We develop a theory of the half-filled Landau level which shows that it is a two-component fluid. One of the components has spin up and the other spin down so that an electron spin resonance can occur. The 1/3 filled level is fully spin polarized one-component fluid. We calculate the spin polarization at the $\nu = 1/2$ as a function of temperature and compare the calculated result with the experimentally measured value. Our theory is in very good agreement with the experimental data.
1. Introduction.

According to the classical Hall effect the transverse resistivity is linearly proportional to the applied magnetic field and the slope of the resistivity versus magnetic field can be used to measure the concentration of electrons,

$$\rho_{xy} = \frac{B_z}{ne^2},$$

where \( n \) is the electron concentration and \( B_z \) is the magnetic field along \( z \) direction. In 1980 von Klitzing, Dorda and Pepper [2] reported that instead of the linear dependence as shown above, a plateau occurs in \( \rho_{xy} \) which can be measured so accurately that the value of \( e^2/h \) can be deduced. Therefore, at the plateau, we have

$$\frac{B}{ne^2} = \frac{h}{ie^2}$$

and hence,

$$i = \frac{n}{(eB/hc)}$$

is an integer. We write the electron density as \( n = n_o/A \), i.e. the number of electrons per unit area so that the above equation becomes,

$$i = \frac{n_o}{A(eB/hc)}$$

which may be arranged as

$$B.A = \frac{n_o}{i} \left(\frac{hc}{e}\right)$$

which is another way of saying that there is a flux quantization, \( \phi_o = hc/e \) and \( n_o \) and \( i \) are integers. Therefore, von Klitzing et al have detected flux quantization while measuring the Hall effect. Very soon it was reported by Tsui, Störmer and Gossard [2] that not only \( i = 1/3 \) but fractional quantization occurs. The value of \( i = 1/3 \) was detected and many other fractions 2/3, 2/5, 3/5 etc were found [3]. Laughlin [4,5] made the theoretical efforts to find the wave function of a fractional charge by introducing incompressibility. In the flux quantization condition 1/3 charge is equivalent to three times the flux

$$\frac{hc}{3e} = \frac{3hc}{e}$$

or the one third area,

$$B.A = \frac{n_o}{3} \frac{hc}{e}$$

Similarly, if we consider the charge per unit area, then 1/3 charge per unit area is equivalent to having one charge in three times the area

$$\frac{\frac{1}{3}e}{A} = \frac{e}{3A}.$$
The cause of incompressibility is not given in Laughlin’s paper and the force which holds
the distances between particles constant is not mentioned. The charge can be 1/3 if area
is $A$ or alternatively, if we give up incompressibility, the charge can be 1 in thrice the
area, $3A$. Laughlin’s wave functions are defined in the complex space in two dimensions
and can not be used to determine those properties which are often measured in a variety
of experiments. There are other theories in which it is suggested that either even number
or odd number of fluxes are attached to the electron to form a “composite fermion”. The
odd number of fluxes such as one flux plus one electron form a boson which obeys the
Bose-Einstein statistics and hence can make a Bose condensate. The even number of
fluxes attached to the electron, form fermions. A mixture of bosons and fermions need
not always make quasiparticles of mixed statistics but it has been reported that flux
tubes attached to electrons will have intermediate statistics. At this time these theories
are not supported by experimental evidence and attachment of flux tubes to electrons is
not confirmed.

We have found [6] that a simple theory explains the experimental data without con-
tradicting the ideas of fractional charge. The fractions which we tabulated in 1985 are in
full agreement with Fig. 18 of Störmer’s Nobel lecture [7]. We have examined consider-
able amount of data in which fractions are experimentally observed and in each and every
case, the observed values are found to be the same as those predicted [8,9]. Some of the
experimentally observed masses are equal and this equality of masses is well explained
by the particle-hole symmetry which in our theory is related to the Kramers conjugate
states [10]. The high Landau levels also fit very well in our theory [11]. The magnetic
moment of the electron is slightly changed due to an effective spin-orbit interaction [12].
It is found that the rate of sweep of magnetic field plays an important role in determin-
ing whether the transverse resistivity should be zero or finite. The field is detected by
nuclear-magnetic resonance. There is obviously a phase factor problem which determines
whether the system prefers a zero or a finite value of $\rho_{xx}$ [13]. There is a phase transition
and Goldstone modes occur due to flux motion, in bilayers [14]. Dementyev et al [16]
have obtained the electron-spin polarization by measuring the $^{71}$Ga nuclear magnetic
resonance in 300 Å wide GaAs wells separated by 3600 Å wide Al$_{0.1}$Ga$_{0.9}$As barriers
at a magnetic field which corresponds to resistivity plateaus at $\nu = 1$ and 1/3 where
$\rho_{xy} = h/\nu e^2$ in the quantum Hall effect. These are very unique NMR measurements
which require a suitable theory [15]. The electron-spin polarization is obtained from the
ratio of Knight shifts of the NMR lines as

$$P(\nu = \frac{1}{2}, T) = \frac{K_s(\nu = \frac{1}{2}, T)f(\frac{1}{2})}{K_s(\frac{1}{3})f(\frac{1}{3})}$$

where $K_s(\nu)$ are the Knight shifts for the fields at which $\nu = 1/2$ which is partially
polarized and $\nu = 1/3$ which is fully polarized and $f(\nu) = 2n/[\omega\rho(o)]$ with $n$ as the
electron density and $\omega$ the well width. Here $\rho(o)$ is the 3-dimensional electron density
at the center of the quantum well. There is an orientation angle $\theta$ between the total
magnetic field and the growth axis of the sample.

