Helicity conservation in inclusive nonleptonic decay $B \to VX$: Test of long-distance final-state interaction

Mahiko Suzuki
Department of Physics and Lawrence Berkeley National Laboratory
University of California, Berkeley, California 94720
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Abstract

The polarization measurement in the inclusive $B$ decay provides us with a simple test of how much the long-distance final-state interaction takes place as the energy of the observed meson varies in the final state. We give the expectation of perturbative QCD for the energy dependence of the helicity fractions in a semiquantitative form. Experiment will tell us for which decay processes the perturbative QCD calculation should be applicable.

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I. INTRODUCTION

It is of crucial importance to know how much long-distance final-state interaction (LDFSI) occurs in $B$ decay. If LDFSI plays a significant role, we have no first-principle method to compute decay amplitudes. Arguments have been presented in favor of short-distance (SD) dominance for the two-body decay in which a fast quark-antiquark pair moves almost collinearly in a colorless lump. Based on this color screening picture [1], the perturbative QCD computation has been carried out for the two-body $B$ decay [2,3]. Even if the SD dominance argument is valid in the infinite $B$ mass limit, a quantitative question exists about the accuracy of the perturbative QCD calculation since the $B$ meson mass is only 5.3 GeV in the real world. When the final mesons are highly excited states, the velocities of the mesons are less fast and the quarks inside them have larger transverse momenta. We expect that the SD dominance is accordingly less accurate in such decays. In the large limit of the excited meson mass, the LDFSI should play a major role in determining the final state. We would like to verify experimentally the SD dominance in the two-body decay and see how the SD dominance disappears as the meson slows down in the inclusive decay.

One of the cleanest ways to test breakdown of the SD dominance or presence of LDFSI directly with experiment is to measure the helicity of a fast flying meson in the final state [4]. Since SD interactions do no flip helicities of light quarks $(u,d,s)$, a fast light meson carries a memory of the quark helicities if no LDFSI enters. Because of the specific form of the weak interaction in the Standard Model, a fast light meson with spin must be polarized in the zero helicity state up to $O(1/M_B^2)$ in probability, when other hadrons fly away approximately to the opposite directions. One can determine the $h = 0$ fraction of the meson by measuring the angular distribution of the its decay products. In fact, this selection rule is so robust that it would be valid even if the right-handed $W$-boson contributes to weak decays. It breaks down most likely by LDFSI, if at all.

Imagine that such polarization measurement is made for the inclusive decay $B \to \rho X$ in which $X$ is a highly excited meson state ($\overline{q}q$) or a multi-quark hadronic state. As the invariant mass $m_X$ increases, it becomes more likely that LDFSI takes place between $\rho$ and $X$. If so, we shall start seeing production of the $\rho$ meson in the $h = \pm 1$ states. By measuring the $\rho$ helicity as a function of $m_X^2$ or equivalently as a function of the $\rho$ energy in the $B$ rest frame, we can determine from experiment how much LDFSI enters the decay as $\rho$ slows down or how much the color screening breaks down.

For the two-body decay, the polarization measurement is possible only when both final mesons have nonzero spins, for instance, $B \to 1^-1^-$. Meanwhile, most decay modes that are easily identifiable and high in branching fraction are $B \to 0^-0^-$ and $1^-0^-$. Nonetheless, the polarization test will have a direct impact on these dominant decay modes of the $B$ meson in the following way. In the charmless $B$ decay, the two-body decays $B \to \pi\pi$ and $\rho\pi$ are among the decay modes of primary interest from the viewpoint of CP violation. If our proposed test reveals that the $h = 0$ state dominates in $B \to \rho\rho$, $\rho\omega$, and so forth, we shall feel more confident with computing the tree and penguin amplitudes of $B \to \pi\pi$, $\rho\pi$ in perturbative QCD. If on the contrary the $h = 0$ dominance is substantially violated in $B \to \rho\rho$, $\rho\omega$, we should not trust the perturbative method of calculation for $B \to \pi\pi$, $\rho\pi$. In this case the only recourse would be to determine the $B \to \pi\pi$ amplitudes by experiment alone [5] without help of theoretical computation. And little could be done for $B \to \rho\pi$. 


with isospin invariance alone. The test proposed here is not for inventing a new method of calculation of decay amplitudes, but for learning from experiment for which decay modes we may perform the perturbative QCD calculation.

II. KINEMATICS OF $B \rightarrow VX$

We consider the inclusive $B$ decay into a vector meson $V$ of $J^P = 1^-$;

$$B(P) \rightarrow V(q,h) + X(p_X),$$

$$\sum a(k_1) + b(k_2) + X(p_X),$$

(1)

where $a$ and $b$ are spinless decay products of $V$ ($m_a \neq m_b$ in general). Here we have $B \rightarrow \rho X$, $K^* X$, and $\phi X$ in mind. The inclusive decay rate is written in the covariant form as

$$\frac{d\Gamma}{d^3k_1d^3k_2} = \sum_{ij} \int \frac{d^3q}{4(2\pi)^3g_0P_02m_V\Gamma_V} \frac{g_{ab}^2}{(2\pi)^4\delta^4(k_1 + k_2 - q)(\epsilon_i \cdot k_1 - k_2)(\epsilon_j^* \cdot k_1 - k_2)e_i^{\mu}\epsilon_j^{\nu}T_{\mu\nu}C_{\mu\nu}},$$

