TEV NEUTRINO PHYSICS AT THE LARGE HADRON COLLIDER

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I argue that TeV neutrino physics might become an exciting frontier of particle physics in the era of the Large Hadron Collider (LHC). The origin of non-zero but tiny masses of three known neutrinos is probably related to the existence of some heavy degrees of freedom, such as heavy Majorana neutrinos or heavy Higgs bosons, via a TeV-scale seesaw mechanism. I take a few examples to illustrate how to get a balance between theoretical naturalness and experimental testability of TeV seesaws. Besides possible collider signatures at the LHC, new and non-unitary CP-violating effects are also expected to show up in neutrino oscillations for type-I, type-(I+II) and type-III seesaws at the TeV scale.

Keywords: TeV seesaws; Collider signatures; Violation of unitarity; Neutrino oscillations.

1. Why TeV Seesaws?

Enrico Fermi elaborated a coherent theory of the beta decay and published it in La Ricerca Scientifica in December 1933 just two months after the Solvay Congress in October 1933. In this seminal paper, Fermi postulated the existence of a new force for the beta decay by combining three brand-new concepts — Pauli’s neutrino hypothesis, Dirac’s idea about the creation of particles, and Heisenberg’s idea that the neutron was related to the proton. Today, we have achieved a standard theory of electroweak interactions at the Fermi scale (∼ 100 GeV), although it is unable to tell us much about the intrinsic physics of electroweak symmetry breaking and the origin of non-zero but tiny neutrino masses. We are expecting that the LHC will soon bring about a revolution in particle physics at the TeV scale (∼ 1000 GeV) — a new energy frontier that we humans have never reached before within a laboratory. Can the LHC help solve the puzzle of neutrino mass generation? We do not yet know the answer to this question. But let us hope so. I personally foresee that TeV neutrino physics might become an exciting direction in the era of the LHC.

Among many theoretical and phenomenological ideas towards understanding why the masses of three known neutrinos are so small [2] the seesaw picture seems to be most natural and elegant. Its key point is to ascribe the smallness of neutrino masses to the existence of some new degrees of freedom heavier than the Fermi scale.
such as heavy Majorana neutrinos or heavy Higgs bosons. Three typical seesaw mechanisms are illustrated in Fig. 1, and some other variations or combinations are possible. The energy scale where a seesaw mechanism works is crucial, because it is relevant to whether this mechanism is theoretically natural and experimentally testable. Between Fermi and Planck scales, there might exist two other fundamental scales: one is the scale of a grand unified theory (GUT) at which strong, weak and electromagnetic forces can be unified, and the other is the TeV scale at which the unnatural gauge hierarchy problem of the standard model (SM) can be solved or at least softened by new physics. Many theorists argue that the conventional seesaw scenarios are natural because their scales (i.e., the masses of heavy degrees of freedom) are close to the GUT scale. If the TeV scale is really a fundamental scale, may we argue that the TeV seesaws are natural? In other words, we are reasonably motivated to speculate that possible new physics existing at the TeV scale and responsible for the electroweak symmetry breaking might also be responsible for the origin of neutrino masses. It is interesting and meaningful in this sense to investigate and balance the “naturalness” and “testability” of TeV seesaws at the energy frontier set by the LHC.

2. Naturalness and Testability

As shown in Fig. 1, the type-I seesaw mechanism gives a natural explanation of the smallness of neutrino masses by introducing three heavy right-handed Majorana
neutrinos, while the type-II seesaw mechanism is to extend the SM by including one $SU(2)_L$ Higgs triplet. One may in general combine the two mechanisms by assuming the existence of both the Higgs triplet and right-handed Majorana neutrinos, leading to a “hybrid” seesaw scenario which will be referred to as the type-(I+II) seesaw mechanism. The gauge-invariant neutrino mass terms in such a type-(I+II) seesaw model can be written as

$$-L_{\text{mass}} = \sum_l Y_L H N_R + \frac{1}{2} m_R N_R + \frac{1}{2} \sum_l Y_{\Delta} \Delta_i \sigma_2 l_L^c + \text{h.c.},$$

