$N_T = 4$ equivariant extension of the
3D topological model of Blau and Thompson

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Abstract

The Blau–Thompson $N_T = 2, D = 3$ non–equivariant topological model, obtained through the so–called ,novel' twist of $N = 4, D = 3$ super Yang–Mills theory, is extended to a $N_T = 4, D = 3$ topological theory. The latter, formally, may be regarded as a topological non–trivial deformation of the $N_T = 2, D = 4$ Yamron–Vafa–Witten theory after dimensional reduction to $D = 3$. For completeness also the dimensional reduction of the half–twisted $N_T = 2, D = 4$ Yamron model is explicitly constructed.

1. Introduction

Topological quantum field theory (TQFT) has become an interesting link between physics and mathematics. It has connected diverse areas and many of the advanced ideas in QFT and string theory with the ones involved in topology (see, e.g., Refs. [1] – [13]). TQFTs with simple, $N_T = 1$, topological symmetry have been widely studied in different space–time dimensions, e.g., the topological sigma models in $D = 2$ [14], the Chern–Simons gauge theory in $D = 3$ [1] and the Donaldson–Witten theory in $D = 4$ [3], namely, from both the perturbative and the non–perturbative point of view. TQFTs with extended, $N_T > 1$, topological symmetry have also been considered, e.g., as effective world volume theories of D3–branes [15], D2–branes [16] and M5–branes [17] in string theory wrapping supersymmetric cycles of higher dimensional compactification manifolds. They provide a promising arena for testing key ideas as $S$–duality [18, 19], large $N$–dynamics of supersymmetric gauge theories and, eventually, the AdS/CFT conjecture [20].

Usually, the topological supersymmetry is realized equivariantly, i.e., prior to the introduction of gauge ghosts the cohomology closes only modulo equations of motions. However, by introducing the ,novel' topological twist of the $N = 4, D = 3$ super Yang–Mills theory (SYM) Blau and Thompson [16] obtained a $N_T = 2, D = 3$ topological model whose topological shift symmetry is strictly nilpotent even prior to the introduction of the gauge ghosts. Such theories are intrinsic for $D = 3$ and, obviously, quite special. After carrying out a dimensional reduction to $D = 2$ one obtains a $N_T = 4$ Hodge–type cohomological theory [21]. In such theories, completely analogous to de Rham cohomology, there exists besides the topological shift operator also a co–shift operator both of which are nilpotent and interrelated by a discrete Hodge–type duality.

Motivated by this interesting possibility we looked for an equivariant $N_T = 4$ extension of the Blau–Thompson model which might provide, after dimensional reduction, another $N_T = 8$ Hodge–type cohomological theory. If such an equivariant extension would exist it had to be equivalent to one of the various non–equivalent $N_T \geq 2, D = 3$ topological theories.

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Diagram 1: Different twisting of $N = 2$, $N = 4$ and $N = 8$ SYM theories and interrelations of various topological theories in $D = 3$ and $D = 4$. 

- $N_T = 1, D = 4$ SYM: Donaldson–Witten model $SO_E(4)$
- $N_T = 2, D = 4$ SYM: $SU_R(2) \otimes SO_E(4)$
- $N_T = 2, D = 3$ SYM: $SU_R(2) \otimes SU_N(2) \otimes SO_E(3)$
- $N_T = 2, D = 3$ super–BF: $SU_N(2) \otimes SO_E'(3)$
- $N_T = 1, D = 3$ SYM: Yamron (half-twisted) model $SU_R(2) \otimes SO_E'(4)$
- $N_T = 1, D = 4$ SYM: Donaldson–Witten model $SO_E(4)$
- $N_T = 1, D = 4$ TFT: Marcus (B) model $SU_R(2) \otimes SO_E'(4)$
- $N_T = 2, D = 4$ SYM: $SU_1(4) \otimes SO_E(4)$
- $N_T = 2, D = 4$ SYM: $SU_R(2) \otimes SO_E'(4)$
- $N_T = 2, D = 4$ SYM: Yamron–Vafa–Witten (A) model $SU_R(2) \otimes SO_E'(4)$
- $N_T = 2, D = 3$ TFT: Blau–Thompson model with non-equivariant cohomology $SU_R(2) \otimes SO_E' (3)$
- $N_T = 2, D = 3$ TFT: equivariant extension of Blau–Thompson model $SU_R(2) \otimes SU_R(2) \otimes SO_E'(3)$
- $N_T = 2, D = 4$ SYM: $SU_I(4) \otimes SO_E(4)$
- $N_T = 2, D = 4$ SYM: $SU_R(2) \otimes SU_R(2) \otimes SU_N(2) \otimes SO_E'(3)$
- $N_T = 1, D = 6$ SYM: $SO_E(6)$
- $N_T = 2, D = 6$ SYM: $SU_R(2) \otimes SO_E(6)$
- $N_T = 2, D = 8$ SYM: $SU_R(2) \otimes SU_R(2) \otimes SU_N(2) \otimes SO_E'(3)$
- $N_T = 2, D = 10$ SYM: $SO_E(10)$

Deformation
Topological twist
Dimensional reduction
A complete group theoretical classification of all the topological twists of \( N = 4 \) and \( N = 8 \) super–Yang–Mills theory (SYM) in \( D = 3 \) has been given in [16]. According to that analysis in both cases there exist exactly two possible topological twists (see Diagram).

In the case \( N = 4 \), \( D = 3 \) SYM the ’standard’ \( \mathcal{N}_T = 2 \) topological twist gives the super–BF model whereas the so–called ’novel’ \( \mathcal{N}_T = 2 \) topological twist by construction leads to a model, henceforth called Blau–Thompson (BT) model, which precisely enjoys the above mentioned property, i.e., which has no bosonic scalar fields and hence no underlying equivariant cohomology.

Also in Ref. [16] it has been shown, without constructing the corresponding models explicitly, that there exist only two (partial) topological twists of \( N = 8 \), \( D = 3 \) SYM, provided one excludes theories involving higher spin fields. These models having an underlying \( \mathcal{N}_T = 4 \) and \( \mathcal{N}_T = 2 \) topological symmetry describe world–volume theories of D2–brane instantons wrapping supersymmetric three–cycles of Calabi–Yau three–folds and \( G_2 \)–holonomy Joyce manifolds.

Moreover, in [16] it has been shown that the \( \mathcal{N}_T = 4 \), \( D = 3 \) model is just the dimensional reduction of either of the two \( \mathcal{N}_T = 2 \), \( D = 4 \) models to \( D = 3 \), namely, the so–called \( A \)–model constructed by Yamron [22] and Vafa–Witten [18] and the \( B \)–model constructed by Marcus [23], whereas the \( \mathcal{N}_T = 2 \), \( D = 3 \) model arises from the dimensional reduction of the ’half–twisted’ \( \mathcal{N}_T = 1 \), \( D = 4 \) theory [22].

In this paper we construct explicitly both the \( \mathcal{N}_T = 2 \) and the \( \mathcal{N}_T = 4 \) model in \( D = 3 \) dimensions. Thereby, we restrict ourselves to Euclidean space–time. In seaching of cohomological Hodge–type theories this restriction is convenient since, after performing a dimensional reduction to \( D = 2 \), the topological co–shift symmetry is partially encoded in the vector supersymmetry. Under that restriction both theories can be characterized uniquely by imposing besides the topological shift symmetry \( Q^a \) (and possibly \( \overline{Q}^a \)) also the vector supersymmetry \( Q_\alpha^a \) (and possibly \( \overline{Q}_\alpha^a \)). Furthermore, the latter model coincides with the \( \mathcal{N}_T = 4 \) equivariant extention of the Blau–Thompson model which we consider at first instance. The equivalence of both models obtains by deforming explicitly the Yamron–Vafa–Witten theory, i.e., the \( A \)–model, after dimensional reduction to \( D = 3 \). In addition, we remark that we have not been able to find an appropiate equivariant extension of the novel topological model preserving the number of topological supercharges \( \mathcal{N}_T = 2 \). On the other hand, due to the completeness of the classification [16], such an extension should really not be possible.

The paper is organized as follows. Following Ref. [16], in Section 2 we recall the structure of the two possible topological twists of \( N = 4 \), \( D = 3 \) SYM, leading to the \( \mathcal{N}_T = 2 \), \( D = 3 \) super–BF model [24] – [26] and the novel \( \mathcal{N}_T = 2 \), \( D = 3 \) Blau–Thompson model. In Section 3 we construct a \( \mathcal{N}_T = 4 \) equivariant extension of the Blau Thompson model. In Section 4 it is shown that this extension coincides with the Yamron–Vafa–Witten theory after carrying out a dimensional reduction to \( D = 3 \), i.e., with the \( \mathcal{N}_T = 4 \) topological twist of \( N = 8 \), \( D = 3 \) SYM. In Section 5 we construct the \( \mathcal{N}_T = 2 \) topological model of \( N = 8 \), \( D = 3 \) SYM which is the extension of the \( \mathcal{N}_T = 2 \), \( D = 3 \) super–BF model by a spinorial hypermultiplet.

