Communication in quantum networks of logical bus topology

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Perfect state transfer (PST) is discussed in the context of passive quantum networks with logical bus topology, where many logical nodes communicate using the same shared media, without any external control. The conditions under which a number of point-to-point PST links may serve as building blocks for the design of such multi-node networks are investigated. The implications of our results are discussed in the context of various Hamiltonians that act on the entire network, and are capable of providing PST between the logical nodes of a prescribed set in a deterministic manner.

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I. INTRODUCTION

Network topology is the study of the geometric arrangement of the various elements in a network (e.g., links, nodes, etc.) [1]. Physical topologies describe how different sites are coupled to each other, whereas logical topologies refer to the flow of information between the various sites in a particular network. The simplest logical topology one may consider is the so-called point-to-point (PP) topology where one node (source) is connected directly to another (destination) node. The logical network topology is not necessarily the same as the physical layout, since the former is bound to the protocols used for communication. For instance, there might exist intermediate physical connections between the source and the destination nodes that do not change the PP character of the logical topology, i.e., any message originating from the source is intended for the destination node only.

A direct generalization of the point-to-point logical topology to multiple nodes is the so-called bus topology, which enables a number of logical nodes to communicate using the same shared media. Physically speaking, various nodes are connected to a common wire (known as backbone or bus). Whenever a node wants to communicate with another node, it sends the message over the common channel. In contrast to the PP topology, the message is visible and, in principle, accessible by all the logical nodes, although the intended recipient is actually the one that accepts and processes the message [2].

Recently, many of the questions typically addressed in conventional networks have been transferred to a quantum setting, where information is imprinted onto the state of quantum systems, and the quantum wires are fully compatible with the hardware in a particular quantum computing implementation [3]. Typically, quantum networks are discrete, consisting of a number of coupled quantum objects (physical nodes), a subset of which can be the logical nodes. Passive quantum networks, where the nodes have fixed energies and are permanently coupled, are of particular interest since they do not require any external control associated with the application of multiple gates and/or measurements. In this spirit, various Hamiltonians have been proposed which allow for PST from one node to another [4, 5, 6, 7, 8, 9, 10, 11]. In such a PP logical topology, the other nodes provide the backbone (or bus), for the communication between the source and the destination nodes. The excitation is shared among all of the nodes throughout its evolution, and it is only after a well-defined time that it can be localized with high probability at the destination node.

This paper goes beyond PP logical topologies, placing the problem of PST in a much broader context involving passive networks with logical bus topology. Many logical nodes need to communicate with each other using the same shared media, and our task is to engineer Hamiltonians, which are capable of transferring successively a quantum state from the source node, to each one of the other logical nodes, in a perfect and deterministic manner [12]. This is in contrast to previous studies on multiport network topologies [13, 14, 15, 16, 17, 18, 19] where some kind of external control is required, to direct selectively the signal to one of the destination nodes only. In our case, the passive network evolves under the influence of a single time-independent Hamiltonian and, in contrast to PP topology, the signal is accessible by all the logical nodes; albeit at different times.

In the following section we formulate the problem under consideration, and describe briefly the methodology we adopt throughout this work.

II. FORMALISM

The system under consideration pertains to a network consisting of $d$ permanently coupled physical sites labeled by $\{0, 1, \ldots, d-1\} \equiv \mathbb{Z}_d$, and let $L$ denote the set of logical nodes with $L \subseteq \mathbb{Z}_d$. The label that we give to each node is arbitrary and does not necessarily correspond to any physical ordering of the network. For example, the physical nodes 0 and $d-1$ could be directly coupled to each other physically. Similarly, if our network was arranged in a linear chain, then the physical nodes 0 and $d-1$ need not be the first and last nodes respectively.

To recapitulate our formalism, it is sufficient to restrict this section to PP logical networks setting $L = \{l_a, l_d\}$,
where this PST protocol is due to the fact that the Hamiltonian, can be used to perfectly transfer a state from one to an eigenvalue $\tau$ defined time $t$. The problem of PST pertains to the quest of Hamiltonians, which allow for the transfer of the state to the destination node $l_d$ with unit efficiency after a well-defined time $\tau$.

Depending on the particular implementation under consideration, the state may involve various degrees of freedom besides the position of the excitation (e.g., spin, angular momentum, etc.). Following previous work in the field [2]–[4], we assume that all of these additional degrees of freedom are preserved throughout the evolution of the system. Moreover, we are interested in Hamiltonians which preserve the total number of excitations in the system, and we thus focus on the transfer of a single excitation from the source to the destination node, within the one-excitation Hilbert subspace.

The corresponding (computational) basis is denoted by $|\kappa\rangle$, where $|\kappa\rangle$ indicates the presence of the excitation at the site $\kappa$, i.e., $|\kappa\rangle \equiv |\psi\rangle \otimes |\psi\rangle \otimes |\psi\rangle \otimes \cdots |\psi\rangle \otimes |\psi\rangle$.

The problem is thus reduced to the quest of PST Hamiltonians, which perform the transformation $|l_s\rangle \rightarrow |l_d\rangle$ at time $t = \tau$.

It has been shown [19] that, for a large class of PST Hamiltonians, the associated unitary evolution $U$ leads to a permutation matrix at time $\tau$ that permutes the source and destinations nodes, i.e.,

$$U(\tau) \equiv e^{-iH\tau} = \hat{\Pi},$$

where

$$\hat{\Pi} = |l_d\rangle\langle l_s| + \hat{\Pi}',$$  \hspace{1cm} (2)

is a permutation in the single excitation sector i.e., the submatrix $\hat{\Pi}'$ is also a permutation in the computational basis. For instance, it is known that a one-dimensional (1D) spin chain with a judiciously chosen XY Hamiltonian, can be used to perfectly transfer a state from one end of the chain to the other [2]–[4]. The success of this PST protocol is due to the fact that the Hamiltonian generates the permutation $\hat{\Pi} = \sum_n |n\rangle\langle d-n|$ [20]. For a network with $d$ sites and PP logical topology, there are $(d-1)!$ different permutations of the form [2], and for each one of these one can construct the corresponding class of Hamiltonians satisfying Eq. (1) (and are thus suitable for PST between the prescribed nodes).

