Study on low intensity aeration oxygenation model and optimization for shallow water

Xiao CHEN\textsuperscript{1}, Zhibin DING\textsuperscript{1}, Jian DING\textsuperscript{1}, Yi WANG\textsuperscript{1}
\textsuperscript{1}PLA Army Engineering University, Nanjing 210007, China
shawchen2817@sina.com

Abstract. Aeration/oxygenation is an effective measure to improve self-purification capacity in shallow water treatment while high energy consumption, high noise and expensive management restrain the development and the application of this process. Based on two-film theory, the theoretical model of the three-dimensional partial differential equation of aeration in shallow water is established. In order to simplify the equation, the basic assumptions of gas-liquid mass transfer in vertical direction and concentration diffusion in horizontal direction are proposed based on engineering practice and are tested by the simulation results of gas holdup which are obtained by simulating the gas-liquid two-phase flow in aeration tank under low-intensity condition. Based on the basic assumptions and the theory of shallow permeability, the model of three-dimensional partial differential equations is simplified and the calculation model of low-intensity aeration oxygenation is obtained. The model is verified through comparing the aeration experiment. Conclusions as follows: (1) The calculation model of gas-liquid mass transfer in vertical direction and concentration diffusion in horizontal direction can reflect the process of aeration well; (2) Under low-intensity conditions, the long-term aeration and oxygenation is theoretically feasible to enhance the self-purification capacity of water bodies; (3) In the case of the same total aeration intensity, the effect of multipoint distributed aeration on the diffusion of oxygen concentration in the horizontal direction is obvious; (4) In the shallow water treatment, reducing the volume of aeration equipment with the methods of miniaturization, array, low-intensity, mobilization to overcome the high energy consumption, large size, noise and other problems can provide a good reference.

1. Introduction

With the rapid development of social urbanization, shallow water engineering, which is represented by landscape water, has been widely popularized and applied. Slow cycle speed, low environmental capacity, weak self-purification capacity, vulnerability to pollution and instability make it difficult to guarantee landscaping function and easy to become a new dead corner of pollutant\textsuperscript{[1]}. In the past decade, about 93\% shallow water in the country are polluted in varying degrees, and eutrophication and black smelly phenomenon is particularly prominent. The key to shallow water treatment is to enhance the self-purification ability of water bodies\textsuperscript{[2]}.

Dissolved Oxygen (DO) is an important indicator of water purification capacity. Artificial aeration oxygenation technology is the most widely used and effective method for repairing shallow water in China. Since the seventies and eighties of the 19th century, the development of aeration technology at home and abroad has become more and more mature. Front scientific and engineering research mainly focus on the improvement of the original equipment, the optimization of the aeration mode and the combination with the new technology. It is of great practical significance to improve the aeration capacity, cut down the energy consumption of equipment and optimize the aeration strategy.
At present, the theoretical research on aeration has some limitations on the improvement and development of engineering application. It is mainly embodied in:(1) Two-film theory, shallow permeability theory and surface renewal theory can only explain the aeration and oxygenation at the micro level, so they can just qualitatively analysis while cannot provide quantitative analysis for engineering practice at macro level. (2) Currently, applicable and feasible computational model for low-strength aeration in shallow water is void.

2. Methods and experiments
Based on two-film theory and method of mathematical physics, three-dimensional mathematical model of aeration in shallow water is established to analysis low-intensity aeration quantitatively. Combined with low intensity conditional aeration oxygen fluent simulation and shallow permeability theory, we verify the assumption that gas-liquid mass transfer occurs in in vertical direction while concentration diffusion occurs in horizontal direction and simply the partial differential equations. Compared the computational model with the aeration tests, we confirm the feasibility and optimization of low intensity aeration/oxygenation in shallow water.

2.1. Theoretical model of aeration oxygenation
Two-film theory holds that there is a gas film and a liquid film at the interface between the gas and the liquid. The resistance of the molecule to the film is caused by the transition from one phase to another through the membrane. Oxygen is insoluble in water. The oxygen molecules are converted from the gas phase to the liquid phase during aeration. The mass transfer rate is proportional to the difference between DO concentration and saturation concentration in the water.

\[
\frac{dC}{dt} = K_{La} (C_s - C)
\]

Where \(K_{La}\) is the total transfer coefficient of oxygen and \(C_s\) is the DO concentration when saturated.

2.1.1. Box model. Shallow water is usually small size in area and depth, which is subject to external pollution. We select a box model \[3\] based on first-order reaction to study aeration in shallow water.

