Free vibration studies on skew sandwich plates by FEM

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Abstract. Present study mainly deals with the extraction of fundamental frequencies using simply supported and clamped isotropic and laminated faced skew sandwich plates with the orthotropic core. Available literature values were referred to validate the fundamental frequencies obtained using the finite element method. The effects of aspect ratio, skew angle, a ratio of the length-to total thickness of the sandwich plate, and the ratio of a thickness of the core to a thickness of face sheet on the free vibration of skew sandwich plates were examined. The present method is also used to study the effect of fiber orientation angle and laminate stacking sequence on free vibration of antisymmetric cross-ply laminated skew sandwich plates are also extracted. CQUAD8 type of element gives converged, accurate results than the CQUAD4 element in the present study. The variation of frequencies is directly proportional with the skew angle.

Keywords: Fundamental Frequency, Dimensionless frequency parameter, skew sandwich plates, and skew angle, antisymmetric laminated sandwich plates.

Nomenclature:

a: Plate length (mm)
b: Plate width (mm)
h: Total thickness of plate (mm)
tc: Core thickness (mm)
tf: Face sheet thickness (mm)
E: Modulus of Elasticity (GPa)
G: Modulus of Rigidity (GPa)
ρ: Density (kg/m³)
ν: Poisson’s Ratio
α: Skew Angle (Degrees)
Kf: Dimensionless Frequency Parameter
ω: Circular Frequency (rad/sec)
C: Core
1. Introduction

Skew sandwich plates now a day frequently used into numerous areas like aeronautical, automobile, civil engineering, and in most structural applications. In skew sandwich plates, the effect of shear deformation is considerably more as compared to laminated composite skew plates, which was the reason behind the widespread applications such plates. Also, skew sandwich plates exhibit less weight, more stiffness, more structural efficiency, and more durable.

Much research was made on sandwich plates on the free vibration behaviour for more than two decades. An expression was derived for natural frequency also conducted experiments to validate analytical results [1]. Skew sandwich plates were considered for static and dynamic analysis using finite element displacement model was carried out [2]. Exact solutions were derived for the problems on honeycomb sandwich structures [3]. The p-Ritz method was worked out for free vibration study on Skew sandwich plates with an orthotropic core and laminated facings [4]. A Higher-order shear deformation theory for free vibration of laminated composite and sandwich plates were presented [5]. The vibration parameters of sandwich plates were predicted by a spline finite strip method [6]. Two new C° assumed strain finite element [7], facet shell element [8]. To extract the fundamental frequencies of various plates, Reddy’s higher-order theory was used. Natural frequencies, displacement and stress eigenvectors were extracted, a semi-analytical [9] and exact analytical solutions based on the propagator matrix method [10] higher-order mixed theory was developed for laminated and sandwich plates. Refined plate theory was proposed on sandwich plates [11].

Free vibration studies on laminated composite and sandwich shells were made by adopting 2D-higher-order shear deformation theories [12]. Free vibration analysis of plates and sandwich plates was discussed using C° iso-parametric finite element model [13]. Global-local higher-order theory was developed on free vibration of laminated composite and sandwich plates [14].

Structural dynamic analysis was performed on a skew sandwich plate with laminated composite faces based on the high-order shear deformation plate theory [15]. The third-order zigzag theory was used to produce an improved discrete Kirchhoff quadrilateral element on the composite and sandwich plates for static analysis [16] and the free vibration response [17]. C¹ plate bending elements proved to be more efficient than the C° plate bending elements to extract exact natural frequencies of sandwich plates [18]. An investigation conducted on the free vibrations of rectangular sandwich plates by applying Hamilton’s principle along with the first-order shear deformation theory [19]. C° finite element model used for bending and free vibration studies of laminated soft core skew sandwich plates [20].

Various shear deformation theories [21] were considered for the comparison based on the displacement fields [22]. Free vibration analysis of laminated composite and sandwich plates using trigonometric shear deformation theory [23]. The bending behavior [24] and free vibration response [25] of composite sandwich plates with multi-layered face sheets has been investigated using a four nodded rectangular finite element formulation based on a layer-wise theory. The effect of openings and additional mass on free vibration analysis of laminated composite sandwich skew plates were studied [26].

The present effort concerned with the investigation of fundamental frequencies of skew sandwich plates with an orthotropic core. CQUAD4 and CQUAD8 elements are employed in the analysis. The correctness of the results from the elements is confirmed from literature results. The effects of aspect ratio, skew angle, fiber orientation angle, and boundary conditions on the free vibrations of skew sandwich plates are studied.

2. Convergence and Validation

2.1. Convergence
The geometrical representation of the sandwich plate is as shown in Figure 1. The skewed sandwich plate with global and local coordinate systems is as shown in Figure 2. The displacement boundary conditions cannot be applied directly, due to inclination of displacements to the skew edges. To prevail over this, a local coordinate system \((x', y')\) normal and tangential to the skew edges is preferred.

