Effective field theory and electro-weak processes

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Abstract. Heavy baryon chiral perturbation theory is applied to one- and two nucleon processes.

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Introduction

A careful and systematic study of low-energy weak- and strong interaction reactions is desirable in order to enhance our understanding of some fundamental astro-physical processes. Since low energy processes are insensitive to details of the short distance structures of the hadrons, we can make use of an effective field theory like Chiral Perturbation Theory (ChPT), which allows a unified approach to weak- and strong interaction processes. The ChPT Lagrangian, which reflects the symmetries and the symmetry breaking pattern of the underlying theory of QCD, also gives a model-independent, gauge-invariant evaluation of radiative QED corrections to these reactions.

We know that the QCD lagrangian is chirally symmetric provided the $u$ and $d$ quarks are massless. Furthermore, it is established that chiral symmetry is spontaneously broken, which implies the existence of massless Goldstone Bosons (pions). The quarks have non-zero masses which however are small compared to the QCD scale, $m_u \simeq m_d \ll \Lambda_{QCD}$. Therefore, a perturbative treatment of the explicit chiral symmetry breaking appears reasonable. In ChPT these considerations are reflected in the hadronic scale, $\Lambda_{ch} \simeq 1 \text{ GeV} \simeq m_N$, being much larger than the corresponding pion mass $m_\pi (\propto \sqrt{m_{\text{quark}}}) \ll \Lambda_{ch}$. ChPT assumes that we consider only low-energy reactions which only allow low momentum probes. As a result we will consider the following (perturbative) expansion parameter in Heavy Baryon Chiral Perturbation Theory (HBChPT): $Q/\Lambda_{ch} \ll 1$, where $Q$ denotes either the typical 4-momentum involved in the process under consideration, or $m_\pi$.

The HBChPT Lagrangian

$L_{ch}$ is written as an expansion in powers of $Q/\Lambda_{ch}$, see e.g. the reviews [1, 2]

$L_{ch} = L^{(1)}_{\pi N} + L^{(2)}_{\pi N} + L^{(2)}_{\pi \pi} + L^{(3)}_{\pi N} + \cdots$

where $L^{(v)}$ contains terms of order $(Q/\Lambda_{ch})^v$. We assume that the terms in the lowest order Lagrangian give the dominant contributions to a process. The higher order terms...
presumably give smaller perturbative corrections. In HBChPT the pions are treated relativistically, whereas the nucleons are treated non-relativistically. In reality we have two simultaneous expansion parameters, \((Q/A_{\text{ch}}) \nu\) and \((Q/m_N) \nu\), which for pragmatic purposes are considered simultaneously.

*The lowest order pion Lagrangian* is:

\[
\mathcal{L}^{(2)}_{\pi\pi} = \frac{f_\pi^2}{4} \text{Tr} \left\{ \nabla_\mu U^\dagger \nabla^\mu U + \chi^\dagger U + \chi U^\dagger \right\},
\]

where \(\nabla_\mu U = \partial_\mu U - i(v_\mu + a_\mu) U + iU(v_\mu - a_\mu)\). Here \(v_\mu\) and \(a_\mu\) are external currents, and \(\chi \propto \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}\). In the evaluations of specific processes the \(U\)-field is expanded in powers of the pion field:

\[
U = uu = \exp \left( i \vec{\tau} \cdot \vec{\phi} / f_\pi \right) \simeq 1 + \frac{i \vec{\tau} \cdot \vec{\phi}}{f_\pi} + \cdots.
\]

This expansion gives the familiar first two terms in \(\mathcal{L}^{(2)}_{\pi\pi}\):

\[
\mathcal{L}^{(2)}_{\pi\pi} = \frac{1}{2} \left( \partial_\mu \vec{\phi} \right)^2 - \frac{1}{2} m_\pi^2 \vec{\phi}^2 + \cdots
\]

