Statistical-model description of $\gamma$ decay from compound-nucleus resonances

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The statistical model of compound-nucleus reactions predicts that the fluctuations of the partial $\gamma$-decay widths for a compound-nucleus resonance are governed by the Porter-Thomas distribution (PTD), and that consequently the distribution of total $\gamma$-decay widths is very narrow. However, a recent experiment [Koehler, Larsen, Guttormsen, Siem, and Guber, Phys. Rev. C 88, 041305(R) (2013)] reported large fluctuations of the total $\gamma$-decay widths in the $^{96}$Mo$(n, \gamma)^{96}$Mo* reaction, contrary to this expectation. Furthermore, in recent theoretical works it was argued that sufficiently strong channel couplings can cause deviations of the partial width distributions from PTD. Here, we investigate whether the combined influence of a large number of nonequivalent $\gamma$-decay channels, each of which couples weakly to the compound-nucleus resonances, can modify the statistics of the partial widths. We study this effect in neutron scattering off $^{95}$Mo within a random-matrix model that includes coupling to the entrance neutron channel and to the large number of $\gamma$ channels. Using realistic coupling parameters obtained from empirical models for the level density and the $\gamma$ strength function, we find that the PTD describes well the distribution of partial widths for all decay channels, in agreement with the statistical-model expectation. Furthermore, we find that the width of the distribution of the total $\gamma$-decay widths is insensitive to wide variations in the parameters of the $\gamma$ strength function, as well as to deviations of the partial-width distributions from the PTD. Our results rule out an explanation of the recent experimental data within a statistical-model description of the compound nucleus.

I. INTRODUCTION

It is widely accepted that low-energy neutron resonance scattering from medium-mass and heavy nuclei is well described by the statistical model [1], in which the compound-nucleus (CN) resonances are described as eigenstates of a Hamiltonian drawn from the Gaussian orthogonal ensemble (GOE) of random-matrix theory. The statistical model predicts that the distribution of the partial widths for each individual reaction channel follows the Porter-Thomas distribution (PTD), i.e., a $\chi^2$ distribution in one degree of freedom. As a result, the total $\gamma$-decay width distribution is expected to be very narrow, resembling a $\chi^2$ distribution in many degrees of freedom.

In recent years, however, some experimental evidence was presented for possible violations of the statistical-model predictions. The distribution of neutron resonance widths obtained from $s$-wave neutron scattering off Pt isotopes was found to be significantly broader than the PTD [2]. Moreover, a recent analysis of the Nuclear Data Ensemble found a statistically significant deviation of the distribution of neutron-resonance widths from the PTD [3]. There have been attempts to explain these findings through non-statistical effects that emerge within the statistical model [4–6]. Other explanations focused on the analysis of the data [7–8]. For the case of Pt isotopes, nearly all of the former explanations were ruled out by recent work [9–10].

Almost all of the experimental and theoretical works were focused on the neutron resonance widths. However, a recent experiment that measured total $\gamma$-decay widths from $s$- and $p$-wave neutron resonances in neutron scattering off $^{95}$Mo found the total $\gamma$-decay width distribution for each spin-parity class of resonances in $^{96}$Mo to be significantly broader than the statistical model predictions [11]. It is important to understand whether such large fluctuations are possible within the framework of the statistical model.

It is known that the nonequivalence (i.e., different coupling strengths) of a set of channels corresponding to a particular CN decay mode leads to a reduction in the effective number of degrees of freedom describing the set and thus to an increase in the total width fluctuations [12]. Individual $\gamma$ channels, each of which is defined by the multipolarity of the transition and the energy of the emitted $\gamma$ ray, are nonequivalent. However, in simulations that were carried out in the experimental analysis, the nonequivalence of the $\gamma$ channels was accounted for through the use of the $\gamma$ strength functions ($\gamma$SF) and of models for the level density. These describe, respectively, the average partial width for a transition of given multipolarity and $\gamma$-ray energy and the number of accessible final states for the $\gamma$ decay. Thus, the nonequivalence of the $\gamma$ channels cannot account for the disagreement between the measured and simulated total $\gamma$-decay width distributions of Ref. [11].

Recent theoretical works have shown that the violation of the orthogonal invariance of the GOE due to the coupling to reaction channels can lead to deviations of the partial width distribution from the PTD [4–6]. However, these works did not include a realistic description of the $\gamma$-decay channels. In medium-weight and heavy nuclei, the number of $\gamma$-decay channels is very large. To avoid the explicit modeling of such a large number of channels, $\gamma$ decay is often modeled by a constant imagi-
nary contribution to the effective Hamiltonian, but this model assumes that the total $\gamma$-decay width distribution is very narrow. Alternatively, using a realistic description of the coupling strength of the $\gamma$ channels, the width distribution for any number of $\gamma$ channels may be calculated using the analytic results of Ref. [5]. However, given the large number of channels, the evaluation of formulas derived in Ref. [5] is impractical. Therefore, it is not known yet whether the combined effect of a large number of weakly coupled $\gamma$-decay channels in the $^{95}$Mo($n, \gamma$)$^{96}$Mo* reaction might modify the distribution of the partial $\gamma$-decay widths.

