Comment to the note ”Counting of discrete Rossby/drift wave resonant triads”, arXiv:1309.0405

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The main purpose of this note is to clarify the following misunderstanding apparent in the note arXiv:1309.0405 by M. Bustamante, U. Hayat, P. Lynch, B. Quinn; [1]: the authors erroneously assume that in the manuscript arXiv:1307.8272 by A. Kartashov and E. Kartashova, [2], resonant triads with real amplitudes are counted whereas it can be seen explicitly from the form of dynamical system that wave amplitudes are complex.

1. Counting of resonant triads.

[1] states that discrete Rossby/drift wave resonant triads counted by [2] must have complex amplitudes. We agree: indeed, the dynamical system (4) for a resonant triad in [2] is written out for complex amplitudes. [1] states also that in [3] the authors counted triads with real amplitudes and that this is the source of discrepancy between the number of triads found by [2] and by [3]. However, [3] regard stream function solutions (3) as the main subject of [3] - the first is a subset of the second. The problem is rather how to estimate what part of the complete set of resonant triads is found by the heuristic algorithm suggested by [3]?

[3] states: "we believe that our new method can be used to obtain the vast majority of the triads within the given box," (p. 2409, [3]) but neither a proof nor any estimate supporting this belief is given. Without such an estimate all results on resonance clustering, dependence of the number of solutions on the box size, etc. presented in [3] are virtually worthless.

2. Theorem of Yamada and Yoneda.

The main result of [4] (Theorem 3 on p.3) states explicitly that the influence of non-resonant terms can be made arbitrarily small through a choice of large enough box size, etc. presented in [3] are virtually worthless. [3] states: "Numerical Method to generate quasi-resonant triads within a given box, starting from exact resonant triads of any size. (...) Then the re-scaled triad \((\alpha K_1, \alpha L_1); (\alpha K_2, \alpha L_2); (\alpha K_3, \alpha L_3)\) is resonant, for any \(\alpha \in \mathbb{R}\). However this triad is not necessarily integer, so we need to approximate the scaled wavevectors to nearby integers, keeping in mind that Eqs. (5) and (6) should be satisfied" (p. 2411).

[2] states that this "algorithm looks for triads with a small frequency detuning in a vicinity of an exact resonant triad" - an adequate explication of the [3] statement. [2] shows that this algorithm is statistically biased while based on the incorrect assumption that detuning value grows monotonously as the box size declines. An example is given, more can be computed by the reader using our on-line program [5]. Moreover, the best quasi-resonance found by this method has detuning about \(2 \cdot 10^{-5}\) - over six orders decimal magnitude worse than the really best (6.8 \(\cdot 10^{-12}\)) and does not by far enter our list of best quasi-resonances (over 3000, cut off at 1.0 \(\cdot 10^{-8}\)).

3. Quasi-resonances

[2] compares a few different methods for finding quasi-resonances, namely: a) the method suggested in [3]; b) search in the neighborhood of the resonant manifold; c) full search; d) random search.

[3] states: "Numerical Method to generate quasi-resonant triads within a given box, starting from exact resonant triads of any size. (...) Then the re-scaled triad \((\alpha K_1, \alpha L_1); (\alpha K_2, \alpha L_2); (\alpha K_3, \alpha L_3)\) is resonant, for any \(\alpha \in \mathbb{R}\). However this triad is not necessarily integer, so we need to approximate the scaled wavevectors to nearby integers, keeping in mind that Eqs. (5) and (6) should be satisfied" (p. 2411).

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4. Conclusions.

What is called "Major error in [2]" by [1] is a simple matter of misunderstanding: [1] did not notice that the dynamical system (4) in [2] is written in complex variables.

5. References.

[1] M. Bustamante, U. Hayat, P. Lynch, B. Quinn. arXiv:1309.0405
[2] A. Kartashov, E. Kartashova. arXiv:1307.8272
[3] M. Bustamante, U. Hayat. arXiv:1210.2036
[4] M. Yamada, T. Yoneda. Physica D 245 (2013): 1-7.
[5] A. Kartashov. http : //www.dynamics – approx.jku.at/portal/?q = node/144 (30 July, 2013)