Two- and three-gluon glueballs of $C = +$

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We study two- and three-gluon glueballs of $C = +$ using the method of QCD sum rules. We systematically construct their interpolating currents, and find that all the spin-1 currents of $C = +$ vanish. This suggests that the “ground-state” spin-1 glueballs of $C = +$ do not exist within the relativistic framework. We calculate masses of the two-gluon glueballs with $J^{PC} = 0^{\pm +}/2^{\pm +}$ and the three-gluon glueballs with $J^{PC} = 0^{\pm +}/2^{\pm +}$. We propose to search for the $J^{PC} = 0^{\pm +}/2^{\pm \mp}/3^{\pm -}$ three-gluon glueballs in their three-meson decay channels in future BESIII, GlueX, LHC, and PANDA experiments.

Keywords: glueball, pomeron, odderon, exotic hadron, QCD sum rules

I. INTRODUCTION

Glueballs, composed of valence gluons, are important for the understanding of non-perturbative QCD [1–3]. There have been tremendous theoretical studies on them in the past fifty years using various models and methods, such as the MIT bag model [4], the flux-tube model [5], the Coulomb Gauge model [6, 7], Regge trajectories [8], holographic QCD [9], Lattice QCD [10–14], and QCD sum rules [15–30], etc. However, experimental efforts in searching for glueballs are confronted with the difficulty of identifying them unambiguously, and there is currently no definite experimental evidence for their existence.

Recently the D0 and TOTEM Collaborations studied $pp$ and $p\bar{p}$ [31] cross sections, which are found to be different with a significance of $3.4\sigma$ [32]. Together with their previous result [33], this significance can be increased to $5.2\sigma$–$5.7\sigma$. The above difference leads to the evidence of a $t$-channel exchanged odderon [34–38], that is predominantly a three-gluon glueball of $C = -$. We refer to Refs. [39–47] and the review [48] for more discussions. Due to these studies, interests in glueballs are reviving recently. Since the above odderon evidence is still indirect, it is crucial and important to directly study the glueball itself.

The lowest-lying two-, three-, and four-gluon glueballs have been systematically investigated in Ref. [49], where the authors constructed their corresponding non-relativistic low-dimension operators. These operators have been successfully used in Lattice QCD calculations. In this paper we systematically study two- and three-gluon glueballs of $C = +$. We shall construct their corresponding relativistic glueball currents, and calculate masses of these glueballs using the method of QCD sum rules. The same approach has been applied in Ref. [50] to study three-gluon glueballs of $C = -$, so a rather complete QCD sum rule study will be done on the lowest-lying glueballs composed of two or three valence gluons. These studies can largely improve our understanding of the gluon degree of freedom as well as the non-perturbative behaviors of the strong interaction at the low energy region.

This paper is organized as follows. We systematically construct relativistic two- and three-gluon glueball currents of $C = +$ in Sec. II. We apply them to perform QCD sum rule analyses in Sec. III, and perform numerical analyses in Sec. IV. The obtained results are summarized and discussed in Sec. V, which are compared with Lattice QCD results [11–14].

II. RELATIVISTIC GLUEBALL CURRENTS

In this section we systematically construct relativistic glueball currents, including the two-gluon glueball currents and the $C = +$ three-gluon glueball currents. We shall do this separately in the following subsections. Note that the two-gluon glueball currents can not reach $C = -$ [51], and the $C = -$ three-gluon glueball currents have been systematically constructed in Ref. [50].

A. Couplings of tensor currents

In the present study we shall use some special tensor currents to study glueballs with non-zero spins $J \neq 0$. These currents have $2 \times J$ Lorentz indices with certain symmetries, and they couple to both positive- and negative-parity glueballs. In this subsection we briefly explain how we deal with them.

We assume $J_{a\beta}$ to be a tensor current with two anti-symmetric Lorentz indices $\mu$ and $\nu$. Taking the current $J_{a\beta} = \bar{c}\sigma_{a\beta}c$ as an example, it can be separated into $(a, \beta = 0, 1, 2, 3$ and $i, j = 1, 2, 3)$:

$$J_{a\beta} = \bar{c}\sigma_{a\beta}c \rightarrow \begin{cases} \bar{c}\gamma^{i}c, \; P = +, \\ \bar{c}\sigma_{0i}c, \; P = -. \end{cases} \quad (1)$$
Accordingly, it couples to both positive- and negative-parity charmonia through

\[ \langle 0 | J_{\alpha \beta} | h_c(\epsilon, p) \rangle = i f_{h_c}^T \epsilon_{\alpha \beta \mu \nu} \epsilon^{\mu \nu} p', \]  
\[ \langle 0 | J_{\alpha \beta} | J/\psi(\epsilon, p) \rangle = i f_{J/\psi} \epsilon_{\alpha \beta \mu \nu} (p_\alpha \epsilon_\beta - p_\beta \epsilon_\alpha), \]

where \( f_{h_c}^T \) and \( f_{J/\psi} \) are relevant decay constants. Given the Lorentz structures of \( J/\psi \) and \( h_c \), they can be completely separated from each other. For example, we can isolate \( h_c \) at the hadron level by investigating the two-point correlation function containing

\[ \langle 0 | J_{\alpha \beta} | h_c \rangle \langle h_c | J_{\alpha' \beta'}^T | 0 \rangle = (f_{h_c}^T)^2 \epsilon_{\alpha \beta \mu \nu} \epsilon_{\alpha' \beta' \mu' \nu'} p' \]
\[ = 2 (f_{h_c}^T)^2 p^2 \left( g_{\alpha \alpha'} g_{\beta \beta'} - g_{\alpha \beta} g_{\beta' \alpha'} \right) + \cdots, \]

since the correlation function of \( J/\psi \) does not contain the above coefficient. It is not so easy to isolate \( J/\psi \) from \( J_{\alpha \beta} \) at the hadron level. Instead, we can investigate its partner current

\[ \tilde{J}_{\alpha \beta} = \epsilon_{\alpha \beta \gamma \delta} \times J^\gamma \delta, \]

which couples to \( J/\psi \) and \( h_c \) just in the opposite ways:

\[ \langle 0 | \tilde{J}_{\alpha \beta} | J/\psi(\epsilon, p) \rangle = i f_{J/\psi}^T \epsilon_{\alpha \beta \mu \nu} \epsilon^{\mu \nu} p', \]
\[ \langle 0 | \tilde{J}_{\alpha \beta} | h_c(\epsilon, p) \rangle = i f_{h_c}^T \epsilon_{\alpha \beta \mu \nu} (p_\alpha \epsilon_\beta - p_\beta \epsilon_\alpha). \]

Accordingly, we can use the two currents \( J_{\alpha \beta} \) and \( \tilde{J}_{\alpha \beta} \) to study and well separate \( J/\psi \) and \( h_c \).

