Research Article

Equation of States and Charmonium Suppression in Heavy-Ion Collisions

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The present article is the follow-up of our work Bottomonium suppression in quasi-particle model, where we have extended the study for charmonium states using quasi-particle model in terms of quasi-gluons and quasi quarks/antiquarks as an equation of state. By employing medium modification to a heavy quark potential thermodynamic observables, viz., pressure, energy density, speed of sound, etc. have been calculated which nicely fit with the lattice equation of state for gluon, massless, and as well massive flavored plasma. For obtaining the thermodynamic observables we employed the debye mass in the quasi-particle picture. We extended the quasi-particle model to calculate charmonium suppression in an expanding, dissipative strongly interacting QGP medium (SIQGP). We obtained the suppression pattern for charmonium states with respect to the number of participants at mid-rapidity and compared it with the experimental data (CMS JHEP) and (CMS PAS) at LHC energy (Pb+Pb collisions, \( \sqrt{s_{NN}} = 2.76 \) TeV).

1. Introduction

The primary goal of heavy-ion experiment at the RHIC and the LHC is to search a new state of matter, i.e., the Quark Gluon Plasma. To study the properties of the Quark Gluon Plasma (QGP) heavy quarks are considered to be a suitable tool. Initially, the heavy quarks can be calculated in pQCD, which are produced in primary hard N N collisions [1]. The charmonia is a bound states of charm (c) and anticharm (\( \bar{c} \)), which is an extremely broad and interesting field of investigation [2]. Charmonium states can have smaller sizes than hadrons (down to a few tenths of a fm) and large binding energies (> 500 MeV) [3]. In ultrarelativistic heavy-ion collisions, it has been realized that early ideas associating with charmonium suppression with the deconfinement transition [4] are less direct than originally hoped for [5–8].

At sufficiently large energy densities, lattice QCD calculations predict that hadronic matter undergoes a phase transition of deconfined quarks and gluons, called Quark Gluon Plasma (QGP). In order to reveal the existence and to analyze the properties of this phase transition several researches in this direction have been done. In the high-energy heavy-ion collision field, the study of charmonium production and suppression is the most interesting investigations, since the charmonium yield would be suppressed in the presence of a QGP due to color Debye screening [4].

In heavy-ion collisions, charmonium suppression study has been carried out first at the Super Proton Synchrotron (SPS) by the NA38 [9–11] and NA60 [12] and then at the Relativistic Heavy-Ion Collider (RHIC) by the PHENIX experiment at \( \sqrt{s_{NN}} = 200 \) GeV [13]. The suppression is defined by the ratio of the yield measured in heavy-ion collisions and a reference, called the nuclear modification factor \( R_{AA} \) [14] and it is considered as a suitable probe to identify the nature of the matter created in heavy-ion collisions. At high temperature, Quantum Chromodynamics (QCD) is believed to be in Quark Gluon Plasma (QGP) phase, which is not an ideal gas of quarks and gluons, but rather a liquid having very low shear viscosity to entropy density (\( \eta/s \)) ratio [15–20].

This strongly suggests that QGP may lie in the nonperturbative domain of QCD which is very hard to address both analytically and computationally. Similar conclusion about QGP and perfect fluidity of QGP have been reached from recent lattice studies and from the AdS/CFT studies [20], spectral functions and transport coefficients in lattice...
QCD [21] and studies based on classical strongly coupled plasmas [22–24], which predict that the equation of state (EoS) is interacting even at \( T \sim 4T_c [25–30]. \)

