Einstein initially objected to the probabilistic aspect of quantum mechanics—the idea that God is playing at dice. Later he changed his ground, and focussed instead on the point that the Copenhagen Interpretation leads to what Einstein saw as the abandonment of physical realism. We argue here that Einstein’s initial intuition was perfectly sound, and that it is precisely the fact that quantum mechanics is a fundamentally probabilistic theory which is at the root of all the controversies regarding its interpretation. Probability is an intrinsically logical concept. This means that the quantum state has an essentially logical significance. It is extremely difficult to reconcile that fact with Einstein’s belief, that it is the task of physics to give us a vision of the world apprehended sub specie aeternitatis. Quantum mechanics thus presents us with a simple choice: either to follow Einstein in looking for a theory which is not probabilistic at the fundamental level, or else to accept that physics does not in fact put us in the position of God looking down on things from above. There is a widespread fear that the latter alternative must inevitably lead to a greatly impoverished, positivistic view of physical theory. It appears to us, however, that the truth is just the opposite. The Einsteinian vision is much less attractive than it seems at first sight. In particular, it is closely connected with philosophical reductionism.
For a long time Einstein strongly objected to the indeterminism of quantum mechanics. As he put it in a letter to Born (written in 1926, in response to Born’s proposal that the wave-function has an essentially probabilistic significance):

Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the ‘old one’. I, at any rate, am convinced that He is not playing at dice. [Born-Einstein letters [1], p. 91]

I think people often find it difficult to understand why Einstein was so emphatic in his rejection of a dice-playing God. Quantum mechanics presents many obstacles to the understanding. But the concept of an objective chance appears, on the face of it, intuitively very natural. At least as judged by the standards of commonsense, if anything is paradoxical, it is the rigid determinism of classical physics, with its apparent denial of human freedom. It seems that Einstein came to feel this himself in the end. One finds him expressing strong objections to the notion of a dice-playing God as late as 1944 (ref. [1], p.149). However in 1954 Pauli reports him as “disput[ing] that he uses as criterion for the admissibility of a theory the question ‘Is it rigorously deterministic?’ ” (ref. [1], p.221).

It seems to me, however, that Einstein gave in too easily. It is precisely the fact that quantum mechanics is a fundamentally probabilistic theory which is at the root of all the controversies regarding its interpretation. Specifically, it is the fact that the wavefunction has a fundamentally probabilistic significance which means that the wavefunction has to collapse consequent on a measurement; and it is that collapse which makes it hard to interpret the quantum state in the way that so many people would like to interpret it, as a physically real entity.

To illustrate the point consider a case where someone—for the sake of definiteness let us call her Alice—has bought a lottery ticket. Suppose that the draw has taken place, and that Alice’s ticket won. However, Alice does not know this. So, even though the reality is that Alice did win the lottery, she herself thinks that the probability of her having won is very small. Now suppose that Alice opens a newspaper, and is surprised to discover that hers is the winning ticket. Then her state of mind will suddenly change, from believing that the probability of her having won is close to zero, to believing that it is close to one (not quite one because the newspaper might have misprinted the winning number). There is, of course, nothing mysterious about this change in Alice’s state of mind. Changing a probability assignment consequent on the acquisition of new information is a very natural and reasonable thing to do.

Now compare this with a quantum mechanical measurement. Before the measurement one integrates the squared modulus of the electron’s wave function and finds that the probability of the electron having $x$ coordinate in the range $1 < x < 2$ is $10^{-7}$. But then one performs the measurement and finds that the $x$ coordinate actually is in the range which one previously considered to be highly improbable. So, just as in the lottery example considered in the last paragraph, one changes one’s assessment of the probability of the electron’s $x$ coordinate being in the range $1 < x < 2$ from $10^{-7}$ to a value close to 1. Since the probability is directly related to the wavefunction this means one must also make an equally sudden and dramatic change to the electron’s wavefunction. That change is just the notorious collapse of the wavefunction, which I think it is probably fair to say has been the cause of more philosophical agonising than any other phenomenon in the history of physics.

