Students’ critical mathematical thinking process based on their cognitive styles

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Abstract. Students' critical mathematical thinking skills are relatively low. Efforts to improve these skills can be made through specific strategies that require information about the critical thinking processes based on students' cognitive styles. This study is qualitative research that aims to describe students' mathematical critical thinking process based on their cognitive styles. The subjects were two students, one field-dependent student and one field-independent student. These students were selected based on the cognitive style test (i.e., GEFT) considering their ability to communicate well. The data obtained from the interview were analyzed to describe the students' critical mathematical thinking process. The results showed that the students with both cognitive styles committed a process of convincing. They tried to reason why they arrived at the solution and convinced others either orally or in writing that every step of the solution was done correctly through a systematic way. Besides, the field-independent student carried out a specializing process in which he investigated the problem carefully, identified the information given, and recognized the information required to solve the problem. In the process of selecting learning methods, teachers need to consider students' cognitive styles so that mathematical critical thinking skills of students with different cognitive styles will increase.

1. Introduction
One of the main focuses of learning mathematics is that students should have mathematical thinking skills. This purpose is in line with the goals of mathematics learning at schools that correspond to the Curriculum 2013, which requires students to have good mathematical thinking skills needed to solve problems. Students' ability to solve mathematical problems involves several factors, including higher-order thinking skills and one of the higher-level thinking skills is critical mathematical thinking skills.

Critical thinking is one of the essential elements to have in 21st-century education. Critical thinking is the way of thinking reflectively and productively that involves evaluating evidence [1]. The one who has critical thinking skills can express his/her opinion based on evidence [2]. Critical thinking is one of the thinking skills that contribute significantly to student motivation and learning outcomes [3]. Correspondingly, the use of critical thinking skills in learning can help students to understand more about the concepts that have been learned [4]. The ability to think critically can be developed by selecting the contents in learning in which the solutions needed involve critical thinking skills [5]. Thus, critical thinking skills are considered as one of the essential abilities that the students must have in learning mathematics.

However, mathematics learning at schools does not fully support the students' mathematical critical thinking skills. Students have not demonstrated satisfying results in solving problems in a way they
could make use of their critical thinking skills [6]. Also, only a few of them meet the criteria of interpretation, analysis, evaluation, conclusion, explanation, and self-regulation in figuring out the critical thinking skill test [7]. This condition shows that students’ critical thinking skills in solving mathematical problems are relatively low; as such, it needs efforts to improve their critical thinking skills.

The learning process experienced by students, as well as their thinking process in solving problems, are different [8]. Thus, teachers need to understand the way the students think to discover the mistakes and the types of errors made by the students [9]. To find out the students’ critical thinking process in solving problems can be undergone by identifying the steps they use. This thinking process is based on the aspects of specializing, generalizing, conjecturing, and convincing [10].

Cognitive style is the ability of students to express the information obtained from their environment. The cognitive styles of students are varied; those refer to the characteristics of students in receiving information, processing, storing, thinking, and being able to use information in solving problems [11]. The differences of the cognitive styles lead to different ways that the students use in solving a problem [12]. The aspects that distinguish an individual's cognitive style are field-dependence and field-independence [13]. Students with the field-dependence cognitive style can think globally, receive and follow existing information, and tend to prioritize external motivation, while those with the field-independence cognitive style are good in analyzing objects separated from their environment, grouping objects, and prioritizing internal motivation [13].

Previous research shows that there are differences in students' thinking processes when solving mathematical problems based on the cognitive styles of field-dependence and field-independence [14]. Also, there has research on the profile of students' critical thinking in problem-solving based on reflective-impulsive cognitive style and mathematical ability [12]. Based on the previous studies, the researchers will examine the students’ critical thinking processes based on their cognitive styles of field-dependence and field-independence.

Based on the explanation above, it is crucial to bear in mind that recognizing students’ critical thinking is an essential aspect of the learning process. Since students’ cognitive styles in solving problems are diverse, this study focuses on investigating students’ critical mathematical thinking process based on their cognitive styles. The research questions of this study are “What are the processes of students’ mathematical critical thinking with the field-dependence cognitive style,” and “What are the processes of students’ mathematical critical thinking with the field-independence cognitive style.”

2. Method

This study is qualitative research that aims to describe students’ mathematical critical thinking process based on the cognitive styles of field-dependence and field-independence. This research was conducted in the seventh grade of a junior high school in Banda Aceh. There were several instruments used in this study, such as the Group Embedded Figures Test (GEFT) to classify students based on their cognitive styles, an interview guide, and a series of questions about critical mathematical thinking to discover students’ critical thinking process.

Based on the cognitive style test using GEFT, 36 seventh grade students were classified into two groups: a group of 28 students with the field-dependence cognitive style and a group of eight students with the field-independence cognitive style. Then, two students were chosen from these two groups: a student from the field-dependence group (i.e., AR) and another student from the field-independence group (i.e., DA). The subjects were selected due to some considerations that they were cooperatively and communicatively active in expressing idea or opinion. Further, those two students were interviewed to investigate their critical thinking processes. To simplify the process of obtaining information during the interview, the researchers asked the students to answer a sort of questions related to critical thinking skills in the form of essay. The questions given were about lines and angles.

Data collection involved the cognitive style test, interview, and critical mathematical thinking test. The data analysis technique consists of three stages: data reduction, data display, and verification [15].
3. Results and discussion

Based on the data analysis, it was found that students’ thinking process with the field-dependence and the field-independence cognitive styles is different.

3.1. Critical thinking process of the field-dependent student

In this section, the critical mathematical thinking process of a field-dependent student (i.e., AR) will be described. AR’s critical thinking process in solving the first problem is shown in Figure 1.

