Evaluations For $\zeta(2)$, $\zeta(4)$, \ldots, $\zeta(2k)$ Based On The WZ Method

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Abstract

Based on the framework of the WZ theory, a new evaluation for $\zeta(2) = \frac{\pi^2}{6}$ and $\zeta(4) = \frac{\pi^4}{90}$ was given respectively, finally, a recurrence formula for $\zeta(2k)$, which is equivalent to the classical formula $B_{2k}(\frac{1}{2}) = (2^{-2k+1} - 1)B_{2k}$, was given.

1 Introduction, Lemmas and Main Results.

We know that there are many evaluations (or proofs) for $\zeta(2) = \frac{\pi^2}{6}$ since the first evaluation belong to Euler, e.g. see [1]-[5] and the related references therein. We also know that there are two recurrence formulas for $\zeta(2k)$ in [5]

$$\zeta(2n) = (-1)^{n-1} \frac{(2\pi)^{2n}}{(2n)!(2^{2n-1} - 1)} \left[ \frac{\pi}{2n} \frac{1}{(2n-1)!} \right] + \sum_{j=1}^{n-1} \frac{(-1)^{j}}{(2j-1)!} \frac{(2n-1)!}{(2^{2j-1} - 1)!} \zeta(2j) \right] \zeta(2) \right]
$$

In this paper, I will give a new evaluation for $\zeta(2) = \frac{\pi^2}{6}$ based on the framework of WZ theory (see [6]-[8]), repeating the process of evaluation can also be applied to evaluating $\zeta(4)$, $\zeta(6)$, \ldots. Finally, through the same process repeatedly, I obtained a recurrence formula for $\zeta(2k)$ which is similar to, but different from the recurrence formulas for $\zeta(2k)$ mentioned above. As $\zeta(2k) = \frac{2^{2k-1} - (-1)^{k-1}}{(2k)!} B_{2k} \pi^{2k}$ (belong to Euler too), and $B_k$ (where $k \in N_0 = N \cup \{0\}$) is called $k$-th Bernoulli number, thus by the recurrence formula for $\zeta(2k)$ in the following theorem, we can obtain a formula for Bernoulli polynomial $B_{2k}$:

$$B_{2k}(\frac{1}{2}) = (2^{-2k+1} - 1)B_{2k},$$

where $B_n(x)$ is the Bernoulli polynomial of order $n$. In fact, there are equivalent. The following theorem is the main result in this paper.

**Theorem.** Given $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$ (where $Re(s) > 1$), then we have $\zeta(2) = \frac{\pi^2}{6}$, $\zeta(4) = \frac{\pi^4}{90}$, more generally with the convention $\sum_{k=1}^{0} a(k) = 0$, for $\zeta(2l)$. (where
$l \in \mathbb{N})$ the following recurrence formula hold

$$\varsigma(2l) = \left(\frac{2^{2l-1}}{1 - 2^l}\right) \left(\frac{(-1)^{l+1} + (-1)^l}{4l} + \frac{\pi^{2l}}{2\Gamma(2l)} + \sum_{j=1}^{l-1} \frac{(-1)^{l-j} \pi^{2(l-j)}}{\Gamma(2(l-j) + 1)} \varsigma(2j)\right)$$

where $\Gamma(z)$ is gamma function.

To prove the theorem, we need the following lemmas.

**Lemma 1.** If given a continuous-discrete WZ pair $(F(x, k), G(x, k))$, that is $(F(x, k), G(x, k))$ satisfy the following so called continuous-discrete WZ equation

$$\frac{\partial F(x, k)}{\partial x} = G(x, k + 1) - G(x, k) \tag{1}$$

then for all $m, n \in \mathbb{N}_0$, for all $h, x \in \mathbb{R}$, we have

$$\sum_{k=m}^{n} F(x, k) - \sum_{k=m}^{n} F(h, k) = \int_{h}^{x} G(t, n + 1)dt - \int_{h}^{x} G(t, m)dt.$$

