I. INTRODUCTION

The measurements of the Cosmic Microwave Background (CMB) anisotropies, most notably by the Wilkinson Microwave Anisotropy Probe (WMAP) mission \(^1\), have truly marked the beginning of the era of precision cosmology. In particular, the shape of the measured temperature and polarisation angular power spectra are in spectacular agreement with the expectations of the standard model of structure formation, based on primordial adiabatic and nearly scale invariant perturbations (see e.g. \(^2\)). More recently, ground based and balloon borne CMB experiments like extended VSA \(^3\), CBI \(^4\), Acatar \(^5\), DASI \(^6\) and BOOMERANG-03 \(^7\), have probed the CMB power spectra at smaller scales, confirming the first unambiguous detection of polarisation.

Moreover, new, complementary, results from the Sloan Digital Sky Survey (SDSS) on galaxy clustering (see e.g. \(^8\)) and, more recently, on Lyman-\(\alpha\) Forest clouds \(^9\) are now further constraining the scenario.

Since all these measurements appear in spectacular agreement with the \(\Lambda\)CDM model, based on a cosmological constant and on cold dark matter, is definitely timely to investigate what space is left for alternative theories.

Perhaps the most exotic alternative to the standard model one could consider is MOdified Newtonian Dynamics (MOND, \(^10\)) where a purely baryonic model with modifications to standard (Newtonian) gravity is suggested. In MOND the departure from Newtonian law \(a = -\nabla \Phi_N\) is given by:

\[
\mu(|a|/a_0)a = -\nabla \Phi_N
\]

\(^{11}\)

where \(\Phi_N\) is the Newtonian potential of the visible matter, \(a_0\) is an acceleration scale while \(\hat{\mu}(x) \approx x\) for \(x \ll 1\) and \(\hat{\mu}(x) \to 1\) for \(x \gg 1\). If \(a_0 \approx 1 \times 10^{-8}\) cm s\(^{-2}\) the Newtonian law is recovered in the solar system where accelerations are large compared to \(a_0\).

The above empirical formula \(^11\) has been originally proposed to explain the fact that rotation curves of disk galaxies become flat outside their central parts. While in the standard dark matter paradigm flat rotation curves are explained by assuming a spherical halo of invisible dark matter around visible disk galaxies, in MOND there is no need to include non baryonic dark matter since, thanks to the Eq.\(^11\) galaxies far out exhibit an approximately spherical Newtonian potential without the inclusion of the dark matter. Attempts were made to confront MOND with clusters of galaxies (see e.g. \(^11\) \(^12\)) and the large scale structure (e.g. \(^13\)) with mixed success.

While the non-baryonic dark matter paradigm is definitely more compelling for its aesthetic simplicity than a modification to Newtonian gravity, the MOND model has been claimed successful in other aspects (see e.g. \(^14\) \(^15\)) and MOND proponents insist that this alternative model for gravity merits serious examination.

The MOND theory has suffered from a lack of a successful relativistic formulation, that would allow one to compare it to observations of CMB and Large Scale Structure. Nevertheless, there were some attempts to confront CMB data \(^16\), which find that the first 2 observed acoustic peaks in the CMB spectrum are compatible with MOND at the price of a substantial neutrino mass, which is barely compatible with current laboratory bounds \(^17\), or at the price of including curvature \(^18\), which is at odds with the inflationary scenario.

A major step in the direction of developing the sim-
pale MOND formula into a more robust theory of gravity has been recently proposed by Bekenstein [19]. In this paper, a relativistic gravitational theory has been presented whose nonrelativistic weak acceleration limit accords with MOND while its nonrelativistic strong acceleration regime is Newtonian.