In this paper, we use our theory to understand the polarization at 1/3 and show that
there is a two-component fluid at $\nu = \frac{1}{2}$ with one component having spin up and the
other down. Therefore, there is an electron spin resonance energy between the up and the down components. We calculate the electron spin polarization at the $\nu = \frac{1}{2}$ and compare it with that experimentally measured by using the NMR at the quantum Hall effect fields at $\nu = \frac{1}{2}$ and $\nu = \frac{1}{3}$. We find that the predicted temperature dependence is in accord with that measured by Dementyev et al [16].

2. Theory.

Polarization at 1/3. According to our theory first given in ref. 6 and later on in refs. [8-15], the quantum Hall plateaus occur at $\nu$ given by two series,

$$\nu_{\pm} = \frac{l + \left(\frac{1}{2}\right) - s}{2l + 1}. \quad (10)$$

For $s = \frac{1}{2}$, we obtain

$$\nu_+ = \frac{l}{2l + 1} \quad (11)$$

and for $s = -\frac{1}{2}$, we get

$$\nu_- = \frac{l + 1}{2l + 1}. \quad (12)$$

These two series are sufficient to explain all of the relevant fractions observed in the experimental data. We take the limit of $l \to \infty$, which gives $\nu_+ = \frac{1}{2}$. Similarly, the $\nu_-$ series at $l \to \infty$ also gives $\nu_- = \frac{1}{2}$. Therefore at $\nu = \frac{1}{3}$, the value approaches from both the right hand side of 1/2 as well as from the left hand side so that the liquid has two components. We call $\nu_+ = 1/2$ as $A^{(+)}$ and $\nu_- = \frac{1}{2}$ as $B^{(-)}$. The values with + sign always have spin +1/2 and the values $\nu_-$ always have spin -1/2. For various values of $l$ $\nu_\pm$ series give,

$$l = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad \infty$$

$$\nu_+ = 0 \quad \frac{1}{3} \quad \frac{2}{5} \quad \frac{3}{7} \quad \frac{4}{9} \quad \frac{5}{11} \quad \frac{1}{2} \quad (13)$$

$$\nu_- = 1 \quad \frac{2}{3} \quad \frac{3}{5} \quad \frac{4}{7} \quad \frac{5}{9} \quad \frac{6}{11} \quad \frac{1}{2}$$

Therefore $\nu = \frac{1}{3}$ is fully spin polarized with polarization $P = 1$. However, there will be entropy and therefore, there is always a possibility that some of the spins are not aligned to the value of +1/2 otherwise the state with $\nu = +1/3$ is fully polarized. Thus we fix the polarization of various states as given above. Any deviation may arise due to thermodynamics. Since $\nu = 1/2$ arises from both +1/2 as well as -1/2, these states are separated in energy by that of the electron spin resonance,

$$k_B T_z = g^* \mu_B B_{tot} \quad (14)$$

where $g^*$ is the usual Lande’s splitting factor, $g^* = 2.0023$ for free electrons and $g^* = -0.44$ in the semiconductor GaAs. If the energy of the $N$ electrons is required, we can multiply the above by $N$. For one electron in the $s = +1/2$ state the energy is $\frac{1}{2} g^* \mu_B B_{tot}$ and in the $s = -1/2$ state it is $-\frac{1}{2} g^* \mu_B B_{tot}$. In the experiment of Dementyev et al, $T_z \simeq 1.63 K$ is a temperature which defines the energy difference between the two
spin states. Usually, three components of the triplet and one singlet should arise but in our formula (13) only two states are found at \( \nu = 1/2 \). Thus we have the fixed value of polarization at \( \nu = 1/3 \) and a two component liquid state at \( \nu = 1/2 \). We will now calculate the polarization at \( \nu = 1/2 \) as per usual simple Maxwell-Boltzman distribution. We consider only a two-level system. Since the particles in these levels are separated by energy given by eq. (14), the population in the two levels will be,

\[
\frac{N_1}{N} = \frac{\exp(\frac{1}{2} g^* \mu_B B/k_B T)}{\exp(\frac{1}{2} g^* \mu_B B/k_B T) + \exp(-\frac{1}{2} g^* \mu_B B/k_B T)}
\]

and

\[
\frac{N_2}{N} = \frac{\exp(-\frac{1}{2} g^* \mu_B B/k_B T)}{\exp(\frac{1}{2} g^* \mu_B B/k_B T) + \exp(-\frac{1}{2} g^* \mu_B B/k_B T)}.
\] (15)

