Nonlinear Static Analysis of Cable Roof Structures with Unified Kinematic Description

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Abstract A finite element analysis technology applicable to the prediction of the static nonlinear response of cable roof structure is presented. The unified kinematic description is employed to formulate the present cable element and different strain definitions such as Green-Lagrange strain, Biot strain and Hencky strain can be adopted. The Newton–Raphson method is used to trace the nonlinear load-displacement path. In the iteration process, the compressive stress of a cable element is not allowed. For the verification of the present cable element, four numerical examples are tackled. Finally, numerical results obtained by using the present cable element are provided as new benchmark test results for cable structures under static loads.

Keywords: Cable Roof Structure, Unified Kinematic Description, Nonlinear Static Analysis, Green-Lagrange Strain, Tangent Stiffness

1. INTRODUCTION

Cable roof structure is one of the important structural systems which can provide a large column-free architectural space. It can basically resist tensile force only and therefore usually needs a pre-stressing force to preserve its form and structural stability. For this reason, it sometimes produces complex structural behaviours against external load.

For the analysis of cable structure, the elastic catenary cable was first presented by O’Brien and Francis (1960). In their study, an iterative method was mainly used to solve a two-dimensional cable structure subjected to static concentrated loads. Later, the elastic catenary element was presented by Peyrot and Goulois (1978, 1979). They used O’Brien’s expression to obtain the flexibility matrix. Then tangent stiffness matrix was produced by taking the inverse of the flexibility matrix and used in the direct stiffness method.

The finite element (FE) analysis technique has been also used for cable analysis. Three types of element have been mainly adopted: 1) straight element, 2) curved element, and 3) curved element with rotational degrees of freedom. Baron and Venkatesan (1971) developed the stiffness matrix of a three-dimensional two-node straight truss element including the effect of stress stiffening. They used the direct stiffness method and a nonlinear iterative scheme to solve cable structures subjected to static concentrated forces. Similarly, other two-node truss elements were developed by Webster (1975) and Broughton et al. (1994). Argyris and Scharpf (1972) developed a FE computer procedure based on the displacement method for the analysis of large pre-stressed networks. The effect of the geometric nonlinear contribution was first reviewed in their study. Gambhir and Batchelor (1977) developed a curved element with rotational degree of freedom for shallow cable nets. The large displacement formulation was used to evaluate the static and dynamic response of three-dimensional cable nets. They described that a cubic displacement field is sufficient for the prediction of the first frequency of shallow nets. However, for globally deep networks, the accuracy can be increased by employing a quintic order of displacement field for the normal component of displacement. Desai et al. (1988) formulated the stiffness matrix of a three-node cable element using a parabolic assumed function. A comparison between two-node element and three-node element was presented. They showed that the three-node cable element is more accurate in a static analysis. Mitsugi (1994) formulated a stiffness matrix for the hyper-cable element which is for a cable connected to intermediate pulleys along its length. Kwan (1998) reviewed the existing technique for static analysis of cable structures and explained nonlinear behaviour of cable networks using linked structure. He also provided the computation time for a cable structure and compared with Lewis’ result (1989). Recently, Thai and Kim (2011) developed a spatial two-node catenary cable element for the nonlinear analysis of cable structures subjected to static and dynamic loadings. The tangent stiffness matrix and internal force vector of the element are derived explicitly based on the exact analytical expressions of elastic catenary. For static analysis, the Newton–Raphson method is adopted for solving the
nonlinear equation of motion. Salehi Ahmad Abad et al. (2013) proposed two elements for three-dimensional FE analysis of cable structures. The first one is the continuous catenary cable (CCC) element which is the extension of the classic catenary cable element. The second element, discrete catenary cable (DCC) element, is introduced by transforming the continuous equations of the CCC element into discrete formulation, giving the capability of dividing the cable into several straight elements with axial behavior.

