Improving students’ mathematical critical thinking through rigorous teaching and learning model with informal argument

H Hamid
Universitas Khairun, Ternate, Indonesia
E-mail: hasan.hamid66@gmail.com

Abstract. The purpose of this study is to analyze an improvement of students’ mathematical critical thinking (CT) ability in Real Analysis course by using Rigorous Teaching and Learning (RTL) model with informal argument. In addition, this research also attempted to understand students’ CT on their initial mathematical ability (IMA). This study was conducted at a private university in academic year 2015/2016. The study employed the quasi-experimental method with pretest-posttest control group design. The participants of the study were 83 students in which 43 students were in the experimental group and 40 students were in the control group. The finding of the study showed that students in experimental group outperformed students in control group on mathematical CT ability based on their IMA (high, medium, low) in learning Real Analysis. In addition, based on medium IMA the improvement of mathematical CT ability of students who were exposed to RTL model with informal argument was greater than that of students who were exposed to CI (conventional instruction). There was also no effect of interaction between RTL model and CI model with both (high, medium, and low) IMA increased mathematical CT ability. Finally, based on (high, medium, and low) IMA there was a significant improvement in the achievement of all indicators of mathematical CT ability of students who were exposed to RTL model with informal argument than that of students who were exposed to CI.

1. Introduction
Students’ ability to think critically is necessary to develop as every human has potential to think critically. This is in line with Cotton [1] who stated that although many people believed that we were born either with or without creative and critical thinking abilities, research had shown that these skills were teachable and learn-able. Watson and Glaser [2] considered that critical thinking includes: (1) attitudes of inquiry that involve an ability to recognize the existence of problems and an acceptance of the general need for evidence in support of what is asserted to be true; (2) knowledge of the nature of valid inferences, abstractions, and generalizations in which the weight or accuracy of different kinds of evidence are logically determined; and (3) skills in employing and applying the above attitudes and knowledge. Furthermore, to facilitate the interpretation of critical thinking, Watson and Glaser [3] introduce the three factors that are key to critical thinking and abbreviated as RED: (1) Recognizing Assumptions; (2) Evaluating Arguments; and (3) Drawing Conclusions. Assumptions are statements that are assumed to be true in the absence of proof. Identifying assumptions helps in discovering information gaps and enriches views of issues. Assumptions can be stated implicitly and explicitly. The ability to recognize assumptions in presentations, strategies, plans, and ideas is a key element in critical thinking. Being aware of assumptions and directly assessing their appropriateness to the situation helps people evaluate the merits of a proposal, policy, or practice. In addition, arguments are assertions
intended to persuade someone to believe or act a certain way. Evaluating arguments is ability to analyze such assertions objectively and accurately. Analyzing arguments aids in determining what weight to put on them and what actions to take. It includes the ability to overcome a confirmation bias as tendency to look for and agree with information that confirms prior beliefs. Emotion plays a key role in evaluating arguments as well. A high level of emotion can cloud objectivity and ability to evaluate arguments accurately. Drawing conclusions consists of arriving at conclusions that logically follow from the available evidence. It includes evaluating all relevant information before drawing a conclusion, judging the plausibility of different conclusions, selecting the most appropriate conclusion, and avoiding overgeneralization beyond the evidence.

Critical thinking in learning mathematics is a cognitive process or mental action in an effort to acquire knowledge of mathematics based on mathematical reasoning. Therefore, critical thinking in mathematics learning includes ability to react to mathematical problems by distinguishing opinion and facts, conclusion and judgments, inductive and deductive arguments, as well as objective and subjective. Thus, critical thinking could be defined as the ability to identify and analyze problems as well as seek and evaluate relevant information to reach the right conclusions.