In general, $\hat{\Pi}$ can be decomposed into disjoint cycles $\hat{\Pi}_i$, i.e., $\hat{\Pi} = \sum_i \hat{\Pi}_i$, and let $\Omega_i$ denote the set of sites (and thus basis states) permuted by $\hat{\Pi}_i$. The sets $\{\Omega_i\}$ are disjoint, while the dimensions of the support of each cycle $\hat{\Pi}_i$ is the cardinality of the corresponding set denoted by $d_i$. The spectrum of each cycle $\hat{\Pi}_i$ is nondegenerate and let $|v_i^{(\lambda_m)}\rangle$ be the eigenvector corresponding to an eigenvalue $\lambda_m$ where $\hat{\Pi}_i|v_i^{(\lambda_m)}\rangle = \lambda_m|v_i^{(\lambda_m)}\rangle$, with

$$|v_i^{(\lambda_m)}\rangle = \frac{1}{\sqrt{d_i}} \sum_{\kappa \in \Omega_i} \lambda_m^n |\kappa\rangle,$$

and

$$\lambda_n = \exp \left( i2\pi \frac{n}{d_i} \right) \text{ for } n \in \mathbb{Z}_{d_i} \equiv \{0, 1, \ldots, d_i - 1\}. \hspace{1cm} (4)$$

The elements of the set $\Omega_i$ are to be arranged in ascending order, and $\zeta_i \in \mathbb{Z}_{d_i}$ is the position of the element $\kappa \in \Omega_i$. Hence, for the permutation $\hat{\Pi}$ the eigenvalue $\lambda_n$ corresponds to $\eta_n$ distinct eigenvectors $\{|v_i^{(\lambda_n)}\}\},$ with $i$ running only on the various cycles having $\lambda_n$ in common.

A class of PST Hamiltonians that satisfy Eq. (1) is of the form

$$H_{\mathbf{x}} = \frac{1}{\tau} \sum_{\lambda_n} \sum_{a=1}^{\eta_n} \varepsilon_{\lambda_n}^{(a)} |y_{\lambda_n}^{(a)}\rangle \langle y_{\lambda_n}^{(a)}|,$$

where $\varepsilon_{\lambda_n}^{(a)} = -\arg(\lambda_n) + 2\pi x_\lambda^{(a)}$ and $\mathbf{x} \in \mathbb{Z}^d \equiv \{(x^{(1)}, \ldots, x^{(d)}) \mid x^{(1)}, \ldots, x^{(d-1)} \in \mathbb{Z}\}$. For a given eigenvalue $\lambda_n$, the $\eta_n$ distinct vectors $\{|y_{\lambda_n}^{(a)}\}\}$ form an orthonormal basis for the corresponding subspace and are of the form

$$|y_{\lambda_n}^{(a)}\rangle = \sum_i \beta_{\lambda_n}^{a,i} |v_i^{(\lambda_n)}\rangle,$$

with $\beta_{\lambda_n}^{a,i} \in \mathbb{C}$ and $\sum_i \beta_{\lambda_n}^{a,i} \beta_{\lambda_n}^{a',i*} = \delta_{a,a'}$. In other words, the rows of the matrix $\hat{B}_{\lambda_n}$ with elements $\beta_{\lambda_n}^{a,i}$ are orthonormal, which implies unitarity of $\hat{B}_{\lambda_n}$ and thus orthonormality of its columns as well, i.e.,

$$\sum_{a} \beta_{\lambda_n}^{a,i} \beta_{\lambda_n}^{a,i*} = \delta_{i,i'}.$$  \hspace{1cm} (7)

For a given PST Hamiltonian of the form (4), i.e., for fixed $\mathbf{x}$, the unitary evolution operator reads

$$U(t) = \sum_{\lambda_n} \sum_{a=1}^{\eta_n} \exp \left( -i\varepsilon_{\lambda_n}^{(a)} t/\tau \right) |y_{\lambda_n}^{(a)}\rangle \langle y_{\lambda_n}^{(a)}|,$$

with $|y_{\lambda_n}^{(a)}\rangle$ given by Eq. (5).

The condition (1) is rather restrictive since, in general, one may derive PST Hamiltonians which do not lead to permutations at time $\tau$, but rather to other unitary operations. Despite this fact, it has been demonstrated that the present theoretical framework is suitable for engineering of passive networks, and many known PST Hamiltonians satisfy condition (1) [20]. The permutation matrix (4) guarantees that the net effect of the evolution (1) is the transfer of the excitation from the source to the destination node at time $\tau$. The present formalism has the advantage that we do not need to make any assumptions about the physical topology of the network. Instead infinitely many Hamiltonians suitable for PST can be specified within the present theoretical framework, and this enables us to find the most suitable Hamiltonian for a given physical realization and topology. Finally, there is
no requirement for the initial overall state of the network (minus the source node) to have a specific form i.e., to be pure or mixed state. Nevertheless, the source node is considered to be initially decorrelated from the rest of the the network, which can be prepared in any state. Moreover, for the sake of simplicity, throughout this work we assume that all the nodes but the source, are initially in their ground state.

We turn now to discuss the problem of PST in the context of logical networks with bus topology. In the classical theory of networks one often assumes that the logical nodes have an equivalence property that is, no matter which logical node the signal starts at, it will be transferred about the predetermined logical network. Similarly, throughout this work we focus on passive quantum networks with equivalent logical nodes, and thus the PST (flow of information) is restricted within the predetermined logical network, irrespective of the source of the state. In the following section we investigate the conditions under which multiple point-to-point PST links can be concatenated to yield PST between multiple nodes.

III. CONCATENATION OF PP PST LINKS

Suppose we are interested in transferring an excitation between three different physical sites. For the sake of convenience we shall label these three logical nodes as \( \{l_0, l_1, l_2\} \), regardless of their physical proximity. Let us also assume that we have solved the problem of pairwise PST, i.e., we have two Hamiltonians \( \hat{h}_1 \) and \( \hat{h}_2 \), which generate the unitary transformations \( \hat{U}_1 \) and \( \hat{U}_2 \), respectively, such that \( \hat{U}_j \) perfectly transfers the state from the node \( l_0 \) to \( l_j \), i.e.,

\[
\hat{U}_j |l_0\rangle = e^{i\varphi_j} |l_j\rangle, \quad \text{for } \varphi_j \in \mathbb{R}. \tag{9}
\]

Our task is to investigate the conditions under which we can define a single time-independent Hamiltonian \( \hat{H} \), which acts on the entire network and generates both of the unitaries at different times, i.e., \( \exp(-i\hat{H}\tau_j) = \hat{U}_j \). In this case, the two unitaries (or else the corresponding PP PST links) are compatible, and can be used as building blocks for the design of passive networks with more involved logical topologies pertaining to the prescribed nodes \( \{l_j\} \).

