All-electrical manipulation of electron spin by the spin-orbit interaction in a semiconductor nanotube: analytical results

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A possibility of controlled manipulation of electron spin states has been investigated for a cylindrical two-dimensional electron gas confined in a semiconductor nanotube/cylindrical nanowire with the Rashba spin-orbit interaction. We present analytical solutions for the two limiting cases, in which the spin-orbit interaction results from (A) the radial electric field and (B) the electric field applied along the axis $z$ of the nanotube. In case (A), the superposition of the two lowest-energy bands corresponding to the opposite spins leads to the precession of electron spin around the nanowire axis. We have found that the direction of the spin precession changes from clockwise to counterclockwise if the energy of the injected electron achieves the value corresponding to the crossing of energy levels associated with the two components of the superposition state. In case (B), we have obtained the damped oscillations of the $z$ spin component with the period that changes as a function of the coordinate $z$. We have also shown that the damped oscillations of the average value of the $z$ spin component form beats localized along the nanowire axis.

I. INTRODUCTION

The electron spin control induced by the electric field is a basic principle for the realization of spintronic devices, including the spin transistor\cite{1,2} and the quantum operations on spin qubits\cite{3,4}. For this reason the spin-orbit interaction\cite{5,6} (SOI), which couples the momentum of the electron with its spin, has attracted the substantial interest in recent years. In the spintronic and quantum computing applications, the Rashba SOI\cite{7} is very promising, since its strength can be controlled by the external electric field\cite{8,9} which opens up a new possibility of manipulating the electron spin state by the gate voltages at zero magnetic field\cite{10,11}. The spin manipulation due to the Rashba SOI has been recently demonstrated in the electric-dipole spin resonance (EDSR) experiments performed in the system of double quantum dots embedded in a gate InAs quantum wire\cite{12,13}. In the EDSR, the Pauli blockade of the current, which occurs when the quantum dots are occupied by the electrons with parallel spins, is lifted by the Rashba SOI generated by the oscillating gate voltage.

Recently, the special attention has been directed towards non-planar low dimensional structures, in which interesting physical effects resulting from the curvature of the surface have been found\cite{14,15}. Many research studies are focused on the cylindrical two-dimensional electron gas (2DEG). The nanostucture containing the cylindrical 2DEG can be fabricated by self-rolling of a thin strained semiconductor planar heterostructure grown by molecular-beam epitaxy\cite{16,17}. This method allows to obtain the free-standing semiconductor nanotubes with the radius that ranges from several nanometers up to several micrometers. The cylindrical 2DEG also appears in the core-shell nanowires, which are produced in the cylindrical substrate with a multilayer overgrowth\cite{18,19}. Another method of the fabrication of the cylindrical 2DEG exploits the electrical neutrality which leads to the formation of the triangular quantum well in the thin region near the surface of the semiconductor nanowire\cite{20}. The electrons confined in this quantum well form the cylindrical 2DEG with radius of few nanometers. The electronic properties of the cylindrical 2DEG are strongly dependent on the curvature, which manifests themselves especially in the presence of the magnetic field. The electron states have been recently calculated by Ferrari et al\cite{21} for the cylindrical 2DEG in the transverse magnetic field. In this system, the electrons are coupled to the magnetic field component perpendicular to the surface, that varies along the circumference of the cylinder. The gradient of magnetic field perpendicular to the surface causes that the electrons propagating in the opposite directions become localized in the opposite sides of the circumference\cite{22}, which leads to the experimentally observed Hall quantization\cite{17,19}.

The interplay between the curvature effects and the SOI in the cylindrical 2DEG has been studied in the recent papers\cite{23,24}. In Ref.\cite{24}, the authors investigated the dimensional dependence of weak localization corrections and spin relaxation in cylindrical nanowires with the Rashba SOI. The spin dependent electric current through the cylindrical nanowire containing a region with the spin-orbit coupling has been investigated by Entin-Wohlman et al\cite{25} who have shown that the tunneling through the region with the SOI causes that each step of the quantized conductance splits into two separate steps with the spin polarization perpendicular to the direction of the current. The spin precession in the cylindrical semiconductor nanowire due to the Rashba spin-orbit coupling has been investigated by Bringer and Schäpers\cite{26}. The numerical approach presented in Ref.\cite{27} takes into account the Rashba SOI generated by the radial electric field, which results from the inhomogeneous radial redistribution of charge near the surface of nanowire. The authors demonstrated that the linear combination of the two electron states with the same total angular momenta but opposite spins leads to the pe-
from the radial electric field with the spin-orbit interaction that originates the summary.

