Modeling the non-spatial structure of *Gmelina arborea* Roxb Stands in the Oluwa Forest Reserve, Nigeria

Nijerya Oluwa koruma ormanında *Gmelina arborea* Roxb meşçerelerinin mekansal olmayan yapı modellemesi

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**ABSTRACT**

Modeling non-spatial forest stand structure is important for prescribing silvicultural treatments and harvesting regimes. It requires identification of a suitable distribution model to provide reliable estimates. Therefore, in this study, we evaluated the performance of various distribution models in describing the structure of the *Gmelina arborea* stands in the Oluwa Forest Reserve, Nigeria. Data were collected from twenty-five sample plots of 0.04 ha in five stands (aged 19, 24, 29, 34, and 39 years). Five distributions, including Weibull, Generalized Weibull, Johnson SB, Logit-logistic (LL), and generalized beta were fitted to diameter data from the individual stands. Model assessment was based on negative log-likelihood, Kolmogorov-Smirnov, Cramer-von Mises, Anderson-Darling, and Bayesian Information Criterion. The results showed that Johnson SB had the smallest rank sum of the fit indices. Kolmogorov-Smirnov and Anderson-Darling values ranged from 0.0333 to 0.0441, and 0.0217 to 0.0522, respectively. As such, the Johnson SB distribution was identified as the most suitable model for the *G. arborea* stands. Generalized beta, LL, Weibull, and generalized Weibull distributions performed equally well. The application of the Johnson SB model with a fitted Näslund height-diameter model provided information on class volume per hectare. This information is important for making decisions on product specification and overall management of the *G. arborea*.

**Keywords:** Diameter distributions, *Gmelina arborea*, probability distribution, stand structure

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**ÖZ**

Mekansal olmayan orman meşçere yapısının modellenmesi, silvikültürel müdahaleler ve ürün düzeyi için önemlidir. Güvenilir tahmin yapayalnızca uygun bir dağıtım modelinin tanımlanması gerekmektedir. Bu nedenle, çalışma Nijerya’nın Oluwa koruma ormanında *Gmelina arborea* meşçerelerinin yapısının açıklanmasında çeşitli dağıtım modellinin performansını değerlendirilmiştir. Veriler beş meşçerede (19, 24, 29, 34 ve 39 yaşlarında) 0.04 ha’lık yirmi beş örnek alanda toplanmıştır. Oluşturulan beş dağılım Weibull, Generalized Weibull, Johnson SB, Logit-logistic (LL) ve genelleştirilmiş beta dağılımı ve meşçerelerin ortalaması çap verilerine göre dizayn edilmiştir. Oluşturulan modelin değerlendirilmesi olusmuş olasılık dağılımı, Kolmogorov-Smirnov, Cramer-von Mises, Anderson-Darling ve Bayesian bilgi kriterlerinde dayanımaktadır. Elde edilen sonuçlar, Johnson SB dağılımının uyum indekslerinde en düşük sıralama sayısı olduğunu göstermiştir. Kolmogorov-Smirnov ve Anderson-Darling değerler sırasıyla 0.0333 ile 0.0441 ve 0.0217 ila 0.0522 arasında değişmektedir. Bu nedenle, Johnson SB dağılımı *G. arborea* meşçereleri için en uygun model olarak belirlenmiştir. Genelleştirilmiş beta, LL, Weibull ve genelleştirilmiş Weibull dağılımları ise egiş derecede performans değerleri göstermiştir. Johnson SB modeline entegre edilmiş bir Näslund yükseklik çap modeli uygulanması, hektar başına elde edilen ürün sınıf hacminin ne olacağı hakkında yeterli düzeyde bilgi vermiştir. Makale sonuçları ile elde edilecek ürün özellikleri ve *G. arborea*’nın genel yönetimi hakkında karar verilmesinde önemli verilerin elde edilmesini sağlamıştır.