*The lowest order heavy nucleon Lagrangian* is:

\[
\mathcal{L}^{(1)}_{\pi N} = \bar{N} \left\{ i (v \cdot D) + g_A (S \cdot u) \right\} N,
\]

where \(D^\mu = \partial^\mu + \frac{1}{2} [u^\dagger, \partial^\mu u] - \frac{i}{2} u^\dagger (v^\mu + a^\mu) u - \frac{i}{2} u (v^\mu - a^\mu) u^\dagger\). If we choose the nucleon velocity \(v^\mu = (1, \vec{0})\), then the nucleon spin is: \(S^\mu = (0, \frac{1}{2} \vec{\sigma})\). By again expanding the \(U\)-field we find the following first three terms:

\[
\mathcal{L}^{(1)}_{\pi N} = \bar{N} \left\{ i \frac{\partial}{\partial t} - \frac{\vec{\tau} \cdot (\vec{\phi} \times \vec{\phi})}{4f_\pi^2} + \frac{g_A}{2f_\pi} \vec{\tau} \cdot (\vec{\sigma} \cdot \nabla \vec{\phi}) \right\} N + \cdots
\]

In effective field theory the lagrangian contains low energy constants (LECs), which parametrize the short-distance physics not probed at long wave-lengths. In principle a LEC should be evaluated from QCD but in practice LECs are determined by reproducing the experimental values of appropriate observables. The nucleon axial coupling constant, \(g_A \simeq 1.27\), in Eq. (2) is an example of a LEC. Once the LECs are determined the theory has predictive power.

*In the next order heavy nucleon Lagrangian* with the expanded \(U\)-field,

\[
\mathcal{L}^{(2)}_{\pi N} = \bar{N} \left\{ \frac{(v \cdot \partial)^2 - \partial^2}{2m_N} + \cdots \right\} N,
\]

we display only the heavy nucleon kinetic operator (“the Schrödinger kinetic operator“) \(\frac{\vec{\phi}^2}{2m_N}\). This nucleon kinetic operator is a “recoil” correction to the leading terms; in other words the heavy nucleon expansion is different from the Foldy-Wouthuysen expansion as discussed in [3]. In the following we will give some examples of one- and two-nucleon electroweak processes which have been evaluated in HBChPT.
Specific processes

The following one-nucleon processes, ordinary muon capture: $\mu^- + p \rightarrow \nu_\mu + n$ (OMC), radiative muon capture: $\mu^- + p \rightarrow \nu_\mu + n + \gamma$ (RMC), and the radiative corrections to $n \rightarrow p + e^- + \nu_e$ and $\bar{\nu}_e + p \rightarrow e^+ + n$ (the CHOOZ process), have all been investigated in HBChPT. Since in all these weak-interaction processes the momentum transfers are small $Q \ll m_W$, the effective interaction lagrangian is the “Fermi” Lagrangian:

$$\mathcal{L}_{\text{Fermi}} = \frac{G_F}{\sqrt{2}} J_\beta(\text{lepton}) \cdot J^\beta(\text{hadron})$$

where $J_\beta(\text{lepton}) = \bar{u}_\nu \gamma_\beta (1 - \gamma_5) u_l$ and $J_\beta(\text{hadron}) = v_\beta^{\text{had}} - a_\beta^{\text{had}}$. Traditionally the hadronic currents $v_\beta^{\text{had}}$ and $a_\beta^{\text{had}}$ are written as:

$$v_\beta^{\text{had}} = \bar{\Psi} \left\{ G_V(q^2) \gamma_\beta + G_M(q^2) \frac{i\sigma_\beta \cdot q}{2m_N} + \text{2nd class} \right\} \Psi$$

$$a_\beta^{\text{had}} = \bar{\Psi} \left\{ G_A(q^2) \gamma_\beta \gamma_5 + G_P(q^2) \frac{q_\beta \cdot q_5}{2m_N} + \text{2nd class} \right\} \Psi.$$

When we expand the nucleon form-factors including the $q^2$ terms, the LECs are determined by the nucleon’s r.m.s. radius, axial radius, anomalous nucleon magnetic moments, i.e. $G_M(q^2) = \kappa_p - \kappa_n$, and the Goldberger-Treiman discrepancy. The pseudoscalar form factor is derived in ChPT (including one-loop corrections) and found to be

$$\frac{G_P(q^2)}{2m_N} = -\frac{2f_\pi g_{\pi NN}}{q^2 - m_\pi^2} - \frac{1}{3} g_A m_N < r_A^2 >,$$  

where the values of all parameters in Eq.(3) have been determined from other reactions. This expression for $G_P(q^2)$ was derived some time ago by Adler and Dothan [4] and Wolfenstein [5]. N. Kaiser used HBChPT to show that the next order corrections to $G_P$ are very small [6]. One challenge exists: Can $G_P(q^2)$ be measured in some process in order to confirm this theoretical prediction?

