SU(3) Analysis of Annihilation Contributions and CP Violating Relations in $B \to PP$ Decays

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Abstract

Several methods proposed to measure the angle $\gamma$ in the KM unitarity triangle assumed that the tree contribution to $B^- \to \pi^- K^0$ is purely due to annihilation contributions and is negligibly small. This assumption has to be tested in order to have a correct interpretation of the experimental data.

In this paper we show that using SU(3) symmetry, the smallness of the tree contribution can be tested in a dynamic model independent way. We also derive several relations between CP violating rate differences for $B \to PP$ decays without assuming the smallness of the annihilation contributions. These relations provide important tests for the Standard Model of CP violation.

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I. INTRODUCTION

Several rare two body decay modes of $B_{u,d}$ mesons have been observed at CLEO [1]. These data have provided interesting information about the Standard Model (SM) [2–5]. With increased luminosities for B-factories at CLEO, KEK and SLAC, more useful information about rare $B_{u,d}$ decays will be obtained. The SM will be tested in detail. At present the study of rare $B_s$ decays are limited by statistics. Only some weak upper limits on the branching ratios have been obtained [6]. However, more data on $B_s$ decays will become available from LHC in the future. These data will help to further test the SM [2,7]. Theoretical predictions are, however, limited by our inability to reliably calculate many hadronic matrix elements related to $B$ decays. This prevents a full test of the Standard Model. In the lack of reliable calculations, attempts have been made to extract useful information from symmetry considerations. $SU(3)$ flavor symmetry is one of the symmetries which has attracted a lot of attentions recently [8–10]. For example, it has been shown that using $SU(3)$ symmetry it is possible to constrain [11] and to determine [4,12] one of the fundamental parameters $\gamma$ in the SM for CP violation by measuring several $B$ meson decay modes.

Some of the methods proposed to measure $\gamma$ depend on the assumption that the tree amplitude to $B^- \rightarrow \pi^- \bar{K}^0$ is negligibly small [10,12]. To correctly interpret the experimental data, the smallness of the tree contribution has to be confirmed experimentally. It is often assumed that the tree amplitude for $B^- \rightarrow \pi^- \bar{K}^0$ receives annihilation contributions only. If this is true, one has to make sure that these contributions are small. Of course one has to make sure that it is true that the decay amplitude is dominated by annihilation contributions. There have been several discussions about constraining the annihilation contributions using $SU(3)$ analysis [13]. In this paper we will use $SU(3)$ symmetry to study further related problems, but look at the problems in a different angle. We will first show how one can use $SU(3)$ relations to test the smallness of annihilation contributions. We then show that the statement that the tree amplitude receives annihilation contributions only for $B^- \rightarrow \pi^- \bar{K}^0$ is not strictly a $SU(3)$ result. We will show how to verify the smallness of the tree amplitude.
for $B^- \to \pi^- K^0$ using several B decay modes. Finally we will use SU(3) symmetry to derive several useful relations regarding CP violating rate differences without any assumption about the size of the annihilation contributions. These relations provide further tests for the SM of CP violation and also the SU(3) symmetry.

II. SU(3) DECAY AMPLITUDES FOR $B \to PP$

The quark level effective Hamiltonian up to one loop level in electroweak interaction for hadronic charmless $B$ decays, including the corrections to the matrix elements, can be written as

$$H_{eff}^q = \frac{4G_F}{\sqrt{2}} \left[ V_{ub} V_{uq}^* (c_1 O_1 + c_2 O_2) - \sum_{i=3}^{12} (V_{ub} V_{uq}^* c_i^{ac} + V_{tb} V_{tq}^* c_i^{tc}) O_i \right].$$

(1)

The operators are defined in Ref. [14]. The coefficients $c_{1,2}$ and $c_{i}^{jk} = c_i^j - c_i^k$, with $j$ indicates the internal quark, are the Wilson Coefficients (WC). These WC’s have been evaluated by several groups [14], with $|c_{1,2}| >> |c_i^j|$. In the above the factor $V_{cb} V_{cq}^*$ has been eliminated using the unitarity property of the KM matrix.

