Non-local double-path Casimir phase in atom interferometers

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received 23 January 2013; accepted in final form 4 March 2013
published online 2 April 2013

PACS 03.65.Yz – Decoherence; open systems; quantum statistical methods
PACS 42.50.Ct – Quantum description of interaction of light and matter; related experiments
PACS 03.75.Dg – Atom and neutron interferometry

Abstract – We present an open quantum system theory of atom interferometers evolving in the quantized electromagnetic field bounded by an ideal conductor. Our treatment reveals an unprecedented feature of matter-wave propagation, namely the appearance of a non-local double-path phase coherence. In the standard interpretation of interferometers, one associates well-defined separate phases to individual paths. Our non-local phase coherence is instead associated to pairs of paths. It arises from the coarse-graining over the quantized electromagnetic field and internal atomic degrees of freedom, which play the role of a common reservoir for the pair of paths and lead to a non-Hamiltonian evolution of the atomic waves. We develop a diagrammatic interpretation and estimate the non-local phase for realistic experimental parameters.

Atom interferometry [1] has become a field of great importance for both basic and applied science, enabling, in particular, the realization of extremely accurate inertial sensors [2,3]. With the advent of the coherent atomic waves guided on chips [4], the investigation of atom-surface interactions has become a frontier for such systems. Already, atom interferometers have been used to probe the van der Waals regime [5]. This experimental effort calls for a complete theory of atom interferometers in the presence of quantum fluctuations of the electromagnetic (EM) field.

In this letter, we layout such theory for a beam of neutral atoms and find an unusual new concept in interferometry: a non-local phase associated to pairs of paths rather than to individual ones. First, we present a theory of atomic phase shifts taking the effect of field and atomic dipole fluctuations separately over each interferometer arm. This method already contains novel dynamical corrections, which cannot be obtained by standard techniques suitable for atoms driven by conservative forces. However, it neglects quantum correlations, mediated by the field, between the atomic wave packets evolving along the separate arms. In order to capture this effect, we develop a theory of atom interferometers based on the influence functional method [6], which allows us to derive the non-Hamiltonian evolution of the external atomic observables after coarse-graining over the quantized electromagnetic field and internal atomic (dipole) degrees of freedom. A non-local phase shift arises as a consequence of the finite correlation time of dipole fluctuations interacting across a pair of interferometer paths. It is absent in the standard Hamiltonian treatment of matter-wave dynamics with conservative forces, which shows that the effect of quantum vacuum and zero-point dipole fluctuations on atomic waves cannot be understood as an effective potential.

The influence functional method also allows one to consider the decoherence effect [7], another important consequence of the non-unitary nature of the matter-wave dynamics. However, in this letter we focus on the non-local real phase shifts beyond the expected loss of contrast in the fringe pattern. Phase shifts induced by the environment were also considered in the context of geometrical phases for spin one-half systems [8].

We consider the Mach-Zehnder atom interferometer depicted in fig. 1(a), with two arms sharing the same origin, and followed by atoms flying above a metallic plate between the instants \( t = 0 \) and \( t = T \). One of the arms is parallel to the plate, and the other has a velocity...
component $v_{\perp}$ perpendicular to the plate. We assume that the atomic motion is non-relativistic. For the typical atomic velocities used in van der Waals experiments [5], the deflection of the average trajectory due to the van der Waals force is negligible. The considered initial time is immediately after the first atomic beam splitter: the initial external atomic quantum state is taken as a coherent superposition of two dilute Gaussian wave packets of common average position $\bf{r}_0$ and different average momenta $\bf{p}_0$ for $k=1,2$. Here we focus on the effect of the plate, and thus calculate the phase accumulated between $t=0$ and $t=T$.

