On the nature of the electroweak phase transition

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Abstract
We discuss the finite-temperature effective potential of the Standard Model, $V_{\text{eff}}(\phi, T)$, with emphasis on the resummation of the most important infrared contributions. We compute the one-loop scalar and vector boson self-energies in the zero-momentum limit. By solving the corresponding set of gap equations, with the inclusion of subleading contributions, we find a non-vanishing magnetic mass for the $SU(2)$ gauge bosons. We comment on its possible implications for the nature of the electroweak phase transition. We also discuss the range of validity of our approximations and compare this with other approaches.
1. In analogy with other symmetry-breaking phenomena in condensed matter physics, the
electroweak gauge symmetry $SU(2) \times U(1)$, which in the Standard Model is spontaneously
broken by the vacuum expectation value $\phi$ of the Higgs field, is expected to be restored at suffi-
ciently high temperatures [1]. Several years after the development of the theoretical framework
for a quantitative treatment of the subject [2], the interest in the electroweak phase transition
has been revived by the observation [3] that the rate of anomalous $B$-violating processes is
unsuppressed at sufficiently high temperatures. This has focused attention on the possibility
that the cosmological baryon asymmetry might be generated by physics at the Fermi scale (for
recent reviews and further references, see e.g. refs. [4]).

One of the basic tools for the discussion of the electroweak phase transition is $V_{\text{eff}}(\phi, T)$,
the one-loop temperature-dependent effective potential, improved by appropriate resumptions
of the most important infrared-dominated higher-loop contributions, which jeopardize the con-
tventional perturbative expansion in the relevant range of temperature and field values [2,3,4].
A recent, explicit computation of the Standard Model $T$-dependent one-loop potential [5] was
soon followed by an improved computation, which resummed the leading, $\phi$-independent, $O(T^2)$
contributions to the effective masses of scalar and vector bosons [5]. Subsequent attempts to
include subleading contributions to the $T$-dependent effective masses [3,6] then originated a
lively theoretical debate [8–15] about the nature of the electroweak phase transition.

In this work we continue the discussion of subleading contributions to the $T$-dependent
effective masses of scalar and vector bosons in the Standard Model. In particular, we compute
the one-loop self-energies in the zero-momentum limit, in the $'t$ Hooft-Landau gauge and in the
approximation $\lambda = g' = 0$. By solving the corresponding set of gap equations, and keeping both
leading and subleading terms in the high-temperature expansion of the self-energies, we find that
a non-vanishing magnetic mass is generated for the $SU(2)$ gauge bosons, $m_T(0) = g^2 T/(3\pi)$.
We discuss its possible implications for the nature of the electroweak phase transition. We
also comment on the range of validity of our calculational framework, and on the possibility of
consistently including subleading contributions in the computation of $V_{\text{eff}}(\phi, T)$.

2. We begin by summarizing some well-known results, which will be useful for the subsequent
discussion. Here and in the following, we work in the $'t$ Hooft-Landau gauge. The spin-0 fields
of the Standard Model are described by the $SU(2)$ doublet

$$
\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_1 + i\chi_2 \\ \phi + h + i\chi_3 \end{pmatrix},
$$

(1)

where $\phi$ is an arbitrary real constant background, $h$ is the Higgs field, and $\chi_a$ ($a = 1, 2, 3$)
are the three Goldstone bosons. In terms of the background field $\phi$, the tree-level potential is
$V_{\text{tree}}(\phi) = -((\mu^2/2)\phi^2 + (\lambda/4)\phi^4)$, with positive $\lambda$ and $\mu^2$, and the tree-level minimum corresponds to $v^2 = \mu^2/\lambda$. The spin-0 field-dependent masses are $m_H^2(\phi) = 3\lambda\phi^2 - \mu^2$ and $m_H^2(\phi) = \lambda\phi^2 - \mu^2$,
so that $m_H^2(v) = 2\lambda v^2 = 2\mu^2$, $m_H^2(v) = 0$. The gauge bosons contributing to the one-loop
effective potential are $W^\pm$ and $Z$, with tree-level field-dependent masses $m_W^2(\phi) = (g^2/4)\phi^2$ and
$m_Z^2(\phi) = [(g^2 + g'^2)/4]\phi^2$. Finally, the only fermion that can give a significant contribution to
the one-loop effective potential is the top quark, with a field-dependent mass $m_t^2(\phi) = (h_t^2/2)\phi^2$,
where $h_t$ is the top-quark Yukawa coupling.