Moreover, Bekenstein’s model provides a specific formalism for constructing cosmological models and testing MOND using cosmological data. Indeed, more recently, Skordis et al. [20] produced the first theoretical prediction for CMB anisotropies and Large Scale Structure in the case of Bekenstein’s model. It has been shown that the Bekenstein model may be put in agreement with the WMAP data and Large Scale Structure observations. Similar to previous results authors find agreement if neutrinos ensure that peak positions are unchanged. The results are obviously of great relevance since the model has no cold dark matter in it and may therefore be considered as an important alternative to the present cosmological scenario, which assumes a fine-tuned cosmological constant and yet to be discovered dark matter particles.

In this brief report we point out that recent, small scale, CMB data already provide discriminating power between these two scenarios. In the standard cold dark matter scenario, the amplitude of the CMB peaks is sensitive to the amount of dark matter because of two effects: increasing the matter density on one hand decreases the radiation driving while on the other hand it increases the depth of potential wells. These two effects nearly cancel out in the amplitude of the second acoustic peak, but conspire to produce a higher amplitude of the third peak (see [21]). The height of the second peak to the first peak therefore contains information on the baryonic content of the Universe, while the ratio of the third peak to the first peak height tells us about the matter density.

A generic prediction of purely baryonic dominated models like MOND is therefore that peaks in the CMB power spectrum should be strictly decreasing in amplitude.

Extraordinary WMAP results on the first two peaks, coupled with the recent small scale CMB data on the third peak now have enough power to discriminate between these two scenarios and to determine the amount of cold dark matter.

The goal of this brief report is to examine the data on the third peak, with special emphasis on the recent measurements of the CMB fluctuations by the Boomerang experiment.

The ultimate test of MOND, would be a full confrontation of the relativistic theory with the data. This is a daunting task, given that the theory is very complicated with yet to be fully understood perturbation theory and several free parameters including a free function. We note, however, that models discussed in [22] are in a complete agreement with generic Λ-CDM predictions in the range ℓ > 200. We therefore assume that this is a generic prediction of the Bekenstein’s theory and proceed by fitting the ℓ > 200 region of the CMB data with the standard Λ-CDM models to see whether models with zero cold dark matter density are compatible with the data. Whether our assumption is a justified one is to be seen, however, we feel it nevertheless provides a first order confrontation of the data with the theory. Our approach is orthogonal to that of [20] in the sense that it provides constraints on any theory that leaves the CMB physics unchanged on scales smaller or roughly equal to that of the first acoustic peak.

II. ANALYSIS

We use the Cosmo-MC package ([22]) to perform parameter estimation on standard flat Λ-CDM models using top-hat priors on the following 6 parameters: \( \omega_B = \Omega_B h^2 \) (the baryon density of the universe), \( \omega_{dm} \) (the dark matter density of the universe), \( \theta \) (ratio of the sound horizon to the angular diameter distance to the surface of last scattering, multiplied by 100), \( \nu_{trac} \) (the fraction of dark matter in form of massive neutrinos), \( n_s \) (spectral index of primordial fluctuations), \( \log 10^{10} A_s \) (the amplitude of primordial scalar fluctuations). The priors used are listed in the Table I.

In our parametrisation the density of the cold dark matter is given by

\[
\Omega_{cdm} = \frac{\omega_{dm} (1 - \nu_{trac})}{h^2}
\]

We intentionally omitted \( \tau \), the optical depth to the last scattering, from our parametrisation as it is completely degenerate with the amplitude in the multipoles of interest.

The following datasets were used in our parametrisation: WMAP [1, 23], VSA [3], CBI [4], Acbar [5], and the latest Boomerang results [6]. In these datasets we have used the default Cosmo-MC distribution datasets for VSA, CBI and Acbar experiments. In all datasets any points with \( \ell < 200 \) were removed and additionally all points with \( \ell < 375 \) were removed.

