Fracture analysis of single edge cracked functionally graded material plate under various loading conditions by extended finite element method

Achchhe Lal¹, B.M Sutaria² and Kundan Mishra³

¹Assistant Professor, Department of Mechanical Engineering, SVNIT, Surat, India
²Associate Professor, Department of Mechanical Engineering, SVNIT, Surat, India
³Research Scholars, Department of Mechanical Engineering, SVNIT, Surat., India

E-mail: lalachchhe@yahoo.co.in

Abstract: Functionally graded material (FGM) is the perfect blend of two (or more) material phases of different materials and functional performance that vary in definite direction within the geometry. It was first introduced by group of scientist in 1984 for shielding and thermal barrier. In the present paper single edge cracked plate under different loadings are used to observe the behavior of FGM plate under different loading conditions by using extended finite element method (XFEM). The basic formulation is carried out in MATLAB environment by XFEM using interaction integral (M-integral) through partition of unity method. In the present study the main focus is to utilize XFEM to find mixed mode stress intensity factor (MMSIF) of edge cracked FGM plate under the action of uniform tensile, shear, combined (tensile and shear) and point load.

1. Introduction

Functionally graded material (FGM) is a new class of composites, where material properties of constituent materials changes smoothly with desired gradation. It was first used for the thermal resisting materials and now it is being utilized for various engineering applications like furnace linear, artificial bones and other medical implants etc. In structural design FGM provides better resistance towards crack growth.

In spite of availability of advanced technology and manufacturing techniques, structural components contain various defects like cracks, which become very severe in various applications where, component with crack under complex loadings. So, it is essential to analyse the behavior of FGM plate with crack under various loadings. Many researchers are working and finding different FGM for better structural integrity. In this direction Kim and Paulino [1] evaluated MMSIF cracked FGM plate where variation of material is the function of geometry. In the present work MMSIF is calculated by utilizing various techniques incorporated with FEM and accuracy of present results are compared with theoretical and experimental results. Man et al. [2] found out the mode I and mixed mode crack solution of stress intensity factor (SIF) for the two-dimensional FGM with crack by weight function method, where it is verified that proposed method is valid for the derivation of weight function in FGMs. In conventional FEM due to limitation in updating of mesh, many researchers have started working on XFEM, where fracture problem is solved by utilising enrichment functions without using re-meshing during evaluation of crack. Lal and Palekar [3] evaluated the stochastic MMSIF of cracked composite plate with random material properties. The proposed work is done by using XFEM.
combined different techniques and the present model is seen to be very accurate for uncertainty analysis. Mohammadi [4] studied fracture mechanics by XFEM, where singular stress field is reproduced by using orthotropic crack tip enrichments. In the present work it is verified that orthotropic XFEM needs lesser DOFs as compared to conventional FEM. Mohammadi [5] presented fracture analysis by XFEM, where MMSIF of cracked FGM plate by using interaction integral through partition of unity method is discussed. Asadpoure et al [6] proposed XFEM with crack-tip displacement fields to evaluate MMSIF of cracked orthotropic plate. In the present study MMSIF is calculated by using interaction integral (M-integral) and the accuracy of the result is compared with analytical and other numerical methods. Kim and Paulino [7] presented interaction Integral (M-integral) method to calculate MMSIF of orthotic FGM plate with straight or curved crack under loading. The proposed work is accurately compared with available analytical and numerical solutions. Ebrahimi et al [8] investigated crack in orthotropic composite plate under uni-axial and bi-axial load by XFEM where re-meshing is not required while progress of crack. In the present study it is observed that crack tip domain size has not any effect on SIF when it reaches one third of crack length. Lal et al [9] investigated stochastic fracture analysis cracked composite beam under tensile, shear and combined loadings. The sensitivity of individual parameter on MMSIF is also investigated and here it is also confirmed that SIF is get reduced when orthotropic angle is above 400. So it is essential to keep orthotropic angle more than 400 for safety point of view. Khatri and Lal [10] presented stochastic fracture analysis of isotropic plate with emanating crack from hole under in plate loadings by utilizing XFEM. It is observed that the SIF for shorter crack length is also very high due to high concentration near crack tip.