The temperature-dependent one-loop effective potential can be calculated according to stan-
dard techniques [3–5]. Here and in the following, field-independent contributions will be system-
tically neglected. The one-loop potential can be decomposed into a zero-temperature and
a finite-temperature part,
\[ V^{(1)}[m_i^2(\phi), T] = V^{(1)}[m_i^2(\phi), 0] + \Delta V^{(1)}[m_i^2(\phi), T], \]

which are finite functions of the renormalized fields and parameters, once infinities are removed from the zero-temperature sector by means of appropriate counterterms. Imposing renormalization conditions that preserve the tree-level values of \( v \) and \( m_h(v) \), we can write
\[ V^{(1)}[m_i^2(\phi), 0] = \sum_{i=h, W, Z} \frac{n_i}{64\pi^2} \left[ m_i^4(\phi) \left( \log \frac{m_i^2(\phi)}{m_i^2(v)} - \frac{3}{2} \right) + 2m_i^2(\phi)m_i^2(v) \right] + \frac{n_\chi}{64\pi^2} m_\chi^4(\phi) \left( \log \frac{m_\chi^2(\phi)}{m_\chi^2(v)} - \frac{3}{2} \right), \]

where \( n_h = 1, n_\chi = 3, n_W = 6, n_Z = 3, n_t = -12 \), and we have used the infinite running of the Higgs mass, from \( p^2 = 0 \) (where the effective potential is defined) to \( p^2 = m_h^2 \) (where the physical Higgs mass is defined), in order to cancel the logarithmic infinity from the massless Goldstone bosons at the minimum \( \{0\} \).

The finite-temperature part of the one-loop effective potential can be written as
\[ \Delta V^{(1)}[m_i^2(\phi), T] = \frac{T^4}{2\pi^2} \left[ \sum_{i=h, \chi, W, Z} n_i J_+(y_i^2) + n_t J_-(y_t^2) \right], \]

where
\[ y^2 = \frac{m^2(\phi)}{T^2}, \quad J_\pm(y^2) = \int_0^\infty dx \, x^2 \log \left( 1 \mp e^{-\sqrt{x^2+y^2}} \right). \]

In view of the following discussion, we write down the high-temperature expansion of eq. (3),
\[ \Delta V^{(1)}[m_i^2(\phi), T] = \sum_{i=h, \chi, W, Z} n_i \left\{ \frac{m_i^2(\phi)T^2}{24} - \frac{m_i^2(\phi)T}{12\pi} - \frac{m_i^4(\phi)}{64\pi^2} \left[ \log \frac{m_i^2(\phi)}{T^2} - 5.4076 \right] \right\} \\
- n_t \left\{ \frac{m_t^2(\phi)T^2}{48} + \frac{m_t^4(\phi)}{64\pi^2} \left[ \log \frac{m_t^2(\phi)}{T^2} - 2.6350 \right] \right\} + \ldots, \]

where the ellipsis stands for terms \( O[m^6(\phi)/T^2] \) or higher. Notice that the \( m^4 \log m^2 \) terms cancel between the \( T = 0 \) and the \( T \neq 0 \) contributions in the high-temperature expansion. Notice also that there is no \( m_t^2(\phi) \) term in the high-temperature expansion of the top-quark contribution to the one-loop potential, since there cannot be fermionic modes of zero Matsubara frequency.

If one tries to use the full \( T \)-dependent one-loop effective potential, one faces a number of difficulties \[ \bullet \] for small values of \( \phi \) [\( \phi^2 < \mu^2/\lambda \) for the Goldstone bosons, \( \phi^2 < \mu^2/(3\lambda) \) for the Higgs boson], the field-dependent squared masses \( m_i^2(\phi) \) and \( m_\chi^2(\phi) \) become negative, leading to a complex one-loop potential \( V_{\text{tree}} + V^{(1)}[m_i^2(\phi), 0] + \Delta V^{(1)}[m_i^2(\phi), T] \). More generally, for small values of the field-dependent masses \( m_i^2(\phi) \) (\( i = h, \chi, W, Z \)), the conventional loop expansion is jeopardized by the (power-like) infrared behaviour of loop diagrams at finite temperature. As will be recalled later, an appropriate resummation of the leading infrared contributions from higher-loop diagrams can remove this illus. Already at this level, however, we can take advantage of the fact that for small values of the Higgs mass, \( m_h^2 \ll m_W^2, m_Z^2, m_t^2 \), the dominant contributions to the effective potential are those coming from top quark and gauge boson loops.
Since the field-dependent masses $m_i^2(\phi)$, $m_Z^2(\phi)$ and $m_T^2(\phi)$ are positive for any $\phi \neq 0$, we can consider, for the sake of discussion, a one-loop finite-temperature potential in which only the $W$, $Z$ and top-quark loops are taken into account. The behaviour of such a potential at the critical temperature is described by the long-dashed line in fig. 1, for the representative choice of parameters $m_h = 60$ GeV, $m_t = 150$ GeV. We can see that for our parameter choice the naive one-loop potential describes a first order phase transition, with the coexistence of two degenerate minima occurring at $T \sim 95$ GeV.

