)} (1)
$$

(17)

$$
\sigma^{(3)} = \sigma_c^{(3)} + \sigma_{fi}^{(3)} (2) + \sigma_{fi}^{(3)} (1),
$$

(18)

where $\sigma_{fi}^{(n)} (i)$ denotes the contribution of $i$ Brink-Axel phonons to the excitation cross-section of $n$ giant resonances phonons [19].

We have calculated the excitation cross-sections, $\sigma^{(1)}$, $\sigma^{(2)}$ and $\sigma^{(3)}$, for various nuclei incident on $^{208}\text{Pb}$ at several bombarding energies, using a three-dimensional (3D) generalization of the model of Ref. [4]. The 3D time evolution equation used in Ref. [19]...
to describe the excitation and decay of the GDR phonons possesses the same form as the one-dimensional equation of Ref. [4]. However, the collective and statistical excited states of the 3D model take into account all possible combinations of the (two) transverse and (one) longitudinal degrees of freedom, which yield 3 coherent one-phonon states, 6 coherent two-phonon states and 10 coherent three-phonon states as well as a multitude of states containing a mixture of coherent and statistical excitations. Decays of the three types of phonons to the statistical background are assumed to occur independently but to each obey Bose-Einstein statistics.

The Coulomb interaction matrix elements used to describe the transverse modes of the GDR excitation in the 3D model are the physically appropriate ones, as given in Ref. [11]. The longitudinal Coulomb interaction matrix element, however, is modified from the form given there. Through a gauge transformation, it is reduced to a term proportional to the longitudinal component of the electric field, in analogy to the transverse terms, but which differs from the expression given in Ref. [11] by a total time derivative. We emphasize that our theory contains the effect of the adiabaticity to all orders. If we neglect the width of the GDR and use perturbation theory, we fully recover the model of Baur and Bertulani[7].

As in Ref. [4], the coupled equations of motion are solved as a function of impact parameter to yield asymptotic occupation probabilities. Effective asymptotic occupation probabilities are defined, for states that decay, as the sum over the probability that decays out of each state during the time evolution. Cross sections are obtained by integrating each probability x differential area over impact parameter and summing over polarizations.

The various contributions to the cross sections are easily extracted from the theoretical calculations. In Table I, we present the coherent and fluctuation contributions to the DGDR cross section, $\sigma_c^{(2)}$ and $\sigma_f^{(2)}(1)$ for various nuclei incident on $^{208}$Pb at several energies. We use a global systematic for the GDR energies and widths: $E_{GDR} = 43.4A^{-0.215}$.

Figure 4: Theoretical multiple giant resonance differential excitation cross section of $^{208}$Pb at a projectile energy of 640 MeV/nucleon.
Table 1: Contributions of the coherent and fluctuation components to the DGDR excitation cross section (in mb) of various projectiles incident on a lead target at two values of the incident energy.

| Projectile | 100 MeV | | | | | 1 GeV | | | |
|------------|---------|---|---|---|---|---|---|---|---|
|            | \( \sigma_{(2)}^c \) | \( \sigma_{(1)}^f \) | \( \sigma_{tot}^f \) | | | \( \sigma_{(2)}^c \) | \( \sigma_{(1)}^f \) | \( \sigma_{tot}^f \) | |
| \(^{40}\text{Ca}\) | 2.17 | 2.19 | 4.36 | | | 7.20 | 0.72 | 7.92 | |
| \(^{120}\text{Sn}\) | 26.48 | 22.94 | 49.42 | | | 72.61 | 6.65 | 79.26 | |
| \(^{132}\text{Xe}\) | 32.19 | 27.57 | 59.76 | | | 88.50 | 8.00 | 96.50 | |
| \(^{165}\text{Ho}\) | 51.13 | 42.60 | 93.73 | | | 138.59 | 12.34 | 150.93 | |
| \(^{208}\text{Pb}\) | 96.95 | 72.87 | 169.82 | | | 234.84 | 19.83 | 254.67 | |
| \(^{238}\text{U}\) | 109.15 | 84.86 | 194.01 | | | 276.53 | 24.04 | 300.59 | |

Table 2: Contributions of the coherent and fluctuation components to the TGDR excitation cross section (in mb) of various projectiles incident on a lead target at two values of the incident energy.

| Projectile | 100 MeV | | | | | 1 GeV | | | |
|------------|---------|---|---|---|---|---|---|---|---|
|            | \( \sigma_{(3)}^c \) | \( \sigma_{(2)}^f \) | \( \sigma_{(1)}^f \) | \( \sigma_{tot}^f \) | | | \( \sigma_{(3)}^c \) | \( \sigma_{(2)}^f \) | \( \sigma_{(1)}^f \) | \( \sigma_{tot}^f \) | |
| \(^{40}\text{Ca}\) | 0.02 | 0.06 | 0.02 | 0.10 | | | 0.11 | 0.02 | 0.00 | 0.13 | |
| \(^{120}\text{Sn}\) | 0.84 | 1.92 | 0.64 | 3.40 | | | 3.03 | 0.47 | 0.04 | 3.54 | |
| \(^{132}\text{Xe}\) | 1.10 | 2.50 | 0.83 | 4.43 | | | 4.07 | 0.62 | 0.05 | 4.74 | |
| \(^{165}\text{Ho}\) | 2.08 | 4.70 | 1.54 | 8.32 | | | 7.76 | 1.17 | 0.09 | 9.02 | |
| \(^{208}\text{Pb}\) | 5.28 | 10.78 | 3.36 | 19.42 | | | 16.68 | 2.40 | 0.18 | 19.26 | |
| \(^{238}\text{U}\) | 6.13 | 13.14 | 4.22 | 23.49 | | | 21.01 | 3.14 | 0.24 | 24.39 | |

MeV and \( \Gamma_{GDR} = 0.3E_{GDR} \)[4]. The energies of the DGDR and TGDR resonances were taken to be two and three times those of the GDR, respectively, since simple anharmonic effects are small [20]. The widths of the DGDR and TGDR were taken in Ref. [19] to be \( \sqrt{2} \) and \( \sqrt{3} \) times those of the GDR widths, respectively, as rigorous sum rule arguments indicate [13].