In the present paper we have studied the possibility of spin manipulation in the semiconductor nanotube with the Rashba SOI that results from the radial electric field and the electric field acting along the nanowire axis. For both cases we have obtained the analytical solutions and discussed them in the context of spin modulation. The present results can be applied to both the semiconductor nanotubes with the few atomic monolayer thickness and cylindrical nanowires with the 2DEG electron gas accumulated near the surface.

The paper is organized as follows: in Sec. II, we describe the theoretical model of the cylindrical 2DEG with the spin-orbit coupling, in Sec. III, we present the results, and the discussion is presented in Sec. IV and Sec. V contains the summary.

II. THEORETICAL MODEL

We consider one-electron states in the cylindrical 2DEG with the spin-orbit interaction that originates from the radial electric field \( F_r \) and homogeneous electric field \( F_z \) applied along the nanotube axis (Fig. 1).

The Hamiltonian of the system takes on the form (see Appendix A)

\[
\hat{H} = \left[ -\frac{\hbar^2}{2m} \left( \frac{1}{r_0^2} \frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2} \right) - eF_z z \right] \hat{I} + \hat{H}_{SO} , \tag{1}
\]

where \( \varphi \) and \( z \) are the cylindrical coordinates (see Fig. 1), \( e \) is the elementary charge, \( r_0 \) is the radius of the nanotube, \( m \) is the electron effective band mass, \( \hat{I} \) is the 2 \( \times \) 2 unit matrix, and \( \hat{H}_{SO} \) is the Hamiltonian of the spin-orbit interaction.

The Hamiltonian \( \hat{H}_{SO} \) of the spin-orbit interaction couples the spin \( \hat{s} \) of the electron with its linear momentum \( \hat{p} \) via the electric field \( \vec{F} = -\nabla V/e \) and can be expressed as (Appendix A)

\[
\hat{H}_{SO} = \frac{e\gamma}{\hbar} \hat{\sigma} \cdot (\vec{F} \times \vec{p})
\]

\[
= \left( \begin{array}{cc}
\hat{\sigma}_x & \hat{\sigma}_y \\
\hat{\sigma}_y & -\hat{\sigma}_x
\end{array} \right) \left( eF_z \frac{\partial}{\partial z} - e\alpha \frac{\partial}{\partial \varphi} \right) \left( eF_z \frac{\partial}{\partial z} + e\alpha \frac{\partial}{\partial \varphi} \right) \tag{2}
\]

where \( \hat{\sigma} \) is the vector of Pauli matrices, \( \hat{s} = (\hbar/2)\hat{\sigma} \) and \( \gamma \) is the coupling constant determined by the band structure of the semiconductor. In the present paper, we will also use the effective coupling constant \( \alpha = -\gamma F_r \) that takes into account both the band structure and the radial electric field effects.

The eigenstate of Hamiltonian (1) is the spinor of the form

\[
\Psi(\varphi, z) = \left( \begin{array}{c}
\psi^\uparrow(\varphi, z) \\
\psi^\downarrow(\varphi, z)
\end{array} \right) = e^{il\varphi} \left( \begin{array}{c}
f(z) \\
g(z)
\end{array} \right) , \tag{3}
\]

where \( l \) is the orbital quantum number. After inserting (3) into the eigenequation of Hamiltonian (1) we obtain

\[
-\frac{\hbar^2}{2m} \frac{d^2 f(z)}{dz^2} + \frac{\hbar^2 l^2}{2mr_0^2} f(z) - eF_2 f(z) + \frac{e\alpha l}{r_0} f(z)
\]

\[
+ eF \left( e\gamma(l + 1) \frac{g(z)}{r_0} + e\alpha \frac{dg(z)}{dz} \right) = Ef(z) , \tag{4a}
\]

\[
-\frac{\hbar^2}{2m} \frac{d^2 g(z)}{dz^2} + \frac{\hbar^2 (l + 1)^2}{2mr_0^2} g(z) - eF_2 g(z) - \frac{e\alpha (l + 1)}{r_0} g(z)
\]

\[
+ eF \left( e\gamma l \frac{f(z)}{r_0} - e\alpha \frac{df(z)}{dz} \right) = Eg(z) , \tag{4b}
\]

where energy \( E \) is measured with respect to the lowest energy of the size-quantized motion in the radial direction. In general, the system of equations (4a)(4b) is not solvable analytically. Nevertheless, we have found that in the two limiting cases, the analytical solutions exist. These are:

(A) Zero axial electric field \( (F_z = 0) \), then the SOI is due to the radial electric field \( F_r \).

