Modelling and Production of artillery firing-tables: case-study

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Abstract. Projectile firing accuracy is an important artillery issue to increase killing probability with minimum number of rounds per target. This can be achieved through constructing a firing-table (FT) with both standard and non-standard conditions. Therefore, lot of rounds need to be shoot for different target ranges as well as meteorological conditions to increase such firing accuracy. Developing an accurate computational FT prior to shoots has the advantages of minimizing the cost and providing preliminary prediction of dispersion. In this study, a computational FT algorithm has been proposed based on the modified-point-mass projectile trajectory model for solving the ballistics problem. A flow-chart is proposed to illustrate the FT production levels and the corresponding procedures. A case study for 155mm-M107-HE projectile is utilized. In order to validate the proposed algorithm, a comparison has been implemented with results obtained from the well-known commercial package PRODAS. Finally, based on real firing data available including the corresponding meteorological conditions, a provisional firing table has been developed.

1. Introduction
Firing tables are the basic document necessary for military combat training and shooting specifications. Without firing table, the effective shooting cannot be performed. Firing table provides the basic data, the shooting deviation correction data, and related data required to command the shooting and obtain effective artillery accuracy. Historically, firing tables were developed continuously over a number of decades. Due to its simplicity, Siacci method were used for computing trajectory problems. In 1917, the French Short Arc methods were implemented to compute such trajectories by Sandy Hook and Aberdeen. Numerical integration methods had been introduced by Aberdeen during 1918 to accurately solve the trajectory problem and hence, firing table calculations.

During the preparation of firing table, it must be specified that there is a representative firing conditions, which is the standard firing conditions. There are many factors affect the flight trajectory, including artillery, projectile, meteorological data and other factors. During real firing, these factors are varying, where it is impossible to specify the actual firing conditions for each shoot, which cannot meet the operational requirements of the firing table. Therefore, the actual firing conditions and standards are not consistent. Therefore, standard firing conditions including standard meteorological, ballistic, and Earth conditions are specified, and hence, the effects of these deviations are corrected. In order to precisely construct firing tables, more than 200 thousands flight trajectory simulation are needed through final phase only [1]. In case of medium range projectiles, the average flight time is
approximately 50 s and hence, approximately two days processing time for normal computer are needed. In 1943, U.S. army developed an electronic device called Electronic Numerical Integrator and Computer (ENIAC) that capable of computing new artillery firing tables up to 1000 times faster than any previously known computer [2]. Different researchers [1, 3-6], proposed the procedure for the processing of measured data available from real fires to produce a provisional table as well as final firing table. Drag and lift factors can reduced by comparing the ballistics parameters computed with the measured ones through fitting process. A graphical firing table mathematical description has been illustrated [7, 8], where the firing parameters as azimuth and elevation angles are obtained based on standard and non-standard conditions. Developing a firing table software can be presented in various configurations based on model complexity starting from point-mass model [1, 3] to six-degree-of-freedom model [9], projectile platform as ground-launched [1, 9-11] or air-launched [12], and nature of meteorological data available [1].

However, with the modernization of the war, the direct use of the firing tables is gradually reduced. Instead, the fire control system is used to automatically, fast and accurate solution of a set of shooting conditions to obtain the needed deviation correction. Therefore, an accurate and fast trajectory model could be developed [12] instead of the well-known six-degree-of-freedom.

In this study, a procedure for developing firing table of high-explosive spinning projectile is illustrated. This research paper is organized as follows, section 2, illustrates the development procedure for ground artillery firing table including trajectory model, standard firing conditions, and the definition of every column through basic and correction tables. The implemented case-study is presented, and finally, results and discussion are illustrated in section 3. In section 4, our conclusion are listed.

2. Methodology
Initially, firing table accuracy is based on the utilized flight model complexity and the number of rounds to be shoot. Construction of FT can be divided into three main phases as, construction of the preliminary FT using a suitable flight model and projectile data available through research and development R&D phase. After achieving the technical and operational requirements of the developed artillery system, a number of field experiments are done to construct the provisional FT which helps to good estimate the projectile impact parameters during the production of final FT, which needs large number of shoots through different elevation angles.

