Spin Chaos Manifestation in a Driven Quantum Billiard with Spin-Orbit Coupling

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The coupling of orbital and spin degrees of freedom is the source of many interesting phenomena. Here, we study the electron dynamics in a quantum billiard driven by a periodic electric field—a mesoscopic rectangular quantum dot—with spin-orbit coupling. We find that the spatial and temporal profiles of the observables demonstrate the transition to chaotic dynamics with qualitative modifications of the power spectra and patterns of probability and spin density. The time dependence of the wavefunctions and spin density indicates spin-charge separation seen in the decay of the spin-charge density correlators. Experimental verification of this spin chaos effect can lead to a better understanding of the interplay between spin and spatial degrees of freedom in mesoscopic systems.

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I. INTRODUCTION

The emergence of stochasticity is of fundamental importance for classical and quantum physics with broad interdisciplinary connections and applications.¹ ² Fascinating examples of irregular dynamics can be found in meso- and nanoscale systems, including quantum dots.³ ⁴ While the understanding of charge transport in such systems is already quite deep, the knowledge of the chaotic spin evolution is still poor. Since the related branch of physics, “spintronics” is among the most promising research fields,⁴ it is of importance and applied interest to study the spin dynamics in mesoscopic systems with coupled charge and spin degrees of freedom. A natural example for these studies is provided by semiconductor structures where the spin-orbit coupling (SOC) plays a significant role in dynamics, including the ability of spin manipulation by electric field.¹⁰,¹¹ It has been predicted that for a sufficiently strong driving field the dynamics of charge and spin can become unexpectedly complicated for electron in a double quantum dot.¹²,¹³ However, it is not known what kind of irregular coupled dynamics one may expect in a mesoscopic structure such as a semiclassical quantum billiard.

Without SOC such structures may demonstrate certain classical traits of the transition to chaos due to the high density of states, including the formation of irregular wavefunctions inside the billiard.¹⁰,¹³,¹⁴,¹⁵ It is known that the eigenstates distributions in rectangular billiards with SOC demonstrate fingerprints of chaos such as the Wigner statistics, not expected for integrable systems.¹⁶ Since chaos driven by an external periodic field is a common phenomenon in nonlinear systems, it is natural to ask whether an irregular motion arises in such a billiard for coupled charge and spin channels under a periodic driving. The problem itself is complicated since there are no explicit criteria for identifying the chaotic regimes in the dynamics of quantum spin observables not having classical counterparts.

In this paper, we present a model for exploring dynamical regimes of coupled charge and spin degrees of freedom for two-dimensional (2D) electrons confined in a rectangular quantum billiard with Rashba SOC and an in-plane magnetic field. We consider charge and spin dynamics driven by a periodic electric field resonating with transitions between two nearest semiclassical size quantization levels. We observe strong indications of transition to irregular dynamics both for spatial coordinates and spins as studied using the Floquet stroboscopic technique.² ⁴,¹⁸,¹⁹ Although the quantum nature of our systems hampers pure classical manifestation of chaos like the Lyapunov exponents, the scenarios for both charge and spin evolution suggest the transition to irregular regimes. We found that the most sensitive observables are the densities of charge and spin, where the textures of different shapes determined by the driving frequency and amplitude are formed. The results can be useful for the understanding of the quantum chaos involving spin as well as for the design of semiconductor-based spintronics devices.

II. MODEL

Our Hamiltonian, $H(t) = H_0 + V(t)$, consists of the unperturbed part $H_0$ describing an electron confined in the 2D rectangular billiard with sides $a$ and $b$, hard-wall boundary conditions, Rashba SOC and Zeeman interaction to the in-plane magnetic field $B_z$:
where we take $H_{so} = \alpha_R (\sigma_x k_x - \sigma_y k_y)$ in the Rashba form. Here $m$, $\alpha_R$, and $g$ are the electron effective mass, Rashba SOC constant, and $g$-factor, respectively. The in-plane magnetic field lifts the Kramers degeneracy, however, does not cause the diamagnetic coupling. Since the $\sigma_z$ operator is present in both the SOC and Zeeman terms, the Zeeman splitting is coupled to the orbital quantization making it orbital state-dependent. The spin-coordinate entangled wavefunction of the $n$-th state of $H_0$ can be constructed as a superposition of the orbital wavefunctions multiplied by the two-component spinors,

$$
\psi_n(r) = \sum_{l_x,l_y} \left( c_{l_x,l_y}^\uparrow \psi_{l_x,l_y}^\uparrow + c_{l_x,l_y}^\downarrow \psi_{l_x,l_y}^\downarrow \right),
$$

where the $n$-dependent coefficients $c_{l_x,l_y}^\uparrow$ and $c_{l_x,l_y}^\downarrow$ are determined from the matrix eigenvalue problem, and $r = (x,y)$. The SOC leads to the spin-coordinate entanglement of the state $\psi_n(r)$, while at $\alpha_R = 0$ the corresponding state is the product of the orbital state and eigenstate of $\sigma_z$.

