Shear Measurement Bias Due to Spatially Varying Spectral Energy Distributions in Galaxies

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Abstract

Galaxy color gradients (CGs)—i.e., spectral energy distributions that vary across the galaxy profile—will impact galaxy shape measurements when the modeled point-spread function (PSF) corresponds to that for a galaxy with spatially uniform color. This paper describes the techniques and results of a study of the expected impact of galaxy CGs on weak lensing measurements with the Large Synoptic Survey Telescope (LSST) when the PSF size depends on wavelength. The bias on cosmic shear measurements from CGs is computed both for parametric bulge+disk galaxy simulations and for more realistic chromatic galaxy surface brightness profiles based on Hubble Space Telescope V- and J-band images in the All-Wavelength Extended Groth Strip International Survey (AEGIS). For the parametric galaxies, and for the more realistic galaxies derived from AEGIS galaxies with a sufficient signal-to-noise ratio that CG bias can be isolated, the predicted multiplicative shear biases due to CGs are found to be at least a factor of two below the LSST full-depth requirement on the \textit{total} systematic uncertainty in the redshift-dependent shear calibration. The analysis code and data products are publicly available (https://github.com/sowmyakth/measure_cg_bias).

Unified Astronomy Thesaurus concepts: Weak gravitational lensing (1797); Cosmology (343); Astronomical techniques (1684); Observational cosmology (1146); Gravitational lensing shear (671)

1. Introduction

Weak gravitational lensing produces coherent alignment of images of distant galaxies due to the intervening tidal gravitational field (Bartelmann & Schneider 2001). Correlations in these distortions can be used to determine the statistical properties of the intervening mass distribution as a function of redshift. These perturbations are small when compared to the initially random orientations of galaxies, requiring a large number of galaxies for meaningful cosmological measurements. While the Large Synoptic Survey Telescope (LSST) survey, encompassing billions of galaxies, will dramatically improve the statistical power of weak lensing observations, the systematic errors must be carefully examined and controlled (LSST Science Collaboration 2009; LSST DESC et al. 2018).

In addition to cosmic shear, the observed shapes of galaxies are distorted by the atmosphere, the telescope optics, and sensor effects—all of which contribute to the point-spread function (PSF). The observed images of the galaxies are thus a convolution of the true image and the observing PSF. This makes it vital for weak lensing measurements to model the convolving PSF and correct for it using stars. Additional complexity arises when the distortions caused by the PSF depend on the observing wavelength (Cypriano et al. 2010). The effects of chromatic atmospheric PSFs on weak lensing measurements have been studied by Meyers & Burchat (2015) who conclude that while the predicted shear bias is significant compared to the LSST requirements, the bias can be reduced sufficiently to meet the LSST requirements if the PSF is corrected using multiband photometry and machine learning techniques. However, these corrections are based on the assumption that the spatial and wavelength dependencies of the galaxy surface brightness profile are independent. If instead the galaxy has a spectral energy distribution (SED) that varies across its profile—a “color gradient (CG)”—then the distortions due to the chromatic PSF will be different for different points on the galaxy. PSF corrections that are applied assuming a position-independent SED do not correct for potential CG effects. Thus, it is necessary to quantify the size of this bias compared to the LSST requirements on bias for shear estimators.

Previous studies of shear bias due to CGs have focused on the space-based Euclid survey (Amiaux et al. 2012), where the wavelength dependence of the PSF is dominated by diffraction, and the wavelength range for the (single) optical filter is \(\approx 550–900\) nm. These studies have concluded that shear bias due to CGs in surveys like Euclid can be significant compared to the requirements on systematic errors (Voigt et al. 2012; Semboloni et al. 2013; Er et al. 2018). We use and extend a method similar to Semboloni et al. (2013) (hereafter S13) to estimate the impact of CG bias on weak lensing measurements with the LSST, where the wavelength dependence of the PSF is dominated by atmospheric effects, and the wavelength range for each filter is \(\approx 150\) nm.

We first use a reference galaxy with extreme CGs to demonstrate the presence of CG bias as well as its dependence on atmospheric seeing and shape measurement algorithms. We then measure the expected size of this bias on LSST weak lensing measurements by extending the study to a more representative sampling of CGs. This is done with two different approaches, each with advantages and disadvantages:

(a) We assemble a parametric catalog of galaxies with a range of CGs that could be observed with LSST. This method allows us to simulate galaxy images with infinite
The parametric galaxy simulations for method (a) are generated from the LSST Catalog Simulator, CatSIM (Connolly et al. 2014), which contains astrophysical sources with properties that are representative of what the LSST will observe at its 10 yr coadded depth. Real galaxy images for method (b) were obtained from Hubble Space Telescope (HST) V- and I-band observations in the All-wavelength Extended Groth strip International Survey (AEGIS).

The work is presented as follows. In Section 2, we describe the sources of CG bias and discuss how the galaxy’s intrinsic properties, atmospheric seeing, and shape measurement algorithms affect the observed bias. In Section 3, we describe a method to isolate CG bias for a galaxy with CG by comparing its shear response to that of an “equivalent” galaxy without CGs. In Section 4, we apply this method to the reference galaxy with extreme CGs and to parametric simulations from CatSIM (approach (a)) for a more realistic estimate. In Section 5, we apply the method to the real HST galaxy images as seen by the LSST (approach (b)). We then summarize the results and their significance.

To help the reader follow the description of the analysis, we compile in Tables 1 and 2 the acronyms and symbols used in this paper, along with the section in which they are defined in the context of this study.

| Acronym | Description | More Details |
|---------|-------------|--------------|
| PSF     | Point-spread function: response of imaging system to a point source. | Section 2.1 |
| SED     | Spectral energy distribution: energy emitted as a function of wavelength. | Section 2.2 |
| CG      | Color gradient: varying SED across spatial profile. | Section 2 |
| SBP     | Surface brightness profile: luminosity of a galaxy as a function of position and wavelength. | Section 4.2 |
| HLR     | Half-light radius: radius within which half the galaxy flux is contained. | Section 4.3.1 |
| FWHM    | Full width at half maximum: for circularly symmetric SBP, diameter of circle at which the surface brightness is half the peak surface brightness. | Section 4.1 |
| CATSIM  | Catalog Simulator: catalog managed by the LSST Systems Engineering group. | Section 4.4 |
| CRG     | Galaxies simulated with the ChromaticRealGalaxy module in GaSim. | Section 5.1 |
| S/N     | Signal-to-noise ratio: ratio of measured flux to uncertainty on the flux. | Section 5.2 |

signal-to-noise ratio (S/N) and measure bias from CG only. However, the accuracy of the prediction for real surveys depends on the extent to which the simulation represents CGs in real galaxies.

(b) We use observed high-resolution galaxy images to estimate CGs. This method allows us to leverage information from real galaxies, potentially leading to more realistic bias estimates. However, when the input galaxy images are noisy (as they are for real galaxies), the effect of noise cannot be removed completely and the estimated bias is a combination of CG bias and residual noise bias (see Refregier et al. 2012, for example, for details on impact of noise bias in shear measurements).

We explore in detail below how the three conditions produce CG bias. In this study, we restrict ourselves to analyzing the effects of CG bias for moment-based shape measurement algorithms that use weight functions matched to galaxy profiles. However, CGs have also been shown to lead to shape measurement errors when “fitting methods” are used, as described in Voigt et al. (2012) where the profiles themselves weight the different regions of the image. Both Voigt et al. (2012), using fitting methods, and Semboloni et al. (2013), using moment-based shape estimation, showed that the CG bias could be substantial in the Euclid survey—exceeding nominal requirements for the multiplicative bias in the shear.

### 2.2. Effective PSF for Correction

The observed image of the galaxy, $I_{CG}^{obs}(x)$, can be described as the convolution (denoted by *) of the galaxy profile $I_{CG}(x, \lambda)$ with the chromatic PSF, $\Pi(x, \lambda)$, weighted by the band transmission function, $T(\lambda)$ (which includes contributions from the atmosphere, optics, filter, and sensor) and integrated over wavelength:

$$I_{CG}^{obs}(x) = \int I_{CG}(x, \lambda) \ast \Pi(x, \lambda) T(\lambda) \, d\lambda.$$  

Because the PSF is chromatic, its effect on the observed galaxy shape will depend on the observing wavelengths. Similarly,
any PSF correction to be applied to a galaxy shape will require knowledge of the galaxy SED over the observing band. If CGs are ignored then the PSF correction will assume an “effective” SED that is uniform across the galaxy profile in place of the true SED of the galaxy that is position dependent. The effective SED, $S_{\text{eff}}(\lambda)$, is defined as the spatially integrated flux of the galaxy as a function of wavelength:

$$S_{\text{eff}}(\lambda) = \int f_{\text{CG}}(x, \lambda) \, dx.$$  

The PSF for correction will also be an “effective” PSF, $\Pi_{\text{eff}}(x)$, corresponding to the image of a point source with SED, $S_{\text{eff}}(\lambda)$:

$$\Pi_{\text{eff}}(x) = \int S_{\text{eff}}(\lambda) \Pi(x, \lambda) T(\lambda) \, d\lambda,$$

where we have implicitly evaluated the convolution with a delta-function point source. Semboloni et al. (2013) pointed out that CGs do not produce a bias if the integrals are allowed to extend to infinity in each direction, which is impossible in practice. All realistic methods use some kind of spatial weighting (even if only to impose zero weight beyond the region of the “postage stamp”), which means that different regions of the galaxy with different colors and PSFs are weighted differently, introducing a potential bias.

### 2.3. Weight Function in Shape Measurements

The galaxy shapes can be expressed as complex ellipticities $e$ computed using quadrupole moments (Seitz & Schneider 1997):

$$e_1 + ie_2 = \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2(Q_{11}Q_{22} - Q_{12}^2)^{1/2}},$$  

where $Q_{ij}$ are the second moments of the galaxy image.

