Supporting Information

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High-Temperature Thermal Transport in Porous Silica Materials: Direct Observation of a Switch from Conduction to Radiation

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Synthesis and Characterization of SiO$_2$ and TiO$_2$ Hollow Particles

Materials. Ammonium hydroxide solution (NH$_4$OH, Sigma-Aldrich GmbH, 30-33 %), 2,2’-azobis-(isobutyramidine) dihydrochloride (AIBA, Sigma-Aldrich GmbH, 97 %), ethanol abs. (Sigma-Aldrich GmbH, ≥ 99.8 %), 2-methacryloxyethyltrimethylammonium chloride (MTC, Sigma-Aldrich GmbH, 75 % soln. in water), polyvinylpyrrolidone (PVP, Sigma-Aldrich GmbH), styrene (Sigma-Aldrich GmbH, > 99 %), tetraethyl orthosilicate (TEOS, Sigma-Aldrich GmbH, 98 %) titanium butoxide (TBT, Sigma Aldrich GmbH, 97 %) were used as received. Millipore water was taken from a Millipore Direct Q3UV unit (Merck Millipore). 2,2’-Azobis(2-methylpropionitril) (AIBN, Sigma-Aldrich GmbH) was recrystallized from ethanol before use.

Microscopy. Scanning electron microscopy (SEM) was performed using a Zeiss Ultraplus instrument using acceleration voltages of 3 kV. InLens and Everhard-Thornley detectors were used. A MATLAB circle detection function was used to evaluate the diameter of the particles. Transmission electron microscopy (TEM) measurements were performed with a JEOL JEM-2200FS field emission energy filtering transmission electron microscope (FE-EFTEM) operated at an acceleration voltage of 200 kV. Zero-loss filtered micrographs (∆E ~ 0 eV) were recorded with a bottom mounted CMOS camera system (OneView, Gatan) and processed with DM 3.3 image processing software (Gatan).

Polystyrene template particles. 217 nm particles (see Fig. S1) were synthesized via emulsifier free emulsion polymerization.[1] For the synthesis, 1.8 g PVP (360 kg/mol), 235 ml Millipore
water, 26 ml styrene, and 25 µl MTC were added to a 500 ml three-neck flask equipped with a
gas inlet and reflux condenser. The emulsion was stirred at a stirring speed of 850 rpm using a
large egg-shaped magnetic stirrer bar and heated to 70 °C under a slight argon flow. After
60 min 0.6 g AIBA, dissolved in 5 ml Millipore water, was added to initiate the polymerization.
After the nucleation the stirring speed was reduced to 450 rpm and the reaction was allowed to
continue over night. The polymerization was stopped by exposing the dispersion to ambient air
and filtrated using a 125 µm nylon filter sieve. No further purification was required.

The 845 nm particles (Figure S1) were synthesized via dispersion polymerization.[2] 12 g PVP
(40 kg/mol), 194 mL of ethanol, 40 ml Millipore water, 12 ml styrene and 1.2 g AIBN were
added to a 1 l three-necked flask equipped with a reflux condenser and a gas inlet. The solution
was degassed while stirring with an egg-shaped stirring bar with a speed of 150 rpm. After
30 min, the mixture was heated to the reaction temperature of 70 °C. After 90 min 194 ml
ethanol, 12 ml styrene, and 450 µl MTC were premixed in an Erlenmeyer flask and added to
the reaction. The reaction was carried out overnight under a slight argon flow. The
polymerization was stopped by exposing the dispersion to ambient air and filtrated using a
125 µm nylon filter sieve. No further purification was required.
**Figure S1.** Histograms and SEM images of polystyrene particles that were used as template for the SiO$_2$ and TiO$_2$ hollow particles.

**SiO$_2$ and TiO$_2$ hollow particles.** A modified Stöber process was used for the synthesis of the silica shell to get PS@SiO$_2$ core-shell particles.$^{[3]}$ For the synthesis, 40 ml (60 ml) dispersion of the 217 nm (845 nm) particles were diluted with 280 ml (450 ml) ethanol and 1/10 of 13.44 ml (1.8 ml) TEOS and stirred at 400 rpm at room temperature. After 20 min equilibration time, 20.8 ml (31 ml) NH$_4$OH solution was added. The remaining TEOS was added in nine more steps in time intervals bigger than 15 min and the reaction was stirred at room temperature over night. The particles were centrifuged and washed twice with ethanol and three times with Millipore water for purification.