At the hadronic level, the decay amplitude can be generically written as

$$A = \langle \text{final state} | H_{eff}^q | B \rangle = V_{ub} V_{uq}^* T(q) + V_{tb} V_{tq}^* P(q),$$

(2)

where $T(q)$ contains contributions from the tree as well as penguin due to charm and up quark loop corrections to the matrix elements, while $P(q)$ contains contributions purely from penguin due to top and charm loops. The relative strength of the amplitudes $T$ and $P$ is predominantly determined by their corresponding WC’s in the effective Hamiltonian. For $\Delta S = 0$ charmless decays, the dominant contributions are due to the tree operators $O_{1,2}$ and the penguin operators are suppressed by smaller WC’s. Whereas for $\Delta S = -1$ decays, because the penguin contributions are enhanced by a factor of $V_{tb} V_{ts}^* / V_{ub} V_{us}^* \approx 55$ compared with the tree contributions, penguin effects dominate the decay amplitudes. In this case the electroweak penguins can also play a very important role [15], in particular when study CP violation in B decays [16]. One should carefully keep track of the different contributions.
The operators $O_{1,2}, O_{3-6,11,12},$ and $O_{7-10}$ transform under SU(3) symmetry as $\bar{3}_a + \bar{3}_b + 6 + \overline{15}, \bar{3},$ and $\bar{3}_a + \bar{3}_b + 6 + \overline{15},$ respectively. These properties enable us to write the decay amplitudes for $B \rightarrow PP$ in only a few SU(3) invariant amplitudes.

For the $T(q)$ amplitude, for example, we have 

$$T(q) = A_3^T B_i H(\bar{3})^i (M_1^k M_1^l) + C_3^T B_i M_1^j M_1^k H(\bar{3})^j$$

$$+ A_6^T B_i H(6)^{ij} M_1^j M_1^k + C_6^T B_i M_1^j H(6)^{jk} M_1^l$$

$$+ A_{15}^T B_i H(\overline{15})^{ij} M_1^j M_1^k + C_{15}^T B_i M_1^j H(\overline{15})^{jk} M_1^l , \quad (3)$$

where $B_i = (B_u, B_d, B_s) = (B^- , \bar{B}^0 , \bar{B}^0_s)$ is a SU(3) triplet, $M_i^j$ is the SU(3) pseudoscalar octet, and the matrices $H(i)$ contain information about the transformation properties of the operators $O_{1-12}.$

For $q = d,$ the non-zero entries of the matrices $H(i)$ are given by

$$H(\bar{3})^2 = 1 , \quad H(6)^{12} = H(6)^{23} = 1 , \quad H(6)^{21} = H(6)^{32} = -1 , \quad H(\overline{15})^{12} = H(\overline{15})^{21} = 3 , \quad H(\overline{15})^{22} = -2 , \quad H(\overline{15})^{32} = H(\overline{15})^{23} = -1 . \quad (4)$$

And for $q = s,$ the non-zero entries are

$$H(\bar{3})^3 = 1 , \quad H(6)^{13} = H(6)^{32} = 1 , \quad H(6)^{31} = H(6)^{23} = -1 , \quad H(\overline{15})^{13} = H(\overline{15})^{31} = 3 , \quad H(\overline{15})^{33} = -2 , \quad H(\overline{15})^{32} = H(\overline{15})^{23} = -1 . \quad (5)$$

Due to the anti-symmetric property of $H(6)$ in exchanging the upper two indices, $A_6$ and $C_6$ are not independent [4]. For individual decay amplitude, $A_6$ and $C_6$ always appear together in the form $C_6 - A_6.$ We will absorb $A_6$ in the definition of $C_6.$ In terms of the SU(3) invariant amplitudes, the decay amplitudes for various B meson decays are given by
\[ \Delta S = 0 \]

\[
T^{B_u}_{\pi^+\pi^0}(d) = \frac{8}{\sqrt{2}} C^T_{15};
\]

\[
T^{B_u}_{\pi^-\eta}(d) = \frac{2}{\sqrt{6}} (C^T_3 - C^T_6 + 3A^T_{15} + 3C^T_{15});
\]

\[
T^{B_u}_{K^-\eta^0}(d) = C^T_3 - C^T_6 + 3A^T_{15} - 3C^T_{15};
\]

\[
T^{B_u}_{\pi^+\pi^-}(d) = 2A^T_1 + C^T_3 + C^T_6 + A^T_{15} + 3C^T_{15};
\]

\[
T^{B_u}_{\pi^0\pi^0}(d) = \frac{1}{\sqrt{2}} (2A^T_3 + C^T_3 + C^T_6 + A^T_{15} - 5C^T_{15});
\]

\[
T^{B_u}_{K^-K^0}(d) = 2A^T_1 + C^T_6 - 3A^T_{15} - C^T_{15};
\]

\[
T^{B_u}_{\pi^0\eta}(d) = \frac{1}{\sqrt{3}} (-C^T_3 + C^T_6 + 5A^T_{15} + C^T_{15});
\]