The following theorem provides necessary and sufficient conditions for the compatibility of two unitary operators.

**Theorem 1.** Two different unitary operators \( \hat{U}_1 \) and \( \hat{U}_2 \) are compatible if and only if

(i) There exist \( \tau_1, \tau_2 \in (0, \infty) \) : \( \hat{U}_1^{\tau_2} = \hat{U}_2^{\tau_1} \) for \( \tau_1 \neq \tau_2 \), and for all \( t \in (0, \infty) \) \( \hat{U}_1^t \neq \hat{U}_2^t \).

(ii) The two unitary operators commute, i.e. \( [\hat{U}_1, \hat{U}_2] = 0 \).

**Proof of Theorem 1.** We begin by showing that these two conditions are sufficient for the two unitary operators to be compatible. We first assume that we have two Hamiltonians \( \hat{h}_1 \) and \( \hat{h}_2 \) that generate two unitary operators \( \hat{U}_1 \) and \( \hat{U}_2 \), which satisfy conditions (i) and (ii). Condition (ii) ensures that the two unitaries have a common eigenbasis (with eigenprojectors \( \{\hat{E}_i\} \)) and can be thus simultaneously diagonalized i.e.,

\[
\hat{U}_1 = \sum_i \alpha_i \hat{E}_i, \quad \hat{U}_2 = \sum_i \gamma_i \hat{E}_i. \tag{10}
\]

Condition (i) then implies that \( (\alpha_i)^{\tau_2} = (\gamma_i)^{\tau_1} \) for all \( i \), and setting \( \alpha_i = \lambda_i^{\tau_1} \) we also obtain \( \lambda_i^{\tau_2} = \gamma_i \). We can now define the normal operator \( \hat{U} = \sum_i \lambda_i \hat{E}_i \), where \( \hat{U}^{\tau_1} = \hat{U}_1 \) and \( \hat{U}^{\tau_2} = \hat{U}_2 \). In order for the operator \( \hat{U} \) to be unitary we must have that \( |\lambda_i|^2 = 1 \) for all \( i \). In our case, however, this is ensured by the equalities \( |\lambda_i|^{2\tau_1} = |\alpha_i|^2 = 1 \) and \( |\lambda_i|^{2\tau_2} = |\beta_i|^2 = 1 \), which enable us to choose \( |\lambda_i| = 1 \). Recalling that all unitary operators can be expressed in the form \( \hat{U} = \exp(-i\hat{H}t) \), where \( \hat{H} \) is Hermitian, it is clear that \( \hat{H} \) will act as a Hamiltonian on the entire network, generating the two unitaries \( \hat{U}_1 \) and \( \hat{U}_2 \) at times \( \tau_1 \) and \( \tau_2 \), respectively.

We shall now show that if two different unitary operators \( \hat{U}_j \) can be generated from a single Hamiltonian \( \hat{H} \), then conditions (i) and (ii) will be satisfied. Let \( \tau_1 \) and \( \tau_2 \) be two different times and define \( \hat{U}_j = \exp(-i\hat{H}\tau_j) \), where \( j = 1, 2 \). By a trivial application of the standard operator ordering theorems, we find that \( \exp(-i\hat{H}\tau_1 \tau_2) = \hat{U}_1^{\tau_2} = \hat{U}_2^{\tau_1} \). If \( \tau_1 = \tau_2 \), then \( \hat{U}_1 = \hat{U}_2 \) (which contradicts our initial assumption) and thus we must have \( \tau_1 \neq \tau_2 \). It is obvious that no time \( t \) can exist such that \( \hat{U}_1^t = \hat{U}_2^t \) for \( \hat{U}_1 \neq \hat{U}_2 \). Condition (i) is therefore a necessary condition for the two unitary transformations to be compatible. Finally, condition (ii) is also a necessary condition since \( \exp(-i\tau_1 \hat{H}), \exp(-i\tau_2 \hat{H}) = 0 \). \( \square \)

The previous discussion pertains to networks with three logical nodes \( \{l_0, l_1, l_2\} \) and the PST from \( l_0 \) to \( l_j \) occurs at well-defined time \( \tau_j \), with \( \tau_2 > \tau_1 \). It is straightforward to extend the notion of compatibility to larger sets of unitary operators \( \{\hat{U}_j\} \), and thus to generalize the previous discussion to passive networks with \( |L| > 3 \) logical nodes.

**Corollary 1.** The unitary operators of a given set \( \{\hat{U}_j \mid \hat{U}_j \neq \hat{U}_{j'} \text{ for } j \neq j' \text{ and } j, j' \in \mathbb{N}\} \) are compatible iff any pair of these operators satisfies the conditions of theorem 1.

If the compatibility conditions are satisfied, there exists a single Hamiltonian to implement the unitary operators \( \{\hat{U}_j\} \) at well-defined distinct times \( \{\tau_j \in (0, \infty) \mid \tau_j > \tau_{j'} \text{ for } j > j'\} \). The above theorem and corollary are very general and applicable to various kinds of PP PST links, including the schemes of Refs. To gain, however, further insight into the problem of concatenating a
number of given PST links, and the possible limitations one may face while looking for solutions, we have to become more specific. To this end, we assume that one of the unitaries, let us say $U_2$, is a permutation matrix $\hat{\Pi}$ that permutes the logical nodes $l_0$ and $l_2$. In any case, this choice cannot be considered very restrictive since many known PST schemes can be obtained in this context \cite{26}. The network is initially in the state $|l_0\rangle$ and in order for $U$ and $U_1$ to be compatible, they must satisfy the conditions of theorem 1. As we will see now, however, the structure of the permutation $\hat{\Pi}$ (i.e., whether or not it can be decomposed into smaller closed cycles) implies additional constraints on the logical networks one may consider.

Suppose that $\hat{\Pi}$ can be decomposed into closed subcycles $\{\Pi_j\}$ that cannot be decomposed further. By definition, $\hat{\Pi}$ permutes the nodes $l_0$ and $l_2$ and thus, these sites must belong to the same cycle that we denote by $\Pi_0$. The question is whether $l_1$ must also belong to the same cycle, in order for the PST within the logical network $\{l_0, l_1, l_2\}$ to be possible.