When computing second moments, pixel values at larger distances from the centroid have more impact on the measurement than pixel values close to the centroid. In real galaxy shape measurements, the moments are computed from noisy images, where the noise will dominate the pixel values (Melchior et al. 2011). Thus in order to prevent noise divergence in the calculation of second moments, it is common practice to employ weight functions of finite width to limit the integration. A weight function whose centroid, size, and ellipticity are matched to the source galaxy image optimizes the significance of the measurement. A Gaussian weight is generally preferred due to its rapid convergence to zero at large radii and the absence of singularities, as well as general mathematical convenience (Hirata & Seljak 2003).

By its very nature, the weight function gives more significance to the central region of the galaxy. For a galaxy with a spatially independent SED, this does not produce an error as long as one correctly accounts for the weight function in the shape estimate. However, if there exists a spatial dependence of the galaxy SED, regions with different colors are weighted differently, potentially resulting in a bias in shape estimation that depends on the size and profile of the weight function and the CG.

A possible method for reducing the impact of CG bias would be to weight the galaxy profile in the computation of the effective SED (Equation (3)) with the same weight function used to measure the galaxy shape. We do not explore this here because, in this paper, our goal is to understand and quantify the size of shear measurement bias due to CGs assuming we do not attempt to correct for galaxy CG; we do not explore optimal methods for mitigating the bias if it is necessary to do so.

### 2.4. Illustration of CG Bias for a Simple Bulge + Disk Galaxy

We illustrate in Figure 1 the impact of the aforementioned three conditions by comparing the measured shape of a galaxy with and without CGs. The top row depicts this measurement.
for a galaxy with no CG when observed with a chromatic PSF and with moments computed using a weight function. The true galaxy (green ellipse) is convolved by the PSF (green circle) and integrated over the observing bandpass to produce the observed galaxy image. The radius of the PSF circle is proportional to the PSF size. As a result of convolution with the PSF, the observed galaxy image appears rounder and larger than the true galaxy. To obtain the correct measured shape the effective PSF must be measured. In the absence of CG, the effective SED is the same as the uniform galaxy SED and the effective PSF correctly encapsulates the PSF distortions. The yellow-dotted circle denotes the weight function for computing moments, which are corrected using the effective PSF in order to retrieve the correct measured shape (gray ellipse).

The bottom row illustrates the same process but for a galaxy with CGs. For simplicity we demonstrate this for a galaxy with radially dependent CG, where the galaxy has a compact central bulge (red ellipse) and an extended disk (blue ellipse). Both the bulge and the disk have the same centroid and shape, but have different size and color with the bulge being smaller and redder than the disk. Because the size of the PSF is color dependent, the distortion caused by the PSF on the two components is different. We assume here that the PSF is smaller for longer wavelengths (as expected for the LSST). Therefore the bulge PSF (red circle) is smaller than the PSF acting on the disk (blue circle). The observed image will be the sum of the convolution of the two components with their respective PSFs. Because the disk PSF is larger, it will have a stronger “rounding” effect on the disk component in comparison to the smaller bulge PSF, making the disk rounder than the bulge in the observed image. The effective SED is an average of the disk and bulge SED, resulting in the effective PSF having a size in between the bulge and disk PSFs.

As before, the weight function is denoted by the yellow-dotted circle. Because the weight function effectively gives more importance to the center, the bulge will be weighted more than the disk in the shape measurement. The correction PSF (effective PSF) is larger than the actual PSF that acted on the bulge. The PSF correction step will thus overestimate the ellipticity of the galaxy.

The extent of this error in the measured shape depends on the intrinsic CG of the galaxy. CG depends on galaxy morphology, which is correlated with redshift (La Barbera et al. 2010; Kennedy et al. 2016). Therefore, CGs can lead to redshift-dependent shape measurement biases.

3. Isolating and Quantifying CG Bias

Shape measurement algorithms can lead to a range of different shape measurement errors that depend on noise, galaxy properties, etc.; see Mandelbaum et al. (2018). To isolate CG bias, we use a technique described in S13: compare the shear measured from a galaxy with CG to that of an “equivalent” galaxy with no CG.

The technique is illustrated in Figure 2. We create a pair of galaxies—one galaxy with a spatially dependent SED (on the left), and an equivalent galaxy with uniform SED (on the right). The SED and surface brightness profile (SBP) of the equivalent galaxy is chosen so that the two galaxies appear identical when convolved with the same chromatic PSF and observed through the same filter (top row). This leads to all non-CG biases being the same for the two galaxies. As illustrated in the bottom row in the figure, if we apply the same shear to each galaxy and convolve it with the same chromatic PSF, the measured shear is no longer the same. We quantify the CG shear bias as this difference between the measured shear estimators (ellipticities). More specifically, in the absence of shear although the observed image of the galaxy with CG and the equivalent galaxy are the same, the SBPs for the two galaxies are not identical. Therefore, the response to the same shear $g$ will be different for the two galaxies, causing the PSF-convolved images and the shear measured from the two observed images.
to be different. The difference in the two shear values ($\hat{g}_{CG}$ and $\hat{g}_{no\ CG}$) is an estimate of the CG bias.

### 3.1. Isolating Shear Bias Due to CG

When a small shear $g$ is applied to a galaxy with no CG, the measured shear, $\hat{g}_{no\ CG}$, can be approximated as

$$\hat{g}_{no\ CG} = (1 + m_a)g + c_o,$$

where $m_a$ and $c_o$ are the multiplicative and additive bias terms. We assume that when the same small shear $g$ is applied to a galaxy with CG, the bias on the measured shear, $\hat{g}_{CG}$, due to CGs can be expressed by two new terms $m_{CG}$ and $c_{CG}$:

$$\hat{g}_{CG} = (1 + m_{a} + m_{CG})g + (c_{o} + c_{CG}).$$

If the galaxy with no CG is the equivalent galaxy described earlier, then taking the difference of the two measured shears will remove all other systematic bias contributions isolating contributions from CGs only:

$$\Delta g = \hat{g}_{CG} - \hat{g}_{no\ CG} = m_{CG}g + c_{CG}. \tag{8}$$

### 3.2. LSST Requirements on Shear Calibration

As documented in Sections 5.2, D2.1 and D2.3 in version 1 of the LSST Dark Energy Science Collaboration (DESC) Science Requirements Document (SRD) (LSST DESC et al. 2018), the requirement on the total systematic uncertainty in the redshift-dependent shear calibration is that it not exceed 0.003. Combined with other requirements in the LSST DESC SRD, LSST will then be able to achieve its design constraints on dark energy (Albrecht et al. 2006; LSST DESC et al. 2018).

The SRD analysis uses five photometric redshift bins, each with a width of $\Delta z = 0.2$, in the redshift range of $0.2 \leq z \leq 1.2$. A linear parameterization is suggested for the redshift dependence of the multiplicative shear bias, $m(z)$:

$$m(z) = m_2 \left( \frac{2z - z_{\text{max}}}{z_{\text{max}}} \right) + m_{\text{avg}}, \tag{9}$$

where $z_{\text{max}}$ is the redshift value at the center of the highest redshift bin, $m_{\text{avg}} = m(z_{\text{max}}/2)$ is the average value of $m$ in the range of $z \in [0, z_{\text{max}}]$, and $2m_2 = m(z_{\text{max}}) - m(0)$ is the total variation in $m$ in the range of $z \in [0, z_{\text{max}}]$. The 0.003 requirement is then on the total systematic uncertainty in the redshift-dependent shear calibration $m$ due to all contributions to multiplicative shear bias. In principle, there is some higher-order dependence on the redshift-dependent function adopted for $m(z)$. In the absence of an explicit requirement for the case of nonlinear dependence of $m(z)$ on $z$, we map the requirement of $2(0.003) = 0.006$ on the uncertainty on $2m_2 = m(z_{\text{max}}) - m(0)$ onto a requirement on the uncertainty on the maximum span of $m(z)$ over the redshift range $[0, 1.2]$.

### 3.3. Creating an Equivalent Galaxy with no CG

The method described in Section 3.1 for isolating CG bias assumes we can create an equivalent galaxy with no CG with the properties shown in Figure 2. We describe below a method adapted from Section 2 of S13 to create an equivalent galaxy with no CG.

In the absence of CGs, the galaxy SBP can be factored into a product of spatial and spectral components $a(x)$ and $S(\lambda)$:

$$f(x, \lambda) = a(x)S(\lambda).$$

The true image of the galaxy, $I^o(x)$, through the transmission function $T(\lambda)$ can be written as

$$I^o(x) = \int f(x, \lambda) \ T(\lambda) \ d\lambda. \tag{11}$$

However, as shown in Equation (2), the observed galaxy is a convolution of the true galaxy and the PSF, in the observed band:

$$I^{\text{obs}}(x) = \int f(x, \lambda) * \Pi(\lambda, x) \ T(\lambda) \ d\lambda. \tag{12}$$

In Fourier space, convolution becomes multiplication and the expression for the observed galaxy image becomes

$$\tilde{I}^{\text{obs}}(k) = \int \tilde{a}(k) S(\lambda) \tilde{\Pi}(k, \lambda) T(\lambda) d\lambda = \tilde{a}(k) \int S(\lambda) \tilde{\Pi}(k, \lambda) T(\lambda) d\lambda, \tag{13}$$

while the unconvolved image in Fourier space is

$$\tilde{I}^o(k) = \int \tilde{f}(k, \lambda) T(\lambda) d\lambda = \int \tilde{a}(k) S(\lambda) T(\lambda) d\lambda. \tag{14}$$

Solving Equation (13) for $\tilde{a}(k)$ and substituting in Equation (14) we get

$$\tilde{I}^o(k) = \int \frac{\tilde{I}^{\text{obs}}(k)}{\int S(\lambda) \tilde{\Pi}(k, \lambda) T(\lambda) d\lambda} S(\lambda) T(\lambda) d\lambda. \tag{15}$$

The denominator is the effective wavelength-independent PSF, $\Pi^{\text{eff}}(k)$, computed in Equation (4), where the effective SED is identical to the uniform galaxy SED. Thus, the SBP of the galaxy at a particular wavelength $\lambda_{\text{ref}}$ can be written as

$$\tilde{I}^{\text{eff}}(k, \lambda_{\text{ref}}) = \frac{S(\lambda_{\text{ref}})}{\Pi^{\text{eff}}(k)} I^{\text{obs}}(k). \tag{16}$$

In other words, in the absence of CGs the galaxy SBP at a given wavelength is the PSF-convolved galaxy image, deconvolved by the effective PSF.

