Supersymmetric Lorentz invariant deformations of superspaces

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Abstract

Lorentz invariant supersymmetric deformations of superspaces based on Moyal star product parametrized by Majorana spinor $\lambda_a$ and Ramond Grassmannian vector $\psi_m = -\frac{i}{2}(\bar{\theta}\gamma_m\lambda)$ in the spinor realization are proposed. The map of supergravity background into composite supercoordinates: $(B^{-1}_{mn}, \Psi^a_m, C_{ab}) \leftrightarrow (i\psi_m\psi_n, \psi_m\lambda^a, \lambda_a\lambda_b)$ valid up to the second order corrections in deformation parameter $h$ and transforming the background dependent Lorentz noninvariant (anti)commutators of supercoordinates into their invariant Moyal brackets is revealed. We found one of the deformations to depend on the axial vector $\psi_1 = \frac{i}{2}(\bar{\theta}\gamma_m\gamma_5\lambda)$ and to vanish for the $\theta$ components with the same chiralities. The deformations in the (super)twistor picture are discussed.

1 Introduction

Studying noncommutative geometry attracts a great interest. Much attention has been paid to the role of the constant background fields of supergravity - $B_{mn}$, the graviphoton $C_{ab}$ and the gravitino $\Psi^a_m$ - as the sources of the superspace deformations. The presence of the constant background in (anti)commutators of the (super)coordinate operators has stated the problem of the Lorentz symmetry breaking introduced by the deformations. The proposal to overcome this problem by the transition to a twisted Hopf algebra interpretation was recently advanced and its supersymmetric generalization was developed in. Another possibility arises from, where the Hamiltonian and quantum structures of the twistor-like model of super p-brane embedded in $N = 1$ superspace extended by tensor central charge coordinates were studied. The Lorentz covariant supersymmetric non(anti)commutative Dirac bracket relations among the brane (super)coordinates with their r.h.s. parametrized by auxiliary spinor variables were derived there. It hints on a hidden spinor structure associated with the Penrose twistor picture behind the non(anti)commutativity. To this end we start here with a spinor extension of the $N = 1 D = 4$ superspace $(x_m, \theta_a)$ by one commuting Majorana spinor $\lambda_a$ and construct Lorentz invariant supersymmetric Poisson and Moyal brackets generating non(anti)commutative relations for the (super)coordinates. The r.h.s of the $x_m$ brackets among themselves and $x_m$ with $\theta_a$ contain Ramond Grassmannian vector $\psi_m$ known from the theory of spinning strings and particles. The
Ramond vector $\psi_m$ appears here in the spinor realization $\psi_m = -\frac{1}{2}(\bar{\theta}\gamma_\mu\lambda)$ bilinear in $\lambda_a, \theta_a$ revealed in [35]. The vector $\psi_m$ is associated with the spin degrees of freedom in the structure of deformed superspace. We revealed a correspondence between the constructed Lorentz invariant Moyal brackets and the above mentioned (anti)commutators depending of the constant supergravity background and string length $\sqrt{\sigma}$. This correspondence is schematically illustrated by the map: $B^{-1}_{mn} \leftrightarrow i\psi_m\psi_n$, $C_{ab} \leftrightarrow \lambda_a\lambda_b$, $\Psi^a_m \leftrightarrow \psi_m\lambda^a$ transforming the field dependent (anti)commutators into the Moyal brackets. We found that the map is valid up to the second order corrections in the deformation parameter $h$ and it works in more sophisticated cases considered below. We studied the null twistor realization of the brackets and observed the dependence of the non(anti)commutativity effect on the choice of effective variables used to describe the primary degrees of freedom. This observation gives a sudden example of possible couplings between commutative and noncommutative geometries in superspaces. For the second of the studied Poisson/Moyal brackets we found a compact example of possible couplings between commutative and noncommutative geometries variables used to describe the primary degrees of freedom. This observation gives a sudden of possible couplings between commutative and noncommutative geometries.

The addition of the dilatation operator $\Delta = \nu_\alpha \partial_{\bar{\nu}_\alpha} + \bar{\nu}_\dot{\alpha} \partial_{\nu_\dot{\alpha}}$.

2 Lorentz invariant splitting of SUSY algebra

The $D = 4N = 1$ supersymmetry transformations in the presence of the twistor-like Majorana spinor $(\nu_\alpha, \bar{\nu}_{\dot{\alpha}})$ are given by the relations [25]

$$\delta \theta_\alpha = \varepsilon_\alpha, \quad \delta x_{a\dot{a}} = 2i(\varepsilon_\alpha \bar{\theta}_{\dot{a}} - \theta_\alpha \bar{\varepsilon}_{\dot{a}}), \quad \delta \nu_\alpha = 0, \quad (1)$$

and the correspondent supersymmetric derivatives $\partial^{a\dot{a}} \equiv \frac{\partial}{\partial x_{a\dot{a}}}$ and $D^\alpha, \bar{D}^{\dot{a}}$ are

$$D^\alpha = \frac{\partial}{\partial \theta_\alpha} - 2i\bar{\theta}_{\dot{a}} \partial^{\alpha\dot{a}}, \quad \bar{D}^{\dot{a}} \equiv -(D^\alpha)^* = \frac{\partial}{\partial \bar{\theta}_{\dot{a}}} - 2i\theta_\alpha \partial^{\dot{a}\alpha}, \quad [D^\alpha, \bar{D}^{\dot{b}}] = -4i\partial^{\alpha\dot{a}}. \quad (2)$$

The spinor coordinates $(\nu_\alpha, \bar{\nu}_{\dot{a}})$ and the light-like vector $\varphi_{a\dot{a}} = \nu_\alpha \bar{\nu}_{\dot{a}}$ composed from them may be used to construct the Lorentz invariant differential operators $D, \bar{D}, \partial$

$$D = \nu_\alpha \partial^\alpha, \quad \bar{D} = \bar{\nu}_{\dot{a}} \partial^{\dot{a}}, \quad \partial = \varphi_{a\dot{a}} \partial^{a\dot{a}} \quad (3)$$

which form a supersymmetric subalgebra of the algebra of the invariant derivatives

$$[D, \bar{D}]_+ = -4i\partial, \quad [D, D]_+ = [\bar{D}, \bar{D}]_+ = 0, \quad [D, \partial] = [\bar{D}, \partial] = [\partial, \partial] = 0. \quad (4)$$

