DEFORMATIONS AND GEOMETRIC COSETS

P.M. Petropoulos

Centre de Physique Théorique, Ecole Polytechnique†
91128 Palaiseau Cedex, FRANCE

Abstract

I review some marginal deformations of $SU(2)$ and $SL(2, \mathbb{R})$ Wess–Zumino–Witten models, which are relevant for the investigation of the moduli space of NS5/F1 brane configurations. Particular emphasis is given to the asymmetric deformations, triggered by electric or magnetic fluxes. These exhibit critical values, where the target spaces become exact geometric cosets such as $S^2 \equiv SU(2)/U(1)$ or $AdS_2 \equiv SL(2, \mathbb{R})/U(1)_{\text{space}}$. I comment about further generalizations towards the appearance of flag spaces as exact string solutions.

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† Unité mixte du CNRS et de l’Ecole Polytechnique, UMR 7644.
1 Framework and summary

Various brane configurations such as D3, NS5, NS5/F1 or D1/D5, ... have attracted much attention over the recent years. They provide interesting supergravity or string backgrounds, which turn out to be laboratories for exploring AdS/CFT correspondence, black-hole physics or little-string theory.

The near horizon geometries of such configurations are remarkable spaces: spheres or anti-de-Sitter spaces in various dimensions. In some situations, they give rise to target spaces of exact two-dimensional conformal models. In those cases, they allow for analyzing string theory beyond the usual supergravity approximation.

An important and yet not fully unravelled subject is the analysis of the moduli space of those brane set-ups. Whenever a two-dimensional conformal interpretation is available, this moduli space can be explored by means of marginal deformations. Marginal deformations are exact and controllable theories, with a good handle over their spectrum, partition function and amplitudes.

Wess–Zumino–Witten models allow for a large class of marginal deformations. The better known among those, referred to as “symmetric deformations” in the following, connect the original WZW model to a $U(1)$-gauged version of it (see also [2] for further references and a more general framework). When the WZW model under consideration is embedded in a wider string set-up, one can introduce further “asymmetric” deformations. Those, originally introduced in the $SU(2)$ WZW model [3, 4, 5], were triggered by a magnetic field in the framework of heterotic string. It has been realized very recently, that such a deformation could be consistently pushed up to a critical value of the magnetic field, with the original three-sphere target space being continuously driven to a two-sphere times a line (free non-compact boson), with a full control over the spectrum and the partition function [6].

The above result brought up again the question of how to construct truly geometric cosets as target spaces of exact conformal field theories. Although some results were available in the literature [7, 8, 9, 10, 11], these were mostly based on asymmetric orbifold or gauging construction, and no systematic method existed for reaching geometric cosets.

The problem at hand has been successfully revisited in [6, 12], generalizing the aforementioned result about the two-sphere. By performing marginal asymmetric deformations on a group-$G$ WZW, one can reach cosets of the type $G/U(1)^n$, where $n \leq \text{rank } G$. No dilaton is needed, whereas in general the NS three-form can survive, together with electric or magnetic fluxes.

Some specific situations turn out to be particularly interesting. One is the $SL(2,\mathbb{R})$ WZW model, where the group is non-compact. Its target space is AdS$_3$, and three different marginal deformations appear, generated by space-, time- or light-like vectors. The latter does not exhibit any critical behavior, whereas the formers lead either to AdS$_2$ or $H_2$. This result is important because it shows that the near-horizon geometry of the four-dimensional Bertotti–Robinson solutions [13, 14] (i.e. a Reissner–Nordström black hole with both electric and magnetic backgrounds), $\text{AdS}_2 \times S^2$, is the target space of an exact two-dimensional
conformal field theory that is continuously connected to the near-horizon geometry of the NS5/F1 brane configuration. The exact spectrum of string excitations is available for this model (in a previous approach [11], the case of RR backgrounds has been studied using a hybrid method of Green–Schwarz/Neveu–Schwarz–Ramond; it has led to $O(\alpha')$ results only).

The situation of the compact group $SU(3)$ is also interesting. The coset $SU(3)/U(1)^2$ appears actually with two different metrics. The first corresponds to the Kähler structure of the “flag space”, recognized long time ago [15] to be a string solution, at $O(\alpha')$ though, whereas in [12] it is shown to be exact since it originates from a marginal deformation. The second metric is accompanied by a Neveu–Schwarz form and is no longer Kähler; the string background is still exact\(^1\). For both, spectra and partition functions are available.