The \(x \) and \(y \) levels are considered infinite in the direction of the horizontal direction, and the depth is \(h \) in vertical direction, assuming that the water body is in steady state, and the oxygen concentration simulation curve of each point in the steady state is equal to:

\[
C(x, y, z) = az^2 + bz + c \quad [4]
\]

At time \(t \), the DO concentration at the point \(M\) \((x, y, z)\) of the water body consists of two parts: the DO diffusion concentration \(C_1\) at the time of \(t-\tau\), and the mass transfer concentration \(C_2\) at the time of \(t\).

\[
\begin{align*}
C_1 &= C_{1x} + C_{1y} + C_{1z} + \delta(x, y, z) \cdot K_{La} (C_s - C) \\
C_1|_{t=0} &= C_b(x, y, z) \\
-\infty < x, y < +\infty, 0 < z < h
\end{align*}
\]

The relationship between the logarithm of the concentration logarithm and the time \(t\) under different gas flows can be better fitted into a linear relationship.

\[
\frac{dC_2}{dt} = K_{La} (C_s - C_1 - C_2) \\
C_2 = (C_s-C_1) \cdot (1-e^{-K_{La}t}) \\
C = C_1 + C_2
\]

where \(\delta(x, y, z) = \begin{cases} \infty, (x, y, z) = (0,0,0) \\
0, (x, y, z) \neq (0,0,0)
\end{cases}\)
The three-dimensional partial differential linear equations can explain the process of oxygen mass transfer, but the process of solving the equation is too complicated. Combined with the actual aeration technology, we consider to simplify the equation by degrading the dimension.

2.1.2. Basic assumptions. In practical engineering applications and experiments, bubbles from the aerator will form a regular gas column in the vertical direction, and the diameter of the gas column is much smaller than the size of the aeration tank. To simplify the calculation of the equation, the following basic assumptions are proposed:

(1) In the low-intensity aeration, the gas column of aeration is considered stable, so the turbulence can be neglected. (2) Gas-liquid mass transfer in vertical direction and concentration diffusion in horizontal direction during the process of aeration. (3) The horizontal dissolved oxygen distribution is pure concentration diffusion.

2.2. Basic hypothesis test
The CFD simulation method is used to test the rationality of the basic assumptions. The Fluent simulation of the gas-liquid two-phase flow in the aeration tank was carried out, and the basic assumptions were verified by the gas content cloud distribution.

2.2.1. Fluent simulation. The gas-liquid phase flow in the aeration tank follow the momentum and mass conservation. In order to simplify the complex calculation of gas-liquid phase flow, the following assumptions are made for two-phase flow:

(1) The gas phase is discrete, the liquid phase is continuous, and the gas-liquid is mixed and continuous. (2) Using the method of volume averaging to establish the governing equations of the unsteady state of each phase. (3) Gas phase and liquid phase are incompressible fluid, regardless of surface tension, ignoring the impact of gravity on the gas, and only the role of drag force is considered between two phases. (4) Gas-liquid phase follows the quality of conservation, and there is no energy exchange with the outside.\[5\]

Based on the above assumptions, we select Euler model to establish the control equation:

Continuity equation:

\[
\frac{\partial (\alpha_k \rho_k)}{\partial t} + \nabla \cdot (\alpha_k \rho_k V_k) = 0
\]

\[
\sum_{K=1, K=G, L} \alpha_k = 0
\]

Momentum equation:

\[
\frac{\partial (\alpha_k \rho_k V_k)}{\partial t} + \nabla \cdot (\alpha_k \rho_k V_k V_k) = -\alpha_k \nabla P_k - M_{\text{drag}} + \alpha_k \rho_k g + \nabla \cdot (\alpha_k \tau_k)
\]

Where \( \alpha_k \), \( \rho_i \), \( V_i \), \( P_k \) respectively represent the gas content, the density, the velocity, the pressure of phase K, L is the liquid phase and G is the gas phase, \( M_{\text{drag}} \) represents the drag forces between two phases.

