TIDAL DISRUPTION AND IGNITION OF WHITE DWARFS BY MODERATELY MASSIVE BLACK HOLE

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ABSTRACT

We present a numerical investigation of the tidal disruption of white dwarfs by moderately massive black holes, with particular reference to the centers of dwarf galaxies and globular clusters. Special attention is given to the fate of white dwarfs of all masses that approach the black hole close enough to be disrupted and severely compressed to such an extent that explosive nuclear burning can be triggered. Consistent modeling of the gas dynamics together with the nuclear reactions allows for a realistic determination of the explosive energy release. In the most favorable cases, the nuclear energy release may be comparable to that of typical Type Ia supernovae. Although the explosion will increase the mass fraction escaping on hyperbolic orbits, a good fraction of the debris remains to be swallowed by the hole, causing a bright soft X-ray flare lasting for about a year. Such transient signatures, if detected, would be a compelling testimony for the presence of a moderately massive black hole (below $10^7 M_{\odot}$).

Key words: accretion, accretion disks – black hole physics – globular clusters: general – hydrodynamics – nuclear reactions, nucleosynthesis, abundances

Online-only material: color figures

1. INTRODUCTION

White dwarfs, the end point of stellar evolution for stars with masses from about 0.07 to 10 solar masses, are extremely common (Weidemann 1990). And we do not have to look far away, either. There are several billions of them in the halo of our very own Milky Way (Reid 2005). They are not only observed in isolation but in binary systems, with normal stellar companions, and, less frequently, with compact stellar companions. At a distance of tens of kiloparsecs, a number of globular clusters (Brodie & Strader 2006; Moehler & Bono 2008) have a high enough central density to let white dwarfs interact and collide with other stars, or, if developed, with a central massive black hole (Gerssen et al. 2002, 2003; Gebhardt et al. 2002, 2005). And at a distance of less than 10 kpc, our galactic nucleus, a central massive black hole of a few million solar masses (Ghez et al. 1998), is surrounded by swarms of all kind of stars, some of them white dwarfs and all of them prone to collisions. In such dense environments, stars of all varieties can exchange mass, disrupt each other or merge, and their merger products can get involved in similar interactions. Because of this reason, the innermost $<0.1$ pc of our Galaxy can be considered an efficient stellar collider (Alexander 2005). Stellar dynamics is perfectly adequate in modeling the motions of stars as point masses moving under the influence of gravity, even in dense stellar systems, unless individual stars closely approach each other or get so close to the central black hole that they become vulnerable to tidal distortions. Once this happens, the internal structure of the stars becomes important for the further dynamical evolution. This paper explores the observational manifestations of such phenomena, with particular reference to white dwarfs.

A white dwarf interacting with a neutron star or a black hole cannot be treated as a point mass if it gets so close to the compact remnant that it becomes tidally deformed. The tidal interaction between a star and a black hole is mainly characterized by three length scales: the stellar radius, $R_{\text{wd}}$, the gravitational radius of the black hole, $R_{\text{g}} = 2 GM_{\text{BH}}/c^2 \simeq 3 \times 10^{11} M_{\text{BH,6}}$ cm, where $M_{\text{BH,6}}$ denotes the hole’s mass in units of $10^6 M_{\odot}$, and the tidal radius, $R_{\text{t}}$. The tidal radius, defined as the distance within which a star gets disrupted, obviously depends on the type of star being considered. For a white dwarf it is roughly given by

$$R_{\text{t}} \simeq 1.2 \times 10^{11} M_{\text{BH,6}}^{1/3} \left( \frac{R_{\text{wd}}}{10^6 \text{ cm}} \right) \left( \frac{M_{\text{wd}}}{0.6 M_{\odot}} \right)^{-1/3} \text{ cm.} \quad (1)$$

Essentially, $R_{\text{t}}$ is the distance from the hole at which $M_{\text{BH}}/R_{\text{t}}^2$ equals the mean density of the passing star. $R_{\text{g}}$ is, in order of magnitude, the same as the Roche radius, which is a precisely defined quantity, but only strictly applicable to a star in a circular orbit with synchronized spin. The strength of the tidal encounter is measured by the dimensionless parameter

$$\eta = \left( \frac{M_{\text{wd}}}{M_{\text{BH}}} \frac{R_{\text{p}}}{R_{\text{wd}}^3} \right)^{1/2}, \quad (2)$$

which is simply the square root of the ratio of the surface gravity of the star to the tidal gravity at the surface when the star is at pericenter distance $R_{\text{p}}$. Most literature on tidal disruption uses $\beta = R_{\text{t}}/R_{\text{p}} = \eta^{-2/3}$ to measure the encounter strength. When $\eta \leq 1$, the star is disrupted in a single flyby. The energy required to tear the star apart (that is, the star’s self-binding energy) is supplied at the expense of the orbital energy, which at $R_{\text{t}}$ is larger by $(M_{\text{BH}}/M_{\text{wd}})^{2/3}$.

For white dwarfs, the ratio between the tidal disruption radius and the gravitational radius is

$$\beta_{\text{wd}} = \left( \frac{R_{\text{t}}}{R_{\text{g}}} \right) \simeq 0.4 M_{\text{BH,6}}^{-2/3} \left( \frac{R_{\text{wd}}}{10^6 \text{ cm}} \right) \left( \frac{M_{\text{wd}}}{0.6 M_{\odot}} \right)^{-1/3}. \quad (3)$$

Note that this is indeed inside the gravitational radius for black hole masses exceeding

$$M_{\text{BH,lim}} \simeq 2.5 \times 10^5 \left( \frac{R_{\text{wd}}}{10^6 \text{ cm}} \right)^{3/2} \left( \frac{M_{\text{wd}}}{0.6 M_{\odot}} \right)^{-1/2} M_{\odot}. \quad (4)$$

...
For this reason, white dwarfs only experience disruptive physical conditions when approaching a moderately massive black hole (Luminet & Pichon 1989). If \( M_{\text{BH}} \ll M_{\text{BH,lim}} \), it is however, sufficiently far outside \( R_e \) that the disruption can be approximated as a Newtonian process, and it makes little difference whether the hole is described by a Schwarzschild or a Kerr metric. When the central hole has a mass \( M_{\text{BH}} \gg M_{\text{wd}} \), the size of the star remains smaller than \( R_e \). A white dwarf cannot thus be disrupted (Figure 1) without entering the strong relativistic domain if

\[
\beta > \min \left[ \left( \frac{M_{\text{BH}}}{M_{\text{wd}}} \right)^{1/3}, \left( \frac{M_{\text{BH,lim}}}{M_{\text{BH}}} \right)^{2/3} \right].
\]

The type of the black hole (Schwarzschild or Kerr?) then has an important quantitative effect, as does (for a Kerr hole) the orientation of the stellar orbit relative to the spin axis.