The superalgebra [4] may be splitted into two invariant and (anti)commuting subalgebras $(D_-, \partial)$ and $(D_+, \partial)$

$$[D_\pm, D_\pm]_+ = \mp 8i\partial, \quad [D_+, D_-]_+ = 0, \quad [D_\pm, \partial] = [\partial, \partial] = 0 \quad (5)$$

formed by the supersymmetric derivatives $\partial$ and $D_\pm$

$$D_\pm \equiv D \pm \bar{D}. \quad (6)$$

The addition of the dilatation operator $\Delta$

$$\Delta = \nu_\alpha \frac{\partial}{\partial \nu_\alpha} + \bar{\nu}_{\dot{a}} \frac{\partial}{\partial \bar{\nu}_{\dot{a}}} \quad (7)$$
changing the scale of the spinor \((\nu_\alpha, \bar{\nu}_\alpha)\) extends the supersubalgebras \(\mathfrak{s}_\alpha\) to the superalgebras formed by the invarint derivatives \((D_-, \partial, \Delta)\) and \((D_+, \partial, \Delta)\)

\[
[D_\pm, D_\pm]_+ = \mp 8i\partial, \quad [\Delta, D_\pm] = D_\pm, \quad [\Delta, \partial] = 2\partial, \\
[D_+, D_-]_+ = [D_\pm, \partial] = [\partial, \partial] = [\Delta, \Delta] = 0.
\]

(8)

Our proposal is to use the Lorentz invariant supersymmetric differential operators \(\mathfrak{s}\) as building blocks for the construction of Lorentz invariant supersymmetric Poisson and Moyal brackets among the (super)coordinates corresponding to (anti)commutators of the supercoordinate operators in quantum theory.

3 Supersymmetric Lorentz invariant Poisson bracket

At first let us study a simple example of the Lorentz invariant and supersymmetric Poisson bracket producing non(anti)commutative relations among the superspace coordinates \(x_{a\dot{a}}, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}\). Such a Poisson bracket may be constructed from the three differential operators \((D_-, \partial, \Delta)\) generating the (-)- superalgebra \(\mathfrak{s}\)

\[
\{F, G\} = F\left[-\frac{i}{4} D_- D_- + (\partial\Delta - \Delta\partial)\right]G,
\]

(9)

where \(\{,\}_{\text{P.B.}} \equiv \{,\\}\) and \(F(x, \theta, \bar{\theta}, \nu, \bar{\nu}), G(x, \theta, \bar{\theta}, \nu, \bar{\nu})\) are generalized superfields depending on the superspace coordinates \((x, \theta, \bar{\theta})\) and the commuting Weyl spinors \(\nu, \bar{\nu}\). More information on the mathematical definitions used here may be found in [36, 37].

As a result of (9), the twistor-like coordinates form zero P.B.’s. among themselves

\[
\{\nu_\alpha, \nu_\beta\} = \{\nu_\alpha, \bar{\nu}_\beta\} = \{\bar{\nu}_\alpha, \bar{\nu}_\beta\} = 0
\]

(10)

and with the Grassmannian spinors \(\theta_\alpha, \bar{\theta}_{\dot{\alpha}}\)

\[
\{\nu_\alpha, \theta_\beta\} = \{\nu_\alpha, \bar{\theta}_{\dot{\beta}}\} = \{\bar{\nu}_\alpha, \theta_\beta\} = \{\bar{\nu}_\alpha, \bar{\theta}_{\dot{\beta}}\} = 0.
\]

(11)

However, they have non zero P.B.’s. with the space-time coordinates \(x_{a\dot{a}}\)

\[
\{x_{a\dot{a}}, \nu_\beta\} = \varphi_{a\dot{a}}\nu_\beta, \quad \{x_{a\dot{a}}, \bar{\nu}_\beta\} = \bar{\varphi}_{a\dot{a}}\bar{\nu}_\beta,
\]

(12)

The P.B.’s. among the super coordinates \(x_{a\dot{a}}\) and \((\theta_\alpha, \bar{\theta}_{\dot{\alpha}})\) are as follows

\[
\begin{align*}
\{x_{a\dot{a}}, x_{\beta\dot{\beta}}\} & = -i\psi_{a\dot{a}}\psi_{\beta\dot{\beta}}, \\
\{x_{a\dot{a}}, \theta_\beta\} & = \frac{i}{2}\psi_{a\dot{a}}\nu_\beta, \quad \{x_{a\dot{a}}, \bar{\theta}_{\dot{\beta}}\} = -\frac{i}{2}\bar{\psi}_{a\dot{a}}\bar{\nu}_{\beta}, \\
\{\theta_\alpha, \theta_\beta\} & = \frac{i}{4}\varphi_{a\dot{a}}, \quad \{\theta_\alpha, \bar{\theta}_{\dot{\beta}}\} = -\frac{i}{4}\bar{\varphi}_{a\dot{a}}\bar{\nu}_{\beta}, \quad \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = \frac{i}{4}\bar{\varphi}_{\dot{a}\dot{b}}
\end{align*}
\]

(13)

where \(\psi_{a\dot{a}}\) is a Grassmannian vector and \(\varphi_{a\dot{a}}, \bar{\varphi}_{\dot{a}\dot{b}}\) are composed symmetric spin-tensors

\[
\begin{align*}
\psi_{a\dot{a}} & \equiv i(\nu_\alpha\bar{\theta}_{\dot{a}} - \theta_\alpha\bar{\nu}_{\dot{a}}), \quad \psi_{a\dot{a}}\varphi_{a\dot{a}} = 0, \quad \varphi_{a\dot{a}} \equiv \nu_\alpha\nu_\beta, \quad \bar{\varphi}_{\dot{a}\dot{b}} \equiv \bar{\nu}_{\dot{a}}\bar{\nu}_{\dot{b}},
\end{align*}
\]

(14)

with the following transformation rules under the supersymmetry \(\Pi\)

\[
\delta \varphi_{a\dot{a}} = \delta \bar{\varphi}_{\dot{a}\dot{b}} = 0, \quad \delta \psi_{a\dot{a}} = -i(\varepsilon_\alpha\bar{\nu}_{\dot{a}} - \varepsilon_{\dot{a}}\nu_\alpha).
\]

(15)
The appearance in (13) of the Ramond vector $\psi_{a\dot{a}}$ (14) associated with the spin degrees of freedom hints on a spin structure behind the coordinate’s non(anti)commutativity. The bilinear spinor representation for $\psi_{a\dot{a}}$ (14) was previously found in [33] as the general solution of the Dirac constraints $p^{a\dot{a}}\psi_{a\dot{a}} = 0 = p^{a\dot{a}}p_{a\dot{a}}$ for massless spinning particle [33, 34]. This spinor representation has established equivalence between spinning and Brink-Schwarz superparticles. Thus, we find the desired component Poisson brackets (10-13) which are covariant under the Lorentz and supersymmetry transformations.