Furthermore, the total $\gamma$-decay width distribution depends not only on the partial $\gamma$-decay width fluctuations but also on the level density and $\gamma$SF. Simulations of the distributions of total $\gamma$-decay widths for a given spin-parity class of resonances use empirical formulas for the $\gamma$SF and for the level density. In Ref. [11], several different $\gamma$SF models were used to generate statistical-model results, but the systematic dependence of the simulated distributions on the model parameters was not studied. It is important to understand this dependence in order to know how sensitive the total $\gamma$-decay width distributions are to the underlying partial width fluctuations.

Here, we investigate the role of the $\gamma$-decay channels in the statistical model. First, we study the effect of the $\gamma$ channels on the fluctuations of the partial widths. To facilitate the numerical simulations, we group $\gamma$ channels of the same multipolarity that are close in energy into a single ‘representative’ channel. We expect that such coarse-graining of the channels does not change the qualitative results if a sufficient number of representative channels is used. We use empirical parameterizations of the level density, $\gamma$SF, and neutron strength function to determine the average channel couplings. We then calculate the distributions of the partial neutron widths and of the partial $\gamma$-decay widths by using a large number of GOE realizations of the CN Hamiltonian. We find no deviation from the PTD and thus confirm the traditional expectation of the statistical model.

Next, we address the distribution of total $\gamma$-decay widths, focusing attention on the widths and peak locations of these distributions. We systematically vary the parameters of the $\gamma$SF, assuming the partial widths are described by PTD. We find virtually no change in the width of the total $\gamma$-decay width distribution for a broad range of the $\gamma$SF parameters. Furthermore, although the peak location may be reproduced for any individual spin-parity class of resonances by parameter adjustments, we cannot obtain agreement of the peak locations with experiment for all spin-parity classes through such adjustments. This result indicates a serious shortcoming of the empirical $\gamma$SF expressions for the $^{96}$Mo compound nucleus.

Finally, we investigate whether the total $\gamma$-decay width distribution is sensitive to deviations in the partial $\gamma$-decay width distribution from PTD, which can occur for sufficiently strong coupling of the neutron channel. We find that these modified fluctuations in the partial widths have virtually no effect on the total $\gamma$-decay width distribution.

In the following, we comment on a limitation of our study. Our goal is to investigate whether the experimental findings of Ref. [11] can be explained within the GOE statistical model of the CN as defined in Ref. [1]. Although the GOE statistical model forms the basis of the statistical theory of CN reactions and is widely used in applications, the GOE effectively assumes random n-body interactions, where $n$ is the number of particles. From a physical point of view, it would be preferable to use more realistic statistical models for the CN that account for the predominantly two-body character of the residual nuclear interaction. Such models are the $k$-body embedded ensemble EGOE($k$) with $k = 2$ [13] and the two-body random ensemble (TBRE) [14]; see Refs. [15–17] for reviews. In contrast to the GOE, however, the spectral fluctuation properties of these models cannot be determined analytically. The limited information that exists has been gained numerically [15–18]. In Ref. [15], the distribution of transition strengths for EGOE(2) was found to be well described by the PTD. Some evidence exists of PTD violation in the TBRE [15–20], but this violation mainly occurs in the tails of its spectrum [18] and is ascribed to a lack of complete mixing of the basis states. That interpretation is consistent with results of shell-model calculations for $sd$-shell nuclei [21–22], which showed that the shell-model eigenvector components were practically Gaussian (non-Gaussian) in the center (tails) of the spectrum. In actual nuclei, such tails would comprise the ground state and a number of low-lying excited states, whereas the CN resonances we consider in this work are far from these tails. Shell model calculations of electromagnetic transition strength distributions in $sd$-shell [23] and in $pf$-shell [24] nuclei were also found to follow the PTD. In view of these facts, using the EGOE(2) or TBRE for CN calculations does not seem to be a pressing need. Moreover, and most importantly, the GOE model for the CN yields analytical results, which are very useful for numerical implementation in applied codes, whereas the EGOE(2) and TBRE have so far not provided such analytical expressions. Therefore, we limit our studies here to the GOE description of the CN.

The outline of this article is as follows. In Sec. [11] we present our model for studying the statistics of the partial widths with the large number of $\gamma$-decay channels taken into account. In Sec. [11], we discuss the physical parameters used to apply this model to the $n+^{95}$Mo reaction. In Sec. [IV] we show that the PTD provides an excellent

1 The residual interaction in nuclei is sufficiently strong for EGOE(2) to be a good approximation to the more realistic mixed ensemble EGOE(1) + EGOE(2).

2 A shell-model Hamiltonian can be considered as one particular realization of EGOE(1) + EGOE(2).
description of the partial width statistics for the reaction considered. In Sec. [V] we show the effect of varying the \( \gamma \)SF parameters on the simulated total \( \gamma \)-decay width distribution. In Sec. [VI] we study the effect of modified partial \( \gamma \)-decay width distributions on the total \( \gamma \)-decay width distribution. Finally, in Sec. [VII] we summarize our results.

