Computational treatment of the neutrino signals produced in nuclear detectors

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Abstract.
We apply the convolution (folding) method in neutrino physics studies. We focus on its use to explore the response of some nuclear detectors to the energy spectra of laboratory neutrinos. After calculating the neutrino-nucleus cross sections (within the context of a nuclear model) for a neutrino detector, the obtained cross section values must be folded with a specific neutrino-energy distribution. In the present work we use the $\nu$-distribution of pion-muon decay at rest neutrino beams created in muon factories like the Fermilab, J-PARC, and other laboratories. Due to the fact that neutrino-nucleus interactions are very weak, the evaluated cross sections are small ($\sim 10^{-42}$ cm$^2$). Thus, one needs a very fine convolution tool to obtain accurate description of the $\nu$-signals (laboratory, supernova neutrinos, etc.) recorded at some nuclear detectors.

1. Introduction
In recent years, there are two neutrino-beam facilities for measuring neutrino-nucleus cross sections: (i). The pion-muon decay at rest (DAR) neutrino facilities existing at Fermilab, USA, at J-PARC, Japan, etc., or proposed to operate near spallation neutron sources (ORLaND experiment, at ORNL, and European Spallation Source, Sweden) [1]. There, the neutrino beams may be intense neutrino pulses of energy spectra that could be unique for terrestrial studies of astrophysical reactions (solar neutrino reactions, etc.) and also suitable for investigation of other neutrino-mediated processes. (ii). The beta-beam neutrino sources which produce neutrino beams of low and intermediate energies through the use of boosted radioactive nuclei that decay by emitting $\beta^\pm$ particles and (anti)neutrinos. The neutrino beams of such facilities are intense, collimated and pure, appropriate for searching standard and non-standard $\nu$-physics as well as for interpreting low-energy neutrino signals. For example, the analysis of supernova neutrino energy spectra, whenever observed, could be realized through proper measurements on low-energy beam spectra originating from boosted radioactive ions $^6$He, $^{18}$Ne, and other ions.

With the above neutrino-beam facilities the physics research that could be undertaken, is associated with the open issues of low- and intermediate-energy neutrinos within nuclear, particle physics, and astrophysics. Within this research there are some well motivated studies on prominent isotopes. They have the potential to act as neutrino detectors. Through these studies we may also investigate the structure properties of nuclei.

The energy spectra of the laboratory neutrinos (see Fig. 1) generated from muons decaying at rest (also the low-energy $\beta$-beam neutrinos), are similar to those of the neutrinos created in
Figure 1. Energy-spectra of $\nu_e$ and $\bar{\nu}_\mu$ neutrino beams, originating from the muon-decay reaction Eq. (1). The $\nu$ energy-distributions are described by the expressions of Eqs. (2).

the core collapse supernova, except in their high energy region where the Supernova neutrinos have a long tail in contrast to the sharp cutoff present in the spectrum of the laboratory neutrino beams. Despite this difference in the high energy region, the similarity in the energy range and spectrum of the supernova neutrinos with those coming from the muons decaying at rest, opens up the possibility of connecting the ground based neutrino-nucleus experiments with the study of supernova neutrino nuclear cross sections.

Such a type of $\nu$-simulations, which could be useful in calibrating the terrestrial detectors proposed for the detection of supernova neutrinos, are going to be discussed in the present paper. This study is also useful in understanding the mechanism of r-process nucleosynthesis leading to the formation of heavy elements in the interstellar medium [3].

2. Energy-spectra of neutrino sources

2.1. Pion-muon decay at rest neutrino energy distributions

In the operating pion-muon decay at rest neutrino sources (Fermilab, at USA, J-PARC, at Japan) and the expected to operate neutrino facilities at the Neutron Spallation Sources (ORNL, USA, and Lund, Sweden), $\nu_e$ neutrinos, and $\bar{\nu}_\mu$ anti-neutrinos are produced from the decay of muons according to the reaction

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \; .$$

Because the decaying muons result from the decay of slow pions ($\pi^+ \rightarrow \mu^+ + \nu_\mu$), they have relatively low energies. In these facilities, the energy-spectra of $\nu_e$ and $\bar{\nu}_\mu$ neutrinos (see Fig. 1) are approximately described by the normalized distributions [1, 2]

$$\eta_{\nu_e}(\varepsilon_\nu) = 96\varepsilon_\nu^2 M_\mu^{-4} (M_\mu - 2\varepsilon_\nu) \; , \quad \eta_{\bar{\nu}_\mu}(\varepsilon_\nu) = 16\varepsilon_\nu^2 M_\mu^{-4} (3M_\mu - 4\varepsilon_\nu) \; ,$$

(2)

where $M_\mu = 105.6$ MeV, is the muon rest mass. The maximum energy of $\nu_e$ and $\bar{\nu}_\mu$ in the later equations is $\varepsilon_{\nu,\bar{\nu}}^{\text{max}} = 52.8$ MeV = $M_\mu/2$ [3, 4, 5]. The distribution of these $\nu_e$ neutrinos is known as Michel energy spectrum. It is worth noting that, the pion-muon decay at rest neutrino beams are not completely pure as, for example, the $\beta$-beam neutrinos.

From a simulation performance point of view, the analytic expressions of Eqs. (2) are convenient for the required numerical integration in the convolution procedure [4, 5]. On the other hand, their energy range and shape roughly resembles that of SN neutrino (alternative distributions are discussed in Refs. [3, 6]).
Figure 2. Total cross sections (in $10^{-42} \text{cm}^2$) for neutrino scattering on $^{130}\text{Te}$ isotope. In the inset contributions coming from the discrete spectrum are shown, while the solid line illustrates the contribution of the continuum nuclear spectrum.

