Calculation of the speed descriptions of the upward swirling gas flow in conditions of lateral wind action

A G Obukhov

Tyumen Industrial University, 38, Volodarskogo Str., Tyumen, 625000, Russia

E-mail: agobukhov@inbox.ru

Abstract. The paper presents the results of numerical simulation of lateral wind impact on three-dimensional non-stationary air flows in ascending vortex flow of artificially created tornado in the stationary operating mode. The mathematical model represents a complete system of Navier-Stokes equations taking into account the viscosity and thermal conductivity of the moving gas, as well as the effects of gravity and Coriolis. Using the explicit difference scheme and the corresponding choice of initial and boundary conditions, the solutions of the complete system of Navier-Stokes equations in the computational domain representing a rectangular parallelepiped are numerically obtained. All components of gas flow speed are calculated at fixed time points and instantaneous current lines are built taking into account the constant horizontal wind speed. The calculations showed that the result of the wind action on the ascending vortex flow is an unsymmetrical change in the circumferential velocity, its uneven deformation at opposite sections, and the total displacement of the vortex in the wind direction. Besides, the displacement and the curvature of the vertical vortex part in wind direction are established, the “separation” of some instantaneous current lines from vertical rotating part and the area in the vortex center increasing in diameter, free from current lines is fixed. In the studied time change range, the wind resistance of the air vortex and stable operation of the used computational scheme are observed.

1. Introduction

The relevance of numerical modeling of complex three-dimensional non-stationary flows of viscous compressible heat-conducting gas in natural atmospheric vortices and gas pipelines is caused by the lack of convincing theory and numerical algorithms that adequately describe such flows. The creation of such a theory and calculation schemes is a serious task both in terms of mathematical justification and from a computational point of view. The results of the studies based on adequate models can be used as the basis for minimizing the negative destructive effects of atmospheric vortices, as well as for improving the efficiency of gas pipeline systems and reducing energy costs, as they are more appropriate to their actual operating conditions. At the same time, the analytical studies allow establishing the mathematical solutions to the considered problems. Numerical modeling provides specific numerical characteristics of the studied flows, including for their comparison with operational data and with experimental observations.

The pattern of occurrence and stable functioning of the ascending vortex air flow [1] implies the presence of a sufficiently long-term ascending air flow and the accompanying radial air flow, as well as the effect of the Coriolis force imparting substantial circumferential velocity to air particles at the bottom.
The fact of vortex occurrence and its direction is strictly mathematically proved by the corresponding theorems [2, 3] and confirmed experimentally [4, 5].

The observations of natural ascending vortex flows [6] made it possible to hypothesize the existence of a boundary separating the external quiet air from the air moving in the ascending vortex flow. In gas dynamics such boundary is called a contact surface [7, 8]. Therefore, in order to create a stable upward vortex flow under laboratory conditions, an air-impermeable vertical cylindrical pipe with an exhaust fan guiding air through the pipe from bottom to top is used as the contact surface [3, 9].

Based on experimental results, all gas-dynamic parameters [3, 10], including speed and energy characteristics of the three-dimensional non-stationary ascending vortex gas flow of a specific scale caused by vertical blowdown, were numerically modeled and calculated. The numerical calculations of this gas flow made it possible to give specific proposals and recommendations for potential large-scale experiment to twist large air masses – artificial tornado.

The previously obtained numerical results [10-12] form the basis for further study in the gasdynamic theory of destructive atmospheric vortices. In particular, the numerical simulation of the phenomenon of the curvature of the vertical part of the ascending vortex flow – an “arm” of visually observed natural tornadoes – seems quite interesting [6]. This effect clearly indicates significant stability of the atmospheric vortex and does not yet have quite adequate scientific explanation. The purpose of this study is numerical simulation of artificial tornado and calculation of its speed characteristics under lateral wind impact.