In the following, I will present a reminder of the symmetric deformations in Sec. 2, with particular emphasis to the $SU(2)$ and $SL(2,\mathbb{R})$ WZW models and $U(1)$-gauged WZW models. Section 3 is devoted to the investigation of the asymmetric deformations of those models, stressing in particular the appearance of geometric cosets such as $S^2$, $H_2$ or $AdS_2$. For generalizations to other groups ($SU(3)$ or $Usp(4)$), I refer to [12]. A few extra remarks are collected in Sec. 4.

2 Symmetric marginal deformations and gauged WZW models

The target space of WZW models is a group manifold with isometry group $G \times G$. At the level of the conformal field theory, this symmetry is promoted to an affine algebra, realized at level $k$, and generated by holomorphic and antiholomorphic currents $J^i(z)$ and $\bar{J}^j(\bar{z})$.

A symmetric deformation of the original model is provided by

$$S_k(\lambda) = S_k + \lambda \int d^2z \, J^i \bar{J}^j. \quad (2.1)$$

It was shown in [17] that the resulting theory is exactly conformal. This deformation is gravitational: it acts on the metric, the antisymmetric NS tensor, and introduces a dilaton background. It also reduces the symmetry to the Cartan subgroup of $G$, both on the left and right side.

An interesting feature of the gravitational deformation is the behavior at large values of $\lambda$. In that limit, the sigma model factorizes into an $\mathbb{R}$-line times a gauged WZW model, $G_k/U(1)$. The target space of the latter is not a geometric coset space. Actually, although the theory is exactly conformal, the background fields cannot be put in a closed form but can only be determined order by order in $\alpha'$.

I will now illustrate these features in the case of the $SU(2)$ and $SL(2,\mathbb{R})$ WZW models.

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\(^1\)Cosets with torsion have been studied in the past as $O(\alpha')$ solutions [16].
2.1 The three-sphere

The three-sphere is the group manifold of $SU(2)$. In Euler-angle parameterization, the metric and the two-form potential read:

\[ ds^2 = \frac{L^2}{4} \left[ d\beta^2 + \sin^2 \beta \, d\alpha^2 + (d\gamma + \cos \beta \, d\alpha)^2 \right], \]

\[ B = \frac{L^2}{4} \cos \beta \, d\alpha \wedge d\gamma, \]

with $L = \sqrt{k+2}$.

The group at hand has rank one and there is only one line of gravitational deformation: all choices for $J_i \bar{J}^i$ are equivalent due to the $SU(2)_L \times SU(2)_R$ symmetry.

At large values of the deformation parameter, one reaches the gauged WZW $SU(2)_k/U(1)$ times a free line. The $SU(2)_k/U(1)$ model is an exact conformal field theory described in terms of compact parafermions. In terms of background fields, it has no antisymmetric tensor but has a dilaton $e^{-\Phi} \sim \cos \theta$, whereas the metric reads (at $O(\alpha')$):

\[ ds^2 = k \left[ d\theta^2 + \tan^2 \theta \, d\psi^2 \right]. \]

$(0 \leq \theta \leq \pi/2$ and $0 \leq \psi \leq 2\pi)$. This is the bell geometry. The residual symmetry is $U(1) \times U(1)$.

2.2 Anti de Sitter in three dimensions

Anti de Sitter in three dimensions is also a group manifold, of a non-compact group though, $SL(2, \mathbb{R})$. Metric and antisymmetric tensor read (in hyperbolic coordinates):

\[ ds^2 = \frac{L^2}{4} \left[ dr^2 - \cosh^2 r \, d\tau^2 + (dx + \sinh r \, d\tau)^2 \right], \]

\[ H_{[3]} = \frac{L^2}{4} \cosh r \, dr \wedge d\tau \wedge dx, \]

with $L$ related to the level of $SL(2, \mathbb{R})_k$ as usual: $L = \sqrt{k+2}$. In the case at hand three different\(^2\) lines of gravitational deformations arise due to the presence of time-like ($J^3$, $\bar{J}^3$), space-like ($J^1$, $\bar{J}^1$, $J^2$, $\bar{J}^2$), or null generators \([18, 19, 20]\). The residual symmetry is again $U(1) \times U(1)$ that can be time-like, space-like or null depending on the deformation under consideration.

The elliptic deformation is driven by $J^3 \bar{J}^3$ bilinear. At infinite $\lambda$, a time-like direction decouples and we are left with the\(^3\) $SL(2, \mathbb{R})_k/U(1)_{\text{time}}$. The target space of the latter is the

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\(^2\)Mixing left and right currents of different kind in the bilinear is forbidden because it generates anomalies.

