Fatigue life prediction of flexible bearings based on ABAQUS and nCode-DesignLife

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Abstract. Flexible bearings are critical components of harmonic drives. In service, the corresponding positions of the long and short axes of the inner and outer rings of a flexible bearing regularly change. This leads to a fatigue fracture of the outer ring because of the cyclic alternating load. In this study, a multi-body contact model and a dynamic model were established using ABAQUS to analyze the load spectrum of a flexible bearing and a wave generator. The dynamic analysis results were employed to simulate the fatigue life of the inner and outer rings using nCode-DesignLife. The results show that the inner ring underwent significant fatigue damage at the long-axis position, and the corresponding fatigue life was low. As the damage on the outer ring was due to continuous alternating deformation, the inner and outer surfaces of the outer ring were significantly damaged. Besides, the outer ring underwent fatigue damage prior to the inner ring under the same working conditions. The results provide theoretical references and practical guidance for the design and life estimation of flexible thin-wall bearings.

1. Introduction

Flexible bearing, which are intermediate deformation components used in harmonic drives, are key elements affecting the work performance, load carrying capacity, and service life of harmonic drives [1-2].

Current studies on harmonic drives have reduced the flexible bearing and wave generator to a rigid cam, ignoring the complicated internal contact state and kinematic characteristics. The failure of the flexible bearing is one of the critical factors leading to the failure of the harmonic drive [3]. The accuracy of the results of most studies that predicted the service life of flexible bearings using the Lundberg–Palmgren theory, which requires that the outer ring of the bearing be installed on a rigid bearing chock and with the load remaining unchanged [4-5], is questionable.

Considering the particularity of the pre-deformation and loading conditions, a dynamic model of a flexible bearing is established to analyze the stress distribution and load spectrum. Based on the dynamic analysis results, a fatigue analysis software was used to simulate the fatigue life of the flexible bearing. The research results have strong engineering application value for the design and optimization of flexible thin-wall bearings.
2. A dynamic model of flexible bearing
The explicit dynamics method was used to simulate the rotation of the flexible bearing under loading conditions. First, the loading model of the flexible bearing is analyzed, and an explicit dynamic analysis model of the flexible bearing is then established.

2.1. Analysis of loading model of flexible bearing
According to the principle of harmonic drives, the flexible bearing is associated with the flexspline, and the loading is relatively complicated. The rigid and flexible wheels are meshed in the long-axis region and are distributed in a mostly symmetrical manner [6-7]. Figure 1 shows the distribution of the loading.

In figure 1, the load distribution of the flexible bearing is approx. \( q_r = q_i \), and in the region where \( \phi_i \) is located, the meshing force between the flexspline and the rigid wheel can be calculated as follows.

\[
\begin{align*}
\bar{q}_r &= \bar{q}_t \cos \left( \frac{\pi (\phi - \phi_1)}{2 \phi_2} \right) \\
\bar{q}_t &= \bar{q}_r \tan \alpha
\end{align*}
\]  

(1)

where \( \bar{q}_r \) and \( \bar{q}_t \) are circumferential and radial components, respectively, of the meshing force; \( \alpha \) is the meshing angle of the flexible wheel.

In a harmonic reducer, if the torque experienced by the flexible wheel is \( T \), then \( T \) and \( \bar{q}_{t \text{max}} \) have the following relationship.

\[
\begin{align*}
T &= 4 \int_{\phi_1}^{\phi_2} b_R \left( \frac{d_g}{2} \right)^2 \bar{q}_{t \text{max}} \cos \left( \frac{\pi (\phi - \phi_1)}{2 \phi_2} \right) d\phi \\
\bar{q}_{t \text{max}} &= \frac{\pi T}{(2 \phi_2 d_g^2 b_R)}
\end{align*}
\]  

(2)

(3)
According to the thin-walled ring theory, the deformation of the ring due to the tangential load is the same that due to the corresponding radial load. Hence, the relationship between the radial load $q_r$ and the tangential load $q_t$, can be given as follows.

$$q_{rt} = \int q_t \, d\varphi$$

(4)

The following relationship can be obtained by performing the Fourier series expansion on $q_t$ in the above equation.

$$\bar{q}_t = q_{tmax} \cos[\pi(\varphi - \varphi_1) / 2\varphi_2] = \frac{4\varphi_2}{\pi^2} q_{tmax} + 8\varphi_2 q_{tmax} \sum_{k=2}^{\infty} \frac{\cos k\varphi_2 \cos k(\varphi - \varphi_1)}{\pi^2 - 4k^2\varphi_2^2}$$

(5)

$$= q_{t0} + \sum_{k=2}^{\infty} q_{tk} \cos k(\varphi - \varphi_1)$$

Here, $q_{t0} = \frac{4\varphi_2}{\pi^2} q_{tmax}$, $q_{tk} = \frac{8\varphi_2 q_{tmax} \cos k\varphi_2}{\pi^2 - 4k^2\varphi_2^2}$, and $k$ takes 2, 4, 6...