**Lemma 2.** If for $a, x \in \mathbb{R}$, $a < x$, $f(x)$ is integrable on the interval $(a, x)$, then we have

$$\int_{a}^{x} \left(\int_{a}^{t_{1}} \cdots \left(\int_{a}^{t_{k-1}} f(t_{k})dt_{k}\right) \cdots dt_{2}\right)dt_{1} = \frac{1}{\Gamma(k)} \int_{a}^{x} (x - t)^{k-1} f(t)dt.$$

**Lemma 3.** For all $n \in \mathbb{N}$, we have

$$\sum_{k=1}^{n} \cos kx = -\frac{1}{2} + \frac{1}{2} \frac{\sin [(2n + 1)x/2]}{\sin(x/2)}.$$

**Lemma 4.** For all $n \in \mathbb{N}_0$, we have

$$\int_{0}^{\pi} \frac{\sin [(2n + 1)x/2]}{\sin(x/2)} dx = \pi.$$

**Lemma 5.** For all $s \geq 1$, we have

$$\lim_{n \to +\infty} \int_{0}^{\pi} \frac{x^{s} \sin [(2n + 1)x/2]}{\sin(x/2)} dx = 0.$$

As the proof of **Lemma 1** is easy, we omit the details of proof here. **Lemma 2** can be seen in [9]-[10], and **Lemma 3** can be seen in [11]. The proof of **Lemma 5** will be given below.
2 Proof of Lemma 5.

Let $t = \frac{x}{2}$, then we have

$$\int_{0}^{\pi} \frac{x^s \sin \left(\frac{(2n+1)x}{2}\right)}{\sin \left(\frac{x}{2}\right)} dx = 2^{s+1} \int_{0}^{\frac{x}{2}} \frac{t^s \sin \left(\frac{(2n+1)t}{2}\right)}{\sin (t)} dt.$$ 

Let $f(t) = \begin{cases} t^s \csc t & 0 < t < \frac{x}{2} \\ 0 & t = 0 \end{cases}$, it is an easy exercise of calculus that when $s \geq 1$, $f(t)$ is differentiable and monotone (increasing) on $[0, \frac{x}{2}]$, then by the Second Mean Value Theorem for Integrals, we know that there exist $\xi$ on $[0, \frac{x}{2}]$ such that

$$\left| \int_{0}^{\frac{x}{2}} \frac{t^s}{\sin t} \sin \left(\frac{(2n+1)t}{2}\right) dt \right| = \left| f(0 + 0) \int_{0}^{\xi} \sin((2n+1)t) dt + f \left(\frac{\pi}{2} - 0\right) \int_{\xi}^{\frac{x}{2}} \sin((2n+1)t) dt \right|$$

$$= \left(\frac{\pi}{2}\right)^s \left| \frac{1}{2n+1} (-\cos((2n+1)t)) \right|_{\xi}^{\frac{x}{2}}$$

$$\leq \left(\frac{\pi}{2}\right)^s \frac{2}{2n+1}.$$ 

By the result above, we can conclude that

$$\lim_{n \to +\infty} \int_{0}^{\pi} \frac{x^s \sin \left(\frac{(2n+1)x}{2}\right)}{\sin \left(\frac{x}{2}\right)} dx = \lim_{n \to +\infty} 2^{s+1} \int_{0}^{\frac{x}{2}} \frac{t^s \sin \left(\frac{(2n+1)t}{2}\right)}{\sin t} dt = 0,$$

the proof of Lemma 5 was completed.

