Nonlinear $\mathcal{N} = 2$ Supersymmetry and D2-brane Effective Actions

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Abstract: D$p$-branes acquire effective nonlinear descriptions whose bosonic parts are related to the Born-Infeld action. This nonlinearity has been proven to be a consequence of the partial $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry breaking, originating from the solitonic nature of the branes. In this work, we focus on the effective descriptions of D2-branes, which play important roles in the Type IIA string theory. Using the Goldstone multiplet interpretation of the action and the method of nilpotent superfields, we construct a 3D superspace description which makes the first supersymmetry manifest and realizes the second, spontaneously broken, supersymmetry nonlinearly. We find that appropriate Goldstone multiplets and their effective superspace actions are generated by the $\mathcal{N} = 2, D = 3$ vector and tensor multiplets after expanding them around a nontrivial vacuum and enforcing constraints that eliminate additional degrees of freedom. We show both descriptions are related by a duality transformation which results in the inversion of a dimensionless parameter. The explicit bosonic and fermionic parts of the effective spacetime action are derived. Finally, we consider the deformation of the superspace action by the characteristic Chern-Simons-like mass term of vector multiplet in 3D.
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### 1 Introduction

The deep connection between partial supersymmetry breaking and nonlinear realizations of extended supersymmetries has been studied extensively [1–11]. One of the most transparent demonstrations of this connection is the existence of D$p$-branes which are solitonic solutions of string theory [12, 13]. The introduction of a boundary to the world-sheet theory breaks some of the spatial translational symmetries and half of the supersymmetries [14] which implies that D-branes correspond to BPS saturated soliton states.

The result is these solutions acquire a lower-dimensional effective description where the bosonic part is given by the DBI action for the corresponding collective coordinates and the world volume gauge field [13, 15–18]. This action is closely related to the nonlinear Born-Infeld type of action of open string theory [19–22] because the DBI action for all $p$-branes ($p < 9$) always corresponds to the dimensional reduction of the ten-dimensional (D9-brane) BI action [17].

The existence of these effective nonlinear descriptions is no accident and follows directly from the partial supersymmetry breaking $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$. In general, the memory of a
spontaneously broken symmetry does not fade completely and is captured through nonlinear realizations of that symmetry. In this fashion, the effective description of Dp-branes must be consistent with the linear representations of the surviving supersymmetry and also with the nonlinear realizations of the second broken supersymmetry. Such nonlinear realizations of supersymmetry are defined via nonlinear constraints which generate precisely the Born-Infeld type actions of D-branes.

For the D3-brane, the manifestly $\mathcal{N}=1$ supersymmetric effective action was first found in [2, 3]. However, as it was shown, demanding the surviving supersymmetry to be manifest is not enough to uniquely determine the superspace action. Later in [7] the 4D, $\mathcal{N}=1$ supersymmetric BI action was derived with the interpretation of being the massless Goldstone supermultiplet associated with the partial breaking of the second supersymmetry. In addition to the $\mathcal{N}=1$ superspace description, demanding the invariance of the theory under the second non-linearly realized supersymmetry fixed all previous ambiguities and gave a unique answer for the superspace Lagrangian.

A general method for constructing nonlinear realizations of a symmetry is to start with a linear representations of that symmetry and impose consistent nonlinear constraints. For the case of supersymmetry this led to the constrained superfield approach of [1] which can be applied universally to find nonlinear actions. In [9] using the constrained superfield approach it was shown that the results of [7] directly follow from the $\mathcal{N}=2$ free Maxwell theory by (i) expanding around a non-trivial vacuum that breaks the second supersymmetry and (ii) imposing the standard nonlinear –nilpotency– constraint which reduces the field content of the $\mathcal{N}=1$ theory to the Maxwell-Goldstone multiplet by relating the $\mathcal{N}=1$ chiral and vector multiplet components.

Interestingly, it was shown in [10, 23] that adding a Fayet-Iliopoulos (FI) term to the superspace action of [7, 9] resulted in a theory which remained invariant under the second non-linearly realized supersymmetry and $\mathcal{N}=1$ manifest. Of course, the presence of the FI term gives a VEV to the auxiliary component of the vector multiplet and therefore it spontaneously breaks the first supersymmetry. Nevertheless, the deformed theory still enjoys an unbroken $\mathcal{N}=1$ symmetry which corresponds to a different choice of an $\mathcal{N}=1$ subsector of the $\mathcal{N}=2$ theory.

It is often common practice, once an action is found, to use dualities —symmetries between the equations of motion and their Bianchi identities— to map it to a dual action with equivalent on-shell dynamics. The duality transformations can be applied to a large class of (non-Gaussian) actions which depend only on the field strength of a p-form algebraically. For the special case of $p = (d - 2)/2$ it may happen that the dual action is the same functional\footnote{Up to additional transformations of background fields and parameters.} of the dual field strength as the original action for the original field strength. The Born-Infeld action falls in this category of self-dual theories with the duality transformation being the standard electric-magnetic duality. The supersymmetric extension of the BI action as was shown in [7, 9] preserves this self-duality property. By turning on the background dilaton and
axion fields\textsuperscript{2}, the U(1) duality rotation is extended to SL(2,R) which is the self-duality group of the effective D3-branes \textsuperscript{24}. The supersymmetric theory with the FI deformation can be understood as a special case of the supersymmetric extension of this generalized BI action \textsuperscript{23}.

Similar to the special role of the D3-brane in IIB theory, D2-brane plays a significant role in IIA theory. What makes them particularly interesting is that they can be interpreted as the (M-theory) eleven-dimensional supermembrane \textsuperscript{15, 16, 25}. One method of deriving the $\mathcal{N} = 1, D = 3$ supersymmetric extension of the effective Born-Infeld action associated with the D2-brane is to dimensionally reduce the 4D results of \textsuperscript{7} down to 3D in a way that breaks half the supercharges. This is the path followed in \textsuperscript{26, 27} via the use of the coset approach \textsuperscript{28, 29} which provides a systematic method of constructing nonlinear realizations and studying properties of Goldstone fields. Specifically, they considered appropriate factorizations of elements of the coset space $\{\mathcal{N} = 1, D = 4 \text{ Super-Poincare}\}/SO(1,2)$ which reflect the spontaneously breaking of one translational symmetry and one supersymmetry.

In this work, we derive the $\mathcal{N} = 1, D = 3$ superspace actions for the effective 3D Born-Infeld theory by considering appropriate, manifestly $\mathcal{N} = 2, D = 3$ theories which via the constraint superfield approach are broken down to $\mathcal{N} = 1, D = 3$, massless multiplets. under supersymmetry. We identify two such $\mathcal{N} = 2$ multiplets that give rise to viable $\mathcal{N} = 1$ goldstone multiplets, the vector multiplet and the tensor multiplet. For both cases, we derive explicitly the bosonic and fermionic parts of the effective Goldstone spacetime action. We show that in three dimensions these two effective descriptions are related by a duality transformation which generates an inversion $\lambda \to \tilde{\lambda} = 1/\lambda$ of a characteristic dimensionless parameter $\lambda$. It is well known that vector supermultiplets in three dimensions can have a gauge invariant, Chern-Simons-like mass term. We show that this term generates a one-parameter deformation of the Maxwell-Goldstone superspace action which respects the non-linearly realized supersymmetry but explicitly breaks the first, linear, supersymmetry.