The bag model, confinement models, and quasi-particle models are the several models for studying the EoS of strongly interacting quark gluon plasma [31, 32], etc. Here in our analysis we are using quasi-particle debye mass [33] where equation of state was derived with temperature dependent parton masses and bag constant [34, 35], with effective degrees of freedom [36], etc. All of them claim to explain lattice results, either by adjusting free parameters in the model or by taking lattice data on one of the thermodynamic quantity as an input and predicting other quantities. However, physical picture of quasi-particle model and the origin of various temperature dependent quantities are not clear yet [37]. In strongly interacting QGP [38–40], one considers all possible hadrons even at \( T > T_c \) and try to explain nonideal behavior of QGP near \( T_c \). Recently, an equation of state for strongly coupled plasma has been inferred by utilizing the understanding from strongly coupled QED plasma [41] which fits lattice data well. It is implicitly assumed that once the charmonium dissociates, the heavy quarks hadronize by combining with light quarks only [42]. About 60% of the observed \( J/\psi \)’s are directly produced in a hadronic collisions, the remaining stemming from the decays of \( \chi_c \) and \( \psi' \), excited charmonium states. Since each \( c\bar{c} \) bound state dissociates at a different temperature, a model of sequential suppression was developed, with the aim of reproducing the charmonium suppression pattern in the heavy-ion collision [43–47]. A suppressed yield of quarkonium in the dilepton spectrum, measured in experiments [48, 49], was proposed as a signature of QGP formation. To determine quarkonium spectral functions at finite temperature there are mainly two theoretical lines of studies which are potential models [50–52] and lattice QCD [21, 53–55].

The central theme of our work is that the potential which we are considering in the deconfined phase could have a nonvanishing confining (string) term, in addition to the Coulomb term [56] unlike Coulomb interaction alone in the aforesaid model [32]. By incorporating this potential we had calculated the thermodynamic variables, viz., pressure, energy density, speed of sound, etc. Our results match nicely with the lattice results of gluon [25–28] and 2-flavor (massless) as well as 3-flavor (massless) QGP [57, 58]. There is also an agreement with (2+1) (two massless and one is massive) and 4 flavoured lattice results too. Motivated by the agreement with lattice results, we employ our equation of state (using quasi-particle Debye mass) to study the Charmonium suppression in expanding plasma in the presence of viscous forces. Here in this work we are not considering the bulk viscosity. This issue will be taken into consideration in near future. \( R_{AA} \) of prompt and nonprompt \( J/\psi \) has been measured separately by CMS in bins of transverse momentum, rapidity, and collision centrality [14]. We have compared our results with the experimental data (CMS JHEP) [14] and (CMS PAS) [59] in Pb+Pb collision at LHC energy and found \( (S^{\text{med}}) \) is closer to the experimental results.

In our previous work [60], we had calculated the plasma parameter, pressure, energy density, and speed of sound for only 3-flavor QGP and finally studied the sequential suppression for bottomonium states at the LHC energy in a longitudinally expanding partonic system for only \( \eta/s = 0.08 \) because the experimental data is available only for ADS/CFT case. In this present article we have extended our previous work for charmonium states for all 3-flavors by using quasi-particle model in terms of quasi-gluons and quasi quarks/antiquarks as an equation of state. Here, we had considered three values of the shear viscosity-to-entropy density ratio to see the effects of nonzero values of the shear viscosity on the expansion. The first one is from perturbative QCD calculations, where \( \eta/s = 0.3 \) near \( T \sim T_c \). The second one is from AdS/CFT studies, where \( (\eta/s) = 1/4\pi \approx 0.08 \). Finally we consider \( \eta/s = 0 \) (for the ideal fluid) for the sake of comparison. These three ratios have been used only for the charmonium states for both EoS1 and EoS2.

The paper is organized as follows. In Section 2 we briefly discuss our recent work on medium modified potential in isotropic medium and we study the effective fugacity quasi-particle model (EFPQM). In Section 3 we studied binding energy and dissociation temperature of \( J/\psi, \psi' \), and \( \chi_c \) state considering isotropic medium. Using this effective potential and by incorporating quasi-particle debye mass, we have then developed the equation of state for strongly interacting matter and have shown our results on pressure, energy density, speed of sound, etc., along with the lattice data in Section 4. In Section 5, we have employed the aforesaid equation of state to study the suppression of charmonium in the presence of viscous forces and estimate the survival probability in a longitudinally expanding QGP. Results and discussion will be presented in Section 6 and finally, we conclude in Section 7.