(B) Zero radial electric field \( (F_r = 0) \), i.e., \( \alpha = 0 \), then the SOI is due to the electric field \( F_z \) applied along the nanotube axis.

III. RESULTS

In this section, we present the analytical solutions for the InAs nanotube/cylindrical nanowire, for which we take on the following values of the parameters: electron band effective mass \( m = 0.026m_0 \), where \( m_0 \) is the free electron mass, the radius of the nanotube \( r_0 = 50 \) nm, and the spin-orbit interaction constants \( \gamma = 1.17 \) nm\(^2\) and \( \alpha = 10 \) meVnm. We discuss the results obtained for two limiting cases (A) and (B). In both the cases, the analytical solutions are considered in context of a possible control of spin precession.
A. Effect of radial electric field

If the axial electric field is equal to zero \( F_z = 0 \) and the electric field has only the radial component, the solution of Eqs. (4a) and (4b) takes on the form

\[
\begin{pmatrix}
  f(z) \\
  g(z)
\end{pmatrix} = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} e^{i k z},
\]

where \( k \equiv k_z \) is the \( z \) component of the wave vector and \( u_0 \) and \( v_0 \) are the amplitudes of the spin-up and spin-down spinor elements.

If we introduce energy \( \varepsilon \) of the electron with the subtracted kinetic energy related to its motion along the \( z \) axis, i.e., \( \varepsilon = E - \hbar^2 k^2 / (2m) \), where \( E \) is the total energy, the system of equations (4a) and (4b) reduces to

\[
V_i u_0 + \frac{e\alpha l}{r_0} u_0 + i e k\alpha v_0 = \varepsilon u_0 ,
\]

(6a)

\[
V_{i+1} v_0 + \frac{e\alpha (l+1)}{r_0} v_0 - i e k\alpha u_0 = \varepsilon v_0 ,
\]

(6b)

where \( V_i = \hbar^2 l^2 / (2m r_0^2) \).

The system of equations (6a, 6b) has non-trivial solutions only if the following characteristic equation is satisfied

\[
\varepsilon^2 + \left[ \frac{e\alpha}{r_0} (V_i + V_{i+1}) \right] \varepsilon + V_i V_{i+1} - V_i \frac{e\alpha (l+1)}{r_0} + V_{i+1} \frac{e\alpha l}{r_0} - e^2 k^2 \alpha^2 = 0 .
\]

(7)

Eq. (7) possesses two solutions \( \varepsilon^\pm \) that can be obtained by the simple analytical calculation. The difference between them \( \Delta \varepsilon = \varepsilon^+ - \varepsilon^- \) corresponds to the spin-orbit energy splitting.

Using the normalization condition \( |u_0|^2 + |v_0|^2 = 1 \) we obtain the coefficients \( u_0^\pm \) and \( v_0^\pm \) that correspond to the energies \( \varepsilon^\pm \)

\[
|u_0^\pm|^2 = \frac{|e\alpha|^2}{|V_i + \frac{e\alpha l}{r_0} - \varepsilon^\pm|^2 + |e\alpha|^2} ,
\]

(8)

\[
|v_0^\pm|^2 = \frac{|e\alpha|^2}{|V_i + \frac{e\alpha l}{r_0} - \varepsilon^\pm|^2 + |e\alpha|^2} .
\]

(9)

In Fig. 2(a), we have plotted the energy \( \varepsilon^\pm \) as a function of wave vector \( k \) for several orbital quantum numbers \( l \). The energy band with orbital momentum \( hl \) and energy \( \varepsilon^\pm \) is denoted by \( l^\pm \). Fig. 2(a) shows that the spin-orbit splitting energy \( \Delta \varepsilon \) increases with the increasing kinetic energy of the electron. We see that for given orbital quantum number \( l \) the energy \( \varepsilon^+ \) increases with increasing wave vector \( k \), while the energy \( \varepsilon^- \) decreases as a function of \( k \). These dependencies lead to the crossing of the energy levels depicted in Fig. 2(a). Fig. 2(b) displays total energy \( E \) as a function of wave vector \( k \).