![Figure 1. Projectile Coordinate System and Ballistic Directions.](image1)

![Figure 2. Flow chart for the development procedure of computation FT.](image2)
2.1. Flight Trajectory Model [13]

Due to the complexity of rigid body 6-DOF trajectory model, a modified point mass (4-DOF) model has been utilized referred to earth fixed coordinate \((X_1, X_2, X_3)\) as shown in figure. The point mass motion of the projectile was modified to include projectile Magnus effect, axial spin and an estimation for the yaw of repose, where the projectile epicyclical motion was neglected. The projectile center of gravity equations of motion based on the modified point mass model are

\[
\dot{\mathbf{X}} = \mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}^T \tag{1}
\]

\[
\dot{\mathbf{V}} = -\rho \mathbf{v} \mathbf{a}_D \mathbf{C}_D + \rho \mathbf{a}_L \mathbf{C}_L^* v^2 \mathbf{a}_R + \rho \mathbf{a}_N \mathbf{C}_N^p \rho \mathbf{a} \times \mathbf{a}_R \] + \mathbf{g} \tag{2}
\]

Where, \(a_1 = \left( \frac{S_{ref}}{2m} \right) \); \(a_2 = \left( \frac{S_{ref} d}{2m} \right) \); \(\mathbf{V}\) is the projectile velocity vector in earth fixed coordinate; \(\mathbf{V}_t = (\mathbf{V} - \mathbf{W})\) is the projectile total velocity vector with respect to air speed \(\mathbf{W}\); \(\mathbf{v}\) is the projectile total aerodynamic speed; \(\rho\) is the air density; \(\mathbf{m}, \mathbf{d}, \mathbf{S}_{ref}\) are the projectile mass, reference diameter and reference area respectively; and the coefficients \(\mathbf{C}_D\), \(\mathbf{C}_L\), and \(\mathbf{C}_N\) are the aerodynamic drag, lift, and Magnus force coefficients respectively.

For symmetrical spinning projectiles, the projectile spinning rate can be obtained using

\[
\dot{\mathbf{p}} = \frac{\rho S_{ref} \mathbf{v}^2 \mathbf{a}_R}{2I_x} - \rho C_I^p \] \tag{3}
\]

Where, \(I_x\) is the projectile axial moments of inertia, \(C_I^p\) is the aerodynamic spin damping moment coefficients and the projectile initial spin rate can be computed by

\[
p_0 = \frac{2 \pi v_o}{\eta d} \] \tag{4}
\]

Where, \(\eta\) is the rifling twist rate at gun muzzle and \(v_o\) is the muzzle velocity.

The projectile repose angle \(\alpha_R\) is defined by,

\[
\mathbf{a}_R = \frac{2I_x \rho}{\rho S_{ref} \mathbf{v}^2 C_m} \left[ \mathbf{V} \times \mathbf{V}_t \right] \] \tag{5}
\]

And, the gravitational acceleration \(\mathbf{g}\) is given by

\[
\mathbf{g} = -g \begin{bmatrix} 0 \\ 0 \\ \frac{R_e}{R_e + X_2}^2 \end{bmatrix}^T \] \tag{6}
\]

Where, \(g = 9.80665\); and \(R_e\) is the average radius of the earth (= 6370 km).

2.2. Standard Firing Conditions

During the preparation of firing table, it must be specified that there is a representative firing conditions, including:

2.2.1. Standard Meteorological Conditions. To determine the atmospheric conditions at zero altitude (sea-level) as air temperature \(T_o = 15^\circ C\), pressure \(p_o = 760 \text{ mmHG}\), and density \(\rho_o = 1.225 \text{ kg/m}^3\). Standard atmospheric conditions as function of altitude are based on U.S. standard atmosphere 1962 [14]. It has been concluded in [15] that using standard atmosphere is acceptable but sometimes a more realistic model defining a particular area of the globe is needed specially those areas have very low/high altitudes compared to sea-level. Finally, the wind speed during whole trajectory is assumed zero.
2.2.2. **Standard Ballistic Conditions.** Include table standard projectile weigh value. Although due to production errors, the projectiles weight is not the same, where they values obey a normal distribution with a mean value named as *standard weight value*. The projectiles weight greater/lower than the standard value are divided into squares with the sign +/-, where firing data are corrected according to the number of signs on projectile. The standard charge/propellant temperature is set to +15°C. For standard projectile weight and charge temperature, the projectile muzzle velocity will change with howitzer shooting number, namely for a certain gun, as the shooting number increases, the velocity will slowly change [16]. Therefore, a standard muzzle velocity value has to be measured from a standard gun.

2.2.3. **Standard Earth Condition.** In the Earth’s inertial (non-rotating) coordinate system, the Coriolis acceleration \( \ddot{\Omega} \) and the centrifugal acceleration are zero as illustrated in equation (2).

2.3. **Artillery Computational (Preliminary) Firing Table Model**

Preliminary firing table are consist of different columns in this section we are going to speak about each column characteristics, definition and the equations that enables us to have this column. This preliminary firing tables consists of basic data and correction data.

As illustrated in Fig. 2, for known case study design parameters and nominal flight conditions (including standard atmosphere), number of flight trajectories parameters can be generated for given range of minimum and maximum elevating angles \( \text{QE} \) (namely, trajectory bundle generation) with step value \( \Delta \text{QE} \). It’s required to set FT-key-value to be the resulted ground range instead of \( \text{QE} \), therefore, a linear interpolation will be utilized to obtain the equi-spaced range values and other flight parameters as *Basic data* table. Finally, based on perturbation model, the correction parameters needed to compensate the impact of non-standard firing conditions is reduced as illustrated in 2.3.2.