### 2. Medium Modified Effective Potential and Fugacity Quasi-Particle Model

The interaction potential between a heavy quark and antiquark gets modified in the presence of a medium. The static interquark potential plays vital role in understanding the fate of quark-antiquark bound states in the hot QCD/QGP medium. In the present analysis, we preferred to work with the Cornell potential [3, 61] that contains the Coulombic as well as the string part given as

\[
V(r) = -\frac{\alpha}{r} + \sigma r. \tag{1}
\]

Here, \( r \) is the effective radius of the corresponding quarkonia state, \( \alpha \) is the strong coupling constant, and \( \sigma \) is the string tension. The in-medium modification can be obtained in the Fourier space by dividing the heavy-quark potential from the medium dielectric permittivity, \( \epsilon(k) \) as

\[
\tilde{V}(k) = \frac{\tilde{V}(k)}{\epsilon(k)}, \tag{2}
\]

where \( \tilde{V}(k) \) is the Fourier transform of \( V(r) \), shown in (1), given as

\[
\tilde{V}(k) = -\sqrt{\frac{2}{\pi}} \left( \frac{\alpha}{k^2} + 2\sigma k^2 \right). \tag{3}
\]
and $\epsilon(k)$ is the dielectric permittivity which is obtained from the static limit of the longitudinal part of gluon self-energy [62]

$$
\epsilon(k) = \left(1 + \frac{\Pi_L(0,k,T)}{k^2}\right) \equiv \left(1 + \frac{\gamma_D}{k^2}\right).
$$

(4)

Next, substituting (3) and (4) into (1) and evaluating the inverse FT, we obtain $r$-dependence of the medium modified potential [63]:

$$
V(r,T) = \left(\frac{2\sigma}{m_D^2} - \alpha\right) \frac{(-m_Df)}{r} - \frac{2\sigma}{m_D^2r} + \frac{2\sigma}{m_D^2} - \alpha m_D.
$$

(5)

In the limiting case $r >> 1/m_D$, the dominant terms in the potential are the long range Coulombic tail and $\alpha m_D$. The potential will be shown as

$$
V(r,T) \sim -\frac{2\sigma}{m_D^2r} - \alpha m_D.
$$

(6)

Now we employ the Debye mass computed from the effective fugacity quasi-particle model (EQPM) [64, 65] to determine the dissociation temperatures for the charmonium states in isotropic medium computed for EoS1 and EoS2, respectively, and develop the equation of state for strongly interacting matter. The Debye mass, $m_D$, is defined in terms of the equilibrium (isotropic) distribution function as

$$
m_D^2 \equiv -\frac{g}{2} \int \frac{d^3p}{(2\pi)^3} \frac{df_{eq}(\vec{p})}{d\vec{p}}.
$$

(7)

where $f_{eq}$ is taken to be a combination of ideal Bose-Einstein and Fermi-Dirac distribution functions as [66] and is given by

$$
f_{eq} = 2N_c f_g(\vec{p}) + 2N_f \left(f_q(\vec{p}) + f_{\bar{q}}(\vec{p})\right),
$$

(8)

Here, $f_g$ and $f_q$ are the quasi-parton thermal distributions, $N_c$ denotes the number of colors, and $N_f$ the number of flavors.

Now, we obtain quasi particle debye mass for full QCD/QGP medium by considering quasi-parton distributions and EoS1 is the $O(g)$ hot QCD [67–69] and EoS2 is the $O(g^2 \ln(1/g))$ hot QCD EoS [70] in the quasi-particle description [64, 65], respectively.

### 3. Binding Energy and Dissociation Temperature

Binding energy is defined as the distance between peak position and continuum threshold at finite temperature. The medium modified potential has similar appearance to the hydrogen atom problem [71]. Therefore to get the binding energies with medium modified potential we need to solve the Schrödinger equation numerically. The solution of the Schrödinger equation gives the eigenvalues for the ground states and the first excited states in charmonium ($I/\psi$, $\psi'$, etc.) and bottomonium ($Y$, $Y'$, etc.) spectra:

$$
E_n = -\frac{1}{n^2} \frac{m_Q g^2}{m_D^4},
$$

(9)

where $m_Q$ is the mass of the heavy quark.