We now extend the above analysis to a galaxy with CG. We approximate the SBP of the equivalent galaxy as Equation (16) applied to the observed image of a galaxy with CG with the same effective SED but no CG. Thus the SBP of the effective galaxy with no CG is

$$\tilde{g}_{no\ CG}(k, \lambda) = \frac{S_{\text{eff}}(\lambda) \tilde{I}^{\text{obs}}_{CG}(k)}{\Pi_{\text{eff}}^{\text{CG}}(k)}, \tag{17}$$

where $S_{\text{eff}}(\lambda)$ is the effective SED computed in Equation (3). In the absence of an applied shear, the observed image corresponding to a galaxy with the SBP of Equation (17) is identical to $\tilde{I}^{\text{obs}}_{CG}(k)$. A flowchart illustrating the entire method is also shown in Figure 3.

We use the modular galaxy simulation toolkit GalSim (Rowe et al. 2015) to perform the integrations, convolutions, and deconvolutions described above to simulate both the
galaxy with CG and the equivalent galaxy with no CG. While
the galaxy image with CG, \( I_{CG}^{obs}(x) \) in Equation (2), is drawn
with the LSST pixel scale, the images in Fourier space, \( I_{CG}^{obs}(k) \)
and \( \tilde{I}_{CG}(k) \) in Equation (17), are drawn with four times the
resolution in order to reduce errors produced by finite pixel size
during deconvolution.

### 3.4. Shear Estimation from Galaxy Shapes

We described how galaxy shapes are measured from galaxy images
in Section 2.3, quantifying the ellipticities from the second moments
with intrinsic shape noise. In this section we describe
the procedure used to estimate the shear from galaxy shapes.

The reduced shear \( g = g_1 + ig_2 \) applied to an unlensed source
with intrinsic shape \( e^{intr} \) is related to the observed ellipticity \( e \)
by the transformation (Seitz & Schneider 1997)

\[
    e = e^{intr} + g \frac{e^{intr}}{1 + g e^{intr}}, \quad \text{if } |g| \leq 1.
\]  

Assuming that the intrinsic orientation of the galaxies is random,
the mean of their observed sheared ellipticities is the
reduced shear, \( \langle e \rangle = g \). Because the weak lensing shear is so small compared to the intrinsic, randomly oriented galaxy ellipticities (shape noise), averaging over very large ensembles of galaxies is necessary to achieve small statistical uncertainties (Mandelbaum et al. 2018). We employ a technique to suppress
shape noise without generating a large number of galaxies by
simulating six galaxies with equidistant intrinsic ellipticity on a
ring around 0—"ring test" (Nakajima & Bernstein 2007) to measure shear (see, for example, Section 6.1 in Meyers & Burchat 2015).

### 3.5. Moment-based Shape Estimation with GalSim

GalSim is used to perform shape estimation, in addition to the image simulation described above. The shapes are computed with "adaptive" moments that use an elliptical Gaussian weight function, the shape of which is matched to that of the PSF-convolved galaxy image. The measured moments of the galaxy are corrected for the PSF to produce the ellipticity of the intrinsic galaxy image (Bernstein & Jarvis 2002).

For most of this analysis, we use the GalSim implementation of the "re-Gaussianization" algorithm (REGAUSS), which treats the deviations of the PSF from Gaussianity perturbatively
in real space, without first measuring moments (Hirata & Seljak 2003).

We found that, when compared to other shape estimation
algorithms in GalSim, the REGAUSS algorithm makes more
accurate shape measurements. The error in estimated shape is approximately \( 10^{-6} \) for a simulated Gaussian galaxy with a
Gaussian PSF drawn with LSST pixel scale; the bias is larger
for more complex situations.

Our method (described above) of using an equivalent galaxy
without CGs to isolate the contribution from CGs reduces
the impact of the shape estimation algorithm on the estimated CG
bias. For the reference galaxy simulated in a redshift range
of 0–1.2, we estimate the CG bias using three other adaptive
moment estimation and PSF correction algorithms implemented
in GalSim: KSB, LINEAR (Hirata & Seljak 2003) and BJ
(Bernstein & Jarvis 2002). The difference in CG bias estimates
for REGAUSS, KSB and BJ is less than \( 1 \times 10^{-4} \) for all
redshifts. For LINEAR and REGAUSS, the difference is also
less than \( 1 \times 10^{-4} \), except for \( z \approx 0.2 \), when the difference
in bias is just under \( 4 \times 10^{-4} \).

As discussed in Section 2.3, the weight function used in the
shape measurement has a very significant effect on CG bias.
Therefore, it is important to study the effect of the size of the
weight function on the measured CG bias. The implementation
of REGAUSS in GalSim does not have the provision to switch
to or fix the size of the weight function computed with
adaptive moments. Therefore, to study the impact of the weight
function size, we use the GalSim implementation of the KSB
shape measurement algorithm where the size of the circular
weight function can be fixed by the user.

While we found that REGAUSS robustly yields correct
results for noise-free parametric simulations and real galaxies
with high S/N, we found a high failure rate for low-S/N
galaxies. This is because the adaptive moment algorithm is not
robust to noise and therefore does not always converge to a
solution. This resulted in the elimination of a significant fraction of small, faint galaxies from our analysis sample.

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1. Drawn refers to the `GalSim drawImage` function, which draws an object profile onto an image.

8. The GalSim KSB algorithm is a specific implementation of the KSB method (Kaiser et al. 1995; Luppino & Kaiser 1997), as described in Appendix C of Hirata & Seljak (2003).

9. We use the same definition of S/N as that used in the GREAT3 challenge (Mandelbaum et al. 2014), where S/N is defined as the ratio of measured flux to uncertainty on the flux within an elliptical Gaussian aperture matched to the size and shape of the PSF-convolved galaxy image.
Although these galaxies are not likely to be used in weak lensing analyses, we recommend that shape estimation methods that are more robust to noise be used in future CG bias studies.

4. Estimating CG Bias with Parametric Galaxies

We first evaluate the impact of CGs on shear measurements with simulated parametric galaxies. Because the galaxies can be simulated with no noise, this enables us to isolate the effect of CG bias without being contaminated by noise bias.

4.1. Observing Conditions

The galaxy images are simulated in the two main LSST lensing bands, r and i, with the measurements performed in the r band, unless otherwise noted. In Figure 4, we show the transmission functions for the LSST r and i bands, as well as the HST V and I bands used in the analysis of AEGIS galaxies, described below.

As described in Equation (1), we model the LSST PSF as a circular Gaussian profile with a wavelength-dependent size that is seeing limited with scaling exponent $\alpha = -0.2$. This value is valid for purely Kolmogorov turbulence—i.e., an infinite outer scale—in the limit of long exposure times. For this analysis we set the PSF size to $\sigma^\prime = 0''297$ (FWHM = $0''7$) at $\lambda^\prime = 550$ nm. This value was chosen to match the expected LSST median zenith seeing in the r band (Ivezic et al. 2008). We study the impact of the scaling exponent and PSF size on the measured bias. We limit the scope of this study to only wavelength-dependent PSF size and do not include any wavelength-dependent PSF ellipticity effects, such as atmospherical differential chromatic refraction (DCR).

4.2. Parametric Galaxy with CGs

We model galaxies with CG as concentric bulge + disk components with different SEDs, with the following SBP:

$$f_{CG}(x, \lambda) = a_b(x)S_b(\lambda) + a_d(x)S_d(\lambda),$$

(19)

where $a_{b,d}(x)$ are the spatial profiles and $S_{b,d}(\lambda)$ the SEDs of the bulge and disk components denoted by the subscripts $b$ and $d$, respectively. We use this parameterization of galaxies with CG for two different samples: (1) the reference galaxy with extreme CGs and (2) the CATSIM catalog of galaxies.

4.3. Reference Parametric Bulge + Disk Galaxy with Extreme CGs

We first demonstrate the presence of CG bias for a simulated reference galaxy with extreme CGs: a superposition of a small red bulge and an extended blue disk. The parameters of the galaxy are chosen to be identical to the "B" type galaxy in S13, which allows us to compare results for the same galaxy model. Indeed, we find that our estimates of CG shear bias for the reference galaxy with a Euclid PSF and bandpass are in agreement with the values estimated in S13.

4.3.1. Parameters for Simulating Extreme-CG Galaxy

The spatial components of the bulge and disk SBP, $a_b(x)$ and $a_d(x)$, are modeled as Sérsic profiles (Sérsic 1963), with indices $n_b = 1.5$ and $n_d = 1.0$ for the bulge and disk, respectively. The bulge is compact with a half-light radius (HLR) of $0''.17$. The spatial component of the bulge accounts for 25% of the total galaxy flux. The extended disk is much larger with an HLR of $1''.2$. Both, the bulge and disk, have the same centroid positions and ellipticities, $e_{bulge} = (0.3, 0.3)$.

The SEDs of the bulge and disk are shown in Figure 4 with the redder bulge assigned a CWW-E spectrum (solid-red line) and the disk a CWW-Im spectrum (blue-dashed line), which has strong emission lines. CWW here refers to template SEDs created from spectroscopic data of nearby galaxies of different types measured by Coleman et al. (1980) and extended below 1400 A and beyond 10000 A by Bolzonella et al. (2000) using evolutionary models of Bruzual & Charlot (1993). To further investigate the impact of intrinsic galaxy CGs, we measure the CG bias for the reference galaxy with extreme CGs at different redshifts and for two additional disk SEDs: CWW-Sbc and CWW-Scd (magenta- and green-dashed lines, respectively). The SEDs in Figure 4 are normalized to 0.2 at 550 nm in the rest frame and then redshifted to 0.3. The bulge and disk parameters are summarized in Table 3.

