Reversed granular Weissenberg effect in ensembles of sheared non-convex granular particles

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Studies of granular materials, both theoretical and experimental, are often restricted to convex grain shapes. We demonstrate that a non-convex grain shape can lead to a qualitatively novel macroscopic dynamics. Spatial crosses (hexapods) are continuously sheared in a split-bottom container. Thereby, they develop a secondary flow profile that is completely opposite to that of rod-shaped or lentil-shaped convex grains in the same geometry. The crosses at the surface migrate towards the rotation center and sink there, mimicking an ‘inverse Weissenberg effect’. The observed surface flow field suggests the existence of a radial outward flow in the depth of the granular bed, thus forming a convection cell. This flow field is connected with a dimple formed in the rotation center. The effect is strongly dependent on the particle geometry and the height of the granular bed.

Granular materials had fundamental importance in human civilization since millennia, but still, their dynamical and structural behavior is much less understood than that of ordinary solids, liquids or gases, and it is often quite counter-intuitive. Ensembles of hard spherical grains have been studied extensively, and important progress was, and still is, achieved regarding, e. g., packing [1]–[3], shear characteristics and flow [4]–[6], jamming [7]–[8], and internal stress distributions [9]–[11]. Recent research has been increasingly dedicated to shape-anisotropic (e. g. [12]–[14]) and soft grains [15]–[19].

We focus here on nonconvex particles that exhibit astonishing new features. Research on such particles has been performed only scarcely. A review of packing problems of particles with various shapes was given by Torquato and Stillinger [20]. Alonso-Marroquin [21] developed a simulation tool for 2D nonconvex objects that was extended to 3D by Galindo-Torres [22]. Azéma [24] numerically investigated stress response and shear strength, defining and analyzing a ‘level of convexity’. Saint-Cyr [24] simulated force chains in 2D, controlling nonconvexity by the choice of special grain geometries. The packing fraction was shown to grow first and then to decay with nonconvexity. In further simulations, stickiness was included [25]. The stress response to cyclic shear was studied by Athanasiadis [26]. Galindo-Torres [22] computed the influence of nonconvexity on friction, and Han [27] simulated nonconvex grains flowing down an inclined plane. Sheared ensembles of U-shaped particles in 2D were simulated by Marschall [25].

Some recent studies dealt with crosses: Huet [29] performed simulations and experiments of the collapse of heaps of 2D crosses. Their packing in 2D was investigated theoretically [30]–[32] and experimentally [33]–[34]. Spatial crosses, also referred to as hexapods, recently received attention. Conzelmann [35] computed their packing and the distribution of contact forces. An experimental method to reveal the local structure of aggregates of spatial crosses with X-ray scanning was presented by Barès [36]. Kuhn described the motion of contacts during slow loading [37], and Zhao [38] reported shearing of crosses with different aspect ratios $\rho = \ell/d$ (maximum extension $\ell$ divided by the arm width $d$, see Fig. 1). Ratios between 1 (sphere) and 10 were considered. The crosses exhibit yield stress when $\rho$ is small ($< 3.3$). Stiffening sets in at $\rho \approx 6.7$. In addition to fundamental interest in such particle shapes, stars and hexapods may be of practical value in granular architecture [39]–[42].

We shear ensembles of spatial crosses (Fig. 1), employing a split-bottom container [43]–[44] with an inner radius of the cylindrical vessel of 190 mm, and a radius $R_0 = 125$ mm of the rotating bottom disk (sketched in Fig. 2). This disk is coated with a grid structure that prevents the crosses from sliding. The $\rho = 3$ crosses have an extension $\ell = 9.6$ mm and an arm width $d = 3.2$ mm. With 50,000 particles available, fill heights $H$ up to 140 mm can be realized. The bottom disk is rotated by a motor with angular velocity $\omega_0$ at controlled rates between 3 rpm and 6 rpm. The top of the granular bed is observed with a stereo camera (Intel D435) that simultaneously captures images of the granular bed and the spatially resolved depth information (1280 px × 720 px).

![Image](image-url)

**FIG. 1.** a) Geometry of the crosses. Top: aspect ratio $\rho = 3$, bottom: $\rho = 4$. b) Shear geometry. The central bottom disk is rotated with a controlled speed. The left (red) and right (blue) dashed lines sketch the central cross sections of the granular bed for the case of elongated or flattened convex particles (left) and for the spatial crosses studied here (right). Arrows indicate the sense of convection.
The in-plane resolution is 0.33 mm/pixel and the depth resolution was better than 0.5 mm. Typically, we took 10 images per full revolution of the bottom disk.