where $M_R$ is the mass matrix of right-handed Majorana neutrinos, and

$$\Delta = \begin{pmatrix} H^- & \sqrt{2} H^0 \\ \sqrt{2} H^0 & -H^- \end{pmatrix}$$

denotes the $SU(2)_L$ Higgs triplet. After spontaneous gauge symmetry breaking, we obtain the neutrino mass matrices $M_D = Y_{\nu}/\sqrt{2}$ and $M_L = Y_{\Delta} v_{\Delta}$, where $\langle H \rangle \equiv v/\sqrt{2}$ and $\langle \Delta \rangle \equiv v_{\Delta}$ correspond to the vacuum expectation values of the neutral components of $H$ and $\Delta$. Then Eq. (1) can be rewritten as

$$-L'_{\text{mass}} = \frac{1}{2} (\nu_L N_R) \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} (\nu_L^c N_R^c) + \text{h.c.}.$$  

The $6 \times 6$ neutrino mass matrix in Eq. (3) is symmetric and can be diagonalized by the following unitary transformation:

$$\begin{pmatrix} V R \\ S U \end{pmatrix}^\dagger \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} V R \\ S U \end{pmatrix}^* = \begin{pmatrix} \widetilde{M}_\nu & 0 \\ 0 & \widetilde{M}_N \end{pmatrix},$$

where $\widetilde{M}_\nu = \text{Diag}\{m_1, m_2, m_3\}$ with $m_i$ being the masses of three light neutrinos $\nu_i$ and $\widetilde{M}_N = \text{Diag}\{M_1, M_2, M_3\}$ with $M_i$ being the masses of three heavy neutrinos $N_i$. Note that $V^\dagger V + S^\dagger S = VV^\dagger + RR^\dagger = 1$ holds as a consequence of the unitarity of this transformation. Hence $V$, the flavor mixing matrix of three light neutrinos, must be non-unitary if $R$ and $S$ are non-zero.

### 2.1. Type-I seesaw

The type-I seesaw scenario can be obtained from Eqs. (1)—(4) by switching off the Higgs triplet. In this case, $M_L = 0$ and $R \sim S \sim M_D/M_R$ hold, leading to the approximate seesaw formula

$$M_\nu \equiv V \widetilde{M}_\nu V^T \approx -M_D M_R^{-1} M_D^T.$$  

The deviation of $V$ from unitarity is measured by $RR^\dagger/2$ and has been neglected in this expression. Let us consider two interesting possibilities:

1. $M_D \sim \mathcal{O}(10^2)$ GeV and $M_R \sim \mathcal{O}(10^{15})$ GeV to get $M_\nu \sim \mathcal{O}(10^{-2})$ eV. In this conventional and natural case, $R \sim S \sim \mathcal{O}(10^{-13})$ holds. Hence the non-unitarity of $V$ is only at the $\mathcal{O}(10^{-26})$ level, too small to be observed.
(2) $M_D \sim \mathcal{O}(10^2)$ GeV and $M_R \sim \mathcal{O}(10^3)$ GeV to get $M_\nu \sim \mathcal{O}(10^{-2})$ eV. In this unnatural case, a significant “structural cancellation” has to be imposed on the textures of $M_D$ and $M_R$. Because of $R \sim S \sim \mathcal{O}(0.1)$, the non-unitarity of $V$ can reach the percent level and may lead to some observable effects.

Now let us discuss how to realize the above “structural cancellation” for the type-I seesaw mechanism at the TeV scale. Taking the flavor basis of $M_R = \tilde{M}_N$, one may easily show that $M_\nu$ in Eq. (5) vanishes if

$$M_D = m \begin{pmatrix} y_1 & y_2 & y_3 \\ \alpha y_1 & \alpha y_2 & \alpha y_3 \\ \beta y_1 & \beta y_2 & \beta y_3 \end{pmatrix} \text{ and } \sum_{i=1}^{3} \frac{y_i^2}{M_i} = 0$$

