Study on the Characteristics and Influence Factors of Spontaneous Imbibition in the Fractured Reservoirs

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Abstract. Spontaneous imbibition is an important mechanism for oil recovery in the natural fractured reservoirs. It’s crucial to study the characteristics and influence factors of the imbibition process. We present analytical solutions for the one-dimensional incompressible imbibition process. Based on analytical solution, we study the effect of some factors on the imbibition process. It is also found in this paper that the larger the viscosity ratio is, the smaller the critical saturation is. A new scaling group is also suggested in this paper to fit experiment data.

Key words: capillary pressure, imbibition, counter-current flow, fractured reservoirs.

1. Introduction
The naturally fractured reservoirs account for a large proportion of world hydrocarbon reserves [1]. To improve oil recovery from fractured reservoirs, it is important to understand the physical processes determining the interaction and fluid transfer between the matrix and the fracture [2]. Spontaneous imbibition can be the primary mechanism of oil recovery from matrix blocks in water-wet fractured reservoirs [3]. This kind of counter-current imbibition process has been widely concerned by scholars, and the variation of its recovery curve with time has become the focus of research [1, 4-6].

The imbibition process is complicated and the influence of some factors need to be study. Many scholars have studied the imbibition process by core experiments and fitted those using empirical formulas [7-9]. However, the empirical formulas are generally simple functions, which cannot accurately describe the process of imbibition, some scholars have carried out the study of imbibition process by using numerical methods [5]. Numerical methods are widely used to study the imbibition process while very fine grids are required. In this paper, an analytical solution is present to describe the imbibition process. The solution is verified by refined numerical results. The effect of the relative permeability and the viscosity is studied. And finally a new scaling group is developed.

2. Mathematical model
Consider a 1D two-phase incompressible imbibition flow. The matrix block with length of L is full of oil, one side of which is exposed in the fracture, which is the open boundary of pure water, and the other boundary is set as no flow. Because of the large capillary pressure in the matrix, the water in the fracture is drawn into the matrix, and the oil in the matrix is driven out of the matrix. The matrix saturation equation in this process can be expressed as follows:
where $S$ is normalized water saturation $S = \left( S_w - S_{wi} \right) / \left( 1 - S_{wi} - S_{or} \right)$, $S_{wi}$ and $S_{or}$ are irreducible water saturation and residual oil saturation respectively, $K$ is absolutely permeability, $\phi$ is porosity, $P_c$ is capillary pressure, $k_r$ and $k_o$ are water and oil relative permeability respectively, $\mu_w$ and $\mu_o$ are water and oil viscosity. 

In order to simplify the study, the following relative permeability and capillary pressure are used in this paper:

$$k_{rw} = k_{rw}^{\text{max}} S^a, k_{ro} = k_{ro}^{\text{max}} (1 - S)^b, P_c = \sigma \sqrt{\frac{\phi}{K}} J(S)$$

(2)

where $a$ and $b$ are positive, $J(S)$ is dimensionless capillary pressure function, $k_{rw}^{\text{max}}$ and $k_{ro}^{\text{max}}$ are the maximum value of water phase and oil phase relative permeability respectively.

Define dimensionless length and time

$$x_D = \frac{x}{L}$$

(3)

$$t_D = t \sqrt{\frac{K}{\phi} \left( 1 - S_{wi} - S_{or} \right) / \mu_w L^2}$$

(4)

Then the equation (1) can be rewritten as:

$$\frac{\partial S}{\partial t_D} = \frac{\partial}{\partial x_D} \left[ - \frac{S^a (1 - S)^b}{M S^a + (1 - S)^b} J'(S) \frac{\partial S}{\partial x_D} \right]$$

(5)

Where $M = \mu_o k_{rw}^{\text{max}} / \mu_w k_{ro}^{\text{max}}$ is generalize viscosity ratio.

