Scaling Transformation for Nonlocal Interactions

Hai-Jun Wang
Center for Theoretical Physics and School of Physics, Jilin University, Changchun 130012, China

More often than not, previous investigations on nonlocal interaction tried to fit certain results to those of renormalization. Reversely, in this paper we extract a scaling transformation for nonlocal interaction in the light of renormalization group method (RGM). By RGM the one-loop renormalized S-matrix keeps exactly its tree-level form except for the running of physical parameters (charge and mass etc.) with respect to a scaling parameter. The symmetric method here falls into the stream of using local method to study nonlocality. Foremost the similarity between the dispensable local-form of nonlocal interaction in momentum representation and the Hamiltonian in one-loop renormalized S-matrix suggests that the scale transformation may underlie the nonlocal interaction. Then by referencing the process of Lorentz transformation applied to Dirac equation, we apply the scale transformation to a scale-invariant interaction vertex. Consequently the scale transformation $e^{\mp \gamma^5}$ in spinor representation and the scale-invariant vertex $\gamma^\mu(1 \pm \gamma^5)$ are obtained. Make the transformation $e^{\mp \gamma^5}$ performed repeatedly on the vector vertex-form $\gamma^\mu$ one obtains a varying vertex-form $\gamma^\mu(a + b\gamma^5)$, i.e. normal vector vertex-form $\gamma^\mu$ would evolve to include the part of $\gamma^\mu(1 \pm \gamma^5)$ due to scale(size)-variation of extended particle. In this sense, the invariant vertex-form $\gamma^\mu(1 \pm \gamma^5)$ can be viewed as the extremes of vector interaction under scaling transformations, and the varying coefficient a before $\gamma^\mu$ is assumed relating compatibly to running charges. Subsequently we study the conservation law for the extreme vertices $\gamma^\mu(1 \pm \gamma^5)$ and find out that the usually defined angular momentum is no longer a conservative quantity. This non-conservation paves the way for a nonperturbative understanding to the puzzle that the nucleon’s spin is not equal to the sum of its constituent parts. Finally, it is found that the general vertex form $\gamma^\mu(a + b\gamma^5)$ would contribute to polarized scattering amplitude with special terms, based on which we present a Gedanken experiment (an ad hoc Electron-Hydrogen scattering) to test nonlocal effects perturbatively.

I. INTRODUCTION

Nowadays, on account of the developments of string theory, Lattice QCD\cite{ref1} and the necessity of describing nonperturbatively the intermediate strong interaction between extended hadrons, the construction of a consistent nonlocal theory is still called for \cite{ref3, ref4, ref6, ref7, ref8, ref14, ref16, ref19}. The pioneering study of nonlocal interaction dates back to the 1930’s \cite{ref20} when quantum field theory was in its infancy. And the phenomenology of nonlocal interaction commenced with the primary attempts to describe the interaction between extended particles (such as hadrons \cite{ref21}), whilst to cope with the divergence appearing in local quantum field theories (LQFT). The development afterward purported mainly to give a consistently convergent theory in order to underlie the named “effective field theory”, whereof some form factors were usually employed \cite{ref3, ref7, ref22, ref23, ref24, ref25, ref33, ref36, ref37, ref45, ref46, ref47, ref48, ref49, ref50, ref51, ref52, ref53, ref54}. Whereas in such context one encounters the difficulty of unitarity and causality in formulating the S matrix \cite{ref24, ref33} using Feldman-Yang \cite{ref35} method or conventionally canonical quantization method \cite{ref24}. Some promising progresses on this issue \cite{ref6, ref7, ref35, ref40} in one way or another showed their accordance with the renormalization methods \cite{ref41, ref44}.

The two major stages of nonlocal-interaction research focused succeedingly on a scalar model (Kristensen-Møller’s interaction form $g \int F(x_1, x_2, x_3) \psi^\dagger(x_1) \varphi(x_2) \psi(x_3) dx_1 dx_2 dx_3$) \cite{ref22, ref23, ref33, ref45} and on vector model—gauge interaction with Moffat factor $\exp[\frac{\beta^2 - m^2}{\Lambda^2}]$ \cite{ref4, ref5, ref6, ref7, ref34, ref36}, which we call it Moffat paradigm in this paper. The objective of Moffat paradigm mainly focuses on the localizable nonlocal theories \cite{ref46, ref48}, and the characteristics of which are summarized into a nonlocal regularization factor $E_m = \exp[\frac{\beta^2 - m^2}{\Lambda^2}]$ \cite{ref5}. The factor plays an essential role in developing ramifications of nonlocal field theory thereafter \cite{ref11, ref17, ref49, ref58}. The Moffat paradigm can be viewed as a new nonlocal regularization method to conventional LQFT \cite{ref36, ref39, ref50, ref61}, by putting the renormalization constant ahead of wave functions, physical parameters and Green functions, prior to the processes of regularization. It is proved to be perturbatively unitary to all orders \cite{ref32}, and leads to usual renormalization results \cite{ref32, ref60}.

In this paper, in the light of Moffat paradigm \cite{ref5}, scaling properties of renormalization group (RGM) \cite{ref41, ref44}, and the dispensable local-form of nonlocal interaction in momentum representation, we shall admit a partial scaling invariance for nonlocal interaction $g \int F(x_1, x_2, x_3) A_\mu(x_2) \psi(x_1) \gamma^\mu \psi(x_3) dx_1 dx_2 dx_3$ and study what is its inference. To our best knowledge, the relationship between scaling and nonlocality has never been taken into account in previous literature.
Though in common with the results of renormalization, the aforementioned nonlocal theories [4, 5, 7, 30, 31, 35–39, 59, 61] didn’t link obviously the scaling property of renormalization with nonlocality. In view of previous nonlocal theories, we suppose that the difficulties these theories once encountered might be ascribed to the scaling property, for instance the conservation laws violation in [33, 34], and the off-shell causality problem in [11, 62, 63]. However, in this paper we are not deeply involved in these issues, each of which deserves separate discussions.

The scaling parameter enters in RGM by the boundary condition \( \beta' = \beta = \mu (\nu^2 = \mu^2) \), where \( \nu = \nu^{-1} m \), with \( \lambda \) the scaling parameter [44]. Except for running masses and charges etc., by RGM the S matrix Lorentz-invariantly keeps its tree-level form under the scaling variation [42, 44]. This unvaried manner could be transferred to that of nonlocal interaction [Please refer to section 2], interpreted via a coordinate scaling transformation \( x' = \lambda x \) [64], which corresponds to particle’s shrinkage or inflation (like zooming in or out). From a classical point of view, while a “soft” body (with definite mass \( m \)) rotating, its shrinkage or inflation would not alter its total orbital angular momentum. However for a micro-quantum particle, its shrinkage or inflation occurs only when it absorbs or releases a certain amount of energy. This energy exchange could break the angular-momentum conservation. Such case is similar to the situation when we extend the three-dimensional rotation to four-dimensional rotation involving Lorentz boosts, whereby we find the 3-dimensional orbital angular momentum is not a conservative quantity any longer, unless we further include the spin angular momentum. In such consequence we call the scaling transformation an extrapolation of Lorentz group. Quantum mechanically, this extrapolation should be manifested by corresponding transformation of spinors. We thus make the scaling transformation acts on spinors, in closely analogous to the Lorentz group on spinors—to reserve a nonlocal interaction form so as to derive the invariant vertex-form and the equivalence transformation in spinor representation. There was literature to investigate the scaling properties of extended particles [65], without involving interaction albeit using group property. The newly similar consideration of the scaling symmetry is Ref. [10], in which the authors discuss the scaling symmetry in 2-dimension system by using the light-cone quantum field method. Their work doesn’t assume Lorentz invariance either (different from it, we are treating an extrapolation of Lorentz group.). Other works on scaling symmetry as far as we know are all within the framework of the renormalization [40–43, 66–68]. In earlier literature the scale transformation was rarely applied effectively to spinors, but scalar-like fields instead [69, 70].

The rest of the paper is arranged as follows. Sect. II is dedicated to explaining the rationale of introducing the scaling transformation to nonlocal interaction. Referencing the process of Lorentz transformation being applied to Dirac equation, in Sect. III we derive a scaling invariant vertex-form as well as the corresponding scaling transformation for spinors. Subsequently in Sect. IV we discuss the conservation law for the derived scaling-invariant vertex-form under the new transformation, and the possibility that it relates to the nucleon’s polarizations is posed. In Sect. V according to the characteristics of the generally local vertex-form (involving nonlocal factors) applied to polarized scattering between fermions, a mechanism is designed to examine the predictions on size effect. Conclusions and discussions are presented finally.

## II. THE SCALING TRANSFORMATION FOR NONLOCAL INTERACTION

This section is dedicated to explaining the rationale of introducing the scaling transformation for nonlocal interaction by directly equalizing the nonlocal Hamiltonian and the corresponding terms from renormalized S-matrix. Foremost we choose the form of nonlocal interaction by referencing forms from both Marnelius [33] and Efimov [29, 30],

\[
\mathcal{L}_I(x) = g \int d^4 \xi d^4 \eta F(\xi, \eta) A_\mu(x + \xi) j^\mu(x + \eta),
\]

which represents the nonlocal (local) interaction vertex of a local boson and a local (nonlocal) fermion current [Fig.1] or the both. One could realize that the quintessential part of the nonlocal interaction is its form factor \( F(\xi, \eta) \). And \( \mathcal{L}_I(x) \) enters into the S matrix as

\[
S = T \{ e^{i \int \mathcal{L}_I(x) dx} \},
\]

where \( T \) means time-ordered product. The nonlocal Hamiltonian corresponding to \( \mathcal{L}_I(x) \) in momentum representation is

\[
\mathcal{H}_{\text{Nonlocal}} = -\mathcal{L}_{\text{Nonlocal}} = -gF(q^2) A_\mu(q^2) j^\mu(p, p'),
\]
where \( q = p' - p \). Caution should be practiced that the Eq. (3), as Fourier transform of \( \mathcal{L}_f(x) \), whether it exists or not depends on how simple the distribution function \( F(\xi, \eta) \) is. That means Eq. (1) is always general, but Eq. (3) not. Basically we get the form factor \( F(q^2) \) by successively approximate steps. \( F(q^2) \) can accurately take the information of \( F(\xi, \eta) \) only when the interaction form is Coulomb like and meanwhile Born approximation is perturbatively available. The arguments what follows is assumed not limited by this dispensable form factor \( F(q^2) \), though it is our starting point.

