Correlation between Decay Rate and Amplitude of Solar Cycles as Revealed from Observations and Dynamo Theory

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Abstract Using different proxies of solar activity, we have studied the following features of solar cycle. (i) A linear correlation between the amplitude of cycle and its decay rate, (ii) a linear correlation between the amplitude of cycle \(n\) and the decay rate of cycle \((n-1)\) and (iii) an anti-correlation between the amplitude of cycle \(n\) and the period of cycle \((n-1)\). Features (ii) and (iii) are very useful because they provide precursors for future cycles. We have reproduced these features using a flux transport dynamo model with stochastic fluctuations in the Babcock-Leighton \(\alpha\) effect and in the meridional circulation. Only when we introduce fluctuations in meridional circulation, we are able to reproduce different observed features of solar cycle. We discuss the possible reasons for these correlations.

Keywords: Solar Cycle, Observations; Solar Cycle, Models; Magnetic fields, Models

1. Introduction

Solar cycles are asymmetric with respect to their maxima, the rise time being shorter than the decay time. While the cycle amplitude (peak value) and the duration have cycle-to-cycle variations, we find some correlations among different quantities connected with the solar cycle. Since 1935, it has been realized that the stronger cycles take less time to rise than the weaker ones (Waldmeier 1935). This anti-correlation between rise times and peak values of the solar cycle is popularly known as the Waldmeier effect. Karak and Choudhuri (2011) have defined this aspect of the Waldmeier effect as WE1, whereas the correlation between the rise
rates and the peak values is called WE2 (see also [Cameron and Schüssler, 2008]). Although WE2 is a more robust feature of the solar cycle, [Karak and Choudhuri, 2011] have shown that both WE1 and WE2 exist in many proxies of the solar cycle. WE2 provides a valuable precursor for predicting solar cycles because one can predict the strength of a cycle once it has just started (see [Lantos, 2000; Kane, 2008]).

The declining phase of the cycle also provides important clues for understanding long-term variations. We find that stronger cycles not only rise rapidly but also fall rapidly (shorter decay time). This results in a good correlation between the decay rate and amplitude of the same cycle. However, defining the decay rate differently, [Cameron and Schüssler, 2008] did not find a significant correlation between the decay rate and amplitude. Furthermore, we also find a strong correlation between the decay rate of the current cycle and the amplitude of the next cycle. The decay time, however, is found to have no correlation with the amplitude of the same cycle. Another important feature observed is that the amplitude of the cycle is inversely correlated with the period of the previous cycle [Hathaway, Wilson, and Reichmann, 2002; Solanki et al., 2002]. These two correlations again provide promising precursors to predict the strength of the future cycle [Solanki et al., 2002; Watari, 2008].

Apart from showing these correlations from observational data, we also attempt to provide theoretical explanations for them. A dynamo mechanism operating in the solar convection zone is believed to be responsible for producing the solar cycle. It is generally accepted that the strong toroidal field (responsible for the formation of bipolar sunspots) is produced from the poloidal field by differential rotation in the solar convection zone [Parker, 1955a]. This is the first part of solar dynamo theory. Due to magnetic buoyancy [Parker, 1955b], the flux tubes of toroidal field erupt out through the surface to form bipolar sunspot regions. These bipolar sunspots acquire tilts due to the action of the Coriolis force during their journey through the convection zone, giving rise to Joy’s law [D’Silva and Choudhuri, 1993]. To complete dynamo action, the toroidal field has to be converted back into the poloidal field. One possible mechanism for generating the poloidal field is the Babcock-Leighton (B-L) process [Babcock, 1961; Leighton, 1969], for which we now have strong observational support [Dasi-Espuig et al., 2010; Kitchatinov and Olemskov, 2011; Muñoz-Jaramillo et al., 2013]. In this process, the fluxes of tilted bipolar active regions spread on the solar surface through different processes (diffusion, meridional circulation, differential rotation) to produce the poloidal field. A model of the solar dynamo that includes a coherent meridional circulation and this B-L mechanism for the generation of the poloidal field is called the flux transport dynamo model. This model was proposed in the 1990s [Wang, Sheeley, and Nash, 1991; Durney, 1995; Choudhuri, Schüssler, and Dikpati, 1995] and has been successful in reproducing many observed regular as well as irregular features of the solar cycle [Charbonneau and Dikpati, 2000; Küker, Rüdiger, and Schuck, 2001; Nandy and Choudhuri, 2002; Chatterjee, Nandy, and Choudhuri, 2004; Guerrero and Muñoz, 2004; Choudhuri and Karak, 2009; Hotta and Yokoyama, 2010; Karak and Choudhuri, 2013]. Recently Charbonneau (2010) and Choudhuri (2011) have reviewed this dynamo model.
An important ingredient in flux transport dynamo is the meridional circulation, which is not completely constrained either from observations or from theoretical studies. Until recently not much was known about the detailed structure of the meridional circulation in the convection zone (Zhao et al., 2013; Schad, Timmer, and Roth, 2013). Therefore, most of the dynamo models use a single-cell meridional circulation in each hemisphere. However, very recently Hazra, Karak, and Choudhuri (2014) have shown that a complicated multi-cellular meridional circulation also retains many of the attractive features of the flux transport dynamo model if there is an equator-ward propagating meridional circulation near the bottom of the convection zone or if there is an equator-ward turbulent pumping (Guerrero and de Gouveia Dal Pino, 2008).

