The light MSSM neutral Higgs boson production associated with an electron and a jet at the LHeC

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Abstract

We study the light $CP$-even neutral Higgs boson production in association with an electron and a jet at the possible CERN large hadron-electron collider within the minimal supersymmetric standard model. We investigate the possible supersymmetric effects on this process and compare our standard model numerical results with those in previous work. We present the leading-order and QCD next-to-leading-order corrected total cross sections and the distributions of the transverse momenta of the final electron, the light neutral Higgs boson, and jet in the minimal supersymmetric standard model. Our results show that the scale dependence of the leading-order cross section is obviously reduced by the QCD next-to-leading-order corrections. The K factor of the QCD correction to the total cross section at the large hadron-electron collider varies from 0.893 to 1.048 when the factorization/renormalization scale $\mu$ goes up from $0.2m_Z$ to $3.8m_Z$ in our chosen parameter space.

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I. Introduction

One of the most significant tasks for high-energy experiments is to search for scalar Higgs particles \[1-5\]. Although the standard model (SM) \[6\] has achieved impressive experimental success, the Higgs boson, which is predicted by the SM for spontaneous electroweak symmetry breaking, remains a mystery. Moreover, there exists the problem of the quadratically divergent contributions to the corrections to the Higgs boson mass, which is the so-called naturalness problem. Alternative conceptional difficulties, such as the hierarchy problem, the necessity of the tuning and the nonoccurrence of gauge coupling unification at high energies, suggest that the SM is probably the low-energy limit of a more fundamental theory.

As the most hopeful extensions of the SM, the supersymmetric (SUSY) models can solve such problems mentioned above. The minimal supersymmetric standard model (MSSM) \[7,8\] is the simplest one among all the SUSY extensions of the SM. In this model, two Higgs doublets $H_1$ and $H_2$ give masses to up- and down-type fermions. The Higgs sector consists of three neutral Higgs bosons, one $CP$-odd particle ($A^0$), two $CP$-even particles ($h^0$ and $H^0$), and a pair of charged Higgs bosons ($H^\pm$). However, these Higgs bosons have not been directly explored experimentally until now. The LEP experiments provided lower mass bounds as: for the SM Higgs boson $m_{H^0} > 114.4$ GeV (at 95% C.L.), and for the MSSM bosons $m_{h^0} > 92.8$ GeV and $m_{A^0} > 93.4$ GeV for $\tan \beta > 0.4$ (at 95% C.L.) \[9,10\].

Recently, a possible high-energy collider in $e^-p$ collision mode at the LHC, the large hadron-electron collider (LHeC), has been sketched \[11,12\]. There will exist a rich physics program \[13\]. The LHeC can be used to accurately determine the parton dynamics and the momentum distributions of quarks and gluons in the proton, and furthermore it may play a significant role in the discovery and interpretation of new physics. The incoming proton beam at the LHeC has an energy $E_p = 7$ TeV and the energy of incoming electron is considered as $E_e = 50 - 200$ GeV according to several scenarios, with the center-of-mass system energy of $\sqrt{s} = 2\sqrt{E_pE_e} \approx 1.18 - 2.37$ TeV. It seems that the LHeC provides a cleaner environment than a hadron-hadron collider in accessing the couplings of the Higgs boson to gauge bosons.

The production channel $e^-p \rightarrow e^-h^0j + X$, a neutral current (NC) process at the LHeC,
attracted the physicist’s attentions. In Ref. [14] it is pointed out that the electron reconstruction in the NC process is superior with respect to that of the missing neutrino in the charged current process, $e^- p \rightarrow \nu_e h^0 j + X$, and the NC process has the potential to increase the overall Higgs boson signal efficiency, and there they studied the use of forward jet tagging as a means to secure the observation of the Higgs boson in the $H^0 \rightarrow b\bar{b}$ decay mode and to significantly improve the purity of the signal. The QCD next-to-leading-order (NLO) corrections to the SM Higgs productions of $e^- p \rightarrow e^- H^0 j + X$ and $e^- p \rightarrow \nu_e H^0 j + X$ processes at the LHeC were calculated by B. Jäger in Ref. [15]. Moreover, not only does this channel provide a spectacular signature ($e^- b\bar{b}j$), but also the lightest Higgs $h^0$ production in MSSM via vector boson fusion with unusual visible decays is possible [16]. The coupling strength of the lightest Higgs $h^0$ with $Z^0 Z^0$ is distinguished from the SM Higgs one with an additional factor $\sin(\beta - \alpha)$, where $\beta$ is related to the ratio of the vacuum expectation values and $\alpha$ is the mixing angle of the two CP-even Higgs states. Therefore, we may disentangle between the SM Higgs and the light MSSM CP-even Higgs by measuring the cross section for $e^- p \rightarrow e^- h^0 j + X$ at the LHeC when $|\sin(\beta - \alpha)|$ is smaller than 1. Besides, in order to find new physics it requires sufficiently precise predictions for the new physics signals and their backgrounds with multiple final particles which cannot be separated in experimental data entirely. Therefore, the higher order QCD predictions for these reactions are necessary.