3. We now wish to improve the one-loop result by including the most important higher-loop contributions \(^4\). As can be shown by simple power-counting arguments, the higher-loop diagrams with the worst infrared behaviour are the so-called daisy diagrams. They are generated by iterated self-energy insertions, in the infrared limit $p^0 = 0$, $\vec{p} \to 0$, on the one-loop diagrams carrying the modes of zero Matsubara frequency. The $n = 0$ contributions to the finite-temperature part of the one-loop effective potential are nothing else than the $m^3 T$ terms in the high-temperature expansion of eq. (6). Resumming the daisy diagrams amounts then, as we shall see, to performing the replacements $m_i^2(\phi) \to \overline{m}_i^2(\phi, T)$ in the $m_i^3(\phi) T$ terms of eq. (3), where the effective masses $\overline{m}_i^2(\phi, T)$ can be computed once the one-loop $T$-dependent self-energies are known. We then begin by computing the $T$-dependent self-energies, for the different bosonic fields that contribute to the one-loop effective potential, in the infrared limit $p^0 = 0$, $\vec{p} \to 0$. For simplicity, we compute them in the approximation $\lambda \to 0$ (i.e. $m^2_h \ll m^2_W, m^2_Z, m^2_\tau$) and $g' \to 0$, and we neglect their $T = 0$ contributions. These approximations, which have been adopted in recent analyses \(^2\), allow us to work with diagonal one-loop self-energies, and are expected to reproduce the main qualitative features of the result correctly.

The self-energies for the scalar fields $h$ and $\chi$, denoted by $\Pi_h$ and $\Pi_\chi$, are given by

$$\Pi_h = \frac{3g^2}{8\pi^2} \left\{ T^2 \left[ I_+(y^2_L) + 2I_+(y^2_T) \right] + \frac{g^2\phi^2}{2} \left[ I'_+(y^2_L) + 2I'_+(y^2_T) \right] \right\} + \frac{3h^2T^2}{\pi^2} \left[ I_-(y^2_L) + \frac{h^2\phi^2}{T^2} I'_-(y^2_T) \right],$$

(7)

and

$$\Pi_\chi = \frac{3g^2T^2}{8\pi^2} \left[ I_+(y^2_L) + 2I_+(y^2_T) \right] + \frac{3h^2T^2}{\pi^2} I'_-(y^2_T),$$

(8)

where $I_\pm(y^2) = \pm2[dI_\pm(y^2)/dy^2]$, $I'_\pm(y^2) = dI_\pm/dy^2$.

In the chosen approximation, the $W$ and the $Z$ are degenerate in mass \(^4\), and their propagator can be decomposed into a transverse and a longitudinal part as

$$D_{\mu\nu}(p) = \frac{T_{\mu\nu}}{p^2 - m_T^2} + \frac{L_{\mu\nu}}{p^2 - m_L^2},$$

(9)

where $(i, j = 1, 2, 3)$

$$T_{\mu\nu} = -g_{\mu i} \left( g^{ij} + \frac{p_i p_j}{p^2} \right) g_{j\nu}, \quad L_{\mu\nu} = \frac{p_\mu p_\nu}{p^2} - g_{\mu\nu} - T_{\mu\nu},$$

(10)

satisfy the orthogonality conditions $T_{\mu\nu}T_{\nu\lambda} = -T_{\mu\lambda}$, $L_{\mu\nu}L_{\nu\lambda} = -L_{\mu\lambda}$, $T_{\mu\nu}L_{\nu\lambda} = 0$. The transverse and longitudinal masses are equal at the tree level, $m_T^2(\phi) = m_L^2(\phi) = g^2\phi^2/4 \equiv m^2(\phi)$. However, they will get different one-loop contributions at finite temperature, since for $T \neq 0$ the

\(^4\)To compensate for this fact, we shall adopt in the following the numerical value $g^2 = 4\frac{2m_h^2 + m_t^2}{3m^2}$. 



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longitudinal and transverse components act in practice as independent degrees of freedom in Feynman diagrams. We will then decompose the gauge boson self-energy, according to (9), as

$$\Pi_{\mu
u} = L_{\mu
u}\Pi_L + T_{\mu
u}\Pi_T,$$

where, in the infrared limit, $$\Pi_L(0) = \Pi_0^L(0), \Pi_T(0) = (1/3)\sum_i\Pi_0^T(0).$$

