The Application Research of GM (1,1) Model with \( X^{(1)}(n) \) as Initial Value in Deformation Prediction Combined with Accumulation and Logarithm Transformation

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Abstract. Mining can easily lead to subsidence. If the subsidence is serious, the ecological environment will break down, and even worse, bring dangers to human. Therefore, the prediction of mining subsidence is of great significance for assessing possible dangers and ecological restoration at a later stage. In order to overcome the theoretical defects in the traditional GM(1,1) model and deal with the low prediction accuracy of deformation data due to fluctuation and oscillation, the original data sequence is accumulated and the logarithmic transformation is performed after adding constant to the accumulated sequence. Then according to the transformed sequence the GM (1,1) model with \( X^{(1)}(n) \) as the initial value is established to make fitting and prediction. At last, based on the measured deformation data, we can not only obtain a satisfactory results for monotone concave sequence, but also improve the prediction accuracy of the fluctuation data by combining with the accumulation operation. So the results of the example meet the requirements of prediction accuracy. The example shows that the method has practical value in engineering.

Keywords. Mining subsidence, GM (1,1) model, Logarithmic transformation, Data prediction, Fluctuation and oscillation

1. Introduction
It is necessary for us to study the rule of the surface and rock deformation, especially when we mine. At present, the regular observation of the surface is often used. Then we analyse the observation data and obtain the rule of it. And the data prediction is conducive to grasp the change trend which can be good for formulating the countermeasures [1]. There are many data prediction methods, and the grey system theory proposed by Professor Deng Julong is one of them [2]. In the application of the traditional GM (1,1) model prediction, Luan Yuanchong [3] used different dimensions to establish the traditional GM (1,1) model and predicted the surface subsidence deformation of mines with it, which has a good effect. Cui Rui [4] respectively established the traditional GM (1,1) model for the three-dimensional vector of gold deformation to predict the three-dimensional vector at different time and achieved good results. Guo Guangli [5, 6] used the traditional GM (1,1) model to analyze and predict the settlement deformation of gangue foundation buildings. Liu Hechun [7-10] also achieved good results in the application of GM (1,1) model on the prediction of iron ore subsidence. Based on the theoretical defects of the traditional GM (1,1) and the low prediction accuracy caused by the fluctuation of the deformation data, this study proposes a new method combined with Dang Yaoguo [11] who established GM (1,1) model with \( X^{(1)}(n) \) as the initial value. The detailed steps are as follows.
Firstly, the original data sequence is accumulated to obtain monotone increasing concave sequence. Then the \( \ln(x+C) \) transformation is performed. Finally, the result sequence is used to establish the GM (1,1) model with \( X^{(1)}(n) \) as the initial value. And the approximate solution method for the optimal constant \( C \) is given. The data in reference [7] is used to verify the method, which verifies that the monotone concave sequence can obtain ideal results and improve the problem of low prediction accuracy caused by data fluctuation.

2. Improved GM (1,1) Model

2.1. Model Principle and Modeling Steps

Assuming that there is a original data sequence \( X' \), \( X' = (x'(1), x'(2), \ldots, x'(n)) \). \( X \) is the i cumulative sequence of \( X' \), \( i \in N + \), and the value of i can make \( X \) be the minimum of monotone increasing concave sequence. Then get another data sequence \( X, X = (x(1), x(2), \ldots, x(n)) \). And assume data sequence \( X^{(0)}, X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \).

\[
x^{(0)}(k) = \ln(x(k) + c) \tag{1}
\]

\( k=1, 2, \ldots, n \), \( c \) is the constant value.

\( X^{(1)} \) is the 1-AGO sequence of \( X^{(0)}, X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)) \). Then the adjacent mean generating sequence of \( X^{(1)} \) is \( Z^{(1)}, Z^{(1)} = (z^{(1)}(2), z^{(1)}(3), \ldots, z^{(1)}(n)) \). The GM(1,1) grey model is as follow.

\[
x^{(0)}(k) + ac^{(1)}(k) = b \tag{2}
\]

\[
\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{3}
\]

Equation (3) is the whitening equation of equation (2).

If \( X^{(0)} \) is a non-negative sequence, then we can get the value of \( Y \) and \( B \).

\[
Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, \quad B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}
\]

It can be solved by the least square method.

\[
\hat{a} = [a \quad b]^T = (B^T B)^{-1} B^T Y \tag{4}
\]

\( \hat{a} \) is introduced into equation (4) to solve the differential equation, then the prediction of the accumulated value is obtained. The following equation can be solved.

\[
\hat{x}^{(0)}(k+1) = (1 - e^{\hat{a}})(x^{(0)}(1) - \frac{b}{a})e^{-ak} \tag{5}
\]

The equation (5) is the prediction formula of grey GM(1,1) model. When \( k+1 \leq n \), the result is fitting value, otherwise, it is predicting value.

According to the reference [7], the general solution of equation (3) is as follow.

\[
\hat{x}^{(1)}(t) = \frac{b}{a} + u e^{-ak} \tag{6}
\]
On the basis of \( X^{(i)}(k) \big|_{k=n} = X^{(i)}(n) \), the value of constant \( c \) is as bellow.

\[
c = (x^{(i)}(n) - \frac{b}{a})e^{an}
\]

(7)

Then we can get the following results.

\[
X^{(i)}(k) = (x^{(i)}(n) - \frac{b}{a})e^{-a(k-n)} + \frac{b}{a}
\]

(8)

\[
X^{(0)}(k+1) = X^{(i)}(k+1) - X^{(i)}(k)
\]

(9)

In the above equations, the value of \( k \) is as bellow, \( k=1, 2, \ldots \). The equation (8) and (9) are the GM (1, 1) model with \( x^{(i)}(n) \) as the initial value for fitting and predicting.

