Remote state preparation serves as a useful way for transmitting information encoded in quantum states between distant places without physically sending themselves. While an ideal remote state preparation scheme requires a pure maximally entangled channel, the reality is that shared entanglement is always severely degraded due to various decoherence mechanisms, seriously deteriorating the performance of the remote state preparation as a result. Herein, we proposed a distinctive remote state preparation scheme where a general quantum state is prepared deterministically via a generally entangled channel. The success probability of preparing a d-dimensional quantum state is maintained at 100% no matter how weak the entanglement of the quantum channel is. Our work contributes to mitigating the effect of decoherence to deliver quantum information more efficiently and paves the way for realizing quantum communication practically.

1 Introduction

Remote state preparation (RSP) is an allimportant quantum communication task where a quantum state is prepared in a remote place via quantum channels [1–3]. It may be considered as teleportation of a known quantum state [4–7]. There have been numerous researches focused on RSP in recent years, such as exploitation of the resource in RSP [3, 8, 9], remote preparations of mixed states [10–12], joint remote state preparation (JRSP) [13], quantum discord for RSP [14–16] and experimental demonstrations [17–21]. The initial RSP schemes [1–3] for preparing a general quantum state are probabilistic with the aid of one c-bit and maximally entangled quantum resources, which is named probabilistic remote state preparation (PRSP). Many researchers have proposed a variety of solutions for this defective preparation and designed some deterministic remote state preparation (DRSP) schemes [13, 22–25]. With the above RSP schemes, Alice (sender) has the ability to deterministically prepare a general quantum state for Bob (receiver) through maximally entangled channels. However, the reality is that shared entanglement is often severely degraded due to various decoherence mechanisms. When the maximally entangled channel utilised in quantum communication tasks degenerates to a generally entangled channel, these schemes therewith are no longer competent to prepare quantum states deterministically and there still exists the problem of information leakage in the process of transferring quantum information.

To date, it is still difficult to prepare high-precision maximally entangled quantum states experimentally [26–28]. The creation and manipulation of entanglement are extremely demanding tasks, as these operations require precise quantum control and isolation from the environment [29, 30]. In contrast to maximally entangled quantum channels, non-maximally entangled quantum channels are easier to be prepared experimentally. If communicators employ non-maximally entangled channels to transfer quantum information, the experimental feasibility will be improved greatly. However, the success probability of RSP is degraded to less than 50% when quantum channels are changed from maximally entangled quantum
channels to non-maximally entangled ones. Conventional RSP protocols hardly achieve the goal that a general quantum state is prepared deterministically via non-maximally entangled quantum channels. Consequently, it is worth considering how to increase the success probability of transferring known quantum information via generally entangled quantum states to reduce the complexity of quantum information processing. A fresh RSP scheme needs to be designed in which any quantum information processing. A fresh RSP scheme needs to be designed in which any quantum state can be deterministically prepared elsewhere via a non-maximally entangled quantum channel.

Herein, we conceived an ingenious DRSP scheme where arbitrary quantum state is prepared deterministically via a pure non-maximally entangled quantum channel which is easily manufactured in actual experiments. The success probability of RSP was increased to 100% with the aid of auxiliary particles. This DRSP protocol is still achievable no matter how weak the entanglement of the quantum channel is, which reduces the complexities and difficulties of preparing quantum resources in the actual manipulation of quantum communication tasks. More noteworthy to mention is the simplicity and feasibility of our measurement methods. The more easier operable projection measurement (PM) is utilised instead of the redundant positive operator-valued measurement complicated to demonstrate in practice [13]. Besides, two communicators only need to communicate once to transfer the message with certainty, and not multiple times communication as in the previous tedious scheme to increase the successful communication probability [31]. Overall, our DRSP scheme has the advantages of high efficiency, low requirement for quantum resources, simple measurement method, and highly eavesdropping resistance. The essay has been organized in the following way. The chapter 2 gives a modified deterministic preparation of a two-dimensional quantum state scheme where the quantum channel is a maximally entangled quantum state. In this scheme, any given quantum state can be deterministically prepared with the aid of auxiliary particles and a partially entangled quantum channel. RSP always succeeds no matter how weak the entanglement of the quantum channel is. Our scheme is also applicable to the preparation of a d-dimensional quantum state via a general quantum channel. The chapter 3 begins by generalizing our RSP scheme to the more general case. Finally, the conclusion gives a summary and critique of the findings.