In our analysis, the quark masses $m_{I/\psi}$, as $m_{I/\psi} = 3.09$ GeV, $m_{\psi'} = 3.68$ GeV, and $m_{\chi_c} = 3.73$ GeV, as calculated in [72] and the string tension ($\sigma$) is taken as 0.184 GeV$^2$.

We listed the values of dissociation temperature in Tables 1 and 2 for the charmonium states $I/\psi$, $\psi'$, and $\chi_c$ for EoS1 and EoS2, respectively, and also have seen that $\psi'$ dissociates at lower temperatures as compared to $I/\psi$ and $\chi_c$ for both the EoS.

### 4. Equation of States of Different Flavors in Quasi-Particle Picture

An extensive study of strong-coupled plasma in QED with proper modifications to include colour degrees of freedom and the strong running coupling constant gives an expression for the energy density as a function of the plasma parameter can be written as

$$
\epsilon = (3 + u_{ex}(\Gamma)) nT,
$$

(10)

Now, the scaled-energy density is written in terms of ideal contribution

$$
\epsilon(\Gamma) \equiv \frac{\epsilon}{\epsilon_{SB}} = 1 + \frac{1}{3} u_{ex}(\Gamma),
$$

(11)

At sufficiently high temperature one must expect hadrons to melt, deconfining quarks and gluons. The exposure of new
(color) degrees of freedom would then be manifested by a rapid increase in entropy density, hence in pressure, with increasing temperature, and by a consequent change in the equation of state (EOS) [15–17]. In this section we will find the pressure, energy density and speed of sound for pure gauge, 2-flavor, 3-flavor, (2+1)-flavor, and 4-flavors QGP for EoS1 and EoS2. To begin with first of all, we will calculate the energy density $\epsilon(T)$ from (11) and using the thermodynamic relation,

$$\epsilon = T \frac{dP}{dT} - P,$$

and we calculated the pressure as

$$P = \left( \frac{P_0/T_0 + 3af \int_{T_0}^{T} d\tau \tau^2 e(\Gamma(\tau))}{T^3} \right).$$