**Anahtar Kelimeler:** Çap dağılımları, *Gmelina arborea*, olasılık dağılımı, meşçere yapısı

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**INTRODUCTION**

Diameter distribution is a component of forest growth and yield, representing non-spatial forest stand structure. Forest structure affects biodiversity, biomass production, and even habitat function (Gadow et al., 2011). Prediction of stem diameter distributions requires the application of probabili-
Diameter at breast height (dbh) is one of the most common and simple parameters to measure (West, 2015). Its frequency distribution provides initial information on the structure of the forest stand. Use of probability density functions to describe forest structure is well described in the forest literature. For example, Zhang et al. (2003), Gorgoso-Varela and Rojo-Alboreca (2014) each compared different methods for fitting Johnson SB and Weibull functions to forest stands. Wang and Rennolls (2005) also evaluated the performance of four-parameter functions to describe the structure of Chinese fir stands. Furthermore, Palahi et al. (2007) and Gorgoso and Gorgoso (2012) applied the beta, Johnson SB, and Weibull models to describe the structure of forest stands in northeast and northwest Spain, respectively. Other studies that have applied probability density functions to describe the structure of forest stands include Nord-Larson and Cao (2006), Poudel and Cao (2013), Mirzaei et al. (2016), Ogana et al. (2017), and Mayrinck et al. (2018). In addition to using ground inventory data, forest stand diameter distributions can also be derived from LiDAR data. Recently, Arias-Rodil et al. (2018) estimated the diameter distributions in Pinus radiata D. Don stands using a moment-based parameter recovery method. These studies employed various density functions, including 2-parameter functions (lognormal, gamma, and Weibull), 3-parameter functions (Weibull), and 4-parameter functions (beta, generalized Weibull, Johnson SB, and Logit-Logistic [LL]). Offering more flexibility and greater coverage in the skewness and kurtosis plane (Wang and Rennolls, 2005), 4-parameter distributions can describe different shapes of forest stand structure. However, few studies have applied 4-parameter functions to describing the structure of forest stands in Nigeria.

Gmelina arborea Roxb. is a fast-growing tree species belonging to the family of Verbenaceae. G. arborea originating from Southeast Asia (Pakistan and Sri Lanka to Myanmar) (Jensen, 1995). It is an important tree species that occupies large hectares of land in Nigeria (Ogana et al., 2017), grown for timber and as a source of raw material for the pulp and paper industries (Ajayi et al., 2004). Of all the exotic timber species grown in plantations, Gmelina arborea has shown greatest practical promise (Ekpa et al., 2014). Therefore, the objective of this study was to evaluate the ability of selected parametric density functions to describe the structure of the Gmelina arborea stand in Nigeria with a view to enhancing sustainable resource management. The functions were fitted using maximum likelihood estimation across multiple age groups.

MATERIALS AND METHODS

Study Area and Data

The data used in this study were plot data of a Gmelina arborea plantation in the Oluwa Forest Reserve, located in the humid tropical zone of Southwestern Nigeria. The Oluwa Forest Reserve, in the Odigbo Local Government Area, is situated between latitude 6°55’ and 7°20’N and longitude 3°45’ and 4°32’E, and occupies an area of 87,816 ha. Oluwa has an annual rainfall in the range 1700 to 2200 mm, an average annual temperature of 26°C, and a mean elevation of 123 m above sea level (Onyekwelu et al., 2006). The plantation trail in Oluwa began in the early 20th century, but establishment of larger scale plantation started in the 1960s. Over the years, Gmelina arborea has emerged as the dominant plantation species in Oluwa, accounting for about 89% of the total plantation (Onyekwelu, 2001). In this work, five age series were considered: 39, 34, 29, 24, and 19 years. A total of 25 sample plots of 20 m×20 m in size from the five stands were included. Diameter measurements of individual trees (over bark) at breast height (1.3 m above ground, dbh) were obtained. The inventory data were used to calculate the number of trees per ha (N trees/ha), basal area per ha (sum basal per plot divide by plot size), quadratic mean diameter (dg, cm), and dominant diameter (average diameter of the 100 largest trees per ha, Do in cm). The dg was computed using equation (1). Descriptive statistics for the data are presented in Table 1.

\[
dg = \sqrt{\frac{4000 G}{\pi N}}
\]

where dg is the quadratic mean diameter, G is the basal area per ha, and N is number of trees per ha.

Model specification

Four 4-parameter distributions, including generalized beta (GBD), generalized Weibull (Gweibull), Johnson SB, and LL were used in this study. The 3-parameter Weibull distribution was also evaluated.

