A Theoretical Contact Mechanics Model of Machine Joint Interfaces Based on Fractal Theory

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Abstract. To obtain more accurate contact mechanics model of joint interfaces theoretically, A theoretical contact mechanics model of joint interfaces based on fractal theory was proposed. An improved 3D WM fractal function was used to characterize the contact surface, contact load and contact area equations of asperities in elastoplastic deformation regime were established, solutions for the relationships of area-displacement and force-displacement in the elastoplastic deformation regime was done based on Hertz contact theory and fractal theory, and the present model was proven to be effective by comparing the present model to other four classical contact models and test data. Furthermore, simulations and numerical calculation results reveal nonlinear relation between the influence factors and the contact area.

1. Introduction

It is important to study and model the deformation behavior of asperities and rough surfaces using adequate parameters [1]. Hertz elastic contact theory on sphere contact was put forward for the first time in 1882, and become the basis of classical contact mechanics. Chang et al. [2-4] proposed CEB contact model based on volume conservation of an asperity, this model connected the elastic deformation and the plastic deformation, which widened the using range of Hertz contact model. Jackson and Green [5] established the finite element model on elastoplastic contact of hemispheroid, that is JG contact model. The set of cubic Hermite polynomials were used to enforce continuity between the elastic and fully plastic regimes by Brake [6], called Brake model. GW contact model was presented by Greenwood and Williamson [7] on contact behavior of joint interfaces in 1966. Majumdar and Bhushan [8] analyzed the contact behavior of the rough surface based on WM function, and this MB model opened the door to build fractal contact theory.

The main purpose of this study is to reveal the contact mechanical behavior of machine joint interfaces. Based on contact mechanics theory and fractal theory as well as considering elastic, elastoplastic and fully plastic deformations of contacting asperities, a fractal contact model of joint interfaces was proposed and studied. Moreover, the present model was compared to other four classical contact models and test data, and numerical calculation are employed to reveal nonlinear relation between the influence factors and the contact area of joint interfaces.

2. Contact model of asperities

2.1. Characterization of surface topography
Weierstrass-Mandelbrot (W-M) function [9] is widely used to characterize rough surface. A revised W-M function can be represented as

\[ z(x) = L(G/L)^D \left( \ln \gamma \right)^{1/2} \sum_{n=0}^{\infty} \gamma^{(D-2)n} \left[ \cos \phi_n - \cos \left( 2\pi \gamma^n x/L - \phi_n \right) \right] \]  

(1)

Where, \( z(x) \) is the surface profile height, \( x \) is the lateral distance, \( L \) is the sample length, \( D \) is the fractal dimension, \( G \) is the fractal roughness parameter, \( \gamma \) is the scaling parameter, \( \phi_n \) is a random phase, and \( n \) is a frequency index ranging from \( n_{\text{min}} \) which represents the lowest level corresponding to the lowest cutoff frequency to \( n_{\text{max}} \) which represents the highest level corresponding to the highest cutoff frequency.

The new size distribution function of microcontacts \( n(a') \) was given as [10],

\[ n(a') = D \cdot \left( \frac{2^D}{2^D-2} \cdot \frac{D(D-2)}{2} \right)^{-(D-2)/2} \]  

(2)

where, \( a' \) is the truncated area of microcontact with the radius \( r' \), \( a' \) is the truncated area of the largest microcontact, and \( D \) is the domain extension factor for microcontact size distribution, and the values of \( D \) are the functions of fractal dimension \( D \).

2.2 Contact mechanics model
As is known that two rough contact surfaces can be modeled as an equivalent rough surface in contact with a rigid smooth surface. Any asperity in contact exist in one of three states, which is elastic, elastoplastic or fully plastic. The critical yield interference \( \delta_{pc} \) of the asperity demarcates the transition between elastic deformation regime and elastoplastic deformation regime, and the critical plastic interference \( \delta_{pc} \) of the asperity demarcates the transition between elastoplastic deformation regime and plastic deformation regime, i.e., elastic deformation of a asperity takes place when \( \delta \leq \delta_{pc} \), elastoplastic deformation takes place when \( \delta_{pc} \leq \delta \leq \delta_{pc} \), and plastic deformation takes place when \( \delta \geq \delta_{pc} \). The critical yield interference \( \delta_{pc} \) and the critical plastic interference \( \delta_{pc} \) can be written as

\[ \delta_{pc} = \left( \frac{\pi KH}{2E} \right) \frac{R}{\bar{p}} \]  

\[ \delta_{pc} = \left( \frac{\bar{p}_{pc}}{\bar{p}_{pc}} \right) ^2 \delta_{pc} \]  