In a similar setup, an interesting phenomenon was observed earlier: for sheared spherical grains, the granular bed remains practically flat, while for elongated cylindrical grains \[45, 46\] as well as for flat lentils \[46\], shear causes the formation of a heap in the rotation center. This heaping is connected with a secondary flow in the granular bed: a convection directed radially outward at the bed surface, and towards the center at the bottom. Figure 1b in the left half sketches a cross-section of the granular bed and the convective flow found in earlier experiments with rods, rice grains and lentils. The heaping effect was found to be particularly efficient when the fill height \(H\) of the container was roughly 0.6 times the bottom disk radius \(R_0\). At this height, the material at the top above the bottom disk rotated with about half the disk rotation rate. When the fill level was lower, the material rotated faster, but the heap formed much slower with a substantially lower height. When the granular bed was much higher than \(0.6R_0\), the material above the bottom disk rotated slower, the heap also formed much slower and the heap height was also reduced considerably \[45, 46\].

We anticipate that the experiments with spatial crosses show exactly the opposite phenomenon. This is demonstrated for different fill heights \(H\). The observed flow and surface profile are sketched qualitatively in Fig. 1b, right.

First, we retrieved the tangential flow profile at the granular bed surface, in order to map the shear zone. For that purpose, some black crosses were distributed on top. By tracing these markers, the local flow field can be probed. Figure 2a shows the initial image subtracted (markers appear white) from that taken after 5 rotations (black markers) for \(H/R_0 = 0.8\). In the central part (radius \(\approx 5\) cm), the granular surface rotates nearly uniformly, but with only 2.5 % of \(\omega_0\) (Fig. 2b). In addition to the rotation, a slow mean radial flow inward of the order of 1 mm/rotation is evident (Fig. 2b). With increasing distance from the center, where the shear zone reaches the surface (radius between \(\approx 5\) cm and \(\approx 15\) cm), the surface rotation decreases monotonously, the particles migrate diffusively. The inward motion ceases. Outside beyond the bottom disk edge, there is hardly any surface motion.

![Figure 2](image)

**FIG. 2.** Superimposed frames: initial image inverted, markers in white, and images after 5 rotations (a) and 15 rotations (b) with marked crosses in black, \(H = 100\) mm \((0.8R_0)\), \(\omega_0 = 6\) rpm, \(\rho = 3\). c) same construction after 15 rotations for \(H = 90\) mm \((0.72R_0)\). Image sizes 24 cm \(\times\) 24 cm. The tracer crosses in the central part move inward, crosses in the shear zone and in the outer parts on average maintain their radial positions. The intersection of the fine horizontal and vertical lines marks the rotation center.

The radial motion of the tracers is shown in Fig. 4. It is considerably slower than the tangential displacement, and partly obscured by the diffusive motion of the in-
dividual tracers. Again, radial displacements were determined as averages of 8 to 20 rotations. It is seen that the overall radial flow velocities increase considerably with lower fill heights, while they decrease in higher granular beds. Nevertheless, in the central region there is a clear trend to negative \( v_r \), i.e. flow towards the rotation center, in the data shown in Fig. 4. In the \( H = 120 \) mm sample, all values except one within a radial distance of about 100 mm (0.8 \( R_0 \)) are negative. They are comparably small, of the order of -0.1 mm/rotation. In the \( H = 100 \) mm bed, much faster transport inward in radial direction, with velocities up to -0.6 mm/rotation. When the fill height \( H \) is further decreased, the diffusive character of radial motion increases, but there is still a net transport towards the center in the region within \( R < 0.5R_0 \). With lower \( H/R_0 \) down to about 0.5, the diffusive motion becomes prevalent. A net inward flow is observed only in a central area within \( R < 0.25R_0 \) and net outward flow dominates elsewhere.

In general, the trend is that at large \( H/R_0 \), there is less diffusion and a prevalence of slow directed flow towards the center. For intermediate fill levels, both the diffusion and directed flow intensify. A separation of a region close to the container center with dominating inward surface flow and an outer region with prevalent outward surface flow is seen. For lower fill heights, inward flow area shrinks and the diffusive motion and outward flow dominate the movement of particles on the surface. Eventually, for very low fill heights (\( H < 0.5R_0 \)), the radial transport ceases. Below \( H/R_0 < 0.4 \), the center rotates nearly like a solid block, and radial flow and diffusion in the central part vanish.