2. DRSP of a two-dimensional quantum state

In this paper, we proposed a modified DRSP scheme where a general quantum state is prepared deterministically with the aid of the quantum channel and auxiliary particles. No matter how weak the entanglement of the quantum channel is, RSP would not be a failure. Next, we describe our DRSP scheme in more detail. Suppose Alice aims to help Bob remotely prepare a two-dimensional quantum state

$$|\Psi\rangle = x_0|0\rangle + x_1|1\rangle = x_0|0\rangle + |x_1|e^{i\phi}|1\rangle,$$

(1)

where $x_0$ is a real number, $x_1$ is a complex number and $x_0^2 + |x_1|^2 = 1$. The quantum channel shared by Alice and Bob is the entangled state of qubits A and B. Alice possesses qubit A and Bob possesses qubit B. According to the Schmidt decomposition, any pure entangled state of two qubits can be written as follows,

$$|\varphi\rangle_{AB} = (\alpha|00\rangle + \beta|11\rangle)_{AB},$$

(2)

where $\alpha$, $\beta$ are complex numbers, $\alpha^2 + \beta^2 = 1$ and $|\alpha| < |\beta|$. When $\alpha$ is equal to $\beta$, the quantum channel is a pure maximally entangled channel. Otherwise, the quantum channel is a pure non-maximally entangled one. It is well known that quantum entanglement is an essential resource in quantum communication tasks unable to be implemented in classical theory, such as quantum teleportation, quantum dense coding (QDC), and quantum key distribution (QKD). A challenge in the full realization of long-distance quantum communication is to overcome the problems caused by decoherence and exponential photon loss in the noisy quantum channel. The unavoidable environmental noise derived from the transmission of entangled pairs in noisy channels degrades entanglement quality. In addition, the efficiency of single-photon detection affects the efficiency of entanglement purification. These losses increase the quantum bit error rate (QBER) destructive for preparing
Entangled states. Secure QKD requires that the QBER is less than 11% to generate an effective key rate [32]. There exist various solutions in the experiment to eliminate the error brought by the lossy channel. Guo et al. proposed a high-efficiency and long-distance entanglement purification using only one pair of hyperentangled states, the fidelity of entanglement distributed over 11 km multicore fiber (noisy channel) can reach 0.887 and the QBER was decreased to 0.062 [33]. Pan et al. realized the first free-space Measurement-device-independent quantum key distribution over a 19.2-km urban atmospheric channel, where QBER for the $X_{1} - X_{1}$ base can be controlled at 0.23% [34]. They also demonstrated a field-test QKD over 428 km of deployed commercial fiber and two users are physically separated by about 300 km in a straight line. Their secure key rate of QKD is increased to $4.80 \times 10^{-8}$/ pulse with the aid of actively odd-parity pairing [35, 36]. Here we assume that Alice and Bob share a pure general entangled quantum channel for preapring quantum information remotely.

**Step (I)** Alice introduces auxiliary qubit $e$ set on $|0\rangle_e$ and performs CNOT gate $[37, 38]$ $C_{AE}$ on qubits $A$ and $e$.

$$|\varphi_{1}\rangle = C_{AE}(|0\rangle_e \otimes |\varphi\rangle_{AB})$$

$$= (\alpha|00\rangle + \beta|11\rangle)_{eAB}. \quad (3)$$

The mathematical form of CNOT gate is

$$C_{ij} = i(|0\rangle\langle0|)_{ij} \otimes (|0\rangle\langle0| + |1\rangle\langle1|)_{ij}$$

$$+ i(|1\rangle\langle1|)_{ij} \otimes (|0\rangle\langle0| + |0\rangle\langle1|)_{ij}, \quad (4)$$

where $i$ is controll qubit and $j$ is target qubit. When the quantum state of controll qubit $i$ is $|0\rangle$, the quantum state of target qubit $j$ is unchanged; when the quantum state of controll qubit $i$ is $|1\rangle$, Pauli operator $\sigma_x$ is operated on target qubit $j$. CNOT gate is easy to achieve in the actual demonstration with the current quantum technology. There have been numerous methods to realize the CNOT gate in the experiment: linear optics [39, 40], nonlinearity effect [41, 42], superconducting charge qubits [43], electromagnetically induced transparency [44], and so on.