(3)

where, \( H \) is the hardness of the softer material, and \( K \) is a hardness coefficient, \( \bar{p}_{pc} \) is the fully plastic indentation pressure, and \( \bar{p}_{pc} \) denotes a mean pressure to initiate yield.

Therefore, the critical truncated yield microcontact area \( a'_{pc} \) and the critical truncated plastic microcontact area \( a'_{pc} \) can be, respectively, written as,

\[ a'_{pc} = \left[ 2^{(7-2D)/2^D} G^{2(D-1)} \ln \left( \frac{2E/\pi KH}{\bar{p}} \right)^2 \right] ^{((D-1)/2)} \]  

(4)

\[ a'_{pc} = \left[ 2^{(7-2D)/2^D} G^{2(D-1)} \ln \left( \frac{2E/\pi KH}{\bar{p}_{pc}} \right)^2 \left( \bar{p}_{pc} / \bar{p}_{pc} \right) \right] ^{((D-1)/2)} \]  

(5)

Thus, the relationship between \( a'_{pc} \) and \( a'_{pc} \) can be derived from equation (4) and (5),

\[ a'_{pc} / a'_{pc} = \left( \frac{\bar{p}_{pc}}{\bar{p}_{pc}} \right) ^{2(D-1)} \]  

(6)
In the elastic deformation regime, based on the Hertz contact theory, the microcontact area \( a_e \) and the elastic contact load \( F_e \) of the contact asperities can be, respectively, expressed as

\[
a_e = \pi R \delta_\nu \left( \delta / \delta_\nu \right)^{1/3}, \quad F_e = 2\pi KH \delta_\nu \left( \delta / \delta_\nu \right)^{3/2} / 3 \tag{7}
\]

In the plastic deformation regime, the microcontact area \( a_p \) and the plastic contact load \( F_p \) of the contact asperities can be, respectively, expressed as

\[
a_p = 2\pi R \delta, \quad F_p = H \delta_p \tag{8}
\]

In the elastic and plastic deformation regimes, the real microcontact area and truncated area can be connected by the following two equations,

\[
a_e = a_p / 2, \quad a_p = a_e \tag{9}
\]

From equation (7), the following relationships can be deduced,

\[
a_e = \pi R \delta_\nu \left( \delta / \delta_\nu \right)^{1/3}, \quad F_e = 2\pi KH \delta_\nu \left( \delta / \delta_\nu \right)^{3/2} / 3 \tag{10}
\]

Similarly, the form of the above two equations can be considered in the elastoplastic deformation regime. The exponential forms are also used to express the relationships of microcontact area \( a_{ep} \) and the microcontact load \( F_{ep} \) in the elastoplastic deformation regime,

\[
a_{ep} = \pi R \delta_\nu \left( \delta / \delta_\nu \right)^{b}, \quad F_{ep} = 2\pi KH \delta_\nu \left( \delta / \delta_\nu \right)^{d} / 3 \tag{11}
\]

where, \( b \) and \( d \) are constants. To derive these two constants, it is assumed that the contact area and contact load are both changing constantly in the whole stages of deformation. Therefore, based on this assumption, the boundary conditions can be established at the end of the elastic regime and the beginning of the plastic regime, i.e.,

\[
[\delta_{\nu}]_{b=\delta_{\nu}} = a_{\nu} \left|_{b=\delta_{\nu}} \right., \quad a_{\nu} \left|_{b=\delta_{\nu}} \right. = a_\nu \left|_{b=\delta_{\nu}} \right., \quad F_{\nu} \left|_{b=\delta_{\nu}} \right. = F_{\nu} \left|_{b=\delta_{\nu}} \right., \quad F_{ep} \left|_{b=\delta_{\nu}} \right. = F_{ep} \left|_{b=\delta_{\nu}} \right. \tag{12}
\]

