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Numerical investigation on magnetohydrodynamics mixed convection flow past a magnetic sphere

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Abstract. This study deals with magnetohydrodynamics mixed convection flow past a magnetic sphere. The governing equations consist of continuity, momentum and energy equation. All of the equations are analysed by using boundary layer concept. Stream function and similarity variable are used to transform these equations. Finite difference method implicit scheme is used to solved the equations numerically. The velocity and temperature profiles are determined for various non-dimensional parameters such as Prandtl number, magnetic parameter and mixed convection parameter.

1. Introduction
The study of fluid flow affected by magnetic field is called as magnetohydrodynamics. In industry and engineering application, the studies of this flow past one or more obstacles are conducted using analytical or numerical method. The flow can involve whether the presence of buoyancy force or not which affects its convection form. Molla et al. (2005) studied steady magnetohydrodynamics flow past a sphere where presence of heat generation express natural convection in this flow [1]. Beg et al. (2009) also conducted almost same things with what Molla et al. did [2]. The differences are that fluid is saturated porous medium and non-dimensional parameter used is different. Türkyılmazoğlu (2011) investigated analytically and numerically of magnetohydrodynamics flow and heat transfer past a porous rotating sphere near the equator. Chamka and Ahmed (2012) used a rotating sphere with different wall condition to study unsteady magnetohydrodynamics flow in forward stagnation point [4]. Widodo et al. studied behaviour of unsteady magnetohydrodynamics forced convection flow past a sphere at stagnation points $x = 0$ by using various Prandtl number and magnetic parameter [5].

The problem in this research is the unsteady of mixed convection magnetohydrodynamics boundary layer flow over a magnetic sphere. The temperature of the sphere is higher than the temperature of fluid. This condition is called as assisting convection $\alpha > 0$. The fluid velocity and temperature profiles in rear stagnation point $x = 0$ is studied. The profiles are determined by non-dimensional parameters such as Prandtl number, magnetic parameter and mixed convection parameter.
2. Numerical Methods

The fluid flows upward past a magnetic sphere with upstream velocity \( U_\infty \) and constant temperature \( T_\infty \) as shown in Figure 1. The magnetic sphere is brought into two-dimensional plane. The coordinate \( \bar{x} \) represents the distance along sphere surface from the rear stagnation point \( x = 0^\circ \) while \( \bar{y} \) represents the distance normal to sphere surface. The sphere has magnetic field strength \( B_0 \) which \( B_0 \) is applied in -\( z \) component with radius \( a \). The gravity \( g \) direction is in opposite of upstream flow. The sphere temperature \( T_w \) is constant.

![Figure 1. Physical Object and Coordinate System](image)

From the problem described above, the dimensional governing equations, i.e. continuity, momentum and energy equation, are given by

\[
\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0
\]

(1)

\[
\rho \left( \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \right) = -\frac{\partial \bar{p}}{\partial \bar{x}} + \mu \left( \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \right) + \sigma B_0 \bar{u} - \rho g (\bar{T} - T_\infty) \sin \left( \frac{\bar{x}}{a} \right)
\]

(2)

\[
\rho \left( \frac{\partial \bar{v}}{\partial t} + \bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} \right) = -\frac{\partial \bar{p}}{\partial \bar{y}} + \mu \left( \frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} \right) + \sigma B_0 \bar{v} - \rho g (\bar{T} - T_\infty) \cos \left( \frac{\bar{x}}{a} \right)
\]

(3)

\[
\rho C_p \left( \frac{\partial \bar{T}}{\partial t} + \bar{u} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} \right) = k \left( \frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \right)
\]

(4)

with appropriate boundary condition

\[
i < 0: \bar{u} = \bar{v} = 0, \bar{T} = T_\infty \text{ for any } \bar{x}, \bar{y}
\]

\[
i \geq 0: \bar{u} = \bar{v} = 0, \bar{T} = T_w \text{ for } \bar{y} = 0
\]

\[
\bar{u} = \bar{u}_e, \bar{T} = T_\infty \text{ as } \bar{y} \to \infty
\]

where \( \rho \) is fluid density, \( \mu \) is fluid viscosity, \( g \) is gravity, \( \beta \) is coefficient of thermal expansion, \( C_p \) is specific heat, and \( k \) is heat flux. In addition, the value of \( r \) is defined as \( \bar{r}(\bar{x}) = a \sin(\bar{x}/a) \), and the value of \( \bar{u}_e \), local free stream, is defined for sphere case as \( \bar{u}_e = (3/2) \sin(\bar{x}/a) \).