The constructed P.B.’s. satisfy the graded Jacobi identities having the standard form

$$\{\{A, B\}, C\} + (-1)^{(b+c)a}\{\{B, C\}, A\} + (-1)^{(a+b)c}\{\{C, A\}, B\} = 0,$$

(16)

where $a, b, c = 0, 1$ denote the Grassmannian gradings of $A, B$ and $C$ respectively.

The P.B.’s. among the supercoordinates and the composite objects $\psi$ and $\varphi$ are

$$\{\psi_{a\dot{a}}, \psi_{\beta\dot{\beta}}\} = -i\varphi_{a\dot{a}}\varphi_{\beta\dot{\beta}},$$

$$\{x_{a\dot{a}}, \psi_{\beta\dot{\beta}}\} = \varphi_{a\dot{a}}\psi_{\beta\dot{\beta}} + \varphi_{\beta\dot{\beta}}\psi_{a\dot{a}},$$

(17)

$$\{\psi_{a\dot{a}}, \theta_{\beta}\} = \frac{1}{2}\varphi_{a\dot{a}}\nu_{\beta},$$

$$\{\psi_{a\dot{a}}, \bar{\theta}_{\dot{\beta}}\} = -\frac{1}{2}\varphi_{a\dot{a}}\bar{\nu}_{\dot{\beta}},$$

$$\{x_{a\dot{a}}, \varphi_{\beta\dot{\beta}}\} = 2\varphi_{a\dot{a}}\varphi_{\beta\dot{\beta}},$$

$$\{x_{a\dot{a}}, \bar{\varphi}_{\beta\dot{\beta}}\} = 2\varphi_{a\dot{a}}\bar{\varphi}_{\beta\dot{\beta}},$$

$$\{x_{a\dot{a}}, \varphi_{\gamma\dot{\gamma}}\} = 2\varphi_{a\dot{a}}\varphi_{\gamma\dot{\gamma}},$$

$$\{x_{a\dot{a}}, \bar{\varphi}_{\gamma\dot{\gamma}}\} = 2\varphi_{a\dot{a}}\bar{\varphi}_{\gamma\dot{\gamma}}.$$ Using these Poisson brackets together with the P.B.’s. (10,13) we obtain

$$\{\{\psi_{a\dot{a}}, \psi_{\beta\dot{\beta}}\}, \psi_{\gamma\dot{\gamma}}\} = 0,$$

$$\{\{\theta_{\alpha}, \theta_{\beta}\}, \theta_{\gamma}\} = ... = \{\{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\}, \bar{\theta}_{\dot{\gamma}}\} = 0$$

(18)

proving the graded Jacobi identity for the $3\psi$ and $3\theta$ Jacobi cycles

$$\text{Cycle}\{\{\psi_{a\dot{a}}, \psi_{\beta\dot{\beta}}\}, \psi_{\gamma\dot{\gamma}}\} = \text{Cycle}\{\{\theta_{\alpha}, \theta_{\beta}\}, \theta_{\gamma}\} = ... = \text{Cycle}\{\{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\}, \bar{\theta}_{\dot{\gamma}}\} = 0.$$ (19)

The vanishing of the $3x$ Jacobi cycle: $\text{Cycle}\{\{x_{a\dot{a}}, x_{\beta\dot{\beta}}\}, x_{\gamma\dot{\gamma}}\} = 0$ follows from the relation

$$\{\{x_{a\dot{a}}, x_{\beta\dot{\beta}}\}, x_{\gamma\dot{\gamma}}\} = 2i(\psi_{a\dot{a}}\psi_{\beta\dot{\beta}})\varphi_{\gamma\dot{\gamma}} + i(\psi_{a\dot{a}}\varphi_{\beta\dot{\beta}} - \varphi_{a\dot{a}}\varphi_{\beta\dot{\beta}})\psi_{\gamma\dot{\gamma}}.$$ (20)

The same result are preserved for other Jacobi cycles proving selfconsistency of the introduced P.B. (21) that opens a way for the corresponding invariant Moyal bracket.

### 4 Lorentz invariant supersymmetric Moyal bracket

A transition to quantum picture based on the P.B. (21) may be done using the Weyl-Moyal correspondence establishing one to one correspondence among quantum field operators and their symbols acting on the commutative space-time. Then the quantum dynamics encodes itself in the change of usual product of the Weyl symbols by their star product

$$F\star G = F e^{i\beta \{\hat{D}_{-}, \hat{D}_{-} + (\hat{\nabla}_{-} - \hat{\Delta}_{-})\}} G,$$

(21)

where $\nabla \equiv 4i\partial$ and $\hbar$ is a quantum deformation parameter associated with the expansion

$$F\star G = FG + \left(\frac{\hbar}{8}\right) F \left[\hat{D}_{-} \hat{D}_{-} + (\hat{\nabla}_{-} - \hat{\Delta}_{-})\right] G$$

$$+ \frac{1}{2!}\left(\frac{\hbar}{8}\right)^2 F \left[\hat{D}_{-} \hat{D}_{-} + (\hat{\nabla}_{-} - \hat{\Delta}_{-})\right]^2 G + .....$$ (22)
The power series expansion in $h$ \(22\) is presented in the arrow ordered form as

\[
F \star G = FG + \left(\frac{-ih}{8}\right) F [D \rightarrow \overrightarrow{D} + (\overrightarrow{\nabla} \Delta - \Delta \overrightarrow{\nabla})] G \\
+ \left(\frac{1}{4i} \left(\frac{-ih}{8}\right)^2\right) F \left[ -11 \overrightarrow{\nabla} + 3 \overrightarrow{D} \overrightarrow{\nabla} \overrightarrow{D} + 2(\overrightarrow{\nabla} \overrightarrow{D} \overrightarrow{\nabla} \overrightarrow{D} - \Delta \overrightarrow{D} \overrightarrow{D} \overrightarrow{\nabla}) \right] \overrightarrow{\nabla} \\
- 3(\Delta \overrightarrow{\nabla} + \overrightarrow{\nabla} \Delta) + \Delta^2 (\Delta + 2 \Delta) + \Delta^2 (\overrightarrow{\nabla} + 2 \overrightarrow{\nabla})^2 - 2 \Delta \overrightarrow{\nabla} \overrightarrow{\nabla} \Delta] G + \ldots, \tag{23}
\]

where we omit the higher order terms in $h$. Using the expansion \(23\) we find the second order corrections to be vanishing for the following $\star$-products of the supercoordinates:

\[
x_{a\dot{a}} \star \nu_\beta = x_{a\dot{a}} \nu_\beta + \frac{h}{2} \varphi_{a\dot{a}} \nu_\beta + \mathcal{O}(h^3), \\
x_{a\dot{a}} \star \bar{\nu}_\beta = x_{a\dot{a}} \bar{\nu}_\beta + \frac{ih}{4} \psi_{a\dot{a}} \nu_\beta + \mathcal{O}(h^3), \\
x_{a\dot{a}} \star \theta_\beta = x_{a\dot{a}} \theta_\beta + \frac{ih}{4} \psi_{a\dot{a}} \nu_\beta + \mathcal{O}(h^3), \\
x_{a\dot{a}} \star \bar{\theta}_\beta = x_{a\dot{a}} \bar{\theta}_\beta - \frac{ih}{4} \psi_{a\dot{a}} \bar{\nu}_\beta + \mathcal{O}(h^3), \\
\theta_\alpha \star \theta_\beta = \theta_\alpha \theta_\beta + \frac{ih}{8} \varphi_{a\dot{a}} \nu_\beta + \mathcal{O}(h^3), \\
\theta_\alpha \star \bar{\theta}_\beta = \theta_\alpha \bar{\theta}_\beta - \frac{ih}{8} \varphi_{a\dot{a}} \bar{\nu}_\beta + \mathcal{O}(h^3), \\
\bar{\theta}_\dot{\alpha} \star \bar{\theta}_\dot{\beta} = \bar{\theta}_\dot{\alpha} \bar{\theta}_\dot{\beta} + \frac{ih}{8} \varphi_{\dot{a}\dot{a}} \bar{\nu}_\dot{\beta} + \mathcal{O}(h^3). \tag{24}
\]

Moreover, the star products of the Majorana spinor components ($\nu_\alpha, \bar{\nu}_{\dot{\alpha}}$) coincide with their usual products in all orders in $h$. We assume that the higher order corrections in the star products \(21\) can be also equal zero. On the contrary, the second order corrections in the star products of the $x_{a\dot{a}}$ components are nonzero

\[
x_{a\dot{a}} \star x_{\beta\dot{\beta}} = x_{a\dot{a}} x_{\beta\dot{\beta}} - \frac{i}{2} \psi_{a\dot{a}} \psi_{\beta\dot{\beta}} - \frac{11 h^2}{24} \varphi_{a\dot{a}} \varphi_{\beta\dot{\beta}} + \mathcal{O}(h^3), \tag{25}
\]

but their contributions in the corresponding Moyal brackets are zero, because of the commutativity $\varphi_{a\dot{a}} \varphi_{\beta\dot{\beta}} = \varphi_{\beta\dot{\beta}} \varphi_{a\dot{a}}$. Consequently, the second order corrections in the Lorentz invariant and supersymmetric Moyal brackets \(24-25\) are equal to zero

\[
[x_{a\dot{a}}, x_{\beta\dot{\beta}}]_\star = x_{a\dot{a}} \star x_{\beta\dot{\beta}} - x_{\beta\dot{\beta}} \star x_{a\dot{a}} = -i h \psi_{a\dot{a}} \psi_{\beta\dot{\beta}} + \mathcal{O}(h^3), \\
[x_{a\dot{a}}, \nu_\beta]_\star = h \varphi_{a\dot{a}} \nu_\beta + \mathcal{O}(h^3), \\
[x_{a\dot{a}}, \bar{\nu}_\beta]_\star = h \varphi_{a\dot{a}} \bar{\nu}_\beta + \mathcal{O}(h^3), \\
[x_{a\dot{a}}, \theta_\beta]_\star = \frac{ih}{2} \psi_{a\dot{a}} \nu_\beta + \mathcal{O}(h^3), \\
[x_{a\dot{a}}, \bar{\theta}_\beta]_\star = -\frac{ih}{2} \psi_{a\dot{a}} \bar{\nu}_\beta + \mathcal{O}(h^3), \\
[\theta_\alpha, \theta_\beta]_\star = \frac{ih}{4} \varphi_{a\dot{a}} \nu_\beta + \mathcal{O}(h^3), \\
[\theta_\alpha, \bar{\theta}_\dot{\beta}]_\star = \frac{ih}{4} \varphi_{\dot{a}\dot{a}} \bar{\nu}_{\dot{\beta}} + \mathcal{O}(h^3). \tag{26}
\]

The Moyal brackets generated by the P.B.’s \(10,13\) replace the (anti)commutators of the coordinate operators used in the standard quantum picture.

## 5 Brackets and twistors

The unification of the Weyl spinors $\nu_\alpha, \bar{\nu}_{\dot{\alpha}}$ with the spinors $\omega^\alpha, \bar{\omega}_{\dot{\alpha}}$ defined as

\[
\omega_\alpha = x_{a\dot{a}} \bar{\nu}^\alpha, \quad \bar{\omega}_{\dot{\alpha}} = x_{a\dot{a}} \nu^\alpha \tag{27}
\]

yields the null twistor $Z^A = (i \omega^\alpha, \bar{\nu}_{\dot{\alpha}})$ and its complex conjugate $\bar{Z}_A = (\nu_\alpha, -i \bar{\omega}^\dot{\alpha})$ connected by the condition $Z^A \bar{Z}_A = 0 \tag{27}$. The Eqs. \(10\) and \(12\) result in the P.B. commutativity among the twistor components $\omega_\alpha$ and $\nu_\alpha, \bar{\nu}_{\dot{\alpha}}$

\[
\{\omega_\alpha, \nu_\beta\} = \{\omega_\alpha, \bar{\nu}_{\dot{\beta}}\} = \{\bar{\omega}_{\dot{\alpha}}, \nu_\beta\} = \{\bar{\omega}_{\dot{\alpha}}, \bar{\nu}_{\dot{\beta}}\} = 0, \tag{28}
\]
because of the orthogonality conditions
\[ \varphi_{\alpha\dot{\alpha}}\nu^\alpha = \varphi_{\alpha\dot{\alpha}}\bar{\nu}^{\dot{\alpha}} = 0. \] (29)

The P.B.'s. among the components of the Majorana spinor \((\omega_\alpha, \bar{\omega}_\dot{\alpha})\) with the same chirality
\[ \{\omega_\alpha, \omega_\beta\} = i\bar{\eta}^2\varphi_{\alpha\beta} \equiv 0, \quad \{\bar{\omega}_\dot{\alpha}, \bar{\omega}_\dot{\beta}\} = i\bar{\eta}^2\bar{\varphi}_{\dot{\alpha}\dot{\beta}} \equiv 0 \] (30)
become zero, because \(\eta^2 = \bar{\eta}^2 = 0\), where \(\eta\) and \(\bar{\eta}\) are grassmannian scalars defined by
\[ \eta \equiv \theta_\alpha\nu^\alpha, \quad \bar{\eta} \equiv \bar{\theta}_{\dot{\alpha}}\bar{\nu}^{\dot{\alpha}}. \] (31)

