Efficient Partial Credit Grading of Proof Blocks Problems

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Abstract. Proof Blocks is a software tool which allows students to practice writing mathematical proofs by dragging and dropping lines instead of writing proofs from scratch. In this paper, we address the problem of assigning partial credit to students completing Proof Blocks problems. Because of the large solution space, it is computationally expensive to calculate the difference between an incorrect student solution and some correct solution, restricting the ability to automatically assign students partial credit. We propose a novel algorithm for finding the edit distance from an arbitrary student submission to some correct solution of a Proof Blocks problem. We benchmark our algorithm on thousands of student submissions from Fall 2020, showing that our novel algorithm can perform over 100 times better than the naive algorithm on real data. Our new algorithm has further applications in grading Parson’s Problems, as well as any other kind of homework or exam problem where the solution space may be modeled as a directed acyclic graph.

Keywords: Mathematical proofs · Automated feedback · Scaffolding.

1 Introduction

Traditionally, classes which cover mathematical proofs expect students to read proofs in a book, watch their instructor write proofs, and then write proofs on their own. Students often find it difficult to jump to writing proofs on their own, even when they have the required content knowledge [27]. Additionally, because proofs need to be graded manually, it often takes a while for students to receive feedback on their work.

Proof Blocks is a software tool that allows students to construct a mathematical proof by dragging and dropping instructor-provided lines of a proof instead of writing from scratch (similar to Parson’s Problems [25] for writing code—see Figure 1 for an example of the user interface of Proof Blocks). This tool scaffolds students’ learning as they transition from reading proofs to writing proofs while also providing instant machine-graded feedback. To write a Proof Blocks problem, an instructor specifies lines of a proof and their logical dependencies. The autograder accepts any ordering of the lines that satisfies all logical dependencies (see [1] for more details on the implementation).

The initial version of Proof Blocks lacked a way for students to receive partial credit. Receiving partial credit for their work is a significant concern for students
Fig. 1. An example of the Proof Blocks student-user interface. The instructor wrote the problem with 1, 2, 3, 4, 5, 6 as the intended solution, but the Proof Blocks autograder will also accept any other correct solution as determined by the dependency graph shown. For example, both 1, 2, 4, 3, 5, 6 and 1, 2, 3, 5, 4, 6 would also be accepted as correct solutions. Line 7 is a distractor which does not occur in any correct solution.

A better way to assign partial credit is based on the minimum edit distance from the student submission to some correct solution. This gives a more holistic measure of the similarity of the student submission to some correct solution. However, because of the large solution space, it is computationally expensive to exhaustively check all possible solutions. Our empirical analysis shows that this naïve approach will not scale to many students needing feedback at the same time (as in an active learning setting in a large classroom), or to problems taking exams in a computerized environment \[6,12\]. A simple partial credit scheme would be based on the location of the first incorrect line of the proof. For example, if the first 3 lines of a student’s proof were correct, and the correct proof had 10 lines, the student would receive 3/10. This doesn’t always work well. For example, what if a student has the first line of the proof wrong, but every other line was correct? Under this simple partial credit scheme, this answer would receive no partial credit, despite being almost completely correct.

A better way to assign partial credit is based on the minimum edit distance from the student submission to some correct solution. This gives a more holistic measure of the similarity of the student submission to some correct solution. However, because of the large solution space, it is computationally expensive to exhaustively check all possible solutions. Our empirical analysis shows that this naïve approach will not scale to many students needing feedback at the same time (as in an active learning setting in a large classroom), or to problems
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of longer than 8 lines. To solve these problems in scaling, we propose a novel algorithm which calculates the minimum edit distance by directly manipulating the student submission into a correct solution and runs asymptotically faster than the naïve solution.

