Charm meson couplings in Hard-Wall holographic QCD

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Abstract In phenomenology of quantum chromodynamics (QCD), charmed meson vertices are useful tools to study the final-state re-scattering effects in the hadronic B decays. The strong couplings of charm mesons vertices, are related to the basic parameters in the heavy quark effective Lagrangian. Here, the four-flavor Hard-Wall holographic QCD is illustrated to evaluate the couplings of \( (D^{(-\pi)}, D^0, a_1^\pm) \), \( (D^{(-\rho)}, D^0, b_1) \), \( (D_1^{(-\pi)}, \bar{D}_1^0, K_1^-) \), \( (D_2^{(+-\pi)}, \bar{D}_2^0, K_2^-) \), \( (D_3^{(+-\rho)}, D_1^+, K_1^{*0}) \), \( (D_4^{(+-\rho)}, D_1^0, K_1^{*0}) \), \( (D_2^{(+-\pi)}, D_1^0, K_1^{*0}) \), \( (D_5^{(+-\pi)}, D_1^0, K_1^{*0}) \) vertices. Moreover, the values of the masses of \( D^{(0)} \), \( D^{(-\pi)} \), \( \rho \), \( D_1^\pm \), \( K_0 \), \( \eta_c \), \( D_1^{(*)} \) and \( \chi_3^\pm \) as well as the decay constant of \( \pi^- \), \( D^{(+-\pi)} \), \( K^- \), \( \rho^- \), \( a_1^{\pm} \) and \( D_3^{(+-\pi)} \) are estimated in this study. A comparison is also made between our results and the experimental values of the masses and decay constants. Our results for strong couplings are also compared with the three point sum rule (3PSR) and light-cone QCD sum rule (LCSR) predictions.

1 Introduction

In recent investigations, the strong interaction of charmed hadrons among themselves and with other particles have received remarkable attention. In phenomenology of the high energy physics, charm meson vertices play a perfect role in meson interactions. The charmed meson vertices help us to investigate the final-state interactions in hadronic B decays. In [1–9], charm mesons are considered as the intermediate states which lead the long distance effect on the values of the branching ratios for non-leptonic \( B \) meson decays. On the other hand, the strong couplings between charm mesons and other hadrons, can help us to study the production of \( J/\psi, \psi(2s), \ldots \), in heavy ion collisions and absorption of these states in hadronic matter such as nucleons and light mesons [10, 11]. Now a days, different theoretical methods are used to consider vertices involving charm mesons. \( D^+ \, D^- \, \pi \), \( D^* \, D^- \, \gamma \), \( D \, D \, \rho \), \( D^* \, D^- \, \rho \) vertices are analyzed via lattice QCD approach in [12–15]. Moreover, \( D^* \, D^* \, \rho \) [16], \( D^* \, D^* \, \pi \) [17, 18], \( D \, D \, \pi \) [19], \( D^* \, D \, \rho \) [20], \( D \, D \, J/\psi \) [21], \( D^* \, D \, J/\psi \) [22], \( D^* \, D_s \, K \), \( D^* \, D_s \, K \), \( D^* \, D_s \, D \), \( D^* \, D_s \, \rho \) [23], \( D^* \, D^* \, P \), \( D^* \, D \), \( D \) [24], \( D^* \, D^* \, \pi \) [25], \( D \, D \, D \, K \), \( D \, D \, D \, K \) [26], \( D \, D \, D \) [11], \( D \, D \, V \), \( D \, D \, V \, D \) [27, 28], \( D \, D \, D \, \pi \), \( D \, D \, D \, \pi \) [29] and \( D \, D \, D \, A \) [30], vertices are often studied via the three point sum rule (3PSR) and the light cone QCD sum rule (LCSR) methods.

In recent years, a relatively new approach named the anti-de Sitter space/quantum chromodynamics (AdS/QCD) correspondence has been utilized to predict the form factors and couplings for the hadronic systems. This method is inspired from correspondence between a type IIB string theory and super Yang–Mills theory in the large \( N \) limit with \( N = 4 \) [31–33]. In this approach, corresponding to every field in the AdS\(_5\) space, an operator is defined in 4 dimensional gauge theory, and the correlation functions involving \( n \) currents are related to the 5D action by functional differentiation with respect to their \( n \) sources [32–35]. Utilizing AdS/QCD correspondence approach interesting results are reported as the masses, couplings, electromagnetic and gravitational form factors of mesons [36–49]. This method is also utilized to predict \( K_{13} \) transition form factors in [50]. In addition, the strong couplings \( g_{\rho \rho \rho \rho}, g_{\rho \rho \rho \rho \rho}, g_{\rho \rho \rho \rho \rho}, g_{\rho \rho \rho \rho \rho}, g_{\rho \rho \rho \rho \rho}, g_{\rho \rho \rho \rho \rho} \) and \( g_{\rho \rho \rho \rho \rho} \) are analyzed in a Hard-Wall holographic QCD in [51].

In Hard-Wall model of AdS/QCD the radial coordinate \( z \) varies between two cut-offs called the UV and the IR boundary. The lower bound of \( z \), which is inserted to avoid singularity of the 5D AdS metric, is located at \( \epsilon (\epsilon \to 0) \) and the higher value is located at \( z = z_0 \) is called the IR boundary. The IR cut-off is necessary to provide confinement property
of QCD and is considered as the free parameter of the mentioned model.

Our goal of this study is to extract the couplings of $(D, D, A), (D^*, D, A), (D, D, V), (D^*, D, V), (D_1, D_1, P), (\bar{\psi}, D, D, A), (\bar{\psi}, D^*, D, A), (\bar{\psi}, D, P)$ and $(\bar{\psi}, D^*, D, P)$ in Hard-Wall holographic QCD with four flavors. The paper is organized as follows: In Sect. 2, our model including pseudoscalar, vector and axial vector mesons is introduced. In Sect. 3, the wave functions and the decay constant of studied mesons are extracted from our model. The strong couplings for three and four-meson vertices derived in Sects. 4 and 5 is reserved for numerical analysis. Our prediction for masses, decay constants, wave functions and the strong couplings are presented in this section and for a better analysis, a comparison is made between our estimations and the results of other methods. Section 6 is reserved for our conclusion and discussions.

2 The AdS/QCD model involving pseudoscalar, vector and axial vector mesons

In this section we introduced our model in 5 dimensions involving pseudoscalar, vector and axial vector mesons. Here, the metric of 5 dimensional anti-de Sitter space is chosen in Poincaré coordinates as:

$$ds^2 = \frac{R^2}{z^2} (-dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu),$$

where $R$ is the warp factor and in pure AdS is chosen as $R = 1$ [52]. Moreover, $\mu, \nu = 0, 1, 2, 3$ and $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ is the usual Minkowski metric in 4 dimensions. In Hard-Wall model, the radial coordinate $z$ varies in the range $(\varepsilon, z_0)$, where the lower bound $z = \varepsilon$ (with $\varepsilon \to 0$) gives the asymptotic feature of QCD and the IR cut-off $z_0 \approx 1/\Lambda_{QCD}$ is used to simulate QCD confinement.

We will consider the 5D action proposed in Ref. [53]. In this action the $N_f$ gauge fields $L^{\mu,a}, R^{\mu,a}$ and a scalar field $X$ correspond to 5D fields for current operators $J^{\mu,a}_{L/R} = \tilde{q}_{L/R} \gamma^{\mu} t^a q_{L/R}$ and $\tilde{q}_{L} q_{R}$ from 4D theory, respectively. In $J^{\mu,a}_{L/R}$ definition, $q$ is quark field and $q_{L/R}$ is $(1 \pm \gamma_5)q$ are the left handed (L) and the right handed (R) quarks. Moreover, $t^a$ (with $a = 1, \ldots, N_f^2 - 1$) are the generators of the SU($N_f$) group which are related to the Gell-Mann matrices $\lambda^a$ by $\lambda^a = 2 t^a$. We take $N_f = 4$ so, the 5D action with SU(4)$_L \otimes$ SU(4)$_R$ symmetry can be written as:

$$S = \int d^5x \sqrt{g} \text{Tr} \left\{ (D_M X) (D_M X)^\dagger + 3 |X|^2 - \frac{1}{4g^2} (L_{MN} L_{MN} + R_{MN} R_{MN}) \right\},$$

where $D_M X = \partial_M X - iL_M X + iX R_M$ is the covariant derivative of the scalar field $X$. In addition, the strength of the non-Abelian $L$ and $R$ fields are defined as:

$$L_{MN} = \partial_M L_N - \partial_N L_M - i[L_M, L_N],$$

$$R_{MN} = \partial_M R_N - \partial_N R_M - i[R_M, R_N],$$

with $L_M = L^a_M t^a$ and $R_M = R^a_M t^a$. The left and right hand gauge fields can also be written in terms of the vector ($V$) and the axial vector field $A$, in the form $L = V + A$ and $R = V - A$. The scalar field $X$ can be expanded as:

$$X = e^{i\pi a^a} X_0 e^{i\pi a^a},$$

where $X_0$ is the classical part and $\pi$ contains the fluctuations. With flavor symmetry, $X_0$ is a multiple of the unit matrix and $X = e^{i\pi a^a} X_0$ can be obtained. This choice for the scalar field is used in [54] with $N_f = 2$, and flavor symmetry is assumed to estimate masses and decay constants for the light and strange mesons. Their model predicts good results for the more excited strange mesons observables. In [52] the part of the action that mixes the axial vectors with the pseudoscalars is just considered and the U(1) problem is studied. All parameters in the mentioned model can be determined by the experimental masses of the $\pi^0$, $K^0$ and $\rho$ mesons, and the pion decay constant $f_\pi$.