We apply the above process to generally investigate the current \( J_{\alpha_1 \cdots \alpha_N, \beta_1 \cdots \beta_N} \), which has \( 2N = 2J \) Lorentz indices with certain symmetries, e.g., the spin-2 current \( J_{\alpha_1 \alpha_2 \beta_1 \beta_2} \) has four Lorentz indices, satisfying

\[ J_{\alpha_1 \alpha_2 \beta_1 \beta_2} = -J_{\beta_1 \beta_2, \alpha_1 \alpha_2} = -J_{\alpha_2 \beta_1, \alpha_1 \beta_2} = J_{\alpha_2 \alpha_1, \beta_1 \beta_2}. \]

Its coupling can be written as:

\[ \langle 0 | J_{\alpha_1 \cdots \alpha_N, \beta_1 \cdots \beta_N} | X \rangle = i f x S(\epsilon^{\alpha_1 \alpha_2 \cdots \alpha_N} p_{\alpha_1} \cdots \epsilon^{\beta_1 \beta_2} p_{\beta_1} \cdots) N \epsilon_{\mu_1 \cdots \mu_N}, \]

where \( X \) is the corresponding state having the same parity as \( J_{\alpha_1 \cdots \alpha_N, \beta_1 \cdots \beta_N} \) \((i_1 \cdots i_N = 1, 2, 3)\); \( S \) denotes symmetrization and subtracting trace terms in the two sets \( \{ \alpha_1 \cdots \alpha_N \} \) and \( \{ \beta_1 \cdots \beta_N \} \) simultaneously, with

\[ [\cdots] N = \epsilon^{\alpha_1 \beta_1 \cdots \alpha_N} p_{\alpha_1} \cdots \epsilon^{\beta_1 \cdots \beta_N} p_{\beta_1} \cdots. \]

Note that the current \( J_{\alpha_1 \cdots \alpha_N, \beta_1 \cdots \beta_N} \) can also couple to the other state \( X' \) having the parity opposite to \( X \), but this state \( X' \) can not be easily isolated at the hadron level, so we do not consider it in the present study.

\section*{B. Two-gluon glueball currents}

In this subsection we use the gluon field strength tensor \( G_{\mu \nu} \) to construct two-gluon glueball currents, with \( a \) the color index and \( \mu, \nu \) the Lorentz indices. We also need \( G^{\alpha \mu}_{\rho \nu} = G^a_{\mu \rho} \times \epsilon_{\mu \rho \nu} / 2 \) to denote the dual gluon field strength tensor, and \( f_{abc} \) to denote the totally antisymmetric SU(3)\(_C\) structure constant. In the present study we only consider local glueball currents without explicit derivatives, although \( G^a_{\mu \nu} \) and \( \tilde{G}^a_{\mu \nu} \) contain covariant derivatives inside themselves.

In Ref. [49] the authors use the chromoelectric and chromomagnetic fields \((i, j = 1, 2, 3)\),

\[ E_i = G_{i0} \quad \text{and} \quad B_i = -\frac{1}{2} \epsilon_{ijk} G^{jk}, \]

to write down all the non-relativistic low-dimension two-gluon glueball operators:

\[ 0^{++} \quad \tilde{E}^2_a \pm \tilde{E}^2_a, \]
\[ 0^{++} \quad \tilde{E}_a \times \tilde{B}_a, \]
\[ 1^{-} \quad \tilde{E}_a \times \tilde{B}_a, \]
\[ 2^{++} \quad S'[E^a_b E^b_a \pm B^a_e B^a_e], \]
\[ 2^{++} \quad S'[E^a_b B^a_e - B^b_e E^b_a], \]

where \( S' \) denotes symmetrization and subtracting trace terms in the set \( \{ij\} \).

We construct their corresponding relativistic currents in order to perform QCD sum rule analyses:

\[ J_0 = g^2_{E} G^a_{\mu \nu} C^a_{\mu \nu}, \]
\[ J_0 = g^2_{E} G^a_{\mu \nu} C^a_{\mu \nu}, \]
\[ J^{\alpha \beta}_1 = g^2_{E} G^a_{\mu \nu} C^a_{\mu \nu} - \{\alpha \leftrightarrow \beta\}, \]
\[ J^{\alpha \beta}_2 = S[g^2_{E} G^a_{\mu \nu} G^a_{\mu \nu}], \]
\[ J^{\alpha \beta}_2 = S[g^2_{E} G^a_{\mu \nu} G^a_{\mu \nu}], \]

We shall explicitly prove in Appendix A that the third current \( J^{\alpha \beta}_1 \) vanishes, suggesting that the “ground-state” two-gluon glueball of \( J^{PC} = 1^{++} \) does not exist within the relativistic framework.

The former two currents \( J_0 \) of \( J^{PC} = 0^{++} \) and \( \tilde{J}_0 \) of \( J^{PC} = 0^{++} \) couple to the \( J^{PC} = 0^{++} \) and \( 0^{-} \) two-gluon glueballs \((\text{GG} \}; J^{PC}), \) respectively:

\[ 0^{++} \quad \langle 0 | J_0 | \text{GG} ; 0^{++} \rangle = f_0^{++}, \]
\[ 0^{++} \quad \langle 0 | \tilde{J}_0 | \text{GG} ; 0^{++} \rangle = f_0^{++}, \]

where \( f_0^{++} \) and \( f_0^{++} \) are decay constants. Besides, the current \( J_0 \) has a partner,

\[ J_0 = g^2_{E} G^a_{\mu \nu} C^a_{\mu \nu}, \]

whose sum rule result is the same as that of \( J_0 \).

