On the proper treatment of massless fields in Euclidean de Sitter space

Arvind Rajaraman

Department of Physics and Astronomy, University of California, Irvine, CA 92697, USA
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We analyze infrared divergences arising in calculations involving light and massless fields in de Sitter space. We show that these arise from an incorrect treatment of the constant mode of the field, and show that a correct quantization leads to a well-defined and calculable perturbation expansion. We illustrate this by computing the first nontrivial loop correction in a theory of a massless scalar field with a quartic interaction.

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I. INTRODUCTION

There has been much recent interest in the possibility of probing nongaussianities in the CMB at the WMAP and PLANCK experiments [1–3]. Such nongaussianities occur in theories where the inflaton has significant self-interactions (and more generally if it has interactions with other fields participating in the inflationary epoch). It is thus of great importance to calculate the prediction for nongaussianities in a theory with inflaton interactions. Unfortunately, the existence of light fields leads to infrared divergences which make such calculations suspect (see, for example, [4–7]).

These divergences often arise because of the infinite expanding volume of de Sitter space. The volume of the metric grows exponentially in the global time coordinate, which leads to an effective growth in the coupling at late times. While this is compensated to some extent by the falloff of the wave functions, it can be shown that even tree level scattering amplitudes grow with time [5, 8, 9]. In loop diagrams, this same effect leads to late-time divergences which have been argued to signal the breakdown of de Sitter field theory [10], perhaps leading to a decay to another vacuum [11]. On the other hand, these divergences can be eliminated entirely by first taking the Euclidean continuation of the theory, which transforms de Sitter space to the Euclidean sphere. The sphere, being compact, does not have large volume divergences. Correlation functions are therefore infrared finite, and can be continued back to the Lorentzian theory to obtain well defined results [8]. This suggests that the late time divergences are unphysical, and arise from the breakdown of the in-out

*Electronic address: arajaram@uci.edu
formalism in Lorentzian de Sitter space in global coordinates \[12\]. We henceforth assume that the true definition of the de Sitter theory is by a continuation from the theory on the Euclidean sphere.

Even though the Euclidean formalism eliminates divergences coming from the infinite volume of de Sitter space, light or massless particles can still lead to infrared divergences. For example, in the theory of a scalar field $\phi$ with a mass $m$ and an interaction $\lambda \phi^4$, a $k$-loop diagram has a factor $\left( \frac{\lambda H^2}{m^2} \right)^k$ \[13\]. For $m^2 \ll \lambda H^2$, the loop diagrams are larger than the tree contribution, and perturbation theory seems to break down. This is a problem for theories of slow roll inflation in which the inflaton is very light field; gravitons also behave similarly to massless scalars, and may potentially lead to divergences. Understanding the massless limit is therefore important for calculations in perturbation theory.

In this note, we reexamine the $\lambda \phi^4$ model in de Sitter space. Our main result is that even in the limit of very small masses ($m^2 \ll \lambda H^2$), the perturbation theory of this model is under control. We show that the $k$-loop diagram scales as $(\sqrt{\lambda})^k$ in the limit $m^2 \to 0$. The apparent infrared divergences thus change the expansion parameter from $\lambda$ to $\sqrt{\lambda}$; nevertheless, for small $\lambda$, the perturbation theory is well defined. For the formalism on the Euclidean sphere, these corrections can be computed explicitly.

In the next section, we begin by reviewing the basic features of de Sitter space, and discuss how light fields lead to the appearance of infrared divergences. We then show that in the Euclidean formalism, the divergences are entirely due to the incorrect treatment of one mode. We show in section III that the correct treatment of this mode removes the divergences and makes tree level correlation functions finite. In section IV we extend this to loop diagrams, and show that they are also well defined and calculable; we illustrate this by computing the first nontrivial loop correction to the two-point correlation function of the field in the massless theory. We then discuss how our methods may be modified for the in-in formalism, and finally end with a discussion of our results.