II. STATISTICAL MODEL OF CN RESONANCES

In the absence of direct reactions, the scattering matrix (\( S \) matrix) for CN reactions is given by

\[
S_{cc'}(E) = \delta_{cc'} - 2\pi i \sum_{\mu\nu} W_{\mu c} (E - H^{\text{eff}})_{\mu\nu}^{-1} W_{\nu c}^*, \tag{1}
\]

where \( c, c' \) denote reaction channels, and \( \mu, \nu \) denote the internal CN states. Eq. (1) depends on the effective non-Hermitian Hamiltonian \( H^{\text{eff}} \) that governs the CN resonances and is given by

\[
H^{\text{eff}} = H^{\text{GOE}} + \sum_c \mathcal{P} \int dE' W_{\mu c}(E') W_{\nu c}(E') \left( \frac{E - E'}{2} \right) - i\pi \sum_{\mu\nu} W_{\mu c}(E) W_{\nu c}(E). \tag{2}
\]

Here \( H^{\text{GOE}} \) is a GOE random matrix, \( W_{\mu c}(E) \) denotes the coupling of the state \( \mu \) of a fixed basis of the internal state space to the channel \( c \) at the incident neutron energy \( E \), and \( \mathcal{P} \) is the principal-value integral. The coupling constants \( W_{\mu c}(E) \) form an \( N \times \Lambda \) real matrix \( W(E) \), where \( N \) is the dimension of the internal space of CN resonances and \( \Lambda \) is the number of open channels.

Ignoring direct reactions, the coupling matrix \( W \) in the basis of physical channels \( c \) satisfies

\[
(W^T W)_{cc'} = \delta_{cc'} \kappa_c \lambda / \pi, \tag{3}
\]

where \( \kappa_c \) are dimensionless parameters determining the strength of the coupling and \( \lambda = ND/\pi \) is the GOE energy scale parameter with \( D \) being the average spacing of GOE eigenstates in the middle of the spectrum \( \Pi \). We choose \( c = 1 \) to be the neutron channel and \( c > 1 \) to be the \( \gamma \) channels.

According to Eq. (3), the \( \Lambda \) vectors \( \tilde{W}_c \sqrt{\kappa_c \lambda / \pi} \) (\( c = 1, \ldots, \Lambda \)) are orthonormal. We choose these and additional \( N - \Lambda \) orthonormal vectors that are orthogonal to them as a basis for the CN states. The GOE is invariant under such an orthogonal transformation. In this basis, the effective Hamiltonian takes its canonical form

\[
\hat{H}^{\text{eff}} = H^{\text{GOE}} + \delta_{\mu\nu} V_\mu, \tag{4}
\]

where the first \( \Lambda \) elements of the diagonal term on the r.h.s. are

\[
V_c = \lambda \left( \frac{1}{\pi} \mathcal{P} \int_0^\infty dE' \frac{\kappa_c}{E - E'} - i\kappa_c \right). \tag{5}
\]

for \( c = 1, \ldots, \Lambda \), and \( V_\mu = 0 \) for \( \mu > \Lambda \).

The principal-value integral in Eq. (4) describes a real diagonal shift to the GOE Hamiltonian. For the reasons explained in Sec. [II] we neglect it for the neutron channel and all the \( \gamma \) channels. Consequently, the non-statistical diagonal shifts to the GOE Hamiltonian in Eq. (4) become purely imaginary

\[
V_c = -i\kappa_c \lambda. \tag{6}
\]

A. Partial widths

The effective Hamiltonian in Eq. (4) provides the most convenient way to study partial widths of the CN resonances to decay into individual channels within the framework of the statistical model. We consider the limit of isolated resonances. The resonance energies and widths are determined, respectively, by the real and imaginary parts of the eigenvalues of \( H^{\text{eff}} \).

Rewriting the \( S \) matrix in Eq. (1) in the basis used in Eq. (4), we obtain

\[
S_{cc'} = \delta_{cc'} - 2i\lambda (\kappa_c \kappa_c')^{1/2} \left( E - \hat{H}^{\text{eff}} \right)_{cc'}^{-1}. \tag{7}
\]

The matrix \( \hat{H}^{\text{eff}} \) is a complex symmetric matrix and can be diagonalized by a complex orthogonal transformation \( U \), yielding

\[
\left( U^T \hat{H}^{\text{eff}} U \right)_{\mu\nu} = \delta_{\mu\nu} \left( E_\mu - i \frac{\Gamma_\mu}{2} \right). \tag{8}
\]

Under this transformation, the diagonal \( S \)-matrix element \( S_{cc} \) becomes

\[
S_{cc} = 1 - 2\lambda \sum_{\mu} \frac{\kappa_c U_{c\mu}^2}{E - E_\mu + i(\Gamma_\mu/2)}. \tag{9}
\]

The partial width \( \Gamma_{\mu c} \) for resonance \( \mu = 1, \ldots, N \) to decay into channel \( c \) is thus given by

\[
\Gamma_{\mu c} = \frac{2\lambda \kappa_c |U_{c\mu}|^2}{(U^T U^*)_{c\mu}} \tag{10}
\]

where the term \((U^T U^*)_c = \sum_{\mu} |U_{c\mu}|^2 \) in the denominator is the squared norm of the column vector \( \mu \) of the matrix \( U \), known as the Peternann factor \( U \). The inclusion of the Peternann factor in the definition of the partial width ensures that the sum of all partial widths is equal to the total resonance width, i.e., \( \sum_c \Gamma_{\mu c} = \Gamma_\mu \). In the limit of isolated resonances, \( U \) is a real matrix and \((U^T U^*)_c = 1. \) Once the values of the coupling parameters \( \kappa_c \) are specified, one can diagonalize a large number of realizations of the effective Hamiltonian and determine the partial width distributions for the various channels.