The constant $K$ is adjusted by the torque $T$ of the output shaft. Therefore, only the variation component in the integral equation is required.

$$\bar{q}_{rt} = \int \bar{q}_t \, d\varphi = \int \sum_{2,4,6,\ldots} \bar{q}_{tk} \cos k(\varphi - \varphi_1) \, d\varphi$$

(6)

$$\bar{q}_{rt} = \sum_{2,4,6,\ldots} \bar{q}_{tk} \sin k(\varphi - \varphi_1) \, d\varphi + C$$

(7)

Because the wave generator only bears the positive pressure transmitted from the flexible wheel, the integral constant $C$ can be determined using the one-way contact condition.

$$C = -\sum_{2,4,6,\ldots} \bar{q}_{tk} \sin k(\gamma - \varphi_1) \, d\varphi$$

(8)

The total radial load on the outer ring of the bearing can be obtained as the sum of the equivalent radial load and the radial load of the meshing force, as follows.

$$\bar{q}_{rh} = \bar{q}_r + \bar{q}_{rt} = \bar{q}_r + \sum_{2,4,6,\ldots} \bar{q}_{tk} \sin k(\varphi - \varphi_1) \, d\varphi + C$$

(9)

where $\bar{q}_r$ and $\bar{q}_n$ are total and equivalent radial loads, respectively; $\bar{q}_r$ is the radial load of the meshing force.

The distribution of the actual load acting on the flexible bearing is indicated using the red dotted line shown in figure 1. The effect of partial loading is ignored, and the load is simplified in this study, as shown in figure 2. The load distribution is as follows:

$$\bar{q}_{rh} = \bar{q}_{rmax} \cos(\pi\varphi / \varphi_4)$$

(10)

2.2. A dynamic model of flexible bearing

The flexible bearing model CSF_25_80 was established. Table 1 lists its geometric parameters.
Table 1. Bearing parameters.

| Main parameters                        | Value  |
|----------------------------------------|--------|
| Inner diameter of bearing [mm]         | 45.1   |
| Thickness of inner ring [mm]           | 3      |
| Width of the inner ring [mm]           | 6.35   |
| Outer diameter of bearing [mm]         | 58.3   |
| Thickness of outer ring [mm]           | 3      |
| Width of the outer ring [mm]           | 9      |
| Diameter of the ball [mm]              | 5.556  |
| Long-axis radius of standard cam [mm]  | 22.95  |
| Short-axis radius of standard cam [mm] | 22.15  |
| Number of balls                        | 23     |

The model is subdivided into full hexahedral meshes via scanning and mapping, and the contact area is refined to improve the analysis accuracy and solution speed.

It is assumed that the inner ring rotates with the camshaft, the outer ring undergoes only radial elastic deformation, and the applied load rotates with the shaft. The applied radial load is 1000 N, and the span angle ($\phi_4$) is 120°. The rotation speed of the camshaft is 1000 rev/min.

2.3 Analysis of dynamic simulation results of flexible bearing

The groove bottom circles of the inner and outer raceways are selected as the analysis paths (the start and end points of the paths are both at the 180° azimuth, as shown in figure 2; the paths are selected in the clockwise direction). Figure 3 shows the displacement curve (in the Y-axis direction, shown in figure 2) of the paths when the assembly and loading are complete.

![Figure 3. Displacement distribution on the bottom circle of the inner and outer rings.](image)

Figure 3 indicates that the inner ring undergoes the most considerable radial deformation along the long axis and the smallest radial deformation along the short axis; the radial deformation of the outer ring is similarly distributed. As the inner ring is directly assembled on the elliptical shaft and the outer ring is forced to deform when contacted with the rolling elements, the radial deformation of the inner ring is slightly higher than that of the outer ring at the proximal short-axis position.