## II. SCALAR FIELD THEORY IN DE SITTER SPACE

There are many possible coordinatizations of de Sitter space \[14\]. We shall be concerned mainly with the coordinatization in global coordinates. In these coordinates, the metric of $D$-dimensional de Sitter (denoted $dS_D$) can be written as

$$ds^2 = R^2(-dt^2 + \cosh^2 t \, d\Omega^2)$$ (1)
where $d\Omega^2$ is the metric on the $d$-sphere with $D = d + 1$. These coordinates cover all of de Sitter space.

In these coordinates, the volume of de Sitter grows exponentially towards both past and future infinity. This exponential growth can lead to infrared divergences in calculations in field theory, since the effective coupling is scaled by the volume. A simple example of this can be seen by considering a massive scalar field theory in de Sitter space [15]. This theory can be solved exactly, and the modes can be solved for. On the other hand, one can attempt to treat the mass as a perturbation by first finding the propagator of the massless theory, and finding the corrections to the propagator as a function of the mass. Curiously, the corrections are found to be divergent at each order, even though the answer is known to be finite. This can be shown to be due to a failure of the standard in-out formalism of field theory [15].

To find a well defined formalism for quantum field theory in de Sitter space, we perform a Wick rotation of the metric. After a rotation $t \to i(\tau - \pi/2)$, the Euclidean metric is found to be

$$ds^2 \equiv R^2 g_{\mu\nu} dx^\mu dx^\nu = R^2(d\tau^2 + \sin^2 \tau d\Omega^2)$$

(2)

To obtain a regular metric, $\tau$ must be compactified by the identification $\tau \to \tau + 2\pi$. $g_{\mu\nu}$ is then the metric on the $D$-sphere of unit radius.

We will consider a scalar field of mass $m$ with a quartic interaction in de Sitter space. Our goal will be to calculate correlation functions in this theory, with particular emphasis on the case of very small masses. The Euclidean action is

$$S_E = \int d^D x R^D \sqrt{g}(R^{-2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2 + \lambda \phi^4)$$

(3)

As usual we begin by solving the quadratic part of the action. The equation of motion is

$$\nabla^2 \phi + m^2 R^2 \phi = 0$$

(4)

where $\nabla^2 \equiv \frac{1}{\sqrt{g}} \partial_\mu \sqrt{g} g^{\mu\nu} \partial_\nu$ is the Laplacian on the unit $D$-dimensional sphere. This equation can be solved in terms of spherical harmonics on the $D$-dimensional sphere.

The $D$-dimensional spherical harmonics have been discussed by several authors (see e.g. [16]). While on the 2-sphere, the harmonics are labeled by two integers $L, m$ with $|m| < L$, the harmonics on the $D$-sphere are labeled by a vector of integers $\vec{L} = (L, L_d...L_1)$ with $L \geq L_d \geq ...L_2 \geq |L_1|$. $L$ is the total angular momentum. We denote the corresponding spherical harmonics as $Y_{\vec{L}}$. These satisfy

$$\nabla^2 Y_{\vec{L}} = -L(L + d)Y_{\vec{L}}$$

(5)
These spherical harmonics also satisfy the orthogonality relations

\[
\sum_{L} Y_L^r(x) Y_L^{r*}(y) = \sqrt{g^{D}}(x - y)
\]

(6)

\[
\int_{S^D} d^D x \sqrt{g} \, Y_L^r(x) Y_M^{r*}(x) = \delta_{LM}
\]

(7)

Much of our discussion will center around the $L = 0$ solution, which we will call the "zero-mode".

The spherical harmonics form a complete set of states. The field $\phi$ can therefore be expanded in modes on the sphere as

\[
\phi = \sum_{L} \phi_L Y_L
\]

(8)

In terms of these modes, the quadratic part of the action becomes

\[
S_2 = R^{d-1} \sum_{L} (L(L + d) + m^2 R^2) |\phi_L|^2
\]

(9)

The exact Euclidean two-point function is defined by the path integral to be

\[
\langle \phi(x)\phi(y) \rangle = \frac{\int \mathcal{D}\phi \, \phi(x) \phi(y) \exp(-S_E(\phi))}{\int \mathcal{D}\phi \, \exp(-S_E(\phi))}
\]

(10)