\[
T^{B_u}_{\eta^0\pi^0}(d) = \frac{1}{\sqrt{2}} (2A^T_3 + 3C^T_3 + C^T_6 - A^T_{15} + C^T_{15});
\]

\[
T^{B_u}_{K^+\pi^+}(d) = C^T_3 + C^T_6 - A^T_{15} + 3C^T_{15};
\]

\[
T^{B_u}_{K^0\eta^0}(d) = -\frac{1}{\sqrt{2}} (C^T_3 + C^T_6 - A^T_{15} - 5C^T_{15});
\]

\[
T^{B_u}_{K^0\eta^0}(d) = -\frac{1}{\sqrt{6}} (C^T_3 + C^T_6 - A^T_{15} - 5C^T_{15});
\]

\[ \Delta S = -1 \]

\[
T^{B_u}_{\pi^+K^0}(s) = C^T_3 - C^T_6 + 3A^T_{15} - 3C^T_{15};
\]

\[
T^{B_u}_{\pi^-K^0}(s) = \frac{1}{\sqrt{2}} (C^T_3 - C^T_6 + 3A^T_{15} + 7C^T_{15});
\]

\[
T^{B_u}_{\eta^0K^0}(s) = \frac{1}{\sqrt{6}} (-C^T_3 + C^T_6 - 3A^T_{15} + 9C^T_{15});
\]

\[
T^{B_u}_{\pi^+K^0}(s) = C^T_3 + C^T_6 - A^T_{15} + 3C^T_{15};
\]

\[
T^{B_u}_{\pi^0\pi^0}(s) = -\frac{1}{\sqrt{2}} (C^T_3 + C^T_6 - A^T_{15} - 5C^T_{15});
\]

\[
T^{B_u}_{\pi^0\pi^0}(s) = -\frac{1}{\sqrt{6}} (C^T_3 + C^T_6 - 3A^T_{15} - 5C^T_{15});
\]

\[
T^{B_u}_{\pi^0\pi^0}(s) = 2(A^T_3 + A^T_{15});
\]

\[
T^{B_u}_{\pi^-\pi^+}(s) = \sqrt{3} (A^T_3 + A^T_{15});
\]

\[
T^{B_u}_{K^0\pi^+}(s) = 2A^T_3 + 3C^T_3 + C^T_6 + A^T_{15} + 3C^T_{15};
\]

\[
T^{B_u}_{K^0\pi^+}(s) = 2A^T_3 + C^T_3 + C^T_6 - A^T_{15} - 3C^T_{15};
\]

\[
T^{B_u}_{K^0\pi^+}(s) = 2A^T_3 + C^T_3 - C^T_6 - 3A^T_{15} - C^T_{15};
\]

\[
T^{B_u}_{K^0\pi^+}(s) = \frac{2}{\sqrt{3}} (C^T_6 + 2A^T_{15} - 2C^T_{15});
\]

\[
T^{B_u}_{K^0\pi^+}(s) = \sqrt{2} (A^T_3 + 2C^T_3 - A^T_{15} - 2C^T_{15}).
\]

The amplitudes for \( P(q) \) in terms of SU(3) invariant amplitudes can be obtained in a similar way. We will indicate the corresponding amplitudes by \( A_i^P \) and \( C_i^P \).

Many analysis have been carried out using SU(3) classification of quark level diagrams [10]. In most cases such an analysis will obtain the same results as the use of SU(3) invariant amplitudes. However, in some cases the classification according to quark level diagrams without care would lose some vital information. An interesting example is the tree amplitude for \( B^- \rightarrow \pi^- \bar{K}^0 \). Using quark level diagram analysis, when the annihilation contributions are neglected, the tree operators do not contribute to this decay. This implies, in the SU(3) invariant amplitude language, that

\[
C^T_3 - C^T_6 - 3A^T_{15} = 0. \tag{6}
\]

This, however, is not generally true as has been confirmed by model calculations [17,18]. In the quark level diagram classification, there are only four independent amplitudes whereas the general SU(3) invariant classification, there are five independent amplitudes [4]. Some information related to different combinations of quark level diagrams and their phases have
been lost in the naive quark level diagram analysis. Specifically, four quark operators containing $\bar{d}\Gamma_1 d\bar{q}\Gamma_2 b$ and $\bar{s}\Gamma_1 sq\Gamma_2 b$ types of terms, where $\Gamma_i$ indicate appropriate Dirac matrices, appear in SU(3) invariant amplitudes do not appear in the naive tree quark diagram analysis. For this reason, we will use the SU(3) invariant amplitude to carry out our analysis.