This paper makes the following contributions:

– An efficient algorithm for calculating the minimum edit distance from an arbitrary sequence to some topological ordering of a directed acyclic graph
– Applications of this algorithm to grading mathematical proofs and other kinds of homework and test questions
– Mathematical proofs that the algorithm is correct, and has asymptotic complexity that allows it to scale to grading many student submissions at once, even for large problems
– Benchmarking results on 4,457 student submissions showing that the efficient algorithm performs over 100 times faster than the naïve approach

2 Related Work

2.1 Software for Learning Mathematical Proofs

Work in intelligent tutors for mathematical proofs goes back to work by John Anderson and his colleagues on The Geometry Tutor [3,216]. More recently, researchers have created tutors for propositional logic, most notably Deep Thought [23,22] and LogEx [21,20,19,18]. A number of other tools have been created to help students learn to construct mathematical proofs with the aid of a computer. Polymorphic Blocks [17] is a novel user interface which presents propositions as colorful blocks with uniquely shaped connectors. The Incredible Proof Machine [9] guides students through constructing proofs as graphs. Jape [8] is a “Proof calculator,” which guides students through the process of constructing formal proofs in mathematical notation with the help of the computer, but requires the instructor to implement the logics in a custom language. MathsTiles [7] is a block-based programming interface for constructing proofs for the Isabelle/HOL proof assistant. Having an open-ended environment where students could construct arbitrarily complex proofs seems like an advantage, but user studies showed that students were only successful if they were provided a small instructor-procured subset of blocks.

Most of these tools cover only small subset of the material typically covered in a discrete mathematics course, for example, only propositional logic. Those tools that are more flexible require learning complex theorem prover languages. In contrast, Proof Blocks enables instructors to easily provide students with proof questions on any topic.

2.2 Edit-distance Based Grading

To our knowledge, no one has ever used edit-distance based grading as a way of providing feedback for mathematical proofs, but edit-distance based grading algorithms have been used in other contexts.

Chandra et al. [10] use edit distance to assign partial credit to incorrect SQL queries submitted by students, using reference solutions provided by the instructor. Edit distance based methods, often backed by a database of known
correct solutions, have also been used to give feedback to students learning to program in general purpose programming languages \cite{24,13}.

One difference between these and our methods is that in programming contexts, the solution space is very large, and so the methods work based on edit distance to some known correct solution (manually provided by the instructor or other students). Because we model mathematical proofs as DAGs, we are able to constrain the solution space to be small enough that our algorithm can feasibly check the shortest edit to any correct solution.

Alur et al. \cite{1} provide a framework for automatically grading problems where students must construct a deterministic finite automata (DFA). They use edit distance for multiple purposes in their multi-modal grading scheme.

3 Proof Blocks

To write a Proof Blocks problem, an instructor provides the proof lines and the logical dependencies between the lines of the proof. These logical dependencies form a directed acyclic graph (DAG). The autograder gives the student points if their submission is a topological sort of the dependency graph. On exams or during in-class activities, students are often given multiple tries to solve a problem, so it is critical that they receive their feedback quickly. For additional details about the instructor and student user interfaces, as well as best practices for using Proof Blocks questions, see \cite{4}. Prior work has shown that Proof Blocks are effective test questions, providing about as much information about student knowledge as written proofs do \cite{5}.

4 The Partial Credit Algorithm

4.1 Mathematical Preliminaries

*Graph Theory.* Let $G = (V, E)$ be a graph. Then a subset of vertices $C \subseteq V$ is a vertex cover if every edge in $E$ is incident to some vertex in $S$. The minimum vertex cover (MVC) problem is the task of finding a vertex cover of minimum cardinality. In defining our algorithms, with will assume the availability of a few classical algorithms for graphs: ALLTOPOLOGICALORDERINGS($G$) to return a set containing all possible topological orderings of a graph $G$ \cite{15}, EXISTSPATH($G, u, v$) returns a boolean value to denote if there is a path from the node $u$ to the node $v$ in the graph $G$, and MINIMUMVERTEXCOVER($G$) to return an MVC of a graph $G$ by exhaustive search.