**Theorem 2.** Let $\hat{H}$ be a Hamiltonian governing the evolution of an excitation in a passive quantum network, such that $\hat{H} = \exp(-i\hat{H}_{T_2})$, where $\hat{H}$ permutes the nodes $l_0$ and $l_2$. To achieve PST to an intermediate site $l_1$ at an earlier time $t_1 < t_2$, $l_1$ must belong to the same closed cycle of $\hat{H}$ as $l_0$ and $l_2$. 

Before we prove this statement let us discuss its physical implications. If we want to perfectly transfer the state to several different logical nodes at different times, then all of these logical nodes must belong to the same cycle of $\hat{\Pi}$. The choice of logical nodes thus places a restriction on our choice of $\hat{\Pi}$, and vice-versa. This will in turn impose physical constraints on the type of Hamiltonians that one can derive (e.g., see \cite{26}). On the other hand, for the design of a universal bus, that transfers the excitation successively to every node of the network, $\hat{\Pi}$ must be a one-cycle permutation (see also Sec. IVA). Theorem 2 also implies that we cannot achieve PST between two physical nodes that belong to different cycles of $\hat{\Pi}$. This means that the probability of finding the state at a node that is not in the same cycle as the source node $l_0$ will be strictly less than one. A question therefore is whether the transfer can be achieved with high probability. The following observation, which is proved in the appendix \cite{A} partially answers this question.

**Observation 1.** Let two logical nodes $l_0$ and $l_1$ belong to different closed cycles $\Pi_0$ and $\Pi_1$ respectively, of a permutation $\hat{\Pi}$. It can be shown that

$$P_m(t) \leq \frac{1}{d_0 d_1} \left(\min\{d_0, d_1\}\right)^2,$$

where $d_j$ is the dimension of the support of $\hat{\Pi}_j$, and $P_m(t) = |\langle m|\hat{U}(t)|l_0\rangle|^2$ is the probability for the excitation to occupy the node $m$ at time $t$, where $\hat{U}(t) = e^{-i\hat{H}t}$ and $\hat{U}(\tau) = \hat{\Pi}$, for some $\tau > 0$.

Inequality (11) provides a useful upper bound (strictly less than 1) on the probability $P_m(t)$ only when $d_0 \neq d_1$. On the other hand, one may expect $P_m(t) = 1$ only for $d_0 = d_1$, but our numerical investigations show that even in this case the probability is generally much less than 1.

**Proof of Theorem 2.** Assuming that the nodes $l_0$ and $l_1$ belong to different cycles (denoted by $\Pi_0$ and $\Pi_1$, respectively), and given the existence of $\hat{\Pi} : |l_0\rangle \rightarrow |l_2\rangle$, we will prove by contradiction that there exists no unitary operator $\hat{U}_1$ which is compatible with $\hat{\Pi}$ and satisfies $U_1|l_0\rangle = e^{i\varphi_1}|l_1\rangle$. It is sufficient, for our purposes, to focus on the case of $d_0 = d_1$ since, according to inequality (11), we cannot achieve PST from $l_0$ to $l_1$ when $d_0 \neq d_1$.

Suppose that there exists such a unitary operator $\hat{U}_1$ which satisfies Eq. (11), and can thus be expressed in the form

$$\hat{U}_1 = e^{i\varphi_1}|l_1\rangle\langle l_0| + \hat{W},$$

where unitarity of $\hat{U}_1$ implies that $\hat{W}^\dagger|l_1\rangle = 0$ and $\hat{W}|l_0\rangle = 0$. In some sense, this is a first guess of $\hat{U}_1$, that we can refine further using the fact that $\hat{\Pi}$ and $\hat{U}_1$ are compatible. According to theorem 1, this means that $[\hat{U}_1, \hat{\Pi}] = 0$, which implies that $\hat{U}_1\left(\hat{\Pi}_0|l_0\rangle\right) = e^{i\varphi_1}\hat{\Pi}_1|l_1\rangle$. In view of this additional information, Eq. (12) can be rewritten as $\hat{U}_1 = e^{i\varphi_1}\left(|l_1\rangle\langle l_0| + \hat{\Pi}_1|l_1\rangle\langle l_0|\hat{\Pi}_0^\dagger\right) + \hat{W}_1$, where $\hat{W}_1|l_1\rangle = \hat{\Pi}_1|l_1\rangle|l_0\rangle = 0$ and $\hat{W}_1|l_0\rangle = \hat{W}_1\hat{\Pi}_0|l_0\rangle = 0$. Given, however, that $[\hat{U}_1, \hat{\Pi}] = 0$, we also have that $U_1$ commutes with any power of $\hat{\Pi}$. Hence, in general, for $d_0 \geq 1$ we can perform $d_0$ iterations of the above refinement with the $j$th iteration pertaining to $[\hat{U}_1, \hat{\Pi}^j] = 0$. The final result is

$$\hat{U}_1 = e^{i\varphi_1}\sum_{j=0}^{d_0-1} \hat{\Pi}_1^j|l_1\rangle\langle l_0|\hat{\Pi}_0^j\hat{\Pi}_1^j + \hat{W}_1,$$

with $\hat{W}_1|\xi_1\rangle = 0$ and $\hat{W}_1|\xi_0\rangle = 0$, $\forall|\xi_1(0)\rangle \in \Omega_1(1)$. So, in order for the nonsingular $\hat{U}_1$ to commute with $\hat{\Pi}$, the map $\hat{U}_1 : \Omega_0 \rightarrow \Omega_1$ must be bijective.

On the other hand, $[\hat{U}_1, \hat{\Pi}] = 0$ implies that $\hat{U}_1$ and $\hat{\Pi}$ are simultaneously diagonalizable, and we can construct a common eigenbasis. The permutation $\hat{\Pi}$ has a degenerate spectrum since, by definition, it consists of several, let us say $a$, cycles $\{\Pi_i\}$, with $\Pi_0$ and $\Pi_1$ involving the nodes $l_0$ and $l_1$, respectively. All the cycles have at least one eigenvalue in common, namely $\lambda_0 = 1$, and let $|\psi^{(\lambda_0)}_i\rangle$ be the corresponding eigenvector for cycle $\Pi_i$, given by Eq. (3) for $n = 0$. In the subspace that is spanned by these $a$ distinct eigenvectors, we can construct an orthonormal eigenbasis of $\hat{\Pi}$ (and thus of $\hat{U}_1$), with elements $\{|\psi^{(\lambda_0)}_i\rangle\}$ given by Eq. (3).