From equations (7), (8), (11) and (12), the value of \( b \) and \( d \) can be got,

\[
b = 1 + \log_{\delta_\nu} / (2), \quad d = 1 + \log_{\delta_\nu} / (3/K) \tag{13}
\]

Thus, the relationships of microcontact area \( a_{ep} \) and the microcontact load \( F_{ep} \) in the elastoplastic deformation regime can be easily obtained as,

\[
a_{ep} = a^{(b-1)(1-D)} / \left[ 2a^{(b-1)(1-D)} \right], \quad F_{ep} = 4ER^{d/2} \delta_\nu^{d/3} \left( a / a_{ep} \right)^{d/3} / 3 \tag{14}
\]

2.3. Verification of the contact model

In order to verify the validity of the present contact model, the following two methods will be used to prove it. One way is to compare present model to other classical contact model; the other way is to compare present model to the experimental data. In the present paper, the other four kinds of classical contact model including Hertz contact model, CEB contact model [2], JG contact model [5] and Brake contact model [6] are used to contrast with present model, in the meantime, present model is also compared with the test data in the literature [11]. For the convenience of comparative analysis, the critical load \( F_w \), dimensionless interference \( \delta^* \), dimensionless microcontact load \( F^* \) and dimensionless microcontact area \( a^* \) are, respectively, defined as,
\[ F_{\text{yc}} = 4ER_{\text{yc}}^2 \delta_{\text{yc}}^{\frac{3}{2}} / 3, \quad \delta^* = \delta / \delta_{\text{yc}}, \quad F^* = F / F_{\text{yc}}, \quad a^* = a / a_{\text{yc}} \] (15)

Table 1 illustrated the experimental data of the contact sphere, the values of material properties and geometric parameters are given in Table 1.

| Material properties                        | Material name: copper | Material name: silicon carbide |
|--------------------------------------------|-----------------------|-------------------------------|
| Density \( \rho_1 \) [kg/m\(^3\)]         | 3200                  | Density \( \rho_2 \) [kg/m\(^3\)] | 7850 |
| Elasticity modulus \( E_1 \) [GPa]         | 430                   | Elasticity modulus \( E_2 \) [GPa] | 120  |
| Poisson's ratio \( \nu_1 \)                | 0.17                  | Poisson's ratio \( \nu_2 \)     | 0.35 |
| Hardness \( H_1 \) [GPa]                   | 28                    | Hardness \( H_2 \) [GPa]        | 1.2  |
| Yield strength \( \sigma_{y1} \) [GPa]     |                        | Yield strength \( \sigma_{y2} \) [GPa] | 0.3  |
| Radius \( R_1 \) [m]                       | +\( \infty \)         | Radius \( R_2 \) [m]            | 0.0015 |

Figure 1 shown the present model compared with experimental data and other four classical contact models including Hertz model, CEB model, JG model and Brake model. Variations of the dimensionless area with the dimensionless interference is illustrated in Figure 1(a), as is demonstrated in Figure 1(a), the dimensionless area is increases along with the increment of dimensionless interference. When interference is higher, the theoretical calculating value of area in Hertz contact model is far lower than experimental data because of the elastoplastic and plastic deformation process are neglected in Hertz model, in the meantime, the theoretical calculating value of area in JG contact model is also far lower than experimental data but slightly larger than Hertz model. on the other hand, the theoretical calculating value of area in the models including present model, CEB model and Brake model agree with the experimental data. Thus, the relationship between dimensionless area and the dimensionless interference have higher precision.

Variations of the dimensionless area with the dimensionless contact load is illustrated in Figure 1(b), as is demonstrated in Figure 1(b), the dimensionless area is increases along with the increment of dimensionless load. Similarly, when contact load is higher, the theoretical calculating value of area in Hertz contact model and JG model are far lower than experimental data because of the elastoplastic and plastic deformation process are also neglected in these two models, meanwhile, the theoretical calculating value of area in CEB contact model is far larger than experimental data. These three contact models have big error, however, the theoretical calculating value of area only in the present model and Brake model agree with the experimental data. Therefore, the relationship between dimensionless area and the dimensionless load have higher precision.
Figure 1. Present model compared with experimental data and other four classical contact models.