(2)

where $\Gamma_V$ is the decay width of $V$, $g_{ab}$ is the decay coupling constant of $V$ defined by $L_{int} = ig_{ab}(\phi_a^* \phi_b)\pi^\mu$ and the subscript of the polarization vector $\epsilon$ refers to three helicity states of $V$. The covariant tensor $T_{\mu\nu}$ is the inclusive structure function defined by

$$T_{\mu\nu}(m_X^2) = 4q_0P_0 \sum_X (2\pi)^4\delta^4(q + p_X - P)|B(P)|H_{int}|V(q,j)X\rangle\langle V(q,i)X|H_{int}|B(P)\rangle,$$

(3)

where the states are normalized as $\langle p|p'\rangle = (2\pi)^3\delta(p - p')$ without $2E_p$. The general tensor form of $T_{\mu\nu}$ is

$$T_{\mu\nu} = -g_{\mu\nu}A(m_X^2) + \frac{1}{M_B^2}P\mu P\nu B(m_X^2) + \frac{i}{M_Bm_V}\varepsilon_{\mu\nu\kappa\lambda}P^{\kappa}q^{\lambda}C(m_X^2),$$

(4)

where $m_X^2 = (P - q)^2$ and the antisymmetric unit tensor is defined as $\varepsilon_{0123} = -1$. The scalar structure functions $A \sim C$ are the absorptive parts of the analytic functions of variable $m_X^2$ that are regular except on the segments of the real axis in the complex $m_X^2$ plane if $V$ is treated as (approximately) stable. In particular, $A \sim C$ are nonsingular ($\neq \infty$) in the physical region of the decay.

The helicity amplitudes $H_h$ for $B \rightarrow V_h X$ in the $B$ rest frame can be expressed in terms of $A \sim C$ as

$$H_0 = A + \frac{q^2}{m_V}B,$$

$$H_{\pm 1} = A \mp \frac{|q|}{m_V}C.$$

(5)

In contracting $T_{\mu\nu}$ with $\epsilon$, we must not make the approximation $\epsilon^\mu(q) \simeq q^\mu/m_V$ as we often do in the exclusive two-body decay $B \rightarrow V_1V_2$ where $g_{\mu\nu}C_{\mu\nu} \simeq (q_1 \cdot q_2)/m^2$. Because $g_{\mu\nu}C_{\mu\nu} = -1$ while $g_{\mu\nu}q^{\mu}q^{\nu}/m_V^2 = +1$ in the inclusive decay kinematics.
Carrying out the summation over the helicities in Eq. (3) with Eq. (4), we obtain the differential decay rate with respect to the direction of \( \mathbf{k}_1 \) and the energy of \( V \). The result is:

\[
\frac{d\Gamma(B \to VX \to abX)}{dq_0d\cos \theta} \bigg|_{\text{at rest}} = \frac{g_{ab}^2|q||\mathbf{k}_{cm}|^3}{32\pi^3m_V^2\Gamma_V} \left[ A(m_X^2) + \frac{\mathbf{P}^2}{M_B^2}B(m_X^2)\cos^2 \theta \right],
\]

where \( q_0 \) is the energy of \( V \) in the rest frame of \( B \), which is related to \( m_X \) by \( m_X^2 = M_B^2 + m_{V_2}^2 - 2M_Bq_0 \) so that \( d\Gamma/dq_0 = 2M_B\Gamma/dm_X^2 \). \( \mathbf{k}_{cm} \) is the momentum of \( a \) in the rest frame of \( V \), \( \mathbf{P} \) is the momentum of \( B \) measured in the rest frame of \( V \), and \( \theta \) is the angle of \( \mathbf{k}_{cm} \) measured from the direction of \( \mathbf{P} \), namely, \( (\mathbf{P} \cdot \mathbf{k}_{cm}) = |\mathbf{P}||\mathbf{k}_{cm}|\cos \theta \).

We make two remarks on Eq. (6). Since the decay products \( a \) and \( b \) are spinless, the structure function of the \( V \rightarrow ab \) decay, \( g_{ab}^2(k_1 - k_2)^\mu(k_1 - k_2)^\nu \), is symmetric under \( \mu \leftrightarrow \nu \) so that the function \( C(m_X^2) \) does not enter the differential decay rate. It means according to Eq. (3) that we cannot separate the \( h = -1 \) decay from the \( h = +1 \) decay in this process. In order to distinguish between \( h = \pm 1 \), we would have to choose a decay in which \( J \neq 0 \) for \( a \) or \( b \) and to measure the helicity of \( a \) or \( b \) through its decay. For instance, the triple product \( q \cdot (k_1 \times k'_2) \) in the sequence of decays \( B \to a_2(q)X \to \pi(k_1)\rho(k_2)X \to \pi(k_1)\pi(k_1)X \) contains such an information.\(^1\) The other comment is on the slow limit of \( V \). In the limit of \( q \to 0 \) in Eq. (3), distinction among three different helicity states of \( V \) disappears for an obvious reason and all helicity functions \( H_h (h = 1, 0, -1) \) are given by \( A(m_X^2) \) since \( B(m_X^2) \) and \( C(m_X^2) \) stay finite there:

\[
H_1 + H_{-1} \to 2H_0, \quad H_1 - H_{-1} \to 0 \quad \text{as} \quad q \to 0.
\]

In this limit only the \( A(m_X^2) \) function survives in the differential decay rate of Eq. (6), as we expect, since \( q \to 0 \) means \( \mathbf{P} \to 0 \).