In view of Eqs. (13), and given that the sets $\Omega_j$ are disjoint, we have $\hat{U}_1|\psi^{(\lambda_0)}_a\rangle = \hat{\Pi}^a|\psi^{(\lambda_0)}_1\rangle + O(|k\rangle)$ for $k \neq i$,
\( \Omega_0 \cup \Omega_1 \)). Recalling that the state \( |y^{(a)}_{\lambda_0}\rangle \) is also an eigenvector of \( \hat{U}_1 \) (with eigenvalue 1), we find that \( e^{i\varphi_{I_{a}}^{(\lambda_0)}} = \beta_{a,0}^{(\lambda_0)} \) \( \forall a \), and the orthogonality condition (1) yields \( \sum_a |\beta_{a,0}^{(\lambda_0)}|^2 = 0 \). This, however, is not possible because by construction, at least one of the vectors \( \{y^{(a)}_{\lambda_0}\} \) will satisfy \( |\langle y^{(a)}_{\lambda_0}|y^{(a)}_{\lambda_0}\rangle| \neq 0 \) and thus not all \( |\beta_{a,0}^{(\lambda_0)}| = 0 \). We have proved by contradiction, therefore, that PST from node \( l_0 \) to node \( l_1 \), with the two nodes being parts of different cycles of \( \hat{\Pi} \), is impossible.

In closing this section, we would like to point out an additional constraint on the structure of the networks under consideration, imposed by the assumption of the final unitary being a permutation. Using Eqs. (6), (4) and (3), one can express the Hamiltonian (1) in the computational basis, where it is apparent that all the physical nodes of the network that belong to the same cycle of the permutation must have the same energy.

### IV. Hamiltonians for PST Between More Than Two Logical Nodes

In this section, by employing the theorems of the previous section, we demonstrate how one can construct Hamiltonians that achieve PST within a logical network pertaining to more than two nodes.

#### A. The universal bus

Suppose we are looking for Hamiltonians that transfer the excitation successively to each one of the \( d \) sites of a passive network, within a well-defined time \( \tau \). We refer to this logical network as the universal bus, since it encompasses the entire physical network. The order in which the excitation is transferred along the network will be as follows \( 0 \rightarrow 1 \rightarrow \ldots \rightarrow d-1 \). This ordering does not affect the generality of our arguments because the labeling of the sites is arbitrary. We do not allow, however, for the same site to get occupied twice within the prescribed time of the transfer.

According to theorem 2, the Hamiltonian of a universal bus can be associated only with a one-cycle permutation \([26]\). A case in point is the permutation

\[
\hat{\Pi}_{ab} = |d-1\rangle\langle 0| + \sum_{m\in S_d^*} |m\rangle\langle m+1|,
\]

with \( m \in S_d^* \equiv S_d \setminus \{0,d-1\} \), and its eigenvalues and eigenvectors given by Eqs. (4) and (3), respectively for \( d_i = d \) and \( \Omega_1 = S_d \). The set of Hamiltonians that transfer the excitation from node 0 to node \( d-1 \) at time \( \tau \), and the corresponding unitary transformation can be obtained from Eqs. (6) and (5) respectively, after dropping the index \( a \) and the inner summations. An important point to note is that any choice for the vector \( \mathbf{x} = (x_1, \ldots, x_n) \), with \( n \in S_d \), will lead to a Hamiltonian that satisfies \( \exp(-i\hat{H}\tau) = \hat{\Pi}_{ab} \), but we are interested in the choices that transfer the state to every node of the network.

For the transfer of the excitation from node 0 to node \( m \in S_d^* \), the corresponding matrix element is given by

\[
\langle m|\hat{U}(t)|0\rangle = \frac{1}{d} \sum_{j=0}^{d-1} \exp \left[ \frac{2\pi i t' (m + j)}{d} - 2\pi i x_j t' \right],
\]

where \( t' = t/\tau \) and \( m \in S_d^* \). To achieve PST at time \( t \), we ask for vectors \( \mathbf{x} \), such that \( \langle m|\hat{U}(t)|0\rangle = e^{i\varphi_m} \), or equivalently

\[
\exp \left[ \frac{2\pi i t' (m + j)}{d} - 2\pi i x_j t' \right] = e^{i\varphi_m}, \quad \forall n \in S_d.
\]

Instead of trying to find all of the possible solutions to this problem, we will instead look for a simple, but infinite class of solutions. Let us impose the restriction that the occupancy of the \( m \)th node must occur at time \( \tau_m = m\tau/(d-1) \). Then, it is straightforward to show that one class of solutions to (15) is

\[
x_j = c_m + n + (d-1)f(n),
\]

where \( c_m \in \mathbb{R} \) and \( f : S_d \rightarrow \mathbb{Z} \). One can easily verify that Eq. (17) leads to the state being perfectly transferred to the \( m \)th node at time \( \tau_m \). Different choices for \( f(n) \) and \( c_m \) will lead to different spectra for the Hamiltonian, and to different dynamics during the process of transferring the state between any two nodes of the network. We have thus found an infinite class of universal-bus Hamiltonians, for passive quantum networks with resonant physical nodes. This is in contrast to other solutions for “all-to-all” networks, which require to switch on and off couplings or energy shifts for the nodes, in order to achieve PST within a prescribed set of logical nodes \([13]\).

#### B. Transfer to a subset of nodes

Let us now consider logical networks with bus (but not universal) topology i.e., the set of logical nodes is \( \{l_j\} = \mathbb{L} \subset S_d \). We will construct Hamiltonians, which are capable of transferring successively an excitation initially occupying one of the logical nodes, to each one of the other logical nodes in a perfect and deterministic manner. As before, one can look for solutions where the occupation times for any two logical nodes \( l_j \) and \( l_{j'} \), with \( j > j' \), satisfy \( \tau_j > \tau_{j'} \). We can impose here an additional constraint namely, PST should occur for the nodes of the logical network only. This last requirement automatically excludes the universal bus, since it will transfer the excitation successively to any node of the physical network.

