Lepton Flavour Violation from SUSY–GUTs: 
Where do we stand for MEG, PRISM/PRIME and a Super Flavour factory

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We analyse the complementarity between Lepton Flavour Violation (LFV) and LHC experiments in probing the Supersymmetric (SUSY) Grand Unified Theories (GUT) when neutrinos got a mass via the see–saw mechanism. Our analysis is performed in an SO(10) framework, where at least one neutrino Yukawa coupling is necessarily as large as the top Yukawa coupling. Our study thoroughly takes into account the whole RG running, including the GUT and the right handed neutrino mass scales, as well as the running of the observable neutrino spectrum. We find that the upcoming (MEG, SuperKEKB) and future (PRISM/PRIME, Super Flavour factory) LFV experiments will be able to test such SUSY framework for SUSY masses to be explored at the LHC and, in some cases, even beyond the LHC sensitivity reach.

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I. INTRODUCTION

In this paper we address the issue of detecting low–energy supersymmetry (SUSY) at present, upcoming and planned Lepton Flavour Violation (LFV) experiments. Moreover we evaluate the complementarity between these experiments and the CERN Large Hadron Collider (LHC) experiments as probes of supersymmetric grand–unified (SUSY–GUT) scenarios.

The study of Flavour Changing Neutral Current (FCNC) processes, which are suppressed by the Glashow–Iliopolous–Maiani mechanism [1] in the Standard Model (SM) of particle physics, has been considered for a long time a powerful tool in order to shed light on new physics, especially for testing low–energy supersymmetry. Indeed, taking into account the fact that neutrinos have mass and mix [2–6], the Standard Model predicts Lepton Flavour Violating processes in the charged sector to occur at a negligible rate (e.g. $BR(\mu \rightarrow e \gamma) \sim O(10^{-54})$ [7, 8]). Given the future experimental sensitivities to LFV processes (Table I), the discovery of such processes will open a window to new physics.

It is known that a generic low–energy SUSY model (i.e. a model with arbitrary mixings in the soft breaking sector) would induce unacceptably large flavour violating effects [20]. The unobserved departures from SM in FCNCs makes it reasonable to assume flavour universality in the mechanism that breaks SUSY. On the other hand, even taking flavour universal SUSY breaking boundary conditions, renormalization effects can generate sizable flavour mixings in the running of the soft parameters from the SUSY breaking mediation scale down to the SUSY decoupling scales. In the leptonic sector, the relevance of such effects strongly depends on the neutrinos’ parameters.

The existence and smallness of neutrinos’ masses can be simply explained by the see–saw mechanism [21], by introducing Right–handed Neutrino (RN) fields, that are singlets under the SM gauge transformations. Since there is no gauge symmetry that protects them, the RNs can get a large Majorana mass ($M_R$), breaking the conservation of lepton number. When they are integrated out, they will give rise to an effective light neutrino Majorana mass matrix

$$m_\nu = -Y_\nu \hat{M}_R^{-1} Y_{\nu}^T \langle H_u \rangle^2,$$

where $(Y_\nu)_{ij}$ are the Yukawa couplings between left and right handed neutrinos and $\langle H_u \rangle$ is the Vacuum Expectation Value (VEV) acquired by the up sector Higgs field.

It is known [22] that the marriage between see–saw and SUSY can generate observable LFV rates in the charged lepton sector. In their renormalization group (RG) evolution, the slepton soft masses ($m^2_{L}^{ij}$) acquire LFV entries

| Process                  | Present bound       | Future sensitivity |
|-------------------------|---------------------|--------------------|
| $\text{BR}(\mu \rightarrow e \gamma)$ | $1.2 \times 10^{-11}$ | $O(10^{-14})$        |
| $\text{BR}(\mu \rightarrow e e e)$ | $1.1 \times 10^{-12}$ | $O(10^{-13})$        |
| $\text{CR}(\mu \rightarrow e$ in Tt) | $4.3 \times 10^{-12}$ | $O(10^{-18})$        |
| $\text{BR}(\tau \rightarrow e \gamma)$ | $3.1 \times 10^{-7}$  | $O(10^{-8})$        |
| $\text{BR}(\tau \rightarrow e e e)$ | $2.7 \times 10^{-7}$  | $O(10^{-8})$        |
| $\text{BR}(\tau \rightarrow \mu \gamma)$ | $6.8 \times 10^{-8}$  | $O(10^{-8})$        |
| $\text{BR}(\tau \rightarrow \mu \mu \mu)$ | $2 \times 10^{-7}$  | $O(10^{-8})$        |

*Planned or discussed experiment, not yet under construction

TABLE I: Present bounds and expected experimental sensitivities on LFV processes [9–19].

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that are proportional to \((Y_e Y_
u^\dagger)\)

\[
(m_{ij}^2)_{i\neq j} = -\frac{3m_0^2 + A_0^2}{16\pi^2} \sum_k Y_{\nu i}^\dagger Y_{\nu k} \ln \left( \frac{M_X^2}{M_{Rk}^2} \right)
\]  

(2)

where \(M_{Rk}\) is the mass of the \(k\)-th right handed neutrino \((i, j, k\) being generation indices), \(m_0\) and \(A_0\) are the universal supersymmetry breaking scalar masses and scalar trilinear couplings respectively. Since the see–saw equation (1) allows large neutrinos’ Yukawa couplings, sizable effects can stem from this running. From the above it is obvious that any estimate of \((m_{ij}^2)_{i\neq j}\) would require a complete knowledge of the neutrino Yukawa matrix \(Y_{\nu ij}\) which is not fixed by the see–saw equation, even with an improved knowledge of the neutrino oscillation parameters, as in (1) there is a mismatch between the number of unknowns and that of low energy observables. This could indeed pose a problem compared to the quark–squark FCNCs sector, where it is possible to make firm predictions of FV entries due to RG evolution in a flavour universal boundary condition. On the other hand, in the quark sector the disentangling of FV effects stemming from SUSY from those coming from the SM is more problematic, as both are driven by the CKM mixing matrix and happen to be roughly of the same order of magnitude. In the charged lepton sector we have the opposite scenario: SM contributions are well below any envisageable experimental sensitivity, but it is not possible to predict SUSY induced FCNC rates without resorting to an ansatz with regard to the form of the Yukawa matrix \(Y_{\nu i}\) or the general framework of the theory. Many groups [24] have addressed this issue in different frameworks.

In this paper we inspect LFV in a \(SO(10)\) supersymmetric grand unified (SUSY–GUT) framework. The choice of the GUT scenario stems from the fact that the possible detection of SUSY particles at the LHC will provide an indirect evidence for such a scenario. Moreover in \(SO(10)\) theories the see–saw mechanism is naturally present and the neutrino Yukawa couplings are related to those of the up quarks, making them naturally large [24], so that sizable LFV entries will stem from the (2) RG evolution. Even if the \(SO(10)\) framework gives us some hints about the unknown neutrino Yukawa matrix \(Y_{\nu i}\), telling us that the eigenvalues are related to the ones of the up Yukawa matrix \(Y_u\), it still leaves uncertainty about the size of mixing angles in \(Y_{\nu i}\), as the knowledge of the low–energy neutrino parameters (masses and mixings) is not sufficient to set the matrices that diagonalize \(Y_{\nu i}\). Following the scheme of previous works [24–26], we bypass the ignorance about the mixings by considering two extremal benchmark cases. Such cases are intended as boundary conditions at high scale. As a minimal mixing case we take the one in which the neutrino and the up Yukawa unify at the high scale, so that the mixing is given by the CKM matrix; this case is named ‘CKM–case’. As a maximal mixing scenario we take the one in which the observed neutrino mixing is coming entirely from the neutrino Yukawa matrix, so that \(Y_{\nu i} = U_{PMNS} Y_{\nu i}^{\text{diag}}\), where

\(U_{PMNS}\) is the Pontecorvo–Maki–Nakagawa–Sakata matrix; in this case the unknown \(U_{\nu 3}\) PMNS matrix element turns out to be crucial in evaluating the size of LFV effects. The maximal case is named ‘PMNS–case’.

The aim of the present work is to study, in a minimal supergravity (mSUGRA) scenario \(^1\), the influence on LFV from the RG running both above and below the unification scale \(M_{GUT}\). A preliminary version of this analysis is already present in the literature [26]. However that work considered the LFV contributions from the see–saw structure only. It is well known that in a \(SO(10)\) scenario there are other contribution stemming from the grand unified structure [27], as on general grounds the SUSY breaking mediation scale and the GUT scale do not coincide. In the present work we detail the various contributions and their relative relevance as source of LFV in the \(SO(10)\) framework. We analyse in detail the impact of LFV experiments in the parameter space region that will be probed by the LHC, both in the minimal (CKM–like) and in the maximal (PMNS–like) cases and for different values of \(\tan \beta\). Such a \textit{vis–a–vis} analysis allows us to address the issue of the complementarity between LHC and LFV experiments as probes of SUSY–GUTs.

We argue that, even in presence of a discovery machine like the LHC, flavour physics experiments will still play an important role in the hunt for new physics. Indeed, LHC evidences alone will hardly discriminate among the many possible SUSY realizations, while flavour physics should be, in this sense, more sensitive. Moreover, several flavour physics experiments are currently running or under construction (such as B–factories, the SuperKEKB upgrade [10] and the upcoming MEG [11] experiment at PSI), and thus some hints of new physics before the LHC era are also possible. Furthermore, there are discussions and plans on very sensitive LFV experiments beyond the LHC era, such as the PRISM/PRIME experiment at J–PARC [17] and a Super Flavour factory [19]. It is thus timely to ask what will be the capability of such experiments to discriminate between different SUSY–GUT realizations, in the case that the LHC gets a positive evidence for SUSY. Let us note that, even in the case that nothing is seen at the LHC, taking into account that SUSY effects decouple slowly with increasing SUSY masses, flavour physics could still exhibit some indirect SUSY evidence.

The main results of our analysis are:

- The maximal PMNS case is already ruled out by the current MEGA bound on \(\mu \rightarrow e \gamma\) in the case the squark masses are lighter than 1.5 TeV. MEG will improve the situation by testing it well beyond the reach of LHC sensitivity.

\(^1\) We are taking the soft trilinear mass scale \(A_0\) as a free parameter, not linked to the Higgs sector \(B\) parameter as in a strict mSUGRA scenario.
• If the unknown \( U_{e3} \) angle is very small, at present the PMNS case is constrained only in the high \( \tan \beta \) region, by \( B \)-factories \( BR(\tau \rightarrow \mu \gamma) \) bounds, to lay in the region of squark masses bigger than 800 GeV. In the future, MEG will be able to test this scenario in all the LHC accessible SUSY parameter space if \( \tan \beta \) is high. For small \( \tan \beta \) the best probe will come from SuperKEKB \( \tau \rightarrow \mu \gamma \) BR bounds, testing it for squark masses up to 700 GeV.