Finally, let us express the differential decay rate in terms of \( H_h \), noting that \( |\mathbf{P}|/M_B = |\mathbf{q}|/m_V \) by the transformation between the \( B \) rest frame and the \( V \) rest frame. The result is

\[
\frac{d\Gamma(B \to VX \to abX)}{dq_0d\cos \theta} \bigg|_{\text{at rest}} = \frac{g_{ab}^2|q||\mathbf{k}_{cm}|^3}{32\pi^3m_V^2\Gamma_V} \left[ H_0 \cos^2 \theta + \frac{1}{2}(H_1 + H_{-1})\sin^2 \theta \right].
\]

We are able to separate between the longitudinal \( (h = 0) \) and transverse \( (h = \pm 1) \) polarization decay with the angular distribution of Eq. (8). Experiment will show us how the \( h = 0 \) dominance goes away as \( m_X \) increases in the inclusive decay \( B \to VX \). If the transverse polarization appears beyond the corrections to be discussed in the subsequent sections, it will be a clear evidence for LDFSII.

III. LONGITUDINAL POLARIZATION DOMINANCE

For the weak interaction of the Standard Model, the zero-helicity function \( H_0 \) should dominate over all other \( H_h \) for small \( m_X \), if the strong interaction corrections are entirely

\(^1\) Such measurement was actually proposed to determine the photon helicity in \( B \to \gamma K_1 \to \gamma K\pi \)\(^2\). The strong phases due to the overlapping resonances are needed to detect the triple product.

\(^2\)
of short distances except at hadron formation. We explain this rule for two-body decays \([\bar{q}q]\), discuss the mass and orbital motion corrections to the rule, and extend it to the inclusive decay \(B \rightarrow VX\). Our argument is based on the standard assumptions made in the perturbative calculation including the light-cone formulation of mesons in \(\bar{q}q\). The helicity selection rule should break down for sufficiently large values of \(m_X\). At which value the rule starts showing a significant departure from the \(h = 0\) dominance will provide us with a quantitative measure of accuracy of the perturbative QCD calculation. We first discuss the charmless decay and then move on to the decays with charm.

A. Meson helicity and helicities of \(\bar{q}q\)

In the nonleptonic \(B\) decay a pair of \(\bar{q}q\) is produced by weak interaction nearly in parallel to form an energetic meson. In the case of a vector meson \((^3S_1)\), we may approximate the \(\bar{q}q\) pair to be literally in parallel by ignoring a tiny \(^3D_1\) component. For excited mesons such as \(J^P = 2^+ (^3P_2)\), the transverse motion of \(q\) and \(\bar{q}\) must be taken into account. It gives rise to an orbital angular momentum \(l\) between \(q\) and \(\bar{q}\) as well as to the meson mass. This angular momentum is part of the meson spin. By simple kinematics, however, the state of \(l_z = 0\) dominates over all others when a meson moves fast. That is, to the lowest order we may leave out the orbital motion of \(\bar{q}q\) inside a meson even for an excited meson state with \(l = 0\). Let us make this statement quantitative.

In the classical picture, the orbital angular momentum vector is squashed to the plane perpendicular to the meson momentum when a meson moves fast. To see it in quantum theory, let us expand the plane wave \(e^{ip \cdot r}\) of a quark in the spherical harmonics for \(p\) off the direction of the meson momentum \(q = |q|\hat{z}\). Defining the directions of the vectors as

\[
\begin{align*}
\mathbf{r} &= r(\sin \vartheta \cos \phi, \sin \vartheta \sin \phi, \cos \vartheta), \\
\mathbf{p} &= |\mathbf{p}|(\sin \vartheta' \cos \varphi', \sin \vartheta' \sin \varphi', \cos \vartheta'), \\
\hat{r} \cdot \hat{p} &= \cos \gamma.
\end{align*}
\]

We obtain by use of the well-known formulae the expansion of the plane wave in the form

\[
e^{i\mathbf{p} \cdot \mathbf{r}} = \sum_l (2l + 1)^{\frac{3}{2}} J_l(|\mathbf{p}|r) P_l(\cos \gamma),
\]

\[
= 4\pi \sum_l l^l j_l(|\mathbf{p}|r) \sum_{m=-l}^l Y^*_l m(\vartheta', \varphi') Y_{lm}(\vartheta, \varphi).
\]

(9)

Treating \(\vartheta' \simeq |\mathbf{p}_T|/|\mathbf{p}|\) as small, we expand \(Y^*_l m(\vartheta', \varphi')\) around \(\vartheta' = 0\). Then Eq.(10) turns into

\[
e^{i\mathbf{p} \cdot \mathbf{r}} \simeq \sum_l \sqrt{4\pi(2l + 1)^{\frac{3}{2}}} j_l(|\mathbf{p}|r) \sum_{m=-l}^l \frac{(-1)^{m+|m|}}{2^{|m|}|m|!} \frac{(l + |m|)!}{(l - |m|)!} e^{-im\varphi'} e^{im\varphi} Y_{lm}(\vartheta, \varphi).
\]

(11)

In the sum over \(l_z\) (denoted by \(m\) above), the amplitudes of \(l_z \neq 0\) are suppressed by \(\vartheta' l_z \simeq \left(\frac{m_T}{E}\right)l_z\) where \(m_T\) stands for the transverse meson mass \(\simeq \sqrt{3} \times \) meson mass. Repeat the argument for \(\bar{q}\). Projecting the \(\bar{q}q\) state with the quark distribution function of meson, we find that the meson helicity consists entirely of the quark helicity \(h_q + h_{\bar{q}}\) in the fast limit. The contribution of the \(l_z \neq 0\) states generates a correction of \(O((1/2m_T/E)^{|l_z|})\) in amplitude for an excited meson and a multi-meson state.
B. Helicity selection rule; charmless decay