III. TEST THE SMALLNESS OF ANNIHILATION CONTRIBUTIONS

The amplitudes $A_{3,15}$ correspond to annihilation contributions. Here we refer the amplitudes with one of the light quark in the effective Hamiltonian corresponds to the light quark inside the B mesons to be annihilation amplitudes. The amplitudes $A_{3,15}$ are annihilation amplitudes can be understood by noticing that the light quark index in the $B$ mesons are contracted with the Hamiltonian [19]. The A and E type of contributions in the quark diagram classification are linear combinations of $A_3$ and $A_{15}$. It has been argued that these contributions are small based on model calculations [10]. At present the annihilation contributions can not be reliably calculated. In view of this, it is important to be able to test the smallness of the annihilation contributions experimentally.

In this section we show that using SU(3) relations, the size of the annihilation contributions can be measured independent dynamic models for the matrix elements and therefore the smallness of these amplitudes can be tested. Two types of tests can be carried out. One of them is to test the smallness of the annihilation contributions of the SU(3) invariant amplitudes, and another is to test the smallness of tree contribution to $B^- \to \pi^- K^0$.

The best way to test the smallness of the annihilation contributions is to use processes involving only $A_{3,15}$. From discussions of the previous section, we find that there are only three such processes. They are: a) $\bar{B}^0 \to K^+ K^-$; b) $B_s \to \pi^- \pi^+$; And c) $B_s \to \pi^0 \pi^0$. Their decay amplitudes are given by

$$A(B_d \to K^+ K^-) = 2V_{ub}V_{ud}^*(A_3^T + A_{15}^T) + 2V_{tb}V_{td}^*(A_3^P + A_{15}^P),$$
$$A(B_s \to \pi^- \pi^+) = 2V_{ub}V_{us}^*(A_3^T + A_{15}^T) + 2V_{tb}V_{ts}^*(A_3^P + A_{15}^P).$$
\[ A(B_s \to \pi^0\pi^0) = \frac{1}{\sqrt{2}} A(B_s \to \pi^+\pi^-). \] (7)

It is clear that these decays receive annihilation contributions only. However, there is a crucial difference between a), and b) and c). The decay amplitude for a) is dominated by the tree contribution and the amplitudes for b) and c), being \( \Delta S = -1 \) processes, are dominated by penguin contributions. If annihilation contributions are small, these processes will all have small branching ratios. At present, these three modes have not been observed. The best constraint is from \( \bar{B}_0 \to K^+K^- \) with an upper bound on the branching ratio \( 0.24 \times 10^{-5} \) at the 90% confidence level from CLEO \([1]\). However this still allow substantial annihilation contributions. The annihilation contributions to the tree amplitude to \( B^- \to \pi^-\bar{K}^0 \) can reach 10% of the total amplitude. We have to wait more data to verify the smallness of the annihilation contributions. Conclusions drawn with such assumption should be viewed with caution.

One should be aware that even the annihilation contributions are small, it does not mean that the tree amplitude for \( B^- \to \pi^-\bar{K}^0 \) is small. One has also to verify that the tree amplitude receives annihilation contributions only. Let us now study how this can be verified. From the SU(3) decay amplitudes listed in the previous section, we see that the tree contribution to this process is given by

\[ \mathcal{T}_{B_u}^{\pi^-\bar{K}^0}(s) = C_3^T - C_6^T + 3A_{15}^T - C_{15}^T. \] (8)

This is not a pure annihilation process as for \( \bar{B}^0 \to K^+K^- \), \( B_s \to \pi^-\pi^+ \), and \( B_s \to \pi^0\pi^0 \). The tree amplitude to \( B^- \to \pi^-\bar{K}^0 \) is pure annihilation contribution only in the factorization calculation where \( C_3^T - C_6^T - C_{15}^T = 0 \). In order this to be true, not only the magnitude of the invariant amplitudes should be arranged, but also the phases of these amplitudes must be arranged to have the cancellation. However, from our experience with K and D systems, we know that different SU(3) (or isospin) amplitudes develop different phases. It is quite possible that the same situation happens in B system \([20,21]\). To have a better understanding of the situation, let us perform a calculation of the tree decay amplitude for \( \mathcal{T}(B^- \to \pi^-\bar{K}^0) \)
in the factorization approximation neglecting the annihilation contributions, but
with insertions of possible final state interaction phases for different amplitudes. We have

\[ T(B^{-} \rightarrow \pi^{-} \bar{K}^{0}) = V_{ub}V_{us}^{*}(e^{i\delta_{1}T_{1}} - e^{i\delta_{3}T_{3}}), \]

