Assessment regarding the use of the computer aided analytical models in the calculus of the general strength of a ship hull

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Abstract. Solving a general strength problem of a ship hull may be done using analytical approaches which are useful to deduce the buoyancy forces distribution, the weighting forces distribution along the hull and the geometrical characteristics of the sections. These data are used to draw the free body diagrams and to compute the stresses. The general strength problems require a large amount of calculi, therefore it is interesting how a computer may be used to solve such problems. Using computer programming an engineer may conceive software instruments based on analytical approaches. However, before developing the computer code the research topic must be thoroughly analysed, in this way being reached a meta-level of understanding of the problem. The following stage is to conceive an appropriate development strategy of the original software instruments useful for the rapid development of computer aided analytical models. The geometrical characteristics of the sections may be computed using a bool algebra that operates with ‘simple’ geometrical shapes. By ‘simple’ we mean that for the according shapes we have direct calculus relations. In the set of ‘simple’ shapes we also have geometrical entities bounded by curves approximated as spline functions or as polygons. To conclude, computer programming offers the necessary support to solve general strength ship hull problems using analytical methods.

1. Introduction
Computer based models are nowadays the most extensively used methods of investigation. In this way the commercial software applications are used for mathematical solutions [1], for computer aided design, for structural analysis based on the finite element method, for computer aided manufacturing and for experimental studies. These solutions are based on the main features of a computer: speed and accuracy. These strengths, together with the computer programming skills, are becoming important resources for the development of original investigation methods. A direction may be the development of interfaces for the profound integration of the various types of studies in a hybrid model, [2], and other direction may be the development of original software instruments for research. One can notice that the analytical methods developed until now are trying to minimise the overall volume of calculi because at the time when they were conceived the computer wasn’t invented yet. In this way the classic theory is based on assumptions that proved to be useful, along time the results being relatively accurate. However, using the nowadays advanced instruments the basic concepts may change and new calculus methods may be developed in engineering.
Note: the following ideas are an exploratory approach regarding the computer aided analytical calculus in the ship hull’s strength problems and we are going to publish follow-up papers that will present substantiated solutions.

2. Problem formulation

The development of a structural model must rely on a series of lower level models, such as: the model of the geometry, the model of the loads, the model of the supports and the model of the material. These basic models may be found, at an abstract level, in both analytical and numerical models based on the finite element method, in each type of model being used the appropriate formulation.

Educational analytical models of a ship hull were developed so far, [3], the main goal of these approaches being the testing of the calculus methodology. Our goal is to identify the sources of inaccuracy and to conceive solutions, if possible computer based solutions, to improve the precision of such model.

Regarding the geometry of the structure, the main procedure is presented in the following figure, [3], the ship hull being divided in sub-domains in order to have lower complexity problems when the weighing forces and the Archimedes’ forces, i.e. buoyancy forces are evaluated. There must be reminded that in all the loading scenarios the centre of gravity is on the same vertical as the longitudinal centre of buoyancy (no trim and heel angles – even keel condition).

![Figure 1. Calculus scheme of a ship hull.](image1)

![Figure 2. Diagram of buoyancy forces distribution.](image2)

![Figure 3. Diagram of the weighting forces distribution.](image3)

The Archimedes’ forces, $F_i$, may be evaluated by calculating the immersed volume of the hull, by using for example the immersed transverse sections areas for various inclined waterlines (even keel as a particular case), the graphical representations of the results being designated as Bonjean curves. To
compute the Bonjean curves, there is not required any data regarding the ship’s compartments but only the geometry of the hull’s outer surface.

Using the $F_i$ Archimedes’ force along a $L_i$ length interval, we compute the $f_i$ equally distributed buoyancy forces

$$f_i = \frac{F_i}{L_i},$$

that are represented in figure 2.