In this study, the cable element is proposed on the basis of the unified kinematic description. The linear shape function is introduced to derive the tangent stiffness matrix of two-node cable element. Strain definitions such as Green-Lagrange strain, Biot strain and Hencky strain can be incorporated to produce the tangent stiffness matrices in the present FE formulation. The Newton–Raphson method is adopted for tracing nonlinear load-displacement path. In particular, the compressive stress of a cable element is not allowed by forcing its values to zero. For the verification of the implementation of the present cable element, four examples, which have been studied by many authors, are thoroughly analyzed. The numerical results obtained by using the present cable element on the basis of unified kinematic description are provided as new benchmark test results for cable structures under static loads.

2. UNIFIED KINEMATIC DESCRIPTION

2.1 Geometry mapping

The deformation of a body can be described by using the mapping between the initial and deformed configurations. The point in the initial configuration is denoted as \( \mathbf{X} \) and the point in the deformed configuration is denoted as \( \mathbf{x} \).

If there is a mapping from the initial configuration to the deformed configuration, the following relationship can be achieved as

\[
\phi(\mathbf{X}, t) = \mathbf{x}
\]  

and the displacement mapping can be written as

\[
u(\mathbf{X}, t) = \phi(\mathbf{X}, t) - \mathbf{X}
\]

The deformation gradient can be defined as

\[
\mathbf{F} = \frac{\partial \phi}{\partial \mathbf{X}} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}
\]

2.2 Strain definitions

Three different definitions of strain (\( \varepsilon \)) such as Green-Lagrange (\( \varepsilon^G \)), Biot (\( \varepsilon^B \)) and Hencky (\( \varepsilon^H \)) strains can be introduced to represent nonlinear structural behaviours as follows

\[
\varepsilon^G = \frac{1}{2} (\mathbf{F}^T \mathbf{F} - \mathbf{I})
\]

\[
\varepsilon^B = \mathbf{F} - \mathbf{I}
\]

\[
\varepsilon^H = \log(\mathbf{F})
\]

In one-dimensional problem, the deformation gradient of (3) can be written as

\[
\mathbf{F} = \frac{L}{L_0}
\]

where \( L \) and \( L_0 \) are the length of deformed bar and the length of the initial bar respectively as illustrated in Figure 2.

Therefore, three strain definitions for one-dimensional cable can be written as

\[
\varepsilon^G = \frac{1}{2} \left( \frac{L}{L_0} \right)^2 - 1
\]

\[
\varepsilon^B = \frac{L}{L_0} - 1
\]

\[
\varepsilon^H = \log \left( \frac{L}{L_0} \right)
\]

The strain magnitude with respect to the value of deformation gradient is illustrated in Figure 3.
2.3 Stress definition

The stress of the cable in the current configuration can be written as

\[ s = s_0 + E \varepsilon \]  

where \( \varepsilon \) is the axial strain, \( s \) is the current stress which is conjugate of \( \varepsilon \) and \( s_0 \) is the stress in the initial configuration. Stress definition of (1) can be rewritten in force term

\[ N = sA_0 = N_0 + EA_0 \varepsilon . \]  

2.4 Strain energy

Strain energy of the cable in current configuration can be written as

\[ U = \int_{B_0} \sigma \, dV = \sigma_0 V_0 \]
\[ = \left( s_0 + \frac{1}{2} E \varepsilon^2 \right) A_0 L_0 = L_0 \left( N_0 + \frac{1}{2} EA_0 \varepsilon^2 \right) \]  

where \( \sigma \) is the energy density.

2.5 Internal force

Internal force can be obtained by the differentiation of the strain energy with respect to the nodal displacement \( u \).

\[ q = \frac{\partial U}{\partial u} = \frac{\partial U}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial u} \]  

2.6 Tangent stiffness

Tangent stiffness of the cable can be obtained by the following expression.

\[ K = \frac{\partial q}{\partial u} = \frac{\partial^2 U}{\partial u^2} \]  

3. CABLE FINITE ELEMENT

3.1 Element Kinematics

In this study, two-node cable element is formulated and implemented to analyze the cable roof structures. The geometry of cable element is illustrated in Figure 4.

The position vector of nodal points in the current configuration can be written as

\[ x = (x_1, x_2)^T = (x_1, y_1, z_1, x_2, y_2, z_2)^T \]  

where \( x_1 \) and \( x_2 \) are the position vectors of node 1 and node 2 respectively.