Two processes can determine $G_P$, OMC and RMC. The $\mu^- p$ capture rate has recently been measured at PSI by Andreev et al. [7]. Instead of the standard liquid Hydrogen target they [7] used a gas target in order to minimize the molecular complications in the capture process, see e.g. Refs. [8, 9]. The initial results are consistent with the ChPT prediction. Forthcoming final experimental results are expected at 1% accuracy. The radiative muon capture has the advantage that $q$ changes with the photon energy, $E_\gamma$, meaning RMC could determine $G_P(q^2)$ via the pion-pole dominance of Eq.(3). A TRIUMF team was able to measure the extremely low RMC rate, $d\Gamma/dE_\gamma$, for photon energies $E_\gamma > 60$ MeV [10, 11]. It was a big surprise that the RMC experimental results disagreed with the HBChPT prediction.
The advantage of the systematic ChPT expansion can be illustrated by the following order by order expression for the $\mu^- p$ spin-singlet capture rate taken from Ref. [12]:

$$\Gamma = \left(957 - \frac{245 \text{GeV}}{m_N} + \left[\frac{30.4 \text{GeV}^2}{m_N^2} - 43.17\right]\right) \text{s}^{-1}$$

The near cancellation of the two terms in the square bracket, originating from “recoil” ($1/m_N^2$) and $q^2$ form-factor contributions, testifies to the value of the systematic perturbative expansion of HBChPT.

The radiative corrections to neutron $\beta$-decay and the CHOOZ process are of critical importance since in the coming decade the processes $n \rightarrow p + e^- + \bar{\nu}_e$ and $\bar{\nu}_e + p \rightarrow e^+ + n$ will be measured very precisely. The precise measurements of neutron $\beta$-decay aim at an accurate value for $V_{ud}$. To extract $V_{ud}$ requires an updated understanding of the radiative corrections (RC). The second reaction, the CHOOZ process [13], will be used to determine neutrino oscillation parameters. Why a new investigation of these RC? A systematic reevaluation of RC [14] to the CHOOZ process is possible within HBChPT, which allows a model-independent, gauge-invariant evaluation of RC. The short distance physics is again well defined in the HBChPT lagrangian by the radiative LECs, which are determined in, e.g., neutron beta-decay RC evaluation [15].

The two-nucleon processes to be discussed are connected to fundamental astrophysical reactions; muon capture on the deuteron: $\mu^- + d \rightarrow \nu_\mu + n + n$, the charged- and neutral currents (CC and NC) of the Sudbury Neutrino Observatory (SNO) reactions: $\nu_e + d \rightarrow e^- + p + p$ and $\nu_x + d \rightarrow \nu_x + p + n$, and the radiative pion capture on the deuteron: $\pi^- + d \rightarrow \gamma + n + n$ or the crossing symmetric process $\gamma + d \rightarrow \pi^+ + n + n$. The ChPT evaluation of these reactions include one unknown axial two-nucleon LEC, $\hat{d}^R$, which also enters in the evaluations of the following few-nucleon reactions [16]: triton $\beta$-decay: $^3H \rightarrow^3He + e^+ + \nu_e$, solar $pp$ fusion: $p + p \rightarrow d + e^+ + \nu_e$, the solar Hep process: $^3He + p \rightarrow^4He + e^+ + \nu_e$, and the modern three-nucleon potential ($\hat{d}^R$ is related to $c_{D_3}$, one of the two unknown LEC parameters in the ChPT-derived three-nucleon potential [17]). The Hep process produces the highest energy solar neutrinos and has to be carefully evaluated [18] in order to extract accurately the $^8$Be solar neutrino spectrum detected at, e.g., SuperKamiokande and SNO. A precise evaluation of Hep is however difficult since leading contributions almost cancel as discussed in e.g. [19].