\[ T_{1} = T_{3} = \frac{1}{3}i \frac{G_{F}}{\sqrt{2}}[(c_{1} + \frac{c_{2}}{N})f_{\pi}^{B_{K}}(m_{\pi}^{2})(m_{B}^{2} - m_{K}^{2}) + (\frac{c_{1}}{N} + c_{2})f_{K}^{B_{\pi}}(m_{K}^{2})(m_{B}^{2} - m_{\pi}^{2})], \]  \[ (9) \]

where \( N \) is the number of colors. We have used the following definitions for the decay
constants and form factors

\[ < P | \bar{q}\gamma_{\mu}(1 - \gamma_{5})u|0 > = i f_{P}P_{\mu}, \]

\[ < P(k) | \bar{q}\gamma_{\mu}b|\bar{B}^{0}(p) > = (k + p)_{\mu}f_{B_{P}}^{B_{P}} + (m_{P}^{2} - m_{B}^{2})\frac{q_{\mu}}{q^{2}}(f_{B_{P}}^{B_{P}}(q^{2}) - f_{0}^{B_{P}}(q^{2})), \]  \[ (10) \]

where \( q = p - k \). The first term \( e^{i\delta_{1}T_{1}} \) in the amplitude \( T(B^{-} \rightarrow \pi^{-} \bar{K}^{0}) \) is equal to \( C_{3}^{T} - C_{6}^{T} \) which is an \( I = 1/2 \) amplitude while the second term \( e^{i\delta_{3}T_{3}} \) is equal to \( C_{15}^{T} \) which is an \( I = 3/2 \) amplitude. We see that the cancellation happens only when \( \delta_{1} = \delta_{3} \) which is an additional assumption about the dynamics beyond SU(3) symmetry. It has been shown that present data does not exclude large final phase difference \( \delta_{1} - \delta_{3} \) \[ 3,21 \]. The smallness of \( C_{3} - C_{6} - C_{15} \) has to be tested experimentally.

To have a model independent test of this cancellation, that is, \( C_{3} - C_{6} - C_{15} = 0 \), one needs to find processes which depend on the same combination of the SU(3) invariant amplitudes as the tree amplitude for \( B^{-} \rightarrow \pi^{-}\bar{K}^{0} \). To this end we carry out an analysis similar to Refs. \[ 18 \] for \( B^{-} \rightarrow K^{-}K^{0} \) using the parametrization of the SU(3) decay amplitudes in the previous section. We have

\[ A(B^{-} \rightarrow K^{-}K^{0}) = V_{ub}V_{ud}^{*}T_{K^{-}K^{0}}^{B_{u}}(d) + V_{tb}V_{td}^{*}P_{K^{-}K^{0}}^{B_{u}}(d). \]  \[ (11) \]

As have been mentioned earlier that the relative strength of the T and P amplitudes is predominantly determined by their WC’s, to a good approximation \( A(B^{-} \rightarrow K^{-}K^{0}) \approx V_{ub}V_{ud}^{*}T_{K^{-}K^{0}}^{B_{u}}(d) \). In the SU(3) limit

\[ T_{K^{-}K^{0}}^{B_{u}}(s) = T_{K^{-}K^{0}}^{B_{u}}(d) = C_{3} - C_{6} + 3A_{15} - C_{15}. \]  \[ (12) \]
Once the branching ratio for $B^- \to K^-K^0$ is measured, we have information about the size of $|T_{K^0_{u\pi^-}}^B|$. If experimentally, the branching ratio $B^- \to K^-K^0$ indeed turns out to be small, this would confirm the smallness of $C_3-C_6-C_{15}$ if annihilation contributions are also found to be small from the branching ratio measurements for $\bar{B}^0 \to K^+K^-$, $B_s \to \pi^+\pi^-$ and $B_s \to \pi^0\pi^0$. In this case conclusions drawn with the assumption, $T_{\pi^-K^0}^{B_s}(s) = 0$ would be good ones. Otherwise the results obtained with this assumption can not be trusted. Unfortunately, at present experimental upper bound, with $Br(B^- \to K^-K^0) < 0.93 \times 10^{-5}$ at 90% confidence level from CLEO [1], still allow large tree contributions to $B^- \to \pi^-\bar{K}^0$.

We stress that the smallness for annihilation contributions and the smallness of the tree amplitude for $B^- \to \pi^-\bar{K}^0$ are two independent assumptions and should be tested separately as discussed in the above. These tests have important implications for the determination of the angle $\gamma$ in the KM unitarity triangle because some of the methods proposed require that the tree amplitude is small such that $A(B^- \to \pi^-K^0) = \bar{A}(B^+ \to \pi^+K^0)$. At present this is not well tested. We have to wait experiments in the future to tell us more.

