Nonlocal condensates and pion form factor

A. V. Pimikov, A. P. Bakulev, N. G. Stefanis

aBogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Moscow region, Russia
bInstitut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany

We review a nonlocal-condensate approach and its application in QCD sum rules for the spacelike electromagnetic pion form factor. It is shown that the nonlocality of the condensates is a key point to include nonperturbative contributions to the pion form factor.

The spacelike electromagnetic pion form factor (FF) describes the scattering of charged particles off the pion by exchanging a photon. It is defined by the following matrix element:

\[ \langle \pi^+(P')|J_\mu(0)|\pi^+(P)\rangle = (P + P')_\mu F_{\pi}(Q^2), \]

where \( J_\mu \) is the electromagnetic current and \( q^2 = (P' - P)^2 = -Q^2 < 0 \) in the spacelike region. For asymptotically large momenta, one can apply the factorization theorem so that the FF will be represented via the inverse moment of the leading-twist pion distribution amplitude (DA). The precise value of \( Q^2 \) at which this perturbative term starts to prevail cannot be determined accurately. The estimates for the crossover momentum scale range from 100 GeV^2 [1,2,3] down to values around 20 GeV^2 [4,5]. But even this latter relatively small momentum is hopelessly far away from the capabilities of any operating or planned accelerator facility.

1. QCD sum rules approach

At intermediate momentum transfers factorization fails; therefore one needs to apply nonperturbative approaches. One of these methods was suggested in 1979 by Shifman, Vainshtein, and Zakharov [6] and was called QCD Sum Rules (SR)s. To extract information about the pion form factor in the QCD SR approach, one needs to investigate the Axial-Axial-Vector (AAV) correlator of three currents:

\[ \int d^4x d^4y e^{i(qx - P'y)} \langle 0|T\left[J_5^+(y)J^\mu(x)J_5(0)\right]|0\rangle, \]

where \( J^\mu(x) = e_u \bar{u}(x)\gamma^\mu u(x) + e_d \bar{d}(x)\gamma^\mu d(x) \) is the electromagnetic current and \( J_5(x) = \bar{d}(x)\gamma_5 u(x) \) is the axial-vector current. For simplicity, let us describe how this method works using as an example the two-point correlator

\[ \Pi(Q^2) = \int d^4x e^{iqx} \langle 0|T|J(x)J(0)|\rangle|0\rangle. \]

There are two ways to calculate this correlator. The first one is based on the dispersion relation

\[ \Pi_{\text{had}}(Q^2) = \int_0^\infty \rho_{\text{had}}(s) \frac{ds}{s + Q^2} + \text{subtractions}, \]

where physical observables (masses \( m_h \) and decay constants \( f_h \)) can be introduced in the calculation by a model spectral density as the sum of the first resonance contribution plus the contribution of the continuum beginning at the threshold \( s_0 \):

\[ \rho_{\text{had}}(s) = f_h^2 \delta(s - m_h^2) + \rho_{\text{pert}}(s) \theta(s - s_0). \]

Higher states are taken into account by the perturbative spectral density \( \rho_{\text{pert}}(s) \) on account of...
the quark-hadron duality. The second approach employs the operator product expansion (OPE):

$$\Pi_{\text{OPE}}(Q^2) = \Pi_{\text{pert}}(Q^2) + \sum_n C_n \left( \frac{|0 : O_n : |0\rangle}{Q^{2n}} \right).$$

Vacuum expectation values $\langle O_n \rangle$ of the normal product of quark and gluon fields are not vanishing but constitute (nonperturbative) condensates. Demanding the agreement between the results of these two calculations, we obtain the following SR

$$\Pi_{\text{had}}(Q^2, m_h, f_h) = \Pi_{\text{OPE}}(Q^2)$$

that allows us to extract the introduced hadronic parameters—masses and decay constants in the case of the two-point SR—from the condensates $\langle O_n \rangle$. On the other hand, the three-point SR [78] helps us to study hadronic form factors.