In this paper, we calculate the full QCD NLO corrections to the process $e^- p \rightarrow e^- h^0 j + X$ at the LHeC and estimate the capability of the LHeC to access the light MSSM $CP$-even Higgs boson in the $e^- h^0 j$ production. The numerical results at the leading-order (LO) are compared with those in Ref. [14]. The paper is organized as follows: We describe the technical details of the related LO and QCD NLO calculations in both the SM and the MSSM in Secs. II and III, respectively. In Sec. IV we give some numerical results and discussions about the QCD NLO corrections in the MSSM. Finally, a short summary is given.
II. LO cross sections

The LO and QCD NLO calculations are carried out in ’t Hooft-Feynman gauge. The FEYNARTS 3.4 package \[17\] is adopted for generating Feynman diagrams and subsequently converting them to corresponding amplitudes. The FORMCALC 5.4 program \[18\] is applied to reduce the amplitudes.

In calculating the $e^- p \rightarrow e^- h^0 j + X$ process in the MSSM, we neglect the u-, d-, c-, s-quark masses ($m_u = m_d = m_c = m_s = 0$), and do not consider the partonic processes with incoming (anti)bottom-quark due to the heavy (anti)bottom-quark suppression in parton distribution functions (PDFs) of proton. That means we involve the contributions of the following partonic processes in our LO calculations:

\[
e^-(p_1) + q(p_2) \rightarrow e^-(p_3) + h^0(p_4) + q(p_5), \quad (q = u, \bar{u}, d, \bar{d}, c, \bar{c}, s, \bar{s}), \quad (2.1)
\]

where $p_i (i = 1, ..., 5)$ represent the four-momenta of the incoming electron, partons, and the outgoing electron, $h^0$-boson and jet, respectively. The LO Feynman diagram for the partonic processes (2.1) is depicted in Fig.1.

The expression of the LO cross section for the partonic process $e^- q \rightarrow e^- h^0 q$ can be written in the form as

\[
\hat{\sigma}_{LO}(\hat{s}, e^- q \rightarrow e^- h^0 q) = \frac{1}{2\hat{s}} \int \sum |M_{LO}|^2 d\Omega_3, \quad (q = u, \bar{u}, d, \bar{d}, c, \bar{c}, s, \bar{s}), \quad (2.2)
\]

where $\hat{s}$ is the partonic center-of-mass (c.m.) energy squared, the summation is taken over the spins and colors of final states, and the bar over the summation means taking average over the intrinsic degrees of freedom of initial particles, and $d\Omega_3$ is the three-body phase-space...
element for the $e^{-} q \rightarrow e^{-} h^{0} q$ subprocess. $\mathcal{M}_{LO}$ in Eq. (2.2) is the tree-level amplitude for the partonic process $e^{-} q \rightarrow e^{-} h^{0} q$. The coupling of the SM Higgs boson ($H^{0}$) to the $Z^{0}$ pair can be expressed as $g_{HZZ}^{SM} = \frac{ie}{c_{W} s_{W}} g^{\mu \nu}$, while the light $CP$-even SUSY Higgs boson to the $Z^{0}$ pair is expressed as $g_{hZZ}^{MSSM} = \frac{ie}{c_{W} s_{W}} \sin(\beta - \alpha) g^{\mu \nu}$.

The LO total cross section for the $e^{-} p \rightarrow e^{-} h^{0} j + X$ process at the LHeC can be expressed as

$$\sigma_{LO}(e^{-} p \rightarrow e^{-} h^{0} j + X) = \int dx \sum_{q=u,\bar{u},d,\bar{d}} \left[ G_{q/p}(x,\mu_{f}) \delta_{LO}(xs,\mu_{f},e^{-} q \rightarrow e^{-} h^{0} q) \right].$$

There $\mu_{f}$ is the factorization scale, $s$ is the total c.m. energy squared of the electron-proton collision, $x$ describes the four-momentum fraction of parton $q$ in an incoming proton with the definitions of $x = \frac{P^{2}}{s}$, and $P$ is the four-momentum of the incoming proton. $G_{q/p}$ ($q = u, \bar{u}, d, \bar{d}, c, \bar{c}, s, \bar{s}$) represent the PDFs of parton $q$ in proton $p$.