The question is: why should the discontinuous change in the electron’s wave function be considered any more puzzling than the no less radical change in Alice’s assessment of the likelihood of her having won the lottery? The short answer to
this question is that there is no temptation to regard the change in Alice’s beliefs as anything more than a change in Alice’s state of mind. On the other hand there is a very strong temptation to regard the electron’s wave function as a physically real entity. Consequently, there is a strong temptation to think that if the electron’s wave function were to change, merely as a consequence of the experimenter acquiring new information, it would mean that reality itself had changed, merely as a consequence of the experimenter acquiring new information. However physicists are, on the whole, reluctant to believe in spiritual phenomena. So the usual response is to try to twist the interpretation of quantum mechanics in such a way as to make it seem, either that the wavefunction does not really collapse (as in, for example, the Bohm or Everett interpretations), or else that it does collapse but not as a consequence of the change in the experimenter’s state of mind (as in spontaneous collapse theories).

At this point it will be convenient to introduce a piece of terminology. I will say that a probability is **epistemic** if it is conceived in the same way that one naturally conceives the probability of Alice having won the lottery, as representing, not a piece of mind-independent physical reality, but only a cognitive agent’s expectations regarding that reality. Phrased in these terms the short answer to the question posed at the beginning of the last paragraph reads: the discontinuous change in the probability of the electron being located in a particular region tends to be seen as problematic because there is a strong temptation to see the probability as, not merely epistemic, but objectively real.

However, the short answer is not completely satisfactory because it fails to make clear why it is only with the development of quantum mechanics that one finds this strong, almost overwhelming temptation to regard probabilities as objectively real entities. After all, probability played a major role in the physics of the nineteenth century. Maxwell, in particular, was very clear that probabilities are to be conceived as logical constructs, rather than objective realities:

> They say that Understanding ought to work by the rules of right reason. These rules are, or ought to be, contained in Logic; but the actual science of Logic is conversant at present only with things either certain, impossible, or entirely doubtful, none of which (fortunately) we have to reason on. Therefore the true logic for this world is the calculus of probabilities, which takes account of the magnitude of the probability which is, or ought to be, in a reasonable man’s mind. [quoted Jeffreys [2], p.1]

This prompts the question: if Maxwell, who was one of those chiefly responsible for the classical theory of statistical mechanics, could cheerfully accept that the probability of a molecule being in a particular region is to be conceived in purely epistemic terms, why is it that his 20th and 21st century successors are so wedded to the opposite, objectivist point of view? Actually, it is worth noting that Einstein himself was happy enough with probabilities conceived in these Maxwellian, epistemic terms. Indeed, some of Einstein’s most important contributions to theoretical physics relied on a masterly deployment of probabilistic ideas. It seems that Einstein was perfectly willing to play dice himself. His objection was only to the idea that God might be playing them too. The question is: why? Why should behaviour which is acceptable in Einstein suddenly become unacceptable when imputed to God?

I think the answer to that question must be that the theories of classical physics were not fundamentally probabilistic theories. To be sure, the classical physicists found themselves compelled to use probabilistic reasoning. However, probabilistic concepts were not embedded in their theories at the most fundamental level.
Maxwell could easily accept that the probability distributions with which he worked were not to be conceived as objective realities because there were numerous other quantities in the theories of classical physics which could be regarded as directly corresponding to physical realities. The problem his 20th and 21st successors face is that quantum mechanics, by contrast, is probabilistic at the fundamental level. In quantum mechanics the only obvious candidate for a mind-independent physical reality is the wave function. So if one takes an epistemic view of the wave function one seems to be driven into the position that nothing at all in the theory corresponds to a mind-independent physical reality. It seems as though one is in danger of losing one’s grip on physical reality altogether. And that, I think, is what leads to all the attempts (the Bohm interpretation, the Everett interpretation, spontaneous collapse theories, etc.) to interpret the quantum state as an objectively existent physical entity.

