THE THREE-POINT CORRELATION FUNCTION OF GALAXIES DETERMINED FROM THE LAS CAMPANAS REDSHIFT SURVEY

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Received 1998 February 2; accepted 1998 March 19

ABSTRACT

We report the measurement of the three-point correlation function (3PCF) of galaxies for the Las Campanas Redshift Survey (LCRS). We have not only measured the 3PCF in redshift space, but we have also developed a method to measure the projected 3PCF that has simple relations to the real-space 3PCF. Both quantities have been measured as a function of triangle size and shape with only a fractional uncertainty in each individual bin. Various tests derived from mock catalogs have been carried out to assure that the measurement is stable and that the errors are estimated reliably. Our results indicate that the 3PCFs both in redshift space and in real space have small but significant deviations from the well-known hierarchical form. The 3PCF in redshift space can be fitted by \(Q_{red}(s, u, v) = 0.5 \times 10^{(0.2 + 0.1x(1 + 1)^{0.2})^{0.2}}\) for \(0.8 < s_{12} < 8 \ h^{-1} \) Mpc and \(s_{13} < 16 \ h^{-1} \) Mpc, and the projected 3PCF by \(Q_{proj}(r_p, u, v) = 0.7r_p^{-0.3}\) for \(0.2 < r_{p12} < 3 \ h^{-1} \) Mpc and \(r_{p31} < 6 \ h^{-1} \) Mpc (s and \(r_p\) are in units of \(h^{-1}\) Mpc), although a systematic weak increase of \(Q_{proj}(r_p, u, v)\) with \(v\) at \(r_p > 1 \ h^{-1} \) Mpc is noted. The real space \(Q(r, u, v)\) for \(0.2 < r_{12} < 3 \ h^{-1} \) Mpc and \(r_{13} < 6 \ h^{-1} \) Mpc can be well described by half the mean 3PCF predicted by a cold dark matter (CDM) model with \(\Omega_0 = 0.2\). The general dependence of the 3PCF on triangle shape and size is in qualitative agreement with the CDM cosmogonic models. Quantitatively the 3PCF of the models may depend on the biasing parameter and the shape of the power spectrum, in addition to other model parameters. Taking our result together with the constraints imposed by the two-point correlation function and the pairwise velocity dispersion of galaxies also obtained from the LCRS, we find that we have difficulties in producing a simple model that meets all constraints perfectly. Among the CDM models considered, a flat model with \(\Omega = 0.2\) meets the 2PCF and PVD constraints, but gives higher values for the 3PCF than observed. This may indicate that more sophisticated bias models or a more sophisticated combination of model parameters must be considered.

Subject headings: cosmology: theory — dark matter — galaxies: clusters: general — galaxies: distances and redshifts — large-scale structure of universe

1. INTRODUCTION

Correlation functions are very powerful statistics to describe the large-scale structures in the universe (Peebles 1980, hereafter P80). The lowest order, the two-point correlation function (2PCF) \(\xi(r)\), has been widely used to measure the clustering strength of galaxies and to confront models of cosmic structure formation. Quite a number of large galaxy catalogs, both angular and redshift, have been used to determine the 2PCF, and this statistic has now been established quite well (Jing, Mo, & Börner 1998, hereafter JMB98; Lin et al. 1998, 1997; Tucker et al. 1997; Ratcliffe et al. 1998; Baugh 1996; Hermit et al. 1996; Strauss & Willick 1995 for a review before 1995). This statistic has produced several constraints on theoretical models already despite the fact that there are many ingredients to a specific model that can be optimally adapted to the properties of the galaxy sample. The cosmological parameters, like the initial power spectrum of the DM component and the bias, i.e., the difference in the clustering of galaxies and DM particles, can all be adjusted to some extent.

The three-point correlation function (3PCF) \(\xi(r_{12}, r_{23}, r_{31})\) is a further statistic useful in characterizing the clustering of galaxies (P80). Its measurement can give additional constraints for cosmogonic models. The determination of the 3PCF was pioneered by Peebles and his coworkers in the seventies. Based on their careful analysis of the Lick and Zwicky angular catalogs of galaxies they propose a so-called “hierarchical” form

\[
\xi(r_{12}, r_{23}, r_{31}) = Q[\xi(r_{12})\xi(r_{23}) + \xi(r_{23})\xi(r_{31}) + \xi(r_{31})\xi(r_{12})]
\]

(1)

with the constant \(Q \approx 1.29 \pm 0.2\). This form is valid for scales \(r \lesssim 3 \ h^{-1} \) Mpc (P80). The analysis of the ESO-Uppsala catalog of galaxies (Lauberts 1982) by Jing, Mo, & Börner (1991) supports this result. The 3PCF was also examined for the CfA, Anglo-Australian Telescope (AAT), and Kirshner-Oemler-Schechter-Sheetman (KOS) redshift samples of galaxies (Peebles 1981; Bean et al. 1983; Efstathiou & Jedrzejewski 1984; Hale-Sutton et al. 1989). Because all these redshift samples are small (with <2000 galaxies), these authors were not able to examine the validity of the hierarchical form in redshift space. Instead they just forced a fit of the hierarchical form and obtained the value of \(Q\). The \(Q\) value of redshift samples obtained in this way is around 0.6 (Efstathiou & Jedrzejewski 1984), much smaller than the value advocated by Peebles and his coworkers. The difference may partially be attributed to the redshift distortion effect that reduces the \(Q\) value (Matsubara 1994). The skewness analysis of the 1.2 Jy IRAS survey of galaxies has given a similar \(Q\) value (Bouchet et al. 1993).

