NONSYMMETRICAL SINUSOIDAL THREE-PHASE SYSTEMS

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Abstract. This article deals with the theoretical aspects of the three-phase systems sinus lop-sided, especially the calculation of active, reactive power and the unbalanced and oscillating power systems. The unbalanced power or improperly called “lop-sided” introduced for pragmatic reasons to explain the loss of power caused by the unbalanced powers replaces the deficit of the three-phase system cumulated as power of the phases which are partial compensated under the three-phase systems. The unbalanced power called the “lop-sided power” should not be confused as it is sometimes with the oscillating (fluctuating) powers from which it differs by the sign of nonorthogonality component.

1. Introduction

For this analysis, symmetrical components phasor [1] and representative vectors (space-temporally phasor [2-4] are used sometimes improperly called symmetrical components [5-6] also.

Symmetrical components phasors, homopolars, direct and inverted (with 0, 1 respectively 2 indexes) have been defined for the three-phase system of the tension phasor (index a, b, c) thus:

\[ u_0 = \frac{1}{3}(u_a + u_b + u_c) \]
\[ u_1 = \frac{1}{3}(u_a + au_b + a^2 u_c) \]
\[ u_2 = \frac{1}{3}(u_a + a^2 u_b + au_c) \]

In these expressions \( a = e^{j2\pi/3} \) and \( a^2 = e^{-j2\pi/3} \) are rotation operators of phasors multiplied with \( \frac{2\pi}{3} \) respective \( -\frac{2\pi}{3} \).

Similar relations can be written for current phasors. The spatial phasors associated to rotating magnetic field of three-phase machines and induced tension by this, has similar expressions with direct and inverted symmetrical components but are functions of instantaneous values of phases tensions for unlike the symmetrical components phasors which are functions of phasors which are associated to phases tensions.

\[ u^+ = \frac{2}{3}(u_a + au_b + a^2 u_c) \equiv (u^-)^* \]
\[ u^- = \frac{2}{3}(u_a + a^2 u_b + au_c) \equiv (u^+)^* \]
Operators $a$ and $a^2$ have here the signification of directional versor [4]. From these result the relations with the symmetrical components phasors with which they must not be confused.

$$u^+ = u_1 + u_2^*$$
$$u^- = u_1^* + u_2$$

The same relations are obtained from phasors associated to currents.

2. Active and reactive powers of the sinusoidal three-phase system

The instantaneous complex power of three-phase systems without homopolar component of tension ($u_0 = 0$) equal with the sum of instantaneous values of direct and inverted complex powers of the phases has the following mathematical expression:

$$\bar{z} = [u_a \quad u_b \quad u_c] \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = u_1 [1 \quad a^2 \quad a] \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + u_2 [1 \quad a \quad a^2] \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

(3)

$$\bar{z} = u_1 (i_a + a^2 i_b + a i_c) + u_2 (i_a + a i_b + a^2 i_c) = \frac{3}{2} u_1 i_1^+ + \frac{3}{2} u_2 i_2^+ =$$

$$= \frac{3}{2} u_1 (i_1^* + i_2) + \frac{3}{2} u_2 (i_2^* + i_1) = 3 (U_1 I_1^* + U_1 I_2 e^{2 j \omega t} + U_2 I_2^* + U_2 I_1 e^{2 j \omega t})$$

(4)

$$= \frac{3}{2} [U_1 I_1^* + U_2 I_2^* + (U_1 I_2 + U_2 I_1) e^{2 j \omega t}] = S_1 + S_2 + \ddot{S}$$

The tensions $U_{1,2}$ and currents $I_{1,2}$ are represented in simplified complex of the respective phasors

$$u_{1,2} = \sqrt{2} U_{1,2} e^{j \omega t}, \quad i_{1,2} = \sqrt{2} I_{1,2} e^{j \omega t}$$