Since we want to do a theoretical study of the irregularities in the solar cycle, let us consider the sources of irregularities in the flux transport dynamo model that make different solar cycles unequal. At present we know two major sources: (i) variations in the poloidal field generation due to fluctuations in the B-L process (Choudhuri, Chatterjee, and Jiang, 2007; Goel and Choudhuri, 2009) and (ii) variations in the meridional circulation (Karak, 2010; Karak and Choudhuri, 2011). Direct observations of the polar field during last three cycles (Svalgaard, Cliver, and Kamide, 2005), as well as its proxies such as the polar faculae and the active network index available for about last 100 years (Muñoz-Jaramillo et al., 2013; Priyal et al., 2014), indicate large cycle-to-cycle variations of the polar field. The poloidal field generation mechanism mainly depends on the tilts of active regions, their magnetic fluxes and the meridional circulation, all of which have temporal variations. Particularly the scatter of tilt angles around the mean, caused by the effect of convective turbulence on rising flux tubes (Longcope and Choudhuri, 2002), has been studied by many authors (Wang and Sheeley, 1989; Dasi-Espuig et al., 2010). Recently Jiang, Cameron, and Schüssler (2014) found that the tilt angle scatter led to a variation in the polar field by about 30% for cycle 17. In fact, even a single big sunspot group with large tilt angle and large area appearing near the equator can change the polar field significantly (Cameron et al., 2013). On the other hand, for the meridional circulation, we have some surface measurements for about last 20 years, showing significant temporal variations (Hathaway and Rightmire, 2010). Although our theoretical understanding of the meridional circulation is very limited, a few existing spherical global convection simulations do show significant variations in the meridional circulation (Passos, Charbonneau, and Beaudoin, 2012; Karak et al., 2014). Introducing randomness in the poloidal field generation and in the meridional circulation, Karak and Choudhuri (2011) have been able to reproduce the Waldmeier effect in their high diffusivity dynamo model. When the meridional circulation becomes weaker, the cycle period and hence the rise time becomes longer. The longer cycle period allows the turbulent diffusion to act for a longer time, making the cycle amplitude weaker (Yeates, Nandy, and Mackay, 2008; Karak, 2010) and leading to the Waldmeier effect. The variation of the meridional circulation is crucial in reproducing this effect.

The motivation of the present work is to explore how the decay rates of cycles are related to their amplitudes in a flux transport dynamo model, with the aim of explaining the observed correlations mentioned earlier. The presentation of
the paper is following. In the next section, we summarize some of the features of solar cycle that are often considered as precursors of the solar cycle. In Section 3, we present a brief summary of our flux transport dynamo model and then in Sections 4 we introduce suitable stochastic fluctuations in the poloidal field and the meridional circulation, in order to reproduce various observed features of the solar cycle. Finally the last section summarizes our conclusions.

2. Observational studies

We have used three different observational data sets: (i) Wolf sunspot number\(^1\) (cycles 1–23), (ii) sunspot area\(^2\) (cycles 12–23), and (iii) 10.7 cm radio flux\(^3\) (available only for last 5 cycles). These parameters are very good proxies of magnetic activity and are often used to study the solar cycle (Hathaway et al. 2002). To minimize the noise while keeping the underlying properties unchanged, we smooth these monthly data using a Gaussian filter having FWHM of 1 year. We also average the data with FWHM of 2 years to check how the results change with the filtering.