We select cuboid aeration tank as simulation subject and select micro-porous aeration diffusors as aerator. In order to simplify the calculation, according to the similarity principle, the simulation study of a smaller cuboid area is selected in the aeration tank. The specific parameters and grid division are respectively shown in table 1 and figure 1.
Table 1. Simulation parameters and conditions.

| Parameters               | Value       |
|--------------------------|-------------|
| Aera l×w×h//m            | 1×1×2       |
| Air velocity//m/s        | 5, 10, 20   |
| Temperature //℃          | 20          |
| Diameter of aerator Dv//m| 0.25        |
| Average pore diameter Dp//m| 0.004      |

The velocity inlet at the bottom and pressure outlet degassing which only allow gas to go through on the top are adopted as the aerator boundary conditions. The wall is no slip boundary. Euler-multiphase flow and k-ε model is selected. In PBM mode, the air flow of the aeration column is controlled to be uniform and the bubble size does not change. The pressure velocity coupling adopts phase-coupled simple, initial time step delta t = 0.001 s, and then the delta t = 0.1s. The time derivative of the discrete format takes the first step, the spatial derivative selects the quick format, the pressure relaxation factor is set to 0.25, the momentum relaxation factor is set to 0.75, and the remaining parameters are the default values.

2.2.2. Analysis on simulation results. Set the intake air velocity v = 5m/s, and get contours of volume fraction in central section.

Figure 2. Contours of volume fraction in central section.

At t = 0.1s, the gas column is formed, t = 1s, the column is in the vertical direction to the centre of the water, t = 2.3s, the gas column reaches the surface and stabilizes, and the gas column remains stable when t = 5s. When air flow enters stable water, the air flow overcomes the vertical rise of the water pressure and forms a stable column due to the vertical velocity component.

According to contours of volume fraction in central section, conclusions are as follows: (1) During the process of aeration underwater, the gas holdup distributes in z axis and is presented as the regular column.(2) Oxygen mass transfer process of oxygenation can be decomposed into the central axial gas-liquid mass transfer and the concentration of dissolved oxygen in the horizontal direction.(3) The amount of dissolved oxygen in the initial mass transfer process is much larger than the output of the concentration diffusion. Before the central axis reaches saturation, the concentration of diffusion in the horizontal direction is not taken into account.

2.3. Calculation model of low intensity aeration oxygenation
In order to control the influence of variables and simplify the model, the effect of temperature and water quality on the DO concentration distribution in water was studied under the condition of 20 ℃ using deoxidized water as the research object.

The theory of shallow permeability theory holds that: (1) For oxygen mass transfer, the gas-liquid phase contact time is so short that it can be regarded as the instantaneous mass transfer, so free diffusion depth of the gas in the water is very shallow; (2) when bubbles move in the water, the body is constantly updated, and the mass transfer process is an unsteady process that changes over time.

![Figure 3. oxygenation calculation model (not to the scale)](image)

On the basis of the theory of shallow permeability and the results of Fluent simulation, the mathematical model of aeration oxygenation can be approximated as the turbulence in the vertical direction along the z axis and the laminar flow in the x and y horizontal directions. The DO in the turbulence mainly comes from the mass transfer of gas and liquid, and the mass transfer is mainly affected by the liquid film resistance in the water. The concentration of the DO in the laminar flow is affected by the concentration difference. In the deoxidized water, the DO concentration at each point in the water is 0, the aeration is oxygenated at t = 0, the gas flow is Q₀, the total cross-sectional area of the aerator is A, and the aeration velocity is v.

### 2.3.1. gas liquid mass transfer

The theory of shallow permeability regards the transfer as instantaneous gas-liquid contact effect, so oxygen can pass as in semi-infinite unsteady in the liquid film diffusion, under the condition of pressure aeration, liquid film on the vertical direction is continuous throughout each other.

According to the second law of Fick, the diffusion equation, initial conditions and boundary conditions of turbulent gas-liquid concentration in the z axis of the aeration point centre are given:

\[
\begin{align*}
C_1 &= DC_{zz} \\
t &= 0, z > 0, C = C_b \\
t > 0, z = 0, C = C_z \\
t > 0, z = \infty, C = C_b
\end{align*}
\]

Where D is the diffusion coefficient of air in water. The aeration column strikes the water to moves upward and accelerates the transfer of oxygen molecules in the water. \(D = \pi D_f v\)

Solution equation:

\[
\frac{\partial C}{\partial z} = \frac{C_s - C_b}{(\pi D_f)^{1/2}} \left[-\exp\left(\frac{-z^2}{4Dt}\right)\right]
\]

The initial concentration of DO initial concentration of deoxygenated water is 0, then \(C_b = 0\).