simultaneously hold. Tiny neutrino masses can be generated from tiny corrections to the texture of $M_D$ in Eq. (6). For example, $M_D' = M_D - \epsilon X_D$ with $M_D$ given above and $\epsilon$ being a small dimensionless parameter (i.e., $|\epsilon| \ll 1$) will yield

$$M_\nu' \approx -M_D' M_R^{-1} M_D^{T} \approx \epsilon \left( M_D M_R^{-1} X_D^T + X_D M_R^{-1} M_T \right),$$

from which $M_\nu' \sim \mathcal{O}(10^{-2})$ eV can be obtained by adjusting the size of $\epsilon$. We learn the following lessons from this simple exercise:

- Two necessary conditions must be satisfied in order to test a type-I seesaw model at the LHC: (a) $M_i$ are of $\mathcal{O}(1)$ TeV or smaller; and (b) the strength of light-heavy neutrino mixing (i.e., $M_D/M_R$) are large enough. Otherwise, it would be impossible to produce and detect $N_i$ at the LHC.
- The collider signatures of $N_i$ are essentially decoupled from the mass and mixing parameters of three light neutrinos $\nu_i$. For instance, the small parameter $\epsilon$ in Eq. (7) has nothing to do with the ratio $M_D/M_R$.
- The non-unitarity of $V$ might lead to some observable effects in neutrino oscillations and other lepton-flavor/number-violating processes, provided $M_D/M_R \lesssim \mathcal{O}(0.1)$ holds. More discussions will be given later.
- The clean LHC signatures of heavy Majorana neutrinos are the $\Delta L = 2$ like-sign dilepton events such as $pp \rightarrow W^{*\pm} W^{*\pm} \rightarrow \mu^{\pm}\mu^{\pm}jj$ (a collider analogue to the neutrinoless double-beta decay) and $pp \rightarrow W^{*\pm} \rightarrow \mu^{\pm}N_i \rightarrow \mu^{\pm}\mu^{\pm}jj$ (a dominant channel due to the resonant production of $N_i$).

Some naive numerical calculations of possible LHC events for a single heavy Majorana neutrino have been done in the literature, but they only serve for illustration because such a minimal version of the type-I seesaw scenario is actually unrealistic.

### 2.2. Type-II seesaw

The type-II seesaw scenario can be obtained from Eqs. (1)—(4) by switching off the right-handed Majorana neutrinos and taking account of a simple potential of the Higgs doublet and triplet:

$$V = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 + \frac{1}{2} M_\Delta^2 \text{Tr} (\Delta^\dagger \Delta) - [\lambda_\Delta M_\Delta H^T i\sigma_2 \Delta H + \text{h.c.}] .$$

(8)
When the neutral components of $H$ and $\Delta$ acquire their vacuum expectation values $v$ and $v_\Delta$, respectively, the electroweak gauge symmetry is spontaneously broken. The minimum of $V$ is achieved at $v = \mu / (\lambda - 2\lambda_\Delta^2)^{1/2}$ and $v_\Delta = \lambda_\Delta v^2 / M_\Delta$, where $\lambda_\Delta$ has been assumed to be real. Note that $v_\Delta$ may modify the SM masses of $W^\pm$ and $Z^0$ in such a way that $\rho \equiv M_W^2 / (M_Z^2 \cos^2 \theta_W) = (v^2 + 2v_\Delta^2) / (v^2 + 4v_\Delta^2)$ holds. By using current experimental data on the $\rho$-parameter, we get $\kappa \equiv \sqrt{2} v_\Delta / v < 0.01$ and $v_\Delta < 2.5$ GeV. Given $M_\Delta \gg v$, an approximate seesaw formula turns out to be

$$M_\nu \equiv M_L = Y_\Delta v_\Delta \approx \lambda_\Delta Y_\Delta v^2 / M_\Delta,$$

(9)
as shown in Fig. 1. Note that the last term of Eq. (8) violates both $L$ and $B - L$, and thus the smallness of $\lambda_\Delta$ is naturally allowed according to 't Hooft’s naturalness criterion (i.e., setting $\lambda_\Delta = 0$ will increase the symmetry of the theory). Given $M_\Delta \approx \mathcal{O}(1)$ TeV, for example, the seesaw works to generate $M_\nu \sim \mathcal{O}(10^{-2})$ eV provided $\lambda_\Delta Y_\Delta \sim \mathcal{O}(10^{-12})$ holds. The neutrino mixing matrix $V$ is exactly unitary in the type-II seesaw mechanism, simply because the heavy degrees of freedom do not mix with the light ones.