For the longitudinal polarization, we obtain

$$\Pi_L = \frac{g^2T^2}{\pi^2} \left\{ 2\left[I_+(y_L^2) + 2I_+(y_T^2)\right] - 2\left[y_L^2I_+(y_L^2) + 2y_T^2I_+(y_T^2)\right] - \frac{3}{y_L} [J_+(y_L^2) - J_+(0)] 
- \frac{1}{8} I_+(0) + \frac{5I_+(y_L^2)}{8} - y_L^2 I_+(y_L^2) + \frac{3}{2} J_+(y_L^2) - J_+(0) - \frac{1}{2} y_L^2 [I_+(y_L^2) - y_L^2 I_+(y_L^2)] \right\} \tag{11}$$

and for the transverse polarization

$$\Pi_T = \frac{g^2T^2}{\pi^2} \left\{ -\frac{1}{3} I_+(y_L^2) - \frac{2}{3} I_+(y_T^2) + \frac{1}{2} I_+(0) + \frac{J_+(y_L^2) - J_+(0)}{y_L^2} + \frac{I_+(y_L^2) + I_+(y_T^2)}{8} - \frac{J_+(y_T^2) - J_+(0)}{y_T^2} \right\}
+ \frac{3g^2}{4} I^r(y_T^2) \left\{ I_+(y_T^2) - I_+(y_T^2) - \frac{J_+(y_T^2) - J_+(0)}{y_T^2} \right\} \tag{12}$$

The high-temperature expansion of the self-energies in eqs. (7), (8), (11), (12) reads

$$\Pi_h[m_\phi^2(\phi), T] = \left( \frac{3}{16} g^2 + \frac{1}{4} \frac{h_T^2}{T^2} \right) - \frac{3g^2}{16\pi} \left( m_L + 2m_T \right) T - \frac{3g^4\phi^2}{32\pi} \left( \frac{1}{m_T} + \frac{1}{2m_L} \right) T + \ldots, \tag{13}$$

$$\Pi_\chi[m_\phi^2(\phi), T] = \left( \frac{3}{16} g^2 + \frac{1}{4} \frac{h_T^2}{T^2} \right) - \frac{3g^2}{16\pi} \left( m_L + 2m_T \right) T + \ldots, \tag{14}$$

$$\Pi_L[m_\phi^2(\phi), T] = \frac{11}{6} g^2 T^2 - \frac{g^2}{16\pi} \left( m_h + 3m_\chi + 16m_T \right) T - \frac{g^4\phi^2}{16\pi m_L + m_h} + \ldots, \tag{15}$$

$$\Pi_T[m_\phi^2(\phi), T] = \frac{g^2}{3\pi} m_T T + \frac{g^2}{12\pi} \left( \frac{m_h + m_\chi}{4} - \frac{m_h m_\chi}{m_h + m_\chi} \right) T - \frac{g^4\phi^2}{24\pi m_T + m_h} + \ldots. \tag{16}$$

Using the one-loop self-energies given above, one can improve the one-loop result of eq. (2) by considering the daisy vacuum diagrams. Using the previous decompositions of the propagator and of the self-energy, we can write

$$D_{\mu\nu} \Pi' = -L_{\mu\lambda} \frac{\Pi_L}{p^2 - m_L^2} - T_{\mu\lambda} \frac{\Pi_T}{p^2 - m_T^2}. \tag{17}$$

The contribution to the effective potential of the daisy diagrams obtained from gauge-boson loops is then

$$-\frac{T}{2} \int \frac{d^3p}{(2\pi)^3} \text{Tr} \sum_{N=1}^\infty \frac{1}{N} \left[ \frac{L_{\mu\nu}}{p^2 + m_L^2} + T_{\mu\nu} \frac{\Pi_T}{p^2 + m_T^2} \right] N^N, \tag{18}$$

where the trace is over Lorentz indices (in the approximation $g' = 0$, the one-loop self-energy matrix of neutral gauge bosons is diagonal whereas, for $g' \neq 0$, mixing effects should be properly taken into account). Using again the orthogonality properties of $L_{\mu\nu}$ and $T_{\mu\nu}$, the final result is

$$\Delta V^{\text{daisy}} = -\frac{T}{2} \sum_{i=h,\chi,L,T} n_i \int \frac{d^3p}{(2\pi)^3} \sum_{N=1}^\infty \frac{1}{N} \left[ -\frac{\Pi_i}{p^2 + m_i^2} \right] N^N. \tag{19}$$
Summarizing, the effective potential improved by the daisy diagrams is\footnote[5]{As for the top quark contribution to $V_{\text{eff}}(\phi, T)$, we did not perform any resummation, since fermion loops do not suffer from the infrared problems associated with the zero Matsubara frequencies.}

$$V_{\text{eff}} = V_{\text{tree}} + V^{(1)}[m_i^2(\phi), T] - \frac{T}{12\pi} \sum_{i=h,\chi,L,T} n_i \left[ m_i^2(\phi, T) - m_i^2(\phi) \right], \quad (20)$$

where

$$m_i^2(\phi, T) = m_i^2(\phi) + \Pi_i[m_j^2(\phi), T]. \quad (21)$$

For consistency, in evaluating \([8,11,12]\) the daisy-improved effective potential of eq. (20), one has to keep only the $T^2$ terms in the self-energies of eqs. (13)–(16). This amounts to using the following set of $T$-dependent masses

$$m_h^2 - m_h^2 = m_h^2 - m_h^2 = \frac{1}{4} \left( \frac{3}{4} g^2 + h_i^2 \right) T^2, \quad m_L^2 = m^2 + \frac{11}{6} g^2 T^2, \quad m_T^2 = m^2. \quad (22)$$