Within the standard formalism of atom interferometers, the external atomic observables, namely the center-of-mass position $\bf{r}_a$ and momentum $\bf{p}_a$ are supposed to evolve according to the Hamiltonian $\hat{H}_E = \frac{\bf{p}_a^2}{2m} + V(\bf{r}_a)$. $V(\bf{r})$ is a quadratic and separable potential, and $m$ is the atomic mass. The internal atomic degrees of freedom (d.o.f.), initially in the ground state, follow a general Hamiltonian $\hat{H}_D$. The $ABCD$ theorem for atomic waves [9] then shows that dilute Gaussian atomic wave packets acquire a phase proportional to the action $S(\bf{r}_0, \bf{p}_0, t)$ along the classical trajectory: $S(\bf{r}_0, \bf{p}_0, t) = \int_0^t dt' \left[ \frac{\bf{p}_a(t')^2}{2m} - (\hat{H}_D)(t') - V(\bf{r}(t')) \right]$. Extensions for interacting samples [10] in the paraxial approximation [11] have been recently developed.

Here we go beyond the standard $ABCD$ approach by taking into account quantum dipole and EM fluctuations. We first describe the effect of the quantum EM field within a local description of the interferometer. We consider the full Hamiltonian $\hat{H} = \hat{H}_E + \hat{H}_D + \hat{H}_F + \hat{H}_AF$ with the field Hamiltonian $\hat{H}_F = \sum \lambda \hbar \omega(\hat{a}_\lambda^\dagger \hat{a}_\lambda + 1/2)$ given as a sum over normal modes $\lambda$ (frequency $\omega$), annihilation and creation operators $\hat{a}_\lambda$ and $\hat{a}_\lambda^\dagger$. The interaction Hamiltonian in the electric dipole approximation is $\hat{H}_AF = -\bf{d} \cdot \bf{E}(\bf{r}_a)$ ($\bf{d} = $ dipole operator). We assign to each interferometer path $k$ a phase corresponding to the average dipole interaction energy for the Gaussian atomic packet following this prescribed path, i.e. $\phi_{\text{loc}}^{(k)} = \frac{1}{\hbar} \int_0^T dt (\bf{d} \cdot \bf{E}(\bf{r}_a) + \bf{E}(\bf{r}_a) \cdot \bf{d})(t)$. To derive this phase, we use linear response theory and treat the dipole interaction Hamiltonian $\hat{H}_AF$ as a perturbation [12]. The phase is thus calculated to first order in the atomic polarizability. Our results hold as long as the atom-surface distance is much larger than the typical atomic size, a condition also necessary for the validity of the electric dipole approximation.

The phase $\phi_{\text{loc}}^{(k)}$ is then expressed as a propagation integral involving two contributions, namely the response of the atomic dipole to the on-atom electric field fluctuations and the response of the on-atom electric field to the dipole fluctuations. The fluctuations of the dipole and electric field are captured by Hadamard Green’s functions, whereas the susceptibilities (polarizability for the atom) correspond to retarded Green’s functions [13]. The retarded $(R)$ and Hadamard $(H)$ Green’s functions read, respectively,

\begin{align}
G_{O,ij}^R(x,x') &= \frac{i}{\hbar} \theta(t-t') \langle [\hat{O}_i^j(x), \hat{O}_k^j(x')] \rangle, \\
G_{O,ij}^H(x,x') &= \frac{1}{\hbar} \langle [\hat{O}_i^j(x), \hat{O}_k^j(x')] \rangle,
\end{align}

with the operator $\hat{O} = \hat{d} \cdot \hat{\bf{E}}$, where $\hat{O}^j$ corresponds to the free-evolving Heisenberg operator, and with the Cartesian indices $i,j=1,2,3$. $\theta(t-t')$ denotes the Heaviside step function. For the dipole operator $\hat{O} = \hat{d}$, the arguments of the Green’s functions are two instants $(x,x') \equiv (t,t')$. For the electric field operator $\hat{O} = \hat{\bf{E}}$, these arguments are two four-vectors $(x,x') \equiv (\bf{r},t;\bf{r}',t')$. By isotropy of the free-evolving dipole operators, we have $G_{d,ij}^R(t,t') = G_{d,ij}^H(t,t') \delta_{ij}$. Consequently, only the trace of the electric field Green’s functions with respect to their tensorial components, noted from now on $G_{\bf{E}}^{(H)}(x,x') = \Sigma_{ij} G_{\bf{E}}^{(H)}(x,x')$, shall contribute to the effects discussed hereafter.