However, I think it is fair to say that these attempts are all highly speculative. Each such approach has its associated group of enthusiasts. However, the enthusiasts for one approach are unable to find arguments sufficient to persuade, either the enthusiasts for any of the others, or the much larger group of the conceptually uncommitted. At least at present the decision to opt for (say) the Bohm interpretation, rather than (say) the Everettian, seems to rest on nothing more than personal taste. That situation might conceivably change. If, for example, Valentini’s hopes [3, 4] were fulfilled we would have solid empirical reasons for preferring one objectivist interpretation over another. But in the present state of knowledge it is difficult to avoid the suspicion that the question is empirically undecidable. Of course, “empirically undecidable” does not mean “necessarily false”. Perhaps the world is in reality Bohmian. There is certainly nothing we know to exclude that hypothesis. The trouble is that there seems to be nothing to support it either. Not only are there no observations which would clinch the question. There does not even seem to be any moderately persuasive reason for thinking the hypothesis likely.

I intend no disrespect to the proponents of the various objectivist interpretations by these remarks. On the contrary it appears to me that Bohm, Everett and others have made very important contributions to our understanding of these questions. Quantum mechanics is a deeply puzzling subject, and I think that if one wants to understand it better one needs to look at it from every possible angle. So I would certainly not dispute the value of the work done by Bohm, Everett and others. However, the fact is that science is concerned specifically with those questions where it is possible to find clearly stateable, cogent reasons for belief or unbelief. Until such reasons are forthcoming I do not see how any of the objectivist interpretations can be considered a satisfactory solution to the problem. At the least I think it must be worth exploring alternative, non-objectivist ways of thinking about the quantum state.

It is a striking fact that, although different people take very different views as to what the quantum state may be in ultimate reality, when it comes to the problem of making experimental predictions everyone calculates in exactly the same way. In particular, everyone collapses the wave function (proponents of the Everett interpretation believe that there is a state vector of the universe which does not collapse; however, the wave function that an Everettian writes down on paper, for the purposes of making an experimental prediction, collapses in just the same way as the one that a Copenhagenist writes down). At least so far as the empirical predictions are concerned the significance of the quantum state begins and ends with the fact that it specifies a set of probabilities. Of course, this does not logically
exclude the hypothesis that the quantum state has some other significance in ultimate reality. However, in the absence of any compelling evidence as to what that significance might be, it seems to me that the most natural and straightforward course is to adopt the hypothesis that the quantum state simply is a compendium of probabilities, and to see what follows from that.

If that is accepted the next question we have to address is, how to interpret a probability statement. Here we run into the difficulty that the theory of probability is troubled by a controversy which is even more long-standing than the 80-year old controversy about the interpretation of the wave function. At the beginning of the last century Poincaré [5] (p.186) described probability as an “obscure instinct”. In the 100 years that have elapsed since then there has been much discussion. However, the effect has, if anything, only been to intensify the disagreements. Broadly speaking there are two schools of thought. On the one hand there is the objectivist school of thought (represented by, for example, von Mises [6, 7], Fisher [8] and Popper [9, 10]) which holds that a classical probability distribution should be regarded as an objectively real physical entity, which is what it is independently of anything that we might know or think about it. On the other hand there is the epistemic school of thought (represented by, for example, Laplace [11], de Finetti [12], Jeffreys [2], Savage [13] and Jaynes [14, 15]) which holds that a probability distribution has an essentially logical significance. For a broad overview of the questions at issue see, for example, Gillies [16] and Howson and Urbach [17]. For the connection to quantum mechanics see Jaynes [14, 15], Fuchs [18, 19], Caves et al [20, 21] and Appleby [22, 23].

Now one might say that philosophy would no longer be philosophy if people ever came to agree about something. However, this particular dispute is unlike many other conceptual disputes in that it has some immediate, and very important practical consequences. For, associated with the two different schools of thought about the content of probability statements, there are two very different statistical methodologies. The objectivist view—the desire to interpret probabilities as mind-independent physical entities which can be measured rather in the way that a mass can be measured—motivated Fisher and others to develop what is now the orthodox statistical methodology, described in every textbook. By contrast the epistemic point of view is associated with the statistical methodology originally proposed by Bayes, and greatly extended by Laplace. These different statistical methodologies will, in general, lead to different practical conclusions.

