OBSERVABLE ZERO-SOUND IN STRONGLY CORRELATED METALS

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Abstract

The slow zero-sound mode expected near the Mott transition in strongly interacting two-dimensional Fermi systems that are neutral is shown to persist as the physical sound mode in the case that the fermion carries electronic charge and is embedded in a positive ionic background. The latter sound velocity softens completely precisely at the Mott transition, indicating that a zone-center structural transition will occur in the system. We suggest that this phonon-softening mechanism is related to the structural transitions commonly observed in the oxide superconductors.

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The realization that the conduction electrons in the Copper-Oxygen planes common to all high-temperature superconductors experience strong repulsive interactions has motivated the ongoing theoretical study of strongly interacting fermions in two dimensions. The simplest theoretical description of this physics for electron densities approaching the antiferromagnetic Mott transition is given by the $t-J$ model on the square-lattice, where double occupancy of electrons at each site is strictly excluded. The latter constraint can be elegantly accounted for by the introduction of an auxiliary slave-boson field, $b_i$, such that the original electron field, $c_{i\sigma}$, is replaced by the composite field, $c_{i\sigma}b_i^\dagger$, along with the new constraint, 

$$\sum_\sigma c^\dagger_{i\sigma}c_{i\sigma} + b_i^\dagger b_i = 1. \quad (1)$$

Certain mean-field treatments based on this formulation of the $t-J$ model that allow for dynamics in the slave-boson field lead to spin/charge separation of the strongly correlated electron. As a result, the latter break-up of electronic quantum numbers manages to reconcile the experimental observation of an electron-type Fermi surface in conjunction with hole-type charge transport in the normal state of the oxide superconductors. Such mean-field spin/charge separated treatments, however, over-estimate the size of the Hilbert space allowed by the constraint against double occupancy (1); e.g., the constraint against double occupancy at a particular site is violated.

Beyond the mean-field approximation, however, there exist fluctuations of the statistical gauge field, $A_\mu$, generated by the internal gauge symmetry $(c_{i\sigma}, b_i) \rightarrow (e^{i\theta_i}c_{i\sigma}, e^{i\theta_i}b_i)$ that mediate interactions between the spin/charge separated species. In particular, slave-boson scattering off of such gapless transverse gauge-field excitations results ultimately in a linear-in-$T$ prediction for resistivity in the fluxless (“strange”) metallic phase of the $t-J$ model that is consistent with transport measurements in the normal phase of the oxide superconductors. It has been pointed out by the author, however, that a gapless pole is also shown by the longitudinal component of the statistical gauge-field, which is directly tied to the constraint against double occupancy (1). Physically, this mode is a slow zero-sound mode with a velocity given by $c_0 \sim (t/Jx)^{1/2}$, where $x$ denotes the concentration of holes. (Throughout this paper, we set $\hbar$, $k_B$, and the lattice constant, $a$, to unity.) Unlike the case of a conventional Fermi-liquid, where sound excitations that are slower than the Fermi velocity become over-damped by the particle-hole continuum, the slow
zero-sound mode in question remains under-damped because of spin/charge separation.\textsuperscript{8} Recently, Hlubina et al. have shown that including such longitudinal gauge-field fluctuations, in general, results in an entropy reduction.\textsuperscript{6} In fact, they found that the addition of the spin/charge separated quasi-particle entropy with the entropy “generated” by the gauge-fields agrees quantitatively with a calculation of the entropy for the $t-J$ model on the square-lattice that is based on a high-temperature expansion.\textsuperscript{6} Therefore, the introduction of the longitudinal gauge-field cures the Hilbert space size problem existing in the mean-field approximation mentioned above, thereby demonstrating that the presently discussed gauge-field theory is a consistent low-energy effective theory of strongly interacting electrons in two dimensions. Note that a very similar slow zero-sound mode also appears in the anyon superconductor saddle-point of the $t-J$ model near half-filling.\textsuperscript{9}

In this Letter, we show that the above-cited slow zero-sound mode existing in neutral two-dimensional (2D) Fermi systems near the Mott transition\textsuperscript{5,9} persists when such fermions are promoted to physical electrons embedded in a positive ionic background. In particular, we compute the sound velocity for a strongly interacting electron liquid embedded in a featureless jellium background and find that it interpolates continuously between the standard Bohm-Staver result for first sound at hole densities far from the Mott transition\textsuperscript{10} and the slow zero-sound velocity at hole densities approaching the Mott transition ($x \to 0$). We therefore argue that a zone-center structural transition will be induced at the Mott transition, where the sound speed softens completely. We also compute the renormalized velocity of acoustic phonons coupled to the same strongly interacting electron liquid via the standard electron-phonon interaction and find very similar results. (It is implicitly assumed in the latter calculation that the long-range Coulomb interaction is screened by some other type of free charge carriers in the system.)