PS@TiO$_2$ core-shell particles were prepared using a method described by Cheng et al. and Lechner et al.$^{[2]}$. 40 ml dispersion of 845 nm particles and 234 ml ethanol were added to an Erlenmeyer flask equipped with a septum. The dispersion was stirred at 350 rpm with a magnetic stirrer bar. After 10 min of degassing, a solution of 4.8 ml TBT in 20 ml ethanol was
added within 30 min using a syringe pump. After another 30 min the stirrer was stopped and the dispersion was allowed to age for 24 h. The core-shell particles were washed three times with ethanol for purification.

In order to remove the PS core of the core-shell particles, the assembled samples were calcinated either in air or inert argon gas atmosphere using a modified temperature profile of Schroden et al.\textsuperscript{[2b, 4]} A heating rate of 2 K/min was used for all heating steps. The samples were heated to 300 °C, followed by an isothermal step of 2 h. They were then heated to 400 °C, followed by an isothermal step of 5 h. After that the samples were heated to 500 °C or 925 °C, followed by an isothermal step of 5 h. Finally, the samples were cooled down to room temperature. The resulting hollow particles can be seen in Figure S2.

| SiO\textsubscript{2} small | SiO\textsubscript{2} big | TiO\textsubscript{2} |
|---------------------------|------------------------|----------------|
| $d_{\text{int}} = 278 \pm 4$ nm | $d_{\text{int}} = 882 \pm 8$ nm | $d_{\text{tot}} = 734 \pm 14$ nm |
| $d_{\text{shell}} = 29 \pm 2$ nm | $d_{\text{shell}} = 31 \pm 2$ nm | $d_{\text{shell}} = 37 \pm 3$ nm |

Figure S2. TEM images of SiO\textsubscript{2} and TiO\textsubscript{2} hollow particles after calcination at 500 °C.
**Figure S3** shows sideview SEM images of the calcinated colloidal glasslike structures. All structures stayed stable at calcination temperatures up to 925 °C. The broken shells of the TiO$_2$ particles came from cutting the sample in half with a knife.

**Figure S3.** Sideview SEM images of all colloidal glasslike structures after calcination at 500 °C or 925 °C.
**X-ray powder diffraction of TiO$_2$ samples**

X-ray powder diffraction was performed to investigate the titania phase of the hollow particles before and after LFA measurements at 925 °C. X-ray powder diffraction patterns for the TiO$_2$ samples were recorded on an Empyrean diffractometer in Bragg-Brentano-geometry (PANalytical B.V.; the Netherlands) using Cu-K$_\alpha$ radiation ($\lambda = 1.54187$ Å). The samples were first calcinated in air atmosphere at 500 °C and then calcinated in inert gas atmosphere up to 925 °C. The results are shown in **Figure S4**. Both samples consist of 100 % anatase.

**Figure S4.** XRD spectra of TiO$_2$ hollow particles calcinated under different conditions, as well as peak evaluation for the 500 °C particles calcinated in air.
Transparent fit of radiation data

Figure S6 shows measurement data with the transparent fit of the Prothues software (Netzsch) for hollow and solid particle and bulk quartz samples. It can be seen that the fit model was working perfectly for the bulk sample. However, it failed to fit the particle samples. The fit started getting inaccurate at 625 °C for the solid particle samples and 525 °C for the hollow particle samples. At 825 °C (solid) and 725 °C (hollow), the fit model could not fit the data anymore. This fit problem originated from the different peak shapes of the radiation peak, as shown in Figure S5. The bulk signal had a sharp initial temperature increase, followed by a decrease. The hollow particle signal had a diffusive-like shape. The solid particle signal is in between with a sharp initial increase, followed by a diffusive-like increase.