There are totally seven physical Higgs bosons in this model: doubly-charged $H^{++}$ and $H^{--}$, singly-charged $H^\pm$ and $H^-$, neutral $A^0$ (CP-odd), and neutral $h^0$ (CP-even), where $h^0$ is the SM-like Higgs boson. Except for $M_{H^\pm}^2 \approx 2\mu^2$, we get a quasi-degenerate mass spectrum for other scalars: $M_{H^{\pm \pm}}^2 = M_{H^0}^2 \approx M_{A^0}^2$, $M_{H^{\pm \pm}}^2 = M_{H^0}^2 (1 + \kappa^2)$, and $M_{A^0}^2 = M_{H^0}^2 (1 + 2\kappa^2)$. As a consequence, the decay channels $H^{\pm \pm} \rightarrow W^\pm H^\pm$ and $H^{\pm \pm} \rightarrow H^\pm H^\pm$ are kinematically forbidden. The production of $H^{\pm \pm}$ at the LHC is mainly through $q\bar{q} \rightarrow \gamma^* Z^* \rightarrow H^{\pm \pm} H^0$ and $q\bar{q} \rightarrow W^* \rightarrow H^{\pm \pm} H^\pm$ processes, which do not depend on the small Yukawa couplings.

The typical collider signatures in this seesaw scenario are the lepton-number-violating $H^{\pm \pm} \rightarrow l_\alpha^+ l_\beta^-$ decays as well as $H^\pm \rightarrow l_\alpha^+ l_\beta^-$ and $H^- \rightarrow l_\alpha^- v$ decays. Their branching ratios

$$\mathcal{B}(H^{\pm \pm} \rightarrow l_\alpha^+ l_\beta^-) = \frac{(2 - \delta_{\alpha\beta}) |(M_L)_{\alpha\beta}|^2}{\sum_{\rho,\sigma} |(M_L)_{\rho\sigma}|^2}, \quad \mathcal{B}(H^\pm \rightarrow l_\alpha^+ \nu) = \frac{\sum_{\beta} |(M_L)_{\alpha\beta}|^2}{\sum_{\rho,\sigma} |(M_L)_{\rho\sigma}|^2} \tag{10}$$

are closely related to the masses, flavor mixing angles and CP-violating phases of three light neutrinos, because $M_L = V \tilde{M}_{L} V^T$ holds. Some numerical analyses of such decay modes together with the LHC signatures of $H^{\pm \pm}$ and $H^\pm$ bosons have been done by a number of authors (see, e.g., Refs. 8 and 9).

### 2.3. Type-($I+II$) seesaw

A type-($I+II$) seesaw mechanism can be achieved by combining the neutrino mass terms in Eq. (1) with the Higgs potential in Eq. (8). The seesaw formula is

$$M_\nu \equiv V \tilde{M}_\nu V^T \approx M_L - M_D M_R^{-1} M_D^T \tag{11}$$
in the leading-order approximation, where the small deviation of $V$ from unitarity has been omitted and the expression of $M_L$ can be found in Eq. (9). Hence the type-I and type-II seesaws can be regarded as two extreme cases of the type-(I+II) seesaw. Note that two mass terms in Eq. (11) are possibly comparable in magnitude. If both of them are small, their contributions to $M_\nu$ should essentially be constructive; but if both of them are large, their contributions to $M_\nu$ must be destructive. The latter case unnaturally requires a significant cancellation between two big quantities in order to obtain a small quantity, but it is interesting in the sense that it may give rise to observable collider signatures of heavy Majorana neutrinos.

