Squeezed States and Affleck Dine Baryogenesis

K. V. S. Shiv Chaitanya,\(^*\) and Bindu A Bambah\(^{†}\)

School of Physics, University of Hyderabad,
Hyderabad, A.P. 500 046, India.

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Quantum fluctuations in the post inflationary Affleck-Dine baryogenesis model are studied. The squeezed states formalism is used to give evolution equations for the particle and anti-particle modes in the early universe. The role of expansion and parametric amplification of the quantum fluctuations on the baryon asymmetry produced is investigated.

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I. INTRODUCTION

The dominance of matter over antimatter is known as baryon asymmetry. The generation of baryon asymmetry from an initially symmetric universe is an open problem. Baryon asymmetry is quantified by the ratio \( \eta = \frac{n_B}{n_\gamma} \), where \( n_B \) is the number of baryons and \( n_\gamma \) is the number of photons in the universe. The present value of the asymmetry in the universe is \( \eta \approx 10^{-10} \). The three conditions postulated by Sakharov\(^{1}\) to generate baryon asymmetry are baryon number violation, CP violation and out of equilibrium evolution of the universe. Many theories have been proposed to explain baryon asymmetry. Some of them are GUT baryogenesis\(^{2}\), electroweak baryogenesis\(^{2}\), leptogenesis\(^{3}\) and the Affleck Dine(AD) mechanism\(^{4}\). Most of these are still inadequate in explaining the value of \( \eta \) and a lot of work remains to be done to get a complete theory of baryon asymmetry generation. In view of the inefficient baryon production in GUTS, Affleck and Dine\(^{4}\) focused on the supersymmetric extension of GUTS to generate a new mechanism for baryon production based on flat directions. In the minimal supersymmetric standard model(MSSM) the number of degrees of freedom are increased by virtue of the fact that bosons and fermions have supersymmetric counterparts. The increase in the number of degrees of freedom results in directions in field space which have virtually no potential. These are known as flat directions and are made up of squarks or sleptons, so they carry baryon or lepton number. During inflation the squarks and sleptons are free to fluctuate along these directions as it costs little energy and can form condensates with a large baryon or lepton number. Supersymmetry breaking lifts these flat directions and sets a scale for the potential. Supersymmetry breaking can introduce terms that violate baryon number and CP. In the Affleck Dine model, the cosmological constant in early universe breaks the supersymmetry spontaneously during inflation and this gives rise to \( B - L \) violation, satisfying Sakharov’s first condition. The scalar fields through the interaction with the inflaton field generate C and CP violation, thus satisfying Sakharov’s second condition. The expectation values of massless scalar fields can start out displaced from true minimum, oscillations around the minimum occur when the Hubble constant becomes comparable to their effective mass resulting in coherent production of scalar fields manifested as a condensate of light scalar particles. This condensate stores baryonic charge and when inflation is over its decay produces nonzero baryon asymmetry.

Dine et al.\(^{5}\) showed that baryon asymmetry can be generated for a scalar field Lagrangian with an interaction term of a general quartic type with complex couplings of the form

\[
L_I = - \lambda |\phi|^4 + \epsilon \phi^3 \phi^\dagger + \varphi \phi^4 + C.C.,
\]

where, \( \lambda, \epsilon \) and \( \varphi \) are of the order of \( M_\xi^2/M_s^2 \), where \( M_s \) is the grand unification scale and \( M_\xi \) is the supersymmetry breaking scale. The baryon number per particle at very large times \( (t \gg m_\phi^{-1}) \) in both the radiation and matter dominated eras is given by

\[
r \approx \frac{\text{Im}(\epsilon + \varphi)}{m_\phi^2}.
\]

This mechanism is too efficient and produces a baryon asymmetry that is too large. Attempts to dilute this have been proposed, but most of these models use classical arguments, where additional entropy is released after baryogenesis through decay of the inflaton field\(^{5,6}\). Other models introduce nonrenormalizable terms\(^7,8\). In \( \xi^5 \), a preliminary perturbative analysis of out of equilibrium quantum fluctuations in the AD model has been shown to lead to some amount of reduction in the asymmetry. In this paper we do a comprehensive study of quantum fluctuations in a non-perturbative fashion, using the squeezed state formalism, which allows the analysis of quantum effects and expansion on Affleck Dine baryogenesis.

\(^*\)Electronic address: phbl3ph13@uohyd.ernet.in
\(^{†}\)Electronic address: bbsp@uohyd.ernet.in
II. THE FORMALISM

To carry out our analysis for AD baryogenesis for quantum fluctuations arising post inflation, we choose a Lagrangian with complex quartic couplings of the form [3]
\[
S = \int dt \sqrt{-g} |g_{\mu\nu} (\partial^\mu \phi^i) (\partial^\nu \phi) - m_{\phi}^2 \phi^i \phi - i \lambda (\phi^4 - \phi^4)|
\]
where $\phi$ is a complex scalar field, $m_{\phi}$ is the mass of the scalar field and $\lambda$ is a dimensionless real coupling constant. The baryon number violation comes from the breaking scale, $\epsilon$ is a real parameter which characterises CP violation and $M_S$ is the grand unification scale. The background metric is considered to be the flat FRW metric.
\[
ds^2 = dt^2 - a^2(t) dx^2
\]
where $a(t)$ is the expansion parameter. The classical equation of motion of the field is,
\[
\ddot{\phi} + 3H \dot{\phi} + m_{\phi}^2 \phi^i = 4i \lambda \phi^4_{\phi} (2)
\]
\[
\ddot{\phi}^i + 3H \dot{\phi}^i + m_{\phi}^2 \phi^i = 4i \lambda \phi^4_{\phi} (3)
\]
where $H$ is Hubble’s constant. The initial conditions at $t = t_0$ are given by
\[
\phi(t = t_0) = \phi_0, \quad \dot{\phi}(t = t_0) = 0. (4)
\]
The baryon number per particle for large times in this model is given by
\[
r \approx \frac{\lambda \phi_0^2}{m_{\phi}}. (5)
\]
in concurrence with the classical AD result.