The complaint of the Bayesians about the orthodox statistical methodology has always been that it is (in the words of de Finetti [12], p.245) “ad hoc” and “arbitrary”. Jeffreys makes the point with characteristic irony when he says of Fisher (one of the founding fathers of the orthodox methodology)

\[
\text{I have in fact been struck repeatedly in my own work, after being led on general principles to the solution of a problem, to find that Fisher had already grasped the essentials by some brilliant piece of common sense [Jeffreys [2], p. 393].}
\]

This is, in a way, a compliment. However, the compliment is distinctly back-handed: for what Jeffreys is really saying is that Fisher, notwithstanding his confusions and inconsistencies, often contrives to get the right answer owing to the power of his intuition. It is rather as if a physicist were to congratulate a snooker player on his ability to pot a ball notwithstanding his ignorance of Newtonian mechanics; or to congratulate a fish on its ability to swim notwithstanding its ignorance of the principles of hydrodynamics. I have argued elsewhere [22, 23] that that criticism is amply justified. Generally speaking what drives the Bayesian school of thought is a desire for clarity and logical cogency. By contrast the orthodox statistical
methodology is driven by what Jaynes describes as an ideological conviction that, if statistics is to be scientific, then probability distributions must be conceived as objectively real entities. To attain that ideological end orthodox statisticians are willing to make whatever sacrifice of logical coherence seems necessary.

The problem orthodox statisticians face is that, however sophisticated the technical superstructure may become, what is at the bottom of the pyramid is the ordinary primitive intuition of one event being more or less probable than another. Furthermore every statistical argument has to rely on that intuition if it is to make contact with reality. One may cover hundreds of pages with intricate calculations. But the question one is ultimately asking, and must answer if there is to be any point to the calculations, is always very simple. It is a question of the form: how probable is it that $X$? Would it be wise to bet on $X$? And however words may be used by professional statisticians in their private reasoning processes, the sense of the word “probable” as it is used in the statement of the final conclusion is always the primitive sense, which a child of 7, who knows nothing of the formal apparatus of probability theory, can comprehend. In particular it is a sense of the word “probable” which applies to single cases. If I want to know whether a drug is more likely to cure me than to kill me then, although I might make use of data regarding the fate of other patients, the question I want to answer is a single-case question: what will probably happen to me if I take the drug? Shall I gamble with my life? (not the lives of a statistical ensemble, but my singular, personal life).

Of course the primitive notion of probability, that a child of 7 can understand, is non-quantitative. It cannot be embedded in a formal mathematical theory without a very considerable degree of theoretical elaboration. The relation between the primitive notion of probability and the formal mathematical one is, in some ways, similar to the relation between the common sense notions of mass and force, and the concepts going by the same names which feature in Newtonian mechanics. So I am certainly not meaning to identify the primitive, common sense notion with the formal mathematical concept of probability. But what I think is undeniably the case is that the formal concept is a development of the primitive one (a very considerable development, no doubt, but a development nonetheless). Furthermore, the formal concept depends on the primitive notion for its empirical applicability. If the formal concept is developed to the point where it loses all connection with the primitive notion, then the theory will lose all its practical utility.