### III. QCD NLO corrections in the MSSM

#### III.1 Virtual corrections

In order to compare the results in the MSSM with those in the SM we present the QCD NLO calculations in both models. In the NLO calculations, we adopt the dimensional regularization in $D = 4 - 2\epsilon$ dimensions to isolate the ultraviolet (UV) and infrared (IR) singularities. The wave functions of the external fields are renormalized under the on-shell renormalization scheme. The virtual correction to the subprocess $e^{-} q \rightarrow e^{-} h^{0} q$ involves both soft and collinear IR singularities. In our calculation we introduce the following counterterms for related wave functions in the SM and the MSSM:

$$\psi_{q,L}^{0} = \left(1 + \frac{1}{2} \delta Z_{q,L}^{SM(MSSM)} \right) \psi_{q,L},$$
$$\psi_{q,R}^{0} = \left(1 + \frac{1}{2} \delta Z_{q,R}^{SM(MSSM)} \right) \psi_{q,R}, \quad (q = u, d, c, s). \quad (3.1)$$

The wave-function renormalization constants of the massless quarks ($q = u, d, c, s$) in the SM are written as

$$\delta Z_{q,L}^{SM} = \delta Z_{q,R}^{SM} = - \frac{\alpha_{s}}{3\pi} [\Delta_{UV} - \Delta_{IR}], \quad (3.2)$$
Figure 2: The SM QCD vertex diagram for the partonic process $e^-q \rightarrow e^-h^0q$ ($q = u, \bar{u}, d, \bar{d}, c, \bar{c}, s, \bar{s}$).

Figure 3: The representative pure SUSY QCD one-loop Feynman diagrams for the partonic process $e^-q \rightarrow e^-h^0q$, where $\tilde{q} = \tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}$ and the lower indexes $i, j, k = 1, 2$.

where $\Delta_{UV} = 1/\epsilon_{UV} - \gamma_E + \ln(4\pi)$ and $\Delta_{IR} = 1/\epsilon_{IR} - \gamma_E + \ln(4\pi)$. The explicit expressions for the one-loop QCD wave-function renormalization constants of the massless quarks ($q = u, d, c, s$) in the MSSM have the forms as

$$\delta Z_{q,L}^{MSSM} = -\frac{\alpha_s}{3\pi} \left[ \Delta_{UV} - \Delta_{IR} \right] + \frac{2\alpha_s}{3\pi} \left[ B_1(0, m_{\tilde{g}}, m_{\tilde{q}_1}) \cos^2 \theta_{\tilde{q}} + B_1(0, m_{\bar{g}}, m_{\tilde{q}_2}) \sin^2 \theta_{\tilde{q}} \right], \quad (3.3)$$

$$\delta Z_{q,R}^{MSSM} = -\frac{\alpha_s}{3\pi} \left[ \Delta_{UV} - \Delta_{IR} \right] + \frac{2\alpha_s}{3\pi} \left[ B_1(0, m_{\tilde{g}}, m_{\tilde{q}_1}) \sin^2 \theta_{\tilde{q}} + B_1(0, m_{\bar{g}}, m_{\tilde{q}_2}) \cos^2 \theta_{\tilde{q}} \right], \quad (3.4)$$

where the definitions for the two-point integrals are adopted from Ref. [19], and $\theta_{\tilde{q}}$ is the mixing angle of scalar quarks ($\tilde{q}_L, \tilde{q}_R$),

$$\tilde{q}_L = \tilde{q}_1 \cos \theta_{\tilde{q}} - \tilde{q}_2 \sin \theta_{\tilde{q}}, \quad \tilde{q}_R = \tilde{q}_1 \sin \theta_{\tilde{q}} + \tilde{q}_2 \cos \theta_{\tilde{q}}. \quad (3.5)$$

The one-loop level Feynman diagrams include self-energy, vertex, box (4-point) and counterterm Feynman graphs. We depict the SM QCD vertex diagram in Fig.2 and the representative pure SUSY QCD (pSQCD) one-loop diagrams are drawn in Fig.3.
III..2 Real gluon and light-(anti)quark emission corrections