• The minimal CKM case is at present unconstrained. MEG will be able to test it in the high \( \tan \beta \) region and for squark masses lighter than 800 GeV; this scenario will evade detection by SuperKEKB. The low \( \tan \beta \) minimal mixing case will remain unconstrained.

• The proposed post–LHC era PRISM/PRIME and Super Flavour factory experiments will much improve the situation. PRISM/PRIME would supersede MEG by testing, by mean of \( \mu \rightarrow e \) conversion in Ti, all the scenarios in all the LHC accessible SUSY parameter space. A Super Flavour factory would be highly complementary, being able to detect the LFV \( \tau \rightarrow \mu \gamma \) process up to 1 TeV squark masses.

The paper is organized as follows: in section II we motivate, in the context of SUSY–GUTs, our \( Y_\nu \sim Y_\tau \) ansatz; in section III we proceed to estimate the LFV RG induced soft masses and the branching ratios that stem from them. The numerical routine is presented in section IV and the results are discussed in section V: we evaluate LFV rates for the parameter space region within the reach of the LHC and comment on the complementarity between direct SUSY searches and LFV experiments. In section VI we predict LFV rates at the SPS benchmark points in our SUSY–GUT frameworks, and argue on the possibility of future experiments to test these scenarios. In section VII we give a summary of our findings and draw the conclusions. Last, in the appendix we fix the notation and give the \( SU(5)_{RN} \) RG equations.

II. SEE–SAW AND SUSY \( SO(10) \)

An \( SO(10) \) SUSY–GUT framework naturally incorporates the see–saw mechanism. This is because the matter representation is a 16-dimensional spinor containing right handed neutrinos which are absent by choice in the Standard Model spectrum. Further models of \( SO(10) \) have two salient features which make them interesting to study SUSY see–saw:

(i) Firstly, they unify the Dirac neutrino Yukawa couplings \( (Y_\nu) \) and the up-type Yukawa couplings \( (Y_u) \). Though this unification is exact for smaller Higgs representations, like the \( 10s \), it can be shown that even in the presence of larger representations like \( 120 \) or \( 126 \), at least one of the neutrino Yukawa couplings is as large as the top Yukawa. This can be shown by a simple analysis of the resultant mass matrices [24].

(ii) Secondly, as mentioned in the introduction, unlike the quark sector, the leptonic sector has the see–saw mechanism that makes it distinct. Particularity, the observed large neutrino mixing doesn’t necessarily mean a large ‘left’ mixing to be present in the neutrino Dirac Yukawa couplings: in fact even CKM-like small mixings in the neutrino Yukawa couplings can lead to large neutrino mixing [28]. This is best depicted by the Casas–Ibarra parametrization [29] that solves the see–saw equation (1) for the neutrino Yukawa matrix \( Y_\nu \)

\[
Y_\nu = \frac{1}{(H_u)}U_{PMNS}D_\nu R D_N
\]

where \( D_N \) and \( D_\nu \) are the square root of the diagonal right handed Majorana and the low energy neutrino mass matrices respectively. The unknown complex orthogonal matrix \( R \) parametrizes the uncertainty of the mixing between Majorana and Dirac right handed eigenstates. This means that if \( R \) is the identity the Dirac neutrino Yukawa matrix inherits the PMNS mixing structure, whereas small CKM–like \( Y_\nu \) mixings reflect in a non trivial structure of the misalignment matrix \( R \). With this in mind, most flavour models have either of the two situations of (a) small left mixing in \( Y_\nu \) or (b) large left mixing in \( Y_\nu \) (which can also be understood as mixing from the charged lepton sector). In an \( SO(10) \) GUT, where there is a unification of \( Y_\tau \) and \( Y_\nu \), both these situations can be realised by choosing appropriate Higgs representations. We have christened these two cases as the CKM-case and the PMNS case respectively. These two cases have the following relations between the Yukawas, in the basis where charged lepton and down quark mass matrices are diagonal

\[
Y_\nu = Y_\tau \quad (\text{CKM case}) \\
Y_\nu = U_{PMNS} Y_u^{\text{diag}} \quad (\text{PMNS case}).
\]

Both the above situations can be realised in \( SO(10) \) without spoiling the relation between neutrino Yukawa and the top Yukawa. The CKM case can be realised by a simple superpotential involving only ten-plets [30]

\[
W_{SO(10)} = (Y_u)_{ij} 16,16,10 + (Y_d)_{ij} 16,16,10d + (Y_R)_{ij} 16,16,126
\]

where \( i \) and \( j \) are generation indices. The PMNS case is a bit more complicated as it can come from either a renormalisable or non-renormalisable couplings. For example, Chang, Masiero and Murayama [31] have proposed the following superpotential

\[
W_{SO(10)} = (Y_u)_{ij} 16,16,10 + (Y_d)_{ij} 16,16,16 \frac{45}{M_{Planck}} 10 + (Y_R)_{ij} 16,16,126
\]

which leads to the PMNS like situation. Note that both the superpotentials we have mentioned here are just scenarios but not complete fermion mass models. For our
purposes, these two scenarios serve as the benchmark points in the see-saw parameter space.

The LFV soft mass entries are generated in the RG evolution from the universality scale down to the SUSY decoupling scale \( M_{\text{SUSY}} = 1 \, \text{TeV} \). The breaking of SUSY \( SO(10) \) down to the SM can be achieved in several different ways [32]. Within the two benchmark scenarios chosen above, we envisage a breaking chain (Fig. 1) of \( SO(10) \rightarrow SU(5)_{\text{RN}} \rightarrow MSSM_{\text{RN}} \rightarrow MSSM \). Such a breaking can be achieved if the singlet under \( SU(5) \) component of a 16 or of a 126 attains a VEV. The scale of \( SU(5)_{\text{RN}} \) is taken to be the scale of the gauge coupling unification \( M_{\text{GUT}} \sim 2 \times 10^{16} \, \text{GeV} \). The \( SO(10) \) scale is considered to be slightly higher about \( M_X \sim 10^{17} \). This scale can be considered to be the string unification scale, \( M_{\text{string}} \), which roughly turns out to be a factor 20–25 from the gauge coupling unification scale after considering string loop effects [33]. One interesting aspect is that, while we set the scale of the right handed neutrinos from low energy neutrino data and \( Y_{\nu} \) (as we will detail in section IV), it turns out that the required right handed neutrino masses are close to the GUT scale, which fits our scheme naturally.

\[
\begin{array}{cccc|c}
M_{\text{Planck}} & M_X & M_{\text{GUT}} & M_{\text{RS}} & M_{\text{SUSY}} \\
\hline
SO(10) & SU(5)_{\text{RN}} & MSSM_{\text{RN}} & MSSM & SM
\end{array}
\]

FIG. 1: Schematic picture of the energy scales involved in the model.

Before proceeding into the next section where we detail the various lepton flavour violating terms generated in these two schemes, we would like to make some comments on the recent progress in \( SO(10) \) model building. In the recent years a new view regarding \( SO(10) \) model building is being developed, where construction of more realistic and complete models is being pursued [34]. In these ‘minimal’ complete models, it is perhaps for the first time possible to compute the entire \( SO(10) \) spectrum, study realistically precision observables such as running of fermion mass spectrum including threshold effects, gauge coupling running, proton decay, etc [35]. In the present work, we are more concerned with the effect of \( SO(10) \) see-saw couplings on the soft supersymmetry breaking sector of the theory. We do not resort to a complete model building of \( SO(10) \), but just consider schemes of \( SO(10) \). This is sufficient for our purposes, as we aim to compute the flavour violating entry in the slepton and sneutrino mass matrices at the weak scale generated through the see-saw mechanism within both these schemes. We use 1-loop RGE equations for this purpose and perform scatter plots in the SUSY breaking parameter space.

### III. LFV SOURCES IN SUSY–SO(10)

In the present section, we elaborate on the various contributions to the lepton flavour violating entries in the \( SO(10) \) SUSY–GUT framework. Perhaps the best way to understand them is in terms of the low-energy parameters. We use the so-called Mass Insertion (MI) [36] notation to denote the various flavour violating entries of the slepton mass matrix. These flavour violating entries are zero at the high scale, where SUSY breaking soft scalar masses are universal. At the weak scale, the universality is broken by the RG evolution and the \( 6 \times 6 \) slepton squared-masses matrix \( M_{\tilde{e}}^2 \) takes the form

\[
M_{\tilde{e}}^2 = \begin{pmatrix}
m_\tilde{e}_e^2 (1 + \delta_{LL}) + Y_{\nu} Y_{\nu}^\dagger \tilde{\mu}^2 + O(g^2) \\
v_d (A_e - Y_{\nu} \mu \tan \beta) + \delta_{RL} \tilde{\mu}^2 \\
m_\tilde{e}_\mu^2 (1 + \delta_{RR}) + Y_{\nu} Y_{\nu}^\dagger v_d^2 + O(g^2)
\end{pmatrix},
\]

where the flavour violation is coded in the \( \delta s \) given by

\[
\delta_{ij} = \frac{\Delta_{ij}}{m_{\tilde{e}_i}^2}
\]

with \( m_{\tilde{e}_i}^2 \) being the geometric mean of the slepton squared masses [37] and \( \Delta_{i\neq j} \) flavour non-diagonal entries of the slepton mass matrix generated at the weak scale by RG evolution. The mass insertions are divided in to the LL/LR/RL/RR types, according to the chirality of the corresponding SM fermions. Detailed bounds on each of these types of \( \delta s \) already exist in the literature [38, 39]. Note that these bounds are obtained by considering one \( \delta \) at time to be the source of the flavour violating effects, and putting all the other \( \delta s \) to zero.

In our case, these \( \Delta_{ij} \) are generated by RG evolution either through the see-saw mechanism or through the GUT evolution. This means that there exist several \( \delta s \) at the same time, so that the interplay between them should be evaluated. However, as an illustration, we compare these resultant \( \Delta s \) generated by RGEs with the existing MI bounds, conveying us the power of each individual contribution at the weak scale in constraining the SUSY breaking parameter space. We further elaborate the cases where double mass-insertions could be impor-
tantly compared to the single mass insertions.

![Graph](image)

**FIG. 2:** Points in the \((M_{1/2}, \delta_{12})_{LL}\) and \((M_{1/2}, \delta_{12})_{RR}\) planes that fulfill the \(\mu \rightarrow e\gamma\) branching ratio experimental limit. The plots are for \(\tan \beta = 10\) and \(m_0 = 500\) GeV.

For the discussion of this section we choose a point in the SUSY parameter space: \(m_0 = 500\) GeV; \(M_{1/2} = 500\) GeV; \(A_0 = 0\); \(\tan \beta = 10\) (some comments will be made also on an high tan \(\beta\) scenario, with \(t_{3/2} = 40\)). The SUSY spectrum for this point is given in Table II. As can be seen from Fig. 2 there is no fine-tuned cancellation in LFV amplitudes at this point, so that we can take it as a ‘safe’ benchmark point.

