The Price of Neutrino Superluminality continues to rise

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We revisit the model building challenges that one faces when trying to reconcile the OPERA claim of neutrino superluminality with other observational constraints. The severity of the supernova bound and of the kinematical constraints of Cohen-Glashow type lead us to focus on scenarios where all types of particles are superluminal inside matter. In contrast to the Dvali-Vikman proposal, this matter effect needs to be very short-ranged to avoid constraints from experiments on the Earth’s surface in low-density environments. Due to this short range, the interaction underlying such a matter effect would have to be far stronger than permitted by fifth-force bounds. As a conceivable way out we suggest to make the matter effect “binary”, i.e., dense matter does not directly trigger superluminality, but merely induces the transition to a different phase of some weakly coupled hidden sector. This phase exhibits spontaneous Lorentz violation or at least a stronger than usual mediation of some residual Lorentz violation to all matter. The effect has not been observed before since we have never before been able to measure the velocity of high-energy particles in dense matter with sufficient precision.

I. INTRODUCTION

Earlier this year, the OPERA collaboration made the surprising claim [1] of an early arrival time of CNGS muon antineutrinos traversing 730 kilometers of rock on their way from CERN to Gran Sasso. This corresponds to a neutrino superluminality of

$$\Delta \equiv v^2_{\nu} - 1 \sim 5 \times 10^{-5}$$

(1)

at 6σ significance. The CNGS neutrinos have an average energy of 17 GeV with a broad distribution reaching up to several tens of GeV, and a separate measurement of neutrinos above and below 20 GeV has revealed no significant energy-dependence of the superluminality in this energy range. This result was preceded by a weaker claim from MINOS [2], and was recently confirmed in a followup investigation by OPERA using short bunches. For the purpose of this note, we take the experimental result at face value and confront the model building challenges which it entails. In other words, we ignore any possible experimental problems and focus on the intellectual challenge of devising at least an effective field-theoretic framework in which the effect can be consistently described. Note that the corresponding effect in standard general relativity which arises due to the presence of the earth, is smaller than the observed effect by a factor of 10^{-5} and thus negligible [3,5].

Since the publication of [1], a large number of preprints has appeared on the arXiv addressing or attacking aspects of the claim or proposing models. The latter can roughly be categorized into models of explicit Lorentz violation [6], geometric solutions in extra dimensions [7], spontaneous Lorentz breaking [8], deformed special relativity [9], environmental superluminality [3,10,11], and combinations of these ideas.

The first obvious phenomenological challenge is the observation of neutrinos from the supernova SN1987A a few hours before the optical confirmation [6]. The naive assumption of a constant and energy-independent superluminality \Delta would have had these neutrinos reach Earth years before the photons. There are several ways out of this problem:

- The superluminality is sufficiently energy dependent such that the low-energy neutrinos from SN1987A see very little of it [12,16].
- It is flavor dependent and electron neutrinos are not superluminal, while muon neutrinos are. This appears to seriously interfere neutrino oscillations [17].
• The velocity of neutrinos is location dependent, e.g., a function of the matter density. This way, neutrinos are barely superluminal in interstellar space, whereas the CNGS neutrinos travelling close to or inside Earth, are \[3, 10, 11\].

The to date most severe direct constraints come from the modified kinematics of decays involving superluminal particles, in particular the argument by Cohen and Glashow (CG) that superluminal neutrinos can radiate non-superluminal particles, such as \(\nu \rightarrow e^+ e^- \bar{\nu}\), and thus quickly lose energy \[15, 20\]. The energy threshold in terms of the superluminality is given by

\[E_\nu > \frac{\sum m_i}{\sqrt{\Delta}}, \quad (2)\]

where \(m_i\) are the masses of the final state particles. Likewise, high-energy mesons would not have any phase space left to decay when neutrinos are in the final state, thus eliminating both the production of atmospheric neutrinos \[21, 22\] and of the CNGS neutrino beam itself. One can try to evade this problem as follows:

• Lorentz symmetry is not broken but deformed, alternative momentum and energy conservation relations hold, and the CG effect is avoided \[9, 23\]. Our only excuse for not following this line of thinking at the moment is our insufficient understanding of the underlying field-theoretic framework.