The simplest condensate is the so-called quark condensate. Consider the vacuum expectation value of the $T$-product of two quark fields:

$$\langle 0 | T(\bar{q}B(0))q_A(x) | 0 \rangle$$

where $A$, $B$ are Dirac indices. The second term here corresponds to the usual propagator due to the Wick theorem, while the first one is the quark condensate. From this equation one can see that the quark condensate is an additional nonperturbative contribution to the quark propagator.

In perturbation theory the vacuum coincides with the ground state of the free-field theory; hence the expectation value of the normal product is zero. Therefore, there are no condensate terms in perturbation theory. However in the physical vacuum this is not the case. For this reason, in the standard QCD SR approach, the nonzero quark condensate $\langle \bar{q}q \rangle = \langle \bar{q}A(0)q_A(0) \rangle$ appears. The value of this constant was defined through comparison with experimental data for the $J/\psi$-meson [6]. Assuming a small coordinate dependence, the quark condensate can be represented by the first two terms of the Taylor expansion:

$$\langle \bar{q}B(0)q_A(x) \rangle = \frac{\delta_{AB}}{4} \left( \langle \bar{q}q \rangle + \ldots \right) + i \frac{\bar{x}_{AB}}{4} \left[ \frac{2\alpha_s\pi\langle \bar{q}q \rangle^2}{81} + \ldots \right],$$

where we kept the scalar and vector parts apart. Note that the condensates in this representation are local.

2. Pion FF in the QCD SR approach

Unfortunately, the local approximation [2] is not reasonable for studying form factors (FF) and distribution amplitudes, as it was stated in [9101112]. The reason is the unphysical behavior of the local condensate [2], at large $x^2$, entailing a constant scalar and a vector part that is even growing with the distance between the quarks $x^2$. As a result, the nonperturbative part of the OPE linearly increases with the momentum $Q^2$ in the case of the FF (or with the moment $N$ in the case of the DA): $(c_1 + Q^2/M^2)$, where $c_1$ is a dimensionless constant (not depending on $Q^2$) and $M^2$ is the Borel parameter. At the same time, the perturbative part decreases, while the nonperturbative one increases with $Q^2$, hence generating an inconsistency of the SR at intermediate and large $Q^2$. Therefore, we can not rely upon the obtained SR for the pion FF for momentum values $Q^2 > 3$ GeV$^2$ [12].

In order to improve the $Q^2$ dependence, one needs to modify the model of the quark-condensate behavior at large distances. Indeed, lattice simulations [1314] and instanton models [1516] indicate a decreasing of the scalar quark condensate with increasing interquark distance, thus confirming the approach of nonlocal condensates (NLC)s [9]. To further improve the condensate contribution, one may calculate terms which contain higher-dimension operators of the form $\langle \bar{q}(0)D^2q(0) \rangle$, $\langle \bar{q}(0)(D^2)q(0) \rangle$, etc., originating from the Taylor expansion of the original nonlocal condensate, i.e., $\langle \bar{q}_B(0)q_A(x) \rangle$. The resulting total condensate contribution decreases for large $Q^2$. However, each term of the standard OPE has the structure $(Q^2/M^2)^n$, and one should, therefore, resum them to get a meaningful result. The

---

2Hereafter we write $\langle O_n \rangle \equiv |0 : O_n : |0\rangle$. A. V. Pinikov et al.
Nonlocal condensates and pion form factor

The described NLC QCD SR approach provides the basis of the theoretical framework for the calculation of the pion form factor proposed in our recent paper [12]. This method yields predictions for the spacelike pion form factor (see Fig. 1) that compare well with the experimental data of the Cornell [18] (triangles) and the JLab Collaborations [19] (diamonds) in the momentum

The same techniques should be applied to deal also with the mixed quark-gluon condensate: \( \langle \bar{q}_B(0)(-gA_\nu(y) t^\nu)q_A(x) \rangle \). There are two models for this condensate: the minimal and

...
region currently accessible to experiment. These predictions cover also the range of momenta to be probed by the 12 GeV$^2$ upgraded CEBAF accelerator at the Jefferson Lab in the near future. This planned high-precision measurement of the pion FF at JLab will certainly help to check the quality of the discussed NLC models.