If we replace the action by $S_2$, and carry out the usual procedure of path integration over bosonic fields, we find the Euclidean correlation function

\[
\langle \phi(x)\phi(y) \rangle = \sum_{L} \frac{Y_L^r(x) Y_M^{r*}(y)}{R^{d-1}(L(L + d) + m^2 R^2)}
\]

(11)

The correlation function can also be written directly in position space as \[8\]

\[
\langle \phi(x)\phi(y) \rangle = \frac{1}{4\pi^{d/2+1}} \frac{\Gamma(d/2)\Gamma(-\sigma)\Gamma(d+\sigma)}{\Gamma(d)} \, _2F_1 \left(-\sigma, d + \sigma; (d + 1)/2; \frac{1 + Z_{xy}}{2}\right)
\]

(12)

where $Z_{xy}$ is the chord distance between the points $x, y$ when the sphere is embedded in a flat $R^{D+1}$, and we have defined $\sigma = -\frac{d}{2} \sqrt{\frac{d^2}{4} - m^2 R^2}$. The de Sitter correlation function can be obtained by rotating the Euclidean correlation function to Lorentzian signature with an appropriate $i\epsilon$ prescription \[8\].

As we have mentioned, since the sphere is compact, there are no divergences arising from integrations over the volume of the space (in contrast to de Sitter which is noncompact). All Euclidean correlation functions are hence finite, and can be continued back to Lorentzian signature to obtain finite results in de Sitter space. This is however only true if the fields are massive; when $m^2 \to 0$, we get a divergence both in tree and loop amplitudes. At tree level, this is most easily
seen by looking at the correlation function. For small masses, we find $\sigma \to -\frac{m^2}{\sigma}$ and the propagator approaches the limit

$$\langle \phi(x)\phi(y) \rangle \to -\frac{1}{4\pi^{d/2+1}} \frac{\Gamma(d/2)}{\sigma}$$

(13)

which is divergent as $m^2 \to 0$.

The same problem will occur in loop diagrams. The one-loop diagram has both a ultraviolet and an infrared divergence, but since it only contributes to the renormalization of the mass, it is completely canceled by the mass counterterm. The leading divergence then comes from the sunset diagram shown here.

![Diagram](image)

The internal propagators are divergent in the massless limit, and this induces a divergence in the loop integral. More generally, a $k$-loop diagram has a factor $(\frac{\lambda H^2}{m^2})^k$, and for $m^2 \ll \lambda H^2$, the perturbation theory breaks down.

III. A PROPER TREATMENT OF THE ZERO-MODE

Looking at the formula for the correlation function, it is clear that the problem comes from the zero-mode. For zero masses, the $L = 0$ term gives a divergent contribution to the sum (11), which then leads to the divergence in (13). In loop diagrams, each zero-mode propagator comes with a factor of $\frac{1}{m^2}$, and for the maximal number of zero-mode propagators, we find a scaling $(\frac{\lambda}{m^2})^k$ for a $k$-loop diagram.

The problem with the zero-mode may be traced to the fact that the terms in the action quadratic in $\phi_0$ are vanishing when $m = 0$. For small masses, the fluctuations are proportional to an inverse power of the mass, which can be large. In the limit of zero masses, fluctuations of the zero-mode are unsuppressed, and the path integral diverges.

For the free theory, this statement is exact but uninteresting since the fluctuations cannot be observed. However, in the interacting theory, it cannot be the case that the fluctuations are unsuppressed; for instance, the $\phi^4$ term would limit the fluctuations of the field even if the mass was zero. Indeed, the large fluctuation of the zero-mode indicates that the terms which are higher order in the zero-mode cannot be neglected. The breakdown of perturbation theory in the small mass limit is thus understood as coming from an incorrect treatment of the zero-mode; we tried
to quantize this mode using the quadratic terms alone, while we should have kept the interaction terms.

More precisely, we only need to keep the $\phi^4_0$ term at leading order; i.e. terms like $\phi^2_0 \phi^2_L \neq 0$ representing interactions between the zero-modes and the nonzero-modes can be treated as perturbations. This is roughly because each of the nonzero-modes has a fluctuation which is small compared to the fluctuation of the zero-mode. The terms involving interactions of only nonzero-modes can also be treated as perturbations for the same reason. We shall explicitly justify this below.