• Superluminality is achieved through a nontrivial dispersion relation which manages to suppress the CG effect, see for example \[24\]. However, we need to suppress the CG effect also for energies \(E \gg 17\) GeV, e.g., to allow for high-energy neutrinos traversing the Earth, as they are observed in “upward” events by IceCube \[25\] (see also \[19\]). This appears to be difficult.

• All particle species are equally superluminal, which amounts to a rescaling of the energy and leads to an effectively Lorentz-invariant kinematics with this rescaled energy. Note that (2) implies that superluminal neutrinos above \(\sim 40\) GeV have enough energy to produce pion pairs via neutral currents, \(\nu \rightarrow \nu \pi \pi\), making mere electron superlumlinality insufficient to avoid the CG effect.

In the following we retain standard energy-momentum conservation in field theory. Thus, from the considerations above, we are compelled to consider the following picture: Neutrinos are superluminal only when close to dense matter. The effect originates from a modified dispersion relation which they share with all other particle species. The severe constraints, e.g., on electron superluminality in synchrotrons \[26, 27\], demand that this effect has a very short range, i.e., that superluminality goes away millimeters or less outside of solids.

II. MATTER-DEPENDENT SUPERLUMINALITY

An elegant way to produce neutrino superluminality close to the Earth in a completely Lorentz-invariant setting was proposed by Dvali and Vikman \[10\]. DV exploit the effect that the mere presence of the Earth constitutes an effective violation of Lorentz invariance, which is then communicated to the neutrinos via a tensor fifth force. The inverse mass of this tensor \(h_{\mu \nu}\) is dialed to a value between the radius of the Earth and the size of the solar system such that it is effectively massless on Earth. The proposed couplings of the tensor to other fields are via the energy-momentum tensor, but are nonuniversal between neutrinos and other particles:

\[L \supset \frac{h_{\mu \nu}}{M_*} \bar{\nu} \gamma^\mu \gamma^\nu \nu + \frac{h_{\mu \nu}}{M} T^\mu_{\bar{\psi} \psi} . \quad (3)\]

The net superluminality on the Earth’s surface is given by the dispersion relation \(p_\nu c^2 = \vec{p}^2 \Delta\), where \[10\]

\[v_\nu - c = \Delta / 2 = -\frac{M_E}{4\pi M_* M R_E} . \quad (4)\]

Clearly, we need \(M_* M < 0\) in order to produce superluminality. This model, as it stands, avoids the constraints from SN1987A, but not the CG effect. Also, it requires considerable fine tuning of counterterms in order to cancel the 1-loop contributions e.g., to electron superluminality \[28\]. One could consider making the tensor heavy at the inverse micron scale in order to allow universal superluminality (which relaxes the fine tuning and avoids CG at the same time), but apart from requiring a rather low scale \(M_* M\), the relative sign between the couplings seems to make this impossible - there is no obvious way to modify hadron dispersion relations in the direction of superluminality while hadrons at the same time provide the source of the tensor field. One way out might be to couple to \(B - L\) or \(B\), but
we are not aware of a viable realization of this idea using tensors. This problem seems to indicate that we need a further ingredient in order to be compatible with SN1987A and the CG effect. We aim to disentangle the source of Lorentz violation and the origin of matter dependence. Somewhat related ideas have appeared, partially in the form of comments, in [29, 30].