The layout of this paper is the following. In section 2 we construct the 3D Maxwell-Goldstone multiplet by following the arguments of Bagger-Galperin \textsuperscript{7}. Subsequently, we show that this Goldstone multiplet is a result of a nilpotent, $\mathcal{N} = 2$ superfield generated by the expansion of the $\mathcal{N} = 2$ Maxwell theory around a nontrivial vacuum. Starting from the effective, superspace action we derive the explicit spacetime effective action. As expected, the bosonic part gives the 3D Born-Infeld action. The fermionic part of the Lagrangian has a more complicated structure with the form of polynomial terms times nonpolynomial factors which correspond to derivatives of the 3D Cecotti-Ferrara functions. In section 3, we consider the Chern-Simons mass term of the vector multiplet as a generator of a one-parameter deformation of the Maxwell-Goldstone action and show that such a deformation is consistent with the second, nonlinear supersymmetry but violates the first supersymmetry. Section 4 is devoted to the study of the Tensor-Goldstone multiplet and has a similar flow of narrative as

\textsuperscript{2}One should also turn on the background two-form $C_2$ and four-form $C_4$ of the R-R sector and the NS-NS two form $B_2$. However, they are not necessary for observing the presence of the SL(2,R) duality group.

\textsuperscript{3}We are grateful to E. Ivanov for bringing these papers to our attention.
We close by showing that Maxwell-Goldstone and Tensor-Goldstone multiplets as well as their superspace actions, map to each other under duality transformations in section 5. This duality generates an inversion of the dimensionless parameter $\lambda$ constructed out of the dimensionful parameters that appear in their effective actions. We conclude with the summary and two appendices.

2 3D, $\mathcal{N} = 1$ Maxwell-Goldstone Multiplet

2.1 Review of 3D, $\mathcal{N} = 1$ super-Maxwell Multiplet

In three dimensions, the $\mathcal{N} = 1$ vector multiplet is described by the spinorial superfield strength $W_\alpha$ constrained by the Bianchi identity:

$$D^\alpha W_\alpha = 0 \Rightarrow D^2 W_\alpha = i \partial_\alpha \beta W_\beta , \quad (2.1)$$

which can be solved by expressing the superfield strength $W_\alpha$ in terms of an unconstrained —prepotential— spinorial superfield $\Gamma_\alpha$:

$$W_\alpha = \frac{1}{2} D^\beta D_\alpha \Gamma_\beta . \quad (2.2)$$

The prepotential $\Gamma_\alpha$ is not unique and defines an equivalence class $[\Gamma_\alpha]$ via the following gauge transformation

$$\delta \Gamma_\alpha = D_\alpha K , \quad (2.3)$$

where $K$ is an arbitrary scalar superfield. The superspace action takes the form

$$S = \frac{1}{g^2} \int d^3x \, d^2\theta \, W^2 , \ W^2 := \frac{1}{2} W^\alpha W_\alpha , \quad (2.4)$$

which results in the spacetime action

$$S = \frac{1}{g^2} \int d^3x \left\{ i \lambda^\alpha \partial_\alpha \beta \lambda_\beta - \frac{1}{2} f^\alpha \beta f_\alpha \beta \right\} , \quad (2.5)$$

where the component fields $\lambda_\alpha$ and $f_{\alpha \beta}$ are defined as $\lambda_\alpha = W_\alpha|_{\theta = 0}$ and $f_{\alpha \beta} = D_\alpha W_\beta|_{\theta = 0}$ respectively and $g$ is a dimensionless constant. Due to constraint (2.1), it is straightforward to see that the component field $f_{\alpha \beta}$ is symmetric in the two spinorial indices ($f_{\alpha \beta} = f_{\beta \alpha}$) and is the spinor form of the usual 3D Faraday tensor.

2.2 Maxwell-Goldstone Multiplet

Our aim is to interpret the above supermultiplet as the Goldstone multiplet that accommodates the Goldstino associated with the spontaneous breaking of the second supersymmetry in 3D precisely as the D2-brane of type IIA string theory is expected to do. Following Bagger and Galperin [7], we search for the most general transformation $\delta^*_{\lambda}$ of $W_\alpha$ which is consistent with constraint (2.1). In order for this transformation to be understood as a second supersymmetry transformation, it must involve the second supersymmetry partners of $W_\alpha$. Due to the spontaneous breaking of this supersymmetry, one of the partners will acquire a non-trivial
VEV which will generate the characteristic shift in the transformation of $W_\alpha$. After redefining the second supersymmetry parameter and the remaining partner superfield, we find the most general transformation is

$$
\delta^*_\eta W_\alpha = \eta_\alpha - \frac{1}{2\kappa} (D^\beta D_\alpha X) \eta_\beta , 
\tag{2.6a}
$$
$$
\delta^*_\eta X = \frac{2}{\kappa} \eta^\alpha W_\alpha . 
\tag{2.6b}
$$

The dimensionful parameter $\kappa \ (|\kappa| = 3/2)$ corresponds to the non-trivial VEV and the overall coefficient in (2.6b) is determined by the compatibility of (2.6) with supersymmetry algebra\(^4\). The second term on the right hand side of (2.6a) appears to have a different structure than the expected 3D reduction of the 4D answer. However, by using the algebra of the 3D spinorial covariant derivatives it can be shown that they are equivalent.

Furthermore, in order to prohibit the remaining partner superfield $X$ to carry independent degrees of freedom and have the 3D Maxwell multiplet be the Goldstone multiplet corresponding to the partial breaking of supersymmetry, we impose the following nonlinear constraint:

$$
\kappa X = W^\alpha W_\alpha + \frac{1}{2} (D^2 X) X \quad \Rightarrow \quad X = \frac{W^\alpha W_\alpha}{\kappa \left(1 - \frac{1}{2\kappa} D^2 X\right)} . 
\tag{2.7}
$$

This constraint is consistent with (2.6) and also compatible with the 3D reduction of the 4D constraint of [7] by recalling that in 3D there is no notion of chirality. Using $W_\alpha W_\beta W_\gamma = 0$, constraint (2.7) can be solved in a similar fashion as in [7] and express $X$ in terms of $W^2$ and $D^2W^2$:

$$
X = \frac{2}{\kappa} W^2 \left[1 + \frac{T}{1 - T + \sqrt{1 - 2T}}\right] , 
\tag{2.8}
$$

where $T = \frac{2}{\kappa^2} D^2 W^2$. The superspace action for the 3D Maxwell-Goldstone multiplet is

$$
S = \tau \int d^3 x \, d^2 \theta \, X = \frac{2\tau}{\kappa} \int d^3 x \, d^2 \theta \, W^2 \left[1 + \frac{T}{1 - T + \sqrt{1 - 2T}}\right] . 
\tag{2.9}
$$

This is manifestly invariant under the first supersymmetry and also respects the second supersymmetry because due to (2.6b) and (2.1) the spacetime Lagrangian transforms as a total derivative

$$
\delta^*_\eta D^2 X = \frac{2i}{\kappa} \partial_\alpha \partial^\beta (\eta^\alpha W_\beta) . 
\tag{2.10}
$$

It is clear that (2.9) is a supersymmetric Born-Infeld type of action and we will show its bosonic part is the corresponding BI effective action for the D2-brane. The superspace effective action (2.9) and constraint (2.7) match the results found in [27].