The fundamental weak interaction is dressed or improved into the effective decay operators by the renormalization group down to the scale $m_b$. In the Standard Model, the chiral structure of the decay operators relevant to the charmless decay are $(\bar{b}_L q_L)(\bar{q}_L q_L) + h.c.$ and $(\bar{b}_L q_L)(\bar{q}_R q_R) + h.c.$, where $q$ stands for a light quark. The short-distance interaction below the scale $m_b$ does not generate any new chiral structure. It can add $q_L q_L + q_R q_R$ through quark pair emission by a hard gluon. The chirality of the spectator quark is indefinite so that it can be either in helicity $+\frac{1}{2}$ or $-\frac{1}{2}$ when it forms a meson.

Let us start with the two-body charmless decay $B \to VM$ ($J \geq 1$ for $M$ too). When one of $V$ and $M$ is formed with $\bar{q}_L q_L$ or with $\bar{q}_R q_R$, this meson is in the $h = 0$ state. The angular momentum conservation along the decay momenta in the $B$ rest frame requires that the helicity of the other meson must also be zero (Fig.1a). Therefore $H_0$ dominates in this case. Alternatively with $(\bar{b}_L q_L)(\bar{q}_R q_R)$, if $\bar{q}_L q_R (h = +1)$ is combined to form one meson, the other meson must be made of the spectator $q_{spec}$ and $\bar{q}_R (h = -\frac{1}{2})$. Then the net helicity of the second meson can be only 0 or $-1$, which does not match the helicity $h = +1$ of the first meson (Fig.1b). Therefore the only two-meson state compatible with helicities and the overall angular momentum conservation is $V_{h=0} M_{h=0}$. This argument is valid only in the limit that the massless $q$ and $\bar{q}$ move strictly in parallel and there is no relative motion between them inside the meson.

C. Mass corrections

The relative motion of $q\bar{q}$ generates a correction to this helicity selection rule. Since the motion of light quarks makes up the entire mass of a nonflavored meson, this correction should be $O(|\mathbf{p}_T|/E) = O(\frac{1}{2} m/E)$ in amplitude, where $m$ is the meson mass and $E \approx \frac{1}{2} M_B$ for two-body decays. When either mass of $V$ and $M$ is large, the correction is large and accuracy of the rule is reduced accordingly. Let us examine this correction.

In the case of the $B(\bar{b}q)$ meson decaying through the interaction $(\bar{b}_L q_L)(\bar{q}_L q_L)$, the quarks in the final state are $\bar{q}_L q_L \bar{q}_R q_{spec}$ where $q_{spec}$ stands for the spectator. The $h = +1$ state of the meson $(\bar{q}_L q_L)$ can arise from a small opposite helicity component of a single $q_L$ while $h = +1$
is allowed for the other meson \((\bar{q}_L q_{spec})\) thanks to the indefinite helicity of \(q_{spec}\). On the other hand, formation of the \(h = -1\) mesons state requires the small opposite helicity components of two \(\bar{q}_L\)'s, one in \(V\) and one in \(M\) (Fig.1a). Consequently, \(H_1\) arises as the first-order correction while \(H_{-1}\) can arise only as the second-order correction. The same conclusion follows when \(B\) decays through \((\bar{b}_L q_L)(\bar{q}_R q_R)\) (Fig.1b). If we define the longitudinal and transverse fractions of helicity decay rates by

\[
\Gamma_L = \frac{H_0}{H_1 + H_0 + H_{-1}}, \quad \Gamma_T = 1 - \Gamma_L,
\]

the mass corrections are expressed as \(\Gamma_T = O(m^2/M_B^2)\) and \(\Gamma_L = 1 - O(m^2/M_B^2)\) in the case of the two-body decay \(B(\bar{b}q) \rightarrow VM\). Here \(m\) is the mass of the meson which does not receive the spectator quark or its descendant. The reason is obvious from the preceding argument: It is the meson formed by the energetic \(\bar{q}q\) originating from the effective decay interaction that primarily determines the helicity state, since the helicity of the other side that receives the spectator has a twofold uncertainty due to the indefinite spectator helicity. The helicity of the meson carrying the spectator is constrained by the overall angular momentum conservation. In the case of the \(\bar{B}(\bar{b}q)\) meson, the mass corrections to \(H_1\) and \(H_{-1}\) are interchanged in the same argument.

We should recall that there is also the \(l = 0\) correction of \(O(m^2/M_B^2)\) in probability in the case that a meson has \(l \neq 0\). This correction contributes to \(H_1\) and \(H_{-1}\) in the same order, namely \(\vartheta^2\). In terms of \(\Gamma_T \approx H_1 + H_{-1}\) the correction takes the same form for excited mesons.

It is easy to see here that the \(h = 0\) dominance holds even if the right-handed current enters weak interaction. Only \(H_1 > H_{-1}\) or \(H_{-1} > H_1\) in the mass correction depends on \(V - A\) or \(V + A\). In order to violate the \(h = 0\) dominance, we would need such an exotic weak interaction as \(b \rightarrow q_L \bar{q}_R q_L q_{spec}\). If the \(h = 0\) dominance breaks down, therefore, the most likely source is LDFSI.