2. Mathematical model

A complete system of Navier-Stokes equations is used as a mathematical model describing complex three-dimensional non-stationary flows of compressible viscous heat-conducting gas (atmospheric air), which in dimensionless variables taking into account the action of gravity forces and the Coriolis force in the vector form can be presented as follows [3]:

\[
\begin{align*}
\rho_t + \mathbf{V} \cdot \nabla \rho + \rho \text{div} \mathbf{V} &= 0, \\
\mathbf{V}_t + (\mathbf{V} \cdot \nabla) \mathbf{V} + \frac{T}{\rho} \nabla \rho + \frac{1}{\gamma} \nabla \Delta T &= \mathbf{g} - 2\mathbf{\Omega} \times \mathbf{V} + \frac{\mu_0}{\rho} \left[ \frac{1}{4} \mathbf{V} (\text{div} \mathbf{V}) + \frac{3}{4} \Delta \mathbf{V} \right], \\
T_t + \mathbf{V} \cdot \nabla T + (\gamma - 1) \Delta \mathbf{V} &= \frac{\kappa_0}{\rho} \Delta T + \frac{\mu_0 (\gamma - 1)}{2\rho} \left[ \left( u_x - v_y \right)^2 + \left( u_y - v_x \right)^2 \right] + \frac{3}{2} \left[ \left( u_p + v_x \right)^2 + \left( u_x + w_z \right)^2 + \left( v_y + w_z \right)^2 \right].
\end{align*}
\]

In this system:
\( t \) – time;
\( x, y, z \) – Cartesian ordinates;
\( \rho \) – gas density;
\( \mathbf{V} = (u, v, w) \) – gas speed vector with projections on the corresponding Cartesian axes;
\( T \) – gas temperature;
\( \mathbf{g} = (0, 0, -g) \) – gravity acceleration vector;
\( \gamma = 1.4 \) – polytropic exponent for air;
\( -2\mathbf{\Omega} \times \mathbf{V} = (av - bw, -au, bu) \) – Coriolis acceleration vector;
\( \mathbf{a} = 2\Omega \sin \psi, \quad \mathbf{b} = 2\Omega \cos \psi, \quad \Omega = \left[ \Omega \right] ; \quad \Omega \) – angular speed vector of the Earth rotation;
\( \psi \) – latitude of the point \( \psi \) – beginning of the Cartesian axes \( xyzO \) rotating with the Earth;
\( \mu_0 = 0.001, \quad \kappa_0 \approx 1.46\mu_0 \) – constant values of dimensionless viscosity and thermal conductivity coefficients.
The computational domain (Figure 1) represents a rectangular parallelepiped with the lateral length $x^0 = 1$, $y^0 = 1$ and $z^0 = 0.04$ along the axes $Ox$, $Oy$ and $Oz$, respectively, that is filled with a three-dimensional grid of intersection nodes of the three plane families $x = x_i$, $y = y_j$, $z = z_k$, where $x_i = i \cdot \Delta x$, $y_j = j \cdot \Delta y$, $z_k = k \cdot \Delta z$, $0 \leq i \leq L$, $0 \leq j \leq M$, $0 \leq k \leq N$. The differential steps on three spatial variables are as follows: $\Delta x = x^0 / L$, $\Delta y = y^0 / M$, $\Delta z = z^0 / N$, $L = 200$, $M = 200$, $N = 20$.

![Figure 1. Computational domain](image)

The values of the desired five functions characterizing the air flow in the artificially created tornado at the moment $t = t_\circ$ of its setting to the stationary operating mode are taken for initial conditions in all internal nodes of the computational domain [11, 12]:

$$u = u(x, y, z, t_\circ), \quad v = v(x, y, z, t_\circ), \quad w = w(x, y, z, t_\circ), \quad T = T(x, y, z, t_\circ), \quad \rho = \rho(x, y, z, t_\circ).$$

The boundary conditions on the faces of the computational domain are set as follows:

The density on four side faces of the computational parallelepiped $x = 0$, $x = x^0$, $y = 0$, $y = y^0$ is taken equal to the values from the stationary distribution of atmospheric air density [3]

$$\rho|_{x=0,x=x^0} = \rho_0(x, y, z), \quad \rho|_{y=0,y=y^0} = \rho_0(x, y, z),$$

$$\rho_0(z) = (1 - kz)^{-1}, \quad k = \frac{ix_{00}}{T_{00}}, \quad l = 0.0065 \text{ K/m}, \quad x_{00} = 50 \text{ m}, \quad T_{00} = 288^\circ \text{K}, \quad \nu = \frac{\gamma g}{k}.$$

The density on the lower $z = 0$ and upper face $z = z^0$ is subject to flow continuity. This means that the density per region boundary belong to linear extrapolation normal to the given boundary surface from the inside of the computational domain [3].