Much of our effort in this paper is therefore dedicated to investigate the disruption of white dwarfs by moderately massive black holes, with particular reference to the centers of dwarf galaxies (e.g., Mathur et al. 2008), globular clusters (e.g., Portegies Zwart et al. 2004; Baumgardt et al. 2004) and the intermediate-mass black hole candidates in active galactic nuclei (Greene & Ho 2004). Special attention is given to the fate of white dwarfs of all masses that approach a black hole close enough to be disrupted and severely compressed to such extent that explosive nuclear burning can be triggered. Tidal disruption of white dwarfs has previously been difficult to model due to its three-dimensional nature and the delicate interplay between gravity, gas dynamics and nuclear reactions. Crucial aspects of the physics of white dwarf disruption and ignition were first understood by Luminet & Pichon (1989), mainly based on the affine model earlier introduced by Carter & Luminet (1983). Luminet & Pichon (1989) suggested that for very close encounters (\( \eta \ll 1 \)), relativistic effects and compression into the orbital plane could trigger significant nuclear energy release in the core (some authors, particularly Wilson & Mathews 2004 and Dearborn et al. 2005, believe such considerations could also apply to weakly disrupted encounters for which general relativistic effects can overwhelm the stabilizing tidal forces and still cause relativistic compression). The affine model of Carter and Luminet approximates the white dwarf as a time-dependent ellipsoid, with a fixed density profile. Such a model is of limited utility if the true matter distribution deviates from the assumed shape and density profile. To study the tidal disruption problem, we use a three-dimensional smoothed particle hydrodynamics method (SPH) to solve the equations of hydrodynamics. This method makes no restrictive assumptions on the hydrodynamic degrees of freedom and allows us to study the three-dimensional encounter with full generality. Due to its Lagrangian nature SPH is perfectly suited to follow tidal disruption processes during which the corresponding geometry, densities, and timescales are changing violently (Rosswog et al. 2008a, 2008b). What is more, our consistent modeling of gas dynamics together with nuclear energy generation allows for a realistic determination of the explosive energy release.

In this paper, a comprehensive approach is adopted. We follow the tidal disruption and compression leading up to the ignition of the white dwarf (when triggered); the accompanying nucleosynthesis and expansion of the debris; and the subsequent accretion of the bound debris. Some of the questions at the forefront of attention include the effects of initial composition and mass of the white dwarf, the passage distance, and the black hole’s mass on ignition and disruption, and the degree to which the post-disruption dynamics is modified by nuclear energy release. We address all of these issues here.

A detailed description of the numerical methods and the initial models is given in Section 2. Detailed hydrodynamic simulations of the disruption of white dwarfs of various masses, initial composition, and passage distances are presented in Sections 3–5, the role of the mass of the black hole in shaping the evolution and ignition of the disrupted white dwarf is discussed in Section 4. The resulting gravitational wave signals are shown in Section 6. Discussion and conclusions are presented in Section 7.

2. NUMERICAL METHODS AND INITIAL MODEL

The observational consequences of stellar disruption depend on what happens to the debris (Rees 1988; Evans & Kochanek 1989; Kobayashi et al. 2004). To quantify this, we have performed detailed three-dimensional, hydrodynamical calculations. The gas dynamics is coupled with a nuclear network to explore the effects of nuclear energy generation during the strong compression phase. The reader is referred to Rosswog et al. (2008a) for a complementary description of the numerical methods employed in calculating the disruptive event.

2.1. Hydrodynamics

The SPH formulation used in this study follows closely the one described in Benz (1990), a brief derivation of the equations can be found in Rosswog & Price (2008). Forces from self-gravity are calculated using a binary tree (Benz et al. 1990b) while the gravitational forces from the central black hole are calculated using a Paczyński–Wiita pseudopotential (Paczyński & Wiita 1980) with a polynomial extension (Rosswog 2005) to avoid the singularity at the Schwarzschild radius. We have taken particular care to avoid artifacts from the use of artificial viscosity (AV). The so-called Balsara switch (Balsara 1995) is implemented to avoid spurious shear forces. More importantly, we use time-dependent viscosity parameters (Morris & Monaghan 1997), so that far from shocks artificial viscosity is essentially absent, but near a shock front the associated viscous parameters
rise to values that are able to avoid postshock oscillations (see Figure 1 of Rosswog et al. 2008a for an illustration). Once the shocks have passed, the parameters decay again. Details of the AV implementation and tests can be found in Rosswog et al. (2000).

A MacCormack predictor-corrector method is used to evolve the fluid in time. The method is implemented with individual time steps, i.e., each particle $i$ is advanced on its own time step $\Delta t_i = 2^n \times \Delta t_{\text{min}}$, where $\Delta t_{\text{min}}$ is the smallest step of all the particles. $n_i$ is chosen to be the largest integer satisfying the condition $\Delta t_i < \Delta t_{i,\text{des}}$, where $\Delta t_{i,\text{des}}$ is the desired time step for particle $i$. With this time marching implementation the total energy is conserved to better than $4 \times 10^{-3}$ and the total angular momentum to better than $2 \times 10^{-4}$. Note that this could, in principle, be improved even further by taking into account the so-called “grad-h”-terms (Springel & Hernquist 2002; Monaghan 2002) and extra terms arising from adapting gravitational smoothing terms (Price & Monaghan 2007). A comparison between the formulation used here and that using “grad-h”-terms can be found in Rosswog & Price (2007).

2.2. Equation of State

We use the HELMHOLTZ equation of state (EOS), developed by the Center for Astrophysical Thermonuclear Flashes at the University of Chicago. The EOS allows to freely specify the chemical composition of the gas and it can be coupled to a nuclear reaction network. The electron-/positron equation of state is calculated without approximation. In other words, it makes no assumption about the degree of degeneracy or relativity and the exact expressions are integrated numerically. The nuclei in the gas are treated as a Maxwell–Boltzmann gas, the photons as blackbody radiation. The EOS is used in tabular form with densities in the range $10^{-10} \leq \rho Y_e \leq 10^{11}$ g cm$^{-3}$ and temperatures varying between $10^4$ and $10^{11}$ K. A sophisticated, biquintic Hermite polynomial interpolation is used to enforce the thermodynamic consistency (i.e., the Maxwell relations) at interpolated values (Timmes & Swesty 2000).

2.3. Nuclear Burning

To account for the feedback onto the fluid from nuclear transmutations we use a minimal nuclear reaction network developed by Hix et al. (1998). It couples a conventional $\alpha$-network stretching from He to Si with a quasi-equilibrium-reduced $\alpha$-network. The QSE-reduced network neglects reactions within small equilibrium groups that form at temperatures above $3.5 \times 10^8$ K to reduce the number of abundance variables needed. Although a set of only seven nuclear species is used, this network reproduces all burning stages from He burning to nuclear statistical equilibrium (NSE) accurately. For details and tests we refer to Hix et al. (1998).

The network is coupled to the hydrodynamics in an operator splitting fashion, i.e., hydrodynamics and nuclear burning—which may in extreme cases require vastly different time steps—are integrated separately. In a first step, the hydrodynamic equations are integrated with the above described predictor-corrector scheme to obtain new quantities at time step $t^{n+1}$. In this step we ignore the nuclear source term in the energy equation, the new value for the specific energy of particle $a$ is denoted by $\tilde{u}_a^{n+1}$. The specific energy has to be corrected for the energy release that occurred between $t^n$ and $t^{n+1}$

$$
\epsilon_{a,n \to n+1} = -N_A \sum_j m_j c^2 \int_{t^n}^{t^{n+1}} \frac{dY_{j,a}}{dt} (\rho_a(t), T_a(t), Y_{k,a}(t)) \, dt
$$