*Edit Distance.* For our purposes, we use the Least Common Subsequence (LCS) edit distance, which only allows deletion or addition of items in the sequence (it does not allow substitution or transposition). This edit distance is a good fit for our problem because it mimics the affordances of the user interface of Proof Blocks. Throughout the rest of the paper, we will simply use “edit distance” to refer to the LCS edit distance. We denote the edit distance between two sequences $S_1$ and $S_2$ as $d(S_1, S_2)$.

Formally defined, given two sequences $S_1, S_2$, the edit distance is the length of the shortest possible sequence of operations that transforms $S_1$ into $S_2$, where the operations are: (1) **Deletion** of element $s_i$: changes the sequence
s_1, s_2, \ldots s_{i-1}, s_i, s_{i+1}, \ldots s_n \) to the sequence \( s_1, s_2, \ldots s_{i-1}, s_i, t, s_{i+1}, \ldots s_n \). (2) **Insertion** of element \( t \) after location \( i \): changes the sequence \( s_1, s_2, \ldots s_i, s_{i+1}, \ldots s_n \) to the sequence \( s_1, s_2, \ldots s_i, t, s_{i+1}, \ldots s_n \). We assume the ability to compute the edit distance between two sequences in quadratic time using the traditional dynamic programming method [26]. We also identify a topological ordering \( O \) of a graph \( G \) with a sequence of nodes so that we can discuss the edit distance between a topological ordering and some other sequence \( d(S, O) \).

### 4.2 Problem Definition

Before defining our grading algorithms rigorously, it will first help to set up some formalism about proof blocks problems. A **proof blocks problem** \( P = (C, G) \) is a set of **proof lines** \( C \) together with a directed acyclic graph (DAG) \( G = (V, E) \), which defines the logical structure of the proof. Both the proof lines and the graph are provided by the instructor who writes the question (see [4] for more details on question authoring). The set of vertices \( V \) of the graph \( G \) is a subset of the set of proof lines \( C \). Proof lines which are in \( C \) but not in \( V \) are proof lines which are not in any correct solution, and we call these **distractors**, a term which we borrow from the literature on multiple-choice questions. A **submission** \( S = s_1, s_2, \ldots s_n \) is a sequence of distinct proof lines, usually constructed by a student who is attempting to solve a proof blocks problem. If a submission \( S \) is a topological ordering of the graph \( G \), we say that \( S \) is a **solution** to the proof blocks problem \( P \).

To be precise about the partial credit we want to assign, if a student submits a submission \( S \) to a proof blocks problem \( P = (C, G) \), we want to assign partial credit with the following properties: (1) students get 100% only if the submission is a solution (2) partial credit declines linearly with the number of edits needed to convert the submission into a solution (3) is is guaranteed to be in the range \( 0 \leq d \leq 100 \). To satisfy these desirable properties, we assign partial credit as follows:

\[
score = 100 \times \frac{\max(0, |V| - d^*)}{|V|},
\]

where \( d^* \) is the minimum edit distance from the student submission to some correct solution of \( P \), that is: \( d^* = \min \{ d(S, O) \mid O \in \text{AllTopologicalOrderings}(G) \} \). This means, for example, that if a student’s solution is 2 deletions and 1 insertion (3 edits) away from a correct solution, and the correct solution is 10 lines long, the student will receive 70%. If the edit distance is greater than the length of the solution, we simply assign 0, as assigning negative points does not make sense.

### 4.3 Baseline Algorithm

The most straightforward approach to calculating partial credit as we define it is to iterate over all topological orderings of \( G \) and for each one, calculate the edit distance to the student submission \( S \). We formalize this approach as Algorithm [1]. While this is effective, this algorithm is computationally expensive.

