Finite-time energy-to-peak control for Markov jump systems with pure time delays

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Abstract
This study investigates finite-time energy-to-peak control for pure-time-delay Markov jump systems. The main objective is to obtain some theorems such that the corresponding pure-time-delay Markov jump systems are finite time energy-to-peak stable or stabilizable. First, based on mathematical transformation, the pure-time-delay Markov jump systems are described in a model description that includes the current system state and several distributed time-delay items. Second, according to linear matrix inequality (LMI) theory, a positive energy functional is constructed, which includes a triple integral item. Then, after some mathematical operations, some sufficient conditions are obtained for Markov jump systems to be finite-time energy-to-peak stable or stabilizable. The obtained results are expressed in LMIs, which can be conveniently solved by computers. Finally, examples are given to show the usefulness of the obtained theorems.

Keywords
Energy-to-peak control, pure time delay, Markov jump system, finite-time stability theory

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Introduction
Because signal transmission and processing need to consume time, time delays almost exist in all real systems. If some of those time delays are not considered properly during system analysis or synthesis, the system may be performance-decreased or destroyed. In order to decrease the influence of time delays on system performance, many efforts have been made by scholars in recent years, and some achievements were gotten, for example, Sun et al.¹ demonstrated the global stability of the joint space of an excavator by applying a time-delay system control method. By using a time delay estimation algorithm, Mazare et al.² proposed a method to design a fault-tolerant controller for a variable-speed wind turbine. By solving some special mathematical equations, Hu and Lu³ designed a time-delay system control strategy for multi-DOF systems with strongly nonlinear characters. Additional results can be found in references.⁴–⁶ However, there is a class of special time-delay systems, called pure time delay systems (PTDSs), which exists in many engineering fields. Compared with regular time delay systems, PTDSs have no current state items. Thus, the system analysis and synthesis methods used for regular time delay systems cannot be applied to PTDSs directly, and trying to obtain some results for PTDSs is necessary and meaningful. Fortunately, during the past several years, some scholars have done some works on this issue. For example, Elshenhab and Wang⁷ considered linear fractional systems with pure time delay in reference, and the solutions of the corresponding systems were obtained by using some delay-based matrix functions and system transform. Liu et al.⁸ presented some exact solutions for a pure delay fractional equation, and some conditions were obtained for the system to be stable. More achievements in this issue can be found in references.⁹–¹¹ However, LMI-based achievements regarding pure time delay systems are still few, and obtaining some LMI-based results for PTDSs is still required.

The literature first mentioned about LMI dates back to 100 years ago.¹² Lyapunov proposed some stability theorems for differential equations in 1890. Then, the
Lyapunov stability theorems were applied to some classical control problems in the 1940s by Lur’e et al., and the embryonic form of the LMI was formed. In the following years, based on the scholars’ efforts, the LMI technique was advanced greatly, and many achievements regarding the use of LMI to solve control problems were obtained; for example, some LMI-based conditions were achieved by Basu et al. to solve the output regulation problem of a linear regular system. Torres-Pinzn et al. presented the design of an LMI-based fuzzy controller for DC-DC converters, and the theoretical predictions were verified by using a 60 W prototype. Based on the LMI technique, Hao et al. addressed some sufficient conditions for obtaining a fault-tolerant controller for unmanned marine vehicles. Chen et al. discussed the controller design of uncertain linear systems, and some LMI-based conditions were derived for the existence of sliding-mode controllers. Additional results can be found in references.

On the other hand, changes in the working surroundings or breakdown of system components always exist in real systems; thus, some system state jumps often occur in a real system. If these jumps are not considered correctly, the system may be performance-decreased or destroyed. It is worth mentioning that a class of jump systems, called Markov jump system (MJS), widely exists in many engineering fields. In order to deal with the Markov jump, many scholars have attempted to conduct research on this issue, and many achievements have been made in the last several decades. For example, Liu et al. addressed a neural network event-triggered scheme for nonlinear MJSs and some theorems that can guarantee fault sensitivity and disturbance attenuation in certain frequency ranges were derived. He et al. dealt with the attack defense control for MJSs, and an adaptive control technique was obtained. Shu et al. addressed the robust controller design for fuzzy MJSs by constructing a special energy functional. Additional results regarding MJSs can be found in references. Thus, doing a system control design for pure time-delay systems with system state jumps considered is also necessary.

It is well known that the energy-to-peak control, as a type of disturbance-resist control method, is widely used in many control engineering fields to constrain the influence of unexpected disturbances on the system. For pure time-delay MJSs, a controller design with the disturbance-resist performance considered is also necessary. During the past several decades, many results about energy-to-peak control also have been achieved. For example, Xie et al. studied the energy-to-peak control for the actuator-saturated time-varying systems, and a heuristic-algorithm-based condition was derived. Chang et al. studied the filter design for a class of singular systems by using energy-to-peak control methods. Additional results can be found in references. Furthermore, it is often that the peak responses of the system states destroy the corresponding system. Thus, doing a system control design with the maximum state response constrained is necessary and important. Fortunately, Russian literature introduced a control method, called finite-time stability (FTS), which can constrain systems’ states in a given domain in a certain time interval. During the past several decades, some results regarding FTS have also been achieved; for example, Yang et al. discussed the FTS of neural network systems with proportional delay, and a less conservative criterion was obtained. Feng et al. considered the input-output FTS for a type of switched system, and some sufficient theorems were obtained for the system. The readers can refer to references for more results about FTS. Thus, if the FTS is introduced in the control of MJSs with PTDs, some improved performances can be expected to be obtained.

This study mainly considered the finite-time energy-to-peak control for a class of Markov jump PTDSs. The main contributions include the following aspects: (1) by using the system transformation, the Markov jump PTDSs are described with some current state items and distributed time-delay items; (2) based on a functional candidate and some mathematical operations, some LMI-based theorems are obtained for the Markov jump PTDSs; (3) if the Markov jump PTDS is unstable, some stabilizing controllers can be obtained by using the obtained theorems, and the finite-time energy-to-peak stability of the controlled system is guaranteed. Moreover, to further illustrate the usefulness of the theorems obtained in this paper, some examples are provided in the end.