Remarks: 1. It is worth mentioning that we can prove Lemma 5 by Riemann-Lebesgue lemma directly as follows. Let $f(t) = \begin{cases} \left(\frac{t}{x}\right)^s \csc \left(\frac{t}{x}\right) & 0 < t < \pi \\ 0 & t = 0 \end{cases}$, it is an easy exercise of calculus that $f(t)$ is continuous on $[0, \pi]$, of course, $f(t)$ is Riemann integrable on $[0, \pi]$, then by Riemann-Lebesgue lemma, we have

$$\lim_{n \to +\infty} \int_{0}^{\pi} f(x) \sin \left(\frac{(2n+1)x}{2}\right) dx = 0,$$

because

$$\int_{0}^{\pi} f(x) \sin \left(\frac{(2n+1)x}{2}\right) dx = \int_{0}^{\pi} \left(\frac{x}{2}\right)^s \sin \left(\frac{(2n+1)x}{2}\right) \sin \left(\frac{x}{2}\right) dx,$$

$$= \int_{0}^{\frac{x}{2}} \frac{t^s \sin \left(\frac{(2n+1)t}{2}\right)}{\sin t} dt,$$

$$\leq \left(\frac{\pi}{2}\right)^s \frac{2}{2n+1}.$$
finally, we have
\[
\lim_{n \to +\infty} \int_{0}^{\pi} x^n \sin \left( \frac{(2n+1)x}{2} \right) \frac{dx}{\sin \left( \frac{x}{2} \right)} = 0.
\]

2. It is also worth mentioning that when \( s \geq 2 \), we can prove Lemma 5 by using integration by parts, but when \( 1 \leq s < 2 \), the method can’t be used.

3 Proof of The Theorem.

(A) Proof of \( \varsigma(2) = \frac{x^2}{6} \). Setting \( F_1(x, k) = \frac{\cos(kx)}{k^2} \), \( G_1(x, k) = \sum_{j=1}^{k-1} -\frac{\sin(jx)}{j} \), then it is easy to verify that \((F_1(x, k), G_1(x, k))\) is a continuous-discrete WZ pair, that is, they satisfy the equation (1). Now let \( h = 0 \), \( m = 1 \), with the convention \( \sum_{k=1}^{0} a(k) = 0 \), by using Lemma 1 we get
\[
\sum_{k=1}^{n} \cos(kx) \frac{1}{k^2} - \sum_{k=1}^{n} \frac{1}{k^2} = \int_{0}^{x} G_1(t, n + 1) dt \tag{2}
\]
To evaluate \( G_1(x, n + 1) = \sum_{j=1}^{n} -\frac{\sin(jx)}{j} \), we also use Lemma 1. Now set
\[
F_2(x, k) = -\frac{\sin(kx)}{k}, \quad G_2(x, k) = \sum_{j=1}^{k-1} -\cos(kx),
\]
then it is easy to verify that \((F_2(x, k), G_2(x, k))\) is a continuous-discrete WZ pair, and for all \( k \in \mathbb{N} \), the following result hold \( F_2(0, k) = 0 \). With the convention \( \sum_{k=1}^{n} a(k) = 0 \), by using Lemma 1 and Lemma 3, we obtain
\[
\sum_{k=1}^{n} \frac{-\sin(kx)}{k} = \int_{0}^{x} G_2(t, k) dt = \int_{0}^{x} \left\{ \frac{1}{2} - \frac{\sin \left[ \frac{(2n+1)t}{2} \right]}{2 \sin(t/2)} \right\} dt \tag{3}
\]
By using (2), (3) and Lemma 2, we obtain
\[
\sum_{k=1}^{n} \frac{\cos(kx)}{k^2} - \sum_{k=1}^{n} \frac{1}{k^2} = \int_{0}^{x} G_1(t, n + 1) dt
\]
\[
= \int_{0}^{x} \left\{ \int_{0}^{t} \left[ \frac{1}{2} - \frac{\sin \left[ \frac{(2n+1)t}{2} \right]}{2 \sin(t_2/2)} \right] dt_2 \right\} dt_1
\]
\[
= \frac{1}{2} \int_{0}^{x} (x - t) dt - \frac{x}{2} \int_{0}^{x} \frac{\sin \left[ \frac{(2n+1)t}{2} \right]}{\sin(t/2)} dt
\]
\[
+ \frac{1}{2} \int_{0}^{x} t \frac{\sin \left[ \frac{(2n+1)t}{2} \right]}{\sin(t/2)} dt
\]
\[
= I_1(x) + I_2(x) + I_3(x).
\]
Recalling $\sum_{k=1}^{+\infty} \frac{(-1)^k}{k^{p}} - \zeta(s) = (-2 + \frac{1}{2^{s-1}}) \zeta(s)$, let $x = \pi$ at first, and then let $n \to +\infty$, we conclude that