$$
= -N_A \sum_{j=1} m_j c^2 (Y_{j,a}^{n+1} - Y_{j,a}^n),
$$

where $N_A$ is Avogadro’s constant, $m_j c^2$ is the mass energy of nucleus $j$ and $Y_{j,a}$ is the abundance of nucleus $j$ in particle $a$. We use $\rho_a(t) \approx \rho_a(t^n) + \frac{\rho_a(t^n)}{\rho_a(t^n) - \rho_a(t^n)}$ and $T_a(t) \approx \frac{1}{\rho_a(t^n)} (T_a(t^{n+1}) - T_a(t^n))$, where $T_a(t^{n+1})$ is the temperature derived from $\tilde{u}_a^{n+1}$, to integrate the abundances $Y_{j,a}$ via the implicit backward Euler method (the network integration is described in detail in Hix et al. 1998). The final value for the specific energy at time $t^{n+1}$ is given by

$$
u_a^{n+1} = \tilde{u}_a^{n+1} + \epsilon_{a,n \to n+1}.
$$

The EOS is then called again to make all thermodynamic quantities consistent with this new value $\nu_a^{n+1}$. Once the derivatives have been updated, the procedure can be repeated for the next time step. For details of the time step criteria we refer the reader to Rosswog et al. (2008a). As will be shown below, the peak compression occurs at a spatially fixed point (see density panels in Figure 6). The white dwarf fuel is fed with free-fall velocity

$$v_f = (2GM_{\odot}/R_{\odot})^{1/2} = 1.6 \times 10^3 \text{ km s}^{-1} (M_{\odot}/1000 M_{\odot})^{1/2} (R_{\odot}/10^4 \text{ km})^{-1/2}$$

into this compression point. This is many orders of magnitude larger than typical flame propagation speeds, therefore flame propagation effects can be safely neglected for this investigation.

2.4. Initial Model

It is a vital ingredient for any calculation to start out from initial conditions that are as accurate as possible. In order to build stars in hydrostatic equilibrium, it is therefore necessary to find SPH particle distributions whose number density reflects the equilibrium mass density distribution. To this end we solve the spherically symmetric Lane–Emden equations for a star of given mass and composition to find a one-dimensional density profile. For simplicity, we assume that each particle distributes the desired number of particles, $N_i$, inside a unit sphere according to a close-packed configuration. Subsequently, we map this distribution into the volume of the star that is to be constructed. As an example, the particle distributions before and after mapping of a 0.2 $M_{\odot}$ white dwarf are shown in Figure 2.

The mapping of the initial configuration is done so that the SPH-particle number density in the star, $n$, yields

$$\rho_{LE}(r) = n \cdot m(r),
$$

where $m$ is the mass of each particle and $\rho_{LE}$ is the density profile obtained by solving the Lane–Emden equation. We

\footnote{Note that the temperature evolution along a hydrodynamical time step is different from the description in Rosswog et al. (2008a). In practice, we only see tiny differences between both approaches.}
Table 1
Summary of the Performed Runs

| Run | M<sub>wd</sub> | M<sub>BH</sub> | β  | Grav. | SPH Particles | log(E<sub>burn</sub>) | “Fe” (M<sub>⊙</sub>) | Comments |
|-----|-------------|-------------|----|-------|--------------|----------------|----------------|----------|
| 1   | 0.2         | 1000        | 12 | N     | 4034050      | 50.46          | 0.025         | Expl.    |
| 2   | 0.2         | 1000        | 12 | PW    | 4034050      | 50.44          | 0.034         | Expl.    |
| 3   | 0.2         | 1000        | 12 | PW    | 200452       | 50.44          |              |          |
| 4   | 0.2         | 100         | 5  | PW    | 100027       | 49.57          | <10<sup>-10</sup> | Expl. BH influence, expl. |
| 5   | 0.2         | 500         | 5  | PW    | 100027       | 49.64          | <10<sup>-10</sup> | Expl. BH influence, expl. |
| 6   | 0.2         | 1000        | 5  | PW    | 100027       | 49.76          | <10<sup>-10</sup> | Expl. BH influence, expl. |
| 7   | 0.2         | 5000        | 5  | PW    | 100027       | 49.93          | <10<sup>-10</sup> | Expl. BH influence, expl. |
| 8   | 0.6         | 500         | 5  | N     | 502479       | 50.68          | 0.18          | Expl.    |
| 9   | 0.6         | 500         | 5  | N     | 502479       | 50.62          | 0.13          | Hot, initial WD |
| 10  | 0.6         | 1000        | 0.9| N     | 1006446      | 0.00           | 0             | No nuclear burning |
| 11  | 0.6         | 10000       | 1.5| PW    | 502479       | 45.07          | <10<sup>-10</sup> |          |
| 12  | 1.2         | 100         | 3.5| N     | 100027       | 51.01          | 0.58          | Expl.    |
| 13  | 1.2         | 500         | 2.6| PW    | 502479       | 51.16          | 0.66          | Expl.    |
| 14  | 1.2         | 1000        | 1.5| PW    | 502479       | 49.63          | 0.014         |          |
| 15  | 1.2         | 1000        | 1.5| N     | 502479       | 51.10          | 0.63          | Expl.    |

Notes. M<sub>wd</sub> and M<sub>BH</sub> are the masses of the white dwarf and the black hole, respectively, β is the ratio of tidal radius and pericentre distance. The type of gravity is indicated by N (Newtonian) or PW (Paczynski–Wiita). Column 6 states the number of SPH particles used in the simulation, E<sub>burn</sub> is the energy generated in burning processes. To put this number into context we briefly state the gravitational binding energies (in ergs) of the different white dwarfs: log(E<sub>bin</sub>, 0.2 M<sub>⊙</sub>) = 49.13, log(E<sub>bin</sub>, 0.6 M<sub>⊙</sub>) = 50.12, log(E<sub>bin</sub>, 1.2 M<sub>⊙</sub>) = 50.98. “Fe” labels the mass in iron-group elements, “expl.” in the comment column indicates that the produced nuclear energy exceeds the WD gravitational binding energy.

Figure 2. Building initial conditions by solving the spherically symmetric Lane–Emden equations for a star of given mass and composition to find a one-dimensional density profile. The particles are then distributed inside a unit sphere according to a close-packed prescription. Left panel: close-packed particle distributions in a unit sphere. This distribution is then mapped into the volume of the star that is to be constructed. Right panel: particle distribution after mapping onto the density profile of a 0.2 M<sub>⊙</sub> white dwarf. Shown are the projections of the particle positions onto the xy-plane. This particle distribution is then finally “relaxed” into its true numerical equilibrium.

denote quantities referring to the star (unit sphere) with (un-) primed variables. If the unit sphere has a constant density ρ<sub>0</sub> and contains N particles, each with mass m = M<sub>wd</sub>/N, the mass in the shell between r<sub>n-1</sub> and r<sub>n</sub> is M<sub>n</sub> ≃ 4πr<sup>2</sup><sub>n-1</sub>Δrρ<sub>0</sub>. As illustrated in Figure 3, the image of this shell (in the star) contains the same mass, but now between r<sub>n-1</sub>′ and r<sub>n</sub>′, so that M<sub>n</sub>′ ≃ 4πr<sup>2</sup><sub>n-1</sub>′Δr′ρ<sub>n-1</sub>. The condition M<sub>n</sub>′ = M<sub>n</sub> then yields

\[ Δr′_n = \left( \frac{r_{n-1}}{r_{n-1}′} \right)^2 \left( \frac{ρ_0}{ρ_{n-1}} \right) Δr. \] (10)

The resultant configuration is very close to hydrostatic equilibrium. To find the “real” numerical equilibrium state we relax this configuration with the full hydrodynamics code by applying an artificial, velocity-dependent damping force as, for example, in Rosswog et al. (2004).