**Dynamic models**

We consider the Markov jump PTDSs:

\[
\dot{x}(t) = \sum_{i=1}^{m} A_i(\delta_i)x(t-\tau_i) + B(\delta_i)u(t) + B_k(\delta_i)\omega(t),
\]

\[
Z(t) = C(\delta_i)x(t),
\]

where \(x(t-\tau_i) \in \mathbb{R}^n\) is the system state with delay time \(\tau_i\); \(u(t) \in \mathbb{R}^p\) is the system control input; \(\omega(t) \in \mathbb{R}^q\) represents the external disturbance; \(A_i(\delta_i), B(\delta_i), B_k(\delta_i),\) and \(C(\delta_i)\) are the system matrices. \(\delta_i\) is a Markov process parameter, which takes values in the space \(S = \{1, 2, 3,...,N\}\). Then, we use the matrix \(\Pi = \{\pi_{ik}\}\) \((i, k \in S)\) to denote the system transition, and \(\pi_{ik}\) is the transition rate, which has \(\pi_{ik} \geq 0\) for \(k \neq i\), and \(\pi_{ii} = -\sum_{k=1, k \neq i}^N \pi_{ik}\). Then, we have \(P(\delta_{i+\epsilon} = k|\delta_i = i) = \)

\[
\begin{cases} 
\pi_{ik} + \omega(\epsilon), i \neq k \\
1 + \pi_{ik} + \omega(\epsilon), i = k
\end{cases}, \quad \epsilon > 0, \quad \text{and} \quad \lim_{\epsilon \to 0} \omega(\epsilon) = 0.
\]

Assume that delay time within the control channel is \(\tau_0\). Then, the controller is described as

\[
u(t) = F(\delta_i)x(t-\tau_0),\]

(2)
where $F(\delta_t)$ is the controller gain. Based on the transformation $\int_{t-\tau}^{t} \dot{x}(s) ds = x(t) - x(t - \tau)$, system (1) and controller (2) can be expressed as

$$
\dot{x}(t) = \sum_{j=1}^{m} A_j(\delta_t) x(t) + \sum_{j=1}^{m} B_j(\delta_t) x(t) + \int_{t-\tau}^{t} \dot{x}(\theta) d\theta + B(\delta_t) u(t) + B_w(\delta_t) \omega(t),
$$

$$
Z(t) = C(\delta_t) x(t),
$$

$$
u(t) = F(\delta_t) x(t) - F(\delta_t) \int_{t-\tau}^{t} \dot{x}(s) ds.
$$

Then, we substitute equation (4) into equation (3), and obtain the closed-loop form:

$$
\dot{x}(t) = \sum_{j=1}^{m} A_j(\delta_t) x(t) + B(\delta_t) F(\delta_t) x(t)
- \sum_{j=1}^{m} A_j(\delta_t) \int_{t-\tau}^{t} \dot{x}(\theta) d\theta - B(\delta_t) F(\delta_t)
\int_{t-\tau}^{t} \dot{x}(\theta) d\theta + B_w(\delta_t) \omega(t),
$$

$$
Z(t) = C(\delta_t) x(t),
$$

$$
x(t) = \Phi(t), \quad \forall t \in [-\tau, 0], \tau = \max\{\tau_0, \tau_2 \cdots, \tau_m\}.
$$

We use the index $i$ to denote the current model, then, $A_i(\delta_t), B_i(\delta_t), F_i(\delta_t), B_w(\delta_t),$ and $C_i(\delta_t)$ are denoted by $A_{ij}, B_i, F_i, B_{wi},$ and $C_i$, respectively.

The focus of this paper is to obtain some conditions such that system (1) satisfies: (i) When $\omega(t) = 0$, system (1) is FTS. (ii) Under 0 initial conditions $\Phi(t) = 0, \forall t \in [-\tau, 0]$, system (1) has the performance $E\{\|Z\|_\infty\} \leq \gamma\|\omega\|_2$ for all nonzero $\omega \in L^2[0, \infty]$.

**Definition 1.** It is said that the system (1) is finite-time energy-to-peak stable in regard to $(c_1, c_2, c_3, R, T, \gamma, d)$, if the system has

$$
\sup_{s \in [-\tau, 0]} \{\Phi^T(s) R \Phi(s)\} \leq c_1^2, \quad \sup_{s \in [-\tau, 0]} \{\dot{\Phi}^T(s) R \dot{\Phi}(s)\} \leq c_2^2 \Rightarrow E\{x(t)^T R x(t)\} \leq c_3^2 \text{ and } E\{|Z|_\infty\} \leq \gamma\|\omega\|_2
$$

for any $t \in [0, T]$, $\int_0^t \omega(t) \omega(t) dt \leq d$, where $0 < c_1 < c_2, c_3 > 0, R > 0, T > 0, \gamma > 0, d \geq 0$.

**Definition 2.** It is said that the system (1) is finite-time energy-to-peak stabilizable in regard to $(c_1, c_2, c_3, R, T, \gamma, d)$, if there exists a controller gain $F(\delta_t)$ such that the controlled system is finite-time energy-to-peak stable.

**Lemma 1.** If there are any matrix $Y > 0$, any scalars $k_1, k_2 > k_1$, and a function $\varphi_{1, 2}: [k_1, k_2] \rightarrow \mathbb{R}^n$, then $(k_2 - k_1) \int_{k_1}^{k_2} \varphi_1(t) \varphi_2(t) ds \geq \int_{k_1}^{k_2} \varphi_1^2(t) ds Y \int_{k_1}^{k_2} \varphi_2^2(t) ds$.