The two real components (three-phase active power) are thus obtained in one vectorial

$$P = 3 U_1 I_1 \cos \varphi_1 + 3 U_2 I_2 \cos \varphi_2$$

(5)

and the “imaginary” reactive power

$$Q = 3 U_1 I_1 \sin \varphi_1 + 3 U_2 I_2 \sin \varphi_2$$

(6)

as well as the oscillating power of three-phase system

$$\ddot{S} = \text{Re} \ddot{z} = 3 U_1 I_2 \cos (2 \omega t + \alpha_1 + \alpha_2 - \varphi_2) + 3 U_2 I_1 \cos (2 \omega t + \alpha_2 + \alpha_1 - \varphi_1)$$

(7)

The amplitude of these oscillating or fluctuant power is

$$S_{osc} = 3 \sqrt{(U_1 I_2)^2 + (U_2 I_1)^2 + 2 U_1 U_2 I_2 I_1 \cos(\varphi_1 - \varphi_2)}$$

(8)

Reported to the algebraic sum of direct and inverted apparent powers, it was considered an indicator of “unbalanced” system [7].

Other expressions of sinusoidal non-symmetrical power system which use Park spatial vectors or biphasic components of this [8-17] offer one exact mathematical expression of the active power.
The deficiency of the expression that contains spatial vectors exclusive (“Park”) are explained through the lock of originality for the solution for instantaneous active power

\[ p = \left[ u_1 u_2 \right] \left[ i_1 u_c \right] = \frac{1}{2} \left[ u^+ - \left[ \begin{array}{ccc} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{array} \right] i_1 \right] = \frac{1}{2} \left[ a^2 u^+ - i_1 \right] = \frac{3}{4} \left( u^+ i^- - u^- i^+ \right) = \frac{3}{2} \text{Re} u^+ i^- = \frac{3}{2} \text{Re} u^- i^+ \]

\[ p = \frac{3}{2} \text{Re} u^+ i^- = \frac{3}{2} \text{Re} \left( u_1 + u_2 \right) \left( i_1 + i_2 \right) = \]

\[ = 3 \text{Re} \left( U_1^* L_1^* + U_2^* L_2^* + U_1^* L_2 e^{j2\omega t} + U_2^* L_1 e^{-j2\omega t} \right) \]

respectively

\[ p = \frac{3}{2} \text{Re} u^- i^+ = \frac{3}{2} \text{Re} \left( u_1 + u_2 \right) \left( i_1 + i_2 \right) = \]

\[ = 3 \text{Re} \left( U_1^* L_1^* + U_2^* L_2^* + U_1^* L_2 e^{-j2\omega t} + U_2^* L_1 e^{j2\omega t} \right) \]

but arbitrary, incorrect for reactive

\[ Q = \text{Im} \left( U_1^* L_1^* + U_2^* L_2^* \right) = 3 U_1 I_1 \sin \phi_1 - 3 U_2 I_2 \sin \phi_2 = Q_1 - Q_2 \]  \quad (9)

respectively

\[ Q = \text{Im} \left( U_1^* L_1^* + U_2^* L_2^* \right) = -3 U_1 I_1 \sin \phi_1 + 3 U_2 I_2 \sin \phi_2 = -Q_1 + Q_2 \]  \quad (10)

as well as for complex instantaneous power

\[ s = \frac{3}{2} u^- i^+ = \frac{3}{2} \left( u_1 + u_2 \right) \left( i_1 + i_2 \right) \]

\[ s = \frac{3}{2} u^+ i^- = \frac{3}{2} \left( u_1 + u_2 \right) \left( i_1 + i_2 \right) \]  \quad (11)

The persistent error in numerous theses in specialized literature of the expression that contains spatial vectors exclusive (“Park”) are explained through the lock of originality for the solution for instantaneous active power

\[ p = \frac{3}{4} \left( u^+ i^- + u^- i^+ \right) = \frac{3}{2} \text{Re} u^+ i^- = \frac{3}{2} \text{Re} u^- i^+ \]  \quad (12)

Any of these two expressions offer concrete solutions for active power but different, incorrect and ambiguous for reactive power.