2.1. Correlation between the decay rate and the cycle amplitude

We have calculated the decay rate at three different phases of the descending phase of the cycle, namely, early phase, late phase and entire phase. For the early phase, the decay rate is taken as the slope between two points with a separation of 1 year with the first point one year after the cycle peak, whereas for the late phase the second point is taken 1 year before the cycle minimum. Here we exclude one year after the maximum when computing decay rate for the early phase because sometimes the cycle peaks are not so prominent. While computing the decay rate for the late phase we also exclude 1 year before the minimum just to avoid the effect of overlapping between two cycles during solar minimum. Finally, the decay rate of the entire decay part (i.e., entire phase) is taken as the average of the individual decay rates computed at 4 different locations with a separation of one year starting from early phase to the late phase. In Figures 1(a), (b) and (c), we show the correlations of the cycle amplitudes with the decay rates of the entire phase computed from sunspot number, sunspot area and 10.7 cm radio flux data, respectively.

We would like to point out that Cameron and Schüssler (2008) have computed the decay rate from the intervals of two fixed values of solar activity and they did not get significant correlation between the decay rate and the amplitude (see right column, Fig. 2 of their paper). The reason of not finding significant correlation is that they have calculated the decay rate in the late phase of the cycle, i.e. near the tail of the cycle where the rate of decay is really very small.

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\(^1\)http://solarscience.msfc.nasa.gov/greenwch/spot_num.txt
\(^2\)http://solarscience.msfc.nasa.gov/greenwch/sunspot_area.txt
\(^3\)http://www.ngdc.noaa.gov/stp/solar/flux.html
We find that their values are comparable with our decay rates computed in the late phase. In 4th and 5th columns of Table 1 we have listed our values and the values computed following Cameron and Schüssler (2008) method (hereafter referred as CS08). It is interesting to note that even for the radio flux data for which we have only 5 data points, we get strong correlation; see Table 1 for details. Therefore we can see that if we determine the decay rates from the entire phase of the solar cycle or the early phase, we find strong correlation with the amplitude. Thus, to determine the decay rate from descending part of the solar cycle, we need to consider the entire decay phase of the cycle, which provides a better estimate than CS08.

2.2. Correlation between the decay rate and the next cycle amplitude

Next we find that there is a significant correlation between the amplitude of cycle and the decay rate of the previous cycle. Again we find this correlation for all the data sets considered here (see Table 1). However in Figure 2(a) we show this correlation only for sunspot number. Note that here the decay rates have been calculated from the entire decay phase as discussed in Section 2.1. This correlation suggests that the decay rate of a cycle carries some information of the strength of the next cycle. It is interesting to note that when we look at

![Figure 1. Scatter plots of the decay rate and the amplitude of the same cycle computed from (a) sunspot number, (b) sunspot area, and (c) 10.7 cm radio flux data. In all these cases the original monthly data are smoothed using a Gaussian filter with FWHM of 2 years. The straight line in each plot is best linear fit of data. The correlation coefficients (r) and the significance levels are also given in each plot.](image-url)
Table 1. Correlation coefficients between different quantities of the solar cycle.

| Data set       | FWHM | Correlation coefficients of the decay rate with the amplitude of | Correlation between the amplitude and the previous cycle period |
|----------------|------|---------------------------------------------------------------|---------------------------------------------------------------|
|                |      | Same cycle                                                  | Next cycle                                                   |
|                |      | Entire phase | Late decay phase | Early phase | Entire phase | Late phase |
| Sunspot number | 1 yr | 0.79       | 0.21             | 0.22       | 0.67       | 0.55       | 0.61       | -0.64          |
|                | 2 yr | 0.86       | 0.45             | –          | 0.86       | 0.65       | 0.85       | -0.67          |
| Sunspot area   | 1 yr | 0.84       | 0.20             | 0.11       | 0.69       | 0.14       | 0.37       | -0.49          |
|                | 2 yr | 0.91       | 0.53             | –          | 0.92       | 0.39       | 0.66       | -0.60          |
| Radio flux     | 1 yr | 0.86       | -0.11            | 0.14       | 0.93       | -0.42      | 0.64       | 0.11           |
|                | 2 yr | 0.82       | 0.24             | –          | 0.95       | -0.43      | 0.46       | 0.09           |

Figure 2. Scatter plots showing the correlation of the amplitude vs. the decay rate of the previous cycle computed from sunspot number data (smoothed with FWHM of 2 year). In (a) the decay rate is computed from the entire decay phase, whereas in (b) it is at late decay phase.

