The Coulomb glory effect in collisions of antiprotons with heavy nuclei: relativistic theory

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Abstract
Collisions of antiprotons with bare uranium nuclei are studied for scattering angles near 180° in the framework of relativistic theory. The Coulomb glory phenomenon is investigated at antiproton energies in the range 100 eV–2.5 keV. The vacuum polarization effect and the anomalous magnetic moment of the antiproton are taken into account. Estimations of possible influence of such effects as radiative recombination and antiproton annihilation are given.

1. Introduction
New facilities for antiproton and ion research at GSI will give an opportunity to observe the Coulomb glory phenomenon, which was predicted by Demkov and coauthors [1, 2]. The effect consists of a prominent maximum of the differential cross section (DCS) in the backward direction at a certain energy of the incident particle, provided the interaction with the target is represented by a screened Coulomb attraction potential. In our previous papers [3, 4] we investigated the backward scattering of antiprotons by highly charged and neutral uranium (Z = 92) in the framework of non-relativistic quantum mechanics. It was shown that the Coulomb glory effect takes place due to the screening of the nuclear Coulomb attraction by the electrons. At the energy where the strongest effect is predicted, the DCS in the backward direction may exceed the corresponding Rutherford DCS by several times, depending on the number of electrons in the ion [3].

In collisions of antiprotons with bare uranium, the Coulomb glory is also present because of the screening properties of the vacuum polarization (VP) potential. In this case, the effect predicted was not as large but still noticeable (about 5% [4]).

In this paper, we investigate the Coulomb glory effect in collisions of antiprotons with bare uranium nuclei in the framework of relativistic quantum theory. Since the kinetic energy of antiprotons in the Coulomb glory region is rather low (does not exceed a few keV), one can expect that the relativistic corrections may not alter the results significantly. This is certainly the case for the uranium ions where the effect itself is very large. However, for bare nuclei the situation is not that straightforward. Although the ratio of the velocity of antiprotons to the velocity of light v_p/c is quite small in the energy range under consideration (≤ 10⁻³), relativistic effects appear important for scattering in the backward direction. As was shown in [5], in the low-energy limit the DCS of the relativistic Coulomb scattering dσ_C/dΩ(θ = 180°) can be expressed as follows:

$$\frac{dσ^C}{dΩ}(\theta = 180°) = \left[ 1 + \sqrt{2π \frac{v_p}{c} \left( \frac{Z}{c} \right)^3} \right] \frac{dσ^B}{dΩ}(\theta = 180°),$$

where dσ_B/dΩ is the DCS obtained in the (relativistic) first Born approximation. We note that for small values of the parameter v_p/c, the Born DCS dσ_B/dΩ is nearly equal to the non-relativistic Rutherford DCS. According to equation (1), the contribution to the DCS due to relativistic effects can be as large as 10–20%, depending on the energy of antiprotons in the interval 100 eV–2.5 keV. Of course, any
estimation based on equation (1) is valid for the pure Coulomb potential (that is, point nucleus charge distribution) only. For a finite size nucleus, the situation is somewhat different. This especially concerns antiproton scattering, where the nuclear size effect can be important. In this case, as our calculations especially concerns antiproton scattering, where the nuclear potential (that is, point nucleus charge distribution) only. For

of the unpolarized beam in the central field is given by the

2. Basic formulae

In the relativistic quantum theory, the DCS of the scattering of the unpolarized beam in the central field is given by the following equation [6]:

$$\frac{d\sigma}{d\Omega} = |A|^2 + |B|^2,$$

where

$$A(\theta) = \frac{1}{2ip} \sum_{l=0}^{\infty} \left[ \left\{ (l+1)\exp(2i\delta_{l+1/2,l}) - 1 \right\} P_l(\cos \theta) + \left\{ \exp(2i\delta_{l-1/2,l}) - 1 \right\} P_l(\cos \theta) \right],$$

$$B(\theta) = \frac{1}{2p} \sum_{l=1}^{\infty} \left[ \exp(2i\delta_{l-1/2,l}) - \exp(2i\delta_{l+1/2,l}) \right] P_l(\cos \theta).$$

Here \( p \) is the momentum of the antiproton, \( P_l(\cos \theta) \) are the Legendre polynomials and \( P_l(\cos \theta) \) are the associated Legendre functions. The phase shifts \( \delta_{l,j} \) corresponding to the total angular momentum \( j \) and orbital momentum \( l \) can be expressed as a sum of the phase shifts \( \delta_{l,j} \) produced by the short-range part of the scattering potential, and the Coulomb phase shift \( \delta_{l,j} \):

$$\delta_{l,j} = \delta_{l,j}^{\text{SR}} + \delta_{l,j}^{\text{C}}.$$

The Coulomb phase shifts can be represented as follows [6]:

$$\delta_{l,j}^{\text{C}} = \xi - \arg \Gamma(\gamma + 1 - iv) - \frac{\pi \gamma}{2} + \frac{\pi l}{2},$$

where \( \gamma = -Ze^2/\hbar r \) is the Coulomb parameter, \( e \) is the total energy of the antiproton, \( \exp(-2i\xi) = (\gamma + iv)/\sqrt{\gamma^2 - (Z/e)^2} \), and the quantum number \( \kappa = (-1)^{(l+1)/2}(j + 1/2) \).