In principle, the coupling parameters \( \kappa_c \) are determined from the average \( S \) matrix

\[
\langle S \rangle_{cc'} = \delta_{cc'} \frac{1 - \kappa_c}{1 + \kappa_c}. \tag{11}
\]
This requires a realistic optical-model calculation. Instead, we determine approximate values \( \kappa_c \) from the partial widths obtained in first-order perturbation theory. In this case, \( U_{c\mu} \) are the elements of GOE eigenvectors and as such are independent Gaussian random variables with zero mean and variance of \( 1/N \). Taking the GOE average of Eq. (10), we find

\[
\kappa_c = \frac{\pi}{2} \frac{\langle \Gamma_{\mu c} \rangle}{D_{J^\pi}}, \tag{12}
\]

where \( \langle \Gamma_{\mu c} \rangle \) is the average partial width to decay into channel \( c \), and \( D_{J^\pi} \) is the average spacing of CN resonances with spin-parity \( J^\pi \). The average partial widths can be estimated using empirical parameterizations of the strength functions for the neutron and \( \gamma \) channels, and of the level density.

B. Representative \( \gamma \) channels

Each \( \gamma \)-decay channel \( f \) is specified by the multipolarity and type (i.e., electric or magnetic) of the emitted \( \gamma \) ray and by the final state (energy \( E_f \) and spin-parity values \( J_f^\pi \)). The number of final states to which each resonance may decay is governed by the level density \( \rho(E_f, J_f^\pi) \). The average partial width to decay from a resonance \( \mu \) of energy \( E_\mu \) and spin-parity \( J^\pi \) to a channel \( f \) is divided by the average resonance spacing, is given by

\[
\frac{\langle \Gamma_{\gamma f XL} \rangle}{D_{J^\pi}} = E_\gamma^{2L+1} f_{XL}(E_\gamma), \tag{13}
\]

where \( E_\gamma = E_\mu - E_f \) is the energy of the emitted \( \gamma \) ray; \( XL \) specifies the type and multipolarity of the transition; and \( f_{XL}(E_\gamma) \) is the corresponding \( \gamma \)SF. The average total \( \gamma \)-decay width \( \langle \Gamma_{\gamma f} \rangle \) of resonance \( \mu \) is obtained by summing \( \langle 13 \rangle \) over the allowed final states

\[
\langle \Gamma_{\gamma f} \rangle = \sum_{XL} \sum_f \langle \Gamma_{\gamma f XL} \rangle = D_{J^\pi} \sum_{XL} \int_0^{E_\mu} dE_\gamma E_\gamma^{2L+1} f_{XL}(E_\gamma) \sum_{J_f^\pi} \rho(E_\mu - E_\gamma, J_f^\pi). \tag{14}
\]

Here we consider only dipole transitions \( L = 1 \) (both electric and magnetic) as these give the main contributions to the total width. As mentioned above, because of the large density of final states, it is impractical to include all of the final states accessible by dipole \( \gamma \)-ray emission. Instead, in our model each representative \( \gamma \)-decay channel \( c \) describes a group of physical \( \gamma \) channels \( f \) that are close in final energy \( E_f \). In practice, we generate a set of representative final levels whose average density is proportional to the actual level density. We set \( \kappa_c \) for each representative channel \( c \) to be

\[
\kappa_c = \frac{\pi}{2} \sum_{f \in c} \frac{\langle \Gamma_{\gamma f XL} \rangle}{D_{J^\pi}}, \tag{15}
\]

which is obtained from Eq. (12) by summing over all physical channels \( f \) in \( c \). We choose the summation in Eq. (15) such that the density of representative channels \( c \) is related to the density of physical channels \( f \) by an energy-independent constant \( G = (\Lambda - 1)/\Lambda_{\gamma f} \), where \( \Lambda - 1 \) is the total number of representative \( \gamma \) channels in our model and \( \Lambda_{\gamma f} \) is the total number of physical \( \gamma \) channels. Finally, we normalize the coupling parameters \( \kappa_c \) to satisfy

\[
\sum_{c=2}^{\Lambda} \kappa_c = \frac{\pi}{2} \frac{\langle \Gamma_{\gamma f\mu; \text{exp}} \rangle}{D_{J^\pi}}, \tag{16}
\]

where \( \langle \Gamma_{\gamma f\mu; \text{exp}} \rangle \) is the average total width determined from the experiment [11].

A proper method of coarse graining should yield the same physical results at any scale. Our method does not guarantee this; for sufficiently small \( \Lambda \), our model could yield effects that vanish as \( \Lambda \) is increased. However, we claim that, for large enough \( \Lambda \), the model results will be qualitatively the same as the physical results. Our argument is as follows. Below some coupling strength, each individual channel may be treated perturbatively. All physical \( \gamma \) channels lie below this bound. As discussed above, no single \( \gamma \) channel is strong enough to perturb the GOE dynamics. If we choose \( \Lambda \) such that the strongest representative \( \gamma \)-decay channel may be treated perturbatively, then the qualitative behavior caused by the set of representative \( \gamma \) channels should be similar to the physical case.