Let me briefly describe a type-(I+II) seesaw model and comment on its possible LHC signatures. First, we assume that both $M_i$ and $M_\Delta$ are of $O(1)$ TeV. Then the production of $H^{\pm\pm}$ and $H^\pm$ bosons at the LHC is guaranteed, and their lepton-number-violating signatures will probe the Higgs triplet sector of the type-(I+II) seesaw mechanism. On the other hand, $O(M_D/M_R) \lesssim O(0.1)$ is possible as a result of $O(M_R) \sim O(1)$ TeV and $O(M_D) \lesssim O(\nu)$, such that appreciable signatures of $N_i$ can be achieved at the LHC. Second, the small mass scale of $M_\nu$ implies that the relation $O(M_\nu) \sim O(M_D M_R^{-1} M_0^2)$ must hold. In other words, it is the significant but incomplete cancellation between $M_\nu$ and $M_D M_R^{-1} M_0^2$ terms that results in the non-vanishing but tiny masses for three light neutrinos. We admit that dangerous radiative corrections to two mass terms of $M_\nu$ require a delicate fine-tuning of the afore-mentioned cancellation.

But this scenario allows us to reconstruct $M_L$ via the excellent approximation $M_L = V \bar{M}_N V^T + R \bar{M}_N R^T \approx R \bar{M}_N R^T$, such that the elements of the Yukawa coupling matrix $Y_\Delta$ read

$$
(Y_\Delta)_{\alpha\beta} = \frac{(M_L)_{\alpha\beta}}{v_\Delta} \approx \sum_{i=1}^{3} \frac{R_{\alpha i} R_{\beta i} M_i}{v_\Delta},
$$

where the subscripts $\alpha$ and $\beta$ run over $e$, $\mu$ and $\tau$. This result implies that the leptonic decays of $H^{\pm\pm}$ and $H^\pm$ bosons depend on both $R$ and $M_i$, which actually determine the production and decays of $N_i$. Thus we have established an interesting correlation between the singly- or doubly-charged Higgs bosons and the heavy Majorana neutrinos. To observe the correlative signatures of $H^\pm$, $H^{\pm\pm}$ and $N_i$ at the LHC will serve for a direct test of this type-(I+II) seesaw model.

To illustrate, here I focus on the minimal type-(I+II) seesaw model with a single heavy Majorana neutrino where $R$ can be parametrized in terms of three rotation angles $\theta_{1i}$ and three phase angles $\delta_{1i}$ (for $i = 1, 2, 3$). In this case, we have

$$
\omega_1 = \frac{\sigma(pp \to \mu^+ \mu^- W^- X)|_{N_i}}{\sigma(pp \to \mu^+ \mu^- H^0 X)|_{H^{++}}} \approx \frac{\sigma_N}{\sigma_H} \frac{s_{14}^2 + s_{24}^2 + s_{34}^2}{4},
$$

$$
\omega_2 = \frac{\sigma(pp \to \mu^+ \mu^- W^- X)|_{N_i}}{\sigma(pp \to \mu^+ \mu^- H^0 X)|_{H^{++}}} \approx \frac{\sigma_N}{\sigma_{\text{pair}}} \frac{s_{14}^2 + s_{24}^2 + s_{34}^2}{4},
$$

for $s_{1i} \equiv \sin \theta_{1i} \lesssim O(0.1)$, where $\sigma_N \equiv \sigma(pp \to l_i^+ N_i X)/|R_{1i}|^2$, $\sigma_H \equiv \sigma(pp \to H^{++} X)$ and $\sigma_{\text{pair}} \equiv \sigma(pp \to H^{++} H^- X)$ are three reduced cross sections.
Fig. 2 illustrates the numerical results of $\omega_1$ and $\omega_2$ changing with $M_1$ at the LHC with an integrated luminosity of 300 fb$^{-1}$, just to give one a ball-park feeling of possible collider signatures of $N_1$ and $H^{\pm\pm}$ and their correlation in our model.