The hierarchical form (eq. [1]) is purely empirical. There is no solid theoretical argument supporting this form. In contrast, the second-order perturbation theory predicts that \(Q\) depends on the shape of the triangle and on the slope of...
the linear power spectrum (Fry 1984) in the linear regime. For a cold dark matter–like (CDM-like) power spectrum with a slope that changes with the scale, \(Q\) then varies with the size and shape of a triangle (Jing & Börner 1997). Even in the strongly nonlinear regime where the hierarchical form was established, the CDM models do not seem to obey this form as demonstrated by Matsubara & Suto (1994) based on N-body simulations. Recently, Yano & Gouda (1997) have reexamined, based on the Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) equation, the stable clustering (strongly nonlinear) problem and pointed out that the hierarchical form holds if the clustering is stable. The question is whether the condition of stable clustering can be achieved in the real universe (Jain 1997).

The 3PCF of galaxies carries much useful information that is important for cosmogonic models. The theories based on CDM models predict that the 3PCF of galaxies depends on the shape of the linear power spectrum (Fry 1984; Jing & Börner 1997) and the galaxy biasing relative to the underlying mass (Davis et al. 1985; Gaztañaga & Freiman 1994; Mo, Jing, & White 1997; Matarrese, Verde, & Heavens 1997; Catelan et al. 1998). It might also be sensitive to a possible non-Gaussianity of the initial density fluctuation (Fry & Scherrer 1994). Furthermore, the 3PCF must be determined accurately if one wants to use the cosmic virial theorem to obtain the mean density of the universe.

We have carried out a detailed analysis of the Las Campanas Redshift Survey (Shectman et al. 1996), and in this paper we report the measurement of the 3PCF of galaxies in that survey. We have not only measured the 3PCF in redshift space, but we have also developed a method to measure the projected 3PCF that has simple relations to the real-space 3PCF (§ 3.1). Our methods are checked very carefully with the help of mock catalogs generated from N-body simulations, and the physical meaning of these two quantities is also investigated (§ 3.2). Our statistical results are compared with previous work, with the emphasis on a critical examination of the hypothesis of the hierarchical form (§ 3.4). As we will see, the hierarchical form does not seem to be a good prescription even in the strong clustering regime. We will present a new fitting formula in § 3.3. Implications for cosmogonic models are discussed in § 4.

2. OBSERVATIONAL SAMPLE AND MOCK CATALOGS

The sample used for our analysis is the Las Campanas Redshift Survey (LCRS; Shectman et al. 1996). This is the largest redshift survey that is now publicly available. Our sample consists of all galaxies with recession velocities between 10,000 and 45,000 km s\(^{-1}\) with absolute magnitudes in the LCRS hybrid R band) between \(-18.0\) and \(-23.0\). There are 19,558 galaxies in this sample, of which 9480 are in the three north slices and the rest are in the three south slices. The survey is a well-calibrated sample of galaxies, ideally suited for statistical studies of large-scale structure. All known systematic effects in the survey are well quantified and documented (Shectman et al. 1996; Lin et al. 1996), and so most can be corrected easily in statistical analyses. The only exception is the “fiber collision” limitation that prevents two galaxies in one \(\sim 1.5 \times 1.5\) deg\(^2\) field from being observed when they are closer than 55” on the sky because it is impossible to put fibers on both objects simultaneously. Here we will use extensively mock catalogs generated from N-body simulations to quantify this effect.

The real-space 2PCF and the pairwise velocity dispersion (PVD) have been determined for the LCRS by JMB98. The redshift-space 2PCF and power spectrum for this sample were presented by Tucker et al. (1997) and Lin et al. (1997), respectively. All these studies have shown that the LCRS is large enough to accurately measure these low-order statistical quantities. In particular, JMB98 have carried out a detailed comparison between the observed 2PCF and PVD and the predictions of currently favored CDM cosmogonies. They have used a large set of mock samples to adequately compare models and observations. The construction of mock catalogs from the simulations, i.e., photometric catalogs subject to the same selection effects as the real observations, is a very important aspect of their analysis because only in this way could the statistical significance of the results be asserted. Three spatially flat models have been considered in JMB98, with \((\Omega_0, \lambda_0, \Gamma, \sigma_a) = (0.2, 0.8, 0.2, 1), (0.3, 0.7, 0.2, 1),\) and \((1.0, 0.0, 0.5, 0.62),\) where \(\Omega_0\) is the density parameter, \(\lambda_0\) is the cosmological constant, \(\Gamma = \Omega_0 \ h\) and \(\sigma_a\) are the shape parameter and normalization of the CDM power spectrum (Bardeen et al. 1986). All of the models give a steeper 2PCF and a higher PVD on small scales than the data. Thus, unless galaxies are biased with respect to the mass with a scale-dependent bias, all these models can be ruled out. Unfortunately physical models for a density or a (not so desirable, but perhaps unavoidable) velocity bias are not on firm grounds. Therefore in JMB98 a simple but plausible phenomenological model for the bias has been suggested. To suppress the number of pairs in the dark matter (DM) distribution at small separations, it is assumed that the number of galaxies per unit dark matter mass \(N/M\) is smaller in massive halos than in less massive ones. If a behavior such as \(N/M \propto M^{\alpha}\) with \(\alpha = -0.08\) is used for clusters of mass \(M_{\odot}\), the predictions of some CDM models are consistent with the observational results. The best agreement was achieved for the flat \(\Omega_0 = 0.2\) model.