Here $P_0$ is the pressure at some reference temperature $T_0$. This temperature has been fixed with the values of pressure at critical temperature 275 MeV, 175 MeV, 155 MeV, and 205 MeV for a particular system—pure gauge, 2-flavor, 3-flavor, and 4-flavor QGP, respectively. For the sake of comparison with the results of Bannur EoS we took the same value of critical temperature as used in Bannur Model. Now, the speed of sound $c_s$ can be calculated once we know the pressure $P$ and energy density $\epsilon$. In Figures 1 and 2, we have plotted the variation of pressure ($P/T^4$) with temperature ($T/T_c$) using EoS1 and EoS2 for pure gauge, 2-flavor, and 3-flavor QGP along with Bannur EoS [32] and compared it with lattice results [25–28,32]. For each flavor, $g^2_\chi$ and $\Lambda_T$ are adjusted to get a good fit to lattice results in Bannur Model. However, in our calculation we have fixed $P_0$ from the lattice data at the critical temperature $T_c$ for each system as mentioned above, and there is no quantity to be fitted for predicting lattice results as done in Bannur case. Now, energy density $\epsilon$, speed of sound $c_s^2$, etc., can be derived since we
had obtained the pressure, $P(T)$. In Figures 3 and 4, we had plotted the energy density ($\varepsilon/T^4$) with temperature ($T/T_c$) using EoS1 [67–69] and EoS2 for pure gauge, 2-flavor, and 3-flavor QGP along with Bannur EoS [32] and compared it with lattice result [25–28, 32]. We observe that reasonably good fit is obtained without any extra parameters for all three systems. As the flavor increases, the curves shift to left. In Figures 5 and 6, the speed of sound, $c_s^2$ is plotted for all three systems, using EoS1 and EoS2 for pure gauge, 2-flavor, and 3-flavor QGP along with Bannur EoS [32]. Since lattice results are available for only pure gauge, therefore comparison has been checked for the above-mentioned flavor only. Our flavored results match excellently with the lattice results. We observe that as the flavor increases $c_s^2$ becomes larger for both EoS1 and EoS2. All three curves show similar behaviour, i.e., sharp rise near $T_c$ and then flatten to the ideal value (1/3). However, in the vicinity of critical temperature, fits or predictions may not be good, especially for energy density $\varepsilon$ and $c_s^2$ which strongly depends on variations of pressure $P$ with respect to temperature $T$. However, except for small region at $T = T_c$, our results are very good for all regions of $T > T_c$. It is interesting to note that Peshier and Cassing [73] also obtained similar results on the dependence of plasma parameter $\Gamma$ in quasi-particle model and concluded that QGP behaves like a liquid, not weakly interacting gas. Now for the realistic case $u$ and $d$ quarks have very small masses (5-10 MeV), strange quarks are having masses 150-200 MeV, and charm quark is with mass 1.5 GeV. Let $g_f$ count the effective number of degrees of freedom of a massive Fermi gas. For a massless gas we have, of course, $g_f = n_f$. In Figures 7–10, we have shown our results on (2+1)-flavors and 4-flavors QGP using EoS1 and EoS2 for pure gauge, 2-flavor, and 3-flavor QGP and compared it with Bannur EoS along with lattice data [74, 75] and replotted the variation of $P(T)/T^4$ and energy density $\varepsilon(T)/T^4$ with temperature $T/T_c$ for all systems. This has concluded that in the massless limit the deviations of...
Figure 5: Plots of $c_s^2$ as a function of $T/T_c$ for Bannur EoS, Our EoS (using quasi-particle Debye mass) for pure gauge (extreme left figure), 2-flavor QGP (middle figure), and 3-flavor QGP (extreme right figure) for EOS1 [67–69].

Figure 6: Plots of $c_s^2$ as a function of $T/T_c$ for Bannur EoS, Our EoS (using quasi-particle Debye mass) for pure gauge (extreme left figure), 2-flavor QGP (middle figure), and 3-flavor QGP (extreme right figure) for EOS2 [70].

Figure 7: Variation of $P/T^4$ as a function of $T/T_c$ for Bannur, Our EoS (using quasi-particle Debye mass), and lattice results [25–28, 32] for two massless and one massive (2+1) extremely left, middle, and extremely right figure for 4-flavour QGP for two different masses, $m/T=0.4$ and 0.2, respectively, for EOS1 [67–69]. The notations are the same as in Figure 1.
Figure 8: Variation of $P/T^4$ as a function of $T/T_c$ for Bannur, Our EoS (using quasi-particle Debye mass), and lattice results [25–28, 32] for two massless and one massive (2+1) extremely left, middle, and extremely right for 4-flavour QGP for two different masses, $m/T=0.4$ and 0.2, respectively, for EoS2 [70]. The notations are the same as in Figure 1.

Figure 9: Variation of $\epsilon/T^4$ as a function of $T/T_c$ for Bannur, Our EoS (using quasi-particle Debye mass), and lattice results [25–28, 32] for two massless and one massive (2+1) extremely left, middle, and extremely right for 4-flavour QGP for two different masses, $m/T=0.4$ and 0.2, respectively, for the EoS1 [67–69] where the notations are the same as in Figure 1.

Figure 10: Variation of $\epsilon/T^4$ as a function of $T/T_c$ for Bannur, Our EoS (using quasi-particle Debye mass), and lattice results [25–28, 32] for two massless and one massive (2+1) extremely left, middle, and extremely right for 4-flavour QGP for two different masses, $m/T=0.4$ and 0.2, respectively, for the EoS2 [70] where the notations are the same as in Figure 1.
pressure from the ideal gas value are larger in the presence of a heavier quark. This is in qualitative agreement with the observations. We also calculate the thermodynamical quantities, viz., pressure, screening energy density ($\epsilon_s$), the speed of sound, etc. to study the hydrodynamical expansion of plasma and finally to estimate the suppression of $J/\psi$ in nuclear collisions.