**Step (II)** Alice performs unitary operation $U_{AE}$ on qubits $A$ and $e$. The quantum state of the system was unitarily transformed into

$$|\varphi_{2}\rangle = U_{AE}|\varphi_{1}\rangle$$

$$= \alpha(|00\rangle + |11\rangle)_{eAB}$$

$$+ \sqrt{\beta^2 - \alpha^2} |01\rangle_{eAB}; \quad (5)$$

where

$$|\varphi_{2}\rangle = \sum_{i=0}^{1} \alpha_{i}|i\rangle_{A}_{eAB}$$

$$\sum_{i=0}^{1} \beta_{i}|i\rangle_{B}_{eAB}.$$

**Step (III)** Alice introduces auxiliary qubit $f$ set on $|0\rangle_f$, then she performed $C_{AEf}$ on qubits $A$, $e$ and $f$.

$$|\varphi_{3}\rangle = C_{AEf}(|\varphi_{2}\rangle \otimes |0\rangle_f)$$

$$= \alpha(|0\rangle_f(|00\rangle + |11\rangle)_{eAB}$$

$$+ \sqrt{3\beta^2 - \alpha^2} |01\rangle_{f} |01\rangle_{eAB}; \quad (7)$$

Figure 1: The anterior schematic of DRSP protocol via a non-maximally entangled quantum channel. One of two communicators, Alice, possesses qubits $A$, $e$, $f$ and $g$. The other, Bob, owns qubit $B$. Qubits $A$ and $B$ are entangled initially and their entangled quantum state serves as the quantum channel $|\varphi\rangle_{AB}$ expressed in Eq. (2). The solid line plotted in the figure represents a qubit, while the dashed line represents a classical bit. Rectangles represent the operations required for DRSP. The green rectangular represents a CNOT gate, the yellow rectangular represents a controlled-U gate showed in Eq. (6), the blue rectangular represents the projection measurement (PM), the pink rectangular represents a Hadamard gate.
where

\[
C_{Aef} = A_{ef}(|101\rangle\langle100| + |100\rangle\langle101| + |000\rangle\langle000|
+ |001\rangle\langle001| + |111\rangle\langle111| + |011\rangle\langle011|)
+ |110\rangle\langle110| + |010\rangle\langle010|)_{Aef}.
\]

The operation \(C_{Aef}\) is a three-dimensional CNOT operation where qubits A and e are control qubits, qubit f is a target qubit. When the quantum states of qubits A and e are 00, 01, 11, the quantum state of qubit f is not changed. When the quantum states of qubits A and e are 10, qubit f is flipped from 0 to 1, or from 1 to 0. This operation is feasible with state-of-the-art technology. According to Deutsch Theorem, any arbitrary d-dimensional unitary transformation can always be decomposed into \(2d^2 - d\) products of two-dimensional unitary transformations. In other words, any high-dimensional unitary transformation can always be formed by multiple two-dimensional unitary transformations. Therefore, the three-dimensional unitary operation \(C_{Aef}\) involved in our scheme can be replaced by a combination of multiple two-dimensional quantum gates which are easy to demonstrate experimentally. Besides, recent years have seen a plethora of high-dimensional multiphotonic experiments [45–47], indicating that large classes of quantum states are accessible. Zeilinger et al. presented general multiphoton high-dimensional transformations involving the crucial CNOT gate that rely solely on known experimental techniques [48]. Paesani et al. designed a scheme for universal high-dimensional quantum computation with linear optics [49]. Also, non-commuting single-qubit superadiabatic, geometric quantum gates on the electron spin of the nitrogen-vacancy center in diamond under ambient conditions were implemented experimentally [50]. Therefore, there are many ways to implement high-dimensional quantum gates including our three-dimensional CNOT gate experimentally.