$$\lim_{n \to +\infty} \left[ \sum_{k=1}^{n} \frac{\cos(k\pi)}{k^2} - \sum_{k=1}^{n} \frac{1}{k^2} \right] = \sum_{k=1}^{+\infty} \frac{(-1)^k}{k^{2}} - \zeta(2) = -\frac{3}{2} \zeta(2).$$

After some computations, we obtain

$$I_1(\pi) = \frac{1}{2} \int_{0}^{\pi} (\pi - t) dt = \frac{\pi^2}{4}.$$ 

By Lemma 4, we obtain

$$I_2(\pi) = -\frac{\pi}{2} \int_{0}^{\pi} \frac{\sin[(2n+1)t/2]}{\sin(t/2)} dt = -\frac{\pi^2}{2}.$$ 

Now by Lemma 5, we obtain

$$\lim_{n \to +\infty} I_3(\pi) = \lim_{n \to +\infty} \frac{1}{2} \int_{0}^{\pi} \frac{t \sin[(2n+1)t/2]}{\sin(t/2)} dt = 0.$$ 

Finally we conclude that $-\frac{3}{2} \zeta(2) = \frac{\pi^2}{4} - \frac{\pi^2}{2} = -\frac{\pi^2}{4}$, that is $\zeta(2) = \frac{\pi^2}{6}$.

(B) Proof of $\zeta(4) = \frac{\pi^4}{90}$. Setting

$$F_1(x, k) = \frac{\cos(kx)}{k^4}, \quad G_1(x, k) = \sum_{j=1}^{k-1} \frac{-\sin(jx)}{j^3},$$

$$F_2(x, k) = \frac{-\sin(kx)}{k^3}, \quad G_2(x, k) = \sum_{j=1}^{k-1} \frac{-\cos(jx)}{j^2},$$

$$F_3(x, k) = \frac{-\cos(kx)}{k^2}, \quad G_3(x, k) = \sum_{j=1}^{k-1} \frac{\sin(jx)}{j},$$

$$F_4(x, k) = \frac{\sin(kx)}{k}, \quad G_4(x, k) = \sum_{j=1}^{k-1} \cos(jx),$$

It is easy to verify that for $j = 1, 2, 3, 4$, $(F_j(x, k), G_j(x, k))$ satisfy equation (11). Setting $H_n^{(l)} = \sum_{k=1}^{n} \frac{1}{k^l}$, completely analogous to the proof of $\zeta(2) = \frac{\pi^2}{6}$ (some details are omitted here), we get

5
\[
\sum_{k=1}^{n} \frac{\cos(kx)}{k^4} - H_n^{(4)} = \frac{1}{\Gamma(4)} \left\{ \left[ \int_{0}^{x} -\frac{1}{2} (x-t)^{3} dt + \frac{x^3}{2} \int_{0}^{x} \frac{\sin[(2n+1)t/2]}{\sin(t/2)} dt \right] \right\} \\
+ \frac{1}{\Gamma(4)} \left\{ \sum_{k=1}^{2} \left( \frac{3}{k} \right) \frac{x^k}{2} \int_{0}^{x} (-t)^{3-k} \sin[(2n+1)t/2] \sin(t/2) dt \right\} \\
- \frac{1}{\Gamma(4)} \int_{0}^{x} (x-t)H_n^{(4)} dt \\
= I_1(x) + I_2(x) + I_3(x).
\]