**Zero-sound + Jellium.** The longitudinal dielectric constant for any electronic liquid embedded in a positive jellium background may be expressed as

$$
\epsilon(\vec{k}, \omega) = 1 - \frac{\Omega_i^2}{\omega^2} + \frac{\Omega_e^2}{c_0^2 k^2 (1 + i n_1 \rho_F F)^{-1}} (1 - \omega^2) 
$$

in the long wavelength limit, where $\Omega_e$ and $\Omega_i$ denote the plasma frequencies of the electron liquid and of the positive jellium background, respectively, and where the zero-sound velocity, $c_0$, is related to the Fermi-Thomas wave-vector of the electron liquid,
\[ k_{FT} = (4\pi e^2 \rho_F)^{1/2} \], by
\[ \Omega_e = c_0 k_{FT}. \] (3)

Above, \( \rho_F \) represents the density of states at the (pseudo) Fermi surface. Also, the prefactor, \( \nu_1 \), to the Landau damping term in eq. (2) is of order unity for \( |\omega/k| < v_F \), where \( v_F \) denotes the (pseudo) Fermi velocity, while it vanishes otherwise. In the case of the spin/charge separated “strange” metal phase of the square-lattice \( t - J \) model at hole concentrations near half-filling \( (x \to 0) \), the electronic plasma frequency is given by that of the holes, \( \Omega^2_e = (32/\pi)(e^2/d)tx \), while the density of states at the pseudo Fermi surface is given by \( \rho_F \sim (tJx)^{-1} \). (We presume that an infinite stack of such \( t - J \) models, each separated by a distance \( d \), fills three-dimensional space.) Hence, eq. (3) implies that the zero-sound velocity is approximately \( c_0 \sim (tJx)^{1/2} \) in this case, as we quoted earlier. The spectrum for the collective charge-modes of this system is determined by the characteristic equation \( \epsilon(\vec{k}, \omega) = 0 \), which yields the following two solutions:

\[ \omega_-(\vec{k}) = c_s k \left( 1 + \frac{i}{2} \nu_1 \rho_F c_s \right), \] (4a)
\[ \omega_+(\vec{k}) = \left[ \Omega^2_e + \Omega^2_1 + c_0^2 \left( 1 + \frac{c_1^2}{c_0^2} \right)^{-1} k^2 \right]^{1/2}, \] (4b)

where the sound speed is given by
\[ c_s = (c_0^{-2} + c_1^{-2})^{-1/2}, \] (5)

and where the bare first-sound speed, \( c_1 \), is related to the plasma frequency of the ionic background by
\[ \Omega_1 = c_1 k_{FT}. \] (6)

In deriving the dispersion for the sound mode (4a), it has been assumed that the Landau damping is a small perturbation, \( \nu_1 \rho_F c_s \ll 1 \), which in the case of the “strange” metal phase of the \( t - J \) model implies that \( tx \ll J \).\(^5\)

Above, we have obtained the acoustic sound mode (4a) and the plasma oscillation (4b) generally expected in a two-component plasma.\(^{11}\) In the case of a conventional metal, where the first-sound speed is much smaller than the zero-sound speed \( (c_1 \ll c_0) \), we recover the well-known result that the zero-sound mode is “pushed-up” to the plasma frequency,\(^7\) while
that the sound speed is given by the Bohm-Staver result,\(^\text{10}\) \(c\). (Note that transverse zero-sound is predicted to exist both in charged Fermi-liquids\(^\text{12}\) and in conventional metals.\(^\text{13}\)

On the other hand, for hole-densities approaching the Mott transition such that \(\Omega_e \ll \Omega_i\), the sound speed \((5)\) is then approximately that of zero-sound; i.e., \(c_s \approx c_0\). Hence, precisely at the Mott transition, where \(\Omega_e = 0\) and \(c_0 = 0\), Eq. \((4a)\) indicates that the acoustic phonon mode softens completely, suggesting that a zone-center structural transition will occur in the system.