Consider a network consisting of five physical nodes \((i.e., d=5)\), and three logical nodes \( \mathbb{L} = \{0, 2, 4\} \). According to theorem 2, if we are looking for PST Hamiltonians, which at time \( \tau \) lead to a permutation \( \hat{\Pi} \) that
permutes the nodes 0 and 4, then all three logical nodes have to belong to the same cycle of $\Pi$. Moreover, to restrict the flow of information within the logical network only, we should avoid choosing permutations for which nonlogical nodes are in the same cycle as the logical ones.

A permutation that satisfies the above requirements is

$$\Pi = |4\rangle\langle 0| + |0\rangle\langle 2| + |2\rangle\langle 4| + |1\rangle\langle 3| + |3\rangle\langle 1|,$$  

(18)

which can be decomposed into two cycles $\Pi_0 = |4\rangle\langle 0| + |0\rangle\langle 2| + |2\rangle\langle 4|$ and $\Pi_1 = |1\rangle\langle 3| + |3\rangle\langle 1|$. The eigenvectors and eigenvalues of these cycles can be obtained from Eqs. 3 and 11 respectively, for $\Omega_0 = \{0, 2, 4\}$, $\Omega_1 = \{1, 3\}$, $d_0 = 3$ and $d_1 = 2$. The spectrum of $\Pi$ is degenerate, since the two cycles have a common eigenvalue $\lambda_0$, while an eigenbasis can be constructed along the lines of Sec. IV A.

Taking into account the orthogonality condition one obtains

$$|\beta^{(1)}\rangle = \alpha|v_0(0)\rangle + \beta|v_1(0)\rangle,$$  

(19a)

$$|\beta^{(2)}\rangle = \beta^*|v_0(0)\rangle - \alpha^*|v_1(0)\rangle,$$  

(19b)

$$|\gamma^{(1)}\rangle = |v^*(1)\rangle,$$  

(19c)

$$|\gamma^{(1)}_{k\pi/3}\rangle = |v_0(k2\pi/3)\rangle \text{ for } k = 1, 2,$$  

(19d)

where $\beta^{(1)} = \alpha$, $\beta^{(1)} = \beta$, $\beta^{(2)} = \beta^*$, $\beta^{(2)} = -\alpha^*$, and $|\alpha|^2 + |\beta|^2 = 1$. Note that, to suppress notation, the vectors as well as the components of the integer vector $x$ in the following, are labeled according to $\arg(\lambda_n)$, and not the actual eigenvalue $\lambda_n$.

A class of PST Hamiltonians that transfer the excitation from node 0 to node 4 at time $\tau$ is given by Eq. 5, and the corresponding evolution operator is of the form Eq. 13. We shall now investigate the choices of the spectrum $\{x_{\lambda_n}\}$, and thus of the integer vector $x$, which enable us to achieve also PST from node 0 to node 2, at an earlier time $t < \tau$. To this end, we focus on the matrix element $\langle 2|\hat{U}(t)|0\rangle$ for which, using Eq. 19, we find

$$\langle 2|\hat{U}(t)|0\rangle = \left[\frac{|\alpha|^2}{3} \exp(-2\pi i x_0^{(1)} t') + \frac{1}{3} \exp\left[\frac{2\pi i(t' + 1)}{3} - 2\pi i x_{2\pi/3}^{(1)} t'\right]\right]$$  

$$+ \left[\frac{|\beta|^2}{3} \exp(-2\pi i x_{2\pi/3}^{(2)} t') + \frac{1}{3} \exp\left[\frac{4\pi i(t' + 1)}{3} - 2\pi i x_{4\pi/3}^{(1)} t'\right]\right].$$  

(20)

where $t' = t/\tau$. We also have for the matrix elements pertaining to non-logical nodes

$$\langle 1|\hat{U}(t)|0\rangle = \langle 3|\hat{U}(t')|0\rangle = \frac{3}{\sqrt{6}} \exp(-i2\pi x_0^{(1)} t')$$  

$$- \frac{\alpha^* \beta}{\sqrt{6}} \exp(-i2\pi x_0^{(2)} t').$$  

(21)

Equation (20) shows that we are free in choosing $x_{\pi}^{(1)}$, as this will not affect the probability of transferring the excitation to node 2.

To proceed further we can specify a time at which the state will be transferred from node 0 to node 2, let us say $\tau_2 = \tau/2$ or equivalently $t' = 1/2$. Hence, we ask for $\langle 2|\hat{U}(\tau_2)|0\rangle = e^{i\pi/2}$. This requirement will constrain the allowed values of the integers $\{x_{\lambda_n}\}$, that appear in Eq. (20). We can distinguish between two cases. (i) For $|\alpha| = 0$ or 1, either $|\alpha|^2 \exp(-2\pi i x_0^{(1)} t')/3$ or $|\beta|^2 \exp(-2\pi i x_0^{(2)} t')/3$ will be zero and thus, without loss of generality, we can set $x_0^{(1)} = x_0^{(2)}$. Working similarly to Sec. IV A we find that the transfer is facilitated by choosing

$$x_0^{(1)} = c + \nu + 2f(\nu), \text{for } \nu \in \{0, 1, 2\} \equiv \mathbb{Z}_3,$$  

(22)

where $c$ is a constant and $f : \mathbb{Z}_3 \rightarrow \mathbb{Z}$. (ii) For $0 < |\alpha| < 1$, in addition to $\langle \left(2|\hat{U}(\tau_2)|0\rangle\rangle = 1$ we require that $\langle 1(3)|\hat{U}(\tau_2)|0\rangle = 0$, which implies that $x_0^{(1)} = x_0^{(2)} + q$ for even integers $q \in \mathbb{N}$. As in case (i), it can be readily checked that a suitable choice for the other components of $x$ is provided by Eq. (22). It should be emphasized also that according to observation 1, the occupation probabilities of the non-logical nodes (1 and 3), cannot exceed $2/3$ for any $t < \tau$, and thus PST occurs within the logical network only. In the appendix B we discuss a possible physical realization of this PST Hamiltonian in the context of spin chains. Finally, it is straightforward to generalize the previous derivation to a case where the PST from node 0 to node 2 has to occur at a time $\tau_m = \tau/m$, for $m > 2$.