**Theorem 1.** The time complexity of Algorithm [1] is \( \mathcal{O}(m \cdot n \cdot n!) \) in the worst case, where \( n \) is the size of \( G \) and \( m \) is the length of the student submission after distractors are thrown out.

**Proof.** The algorithm explicitly enumerates all \( \mathcal{O}(n!) \) topological orderings of \( G \). For each ordering, the algorithm forms the associated proof sequence, and
computes the edit distance to the student submission, which requires quadratic
time.

\begin{algorithm}[H]
\caption{Baseline Algorithm}
\begin{algorithmic}[1]
\Input \( S \) The student submission being graded
\Input \( P \) The proof blocks problem written by the instructor
\Output \( n \) The minimum number of edits needed to transform \( S \) into a solution
\Procedure{GetMinimumEditDistance}{\( S = s_1, s_2, \ldots, s_n, P = (C, G) \)}
\State \textbf{Brute force calculation of} \( d^* \):
\State \Return \( \min \{ d(S, O) \mid O \in \text{AllTopologicalOrderings}(G) \} \)
\EndProcedure
\end{algorithmic}
\end{algorithm}

4.4 Optimized (MVC-based) implementation of Partial Credit Algorithm

We now present a faster algorithm for calculating the proof blocks partial credit, which operates by reducing the problem to the \textit{minimum vertex cover} (MVC) problem over a subset of the student’s submission.

Rather than iterate over all topological orderings, this algorithm works by directly manipulating the student’s submission until it becomes a correct solution. In order to do this, we define a few more terms. We call a pair of proof lines \((s_i, s_j)\) in a submission a \textit{problematic pair} if line \( s_j \) comes before line \( s_i \) in the student submission, but there is a path from \( s_i \) to \( s_j \) in \( G \), meaning that \( s_j \) must come after \( s_i \) in any correct solution.

We define the \textit{problematic subgraph} to be the graph where the nodes are the set of all proof lines in a student submission that appear in some problematic pair, and the edges are the problematic pairs. We can then use the problematic subgraph to guide which proof lines need to be deleted from the student submission, and then we know that a simple series of insertions will give us a topological ordering. The full approach is shown in Algorithm 2, and the proof of Theorem 2 proves that this algorithm is correct.

4.5 Worked example of Algorithm 2

For further clarity, we will now walk through a full example of executing Algorithm 2 on a student submission.

Take, for example, the submission shown in Figure 1. In terms of the proof line labels, this submission is \( S = 1, 3, 4, 5, 2, 7 \). In this case, proof line 2 occurs after proof lines 3, 4, and 5, but because of the structure of the DAG, we know that it must come before all of those lines in any correct solution. Therefore, the problematic subgraph in this case is problematicSubgraph = \( \{ \{2, 3, 4, 5\}, \{(2, 3), (2, 4), (2, 5)\} \} \). The minimum vertex cover here is \( \{2\} \), because that is the smallest set which contains at least one endpoint of each edge in the graph. Now we know that the number of deletions needed is \( 1 + 1 = 2 \) (vertex cover of size one, plus one distractor line picked, see Algorithm 2 line 11)), and the number of insertions needed is 2 (line 2 must be reinserted in the correct position after being deleted, and line 6 must be inserted). This gives
Algorithm 2 Novel Algorithm using the MVC

1: **Input**
2: \( S \) The student submission being graded
3: \( P \) The proof blocks problem written by the instructor
4: **Output**
5: \( n \) The minimum number of edits needed to transform \( S \) into a solution
6: **procedure** GETMINIMUMEDITDISTANCE\((S = s_1, s_2, \ldots, s_n, P = (C, G))\)
7: Construct the problematic subgraph:
8: \( E_0 \leftarrow \{(s_i, s_j) \mid i > j \text{ and } \exists \text{Path}(G, s_i, s_j)\} \)
9: \( V_0 \leftarrow \{s_i \mid \text{there exists } j \text{ such that } (s_i, s_j) \in E_0 \text{ or } (s_j, s_i) \in E_0\} \)
10: problematicSubgraph \( \leftarrow (V_0, E_0) \)
11: Find insertions and deletions needed:
12: mvcSize \( \leftarrow |\text{MINIMUMVERTEXCOVER}(\text{problematicSubgraph})|\)
13: deletionsNeeded \( \leftarrow |\{s_i \in S \mid s_i \notin G\}| + \text{mvcSize} \)
14: insertionsNeeded \( \leftarrow |V| - (n - \text{deletionsNeeded}) \)
15: **return** deletionsNeeded + insertionsNeeded
16: **end procedure**