We use this procedure for all but the heaviest white dwarfs (1.2 M<sub>⊙</sub>). Since their adiabatic exponent is already approaching the critical 4/3, these stars are very centrally condensed and, as a result, the bulk of the SPH particles resides very close to each other in the stellar center. This would, even for a moderate
resolution of a few hundred thousand particles, result in most of the particles running at time steps that are orders of magnitude smaller than the dynamical time of the star. To alleviate this computational burden in the heaviest white dwarfs we allow for slightly different particle masses (about a factor of 10 from the center to the stellar surface).

Throughout this simulation set we assume a uniform nuclear composition across the white dwarfs. Stars with masses \(< 0.6 \, M_\odot\) are instantiated as pure helium, more massive stars are modeled as 50% carbon and 50% oxygen. Note that helium core burning may produce composition gradients throughout the stars with oxygen being more abundant in the stellar core. Since it is mainly the core that becomes ignited during the compression, see Section 3, such a gradient may slightly reduce the nuclear energy that can be released in principle.

Common to all calculations is the initiation of the calculations with the white dwarf being placed safely outside \(R_{\tau}\). Initial separations are at least twice, in most cases three times the tidal radius to allow the initially spherical star to adjust properly to the changing tidal potential as it approaches the black hole. All stars are set onto parabolic orbits so that \(R_p\) is larger than \(R_g\).

The performed runs are summarized in Table 1.

3. EVENTS IN THE LIFE OF A TIDALLY DISRUPTED WHITE DWARF

Since the focus of this study is closely related to the nuclear energy release during strong encounters, practically all following calculations refer to cases with penetration factors \(\beta\) clearly beyond unity, see Table 1. To illustrate the evolution of a white dwarf that is just marginally disrupted, we show in Figure 4 a 0.6 \(M_\odot\) CO white dwarf that passes a 1000 \(M_\odot\) black hole with a penetration factor of \(\beta = 0.9\) (run 10, Table 1; keep in mind that the tidal radius as defined above is approximate and its exact value depends on the internal structure of the disrupted star). The upper panel shows snapshots at \(t = 0.34, 3.43, 6.86, 10.29, 13.72, 17.15, 20.58\), and 24.01 s after the simulation start at a separation of 3 \(R_\tau\). The lower panel shows late stages at \(t = 138.24\) and 380.53 s. The circle indicates in both cases the location of the tidal radius. At about 1.5 \(R_\tau\) the star becomes noticeably deformed and substantially spun up by the time it reaches pericenter (snapshot 3, upper panel). While receding from the black hole large, puffed-up lobes form at the extremes of the star. The inner lobe returns to the black hole (see \(t = 138.24\) s, lower panel) while the outer is ejected to infinity, both are connected by a well-defined and homogenous spaghetti-like tube of white dwarf debris.

The opposite limit, an extremely strong encounter, is illustrated in Figure 5. The snapshots are taken from our numerical simulations of a 0.2 \(M_\odot\) white dwarf approaching a 10^3 \(M_\odot\) black hole on a parabolic orbit with pericenter distance well within the tidal radius (\(R_p = R_\tau / 12\); run 1 in Table 1). While falling inwards toward the hole, the white dwarf develops a quadrupole distortion, which attains an amplitude of order unity by the time of disruption at \(R \sim R_\tau\).

When the star is still far from the black hole, it remains close to its initial stationary equilibrium state as characterized by the usual virial relation. This is because the timescale characterizing the rate of change of the tidal force will initially be very long compared with the intrinsic timescale characterizing the corresponding quadrupole oscillations of the star. The tidal bulge raised on the star by the black hole becomes an order unity distortion near the tidal radius. The resultant gravitational torque spins it up to a good fraction of its corotation angular velocity by the time it gets disrupted. The large surface velocities and the order unity tidal bulge combine to overcome the star’s self gravity and lead to the disruption of the star.

Our present investigation is concerned with cases of deeper penetration within the tidal radius so that the core as well as the envelope are affected. The behavior of a white dwarf passing well within the tidal radius exhibits special features (Carter
Figure 5. 0.2 $M_\odot$ white dwarf (modeled with more than 4 x $10^6$ SPH particles) approaching a $10^3 M_\odot$ black hole on a parabolic orbit with pericenter distance $R_p = R_\tau / 12$ is distorted, spun up during infall and then tidally disrupted. Shown are density cuts at various instants along the orbital $xy$-plane. The color bar gives the amplitude of log $\rho = [\log \rho_{\text{max}}, \log \rho_{\text{min}}]$ in cgs units: [4.0, 5.3] for $t = 6.2$ s, [3.6, 5.4] for $t = 6.8$ s, and [0, 3.4] for $t = 10.3$ s.

(A color version of this figure is available in the online journal.)

& Luminet 1983; Bicknell & Gingold 1983). As illustrated in Figure 6, the degenerate star is not only elongated along the orbital direction but also severely compressed perpendicular to the orbital plane. This anisotropy can be understood as arising from the fact that the principal tidal axes within the orbital plane will rotate prior to pericenter passage so that the corresponding effects of elongation and compression will roughly cancel out, whereas the third principal axis retains a fixed direction so that compression orthogonal to the orbital plane is uncontested (see, e.g., the appendix of Brassart & Luminet 2008). Each section of the star is squeezed through a point of maximum compression at a fixed point on the star’s orbit. This takes place on a timescale comparable to the crossing time of the star through periastron,$\delta t \sim \frac{R_{wd}}{v_p} \simeq 0.2 \left( \frac{M_{wd}}{0.6 M_\odot} \right)^{-1/6} \left( \frac{R_{wd}}{10^9 \text{ cm}} \right)^{3/2} \times \left( \frac{M_{BH}}{10^3 M_\odot} \right)^{-1/3} \text{s}, \tag{11}$(11)

where

$$v_p \sim (R_g/R_\tau)^{1/2}c \simeq 5 \times 10^9 \left( \frac{M_{wd}}{0.6 M_\odot} \right)^{1/6} \left( \frac{R_{wd}}{10^9 \text{ cm}} \right)^{-1/2} \times \left( \frac{M_{BH}}{10^3 M_\odot} \right)^{1/3} \text{ cm s}^{-1} \tag{12}$$

is the orbital velocity at periastron.

During this very short-lived phase, the star attains its maximum degree of compression (here by a factor $\sim 100$) although its orbital-plane section increases only by a factor of a few due to the continually changing directions of the two principal tidal axis (Figure 6). The distortions $\Delta R_{wd}/R_{wd} \sim 1$ impart supersonic bulk flow velocities of order

$$\frac{R_{wd}}{\delta t} \sim v_p \ll c_s \sim 2.8 \times 10^8 \left( \frac{R_{wd}}{10^9 \text{ cm}} \right)^{-1/2} \times \left( \frac{M_{wd}}{0.6 M_\odot} \right)^{1/2} \text{ cm s}^{-1} \tag{13}$$

during the drastic compression of the stellar material. This compression is thus halted by a shock (Kobayashi et al. 2004; Brassart & Luminet 2008), raising the matter, which then rebounds perpendicular to the orbital plane, to a higher adiabat (Figure 7). As a result, the temperatures increase sharply and trigger explosive burning (of He for the case shown in Figure 6).