The length of bar in current configuration is as follows

\[ L = \sqrt{L_x^2 + L_y^2 + L_z^2} \]  

where the projected length into the global axes are

\[ L_x = x_2 - x_1; \quad L_y = y_2 - y_1; \quad L_z = z_2 - z_1. \]  

3.2 Displacement

The displacement is obtained by subtracting the position vectors \( x \) and \( X \).

\[ u = x - X = (u_1, \nu_1, w_1, u_2, \nu_2, w_2)^T \]  

where \( u_a, \nu_a, w_a \) are the displacement at the node a in x-, y-, z-direction respectively.

3.3 Internal force

Internal force of the cable can be obtained by using (14)

\[ q = \frac{\partial U}{\partial u} = \frac{\partial U}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial u} = \frac{\partial U}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial L} = L_0 \frac{\partial \varepsilon}{\partial L} \]  

where the terms \( \frac{\partial U}{\partial e} \) and \( \frac{\partial L}{\partial u} \) are calculated as

\[
\frac{\partial U}{\partial e} = L_0(N_0 + EA_0) = L_0 N
\]

\[
\frac{\partial L}{\partial u} = \begin{pmatrix} \frac{\partial L}{\partial u_1} & \frac{\partial L}{\partial u_2} \\ \frac{\partial L}{\partial w_1} & \frac{\partial L}{\partial w_2} \end{pmatrix} = \begin{pmatrix} -L_x/L & -L_y/L \\ -L_z/L & L_x/L \end{pmatrix} = \begin{pmatrix} -\Theta \\ \Theta \end{pmatrix} = \Theta.
\]

3.4 Tangent stiffness matrix

Substituting (20) into (15), the tangent stiffness of the cable in discretized domain can be calculated as

\[
K = \frac{\partial \mathbf{u}}{\partial \mathbf{u}} = \frac{\partial}{\partial \mathbf{u}} \left( L_0 \mathbf{N} \frac{\partial e}{\partial \mathbf{u}} \right).
\]

Using (23), the tangent stiffness matrix can be obtained in the following form

\[
K = K_m + K_g
\]

where the material stiffness matrix \( K_m \) and geometrical stiffness matrix \( K_g \) are

\[
K_m = EA_0L_0 \left( \frac{\partial e}{\partial L} \right)^2 \Theta \Theta^T
\]

\[
K_g = NL \left[ \frac{1}{L} \frac{\partial e}{\partial L} \bar{I} + \left( \frac{\partial^2 e}{\partial L^2} - \frac{1}{L} \frac{\partial e}{\partial L} \right) \Theta \Theta^T \right]
\]

in which \( \bar{I} \) is

\[
\bar{I} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}
\]

Since the unified kinematic description is used here, the tangent stiffness matrix of (25) and (26) will be led into the equations summarized in Table 1.

| Strains \( e \) | \( \frac{\partial e}{\partial L} \) | \( \frac{\partial^2 e}{\partial L^2} \) | \( K_m \) | \( K_g \) |
|----------------|-----------------|-----------------|-----------------|-----------------|
| \( e^c \) | \( \frac{1}{L} \) | \( \frac{1}{L_0} \) | \( \frac{EA_0L^2}{L_0} \Theta \Theta^T \) | \( \frac{N}{L_0} \bar{I} \) |
| \( e^\theta \) | \( \frac{1}{L_0} \) | 0 | \( \frac{EA_0}{L} \Theta \Theta^T \) | \( \frac{N}{L} [I - \Theta \Theta^T] \) |
| \( e^H \) | \( \frac{1}{L} \) | \( \frac{1}{L_0} \) | \( \frac{EA_0L}{L_2} \Theta \Theta^T \) | \( \frac{NL_0}{L_2} [I - 2\Theta \Theta^T] \) |

4. NUMERICAL EXAMPLES

In order to verify the present cable element, a single cable under central point load is first tested. Furthermore, three examples are analyzed and the results are compared to reference solutions (Jayaraman and Knudson, 1981; Michalos and Birnstiel, 1960; O’Brien and Francis, 1964; Saafan, 1970; West and Kar, 1973). Note that all the present solution is obtained with the tolerance \( Tol = 10^{-10} \) for the displacement residual in Newton-Rapson method.