**Step (IV)** Alice measures the quantum state of auxiliary qubit f under the orthogonal basis \(\{|0\rangle, |1\rangle\}\). If the measurement outcome is \(|0\rangle_f\), Bob is certainly able to obtain the quantum state \(|\Psi\rangle = x_0|0\rangle + x_1|1\rangle\) (Eq. 1) with a 100% probability of success according to Nguyen’s scheme [24]. If the measurement outcome is \(|1\rangle_f\), the system is collapsed into

\[
|\varphi_4\rangle = |011\rangle_{eAB}.
\]

**Step (V)** Alice performs Hadamard gate [51, 52] \(H_A\) on qubit A, then performs CNOT gate \(C_{Ae}\) on qubits A and e.

\[
|\varphi_5\rangle = C_{Ae}H_A|\varphi_4\rangle
= \frac{1}{\sqrt{2}}(|001\rangle + |111\rangle)_{AeB},
\]

where

\[
H = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)(0\rangle + (|0\rangle - |1\rangle)(1).\]

**Step (VI)** Alice introduces auxiliary qubit g set on \(|0\rangle_g\) and performed \(C_{eg}\) on qubits e and g. The procedure consisting of Step (I-VI) is depicted by the quantum circuit shown in Fig. 1.

\[
|\varphi_6\rangle = C_{eg}|\varphi_5\rangle
= (|0010\rangle + |1111\rangle)_{AeBg}.
\]

**Step (VII)** Joint operations on qubits g and B are required for achieving the objective of preparing quantum information deterministically. It is an advisable approach for faithful RSP without increasing resources consumption of auxiliary particles. As qubits g and B are located in two different spatial positions far away from each other, the joint operations performed on them are hardly realizable with the current experimental techniques. Intending to address this challenge, we add a feasible process of transferring qubit g from Alice to Bob. Subsequently, Bob can perform joint operation \(C_{gB}\) on qubits g and B.

\[
|\varphi_7\rangle = C_{gB}|\varphi_6\rangle
= (|0010\rangle + |1101\rangle)_{AeBg}.
\]

This solution is also adopted in quantum communication: a theoretical RSP scheme [?] contains transmitting four qubits when establishing an entanglement channel between five communicators: one sender Alice1 firstly prepares the quantum channel of a five-qubit cluster state, then sends four of these five qubits to the other sender and three receivers respectively. Experimentally, a in principle unconditional teleportation protocol that requires only a single photon as an ex-anteprepared resource was demonstrated in the atom-cavity systems with the aid of transmitting a photon [53]. It also can be implemented with different carriers of quantum information coupled to resonators such as vacancy...
centers in diamond [54], rare-earth ions [55], superconducting qubits [56], or quantum dots [57].

**Step (VIII)** Alice who knows the prepared information measures qubit A based on the orthogonal basis \{|u_0\rangle, |u_1\rangle\}, where
\[
|u_0\rangle = x_0|0\rangle + |x_1|1\rangle,
|u_1\rangle = |x_1|0\rangle - x_0|1\rangle.
\]

Under this orthogonal basis, the quantum state \(|\varphi_0\rangle\) can be written as follows,
\[
|\varphi_7\rangle = |u_0\rangle_A(x_0|010\rangle + |x_1|101\rangle)e_{Bg} + |u_1\rangle_A(|x_1|010\rangle - x_0|101\rangle)e_{Bg}.
\]

It is easily obtained from the above equation that there are two kinds of measurement outcomes of qubit A. According to the different measurement outcomes, the subsequent operations are different. Next, we divide them into the following two cases to discuss.

### 2.1 The measurement outcome of \(|u_0\rangle_A\)

If the outcome of the measurement is \(|u_0\rangle_A\), \(|\varphi_7\rangle\) is collapsed into
\[
|\varphi_8\rangle = (x_0|010\rangle + |x_1|101\rangle)e_{Bg}.
\]

**Step (IX)** Bob performs \(C_{Bg}\) on qubits B and g.
\[
|\varphi_9\rangle = C_{Bg}|\varphi_8\rangle = |1\rangle_g|x_0|01\rangle + |x_1|10\rangle)e_{B}.
\]

**Step (X)** Alice performs unitary transformation \(V_e\) on qubit e, where \(V_e = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + e^{2i\theta}|1\rangle|1\rangle)e_e\). Then she measures qubit e based on the orthogonal basis \{\(|\nu_0\rangle, |\nu_1\rangle\}\}, where
\[
|\nu_0\rangle = |0\rangle + e^{i\theta}|1\rangle,
|\nu_1\rangle = e^{-i\theta}|0\rangle - |1\rangle.
\]

Based on the orthogonal basis \{\(|\nu_0\rangle, |\nu_1\rangle\}\}, \(|\varphi_9\rangle\) can be written as
\[
|\varphi_9\rangle = \frac{1}{2}|1\rangle_g(\nu_0|x_0|1\rangle + x_1e^{i\theta}|0\rangle)_B + |\nu_1|x_0|1\rangle - x_1e^{i\theta}|0\rangle)_B.
\]

If the measurement outcome is \(|\nu_0\rangle\), Bob performs Pauli operator \(\sigma_x\) to get the quantum state (Eq. (1)) after measurement on qubit g; if the measurement outcome is \(|\nu_1\rangle\), Bob performs Pauli operator \(\sigma_x\). The procedure consisting of Step (VII-X) is depicted by the quantum circuit shown in Fig. 2.