For networks with $d > 5$ nodes, it is rather difficult to obtain analytic results, but one can resort to numerical investigations. In Fig. 11 we present results pertaining to a network consisting of 11 physical nodes, and two different choices of logical nodes. For Fig. 11a, the set of logical nodes is $\mathbb{L}_a = \{0, 2, 4, 6, 8, 10\}$, whereas for Fig. 11b, $\mathbb{L}_b = \{0, 3, 6, 10\}$. To construct Hamiltonians that allow for PST within these sets of logical nodes, one has to work as before. Following theorem 2 and observation 1, in both cases we have chosen a permutation which has a cycle that contains only the logical nodes e.g., $\Pi_a = \{10\} \{0 + 8\} \{0 + 6\} \{8 + 4\} \{6 + 2\} \{4 + 0\} \{2 + 9\} \{1 + 7\} \{9 + 5\} \{7 + 3\} \{5 + 1\} \{3\}$, and $\Pi_b = \{10\} \{0 + 6\} \{10 + 3\} \{6 + 0\} \{3 + 9\} \{1 + 8\} \{9 + 7\} \{8 + 5\} \{7 + 4\} \{5 + 2\} \{4 + 1\} \{2\}$. Each one of these permutations consists of two cycles and let $d_{(0)}$ denote the cardinality of the cycle containing the node 0(1), respectively. The class of Hamiltonians that transfer the excitation from node 0 to node 10 at time $t = \tau$ is given by Eq. 9, and one can choose the integer vector $x$, and thus the spectrum $\{x_{\lambda_n}\}$, so that the excitation is transferred successively to the other nodes of the logical network $\mathbb{L}_a(\mathbb{b})$ at well defined times $\tau_{\alpha(\beta)}$, for $m \in \mathbb{L}_a(\mathbb{b}) \setminus \{0, 10\}$. When asking for these times to be equally spaced i.e., $\tau_{\alpha(\beta)} = m\tau/(||\mathbb{L}_a(\mathbb{b})|| - 1)$, one can choose: (a)
\(x_{2πn_0/d_0}^{(1)} = 11 - n_0\), \(x_{2πn_1/d_1}^{(1)} = 5 - n_1\), \(x_0^{(1)} = 11\), and \(x_0^{(2)} = 5\); (b) \(x_{2πn_0/d_0}^{(1)} = 11 - n_0\), \(x_{2πn_1/d_1}^{(1)} = 11 - n_1\), and \(x_0^{(1,2)} = 11\), where \(n_0(1) \in \mathbb{Z}_{d_0(d_1)} \setminus \{0\}\). The evolution of the corresponding occupation probability distributions (defined in observation 1) are depicted in Figs. 1(a-b).

V. SUMMARY

We have extended the problem of PST to passive quantum networks of bus logical topology, where the information flow is restricted within a prescribed set of logical nodes. The excitation evolves under the influence of a single Hamiltonian which acts on the entire network, and is transferred successively to each one of the logical nodes in a perfect and deterministic manner. We have provided necessary and sufficient conditions for the engineering of such networks, by concatenating a number of point-to-point PST links of any type. Our theory is not subject to any a priori restrictions on the physical topology of the network, or its initial state, and provides new ways for quantum network engineering beyond point-to-point logical topology.

The network engineering has been demonstrated in the context of a set of point-to-point PST links, where the transformation associated with one of them is a permutation. Such an assumption automatically imposes certain restrictions on the physical topology of the network, but is not very restrictive since many known PST Hamiltonians can be obtained in this framework. Although our formalism allows for the construction of Hamiltonians of any kind, which can be rather tedious, we restricted our network engineering to Hamiltonians that involve complex couplings. In spin networks, the adjustment of geometric phases is possible by looping around magnetic fields along the relevant sections [28], while for optical networks one may use phase shifters.

A number of interesting questions, such as the design of passive quantum networks with other logical topologies, the problem of time-limited PST, and the transfer of entangled states (encoded on two or more nodes) in networks of bus topology, deserve further investigation.

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APPENDIX A: PROOF OF OBSERVATION 1

To prove inequality (11), we start from Eq. (8) which provides the operator \(\hat{U}(t) = \exp(-i\hat{H}t)\) with \(\hat{U}(\tau) = \hat{\Pi}\). Using the triangle inequality we have for the matrix element of interest

\[ |\langle l_1 | \hat{U}(t) | l_0 \rangle| \leq \sum_{\lambda_n,a} |\langle l_1 | y_{\lambda_n}^{(a)} \rangle \langle y_{\lambda_n}^{(a)} | l_0 \rangle| \leq \sum_{\lambda_n,a} \left( \sum_a |\langle l_1 | y_{\lambda_n}^{(a)} \rangle| \right) \left( \sum_a |\langle y_{\lambda_n}^{(a)} | l_0 \rangle| \right) \]

(A1)

Given that \(l_{0(1)} \in \tilde{\Pi}_{0(1)}\), it is clear that the matrix elements \(|\langle l_{0(1)} | y_{\lambda_n}^{(a)} \rangle|\) vanish unless \(\lambda_n\) is an eigenvalue of \(\tilde{\Pi}_{0(1)}\), respectively. Let \(\sigma\) denote the set of eigenvalues that \(\tilde{\Pi}_0\) and \(\tilde{\Pi}_1\) have in common. Then, using Eqs. (4) and (8), we have

\[ |\langle l_1 | \hat{U}(t) | l_0 \rangle| \leq \frac{1}{\sqrt{d_1d_2}} \sum_{\lambda_n,a} \sum_{\sigma} |\beta_{a,0}^{(\lambda_n)}|^2 |\beta_{a,1}^{(\lambda_n)}|^2 \leq \frac{1}{\sqrt{d_1d_2}} \sum_{\lambda_n,a} \sum_{\sigma} |\beta_{a,0}^{(\lambda_n)}|^2 \sum_{\sigma} |\beta_{a,1}^{(\lambda_n)}|^2 \]