Now let's analyze the role of form factor \( F(q^2) \) in renormalized \( S \)-matrix. Recall that the essential conclusion of RGM [41] [42] is that it keeps lowest tree-level form of the \( S \)-matrix [43], after involving one-loop contributions from Feynman graphs. Since in RGM it happens that all the evolution exponential factors ahead of wave-functions, propagators, Green functions cancel each other, whilst making the physical parameters (like charges and masses etc.) of fermions become running ones with respect to a scaling parameter \( \mu \). To speak alternatively, if the original \( S \)-matrix is \( S^{(2)}(p, p', q, e, m, \cdots) = \frac{(-i)^5}{2^{15}} \int d^4x_1 d^4x_2 T\{\mathcal{H}_f(x_1), \mathcal{H}_f(x_2)\} \), then after renormalization it becomes to be \( S^{(2)}(p, p', q, e'(\mu), m'(\mu), \cdots) \) where charges and masses turn out to be effective (running) ones. Reversing the renormalization formalism, i.e. from renormalized tree-level \( S \)-matrix back to Hamiltonian, as from Eq. (2) to Eq. (3), we note that the conclusion of RGM somewhat identifies multiplying a form factor ahead of the \( \mathcal{H}_f \) (in fact just the running coefficient of charge) so that \( F(q^2) \mathcal{H}_f \rightarrow \mathcal{H}_{\text{nonlocal}} \). And this Hamiltonian allow us to directly compute scattering amplitude under Born approximation without considering loops’ corrections. So based on the above discussion and regardless of the running mass \( m \) (or including it by changing the mass \( m \) to a renormalized \( m^* \) at nonlocal side), we can equalize the resultant Hamiltonian extracted from renormalized \( S \)-matrix and the nonlocal interaction in the momentum-representation,

\[
T\{e^{-i \int \mathcal{H}_f(x) dx}\} \Rightarrow \mathcal{H}_{\text{nonlocal}} = -g F(q^2) A_\mu(q^2) j^\mu(p, p') \Rightarrow g \int d\xi d\eta F(\xi, \eta) A_\mu(\xi, \eta) \frac{d}{dt}(x + \xi) j^\mu(\xi, \eta).
\]

In such a consequence we recognize the correlation between the scaling parameter and nonlocal interaction.

Now let’s introduce first the scaling transformation then link it with above formalism. The scaling transformation here means an intuitive way to describe the extended particle shrinking or inflating uniformly within a little bit of small scale. The shrinkage (dilation) corresponds to a coordinate inflation (shrinking) transformation due to the reason that a spatial transformation for a body can be interpreted in two equivalent ways. We are familiar with such instance while regarding the rotation of a rigid body, we can consider in one way that the body remains still while the coordinate rotates, or alternatively in another way that the coordinate system remains still while the body rotates. Analogously, concerning the dilation of an extended particle, we may infer that it is the coordinates’ shrinkage that causes the particle inflating, provided that the origo of the coordinate system locates at the center of the extended particle. Generally, we may put the coordinate transformation in a four-dimensional form as [15]

\[
\begin{pmatrix}
  t' \\
  x'_1 \\
  x'_2 \\
  x'_3
\end{pmatrix} = \begin{pmatrix}
  t \\
  x_1 \\
  x_2 \\
  x_3
\end{pmatrix} + \delta \begin{pmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
  t \\
  x_1 \\
  x_2 \\
  x_3
\end{pmatrix},
\]

(5)

where the constant \( \delta \) is responsible for the scale of inflation (or shrinkage). The above addition form is obviously equivalent to the following product form

\[
\begin{pmatrix}
  x' \\
  x'_1 \\
  x'_2 \\
  x'_3
\end{pmatrix} = (1 + \delta) \begin{pmatrix}
  x \\
  x_1 \\
  x_2 \\
  x_3
\end{pmatrix},
\]

(6)

which is the very dilation transformation [10] with the intuitive form \( x'_\mu = \lambda x_\mu \). Particularly, we use contra-variant indices to mark the inverse transformation, \( x'^\mu = \lambda^{-1} x^\mu \). i.e.,

\[
\begin{pmatrix}
  t' \\
  x'^1 \\
  x'^2 \\
  x'^3
\end{pmatrix} = (1 - \delta) \begin{pmatrix}
  t \\
  x^1 \\
  x^2 \\
  x^3
\end{pmatrix}.
\]

(7)

One notes that a point particle would not be affected by above transformation. Most material micro-particles (for instance, hadrons) however, have a limited size. The size could be of its Compton wavelength if the particle is nearly
point-like (like electron). Nowadays it is accepted that there is no true point in quantum world, and the so-called "point" is just an abstract conception from mathematical point of view.

We would like to point out that the scaling transformation mentioned above is just a manifestation of scaling parameter in RGM \([11, 44]\). Recall that the parameter \(\mu\) is introduced into RGM by boundary condition \(p = p' = \mu\), and it is not limited to its on-shell value \(\mu = m\), where \(m\) is the rest mass. Being off shell, the momentum \(p = \mu\) does not characterize a naked particle any longer, but an extensive particle with higher-order virtue processes intimately occurring around. (Conventionally we call such extensive particle dressed particle, or particle dressing cloud.) We note following de Broglie relationship that the equation \(\lambda = \frac{\mu}{m}\) naturally corresponds to coordinates transformation \(x'_{\nu} = \lambda x_{\nu}\). According to the understanding to Compton wavelength \([74]\), the \(\lambda\) is inversely proportional to \(\mu\), and approximately proportional to the radii of the cloud. The smaller the \(\lambda\) is, the more the processes occur in the cloud. Following the "cloud" picture, it is tempting to choose the origo of coordinate system \(x_{\nu}\) to be fixed at the center of mass of the extended particle. (with cloud fig picture 2)

From the viewpoint of renormalization, the occurrence of \(F(q^2)\) in the final results indicates that the \(S\)-matrix as well as Hamiltonian \(\mathcal{H}_I\) are not \(\mu\)-parameter independent. Therefore it suggests that the vertex-form in \(j_{\nu}(p, p') = \tilde{\psi}(p)\Gamma_{\nu}\tilde{\psi}(p')\) should vary with \(\mu\)-parameter indirectly,

\[
A^\nu(q^2)j_{\nu}(p, p') = A^\nu(q^2)\tilde{\psi}(p)\gamma_{\nu}\tilde{\psi}(p') \rightarrow \text{renormalization} \quad A^\nu(q^2)\tilde{\psi}(p)\tilde{\psi}(p') = F(q^2)A^\nu(q^2)\tilde{\psi}(p)\gamma_{\nu}\tilde{\psi}(p').
\]

(8)

We thus attempt to extend the interaction vertex structure to equivalently describe the renormalization effect at the tree level. Traditionally we don’t think the vertex-form \(\Gamma_{\nu}\) would deviate very much from original vertex-form \(\gamma_{\nu}\) after renormalization that, mostly we still use the same vertex-form \(\gamma_{\nu}\) with a form factor ahead of it. If the form factor is almost equal to 1 and can be approximately omitted, then we call the vertex \(\tilde{\psi}(p)\gamma_{\nu}\tilde{\psi}(p')\) scaling invariance. Actually, there is an invariant vertex \(\tilde{\psi}(p)\Gamma_{\nu}\tilde{\psi}(p')\) under the scaling transformation, other than the vector vertex, which will be considered in detail in next section. According to our aforementioned equivalence of renormalized interaction \(\mathcal{H}_I\) and the nonlocal interaction in momentum space, Eq. (5) suggests that nonlocal interaction is \(\mu\)-parameter dependent and thus not scaling invariant either.

Although neither the renormalized \(S\)-matrix nor nonlocal interaction is scale invariant, it turns out that the scaling symmetry underlies the formalism via a special manner. In the next section, in order to obtain the scale transformation for spinors, we apply the coordinate-form scaling transformation Eq. (5) to nonlocal interaction Eq. (3) by referencing how the Lorentz transformation is applied to Dirac equation. As a byproduct we also obtain the scale-invariant vertex-form \(\Gamma_{\nu}\). By definition the vertex-form \(\Gamma_{\nu}\) possesses the same form for local and nonlocal particles, namely its nonlocal properties are equal to the sums of its shreds. As for the scale transformation for spinors, while applying it back to the vertex-form \(\gamma_{\nu}\), it is noted that, as anticipated, it doesn’t conserve the \(\Gamma_{\nu}\) but gives a varying vertex-form, in which the coefficient ahead of \(\gamma_{\nu}\) can be viewed somewhat as a running charge.

The meaning of scale invariance in this paper thus follows the one conventionally defined, which is relevant to renormalization and Ward identities. For instance in Ref. [66], the dimension is analyzed in details and violations of scale invariance contribute to anomalous dimensions \(\gamma_G(g)\) of Green-function. \([67]\). The scale invariance there means that the anomaly \(\gamma(g)\) is equal to null. And it is noted that in our case scale invariance is identical with the conventional case while \(\gamma(g) = 0\). Since if \(\gamma(g) = 0\) then there is no charge or mass running. The total effect of a scale-invariant interaction is equal to that of the summation of its local constituents. The scale invariance here is thus an ideal case not matching any true interaction. However, we will recognize its significance in latter sections. The gauge symmetry and thus Ward identity are unable to be included in our consideration. We just make primary analysis from a narrow point of view.

We would like to stress that the conventional dilation transformations relating to extended objects are somehow included in special relativity (SR)—for instance the time dilation and length contraction. Our scaling transformation differs from Lorentz transformation in treating all the four components of \((t, \vec{x})\) on the same foot, as \(x'_\nu = \lambda x_{\nu}\) (\(\nu = 0, 1, 2, 3\)), and demanding the origo of coordinate system to locate within the extended particle. In this sense the scaling transformation \(x'_\nu = \lambda x_{\nu}\) does commutate with Lorentz transformation and thus reserves Lorentz invariance,
which actually results in extrapolating the Lorentz transformation. Whereas most previous investigations on Lorentz property of nonlocal system were along the opposite direction, mainly focused on reducing the degree of freedom of Lorentz group. The minimal Plank length $l_p$ falls into such class—an intriguing ramification of nonlocal theory, which was first put forward by Heisenberg.

### III. THE SCALING TRANSFORMATION FOR SPINORS

In order to derive the scaling transformation for spinors, in this section we apply the coordinate-form scale-transformation Eq. (3) to a nonlocal interaction $A^\nu(q^2)\bar{\psi}(p)\Gamma_\nu \psi(p')$, and demand it invariant (assuming that the form factor has been involved in the vertex $\Gamma_\nu$), by referencing the case that Lorentz transformation is applied to Dirac equation, like

$$\gamma^\mu S_{\mu\nu} \psi'(y) = m S \psi(y),$$

in which $S \psi'(y) = \psi(y)$, $S$ is the Lorentz transformation in spinor form to act on $\psi(y)$, and $\psi'(y)$ is to couple with $\gamma^\mu$, which appears in the alternative form of Eq. (3). $S_{\mu\nu} \psi'(y) = m \psi(y)$ (or $\gamma^\mu = S^{-1} \gamma^\mu S = \Lambda^\mu_{\nu} \gamma^\nu$ and $\psi(y) S^{-1} \gamma^\mu S \psi(y) = \psi(y) \Lambda^\mu_{\nu} \gamma^\nu \psi(y)$). Yukawa and Yennie once contemplated that nonlocality should add "inner motion" degree of freedom to the system (though later it proved to be wrong with their interpreting manner). Coincidentally we recognize that the above scaling transformation could provide a new intrinsic degree. Based on the experience from renormalization, at least the parameter $\lambda$ could provide a new inner motion degree of freedom to the system (though later it proved to be wrong with their interpreting manner). Now let’s study how the scaling transformation of coordinate-form leads to that of spinor-form. In what follows our discussions are independent of the representations, coordinate or momentum.