**Figure 1.** Geometry details of Sandwich Plate

**Figure 2.** Finite Element Mesh of Skew sandwich plate with a local and global coordinate system.

Total elements in a plate model are optimized to get exact and consistent values. Consequently, it is essential to analyze the convergence of the values. The convergence was made on simply supported and clamped skew sandwich plates having an aspect ratio, length to thickness ratio and the ratio of a thickness of core to facing for skew angles 0°, 15°, 30° and 45° using both elements. The converged detailed results are delivered in Table 1.

The material properties used are, for face sheets \(E = 68.948\) GPa, \(G = 25.924\) GPa, \(\nu = 0.33\), \(\rho = 2768.0\) kg/m³ and for core \(G_{13} = 0.05171\) GPa, \(G_{13} = 0.13445\) GPa, \(\rho = 121.83\) kg/m³ [1].

### 2.2. Validation

Validation of the results from the elements used in the present study is made by matching up to the values for the natural frequency found in the present study to the available literature values. The comparison is shown in Table 2, for a simply supported skew sandwich plate. The material constants employed are similar to those used for the convergence study. The values found in the study are in good harmony with literature results.

In Table 3 the dimensionless frequency parameter \((K_f)\) for skew sandwich plates with graphite-epoxy laminate face sheet and Al honeycomb core are compared with literature results with simply supported edge condition. Face sheets having two layers with anti-symmetric layup sequence are employed. The material properties adopted for face sheet are, \(E_1 = 131\) GPa, \(E_3 = 3.275\) GPa, \(G_{13} = 3.275\) GPa, \(\nu_{13} = 0.25\) and \(\rho = 1627\) kg/m³ and for core \(G_{13} = 0.1173\) GPa, \(G_{23} = 0.2415\) GPa and \(\rho = 2386\) kg/m³[13]. In Table 3 the results are compared in Dimensionless form using the formula \(K_f = 100oaoa\sqrt{(p/E_1)F}\). From Table 1 to Table 3 it is observed that CQUAD8 element gives accurate and converge results as than CQUAD4 element. From now CQUAD8 is adopted in the work.

### 3. Results and discussion

#### 3.1. Sandwich plates with isotropic face sheet and aluminum honeycomb core

The present study considers a variety of parameters such as aspect ratio, ratio of length to thickness of sandwich plates, ratio thickness of face sheet to thickness of core, skew angle and boundary conditions.
The results from the numerical methods are presented in the Table 4 for different aspect ratio and skew angle for all sides simply supported and clamped edge condition. The graphical presentation of the results in the Table 4 is as shown in Fig 3 and Fig 4.

Table 1. Convergence for fundamental frequencies (Hz) of simply supported skew sandwich plates (a/b=1, a/h=10, t_c/t_f =10).

| Element Density | Element Type | S-S-S-S | Skew Angle (α) | C-C-C-C | Skew Angle (α) |
|----------------|--------------|---------|----------------|---------|----------------|
| Present (10 x 10) | CQUAD 4 | 2493.991 | 2570.462 | 2827.037 | 3369.724 | 3017.56 | 3779.85 |
| Present (10 x 10) | CQUAD 8 | 2519.097 | 2596.492 | 2856.251 | 3405.989 | 3052.65 | 3717.40 | 3339.14 | 3824.81 |
| Present (14 x 14) | CQUAD 4 | 2507.189 | 2584.146 | 2842.358 | 3388.675 | 3036.29 | 3100.64 | 3720.98 | 3803.53 |
| Present (14 x 14) | CQUAD 8 | 2520.018 | 2597.446 | 2857.271 | 3407.130 | 3054.26 | 3119.01 | 3820.76 | 3826.51 |
| Present (18 x 18) | CQUAD 4 | 2512.631 | 2589.787 | 2848.699 | 3396.465 | 3044.04 | 3108.55 | 3812.99 | 3813.30 |
| Present (18 x 18) | CQUAD 8 | 2520.395 | 2597.836 | 2857.720 | 3407.607 | 3054.93 | 3120.02 | 3826.51 | 3827.21 |
| Present (22 x 22) | CQUAD 4 | 2515.387 | 2592.644 | 2851.877 | 3400.404 | 3047.97 | 3112.56 | 3828.77 | 3828.26 |
| Present (22 x 22) | CQUAD 8 | 2520.586 | 2598.034 | 2857.901 | 3407.851 | 3055.26 | 3120.02 | 3827.56 | 3827.56 |
| Present (26 x 26) | CQUAD 4 | 2516.973 | 2594.288 | 2853.718 | 3402.667 | 3050.23 | 3114.87 | 3821.11 | 3821.11 |
| Present (26 x 26) | CQUAD 8 | 2520.696 | 2598.147 | 2858.040 | 3407.993 | 3055.46 | 3120.21 | 3827.77 | 3827.77 |
| Present (30 x 30) | CQUAD 4 | 2517.968 | 2595.319 | 2854.872 | 3404.086 | 3051.65 | 3116.32 | 3822.89 | 3822.89 |
| Present (30 x 30) | CQUAD 8 | 2520.765 | 2598.218 | 2858.119 | 3408.082 | 3055.58 | 3120.33 | 3827.90 | 3827.90 |
| Present (34 x 34) | CQUAD 4 | 2518.634 | 2596.009 | 2855.666 | 3405.033 | 3052.60 | 3117.29 | 3824.09 | 3824.09 |
| Present (34 x 34) | CQUAD 8 | 2520.811 | 2598.266 | 2858.193 | 3408.143 | 3055.66 | 3120.42 | 3827.98 | 3827.98 |
| Present (38 x 38) | CQUAD 4 | 2519.100 | 2596.492 | 2856.207 | 3405.698 | 3053.27 | 3117.97 | 3824.93 | 3824.93 |
| Present (38 x 38) | CQUAD 8 | 2520.843 | 2598.299 | 2858.230 | 3408.186 | 3055.72 | 3120.47 | 3828.04 | 3828.04 |

