LARGE N QCD AND q-DEFORMED QUANTUM FIELD THEORIES

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A construction of master field describing multicolour QCD is presented. The master fields for large N matrix theories satisfy to standard equations of relativistic field theory but fields are quantized according q-deformed commutation relations with q = 0. These commutation relations are realized in the Boltzmannian Fock space. The master field for gauge theory does not take values in a finite-dimensional Lie algebra, however, there is a non-Abelian gauge symmetry and BRST-invariance.

1 Introduction

The large N limit in QCD where N is the number of colours enables us to understand qualitatively certain striking phenomenological features of strong interactions. To perform an analytical investigation one needs to compute the sum of all planar diagrams. Summation of planar diagrams has been performed only in low dimensional space-time.

It was suggested that there exists a master field which dominates the large N limit. There was an old problem in quantum field theory how to construct the master field for the large N limit in QCD. This problem has been discussed in many works. The problem has been reconsidered by using methods of non-commutative probability theory. Gopakumar and Gross and Douglas have described the master field using a knowledge of all correlation functions of a model.

The problem of construction of the master field has been solved in. It was shown that the master field satisfies to standard equations of relativistic field theory but it is quantized according to q-deformed relations

\[ a(k)a^*(k') - qa^*(k')a(k) = \delta(k - k'), \]  

(1)

with q = 0. Relations (1) are related with quantum groups. About attempts to construct a field theory based on see and refs therein. Operators a(k), a^*(k') for q = 0 have a realization in the free (Boltzmannian) Fock space. Quantum field theory in Boltzmannian Fock space has been considered in papers. Some special form of this theory realizes the master field for a subset of planar diagrams, for the so called half-planar (HP) diagrams and gives an analytical summations of HP diagrams. HP diagrams are closely related with so called parquet approximation.
2 Free Master Field

We consider the Minkowski space-time. We use a notation $M^{(in)}$ for the free Minkowski matrix field. For two-point Wightman functions one has

$$<0|M^{(in)}_{ij}(x)M^{(in)}_{pq}(y)|0> = \delta_{iq}\delta_{jp}D^{\pm}(x-y). \quad (2)$$

A free scalar Boltzmannian field $\phi^{(in)}(x)$ is given by

$$\phi^{(in)}(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{\sqrt{2\omega(k)}} (a^*(k)e^{ikx} + a(k)e^{-ikx}), \quad \omega(k) = \sqrt{k^2 + m^2}. \quad (3)$$

It is an operator in the Boltzmannian Fock space with relations

$$a(k)a^*(k') = \delta^{(3)}(k-k') \quad (4)$$

and vacuum $|\Omega_0\rangle$, $a(k)|\Omega_0\rangle = 0$. An $n$-particle state is created from the vacuum $|\Omega_0\rangle = 1$ by the usual formula $|k_1, ..., k_n\rangle = a^*(k_1)...a^*(k_n)|\Omega_0\rangle$. There is no symmetric under permutation of $k_i$.

The following basic relation takes place

$$\lim_{N\to\infty} \frac{1}{N^{1+\frac{d}{2}}} <0|\text{tr}((M^{(in)}(y_1))^{p_1}...(M^{(in)}(y_r))^{p_r})|0> \quad (5)$$

$$= (\Omega_0)(\phi^{(in)}(y_1))^{p_1}...(\phi^{(in)}(x_r))^{p_r}|\Omega_0\rangle, \quad k = p_1 + ... + p_r. \quad (6)$$

3 Master Field for Interacting Matrix Scalar Field

To construct the master field for interacting quantum field theory we have to work in Minkowski space-time and use the Yang-Feldman formalism. Let us consider a model of an Hermitian scalar matrix field $M(x) = (M_{ij}(x))$, $i, j = 1, ..., N$ in the 4-dimensional Minkowski space-time with the field equations

$$(\Box + m^2)M(x) = J(x), \quad J(x) = -\frac{g}{N}M^3(x), \quad (6)$$

$g$ is the coupling constant. One has the Yang-Feldman equation

$$M(x) = M^{(in)}(x) + \int D^{ret}(x-y)J(y)dy \quad (7)$$

where $D^{ret}(x)$ is the retarded Green function for the Klein-Gordon equation and $M^{(in)}(x)$ is a free Bose field. The $U(N)$-invariant Wightman functions are

$$W(x_1, ..., x_k) = \frac{1}{N^{1+\frac{d}{2}}} <0|\text{tr}(M(x_1)...)M(x_k))|0> \quad (8)$$
where $|0\rangle$ is the Fock vacuum for the free field $M^{(in)}(x)$.

We define the master as a solution of the equation
\[
\phi(x) = \phi^{(in)}(x) + \int D^{ret}(x-y) j(y)dy, \quad j(x) = -g\phi^3(x).
\] (9)

The master field $\phi(x)$ does not have matrix indexes.

At every order of perturbation theory in the coupling constant one has the following relation
\[
\lim_{N\to\infty} \frac{1}{N^{1+\frac{D}{2}}} <0| \text{tr}(M(x_1)...M(x_k))|0> = (\Omega_0|\phi(x_1)...\phi(x_k)|\Omega_0) \]
(10)
where the field $M(x)$ is defined by (7) and $\phi(x)$ is defined by (9).

4 Gauge field

The $U(N)$-invariant Wightman functions corresponding to $SU(N)$ gauge theory with Lagrangian
\[
L = \text{tr} \left\{ -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2\alpha} (\partial_\mu A_\mu)^2 + \bar{c} \partial_\mu \nabla_\mu c \right\}
\] (11)
are defined as
\[
W(x_1,...,x_k) = \frac{1}{N^{1+\frac{D}{2}}} <0| \text{tr}(\psi_{i_1}(x_1)...)\psi_{i_k}(x_k))|0>,
\] (12)
where $\psi_i = (A_\mu, c, \bar{c})$, $A_\mu$ is the gauge field for the $SU(N)$ group, $c$ and $\bar{c}$ are the Faddeev-Popov ghost fields; $\alpha$ is a gauge fixing parameter.

According to the limit of functions (12) when $N \to \infty$ can be expressed in terms of the master fields $B_\mu(x)$, $\eta(x)$ and $\bar{\eta}(x)$ corresponding to $A_\mu(x)$, $c(x)$ and $\bar{c}(x)$, respectively. The master fields satisfy to equations
\[
D_\mu F_{\mu\nu} + \frac{1}{\alpha} \partial_\nu \partial_\mu B_\mu + g \partial_\nu \bar{\eta} + g \eta \partial_\nu \eta = 0,
\]
\[
\partial_\mu (D_\mu \eta) = 0, \quad D_\mu (\partial_\mu \bar{\eta}) = 0
\] (13)
where $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + g[B_\mu, B_\nu]$, $D_\mu \eta = \partial_\mu \eta + g[B_\mu, \eta]$. These equations have the form of the Yang-Mills equations, however, the master fields $B_\mu$, $\eta$, $\bar{\eta}$ do not have matrix indexes and they do not take values in a finite dimensional Lie algebra. The gauge group for the field $B_\mu$ is an infinite dimensional group of unitary operators in the Boltzmannian Fock space. Eqs. (13) are invariant under the following BRST transformations
\[
\delta B_\mu = D_\mu \eta \epsilon, \quad \delta \eta = \eta^2 \epsilon, \quad \delta \bar{\eta} = -\frac{1}{\alpha} \partial_\mu B_\mu \epsilon,
\]
where $\epsilon$ is a constant infinitesimal parameter, $\eta \epsilon + \epsilon \eta = 0$, $\bar{\eta} \epsilon + \epsilon \bar{\eta} = 0$. 
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