### 2.2 The measurement outcome of \(|u_1\rangle_A\)

If the measurement outcome is \(|u_1\rangle_A\), \(|\varphi_7\rangle\) is collapsed into
\[
|\varphi_8\rangle = (|x_1|010\rangle - x_0|101\rangle)e_{Bg}.
\]

**Step (IX)** Bob performs \(C_{Bg}\) on qubits B and g.
\[
|\varphi_9\rangle = C_{Bg}|\varphi_8\rangle = |1\rangle_g(|x_1|01\rangle - |x_0|10\rangle)e_{B}.
\]

**Step (X)** Alice performed unitary transformation \(V_e\) on qubit e. Then she measured qubit e based on the orthogonal basis \{\(|\nu_0\rangle, |\nu_1\rangle\}\}.
\[
|\varphi_9\rangle = \frac{1}{2}|1\rangle_g(\nu_0|x_0|1\rangle + x_1e^{i\theta}|0\rangle)_B + |\nu_1|x_0|1\rangle + x_1e^{i\theta}|0\rangle)_B.
\]

If the measurement outcome is \(|\nu_0\rangle\), Bob performs Pauli operator \(\sigma_x\) to get the quantum state (Eq. (1)) after measurement on qubit g; if the measurement outcome is \(|\nu_1\rangle\), Bob performs Pauli operator \(\sigma_x\).
3 DRSP of a d-dimensional quantum state

Our scheme is also applicable to preparing an d-dimensional quantum state

$$|\Psi\rangle = \sum_{j=0}^{d-1} x_j |j\rangle = \sum_{j=0}^{d-1} |x_j e^{i\theta_j}|j\rangle.$$ (23)

The general quantum channel of two qudits can be written as

$$|\varphi\rangle_{AB} = \sum_{k=0}^{d-1} a_k |kk\rangle_{AB},$$ (24)

**Step (I)** Alice introduces auxiliary qudit e set on $|0\rangle_e$ and performs GCNOT gate $GC_{Ae}$ on qudits A and e.

$$|\varphi_1\rangle = GC_{Ae}|\langle 0|_e \otimes |\varphi\rangle_{AB}\rangle = a_0 |\langle 00\rangle_{eAB} + \sum_{k=1}^{d-1} a_k |1kk\rangle_{eAB},$$ (25)

where

$$GC_{Ae} = A(|\langle 0|_e \otimes |e\rangle_{00} + |1\rangle_{e}|1\rangle_e)$$

$$\sum_{k=1}^{d-1} A(|\langle k|_e \otimes |e\rangle_{0k} + |0\rangle_{e}|1\rangle_e)$$

$$+ \sum_{k=2}^{d-1} |\langle k|_e \rangle_e.$$ (26)

**Step (II)** Alice performs unitary operation $GU_{Ae}$ on qudits A and e. The quantum state of system is unitarily transformed into

$$|\varphi_2\rangle = GU_{Ae}|\varphi_1\rangle$$

$$= a_0 |\langle 00\rangle_{eAB} + \sum_{k=1}^{d-1} |1kk\rangle_{eAB}$$

$$+ \sum_{k=1}^{d-1} \sqrt{a_k^2 - a_0^2} |0kk\rangle_{eAB},$$ (27)

where

$$GU_{Ae} = Ae[|\langle 00\rangle_{e} + |01\rangle_{e}|1\rangle_e]$$

$$+ \sum_{k=1}^{d-1} \left( \frac{a_k}{a_k} |\langle 1\rangle_{e} + \sqrt{1 - \frac{a_k^2}{a_0^2}} |\langle 0\rangle_{e} | \langle k\rangle_{e} \right).$$ (28)