\[
\frac{\partial C}{\partial z} = \frac{C_s}{(\pi D_f)^{1/2}} \left[-\exp\left(\frac{-z^2}{4Dt}\right)\right]
\]

\[
C(z,t) = \lim_{\Delta z \to 0} \int_{z}^{z+\Delta z} \frac{C_s}{(\pi D_f)^{1/2}} \left[-\exp\left(\frac{-z^2}{4Dt}\right)\right] dz
\]
2.3.2. Concentration diffusion. At time of T, the concentration of all points in the z axis reaches CS, and z is:

$$C(0,0,z,t) = C_{S,z}, 0 \leq z \leq h, t \geq T$$

The diffusion equation of DO concentration in the horizontal direction of x and y is:

$$\frac{\partial C}{\partial t} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + C(z,t) \delta(x,y)$$

$$C \Big|_{z=0} = C_{S,z}, C \Big|_{z=h} = 0, t > 0$$

$$C \Big|_{r=0} = 0, 0 \leq z \leq h$$

The diffusion coefficient of oxygen in water is

$$D_{o-w} = 2.1 \times 10^{-5} \text{cm}^2 / \text{s}$$

Horizontal x and y have symmetry, selecting a radial section (xoz plane), establish new cartesian coordinate system and two-dimensional partial differential equation, to analyse the concentration of the horizontal diffusion.

When z=h, the diffusion equation is

$$\frac{\partial C}{\partial t} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + C(h,t) \delta(x)$$

$$C \Big|_{r=0} = \delta(x)$$

$$C(x,t) = \frac{C(z,t)}{a^2 t} \left[ \text{erf} \left( \frac{-x}{2a^2 t} \right) + \frac{a^2 t}{x} \right]$$

where $\text{erf}(x) = \int_0^x e^{-t^2} dt$

According to the symmetry of x and y, the concentration of DO in the aeration pool is:

$$C(x,y,z,t) = \frac{C(z,t)}{a^2 t} \left[ \text{erf} \left( \frac{-(x^2 + y^2)^{1/2}}{2a^2 t} \right) + \frac{a^2 t}{x^2 + y^2} \right]$$

2.4. Aeration experiment

Instruments and chemical reagents: DO metre, sodium thiosulfate, air pump, controller, DO probe.

![Figure 4. Schematic diagram of aeration experiment](image)

A certain height of water was injected into the aeration reactor (1m x 1 m x 2 m) and initial dissolved oxygen was measured. In the aeration tank, as shown in the figure 4, the dissolved oxygen electrode is arranged at equal distance. Deoxidizer anhydrous sodium sulphite and oxidizer anhydrous sodium sulphite are utilized to deoxidize to zero. The reagents were placed directly in the aeration reactor and
slowly stirred for half a minute to diffuse to complete mixing. In the aeration tank, we equidistantly arranged dissolved oxygen meter detection, preheat 15-30 minutes, and then corrected. When dissolved oxygen is reduced to zero and stabilized, the aeration begins. Control the air pump to maintain a constant intake air velocity, determine the dissolved oxygen concentration at different locations (shown as figure 4), and record. Change the intake rate, repeat the experiment, record the data.

3. Results and discussions

3.1. model verification

3.1.1. vertical direction. Take the central of the aeration tank for example, the theoretical and measured value of DO concentration curves are shown as figure 5. At the initial time, only the DO concentration of the aeration point reach saturation, and the rest everywhere is 0. After the gas column is stabilized, the DO concentration at each point in the z-axis direction is gradually saturated from the aeration point to the top of the liquid surface. With the mass transfer, the oxygen content in the air gradually becomes lower, the DO concentration growth rate is positively correlated with the water depth, the highest at the bottom of the water, the lowest at the top of the water. The correlation between the theoretical curve and the test data is good at the initial time. One possible influence factor is the delayed error of the test of the dissolved oxygen meter, so the measured values are higher than the theoretical values. And another factor is that the concentration of the dissolved oxygen at a certain point is actually a certain range of dissolved oxygen concentration, so the measured values fluctuate up and down near the theoretical value. Besides, the water flow may have impacts on the measured values. The correlation between the theoretical and measured values is decreased as time grows, but the trends keep the same.