We now study the quantum effects on 'r'. We rewrite eqn.[1] in terms of conformal time $\eta = \int \frac{dt}{a(t)}$ as
\[
S = \int d\eta d^3x \frac{a^4}{a^2} \left[ (\frac{\partial \phi^i}{\partial \eta})(\frac{\partial \phi}{\partial \eta}) - (\nabla \phi^i)(\nabla \phi) \right] - m_{\phi}^2 \phi^i \phi - i \lambda (\phi^4 - \phi^4_{\phi}). (6)
\]
Defining $\chi = a(\eta) \phi$, we get
\[
S = \int d\eta d^3x \left[ (\frac{\partial \chi^i}{\partial \eta})(\frac{\partial \chi}{\partial \eta}) - (\nabla \chi^i)(\nabla \chi) \right] - (m_{\phi}^2 a^2 - a'' \frac{a}{a}) \chi^i \chi + i \lambda (\chi^4 - \chi^4_{\phi}). (7)
\]
Decomposing $\chi$ into two real scalar fields,
\[
\chi = \frac{1}{\sqrt{2}} (\chi_1 + i \chi_2) (8)
\]
\[
\chi^i = \frac{1}{\sqrt{2}} (\chi_1 - i \chi_2), (9)
\]
and substituting into the action, we get
\[
S \approx \int d\eta d^3x \frac{1}{2} (\chi_1^2 - \chi_2^2)^2 + \frac{1}{2} \chi_1^2 - \frac{1}{2} (\chi_2^2 - \chi_2^2)^2 - m_{\phi}^2 (\chi_1^2 + \chi_2^2) - 2\lambda \chi_1 \chi_2 (\chi_1^2 - \chi_2^2), (10)
\]
where prime denotes derivative with respect to conformal time $\eta$ and $m_{\phi}^2 = m_{\phi}^2 a^2 - \frac{a''}{a}$. Using the background field method to study quantum effects [10, 11, 12], we assume field $\chi_i$, $i = 1, 2$, has background classical component $\chi_{i0}$ and a quantum(order $\frac{\hbar}{2\pi}$) field $\tilde{\chi}_i$:
\[
\chi_i = \chi_{i0} + \tilde{\chi}_i (11)
\]
where $\chi_{i0}$ satisfies the classical equation of motion,
\[
\frac{\delta S}{\delta \chi_i} |_{(\chi_i = \chi_{i0})} = 0 (12)
\]
and the field $\tilde{\chi}_i$ represents quantum fluctuations around the classical solution. We expand the action in terms of Taylor series
\[
S[\chi_i, \chi_j] = S[\chi_i, \chi_j] + \frac{\delta S[\chi_i, \chi_j]}{\delta \chi_j} |_{(\chi_i = \chi_{i0})} \frac{\delta S[\chi_i, \chi_j]}{\delta \chi_j} |_{(\chi_i = \chi_{i0})} (13)
\]
Since $\chi_{i0}$ satisfies the classical equation of motion, the second term is zero and the contribution of the quantum fluctuations comes from the quadratic and higher order terms.
\[
S = \int d\eta d^3x \frac{1}{2} (\tilde{\chi}_1^2 - \frac{1}{2} (\nabla \tilde{\chi}_1)^2 + \frac{1}{2} (\tilde{\chi}_2^2 - \frac{1}{2} (\nabla \tilde{\chi}_2)^2 - m_{\phi}^2 (\tilde{\chi}_1^2 + \tilde{\chi}_2^2) - 2\lambda (\rho \tilde{\chi}_1 \tilde{\chi}_2 + \delta (\tilde{\chi}_1^2 - \tilde{\chi}_2^2)), (14)
\]
where $\rho = 3(\chi_{10}^2 - \chi_{20}^2)$ and $\delta = 3\chi_{10}\chi_{20}$.