We will use 10 mock catalogs of this model to test our statistical methods and quantify the “fiber collision” effect. Since this model has reproduced the LCRS 2PCF and PVD, we believe these mock catalogs are very suitable for this purpose. We shall also use these mock samples for model testing, as an example to illustrate the power of the three-point correlation function in discriminating between models that have similar two-point correlations. Since the model is a typical CDM model, we will generalize the discussion to other CDM models.

3. THE THREE-POINT CORRELATION FUNCTION

3.1. Definitions and Statistical Methods

The three-point correlation function (3PCF) \(\xi(r_{12}, r_{23}, r_{31})\) is defined, through the joint probability \(dP_{123}\) of finding one object simultaneously in each of the three volume elements \(dr_{1}, dr_{2},\) and \(dr_{3}\) at positions \(r_{1}, r_{2},\) and \(r_{3}\), respectively, as follows (P80):

\[
dP_{123} = \bar{n}(r_{1})\bar{n}(r_{2})\bar{n}(r_{3})[1 + \xi(r_{12}) + \xi(r_{23}) + \xi(r_{31})]dr_{1}dr_{2}dr_{3},
\]

where \(r_{ij} = |r_{i} - r_{j}|\) and \(\bar{n}(r)\) is the mean density of galaxies at \(r\). This definition can be applied straightforwardly to redshift surveys of galaxies to measure the 3PCF \(\xi(s_{12}, s_{23}, s_{31})\) of galaxies in redshift space (at this point we neglect the
anisotropy induced by the redshift distortion that will be considered later). Here and below we use \( r \) to denote the real space and \( s \) the redshift space.

The 3PCF of galaxies can be measured from the counts of different triplets (P80). For this purpose, a sample of randomly distributed points, which has exactly the same boundaries and the same observational selection effects as the real survey, is generated. Four types of distinct triplets with triangles in the range \((s_{12}\pm\frac{1}{2}\Delta s_{12}, s_{23}\pm\frac{1}{2}\Delta s_{23}, s_{31}\pm\frac{1}{2}\Delta s_{31})\) are counted: the count \( DDD(s_{12}, s_{23}, s_{31}) \) of triplets formed by three galaxies, the count \( DRR(s_{12}, s_{23}, s_{31}) \) of triplets formed by two galaxies and one random point, the count \( RRR(s_{12}, s_{23}, s_{31}) \) of triplets formed by one galaxy and two random points, the count \( RR(s_{12}, s_{23}, s_{31}) \) of triplets formed by three random points. Following the definition (eq. [2]), we shall use the following estimator

\[
\zeta(s_{12}, s_{23}, s_{31}) = \frac{27 RRR(s_{12}, s_{23}, s_{31}) DDD(s_{12}, s_{23}, s_{31})}{31 DRR(s_{12}, s_{23}, s_{31})} - \frac{9 RRR(s_{12}, s_{23}, s_{31}) DDR(s_{12}, s_{23}, s_{31})}{31 DRR(s_{12}, s_{23}, s_{31})} + 2
\]

(3)

to measure the 3PCF of the galaxies in redshift space. The above formula is slightly different from the estimator used by Groth & Peebles (1977). Here we have extended the argument of Hamilton (1993) for the 2PCF to the case of the 3PCF. The coefficients 27 and 9 are due to the fact that only distinct triplets are counted in this paper. Since the early work of Peebles and coworkers (P80) indicates that the 3PCF of galaxies is approximately hierarchical, it is convenient to express the 3PCF in a normalized form \( Q_{3PCF}(s_{12}, s_{23}, s_{31}) \):

\[
Q_{3PCF}(s_{12}, s_{23}, s_{31}) = \frac{\zeta(s_{12}, s_{23}, s_{31})}{\zeta(s_{12}) \zeta(s_{23}) \zeta(s_{31}) + \zeta(s_{23}) \zeta(s_{31}) + \zeta(s_{31}) \zeta(s_{12})}. \]

(4)

It is also convenient to use the variables introduced by Peebles (P80) to describe the shape of the triangles formed by the galaxy triplets. For a triangle with the three sides \( s_{12} \leq s_{23} \leq s_{31} \), \( s \), and \( v \) are defined as

\[
s = s_{12}, \quad u = \frac{s_{23}}{s_{12}}, \quad v = \frac{s_{31} - s_{23}}{s_{12}}.
\]