The Weibull distribution: The probability density function (pdf) and cumulative distribution function (cdf) of the 3-parameter Weibull distribution (Weibull, 1951) are expressed as:

\[
f(x) = \frac{a}{\beta} \left(\frac{x - \gamma}{\beta}\right)^{a-1} e^{-\left(\frac{x - \gamma}{\beta}\right)^a}
\]

\[
F(x) = 1 - e^{-\left(\frac{x - \gamma}{\beta}\right)^a}
\]

where \( f(x) \) is the probability density function (pdf); \( F(x) \) is the cumulative distribution function (cdf); \( a \) is the shape parameter (\( a > 0 \)); \( \beta \) is the scale parameter (\( \beta > 0 \)); \( \gamma \) is the location parameter.

Johnson SB distribution: The Johnson SB function (Johnson 1949) is given by:

\[
f(x) = \frac{\delta}{\sqrt{2\pi}} \left(\frac{x - \xi}{\lambda}\right) e^{-\frac{1}{2} \left(\frac{x - \xi}{\delta}\right)^2}\left(1 + 3 + \frac{(x - \xi)^2}{6}\right)^{-\frac{1}{2}}
\]

where \( \xi \) is the location parameter, \( \lambda \) is the scale parameter, and \( \delta \) are the shape parameters (i.e., asymmetry and kurtosis parameters, respectively); \( \xi < x < \xi + \lambda \), \( -\infty < \xi + \lambda < +\infty \), \( -\infty < \lambda \). This distribution has no closed-form cdf; hence, numerical integration can be used.
The generalized beta distribution (GBD): The pdf of the GBD is expressed as:

\[ f(x) = \frac{1}{(b-a)B(\alpha, \beta)} \left( \frac{x-a}{b-a} \right)^{\alpha-1} \left( 1 - \frac{x-a}{b-a} \right)^{\beta-1} \]  
Eq. (5)

\[ B(\alpha, \beta) = \frac{\Gamma(\alpha) \Gamma(\beta)}{\Gamma(\alpha + \beta)} \]  
Eq. (6)

where \( \alpha, \beta \) are the shape parameters; \( a \) is location and \( b \) is the scale parameter, \( \Gamma \) is the gamma function. \( a < x < b, \alpha > 0, \beta > 0 \). As with the Johnson SB distribution, numerical integration was used for GBD because it has no closed-form cdf.

The LL distribution (Wang and Rennolls 2005): The pdf and cdf of LL are given by:

\[ f(x) = \frac{\mu}{\sqrt{2\pi\sigma^2}} \exp \left( -\frac{(x-\mu)^2}{2\sigma^2} \right) \]  
Eq. (7)

\[ F(x) = \frac{1}{2} \left[ 1 + \exp \left( \frac{x - \mu}{\sigma\sqrt{2}} \right) \right] \]  
Eq. (8)

where \( f(x) \) is the probability density function, \( F(x) \) is the cumulative distribution function, and \( x \) is diameter/height. The parameters \( \mu \) (mu) and \( \sigma \) (sigma) are the shape parameters. Other parameters are as previously defined in equation (3).

The generalized Weibull (GW) distribution (Wang, 2005): The pdf and cdf of the GW distribution are expressed as:

\[ f(x) = \frac{k}{\beta} \left( \frac{x}{\beta} \right)^{k-1} \exp \left[ -\left( \frac{x}{\beta} \right)^k \right] \]  
Eq. (9)

\[ F(x) = \left( 1 - \exp \left[ -\left( \frac{x}{\beta} \right)^k \right] \right) \]  
Eq. (10)

where \( k \) is the shape parameter (\( k > 0 \)); \( \beta \) is the scale parameter (\( \beta > 0 \)); \( \gamma \) is the location parameter.

### Fitting Method and Assessment

The maximum likelihood estimation method was used to fit the distribution models to the data, applied using the Nelder-Mead optimization algorithm (Nelder and Mead, 1965) in the “optim” function in R statistical software (R Core Team, 2017). The distribution models were assessed based on negative log-likelihood (−Λ), Kolmogorov-Smirnov (KS), Cramer-von Mises (CvM), Anderson-Darling (AD), and Bayesian Information Criterion (BIC). The smaller the values, the better the distribution fits the data set. Subsequently, the best-performing function was applied to estimate the density and volume per ha for different diameter.