2. The two topological twists of \( N = 4 \), \( D = 3 \) SYM theory:
Super–BF and Blau–Thompson model

In this section we briefly recall the two possible topological twists of \( N = 4 \) SYM in \( D = 3 \) dimensional Euclidean space–time which obtains by dimensional reduction of \( N = 1 \), \( D = 6 \) SYM, either directly or via \( N = 2 \), \( D = 4 \) SYM, to \( D = 3 \) (cf., upper half of the Diagram). As pointed out in Ref. [27], this theory has a global \( (SU(2)_R \otimes SU(2)_N) \otimes SU(2)_E \) symmetry, where the \( SU(2)_R \) group primarily results from the symmetry of the fermions of \( N = 1 \) SYM in \( D = 6 \).
After dimensional reduction the gauge multiplet of $N = 4$ SYM in $D = 3$ contains three scalar fields which transform in the vector representation under the group $SU(2)_{N}$, the internal Euclidean symmetry group arising from the decomposition $\text{Spin}(6) \rightarrow SU(2)_{N} \otimes SU(2)_{E}$. The symmetry group $SU(2)_{E}$ is the Euclidean rotation group in $D = 3$.

There are only two essentially different possibilities to construct topological models with $N_{T} = 2$ scalar topological supercharges, arising from twisting $N = 4$ SYM in $D = 3$ \[16\].

(A) The super–BF model

The standard twist consists in replacing $SU(2)_{E} \otimes SU(2)_{R}$ through its diagonal subgroup. This leads to the universal gauge multiplet \{$A_{\alpha}, \psi_{\alpha}^{a}, \phi^{ab}, \eta^{a}$\}, $a = 1, 2$; $\alpha = 1, 2, 3$, of the $N_{T} = 2$, $D = 3$ super–BF model \[24\] – \[26\], which is built up from the gauge field $A_{\alpha}$, a $SU(2)_{N}$ doublet of Grassmann–odd topological ghost–antighost vector fields $\psi_{\alpha}^{a} = \{\psi_{\alpha}, \chi_{\alpha}\}$, a $SU(2)_{N}$ triplet of Grassmann–even ghost–for–ghost scalar fields $\phi^{ab} = \{\phi, \tau, \phi\}$, where $\tau$ plays the role of a Higgs field, and a $SU(2)_{N}$ doublet of Grassmann–odd scalar fields $\eta^{a} = \{\lambda, \eta\}$, respectively. (Let us recall, that $\phi^{ab}$ is symmetric, $\phi^{ab} = \phi^{ba}$.) In order to close the topological superalgebra (see Eq. (2.4) below) it is necessary to introduce the bosonic auxiliary vector field $B_{\alpha}$. All the fields are in the adjoint representation and take their values in the Lie algebra Lie($G$) of some compact gauge group $G$.

The twisted action of this $D = 3$ super–BF (Casson or Euler character) model \[24\] with a $N_{T} = 2$ off–shell equivariantly nilpotent topological supersymmetry $Q^{a}$ is given by

$$S_{BF} = \int d^{3}x \text{tr} \left\{ e^{\alpha\beta\gamma} B_{\gamma} F_{\alpha\beta} + e^{\alpha\beta\gamma} \epsilon_{ab} \psi_{\gamma}^{a} D_{\alpha} \psi_{\beta}^{b} - 2 \epsilon_{ab} \eta^{a} D^{a} \psi_{\alpha}^{b} - 2 \eta^{a} [\phi^{ab}, \eta^{b}] \right. $$

$$-2 \psi^{a\alpha} [\phi^{ab}, \psi_{\alpha}^{b}] + \phi^{b} D^{2} \phi^{ab} - [\phi^{ab}, \phi^{cd}] [\phi^{ab}, \phi^{cd}] - 2 D^{a} B_{\alpha} \right\},$$

(1)

where $F_{\alpha\beta} = \partial_{\alpha} A_{\beta} - \partial_{\beta} A_{\alpha} + [A_{\alpha}, A_{\beta}]$ and $D_{\alpha} = \partial_{\alpha} + [A_{\alpha}, \cdot ]$ is the YM field strenght and the covariant derivative in the adjoint representation, respectively; $\epsilon_{\alpha\beta\gamma}$ is the totally antisymmetric Levi–Civita tensor in $D = 3$, $\epsilon_{123} = 1$, and $\epsilon_{ab}$ is the invariant tensor of the group $SU(2)_{N}$, $\epsilon_{12} = 1$. The internal index $a$, which labels the different $N_{T} = 2$ charges, is raised and lowered as follows: $\varphi_{a} = \varphi^{b} \epsilon_{ba}$ and $\varphi^{a} = \epsilon^{a} \varphi_{b}$ with $\epsilon^{ac} \epsilon_{bc} = -\delta^{a}_{b}$.

The action \[1\] can be cast into the $Q^{a}$–exact form

$$S_{BF} = \frac{1}{2} \epsilon_{ab} Q^{a} Q^{b} X_{BF},$$

where

$$X_{BF} = S_{CS} + \int d^{3}x \text{tr} \left\{ \epsilon_{ab} \psi^{a\alpha} \psi_{\alpha}^{b} + \epsilon_{ab} \eta^{a} \eta^{b} \right\},$$

$$S_{CS} = \int d^{3}x \text{tr} \left\{ \epsilon^{\alpha\beta\gamma} (A_{\gamma} \partial_{\alpha} A_{\beta} + \frac{\gamma}{3} A_{\alpha} A_{\beta} A_{\gamma} \right\}$$

being the Chern–Simons action. Let us notice, that the gauge boson $X_{BF}$ is not uniquely specified, namely, substituting for $\epsilon_{ab} \psi^{a\alpha} \psi_{\alpha}^{b}$ the expression $\frac{2}{3} \epsilon_{ab} \phi_{cd} [\phi^{ac}, \phi^{bd}]$ gives the same action. Since the BF–term $\epsilon^{\alpha\beta\gamma} B_{\gamma} F_{\alpha\beta}$ in \[1\] has an on–shell first–stage reducible gauge symmetry, $\delta_{G}(\omega) B_{\gamma} = -D_{\gamma} \omega$, the gauge–fixing terms can be derived also by the Batalin–Vilkovisky procedure \[13\], \[28\]. Let us notice, that the gauge–fixed action of the $D = 3$ super–BF theory can also be obtained by a dimensional reduction \[23\] of the $N_{T} = 1, D = 4$ Donaldson–Witten theory \[3\] which results by twisting the $N = 2, D = 4$ SYM (see Diagram).

The off–shell equivariantly nilpotent topological shift symmetry $Q^{a}$ takes the form

$$Q^{a} A_{\alpha} = \psi_{\alpha}^{a},$$

$$Q^{a} \phi^{bc} = \frac{1}{2} \epsilon^{abc} \psi^{c}, \quad Q^{a} \eta^{b} = -\epsilon_{cd} [\phi^{ac}, \phi^{bd}],$$

$$Q^{a} \psi^{b}_{\alpha} = D_{\alpha} \phi^{ab} + \epsilon^{ab} B_{\alpha}, \quad Q^{a} B_{\alpha} = -\frac{1}{2} D_{\alpha} \eta^{a} - \epsilon_{cd} [\phi^{ac}, \psi^{d}_{\alpha}];$$

(2)
it agrees with that of Ref. [18, 29] (up to the ubiquitous factor of $1/2$ in front of $\eta^a$) and [14]. In addition, by restricting to flat Euclidean space–time the action (1) is invariant also under the following vector supersymmetry $Q^a_\alpha$,

\[
\begin{align*}
\tilde{Q}^a_\alpha A_\beta &= \delta_{\alpha\beta} \eta^a + \epsilon_{\alpha\beta\gamma} \psi^\gamma a, \\
\tilde{Q}^a_\alpha \phi^{bc} &= -\frac{1}{2} \epsilon^{abc} \psi^c - \frac{1}{2} \epsilon^{aac} \psi^b, \\
\tilde{Q}^a_\alpha \bar{\eta}^b &= D_\alpha \phi^{ab} + \epsilon^{ab} B_\alpha, \\
\tilde{Q}^a_\alpha \psi^b_\beta &= -\epsilon^{ab} F_{\alpha\beta} + \epsilon^{ab} \epsilon_{\alpha\beta\gamma} B^\gamma - \epsilon_{\alpha\beta\gamma} D^\gamma \phi^{ab} + \delta_{\alpha\beta} \epsilon_{cd}[\phi^{ac}, \phi^{bd}], \\
\tilde{Q}^a_\alpha B_\beta &= D_\alpha \psi^a_\beta - \frac{1}{2} D_\beta \psi^a_\alpha + \epsilon_{\alpha\beta\gamma} D^\gamma \eta^a + \epsilon_{cd}[\phi^{ac}, \epsilon_{\alpha\beta\gamma} \psi^\gamma d - \delta_{\alpha\beta} \eta^d].
\end{align*}
\]

By a straightforward calculation it can be verified that the four supercharges $Q^a$ and $\tilde{Q}^a_\alpha$, together with the generator $P_\alpha = i\partial_\alpha$ of space–time translations, obey the following topological superalgebra:

\[
\begin{align*}
\{Q^a, Q^b\} &= -2\delta_G(\phi^{ab}), \\
\{Q^a, \tilde{Q}^b_\alpha\} &= \epsilon^{ab}(-iP_\alpha + \delta_G(A_\alpha)), \\
\{\tilde{Q}^a_\alpha, \tilde{Q}^b_\beta\} &= -2\delta_{\alpha\beta} \delta_G(\phi^{ab}),
\end{align*}
\]

where the symbol $\doteq$ means that the corresponding relation is satisfied only on–shell, i.e., by taking into account the equation of motions. Since both the supersymmetries $Q^a$ and $\tilde{Q}^a_\alpha$ are realized nonlinearly, the superalgebra (3) closes only modulo the field–dependent gauge transformations $\delta_G(\omega)$, $\omega = \{A_\alpha, \phi^{ab}\}$, which are defined by $\delta_G(\omega) A_\alpha = -D_\alpha \omega$ and $\delta_G(\omega) \varphi = [\omega, \varphi]$. $\varphi = \{\phi^{ab}, \eta^a, \psi^a_\alpha, B_\alpha\}$. Let us emphasize that the form of the action (1) is not completely specified by the topological supersymmetry $Q^a$, i.e., it is not the most general action compatible with the gauge and the $Q^a$–invariance. Nevertheless, it turns out to be uniquely characterized by imposing the vector supersymmetry $\tilde{Q}^a_\alpha$. The conditions $Q^a S_{\text{BF}} = \tilde{Q}^a_\alpha S_{\text{BF}} = 0$ fix all the relative numerical coefficients of the action (2), allowing, in particular, for a single coupling constant.