A formula considered “general” for reactive power from three-phase systems [18] that cures the deficiency of the expression \( s = \frac{3}{2} u^- i^+ \) to reproduce incorrectly the three-phase reactive power of non-symmetrical systems uses one representative tension vector rotated with \(-\pi/2\) in rapport with direct succession vector and with \(+\pi/2\) in rapport with the inverted conjugated succession

\[ u' = -j (u_1 - u_2^*) \]  \quad (13)

thus obtain the following expression

\[ s' = \frac{3}{2} u' i^* = \frac{3}{2} u' i^- = \frac{3}{2} (-j) (u_1 i_1 + u_2 i_2 - u_1 i_2 - u_2 i_1) \]  \quad (14)

The three-phase complex power (average in one period) results in:

\[ S = 3 \left( -j \left( U_1^* L_1^* - U_2^* L_2^* \right) \right) = 3 \left( Q_1 + Q_2 \right) - 3 j (P_1 - P_2) \]  \quad (15)

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In consequence, reactive power $Q = \text{Re} \bar{s}$ is reproduced correctly while active power $P = \text{Im} \bar{s}$ is wrong.

In conclusion, the only one correct expression at three-phase complex power for unbalanced system, single and complete [19-22] is the following

$$s = \frac{3}{2} [u_1 i_1^- + u_2 i_2^+] = \frac{3}{2} [u_1 i_1^* + u_2 i_2^* + u_1 i_1 + u_2 i_2]$$

or

$$p = \frac{3}{4} (u_1^+ i_1^- + u_2^+ i_2^-)$$

Following a correct grouping of the pairs of terms complexly conjugated

$$p = \frac{3}{4} [(u_1 i_1^- + u_2 i_2^+) + (u_1^* i_1^+ + u_2^* i_2^-)]$$

(16)

With the election of the first two terms of this expression

$$p = \frac{3}{2} \text{Re} [(u_1 i_1^- + u_2 i_2^+)] = \text{Re} \bar{s}$$

(18)

results in a correct solution for complex power $\bar{s}$ and therefore for the imaginary parts (reactive power) and for real parts (active power) of it.

As in case with the three-phase systems symmetrical voltages and currents where the total instantaneous power does not have the characteristic reactive power of each phase (because of the compensation between system phases), similarly to systems with non-symmetrical three-phase voltages and currents, the total instantaneous power does not contain the other specific power of phases.

Therefore, before analyzing the various proposals for the definition of effective power of three-phases systems, we express the instantaneous physical powers of phases using symmetrical components of voltages and currents and not representative vectors (Park or Clark) as is usually done incorrectly [8-17].

In the absence of homopolar components, with relations between the tensions and currents phasors of phases and symmetrical components

$$\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ a^2 & a \\ a & a^2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ a^2 & a \\ a & a^2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

obtained [19-21] following the expression of complex instantaneous power for each phases (a, b, c) of the system.

$$\bar{s}_{a,b,c} = (u \cdot i)_{a,b,c} = \frac{(u \cdot \frac{i + i^*}{2})_{a,b,c}}{2}$$

$$= \frac{1}{2} [u_1 (i_1^* + i_2^*) + u_2 (i_2^* + i_1^*) + u_1 (i_1^* + i_2^*) + u_2 (i_2 + i_1^*)]_{a,b,c}$$

or
where $s = \frac{3}{2}(u_1^* i_1 + u_2^* i_2 + u_1 i_1 + u_2 i_2)$ is the complex instantaneous power of three-phase system, common to the three phases.

If the phasors of symmetrical components of tensions and currents are explained the following expressions of instantaneous complex power are obtained

$$s = 3(U_1 i_1^* + U_2 i_2^* + (U_1 i_1 + U_2 i_2))e^{j2\omega t}$$

$$s_a = \frac{s}{3} + U_1 i_1^* + U_2 i_2^* + u_1 i_1 + u_2 i_2$$

$$s_b = \frac{s}{3} + aU_1 i_1 + aU_2 i_1 + a^2 U_1 i_2 + a^2 U_2 i_2$$

$$s_c = \frac{s}{3} + a^2 U_1 i_1 + aU_2 i_1 + a^2 U_1 i_2 + aU_2 i_2$$

The expressions of medium powers (measured) components of classical vectorial complex power of non-symmetrical three-phase system are resulted

$$s = 3(P_1 + P_2) + j(Q_1 + Q_2) = 3(U_1 i_1^* + U_2 i_2^*) =$$

$$= 3(U_1 i_1 \cos \varphi_1 + U_2 i_2 \cos \varphi_2) + j3(U_1 i_1 \sin \varphi_1 + U_2 i_2 \sin \varphi_2)$$

as well as the (measured) medium powers, components of complex power for each phase.