In the present paper MMSIF of edge cracked FGM plate under the action of point load, uniformly distributed tensile, shear, combined is evaluated for different fracture parameter like crack length and crack angle. XFEM based fracture analysis is utilized in the present study and this approach is also validated with other numerical technique in the literature.

2. Problem formulation

Conventional FEM has limitations to handle fracture problem, where it is necessary to update mesh. To overcome this difficulty XFEM provides better approach to solve fracture problems. The displacement vector in XFEM framework is given below.

\[ u^e(x) = \sum_{i=1}^{n} N_i(x) u_i + \sum_{i=1}^{n} N_i(x) H_i(x) a_i + \sum_{i=1}^{n} N_i(x) \sum_{\alpha=1}^{\alpha_1} \Phi_i^\alpha(x) b_i^{\alpha 1} + \sum_{i=1}^{n} N_i(x) \sum_{\alpha=1}^{\alpha_2} \Phi_i^\alpha(x) b_i^{\alpha 2} \]  (1)

Where \( u_i, a_i, b_i^{\alpha 1} \) and \( b_i^{\alpha 2} \) are the conventional degrees of freedom (dof), dof for enrichment function, crack tip and crack face asymptotic functions. Figure 1 shows a body of area \( \Omega \) and its outer boundary \( \Gamma \) with a crack. The body is having volume \( \tilde{b} \) body loads \( b \) and the surface load at the boundary \( \Gamma_\tau \). Boundary surface is represented as \( \Gamma = \Gamma_u + \Gamma_t + \Gamma_c \), where \( u, t \) and \( c \) are displacement, traction and discontinuity. It is assumed that surface with crack is traction free.

![Figure 1. An arbitrary body with edge crack with displacement \( \bar{u} \) and traction \( \bar{t} \)](image-url)
dofs = size(n) + size(n_c) + size(n_t) \text{ Where, } n_c = n_t + n_dofs \quad (2)

Here, \( n_c, n_t \) and \( n_dofs \) are the nodes for FEM mesh, crack face and crack tip respectively. \( H(x) \) denotes the Heaviside function for the enrichment of crack which values are +1 or -1. Crack tips asymptotic functions are represented by \( \Phi_a^1 \) and \( \Phi_a^2 \). The displacement function or asymptotic function near crack tip is represented as:

\[
\Phi_a^1 = \sqrt{r} \sin \left(\frac{\theta}{2}\right), \quad \Phi_a^2 = \sqrt{r} \cos \left(\frac{\theta}{2}\right) \quad (3)
\]

\[
\Phi_u^3 = \sqrt{r} \sin \theta \cos \left(\frac{\theta}{2}\right), \quad \Phi_u^4 = \sqrt{r} \sin \theta \cos \left(\frac{\theta}{2}\right) \quad (4)
\]

The relationship between SIF and J-integral in 2D can be represented as,

\[
J = \frac{K^2_1 + K^2_2}{E_{eff}} \quad \text{Where, } E_{eff} = \begin{cases} E & \text{for plane stress} \\ \frac{E}{1-\nu^2} & \text{for plane strain} \end{cases}
\]

(5)

The integral for the body with crack is given in equation (6), where \( \Gamma \) is the boundary surface, \( W \) is the strain energy, \( \delta_{ij} \) is Kronecker delta, \( n_j \) is the outward unit normal to \( \Gamma \) for \( j^{th} \) component, \( \sigma_{ij} \) is stress tensor and \( u_i \) is displacement vector. A crack body is represented as two states and the sum of two states is represented by equation (7).

\[
J = \int_{\Gamma} \left( W \delta_{ij} - \sigma_{ij} \frac{\partial u_i}{\partial x_j} \right) n_j \, d\Gamma \quad (6)
\]

\[
J^{(1,2)} = \int_{\Gamma} \left[ \frac{1}{2} \left( \sigma^{(0)} + \sigma^{(2)} \right) \varepsilon^{(0)} + \varepsilon^{(2)} \delta_{ij} - \left( \sigma^{(0)} + \sigma^{(2)} \right) \frac{\partial (u_i^{(0)} + u_i^{(2)})}{\partial x_j} \right] n_j \, d\Gamma \quad (7)
\]