We assume that the width of the atomic wave packet is small compared to the relevant EM field wavelengths, which allows us to replace the on-atom electric field operator at a given time by the electric field evaluated at the corresponding average atomic position $\bf{r}_a(t)$ on the
considered path $k$. One can then write the local Casimir phase as
\[
\phi_{\text{loc}}^{(k)} \approx \frac{1}{4} \int_0^T dt \int_0^T dt' \left[ G_H^H (t, t') G_R^R (r_k (t), r_k (t')) + G_R^R (t, t') G_H^H (r_k (t), r_k (t')) \right]
\]
(3)
with the four-vectors $r_k (t) \equiv (r_k (t), t)$. The first term in (3) represents the contribution of dipole fluctuations modifying the on-atom electric field (radiation reaction). The second term accounts for the polarization of the atom by EM field fluctuations since the retarded dipole Green’s function $G_R^R (t, t')$ represents the atomic linear polarizability in the time domain.

We derive the electric field Green’s functions defined by eqs. (1), (2) by taking the full normal mode decomposition of the electric field operator in the presence of a planar perfect conductor placed at $z = 0$. The Hadamard Green’s function can be obtained from the retarded one thanks to the fluctuation-dissipation theorem, thus we focus on the latter. This function can be written as the sum of free-space and scattering contributions $G_{E}^R (x, x') = G_{E}^{R,0} (x, x') + G_{E}^{R,S} (x, x')$ depending on the time difference $\tau = t - t'$ and on the positions $(r, r')$. Our result for the free-space contribution $G_{E}^{R,0} (x, x')$ depends only on the relative distance $|r - r'|$ and is consistent with known expressions for the electric field commutators [14]. The scattering contribution $G_{E}^{R,S} (x, x')$ depends on the propagation distance $|R_1| = |r - r'|$ between the point $r$ and the image $r'_1$ of the source point $r'$ with respect to the plate:
\[
G_{E}^{R,S} (x, x') = \frac{\Theta (\tau)}{2 \pi \epsilon_0} \frac{\partial^2}{\partial z \partial \tau^2} \left( \frac{\delta (\tau - |R_1|/c)}{|R_1|} \right).
\]
(4)

As an independent check, we use (4) to evaluate the local phase given by (3) for the two paths shown in fig. 1(a). For the path $k = 1$ parallel to the plate, we find, in agreement with the $ABCD$ approach, the van der Waals phase $\phi_{\text{loc}}^{(1)} = -V_{\text{vdW}} (z_0) T / h$ for long interaction times $T$, where $V_{\text{vdW}} (z_0)$ is the van der Waals (Casimir-Polder for long distances) potential at the atom-surface distance $z_0$. On the other hand, eq. (3) already contains non-trivial dynamical Casimir effects [15], beyond the plain $ABCD$ integration of the van der Waals potential taken at the instantaneous position along the classical trajectory, particularly in the case of a time-dependent atom-surface distance as in the trajectory $r_2 (t)$ shown in fig. 1(a), for which (3) and (4) lead to a dynamical correction proportional to $v_J / c$.