The relevant real emission partonic processes can be grouped as (1) $e^-q \rightarrow e^-h^0qg$, (2) $e^-g \rightarrow e^-h^0q\bar{q}$. There the quark notation, $q$, represents $u-, \bar{u}-, d-, \bar{d}-, c-, \bar{c}-, s- \text{ and } \bar{s}-$ quark. The real gluon/light-(anti)quark emission partonic channels (1) and (2) at the tree-level contain soft and collinear IR singularities. After the summation of the virtual corrections with all the real parton emission corrections, the numerical result is soft IR-safe, while collinear divergences still remain. It will be totally IR-safe when we include the contributions from the collinear counterterms of the PDFs. The IR finiteness can be verified numerically in our numerical calculations.

The IR singularities of the real parton emission subprocesses can be isolated by adopting the two cutoff phase-space slicing method [20]. In Figs.4 and 5 we present the Feynman diagrams for the real gluon emission subprocess $e^- (p_1) q (p_2) \rightarrow e^- (p_3) h^0 (p_4) q (p_5) g (p_6)$ and real light-(anti)quark emission subprocess $e^- (p_1) g (p_2) \rightarrow e^- (p_3) h^0 (p_4) q (p_5) \bar{q} (p_6)$, respectively. In adopting the two cutoff phase-space slicing method we introduce an arbitrary small soft cutoff $\delta_s$ to separate the $2 \rightarrow 4$ phase-space into two regions, $E_6 \leq \delta_s \sqrt{\hat{s}}/2$ (soft gluon region) and $E_6 > \delta_s \sqrt{\hat{s}}/2$ (hard gluon region), and another cutoff $\delta_c$ to decompose the hard region into a hard collinear (HC) region with $p_2 (p_5).p_6 < \delta_c \hat{s}/2$ and hard noncollinear ($\bar{HC}$) region with $p_2 (p_5).p_6 \geq \delta_c \hat{s}/2$.

Then the cross sections for the real emission subprocesses $e^- (q, g) \rightarrow e^- h^0 q (g, \bar{q})$ can be written as

$$\hat{\sigma}_R = \hat{\sigma}^S + \hat{\sigma}^H = \hat{\sigma}^S + \hat{\sigma}^{HC} + \hat{\sigma}^{\bar{HC}}.$$  

(3.6)

IV. Numerical Results and Discussion

In our numerical calculations we take one-loop and two-loop running $\alpha_s$ in the LO and NLO calculations, respectively [9]. The QCD parameters are taken as $N_f = 5$, $\Lambda_{LO}^5 = 165 \text{ MeV}$ and $\Lambda_{MS}^5 = 226 \text{ MeV}$. We take the renormalization and factorization scales to be a common value as $\mu \equiv \mu_r = \mu_f$ and choose the energy scale to be at the $Z^0$ mass (i.e., $\mu = \mu_0 = m_Z$) by
Figure 4: The tree-level Feynman diagrams for the gluon emission partonic process $e^- q \rightarrow e^- h^0 q g$ ($q = u, \bar{u}, d, \bar{d}, c, \bar{c}, s, \bar{s}$).

Figure 5: The tree-level Feynman diagrams for the light-quark emission partonic process $e^- g \rightarrow e^- h^0 q \bar{q}$ ($q = u, d, c, s$).

default, which characterizes the typical momentum transfer in the process $e^- p \rightarrow e^- h^0 j + X$.

The relevant SM parameters are taken as $m_e = 0.511$ MeV, $m_b = 4.2$ GeV, $m_t = 171.2$ GeV, $m_W = 80.398$ GeV, $m_Z = 91.1876$ GeV and $G_F = 1.16637 \times 10^{-5}$ GeV$^{-2}$ [9], and thus we get $\alpha = 1/132.34$ by adopting the relation of $\alpha = \frac{\sqrt{2}}{\pi} G_F m_W^2 s_{W}^2$. We use the PDFs of CTEQ6L1 and the CTEQ6M in the LO and NLO calculations, respectively [21].