Therefore, the final modeling steps can be obtained. Firstly, transform the original sequence according to equation (1). Then, the parameters are solved according to (4). At last, the fitting and predicting are carried out according to equation (8) and (9).

2.2. Approximate Selection of Constants \( c \) in Logarithmic Transformation

The logarithmic transformation changes the smoothness of the sequence, so the inequality \( x+c>e \) is required and the lower limit of \( c \) is determined. On the basis of decimal digit, the minimum value of \( c \) can be determined. With the increase of \( c \), the fitting and forecasting sequence tend to be straight line gradually, which leads to the descending trend of fitting value \( \hat{x}_n \) and the later prediction series. And the model function is exponential growth. In order to determine the approximate solution \( \hat{c} \) of the optimal value \( c \), there are two restricted conditions.

\[
\begin{align*}
& x_n - \hat{x}_n > 0 \\
& |\hat{x}_{n+1} - \hat{x}_{n+1}| \leq \varepsilon
\end{align*}
\]

(10)

Among the above equations, \( x_n \) is the last one of the original modeling sequence. \( \hat{x}_n \) is the fitting value of the \( n \)th term of the model after \( c \) is selected. \( \hat{x}_{n+1} \) and \( \hat{x}_{n+1}^* \) are the first predicted values of the model after the transformation of \( c \) and \( c+1 \) respectively. \( \varepsilon \) is the limit value, \( \varepsilon \leq 0.5 \). \( \hat{c} \) is the approximate value of the optimal value \( c \), and it can be taken to integer or decimal place.

3. Analysis of Application Examples

Based on the first eight stages’ measured data of CD3\# and CD9\# of Zhongxihaozhuang iron mine in reference [7], a model was established to predict the accumulated settlement values of the ninth and tenth periods. C\# language is used to develop the program. The process and result are of double precision. The prediction results are as follows (the data are the same as those in the original literature, which are taken to two decimal places).

Among them, the cumulative times of CD3\# point is 1, and the value of \( \hat{c} \) is 171. According to the one time accumulated value and predicted value, the posterior variance ratio is calculated as 0.005. The result was grade 1. The cumulative times of CD9\# point is 2, and the value of \( \hat{c} \) is 160. According to the quadratic accumulated value and predicted value, the posterior variance ratio is calculated as 0.067. The result was grade 1, too. It can be seen that the fitting accuracy of cumulative sequence is high (as shown in figure 1). Referring to the table 1, it can be seen that the fitting accuracy of the original sequence CD3\# point is higher than that of CD9\# point. The reason still lies in the fact that the regressive value is an increasing trend or an increasing trend after decreasing, which is not fit
well with the fluctuation data. But it is better for monotone increasing concave sequence or approximate monotone increasing concave sequence.

### Table 1. The comparison of measured and predicted values(unit: mm).

| Period | CD3 Point | CD9 Point |
|--------|-----------|-----------|
|        | Measured value | Predicted value | Residual error | Relative error % | Measured value | Predicted value | Residual error | Relative error % |
| 1      | 0          | 0.05      | -0.05         | —               | 0             | 0.28        | -0.28         | —               |
| 2      | 9.2        | 9.24      | -0.04         | -0.43           | 6.34          | -0.13       | 6.47          | 102.05          |
| 3      | 9.71       | 9.49      | 0.22          | 2.27            | 6.68          | 19.87       | -13.19        | -197.46         |
| 4      | 10.12      | 10.10     | 0.02          | 0.20            | 6.53          | 3.05        | 3.48          | 53.29           |
| 5      | 10.34      | 10.74     | -0.4          | -3.87           | 6.78          | 3.59        | 3.19          | 47.05           |
| 6      | 11.78      | 11.43     | 0.35          | 2.97            | 6.92          | 4.23        | 2.69          | 38.87           |
| 7      | 12.05      | 12.18     | -0.13         | -1.08           | 6.89          | 5.01        | 1.88          | 27.29           |
| 8      | 13.04      | 12.97     | 0.07          | 0.54            | 7.12          | 5.95        | 1.17          | 16.43           |
| 9      | 14.06      | 13.83     | 0.23          | 1.64            | 7.2           | 7.08        | 0.12          | 1.67            |
| 10     | 14.75      | 14.75     | 0           | 0               | 7.23          | 8.45        | -1.22         | -16.87          |

By analyzing the prediction accuracy of the 9th and 10th periods, the prediction accuracy of CD3 point is higher than that of CD9, but the latter can also meet the requirements of engineering practice. However, the change rate of deformation curve is not consistent with that of prediction curve, which leads to the trend of separation after similar fitting. Therefore, the metabolic method can be used to update the original data series. However, with regard to the long-term oscillation series, it is not suitable for long-term prediction, and the prediction results may not be accurate, which needs further study.

### 4. Conclusion

In this paper, a prediction method of mining subsidence is proposed. The original sequence is accumulated and combined with logarithmic transformation. And the GM (1,1) model with \( x^{(1)}(n) \) as the initial value is established for the transformed data. Then the method is verified by the measured data of mining area, which proves that the prediction accuracy is perfect. In addition, it solves the problem that the prediction accuracy of wave oscillation sequence is not high. The method can also meet the requirements of practical application. The method proposed in this paper can provide a basis for later settlement prediction and analysis. It has higher value in engineering application. Of course,
the empirical formula and empirical parameters are insufficient in determining the additive constant \( C \), and there is also a gap in the multi-period prediction of volatility sequence, which needs further research.

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