(B) The Blau–Thompson model

The second twist of $N = 4$ SYM in $D = 3$ consists in replacing $SU(2)_E \otimes SU(2)_N$ through its diagonal subgroup. This yields the novel $N_T = 2$ topological twist introduced by Blau and Thompson [11]. The gauge multiplet $\{A_\alpha, V_\alpha, \psi^a_\alpha, \bar{\eta}^a\}$ of this topological model is built up from the gauge field $A_\alpha$, a bosonic vector field $V_\alpha$, a $SU(2)_R$ doublet of Grassmann–odd topological ghost–antighost vector fields $\psi^a_\alpha = \{\psi_\alpha, \chi_\alpha\}$ and a $SU(2)_R$ doublet of Grassmann–odd scalar fields $\bar{\eta}^a = \{\lambda, \bar{\eta}\}$. In order to close the topological superalgebra (see Eq. (3) below) it is necessary to introduce a further set of bosonic auxiliary fields, namely two vector fields $B_\alpha$, $\bar{B}_\alpha$ and a scalar field $Y$, respectively.

The twisted action of this topological model is given by (14)

\[
S_{\text{BT}} = \frac{1}{2} \int d^3x \sum \left\{-i\epsilon^{\alpha\beta\gamma} B_\gamma F_{\alpha\beta} (A + iV) - i\epsilon^{\alpha\beta\gamma} \epsilon_{ab} \psi^a_\alpha D_\alpha (A + iV) \psi^b_\beta - 4B^a \bar{B}_a - 4Y^2 \right. \\
+ \left. i\epsilon^{\alpha\beta\gamma} \bar{B}_\gamma F_{\alpha\beta} (A - iV) - 2\epsilon_{ab} \bar{\eta}^a D^a (A - iV) \psi^b_\alpha - 4Y D^a (A) V_\alpha \right\},
\]

and can be rewritten as sum of a BF–like topological term and a $Q^a$–exact term,

\[
S_{\text{BT}} = \frac{1}{2} \int d^3x \sum \left\{i\epsilon^{\alpha\beta\gamma} \bar{B}_\gamma F_{\alpha\beta} (A - iV) \right\} + \frac{1}{2} \epsilon_{ab} Q^a Q^b X_{\text{BT}},
\]

with the gauge boson

\[
X_{\text{BT}} = -\frac{1}{4} iS_{\text{CS}} (A + iV) - \int d^3x \sum i\bar{B}^a V_\alpha + \frac{1}{2} \epsilon_{ab} \bar{\eta}^a \bar{\eta}^b \right\}.
\]
Here, the Chern–Simons action $S_{CS}(A + iV)$ is formed by the complexified gauge field $A_\alpha + iV_\alpha$. A striking, but somewhat unusual feature of this model is that there are no bosonic scalar fields and hence no underlying equivariant $Q^a$-cohomology (after dimensional reduction the three scalar fields are combined to form the vector field $V_\alpha$). Another special feature is that $A_\alpha - iV_\alpha$ is $Q^a$-invariant. Thus, as pointed out in [16], any gauge invariant functional of $A_\alpha - iV_\alpha$, constrained by $F_{\alpha\beta}(A - iV) = 0$, is a good observable (e.g., bosonic Wilson loops). Moreover, since this twisted model differs from the $D = 3$ super–BF model by an exchange of $SU(2)_R$ and $SU(2)_N$, in Ref. [16] it has been speculated, that it can be regarded as providing a mirror description of the Casson model.

Let us now give the transformation laws which leave the action (5) invariant. The off–shell nilpotent topological supersymmetry $Q^a$ takes the form [16]

$$
\begin{align*}
Q^a A_\alpha &= \psi^a_\alpha, \\
Q^a \psi^b_\alpha &= 2\epsilon^{ab}B_\alpha, \\
Q^a B_\alpha &= 0, \\
Q^a Y &= 0,
\end{align*}
$$

(6)
i.e., prior to the introduction of gauge ghosts, the $N_T = 2$ topological supersymmetry $Q^a$ is not equivariant, but rather strictly nilpotent. In addition, by restricting to flat Euclidean space–time, the action (5) is left invariant under the following vector supersymmetry $Q^a_\alpha$,

$$
\begin{align*}
Q^a_\alpha A_\beta &= \delta_{\alpha\beta}\bar{\psi}^a - i\epsilon_{\alpha\beta\gamma}\psi^\gamma, \\
Q^a_\alpha \bar{\psi}^b &= 2\epsilon^{ab}B_\alpha, \\
Q^a_\alpha \bar{B}_\beta &= -i\epsilon_{\alpha\beta\gamma}D^\gamma(A + iV)\bar{\psi}^a, \\
Q^a_\alpha V_\beta &= -i\delta_{\alpha\beta}\bar{\psi}^a + \epsilon_{\alpha\beta\gamma}\psi^\gamma, \\
Q^a_\alpha \psi^b_\beta &= -2\epsilon^{ab}F_{\alpha\beta}(A) - 2i\epsilon^{ab}D_\alpha(A)\psi^\gamma_\beta + 2i\epsilon^{ab}\epsilon_{\alpha\beta\gamma}\bar{B}^\gamma - 2i\epsilon_{\alpha\beta\gamma}\epsilon^{ab}Y, \\
Q^a_\alpha \bar{B}_\beta &= 2D_\alpha(A)\psi^a_\beta - D_\beta(A + iV)\psi^a_\alpha + i\epsilon_{\alpha\beta\gamma}D^\gamma(A - iV)\bar{\psi}^a, \\
Q^a_\alpha Y &= iD_\alpha(A - iV)\psi^a.
\end{align*}
$$

(7)
The scalar and the vector supercharges, $Q^a$ and $Q^a_\alpha$, together with the generator $P_\alpha$ of space–time translations, satisfy the following topological superalgebra:

$$
\begin{align*}
\{Q^a, Q^b_\alpha\} &= 0, \\
\{Q^a, Q^b_\alpha\} &= \epsilon^{ab}(-iP_\alpha + \delta_{G}(A_\alpha - iV_\alpha)), \\
\{Q^a_\alpha, Q^b_\beta\} &= 2i\epsilon^{ab}\epsilon_{\alpha\beta\gamma}(-iP^\gamma + \delta_{G}(A^\gamma - iV^\gamma)).
\end{align*}
$$

(8)
As before, all relative numerical factors of the action (5), except for an overall unique coupling constant, are fixed by imposing the requirements $Q^a S_{BT} = Q^a_\alpha S_{BT} = 0$.

3. $N_T = 4$ equivariant extension of Blau–Thompson model

After having characterized the two possible topological twists of $N = 4$, $D = 3$ SYM let us turn to the question whether the topological model constructed by Blau and Thompson can be regarded as deformation of another one with underlying $N_T \geq 2$ equivariant cohomology. Under the requirement of preserving the number of topological supercharges, $N_T = 2$, we have not been able to find an appropriate equivariant extension of this model. However, if the condition $N_T = 2$ is relaxed, it is not difficult to show that this model, formally, can be recovered by
deforming a cohomological theory with an extended $N_T = 4$ topological supersymmetry. Such a theory has an additional global symmetry group, which will be denoted by $SU(2)_{R}$. In Section 4 it will be shown that this cohomological theory coincides with the $N_T = 4$ topological twist of $N = 8$ SYM in $D = 3$.

The construction of the $N_T = 4$ equivariant extension of the Blau–Thompson topological model is governed by the following strategy:

First, after eliminating, by the use of the equations of motion, in the action $S[3]$ the auxiliary fields $B_α$, $B_α$ and $Y$ the bosonic part of the resulting action involves the complexified gauge fields $A_α ± iV_α$. This bears a strong resemblance to the so–called topological $B$–twist of $N = 4$, $D = 4$ SYM studied by Markus [23] (recalling that $A_α$, $V_α$ and $Y$ are anti–hermitean). Hence, $B_α$ should be regarded as the anti–hermitean conjugate of $B_α$.