Once this stage completed, one should evaluate the $Q_i$ weighting forces produced by the structure itself, by the on-board equipment and by the cargo manoeuvres. Similarly to (1), we compute the $q_i$ equally distributed weighting forces

$$q_i = \frac{Q_i}{L_i},$$

that are represented in figure 3. The position of the centroid of the sub-graph area represents the position of the centre of gravity of the ship. The position of the centroid of the sub-graph area represents the position of the hull centre corresponding to the ship’s buoyancy.

**Figure 4.** The diagram of loads distribution resulted from the difference between the diagram of weight distribution and the diagram of Archimedes forces distribution.

The diagram of load distribution, figure 4, is obtained by summing the effects of distributed weights and distributed Archimedes forces. Thence, for each theoretical compartment, the two diagrams are overlapped, by subtracting the areas of distributed forces. The equally distributed force along the ‘$i$’ interval is:

$$p_i = q_i - f_i.$$  

(3)
In figure 4 we must remark that the area of sub-graph of diagram of load distribution placed over the zero line represents the weight surplus and is equal to the area placed under the zero line which represents the surplus of Archimedes forces. This feature represents the equilibrium condition of the forces along the vertical direction:

\[ \sum_{i=1}^{N} V_i = 0 \]  
(4)

Regarding figure 4 we also remark that the centre of gravity of the area placed over the zero line is located on the same vertical with the centre of gravity of the area placed under the zero line. This aspect represents the condition that the ship in equilibrium does not rotate around any point, hence the bending moments sum in respect to any point is zero:

\[ \sum_{i=1}^{N} M^K_i = 0 \]  
(5)

where \( K \) is a point of reference randomly located along the ship hull.

Using the diagram of loads distribution we may draw the free body diagrams of the ship hull and we identify the section where the \( T_z \) shear force is zero, i.e. the section where the \( M_y \) bending moment is reaching an extreme value, [4].

The calculus of the normal stresses may be done using Navier’s relation:

\[ \sigma^{M_y}(z) = \frac{M_y}{I_y} \cdot z \]  
(6)

where \( I_y \) is the second moment of area in a given section with respect to the horizontal axis.

The calculus of the shear stresses may be done using Juravski’s relation

\[ \tau^{T_y} = \left( \frac{T_z}{I_y} \right) \cdot \frac{S_y}{b_y} \]  
(7)

where \( S_y \) is the first moment of area with respect to the horizontal axis of the region of the cross section where the current calculus point is located and \( b_y \) is the width of the section in this point.

Taking into account the theoretical background previously presented, the problem is to conceive the most effective calculus methodology for ship strength problems, based on analytical aspects.

3. Discussion

The calculus methodology may look simple for the experienced structural analysts but the details of the analytical model calculus will modify this initial opinion.

The definition of the ship hull geometry must be done in a flexible way, the more structural elements taken into consideration, the more accurate results being reached. This means that we may start with a relative simple geometry and then improve it at later stages.

As mentioned before, the Archimedes’ forces may be computed by considering only the geometry of the ship hull’s outer surface, the width of the shell being considered null.

The total weight includes the weight of the ship and the deadweight tonnage that consists of the sum of the weights of cargo, fuel, fresh water, ballast water, provisions, passengers and crew. In order to compute the weighting forces, the ship hull must be divided in compartments that are structurally defined. There must be computed the volumes of the tanks, of the storage compartments, in this stage being also used theoretical values of the plates’ widths. The width of the plates is decided according to the ship’s registry rules, by using analytical relations of by parametric design followed by finite element analysis of the structure.
The design of the ship hull may be done in several ways, i.e. a common method is starting by considering the transversal frames of the hull. The transversal frames must be dimensioned and placed at 500 mm up to 1000 mm distance one from the other. Usually, for the general design of the ship hull there are considered 20 transversal sections. According to the particular type of vessel, the appropriate intuitive rules and calculus relations are applied, i.e. the total weight of a ship hull which uses plate stiffeners consists of 0.2% the weight of the shell and 33% the weight of the frames, girders, strakes and other structural components. An important idea to create a light structure is to use many structural elements placed close one to the other and a thin shell.