(5)

Clearly, \( u \) and \( v \) characterize the shape and \( s \) the size of a triangle. We take equal logarithmic bins for \( s \) and \( u \) with the bin intervals \( \Delta \log s = \Delta \log u = 0.2 \), and equal linear bins for \( v \) with \( \Delta v = 0.2 \). For our analysis, we take the following ranges for \( s \), \( u \), and \( v \): \(0.63 \leq s \leq 10^{-1} \text{ Mpc} \) (6 bins), \(1 \leq u \leq 4 \) (3 bins), and \(0 \leq v \leq 1 \) (5 bins).

A sample of 25,000 random points is first generated. The counts \( RRR \) are less than \( \sim 5 \) for small triangles \( s < 1^{-1} \text{ Mpc} \). In order to suppress the fluctuation induced by the random samples, we have recalculated the counts \( RRR \) for \( s_{31} \leq 4^{-1} \text{ Mpc} \) by generating a random sample 10 times larger, which ensures that the counts \( RRR \) are at least \( \sim 300 \) of the interested triangle configurations. We scaled these counts to 25,000 random points and also use these counts to get \( DRR \) on the small scales since \( RRR/DRR \) is constant. However, it is not easy to search triplets for so many points. We have generalized the ordinary linked-list technique of \( P^3M \) simulations (Hockney & Eastwood 1980) to spherical coordinates to count the triplets. The linked-list cells are specified by the spherical coordinates, i.e., the right ascension \( \alpha \), the declination \( \delta \), and the distance \( s \). With this short-range searching technique we can avoid the triplets out of the range specified, thus making counting triplets very efficient.

The 3PCF in redshift space \( Q_{3PCF}(s, u, v) \) depends both on the real space distribution of galaxies and on their peculiar motions. Although this information contained in \( Q_{3PCF}(s, u, v) \) is also useful for the study of the large-scale structures (see § 4), it is apparent that \( Q_{3PCF}(s, u, v) \) is different from \( Q(r, u, v) \) in real space. In analogy with the analysis for the two-point correlation function, we have determined the projected three-point correlation function \( \Pi(r_{12}, r_{23}, r_{31}) \). We define the redshift-space three-point correlation function \( \zeta_2(r_{12}, r_{23}, r_{31}, \pi_{12}, \pi_{13}) \) through

\[
dP_1^{23} = n(s_1)n(s_2)n(s_3)[1 + \zeta_2(r_{12}, \pi_{12}) + \zeta_2(r_{23}, \pi_{23}) + \zeta_2(r_{31}, \pi_{31}) + \zeta_2(r_{12}, r_{23}, r_{31}, \pi_{12}, \pi_{13})] ds_1 ds_2 ds_3,
\]

(6)

where \( dP_1^{23} \) is the joint probability of finding one object simultaneously in each of the three volume elements \( ds_1, ds_2, \) and \( ds_3 \) at positions \( s_1, s_2, \) and \( s_3 \); \( \zeta_2(r, \pi) \) is the redshift-space two-point correlation function; \( r_{ij} \) and \( \pi_{ij} \) are the separations of objects \( i \) and \( j \) perpendicular to and along the line of sight, respectively. The projected 3PCF \( \Pi(r_{12}, r_{23}, r_{31}) \) is then defined as

\[
\Pi(r_{12}, r_{23}, r_{31}) = \int \zeta_2(r_{12}, r_{23}, r_{31}, \pi_{12}, \pi_{13}) d\pi_{12} d\pi_{23}.
\]

(7)

Because the total amount of triplets along the line of sight is not distorted by the peculiar motions, the projected 3PCF \( \Pi(r_{12}, r_{23}, r_{31}) \) is related to the 3PCF in real space \((\zeta(r_{12}, r_{23}, r_{31})\)

\[
\Pi(r_{12}, r_{23}, r_{31}) = \int \zeta_2(r_{12}, r_{23}, r_{31}, \pi_{12}, \pi_{13}) d\pi_{12} d\pi_{23}.
\]

(8)

Similarly to \( \zeta(s_{12}, s_{23}, s_{31}) \), we measure \( \zeta_2(r_{12}, r_{23}, r_{31}, \pi_{12}, \pi_{13}) \) by counting the numbers of triplets \( DDDD(r_{12}, r_{23}, r_{31}, \pi_{12}, \pi_{13}), \) \( DRR(r_{12}, r_{23}, r_{31}, \pi_{12}, \pi_{13}), \) \( RRR(r_{12}, r_{23}, r_{31}, \pi_{12}, \pi_{13}), \) \( RDR(r_{12}, r_{23}, r_{31}, \pi_{12}, \pi_{13}), \) \( DDR(r_{12}, r_{23}, r_{31}, \pi_{12}, \pi_{13}), \) \( RRR(r_{12}, r_{23}, r_{31}, \pi_{12}, \pi_{13}), \) \( RRR(r_{12}, r_{23}, r_{31}, \pi_{12}, \pi_{13}) \) formed by galaxies and/or random points with the projected separations \( r_{12}, r_{23}, \) and \( r_{31} \) and radial separations \( \pi_{12} \) and \( \pi_{23} \). We will use \( r_p, u, \) and \( v \):

\[
r_p = r_{12}, \quad u = u_{12}, \quad v = v_{12}.
\]