The resulting effective potential at the critical temperature is displayed as the short-dashed line in fig. 1, for the same parameter choices as before. One can notice that the effective potential corresponding to eqs. (20) and (22) still describes a first-order phase transition. However, one can also observe \([11]\) the characteristic reduction by a factor of $\sim 2/3$ in the non-trivial vacuum expectation value of the field $\phi$, at the temperature $T_c \sim 97$ GeV at which the two minima become degenerate. This point can be qualitatively understood by looking at the coefficient of the $\phi^3$ term in the high-temperature expansion of the effective potential.

4. To improve over the previous approximation, one could try to keep also the subleading terms in the high-temperature expansion of the self-energies in eqs. (13)–(16), obtaining the effective masses

$$\begin{align*}
\overline{m}_h^2 &= m_h^2 + \frac{1}{4} \left( \frac{3}{4} g^2 + h_i^2 \right) T^2 - \frac{9}{16\pi} g^3 \phi T, \\
\overline{m}_\chi^2 &= m_\chi^2 + \frac{1}{4} \left( \frac{3}{4} g^2 + h_i^2 \right) T^2 - \frac{9}{32\pi} g^3 \phi T, \\
\overline{m}_L^2 &= m^2 + \frac{11}{6} g^2 T^2 - \frac{5}{8\pi} g^3 \phi T, \\
\overline{m}_T^2 &= m^2 + \frac{1}{12\pi} g^3 \phi T.
\end{align*} \quad (23)$$

Doing so, one would obtain linear terms in $\phi$ from the effective potential of eq. (20). From the gauge boson sector, one would get $\sqrt{33/2[5/(64\pi^2)]} g^4 \phi T^3$, a positive linear term near the origin, as observed in ref. \([4]\). Including the Higgs and Goldstone boson sectors, one would obtain the additional linear term $[45/(512\pi^2)] g^3 \sqrt{(3/4) g^2 + h_i^2} \phi T^3$. Both results quoted above originate from subleading contributions to the self-energies. They are a consequence of the set of (daisy) diagrams considered to improve the one-loop result: as shown in ref. \([13]\), the daisy resummation is an inconsistent procedure at subleading order.

Naive power-counting arguments would suggest that, to include subleading corrections to the effective potential, one has to consider the full set of super-daisy diagrams, where all bubbles belonging to daisy diagrams have bubble-corrected propagators. This amounts to solving a system of gap equations, of the form

$$\overline{m}_i^2 = m_i^2 + \Pi_i(\overline{m}_h, \overline{m}_\chi, \overline{m}_L, \overline{m}_T), \quad (i = h, \chi, L, T), \quad (24)$$
and to writing down for the effective potential the formal expression (20), but now with the masses $m_i^2$ given by the solution of eq. (24).

In the improved theory, the expansion of the effective potential is controlled by the set of parameters

$$\alpha_i = \frac{g^2 T^2}{2\pi m_i^2}, \quad \beta_i = \frac{g^2 T}{2\pi m_i}, \quad \gamma = \frac{\phi^2}{T^2},$$

where $i = h, \chi, L, T$ and $m_i^2$ are the improved masses that solve the gap equations (24). Accordingly, we can write the solution to the gap equations (24) to leading order $O(\alpha_i)$ for $i = h, \chi, T$ as

$$m_h^2 = m_h^2(\phi) + \left(\frac{3}{16}g^2 + \frac{1}{4}h^2\right)T^2,$$
$$m_\chi^2 = m_\chi^2(\phi) + \left(\frac{3}{16}g^2 + \frac{1}{4}h^2\right)T^2,$$
$$m_L^2 = m_L^2(\phi) + \frac{11}{6}g^2T^2,$$

and

$$m_T^2 = m_T^2(\phi) + \frac{g^2T}{3\pi}m(\phi) + \frac{g^2T (m_h - m_\chi)^2}{48\pi m_h + m_\chi} - \frac{g^4\phi^2}{24\pi} \frac{T}{m_T + m_h}. \quad (27)$$

Notice that the solution (27) for $m_T$ improves over the tree-level solution $g\phi/2$ by the inclusion of $O(\beta, \alpha_T \beta \gamma)$ terms. The latter provide the leading plasma contributions to $m_T$ and should therefore be kept.