$$s_a = \frac{s}{3} + U_1 i_2 \cos \varphi_1 + U_2 i_1 \cos \varphi_2 + j(U_1 i_2 \sin \varphi_1 + U_2 i_1 \sin \varphi_2)$$

$$s_b = \frac{s}{3} + U_1 i_2 \sin \left(\frac{\varphi_1 + 2\pi}{3}\right) + U_2 i_1 \cos \left(\frac{\varphi_2 + 2\pi}{3}\right) +$$

$$+ j \left[ U_1 i_2 \sin \left(\frac{\varphi_1 + 2\pi}{3}\right) + U_2 i_1 \sin \left(\frac{\varphi_2 + 2\pi}{3}\right) \right]$$

$$s_c = \frac{s}{3} + U_1 i_2 \cos \left(\frac{\varphi_1 - 2\pi}{3}\right) + U_2 i_1 \cos \left(\frac{\varphi_2 + 2\pi}{3}\right) +$$

$$+ j \left[ U_1 i_2 \sin \left(\frac{\varphi_1 - 2\pi}{3}\right) + U_2 i_1 \sin \left(\frac{\varphi_2 + 2\pi}{3}\right) \right]$$

It can be observed that the active powers associated to the phases
\[ P_a = U_1 I_1 \cos \phi_1 + U_2 I_2 \cos \phi_2 + U_1 I_2 \cos (\phi_1 + \frac{2\pi}{3}) + U_2 I_1 \cos \phi_2 \]

\[ P_b = U_1 I_1 \cos \phi_1 + U_2 I_2 \cos \phi_2 + U_1 I_2 \cos \phi_2 + U_2 I_1 \cos \phi_1 - \frac{2\pi}{3} \]

\[ P_c = U_1 I_1 \cos \phi_1 + U_2 I_2 \cos \phi_2 + U_1 I_2 \cos \phi_2 - \frac{2\pi}{3} + U_2 I_1 \cos \phi_2 + \frac{2\pi}{3} \]

(27)

contain components of mutual unbalanced which is compensated within of total (sum) power. So the total three-phase power is deficient because it does not contain all powers available for each phase.

Because of mutual unbalanced components, the powers associated phases may have different values, some phases with greater or smaller powers, positive or negative in relation to other phases.

The same remark resulted for reactive powers (measured by wattmeter) through the use of some artificial tensions dephased with \( \pi/2 \) in relation with real tensions as well as for “vectorial” complex power.

\[ Q_a = U_1 I_1 \sin \phi_1 + U_2 I_2 \sin \phi_2 + U_1 I_2 \sin \phi_2 + U_2 I_1 \sin \phi_1 - \frac{2\pi}{3} \]

\[ Q_b = U_1 I_1 \sin \phi_1 + U_2 I_2 \sin \phi_2 + U_1 I_2 \sin \phi_2 + U_2 I_1 \sin \phi_2 - \frac{2\pi}{3} \]

\[ Q_c = U_1 I_1 \sin \phi_1 + U_2 I_2 \sin \phi_2 + U_1 I_2 \sin \phi_2 - \frac{2\pi}{3} + U_2 I_1 \sin \phi_2 + \frac{2\pi}{3} \]

(28)

Therefore they sought other practical expressions of apparent powers, active and reactive and for “unbalanced” to correct this deficiency.

As the sinusoidal monophase system, the movement of power does not have a special physical significance because the power components do not have the property of identifiable substances whose “flowing” could be tracked [23]. However, it stressed the usefulness of conservation property of some power.

Decomposition of the unbalanced regime in balanced components as well as a non-sinusoidal periodic in harmonic components has an important role regarding the methodological point of view for observing the different effects in relation to symmetrical sinusoidal reference and for achievement of compensating filters.