2.3.1. **Basic data**, it consist of 9 columns as illustrated in Table 1. All trajectory parameters are calculated at standard conditions.

| No. | Unit | Definition |
|-----|------|------------|
| 1   | m    | Range, the resultant ground range (sea-level) as \( R_{\text{std}} = \sqrt{X_1^2 + X_3^2} \). |
| 2   | mils | Elevation angle QE, the gun QE corresponds to required ground range. |
| 3   | s    | Fuse setting FS for graze burst, time to be adjusted to have burst earlier to ground impact as in case of air burst. |
| 4   | s    | Change in fuse setting time \( \Delta \text{FS} \) for 10 m decrease in fuse height burst. |
| 5   | m    | Change in projectile range for 1 mils change in elevation angle, \( \Delta R_{(1\text{mil})} \). |
| 6   | mils | Fork, the change in the elevation angle necessary to produce a change in ground range equivalent to four times the range probable error, \( 4\sigma \). |
| 7   | s    | Time of flight, TOF. |
| 8   | mils | Azimuth correction due to drift \( AZ_{\text{std}} \), the change in the traverse angle needed to discard the effect of drift in case of spin stabilized projectiles (drift must be compensated to left in case of right hand twist barrel). |
| 9   | mils | Azimuth correction due to cross-wind \( AZ_{\text{CW}} \), the change in azimuth angle needed to compensate for 1 knot cross wind either from right or left, where the correction azimuth angle is opposite to wind direction. |
2.3.2. **Correction data.** introduces correction factors which are needed to account for nonstandard firing conditions as a unit increase or decrease to some flight parameters through columns 10 to 19 as follows, 

- Column 10 and 11, represent range correction $\Delta R_{\Delta v_o}$ in [m] due to decrease/increase in projectile muzzle velocity as $\Delta v_o = \pm 1 \text{ m/s}$ resulting new range $R_c = \sqrt{X_{1c}^2 + X_{3c}^2}$. It will be added to compensate the deviation in muzzle velocity from the standard value, where

$$\Delta R_{\Delta v_o} = |R_{\text{std}} - R_c|$$  \hspace{1cm} (7)

- Column 12 and 13, represent range correction $\Delta R_{\Delta W}$ in [m] due to head/tail range wind as $\Delta W = \pm [1 \hspace{0.5cm} 0 \hspace{0.5cm} 0]$ knot resulting $R_c$.

- Columns 14 and 15, represent range correction $\Delta R_{\Delta T_o}$ in [m] due to increase/decrease air temperature (at sea-level) by $\Delta T_o = \pm (0.01 \hspace{0.5cm} T_o)$ which will affect the sonic speed and resulting $R_c$.

- Columns 16 and 17, represent range correction $\Delta R_{\Delta \rho_o}$ in [m] due to increase/decrease air density (at sea-level) by $\Delta \rho_o = \pm (0.01 \hspace{0.5cm} \rho_o)$ which will affect air drag and resulting new range $R_c$.

- Columns 18 and 19, as projectile weight deviated from its standard value by $\Delta m$ resulting $R_c$, where one square represents $\Delta m = \frac{0.02}{3} m_o$.

2.4. **Provisional Firing Table Model**

Through the second phase of the FT production procedure, real shoots will be utilized to improve the computational-FT accuracy as a provisional FT. Therefore, data for three elevation angles QE are collected corresponding to minimum, medium, and maximum ground range which can be implemented through the R&D phase. These data includes, projectile, range, drift, maximum altitude/flight time, and the corresponding meteorological data. This type of data is mandatory for the development of this and next step through construction of FT. Hence, the provisional FT is a good estimation for the weapon’s range-elevation relationship to carry-out firing tests through the final-FT production phase. A meteorological data has to be measured before firings and to be updated every hour. At the end of experiments final meteorological observation has to be obtained [1]. In this paper, a computer program has been developed to assess the relationship between range and elevation with both standard and non-standard conditions. In order to fit the simulated and real shoot under same meteorological data, three main factors have to be iteratively estimated namely ballistic coefficient $BC$, lift factor $f_L$, and Magnus force fitting factor $Q_M$ as defined in [6].