We consider antiprotons moving in the central field \( V(r) \) and take into account the anomalous magnetic moment of the antiproton. In this case, the Dirac equation appears in the following form (see, e.g., [7, 8]):

$$\left[ c(\alpha \cdot p) + m_pc^2 \gamma + V(r) - \frac{i\kappa_0}{2m_pc} V'(r) \beta (\alpha \cdot \hat{r}) \right] \psi(r) = \epsilon \psi(r),$$

where \( \kappa_0 = 1.792 \, 847 \, 34 \) is the anomalous magnetic moment, \( V'(r) \) stands for the first derivative of the potential \( V(r) \), \( \alpha \) and \( \beta \) are the Dirac matrices and \( \hat{r} \) denotes the unit vector in the \( r \) direction. Equation (7) has solutions corresponding to the definite total angular momentum \( j \) and its projection \( m \):

$$\psi(r) = \frac{1}{r} \left( G(r)\Omega_{ljm}(\hat{r}) \right),$$

Here \( \Omega_{ljm}(\hat{r}) \) are the spherical spinors, \( l = j \pm \frac{1}{2} \) and \( l' = 2j - l \). The radial functions \( F(r) \) and \( G(r) \) satisfy the set of the radial Dirac equations:

$$c\frac{dF(r)}{dr} - \frac{ck}{r} F(r) + [\epsilon - m_pc^2 - V(r)] F(r) = 0,$$

$$c\frac{dG(r)}{dr} + \frac{ck}{r} G(r) - [\epsilon + m_pc^2 - V(r)] G(r) = 0.$$

The phase shifts \( \delta_{l,j}^{C} \) can be obtained from asymptotic expansions of \( F(r) \) and \( G(r) \) as \( r \to \infty \). An alternative way to calculate \( \delta_{l,j}^{C} \) is the variable phase method [9–11]. Within this approach, the variable phase \( \delta_{l,j}^{C} \) is a solution of a first-order differential equation:

$$\frac{d}{dr} \delta_{l+1/2,j}^{C}(p,r) = -\left( \frac{v(r)}{c} \right)^2 \left( \frac{v + m_pc^2}{v - m_pc^2} [\cos \delta_{l+1/2,j}^{C}(p,r) F_{l}^{C}(p,r)] + \tan \delta_{l+1/2,j}^{C}(p,r) \right)^2$$

$$+ v(r) \left( \frac{\epsilon - m_pc^2}{\epsilon + m_pc^2} [\cos \delta_{l+1/2,j}^{C}(p,r) F_{l}^{C}(p,r)] + \tan \delta_{l+1/2,j}^{C}(p,r) \right)^2,$$

where \( \delta_{l+1/2,j}^{C}(p,r) \) are the regular and irregular Dirac wavefunctions for the pure Coulomb potential, respectively, and \( v(r) \) is the short-range part of the scattering potential:

$$v(r) = V(r) + \frac{Z}{r} = V_0(r) + V_U(r) + V_{WK}(r) + \frac{Z}{r}. $$

Here \( V_0(r) \) is the electrostatic potential of interaction with the finite nucleus and \( V_U(r) \) is the Uehling potential which is given by the lowest order term in the expansion of the one-electron-loop VP in powers of the Coulomb electron–nucleus interaction. The Wichmann–Kroll potential \( V_{WK}(r) \) accounts for the higher order terms in the expansion of the vacuum loop in powers of the Coulomb electron–nucleus interaction [12]. The explicit forms of the potentials \( V_0(r), V_U(r) \) and \( V_{WK}(r) \) can be found elsewhere [3, 4, 13–15].