The dilation transformation Eq. (3) can be formally cast into the form $x'_{\mu} = \Lambda^\nu_{\mu} x_{\nu} = (\delta^\nu_{\mu} + \omega^\nu_{\mu}) x_{\nu}$, where $\omega^\nu_{\mu}$ stands for the latter part of the right hand of Eq. (3). Mimicking Lorentz transformation $x'_{\mu} = \Lambda^\nu_{\mu} x_{\nu} = (\delta^\nu_{\mu} + \omega^\nu_{\mu}) x_{\nu}$, the scaling transformation can be associated with spinors straightforwardly. As for Lorentz transformation, the transformation matrix $(\Lambda_{\mu}^\nu)$ for $j^\mu(y)$ corresponds to a complex transformation $S$ for $\psi(y)$ so that the effect of the transformed result $\psi(y) S^{-1} \gamma^\mu S \psi(y)$ is equivalent to $\psi(y) \Lambda^\mu_{\nu} \gamma^\nu \psi(y)$. Our goal in this section is to search for the corresponding vertex-form $\Gamma^\mu$ as well as transformation $S'$ for scaling transformation Eqs. (6), so that $S'^{-1} \Gamma^\mu S' = \Lambda^\mu_{\nu} \Gamma^\nu$. Before searching $\Gamma^\mu$ and $S'$, let’s first recall how the Lorentz transformation leads to the spinor’s transformation. Usually, we perform the Lorentz transformation on the vectors $A_{\mu}$ and $\gamma^\mu$. Obviously this combination leads to invariant formalism. We follow the convention that the same set of $\{\gamma^\mu\}$ is used in different coordinate systems, which naturally yields an equivalence transformation $S$ with the properties [83]

$$S^{-1} \gamma^\mu S = \Lambda^\mu_{\nu} \gamma^\nu,$$

where $\Lambda^\mu_{\nu}$ stands for the tensors’ components of the Lorentz transformation. Substitute the Eq. (10) into $A_{\mu}(x) \bar{\psi}(x) \gamma^\mu \psi(x)$ and note its invariance

$$A'_{\mu}(y) \bar{\psi}(y) S^{-1} \gamma^\mu S \psi(y) = A_{\mu}(y) \bar{\psi}(y) \gamma^\mu \psi(y).$$

The Eq. (11) shows that if we use the same $\gamma$-matrix in different coordinate systems, the field $\psi(y)$ must undertake a corresponding transformation like $\psi(y) \rightarrow S \psi(y)$.

Now let’s introduce the explicit form $\Lambda^\nu_{\mu} = e^\nu_{\mu} + \omega^\nu_{\mu}$ of Lorentz transformation, where $\omega^\nu_{\mu}$ comes from the generators of Lorentz group with $\omega^\nu_{\mu} = g^{\nu\lambda} \epsilon_{\lambda\mu}$ [83], and $\epsilon_{\lambda\mu}$ is an infinitesimal antisymmetric tensor. Substitute this explicit form into Eq. (10) and make some changes, we have

$$\gamma_{\mu} S - S \gamma_{\mu} = [\gamma_{\mu}, S] = S \omega^\nu_{\mu} \gamma_{\nu}.$$

Furthermore, if we take into account the infinitesimal parameter for the transformation $S$

$$S = 1 + \epsilon_{\mu\nu} S^{\mu\nu},$$

then to the first order, Eq. (12) reads

$$[\gamma_{\mu}, \epsilon_{\rho\sigma} S^{\rho\sigma}] = \omega^\nu_{\mu} \gamma_{\nu},$$

which actual results in extrapolating the Lorentz transformation. Whereas most previous investigations on Lorentz property of nonlocal system were along the opposite direction, mainly focused on reducing the degree of freedom of Lorentz group. The minimal Plank length $l_p$ falls into such class—an intriguing ramification of nonlocal theory, which was first put forward by Heisenberg.
accordingly one finds the solution of Eq. (14) could be

\[ S^{\mu\nu} = \frac{1}{2} \gamma^\mu \gamma^\nu. \] (15)

The above formalism is based on the conventional Lorentz generators. If we are concerned about the additional generator as that in Eq. (15), then the vertex form reads

\[ A'_\mu(x, y)\bar{\psi}(y)\Pi^\mu \psi(y), \] (16)

in which \( \Pi^\mu \) is a general form of vertex (i.e. the linear combination of \( \gamma \) matrices). To reserve the vertex \( \Pi^\mu \) under scaling transformation we have to alter the consequence of Eq. (14) correspondingly,

\[ \Pi^\mu, \bar{\varepsilon}_{\mu\nu} \bar{S}^{\mu\nu} = \bar{\omega}^\nu \Pi^\mu, \] (17)

where \( \bar{\omega}^\nu \) now becomes the unit matrix in Eq. (15) multiplied with infinitesimal constants \( \bar{\varepsilon}_{\mu\nu} \), which may not be antisymmetric any longer. Then it is found that when

\[ \Pi^\mu = \gamma^\mu (1 + \gamma_5), \quad \bar{\varepsilon}_{\mu\nu} \bar{S}^{\mu\nu} = \frac{1}{2} \gamma_5, \] (18)

Eq. (17) can be satisfied, and the selected \( \bar{\omega}^\nu \) could trivially be \( \varepsilon \delta^\nu_\mu \) (\( \varepsilon \) is extracted from \( \bar{\varepsilon}_{\mu\nu} \)). The above conclusion Eq. (18) is equivalent to and evidenced by the following relations

\[ e^{-\frac{\bar{\varepsilon}_{\mu\nu}}{2} \varphi \gamma^\mu e^{\bar{\varepsilon}_{\mu\nu}} \varphi} \simeq (1 - \frac{u}{2} \gamma_5) \gamma^\mu (1 + \frac{u}{2} \gamma_5) \simeq \gamma^\mu + u \gamma^\mu \gamma_5, \] (19)

\[ e^{-\frac{\bar{\varepsilon}_{\mu\nu}}{2} \varphi \gamma^\mu \gamma_5 e^{\bar{\varepsilon}_{\mu\nu}} \varphi} \simeq (1 - \frac{u}{2} \gamma_5) \gamma^\mu \gamma_5 (1 + \frac{u}{2} \gamma_5) \simeq \gamma^\mu \gamma_5 + u \gamma^\mu, \] (20)

\[ e^{-\frac{\bar{\varepsilon}_{\mu\nu}}{2} \varphi \gamma^\mu (1 \pm \gamma_5) e^{\bar{\varepsilon}_{\mu\nu}} \varphi} \simeq (1 - \frac{u}{2} \gamma_5) \gamma^\mu (1 \pm \gamma_5) (1 + \frac{u}{2} \gamma_5) \simeq (1 \pm u) \gamma^\mu (1 \pm \gamma_5), \] (21)

where \( u \) is an infinitesimal constant. The coefficients \( (1 \pm u) \) of Eq. (21) can be contracted to be 1 with \( (1 \mp u) \) ahead of \( A'_\mu(x, y) \) due to the same dilation (or shrinkage) transformation, by which the term \( A'(q^2)\bar{\psi}(p)\Gamma^\nu \psi(p') \) becomes invariant. From now on we will call such a scaling invariant vertices \( \gamma^\mu (1 \pm \gamma_5) \) nonlocal vertices, which display the same local and global (nonlocal) structure in interaction, i.e., display the uniform structure in both local sense and nonlocal sense. Therefore it is noted that if the vertex-form in integrand of Eq. (1) is the form \( \gamma^\mu (1 \pm \gamma_5) \), then the vertex form reads \( \gamma^\mu (1 \pm \gamma_5) \), i.e. the interaction vertex is \( A'(y)\bar{\psi}(y)\gamma^\mu (1 \pm \gamma_5)\psi(y) \), the integral result will be of the form \( A'_\mu(y)\bar{\psi}(y)\gamma^\mu (1 \pm \gamma_5)\psi(y)\Delta V \), where \( \Delta V \) is the integral volume.

Except the nonlocal vertices \( \gamma^\mu (1 \pm \gamma_5) \), the scaling transformation doesn’t reserve any other vertex form. Though the vector vertex-form \( \gamma^\mu \) is not conserved consequently, we are very interested in it for it is the starting point and the end of this research. In order to understand its properties in scaling transformation, let’s tidy up the Eq. (19) as follows

\[ e^{-\frac{\bar{\varepsilon}_{\mu\nu}}{2} \varphi \gamma^\mu e^{\bar{\varepsilon}_{\mu\nu}} \varphi} \simeq \gamma^\mu + u \gamma^\mu \gamma_5 = (1 - u) \gamma^\mu + u \gamma^\mu (1 + \gamma_5) = (1 + u) \gamma^\mu - u \gamma^\mu (1 - \gamma_5), \] (22)

where it is divided into two terms: the vector vertex-form and the nonlocal vertex-form. Repeating separately the first equility and the second, we may get an expression after \( N \) steps of transformation

\[ (e^{-\frac{\bar{\varepsilon}_{\mu\nu}}{2} \varphi \gamma^\mu e^{\bar{\varepsilon}_{\mu\nu}} \varphi})^N \approx (1 - u)^N \gamma^\mu + Nu \gamma^\mu (1 + \gamma_5), \] (23)

or

\[ (e^{-\frac{\bar{\varepsilon}_{\mu\nu}}{2} \varphi \gamma^\mu e^{\bar{\varepsilon}_{\mu\nu}} \varphi})^N \approx (1 + u)^N \gamma^\mu - Nu \gamma^\mu (1 - \gamma_5), \] (24)

in which we have omitted some terms in intermediate process since \( u \) is an infinitesimal constant. The two equations display the obvious symmetry that when make \( u \rightarrow -u \) and \( \gamma_5 \rightarrow -\gamma_5 \) simultaneously, then Eq. (23) \( \rightarrow \) (24). The two forms appear different which may mislead us to regard they are truly different. However, we find their extremes
do not depend on the seemingly different expressions of Eq. (23) or Eq. (24), but just on the value of $u$ and steps $N$. For instance, when $Nu \rightarrow 1$, both Eq. (23) and Eq. (24) yield

$$(e^{-\tilde{\tau} \gamma_5})^N \gamma^\mu (e^{\tilde{\tau} \gamma_5})^N \rightarrow \gamma^\mu (1 + \gamma_5),$$

and for $Nu \rightarrow -1$ they both yield

$$(e^{-\tilde{\tau} \gamma_5})^N \gamma^\mu (e^{\tilde{\tau} \gamma_5})^N \rightarrow \gamma^\mu (1 - \gamma_5).$$

In above equations we have employed the approximation $(1 \pm u)^N \sim 1 \pm Nu$. From Eq. (23) and Eq. (26), we note that the nonlocal vertices $\gamma^\mu (1 \pm \gamma_5)$ originate from when we perform scaling transformations on $A_\mu(y)\bar{\psi}(y)\gamma^\mu \psi(y)$. The vertex extremes $\gamma^\mu (1 \pm \gamma_5)$ are obtained if we repeatedly carry the same transformation and fix the signs of constant $u$. In the next section, we will study the conservation laws at these extremes.