Second, we introduce a $SU(2)_{R}$ doublet of Grassmann–odd topological ghost–antighost vector fields $ψ^a_α = \{ψ_α, \bar{χ}_α\}$ and a $SU(2)_{R}$ doublet of Grassmann–odd scalar fields $η^a = \{λ, η\}$, which should be regarded as the hermitean conjugate of $ψ^a_α$ and $η^a$, respectively. Then, we construct an $SU(2)_{R} \otimes SU(2)_{R}$ invariant action by adding to $S[3]$ appropriate $ψ^a_α$ and $η^a$–dependent terms.

Third, we introduce a $SU(2)_{R} \otimes SU(2)_{R}$ quartet of Grassmann–even ghost–for–ghost scalar fields $ζ^{ab} = \{ϕ, τ + iϕ, τ − iϕ, φ\}$ and $\tilde{ζ}^{ab} ≡ ζ^{bc}$(2) where $τ ± iϕ$ plays the role of a complexified Higgs field.

Finally, we complete the action by adding suitable $ζ^{ab}$– and $\tilde{ζ}^{ab}$–dependent terms analogous to the $φ^{ab}$–dependent terms in the action $S[4]$ of the super–BF model.

Proceeding in that way one gets the following $N_T = 4$ equivariant extension of the action $S[3]$, $S^{(N_T=4)} = \frac{1}{2} \int d^3 x Tr\left\{− ie^{αβγ}B_γ F_{αβ}(A + iV) − ie^{αβγ}ε_{ab}ψ^a_α D_α(A + iV)ψ^b_βight.$

$− 2ε_{ab}η^a D^α(A − iV)ψ^b_α + 2ζ_{ab}\{η^a, η^b\} + 2ζ_{ab}\{ψ^a_α, ψ^b_α\}$

$+ ζ_{ab} D^2(A + iV)ζ^{ab} − [ζ_{ab}, ζ_{cd}][ζ^{ab}, ζ^{cd}] − 4B^α B_α$

$+ ie^{αβγ} B_γ F_{αβ}(A − iV) + ie^{αβγ}ε_{ab}ψ^a_α D_α(A − iV)ψ^b_β$

$− 2ε_{ab}η^a D^α(A + iV) \tilde{ψ}^b_α + 2ζ_{ab}\{η^a, η^b\} + 2ζ_{ab}\{ψ^a_α, \tilde{ψ}^b_α\}$

$+ \tilde{ζ}_{ab} D^2(A − iV)ζ^{ab} − [ζ_{ab}, ζ_{cd}][ζ^{ab}, ζ^{cd}] − 4Y D^α(A) V_α − 4Y^2\right\}$, (9)

which, by construction, is manifestly invariant under hermitean conjugation or, equivalently, under the discrete symmetry

$$(A_α, V_α, B_α, B_α, Y) \to (A_α, −V_α, −B_α, −B_α, −Y),$$

$$(ψ^a_α, \tilde{ψ}^a_α, η^a, \tilde{η}^a, ζ^{ab}, \tilde{ζ}^{ab}) \to (iψ^a_α, i\tilde{ψ}^a_α, iη^a, i\tilde{η}^a, ±ζ^{ab}, ±\tilde{ζ}^{ab}),$$ (10)

exchanging the scalar and the vector supercharges with their conjugate ones. Thus, the underlying equivariant cohomology should be a $N_T = 4$ supersymmetry. This is indeed the case. By an explicit calculation one establishes that the action $S[3]$ is invariant under the following off–shell equivariantly nilpotent topological supersymmetry $Q^a$,

$$Q^a A_α = ψ^a_α, \quad Q^a V_α = −iψ^a_α,$$

$$Q^a ζ^{bc} = ε^{ac}η^b, \quad Q^a \tilde{ζ}^{bc} = ε^{ab}η^c,$$

$$Q^a ψ^b_α = 2ε^{ab}B_α, \quad Q^a \tilde{ψ}^b_α = 2D_α(A − iV)ζ^{ab},$$

$$Q^a η^b = 0, \quad Q^a \tilde{η}^b = −2ε^{ab}Y − 2ε_{cd}[ζ^{ac}, ζ^{bd}],$$

$$Q^a B_α = 0, \quad Q^a \tilde{B}_α = −D_α(A − iV)\tilde{η}^a − 2ε_{cd}[ζ^{ac}, \tilde{ψ}^d_α],$$

$$Q^a Y = iε_{cd}[ζ^{ac}, η^d],$$ (11)
which, formally, may be regarded as deformation of the topological supersymmetry displayed in \([3]\). This deformation is, of course, topological non–trivial since some of the fields, namely \(\eta^a\), \(\psi^a\), \(\zeta^{ab}\) and \(\bar{\zeta}^{ab}\), must be deformed equal to zero. In addition, applying the discrete symmetry \([10]\) on \(Q^a\), which maps \(Q^a\) to \(iQ^a\), one gets a further one, namely the conjugate topological supersymmetry \(\bar{Q}^a\),

\[
\begin{align*}
\bar{Q}^a A_\alpha &= \bar{\psi}_\alpha^a, & \bar{Q}^a V_\alpha &= i\bar{\psi}_\alpha^a, \\
\bar{Q}^a \bar{\zeta}^{bc} &= \epsilon^{ac} \bar{\eta}^b, & \bar{Q}^a \zeta^{bc} &= \epsilon^{ab} \bar{\eta}^c, \\
\bar{Q}^a \bar{\psi}^b_\alpha &= 2\epsilon^{ab} \bar{B}_\alpha, & \bar{Q}^a \psi^b_\alpha &= 2D_\alpha(A + iV)\zeta^{ab}, \\
\bar{Q}^a \bar{\eta}^b_\alpha &= 0, & \bar{Q}^a \eta^b_\alpha &= 2i\epsilon^{ab}Y - 2\epsilon_{cd}[\zeta^{ac}, \zeta^{bd}], \\
\bar{Q}^a \bar{B}_\alpha &= 0, & \bar{Q}^a B_\alpha &= -D_\alpha(A + iV)\eta^a - 2\epsilon_{cd}[\zeta^{ac}, \psi^d_\alpha], \\
\bar{Q}^a Y &= -i\epsilon_{cd}[\zeta^{ac}, \bar{\psi}^d_\alpha], & \bar{Q}^a Y &= -i\epsilon_{cd}[\zeta^{ac}, \psi^d_\alpha],
\end{align*}
\]  

proving, as promised, that the action \([3]\) actually possesses a \(N_T = 4\) topological supersymmetry.

By restricting to flat Euclidean space–time one can convince oneself by a rather lengthy calculation that the action \([3]\) is also invariant under the following vector supersymmetry \(Q^a\):

\[
\begin{align*}
Q^a A_\beta &= \delta_{\alpha\beta} \eta^a + i\epsilon_{\alpha\beta\gamma} \bar{\psi}^{\gamma a}, \\
Q^a \bar{\zeta}^{bc} &= -\epsilon^{ab} \bar{\psi}^c, \\
Q^a \psi^b_\beta &= -2i\epsilon_{\alpha\beta\gamma} D^\gamma (A - iV)\zeta^{ab}, \\
Q^a \eta^b_\alpha &= 2\epsilon^{ab} \bar{B}_\alpha, \\
Q^a B_\beta &= i\epsilon_{\alpha\beta\gamma} D^\gamma (A - iV)\eta^a, \\
Q^a V_\beta &= i\delta_{\alpha\beta} \eta^a + \epsilon_{\alpha\beta\gamma} \bar{\psi}^{\gamma a}, \\
Q^a \zeta^{bc} &= -\epsilon^{ac} \bar{\psi}^b, \\
Q^a \bar{\psi}^b_\beta &= -2\epsilon^{ab} F_{\alpha\beta}(A) + 2i\epsilon^{ab} D_\alpha(A)V_\beta - 2i\epsilon^{ab} \epsilon_{\alpha\beta\gamma} B^\gamma + 2\delta_{\alpha\beta} \epsilon_{cd}[\zeta^{ac}, \zeta^{bd}] + 2i\delta_{\alpha\beta} \epsilon^{ab} Y, \\
Q^a \bar{\eta}^b_\alpha &= 2D_\alpha(A + iV)\zeta^{ab}, \\
Q^a B_\beta &= 2D_\alpha(A)\bar{\psi}^a_\beta - D_\beta(A - iV)\bar{\psi}^{\gamma a}_\alpha - i\epsilon_{\alpha\beta\gamma} D^\gamma (A + iV)\eta^a - 2\epsilon_{cd}[\zeta^{ac}, \delta_{\alpha\beta} \eta^d] + i\epsilon_{\alpha\beta\gamma} \psi^{\gamma d}. \\
Q^a Y &= -iD_\alpha(A + iV)\eta^a - i\epsilon_{cd}[\zeta^{ac}, \psi^d_\alpha],
\end{align*}
\]