\(^4\) We follow the conventions of Superspace [30]. For details see appendix B.
Following [1, 9] the above results can be easily understood from the viewpoint of the manifestly \( N = 2 \) theory. The 3D, \( N = 2 \) vector multiplet is described by a scalar superfield \( \mathcal{W}(x, \theta, \bar{\theta}) \) which satisfies the following irreducibility conditions:

\[
D^2 \mathcal{W} = \tilde{D}^2 \mathcal{W}, \quad D^\alpha \tilde{D}_\alpha \mathcal{W} = 0, \quad (2.11)
\]

where the tilded Grassmann coordinates (\( \tilde{\theta}_\alpha \)) and covariant derivatives (\( \tilde{D}_\alpha \)) correspond to the second supersymmetry. By expanding the \( N = 2 \) superfield \( \mathcal{W} \) in terms of \( N = 1 \) component superfields

\[
\mathcal{W}(x, \theta, \bar{\theta}) = \Phi(x, \theta) + \tilde{\theta}^\alpha W_\alpha(x, \theta) - \tilde{\theta}^2 F(x, \theta), \quad (2.12)
\]

we can solve (2.11) to find that

\[
F = D^2 \Phi, \quad D^2 W_\alpha = i \partial_\alpha \beta W_\beta, \quad D^\alpha W_\alpha = 0. \quad (2.13)
\]

Moreover, superfields \( \Phi, W_\alpha, \) and \( F \) transform under the second supersymmetry as follows:

\[
\begin{align*}
\delta^e \Phi &= - \epsilon^\alpha W_\alpha, \\
\delta^e W_\alpha &= \epsilon_\alpha F - i \epsilon^\beta \partial_\beta \Phi = D^\beta D_\alpha \Phi \epsilon_\beta, \\
\delta^e F &= - i \epsilon^\alpha \partial_\alpha \beta W_\beta,
\end{align*}
\]

which are consistent with (2.13). In order to break this second supersymmetry we follow the constrained superfield approach [1]. We (i) expand the \( N = 2 \) superfield \( \mathcal{W} \) around a background superfield which breaks the second supersymmetry and (ii) we impose the usual nilpotence condition in order to remove ‘radial’ superfields:

\[
\mathcal{W} = \langle \mathcal{W} \rangle + \mathcal{W}, \quad W^2 = 0. \quad (2.15)
\]

The condensate \( \langle \mathcal{W} \rangle \) is Lorentz and \( N = 1 \) invariant with a non-trivial \( \tilde{\theta} \) dependence.

\[
\langle \mathcal{W} \rangle = \kappa \tilde{\theta}^2 \Rightarrow \mathcal{W} = - \frac{1}{2} X + \tilde{\theta}^\alpha W_\alpha - \tilde{\theta}^2 \left( - \frac{1}{2} D^2 X + \kappa \right), \quad (2.16)
\]

where \( X = -2\Phi \). The second supersymmetry transformations take the form

\[
\begin{align*}
\delta^e X &= 2 \epsilon^\alpha W_\alpha = \frac{2}{\kappa} \epsilon^\alpha W_\alpha, \\
\delta^e W_\alpha &= \epsilon_\alpha \kappa - \frac{1}{2} (D^\beta D_\alpha X) \epsilon_\beta = \epsilon_\alpha - \frac{1}{2\kappa} (D^\beta D_\alpha X) \epsilon_\beta,
\end{align*}
\]

where \( \epsilon_\alpha \) is the \( \kappa \) scaled supersymmetry parameter (\( \epsilon_\alpha = \epsilon_\alpha \kappa \)). These transformations match exactly (2.6). The nilpotence condition \( W^2 = 0 \) imposes the following nonlinear constraints:

\[
X^2 = 0, \quad X W_\alpha = 0, \quad \kappa X = W^\alpha W_\alpha + \frac{1}{2} XD^2 X. \quad (2.18)
\]

which agree with (2.7).
2.3 Supersymmetric 3D Born-Infeld action in components

Starting from the superspace action (2.9), we extract the spacetime component action. It can be written in the following way

\[ S = \tau \kappa \int d^3x \left\{ T|_{\theta=0} + \frac{2}{\kappa^2} \int d^2\theta \, \Psi(T) \, W^2 \right\}, \tag{2.19} \]

which makes it easy to see that it is a member of the Cecotti-Ferrara class of actions [3] after a dimensional reduction to 3D. In this case, the function \( \Psi(x) \) is fixed to be:

\[ \Psi(x) = \frac{x}{1 - x + \sqrt{1 - 2x}}. \tag{2.20} \]

By performing the \( \theta \) integral we find the bosonic part of the spacetime action to be

\[ S_B = \tau \kappa \int d^3x \left( 1 - \sqrt{1 - 2s} \right), \tag{2.21} \]

where

\[ s = -\frac{1}{\kappa^2} f^{\alpha\beta} f_{\alpha\beta} \tag{2.22} \]

and corresponds to the BI effective action of D2-brane. The fermionic part of the spacetime action is determined to be

\[ S_F = \frac{\tau}{\kappa^3} \int d^3x \left\{ \Psi'(T) \left[ 4i (f^{\alpha\delta} \lambda_{\delta}) \partial_{\alpha\beta} (f^{\beta\gamma} \lambda_{\gamma}) \right. \right. \]

\[ + \left. \left. \left( 2 (\Box \lambda_{\gamma}) \lambda_{\gamma} + (\partial^{\alpha\beta} \lambda_{\gamma}) (\partial_{\alpha\beta} \lambda_{\gamma}) \right) \lambda^\sigma \lambda_{\sigma} \right] \right\} + \frac{2}{\kappa^2} \Psi''(T) \left[ \partial^{\alpha\beta}(f^{\beta\gamma} \lambda_{\gamma}) \right] \left[ \partial_{\alpha\delta}(f^{\delta\epsilon} \lambda_{\epsilon}) \right] \lambda^\sigma \lambda_{\sigma} \right\}, \tag{2.23} \]

where

\[ T| = \frac{2}{\kappa^2} \left[ -\frac{1}{2} f^{\alpha\beta} f_{\alpha\beta} - i \lambda^\alpha (\partial_{\alpha\beta} \lambda^\beta) \right], \tag{2.24} \]

\[ \Psi'(T|) = \frac{1}{\sqrt{1 - 2T|}} \frac{1}{1 - T| + \sqrt{1 - 2T|}}, \tag{2.25} \]

\[ \Psi''(T|) = \frac{2 - 3T| + 2\sqrt{1 - 2T|}}{(1 - 2T|)^{3/2} (1 - T| + \sqrt{1 - 2T|})^2}. \tag{2.26} \]