**D. Inclusive charmless decay**

The argument in the preceding section can be immediately extended to the inclusive decay \(B \rightarrow VX\) in the case that \(X\) is described as excited \(q\bar{q}\) states. When a \(q\bar{q}\) pair is created almost collinearly by a hard gluon and turns \(X\) into a \(q\bar{q}q\bar{q}\) state, the added pair \(\bar{q}_L q_L\) or \(\bar{q}_R q_R\) has net helicity zero and does not contribute to the helicity of \(X\) (Fig.2a). In this case the previous argument of the \(h = 0\) dominance is unaffected. It can happen alternatively that the hard \(q\) and \(\bar{q}\) are emitted back to back. Imagine, for instance, that \(q_R\) enters \(V\) and \(\bar{q}_R\) goes into \(X\) so that \(V \sim \bar{q}_L q_L\) and \(X \sim \bar{q}_R q_R q_{spec}\) (Fig.2b). Then

\[\text{\footnotesize 2 Such a mass correction can be seen in the } U(6) \times U(6) \text{ model calculation of the charmless decay } B \rightarrow 1^- 1^- \text{ by Ali et al. Cheng et al. recently referred to this correction in their improved factorization calculation of } B \rightarrow J/\psi K^* \text{. Many other model calculations in the past based on the factorization, however, do not follow this pattern of mass corrections since vector and axial-vector form factors were introduced without chiral constraints.} \]
the net helicities are \( h = +1 \) for \( V \) and \( h = 0, -1 \) for \( X \), so the additional hard pair of \( \bar{q}q \) cannot realize \( V_{h=\pm1}X_{h=\pm1} \). We can easily see that the helicities of \( V \) and \( X \) do not match for \( h = \pm1 \) even when \( V \) receives \( q_{\text{spec}} \). The only helicity final state compatible with the overall angular momentum conservation is still \( V_{h=0}X_{h=0} \) in the collinear limit. Therefore, the preceding argument for the two-body decay \( B \to VM \) is carried over to the inclusive decay \( B \to VX \).

![Fig.2a](image1.png) ![Fig.2b](image2.png)

**FIG. 2.** The helicities in the inclusive \( B(bq) \) decay where an additional hard pair of \( \bar{q}q \) is produced and leads to the final state \( \bar{q}q \bar{q}q_{\text{spec}} \).

However, the collinear quark limit becomes a poor approximation as \( m_X \) increases in the inclusive decay. The transverse quark momenta \( p_T \) in \( X \) become large with respect to \( p_X \) so that the corrections grow with \( m_X \). The mass correction depends on whether the spectator \( q_{\text{spec}} \) enters \( V \) or \( X \). For the same reason as in the two-body decay, the final helicity state is determined primarily by the meson \( (V) \) or the group of mesons \( (X) \) that does not receive \( q_{\text{spec}} \) of indefinite helicity. Making an appropriate substitution in the mass corrections for the two-body decay, we obtain for \( m_X \gg m_V \)

\[
(1 - \Gamma_L)_{\text{mass}} \approx \frac{m_V^2 M_B^2}{(M_B^2 - m_X^2)^2}, \quad (q_{\text{spec}} \text{ in } X),
\]

\[
(1 - \Gamma_L)_{\text{mass}} \approx \frac{m_X^2 m_B^2}{(M_B^2 + m_X^2)^2}, \quad (q_{\text{spec}} \text{ in } V).
\]

(13)

The right-hand sides indicate the orders of magnitude. It is difficult even within perturbative QCD to compute their coefficients with good accuracy since they depend on the quark distributions inside mesons and other details. The coefficients are highly dependent on individual decay modes. Nonetheless, the rise of \( \Gamma_T \) with \( m_X^2 \), particularly in the case that \( X \) is produced without the spectator, is an important trend. It simply means that the “small opposite helicity component” of \( O(m_X/E_X) \) ceases to be small when \( m_X \) becomes large.

The orbital motion inside \( X \) is not restricted to \( l = 0 \). Therefore \( l_z \) of \( X \) can make up for violation of the overall angular momentum conservation when \( V \) is formed with \( \bar{q}Lq_R \) \((h = +1)\) or \( \bar{q}Rq_L \) \((h = -1)\). In terms of the helicity fraction, the \( l_z \) correction to \( X \) generates the leading correction that grows rapidly with \( m_X \);

\[
(1 - \Gamma_L)_{l_z} \approx \frac{m_X^2 m_B^2}{(M_B^2 + m_X^2)^2}.
\]

(14)
When $\Gamma_T = 1 - \Gamma_L$ becomes a substantial fraction of unity, LDFSI is clearly important. As $m_X$ approaches the kinematical upper limit corresponding to $q = 0$, $\Gamma_L$ should reach $1/3$ according to the limiting behavior of Eq.(7):

$$\Gamma_L \to \frac{1}{3}, \quad \text{as} \quad m_X \to m_{\text{max}}.$$  

(15)

Future experiment on the inclusive decay will determine $\Gamma_L$ as a function of $m_X$ interpolating between $1 - O(m_V^2/M_B^2)$ and $1/3$, as sketched qualitatively in Fig.3. We should keep in mind that the corrections presented here are the expectation based on perturbative QCD. It is only a theoretical prediction that should be tested by experiment. While the helicity test of the charmless decay is of primary interest, no experimental data exist on $\Gamma_{T,L}$ for any charmless decay mode at present.