III. EFFECTIVE MODELS

To make our argument as general as possible, we introduce a spurion-like Lorentz-violating dimensionless symmetric tensor which we choose diagonal in the Earth frame for simplicity, $\theta_{\mu\nu} = \text{diag}(\alpha, \beta, \beta)$. This tensor could be sourced by the Earth, could represent a cosmological background, or could be of yet another unknown origin. The scalar field $\phi$ is sourced by matter and thus produces the matter dependence. For the sake of concreteness, we consider the simple effective Lagrangian

$$\mathcal{L} \supset \frac{\phi}{M} \theta_{\mu\nu} T^{\mu\nu} + \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{m^2}{2} \phi^2.$$  \hspace{1cm} (5)

where $T^{\mu\nu}$ is now the complete energy-momentum tensor. In the spirit of [10], $\langle \phi \rangle \theta_{\mu\nu} / M$ can be viewed as a perturbation away from Minkowski of the effective metric in which particles inside matter propagate, thus modifying their dispersion relations. At the same time, this term sources the field $\phi$. We choose $m^{-1} < 10^{-3}$ meters, which implies that inside Earth (more than a distance of $m^{-1}$ away from the surface), we have

$$\langle \phi \rangle_E = \frac{\theta_{\mu\nu} (T^{\mu\nu})_E}{M m^2} = \frac{\alpha \rho E}{M m^2}.$$ \hspace{1cm} (6)

We can now reinsert this into $\mathcal{L}$ in order to obtain an effective Lagrangian for high energy particles inside matter,

$$\mathcal{L}_{\text{eff}} \supset \epsilon \left( \alpha^2 T^{00} - \alpha \beta \eta_{ij} T^{ij} \right), \quad \epsilon \equiv \frac{\rho E}{M^2 m^2}.$$ \hspace{1cm} (7)

There are several ways to derive $\Delta$ from (7). One can treat it as a contribution to the effective metric, or one can use it as an operator insertion into the decay processes. For neutrinos for example, the corresponding Feynman rule reads $i \epsilon (\alpha^2 \bar{\nu} p^0 - \alpha \beta \gamma^i p)$. When Dyson-resummed for the external legs, this yields the modified, superluminal propagator. Both yield the same result, namely (for a massless particle)

$$(1 + \alpha^2) p_0^2 - (1 - \epsilon \alpha \beta) p^2 = 0$$ \hspace{1cm} (8)

and thus $\Delta \approx -\epsilon (\alpha \beta + \alpha^2)$. What we have gained is the freedom to choose $\theta$ such that $\alpha \beta + \alpha^2 = -1$, and thus $\Delta = \epsilon$. Note that this was impossible in the DV model with universal couplings since there, the corresponding expression would be a square and hence positive. We now get a relation between the range $m^{-1}$ and the coupling scale $M^{-1}$,

$$M = \sqrt{\frac{\rho E}{\Delta}} m^{-1} \sim 3 \cdot 10^9 \text{ GeV} \times m^{-1} / \text{meter}. \hspace{1cm} (9)$$

Thus, we obtain a Yukawa-type short-range modification of gravity with a suppression scale $M \sim 3 \times 10^3$ GeV for $m^{-1} \sim \mu m$, in gross conflict with precision experiments such as those by Lamoreaux et al. (11), fig. 28). Extreme choices which one might consider in order to escape this are the nanometer range, but then GeV suppressed interactions of a light scalar would have shown up in particle physics, or in the centimeter range, where however the suppression of the coupling is not strong enough to evade Eötvös-type experiments. This model makes some assumptions, but the general problem of scales will be the same for any source of Lorentz violation, and will not be radically different for DV type models involving vector bosons rather than a tensor. Modifications of the scalar model by $D > 5$ operators do not seem promising. There is the possibility to add a term of the form $\phi^2 T$ in order to reduce the effective mass of the scalar inside matter, but this must be finely tuned, and it would lead to a runaway potential in more dense matter. As yet another option, one might choose $|\alpha| \ll 1$ and $|\beta| \gg 1$, but the required $\beta$-values appear to be too extreme.