The problem this poses for orthodox statisticians (the insuperable problem, as it seems to me) is that the primitive notion of probability is, obviously and unavoidably, epistemic in its character. Consider the example I discussed earlier, where as it happens Alice’s lottery ticket won, but she does not know it. In those circumstances Alice believes that it is most unlikely that she won. Most people would consider she was right to think it most unlikely. And yet the fact of the matter is that she did win. It can be seen that we have here two statements having radically different logical characters. On the one hand there is the epistemic statement:

Alice is most unlikely to have won the lottery

On the other hand there is the factual statement (which, although Alice does not know it, describes the actual state of affairs)

Alice did win the lottery

The second statement is a proposition about the lottery draw, as it exists independently of what Alice knows or does not know. The first statement, by contrast, is as much about Alice, and her limited information, as it is about the lottery. Specifically, it is a logically evaluative statement about what Alice, in her epistemic situation, can reasonably expect. This epistemic character becomes apparent when one considers what happens when Alice opens the newspaper and discovers that her
ticket did win. In that case her belief state changes discontinuously. Rather in the way that the wave function changes discontinuously consequent on a measurement, Alice switches from thinking that she almost certainly did not win the lottery to thinking that she almost certainly did win (not quite certainly because, for example, the newspaper might have misprinted the winning number or, for example, because she might be hallucinating). There is nothing mysterious, or philosophically offensive about this discontinuous change. It simply reflects the fact that the statement was epistemic, and epistemic statements naturally are subject to revision, consequent on the acquisition of new information. It is also worth noting that Alice’s discovery, that her ticket did win, will not lead her to think that she was wrong to believe that she probably had not won. On the contrary, she will continue to think that she was right to believe that she probably had not won. Naturally so: for the belief that she probably had not won did not represent a mistaken belief about the physical world. Rather it represented a logically evaluative belief about what she, in her previous epistemic situation, could reasonably expect.

If this is accepted, and if the point I made earlier (that the mathematical theory of probability relies on the primitive notion of probability to make contact with reality) is also accepted, then it follows that the project of the orthodox statisticians (to construct a completely objective theory of statistical inference) is likely to run into insuperable obstacles. I believe that a more detailed examination \[2, 12, 14, 15, 22, 23\] of the question confirms that proposition. As a result the orthodox statisticians are rather in the position that one would be in if one insisted on trying to construct a theory of sound based on the assumption that sound is a form of electromagnetic radiation; or a theory of number based on the assumption that a number is, not an abstract logical entity, but a concrete physical object. It is only possible to produce a simulacrum of success by bending the facts and twisting the logic.

I believe that similar considerations apply to the quantum state. Quantum mechanics is deeply and intriguingly different from classical probability theory. It is also much farther removed from common sense ways of thinking. Nevertheless it has certain basic features in common. In particular quantum probabilities, like classical probabilities, are unavoidably epistemic in character: as is clearly signalled by the discontinuous change in the quantum state which occurs consequent on a measurement outcome.

This brings us back to the problem which troubled Einstein. It is easy to take an epistemic view of the probabilities in classical statistical mechanics because classical statistical mechanics presents us with various other mathematical constructs which can be thought of as the depictions of actually existent physical entities. But the same is not true of quantum mechanics. Consequently, it may seem that taking an epistemic view of the quantum state amounts to giving up on the project of understanding physical reality altogether. It may look, on the face of it, as though quantum mechanics thus interpreted leads, if not to idealism, or to pure solipsism, then at any rate to a depressingly positivistic view in which physics begins and ends with the task of predicting detector “clicks”. I believe it is that perception which motivated Einstein’s search for a more complete description, and which continues to motivate the various objectivist interpretations of the quantum state.

In response to that objection let me begin by saying that it is, to my mind, a very reasonable objection. Indeed, for most of my research career my sympathies have been with the opponents of the Copenhagen interpretation. To a considerable extent they still are. Bell complains that the Copenhagen interpretation is “unprofessionally vague and ambiguous” and that quantum mechanics, when interpreted
in Copenhagen terms, seems to be “exclusively concerned with ‘results of measurement’ and [seems to have] nothing to say about anything else” (Bell [24], pp. 173 and 117). I think he is right on both counts. I share Bell’s conviction that the aim of physics is to understand nature, and that counting detector “clicks” is not intrinsically any more interesting than counting beans. The day I become convinced that physics does not in fact provide us with anything more than procedures for predicting detector “clicks” will be the day I abandon physics in favour of some more stimulating activity. Nevertheless I have gradually come to feel that the most promising way forward lies, not in the Copenhagen Interpretation as such, but in a greatly improved version of it to which Bell’s criticisms would not apply. If I am asked to accept Bohr as the authoritative voice of final truth then I cannot assent. But if his writings are approached in a more flexible spirit, as a source of insights which are but dimly apprehended, then they suggest a line of thought which I feel might, if further developed, be very fruitful.