![FIG. 3. The qualitative behavior of $\Gamma_L$ against $m_X^2$. While $\Gamma_L = 1/3$ at $m_X = m_{\text{max}}$ is a kinematical constraint, the behavior of $\Gamma_L$ near the small end of $m_X$ is only the expectation of perturbative QCD.](image)

One problem exists in performing an inclusive measurement of the charmless decay $B \to VX$. One has to make sure that $X$ does not contain charm nor hidden charm. Since the charmless decays are the rare decays, the region above the charm threshold for $m_X$ is overwhelmed by the background that is much higher in branching. In practice, the charmless inclusive decay will be analyzed only in the region separated from the charm background by kinematics, that is,

$$m_X < m_D.$$  

(16)

Above $m_D$, the dominant process is $B \to VX_\tau$ where $X_\tau$ contains an anticharmed meson. Fortunately, Eq.(16) is the mass range where many interesting results will be extracted from the charmless decay. For the decays into $X_\tau$, the helicity selection rule holds in a manner

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3 It is possible that $X$ consists of a widely separated pair of mesons interacting only through SDFSI. In this case, the final state is a three-jet state and the decay may be a SD process calculable by perturbative QCD for $m_B \to \infty$. However, such a contribution is suppressed by $O(\alpha_s/\pi)$ and not expected to be a significant portion of the inclusive decay. One should be able to check by actually examining the final states whether it is the case or not.
almost identical to the charmless decay. We shall see that the Figure 3 applies to $B \rightarrow VX_{\tau}$ as well. Therefore, separate tests of the rule will be possible with $B \rightarrow VX_{\tau}$ in the range above $m_X = m_D$.

As for $V$, reconstruction of $\rho$ from $\pi\pi$ may encounter an excessive combinatorial background. If this happens, $\phi$ will be a clean alternative for $V$ in the environment of BaBar and Belle.\footnote{The author owes to R.N. Cahn for this remark.} As a last resort, we can work on fully reconstructed $B$ events with reduced statistics.

IV. DEAY INTO CHARMED X OR CHARMED V

We extend the argument for the charmless decay to the charmed meson production decay $B \rightarrow VX_{\tau}$ and $B \rightarrow V_{\tau}X$. We ignore here the small contribution from the penguin-type processes for this class of decays. When $V$ is formed without involving the spectator, $V$ carries $h = 0$ of $q_Lq_L$ up to the small mass correction given by the first line of Eq.(13). The $h = 0$ dominance remains true even when an extra $qq$ pair is produced: Imagine, for instance, that $q_R$ and $q_R$ are produced secondarily by a hard gluon and enter both $V$ and $X$. Then $V = \overline{q}_Lq_R$ and $X = \overline{c}_Lq_L\overline{q}_Rq_{spec}$ can satisfy the overall angular momentum conservation only with help of $l_z = +1$ or the opposite component of $q_L$ or $\overline{q}_R$. In the case of $V = \overline{q}_Rq_L$ and $X = \overline{c}_Lq_Rq_{spec}$, both $l_z = -1$ and the opposite helicity of $\overline{c}_L$ are needed.\footnote{The opposite helicity content of $c_L$ is larger; $(m_c^2 + p_T^2)^{1/2}/E_c$ instead of $|p_T|/E_c$.}

In the two-body decay where $X_{\tau}$ is $\overline{D}^*$ ($l = 0$) and $q_{spec}$ enters $D^*$, therefore, the correction to the $h = 0$ rule is dominated by the mass correction to $V$, 

$$1 - \Gamma_L \approx \frac{m_V^2}{(M_B - m_X)^2}. \tag{17}$$

This correction will apply to $B^0/\overline{B}^0 \rightarrow \rho^\pm D^{*\mp}$ since the quark distribution function disfavors formation of $\rho^\pm$ with the spectator. Because of the large branching, experiment already measured the helicity fractions with good accuracy for the two-body decay $B^0/\overline{B}^0 \rightarrow \rho^\pm D^{*\mp}$ many years ago. The experimental result was in agreement with the $h = 0$ dominance \footnote{The author owes to R.N. Cahn for this remark.};

$$\Gamma_L = 0.93 \pm 0.05 \pm 0.05. \tag{18}$$

The deviation from unity of $\Gamma_L$ is consistent with the mass correction ($\approx 0.03$) that we expect from Eq.(17). Even when $X_{\tau/c}$ is a higher state of $l \neq 0$, the correction to the $h = 0$ rule is determined by $\rho^\mp$ and grows rather slowly with $m_X$ according to Eq.(17) since $q_{spec}$ enters $X_{\tau/c}$ in the dominant process of $B^0/\overline{B}^0 \rightarrow \rho^\pm X_{\tau/c}$.

The correction is a little different for the so-called color-disfavored decays. Take $B^0(\bar{b}d) \rightarrow \rho^0\overline{D}^{*0}$ as an example: The $\rho^0$ meson must be formed with the spectator when the decay occurs through the dominant operator for this decay. The final helicity is constrained by $D^{*-}$ and the correction is $1 - \Gamma_L \approx m_X^2M_B^2/(M_B^2 + m_X^2)^2$. Therefore we expect that the correction is larger in $B^0 \rightarrow \rho^0\overline{D}^{*0}$ than in $B^0 \rightarrow \rho^+D^{*-}$;
\[ \Gamma_T(B^0 \to \rho^0 D^0) > \Gamma_T(B^0 \to \rho^+ D^{*-}). \] (19)

The recent measurement \[13\] of the factorization-disfavored two-body decays, \( B^0 \to D^{(*)0} X^0 \) \((X^0 = \pi^0, \omega, \eta)\) seems to show that the branching fractions for these decays are larger than their lowest-order perturbative QCD calculations \[3\]. The helicity analysis of \( B^0 \to \rho^0 X^0 \) and \( K^{-0} X^0 \) will help us toward better understanding of how much LDFFSI is involved here.