\(^3\)The deformation parameter has two T-dual branches (positive or negative $\lambda$) the extreme values of deformation corresponds to the axial or vector gaugings. The vector gauging leads to the trumpet. For the $SU(2)_k/U(1)$, both gaugings correspond to the bell.
cigar geometry (also called Euclidean two-dimensional black-hole)

\[ e^{-\Phi} \sim \cosh \rho, \quad (2.8) \]

\[ ds^2 = k \left[ d\rho^2 + \tanh^2 \rho \, d\psi^2 \right], \quad (2.9) \]

\((0 \leq \rho < \infty \text{ and } 0 \leq \psi \leq 2\pi).\)

Similarly, by using \(J^2\bar{J}^2,\) one generates the hyperbolic deformations. This allows to reach the Lorentzian two-dimensional black-hole times a free space-like line.

Finally, the bilinear \((J^1 + J^3) \left( \bar{J}^1 + \bar{J}^3 \right)\) generates the parabolic deformation. At extreme values of the deformation parameter, a whole light-cone decouples and we are left with a single direction and a dilaton field (Liouville model) \([20]\).

### 3 Asymmetric marginal deformations and geometric cosets

In the framework of string theory, group-\(G\) WZW models are usually embedded in wider structures. This offers new possibilities for the choice of bilinears that trigger deformations. Asymmetric deformations are generated by \(J^j \bar{J}_{\text{gau}},\) with \(J^j\) a current of the left chiral algebra \(G_k\) of the WZW model, and \(\bar{J}_{\text{gau}}\) a right \((0,1)\) current of some other sector of the theory. In heterotic strings e.g., this could be a current from the gauge sector and this is the situation I will have in mind in the following, with the deformation

\[ \delta S = \frac{\sqrt{k k_{\text{gau}}} H}{2\pi} \int d^2 z \left( J^j + i \Upsilon \Psi \right) \bar{J}_{\text{gau}}, \quad (3.1) \]

where the fermion bilinear \(\Upsilon \Psi\) is introduced in order to preserve the worldsheet \(N = (1,0)\) supersymmetry.

From the target-space point of view, such a deformation generates electric or magnetic background fluxes. It is worth stressing here that introducing such backgrounds in flat space is not a marginal deformation, whereas it is marginal in the framework of WZW, thanks to the gravitational background \([3, 10, 12]\). The latter is altered by the presence of the electric/magnetic field that induces a back-reaction. Both gravitational and antisymmetric-tensor back-reactions, and background electric/magnetic fields are directly read off by performing a Kaluza–Klein type of reduction. No dilaton appears. Furthermore, contrary to what happens to the purely gravitational deformation (the symmetric one), the background fields one obtains at first order are exact to all orders in \(\alpha'\), provided \(k \rightarrow k + g^*\) (\(g^*\) is the dual coxeter number of \(G\)) \([21]\). The trivialization of the higher-order corrections is here due to the high degree of residual symmetry: \(U(1) \times G\) (instead of \(U(1) \times U(1)\) as it appears in the symmetric deformation).

Notice finally that electric and magnetic fluxes usually brake most of target-space supersymmetry, which is in general avoided with purely gravitational (left-right symmetric) perturbations. Let me now turn to the description of the \(SU(2)_k\) and \(SL(2, \mathbb{R})_k\) electric and magnetic deformations.

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\(^4\)This is also what happens for the undeformed WZW model.
3.1 The squashed three-sphere

As for the symmetric deformation, the $SU(2)_k$ WZW model makes it possible for a single asymmetric deformation only. The resulting geometry is a squashed sphere with NS and magnetic backgrounds (in Euler coordinates):

$$ds^2 = \frac{k}{4} \left[ d\beta^2 + \sin^2 \beta \, d\alpha^2 + (1 - 2H^2) \left( d\gamma + \cos \beta \, d\alpha \right)^2 \right],$$

$$A = \sqrt{\frac{2k}{k_{\text{gau}}}} H \left( d\gamma + \cos \beta \, d\alpha \right),$$

$$H_{[3]} = dB - \frac{k_{\text{gau}}}{4} A \wedge dA = \frac{k}{4} \left( 1 - 2H^2 \right) \sin \beta \, d\alpha \wedge d\beta \wedge d\gamma.$$

The residual isometry is $U(1) \times SU(2)$ and the curvature reads:

$$R = \frac{2}{k}(3 + 2H^2).$$

The metric is an $S^1$ fibration over an $S^2$ base, as it is for the ordinary three-sphere. However, the radius of the fiber is altered by the magnetic field. At the critical value $H_{\text{max}}^2 = 1/2$, it shrinks to zero and a line decouples. The rest of the target space is a two-sphere plus a magnetic monopole. A two-sphere is a geometric coset: $S^2 \equiv SU(2)/U(1)$, which therefore emerges as an exact string background.