3.1.2. horizontal direction. Take the theoretical and measured values of a cut xoz at the depth of 1m in the centre of the aeration tank to verify the horizontal model. The curves of values are presented as figure 6. The theoretical curves and the simulation of the test data are well correlated, but the correlation

Figure 5. DO concentration distribution in the vertical direction
decreases as time grows and the trends are same. Besides, the temperature may influence the diffusion of the oxygen in the horizontal direction. With the extension of the aeration time, the dissolved oxygen concentration in the centre axis reaches the saturation state, and begins to diffuse in the horizontal direction. With the mass transfer of oxygen, the concentration of oxygen in the horizontal direction of the oxygen input from deep to shallow gradually smaller. When the intake air velocity is reduced, the residence time in the vertical direction is prolonged and the time of mass transfer is prolonged.

![Figure 6. DO concentration distribution in the horizontal direction (at the depth of 1m)](image)

According to the integration of the vertical and horizontal test of the low intensity aeration, the basic assumption is well verified and the calculation model can well interpret the process of the low intensity aeration in the shallow water, so the possibility and feasibility of the low intensity aeration in shallow water are verified, which is consistent with relevant tests.

3.2. Discussions

3.2.1. The influence of intake velocity. At the basic of calculation model of aeration, we study the influence of different air intake velocities. We take the value of 5, 10, 20 m/s to compare the effect of aeration according to the DO index. The vertical and horizontal values are shown as the figure 7.

The aeration velocity affects the velocity of the aeration centre in the vertical direction DO to reach the saturation concentration. When the gas flow rate increases, the oxygen supply increases and the regeneration speed increases. The time when the gas reaches the saturation is reduced. With the passage of time, the effect of different aeration speed on DO concentration gradually became smaller and the concentration distribution curve showed a coincidence trend. The effect of aeration velocity on the DO concentration of the surface of the water body is greater than on the DO concentration at the bottom of the water body. The increase in aeration rate makes that the gas residence time decreases so that the oxygen transfer efficiency decreases.

The DO concentration distribution curves in the horizontal direction are almost coincident with each other, indicating that the effect of the aeration velocity on the diffusion in the horizontal direction is
negligible under the condition of low strength. With the passage of time, the DO concentration in the horizontal direction gradually increases and the curve is more moderate. The growth rate is inversely proportional to the distance from the aeration centre. When the time is infinite, the concentration in the horizontal direction can be equal concentration. Under low-intensity conditions, the long-term aeration oxygenation is theoretically feasible to enhance the self-purification capacity of shallow water bodies. The concentration of diffusion in the horizontal direction to reach the effective DO concentration range is limited, with time is very slow, and then take into account the DO in the water consumption, single point aeration oxygenation in theory, although feasible, but in the actual operation cannot be taken.

Figure 7. Velocity’s impact on the DO concentration in vertical and horizontal and Effects of single point aeration and dispersion aeration on the horizontal DO concentration (at depth of 1 m)

3.2.2. Aeration optimization. We put forward two aeration methods in the condition of keeping the total input gas flow constant: method 1 (single point centre aeration, inlet velocity of 10 m/s) and method 2 (two-point dispersion aeration, each centre on each side of aeration point, inlet velocity of 5 m/s) to analyse the impact on the horizontal diffusion. Values are shown in the right corner of the figure 7.

The DO concentration in the vertical direction of the aeration point is the highest at the centre and decreases rapidly toward both sides. When the total inlet gas volume is the same, the growth rate of the DO concentration in the double aeration point is larger than that on the outside of the aeration point, which is greater than the single point aeration. When the aeration is carried out, the effective DO concentration range is far greater than the effective DO concentration at single point aeration. Multipoint distribution aeration increases the total concentration of diffusion distance.

4. Conclusions
When aeration is conducted in shallow water, the concentration distribution in the vertical direction is in well correlative with the distribution of the concentration diffusion equation. The calculation model can well reflect the aeration process.

Increasing the aeration intensity, the DO concentration in the vertical direction reaches the saturation value more quickly, and the time of gas residence, the gas-liquid exchange time and the oxygen transfer efficiency will decrease accordingly.

The DO concentration in the horizontal direction presents few differences with different velocity, and the aeration intensity has little effect on the concentration diffusion in the horizontal direction.

In the case of the same total aeration intensity, the multipoint distribution has obvious effect on the diffusion of oxygen concentration in the horizontal direction, prolonging the residence time and increasing the oxygen transfer efficiency. So it is feasible to conduct the aeration at low intensity, which is consistent with the conclusion of relevant research [6].
In the shallow water treatment, reducing the volume of aeration equipment, miniaturization, array, low strength, mobile processing equipment to overcome the high energy consumption, large size, noise and other issues can provide a good reference.

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