This correlation with the decay rate computed in the late phase, the correlations become even stronger; see Figure 2(b). In 7th and 8th column of Table 2, we show both correlations for all three data sets. These results suggest that particularly the late phase of the cycle carries more information of the forthcoming cycle.

Cameron and Schüssler (2007) (also see Brown, 1976) have observed similar feature that the activity level during the solar minimum is an indicator for the
Correlation of the amplitude of the solar cycle with decay rate and period

strength of the next solar cycle and argued that this is caused by overlap between two cycles during solar minimum.

In all our theoretical calculations (subsequent section), while studying the correlation between the amplitude and the decay rate of the same cycle, we shall consider the decay rate of the entire phase, but for the correlation with the next cycle we shall consider only the late-phase decay rate.

Figure 3. Scatter plot of $n$th cycle amplitude and the amplitude of the next $n+1$ cycle from sunspot number data (smoothed with FWHM of 2 years).

Since the decay rate of the cycle $n$ is correlated both with the amplitude of cycle $n$ (Figure 1) and the amplitude of cycle $n + 1$ (Figure 2), one question that naturally arises is whether the amplitude of cycle $n$ and the amplitude of cycle $n + 1$ are themselves correlated. We show a correlation plot between these amplitudes in Figure 3, demonstrating that there is not a significant correlation. The challenge before a theoretical model is, therefore, to explain how the decay rate of cycle $n$ is correlated both with the amplitude of cycle $n$ and the amplitude of cycle $n + 1$, while these amplitudes themselves do not have a strong correlation.

2.3. Correlation between the cycle period and the next cycle amplitude

Finally, we also find that the shorter cycles are followed by stronger cycles and vice versa. This produces an anti-correlation between the amplitude of a cycle and the period of the previous cycle. Figure 4 shows this correlation from sunspot number data (smoothed using a Gaussian filter with FWHM of 2 years). The correlation coefficients from other data are listed in Table 1. For all data we have taken the period of the cycle just as the time difference between two successive minima.
3. Theoretical framework of the Dynamo model

We carry out our theoretical studies using the flux transport dynamo model originally presented by Chatterjee, Nandy and Choudhuri (2004). In this model, the evolution of the axisymmetric two-dimensional magnetic field is governed by following two equations:

$$\frac{\partial A}{\partial t} + \frac{1}{s} (\mathbf{v} \cdot \nabla)(sA) = \eta_p \left( \nabla^2 - \frac{1}{s^2} \right) A + S_{BL}(r, \theta; B),$$

$$\frac{\partial B}{\partial t} + \frac{1}{r} \left[ \frac{\partial}{\partial r}(rv_r B) + \frac{\partial}{\partial \theta}(v_\theta B) \right] = \eta_t \left( \nabla^2 - \frac{1}{s^2} \right) B + s(B_p \cdot \nabla)\Omega + \frac{1}{r} \frac{\partial \eta_t}{\partial r} \frac{\partial B}{\partial r},$$

where $s = r \sin \theta$, $B(r, \theta)$ is the toroidal component of the magnetic field, $A(r, \theta)$ is the vector potential of the poloidal field, $\mathbf{v} = v_r \hat{r} + v_\theta \hat{\theta}$ is the velocity of the meridional flow, $\Omega$ is the internal angular velocity of the Sun and $\eta_t$, $\eta_p$ are the turbulent diffusivities of the toroidal and the poloidal fields. Since the detailed discussion of the parameters and boundary conditions are given in Chatterjee, Nandy and Choudhuri (2004) and Karak and Choudhuri (2011), here we do not discuss them again. We only make a few remarks about magnetic buoyancy and about the term $S_{BL}(r, \theta; B)$ appearing in (1), which captures the longitude averaged B-L mechanism.

Let us discuss how the magnetic buoyancy is treated in this model. When the toroidal field above the tachocline ($r = 0.71 R_\odot$) at any latitude exceeds
a certain value, a fraction of it is reduced there and the equivalent amount of this field is added on the solar surface. Then this local toroidal field near the surface is multiplied by a factor $\alpha$ to give the poloidal field. The source term in Equation (1), therefore, is

$$S_{BL}(r, \theta; B) = \alpha B(r, \theta, t),$$

where

$$\alpha = \frac{\alpha_0}{4} \cos \theta \left[ 1 + \text{erf} \left( \frac{r - 0.95 R_\odot}{0.03 R_\odot} \right) \right] \left[ 1 - \text{erf} \left( \frac{r - R_\odot}{0.03 R_\odot} \right) \right],$$

with $\alpha_0 = 30 \text{ m s}^{-1}$. Now our job is to use this model to study the observed features of solar cycle reported in previous sections. To study any irregular feature of the solar cycle, we have to make the cycles unequal by introducing randomness in this regular dynamo model, as we discuss in the following sections.