Using the Legendre transformation, we get the effective Hamiltonian of the fluctuations to be
\[
H = \int d\eta d^3x \left[ \frac{\tilde{\chi}_1^2}{2} + \frac{(\nabla \tilde{\chi}_1)^2}{2} + \frac{m_{\phi}^2 \tilde{\chi}_1^2}{2} + \frac{\tilde{\chi}_2^2}{2} + \frac{(\nabla \tilde{\chi}_2)^2}{2} + \frac{2}{2} (\tilde{\chi}_1^2 - \tilde{\chi}_2^2) \right] (15)
\]
here $\tilde{\chi}_i$ are the canonical momenta of the $\tilde{\chi}_i$ fields. Carrying out the mode expansion of the fields, we get
\[
\tilde{\chi}_1 = \int \frac{dk}{2\pi} \left[ a_k e^{ikx} + a_k^* e^{-ikx} \right], (16)
\]
\[
\tilde{\chi}_2 = \int \frac{dk}{2\pi} \left[ b_k e^{ikx} + b_k^* e^{-ikx} \right], (17)
\]
where,
\[
k \cdot x = k_i x^i = \omega \eta - k_i x_i, (18)
\]
\[
dk = \frac{d^3k}{(2\pi)^3 2\omega^{\frac{3}{2}}}, \quad \omega^2 = k^2 + m_{\phi}^2.
\]
The mode Hamiltonian is
\[
H = \int \frac{d^3k}{(2\pi)^3}\left(\frac{\omega}{2} + \frac{\lambda\delta}{2\omega}(a_k^\dagger a_k + a_{-k}a_{-k}^\dagger) + \frac{\lambda\rho}{2\omega}(b_k^\dagger b_k + b_{-k}b_{-k}^\dagger)\right)
\]
\[+ \frac{\lambda\rho}{2\omega}(a_k b_k + a_{-k}b_{-k}^\dagger + a_k^\dagger b_{-k} + a_{-k}^\dagger b_{-k}^\dagger)
\]
\[+ \frac{\lambda\delta}{2\omega}(a_k a_{-k}^\dagger + a_{-k}a_k^\dagger - \lambda\delta(b_k b_{-k} + b_{-k}b_k))\]
\]

To see the symmetries of the Hamiltonian, we define the following generators
\[
N_1 = \frac{1}{2}(a_k^\dagger a_k + a_{-k}a_{-k}^\dagger), \quad N_2 = \frac{1}{2}(b_k^\dagger b_k + b_{-k}b_{-k}^\dagger)
\]
\[J_+ = \frac{1}{2}(a_k^\dagger b_k + a_{-k}b_{-k}^\dagger), \quad J_- = \frac{1}{2}(b_k^\dagger a_k + b_{-k}a_{-k}^\dagger),
\]
\[J_0 = \frac{1}{2}(N_1 - N_2),
\]
\[K_+ = a_k b_{-k}, \quad K_- = b_{-k}^\dagger a_k^\dagger, \quad K_0 = \frac{1}{2}(N_1 + N_2 + 1),
\]
\[L_{1-} = a_{-k}a_k^\dagger, \quad L_{1+} = a_k a_{-k}^\dagger, \quad L_{10} = \frac{1}{2}(N_1 + 1),
\]
\[L_{2-} = b_{-k}b_{-k}^\dagger, \quad L_{2+} = b_{-k}b_{-k}^\dagger, \quad L_{20} = \frac{1}{2}(N_2 - 1).
\]

We can show that \((J_+, J_-, J_0)\) satisfy an \(su(2)\) algebra and \((K_+, K_-, K_0), (L_{1+}, L_{1-}, L_{10}), (L_{2+}, L_{2-}, L_{20})\) satisfy \(su(1,1)\) algebras.

In terms of these generators and the number operators, the Hamiltonian is
\[
H = \int \frac{d^3k}{(2\pi)^3}\left(\frac{\omega}{2} + \frac{\lambda\delta}{2\omega}N_1 + \frac{\omega}{2} - \frac{\lambda\delta}{2\omega}N_2\right)
\]
\[+ \frac{\lambda\rho}{2\omega}(J_+ + J_- + K_+ + K_-)
\]
\[+ \frac{\lambda\delta}{2\omega}(L_{1+} + L_{1-}) - \lambda\delta(2L_{2+} + 2L_{2-})\]
\]
and in this form explicitly displays \(su(1,1)\) and \(su(2)\) symmetries.

We diagonalize this Hamiltonian in two steps. First we use a unitary rotation transformation and then a squeezing transformation\([13]\). The first transformation is given by
\[
H_1 = U^\dagger(R_1)HU(R_1),
\]
where
\[
U(R_1) = \exp[\theta(J_+ e^{2i\xi} + J_- e^{2i\eta})]
\]
The operator \(U(R_1)\) provides the following transformation relations:
\[
U^\dagger(R_1) \left( \begin{array}{c} a_k \\ b_k \end{array} \right) U(R_1) = \left( \begin{array}{cc} \cos(\theta) & e^{2i\xi}\sin(\theta) \\ -e^{-2i\xi}\sin(\theta) & \cos(\theta) \end{array} \right) \left( \begin{array}{c} a_k \\ b_k \end{array} \right)
\]
\[= \left( \begin{array}{c} A_k \\ B_k \end{array} \right),
\]
the angle \(\theta\) is defined from the relation \(\sin(2\theta) = \frac{\lambda\delta}{\sqrt{\rho^2 + \delta^2}}\). The creation and annihilation operators \(A_k\) and \(B_k\) are
\[
A_k = a_k \cos(\theta) + b_k e^{2i\xi} \sin(\theta)
\]
\[B_k = b_k \cos(\theta) - a_k e^{2i\xi} \sin(\theta),
\]
and their complex conjugates.