Combining \(\bar{Q}^a\) with the discrete symmetry \([10]\), which maps \(Q^a\) into \(iQ^a\), gives rise to the conjugate vector supersymmetry \(Q^a\),

\[
\begin{align*}
Q^a A_\beta &= \delta_{\alpha\beta} \eta^a - i\epsilon_{\alpha\beta\gamma} \bar{\psi}^{\gamma a}, \\
Q^a \bar{\zeta}^{bc} &= -\epsilon^{ab} \bar{\psi}^c, \\
Q^a \psi^b_\beta &= 2i\epsilon_{\alpha\beta\gamma} D^\gamma (A + iV)\bar{\zeta}^{ab}, \\
Q^a \eta^b_\alpha &= 2\epsilon^{ab} \bar{B}_\alpha, \\
Q^a B_\beta &= -i\epsilon_{\alpha\beta\gamma} D^\gamma (A + iV)\bar{\eta}^a, \\
Q^a V_\beta &= -i\delta_{\alpha\beta} \bar{\eta}^a + \epsilon_{\alpha\beta\gamma} \bar{\psi}^{\gamma a}, \\
Q^a \zeta^{bc} &= -\epsilon^{ac} \bar{\psi}^b, \\
Q^a \bar{\psi}^b_\beta &= -2\epsilon^{ab} F_{\alpha\beta}(A) - 2i\epsilon^{ab} D_\alpha(A)V_\beta + 2i\epsilon^{ab} \epsilon_{\alpha\beta\gamma} \bar{B}^\gamma + 2\delta_{\alpha\beta} \epsilon_{cd}[\zeta^{ac}, \zeta^{bd}] - 2i\delta_{\alpha\beta} \epsilon^{ab} Y, \\
Q^a \bar{\eta}^b_\alpha &= 2D_\alpha(A - iV)\zeta^{ab}, \\
Q^a B_\beta &= 2D_\alpha(A)\bar{\psi}^a_\beta - D_\beta(A + iV)\bar{\psi}^{\gamma a}_\alpha + i\epsilon_{\alpha\beta\gamma} D^\gamma (A - iV)\bar{\eta}^a - 2\epsilon_{cd}[\zeta^{ac}, \delta_{\alpha\beta} \eta^d] - i\epsilon_{\alpha\beta\gamma} \psi^{\gamma d}. \\
Q^a Y &= iD_\alpha(A - iV)\bar{\eta}^a + i\epsilon_{cd}[\zeta^{ac}, \psi^d_\alpha],
\end{align*}
\]
which, formally, may be regarded as deformation of the vector supersymmetry given in (8).
The eight supercharges $Q^a, Q^b, Q^a_{\alpha}$ and $Q^b_{\alpha}$, together with the generator $P_{\alpha}$ of space–time
translations, obey the following topological superalgebra,

$$
\begin{align*}
\{Q^a, Q^b\} &= 0, \\
\{Q^a, \bar{Q}^b\} &= 0, \\
\{Q^a, Q^b_{\alpha}\} &= 0, \\
\{Q^a_{\alpha}, \bar{Q}^b_{\beta}\} &= 0, \\
\{Q^a_{\alpha}, Q^b_{\beta}\} &= -4e^{ab}\delta_{\alpha\beta}\delta_G(\zeta^{ab}), \\
\{Q^a_{\alpha}, \bar{Q}^b_{\beta}\} &= -4e^{ab}\delta_{\alpha\beta}\delta_G(\zeta^{ab}).
\end{align*}
$$

which is just the $N_T = 4$ equivariant extension of the topological superalgebra (8).

In summary, so far we have seen that the novel $N_T = 2$ topological twist of $N = 4$, $D = 3$ SYM can be regarded, formally, as restriction of a $N_T = 4$ topological gauge theory.

4. Dimensional reduction of $N_T = 2$, $D = 4$ topological YM theory to $D = 3$

Now, our aim is to show that the $N_T = 4$ equivariant extension of the Blau–Thompson model
introduced in Section 3 is precisely one of the two essentially different topological twists of
$N = 8$, $D = 4$ SYM which, on the other hand, is just the dimensional reduction of either of the
two $N_T = 2$, $D = 4$ models to $D = 3$, namely, either the $A$–model [18, 22] or the $B$–model [23].
However, before proceeding, let us briefly recall the two possible topological twists of $N = 8$,
$D = 3$ SYM and its relation to the three essentially different topological twists of the $N = 4$,
$D = 4$ SYM.

The $N = 8$, $D = 3$ SYM obtains by dimensional reduction of $N = 1$, $D = 10$ SYM, either
directly or via $N = 4$, $D = 4$ SYM, to $D = 3$ (cf., lower half of the Diagram). The global
symmetry group of $N = 8$, $D = 3$ SYM is $SU(2)_E \otimes Spin(7)$. In decomposing $Spin(7)$, with
respect to the twist procedure, some restrictions have to be required [18]:
First, the twisted theory should contain at least one scalar topological supercharge.
Second, among the spinor representations of $Spin(7)$ no ones with spin $\geq 2$ should appear.
Third, for a full topological twist only half–integral spins should appear among the spinor
representations.
Under these restrictions $Spin(7)$ decomposes as $Spin(7) \rightarrow SU(2)_R \otimes SU(2)_L \otimes SU(2)_N$, so that
the maximal residual global symmetry group is $SU(2)_R \otimes SU(2)_L$.

Now, choosing the diagonal subgroup of $SU(2)_E \otimes SU(2)_N$ one gets a twisted theory with
an underlying $N_T = 4$ equivariant cohomology. Below, it will be shown that the action of this
model is precisely the one given in (8).

The other topological twist is obtained by taking the diagonal subgroup of $SU(2)_E \otimes SU(2)_R$
and gives rise to a $N_T = 2$ theory with global symmetry group $SU(2)_N \otimes SU(2)_L$. The action
of that theory will be explicitly constructed in Section 5.

Furthermore, there are three topological twists of $N = 4$, $D = 4$ SYM, namely the $A$–model,
which is the $N_T = 2$ equivariant extension of the $N_T = 1$, $D = 4$ Donaldson–Witten model
[8], the $B$–model, which formally can be regarded as a deformation [16] of the $N_T = 2$, $D = 4$
super–BF model [28], and the half–twisted $N_T = 1$, $D = 4$ model [22], the Donaldson–Witten
theory coupled to a spinorial hypermultiplet. Therefore, one might have expected that there
are at least three topological twists of $N = 8$, $D = 3$ SYM. But, as pointed out in [16], the
dimensional reduction of either of the two $N_T = 2$, $D = 4$ theories, i.e., the $A$– and the $B$–model,
give rise to equivalent $D = 3$ topological gauge theories, so that, under the above–mentioned
restrictions, there are only two different twists.
Now we want to show that the dimensional reduction of the $N_T = 2, D = 4$ Yamron–Vafa–Witten theory with global symmetry group $SU(2)_R$, i.e., the A-model, leads precisely to the action given in Eq. (16). The gauge multiplet of this theory consists of the gauge field $A_\mu$, a Grassmann–even self–dual field $M_\mu$, a $SU(2)_R$ doublet of Grassmann–odd self–dual ghost–antighost tensor fields $\chi^a_{\mu} = \{ \psi^a_{\mu}, \chi^b_{\mu} \}$, a $SU(2)_R$ doublet of Grassmann–odd ghost–antighost vector fields $\psi_{\mu}^a = \{ \psi_{\mu}, \chi^b_\mu \}$, a $SU(2)_R$ doublet of Grassmann–odd scalar fields $\eta^a = \{ \lambda, \eta \}$, and a $SU(2)_R$ triplet of Grassmann–even ghost–for–ghost complex scalars $\phi^{ab} = \{ \phi, \tau, \phi \}$. For the closure of the topological superalgebra (see Eq. (19) below) it is necessary to introduce a set of bosonic auxiliary fields, namely the self–dual tensor field $G_\mu$ and the vector field $H_\mu$. All the fields are in the adjoint representation and take their values in the Lie algebra $Lie(G)$ of some compact gauge group $G$.

The action of the Yamron–Vafa–Witten model, with an $N_T = 2$ off–shell equivariantly nilpotent topological shift symmetry $Q^a$, is given by [18] (see also [21])

\[
S_{YWW} = \int d^4X \left\{ G_{\mu\nu} F_{\mu\nu} - \frac{1}{4} G_{\mu\nu}\left[ M_\mu^{\lambda}, M_\nu^{\nu} \right] + 2 \epsilon_{ab} \chi^{a\mu a}_\mu D_\mu \psi^b_\mu - \frac{1}{2} \epsilon_{ab} \chi^{b\mu b}_\mu \left[ M_\mu^{\mu}, \chi_\mu^{\nu} \right] - \epsilon_{ab} \psi^{a\mu a}_\mu \left[ M_\mu^{\mu}, \psi^{b\mu b}_\mu \right] - 2 \epsilon_{ab} \eta^b_\mu D_\mu \psi^a_\mu - \frac{1}{2} \epsilon_{ab} \eta^b_\mu \left[ M_\mu^{\mu}, \psi^{b\mu b}_\mu \right] - \frac{1}{2} G_{\mu\nu} G_{\mu\nu} + 2 \phi_{ab} \left[ \eta^a_\mu, \eta^b_\mu \right] + 2 \phi_{ab} \left[ \psi^{a\mu a}_\mu, \psi^{b\mu b}_\mu \right] + \frac{1}{2} \phi_{ab} \left[ \chi^{a\mu a}_\mu, \chi^{b\mu b}_\mu \right] + \phi_{ab} D^2 \phi^{ab} - \frac{1}{4} \left[ \phi_{ab}, M_\mu^{\mu} \right] \left[ \phi^{ab}, M_\mu^{\mu} \right] - \left[ \phi_{ab}, \phi_{cd} \right] \left[ \phi^{ab}, \phi^{cd} \right] + 2 H_\mu^\nu D_\mu M_\mu^{\nu} - 2 H_\mu^{\mu} H_\mu \right\},
\]

