Research on tooth profile modification of two spur gear driven system

Zhang Bin¹,a

¹Department of Computer, Jiangsu Automation Research Institute, 222061, China

a Corresponding author: chnzhangbin@163.com

Abstract. The gear system of a certain type of space drive mechanism generates vibration during the actual operation, which leads to a decrease in transmission accuracy and seriously affects the normal operation of the system. A nonlinear dynamic model was established for this problem, and the tooth profile error and tooth profile modification were considered in the model. The dynamic characteristics of the space drive mechanism gear system were analyzed by the dynamic model. The influence of the profile modification parameters on the dynamic load coefficient and load transmission error was analyzed. The purpose is to reduce the vibration amplitude, reduce the transmission error and improve the transmission accuracy during gear transmission through the profile modification. The analysis results in this paper enrich the research on tooth profile modification of gear system, and provide a theoretical basis for the subsequent design of gear system and the improvement of vibration and noise during operation.

1. Introduction

Gear systems are widely used in machine tools, aerospace and marine applications, which played a key role in the mechanical transmission process. In recent years, a large number of scholars, universities and research institutions have studied the transmission characteristics of gear systems and obtained a series of research results [1-4]. The gear system generates vibration and noise during operation, causing transmission errors, which can be effectively alleviated by the profile modification. In the process of researching the gear system, it is often necessary to establish a dynamic model and then perform numerical solution to analyze the dynamic characteristics of the gear system [5, 6].

The tooth profile deviation is one of the main causes of the gear system transmission error. It is of great significance to study the tooth profile deviation and profile modification [7]. Liu Hongqi [8] studied the curve optimization law and design method of tooth profile modification, and proposed three principles of tooth profile modification curve optimization based on error excitation. To carry out tooth profile modification, Liu Yongshen [9] gradually approached the involute curve by means of circular arc transformation curve, and strived to find a tooth profile modification method to solve practical engineering problems.

The space drive mechanism deceleration system has the characteristics of ultra-low speed and large inertia, and the transmission accuracy is high [10]. The two-stage spur gear reduction system of the space drive mechanism studied in this paper has generated vibration phenomenon during the operation. In order to solve this problem, our research team has carried out a lot of simulation analysis and experimental research. This paper focuses on the research on tooth profile modification. Firstly, the 14-degree-of-freedom dynamics model of the two-stage spur gear system is established. Then the calculation formulas of the tooth profile deviation and the tooth profile modification are pointed out.
The formula is brought into the dynamic equation and then a series of dynamic characteristics are analyzed.

2. Dynamic model of space driven gear system

The two-stage spur gear transmission system is simplified, and the simplified dynamic model is shown in Fig. 1. The power transmission process is shown by the blue line arrow in Fig. 1. The gear parameters of the space drive reduction mechanism are shown in Table 1.

![Fig. 1 Transmission schematic diagram of gear system.](image)

Table 1 Gear parameter table of two-stage gear system.

| Parameters of gear pair 1 | Parameters of gear pair 2 |
|--------------------------|--------------------------|
| Gear 1 teeth             | 18                       |
| Gear 2 teeth             | 90                       |
| Gear 3 teeth             | 18                       |
| Gear 4 teeth             | 360                      |
| Tolerance of tooth shape error of gear 1 | 0.008mm |
| Tolerance of tooth shape error of gear 3 | 0.008mm |
| Tolerance of tooth shape error of gear 2 | 0.008mm |
| Tolerance of tooth shape error of gear 4 | 0.006mm |

Considering the time-varying meshing stiffness of the two-stage gear system, the radial displacement of the gear at the bearing, the tooth profile deviation, etc., the dynamic model of the system was established under the following assumptions.

1) Ignoring bearing friction, only a load resistance torque at the load end is added.
2) It is assumed that the input torque of the motor is a fixed value, and the torque fluctuation is not considered.
3) The gear meshing force is always along the meshing line direction.