### Table I: Flat priors on the cosmological parameters.

| Parameter | Prior |
|-----------|-------|
| \( \omega_B \) | BBN (see text) |
| \( \omega_{dm} \) | (0,0,99) |
| \( \theta \) | (0.5,10) |
| \( \nu_{trac} \) | (0,1) |
| \( n_s \) | (0.5,1.5) |
| \( \log 10^{10} A_s \) | (2.5) |
| \( n_{run} \) | (-0.2,0.2) |
an additional prior on the Big Bang Nucleosynthesis (BBN) prior of about 0.3. In the Figure 1, we plot the data we used (note that some points were omitted from the plot - see caption) and a few theoretical predictions. The solid line model corresponds to a standard ΛCDM flat model and fits the data very well. The dashed and dotted models were cherry-picked from the models that have the highest likelihood of models that satisfy Ω_m < 0.01 in each individual MCMC chain (using WMAP data alone). The dashed model illustrates the common wisdom that it is possible to construct models that have nearly identical peak positions and heights of even peaks with zero dark matter. Finally, the dotted model shows an example of a zero dark matter model that is allowed by the WMAP data but obviously at odds with measurements of the small scale power.

In the Figure 2 we plot the marginalised probability distribution for Ω_cdm for the WMAP dataset and the WMAP dataset after inclusion of all the other CMB data. We note that the WMAP alone admits the zero cold dark matter solutions, in agreement with previous investigations [16, 20]. The addition of other datasets, however, strongly rejects this region of the parameter space, without resorting to non-CMB experiments.

In order to quantify this further, we use two statistical tests. Firstly, we compare two basic models, namely the flat CDM model with Ω_cdm between 0 and 1 and the MOND model for which Ω_cdm = 0 using Bayesian model comparison [25]. In order to do this we compute the logarithm of evidence ratio with a version of the Savage-Dickey test, equivalent to that described in the Appendix of [26]. Secondly we estimate the number of “sigmas” in a frequentist manner by comparing the likelihoods of the Ω_cdm = 0 point and the most likely point in the marginalised probability distribution for Ω_cdm. The likelihood ratio can be converted to a number of standard deviations using the prescription n_σ = √2 L_{MOND}/L_{MAX}, which returns the expected result in the Gaussian case. In such short communication it is impossible to compare the two methods in depth, but we note that frequentist ap-
Finally, we have also considered running of the spectral index, defined by $n_{\text{run}} = d n_s / d \ln k$ with pivot scale set to $k = 0.05 \text{ Mpc} h^{-1}$. Using a top-hat prior between -0.2 and 0.2 on $n_{\text{run}}$ we find that this parameter is completely unconstrained by the WMAP data and only weakly constrained by the all CMB data. However, the very high $\ell$ points from Acbar seem to favour negative running indices, thus even lowering the third peak required by MOND. Consequently we find that the bound on $\Omega_{\text{cdm}}$ is unaffected when all CMB data are included.

| Dataset                  | $\Delta \log E$ | $n_s$ |
|--------------------------|------------------|-------|
| WMAP                     | 0.1              | 1.5   |
| WMAP/VSA/ACBAR/CBI       | 2.1              | 2.6   |
| WMAP/VSA/ACBAR/CBI/BOOM  | 3.6              | 3.1   |
| +HST                     | 5.2              | 3.6   |
| +HST +SN                 | 10.5             | 5.1   |
| ALL CMB w/o BBN          | 2.2              | 2.6   |
| -BBN +HST                | 3.2              | 3.3   |
| -BBN +HST +SN            | 4.3              | 3.6   |

**TABLE II:** This table shows results of the two statistical tests described in the text for a range of datasets considered. The sign convention is such that higher number implies higher statistical evidence in favour of cold dark matter models. The $\Delta \log E$ is trivially interpreted as the logarithm of probability ratio. See text for discussion.

proaches near always give “detection” confidences. Bayesian evidence ratio has an advantage that it directly encodes the probability ratio.