3. Unitarity Violation

It is worth emphasizing that the charged-current interactions of light and heavy Majorana neutrinos are not completely independent in either the type-I seesaw or the type-(I+II) seesaw. The standard charged-current interactions of $\nu_i$ and $N_i$ are

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \left[ (e \mu \tau)_L V^\mu_L \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} W^\mu_L + (e \mu \tau)_L R^\mu \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_L W^\mu_L \right] + \text{h.c.,} \quad (14)$$

where $V$ is just the light neutrino mixing matrix responsible for neutrino oscillations, and $R$ describes the strength of charged-current interactions between $(e, \mu, \tau)$ and $(N_1, N_2, N_3)$. Since $V$ and $R$ belong to the same unitary transformation done in Eq. (4), they must be correlated with each other and their correlation signifies an important relationship between neutrino physics and collider physics.

It has been shown that $V$ and $R$ share nine rotation angles ($\theta_{i4}$, $\theta_{i5}$ and $\theta_{i6}$ for $i = 1, 2$ and 3) and nine phase angles ($\delta_{i4}$, $\delta_{i5}$ and $\delta_{i6}$ for $i = 1, 2$ and 3)\[14\] To see
this point clearly, let me decompose $V$ into $V = AV_0$, where

$$
V_0 = \begin{pmatrix}
  c_{12}c_{13} & \hat{s}_{12}c_{13} & \hat{s}_{13} \\
  -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}_{23} & c_{12}c_{23} - \hat{s}_{12}\hat{s}_{13}\hat{s}_{23} & c_{13}\hat{s}_{23} \\
  \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} & \hat{s}_{12}\hat{s}_{13}\hat{s}_{23} + c_{13}\hat{s}_{13} \\
\end{pmatrix}
$$

(15)

with $c_{ij} \equiv \cos \theta_{ij}$ and $\hat{s}_{ij} \equiv e^{i\delta_{ij}} \sin \theta_{ij}$ is just the standard parametrization of the $3 \times 3$ unitary neutrino mixing matrix (up to some proper phase rearrangements).

Because of neutrino oscillation data and precision electroweak data, we expect to be a small effect (at most at the percent level as constrained by current neutrino oscillation experiments) with medium-baseline neutrino oscillation experiments.

Induced by the non-unitarity of $V$, oscillations may serve as a good tool to probe possible signatures of CP violation.

Denominator of Eq. (17) will become unity and the conventional formula of $P_{\alpha\beta}$ will be reproduced. It has been observed in Refs. 14 and 18 that $3$ unitary neutrino mixing matrix (up to some proper phase rearrangements).

Because of the neutrino beam energy and the baseline length. If $V$ is exactly unitary (i.e., $A = 1$ and $V = V_0$), the denominator of Eq. (17) will become unity and the conventional formula of $P_{\alpha\beta}$ will be reproduced. It has been observed in Refs. 14 and 18 that $\nu_\mu \to \nu_\tau$ and $\bar{\nu}_\mu \to \bar{\nu}_\tau$ oscillations may serve as a good tool to probe possible signatures of CP violation induced by the non-unitarity of $V$.

The probability of $\nu_\alpha \to \nu_\beta$ oscillations in vacuum, defined as $P_{\alpha\beta}$, is given by:

$$
P_{\alpha\beta} = \frac{\sum_i |V_{\alpha i}|^2 |V_{\beta i}|^2 + 2 \sum_{i<j} \text{Re} (V_{\alpha i}V_{\beta j}^* V_{\alpha j}^* V_{\beta i}^*) \cos \Delta_{ij} - 2 \sum_{i<j} J^{ij}_{\alpha\beta} \sin \Delta_{ij}}{(V V^\dagger)_{\alpha\alpha} (V V^\dagger)_{\beta\beta}},
$$