4. Results of theoretical modeling

4.1. Fluctuations in the poloidal field generation

We have discussed in the Introduction that the Sun does not produce equal amount of poloidal field at the end of every cycle and that the generation of the poloidal field involves randomness. Therefore, similar to adding stochastic fluctuations in the traditional mean-field alpha (Choudhuri, 1992), adding stochastic fluctuations in the B-L $\alpha$ has become a standard practice in the flux transport dynamo community (Charbonneau and Dikpati, 2000; Jiang, Chatterjee, and Choudhuri, 2007; Karak and Nandy, 2012). In the present work, first we introduce stochastic noise in the B-L $\alpha$ in the following way:

$$\alpha_0 \rightarrow \alpha_0 + \sigma(t, \tau) \alpha'_0,$$

where $\tau$ is the coherence time during which the fluctuating component remains constant and $\sigma$ is a uniformly distributed random number in the interval [-1, 1]. Considering the typical decay time of the active regions by surface flux transport process, we fix the coherence time within 0.5 – 2 months. To see a noticeable effect, we add 75% fluctuations in $\alpha$ (i.e., $\alpha'_0/\alpha_0 = 0.75$) with coherence time of 1 month. From this stochastically forced model we have to calculate a measure of the theoretical sunspot number. We consider the magnetic energy density ($B^2$) of toroidal field at latitude $15^\circ$ at the base of the convection zone ($r = 0.7 R_\odot$) as a proxy of sunspot number (this was done by Charbonneau and Dikpati, 2000). Note that absolute value of the theoretical sunspot number does not have any physical meaning. Therefore, we scale it by an appropriate factor to match it with the observed sunspot number. From the time series of theoretical sunspot number, we calculate the cycle periods and decay rates in the same way as we have done for the observational data.
Figure 5. Results from stochastically forced dynamo model with B-L A fluctuations: Scatter plots showing the correlations between (a) the decay rate and the amplitude of cycle n, (b) the decay rate of cycle n and the amplitude of cycle n + 1, (c) the period of cycle n and the amplitude of cycle n + 1.

In Figure 5(a) we show the correlation between the decay rates and the amplitudes of the same cycles. We see a positive correlation as in the observed data presented in Figure 1. It is easy to understand the reason behind getting this positive correlation. Since we have kept meridional circulation fixed, the periods of the solar cycle do not vary much but the cycle strengths do vary due to the fluctuations in the poloidal field generation. Therefore, when the amplitude of a cycle increases while its period remains approximately fixed, the cycle has to decay rapidly. Hence we find that the stronger cycles decay faster than the weaker cycles, producing the positive correlation seen in Figure 5(a). However, we see in Figure 5(b) that there is not much correlation between the decay rate of the cycle n and the amplitude of the next cycle n + 1 and we are unable to explain the observed correlation seen in Figure 2. Note that for Figure 5(a) the decay rates are calculated from the entire decaying part of the cycle which is more appropriate definition of the decay rate as we argued in Section 2, whereas for Figure 5(b) it is computed at the late decay phase because observationally we find strong correlation when decay rate is computed in late decay phase only. Finally we see in Figure 5(c) that in this study the observed anti-correlation between the period of cycle n and the amplitude of cycle n+1 (shown in Figure 4) is also not reproduced. Note that the period does not vary too much when the meridional circulation is kept constant.
To sum up, when we introduce fluctuations in the poloidal field generation mechanism, we can explain the observed correlation between the decay rate and the amplitude of the cycle shown in Figure 1, but we cannot explain the other observed correlations presented in Figures 2 and 4.