3 Mass Deformation

A special property of the 3D vector multiplet which makes it very different from its 4D analogue is the existence of a gauge invariant multiplet mass term

\[ S_m = \frac{m}{2} \int d^3x \, d^2\theta \, \Gamma^\alpha W_{\alpha}. \tag{3.1} \]

\(^5\)For details look in appendix A.
The gauge invariance of this term is based on the Bianchi identity (2.1) generated by varying (3.1) via (2.3). This term is manifestly invariant under the first, linear realized, supersymmetry and we want to study its transformation under the second supersymmetry. That requires to have knowledge about the transformation of the prepotential $\Gamma_\alpha$ under the second supersymmetry.

We introduce a new superfield $\Delta_\alpha$ defined as follows

$$D^\alpha \Delta_\alpha = X .$$  \hspace{1cm} (3.2)

This definition does not determine $\Delta_\alpha$ uniquely, as it enjoys the gauge transformation

$$\delta \Delta_\alpha = D^\beta D_\alpha K_\beta , \quad \delta K_\alpha = D_\alpha K .$$  \hspace{1cm} (3.3)

Using (2.2) and (3.2), we find that transformations (2.6) induce the following second supersymmetry transformations for superfields $\Gamma_\alpha$ and $\Delta_\alpha$

$$\delta^* \Gamma_\alpha = \frac{1}{\kappa} \eta^\beta D_\beta \Delta_\alpha + \Phi_\alpha^{(sp)} ,$$  \hspace{1cm} (3.4a)

$$\delta^* \Delta_\alpha = - \frac{1}{\kappa} \eta^\beta D_\beta \Gamma_\alpha ,$$  \hspace{1cm} (3.4b)

where $\Phi_\alpha^{(sp)} = -2\eta_\alpha \theta^2$ is the special solution of the inhomogeneous equation $D^\beta D_\alpha \Phi_\beta^{(sp)} = 2\eta_\alpha$. The full solution of the homogeneous equation $D^\beta D_\alpha \Phi_\beta = 0$ corresponds to a gauge transformation of $\Gamma_\alpha$ and thus can be dropped since (3.4a) is defined modulo gauge transformation terms. Under the above transformations (3.4), the mass term (3.1) is not invariant

$$\delta^* S_m = \frac{m}{g^2} \int d^3 x d^2 \theta \Gamma_\alpha \eta_\alpha = \frac{2m}{g^2} \int d^3 x \lambda^\alpha \eta_\alpha ,$$  \hspace{1cm} (3.5)

where $D^2 \Gamma_\alpha |_{\theta=0} = 2W_\alpha |_{\theta=0} = 2\lambda_\alpha$. However, it is easy to check that the integrand $2\lambda^\alpha \eta_\alpha$ corresponds to the second supersymmetry transformation (2.6b) of $X|_{\theta=0}$. Therefore, there exists a one-parameter family of deformations of the Maxwell-Goldstone action

$$S_\xi = \frac{\xi m}{2} \int d^3 x d^2 \theta \left\{ \Gamma^\alpha W_\alpha - 2\kappa \theta^\alpha \Delta_\alpha \right\} ,$$  \hspace{1cm} (3.6)

which respects the second supersymmetry transformations (3.4) and gauge transformations (2.3) and (3.3). However, the second term of $S_\xi$

$$\int d^3 x d^2 \theta \theta^\alpha \Delta_\alpha = \int d^3 x X|_{\theta=0}$$  \hspace{1cm} (3.7)

explicitly breaks the first supersymmetry. Using (2.8), it is straightforward to find that

$$X|_{\theta=0} = \frac{1}{\kappa} \lambda^\alpha \lambda_\alpha \left( 1 + \Psi(T) \right) ,$$  \hspace{1cm} (3.8)

it does not include a bosonic part and under a first supersymmetry transformation (3.7) is not invariant.
4 3D, $\mathcal{N}=1$ Tensor-Goldstone Multiplet

The choice of the Goldstone multiplet which will accommodate the goldstino field generated by the partial supersymmetry breaking is not unique. In four dimensions, it has been shown that chiral multiplets, vector multiplets, or tensor multiplets can play that role [7–9, 31]. In this section, we explore the use of a 3D tensor multiplet for the role of the Goldstone multiplet.

4.1 Review of 3D, $\mathcal{N}=1$ Tensor multiplet

In three dimensions, the tensor multiplet is described by a spinorial superfield $U_\alpha$ which satisfies the following constraint:

$$D^\alpha D_\beta U_\alpha = 0 \Rightarrow D^2 U_\alpha = -i \partial_\alpha \beta U_\beta .$$

These can be solved by expressing $U_\alpha$ in terms of an unconstrained scalar superfield $G$:

$$U_\alpha = D_\alpha G .$$

The superspace action describing the free dynamics of tensor multiplet takes the form

$$S = -\frac{1}{g^2} \int d^3 x \, d^2 \theta \, U^2 ,$$

which generates the spacetime action

$$S = \frac{1}{g^2} \int d^3 x \left\{ -\frac{1}{2} (\partial^\alpha \beta \phi) (\partial_\alpha \beta \phi) + H^2 - i \chi^\alpha (\partial_\alpha \beta \chi^\beta) \right\} ,$$

$$= \frac{1}{g^2} \int d^3 x \left\{ \frac{1}{2} \hat{f}_\alpha \hat{f}_\alpha - i \chi^\alpha (\partial_\alpha \beta \chi^\beta) \right\} ,$$

where we define the component fields by the following projection

$$U_\alpha| = \chi_\alpha , \quad D_\alpha U_\beta| := \hat{f}_\alpha \beta = i \partial_\alpha \beta \phi - C_\alpha \beta H , \quad D^2 U_\alpha| = i \partial_\alpha \beta \chi^\beta .$$