The typical level splitting in a $\mu$m-size billiard, being significantly lower than in a nanoscale quantum dot, leads to a better possibility to reproduce the quantum-classical correspondence in the regular and stochastic evolution. Berggren and Ouchterlony proved that the SOC leads to the non-Poissonian level statistics, indicating the presence of quantum chaos \[22\]. However, this well-known picture of static eigenstate description leaves open the question on the dynamical characteristics of the associated evolution. In particular, on the difference between simply irregular and chaotic behavior for coupled spin and charge degrees of freedom, which we consider in our paper. The external driving is needed since it allows manipulating the perturbation amplitude in a controlled manner, which is required in most of the applications to transfer and keep the system in a required state in the presence of inevitable momentum and spin relaxation. We will see that the quantum-classical correspondence with the generation of the chaotic behavior of observables will be maintained mainly in the initial period of the evolution, and after that more regular quantum dynamics is achieved both for charge and spin degrees of freedom. This is in agreement with general properties of quantum chaos \[1\,3\].

The driving term $V(t) = e \mathcal{E}_0 x \cos \omega t$, where $e$ is the fundamental charge, is chosen as the monochromatic uniform electric field of amplitude $\mathcal{E}_0$ which induces the local resonance between the level pair split by $E_{n_0} - E_{n_0-1} = \hbar \omega_0$ for a given $n_0$. Due to the spin-coordinate entanglement in Eq. \[2\], the electric field causes transitions between states with different expectation values of spin, and leads to nontrivial dynamics as discussed below.

To characterize the couplings in the system, we apply a perturbative approach and find the dimensionless parameters for spin-orbit coupling and electric field strength. We begin with the unperturbed spectrum:

$$
E(l_x,l_y,\sigma_z) = \frac{\pi^2 \hbar^2}{2m} \left( \frac{l_x^2}{a^2} + \frac{l_y^2}{b^2} \right) + \frac{1}{2} g \mu_B \sigma_z B_x
$$

and take the main semiclassical term (assuming $l_x \gg 1, l_y \gg 1$) in energy difference of the same-spin states

$$
\Delta E \equiv E(l_x+1, l_y, \sigma_z) - E(l_x, l_y, \sigma_z) = \frac{\pi^2 \hbar^2}{ma^2} l_x.
$$

For spin-orbit coupling we take the ratio

$$
f_{so} \equiv \frac{|(l_x+1, l_y, 1)|_{H_{so}} |(l_x, l_y, -1)|}{\Delta E \pi l_x^{-1}} = \frac{2 \alpha_R a m a^2}{\hbar^2},
$$

that is, essentially, the ratio of the billiard size to the spin precession length $\hbar^2/ma_R$ as a dimensionless strength of spin-orbit coupling. For the driving field we proceed similarly and define

$$
f_E \equiv \frac{e \mathcal{E}_0 |(l_x+1, l_y, 1)|_{H_{so}} |(l_x, l_y, 1)|}{\Delta E l_x^{-1}} = \frac{2}{\pi^4} e \mathcal{E}_0 a m a^2 \hbar^2.
$$

### III. LEVEL VARIANCE EVOLUTION AND FOURIER POWER SPECTRA

We begin by solving the nonstationary Schrödinger equation with the driving $V(t)$ in the basis of the eigenstates \[2\] to obtain the spinor wavefunction

$$
\Psi(r,t) = \sum_n C_n(t) \psi_n(r),
$$

where the time-dependent coefficients $C_n(t)$ are solutions of a system of ordinary differential equations. The initial condition is taken as the single level occupancy, $C_n(0) = \delta_{n,n_0}$. The initial single level state can be prepared in a mesoscopic billiard by resonant tunneling of an electron with required energy, entering from attached leads \[28\]. If the initial state is a superposition of the eigenstates, its dynamics can be found as the corresponding superposition of the time-dependent states demonstrated below.