On further solving, we get,

\[
J^{(1,2)} = J^{(1)} + J^{(2)} + \frac{2}{E_{eff}} (K^{(1)}_I K^{(2)}_I + K^{(1)}_II K^{(2)}_II) \quad (8)
\]

Now, comparing the Equations. (5) and (8) we get,

\[
I^{(1,2)} = \frac{2}{E_{eff}} (K^{(1)}_I K^{(2)}_I + K^{(1)}_II K^{(2)}_II) \quad (9)
\]

The SIF in XFEM for two states \( K^{(1)}_I \) and \( K^{(1)}_II \) are evaluating by putting \( K^{(2)}_I = 1 \) and \( K^{(2)}_II = 0 \) and \( K^{(2)}_I = 0 \) and \( K^{(2)}_II = 1 \). in equation (9), we get,

\[
K^{(1)}_I = \frac{M^{(1,\text{Mode I})}_{eff}}{2}, \quad K^{(1)}_II = \frac{M^{(1,\text{Mode II})}_{eff}}{2} \quad (10)
\]

Where, \( I^{(1,\text{Mode I})} \) and \( I^{(1,\text{Mode II})} \) are interaction integrals.

The following normalized mean MMSIF for first and second mode SIF under tensile loading are represented as

\[
K_i = \bar{K}_i / \sigma \sqrt{a} \quad \text{and} \quad K_{ii} = \bar{K}_{ii} / \sigma \sqrt{a} \quad (11)
\]
3. Result and Discussion

FGM plate with width W and length L = 4W under different loadings is shown. Material property of FGM plate is the function of distance along width. The material properties utilized for this present study are $E_1= 1$, $\nu=0.3$ and unit uniform tensile, shear and point loads are considered. Figure 2 (a-d) shows the geometry of FGM plate with (a) single edge crack plate under uniaxial uniform tensile and shear stress, (b) crack enrichments, (c) deformation contour, and (d) stress contour.

$$E(x) = E_1 e^{\beta x}, \beta = \ln(2 / \beta), \quad G_{12} = \frac{0.5 E}{1 + \nu}$$  \hfill (12)

The current approach is well validated from the literature, which is shown in figure 3. Figure 3(a) shows the FGM plate with uniform tensile load and Figure 3(b) shows the normalized $K_I$ of edge cracked FGM plate under uniform tensile load.

![Figure 2](image-url)

**Figure 2.** Geometry of FGMs plate with (a) edge crack under combined (Tensile and Shear) (b) crack enrichments, (c) deformation contour and (d) stress contour

Table 1 shows the effect of crack length on edge cracked FGM plate with various loadings. The present study is carried out for Crack angle ($\alpha$) = 45$^\circ$ and $E_2/E_1=0.45$. It is observed that as crack length increases from 0.4 to 0.6 MMSIF $K_I$ and $K_{II}$ also increase for all loading conditions. For particular crack length maximum MMSIF $K_I$ and $K_{II}$ are observed for combined loading and minimum for uniform tensile loading. The percentage change in $K_I$ when crack length varies from 0.4 to 0.6 for uniform tensile, shear, combined and point loadshare 77, 41, 43 and 75 respectively.
Table 1. Effect of crack length on the normalized mean $K_I$ and $K_{II}$ of edge cracked FGM plate with different loadings.

| $a$  | SIF   | Tensile (UDL) | Shear   | Combined | Point Load |
|------|-------|---------------|---------|----------|------------|
| 0.4  | $K_I$ | 2.1490        | 32.4447 | 34.5937  | 2.3078     |
|      | $K_{II}$ | 0.8126      | 8.2275  | 9.0401   | 0.8593     |
| 0.5  | $K_I$ | 2.7843        | 37.5495 | 40.3338  | 2.9681     |
|      | $K_{II}$ | 1.0191      | 8.8253  | 9.8445   | 1.0695     |
| 0.6  | $K_I$ | 3.8150        | 45.8914 | 49.7064  | 4.0403     |
|      | $K_{II}$ | 1.2883      | 9.5875  | 10.8758  | 1.3435     |