![Figure S6.](image)

Figure S5. Peak shape of radiation peak of hollow and solid particles, and bulk quartz glass from 425 to 925 °C.
**Figure S6.** Transparent fit on LFA data of hollow and solid particles, and bulk quartz. The colored lines show the measurement signal at 425 – 925 °C, the dashed lines show the corresponding fit.
Double-diffusive model of LFA data of hollow particle samples

From the double-diffusive model, it was possible to determine the thermal diffusivity of the radiation and conduction process of the hollow particle samples. The results are shown in Figure 3 and Figure S7. The radiation diffusivity was increasing with increasing temperature. An increasing radiative thermal conductivity is known from the geophysical science and measurements and calculations on olivine samples, where the conductivity is calculated from high-temperature absorption data.[5] The conductive thermal diffusivity is almost constant in both samples. It has to be noted that the values do not represent the actual values because of two reasons. First, at higher temperatures, the conduction peak has almost vanished and the fit may be inaccurate. Second, the double-diffusive model is simplified and does not take into account reflection of heat at the sample-graphite-boundary. Therefore, an accurate evaluation of the conductive thermal diffusivity was not achieved with this fit.

Figure S7. Double diffusive model applied to temperature-dependent measurements of big hollow particle samples.
Figure S8. Thermal diffusivity of radiation and conduction of hollow particle samples, calculated by the double diffusive model.
TiO2 hollow particle sample

Similar to the 8 µm solid SiO2 particles, the TiO2 hollow particles have almost no transmission in the IR region (Figure S9). Because of the missing bandgap between 3 and 5 µm, radiation does not contribute to the thermal transport in the sample. Therefore, no radiation peak is visible in the measurements (Figure S10). The sample consisting of particles with a diameter of 734 nm and a shell thickness of 37 nm, has a density of 0.385 g/cm³, a thermal diffusivity of 0.613 mm²/s, and a thermal conductivity of 0.163 W/mK at 25 °C.

Figure S9. Transmission and absorbance spectra of TiO2 hollow particles in the IR region.

Figure S10. Temperature-dependent LFA measurement signals of TiO2 hollow particle samples that were calcinated at 500 °C in air or inert atmosphere before the measurement.
Inert calcined SiO₂ hollow particle samples

Figure S11 shows a picture of whole samples of SiO₂ hollow particles after calcination in air and inert atmosphere. The color of the inert calcinated samples comes from carbon residues in the hollow particles, as described by Wang et al.\textsuperscript{[6]} The samples have totally different optical properties in the visible region, however in the IR region the transmission is barely reduced (Figure S12). Therefore, the thermal behavior, especially the intensity of the radiation peak, does not change, which can be seen in Figure S13.

Figure S11. Picture of colloidal glasslike assemblies of SiO₂ hollow particles after calcination at 925 °C in air and inert atmosphere.

Figure S12. Optical properties of air and inert calcinated colloidal assemblies of hollow SiO₂ particles in the UV-Vis and IR region.
Figure S13. Temperature dependent LFA measurement signals of SiO$_2$ hollow particle samples that were calcined in argon atmosphere at 925 °C before the measurement. The time is normalized by the characteristic thermal response time $t_c$. 
Mean particle distance for multilayer model

To get the mean interparticle distance in colloidal glasses, an MD simulation of particles with a diameter of 1 µm with a compression rate of $10^{-2}$ has been performed. The resulting packing density was 61.1 %, which is comparable to the experimental data. The 3D radial distribution function of the simulated structure was calculated, according to Kopera et al.⁷ An interparticle distance of 63 nm was determined. Thus, we assumed to have a mean distance of 6 % of the total particle diameter for all samples.

Figure S14. 3D radial distribution function of a colloidal glass with a packing density of 61.1 % and particles with a diameter of 1 µm.
Transmission coefficient: influence of number of particle layers

In Figure S15 we show the Transmission coefficient $T_p^{12}$ for the p-polarized waves (which include possible surface wave contributions) in the $\lambda - \kappa$ plane choosing $\delta = 20$ nm and the multilayer structure, which models the structure of solid 353 nm silica particles. It can be seen how the number of the Fabry-Pérot modes is increasing when increasing the number of layers from $N = 9$ ($d = 2.5$ $\mu$m) to $N = 3121$ ($d = 827$ $\mu$m) forming Bloch band structures $^8$ due to the periodicity of the structure. The same behavior can be expected for highly ordered nanoparticle structures. It can be further seen that when increasing the number of layers from $N = 9$ to $N = 3121$, only waves in the transparency window of glass for $\lambda < 5$ $\mu$m can reach the second graphite layer, and all the other waves are damped out due to the losses in the glassy part of the structure. The waves reaching the second graphite layer also include evanescent contributions because the wavevector in glass can be larger than in vacuum. In the transparency window, the lateral wave vector $\kappa$ is limited by $\sqrt{R(\varepsilon_{SiO_2})k_0} \approx 1.4k_0$ due to the material and those waves which can tunnel between the layers of the multilayer structure will finally heat up the graphite layer. It is interesting to note that there is no significant surface wave contribution for $h^{12}$ in the restrahlen band $\lambda \in [8\mu$m: 9.3$\mu$m]. This is due to the fact that damping of the coupled surface modes is too strong to ensure an efficient coupling over long distances.
Figure S15. Transmission coefficients of a model representing solid SiO$_2$ particle glasses with an increasing number of layers, N, from 9 to 3121.
Heat transfer coefficient