By turning on a \(\delta\) at time, we get the present and future bounds on each single LFV \(\delta\), as given in Table III. From the table and from Fig. 2 it is clear that the RR \(\delta_s\) are less constrained than the LL ones by the LFV branching ratio bounds; this is due to two reasons: (i) the amplitudes involving only \(\delta_{RR}\) MI do not have chargino contributions and (ii) in certain regions of the parameter space, there could be cancellations between the bino and the higgsino–bino–higgsino contributions [39]. It is also clear that the LR entries are much suppressed. This is mainly due to the fact that, as can be seen by comparing Eq. (8) and (9), in the normalization procedure (9) the \((\delta_{LR})_{12}\) entry pays a factor \(M_{\tilde{t}}\), where \(m_i\) is the mass of the \(i\)-th lepton.

### A. LL insertions from the running

To compute the \(\Delta\)’s from the RGEs, in this section we use the leading log approximation. Taking the soft masses to be flavour universal at the input scale, off diagonal entries in the LL sector are generated by right handed neutrinos running in the loops; in our framework where \(Y_{\nu_3} = Y_1\) we can estimate

\[
(\Delta_{LL})_{i\neq j} = -\frac{3m_0^2 + A_0^2}{16\pi^2} Y_1^2 V_{i3} V_{j3} \ln \left(\frac{M_X^2}{M_{R_3}^2}\right)
\]

where \(V\) can be either \(V_{CKM}^T\) or \(U_{PMNS}\), depending on the case.

#### 1. CKM case

To use the leading log expression (10) we need to know the mass of the heaviest right handed neutrino. By using the see–saw formula (1) we can estimate it to be in the CKM case [26]:

\[
M_{R_3} \approx \frac{m_{\nu_1}^2}{4 m_{\nu_1}};
\]

taking \(m_{\nu_1} \approx \text{10}^{-3}\) eV we get \(M_{R_3} \approx 10^{15} - 10^{16}\) GeV.

The induced off–diagonal entries relevant to \(\ell_i \rightarrow \ell_j, \gamma\) are of the order of (putting \(A_0\) to zero)

\[
(\delta_{LL})_{\mu e} = -\frac{3}{8\pi^2} Y_1^2 V_{i3} V_{j3} \ln \left(\frac{M_X}{M_{R_3}}\right)
\]
\[
(\delta_{LL})_{\tau \mu} = -\frac{3}{8\pi^2} Y_1^2 V_{i3} V_{j3} \ln \left(\frac{M_X}{M_{R_3}}\right)
\]
\[
(\delta_{LL})_{\tau e} = -\frac{3}{8\pi^2} Y_1^2 V_{i3} V_{j3} \ln \left(\frac{M_X}{M_{R_3}}\right)
\]

In these expressions, the CKM angles are small but one would expect the presence of the large top Yukawa coupling \(Y_1\) to compensate such a suppression, giving rise to sizable \(\delta s\). We see from Table IV that all the \(\delta_s\) will be outside the reach of planned experiments. Let us note that the \(\mu e\) sector entry is almost at the boundary of MEG sensitivity: from Fig. 2 it is clear that MEG will test it for \(M_{1/2} \lesssim 250\) GeV.

### Table II: Masses (in GeV) of the lightest SUSY particles

| Mass | \(\tan \beta = 10\) | \(\tan \beta = 40\) |
|------|----------------|----------------|
| \(m_{\ell_1}\) | 574 | 447 |
| \(m_{\ell_2}\) | 845 | 838 |
| \(m_{\ell_3}\) | 1225 | 1225 |
| \(m_{\chi_1^0}\) | 234 | 234 |
| \(m_{\chi_1^\pm}\) | 431 | 432 |
| \(m_h\) | 123 | 124 |
TABLE III: Bounds on the $\delta$s from the present and future $BR(\ell_i \to \ell_j \gamma)$ experimental limits; by unstrained we mean that the $\delta$ is $O(1)$. In the $\tau\mu$ sector the future sensitivity is given both for SuperKEKB and for the proposed Super Flavour factory. The $\delta$s are calculated at $\tan \beta = 10$, $m_0 = 500$ GeV and $M_{1/2} = 500$ GeV; as can be seen from Fig. 2 no particular cancellation is occurring at this point.

| Process | Present | Future | Present | Future |
|---------|---------|--------|---------|--------|
| $\mu \to e \gamma$ | $1.4 \cdot 10^{-3}$ | $8.5 \cdot 10^{-5}$ | $1.4 \cdot 10^{-2}$ | $9 \cdot 10^{-4}$ |
| $\tau \to \mu \gamma$ | $2.4 \cdot 10^{-1}$ | $1.3 \cdot 10^{-1} / 3 \cdot 10^{-2}$ | uncon. | uncon. / $2.9 \cdot 10^{-1}$ |
| $\tau \to e \gamma$ | $4.7 \cdot 10^{-1}$ | $1.4 \cdot 10^{-1}$ | uncon. | $5.9 \cdot 10^{-2}$ |

TABLE IV: CKM case: leading log estimates of off-diagonal entries in the slepton mass matrices. The bounds are calculated at $\tan \beta = 10$, $m_0 = 500$ GeV and $M_{1/2} = 500$ GeV. For the $\tau\mu$ sector we give the sensitivity for both SuperKEKB and a Super Flavour factory.

| gen. | $|\delta_{LL}|$ | Present bound | Future sensitivity |
|------|----------------|---------------|--------------------|
| $\mu e$ | $3.4 \cdot 10^{-5}$ | $1.4 \cdot 10^{-3}$ | $8.5 \cdot 10^{-5}$ |
| $\tau\mu$ | $6.2 \cdot 10^{-3}$ | $2.4 \cdot 10^{-1}$ | $1.3 \cdot 10^{-1} / 3.0 \cdot 10^{-2}$ |
| $\tau e$ | $8.5 \cdot 10^{-4}$ | $4.7 \cdot 10^{-1}$ | $1.4 \cdot 10^{-1}$ |

2. PMNS case

In the PMNS case the $R$ matrix is the identity; the see–saw formula (1) can be straightforwardly inverted to get

$$\tilde{M}_R = \text{diag} \left( \frac{m_{\nu_1}^2}{m_{\nu_2}}, \frac{m_{\nu_2}^2}{m_{\nu_3}}, \frac{m_{\nu_3}^2}{m_{\nu_1}} \right).$$

(13)

Taking the neutrino spectrum to be hierarchical so that $m_{\nu_3} \approx \sqrt{\Delta m_{\text{atm}}^2}$ we can estimate the third right handed neutrino to have mass $M_{R_3} \sim 10^{14}$. Plugging the value in the equation (10)

$$\begin{align*}
(\delta_{LL})_{\mu e} &= -\frac{3}{8\pi^2} Y_{\tau e}^2 U_{e3}^* U_{\mu3} \ln \frac{M_X}{M_{R_3}} \\
(\delta_{LL})_{\tau \mu} &= -\frac{3}{8\pi^2} Y_{\tau e}^2 U_{e3} U_{\tau3} \ln \frac{M_X}{M_{R_3}} \\
(\delta_{LL})_{\tau e} &= -\frac{3}{8\pi^2} Y_{\tau e}^2 U_{e3} U_{\tau3} \ln \frac{M_X}{M_{R_3}}
\end{align*}$$

(14)

and taking $U_{e3} = 0.07$ at about half of the current CHOOZ bound we get the estimates in Table V. We see from the table that the $\mu e$ sector is already ruled out by the present bound and that the upcoming bound will be able to test it up to

$$U_{e3} = 0.07 \cdot 8.5 \cdot 10^{-5} / 1.4 \cdot 10^{-3} \sim 10^{-3}.$$  

(15)

Moreover the $\tau\mu$ sector will be probed by the SuperKEKB machine and thoroughly tested by the proposed Super Flavour factory.

B. LR/RL insertions from the running

The flavour violating terms in the LR sector are given by the off–diagonal terms of the slepton’ soft trilinear $(A_e)_{ij}$; the RG generated entries in (8) are

$$\begin{align*}
(\Delta_{LR})_{i \neq j} &= (H_\mu)^i_j [(A_e)_{ij} (M_X \to M_{\text{GUT}}) \\
&+ (A_e)_{ij} (M_{\text{GUT}} \to M_R)] \\
&= \frac{3 m_i A_0}{32 \pi^2} \sum_k Y_{\nu i} Y_{\nu j}^* \ln \left( \frac{M_X^2}{M_{R k}^2} \right) \\
&- \frac{9 m_i A_0}{32 \pi^2} \sum_k Y_{\nu i} Y_{\nu j}^* \ln \left( \frac{M_X^2}{M_{\text{GUT}}^2} \right)
\end{align*}$$

(16)

where $m_i$ is the mass of the $i$-th lepton and the last line is coming from the fact that the left handed sleptons and the $d^c$ squarks are hosted together in the $5$ of $SU(5)$. Taking into account only the third generation order one Yukawa coupling we have

$$\begin{align*}
(\Delta_{LR})_{ij} &= -\frac{3 A_0}{32 \pi^2} \left[ m_i Y_{\nu i} Y_{\nu j}^* \ln \left( \frac{M_X^2}{M_{R_3}^2} \right) \\
&+ 3 m_j Y_{\nu i} Y_{\nu j}^* \ln \left( \frac{M_X^2}{M_{\text{GUT}}^2} \right) \right]
\end{align*}$$

(17)

so that the see–saw driven contribution and the GUT driven one give the dominant contribution to different (transposed) entries. Let us note that these entries are roughly equal, as the color factor 3 is almost compensated by the longer running of the see–saw driven entries. As a consequence, the remarks we are going to do about the
$i j$ entry will at the same time apply to the transposed $j i$ one. Doing a comparison with (10) we have

$$\langle \Delta LL \rangle_{ij} = \frac{3m_i A_0}{3 m_0^2 + A_0^2} \langle \Delta LL \rangle_{ij}$$

switching to the adimensional $\delta$ we get

$$|\langle \delta LL \rangle_{ij}| = \frac{3[a_0]}{3 + a_0^2} m_i m_0 |\langle \delta LL \rangle_{ij}| < \frac{m_i}{m_0} |\langle \delta LL \rangle_{ij}|,$$

where $a_0 = A_0/m_0$. A crucial point is that the RG generated LR insertion (19) is not the main contribution to the LR flavour violating insertion [40]. Indeed it is possible to build an effective LR flavour violating insertion by combining together the LL RG generated flavour violating entry (10) with a LR flavour conserving $m_i \mu \tan \beta$ chirality flip

$$\langle \delta LL \rangle_{ij}^{\text{eff}} = \frac{m_i \mu \tan \beta}{m_i^2} \langle \delta LL \rangle_{ij}.$$  

Comparing Eq. (19) to (20)

$$|\langle \delta LL \rangle_{ij}| / |\langle \delta LL \rangle_{ij}^{\text{eff}}| < \frac{\hat{m}_i^2}{m_0 |\mu| \tan \beta} \approx (\tan \beta)^{-1}$$

we see that the effective LR insertion stemming from the RG generated LL one is enhanced by a factor $\tan \beta$ with respect to the RG generated pure LR insertion, so that the effective insertion always dominates.