(A2)

where the second part is due to Cauchy-Schwarz inequality. Finally, using Eq. (7) we obtain \(|\langle l_1 | \hat{U}(t) | l_0 \rangle| \leq \sum_{\lambda_n,a} ...
The formalism we have adopted throughout this work is rather general, and can be easily adjusted to a particular physical implementation of the network. For instance, to make the example of Sec. IV B more concrete, let us consider the situation where the network is a spin chain, with $d = 5$ and $\mathbb{L} = \{0, 2, 4\}$. In this case each node possesses two degrees of freedom, corresponding to the individual spins being either up or down along a particular direction, i.e. $|\pm\rangle$, with $|\mp\rangle \equiv |\pm\rangle \otimes |\mp\rangle$ denoting the ground state of the chain. The source node (0) is considered to be initially de-correlated from the rest of the spins \[24\], and let the state of the entire chain be $|\Psi(0)\rangle = |\psi\rangle_0 |\mp\rangle^4$, where $|\psi\rangle = b_1|+\rangle + b_2|\rangle$ (see also related discussion towards the end of Sec. II). The state of the source site has been transferred successively to the sites 2 and 4, at the prescribed times $\tau_2 = \tau/2$ and $\tau_4 = \tau$, respectively. This means that the state of the entire chain has to transform according to $|\Psi(0)\rangle \rightarrow |\Psi(\tau_2)\rangle \rightarrow |\Psi(\tau_4)\rangle$, where $|\Psi(\tau_4)\rangle = |\psi\rangle_4 |\mp\rangle^4$. The initial state, however, is basically a superposition of the ground state and the first-excited state of the chain, i.e., $|\Psi(0)\rangle = b_1|+\rangle_0 |\mp\rangle^4 + b_2|\rangle_0 |\mp\rangle^4$. Thus, given that the ground state does not evolve in time, for the transfer of $|\Psi\rangle$ in time, it is sufficient to consider PST in the one-excitation subspace, i.e., $|0\rangle \rightarrow |2\rangle \rightarrow |4\rangle$, where

$$|m\rangle = |\mp\rangle^m |+\rangle_m |\rangle^4-m.$$  

The solution of this problem has been discussed in Sec. IV B in the context of a PST Hamiltonian that satisfies Eq. (1) with $\Pi$ given by Eq. (14). A class of PST Hamiltonians that transfer the excitation from spin 0 to spin 4 at time $\tau$ is given by Eq. (17), with $|y_{mn}^{(a)}\rangle$ given by Eq. (16). This class of Hamiltonians can be expressed in terms of the the Pauli spin operators $\hat{X}_j$ and $\hat{Y}_j$ acting on the $j$th spin, with $\hat{X}_j = |\mp\rangle_j (|+\rangle_j + |\rangle_j |+\rangle_j)$ and $\hat{Y}_j = i(-)|\rangle_j (|+\rangle_j - |\rangle_j )$, using the convention (11). It is convenient to split the analysis into two different cases i.e., to couplings between spins that belong to the same cycle, and couplings between spins that belong to different cycles.

A simple calculation shows that the spins 1 and 3 (non-logical nodes), which belong to the same cycle $\Pi_1$, are coupled by a term in the Hamiltonian that has the following form

$$\frac{1}{4\tau} \left( |\beta|^2 \varepsilon_0^{(1)} + |\alpha|^2 \varepsilon_0^{(2)} + \varepsilon_0^{(1)} \right) \left( \hat{X}_1 \hat{X}_3 + \hat{Y}_1 \hat{Y}_3 \right).$$  

(B2)

The couplings between the other three spins (logical nodes) that belong to the cycle $\Pi_0$, have the form

$$\frac{J_{mn}}{2} \left( \hat{X}_m \hat{X}_n + \hat{Y}_m \hat{Y}_n \right) + \frac{J_{'mn}}{2} \left( \hat{X}_m \hat{Y}_n - \hat{Y}_m \hat{X}_n \right).$$  

(B3)

with $m, n \in \Omega_0$ and $m > n$. The expressions for the coupling constants $J_{mn}$ and $J_{'mn}$ are

$$J_{20} = J_{40} = J_{42} = \frac{1}{3\tau} \left[ |\alpha|^2 \varepsilon_0^{(1)} + |\beta|^2 \varepsilon_0^{(2)} - \frac{\varepsilon_2^{(1)} + \varepsilon_4^{(1)}}{2} \right]$$  

(B4)

$$J_{'20} = -J_{40} = J_{42}' = \frac{1}{2\sqrt{3}\tau} \left[ \varepsilon_2^{(1)} - \varepsilon_4^{(1)} \right].$$  

(B5)

The energies of each node can be adjusted by changing the values of the integers $x_j^{(a)}$, as well as on the parameters $\alpha$ and $\beta$. The coupling constants can thus be adjusted while still obtaining Hamiltonians that perform the desired PST. For instance, taking $x_0^{(1)} = x_0^{(2)}$ will result in vanishing couplings between spins that belong to different cycles.

Finally, setting for convenience the ground state energy equal to zero, one readily obtains for the diagonal terms of the Hamiltonian

$$\langle 0|\hat{H}|0\rangle = \langle 2|\hat{H}|2\rangle = \langle 4|\hat{H}|4\rangle$$  

$$= \frac{1}{3\tau} \left[ \varepsilon_0^{(1)} |\alpha|^2 + \varepsilon_0^{(2)} |\beta|^2 + \varepsilon_2^{(1)} + \varepsilon_4^{(1)} \right]$$  

(B6)

$$\langle 1|\hat{H}|1\rangle = \langle 3|\hat{H}|3\rangle = \frac{1}{2\tau} \left[ \varepsilon_0^{(1)} |\beta|^2 + \varepsilon_0^{(2)} |\alpha|^2 + \varepsilon_3^{(1)} \right].$$  

(B7)

It can be seen that the nodes belonging to the same cycle have the same energy, when they are in the state $|+\rangle$. The energies of each node can be adjusted by changing the values of the integers $x_j^{(a)}$.  

\[1\] See, for example, Steinke S 2003 Network Tutorial (San Francisco: CMP Books) p 99; P Ciccarelli and P Faulkner 2004 Networking Foundations:Technology Fun-
In a classical setting, this is achieved by means of the so-called internet-protocol (IP) addresses. As the message arrives at each node, the corresponding device checks the destination address contained in the message to see if it matches its own address. If the two addresses match the device processes the message, otherwise it does nothing. Clearly, no addressing is required in a point-to-point topology since any data transmitted from one port is intended for the other port.

For recent reviews, see S. Bose, *Contemp. Phys.* **48**, 13 (2007); D. Burgarth, *Eur. Phys. J. Special Topics* **151**, 147 (2007); A. Kay, e-print, e-print [arXiv:0903.4274](http://arxiv.org/abs/0903.4274) (2009).

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