In order to obtain the spin precession in the cylindrical nanowire we consider the quantum state, which is the superposition of one-electron states with the opposite spins and different orbital quantum numbers. Assuming that the wave function corresponding to orbital quantum number \( l = i \) and energy \( \varepsilon^\pm \) is denoted by \( \psi_i^\pm (\varphi, z) \), the superposition of states with orbital quantum numbers \( l = i \) and \( l = j \) can be expressed using the Bloch sphere representation as follows:

\[
\Psi_{i, j^\pm} (\varphi, z) = \cos \left( \frac{\theta}{2} \right) \psi_i^\pm (\varphi, z) + \sin \left( \frac{\theta}{2} \right) e^{i\phi} \psi_j^\pm (\varphi, z) ,
\]

(10)

where the angles \( \theta \) and \( \phi \) determine the quantum state on the Bloch sphere. In our calculations, we set the values of \( \theta \) and \( \phi \) in order to determine the spin state of the electron injected at \( z = 0 \).

Let us consider the two states: (a) \( \Psi_{-1, 0^-} (\varphi, z) \) being the superposition of the two lowest-energy states, and (b) \( \Psi_{0, 1^-} (\varphi, z) \) being the superposition of the two higher-energy states with the energy levels that cross at certain \( k \) (see Fig. 2). For the fixed energy of electron, the wave vectors for the states which are the components of the superposition are different, e.g., for energy \( E = 1 \) meV the wave vectors related to states \( \psi_0^+ (\varphi, z) \) and \( \psi_1^- (\varphi, z) \) (components of the superposition state \( \Psi_{0, 1^-} (\varphi, z) \)) are denoted by \( k_0^+ \) and \( k_1^- \) in Fig. 2(b). The difference

![FIG. 2. (Color online) (a) Energy \( \varepsilon \) as a function of wave vector \( k \) and orbital quantum number \( l \). (b) Total energy \( E \) as a function of \( k \). The energy band with orbital momentum \( hl \) and energy \( \varepsilon^\pm \) is denoted by \( l^\pm \). Wave vectors \( k_0^+ \) and \( k_1^- \) correspond to states \( \psi_0^+ (\varphi, z) \) and \( \psi_1^- (\varphi, z) \), which are the components of \( \Psi_{0, 1^-} (\varphi, z) \).]
\[ \Delta k = |k_1^\pm - k_2^\pm| \]

of the wave vectors causes the orientation of the average spin vector in the superposition state changes continuously along the z axis (cf. right parts of Fig. 3). Throughout the present paper, such changes of the spin orientation will be referred as the spin precession. The precession length can be expressed as \( \lambda_{SO} = 2\pi/\Delta k \). In Fig. 3 we present the spin precession in the nanowire for both the states (a) \( \Psi_{-1,-0}^\pm(\varphi, z) \) and (b) \( \Psi_{0,-1}^\pm(\varphi, z) \). For \( z = 0 \) the parameters \( \theta \) and \( \phi \) have been chosen so that the spin of injected electron is directed parallel to the x axis. We observe that the spin precession in state \( \Psi_{-1,-0}^\pm(\varphi, z) \) has much longer precession length \( \lambda_{SO} \) and the opposite direction in comparison to the spin precession in the state \( \Psi_{0,-1}^\pm(\varphi, z) \). Moreover, the average value of the z-spin component does not change along the entire length of the nanowire, which means that the spin precession results from the changes of the transverse spin components. Since the difference \( \Delta k \) of the wave vectors changes as a function of the electron energy, the precession length \( \lambda_{SO} \) depends on the energy of the injected electrons. Fig. 4 displays the precession of the average spin component \( \langle s_z \rangle \) in the nanowire as a function of energy for both the considered states (a) \( \Psi_{-1,-0}^\pm(\varphi, z) \) and (b) \( \Psi_{0,-1}^\pm(\varphi, z) \). For the state \( \Psi_{-1,-0}^\pm(\varphi, z) \) the precession length \( \lambda_{SO} \) decreases with the increasing energy, which is related to the monotonically increasing difference \( \Delta k \) between the states \( \psi_0^\pm(\varphi, z) \) and \( \psi_1^\pm(\varphi, z) \) [cf. upper part of Fig. 4(a)]. On the other hand, for state \( \Psi_{0,-1}^\pm(\varphi, z) \) the precession length \( \lambda_{SO} \) first increases with the increasing energy up to \( E \approx 2.05 \text{ meV} \), becomes divergent at \( E \approx 2.05 \text{ meV} \) and next decreases. This behavior results from the energy level crossing that corresponds to the states \( \psi_0^\pm(\varphi, z) \) and \( \psi_1^\pm(\varphi, z) \), which are the components of the superposition state \( \Psi_{0,-1}^\pm(\varphi, z) \) (Fig. 2). For the energy level crossing the difference between the wave vectors of both states vanishes (\( \Delta k = 0 \)), which means that \( \lambda_{SO} \to \infty \). Consequently, the spin does not exhibit the precession. We have found that the spin of the electron with the energy lower and higher than the energy level crossing precesses in the opposite directions. Fig. 5 is the illus-
energy is important for the spin control in spintronic applications.