(17)

where $\Delta_{ij} \equiv \Delta m_{ij}^2 L/(2E)$ with $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$, $E$ being the neutrino beam energy and $L$ being the baseline length. If $V$ is exactly unitary (i.e., $A = 1$ and $V = V_0$), the denominator of Eq. (17) will become unity and the conventional formula of $P_{\alpha\beta}$ will be reproduced. It has been observed in Refs. 14 and 18 that $\nu_\mu \to \nu_\tau$ and $\bar{\nu}_\mu \to \bar{\nu}_\tau$ oscillations may serve as a good tool to probe possible signatures of CP violation induced by the non-unitarity of $V$. To illustrate this point, we consider a short- or medium-baseline neutrino oscillation experiment with $|\sin \Delta_{13}| \sim |\sin \Delta_{23}| \gg |\sin \Delta_{12}|$, in which the terrestrial matter effects are expected to be insignificant or negligibly small. Then the dominant CP-conserving and CP-violating terms of $P(\nu_\mu \to \nu_\tau)$ and $P(\bar{\nu}_\mu \to \bar{\nu}_\tau)$ can simply be obtained from Eq. (17):

$$
P(\nu_\mu \to \nu_\tau) \approx \sin^2 \theta_{23} \sin^2 \frac{\Delta_{23}}{2} - 2 \left(J^{23}_{\mu\tau} + J^{13}_{\mu\tau}\right) \sin \Delta_{23},$$

$$
P(\bar{\nu}_\mu \to \bar{\nu}_\tau) \approx \sin^2 \theta_{23} \sin^2 \frac{\Delta_{23}}{2} + 2 \left(J^{23}_{\mu\tau} + J^{13}_{\mu\tau}\right) \sin \Delta_{23},$$

(18)
where the good approximation $\Delta_{13} \approx \Delta_{23}$ has been used in view of the experimental fact $|\Delta m^2_{13}| \approx |\Delta m^2_{23}| \gg |\Delta m^2_{12}|$, and the sub-leading and CP-conserving “zero-distance” effect has been omitted. For simplicity, I take $V_0$ to be the exactly tri-bimaximal mixing pattern (i.e., $\theta_{12} = \arctan(1/\sqrt{2})$, $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ as well as $\delta_{12} = \delta_{13} = \delta_{23} = 0$) and then arrive at

$$2 (J_{\mu\tau}^{23} + J_{\mu\tau}^{13}) \approx \sum_{l=4}^{6} s_{2l} s_{3l} \sin (\delta_{2l} - \delta_{3l}).$$

(19)

Given $s_{2l} \sim s_{3l} \sim O(0.1)$ and $(\delta_{2l} - \delta_{3l}) \sim O(1)$ (for $l = 4, 5, 6$), this non-trivial CP-violating quantity can reach the percent level. A numerical illustration of the CP-violating asymmetry between $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ oscillations has been presented in Ref. [14] from which one can see that it is possible to measure this asymmetry in the range $L/E \sim (100 \cdots 400)$ km/GeV if the experimental sensitivity is $\leq 1\%$. A neutrino factory with the beam energy $E$ being above $m_\tau \approx 1.78$ GeV may have a good chance to explore the non-unitary effect of CP violation.

When a long-baseline neutrino oscillation experiment is concerned, however, the terrestrial matter effects must be taken into account because they might fake the genuine CP-violating signals. As for $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ oscillations discussed above, the dominant matter effect results from the neutral-current interactions

$$2 (J_{\mu\tau}^{23} + J_{\mu\tau}^{13}) = \sum_{l=4}^{6} s_{2l} s_{3l} \sin (\delta_{2l} - \delta_{3l}) + A_{NC} L \cos (\delta_{2l} - \delta_{3l}) \cos (\delta_{2l} - \delta_{3l}),$$

(20)

where $A_{NC} = G_F N_n / \sqrt{2}$ with $N_n$ being the background density of neutrons, and $L$ is the baseline length. It is easy to find $A_{NC} L \sim O(1)$ for $L \sim 4 \times 10^3$ km.

4. Concluding Remarks

We hope that the LHC might open a new window for us to understand the origin of neutrino masses and the dynamics of lepton number violation. To be more specific, a TeV seesaw might work (naturalness?) and its heavy degrees of freedom might show up at the LHC (testability?). A bridge between collider physics and neutrino physics is highly anticipated and, if it exists, will lead to rich phenomenology.

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