For numerical calculations we consider a GaAs billiard with (if not stated otherwise) $\alpha_R = 5$ meVnm, $a = 2.1 \mu$m and $b = 1.5 \mu$m (same as in Ref.\[22\]). Here $\hbar^2/ma^2 = 0.26 \mu eV$, and $f_{so} \approx 2.0$. We assume a magnetic field $B_x = 500$ Gs with the Zeeman splitting of $1.3 \mu eV$. The
expression of the Hilbert space diffusion \[2, 5, 23\]. As the mean level number¯
ing the evolution in the Hilbert space can be chosen
evolution, where \(\Delta\) time, this stage transforms into the stabilized quantum
counterpart of the classical chaotic dynamics. After some
creasing number of levels being involved, and represents
corresponds to the diffusion in the Hilbert space with in-
ber \(\Delta\)
is known that during the initial stages of the develop-
(\(\nu_0\) typical driving frequency here is \(\nu = \nu_0/2\pi = 0.78\) GHz,
and the initial state is on the level \(n_0 = 200\).
We calculate the evolution of quantum observables
using the wavefunction \(\text{[2]}\). The parameters describ-
ing the evolution in the Hilbert space can be chosen
as the mean level number \(\hat{L}(t) = \sum L|\langle C_L(t)|\|^2 \)
and its dispersion \(\langle \Delta^2 L(t)\rangle = \sum L (L - \hat{L}(t))^2 |\langle C_L(t)|\|^2\). It
is known that during the initial stages of the develop-
ment of quantum chaos the variance of the level number
\(\Delta L(t) \equiv \langle \Delta^2 L(t)\rangle^{1/2}\) grows with time \(\text{[2]}\). This
represents the chaotic behavior.

The stroboscopic dynamics of level variance \(\Delta L(t)\) is
shown in Fig.\(\text{[2]}\) for 500 periods \(T\), of driving (a) for the
moderate field \(\mathcal{E}_0 = 0.14\) V/cm \((f_\text{e} \approx 2.3)\) and (b) for
stronger field \(\mathcal{E}_0 = 0.70\) V/cm \((f_\text{e} \approx 11.5)\). It is clear
that the initial fast growth in the number of involved lev-
els ceases after \(N = 10 \ldots 30\) periods of driving field, and
after that \(\Delta L(t)\) demonstrates the oscillating behavior
around the average \(\Delta L_{\text{av}} \approx 9\) in Fig.\(\text{[2]}\)a and \(\Delta L_{\text{av}} \approx 46\)
in Fig.\(\text{[2]}\)b). This behavior corresponds to the expected
growth in the number of participating states with in-
creasing driving amplitude. We may conclude that the
dynamics rather quickly reaches a stabilized regime without
further diffusion in the Hilbert space, indicating that even
a large billiard with hundreds of levels involved in the
dynamics behaves essentially as a quantum system without
the long-lasting Hilbert space diffusion.

Quantum evolution can be described in terms of the
power spectrum, defined for an observable \(\xi(t)\) as

\[
I_\xi(\omega) = \left| \int_{-\infty}^{+\infty} \xi(t) e^{-i\omega t} dt \right|^2. \tag{8}
\]

Figures\(\text{[2]}\).c),(d) show power spectra for the spin com-
ponent \(\sigma_z\) for the same driving fields, plotted in di-
ensionless units for mutual comparison. While for Fig.\(\text{[2]}\)c)
the spectrum has the form of discrete harmonics, the
stronger driving field leads to qualitatively richer spec-
trum as shown in Fig.\(\text{[2]}\)d) where the lower band is filled
continuously. According to the basic concepts of quan-
umin.\(\text{[1]}\) this may be considered as the onset of chaos.
It should be mentioned that the long-period patterns in level variations visible in Fig.\(\text{[2]}\)a),(b) provide
sizable contributions into the Fourier power spectra for the
spin component in Fig.\(\text{[2]}\)c),(d) as the strong peaks
at the left part of the frequency axis near \(\omega = \omega_0\). How-
ever, the main difference between the regular and chaotic
dynamics is in the mid- and high-frequency part of the
spectrum, as it can be seen by comparing Figs\(\text{[2]}\)c) and
(d), where a densely filled frequency band in Fig.\(\text{[2]}\)d)
represents the chaotic behavior.