In general, using equation of motions and turning off all fields except $X_0(z)$, one obtains:

$$2X_0(z) = \xi M z + \sum \zeta z^3,$$

where $M$ and $\Sigma$ are the quark-mass and the quark condensates $\langle \bar{q} q \rangle$ matrices, respectively. For $N_f = 4$ we take $M = \text{diag}(m_u, m_d, m_s, m_c)$ and $\Sigma = \text{diag}(\sigma_u, \sigma_d, \sigma_s, \sigma_c)$. Moreover in Eq. (5), $\xi = \sqrt{N_f/2\pi}$ is the normalization parameter introduced in Ref. [55]. Note that for the light-quark sectors in the SU(2) isospin symmetry, $m_d = m_u$ and $\sigma_u = \sigma_d$ are assumed in [50,51]. Equation (5) is used in Refs. [50,56–58] and in this work we shall use it.

3 Wave functions, masses and the decay constants for the pseudoscalar, vector and axial vector mesons

Expanding the action in Eq. (2) up to second order in vector ($V$), axial vector ($A$) and pseudoscalar field ($\pi$), we obtain:

$$S = \int d^5x \frac{1}{g^2} \sum_{a=1}^{15} \frac{-1}{2g_s z} \gamma_{\eta_{MM'}} \gamma_{NN'} (\partial_M V_{N'} - \partial_N V_{M'}) \times (\partial_M V_{N'a} - \partial_N V_{M'a}) + \frac{M_{V}^2}{2z^2} \gamma_{MM'} \gamma_{NN'} V_{M'a} V_{N'a} \times \frac{-1}{4g^2} \gamma_{MM'} \gamma_{NN'} (\partial_M A_{N'} - \partial_N A_{M'}) \times (\partial_M A_{N'a} - \partial_N A_{M'a})$$
In this subsection we study wave functions of vector, axial vector and pseudoscalar mesons. We start with the vector field, which satisfies the following equation of motion:

\[
\eta_{\mathcal{LM}} \partial_M \left( \frac{1}{z} \left( \partial_L V_N^a - \partial_N V_L^a \right) \right) + \frac{\alpha^a(z)}{z} V_N^a = 0.
\]

(9)

In this equation, \( z^2 \alpha^a(z) = g_A^2 M^2 V^a \) and \( V_N^a = (V^0 \mu, V^i \xi) \) is the vector field where \( V^a = V^a_{\mu} + V^a_{\perp} \). For the transverse part \((V^a_{\perp})\), choosing \( \partial \mu V^a_{\perp}(x, z) = 0 \), the following result is obtained:

\[
\left( \partial_z \frac{1}{z} \partial_z + \frac{q^2 - \alpha^a}{z} \right) V^a_{\mu\perp}(q, z) = 0.
\]

(10)

Here, \( q \) is the Fourier variable conjugate to the 4 dimensional coordinates, \( x \). The transverse part of the vector field can be written as \( V^a_{\mu\perp}(q, z) = V^0_{\mu}(q) \lambda^a(q^2, z) \) where \( V^0_{\mu} \) and \( \lambda^a \) are boundary values at UV and bulk-to-boundary propagator, respectively. \( \lambda^a(q^2, z) \) satisfies the same equation as \( \lambda^a q_{\mu\perp}(q, z) \) with the boundary conditions \( \lambda^a(q^2, \epsilon) = 1 \) and \( \partial_z \lambda^a(q^2, z_0) = 0 \).

The longitudinal parts of the vector field, defined as \( V^a_{\mu\parallel} = \partial_\mu A^a \) and \( V^a_{\perp} = -\partial_\mu \tilde{A}^a \), are coupled as follows:

\[
-q^2 \partial_z \tilde{A}^a(q^2, z) + \alpha \partial_z A^a(q^2, z) = 0.
\]

(11)

where we define \( \xi^a = \tilde{A}^a - A^a \). The boundary conditions to evaluate \( \phi^a \) and \( \pi^a \) are \( \tilde{A}^a(q, \epsilon) = 0, \tilde{A}^a(q, \epsilon) = -1 \) and \( \partial_\epsilon \tilde{A}^a(q^2, z_0) = \partial_\epsilon \pi^a(q^2, z_0) = 0 \).

In general form of differential equations Eqs. (10) and (11), \( \lambda^a(q^2, z), \phi^a(q^2, z) \) and \( \pi^a(q^2, z) \) can be solved numerically. We expect that, normalizable modes of Eqs. (10) describe the vector mesons while, Eqs. (11) and (12) are utilized to study the scalar ones. The scalar mesons are not considered in our calculations.

To obtain the wave functions of the axial vector and pseudoscalar mesons, the variation over the axial vector field \( A^a_{\mu\perp} = (A^a_{\mu \perp}, A^a_{\perp}) \) of Eq. (6), is taken. The transverse part of the axial vector field satisfies the following equation of motion:

\[
\left( \partial_z \frac{1}{z} \partial_z + \frac{q^2 - \beta \alpha}{z} \right) A^a_{\mu\perp}(q, z) = 0,
\]

(13)

where \( z^2 \beta \alpha(z) = g_A^2 M^2 A_{\perp} \). Moreover, the gauge choices \( \partial_\mu A^a_{\mu\perp}(x, z) = 0 \) and \( A^a_{\perp} \) are imposed in the Fourier transform. Note that \( A^a_{\mu\perp} = A^a_{\mu\perp} + \partial_\epsilon \phi^a \) is used to separate the transverse and longitudinal parts of the axial vector field. The transverse part \( A^a_{\mu\perp} \), can be written as \( A^a_{\mu\perp}(q, z) = A^a_{\mu\perp}(q) A^a_{\perp}(q^2, z) \). To obtain \( A(q^2, z) \), we set \( A^a(q^2, \epsilon) = 1 \) for the UV boundary and for the IR boundary we choose Neumann boundary condition \( A^a(q^2, z_0) = 0 \). This part describes the axial vector states.

The longitudinal part of the axial-vector field \( \phi^a \) and the \( \pi^a \) describe the pseudoscalar fields and satisfy the following equations:

\[
-q^2 \partial_\epsilon \phi^a(q^2, z) + \beta \alpha(z) \partial_\epsilon \pi^a(q^2, z) = 0,
\]

(14)

\[
\partial_\epsilon \left( \frac{1}{z} \partial_z \phi^a(q^2, z) \right) - \frac{\beta \alpha(z)}{z} \left( \phi^a(q^2, z) - \pi^a(q^2, z) \right) = 0,
\]

(15)

where the boundary conditions are \( \phi^a(q^2, \epsilon) = 0, \pi^a(q^2, \epsilon) = -1 \) and \( \partial_\epsilon \phi^a(q^2, z_0) = \partial_\epsilon \pi^a(q^2, z_0) = 0 \).

Table 1 The values of \( M^2_V \) and \( M^2_A \) with \( v_\epsilon(z) = \xi q^2 + \frac{1}{\xi} \)

| \( a \) | \( M^2_V \) | \( M^2_A \) | \( a \) | \( M^2_V \) | \( M^2_A \) | \( a \) | \( M^2_V \) | \( M^2_A \) |
|---|---|---|---|---|---|---|---|---|
| (1, 2) | \( \frac{1}{2}(v_\epsilon - v_0)^2 \) | \( \frac{1}{2}(v_\epsilon + v_0)^2 \) | (6, 7) | \( \frac{1}{2}(v_\epsilon - v_0)^2 \) | \( \frac{1}{2}(v_\epsilon + v_0)^2 \) | (11, 12) | \( \frac{1}{2}(v_\epsilon - v_0)^2 \) | \( \frac{1}{2}(v_\epsilon + v_0)^2 \) |
| 3 | 0 | \( \frac{1}{2}(v_0^2 + v_\epsilon^2) \) | 8 | 0 | \( \frac{1}{2}(v_0^2 + v_\epsilon^2 + 4v_\epsilon)^2 \) | (13, 14) | \( \frac{1}{2}(v_0 - v_\epsilon)^2 \) | \( \frac{1}{2}(v_0 + v_\epsilon)^2 \) |
| (4, 5) | \( \frac{1}{2}(v_\epsilon - v_0)^2 \) | \( \frac{1}{2}(v_\epsilon + v_0)^2 \) | (9, 10) | \( \frac{1}{2}(v_\epsilon - v_0)^2 \) | \( \frac{1}{2}(v_\epsilon + v_0)^2 \) | 15 | 0 | \( \frac{1}{12}(v_0^2 + v_\epsilon^2 + v_\epsilon^2 + 9v_\epsilon^2) \) |
We finish this subsection by writing the SU(4) vector $V$, axial vector $A$ and pseudoscalar $\pi$ meson matrices terms of the charged states as:

$$ V = V^{a\mu} = \frac{1}{\sqrt{2}} \left( \begin{array}{cccc} a_0^0 + \alpha_0^0 + \beta_0^0 \sqrt{2} & \rho^+ & K_{++} & \bar{D}_0 \\ \rho^- & -\rho^0 + \alpha_0^0 + \beta_0^0 \sqrt{2} & K_{++} & \bar{D}_0 \\ K^- & -\frac{\sqrt{2}}{2} \omega' + \frac{\sqrt{2}}{\sqrt{12}} & D_{++} & -\frac{3}{\sqrt{12}} \psi \\ D^0 & D^{++} & D_+ & -\frac{3}{\sqrt{12}} \chi \sqrt{12} \end{array} \right) $$