The typical temperature of the shocked star and the thermal energy produced by shock heating can be roughly estimated from the virial theorem as

$$T \sim 6 \times 10^8 \left( \frac{R_{wd}}{10^9 \text{ cm}} \right)^{-1} \left( \frac{M_{wd}}{0.6 M_\odot} \right) \text{ K}, \tag{14}$$

and

$$E_{\text{therm}} \sim E_G \sim 10^{50} \left( \frac{R_{wd}}{10^9 \text{ cm}} \right)^{-1} \left( \frac{M_{wd}}{0.6 M_\odot} \right)^2 \text{ erg}. \tag{15}$$

Adiabatic compression alone can only increase the stellar surface and interior layer temperatures by a modest factor. If adiabatic compression was the only source of heating, the response to the flow to the varying potential would be $\sim R_{wd}/c_s \sim 4$ s, which is significantly longer than the dynamic crossing time.

The evolution of the abundances (for the simulation shown in Figure 5) is shown in Figure 8, within the 12 s during which the entropy increases the fastest. During the 1 s long
period of compression, the temperature increases beyond $3 \times 10^9$ K, approaching but not quite reaching NSE. During this brief period of compression, nuclei up to and beyond Si are synthesized. The initial composition was pure He and the final mass fraction in iron-group nuclei is about 15%. This result should be taken as a modest underestimate, since the seven species nuclear network only provides an approximation to the detailed nuclear processes. Postprocessing calculations, using a 300 isotope nuclear network over thermodynamic particle histories resulting from these calculations, show significant nuclear flow above silicon, for helium-rich portions of the gas with peak temperatures above $2 \times 10^9$ K. As a result, heavier elements (like calcium, titanium and chromium) would be made, accompanied by a modest increase in the energy generation. It is thus safe to conclude that the white dwarf is tidally ignited and that a sizable mass of iron-group nuclei is injected into the outflow.

The variation of the specific energy in the released gas, in the absence of explosive energy input, is determined mainly by the relative depth of a mass element across the disrupted star in the potential well of the black hole (Rees 1988). This is much larger than the binding energy and the kinetic energy generated by spin-up near pericenter. Even though the mean specific binding energy of the debris to the hole is comparable with the self-binding energy of the original star, the spread about this mean is larger by a factor $(M_{\text{BH}}/M_{\text{wd}})^{1/3}$. Nuclear energy released during the drastic compression and distortion of the stellar material, further enhances the spread in specific energies at pericenter (Figure 9), and, as a result, the mass fraction escaping on hyperbolic orbits is increased from $\sim 50\%$ to $\sim 65\%$ of the initial mass of the white dwarf. The mass fraction that is ejected rather than swallowed, though less spectacular than typical Type Ia supernovae (Hillebrandt & Niemeyer 2000), should have many distinctive observational signatures. First, the explosion itself should be different, since the disrupted, degenerate stars should be, on average, lighter than those exploding as Type Ia supernovae. Second, the spectra should exhibit large Doppler shifts, as the ejected debris would be expelled with speeds $\geq 10^4$ km s$^{-1}$. Finally, the optical light curve should be rather unique as a result of the radiating

Figure 6. Tidal deformation of the white dwarf before and after passage through pericenter, as the star attains its maximum degree of compression (same simulation as referred to in Figure 5). The panels in the upper rows show cuts of temperature (in units of $10^6$ K) and density (in cgs units) through the orbital $xy$-plane. The color bar gives the values of $T = [T_{\text{min}}, T_{\text{max}}]$ (log $\rho = [\log \rho_{\text{min}}, \log \rho_{\text{max}}]$): [0, 330] ($[3.6, 5.4]$) for $t = 6.8$ s, [0, 1280] ($[1.0, 6.0]$) for $t = 7.0$ s, and [0, 3200] ($[1.0, 6.0]$) for $t = 7.2$ s. The panels in the lower two rows show the temperature (in units of $10^6$ K) and column density (in cgs units) distributions in the $xz$-plane (averaged along the $y$-direction). The color bar gives the values of $T = [T_{\text{min}}, T_{\text{max}}]$ (log $\Sigma = [\log \Sigma_{\text{min}}, \log \Sigma_{\text{max}}]$): [0, 220] ($[12.0, 15.7]$) for $t = 6.8$ s, [0, 720] ($[12.0, 16.0]$) for $t = 7.0$ s, and [0, 1700] ($[12.0, 16.5]$) for $t = 7.2$ s. The dimension of the bar scale is $10^9$ cm. (A color version of this figure is available in the online journal.)

Figure 7. Evolution of the entropy $\varepsilon$ of the central portion of the disrupted white dwarf just after it attains its maximum degree of compression (same simulation as referred to in Figures 5 and 6). This compression is halted by a shock, raising the matter to a higher adiabat. The panels show entropy (in cgs units) cuts through the orbital $xy$-plane. (A color version of this figure is available in the online journal.)
material being highly squeezed into the orbital plane (one thus expects different timescales for conversion of nuclear energy to observable luminosity when compared with normal Type Ia events).

Although the explosion will increase the fraction of ejected debris, a good fraction (~35%) remains to be accreted on to the hole. The returning gas does not immediately produce a flare of activity from the black hole. First material must enter quasi circular orbits and form an accretion torus (Rees 1988; Evans & Kochanek 1989). Only then will viscous effects release enough binding energy to power a flare. The bound orbits are very eccentric, and the range of orbital periods is large. For white dwarfs, the orbital semimajor axis of the most tightly bound debris is

\[
a \sim 300 \left( \frac{M_{\text{BH}}}{10^3 M_\odot} \right)^{-1/3} \left( \frac{R_{\text{wd}}}{10^9 \text{ cm}} \right)^{-1/2} \left( \frac{M_{\text{wd}}}{0.6 M_\odot} \right)^{-2/3} R_g,
\]

and the period is only

\[
t_a \sim 150 \left( \frac{a}{300 R_g} \right)^{3/2} \left( \frac{M_{\text{BH}}}{10^3 M_\odot} \right)^{-1/2} \text{ s}.
\]

If the gaseous debris suffered no internal dissipation due to high viscosity or shocks, it would, after one or two orbital periods, form a highly elliptical disc with a big spread in apocenter distances between the most and least bound orbits,
Figure 10. Column density (in cgs units) and temperature (in units of $10^6$ K) distributions in the orbital plane of the bound debris minutes after disruption. The most tightly bound debris would transverse an elliptical orbit with major axis $\sim 300 R_\odot$ before returning to $R \approx R_\tau$ (same simulation as referred to in Figures 5–9).

(A color version of this figure is available in the online journal.)

Figure 11. Rate at which the 0.2 $M_\odot$ white dwarf debris returns to the vicinity of the black hole (same simulation as referred to in Figures 5–10).

but where at pericenter, $R_p$, the radial focusing of the orbits acts as an effective nozzle (Figures 4 and 10). After pericenter passage, the outflowing gas is on orbits which collide with the infalling stream near the original orbital plane at apocenter, giving rise to an angular momentum redistributing shock (Figure 10) much like those in cataclysmic variable systems. The debris raining down would, after little more than its free-fall time, settle into a disc. This orbiting debris starts to form when the most tightly bound debris falls back. The simulation shows that the first material returns at a time $\leq t_a$ with an infall rate of about $10^2 M_\odot \text{yr}^{-1}$ (Figure 11). Such high infall rates are expected to persist, relatively steadily, for at least a few orbital periods, before all the highly bound material rains down. The vicinity of the hole would thereafter be fed solely by injection of the infalling matter. The early mass infall rate is sensitive to the stellar structure (Lodato et al. 2009; Ramirez-Ruiz & Rosswog 2008), at late times, $t \geq t_{fb} \approx 600$ s, it drops off as $t^{-5/3}$ (Rees 1988; Phinney 1989). Once the torus is formed, it will evolve under the influence of viscosity, radiatively cooling winds and time-dependent mass inflow.