B. Effect of axial electric field

In this subsection, we study the influence of the longitudinal electric field on the spin precession in the nanotube. We consider the nanotube in which the homogeneous electric field $E_z$ is applied parallel to the $z$ axis. If we assume that total energy $E$ is a sum of kinetic energy $E_k$ of the longitudinal motion and energy $\varepsilon$ of the spin-orbit interaction, i.e., $E = E_k + \varepsilon$, the solution of equation system (4a) and (4b) takes on the form

$$
\begin{pmatrix}
  f(z) \\
  g(z)
\end{pmatrix} =
\begin{pmatrix}
  u_0 \\
  v_0
\end{pmatrix} \mathcal{A}(-\xi),
$$

(11)

where $\mathcal{A}(\xi)$ is the Airy function and

$$
\xi = \left(\frac{2meE_z}{\hbar^2}\right)^{1/3} \left(z + \frac{E_k}{eF_z}\right).
$$

(12)

In this case, the system of equations (4a) and (4b) reduces to

$$
V_l u_0 + e\gamma l + 1 r_0 eF_z v_0 = \varepsilon u_0, \quad (13a)
$$

$$
V_{l+1} v_0 + e\gamma l + 1 r_0 eF_z u_0 = \varepsilon v_0. \quad (13b)
$$

System of equations (13a, 13b) possesses the non-trivial solutions only for the following two different values of energy

$$
\varepsilon^\pm = \frac{1}{2} \left[ (V_l + V_{l+1}) \pm \right.
$$

$$
\sqrt{(V_l - V_{l+1})^2 + 4\gamma^2 e^4 F_z^2} \frac{(l+1)}{r_0^2}. \quad (14)
$$

Using the normalization condition $|u_0|^2 + |v_0|^2 = 1$ we derive the analytical expressions for coefficients $u_0^\pm$ and $v_0^\pm$ that correspond to the energy values $\varepsilon^\pm$, respectively,

$$
|u_0^\pm|^2 = \frac{\beta^2}{(V_l - \varepsilon^\pm)^2 + \beta^2}, \quad (15)
$$

$$
|v_0^\pm|^2 = \frac{\beta^2}{(V_l - \varepsilon^\pm)^2 + \beta^2}, \quad (16)
$$

where $\beta = \gamma e^2 F_z(l+1)/r_0$.

In Fig. 6 we display the spin-orbit interaction energy $\varepsilon$ as a function of electric field $F_z$ for several $l$-bands. We see that for the electric field $F_z = 0$ each state is fourfold degenerate, which results from the following symmetries: the $z$-parity symmetry ($z \leftrightarrow -z$) leading to the degeneracy of states $\psi^+_l(\varphi, z)$ and $\psi^-_{l+1}(\varphi, z)$ and the time reversal symmetry leading to the degeneracy of states with angular quantum number $l$ and $-(l+1)$ with the opposite spins (Kramers degeneracy). If the electric field is switched on, the $z$-parity symmetry is broken and the degeneracy of states $\psi^+_l(\varphi, z)$ and $\psi^-_{l+1}(\varphi, z)$ is lifted. However, the electric field does not break the time reversal symmetry, which means that the states $\psi^+_l(\varphi, z)$ and $\psi^-_{l+1}(\varphi, z)$ remain degenerate. We note that the two lowest-energy bands, described by $\psi^0_{-l}(\varphi, z)$ and $\psi^0_{l}(\varphi, z)$ are degenerate in the non-zero electric field. This means that the electric field applied along the nanowire axis does not lead to the spin precession in the superposition state $\Psi_{-l,0}(\varphi, z)$, which is in contrast to the case of $\alpha \neq 0$, in which the spin precession due to the spin-orbit splitting occurs (see Subsection III.A).