us a least edit distance \((d^*)\) of 4, and so the partial credit assigned would be

\[
\text{score} = 100 \times \frac{\max(0, |V| - d^*)}{|V|} = 100 \times \frac{5-4}{6} \approx 33\%.
\]

4.6 Proving the Correctness of Algorithm 2

First we’ll show that the Algorithm constructs a feasible solution, and then we will show that it is minimal.

**Lemma 1** (Feasability). Given a a submission \( S = s_1, s_2, \ldots, s_n \), there exists an edit \( E \) from \( S \) to some solution of \( P \) such that the length of \( E \) is the value returned by Algorithm 2.

**Proof.** Given the MVC computed on line 10 of Algorithm 2, delete all proof lines in the MVC from \( S \), as well as all distractors in \( S \), and call this new submission \( S' \). Now \( S' \) is a submission such that it contains no distractors, and its problematic subgraph is empty.

Now, for all \( i \) where \( 1 \leq i < n \), add the edge \((s_i, s_{i+1})\) to the graph \( G \), and call this new graph \( G' \). Because there are no problematic pairs in \( S' \), we know that adding these new edges does not introduce any new cycles, so \( G' \) is a DAG. Now, a topological ordering \( O \) of the graph \( G' \) will be a topological ordering of \( G \) with the added constraint that all proof lines which appeared in the submission \( S' \) are still in the same order with respect to one another. Then since there are no distractors in \( S' \), \( S' \) will be a subsequence of \( O \). Thus, we can construct \( O \) simply by adding proof lines to \( S \).

The length of this edit sequence is exactly what Algorithm 2 is computing.

**Lemma 2** (Minimality). Let \( E' \) be any edit from the submission \( S \) to some correct solution of \( P \). Then the length of \( E' \) is greater than or equal to the output of Algorithm 2.

\[\]
Proof. Let $E$ be the edit sequence constructed in Lemma 1. We will show that the number of deletions and the number of insertions in $E'$ is greater than or equal to the number of deletions and insertions in $E$.

Because we are using least common subsequence edit sequence, if there is any problematic pair $(s_i, s_j)$ in the student submission, one of $s_i$ and $s_j$ must be deleted from the submission to reach a solution. Because there is no substitution or transposition allowed, and because each proof line may only occur once in a sequence, there is no other way for the student submission to be transformed into some correct solution unless $s_i$ or $s_j$ is deleted and then re-inserted in a different position.

Therefore, the set of proof lines deleted in the edit sequence $E'$ must be a vertex cover of the problematic subgraph related to $S$ In $E$, we delete only the proof lines in the minimum vertex cover of the problematic subgraph. In both cases, all distractors must be deleted. So, the number of deletions in $E'$ is greater than or equal to the number of deletions in $E$.

The number of insertions in any edit sequence must be the number of deletions, plus the difference between the length of the submission and the size of the graph, so that the final solution will be the correct length. Then since the number of deletions in $E'$ is at least as many as there are in $E$, the number of insertions in $E'$ is also at least as many as the number of insertions in $E$.

Combining what we have shown about insertions and deletions, we have that $E'$ is at least as long of an edit sequence as $E$.

Combining Lemma 1 and 2 gives us the following theorem:

**Theorem 2.** Algorithm 2 computes $d^*$ — the minimum edit distance from the submission $S$ to some topological ordering of $G$.