**Zero-sound + Phonons.**\(^\text{5}\) Consider now the case of the very same strongly interacting electron liquid with a dynamic compressibility [see the last term in eq. \((2)\)] given by

\[
\kappa(\vec{k}, \omega) = \rho_F \left[ 1 + i\nu_1 \rho_F \frac{\omega}{k} \right]^{-1} - \frac{\omega^2}{c_0^2 k^2} \tag{6}
\]

that couples to acoustic phonons via the conventional electron-phonon interaction. Then standard diagrammatic techniques based on Dyson sums yield that the renormalized phonon propagator, \(D(\vec{k}, \omega)\), satisfies

\[
D^{-1} = D_0^{-1} + g^2 \kappa, \tag{7}
\]

where \(D_0(\vec{k}, \omega) = c_1^2 k^2 (\omega^2 - c_1^2 k^2)^{-1}\) is the bare phonon propagator and \(g\) denotes the coupling-constant for the electron-phonon interaction.\(^\text{14}\) Substituting \((6)\) into \((7)\), we find that the renormalized phonon propagator, \(D(\vec{k}, \omega)\), has poles of the form \([\omega - c_\pm k - i\gamma_\pm(\vec{k})]^{-1}\), with phonon velocities and damping rates given by

\[
c_\pm = \left\{ \frac{1}{2} \left( c_0^2 + c_1^2 \right) \pm \frac{1}{2} \left[ (c_0^2 - c_1^2)^2 + 4\rho_F^2 g^2 c_0^2 c_1^2 \right]^{1/2} \right\}^{1/2}, \tag{8a}
\]

\[
\gamma_\pm(\vec{k}) = \frac{1}{2} \nu_1 \rho_F c_0^2 k \left\{ \frac{1}{2} \pm \frac{1}{2} \left[ (c_0^2 - c_1^2)^2 + 4\rho_F^2 g^2 c_0^2 c_1^2 \right]^{1/2} \right\}. \tag{8b}
\]

In deriving the dispersion for the phonon modes \((8a,b)\), it has been assumed that the Landau damping is a small perturbation; i.e., \(\gamma(\vec{k}) \ll c_\pm k\). This again requires that \(\nu_1 \rho_F c_s \ll 1\), which implies that \(tx \ll J\) for the case of the “strange” metal phase of the \(t-J\) model.\(^\text{5}\)

We see above in eqs. \((8a)\) and \((8b)\) that the coupled electron-phonon system results in two acoustic modes that are weakly damped. As shown in Fig. 1, the two branches of
velocity exhibit level-repulsion characteristic of reactively coupled modes. In the conventional metallic regime, where \( c_1 \ll c_0 \), while we do recover the well-known result \( c_- \approx c_1(1 - \rho F g^2)^{1/2} \) for the renormalized phonon velocity,\(^{14}\) we also obtain an observable zero-sound branch with velocity \( c_+ \approx c_0 \). The latter result appears to contradict our previous calculation in the jellium system, which found that the zero-sound mode was promoted to the plasma frequency in this regime. However, it can be shown that the presently obtained acoustic modes can be recovered in the former jellium model as long as the vacuum dielectric constant, \( \epsilon_0 = 1 \), in Eq. (2) is replaced by one with static screening,\(^{15}\) i.e., \( \epsilon_0 = 1 + k'^2 F_T / k^2 \). Such static screening could result from the existence of some other type of charge-carriers in the system.\(^5\) On the other hand, in the limit approaching the Mott transition, where \( c_0 \ll c_1 \), eq. (8a) indicates that the renormalized phonon velocity, \( c_- \), evolves continuously into the zero-sound velocity, \( c_0 \) (see Fig. 1). Hence, just as in the previous case of the jellium model, this phonon will soften completely at the Mott transition, \( c_0 = 0 \), suggesting that a zone-center structural transition will occur in the system. Last, we take this opportunity to point out that the preceding result corrects previous work by the author,\(^5\) where it was erroneously asserted that complete phonon softening occurs when \( c_0 = c_1 \).