To make the above statements quantitative, we now proceed to quantize the theory keeping the $\phi^4_0$ term. We will be working in the limit where $m^2 \ll \lambda H^2$, which is the parameter region where the infrared divergences are important. These masses are parametrically small, and can be treated in perturbation theory (assuming of course that the $m \rightarrow 0$ limit exists). We will therefore begin by taking the mass to be zero. We will also henceforth set $R = 1$ by a rescaling of the fields.

The leading order action is then

$$S_0 = \lambda \phi^4_0 |Y_0|^2 + \sum_{L \neq 0} L(L + d) |\phi_L|^2$$

$$= \lambda \phi^4_0 \frac{\Gamma((d + 2)/2)}{2 \pi^{(d+2)/2}} + \sum_{L \neq 0} L(L + d) |\phi_L|^2$$

where we used the orthonormality condition to determine $Y_0(x)$ (we have also chosen a phase convention where it is real).

Note that this is now not a Gaussian theory, which would normally make it impossible to work with. In fact, in addition to being nonlinear, the zero-mode is strongly coupled; $\lambda$ can be scaled out of the $\phi^4_0$ interaction. In this case, only one mode is involved in the nonlinear interaction. We can therefore hope to solve this single mode exactly as in quantum mechanics (indeed it is even simpler than quantum mechanics, since there is no time dependence either).

To illustrate this, we find the exact two point function in this theory. The two-point function is now

$$\langle \phi(x) \phi(y) \rangle = \frac{\int D\phi \phi(x) \phi(y) \exp(-S_0(\phi))}{\int D\phi \exp(-S_0(\phi))}$$

$$= \frac{\int D\phi_0 \phi^2_0 Y_0(x) Y_0(y) \exp(-\lambda_{eff} \phi^4_0)}{\int D\phi_0 \exp(-\lambda_{eff} \phi^4_0)} + \sum_{L \neq 0} \frac{Y_L(x) Y^*_L(y)}{L(L + d)}$$

$$= \frac{c_2 Y_0(x) Y_0(y)}{\sqrt{\lambda_{eff}}} + \sum_{L \neq 0} \frac{Y_L(x) Y^*_L(y)}{L(L + d)}$$

$$= \frac{c_2 Y_0(x) Y_0(y)}{\sqrt{\lambda_{eff}}} + \sum_{L \neq 0} \frac{Y_L(x) Y^*_L(y)}{L(L + d)}$$
where we have introduced the constant
\[
c_2 = \frac{\int dx \, x^2 \exp(-x^4)}{\int dx \exp(-x^4)} = \frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})}
\] (19)
and we have defined
\[
\lambda_{eff} = \lambda \frac{\Gamma((d + 2)/2)}{2\pi^{(d+2)/2}}
\] (20)

We thus have an exact solution for the correlation function, which is finite even in the massless limit. In particular, the $\vec{L} = 0$ mode gives a finite contribution to the propagator in this limit.

When the mass term is nonzero and small, the correlation function is corrected. This can be done by treating the mass term as a perturbation in the action and resumming the corrections in the standard way. The result is to shift the poles in the correlation function, which now becomes
\[
\langle \phi(x)\phi(y) \rangle = \frac{Y_{\vec{0}}(x)Y_{\vec{0}}(y)}{\sqrt{\lambda_{eff} c_2} + m^2} + \sum_{\vec{L} \neq 0} \frac{Y_{\vec{L}}(x)Y_{\vec{L}}(y)}{L(L+d) + m^2}
\] (21)

Comparing (11) and (18), the correlation function in position space is now
\[
\langle \phi(x)\phi(y) \rangle = \frac{1}{4\pi^{d/2+1}} \frac{\Gamma(d/2)\Gamma(-\sigma)\Gamma(d+\sigma)}{\Gamma(d)} \ _2F_1(-\sigma, d+\sigma; (d+1)/2; \frac{1 + Z_{xy}}{2})
\] \[
- \frac{\Gamma(d+2)}{2\pi^{(d+2)/2}} \left( \frac{1}{m^2} - \frac{1}{\sqrt{\lambda_{eff} c_2} + m^2} \right)
\] (22)
which in the limit $m^2 \to 0$ goes to the finite limit
\[
\langle \phi(x)\phi(y) \rangle \to \frac{c_2}{\sqrt{\lambda_{eff}}}
\] (23)
This is our main result; the proper quantization of the zero-mode has modified the correlation function in such a way as to render the massless limit finite.