**Lemma 2.** If there are a matrix $\Gamma = I^T > 0$, and an integrable function $\{\omega(\alpha) | \alpha \in [h_1, h_2]\}$, then

$$
\int_{h_1}^{h_2} \omega(T(\alpha) \Gamma \omega(\alpha) d\alpha \geq \frac{1}{h_2 - h_1} \int_{h_1}^{h_2} \omega(T(\alpha) d\alpha \Gamma
+ \frac{3}{h_2 - h_1} \int_{h_1}^{h_2} \omega(\alpha) d\alpha \Sigma \omega(\alpha) d\alpha d\beta.
$$

where $\Sigma = \int_{h_1}^{h_2} \omega(\alpha) d\alpha - \frac{2}{h_2 - h_1} \int_{h_1}^{h_2} \omega (\alpha) d\alpha d\beta$.

**Lemma 3.** Let $\rho(t)$ be a nonnegative function such that $\rho(t) \leq a + b \int_0^t \rho(s) ds$, $0 \leq t \leq T$ for some constants $a, b > 0$, then, we have $\rho(t) \leq a \exp(bt), 0 \leq t \leq T$.

**Main results**

**Theorem 1.** There are delay times $\tau_j > 0$ ($j = 1, \cdots, m$) and constant $\gamma > 0$ such that the system (1) is finite-time energy-to-peak stable with respect to $(c_1, c_2, c_3, R, T, \gamma, d)$, if there are any matrices $P_i = P_i^T > 0, V_j = V_j^T > 0$ ($j = 1, \cdots, m$), $Q_i = Q_i^T > 0$ ($j = 1, \cdots, m$), $W_j = W_j^T > 0$ ($j = 1, \cdots, m$), matrices $S_i$, and scalars $\eta_i > 0, \alpha_i > 0, \lambda_i, \lambda_2, \lambda_3, \lambda_4, \beta_i, \beta_j$ ($j = 1, \cdots, m$), $\beta_{ij} (j = 1, \cdots, m)$ satisfying the following LMI

$$
\begin{bmatrix}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \cdots & \Xi_{1(m+2)} & \Xi_{1(m+3)} & \cdots & \Xi_{1(2m+2)} & S_i B_{si} \\
* & \Xi_{22} & -\beta_2 S_i A_2 & \cdots & -\beta_{2m} S_i A_2 & \Xi_{2(m+3)} & \cdots & \Xi_{2(2m+2)} & \beta_{1i} S_i B_{si} \\
* & * & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
* & * & * & \cdots & 0 & \cdots & \cdots & \cdots & \cdots \\
* & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
* & * & * & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
* & * & * & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
* & * & * & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
* & * & * & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
* & * & * & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
* & * & * & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
* & * & * & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\end{bmatrix} < 0,
$$

($7$)
where

\[
\begin{align*}
\Xi_{i11} &= \sum_{j=1}^{m} \left( \tau_j^2 V_j - 12Q_j - \tau_j^2 W_j + S_j A_{ij}^T + A_{ij}^T S_j^T \right) \\
&\quad + \alpha P_i + \sum_{k=1}^{N} \sigma_{ik} P_k, \\
\Xi_{i12} &= P_i - S_i + \beta_{\|i\|} \sum_{j=1}^{m} A_{ij}^T S_j^T, \\
\Xi_{i22} &= \sum_{j=1}^{m} \left( \tau_j^2 Q_j + \frac{1}{4} \tau_j^4 W_j \right) - \beta_{\|i\|} S_i - \beta_{\|i\|} S_i^T, \\
\Xi_{i33} &= -V_i - W_i - \frac{12}{\tau_1} Q_1, \\
\Xi_{i13} &= \frac{12}{\tau_1} Q_1 + \tau_1 W_1 + \beta_{\|2\|} \sum_{j=1}^{m} A_{ij}^T S_j^T, \\
\Xi_{i(m+2)} &= \frac{12}{\tau_m} Q_m + \tau_m W_m + \beta_{\|2m\|} \sum_{j=1}^{m} A_{ij}^T S_j^T, \\
\Xi_{i(m+2)(m+2)} &= -V_m - W_m - \frac{12}{\tau_m} Q_m, \\
\Xi_{i(m+3)} &= 6Q_1 - S_i A_{i1} + \beta_{\|3\|} \sum_{j=1}^{m} A_{ij}^T S_j^T, \\
\Xi_{i(2m+3)} &= -\beta_{\|3\|} S_i A_{i1} - \beta_{\|3\|} S_i^T, \\
\Xi_{i(3m+3)} &= -6 \frac{1}{\tau_1} Q_1 - \beta_{\|2\|} S_i A_{i1}, \\
\Xi_{i(m+3)(m+3)} &= -4Q_1 - \beta_{\|3\|} S_i A_{i1} - \beta_{\|3\|} A_{i1}^T S_i^T, \\
\Xi_{i(2m+2),i} &= 6Q_{\|m\|} - S_i A_{i}\|m\| + \beta_{\|3m\|} \sum_{j=1}^{m} A_{ij}^T S_j^T, \\
\Xi_{i(2m+2),i} &= -\beta_{\|3m\|} S_i A_{i}\|m\| - \beta_{\|3m\|} A_{i1}^T S_i^T, \\
\Xi_{i(3m+2),i} &= -4Q_{\|m\|} - \beta_{\|3m\|} S_i A_{i}\|m\| - \beta_{\|3m\|} A_{i1}^T S_i^T.
\end{align*}
\]