The theoretical results of $\nu$-nucleus cross sections (see Fig. 2 for the case of $^{130}\text{Te}-\nu$ scattering) can be connected with the neutrino experiments and the neutrino sources discussed before. To this end, we have to carry out the folding (convolution) procedure on the calculated cross sections with the distribution $\eta(\varepsilon_{\nu})$ of the neutrino source of interest. In this way we estimate the response of several nuclear isotopes to the corresponding neutrino spectrum.

3. The convolution procedure with laboratory neutrino distribution

The signal of a nuclear $\nu$-detector (throughout its energy spectrum as a function of the incoming energy $\varepsilon_{\nu}$ created by a specific neutrino distribution $\eta(\varepsilon_{\nu})$ is written as (see Ref. [7])

$$\sigma_{\text{fold}}(\varepsilon_{\nu}) = \sum_{\omega=\varepsilon_{\text{thres}}} \sigma(\omega, \varepsilon_{\nu}) \eta(\varepsilon_{\nu}).$$

(3)

For the total $\nu$-nucleus cross section $\sigma_{\text{tot}}(\varepsilon_{\nu})$, the signal on the detector of the $\nu$ spectral distribution $\eta(\varepsilon_{\nu})$, is evaluated by

$$\sigma_{\text{fold}}^{\text{sign}}(\varepsilon_{\nu}) = \sigma_{\text{tot}}(\varepsilon_{\nu}) \eta^{\text{sign}}(\varepsilon_{\nu}).$$

(4)

As a concrete example, in Fig. 3 we present results based on the latter expression for the cross sections of $\nu$-scattering on $^{130}\text{Te}$ isotope. The folding is performed with the normalized energy distributions given by the expressions of Eq. (2). More specifically, with the distribution $\eta_{\nu_e}(\varepsilon_{\nu})$ we obtain the signal of Fig. 3(left) and with the $\eta_{\nu_{\mu}}(\varepsilon_{\nu})$ we obtain the signal of Fig. 3(right).

It is worth remarking that, difficulties arise due to the fact that, the nuclear excitation spectrum is partly discrete for low-lying transitions $\omega \leq 10\text{MeV}$ (see the inset of Fig.2) and partly continuum, i.e. above nucleon emission threshold, namely for $\varepsilon_{\nu} \gtrsim 10 \text{ MeV}$ (see Fig. 2). This imposes the use of discrete convolution for the discrete spectrum, and numerical integration for the continuum spectrum or the use of proper interpolation techniques [3, 6, 7]. We note that, in most of the previous similar convolutions, the discrete spectrum is not included in the integrals of Eqs. (3) and (4), but also in other convolution integrals. The latter observable is obtained by inserting the total neutrino-nucleus cross sections, $\sigma_{\text{tot}}(\varepsilon_{\nu})$, in the convolution integral [2, 5, 7]...
which is a useful quantity like the known as flux averaged total cross section, $\langle \sigma_{\text{tot}} \rangle$, for various neutrino studies.

$$\langle \sigma_{\text{tot}} \rangle = \int_{\varepsilon_{\text{thr}}(\varepsilon_{\nu})}^{\infty} \sigma_{\text{tot}}(\varepsilon_{\nu}) \eta(\varepsilon_{\nu}) d\varepsilon_{\nu}, \tag{5}$$

($\varepsilon_{\text{thr}}$ is the threshold energy of the nuclear detector in question). For the sake of completeness, we mention that, we furthermore distinguish two special cases of the convolutions of Eqs. (3), (4) and (5): the radiochemical ($\sigma_{\text{rad}}^{\text{incoh}}$) which includes only the particle bound transitions (discrete spectrum), and the coherent flux-averaged cross sections ($\sigma_{\text{coh}}^{\text{fold}}$) which includes the ground-state to ground state transitions. These quantities are important to be computed for various neutrino sources.

4. Summary and Conclusions

In this work, we discussed the computational treatment of neutrino-nucleus cross section calculations performed with the use of the convolution method. In this way, the signal created at nuclear $\nu$-detectors can be simulated by employing various $\nu$-energy distributions. As a special case, the convolution involved in Eqs. (4) and (5) was carried out by employing the MERLIN package [8] which has been successfully used in similar studies in Refs. [3, 4, 5].

5. Acknowledgments

One of us (T.S.K) wishes to thank Dept. of Information Engineering, TEI of W. Macedonia for financial support.

References

[1] Louis W C 2009 Prog. Part. Nucl. Phys. C 63 51.
[2] Kosmas T S and Oset E 1996 Phys. Rev. C 53 1409.
[3] Tsakstara V and Kosmas T S 2011 Phys. Rev. C 84 064620 and references therein.
[4] Tsakstara V and Kosmas T S 2011 Phys. Rev. C 83 054612.
[5] Tsakstara V and Kosmas T S 2012 Phys. Rev. C 86 044618.
[6] Jachowicz N, McLaughlin G C, Volpe C 2008 Phys. Rev. C 77, 055501.
[7] Tsakstara V, Kosmas T S, Sinatkas J et. al. 2007 Proc. Int. Conf. on Computational Methods in Science and Engineering (Corfu) vol 2 (AIP) p 1383 and references therein.
[8] Lagaris I E, Likas A, Fotiadis D 1997 Comput. Phys. Commun. 104 1; Kosmas T S, Lagaris I E 2002 J. Phys. G 28 2907.