The radiative heat flux in the multilayer systems was calculated by using the theory of fluctuational electrodynamics.

The heat transfer coefficient describing the radiative heat flow $h^{12}$ from the graphite layer 1 at temperature $T_1$ and the heat flow $h^{23}$ from the intermediate film 2 at temperature $T_2$ towards the second graphite layer 3 at temperature $T_3$ are given by the expressions:\[^{[10]}\]

$$h^{12/23} = \int_0^\infty \frac{d\omega}{2\pi} \sum_{j=s,p} \int \frac{d^2\kappa}{(2\pi)^2} \frac{\partial \Theta(T)}{\partial T} J_{j}^{12/23}(\omega, \kappa).$$

Here $\Theta(T) = \hbar \omega/(\exp(\hbar \omega/k_B T) - 1)$ is the mean energy of a harmonic oscillator at temperature $T$; $k_B$ is the Boltzmann and $\hbar$ the reduced Planck constant. The transmission coefficients $J_{j}^{12/23}$ ($j = s, p$) are the functions of the frequency $\omega$ and the lateral wavevector $\kappa$ for s- and p-polarized waves and can have values between 0 and 1. They contain the contributions of propagating waves ($\kappa < k_0$) but also evanescent waves ($\kappa > k_0$) in vacuum introducing the vacuum wavevector $k_0 = \omega/c$ and the light velocity $c$. The complex expressions can be found in Messina et al.\[^{[9]}\] However, they mainly depend on the distances $\delta$ and the layer thickness $d$ as well as on the optical properties of the three layers. For example they depend on the corresponding amplitude transmission and reflection coefficients so that they can be evaluated as soon as the optical properties of the three layers are known. For the intermediate multilayer structure, we used the standard S-matrix method to calculate the needed reflection and transmission coefficients. The optical data for the permittivity of silica and graphite are taken from literature values in Ref.\[^{[10]}\]
**Figure S16.** Sketch of the three-layer structure. Layer 1 and 3 are graphite sheets, and the intermediate layer 2 is the nanoparticle glass of thickness $d$.

The radiative heat flux $\Phi$ (W/m$^2$) to the graphite layer 3 is within this model simply given as

$$\Phi = h^{12}(T_1 - T_2) + h^{23}(T_2 - T_3).$$

That means that in the first instance where the graphite layer 1 is heated by the laser flash, we have $T_1 > T_2 = T_3$ so that $h^{12}$ describes the radiative coupling between the first and second graphite layers. Therefore the modes contributing to the radiative heat flow between the two graphite layers are mainly determined by $\tau_j^{12}$ ($j = s, p$).
Influence of coupling surface modes on transmission of heat radiation

To clarify the role of the surface mode contribution to the radiative heat flux, we have made exact numerical calculations using the open-source program SCUFF-EM developed by Homer Reid at MIT. It uses a boundary element method to determine the transmission functions $F_{ij}(\omega)$ between two objects $i$ and $j$ so that the exchanged power $P$ between these objects due to thermal radiation (far- and near-field) can then be determined by

$$P_{ij} = \int_0^\infty d\omega \left( \Theta(T_i) - \Theta(T_j) \right) F_{ij}(\omega).$$

In Figure S17 we show the transmission functions for the heat flux between two, three, and four nanoparticles of diameter 800 nm and an interparticle distance of 50 nm. As can be expected, the heat flux between adjacent silica nanoparticles is dominated by the surface mode resonance at $\lambda \approx 9 \, \mu m$, and it becomes weaker and weaker when considering the coupling between nanoparticles with one or two nanoparticles in between them. When replacing the outer nanoparticles in the chain of 3 and 4 nanoparticles with a graphite particle, it can be nicely seen that the transmission of heat radiation will be mainly in the transparency region below 8 $\mu m$, i.e., below the reststrahlen band. The surface mode resonance contribution becomes already relatively weak for a chain of 4 nanoparticles, and it can be expected that it is negligible for chains of 3000 to 4000 nanoparticles.