Figure 4 shows the distribution of the contact force of each rolling element when the assembly and loading are complete. The analysis results show that the rolling elements in the vicinity of the long axis...
are subjected to a large load, whereas those in the vicinity of the short axis are not subjected to any loading. Among the twenty-three rolling elements, thirteen are loaded, and the load distribution is approximately symmetric concerning the short axis of the elliptical axis.

![Figure 5. Displacement evolution curve of the outer and inner ring nodes.](image1)

![Figure 6. Stress evolution curve of the inner and outer ring nodes.](image2)

The nodes in the 90° azimuth of the inner and outer ring raceway grooves, shown in figure 2, are selected as the research nodes. Figures 5 and 6 show the displacement curves (along the Y-axis direction) and stress curves of the chosen nodes during operation, respectively.

Figure 5 shows that the spatial displacement of the inner ring node reciprocates once from 0.06 s to 0.12 s (elliptical axis rotates one week), the displacement of the outer ring node undergoes four cyclic alternations between −0.4 mm and 0.4 mm, and the change in the outer ring node displacement is in the form of a regular cosine curve over time.

At 0.09, 0.12, 0.15, and 0.18 s, when the long axis passes through the selected node of the outer ring, the stress increases sharply because of the combined action of deformation and loading. As the inner ring is assembled with the camshaft, the shape of the deformation remains unchanged during the movement. When the rolling ball passes through the position of the node, the contact stress mutates.

3. Simulation analysis of fatigue life
The failure of a flexible bearing can be due to many reasons. The contact stress and cyclic bending stress in service are two main factors causing fatigue failure [8].

3.1. Determination of fatigue analysis parameters for flexible bearing
According to the fatigue analysis principle of nCode-DesignLife [9-10] and based on the dynamic analysis results, the loading spectra of the inner and outer rings can be obtained. Their material properties were defined according to reference [11]. The S-N curve equation for GCr15 steel under an experimental condition with 99% reliability can be written as follows.

$$\log(\lambda) = 43.3934 - 12.4999 \log(S)$$  \hspace{1cm} (11)

where \(S\) is cyclic stress and \(\lambda\) is the time to fracture under cyclic loading.

3.2. Results and analysis
Figures 7 and 8 show the fatigue damage and life cloud diagrams of the inner and outer rings, respectively.
Figure 7. Inner ring fatigue damage cloud diagram (a)—L and outer ring fatigue damage cloud diagram (b)—R.

Figure 7(a) shows that the region with a higher fatigue damage on the inner ring is mainly concentrated on the surface near the long and short axis of the cam outline, and the damage is more severe in the long-axis raceway than that in other areas. The damage on the inner ring is maximum at node N861 with a damage size of $2.799 \times 10^{-10}$. As shown in figure 7(b), the fatigue damage on the outer ring is mainly concentrated on the surface area, and the damage on the outer and inner surfaces near the raceway is more significant. The damage on the outer ring is maximum at node N1613 with a damage size of $1.38 \times 10^{-9}$.

The fatigue life of the components obtained using the nCode-DesignLife software is defined as the number of cycles corresponding to the load history. A working cycle is equal to 0.12 s.

Figure 8. Inner race fatigue life cloud diagram (a)—L and outer race fatigue life cloud diagram (b)—R.

Figure 8(a) shows that the lowest fatigue life position of the inner and outer rings corresponds to the highest fatigue damage position. The minimum fatigue cycle number for the inner ring is $3.573 \times 10^9$. 
Figure 8(b) shows that the minimum fatigue cycle number for the outer ring is $7.262 \times 10^8$. The minimum fatigue life of the inner and outer rings under the same working conditions is 24206.67 h.

4. Conclusions

The conclusions drawn from this study are as follows.

✓ When the flexible bearing was installed on the camshaft, the inner ring deformed only once and then rotated synchronously with the camshaft. The relative position of any node on the outer ring regularly changed in service. As the change in the displacement could be represented in the form of a regular cosine curve over time, the outer ring was always subjected to alternating bending.

✓ The fatigue damage on the inner ring was mainly due to the contact stress, whereas the outer ring simultaneously underwent contact fatigue failure and bending fatigue failure. In service, the outer ring underwent fatigue failure before the inner ring. Through simulation, the maximum fatigue life of the selected flexible bearing was found to be 24206.67 h under rated conditions.

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