IV. CALCULATIONS IN THE MASSLESS THEORY

We now show explicitly that the proper treatment of the zero-mode leads to finite results for tree and loop diagrams. We shall work in the massless limit; small masses may be treated in a perturbation expansion.

It is immediate that the tree level diagrams are all finite. As we have shown above the propagator is finite in the massless limit. Since the sum over the higher harmonics is convergent, any product of propagators will yield a finite result.

The finiteness of the loop diagrams is not as clear, and requires more analysis.
Consider the sunset diagram in the previous section. If the external states are zero-modes ($\vec{L} = 0$), there is a contribution to the loop where all the internal lines are also zero-mode propagators. This contribution has two factors of $\lambda$ from the vertices, and four factors of $\frac{1}{\sqrt{\lambda}}$ from the propagators, and is therefore an $O(1)$ correction to the correlation function. This is a direct indication that the zero-modes are strongly coupled; the loops are as important as the leading order diagram. Fortunately, we have already solved this problem; we have found above the exact two point correlation function of the zero-modes i.e. the resummation of these diagrams has already been performed in obtaining the leading order correlation function (21).

We now consider the case where the external legs are not zero-modes ($\vec{L} \neq 0$). The leading effect then comes from diagrams where two of the internal lines are zero-mode propagators (momentum conservation prevents all three from being zero-mode propagators). This contribution once again has two factors of $\lambda$ from the vertices, but now there are only 2 factors of $\frac{1}{\sqrt{\lambda}}$ from the zero-mode propagators. The correction therefore scales as $O(\lambda)$, and is a parametrically small correction to the correlation function.

The interactions of zero-modes and nonzero-modes can therefore be treated in perturbation theory, confirming the argument above. However, to explicitly find the $O(\lambda)$ correction to the correlation function, we should check whether higher order diagrams are suppressed. In fact, as we now see, this is not the case.

Higher order diagrams will involve both interactions between zero-modes and nonzero-modes, which can be treated in perturbation theory, as well as zero-mode self interactions, which must be treated exactly. We show below such a representative diagram which contributes to corrections to a nonzero-mode correlation function (we have denoted the nonzero-mode correlation function by a thicker line). There are four vertices and 6 zero-mode propagators, so the diagram scales as $O(\lambda)$, which is the same scaling as the sunset diagram correction. This means that we have to resum all these higher loop diagrams to get the $O(\lambda)$ correction to the nonzero-mode correlation function. This is again because the zero-modes are strongly coupled.

The full set of diagrams that we need to resum involves further vertices involving only zero-modes. It is easy to see that the full set of such diagrams is of the form shown in the diagram below.
The blob is any interaction of the zero-modes alone. While it is impossible to calculate these diagrams perturbatively, we can write the full set of diagrams as nonzero-mode propagators convolved with a four point function of zero-modes

\[ 288\lambda^2 \int d^3x_1 d^3x_2 |Y_L(x_1)|^2 |Y_L(x_2)|^2 Y^2_0(x_1) Y^2_0(x_2) \langle \phi_L \phi_L^* \rangle^3 \times \langle \phi^2_0 \phi^2_0 \rangle \]

We evaluate

\[ \langle \phi^4_0 \rangle = \int \mathcal{D}\phi_0 \phi_0^4 \exp(-\lambda_{eff} \phi_0^4) \int \mathcal{D}\phi_0 \exp(-\lambda_{eff} \phi_0^4) = \frac{c_4}{\lambda_{eff}} \]

where

\[ c_4 = \int dx x^4 \exp(-x^4) \int dx \exp(-x^4) = \frac{1}{4} \]

The final correction to the correlation function is then of order \( \lambda \). The corrected correlation function is

\[ \langle \phi(x) \phi(y) \rangle = \frac{c_2 Y_0(x) Y_0(y)}{\sqrt{\lambda_{eff}}} + \sum_{L \neq 0} Y^*_L(x) Y^*_L(y) \left( \frac{1}{L(L+d)} + 288\lambda_{eff} \frac{c_4}{L^3(L+d)^3} \right) \]