Next we will treat the computational complexity of Algorithm 2.

**Theorem 3.** The time complexity of Algorithm 2 is $O(m^2 \cdot 2^n)$, where $m$ is the length of the student submission after distractors are thrown out.

Proof. Constructing the problematic subgraph requires using a breadth first search to check for the existence of a path between each proof line and all of the proof lines which precede it in the submission $S$, which can be completed in polynomial time. Naively computing the MVC of the problematic subgraph has time complexity $O(m^2 \cdot 2^n)$. Asymptotically, the calculation of the MVC will dominate the calculation of the problematic subgraph, giving an overall time complexity of $O(m^2 \cdot 2^n)$.

Remark 1. The complexity of Algorithm 2 is due to the brute force computation of the MVC, however, there exists a a $O(1.2738^k + kn)$-time fixed parameter tractable (FPT) algorithm where $k$ is the size of the minimum vertex cover [11]. While we focus in this paper on the brute force MVC method since it is sufficient for our needs, using the FPT method may give further speedup, especially considering the often small size of $k$ in real use cases (see Table 1).
5 Benchmarking Algorithms on Student Data

5.1 Data Collection

We collected data from homework, practice tests, and exams from the Discrete Mathematics course in the Computer Science department at Large Research University during the Fall 2020 semester. During this semester all activity was held online due to the COVID-19 pandemic. In total, 393 signed up for the class, from which we collected 9,610 submissions to Proof Blocks problems. Proof Blocks questions were on many mathematical topics including Number theory, Cardinality, Functions, Graphs, and Algorithm analysis. Some Proof Blocks problems were edited during the semester in such a way that altered the structure of the problem. In these cases, we simply discarded submissions to altered versions of the question. We also discarded all submissions which were correct solutions, as we are only concerned in this paper with partial credit grading. This left us with a total of 4,457 submissions for our benchmarking data set. Some questions only appeared on optional practice exams, while others appeared on exams. Also, more difficult questions received more incorrect submissions. This explains the large discrepancy between the number of submissions to certain questions seen in Table 1.

5.2 Benchmarking Details

For Algorithm 1, we used the NetworkX Python library [14] to generate all topological orderings of $G$ and used the standard dynamic programming algorithm for LCS edit distance to calculate the edit distance between each submission and each topological ordering.

We implemented Algorithm 2 in Python, again using NetworkX to store the graph. Though faster implementations of the MVC algorithm exists, we simply used the naive method of iterating over all subsets of the graph, starting from the smallest to the largest, until finding one which is a vertex cover. Benchmarks were run on an Intel i5-8530U CPU with 16GB of RAM.

5.3 Results

Table 1 shows the benchmarks and statistical comparisons of our novel MVC-based algorithm (Algorithm 2) and the baseline algorithm (Algorithm 1). Algorithm 1 performed about 2 times as fast as Algorithm 2 when there was one solution—more trivial cases when both Algorithms took less than a third of a millisecond. Algorithm 2 performed about 5 to 200 times faster than Algorithm 1 when there were 20 or more possible solutions. In Figure 2 (A), we show that the run time of Algorithm 1 scales exponentially with the number of topological orderings of the proof graph, while in (B) we show that the run time of Algorithm 2 scales exponentially with the length of the proof. This is a critical difference, because the number of topological orderings of a DAG can be $n!$ for a graph with $n$ nodes. Thus, a relatively short Proof Blocks problem could have a very long grading time with Algorithm 1 while with Algorithm 2 we know that we can guarantee a tight bound on grading time given the problem size.

