QCD and Heavy Ions

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This short paper is an attempt to describe a theorist’s view of the goals of relativistic heavy ion program which has just entered the collider era. These goals are centered around understanding the properties and the critical behavior of Quantum Chromo–Dynamics (QCD) in the non–linear regime of high color field strength and high parton density. Some of the current theoretical challenges are highlighted, and the place of heavy ion research in the broader context of modern particle and nuclear physics is discussed.

1. WHAT IS QCD?

Strong interaction is, indeed, the strongest force of Nature. It is responsible for over 80% of the baryon masses, and thus for most of the mass of everything on Earth and in the Universe. Strong interactions bind nucleons in nuclei, which, being then bound into molecules by much weaker electro-magnetic forces, give rise to the variety of the physical World. Quantum Chromo–Dynamics is the theory of strong interactions, and its practical importance is thus undeniable. But QCD is more than a useful tool – it is a consistent and very rich field theory, which continues to serve as a stimulus for, and testing ground of, many exciting ideas and new methods in theoretical physics.

1.1. The structure of QCD

So what is QCD? From the early days of the accelerator experiments it has become clear that the number of hadronic resonances is very large, suggesting that all hadrons may be classified in terms of a smaller number of (more) fundamental constituents. A convenient classification was offered by the quark model, but QCD was not born until the hypothetical existence of quarks was not supplemented by the principle of local gauge invariance, previously established as the basis of electromagnetism. The resulting Lagrangian has the form

\[ \mathcal{L} = -\frac{1}{4} G^{a}_{\mu\nu} G^{a}_{\mu\nu} + \sum_{f} \bar{q}_{f}^{a}(i\gamma_{\mu} D_{\mu} - m_{f})q_{f}^{a}; \] (1)

the sum is over different colors \( a \) and quark flavors \( f \); the covariant derivative is \( D_{\mu} = \partial_{\mu} - igA^{a}_{\mu}t^{a} \), where \( t^{a} \) is the generator of the color group \( SU(3) \), \( A^{a}_{\mu} \) is the gauge (gluon)

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field and $g$ is the coupling constant. The gluon field strength tensor is given by

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu,$$  \hspace{1cm} (2)

where $f^{abc}$ is the structure constant of $SU(3)$: $[t^a, t^b] = i f^{abc} t^c$.

\section*{1.2. Asymptotic freedom}

\textit{Screening and anti–screening of charge.} Due to the quantum effects of vacuum polarization, the charge in field theory can vary with the distance. In electrodynamics, summation of the electron–positron loops in the photon propagator leads to the following expression for the effective charge, valid at $r \gg r_0$:

$$\alpha_{em}(r) \simeq \frac{3\pi}{2 \ln(r/r_0)}.$$  \hspace{1cm} (3)

This formula clearly exhibits the “zero charge” problem \cite{1} of QED: in the local limit $r_0 \to 0$ the effective charge vanishes at any finite distance away from the bare charge due to the screening. Fortunately, because of the smallness of the physical coupling, this apparent inconsistency of the theory manifests itself only at very short distances $\sim e^{\alpha_{em}/N_c}$, $\alpha_{em} \simeq 1/137$. Such short distances are (and probably will always remain) beyond the reach of experiments, and one can safely use QED as a truly effective theory.

As it has been established long time ago \cite{2}, QCD is drastically different from electrodynamics in possessing the remarkable property of “asymptotic freedom” – due to the fact that gluons carry color, the behavior of the effective charge $\alpha_s = g^2/4\pi$ changes from the familiar from QED screening to anti–screening:

$$\alpha_s(r) \simeq \frac{3\pi}{(11 N_c/2 - N_f) \ln(r_0/r)};$$  \hspace{1cm} (4)

as long as the number of flavors does not exceed 16 ($N_c = 3$), the anti–screening originating from gluon loops overcomes the screening due to quark–antiquark pairs, and the theory, unlike electrodynamics, is weakly coupled at short distances: $\alpha_s(r) \to 0$ when $r \to 0$.