Note that the second projection in (4.5) is defined in a way as an analog of $D_\alpha W_\beta|$ projection, while $\hat{f}_\alpha \beta$ is no longer symmetric in the two spinorial indices due to the different Bianchi identity. Therefore it can be decomposed into a symmetric and anti-symmetric part labeled by component fields $\phi$ and $H$ respectively, derived from the projection of the gauge invariant superfield $G$:

$$G| = \phi , \quad D_\alpha G| = \chi_\alpha , \quad D^2 G| = H .$$

Furthermore, in 3D, tensor and vector multiplets are related by a duality transformation. This can be easily seen by the following auxiliary action

$$S = \frac{1}{g^2} \int d^3 x \, d^2 \theta \left\{ W^2 + \Lambda^\alpha \left( W_\alpha - \frac{1}{2} D^\beta D_\alpha \Gamma_\beta \right) \right\} ,$$

$$= \frac{1}{g^2} \int d^3 x \, d^2 \theta \left\{ W^2 + \Lambda^\alpha \left( W_\alpha - \frac{1}{2} D^\beta \partial_\alpha \beta \right) \right\} ,$$

$$= \frac{1}{g^2} \int d^3 x \, d^2 \theta \left\{ W^2 + \Lambda^\alpha \left( W_\alpha - \frac{1}{2} D^\beta \partial_\alpha \beta \right) \right\} .$$
where superfields $W_\alpha$, $\Gamma_\alpha$ and $\Lambda_\alpha$ are unconstrained. The Lagrange multiplier $\Lambda_\alpha$, once integrated out, identifies $W_\alpha$ with the vector multiplet superfield strength (2.2) and enforces constraints (2.1). As a result we get the free vector multiplet action (2.4). On the other hand, by integrating out the superfield $\Gamma_\alpha$ we get that the superfield $\Lambda_\alpha$ satisfies the following equation of motion

$$D^\alpha D_\beta \Lambda_\alpha = 0 \quad (4.8)$$

hence $\Lambda_\alpha$ becomes a tensor multiplet superfield. Finally, by integrating out the superfield $W_\alpha$, we find its equation of motion to be $W_\alpha = -\Lambda_\alpha$ which when substituted back gives the tensor multiplet action (4.3).

4.2 Tensor-Goldstone Multiplet

Interpreting the 3D tensor multiplet as the Goldstone multiplet corresponding to the breaking of the second supersymmetry requires to find a transformation of $U_\alpha$ compatible with the constraint (4.1) which includes a constant shift term and has the interpretation of supersymmetry —must involve partners and be consistent with the susy algebra. The most general transformation of this type is:

$$\delta^*_\eta U_\alpha = \eta_\alpha - \frac{1}{2\kappa} (D_\alpha D^\beta \tilde{X}) \eta_\beta \quad (4.9a)$$

$$\delta^*_\eta \tilde{X} = \frac{2}{\kappa} \eta^\alpha U_\alpha \quad (4.9b)$$

Notice that (4.9) and (2.6) differ in the order in which the spinorial covariant derivatives act on the partner superfield. As usual, in order to remove the independent degrees of freedom in $\tilde{X}$, we impose a non-linear constraint which expresses $\tilde{X}$ as a function of $U_\alpha$ and its derivatives. The compatibility of this constraint with (4.9) determines it to be the following:

$$\tilde{\kappa} \tilde{X} = U^\alpha U_\alpha - \frac{1}{2} (D^2 \tilde{X}) \tilde{X} \Rightarrow \tilde{X} = \frac{U^\alpha U_\alpha}{\tilde{\kappa} \left(1 + \frac{1}{2\kappa} D^2 \tilde{X}\right)} \quad (4.10)$$

Similar to (2.7), using $U_\alpha U_\beta U_\gamma = 0$, this constraint can be solved in order to express $\tilde{X}$ in terms of $U^2$ and its derivative $\tilde{T} = \frac{2}{\kappa^2} D^2 U^2$:

$$\tilde{X} = \frac{2}{\kappa} U^2 \left[1 - \frac{\tilde{T} \sqrt{1 + 2\tilde{T}}}{1 + \tilde{T} + \sqrt{1 + 2\tilde{T}}} \right] \quad (4.11)$$

The superspace action for the 3D Tensor-Goldstone multiplet is

$$S = -\tau \int d^3x d^2\theta \tilde{X} = -\frac{2\tau}{\kappa} \int d^3x d^2\theta U^2 \left[1 - \frac{\tilde{T} \sqrt{1 + 2\tilde{T}}}{1 + \tilde{T} + \sqrt{1 + 2\til{T}}} \right] \quad (4.12)$$

It is manifestly invariant under the first supersymmetry and it is straightforward to check its invariance under the second supersymmetry. Due to (4.9b) and (4.1) the spacetime Lagrangian transforms under the second supersymmetry as a total derivative

$$\delta^*_\eta D^2 \tilde{X} = -\frac{2i}{\kappa} \partial_\alpha (\eta^\alpha U_\beta) \quad (4.13)$$
The action (4.12), as well as the constraint (4.10), match the results of [26]. We now show that these results emerge from the partial supersymmetry breaking procedure of the $\mathcal{N} = 2$ tensor multiplet in 3D. Such a multiplet is being described by a scalar $\mathcal{N} = 2$ superfield $\mathcal{U}(x, \theta, \tilde{\theta})$ which satisfies the following irreducible conditions:

$$D^2 \mathcal{U} = - \tilde{D}^2 \mathcal{U} , \ D^\alpha D_\beta \tilde{D}_\alpha \mathcal{U} = 0 \ .$$ (4.14)

We solve these constraints by expanding the $\mathcal{N} = 2$ superfield $\mathcal{U}$ in its $\mathcal{N} = 1$ superfield components

$$\mathcal{U}(x, \theta, \tilde{\theta}) = \Phi(x, \theta) + \tilde{\theta}^\alpha U_\alpha(x, \theta) - \tilde{\theta}^2 F(x, \theta) \ ,$$ (4.15)

which satisfy the following relations

$$F = - D^2 \Phi , \ D^2 U_\alpha = -i \partial_\alpha \beta U_\beta , \ D^\alpha D_\beta U_\alpha = 0 \ .$$ (4.16)

Their transformations under the second supersymmetry are

$$\delta^*_\epsilon \Phi = - \epsilon^\alpha U_\alpha ,$$ (4.17a)

$$\delta^*_\epsilon U_\alpha = \epsilon_\alpha F - i \epsilon_\beta \partial_\beta \alpha \Phi = D_\alpha D^\beta \Phi \epsilon_\beta ,$$ (4.17b)

$$\delta^*_\epsilon F = - i \epsilon^\alpha \partial_\alpha \beta U_\beta .$$ (4.17c)

Applying on this supermultiplet the constrained superfield approach in order to break the second manifest supersymmetry, we expand $\mathcal{U}$ around a non-trivial vacuum that preserves only the first supersymmetry and at the same time we eliminate the remaining partner superfield by imposing the nilpotency condition:

$$\mathcal{U} = \langle \mathcal{U} \rangle + \mathcal{U} \ , \ \mathcal{U}^2 = 0 \ .$$ (4.18)

The condensate $\langle \mathcal{U} \rangle$ is Lorentz and $\mathcal{N} = 1$ invariant with a non-trivial $\tilde{\theta}$ dependence.