**Proof:** Choose a suitable functional candidate as

\[
V(t,i) = V_1(t,i) + V_2(t,i) + V_3(t,i),
\]

where

\[
\begin{align*}
V_1(t,i) &= x(t)^T P_i x(t), \\
V_2(t,i) &= \sum_{j=1}^{m} \int_{t_j - \epsilon}^{t_j} x^T(s) V_j x(s) ds dc, \\
V_3(t,i) &= \sum_{j=1}^{m} \int_{t_j - \epsilon}^{t} \dot{x}^T(s) Q_j x(s) ds dc, \\
V_4(t,i) &= \sum_{j=1}^{m} \int_{t_j - \epsilon}^{t} \dot{x}^T(s) W_j x(s) ds dc dt,
\end{align*}
\]
and \( P_i = P_i^T > 0 \), \( U_j = U_j^T > 0 \) \( (j = 1, \ldots, m) \), \( Q_j = Q_j^T > 0 \) \( (j = 1, \ldots, m) \). By using the weak infinitesimal generator \( \mathcal{A} \), we have

\[
\mathcal{V}_1(t, i) \leq 2x^T(t)P_i \dot{x}(t) + \sum_{k=1}^{N} \pi_k \dot{x}^T(t)P_k \dot{x}(t),
\]

\[
\mathcal{V}_2(t, i) \leq \sum_{j=1}^{m} \left( \tau_j^2 \dot{x}^T(t)Q_j \dot{x}(t) - 2 \tau_j \int_{t_j}^{t} \dot{x}^T(s)Q_j \dot{x}(s) ds \right).
\]

\[
\mathcal{V}_3(t, i) = \sum_{j=1}^{m} \left( \tau_j^2 \dot{x}^T(t)Q_j \dot{x}(t) - \tau_j \int_{t_j}^{t} \dot{x}^T(s)Q_j \dot{x}(s) ds \right).
\]

\[
\mathcal{V}_4(t, i) = \sum_{j=1}^{m} \frac{1}{2} \tau_j^2 \dot{x}^T(t)W_j \dot{x}(t) - \sum_{j=1}^{m} \frac{1}{2} \tau_j^2 \int_{t_j}^{t} \dot{x}^T(s)W_j \dot{x}(s) ds.
\]

Furthermore, based on Lemma 1, the following equations can be gotten:

\[
- \tau_j \int_{t_j}^{t} \dot{x}^T(s)Q_j \dot{x}(s) ds \leq - \int_{t_j}^{t} \dot{x}^T(s)Q_j \dot{x}(s) ds \leq - \int_{t_j}^{t} \dot{x}^T(s)Q_j \dot{x}(s) ds
\]

\[
- \sum_{j=1}^{m} \frac{1}{2} \tau_j^2 \int_{t_j}^{t} \dot{x}^T(s)W_j \dot{x}(s) ds \leq
\]

\[
- \sum_{j=1}^{m} \left( \tau_j \dot{x}^T(t) - \int_{t_j}^{t} \dot{x}^T(s) ds \right)
\]

\[
W_j \left( \tau_j \dot{x}(t) - \int_{t_j}^{t} \dot{x}(s) ds \right).
\]

By utilizing Lemma 2, the following Eq. (21) is obtained:

\[
- \tau_j \int_{t_j}^{t} \dot{x}^T(s)Q_j \dot{x}(s) ds \leq - \int_{t_j}^{t} \dot{x}^T(s)Q_j \dot{x}(s) ds \leq - \int_{t_j}^{t} \dot{x}^T(s)Q_j \dot{x}(s) ds - 3 \Omega_2^T Q_j \Omega_2,
\]

where \( \Omega_2 = \int_{t_j}^{t} \dot{x}(s) ds - \frac{1}{2} \int_{t_j}^{t} \int_{t_j}^{s} \dot{x}(s) ds dB = \int_{t_j}^{t} \dot{x}(s) ds - 2x(t) + \frac{1}{2} \int_{t_j}^{t} \dot{x}(s) ds dB. \) Considering the open-loop form of system (1), we can get the following Eq. (22) by setting \( B(\delta_i) = 0 \).

\[
\left( x^T(t) + \beta_1 \dot{x}^T(t) + \sum_{j=1}^{m} \beta_2 j \int_{t_j}^{t} x(\theta) d\theta + \sum_{j=1}^{m} \beta_2 j \int_{t_j}^{t} \dot{x}(\theta) d\theta \right) \times \left( \sum_{i=1}^{m} A_i x(t) - \sum_{i=1}^{m} A_i \right) = 0,
\]

where \( S_i \) is a suitable matrix, \( \beta_1, \beta_2, \) and \( \beta_2 j \) are constants. By combining the equations (16)–(22), the following Eq. (23) can be obtained.

\[
\mathcal{V}(t, i) - \alpha \mathcal{V}(t, i) - \eta \omega_i(t) \mathcal{V}(t, i) \leq \xi(t)^T \Xi \xi(t),
\]

where \( \xi(t) = [x(t) \dot{x}(t) \int_{t_1}^{t} \dot{x}(s) ds \cdots \int_{t_m}^{t} \dot{x}(s) ds \int_{t_1}^{t} \cdots \int_{t_m}^{t} \dot{x}(s) ds \cdots \int_{t_m}^{t} \dot{x}(s) ds \omega(t)] \). Based on LM1 (7), it is easy to obtain

\[
\mathcal{V}(t, i) \leq a \mathcal{V}(t, i) + \eta \omega_i(t) \omega(t).
\]

By doing an integration on both sides of (24) from 0 to \( t \), \( t \in [0, T] \), we get

\[
\mathcal{V}(t, i) \leq \mathcal{V}(0, i) + \alpha \int_{0}^{t} \mathcal{V}(s, i) ds + \eta \int_{0}^{t} \omega_i(s) \omega(s) ds.
\]

According to Lemma 3, it has

\[
\mathcal{V}(t, i) \leq \mathcal{V}(0, i) e^{\alpha t} + \eta e^{\alpha t} \int_{0}^{t} \omega_i(s) \omega(s) ds.
\]

According to equation (14), we have

\[
V(t, i) \geq x^T(t) P_i x(t) \geq \lambda_{min} (R^{-1/2} P_i R^{-1/2}) x^T(t) R x(t),
\]

where \( R > 0 \). Then

\[
x^T(t) R x(t) \leq \frac{1}{\lambda_{min} (R^{-1/2} P_i R^{-1/2})} V(t, i).
\]

