NUMERICAL STUDY OF THE EQUATION OF STATE FOR TWO FLAVOR QCD AT NON-ZERO BARYON DENSITY *

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We discuss the equation of state (EoS) for two flavor QCD at non-zero temperature and density. Derivatives of lnZ with respect to quark chemical potential $\mu_q$ are calculated up to sixth order. From this Taylor series, the pressure, quark number density and associated susceptibilities are estimated as functions of temperature and $\mu_q$. It is found that the fluctuations in the quark number density increase in the vicinity of the phase transition temperature and the susceptibilities start to develop a pronounced peak as $\mu_q$ is increased. This suggests the presence of a critical endpoint in the ($\mu_q, T$) plane. Moreover, we comment on the hadron resonance gas model, which explains well our simulation results below $T_c$.

1. Introduction

It is important to study QCD at high temperature and density by numerical simulations of lattice QCD. In particular, studies of the equation of state (EoS) can provide basic input for the analysis of the experimental signatures for quark-gluon plasma formation. We study the EoS in the low but non-zero baryon number density regime, interesting for heavy-ion collisions. The simulation of QCD at non-zero $\mu_q$ is known to be difficult. However, studies based on a Taylor expansion with respect to $\mu_q$ turned out to be an efficient technique to investigate the low density regime.

Thermal fluctuations near the critical temperature $T_c$ are important information for the understanding of the QCD phase diagram. Baryon number fluctuations are expected to diverge at the endpoint of the first order phase transition.

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chiral phase transition line. The electric charge fluctuation in heavy-ion collisions is one of the most promising experimental observables to identify the critical endpoint. The fluctuations of quark number, isospin number and electric charge are estimated by the corresponding susceptibilities, $\chi_\mu$, $\chi_I$ and $\chi_C$, which are given by $\chi_\mu = \partial^2 p/\partial \mu_\mu^2$ and $\chi_I = \partial^2 p/\partial \mu_I^2$, where $\mu_\mu = (\mu_u + \mu_d)/2$, $\mu_I = (\mu_u - \mu_d)/2$, and $\mu_{u[d]}$ is the chemical potential for the $u[d]$ quark. For $\mu_u = \mu_d$, the charge susceptibility is $\chi_C = \chi_\mu/36 + \chi_I/4$.

In this study, we investigate the pressure $p$, quark number susceptibility $\chi_\mu$ and isovector susceptibility $\chi_I$ by calculating the Taylor expansion coefficients of $p$, $c_n$ and $c^I_n$, up to sixth order in $\mu$, which are defined by $p/T^4 = (\ln Z)/(VT^3) = \sum_{n=0}^\infty c_n(\mu_q/T)^n$ and $\chi_I(\mu_I/T^2) = \sum_{n=2}^\infty n(n-1)c_n[c^I_n(\mu_q/T)^{n-2}$ for $\mu_u = \mu_d$. Here $c_n = (\partial^n \ln Z/\partial \mu_q^n)/n!$ and $c^I_n = (\partial^n \ln Z/\partial \mu_q^{n-2}\partial \mu_I^2)/n!$. Moreover, we compare these results with the prediction from the hadron resonance gas model $^{3,4}$.

2. Quark gluon gas or hadron resonance gas

We expect the equation of state approaches that of a free quark-gluon gas (Stefan-Boltzmann (SB) gas) in the high temperature limit. The coefficients in the SB limit for $N_f = 2$, $\mu_I = 0$ are well known as $c_2 = c^I_2 = 1$, $c_4 = c^I_4 = 1/(2\pi^2)$ and $c_n = c^I_n = 0$ for $n \geq 6$.

On the other hand, in the low temperature phase QCD is well modelled by a hadron resonance gas. If the interaction between these hadrons can be neglected, the pressure is obtained by summing over the contributions from all resonance states of hadrons. The contribution to $p/T^4$ from a particle which has mass $m_i$, baryon number $B_i$ and third component of isospin number $I_{3i}$ is given by

$$\frac{pm_i}{T^4} = \frac{1}{2\pi^2} \left(\frac{m_i}{T}\right)^2 \sum_{l=1}^{\infty} \frac{\eta^{l+1}}{l^2} K_2 \left(\frac{lm_i}{T}\right) \exp \left[\frac{3B_i\mu_q + 2I_{3i}\mu_I}{T}\right],$$  \hspace{1cm} (1)

where $K_2$ is the modified Bessel function, and $\eta$ is 1 for mesons and $-1$ for baryons. Moreover, since $m_i/T \gg 1$ for all baryons and $K_2(x) \approx \sqrt{\pi/2x} \exp(-x)$, the first term of $l$ is dominant in the baryon sector. Therefore, the pressure can be written by $p/T^4 = G(T) + F(T) \cosh(3\mu_q/T)$ for $\mu_I = 0$, where $G(T)$ and $F(T)$ are the mesonic and baryonic components of $p/T^4$ at $\mu_q = 0$, respectively. Similarly, we obtain $\chi_\mu/T^2 = 9F(T) \cosh(3\mu_q/T)$ and $\chi_I/T^2 = G^I(T) + F^I(T) \cosh(3\mu_q/T)$. Here, the mesonic component for $\chi_\mu$ is zero because mesons ($B_i = 0$) are independent of $\mu_q$. Therefore, $c_4/c_2 = 3/4$, $c_6/c_4 = c^I_6/c^I_4 = 3/10$ in the region where the non-interacting hadron resonance gas provides a good approximation.
We investigate these coefficients. Simulations are performed on a $16^3 \times 4$ lattice for 2 flavor QCD. The $p^4$ improved action is employed at $ma = 0.1$, which gives $m_{PS}/m_V = 0.7$. The number of configurations is 1000-5000 for each $T$. We use the random noise method. The results for $c_{n+2}/c_n$ and $c_{I+2}/c_{I_n}$ are shown in Fig. 1. We find these results are consistent with the prediction from the hadron resonance gas model for $T/T_c \leq 0.96$ and approach the SB values, i.e. $c_4/c_2 = c_{I_4}/c_{I_2} = 1/2\pi^2$, $c_6/c_4 = c_{I_6}/c_{I_4} = 0$, in the high temperature limit. These results suggest that the models of free quark-gluon gas and hadron gas seem to explain the behavior of thermodynamical quantities well except in the narrow regime near $T_c$.

