Mathematical simulation of the number of fires in Russian Federation on the kind of object

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Abstract. An attempt was undertaken to find the kind of the functional dependence between the number of fires in the Russian Federation and the kind of an object. Model of Verhulst was employed in the process of regression analysis previously used in biology for description of the process of the animals’ population growth. Modification of Verhulst model assuming variable rate of growth was found to allow approximation of the studied dependence with 100% accuracy.

1. Introduction

Now there are no studies concerning mathematical simulation of the available data on the dependence of the number of fires in Russian Federation on the kind of a protection object [1-3].

Available data concerning the number of fires in a dependence on the kind of the object are arranged according to the order of disposition of the objects’ types (table 1), that provides a curve similar to that one describing population growth of the animals in biology.

In this situation, we used rank variable X to define the kind of the object. It takes discrete integral values within the range of 1 to 20. This variable numerically coincides with the number of the object’s kind in the list (table 1).

For the years from 2014 to 2017 similar situation can be observed [3].

Let us consider the possibility of applying the law of the biological population growth for the approximation of the dependence of the number of fires on the objects’ kind.

Table 1. Number of the fires in Russia by the kinds of the objects.

| X | Object of the fire                                                                 | 2014 | 2015 | 2016 | 2017 | 2018 |
|---|-----------------------------------------------------------------------------------|------|------|------|------|------|
| 1 | All other buildings, constructions and public buildings                            | 151  | 141  | 144  | 115  | 97   |
| 2 | Public health buildings and institutions of social services for population         | 192  | 171  | 153  | 164  | 211  |
| 3 | Buildings, constructions and premises for cultural and entertainment activities and for religious rites | 266  | 262  | 247  | 233  | 272  |
4. Buildings of training and educational purpose
5. Buildings for temporary residence of the people
6. Buildings and constructions for agricultural purpose
7. All other objects of the fire
8. Erected (reconstructed) buildings (constructions)
9. Constructions, plants for industrial purpose
10. Office building
11. Buildings, residences for support Manning of the population
12. Wearable stuff (on the man)
13. Warehouses and constructions
14. Separately arranged structure (site hut, trailer, shed, service utility, booth and so on)
15. Buildings, constructions and premises in the mercantile business
16. Production buildings
17. Unexploited building (construction)
18. Place of the outdoor storage of the substances, materials, place of the lands and other open territories
19. Vehicles
20. Housing stock buildings and outhouses

2. Results and discussion
The number of the individuals $P(t)$ in the population is described by Verhulst function [6-8]:

$$P(t) = \frac{K P_0 G}{K + P_0 (G - 1)} \times G = \exp (rt)$$  \hfill (1)

where $P_0$ – is the initial number of the population, $K$ is a capacity of the environment (maximum possible number of the population), $r$ is a rate of reproduction.

If one uses approximation for the number of the fires the meaning of the constants in equation (1) changes. Let $P_0$ is a minimal number of the fires. Assume that $P_0 = 1$. If it is adopted that $P_0 = 0$, then all the other values of $P(t) = 0$, and this does not happen. Next, $K$ is the maximum value of the fires.

One can expect that $K$ is equal or greater from the maximum values from the really happened ones (table 1). Parameter $r$ is the rate of changes in the number of fires. Parameters $K$ and $r$ will be determined by the choice.

According to their meanings these parameters should be integer ones. As a result of the choice for the number of fires the following approximation should be employed

$$Y_m = \frac{K P_0 G}{K + P_0 (G - 1)}, G = \exp (rX)$$  \hfill (2)

Optimal values for the parameters $K$ and $r$ were found with the use of the feature “Search of a solution in Microsoft Excel program. They should provide minimum of the mean value of the squared error. The error is understood as a value of $e = Y_m - Y$, where $Y_m$ - is a model value while $Y$ is a real value.
For the year of 2018 minimum was found at the following values of parameters $K = 93384$ and $r = 0.4115$, $P_0 = 21$. The mean value of the error was of 1455, while the man value of the squared error is 187353650 (table 2).

**Table 2.** Verhulst model for the year of 2018.

| X  | Y   | Ym  | e   | $e^2$ |
|----|-----|-----|-----|-------|
| 1  | 97  | 32  | -65 | 4266  |
| 2  | 211 | 48  | -163| 26631 |
| 3  | 272 | 72  | -200| 39949 |
| 4  | 276 | 109 | -167| 27955 |
| 5  | 277 | 164 | -113| 12748 |
| 6  | 522 | 247 | -275| 75402 |
| 7  | 703 | 373 | -330| 109000|
| 8  | 765 | 562 | -203| 41408 |
| 9  | 777 | 845 | 68  | 4592  |
| 10 | 799 | 1269| 470 | 220866|
| 11 | 988 | 1902| 914 | 835015|
| 12 | 1233| 2840| 1607| 2583978|
| 13 | 1402| 4221| 2819| 7947308|
| 14 | 1772| 6227| 4455| 19843684|
| 15 | 2632| 9088| 6456| 41678631|
| 16 | 2813| 13067|10254|105141874|
| 17 | 3122| 18408|15286|233647781|
| 18 | 3385| 25245|21860|477857155|
| 19 | 16410|33488|17078|291648528|
| 20 | 93383|42734|-50649|2565326219|
| mean | | 1455| | 187353650 |

Pearson coefficient of the linear correlation between the model and real values is equal to $R = 0.7550$. The square of this value gives the value of the determination coefficient $R^2 = 0.57$. 