Setting \(x = \pi\), by Lemma 4 we obtain \(I_1(\pi) = \frac{\pi^4}{16}\), and by Lemma 5 we obtain \(\lim_{n \to +\infty} I_2(\pi) = 0\), recall \(\lim_{n \to +\infty} H_n^{(4)} = \zeta(2) = \frac{\pi^2}{6}\), we obtain \(\lim_{n \to +\infty} I_3(\pi) = -\frac{\pi^2}{12}\). Recall \(\lim_{n \to +\infty} H_n^{(4)} = \zeta(4)\), we get

\[
\lim_{n \to +\infty} \left[ \sum_{k=1}^{n} \frac{\cos(k\pi)}{k^4} - H_n^{(4)} \right] = \sum_{k=1}^{+\infty} \frac{(-1)^k}{k^4} - \zeta(4) = \left( -2 + \frac{1}{2^4} \right) \zeta(4).
\]

Finally we obtain \(\zeta(4) = \frac{\pi^4}{96}\).

(C) Proof of \(\zeta(2l) = \left( \frac{x^{2l-1}}{2^{2l-1}} \right) \left\{ \left[ \frac{(1-\frac{1}{2})^{2l-1}}{4l} + \frac{(1-\frac{1}{2})^{2l-1}}{4l} \right] \pi^{2l} + \frac{1}{4l} \sum_{j=1}^{l-1} \frac{(-1)^{l-j}2^{2l-1-j}}{1(2(l-j)+1)} \zeta(2j) \right\} \).

The result can be proved in the above framework of proving \(\zeta(2) = \frac{\pi^2}{6}\) and \(\zeta(4) = \frac{\pi^4}{96}\), some details are omitted here. For convenience, setting

\[
H_n^{(l)}(x) = \sum_{k=1}^{n} \frac{\cos(kx)}{k^l}, \quad H_n^{(l)}(0) = H_n^{(l)}, \quad H_n^{(l)}(\pi) = \sum_{k=1}^{n} \frac{(-1)^k}{k^l},
\]

\[
I_j(f)(x) = \frac{1}{\Gamma(j)} \int_{0}^{x} (x-t)^{j-1} f(t) dt,
\]

with the convention that \(I_0(f)(x) = f(x)\), where \(j \in N_0\). Now it is easy to verify that for all \(j \in N_0\), \(I_j\) own the following properties

\[
I_j(f + g)(x) = I_j(f)(x) + I_j(g)(x), \quad I_j(cf)(x) = cI_j(f)(x),
\]

where \(c\) is a constant having nothing to do with \(t\), the variable of integral. Also setting

\[
f(t) = -\frac{1}{2} + \frac{1}{2} \frac{\sin[(2n+1)t/2]}{\sin(t/2)},
\]

we obtain

\[
H_n^{(2l)}(x) = (-1)^l I_{2l}(f)(x) + \sum_{j=1}^{l} (-1)^{l-j} I_{2l-2j}(H_n^{(2j)})(x).
\]
Let \( x = \pi \), then we get the following result

\[
\sum_{k=1}^{n} \frac{(-1)^k}{k^{2l}} - H_n^{(2l)} = (-1)^l I_{2l} \left( -\frac{1}{2} \right) (\pi) + (-1)^l I_{2l} \left( \frac{\sin [(2n + 1)\pi/2]}{2 \sin (\pi/2)} \right) (\pi)
\]

\[
+ \sum_{j=1}^{l-1} (-1)^{l-j} I_{2(l-j)} (H_n^{(2j)})(\pi)
\]

\[= I + II + III.\]