The Hamiltonian \(H_1\) in terms of \(A_k^\dagger, B_k^\dagger, A_k\) and \(B_k\) is given by
\[
H_1 = \int \frac{\omega^2}{(2\pi)^3} m_1[A_k^\dagger A_k + A_{-k}^\dagger A_{-k} - m_2B_k^\dagger B_k + B_{-k}^\dagger B_{-k}] + m_2B_k^\dagger B_{-k} + B_{-k}^\dagger B_k + n_1[A_k A_{-k} + A_{-k}^\dagger A_k^\dagger]
\]
\[+ n_2[B_k B_{-k} + B_{-k}^\dagger B_k^\dagger]
\]
where, \(m_2 = \frac{\lambda\delta}{2\omega}\), \(m_1 = \frac{\lambda\rho}{2\omega}\), \(n_1 = -\frac{\lambda\delta}{\sqrt{\rho^2 + \delta^2}}\) and \(n_2 = \frac{\lambda\delta}{\sqrt{\rho^2 + \delta^2}}\).

We again define new generators \((D_{1+}, D_{1-}, D_{10}), (D_{2+}, D_{2-}, D_{20})\) satisfying \(su(1,1)\) algebras.
\[
D_{1+} = A_k^\dagger A_{-k}^\dagger, \quad D_{1-} = A_k A_{-k} + A_{-k}^\dagger A_k^\dagger + 1),
\]
\[D_{10} = \frac{1}{2}A_k^\dagger A_k + A_{-k}^\dagger A_{-k} + 1),
\]
\[D_{2+} = B_k^\dagger B_k^\dagger, \quad D_{2-} = B_k B_{-k},
\]
\[D_{20} = \frac{1}{2}B_k^\dagger B_{-k} + B_k B_{-k} + 1),
\]
and rewrite the Hamiltonian in terms of the new generators
\[
H_1 = \int \frac{d^3k}{(2\pi)^3} \omega^2[[m_1D_{10} + m_2D_{20}]
\]
\[+ n_1[D_{1+} + D_{2-}] + n_2(D_{2+} + D_{1-})],
\]
showing \(su(1,1)\) symmetry. We can diagonalize the Hamiltonian using squeezing (Bogolubov) transformation
\[
H_f = S(\zeta_2)^\dagger S(\zeta_1)^\dagger H_1 S(\zeta_1)S(\zeta_2)
\]
where
\[
S(\zeta_1) = \exp[\zeta_1 D_{1+} - \zeta_1^* D_{1-}], \quad S(\zeta_2) = \exp[\zeta_2 D_{2+} - \zeta_2^* D_{2-}], \quad \zeta_1 = r_1 \exp[\gamma_1] \quad \text{and} \quad \zeta_2 = r_2 \exp[\gamma_2].
\]

The effect of the operators \(S(\zeta_1)\) and \(S(\zeta_2)\) on \(A_k\) and \(B_k\) is
\[
A_k(k, \eta) = \mu_1 A_k + \nu_1 A_{-k}^\dagger,
\]
\[A_{k}^\dagger(k, \eta) = \mu_1^* A_k + \nu_1^* A_{-k}^\dagger,
\]
\[B_k(k, \eta) = \nu_2 B_k + \mu_2 B_{-k}^\dagger,
\]
\[B_k^\dagger(k, \eta) = \nu_2^* B_k^\dagger + \mu_2^* B_{-k}.
\]
where $\mu_1 = \cosh(r_1) = \frac{m_1}{\sqrt{m_1^2 - n_1^2}}$, $\nu_1 = e^{-i\gamma} \sinh(r_1) = e^{-i\gamma} \frac{m_1}{\sqrt{m_1^2 - n_1^2}}$, $\mu_2 = \cosh(r_2) = \frac{m_2}{\sqrt{m_2^2 - n_2^2}}$ and $\nu_2 = e^{-i\gamma} \sinh(r_2) = e^{-i\gamma} \frac{m_2}{\sqrt{m_2^2 - n_2^2}}$. Thus the final diagonalized Hamiltonian after two unitary transformations is

$$H_f = \int \frac{d^3k}{(2\pi)^3} \Omega_+ [A_1^*(k,\eta) A_2(k,\eta) + 1] + \Omega_- [B_1^*(k,\eta) B_2(k,\eta) + 1],$$

where $\Omega_+ = \sqrt{m_1^2 - n_1^2} = \sqrt{\omega^2 - 2\Omega^2}$ and $\Omega_- = \sqrt{m_2^2 - n_2^2} = \sqrt{\omega^2 + 2\Omega^2}$.

The vacuum state of $H_f$ at time $\eta$ is given by $|0(\eta), 0(\eta)\rangle >$ and vacuum state of $H$ at initial time is given by $|0, 0\rangle >$, which are related by

$$|0(t), 0(t)\rangle = e^{\frac{\omega^2 t^2}{8D^2}}\langle [\xi, D_1, -\zeta_1 D_2, [\xi, D_1, -\zeta_1 D_2]] |0, 0\rangle >$$

(38)

We see that the vacuum at later times is populated with particles and anti-particles with respect to vacuum state at initial time. We can estimate the number of particles and anti-particles from the relationship between the creation and annihilation operators at initial time given by $a_s$ and $b_s$, and the final creation and annihilation at later time given by operators $A_s$ and $B_s$.

$$A_s(k,\eta) = (\mu_1 \cos(\theta) + (\nu_1 \sin(\theta) e^{2i\xi}) b_k^\dagger + (\mu_1 \sin(\theta) e^{2i\xi}) b_k^\dagger, (39)$$

$$B_s(k,\eta) = (\nu_2 \cos(\theta) + (\nu_2 \sin(\theta) e^{2i\xi}) b_k^\dagger + (\mu_2 \sin(\theta) e^{2i\xi}) b_k^\dagger (40)$$