4.1 Single cable with central point load

In this example, we consider a single horizontal cable which is pre-stressed by \( N_0 = 10 \). The cable is kinematically indeterminate structure. The cable is fixed at both end supports and it is subjected to a central point load. In the analysis, the elastic modulus \( E = 1000 \) and section area \( A = 1 \) are used. For the FE analysis, three nodes and two cable elements are used to discretize the cable structure. All units are assumed to be consistent.

Figure 5. Single horizontal cable subjected to point load

The relationship between the central point load \( P \) and the deflection \( w \) (Kollr, 2003) can be written as

\[
P = 8EA \frac{w^3}{L^3} + 4N_0 \frac{w}{L}
\]

where \( P \) is the applied central load, \( w \) is the deflection, \( N_0 \) is the initial stress. The present result is compared with analytical solution of (28) and illustrated in Figure 6. In this example, it is turned out to be that the present numerical result has an excellent agreement with the analytical solution.

Figure 6. Force-displacement path at loading point
4.2 Poskitt cable truss

A two-dimensional structure shown in Figure 7 is considered. It consists of an upper cable and a lower cable and 14 vertical hangers. Hangers are equally spaced and all displacement in the out-of-direction is prevented. A point load \( P = \lambda P_0 = \lambda \times 115.4 \text{ N} \) in the vertical direction is applied at the lower cable of the fifth hanger from the left support. A mesh of 44 cable elements with 32 nodes is used. The nodal coordinates are summarized in Table 2. Material properties are assumed: elastic modulus \( E = 12800 \text{ kN/cm}^2 \) and the section area of the upper and lower cables \( A = 0.01866 \text{ cm}^2 \). The section area of hangers is \( A = 0.0080654 \text{ cm}^2 \). Both ends of cable structure are fixed. Pre-tensioning stress is applied to the lower and upper cables about 0.6896 kN. In this example, 22 load steps (\( \chi \)) are used to produce the load-displacement path as illustrated in Figure 8. Noted that there is no load-displacement path information of this problem in other references, we therefore provide it here.

The vertical displacement of lower cable at the load \( P_0 = 115.4 \text{ N} \) is illustrated in Figure 9. The horizontal displacement of lower cable and upper cable are provided in Figure 10.

Since the present results are almost identical to the results in References (Poskitt, 1967; Meek, 1991; Lewis, 2003), the present result only is therefore provided in Figures 9 and 10.

| Node | Nodal coordinates | Node | Nodal coordinates |
|------|------------------|------|------------------|
|      | \( x \) \( y \) |      | \( x \) \( y \) |
| 1    | 0                | 17   | 1.6184          |
| 2    | 0                | 18   | 1.6184          |
| 3    | 0.2023           | 19   | 1.8207          |
| 4    | 0.2023           | 20   | 1.8207          |
| 5    | 0.4046           | 21   | 2.0230          |
| 6    | 0.4046           | 22   | 2.0230          |
| 7    | 0.6069           | 23   | 2.2253          |
| 8    | 0.6069           | 24   | 2.2253          |
| 9    | 0.8092           | 25   | 2.4276          |
| 10   | 0.8092           | 26   | 2.4276          |
| 11   | 1.0115           | 27   | 2.6299          |
| 12   | 1.0115           | 28   | 2.6299          |
| 13   | 1.2138           | 29   | 2.8322          |
| 14   | 1.2138           | 30   | 2.8322          |
| 15   | 1.4161           | 31   | 3.0345          |
| 16   | 1.4161           | 32   | 3.0345          |

Figure 7. The geometry of Poskitt truss
4.3 Pre-stressed cable net under vertical load

A pre-stressed cable net is considered. It is first studied by Saafan (1973) and subsequently by West and Kar (1970) and Jayaraman and Knudson (1981). Later, Tibert (1999) provided a comprehensive review on this subject. The geometry of the cable net is illustrated in Figures 11 and 12. The initial data for the analysis are given in Table 3. The cable net is discretized with twelve elements and twelve nodes. Four point loads \( P = 4\lambda P_0 = \lambda \times 8.0 \text{ kip} \) are applied at the nodes 4, 5, 8 and 9 in the vertical direction. Since the geometry of cable net and loadings are all symmetry, the vertical displacements at all nodes having the point load will be therefore the same values and lateral displacements in x- and y-direction should be the same.