To facilitate students’ critical thinking it is necessary to design precise and meaningful learning situations that give students opportunity to explore cognitive aspects. In consequence, students will be able to interpret and find the right concepts and strategies to facilitate them in learning Real Analysis. The learning model implemented in this study is Rigorous Teaching and Learning (RTL). Rhodes [4] revealed that there are three characteristics of students involved in rigorous learning. Furthermore, Rhodes [4] and Reyes [5] described indicators of rigorous teaching and learning models on academic rigor. It shows the level of challenges that integrate knowledge content, high-level thinking, active engagement, and strategies to design strict learning will enable students to interact with content strictly. This is in line with what Jackson [6] explained on strict learning strategies that describe the way teachers help students learn, manipulate, reorganize, adapt with new situations, and apply learned context. The use of informal arguments in this study is based on the concepts of Strauss and Corbin who acknowledged three activities: Syntactifying, rewarrenting and elaborating. These three activities could be drawn in the following scheme [7]:

![Figure 1. Three Translational Activities](image)

Syntactifying occurs when a student seeks to take a statement in an informal argument given in what is considered to be a strict term and translate it into what is considered a more appropriate representation system for evidence. Such actions include removing references to diagrams used in informal arguments and replacing them with conventional mathematical terms, or introducing logical algebraic notation. In the case of the Toulmin’s scheme, we may assume syntactifying as translating data (D), claims (C), and/or warranty (W) from arguments into new data (D’), claims (C’), and/or guarantees (W’). In other representational systems, it is shown without intending to change the meaning of D, C, or W.
Rew warranting occurs when students try to find a deductive reason for the claim that their informal argument is justified by non-deductive means. In the case of the Toulmin’s scheme, we may consider re warranting as a substitute for a reasonable guarantee (\(W\)) (i.e., assurance that the student believes it is possible to produce truth) with a valid warranty (\(W^\circ\)) (i.e., warranty that the student believes is deemed legitimate by Community mathematics).

Elaborating occurs when students try to add in more detail to evidences that do not appear in their informal arguments. This happens in several ways: students will justify the assertion that they are taken for granted in their informal arguments by making explicit assurances that were initially implicit (\(W_i\)) in their informal argument, or rather justifying their data (\(D\)) (i.e., students trying to justify the facts taken for granted).

2. Methods
This study employed mixed method. The strategy used is a concurrent embedded strategy [8] that is a research model that combines quantitative and qualitative research methods, and quantitative and qualitative data collected simultaneously. The quantitative method in this study is a primary method, while the qualitative method is a secondary method. The findings of each methods are combined to formulate a comprehensive interpretation of students’ mathematical critical thinking (CT) using the Rigorous Teaching and Learning model with utilizing informal arguments. The reason for choosing this method is to make this research more objective, complete and reliable.

The design of this study was quasi-experimental research using experimental group and control group, known as pretest-posttest control group design [9]. The reason for selecting quasi-experimental design is that intact classrooms are used and there are not random assignment. The data collected in this study are quantitative and qualitative data. Quantitative data were obtained from IMA test results, pre-test or post-test mathematical critical thinking (CT) ability. While the qualitative data were obtained from analysis of students works concerning the test of mathematical CT ability, observation and interview.

The data in this study are processed and analyzed descriptively and inferentially. Stages in descriptive data analysis to see the improvement that is by calculating the normalized gain value \(\langle g \rangle\) from pretest and posttest mathematical CT, using the following formula [10]:

\[
\langle g \rangle = \frac{S_f - S_i}{100 - S_i}
\]

Where:
\(S_f\) = final test (post-test)
\(S_i\) = initial test (pre-test)

| Interval          | Criteria |
|-------------------|----------|
| \(\langle g \rangle < 0,3\) | Low      |
| \(0,3 \leq \langle g \rangle < 0,7\) | Medium   |
| \(\langle g \rangle \geq 0,7\)   | High     |

The research hypothesis were tested to understand whether using parametric statistical analysis or nonparametric statistical analysis. Statistic test which is used in study comprise t-test, t’-test, Levene test, Mann-Whitney U test, Kruskal-Wallis test, and Analyzing Estimated Marginal Means through graph.