(16)

and can be cast into the $Q^a$–exact form

\[
S_{YWW} = \frac{1}{2} \epsilon_{ab} Q^a Q^b X_{YWW},
\]

with the gauge boson

\[
X_{YWW} = \int d^4X \left\{ M_\mu^{\mu} F_{\mu\nu} - \frac{1}{12} M_\mu^{\mu} \left[ M_\mu^{\lambda}, M_\nu^{\nu} \right] - \frac{1}{2} G_{\mu\nu} M_\mu^{\nu} + \epsilon_{ab} \psi^{a\mu a}_\mu \psi^{b\mu b}_\mu + \epsilon_{ab} \psi^{a\mu a}_\mu \psi^{b\mu b}_\mu \right\}.
\]

The complete set of symmetry transformations which fix all the relative numerical factors in (16), except for an overall coupling constant, is given by the topological supersymmetry $Q^a$,

\[
\begin{align*}
Q^a A_\nu &= \psi^a_\mu, \\
Q^a \phi^{bc} &= \frac{1}{2} \epsilon^{ab} \eta^c + \frac{1}{2} \epsilon^{ac} \eta^b, \\
Q^a \psi^b_\mu &= D_\mu \phi^{ab} + \epsilon^{ab} H_\mu, \\
Q^a \chi^{b\mu}_\mu &= \left[ M_\mu^{\mu}, \phi^{ab} \right] + \epsilon^{ab} G_{\mu\nu}, \\
Q^a M_\mu^{\nu} &= \chi^{a\mu}_\nu, \\
Q^a \phi^{bc} &= -\epsilon_{cd} \left[ \phi^{ac}, \phi^{bd} \right], \\
Q^a \psi^b_\mu &= -\epsilon_{cd} \left[ \phi^{ac}, \psi^{bd}_\mu \right], \\
Q^a \chi^{b\mu}_\mu &= \left[ M_\mu^{\mu}, \phi^{ab} \right] + \epsilon^{ab} G_{\mu\nu}, \\
Q^a \phi^{bc} &= -\epsilon_{cd} \left[ \phi^{ac}, \phi^{bd} \right] + \epsilon^{ab} G_{\mu\nu} - \left[ M_\mu^{\mu}, \phi^{ab} \right], \\
Q^a H_\mu &= -\frac{1}{2} D_\mu \eta^a - \epsilon_{cd} \left[ \phi^{ac}, \psi^{bd}_\mu \right], \\
Q^a G_{\mu\nu} &= -\frac{1}{2} \left[ M_\mu^{\mu}, \eta^a \right] - \epsilon_{cd} \left[ \phi^{ac}, \chi^{b\mu}_\mu \right],
\end{align*}
\]

(17)

and by the vector supersymmetry $Q^a_{\mu}$,

\[
\begin{align*}
Q^a_{\mu} A_\nu &= \delta_{\nu\mu} \eta^a + \chi^{a\mu}_\mu, \\
Q^a_{\mu} M_\rho &= -\delta_{\mu\rho} \psi^{a\sigma}_\sigma - \epsilon_{\mu\nu\rho\sigma} \psi^{a\nu\sigma}_\sigma, \\
Q^a_{\mu} \phi^{bc} &= -\frac{1}{2} \epsilon^{ab} \psi^{c}_\mu + \frac{1}{2} \epsilon^{ac} \psi^{b}_\mu, \\
Q^a_{\mu} \psi^b_\mu &= D_{\mu} \phi^{ab} + \epsilon^{ab} H_\mu, \\
Q^a_{\mu} \psi^b_\mu &= -\epsilon^{ab} F_{\mu\nu} + \delta_{\mu\nu} \epsilon_{cd} \left[ \phi^{ac}, \phi^{bd} \right] + \epsilon^{ab} G_{\mu\nu} - \left[ M_\mu^{\mu}, \phi^{ab} \right], \\
Q^a_{\mu} H_\mu &= D_{\mu} \psi^a_\nu - \frac{1}{2} D_{\nu} \psi^{a\sigma}_\sigma + \epsilon_{cd} \left[ \phi^{ac}, \chi^{b\mu}_\mu - \delta_{\mu\nu} \eta^a \right] - \left[ M_\mu^{\mu}, \eta^a \right] + \left[ M_\mu^{\mu}, \eta^a \right], \\
Q^a_{\mu} \chi^{b\mu}_\rho &= \delta_{\mu\rho} \phi^{ab} + \epsilon_{\mu\nu\rho\sigma} D^\nu \phi^{ab} - \epsilon^{ab} \delta_{\mu\rho} H_\sigma - \epsilon^{ab} \epsilon_{\mu\nu\rho\sigma} H^{\nu} - \epsilon^{ab} D_{\mu} M_\rho, \\
Q^a_{\mu} G_{\rho\sigma} &= D_{\mu} \chi^{a\rho}_\sigma - \delta_{\mu\rho} \delta_{\nu\sigma} \eta^a - \epsilon_{\mu\nu\rho\sigma} D^\nu \eta^a - \epsilon_{cd} \left[ \phi^{ac}, \delta_{\mu\rho} \psi^{b\sigma}_\sigma + \epsilon_{\mu\nu\rho\sigma} \psi^{b\sigma}_\sigma \right] + \frac{1}{2} \left[ \psi^a_\mu, M_\rho \right],
\end{align*}
\]

(18)
which, together with the space–time translations $P_\mu$, obey the topological superalgebra

$$\{Q^a, Q^b\} = -2\delta_G(\phi^{ab}),$$
$$\{Q^a, Q^b_\mu\} = \epsilon^{ab}(-iP_\mu + \delta_G(A_\mu),$$
$$\{\bar{Q}^\alpha, \bar{Q}^\beta_\nu\} = -2\delta_{\mu\nu}\delta_G(\bar{\phi}^{\alpha\beta}) - \epsilon^{\alpha\beta}\delta_G(M_{\mu\nu}).$$

(19)

Our next goal is to show how the action (19) is obtained from (16) by a dimensional reduction. To this end, we perform a $(3 + 1)$–decomposition of the action (16), i.e., we split the space–time coordinates into $x^\mu = \{x^0, x^4\}$, $\alpha = 1, 2, 3$, where $x^0$ and $x^4$ denote the spatial and the temporal part, respectively. As a next step, we assume that no field depends on the $x^4$ coordinate. To this end, we perform a $(3 + 1)$–decomposition of the action (16), i.e., we split the space–time coordinates into $x^\mu = \{x^0, x^4\}$, $\alpha = 1, 2, 3$, where $x^4$ denotes the temporal part, respectively. As a next step, we assume that no field depends on $x^4$, i.e., $\partial_4 = 0$, so that the integration over $x^4$ factors out and, therefore, can be ignored. Furthermore, we rename the temporal part of $A_\mu$, $\psi^a_\mu$ and $H_\mu$, according to

$$A_4 = 2\rho, \quad \psi^a_4 = \tilde{\eta}^a, \quad H_4 = Y,$$

reserving the notation $A_\alpha$ and $\psi^a_\alpha$ for the spatial part of $A_\mu$ and $\psi^a_\mu$, and identify $M_{\mu\nu}, \chi_{\mu\nu}, G_{\mu\nu}$ and the spatial part of $H_\mu$ with

$$M_{\alpha\beta} = \epsilon_{\alpha\beta\gamma}V^\gamma, \quad M_{\alpha 4} = V_\alpha, \quad \chi^{\alpha}_\alpha = \epsilon_{\alpha\beta\gamma}\tilde{\psi}^{\alpha\gamma}_\alpha, \quad \chi^{\alpha}_4 = \bar{\psi}^\alpha_\alpha,$$
$$G_{\alpha\beta} = \epsilon_{\alpha\beta\gamma}\tilde{B}^\gamma + \epsilon_{\alpha\beta\gamma}D^\gamma\rho, \quad G_{\alpha 4} = \tilde{B}_\alpha + D_\alpha\rho, \quad H_\alpha = B_\alpha - [V_\alpha, \rho],$$

respectively. Here, the $\rho$–dependent shifts in $G_{\alpha\beta}, G_{\alpha 4}$ and $H_\alpha$ ensure that after carrying out the dimensional reduction the hermitean conjugate $\bar{Q}^a$ of the scalar supercharge $Q^a$ coincides with the temporal part $\bar{Q}^a_4$ of the vector supercharge $\bar{Q}^a_\mu$, i.e., $\bar{Q}^a = \bar{Q}^a_4$. Then, after squeezing (16) to $D = 3$, we arrive at the following reduced action

$$S^{(N_T=4)} = \int d^3x \text{tr}\left\{\epsilon^{\alpha\beta\gamma}\tilde{B}_\gamma F_{\alpha\beta} - \epsilon^{\alpha\beta\gamma}\tilde{B}_\gamma[V_\alpha, V_\beta] + 2B^\alpha\tilde{B}_\alpha + 2\epsilon_{\alpha\beta\gamma}\bar{\psi}^\alpha_\gamma D_\alpha\psi^b_\beta$$
$$+ \epsilon_{\alpha\beta\gamma}V^\gamma(\{\psi^a_\alpha, \psi^b_\beta\} - \{\psi^\alpha_\beta, \psi^\beta_\alpha\}) + 2\epsilon_{\alpha\beta\gamma}B_\gamma D_\alpha V_\beta - 2B^\alpha B_\alpha$$
$$- 2\epsilon_{\alpha\beta}\eta^\alpha D^\alpha\psi_\beta + \eta^\alpha D^\alpha\psi^\alpha_\beta + 2\epsilon_{\alpha\beta}\tilde{\psi}^\alpha_\gamma D_\gamma\phi_\beta + 4\epsilon_{\alpha\beta}(\eta^\alpha, \bar{\psi}^\alpha_\beta) + 4\rho\epsilon(\eta^\alpha, \bar{\psi}^\alpha_\beta)$$
$$+ 2\phi(\eta^\alpha, \bar{\psi}^\alpha_\beta) + 2\epsilon_{\alpha\beta}\bar{\psi}^\alpha_\beta D^2\phi_\beta + 2\phi D^2\phi + 2\rho D^2\rho - [V_\alpha, \phi_\beta][V_\alpha, \phi_\beta]$$
$$- 2[V_\alpha, \rho][V_\alpha, \rho] - [\phi_\alpha, \phi_\beta][\phi_\alpha, \phi_\beta] - 4[\rho, \phi_\beta][\rho, \phi_\beta] - 2Y\phi_\alpha^2 - 2Y^2\right\}$$