IV. CP ASYMMETRY RELATION BETWEEN $B$ DECAYS

From the previous discussions, we see that predictions with certain dynamic assumptions about the amplitudes suffer from possible uncertainties and need to be tested. It is desirable that tests for the SM can be performed in a dynamic model independent way. In this section we will derive several such relations which can be used to test the Standard Model. These relations are related to CP violating rate difference defined as

$$\Delta(B \to PP) = \Gamma(B \to PP) - \Gamma(\bar{B} \to \bar{P}\bar{P}).$$

SU(3) symmetry relates $\Delta S = 0$ and $\Delta S = -1$ decays. One particularly interesting class of relations are the ones with $T(d) = T(s) = T$ and $P(d) = P(s) = P$. For this class of decays, we have [19, 22]

$$A(d) = V_{ub}V_{ud}^*T + V_{tb}V_{td}^*P,$$
\[ A(s) = V_{ub}V_{us}^{*}T + V_{tb}V_{ts}^{*}P. \]  

(14)

Due to different KM matrix elements involved in \( A(d) \) and \( A(s) \), although the amplitudes have some similarities, the branching ratios are not simply related. However, when considering rate difference, \( \Delta(B \rightarrow PP) \), the situation is dramatically different. Because a simple property of the KM matrix element [23],

\[ \text{Im}(V_{ub}V_{ud}^{*}V_{tb}V_{td}) = -\text{Im}(V_{ub}V_{us}^{*}V_{tb}V_{ts}), \]

we find that in the SU(3) limit,

\[ \Delta(d) = -\Delta(s), \]

(15)

where \( \Delta(i) = (|A(i)|^2 - |\bar{A}(i)|^2)\lambda_{ab}/(8\pi m_B) \) is the CP violating rate difference defined earlier and

\[ \lambda_{ab} = \sqrt{1 - 2(m_a^2 + m_b^2)/m_B^2 + (m_a^2 - m_b^2)^2/m_B^4} \]

with \( m_{a,b} \) being the masses of the two particles in the final state.

In the SU(3) limit we find the following equalities:

1) \( \Delta(B^{-} \rightarrow K^{-}K^{0}) = -\Delta(B^{-} \rightarrow \pi^{-}\bar{K}^{0}) \),

2) \( \Delta(\bar{B}^{0} \rightarrow \pi^{-}\pi^{+}) = -\Delta(B_{s} \rightarrow K^{-}K^{+}) \),

3) \( \Delta(\bar{B}^{0} \rightarrow K^{-}K^{+}) = -\Delta(B_{s} \rightarrow \pi^{-}\pi^{+}) \\
    = -2\Delta(B_{s} \rightarrow \pi^{0}\pi^{0}) \),

4) \( \Delta(\bar{B}^{0} \rightarrow \bar{K}^{0}K^{0}) = -\Delta(B_{s} \rightarrow K^{0}\bar{K}^{0}) \),

5) \( \Delta(\bar{B}^{0} \rightarrow \pi^{+}K^{-}) = -\Delta(B_{s} \rightarrow K^{+}\pi^{-}), \)

6) \( \Delta(\bar{B}^{0} \rightarrow \pi^{0}\bar{K}^{0}) = -\Delta(B_{s} \rightarrow K^{0}\pi^{0}) \\
    = 3\Delta(\bar{B}^{0} \rightarrow \eta_{8}\bar{K}^{0}) = -3\Delta(B_{s} \rightarrow K^{0}\eta_{8}). \)

(16)

Note that in the SU(3) limit, beside the above relations there are several other relations for the branching ratios, that is, some of the decay amplitudes are actually equal in the SU(3) limit. We have

\[ \Gamma(B_{s} \rightarrow \pi^{+}\pi^{-}) = 2\Gamma(B_{s} \rightarrow \pi^{0}\pi^{0}), \]

\[ \Gamma(\bar{B}^{0} \rightarrow \pi^{0}\bar{K}^{0}) = 3\Gamma(\bar{B}^{0} \rightarrow \eta_{8}\bar{K}^{0}), \]

\[ \Gamma(B_{s} \rightarrow K^{0}\pi^{0}) = 3\Gamma(B_{s} \rightarrow K^{0}\eta_{8}). \]  

(17)
The last two equalities for the decay rate involve $\eta_8$ which mixes with $\eta_1$. It will be difficult to carry out these tests. The branching ratio for the first one may be small due to pure annihilation contributions, although it has to be tested independently. This test will also be difficult to carry out.