![Figure 1. AR’s work on the first problem](image)

The analysis results revealed that AR was not able to undergo the specializing process. It was represented in her work in which she did not thoroughly understand the question. Also, she did not notice the necessary information given in the test. It was evident from her answer sheet; she could not provide the correct information needed. The student also got confused to answer the question when the teacher asked, “What information do you need to figure out the question?”. In other words, she was considered unable to find the information needed to solve the problem.

Moreover, AR could not undertake the process of conjecturing. It was shown from her response when asked, “How do you do the test?”. This student could not explain the strategy that can be used to deal with the problem. Thus, she was considered unable to find the method of solving the problem. In addition to that, the student could not reflect on the idea of accomplishing the problem, for example, determining which part was difficult, and what she could learn from the process of solving the problem.

AR’s critical thinking process in solving the second problem is shown in Figure 2.

![Figure 2. AR’s work on the second problem](image)

In this second problem, AR could formulate the statement in the question into a mathematical model to complete the answer and gave the relevant justifications. When the teacher asked her, “What are the stages you use in analyzing the statement given?”. The student explained, “First thing to do is changing the statement into the mathematical form.” She continued, “Right angle means 90 − α,
straight angle means $180 - \alpha$ so that the first statement can be described as $\alpha = 2(90 - \alpha) - (180 - \alpha)$. and the second statement is $180 - \alpha = \alpha + 2(90 - \alpha)$.” “What do you do after defining it into the mathematical form?” the teacher then inquired her again. The student answered, “We can directly answer the question. The first statement is wrong because its deviation is not $\alpha$, while the second statement is correct because its result is the same.” In this excerpt, the student tried to convince other people of every stage of her solution orally. Therefore, this student was considered doing the process of convincing. However, in answering the question, the student made a mistake in algebra. Being asked about this, she was very sure that the answer she gave was correct without re-checking the accuracy of her calculation. Hence, this student was considered not doing the process of generalizing.

Based on the result of data analysis, it can be concluded that AR with her field-dependence cognitive style could only do the process of convincing in solving the problems.

3.2. Critical thinking process of the field-independent student

In this section, the critical mathematical thinking process of a field-independent student (i.e., DA) will be explained. DA’s critical thinking process in solving the first problem is shown in Figure 3.

When the teacher asked this student, “What do you understand about this question?”. He replied, “In this question, there is one shifted point which makes the location of the line changes.” Then, the teacher asked him again, “In your opinion, what information do you need to solve the problem?”. “We can use the formula of a straight angle,” the student answered and then explained how the angle size changes because of the shifting point. In his explanation, the student stated; "For a straight angle, if one angle narrows, another angle would get bigger because a straight angle always adds up to 180°." However, when he was inquired about the size of the previous angle, which also had a straight angle, the student looked confused and could not reveal the answer. Then, when he was questioned about the relationship between the angles if there were two parallel lines cut by another line, DA successfully mentioned the whole relationship. While describing this relationship, this student realized that there was a relationship between angles and lines in the problem solved. Then, he tried to tackle the second problem about the concept of the relationship between lines and angles appropriately. Thus, it can be concluded that this student performed the process of specializing.

DA’s critical thinking process in solving the second problem is shown in Figure 4.
In this problem, DA could express the statement in the question into a mathematical model followed by the relevant justifications. When the teacher asked him, "What steps do you use to determine the answer of the statement?". The student explained, "We can assume the size of the acute angle as $\alpha$ so that the right angle is $90 - \alpha$ and the straight angle is $180 - \alpha$." The student added, "From the previous comparison, the first statement resulted as $\alpha = 2(90 - \alpha) - (180 - \alpha)$ and the second statement is $180 - \alpha = \alpha + 2(90 - \alpha)$." Then, the teacher confirmed the student, "How do you determine whether the statement is true or false?". He replied, "The first statement is false, while the second one is correct." After expressing his opinion, the student tried to convince others in every step of the solutions verbally. Thus, it can be concluded that the student did a process of convincing. However, in solving the problem, he made mistakes in the algebraic calculation. Being asked about this, the student confidently stated that the answer he gave was correct without re-checking its accuracy. Therefore, it can be construed that the student did not do the generalizing process.

To sum up, the analysis results showed that DA with his field-independence cognitive style was considered to perform the process of convincing and specializing in solving problems.

Based on the data analysis on the two students, AR and DA, it can be stated that there are differences between field-dependent and field-independent students in solving the problems [15]. Those differences can be seen in their critical thinking process when solving the problems. The student with the field-dependence cognitive styles performed a process of convincing, while the one with the field-independence cognitive style demonstrated the process of specializing and convincing. Thus, it can be concluded that in understanding and solving the problems given, the thinking process of the field-independent student is better than that of the field-dependent student. This finding is in line with the result of Ngilawajan’s research which reported that field-independent students are better than field-dependent students in processing information and understanding problems [14]. In this present study, the thinking processes that emerged were the convincing and specializing process. It indicates that not all thinking processes come up while students are solving the problem [16]. Therefore, students' thinking abilities are relatively low [17].
4. Conclusion
Based on the results and discussion described above, it can be concluded that the students with both cognitive styles carried out the process of convincing when justifying their solutions. They were also able to convince other people either verbally or in writing that each step of completions was done correctly in a systematic way. Besides, the field-independent student also demonstrated the specializing process of understanding the questions carefully, identifying the information given in the questions, and figuring out the information needed to solve the problems. Suffice to say that students’ critical mathematical thinking processes differ in terms of their cognitive styles. The selection of appropriate learning methods can affect students’ mathematical critical thinking skills. Therefore, selecting the variety of learning methods in the classroom needs to consider students’ cognitive styles so that students’ critical thinking skills with different cognitive styles could improve as a whole.

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