For temperature on all lateral faces the values from the stationary distribution of free air temperature are specified

$$T|_{x=0,x=x^0} = T_0(x, y, z), \quad T|_{y=0,y=y^0} = T_0(x, y, z), \quad T_0(z) = 1 - kz.$$

The temperature at the lower $z = 0$ and upper face $z = z^0$ corresponds to the symmetry condition. At the same time, the temperature values are calculated from the condition of equality to zero of their derivative by normal to this face.

Impermeability conditions are set at the lower and upper faces. The third velocity component equals zero $w|_{z=0,z=z^0} = 0$, and the first and second velocity vector components are determined from
the symmetry condition, i.e. they are considered to be zero of their derivative normal to the face. Besides, through a square hole with size \(0.1 \times 0.1\) in the center of the upper face, a vertical speed \(w = 0.03\) is set, which simulates the vertical purge with air at a speed of 10 m/s through a pipe with a diameter of 5 m.

On the side faces of the computational domain, the continuity conditions are set for all velocity vector components normal to the faces, and two tangential velocity components are calculated from the symmetry condition. Additionally, a normal velocity component equal to wind speed \(u = 0.03\) is set on the left (west) face \(x = 0\). Thus, a horizontally directed wind action on the ascending vortex flow of artificially created tornado is simulated.

The values of functions from two consecutive time layers are used to approximate the derivatives over time, and the central differences of function values are used to approximate the derivatives over spatial variables.

The three-dimensional non-stationary flow is calculated according to a clear difference scheme of transition from one \(n\) time layer to the next \(n + 1\) time layer with constant specified step \(\Delta t\). The values of all required functions at all internal points of a rectangular parallelepiped are calculated. The required function values are then defined at all internal points of each of the six faces. The values of all the required functions at the inner points of all twelve edges of the rectangular parallelepiped are found as the arithmetic mean of the two intermediate values determined by linear interpolation according to the values of functions at two points on the normals to the edge at each face.

The calculations were performed with the following input parameters: scaling dimensional values of density, speed, distance and time are equal accordingly to \(\rho_{00} = 1.2928\ \text{kg/m}^3\), \(u_{00} = 333\ \text{m/s}\), \(x_{00} = 50\ \text{m}\), \(t_{00} = x_{00}/u_{00} = 0.15\ \text{s}\). The differential steps – as three spatial variables \(\Delta x = \Delta y = 0.005\), \(\Delta z = 0.002\), and the time step – as \(\Delta t = 0.001\).

### 3. Results

The calculation analysis of speed characteristics of air flow in artificial tornado at the presence of the wind load on the lower half of its bottom part is given. Figures 2-7 show the diagrams of three velocity components for height \(z = 0.2\ \text{m}\) and for two fixed moments of time \(t_1 = 1\ \text{s},\ t_2 = 10\ \text{s}\). The numbers of the computational grid nodes are placed along axes \(Ox\) and \(Oy\), along the axis \(Oz\) – gas velocity in dimensionless values.

Figures 2-3 show the calculated surfaces of the first gas velocity component as a graphical representation of functions of two variables \(u(x, y)\) for the above height and fixed times.
In the figures given, the diagrams of the first speed component \( u \) reflect the presence of the specified wind speed 0.03 (dimension value \( 10 \) m/s) on the west (left) face \( x = 0 \) of the computational domain. The horizontal axially directed movement of air loses its speed quite quickly to the values 0.008 (dimensional value \( 2.7 \) m/s) when interacting with an axis flow (Figure 2). The oncoming movement of air in the wind flow and the northern part of the vortex results in their interaction, as a result of which the wind speed is subtracted from the circumferential velocity of the vortex, which as a result takes a negative value \(-0.01\) (dimensional value \(-3.33 \) m/s), and the vortex is completely displaced eastwards (Figure 3).

At the same time, the air movement in the wind flow coincides with the air movement in the southern part of the vortex. As a result, the wind speed is added with the circumferential velocity of the vortex in the southern part, reaching the total velocity of \( 0.02 \) (dimensional value \( 6.66 \) m/s).

Thus, the result of the wind action on the ascending vortex flow is its uneven deformation in the northeast direction, an unsymmetrical change in the circumferential velocity, and the total displacement of the vortex in the wind direction.

\[ \text{Figure 4. Speed } v \text{ for } t_1 = 1 \text{s} \]

\[ \text{Figure 5. Speed } v \text{ for } t_2 = 10 \text{s} \]

Figures 4-5 show the calculated surfaces of the second gas velocity component as a graphical representation of the functions of the two variables \( v(x, y) \) for the above-mentioned height and fixed times. The figures illustrate the behavior of the second component \( v \) of the gas flow rate.