3. Contact model of joint interfaces

3.1. Contact load model of joint interfaces

The total load of joint interfaces \( F_{ns} \) includes total elastic load \( F_{ne} \), total elastoplastic load \( F_{nep} \) and total fully plastic load \( F_{np} \). So, the total load of joint interfaces can be obtained by following calculation,

\[
F_{ns} = F_{ne} + F_{nep} + F_{np} = \int_{a_{n}}^{a_{e}} F_n \, n(a') \, da' + \int_{a_{n}}^{a_{p}} F_{nep} \, n(a') \, da' + \int_{a_{p}}^{a_{e}} F_{np} \, n(a') \, da' \quad (16)
\]

Dimensionless total load of joint interfaces can be defined as

\[
F_{ns}^* = F_{ns} / (Ea_n)
\]

where, \( a_n \) is the nominal contact area of joint interfaces.

3.2. Contact load model of joint interfaces.

The total truncated area of joint interfaces \( A'_r \) can be got by Integral algorithm,

\[
A'_r = \int_{a_{r}}^{a_{e}} a' n(a') \, da'
\]

(18)

Similarly, The total real contact area of joint interfaces \( A'_r \) also includes three parts including total real elastic contact area \( A_{re} \), total real elastoplastic contact area \( A_{rep} \) and total real fully plastic contact area \( A_{rp} \). So, the total real contact area of joint interfaces can be obtained as,

\[
A_r = A_{re} + A_{rep} + A_{rp} = \int_{a_{re}}^{a_{e}} a_{re} n(a') \, da' + \int_{a_{rep}}^{a_{e}} a_{rep} n(a') \, da' + \int_{a_{rp}}^{a_{e}} a_{rp} n(a') \, da'
\]

(19)

Dimensionless total real contact area of joint interfaces can be also defined as

\[
A'_r = A_r / A_n
\]

(20)

4. Simulation and calculation of Contact model

Fractal parameters and contact load are two main influencing factors to real contact area when two rough surface contact into each other. Variations of the dimensionless total real contact area \( A'_r \) with the fractal parameters are illustrated in Figure 2, and Figure 2(a) shows the variations of the dimensionless total real contact area \( A'_r \) with the fractal dimension \( D \). As is shown in Figure 2(a), real contact area \( A'_r \) is only a small part of nominal contact area \( A_n \), meanwhile, there is a convex nonlinear relation between \( A'_r \) and \( D \), \( A'_r \) increases along with the increment of \( D \) at first, then decreases when reaching the maximum point. The maximum point appears near the value of \( D \approx 1.5 \). Figure 2(b) shows the Variations of the dimensionless total real contact area with the fractal roughness parameter \( G' \), and \( A'_r \) decreases along with the increment of \( G' \). So, the rougher of the contact surface, the larger of the real contact area, and reducing the surface roughness is helpful to increase the contact area.
Figure 2. Variations of the dimensionless total real contact area with the fractal parameters. Variations of the dimensionless total real contact area $A_r^*$ with the dimensionless total contact load $F_n^*$ is demonstrated in Figure 3. As is illustrated in Figure 3(a) and (b), $A_r^*$ increases near linearly with the increment of $F_n^*$, meanwhile, $A_r^*$ increases along with the increment of $G^*$.

Figure 3. Variations of the dimensionless total real contact area with the dimensionless total contact load

5. Conclusions

Based on fractal theory and contact mechanics theory as well as considering elastic, elastoplastic and fully plastic deformations of contacting asperities, a fractal contact model of joint interfaces was proposed and studied. It was verified by comparing the present to the other four classical contact model and test data that the two basic relationships including area-displacement relationship and force-displacement relationship of asperities have higher precision. Besides, it was concluded from simulations and calculations results that dimensionless total real contact area increases along with the increment of fractal dimension at first, then decreased when reaching the maximum point, and dimensionless total real contact area decreased along with the increment of fractal roughness parameter. In the meantime, dimensionless total real contact area increased near linearly with the increment of dimensionless total contact load of machine joint interfaces.

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