Table 2. Fundamental frequencies (Hz) of skew sandwich plates with simply supported

| Authors | 0° | 15° | 30° | 45° |
|---------|----|----|----|----|
| Raville [1] | 23.0000 | - | - | - |
| Zhou [27] | 23.2900 | - | - | - |
| Bardell [28] | 23.0500 | - | - | - |
| Yuan [6] | 23.4100 | - | - | - |
| FOST | 23.5994 | - | - | - |
| HOST | 23.4841 | - | - | - |
| Araújo [30] | 23.5000 | - | - | - |
| Ferreira [29] | 23.2600 | - | - | - |
| Bo Liu [31] | 23.2591 | - | - | - |
| Shenoi [7] | 23.6300 | - | - | - |
| HOST12 | 23.4258 | - | - | - |
| HOST11 | 23.4268 | - | - | - |
| HOST9 | 23.4265 | - | - | - |
| HOST | 23.4251 | - | - | - |
| Voyiadjis [15] | 23.3190 | 24.8550 | 29.8300 | 41.4260 |
| Host | 23.5279 | 24.9315 | 30.0623 | 43.1677 |
| A. K. Garg [12] | 23.4268 | - | - | - |
| HOST12 | 23.4258 | - | - | - |
| HOST11 | 23.4268 | - | - | - |
| HOST9 | 23.4265 | - | - | - |
| HOST | 23.4251 | 24.8438 | 29.9153 | 42.7936 |
| A. K. Garg [13] | 23.4268 | - | - | - |
| HOST12 | 23.4258 | - | - | - |
| HOST11 | 23.4268 | - | - | - |
| HOST9 | 23.4265 | - | - | - |
| HOST | 23.4251 | 24.8438 | 29.9153 | 42.7936 |

Table 3. Dimensionless frequency parameter (Kf) for simply supported graphite epoxy laminated composite skew sandwich plates with orthotropic core.

| AUTHORS | Skew Angle (α) |
|---------|---------------|
| R. Khare [8] | 23.5994 |
| Host | 23.5279 |
| A. K. Garg [12] | 23.5279 |
| HOST12 | 23.5251 |
| HOST11 | 23.5251 |
| HOST9 | 23.5251 |
| HOST | 23.5251 |
| A. K. Garg [13] | 23.5251 |
| HOST12 | 23.5251 |
| HOST11 | 23.5251 |
| HOST9 | 23.5251 |
| HOST | 23.5251 |

4
From the results, keeping the ratio of a/h and t_c/t_f as constant, the following observations were made.

- With increment in the aspect ratio, fundamental frequency diminishes for a given skew angle.
- With increment in the skew angle, fundamental frequency increases for a given aspect ratio.
- The value of natural frequency is high for all sides’ clamped edge as compared to the simply supported.

**Table 4. Fundamental frequencies [Hz] of skew sandwich plates (a/h=10, t_c/t_f=10)**

| a/b   | All Sides Simply Supported | All Sides Clamped |
|-------|----------------------------|-------------------|
|       | Angle of Skew (α)          | Angle of Skew (α) |
|       | 0  15°  30°  45°           | 0  15°  30°  45° |
| 1     | 2532.05 2592.03 2794.67 3232.52 | 2797.83 2850.8 3032.62 3436.39 |
| 1.5   | 1509.04 2200.22 2269.83 2685.75 | 1795.36 2012.27 2398.76 2614.63 |
| 2     | 1010.33 1969.91 2108.33 2462.06 | 1283.64 2145.97 2398.76 2614.63 |
| 2.5   | 724.62 1624.88 1859.73 2345.92 | 972.97 1787.48 2012.27 2482.03 |

The results showing the effect of ratio of (t_c/t_f) of the sandwich skew plate on the natural frequency are presented in the Fig 5 and Fig 6. The aspect ratio (a/b=1) and ratio of (a/h=10) of the sandwich skew plate kept constant. From the figures the following observations were made,

- With increment in the ratio of (t_c/t_f), fundamental frequency increases for a given skew angle.
- With increment in the skew angle, fundamental frequency increases for a given ratio of (t_c/t_f).
- The value of natural frequency is high for all sides’ clamped edge as compared to the simply supported.