3. Conclusions

We presented an analysis of the spacelike pion form factor using the SR in connection with two different models for the nonlocal condensate. The local condensates used in the pioneering studies \[7,8\] lead to an unphysical increasing of the nonperturbative contribution to the pion form factor at large $Q^2$, the reason being the unnatural behavior of the local condensates \[2\] at large $x^2$. This behavior restricts the region of the QCD SR applicability to $Q^2 \lesssim 3 \text{ GeV}^2$. In contrast to the local condensates, the nonlocal ones decrease at large $x^2$, hence inducing the decay of the nonperturbative terms at large $Q^2$. This makes the QCD SRs stable and enlarges the region of its applicability towards momenta as high as 10 GeV$^2$.

4. Acknowledgments

This work was supported in part by the Russian Foundation for Fundamental Research, grants No. 07-02-91557, 08-01-00686, and 09-02-01149, the BRFBR–JINR Cooperation Programme, contract No. F08D-001, the Deutsche Forschungsgemeinschaft (Project DFG 436 RUS 113/881/0-1), and the Heisenberg–Landau Program under grant 2009. A. V. P. acknowledges support from the Program “Development of Scientific Potential in Higher Schools” (projects 2.2.1.1/1483, 2.1.1/1539).

REFERENCES

1. N. Isgur and C. H. Llewellyn Smith, Phys. Rev. Lett. 52 (1984) 1080.
2. R. Jakob and P. Kroll, Phys. Lett. B315 (1993) 463.
3. Victor Braguta, Wolfgang Lucha, and Dmitri Melikhov, Phys. Lett. B661 (2008) 354.
4. N. G. Stefanis, W. Schroers, and H.-C. Kim, Phys. Lett. B449 (1999) 299; Eur. Phys. J. C18 (2000) 137.
5. A. P. Bakulev, K. Passek-Kumerički, W. Schroers, and N. G. Stefanis, Phys. Rev. D70 (2004) 033014; Phys. Rev. D70 (2004) 079906, Erratum.
6. M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B147 (1979) 385; ibid. 448; ibid. 519.
7. V. A. Nesterenko and A. V. Radyushkin, Phys. Lett. B115 (1982) 410.
8. B. L. Ioffe and A. V. Smilga, Phys. Lett. B114 (1982) 353.
9. S. V. Mikhailov and A. V. Radyushkin, JETP Lett. 43 (1986) 712; Sov. J. Nucl. Phys. 49 (1989) 494; Phys. Rev. D45 (1992) 1754.
10. A. P. Bakulev and A. V. Radyushkin, Phys. Lett. B271 (1991) 223.
11. S. V. Mikhailov, Phys. Atom. Nucl. 56 (1993) 650.
12. A. P. Bakulev, A. V. Pimikov, and N. G. Stefanis, Phys. Rev. D79 (2009) 093010.
13. M. D’Elia, A. Di Giacomo, and E. Meggiolaro, Phys. Rev. D59 (1999) 054503.
14. A. P. Bakulev and S. V. Mikhailov, Phys. Rev. D65 (2002) 114511.
15. A. E. Dorokhov, S. V. Esabegian, and S. V. Mikhailov, Phys. Rev. D56 (1997) 4062.
16. M. V. Polyakov and C. Weiss, Phys. Lett. B387 (1996) 841.
17. A. P. Bakulev and A. V. Pimikov, Phys. Part. Nucl. Lett. 4 (2007) 377.
18. C. J. Bebek et al., Phys. Rev. D9 (1974) 1229; ibid. D13 (1976) 25; ibid. D17 (1978) 1693.
19. G. M. Huber et al., Phys. Rev. C78 (2008) 045203.