Since we observe a net inward motion of surface particles, at a stationary surface profile (see below), there must be a net transport radially outward in the deeper layers of the granular bed. Thus, it is reasonable to assume a convection as sketched in Fig. 4. The maximum convection speed is found at fill heights between 0.6 \( R_0 \) and 0.8 \( R_0 \).

Analogous to the earlier findings for rods, where heap-ing in the center and outward surface flow was found, the radially inward surface flow of crosses is related to the formation of a dimple in the rotation center. We show examples of height profiles recorded with the stereo camera in Fig. 5. Initially, the granular bed is flat within \( \approx 5 \) mm. Tracer crosses arranged initially in cross-shape can be identified as bright four lines. After a few rotations, the granular bed expands above the shear zone, indicated by a height change of about 5% to 10%. This is the effect of Reynolds dilatancy [47]. The material cannot expand in the plane quickly, so the local surface is elevated. After 15 rotations, this effect is even stronger, and the elevation shifts outward. Remarkably, a dimple is formed in the middle. This region has a diameter of roughly 3 to 4 cm. Marked crosses that are dragged into this sink disappear from the surface.

Figure 6 shows the space-time plot of the azimuthally averaged surface profiles for crosses with \( \rho = 3 \), \( H = 0.8 R_0 \), \( \omega_0 = 6 \) rpm. a) is the initial transient, and b) shows the situation after a stationary profile has established.
nomenon was observed when the rotation sense of the distances from the rotation center. The graphs show quantitative data for three selected rotation was reversed after 20 rotations in the plot (black arrow). The graphs start at roughly the same level, then the sense of rotation: The space-time plot on the right has the same color code as Fig. 6. It shows 40 rotations, starting from a stationary profile after 480 rotations. The sense of Fig. 8 shows the dynamics of the height profile starting with the stationary convection and dimple shape after 500 forward rotations. Again, the same radii were selected as in Fig. 7 plus one additional radial position. As can be seen, the elevation of the granular bed collapses within less than 1/10 rotation of the bottom disk by about 3 - 5%. This is seen particularly well in the space-time plot at the right hand side. The central dimple is not noticeably affected, because the height fluctuations are much larger than 5%, but one may speculate that a compaction is effective there as well. The original, stationary state is recovered after a transient phase. In the range $0.25R_0 < R < 0.6R_0$, the recovery takes 3 to 5 bottom disk rotations. In the outer parts of the granular bed, above the bottom disk edge, this recovery takes much longer, as in the initial phase of the experiment.

In summary, spatial crosses with an aspect ratio $\rho = 3$ generate a convective secondary flow when sheared in a split-bottom container. This convection is connected with the formation of a dimple whereas rods or lentils exhibit a completely opposite behavior. We have performed similar experiments with thinner spatial crosses of the same volume and aspect ratio $\rho = 4$ ($\ell = 14.8$ mm, $d = 3.7$ mm). In these experiments, the same features as for the thicker $\rho = 3$ crosses are qualitatively confirmed. Dimple formation as well as surface flow are found under comparable geometrical conditions. On the other hand, similar experiments performed with crosses of aspect ratio $\rho = 6$ show no noticeable convective flow nor a central dimple. In these experiments, similar parameters were chosen as for the $\rho = 3$ crosses. It is possible that at other fill levels and on much longer timescales (more revolutions) these effects might be recognized. The central surface rotation rate $\omega_c$ is shown in Fig. 3. At comparable fill heights, it is much slower than for the $\rho = 3$ crosses. Regarding the level of convexity, it is intuitively clear that this feature is larger for the $\rho = 3$ crosses than for $\rho = 6$. Thus, it is somewhat counterintuitive, if one attributes the observed phenomena to the nonconvex shapes of the spatial crosses, that the effect is stronger for the thicker crosses with $\rho = 3$.

A physical explanation of the observed phenomena is still pending. Possibly, the reason is related to the strong Reynolds dilatancy of the crosses. This effect is much smaller for rod-like particles where it is at least partially compensated by a compaction through shear-alignment. It may be easier for a spatial cross in the depth of the granular bed to enter the shear zone from the side because of the lower packing fraction. This would lead to an absorption of crosses by the shear zone, related to a subsurface net flow into the shear zone. Such a flow would create voids in the center of the container where
crosses are pulled down, leaving a dimple and creating an inward surface flow. A better understanding requires a more comprehensive investigation of this phenomenon, including non-invasive 3D imaging with X-ray Computed Tomography or Nuclear Magnetic Resonance. In addition, the development of numerically efficient and accurate DEM simulation of the system under consideration would be highly desirable.

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