The latter two currents \( J^{\alpha \beta}_1 \) and \( J^{\alpha \beta}_2 \) couple to the \( J^{PC} = 2^{++} \) and \( 2^{-} \) glueballs through:

\[ \langle 0 | J^a_{2^{++}} | \text{GG} ; 2^{++} \rangle = i f_{2^{++}} S[\epsilon^{\alpha \beta \mu \nu} p_{\mu \nu}]^2 \epsilon_{\mu_1 \mu_2}, \]
\[ \langle 0 | \tilde{J}^a_{2^{-}} | \text{GG} ; 2^{++} \rangle = i f_{2^{++}} S[\epsilon^{\alpha \beta \mu \nu} p_{\mu \nu}]^2 \epsilon_{\mu_1 \mu_2}. \]
The current $J_2^{a_2a_2,β_2β_2}$ also has a partner,
\[ J_2^{a_1a_2,β_1β_2} = S_2 [g_s^2 \tilde{G}^{a_1β_1}_a \tilde{G}^{a_2β_2}_a], \]  
(23)
whose sum rule result is the same as that of $J_2^{a_1a_2,β_1β_2}$.

C. Three-gluon glueball currents of $C = +$

In this subsection we use $G_μν$ and $\tilde{G}_μν$ to construct three-gluon glueball currents of $C = +$. Some of their corresponding non-relativistic operators have been constructed in Ref. [49]:
\[ 0^+ \quad f^{abc}(\vec{E}_a \times \vec{E}_b) \cdot \vec{B}_c, \]
\[ 0^+ \quad f^{abc}(\vec{E}_a \times \vec{E}_b) \cdot \vec{E}_c, \]
\[ 1^+ \quad f^{abc}(\vec{B}_a \cdot \vec{E}_b) \vec{E}_c, \]
\[ 1^+ \quad f^{abc}(\vec{B}_a \cdot \vec{B}_b) \vec{B}_c, \]
\[ 2^+ \quad f^{abc} S'(\vec{B}_a \times \vec{B}_b) \vec{B}_c + \cdots, \]
\[ 2^+ \quad f^{abc} S'(\vec{E}_a \times \vec{E}_b) \vec{E}_c + \cdots. \]  
(24)
We further construct their corresponding relativistic currents as follows:
\[ \eta_0 = f^{abc} g_s^3 G_μα_1 β_1 G_νμ G_ρ_β_2 ; \]
\[ \hat{\eta}_0 = f^{abc} g_s^3 \tilde{G}_μα_1 β_1 G_νμ G_ρ_β_2 ; \]
\[ \eta_1^{αβ} = f^{abc} g_s^3 G_μα_1 β_1 G_νμ G_ρβ_2 , \]
\[ \hat{\eta}_1^{αβ} = f^{abc} \tilde{G}_μα_1 β_1 G_νμ G_ρβ_2 , \]
\[ \eta_2^{α1α_2,β_1β_2} = f^{abc} S[g_s^3 \tilde{G}_μα_1 β_1 G_νμ G_ρβ_2 - \{α_2 \leftrightarrow β_2\}], \]
\[ \hat{\eta}_2^{α1α_2,β_1β_2} = f^{abc} S'[g_s^3 \tilde{G}_μα_1 β_1 G_νμ G_ρβ_2 - \{α_2 \leftrightarrow β_2\}]. \]  
(25)

We shall explicitly prove in Appendix A that the third and fourth currents $\eta_1^{αβ}$ and $\hat{\eta}_1^{αβ}$ both vanish, suggesting that the “ground-state” three-gluon glueballs of $J^{PC} = 1^+$ do not exist within the relativistic framework.

The former two currents $\eta_0$ of $J^{PC} = 0^+$ and $\hat{\eta}_0$ of $J^{PC} = 0^-$ couple to the $J^{PC} = 0^+$ and $0^-$ three-gluon glueballs [GGG; $J^{PC}$], respectively:
\[ \langle 0 | \eta_0 | GGG; 0^{++} \rangle = \int_{s_0} f_0^{++}, \]
\[ \langle 0 | \hat{\eta}_0 | GGG; 0^{-+} \rangle = \int_{s_0} f_0^{-+}. \]
(31)

The latter two currents $\eta_2^{α1α_2,β_1β_2}$ and $\hat{\eta}_2^{α1α_2,β_1β_2}$ couple to the $J^{PC} = 2^+$ and $2^-$ glueballs through:
\[ \langle 0 | \eta_2^{α1α_2,β_1β_2} | GGG; 2^{++} \rangle = i f^{α_2α_2}_2 S [\epsilon^{α_1β_1 μ_1 ν_1} p_{μ_2} ] \epsilon_{μ_1μ_2}, \]
\[ \langle 0 | \hat{\eta}_2^{α1α_2,β_1β_2} | GGG; 2^{−−} \rangle = i f^{α_2α_2}_2 S [\epsilon^{α_1β_1 μ_1 ν_1} p_{μ_2} ] \epsilon_{μ_1μ_2}. \]
(33)

III. QCD SUM RULE ANALYSES

In this section we use the two-gluon glueball currents $J_0$, $\hat{J}_0$, $J_2^{α1α_2,β_1β_2}$, and $\hat{J}_2^{α1α_2,β_1β_2}$ as well as the three-gluon glueball currents $\eta_0$, $\hat{\eta}_0$, $\eta_2^{α1α_2,β_1β_2}$, and $\hat{\eta}_2^{α1α_2,β_1β_2}$ to perform QCD sum rule analyses. This method has been widely applied in the field of hadron phenomenology [52, 53] to study various exotic hadrons [54–56]. Especially, all the above spin-2 currents have four Lorentz indices with certain symmetries, so that they couple to both positive- and negative-parity glueballs simultaneously. We refer to Ref. [50] for detailed discussions.