From the point of view of the spectrum, a whole tower of states, coupled to the magnetic field, become infinitely massive at $H_{\text{max}}^2$, and decouple from the remaining. From the analysis of the spectrum in the presence of the magnetic field, we can learn another interesting feature: for $H_{\text{lower}}^{\text{crit}} < H < H_{\text{upper}}^{\text{crit}} < H_{\text{max}}$, infinitely many tachyons appear, which create an instability [5]. Tachyonic instabilities are often observed in open or closed string theories in the presence of electric or magnetic fields [22, 23]. For our purpose however, the existence of a range of values for the magnetic field where tachyons are present is of little relevance since the critical value of $H$, where the two-sphere decouples, is outside the dangerous range.

Let me finally quote that the expression for the partition function can be found in [6].

3.2 A variety of squashed anti de Sitter’s

I now turn the $SL(2,\mathbb{R})$ case. As previously, three asymmetric deformations are available: the elliptic, the hyperbolic and the parabolic.

The elliptic deformation is generated by a bilinear where the left current is an $SL(2,\mathbb{R})_k$ time-like current. The background field is magnetic and the residual symmetry is $U(1)_{\text{time}} \times SL(2,\mathbb{R})$. The metric reads (in elliptic coordinates):

$$ds^2 = \frac{k}{4} \left[ d\rho^2 + \cosh^2 \rho \, d\phi^2 - (1 + 2H^2) \left( dt + \sinh \rho \, d\phi \right)^2 \right],$$

5This is in $\alpha'$ units, thus the maximum magnetic field is at the Planck scale.
where $\partial_t$ is the Killing vector associated with the $U(1)_{\text{time}}$. This AdS$_3$ deformation was studied in [24] as a *squashed anti de Sitter*. It has curvature

$$R = -\frac{2}{k} (3 - 2H^2). \quad (3.7)$$

Here, it comes as an *exact string solution* (provided $k \to k + 2$) together with an NS three-form and a magnetic field:

$$H[3] = dB - \frac{k_{\text{gau}}}{4} A \wedge dA = -\frac{k}{4} (1 + 2H^2) \cosh \rho \, d\rho \wedge d\phi \wedge dt, \quad (3.8)$$

$$A = H \sqrt{\frac{2k}{k_{\text{gau}}}} (dt + \sinh \rho \, d\phi). \quad (3.9)$$

For $H^2 > 0$, the above metric is pathological because it has topologically trivial closed time-like curves passing through any point of the manifold. Actually, for $H^2 = 1/2$ we recover exactly Gödel space, which is a well-known example of pathological solution of Einstein–Maxwell equations. Notice that, as long as geometry is concerned, $H^2$ needs not be positive. In particular, for $H^2 < 0$, the closed time-like curves disappear and the space–time is well defined. Unfortunately, the corresponding string theory is no longer unitary because the magnetic field becomes imaginary (see Eq. (3.9)).

From the string-theory point of view, the existence of closed time-like curves in the range $H^2 > 0$ translates into the appearance of tachyons. Those can be eliminated provided an extra, purely gravitational deformation, is switched on [25].

As it stands in expression (3.6), the squashed anti de Sitter is obtained starting with AdS$_3$ as an $S^1$ fibration over $H_2$, and acting on the radius of the fiber. A critical value for the magnetic field appears, where the fiber degenerates: $H_{\text{min}}^2 = -1/2$. We are left in this limit with a two-dimensional hyperbolic plane $H_2$, which is again a geometric coset $SL(2, \mathbb{R})/U(1)_{\text{time}}$, much like in the $SU(2)$ WZW model studied previously.

This latter result shows that one can explicitly construct an exact conformal sigma model with $H_2$ as target space, which is however non-unitary since it appears in the region of imaginary magnetic field.