Apart from these extremes, the vertex-form would mostly be at its mixture form like $a\gamma^\mu + b\gamma^\mu \gamma_5$ after carrying some steps of scaling transformation. The initially pure vector-form $\gamma^\mu$ would somehow evolve to a mixture form $a\gamma^\mu + b\gamma^\mu \gamma_5$, and the additional coefficient $a$ will be identified with form factor of vector interaction, thus equivalent to the running of coupling constant. In the next section we will envisage that this form factor is consistent with RGM. Since for an unpolarized scattering (averaged), the second part $b\gamma^\mu \gamma_5$ will contribute nothing to the final cross-section. Whereas for polarized scattering, this part will display its contribution. If regarding the vertex-form $\gamma^\mu (1 \pm \gamma_5)$ take all nonlocal information, we infer that the nonlocal effect is also partially included in coefficient $a$.

We note that both the transformation set $\{e^{\tilde{\tau} \gamma_5 y}\}$ and $e^{\tilde{\tau} \gamma_5}$ change the vertex $\bar{\psi}(y)\gamma^\mu \psi(y)$, for example one of the elements in set $\{e^{\tilde{\tau} \gamma_5 y}\}$ yields

$$e^{-\tilde{\tau} \gamma_0 y}\gamma_0 = \gamma_0 \cosh u + \gamma_3 \sinh u,$$

in which the effect can be cancelled by $A_\mu(x, y)$ via $S^{-1}\gamma^\mu S = \Lambda^\mu_{\nu} \gamma^\nu$ and $\Lambda^\mu_{\nu} A_\mu = A_\nu$, so that $A_\mu(y)\bar{\psi}(y)\gamma^\mu \psi(y)$ keeps invariant. However, $S' = e^{\tilde{\tau} \gamma_5}$ doesn’t reserve both the vertices $\bar{\psi}(y)\gamma^\mu \gamma_5 \psi(y)$ and $\bar{\psi}(y)\gamma^\mu \psi(y)$, and the equality $S'^{-1}\Gamma' S' = \Lambda_{\nu}' \Gamma'_{\nu}$ is not met unless the vertex-form $\Gamma^\mu$ possesses the special form $\gamma^\mu (1 \pm \gamma_5)$. Therefore after scaling transformation $\bar{\psi}(y)\gamma^\mu \gamma_5 \psi(y)$ and $\bar{\psi}(y)\gamma^\mu \psi(y)$ cannot be recovered by multiplying $A_\mu(x, y)$. That displays the essential difference of transformation $e^{\tilde{\tau} \gamma_5}$ from the set $\{e^{\tilde{\tau} \gamma_5 y}\}$.

Consequently, we have envisaged that the interaction vertex $A_\mu(x, y)\bar{\psi}(y)\gamma^\mu (a + b \gamma_5) \psi(y)$ is not invariant under the extended generator set $\{\frac{1}{2} \gamma_5, \{\gamma^\mu \}}$, unless the vertex evolves to $A_\mu(x, y)\bar{\psi}(y)\gamma^\mu (1 \pm \gamma_5) \psi(y)$, though such form of interaction does not match any true interaction. The general vertex $A_\mu(x, y)\bar{\psi}(y)\gamma^\mu (a + b \gamma_5) \psi(y)$ may be preserved if the generator set $\{\frac{1}{2} \gamma_5, \{\gamma^\mu \}}$ is further extended to include more $\gamma-$matrices like $\gamma^\mu \gamma^\nu \gamma^\rho$ etc., on which relevant work is in progress. However, from an alternative point of view, actually under the transformation $\{e^{\tilde{\tau} \gamma_5}, \{e^{\tilde{\tau} \gamma_5 y}\}\}$ the general form $a\gamma^\mu + b\gamma^\mu (1 + \gamma_5)$ remains invariant, apart from the variations of the coefficients $a$ and $b$. In this sense we regard the generator set $\{\frac{1}{2} \gamma_5, \{\frac{1}{2} \gamma^\mu \gamma^\nu \}}$ hold the form $a\gamma^\mu + b\gamma^\mu (1 + \gamma_5)$, with $a, b$ running in analogy to renormalization case.

Due to the Special Relativity (SR), whence the length of a ruler and the time of clock would vary with the relative velocity between different observers, we find out that the product of transformations $e^{\tilde{\tau} \gamma_5 y}$ and $e^{-\tilde{\tau} \gamma_5 y}$ acting on the wave function $\psi(y)$ successively (a boost followed by a rotation) also provides the transformation term $\gamma_5$ in their second order expansion. But the transformation involving $\gamma_5$ in our case is at its leading order. The essential reason is that our transformation stems from volume (bulk, size) effect, in contrast to SR. In addition, we choose the origin of the coordinates located at the extended particle or the vicinity around the core. These characteristics extrapolate Lorentz transformation, but don’t violate Lorentz invariance.

### IV. THE CONSERVATION LAW FOR THE SCALE-INARIANT INTERACTION $A_\mu(x, y)\bar{\psi}(y)\gamma^\mu (1 \pm \gamma_5) \psi(y)$

The significance of studying the conservation law for these extreme vertices is that according to Eq. (23), to repeatedly perform the transformation successively until $Nu \rightarrow 1$ ($Nu \rightarrow -1$), the incident particle would approach to a very high energy (a very low energy) and its wave shrinks (inflates) to a very small scale (a very large scale), the extreme scale. So at a very high (low) energy scale, it is impossible for the particles to shrink (inflate) more, and its interaction vertex approaches almost the one like $A_\mu(x, y)\bar{\psi}(y)\gamma^\mu (1 \pm \gamma_5) \psi(y)$. To put it in another way, this interaction vertex may appear to systems of two fundamental particles colliding at a very high (low) energy, and the vertex maybe exist just for a very short moment. It also means that the particle’s charge approaches to an almost steady value with respect to such transformations. That is, such vertex form would not further change its coupling constant under the scaling transformation according to the explanation of the previous sections. For not matching any
true interactions, $\gamma^\mu(1 \pm \gamma_5)$ just make their sense relatively to be their counterpart $\gamma^\mu$, for they have been evolved from the vertex-form $\gamma^\mu$. In what follows we explain what might be the conservation law of such an interaction system at its extreme state. The deviation of above vertex-form from vector vector $\gamma^\mu$ suggests that new conservative current may appear $^{84}$.

From a classical perspective, while a soft body (with definite mass $m$) shrinking or inflating, its moment of inertia $I = m \int \rho_m(r) r^2 dr$ ($\rho_m(r)$ is the mass density) would vary, with the total orbital angular momentum conserved. However for a micro-quantum particle, it inflates or shrinks only when it releases or absorbs proper amount of energy. It is believed that this energy exchange would definitely break the angular momentum conservation, at least for polarization (spin) part. Quantum mechanically, this breaking should be manifested by the transformation of spinors. Such case is similar to that when we extend the three-dimensional rotation to four-dimensional rotation involving Lorentz boosts, we find the 3-dimensional orbital angular momentum is not a conservative quantity any longer, unless we extend the concept further to include the spin angular momentum. Here we would extrapolate the four dimensional rotation on account of the scaling-invariant interaction $A_\mu(x, y) \psi(y) \gamma^\mu(1 \pm \gamma_5) \psi(y)$.

In the $S$-matrix of Heisenberg picture, only interaction term is present, whereas in Lagrangian there are at least another two additional terms involved, namely the kinetic term $\bar{\psi} \gamma^\mu \psi$ and mass term $m \bar{\psi}(y) \psi(y)$. If we intend to discuss the scaling invariance of Lagrangian, we should clarify how these two terms behave under the scaling transformation. As for the kinetic term of an extended particle, we prefer to view its momentum the same as that for a point particle—-because it is viewed as an entirety while moving freely. It possesses the form $\bar{\psi} \gamma^\mu \psi$ rather than $\bar{\psi} \gamma^\mu (a + b \gamma_5) \psi$, since we don’t need to define a new momentum in the evolving process of interaction vertex from $A_\mu(x, y) \psi(y) \gamma^\mu \psi(y)$ to $A_\mu(x, y) \bar{\psi}(y) \gamma^\mu(1 \pm \gamma_5) \psi(y)$, by integration or summation. Thus it keeps invariant form under scaling transformation, Eq. (31) or Eq. (32). The invariance of mass term is ensured by the following relation,

$$\begin{align}
[e^{\frac{\gamma_5}{2}} \psi(x)]^\dagger &= \psi(x)^\dagger (e^{\frac{\gamma_5}{2}})^\dagger = \psi(x)^\dagger \gamma_0 \gamma_0 (e^{\frac{\gamma_5}{2}}) = \bar{\psi}(x) e^{-\frac{\gamma_5}{2}} \gamma_0,
\end{align}$$

so

$$\begin{align}
[e^{\frac{\gamma_5}{2}} \psi(x)]^\dagger \gamma_0 &= \bar{\psi}(x) e^{-\frac{\gamma_5}{2}},
\end{align}$$

which is consistent with

$$\begin{align}
\bar{\psi}'(y) \psi'(y) &= \bar{\psi}(y) S^{-1} S \psi(y) = \bar{\psi}(y) \psi(y) .
\end{align}$$

Since only variations are involved in this section, it makes no difference to use Lagrangian in coordinates’ representation, as follows

$$\begin{align}
\mathcal{L} = i \bar{\psi}(y) \gamma^\mu \partial_\mu \psi(y) + m \bar{\psi}(y) \psi(y) + g \int \int dx dy F(x, y) A_\mu(x) \bar{\psi}(y) \gamma^\mu(1 + \gamma_5) \psi(y).
\end{align}$$