We can see from eqs. (13) and (14) that at the origin, $\phi = 0$, $\Pi_h(m_h^2(0), T) = \Pi_\chi(m_\chi^2(0), T)$ and therefore, since $m_h^2(0) = m_\chi^2(0)$, the general solutions to the gap equations (24) satisfy the condition $m_h(0) = m_\chi(0)$. Using now, from eq. (16), the fact that $\Pi_T(m_T^2(0), T) = \frac{g^2}{3\pi}m_T T$, the gap equation for $m_T$ at $\phi = 0$ can be simply written as

$$m_T^2 = \frac{g^2T}{3\pi}m_T,$$

with the two solutions

$$m_T(0) = 0 \quad (29)$$

and

$$m_T(0) = \frac{g^2T}{3\pi}. \quad (30)$$

Solution (29) coincides with (27) at $\phi = 0$. It is reached when solving the gap equation for $m_T$

$$m_T^2 = m_T^2(\phi) + \frac{g^2T}{3\pi}m_T + \frac{g^2T (m_h - m_\chi)^2}{48\pi m_h + m_\chi} - \frac{g^4\phi^2}{24\pi} \frac{T}{m_T + m_h} \quad (31)$$

to first order in $\beta, \alpha_T \beta \gamma$. In that case the validity of the perturbative expansion in the improved theory breaks down near the origin, since $m_T \rightarrow \infty$ when $\phi \rightarrow 0$. However, by going to all orders

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6 When solving the gap equations (24), the results obtained by using our approximate expressions for the self-energies, eqs. (13)–(16), disagree with those of ref. [14]. We can identify two sources of disagreement. First, ref. [14] does not distinguish between transverse and longitudinal vector boson masses on the right-hand side of eq. (24). Second, when solving the gap equations for $m_T$ and $m_T$, ref. [14] neglects the $g^2T^2$ and $h^2T^2$ corrections to the Higgs and Goldstone boson masses, which make them different from zero even in the limit $\lambda \rightarrow 0$. At subleading order this is not correct, since the masses appearing in the self-energies of eq. (24) are the unknown solutions of the gap equations.
in $\bar{\beta}$ one can reach the (non-perturbative) solution (30) and avoid the singularity of $\bar{\beta}_T$ (make the improved perturbative expansion more reliable) at the origin.

The solution to the gap equation (31), which is continuously connected to (30), is given by

$$m_T = 2\sqrt{Q(\phi)} \cos \left[ \frac{1}{3} \theta(\phi) \right] + \frac{1}{3} \left( \frac{g^2 T}{3\pi} - m_h \right),$$

(32)

where the functions $Q(\phi)$ and $\theta(\phi)$ are defined as

$$Q(\phi) = \frac{1}{9} (a^2 + 3b) \quad \cos \theta(\phi) = \frac{1}{2} \frac{9a b + 2a^3 - 27c}{(a^2 + 3b)^{3/2}},$$

(33)

with

$$a = \frac{g^2 T}{3\pi} + 2m_h, \quad c = \frac{g^4}{24\pi} \phi^2 T,$$

(34)

$$b = m^2(\phi) + \frac{g^2 T}{48\pi} \left( \frac{m_h - m_\chi}{m_h + m_\chi} \right)^2 - \left( \frac{g^2 T}{3\pi} + m_h \right) m_h.$$

(35)

For temperatures close to the critical temperature, $T \sim 100$ GeV, our solution would give $m_T(0) \sim 5$ GeV, with $m^2_T(\phi)$ quickly approaching $m^2(\phi)$ for increasing values of $\phi$. It is also interesting to consider the parameters $\bar{\beta}_i(\phi)$ ($i = h, \chi, L, T$), which can help to identify the region where our approximations are reliable. Whilst $\bar{\beta}_{h,\chi,L} \ll 1$ for all values of $\phi$, near $\phi = 0$ it is $\bar{\beta}_T \sim 1.5$, so that our improved perturbative expansion is no longer reliable, and unaccounted for non-perturbative effects could significantly change our results. However, for increasing $\phi$, also $\bar{\beta}_T$ moves fast into the perturbative regime.

Considering $O(\bar{\beta}, \bar{\alpha}_i\bar{\beta}\gamma)$, subleading corrections to $\bar{m}_h$, $\bar{m}_\chi$, $\bar{m}_L$ would amount to adding

$$\Delta m^2_h = -\frac{3 g^2}{16\pi} (\bar{m}_L + 2\bar{m}_T) T - \frac{3 g^4 \phi^2}{32\pi} \left( \frac{1}{\bar{m}_T} + \frac{1}{2\bar{m}_L} \right) T,$$

$$\Delta m^2_\chi = -\frac{3 g^2}{16\pi} (\bar{m}_L + 2\bar{m}_T) T,$$

$$\Delta m^2_L = -\frac{g^2}{16\pi} (\bar{m}_h + 3\bar{m}_\chi + 16\bar{m}_T) T - \frac{g^4 \phi^2}{16\pi} \frac{T}{\bar{m}_L + \bar{m}_h},$$

(36)

where $\bar{m}_i$ ($i = h, \chi, L, T$) are the solutions in eqs. (26) and (32)–(35). Notice that the total squared masses $m^2_i + \Delta m^2_i$ ($i = h, \chi, L, T$; $\Delta m^2_T = 0$), as given by eqs. (26), and (32)–(35), do not have any linear term in $\phi$, unlike (23), and so would not give rise to any linear term in the effective potential, as anticipated.