### Table 1. Descriptive statistics of stand variable

|  | Statistics | Dmin (cm) | Dmax (cm) | dg (cm) | N (tree/ha) | G (m²/ha) | Do (cm) |
|---|---|---|---|---|---|---|---|
| 1 | Mean | 8.7 | 41.9 | 25.4 | 870 | 46.5 | 39.6 |
|  | SD | 2.00 | 5.28 | 4.23 | 164.32 | 20.5 | 4.54 |
|  | Min | 6.5 | 35.0 | 20.7 | 625 | 21.02 | 34.4 |
|  | Max | 10.6 | 46.5 | 31.0 | 1075 | 69.76 | 44.6 |
| 24 | Mean | 6.4 | 47.5 | 25.5 | 1150 | 59.2 | 43.8 |
|  | SD | 2.39 | 5.56 | 1.83 | 93.54 | 13.16 | 38.2 |
|  | Min | 4.0 | 40.3 | 23.2 | 1075 | 45.4 | 39.5 |
|  | Max | 10.2 | 54.5 | 27.7 | 1300 | 78.1 | 47.9 |
| 29 | Mean | 5.6 | 44.2 | 23.8 | 1435 | 63.79 | 40.5 |
|  | SD | 1.52 | 4.34 | 1.77 | 123.24 | 6.91 | 2.07 |
|  | Min | 3.0 | 40.6 | 21.5 | 1225 | 52.7 | 38.9 |
|  | Max | 7.0 | 51.4 | 26.2 | 1525 | 71.1 | 42.8 |
| 34 | Mean | 4.6 | 38.6 | 21.9 | 855 | 34.97 | 35.6 |
|  | SD | 1.04 | 5.46 | 3.34 | 290.68 | 19.63 | 6.14 |
|  | Min | 3.0 | 32.5 | 17.9 | 425 | 10.66 | 26.9 |
|  | Max | 5.8 | 46.5 | 26.1 | 1175 | 55.18 | 43.2 |
| 39 | Mean | 8.6 | 46.9 | 28.2 | 950 | 58.74 | 43 |
|  | SD | 1.61 | 3.09 | 1.46 | 190.39 | 6.61 | 0.98 |
|  | Min | 6.3 | 44.0 | 25.8 | 775 | 49.60 | 41.4 |
|  | Max | 10.5 | 51.5 | 29.4 | 1275 | 66.69 | 44.0 |

dg: quadratic mean diameter; Do: dominant diameter; SD: standard deviation; Dmax: maximum diameter; Dmin: minimum diameter; G: basal area per ha; N: number of trees per ha
classes. An appropriate height-diameter model was first fitted to estimate the mean from which volume was derived.

RESULTS AND DISCUSSION

The estimated parameters of the distributions are presented in Table 2. The location parameter γ of the Weibull distribution ranged from 1.321 to 5.027, the scale parameter β ranged from 21.557 to 28.585, and the shape parameter α ranged from 1.727 to 2.781. The values for the two shape parameters of GW, k and α ranged from 0.277 to 0.661 and −46.338 to 6.458, respectively.

For Johnson SB, γ and σ estimates ranged from 0.195 to 0.294, and 0.839 to 1.148, respectively. For LL (μ, σ) and GBD (α, β), shape parameter values ranged from −0.662 to −0.193, and 0.529 to 0.733, and from 1.306 to 2.057 and 1.759 to 2.202, respectively. No fit was recorded for GBD in the stand aged 39 years.

The diameter distributions of the *Gmelina arborea* stands in Oluwa Forest Reserve are presented in Figures 1 to 5. The graphs show the observed relative frequency of trees with the fitted Weibull, GW, Johnson SB, LL, and GBD distributions by diameter classes for stands ages 19, 24, 29, 34, and 39 years. Diameter distribution shapes were slightly positively skewed in stands aged 29 and 34 years. The Weibull distribution overestimated the rela-