4.2. Fluctuations in the meridional circulation

Next we introduce the other important source of fluctuations in the flux transport dynamo model, namely, variations of the meridional circulation. Although we have some observational results of the meridional circulation variations near the solar surface for the last 15 – 20 years, we do not have long data to conclude the nature of long-term variations (Chou and Dai, 2001; Hathaway and Rightmire, 2010). However, there are indirect evidences for the variation of the meridional circulation over a long time (Lopes and Passos, 2008; Karak, 2010; Passos and Lopes, 2012). Particularly, Karak and Choudhuri (2011) have used the durations of the past cycles to argue that the meridional circulation has long-term variations with the coherence time of probably 20 – 45 years. There can also be short-term variations in the meridional circulation whose time scale may be related to the convective turnover time of the solar convection zone. Such variations with the time scale from a few months to a year are also observed in global magnetohydrodynamic simulations (Karak et al., 2014). In this work, we vary the amplitude of the meridional circulation in the same way as we have done for the \( \alpha \) term but with a different coherence time. We show the results of simulations with 30% fluctuations in the meridional circulation with coherence time of 30 years. How the coherence time affects the results will be discussed later. With 30% level of fluctuations, we get variations of the amplitude and of the period in our theoretical model comparable to the observational data. As in § 4.1, we take the time series \( (B^2) \) at latitude 15° at the base of the convection zone as our proxy of sunspot activity and calculate the required correlations from it. The relevant correlation plots are shown in Figure 6. We see in Figure 6(a) that now the correlation between the decay rates and the cycle amplitudes has improved. Importantly, the other correlations are also correctly reproduced in Figure 6(b,c) and can be compared with the observational plots Figure 2(b) and Figure 4. These correlations did not appear at all when the fluctuations in poloidal field generation was introduced (cf. Figures 5(b,c)). To show how the correlations change on changing the correlation time or the level of fluctuations, we tabulate the values of correlations coefficients under different situations in Table 2. Each correlation coefficient is calculated from a run of 50 cycles. It should be kept in mind that there is some statistical noise in the values of correlation coefficients. If the correlation coefficient for exactly the same set of parameters is calculated from different independent runs, the values for different runs will be a little bit different. Keeping this in mind, we note that there is no clear trend of the correlation coefficients increasing or decreasing with increasing levels of fluctuations (other things being the same). However, all the correlation coefficients tend to decrease on decreasing the coherence time.

It is not difficult to understand how the correlation in Figure 6(a) arises. For a stronger cycle, the sunspot number has to decrease by a larger amount during the
Figure 6. Same as Figure 5 but with meridional circulation fluctuations.

decay phase, making the decay rate faster. However, to understand the physical reason behind the other two correlations seen in Figure 6(b,c), more subtle arguments are needed. Karak and Choudhuri (2011) extended the arguments of Yeates, Nandy, and Mackay (2008) and pointed out that a weaker meridional circulation, which makes the cycles longer, will have two effects. Firstly, the differential rotation has more time to generate more toroidal field and tends to make the cycles stronger. Secondly, the turbulent diffusivity gets more time to act on the fields and tends to make the cycles weaker. When the diffusivity is high (as in our model), the second effect dominates over the first and the longer cycles are weaker (the opposite is true for dynamo models with low diffusivity). Karak and Choudhuri (2011) showed that this led to an explanation of the Waldmeier effect for dynamo models with high diffusivity. We now point out that this tendency (longer cycles tending to be weaker) is also crucial in our understanding of the correlations seen in Figure 6(b,c).

If the meridional circulation keeps fluctuating with a coherence time of 30 years, it would happen very often that the meridional circulation would have a certain value during a cycle (say cycle $n$) and the early rising phase of the next cycle (say cycle $n + 1$). This is less likely to happen when the coherence time is reduced. Suppose the meridional circulation is weaker during cycle $n$ and the
Correlation of the amplitude of the solar cycle with decay rate and period

rising phase of cycle \( n + 1 \). Then cycle \( n \) will tend to be longer and to have a weaker decay rate. The following cycle \( n + 1 \) will have a tendency of being weaker. This will produce the correlations seen in Figure 6(b,c). On decreasing the coherence time, it will happen less often that the meridional circulation will be the same during cycle \( n \) and the rising phase of the next cycle \( n + 1 \). Hence the correlations degrade on decreasing the coherence time of the meridional circulation.