\textit{Does $\alpha_s$ ever get large?} Asymptotic freedom ensures the applicability of QCD perturbation theory to the description of processes accompanied by high momentum transfer $Q$. However, as $Q$ decreases, the strong coupling $\alpha_s(Q)$ grows, and the convergence of perturbative series is lost. How large can $\alpha_s$ get? The analyzes of many observables suggest that the QCD coupling may be “frozen” in the infrared region at the value $\langle \alpha_s \rangle_{IR} \simeq 0.5$ (see \cite{3} and references therein). Gribov’s program \cite{4} relates the freezing of the coupling constant to the existence of massless quarks, which leads to the “decay” of the vacuum at large distances similar to the way it happens in QED in the presence of “supercritical” charge $Z > 1/\alpha$. One may try to infer the information about the behavior of the coupling constant at large distances by performing the matching of the fundamental theory onto the effective chiral Lagrangian \cite{5}. The results of \cite{5} lead to the coupling constant which freezes at the value

$$\langle \alpha_s \rangle = \frac{6\sqrt{2} \pi}{11 N_c - 2 N_f} \sqrt{\frac{N_f^2 - 1}{N_c^2 - 1}};$$  \hspace{1cm} (5)

numerically, for QCD with $N_c = 3$ and $N_f = 2$ one finds $\langle \alpha_s \rangle_{IR} \simeq 0.56$. It remains to be seen if a consistent perturbative scheme can be built on the basis of this approach \cite{6}.
1.3. Chiral symmetry

In the limit of massless quarks, QCD Lagrangian (1) possesses an additional symmetry $U_L(N_f) \times U_R(N_f)$ with respect to the independent transformation of left– and right–handed quark fields $q_{L,R} = \frac{1}{2}(1 \pm \gamma_5)q$:

$$q_L \rightarrow V_L q_L; \quad q_R \rightarrow V_R q_R; \quad V_L, V_R \in U(N_f);$$

(6)

this means that left– and right–handed quarks are not correlated. Even a brief look into the Particle Data tables, or simply in the mirror, can convince anyone that there is no symmetry between left and right in the physical World. One thus has to assume that the symmetry (6) is spontaneously broken in the vacuum. The flavor composition of the existing eight Goldstone bosons (3 pions, 4 kaons, and the $\eta$) suggests that the $U_A(1)$ part of $U_L(3) \times U_R(3) = SU_L(3) \times SU_R(3) \times U_V(1) \times U_A(1)$ does not exist. This constitutes the famous “$U_A(1)$ problem”.

1.4. The origin of mass

There is yet another problem with the chiral limit in QCD. Indeed, as the quark masses are put to zero, the Lagrangian (1) does not contain a single dimensionful scale – the only parameters are pure numbers $N_c$ and $N_f$. The theory is thus apparently invariant with respect to scale transformations, and the corresponding scale current is conserved: $\partial_\mu s_\mu = 0$. However, the absence of a mass scale would imply that all physical states in the theory should be massless!

1.5. Quantum anomalies and classical solutions

Both apparent problems – the missing $U_A(1)$ symmetry and the origin of hadron masses – are related to quantum anomalies. Once the coupling to gluons is included, both flavor singlet axial current and the scale current cease to be conserved; their divergences become proportional to the $\alpha_s G_\mu\nu G^{\mu\nu}$ and $\alpha_s G_\mu\nu G^{\mu\nu}$ gluon operators, correspondingly. This fact by itself would not have dramatic consequences if the gluonic vacuum were “empty”, with $G_\mu\nu^{\mu\nu} = 0$. However, it appears that due to non–trivial topology of the $SU(3)$ gauge group, QCD equations of motion allow classical solutions even in the absence of external color source, i.e. in the vacuum. The well–known example of a classical solution is the instanton, corresponding to the mapping of a three–dimensional sphere $S^3$ into the $SU(2)$ subgroup of $SU(3)$; its existence was shown to solve the $U_A(1)$ problem.

1.6. Confinement

The list of the problems facing us in the study of QCD would not be complete without the most important problem of all – why are the colored quarks and gluons excluded from the physical spectrum of the theory? Since confinement does not appear in perturbative treatment of the theory, the solution of this problem, again, must lie in the properties of the QCD vacuum.

1.7. Understanding the Vacuum

As was repeatedly stated above, the most important problem facing us in the study of all aspects of QCD is understanding the structure of the vacuum, which, in a manner of saying, does not at all behave as an empty space, but as a physical entity with a complicated structure. As such, the vacuum can be excited, altered and modified in physical processes $[7]$; this brings us to the main topic of this talk.
2. WHY STUDY QCD WITH HEAVY IONS?

Most of the applications of QCD so far have been limited to the short distance regime of high momentum transfer, where the theory becomes weakly coupled and can be linearized. While this is the only domain where our theoretical tools based on perturbation theory are adequate, this is also the domain in which the beautiful non–linear structure of QCD does not yet reveal itself fully. On the other hand, as soon as we decrease the momentum transfer in a process, the dynamics rapidly becomes non–linear, but our understanding is hindered by the large coupling. Being perplexed by this problem, one is tempted to dream about an environment in which the coupling is weak, allowing a systematic theoretical treatment, but the fields are strong, revealing the full non–linear nature of QCD. I am going to argue now that this environment can be created on Earth with the help of relativistic heavy ion colliders.