In order to transform Eq. (1) – (4) be non-dimensional equations, dimensionless variables are introduced as follow
Boundary layer theory is also applied to nondimensional governing equations system such that the system is reduced to be

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{\partial^2 u}{\partial x^2} + v \frac{\partial^2 u}{\partial y^2} + M (u - u_e) - \alpha T \sin x
\]

(5)

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2}
\]

(6)

with conditions for nondimensional governing equations as below

\[
t < 0 : u = 0, T = 0 \text{ for any } x, y
\]

\[
t \geq 0 : u = v = 0, T = 1 \text{ for } y = 0
\]

\[
u = u_e, T = 0 \text{ as } y \rightarrow \infty
\]

The dimensionless parameters shown in the system are defined as

\[
Re = \frac{U_{\infty} a}{v}, \quad Pr = \frac{\nu C_p}{k}, \quad M = \sigma B_0^2 a, \quad \alpha = \frac{Gr}{Re^2}, \quad Gr = \frac{g \beta(T_w - T_e)a^3}{\nu}
\]

where Pr is Prandtl number, M is magnetic parameter and \( \alpha \) is mixed convection parameter. In the two-dimensional flow, the velocity in each \( x \) and \( y \) component is able to be expressed as stream function as follow

\[
u = 1 \frac{\partial \psi}{\partial y} \quad \text{and} \quad \psi = -1 \frac{\partial \psi}{\partial x}
\]

(8)

By substituting Eq. (8) into Eq. (5) and (6), then

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r^2}{r} \frac{\partial \psi}{\partial y} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\partial \psi}{\partial y} \right)^2 + \frac{1}{r} \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} + \frac{1}{r} \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial x^2} = u_e \frac{\partial \psi}{\partial x} + v \frac{\partial \psi}{\partial y} + M \left( \frac{1}{r} \frac{\partial \psi}{\partial y} - 1 \right) + \alpha T \sin x
\]

(9)

\[
\frac{\partial T}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial y} \right) - \frac{1}{r} \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial y} = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2}
\]

(10)

with respect to boundary conditions

\[
t < 0 : \psi = \frac{\partial \psi}{\partial y} = 0, T = 0 \text{ for any } x, y
\]

\[
t \geq 0 : \psi = \frac{\partial \psi}{\partial y} = 0, T = 1 \text{ for } y = 0
\]

\[
\frac{\partial \psi}{\partial y} = ru_e, T = 0 \text{ as } y \rightarrow \infty
\]

In order to obtain the solution for small time case \( t \leq t^* \), the non-similarity variables applied into governing equation system are

\[
\psi = t^{1/2} u_e(x) r(x) f(x, \eta, t), \quad \eta = \frac{y}{t^{1/2}}, \quad T = s(x, \eta, t)
\]

The non-similarity variable above is substituted into Eq. (9) and (10), then the governing equation system for small time case becomes
\[
\frac{\partial^3 f}{\partial \eta^3} + \eta \frac{\partial^2 f}{\partial \eta^2} + \frac{1}{r} \frac{\partial u_e}{\partial x} \left( 1 - \left( \frac{\partial f}{\partial \eta} \right)^2 \right) + f \frac{\partial^2 f}{\partial \eta^2} + \frac{r}{\partial x} \left( \frac{\partial f}{\partial \eta} \right) + \frac{1}{r} \frac{M}{\partial \eta} \left( \frac{\partial f}{\partial \eta} - 1 \right) - \frac{1}{u_e} \frac{\lambda_s}{\sin x} = 0,
\]

\[
\frac{\partial^2 s}{\partial \eta^2} + Pr \eta \frac{\partial s}{\partial \eta} + Pr \frac{1}{r} \frac{\partial u_e}{\partial x} \frac{\partial \eta}{\partial \eta} = Pr \left( \frac{\partial s}{\partial \eta} + \frac{1}{u_e} \left( \frac{\partial s}{\partial y} \frac{\partial y}{\partial \eta} - \frac{\partial s}{\partial y} \frac{\partial y}{\partial \eta} - \frac{\partial s}{\partial y} \frac{\partial y}{\partial \eta} \right) \right)
\]
subject to boundary condition as below
\[
t < 0 \rightarrow f = \frac{df}{d\eta} = 0, s = 1 \text{ for any } x, \eta
\]
\[
t \geq 0 \rightarrow f = \frac{df}{d\eta} = 0, s = 0 \text{ for } \eta = 0
\]
\[
\frac{df}{d\eta} = 1, T = 0 \text{ as } \eta \rightarrow \infty
\]