C. Principal-value integral

The principal-value integral on the r.h.s. of Eq. (2), also known as the Thomas-Ehrman shift, contributes a real non-statistical term to the effective Hamiltonian and thus appears to be a possible source of deviations from GOE statistics. In Ref. [6], the real shift due to the neutron channel was proposed as a possible explanation of the deviation from the PTD observed in Ref. [2]. Assuming an energy-independent coupling, it was shown that a real shift in the single-channel case leads to an energy dependence of the average partial width on the scale of the entire spectrum but locally the fluctuations are still described by the PTD [9]. Recent work showed that the real shift does not affect the PTD of the normalized widths even when a realistic energy dependence of the couplings is included [10]. In the calculations that follow, we thus ignore the real shifts in all channels.

III. APPLICATION TO n+\(^{95}\text{Mo}\)

Here we study the \(^{95}\text{Mo}(n, \gamma)^{96}\text{Mo}^*\) reaction using the statistical model discussed in Sec. [11]. The ground state of \(^{95}\text{Mo}\) has spin-parity \( 5/2^+ \). Therefore, the CN resonances in \(^{96}\text{Mo}^*\) have spin-parity of \( J^\pi = 2^+, 3^+ \) for
s-wave neutrons and $J^\pi = 1^-, 2^-, 3^-, 4^-$ for $p$-wave neutrons. We study each of these cases.

## A. Level density

Within our model, the calculation of the statistics of $\gamma$-decay widths requires realistic parameterizations of the level density and the $\gamma$SF of the compound nucleus $^{96}$Mo. For the level density, we use the back-shifted Fermi gas formula [27], also known as the back-shifted Bethe Formula (BBF), together with the spin-cutoff model [28] and the assumption of equal densities for both parities. We have

\[
\rho(E, J^\pi) = f(J) \frac{\sqrt{\pi}}{24 a^{1/4}} \frac{e^{2\sqrt{\alpha(E-\Delta)}}}{(E-\Delta)^{5/4}},
\]

where $a$ and $\Delta$ are, respectively, the single-particle level density and backshift parameters, and $f(J) = \rho(E, J)/\rho(E)$ is the spin distribution

\[
f(J) = \frac{(2J+1)}{2\sqrt{2\pi\sigma_c^2}} e^{-\frac{4(J+1)}{2\sigma_c^2}}.
\]

The parameter $\sigma_c$ in [15] is known as the spin-cutoff parameter, for which we use [29, 30]

\[
\sigma_c^2 = 0.0888 A^{2/3} \sqrt{a(E-\Delta)}
\]

with $A$ being the mass number. The values for $a$ and $\Delta$, determined by fitting the BBF to level counting data at low energies and the neutron resonance data at the neutron threshold energy [31], are given in Table I.

| Parameter | Value |
|-----------|-------|
| $a$ (MeV$^{-1}$) | 11.41 |
| $\Delta$ (MeV) | 0.85 |
| $E_G$ (MeV) | 16.2 |
| $\Gamma_G$ (MeV) | 6.01 |
| $\sigma_G$ (mb) | 185.0 |
| $\Delta_G$ (MeV) | 2.55 |
| $E_{SF}$ (MeV) | 8.95 |
| $\Gamma_{SF}$ (MeV) | 4.0 |
| $\sigma_{SF}$ (mb) | 0.4 |

We use the level density to generate a spectrum of final states, which is necessary to calculate the average partial widths for the $\gamma$ transitions [see Eq. (13)]. To generate these final states, we follow a similar procedure to that used to produce each realization in the DICEBOX code [32]. Below a threshold energy $E_{th} = 2.79$ MeV, we include a complete set of experimentally measured discrete levels [33]. Above $E_{th}$, we draw energies that follow the corresponding level density. The total number $N_{J_f}$ of final energies we draw for spin-parity class $J_f^\pi$ is given by

\[
N_{J_f} = \int_{E_{th}}^{S_n} \rho(E, J_f^\pi) dE,
\]

where the allowed final spins and parities $J_f^\pi$ are determined by the selection rules for $E1$ and $M1$ transitions, and $S_n$ is the neutron separation energy. In contrast to the DICEBOX approach, we do not average over realizations. Instead, we use only one fixed set of final energies for each spin-parity class, neglecting the fluctuations of the final states. To create the representative channels described in Sec. II B from these final states, we calculate the average partial width $\Gamma_{\gamma\pi}(XL)$ for each of these final energies. We then collect the final energies into groups, each of which consists of the same number of neighboring final energies. This group corresponds to a representative channel $c$. We calculate the parameters $\kappa_c$ by using Eq. (15) for each group.

### B. $\gamma$SF

We use the $E1$ and $M1$ $\gamma$SF of Refs. [28, 30]. For the $E1$ $\gamma$SF, we use the generalized lorentzian (GLO) model given in Eq. (6) of Ref. [30]

\[
f_{E1}(E_\gamma) = \frac{\sigma_{G1E}(E_{\gamma}, T)}{3(\pi\hbar)^2} \left[ \frac{E_\gamma \Gamma_{1E}(E_{\gamma}, T)}{(E_\gamma^2 - E_G^2)^2 + E_\gamma^2 \Gamma_{1E}(E_{\gamma}, T)^2} \right] + \frac{0.74\pi\hbar E_{\gamma}^2 T^2}{E_G^2}.
\]

Here $T$ is a temperature parameter given by $T^2 = (S_n - E_{\gamma} - \Delta_G)/a$. $S_n = 9.154$ MeV is the neutron separation energy in $^{96}$Mo, and

\[
\Gamma_{1E}(E_{\gamma}, T) = \Gamma_G - \frac{E_\gamma^2 + 4\pi^2 T^2}{E_G^2}.
\]