4.3.2. Verification of Method to Isolate CG Bias

We verify our proposed method for isolating CG bias described in Section 3 by simulating an equivalent galaxy with no CG, for the reference galaxy, and then applying the same shear and PSF to both the original galaxy and the equivalent galaxy. The shears recovered with a ring test for each galaxy are compared. If this difference in shears, $\Delta g$, shows a linear dependence on the applied shear, then we have confirmation that the method described in Section 3.1 is able to isolate CG bias. The slope and intercept of the linear model, $\Delta g = mCGg + cCG$, correspond to multiplicative and additive bias, respectively, from CGs only.

The galaxy shapes are estimated with the REGAUSS method of GalSim. Because the bias values are found to be similar for the two shear components $g_1$ and $g_2$, we report results here for only $g_1$. In the top-left and top-center panels of Figure 5, we plot as a function of applied shear $g$ the difference between measured

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11 We chose the bulge Sérsic index to match the value in S13, rather than using the more conventional value $n_b = 4$. We find that the predicted CG shear bias $mCG$ is up to 26% smaller in magnitude when the reference galaxy bulge Sérsic index is taken to be 4 rather than 1.5.
shear $\delta_{\text{CG}}$ and applied shear for a galaxy with CG (left), and the difference between measured shear $\delta_{\text{me}}$ and applied shear for the equivalent galaxy with no CG (center). The blue dots correspond to these differences and the red line connects the result of a linear fit to these values. The bottom plot in each panel corresponds to the difference between the linear fit and the points. We see that for this range of applied shear ($g \leq 0.03$), the measured shear is linearly dependent on the applied shear. The multiplicative and additive biases, corresponding to the slope ($mm = m_c + m_{\text{CG}}$) and intercept ($cc = c_o + c_{\text{CG}}$) of the fits, are found to be $m = 1.01 \times 10^{-2}$ and $c = -1.86 \times 10^{-7}$ for the original galaxy with CG, and $m_c = 1.17 \times 10^{-2}$ and $c_o = -1.87 \times 10^{-7}$ for the equivalent galaxy with no CG.

We proposed in Section 3.1 that $m_c$ includes the shear bias from all sources except CG and that the bias from CG can be isolated as $m_{\text{CG}} = m - m_c$. We test this hypothesis by plotting in the top-right panel of Figure 5 the difference $\Delta g = \delta_{\text{CG}} - \delta_{\text{me}}$ as a function of applied shear. The small differences between the linear fit and the points, shown in the bottom-right panel, validate the linear relationship assumed in Equation (8). The slope of the linear fit is an estimate of bias from only CG: $m_{\text{CG}} = 1.03 \times 10^{-3}$. The sign of $m_{\text{CG}}$ is positive; i.e., the measured shear is larger than the true shear, in agreement with the example in Section 2.4.

We now apply this method to study the dependence of CG bias on intrinsic CG and observing conditions. Due to the small magnitudes of the additive bias in our simulations, which is expected for a circular PSF with circularly symmetric wavelength-dependence, we focus our analysis on studying the impact of CGs on only the shear multiplicative bias.

### 4.3.3. Results of CG Bias Measurements on an Extreme-CG Galaxy

To illustrate the impact of intrinsic galaxy CGs on CG bias we estimate the CG bias for the reference galaxy with extreme CGs. The estimated bias values here are not representative of the bias expected for shear measurements from all galaxies seen by LSST; rather, we expect these bias values to be outliers for a more representative sample.

We plot in Figure 6 the estimated value of $m_{\text{CG}}$ for three different disk SEDs, with the same bulge SED (CWW-E), for different redshifts. The CG biases are estimated from the shear recovered when a constant shear of $g = (0.01, 0.01)$ is applied to the galaxy. The bias is larger for the CWW-Im disk SED, which has emission lines. Because CG bias arises due to the difference in bulge and disk color in the observing bandpass, galaxies with the same bulge and disk SEDs can have different CG bias at different redshifts, depending on their SED profiles. The yellow-dashed lines show the LSST DESC requirement of 0.003 on the total systematic uncertainty in the redshift-dependent shear calibration. The estimated CG bias for all three disk SEDs and redshifts are observed to be lower than the LSST requirement, with the mean CG bias at all redshifts and disk SEDs being $m_{\text{CG}} = 8.65 \times 10^{-4}$.

In S13, the magnitude of the CG bias for the reference galaxy in the same redshift range, with Euclid’s expected PSF size and wavelength dependence, was estimated to be in the range $4 \times 10^{-4} - 1.5 \times 10^{-3}$, which is not dissimilar to our predictions for LSST for this extreme case. The observing bandpass for Euclid covers a wider wavelength range (550–900 nm) than those for LSST and the slope of the PSF chromaticity is steeper, leading to higher sensitivity to CGs in Euclid. However, the Euclid PSF is smaller (FWHM = 0′′15) than the LSST PSF, offsetting the other effects.

### 4.3.4. Dependence on Weight Function

As described in Section 2.3, CG bias occurs in the presence of a weight function that assigns more weight to certain parts of the galaxy image. In the absence of a weight function, we expect the CG bias to be zero (Semboloni et al. 2013). To study the effect of our choice of weight function, we estimate CG bias while varying the size of the circular Gaussian weight function using the KSB method for galaxy shape measurement in GalSim. The results of the analysis for the reference galaxy with CWW-Im disk SED at redshift 0.3 is shown in Figure 7. The values on the horizontal axis correspond to the ratio of the HLR of the weight function to the bulge+disk galaxy HLR (0′′949). As expected, the CG bias decreases with increasing size of the weight function, with the bias approaching zero for large weight sizes. The yellow circle denotes the CG bias for a Gaussian weight function with HLR equal to the size of the galaxy as used in S13. For comparison we show that the CG bias computed with REGAUSS for the reference galaxy (blue star) is larger. This is because the GalSim REGAUSS algorithm uses adaptive moments to match an elliptical Gaussian weight to the galaxy image. Because the reference galaxy is the sum of two Sérsics, the Gaussian profile matched by REGAUSS has an HLR = 0′′7, smaller than the PSF-convolved galaxy HLR of 0′′949. Thus, this analysis uses a smaller weight function than in S13, leading to larger biases.

### 4.3.5. Dependence on PSF

We illustrate the effect of the PSF chromaticity on CG bias in Figure 8. The left panel shows the dependence on the PSF size scaling exponent $\alpha$. A scaling exponent of zero is an achromatic PSF and thus has no CG bias. The sign of the exponent determines the sign of the bias, with bias being positive for a negative exponent (ground-based survey) and negative for positive exponents (space-based surveys). The CG bias is highly sensitive to the chromatic nature of the PSF, with a change of 0.1 in the value of $\alpha$ changing the CG bias by 20% of the LSST requirement. The panel on the right in Figure 8 shows the dependence of CG bias on the size of the PSF. For small PSF sizes, the image distortions and PSF corrections are

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### Table 3: Parameters of Simulated Galaxies for Bulge/Disk Components

| Name                      | Sérsic Index | SED           | HLR (arcsec) | Flux Proportion (%) |
|---------------------------|--------------|---------------|--------------|--------------------|
| Reference galaxy with extreme CGs | 1.5/1        | E/(Im, Scd, Sbc) | 0.17/1.2    | 25/75              |
| CATSim galaxies (mean)    | 4/1          | Template SEDs  | 0.2/0.4     | 20/80              |

**Note.** Values correspond to bulge/disk components of the reference galaxy and CATSim galaxies. The entries for the CATSim galaxies are the mean bulge and disk parameters of the entire sample. The CATSim SEDs are drawn from an ensemble of template SEDs.
small, resulting in smaller CG bias. When the PSF is larger than the bulge \( \sigma \approx 0''15 \), the CG bias comes into play and \( m_{CG} \) remains constant even at larger PSF sizes. The blue star is the same as in Figure 6 and shows the bias for the reference galaxy PSF with \( \alpha = -0.2 \) and \( \sigma'' = 0''297 \) at \( \lambda'' = 550 \) nm.

We also found that the predicted CG shear bias \( m_{CG} \) was up to 20% smaller when the reference galaxy is convolved with a Kolmogorov PSF, rather than a Gaussian PSF, with the same FWHM value and the same wavelength-dependent scaling exponent. Because the aim of this analysis is to place conservative upper limits on CG bias estimates, we use the nominal Gaussian PSF for the rest of the analysis.

4.4. CATSIM Catalog of Parametric Galaxies

Although the CG bias for the reference galaxy computed above is small, it is not negligible in comparison to the LSST requirement for all systematic effects. Because we also observe a large dependence of CG bias on the galaxy intrinsic properties, CG bias cannot be outright ignored. Therefore, to more accurately estimate the expected CG bias for LSST, we calculate the bias using a catalog of galaxy characteristics (redshift, size, magnitude, and shape) expected in a 10 yr LSST weak lensing data set.

We use a simulated catalog prepared by the LSST Catalog Simulator, CATSIM (Connolly et al. 2014), containing astrophysical sources with properties that are representative of what the LSST will observe to its coadded depth. The catalog covers a \( 45 \times 45 \) degree footprint on the sky with realistic galaxy morphologies, apparent colors and spatial distributions, and redshifts. The galaxy simulation is based on dark matter halos from the Millennium Simulation (Springel et al. 2005) and a semi-analytic baryon model described in De Lucia et al. (2006). Galaxy morphologies are modeled using two Sérsic profiles, where the bulge and disk components have Sérsic indices 4 and 1, respectively. For all sources, an SED is fit to the galaxy colors using a sophisticated fitting algorithm, independently for the bulge and disk, that includes inclination-dependent reddening. The catalog contains galaxies with \( r \)-band AB magnitudes brighter than 28.

4.4.1. Parameters of CATSIM Galaxies

For our analysis, we select galaxies from 858,502 galaxies in the one-square-degree CATSIM catalog. The selected galaxies satisfy the following criteria, where the number in parentheses is the number of galaxies satisfying the cumulative criteria:

1. Composed of bulge and disk components—i.e., there is a nonzero CG (245,359).
2. Redshift less than 1.2 (76,201).
3. \( i \)-band magnitude <25.3 (58,310).
4. Bulge and disk semimajor axis each less than 3'' (\( \approx 15 \) pixels) (57,943).