$$ A = A^{a\mu} = \frac{1}{\sqrt{2}} \left( \begin{array}{cccc} a_0^0 + \alpha_0^0 + \beta_0^0 \sqrt{2} & f_1 f_1 + \chi_{11} \sqrt{2} & K_{1A}^+ + K_{1B}^+ & \bar{D}_1 \\ a_0 + \beta_0^0 + \frac{\sqrt{2}}{6} + f_1 f_1 & -a_0^0 + \beta_0^0 + \frac{\sqrt{2}}{6} + \chi_{11} \sqrt{2} & K_{1A}^+ + K_{1B}^+ & \bar{D}_1 \\ K_{1A}^- + K_{1B}^- & -\frac{\sqrt{2}}{2} (f_1 + f_1') + \frac{\chi_{11}}{\sqrt{12}} & D_{++} & -\frac{3}{\sqrt{12}} \chi \sqrt{12} \end{array} \right) $$

$$ \pi = \pi^{a\mu} = \frac{1}{\sqrt{2}} \left( \begin{array}{cccc} \pi_0^0 + \eta_6 + \eta_6 \sqrt{2} & \pi^+ & K^+ & \bar{D}_0 \\ \pi^- & -\pi_0^0 + \eta_6 + \eta_6 \sqrt{2} & K^+ & \bar{D}_0 \\ K^- & K_0 & -\frac{\sqrt{2}}{2} \eta_0 + \frac{\eta_6}{\sqrt{12}} & D_+ \\ D^0 & D^{++} & D_+ & -\frac{3}{\sqrt{12}} \eta_0 \end{array} \right) $$

It should be noted that $K_{1A}$ and $K_{1B}$ are not physical states. The physical states of $K_1(1270)$ and $K_1(1400)$ mesons are related to these states in terms of a mixing angle $\theta_K$ as follows:

$$ K_1(1270) = \sin \theta_K K_{1A} + \cos \theta_K K_{1B} $$
$$ K_1(1400) = \cos \theta_K K_{1A} - \sin \theta_K K_{1B} $$

(16)

The mixing angle $\theta_K$ can be determined by the experimental data. There are various approaches to estimate the mixing angle. The result $35^\circ < |\theta_K| < 55^\circ$ was found in Ref. [59], while two possible solutions with $|\theta_K| \approx 33^\circ$ and $57^\circ$ were obtained in Ref. [60].

3.2 Decay constants

To evaluate the decay constant of the vector mesons in the above mentioned model, the two-point functions are needed. According to AdS/QCD correspondence, two-point functions can be calculated by evaluating the action, Eq. (6) with the classical solution and taking the functional derivative over $V_\mu^0$ twice as:

$$ \langle 0 | J_{V \perp}^{a\mu}(x) J_{V \perp}^{b\nu}(y) | 0 \rangle = -i \frac{\delta^2 S(V\nu)}{\delta V^{a\mu}_\perp(x) \delta V^{b\nu}_\perp(y)} $$

(17)

In the LHS of Eq. (17), we insert one complete set intermediate states with the same quantum numbers as the meson currents, and use the vector mesons decay constants definition as:

$$ \langle 0 | J_{V \perp}^{a\mu}(x) J_{V \perp}^{b\nu}(p, \epsilon) | 0 \rangle = f_V \epsilon^\nu \delta^{a\nu}, $$

(18)

where $f_V$ and $\epsilon$ are the decay constant and the polarization vector for vector meson $V(p, \epsilon)$, respectively. After performing the Fourier transformation, we can find:

$$ i \int d^4 x e^{i p x} \langle 0 | T \{ J_{V \perp}^{a\mu}(x) J_{V \perp}^{b\nu}(0) \} | 0 \rangle = \sum_n \frac{f_V^2}{\sqrt{2} m_V} \Pi^{a\mu b\nu} $$

(19)

where $\Pi^{a\mu b\nu} = (n^{\mu \nu} - p^{\mu} p^{\nu}/p^2)$ is transverse projector. In the RHS of Eq. (17), $S(V\nu)$ contains two vector mesons and can be obtained by inserting the solution for $V_\mu^0$ back into the action. After applying Fourier transformation, in the final result, only the contribution of the surface term at $z = \epsilon$ remains as:

$$ S(V\nu) = \int \frac{d^4 p}{(2\pi)^4} V^{a\nu}_\perp(p) V^{b\nu}_\perp(0) \left( -\partial_z V(p, z) \right) $$

(20)

On the other hand, using Green’s function formalism to solve Eq. (10), the bulk-to-boundary propagator can be written as a sum over vector mesons poles:

$$ Y^{a\mu}(q^2, z) = \sum_n \frac{-g s f_V \psi_{Vn}^a(z)}{q^2 - m_V^2} $$

(21)

where boundary conditions for the $n^{th}$ vector meson’s wave function are $\psi_{Vn}(0) = 0$ and $\partial_z \psi_{Vn}(z_0) = 0$. Moreover the normalization condition is $\int (dz/z) \psi_{Vn}^2 = 1$. Using Eqs. (17)–(21), the decay constant of the $n^{th}$ mode of the
vector meson is obtained as:

\[ f_{Vn}^d = \frac{\partial \psi_{Vn}^a}{gs \, z} \bigg|_{z=\epsilon} \]  

For the axial vector and the pseudoscalar states, the decay constants are defined as:

\[ \langle 0 \mid J_{\alpha}^{A} \mid \phi_{n}^d(p, \epsilon) \rangle = f_{A} \epsilon^{\alpha} \delta_{bb}, \]
\[ \langle 0 \mid J_{\alpha}^{d} \mid \phi_{n}^d(p) \rangle = if_{d} \epsilon^{\alpha} \delta_{dd}. \]

To evaluate the decay constants of the vector mesons and the pseudoscalar ones, the following Green’s functions are used:

\[ \mathcal{A}^a(q^2, z) = \sum_n \frac{-gs f_{Vn} \psi_{Vn}^a(z)}{q^2 - m_{\pi}^2}, \]
\[ \phi^a(q^2, z) = \sum_n \frac{-gs m_{\pi}^2 f_{Vn} \phi_{n}^a(z)}{q^2 - m_{\pi}^2}, \]
\[ \pi^a(q^2, z) = \sum_n \frac{-gs m_{\pi}^2 f_{Vn} \pi_{n}^a(z)}{q^2 - m_{\pi}^2}, \]  

where for the \( \psi_{Vn}^a(z) \) the boundary conditions are similar to \( \psi_{Vn}^a(z) \). For the pseudoscalar meson’s wave functions, \( \phi_{n}^a(\epsilon) = \pi_{n}^a(\epsilon) = 0 \) and \( \partial_{\epsilon} \phi_{n}^a(\epsilon) = \partial_{\epsilon} \pi_{n}^a(\epsilon) = 0 \) are the boundary conditions. The similar method is used to calculate the vector mesons decay constants, the following results can be obtained for the axial vector mesons and the pseudoscalar states decay constant, respectively:

\[ f_{A}^d = \frac{\partial \psi_{A}^a}{gs \, z} \bigg|_{z=\epsilon} \]
\[ f_{n}^a = \frac{\partial \phi_{n}^d}{gs \, z} \bigg|_{z=\epsilon}. \]  

### 4 Strong coupling constants from three and four point functions

In this section, we study the triplet and quadratic vertices including charm, vector, axial vector and pseudoscalar mesons. The corresponding diagrams for triplet vertices are given in Fig. 1. The vertices \( (D^{+}, \bar{D}^{0}, a_+^{1}), (D^{-}, \bar{D}^{0}, b_+^{1}), (D^{+}, \bar{D}^{0}, a_+^{1}), (D^{-}, \bar{D}^{0}, b_+^{1}), (D^{-}, \bar{D}^{0}, a_+^{1}), (D^{+}, \bar{D}^{0}, b_+^{1}) \) can be described with diagrams (a) while diagram (b) is used to explain \( (D^{+}, \bar{D}^{0}, K_{1A}^{0}) \), \( (D^{+}, \bar{D}^{0}, K_{1A}^{0}) \) and \( (D^{+}, \bar{D}^{0}, K_{1A}^{0}) \) can be described with diagrams (a) while diagram (b) is used to explain \( (D^{+}, \bar{D}^{0}, K_{1A}^{0}) \).
In the following two subsections the strong couplings \( \star \) and \( \perp \) for charm, axial vector, vector and pseudoscalar mesons are derived. The following definitions:

\[
\begin{align*}
\langle D(p_1) | A(p_2, \varepsilon') D(p_3) \rangle &= 2 \langle \varepsilon' | p_3 \rangle \delta_{DDA}, \\
\langle D^*(p_1, \varepsilon) | A(p_2, \varepsilon') D(p_3) \rangle &= \left[ \langle \varepsilon' | p_3 \rangle - \langle \varepsilon | p_3 \rangle \langle \varepsilon' | p_1 \rangle \right] \delta_{D^*DA}, \\
\langle D(p_1) | V(p_2, \varepsilon) D(p_3) \rangle &= 2 \langle \varepsilon | p_3 \rangle \delta_{DDV}, \\
\langle D^*(p_1, \varepsilon) | V(p_2, \varepsilon) D(p_3) \rangle &= \left[ \langle \varepsilon' | p_3 \rangle - \langle \varepsilon' | p_1 \rangle \langle \varepsilon | p_3 \rangle \right] \delta_{D^*DV}, \\
\langle D^*(p_1, \varepsilon) | P(p_2) D(p_3, \varepsilon') \rangle &= \left[ \langle \varepsilon' | p_3 \rangle - \langle \varepsilon | p_3 \rangle \langle \varepsilon' | p_1 \rangle \right] \delta_{D^*DP}, \\
\langle D(p_1) | V(p_2, \varepsilon) D(p_3) \rangle &= 2 \langle \varepsilon | p_3 \rangle \delta_{DDV}, \\
\langle D^*(p_1, \varepsilon) | V(p_2, \varepsilon) D(p_3, \varepsilon') \rangle &= \left[ \langle \varepsilon' | p_3 \rangle - \langle \varepsilon' | p_1 \rangle \langle \varepsilon | p_3 \rangle \right] \delta_{D^*DP},
\end{align*}
\]

with \( p_1 = p_2 + p_3 \) are also used for the \( (D, D, A), (D^*, D, A), (D, D, V), (D^*, D, V), (D^*, D^*, V) \) and \( (D_1, D_1, P) \) couplings \cite{130, 131, 132}. Where as emphasized in Eqs. (18) and (23), \( \varepsilon \) denotes the polarization vector of the vector meson \( V \) and \( D^* \) while \( \varepsilon' \) is used for axial vector mesons \( A \) and \( D_1 \).