5. Survival Probability of $c\bar{c}$ States

To obtain the charmonium survival probability for an expanding QGP/QCD medium in the presence of viscous forces, the solution of equation of motion gives the time $\tau_s$, which is estimated when the energy density drops to the screening energy density $\epsilon_s$ as

$$\tau_s (r) = \tau \left[ \frac{\epsilon_s (r) - 4a/3r_s^3}{\epsilon_s - 4a/3r_s^3} \right]^{1/2(r_s^2)}$$

(14)

where $\epsilon_s (r) = \epsilon (\tau_s, r)$ and $r_s^2 = (1 - c^2) r_F^2$. The critical radius $r_s$ is seen to mark the boundary of the region where the quarkonium formation is suppressed, can be obtained by equating the duration of screening $\tau_s (r)$ to the formation time $t_F = r_F/\tau_F$ for the quarkonium in the plasma frame, and is given by

$$r_s = R_F (1 - A)^{1/2} (1 - A).$$

(15)

The quark-pair will escape the screening region (and form quarkonium) if its position $r$ and transverse momentum $p_T$ are such that

$$r + \frac{r_F p_T}{M} \geq r_s.$$  

(16)

Thus, if $\phi$ is the angle between the vectors $r$ and $p_T$, then

$$\cos \phi \geq \left[ (r_s^2 - r_F^2) M - r_F^2 p_T^2 / M \right] / [2r_F p_T].$$

(17)

Here we choose $\alpha = 0.5$ in our calculation as used in [76]. Therefore the survival probability for the charmonium in QGP medium can be expressed as [76–78]

$$S\left(p_T, N_{part}\right) = \frac{2(\alpha + 1)}{\pi R_F^2} \int_0^{p_T} d\mathbf{r} r_{\phi_{max}} (r) \left( \frac{1 - r_s^2}{R_s^2} \right)^\alpha,$$

(18)

where $\phi_{max}$ is the maximum positive angle [79]. In nuclear collisions, the $p_T$-integrated inclusive survival probability of $J/\psi$ in the QGP/QCD medium becomes [21, 80]

$$\langle S^{incl} \rangle = 0.6 \langle S^{dir} \rangle_{J/\psi} + 0.3 \langle S^{dir} \rangle_{X_c} + 0.1 \langle S^{dir} \rangle_{\psi'}.$$  

(19)

6. Results and Discussion

Now we will discuss the physical understanding of charmonium suppression due to screening in the deconfined medium produced in relativistic nucleus-nucleus collisions. This involves a competition of various time-scales involved in expanding plasma. From Tables 1 and 2 we observe that the value of $\epsilon_s$ is different for different charmonium states and varies from one EoS to another. If $\epsilon_s \geq \epsilon_s$, then there will be no suppression at all; i.e., survival probability, $S(p_T)$, is equal to 1. With this physical understanding we analyze our results, $(S(p_T))$ as a function of the number of participants $N_{part}$ in an expanding QGP. At RHIC energy, $J/\psi$ yields have resulted from a balance between annihilation of $J/\psi$’s due to hard, thermal gluons [81, 82] along with colour screening [76, 77] and enhancement due to coalescence of uncorrelated $c\bar{c}$ pairs [83–86] which are produced thermally at deconfined medium. A detailed investigation of the scaling properties of $J/\psi$ suppression as a function of several centrality variables would give valuable insights into the origin of the observed effect [12]. However, recent CMS data do not show a fully confirmed indication of $J/\psi$ enhancement except for the fact that $p_T^2$ of the data and shape of rapidity-dependent nuclear modification factor $R_{AA}(y)$ [13, 14, 59, 87] show some characteristics of coalescence production.