Now we can examine what is the conservation law for Eq. (31) under the transformation set $\{ e^{\frac{\gamma_5}{2}} \gamma^\nu, e^{\frac{\gamma_5}{2}} \}$. Before doing that, let’s first recall the customarily conserved quantities under the usual spatial transformation, i.e. the translations and rotations. These 4-dimensional spatial transformations with infinitesimal forms are

$$\begin{align}
x_\alpha &\to x_\alpha' = x_\alpha + \delta \epsilon_\alpha x_\beta + \delta \alpha ,
\end{align}$$

where $\delta \alpha$ is an infinitesimal displacement and $\epsilon_\alpha$ is an infinitesimal antisymmetric tensor for rotation in 4-dimension, $\epsilon_\alpha \epsilon_\beta = -\epsilon_\beta \epsilon_\alpha$. This transformation guarantees the invariance of $x_\alpha x^\alpha$ while $\delta \alpha = 0$. The above spatial transformation corresponds to the transformation for quantum fields as

$$\begin{align}
\psi_r(x) &\to \psi'_r(x) = \psi_r(x) + \frac{1}{2} \epsilon_\alpha \epsilon_\beta S_{\alpha \beta}^\gamma \psi_s(x),
\end{align}$$

in which the matrices elements $S_{\alpha \beta}^\gamma$ are from the spinor representation of Lorentz group $^{83}$, and in the second term both of the repeated indices stand for summations, and $\psi_s(x)$’s are components in $\psi^T(x) = (\psi_1(x), \psi_2(x), \psi_3(x), \psi_4(x))$. Take $\delta \alpha = 0$ and impose additionally the invariance of the Lagrangian

$$\begin{align}
\mathcal{L}(\psi_r(x), \psi_{r, \alpha}(x)) = \mathcal{L}(\psi'_r(x), \psi'_{r, \alpha}(x')) ,
\end{align}$$

one gets a general conserved current (known as the Northern current) $^{83}$,

$$\begin{align}
J^\alpha = \frac{1}{2} \epsilon_\beta \epsilon_\gamma S_{\alpha \beta}^\gamma ,
\end{align}$$
where
\[ \psi^{\alpha\beta\gamma} = \frac{\partial L}{\partial \psi_{r,\alpha}} S_{r,s}^{\beta\gamma} \psi_s + [x^{\beta} \Im^{\alpha\gamma} - x^{\gamma} \Im^{\alpha\beta}] , \] (36)

and
\[ \Im^{\alpha\beta} = \frac{\partial L}{\partial \psi_{r,\alpha}} \frac{\partial \psi_r(x)}{\partial x_\beta} - \mathcal{L} \dot{\psi}^{\alpha\beta} . \] (37)

The current Eq. (35) leads to angular momentum operator in four dimensions by
\[ M^{\alpha\beta} = \int d^3x \psi^{\alpha\beta\gamma} = \int d^3x \{ [x^{\alpha} \Im^{\beta\gamma} - x^{\beta} \Im^{\alpha\gamma}] + \pi_r(x) S_{r,s}^{\alpha\beta}(x) \} , \] (38)
in which \( \pi_r(x) = \frac{\partial \mathcal{L}}{\partial x_\alpha} \) is a conjugate field of \( \psi_r(x) \).

Let’s add the new ingredient \( \bar{\beta} \beta \gamma \) of Eq. (18) into the form of Eq. (33), and we immediately recognize that Eq. (37) bears no change but Eq. (36) would undergo certain variations owing to the change of \( S_{r,s}^{\beta\gamma} \), as \( \varepsilon_{\beta\gamma} S_{r,s}^{\beta\gamma} \rightarrow \varepsilon_{\beta\gamma} S_{r,s}^{\beta\gamma} + \varepsilon_{\beta\gamma} S_{r,s}^{\beta\gamma} \). The conserved current varies correspondingly
\[ \dot{\gamma}^\alpha = \frac{1}{2} \varepsilon_{\beta\gamma} \dot{\psi}^{\alpha\beta\gamma} + \frac{1}{2} \varepsilon_{\beta\gamma} \dot{\psi}^{\alpha\beta\gamma} , \] (39)
with
\[ \varepsilon_{\beta\gamma} \dot{\psi}^{\alpha\beta\gamma} = \frac{\partial L}{\partial \psi_{r,\alpha}} \varepsilon_{\mu
u} S_{\mu\nu}^{\beta\gamma} \psi_s = \frac{\partial L}{\partial \psi_{r,\alpha}} \left( \frac{1}{2} \varepsilon_{\gamma_5} \right) r_s \psi_s . \] (40)

Since the part \( \varepsilon_{\mu\nu} S_{\mu\nu} = \frac{1}{2} \varepsilon_{\gamma_5} \) is symmetric, if we want to combine it with the anti-symmetric part \( \varepsilon_{\beta\gamma} S_{r,s}^{\beta\gamma} \) and extract a common factor \( \varepsilon_{\beta\gamma} \), we have to multiply a factor \( \frac{1}{6} \varepsilon_{\beta\gamma} e^{\beta\gamma} \) ahead of the \( \varepsilon_{\mu\nu} S_{\mu\nu} \). Then Eq. (39) yields
\[ \dot{\gamma}^\alpha = \frac{1}{2} \varepsilon_{\beta\gamma} \dot{\psi}^{\alpha\beta\gamma} + \frac{1}{24} \varepsilon_{\beta\gamma} e^{\beta\gamma} \frac{\partial L}{\partial \psi_{r,\alpha}} (\varepsilon_{\gamma_5}) r_s \psi_s , \] (41)
and we should caution that in the second term only the product of the constants \( \varepsilon_{\beta\gamma} \) and \( \varepsilon \) is equivalent to infinitesimal constant \( \varepsilon_{\beta\gamma} \) of the first term, in despite of that we use the same denotations. The left constant \( \varepsilon^{\beta\gamma} \) is the normal antisymmetric constant satisfying
\[ \varepsilon^{\beta\gamma} = \begin{cases} 1 & \text{if } \beta, \gamma \text{ are different} \\ 0 & \text{if } \beta, \gamma \text{ are the same} \end{cases} . \] (42)

Thus apart from the infinitesimal parameter \( \varepsilon_{\beta\gamma} \), the remaining tensor similar to Eq. (36) becomes
\[ \tilde{\psi}^{\alpha\beta\gamma} = \frac{\partial L}{\partial \psi_{r,\alpha}} (S_{r,s}^{\beta\gamma} + \frac{1}{12} \varepsilon^{\beta\gamma} (\varepsilon_{\gamma_5}) r_s \psi_s + [x^{\beta} \Im^{\alpha\gamma} - x^{\gamma} \Im^{\alpha\beta}] , \] (43)
the constant \( \varepsilon \) is responsible for the quotient between the two constants \( \varepsilon_{\beta\gamma} \) and \( \varepsilon_{\mu\nu} \), which is assumed to be adjustable.

Now it is evident how the angular momentum Eq. (36) varies correspondingly with the transformation. However, we might have neglected an important factor stated at the beginning of this section, i.e. the inflation or shrinkage of a quantum particle needs the loss or injection of external energy. If the energy really works, it should also contribute to the second term in Eq. (13), i.e. \( x^{\beta} \Im^{\alpha\gamma} - x^{\gamma} \Im^{\alpha\beta} \), via non-zero components \( \Im^{\alpha} \). And thus the total angular momentum \( M^{\alpha\beta} \) might be conservative due to this consideration. However, if we just observe (detect) the spin polarization, obviously the spin part \( \frac{\partial L}{\partial \psi_{r,\alpha}} \frac{1}{12} \varepsilon^{\beta\gamma} (\varepsilon_{\gamma_5}) r_s \psi_s \) would entail somewhat “non-conservation” between the total and its parts, even if we might have recognized the extra contribution from \( x^{\beta} \Im^{\alpha\gamma} - x^{\gamma} \Im^{\alpha\beta} \). In this sense we conclude that the nonlocal interaction entails the new internal freedom and becomes the particle intrinsic local properties. Meanwhile it leads to the extrapolation of conventional spin angular-momentum. To put it in other words, the particles with shape and those point-like seem to follow different conservation laws. For the extensive particles it is necessary to involve this correction term \( \frac{\partial L}{\partial \psi_{r,\alpha}} \frac{1}{12} \varepsilon^{\beta\gamma} (\varepsilon_{\gamma_5}) r_s \psi_s \) in spin part and the new \( \Im^{\alpha} \) contribution due to energy exchange in orbital part. Thus when an extended particle (like proton) is smashed we should not evaluate the polarizations of its initial state (of proton) and its final state (smashed shreds of proton) in the conventional way, since the initial state (proton) and the final states (smashed proton) all have their non-point size and thus the initial
polarization should not be the sum of its final states (smashed proton). Regarding this different conservation law may help us alleviate the spin crisis appearing in the electron-nucleon polarized scattering experiment [85–91].

It has been a long-standing puzzle how the nucleon spin originates from its constituent parts, namely, the angular momentum of quarks and gluons. The conflict arose from the estimation of the total spin of proton based on the experimental value of the antisymmetric structure function $g_1$. The total spin $\Sigma$ (defined to be $\Sigma = \Delta u + \Delta d + \Delta s$, $\Delta u$ is the fraction of $u$ quark in proton’s spin, and the same to $\Delta d$ and $\Delta s$) of proton relates to the structure function $g_1$ by generalizing Bjorken’s sum rule [52], namely,

$$
\frac{1}{12} \int_0^1 g_1^2(x, Q^2) = \frac{1}{12} [g_A^{(3)} + 1/3 g_A^{(8)}] + \frac{1}{9} \Sigma,
$$

(44)

where $g_A^{(3)} = \Delta u - \Delta d$, $g_A^{(8)} = \Delta u + \Delta d - 2\Delta s$ are separately the iso-vector, $SU(3)$ octet. $g_A^{(3)}$, $g_A^{(8)}$ have been very well determined respectively from neutron $\beta$-decay and semi-leptonic hyperon decay [96]. After involving the radiative correction in perturbative QCD, the above relation can be precisely interpreted as

$$
\int_0^1 g_1^2(x, Q^2) = \frac{c_1(Q^2)}{12} [g_A^{(3)} + 1/3 g_A^{(8)}] + \frac{c_2(Q^2)}{9} \Sigma,
$$

(45)

where the coefficients $c_1(Q^2)$ and $c_2(Q^2)$ come from QCD perturbative corrections. With above knowledge, hitherto it is well known that the value of $\Sigma$ only amounts to 1/3 of total proton’s spin.

People have once conceived gluons’ spin may contribute much to proton’s spin, but recent experimental analysis [98] support merely small fraction of gluon’s contribution, though the experiment is conducted just within a narrow range of $x$ scope. Another recent flurry has been the focus on the decomposition of angular momentum of quark and gluon into spin part and orbital angular momentum part based on the gauge-invariant QCD dynamics [90, 96, 99–103]. But the ways of treating angular momentum of bounded quarks are so controversial that hitherto there has been no widely accepted scheme. In a very recent paper, Ji et al [88] refined the sum rule using generalized parton distribution (GPD) method, which may improve the further evaluation of $\Sigma$.