IV. Poincaré Sections

In order to gain insight into mutual impact of orbital
and spin motion, we plot the evolution of expectation
values in the pair of canonical variables \((x(t), p_x(t))\) and
in the non-canonical pairs \((S_\beta(t), S_x(t))\), where \(\beta\) and \(\gamma\) are Cartesian coordinates. For a periodic driving it is
of interest to consider the Poincaré sections at stro-
boscopic times \(t = NT\) (with \(N\) an integer) such as
\((x(N), p_x(N))\) or \((S_\beta(N), S_x(N))\). To obtain the ev-
olution with high accuracy, we use the Floquet strobos-
topic technique which requires the direct integration of
the time-dependent Schrödinger equation only at a sing-
le period \(T\) of the perturbation \(V(t)\). After that
the state of the system at any \(t = NT\) can be obtained by
a finite algebraic procedure \(\text{[18, 22]}\). In Fig.\(\text{[2]}\) we show
the stroboscopic Poincaré plots for the same fields as in
Fig.\(\text{[2]}\).

Due to the Rashba SOC the coordinate degree of free-
dom evolves in the same manner as the spin one, indi-
cating an irregular regime of dynamics like a “stochastic
sea” \(\text{[1, 2]}\) without any “islands of regularity” with peri-
odic orbits. This example of dynamics may serve as a tool
for observing the spin chaos in quantum systems. In the
absence of SOC, the states would remain the eigenstates
of \( \sigma_x \), and the Poincaré sections would be reduced to the points \( S_x(N) = \pm 1 \) and \( S_y(N) = S_z(N) = 0 \). Small expectation values of spin components are due to the spin-coordinate entanglement of the states in Eq. (4). It can be seen that the spin dynamics is strongly sensitive to the number of levels involved each having a different spin polarization, so the amplitudes for the mean values also grow at higher electric fields. By looking at Fig. 2 one also notices that the dynamics in coordinate space is not so sensitive to the driving field strength as the evolution of spin variables. This can be explained by taking into account the structure of eigenstates and the initial condition of our evolution model which is an eigenstate with a rather high number of spatial harmonics, and is completely delocalized in the billiard. The volume spanned by the evolution of the \( (x, p_x) \) pair of mean values does not expand greatly with increasing driving strength since these variables have comparable expectation values for two different numbers of levels effectively involved into the dynamics for weak (Fig. 1(a)) and moderate (Fig. 1(b)) driving amplitudes.

It should be mentioned also that the average value of spin for the weaker driving is shifted from zero more visibly, as it can be seen in Fig. 2. We explain this behavior by distinct, but generally alternating spin polarizations of the basis states, where the greater number of states involved at stronger driving leads to more effective canceling of non-zero contributions to the midpoint of spin density stroboscopic ensemble.

V. SPIN TEXTURES AND SPIN-CHARGE SEPARATION

A typical experiment with scanning of the billiard with electron gas measures local spin density integrated over the “spot” under the probe [24]. Thus, the evolution of local spin density in the billiard can be of interest for further experimental advances in exploring and controlling various regimes of the driven spin dynamics. We then look at the spatial distributions of spin density in the whole billiard, or spin textures, considered at the stroboscopic time \( t = NT \) with arbitrary \( N \), together with the charge density contained in the spinor components.

The charge- \( \rho(r, N) \) and spin density components \( S_\beta(r, N) \) are found with the wavefunction \( \Psi \) as

\[
\rho(r, N) = \Psi^\dagger(r, N) \Psi(r, N),
\]

\[
S_\beta(r, N) = \Psi^\dagger(r, N) \sigma_\beta \Psi(r, N),
\]

respectively. In Fig. 3 we show the probability distributions for the charge and the \( S_z \) component for the initial eigenstate \( (n = 200) \) with high number of spatial harmonics, and after \( N = 500 \) periods of driving. The other components of spin density as well as the probability density have similar patterns. The charge and spin densities calculated at \( N < 500 \) indicate that the picture is stabilized after several hundreds of driving periods, similar to the mean level dynamics in Fig. 1(a),(b). We have found that the distributions presented in Figs. 3(b),(c),(e),(f) are formed in the evolution in the electric field as the interplay of two distinct patterns, which modify the regular structure of the initial state in Fig. 3(a),(d). One pattern is the average-scale and large-scale structure with regular spatial oscillations stemming from the limited number of basis states effectively involved into dynamics. The other pattern of spin density in Fig. 3 has an irregular and spatially chaotic character, mostly on the small and medium scales with formation of peaks with variable height. The amplitude of small scale irregular contribution to the spin density grows with increasing driving strength. This is an indication of the chaotic regime which is induced in our system at strong driving field. We have observed the formation of similar two-scale density distributions for both charge and spin also at longer times compared to the snapshot in Fig. 3. The formation of this stable picture can be attributed to the onset of the quasi-stationary profile of the