Define $D(S) = -S^a (1 - S)^b J'(S) / \left[ M S^a + (1 - S)^b \right]$, then the dimensionless equation (5) can be rewritten as:

$$\frac{\partial S}{\partial t_D} = \frac{\partial}{\partial x_D} \left[ D(S) \frac{\partial S}{\partial x_D} \right]$$

(6)

Set $\xi = \alpha x_D / \sqrt{t_D}$, $\alpha$ is an undetermined constant, $\xi \in [0, 1]$, $\xi = 1$ represents waterflood frontiers. It is not difficult to prove there is similarity solution $S(x_D, t_D) = \tilde{S}(\xi)$. By substituting the similarity solution into equation (5) we have

$$\frac{d}{d\xi} \left[ D(S) \frac{dS}{d\xi} \right] + \frac{\xi}{2\alpha} \frac{dS}{d\xi} = 0$$

(7)

This is a second order ordinary differential equation with variable coefficients. In addition to two boundary conditions $S\big|_{\xi=0} = 1$ and $S\big|_{\xi=1} = 0$, we need to supplement another one. Considering that the flux at the waterflood frontiers is 0, then we have:
\[
D(S) \frac{\partial S}{\partial x_D} \bigg|_{\xi=0} = \alpha D(S) \frac{dS}{d\xi} \bigg|_{\xi=1} = 0
\]  

(8)

So the boundary conditions of (7) are:

\[
S|_{\xi=0} = 1, S|_{\xi=1} = 0, D(S) \frac{dS}{d\xi} |_{\xi=1} = 0
\]  

(9)

We can obtain \(S(\xi)\) and \(\alpha\) by solving the equation.

Now back in the space \(S(x,t)\), the inlet water flux can be expressed as:

\[
q_{in} = -D(S) \frac{\partial S}{\partial x_D} \bigg|_{x=0} = -\frac{\alpha}{\sqrt{t_D}} D(S) \frac{dS}{d\xi} \bigg|_{\xi=0}
\]  

(10)

For simplicity, set \(-2\alpha D(S) \frac{dS}{d\xi} \bigg|_{\xi=0} = \gamma\), integrate formula (6), we can get:

\[
\int_0^1 \frac{\partial S}{\partial x_D} dx_D = q_{in} = \frac{\gamma}{2\sqrt{t_D}}
\]  

(11)

Then the average water saturation is related to time as follows:

\[
\bar{S} = \gamma \sqrt{t_D}
\]  

(12)

The critical time is defined as the time when the waterflood frontier reaches the boundary, and at this time \(\xi = 1\) correspond to \(x_D = 1\). Therefore, \(\alpha / \sqrt{t_D} = 1\) and the critical time and critical saturation are:

\[
t_D^* = \alpha^2
\]  

(13)

\[
S^* = \gamma \alpha
\]  

(14)

A new scaling group is suggested as:

\[
t_{D,\text{new}} = \sqrt{\frac{K}{\phi \left(1-S_{wi}-S_{or}\right) \mu_w \eta_w L^2}} \left(\frac{k_{rw} \text{max}}{k_{rw}}\right)
\]  

(15)

3. Numerical test

In this section we will test the analytic solution. The proposed solution is compared with the reference numerical solution obtained on the enough refined grids. Consider a spontaneous imbibition process with OEO (One-End-Open) boundary condition as same as used by Zhang et al. (1996). From Figure 1 it can be seen that analytical solution is consistent with the reference one. Furthermore, the linear relation between the average saturation and the square root of dimensionless time is reproduced, which is consistent with the conclusion drawn by Tavassoli et al. [5].
Figure 1. Comparison of the average saturation in the matrix

From equation (4), we can see that the imbibition rate is proportional to the interfacial tension and inversely proportional to the length of the system squared, as demonstrated experimentally. Another important factor is viscosity. Using the analytical solution, we can obtain the relation between the viscosity ratio and parameter $\gamma$ which represents the imbibition rate. In Figure 2, variation curves of the critical saturation with viscosity ratio $M$ is shown. It indicates that the higher viscosity ratio is, the slower the imbibition process is.

Figure 2. Parameter $\gamma$ with different $M$

4. Conclusion
In this paper, we use the mathematical method to study the one-dimensional imbibition problem. By studying the dimensionless governing equation, the dimensionless analytical similarity solution is obtained for the early time of imbibition process, and the accurate critical time and critical saturation are then obtained. Examples show that the proposed solution in this paper is in good agreement with numerical results. Some factors are also studied by using this analytical solution and this method can also be applied in the development of the fractured reservoirs in the future.
Acknowledgments
This work is supported by National Science and Technology Major Project (Grant Number No. 2017ZX05072005).

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