| Stand age | Distributions | Parameters |
|-----------|---------------|------------|
|           | Weibull       | α | β | γ |   |
| 19        | 1.981         | 21.557 | 5.027 |
| 24        | 2.040         | 23.213 | 2.971 |
| 29        | 1.898         | 20.227 | 4.450 |
| 34        | 1.727         | 19.554 | 2.207 |
| 39        | 2.781         | 28.585 | 1.321 |
|           | GW            | k | α | β | γ |   |
| 19        | 0.277         | 6.458 | 30.606 | 4.325 |
| 24        | 109.788       | -42.049 | 20.588 | 1.432 |
| 29        | 83.241        | -46.338 | 27.142 | 1.720 |
| 34        | 0.464         | 2.957 | 25.188 | 2.601 |
| 39        | 0.661         | 5.288 | 28.124 | 2.770 |
|           | Johnson SB    | γ | σ | λ | ξ |   |
| 19        | 0.294         | 0.839 | 43.421 | 5.403 |
| 24        | 0.728         | 1.148 | 59.700 | 1.621 |
| 29        | 0.772         | 1.096 | 54.394 | 3.127 |
| 34        | 0.535         | 0.856 | 46.331 | 2.010 |
| 39        | 0.195         | 1.111 | 52.492 | 2.411 |
|           | LL            | μ | σ | λ | ξ |   |
| 19        | -0.193        | 0.650 | 47.830 | 3.644 |
| 24        | -0.549        | 0.573 | 55.988 | 2.958 |
| 29        | -0.662        | 0.628 | 52.429 | 5.010 |
| 34        | -0.596        | 0.733 | 46.781 | 2.815 |
| 39        | -0.464        | 0.529 | 59.524 | 5.136 |
|           | GBD           | α | β | b | a |   |
| 19        | 1.349         | 1.759 | 47.573 | 6.294 |
| 24        | 2.057         | 3.831 | 61.276 | 3.266 |
| 29        | 1.686         | 3.331 | 56.499 | 5.114 |
| 34        | 1.306         | 2.202 | 47.836 | 2.873 |
| 39        | No fit        | No fit | No fit | No fit |
tive frequency in the diameter class of 15–30 cm more than the other distributions in the youngest stand (aged 19 years). The older stands were well represented by the density functions.

To evaluate the performance of the distributions in describing the structure of the *G. arborea* stand, five fit indices were computed, including negative log-likelihood (−Λ), KS, CvM, AD, and BIC, as presented in Table 3. The results showed that GBD and Johnson SB had the best fit for age 19 years, achieving −Λ, KS, CvM, AD, and BIC values of −692.332, 0.0356, 0.0394, 0.3134, and 1406, and of −692.586, 0.0333, 0.0321, and 1406, respectively. Next best results were obtained with GW and LL. Weibull had the worst fit. For age 24 years, Johnson SB performed best, with −Λ, KS, CvM, AD, and BIC values of −636.709, 0.0441, 0.0457, 0.2541 and 1294, respectively. This was followed by GBD, Weibull, and LL; GW had the worst fit for this stand. For the stand aged 29 years, Weibull had the lowest values for most of the indices, and as such, ranked best. Its indices values were −1046.68, 0.0363, 0.0546, 0.4314, and 2110. LL, Johnson SB, GBD, and GW followed in that order. GBD had the best performance for the stand aged 34 years, with −Λ, KS, CvM, AD, and BIC values of −838.720, 0.0367, 0.0343, 0.2524, and 1699, respectively. It was followed by LL and Johnson SB. GBD and Weibull distributions had the worst fit. For the stand aged 39 years, Weibull had the lowest values for most of the indices, and as such, ranked best. Its indices values were −1043.039, 0.0363, 0.0546, 0.4314, and 2110, respectively. Weibull, GW, and LL followed in that order. No fit was observed for GBD in this stand.

To determine the most suitable distribution across all age groups, ranks were allocated to the distributions based on
the fit indices. The lowest value (1) was assigned to the distribution with the smallest index, whereas the highest value (5) was assigned to distribution with the largest index value. These ranks were summed and plotted against stand age, as shown in Figure 6. Observation of the trend lines in that graph showed Johnson SB to have the lowest peak with respect to both the minima and maxima points of the five distributions. It was thus regarded as the most suitable model for describing the diameter distribution of the G. arborea stands. GW, with the highest peak, displayed the worst performed distribution in the stands.

Details of how the Johnson SB distribution was applied to estimate number of trees and volume per ha of the Gmelina stands are provided in Appendix 1.