FIG. 2: (Color online) stroboscopic Poincaré plots: (a,d) coordinate mean values \( (x(N), p_x(N)) \) and (b) and (e) spin mean values \( (S_x(N), S_y(N)) \) and (c) and (f) - spin mean values \( (S_y(N), S_x(N)) \), shown for the driving field strength ((a)-(c)) \( \mathcal{E}_0 = 0.14 \) V/cm and ((d)-(f)) \( \mathcal{E}_0 = 0.70 \) V/cm. The coordinate degree of freedom evolves in the same manner as the spin degree of freedom due to Rashba SOC, both indicating an irregular regime of dynamics. Momentum \( p_x \) is measured in units of \( \hbar/\mu m \).
FIG. 3: (Color online) Probability density distributions for (a)-(c) the charge and (d)-(f) the $S_z$ component of spin density in the billiard (a),(d) for the initial state taken as an eigenstate of the billiard with high number of spatial harmonics, and after $NT = 500$ periods of driving field with amplitude (b),(e) $E_0 = 0.14$ V/cm, and (c),(f) $E_0 = 0.70$ V/cm. The spatial distributions for the $S_z$ (and other spin components) have the regular component on the medium scale and the irregular peaked contribution on small scale for both initial state (a),(d) and after the driven evolution (b,c,e,f).

FIG. 4: (Color online) The magnitude of the time-dependent correlator between the charge density and the $S_x$ component of spin density for $E_0 = 0.14$ V/cm; $\alpha_R = 0.5$ meVnm (solid line), $\alpha_R = 1.5$ meVnm (dashed line), and $\alpha_R = 5.0$ meVnm (dashed-dotted line). The correlators fall below 0.2 when SOC strength is high, and the spin-charge separation occurs.

In our billiard the chaos is generated by the SOC rather than by the geometry [22]. After turning on the driving this initially chaotic structure of the spectrum and the eigenstates determines the dynamical variables and density distributions such as the spin textures in Fig.3. One of the tools for checking the degree of spin-charge separation is the correlators between the charge- and the spin density components taken on the spatial grid with $N$ points and treated as statistical variables. We then calculate the correlators of quantities $q$ and $v$ as $r_{q,v} = K_{q,v}/w_q w_v$, where

$$K_{q,v} = \frac{\sum_i q_i v_i}{N} - m_q m_v$$  \hspace{1cm} (11)

is the correlation coefficient, $m_{q,v}$ is the mean value, and $w_{q,v}$ is the corresponding variance. An example of such a time-dependence is shown in Fig.4. The amplitudes of correlators are below 0.2 when SOC is strong mixing several energy levels, and the spin-charge separation occurs. This result can be viewed as another consequence of the strong SOC and driving producing the time-dependent entanglement of the charge and spin degrees of freedom.

VI. CONCLUSIONS

We have studied the electron dynamics in a quantum billiard with spin-orbit coupling and driven by a monochromatic electric field. It was found that the spatial and time resolved patterns for probability and spin densities demonstrate the onset of chaotic dynamics with qualitative modifications of the power spectra and spatial patterns. In particular, we have identified new regimes of quantum chaos in this system described by two-scale spatial charge and spin density distributions. The onset of
spin-charge separation effect is predicted by the dynamics of the spin and charge density correlators. Our predictions can be important for the understanding of the coupled spin-charge transport through mesoscopic billiards driven by a uniform electric field, where chaos can arise for both spin and charge current observables. The stationary density distributions seen in the absence of the driving can be verified in the tunneling experiments similar to those presented in Ref. [24].