We have realized that there is also a memory effect, which enhances the correlations explained in the previous paragraph. To illustrate this memory effect, we make a run of our dynamo code in which the meridional circulation is decreased suddenly during a sunspot minimum and then brought back to its original value during another sunspot minimum a few cycles later. The meridional circulation and the resulting sunspot activity are plotted in Figure 7. The periods of successive cycles are also indicated in the middle panel of Figure 7. On changing the meridional circulation, it is found that the periods of cycles begin changing almost immediately. However, there seems to be a memory effect as far as the amplitudes of the cycles are concerned. Even after the meridional circulation changes, the amplitude of the next cycle is very similar to the amplitude corresponding to the earlier value of the meridional circulation. This memory effect will certainly enhance the correlations we are discussing. Suppose the meridional circulation is weaker during the cycle \( n \), making its period longer and decay rate weaker. Even if the meridional circulation becomes stronger by the rising phase of the next cycle \( n + 1 \), the memory effect will ensure the amplitude of the cycle \( n + 1 \) will still be weak, thereby producing the correlation.

Figure 7. Plots showing how the variation of meridional circulation, measured by \( v_0 \), with time (upper panel) changes the period of the cycle (middle panel) and strength of the magnetic field (shown by \( B^2 \) in the lower panel).
At this point, we would like to mention a misconception behind the correlation between cycle $n$ period and cycle $n+1$ amplitude. It may be thought that the overlap between two cycles during solar minimum is the cause of this correlation. If the next cycle is stronger, then it starts early and the overlap with the present cycle is more. This makes the present cycle shorter. However we believe that this is not the source of this correlation because if this is so, then we would have seen this correlation in Figure 3(c) also, where cycle strengths were varied by fluctuations in the poloidal field generation. So the overlap is not the reason behind this correlation and we only get this in high diffusivity dynamo model with fluctuating meridional circulation.

4.3. Fluctuations in the poloidal field generation and the meridional circulation

Finally we add fluctuations in both the poloidal field generation process and the meridional circulation of the regular model, which is the realistic scenario. We add the same amount of fluctuations in poloidal field generation and in meridional circulation that we had added earlier in the individual cases (i.e., 75% fluctuations in the poloidal field generation with a coherence time of 1 month and 30% fluctuations in the meridional circulation with a coherence time of 30 years). The results are shown in Figures 8. In this figure, we see that the scatters in the correlation plots are very close to what we find in actual observations. It is perhaps not a big surprise that all the correlations are reproduced correctly, because they were already reproduced on introducing fluctuations in meridional circulation alone.

A correct theoretical model also should explain the lack of correlation seen in Figure 3 between peaks of two successive cycles. Figure 3(a) shows the correlation between the amplitude of cycle $n$ and the amplitude of cycle $n+1$ for the same level of fluctuations which were used to generate Figure 8 whereas Figure 3(b) gives the same correlation when the fluctuation is B-L $\alpha$ is raised to 100% from 75%. It is seen that the correlations between these amplitudes is weak and becomes weaker still on increasing the fluctuation in the B-L $\alpha$. A physical interpretation is not difficult to give. A coherence time of 30 years in meridional circulation implies that very often the meridional circulation will be the same during two successive cycles, trying to produce a correlation between the cycles. On the other hand, a fluctuation in the B-L $\alpha$ will definitely try to reduce the correlation. Certainly this fluctuation would try to reduce the correlations seen in Figure 8 as well. However, for our choice of parameters, we are able to theoretically reproduce the three observed correlations as seen in Figure 8 whereas the correlation between successive cycles is much weaker in conformity with observations.

5. Conclusion

We have observed three important features of solar cycle – i) a linear correlation between the amplitude of cycle and its decay rate, ii) a linear correlation between the amplitude of cycle $n$ and the decay rate of cycle $n-1$ and iii) an anti-correlation between the amplitude of cycle $n$ and the period of cycle $n-1$. We
Correlation of the amplitude of the solar cycle with decay rate and period