3. Power of the unbalanced sinus lop-sided three-phase system

The apparent effective power” is suggested by the expressions of actual values tensions and currents as functions of the harmonic expressions of the non-sinusoidal single phase systems, similarly for expressions corresponding to the phases of unbalanced three-phase systems. Defined [25] and adopted [24][26] an expression of “actual apparent” power

\[ S_e = 3U_e I_e \]

(29)

with

\[ 3U_e^2 = U_a^2 + U_b^2 + U_c^2 = 3(U_1^2 + U_2^2 + U_0^2) \]

\[ 3I_e^2 = I_a^2 + I_b^2 + I_c^2 = 3(I_1^2 + I_2^2 + I_0^2) \]

To simplify, in the absence of homopolar components, if using Lagrange identity similar to that used by C Budeanu at non-sinusoidal monophase systems by explaining the active and reactive components of currents on orthogonal relationship of power is obtained.
\[
\left( \frac{S_e}{3} \right)^2 = (U_1^2 + U_2^2)(I_1^2 + I_2^2) = \\
= (U_1^2 + U_2^2)(I_1^2 \cos^2 \phi_1 + I_1^2 \sin^2 \phi_1 + I_2^2 \cos^2 \phi_2 + I_2^2 \sin^2 \phi_2) = \\
= (U_1 \cos \phi_1 + U_2 \cos \phi_2)^2 + (U_1 \sin \phi_1 + U_2 \sin \phi_2)^2 + \\
+ (U_1 \cos \phi_2 - U_2 \cos \phi_1)^2 + (U_1 \sin \phi_2 - U_2 \sin \phi_1)^2
\]

or
\[
S^2_e = P^2 + Q^2 + N_p^2 + N_q^2
\tag{30}
\]

In addition to the powers

active \quad P = (U_1 \cos \phi_1 + U_2 \cos \phi_2)

reactive \quad Q = 3(U_1 \sin \phi_1 + U_2 \sin \phi_2)

that exist in the complex classic expression (vectorial) with symmetrical components, new orthogonal “unbalanced” components appear

active \quad N_p = 3(U_1 \cos \phi_2 - U_2 \cos \phi_1)

reactive \quad N_q = 3(U_1 \sin \phi_2 - U_2 \sin \phi_1)

that forming apparent unbalanced power similar to deforming power
\[
N^2 = N_p^2 + N_q^2 = 9 \left[ (U_1 I_2)^2 + (U_2 I_1)^2 - 2U_1 U_2 I_1 I_2 \cos(\phi_1 - \phi_2) \right]
\tag{32}
\]

For a balanced three-phase system with equal impedances of phases
\[
Z = \frac{U_1}{I_1} = \frac{U_2}{I_2}, \quad \phi_1 = \phi_2
\]

the unbalanced power is cancelled \((N=0)\) resulting a decrease of total losses of power by the interconnection line.
\[
\Delta P = 3r(I_a^2 + I_b^2 + I_c^2) = 9rI_e^2 = r \frac{S_e^2}{U_e^2} = r \frac{P^2 + Q^2 + N^2}{U_e^2}
\tag{33}
\]

The “unbalanced” power or improperly called “non-symmetrical” \([24][27]\) introduced for pragmatic reasons to explain the loss of power caused by the unbalanced regime replaces the deficit of the cumulative three-phase powers like powers of phases which are partially compensated under the three-phase systems.

Even if in calculations and judgment the powers of unbalanced three-phase systems (and not algebraic sum of them like three-phase) the utility of effective three-phase power is justified because, “unbalanced power” will be cancelled simultaneously with cancellation of the unbalanced (the identity of impedances phases system).