The related SUSY parameters, such as the mixing angle of the MSSM Higgs fields $\alpha$ and masses of the light $CP$-even neutral Higgs boson, gluino, and scalar quarks, are obtained from the FormCalc program, except otherwise stated. The input parameters for the FORMCALC program are $M_S$, $M_2$, $A_f$, $m_{A^0}$, $\mu$ and $\tan \beta$. There $M_Q = M_U = M_D = M_S$ and the soft trilinear couplings for squarks $\tilde{q}$ being equal, i.e., $A_q = A_t = A_f$ are assumed, and the grand unification theory relation $M_1 = (5/3) \tan^2 \theta_W M_2$ is adopted for simplification. In our numerical calculation, we set $M_S = 400$ GeV, $M_2 = 110$ GeV, $m_{A^0} = 150$ GeV, $\mu = -200$ GeV, $A_f = 800$ GeV, $\tan \beta = 3$, and $m_{\tilde{g}} = 230$ GeV in default. Then we get $\sin(\beta - \alpha) = 0.9347$, $m_{\tilde{u}_1} = 198.17$ GeV, $m_{\tilde{u}_2} = 579.67$ GeV, $m_{\tilde{d}_1} = m_{\tilde{d}_2} = 397.07$ GeV, $m_{\tilde{\ell}_1} = m_{\tilde{\ell}_2} = 398.76$ GeV, $\theta_{\tilde{u}} = \theta_{\tilde{d}} = \frac{\pi}{2}$, $m_{\tilde{s}_1} = m_{\tilde{s}_2} = 400.62$ GeV, $m_{\tilde{\ell}_1} = m_{\tilde{\ell}_2} = 403.52$ GeV, $\theta_{\tilde{d}} = \theta_{\tilde{s}} = 0$ and
\( m_{h^0} = 98.36 \, \text{GeV} \). In the FormCalc program the radiative corrections to the MSSM Higgs boson masses up to two-loop contributions are involved, and the expressions related to the mass of the light neutral \( CP \)-even Higgs boson in Ref. \[22\] are adopted, where the input parameters of \( m_b, m_t, m_{\tilde{t}_1} \) and \( m_{\tilde{t}_2} \) are necessary.

The verifications for the total QCD NLO correction being independent of the two cutoffs \( \delta_s \) and \( \delta_c \) are made. We calculate the total QCD NLO corrections to the \( e^- p \rightarrow e^- h^0 j + X \) process in the MSSM at the LHeC with the cutoffs \( \delta_s \) running from \( 10^{-5} \) to \( 10^{-3} \), \( \delta_c = \delta_s/200 \), and \( \mu = \mu_0 = m_Z \). The results show that although the three-body correction \( \Delta \sigma^{(3)} = \sigma^{V} + \sigma^{S} + \sigma^{HC} \) and four-body correction \( \Delta \sigma^{(4)} = \sigma^{HC} \) depend strongly on the cutoff \( \delta_s (\delta_c) \), the final total QCD NLO correction \( \Delta \sigma_{NLO} \), which is the summation of the three-body and four-body terms, i.e., \( \Delta \sigma_{NLO} = \Delta \sigma^{(3)} + \Delta \sigma^{(4)} \) is independent of the two cutoffs within the statistic errors. The independence of the full QCD NLO corrections to the \( e^- p \rightarrow e^- q (q = u, \bar{u}) \rightarrow e^- h^0 j + X \) process on the cutoffs \( \delta_s \) and \( \delta_c \) provides an indirect check for the correctness of the calculations. In further numerical calculations, we fix \( \delta_s = 8 \times 10^{-4} \) and \( \delta_c = \delta_s/200 \).

We made the comparison of our LO numerical results for the process \( e^- p \rightarrow e^- H^0 j + X \) in the SM at the LHeC with the corresponding results read out from Fig.2 in Ref. \[14\], and find that they are coincident with each other within the statistic errors.

In the following LO and NLO numerical calculations, we adopt the massless four-flavor scheme and put the restriction of \( p^j_T > p^{cut}_{T,j} \) on the jet transverse momentum for one-jet events. For the two-jet events (originating from the real corrections), we apply the jet algorithm in the definition of the tagged hard jet with \( R = 1 \), i.e., if final state two partons satisfy \( \sqrt{\Delta \eta^2 + \Delta \phi^2} < 1 \) (where \( \Delta \eta \) and \( \Delta \phi \) are the differences of rapidity and azimuthal angle between the two jets), we merge them into a single jet. We use the so-called "inclusive" scheme and keep events with one or two jets. We require that there is one jet with \( p^j_T > p^{cut}_{T,j} \), and set \( p^{cut}_{T,j} = 30 \, \text{GeV} \) by default in following calculations. Furthermore, to reduce the background of the Higgs signals, we require the final electron with the following cuts

\[
p^e_T > 30 \, \text{GeV}, \quad |\eta^e| < 5. \tag{4.1}
\]