The $M1$ strength function is given by

\[
f_{M1}(E_\gamma) = \frac{1}{3(\pi\hbar)^2} \frac{\sigma_{SF} E_{\gamma}^2 \Gamma_{2E}^2}{(E_\gamma^2 - E_{SF}^2)^2 + E_\gamma^2 \Gamma_{2E}^2} + C.
\]

The Lorentzian term on the r.h.s. of Eq. (23) describes the spin-flip term [see Eq. (5) of Ref. [30]], and the constant $C$ is the single-particle term. The values of the parameters in Eqs. (21) and (23) are given in Table II. In Fig. 1 we show the $E1$ and $M1$ $\gamma$SF of Eqs. (21) and (23), respectively, for $^{96}$Mo$^*$. We determine the coupling $\kappa_n$ in the neutron channel from the neutron strength function. For $s$-wave resonances, we take the neutron strength function parameter $S_0 = 0.47 \times 10^{-4}$ eV$^{-1/2}$ from the RIPL-3 database [34]. The average partial width for these resonances is then given by

\[
\langle \Gamma_{\mu\nu} \rangle / D_{\mu\nu} = S_0 \sqrt{\hbar E}
\]

where $E$ is the energy of the incoming neutron. We ignore the energy dependence of the average neutron width, and take its value for $E = 10$ eV, which is at the highest end of the experimental range of Ref. [11] (see Fig. 5 of this reference). The coupling constant $\kappa_n$ is then determined from (12). For simplicity, we use for $p$-wave resonances the same coupling as for the $s$-wave resonances (see Sec. IV).
IV. PARTIAL WIDTH DISTRIBUTIONS

In our simulations, we used 100 realizations of a GOE matrix of dimension $N = 1000$ and $\Lambda = 401$ channels. These channels consist of one neutron channel, 200 $E1$ representative channels, and 200 $M1$ representative channels. For each GOE realization, we diagonalized the Hamiltonian in Eq. (4) to determine its eigenstates, which compose the columns of the matrix $U$ in Eq. (5). We took the eigenstates $\mu$ from the middle half of the spectrum to avoid unphysical effects due to the finite bandwidth of the GOE matrices. According to Eq. (10), the partial width $\Gamma_{\mu c}$ of resonance $\mu$ to decay into channel $c$ is proportional to $|U_{c\mu}|^2/(U^TU^*)_{\mu\mu}$. This term is equivalent to the projection of the normalized complex eigenvector $|\mu\rangle$ of the effective Hamiltonian onto the channel vector $|c\rangle$, i.e., $|\langle c|\mu\rangle|^2 = |U_{c\mu}|^2/(U^TU^*)_{\mu\mu}$. We define

$$g_{\mu c} = |\langle c|\mu\rangle|^2/|\langle \mu|\mu\rangle|^2,$$  \hspace{0.5cm} (24)

where the bar indicates the average value of the entire data set. According to Eq. (10), the fluctuations of the partial widths $\Gamma_{\mu c}$ are determined by the fluctuations of $g_{\mu c}$. In the following, we will refer to the normalized squared projections $g_{\mu c}$ simply as the widths. We study both the energy-dependent average widths $\langle g_{\mu c} \rangle$ and the fluctuations of the reduced widths $\hat{g}_{\mu c} = g_{\mu c}/\langle g_{\mu c} \rangle$. If the couplings to the channels do not significantly perturb the GOE behavior of the resonances, then the average squared projection for any channel in our model will be constant, i.e., independent of the real resonance energy. Moreover, the fluctuations of the squared projections will follow the PTD.

In Fig. 2 we show the average partial width $\langle g_{\mu c} \rangle$ for the neutron channel and the most strongly coupled $\gamma$ channel for initial resonances with spin-parity $1^-$. The average width is a constant across the spectrum, in agreement with the GOE expectation. The average widths are the same for the neutron and $\gamma$ channels because of the normalization in Eq. (24). The histograms in Fig. 3 show the distribution of $y = \ln x$, where $x = \hat{g}/\langle \hat{g} \rangle$ for the neutron channel and most strongly coupled $\gamma$ channel. The PTD for $y$ (solid line)

$$P(y) = \sqrt{\frac{x}{2\pi}} e^{-x/2}$$  \hspace{0.5cm} (25)

is seen to be in excellent agreement with the model calculations.

We found similar results for other $\gamma$ channels (besides the most strongly coupled one), and for other spin-parity values of the initial resonances. These results, as well as the computer codes used for the calculation, are provided in the Supplemental Material [35]. For the $p$-wave resonances, we should in principle use a weaker coupling for the neutron channel. However, for simplicity we used the $s$-wave neutron channel coupling. Since we find no deviation from the usual statistical behavior for this stronger coupling, we conclude that there will be no deviation for the more realistic $p$-wave coupling.

V. VARIATION OF THE $\gamma$ STRENGTH FUNCTION

Statistical-model results were generated in Ref. [11] for various combinations of $\gamma$SF and level-density models. We do not undertake a similarly thorough investigation here. Rather, we intend to establish whether it is possible to reproduce either the peak locations or the widths of the experimental total $\gamma$-decay width distributions within large variations of the parameters of the $\gamma$SF defined in Sec. III B. We focus on the strength function because it is less well determined than the level density.