C. RR insertions from the running

The $SU(5)_{\text{RN}}$ running from the soft breaking scale $M_X$ to the GUT scale already breaks the universality of the soft spectrum by generating LFV entries at $M_{\text{GUT}}$. The renormalization group equation for $SU(5)_{\text{RN}}$ are calculated at the 1-loop level in the Appendix; the most interesting consequence of the $SU(5)_{\text{RN}}$ running is that, as both $Q$ and $e^\nu$ are hosted in the 10, the CKM matrix mixing the left handed quarks will give rise to off diagonal entries in the running of the right handed slepton soft masses [27, 41–44]

$$\langle \Delta RR \rangle_{i \neq j} = \frac{3 m_0^2 + a_0^2}{16 \pi^2} V_{ik} V_{j} \ln \left( \frac{M_X^2}{M_{\text{GUT}}^2} \right),$$

where we have used the fact that $Y_i \approx 1$ and $V_{ik}$, $k = i, j$ is the $tk$ entry of the CKM matrix. Let us note that this effect is due to the GUT structure, and is so independent on any ansatz on the form of the neutrino Yukawa matrix: in this sense the $\delta_{RR}$ and the BRs stemming from them form a guaranteed minimum on the LFV effects from a SUSY–GUT.

In Table VI we present a comparison of all the $\delta$’s, other than the LR ones, for the two cases of minimal and maximal mixing. We see that in the PMNS case the main source of LFV violation are the LL insertions, whereas in the CKM case the $SU(5)_{\text{RN}}$ running gives rise to a sizable right slepton off-diagonal mass

$$(m_{\tilde{l}}^2)_{i \neq j} \approx 2 (m_{\tilde{l}}^2)_{i = j},$$

which could give a significant contribution to LFV amplitudes. However, as we mentioned at the beginning of this section, the $\delta_{RR}$ contribution to the branching ratio is suppressed by the cancellations in the neutralino sector. In Fig. 3 we point out that the full BR in the CKM case is of the same order of magnitude of the one calculated by taking into account the LL entries only; on the other hand, the ratio between the $\delta_{LL}$ and the $\delta_{RR}$ generated BRs is more than a factor 10.

![Contributions to $\text{BR}(\mu \rightarrow e \gamma)$](image)

FIG. 3: Comparison of the $\mu \rightarrow e \gamma$ BRs occurring from a full CKM case with those from just the LL entries, the GUT generated RR entries and the double mass insertions. The plots are done at $\tan \beta = 40$, $m_0 = 500$ GeV and varying $M_{1/2}$ between 200 and 650 GeV. Details about the numerical procedure will be given in the next section.

Following these results we can safely neglect the RR contributions and estimate the branching ratios according to the formula [45] (note that this already incorporates the effective LR insertion)

$$\text{BR}(l_i \rightarrow l_j \gamma) = \frac{3}{G_F} \alpha \frac{(\delta_{LL})_{ij}^2}{m_{\tilde{\nu}}^4} \tan^2 \beta \approx 4.5 \cdot 10^{-6} \left( \frac{500 \text{GeV}}{m_{\text{SUSY}}} \right)^4 (\delta_{LL})_{ij}^2 \left( \frac{t_\beta}{10} \right)^2$$

| gen. | $|\delta_{LL}|$ | $|\delta_{LL}|$ | $|\delta_{RR}|$ |
|------|---------------|---------------|---------------|
| $\mu e$ | $3.4 \cdot 10^{-5}$ | $1.2 \cdot 10^{-2}$ | $7.8 \cdot 10^{-5}$ |
| $\tau \mu$ | $6.2 \cdot 10^{-3}$ | $1.2 \cdot 10^{-1}$ | $1.4 \cdot 10^{-2}$ |
| $\tau e$ | $8.8 \cdot 10^{-4}$ | $1.2 \cdot 10^{-2}$ | $2 \cdot 10^{-3}$ |
TABLE VII: Estimates of the branching ratios versus present bounds and future sensitivities. $m_0$ and $M_{1/2}$ are taken to be 500 GeV.

| gen. $\tan \beta$ | 40 CKM | $t_\beta$ = 10 MNS Exp. bound | Fut. sensit. |
|-------------------|---------|-------------------------------|-------------|
| $\mu$             | $6 \times 10^{-14}$ | $5 \times 10^{-10}$ | $1.2 \times 10^{-11}$ | $10^{-13}$ | $10^{-14}$ |
| $\tau \mu$        | $2 \times 10^{-9}$ | $5 \times 10^{-8}$ | $6.8 \times 10^{-8}$ | $10^{-8}$ |
| $\tau e$          | $4 \times 10^{-11}$ | $5 \times 10^{-10}$ | $3.1 \times 10^{-7}$ | $10^{-8}$ |

where $m_{SUSY}$ is linked to the high energy inputs parameters $m_0$ and $M_{1/2}$ by the best fit relation [46]

$$m_{SUSY}^4 = 0.5 \left( \frac{M_{1/2}^2}{m_0^2} \right) \left( m_0^2 + 0.6 M_{1/2}^2 \right)^2 .$$ (25)

The estimated $BR(\ell_i \rightarrow \ell_j \gamma)$ are given in Table VII. We see that for $m_0$ and $M_{1/2}$ at 500 GeV the PMNS case is already ruled out even at small $\tan \beta$ by the $\mu \rightarrow e \gamma$ branching ratio, so that we expect the upcoming MEG experiment to be able to test it even for soft masses as big as 5 TeV. On the other hand, we expect that the MEG experiment will be able to test the small mixing angle case only for high values of $\tan \beta$. As for the $\tau$ sector, we see that the only channel that offers interesting rates is the $U_{e3}$ independent $\tau \rightarrow \mu \gamma$ process: the SuperKEKB bound of $10^{-8}$ will be able to test the PMNS case even in the small $\tan \beta$ region, whereas a Super Flavour factory, with a planned sensitivity of at least $O(10^{-9})$, is expected to address even the issue of small mixing angles, provided that $\tan \beta$ is large.

It is interesting to note from Fig. 3 that the subleading contribution to the $\mu \rightarrow e \gamma$ process is not arising from a pure $(\delta_{RR})_{\mu \mu}$ insertion but from the double FV mass insertion $(\delta_{LL})_{e \tau} (\delta_{RR})_{\tau \mu} + (e \rightarrow \mu)$: this is an effective LR MI, which is enhanced by the flavour conserving $m_{\tau \mu} \tan \beta$ contribution. This allows us to give a rough estimate for the subleading contribution to the $BR$ to be

$$BR(\mu \rightarrow e \gamma)_{2\delta} = \left( \frac{h_\tau}{h_\mu} \right)^2 (\delta_{RR})_{\mu \tau}^2 BR(\tau \rightarrow \mu \gamma),$$ (26)

where the suffix $2\delta$ represents the 2 flavour violating effective MI.

IV. NUMERICAL ANALYSIS OF LFV PROCESSES

A. Parameter space of SUSY–GUTs

As mentioned above, we consider mSUGRA boundary conditions for the soft masses. At the high scale, the parameters of the model are the universal scalar mass $m_0$, universal trilinear couplings $A_0$, unified gaugino masses $M_{1/2}$. In addition to these there are the two Higgs potential parameters $\mu$ and $B$ and the undetermined ratio of the Higgs VEVs, $\tan \beta$. The entire supersymmetric mass spectrum is determined once these parameters are given. However, all these parameters are not independent. Incorporating electroweak symmetry breaking gives rise to two conditions, reducing the number of independent parameters by two. In our case, we determine $\mu$ and $B$ by electroweak symmetry breaking conditions. The two conditions of the electroweak symmetry breaking are

$$|\mu|^2 = \frac{m_{H_u}^2 - m_{H_d}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} m_Z^2 ,$$
$$\sin 2\beta = \frac{2 B |\mu|}{m_{H_u}^2 + m_{H_d}^2 + 2 |\mu|^2} ,$$ (27)

where $m_{H_u}^2$ and $m_{H_d}^2$ are the up and down type Higgs soft mass squared parameters determined at the weak scale, using the RG equations from the $M_X$ scale to the weak scale. At the weak scale, all the supersymmetric soft parameters are thus known, enabling us to compute the complete supersymmetric mass spectrum.

We impose two main ‘theoretical’ constraints on the SUSY parameter space: (a) Radiative ElectroWeak Symmetry Breaking (REWSB) [47] should take place. (b) No tachyonic particles and that the Lightest Supersymmetric Particle (LSP) should be a neutralino. The experimental constraints are detailed in the next subsection. In contrast to the MSSM, both these constraints are significantly modified in the $SO(10)$ framework we discuss in the present paper. As it is well known, in the MSSM, radiative electroweak symmetry breaking is driven by the top Yukawa coupling. In the $SO(10)$ model we are considering, two further effects are present: (i) the range of the logarithmic running is a bit larger as $M_X$ is taken to be $\sim 5 \times 10^{17}$ compared to typical MSSM studies, which consider the scale to be $\sim 2 \times 10^{16}$; (ii) the neutrino Yukawa couplings $Y_\nu$, one of which is necessarily as large as the top Yukawa coupling, also contribute to driving the up–type Higgs soft mass squared $m_H^2$ negative in the running from the scales $M_X$ to $M_{R_3}$. These two contributions can significantly alter the parameter space which is viable under the electroweak breaking constraint.

A similar effect takes place for the region of the parameter space in which the lightest slepton, which is typically the right-handed stau $\tilde{\tau}_R$, is the LSP. In contrast to MSSM, in the GUT framework, the stau also receives corrections from the ‘pre–GUT scale’ running from the scale $M_X$ to the $M_{GUT}$. In $SU(5)$, as we consider in the present scenario, the stau sits in the ten-plet 10 which also hosts the strongly interacting sector, leading to ‘strong’ contributions through the gaugino loops. A leading log estimate of these contributions for $m_0 \approx 0$

\footnote{We do not impose any additional constraints requiring unification of the Yukawa couplings in the present work.}
is given by

$$m_{\tilde{t}}^2(M_{\text{GUT}}) = \frac{96}{80\pi^2} M_{1/2}^2 \ln \left( \frac{M_X}{M_{\text{GUT}}} \right) \approx 0.4 M_{1/2}^2 .$$

(28)

These positive contributions can thus off-set the negative contributions from the Yukawa running. Both these effects are best demonstrated in the Fig. 4, where the difference in the allowed parameter space of the MSSM and of the $SO(10)$ framework is evident.

![MSSM and SUSY-GUT parameter spaces](image)

FIG. 4: Comparison of MSSM and SUSY-GUT parameter spaces. The colored areas are ruled out: green one correspond to points where the vacuum is not viable (either because of no REWSB or tachyonic particles) The plots are for tan $\beta = 30$, $A_0 = 0$ and $m_t = 173$ GeV.