The GalSim REGAUSS shape measurement algorithm fails for a fraction of galaxies with CG or their equivalent galaxies

12 We impose this maximum size to avoid GalSim run-time errors caused by Fourier transform arrays being too large in the ChromaticRealGalaxy (CRG) modeling described later in Section 5.1.
HLR is equal to the galaxy HLR computed with adaptive moments algorithm. We show the value of the CG bias when the weight function is performed using the GalSim weight function size to galaxy size (CWW-Sbc, CWW-Scd) successful.

Figure 6. Multiplicative shear bias due to CG, \( m_{\text{CG}} \), as a function of redshift for a reference galaxy with CWW-E bulge SED and three different disk SEDs: CWW-Im, CWW-Sbc, CWW-Scd (left to the right). The star denotes the CG bias for the galaxy with CWW-Im disk SED at redshift of 0.3.

![Figure 6](image)

Figure 7. Multiplicative shear bias from CG, \( m_{\text{CG}} \), as a function of the ratio of weight function size to galaxy size (red dots) with shape measurements performed using the GalSim implementation of the KSB shape estimation algorithm. We show the value of the CG bias when the weight function is computed with adaptive moments (blue star) and when the weight function HLR is equal to the galaxy HLR (yellow circle).

![Figure 7](image)

with no CG. While the CG bias estimates shown in this section are for noise-free CATSIM simulations, we also study the impact of Poisson noise in the image on our CG bias estimates, as explained in detail in Section 5.2 and Appendix A.2. As described earlier in Section 3.5, GalSim shape estimation methods have a high failure rate for small noisy galaxy images. We exclude such galaxies from our final sample if the shape measurement failed for any of the six galaxy shape measurements in the ring test of the galaxy with CG or its equivalent galaxy with no CG. Our final results are based on 45,534 CATSIM galaxies where the shape measurement was always successful.

In Figure 9 we show the distribution of the intrinsic parameters of the selected galaxies. The top-left panel shows a histogram of the galaxy AB magnitudes in the \( r \) and \( i \) bands. With the criteria being applied to select galaxies in the “Gold Sample” \((i < 25.3)\) (LSST Science Collaboration 2009), the sample includes galaxies with \( r \)-band magnitudes up to 26.5. The top-right plot shows the distribution of galaxy redshifts. The bottom-left panel shows the distribution of bulge and disk sizes, with a mean HLR of \(0''/2\) and \(0''/4\), respectively. A large fraction of the selected galaxies are small, with a large number of galaxies with HLR comparable to an LSST pixel \((0''/2)\). Most galaxy SBPs are composed of a small compact bulge and an extended disk. The CATSIM galaxy bulges have a steeper profile in the center when compared to the reference galaxy, while the disks are smaller on average (see Table 3). The distribution of the difference between the disk and bulge magnitudes is shown in the bottom-right panel. Most of the galaxy flux is contained in the disk, with the disk being on average 2.5 mag brighter than the bulge.

The galaxies selected here are not completely representative of the galaxies in the LSST lensing sample. However, the criteria are conservative in that they are not expected to reduce the estimated CG bias on LSST shear estimates. While the criteria could potentially introduce selection bias, the focus of this study is to isolate the impact of only CGs.

4.4.2. Results of CG Shear Bias Measurements for CATSIM Galaxies

For each galaxy in the CATSIM catalog we draw an equivalent galaxy with no CG and measure the shear response when a shear of \( g = (0.01,0.01) \) is applied to both galaxies. A histogram of the estimated CG bias \( m_{\text{CG}} \) in the \( r \) and \( i \) bands is shown in Figure 10, with their means shown as vertical-dashed lines. While the means of the distributions are consistent with zero, there exist galaxies in the tails of the distribution with large bias. Approximately 5% of the galaxies have \( |m_{\text{CG}}| \geq 10^{-3} \). For these galaxies, the bulge and disk have similar brightness, but different colors. The bulge-to-disk flux ratio of the entire sample is \( \approx 0.1 \), compared to \( \approx 0.4 \) for the galaxies in the tails and 0.33 for the extreme-CG galaxies described in Section 4.3.1. The median difference in \( V-I \) color of bulge and disk components is \( \approx 0.4 \) for the entire sample, compared to \( \approx 0.7 \) for the galaxies in the tails. The mean and median multiplicative CG biases are \((2.69 \pm 0.03) \times 10^{-5} \) and \(6.17 \times 10^{-5} \) in the \( r \) band, and \((3.72 \pm 0.01) \times 10^{-5} \) and \(2.58 \times 10^{-5} \) in the \( i \) band—in each case, much smaller than LSST requirements.

5. Estimating CG Bias with Chromatic Real Galaxies

To place more robust limits on the expected CG shear bias for LSST we estimate CG bias for a large sample of realistic galaxies with CGs. We use galaxy images from the HST to
obtain realistic galaxy morphologies and then draw them as seen by LSST using the CRG module in GalSim.

In a separate note,13 we describe the data reduction process used to produce postage-stamp images of 27k galaxies observed by the AEGIS (Davis et al. 2007) in the HST V (F606W) and I (F814W) bands. Our goal is to use the V- and I-band images of galaxies in the AEGIS catalog to create their profiles in r- and i-LSST bands, while preserving their chromatic features. We can thus estimate the CG bias when these galaxies are imaged by LSST.

5.1. CRG Model

CRG14 models multiband images of individual galaxies as chromatic PSF convolutions (and integrations over wavelength) with a sum of profiles separable into spatial and spectral components. This decomposition can be thought of as a constrained chromatic deconvolution of the multiband images by the associated PSFs.

The pre-convolution chromatic surface brightness profile of the galaxy, \( f_{\text{CG}}(x, \lambda) \), is modeled as a sum of two or more separable chromatic surface brightness profiles, each with a particular asserted SED:

\[
f_{\text{CG}}(x, \lambda) = \sum_j a_j(x) S_j(\lambda),
\]

where \( S_j(\lambda) \) is the jth SED asserted as part of the decomposition, and \( a_j(x) \) is the spatial component of the jth separable chromatic profile.

The observed image in the ith band is

\[
I_i^{\text{obs}}(x) = \int T_i(\lambda)[\Pi(x, \lambda) * f(x, \lambda)] d\lambda + \eta_i(x) = \int T_i(\lambda)S_j(\lambda)[\Pi(x, \lambda) * a_j(x)] d\lambda + \eta_i(x),
\]

where \( \eta_i(x) \) corresponds to (potentially spatially correlated) Gaussian noise in the ith band image. The noise \( \eta_i(x) \) is related to the noise covariance function via \( \langle \eta_i(x) \eta_k(x_m) \rangle = \xi_i(x_i - x_m) \), where angle brackets indicate averaging over realizations of the noise.

The convolution is easier to work with in Fourier space where it becomes a mode-by-mode product. The model in Fourier space is

\[
I_i^{\text{obs}}(k) = \int T_i(\lambda) \sum_j S_j(\lambda) \tilde{\Pi}(k, \lambda) \tilde{a}_j(k) d\lambda + \tilde{\eta}(k) = \sum_j \tilde{\Pi}_j^{\text{eff}}(k) \tilde{a}_j(k) + \tilde{\eta}(k),
\]

where the effective PSF for the ith band and jth SED is

\[
\tilde{\Pi}_j^{\text{eff}}(k) = \int T_i(\lambda) S_j(\lambda) \tilde{\Pi}(k, \lambda) d\lambda.
\]

CRG solves for the (complex-valued) \( \tilde{a}_j(k) \) while propagating the statistics of the noise.

5.2. Limitations of Using Real Galaxy Images

We use CRG to model real HST galaxies in an LSST observing band and then convolve the images with the expected LSST PSF and add the sky noise expected at 10 yr LSST depth. We assume here that the observing conditions for HST and LSST are known perfectly. For real images where the SED of the galaxies are unknown, we approximate the SEDs of the CRG components as polynomials. Thus, the accuracy with which the CRG algorithm reproduces chromatic features depends on how different the bandpasses of input and target surveys are, and the assumed model of the position-dependent SEDs. Since we use only two images of the HST galaxy (V and I bands), the SEDs are assumed to be polynomials of order 0 and 1 (linear SED).

The simulations of parametric galaxies analyzed in Section 4 are noise-free. Bias from CG is computed as the difference in the shear estimator measured from a galaxy with CG and the equivalent galaxy with no CG. The assumption was that biases from all sources other then CGs would act equally on both measurements and thus cancel in the difference, giving the bias from CG only. However, each real galaxy image includes noise in the signal in each pixel15 (pixel noise) while the image of the equivalent galaxy with no CG has different noise in each pixel. This results in unequal noise bias in the shear measurements for the two galaxies. Therefore, the difference in shear measurements

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13 http://github.com/sowmyakth/measure_cg_bias/raw/master/pdfs/Reducing_AEGIS_gal.pdf

14 http://github.com/GalSim-developers/GalSim/blob/master/devel/modules/CGNotes.pdf

15 We assume that fluctuations in pixel values are dominated by Poisson fluctuations in a large number of detected electrons so that we can neglect the relatively small noise contributions expected from dark current and readout noise.
no longer isolates bias from only CG; rather it is a combination of CG bias and residual noise bias. Therefore, before using CRG to model real HST galaxies, we test CRG on simulated parametric galaxies where the truth is known. The detailed procedure used to test the impacts of the linear approximation for the SED and pixel noise, while using CRG to model the reference galaxy and the CATSIM galaxies is described in the Appendix. We summarize the results here.

1. For reference galaxy with extreme CGs the redshift-averaged error in CG shear bias caused by modeling the galaxy SEDs as linear is $O(10^{-4})$; however, the linear SED model tends to smooth the variation in bias with redshift, leading to a total variation in bias over the redshift range $[0,1.2]$ that is a factor of $\approx 2$ less for the linear SED compared to the true SED.