$$\langle \mathcal{U} \rangle = \tilde{\kappa} \tilde{\theta}^2 \Rightarrow \mathcal{U} = - \frac{1}{2} \tilde{X} + \tilde{\theta}^\alpha U_\alpha - \tilde{\theta}^2 \left( \frac{1}{2} D^2 \tilde{X} + \tilde{\kappa} \right)$$ (4.19)

where $\tilde{X} = -2\Phi$. The second supersymmetry transformations take the form

$$\delta^*_\epsilon \tilde{X} = 2 \epsilon^\alpha U_\alpha = \frac{2}{\tilde{\kappa}} \epsilon^\alpha U_\alpha \ ,$$ (4.20a)

$$\delta^*_\epsilon U_\alpha = \epsilon_\alpha \tilde{\kappa} - \frac{1}{2} (D_\alpha D^\beta \tilde{X}) \epsilon_\beta = \epsilon_\alpha - \frac{1}{2\tilde{\kappa}} (D_\alpha D^\beta \tilde{X}) \epsilon_\beta ,$$ (4.20b)

where $\epsilon_\alpha$ is the $\tilde{\kappa}$-rescaled supersymmetry parameter. The nilpotency condition $\mathcal{U}^2 = 0$ generates the following nonlinear constraints

$$\tilde{X}^2 = 0 \ , \ \tilde{X} U_\alpha = 0 \ , \ \tilde{\kappa} \tilde{X} = U^\alpha U_\alpha - \frac{1}{2} (D^2 \tilde{X}) \tilde{X} \ ,$$ (4.21)

which match with (4.10).
4.3 Spacetime action for Tensor-Goldstone multiplet

The superspace action (4.12) can be written in the three-dimensional Ceccoti-Ferrara form

\[ S = - \tilde{\tau} \tilde{\kappa} \int d^3 x \left\{ \tilde{T} \big|_{\theta=0} + \frac{2}{\kappa^2} \int d^2 \theta \ \Psi(-\tilde{T}) U^2 \right\} . \]  

(4.22)

where \( \Psi(x) \) is the same function as in the Maxwell-Goldstone case (2.20). By performing the grassman integral we extract the bosonic part of the spacetime action:

\[ S_B = \tilde{\tau} \tilde{\kappa} \int d^3 x \left( 1 - \sqrt{1 + 2 |\tilde{T}|} \right) . \]  

(4.23)

The fermionic part of the action is

\[ S_F = \frac{\tilde{\tau}}{\kappa^3} \int d^3 x \left\{ \Psi'(-\tilde{T}) \left[ 4i \partial_{\alpha\beta}(\hat{f}^{\beta\gamma} \chi_{\gamma}) \hat{f}^{\alpha\delta} \chi_{\delta} \right. \\
+ \left. \left( 2 \left( \Box \chi_{\gamma} \right) \chi_{\gamma} + (\partial^{\alpha\beta} \chi_{\gamma})(\partial_{\alpha\beta} \chi_{\gamma}) \right) \chi^\sigma \chi_\sigma \right] \right\} , \]  

(4.24)

where

\[ |\tilde{T}| = \frac{2}{\kappa^2} \left[ -\frac{1}{2} \hat{f}^{\alpha\beta} \hat{f}_{\alpha\beta} + i \chi^\alpha (\partial_{\alpha\beta} \chi^\beta) \right] . \]  

(4.25)

5 Tensor-Goldstone and Maxwell-Goldstone Duality

In 4D, the self-duality of the Born-Infeld action was extended to the supersymmetric BI action based on the self-duality of the Maxwell-Goldstone multiplet. In 3D, as mentioned previously, the vector multiplet is no longer self-dual but it maps to the tensor multiplet. We will show that this duality map survives between the Maxwell-Goldstone and Tensor-Goldstone multiplets.

A very transparent method for studying the duality properties of these multiplets is\(^6\) to consider unconstrained superfields and impose all nonlinearities and constraints via Lagrange multipliers. Therefore, the Maxwell-Goldstone action can be written in the form

\[ S = \int d^3 x \ d^2 \theta \left\{ \Lambda \ \left[ W^\alpha W_\alpha + \frac{1}{2} X D^2 X - \kappa X \right] + \tau X \right\} , \]  

(5.1)

where \( \Lambda \) and \( X \) are unconstrained scalar superfields. When the Lagrange multiplier \( \Lambda \) is integrated out, it imposes the susy breaking constraint (2.7) and the above action becomes identical to (2.9). Action (5.1) can also be motivated by the free \( N = 2 \) action (the sum of kinetic energy terms for \( W^\alpha \) and \( X \)) plus a constraint term with a Lagrange multiplier.

\(^6\)See [9, 24].
Furthermore, using (4.7) and (4.8), we can relax the $\mathbf{D}^\alpha W_\alpha = 0$ constraint of the vector multiplet $W_\alpha$ by adding a duality term with a Lagrange multiplier which must be a tensor supermultiplet spinorial superfield $U_\alpha$ ($\mathbf{D}^\alpha \mathbf{D}_\beta U_\alpha = 0$)

$$S = \int d^3x d^2\theta \left\{ \Lambda \left[ W^{\alpha}W_\alpha + \frac{1}{2} X D^2 X - \kappa X \right] + \tau X + gU^{\alpha}W_\alpha \right\} . \quad (5.2)$$

Integrating out $U_\alpha$ restores the vector constraint $\mathbf{D}^\alpha W_\alpha = 0$. However, because $W_\alpha$ is now unconstrained and appears algebraically we can choose to integrate it out first. The result is the following action

$$S = \int d^3x d^2\theta \left\{ \tilde{\Lambda} \left[ U^{\alpha}U_\alpha - \frac{1}{2} \tilde{X} D^2 \tilde{X} - \tilde{\kappa} \tilde{X} \right] - \tilde{\tau} \tilde{X} \right\} , \quad (5.3)$$

where

$$\tilde{\Lambda} = -\frac{g^2}{4\Lambda} , \quad X = \frac{g \tilde{X}}{2\Lambda} , \quad \tilde{\kappa} = \frac{2\tau}{g} , \quad \tilde{\tau} = \frac{g \kappa}{2} . \quad (5.4)$$

Action (5.3) corresponds to the Tensor-Goldstone multiplet action (4.12), since the Lagrange multiplier $\tilde{\Lambda}$ imposes constraint (4.10). Moreover, the relation between Maxwell-Goldstone parameters $(\kappa, \tau)$ and the corresponding Tensor-Goldstone parameters $(\tilde{\kappa}, \tilde{\tau})$ is such that the dimensionless parameter $\lambda = \frac{\kappa}{\tau}$ undergoes a standard inversion

$$\lambda \rightarrow \tilde{\lambda} = \frac{4}{g^2 \lambda} . \quad (5.5)$$

Notice that this duality is not only a property of the Goldstone actions but it is also true for the kinetic energy terms of the two $\mathcal{N} = 2$ multiplets. Specifically, if one chooses the Lagrange multiplier $\Lambda$ to be the constant $\Lambda = \frac{\tau}{\kappa}$, then the linear $X$ terms in (5.1) drop and we recover the free $\mathcal{N} = 2$ multiplet written in terms of $\mathcal{N} = 1$ superfields. Under the duality (5.4) this action maps to the free $\mathcal{N} = 2$ tensor multiplet (5.3) with the choice $\tilde{\Lambda} = -\frac{\tilde{\tau}}{\tilde{\kappa}}$.