![Figure 1. Comparison of the actual values (Y) with the results of Verhulst model (Ym) for 2018 year.](image)
It means that Verhulst model can explain 57.00% of the actual values.

Comparison of the plots for the actual and simulated number of the fires in Russia demonstrated that Verhulst model does not provide quite reliable description of the situation (figure 1).

Results for the years of 2014 - 2017 are quite similar and they differ only by the values of constants K, P₀, r (table 3).

Note that Verhulst model provides not so great value for the determination coefficient. It is within the range of 57.00 – 59.49%. Therefore, Verhulst model seems to be inappropriate for the approximation of the number of fires in Russia over the kinds of the objects.

**Table 3.** Constants in Verhulst model.

| Year | K     | P₀ | r   | R   |
|------|-------|----|-----|-----|
| 2014 | 103580| 22 | 0.414961 | 59.32 |
| 2015 | 100499| 20 | 0.41812  | 59.49 |
| 2016 | 96815 | 23 | 0.408695 | 58.25 |
| 2017 | 93002 | 18 | 0.419802 | 58.50 |

Let us try an approximation.

\[ Y_\text{r} = \frac{KP_0 G}{K + P_0 (G - 1)}, G = \exp(r_1 X_i) \]  \hspace{1cm} (3)

For the every discrete value of Xᵢ (in the range of 1 to 20) there exists its own value of the growth rate rᵢ (table 4).

**Table 4.** Rate of growth for different kind of the objects.

| X   | 2014     | 2015     | 2016     | 2017     | 2018     |
|-----|----------|----------|----------|----------|----------|
| 1   | 1.927484 | 1.954233 | 1.83557  | 1.855604 | 1.531003 |
| 2   | 1.084048 | 1.073718 | 0.948144 | 1.105533 | 1.154686 |
| 3   | 0.831604 | 0.858341 | 0.79207  | 0.854327 | 0.854657 |
| 4   | 0.585074 | 0.66921  | 0.561602 | 0.620597 | 0.644653 |
| 5   | 0.452528 | 0.503994 | 0.466108 | 0.509124 | 0.516448 |
| 6   | 0.556598 | 0.553854 | 0.537141 | 0.579786 | 0.536421 |
| 7   | 0.587365 | 0.525371 | 0.499831 | 0.525849 | 0.502595 |
| 8   | 0.475209 | 0.487291 | 0.446524 | 0.461355 | 0.450419 |
| 9   | 0.416632 | 0.423441 | 0.414864 | 0.419827 | 0.402116 |
| 10  | 0.36972  | 0.382661 | 0.366132 | 0.372407 | 0.364721 |
| 11  | 0.355751 | 0.359866 | 0.3575   | 0.363194 | 0.351052 |
| 12  | 0.365708 | 0.374326 | 0.356205 | 0.362791 | 0.340479 |
| 13  | 0.320227 | 0.322452 | 0.313508 | 0.337556 | 0.32431 |
| 14  | 0.314458 | 0.325038 | 0.309996 | 0.326333 | 0.318162 |
| 15  | 0.334326 | 0.336892 | 0.322189 | 0.335688 | 0.323956 |
| 16  | 0.311122 | 0.313526 | 0.299431 | 0.317013 | 0.30799 |
| 17  | 0.286418 | 0.303633 | 0.292243 | 0.308123 | 0.296205 |
| 18  | 0.283716 | 0.297997 | 0.286478 | 0.295953 | 0.284405 |
| 19  | 0.378659 | 0.377879 | 0.366021 | 0.373319 | 0.360745 |
| 20  | 1.000247 | 1.001993 | 0.956611 | 0.999509 | 0.99221 |

An absolutely precise reproducing of the actual values (table 5) occurs at the values of constants K, P₀, (table 6) and the rates of rᵢ.
Table 5. Verhulst model with a variable growth rate for the period of 2018.

| X  | Y   | Ym  |
|----|-----|-----|
| 1  | 97  | 97  |
| 2  | 211 | 211 |
| 3  | 272 | 272 |
| 4  | 276 | 276 |
| 5  | 277 | 277 |
| 6  | 522 | 522 |
| 7  | 703 | 703 |
| 8  | 765 | 765 |
| 9  | 777 | 777 |
| 10 | 799 | 799 |
| 11 | 988 | 988 |
| 12 | 1233| 1233|
| 13 | 1402| 1402|
| 14 | 1772| 1772|
| 15 | 2632| 2632|
| 16 | 2813| 2813|
| 17 | 3122| 3122|
| 18 | 3385| 3385|
| 19 | 16410| 16410|
| 20 | 93384| 93383|

In this case an error of the model is equal to zero.

Table 6. Constants in Verhulst model with a variable rate of growth.

| K | P0 |
|---|----|
| 2014 | 103580 | 22 |
| 2015 | 100499 | 20 |
| 2016 | 96815  | 23 |
| 2017 | 93002  | 18 |
| 2018 | 93384  | 21 |

3. Conclusions
It means that absolutely precise description of dependence for the number of fires in Russian Federation on the object’s kind is provided by Verhulst model with a variable rate of growth,
Thus, representations concerning the population growth used in biology proved to be productive ones when searching for the approximation of the dependence for the number of fires in Russian Federation on the kind of an object.

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