According to (14), we can get

\[
V(0, i) = x^T(0) P_i x(0) + \sum_{j=1}^{m} \tau_j \int_{t_j}^{0} \int_{t_j}^{s} \dot{x}^T(s) Q_j \dot{x}(s) ds ds
\]

\[
\leq \lambda_{max} (R^{-1/2} P_i R^{-1/2}) x^T(0) R x(0)
\]

\[
+ \sum_{j=1}^{m} \frac{1}{2} \lambda_{max} (R^{-1/2} U_j R^{-1/2}) x^T(0) R x(0)
\]

\[
+ \sum_{j=1}^{m} \frac{1}{2} \lambda_{max} (R^{-1/2} Q_j R^{-1/2}) \dot{x}^T(t) R \dot{x}(t)
\]

\[
\leq \lambda_{max} (R^{-1/2} P_i R^{-1/2}) c_1^2
\]

\[
+ \sum_{j=1}^{m} \frac{1}{2} \lambda_{max} (R^{-1/2} U_j R^{-1/2}) c_1^2
\]

\[
+ \sum_{j=1}^{m} \frac{1}{2} \lambda_{max} (R^{-1/2} Q_j R^{-1/2}) c_2^2.
\]
Furthermore, it holds
\[ \int_0^t \omega^T(s) \eta \omega(s) ds < \eta d. \] (30)

In view of (26)–(30), it yields
\[ E\{x^T(t) Rx(t)\} \leq \frac{e^{\alpha t}}{\lambda_{\text{min}}(R^{-1/2} P_l R^{-1/2})} \]
\[ + \sum_{j=1}^m \frac{\tau_j^2}{2} \lambda_{\text{max}}(R^{-1/2} Q_j R^{-1/2}) c_1^2 + \eta d. \] (31)

By considering the conditions (8)–(13), we can obtain
\[ E\{x^T(t) Rx(t)\} \leq c_3^2. \] Then, we consider the system
\[ Z(t) = x^T(t) C_i^T C_i x(t) \]
\[ < \frac{\gamma^2}{\eta} e^{-\alpha T} P_l x(t) \]
\[ \leq \frac{\gamma^2}{\eta} e^{-\alpha T} V(t, i) \]
\[ \leq \frac{\gamma^2}{\eta} \int_0^t \omega^T(t) \omega(t) dt, \]

that is,
\[ E\|Z(t)\|_2 < \gamma\|\omega(t)\|_2. \]

According to Definition 1, we have system (1) is finite-time energy-to-peak stable.

**Theorem 2:** There are delay times \( \tau_j > 0 \) \((j = 1, \ldots, m)\) and constant \( \gamma > 0 \) such that the system (1) is finite-time energy-to-peak stabilizable in regard to \((c_1, c_2, c_3, R, T, \gamma, d)\), if there are any matrices \( P_l, P_l^T > 0, V_j = V_j^T > 0 \) \((j = 1, \ldots, m)\), \( Q_j = Q_j^T > 0 \) \((j = 1, \ldots, m)\), \( W_j = W_j^T > 0 \) \((j = 1, \ldots, m)\), matrix \( S_i, G_i \) and scalars \( \eta > 0, \alpha > 0, \lambda_1, \lambda_2, A_1, A_2, \lambda_4, \beta_{11}, \beta_{12}, \beta_{2j} \) \((j = 1, \ldots, m)\), \( \beta_{2j} \) \((j = 1, \ldots, m)\) satisfying the following LMIs

\[ \Xi_i = \begin{bmatrix}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \ldots & \Xi_{1(m+3)} & \Xi_{1(m+4)} & \ldots & \Xi_{1(m+5)} & \ldots & \Xi_{1(2m+4)} & B_{w_1} \\
* & \Xi_{22} & -\beta_1 A_{11} & -\beta_2 A_{12} & \ldots & -\beta_2 A_{1m} & \ldots & -\beta_2 A_{1m} & \ldots & -\beta_2 A_{1m} & B_{w_1} \\
* & * & \Xi_{33} & 0 & 0 & \ldots & -\beta_2 A_{21} & G_1 & \ldots & -\beta_2 A_{21} & \ldots & -\beta_2 A_{21} \\
* & * & * & \Xi_{44} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
* & * & * & * & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
* & * & * & * & * & \Xi_{55} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
* & * & * & * & * & * & \Xi_{66} & \ldots & \ldots & \ldots & \ldots & \ldots \\
* & * & * & * & * & * & * & \Xi_{77} & \ldots & \ldots & \ldots & \ldots \\
* & * & * & * & * & * & * & * & \Xi_{88} & \ldots & \ldots & \ldots \\
* & * & * & * & * & * & * & * & * & \Xi_{99} & \ldots & \ldots \\
* & * & * & * & * & * & * & * & * & * & \Xi_{101} & \ldots \\
\end{bmatrix} < 0 \] (33)

\[ \begin{bmatrix}
-e^{-\alpha T} P_l \\
\eta S_i^T C_i^T \\
-\eta^2 R_i \\
\end{bmatrix} < 0, \] (34)

\[ \begin{bmatrix}
P_l & \lambda_1 S_i R \\
* & -\lambda_2 R_i \\
\end{bmatrix} < 0, \] (35)

\[ P_l - \lambda_2 S_i R - \lambda_2 R S_i^{-1} + R_i < 0, \] (36)

\[ U_j - \lambda_3 S_i R - \lambda_3 R S_i^{-1} + R_i < 0, \] (37)

\[ Q_j - \lambda_4 S_i R - \lambda_4 R S_i^{-1} + R_i < 0, \] (38)

under 0 initial conditions \( \Phi(t) = 0, \forall t \in [-\tau, 0] \). By doing an integration on both sides of (24) from 0 to \( t \), \( t \in [0, T] \), we obtain the following equation (32) by Lemma 2.