The torsional vibration model of the 14-DOF two-stage spur gear transmission system is established, as shown in Fig. 2, and then the nonlinear dynamic equation can be established according to Newton's second law and related mechanics knowledge, as shown in formula (1) and formula (2). Detailed kinetic equations are shown in other work [1] of our team.

\[ m\ddot{x} + c_v \dot{x} + k_v x = A\dot{\theta} + B\theta' \]  \hspace{1cm} (1)

\[ J\ddot{\theta} + C\theta + K\theta = M + A\dot{Y} + B'Y \]  \hspace{1cm} (2)
3. tooth profile deviation and profile modification

3.1. tooth profile deviation
The tooth profile deviation of each tooth is similar during actual machining. Therefore, the tooth profile deviation of one tooth can be derived first, and then extended to a periodic function to approximate the tooth profile deviation. Mucchi et al (literature [10] p. 140) proposed a formula for describing the tooth profile deviation, as shown in equation (3).

$$e_p(s) = f_{ho} \frac{s - s_0}{s_f - s_0} + \frac{f_r}{2} \sin(2\pi f_r \frac{s - s_0}{s_f - s_0})$$

(3)

In the formula,
- $s$ —— Length of meshing line;
- $s_f$ —— Maximum radius of curvature of the tooth profile;
- $s_0$ —— Minimum radius of curvature of the tooth profile;
- $f_r$ —— Ratio of the total length of each tooth profile to the tooth profile deviation period.

Take $f_{ho}$ and $f_r$ as the tolerance of the tooth profile deviation. The specific value can be seen in Table 1. To simplify the calculation, $f_r$ is taken as 1. Given an initial phase, the tooth profile deviation curves of the four gears can be made according to Table 1 and equation (3), as shown in Fig. 3.

![Fig. 3 Tooth profile deviation of four gears.](image)

3.2. Tooth profile modification
In the production process of the gear, the tooth profile will inevitably produce errors, resulting in an unstable transmission process. To reduce the stress on the teeth, avoid the tip contact, and make the contact between the teeth as smooth as possible, it is a common practice in gear design to introduce
the profile modification at the root or the top of the tooth. In the design process of the spur gear, it is necessary to design the shaping parameters according to the magnitude of the load torque. Depending on the amount of modification, the profile modification can be divided into short modification and long modification. According to the shape of the modification, it can be divided into linear modification and parabolic modification. The short modification is generally used near the position where the number of meshing teeth changes, and is usually used for a gear system reducer with a small load torque, while the long modification usually used for a gear system reducer with a large load torque. Short modification only affects the double-toothed meshing zone, while long modification has an effect on both the single-tooth meshing zone and the double-toothed meshing zone.

In the simulation process, the implementation of the profile modification is similar to the tooth profile deviation. A positive tooth profile deviation is equivalent to material removal of the theoretical profile, while a negative value indicates the opposite. The profile modification can be defined by the maximum amount of modification, the length of the modification, and the type of practice (Ref. [10], p. 140), as shown in equation (4), equation (5), and Fig. 4. To ensure the strength of the gear, only the top modification was considered in this paper, and the root modification was neglected.

![Fig. 4 Schematic diagram of tooth profile modification.](image)

In the formula,

- \( R_{TC} \) —— Maximum modification of tooth top;
- \( R_{BC} \) —— Maximum modification of tooth root;
- \( T_\Delta \) —— Tooth top length;
- \( B_\Delta \) —— Tooth root length;
- \( n \) —— A value of 1 is a linear modification, and a value of 2 is a parabolic modification.

4. Analysis of dynamic characteristics of tooth profile modification

4.1. Comparison of two-stage gear pair

To study the dynamic characteristics of the two-stage gear system of the space drive mechanism, the linear profile modification method is used to simulate the tooth profile of the first and second gear pairs respectively, and the load transmission error response is obtained through calculation. The gear system with profile modification is compared with a gear system without profile modification, as shown in Fig. 5. It can be seen from the load transmission error curve shown in Fig. 5 that the transmission error of the gear system exhibits a periodic change, and the influence of the profile modification at the peak-to-peak value of the transmission error is large, and the influence at other positions is not obvious. It can be seen from Fig. 5 that when the tooth profile modification is performed only on the second-stage gear pair, the peak-to-peak value of the load transmission error is significantly reduced; when the tooth profile modification is performed only on the first-stage gear pair, the peak-to-peak value of the load transmission error is slightly reduced and not very obvious. This is because the change of the first stage gear pair needs to be transmitted through the second stage gear pair, and the transmission process needs to experience the influence of the second stage gear pair flank clearance and the like, which weakens the response caused by the first stage gear pair change. The second stage gear pair is directly connected to the load, which can directly affect the load response. Since the first-stage gear...
pair has little influence on the dynamic characteristics of the gear system, in the subsequent research and analysis, to simplify the calculation, only the second-stage gear pair is taken as the research object.