In our analysis, we have employed the quasi-particle debye mass to determine the dissociation temperatures for the charmonium states ($J/\psi, \psi', X_c$, etc.) in isotropic medium computed in Tables 1 and 2 for EoS1 and EoS2, respectively. On that dissociation temperature we had calculated the screening energy densities, $\epsilon_s$, and the speed of sound $c_s$ which are also listed in Tables 1 and 2 for both EoS1 and EoS2. These values will be used as inputs, to calculate $\langle S(p_T) \rangle$.

We have shown the variation of $p_T$-integrated survival probability in the range allowed by invariant $p_T$ spectrum of $J/\psi$ in CMS experiment with $N_{part}$ at mid-rapidity and compared with the experimental data (CMS JHEP) [14] in Figures 11 and 13 and (CMS PAS) [59] in Figures 12 and 14. For this we had used the values of $T_{dir}$’s and related parameters from Tables 1 and 2 using SIQGP equation of state for both EoS1 and EoS2.

We find that the survival probability of sequentially produced $J/\psi$ is slightly higher compared to the directly produced $J/\psi$ and is closer to the experimental results. The smaller value of screening energy density $\epsilon_s$ causes an increase in the screening time and results in more suppression to match with the CMS results at LHC. We have also plotted the pressure, energy density, and speed of sound for pure gauge, 2-flavor, 3-flavor, (2+1)-flavors, and 4-flavors QGP for both EoS1 and EoS2 in Figures 1–10 where we have employed QP EoS (QP EoS is the equation of state calculated by using quasi-particle debye mass) along with the Bannur EoS. Here we observe that the results of various equations of states coming by incorporating the quasi-particle Debye mass increase sharply.

7. Conclusion

We studied the equation of state for strongly interacting quark-gluon plasma in the framework of strongly coupled plasma with appropriate modifications to take account of color and flavor degrees of freedom and QCD running
coupling constant. In addition, we incorporate the nonperturbative effects in terms of nonzero string tension in the deconfined phase, unlike the Coulomb interactions alone in the deconfined phase beyond the critical temperature. Our results on thermodynamic observables, \( \nu \), pressure, energy density, speed of sound, etc., nicely fit the results of lattice equation of state with gluon, massless, and as well massive flavored plasma. In Figures 1–10 we see that the results coming out by using quasi-particle Debye mass increase sharply as the temperature increases. Now by using quasi-particle Debye mass we estimated the centrality dependence of charmonium suppression in an expanding dissipative strongly interacting QGP produced in relativistic heavy-ion collisions as shown in Figures 11–14 for both EoS1 and Eos2. We find that the survival probability of sequentially produced \( J/\Psi \) is slightly higher compared to the directly produced \( J/\Psi \) and is closer to the experimental results. The smaller value of screening energy density \( \epsilon_s \) causes an increase in the screening time and results in more suppression to match with the experimental results.

At LHC energies, the inclusive \( J/\Psi \) yield contains a significant nonprompt contribution from \( b \)-hadron decays [88, 89]. For the lower value of \( \eta/s \) we observe that our predictions are closer to the experimental ones.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.
Figure 13: The variation of $p_T$ integrated survival probability (in the range allowed by invariant $p_T$ spectrum of $J/\psi$ by the CMS experiment) versus number of participants at mid-rapidity for the EoS2 [70]. The experimental data (CMS JHEP) [14] are shown by the squares with error bars whereas circles and diamonds represent with ($\langle S_{\text{dir}} \rangle$) without ($\langle S_{\text{incl}} \rangle$) sequential melting using the values of $T_D$'s and related parameters from Table 2 using SIQGP equation of state.

Figure 14: The variation of $p_T$ integrated survival probability (in the range allowed by invariant $p_T$ spectrum of $J/\psi$ by the CMS experiment) versus number of participants at mid-rapidity for the EoS2 [67–69]. The experimental data (CMS PAS) [59] are shown by the squares with error bars whereas circles and diamonds represent with ($\langle S_{\text{dir}} \rangle$) without ($\langle S_{\text{incl}} \rangle$) sequential melting using the values of $T_D$'s and related parameters from Table 2 using SIQGP equation of state.

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