The power of “unbalanced” \(N\) called the “lop-sided power” \([24][28]\) should not be confused as it is sometimes \([28-29]\) with the oscillating (fluctuating) powers \(S_{osc}\) \([7]\) from which it differs by the sign of non-orthogonality components.
\[
\left( \frac{N}{3} \right)^2 = U_1^2 I_2^2 + U_2^2 I_1^2 - 2U_1 U_2 I_1 I_2 \cos(\phi_1 - \phi_2)
\]
\[
\left( \frac{S_{osc}}{3} \right)^2 = U_1^2 I_2^2 + U_2^2 I_1^2 + 2U_1 U_2 I_1 I_2 \cos(\phi_1 - \phi_2)
\]
Fluctuating power $S_{osc}(8)$ will be cancelled $U_1/I_1 = U_2/I_2$ only if $\phi_1 - \phi_2 = \pi$ theoretical condition cannot be realized physically with passive circuits when the unbalanced power $N$ is cancelled for purely balanced receiver, with $\phi_1 = \phi_2$.

Expressed in relative units by reporting at direct apparent power
\[
\left( \frac{S_1}{3} \right)^2 = \left( \frac{U_1 I_1}{3} \right)^2
\]
a function of factors of non-symmetrical tension and circuits (factors as the functioning of the unbalanced) is obtained.
\[
\left( \frac{N}{S_1} \right)^2 = \left( \frac{I_2}{I_1} \right)^2 + \left( \frac{U_2}{U_1} \right)^2 - 2 \left( \frac{I_2}{I_1} \right) \left( \frac{U_2}{U_1} \right) \cos(\phi_1 - \phi_2)
\]
\[
\left( \frac{S_{osc}}{S_1} \right)^2 = \left( \frac{I_2}{I_1} \right)^2 + \left( \frac{U_2}{U_1} \right)^2 + 2 \left( \frac{I_2}{I_1} \right) \left( \frac{U_2}{U_1} \right) \cos(\phi_1 - \phi_2)
\]

The effective apparent power with his active, reactive, deforming and unbalanced components was previously deduced [24] as the maximum power transmitted for loss of power by transmission line (proportional to the squares effective currents) data. The expressions obtained include both disturbing as the unbalanced power.

A simplified expression of the effective apparent power $S_e$ to highlight the quality factors of unbalanced and distorting functioning is obtained in relative units by reporting at direct fundamental apparent power $3U_1 I_1$.
\[
S_e^2 = 3U_e^2 \cdot 3I_e^2 = 3 \left[ U_1^2 + U_2^2 + U_H^2 \right]^2 \left[ I_1^2 + I_2^2 + I_H^2 \right]^2
\]
\[
\left( \frac{S_e}{3U_1 I_1} \right)^2 = 1 + \left( \frac{U_2}{U_1} \right)^2 + \left( \frac{U_H}{U_1} \right)^2 \left[ 1 + \left( \frac{I_2}{I_1} \right)^2 + \left( \frac{I_H}{I_1} \right)^2 \right]
\]

$U_H$ and $I_H$ are the effective value of all voltage respectively current harmonics direct and inverted components.

4. Conclusions

Using formal representative spatial vectors lead to erroneous expressions of reactive power from the three-phase lop-sided. Using rotating phasors (Fresnel) leads to the correct expressions.

Correct expression of complex three-phase power for the unbalanced, single and complete system is the following
\[
p = \frac{3}{4} (u^+ i^- + u^- i^+) = \frac{3}{4} \left[ (u_1 + u_2^*) i^- + (u_1^* + u_2) i^+ \right]
\]

with
\[
p = \frac{3}{2} \text{Re} \left( u_1 i^- + u_2 i^+ \right) = \text{Re} \delta \quad \text{and} \quad q = \frac{3}{2} \text{Im} \delta = 3 \left( U_1 I_1 \sin \phi_1 + U_2 I_2 \sin \phi_2 \right)
\]

which provides a fair solution for complex power and as well for the real (active power) and for the imaginary (reactive power) parts.

The unbalanced power $N$ called the "lop-sided power" should not be confused as it is sometimes with the oscillating (fluctuating) power $S_{osc}$ from which it differs by the sign of non-orthogonality.

Only the analysis of quantities (current, voltage, and power) characteristic to phases of the three-phase systems presents theoretical and practical importance to achieve balance of the unbalanced systems as well as to analyze the current and powers that load the phases of the system.
The analysis of the entire three-phase system provides little indications due to the compensation between the phases of active and reactive powers of the phases.

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