Investigation of the maximum error of motion control

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Abstract. The task of synthesizing an automatic motion control system based on the criterion of the minimization of maximum control error is set. The maximum resulting error is defined as the sum of the dynamic component of the error and the component of the error from the broadband interference disturbance. The task is especially relevant for controlling the landing of aircraft, a wide range of guidance tasks, flight control in the mode of following the terrain, in the mode of stabilizing the low altitude of the WIG-craft. The maximum possible value of the dynamic error is strictly estimated from the given maximum values of several derivatives of the input action. These numerical data are easy to find experimentally or based on a priori theoretical research. As for the second component of the error from the input white noise, its quasi-maximum value is assumed to be equal to five r.-m.-s. values. The objective function of the system parameters optimization is investigated analytically for the extremum. The potential control accuracy is determined. In contrast to optimization by the criterion of minimum r.-m.-s. error, the optimal bandwidth of the system is narrower that is better for suppression the interference error.

1. Introduction

The problem of synthesis of automatic motion control system under an any accuracy level criteria involves setting the class of the considered systems, the properties of the desired trajectory (nominal effects), and the properties of the navigation error of the meter (disturbance shown to the system input), as well as the main indicator of the quality (accuracy) of the control that you want to restrict by a certain acceptable value or even provide its extremalization. The amount of known information about the properties of impacts should allow calculating the quality indicator or at least evaluating it from above for all the considered variants of the system. For linear systems, an analytical estimate is usually possible for each variant of the transfer function of the system; for nonlinear systems mathematical modeling and simulation has to be used.

The long period of undivided dominance in the field of dynamic filtering of the Wiener and Kalman methods of statistical synthesis with a full spectral-correlation description of the effects led to the fact that the quadratic accuracy criterion was widely used. The root-mean-square control error \( \sigma_e \) has become the main indicator of control accuracy, and its calculation in any linear system for each action is easily performed. The law of the impact distribution is not taken into account, and the law of the error distribution, due to the central limit theorem, tends to be normal and is described by a Gaussian non-finite curve. Sometimes the almost maximum error (3-5) \( \sigma_e \) is considered. This approach to statistical synthesis is well developed methodically and is also used in many modified versions in the so-called post-Kalman filters [1-4].

However, from the point of view of practical engineers, it has two very significant drawbacks:
• neither the spectral, nor even the correlation characteristics of the effects, especially the driving
effect, are precisely known, and we have to consider their extremely simplified models such as
exponential-correlation noise;
• in engineering practice, it is important to limit the maximum error value rather than the r.-m.-s.
value; this is especially important when landing an aircraft, docking aerospace vehicles, and in
many other tasks.

The definition of the maximum error can be given when considering the error probability density as
a finite function - this is the maximum value of the argument at which the probability density is not yet
zero. To accurately calculate the maximum error, one actually needs to choose the most unfavorable
combination of effects that maximizes the error. This is not a stochastic problem, but a completely
deterministic one, for which spectral-correlation characteristics are useless. But some a priori
information about the impacts must be known. It can be argued that the most reliable numerical data on
the properties of the driving force \( g(t) \) are the maximum values of several of its derivatives, usually two
or three, i.e. the maximum values of displacement, velocity, and acceleration included in the inequalities

\[
|g^{(i)}(t)| \leq g_{M,i}, \quad i = K, N, \quad 0 \leq K \leq N. \tag{1}
\]

As already noted, most often \( K = 0, \quad N = 2, \quad i = 0, 2 \).

Before putting the synthesis problem of the automatic control system corresponding to the defined
accuracy requirements, it is necessary to be convinced that the available a priori information about the
properties of external actions is enough for an estimation of accuracy for any considered variants of the
system. And it is desirable, that the deriving of numerical value for accepted accuracy factor could be
produced simply enough or, at least, could be easily algoirthmized. Therefore, the next paragraph is
devoted to the methods of accuracy analysis.

2. Maximal error estimation
Here the maximal value of stationary centered error is accepted as the fundamental factor of control
accuracy. Reference action \( g(t) \) and interference \( v(t) \), reduced to a system input, are implicated as their
casual centered components. These components can be either stationary, or have stationary casual
increments [5].

The linear systems with the frequency transfer functions of the open loop are considered

\[
W(j\omega) = \frac{1 + b_1 j\omega + ... + b_{n-1} (j\omega)^{n-1}}{a_k (j\omega)^k + a_{k+1} (j\omega)^{k+1} + ... + a_n (j\omega)^n}.
\tag{2}
\]

The parameters \( \{a_i\}_k, \{b_j\}_{n-1} \in [0, \infty) \) are considered constant and known.

At the unit principal feed-back the frequency transfer functions for the closed loop system on
controlled output and on error are determined by the formulas

\[
H_c(j\omega) = \left[1 + W(j\omega)\right]^{-1}, \tag{3}
\]

\[
H(j\omega) = W(j\omega)\left[1 + W(j\omega)\right]^{-1}. \tag{4}
\]

The resultant error is represented as the sum of two its components

\[
e = e_g + e_v. \tag{5}\]

The dynamic error \( e_g \) and error from interference \( e_v \) are formed accordingly at transiting of the
assigning action through the filter with transfer function $H_e(s)$ and disturbing action through the filter with transfer function $H(s)$ as shown in figure 1.

\[
\begin{align*}
&\text{Figure 1. Formation of the resulting error.}
\end{align*}
\]

3. Calculation of the maximum error when limiting the first derivative of the reference action

Let it be known that its $K$-th derivative obeys the inequality

\[
\left| g^{(K)}(t) \right| \leq g^K_M, \quad K \geq 0. \tag{6}
\]

This corresponds to constraint (1) when $N = K$.