Fig 7 and Fig 8 shows the results of effect of ratio of (a/h) of the skew sandwich plate on natural frequency. The aspect ratio (a/b=1) and ratio of (t_c/t_f=10) of the sandwich skew plate kept constant. From the figures the following observations were made,

- With increment in the proportion of (a/h), fundamental frequency diminishes for a given skew angle.
- With increment in the skew angle, fundamental frequency increases for a given ratio of (a/h).
- The value of natural frequency is high for all sides’ clamped edge as compared to the simply supported.
3.2. Sandwich plates with orthotropic face sheet and honeycomb core

The numerical study is made for sandwich skew plates with orthotropic face sheet and aluminium honeycomb core. The material constants employed are, face sheet, $E_1=206.84$ GPa, $E_2=E_3=5.1711$ GPa, $\nu_{12}=0.25$, $\nu_{23}=0.3$ and $\rho=1603.1$ kg/m$^3$. And for core, $E=627.9$ MPa $G_{13}=117.21$ MPa, $G_{23}=241.32$ MPa, $\rho=2351.2$ kg/m$^3$.The results are presented in a Dimensionless form using the formula $K_f = \frac{100\omega a}{\sqrt{(\rho/E_1)r}}$. The face sheet is made-up of two layers anti-symmetric laminate with core in the centre. Dimensionless frequency parameters for different aspect ratio and boundary conditions are obtained by varying the skew angle. The values found are as shown in Table 5. The graphical representation of the same is made in Fig 9 and Fig 10.
Figure 5. Fundamental Frequency v/s ratio of $t_c/t_f$ with all sides simply supported.

Figure 6. Fundamental Frequency v/s ratio of $t_c/t_f$ with all sides clamped
Figure 7. Fundamental Frequency v/s ratio of a/h of skew sandwich plate for all sides simply supported

Figure 8. Fundamental Frequency v/s ratio of a/h of skew sandwich plate for all sides clamped edges
Table 5. Dimensionless frequency parameters of laminated antisymmetric skew sandwich plates
\((0^0/90^0/C/0^0/90^0)\) (\(a/h=10, t_c/t_f=10\))

| a/b | Angle of Skew (\(\alpha\)) | All Sides Simply Supported | All Sides Clamped |
|-----|---------------------------|---------------------------|-------------------|
|     | 0^0 | 15^0 | 30^0 | 45^0 | 0^0 | 15^0 | 30^0 | 45^0 |
| 1   | 9.1174 | 9.3786 | 10.2724 | 12.2037 | 10.6262 | 10.8547 | 11.6391 | 13.3712 |
| 1.5 | 11.842 | 12.0111 | 13.0143 | 15.7942 | 14.2666 | 14.3657 | 15.1059 | 17.4253 |
| 2   | 14.8035 | 14.7184 | 15.7272 | 19.4847 | 18.1454 | 18.3954 | 18.7866 | 21.4101 |
| 2.5 | 17.0208 | 17.3233 | 18.4786 | 23.2834 | 19.9568 | 21.2636 | 21.5831 | 25.4219 |

Figure 9. Dimensionless frequency Parameter v/s Aspect ratio for simply supported anti-symmetric cross ply Graphite/Epoxy laminated skew sandwich plates.

Figure 10. Dimensionless frequency Parameter v/s Aspect ratio for clamped anti-symmetric cross ply Graphite/Epoxy laminated skew sandwich plates.
4. Conclusions
The following points are noticed after carrying out numerical study.

• Both CQUAD4 and CQUAD8 elements have good agreement with the available literature results. But CQUAD8 element yields more accurate results. The same is true for both isotropic and orthotropic skew sandwich plates.

• With increment in aspect ratio, fundamental frequency values diminish in all the cases.

• As skew angle increased the fundamental frequency value also increases in all the cases.

• The ratio \( \frac{t_c}{t_f} \) has major influence till \( \frac{t_c}{t_f} = 20 \) for simply supported and \( \frac{t_c}{t_f} = 40 \) for clamped boundary condition, there after the influence is negligible.

Natural frequency values decrease considerably for in the influence of ratio of length to total thickness of the skew sandwich plates.

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