![FIG. 6. (Color online) Spin-orbit interaction energy $\varepsilon$ as a function of electric field $F_z$. The results for the states with orbital quantum number $l$ and energy $\varepsilon^\pm$ are marked by $l^\pm$.](image)

In order to study the influence of the homogeneous electric field applied along the axis of nanotube on the spin of the electron, let us consider the superposition of
states with the same orbital quantum number and opposite spin. Fig. (7a) displays the average value of the z spin component as a function of the position in the nanotube calculated for the state $\Psi_{0^+,0^-}(\varphi, z)$. If the total energy $E$ of the electron is fixed, the difference between energies $\varepsilon^+$ and $\varepsilon^-$ causes that the kinetic energies $E_k$ corresponding to the states $\psi_0^+(\varphi, z)$ and $\psi_0^-(\varphi, z)$, which form the superposition state $\Psi_{0^+,0^-}(\varphi, z)$, are different. Since the kinetic energy appears in the argument of the Airy function in solution (11), the kinetic energy difference causes that the corresponding states are shifted in phase. This shift leads to the oscillatory changes of the average z spin component in the superposition state $\Psi_{0^+,0^-}(\varphi, z)$. Figures 7 and 8 show that the spin of the electron is a periodic function of the position measured along the nanowire axis. Fig. 8 allows us to distinguish the periodic features that are closely related to the properties of the Airy function and the fact that for $z \to -\infty$ potential energy $-eFz \to \infty$, which means that the considered system is open only on one side. Fig. 9(a) shows that the average value of the z spin component monotonically decreases with coordinate $z$, which results from the monotonically decreasing Airy function $A(\xi)$. The even more interesting feature is the appearance of beats for the damped oscillations of z spin component. Fig. 7(a) shows that in some points on the nanowire axis the average z spin component vanishes and in the other points $\langle s_z \rangle$ exhibits the local maxima.

In order to get a more physical insight into this phenomena, in Fig. 8 we present the probability density calculated for states $\psi_0^+(\varphi, z)$ and $\psi_0^-(\varphi, z)$, together with the average value of the z-spin component. We see that the average z spin component vanishes in the position, in which $|\psi_0^+(\varphi, z)|^2$ and $|\psi_0^-(\varphi, z)|^2$ electron probability densities oscillate in phase. Since the oscillation period of $|\psi_0^+(\varphi, z)|^2$ and $|\psi_0^-(\varphi, z)|^2$ increases with increasing coordinate $z$, which results from the property of the Airy function) the positions, at which the average value of the z-spin component vanishes, are not equidistant. For the same reason the spin oscillation period is not constant but changes along the z axis [cf. Figs. 7(b) and 7(c)]. The phase shift between both the components of the wave function depends on the spin-orbit interaction. Therefore, the position of the beat changes if the electron energy or/and the electric field are changed. In Fig. 9 the average value of z spin component is plotted as a function of axial electric field $F_z$. The white bands correspond to the positions of the beats. We see that the zeros and maxima (minima) of the average spin component can be shifted along the nanowire axis if we change the external electric field. This property opens up a possibility to control of the electron spin by tuning the bias voltage.