2. For simulated HST-like CATSIM galaxies (modeled by CRG), the value of the CG bias estimate exhibits a strong dependence on the S/N. The measured bias diverges from the noise-free CG bias estimate by $O(10^{-3})$ for the HST $I$-band $S/N \leq 100$.

3. For simulated HST-like CATSIM galaxies with an $I$-band $S/N > 200$, the mean bias is $m_{CG} = (-2.53 \pm 0.27) \times 10^{-4}$ in the $r$ band and $m_{CG} = (0.47 \pm 0.10) \times 10^{-4}$ in

\begin{figure}[h]
\centering
\includegraphics[width=\linewidth]{figure9.pdf}
\caption{Distributions of galaxy parameters for selected CATSIM galaxies. Top left: $i$- and $r$-band AB magnitudes. Top right: galaxy redshifts. Bottom left: HLR (in arcseconds) of the bulge and disk components. Bottom right: difference between disk and bulge magnitudes.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\linewidth]{figure10.pdf}
\caption{Histograms of CG multiplicative shear bias in the $r$ and $i$ bands for CATSIM galaxies. The almost completely overlapping vertical-dashed lines denote the mean in each band.}
\end{figure}
the $i$ band. The mean CG bias for the noise-free parametric simulations of the same sample of galaxies is $m_{CG} = (-0.02 \pm 0.03) \times 10^{-4}$ in the $r$ band and $m_{CG} = (0.25 \pm 0.02) \times 10^{-4}$ in the $i$ band.

Thus, by using noisy images of CATSIM galaxies, we find that our method of using an “equivalent galaxy” to minimize the impact of effects other than CGs does not completely eliminate the impact of “pixel noise” for smaller values of S/N. Therefore, we expect the results from (noisy) AEGIS galaxies to be impacted by residual noise bias for small S/N. For high S/N galaxies in CATSIM, the impact of pixel noise is statistically significant; however, this residual noise bias is smaller than the requirement on multiplicative bias by at least a factor of a few in the $r$ band, and by an order of magnitude in the $i$ band. Therefore, we conclude that, for high S/N AEGIS galaxies, our method will be sensitive to CG biases that are significant compared to the requirements.

5.3. Estimating CG Bias with AEGIS Galaxies

Having demonstrated that CRG is able to reproduce CG bias results for galaxies with high S/N, we now apply our analysis to real galaxies with CGs. A flowchart depicting the methodology to estimate CG bias from real HST galaxies in the AEGIS survey is shown in Figure 11. The $V$- and $I$-band images of galaxies that pass certain selection criteria are input to CRG to model the galaxy SBP. Along with the PSF-convolved images, CRG also requires as input the noise correlation functions and PSF images in the $V$ and $I$ bands. The modeled chromatic SBP of each galaxy is then used to estimate CG bias when seen by LSST as described in Section 2.1.

5.3.1. Creating the AEGIS Catalog

The AEGIS imaging data is composed of a mosaic pattern of $21 \times 3 = 63$ contiguous HST “tiles” covering an effective area of $10/1 \times 70/5 = 710.9$ arcmin$^2$, with a four pointing dither pattern for each tile (Davis et al. 2007). The dithered observations for each pointing and band were combined (“drizzled”) using MultiDrizzle to produce coadded images with a pixel scale of 0’03 (Rhodes et al. 2007).

The procedure used to create the catalog, based on Emi (2018), is similar to that used for the weak lensing catalog for the HST ACS COSMOS survey (Leauthaud et al. 2007). We use version 2.8.6 of the SExtractor (Bertin & Arnouts 1996) package to produce a source catalog of positions and various photometric parameters. Detection was performed on a coaddition image of both bands, and photometric measurements then performed on each band at the previously detected locations. Unreliable regions such as tile boundaries, diffraction spikes, and “ghosts” due to internal reflections are masked. CRG modeling requires postage-stamp images of individual galaxies. Thus, if flux from a neighboring object lies within a stamp, it is replaced with noise. If the overlap with the neighbor is too significant, then the galaxy is not selected for analysis.

The final catalog consists of 26,517 galaxies that satisfy the following criteria:

1. The object was detected in both the $V$ and $I$ bands.
2. The object is not classified as a star in either band.
3. The object is not in a masked region in either band.
4. The object has magnitude brighter than 25.2 in the $I$ band.
5. The postage stamp does not contain flux from a neighboring object (no significant overlap).

For 24,635 of these galaxies, shapes were successfully measured in the ring test for the galaxy and for its equivalent with no CG.

We model the PSF by comparing stars in a given HST tile to star fields drawn from Tiny Tim ray-tracing software (Krist 1993). The variations in the size and ellipticity of the PSF across the focal plane of the HST ACS is dominated by the effective focus, which changes due to thermal expansion and contraction of the HST. Tiny Tim simulates the variation of the PSF across the field for different focal lengths. The PSF variation across an image is characterized by comparing the measured PSF ellipticity for stars in a field to the Tiny Tim predictions for different focus offsets, and finding the focus that best matches the measured and predicted PSFs across the field (Leauthaud et al. 2007). The PSF for each galaxy is taken to be the Tiny Tim model PSF image whose location in the Tiny Tim grid is closest to that galaxy’s position in the focal plane, for the best-fit focus offset.

The correlated noise in the two bands is estimated from empty regions in the AEGIS fields. A detailed description of how individual galaxy images, their corresponding PSFs and the noise correlation functions are determined can be found in the note described in Section 5. We also include in our final catalog the spectroscopic redshifts for 3763 matched galaxies from the DEEP2 galaxy redshift survey, Data Release 4 (Newman et al. 2013).

5.3.2. Characteristics of AEGIS Catalog

Characteristics of the selected AEGIS galaxies are shown in Figure 12. The left panel shows the distribution of magnitudes in the $V$ and $I$ bands. The selection criteria ensure that all galaxies have $I < 25.2$. The center panel shows the distribution of galaxy HLR in arcseconds: 32% of the galaxies have HLR less than or equal to the LSST pixel scale of 0’/2. The magnitude and HLR were estimated with SExtractor. The

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16 https://www.astromatic.net/software/sExtractor
right panel shows the redshift distribution for the 14% of the galaxies for which spectroscopic redshift estimates are available in the DEEP2 Galaxy Redshift Survey.

Since CG bias estimates show a large dependence on galaxy S/N, we show the distribution of galaxy S/N in the HST V and I bands in Figure 13 (top left panel); the mean S/Ns are 76 and 88 in the V and I bands, respectively. The top-right panel in Figure 13 shows the correlation of I-band S/N with magnitude. A selection criterion of HST I-band S/N > 200 excludes galaxies with I-band magnitude fainter than 23.2. In the bottom panels, we plot the S/N in the I band as a function of V–I color (left), and redshift (right). The selection cut of the I-band S/N > 200 eliminates 97% of galaxies with z > 1.2 from the sample.

5.3.3. Results of CG Bias Analysis of AEGIS Galaxies

The results of the CG bias analysis of AEGIS galaxies when observed in the LSST r band are shown in Figure 14. The plot on the left shows the distribution of estimated bias. The mean multiplicative shear bias is \( m_{\text{CG}} = (-3.3 \pm 1.6) \times 10^{-3} \) for all galaxies in the sample. However, as shown in the center panel, the large spread in the estimated bias is due to the low-S/N galaxies. In the right panel, we show the mean value of \( m_{\text{CG}} \) for galaxies with an I-band S/N > 200. The S/N and color dependence is depicted for the 1900 galaxies with an I-band S/N > 200, while the redshift dependence is plotted for 1043 of these galaxies for which spectroscopic redshift estimates are available. The dots denote the mean value for each bin, while the error bars denote the statistical uncertainty on the mean. The CG bias values do not show a statistically significant dependence on galaxy color. The bias values show a small dependence on the S/N; however, the magnitudes of the mean biases are lower than the LSST requirement.

Similar to the results for CatSIM galaxies in Section 4.4, the CG biases computed here for the AEGIS galaxies could be affected by selection effects leading to a sample that is not representative of the LSST sample that would be selected for a shear analysis. Overcoming this limitation will require further studies of galaxy CG.

We compare CG bias estimates for the AEGIS galaxies when observed in the LSST i and r bands in Figure 16. We show bias estimates for 24,635 galaxies from the AEGIS sample for which bias estimates were successfully computed in both bands. The left panel shows the distribution of bias estimates. The i-band CG bias estimate distribution has smaller tails than the r band, with a mean \( m_{\text{CG}} = (1.7 \pm 0.5) \times 10^{-3} \) for the entire sample. The right panel plots the mean CG bias for galaxies above a minimum I-band S/N. The mean bias quickly converges to zero, once the low-S/N galaxies are excluded from the sample. For galaxies with an I-band S/N > 200, the mean multiplicative shear bias in the i band is \( m_{\text{CG}} = (7.8 \pm 3.6) \times 10^{-5} \).
6. Limitations of this Study

We summarize here the limitations of this analysis and how it could be extended to future studies.

1. Our method for isolating CG bias breaks down when galaxy SBPs are modeled from noisy images. This limits our analysis of real images to high-S/N galaxies. Due to this limitation, we are only able to measure CG bias for real galaxies with redshift \( z < 1 \). Estimating CG bias for higher redshift galaxies will require a statistically significant sample of high-S/N real images or high fidelity simulations of noise-free galaxies with realistic morphologies.

2. Our methodology assumes a linear response to the applied shear, which is true for weak lensing measurements. However, for lensing by galaxy clusters and in the strong lensing regime when this approximation no longer holds, CG bias could result in non-negligible biases in shear estimates.

3. We studied one source of PSF chromaticity: wavelength-dependent PSF size. However, several chromatic effects, including DCR, refraction in optics, and sensor effects, can also produce a wavelength-dependent PSF ellipticity.