IV. TWO PHASE MODELS

We want to retain the idea of a matter effect, but gain more freedom concerning the coupling strength of the underlying scalar field. This might be doable in a two-phase model, where the matter and the vacuum phase have
roughly independent Lagrangian parameters. As a very simple attempt at such a model using renormalizable couplings, consider the scalar potential

$$V(\phi) = \frac{\lambda}{4} \left[ \left( \phi - \frac{\mu}{\sqrt{\lambda}} \right)^2 - \frac{\mu^2}{\lambda} \right]^2 + \phi^2 \mu^2 \delta$$

which, for $0 < \delta \ll 1$, is simply a deformed double well with the global minimum $V(0) = 0$ and a local one $V(\Delta\phi) = 4\delta\mu^4/\lambda$ at $\Delta\phi \equiv 2\mu/\sqrt{\lambda}$. We assume that the presence of matter density tilts this potential, making the “superluminal” minimum at $\Delta\phi$ the global one. In order to achieve this, we postulate the Lagrangian

$$\mathcal{L} \supset -V(\phi) - \frac{\phi}{\Lambda_{LV}} \theta_{\mu\nu} T^{\mu\nu} + \frac{\phi}{\Lambda_{LI}} T_{\mu}^{\mu}$$

where we have a Lorentz invariant coupling producing the deformation of the potential, and the Lorentz violating one inducing the superluminality.

Let $b$ denote the thickness of a domain wall separating the two phases. We call the height of the corresponding potential barrier $V$. The potential energy density contribution to the surface tension of the wall is approximately $\sigma \sim V^2 b$, since the scalar field has to pass over the maximum inside the domain wall. Likewise, the kinetic energy contribution is approximately $\sigma \sim b(\Delta\phi/b)^2$. If we assume the optimal solution $V b \sim \Delta \phi^2/b$, we obtain an expression for the wall thickness, $b \sim \Delta\phi/\sqrt{V}$. Two competing effects, the difference in energy density and the surface tension, give us a critical bubble size inside or outside of matter via $dE = 4\pi \Delta V R^2 dR - 8\pi \sigma RdR$. In our order of magnitude estimate, the critical bubble size inside(outside) of matter is thus $R_c \sim \sigma/\Delta V^{(i)} \approx bV/\Delta V^{(i)} \sim \Delta\phi\sqrt{V}/\Delta V^{(0)}$. Here, $\Delta V$ and $\Delta V'$ denote the potential energy differences between the two local minima inside and outside of matter respectively.

We can now make order of magnitude estimates for the parameters. The wall thickness $b$ in this model is simply given by $b \sim \Delta\phi/\sqrt{V} \sim \mu^{-1}$, and the height of the potential well by $V \sim V(\Delta\phi/2) \sim \mu^4/\lambda$, and thus the surface tension is $\sigma \sim \mu^4/\lambda$. We demand that the range of the fifth force, and thus the wall thickness, are small enough to evade the experimental constraints from short range measurements [31], and choose $\mu \sim (10^{-10} \text{ meter})^{-1} \sim 10^{-5} \text{ GeV}$. We can see from (10) and the required amount of superluminality, that $\Delta\phi/\Lambda_{LV} \sim 5 \cdot 10^{-5}$ and thus $\Lambda_{LV} \sim 1 \text{ GeV}/\sqrt{\lambda}$, telling us that $\lambda < 10^{-6}$ to evade collider bounds. The surface tension is thus $\sigma > 10^6 \mu^4 \sim 10^{20} \text{ GeV/meter}^2$, which is a very large surface energy density even in macroscopic terms. While we have not checked all variants of this model, we suspect that the framework described above is too restrictive.

Hence, if we want to keep this idea of a phase transition, we need to allow for more general potentials as well as more general couplings to matter. Instead of working with a simple renormalizable potential as before, we postulate a general potential with two local minima which is tilted by the presence of energy density such that the superluminal vacuum becomes the true one. We study this scenario with a Lagrangian of the form

$$\mathcal{L} \supset -V(\phi) - \frac{\Delta V'}{\Delta\phi} \phi - f(\phi) T^{\mu}_{\mu} - g(\phi) \theta_{\mu\nu} T^{\mu\nu}$$

in which we have introduced a Lorentz invariant coupling and a Lorentz violating one with coefficient functions $f$ and $g$ which we choose in order to make the model phenomenologically viable. We assume a generic potential with $\Delta\phi$ the size of the Vev given by the distance of the two local minima and $V$ the approximate potential barrier. For
simplicity, we assume \( V(0) = V(\Delta \phi) = 0 \), while \( \Delta V \) and \( \Delta V' \), the energy differences between the standard vacuum and the superluminal vacuum, arise from the choice of \( f \). This is illustrated in Figure[1].