To obtain these strong coupling constants, we start with the correlation function including the currents of 3 considered particles. In AdS/QCD approach these 3-point functions can be obtained by functionally differentiating the 5-D action with respect to their sources, which are taken to be boundary values of the 5-D fields that have the correct quantum numbers as \cite{133, 134, 135}:

\[
(0)\langle T \left[ J^{aa}_{A} \langle x \rangle J^{bb}_{A} \langle y \rangle J^{bc}_{A} \langle w \rangle \right]|0\rangle = -\frac{\delta^3 S(DDA)}{\delta A^a_{\mu} \delta A^b_{\perp \mu} \delta A^c_{\parallel \mu} (w)} (0)\langle T \left[ J^{aa}_{A} \langle x \rangle J^{bb}_{A} \langle y \rangle J^{bc}_{A} \langle w \rangle \right]|0\rangle = -\frac{\delta^3 S(D^*DA)}{\delta V^a_{\mu \nu} (x) \delta V^b_{\mu \nu} (y) \delta A^c_{\parallel \mu} (w)} (D^*DA vertex),
\]

\[
(0)\langle T \left[ J^{aa}_{V} \langle x \rangle J^{bb}_{V} \langle y \rangle J^{bc}_{V} \langle w \rangle \right]|0\rangle = -\frac{\delta^3 S(D^*DV)}{\delta V^a_{\mu \nu} (x) \delta V^b_{\mu \nu} (y) \delta A^c_{\parallel \mu} (w)} (D^*DV vertex),
\]

\[
(0)\langle T \left[ J^{aa}_{A} \langle x \rangle J^{bb}_{V} \langle y \rangle J^{bc}_{V} \langle w \rangle \right]|0\rangle = -\frac{\delta^3 S(D^*DV)}{\delta A^a_{\mu} \delta V^b_{\mu \nu} (y) \delta A^c_{\parallel \mu} (w)} (D^*DV vertex),
\]

\[
(0)\langle T \left[ J^{aa}_{A} \langle x \rangle J^{bb}_{V} \langle y \rangle J^{bc}_{V} \langle w \rangle \right]|0\rangle = -\frac{\delta^3 S(D^*DV)}{\delta A^a_{\mu} \delta V^b_{\mu \nu} (y) \delta A^c_{\parallel \mu} (w)} (D^*DV vertex),
\]

\[
(0)\langle T \left[ J^{aa}_{A} \langle x \rangle J^{bb}_{V} \langle y \rangle J^{bc}_{V} \langle w \rangle \right]|0\rangle = -\frac{\delta^3 S(D^*DV)}{\delta A^a_{\mu} \delta V^b_{\mu \nu} (y) \delta A^c_{\parallel \mu} (w)} (D^*DV vertex),
\]

where \( S(123) \) is the relevant part of the 5-D action for \( (1, 2, 3) \) vertex. To make a relation between the correlation functions and their corresponding vertexes, we insert three complete sets of intermediate states with the same quantum numbers as the meson currents into the correlation function. In the next step, the matrix elements are defined in Eqs. (18), (23) and (24) are used and the results can be obtained as:

\[
(0)\langle T \left[ J^{aa}_{A} \langle x \rangle J^{bb}_{A} \langle y \rangle J^{bc}_{A} \langle w \rangle \right]|0\rangle = -\frac{\delta^3 S(DDA)}{\delta A_{\mu}^a (x) \delta A_{\mu}^b (y) \delta A_{\perp \mu}^c (w)} (0)\langle T \left[ J^{aa}_{A} \langle x \rangle J^{bb}_{A} \langle y \rangle J^{bc}_{A} \langle w \rangle \right]|0\rangle = -\frac{\delta^3 S(D^*DA)}{\delta V_{\mu \nu}^a (x) \delta V_{\mu \nu}^b (y) \delta A_{\parallel \mu}^c (w)} (D^*DA vertex),
\]

\[
(0)\langle T \left[ J^{aa}_{V} \langle x \rangle J^{bb}_{V} \langle y \rangle J^{bc}_{V} \langle w \rangle \right]|0\rangle = -\frac{\delta^3 S(D^*DV)}{\delta V_{\mu \nu}^a (x) \delta V_{\mu \nu}^b (y) \delta A_{\parallel \mu}^c (w)} (D^*DV vertex),
\]

\[
(0)\langle T \left[ J^{aa}_{A} \langle x \rangle J^{bb}_{V} \langle y \rangle J^{bc}_{V} \langle w \rangle \right]|0\rangle = -\frac{\delta^3 S(D^*DV)}{\delta A^a_{\mu} \delta V^b_{\mu \nu} (y) \delta A_{\parallel \mu}^c (w)} (D^*DV vertex),
\]

\[
(0)\langle T \left[ J^{aa}_{A} \langle x \rangle J^{bb}_{V} \langle y \rangle J^{bc}_{V} \langle w \rangle \right]|0\rangle = -\frac{\delta^3 S(D^*DV)}{\delta A^a_{\mu} \delta V^b_{\mu \nu} (y) \delta A_{\parallel \mu}^c (w)} (D^*DV vertex),
\]
\[ \times \left( \langle 0 | T \left[ J_{AA}^{\mu a}(x) J_{V_\perp}^{\mu b}(0) J_{A}^{\mu c}(w) \right] | 0 \rangle \right), \]

\[ \langle D^* (p_1, e_1) | V (p_2, e_2) D^* (p_3) \rangle \]

\[ = \Omega_{\alpha}(3) \Lambda (D^* V D) \epsilon_{1 \mu}^{\alpha} \epsilon_{2 \nu}^{\mu} \hat{L} \times \left( \langle 0 | T \left[ J_{V_\perp}^{\mu a}(x) J_{V_\perp}^{\mu b}(0) J_{A}^{\mu c}(w) \right] | 0 \rangle \right), \]

\[ \langle D^* (p_1, e_1) | V (p_2, e_2) D^* (p_3, e_3) \rangle \]

\[ = \Lambda (D^* V D^*) \epsilon_{1 \mu}^{\alpha} \epsilon_{2 \nu}^{\mu} \epsilon_{3 \lambda}^{\nu} \hat{L} \times \left( \langle 0 | T \left[ J_{V_\perp}^{\mu a}(x) J_{V_\perp}^{\mu b}(0) J_{V_\perp}^{\mu c}(w) \right] | 0 \rangle \right). \]

\[ \langle D_1 (p_1, e_1') | P (p_2) D_1 (p_3, e_2') \rangle \]

\[ = \Omega_{\alpha}(2) \Lambda (D_1 P D_1) \epsilon_{1 \mu}^{\alpha} \epsilon_{2 \nu}^{\mu} \hat{L} \times \left( \langle 0 | T \left[ J_{A_\perp}^{\mu a}(x) J_{A_\perp}^{\mu b}(0) J_{A}^{\mu c}(w) \right] | 0 \rangle \right), \]

where,

\[ \hat{L} = \int d^4 x \, d^4 w \, e^{i p_{1 \mu} x - i p_{3 \mu} w}, \quad \Omega_{\alpha}(i) = \frac{p_{1 \mu}}{p_{1 \mu}}, \quad \Omega_{\alpha}(i) \Omega_{\beta}(j). \] (35)

Moreover,

\[ \Lambda (O_1 O_2 O_3) = \left( \frac{p_1^2 - m_1^2}{f_{O_1}} \right) \left( \frac{p_2^2 - m_2^2}{f_{O_2}} \right) \left( \frac{p_3^2 - m_3^2}{f_{O_3}} \right). \] (36)

is defined for the \( \langle O_1 | O_2 O_3 \rangle \) matrix element where \( f_{O_i} \) is the decay constant of the \( i \)th particle. Moreover, in the final result, the limit \( (p_1^2, p_2^2, p_3^2) \to (m_1^2, m_2^2, m_3^2) \) is taken for considered vertex.