Taking this view does not amount to the abandonment of physical realism. If anything it is exactly the other way round. The various objectivist interpretations (Bohm, Everett, …) do indeed provide us with numerous beguiling images of how things might, conceivably stand in ultimate reality. The trouble is that there does not seem to be any way to decide which, if any of these alternative pictures corresponds to the truth. As I stated earlier this situation could change. If, for example, Valentini’s hopes [3, 4] were fulfilled we might have solid empirical grounds for regarding the quantum state as a physically real entity. But in the present state of our knowledge objectivist interpretations of the quantum state are purely speculative. It is, of course, true that every well-attested scientific theory started out as a piece of speculation. However, although speculation undoubtedly has its place in science, I think there is something very discouraging about a theory which, one may reasonably fear, is never going to get beyond the level of pure speculation. The aim of science is, after all, not merely to speculate, but to make empirically well-grounded statements about physical reality. It is not inconceivable that one or other of the objectivist approaches will eventually be pushed to the point where it meets that requirement. However, my own feeling is that a more promising approach is to try to make better sense of the wave function when it is conceived epistemically. Whether that judgment is correct only time will tell. But what is certain is that the motive is to better understand what quantum mechanics is telling us about the world.

It is, however, true that, although the epistemic approach does not lead to the abandonment of physical realism, it forces us to think of physical reality in a very different way from the one to which we have become classically accustomed. For 200 years after the time of Newton physics was inspired by the dream of constructing a perfectly faithful depiction of the world. A map, each point of which is in one-one correspondence with an element of physical reality. A map, furthermore, which leaves nothing out—a map in which every feature of physical reality has its representative, and in which it never happens that a simple feature of the map corresponds to a complex feature of the actual world. When Einstein talks of God I believe it is this that he has in mind: a vision of the world apprehended sub specie aeternitatis, as God might see it looking down on things from above. I think it must be true that the epistemic interpretation requires us to give up on that. And I think it must also be true that that to anyone of Einstein’s philosophical persuasion that is going to feel like a major sacrifice. However, I would suggest that if one takes the trouble to think through the implications carefully one may start to feel that the sacrifice is not so great as it initially looks. In fact one may even begin to feel that it is not a sacrifice at all, but a liberation.
The epistemic interpretation requires us to abandon the idea that a cat’s wave function is in one-one correspondence with the cat’s ultimate reality. But that does not mean it requires us to give up the idea that quantum mechanics tells us something important about the cat. Quite the contrary. Quantum mechanics makes a large number of remarkably detailed statements about, for example, the cat’s molecular biology. The epistemic interpretation leaves all of those statements completely intact. The statements are, to be sure, only probabilistic. In other words they are statements about what we, with our limited information, can reasonably expect. However, I would question whether that acknowledgment represents the major intellectual defeat that Einstein would take it to be. It is not, after all, as though there has ever really been any question of apprehending the world sub specie aeternitatis. God may know, with absolute certainty, the position of a classical particle. But we never have. It is a truism that science does not give us complete certainty. The propositions of science are and always have been, without exception, statements that something is more or less probable. Quantum mechanics, interpreted epistemically, changes nothing in that respect.

If Einstein’s hopes had been fulfilled we would be in the ostensibly happy position of seeing right through to the cat’s metaphysical bottom. At any rate, we would be able to imagine that we had achieved that feat. By contrast, the epistemic interpretation obliges us to concede to the cat a degree of metaphysical privacy. Is that such a bad thing? Einstein’s vision, of the world apprehended sub specie aeternitatis, is closely connected with philosophical reductionism. And, however it may be with cats, the idea that a human being simply reduces to a configuration of classical fields (or whatever) is not very plausible. A large part of the philosophy of the mind consists of various rather unconvincing attempts to understand how the brain, conceived in reductionist terms, can give rise to consciousness. One of the reasons I am interested in the epistemic point of view is that I feel that when properly developed it may lead to a much more satisfactory, non-reductionist way of thinking about the mind-brain relationship.