Now let’s focus on the coefficients of $c_1(Q^2)$ and $c_2(Q^2)$ in Eq. (45), whose accurate values are based on the perturbative calculation in QCD. For instance, the coefficients $c_1(Q^2)$ reads

$$
c_1(Q^2) = 1 - \left(\frac{C_s}{\pi}\right) - 3.58333\left(\frac{C_s}{\pi}\right)^2 + \cdots,
$$

(46)

in which $C_s(Q^2)$ may have a running value with respect to $Q^2$, roughly around $0.1^20.3$. Because the scale transformation is somehow derived from the renormalization group, including the running of charges etc., one may be aware that the corrections in Eq. (46) are to some extent equal to the scaling transformation. And the effect from Eq. (46) might also be consistent with that interpreted by Eq. (43). Though the corrections have not made the coefficients $c_1(Q^2)$ and $c_2(Q^2)$ deviate so much from 1, we know the corrections actually affect the value of $\Sigma$. When the scale approaches to the nonperturbative regime and $C_s(Q^2)$ becomes larger, the expression of Eq. (46) however, may lose its validation. Whereas we note our scaling transformation happens to be responsible for the shift between the perturbative and nonperturbative regimes since the dilation (shrinkage) occurs accompanying the loss (injection) of energy. We thus speculate that the scaling transformation might be helpful to transform the spin value from perturbative scale to nonperturbative scale, or vice versa. In this sense the transformation method could be a way of explaining the spin crisis of proton, and further investigation is in progress.

V. THE IMPACT OF VERTEX $F(q^2)A_{\mu}(k)\bar{\psi}(p)\gamma^\mu(a + b \gamma_5)\psi(p)$ ON POLARIZED SCATTERING

In this section we will discuss that after finite steps of scaling transformations (corresponding to a certain incident energy), what the contribution of the evolving vertex $F(q^2)A_{\mu}(k)\bar{\psi}(p)\gamma^\mu(a + b \gamma_5)\psi(p)$ to the scattering processes. (Though we have carried discussions in momentum representation, in the next section we would provide an interpretation method to make the results here independent of representations.) Analogous to the inelastic e-p scattering, where the assumed vertex $A_{\mu}B_{\sigma}p^\sigma$ yields some observed structures $W^{\mu\nu}$ in cross-section [104], here we are concerned about what the structures the evolving vertex-form $\gamma^\mu(a + b \gamma_5)$ leads to. Although we work following the analogy, we should caution that we focus on elastic scattering, rather than inelastic scattering. In the end of this section we arrive the conclusions that the part $b \gamma^\mu\gamma_5$ contributes nothing to the normal unpolarized cross-section of elastic scattering, so effectively doesn’t change the original structure form. However, for the polarized scattering, there appears exceptional terms additional to the original structure function.

Firstly, let’s carry out the structure function of unpolarized cross-section by averaging over the initial spins and summing over the final spins [104]. Without losing generality, let’s suppose that it is the very case for an electron...
incident on nucleon [picture 4, Feynman graph]. By conventional steps, one finds that the evolving vertex-form 
\( \gamma^\mu (a + b \gamma_5) \) yields the scattering tensor,

\[
W_{\mu \nu}^{(p)} = \frac{1}{2} Tr\left[ q_2 + M \gamma_\mu + \frac{p_2 + M}{2M} \gamma_\nu (a + b \gamma_5) \right],
\]

(47)

and apart from the coefficient \( \frac{1}{2} \), the trace can be separated into four terms

\[
W_{\mu \nu}^{(p)} = W_{\mu \nu}^{(1)} (a^2) + W_{\mu \nu}^{(2)} (a b) + W_{\mu \nu}^{(3)} (b a) + W_{\mu \nu}^{(4)} (b^2)
\]

(48)

\[
= \frac{1}{2} a^2 \left[ \frac{q_2 + M}{2M} \gamma_\mu + \frac{p_2 + M}{2M} \gamma_\nu \right] + \frac{1}{2} a b \left[ \frac{q_2 + M}{2M} \gamma_\mu + \frac{p_2 + M}{2M} \gamma_\nu \right] + \frac{1}{2} b^2 \left[ \frac{q_2 + M}{2M} \gamma_\mu + \frac{p_2 + M}{2M} \gamma_\nu \right] + \frac{1}{2} b \left[ \frac{q_2 + M}{2M} \gamma_\mu + \frac{p_2 + M}{2M} \gamma_\nu \right].
\]

and the result of the first term is well-known, it yields

\[
W_{\mu \nu}^{(1)} (a^2) = \frac{a^2}{2M^2} \left[ q_{2\mu} p_{2\nu} + q_{2\nu} p_{2\mu} - (q_2 \cdot p_2 - M^2) g_{\mu \nu} \right],
\]

(49)

the second term is

\[
W_{\mu \nu}^{(2)} (a b) = \frac{a b}{M^2} \epsilon_{\mu \nu \rho \sigma} q^\rho_2 p^\sigma_2,
\]

(50)

and the third term results in the same

\[
W_{\mu \nu}^{(3)} (b a) = \frac{a b}{M^2} \epsilon_{\mu \nu \rho \sigma} q^\rho_2 p^\sigma_2.
\]

(51)

The last term has the same form as the first \( W_{\mu \nu}^{(1)} (a^2) \) apart from the coefficient

\[
W_{\mu \nu}^{(4)} (b^2) = \frac{b^2}{2M^2} \left[ q_{2\mu} p_{2\nu} + q_{2\nu} p_{2\mu} - (q_2 \cdot p_2 - M^2) g_{\mu \nu} \right].
\]

(52)

We note the effectively additional terms are from Eqs. (50, 51), whose contribution however, would vanish since the tensor \( \epsilon_{\mu \nu \rho \sigma} \) are antisymmetric, while the tensor of lepton part \( L^{\mu \nu} \) coupled to them is symmetric with respect to indices \( \mu \nu \).

Secondly, as for the polarized cross-section, we do not fix the initial or final spin states and leave the spin operator in the potential. With the same markings as in Fig. 4 and apart from a propagator, we write the polarized amplitude (potential) by using the Dirac spinors as follows

\[
M_{AV}(\vec{p}, \vec{q}, \vec{k}) = \tilde{\psi}_1 \gamma_\mu (a + b \gamma_5) \psi_1 \tilde{\psi}_2 \gamma_\mu (a + b \gamma_5) \psi_2
\]

(53)

\[
= a^2 \tilde{\psi}_1 \gamma_\mu \psi_1 \tilde{\psi}_2 \gamma_\mu \psi_2 + a b \tilde{\psi}_1 \gamma_\mu \psi_1 \tilde{\psi}_2 \gamma_\mu \psi_2 + b^2 \tilde{\psi}_1 \gamma_\mu \psi_1 \tilde{\psi}_2 \gamma_\mu \psi_2,
\]

Only the terms with coefficients \( ab \) and \( ba \) (and henceforth we denote the two terms as \{ab\} and \{ba\}) are new, the results for the other two terms \{aa\} and \{bb\} can be found in Ref. [102]. The term \{ab\} can be tidied up into

\[
M_{\{ab\}} = \vec{U}(\vec{p}_1) \gamma_\mu U(\vec{q}_1) \vec{U}(\vec{p}_2) \gamma_\nu U(\vec{q}_2) - \vec{U}(\vec{p}_1) \gamma_\nu U(\vec{q}_1) \vec{U}(\vec{p}_2) \gamma_\mu U(\vec{q}_2),
\]

(54)

where the indices 1, 2 represent separately the first and the second particles. Substitute the concrete form of Dirac spinor \( U(\vec{p}) = \sqrt{\frac{E - m}{2E}} \left( \frac{1}{E + m} \right) \) and \( \vec{U}(\vec{p}) = U^\dagger(\vec{p}) \gamma_0 \) (where \( E = \sqrt{\vec{p}^2 + m^2} \)) into the above equation, and after lengthy calculation, it yields

\[
M_{\{ab\}} = A \left( \tilde{\sigma}_2 \tilde{\sigma}_1, \tilde{\sigma}_2, \tilde{\sigma}_1 \right) + \frac{\tilde{\sigma}_2 \tilde{\sigma}_1}{E(\vec{p}_1) + m_1} + \frac{\tilde{\sigma}_2 \tilde{\sigma}_1}{E(\vec{q}_1) + m_1} + \frac{\tilde{\sigma}_2 \tilde{\sigma}_1}{E(\vec{p}_2) + m_2} + \frac{\tilde{\sigma}_2 \tilde{\sigma}_1}{E(\vec{q}_2) + m_2}
\]

(55)

\[
+ \frac{\tilde{\sigma}_2 \tilde{\sigma}_1}{E(\vec{p}_1) + m_1} \tilde{\sigma}_2 + \tilde{\sigma}_1 \tilde{\sigma}_2 \left[ \frac{\tilde{\sigma}_2 \tilde{\sigma}_1}{E(\vec{q}_1) + m_1} \tilde{\sigma}_2 + \tilde{\sigma}_1 \tilde{\sigma}_2 \right] \frac{\tilde{\sigma}_2 \tilde{\sigma}_1}{E(\vec{p}_2) + m_2} \frac{\tilde{\sigma}_2 \tilde{\sigma}_1}{E(\vec{q}_2) + m_2}
\]

here we use underlines to denote the inner product and

\[
A = \sqrt{\frac{2}{E(\vec{p}_1)} + \frac{2}{E(\vec{p}_2)} + \frac{2}{E(\vec{q}_1)} + \frac{2}{E(\vec{q}_2)} \approx 1 - \frac{1}{8m_1^2} (q_1^2 + q_2^2) - \frac{1}{8m_2^2} (p_1^2 + q_2^2).}
\]

(56)
The second step follows while using the approximation $E(p') \approx m_1 + \frac{p^2}{2m_1}$ and to the order of $\frac{p^2}{m^2}$. With this approximation, one further gets

$$M_{(ab)} = A\left\{ \frac{1}{2m_2}\vec{s}_2 \cdot (\vec{p}_2 + \vec{q}_2) - \frac{1}{2m_1}(\vec{s}_1 \cdot \vec{p}_1\vec{s}_1 \cdot \vec{s}_2 - \vec{s}_1 \cdot \vec{q}_1\vec{s}_2) \right\},$$

in the second step of the above equation we have used the following relations

$$\sigma_i \sigma_j = \delta_{ij} + i\varepsilon_{ijk}\sigma_k,$$  

and

$$\langle \vec{s} \cdot \vec{A} (\vec{s} \cdot \vec{B}) = A_iB_j\sigma_i\sigma_j = \vec{A} \cdot \vec{B} + i(\vec{A} \times \vec{B}) \cdot \vec{s}. $$

Likewise, we obtain $M_{(ba)}$ as follows

$$M_{(ba)} = A\left\{ \frac{1}{2m_1}\vec{s}_1 \cdot (\vec{p}_1 + \vec{q}_1) - \frac{1}{2m_2}(\vec{p}_2 + \vec{q}_2) \cdot \vec{s}_1 + i(\vec{s}_1 \times \vec{s}_2) \cdot (\vec{p}_2 - \vec{q}_2) \right\}. $$

We note that the terms $M_{(ab)}$ and $M_{(ba)}$ cannot appear in those cross-sections due to any single of five Lorentz-invariant currents $\tilde{\psi}(p)\psi(p), \gamma\psi(p), \psi(p)\gamma\gamma\psi(p), \gamma\gamma\gamma\psi(p), \gamma\gamma\gamma\psi(p)$ unless some of them are mixed. Thus we realize that $M_{(ab)}$ and $M_{(ba)}$ can actually occur if the current is weak current $\psi(p)\gamma^\mu(1 - \gamma_5)\psi(p)$, for instance in the scattering of neutralino incident on electron. On one hand, since the intermediate Z boson is very heavy and thus the scattering involving weak interaction only appear in very high energy, which we may avoid by testing effects of nonlocality in somewhat lower energy regime, as though experiment put forward in next paragraph. Moreover, mostly the vertex we meet in nonlocal current must be $\psi(p)\gamma^\mu(a + b\gamma_5)\psi(p)$ instead of pure extremes $\psi(p)\gamma^\mu(1 \pm \gamma_5)\psi(p)$, which endow us with coefficients $a, b$ to adjust. On the other hand, if the nonlocal extreme vertices physically mixed or entangled with weak interaction, then it must be an interesting topic deserving further investigation.