Moreover, if the data is consistent with the hadron resonance gas model, we can separate $\chi_I$ into the contributions from isotriplet mesons and baryons, since $G^I + F^I = 2c_2^I$ and $9F^I/2 = 12c_4^I$. Isosinglet mesons do not contribute to $\chi_I$ because $I_{3i} = 0$. Also, $9F = 2c_2$ from the equation of $\chi_q$. Furthermore, for the case of $m_\pi \gg T$, the first term ($l = 1$) in Eq.(1) is dominant. This approximation holds for our present simulations where $m_\pi/T \approx 4-5$ but will no longer be valid with physical quark mass values. For our present simulation parameters, the ratio of $\partial^2(p_{mi}/T^4)/\partial(\mu_1/T)^2$ and $p_{mi}/T^4$ equals $(2I_{3i})^2$ for $I_{3i} = \{-1, 0, 1\}$. We thus find for the contribution of the triplet meson part: $\tilde{G} = 3G^I/8$. We then can estimate the contribution of the isosinglet mesons to $p/T^4$ by comparing $F + \tilde{G}$ with the total pressure. In Fig. 2 (left), we plot the baryon component $F = 2c_2/9$ (square), the triplet meson component $\tilde{G} = 3c_4^I/4 - c_4^I$ (circle) and the total (diamond). The solid line is the pressure obtained with the integral method in Ref. 6. The dashed lines in Fig. 2 (left) represent the predictions from the hadron resonance gas model for the mass parameter of our simulations. The resonance states for this quark mass are adjusted by the method.
described in Ref. 3. The simulation results for \( p/T^4 \) obtained with the integral method, together with the triplet meson and baryon contributions, are surprisingly well reproduced by this model calculation. The result of the singlet meson contribution may explain the difference in the pressure obtained from the integral method and that from the Taylor expansion which neglects the singlet part.

3. Pressure and susceptibilities at \( \mu_q \neq 0 \)

Next, we calculate pressure and quark number susceptibility in a range of \( 0 \leq \mu_q/T \leq 1 \), using the data of \( c_n \) and \( c_n^I \): \( \Delta(p/T^4) \equiv p(T, \mu_q)/T^4 - p(T, 0)/T^4 = c_2(\mu_q/T)^2 + c_4(\mu_q/T)^4 + c_6(\mu_q/T)^6 \), \( \chi_q/T^2 = 2c_2 + 12c_4(\mu_q/T)^2 + 30c_6(\mu_q/T)^4 \), and the corresponding equation for \( \chi_I \).

We draw \( \Delta(p/T^4) \) for each fixed \( \mu_q/T \) in Fig. 2 (right) and find that the difference from \( p|_{\mu_q=0} \) is very small in the interesting regime for heavy-ion collisions, \( \mu_q/T \approx 0.1 \) (RHIC) and \( \mu_q/T \approx 0.5 \) (SPS), in comparison with the value at \( \mu_q = 0 \), e.g. the SB value for 2 flavor QCD at \( \mu_q = 0 \): \( p^{SB}/T^4 \approx 4.06 \). The effect of a non-zero quark density on the pressure at \( \mu_q/T = 0.1 \) is only 1%. Comparing with previous result up to \( O(\mu^4_q) \), the effect from \( O(\mu^6_q) \) seems to be small in the range we investigated. Also, the result is qualitatively consistent with that of Ref. 7 obtained by the reweighting method.

The data connected by solid lines in Fig. 3 are the results for \( \chi_q \) (left) and \( \chi_I \) (right) obtained by this method up to \( O(\mu^4_q) \). We also calculated the susceptibilities by another two methods. Dot-dashed lines are from the hadron resonance gas model, using \( F, F^I \) and \( G^I \) we obtained. This turned out to be a good approximation for \( T/T_c \lesssim 0.96 \). Dashed lines are calculated
by the reweighting method using an approximation discussed in Ref. 1. Here the truncation error is $O(\mu_q^6)$ but the effect from higher order terms of $\mu_q$ is partially included. The sign problem in the reweighting method is serious when the complex phase fluctuations of the quark determinant are large at large $\mu_q/T$. Hence we omitted data at which the standard deviation of the complex phase is larger than $\pi/2$. The difference among the three results is caused by the approximation in the higher order terms of $\mu_q/T$.

Since the statistical error of $c_6$ is still large near $T_c$, the peak of $\chi_q$ is not clear. However, as seen in Fig. 1, $c_6$ changes its sign at $T_c$. This means the peak position of $\chi_q$ moves left, which is corresponding to the change of $T_c$ as a function of $\mu_q$. $T_c(\mu_q/T = 1)/T_c(\mu_q/T = 0)$ in Ref. 1 is about 0.93. Moreover, the strong enhancement of $\chi_q$ with increasing $\mu_q$ near $T_c$, obtained in Ref. 2, is observed also in the analysis up to $O(\mu_q^4)$. This suggests the presence of a critical endpoint in the $(T, \mu_q)$ plane. On the other hand, $\chi_I$ in Fig. 3 does not show any singular behavior. This is consistent with the sigma model prediction that only isosinglet degrees of freedom become massless at the critical endpoint $^8$.

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