Here \( N_1 \) and \( N_2 \) are the populations of the lower and upper levels for a positive value of \( g^* \) and \( N = N_1 + N_2 \) is the total number of electrons. In case \( g^* \) is negative, the levels get interchanged. The projection of the magnetic moment of the upper state along the field direction is \( -\frac{1}{2} g^* \mu_B \) and that of the lower state is \( +\frac{1}{2} g^* \mu_B \) which again get interchanged if the sign of \( g^* \) is negative. We make the change of variables as,

\[
x = \frac{1}{2} g^* \mu_B B/k_B T .
\] (16)

The magnetization of \( N \) sites per unit volume is,

\[
M = (N_1 - N_2) \mu_B (\frac{1}{2} g^*) = \frac{1}{2} N g^* \mu_B \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1}{2} N g^* \mu_B \tanh x.
\] (17)

The high-temperature expansion of \( \tanh x = x \) may be found for \( x \ll 1 \) so that,

\[
M = N(\frac{1}{2} g^* \mu_B)^2 B/k_B T .
\] (18)

Thus the magnetization and hence the polarization in the state which has a population of \( N_1 \) varies as the inverse temperature. From this result the susceptibility also varies as \( 1/T \). At \( \nu = 1/2 \), the quantum Hall system behaves like a two level system. Near the peak there is a small region which requires the wave vector dependent response given in ref. [8].

3. Comparison with experimental data.

Dementyev et al [16] have performed the experimental measurements of the electron spin polarization which we compare with our theory. Two cases have been measured. Case I has forty wells of width 300 Å separated by 3600 Å wide Al\(_{0.1}\)Ga\(_{0.9}\)As barriers. The angle between the sample growth axis and the direction of the total magnetic field is \( \theta = 38.4^\circ \), \( B_{tot} = 7.03\)T. The case II has \( \theta = 0.0^\circ \) and \( B_{tot} = 5.52\)T. The spin polarization at \( \nu = 1/2 \) for both the cases is given as a function of temperature with \( T_z = 2.08\)K in case I and \( T_z = 1.63\)K in case II. In both the cases the polarization varies as \( 1/T \) as predicted by our theory for \( T > 0.6\)K. There is no analog of the ferromagnetic phase and if any, its transition temperature will be \( T_c = 0.0\)K. Therefore, there is no region where we can
see the effect of the dependence on the wave vector of the modes. For temperatures less than 0.6 K there may be a contribution to the polarization which depends on the wave vector of the response function. However, we know that a Goldstone boson emerges at low temperatures in the two component fluid so that the polarization will obey a critical exponent characteristic of a phase transition. The polarization at \( \nu = 1/2 \) measured by Dementyev et al is shown in Fig.1 as a function of temperature. Two calculated curves are also plotted alongwith the data. It is clear that the data agrees with the predicted behaviour at most of the temperatures, \( 0.6 < T < 3.4 \text{K} \). It is possible that there is a region below 0.6 K where wave vector dependence should be considered. An effort is made to compare the polarization as a function of \( (1 - T/T_c)^\alpha \) in search of an exponent below \( T = 1\text{K} \). It is found that for \( T < 1\text{K} \), \( \alpha = 0.15 \) as shown in the ln-ln plot shown in Fig. 2 for \( 0.26 < T < 0.76 \text{K} \). Apparently, the Knight shifts used to measure the polarization provide this value for the electron spin polarization but such a small value for the exponent of polarization is not expected from the scaling theory. Similar study of one more sample has been reported by Melinte et al [17]. We have extracted some representative points from their measurements and shown them in Fig. 3 alongwith the curve calculated by us. Apparently, our theory is in reasonable agreement with the data.

4. Conclusions.

We conclude that there is a two component fluid at \( \nu = 1/2 \) with one component having spin +1/2 and the other component having spin -1/2. The polarization near or above a temperature of 1 K varies as the inverse temperature. In this respect our theory agrees with the experimental data. It should be noted that there is no evidence of attaching even or odd flux quanta to an electron and there is no evidence for the existence of any composite fermions or composite bosons. Assuming that flux was attached to the electrons, the field \( B \) in (18) will be replaced by,

\[
B^* = B - 2p\rho \phi_o
\]

where \( p \) is an integer so that \( 2p \) is an even number, \( \rho \) is the charge density per unit area and \( \phi_o \) is the unit flux. When we put \( p = 0, 1, 2, 3, \ldots, \text{etc.} \), we get different values of \( B^* \) and hence from (18) different values of the magnetization. Therefore, magnetization should show plateaus for different values of \( p \). In the present problem, the polarization as a function of temperature should show plateaus. No such plateaus are found and hence there is no evidence of attachment of fluxes. If spin is completely free, in the half filled Landau level a four-component fluid with a singlet and three components of the triplet should occur. Such a fluid is not observed in the experiments. In a previous eprint [15] we have shown that Jain’s theory is internally inconsistent. From the present discussions it is clear that the fluxes are not attached to the electrons and four components are not found. Hence Jain’s theory [18] of composite fermions is not applicable to the experimental measurements.
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