IV. DISCUSSION

In the cylindrical nanowire, the 2DEG can be realized by confining the electrons in a narrow asymmetric quantum well created near the surface of nanowire. The reduction of the radial degree of freedom leads to the quantization of the energy associated with the radial motion of the electron. If the potential confining the electrons near the surface is sufficiently strong, all the electrons occupy the lowest-energy state, which results in the creation of the cylindrical 2DEG. The asymmetry of this quantum well causes that the cylindrical 2DEG is subjected to the electric field, which acts perpendicularly to the surface of the cylinder. In order to generate the flow of electrons through the nanowire, the electric field parallel to the nanowire axis has to be applied. Both the electric
fields couple the momentum of the electron with its spin via the Rashba SOI and should be taken into account when describing the electron spin effects in the cylindrical nanotubes and nanowires. The spin precession due to the Rashba spin-orbit interaction resulting from exclusively the radial electric field has been investigated numerically in Ref. [27]. In this paper, we have developed the 2D model of the semiconductor nanotube with the spin-orbit interaction and found the analytical solutions for the two cases in which (A) the SOI results from the radial electric field (the problem studied by the numerical means in Ref. [27]) and (B) the SOI results only from the longitudinal electric field applied along the nanotube axis. We have shown, by the analytical means, that for the spin-orbit interaction induced by the radial electric field, the superposition of the electron states with the different angular momenta leads to the precession of the average value of the transverse spin component around the nanowire axis. These results confirm those of Ref. [27], but in addition we have shown that the direction of the spin precession in the linear combination of the one-electron states with the different angular momenta depends on the electric field. If the energy of the injected electron is aligned with the energy of the crossing of energy levels which form the superposition state, the direction of spin precession changes from clockwise to counterclockwise. This opens up a new possibility to control the spin precession by changing the energy of injected electrons. Moreover, in our paper we have investigated the spin modulation resulting from the electric field applied along the z axis. This axially directed electric field, which is necessary for the current flow, was neglected in Ref. [27]. We have found that the longitudinal electric field generates the spin beats formed along the nanotube/nanowire axis. These beats can be changed by the electric field. Therefore, our results suggest that the spin precession due to the radial electric field can be significantly modified by the longitudinal axial electric field.

V. CONCLUSIONS

In the present paper, we have studied the modulation of the electron spin in the semiconductor nanotube/nanowire with the Rashba spin-orbit interaction. We have succeeded to obtain the analytical solutions for two limiting cases for which the spin-orbit interaction is generated by (A) the radial electric field and (B) the electric field applied along the z axis.

If the spin-orbit interaction is only due to the radial electric field (case A), the superposition of the electron states with the different angular momenta leads to the precession of the average value of the transverse spin component. We have found that this precession changes its direction from clockwise to counterclockwise at the crossing of the energy levels associated with both the wave function components. It is also interesting that the spin precession occurs in the superposition of the two lowest-energy bands $\Psi_{-1} \cdot (\varphi, z)$. This state can be realized if the radius of the nanowire is sufficiently small. Our calculations show that the energy separation between these two states and the higher bands increases with the decreasing radius of nanowire. Therefore, if the nanowire is thin enough, only these lowest-energy bands are occupied and the spin precession can be observed experimentally.

The solutions for the system with the spin-orbit interaction generated by the electric field applied along the axis of nanowire (case B) show that for the superposition of states with the same angular momenta the z spin component oscillates as a function of z coordinate with the period that depends on the electric field. We have also found that the longitudinal electric field generates the spin beats formed along the nanotube/nanowire axis. The position of these beats and consequently the maximal value of average spin can be controlled by the bias voltage.

In summary, we have proposed the all-electrical mechanism of the electron spin manipulation based on the spin-orbit interaction in semiconductor nanotubes and cylindrical nanowires.

ACKNOWLEDGMENTS

This work has been partly supported by the National Science Centre, Poland, under grant DEC-2011/03/B/ST3/00240. P. W. has been supported by the Polish Ministry of Science and Higher Education and its grants for Scientific Research.

Appendix A: Derivation of the effective 2D Hamiltonian

We consider the cylindrical nanotube with the radius $r_0$ and the thickness in radial direction equal to $2\Delta r$ (in the case of cylindrical nanowire $2\Delta r$ is the thickness of charge accumulation layer). The three-dimensional (3D) Hamiltonian for the single electron in the cylindrical nanotube/nanowire with the spin-orbit interaction has the form

$$
\hat{H}_{3D} = -\frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial}{\partial \varphi} + \frac{\partial^2}{\partial z^2} \right) + V(r, \varphi, z) + \hat{H}_{SO}^{3D},
$$

where $r$, $\varphi$, and $z$ are the cylindrical coordinates, $V(r, \varphi, z)$ is the potential energy, and $\hat{H}_{SO}^{3D} = (\gamma/\hbar)\hat{\sigma} \cdot (\nabla V \times \vec{p})$ is the 3D spin-orbit interaction Hamiltonian.