Finally, as mentioned in the introduction, we would like to do a complementarity study between the region of the parameter space probed at the LHC vis-a-vis the LFV experiments. For this, we first need to determine the region of the parameter space probed by the LHC considering various detection channels, putting the appropriate background cuts, detector response functions etc. We do not intend to do such detailed analysis in this work. For mSUGRA, it is already present in the literature [48]. The typical estimate for the mass of the gluino and squarks to be detected at the LHC is about 2-3 TeV. We define the parameter space region that allows a squark mass to be below 2.5 TeV to be the region probed by the LHC. In Fig. 4 the contours for the masses are shown in the $(m_0,M_{1/2})$ plane. We call this region the LHC accessible region. However, we also further consider other regions of the parameter space which, though not accessible at the LHC, can be relevant for the reach of flavour physics experiments. With this in mind, we scan the total parameter space in the following ranges

$$m_0 \in (0,5000) \text{ GeV}$$
$$M_{1/2} \in (0,1500) \text{ GeV}$$
$$A_0 \in (-3 \cdot m_0, +3 \cdot m_0)$$
$$\tan \beta = 10, 40$$
$$\text{sign} \mu \in \{+,-\}$$

B. Integration procedure

![Diagram of the running routine](image)

FIG. 5: Pictorial explanation of the running routine. See the text for the details.

In the present section, we detail the integration procedure we have incorporated in our work. A schematic diagram is presented in Fig. 5. As inputs at the weak scale, we consider the Yukawa couplings of the up-type quarks, down-type quarks, charged leptons, the CKM mixing matrix and $\tan \beta$. We employ a hierarchical scheme for the neutrino masses. The lightest neutrino is taken to be around $10^{-3}$ eV. The other two neutrino masses are determined by the square-roots of the solar and atmospheric mass squared differences respectively. The leptonic mixing matrix $U_{PMNS}$ has two large mixing angles, and the unknown third mixing angle $\theta_{13}$ is left as a free parameter. Unless otherwise stated, we take $\theta_{13} = 0.07$, half of the current upper limit from the CHOOZ experiment. We use 1-loop RGEs to run all the Yukawa couplings up to the high scale. For the neutrino masses and mixing we use the RGE given in the literature [49, 50].
As a first step, we run the neutrino mass matrix and the Yukawa and gauge couplings up to the right handed neutrino masses, using an estimated \( M_R \), given by (11) in the CKM case and by (13) in the PMNS one. At that scale we assign the neutrino Yukawa matrix: in the CKM case evaluate it to be \( Y_{\nu} = Y_u \); in the PMNS case we first extract \( U_{PMNS}(M_R) \) and then define \( Y_{\nu} = U_{PMNS}(M_R) Y_u^{\text{diag}} \). We then run up to the \( M_Z \) scale and redefine \( Y_{\nu}(M_Z) \) to be equal to \( Y_u(M_Z) \) in the CKM case, or to \( U_{PMNS}(M_Z) Y_u^{\text{diag}} \) in the PMNS case. Once that the neutrino Yukawa matrix is known at \( M_Z \) we are able to use the see-saw formula (1) in order to calculate the right handed neutrinos mass matrix and, thus, the energies at which each heavy neutrino should decouple; we have only to use again the RGEs down to the estimated \( M_R \), and do the calculation ³. We thus use the iterative method to check if our results are right.

With this information, high energy inputs and the intermediate energy scales, we are now ready to compute the running of the soft spectrum from the high scale to the weak scale. We do this using 1-loop RGEs [51]. At the weak scale, we compute the full \( 6 \times 6 \) mass matrices of all the scalars and the neutralino and chargino mass matrices. In the Higgs sector we employ the full 1–loop of all the scalars and the neutralino and chargino mass matrices. 

For every point which passes through all these constraints, we compute leptonic flavour violating decay rates by using the exact mixing matrices [43] as well as masses for the sleptons, neutralinos and charginos.

V. RESULTS

From our leading log estimates we expect that the most promising sectors for finding SUSY–GUT induced LFV are the \( \mu e \) and the \( \tau \mu \) ones. Given that the planned sensitivities (Table I) to all the LFV processes will be of the same order \( (\sim \mathcal{O}(10^{-13} - 10^{-14}) \) in the \( \mu e \) sector and \( \sim \mathcal{O}(10^{-8}) \) in the \( \tau \mu \) one), we concentrate on the two body decays, \( \mu \to e \gamma \), to be probed by the MEG experiment at PSI, and \( \tau \to \mu \gamma \), that is under study at Beauty factories. Indeed, the three body decays are weaker probes of SUSY–GUTs, as the leading penguin contribution leads to a BR that is suppressed by a factor \( \sim \alpha \) with respect to the two body decay. The \( \mu \to e \) conversion in Nuclei process suffers from a similar suppression, but due to the well defined experimental signal the PRISM/PRIME aims to a huge improvement in the sensitivity to offset this factor.

In this section we display the results from the numerical routine for the processes of interest. All the plots are done for positive \( \mu \) as there is no sensible difference with the negative \( \mu \) case as far as lepton flavour violating processes are concerned ⁴.

### A. The MEG experiment at PSI

Given the planned astonishing sensitivity of the upcoming MEG [11] experiment at PSI, we expect that the muon decay \( \mu \to e \gamma \) will be a very interesting probe of LFV in a SUSY–GUT scenario. This statement is quantified in Figs. 6 and 7: the PMNS case high \( \tan \beta \) scenario is already ruled out by the current MEGA [12] bound on the BR(\( \mu \to e \gamma \)); the low \( \tan \beta \) regime is already severely constrained for not too high \( M_1/2 \) and will be completely probed by the upcoming MEG experiment. The CKM case, instead, is below the present bounds in all the parameter space, but a sensible portion of the high \( \tan \beta \) regime will be within the reach of MEG sensitivity (Fig. 7). 

| Exp. | \( t_\beta = 40 \) | \( t_\beta = 10 \) | \( t_\beta = 40 \) | \( t_\beta = 10 \) |
|------|----------------|----------------|----------------|----------------|
| MEGA | LHC | 2 TeV | no | no |
| MEG  | all | all | 1.3 TeV | no |

This allows us to draw the conclusion that (Table VIII), for not too big values of the soft breaking parameters (i.e.: \( (m_0, m_\tilde{g}) \leq 1 \) TeV), the MEG experiment will be able to find evidence of SUSY induced lepton flavour violation, unless we are in a low \( \tan \beta \), small mixing SUSY–GUT; as a consequence, if the LHC finds supersymmetry to be at the TeV scale but \( \mu \to e \gamma \) escapes MEG detection, this will be the only viable SUSY SO(10) see–saw scenario. Moreover, as depicted in Fig. 8, in the PMNS case the sensitivity of MEG will outreach that of the LHC, being able to probe soft masses as high as

³ This last step is necessary only in the CKM case, as the relations (13) are exact at the scale \( M_R \).

⁴ Let us note that the \( \mu < 0 \) scenario is strongly disfavored by bounds on the FCNC \( b \to s \gamma \) amplitude and by the SUSY corrections to \( (g_\mu - 2) \)
cess arise from the fact that the dominant LFV insertion of magnitude. The main theoretical interest for such improvement of the present bound by nearly two orders $U_{a}$ sensitivity of at least $\beta$ realiz: let us note that this machine is planned to reach the case that the planned Super Flavour factory [19] will be a constraint the CKM one.

B. B factories, SuperKEKB and Super Flavour factory

The $\tau\mu$ sector poses promising prospects of discovery of SUSY–GUT induced lepton flavour violation in the case that the planned Super Flavour factory [19] will be realized: let us note that this machine is planned to reach a sensitivity of at least $BR(\tau \to \mu \gamma) \sim O(10^{-9})$, with an improvement of the present bound by nearly two orders of magnitude. The main theoretical interest for such process arise from the fact that the dominant LFV insertion $(0_{\mu\ell\ell})_{\tau\mu}$ does not depend on the unknown PMNS angle $U_{e3}$.

As far as Beauty factories [14–16] are concerned, we see from Fig. 9, that even with the present bound it is possible to rule out part of the PMNS high tan $\beta$ regime; the planned accuracy of the SuperKEKB [10] machine $\sim O(10^{-8})$ will allow to test much of high tan $\beta$ region and will start probing the low tan $\beta$ PMNS case, with a sensitivity to soft masses as high as $(m_{0}, m_{\delta}) \lesssim 900$ GeV. The situation changes dramatically if one takes into account the possibility of a Super Flavour factory (Fig. 9, 10): taking the sensitivity of the most promising $\tau \to \mu \gamma$ process to $\sim O(10^{-9})$, the PMNS case will be nearly ruled out in the high tan $\beta$ regime and severely constrained in the low tan $\beta$ one; as for the CKM case it would be tested in the $(m_{0}, m_{\delta}) \lesssim 900$ GeV region, provided that tan $\beta$ is high.

The conclusions (Table IX) are that with the planned improvements of the KEK facility the $U_{e3}$ independent $\tau \to \mu \gamma$ process will allow us to test much of the PMNS scenario. A Super Flavour factory would much improve the situation, as it would be able to almost completely probe the PMNS case and to test the minimal mixing, high tan $\beta$ scenario up to soft masses of 600 GeV.

TABLE IX: Reach in $(m_{0}, m_{\delta})$ of the present and planned experiment from their $\tau \to \mu \gamma$ sensitivity.

| Exp.       | $t_{\beta} = 40$ | $t_{\beta} = 10$ | $t_{\beta} = 40$ | $t_{\beta} = 10$ |
|------------|-----------------|-----------------|-----------------|-----------------|
| PMNS       |                 |                 |                 |                 |
| BaBar, Belle | 1.2 TeV         | no              | no              | no              |
| SuperKEKB  | 2 TeV           | 0.9 TeV         | no              | no              |
| Super Flavour $^a$ | 2.8 TeV | 1.5 TeV         | 0.9 TeV         | no              |

$^a$Post–LHC era proposed/discussed experiment
an experiment in the PMNS case, if interesting to ask what is the probing capability of such whereas MEG will sure be operating, it is nevertheless a Super Flavour factory is just a proposed experiment, the present MEGA bound. The upcoming MEG sensitivity will test all the points.

C. Probing the PMNS case with $U_{e3} \approx 0$ at MEG

We have seen that if a Super Flavour factory will be built, the $\tau \rightarrow \mu \gamma$ process will be highly complementary to the $\mu \rightarrow e \gamma$ one as a probe of SUSY–GUT scenarios, with the added bonus of being $U_{e3}$ independent. As a Super Flavour factory is just a proposed experiment, whereas MEG will sure be operating, it is nevertheless interesting to ask what is the probing capability of such an experiment in the PMNS case, if $U_{e3}$ happens to be vanishing small, or even 0.