We take the current $J_0$ defined in Eq. (14) as an example, and calculate its two-point correlation function
\[ \Pi(q^2) \equiv i \int d^4xe^{iqx} \langle 0 | T[J_0(x)\bar{J}_0(0)] | 0 \rangle, \]  
(35)
separately at hadron and quark-gluon levels.

At the hadron level we express Eq. (35) using the dispersion relation as
\[ \Pi(q^2) = \int_{s_0}^{\infty} \frac{\rho(s)}{s - q^2 - i\varepsilon} ds, \]  
(36)
with $\rho(s) = \text{Im}\Pi(s)/\pi$ the spectral density. It is parameterized using one pole dominance for the ground state $X$ as well as the continuum contribution,
\[ \rho(s) = \sum_n \delta(s - M_n^2) \langle 0 | J_0(n) | n, J_0 \rangle \]
\[ = f_X^2 \delta(s - M_X^2) + \text{continuum}. \]  
(37)

At the quark-gluon level we insert Eq. (14) into Eq. (35), and calculate it using the method of operator product expansion (OPE). After performing the Borel transformation to Eq. (36) at both hadron and quark-gluon levels, we approximate the continuum using the spectral density above a threshold value $s_0$ and obtain
\[ \Pi(s_0, M_B^2) \equiv f_X^2 e^{-M_X^2/M_B^2} \int_{s_0} ds \rho(s) ds. \]  
(38)
This equation can be used to further calculate the mass of $X$ through
\[ M_X^2(s_0, M_B) = \int_{s_0} e^{-s/M_B^2} sp(s) ds \int_{s_0} e^{-s/M_B^2} \rho(s) ds. \]  
(39)

Since the gluon field strength tensor $G_{μν}^a$ is defined as
\[ G_{μν}^a = \partial_μ A_{ν}^a - \partial_ν A_{μ}^a + g_s f^{abc} A_{μ}^b A_{ν}^c, \]  
(40)
it can be naturally separated into two parts. As shown in Fig. 1, we depict the former two terms using the single-gluon-line, and the latter one term using the double-gluon-line with a red vertex (see also the diagram
The gluon field strength tensor $G_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} + g_s f^{abc} A_{\mu} A_{b,\nu}$, naturally separated into two parts (a) and (b).

Fig. 2(c–3). Here $A_{\mu}$ is the gluon field, whose propagator is [57]:

$$\langle 0 | T[A_{\mu}(x)A_{\nu}^{\dagger}(y)]0 \rangle = \frac{\delta^{ab} g_{\mu\nu}}{4\pi^2(x-y)^2}$$

$$+ \frac{g_s}{8\pi^2} \ln(-x-y) f^{abc} G_{c,\mu\nu}(0)$$

$$- \frac{g_s}{8\pi^2} \epsilon^\alpha \epsilon^\beta \epsilon^\gamma f^{abc} G_{c,\alpha\beta}(0).$$

We work in the fixed-point gauge so that

$$A_{\mu}(x) \approx -\frac{1}{2} \epsilon^\nu g^\alpha_{\mu}(0).$$

In the present study we consider the Feynman diagrams depicted in Fig. 2 (for three-gluon glueballs), and calculate OPEs up to the dimension eight $(D = 8)$ condensates. We take into account the perturbative term, the two-gluon condensate $(g_s^2 G G)$, the three-gluon condensate $(g_s^2 G^3)$, and the $D = 8$ condensate $(g_s^2 G G)^2$:

$$\Pi_{GGG;0^{++}}(s_0, M_B^2)$$

$$= \int_{s_0}^{s} \left( 32a^2 s^2 + 60a_s^2 (g_s^2 GG) \right) e^{-s/s_B} ds$$

$$+ 24\pi a_s (g_s^2 G^3),$$

$$\Pi_{GGG;2^{++}}(s_0, M_B^2)$$

$$= \int_{s_0}^{s} \left( 2a^2 s^2 - 5a_s^2 (g_s^2 GG) \right) e^{-s/s_B} ds$$

$$+ \frac{\pi a_s (g_s^2 G^3)}{3},$$

$$\Pi_{GGG;0^{-+}}(s_0, M_B^2)$$

$$= \int_{s_0}^{s} \left( 32a^2 s^2 - 40a_s (g_s^2 G^3) \right) e^{-s/s_B} ds,$$

$$\Pi_{GGG;2^{-+}}(s_0, M_B^2)$$

$$= \int_{s_0}^{s} \left( 2a^2 s^2 + \frac{a_s (g_s^2 GG)}{12} \right) e^{-s/s_B} ds$$

$$- \frac{\pi a_s (g_s^2 G^3)}{2}.$$
\[ \xi_2^{\alpha_1\alpha_2,\beta_1\beta_2} = d^{abc}S[g_a^3\tilde{c}^{\alpha_1\beta_1}G_{a,\mu}G_{b,\nu}^2G_{c,\rho}^2 - \{\alpha_2 \leftrightarrow \beta_2\}], \]
\[ \xi_2^{\tilde{c}} = d^{abc}S[g_a^3\tilde{c}^{\alpha_1\beta_1}G_{a,\mu}G_{b,\nu}^2G_{c,\rho}^2], \]
\[ \xi_3^{\tilde{c}} = d^{abc}S[g_a^3\tilde{c}^{\alpha_1\beta_1}G_{a,\mu}G_{b,\nu}^2G_{c,\rho}^2], \]
where \( d^{abc} \) is the totally symmetric \( SU(3)_C \) structure constant. Their sum rule equations are:

\[ \Pi_{(GGG;1-)}(s_0, M_B^2) = \int_0^{s_0} \left( 4\alpha_3^2 \frac{s^4 + 10\alpha_2^2(g_2^2GG)^2}{81\pi} s^2 + \frac{5\alpha_2^2(g_3^2G^3)^2}{36\pi} s \right) e^{-s/M_B^2} ds, \]
\[ \Pi_{(GGG;2-)}(s_0, M_B^2) = \int_0^{s_0} \left( \frac{\alpha_3^2}{324\pi} s^4 - \frac{5\alpha_2^2(g_2^2GG)^2}{108} s^2 + \frac{15\alpha_2^2(g_2^2GG)^2}{128\pi} s \right) e^{-s/M_B^2} ds, \]
\[ \Pi_{(GGG;3-)}(s_0, M_B^2) = \int_0^{s_0} \left( \frac{\alpha_3^2}{2016\pi} s^4 - \frac{5\alpha_2^2(g_2^2GG)^2}{512\pi} s^2 - \frac{\alpha_2^2(g_3^2G^3)^2}{2} s \right) e^{-s/M_B^2} ds, \]

\[ \Pi_{(GGG;1-)}(s_0, M_B^2) = \int_0^{s_0} \left( \frac{5\alpha_3^2}{2016\pi} s^4 + \frac{\alpha_2^2(g_2^2GG)^2}{16} s^2 \right) e^{-s/M_B^2} ds, \]
\[ \Pi_{(GGG;2-)}(s_0, M_B^2) = \int_0^{s_0} \left( \frac{4\alpha_3^2}{81\pi} s^4 - \frac{10\alpha_2^2(g_2^2GG)^2}{9} s^2 + \frac{25\alpha_2^2(g_2^2GG)^2}{36\pi} s \right) e^{-s/M_B^2} ds, \]
\[ \Pi_{(GGG;3-)}(s_0, M_B^2) = \int_0^{s_0} \left( \frac{\alpha_3^2}{324\pi} s^4 + \frac{5\alpha_2^2(g_2^2GG)^2}{108} s^2 + \frac{15\alpha_2^2(g_2^2GG)^2}{128\pi} s \right) e^{-s/M_B^2} ds, \]
\[ \Pi_{(GGG;1-)}(s_0, M_B^2) = \int_0^{s_0} \left( \frac{5\alpha_3^2}{2016\pi} s^4 + \frac{\alpha_2^2(g_2^2GG)^2}{16} s^2 \right) e^{-s/M_B^2} ds, \]
\[ \Pi_{(GGG;2-)}(s_0, M_B^2) = \int_0^{s_0} \left( \frac{4\alpha_3^2}{81\pi} s^4 - \frac{10\alpha_2^2(g_2^2GG)^2}{9} s^2 + \frac{25\alpha_2^2(g_2^2GG)^2}{36\pi} s \right) e^{-s/M_B^2} ds, \]
\[ \Pi_{(GGG;3-)}(s_0, M_B^2) = \int_0^{s_0} \left( \frac{\alpha_3^2}{324\pi} s^4 + \frac{5\alpha_2^2(g_2^2GG)^2}{108} s^2 + \frac{15\alpha_2^2(g_2^2GG)^2}{128\pi} s \right) e^{-s/M_B^2} ds, \]

The above three-gluon glueball currents of \( C = - \) have...
been systematically studied in Ref. [50], but there we did not calculate the Feynman diagrams depicted in Figs. 2(c – 3, c – 4, c – 5). Similar to Eqs. (43)-(50), we find all the \( D = 8 \) terms proportional to \( (g^2_{\text{GG}})^2 \) vanish, so the convergence of the above OPE series are also quite good.

We shall use the above sum rules to perform numerical analyses in the next section.

**IV. NUMERICAL ANALYSES**

In this section we perform numerical analyses using the sum rules given in Eqs. (43)-(50) and Eqs. (57)-(62). The glueball mass \( M_X \) depends significantly on the gluon condensates \( (g^2_{\text{GG}}) \) and \( (g^3_{\text{GG}}) \), both of which are still not precisely known. In the present study we use the following values for these parameters [58, 59]:

\[
\begin{align*}
\langle \alpha_s GG \rangle &= (6.35 \pm 0.35) \times 10^{-2} \text{ GeV}^4, \\
\langle g^3_{\text{GG}} \rangle &= \langle \alpha_s GG \rangle \times (8.2 \pm 1.0) \text{ GeV}^2 .
\end{align*}
\]

Besides, we use the following value for the strong coupling constant at the QCD scale \( \Lambda_{\text{QCD}} = 300 \) MeV [60]:

\[
\alpha_s(Q^2) = \frac{4\pi}{11 \ln(Q^2/\Lambda_{\text{QCD}}^2)} .
\]

As shown in Fig. 3 using the solid curve, we determine the lower limit of \( M_B \) to be \( M_B^2 \geq 3.28 \text{ GeV}^2 \) when setting \( s_0 = 9.0 \) GeV.

The above condition is the cornerstone of a reliable sum rule analysis, where we have taken into account two terms because the OPE is expanded in two directions: the dimension of condensates and the coupling constant \( \alpha_s \). Eqs. (65) and (66) are for two-gluon glueball currents, and the conditions for three-gluon glueball currents are

\[
\begin{align*}
\text{CVG}_A' &= \frac{\Pi^{\alpha=6}(s_0, M_B^0)}{\Pi(s_0, M_B^0)} \leq 5\% , \\
\text{CVG}_B' &= \frac{\Pi^{\alpha=6}(s_0, M_B^0)}{\Pi(s_0, M_B^0)} \leq 10\% .
\end{align*}
\]

As shown in Fig. 3 using the dashed curves, we determine the upper limit of \( M_B \) to be \( M_B^2 \leq 3.70 \text{ GeV}^2 \) when setting \( s_0 = 9.0 \) GeV.