The ambition to “know the mind of God” is not realistic. But I would go further than that. I would question whether the idea is even attractive. Suppose one really could comprehend the universe in its entirety. Might this not be found a little cramping? If the universe really could be comprehended in its entirety it would mean that the universe was as limited as we are. It seems to me that living in such a universe would be rather like trying to swim in water that is only six inches deep. Groucho Marx once said that he would not want to belong to a club that would have him as a member. In a similar vein, my personal feeling is that I would not wish to belong to a universe that I was able to fully comprehend.

Against this vision, of physics as knowing the mind of God, I would like to set another: physics as swimming in water that is a great deal deeper than we are—perhaps even infinitely deep.

References

[1] M. Born and A. Einstein (trans. I. Born), The Born-Einstein Letters (Macmillan, London, 1971).
[2] H. Jeffreys, Theory of Probability, 3rd edition (Clarendon Press, Oxford, 1961).
[3] A. Valentini, Pramana—Journal of Physics 59, 269 (2002).
[4] A. Valentini, Phys. Lett. A 158, 1 (2002).
[5] H. Poincaré, Science and Hypothesis (Dover, New York, 1952). English translation, French original first published 1905.
[6] R. von Mises, Probability, Statistics and Truth (Dover, New York, 1981). Reprint of 2nd English edition, published 1957.
[7] R. von Mises (ed. H. Geiringer), Mathematical Theory of Probability and Statistics (Academic Press, New York, 1964).
[8] R.A. Fisher (ed. J.H. Bennett), *Statistical Methods, Experimental Design, and Scientific Inference* (Oxford University Press, Oxford, 1990).
[9] K.R. Popper, *The Logic of Scientific Discovery* (Hutchinson, London, 1959).
[10] K.R. Popper, *A World of Propensities* (Thoemmes, Bristol, 1990).
[11] P.S. de Laplace (trans. F.W. Truscott and F.L. Emory), *A Philosophical Essay on Probabilities* (Dover, New York, 1951). French original first published 1820.
[12] B. de Finetti (trans. A. Machi and A. Smith), *Theory of Probability* (Wiley, New York, 1975). Italian original first published 1971.
[13] L.J. Savage, *The Foundations of Statistics*, 2nd edition (Dover, New York, 1972).
[14] E.T. Jaynes (ed. R.D. Rosenkranz), *Papers on Probability, Statistics and Statistical Physics* (Reidel, Dordrecht, 1983).
[15] E.T. Jaynes (ed. G.L. Bretthorst), *Probability Theory: the Logic of Science* (Cambridge University Press, Cambridge, 2003).
[16] D. Gillies, *Philosophical Theories of Probability* (Routledge, London, 2000).
[17] C. Howson and P. Urbach, *Scientific Reasoning: the Bayesian Approach* (Open Court, La Salle, 1989).
[18] C.A. Fuchs, *Notes on a Paulian Idea* (Växjö University Press, Växjö, 2003). Also available as e-print quant-ph/0105039.
[19] C.A. Fuchs, “Quantum Mechanics as Quantum Information (and only a little more)”, quant-ph/0205039 (2002).
[20] C.M. Caves, C.A. Fuchs and R. Schack, *Phys. Rev. A* **65**, 022305 (2002).
[21] C.M. Caves, C.A. Fuchs and R. Schack, *J. Math. Phys.* **43**, 4537 (2002).
[22] D.M. Appleby, *Found. Phys.* **35**, 627 (2005).
[23] D.M. Appleby, *Optics and Spectroscopy* **99**, 447 (2005).
[24] J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, first edition (Cambridge University Press, Cambridge, 1987).