This shows that Algorithm 2 is far superior in performance. The mean time of 92 milliseconds for the most complex Proof Blocks problem under Algorithm 1 may not seem computationally expensive, but it does not scale to having
Table 1. Performance of baseline vs. MVC algorithm. (*) denotes a statistically significant difference in algorithm time, $p < 0.001$ in all cases. For all problems with more than 3 possible solutions, the MVC algorithm was significantly faster, with a speedup of up to almost 200 times. This speedup gap will continue to grow as instructors write more complex problems.

| Question Number | Proof Length | Possible Solutions | Distractors | Submissions | Prob. Subgraph Size (mean) | MVC Size (mean) | Baseline Alg. Time (mean ms) | MVC Alg. Time (mean ms) | Speedup Factor |
|-----------------|--------------|--------------------|-------------|-------------|--------------------------|----------------|-----------------------------|------------------------|-----------------|
| 1               | 4            | 1                  | 4           | 291         | 0.5                      | 0.3            | 0.1 (0.03)                  | 0.16 (0.04)            | 0.6*            |
| 2               | 5            | 1                  | 0           | 104         | 5.0                      | 2.1            | 0.12 (0.05)                 | 0.21 (0.05)            | 0.6*            |
| 3               | 6            | 1                  | 0           | 551         | 5.8                      | 3.7            | 0.14 (0.04)                 | 0.31 (0.09)            | 0.5*            |
| 4               | 6            | 3                  | 4           | 19          | 5.0                      | 0.2            | 0.25 (0.07)                 | 0.26 (0.09)            | 1.0             |
| 5               | 6            | 6                  | 5           | 674         | 6.2                      | 0.1            | 0.41 (0.11)                 | 0.25 (0.06)            | 1.6*            |
| 6               | 7            | 20                 | 7           | 488         | 6.5                      | 0.0            | 1.46 (0.49)                 | 0.26 (0.07)            | 5.6*            |
| 7               | 9            | 24                 | 0           | 28          | 9.0                      | 3.2            | 2.23 (0.31)                 | 0.47 (0.09)            | 4.7*            |
| 8               | 9            | 24                 | 0           | 529         | 8.9                      | 3.5            | 2.22 (0.53)                 | 0.47 (0.10)            | 4.7*            |
| 9               | 9            | 35                 | 5           | 376         | 8.5                      | 1.1            | 3.03 (0.97)                 | 0.41 (0.12)            | 7.4*            |
| 10              | 9            | 42                 | 0           | 13          | 8.8                      | 2.2            | 3.49 (0.46)                 | 0.44 (0.08)            | 8.0*            |
| 11              | 10           | 21                 | 0           | 623         | 9.8                      | 4.6            | 2.36 (0.42)                 | 0.57 (0.15)            | 4.1*            |
| 12              | 10           | 96                 | 0           | 145         | 8.3                      | 3.0            | 8.92 (2.38)                 | 0.78 (0.64)            | 11.4*           |
| 13              | 10           | 1100               | 0           | 616         | 9.4                      | 3.1            | 92.04 (20.78)               | 0.47 (0.13)            | 194.9*          |

hundreds of students working on an active learning activity, or taking exams at the same time. Furthermore, this grading time could easily be 10 or even 100 times longer per question if the DAG for the question was more complex by only a couple of nodes or edges.

6 Conclusions and Future Work
In this paper, we have presented a novel algorithm for assigning partial credit. This algorithm can also be used to grade Parson’s Problems, or any other type of problem where the solution space can be modeled as a DAG. We showed with student data that our algorithm far outperforms the baseline algorithm, allowing us to give students their partial credit grades in real time as they work through exams and homework. Starting in the Spring 2022 semester, this will benefit thousands of students immediately, and many more in future semesters.

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Fig. 2. Comparison of grading time for the two grading algorithms. Subplot (A) is a log-log plot showing that we see that the baseline algorithm scales with the number of possible solutions, Subplot (B) is a log-linear plot showing that the MVC Algorithm runtime scales with the length of the proof. This is a critical difference, because the number of topological orderings of a DAG can be $n!$ for a graph with $n$ nodes.

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