Altogether we determine the Borel window to be \( 3.28 \text{ GeV}^2 \leq M_B^2 \leq 3.70 \text{ GeV}^2 \) for the fixed threshold value \( s_0 = 9.0 \) GeV. Then we redo the same procedures by changing \( s_0 \), and find that there exist non-vanishing Borel windows as long as \( s_0 \geq s_0^{\text{min}} = 8.2 \text{ GeV}^2 \). Accordingly, we choose \( s_0 \) to be slightly larger, and determine our working regions to be \( 8.0 \text{ GeV}^2 \leq s_0 \leq 10.0 \text{ GeV}^2 \) and \( 3.28 \text{ GeV}^2 \leq M_B^2 \leq 3.70 \text{ GeV}^2 \), where we calculate the mass of \( \left| \text{GG};0^{-+} \right| \) to be

\[
M_{\text{GG};0^{-+}} = 2.17 \pm 0.11 \text{ GeV} .
\]

Its central value corresponds to \( M_B^2 = 3.49 \text{ GeV}^2 \) and \( s_0 = 9.0 \) GeV, and its uncertainty comes from the threshold value \( s_0 \), the Borel mass \( M_B \), and the gluon condensates listed in Eqs. (63).

We show \( M_{\text{GG};0^{-+}} \) in the left panel of Fig. 4 as a function of the Borel mass \( M_B \), and find it quite stable inside the Borel window \( 3.28 \text{ GeV}^2 \leq M_B^2 \leq 3.70 \text{ GeV}^2 \). We also show it in the right panel of Fig. 4 as a function of the threshold value \( s_0 \). We find there exists a mass minimum around \( s_0 \approx 5 \text{ GeV}^2 \), and the \( s_0 \) dependence is weak and acceptable inside the working region \( 8.0 \text{ GeV}^2 \leq s_0 \leq 10.0 \text{ GeV}^2 \).

Similarly, we use the sum rules given in Eqs. (43)-(50) and Eqs. (57)-(62) to perform numerical analyses, and
calculate masses of two- and three-gluon glueballs systematically. The obtained results are summarized in Table I, where we choose threshold values \( s_0 \) for two-gluon glueballs to be around \( 9 \sim 10 \text{ GeV}^2 \), and those for three-gluon glueballs to be around \( 33 \sim 38 \text{ GeV}^2 \).

**V. SUMMARY AND DISCUSSIONS**

In this paper we study two- and three-gluon glueballs of \( C = + \) using the method of QCD sum rules, including

- the two-gluon glueballs with the quantum numbers \( J^{PC} = 0^{\pm +}, 1^{-+}, \) and \( 2^{\pm +} \);

- the three-gluon glueballs with the quantum numbers \( J^{PC} = 0^{\pm +}, 1^{\pm +}, \) and \( 2^{\pm +} \).

We systematically construct their interpolating currents, and find that all the spin-1 currents of \( C = + \) vanish, suggesting that the “ground-state” spin-1 glueballs of \( C = + \) do not exist within the relativistic framework. This behavior is consistent with Lattice QCD calculations [11–14].

We use spin-0 and spin-2 glueball currents to perform QCD sum rule analyses, and calculate masses of their corresponding spin-0 and spin-2 glueballs. All these spin-2

| Glueball | Current | \( s_0^{\text{min}} \) [GeV^2] | Working Regions | Pole [%] | Mass [GeV] |
|----------|---------|------------------|-----------------|---------|-------------|
| \(|\text{GG}; 0^{++}\)| \( J_0 \) | 7.8 | 9.0 ± 1.0 | 3.70–4.19 | 40–48 | 1.78 ± 0.14 |
| \(|\text{GG}; 2^{++}\)| \( J_2^{(0,2,3,1,2,3)} \) | 8.5 | 10.0 ± 1.0 | 3.99–4.60 | 40–50 | 1.86 ± 0.17 |
| \(|\text{GG}; 0^{-+}\)| \( J_0 \) | 8.2 | 9.0 ± 1.0 | 3.28–3.70 | 40–47 | 2.17 ± 0.11 |
| \(|\text{GG}; 2^{-+}\)| \( J_2^{(0,2,3,1,2,3)} \) | 8.1 | 10.0 ± 1.0 | 3.27–4.20 | 40–55 | 2.24 ± 0.11 |
| \(|\text{GGG}; 0^{++}\)| \( \eta_0 \) | 31.6 | 33.0 ± 3.0 | 7.25–7.61 | 40–44 | 4.46 ± 0.17 |
| \(|\text{GGG}; 2^{++}\)| \( \eta_2^{(0,2,3,1,2,3)} \) | 16.0 | 35.0 ± 3.0 | 4.77–9.04 | 40–90 | 4.18 ± 0.19 |
| \(|\text{GGG}; 0^{-+}\)| \( \tilde{\eta}_0 \) | 17.0 | 33.0 ± 3.0 | 4.48–8.13 | 40–88 | 4.13 ± 0.18 |
| \(|\text{GGG}; 2^{-+}\)| \( \tilde{\eta}_2^{(0,2,3,1,2,3)} \) | 33.1 | 35.0 ± 3.0 | 8.10–8.53 | 40–44 | 4.29 ± 0.20 |
| \(|\text{GGG}; 1^{++}\)| \( \xi_0^{\alpha \beta} \) | 9.0 | 34.0 ± 4.0 | 3.16–9.09 | 40–99 | 4.01 ± 0.26 |
| \(|\text{GGG}; 2^{++}\)| \( \xi_2^{(0,2,3,1,2,3)} \) | 32.7 | 35.0 ± 4.0 | 7.53–8.09 | 40–46 | 4.42 ± 0.24 |
| \(|\text{GGG}; 3^{++}\)| \( \xi_3^{(0,2,3,1,2,3)} \) | 30.2 | 33.0 ± 4.0 | 7.69–8.40 | 40–47 | 4.30 ± 0.23 |
| \(|\text{GGG}; 1^{-+}\)| \( \tilde{\xi}_0 \) | 31.2 | 34.0 ± 4.0 | 5.81–6.77 | 40–51 | 4.91 ± 0.20 |
| \(|\text{GGG}; 2^{-+}\)| \( \tilde{\xi}_2^{(0,2,3,1,2,3)} \) | 19.7 | 36.0 ± 4.0 | 5.80–9.47 | 40–81 | 4.25 ± 0.33 |
| \(|\text{GGG}; 3^{-+}\)| \( \tilde{\xi}_3^{(0,2,3,1,2,3)} \) | 35.8 | 38.0 ± 4.0 | 6.15–7.22 | 40–49 | 5.59 ± 0.33 |