![Figure 11. Cable net: x-y plane view](image)

![Figure 12. Cable net: perspective view ()](image)

![Figure 13. Displacement at node 4](image)

The present result with a particular load (\( P_0 = 8.0 \text{ kip} \)) is summarized with other reference solutions (Saafan, 1970; West & Kar, 1973; Jayaraman & Knudson, 1981; Tibert, 1999) in Table 4. It is found to be that the present result has a good agreement with other solutions.

![Table 4. Displacement with \( P_0=8.0\text{ kip} \) at the node 4](image)

| References               | Displacement at loading point |
|--------------------------|-------------------------------|
|                          | \( u_4 \)          | \( v_4 \)          | \( w_4 \)          |
| Straight bar (Ref1)      | -0.1324                  | -0.1324                  | -1.477                  |
| Straight bar (Ref2)      | -0.1325                  | -0.1324                  | -1.468                  |
| Elastic catenary (Ref3)  | -0.1300                  | -0.1319                  | -1.463                  |
| Straight bar (Ref3)      | -0.1322                  | -0.1322                  | -1.477                  |
| Straight bar (Ref4)      | -0.1322                  | -0.1322                  | -1.477                  |
| Elastic parabola (Ref4)  | -0.1338                  | -0.1338                  | -1.483                  |
| Elastic catenary (Ref4)  | -0.1328                  | -0.1328                  | -1.474                  |
| Associate catenary (Ref4)| -0.1338                  | -0.1338                  | -1.484                  |
| Present                  | -0.1323                  | -0.1323                  | -1.469                  |

Ref1: Saafan (1970). Ref2: West & Kar (1973). Ref3: Jayaraman & Knudson (1981). Ref4: Tibert (1999).
### Table 5. Displacement at specific free nodes (a)