The number of particles (baryons) for each mode

$$N_{kB}(\eta) = \langle B_1^*(k,\eta) B_2(k,\eta) \rangle = |\nu_2|_k|^2, (41)$$

and the number of anti-particles (anti-baryons) for each mode

$$N_{K\bar{B}}(\eta) = \langle A_1^*(k,\eta) A_2(k,\eta) \rangle = |\nu_1|_k|^2. (42)$$

Therefore the baryon asymmetry is

$$\Delta N_k = N_B(\eta) - N_{\bar{B}}(\eta) = \frac{\Omega^6}{\omega^2 (4\Omega^4 - \omega^4)} (43)$$

where $\Omega^2 = \sqrt{\rho_2^2 + \delta^2}$, recalling that $\rho = 3(\chi_2^2 - \chi_2^4)$ and $\delta = 3(\chi_2^4 - \chi_2^6)$. We find that $N_B(\eta) - N_{\bar{B}}(\eta)$ is dependent on vacuum expectation values of real and imaginary parts of scalar field and coupling constant $\lambda$.

The total asymmetry is given by,

$$\int_0^\infty \Delta N_k d^3k = \int_0^\infty k^2 dk \frac{\Omega^6}{\omega^2 (4\Omega^4 - \omega^4)}$$

$$= |\Omega^2 (\sqrt{m_2^2 + 2\Omega^2} - \sqrt{m_2^2 - 2\Omega^2})|^4$$

(41)

It is interesting to see that when $\lambda \ll 1$ and $\chi_2 = \phi_0$ the asymmetry reduces to classical value,

$$(N_B(\eta) - N_{\bar{B}}(\eta)) = \frac{3\lambda \phi_0^2}{2m_2^2} \simeq r. (45)$$

### III. EVOLUTION OF ASYMMETRY PARAMETER:

In order to get some exact results and numerical values for the parameter $r$ after expansion, we consider a (quite realistic) expansion where we can evaluate the Bogolubov coefficients exactly.

For this consider the time evolution of wave function under the action of the Hamiltonian $H_f$ given by (37). Going over to the coordinate representation $P_{(A_B)}$ and $P_{\Pi_{(A_B)}}$ defined by the relations (14), $A_s(k,\eta) = e^{i/2\langle A_s(\eta) b_s(k,\eta) \rangle}$ and $B_s(k,\eta) = e^{i/2\langle A_s(\eta) b_s(k,\eta) \rangle}$ (and their complex conjugates), the Hamiltonian $H_f$ is

$$H_f(\eta) = \int \frac{d^3k}{(2\pi)^3} \sum_{i=A,B} \frac{1}{2} [(\Omega_+)^2 \Omega_+^2 + P_{\Pi_i}^2]$$

(46)

The time evolution of a wave function $\psi(\eta)$ under the action of a Hamiltonian $\dot{H}(\eta)$ is simply

$$H(\eta) \psi(\eta) = i \frac{d}{d\eta} \psi(\eta), (47)$$

From the form of $H_f$ given above, it is clear that it is the direct sum of two independent Hamiltonian $H_A(\eta)$ and $H_B(\eta)$ for each of the $A_s$ and $B_s$ modes. The wave function for each momentum mode evolves as

$$H_A(k,\eta) \psi_A(k,\eta) = i \frac{d}{d\eta} \psi_A(k,\eta), (48)$$

$$H_B(k,\eta) \psi_B(k,\eta) = i \frac{d}{d\eta} \psi_B(k,\eta). (49)$$

Since the Hamiltonians $H_A$ and $H_B$ are time dependent harmonic oscillators, in the coordinate space representation the wave functions $\psi_A(k,\eta)$ and $\psi_B(k,\eta)$ can be represented by gaussian wave functions.

After some algebra, the evolution equations satisfied by the two wave functions for each mode are

$$\psi''_A(k,\eta) + \Omega_+^2 \psi_A(k,\eta) = 0, (50)$$

$$\psi''_B(k,\eta) + \Omega_-^2 \psi_B(k,\eta) = 0. (51)$$

where $\Omega_+ = k^2 + m_2^2 a^2 - \frac{\omega}{a} - 2\lambda \sqrt{\rho^2 + \delta^2}$ and $\Omega_- = k^2 + m_2^2 a^2 - \frac{\omega'}{a} + 2\lambda \sqrt{\rho^2 + \delta^2}$.

We rewrite these equations in the Schroedinger like form in $\eta$

$$\psi_A' + (E + V_1(a)) \psi_A = 0, (52)$$

and

$$\psi_B' + (E + V_2(a)) \psi_B = 0, (53)$$

where

$$E = k^2 + m_2^2.$$
\[ V_1(a) = -2\lambda \sqrt{\rho^2 + \delta^2}, \quad (55) \]
\[ V_2(a) = 2\lambda \sqrt{\rho^2 + \delta^2}. \quad (56) \]

where \( \sqrt{\rho^2 + \delta^2} = 3\sqrt{\chi_{10}^2 + \chi_{20}^2 - \chi_{10}^2 \chi_{20}^2} \). Writing the equations in this form allows us to use the machinery of potential barrier reflection and transmission problems in quantum mechanics. The reflection \((R)\) and transmission \((T)\) coefficients can be related to the squeezing parameter \((r)\) through the relation \(sinh^2(r) = |r|^2 = \frac{\delta}{\chi} \) allowing us to calculate \( N(k) = |r|^2 \). We can also explicitly see the origin of the asymmetry in the amplification of the particle and anti-particle modes. The particles face a potential barrier and the antiparticles a potential well. This results in a differential evolution of the particle and antiparticle modes resulting in baryon asymmetry.