Let us move to the other inclusive measurement where a charmed meson is identified instead of a light meson; \( B \to D^* X \). There is an experimental advantage in reconstructing \( D^* \) through its soft decay into \( D \pi \). The \( D^* \) meson can be formed with or without the spectator. With the spectator (\( \bar{D}^* = \bar{c} \bar{q}_{\text{spec}} \)), the accuracy of the \( h = 0 \) dominance is controlled by the helicity of \( X \), which is determined by \( \bar{q}_L q_L, \bar{q}_L q_L \bar{q}_L q_L, \bar{q}_L q_L \bar{q}_R q_R \cdots \). The correction is given by the second line of Eq. (13) and grows rapidly with \( m_X \). On the other hand, when \( X \) receives the spectator, \( \bar{D}^* = \bar{c} \bar{q}_{\text{spec}} \), the final helicity is determined by \( \bar{q}_L q_L, \bar{q}_L q_L \bar{q}_L q_L, \bar{q}_L q_L \bar{q}_R q_R \cdots \). Then it is \( \bar{D}^* = \bar{c} \bar{q}_{\text{spec}} \), that determines the final helicity. The dominant helicity is again \( h = 0 \) and the correction is given by the first line of Eq. (13), but the magnitude is large because of the larger opposite helicity content in \( \bar{D} \).

Finally we comment on the decays \( B \to V X_{c\bar{c}} \) and \( V_{c\bar{c}} X \). A pair of \( c\bar{c} \) is produced by weak interaction and forms one of charmonia or turns into \( D^{(*)0} D^{(*)} \). \( V \) is most likely formed with the spectator since little phase space is left for production of a fast pair of \( \bar{q} q \). In this case, the helicity content is determined by \( \bar{c} \bar{L} c_L \). Since \( c_L \) and \( \bar{c} \bar{L} \) are heavy and slow, the opposite helicity content of \( O(\frac{1}{2} m_{c\bar{c}} / E_{c\bar{c}}) \) does not give an accurate estimate. Nonetheless let us stretch for the moment the mass correction formula for \( \Gamma_T \) such that the coefficient in front be adjusted to give the kinematical constraint \( \Gamma_T = \frac{2}{3} \) at the maximum value of \( m_X \). Then the prediction on \( \Gamma_T \) would be

\[ \Gamma_T \simeq \frac{8}{3} \times \frac{m_{c\bar{c}}^2 M_B^2}{(M_B^2 + m_{c\bar{c}}^2)^2}, \] (20)

where \( m_{c\bar{c}} \) is the invariant mass of all hadrons but \( V \). \( X_{c\bar{c}} \) is most likely one of charmonia. Detailed measurements were made for the helicity content of \( B \to J/\psi K^* \). For this decay mode, Eq. (20) gives a “correction” of \( \Gamma_L \simeq 0.49 \). The latest result of the helicity analysis by BaBar \[10\] can be expressed as,

\[ \Gamma_L = 0.597 \pm 0.028 \pm 0.024, \] (21)

which is not far from 0.49. However, the agreement is probably fortuitous since the Lorentz factor \( \gamma \) of \( J/\psi \) is only 1.12 in this decay.

In the decay \( B \to J/\psi K^* \), the \( K^* \) meson moves with \( \gamma \simeq 2 \). If we make the approximation of \( K^* \) being fast, \( K^* (\bar{c} \bar{q}_{\text{spec}}) \) can be only in helicity +1 or 0, not in −1. Therefore \( H_{-1} \simeq 0 \) is predicted for \( B \to J/\psi K^* \) if one assumes perturbative QCD for \( K^* \). The transversity angular analysis \[10\] allows two solutions, \( H_1 \gg H_{-1} \) and \( H_1 \ll H_{-1} \), but cannot resolve the twofold ambiguity. At present, experiment still does not exclude the possibility that perturbative QCD is applicable to the light meson side \((K^*)\) of the decay.\[6\]

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6 Following earlier experimental papers \[11\], the BaBar analysis \[10\] quotes only one solution,
The Belle Collaboration recently measured the branching fraction for the factorization-suppressed decay \( B \to \chi_0 K \) \([15]\) at the level comparable with the factorization-favored decays \( B \to \eta_c K, J/\psi K, \) and \( \chi_1 K \). It shows that the simple factorization clearly fails in the decay \( B \to \text{charmonium} \).

The decay \( B \to D^* D^* \) is being analyzed at the B-factories. The branching fraction was reported for \( D^{**} D^{*-} \) \([16]\). After accumulation of more events, the helicity analysis will become feasible. Comparison of this decay with \( B \to J/\psi K^* \) may provide an additional useful information about dynamics in \( b \to c \bar{q} q \).