\[ V(t, i) < \eta e^{\alpha t} \int_0^t \omega^T(s) \omega(s) ds. \] (32)

According to Eq. (8), one gets \( C_i^T C_i < \frac{\gamma^2}{\eta} e^{-\alpha T} P_l \), thus

\[ \begin{bmatrix}
-\alpha \lambda_2^2 c_1^2 + \eta d \\
\tau_1^2 \lambda_3 c_1 \\
* \tau_1^2 \lambda_3 c_1 \\
\alpha \lambda_2^2 c_2 \\
* \tau_1^2 \lambda_3 c_2 \\
\alpha \lambda_2^2 c_2 \\
\alpha \lambda_2^2 c_3 \\
* \tau_1^2 \lambda_3 c_3 \\
\alpha \lambda_2^2 c_3 \\
\alpha \lambda_2^2 c_3 \\
\alpha \lambda_2^2 c_3 \\
\end{bmatrix} < 0, \] (39)
where

\[ \Xi_{i1} = \sum_{j=0}^{m} (\tau_j^2 V_j - 12 Q_j - \tau_j^2 W_j) + \sum_{j=1}^{m} (A_{ij} S_j + S_j^T A^T_{ij}) \]
\[ + B_i G_i + G_i^T B_i^T - \alpha P_1 + \sum_{k=1}^{N} \pi_{ik} P_k, \]

\[ \Xi_{i2} = P_i - S_i + \beta_{i1} \left( G_i^T B_i^T + \sum_{j=1}^{m} S_j^T A^T_{ij} \right), \]

\[ \Xi_{i3} = - V_0 - W_0 - \frac{12}{\tau_0} Q_0, \]

\[ \Xi_{i4} = \frac{12}{\tau_0} Q_0 + \tau_0 W_0 + \beta_{i20} \left( G_i^T B_i^T + \sum_{j=1}^{m} S_j^T A^T_{ij} \right), \]

\[ \Xi_{i5} = 6 Q_0 - B_i G_i \]
\[ + \beta_{i30} \left( G_i^T B_i^T + \sum_{j=1}^{m} S_j^T A^T_{ij} \right), \]

\[ \Xi_{i6} = - \beta_{i1} B_i G_i - \beta_{i1} S_i^T, \]

\[ \Xi_{i7} = - \frac{6}{\tau_0} Q_0 - \beta_{i20} B_i G_i, \]

\[ \Xi_{i8} = - \beta_{i1} B_i G_i - \beta_{i1} S_i^T, \]

\[ \Xi_{i9} = - \frac{6}{\tau_0} Q_0 - \beta_{i20} B_i G_i, \]

\[ \Xi_{i10} = 6 Q_1 - A_{i1} S_1 \]
\[ + \beta_{i31} \left( G_i^T B_i^T + \sum_{j=1}^{m} S_j^T A^T_{ij} \right), \]

\[ \Xi_{i11} = - \beta_{i1} A_{i1} S_1 - \beta_{i30} S_i^T, \]

\[ \Xi_{i12} = - \frac{6}{\tau_2} Q_2 - \beta_{i21} A_{i2} S_i, \]

\[ \Xi_{i13} = - \beta_{i20} A_{i1} S_i - \beta_{i31} S_i^T A_{i1}^T, \]

\[ \Xi_{i14} = - 4 Q_1 - \beta_{i31} A_{i1} S_i - \beta_{i31} S_i^T A_{i1}^T, \]

\[ \Xi_{i15} = 6 Q_0 - A_{i0} S_i \]
\[ + \beta_{i3m} \left( G_i^T B_i^T + \sum_{j=1}^{m} S_j^T A^T_{ij} \right), \]

\[ \Xi_{i16} = - \beta_{i1} A_{i0} S_i - \beta_{i3m} S_i^T, \]

\[ \Xi_{i17} = - \frac{6}{\tau_m} Q_0 - \beta_{i2m} A_{i0} S_i, \]

Then, a controller gain can be obtained by \( F_i = G_i S_i^{-1}. \)

**Proof:** By replacing equations (14) and (22) with the following equations (40) and (41), respectively, and pre- and post-multiplying (33) and (34) with \( \text{diag} \{ S_i^{-T} \cdots S_i^{-T} \} \), \( \text{diag} \{ S_i^{-T} \} \) and their transpose, respectively, Then, Theorem 2 can be obtained by doing a similar operation with Theorem 1.

\[ V(t, i) = V_1(t, i) + V_2(t, i) + V_3(t, i), \]

where

\[ V_1(t, i) = x(t)^T P_i x(t), \]
\[ V_2(t, i) = \sum_{j=0}^{m} \int_{\tau_j - \tau_0}^{\tau_j} x^T(s) V_j x^T(s) ds, \]
\[ V_3(t, i) = \sum_{j=0}^{m} \int_{\tau_j - \tau_0}^{\tau_j} \tilde{x}(s) \tilde{Q}_j \tilde{x}(s) ds, \]
\[ V_4(t, i) = \sum_{j=0}^{m} \frac{1}{\tau_m^2} \int_{\tau_j - \tau_0}^{\tau_j} \tilde{x}(s) W_j \tilde{x}(s) ds d\theta, \]

and \( P_i = P_i^T > 0, \)
\[ Q_j = Q_j^T > 0 \] \( (j = 1, \ldots, m), \)
\[ W_j = W_j^T > 0 \] \( (j = 1, \ldots, m), \)

\[ \left( x^T(t) + \beta_{i1} \tilde{x}^T(t) + \sum_{j=0}^{m} \beta_{i3} \int_{\tau_j}^{\tau_j + \tau_0} x^T(\theta) d\theta \right) \]
\[ + \sum_{j=0}^{m} \beta_{i3} \int_{\tau_j}^{\tau_j + \tau_0} \tilde{x}(\theta) d\theta \]
\[ < 0, \]

\[ \left( \begin{array}{c}
\sum_{j=0}^{m} A_{ij} x(t) + B_i K_i \tilde{x}(t) - \sum_{j=0}^{m} A_{ij} \int_{\tau_j - \tau_0}^{\tau_j} \tilde{x}(\theta) d\theta \\
- B_i K_i \int_{\tau_j - \tau_0}^{\tau_j} \tilde{x}(\theta) d\theta + B_i \omega(t) - \dot{x}(t)
\end{array} \right) = 0 \]