(9)

to quantify a triangle with \( r_{12} \leq r_{23} \leq r_{31} \) on the projected plane. Equal logarithmic bins of intervals \( \Delta \log r_p = \Delta \log u = 0.2 \) are taken for \( r_p \) and \( u \), and equal linear bins of \( \Delta v = 0.2 \) for \( v \). The same ranges of \( u \) and \( v \) are used as for \( \zeta(s, u, v) \), but \( r_p \) is from 0.128 \text{ h}^{-1} \text{ Mpc} to 4 \text{ h}^{-1} \text{ Mpc} (7 bins). The radial separations \( \pi_{12} \) and \( \pi_{23} \) are from \( -25 \text{ h}^{-1} \text{ Mpc} \) to 25 \text{ h}^{-1} \text{ Mpc} with a bin size of \( 1 \text{ h}^{-1} \text{ Mpc} \). The projected 3PCF is estimated by summing up \( \zeta_2(r_p, u, v, \pi_{12}, \pi_{23}) \) at
\[
\Pi(r_p, u, v) = \sum_{i,j} \xi_i(r_p, u, v, \pi_{12}, \pi_{23}) \Delta \pi_{12} \Delta \pi_{23}
\]
(10)

and normalized as
\[
Q_{\text{proj}}(r_p, u, v) = \frac{\Pi(r_p, u, v)}{w(r_{p12})w(r_{p23}) + w(r_{p31})w(r_{p12})},
\]
(11)

where \(w(r_p)\) is the projected two-point correlation function (Davis & Peebles 1983; JMB98)
\[
w(r_p) = \sum_i \xi_i(r_p, \pi^i) \Delta \pi^i.
\]
(12)

An interesting property of the projected 3PCF is that if the three-point correlation function is of the hierarchical form, the normalized function \(Q_{\text{proj}}(r_p, u, v)\) is not only a constant but also equal to \(Q\). Therefore the measurement of \(Q_{\text{proj}}(r_p, u, v)\) can be used to test the hierarchical form that was proposed mainly based on the analysis of angular catalogs.

3.2. N-Body Tests of the Statistical Methods

To test the reliability of our statistical analysis and to demonstrate the effects of the redshift distortion, the projection, and the fiber collisions, we make use of the full simulation and the mock catalogs. Because the mock samples are cluster (under)weighted, we have applied the same weighting to the full simulation to achieve a proper comparison. To calculate the quantities in redshift space for the full simulation, we assume that the third axis is along the line of sight.

In Figure 1 we compare \(Q_{\text{red}}(s, u, v)\) estimated with our statistical method from the mock samples with the true value. The latter is determined from the full simulation by means of the method of Jing & Börner (1997). On the scales from 1 to 10 h\(^{-1}\) Mpc, the two estimated quantities agree fairly well, indicating that the LCRS can yield an unbiased estimate within the error bars of the 3PCF in redshift space. The test is important, considering the fact that the LCRS is essentially two-dimensional with one dimension in the direction of the line of sight.

Figure 2 shows the projected \(Q_{\text{proj}}(r_p, u, v)\) estimated from the mock samples with the method described above. The true \(Q_{\text{proj}}(r_p, u, v)\) can be calculated from the real-space 3PCF \(\xi(r, u, v)\) through equation (8). We determine \(\xi(r, u, v)\) for the full simulation using the method of Jing & Börner (1997) and calculate the integral of equation (8) by linearly interpolating the estimated \(\xi(r, u, v)\). The two estimated quantities agree very well within the error bars, indicating that our method can give a correct estimate of the projected 3PCF. The real-space 3PCF \(Q(r, u, v)\) is also shown in the figure by the thick lines. It decreases with the scale \(r\) as noted previously (Matsubara & Suto 1994; Jing & Börner 1997). The reason for the decrease is that the slope of the power spectrum is more negative on smaller scales (see Jing 1998 for a detailed discussion). The consequence is that as a result of the averaging of \(Q(r, u, v)\) on scales \(r \geq r_p\) (eq. [8]), the projected \(Q_{\text{proj}}(r_p, u, v)\) is also a decreasing function of \(r_p\) but smaller than \(Q(r, u, v)\) for \(r = r_p\) (compare thick and thin lines in the figure). Another interesting point is that \(Q(r, u, v)\) is much higher than its counterpart in redshift space \(Q_{\text{red}}(s, u, v)\) on scales \(\lesssim 10 h^{-1}\) Mpc. This result is well known since the redshift distortion smears out the dense clusters and thus reduces the 3PCF on small scales (e.g., Matsubara & Suto 1994; Matsubara 1994).