| Coherence time (year) | Fluctuations (%) | Correlation of decay rate with cycle amplitude of | Correlation of previous cycle period with amplitude |
|----------------------|------------------|-----------------------------------------------|---------------------------------------------------|
|                      |                  | Same cycle (Entire phase) | Next cycle (Late phase) |
| 30                   | 10               | 0.92                          | 0.92                      | -0.97                          |
|                      | 20               | 0.86                          | 0.92                      | -0.95                          |
|                      | 30               | 0.87                          | 0.89                      | -0.96                          |
|                      | 40               | 0.92                          | 0.96                      | -0.73                          |
|                      | 50               | 0.87                          | 0.91                      | -0.94                          |
| 20                   | 10               | 0.79                          | 0.85                      | -0.95                          |
|                      | 20               | 0.86                          | 0.86                      | -0.98                          |
|                      | 30               | 0.93                          | 0.96                      | -0.97                          |
|                      | 40               | 0.90                          | 0.87                      | -0.88                          |
|                      | 50               | 0.89                          | 0.90                      | -0.97                          |
| 11                   | 10               | 0.78                          | 0.74                      | -0.90                          |
|                      | 20               | 0.88                          | 0.77                      | -0.97                          |
|                      | 30               | 0.90                          | 0.85                      | -0.92                          |
|                      | 40               | 0.82                          | 0.74                      | -0.89                          |
|                      | 50               | 0.82                          | 0.84                      | -0.83                          |
| 5.5                  | 10               | 0.70                          | 0.63                      | -0.87                          |
|                      | 20               | 0.83                          | 0.74                      | -0.86                          |
|                      | 30               | 0.81                          | 0.79                      | -0.84                          |
|                      | 40               | 0.81                          | 0.57                      | -0.85                          |
|                      | 50               | 0.80                          | 0.81                      | -0.78                          |
| 1                    | 10               | 0.57                          | 0.48                      | -0.78                          |
|                      | 20               | 0.58                          | 0.59                      | -0.64                          |
|                      | 30               | 0.61                          | 0.67                      | -0.80                          |
|                      | 40               | 0.73                          | 0.25                      | -0.65                          |
|                      | 50               | 0.69                          | 0.38                      | -0.72                          |
|                      | 75               | 0.64                          | 0.39                      | -0.58                          |
|                      | 100              | 0.65                          | 0.73                      | -0.76                          |
| 0.5                  | 10               | 0.42                          | 0.62                      | -0.80                          |
|                      | 20               | 0.56                          | 0.69                      | -0.78                          |
|                      | 30               | 0.68                          | 0.47                      | -0.74                          |
|                      | 40               | 0.62                          | 0.56                      | -0.67                          |
|                      | 50               | 0.61                          | 0.56                      | -0.79                          |
|                      | 75               | 0.64                          | 0.50                      | -0.81                          |
|                      | 100              | 0.64                          | 0.60                      | -0.87                          |

have seen that all these correlations exist in all the data sets considered here. Last two correlations involve characteristics of one cycle and the amplitude of
the next. So they provide useful precursors for predicting a future cycle. Just by measuring the period and the decay rate of a cycle, we can get an idea of the strength of the next cycle.

We have also explored whether these features can be explained in a B-L type flux transport dynamo model. We have first introduced stochastic fluctuations in the poloidal field generation (B-L $\alpha$ term) and we find that only the correlation between the decay rate and the cycle amplitude is reproduced. However when we added fluctuations in the meridional circulation, we found that all three correlations are reproduced in qualitative agreement with observational data. In our high diffusivity dynamo model, strong meridional circulation makes the period shorter and the decay rate faster, but it also makes the next cycle stronger—especially because the cycle strength displays a memory effect, depending on the meridional circulation a few years earlier. The opposite case happens when meridional circulation becomes weaker. Therefore the fluctuations in the meridional circulation are essential to reproduce the observed features. This study is consistent with earlier studies for modeling the cycle durations and strengths of observed cycles (Karak, 2010), the Waldmeier effect (Karak and Choudhuri, 2011), grand minima (Choudhuri and Karak, 2012) and few others (Passos, 2012).
Figure 9. (a) Scatter plot of the amplitude of cycle $n$ with the amplitude of cycle $n+1$ with 75% fluctuation in B-L $\alpha$. (b) Same as (a) but with 100% fluctuation in B-L $\alpha$.

which indicate that the variable meridional circulation is crucial in modeling many aspects of the solar cycle.

We have pointed out that the period or the decay rate of a cycle may be used to predict the next cycle, since these quantities indicate the strength of the meridional circulation which also determines the amplitude of the next cycle a few years later (due to the memory effect). It seems that the decay rate during the late phase of the cycle is the most reliable precursor for the next cycle, as seen in Figure 2(b)—presumably because the decay rate during this phase is the best indicator of the meridional circulation during the particular interval of time which is most crucial in determining the amplitude of the next cycle. However, fluctuations in the poloidal field generation process degrades all the observed correlations. As a result, even Figure 2(b)—displaying the correlation between the decay rate during the late phase and the amplitude of the next cycle—has considerable scatter, limiting our ability to predict the next cycle in this way.

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Correlation of the amplitude of the solar cycle with decay rate and period

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