| Node (a) | z-Coord. | Kwan(1998) | Thai and Kim(2011) | Present |
|----------|----------|------------|--------------------|---------|
|          |          | $u_a$      | $v_a$              | $w_a$   | $u_a$   | $v_a$ | $w_a$ |
| 1        | 1368     | 0          | 0                  | 0       | -       | -     | -     |
| 2        | 2432     | 0          | 0                  | 0       | -       | -     | -     |
| 3        | 3192     | 0          | 0                  | 0       | -       | -     | -     |
| 4        | 3648     | 0          | 0                  | 0       | -       | -     | -     |
| 5        | 3800     | 0          | 0                  | 0       | -       | -     | -     |
| 6        | 1032     | 15.55      | -4.46              | 81.70   | 15.55   | -4.46 | 81.66 |
| 7        | 1835     | 11.50      | -5.55              | 51.22   | 11.05   | -5.54 | 61.18 |
| 8        | 2408     | 7.38       | -4.20              | 33.31   | 7.38    | -4.19 | 33.28 |
| 9        | 2752     | 5.34       | -3.11              | 17.88   | 5.34    | -3.11 | 17.87 |
| 10       | 2867     | 4.11       | -2.80              | 11.16   | 4.10    | -2.80 | 11.15 |
| 11       | 792      | 14.43      | -3.53              | 97.14   | 14.42   | -3.53 | 97.10 |
| 12       | 1408     | 11.27      | -4.47              | 72.90   | 11.26   | -4.46 | 72.84 |
| 13       | 1848     | 7.25       | -2.97              | 31.98   | 7.25    | -2.97 | 31.94 |
| 14       | 2118     | 5.67       | -2.12              | 10.54   | 5.67    | -2.11 | 10.52 |
| 15       | 2200     | 4.77       | -0.60              | -11.34  | 4.77    | -0.60 | -11.34|
| 16       | 648      | 11.71      | -1.71              | 92.44   | 11.70   | -1.71 | 92.40 |
| 17       | 1152     | 9.55       | -2.11              | 66.94   | 9.54    | -2.11 | 66.89 |
| 18       | 1512     | 6.30       | -1.15              | 20.21   | 6.30    | -1.15 | 20.17 |
| 19       | 1728     | 4.92       | -0.23              | -14.05  | 4.91    | -0.23 | -14.06|
| 20       | 1800     | 4.65       | 0.52               | -35.79  | 4.65    | 0.52  | -35.77|
| 21       | 600      | 10.63      | 0                  | 88.73   | 10.62   | 0     | 88.68 |
| 22       | 1067     | 8.80       | 0                  | 62.83   | 8.79    | 0     | 62.77 |
| 23       | 1400     | 5.83       | 0                  | 13.99   | 5.83    | 0     | 13.95 |
| 24       | 1600     | 4.64       | 0                  | -22.52  | 4.63    | 0     | -22.52|
| 25       | 1667     | 4.55       | 0                  | -45.89  | 4.54    | 0     | -45.87|
| 26       | 600      | 0.92       | 0                  | 5.86    | 0.92    | 0     | 5.86  |
| 27       | 1840     | 3.85       | -0.78              | -30.12  | 3.85    | -0.78 | -30.10|
| 28       | 2867     | 4.11       | 2.80               | 11.16   | 4.10    | 2.80  | 11.16 |
| 29       | 1032     | 5.40       | 1.87               | 32.17   | 5.40    | 1.87  | 32.15 |
| 30       | 2752     | -         | -                  | -       | -       | -     | -     |
| 31       | 2408     | -         | -                  | -       | -       | -     | -     |
| 32       | 1835     | -         | -                  | -       | -       | -     | -     |
| 33       | 1032     | -         | -                  | -       | -       | -     | -     |
| 34       | 2118     | -         | -                  | -       | -       | -     | -     |
| 35       | 1848     | -         | -                  | -       | -       | -     | -     |
| 36       | 1408     | -         | -                  | -       | -       | -     | -     |
| 37       | 792      | -         | -                  | -       | -       | -     | -     |
| 38       | 1728     | -         | -                  | -       | -       | -     | -     |
| 39       | 1512     | -         | -                  | -       | -       | -     | -     |
| 40       | 1152     | -         | -                  | -       | -       | -     | -     |
| 41       | 648      | -         | -                  | -       | -       | -     | -     |
| 42       | 1600     | -         | -                  | -       | -       | -     | -     |
| 43       | 1400     | -         | -                  | -       | -       | -     | -     |
| 44       | 1067     | -         | -                  | -       | -       | -     | -     |
| 45       | 600      | -         | -                  | -       | -       | -     | -     |
4.4 Three-dimensional saddle net

Three-dimensional saddle net with the dimension of $50 \, \text{m} \times 40 \, \text{m}$ is considered. It consists of 142 cable elements which are equally spaced with $5 \, \text{m} \times 5 \, \text{m}$ grid. The equal pre-tension force $N_0 = 60 \, \text{kN}$ is applied to all cables. The external loads with the magnitude of $P = 1 \, \text{kN}$ are applied to the half structure in the positive x- and negative z-directions as illustrated in Figure 14. The equal pre-tension force is applied to all cables. The external loads with the magnitude of are applied to the half structure in the positive x- and negative z-directions as illustrated in Figure 14. The elastic modulus and section area are $E = 147 \, \text{kN/mm}^2$ and $A = 306 \, \text{mm}^2$ respectively. Note that the geometry of saddle net has doubly symmetry in x-, y-direction.

5. CONCLUSIONS

A cable roof structures are analyzed by using a two-node cable element formulated by using unified kinematic description. The tangent stiffness matrix based on Green-Lagrange strain definition is consistently used in numerical tests and the present numerical results have a very good agreement with other reference solutions. From numerical tests, it is observed that the present element with Newton-Raphson method shows a good performance since it can produce a converged solution with a few iterations without any unstable situation. Comprehensive load-displacement paths are newly provided for all numerical examples. In particular, full displacement information of saddle net is described here for future benchmark test. Throughout this study, it is turned out to be that the unified kinematic description can provide a very simple process for the derivation of all terms required in the geometrically nonlinear cable element and it ultimately lead us to formulate an efficient FE cable element for the geometrically nonlinear analysis of two- and three-dimensional cable structures.

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(Received July 20, 2015/Accepted January 25, 2016)