The structure of the G. arborea stands in the Oluwa Forest Reserve was determined using flexible four-parameter and Weibull distributions. The forest literature reports recent application of the generalized beta, GW, LL, Johnson SB, and Weibull distributions (e.g., Gorgoso-Varela and Rojo-Alboreca, 2014; Mayrinck et al., 2018; Ogana et al., 2018). Of the distributions assessed in the current study, Johnson SB offered greatest consistency across the different age groups (19, 24, 29, 34, and 39 years). The flexibility of the Johnson SB distribution in describing various shapes of forest stand structure has been well reported. For example, Mateus and Tomé (2011) observed that the Johnson SB distribution was the only pdf of those evaluated that could assume the range of skewness and kurtosis values observed in the data from Eucalyptus stands. Similar results were observed in Mayrinck et al. (2018) who reported a better performance with the Johnson SB distribution compared to GBD and Weibull for Kha-ya ivorensis. Conversely, Wang and Rennolds (2005) found the LL distribution to be more suitable than the Johnson SB or GBD based on −ΛΛ for Chinese fir stands. The GBD performed relative

Table 3. Fit indices of the five distributions for the different stand ages

| Age | Distribution | −ΛΛ  | KS     | CvM    | AD         | BIC |
|-----|--------------|------|--------|--------|------------|-----|
| 19  | Weibull      | -699.997 | 0.0721 | 0.1787 | 1.0959     | 1416 |
|     | GW           | -694.427 | 0.0449 | 0.0606 | 0.4274     | 1410 |
|     | SB           | -692.586 | 0.0333 | 0.0321 | Inf        | 1406 |
|     | LL           | -696.723 | 0.0473 | 0.0893 | 0.6299     | 1414 |
|     | GBD          | -692.332 | 0.0356 | 0.0394 | 0.3134     | 1406 |
| 24  | Weibull      | -638.006 | 0.0567 | 0.0821 | 0.4612     | 1291 |
|     | GW           | -641.815 | 0.0734 | 0.1535 | 0.9004     | 1304 |
|     | SB           | -636.709 | 0.0441 | 0.0457 | 0.2541     | 1294 |
|     | LL           | -637.326 | 0.0525 | 0.0616 | 0.3560     | 1295 |
|     | GBD          | -636.748 | 0.0477 | 0.0497 | 0.2790     | 1294 |
| 29  | Weibull      | -1046.681 | 0.0363 | 0.0546 | 0.4314     | 2110 |
|     | GW           | -1053.405 | 0.0430 | 0.0889 | 0.6952     | 2129 |
|     | SB           | -1045.686 | 0.0476 | 0.0522 | 0.3322     | 2114 |
|     | LL           | -1044.696 | 0.0421 | 0.0441 | 0.3581     | 2112 |
|     | GBD          | -1044.298 | 0.0481 | 0.0571 | 0.3384     | 2111 |
| 34  | Weibull      | -846.645 | 0.0676 | 0.1670 | 1.1615     | 1710 |
|     | GW           | -841.631 | 0.0694 | 0.1610 | 0.9916     | 1705 |
|     | SB           | -839.626 | 0.0378 | 0.0410 | Inf        | 1701 |
|     | LL           | -840.128 | 0.0396 | 0.0515 | 0.4460     | 1702 |
|     | GBD          | -838.720 | 0.0367 | 0.0343 | 0.2524     | 1699 |
| 39  | Weibull      | -645.635 | 0.0347 | 0.0552 | 0.4239     | 1307 |
|     | GW           | -644.858 | 0.0433 | 0.0480 | 0.3826     | 1310 |
|     | SB           | -643.039 | 0.0341 | 0.0217 | 0.1809     | 1307 |
|     | LL           | -646.690 | 0.0459 | 0.0544 | 0.5920     | 1314 |
|     | GBD          | No fit   | No fit | No fit | No fit     | No fit |

−ΛΛ: negative log-likelihood value; KS: Kolmogorov-Smirnov; CvM: Cramer-von Mises; AD: Anderson-Darling; BIC: Bayesian Information Criterion
well in the current study but could not describe the structure of the G. arborea stand aged 39 years (i.e., the oldest stand). The reason for this is not yet understood.