We have also tested for the fiber collision effect of the LCRS sample as we did for the 2PCF and PVD in JMB98. Although both the 2PCF \(\langle \xi(s) \rangle\) and the 3PCF \(\langle \xi(s, u, v) \rangle\) show some small dependence on this effect, it cancels out completely when we divide these two quantities by one another to form the normalized 3PCF \(Q_{\text{red}}(s, u, v)\) and \(Q_{\text{proj}}(r_p, u, v)\). Therefore, the fiber collisions of the LCRS have little effect on the normalized functions \(Q_{\text{red}}(s, u, v)\) and \(Q_{\text{proj}}(r_p, u, v)\).

In summary, these tests have convinced us that our method is suitable for giving a stable measurement of the 3PCF from the LCRS with reliable error estimates.

3.3. The Statistical Results of the LCRS

We present our results of the 3PCF in redshift space \(Q_{\text{red}}(s, u, v)\) and of the projected 3PCF \(Q_{\text{proj}}(r_p, u, v)\) in Figures 3 and 4, respectively, for the Las Campanas Redshift Survey. The errors of the \(Q\) values are the bootstrap errors that are estimated with the approximate formula of Mo, Jing, & Börner (1992). As we can see from Figure 3, the 3PCF obtained from redshift space is not changing very much with \(s\) or \(u\); it increases somewhat with \(v\). For small \(v\), \(Q_{\text{red}}\) is approximately constant with a value of \(\sim 0.5\), but it increases up to \(\sim 1\) when \(v \approx 1\). Compared with the 3PCF in redshift space, the projected one \(Q_{\text{proj}}(r_p, u, v)\) (Fig. 4) shows quite similar dependences on triangle shape (i.e., \(u\) and \(v\)), but a quite different dependence on the triangle size (i.e., \(s\) or \(r_p\)), although the errors of the projected 3PCF are larger. Its value decreases with \(r_p\) from about 1.2 at \(r_p = 0.2 h^{-1}\) Mpc to 0.5 at \(r_p \approx 2 h^{-1}\) Mpc. Both this decrease with growing \(r_p\) and the weak increase with \(v\) are in contrast to the hierarchical assumption. If the three-point correlation function in real space were hierarchical, the projected one would also be hierarchical and equal to \(Q\) (see eq. [8]). This behavior, however, is qualitatively in agreement with the CDM model predictions, and we will discuss this point in \$4.\n
The hierarchical form (eq. [1]) does not seem to provide an adequate description of our results of the 3PCF in redshift space \(Q_{\text{red}}(s, u, v)\) or of the projected 3PCF \(Q_{\text{proj}}(r_p, u, v)\). We have looked for fitting formulae for both quantities. Our results can be fitted quite well by \(Q_{\text{red}}(s, u, v) = 0.5 \times 10^{(0.2 + 0.1 \langle s \rangle + 1.1 \langle w \rangle^2 - 1)}\) and \(Q_{\text{proj}}(r_p, u, v) = 0.7 \langle s \rangle^{0.3} (s < r_p)\) (Fig. 3, thick line), which are the solid lines in Figure 3 and the thin lines in Figure 4. We have neglected the weak systematic dependence on \(v\) of \(Q_{\text{proj}}(r_p, u, v)\) at \(r_p \gtrsim 1 h^{-1}\) Mpc in fitting this quantity since the dependence is not statistically significant on the scales we probed. We do not intend to give an error estimate for the coefficients in the formulae, since the errors of the \(Q\) values in individual bins are likely to be non-Gaussian distributed and correlated among different bins. These formulae are intended to give a simple, but for most purposes accurate, approximation to our statistical results.

Since the real-space 3PCF \(\xi(r, u, v)\) possibly depends on \(r, u, v\) in a complicated way, the inversion of equation (8) to get \(\xi(r, u, v)\) from the projected \(\Pi(r_p, u, v)\) is certainly unstable. We have noted however that the projected \(Q_{\text{proj}}(r_p, u, v)\) can be modeled very well by half the value of the mock projected \(Q_{\text{proj}}^{\text{mock}}(r_p, u, v)\) (Fig. 4, thin lines), which means that \(Q(r, u, v) = 0.5Q_{\text{CDM}}^0(r, u, v)\), where \(Q_{\text{CDM}}^0(r, u, v)\) is the real-space 3PCF of the CDM model, is a good approx-
imiation to the real-space 3PCF of LCRS galaxies on scales $\lesssim 3 \, h^{-1} \, \text{Mpc}$. These $Q(r, u, v)$ can be easily read out from the $Q_{\text{CDM}}(r, u, v)$ in Figure 2.

### 3.4. Discussion

The high quality of the LCRS survey, owing to its CCD photometry, its complete redshift information, and its large size, has enabled us to give a reliable determination of the 3PCF. This is the first time that the three-point correlation functions both in redshift space and in real (projected) space can be measured as a function of the triangle size and shape, with only a fractional error in each individual bin, for a wide range of triangle configurations. Although our statistical results in real space on scales $\lesssim 1 \, h^{-1} \, \text{Mpc}$ are not far from the hierarchical prediction (eq. [1]) with $Q = 1.3 \pm 0.2$ (Groth & Peebles 1977; GP77), the systematic changes of $Q_{\text{proj}}(r_p, u, v)$ with the triangle configurations clearly point to a more elaborate model for $Q(r, u, v)$. One such model was already proposed in § 3.3. Although our result of $Q_{\text{red}}(s, u, v)$ generally agrees with the previous studies based on much smaller redshift samples (see § 1), our measurement has much better accuracy.