**Remark 1:** It is worth pointing out while we solve the Theorems 1 and 2, there are several parameters needed to be given ahead. The values of the parameters \( c_1 \) and \( c_2 \) can be gotten based on the initial conditions of the corresponding system. The value of \( c_3 \) is given according to the state-constraint requirements, and we can choose \( R = I, T \) is a given upper bound of the time interval, in which, the system is finite time stable. \( \gamma \) is the gain from the energy of the disturbance to the peak response of the controlled output. Furthermore, the scalars \( \alpha, \beta_{i1}, \beta_{i2j} \) \((j = 1, \ldots, m)\), and \( \beta_{ij} \)
(j = 1, 2, ..., m) supply an additional degree of freedom for solving of the Theorems 1 and 2. Some optimization theories (such as neural network and GA etc.) can be used to optimize the scalars $\alpha, \beta_{1}, \beta_{2} (j = 1, 2, ..., m)$, and get the feasible results of Theorems 1 and 2.

**Remark 2.** If Theorem 1 or 2 is solvable, we can obtain the FTS and energy-to-peak performance of system (1). However, while a time-delay system includes current states (see Eq. (42)), Theorems 1 and 2 are unfit because the current states are not considered. Fortunately, we can extend Theorems 1 or 2 to Theorems 3 and 4, which can be used to solve the problem of system analysis and control of the regular time-delay system shown by equation (42).

$$
\dot{x}(t) = A_{0}(\delta_{1}) x(t) + \sum_{j=1}^{m} (A_{j}(\delta_{1})) x(t-\tau_{j}) + B(\delta_{1}) u(t)
$$

$$
u(t) = f(\delta_{1}) x(t-\tau_{0}),
$$

$$Z(t) = C(\delta_{1}) x(t).
$$

(42)

By replacing the $\sum_{j=1}^{m} (A_{j}(\delta_{1})) x(t-\tau_{j})$ in system (1) with $A_{0}(\delta_{1}) x(t) + \sum_{j=1}^{m} (A_{j}(\delta_{1})) x(t-\tau_{j})$, we can obtain Theorems 3 and 4 from Theorems 1 and 2 to stabilize the system (42).

**Theorem 3.** There are delay times $\tau_{j} > 0$ ($j = 1, 2, ..., m$) and constant $\gamma > 0$ such that the system (42) is finite-time energy-to-peak stable in regard to $(c_{1}, c_{2}, c_{3}, R, T, \gamma, d)$, if there are any matrices $P_{i} = P_{i}^{T} > 0, V_{j} = V_{j}^{T} > 0$ ($j = 1, 2, ..., m$), $Q_{i} = Q_{i}^{T} > 0$ ($j = 1, 2, ..., m$), $W_{j} = W_{j}^{T} > 0$ ($j = 1, 2, ..., m$), matrix $S_{i}$ and scalars $\eta > 0, \alpha > 0, \lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{4}, \beta_{1}, \beta_{2i} (j = 1, 2, ..., m), \beta_{3j} (j = 1, 2, ..., m)$ satisfying the following LMI (43) and LMI (8)-(13).
Theorem 3 and 4 provide the conditions for regular time delay systems to be stable. However, it is worth pointing out that if we choose $A_{\theta} = 0$ in Theorems 3 and 4, Theorems 3 and 4 will also be fit for a pure time delay system. However, those achievements obtained in references cannot be used to analyze those MJSs with PTD. Thus, compared with the existing achievements shown in references, Theorems 3 and 4 are more general. On the other hand, Theorems 1–4 are obtained by finite-time stability theory, which are more relaxed than the Lyapunov stability theory. This is also shown by the example in Part 4.

### Illustrative Example

**Example 1.** There is a two subsystems' MJS, which has

\[
\hat{z}_{11} = \sum_{j = 0}^{m} \left( \tau_j^2 V_j - 12 \tau_j W_j + A_{ij} S_i + S_i^T A_{ij}^T \right) + B_{ij} G_j + G_{ij}^T B_{ij}^T - \alpha P_i + \sum_{k = 1}^{N} \pi_{ik} P_k,
\]

\[
\hat{z}_{12} = P_i - S_i + \beta_{11} \left( G_{ij}^T B_{ij}^T + \sum_{j = 0}^{m} S_i^T A_{ij}^T \right),
\]

\[
\hat{z}_{13} = \frac{12}{\tau_0} Q_0 + \tau_0 W_0 + \beta_{12} \left( G_{ij}^T B_{ij}^T + \sum_{j = 0}^{m} S_i^T A_{ij}^T \right),
\]

\[
\hat{z}_{1(m + 3)} = \frac{12}{\tau_m} Q_m + \tau_m W_m + \beta_{12m} \left( G_{ij}^T B_{ij}^T + \sum_{j = 0}^{m} S_i^T A_{ij}^T \right),
\]

\[
\hat{z}_{1(m + 4)} = 6Q_0 - B_{ij} G_j + \beta_{130} \left( G_{ij}^T B_{ij}^T + \sum_{j = 0}^{m} S_i^T A_{ij}^T \right),
\]

\[
\hat{z}_{1(m + 5)} = 6Q_1 - A_{ij} S_i + \beta_{131} \left( G_{ij}^T B_{ij}^T + \sum_{j = 0}^{m} S_i^T A_{ij}^T \right),
\]

\[
\hat{z}_{1(m + 4)} = 6Q_m - A_{ij} S_i + \beta_{132} \left( G_{ij}^T B_{ij}^T + \sum_{j = 0}^{m} S_i^T A_{ij}^T \right).
\]

Then, a controller gain can be obtained by $F_i = G_i S_i^{-1}$.