Now the relevant actions for every 3-point function are needed. For example, to obtain \( S (D_1 A) \), we need to separate two pseudoscalar fields (for \( D \) mesons), and one axial vector field (for \( A \) meson) from three point action or for \( S (D^* A) \), we need a vector field, a pseudoscalar field and one axial vector one. The results are calculated as:

\[ S (D A) = \int d^5 x \left( \frac{f_{abc}}{z^3} \left[ A_{\mu}^{\alpha} (\pi^b \pi^c) - \partial_\mu (\pi^p - \phi^s) A_{\mu}^{\alpha} \pi^s \right] \right), \] (37)

\[ S (D^* A) = \int d^5 x \left( \frac{2 f_{abc}}{2 g_s z} \left[ A_{\mu}^{\alpha} \phi^b V_{\mu b} A_{\mu}^{\alpha} \pi^c \right] + \frac{g_{abc}}{z^3} \left[ V_{\mu b}^{\alpha} A_{\mu}^{\alpha} \pi^c \right] + \frac{g_{abc}}{z^3} \left[ A_{\mu}^{\alpha} V_{\mu b} A_{\mu}^{\alpha} \pi^c \right] \right) \] (38)

\[ S (D D V) = \int d^5 x \left( \frac{f_{abc}}{2 g_s z} \left[ A_{\mu}^{\alpha} \phi^b V_{\mu b}^{\alpha} \phi^c + 2 \partial_\mu \phi^b V_{\mu b}^{\alpha} \phi^c \right] \right. \]

\[ + \left. \frac{g_{abc}}{z^3} \left[ V_{\mu b}^{\alpha} \phi^b V_{\mu b}^{\alpha} \phi^c - \partial_\mu \phi^b V_{\mu b}^{\alpha} \phi^c \right] \right) \]

\[ - \int d^5 x \left( \frac{h_{abc}}{z^3} \left[ (\partial_\mu (\pi^p - \phi^s) V_{\mu b}^{\alpha} \pi^c - \partial_\mu (\pi^p - \phi^s) V_{\mu b}^{\alpha} \pi^c) \right] \right. \]

\[ - \partial_\mu (\pi^p - \phi^s) V_{\mu b}^{\alpha} \pi^c \right), \] (39)

\[ S (D^* D V) = \int d^5 x \left( \frac{h_{abc}}{z^3} \left[ V_{\mu b}^{\alpha} V_{\mu b}^{\alpha} \pi^c + V_{\mu b}^{\alpha} V_{\mu b}^{\alpha} \pi^c \right] \right), \] (40)

\[ S (D^* D^* V) = \int d^5 x \left( \frac{f_{abc}}{z^3} \left[ V_{\mu b}^{\alpha} V_{\mu b}^{\alpha} \pi^c \right] \right), \] (41)

\[ S (D_1 D_1 P) = \int d^5 x \left( \frac{g_{abc}}{z^3} \left[ A_{\mu}^{\alpha} A_{\mu}^{\alpha} \pi^c \right] \right), \] (42)

where,

\[ f_{abc} = -2 i \, \text{Tr} \left( [A_\mu, X_0] [T^b, [T^c, X_0]] \right), \]

\[ g_{abc} = -2 i \, \text{Tr} \left( [A_\mu, X_0] [T^b, [T^c, X_0]] \right), \]

\[ k_{abc} = -2 i \, \text{Tr} \left( [A_\mu, X_0] [T^b, [T^c, X_0]] \right). \] (43)

In all of the actions obtained here, the \( f_{abc} \) terms come from the gauge part and the terms containing \( f_{abc}, g_{abc}, h_{abc} \) and \( k_{abc} \) are from the chiral part of the original action. The values of \( f_{abc} \) are given in [64] and for \( f_{abc}, h_{abc} \) and \( k_{abc} \) the values which are used in numerical part, are collected in Appendix.

It should be noted that in \( S (D_1 A) \), \( S (D^* D V) \) and \( S (D_1 D_1 P) \), the left hand gauge field term; \( (L^{MN} L_{MN}) \) cancels the contribution of the right hand ones; \( (R^{MN} R_{MN}) \); and in the final result, the gauge part has no contribution. Going to Fourier transform space and using the relations [42, 65]:

\[ \phi^0 (p, z) = \phi^0 (p^2, z) \frac{i p_\mu}{p^2} A_{\mu 0}(p), \]

\[ \pi^a (p, z) = \pi^a (p^2, z) \frac{i p_\mu}{p^2} A_{\mu a}(p), \] (45)

\[ A_{\mu 0} (q, z) = A^0 (q^2, z) A_{\mu 0} (q), \]

\[ V_{\mu b} (q, z) = V^b (q^2, z) V_{\mu b} (q), \] (46)

\[ V_{\mu b} (q, z) = - \partial_\mu \pi^b (q^2, z), \]

\[ \partial_\mu \to - i \, (\text{relevant momentum})^\mu, \] (47)

the strong couplings are obtained as:

\[ g_{DD A} = g_s^3 \int_0^{z_0} dz \left( \frac{f_{abc}}{z^3} \left[ \psi_{A}^a (z) \psi^b (z) A_{\mu 0}^c (z) - (\pi^a (z) - \phi^a (z)) \psi_{A}^c (z) \right] \right), \] (48)

\[ g_{DD A} = g_s^3 \int_0^{z_0} dz \left( \frac{f_{abc}}{z^3} \left[ \psi_{A}^a (z) \psi^b (z) \psi_{A}^c (z) \right] \right), \]

\[ + g_s^3 \int_0^{z_0} dz \left( \frac{h_{abc}}{z^3} \left[ \psi_{A}^a (z) \psi_{A}^c (z) \pi^b (z) \right] \right), \] (49)

\[ g_{DD V} = g_s^3 \int_0^{z_0} dz \left( \frac{f_{abc}}{z^3} \left[ \phi^a (z) \psi_{A}^b (z) \phi^c (z) \right] \right), \]

\[ + g_s^3 \int_0^{z_0} dz \left( \frac{h_{abc}}{z^3} \left[ \phi^a (z) \psi_{A}^b (z) \phi^c (z) \right] \right), \]

\[ + \frac{g_s^3}{z^3} \left[ (\phi^a (z) - \pi^a (z)) \psi_{A}^b (z) \phi^c (z) \right] \right). \]
In this subsection we consider \((\psi, D, D, P), (\psi, D^*, D, P), (\psi, D, A)\) and \((\psi, D^*, D, A)\) vertices. To obtain these vertexes couplings, we start with the following 4-point functions:

\[
\langle 0 | T \left[ J^{ia}_{\perp}(x) J^{jb}_{\perp}(y) J^{kc}_{\perp}(w) J^{kd}_{\perp}(u) \right] | 0 \rangle
\]

\[
i \delta^4 s (\psi D D P)
\]

\[
= \frac{1}{\Lambda^2} \left( \frac{1}{\sqrt{3}} \right) \left[ \frac{1}{\Lambda^2} \left( \frac{1}{\sqrt{3}} \right) \right]
\]

(55)

where \(\Lambda = p_1^2 + p_2^2 - p_3^2\).

Note that \(\psi^a(z)\) and \(\psi^b(z)\) are dimensionless but the units of \(\phi^a(z)\) and \(\pi^a(z)\) are GeV\(^{-1}\) (or in the units of \(z\). So, \(g_{D^* D A}\), and \(g_{D^* D V}\) and \(g_{D_P D P}\) are in units GeV\(^{-1}\) and other couplings are dimensionless.

4.2 4-Point functions and charm meson couplings

In this subsection we consider \((\psi, D, D, P), (\psi, D^*, D, P), (\psi, D, A)\) and \((\psi, D^*, D, A)\) vertices. To obtain these vertexes couplings, we start with the following 4-point functions:

\[
\langle 0 | T \left[ J^{ia}_{\perp}(x) J^{jb}_{\perp}(y) J^{kc}_{\perp}(w) J^{kd}_{\perp}(u) \right] | 0 \rangle
\]

\[
i \delta^4 s (\psi D D P)
\]

\[
= \frac{1}{\Lambda^2} \left( \frac{1}{\sqrt{3}} \right) \left[ \frac{1}{\Lambda^2} \left( \frac{1}{\sqrt{3}} \right) \right]
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where \(\Lambda = p_1^2 + p_2^2 - p_3^2\).

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\[
\langle 0 | T \left[ J^{ia}_{\perp}(x) J^{jb}_{\perp}(y) J^{kc}_{\perp}(w) J^{kd}_{\perp}(u) \right] | 0 \rangle
\]

\[
i \delta^4 s (\psi D D P)
\]

\[
= \frac{1}{\Lambda^2} \left( \frac{1}{\sqrt{3}} \right) \left[ \frac{1}{\Lambda^2} \left( \frac{1}{\sqrt{3}} \right) \right]
\]

(55)

where \(\Lambda = p_1^2 + p_2^2 - p_3^2\).

Note that \(\psi^a(z)\) and \(\psi^b(z)\) are dimensionless but the units of \(\phi^a(z)\) and \(\pi^a(z)\) are GeV\(^{-1}\) (or in the units of \(z\). So, \(g_{D^* D A}\), and \(g_{D^* D V}\) and \(g_{D_P D P}\) are in units GeV\(^{-1}\) and other couplings are dimensionless.

4.2 4-Point functions and charm meson couplings

In this subsection we consider \((\psi, D, D, P), (\psi, D^*, D, P), (\psi, D, A)\) and \((\psi, D^*, D, A)\) vertices. To obtain these vertexes couplings, we start with the following 4-point functions:

\[
\langle 0 | T \left[ J^{ia}_{\perp}(x) J^{jb}_{\perp}(y) J^{kc}_{\perp}(w) J^{kd}_{\perp}(u) \right] | 0 \rangle
\]

\[
i \delta^4 s (\psi D D P)
\]

\[
= \frac{1}{\Lambda^2} \left( \frac{1}{\sqrt{3}} \right) \left[ \frac{1}{\Lambda^2} \left( \frac{1}{\sqrt{3}} \right) \right]
\]

(55)

where \(\Lambda = p_1^2 + p_2^2 - p_3^2\).