In the case that $U_{e3} = 0$ equation (10) is no more a good approximation to the running of the off-diagonal LL entries, as we have to resort to the 2nd generation entries:

$$ (\delta_{LL})_{\mu e} = -\frac{3}{8\pi^2} Y_\mu^2 U_{e2} U_{\mu 2} \ln \frac{M_X}{M_{R_2}}. \quad (29) $$

Now

Here the off-diagonal contribution in slepton masses, now being proportional to the square of the charm Yukawa $Y_c$ are much smaller, in fact even smaller than the CKM contribution by a factor

$$ \frac{Y_\mu^2 U_{e2} U_{\mu 2} \ln(M_X/M_{R_2})}{Y_\tau^2 V_{td} V_{ts} \ln(M_X/M_{R_3})} \sim \mathcal{O}(10^{-2}). \quad (30) $$

The point is that the estimate (30) misses and important point. The PMNS case is the case where the $R$ matrix is the identity; but we should keep in mind at what scale we should enforce this. Because the angle $U_{e3}$ runs with the energy scale and $U_{e3} \approx 0$ at the weak scale does not necessarily mean $U_{e3} \approx 0$ at high scale. Even for hierarchical spectra, where the running effects are small, the induced RG effects in the soft spectrum could be large, leading to large enough $\mu \rightarrow e \gamma$. The running effect of the neutrino mixing angle can be estimated by using the neutrino RG [49, 50] equations.

Moreover, as we have seen in section III, in a SUSY–GUT framework we have also sizable subleading contribution to the amplitude of the $\mu \rightarrow e \gamma$ process coming form the $(\delta_{RR})_{e\mu}$ insertion and from the double insertions $(\delta_{RR})_{e\tau} (\delta_{LL})_{\tau\mu}$; the interplay between the RG enhancement of $U_{e3}$ and the amplitudes coming from the
The results for the PMNS mixing with $U_{c3} = 0$ (defined at the weak scale) are shown in Fig. 11. We see that even for low $\tan \beta$ the branching ratio is never lower than that of the CKM case, giving a proof that the CKM case is really a representative of a ‘minimal mixing’ case. We see (Table X) that, given the present experimental LFV rates bounds, the $U_{c3} = 0$ PMNS case is better constrained by the $\tau \to \mu \gamma$ than by the $\mu \to e \gamma$ process. MEG will be able to probe much of this scenario: for high values of $\tan \beta$ almost all the LHC accessible parameter space will be probed, whereas if $\tan \beta$ happens to be small it will be probed up to $(m_0, m_3) \lesssim 1100$ GeV. We thus can state that if $\tan \beta$ is high the MEG experiment will probe the PMNS case better than the $\tau \mu$ sector experiments, irrespectively of the value of $U_{c3}$, and with an accuracy comparable to that of the SuperKEKB machine if $\tan \beta$ is small. On the other hand, a Super Flavour factory would for sure supersede MEG.

D. The PRISM/PRIME experiment at J-PARC

Since the experimental signal is very well defined, the $\mu \to e$ conversion in Nuclei poses very good prospects as
a probe of lepton flavour violating scenarios. In SUSY–GUT frameworks the main contribution to the amplitude comes from the penguin diagram that is also responsible for the FV $\mu \rightarrow e \gamma$ amplitude. There is thus a strong correlation between these two processes, the $\mu \rightarrow e$ conversion being suppressed by a factor $\sim Z\alpha/\pi$ with respect to the flavour violating decay $\mu \rightarrow e\gamma$.

The present bounds on $\mu \rightarrow e$ conversion come from the SINDRUM II experiment at PSI, that gave bounds on conversion rates in different Nuclei. For instance, the bound for the conversion in Titanium ($4.3 \cdot 10^{-12}$) is almost as good as the current MEGA bound on $\mu \rightarrow e\gamma$ ($1.1 \cdot 10^{-11}$) in constraining the SUSY–GUT parameter space, but it will be superseded by the future MEG sensitivity. To achieve a sensitivity to SUSY–GUTs scenarios that is comparable to the MEG experiment, a $\mu \rightarrow e$ conversion experiment in Titanium would need a sensitivity of $\mathcal{O}(10^{-15})$. This would require an high intensity muon source and an experimental apparatus that provides a very good resolution in the energy of the emitted electron, to discriminate with high accuracy the $\mu \rightarrow e$ conversion versus the $\mu$ decay in orbit. The J-PARC experiment PRISM/PRIME [17] addresses these issues by means of an innovative $\mu$ source (Phase Rotated Intense Slow Muons, PRISM), with an intensity of $10^{11} - 10^{12}$ $\mu$/s, and its $\mu \rightarrow e$ conversion in Ti dedicated experiment (PRIME: PRISM $\mu \rightarrow e$ conversion experiment); the planned sensitivity of the experiment is of $4 \cdot 10^{-18}$, with the possibility of improving it by upgrading the PRISM machine intensity to $10^{14} \mu$/s.

Although the experiment has not yet been approved, the construction of the PRISM machine has already begun and should be completed in five years [18]. It is thus timely to ask what will be the power of the post–LHC PRIME experiment to discriminate between the different SUSY–GUT scenarios in the case that the LHC finds evidence for SUSY. As can be seen from Fig. 12 and 13 the PRIME experiment would be able to really test our SUSY–GUT ansatz (Table XI): the high tan $\beta$ case would be tested in both the large and small mixing angles scenarios, even beyond the reach of the LHC. As for the low tan $\beta$ scenario, the PMNS case would be completely tested and much of the CKM case would be within reach: masses as high as $(m_0, m_{\tilde{b}}) \lesssim 2800$ GeV could be probed.

As the PRIME experiment would be a post–LHC era experiment its capability of testing and ruling out so many different SUSY–GUT scenarios is most interesting. It would be an ideal complement to the findings of the LHC in the case that it gets positive evidence for low energy supersymmetry.

VI. LFV RATES AT SPS BENCHMARK POINTS

In this section we discuss the possibility of detecting supersymmetry at the SPS benchmark points [55] by means of LFV experiments. We concentrate on the SPS points defined for mSUGRA/CMSSM framework. These take in to consideration various constraints, including relic density requirements, in addition to what we have considered here. We note that some of these points will be ruled out in the light of new WMAP data if one requires a purely Bino dark matter. As of now, there is no corresponding definition of SPS points within SUSY–GUTs. In the present work, we consider the input values of the mSUGRA SPS points in our SO(10) model and study the impact of flavour violation for that spectra.

5 We note that for all the points, the PMNS framework is ruled out by the present MEGA bound on $\mu \rightarrow e\gamma$. Furthermore, the PRISM/PRIME experiment would be able to test all the scenarios.

5 In some points, we notice the need for modifying these numbers within a SUSY–GUT framework. For example, in SPS 3, the LSP and $\tilde{b}_1$ are no longer degenerate, whereas SPS 4 and SPS 5 are already in conflict with experimental measurements.
The ‘typical’ mSUGRA scenario is represented by SPS points 1a at low tan β and 1b at relatively high tan β.

**SPS 1a**: $m_0 = 100$, $M_{1/2} = 250$, $A_0 = -100$, $t_β = 10$

$m_h = 112$, $m_t = 375$, $m_{\tilde{g}} = 612$

**SPS 1b**: $m_0 = 200$, $M_{1/2} = 400$, $A_0 = 0$, $t_β = 30$

$m_h = 120$, $m_t = 636$, $m_{\tilde{g}} = 980$

where the values are given in GeV and we have also given the values of three low energy observables ($m_h$, $m_t$, $m_{\tilde{g}}$) as obtained from the routine 6. We see that point 1a is already ruled out by the bound on the lightest Higgs mass. We are including it as it lays at the boundary of the experimentally ruled out region, so that a further improved version of our code could give the small correction that is needed to satisfy the present bound. The CKM scenario and the PMNS case at $U_{C3} = 0$ are unscathed by the present bounds. We see (Table XII) that the PMNS $U_{C3} = 0$ scenario will be within reach of both MEG and SuperKEKB, for the two benchmark points, while the CKM case could escape MEG detection, as the predicted BR for both points are at the boundary of the planned sensitivity.

The SPS 2 benchmark point lies in the so-called ‘focus point’ region [56]

**SPS 2**: $m_0 = 1450$, $M_{1/2} = 300$, $A_0 = 0$, $t_β = 10$

$m_h = 124$, $m_t = 940$, $m_{\tilde{g}} = 735$

where all the masses are given in GeV. From Table XII we see that the PMNS $U_{C3} = 0$ scenario will be within reach of the proposed Super Flavour factory; as for the other processes they will escape detection.

The mSUGRA/CMSSM ‘coannihilation region’ [57] has its representative in point SPS 3. In this region a rapid coannihilation between the neutralino LSP and the stau NLSP will give rise to a sufficiently low relic abundance: for this reason, we are also giving $m_{\tilde{\tau}}$ and $m_{LSP}$ as low energy observables (all masses in GeV)

**SPS 3**: $m_0 = 90$, $M_{1/2} = 400$, $A_0 = 0$, $t_β = 10$

$m_h = 119$, $m_t = 631$, $m_{\tilde{g}} = 980$

$m_{\tilde{\tau}} = 270$, $m_{LSP} = 185$

This point will be within reach of the proposed Super Flavour factory (Table XII) in the PMNS $U_{C3} = 0$ scenario.

The mSUGRA scenario at high tan β has it benchmark in point SPS 4

**SPS 4**: $m_0 = 400$, $M_{1/2} = 300$, $A_0 = 0$, $t_β = 50$

where all masses are in GeV. This point is ruled out, because it gives a non-viable vacuum.

The point SPS 5, that corresponds to a scenario of relatively light stop, is ruled out because it predicts a too light Higgs boson

**SPS 5**: $m_0 = 150$, $M_{1/2} = 300$, $A_0 = -1000$, $t_β = 5$

$m_h = 102$, $m_t = 275$, $m_{\tilde{g}} = 735$
TABLE XII: LFV rates for points SPS 1a and SPS 1b in the CKM case and in the $U_{e3} = 0$ PMNS case. The processes that are within reach of the future experiments (MEG, SuperKEKB) have been highlighted in boldface. Those within reach of post-LHC era planned/discussed experiments (PRISM/PRIME, Super Flavour factory) highlighted in italics.

| Process | SPS 1a | SPS 1b | SPS 2 | SPS 3 | Future |
|---------|--------|--------|-------|-------|--------|
| $\mu \rightarrow e \gamma$ | $3.2 \cdot 10^{-14}$ | $3.8 \cdot 10^{-13}$ | $4.0 \cdot 10^{-13}$ | $1.2 \cdot 10^{-12}$ | $1.3 \cdot 10^{-15}$ |
| $\mu \rightarrow e e e$ | $3.3 \cdot 10^{-15}$ | $2.7 \cdot 10^{-15}$ | $2.9 \cdot 10^{-16}$ | $8.6 \cdot 10^{-15}$ | $9.4 \cdot 10^{-18}$ |
| $\tau \rightarrow \mu \nu \nu$ | $2.8 \cdot 10^{-14}$ | $2.4 \cdot 10^{-14}$ | $2.6 \cdot 10^{-15}$ | $7.6 \cdot 10^{-14}$ | $1.0 \cdot 10^{-16}$ |
| $\tau \rightarrow \tau \nu \nu$ | $2.3 \cdot 10^{-12}$ | $6.0 \cdot 10^{-13}$ | $3.5 \cdot 10^{-12}$ | $1.7 \cdot 10^{-12}$ | $1.4 \cdot 10^{-13}$ |
| $\tau \rightarrow \mu \nu \nu$ | $2.7 \cdot 10^{-14}$ | $7.1 \cdot 10^{-15}$ | $4.2 \cdot 10^{-14}$ | $2.0 \cdot 10^{-14}$ | $1.7 \cdot 10^{-15}$ |
| $\tau \rightarrow \mu \nu \nu$ | $5.0 \cdot 10^{-11}$ | $1.1 \cdot 10^{-10}$ | $7.3 \cdot 10^{-11}$ | $1.3 \cdot 10^{-8}$ | $2.9 \cdot 10^{-12}$ |
| $\tau \rightarrow \mu \nu \nu$ | $1.6 \cdot 10^{-13}$ | $3.4 \cdot 10^{-11}$ | $2.2 \cdot 10^{-13}$ | $3.9 \cdot 10^{-11}$ | $8.9 \cdot 10^{-15}$ |
| $\tau \rightarrow \tau \nu \nu$ | $2.8 \cdot 10^{-13}$ | $3.4 \cdot 10^{-13}$ | $2.2 \cdot 10^{-13}$ | $3.9 \cdot 10^{-11}$ | $8.9 \cdot 10^{-15}$ |

where the dimensional parameters are given in GeV.