Figure S17. SCUFF-EM calculation of transmission functions for heat flux between first and last particle in a row of two to four particles.
Influence of material thermal conductivity on radiation

We have used a set of 14 coupled differential equations to describe the temperature evolution of the system. The first and last graphite layers are in this model split into a \( d_{\text{surf}} = 200 \text{ nm} \) thin surface layer (the skin depth of graphite for \( \lambda = 1 \mu\text{m} – 10 \mu\text{m} \) is 100 nm – 200 nm) with temperatures \( T_1 \) and \( T_{14} \) standing for the surface temperatures of the laser flash heated first layer and the measured surface temperature of the last layer, and a layer of thickness \( d_c \) with temperatures \( T_2 \) and \( T_{13} \). The glass film is divided into 10 layers of thickness \( d_{\text{SiO}_2} \) with temperatures \( T_3, T_4, \ldots, T_{12} \). In this model, we assume that the adjacent layers are coupled via thermal conduction and that the graphite layers are coupled to the 3 glass layers by \( h^{23} \) and between each other by \( h^{12} \). Then using Newton’s law of cooling, we have the set of equations

\[
C_{p, \text{surf}} \frac{dT_1}{dt} = h_{\text{cond}, c} (T_2 - T_1) + h_{\text{rad}} (T_m - T_1) + h_{\text{cond}, \text{lat}} (T_m - T_1),
\]

\[
C_p, c \frac{dT_2}{dt} = h^{12} (T_{13} - T_{\text{mean}}) + h^{23} (T_3 - T_2) + h_{\text{cond}, \text{SiO}_2} (T_3 - T_2)
\]

\[
+ h_{\text{cond}, c} (T_1 - T_2) + h_{\text{cond}, \text{lat}} (T_m - T_2),
\]

\[
C_{p, \text{SiO}_2} \frac{dT_3}{dt} = h_{\text{cond}, \text{SiO}_2} (T_2 - T_3) + h_{\text{cond}, \text{SiO}_2} (T_4 - T_3) + h^{23} (T_2 - T_3)
\]

\[
+ h_{\text{cond}, \text{lat}} (T_m - T_3),
\]

\[
C_p, \text{SiO}_2 \frac{dT_4}{dt} = h_{\text{cond}, \text{SiO}_2} (T_3 - T_4) + h_{\text{cond}, \text{SiO}_2} (T_5 - T_4) + h_{\text{cond}, \text{lat}} (T_m - T_4),
\]

\[... = ...
\]

\[
C_{p, c} \frac{dT_{13}}{dt} = h^{12} (T_2 - T_{\text{mean}}) + h^{23} (T_{12} - T_{13}) + h_{\text{cond}, \text{SiO}_2} (T_{12} - T_{13})
\]

\[
+ h_{\text{cond}, c} (T_{14} - T_{13}) + h_{\text{cond}, \text{lat}} (T_m - T_{13}),
\]

\[
C_{p, \text{surf}} \frac{dT_{14}}{dt} = h_{\text{rad}} (T_m - T_{14}) + h_{\text{cond}, c} (T_{13} - T_{14}) + h_{\text{cond}, \text{lat}} (T_m - T_{14}).
\]

In these equations \( T_m \) is the ambient temperature, \( T_{\text{mean}} \) is the mean temperature in the glass medium, and
\( C_{\rho, C, \text{surf}} = C_C \rho_C d_{\text{surf}} \quad C_{\rho, C} = C_C \rho_C d_C \quad C_{\rho, \text{SiO}_2} = C_{\text{SiO}_2} \rho_{\text{SiO}_2} d_{\text{SiO}_2} \)

are the products of the heat capacities of graphite and silica, the corresponding mass densities (Table S1), and the layer thicknesses where \( d_{\text{SiO}_2} = d/10 \), \( d_{\text{surf}} = 200 \text{ nm} \), and \( d_C \) can vary from \( 10 \mu m - 30 \mu m \). The heat transfer coefficients \( h^{12} \) and \( h^{23} \) are defined above \( h_{\text{rad}} \) is the htc of the graphite layer into vacuum, and takes the radiative cooling of the surface layers into account. The heat transfer coefficients caused by thermal conduction in graphite, in silica, or between graphite and silica are defined as

\[
\begin{align*}
    h_C & = \frac{\kappa_C}{d_{\text{surf}} + d_C}, \\
    h_{\text{SiO}_2} & = \frac{\kappa_{\text{SiO}_2}}{d_{\text{SiO}_2}}, \\
    h_{C-SiO_2} & = \frac{1}{h_{\text{SiO}_2}} + \frac{1}{h_C}.
\end{align*}
\]