Recently there have been concerns about the reliability of the high-order correlation functions derived from photographic-plate based galaxy catalogs. The skewness of the galaxies derived from the Automatic Plate Machine (APM) angular catalog (Maddox et al. 1990) and from the Edinburgh/Durham Southern Galaxy Catalogue (EDSGC; Heydon-Dumbleton, Collins, & MacGillivray 1989) is significantly different even on small scales less than $1 \, h^{-1} \, \text{Mpc}$. 

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**Fig. 1.** Normalized 3PCF in redshift space $Q_{\text{red}}(s, u, v)$ of the mock samples (symbols) and of the full simulation (lines). The error bars are the 1 $\sigma$ standard deviation of the measurement for the 10 mock samples. For clarity, the error bars are plotted for $u = 2$ only but those for the other two values of $u$ are very similar.
It is interesting to compare our results with the skewness $S_3(R)$ determined from the large angular catalogs, in particular since the two studies based on the APM and EDSCG catalogs have yielded rather discrepant results. The skewness is related to the 3PCF through an integral as follows:

$$S_3(R) = \frac{\bar{\zeta}(R)}{\zeta(R)} ,$$

$$\bar{\zeta}(R) = \frac{1}{V^2} \int_{\text{sphere} R} dr_1 \, d r_2 \, d r_3 \, \zeta(r_{12}, r_{23}, r_{31}) ,$$

$$\bar{\zeta}(R) = \frac{1}{V^2} \int_{\text{sphere} R} dr_1 \, d r_2 \, \zeta(r_{12}) ,$$

(13)
where $V = (4\pi/3)R^3$. Full information on $Q(r, u, v)$ is necessary to calculate the skewness (eq. [13]), and we use the model proposed in § 3.3. The skewness $S_3(R)$ for the LCRS survey is then about 4.5 at $R = 0.2 \, h^{-1} \text{ Mpc}$ and about 3.5 at $R = 1 \, h^{-1} \text{ Mpc}$, which seem in agreement with the results of Szapudi et al. (1996) based on the EDSGC catalog, but significantly higher than the APM results of Gaztañaga (1994) on the scales less than $1 \, h^{-1} \text{ Mpc}$.

The cosmic virial theorem (CVT) has been widely used to measure the mean density of the universe. If the 3PCF is hierarchical, the CVT can be expressed in its simplified form relating the density parameter $\Omega_0$, $\zeta(r)$, $Q$, and the PVD $\sigma_{12}(r)$. Since our results show that the 3PCF is not hierarchical, this relation becomes much more complicated, and it is necessary to work out the integration over $\zeta(r_{12}, r_{23}, r_{31})$, which might depend on $\zeta(r_{12}, r_{23}, r_{31})$ on very small scales $r \sim 0$. Since $\zeta(r_{12}, r_{23}, r_{31})$ is a decreasing function of the triangle size, previous studies that usually used the $Q$ value at $\sim 1 \, h^{-1} \text{ Mpc}$ might have overestimated the mean density. The size of galaxies, which are usually treated as point sources in the CVT application, may also be important in the estimate of the mean density, especially near $r \sim 0$ (Peebles 1976; Suto & Jing 1997).

We would like to remark here that the significant difference between $Q_{\text{red}}$ and $Q_{\text{proj}}$ at small scales comes from peculiar motions of galaxies. This gives another possibility
to estimate the velocity dispersion of galaxies (Matsubara 1994).

4. A CASE FOR MODEL TESTING

In this section we compare the 3PCF of the LCRS with model predictions. Jing & Börner (1997) and Jing (1998) have recently studied the 3PCF \(Q(r, u, v)\) for a set of CDM models based on second-order perturbation theory and N-body simulations. We found that for fixed \(u\) and \(v\), \(Q(r, u, v)\) is a decreasing function of the size \(r\). In the strongly nonlinear regime \(\xi(r) \gg 1\), \(Q(r, u, v)\) shows a very weak dependence on \(u\) and \(v\). In the weakly nonlinear and linear regimes, \(Q(r, u, v)\) increases significantly with \(v\) for fixed \(r \) and \(u\. All these features are found in the projected \(Q_{\text{proj}}(r_p, u, v)\) of the LCRS galaxies. Therefore the statistical results found from the LCRS survey are all qualitatively consistent with the mass 3PCF based on N-body simulations of cosmological models.