3. **Results and Discussion**

3.1. **Case Study and Model Validation**

Through this study, a case study has been selected to be 155mm-M107 HE projectile. The corresponding mass properties and aerodynamic coefficients, which are calculated using the commercial package PRODAS, are listed in [17]. In order to validate the proposed model, a comparison between the produced FT using the proposed model and the one produced by PRODAS has been implemented. All results $P_{\text{est}}$ are based on the deviation from the approved published 155-M107 firing-table parameters $P_{\text{FT}}$ as

$$\Delta P = P_{\text{FT}} - P_{\text{est}}$$  \hspace{1cm} (8)

As illustrated in figure 3, the estimated elevating angles QE using the proposed model is outperform PRODAS results. In case of flight time error $\Delta \text{TOF}$, it has been noticed much closer between the two curves to have maximum error at maximum range as 2 s less than the tabulated value. Fig. 5 shows the the estimated drift error due to projectile spinning, where the proposed model has better accuracy than PRODAS with maximum error less than 1 mils through all tabulated data.

Again, as seen in Fig. 6, both results obtained have the same manner with tabulated data. The range error resulted due to 1knot head/tail wind are illustrated in Fig. 7 and Fig. 8. But in case of the change
in air temperature by 1%, the estimated range deviated from the tabulated ones as shown in Fig. 9 and Fig. 10.

3.2. Problem description
As illustrated in section 3.1, the accuracy of the developed computational FT is well enough compared to the approved and published 155-M107-HE firing tables. Therefore, to have a realistic engineering problem through the development of provisional FT, different aerodynamic coefficients (i.e. baseline projectile) have been used in order to simulate the status of new projectiles development. In this study we choose the 105-M1-HE as baseline, where its corresponding aerodynamic coefficients are listed in [13]. A comparison has been illustrated between the aerodynamic coefficients for both 105 and 155mm projectiles as shown in Fig. 11-13. According to lift force slope and Magnum force coefficient, a big difference has been observed. Only two shoots groups from past experiments have been collected as
$QE = 141.4$ and $743.2$ mils. Every group consists of seven projectiles. Average projectile mass for each group has been collected. A meteorological data had been measured once before firing including air temperature, pressure, humidity and density, and wind speed and direction. The projectile muzzle velocity had been measured for each shoot, and hence the average velocity for each group were obtained. As shown in Table 2, other measured data including down range and drift are illustrated.

![Fig. 7 Range error due to 1 knot Head-wind](image1)

![Fig. 8 Range error due to 1 knot Tail-wind](image2)

![Fig. 9 Range error due to $\Delta T_o = +0.01 \ T_o$](image3)

![Fig. 10 Range error due to $\Delta T_o = -0.01 \ T_o$](image4)

| Table 2 Firing data collected for the two groups |
|-----------------------------------------------|
| Group no | $QE$  | $v_0$ [m/s] | $m_{avg}$ [kg] | $X_{1 \ avg}$ [m] | $X_{3 \ avg}$ [m] |
|----------|-------|-------------|----------------|-----------------|-----------------|
| 1        | 141.4 | 690.3       | 42.664         | 7943.2          | 27.2            |
| 2        | 743.2 | 692.7       | 43             | 18075.5         | 11.9            |
### 3.3. Results

An iterative process has been implemented in order to fit the measured data for two shoot groups as illustrated before. Only two factors can be estimated as both the total flight time and summit point were not measured. Therefore, the Magnus force fitting factor is neglected $Q_M = 1$. Other estimated fitting coefficient namely ballistic coefficient and lift factor are listed on Table 3. By implanting these factors into our model for computational FT as illustrated before, we will obtain a first draft for provisional FT with improved accuracy as shown in Fig. 14-16. Although, the error still noticeable as we only have one meteorological observation and its only valid for one hour.
There for the results consists of error from the standard table before fitting and after fitting shows in figures below

Table 3 Fitted coefficients obtained

| Group no | QE   | BC    | \(f_L\)  |
|----------|------|-------|-----------|
| 1        | 141.4| 1.012186 | 0.955145 |
| 2        | 743.2| 1.012186 | 0.964955 |

4. Conclusion
As artillery firing table FT is a primary and essential tool for artillery systems developments, this paper illustrates the problem of constructing FT starting from computational FT and hence obtain the provisional one. A flow-chart for computational (preliminary) FT has been proposed including both standard and non-standard firing conditions. A flat Earth approximation has been utilized neglecting Earth’s rotation as well as Coriolis acceleration. In order to validate the proposed model, a case-study for 155mm-M107-HE projectile is used. Results obtained using the proposed model (computational FT) is compared with the well-known commercial package PRODAS and the approved published 155-M107 firing-table. It can concluded that the proposed model has good accuracy as well as PRODAS. Finally, in order to demonstrate the effectiveness of including real fires through provisional FT production, a different aerodynamic model (105mm-HE) has been implemented for 155mm-M107 firing test. Hence, the ballistic coefficient and lift factor have been iteratively estimated. It has been observed that the provisional FT accuracy is improved by implementing available firing tests. This study only includes two shooting groups for two ground range with one meteorological observation. No measurements for the total flight time was not observed. The accuracy of the obtained provisional table still need more improvements using more data for firing angles including the total flight time.
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