A luminosity comparable to the Eddington value, $\sim L_{\text{Edd}} = 10^{31}(M_{\text{BH}}/10^3 M_\odot) \text{ erg s}^{-1}$, can therefore only be maintained for at most a year; thereafter the flare would rapidly fade. It is clear that most of the debris would be fed to the hole far more rapidly than it could be accepted if the radiative efficiency
were high; much of the bound debris must either escape in a radiatively driven outflow or be swallowed inefficiently. The rise and the peak bolometric luminosity can be predicted with some confidence. However, the effective surface temperature (and thus the fraction of luminosity that emerges predominantly in the soft X-ray band) is harder to predict, as it depends on the size of the effective photosphere that shrouds the hole. Such transient signals, if detected, would be a compelling testimony for the presence of moderately massive black holes in the centers of globular clusters and dwarf galaxies.

4. INFLUENCE OF BLACK HOLE’S MASS ON DISRUPTION

The characteristic tidal radius, \( R_t \), for a given white dwarf star is solely determined by the black hole’s mass, while the strength of the tidal encounter is traditionally measured by the dimensionless parameter \( \beta = R_t / R_p \) (provided \( \beta \leq \beta_g \)). The aim of this section is to illustrate, with the help of a few specific calculations, the role of the black hole’s mass in shaping the evolution and ignition of the disrupted white dwarf. To quantify this, we have performed detailed three-dimensional, hydrodynamical calculations of the dynamics of a 0.2 \( M_\odot \) white dwarf approaching black holes of various masses on parabolic orbits with \( \beta = 5 \). For black hole masses \( 10^2 M_\odot \leq M_{BH} \leq 10^4 M_\odot \) and \( \beta = 5 \), a 0.2 \( M_\odot \) white dwarf can be disrupted without entering the strong relativistic domain (Figure 1). However, complete disruption occurs sufficiently close to the hole for a Newtonian approximation to be inadequate (Figure 12).

The behavior of a 0.2 \( M_\odot \) white dwarf approaching black holes of various masses is illustrated in Figure 13. As discussed previously, the effects of the black hole’s mass (for a fixed \( \beta \) and \( M_{wd} \)) will be to move the critical pericentric distance by \( M_{BH}^{1/3} \) and change the star’s crossing time through the maximum compression point by \( M_{BH}^{-1/3} \). As a result, stars approaching black holes of increasing mass will appear more elongated along the orbital plane, as each section of the star is squeezed faster through a point of maximum compression at a fixed point on the star’s increasingly extended orbit. Such stars are not only more elongated but are even more severely compressed into a prolate shape. If the initial white dwarf is made of pure He (as in the 0.2 \( M_\odot \) case), the combustion rate will be determined by the \( 3\alpha \) reaction, on timescale approximately given by (Khokhlov & Ergma 1986)

\[
t_b \approx 9.0 \times 10^{-4} T_9^3 \exp(4.4/T_9) \rho_6^{-2},
\]

where \( T_9 \) is the temperature in units \( 10^9 \) K and \( \rho_6 \) the density in \( 10^6 \) g cm\(^{-3}\). If the timescale on which the white dwarf can react, its dynamical timescale \( t_G = (G\rho_{wd})^{-1/2} \) is much shorter than the burning timescale \( t_b \), the star can expand rapidly enough to quench burning by a reduction of temperature and density. Appreciable burning will therefore only take place if \( t_b \ll t_G \). As illustrated in Figure 14, this comparison of timescales can be used as a simple estimate for whether or not substantial He burning will occur. Promising models for thermonuclear explosions are those that reach high temperatures at high densities (upper right corner). For a fixed \( \beta \), increasing \( M_{BH} \)}
and $M_t = t_s$, raises the nuclear energy release. For black hole masses $0.2 \times 10^5 M_\odot$ on a parabolic orbit with $\beta = 414$ ROSSWOG, RAMIREZ-RUIZ, & HIX Vol. 695

be, however, swallowed whole ($\beta = 5$). TIDAL DISRUPTION FOR A VARIETY OF WHITE DWARFS

In isolation, such low mass stars have life times much longer than the present age of the universe. White dwarfs are thought to be the final evolutionary state of all stars whose mass is below $\sim 10 M_\odot$ (Weidemann 1990). Stars with masses less than about half a solar mass become degenerate before helium ignition, and therefore will end their lifetimes as helium white dwarfs. In isolation, such low mass stars have life times much longer than the present age of the universe. Still, such He white dwarfs are observed, they are thought to be the result of the evolution of a close binary system (e.g., Heber 2002). Stars of low or medium mass, achieve helium burning and become carbon–oxygen white dwarfs (perhaps the most common sort). The observed white dwarf mass distribution is strongly peaked around $0.6 M_\odot$ (Kepler et al. 2007).

When a white dwarf is subject to strong tidal compression, triggering of nuclear processes in the stellar cores depends on initial composition. To investigate this dependence, we have performed calculations with various white dwarf masses/compositions. As outlined above, our white dwarf models are initialized with pure He composition for $M_{wd} < 0.6 M_\odot$ and 50% carbon and 50% oxygen otherwise.

Although the thermodynamical evolution and the nuclear energy release are very sensitive to the initial stellar composition (in this text, we have so far considered only a pure $^4$He 0.2 M\_\odot white dwarf), we found that ignition is in fact a natural outcome for white dwarfs of all masses passing well within the tidal radius. For example, a $C/O$ 1.2 $M_\odot$ white dwarf (here assumed to be composed of 50% carbon and 50% oxygen throughout the star) approaching a $500 M_\odot$ black hole on a parabolic orbit with pericenter distance $\beta = 3.2$ ignites (Figure 16) and, as a result, at least $0.6 M_\odot$ of Fe are synthesized in the flow (Figure 17). Similarly, at least 0.66 $M_\odot$ of Fe are synthesized when a $C/O$ 1.2 $M_\odot$ white dwarf approaches a $10^3 M_\odot$ black hole with $\beta = 3$. By contrast with the previous two cases, explosive energy release is too slow to release much energy on a dynamical timescale when a $C/O$ 1.2 $M_\odot$ white dwarf approaches a $10^3 M_\odot$ black hole with $\beta = 1.5$ and, as a result, only $1.4 \times 10^{-2} M_\odot$ of Fe are synthesized.

0.18 $M_\odot$ of Fe are synthesized when a typical $0.6 M_\odot$ $C/O$ white dwarf approaches a $500 M_\odot$ black hole with $\beta = 5$. To explore how sensitive our models are to the initial temperature, we have rerun this simulation, but now with a hot white dwarf. The initial specific energy of a cold ($T = 5 \times 10^4$ K) equilibrium model was adjusted in a way that the temperature varied linearly between $5 \times 10^4$ K in the center and $5 \times 10^4$ K at the stellar surface. In the subsequent relaxation the star expanded slightly, since the degeneracy was lifted to a small extent. As a result of the reduced degeneracy, lower peak temperatures and densities are reached (see Figure 18), reducing slightly the amount of nucleosynthesis which takes place. We consider the shown results as representative, but the actual range of behavior for given sets of masses and orbits may be wider due to gradients in temperature and nuclear composition which have not been explored systematically here. To summarize, in the most favorable cases, the nuclear energy release is comparable to that of typical Type Ia supernovae.