As an example to quantitatively test models with the 3PCF, we compare the 3PCFs of the LCRS galaxies with the results of the mock samples in Figures 5 and 6. Only in this way could the redshift distortion and the projection effects be accounted for properly. From the figures we find that the qualitative features, i.e., the dependence on \(v\) for fixed \(s\) or \(r_p\), and \(u\), the decrease of \(Q\) with increasing values of \(s\) or \(r_p\), are reproduced quite well in the mock samples.
The values of the data set, however, are consistently lower than the mean model predictions by a factor \( \sim 2 \). Since the \( Q \) values in each bin of the 10 mock samples are not Gaussian distributed (skewed to high values), it is not meaningful to use the standard deviation to quantify the statistical significance. Instead we pick up the lowest of the 10 mock \( Q_{\text{red}} \) or \( Q_{\text{proj}} \) values in each bin and compare it with the observed results. The thick lines in Figures 5 and 6 correspond to these lowest values for \( u = 1.29 \), which should be compared with the open triangles. These lines are still higher than (in most bins) or at least as high as (in a few bins) the observational values, which indicates that the observational values are lower than this model’s predictions at a confidence level \( \gtrsim 90\% \). The underlying model for the mock sample is a CDM universe with \( \Omega_0 = 0.2 \) and \( \lambda_0 = 0.8 \) and with clusters underweighted (see § 2). We have computed the 3PCF for this universe without cluster weighting and found that cluster weighting, like the fiber collision effect, does not change the value of \( Q(r, u, v) \) much. Thus, even if this model fits the 2PCF and the PVD of the LCRS galaxies quite well, it seems not entirely adequate to describe the clustering of galaxies when the 3PCF is considered, unless the observed 3PCF is biased low by the cosmic variance [at \( \lesssim 10\% \) probability]. This might indicate that the gravitational interaction alone is not sufficient to describe the clustering of galaxies, and physical processes of gas and radiation hydrodynamics connected with galaxy formation must be taken into account. A positive bias, i.e., a biasing
parameter $b > 1$, can reduce the 3PCF. But, perhaps this conclusion goes too far—in fact, a slightly higher shape parameter $\Gamma$ will give a better fit because $Q$ becomes smaller if $\Gamma$ is increased. It appears that a model with a new set of parameters is to be sought and the 3PCF determined here should provide a test for such new models in addition to the 2PCF and the PVD.

5. CONCLUSION

The result is clear, and the conclusions are straightforward: We have succeeded in measuring the 3PCF from the LCRS. This is the first time that the three-point correlation function of galaxies has been measured accurately from a redshift survey. Both the 3PCF in redshift space and the projected 3PCF have been measured as a function of the triangle size and shape with only a fractional uncertainty in each individual bin. Various tests have been carried out to ensure that the measurement is stable and that the errors are estimated reliably. Our results indicate that the 3PCFs both in redshift space and in real space have small but significant deviations from the well-known hierarchical form. The 3PCF in the redshift space can be fitted by

$$Q_{\text{3PCF}}(s, u, v) = 0.5 \times 10^{0.2 + 0.1(s/u + 1)^{1/2}}$$

for $0.8 < s < 8 \ h^{-1}\text{Mpc}$ and $s < 16 \ h^{-1}\text{Mpc}$, and the projected 3PCF by

$$Q_{\text{proj}}(r_p, u, v) = 0.7 r_p^{-0.3}$$

for $0.2 < r_p < 3 \ h^{-1}\text{Mpc}$ and $r_p < 6 \ h^{-1}\text{Mpc}$ ($s$ and $r_p$ are in units of $h^{-1}\text{Mpc}$). Although it might not be unique to get three-dimensional real space $Q(r, u, v)$ from the measured projected function.

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**Fig. 6.**—Normalized projected 3PCF of the LCRS survey (open symbols) compared with the mean value of the 10 mock samples (solid symbols). The thick solid lines represent the lowest of the 10 mock $Q_{\text{3PCF}}(s, u, v)$ values in each $u = 1.29$ bin, which can be regarded as the lower model limits at $\sim 90\%$ significance level.
we found that a half of the predicted $Q(r, u, v)$ of the CDM model considered in this paper provides a good description of the LCRS data.

The three-point correlation function gives an additional statistical tool to constrain cosmogonic models. The general dependence of the 3PCF on triangle shape and size is in qualitative agreement with the CDM cosmogonic models. Quantitatively the 3PCF of the models may depend on the biasing parameter and the shape of the power spectrum, in addition to other model parameters. Taking our result together with the constraints imposed by the two-point correlation function and the pairwise velocity dispersion of galaxies also obtained from the LCRS, we find that we have difficulties to produce a simple model that meets all constraints perfectly. Among the CDM models considered, the flat model with $\Omega = 0.2$ meets the 2PCF and PVD constraints, but gives higher values for the 3PCF than observed. This may indicate that more sophisticated bias models or a more sophisticated combination of model parameters must be considered.

We are grateful to Yasushi Suto for helpful discussions, and for the hospitality extended to us at the physics department of Tokyo University. G. B. thanks the Yamada foundation for support during his stay at RESCEU. J. Y. P. gratefully acknowledges the receipt of a JSPS postdoctoral fellowship. Support by SFB375 is also acknowledged. The simulations were carried out on VPP/16R and VX/4R at the Astronomical Data Analysis Center of the National Astronomical Observatory, Japan.

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