We plot the dependence of the LO and QCD NLO corrected total cross sections for the
\[ e^{-p} \rightarrow e^{-h^0}j + X \] process in the MSSM on the renormalization/factorization scale \( \mu \) in Fig.6(a). The corresponding K factor defined as \( K = \frac{\sigma_{NLO}}{\sigma_{LO}} \), versus the energy scale is presented in Fig.6(b). Figure6(a) shows that the LO curve is obviously dependent on the energy scale \( \mu \), although there only the factorization scale is involved in the convolution with the PDFs of the initial parton. If we define the scale uncertainty parameter as \( \eta = \frac{|\sigma(\mu_1) - \sigma(\mu_2)|}{\sigma(\mu_0)} \) in the scale range of \([\mu_1 = \frac{1}{3}\mu_0, \mu_2 = 3\mu_0]\), from Fig.6(a) we can get the uncertainty parameters for the LO and QCD NLO corrected cross sections having the values as \( \eta_{LO} = 10.35\% \) and \( \eta_{NLO} = 1.58\% \), respectively. It is obvious that the dependence of the LO total cross section on the energy scale is significantly reduced by the QCD NLO corrections. We can read out from Fig.6(b) that when the energy scale varies from \( 0.2m_Z \) to \( 3.8m_Z \), the value of the K factor increases from \( 0.893 \) to \( 1.048 \). In the following, we choose \( \mu = \mu_0 \) except otherwise stated.

Figure 6: (a) The dependence of the LO and QCD NLO corrected total cross sections for the \( e^{-p} \rightarrow e^{-h^0}j + X \) process on the renormalization/factorization scale \( \mu = \mu_r = \mu_f \) in the MSSM, where we take \( E_p = 7 \) TeV and \( E_e = 140 \) GeV. (b) The corresponding K factor of Fig.6(a) versus the energy scale \( \mu \) (where we define \( K = \frac{\sigma_{NLO}}{\sigma_{LO}} \)).

We plot the LO and QCD NLO corrected total cross sections for the \( e^{-p} \rightarrow e^{-h^0}j + X \) process in the MSSM as a function of the incoming electron beam energy \( E_e \) running from 50 GeV to 200 GeV in Fig.7(a), that corresponds to the c.m. colliding energy range of \( \sqrt{s} \approx 1.18 - 2.37 \) TeV. The corresponding K factors are depicted as a function of the incoming electron beam energy \( E_e \) in Fig.7(b). In Fig.7(a) the full line is for the QCD NLO corrected
total cross section for the $e^- p \rightarrow e^- h^0 j + X$ process, and the dotted line for the LO cross section. We can see from Figs.7(a) and 7(b) that the QCD NLO corrections reduce slightly the LO total cross sections for the process $e^- p \rightarrow e^- h^0 j + X$ in the plotted incoming electron beam energy range, and the production rate increases with $E_e$. In Fig.7(c) we depict the K factor versus electron beam energy $E_e$, the energy scales $\mu$ being $0.5\mu_0$ and $3\mu_0$ separately. We can see from Fig.7(c) that the K-factor uncertainty, $\Delta K = K(\mu = 3\mu_0) - K(\mu = 0.5\mu_0)$, ranges from 12.79% to 7.13% when $E_e$ goes up from 50 GeV to 200 GeV.

Figure 7: (a) The dependence of the LO and QCD NLO corrected total cross sections for the $e^- p \rightarrow e^- h^0 j + X$ process in the MSSM on the incoming electron beam energy $E_e$ in the MSSM, where we take $\mu = \mu_0$, $E_p = 7$ TeV, and $E_e = 140$ GeV. (b) The corresponding K factor ($K = \frac{\sigma_{\text{NLO}}}{\sigma_{\text{LO}}}$) versus the incoming electron beam energy $E_e$. (c) The K factor versus the incoming electron beam energy $E_e$ with $\mu = 0.5\mu_0$ and $\mu = 3\mu_0$, respectively.