3. Results and Discussion
Data obtained from both pre-test and post-test results and the normalized gain (N-gain) of mathematical CT capability, this data would be analyzed descriptively or inferentially. This data would be analyzed
based on IMA and learning model (RTL and CI). To obtain a deeper picture of the mathematical CT's ability to improve observed aspects, the following explanation is presented descriptively or inferentially.

3.1 Descriptive Data Analysis Improvement of Critical Thinking (CT) Ability Mathematical Student

Data analysis of mathematical CT ability were obtained from both pre-test and post-test results including mean, standard deviation and N-gain based on IMA criteria and learning model. The descriptive statistics of students’ mathematical CT ability is presented in the following table.

| IMA   | Stat. | RTL     | CI     | Total |
|-------|-------|---------|--------|-------|
|       | (g)   | N       | (g)    | N     |
| High  | 0.81  | 8       | 0.79   | 7     | 0.80  | 15 |
|       | 0.09  | 0.74    | 0.08   |       |
| Medium| 0.61  | 26      | 0.51   | 27    | 0.56  | 53 |
|       | 0.11  | 0.12    |        | 0.12  |
| Low   | 0.42  | 9       | 0.36   | 6     | 0.40  | 15 |
|       | 0.11  | 0.06    |        | 0.09  |
| Total | 0.61  | 43      | 0.54   | 40    | 0.58  | 83 |
|       | 0.16  | 0.16    |        | 0.17  |

Description: N = Number of data; (g) = Normalized gains

Table 2 shows that in high IMA criteria, mean improvement of students’ mathematical CT ability using RTL model with informal arguments is higher than that of the CI students. The normalized gain criteria of the RTL and CI models are included in the high criteria. Furthermore, in medium IMA criteria, the increase in mean of students’ mathematical CT ability using RTL model with informal arguments is higher than that of students who received CI, although the criteria of increase in mean for both groups were included in the medium criteria. Whereas, in low medium criteria, the increase in mean of students’ mathematical CT ability using the RTL model with informal arguments was higher than that of the CI students, and the criteria of the increase in mean for both groups were categorized as medium criteria. Table 2 generally shows that the increase in mean of students’ mathematical CT ability using the RTL model with informal arguments was higher than that of CI students. In addition, the criteria of increase in mean for both groups were included in the medium criterion. To obtain clear description of the increase in mean of mathematical CT ability of RTL students and CI students, the following diagram is shown.

![Figure 2. Comparison between increase in mean of student mathematical CT ability across learning model and IMA](image)
3.2 Inferential Analysis (Mathematical CT Improvement)

Table 3. Mean Difference of Mathematical CT Ability Score based on IMA Categories

| Ability    | IMA   | Learning model | Sig. | $\alpha = 0.05$ | Hypothesis          | Test   |
|------------|-------|----------------|------|-----------------|---------------------|--------|
| Mathematical CT | High  | RTL = CI     | 0.293| >               | $H_0$ be accepted   | t-test |
|             | Medium| RTL > CI      | 0.000| <               | $H_0$ rejected      | t-test |
|             | Low   | RTL = CI     | 0.100| >               | $H_0$ be accepted   | t-test |
|             | Total | RTL > CI      | 0.000| <               | $H_0$ rejected      | ANOVA  |

Table 3 shows that for high and low category of IMAs there is no significant difference on increase in mean of mathematical CT ability of RTL students and CI students. Furthermore, from Table 3 it appears that the significant difference can be seen for medium IMA.

Table 4. Mean Difference of Mathematical CT Ability Score based on Learning Model

| Ability    | Learning model | Sig. | $\alpha = 0.05$ | Hypothesis        | Test   |
|------------|----------------|------|-----------------|-------------------|--------|
| Mathematical CT | RTL > CI  | 0.02 | <               | $H_0$ rejected    | t-test |

Table 4 shows that the null hypothesis is rejected. This means that there is a significant mean difference of mathematical CT ability scores of RTL students and CI students.