V. HIGHER SPIN \((J \geq 2)\)

The helicity test can be performed for higher-spin inclusive processes \( B \to MX \to abX \) with \( J \geq 2 \) for \( M \). For \( J = 0 \) for \( a \) and \( b \), the differential decay rate in the \( B \) rest frame takes the form,

\[
\frac{d\Gamma}{dq \cos \theta} \propto |q|^J \sum_{\lambda=-J}^{J} H_{\lambda}(m_X^2)|d_{\lambda,0}^f(\theta)|^2,
\]

where \( \lambda \) is the helicity of \( M \). The momentum \( q \) and the angle \( \theta \) are defined in the same way as in Eq. (6). In the case of \( J \neq 0 \) for \( a \) and/or \( b \), an additional \( \lambda \) dependence enters through the decay \( M \to a + b \). The dominant helicity structure function is \( H_0 \), then \( H_{\pm 1} \) for both \( B \) and \( \overline{B} \) decays, since the \( l_z \) correction to \( M \) contributes to \( H_1 \) and \( H_{-1} \) in the same order. If perturbative QCD is valid, the function \( H_{\lambda} \) with \( |\lambda| \geq 2 \) cannot arise without the \( l_z \) correction. \( H_h \) with \( |h| \geq 2 \) beyond the \( l_z \) correction will be a clear evidence for LDFSI. As \( m_X \) tends to its maximum value, \( \Gamma_L \) should approach \( 1/(2J+1) \). In the decay \( B \to f_2 X \to \pi \pi X \), for instance, the angular dependence \( |d_{2,0}^f|^2 = \frac{3}{4}(1 - \cos^2 \theta)^2 \) appears as \( f_2 \) slows down. Appearance of \( (1 - \cos^2 \theta)^2 \) indicates that the orbital angular momentum of \( \overline{q} \) inside \( f_2 \) becomes important in the \( B \) rest frame. One might think of attributing appearance of \( |h| \geq 2 \) to possible breakdown of the \( \overline{q}q \) description of \( f_2 \). But it is unlikely in the face of the static quark model: the \( \overline{q}q \) description of low-lying mesons works well both in the infinite momentum limit and in the static limit albeit the physical nature of quarks is different between the two limits. As \( q \to 0 \), all \( l_z \) states of \( f_2 \) are equally produced and \( \Gamma_L \) should approach \( 1/5 \).

\[\phi_\parallel - \phi_\perp \simeq \pi,\] which would lead to \( H_1 \ll H_{-1} \) in the ordinary sign convention chosen in its reference \([12]\). It might look as if the BaBar result were in direct conflict with the prediction of perturbative QCD for \( K^* \). In fact, the other solution \( \phi_\parallel - \phi_\perp \simeq 0 \) leading to \( H_1 \gg H_{-1} \) is also allowed by this experiment, though not explicitly quoted as such \([13]\). Therefore no conclusion can be drawn from this experiment as to which is larger between \( H_1 \) and \( H_{-1} \) in \( B \to J/\psi K^* \). The same comment applies to the latest Belle analysis \([14]\).
VI. COMPARISON WITH OTHER TESTS

Various tests have so far been proposed concerning validity of the factorization. The most straightforward is to compute as many decay amplitudes as possible with theoretical resources at hand. In some simple cases we are fortunate to have only a single dominant decay process in the factorization limit. An example is $B^0 \rightarrow D^- \pi^+$. Otherwise a decay amplitude for a given process is sum of competing contributions of more than one decay process. Once short-distance QCD corrections are included, the quark operators producing mesons are nonlocal. Then we need to know not only the decay constants, the wave functions at origin, but also the entire light-cone quark distribution functions in order to obtain a single decay amplitude. Furthermore, the relevant energy scale of the QCD coupling $\alpha_s(E)$ can take different values depending on how and where it appears. Therefore a final number for a total decay amplitude is sensitive to small theoretical uncertainties of each contribution particularly when different terms enter with different signs. These added uncertainties make comparison of theory with experiment less decisive. For this reason we give up here attempting numerical estimate of the coefficients of the corrections to the $h = 0$ dominance rule even for the simplest two-body decay $B \rightarrow 1^-1^-$. 

A while ago Ligeti et al. [17] proposed a test of the factorization in the decay $B \rightarrow \overline{D}^{(*)}X$. They proposed to compare the $m_X$ distribution of this inclusive decay with the $m_\ell \nu$ distribution of the semileptonic decay $B \rightarrow \overline{D}^{(*)}\ell \nu$. It appears to be a clean test. In order for this test to work, however, $X$ must be produced from a single weak current just as $\ell \nu$ is. Therefore, it applies to $B^0 \rightarrow D^{(*)-}X^+$ (and the conjugate) through the dominant decay operator, but not to $B^+ \rightarrow D^{(*)0}X^+$ (and the conjugate) since $X^+$ can pick either the current quark $u$ or the spectator $u$ in the $B^+$ decay. Only the neutral $B$ decay is possibly related to the semileptonic decay. The most important difference from our test is that the comparison with the semileptonic decay tests only validity of the factorization before the perturbative QCD improvement. The SDFSI surely plays a significant role in the final state and breaks down the similarity between the nonleptonic and the semileptonic decay. An alternative to this test was proposed for two-body decays and importance of spin was mentioned [18], but it is not free of the uncertainties and complications in theoretical computation. In contrast, the inclusive helicity measurement tests not just the lowest-order factorization but its perturbative QCD corrections to all orders independent of theoretical details. It will provide us with an important information as to how much long-distance QCD interactions enter a given process and allow us to use it for related processes. A negative side of the helicity test is, of course, the common drawback of LDFSI that after LDFSI is found, we cannot compute phases nor magnitudes of decay amplitudes from the first principle. However, just measuring CP violations beyond the $B^0-\overline{B}$ mixing effect will be important even if we cannot easily relate it to fundamental parameters of theory. Only when LDFSI is significant, do we have a chance to detect a direct CP violation from particle-antiparticle asymmetry. The helicity test will hopefully tell us which decay modes we should go after for search of direct CP violations.
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