The curves for the LO and QCD NLO corrected cross sections for the process $e^- p \rightarrow$
$e^- p \to e^- H^0 j + X$ as a function of $\tan \beta$ are drawn in Fig.8(a), where the corresponding values of $m_{h^0}$ are also shown on the x axis in Figs.8(a) and 8(b). The values of $m_{A^0}$ and of the other parameters are those given above. In Fig.8(a), we can see that both curves go down rapidly in the region of $2 < \tan \beta < 6$ ($85.52 \text{ GeV} < m_{h^0} = 113.14 \text{ GeV}$). Then the curves go up slowly after the values reach their corresponding minimal values at position around $\tan \beta \sim 7.5$. The relevant K-factor ($K = \sigma_{NLO}/\sigma_{LO}$) versus $\tan \beta$ (and $m_{h^0}$) is plotted in Fig.8(b). The K factor generally has a constant value of about 0.99. We further depict two curves for the K factors with $\mu = 0.5 \mu_0$ and $\mu = 3 \mu_0$ separately, as a function of $\tan \beta$ (and $m_{h^0}$) in Fig.8(c). We can read out from Fig.8(c) that the K-factor uncertainty due to the scale $\mu$, defined as $\Delta K = K(\mu = 3 \mu_0) - K(\mu = 0.5 \mu_0)$, is in the range from 4.06% to 6.29% when $\tan \beta$ ($m_{h^0}$) varies from 2 ($92.80 \text{ GeV}$) to 50 ($121.64 \text{ GeV}$).

For the comparison of the results for the processes $e^- p \to e^- H^0 j + X$ in the SM and $e^- p \to e^- h^0 j + X$ in the MSSM at the LHeC, we read out the data in the MSSM from Fig.8(a) at the positions of $\tan \beta = 3, 7, 18, 38$ respectively, and list these results together with the corresponding SM ones in Table 1. All SM parameters, including the mass of the SM Higgs boson, have the same values in both the SM and the MSSM calculations. The relative difference between the cross sections in both models, is defined as $\delta_{NLO} = \frac{\sigma_{NLO}^{MSSM} - \sigma_{NLO}^{SM}}{\sigma_{NLO}^{SM}} \times 100\%$. These numbers are obtained by adopting the values of the renormalization/factorization scale $\mu$ and the input SUSY parameters mentioned above. From the table we can see that the relative difference, $\delta_{NLO}$, between the cross sections in both models can reach the value of $-10.57\%$ up to the QCD NLO, when we take $\tan \beta = 3$ for the MSSM.

In Fig.9(a) we depict the LO and QCD NLO corrected cross sections for the process $e^- p \to e^- h^0 j + X$ as a function of mass $m_{A^0}$ (and $m_{h^0}$). As we know, the light CP-even Higgs boson mass $m_{h^0}$ depends on the CP-odd Higgs boson mass $m_{A^0}$, when the other related SUSY and SM input parameters are fixed. The values of $m_{h^0}$ corresponding to different $m_{A^0}$ values are also shown on the x axis in Figs.9(a) and 9(b). In Fig.9(a), we see that the cross sections increase rapidly in the range of $100 \text{ GeV} < m_{A^0} < 180 \text{ GeV}$ (It corresponds to the range of $80.92 \text{ GeV} < m_{h^0} < 106.02 \text{ GeV}$). After reaching their maximal values at position of $m_{A^0} =$
Figure 8: (a) The LO and QCD NLO corrected total cross sections for the $e^- p \rightarrow e^- h^0 j + X$ process as a function of $\tan \beta$ ($m_{A^0}$ fixed) and the corresponding mass of the light $CP$-even neutral Higgs boson $m_{h^0}$ in the MSSM, where we take $\mu = \mu_0$, $E_p = 7 TeV$, and $E_e = 140 GeV$. (b) The corresponding K factor ($K = \frac{\sigma_{NLO}}{\sigma_{LO}}$) versus $\tan \beta$ and $m_{h^0}$. (c) The K factor versus $\tan \beta$ and $m_{h^0}$ with $\mu = 0.5\mu_0$ and $\mu = 3\mu_0$, respectively.
Table 1: The numerical results of the $\sigma_{\text{LO}}^{\text{MSSM}}$, $\sigma_{\text{NLO}}^{\text{MSSM}}$ for $\tan \beta = 3$, 7, 18, 38 obtained from Fig.8(a), and the corresponding SM results of the $\sigma_{\text{LO},\text{NLO}}^{\text{SM}}$ of the process $e^- p \rightarrow e^- H_0^0 j + X$ are listed in the table, where we take the same SM parameters and the mass of the Higgs boson ($m_{h_0} = m_{H_0}$) in both the SM and the MSSM calculations. $\delta_{\text{NLO}}$ is defined as $\frac{\sigma_{\text{NLO}}^{\text{MSSM}} - \sigma_{\text{NLO}}^{\text{SM}}}{\sigma_{\text{NLO}}^{\text{SM}}} \times 100\%$.