**Step (III)** Alice introduces auxiliary qudit f set on $|0\rangle_f$ and performs $GC_{Aef}$ on qudits A, e and f.

$$|\varphi_3\rangle = GC_{Aef}(|0\rangle_f \otimes |\varphi_2\rangle)$$

$$= a_0 |\langle 00\rangle_{f} + \sum_{k=1}^{d-1} |1kk\rangle_{eAB}$$

$$+ \sum_{k=1}^{d-1} \sqrt{a_k^2 - a_0^2} |1kk\rangle_{eAB},$$ (29)

where

$$GC_{Aef} = Ae_f[|\langle 00\rangle_{f} + |01\rangle_{f}|1\rangle_f]$$

$$+ |\langle 001\rangle_{f} + |\langle 010\rangle_{f}|1\rangle_f$$

$$+ \sum_{k=1}^{d-1} |\langle k01\rangle_{f} + |\langle k00\rangle_{f}|1\rangle_f].$$ (30)

**Step (IV)** Alice measures auxiliary qudit f under the orthogonal basis $\{0\rangle_f, |1\rangle_f\}$. If the measurement outcome is $|0\rangle_f$, Bob can obtain the quantum state (Eq. (23)) with a 100% successful probability according to Nguyen’s scheme [24]. If the measurement outcome is $|1\rangle_f$, the system is collapsed into

$$|\varphi_4\rangle = \sum_{k=1}^{d-1} \sqrt{a_k^2 - a_0^2} |0kk\rangle_{eAB}.$$ (31)

**Step (V)** Alice performs Hadamard gate $GH_A$ on qubit A, then performs $GC_{Ae}'$ on qudits A and e.

$$|\varphi_5\rangle = GC_{Ae}'GH_A|\varphi_4\rangle$$

$$= \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} \sum_{k'=0}^{d-1} e^{2\pi ikk'} |\langle k'|_{eAB} | k\rangle_a$$

$$where$$

$$GH = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} e^{2\pi ikk'} |\langle k'|_{eAB} | k\rangle_a,$$

$$GC_{Ae}' = Ae'[|\langle k'k\rangle_{eAB} + |\langle k'0\rangle_{eAB}|1\rangle_{eAB}$$

$$+ \sum_{k',k=1}^{d-1} |\langle k'\rangle_{eAB} | k\rangle_{Ae}.$$ (33)

**Step (VI)** Alice introduces auxiliary qubit g set on $|0\rangle_g$ and performed $GC_{eg}$ on qudits e and g.

$$|\varphi_6\rangle = GC_{eg}|\varphi_5\rangle \otimes |0\rangle_g$$

$$= \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} \sum_{k'=0}^{d-1} e^{2\pi ikk'} |\langle k'k\rangle_{eAB} | k\rangle_{AeB}$$

$$+ \sum_{k'=1}^{d-1} e^{2\pi ikk'} |\langle k'k1\rangle_{eAB} | k\rangle_{AeB},$$ (34)
where

\[ GC_{eg} = e_g^H |00\rangle\langle00| + |01\rangle\langle10| \]
\[ + \frac{1}{\sqrt{d}} \sum_{k=1}^{d-1} \sum_{k'=1}^{d-1} \sqrt{a_k^2 - a_0^2} (|k'0\rangle\langle k'0|) + \sum_{k'=1}^{d-1} e^{\frac{2\pi i k' k}{d}} |k' k'\rangle \langle k' k'|_{eg} . \] (35)

**Step (VII)** Alice sends auxiliary qubit \( g \) to Bob, then Bob performs \( GC_{gB} \) on qubits \( g \) and \( B \).

\[ |\varphi_7\rangle = GC_{gB} |\varphi_6\rangle \]
\[ = \frac{1}{\sqrt{d}} \sum_{k=1}^{d-1} \sqrt{a_k^2 - a_0^2} (|0000\rangle) \]
\[ + \sum_{k'=1}^{d-1} e^{\frac{2\pi i k' k}{d}} |k' k' k' k'\rangle \langle k' k' k' k'|_{eg} , \] (36)

where

\[ GC_{gB} = |00\rangle\langle00| + |0k\rangle\langle0k| + \sum_{i=1}^{d-1} |0i\rangle\langle0i| \]
\[ + |1k\rangle\langle1k| + |1i\rangle\langle1i| \]
\[ + \sum_{i=0}^{d-1} \sum_{j=0}^{d-1} \sum_{k'=2}^{d-1} |k' i\rangle\langle k' i| . \]