As a conclusion (Table XIII) we can state that the only scenarios that will for sure escape detection are the CKM focus point SPS 2 and CKM coannihilation region SPS 3 cases. The SPS 1a and SPS 1b CKM scenario are at the boundary of MEG sensitivity so that probing these scenarios, though hard, is nevertheless a possibility. The PRISM/PRIME experiment would much improve the situation, as it would be able to test all the scenarios; these results would be complemented by that from a Super Flavour factory.

TABLE XIII: Capability of past, present and future experiment to detect LFV at the SPS benchmark points. When two experiment are able to detect the same process, only the less sensitive experiment is displayed.

| Point | SPS 1a | SPS 1b | SPS 2 | SPS 3 |
|-------|--------|--------|-------|-------|
| CKM   | MEG (maybe) | MEG (maybe) | MEG | MEG |
| PMNS  | MEGA  | SINDRUM II | MEGA  | SINDRUM II |
| PMNS, $U_{e3} = 0$ | MEG | MEGA  | MEGA  | MEGA  |
| $\tau \rightarrow \mu \nu \nu$ | SuperKEKB | SuperKEKB | SuperKEKB | SuperKEKB |
| $\tau \rightarrow \tau \nu \nu$ | SuperFlavour | SuperFlavour | SuperFlavour | SuperFlavour |

VII. CONCLUSIONS

In this paper we addressed the capability of past (MEGA, SINDRUM II), present (BaBar, Belle),

upcoming (MEG, SuperKEKB) and proposed (PRISM/PRIME, Super Flavour factory) Lepton Flavour Violation experiments to probe SUSY–GUT scenarios. We have found that these experiments have strong capabilities to detect SUSY induced LFV, in some cases even outraching the LHC.

The more interesting feature of such experiments is the possibility to give hints about the viable SUSY–GUT scenarios, by constraining the neutrino Yukawa sector. The reach of such experiments as probes of different scenarios are summarized in Table XIV and displayed in Fig. 14, where we compare the scope of $\tau \rightarrow \mu \gamma$ and $\mu \rightarrow e \gamma$ experiments.

TABLE XIV: Reach in $(m_\tau, m_\mu)$ of the past, present and upcoming experiments from their LFV sensitivity. LHC means that all the LHC testable parameter space will be probed; all means that soft masses up to $(m_\mu, m_\tau) \lesssim 5$ TeV will be probed.

| Experiment | PMNS | $t_\beta = 40$ | $t_\beta = 10$ | $t_\beta = 40$ | $t_\beta = 10$ |
|-----------|------|----------------|----------------|----------------|----------------|
| $\mu e$ sector | MEGA | 1.1 TeV | 2.0 TeV | no | no |
| PRISM/PRIME | all | all | 1.3 TeV | no | no |
| $\tau e$ sector | BaBar, Belle | 1.2 TeV | no | no | no |
| SuperKEKB | 2.0 TeV | 0.9 TeV | no | no | no |
| SuperFlavour | 2.8 TeV | 1.5 TeV | 0.9 TeV | no | no |

$^a U_{e3} = 0$

$^b$Post–LHC era, planned/discussed experiment

Suppose that the LHC does find signals of low–energy supersymmetry, then grand unification becomes a very appealing scenario, because of the successful unification of gauge couplings driven by the SUSY partners. Among SUSY–GUT models, an SO(10) framework is much favored as it is the ‘minimal’ GUT to host all the fermions in a single representations and it accounts for the smallness of the observed neutrino masses by naturally in-
chuding the see–saw mechanism. Moreover in the recent years $SO(10)$ SUSY models have spurred much interest as in this framework it is possible to build realistic fermion mass model and to account for the proton lifetime bounds. In this paper we have addressed the issue by a generic benchmark analysis, within the ansatz that there is no fine–tuning in the neutrino Yukawa sector.

From our analysis we can state that lepton flavour violation experiments should be able to tell us much about the structure of such a SUSY–GUT scenario. If they detect LFV processes, by their rate and exploiting the interplay between different experiments, we would be able to get hints of the structure of the unknown neutrinos’ Yukawas. In this sense, the capability of a Super Flavour factory to discriminate between the minimal mixing case and the $U_{e3} = 0$ PMNS case is a most interesting feature.

On the contrary, in the case that both MEG and a future Super Flavour factory happen not to see any LFV process, only two possibilities should be left: (i) a the minimal mixing, low tan $\beta$ scenario; (ii) mSUGRA–$SO(10)$ see–saw without fine–tuned $Y_\nu$ couplings is not a viable framework of physics beyond the standard model. Moreover, if the planned, high sensitivity PRISM/PRIME conversion experiment, able to test even the minimal mixing low tan $\beta$ region, doesn’t manage to find LFV evidences, the latter conclusion should be the most sensible one and there should be no room left for the no fine–tuning framework we studied in this paper. Actually one should remark that LFV experiments will be able to falsify some of the SUSY GUT scenarios even in regions of the SUGRA parameter space that are beyond the reach of LHC experiments. In this sense, the power of LFV experiments of testing/discriminating among different SUSY GUTs models results very interesting and highly complementary to the direct searches at the LHC.

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APPENDIX: NOTATION AND RGE EQUATIONS

1. The model

The model consists in a supersymmetric $SO(10)$ framework with the following breaking pattern

$$SO(10) \rightarrow SU(5)_{RN} \rightarrow MSSM_{RN}$$

(A.1)

where $SO(10)$ is broken at the scale $M_X = 5 \cdot 10^{17}$ GeV which we equate it to the SUSY breaking mediation scale and the GUT scale is $M_{GUT} = 2 \cdot 10^{16}$ GeV. Below the scale $M_X$ the model is given by the following $SU(5)_{RN}$ superpotential

$$W_{SU(5)_{RN}} = Y_{10\ ij} 10_{i} 10_{j} 5^H + Y_{5\ ij} 5_{i} 5_{j} 5^H$$

(A.2)

$$+ Y_{1\ ij} 1\ i 5^H + M_{ij} 1_{i} 1_{j} + \mu 5^H 5^H$$

While the corresponding soft SUSY breaking potential is

$$V_{SU(5)_{RN}} = (A_{10\ ij} 10_{i} 10_{j} 5^H + A_{5\ ij} 5_{i} 5_{j} 5^H$$

FIG. 14: Comparison of $\mu \rightarrow e \gamma$ and $\tau \rightarrow \mu \gamma$ as a probes of SUSY–GUTs scenarios. The plots are done by scanning the LHC accessible parameter space at fixed tan $\beta$. The lines are the present bounds and future sensitivities. Let us note that the interplay between MEG and a Super Flavour factory will leave unscathed only the low tan $\beta$ CKM case.
Majorana mass:

\[ + A_{ij} \tilde{1}_j \tilde{5}_i + \tilde{M}_{ij} 1_{i} 1_{j} + B \mu 5_{i} 5_{j} + \text{h.c.} \]

\[ + m^2_{\tilde{e}_{ij}} \tilde{e}_{ij} + m^2_{\tilde{\nu}_{ij}} \tilde{\nu}_{ij} + m^2_{\tilde{\nu}_{ij}} \tilde{1}_{i} \tilde{5}_{j} + m^2_{\tilde{H}_{ij}} \tilde{1}_{i} \tilde{H}_{j} + M_5 2424 \quad (A.3) \]

After reaching the GUT scale, the theory is broken to the MSSM (plus right handed neutrinos) lagrangian

\[
W_{\text{MSSMRN}} = Y_{ij}^u Q_j U_j^c H_2 + Y_{ij}^d Q_j D_j H_1 + Y_{ij}^e L_i E_i^c H_1 + Y_{ij}^\nu L_i \tilde{\nu}_j + B_{ij} H_1 H_2 + \text{h.c.} \\
V_{\text{MSSMRN}} = \left( A_{ij}^u \tilde{Q}_i \tilde{U}_j^c H_2 + A_{ij}^d \tilde{Q}_i \tilde{D}_j H_1 + A_{ij}^e \tilde{L}_i \tilde{E}_j^c H_2 + \tilde{M}_{ij} \tilde{U}_j \tilde{N}_j + B_{ij} H_1 H_2 + \text{h.c.} \right) \\
+ m^2_{\tilde{Q}_{ij}} \tilde{Q}_i \tilde{Q}_j + m^2_{\tilde{U}_{ij}} \tilde{U}_i \tilde{U}_j + m^2_{\tilde{D}_{ij}} \tilde{D}_i \tilde{D}_j + m^2_{\tilde{E}_{ij}} \tilde{E}_i \tilde{E}_j + m^2_{\tilde{N}_{ij}} \tilde{N}_i \tilde{N}_j + m^2_{\tilde{H}_{i}^c} H_1 + m^2_{\tilde{H}_{i}^c} H_2 + M_1 \tilde{B} \tilde{B} + M_2 \tilde{W} \tilde{W} + M_3 \tilde{G} \tilde{G} \quad (A.5) \]

The matching between SU(5) parameters and MSSM ones, at \( M_{GUT} \) is given by:

\[
Y_{ij}^u = Y_{ij}^u \quad Y_{ij}^d = 4 Y_{ij}^{10} \quad Y_{ij}^e = \frac{1}{\sqrt{2}} Y_{ij}^5 \quad Y_{ij}^\nu = \frac{1}{\sqrt{2}} Y_{ij}^5 \quad (A.6) \]