For large time case \((t > t^*)\), the similarity variables are defined as
\[
\psi = u_e(x) r(x) F(x, Y, t), \quad y = Y, \quad T = S(x, Y, t)
\]
The similarity variables above are substituted into Eq. (9) and (10) such that the equations become
\[
\frac{\partial^3 F}{\partial Y^3} + \frac{u_e}{c_x} \left( 1 - \left( \frac{\partial F}{\partial Y} \right)^2 \right) = \frac{\partial^2 F}{\partial \eta^2} + \frac{1}{u_e} \left( \frac{\partial^2 F}{\partial \eta^2} + \frac{\partial F}{\partial \eta} \frac{\partial \eta}{\partial \eta} - \frac{\partial F}{\partial \eta} \frac{\partial \eta}{\partial \eta} - \frac{\partial F}{\partial \eta} \frac{\partial \eta}{\partial \eta} \right) - \frac{M}{\partial \eta} \left( \frac{\partial F}{\partial \eta} - 1 \right) - \frac{\lambda_s}{u_e} \frac{\sin x}{Y}
\]

\[
\frac{\partial^2 S}{\partial \eta^2} + Pr \frac{1}{r} \frac{\partial u_e}{\partial x} \frac{\partial \eta}{\partial \eta} = Pr \left( \frac{\partial S}{\partial \eta} + \frac{1}{u_e} \left( \frac{\partial S}{\partial y} \frac{\partial y}{\partial \eta} - \frac{\partial S}{\partial y} \frac{\partial y}{\partial \eta} - \frac{\partial S}{\partial y} \frac{\partial y}{\partial \eta} \right) \right)
\]
with boundary condition
\[
F = \frac{\partial F}{\partial Y} = 0, T = 1 \text{ for } Y = 0
\]
\[
F = \frac{\partial F}{\partial Y} = 1, T = 0 \text{ as } Y \rightarrow \infty
\]

3. Results and Discussion
The unsteady magnetohydrodynamics mixed convection flow past a sphere is numerically solved using finite different method implicit scheme. Matlab is used to run the simulation of the program. Various Prandtl number, magnetic parameter, and mixed convection parameter are used to determine velocity and temperature profiles at stagnation point \(x = 0\).

![Figure 2](image_url)

**Figure 2.** (a) velocity and (b) temperature profiles for various Prandtl number with fixed \(M = 1\) and \(\alpha = 1\).
From Figure 2 (a), the velocity decreases as Prandtl number increases. The increase of the value Prandtl number impacts to the increase of fluid density. The higher fluid density makes the fluid flows slowly. The increase of Prandtl number affects to the decrease of temperature of the fluid as shown in Figure 2 (b). As Prandtl number increases, thermal conductivity decreases such that temperature diffuses slowly.

![Figure 2](image)

**Figure 3.** (a) velocity and (b) temperature profiles for various magnetic parameter with fixed $Pr = 1$ and $\alpha = 1$

Figure 3 (a) shows that the velocity of fluid decreases while magnetic parameter increases. It caused Lorentz force received by the fluid. The higher distance fluid position to the surface of the sphere, the lower Lorentz force received by the fluid. The value of Lorentz force is depended on magnetic field strength $B_0$ which corresponds to magnetic parameter. When magnetic parameter increases, Lorentz force also increases which makes the fluid velocity become slower. As it seen in Figure 3 (b), the temperature of the fluid decreases as the magnetic parameter increases. The increases magnetic parameter impacts to the decline of Lorentz force. Hence, fluid velocity decreases. This condition makes the internal energy decrease. The decrease of internal energy impacts to the decrease of the fluid temperature.

![Figure 3](image)

**Figure 4.** (a) velocity and (b) temperature profiles for various mixed convection parameter with fixed $Pr = 1$ and $M = 1$

Figure 4 presents that while mixed convection parameter increases, both velocity and temperature profiles of the fluid increases. Mixed convection parameter represents the buoyancy force which has correlation with momentum equation. The increase of mixed convection parameter causes the decrease of fluid density. Hence, the velocity of the fluid decreases as it shown in Figure 4 (a). The increase of mixed convection parameter impacts to the increase of fluid temperature as it seen in Figure 4 (b). While the mixed convection parameter increases, fluid velocity increases. It causes the internal energy of the fluid also increases. Thus, the temperature of the fluid increases.
4. Conclusion
The numerical investigation on magnetohydrodynamics mixed convection flow past a magnetic sphere was studied. The governing equations are transformed into non-dimensional one. Boundary layer concept is used to analyze the equations. All of the equations developed using stream function and similarity variables. Thus, the equations are solved by finite difference method implicit scheme. Velocity and temperature profile decrease when Prandtl number increases. Velocity and temperature profile also decrease as magnetic parameter increases, but these profiles decrease as mixed convection parameter increases.

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