![FIG. 2. Average widths $\langle g_{\mu c} \rangle$ from Eq. (24) for the neutron channel (black solid line) and the strongest $\gamma$ channel (red dashed line) as a function of the real part of the resonance energy. All energies are in units of the GOE parameter $\lambda$ (see Sec. III).](image-url)
Our method for generating a total \( \gamma \)-width distribution is essentially the same as that of Ref. \[11\] and follows the first step of the DICEBOX approach \[22\]. We use as input the \( \gamma \)SF parameters and a set of final states with allowed values \( J^\pi \) determined by the selection rules and their corresponding level densities. We then calculate a total \( \gamma \)-decay width by summing over the partial widths for transitions to each of the final states \( f \). The partial widths for resonances of spin-parity \( J^\pi \) to decay with \( \gamma \) radiation of multipolarity \( XL \) are given by

\[
\Gamma_{\gamma f XL}^J = \langle \Gamma_{\gamma f XL}^\pi \rangle x_f^2,
\]

where \( x_f \) is drawn from a normal distribution with zero mean and unit variance. \( x_f^2 \) is thus distributed according to the PTD. This procedure is repeated 1000 times to obtain a set of total widths.

We vary the parameters \( E_G \), \( \Gamma_G \), and \( \sigma_G \) of the \( E1 \) \( \gamma \)SF \[21\] by factors of 2 in either direction to make them greater or smaller than their values given in Table \[1\]. These variations dramatically change the strength of the \( E1 \) component of the \( \gamma \)SF, making the \( M1 \) component either more or less significant relative to the \( E1 \) component.

We find that these variations of the \( \gamma \)SF have no significant effect on the widths of the total \( \gamma \) decay width distributions. We show a representative result in Fig. 4 for the \( 1^- \) resonances. We plot the cumulative fraction, i.e., the fraction of total widths greater than a given width \( \Gamma_{\gamma, \text{tot}} \). The simulated partial widths have been normalized such that their sum reproduces the average value of the experimentally observed total width. This normalization does not affect the relative contributions of the various partial widths and thus does not change the width of the distribution. Our results, shown in Fig. 4, exhibit only weak dependence on the parameter \( \Gamma_G \) and are compared with the experimental cumulative fraction measured in Ref. \[11\]. We conclude that the experimental distribution of the total \( \gamma \)-decay width \( \Gamma_{\gamma, \text{tot}} \) is significantly broader than the theoretical distribution obtained in the statistical model, and cannot be reproduced by a reasonable variation of the \( \gamma \)SF parameters.

The average total \( \gamma \) width, i.e., the peak of the total \( \gamma \)-decay width distribution, is sensitive only to the level density and the \( \gamma \)SF [see Eq. \[14\]]. In Ref. \[11\], there were large discrepancies between the simulated and experimental average total \( \gamma \)-decay widths. We find such discrepancies for our choice of level density and \( \gamma \)SF as well. In Table \[3\] we list the average total widths calculated for our baseline parameter values versus the experimental values for all spin-parity classes of resonance.

The single-parameter variations in the \( \gamma \)SF we considered above also influence the average total width. For any given spin-parity class of resonances, we are able to reproduce the average width by varying one parameter. For instance, for the \( 2^+ \) resonances, multiplying the parameter \( \Gamma_G \) of Table \[1\] by a factor \( f_G = 1.13 \) brings the average total width into excellent agreement with the experimental value \( \Gamma_{\gamma, \text{tot}}^2 = 206 \) meV. However, none of these simple parameter adjustments reproduces simultaneously the average total \( \gamma \)-decay widths for all the spin-parity classes.

| \( J^\pi \) | \( \gamma \)-decay width (meV) |
|-----|-----------------|
|     | \( \langle \Gamma_{\gamma, \text{sim}} \rangle \) | \( \langle \Gamma_{\gamma, \text{exp}} \rangle \) |
| \( 2^+ \) | 165.5 | 157.5 |
| \( 3^+ \) | 191.2 |
| \( 1^- \) | 206 (31) | 240 (58) | 670 (225) |

The simulated results are calculated using the baseline parameter values for the strength functions.
We also find that for all choices of the $\gamma$SF parameters described above, the partial width fluctuations follow the PTD, similar to what is shown in Fig. 3. The results for the various cases are included in the Supplemental Material.

VI. SENSITIVITY TO DEVIATIONS FROM PTD

In Sec. [IV] we showed that realistic level-density and $\gamma$SF parameterizations do not lead to any violation of the PTD for the partial $\gamma$-decay widths. It is interesting, however, to find out whether the experimental results may be interpreted as evidence of PTD violation in some channels. If this were to be the case, then it would indicate a problem with the conventional statistical-model approach. The authors of Ref. [11] used a $\chi^2$ distribution with $\nu = 0.5$ degrees of freedom instead of the PTD but could not obtain agreement with the data. However, when the PTD is violated, the partial width distribution is not described well by a $\chi^2$ distribution in $\nu$ degrees of freedom. In this section, we examine the effect of a realistic PTD violation on the simulated total width distribution.