The *hyperbolic deformation* can be studied in a similar fashion, where the left current in the bilinear is an $SL(2, \mathbb{R})_k$ space-like current. In hyperbolic coordinates:

$$ds^2 = \frac{k}{4} \left[ dr^2 - \cosh^2 r \, d\tau^2 + (1 - 2H^2) \left( dx + \sinh r \, d\tau \right)^2 \right], \quad (3.10)$$

where $\partial_x$ generates a $U(1)_{\text{space}}$. The total residual symmetry is $U(1)_{\text{space}} \times SL(2, \mathbb{R})$ and

$$R = -\frac{2}{k} (3 + 2H^2). \quad (3.11)$$

The complete string background now has an NS three-form and an electric field:

$$H[3] = \frac{k}{4} (1 - 2H^2) \cosh r \, dr \wedge d\tau \wedge dx, \quad (3.12)$$

$$A = H \sqrt{\frac{2k}{k_{\text{gau}}}} (dx + \sinh r \, d\tau). \quad (3.13)$$
The background at hand is free of closed time-like curves. The squashed AdS$_3$ is now obtained by going to the AdS$_3$ picture as an $S^1$ fibration over an AdS$_2$ base, and modifying the $S^1$ fiber. The magnitude of the electric field is limited at $H^2_{\text{max}} = 1/2$, where it causes the degeneration of the fiber, and we are left with an AdS$_2$ background with an electric monopole; in other words, a geometric coset $SL(2, \mathbb{R})/U(1)_{\text{space}}$.

The string spectrum of the above deformation is accessible by conformal-field-theory methods. It is free of tachyons and a whole tower of states decouples at the critical values of the electric fields. Details are available in [6].

Let me finally turn to the parabolic deformation, generated by a null $SL(2, \mathbb{R})_k$ current times some internal right-moving current. The deformed metric reads, in Poincaré coordinates:

$$ds^2 = k \left[ \frac{du^2}{u^2} + \frac{dx^+ dx^-}{u^2} - 2H^2 \left( \frac{dx^+}{u^2} \right)^2 \right], \quad (3.14)$$

and the curvature remains unaltered $R = -6/k$. This is not surprising since the resulting geometry is the superposition of AdS$_3$ with a gravitational plane wave. The residual symmetry is $U(1)_{\text{null}} \times SL(2, \mathbb{R})$, where the $U(1)_{\text{null}}$ is generated by $\partial_-$.

The parabolic deformation is somehow peculiar. Although it is continuous, the deformation parameter can always be re-absorbed by a redefinition of the coordinates: $x^+ \rightarrow x^+ / |H|$ and $x^- \rightarrow x^- / |H|$. Put differently, there are only three truly different options: $H^2 = 0, \pm 1$. No limiting geometry emerges in the case at hand.

As expected, the gravitational background is accompanied by an NS three-form (unaltered) and an electromagnetic wave:

$$A = 2 \sqrt{\frac{2k}{k_{\text{gau}}} H} \frac{dx^+}{u^2}. \quad (3.15)$$

4 Comments

A few final comments are in order here. Geometric cosets such as $S^2$, $H_2$ or AdS$_2$ show up in the asymmetric (electric/magnetic) deformations of $S^3$ and AdS$_3$. No sign of dS$_2$ appears however, even in some non-unitary regime (like $H_2$). This is another instance where de Sitter spaces seem naturally incompatible with string theory.

Critical, unitary string theory is compatible with $S^2$ or AdS$_2$, with very specific values of the magnetic or electric fields, appearing as critical values in the framework of deformed $S^3$ or AdS$_3$. To get a better intuition about this phenomenon, one could study the system off criticality: set $S^2$ or AdS$_2$ with a non-critical electric or magnetic fields and analyze the renormalization flow towards the infrared. This could go along with understanding instabilities induced by pair creation in the presence of electric or magnetic fields in those backgrounds, and might be relevant for understanding the radiation of four-dimensional non-extremal charged black holes.

Another interesting observation is that the Killing vectors which are necessary for introducing BTZ identifications are still present in the hyperbolic deformation, for
the non-extremal case, and in the parabolic deformation, for the extremal BTZ black hole. Whether those identifications lead, in the above framework, to a kind of squashed BTZ-like black holes is still an open problem.

Finally, one should quote that the investigations on the moduli space of NS5/F1 brane configurations cannot be complete by implementing only separate deformations of the $S^3$ or AdS$_3$ components. Marginal operators exist, which are combinations of $SU(2)_k$ and $SL(2,\mathbb{R})_k$ generators. Those certainly lead to new deformed geometries, about which very little is known [29, 30].

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