**Remark 3.** Theorems 3 and 4 provide the conditions for regular time delay systems to be stable. However, it is worth pointing out that if we choose $A_{\theta} = 0$ in Theorems 3 and 4, Theorems 3 and 4 will also be fit for a pure time delay system. However, those achievements obtained in references cannot be used to analyze those MJSs with PTD. Thus, compared with the existing achievements shown in references, Theorems 3 and 4 are more general. On the other hand, Theorems 1–4 are obtained by finite-time stability theory, which are more relaxed than the Lyapunov stability theory. This is also shown by the example in Part 4.

**Illustrative Example**

**Example 1.** There is a two subsystems' MJS, which has

mode 1.1: $\dot{x}(t) = \begin{bmatrix} -2 & 0 \\ 0 & -0.5 \end{bmatrix} x(t) + \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix} x(t - \tau)$,

mode 1.2: $\dot{x}(t) = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} -1 & 0 \\ -0.1 & -1 \end{bmatrix} x(t - \tau)$.

This system was considered by Li et al. Reference showed that when $\gamma_{11} = 0.2$, the maximum value of $\tau$ is 0.352, the maximum values of $\tau$ in references are 0.822, 0.848, 1.002, and 1.079, respectively. However, by setting $\gamma_{11} = 0.2$, $\alpha = 1$, $\beta_{11} = \beta_{13} = 1$, and $\beta_{12} = 0.5$, we can obtain the maximum $\tau = 1.119$ by Theorem 3 in this study. Some more comparisons are given in Table 1, and the maximum values of $\tau$ obtained in this study are much higher than those obtained in references. Moreover, we can obtain a satisfactory result by tuning the value of $\alpha$. In other words, the results obtained in this study are less conservative than those in references.
Example 2. We consider the following MJS with PTD:

mode 2.1: \( \dot{x}(t) = \begin{bmatrix} 0.11 & 0.12 \\ 0.1 & 0.13 \end{bmatrix} x(t-0.1) + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} u_1(t) + \begin{bmatrix} 0.1 \end{bmatrix} \omega(t) \),

\[
\begin{align*}
&\quad + \begin{bmatrix} 0.1 & 0.12 \\ 0.1 & 0.13 \end{bmatrix} x(t-0.2) + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} u_2(t) + \begin{bmatrix} 0.1 \end{bmatrix} \omega(t).
\end{align*}
\]

mode 2.2: \( \dot{x}(t) = \begin{bmatrix} -0.31 & 0.11 \\ 0.13 & -0.35 \end{bmatrix} x(t-0.1) + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} u_2(t) + \begin{bmatrix} 0.1 \end{bmatrix} \omega(t) \).

Furthermore, \( C_1 = C_2 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix} \) and \( \tau_0 = 0.25 \). We assume the transition rate matrix

\[
II = \begin{bmatrix} -0.3 & 0.3 \\ 0.7 & -0.7 \end{bmatrix}
\]

Because the subsystems 2.1 and 2.2 both have no current state, theorems in the references \(^{31,43-45}\) cannot deal with this MJS. Assume this MJS is under zero initial condition, and a disturbance signal (see Figure 1: El Centro 1940 earthquake excitation) is introduced to this MJS. This excitation has \( \int_0^{10} \omega^T(t) \omega(t)dt = 0.075 \), thus, it can be chosen that \( d = 0.1 \). Under this excitation, the responses of \( r(t) \) and \( x(t) \) are given in Figures 2 and 3, respectively, which clearly indicate that this MJS under

\[
II = \begin{bmatrix} -0.3 & 0.3 \\ 0.7 & -0.7 \end{bmatrix}
\]

is unstable.

Then, we choose \( \beta_{11} = \beta_{21} = \beta_{12} = \beta_{22} = \beta_{220} = \beta_{221} = \beta_{222} = 1, \gamma = 0.2, c_1 = \sqrt{0.1}, \beta_{130} = \beta_{230} = \beta_{131} = \beta_{231} = \beta_{132} = \beta_{232} = 0.1, c_2 = \sqrt{0.1}, c_3 = \sqrt{0.3}, \alpha = 0.01, d = 5\times10^{-5}, R = I, \) and \( T = 10 \) s, solve Theorem 2, and obtain a finite-time stability controller that has

\[
K_1 = [-7.1061 - 7.7048], K_2 = [-4.6036 - 4.8634].
\]

(45)

Then, we consider \( E\{\|Z\|_\infty\} \leq \gamma \|\omega\|_2 \) of the MJS controlled by the controller shown in equation \( (45) \). The MJS is under zero initial condition, and excited by the disturbance signal shown in Figure 1. After doing a simulation by computer, the system state responses are given in Figure 4, and \( r(t) \) has the same values as that given in Figure 2. From Figures 1 and 4, it is obtained that \( \|\omega\|_2 = \sqrt{\int_0^{10} \omega^T(t) \omega(t)dt} = 0.2739 \) and \( \|Z\|_\infty = 0.0018 \). Then, we have \( \|Z\|_\infty/\|\omega\|_2 = 0.0018/0.2739 = 0.0064 < \gamma \), that is, \( E\{\|Z\|_\infty\} \leq \gamma \|\omega\|_2 \) is satisfied for the controlled MJS.

Conclusions

In this study, the finite-time energy-to-peak control of MJSs with PTD is discussed. First, by utilizing mathematical transformation, the MJSs with PTD are described in a system description with the current state.
and some distributed time delay items. Second, based on a suitable functional candidate that includes a triple integral item and according to the finite-time stability theory, some finite-time energy-to-peak stability criteria are gotten for the MJSSs with PTD. If these criteria are solvable, the corresponding system can be ensured to be finite-time energy-to-peak stable. Finally, some examples are given to illustrate the usefulness of the obtained methods.

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