(20)

which is manifestly invariant under the discrete symmetry

$$(A_\alpha, V_\alpha, B_\alpha, \tilde{B}_\alpha, Y) \rightarrow (A_\alpha, -V_\alpha, -B_\alpha, \tilde{B}_\alpha, -Y),$$
$$(\psi^a_\alpha, \tilde{\psi}^\alpha_\beta, \eta^\alpha, \bar{\eta}^\alpha, \phi_\alpha) \rightarrow (\bar{\psi}^\alpha_\beta, \psi^\alpha_\beta, \bar{\eta}^\alpha, \eta^\alpha, \phi_\alpha, -\rho),$$

(21)

exhibiting that the global symmetry group is actually $SU(2)_R \otimes SU(2)_R$.

Next, after squeezing (17) and (18) to $D = 3$, for the transformation laws generated by the
recalling that
\[ \eta^a, \bar{\phi}^a, \phi^a, Q^a, \bar{Q}^a, \]
and in terms of the complexified supercharges
\[ Q^a = \frac{1}{2} e^{ab} \bar{\eta}^b + \frac{1}{2} e^{ac} \eta^b, \]
\[ Q^a \bar{\eta}^b = -\epsilon_{cd}[\phi^{ac}, \phi^{bd}], \]
\[ Q^a \psi^b = D_a \phi^{ab} - \epsilon^{ab} [V_a, \bar{\rho}] + \epsilon^{ab} B_a, \]
\[ Q^a B_a = -\frac{1}{2} D_a \eta^a + \frac{1}{2} [V_a, \bar{\eta}^a] - \epsilon_{cd}[\phi^{ac}, \bar{\psi}^d] - [\rho, \bar{\psi}^a], \]
\[ Q^a \bar{\psi}^b = [V_a, \phi^{ab}] + \epsilon^{ab} D_a \rho + \epsilon^{ab} \bar{B}_a, \]
\[ Q^a \bar{B}_a = -\frac{1}{2} D_a \bar{\eta}^a - \frac{1}{2} [V_a, \bar{\eta}^a] - \epsilon_{cd}[\phi^{ac}, \bar{\psi}^d] + [\rho, \psi^a], \]
\[ Q^a Y = -[\rho, \bar{\eta}^a] - \epsilon_{cd}[\phi^{ac}, \bar{\eta}^d] \]
and
\[ \tilde{Q}^a A_{\beta} = \delta_{\alpha \beta} \eta^a + \epsilon_{\alpha \beta \gamma} \bar{\psi}^\gamma, \]
\[ \tilde{Q}^a \bar{\psi}^b = -\frac{1}{2} e^{ab} \bar{\psi}^a - \frac{1}{2} e^{ac} \bar{\psi}^b, \]
\[ \tilde{Q}^a \rho = D_a \phi^{ab} - \epsilon^{ab} [V_a, \bar{\rho}] + \epsilon^{ab} B_a, \]
\[ \tilde{Q}^a \bar{\psi}^b = -e^{ab} D_a \bar{\eta}^a - [V_a, \phi^{ab}] + \epsilon^{ab} D_a \rho - \epsilon^{ab} \bar{B}_a, \]
\[ \tilde{Q}^a \bar{B}_a = -e^{ab} D_a \bar{\eta}^a - e^{ab} \bar{B}_a, \]
\[ \tilde{Q}^a \bar{\psi}_\gamma = -e^{ab} D_a \bar{\psi}_\gamma + 2 \epsilon^{ab} \epsilon_{\alpha \beta \gamma} B^\gamma + 2 \epsilon_{\alpha \beta \gamma} \bar{\psi}^\gamma - 2 \epsilon^{ab} \epsilon_{\alpha \beta \gamma} V_a \bar{\eta}^a - \epsilon_{cd}[\phi^{ac}, \bar{\psi}^d] - [\rho, \bar{\psi}^a], \]
\[ \tilde{Q}^a \bar{B}_a = -e^{ab} D_a \bar{\psi}^a - \frac{1}{2} [V_a, \bar{\psi}^a] - \epsilon_{cd}[\phi^{ac}, \bar{\psi}^d] - [\rho, \psi^a], \]
\[ \tilde{Q}^a \bar{Y} = D_a \bar{\psi}_\gamma + [V_a, \bar{\psi}_\gamma] + \epsilon_{cd}[\phi^{ac}, \bar{\psi}^d] - [\rho, \bar{\psi}^a], \]
(22)
respectively. The transformation laws generated by the conjugate supercharges, \( \tilde{Q}^a \) and \( \tilde{Q}^a_\alpha \), are obtained from (22) and (23) by carrying out the replacements (21), mapping \( Q^a \) to \( \tilde{Q}^a \) and \( \tilde{Q}^a_\alpha \) to \( Q^a_\alpha \), respectively.

To make contact with the action and the transformations given in (21), (22) and (23) we express (20), (22) and (23) in terms of the complexified fields \( A_a \pm iV_a, B_a \pm i\bar{B}_a, \psi^a \pm i\bar{\psi}^a, \eta^a \pm i\bar{\eta}^a \) and in terms of the complexified supercharges \( Q^a \pm i\bar{Q}^a, Q^a_\alpha \pm iQ^a_\alpha \), respectively. In addition, we combine \( \phi^{ab} \) and \( \rho \) to form the complex scalar fields
\[ \zeta^{ab} = \phi^{ab} + i\epsilon^{ab} \rho \quad \text{and} \quad \tilde{\zeta}^{ab} = \phi^{ab} - i\epsilon^{ab} \rho, \]
recalling that \( \phi^{ab} \) is symmetric, \( \phi^{ab} = \phi^{ba} \). Then, after carrying out the redefinitions
\[ B_a + i\bar{B}_a \rightarrow B_a, \quad B_a - i\bar{B}_a \rightarrow \bar{B}_a, \]
\[ \eta^a + i\bar{\eta}^a \rightarrow \eta^a, \quad \eta^a - i\bar{\eta}^a \rightarrow \bar{\eta}^a, \quad \psi^a + i\bar{\psi}^a \rightarrow \psi^a, \quad \psi^a - i\bar{\psi}^a \rightarrow \bar{\psi}^a \]
and
\[ Q^a + i \bar{Q}^a \rightarrow Q^a, \quad Q^a - i \bar{Q}^a \rightarrow \bar{Q}^a, \quad \bar{Q}_\alpha^a + i Q_\alpha^a \rightarrow \bar{Q}_\alpha^a, \quad \bar{Q}_\alpha^a - i Q_\alpha^a \rightarrow Q_\alpha^a, \]
it is easily seen that the resulting action and transformations are precisely the ones given in Section 4. Hence, it is proven that by a dimensional reduction of either of the two \( N_T = 2 \), \( D = 4 \) theories one recovers the \( N_T = 4 \) equivariant extension of the Blau–Thompson model proposed in Section 4.

5. Dimensional reduction of half–twisted \( N_T = 1 \), \( D = 4 \) Yamron model

After having described the \( N_T = 4 \) topological twist of \( N = 8 \), \( D = 3 \) SYM, arising from the dimensional reduction of either of the two \( N_T = 2 \), \( D = 4 \) theories, now, for the sake of completeness, we will also explicitly construct the other \( N_T = 2 \) topological model arising either by partially twisting \( N = 8 \), \( D = 3 \) SYM or by dimensional reduction of the half–twisted \( N_T = 1 \), \( D = 4 \) theory \([24]\) (see Diagram).

The global symmetry group of this theory is \( SU(2)_N \otimes SU(2)_R \). The action of this partially twisted theory can be described as the coupling of the \( N_T = 2 \), \( D = 3 \) super–BF theory to a spinorial hypermultiplet \( \{ \lambda_{ab}^A, \zeta_A^a \} \). This hypermultiplet is built up from a \( SU(2)_N \otimes SU(2)_R \) quartet of Grassmann–odd spinor fields \( \lambda_{ab}^A \) and a \( SU(2)_R \) doublet of Grassmann–even spinor fields \( \zeta_A^a \). In order to close the topological superalgebra (see Eq. (29) below), we introduce a \( SU(2)_R \) doublet of bosonic auxiliary spinor fields \( Y_A^a \). The spinor indices are denoted by \( A = 1, 2 \). All the spinor fields are taken in the adjoint representation of the gauge group \( G \).