In the tidal pinching process, explosive nucleosynthesis is likely to proceed only for white dwarfs passing well within the tidal radius (Figure 19). Explosive energy release, as calculated here, appears to be a natural outcome for $0.6 M_\odot$ (1.2 $M_\odot$) white dwarfs approaching moderately massive black holes with $\beta \geq 5$ ($\beta \geq 3$).

5. TIDAL DISRUPTION FOR A VARIETY OF WHITE DWARFS

White dwarfs are thought to be the final evolutionary state of all stars whose mass is below $\sim 10 M_\odot$ (Weidemann 1990). Stars with masses less than about half a solar mass become degenerate before helium ignition, and therefore will end their lives as helium white dwarfs. In isolation, such low mass stars have life times much longer than the present age of the universe. Still, such He white dwarfs are observed, they are thought to be the result of the evolution of a close binary system (e.g., Heber 2002). Stars of low or medium mass, achieve helium burning and become carbon–oxygen white dwarfs (perhaps the most common sort). The observed white dwarf mass distribution is strongly peaked around $0.6 M_\odot$ (Kepler et al. 2007).

Figure 13. $0.2 M_\odot$ white dwarf approaching black holes of various masses on a parabolic orbit with $\beta = 5$. The panels in the show density (in cgs units) cuts through the orbital $xy$-plane. The color bar gives the values of $\log \rho = [\log \rho_{\text{min}}, \log \rho_{\text{max}}]$. For $M_{BH} = 10^5 M_\odot$, $\log \rho = [4.25, 5.3]$ for $t = 14.8 \times 10^3$, $[4.0, 5.4]$ for $t = 15.4$, and $[3.75, 5.8]$ for $t = 16.1$. For $M_{BH} = 5 \times 10^3 M_\odot$, $\log \rho = [4.15, 5.3]$ for $t = 15.6$, $[3.9, 5.65]$ for $t = 16.4$, and $[3.75, 5.8]$ for $t = 17.3$. For $M_{BH} = 10^3 M_\odot$, $\log \rho = [4.1, 5.4]$ for $t = 15.6$, $[3.9, 5.65]$ for $t = 16.3$, and $[3.4, 5.6]$ for $t = 17.2$. For $M_{BH} = 5 \times 10^4 M_\odot$, $\log \rho = [4.0, 5.4]$ for $t = 15.4$, $[3.7, 5.75]$ for $t = 16.0$, and $[3.4, 5.8]$ for $t = 16.3$.

(A color version of this figure is available in the online journal.)

6. GRAVITATIONAL WAVES

A white dwarf approaching the tidal radius will be disrupted in a single flyby. The resulting stellar debris trail is not compact enough to emit strong gravitational waves after leaving $R_p$. The detectable gravitational wave signal will therefore have a burstlike behavior, roughly characterized by an amplitude $h$ and a duration $t \sim 1/f$, where

$$h \approx \frac{G \tilde{Q}}{c^4} \frac{\dot{M}}{D} \sim 7 \times 10^{-20} \beta \left(\frac{D}{10 \text{ kpc}}\right)^{-1} \times \left(\frac{M_{wd}}{0.6 M_\odot}\right)^{4/3} \left(\frac{R_{wd}}{10^9 \text{ cm}}\right)^{-1} M_{BH}^{2/3},$$

(19)

When a white dwarf is subject to strong tidal compression, triggering of nuclear processes in the stellar cores depends on initial composition. To investigate this dependence, we have performed calculations with various white dwarf masses/compositions. As outlined above, our white dwarf models are initialized with pure He composition for $M_{wd} < 0.6 M_\odot$ and 50% carbon and 50% oxygen otherwise.

Although the thermodynamical evolution and the nuclear energy release are very sensitive to the initial stellar composition (in this text, we have so far considered only a pure $^4$He 0.2 M\_\odot white dwarf), we found that ignition is in fact a natural outcome for white dwarfs of all masses passing well within the tidal radius. For example, a $C/O$ 1.2 $M_\odot$ white dwarf (here assumed to be composed of 50% carbon and 50% oxygen throughout the star) approaching a $500 M_\odot$ black hole on a parabolic orbit with pericenter distance $\beta = 3.2$ ignites (Figure 16) and, as a result, at least $0.6 M_\odot$ of Fe are synthesized in the flow (Figure 17). Similarly, at least 0.66 $M_\odot$ of Fe are synthesized when a $C/O$ 1.2 $M_\odot$ white dwarf approaches a $10^3 M_\odot$ black hole with $\beta = 3$. By contrast with the previous two cases, explosive energy release is too slow to release much energy on a dynamical timescale when a $C/O$ 1.2 $M_\odot$ white dwarf approaches a $10^3 M_\odot$ black hole with $\beta = 1.5$ and, as a result, only $1.4 \times 10^{-2} M_\odot$ of Fe are synthesized.

0.18 $M_\odot$ of Fe are synthesized when a typical $0.6 M_\odot$ $C/O$ white dwarf approaches a $500 M_\odot$ black hole with $\beta = 5$. To explore how sensitive our models are to the initial temperature, we have rerun this simulation, but now with a hot white dwarf. The initial specific energy of a cold ($T = 5 \times 10^4$ K) equilibrium model was adjusted in a way that the temperature varied linearly between $5 \times 10^4$ K in the center and $5 \times 10^4$ K at the stellar surface. In the subsequent relaxation the star expanded slightly, since the degeneracy was lifted to a small extent. As a result of the reduced degeneracy, lower peak temperatures and densities are reached (see Figure 18), reducing slightly the amount of nucleosynthesis which takes place. We consider the shown results as representative, but the actual range of behavior for given sets of masses and orbits may be wider due to gradients in temperature and nuclear composition which have not been explored systematically here. To summarize, in the most favorable cases, the nuclear energy release is comparable to that of typical Type Ia supernovae.

In the tidal pinching process, explosive nucleosynthesis is likely to proceed only for white dwarfs passing well within the tidal radius (Figure 19). Explosive energy release, as calculated here, appears to be a natural outcome for $0.6 M_\odot$ (1.2 $M_\odot$) white dwarfs approaching moderately massive black holes with $\beta \geq 5$ ($\beta \geq 3$).
Figure 14. Importance of the central black hole’s mass in determining the behavior of a 0.2 $M_{\odot}$ white dwarf passing within $\beta = 5$ (same simulations as referred to in Figure 13). The evolution of the compressed, and tidally disrupted white dwarf in the $\rho$-$T$ plane. The hottest 10% of the particles are identified and their average temperature (in units of $10^6$ K) is plotted as a function of their average density (in cgs units). These trajectories always start cold and dense (right lower corner) and become hot and during the black hole flyby. If the timescale on which the white dwarf can react (dynamical timescale $t_G$) is longer than the burning timescale ($t_b$, see Equation (18)), the star cannot expand rapidly enough to quench burning.

(A color version of this figure is available in the online journal.)

Figure 15. Energy generated in nuclear burning (in units of the star’s binding energy) as a function of time (same simulations as referred to in Figures 13 and 14). Here time is measured in units of the dynamical timescale ($t_G$) of the initial 0.2 $M_{\odot}$ white dwarf. In addition, the time axis has been shifted so that the maximum $E_b$ occurs at $t = 0$.