Table 5. Post-hoc Test Result of Mean Difference of N-gain of CT based on IMA category

| Learning model | Test Statistics | IMA category | Mean Difference (I-J) | Sig. | Testing Rules |
|----------------|-----------------|--------------|------------------------|------|---------------|
| RTL >> CI      | Bonferroni      | Low >> Medium| -0.162                 | 0.000| Different     |
|                |                 | Low >> High  | -0.405                 | 0.000| Different     |
|                |                 | Medium >> High| -0.243                | 0.000| Different     |

It can be seen from Table 5 that the ability of CT with IMA category for low and medium, low and high pairs, and medium and high based on the learning model identified different differences. Furthermore, from the table above it can be seen that for the low IMA group (I) and the IMA group being (J), the mean difference value (IJ) is negative, i.e. -0.162, this means that the average CT of mathematical students from the IMA group is better than the average IMA group is low with an average difference of 0.162. For the low IMA group (I) and high IMA groups (J), the mean difference value (IJ) is negative, i.e. -0.405, this means that the mean mathematical CT of students from the high IMA group is better than the average low IMA group with an average difference of 0.405. While in the IMA group being (I) and IMA group high (J), the mean difference value (IJ) is negative, i.e. -0.2243, this means that the average CT of mathematical students from the high IMA group is better than the average IMA group being with the average difference of 0.243.
The interactive graphs between learning models (RTL, and CI) and high, medium, and low category IMAs on improving students' mathematical CT skills are shown in the following figure:

![Graph showing interactive graphs between learning models and IMAs](image)

**Figure 3.** Interaction between Model (RTL, and CI) and IMA on Student Mathematical CT Ability

Based on Figure 4, it can be stated that graph of the average line of increase of mathematical CT ability of RTL students is above the average line increase of the mathematical ability of CI students. This indicates that for each level of the IMA category the RTL model has a greater influence on the improvement of students’ mathematical CT ability compared to the CI model. It also appears that the average line graphs for both model groups (RTL and CI) did not change sequence i.e. high-IMA students were better than those with medium and low-IMA categories, as well as for medium IMA categories with low IMA. This indicates that the IMA category has a greater effect on improving mathematical CT ability. Based on the behavior of both graphs that have positive gradient shows that the learning model (RTL and CI) as well as IMA of high, medium and low categories independently give a real influence to the improvement of students' mathematical CT ability. Based on the distance of the graph it appears that for each IMA category tends to be similar and not mutually intersect. Thus, it can be concluded that there is no interaction effect between the model (RTL, and CI) and the IMA categories on improving students' mathematical CT ability.

| No. | Indicators CT               | SMI | Stat | RTL Pre-test | RTL Post-test | (g) | CI Pre-test | CI Post-test | (g) |
|-----|----------------------------|-----|------|--------------|---------------|-----|-------------|--------------|-----|
| 1.  | Recognizing Assumptions   | 14  | \( x \) | 2.05         | 10.63         | 0.72| 2.00        | 10.33        | 0.69|
| 2.  | Evaluating Arguments      | 14  | \( x \) | 2.40         | 10.98         | 0.74| 2.28        | 10.08        | 0.67|
| 3.  | Drawing Conclusions       | 21  | \( x \) | 2.37         | 10.70         | 0.45| 2.33        | 8.73         | 0.34|