Ideally the two-nucleon reactions should be evaluated using transition operators and nucleon wave functions obtained from ChPT. For pragmatic reasons however a hybrid ChPT called $EFT^*$ has been used in the two- and more nucleon processes. In $EFT^*$ we use the one- and two-nucleon transition operators from ChPT, whereas the nuclear wave functions are evaluated using modern “high precision” $NN$ potentials $V_{NN}$, e.g., Argonne $V_{18}$, CD-Bonn, $V_{low-k}$, etc. In $EFT^*$ calculations a Gaussian cut-off $\Lambda_G$ was introduced in the nuclear wave functions in order to limit the contributions from the high momentum components of the wave functions generated by $V_{NN}$. These high momentum components in the nuclear wave functions generated by, e.g., the Argonne $V_{18}$ potential, go beyond the relevant limited momentum range of ChPT, $Q^2 \ll \Lambda_{ch}$. This Gaussian cut-off procedure is therefore in accordance with one of the principal assumptions of
ChPT allowing only a limited low $Q^2$ range. As a consequence however the axial two-nucleon LEC will $\hat{d}^R$ depend on $\Lambda_G$. The observables should be independent of this momentum cut-off, and we find that the measurable rates and cross-sections have less than 1% variations for $500 \text{ MeV} < \Lambda_G < 800 \text{ MeV}$.

Presently $\hat{d}^R$ is is determined from tritium $\beta$-decay rate. It is however desirable to avoid the complexity of a three-nucleon system in determining the two-nucleon axial coupling $\hat{d}^R$, so that two-nucleon processes can be calculated self-consistently within the framework of ChPT. Avoiding the inherent uncertainties of the three-nucleon system will also allow a more reliable evaluation of the uncertainties involved in two-nucleon reactions. The rate of muon capture on a deuteron ($\mu^- d$) is being measured (2009-2011) at PSI by the MuSun collaboration with a projected error of 1.5% [20]. We are presently re-evaluating our $\mu^- d$ ChPT calculation to match this expected experimental precision. Once the $\mu^- d$ capture rate is accurately measured, the following three reaction can be evaluated model independently with the same accuracy: (i) the solar pp fusion reaction, the primary energy source in the sun, (ii) the SNO neutrino-deuteron reactions, which provided convincing evidence for neutrino oscillation, and (iii) the reaction $\pi^- + d \rightarrow \gamma + n + n$ [21] or $\gamma + d \rightarrow \pi^+ + n + n$ [22] which can be used to determine the neutron-neutron scattering length $a_{nn}$, see the review [23] for a discussion. Furthermore, one of the LEC in the three-nucleon potential, $c_D$, which is an axial two-nucleon LEC, is determined once the value of $\hat{d}^R$ is fixed by the $\mu^- d$ capture reaction. In other words, only one unknown three-nucleon LEC, $c_E$, remains in the ChPT three-nucleon potential.

The expected accurate measurements of the $\mu^- p$ and $\mu^- d$ capture rates will require a re-examination of the radiative corrections to these two processes. The estimated radiative corrections are larger than the expected experimental errors from the MuCap and MuSun collaborations and a renewed evaluation of the RC is in progress.

**Supernova Explosion**

Computer simulations of the supernova have not been very successful in generating the explosion. This is possibly due to the neutrino luminosity being too small. We have identified new processes which generate neutrinos in the proto-neutron star at the center of the supernova explosion. These reactions might affect the explosion-simulation due to an (estimated) increased in the neutrino flux [24].

**Conclusions**

The low-energy effective theory, ChPT, allows a systematic evaluation of electro-weak and strong interaction processes. HBChPT predicts accurately the analytic expression for $G_P(q^2)$. The predicted value for $G_P$ in the $\mu^- p$ process is confirmed by recent MuCap data [7]. The published MuCap experimental $\mu^- p$ capture rate is also compatible with the HBChPT prediction. The advantage utilizing ChPT is that ChPT provides analytic expressions for both the $\mu^- p$ and $\mu^- d$ capture operators at each perturbative order, and ChPT permits us to make a reasonable estimate the theoretical uncertainty of the calculated observable. Once we have a measurable quantity evaluated at “order” $(Q/\Lambda_{ch})^v$, an estimated uncertainty is given by the magnitude of the next order contribution $(Q/\Lambda_{ch})^{v+1}$. Two-nucleons reactions (including the energy dependence of
\[ n + p \rightarrow d + \gamma \] which is important in cosmology) are well described by EFT* (one exception is the measured RMC rate versus the photon energy). The \( \mu^- d \) capture process being measured by the MuSun collaboration at PSI will allow a more accurate value for \( \hat{d}_R \). This MuSun measurement will permit us to make more accurate model-independent predictions for the solar \( pp \) fusion and the \( vd \) SNO reactions. However, improved radiative corrections are needed to “compete” with the expected MuCap and MuSun \( \mu^-\) capture data. ChPT is ideally suited for an evaluation of these radiative corrections.

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