Before proceeding, some comments on the previous results are in order. The generation of a non-vanishing transverse mass at $\phi = 0$, eq. (30), does not contradict any general result. It is known [16] that in scalar electrodynamics at finite temperature the transverse gauge-boson mass is equal to zero. However, this theorem does not apply to non-Abelian gauge theories, and it is easy to check that the transverse mass of eq. (30) is originated by the quartic non-Abelian gauge coupling. We should also stress that in our calculational framework the masses $\bar{m}_i$ are gauge-dependent at subleading order and do not have a direct physical meaning, but are just an intermediate result in the computation of the improved effective potential. Calculations similar to ours (even if performed in different gauges) found a non-vanishing transverse mass
in finite-temperature QCD [17]. After the appearance of the first version of the present paper, our result was confirmed [18] by an independent calculation along the same lines. Other techniques for estimating the magnetic mass $m_{\text{mag}}$ have been tried in the literature, with different results. Lattice calculations [19] give estimates of $m_{\text{mag}}$ that (rescaled to the SU(2) case when appropriate) range from $\sim 0.25g^2T$ to $\sim 3g^2T$. Resummation techniques using a gauge-invariant propagator [20] find an approximate lower bound $m_{\text{mag}} \gtrsim 0.6g^2T$. Exact zero-momentum sum rules in $d = 3$ gauge theory lead to the estimate [21] $m_{\text{mag}} \sim 0.23g^2T$. A recent estimate of $m_{\text{mag}}$ based on a semiclassical method [22] also gives a very similar result.

5. We now want to discuss the possibility of improving the effective potential by making use of the previous results. As we already mentioned, an attempt to include subleading corrections coming from higher-loop diagrams is the so-called superdaisy approximation: first one solves the gap equation in the infrared limit at subleading order, then one plugs the solution $m_i$ into the expression (20). This procedure, however, has a number of shortcomings, which we are now going to describe.

The first one is [5,13] that the combinatorics of superdaisy diagrams do not fit the shifting in the $m^3$ term, eq. (20), and have to be compensated by a correction $\Delta V^{\text{comb}}$ to the effective potential. If one works with the leading-order expressions (22) for the effective masses, the correct combinatorics is automatically reproduced by the counting of different two-loop contributions corresponding to just one propagator with zero Matsubara frequency, so $\Delta V^{\text{comb}} = 0$. To subleading order, however, one should also include the two-loop diagrams where all propagators correspond to zero Matsubara frequency: in this case $\Delta V^{\text{comb}} \neq 0$. For example, in the scalar theory discussed in [13] one would find $\Delta V^{\text{comb}} = (-7\lambda)/(64\pi^2) \cdot m^2(\phi)T^2$, in full agreement with the recent results of ref. [25], once the effects of the cubic coupling are taken into account. In principle, one can compute $\Delta V^{\text{comb}}$ at arbitrary order in the expansion parameter $\beta$, as explicitly shown in refs. [25,24]. However, the superdaisy approximation is certainly not accurate beyond subleading order $\beta$ [13], so higher-loop contributions to $\Delta V^{\text{comb}}$ are only of academical interest.

The second, more serious shortcoming of the superdaisy approximation, which was recently stressed in ref. [26], is related to the fact that the masses obtained from the gap equation (24) are calculated in the zero-momentum limit. In fact, diagrams with overlapping momenta give logarithmic contributions to the effective potential that are missed by the superdaisy approximation, but are also corrections belonging to subleading order $\beta$. In spite of some interesting recent attempts [27] to achieve consistency of the superdaisy approximation at subleading order, this still remains, in our opinion, an open problem. In ref. [26] a hybrid method was proposed, where resummation limited to leading order is combined with a full two-loop computation. We shall take here an alternative approach, following the lines of ref. [18]. Given the uncertainties in the determination of the magnetic mass, and the absence of a consistent computation of $V_{\text{eff}}(\phi, T)$ at subleading order $\beta$, we shall attempt a phenomenological parametrization of subleading effects by introducing a constant magnetic mass $m_{T}(0) = \gamma \cdot (g^2T)/(3\pi)$, and by replacing $m^2_T(\phi)$ with $m^2_T(\phi) = m^2_T(0) + m^2(\phi)$ in eq. (20).