We now see that there are two factors that contribute to the time evolution of the particle and anti-particle modes, the time dependence of the background classical solution and the time dependence of the expansion factor \( a(\eta) \).

**A. Slow Expansion**

As a first approximation we consider the case where \( \omega^2 = 0 \), i.e., radiation dominated universe. In this case as seen in \( \ref{eq:1} \) the classical equations for \( \chi_{10} \) and \( \chi_{20} \) are given by

\[ \chi_{10}'' + m_{\eta}^2 \chi_{10} = 6\lambda \chi_{10}^2 \chi_{20} - 2\lambda \chi_{20}^3, \quad (57) \]

and

\[ \chi_{20}'' + m_{\eta}^2 \chi_{20} = -6\lambda \chi_{20}^2 \chi_{10} + 2\lambda \chi_{10}^3 \quad (58) \]

To solve these equations analytically we neglect the cubic term and then we apply the boundary conditions in equation \((4)\) to get the time dependent background solutions

\[ \chi_{10} = \frac{\lambda \phi_0}{(m\eta)^3} sin(m\eta + \epsilon) \quad (59) \]
\[ \chi_{20} = \frac{\phi_0}{m\eta} sin(m\eta). \quad (60) \]

Then to up to first order we have \( \lambda \sqrt{\rho^2 + \delta^2} = \sqrt{\rho^2 + \delta^2} \) where \( \sqrt{\rho^2 + \delta^2} \) is a slowly decreasing amplitude, given by \( \sqrt{\rho^2 + \delta^2} \approx 3\lambda(m\eta)^2 \).

The equations \( \ref{eq:2} \) and \( \ref{eq:3} \) become

\[ \psi_A'' + (k^2 + m_{\phi}^2 - \sqrt{\rho^2 + \delta^2} \sin^2(m\eta)) \psi_A = 0, \quad (61) \]

and

\[ \psi_B'' + (k^2 + m_{\phi}^2 + \sqrt{\rho^2 + \delta^2} \sin^2(m\eta)) \psi_B = 0. \quad (62) \]

These can be written as the Mathieu equations associated which are familiar from the parametric amplification problem in inflation.

\[ \psi_A'' + \omega_{1k}^2(1 + \frac{\sqrt{\rho^2 + \delta^2}}{\omega_{1k}} \cos(\gamma \eta) - \frac{\alpha''}{\omega_{1k}^2}) \psi_A = 0, \quad (63) \]

and

\[ \psi_B'' + \omega_{2k}^2(1 - \frac{\sqrt{\rho^2 + \delta^2}}{\omega_{2k}} \cos(\gamma \eta) + \frac{\alpha''}{\omega_{2k}^2}) \psi_B = 0. \quad (64) \]

. where \( \omega_{1k}^2 = k^2 + m_{\phi}^2 - \sqrt{\rho^2 + \delta^2} \), \( \omega_{2k}^2 = k^2 + m_{\phi}^2 + \sqrt{\rho^2 + \delta^2} \) and \( \gamma = 2m\eta \)

To solve these equations we follow the method given in \( \ref{15} \) and \( \ref{16} \). From the theory of parametric resonance, the resonance is strongest if the frequency is twice \( \omega_{ik} \), hence we put \( \gamma = 2\omega_{ik} + \epsilon \) with \( \epsilon \ll \omega_{ik} \), the resonance condition will be satisfied if \( \sqrt{\rho^2 + \delta^2} > 0 \) or \( |\epsilon| < \sqrt{\rho^2 + \delta^2} \). We define a new variable \( l = \frac{\phi}{\sqrt{\rho^2 + \delta^2}} \) so that resonance occurs \(-1 < l < 1\).

Then by using \( \ref{16} \), the number of particles produced is given by

\[ N_{1k} = \frac{1}{1 - l^2} \sinh^2\left(\sqrt{\rho^2 + \delta^2} \sqrt{\rho^2 + \delta^2} \eta\right), \quad (65) \]

and the number of anti-particles is

\[ N_{2k} = \frac{1}{1 - l^2} \sin^2\left(\sqrt{\rho^2 + \delta^2} \sqrt{\rho^2 + \delta^2} \eta\right). \quad (66) \]

For parametric resonance, it is important that the inflaton stays in resonance band and this is possible as long as its amplitude is slowly varying function of time. The time dependence of the number of particles and antiparticles comes from the slow time variation of the decaying amplitude, which we phenomenologically approximate with \( \rho^2 \approx \sqrt{\rho^2 - \epsilon^2} \) where \( \epsilon \) is damping scale. In the figure (a) and (b) we have plotted \( N_1(k) - N_2(k) \) for various values of the values of \( \sqrt{\rho^2} \). We can see that the value of asymmetry saturates to a finite value of \( \approx 10^{-8} \).