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[1] M.C. Gutzwiller, Chaos in Classical and Quantum Mechanics, Springer-Verlag, New York, 1990.
[2] L.E. Reichl, The Transition to Chaos. Conservative Classical Systems and Quantum Manifestations, 2nd Ed., Springer-Verlag, New York, 2004.
[3] F. Haake, Quantum Signatures of Chaos, Springer-Verlag Berlin Heidelberg, 3rd Ed., 2010.
[4] H.-J. Stöckmann, Quantum Chaos: An Introduction, Cambridge University Press, 1999.
[5] G. Casati, B.V. Chirikov, F.M. Izrailev, and J. Ford, Lect. Notes Phys. 93, 334 (1979)
[6] K. Nakamura and T. Harayama, Quantum Chaos and Quantum Dots, Oxford University Press, New York, 2004.
[7] N. Bushong, Y.V. Pershin, and M. Di Ventra, Phys. Rev. Lett. 99, 226802 (2007).
[8] M. Di Ventra, Electrical transport in nanoscale systems, (Cambridge University Press, Cambridge, 2008).
[9] T. Dietl, D.D. Awschalom, M. Kaminska, and H. Ohno, SpINTRONICS, (Elsevier, Amsterdam), 2008.
[10] Spin physics in semiconductors, ed. by M.I. Dyakonov, Springer-Verlag Berlin Heidelberg, 2008.
[11] I. Zutić, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. 76, 323 (2004).
[12] E. I. Rashba and Al. L. Efros, Phys. Rev. Lett. 91, 126405 (2003).
[13] K.C. Nowack, F.H.L. Koppens, Yu.V. Nazarov, and L.M.K. Vandersypen, Science 318, 1430 (2007).
[14] M. Pioro-Ladriere, T. Obata, Y. Tokura, Y.-S. Shin, T.Kubo, K. Yoshida, T. Taniyama, and S. Tarucha, Nature Physics 4, 776 (2008).
[15] V.N. Golovch, M. Borhani, and D. Loss, Phys. Rev. B 74, 165319 (2006).
[16] L.S. Levitov and E.I. Rashba, Phys. Rev. B 67, 115324 (2003).
[17] D. V. Khomitsky and E. Ya. Sherman, Phys. Rev. B 79, 245321 (2009).
[18] D. V. Khomitsky, L.V. Gulyaev, and E. Ya. Sherman, Phys. Rev. B 85, 125312 (2012).
[19] L. Chotorlishvili, Z. Tklikishvili, A. Komnik, and J. Berakdar, Phys. Lett. A 377, 69 (2012).
[20] K.-F. Berggren, A.F. Sadreev, and A.A. Starikov, Phys. Rev. E 66, 016218 (2002).
[21] K.-F. Berggren, D.N. Maksimov, A.F. Sadreev, R. Höhmann, U. Kuhl, and H.-J. Stöckmann, Phys. Rev. E 77, 066209 (2008).
[22] K.-F. Berggren and T. Ouchterlony, Found. Phys. 31, 233 (2001). For a detailed analysis of semiclassical eigenstates in Rashba billiards, see A. Csortós, J. Cserti, A Pályi, and U. Zülicke, Eur. Phys. J. B 54, 189 (2006). For the analysis of spin-orbit coupling effects on level statistics and electromagnetic response of metal nanoparticles see, for example, L.P. Gor’kov and G.M. Éliashberg, Sov. Phys. JETP 21, 940 (1965).
[23] K.-F. Berggren and T. Ouchterlony, Found. Phys. 31, 233 (2001). For a detailed analysis of semiclassical eigenstates in Rashba billiards, see A. Csortós, J. Cserti, A Pályi, and U. Zülicke, Eur. Phys. J. B 54, 189 (2006). For the analysis of spin-orbit coupling effects on level statistics and electromagnetic response of metal nanoparticles see, for example, L.P. Gor’kov and G.M. Éliashberg, Sov. Phys. JETP 21, 940 (1965).
[24] E.N. Bulgakov and A.F. Sadreev, Phys. Rev. E 70, 056211 (2004).
[25] A.T. Ngo, E.H. Kim, and S.E. Ulloa, Phys. Rev. B 84, 155457 (2011).
[26] J.D. Urbina, M. Wimmer, D. Bauernfeind, D. Espitia, I Adagideli, and K. Richter, Phys. Rev. E 87, 042115 (2013).
[27] A.F. Sadreev, E.N. Bulgakov, and I. Rotter, Phys. Rev. B 73, 235342 (2006); E.N. Bulgakov and I. Rotter, Phys. Rev. E 73, 066222 (2006); I. Adagideli, Ph. Jackoud, M. Scheid, M. Duckheim, D. Loss, and K. Richter, Phys. Rev. Lett. 105, 246807 (2010); D. Waltner, J. Kuipers, Ph. Jacqoud, and K. Richter, Phys. Rev. B 85, 024302 (2012).