4.2. Effect of profile modification parameters on dynamic characteristics of gear system
Keeping the modification length (ML) 0.6mm, the modification amount (MA) of the second-stage gear pair is changed and simulated. The value of modification amount range from 0 to 0.01mm. The variation of the dynamic load factor of the gear with the modification amount under linear modification and parabola modification is shown in Fig. 6. It can be seen from Fig. 6 that as the modification amount increases, the dynamic load coefficient first decreases and then increases. The dynamic load factor under linear modification has a minimum value of 0.93 when the modification amount is 0.004 mm, and the dynamic load factor under parabola modification has a minimum value of 0.97 when the modification amount is 0.006 mm. It can be seen that both modification methods can reduce the dynamic load coefficient of the gear system. When the trimming amount is less than 0.003mm, the parabolic modification is better than the linear shape modification; when the shape modification amount exceeds 0.003mm but less than 0.006mm, the linear shape modification is superior to the parabolic modification; when the shape modification amount exceeds 0.006mm but less than 0.008mm, the parabolic modification is superior to linear shape modification; when the amount of modification exceeds 0.008 mm, linear modification begins to be superior to parabolic modification. There is a critical value in both modes of modification. Once this value is exceeded, the profile modification will increase the dynamic load coefficient and act as a counteraction, so that it cannot effectively reduce vibration.

It is guaranteed that the type of tooth profile is unchanged. Here, the linear modification is taken as an example to change the modification length of the second gear pair for simulation analysis. The analysis lengths of 0.3mm, 0.4mm, 0.6mm and 0.7mm were respectively selected for analysis, wherein the first two lengths were short modification and the latter two were long modification. The simulation results under different modification lengths are shown in Fig. 7. It can be seen from Fig. 7 that with
the increase of the modification amount, the dynamic load coefficients of different modification lengths first increase and then decrease, but the difference between the optimal modification amount and the critical modification amount under different modification lengths is large. With the increase of the length of modification, the optimal modification amounts are 0.004mm, 0.003mm, 0.004mm, and 0.006mm, respectively, and the corresponding minimum dynamic load coefficients are 1.02, 0.98, 0.93, and 0.96. Through analysis and comparison, it is found that the optimal modification amount under long shape modification is obviously larger than the short shape modification, and the minimum dynamic load coefficient is obviously smaller than the short modification. The space-driven gear system studied in this paper has a large load torque, and the long modification is better than the short modification. It is consistent with the experience of Fernández (literature [11], p. 142), which verifies the correctness of this study.

Fig. 7 Change of dynamic load coefficient under different lengths of modification.

The purpose of profile modification of the gear system is to reduce vibration and noise, reduce transmission errors, and improve transmission accuracy. To further analyze the influence of the profile modification on the dynamic characteristics of the gear system, the load transmission error response curves under the optimal modification amount are analyzed respectively, and compared with the unreformed transmission error curve, as shown in Fig. 8. It can be seen from Fig. 8 that the tooth profile modification significantly reduces the peak-to-peak value of the load transmission error compared with the gear system without the profile modification. When the modification length is 0.6 mm and the modification amount is 0.004 mm, the peak-to-peak value of transmission error is the smallest. It is calculated that the peak-to-peak value of the transmission error is reduced by 36% in this modification mode, as shown by the red curve in Fig. 8, in which the transmission accuracy of the system can be significantly improved, and the vibration phenomenon in the transmission process can be reduced.

Fig. 8 Load transmission error curve under different modification parameters.
5. Conclusion
A dynamic numerical model is established for the vibration problem of the gear system deceleration drive mechanism during operation. It is proposed to relieve the vibration phenomenon by tooth profile modification, and the calculation formula of tooth profile deviation and tooth profile modification is pointed out and applied to the dynamic equation.

The dynamic characteristics of the gear system were analyzed. The tooth profile modification of the two-stage gear pair was compared. The changes of the dynamic load coefficient and the load transmission error under different modification parameters were analyzed. The analysis results show that the tooth profile modification of the second gear pair is more helpful to reduce the vibration amplitude. Under the linear modification, the modification length 0.6mm and the shape modification amount 0.004mm are the optimal modification scheme, and the load transmission error peak-to-peak value is reduced by 36%.

This paper enriches the research on tooth profile modification of gear system, and provides a theoretical reference for the subsequent practical design and optimization of gear system.

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