In order to calculate the DCS (2), one has to evaluate the quantities \( A(\theta) \) and \( B(\theta) \) in equations (3) and (4) which contain infinite summations over the angular momenta. For
the slowly decaying (Coulomb-tail) interaction, series (3) is formally divergent because the phaseshifts $\delta_{l\pm 1/2}$ do not decrease with increasing $l$. For the pure Coulomb potential, a regularization procedure was suggested by Mott [16] who obtained also the first numerical results for the relativistic Coulomb scattering. Detailed theoretical and numerical studies of this problem can be found in a number of papers (see, e.g., [17–19]); it can be regarded as well understood and does not pose any challenge nowadays. Once the short-range interaction and contain the parameters $\nu$ and $s$ [21, 22], we have also estimated the influence of the strong interaction between the antiproton and the nucleus and found it negligible. In the non-relativistic limit $c \to \infty$, the present results coincide with the previous calculations [4] which are represented by curve (c) in figure 1.\(^5\) As one can see, the relativistic corrections are very important: they are responsible for overall increase of the DCS by approximately 10% and the shift of the maximum to higher energies (from 300 eV to 600 eV). In figure 2, we present the DCS dependence on the scattering angle at the antiproton energy $E = 600$ eV corresponding to the strongest Coulomb glory effect. Curve (a) shows the DCS resulting from the full interaction, while curve (b) represents the DCS produced by the finite nucleus potential only; curve (c) corresponds to the pure (point charge) Coulomb potential, and curve (d) shows the non-relativistic Rutherford DCS. The relativistic contribution to the DCS at $\theta = 180^\circ$ for the pure Coulomb potential (figure 2, curve (c)) amounts to 14.5% at this energy, in good accordance with the value obtained from the approximate equation (1). The oscillatory dependence of the DCS on the scattering angle has the same nature as in the non-relativistic case where it appears because of the interference between the Coulomb and short-range contributions to the scattering amplitude [23]. Such oscillations were discussed previously in connection with elastic scattering of slow electrons by positive ions [24]. The non-relativistic DCS can be parametrized as follows:

$$ \frac{d\sigma}{d\Omega} = \left( \frac{Z}{4E} \right)^2 \frac{d\sigma}{d\Omega} \quad \text{(14)} $$

Using the scaled DCS, we can compare the results at different energies and determine the energy domain with the largest Coulomb glory effect. In figure 1 we show the scaled DCS at $\theta = 180^\circ$ as a function of the antiproton energy. The maximum Coulomb glory effect is reached at the antiproton energy of about 600 eV and amounts to 15.6% (figure 1, curve (a)). The main contribution comes from the VP potential, as the DCS produced by the finite nucleus potential only (figure 1, curve (b)) is about 10% smaller and does not exhibit a pronounced maximum. We note that the contribution from the anomalous magnetic moment to $d\sigma/d\Omega$ is very small and has an order of magnitude $\sim 10^{-4}$, which is in agreement with a qualitative estimate given by Milstein [20]. With the help of the optical potential method [21, 22], we have also estimated the influence of the strong interaction between the antiproton and the nucleus and found it negligible. In the non-relativistic limit $c \to \infty$, the present results coincide with the previous calculations [4] which are represented by curve (c) in figure 1.\(^5\) As one can see, the relativistic corrections are very important: they are responsible for overall increase of the DCS by approximately 10% and the shift of the maximum to higher energies (from 300 eV to 600 eV). In figure 2, we present the DCS dependence on the scattering angle at the antiproton energy $E = 600$ eV corresponding to the strongest Coulomb glory effect. Curve (a) shows the DCS resulting from the full interaction, while curve (b) represents the DCS produced by the finite nucleus potential only; curve (c) corresponds to the pure (point charge) Coulomb potential, and curve (d) shows the non-relativistic Rutherford DCS. The relativistic contribution to the DCS at $\theta = 180^\circ$ for the pure Coulomb potential (figure 2, curve (c)) amounts to 14.5% at this energy, in good accordance with the value obtained from the approximate equation (1). The oscillatory dependence of the DCS on the scattering angle has the same nature as in the non-relativistic case where it appears because of the interference between the Coulomb and short-range contributions to the scattering amplitude [23]. Such oscillations were discussed previously in connection with elastic scattering of slow electrons by positive ions [24]. The non-relativistic DCS can be parametrized as follows:

$$ \frac{d\sigma}{d\Omega} = \left( \frac{Z}{4E} \right)^2 \left[ \frac{1}{\sin^2 \theta/2} + \frac{4f_z^2}{\nu^2} \right. $$

$$ \left. + \frac{4f_z}{\nu \sin^2 \theta/2} \cos(2\nu \ln \sin \theta/2 + \phi_z) \right] \quad \text{(15)} $$

Here the first term in square brackets corresponds to the Rutherford DCS while the other two terms appear due to the short-range interaction and contain the parameters $f_z$ and $\phi_z$. For low energies of the incident particle ($|\nu| \gg 1$), these

5 Only curve (c) in figure 2 of [4] corresponding to the non-relativistic DCS for the finite nucleus potential appeared incorrect in the paper; it must be shifted down by about 3–5%. All other curves in that figure are correct.
parameters depend on the scattering angle θ but do not depend on the energy [24]. The last term in (15) describes oscillations in the dependence of the DCS on the scattering angle θ. In the low-energy limit, the relative amplitude of the oscillations is small (∼|v|−1), and the frequency is high (∼|v|), except in the vicinity of θ = 180°. The amplitude and phase of the oscillations depend strongly on the short-range parameters fs and φs while the frequency does not; the oscillations become more rapid as the deviation from θ = 180° increases. We should note that the interference oscillations take place for the attractive Coulomb potential only. For the repulsive potential, low-energy particles do not penetrate the Coulomb barrier and do not experience the short-range interaction; this is described by the limit fs → 0 in (15).