Let us consider an external probe $J$ interacting with the nuclear target of atomic number $A$. At small values of Bjorken $x$, by uncertainty principle the interaction develops over large longitudinal distances $z \sim 1/mx$, where $m$ is the nucleon mass. As soon as $z$ becomes larger than the nuclear diameter, the probe cannot distinguish between the nucleons located on the front and back edges of the nucleus, and all partons within the transverse area $\sim 1/Q^2$ determined by the momentum transfer $Q$ participate in the interaction coherently. The density of partons in the transverse plane is given by

$$\rho_A \simeq \frac{xG_A(x, Q^2)}{\pi R_A^2} \sim A^{1/3},$$  \hspace{1cm} (7)

where we have assumed that the nuclear gluon distribution scales with the number of nucleons $A$. The probe interacts with partons with cross section $\sigma \sim \alpha_s/Q^2$; therefore, depending on the magnitude of momentum transfer $Q$, atomic number $A$, and the value of Bjorken $x$, one may encounter two regimes:

- $\sigma \rho_A \ll 1$ – this is a familiar “dilute” regime of incoherent interactions, which is well described by the methods of perturbative QCD;

- $\sigma \rho_A \gg 1$ – in this regime, we deal with a dense parton system. Not only do the “leading twist” expressions become inadequate, but also the expansion in higher twists, i.e. in multi–parton correlations, breaks down here.

The border between the two regimes can be found from the condition $\sigma \rho_A \simeq 1$; it determines the critical value of the momentum transfer (“saturation scale” [8]) at which the parton system becomes to look dense to the probe:

$$Q_s^2 \sim \alpha_s \frac{xG_A(x, Q_s^2)}{\pi R_A^2}. $$ \hspace{1cm} (8)

In this regime, the number of gluons from (8) is given by

$$xG_A(x, Q_s^2) \sim \frac{\pi}{\alpha_s(Q_s^2)} Q_s^2 R_A^2, $$ \hspace{1cm} (9)

Note that since $xG_A(x, Q_s^2) \sim A^{1/3}$, which is the length of the target, this expression in the target rest frame can also be understood as describing a broadening of the transverse momentum resulting from the multiple re-scattering of the probe.
where $Q_s^2 R_A^2 \sim A$. One can see that the number of gluons is proportional to the inverse of $\alpha_s(Q_s^2)$, and becomes large in the weak coupling regime. In this regime, as we shall now discuss, the dynamics is likely to become essentially classical.

Indeed, the condition (8) can be derived in the following, rather general, way. As a first step, let us re-scale the gluon fields in the Lagrangian (11) as follows: $A_\mu^a \rightarrow \tilde{A}_\mu^a = g A_\mu^a$. In terms of new fields, $\tilde{G}_{\mu\nu}^a = g G_{\mu\nu}^a = \partial_\mu \tilde{A}_\nu^a - \partial_\nu \tilde{A}_\mu^a + f^{abc} \tilde{A}_\mu^b \tilde{A}_\nu^c$, and the dependence of the action corresponding to the Lagrangian (11) on the coupling constant is given by

$$S \sim \int \frac{1}{g^2} \tilde{G}_{\mu\nu}^a \tilde{G}_{\mu\nu}^a \, d^4x. \quad (10)$$

Let us now consider a classical configuration of gluon fields; by definition, $\tilde{G}_{\mu\nu}^a$, in such a configuration does not depend on the coupling, and the action is large, $S \gg \hbar$. The number of quanta in such a configuration is then

$$N_g \sim \frac{S}{\hbar} \sim \frac{1}{\alpha_s} \rho_4 V_4, \quad (11)$$

where we re-wrote (11) as a product of four-dimensional action density $\rho_4$ and the four-dimensional volume $V_4$.