**Step (VIII)** Alice measures qudit \( A \) based on the orthogonal basis \( \{|u_0\rangle, |u_1\rangle, \cdots, |u_{d-1}\rangle\} \), where

\[ |u_i\rangle = \frac{1}{\sqrt{d}} \sum_{k'=0}^{d-1} |x_{ki}\rangle e^{\frac{2\pi i k k'}{d}} |k'\rangle . \] (38)

If the outcome is \( |u_i\rangle_A \), the quantum state of system is collapsed into

\[ |\varphi_8\rangle = \frac{1}{d} \sum_{k=1}^{d-1} \sqrt{a_k^2 - a_0^2} (|x_0\rangle |000\rangle) \]
\[ + \sum_{k'=1}^{d-1} e^{\frac{2\pi i k' k}{d}} |x_{k'}\rangle_1 |k' k'\rangle \langle k' k'|_g e_{Bg} . \] (39)

**Step (IX)** Bob performs \( GC_{Bg} \) on qubits \( g \) and \( B \).

\[ |\varphi_9\rangle = GC_{Bg} |\varphi_8\rangle \]
\[ = \frac{1}{d} \sum_{k=1}^{d-1} \sum_{k'=1}^{d-1} \sqrt{a_k^2 - a_0^2} e^{\frac{2\pi i k k'}{d}} |x_{k'}\rangle_1 |k' k'\rangle \langle k' k'|_g e_{Bg} . \] (40)

**Step (X)** Alice performs unitary operation \( V_e^{\theta_j} = \sum_{j=0}^{d-1} e^{i\theta_j} |j\rangle\langle j| \) on qudit \( e \), and measures \( e \) based on the orthogonal basis \( \{|\nu_0\rangle, |\nu_1\rangle, \cdots, |\nu_{d-1}\rangle\} \), where

\[ |\nu_j\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} e^{i\theta_j} |j\rangle . \] If the outcome is \( |\nu_j\rangle \), the quantum state of the system is collapsed into

\[ |\varphi_{10}\rangle = \frac{1}{d} \sum_{k=1}^{d-1} \sqrt{a_k^2 - a_0^2} \sum_{j=0}^{d-1} e^{\frac{-2\pi i (k+1) k'}{d}} |j\rangle \langle j| \] \[ = \sum_{k=0}^{d-1} e^{\frac{-2\pi i (k+1) k'}{d}} |j\rangle \langle j| \text{ on qudit } B \text{ to derive the prepared quantum information (Eq. (23)).} \]

4 conclusion

While there has been a tremendous advancement in quantum computation and long-distance quantum communication recently, degradation of entanglement due to noisy environment remains to be a sticky challenge to address in constructing sizable quantum frameworks. To that end, we reinvestigated the remote state preparation scheme and devised an ingenious DRSP protocol for preparing a general quantum state faithfully. Any given quantum state can be prepared in a remote place with a 100% success probability via a pure non-maximally entangled quantum channel. In contrast to the previous resource-intensive RSP schemes relying on entanglement purification or quantum error correction to enhance the entanglement of quantum channels, it is possible in this proposed DRSP protocol to faithfully prepare a quantum state through a noisy quantum channel with limited quantum resources. This well-designed DRSP protocol applies to preparing high-dimensional quantum states whereas traditional RSP schemes are only suitable for the dimension of 2, 4, or 8 [58, 59]. It is considerable to discuss the effect of the dimension of the quantum channel on the efficiency of preparing quantum information. The quantum channel used in our RSP scheme is a pure two-qubit entangled state. If the quantum channel is changed from a pure two-qubit entangled state to a pure two-qubit entangled state, the efficiency of preparing quantum information becomes higher. In other words, the degree of freedom available to the communicator in the system becomes more, the difficulty to
achieve the same purpose is reduced obviously. Besides, preparing quantum information in high-dimensional quantum channels has significant advantages in robustness against errors [60, 61] and can reduce quantum circuit complexities [62, 63], and high-dimensional error-correction codes have shown advantages in terms of resources [64, 65]. DRSP will be more efficient if Alice and Bob share a higher dimensional pure non-maximally entangled quantum channel. In a nutshell, this investigation provides insights into practically eliminating the influence of decoherence with a handful of qudits and more efficiently realizing noisy quantum communication tasks, such as quantum teleportation, quantum dense coding, and secure direct communication via noisy quantum channels.

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