Next, let us consider I, II and III respectively

\[I = (-1)^l \left( -\frac{1}{2} \right) \frac{1}{\Gamma(2l)} \int_{0}^{\pi} (\pi - t)^{2l-1} dt = \frac{(-1)^{l+1}}{4l} \pi^{2l} \frac{\Gamma(2l)}{\Gamma(2l)}\]

\[II = (-1)^l \frac{1}{2 \Gamma(2l)} \int_{0}^{\pi} (\pi - t)^{2l-1} \frac{\sin [(2n + 1)\pi/2]}{\sin (\pi/2)} dt\]

\[= (-1)^l \frac{1}{2 \Gamma(2l)} \int_{0}^{\pi} \pi^{2l-1} \frac{\sin [(2n + 1)\pi/2]}{\sin (\pi/2)} dt\]

\[+ (-1)^l \frac{1}{2 \Gamma(2l)} \int_{0}^{\pi} \sum_{k=1}^{2l-2} \left( \frac{2l-1}{k} \right) \pi^k (-t)^{2l-1-k} \frac{\sin [(2n + 1)\pi/2]}{\sin (\pi/2)} dt\]

\[= II_1 + II_2.\]

By Lemma 4, we obtain

\[II_1 = (-1)^l \frac{\pi^{2l}}{2 \Gamma(l)},\]

and by Lemma 5 we conclude that \( \lim_{n \to +\infty} II_2 = 0 \). By using the results above, we obtain

\[\lim_{n \to +\infty} II = (-1)^l \frac{\pi^{2l}}{2 \Gamma(l)}.\]

After some computations, we obtain

\[III = \sum_{j=1}^{l-1} \frac{(-1)^{l-j} H_n^{(2j)}}{\Gamma(2l-j)} \int_{0}^{\pi} (\pi - t)^{2(l-j)-1} dt\]

\[= \sum_{j=1}^{l-1} (-1)^{l-j} H_n^{(2j)} \frac{\pi^{2(l-j)}}{\Gamma (2(l-j) + 1)} .\]

Recalling \( \lim_{n \to +\infty} H_n^{(2j)} = \zeta(2j) \), we conclude that
\[
\lim_{n \to +\infty} \text{III} = \sum_{j=1}^{l-1} \frac{(-1)^{l-j} \pi^{2(l-j)}}{\Gamma(2(l-j) + 1)} \zeta(2j).
\]

It is easy to verify that
\[
\lim_{n \to +\infty} \left( \sum_{k=1}^{n} \frac{(-1)^{k}}{k^{2l}} - \sum_{k=1}^{n} \frac{1}{k^{2l}} \right) = \left( -2 + \frac{1}{2^{2l-1}} \right) \zeta(2l).
\]

So finally with the convention \(\sum_{k=1}^{0} a(k) = 0\), we obtain the following recurrence formula for \(\zeta(2k)\)
\[
\zeta(2l) = \left( \frac{2^{2l-1}}{1 - 2^{2l}} \right) \left\{ \left[ \frac{(-1)^{l+1}}{4l} + \frac{(-1)^{l}}{2} \right] \frac{\pi^{2l}}{\Gamma(2l)} + \sum_{j=1}^{l-1} \frac{(-1)^{l-j} \pi^{2(l-j)}}{\Gamma(2(l-j) + 1)} \zeta(2j) \right\}.
\]

The proof of the theorem is completed.

**Remarks:**
1. We can set \(F_{1}(x, k) = \frac{e^{ikx}}{k^{2}}, \quad G_{1}(x, k) = \sum_{j=1}^{k-1} \frac{ie^{jx}}{j}\),
\(F_{2}(x, k) = \frac{ie^{ikx}}{k}, \quad G_{2}(x, k) = \sum_{j=1}^{k-1} -e^{jx}\)
where \(i = \sqrt{-1}\), it is easy to verify that \((F_{j}(x, k), G_{j}(x, k))\) (where \(j = 1, 2\)) is a continuous-discrete WZ pair. Then through the same process above, we can also obtained \(\zeta(2) = \frac{\pi^{2}}{6}\), the details are omitted here. Of course, we can do in the same way for \(\zeta(4), \zeta(6), \ldots\), and for the general case \(\zeta(2k)\) respectively. 2. It is also worth mentioning that the ideas in the proof above can be used to solve other similar problems of summation of infinite series, and I will give the details in another paper.

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