We assume broad resonance such that the Mathieu equation has instability bands with in which parametric resonance occurs, we shall select the first instability region as broad resonance band.

Therefore in the region of broad band resonance we replace the oscillating potential near its zeros with an asymptotically flat potential of the form

\[ |\sqrt{\rho^2} \sin^2(m\eta)| \approx 2|m\eta|^2 \tanh^2(m\eta - \eta) \sqrt{\rho^2}, \quad (67) \]

Then \( \ref{eq:2} \) and \( \ref{eq:3} \) become

\[ \psi_A'' + (k^2 + m_{\phi}^2 - \sqrt{\rho^2} \tanh^2(m\eta - \eta) \sqrt{\rho^2}) \psi_A = 0, \quad (68) \]

and

\[ \psi_B'' + (k^2 + m_{\phi}^2 + \sqrt{\rho^2} \tanh^2(m\eta - \eta) \sqrt{\rho^2}) \psi_B = 0. \quad (69) \]
\[ \varphi = 10^{-2} e^{-\eta / 3} \]

where the number of antiparticles is.

We get the following differential equations for the particle and antiparticle modes

\[ \frac{d^2 \psi_A}{dy^2} + \left[ \kappa_1^2 + \varphi^2 \text{sech}^2(y) \right] \psi_A = 0. \] (70)

\[ \frac{d^2 \psi_B}{dy^2} + \left[ \kappa_2^2 + \varphi^2 \tanh^2(y) \right] \psi_B = 0. \] (71)

where

\[ \kappa_1^2 = \frac{k^2 - \varphi^2}{m_\phi^2} + 1, \]

\[ \kappa_2^2 = \frac{k^2}{m_\eta^2} + 1 \]

\[ y = m_\phi (\eta - \eta_0). \]

Using the transmission and reflection coefficients \([17]\), the number of particles is

\[ n_{1k} = |\nu_{1k}|^2 = \frac{\left( \cos^2(\pi \sqrt{\varphi^2 + \frac{1}{4}}) \right)^2}{\sinh^2(\pi \kappa_1 \varphi)}, \] (73)

the number of antiparticles is

\[ n_{2k} = |\nu_{2k}|^2 = \frac{\left( \cosh(\pi \sqrt{\varphi^2 + \frac{1}{4}}) \right)^2}{\sinh^2(\pi \sqrt{\varphi^2 + \kappa_2^2})}. \] (74)

In the figure (c) and (d) the evolution of particles and antiparticles for different values of \( \varphi = 0.27 \), \( \varphi = 0.28 \), \( \varphi = 0.29 \) are plotted respectively. We can see clearly that the number of particles increases and number of antiparticles decreases due to differential amplification of particle and antiparticle modes.

### B. Rapid Expansion

Now we consider the effect of rapid expansion on the asymmetry parameter. We consider the case when the rate of expansion dominates over the oscillation period of the classical background solution so that \( \rho \) and \( \delta \) can be considered as time independent. We consider

\[ a(\eta) = (a_0 \eta)^{\frac{p}{2}}, \quad \eta < \eta_0 \]

\[ a(\eta) = C(\eta - \eta_0), \quad \eta > \eta_0 \]

where \( \eta_0 = \eta_s - (a_0^2 \eta_s)^{-1} \). It is convenient to set \( a(\eta_0) = 1 \) which sets \( \eta_* = a_0^{-1} \), and thus \( \eta_0 = 0 \) and \( C = a_0^{-1} \) where \( a_0 = -H_0 \) for de Sitter spacetime. Here \( p=\frac{1}{2} \) corresponds to radiation dominated universe and \( p \rightarrow \infty \) corresponds to de Sitter epoch.

Thus \([27]\) is

\[ \frac{d^2 \psi_A}{d\eta^2} + \left[ k^2 - 2\lambda \sqrt{\rho^2 + \delta^2} - \frac{p(2p - 1)}{(p - 1)^2 \eta^2} + \frac{m^2}{H^2 \eta^2} \right] \psi_A = 0 \]

\[ \eta < \eta_0 \] (79)

The equation \([77]\) can be written as

\[ \frac{d^2 \psi_A}{d\tau^2} + \left[ \frac{1}{\tau_1^2} - \frac{q^2}{\varphi_1^2} \right] \psi_A = 0 \] (79)

where \( q^2 = \frac{(3p-1)^2}{4(p-1)^2} - \frac{m^2}{\phi^2} \), \( \tau_1 = \sqrt{\varphi_1 \eta} \) and \( g_1 = k^2 - 2\lambda \sqrt{\rho^2 + \delta^2} \). The solution is given by

\[ \psi_A = e^{\sqrt{\eta} \tau_1} \left[ A_k H_q^{(1)}(\sqrt{\eta} \tau_1) + B_k H_q^{(2)}(\sqrt{\eta} \tau_1) \right] \] (80)

where \( H_q^{(1)} \) and \( H_q^{(2)} \) are Hankel functions.