If it turns out that the annihilation contributions are all small as can be tested in $B^- \rightarrow K^-K^0$, $B_s \rightarrow \pi^+\pi^-$ and $B_s \rightarrow \pi^0\pi^0$, there are additional relations for rate differences. We find

$$1) \approx 4),$$
$$2) \approx 5),$$
$$6) \approx \Delta(\bar{B}^0 \rightarrow \pi^0\pi^0) \quad (18)$$

In the limit that annihilation contributions are small, it is difficult to perform tests related to 1), 3) and 4) because the decay rates involved are all small. The equalities of 2) and 5) provide the best chances to test the SM.

The above non-trivial equalities do not depend on the numerical values of the final state rescattering phases. Of course these relations are true only for the SM with three generations. Therefore they provide tests for the three generation model.

The relations obtained above hold in the SU(3) limit. Let us now study how these relations are modified when SU(3) breaking effects are included. Since no reliable calculational tool exists, in the following we will use factorization approximation neglecting the annihilation contributions to estimate the SU(3) breaking effects for 2) for illustration. We have

$$T_{B_d}^{d\pi^-\pi^+}(s) = i\frac{G_F}{\sqrt{2}} f_{\pi\pi} F_0 \left( \frac{m_\pi^2}{m_B^2-m_\pi^2} \right) \left[ \frac{1}{N_c} c_1 + c_2 + \frac{1}{N} c_{uc}^3 + c_{uc}^4 + \frac{1}{N} c_{uc}^9 + c_{uc}^{10} \right]$$
$$+ \frac{2m_\pi^2}{(m_b-m_u)(m_u+m_d)} \left[ \frac{1}{N_c} c_{uc}^3 + c_{uc}^4 + \frac{1}{N} c_{uc}^7 + c_{uc}^8 \right],$$

$$T_{B_s}^{sK^-\pi^+}(s) = i\frac{G_F}{\sqrt{2}} f_{K\pi} F_0 \left( \frac{m_K^2}{m_B^2-m_K^2} \right) \left[ \frac{1}{N_c} c_1 + c_2 + \frac{1}{N} c_{uc}^3 + c_{uc}^4 + \frac{1}{N} c_{uc}^9 + c_{uc}^{10} \right]$$
$$+ \frac{2m_K^2}{(m_b-m_u)(m_u+m_s)} \left[ \frac{1}{N_c} c_{uc}^3 + c_{uc}^4 + \frac{1}{N} c_{uc}^7 + c_{uc}^8 \right]. \quad (19)$$
The amplitudes $P(d, s)$ are obtained by setting $c_{1,2} = 0$ and replacing $c_{i}^{ac}$ by $c_{i}^{ae}$.

Using the fact $m_{\pi}^{2}/(m_{u} + m_{d}) \approx m_{K}^{2}/(m_{u} + m_{s})$, we obtain

$$
\Delta(\bar{B}^{0} \rightarrow \pi^{+}\pi^{-}) \approx -\frac{(f_{\pi}F_{0}^{B\pi}(m_{\pi}^{2}))^{2}}{(f_{K}F_{0}^{B_{s}K}(m_{K}^{2}))^{2}} \frac{\lambda_{\pi\pi}}{\lambda_{K\pi}} \Delta(B_{s} \rightarrow K^{+}K^{-}),
$$

(20)

In the above the final state interaction phases for different amplitudes have been assumed to be zero. We point out that as long as these phases satisfy SU(3) symmetry relations, the above equation does not change.

Similarly we also have

$$
\Delta(\bar{B}^{0} \rightarrow \pi^{+}\pi^{-}) \approx -\frac{(f_{\pi}F_{0}^{B\pi}(m_{\pi}^{2}))^{2}}{(f_{K}F_{0}^{B_{s}K}(m_{K}^{2}))^{2}} \frac{\lambda_{\pi\pi}}{\lambda_{K\pi}} \Delta(\bar{B}^{0} \rightarrow \pi^{+}K^{-})
$$

$$
\approx \frac{(f_{\pi}F_{0}^{B\pi}(m_{\pi}^{2}))^{2}}{(f_{K}F_{0}^{B_{s}K}(m_{K}^{2}))^{2}} \frac{\lambda_{\pi\pi}}{\lambda_{K\pi}} \Delta(\bar{B}_{s} \rightarrow K^{+}\pi^{-}).
$$

(21)

The form factors are usually assumed to have pole form dependence on $q^{2}$. For the above cases the form factors are approximately equal to their values at $q^{2} = 0$ because the $B$ meson mass is much larger than $\pi$ and $K$ meson masses. For the same reason, $\lambda_{\pi\pi}/\lambda_{\pi\pi, K} \approx 1$.