Table 2. Effect of crack angle on the normalized mean of $K_I$ and $K_{II}$ of edge cracked FGM plate under various loadings.

| $a$ | SIF   | Tensile (UDL) | Shear   | Combined | Point Load |
|-----|-------|---------------|---------|----------|------------|
| 0   | $K_I$ | 2.9506        | 38.7081 | 41.6587  | 3.1481     |
|     | $K_{II}$ | 0.0005       | 1.8327  | 1.8332   | 0.2164     |
| 15  | $K_I$ | 2.8937        | 38.6665 | 41.5601  | 3.0882     |
|     | $K_{II}$ | 0.3554      | 2.0189  | 2.3743   | 0.3744     |
| 25  | $K_I$ | 2.7932        | 37.9929 | 40.7861  | 2.9826     |
|     | $K_{II}$ | 0.5740      | 4.4792  | 5.0532   | 0.6048     |
| 45  | $K_I$ | 2.4586        | 35.1594 | 37.6180  | 2.6306     |
|     | $K_{II}$ | 0.9259       | 8.7667  | 9.6926   | 0.9757     |
Table 2 shows the effect of crack angle on edge cracked FGM plate with various loadings. The present study is carried out for Crack length \( a = 0.45 \) and \( E_2/E_1=0.45 \). It is observed that as crack angle increases from \( 0^\circ \) to \( 45^\circ \) \( K_I \) decreases whereas \( K_{II} \) increases for all loading conditions. For particular crack angle maximum MMSIF \( K_I \) and \( K_{II} \) are observed for combined loading and minimum for uniform tensile loading. The percentage change in \( K_I \) when crack angle varies from 0 to 45 for uniform tensile, shear, combined and point loads are 16, 9.16, 9.69 and 16.43 respectively.

4. Conclusions
In the present study MMSIF of edge crack FGM plate with different loading conditions are presented. The mathematical formulation is carried out by utilizing XFEM. The conclusions from the present study are.

- MMSIF \( K_I \) and \( K_{II} \) are calculated for different crack length and crack angle and it is observed that effect of combined loading is maximum with respect to other loading conditions. This is due to combined effect of opening and shearing mode.
- It is observed that as crack length increases, MMSIF \( K_I \) and \( K_{II} \) also increase.
- Crack length is one of the most critical parameter for the fracture behavior of materials.
- On comparing uniform tensile and central point load, central point load is more critical because whole concentrated load is at center of plate, which is near to the crack whereas in uniform tensile same amount of load is distributed throughout the width of plate.

References
[1] Kim J H and Paulino G H 2002 *Int. J. Numer. Methods Eng.* 53 1903–1935.
[2] Man S, Huaping W, Long L and Chai C 2014 *Int. J. Mech. Mater Des.* 10 65–77.
[3] Lal A and Palekar S P 2015 *Int. J. Mech Mater Des.* 13-2 195-228.
[4] Bayesteh H and Mohammadi S 2013 *Journal of Composites Part B* 44 8-25.
[5] Mohammadi S 2012 *John Wiley and Sons Ltd. Publication*
[6] Asadpoure A, Mohammadi S and Vafai A 2006 *Thin walled Struc.* 44 1031 – 1038.
[7] Kim J H and Paulino G H 2003 *International journal of solids and structures* 40 3967-4001.
[8] Ebrahimi S H, Asadpoure A and Mohammadi S 2007 *Int. J. Civil Eng.* 6-3 198-207.
[9] Lal A, Mulani S B and Kapania R K 2017 *journal of applied mechanics* 9-4 1750061.
[10] Khatri K and Lal A 2018 *Mechanics of Advanced Materials and Structures* 25-9 732-755.