These anticommuting scalars have zero P.B.'s. between themselves, with \(\nu, \omega, \theta\)
\[ \{\eta, \nu_\alpha\} = \{\eta, \omega_\alpha\} = \{\eta, \theta_\alpha\} = 0 \] (32)
and with \(\bar{\nu}, \bar{\omega}, \bar{\theta}\). Single nonzero of the Poisson brackets among the null twistor \(Z^A = (i\omega^\alpha, \bar{\nu}_\dot{\alpha})\), \(\bar{Z}_\dot{A} = (\nu_\alpha, -i\bar{\omega}^{\dot{\alpha}})\) components is
\[ \{\omega_\alpha, \bar{\omega}_\dot{\beta}\} = i\eta\bar{\eta}\varphi_{\alpha\beta}. \] (33)

and it may be written down in the equivalent form \(\{\omega_\alpha, \bar{\omega}_\dot{\beta}\} = 8\eta\bar{\eta}\{\theta_\alpha, \bar{\theta}_{\dot{\beta}}\}\) showing proportionality of the \((\omega, \bar{\omega})\) noncommutativity to the \((\theta, \bar{\theta})\) nonanticommutativity. It fixes the correlation of the twistor deformation with supersymmetry encoded in the Lorentz invariant P.B. (9). This correlation manifests its under reduction of the original superspace to the null supertwistor subspace formed by \(Z^A, \bar{Z}_{\dot{A}}\) connected by the relation: \(Z^A\bar{Z}_{\dot{A}} = 0\) [23]. The null supertwistors are formed by the triads \(Z^A = (iq^\alpha, \nu_\alpha, 2\bar{\eta}), \bar{Z}_{\dot{A}} = (\nu_\alpha, -iq^{\dot{\alpha}}, 2\eta)\), where \(q_\alpha = \omega_\alpha - 2i\eta\theta_\alpha\), whose supersubspace is closed under the supersymmetry transformations. Because of this reduction we find the counterpart of the P.B. (33) to vanish
\[ \{q_\alpha, \bar{q}_{\dot{\beta}}\} = 0 \] (34)
together with any other P.B.'s. among the components of \(Z^A, \bar{Z}_{\dot{A}}\). It means that the supersubspace of null supertwistors \(Z^A, \bar{Z}_{\dot{A}}\) is inert under the deformation associated with the P.B. (9). This effect is a consequence of nonlocal connection [27] between the coordinates and twistors, because the relation (27) is invariant under the shifts: \(x_{a\dot{a}} \rightarrow x_{a\dot{a}} + s\nu_a\bar{\nu}_{\dot{a}}\). This shifts map light-like lines into points and wash off the noncommutativity effect originating from the uncertainty relations for the \(x_m\) components on the Plank scale. This observation gives an example of couplings between commutative and noncommutative geometries in the presence of supersymmetry. So, we observe interesting couplings of twistor structure with supersymmetry, Lorentz invariance and Poisson structure which shed light on general structure of non(anti)commutative superspaces.

6 The Lorentz invariant bracket in higher dimensions

The passage to the Majorana representation in the Poisson brackets [10,13]
\[ \nu_a = \left(\nu_\alpha\right), \quad \theta_a = \left(\theta_\alpha\right), \quad C^{ab} = \left(\begin{array}{cc} \varepsilon^{\alpha\beta} & 0 \\ 0 & \varepsilon_{\dot{\alpha}\dot{\beta}} \end{array}\right), \quad \chi^a = C^{ab}\chi_b. \] (35)
where $C^{ab}$ is the charge conjugation matrix, presents them in the form suitable for the generalization to higher dimensions

$$\{\nu_a, \nu_b\} = 0, \quad \{\theta_a, \nu_b\} = 0, \quad \{x_m, \nu_a\} = \varphi_m \nu_a. \quad (36)$$

The real vectors $x_m$ and $\varphi_m$ in (36) are defined by the relations

$$x_m = -\frac{1}{2}(\tilde{\sigma}_m)_{\dot{a}\dot{b}} x_{\dot{a}\dot{b}}, \quad x_{\alpha\beta} = (\sigma^m)^{\alpha\beta} x_m,$$

$$\varphi_m = -\frac{1}{2}(\tilde{\sigma}_m)_{\dot{a}\dot{b}} \varphi_{\dot{a}\dot{b}} \equiv \frac{1}{4}(\tilde{\nu} \gamma_m \nu), \quad (37)$$

where $\gamma_m$ are the Dirac matrices in the Majorana representation.

To rewrite the rest of the P.B.’s. in the Majorana representation it is convenient to change the Majorana spinor $\nu_a$ by other Majorana spinor $\lambda_a$

$$\lambda_a = \left(\frac{\lambda_a}{\bar{\lambda}_{\dot{a}}}\right) \equiv (\gamma_5 \nu)_a, \quad (\gamma_5)_a^b = \left(\begin{array}{cc} -i\delta_\alpha^\beta & 0 \\ 0 & i\delta_\dot{\beta}^{\dot{a}} \end{array}\right) \quad (38)$$

preserving the form of the P.B.’s. (36). In terms of the real Majorana spinor $\lambda_a$ and the composed vectors $\varphi_m$ and $\psi_m$

$$\varphi_m = \frac{1}{4}(\bar{\lambda}_m \lambda), \quad \psi_m = -\frac{1}{2}(\tilde{\sigma}_m)_{\dot{a}\dot{b}} \psi_{\dot{a}\dot{b}} \equiv -\frac{1}{2}(\tilde{\theta}_m \lambda) \quad (39)$$

the P.B.’s. (10-13) of the primordial coordinates $x_m, \theta_a, \lambda_a$ are presented as follow

$$\{\lambda_a, \lambda_b\} = 0, \quad \{\theta_a, \lambda_b\} = 0, \quad \{x_m, \lambda_a\} = \varphi_m \lambda_a,$$

$$\{x_m, x_n\} = -i\psi_n \psi_m, \quad \{x_m, \theta_a\} = -\frac{1}{2} \psi_m \lambda_a, \quad \{\theta_a, \theta_b\} = -\frac{1}{4} \lambda_a \lambda_b. \quad (40)$$

The P.B.’s. of the composite vectors $\psi_m$ and $\varphi_m$ (39) among themselves and with the primordial coordinates take the form

$$\{x_m, \psi_n\} = \varphi_m \psi_n + \varphi_n \psi_m, \quad \{\psi_m, \theta_b\} = \frac{i}{2} \varphi_m \lambda_b, \quad \{\psi_m, \lambda_a\} = 0,$$

$$\{\psi_m, \psi_n\} = -i\varphi_m \varphi_n, \quad \{\psi_m, \varphi_n\} = 0 \quad (41)$$

and respectively

$$\{x_m, \varphi_n\} = 2\varphi_m \varphi_n, \quad \{\theta_a, \varphi_m\} = \{\lambda_a, \varphi_m\} = \{\varphi_m, \varphi_n\} = 0. \quad (42)$$

The P.B.’s. (10-12) originally derived for $D = 4$ are valid in $D$-dimensional space with $D = 2, 3, 4 (mod 8)$, where the Majorana spinors exist. This procedure restores the vector form of the Moyal brackets (26) in the higher dimensions.