4. The fidelity of CG models in the samples of galaxies used in this study is limited. In CRG, the SEDs for the AEGIS galaxies, as would be seen by LSST, are modeled as simple linear functions of wavelength because only two \( HST \) bands are available. The parametric galaxies in the CATSIM sample are described as simple Sérsic bulge + disk profiles. As our knowledge of CGs as a function of redshift improves, future studies could use more realistic CG models, including active galactic nuclei, star-forming regions, dust lanes, etc.

7. Conclusions

We extend the method employed in Semboloni et al. (2013) for \( Euclid \)-type observations to isolate the effect of CGs on weak lensing shear measurements for \( HST \) observations in the \( r \) and \( i \) bands. The bias originates when shapes of galaxies with CGs, observed with a chromatic PSF, are measured with some form of spatial weighting. The bias depends only weakly on the size of the PSF, but depends linearly on the magnitude of the exponent \( \alpha \) in its wavelength dependence \( \lambda^{\alpha} \) (Figure 8). The bias is very sensitive to the size of the weight function (Figure 7). We estimate CG shear bias for three samples as observed with LSST: reference parametric galaxies with large CGs, an ensemble of parametric galaxies with more realistic CGs and galaxies with realistic morphology. The results are summarized in Table 4.

For noise-free parametric galaxy simulations, the value of half the maximum span of \( m_{CG}(z) \) in the redshift range \([0, 1.2]\)
is $\lesssim 1.5 \times 10^{-3}$ for the reference galaxies with extreme CGs and $\lesssim 10^{-3}$ for CATSIM galaxies. For input AEGIS galaxies with pixel noise, the estimated bias shows a strong dependence on the S/N due to contributions from effects other than CG. However, for AEGIS galaxies with an $HST$ $i$-band S/N $> 200$, the magnitude of the mean estimated bias in the LSST $r$ band is $(0.4 \pm 2.2) \times 10^{-4}$, while the value of half the maximum span of $m_{CG}(z)$ in the redshift range $[0, 1.2]$ is $\lesssim 1.5 \times 10^{-4}$. Therefore, for both the noise-free parametric galaxies and for the AEGIS galaxies with an $i$-band S/N $> 200$, the half-maximum span is less than the LSST full-depth requirement of 0.003 on the total systematic uncertainty in the redshift-dependent shear calibration $m_z$ (Equation (9)). This result is important because shear estimation depends on accurate knowledge of the PSF—even for shear estimation techniques that self-calibrate responsivity to shear (e.g., Metacal, Huff & Mandelbaum 2017)—and the effects of CGs on the per-galaxy PSF could be very challenging to predict and correct. We are thus optimistic that CGs will not be a source of limiting systematic uncertainty for LSST; however, the limitations listed in Section 6 should be investigated as more observations and studies of CGs in galaxies become available.

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J.E.M. and P.R.B. provided motivation for the study and guidance throughout the analysis. S.K. produced the simulated images, prepared the AEGIS-based images, and performed the analysis. J.E.M. wrote the CRG module in GalSim, to simulate LSST galaxies from $HST$ images, and offered technical advice. All authors discussed interpretation of the results. S.K. wrote the initial draft of the manuscript; P.R.B., S.K., and J.E.M. all contributed to the final draft.

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### Table 4

| Sample                                | Half-maximum Span of $m_{CG}(z)$ in the $r$ Band | Mean $m_{CG}$ in the $r$ Band | Mean $m_{CG}$ in the $i$ Band |
|----------------------------------------|-----------------------------------------------|--------------------------------|--------------------------------|
| Reference galaxy with extreme CGs (parametric)$^b$ | $1.5 \times 10^{-3}$ (see Figure 6)          | $(8.7 \pm 0.6) \times 10^{-4}$ | ...                           |
| CATSIM galaxies (parametric)$^b$        | $\lesssim 10^{-4}$ (see Figure 23)            | $(0.268 \pm 0.003) \times 10^{-4}$ | $(0.372 \pm 0.001) \times 10^{-4}$ |
| AEGIS galaxies (CRG)$^c$               | $\lesssim 1.5 \times 10^{-4}$ (see Figure 15) | $(0.4 \pm 2.2) \times 10^{-4}$  | $(0.8 \pm 0.4) \times 10^{-4}$  |

Notes.

$^a$ Half-maximum span of $m_{CG}$ over redshift range $[0, 1.2]$; see Section 3.2 for motivation for requirement of 0.003 on this value.

$^b$ Values computed for noise-free parametric simulations.

$^c$ Values computed for CRG generated from images with an $HST$ $i$-band S/N $> 200$.

Figure 16. Left: histogram of multiplicative shear bias from CG, $m_{CG}$, in the $i$ band (red) and $r$ band (blue) for AEGIS galaxies. Right: mean $m_{CG}$ in the $i$ band (red) and $r$ band (blue) for galaxies with an $i$-band S/N $> \text{min}(S/N)$. The yellow-dashed lines show the LSST DESC requirement of 0.003 on the total systematic uncertainty in the redshift-dependent shear calibration.
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Software: GalSim (Rowe et al. 2015), SExtractor (Bertin & Arnouts 1996), Astropy (Astropy Collaboration et al. 2013, 2018), NumPy (van der Walt et al. 2011), matplotlib (Hunter 2007).

Appendix

Testing CRG

In this section we describe the tests conducted to estimate the accuracy of the CRG algorithm for reproducing chromatic features from noisy images. We simulate parametric galaxies as they would be observed by HST (henceforth called “HST-like” galaxies)—i.e., convolved with an HST-like chromatic PSF and observed in the HST V and I bands with correlated pixel noise. CRG is then used to model the SBP of the HST-like galaxies in order to estimate the CG bias when observed by LSST. We then compare this estimated CG bias to the CG bias estimated in Section 4 from noise-free parametric galaxies simulated directly in LSST bands.

A.1. Testing CRG with the Reference Galaxy with Extreme CGs

We first test the ability of CRG to model the reference galaxy with extreme CGs from V and I band HST-like images of the galaxy. In particular, we investigate two potential limitations:

1. Impact of imperfect SEDs: for real images where the SEDs of the galaxies are unknown, we approximate the SEDs as linear. We test the impact of these approximate SEDs by comparing the estimated CG bias when the CRG is modeled with linear SEDs and with true SEDs.

2. Impact of pixel noise: the simulations analyzed in Section 4 were noise-free. However, real galaxy images include pixel noise. We test the impact of this noise by comparing CG bias estimates from noisy and noise-free HST-like images.

A flowchart describing the CRG test with the reference galaxy with extreme CGs is shown in Figure 17. The red-dashed box A shows the method used in Section 4 to estimate CG bias of the reference parametric galaxy when seen by LSST. Boxes 1 and 2 show the CRG tests to study the impact of imperfect SEDs and pixel noise, respectively.

For Box 1, the HST-like images of the reference galaxy in V and I bands are input to CRG along with the effective HST PSF images. Two chromatic SBP of the galaxy are produced: one for CRG modeled with the true bulge and disk SEDs and the other with linear SEDs. Each of the modeled CRG profiles are then used to estimate CG bias similar to Box A.

We study the impact of noise in the input HST images (Box 2) by drawing the HST-like reference galaxy images with pixel noise in the V and I bands. These noisy images are then input to CRG to model the chromatic SBP with linear SEDs. The chromatic SBPs are then used to estimate CG shear bias. Because the analysis would likely depend on the galaxy S/N, we perform the study for varying noise levels for the input HST-like images.

A.1.1. Parameters of HST-like Images

The bulge and disk SBPs have the same parameters as described in Section 4.3.1. The HST and LSST parameters used in the simulations are summarized in Table 5. The HST-like images are drawn with a pixel scale of 0.03 and the LSST images with pixel scale 0.2. The chromatic PSFs are assumed to have a Gaussian profile with a wavelength-dependent size as described in Equation (1). The chromatic HST PSF is assumed to be diffraction-limited with wavelength-dependent size scaling exponent $\alpha = 1.0$. The PSF size at a reference wavelength of $\lambda = 806$ nm is set equal to $\sigma = 0.031$, which is the mean value from fits to real I-band HST PSF images. The HST noise is modeled using the same noise correlation function described in Section 5.3.2.

A.1.2. Results of CRG Tests with a Reference Galaxy with Extreme CGs

In Figure 18 we show the estimated multiplicative shear bias due to CG, $m_{CG}$, as a function of redshift for the reference galaxy with the three different disk SEDs (CWW-Im, CWW-Sbc, CWW-Scd) and a common CWW-E bulge SED. The estimated bias $m_{CG}$ from parametric simulations is shown as the solid blue line, identical to Figure 6. We show the bias values measured by modeling the reference galaxy using CRG with the true SED (orange squares) and with a linear SED (teal dots). As expected, the biases estimated for the parametric simulation are in excellent agreement with the CRG-modeled SBP with true SEDs. However, the predicted biases for CRG with linear SEDs differ from these values, revealing the limitations in using zero and first-degree polynomials to reproduce the nonlinear features in the bulge and disk SEDs. In particular, CRG with a linear SED tends to smooth the variation in bias with redshift. The difference between the redshift-averaged CG bias estimated with the linear SED and with the true SED is $O(10^{-4})$, and the estimated biases satisfy the LSST DESC requirement of 0.003 on the total systematic uncertainty in the redshift-dependent shear calibration (yellow-dashed lines) for both the linear and true SED models, for the three disk SEDs. However, the total variation in CG bias over the redshift range [0.1, 2] is a factor of 2.6, 1.9, and 2.5 less when calculated with the linear SED rather than the true SED, for the CWW-Im, CWW-Sbc, and CWW-Scd disk SEDs, respectively.

To assess the impact of noise we simulate HST-like images for the reference galaxy SBP with E-type bulge and Im-type disk SED, at a redshift of 0.3, but with different noise levels. As illustrated in Box 3 of Figure 17, we add HST-like noise to the simulated galaxy images before they are modeled with CRG. Because our noise-free analysis already showed that the predicted biases for CRG with true SEDs are identical to those for the parametric simulations, we perform the noise study for CRG with linear SEDs only. Because our ultimate aim is to model AEGIS galaxies with CRG, we add noise levels similar to those in the AEGIS images.