Independent of the specific value for the ratio $r = F_{0}^{B_{s}K}(0)/F_{0}^{B\pi}(0)$, we obtain the following relations:

$$
\Delta(\bar{B}^{0} \rightarrow \pi^{+}\pi^{-}) \approx -\frac{f_{\pi}^{2}}{f_{K}^{2}} \Delta(\bar{B}^{0} \rightarrow \pi^{+}K^{-}),
$$

$$
\Delta(\bar{B}_{s} \rightarrow K^{+}K^{-}) \approx -\frac{f_{K}^{2}}{f_{\pi}^{2}} \Delta(\bar{B}_{s} \rightarrow \pi^{-}K^{+}).
$$

(22)

The first equality in the above has already been obtained before [19]. The ratio $r$ is expected to be about one. If this is indeed the case, one would obtain $\Delta(\bar{B}^{0} \rightarrow \pi^{+}\pi^{-}) \approx \Delta(\bar{B}_{s} \rightarrow K^{+}\pi^{-})$.

It has been shown that the normalized asymmetry, that is, the rate difference divided by the averaged particle and anti-particle branching for $\bar{B}^{0} \rightarrow \pi^{+}K^{-}$, can be as large as 20% [3,21]. Such a large value can be measured in the future at B factories. The Standard Model can be tested using the relations discussed in this section.
V. CONCLUSIONS AND DISCUSSIONS

Several methods proposed to measure the fundamental parameter $\gamma$ in the KM unitarity triangle depend on the assumption that, $A(B^- \to \pi^- K^0) = \bar{A}(B^+ \to K^0 \pi^+)$. In order this assumption to hold it is not sufficient to only require the annihilation contributions to be small. One has also to show that the tree amplitude only receives annihilation contributions. In this paper we have shown that these two conditions can be separately tested at B factories in the near future. Of course one should also keep an open mind the possibility that the annihilation contribution $A_{15}$ is not small, but the total tree contribution $C_T^{3} - C_T^{6} + 3A_{15}^{T} + C_T^{15}$ is small. This can also be tested by measuring $B^- \to K^- K^0$ branching ratio because the dominant contribution to the amplitude is proportional to the tree amplitude for $B^- \to \pi^- K^0$.

We have also derived several useful relations using SU(3) symmetry without any additional dynamic model assumptions about the amplitudes. These relations will provide further tests for the Standard Model of CP violation. The SU(3) symmetry is expected to be broken in reality. Therefore the validity about some of the methods for measuring $\gamma$ and the relations derived in this paper remain to be a problem to be studied.

Let us conclude with a discussion about the validity of SU(3) relations for B meson decays. We have used factorization approximation to provide some idea about how the SU(3) breaking effects affect the results. We stress that these results are only indicative. One should not exclude the possibility that the experimental results obtained will be actually more closer to the SU(3) limit results. Even though we know that SU(3) symmetry is broken in reality, the breaking pattern may be much more subtle than a simple decay constant rescaling as indicated from our factorization calculations in previous sections. To see why this might happen let us consider $B^- \to D^0 \pi^-$ and $B^- \to D^0 K^-$ decays.

We find that in the SU(3) limit the ratio $R = Br(B^- \to D^0 K^-)/Br(B^- \to D^0 \pi^-)$ is equal to $|V_{us}/V_{ud}|^2(\lambda_{DK}/\lambda_{D\pi})$. The value $R = 0.049$ obtained in the SU(3) limit is more closer to the experimental central value of $0.055 \pm 0.015 \pm 0.005$ from CLEO [24] than the
factorization estimate with SU(3) breaking $R \approx (f_K^2/f_\pi^2)|V_{us}/V_{ud}|^2(\lambda_{DK}/\lambda_{D\pi}) \approx 0.07$. Of course the experimental result is consistent with both predictions at the present. The point of this example is that one should be careful about factorization estimate of SU(3) breaking effects. SU(3) relations may turn out to be better than expected. We have to wait more experimental data to provide us with more information.

The above discussion also applies to the relation between the tree amplitude $A_T$ for $B^- \to \pi^-\pi^0$ and the $I = 3/2$ tree amplitude $A_{3/2}^T$ for $B^- \to \pi^0K^-$ and $B^- \to \pi^-\bar{K}^0$ decays. The experimental value may turn out to be closer to the SU(3) limit result than the factorization estimated relation $A_{3/2}^T = (f_K^2/f_\pi^2)|V_{us}/V_{ud}|^2A_T$. This also have important implications for the determination of $\gamma$. Any method to determine $\gamma$ using this relation should be analyzed with care.

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