We plot as solid lines in fig. 1 the improved potentials corresponding to our approach, at...
the critical temperature $T_c \sim 97$ GeV, for the same choice of $m_h$ and $m_t$ as before, and for some representative values of $\gamma$. The result still shows, for $\gamma \lesssim 2$, a first-order phase transition, but weaker than that in the approximation corresponding to the short-dashed line. The reason is that no temperature screening was considered in the latter for $\overline{m}_T$. The subleading-order screening parametrized by our approach translates into a further weakening of the first-order phase transition, which depends on $m_h$ and $m_t$. We then compare the different approximations for various values of the Higgs mass $m_h$. We plot in fig. 2 the ratio $v(T_c)/T_c$ as a function of $m_h$; the different curves have the same meaning as in fig. 1. We can see that, for a fixed value of $\gamma$, the phase transition becomes second order \[v(T_c)/T_c = 0\] for Higgs masses greater than a critical value. Also, for a fixed value of $m_h$, there is a critical value of $\gamma$ beyond which the phase transition becomes second order. For instance, for $m_h = 60$ GeV, $m_t = 150$ GeV and $\gamma = 3$, the phase transition is second order, as can be seen from fig. 1. There is also a mild dependence of $v(T_c)/T_c$ on $m_t$, which was not explicitly exhibited in the figures. For $m_h = 60$ GeV and $m_t = 130$ GeV ($m_t = 170$ GeV) the value of $v(T_c)/T_c$ increases (decreases) by $\sim 20\%$ with respect to its value at $m_t = 150$ GeV. Anyhow, the main result that can be read off fig. 2, i.e. the turnover to a second-order phase transition, has to be taken \emph{cum grano salis}, since it occurs at large values of $m_h$, where our approximation is less reliable, or large values of $\gamma$, which could be provided by non-perturbative effects not accounted for by our calculation. On the other hand, we do not believe that the inclusion of terms of order $\lambda$ and/or $g'$ in the self-energies will qualitatively change the above picture.

6. In conclusion, we have analysed the nature of the phase transition and the structure of the finite-temperature effective potential of the Standard Model, $V_{\text{eff}}(\phi, T)$, including some higher-order infrared-dominated contributions. We have improved over the existing computations by including subleading contributions to the $T$-dependent effective masses of the transverse polarizations of vector bosons, in the zero-momentum approximation. In particular, we have found that in the 't Hooft-Landau gauge a non-vanishing magnetic mass $\overline{m}_T(0) = g^2 T/(3\pi)$ is generated. In the absence of a consistent resummation procedure for the subleading higher-loop contributions to $V_{\text{eff}}(\phi, T)$, we have studied the effects of a non-vanishing magnetic mass, phenomenologically parametrized in terms of a fudge factor $\gamma$. We have proved that, in our approach, subleading corrections do not lead to linear $\phi$-terms in $V_{\text{eff}}(\phi, T)$. We have also found a tendency towards a further weakening of the phase transition, with the possibility of a second-order phase transition for sufficiently large values of $m_h$ and $\gamma$. The computation of other subleading effects, associated with overlapping momenta in two-loop graphs, gives however corrections going in the opposite direction \[26\, 28\]. Both these results should then be taken as indications of the possible size of subleading effects, but none of the two is fully consistent at subleading order. In any case, these corrections do not seem to be sizeable enough to rescue the Standard Model as a candidate for electroweak baryogenesis.

It would be good to have available more reliable calculational methods to deal with non-perturbative effects. Some possibilities currently under investigation are lattice computations at finite temperature \[28\], the $(1/N)$-expansion \[30\], the $\epsilon$-expansion \[31\] and various $d = 3$ effective theory methods \[32\], in particular the average potential method \[33\], but none of them has yet given conclusive results for the Standard Model case and the presently allowed values of $m_t$ and $m_h$.

As a final remark, we would like to recall that the effective potential describes the static properties of the phase transition, and that many other subtle issues have to be addressed when
discussing its dynamical aspects.

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Figure captions

Fig.1: The temperature-dependent effective potential of the Standard Model, at the critical temperature and for $m_h = 60$ GeV, $m_t = 150$ GeV. The long-dashed line corresponds to the naïve one-loop approximation, neglecting the scalar loops; the short-dashed line corresponds to the approximation of eq. (22), as in ref. [8]; the solid lines correspond to the approach discussed in the text, for $\gamma = 1, 1.5, 2, 3$.

Fig.2: The ratio $v(T_c)/T_c$, as a function of the Higgs mass $m_h$, for $m_t = 150$ GeV. The long-dashed line corresponds to the one-loop approximation, the short-dashed line to the approximation of eq. (22), as in ref. [8], and the solid lines to the approach discussed in the text, for $\gamma = 1, 1.5, 2, 2.5, 3$. 