We now turn to a rigorous computation of the phase with the influence functional method [6], which allows us to capture non-local effects. The monitoring of only a subset of the d.o.f.’s — the external atomic motion — calls for a partial trace (or coarse-graining) over the EM field and dipole moment. The time evolution of the reduced density matrix for the external atomic d.o.f. is obtained from closed time path (CTP) integrals [16]. Such path integrals involve simultaneously forward and backward histories of the system. The coarse-graining over the environment d.o.f.’s yields an influence functional connecting these two histories, which will be associated to a pair of interferometer paths. The influence of the environment onto the external atomic waves, captured by the influence functional, is at the origin of a path entanglement inducing the double-path phase discussed below.

First, we specify the actions for the considered quantum d.o.f., namely $S_{E} [r_a] = \int_0^T dt \left( \frac{\mu}{2} \dot{r}^2 (t) - V (r_a (t)) \right)$ for the atomic position $r_a$, $S_{F} [A^\mu] = (\epsilon_0 / 4) \int d^4 x F_{\mu \nu} F_{\mu \nu}$ for the EM field, and the action for the dipolar interaction $S_{IF} [A_\mu, d_i, r_a] = -\int d^4 x J^\mu (x) A_\mu (x)$ defined in terms of the current $J^\mu (d_i, r_a) (x) = -\int d^4 t \dot{r}_i (t) \delta^4 (x - r_a (t))$. The four-dimensional integrals are defined as $\int d^4 x \equiv \int_0^T dt \int d^4 r$ and the four-vector $r_a (t) \equiv (r_a (t), t)$. We have introduced the differential operator $\kappa_{\mu \nu} = \partial_\mu \partial_\nu - \partial_\mu \partial_\nu \eta_{\mu \nu}$ (the Minkowski metric with mostly plus signature) relating the electric field to the vector potential $A_\mu$ by contraction: $E_i (x) = k_\mu A_\mu (x)$. The following discussion is valid for an arbitrary action $S_{IF} [d]$ for the internal atomic d.o.f. We consider the evolution of the density matrix $\rho (r_\alpha, r'_\alpha; d, d', A^\mu, A'^\mu; t)$ and trace over the field and dipole d.o.f. The result can be expressed as a CTP integral over the position involving an influence action $S_{IF} [r_a, r'_a]$
\[
\rho (r_f, r'_f; T) = \int_{\text{CTP}} D r_a e^{i \int_0^T \left( S_{E} [r_a] - S_{E} [r'_a] + S_{IF} [r_a, r'_a] \right)},
\]
(5)
where $r_a$ and $r'_a$ refer to forward and backward histories, respectively. We use the compact notation for the CTP integral over a generic d.o.f. $X$
\[
\int_{\text{CTP}} D X = \int d X_0 d X'_0 \int_{X_0} X'_0 D X \int_{X'_0} X_0 D X' \rho (X_0, X'_0; 0)
\]
(6)
with $\rho (0)$ the initial density matrix.

At this level, the influence action $S_{IF} [r_a, r'_a]$ fully accounts for the effects of the field and the internal atomic dynamics on the external atomic d.o.f. As in the derivation of the local phase, we now assume that the wave packets propagating along the two interferometer arms are very narrow. The final density matrix $\rho (T)$ then contains four sharp peaks in the region $(r_a, r'_a)$ centered around the classical positions $(r_k (T), r'_k (T))$ for $k, l = 1, 2$. The desired phase shift is obtained from the off-diagonal density matrix elements (i.e. $k \neq l$). The main contribution to the path integral comes from the paths in the vicinity of the two stationary paths. Thus, the phase difference due to quantum fluctuations can be evaluated on the classical paths by taking the real part of the influence action $\phi_{IF} = \frac{1}{2} \text{Re} \left[ S_{IF} [r_1, r_2] \right]$ [17] (whereas the imaginary part represents the decoherence due to the plate [7]).
that the influence action $S_{IF}[r_1, r_2]$ depends in general on the entire paths followed from $t = 0$ to $t = T$, and not only on the end-points. Let us detail the procedure to obtain the influence action in the spirit of previous derivations of non-equilibrium forces mediated by a quantum field [15,18]. First, we define an intermediate influence action $S^+_A[d, d', r_1, r'_1]$ corresponding to the trace over the EM field alone:

$$e^\frac{i}{\hbar}S^+_A[d, d', r_1, r'_1] \equiv \int DA_\mu \int_{CTP} DA_\nu \times e^\frac{i}{\hbar} \left( S_{SP}[A, d, r_1] - S_{SP}[A, d', r'_1] \right) \left( S_{SP}[A, d, r_1] + S_{SP}[A, d', r'_1] \right).$$