The results are summarised in the Table II. The exact numbers somewhat depend on the binning width and therefore the numbers in the table are accurate to about 0.1 in both columns. Some row state the constraints upon adding two extra constraints on top of all CMB data. HST label denotes the Hubble Space Telescope (HST) constraint on the Hubble’s constant $H_0$, which is conveniently described as a Gaussian around $h = H_0/100 \text{ km/s/Mpc} = .72$ with 1-σ dispersion of 0.8. The HST data actually has a rather strong effect on our results as the Hubble constant favoured by the $\Omega_{\text{cdm}} = 0$ models is rather low. SN label denotes the gold dataset of the supernovae data. These results must be taken with some caution, because it is not entirely clear that the standard interpretation of these two cosmological probes is applicable in the MONDian setting.

Finally, we have also considered running of the spectral index, defined by $n_{\text{run}} = \frac{d n_s}{d \ln k}$ with pivot scale set to $k = 0.05 \text{ Mpc} h^{-1}$. Using a top-hat prior between -0.2 and 0.2 on $n_{\text{run}}$ we find that this parameter is completely unconstrained by the WMAP data and only weakly constrained by the all CMB data. However, the very high $\ell$ points from Abar seem to favour negative running indices, thus even lowering the third peak required by MOND. Consequently we find that the bound on $\Omega_{\text{cdm}}$ is unaffected when all CMB data are included.

**IV. DISCUSSION AND CONCLUSIONS**

In this brief report we have analysed the latest CMB data in light of the recently renewed interest in the MOND theories of gravity. We simplified (and potentially oversimplified) the theory by assuming that the small scale CMB fluctuations are unmodified by the MOND theory in accordance with recent attempts to model linear fluctuations in the relativistic theory of MOND recently proposed by Bekenstein. Under this assumption we find that the WMAP data alone is fully consistent with the $\Omega_{\text{cdm}} = 0$ required by MOND, in accordance with previous findings. A component of massive sterile neutrinos is required with $\omega_{\nu} > 0.1$, which is marginally consistent with earth-based beta electron decay experiments. Addition of other CMB data constrains the third peak height, which encodes the information on the presence of the cold dark matter. The data before the latest Boomerang dataset weakly favour the cold dark matter but only at 1 in 8 probability ratio for a Bayesian analysis and less than 3-σ for the more conventional approach. The VSA’s measurement of power at the top of the third peak is probably crucial in absence of Boomerang data. Addition of the third peak from latest Boomerang data gives crucial extra information. The Bayesian probability ratio in favour of cold dark matter increases to 1 in 36, while the likelihood ratio breaks the 3-σ “barrier”. We note that using less general models for the CDM setting would make the Bayesian model selection even stronger in favour of cold dark matter models.

Releasing the BBN prior somewhat weakens the constraints. We note however, that there is no obvious mechanism how could a MOND theory evade BBN constraints and that such models are additionally characterised by very low values of $h$ and $n_s$. Marginalised value of $\omega_b \sim 0.025$ for these models.

Finally, we have added two other standard cosmological probes, the HST measurements of the Hubble’s constant and the recent supernovae measurements of the luminosity distance. Taken at the face value, they seem to blow the MOND model into oblivion. Care must be taken, however, in interpreting these two datasets as it is not clear whether it is appropriate to include them in the MONDian scenarios without detailed treatment of possible MONDian effects on the background evolution.

We have shown that running of the spectral index cannot rescue the third peak, at least for $|n_{\text{run}}| < 0.2$.

We note that our results are somewhat prior dependent: the $\Omega_{\text{cdm}}$ parameter is a derived parameter in the standard parametrisation used by the Cosmo-MC package and therefore the implied prior on it is certainly not flat. However, we feel that priors employed are actually a sound set of physical priors and therefore our results should be fairly insensitive to any sensible reparametrisations.

Finally, we note that there is still a possibility that a version of MOND theory with a high third peak is constructed. However, this would seem rather artificial. The ratio of height of third to the first peak encodes the amount of the baryonic drag and in order to get a high third peak one needs some cold dark matter like element. Even if this eventually turns out to be a scalar field in a MONDian theory, the present
data indicate that dark matter theory is at least a very good approximation to the full underlying theory.

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