The properties of an ideal distribution for size class modeling include relative flexibility to describe a wide spectrum of shapes, and ease of parameter estimation and approximation of proportions of trees in diameter classes (Burkhart and Tome, 2012). The Johnson SB distribution possess these properties: It offers various estimation methods, including conditional maximum likelihood, moments, mode, regression, and the Knoebel and Burkhart method. It also provides wide coverage in the skewness and kurtosis plane. In addition, the proportion of trees in size classes can easily be estimated with numerical methods. Although the Weibull distribution offers multiple estimation methods and relative simplicity, it occupies a smaller region in the skewness and kurtosis plane (Wang, 2005). Few estimation methods for the GBD and LL distributions have been applied to forestry.

Various indices were used to evaluate the five distributions. Assessing the performance of fitted distributions based on single index can be misleading (Ogana et al. 2018). For example, if the negative log-likelihood value (−ΛΛ) (deviance statistic) had been used as the sole index, the Weibull distribution would have been selected as the best model, since it had the smallest −ΛΛ value in stands aged 19, 24, 29, and 34 years. This was not the case when rank sum was used to determine the most appropriate distribution model for the stands. Other cdf distance indices, such as AD, KS, and CVM favor more complex distributions; that is, those with a higher number of parameters (Delignette-Muller and Dutang, 2015). All the distributions considered in this study have four parameters (two shapes, scale, and location parameters) except the Weibull distribution with three parameters. Thus, BIC was included as a penalty criterion. For all indices, the Johnson SB distribution seems to be more stable than the others evaluated in this work.

In conclusion, modeling forest stand structure is important for prescribing silvicultural treatments (e.g., thinning either above or below) and harvesting regimes. Effective modeling of stand structure requires a suitable distribution model. In this study, we found the Johnson SB to be more suitable than the generalized beta, GW, LL, and Weibull distributions for the G. arborea stands in the Oluwa Forest Reserve. Application of the Johnson SB model with a fitted Näslund height-diameter model provided information on class volume per hectare. This information is important for making decisions on product specification and overall management of the G. arborea.

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### Appendix 1. Application of the Johnson SB distribution to estimate density and volume by diameter class for the Gmelina stands

| Dbh class (cm) | Class midpoint | Relative Frequency | N trees/ha | Volume (m³/ha) |
|---------------|----------------|--------------------|------------|----------------|
| 2-7           | 4.5            | 0.03619            | 38         | 0.0343         |
| 7-12          | 9.5            | 0.12169            | 128        | 2.4708         |
| 12-17         | 14.5           | 0.1674             | 176        | 18.0855        |
| 17-22         | 19.5           | 0.17478            | 184        | 56.7423        |
| 22-27         | 24.5           | 0.15937            | 168        | 116.4566       |
| 27-32         | 29.5           | 0.13176            | 139        | 182.5771       |
| 32-37         | 34.5           | 0.09861            | 104        | 230.2924       |
| 37-42         | 39.5           | 0.06457            | 68         | 234.4808       |
| 42-47         | 44.5           | 0.03376            | 35         | 178.7935       |
| 47-52         | 49.5           | 0.01091            | 11         | 80.3830        |
| 52-57         | 54.5           | 0.00087971         | 1          | 8.7183         |
| Total         |                |                    | 1,052      | 1109.0346      |

**Models used for development of the table:**

**Johnson SB:**

\[
 f(x) = \frac{1.1227}{\sqrt{2 \pi}} \cdot \frac{56.3343}{(0.9403 + 56.3343 - D)(D - 0.9403)} \cdot e^{-\frac{1}{2} \left[ 0.5794 + 1.1227 \ln \left( \frac{D - 0.9403}{0.9403 + 56.3343 - D} \right) \right]^2} \quad \text{Eq. (A1)}
\]

**Nalund HD:**

\[
 H = \frac{D^2}{(2.850715 + 0.100139D)^2} \quad \text{Eq. (A2)}
\]

**Volume equation:**

\[
 V = 1.534 \times 10^6 D^{1.12} H^{0.545} \quad \text{RMSE = 0.0842, MAE = 0.0019} \quad \text{Eq. (A3)}
\]

where \( f(x) \) = probability density; \( H \) = tree height (m); \( D \) = diameter midpoint (cm); \( V \) = volume (m³)