We observe that the terms in Eqs. (57,60) are all proportional to helicity $\vec{s} \cdot \vec{p}$ (also called handedness term) in one manner or another, and thus the terms spontaneously relate to the handedness of nonlocal interaction as anticipated. Accordingly we would like to present a simple gedanken experiment to test the characteristic of handedness terms. For the feasibility of the experiment, we leave from hadron dynamics and turn to molecular scale to test the prediction of Eqs. (57,60). Imagine that an electron scattered away from a simple atom like hydrogen, which stays in its ground state, so that no orbital angular momentum is involved in the scattering processes. Meanwhile we should control the energy of incident electron to be low enough so that other orbital states of hydrogen cannot be involved, maybe the energy should $E \leq 1eV$ or several $eVs$ (with wavelength less than 2 Å), which is largely lower than any threshold of transitions. Thus the total angular momentum of such hydrogen atom is its spin, and the nucleon magnetic moment is omitted for its small fraction in the total (about 1 in 2000). And the spreading electron cloud of hydrogen atom meets the case of our nonlocal description. Such hydrogen atom could be good test ground for nonlocal predictions.

The proposed experiment is to use polarized electrons colliding on polarized hydrogen (polarized by magnetic field). Different from the previous calculations on polarized electron-electron (e-e) scattering [108] and e-H scattering [107–112], the main results here are shown in Eqs. (57,60), which is characterized by terms like $i(\vec{s}_1 \times \vec{s}_2) \cdot (\vec{p}_2 - \vec{q}_2)$. Such term differs from the normal handedness term $\vec{s} \cdot \vec{p}$ in that it permits the existence of two perpendicular spins $\vec{s}_1$ and $\vec{s}_2$. Whereas the previous test-experiments on spin asymmetry [108] mainly focused on the parallel or anti-parallel difference, as

$$A_{1s,1s}(\theta) = \frac{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\uparrow}}{\sigma_{\uparrow\uparrow} + \sigma_{\uparrow\uparrow}},$$  

or

$$A_{1s,1s}(\theta) = \frac{\sigma_{\uparrow\downarrow} - \sigma_{\downarrow\uparrow}}{\sigma_{\uparrow\downarrow} + \sigma_{\downarrow\uparrow}},$$

where $\sigma_{\uparrow\downarrow}, \sigma_{\downarrow\uparrow},$ or $\sigma_{\uparrow\uparrow}, \sigma_{\downarrow\downarrow}$ mean the parallel or anti-parallel in common sense. However, in our case, we suggest a new asymmetric parameter

$$A_{1s,1s}(\theta) = \frac{\sigma_{\uparrow\uparrow} - \sigma_{\downarrow\downarrow}}{\sigma_{\uparrow\downarrow} + \sigma_{\downarrow\uparrow}},$$

or

$$A_{1s,1s}(\theta) = \frac{\sigma_{\uparrow\downarrow} - \sigma_{\downarrow\uparrow}}{\sigma_{\uparrow\uparrow} + \sigma_{\downarrow\downarrow}}.$$
which has never been investigated in previous theoretical study or experiments. But this term may contribute even smaller fractions to total cross-section, since all of other spin-dependent terms have attributed minor fraction in total cross-section. The calculation details can follow the paper concerning additionally the present interaction. If any experiment gets a nontrivial parameter \( \frac{\sigma_{\text{pp}} - \sigma_{\text{pp}}}{\sigma_{\text{pp}} + \sigma_{\text{pp}}} \) then it proves our predictions. To put precisely, maybe all parameters \( \frac{\sigma_{\text{pp}} - \sigma_{\text{pp}}}{\sigma_{\text{pp}} + \sigma_{\text{pp}}} \) would deviate from original evaluations after considering all of the handedness terms \( \varepsilon \cdot \bar{p} \) in Eqs. \([57, 99]\). In the reference there is a similar term to our case, however there a polarization instead of spin is involved in the expression, so it is different from ours.

VI. CONCLUSIONS AND DISCUSSION

In this paper we have put forward a new kind of scaling transformation for nonlocal interaction, which belongs to an extrapolation of Lorentz transformation. The scaling transformation is constructed on the experience of calculating renormalization. By renormalization group method (RGM), the scaling invariance is formally respected by \( S \)-Matrix at one-loop corrections, despite of the running charges and masses. While tidying the nonlocal interaction in momentum representation, we note its structure being similar to the Hamiltonian of renormalized \( S \)-Matrix. Heuristically we suppose the scaling transformation has the same effect on nonlocal interaction in the perturbative regime. Based on this resemblance and regarding the physics of extended particles, we admit the intuitive form of scaling transformation \( x'_\nu = \lambda x_\nu (\nu = 0, 1, 2, 3) \). Furthermore to carry out the scaling transformation for spinors, a tentative vertex-form is assumed to be scaling invariant by mimicking that the Lorentz transformation conserves vector vertex, \( S^{-1} e^\lambda x_\nu \gamma_5 = \gamma_5 \), where \( S \) corresponds to Lorentz transformation in spinor representation. In this way we obtain both the scaling transformation in spinor representation \( \{ e^{\sum_5} \} \) and the scaling invariant vertices \( \gamma_\mu (1 \pm \gamma_5) \). While the transformation is applied repeatedly back to vector vertex \( e^{-\frac{1}{2} \sum_5} \gamma_5 \gamma_\mu e^{\sum_5} \simeq \gamma_\mu + u \gamma_\mu \gamma_5 = (1 - u) \gamma_\mu + u \gamma_\mu (1 + \gamma_5) = (1 + u) \gamma_\mu - u \gamma_\mu (1 - \gamma_5) \), one finds the varying coefficients for charges ahead of \( \gamma_\mu \), which matches the running coupling constant occurred in RGM. For vertices \( \gamma_\mu (1 \pm \gamma_5) \), it is viewed as extremes of normal vector vertex-form \( \gamma_\mu \) after infinite steps of scaling transformation, \( e^{-\frac{1}{2} \sum_5} \gamma_5 \gamma_\mu (e^{\sum_5})^N \rightarrow \gamma_\mu (1 + \gamma_5) \) for \( Nu \rightarrow 1 \) and \( (e^{-\frac{1}{2} \sum_5})^N \gamma_\mu (e^{\sum_5})^N \rightarrow \gamma_\mu (1 - \gamma_5) \) for \( Nu \rightarrow -1 \). We further discuss the conservation law for these extreme vertices, and surmise this relates to the spin crisis of nucleons since the occurrence of the vertices resulted in ultrahigh (or ultra-low) energy exchange, which truly happens in e-N scattering. For an extended particle involved (participated) in a scattering at certain energy, we have to make a corresponding scaling transformation to interpret its true interaction vertex. Then a general interaction-vertex for two elementary particles via exchanging vector bosons possesses the form \( \gamma_\mu (a + b \gamma_5) \). It has effects on polarized scattering rather than unpolarized scattering. Accordingly we propose a gedanken experiment to test our predictions on nonlocal interaction.

In what follows we will discuss several particular facets of the scaling transformation.

First, we comment on the property of "new set of generator \( \{ \gamma_5, \{ \frac{1}{2} \gamma_\mu \gamma_\nu \} \}\)" with the new element \( \gamma_5 \) added to those of Lorentz group, namely \( \{ \frac{1}{2} \gamma_\mu \gamma_\nu \} \). One notes the commutators between \( \gamma_5 \) and \( \{ \frac{1}{2} \gamma_\mu \gamma_\nu \} \) yields null, \([\gamma_5, \{ \frac{1}{2} \gamma_\mu \gamma_\nu \}] = 0\), which suggests the entering of \( \gamma_5 \) doesn’t give rise to new algebra. And this matches our previous analysis that the scaling transformation commutes with the Lorentz transformation. Furthermore it is recognized that the product of \( \gamma_5 \) with any of \( \{ \frac{1}{2} \gamma_\mu \gamma_\nu \} \) is still in the set \( \{ \frac{1}{2} \gamma_\mu \gamma_\nu \} \), and in the sense of multiplication the set \( \{ \gamma_5, \{ \frac{1}{2} \gamma_\mu \gamma_\nu \} \} \) is a closed set apart from the signs. Reversely the two Lorentz transformations \( e^{\sum_1} \) and \( e^{\sum_2} \gamma^5 \) (where \( u_1 \) and \( u_2 \) are infinitesimal parameters) naturally lead to the entering of \( \gamma_5 \) at the second order expansion, \( \frac{1}{2} u_1 u_2 \gamma^5 \). All of these facts confirm that introducing the generator \( \gamma_5 \) into \( \{ \frac{1}{2} \gamma_\mu \gamma_\nu \} \) doesn’t violate Lorentz transformation, but rather an extrapolation. The derived \( \gamma_5 \) here just promote the position of \( \gamma_5 \) from second order in original Lorentz group to the first order. However, such a promotion actually ruins the anti-symmetry of matrix elements of generators of Lorentz group, subsequently the noncompact property of Lorentz group is also lost. Summarizing the above properties of the new group based on generators \( \{ \gamma_5, \{ \frac{1}{2} \gamma_\mu \gamma_\nu \} \} \), we would like to call it an extended Lorentz group.