### 7 Other supersymmetric Lorentz invariant brackets

Using the Majorana spinor $\nu_a$ one can construct one more supersymmetric and Lorentz invariant Poisson bracket in the addition to the P.B. (9) which is given by

$$\{F, G\} = F \left[ \frac{i}{4} (\vec{D} \vec{D} + \vec{D} \vec{D}) + \frac{1}{2} (\vec{\partial} \vec{\Delta} - \vec{\Delta} \vec{\partial}) \right] G \quad (43)$$
and yields different invariant Poisson brackets for the supercoordinates $x$ and $\theta$

$$\{x_{\alpha\dot{\alpha}}, x_{\beta\dot{\beta}}\} = -i(\varphi_{\alpha\beta}\dot{\theta}^\alpha\dot{\theta}^\beta - \varphi_{\beta\dot{\alpha}}\dot{\theta}^\beta\dot{\theta}^\alpha),
\{x_{\alpha\dot{\alpha}}, \theta^\beta\} = \frac{1}{2}\varphi_{\beta\dot{\alpha}}\dot{\theta}^\beta, \quad \{x_{\alpha\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = \frac{1}{2}\varphi_{\dot{\alpha}\beta}\theta^\alpha,
\{\theta^\alpha, \theta^\beta\} = \{\dot{\theta}^\alpha, \bar{\theta}^{\dot{\beta}}\} = 0, \quad \{\theta^\alpha, \bar{\theta}^{\dot{\beta}}\} = -\frac{i}{4}\varphi_{\alpha\beta}, \tag{44}$$

We see that the new deformation $[13]$ generates the zero P.B.’s for the $\theta^a$ components with the same chirality in contrast to the deformation $[9]$. The P.B.’s $[13]$ are added by

$$\{\nu^\alpha, \nu^\beta\} = \{\nu^\alpha, \bar{\nu}^{\dot{\beta}}\} = \{\bar{\nu}^{\dot{\alpha}}, \bar{\nu}^{\dot{\beta}}\} = 0,
\{\nu^\alpha, \bar{\theta}^{\dot{\beta}}\} = \{\bar{\nu}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = 0, \quad \{x_{\alpha\dot{\alpha}}, \nu^\beta\} = \frac{1}{2}\varphi_{\alpha\beta}\nu^\beta, \quad \{x_{\alpha\dot{\alpha}}, \bar{\nu}^{\dot{\beta}}\} = \frac{1}{2}\varphi_{\alpha\dot{\beta}}\bar{\nu}^{\dot{\beta}}. \tag{45}$$

The P.B. $[13]$ satisfies the Jacobi identities and produces the corresponding Moyal bracket

$$F \ast G = F e^{\frac{i}{\hbar} \left[ \Delta \bar{\Delta} + \bar{\Delta} \Delta - \frac{1}{2} (\bar{\Delta} \Delta - \Delta \bar{\Delta}) \right]} G, \tag{46}$$

where $\nabla \equiv 4i\partial$ and $\hbar$ is a quantum deformation parameter.

Using the conversion formulae from Sect. 6 gives the vector form for the P.B.’s $[14]$

$$\{x^m, x^n\} = -\frac{i}{\hbar}(\chi^m \bar{\chi}^n - \chi^m \bar{\chi}^n),
\{x^m, \theta^\beta\} = -\frac{1}{2}\chi^m \nu^\beta, \quad \{x^m, \bar{\theta}^{\dot{\beta}}\} = -\frac{1}{2}\chi^m \bar{\nu}^{\dot{\beta}},
\{\theta^\alpha, \theta^\beta\} = -\frac{i}{8}(\nu^\alpha \nu^\beta + \nu^{\beta \alpha}), \tag{47}$$

where we introduced the complex Grassmannian vector $\chi^m$ with the real and imaginary parts presented by $\psi_{1m}, \psi_{2m}$ and the chiral components $\theta^{(\pm)}$ and $\nu^{(\pm)}$

$$\chi^m \equiv (\nu^m \bar{\theta}) \equiv -\bar{\nu} \gamma_m \frac{1 + ig_5}{2} \theta \equiv \psi_{1m} + i\psi_{2m},
\bar{\chi}^m \equiv (\chi^m)^* = -\bar{\psi}_{1m} \frac{1 - ig_5}{2} \theta, \quad \psi_{1m} \equiv -\frac{1}{2}(\bar{\theta} \gamma_m \nu), \quad \psi_{2m} \equiv -\frac{1}{2}(\bar{\theta} \gamma_m \gamma_5 \nu), \tag{48}$$

$$\theta^{(\pm)} \equiv \frac{1}{2}(1 \pm ig_5) \theta, \quad \nu^{(\pm)} \equiv \frac{1}{2}(1 \pm ig_5) \nu.$$

Then the P.B.’s $[17]$ are presented in the form directly generalizing the P.B.’s $[10]$

$$\{x^m, x^n\} = -\frac{i}{2}(\psi_{1m} \psi_{1n} + \psi_{2m} \psi_{2n}),
\{x^m, \theta^\alpha\} = -\frac{i}{2}(\psi_{1m} \nu^\alpha + \psi_{2m} \lambda_\alpha),
\{\theta^\alpha, \theta^\beta\} = -\frac{i}{8}(\nu^\alpha \nu^\beta + \lambda_\alpha \lambda_\beta), \tag{49}$$

where $\lambda_\alpha \equiv (\gamma_5 \nu)^\alpha_a$ as in $[88]$. Comparing $[49]$ with $[10]$ we observe that the change of the P.B. $[9]$ by $[13]$ is equivalent to the complexification of the real Grassmannian vector $\psi_m$ $[39]$ accompanied by the appearance of the spinors $\nu^\alpha$ and $(\gamma_5 \nu)^\alpha_a$ in the r.h.s. of $[49]$.