Omitting any details, after dimensional reduction of the half–twisted \( N_T = 1 \), \( D = 4 \) theory the reduced action, with an underlying \( N_T = 2 \) off–shell equivariantly nilpotent topological shift symmetry \( Q^a \), splits up in two \( SU(2)_N \otimes SU(2)_R \) invariant parts
\[ S^{(N_T=2)} = S_{BF} + S_M, \tag{24} \]
where the first part is just the action of the \( N_T = 2 \), \( D = 3 \) super–BF theory given in \([1]\) and the second one is the matter action
\[ S_M = \int d^3 x \text{tr}\left\{ -i \lambda_{ab}^{(\alpha)}(\sigma^\alpha)^{AB} D_\alpha \lambda_{ab}^{(\beta)} - 2i \lambda_{ab}^{(\alpha)}(\sigma^\alpha)^{AB} [\psi_{\alpha}^a, B^b] + 2 \lambda_{ab} [\eta^a, C^{ab}] + 2 \epsilon_{cd} \lambda_A^{ac} [\phi_{ab}, \lambda_B^{bd}] + 2 \epsilon_{cd} \zeta_{AB}^{(a)} C^{ab} [\phi_{ab}, \zeta_{AC}^{(b)}] - 2i \epsilon_{ab} \zeta_{AB}^{(a)} C^{ab} [\phi_{ab}, \zeta_{BC}^{(a)}] - 2i \epsilon_{ab} \zeta_{AB}^{(a)} C^{ab} [\phi_{ab}, \zeta_{BC}^{(a)}] \right\}; \tag{25} \]
here, \( D_\alpha \) is the covariant derivative (in the adjoint representation) and \( \sigma_\alpha \) are the Pauli matrices,
\[ (\sigma_\alpha)_A^C (\sigma_\beta)_B^C = \delta_{\alpha\beta} \epsilon_{AB} + i \epsilon_{\alpha\beta\gamma} (\sigma_\gamma)_A^B, \quad \alpha = 1, 2, 3. \]
The spinor index \( A \) is raised and lowered as follows, \( \varphi_A = \varphi^B \epsilon_{BA} \) and \( \varphi^A = \epsilon^{AB} \varphi_B \) with \( \epsilon^{AC} \epsilon_{CB} = -\delta_A^B \).

The action \([24]\) can be cast in the \( Q^a \)–exact form
\[ S_M = \frac{1}{2} \epsilon_{ab} Q^a Q^b X_M \]
with the gauge boson
\[ X_M = \frac{1}{2} \int d^3 x \text{tr}\left\{ \lambda_{ab}^{(\alpha)} (\sigma^\alpha)^{AB} D_\alpha \zeta_{ab}^{(a)} \right\} . \]
By a straightforward calculation in can be verified that both parts in (24) are separately invariant under the off–shell equivariantly nilpotent topological supersymmetry \( Q^a \) (cf., Eqs. (3)),

\[
\begin{align*}
Q^a A_\alpha &= \psi^a_\alpha, \\
Q^a \phi_{bc} &= \frac{1}{2} \epsilon^{ab} \eta^c + \frac{1}{2} \epsilon^{ac} \eta^b, \\
Q^a \psi_\alpha &= D_\alpha \phi_{ab} + \epsilon^{ab} B_\alpha, \\
Q^a \lambda^c &= -[\phi_{ab} \zeta^c] + \epsilon^{ab} \psi^c A_\alpha, \\
Q^a \zeta^c &= \frac{1}{2} \eta^c \zeta^b - \epsilon^{cd} \lambda^d,
\end{align*}
\]

and under the vector supersymmetry \( \bar{Q}^a \) (cf., Eqs. (3)),

\[
\begin{align*}
\bar{Q}^a A_\beta &= \delta_{\alpha\beta} \eta^a + \epsilon_{\alpha\beta\gamma} \psi^a, \\
\bar{Q}^a \phi_{bc} &= i(\sigma^a)_{AB} \lambda^{Bab}, \\
\bar{Q}^a \psi_\beta &= \frac{1}{2} \epsilon^{ab} \psi^c - \frac{1}{2} \epsilon^{ac} \psi^b, \\
\bar{Q}^a \lambda^c &= -[\phi_{ab} \zeta^c] + \epsilon^{ab} \psi^c A_\alpha, \\
\bar{Q}^a \zeta^c &= \frac{1}{2} \eta^c \zeta^b - \epsilon^{cd} \lambda^d.
\end{align*}
\]

Furthermore, it can be proven that the sum of the two parts in (24) is invariant under the transformations generated by the spinorial supercharges \( Q^a_\alpha \),

\[
\begin{align*}
Q^a_\alpha A_\beta &= \delta_{\alpha\beta} \eta^a + \epsilon_{\alpha\beta\gamma} \psi^a, \\
Q^a_\alpha \phi_{bc} &= \frac{1}{2} \epsilon^{ab} \psi^c - \frac{1}{2} \epsilon^{ac} \psi^b, \\
Q^a_\alpha \lambda^c &= \frac{1}{2} \eta^c \zeta^b - \epsilon^{cd} \lambda^d,
\end{align*}
\]

By the symmetry requirements \( Q^a S_{BF} = \bar{Q}^a S_{BF} = 0 \) and \( Q^a S_M = \bar{Q}^a S_M = 0 \), together with \( Q^a_{\alpha}(S_{BF} + S_M) = 0 \), all the relative numerical coefficients of the action \( S_{BF} + S_M \) are uniquely fixed, except for a single overall coupling constant.

The eight supercharges \( Q^a, \bar{Q}^a \) and \( Q^a_\alpha \), together with the generator \( P_\alpha \) of space–time translations, satisfy the topological superalgebra

\[
\begin{align*}
\{Q^a, Q^b\} &= -2 \delta_G(\phi^{ab}), \\
\{Q^a, \bar{Q}^a_\alpha\} &= \epsilon^{ab}(-iP_\alpha + \delta_G(A_\alpha)), \\
\{Q^a, Q^b_\alpha\} &= -\epsilon^{ab} \delta_G(\zeta^c), \\
\{Q^a_\alpha, Q^b_\beta\} &= -2 \delta_{\alpha\beta} \delta_G(\phi^{ab}), \\
\{Q^a_\alpha, \bar{Q}^a_\beta\} &= -\epsilon^{ab} (\sigma^a)_{AB} \delta_G(\zeta^c), \\
\{Q^a_\alpha, Q^b_\beta\} &= 2 \epsilon^{cd} \epsilon_{AB} \delta_G(\phi^{ab}) - \epsilon^{ab} \epsilon^{cd} (\sigma^a)_{AB} (-iP_\alpha + \delta_G(A_\alpha)).
\end{align*}
\]
Finally, let us notice that the dimensional reduction of the half-twisted \( N_T = 1, D = 4 \) theory can also be described by decomposing \( \text{Spin}(7) \rightarrow G_2 \rightarrow SU(2)_N \otimes SU(2)_R \) \[16\]. In that description, however, only the diagonal subgroup of the global symmetry group \( SU(2)_N \otimes SU(2)_R \) is manifest. The action of this theory coincides with the dimensional reduction of the \( N_T = 1, D = 4 \) Donaldson–Witten theory \[3\] coupled to the standard hypermultiplet \[32\] to \( D = 3 \).

### Conclusions and Remarks

In the present paper we proposed a \( N_T = 4 \) equivariant extension of the Blau–Thompson \( N_T = 2 \) non-equivariant topological model in \( D = 3 \) Euclidean space–time. Furthermore, we showed, by proving the equivalence with the dimensional reduction to \( D = 3 \) of the Yamron–Vafa–Witten \( N_T = 2, D = 4 \) theory, that this extended topological model coincides with one of the two inequivalent topological models which, according to the classification of \[16\], may be constructed by twisting the \( N = 8, D = 3 \) super–Yang–Mills theory. In addition, we constructed explicitly also the other topological model which may be obtained, namely, the \( N_T = 2, D = 3 \) super–BF theory coupled to a spinorial hypermultiplet.

All these models, including the \( N_T = 2, D = 3 \) super–BF model, are considered in the flat Euclidean space–time where, besides the topological shift symmetry also the vector supersymmetry can be constructed. All these symmetries have been given explicitly. The actions of the corresponding theories are shown to be uniquely determined, up to an overall coupling constant, when they are required to be invariant not only with respect to the topological shift symmetry but also with respect to the vector supersymmetry. Furthermore, it is shown that these symmetry operations, together with the generators of space–time translations, fulfill corresponding topological superalgebras.

As already mentioned in the introduction, the Blau–Thompson \( N_T = 2 \) non-equivariant topological model and its \( N_T = 4 \) equivariant extension provide not only the link between the various topological theories arising from twisting \( N = 1, D = 6 \) or \( N = 1, D = 10 \) super–Yang–Mills theory, but also are pre–candidates, after carrying out a dimensional reduction to \( D = 2 \), for Hodge–type cohomological theories in \( D = 2 \). Of course, in searching for all of the Hodge–type cohomological theories in \( D = 2 \) a complete group theoretical classification of the topological models in \( D = 2 \) and their explicit construction would be necessary.

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