The solution for \([28]\) is given by

\[ \psi_A(\eta > \eta_0) = \frac{1}{\sqrt{2k}} [m e^{-i \sqrt{\eta} \varphi} + m e^{i \sqrt{\eta} \varphi}] \] (81)

The Bogolubov coefficients are obtained by matching the wave functions and its first derivative at \( \eta = \eta_0 \)

The number of particles is given by

\[ N_1(k) = |\nu_1|^2 = 4^{q-2} \left( \frac{\sqrt{\eta_0}}{a_0} \right)^{-2q-1} \left( \frac{1}{2} \right)^2 \Gamma^2(q). \] (82)

Using similar methods for \([28]\) the number of antiparticles produced is given by

\[ N_2(k) = |\nu_2|^2 = 4^{q-2} \left( \frac{\sqrt{\eta_0}}{a_0} \right)^{-2q-1} \left( \frac{1}{2} \right)^2 \Gamma^2(q). \] (83)
where \( g_2 = k^2 + 2\lambda\sqrt{\rho^2 + \delta^2} \), \( a_0 \) is the reference scale of \( H_0 \), \( H_0 \) is constant for de Sitter expansion.

First we have considered a case when \( p \to \infty \) which corresponds to de Sitter epoch.

The vacuum fluctuations of massive fields on exact de Sitter background leads to density perturbations only for \( \frac{m^2_{a_0}}{H^2} < 2 \) and \( 2 < m^2 < \frac{9H^2_0}{4} \). The corresponding characteristic values are \( q = 0 \), and \( \frac{q^2}{m^2_{a_0}} = \frac{9}{4} \) for de Sitter case. In the figure (e) \( (N_1(k) - N_2(k)) \) is plotted, with \( q = .6 \), \( q = .7 \), \( q = .8 \), which corresponds to \( \frac{m^2_{a_0}}{H^2} < 2 \) for de Sitter epoch at this value the condensate starts oscillating and gives rise to fluctuations.

In the figure (c) \( N_1(k) - N_2(k) \) is plotted for the different values of \( \phi_0 \) for a fixed \( q = .6 \). From the figure we can see that as the \( \phi_0 \) value decreases the value of asymmetry reduces. In the figure (g) \( N_1(k) - N_2(k) \) is plotted for the different values of \( \lambda \) for a fixed \( q = .6 \) and \( \phi_0 \). From the figure we can see that as the \( \lambda \) value decreases the value of asymmetry reduces. Large occupation number in a given mode means that quasi-particles formed a condensate. Therefore from the figure we can see that once the quantum fluctuations are switched on the value of asymmetry reduces but does not goes to zero.

Now we consider the case when \( p \to \frac{1}{2} \) which corresponds to radiation dominated universe.

In the radiation dominated universe, when \( \frac{m^2_{a_0}}{H^2} < .25 \), then only the vacuum fluctuations of massive fields will be switched on, and the characteristic values for \( q = 0 \), is \( \frac{q^2}{m^2_{a_0}} = \frac{1}{4} \).

In the figure (h) \( N_1(k) - N_2(k) \) is plotted for the different values of \( \phi_0 \) for a fixed \( q = .4 \). corresponds to \( \frac{m^2_{a_0}}{H^2} = .16 \). From the figure we can see that once the quantum fluctuations are switched on the value of asymmetry reduces. In the figure (i) \( N_1(k) - N_2(k) \) is plotted for the different values of \( \phi_0 \) for a fixed \( q = .4 \) for quantum fluctuations, in this case the asymmetry goes to \( 10^{-3} \frac{95\lambda^2}{m^2_{a_0}} \). We conclude that the Affleck Dine mechanism when combined with the CP violating amplification of vacuum fluctuations during inflation can give an acceptable value of baryon asymmetry of universe.

**IV. CONCLUSION**

In this paper we have studied the non-equilibrium quantum effects of Affleck Dine baryogenesis in the post inflationary scenario. In the paper [9] they have studied the model using the nonequilibrium dynamics and used perturbative methods to get the asymmetry, whereas the methodology developed here using squeezed states or Bogolubov transformations has allowed us to derive the general evolution equations for baryon and anti-baryon modes non-perturbatively in an expanding FRW metric. In our evolution equation the effective potential is dependent upon the expansion parameter \( a(\eta) \) and inflaton potential. The amount of particle production in the de Sitter expansion is calculated as the tunnelling through a barrier of potential \( V(a) \). We find that by considering different inflationary scenarios and parametric resonance we can control the asymmetry parameter "r" to a much lower value than in the classical Affleck Dine model.

Of course, we have used a simplified toy model, but the method is general and can work for a more realistic scenario also.

Baryon asymmetry remains an intriguing unsolved issue in physics. In view of the standard model being in sufficient to explain this essential fact about the universe, one has to look beyond the standard model. Supersymmetry is a compelling idea beyond standard model. Hence finding arguments for baryon asymmetry in MSSM is a natural idea. The Affleck Dine mechanism which uses the flat directions in supersymmetric models is therefore a very promising mechanism for baryon asymmetry generation. Furthermore there is also very compelling evidence that universe went through an inflationary phase and particles were generated by reheating processes. Thus combining the two we should be able to get a plausible and viable scenario for baryon asymmetry. Means to reduce the rather over efficient generation of
baryon asymmetry in Affleck Dine mechanism require a thorough study. In this paper we have done a systematic study of the effects of inflation and parametric amplification of quantum fluctuations on the baryon asymmetry generated in post-inflationary Affleck Dine baryogenesis. Since a variant of the Affleck Dine mechanism is also used to account for dark matter as well as in most exotic scenarios of baryogenesis our method should be useful in this context also.

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