The noisy reference galaxy is simulated 10,000 times, each with a different correlated noise realization to obtain an S/N level drawn randomly from the AEGIS catalog. The sampled I-band S/N values are shown in the left panel of Figure 19. The CG bias is then estimated using the CRG model of each of the noisy images.

The measured biases are binned by the S/N in the right panel of Figure 19. The solid dots denote the mean binned values of the
measured CG bias and the error bars correspond to the statistical uncertainty on the means. The estimated CG bias shows a dependence on the S/N even though all points correspond to the same galaxy SBP and thus have the same intrinsic CG. The dashed line at $+0.00124$ corresponds to the estimated CG bias for the reference galaxy at a redshift of 0.3 drawn with no noise and flowchart describing the method for testing the impact of (1) the linear approximation for the SED and (2) pixel noise, on the CRG model using HST-like images of the reference parametric galaxy with extreme CGs. The CG bias estimated from 1 and 2 are then compared to the estimates from noise-free parametric simulations in A. See Appendix A.1 in text for details.

Figure 18. Multiplicative shear bias due to CG, $m_{CG}$, as a function of redshift for a reference galaxy with CWW-E bulge spectrum and three different disk SEDs (from left to right: CWW-Im, CWW-Sbc, CWW-Scd), for three types of noise-free simulations: parametric (solid blue line), CRG with the true SED (orange squares), and CRG with linear SEDs (teal dots). The yellow-dashed line shows the LSST DESC requirement of 0.003 on the total systematic uncertainty in the redshift-dependent shear calibration.

| Survey | Observing Bands | $\alpha$ | PSF $\sigma^{\prime\prime}$ (arcsec) | PSF $\lambda^{\prime\prime}$ (nm) | Pixel Size (arcsec) | Collecting Area ($m^2$) |
|--------|-----------------|--------|-------------------------------|-----------------|-----------------|-----------------|
| HST    | V/I             | +1.0  | 0.071                         | 806             | 0.03            | 4.44            |
| LSST   | r/i             | −0.2  | 0.297                         | 550             | 0.2             | 32.4            |

Note. The PSF is modeled as a circular Gaussian with size $\sigma^{\prime\prime}$ at reference wavelength $\lambda^{\prime\prime}$. The wavelength-dependent PSF size is determined by the scaling exponent $\alpha$ (see Equation (1)).

Table 5. HST and LSST Parameters
modeled by CRG with linear SEDs. At high S/N, the mean value of the $m_{\text{CG}}$ estimate approaches this noise-free CG bias. However, at low S/N the estimate diverges from the noise-free CG bias indicating that our methodology to isolate CG bias is not robust against noise bias. Hence we are limited to estimating CG bias for real galaxies with an HST I-band S/N greater than 200.

A.2. Testing CRG with CATSIM Galaxies

We extend the CRG tests in the presence of noise to an ensemble of galaxies by repeating our analysis in Box 2 of Figure 17 to galaxies in the CATSIM catalog with a range of bulge and disk parameters. Because we concluded above that our methodology is applicable to only high-S/N AEGIS galaxies, we compute the S/N of the HST-like galaxy as well as the S/N at 10 yr LSST depth.

A flowchart describing the procedure is shown in Figure 20. The center side of the flowchart illustrates the method used in Section 4.4.2 to compute CG bias for the parametric CATSIM galaxies as measured by LSST and does not involve modeling with CRG. We also draw the galaxies in the $r$ band with sky noise corresponding to the 10 yr LSST depth and compute their S/Ns.

The left side of the flowchart (in green) illustrates how we test CRG with HST-like CATSIM galaxy images as input. The $V$- and $I$-band galaxy images are input to CRG, along with the HST effective PSF image and the correlation function for pixel noise, to model a chromatic SBP. CG bias is then estimated for these galaxies when observed in the LSST $r$ and $i$ bands. For each noisy HST-like CATSIM galaxy image, the $V$- and $I$-band S/Ns are also computed.

A.2.1. Estimating the S/N of CATSIM Galaxies

The results of the S/N calculations for the galaxies are shown in Figure 21. The top panels show the computed S/N for the CATSIM galaxies when seen by the HST $V$ and $I$ bands.
Figure 21. HST and LSST S/N distributions for CATSIM galaxies. Histograms of S/N for HST V and I bands (top left) and LSST r and i bands (top center). Calculated LSST i-band S/N versus i-band magnitude (top right), and HST I-band S/N versus LSST i-band S/N (bottom left). HST I-band S/N versus redshift (bottom center) and LSST r − i color (bottom right).

(left) and LSST r and i bands (center). 53% of the galaxies have an I-band S/N less than 200. We plot the measured LSST i-band S/N against the true i-band CATSIM catalog magnitude in the top right panel; the lower magnitude (i.e., brighter) galaxies have a higher S/N as expected. In the bottom-left plot, we compare HST I-band S/N and LSST i-band S/N. We do not expect the S/N in the two bands to be the same because the bandpasses, exposure times, collecting areas, and noise levels are different (see Table 5). However, because we have added a constant noise to all the HST and LSST simulations individually, we do expect the two S/Ns to be linearly related. We observe that LSST i-band S/N is ≈0.7× HST I-band S/N. A selection criterion of I-band S/N > 200 for the AEGIS galaxies effectively eliminates all galaxies with LSST i-band S/N less than ≈80.

In the final two panels, we show the correlation between I-band S/N and redshift (center) and r − i color (right). The magnitudes, redshifts, and colors are from the CATSIM catalog and are noise-free. High redshift galaxies have, on average, smaller S/Ns, which is expected because the higher redshift galaxies tend to have lower observed flux. As a consequence, our CG bias estimates from AEGIS galaxies are not as reliable for higher redshift galaxies. There appears to be a weak correlation between S/N and color.

A.2.2. CG Shear Bias Estimates from Noisy CATSIM Galaxies with CRG

Figure 22 plots the results from the CG bias estimates for all the noisy HST-like CATSIM galaxies modeled by CRG and observed in the LSST r band (top) and i band (bottom). For reference we also show the estimates from noise-free parametric simulations (blue). The panels on the left show histograms of the estimated CG bias from noisy HST-like CATSIM images modeled by CRG with true SEDs (orange) and linear SEDs (teal). While the noise-free parametric galaxies exhibit small biases, the estimates from noisy CRG-modeled images show large tails due to contributions from pixel noise. This is more evident in the right panels where we plot the mean CG bias estimates for galaxies with S/N higher than min(S/N) in the HST I band band. Noise impacts the CG bias estimates for both the linear and true SED models at low S/N. With no S/N cut on the HST-like images, the mean CG bias estimates from CRG galaxies modeled with linear SEDs in the r band is $m_{CG} = (-1.7 \pm 0.5) \times 10^{-3}$, while the median of the distribution is $-4.1 \times 10^{-6}$ with a median absolute deviation of $3.3 \times 10^{-3}$. The mean and median bias estimates in the i band are $3.2 \times 10^{-4}$ and $4.1 \times 10^{-6}$, respectively.

The pixel noise causes the bias estimated from HST-like galaxies modeled by CRG with true SEDs to be more negative in both bands, while the impact of imperfect SEDs causes the bias to be more negative in the r band and positive in the i band. The impact of noise bias on the CG bias estimates is less pronounced in the i band. This is because the galaxy images have higher S/N in the i band than in the r band as shown in Figure 21. As more low-S/N galaxies are included, the bias measured with the CRG method approaches the value determined with the parametric analysis. For galaxies with HST I-band S/N > 200, the mean CG bias is $m_{CG} = (-2.5 \pm 0.3) \times 10^{-4}$ in the r band and $m_{CG} = (0.47 \pm 0.10) \times 10^{-4}$ in the i band. The LSST requirement on redshift dependence of the shear bias from all systematic effects is shown as the yellow-dashed line.

Because the ability of CRG to model chromatic features and accurately estimate the magnitude of CG bias depends on the intrinsic properties of the galaxies, we investigate the dependence of the estimated CG bias on the galaxy color and redshift. In Figure 23, we show the dependence of estimated
CG bias on \( r - i \) color (left) and redshift (right). The top plots correspond to noise-free parametric simulations of CATSIM galaxies. The bottom plots illustrate the impact of applying three different minimum S/N cutoffs. The dots denote the difference in mean \( m_{\text{CG}} \) for noise-free simulations (top panel) and mean \( m_{\text{CG}} \) for \( HST \)-like galaxies modeled with CRG (\( \Delta m_{\text{CG}} \)), with an \( HST \) \( I \)-band S/N > 100, 200, and 1000. The difference in the mean bias decreases with increasing S/N cutoffs. Selection effects due to the S/N cutoffs are found to bias the binned means of CG bias estimates for noise-free parametric simulations by \( O(10^{-4}) \). Therefore, our method of using CRG to model galaxies observed by LSST from noisy \( HST \) images is able to reproduce CG bias with a sensitivity of \( O(10^{-4}) \) for galaxies with \( HST \) \( I \)-band S/N > 200.

Figure 22. Left panels: distribution of \( m_{\text{CG}} \) in the \( r \) band (top) and \( i \) band (bottom) for CATSIM galaxies. Right panels: mean \( m_{\text{CG}} \) in the \( r \) band (top) and \( i \) band (bottom) for galaxies with an \( I \)-band \( S/N > \min(S/N) \). Results from parametric simulations are shown for true SEDs in blue, for CRG with linear SEDs in teal, and CRG with true SED input in orange. The yellow-dashed lines show the LSST DESC requirement of 0.003 on the total systematic uncertainty in the redshift-dependent shear calibration.
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Figure 23. Multiplicative shear bias from CG, m_{CG}, as a function of r−i color (left) and redshift (right) for CATSIM galaxies. Top: m_{CG} estimates from noise-free parametric simulations. Bottom: difference in mean m_{CG} estimated from noise-free parametric simulations and HST-like galaxies modeled with CRG (Δm_{CG}) for three different HST I-band S/N cutoffs. The error bars correspond to the statistical uncertainties on the binned means.