(7)

This CTP integral over the vector potential linearly coupled to an external current yields $S^+_A[d, d', r_1, r'_1] = \int d^4 x d^4 x' \left[ J^{\mu \nu}(-x)G_{A\mu\nu}(x, x')J^{\nu \nu}(x') \right] + \frac{i}{4} \int d^4 x d^4 x', d^4 x'' \left[ J^{\mu \nu}(-x)G_{H\mu\nu}(x, x')J^{\nu \nu}(x') \right]$. (8)

Taking standard conventions, we have introduced the semi-sum $J^{\mu \nu} = \frac{1}{2} J^{\mu}[d, r_1] + \frac{i}{2} J^{\mu}[d', r'_1]$ and difference $J^{\mu \nu} = J^{\mu}[d, r_1] - J^{\mu}[d', r'_1]$ variables. The vector potential’s retarded and Hadamard Green’s functions $G_{A\mu
u}^{R,RH}$ are defined as the electric field Green’s functions $G_{E\mu
u}^{R,RH}$ in eqs. (1), (2) with Cartesian coordinates replaced by Lorentz indices.

To obtain the desired influence action $S_{IF}[r_0, r_1]$ we average the EM influence functional given by eqs. (7), (8): $e^\frac{i}{\hbar}S^+_{IF}[r_0, r_1] \equiv \langle e^\frac{i}{\hbar}S^+_A[d, d', r_1, r'_1] \rangle_d$ with $\langle \ldots \rangle_d$ denoting the time-dependent average over the free-evolving dipole d.o.f. As in the derivation of the local phase (3), we take the approximation of small dipolar coupling and expand the influence functionals to first order in the atomic polarizability: $e^\frac{i}{\hbar}S^+_A = 1 + e^\frac{i}{\hbar}S^+_F$ and likewise for $e^\frac{i}{\hbar}S^+_{IF}$. The influence action then reads $S_{IF}[r_0, r_1] = \langle S^+_A[d, d', r_1, r'_1] \rangle_d$ [15]. In order to express the influence action in terms of EM and dipole correlation functions, we expand the currents $J^{\mu \nu}$ in eq. (8) and integrate over the spatial coordinates. The influence action receives a single-path (SP) and a double-path (DP) contribution, i.e., $S_{IF} = S^+_{IF} + S^+_{IF}$. (9) The plain and time-ordered correlations of a free-evolving dipole are defined as $g_{\mu \nu} = \langle \hat{d}_\mu(t)\hat{d}_\nu(t') \rangle_\delta$ and $g^{F}_{\mu \nu} = \frac{1}{2}(\langle \hat{T}\hat{d}_\mu(t)\hat{d}_\nu(t') \rangle_\delta - \langle \hat{d}_\mu(t)\hat{d}_\nu(t') \rangle_\delta)$. (10) The plain and time-ordered correlations of a free-evolving dipole are defined as $g_{\mu \nu} = \langle \hat{d}_\mu(t)\hat{d}_\nu(t') \rangle_\delta$ and $g^{F}_{\mu \nu} = \frac{1}{2}(\langle \hat{T}\hat{d}_\mu(t)\hat{d}_\nu(t') \rangle_\delta - \langle \hat{d}_\mu(t)\hat{d}_\nu(t') \rangle_\delta)$. (10) The plain and time-ordered correlations of a free-evolving dipole are defined as $g_{\mu \nu} = \langle \hat{d}_\mu(t)\hat{d}_\nu(t') \rangle_\delta$ and $g^{F}_{\mu \nu} = \frac{1}{2}(\langle \hat{T}\hat{d}_\mu(t)\hat{d}_\nu(t') \rangle_\delta - \langle \hat{d}_\mu(t)\hat{d}_\nu(t') \rangle_\delta)$. (10) The plain and time-ordered correlations of a free-evolving dipole are defined as $g_{\mu \nu} = \langle \hat{d}_\mu(t)\hat{d}_\nu(t') \rangle_\delta$ and $g^{F}_{\mu \nu} = \frac{1}{2}(\langle \hat{T}\hat{d}_\mu(t)\hat{d}_\nu(t') \rangle_\delta - \langle \hat{d}_\mu(t)\hat{d}_\nu(t') \rangle_\delta)$. (10)
non-relativistic limit, $\tau$ is the same for both diagrams, and so is the displacement parallel to the plate during $\tau$. However, the distance $|r_2(t) - r_1(t')|$ (c) is larger than $|r_1(t) - r_2(t')|$ (b) because the wave packet in arm 2 is moving away from the surface. Since the retarded Green’s function decreases as a function of distance, diagram (b) corresponds to a stronger cross-talk between the quantum dipole fluctuations, leading to a positive phase $\phi_{\text{DP}}^{(2)}$ in eq. (11). Thus, the DP phase is essentially a signature of the asymmetry between diagrams (b) and (c), which is brought into play by the combination of two properties:

- The finite speed of the propagation. Note that $\phi_{\text{DP}}^{(2)}$ vanishes when $c \rightarrow \infty$. In fact, $t' \rightarrow t$ leads to an exact cancelation between the two diagrams since $|r_2(t) - r_1(t)| = |r_1(t) - r_2(t)|$ (this limit corresponds to vertical purple lines in figs. 1(b) and 1(c)).

- The large memory time of dipole fluctuations. If the dipole correlation times were shorter than $z_0/c$, each separate diagram contribution would be suppressed after multiplying by $g_{t',t}$ in eq. (11). In other words, the dipole memory time should be sufficiently large to enable the electromagnetic cross-talk between one arm and the image of the other arm. More generally, very short-living fluctuations lead to a coarse-grained evolution with no coupling between forward and backward histories of the system. Our double-path phase is precisely the signature of such a coupling for the atomic center-of-mass evolution.

According to our convention, positive values for the double-path phase $\phi_{\text{DP}}^{(2)}$ have the same interferometric effect of a standard local phase on arm 1 larger than on arm 2. For the paths shown in fig. 1, $\phi_{\text{DP}}^{(2)}$ adds to the van der Waals local phase difference since arm 1 is closer to the plate than arm 2. However, the sign of $\phi_{\text{DP}}^{(2)}$ is not determined by which path is closer (in average) to the plate, but rather by which path is moving away/towards the plate. For instance, $\phi_{\text{DP}}^{(2)}$ would be negative if path 2 were moving towards the plate.

In order to derive simple analytical results, we model the internal atomic d.o.f. as a harmonic oscillator with a transition frequency $\omega_0$. The dipole correlation function is then proportional to the static atomic polarizability $\alpha(0): g_{t',t} = \frac{1}{2} \alpha(0) \omega_0 e^{-i \omega(t-t')} \alpha''(\omega)$ (the frequency-dependent polarizability $\alpha''(\omega)$ is the Fourier transform of the dipole retarded Green’s function $G_d^R(t') \equiv G_d^R(t' + \tau, \tau')$). In the short-distance limit $z_0, v_\perp T \ll \lambda_0$, eq. (11) leads to

$$\phi_{\text{DP}}^{(2)} = \frac{3 \pi}{4 \lambda_0} \left( \frac{\alpha(0)}{4 \pi \epsilon_0} \right) \left( \frac{1}{z_0} - \frac{1}{z_0 + v_\perp T/2} \right),$$