It is necessary to caution that both the transformations \( e^{\sum_5} \gamma_5 \gamma_\nu \) and \( e^{\sum_5} \gamma_\nu \) change the vertex \( \bar{\psi}(y) \gamma_\mu \gamma_\nu \psi(y) \), for example one of the elements in set \( e^{\sum_5} \gamma_\nu \) yields

\[
e^{-\frac{1}{2} \sum_5} \gamma_5 \gamma_\nu e^{\sum_5} = \gamma_0 \cosh u + \gamma_3 \sinh u ,
\]

in which the effect can be formally cancelled by \( A_\mu (x, y) \) via \( S^{-1} \gamma_0 S = A_\nu A_\mu = A_\mu \), so that \( A_\mu \bar{\psi}(y) \gamma_\mu \gamma_\nu \psi(y) \) keeps invariant. However, \( S' = e^{\sum_5} \) doesn’t reserve both the vertices \( \bar{\psi}(y) \gamma_\mu \gamma_\nu \psi(y) \) and \( \bar{\psi}(y) \gamma_\mu \psi(y) \), and the equality
$S'$ is not met unless the vertex-form $\Gamma'^{\mu}$ possesses the special form $\gamma^{\mu}(1 \pm \gamma_5)$. Therefore after scaling transformation $\tilde{\psi}(y)\gamma^{\mu}\psi(y)$ and $\bar{\psi}(y)\gamma^{\mu}\psi(y)$ cannot be recovered by multiplying $A_\mu(x, y)$. That displays the essential difference between transformation $e^{it\gamma_5}$ and the set $\{e^{it\gamma_5}\}$. That’s why we single out vertex $\bar{\psi}(y)\gamma^{\mu}(1 \pm \gamma_5)\psi(y)$ to consider its conservation law.

Secondly, we would like to explain a little about that the extending and broken cases of Lorentz group with examples of this paper. In this paper, the transformation set $\{\gamma_5, \{1, \gamma\mu\gamma^\nu\}\}$ is performed on points locating within the extended particles. It’s a local transformation for nonlocal interaction, or for the local interaction of extended body. From this perspective, the Lorentz group seems to be enlarged. Just like aforementioned example of orbital angular momentum: when the three-dimensional rotation is extended to four-dimensional rotation involving Lorentz boosts, we find the usual orbital angular momentum is not a conservative quantity any longer, unless the spin angular momentum is included. The original 3-dimensional rotation group is thus enlarged by introducing new degree of freedom. Now, let’s view this problem differently, i.e. concerning transformation of whole extended body. Then it is noted that the original transformations for a local point could not hold for all of points within extended particle simultaneously. For example, 3-dimensional rotational invariance holds for any local point, but when this point is replaced by a disc-like extended body, this rotational invariance is broken. The invariance holds only when the rotating axis is perpendicular to the disc and passes through the disc center specifically. In this global sense, the original rotating group is reduced rather than extended. Most studies on Lorentz transformation in nonlocal interaction similarly fall into this category (reduced class)\cite{75, 117–124}. When concerning nonlocal interaction, as for the local transformations, it is extended, but for global transformation, it is broken. Therefore we are not free in shifting our view from local to nonlocal or vice versa while studying nonlocal interaction.

Thirdly, let’s turn to gauge transformation of the following nonlocal Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 = \bar{\psi}(x)(i\gamma^\mu\partial_\mu - m)\psi(x) + g \int \int dyF(x, y)A_\mu(x)\bar{\psi}(x)\gamma^\mu\psi(x).$$  \hspace{1cm} (61)

There are many works on the gauge invariance of the nonlocal interaction in literature \cite{3, 7, 12, 123}. Hitherto this problem has not yet been resolved completely. The main difficulty lies in the incompatible forms of the first term and the second term. For instance, the conventional gauge transformation

$$\psi(x) \rightarrow \psi'(x) = \psi(x)e^{igf(x)},$$  \hspace{1cm} (62)

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu f(x),$$  \hspace{1cm} (63)

makes no sense for the Lagrangian Eq. (61), because the redundant term from the differentiation of $\bar{\psi}(x)(i\gamma^\mu\partial_\mu - m)\psi(x)$ cannot be cancelled by applying the transformation Eq. (63) to $g \int \int dyF(x, y)A_\mu(x)\bar{\psi}(x)\gamma^\mu\psi(x)$ as usual. Another incompatibility may arise since we have carried the discussions in momentum representation in previous sections, which seems impertinent to the above Lagrangian Eq. (61).

To reconcile the above incompatibilities, one way out is to write the second term in Eq. (61) with local forms. And recall our conclusion that once we write it in local form, the vertex-form $\gamma^\mu$ in current $\bar{\psi}(x)\gamma^\mu\psi(x)$ should be adjusted a little to fit the corresponding energy scale of scattering, $\bar{\psi}(x)\gamma^\mu\psi(x) \rightarrow \bar{\psi}(x)\gamma^\mu(a + b\gamma^5)\psi(x)$. The statement has been validated in momentum representation. Here we propose a method to validate it in coordinate space, i.e. we employ the following approximation in perturbative case \cite{126},

$$d\bar{\psi}(x)d\psi(x) \approx \tilde{\psi}(x)\psi(x),$$  \hspace{1cm} (64)

which is effectively equivalent to $\psi(x) \rightarrow d\psi(x)$. Here the differential form $d\psi(x)$ represents the local field submitting to global $\psi(x)$ as in real space $dx$ to $x$. We assume here that the sign $d$ somehow identifies with variation $\delta$ so that it doesn’t affect normal differential or integral calculation. By such an approximation we can unify the kinetic term and nonlocal interaction term to a common form as follows

$$\mathcal{L}_0 = d\bar{\psi}(x)(i\gamma^\mu\partial_\mu - m)d\psi(x),$$
\[ \mathcal{L}_I = gF(x,y)A_\mu(x)d\bar{\psi}(x)\gamma^\mu(a + b\gamma^5)d\psi(x) . \]

Now the local Lagrangian form is,
\[ d\mathcal{L} = d\bar{\psi}(x)(i\gamma^\mu\partial_\mu - m)d\psi(x) + gF(x,y)A_\mu(x)d\bar{\psi}(x)\gamma^\mu(a + b\gamma^5)d\psi(x) , \] (65)
and the gauge transformations correspondingly shift to
\[ d\psi(x) \rightarrow d\psi'(x) = e^{igf(x)(a + b\gamma^5)}d\psi(x) , \] (66)
\[ d\bar{\psi}(x) \rightarrow d\bar{\psi}'(x) = d\bar{\psi}(x)e^{-igf(x)(a + b\gamma^5)} , \] (67)
\[ A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu f(x) . \] (68)

The above formulae could be a local method to treat the nonlocal interaction between extended particles, and scale transformation could be another local method. The scaling transformation and gauge transformation are obviously independent of each other according to the above argument. And our discussions in previous sections thus is generalized in sense of representations and seems congruous fore-and-aft. The approximation Eq. (64) once was employed in Ref. [127] as an input criterion, however the topic was too metaphysical to apply results there to any realistic physics system.

Finally, although the dynamics used in this paper mostly stems from the perturbative dynamics, it paves a way for our understanding of nonperturbative dynamics. There have been continuous efforts to study nonperturbative interaction ever since the birth of renormalization [128, 129]. To utilize somehow the scale parameter of renormalization to intermediate-strong-interaction was the primary goal of this paper. Furthermore a even stronger motivation is to develop an analytic non-perturbation method to understand strong interaction and strong correlation. The motivation thus led us to applying transformation instead of solely the scale parameter to nonperturbative interactions. Some other nonlocal theories have made efforts to link the nonlocal interaction with renormalization, for instance in Ref. [4, 7, 29, 30]. But none of them used transformation method, which was laid there years before [69, 70]. The appearance of \( \gamma_5 \) in the scaling transformation and the nonlocal vertex-form gives us the confidence that we might have unveiled a truth of nonperturbative dynamics. Because we have the intuition that when the current quark gains its mass non-perturbatively and thus becomes constituent quarks, it accompanies with the chiral-symmetry breaking and the occurrence of \( \gamma_5 \). However, to test anything in nonperturbative regime is difficult, since the low energy QCD is unresolvable analytically. In this paper we have presented an alternative way to test nonlocal effect, i.e., via the polarized scattering of electrons incident on ground-state hydrogen. This experimental method is easy to realize, which may justify our predictions perturbatively.

The discussions of this paper have based on the fundamental interaction with QED as paradigm, and the extension may be available to other interactions between material fermions.

VII. ACKNOWLEDGEMENTS

I am grateful to the hospitality of Prof. Y. B. Dong and Prof. P. Wang when I visited High Energy Institute of Chinese Academy, as well as the fruitful discussions with them. Also thanks for the encouragements from Prof. W. Q. Wang and Prof. W. Han.

[1] Zoltan Fodor and Christian Hoelbling, Rev. Mod. Phys. 84, 449 (2012).
[2] N. K. Timofeyuk and R. C. Johnson, Phys. Rev. Lett. 110, 112501 (2013).
[3] D.A. Eliezer and R.P. Woodard, Nucl. Phys. B 325, 389 (1989).
[4] J. W. Moffat, Phys. Rev. D, 39, 3654 (1989).
[5] D. Evens, J. W. Moffat, G. Kleppe, R. P. Woodard, Phys. Rev. D 43, 499 (1991).
[128] QCD, BREAKING IN. "NON-PERTURBATIVE RENORMALIZATION GROUP." International Conference on Color Confinement and Hadrons in Quantum Chromodynamics: The Institute of Physical and Chemical Research (RIKEN), Japan, 21-24 July 2003. World Scientific Publishing Company Incorporated, 2004.

[129] Berges, Jiří, Nikolaos Tetradis, and Christof Wetterich, Phys. Rep. 363.4 (2002): 223-386.
Schematic for the local interaction \( A_\mu(x) \) between a point particle and a extended particle. Using integration, it is \( A_\mu(x) \int d^4\eta F(\eta) j^\mu(x + \eta) \).
Schematic for nonlocal interaction $A_\mu(x)$ between local particles. Using integration, it is

$$\int d^4 \xi F(\xi) A_\mu(x + \xi) j^\mu(x).$$

The formula eq. (1) represents the both.
Cloud Picture

Caption: The picture schemes the cloud attaching a charged particle. The central part is the charge and around is cloud marked by $p = m$ and $p = \mu$. $p = m$ stands for the on-shell case, and $p = \mu$ represents the off-shell case. We assume in nonlocal interaction while the cloud shrinks or inflates a little the total interaction may vary according to RGM, renormalization results. To speak strictly, the cloud is just the wave part of quantum entity (the other part is its charge part), and cloud radii only makes approximate sense. $p = \mu$ occurs only in interaction process and not a freely stable state for a free moving particle. So the blue part cloud only makes sense in interacting processes.
Caption: Explanation of how we transform a nonlocal interaction to a local one. When a point particle interacts with an extended particle, the interaction is represented by the term $A_{\mu} \gamma^{\mu}$. The transformation allows for a local description of the interaction.
The Feynman graph for calculating the scattering cross-section for point particles.

\[ A_\mu \gamma^\mu (1 + \gamma^5) \]