In order to have a flexible definition of the loads, there may be considered parameterizations of the weighting forces, for instance a flag may control the percentage of the fluid in the tank or of the cargo in its storage area. Other parameter is the density of the fluid or of the current cargo. By the use of these parameters, there may be easily considered several load case scenarios.

Once we have the weighting forces we must solve an equation expressed in a procedural way which will use the equilibrium conditions in order to find the position of the ship hull in the current load case scenario. However, for large volume of calculi the (4) and (5) equilibrium relations must be transformed in an error-minimization relations, i.e.

\[
\begin{align*}
\sum V_i &\leq \varepsilon \\
\sum M_i^k &\leq \varepsilon
\end{align*}
\]

where \( \varepsilon \) is the maximum absolute accepted error.

The calculus of the weighting forces, action that includes the design of the ship hull’s geometry is a laborious design stage and a fine idea is to automatize the calculi.

After the general design of the structural elements is done, there must be computed the geometrical characteristics of the cross sections, i.e. the \( I_y \) second moment of area, \( W_y \) section modulus, \( S_y \) first moment of area and \( b_y \) width of the section. There are several methods to compute the geometrical characteristics and, moreover, to select the minimum set of points where the maximum stresses may be found. In [5] is presented a computer method that considers that the cross section in bounded by spline curves and one integral may be considered to compute all the geometrical characteristics, i.e.

\[
I = \int_a^b \left( \int_{f_i(y)}^y \int f(y,z)dz \right)dy,
\]

where the functions that are the frontiers of the domain are

\[
f_1(y) = a_5 \cdot y^3 + a_2 \cdot y^2 + a_1 \cdot y + a_0, \quad f_2(y) = b_3 \cdot y^3 + b_2 \cdot y^2 + b_1 \cdot y + b_0,
\]

and the function is

\[
f(y,z) = y^u z^v.
\]

The \( u \) and \( v \) exponents must comply with the following conditions: \( u, v \in \mathbb{N} \) and \( u + v \leq 2 \). This means that we have the following pairs:

\[
(u = 0; v = 0), \quad (u = 1; v = 0), \quad (u = 0; v = 1), \quad (u = 2; v = 0), \quad (u = 0; v = 2) \quad \text{and} \quad (u = 1; v = 1).
\]
The first pair leads to the area of the section, the following two offer the first moments of area, the next two pairs are used for the second moments of area and the final one is for the product moment of area.

The first form of the previous integral is

\[ I = \frac{1}{v+1} \int_a^b y^v \left( (b_3 - a_3) y^3 + b_2 - a_2 \right) dy \]

and it can be computed numerically. In [5] is also presented an analytical direct solution of the integral.

Other idea to easily compute the geometrical characteristics of the cross sections is presented in [6] where the cross section is defined using a bool algebra that operates with simple shapes, in this case being used polygons. There must be noticed that any area may be approximated using polygons, i.e. the calculus relations for a polygon are general and they may be used for any shape of a composite cross section. In [6] there are also presented some calculus examples.

4. Conclusions
The analytical approaches previously presented were applied in an initial study dedicated to a heavy lift vessel (light ship condition), the according free body diagrams being presented in the following figure.

![Heavy Lift Vessel](image)

**Figure 5.** The free body diagrams of the analytical model of a heavy lift ship.

Accurate results require a large amount of calculi, being necessary to conceive an integrative strategy of computing based on original software instruments, otherwise the previous aspects remain only theoretical approaches, without practical applications. This strategy should include the development of several software libraries dedicated to the calculus of the buoyancy forces, of the weighting forces and of the geometrical characteristics of the ship hull cross sections.

Using the computer based analytical model, the structural engineers may have a more accurate ‘initial approximation’ of the technical solution to be designed, [7]. Moreover, using the original software instruments several variants of the initial solution may be readily conceived including for the ‘exotic’ types of ships, [8].
5. References
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