where $\lambda_0 = 2\pi c/\omega_0$ is the transition wavelength. For a long path separation, i.e., $z_0 \ll v_\perp T \ll \lambda_0$, the DP phase saturates to a maximal value independent of $v_\perp T$. We compute the saturation value for $^{87}\text{Rb}$ atoms, with the static polarizability $\alpha_{\text{Rb}}(0)/(4\pi \epsilon_0) = 4.72 \times 10^{-29} \text{m}^3$. The dominant contribution to the ground state dipole fluctuations comes from the $5s_{1/2} - 5p_{1/2}$ and $5s_{1/2} - 5p_{3/2}$ transitions, with wavelengths close to $\lambda_0 \approx 0.79 \mu\text{m}$ (they correspond to large dipole transition matrix elements). We take $z_0 = 20 \text{nm}$ similar to the distance used in the experiments of ref. [5]. Such parameters yield a DP phase of $\phi_{\text{DP}}^{(2)} = 3.5 \times 10^{-3} \text{rad}$, hence beyond the sensitivity given by the state-of-the-art atom interferometers but still larger than systematic phases considered in atom gravimeters [3]. This DP phase can be compared with the standard vdW phase $\phi_{\text{vdW}}^{(2)}$ obtained by integration of the potential $V_{\text{vdW}}(z) = -\hbar \omega_0 \alpha(0)/(32 \pi \epsilon_0 z^3)$ along the path 2. The latter is inversely proportional to the normal velocity $v_\perp$ and reads $\phi_{\text{vdW}}^{(2)} = \left( \frac{\pi}{8 \lambda_0^3} \right) \left( \frac{\alpha(0)}{4 \pi \epsilon_0} \right) \left( \frac{1}{z_0^2} - \frac{1}{(z_0 + v_\perp T/2)^2} \right)$.

In the considered limit $\phi_{\text{DP}}^{(2)} \approx 6(v_\perp/c)\phi_{\text{vdW}}^{(2)}$, showing that the non-local DP phase is a first-order relativistic correction to the standard vdW phase, in agreement with our discussion about the role of the finite value of $c$. This explains why the DP phase discussed here is several orders of magnitude smaller than the typical vdW phase $\phi_{\text{vdW}} = 0.2 \text{rad}$ measured in refs. [5]. It is actually hard to isolate the non-local DP phase from the much larger standard vdW phase in the atom interferometer discussed here. Other interferometer setups better suited for that purpose remain to be investigated.

To conclude, we have developed an open quantum system theory of atom interferometers, predicting non-local double-path phase shifts in the propagation of atomic waves. We have shown that the standard atom-optics approach catches only the local phase shifts, which correspond to the single-path terms obtained with the influence functional method. The atomic center of mass is coupled to dipole and EM fluctuations which play the role of a common environment for the two wave packets propagating in the interferometer. The coherence of matter waves and large dipole memory times allow for cross-talks between the dipole fluctuations on each arm, leading to the non-local DP phase. We have developed a diagnostic picture of this one-particle quantum interference effect, which can be interpreted as an asymmetry between diagrams involving simultaneous atomic propagation on distinct paths. The finite speed of light allows quantum fluctuations to probe this asymmetry. Thus, the DP phase can be interpreted as a dynamical relativistic correction. We have shown that the DP phase shift compares to systematics considered for accurate atom interferometers. Our approach can be extended to multiple-path atom interferometers by considering pairs of paths. This is to our knowledge the first evidence of a non-local phase coherence in atom optics.

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The authors are grateful to Christian J. Bordé, Diego A. R. Dalvit, Arnaud Landragin, and Reinaldo de Melo e Souza for stimulating
discussions. This work was partially funded by CNRS (France), CNPq, FAPERJ and CAPES (Brazil).

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