Note that \(\psi^a(z)\) and \(\psi^b(z)\) are dimensionless but the units of \(\phi^a(z)\) and \(\pi^a(z)\) are GeV\(^{-1}\) (or in the units of \(z\). So, \(g_{D^* D A}\), and \(g_{D^* D V}\) and \(g_{D_P D P}\) are in units GeV\(^{-1}\) and other couplings are dimensionless.
\[ - \int d^5 \frac{f^{abcd}}{2z^3} [V^{a\mu} \partial_\mu \psi (\pi^z \psi^d)] + \frac{f^{abcd}}{2z^3} [V^{a\mu} \partial_\mu \psi (\pi^b \psi^d)] + \int d^5 \left( \frac{f^{abcd}}{6z^3} [V^{a\mu} \partial_\mu (\pi^b \psi^d)] + V^{a\mu} \partial_\mu (\pi^b \psi^d) \right). \] (65)

\[ S(\psi DDA) = \int d^5 x \left( - \frac{g^{abcd}}{2z^3} (A_\mu V^{b\mu} \pi^e \pi^d) + \frac{h^{abcd}}{2z^3} [V^{a\mu} A^{b\mu} \pi^e \pi^d] + \frac{k^{abcd}}{z^3} [V^{a\mu} A^{b\mu} \pi^e \pi^d] \right). \] (66)

\[ S(\psi D^* D P) = \int d^5 x \left( \frac{f^{abcd}}{2z^3} [V^{a\mu} V^{b\mu} \partial_\mu \phi^d + V^{a\mu} \partial_\mu \psi (\pi^e \psi^d)] - \frac{f^{abcd}}{2z^3} [V^{a\mu} V^{b\mu} \partial_\mu \phi^d + V^{a\mu} \partial_\mu \psi (\pi^e \psi^d)] \right). \] (67)

\[ S(\psi D^* D A) = \int d^5 x \left( \frac{f^{abcd}}{2z^3} [V^{a\mu} A^{b\mu} \partial_\mu \phi^d + V^{a\mu} A^{b\mu} \partial_\mu \phi^d] + \frac{f^{abcd}}{2z^3} [V^{a\mu} A^{b\mu} \partial_\mu \phi^d + V^{a\mu} A^{b\mu} \partial_\mu \phi^d] \right). \] (68)

with
\[ f^{abcd} = 2 Tr ([r^a, b^b][r^c, r^d]), \]
\[ g^{abcd} = 2 Tr ([r^a, X_0][b^b, [r^c, [r^d, X_0]]]), \]
\[ h^{abcd} = 2 Tr ([r^a, X_0][b^b, [r^c, [r^d, X_0]]]), \]
\[ k^{abcd} = 2 Tr ([r^a, [b^b, X_0]][r^c, [r^d, X_0]]), \]
\[ f^{abcd} = 2 Tr ([r^a, [b^b, X_0]][r^c, [r^d, X_0]]), \]

where \( f^{abcd} \) can be written in terms of structure constant as \( f^{abcd} = - f^{adb} f^{c_bm} \). The values of \( g^{abcd}, h^{abcd}, k^{abcd} \) and \( f^{abcd} \) used in this paper are presented in Appendix. Using Eqs. (45), (46) and (47), and then by functional derivation according to Eqs. (55)–(58), the final results for \( g_\psi D^* D P \), \( g_\psi D^* D A \), and \( g_\psi D^* D A \) couplings are obtained as:

\[ g_\psi D^* D P = g_\psi D^* D A = g_\psi D^* D A = \frac{4}{\pi} \int_0^\infty dz \left( \frac{f^{abcd}}{2z^3} [\psi^a(z) \psi^b(z) \psi^c(z) \pi^d(z)] - \frac{h^{abcd}}{2z^3} [\psi^a(z) \psi^b(z) \psi^c(z) \pi^d(z)] + \frac{k^{abcd}}{2z^3} [\psi^a(z) \psi^b(z) \psi^c(z) \pi^d(z)] \right). \] (70)

5 Numerical analysis

In this section, our numerical analysis is presented for the strong coupling constants \( g_{DDA} \) and \( g_{D^* D A} \), \( g_{DP D P} \), \( g_{DP D A} \), \( g_{PP D P} \), \( g_{PP D A} \), \( g_{PP D A} \), \( g_{PP D A} \) and \( g_{PP D A} \) couplings are obtained in the first step of numerical analysis we must determine the values of \( z_0, m_q \) and \( \sigma_q \) for \( q = (u, d, s, c) \) using experimental values of the masses.

The values of the experimental masses are utilized to fit \( z_0 \), quark masses and quark condensates are presented in Table 2.

To evaluate \( z_0 \), the observable which does not depend on any other parameter is used. For this purpose, we use the vector mesons with \( M_{V}^2 \approx 0 \). Our choice in this part is the mass of the \( \rho^0 \) meson which gives us \( z_0^{-1} = (323 \pm 1) \) MeV.

After estimating \( z_0 \), we use the masses of the light mesons \( \rho^0, a_1^-, \pi^0 \) and \( \pi^- \) to fit \( (m_u, m_d, \sigma_u, \sigma_d) \). In addition, \( (m_s, \sigma_s) \) are determined using the experimental masses of the strange states \( K^- \) and \( K^{*-} \). Finally, the experimental values of \( m_{D^-} \) and \( m_{D^{*-}} \) are utilized to find fitted values of \( (m_c, \sigma_c) \). Numerically, the best global fit for the parameters \( m_q \) in MeV are obtained as: \( m_u = (8.5 \pm 2.5) \), \( m_d = (12.36 \pm 2.45) \), \( m_s = (195.31 \pm 5.89) \) and \( m_c = (1590.56 \pm 8.42) \). Moreover, for the quark condensates \( \sigma_q \) in MeV\(^3\) the best global fit values are \( \sigma_u = (173.65 \pm 2.21) \), \( \sigma_d = (177.42 \pm 3.15) \), \( \sigma_s = (226.20 \pm 3.72) \) and \( \sigma_c = (310.35 \pm 5.65) \).

Having all of these parameters in hand, we can estimate the pseudoscalar, vector and axial vector meson masses. Table 3 includes our predictions and the experimental values of the mesons which are given taken from [66, 67]. As it can be seen from the masses reported in Table 3, the uncertainty for \( \psi \) and \( \omega \) meson masses are lower than the others for the others. For these two vector mesons, the uncertainties comes from \( z_0 \)
parameter, while for the other mesons, the quark masses and quark condensates are also included in the lower and higher bounds of the masses. The mass of the $K^{1-}_{1A}$ state is estimated using sum rules in [68] as $m_{K^{1-}_{1A}} = (1310 \pm 60)$ MeV while, our analysis predicts $m_{K^{1-}} = (1316.52 \pm 7.50)$ MeV.

Our prediction for the decay constants of some mesons are presented in Table 4. The experimental measurements of the considered decay constants are also given in this table. The measured values for $f_{D^-}$ and $f_{D^+}$ are averages from lattice QCD results, taken from Ref. [66]. The decay constants of $\rho^-$ and $a_1^-$ mesons are taken from [69,70], respectively. The other measured values are taken from experimental data.

It should be noted that in our model, there are no differences between the mass and decay constants of $a_1^-$ and $b_1^-$. In addition, the mass and the decay constants of $K_{1A}^-$ and $K_{1B}^0$ are similar to the values obtained for $K_{1B}^-$ and $K_{1B}^0$, respectively.

Now the wave functions for the studied mesons can be evaluated. The wave functions $\psi_V^0, \psi_A^0, \phi_p^0$ and $\pi_p^0$ as functions of $z$ are plotted in Fig. 3 for $z \leq z_0$. Here, $\rho^-, a_1^-, \pi^-$ are selected from the light mesons while, from the strange mesons we plot the wave functions for $K^{*-}, K_{1A}^-$ and $K^{-}$. Moreover, from the charm mesons group the plots are drawn for $D^{*-}$, $D_{1}^{*-}$ and $D^{*-}$ states and the mesons $(D_{s}^{*-}, D_{s1}^{*-}, D_{s2}^{*-})$ and $(\psi, \chi_3, \eta_c)$ are chosen from the charmed-strange and $q \bar{q}$ states, respectively.

In this figure for the light, strange, charm, charm-strange and $q \bar{q}$ mesons, the plots are displayed with short-dot, short-dash, dot, dash and dash-dot dotted lines, respectively. For $(\pi^-, \rho^-, a_1^-, K^-, K^{*-}, D^-, D^{*-})$ the values of the masses, taken from the experimental data are reported in Table 2 while, for the other ground state mesons, the masses are taken from our predictions given in Table 3.

It should be noted that, since the values of $(m_u, \sigma_u)$ are close to those of $(m_d, \sigma_d)$, and the masses of $D^{0}$ and $D^{*-}$ have almost no differences, the plot of $\psi_{D^{*0}}$ is similar to the $\psi_{D^{*-}}$. Similarities of plots of $\rho^0, \pi^0, D^0, D^{*0}$ and $K^0$ are similar to those obtained for $\rho^-, \pi^-, D^-_1, D^-_2$ and $K^-_{1A}$, respectively. For this reason, in Fig. 3 just one of these two choices are displayed.