We now want to exploit our freedom of choosing \( g \) and \( f \) in order to make this scenario compatible with OPERA and fifth force constraints. The perturbative couplings of \( \phi \) to the Standard Model in the vacuum and in matter are given by \( \partial^4 f/\partial \phi^n(0) \), \( \partial^4 g/\partial \phi^n(0) \) and \( \partial^4 f/\partial \phi^n(\Delta \phi) \), \( \partial^4 g/\partial \phi^n(\Delta \phi)^2 \) respectively. We thus want to set these to zero, e.g. by having \( f \) and \( g \) locally constant or of the type \( e^{-1/(x-x_0)^2} \) at \( x_0 = 0 \) and \( x_0 = \Delta /\Lambda \), with a smooth transition between the two values. It is not obvious whether or how such a model with a non-renormalizable non-analytic coupling can be UV completed in the hidden sector, and we postpone this discussion. For simplicity we choose \( \theta_90 = 0 \) in the Earth frame such that \( g \) does not modify the potential while still inducing superluminality. In order to evade superluminality constraints outside matter such as the supernova bound while reproducing the OPERA measurement, we need \( g(0) = 0, g(\Delta \phi) \sim 10^{-5} \). The function \( f \), which sources the deformation of the potential, must have a suitable value \( f(\Delta \phi) < 0 \) to produce the necessary shift \( \Delta V \sim \rho f(\Delta \phi) \) which makes \( \phi = \Delta \phi \) the true vacuum inside matter. Further constraints on the function \( f \) can be derived from the desire to have a negligible contribution of the condensate to the mass of macroscopic objects, i.e. the energy density of the true vacua inside and outside of matter should satisfy \( |\Delta V' + \rho f(\Delta \phi)| \ll \rho \). The critical bubble of the Lorentz conserving vacuum outside of matter should be smaller than e.g. the beam pipe of synchrotrons, which are very sensitive to electron superluminality, while the upper bound on the size of critical bubbles inside the Earth might be less strong. We furthermore require a thickness of the domain wall \( b \gtrsim 10^{-8} \) meters, in order to have a homogeneous phase inside matter.

V. CONCLUSIONS

We have attempted to account for neutrino superluminality, as reported by OPERA, while staying within the familiar framework of low-energy effective field theory. Even if one is prepared to allow for explicit or spontaneous Lorentz violation, this turns out to be surprisingly challenging: On the one hand, the supernova bound strongly suggests that we are dealing with a matter effect. On the other hand, the kinematical constraints of Cohen-Glashow type force one to extend superluminality to all types of particles. However, superluminality of various standard model particles or a varying velocity of photons appears to be completely excluded by a multitude of laboratory experiments. An obvious way out is to make superluminality a universal, matter-induced effect, but with a very short (sub-mm) range. One now faces the problem that only a small volume of rock around each point of the neutrino trajectory contributes to the effect. To compensate for this smallness, the short-ranged mediator field has to couple rather strongly. This tends to be in conflict with fifth-force bounds.

Given this situation, one is naturally led to consider a non-trivial phase structure of some hidden sector: This hidden sector comes in a ‘matter phase’ associated with superluminality and a vacuum phase where no such effect is present. Two couplings of the hidden sector to our world are mandatory: A coupling to energy density, which stabilises the superluminal phase inside matter, and a coupling to all Standard Model kinetic terms, which induces superluminality whenever the hidden sector is in that phase. The crucial point is that, in contrast to the fifth-force-case above, these couplings do not need to be linear in any dynamical scalar field. The constraints are now associated with very different parameters: the energy density difference between the phases, the domain-wall thickness, and its tension. As a further (derived) quantity, the size of critical bubbles relevant in the transition between the phases may be important in certain situations. As discussed in more detail in the main text, there appears to be enough freedom to satisfy at least some of the most obvious constraints. A more explicit construction and a more detailed analysis of this suggested two-phase model is clearly necessary.

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