The P.B. $[13]$ and respectively the Moyal bracket $[46]$ may be generalized to the case of extended supersymmetries with $N > 1$. The corresponding P.B. may be chosen as

$$\{F, G\} = F \left[ \frac{i}{4} (D_i \bar{D}^i + \bar{D}_i D^i) + \frac{1}{2}(\bar{\theta} \Delta - \Delta \bar{\theta}) \right] G, \tag{50}$$
where \( D_\gamma \equiv \nu_\alpha D^\alpha \) and \( \bar{D}_\gamma \equiv \bar{\nu}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} \) with \( i=1,2,\ldots,N \). The P.B.’s. (50) generate the following brackets for the primordial (super)coordinates

\[
\{ x_{\alpha\dot{\alpha}}, x_{\beta\dot{\beta}} \} = -i(\varphi_{\alpha\beta\dot{\alpha}\dot{\beta}} - \varphi_{\dot{\alpha}\dot{\beta}\alpha\beta}),
\]

\[
\{ x_{\alpha\dot{\alpha}}, \theta^i_{\beta} \} = \frac{i}{2} \varphi_{\alpha\beta\dot{\alpha}i}, \quad \{ x_{\alpha\dot{\alpha}}, \bar{\theta}_{\dot{\beta}} \} = \frac{i}{2} \varphi_{\alpha\dot{\alpha}\beta\dot{\beta}},
\]

\[
\{ \theta^i_{\alpha\dot{\alpha}}, \theta^j_{\beta\dot{\beta}} \} = \{ \bar{\theta}^i_{\dot{\alpha}i}, \bar{\theta}^j_{\dot{\beta}j} \} = 0, \quad \{ \theta^i_{\alpha\dot{\alpha}}, \bar{\theta}^j_{\dot{\beta}j} \} = \frac{i}{2} \varphi_{\alpha\dot{\alpha}\beta\dot{\beta}} \delta^i_j.
\]

The rest of the P.B.’s. for the supercoordinates \( x_{\alpha\dot{\alpha}}, \nu_\alpha, \theta^i_{\alpha\dot{\alpha}} \) coincides with the P.B.’s. (49).

### 8 Lorentz invariant brackets with two spinors

Up to now we have studied the Lorentz invariant Poisson and Moyal brackets including only one auxiliary Majorana spinor \( \nu \equiv -\gamma_5 \lambda \) to construct the scalar derivatives \( 8 \) from the supersymmetric derivatives \( D^\alpha, \bar{D}^{\dot{\alpha}} \) \( \alpha, \dot{\alpha} \). Using only these scalars in the Poisson/Moyal brackets restricts the admissible motions in superspace. To extend the set of Lorentz invariant supersymmetric derivatives we need more auxiliary spinors to form the complete spinor basis in \( D \)-dimensional Minkowski space. For \( D = 4 \) it is enough to add only one new Majorana spinor \( (\mu_\alpha, \bar{\mu}_{\dot{\alpha}}) \) forming the complete spinor basis together with \( (\nu_\alpha, \bar{\nu}_{\dot{\alpha}}) \). The pair \( \nu_\alpha, \mu_\alpha \) of Majorana spinors may be identified with the Newman-Penrose dyad \( 27 \) if the relations

\[
\mu^\alpha \nu_\alpha \equiv \mu^\alpha \varepsilon_{\alpha\beta} \nu_\beta = 1, \quad \mu_\alpha \nu^\alpha - \mu^\alpha \nu_\alpha = \varepsilon_{\alpha\beta}
\]

for the Weyl spinors \( \nu_\alpha, \mu_\alpha \) and their complex conjugate are used. Having this spinor basis one can form four real independent Lorentz invariant supersymmetric differential operators

\[
D^{(\nu)} = \nu_\alpha D^\alpha, \quad \bar{D}^{(\nu)} = \bar{\nu}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}}, \quad D^{(\mu)} = \mu_\alpha D^\alpha, \quad \bar{D}^{(\mu)} = \bar{\mu}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}},
\]

two of which \( D^{(\nu)}, \bar{D}^{(\nu)} \) coincide with the operators \( D, \bar{D} \). Their linear combinations

\[
D^{(\mu)}_\pm \equiv D^{(\nu)} \pm \bar{D}^{(\nu)}, \quad D^{(\mu)}_\pm \equiv D^{(\mu)} \pm \bar{D}^{(\mu)},
\]

form four Lorentz invariant and supersymmetric supersubalgebras

\[
[D^{(\nu)}_\pm, D^{(\mu)}_\pm]_+ = \mp 8i \partial^{(\nu)}, \quad [D^{(\nu)}_\pm, \partial^{(\nu)}] = [\partial^{(\nu)}, \partial^{(\nu)}] = 0, \quad \partial^{(\nu)} \equiv (\nu_\alpha \bar{\nu}_{\dot{\alpha}} \partial^{\alpha\dot{\alpha}}),
\]

\[
[D^{(\mu)}_\pm, D^{(\mu)}_\pm]_+ = \mp 8i \partial^{(\mu)}, \quad [D^{(\mu)}_\pm, \partial^{(\mu)}] = [\partial^{(\mu)}, \partial^{(\mu)}] = 0, \quad \partial^{(\mu)} \equiv (\mu_\alpha \bar{\mu}_{\dot{\alpha}} \partial^{\alpha\dot{\alpha}}),
\]

which are connected by the anticommutation relations

\[
[D^{(\nu)}_\pm, D^{(\mu)}_\pm]_+ = \mp 4i \partial^{(\pm)}, \quad \partial^{(\pm)} \equiv (\nu_\alpha \bar{\mu}_{\dot{\alpha}} + \mu_\alpha \bar{\nu}_{\dot{\alpha}}) \partial^{\alpha\dot{\alpha}};
\]

\[
[D^{(\nu)}_\pm, D^{(\mu)}_\pm]_+ = \pm 4i \partial^{(\mp)}, \quad \partial^{(\mp)} \equiv (\nu_\alpha \bar{\mu}_{\dot{\alpha}} - \mu_\alpha \bar{\nu}_{\dot{\alpha}}) \partial^{\alpha\dot{\alpha}}.
\]

It is easy to see that the Lorentz invariant supersymmetric differential operators \( D^{(\nu)}, D^{(\mu)}, \partial^{(\nu)}, \partial^{(\mu)}, \partial^{(\pm)} \) describe the whole class of admissible motions in the superspace. These operators together with the extended dilatation operator \( \Delta' \)