Table 6 shows that all indicators of students' mathematical critical thinking skills improved in real-time learning after being treated with RTL learning models utilizing both informal and conventional (CI) arguments. For the RTL learning model, the mean of the highest increase of students' mathematical critical thinking ability is on the indicator of evaluating arguments. The average increase of the value of \( g \) is 0.74 and is included in the high category. The high increase was obtained by the students on the indicator of recognizing assumptions with a value of 0.72. For the indicator of drawing conclusions, it shows that the increase in mean of students' mathematical critical thinking ability after the treatment of
RTL learning model is 0.45 and it includes in the medium category. Furthermore, for the CI learning model, the highest increase in mean of students' mathematical critical thinking ability was on the indicator of recognizing assumptions with the value of 0.69 and it includes in the medium category. For the indicator of evaluating arguments and drawing conclusions, the value of increase are 0.67 and 0.34 respectively. This suggests that strong prior knowledge acquisition, rich experience, and familiar with problems that require analysis, synthesis, and evaluation will affect students' mathematical critical thinking ability. The results of this study are consistent with Bloom's opinion [11] in which critical thinking relates to high-level cognitive thinking (analysis, synthesis and evaluation).

4. Conclusion
This study generally indicates that students who were exposed to RTL learning model outperformed students who were exposed to conventional instruction on math mathematical CT ability of in learning of Real Analysis. The increase of mathematical CT ability for both group of students was categorized as medium. In the learning of Real Analysis based on IMA (High, Medium, and Low), the increase in mean of the mathematical CT of RTL students is better than that of CI students. There is no interaction effect between Rigorous Teaching and Learning (RTL) model and conventional learning model (CI) with IMA (High, Medium, and Low) to the increase of students' mathematical CT ability scores. The highest increase in students' mathematical critical thinking ability on the RTL model was on the indicator of evaluate arguments and it fell into high category. The highest increase in students' mathematical critical thinking abilities in the CI model was on the indicator of recognize assumptions and it included in medium category. Therefore, to improve the critical mathematical thinking ability of students with IMA (high, medium and low) in Real Analysis course, the Rigorous Teaching and Learning (RTL) model that utilizes informal arguments could be applied as an alternative model of learning.

References

[1] Cotton K 1991 Teaching Thinking Skills [Online] Available at http://www.nwrel.org/scpd/sirs/6cu11.html [Retrieved, December 14, 2014]
[2] Watson G and Glaser E M 2009 Watson-Glaser II Critical Thinking Appraisal, Technical Manual and User’s Guide (San Antonio, TX: Pearson)
[3] Watson G and Glaser E M 2012 Watson-Glaser™ Critical Thinking Appraisal, Technical Manual and User’s Guide (San Antonio, TX: Pearson)
[4] Hart P, Natale L and Starr C 2010 Recognizing Rigorous and Engaging Teaching and Learning [Online] Available: http://billmcbride.pbworks.com/f/Recognizing+Rigorous+and+Engaging+Teaching+and+Learning.doc [Retrieved, September 22, 2014]
[5] Reyes N 2010 Transforming Our Teaching and Learning: The Role of Academic Rigor, Inquiry, and Higher Order Thinking National Center for Teacher Education (Maricopa County Community College District)
[6] Jackson 2010 How To Plan Rigorous Instruction (Virginia, USA: ASCD Alexandria)
[7] Zazkis D, Weber K and Mejia-Ramos J P 2014 Activities that mathematics majors use to bridge the gap between informal arguments and proofs. In P Liljedahl, C Nicol, S Oesterle, D Allan (Eds.) Proceedings of the 38th Conference for the International Group of the Psychology of Mathematics Education 4 417-424 (Vancouver, Canada: PME)
[8] Creswell J W and Clark V L P 2007 Designing and Conducting Mixed Methods research (Thousand Oaks: Sage)
[9] Fraenkel J, Wallen N and Hyun H 2012 How to Design and Evaluate Research in Education 8th (New York: McGraw-Hill Companies, Inc.)
[10] Hake R 1998 Journal of American Association of Physics Teachers 66 1 pp 64
[11] Dehghani M, Sani H J, Pakmehr H, and Malekzadeh A 2011 Procedia Social and Behavioral Sciences 15 pp 2952