FIG. 4: Mass of the two-gluon glueball \(|\text{GG}; 0^{++}\rangle\) as a function of the Borel mass \( M_B \) (left) and the threshold value \( s_0 \) (right), calculated using the current \( J_0 \). In the left panel the short-dashed/solid/long-dashed curves are obtained by setting \( s_0 = 8.0/9.0/10.0 \text{ GeV}^2 \), respectively. In the right panel the short-dashed/solid/long-dashed curves are obtained by setting \( M_B^2 = 3.28/3.49/3.70 \text{ GeV}^2 \), respectively.
 currents have four Lorentz indices with certain symmetries, so that they couple to both positive- and negative-parity glueballs, which need to be further separated at the hadron level. We refer to Ref. [50] for detailed discussions.

We summarize the obtained results in Table II, which are compared with the Lattice QCD results obtained using non-relativistic glueball operators [11–14]. For completeness, we also reanalyze the results of $C = -$ three-gluon glueballs (also called as odderon), which have been previously studied in Ref. [50]. Therefore, a rather complete QCD sum rule study have been done on the lowest-lying glueballs composed of two or three valence gluons. We find that our QCD sum rule results are generally consistent with unquenched Lattice QCD results [14].

To end this paper, we briefly discuss possible decay patterns of two- and three-gluon glueballs. The two-gluon glueballs can decay after exciting two quark-antiquark pairs, and recombine into two mesons. However, it is rather difficult to differentiate them from standard $q\bar{q}$ states. The three-gluon glueballs can decay after exciting three quark-antiquark pairs, and recombine into three mesons. Their possible decay patterns are:

$$
\begin{align*}
0^{--} & \rightarrow VVP, VVV \quad (S\text{-wave}), \\
0^{++} & \rightarrow VPP, VVP, VVV \quad (P\text{-wave}), \\
1^{--} & \rightarrow VPP, VVP, VVV \quad (S\text{-wave}), \\
1^{+-} & \rightarrow PPP, VPP, VVP, VVV \quad (P\text{-wave}), \\
2^{-+} & \rightarrow VPP, VVV \quad (S\text{-wave}), \\
2^{++} & \rightarrow VPP, VVP, VVV \quad (P\text{-wave}), \\
3^{--} & \rightarrow VVV \quad (S\text{-wave}), \\
3^{+-} & \rightarrow VVP, VVV \quad (P\text{-wave}),
\end{align*}
$$

where $P$ and $V$ denote light vector and pseudoscalar mesons, respectively. Considering their limited decay patterns, the $J^{PC} = 0^{--}/2^{-+}/3^{+-}$ three-gluon glueballs may have relatively smaller widths, and we propose to search for them in their $VVV$ and $VP$ decay channels in future BESIII, GlueX, LHC, and PANDA experiments.

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**Appendix A: Spin-1 currents of $C = +$**

In this appendix we prove that the three spin-1 currents $J_1^{\alpha\beta}$, $\eta_1^{\alpha\beta}$, and $\tilde{\eta}_1^{\alpha\beta}$ all vanish. Their definitions are given in Eqs. (15), (27), and (28), respectively. For simplicity, we shall not differentiate the superscript and subscript in the following calculations.

Firstly, we investigate the current $J_1^{\alpha\beta}$. Due to the Lorentz invariant, we simply assume $\alpha = 0$ and $\beta = 1 \cdots 3$; besides, we need the Lorentz indices $\mu = 0/i$, $\rho = 0/k$, and $\sigma = 0/l$, with $i/k/l = 1 \cdots 3$. We obtain:

$$
\begin{align*}
2J_1^{\alpha\beta} &= 2G_0^{\alpha\mu}G_0^{\beta\mu} - \{0 \leftrightarrow \beta\}, \\
&= G_0^{\alpha\mu}G_0^{\beta\rho}G_0^{\mu\rho} - G_0^{\beta\mu}G_0^{\rho\sigma}G_0^{\mu\sigma} \\
&= G_0^{\alpha\mu}G_0^{\beta\kappa}G_0^{\rho\kappa} + G_0^{\alpha\mu}G_0^{\beta\rho}G_0^{\kappa\sigma} - G_0^{\beta\mu}G_0^{\kappa\iota}G_0^{\rho\iota}. \\
\end{align*}
$$

(A1)

After interchanging $i \leftrightarrow k$, the first term turns out to be zero:

$$
G_0^{\alpha\mu}G_0^{\beta\kappa}G_0^{\rho\kappa} = G_0^{\alpha\mu}G_0^{\beta\rho}G_0^{\kappa\sigma} = G_0^{\beta\mu}G_0^{\kappa\iota}G_0^{\rho\iota} \rightarrow 0. \\
$$

(A2)

So does the second term. The third term is non-zero when $\beta = k$ or $\beta = l$. However, for the case $\beta = k$, we can interchange $i \leftrightarrow l$ and obtain (not sum over $\beta$ here):

$$
G_0^{\alpha\mu}G_0^{\beta\rho}G_0^{\kappa\sigma} = G_0^{\beta\rho}G_0^{\kappa\iota}G_0^{\sigma\iota} \rightarrow 0. \\
$$

(A3)

So does the case $\beta = l$. Therefore, the third term is also zero, and the current $J_1^{\alpha\beta}$ vanishes.