In Table II, we present the contributions to the TGDR cross section, \( \sigma_{(3)}^c \), \( \sigma_{(2)}^f \), and \( \sigma_{(1)}^f \). We observe that the cross sections increase dramatically with the charge of the projectile. As is well known, the coherent two-phonon cross sections scales approximately as the projectile charge \( Z \) squared, while the three-phonon one scales as \( Z^3 \). We also observe that the coherent contribution to the cross sections only dominates at relatively high incident energies. At \( E/A = 100 \text{ MeV} \), it is clear from the tables that the fluctuation contribution to the DGDR cross section is about as large as the coherent one, while the fluctuation contribution to the TGDR is about three times larger than the coherent contribution.

The reason for the unexpected larger cross sections at lower energies can be traced to two factors. The average cross section is larger than that of Baur and Bertulani owing to the fact that the inclusion of the width of the one-phonon resonance (GDR) allows the
excitation of that resonance, as well as the DGDR and TGDR, even at very low excitation energies, where the virtual photon spectrum is concentrated at low bombarding energy. This enhancement can easily be missed, if the width is not taken into account, as in the original Weisäcker-Williams approximation[7]. The second reason for the increase in the DGDR and the TGDR cross sections, which is also related to the inclusion of the width, is the Brink-Axel fluctuation contribution, which tends to increase as the bombarding energy is lowered.

8. DECAY OF MULTIPLE GIANT RESONANCES

We turn now to the decay of the DGR and TGR. We first remind the reader of the hybrid direct+fluctuation decay model of DHA [5]. According to this model, which has been extensively used in the analysis of decay data [21,22], the GR decays to a final channel \( f \) in the following manner:

\[
\sigma_f^{(1)} = \sigma^{(1)} \left[ (1 - \mu_1) \frac{\tau_f^{(GR)}}{\sum_j \tau_j^{(GR)}} + \mu_1 \frac{\tau_f^{(CN)} + \mu_1 \tau_f^{(GR)}}{\sum_j \left( \tau_j^{(CN)} + \mu_1 \tau_j^{(GR)} \right)} \right]
\]

\[\equiv \sigma^{(1)} (P_f^\uparrow + P_f^\downarrow), \tag{19}\]

where \( \sigma^{(1)} \) is the one phonon excitation cross section discussed before, while \( \mu_1 = \frac{\Gamma_1^\uparrow}{\Gamma_1} \) and the \( \tau' \)'s are the appropriate transmission coefficients. We have written the probability of populating the final channel \( f \) through direct decay of the GR as

\[P_f^\uparrow = (1 - \mu_1) \frac{\tau_f^{(GR)}}{\sum_j \tau_j^{(GR)}}, \tag{20}\]

and the probability of populating the channel \( f \) through the statistical states as

\[P_f^\downarrow = \mu_1 \frac{\tau_f^{(CN)} + \mu_1 \tau_f^{(GR)}}{\sum_j \left( \tau_j^{(CN)} + \mu_1 \tau_j^{(GR)} \right)} \tag{21}\]

Note that the statistical decay component contains explicit reference to the GR direct transmission, \( \left( \mu_1 \tau_f^{(GR)} \right) \).

A detailed derivation of the direct decay of the DGDR and TGDR was presented in Ref. [19]. Here we only present the final result,

\[
\frac{\sigma_f^{(2)}}{\sigma_{c,f}^{(2)}} = 1 + \frac{1}{2} \frac{\sigma_{fl}^{(2)}(1)}{\sigma_c^{(2)}}, \tag{22}\]

and

\[
\frac{\sigma_f^{(3)}}{\sigma_{c,f}^{(3)}} = 1 + \frac{2}{3} \frac{\sigma_{fl}^{(3)}(2)}{\sigma_c^{(3)}} + \frac{1}{3} \frac{\sigma_{fl}^{(3)}(1)}{\sigma_c^{(3)}}. \tag{23}\]
Thus, a considerably larger direct decay may occur if the fluctuation contributions are important, which may occur at lower bombarding energies. Of course, one could obtain deviation of the direct decay from the harmonic limit (two or three independently decaying phonons), if anharmonic effects were allowed. This, however, will imply deviation of the spectrum of the oscillator from the harmonic sequence, which seems to be borne out neither by experiment [1] nor by calculation [20], notwithstanding the higher order anharmonic effects considered in Ref. [16].

9. CONCLUSIONS

We have developed a description of multiple giant resonance excitation and decay that incorporates incoherent (statistical) contributions to the cross section. The incoherent contributions arise from the excitation of collective phonons on top of the statistical background that results from the decay of previously excited collective phonons, namely, the Brink-Axel mechanism. Semi-classical calculations show that incoherent, B-A, contributions are an important part of the cross section at low to intermediate energies. These contributions are expected to be important in all regions of the mass table. This phenomenon is not limited to the excitation of collective modes in nuclei. It is of potential importance in any system in which multiply excited collective modes couple and decay to a statistical background of states. The decay of multi-phonon giant resonances is discussed within the hybrid, direct + statistical, model of Dias- Hussein- Adhikari (DHA). It is found that the B-A mechanism enhances the direct decay component of multi-phonon giant resonances.

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