V. COMMENTS ON THE IN-IN FORMALISM

We have demonstrated that calculations in the Euclidean theory yield finite and calculable results in the massless limit. It is interesting to ask whether the same statement is true for the in-in formalism for calculating correlation functions. In this formalism, the correlation functions at some time are defined by

\[ \langle Q(t) \rangle = \int D\phi \exp(-i \int_{t_0}^t L(\phi_+) dt) Q(t) \exp(i \int_{t_0}^t L(\phi_-) dt) \langle \phi_+(t) - \phi_-(t) \rangle \]

We take \( L(\phi) = \partial \phi^2 - m^2 \phi^2 - \lambda \phi^4 \). In perturbation theory, one again finds that the \( k \)-loop diagram comes with a factor \( \left( \frac{\alpha}{m^2} \right)^k \). This implies once again that the long wavelength modes are strongly coupled. (This fact has been known for a long time in the inflation literature.)

In the case of the Euclidean sphere, there was only one mode (the \( L = 0 \) mode) that was strongly coupled. Here there are a continuum of modes with small \( k^2 \) which are strongly coupled. This means that we do not have an easily calculable theory; we still need to work in a strongly
coupled field theory as opposed to a theory of one mode. We will therefore be unable to find exact results in this model, unlike the case of the Euclidean theory.

We can nevertheless make some qualitative statements for the correlation functions. We focus here on the two point function in the massless theory. If we are considering the two point function at short wavelengths and we treat the quartic interaction as a perturbation, we recover the standard expression that

$$\langle \phi_k(t)\phi_k(t) \rangle \propto \frac{1}{k^3} \quad (28)$$

On the other hand, for long wavelengths, we may expect the field to develop a mass, just like the zero-mode on the Euclidean sphere. This mass will scale as some power of $\lambda$ i.e. $m^2 \propto \lambda^a$. The $k$-loop diagram will then scale as $(\frac{1}{\lambda})^k$, and will be small as along as the constant $a$ determining the scaling of the mass is small enough i.e. if $a < 1$. It would be interesting to see if a nonperturbative approach can be used to calculate this scaling, and hence to justify the validity of cosmological perturbation theory (a related but different approach has been suggested in [18]).

VI. DISCUSSION AND CONCLUSIONS

We have considered an interacting scalar field in de Sitter space. We showed that apparent infrared divergences which occur when the mass is small are the result of an incorrect quantization of the zero-modes. In the Euclidean theory (which corresponds to field theory on the Euclidean sphere), the correct quantization of the zero-mode leads to a finite and calculable perturbation expansion. Unlike the massless noninteracting theory, where it is well known that no de Sitter invariant vacuum exists, the interacting theory has a vacuum with unbroken de Sitter invariance.

Qualitatively, the inclusion of the interaction terms shifts the location in the pole of the zero-mode by an amount proportional to $\sqrt{\lambda}$. This shift is physically different from a renormalization of the mass parameter, which can be taken to run to zero at low energies. The mass gap is purely due to the resummation of the quartic interactions; it is therefore analogous to the mass gap in QCD which can occur even for massless quarks. In many ways, this result is similar to the analysis of the gap equation in [7]; however we have succeeded in obtaining the gap without the need for any approximation because only one mode is strongly coupled.

We note that in some special theories, several modes may be strongly coupled, which may make the theory uncalculable. An example is the theory of a massless field coupled to a massive field with $L = (\partial \phi)^2 + (\partial \chi)^2 - m^2 \chi^2 - \lambda \phi^2 \chi^2$. Here, the zero-mode of $\phi$ is strongly coupled to all the
modes of $\chi$, and the theory can be seen to be uncalculable. Nevertheless, the field theory does not break down, and correlation functions can be calculated numerically.

Finally, we note that we have been considering a toy model with a scalar field on a fixed de Sitter background. In inflation, the metric is not exactly de Sitter, which makes the Euclidean continuation problematic. It would be very interesting to see whether our methods can be applied to this more realistic case. We hope to return to this in future work.

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