The matching of the soft A-matrices is the same as the Yukawas, whereas for the soft mass matrices is:

\[
m^2_{\tilde{D}} = m^2_{\tilde{10}}; \quad m^2_{\tilde{Q}} = m^2_{\tilde{10}}; \quad m^2_{\tilde{D}} = m^2_{\tilde{5}} \]

\[
m^2_{\tilde{L}} = m^2_{\tilde{3}}; \quad m^2_{\tilde{E}} = m^2_{\tilde{5}}; \quad m^2_{\tilde{N}} = m^2_{\tilde{1}} \quad (A.7) \]

\[
M_1 = M_2 = M_3 = M_5 \quad (A.8) \]

2. SU(5)_{RN} RGE

Conventions:

\[
\tilde{Y} = \frac{Y}{4\pi}; \quad \tilde{A} = \frac{A}{4\pi}; \quad \tilde{\alpha} = \frac{\alpha}{4\pi} = g^2 \quad \text{and} \quad t = \ln \frac{M_{\tilde{5}}}{Q^2} \]

Yukawas:

\[
\frac{d}{dt} \tilde{Y}_{10 \; ij} = \frac{48}{5} \tilde{\alpha}_5 \tilde{Y}_{10 \; ij} - 24 Tr \left( \tilde{Y}_{10}^\dagger \tilde{Y}_{10} \right) \tilde{Y}_{10 \; ij} - \frac{1}{2} Tr \left( \tilde{Y}_{10}^\dagger \tilde{Y}_{10} \right) \tilde{Y}_{10 \; ij} - 48 \tilde{Y}_{10}^\dagger \tilde{Y}_{10} \tilde{Y}_{10 \; ij} - \frac{1}{2} \left( \tilde{Y}_{10}^\dagger \tilde{Y}_{10} \right) \tilde{Y}_{10 \; ij} \quad (A.9) \]

\[
\frac{d}{dt} \tilde{Y}_{5 \; ij} = \frac{42}{5} \tilde{\alpha}_5 \tilde{Y}_{5 \; ij} - Tr \left( \tilde{Y}_{5}^\dagger \tilde{Y}_{5} \right) \tilde{Y}_{5 \; ij} - \frac{3}{2} \left( \tilde{Y}_{5}^\dagger \tilde{Y}_{5} \right) \tilde{Y}_{5 \; ij} - 24 \tilde{Y}_{10}^\dagger \tilde{Y}_{10} \tilde{Y}_{5 \; ij} - \frac{1}{2} \left( \tilde{Y}_{5}^\dagger \tilde{Y}_{5} \right) \tilde{Y}_{5 \; ij} \quad (A.10) \]

\[
\frac{d}{dt} \tilde{Y}_{1 \; ij} = \frac{24}{5} \tilde{\alpha}_5 \tilde{Y}_{1 \; ij} - \frac{1}{2} Tr \left( \tilde{Y}_{1}^\dagger \tilde{Y}_{1} \right) \tilde{Y}_{1 \; ij} - 24 Tr \left( \tilde{Y}_{10}^\dagger \tilde{Y}_{10} \right) \tilde{Y}_{1 \; ij} - 3 \left( \tilde{Y}_{1}^\dagger \tilde{Y}_{1} \right) \tilde{Y}_{1 \; ij} \quad (A.11) \]

Majorana mass:

\[
\frac{d}{dt} M_{ij} = - \frac{5}{2} \left[ \left( \tilde{M}_{10} \tilde{Y}_{1} \right)_{ij} + \left( \tilde{M}_{10}^\dagger \tilde{Y}_{1}^\dagger \right)_{ij} \right] \quad (A.12) \]

Soft masses:

\[
\frac{d}{dt} \left( m^2_{\tilde{\nu}} \right)_{ij} = \frac{48}{5} \tilde{\alpha}_5 M^2_{\tilde{\nu}} \delta_{ij} - \left[ \left( m^2_{\tilde{\nu}} \tilde{Y}_{5}^\dagger \tilde{Y}_{5} \right)_{ij} + \left( \tilde{Y}_{5}^\dagger \tilde{Y}_{5} m^2_{\tilde{\nu}} \right)_{ij} \right] - \frac{1}{2} \left[ \left( m^2_{\tilde{\nu}} \tilde{Y}_{1}^\dagger \tilde{Y}_{1} \right)_{ij} + \left( \tilde{Y}_{1}^\dagger \tilde{Y}_{1} m^2_{\tilde{\nu}} \right)_{ij} \right] \]
\[
-2 \left[ (\hat{Y}_5^\dagger m_{10}^2 \hat{Y}_5)_{ij} + (\hat{Y}_5^\dagger \hat{Y}_5)_{ij} m_{10}^2 + (\hat{A}_5^\dagger \hat{A}_5)_{ij} \right] \\
- \left[ (\hat{Y}_1^* m_{10}^2 \hat{T}_1^T)_{ij} + (\hat{Y}_1^* \hat{T}_1^T)_{ij} m_{10}^2 + (\hat{A}_1^* \hat{A}_1)_{ij} \right] \\
\frac{d}{dt} (m_{10}^2)_{ij} = \frac{72}{5} \hat{\alpha}_5 M_5^2 \delta_{ij} - 24 \left[ (m_{10}^2 \hat{Y}_1^\dagger \hat{Y}_1 m_{10}^2)_{ij} \right] - \frac{1}{2} \left[ (m_{10}^2 \hat{Y}_5^\dagger \hat{Y}_5)_{ij} + (\hat{Y}_5^\dagger m_{10}^2)_{ij} \right] \\
\frac{d}{dt} (m_5^2)_{ij} = \frac{48}{5} \hat{\alpha}_5 M_5^2 \delta_{ij} - 48 \left[ Tr (\hat{Y}_5^\dagger \hat{Y}_5) m_5^2 + Tr (\hat{Y}_5^\dagger m_5^2) \right] \\
\frac{d}{dt} (m_{10}^2) = \frac{48}{5} \hat{\alpha}_5 M_5^2 \delta_{ij} - 2 \left[ Tr (\hat{Y}_5^\dagger \hat{Y}_5) m_{10}^2 + Tr (\hat{Y}_5^\dagger m_{10}^2) \right] \\
\frac{d}{dt} (m_{10}^2)_{ij} = \frac{72}{5} \hat{\alpha}_5 M_5^2 \delta_{ij} - 24 \left[ (m_{10}^2 \hat{Y}_1^\dagger \hat{Y}_1 m_{10}^2)_{ij} \right] - \frac{1}{2} \left[ (m_{10}^2 \hat{Y}_5^\dagger \hat{Y}_5)_{ij} + (\hat{Y}_5^\dagger m_{10}^2)_{ij} \right]
\]

**A-terms:**

\[
\frac{d}{dt} \hat{A}_{10} \ ij = \frac{48}{5} \hat{\alpha}_5 \left( \hat{A}_{10} \ ij - 2 M_5 \hat{Y}_{10} \ ij \right) - 24 Tr (\hat{Y}_1^\dagger \hat{Y}_10) \hat{A}_{10} \ ij - \frac{1}{2} Tr (\hat{Y}_5^\dagger \hat{Y}_5) \hat{A}_{10} \ ij \\
- 48 Tr (\hat{Y}_5 \hat{A}_{10}) \hat{Y}_{10} \ ij - Tr (\hat{Y}_5 \hat{A}_1) \hat{Y}_10 \ ij - 72 \left[ (\hat{Y}_5 \hat{Y}_{10}^\dagger \hat{A}_{10})_{ij} + (\hat{A}_{10}^\dagger \hat{Y}_{10}^\dagger \hat{Y}_5)_{ij} \right] \\
- \frac{1}{2} \left[ (\hat{Y}_5^\dagger \hat{Y}_5)_{ij} + (\hat{A}_{10} \hat{Y}_5^\dagger \hat{Y}_5)_{ij} \right] - (\hat{A}_5 \hat{Y}_5^\dagger \hat{Y}_5)_{ij} \\
\frac{d}{dt} \hat{A}_5 \ ij = \frac{42}{5} \hat{\alpha}_5 \left( \hat{A}_5 \ ij - 2 M_5 \hat{Y}_5 \ ij \right) - Tr (\hat{Y}_5^\dagger \hat{Y}_5) \hat{A}_5 \ ij - 2 Tr (\hat{Y}_5 \hat{A}_5) \hat{Y}_5 \ ij \\
- \frac{5}{2} \left( \hat{Y}_5^\dagger \hat{Y}_5 \hat{A}_5 \ ij \right) - 2 \left( \hat{A}_5 \hat{Y}_5 \hat{Y}_5 \hat{A}_5 \ ij \right) - 24 \left( \hat{Y}_5 \hat{Y}_{10} \hat{A}_5 \ ij \right) \frac{1}{2} \left( \hat{A}_5 \hat{Y}_5 \hat{Y}_5 \hat{A}_5 \ ij \right) \\
- (\hat{Y}_5 \hat{Y}_5^\dagger \hat{A}_5 \ ij) - 48 \left( \hat{A}_{10} \hat{Y}_5 \hat{Y}_5 \hat{Y}_5 \hat{A}_1 \ ij \right) \\
\frac{d}{dt} \hat{A}_1 \ ij = \frac{24}{5} \hat{\alpha}_5 \left( \hat{A}_1 \ ij - 2 M_5 \hat{Y}_1 \ ij \right) - \frac{1}{2} Tr (\hat{Y}_1 \hat{Y}_1) \hat{A}_1 \ ij - 24 Tr (\hat{Y}_1 \hat{Y}_1 \hat{Y}_1) \hat{A}_1 \ ij \\
- 48 Tr (\hat{Y}_1 \hat{A}_{10}) \hat{Y}_1 \ ij - Tr (\hat{Y}_1 \hat{A}_1) \hat{Y}_1 \ ij - \frac{11}{2} \left( \hat{Y}_1 \hat{Y}_1 \hat{A}_1 \ ij \right) \frac{7}{2} \left( \hat{A}_1 \hat{Y}_1 \hat{Y}_1 \hat{A}_1 \ ij \right) \\
- 2 \left( \hat{A}_5 \hat{Y}_5 \hat{Y}_1 \hat{A}_1 \ ij \right) - \left( \hat{Y}_5 \hat{Y}_5^\dagger \hat{Y}_1 \hat{A}_1 \ ij \right)
\]

**μ terms:**

\[
\frac{d}{dt} \mu = 2 \left[ \frac{24}{5} \hat{\alpha}_5 - 12 Tr (\hat{Y}_1 \hat{Y}_1) \hat{Y}_1 \hat{Y}_1 - \frac{1}{2} Tr (\hat{Y}_5^\dagger \hat{Y}_5) \mu \right] \\
\frac{d}{dt} B\mu = - \left[ \frac{48}{5} \hat{\alpha}_5 M_5 + 12 Tr (\hat{A}_1 \hat{Y}_1) \hat{Y}_1 \hat{Y}_1 \hat{A}_1 \hat{Y}_1 + \frac{1}{2} Tr (\hat{A}_5 \hat{Y}_5) \mu \right] + \left( \frac{24}{5} \hat{\alpha}_5 - 12 Tr (\hat{Y}_1 \hat{Y}_1) - \frac{1}{2} Tr (\hat{Y}_5^\dagger \hat{Y}_5) \right) B\mu
\]
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