In the relativistic case, the set of two first-order radial Dirac equations (9) can be converted to a single second-order Schrödinger-like equation containing an effective potential Veff(r) (see, e.g., [9]). The potential Veff(r) depends on the energy and angular momentum; besides V(r), it includes relativistic correction terms which decay faster than the Coulomb potential as r → ∞. Thus even for pure Coulomb relativistic scattering, the equivalent Schrödinger equation contains not only the Coulomb potential, but also the short-range potential terms. Based on the above discussion of the non-relativistic interference oscillations, we can expect similar oscillations in the DCS of relativistic Coulomb scattering of low-energy particles. Indeed, this is the case, as one can see from figure 2, curve (c). The contributions to the scattering amplitude due to the short-range terms in Veff(r) interfere with the non-relativistic Rutherford amplitude giving rise to the oscillations in the DCS. Similar oscillations for the relativistic Coulomb scattering were revealed in the previous calculations [25]. If one accounts for the finite size of the nucleus, the short-range part of Veff(r) changes. Then the amplitude and the phase of the oscillations are changed, too (figure 2, curve (b)). However, the frequency of the oscillations remains relatively unchanged. For the parameters of the antiproton scattering in figure 2, the positions of the maxima and minima in the DCS are well described by formula (15) with the appropriate parameters fs and φs. If the radius of the nucleus gradually decreases, a smooth transition from curve (b) to curve (c) in figure 2 is expected. We note, again, that the interference oscillations in the DCS can be observed in collisions of slow antiprotons with the uranium nuclei (Coulomb attraction) and do not appear in collisions of slow protons (Coulomb repulsion) where the relativistic, finite nucleus, and VP effects are negligible. In the Coulomb glory case, the highest maximum of the DCS appears at θ = 180° (figure 2, curve (a)), and its width is about 5°. The crucial role of the VP potential in the Coulomb glory effect should be emphasized: with the VP potential switched off (figure 2, curve (b)) the height of the maximum at θ = 180° is about three times smaller.

In order to check that inelastic scattering channels do not mask the Coulomb glory phenomenon, we have also evaluated the total cross sections of such processes as radiative recombination (RR) and annihilation of antiprotons. To compare the results, the cross sections of the inelastic processes have been scaled in a similar manner that was used for the elastic scattering DCS. Since the width of the Coulomb glory maximum in the angular dependence of the DCS is about 5°, we define the scale factor σ0 as the Rutherford DCS integrated over the same angular domain in the vicinity of θ = 180°:

$$\sigma_0 = \left( \frac{Z}{4E} \right)^2 \int_{180°-\delta} \frac{d\Omega}{\sin^2 \frac{\theta}{2}}$$

(16)

We have found that the total RR cross section (scaled by σ0) $\hat{\sigma}_a$ does not exceed $10^{-4}$ and, therefore, is small compared to the Coulomb glory effect. A rough estimate of the scaled antiproton annihilation cross section $\hat{\sigma}_a$ can be obtained as follows:

$$\hat{\sigma}_a \sim \frac{\pi R_n^2 |\psi_C(r = 0)|^2}{\sigma_0 |\psi_f(r = 0)|^2},$$

(17)

where $R_n$ is the nuclear charge radius, $\psi_C$ is the non-relativistic wavefunction of an antiproton in the Coulomb field and $\psi_f$ is the wavefunction of a free antiproton. The results obtained by this formula are presented in table 1. They show that the antiproton annihilation should not mask the Coulomb glory effect.

4. Summary

In this paper, the backward scattering of antiprotons by bare uranium nuclei has been investigated in the framework of relativistic theory in the antiproton kinetic energy range 100 eV–2.5 keV. The effects due to VP and finite size of the nucleus, as well as the influence of the anomalous magnetic moment of the antiproton, have been taken into account.
It is the screening property of the one-loop VP potential that is responsible for the Coulomb glory effect with the prominent DCS maximum in the backward direction. Both the non-relativistic and relativistic theories predict this maximum in some range of the antiproton kinetic energy. The relativistic effects, however, significantly alter the non-relativistic Coulomb glory picture. The kinetic energy corresponding to the strongest effect is shifted to higher values (600 eV versus 300 eV in the non-relativistic case), and the DCS in the vicinity of $\theta = 180^\circ$ becomes larger. We have also estimated the role of inelastic processes, such as the RR and annihilation of antiprotons, and found that they should not mask the Coulomb glory phenomenon.

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