Secondly, we investigate the current $\eta_1^{\alpha\beta}$:

$$
\begin{align*}
2\eta_1^{\alpha\beta} &= 2f_{abc}G_0^{\alpha\mu}G_0^{\beta\nu}G_0^{\gamma\rho} - \{\alpha \leftrightarrow \beta\}, \\
&= f_{abc}G_0^{\mu\rho}G_0^{\nu\sigma}G_0^{\gamma\rho}G_0^{\alpha\beta} \\
&= f_{abc}G_0^{\mu\rho}G_0^{\nu\kappa}G_0^{\beta\kappa}G_0^{\alpha\beta} \\
&= -f_{abc}G_0^{\mu\rho}G_0^{\nu\kappa}G_0^{\beta\kappa}G_0^{\alpha\beta} \\
&\rightarrow 0. \\
\end{align*}
$$

(A4)

In the above expressions, we have consequently interchanged $\mu\nu \leftrightarrow \rho\sigma$ and $a \leftrightarrow b$. Similarly, we can prove the current $\tilde{\eta}_1^{\alpha\beta}$ to be zero.

One can construct more spin-1 three-gluon glueball currents of $C = +$, such as:

$$
\begin{align*}
\eta_1^{\alpha\beta} &= f_{abc}G_0^{\alpha\mu}G_0^{\beta\nu}G_0^{\mu\rho} - \{\alpha \leftrightarrow \beta\}, \\
\tilde{\eta}_1^{\alpha\beta} &= f_{abc}G_0^{\alpha\mu}G_0^{\mu\nu}G_0^{\beta\rho} - \{\alpha \leftrightarrow \beta\}. \\
\end{align*}
$$

(A5)

(A6)

It is straightforward to prove the former current $\eta_1^{\alpha\beta}$ to be zero:

$$
\begin{align*}
\eta_1^{\alpha\beta} &= f_{abc}G_0^{\alpha\mu}G_0^{\beta\nu}G_0^{\mu\rho} - \{\alpha \leftrightarrow \beta\} \\
&= f_{abc}G_0^{\alpha\mu}G_0^{\beta\nu}G_0^{\mu\rho} - \{\alpha \leftrightarrow \beta\} \\
&= -f_{abc}G_0^{\beta\nu}G_0^{\mu\rho}G_0^{\alpha\mu} + \{\alpha \leftrightarrow \beta\} \\
&= f_{abc}G_0^{\beta\nu}G_0^{\mu\rho}G_0^{\alpha\mu} - \{\alpha \leftrightarrow \beta\} \\
&\rightarrow 0. \\
\end{align*}
$$

(A7)
TABLE II: Masses of two- and three-gluon glueballs, in units of GeV. Our QCD sum rule results are listed in the 2nd column. Lattice QCD results are listed in the 3rd-6th columns, taken from Refs. [11–13] (quenched) and Ref. [14] (unquenched).

| Glueball | QCD sum rules | Ref. [11] | Ref. [12] | Ref. [13] | Ref. [14] |
|----------|---------------|------------|------------|------------|------------|
| GG; 0^{++} | 1.78^{+0.14}_{-0.17} | 1.71 ± 0.05 ± 0.08 | 1.73 ± 0.05 ± 0.08 | 1.48 ± 0.03 ± 0.07 | 1.80 ± 0.06 |
| GG; 2^{++} | 1.86^{+0.14}_{-0.17} | 2.39 ± 0.03 ± 0.12 | 2.40 ± 0.03 ± 0.12 | 2.15 ± 0.03 ± 0.10 | 2.62 ± 0.05 |
| GG; 0^{-+} | 2.17^{+0.11}_{-0.11} | 2.56 ± 0.04 ± 0.12 | 2.59 ± 0.04 ± 0.13 | 2.25 ± 0.06 ± 0.10 | – |
| GG; 2^{-+} | 2.24^{+0.11}_{-0.11} | 3.04 ± 0.04 ± 0.15 | 3.10 ± 0.03 ± 0.15 | 2.78 ± 0.05 ± 0.13 | 3.46 ± 0.32 |
| GGG; 0^{++} | 4.46^{+0.17}_{-0.19} | – | 2.67 ± 0.18 ± 0.13 | 2.76 ± 0.03 ± 0.12 | 3.76 ± 0.24 |
| GGG; 2^{++} | 4.18^{+0.19}_{-0.42} | – | – | 2.88 ± 0.10 ± 0.13 | – |
| GGG; 0^{-+} | 4.13^{+0.18}_{-0.36} | – | 3.64 ± 0.06 ± 0.18 | 3.37 ± 0.15 ± 0.15 | 4.49 ± 0.59 |
| GGG; 2^{-+} | 4.29^{+0.20}_{-0.22} | – | – | 3.48 ± 0.14 ± 0.16 | – |
| GGG; 1^{+-} | 4.01^{+0.26}_{-0.95} | 2.98 ± 0.03 ± 0.14 | 2.94 ± 0.03 ± 0.14 | 2.67 ± 0.07 ± 0.12 | 3.27 ± 0.34 |
| GGG; 2^{-+} | 4.42^{+0.24}_{-0.29} | 4.23 ± 0.05 ± 0.20 | 4.14 ± 0.05 ± 0.20 | – | – |
| GGG; 3^{-0} | 4.30^{+0.23}_{-0.26} | 3.60 ± 0.04 ± 0.17 | 3.55 ± 0.04 ± 0.17 | 3.27 ± 0.09 ± 0.15 | 3.85 ± 0.35 |
| GGG; 1^{--} | 4.91^{+0.20}_{-0.18} | 3.83 ± 0.04 ± 0.19 | 3.85 ± 0.05 ± 0.19 | 3.24 ± 0.33 ± 0.15 | – |
| GGG; 2^{--} | 4.25^{+0.22}_{-0.33} | 4.01 ± 0.05 ± 0.20 | 3.93 ± 0.04 ± 0.19 | 3.66 ± 0.13 ± 0.17 | 4.59 ± 0.74 |
| GGG; 3^{--} | 5.59^{+0.30}_{-0.32} | 4.20 ± 0.05 ± 0.20 | 4.13 ± 0.09 ± 0.20 | 4.33 ± 0.26 ± 0.20 | – |

It is a bit tricky but one can still prove the latter current \( J^{a}_{\mu \nu} \) to be zero, after explicitly writing out all its Lorentz indices. We have done this using the software Mathematica.

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