\[
\Delta' = (\nu_\alpha \partial^{\alpha}_{\bar{\nu}_{\dot{\alpha}}} + \bar{\nu}_{\dot{\alpha}} \partial^{\alpha}_{\nu_\alpha}) - (\mu_\alpha \partial^{\alpha}_{\bar{\mu}_{\dot{\alpha}}} + \bar{\mu}_{\dot{\alpha}} \partial^{\alpha}_{\mu_\alpha})
\]
preserving the condition \([52]\) may be used as invariant building blocks for the construction of more general Lorentz invariant supersymmetric Poisson and Moyal brackets. Using them one can propose the Lorentz invariant and supersymmetric Poisson bracket
\[
\{ F, G \} = F \left[ - \frac{1}{4} \left( \bar{\alpha}^{(\nu)} \bar{\alpha}^{(\nu)} + \bar{\alpha}^{(\mu)} \right) \right] + c(\bar{\delta}^{(\nu)} + \bar{\delta}^{(\mu)}) \Delta' - \Delta' (\bar{\delta}^{(\nu)} + \bar{\delta}^{(\mu)}) \right] G. \tag{58}
\]
as a generalizations of \([9]\). The P.B. \([48]\) yields the following coordinate P.B.’s.
\[
\begin{align*}
\{ x_m, x_n \} &= -i (\psi_m^{(\nu)} \psi_n^{(\nu)} + \psi_m^{(\mu)} \psi_n^{(\mu)}), \\
\{ x_m, \theta_a \} &= -\frac{i}{2} (\psi_m^{(\nu)} \lambda_{a}^{(\nu)} + \psi_m^{(\mu)} \lambda_{a}^{(\mu)}), \\
\{ \theta_a, \theta_b \} &= -\frac{i}{4} (\lambda_{a}^{(\mu)} \lambda_{b}^{(\nu)} + \lambda_{a}^{(\nu)} \lambda_{b}^{(\mu)}),
\end{align*}
\tag{59}
\]
where the additional spinor \(\lambda_{a}^{(\mu)}\) and Grassmannian vector \(\psi_m^{(\mu)}\) are defined by the relations
\[
\psi_m^{(\nu)} \equiv \psi_n, \quad \lambda_{a}^{(\nu)} \equiv \lambda_{a}, \quad \psi_m^{(\mu)} \equiv \frac{1}{2} (\bar{\theta}_{\gamma m} \lambda_{a}^{(\mu)}), \quad \lambda_{a}^{(\mu)} \equiv (\gamma_{5} \mu)_{a}
\tag{60}
\]
The Majorana spinors \(\lambda_{a}^{(\nu)}\) and \(\lambda_{a}^{(\mu)}\) have zero P.B.’s between themselves and with \(\theta_{a}, \psi_m^{(\nu)}, \psi_n^{(\mu)}\), but non zero P.B.’s. with \(x_m\)
\[
\begin{align*}
\{ x_m, \lambda_{a}^{(\nu)} \} &= c (\varphi_m^{(\nu)} + \varphi_m^{(\mu)} \lambda_{a}^{(\nu)}), \\
\{ x_m, \lambda_{a}^{(\mu)} \} &= -c (\varphi_m^{(\nu)} + \varphi_m^{(\mu)} \lambda_{a}^{(\mu)}),
\end{align*}
\tag{61}
\]
where the real constant \(c\) has to be defined from the analysis of the Jacobi identities. We see that the addition of new spinors permits to define more wide class of Lorentz invariant deformations. From the physical point of view these extensions permit to take into account deformations associated with tenseile branes or massive fields in the addition to the above considered deformations associated with tensionless branes or massless fields. It follows from the results \([40, 41]\), where the transition from tensionless string/brane \([39]\) to tensile one was described in geometrical terms.

9 Discussion

We constructed selfconsistent Poisson and Moyal brackets describing Lorentz invariant supersymmetric deformations of the \(N = 1\) superspace \((x_m, \theta_a)\) equipped by commuting Majorana spinor \(\lambda_{a}\) (equivalently \(\nu \equiv -\gamma_{5} \lambda\)). We proved their selfconsitency and found that the noncommutativity of \(x_m\) with \(x_n\) or \(\theta_a\) is parametrized by composite Ramond vector \(\psi_m \equiv -\frac{1}{2} (\bar{\theta}_{\gamma m} \lambda)\) partially accompanied by the Grassmannian axial vector \(\psi_{1 m} \equiv \frac{1}{2} (\bar{\theta}_{\gamma m} \gamma_{5} \lambda)\). The Ramond vector \(\psi_m\) originating from the spinning string/particle models is associated with the spin structure of the enlarged superspace. The nonanticommutativity of the \(\theta_a\) components among themselves depends only on spin tensors constructed from the auxiliary spinor which may be treated as a component of twistor. It points out that a hidden spinor structure of space-time associated with the Penrose twistor picture could underly the non(anti)commutativity. We found a one to one correspondence between the Lorentz invariant Moyal brackets \([26]\) of the supecoordinates and their background dependent (anti)commutators parametrized by the antisymmetric field \(B_{mn}\), the graviphoton \(C_{ab}\) and the gravitino \(\Psi_m^a\). The map was schematically presented as
\[
B_{mn}^{-1} \leftrightarrow i \psi_m \psi_n, \quad C_{ab} \leftrightarrow \lambda_a \lambda_b, \quad \Psi_m^a \leftrightarrow \psi_m \lambda^a
\tag{62}
\]
and is valid up to the second order corrections in the deformation parameter $h$. The map $(62)$ transforms the background dependent and Lorentz noninvariant (anti)commutators of the supercoordinates into their invariant Moyal brackets $(26)$ restoring desirable Lorentz invariance of the deformations. The map gets a natural explanation in the frame of the Feynman-Wheeler action at-a-distance theory and its superymmetric generalization $(38)$, where the (super)fields were constructed from the (super)space coordinates resulting in the Maxwell and Dirac equations. We studied the null twistor realization of the brackets and observed the dependence of the non(anti)commutativity effect on the choice of supercoordinate variables. We found that transition to the null supertwistor subspace $(28)$ washes off the noncommutativity effect among the null supertwistor components, because of the nonlocal connection between (super)twistors and (super)coordinates. It shows that the non(anti)commutativity of the original coordinates of the superspace may be hidden in their nonlinear combinations describing supersymmetric (anti)commutative hypersurfaces embedded in the primary superspace. This observation attracts an attention to the paper $(42)$, where the new super $(D)$p-brane models with the $OSp(1,2M)$ spontaneously broken symmetry were constructed using supertwistor space. We outlined some generalizations of the studied invariant brackets to the cases of $N$ extended supersymmetry and additional spinor coordinates based on the possibility to construct additional Lorentz invariant supersymmetric derivatives. An attractive feature of the deformation defined by the Moyal brackets originated from the chiral Poisson bracket $(46)$ is the appearance of composite axial vector $\psi_1 \gamma_m = \frac{1}{2} (\bar{\psi} m \gamma_5 \lambda)$ in the pair with Ramond vector $\psi_m$. It reminds on $V - A$ structure of the chiral sigma models, electroweak interactions and parity breaking and sends a signal to think about non(anti)commutative deformations of underlying superspaces as a geometrical source behind the physics of the Standard Model. The microscopic scale of the superspace deformations is fixed by the above introduced deformation parameter $h$. We suppose that effect of the proposed deformations on the structure of minimal supersymmetric SM deserves an attention. The same concerns a possible role of the deformations in the problem of supersymmetric dark matter and dark energy. The work is in progress.

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