## 6 Summary

The existence of solitonic, BPS, solutions (Dp-branes) of type II string theory motivates the study of supersymmetric actions of the Born-Infeld type. These are viewed as low-energy effective descriptions —hence they are not subject to any renormalizability requirements—which correctly capture the spontaneous breaking of half of the supersymmetries. Therefore, these effective supersymmetric actions can be written in terms of linear representations of $\mathcal{N} = 1$ supersymmetry and are also invariant under a second, nonlinear supersymmetry transformation. It has long been understood that broken symmetries lead to nonlinear representations of the broken symmetry, which accommodate the Goldstone modes.

In 4D, such effective supersymmetric BI actions have been constructed [3, 7–9, 31] and studied extensively. It was shown that the Goldstone fermion, corresponding to the spontaneously broken second supersymmetry, could be accommodated in an $\mathcal{N} = 1$ chiral, vector,
or tensor multiplets. For each one of such descriptions, the \( \mathcal{N} = 1 \) manifestly supersymmetric action was constructed and showed that the bosonic part of these actions matched the expected BI action. Moreover, it was shown that the Maxwell-Goldstone multiplet is self-dual and the Tensor and Chiral Goldstone multiplets map to each other under duality transformations. At the component level, these duality properties reproduced the self-duality of the BI action. Finally, it was later shown [10] that the requirement of invariance under a first linear supersymmetry and a second nonlinear supersymmetry does not uniquely determine the action. The addition of a FI term preserves the nonlinearly realized supersymmetry, but it spontaneously breaks the linear supersymmetry. However, the deformed theory still describes a partial and not full supersymmetry breaking, with the surviving supersymmetry to correspond to a different \( \mathcal{N} = 1 \) slice of the \( \mathcal{N} = 2 \) theory.

In this work, motivated by the special role of D2-brane in type IIA string theory, we aim towards the construction of 3D supersymmetric Born-Infeld actions. Effective actions of this type have been obtained by performing dimensional reductions from 4D to 3D which break half of the supersymmetries [26, 27]. We consider manifestly supersymmetric \( \mathcal{N} = 2, D = 3 \) theories that are spontaneously broken down to \( \mathcal{N} = 1 \) Goldstone multiplets by expanding them around a nontrivial vacuum and enforce nilpotence conditions on the surviving superfields à la [1]. We find that the Goldstone multiplet can be described in terms of a vector or a tensor multiplet. For both cases, the constraint superfield approach produces explicitly:

(i) the set of transformations (2.6) and (4.9) which satisfy the supersymmetry algebra and are consistent with the irreducibility conditions of \( \mathcal{N} = 1, D = 3 \) vector \((W_\alpha)\) and tensor \((U_\alpha)\) superfield strengths respectively and (ii) constraints (2.7) and (4.10) which are consistent with the above transformations and eliminate the introduction of new dynamical degrees of freedom. The solution of these constraints is used to write the effective 3D, \( \mathcal{N} = 1 \) superspace action. The above results are in perfect agreement with [26, 27].

Furthermore, for both superspace Goldstone multiplet actions we derive the spacetime components actions. Their bosonic parts (2.21) and (4.23) match the expected 3D BI action, while the fermionic part of the Lagrangians (2.23) and (4.24) are organized into sums of polynomial terms weighted by non-polynomial factors which correspond to derivatives of the 3D Ceccoti-Ferrara functions. Also, we show that under duality transformations the Maxwell-Goldstone action maps to the Tensor-Goldstone multiplet and this duality generates an inversion (5.5) of the dimensionless parameter \( \lambda = \mu \). Finally, we explore the possibility of deforming the Maxwell-Goldstone superspace action by a CS-like mass term. We find that such a term generates a one-parameter deformation (3.6) of the action which is consistent with the second —nonlinearly realized— supersymmetry, but it explicitly breaks the first supersymmetry.

An alternative approach to organizing nonlinearly realized supersymmetries is the \( T\bar{T} \) deformation [32, 33]. This approach has been studied extensively for supersymmetric and non-supersymmetric theories in two and four dimensions. The 4D Born-Infeld action and its supersymmetric extension have been recently understood as a generalized \( T\bar{T} \) deformation [34, 35]. In the future, we would like to investigate if the 3D supersymmetric BI action
can have a similar interpretation. Meaning, is there an operator which depends on the 3D supercurrent and drives a flow resulting in the 3D supersymmetric BI action?

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A Projection from 4D Cecotti-Ferrara Action

In this appendix, we review the Cecotti-Ferrara Lagrangian [3] in 4D, \( \mathcal{N} = 1 \) superspace, and construct the 3D Cecotti-Ferrara action via the dimensional reduction.

First, recall that the 4D, \( \mathcal{N} = 1 \) Cecotti-Ferrara Lagrangian \((A.1)\) reads

\[
\mathcal{L}^{4D}_{CF} = \hat{T} + \int d^2\theta d^2\bar{\theta} \Psi(T, \bar{T}) W^2 \bar{W}^2 ,
\]

where

\[
T = \frac{1}{2} \bar{\nabla}^2 \bar{W}^2 , \quad \bar{T} = \frac{1}{2} \nabla^2 W^2 ,
\]

\[
\Psi(T, \bar{T}) = \frac{1}{1-T+\sqrt{1-2T-T^2}} ,
\]

\[
\hat{T} = \frac{1}{2} (T + \bar{T}) , \quad \bar{T} = \frac{1}{2\epsilon} (T - \bar{T}) .
\]

Before projecting the CF Lagrangian to 3D, first note that in 4D, \( \mathcal{N} = 1 \) superspace, superfields are complex, the chirality and complex conjugate operation are well-defined, and there are two types of superspace covariant derivatives \( \nabla_\alpha \) and \( \bar{\nabla}_\dot{\alpha} \). However, in 3D, \( \mathcal{N} = 1 \) superspace, superfields are real, the chirality and complex conjugate operation are not defined, and there is only one type of superspace covariant derivative \( \nabla_\alpha \). Then, one can project the 4D Lagrangian \((A.1)\) to 3D simply by \(i\) doing \( d^2\theta \) integral first, \( \int d^2\theta \Psi(T, \bar{T}) W^2 \bar{W}^2 = \Psi(T, \bar{T}) W^2 \bar{D}^2 \bar{W}^2; \) \(ii\) setting everything as real, i.e. \( T = \bar{T} = \hat{T}, W = \bar{W}, \) and \( \bar{T} = 0. \) Therefore we have

\[
S^{3D}_{CF} = \int d^3x T + 2 \int d^3x d^2\theta \Psi(T) W^2 ,
\]

where \( \Psi(T) \) takes the same form as \((2.20)\) and \((A.3)\) matches with the supersymmetric Born-Infeld action \((2.19)\).
B Conventions in 3D, \( \mathcal{N} = 1 \) Superspace

In this appendix, we will briefly summarize our conventions and notations, which mostly follow from those of [30]. In three dimensional spacetime, the Lorentz group is \( \text{SL}(2,\mathbb{R}) \) and the corresponding fundamental representation acts on a real (Majorana) two-component spinor \( \psi^\alpha = (\psi^+, \psi^-) \). Vector indices are denoted as \( a = 0, 1, 2 \).