We obtain a partial width distribution that deviates from the PTD using the model of Ref. [6]. In this model, the effective Hamiltonian of Eq. (4) is replaced by

$$H_{\mu\nu}^{\text{eff}} = H_{\mu\nu}^{\text{GOE}} + Z\delta_{\mu1}\delta_{\nu1}.$$  (27)

To obtain a large PTD violation, we use a relatively large imaginary value $Z/\lambda = -0.8i$, as was done in Ref. [6]. We then examine the distributions of the quantities $g_{\mu1}$ and $g_{\mu2}$, i.e., the normalized squared projections of the eigenvectors $|\mu\rangle$ of $H_{\mu\nu}^{\text{eff}}$ onto the first and second basis vectors [see Eq. (24)]. Following the approach described in Sec. [IV] we can identify the basis vectors $|1\rangle$ with the neutron channel and $|2\rangle$ with a $\gamma$ channel. As before, we include only the middle half of the GOE spectrum to avoid edge effects. It was shown in Ref. [6] that the distribution of partial neutron widths $g_{\mu1}$ is substantially different from the PTD in this case. Interestingly, we find that the distribution of the width $g_{\mu2}$ for a $\gamma$ channel is also significantly modified. Thus, sufficiently large non-statistical terms in the effective Hamiltonian can cause a ‘cross-channel’ effect. The resulting distributions of the logarithms of the normalized squared projections are shown in Fig. 4 along with the PTD. The figure makes it clear that neither of the modified distributions is well-described by a $\chi^2$ distribution.

We use these modified distributions to generate partial width fluctuations in our simulation of the total width distribution described in Sec. [V]. Specifically, we replace the quantity $x y^2$ in Eq. (26) with a number drawn from one of these modified distributions. In Fig. 5 we compare the simulated total $\gamma$-decay width distributions obtained when the partial width distribution is either the PTD or one of the above modified distributions with the experimental data for the $1^-$ resonances. The modified partial width distributions widen the total $\gamma$-decay width distribution slightly, but not sufficiently to obtain agreement with the data. Moreover, variations of the $\gamma$SF parameters in the case of the modified distributions also do not broaden significantly the $\gamma$-decay width distributions. Thus, we find no evidence that a modification of the PTD alone can account for the broader total $\gamma$-decay width fluctuations that are observed in the experiment.

The above conclusion is not unexpected for the following reason. The total $\gamma$-decay width is the sum of independently distributed random variables, i.e., the partial width fluctuations $x y^2$ of Eq. (26), each weighted by the appropriate average partial width. If a sufficiently large number of final states contribute roughly equally to the total width, then the central-limit theorem guarantees that the total $\gamma$-decay width distribution will be a Gaussian with a very narrow variance. This conclusion holds regardless of the underlying distribution of the partial widths, provided that the distribution does not violate the assumptions of the theorem. As is evident from our results, partial width distributions that are derived in the framework of the the statistical model are consistent with the central-limit theorem.

The total $\gamma$-decay width distribution can be broad only if there exist a small number of $\gamma$ channels coupled strongly enough to overcome the restriction of the central-limit theorem. This idea is consistent with the ‘doorway’ model of Koehler et al. in Ref. [11], in which the strengths of transitions to all low-lying final states were multiplied by a factor of 25. The results of this model were in good agreement with the experimental data. However, such a drastic increase in the $\gamma$ transition strength to low-lying states is outside the relatively
large range of conventional γSF models that we have explored above and thus demands a physical explanation. In particular, such an enhancement would constitute a violation of the generalized Brink-Axel hypothesis [36], which states that the strength of a γ transition is independent of the details of the initial and final states at low excitation energies. Recently, an experiment measured the photo-absorption strength for 1− states of the 96Mo CN and found agreement with γ decay experiments [36]. This indicates that the generalized Brink-Axel hypothesis holds to a fairly good approximation in this nucleus and casts doubt on the existence of the sort of enhancement discussed above.

VII. CONCLUSION

We have presented a model that is based on a statistical description of the CN but takes into account the many γ channels coupled to the CN in a semi-realistic way. We applied the model to the 95Mo(n,γ)96Mo* reaction. Using empirical parameterizations for the level density and γSF, we found that the PTD provides an excellent description of the partial widths for both the neutron and the γ-decay channels, in agreement with the traditional prediction of the statistical model. This result holds for all spin-parity values of the CN resonances. We conclude that the net effect of the large number of γ-decay channels does not perturb the GOE statistics of the CN and cannot explain the experimental results of Ref. [11]. Although it is usually assumed that the γ-decay channels have little effect on the GOE statistics of the resonances, this has not previously been demonstrated within a realistic model.

Furthermore, we find that the width of the total γ-decay width distribution is insensitive to large parameter variations of the E1 γSF. In particular, the measured width of the distribution of total γ-decay widths cannot be reproduced. We also find that deviations of the partial-width distributions from PTD (which can in principle occur for sufficiently strong coupling of the neutron channel) do not significantly broaden the total γ-decay width distributions. This finding follows from the central-limit theorem and the fact that, for common parameterizations of the level density and γSF, many γ channels contribute similarly to the total width.

The only way to overcome the limitation of the central-limit theorem is to dramatically increase the γ transition strength to a small group of channels, as investigated in Ref. [11]. However, such an enhancement would violate the generalized Brink-Axel hypothesis and consequently contradict recent experimental results [36].

In conclusion, our analysis shows that the results of Ref. [11] cannot be explained within the statistical-model framework. Given the fundamental importance of the statistical model for nuclear-reaction modeling, this discrepancy should motivate further experimental investigations, both to verify the findings of Ref. [11] and to test the GOE description of the compound nucleus.

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