Now we move to 3-particle states couplings defined in Eqs. (48)–(53). Here, to evaluate charm meson couplings to the axial vector mesons, the mass of $b_1^-$ is taken from PDG as $m_{b^-} = (1229.50 \pm 3.20)$ MeV [66]. Moreover, for $K_{1B}^-$ the mass is taken from 3PSR prediction as $m_{K^-_{1B}} = (1340 \pm 80)$ MeV [68]. Our predictions for $g_{D_{1B}^0}$, $g_{D_{1B}^{*0}}$, $g_{D_{1B}^{*0}D_{1B}^{*0}}$, $g_{D_{1B}^{*0}D_{1B}^{*0}V}$ and $g_{D_{1B}^{*0}D_{1B}^{*0}F}$ are reported in Tables 5 and 6. Notice that the main uncertainty in the values of the couplings comes from $\sigma_q$, $(q = u, d, s, c)$ and the meson masses. Also, our numeric analyze shows that the more difference between $a_1^-$ and $b_1^-$ in all of the couplings including these states, is in their uncertainy region of masses.
Fig. 3  Plots of the wave functions \(\psi_1^V(z)\), \(\psi_2^V(z)\), \(\phi_1^P(z)\) and \(\pi_1^P(z)\) for \(V = (\rho^-, K^+, D^{*-}, D_s^-, \psi)\), \(A = (\bar{a}_1, K_{1A}^+, D_{1A}^-, D_{1L}, \chi_{1L})\) and \(P = (\pi^-, K^-, D^-, D_s, \eta_c)\) as functions of the radial coordinate \(z\) in the interval \((\varepsilon, z_0)\).

Table 5  Our predictions for the strong couplings of \((D, D, A)\) and \((D^*, D, A)\) vertices

\[
\begin{array}{ccccccc}
(D, D, A) & (D^-, \bar{D}^0, a_1^-) & (D^-, \bar{D}^0, b_1^-) & (D_s^-, \bar{D}^0, K_{1A}^+) & (D_s^-, \bar{D}^0, K_{1B}^-) & (D_s^+ \bar{D}^0, K_{1A}^+) & (D_s^+, \bar{D}^0, K_{1B}^-) \\
\bar{g}_{D^*, D^*, D^*} & 0.32 \pm 0.04 & 0.37 \pm 0.04 & 0.89 \pm 0.26 & 0.80 \pm 0.21 & 0.92 \pm 0.28 & 0.86 \pm 0.17 \\
(D^*, D, A) & (D^-, \bar{D}^0, a_1^-) & (D^-, \bar{D}^0, b_1^-) & (D_s^-, \bar{D}^0, K_{1A}^+) & (D_s^-, \bar{D}^0, K_{1B}^-) & (D_s^+ \bar{D}^0, K_{1A}^+) & (D_s^+, \bar{D}^0, K_{1B}^-) \\
\bar{g}_{D^*, D^*, D^*} & 1.94 \pm 0.63 & 2.08 \pm 0.52 & 2.27 \pm 0.42 & 2.06 \pm 0.35 & 2.47 \pm 0.64 & 2.12 \pm 0.36 \\
\end{array}
\]

Table 6  Couplings for the \((D, D, V), (D^*, D, V), (D^*, D^* V), (D_1, D_1, P)\) vertices

\[
\begin{array}{ccccccc}
(D, D, V) & (D^-, \bar{D}^0, \rho^-) & (D_s^+, \bar{D}^0, K^-) & (D_s^+, \bar{D}^0, \psi) \\
\bar{g}_{D^*, D^*, D^*} & 1.02 \pm 0.16 & 0.80 \pm 0.06 & 3.03 \pm 0.47 \\
(D^*, D, V) & (D^-, \bar{D}^0, \rho^-) & (D_s^+, \bar{D}^0, K^-) & (D_s^+, \bar{D}^0, \psi) \\
\bar{g}_{D^*, D^*, D^*} & 1.29 \pm 0.24 & 1.06 \pm 0.10 & 5.02 \pm 0.66 \\
(D^*, D^*, V) & (D^-, \bar{D}^0, \rho^-) & (D_s^+, \bar{D}^0, K^-) & (D_s^+, \bar{D}^0, \psi) \\
\bar{g}_{D^*, D^*, D^*} & 2.22 \pm 0.27 & 1.78 \pm 0.21 & 5.32 \pm 0.70 \\
(D_1, D_1, P) & (D_s^+, \bar{D}_1^0, \pi^-) & (D_s^+, \bar{D}_1^0, K^-) & (D_s^+, \bar{D}_1^0, \eta_c) \\
\bar{g}_{D_1, D_1, D_1} & 0.52 \pm 0.11 & 0.83 \pm 0.21 & 1.35 \pm 0.29 \\
\end{array}
\]
The strong coupling constants $g_{D^- D^0 K^-}$ and $g_{D_s^- D_s^0 K^-}$ for $K_1 = K_1(1270)$, $K_1(1400)$ as a function of the mixing angle $\theta_K$ as well as the uncertainty regions.

**Table 7** The charm meson strong couplings in various theoretical approaches. Here, $g_{D^+ D_A}$, $g_{D^+ D_V}$ and $g_{D_s D_s P}$ are in the unit GeV$^{-1}$.

| Coupling constant | LCSR [30] | This work | Coupling constant | LCSR [30] | This work |
|-------------------|-----------|-----------|-------------------|-----------|-----------|
| $g_{D^- D^0 a_1^-}$ | 0.38 ± 0.07 | 0.32 ± 0.04 | $g_{D^- D^0 a_1^-}$ | 1.03 ± 0.25 | 1.94 ± 0.63 |
| $g_{D^- D^0 b_1^-}$ | 1.64 ± 0.15 | 0.37 ± 0.04 | $g_{D^- D^0 b_1^-}$ | 1.90 ± 0.72 | 2.08 ± 0.52 |
| $g_{D_s^- D_s^0 K_{1a}}$ | 1.17 ± 0.49 | 0.89 ± 0.26 | $g_{D_s^- D_s^0 K_{1a}}$ | 1.36 ± 0.78 | 2.27 ± 0.42 |
| $g_{D_s^- D_s^0 K_{1b}}$ | 1.51 ± 0.11 | 0.80 ± 0.21 | $g_{D_s^- D_s^0 K_{1b}}$ | 2.48 ± 0.78 | 2.06 ± 0.35 |

| Coupling constant | LCSR [71] | This work | Coupling constant | LCSR [71] | This work |
|-------------------|-----------|-----------|-------------------|-----------|-----------|
| $g_{D^- D_0 \rho^-}$ | 1.31 ± 0.29 | 1.02 ± 0.16 | $g_{D^- D_0 \rho^-}$ | 0.89 ± 0.15 | 1.29 ± 0.24 |
| $g_{D_s^- D_s^0 \rho^-}$ | 1.61 ± 0.32 | 0.80 ± 0.06 | $g_{D_s^- D_s^0 \rho^-}$ | 1.01 ± 0.20 | 1.06 ± 0.10 |

| Coupling constant | 3PSR [63] | This work | Coupling constant | 3PSR [63,72] | This work |
|-------------------|-----------|-----------|-------------------|-----------|-----------|
| $g_{D^0 D_s \bar{D}_s \psi}$ | 5.80 ± 0.90 | 3.03 ± 0.47 | $g_{D^0 D_s \bar{D}_s \psi}$ | 4.00 ± 0.60 | 5.02 ± 0.66 |
| $g_{D_s^0 D_s \bar{D}_s \psi}$ | 6.20 ± 0.90 | 5.32 ± 0.70 | $g_{D_s^0 D_s \bar{D}_s \psi}$ | 0.17 ± 0.04 | 0.52 ± 0.11 |

**Table 8** Our predictions for the couplings of $(\psi, D_{(0,6)}^0, D^+, \pi^-)$, $(\psi, D_{(0,6)}^0, D^+, \pi^0)$ and $(\psi, D_{(*)}^+ K^0, D^-, K^0)$ vertices. The values of $(\psi, D^+, D, P)$ couplings are reported at $q^2 = 0$.

| Coupling constant | $(D, D, P)$ | $(D^0, D^+, \pi^-)$ | $(D^0, D^0, \pi^0)$ | $(D_s^+, D^-, K^0)$ |
|-------------------|--------------|----------------------|----------------------|----------------------|
| $g_{\psi D D P}$ (GeV$^{-1}$) | 1.28 ± 0.50 | 2.07 ± 0.85 | 0.49 ± 0.13 |

| $(D^*, D, P)$ | $(D^{*0}, D^+, \pi^-)$ | $(D^{*0}, D^0, \pi^0)$ | $(D_s^{*+}, D^-, K^0)$ |
|----------------|------------------------|------------------------|------------------------|
| $g_{\psi D^{*} D P}$ (GeV$^{-1}$) | 1.14 ± 0.08 | 1.05 ± 0.10 | 0.84 ± 0.04 |