We choose Gamma matrices as

\[
\begin{align*}
(\gamma^0)_{\alpha}^{\beta} &= (i\sigma^2)_{\alpha}^{\beta}, \\
(\gamma^1)_{\alpha}^{\beta} &= (\sigma^3)_{\alpha}^{\beta}, \\
(\gamma^2)_{\alpha}^{\beta} &= (\sigma^1)_{\alpha}^{\beta},
\end{align*}
\]

which satisfy the Clifford algebra:

\[
\{\gamma^a, \gamma^b\} = 2\eta^{ab},
\]

where the Minkowski metric is

\[
\eta_{ab} = \eta^{ab} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\]

The gamma matrix has the following trace identity,

\[
(\gamma^a)_{\alpha}^{\beta}(\gamma^b)_{\beta}^{\alpha} = 2\delta^a_b.
\]

We use the spinor metric to raise and lower spinor indices:

\[
\begin{align*}
\psi_{\alpha} &= \psi^\beta C_{\beta\alpha}, \\
\psi^\alpha &= C^{\alpha\beta} \psi_{\beta},
\end{align*}
\]

where the definition of the spinor metric is

\[
C_{\alpha\beta} = -C_{\beta\alpha} = -C^{\alpha\beta} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.
\]

From (B.5) and (B.6), we have the following identities

\[
\begin{align*}
C_{\alpha\beta}C^{\gamma\delta} &= \delta^{[\gamma}_{\alpha} \delta^\delta_{\beta]}, \\
C_{\alpha\beta}C^\alpha\beta &= \delta^\delta_{\beta}, \\
\psi^2 &= \frac{1}{2} \psi^\alpha \psi_\alpha = i\psi^+ \psi^-,
\end{align*}
\]

By using the spinor metric, we know that the gamma matrices are symmetric, namely,

\[
\begin{align*}
(\gamma^a)_{\alpha\beta} &= (\gamma^a)_{\beta\alpha}, \\
(\gamma^a)^{\alpha\beta} &= (\gamma^a)^{\beta\alpha}.
\end{align*}
\]
Below, we list some useful identities of gamma matrices.

\[ A_{[\alpha \beta]} = - C_{\alpha \beta} A^\gamma B_\gamma , \quad \text{(B.12)} \]
\[ \gamma^a \gamma_b = 3 \mathbb{I} , \quad \text{(B.13)} \]
\[ \gamma_a \gamma_b = - \epsilon_{abc} \gamma^c + \eta_{ab} \mathbb{I} , \quad \text{(B.14)} \]
\[ \gamma^b \gamma_a \gamma_b = - \gamma_a , \quad \text{(B.15)} \]
\[ (\gamma^a)_{\alpha \beta} (\gamma_a)_{\gamma \delta} = - \frac{3}{2} \delta^\gamma_{(\alpha} \delta^\delta_{\beta)} - \frac{1}{2} (\gamma^a)_{\alpha} (\gamma_a)_{\beta} , \quad \text{(B.16)} \]
\[ (\gamma^a)_{\alpha \beta} (\gamma_a)_{\gamma \delta} = - \delta^\gamma_{(\alpha} \delta^\delta_{\beta)} = - (\gamma^a)_{(\alpha} (\gamma_a)_{\beta} , \quad \text{(B.17)} \]

where we define \( e^{012} = 1 \).

In the 3D, \( \mathcal{N} = 1 \) superspace, the superspace coordinate is labeled by \( z^A = (x^{\alpha \beta}, \theta^\alpha) \). They satisfy the hermiticity condition \( (z^A)^\dagger = z^A \). Define derivatives as
\[ \partial_{\alpha \beta} x^{\gamma \delta} \equiv [\partial_{\alpha \beta}, x^{\gamma \delta}] = \frac{1}{2} \delta^\gamma_{(\alpha} \delta^\delta_{\beta)} \]
\[ \partial_{\alpha} \theta^\beta \equiv \{\partial_{\alpha}, \theta^\beta\} = \delta^\beta_{\alpha} \quad \text{(B.18)} \]

implying that
\[ [\partial_{\alpha \beta}]^\dagger = - \partial_{\alpha \beta} , \quad [\partial_{\alpha}]^\dagger = \partial_{\alpha} , \quad [\partial^A]^\dagger = - \partial^A . \quad \text{(B.19)} \]

The superspace covariant derivatives are defined as \( D_A = (\partial_{\alpha \beta}, D_{\alpha}) \), where
\[ \partial_{\alpha \beta} = i (\gamma^a)_{\alpha \beta} \partial_a , \quad \text{(B.20)} \]
\[ D_{\alpha} = \partial_{\alpha} + i \theta^\beta \partial_{\alpha \beta} . \]

They satisfy the algebra
\[ \{D_{\alpha}, D_{\beta}\} = 2i \partial_{\alpha \beta} , \quad \text{(B.21)} \]
\[ [\partial_{\alpha \beta}, D_{\gamma}] = 0 . \]

Finally, we list some identities of covariant derivatives, which are useful in the calculations we have encountered throughout this paper.

\[ \partial^{\alpha \gamma} \partial_{\beta \gamma} = \delta^\alpha_{\beta} \Box , \quad \text{(B.22)} \]
\[ D_{\alpha} D_{\beta} = i \partial_{\alpha \beta} - C_{\alpha \beta} D^2 , \quad \text{(B.23)} \]
\[ D^2 D_{\alpha} = - D_{\alpha} D^2 = i \partial_{\alpha \beta} D^\beta , \quad \text{(B.24)} \]
\[ D^\beta D_{\alpha} D_{\beta} = 0 , \quad \text{(B.25)} \]
\[ (D^2)^2 = \Box , \quad \text{(B.26)} \]

where
\[ \Box = \frac{1}{2} \partial^{\alpha \beta} \partial_{\alpha \beta} = \partial^a \partial_a , \quad \text{(B.27)} \]
\[ D^2 = \frac{1}{2} D^a D_{\alpha} . \]
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