**Observability.** We have shown above that slow zero-sound evolves continuously into the first-sound mode in strongly correlated metals near the Mott transition. The conditions for the observability of such a mode are therefore (a) that it exist in the neutral system, and (b) that the plasma frequencies satisfy \( \Omega_e \approx \Omega_i \). For the case of the spin/charge separated “strange” metal phase of the \( t - J \) model in two dimensions,\(^4\) the former requirement reduces to the acoustic-plasmon type existence condition,\(^5,11\)

\[
v_B \ll c_0 \ll v_F, \tag{9}
\]

where \( v_B \) denotes the characteristic velocity of the slave-bosons (holons), and where \( v_F \) denotes the characteristic velocity of the pseudo-fermions (spinons). Near half-filling, since \( v_F \sim J \) and \( c_0 \sim (tJx)^{1/2} \), the righthand side of eq. (9) implies (i) that \( tx \ll J \). Now suppose first that no superfluid transition exists in the spinon component. It can then be shown that the holon Bose-condensation transition\(^{16}\) is suppressed to zero-temperature by chiral spin-fluctuations,\(^{17,18}\) which implies that the characteristic velocity of the holon liquid is given by the thermal velocity, \( v_B \sim (tT)^{1/2} \), at all temperatures \( T \). Hence, the
lethand side of eq. (9) implies (ii) that \( T \ll Jx \), in this case. These results indicate, therefore, that slow zero-sound exists in strongly interacting 2D Fermi systems near the Mott transition and at low-temperature when Cooper pairing instabilities in the spinon fluid are suppressed. If, on the other hand, we suppose that there does indeed exist a superfluid transition in the spinon component at a critical temperature, \( T_F \), then the critical temperature of the physical superconducting transition will be given by the geometric mean \( T_c = (T_F^{-1} + T_B^{-1})^{-1} \), where \( T_B \sim tx \) is the Bose-Einstein condensation temperature of ideal holons.\(^{18}\) In such case, the characteristic velocity of the holons is given by \( v_B \sim (tT_c)^{1/2} \) in the superconducting phase, and the lefthand side of eq. (9) then implies (iii) that \( T_c \ll Jx \). However, for \( t > J \), the latter inequality can only be satisfied at hole concentrations beyond that which optimizes \( T_c \), since \( T_c \cong T_B \sim tx \) for small hole concentrations, \( x \to 0 \). In the context of the oxide superconductors, therefore, the above discussion suggests that slow zero-sound of the type sketched here is observable in the optimally-doped to over-doped regions of the phase diagram, where spin-gap behavior in the normal state is absent.\(^{16-19}\)

It is interesting to remark that structural transitions in the cuprate superconductors are known to occur precisely in the above regime.\(^{20}\) This observation should be tempered, however, by the fact that the oxide superconductors appear to be far from a Mott transition at such hole concentrations (\( x \approx 0.1 \)). In particular, the condition (b) \( \Omega_e^2 \approx c_1^2 k_F^2 T \) for proximity to the Mott transition yields an upper bound of \( x \approx (c_1/ta)^2(2t/J) \sim 0.01 \) for the hole concentration in this case,\(^{5}\) where we have taken \( \Omega_e^2 \sim 10tx(e^2/d) \), along with model parameters \( t/J \sim 5 \), \( t = 0.5 \text{ eV} \), \( a = 4 \text{ Å} \), and a first sound speed \( c_1 \sim 10 \text{ km/s} \) characteristic of such materials.\(^{21}\) Mean-field treatments of the square-lattice \( t-J \) model find,\(^{9}\) however, that the metallic saddle-points become unstable to a Mott insulator at \( tx \approx 0.02J \). Hence, slow zero-sound should be observable within a window of mobile hole concentrations satisfying \( 0.001 \lesssim x \lesssim 0.01 \) in the oxide superconductors.