**Table 9** Couplings for the $(D, D, V)$, $(D^*, D, V)$, $(D^*, D^* V)$ and $(D_1, D_1, P)$ vertices.

| Coupling constant | $(D, D, A)$ | $(D^0, D^+, a_1^-)$ | $(D^0, D^+, b_1^-)$ | $(D_s^+, D^-, K_{1A}^0)$ |
|-------------------|--------------|---------------------|---------------------|------------------------|
| $g_{\psi D D A}$ | 1.27 ± 0.03 | 1.30 ± 0.05 | 0.36 ± 0.10 | 0.38 ± 0.08 |

| $(D^*, D, A)$ | $(D^{*0}, D^+, a_1^-)$ | $(D^{*0}, D^+, b_1^-)$ | $(D_s^{*+}, D^-, K_{1A}^0)$ |
|----------------|------------------------|------------------------|-----------------------|
| $g_{\psi D^{*} D A}$ (GeV$^{-1}$) | 0.12 ± 0.02 | 0.14 ± 0.02 | 0.50 ± 0.12 | 0.58 ± 0.11 |
To evaluate strong couplings for $A = K_1(1270), K_1(1400)$, the following relations are used:

$$g_{D^-_K^0} K_1(1270) = g_{D^-_K^0} K_1^- \sin \theta_K + g_{D^-_K^0} K_1^\pm \cos \theta_K,$$

$$g_{D^-_K^0} K_1(1400) = g_{D^-_K^0} K_1^- \cos \theta_K - g_{D^-_K^0} K_1^\pm \sin \theta_K,$$

$$g_{D^*_K^-} K_1(1270) = r_{1A} g_{D^*_K^-} K_1^- \sin \theta_K + r_{1B} g_{D^*_K^-} K_1^\pm \cos \theta_K,$$

$$g_{D^*_K^-} K_1(1400) = r_{2A} g_{D^*_K^-} K_1^- \cos \theta_K - r_{2B} g_{D^*_K^-} K_1^\pm \sin \theta_K,$$

where,

$$r_{1A} = \frac{m^2_{D^*_K^-} + m^2_{D^0} - m^2_{K_1^-}}{m^2_{D^*_K^-} + m^2_{D^0} - m^2_{K_1^-}(1270)},$$

$$r_{1B} = \frac{m^2_{D^*_K^-} + m^2_{D^0} - m^2_{K_1^-}}{m^2_{D^*_K^-} + m^2_{D^0} - m^2_{K_1^-}(1400)},$$

$$r_{2A} = \frac{m^2_{D^*_K^-} + m^2_{D^0} - m^2_{K_1^-}}{m^2_{D^*_K^-} + m^2_{D^0} - m^2_{K_1^-}(1400)},$$

$$r_{2B} = \frac{m^2_{D^*_K^-} + m^2_{D^0} - m^2_{K_1^-}}{m^2_{D^*_K^-} + m^2_{D^0} - m^2_{K_1^-}(1270)}.$$

The $\theta_K$ dependence of the strong coupling constants $g_{D^*_K^-} D K_1$ and $g_{D^*_K^-} D K_1$ for $K_1(1270)$ and $K_1(1400)$ are displaced in Fig. 4 with solid and dash lines, respectively. The uncertainty regions are also displayed in this figure.

Charm meson couplings to the vector, axial vector and the pseudoscalar mesons are evaluated via different approaches. Table 7, shows the values of the strong couplings calculated

Fig. 5 The strong couplings of $(\psi, D^{*0}, D^0, \pi^-)$, $(\psi, D^{*0}, \bar{D}^0, \pi^0)$ and $(\psi, D^{*+}, D^-, K^0)$ as well as their uncertainty regions on $q^2$

Fig. 6 The $\theta_K$ dependence of the strong coupling constants $g_{\psi D^*_K^- D K_1}$ and $g_{\psi D^*_K^- D K_1}$ for $K_1 = K_1(1270), K_1(1400)$
via LCSR [30,71] and 3PSR [63,72] as well as our predictions.

Now, we consider the strong couplings for quadratic vertices. The values of $g_{\psi D^{(s)}D_P}$ and $g_{\psi D^{(s)}DA}$ are listed in Tables 8 and 9. The reported values of $g_{\psi D^{(s)}D_P}$ at $q^2 = 0$. The strong couplings $g_{\psi D^{(s)}D^+ \pi^-}$ are plotted as functions of $q^2$ in Fig. 5. The values of $q^2_{\text{max}}$ are $(m_{D^+} + m_{\pi^-})^2$, $(m_{D^0} + m_{\pi^0})^2$ and $(m_{D^-} + m_{K^0})^2$ for $(\psi, D^{(s)}, D^0, K^0)$ vertices, respectively.

To evaluate the couplings of $(\psi, D^+_1, D^-, K_0^0(1270))$, $(\psi, D^+_1, D^-, K_0^0(1400))$, $(\psi, D^+_2, D^-, K_0^0(1270))$ and $(\psi, D^+_2, D^-, K_0^0(1400))$ vertices, we use the relations similar to those used in Eqs. (75) and (76). These couplings and the uncertainty regions are plotted as functions of the mixing angle $\theta_K$ in Fig. 6. Our numeric analyze show that the main sources of uncertainties in the four particles vertices are $m_c$ and $\sigma_c$.

6 Conclusion

In summary, the two flavor Hard-Wall holographic model introduced in [53] is extended to four flavors to study the pseudoscalar, vector and axial vector mesons. Our model consists of nine parameters including the Hard-Wall position $z_0$, quark masses $m_q$ and quark condensates $\sigma_q$ with $q = (u, d, s, c)$. These parameters are fitted to the experimental masses of $\rho^0$, $\rho^-$, $\alpha_1^-$, $\pi^0$, $\pi^-$, $K^-$, $K^+$, $D^-$ and $D^+$ mesons. The masses and decay constants of some pseudoscalar, vector and axial vector mesons are evaluated using our model and a comparison is made between our predictions and the experimental data for these observables. Our prediction for the masses of $D_0^{(s)(0)}$, $D_0^{(s)(-)}$, $K_0^0(\psi)$, $D_1^+$ and $\chi_{c1}$ mesons as well as the decay constant of $\pi^-$, $\alpha_1^-$ and $\rho^-$ states, are in good agreement with the experimental data. After analyzing the wave functions, the strong couplings of $(D^{(s)}$, $D$, $\alpha_1$, $\pi^0$, $\pi^-$, $K^-$, $K^+$, $D^-$ and $D^+$) are plotted as functions of the mixing angle $\theta_K$. For these mesons vertices a comparison is also made between our predictions and estimations made by other theoretical approaches. Our numeric analyze show that the main uncertainty in masses, decay constant and the strong couplings comes form the quark condensates $\sigma_q$. The strong couplings evaluated in this paper, can be utilized in estimation the branching fractions of non-leptonic $B$ meson decays.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: There is no experimental data for the couplings are calculated in this study and we compare our result with the previous theoretical approach predictions.]

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Appendix A: values for $g^{abc}$, $h^{abc}$, $l^{abc}$, $k^{abc}$, $h^{abcd}$, $k^{abcd}$ and $l^{abcd}$

In this appendix, we present the nonzero values for $g^{abc}$, $h^{abc}$, $l^{abc}$, $k^{abcd}$, $h^{abcd}$, $k^{abcd}$ and $l^{abcd}$. The values results of the

| Table 10 | The values of $g^{abc}$, $h^{abc}$, $l^{abc}$ and $k^{abc}$ which are used in numerical analyze |
| --- | --- | --- | --- |
| $(a, b, c)$ | $g^{abc}$ | $h^{abc}$ | $l^{abc}$ | $k^{abc}$ |
| (2, 9, 11) | $\frac{1}{2}(u_d + v_d)(u_d + v_c)$ | $\frac{1}{2}(v_u - v_d)(u_u + v_c)$ | $\frac{1}{2}(v_d - u_d)(u_u + v_c)$ | $\frac{1}{2}(v_d - u_u)(u_d + v_c)$ |
| (4, 9, 14) | $\frac{1}{2}(u_u + v_u)(u_u + v_v)$ | $\frac{1}{2}(u_u - u_d)(u_u + v_c)$ | $\frac{1}{2}(v_u - v_d)(u_u + v_c)$ | $\frac{1}{2}(v_u - v_u)(u_u + v_c)$ |
| (6, 11, 14) | $-\frac{1}{2}(u_d + v_d)(v_u + v_v)$ | $-\frac{1}{2}(v_u - u_d)(u_d + v_v)$ | $\frac{1}{2}(v_d - v_u)(u_d + v_v)$ | $\frac{1}{2}(v_d - v_u)(u_d + v_v)$ |
| (9, 10, 15) | $\frac{\sqrt{5}}{6}(v_u + v_v)(u_u + 3v_v)$ | $\frac{\sqrt{5}}{6}(u_u - v_v)(u_u - 3v_v)$ | $-\frac{\sqrt{5}}{6}(v_u + v_v)(u_u + 3v_v)$ | $-\frac{\sqrt{5}}{6}(v_u + v_v)(u_u + 3v_v)$ |

| Table 11 | The values of $g^{abcd}$, $h^{abcd}$, $l^{abcd}$ and $k^{abcd}$ which are used in numerical analyze |
| --- | --- | --- | --- |
| $(a, b, c, d)$ | $-i g^{abcd}$ | $-i h^{abcd}$ | $-i l^{abcd}$ | $-i k^{abcd}$ |
| (9, 15, 2, 11) | $\frac{\sqrt{5}}{6}(u_u + v_v)(v_d + v_v)$ | $\frac{\sqrt{5}}{6}(u_u - v_v)(v_d + v_v)$ | $-\frac{\sqrt{5}}{6}(v_u + v_v)(u_u + 3v_v)$ | $-\frac{\sqrt{5}}{6}(v_u + v_v)(u_u + 3v_v)$ |
| (9, 15, 3, 10) | $\frac{\sqrt{5}}{6}(v_u + v_v)^2$ | $\frac{\sqrt{5}}{6}(u_u - v_v)^2$ | $\frac{\sqrt{5}}{6}(v_u + v_v)(u_u + 3v_v)$ | $\frac{\sqrt{5}}{6}(v_u + v_v)(u_u + 3v_v)$ |
| (13, 15, 7, 11) | $\frac{\sqrt{5}}{6}(v_u + v_v)(v_d + v_v)$ | $\frac{\sqrt{5}}{6}(v_u + v_v)(v_d + v_v)$ | $-\frac{\sqrt{5}}{6}(v_u + v_v)(u_u + 3v_v)$ | $-\frac{\sqrt{5}}{6}(v_u + v_v)(u_u + 3v_v)$ |
factors appeared in 3-point and 4-point functions which are used in numerical analyze are given in Tables (10) and (11), respectively.

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