In particular, constraint (6) belongs to the action

\[
g(t) = g_0 + \dot{g}t + \ldots + \frac{g^{(K-1)}_{0}K^{-1}}{(K-1)!} + g^{(K)}_M \frac{t^K}{K!}. \tag{7}
\]

Under the reference action (7), the error in the steady state will change according to the law

\[
e_{ss}(t) = c_0g(t) + c_1\dot{g}(t) + \ldots + c_{K-1}g^{(K-1)}(t) + \frac{c_K g^{(K)}_M}{K!}, \tag{8}
\]

where $c_i$ - error rates are defined as follows

\[
c_i = \left. \frac{d^i H_e(s)}{ds^i} \right|_{s=0}, \quad i = 0, K. \tag{9}
\]

Since the action described (7) and its lower derivatives can have infinitely large initial values or grow indefinitely with time, the necessary condition for the finiteness of the error is the equality to zero of the error coefficients (9) at $i = 0, K - 1$. That is, the system must have $K$-th order astatism and higher.

Otherwise, the formulation of the problem of calculating the maximum error is meaningless.

For a system with transfer function (3) when $r = K$ from (9) we obtain

\[
c_K = \left. \frac{d^K H_e(s)}{ds^K} \right|_{s=0} = a_K. \tag{10}
\]

Hence, in accordance with (8),

\[
e_{ss}(t) = e_{ss} = a_K g^{(K)}_M. \tag{11}
\]

Moreover, the found value $e_{ss}$ cannot be considered as maximum error of the system, since it is achieved in the steady-state mode of working out the reference action of a narrower class than the action from condition (6).
To accurately determine the maximum error, we will use the method of "accumulation of disturbances" developed by B.V. Bulgakov and will introduce the dimensionless coefficient of error accumulation [6-8]

\[
k(T_e) = \frac{1}{a_K} \int_0^{T_e} |w_{eK}(t)| \, dt.
\]  

(12)

Coefficient (12) shows how many times the maximum error with the most unfavorable reference action exceeds the steady-state value of the error with a constant value of the \( K \)th derivative of the reference action. Then the maximum error is determined as follows

\[
e_M = ke_{ss} = ka_K g_M^{(K)}.
\]  

(13)

Curve \( k(t) \), that called curve of B.V. Bulgakov, can be obtained graphically from the transient response

\[
h_{eK}(t) = \int_0^t w_{eK}(\tau) \, d\tau = L^{-1} \{ H_{eK}(s) / s \}.
\]  

(14)

Figure 2 shows the construction of the \( k(t) \) curve for a third-order system, which has only two extrema of the transient (step) response \( h_{eK}(t) \).

4. An example of system optimization based on the criterion of minimum maximum total error

To explain the essence of the proposed method briefly, we will analyze the simplest case of limiting only the first derivative of the reference action when \( K = N = 1 \).

The following problem of parametric optimization of the angular motion control system is considered.

Let the rate of change of the reference action be limited by the maximum value \( g_M^{(1)} = 10 \) deg/s, broadband disturbance is considered by white noise with an intensity \( S_\omega(\omega) = \omega = 0.001 \) deg\(^2\)-s. The simplest open-loop transfer function of the system has the form

\[
W(s) = \frac{K_1}{s}.
\]
The objective optimization function according to the criterion of the minimum of maximum total error has the form

\[ e_m = e_{gm} + e_{vm} = \frac{g_M^{(1)}}{K_1} + 5\sigma_v = \frac{g_M^{(1)}}{K_1} + 5\left( S_v \cdot \frac{K_1}{2}\right)^{1/2}. \]

To examine the error for an extremum, we obtain the expression

\[ \frac{g_M^{(1)}}{K_1^2} = \frac{5}{2} \left( \frac{S_v}{2}\right)^{1/2} \cdot K_1^{-1/2}. \]

Hence

\[ K_1^{3/2} = \frac{2}{5} \left( \frac{2}{S_v}\right)^{1/2} \cdot g_M^{(1)}, \]

or

\[ K_1 = \left[ \frac{2}{5} \left( \frac{2}{S_v}\right)^{1/2} \cdot g_M^{(1)} \right]^{2/3}. \]

The substituting of the numeric values gives \( K_1 = 31.75 \) s\(^{-1}\).

Then

\[ e_m = e_{gm} + e_{vm} = \frac{10}{31.75} + 5 \left( 1 \cdot \frac{31.75}{2}\right)^{1/2} = 0.945 \text{ deg}. \]

At white noise intensity \( S_v = 0.01 \) deg\(^2\)s the maximal error value will be \( e_m = 2.036 \) deg.

Unfortunately, in the first-order system, the transition characteristic does not have a single extremum, and the calculation of B. Bulgakov coefficient does not allow to get its difference from unity. For higher-order systems, the effect of accumulation of dynamic error is more significant and the forms of the most unfavorable impacts are very interesting.

5. Conclusions

The approaches to calculating the maximum control error from the maximum values of the impact derivatives are proposed: the method of canonical representations (time domain analysis) and the approximative method of analysis in the frequency domain are recommended. The first method allows to find the exact value of the maximum error of motion control, the second-a strict estimate from above of this value, but without finding the worst impact. Having an estimate of the maximum error can be analytically or numerically solve the problem of synthesis of linear systems with given accuracy and maximum simplicity, and the problem of system optimization by the criterion of minimum maximum total error.

A comparison of the optimal systems according to the proposed criterion and the standard quadratic criterion showed that the proposed criterion gives systems with a narrower bandwidth.

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