The values of the thermal conductivities \( \kappa_C \) and \( \kappa_{\text{SiO}_2} \) are taken from literature (Table S1). Since the Kapitza resistance between graphite and silica is extremely small, we have neglected it. \( h_{\text{cond, lat}} \) is a parameter which can be used to model lateral conductive losses in the sample.

This model neglects the thermal radiation channels within the graphite and silica media. That means it is assumed that within the materials conduction is more important than radiation. The numerical results also suggest that \( h^{23} \) has negligible impact for the used parameters underlining the assumption that the radiative coupling between adjacent layers by the surface mode interaction is small compared to the conductive one. Note that in this model, there are only three more or less unknown parameters: the conductivity of the glass medium \( \kappa_{\text{SiO}_2} \) which is actually measured by the LFA (i.e. by fitting the model to the data), the thickness of the graphite layers which are estimated to be \( 20 \mu m - 30 \mu m \) (dependent on the number of coated layers), and the lateral conductivity \( h_{\text{cond, lat}} \).
Table S1. Specific heat capacity, $C_P$, and thermal conductivity, $\kappa$, of quartz glass and graphite. $C_P$ of quartz glass was taken from Smyth et al.\textsuperscript{[12]}, $\kappa$ was calculated using the thermal diffusivity from our measurements on bulk quartz (Figure 2) and a density of 2.201 g/cm$^3$. The $C_P$ of graphite was calculated according to Butland et al.\textsuperscript{[13]}, $\kappa$ was calculated according to McEligot et al.\textsuperscript{[14]}

| Temperature / °C | Quartz $C_P$ / J/gK | Quartz $\kappa$ / W/m*K | Graphite $C_P$ / J/gK | Graphite $\kappa$ / W/m*K |
|------------------|----------------------|-------------------------|----------------------|-------------------------|
| 25               | 0.744                | 1.439                   | 0.713                | 131.338                 |
| 125              | 0.879                | 1.593                   | 0.980                | 121.156                 |
| 225              | 0.981                | 1.700                   | 1.193                | 111.718                 |
| 325              | 1.057                | 1.793                   | 1.360                | 103.023                 |
| 425              | 1.110                | 1.870                   | 1.491                | 95.072                  |
| 525              | 1.153                | 1.952                   | 1.596                | 87.865                  |
| 625              | 1.189                | 2.041                   | 1.681                | 81.402                  |
| 725              | 1.217                | 2.137                   | 1.751                | 75.683                  |
| 825              | 1.242                | 2.238                   | 1.809                | 70.707                  |
| 925              | 1.263                | 2.345                   | 1.857                | 66.476                  |

In Figure S18 we show the simulation of the LFA experiment by plotting the temporal temperature evolution $\Delta T_{14} = T_{14} - T_m$ of the second graphite surface temperature for $T_m = 925 °C$ with initial conditions $T_1 = T_m + 10K$ and $T_2 = T_3 = \cdots = T_{14} = T_m$ at time $t = 0$ ms and $d_c = 20 \mu$m. The values for $h^{12}$ and $h^{23}$ are calculated for the multilayer structure replacing the solid 353 nm nanoparticles with $N = 3121$ layers (so that $d = 827 \mu$m) and $\delta = 20$ nm. To show the impact of the conductivity of the intermediate glass layer, we have varied $k_{SiO_2}$. It can be seen that the relative strengths of the peaks, which are due to heating by thermal radiation or conduction, highly depend on the thermal conductivity of the glass layer.
Figure S18. Multilayer model for heating dynamics calculations of a solid SiO$_2$ colloidal structure with a particle diameter of 353 nm (left). Heating dynamics with lateral losses of the multilayer model at a temperature of 925 °C (right).
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