The effects of non-linear interactions among the gluons become important when $\partial_\mu \tilde{A}_\mu \sim \tilde{A}_\mu^2$ (this condition can be made explicitly gauge invariant if we derive it from the expansion of a correlation function of gauge-invariant gluon operators, e.g., $\tilde{G}^2$). In momentum space, this equality corresponds to

$$Q_s^2 \sim \bar{A}^2 \sim (\tilde{G}^2)^{1/2} = \sqrt{\rho_4}; \quad (12)$$

$Q_s$ is the typical value of the gluon momentum below which the interactions become essentially non-linear.
Consider now a nucleus $A$ boosted to a high momentum. By uncertainty principle, the gluons with transverse momentum $Q_s$ are extended in the longitudinal and proper time directions by $\sim 1/Q_s$; since the transverse area is $\pi R_A^2$, the four–volume is $V_4 \sim \pi R_A^2/Q_s^2$. The resulting four–density from (11) is then

$$\rho_4 \sim \alpha_s \frac{N_g}{V_4} \sim \alpha_s \frac{N_g Q_s^2}{\pi R_A^2} \sim Q_s^4,$$

(13)

where at the last stage we have used the non–linearity condition (12), $\rho_4 \sim Q_s^4$. It is easy to see that (13) coincides with the saturation condition (8), since the number of gluons in the infinite momentum frame $N_g \sim xG(x, Q_s^2)$. This simple derivation illustrates that the physics in the high–density regime can potentially be understood in terms of classical gluon fields. This correspondence allowed to formulate an effective quasi–classical theory [9], which is a subject of vigorous investigations at present (see, e.g., [10]).

In nuclear collisions, the saturation scale becomes a function of centrality; a generic feature of the quasi–classical approach – the proportionality of the number of gluons to the inverse of the coupling constant (11) – thus leads to definite predictions [11] on the centrality dependence of multiplicity, which are so far in accord with the data coming from RHIC [12]. The crucial test of these ideas will come from the data taken at higher energies, where the saturation scale $Q_s^2$ should be larger, and according to the logarithmic running of $\alpha_s$ [4], the centrality dependence of multiplicity should become more flat.

The possible relevance of classical theory raises an interesting question – Weizsäcker-Williams gluon field of a fast nucleus can be found [9,10] from the QCD analog of Maxwell equation

$$\partial_{\mu} G_{\mu\nu} = J_{\nu}$$

(14)

with color charges inside the nucleus acting as an external source for gluons. On the other hand, QCD possesses classical (Euclidean) solutions even in empty space. Can new classical solutions to (14) different from Weizsäcker-Williams fields be found? Does topology play a rôle in relativistic heavy ion collisions? Can topological effects induce violations of discrete symmetries manifesting themselves in the multi–meson correlations [13]? The full answer is lacking despite some recent progress [14,15].

What happens when relativistic heavy ions (looking like dense “gluon walls” at sufficiently high energy [16]) collide? One thing we now know for sure is that collisions at RHIC energies produce on the order of a thousand particles per unit of rapidity [12]. A system with a number of particles that big can be described by statistical methods, and, given the high density of the produced partonic system, an approach to equilibrium is likely [17], leading to a state of matter known as the quark–gluon plasma. The study of the critical behavior of strongly interacting matter and the dynamics of phase transitions – deconfinement, chiral, $U_A(1)$ – is the central goal of the heavy ion program. These topics have been extensively discussed in the literature; recent reviews can be found in [18]. The steady progress has been driven largely by numerical simulations on the lattice; for a comprehensive review, see [19]. The most important problem in the field of heavy ion physics is isolating reliable signatures of collective behavior. Due to the lack of space, I cannot dwell on this issue here, and refer the reader to recent reviews [20] which discuss observables in heavy ion collisions. Quarkonium suppression [21] and jet quenching [22] are among the most promising hard probes of dense partonic matter.
3. NEW FRONTIERS OF QCD

What is the place of relativistic heavy ion program in modern physics? Heavy ion research is aimed at understanding QCD, the fundamental particles of which – quarks and gluons – are already well established; QCD has firmly occupied its place as part of the Standard Model. However, understanding the physical World does not mean only establishing its fundamental constituents; it means, mostly, understanding how these constituents interact and bring to the existence the entire variety of physical objects composing the Universe. Think of electrodynamics – the simplest of all gauge theories – which is responsible for an enormous assortment of materials and substances of different structure. Now try to imagine the beauty and complexity of collective phenomena made possible in the theory where “electrons” carry three different “charges”, “photons” carry eight, and they are all bound by the force two orders of magnitude stronger than electromagnetic forces! Just as the research in condensed matter physics is driven by the ability to perform experiments with different number of atoms, under different conditions of low and high temperature and pressure, further progress in QCD will be largely driven by the studies of hadronic matter under extreme conditions. By increasing the atomic number of the colliding systems and by raising the energy of the collision, we get access to the high parton density, high field strength QCD (see Fig.2).
The heavy ion program thus brings us to the important new frontier of modern physics. I believe we are at the beginning of a long and exciting journey.

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