220 GeV, the LO and QCD NLO corrected cross sections decrease gently. The corresponding K factor versus $m_{A_0}$ (and $m_{h_0}$) is displayed in Fig.9(b). The K factor seems to be stable and has the value around 0.99. We can see that when we fix the energy scale $\mu = \mu_0$, the QCD NLO corrections in the MSSM generally reduce the LO cross section by about 1%, while the pure SUSY QCD (pSQCD) NLO contributions are negligibly small and the relative pSQCD corrections have the values less than 0.01%. But as seen earlier [Fig.8(b)], when $\mu = 0.2m_Z$ (3.8$m_Z$) the QCD NLO relative correction reaches $-10.7\%$ (4.8%).

Figure 9: (a) The LO and QCD NLO corrected total cross sections for the $e^- p \rightarrow e^- h_0^0 j + X$ process as a function of the masses of the CP-odd Higgs boson $A_0$ and the light $CP$-even neutral Higgs boson $h_0$ ($\tan \beta$ fixed) in the MSSM, where we take $\mu = \mu_0$, $E_p = 7$ TeV and $E_e = 140$ GeV. (b) The corresponding K factor ($K = \sigma_{\text{NLO}}/\sigma_{\text{LO}}$) as a function of $m_{A_0}$ and $m_{h_0}$. 

The distributions of the transverse momenta of the final particles at the LO and up to the QCD NLO, and their corresponding K factors for the process $e^-p \rightarrow e^- h^0 j + X$ are depicted in Figs.10(a,b,c), where we define $K = \frac{d\sigma_{NLO}}{dp_T} / \frac{d\sigma_{LO}}{dp_T}$. In Figs.10(a), (b) and (c), the distributions of transverse momenta and K factors are for the final electron, the light $CP$-even neutral Higgs boson and jet, respectively. We can find that there is no obvious distortion induced by the QCD NLO corrections for the $p_T^e$ and $p_T^{h^0}$ distributions, while the shape distortion for the $p_T^{jet}$ distribution is not negligible since the K factor of the $p_T^{jet}$ distribution varies in the range of $0.865 < K_{p_T^{jet}} < 1.049$.

V. Summary

In this paper we calculate the full QCD NLO corrections to the light $CP$-even neutral Higgs boson production associated with an electron and a jet in the MSSM at the possible CERN LHeC. We investigate the uncertainty of the integrated cross sections induced by the factorization/renormalization scale, and present the LO and QCD NLO corrected total cross sections and the distributions of the transverse momenta of final particles. By adopting the definition of the scale uncertainty parameter in the scale range of $[\mu_1 = \frac{1}{3} \mu_0, \mu_2 = 3 \mu_0]$ as $\eta \equiv |\frac{\sigma(\mu_1) - \sigma(\mu_2)}{\sigma(\mu_0)}|$, we obtain the scale uncertainty parameters for the LO and NLO corrected cross sections are 10.35% and 1.58%, respectively. It is clear that the scale dependence of the LO cross section is obviously improved by the QCD NLO corrections. We find that there is no obvious distortion induced by the QCD NLO corrections for the $p_T^e$ and $p_T^{h^0}$ distributions, and the K factor of the QCD correction to the total cross section at the LHeC varies from 0.893 to 1.048 when the factorization/renormalization scale $\mu$ goes up from $0.2 m_Z$ to $3.8 m_Z$ in our chosen parameter space.

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Figure 10: (a) The LO and QCD NLO corrected differential cross sections \( \frac{d\sigma}{dp_T^e} \) and the corresponding K factor \( \left( K = \frac{d\sigma_{NLO}}{dp_T^e} / \frac{d\sigma_{LO}}{dp_T^e} \right) \) for the process \( e^- p \rightarrow e^- h^0 j + X \). (b) The LO and QCD NLO corrected differential cross sections \( \frac{d\sigma}{dp_T^h} \) and the corresponding K factor \( \left( K = \frac{d\sigma_{NLO}}{dp_T^h} / \frac{d\sigma_{LO}}{dp_T^h} \right) \) for the process \( e^- p \rightarrow e^- h^0 j + X \). (c) The LO and QCD NLO corrected differential cross sections \( \frac{d\sigma}{dp_T^{jet}} \) and the corresponding K factor \( \left( K = \frac{d\sigma_{NLO}}{dp_T^{jet}} / \frac{d\sigma_{LO}}{dp_T^{jet}} \right) \) for the process \( e^- p \rightarrow e^- h^0 j + X \).
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