(A color version of this figure is available in the online journal.)

and

$$ f \sim \left( \frac{GM_{BH}}{R_p^3} \right)^{1/2} \sim 0.3 \beta^{3/2} \left( \frac{M_{\text{wd}}}{0.6 M_{\odot}} \right)^{1/2} \left( \frac{R_{\text{wd}}}{10^9 \text{ cm}} \right)^{-3/2} \text{ Hz.} $$

The gravitational wave amplitudes $h_+$ and $h_\times$ shown in Figure 21 are calculated in the quadrupole approximation. The reduced quadrupole moments can be written in terms of the SPH-particle properties (Centrella & McMillan 1993)

$$ I_{jk} = \sum_i m_i \left( x_{ji} x_{ki} - \frac{1}{3} \delta_{jk} r_i^2 \right). $$

The second time derivatives, $I_{jk}$, can then be expressed in terms of the particle properties by simple, direct differentiation. The retarded gravitational wave amplitudes for a distant observer
Figure 16. 1.2 $M_\odot$ white dwarf (modeled with more than $5 \times 10^5$ SPH particles) approaching a 500 $M_\odot$ black hole on a parabolic orbit with pericenter distance $r_{\text{min}} = r_T/3.2$. Shown are temperature (in units of $10^6$ K) and surface density (in cgs units) cuts at various instants along the orbital ($x$, $y$)-plane.

(A color version of this figure is available in the online journal.)

Figure 17. Evolution of the abundances during the disruption and ignition of a 1.2 $M_\odot$ white dwarf approaching a 500 $M_\odot$ black hole (same simulation as referred to in Figure 16).

(A color version of this figure is available in the online journal.)
Figure 18. Comparison of the average properties of the hottest 10% of the SPH particles for an initially cold and warm star (runs 8 and 9 in Table 1).

(A color version of this figure is available in the online journal.)

Figure 19. Evolution of the compressed and tidally disrupted white dwarfs of various masses in the $\rho$–$T$ plane: a 0.2 He $M_\odot$ white dwarf approaching a $10^3 M_\odot$ black hole with $\beta = 12$; a 0.6 C/0 $M_\odot$ white dwarf approaching a $10^4 M_\odot$ black hole with $\beta = 1.5$ and a 1.2 C/0 $M_\odot$ white dwarf approaching a $10^5 M_\odot$ black hole with $\beta = 5$. The hottest 10% of the particles are identified and their average temperature (in units of $10^6$ K) is plotted as a function of their average density (in cgs units). These trajectories always start cold and dense (right lower corner) and become hot and during the black hole flyby. For the 1.2 $M_\odot$ and 0.6 $M_\odot$ cases, the timescale on which the white dwarf can react is similar to the burning timescale and the star expands rapidly enough to quench explosive energy release.

(A color version of this figure is available in the online journal.)

along the $z$-axis at distance $D$ are given by

$$D h_x = \frac{G}{c^2} (\ddot{I}_{xx} - \ddot{I}_{yy})$$

and

$$D h_x = \frac{2G}{c^2} I_{xy}.$$  (23)

Both duration and amplitudes of the gravitational wave bursts that are produced by the disruption calculations (Figure 21) are in agreement with the simple estimates given above.

7. DISCUSSION

This paper presents a computational investigation of the mechanical and nuclear evolution of white dwarfs passing well within the tidal radius of a moderately massive black hole. A comprehensive top to bottom approach is adopted: we follow the tidal disruption and compression leading up to the ignition of the white dwarf; the complex propagation of the nuclear energy release through the star; the resultant gravitational wave signal and the subsequent accretion of the bound debris.

This paper has outlined several potentially observable effects. The detection of a peculiar, underluminous thermonuclear
Figure 20. Frequency and amplitude of gravitational radiation produced by the disruption of white dwarfs approaching an intermediate-mass black hole, assuming $\beta = 1$ and that the cluster hosting the black hole is at $D = 8$ kpc.

Figure 21. Gravitational wave amplitudes. Top panel: gravitational radiation produced by the disruption of a $0.2 M_\odot$ white dwarf approaching a $10^3 M_\odot$ black hole with $\beta = 12$. Bottom panel: gravitational radiation produced by the disruption of a $0.6 M_\odot$ white dwarf approaching a $10^4 M_\odot$ black hole with $\beta = 1.5$.

(A color version of this figure is available in the online journal.)
explosion (Rosswog et al. 2008b) accompanied by a thermal transient signal of predominantly soft X-rays with a peak luminosity $L \sim L_{\text{Edd}} = 10^4 M_{\text{BH,1}} \text{erg s}^{-1}$, fading within a year would, if detected, be a compelling testimony for the existence of a new mechanism by which white dwarf ignition can be achieved. Although the thermodynamical and nuclear energy of the star is sensitive to the initial composition, we found that thermonuclear ignition is a natural outcome for white dwarfs of all masses passing well within the tidal radius, with lighter stars requiring deeper penetration into the tidal radius due to their lower densities. For simplicity, we have instantiated our initial carbon–oxygen white dwarf models ($M_{\text{WD}} \geq 0.6 M_{\odot}$) as homogeneously mixed stars with a 50% mass fraction of each nucleus. While such internal chemical profiles are likely accurately realized in nature in very massive white dwarfs ($\sim 1 M_{\odot}$) (Mazzitelli & Dantona 1986), for lower masses the gravothermal adjustment of the interior during the cooling phase produces oxygen-enhanced stellar cores surrounded by very carbon-rich mantles ($X_C \sim 0.8$). The exact radial distribution depends on the exact value of $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate and the details of how convection proceeds (Mazzitelli & Dantona 1986; Salaris et al. 1997; Straniero et al. 2003), but this general stratification tendency is well established. Thus, the disruption of a standard $0.6 M_{\odot}$ white dwarf should produce a highly carbon-enriched remnant atmosphere. Maybe, the recently detected carbon-rich transient SCP 06F6 accompanied by an X-ray signal (Gaensicke et al. 2008) is already the first example for this class of object.

The gravitational forces from the central black hole are currently calculated using a Paczynski–Wiita pseudopotential but our goal is to incorporate the effects of general relativity, as white dwarfs passing well within the tidal radius cannot be disrupted without entering the strong relativistic regime. The form of the black hole (Schwarzschild or Kerr?) then has an important quantitative effect, as does (for a rotating Kerr hole) the orientation of the stellar orbit relative to the black hole spin axis. The orbits are then not ellipses, but may turn through $2\pi$ or even more (Luminet & Pichon 1989). A fluid element, which in the case of an elliptical orbit would cross the orbital plane just once, may then have two or more traversals. This opens up the possibility of multiple shocks.

For a white dwarf which does not pass close enough to the hole to release much energy on a dynamical timescale, adiabatic cooling would severely reduce the internal radiative content before the debris became translucent (just as a supernova would be optically inconspicuous in the absence of continuing energy injection in the months after the explosion). There would be no transient until, as discussed above, the bound debris fell back onto the hole after $t_b$. The integrated output from this flare could, in principle, amount to a few percent of the white dwarf’s rest mass, but would probably be significantly less, because most of the debris would be fed to the black hole far more rapidly than it could be accepted if the radiative efficiency were high; much would then be swallowed inefficiently or most likely escape in radiatively driven directed outflow: its ram pressure and subsequent heating could inhibit the steady accretion that would otherwise be inevitable in any galaxy or globular cluster harboring a moderately massive black hole.

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