If the potential energy possesses the rotational symmetry, i.e., $V(r, \varphi, z) = V(r, z)$, the spin-orbit interaction
Hamiltonian can be expressed as

\[
\hat{H}^{SD}_{SOI} = i\gamma \left[ \left( \frac{\partial}{\partial z} e^{i\phi} \frac{\partial}{\partial \phi} - \frac{\partial}{\partial \phi} e^{i\phi} \frac{\partial}{\partial z} \right) \right] \left[ \left( \frac{1}{r} \frac{\partial}{\partial r} e^{-i\phi} \frac{\partial}{\partial \phi} - \frac{1}{r} \frac{\partial}{\partial \phi} e^{-i\phi} \frac{\partial}{\partial r} \right) \right] + \left( i e^{i\phi} \left( \frac{\partial}{\partial r} - \frac{\partial}{\partial \phi} \right) \right) - i e^{-i\phi} \left( \frac{\partial}{\partial \phi} - \frac{\partial}{\partial r} \right) \right].
\]

(A2)

In the considered problem, the potential energy \( V(r, z) \) is separable, i.e., \( V(r, z) = V_r(r) + V_z(z) \), where \( V_r(r) \) is responsible for confining the electron near the surface of the nanowire and \( V_z(z) \) is the potential energy of the electron in the axially directed electric field. Assuming that the radial quantum states are spin-degenerate, the one-electron wave function can be take on in the form

\[
\Psi(r, \varphi, z) = \mathcal{R}(r)\psi(\varphi, z),
\]

(A3)

where \( \mathcal{R}(r) \) is the radial wave function and \( \psi(\varphi, z) \) is the spinor.

If the confinement of the electron in the nanotube is strong, we can assume that the electron occupies the radial ground state \( \mathcal{R}_{gs}(r) \). The ground state with the definite parity with respect to \( r = r_0 \) has the following property:

\[
\langle \mathcal{R}_{gs} \rangle \frac{\partial}{\partial r} \langle \mathcal{R}_{gs} \rangle = 0.
\]

(A4)

Multiplying both the sides of Schrödinger equation with Hamiltonian (A1) by \( \mathcal{R}_{gs}^* \) and integrating over \( r \) in range \([r_0 - \Delta r, r_0 + \Delta r]\) we obtain

\[
\left[ -\frac{\hbar^2}{2m} \left( \frac{1}{r^2} \frac{\partial}{\partial r} \right)^2 + \frac{\partial^2}{\partial z^2} + E_0 + V_z(z) \right] \hat{\psi}(\varphi, z) + \hat{H}_{SO}(r, \varphi, z) \psi(\varphi, z) = E \psi(\varphi, z), \quad (A5)
\]

where

\[
\left( \frac{1}{r^2} \right) = \langle \mathcal{R}_{gs} \rangle \frac{1}{r^2} \langle \mathcal{R}_{gs} \rangle,
\]

(E6)

\[
E_0 = \langle \mathcal{R}_{gs} \rangle - \frac{\hbar^2}{2m} \left( \frac{1}{r_0^2} \frac{\partial^2}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} + V_R(r) \right) \langle \mathcal{R}_{gs} \rangle \quad (A7)
\]

\[
\hat{H}_{SO} = \langle \mathcal{R}_{gs} | \hat{H}_{3D,SO} | \mathcal{R}_{gs} \rangle. \quad (A8)
\]

For the strictly 2DEG \( \Delta r \to 0 \), which allows us to perform the integration in (A6). For the normalized \( \mathcal{R}_{gs}(r) \), the integration in (A9) is equal to 1/r_0^2. In the present paper, we treat energy \( E_0 \) as the reference energy and put it equal to 0. Finally, we obtain the effective 2D Hamiltonian in the form

\[
\hat{H} = -\frac{\hbar^2}{2m} \left( \frac{1}{r_0^2} \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial z^2} \right) + V_z(z) \hat{\psi} + \hat{H}_{SO}(A9)
\]

where

\[
\hat{H}_{SO} = \left( \begin{array}{cc}
0 & \frac{\partial}{\partial z} e^{i\phi} \frac{\partial}{\partial \phi} e^{-i\phi} \\
0 & -\frac{\partial}{\partial \phi} e^{-i\phi} \frac{\partial}{\partial \phi} e^{i\phi} \\
\end{array} \right) + \left( \begin{array}{cc}
e^{-i\phi} \frac{\partial}{\partial r} | r=r_0 \frac{\partial}{\partial z} \\
0 & 0 \end{array} \right) - \left( \begin{array}{cc}
e^{i\phi} \frac{\partial}{\partial r} | r=r_0 \frac{\partial}{\partial z} \\
0 & 0 \end{array} \right). \quad (A10)
\]
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