During the first five seconds of the estimated time, the absolute value of this velocity component directed to the south of the computational domain towards the face \( y = 0 \) (left side in the figures) increases within \( 0.01 \pm 0.015 \) (dimensional values \( 3.33 \pm 5.0 \) m/s). This is caused by the fact that during this period of time there is an intensive addition of two perpendicular directed flows – the flow of air in the vortex directed to the south and the specified wind flow directed to the east (Figure 4). The second half of the estimated time is characterized by the growth rate reduction of this velocity component in the specified part of the computational domain with its simultaneous displacement towards the eastern part and an increase in velocity in this part of the vortex directed to the north (Figure 5). The above-described process of changing the second velocity component suggests deformation and displacement of the vortex as a whole in the northeast direction at initially given wind velocity.

Figures 6-7 show the diagrams of the third velocity component \( w \) for the considered time moments. It is typical to have a local increase of this velocity component to a value of \( 0.0025 \) (dimensional \( 0.83 \) m/s) in the center of the computational domain corresponding to the square air flow in the upper face of the computational domain. Besides, the figures show an area of increased speed
values \( w \) to values 0.003 (dimensional 1 m/s) near the left (west) face of the computational domain, which at the distance of the order of 10 calculation nodes (2.5 m) decreases to zero. There is also a slight increase in vertical velocity values in the vicinity of the north-west vertical rib of the computational domain over time, the presence of which can be interpreted as a result of the interaction of opposite air flows.

Numerical calculations of three velocity components make it possible to build instantaneous current lines of complex three-dimensional non-stationary flows in the considered ascending vortex flows of the artificial tornado when exposed to external air flow. Figures 8-9 show instantaneous current lines in lateral projection perpendicular to the abscissa axis. In the figures, the number of instantaneous current lines is the same and is taken from the horizontal surface adjacent to the plane \( z = 0 \).

The figures with instantaneous current lines correspond to the same moments of time \( t_1 = 1 \) s, \( t_2 = 10 \) s. the distances in dimensionless values are set along the axes of coordinates.

As can be seen from the calculations, during the calculated period of time the vortex center in the bottom of the lower part shifted in the east direction (to the right) to a distance of 0.08 (dimensional
value 4 m), and the vortex center at the top of the vertical part in the same direction to a distance of 0.04 (dimensional value 2 m). At the same time, the whole vertical part of the vortex – its “arm” – is subject to bending, convexity in the east direction (to the right). Besides, the figures showing instantaneous current lines clearly show an increase in vortex diameter starting at the bottom and moving to the vertical part thereof.

Figures 10-11 show a plan view of the same instantaneous current lines, from which it can be seen how the vortex is deformed and displaced in the north-east direction under the influence of a given wind flow. The result of such deformation can also be considered the appearance of an increasing area free of current lines in the center of the vortex (Figure 11). Finally, the built instantaneous current lines allow observing how the vortex gradually bends from the south side through the flow of the external wind flow.

Figures 12-13 represent instantaneous current lines at corresponding times at some angle. They give the overall picture of wind action on the vortex and highlight another feature.

Figure 13 shows that some current lines begin to “break off” from the vertical part. The effect can also be seen in Figure 9. This can be explained by both the substantial deformation of the vortex flow in the northeast direction and the different velocity of the circumferential gas flow in different parts of the vortex.
4. Conclusion

The study presents the numerical simulation of lateral wind action on three-dimensional non-stationary air flows in the ascending vortex flow of artificially created tornado in the stationary operating mode. Using the explicit difference scheme and the corresponding choice of initial and boundary conditions, the solutions of the complete system of Navier-Stokes equations in the computational domain representing a rectangular parallelepiped are numerically obtained. All flow speed characteristics and instantaneous current lines under the gravity conditions and the Coriolis force, as well as taking into account constant horizontal wind speed are calculated at fixed moments of time.

Displacement, curvature of the vertical part of vortex in wind direction, increase of vortex diameter in the lower part passing into the vertical part is established. There is a “break” of some current line from the vertical part and the appearance of an increasing area free of current lines in the center of the vortex.

In the studied time change range, wind resistance of air vortex and stable operation of the used computational scheme are observed.

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