In summary, the main result established in this paper is that the slow zero-sound mode that appears in certain spin/charge separated treatments of the problem of strongly interacting electrons in two dimensions near the Mott transition\(^{5,9}\) is observable, in the presence of an ionic background, as the physical sound mode for hole concentrations that satisfy \( \Omega_e \ll \Omega_i \). On the other hand, far from the Mott transition in the conventional
metallic regime, $\Omega_e \gg \Omega_i$, it evolves continuously into the standard Bohm-Staver first-sound mode.\textsuperscript{10} The former result also implies that the sound speed softens completely precisely at the Mott transition, $\Omega_e = 0$, which suggests that a zone-center structural transition will occur in the system. It is useful to compare the present mechanism for electronically driven zone-center structural transitions with those based on strong electron-phonon coupling,\textsuperscript{14} $\rho_F g^2 = 1$, and with mechanisms for zone-edge structural transitions that rely on the appearance of a Peierls distortion in the system.\textsuperscript{22} Last, since the present results rely essentially on the collapse of the Drude weight near the Mott transition, they could well be valid for any strongly interacting Fermi system in two dimensions. Such is the case for example in one dimension, where the velocities for collective charge excitations in the Hubbard model and in the $t-J$ model vanish at half-filling in an identical fashion.\textsuperscript{8}

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References

1. P.W. Anderson, Science 235, 1196 (1987).
2. L.B. Ioffe and A.I. Larkin, Phys. Rev. B 39, 8988 (1989).
3. J.P. Rodriguez and B. Douçot, Europhys. Lett. 11, 451 (1990).
4. N. Nagaosa and P.A. Lee, Phys. Rev. Lett. 64, 2450, (1990); L.B. Ioffe and P.B. Wiegmann, Phys. Rev. Lett. 65, 653 (1990); L.B. Ioffe and G. Kotliar, Phys. Rev. B 42, 10348 (1990).
5. J.P. Rodriguez, Phys. Rev. B 44, 9582 (1991); (E) 45, 5119 (1992).
6. R. Hlubina, W.O. Putikka, T.M. Rice, and D.V. Khveshchenko, Phys. Rev. B46, 11224 (1992).
7. D. Pines and P. Nozières, The Theory of Quantum Liquids, vol. 1 (Addison-Wesley, New York, 1989).
8. H. J. Schulz, Int. J. of Mod. Phys. B5, 57 (1991); M. Ogata, M.U. Luchini, S. Sorella and F.F. Assaad, Phys. Rev. Lett. 66, 2388 (1991).
9. J.P. Rodriguez and B. Douçot, Phys. Rev. B45, 971 (1992).
10. D. Bohm and T. Staver, Phys. Rev. 84, 836 (1950).
11. P.M. Platzman and P.A. Wolff, Solid State Physics (Suppl.) 13 (Academic Press, New York, 1973).
12. V.P. Silin, Zh. Eksp. Teor. Fiz. 35, 1243 (1958) [Sov. Phys. JETP 8, 870 (1959)].
13. L.P. Gorkov and I.E. Dzyaloshinskii, Zh. Eksp. Teor. Fiz. 44, 1650 (1963) [Sov. Phys. JETP 17, 1111 (1963)].
14. A.A. Abrikosov, L.P. Gorkov, and I.E. Dzyaloshinski, Methods of Quantum Field Theory in Statistical Physics, (Dover, New York, 1975).
15. J.P. Rodriguez, unpublished.
16. Y. Suzumara, Y. Hasegawa, and H. Fukuyama, J. Phys. Soc. Jpn. 57, 2768 (1988).
17. N. Nagaosa and P.A. Lee, Phys. Rev. B 45, 966 (1992).
18. J.P. Rodriguez, Phys. Rev. B 49, 3663 and 9831 (1994); J.P. Rodriguez and P. Lederer, Phys. Rev. B 48, 16051 (1993).
19. A. Sokol and D. Pines, Phys. Rev. Lett. 71, 2813 (1993).
20. B. Keimer et al., Phys. Rev. B 46, 14034 (1992).
21. P.B. Allen, Z. Fisk, and A. Migliori, in *Physical Properties of High Temperature Superconductors*, edited by D.M. Ginsberg (World Scientific, Singapore, 1989).

22. S. Barisic and J. Zelanko, *Europhys. Lett.* 8, 765 (1989); R.S. Markiewicz, *J. Phys. Condensed Matter* 2, 6223 (1990).
Figure Caption

Fig. 1. Shown are the two branches, $c_-$ and $c_+$, of the renormalized phonon velocity, $c_s$, as a function of the zero-sound velocity, $c_0$, where the electron-phonon coupling strength is fixed at $\rho_F g^2 = 0.2$ [see eq. (8a)]. The horizontal dashed line represents bare first-sound, $c_s = c_1$, while the diagonal dashed line represents bare zero-sound, $c_s = c_0$. 