1 Introduction

The status of the uncertainty relations varies between the different interpretations of quantum mechanics. At one end of the scale, for Heisenberg they showed the fundamental limits of applicability of the wave and particle pictures separately. At the other end, in hidden variable theories they are merely statistical relationships of no foundational significance. The aim of the current paper is to explore their meanings within a certain neo-Everettian many worlds interpretation. We will also look at questions that have been linked with the uncertainty relations since Heisenberg’s uncertainty principle: those of joint and repeated measurement of non-commuting (or otherwise ‘incompatible’) observables. This will have implications beyond the uncertainty relations, as we will see the fundamentally different way in which statistical statements are interpreted in the neo-Everett theory that we use.

2 Neo-Everettian Many Worlds

It is firstly necessary briefly to describe the interpretation with which we will explore the uncertainty relations. This has been described (with full references) in [1], and is a variant of the many worlds theory started by Everett [2].

The salient points are these. As with all Everett-style theories, the measurement problem is solved by having all possible outcomes of an interaction instantiated. The ‘worlds’ of which there are ‘many’ are not defined by a precise rule, but rather are structures identified for all practical purposes”. They are structures marked out by their usefulness, explanatory relevance and persistence over time, as high-order rather than fundamental ontology. They occur at all levels: there are structures of decoherent branches that persist over large time scales and act in ways that we would call ‘universe’, and within them can be identified other substructures.

To take the old example, within the state of a Schrödinger Cat experiment we would be able to identify two structures that persisted stably over time: one of a cat alive (and unbroken apparatus) and one of a dead cat (and associated apparatus). These are not,
of course, the only structures identifiable in that state (we need only to change basis),
but they persist in a stable state over time scales relevant to the experiment, and indeed
after the diabolical device is opened they become elements in the decoherence basis.
The identification of the cats is not precise: just as, in everyday life, we chose to call
an imprecisely defined and constantly changing bundle of atoms 'a cat', so when we
look at the quantum state of the system we see something that looks like that which we
would normally refer to as 'an alive cat', and something that looks like a dead cat. We	herefore say we have two worlds, one in which there is an alive cat, and one in which
there is a dead one.¹

One unusual feature of this type of any worlds theory is that, while at larger scales
decoherence picks out the relevant structures for us, at very small scales there can be
more than one basis in which we find relevant structures. This will, of course, depend
on what we are looking for and how the system is evolving (again, the identification of
structures is not fundamental).

3 The Uncertainty Relations

Before coming on to the uncertainty relations that we will be looking at, it is worth
saying what we will not be dealing with. We will not be using the 'textbook' form of the uncertainty relations, the Robertson relation for non-commuting observables:

\[ A \cdot B = \frac{1}{2} \hbar [A;B]_{ij} \]  

where

\[ A = \hbar A^2 i \quad B = \hbar B^2 i \]

There are various standard problems with this formulation (summarized in [3]).² For
present purposes the main problems are:

These relations tell us nothing when one of the observables is in an eigenstate
as then the right hand side of the inequality is zero. There are also other situations
when they give no information (for example, in a spin-half system, because
\[ [S_x;S_z] = i\hbar S_y, \text{ if } S_y = 0 \text{ then the relations tell us nothing about } S_x \text{ and } S_z. \]
This is even though they are non-commuting observables of the system, which are
considered the objects of the uncertainty relations on this view. They are not, therefore, fully general inequalities.

¹These worlds at this point only extend as far as the bounds of the apparatus - the theory is local.
²The stronger relation owing to Schrödinger,

\[ (A)^2 (B)^2 \quad \frac{1}{4} \hbar [A;B]_{ij}^2 + \frac{1}{4} \hbar [A A i; B B i]_{ij}^2 \]

will also not be dealt with, as it suffers from the same problems.
The relations work only for Gaussian (or near-Gaussian) distributions. The interpretation of the quantities $A$ and $B$ as spreads only works for such distributions.

Neither will we look at any uncertainty relations including time (given the status of time these will be very different objects from other uncertainty relations. Lastly, we will not be looking at entropic uncertainty relations, for the very reason that Deutsch [4] champions them: they deal not with findam entals (which we wish to look at to analyse from a neo-Everettian perspective) but with operational situations.

We will be using the uncertainty relations owing to Unk [5]:

\[ W(A) \propto \text{arccos} \frac{1}{1 + W(A)} \]  

(1)

The quantities are defined as follows: $\hat{A}$ is an operator that generates a group of states $j_i$ by

\[ j_i = e^{\frac{i\hat{A}}{\hbar}} j_i \]

is the smallest value of $\theta$ such that

\[ j_h j + \theta j = 1 \]

$W(\hat{A})$ is the smallest interval in the distribution $h j_0 i$ (where $\hat{A}j_0 i = aj_0 i$) in which a fraction $< 1$ is contained:

\[ \int_{-\frac{W}{h}}^W \frac{dy}{h} j_0 i j da = \frac{W}{W} \]

Finally it is a requirement of the relation that $2 \leq 1$.

If we chose such that states become more-or-less distinguishable once their overlap reaches it, the quantity is therefore the expected error in estimated a given state: states in the range $j_i$ to $j_{i+1}$ are indistinguishable by any measurement.

The ‘minimal interpretation’ [3] of these identities is that they are relations between constraints on different probability distributions of the same state, for groups and their generators. They do not fully define the distributions, but give conditions they must necessarily follow. gives a constraint on the $h j_i$ probability distribution: the more spread out the probability distribution, the less sharply we would be able to estimate the state, even given multiple measurements on an ensemble of similarly prepared states.

\[^3\text{The most famous of these in the context of uncertainty relations are position and momentum, each of which acts as the generator of the other group. The set of spatial translations is generated by} \]

\[ 1 \frac{i}{\hbar} px^0 \]

and that of translations in momentum space by

\[ 1 \frac{i}{\hbar} x \frac{p}{n} p^0 \]
To go beyond this minimal interpretation in the direction of our many worlds theory we will need to make an assumption about probability. The interpretation of probability is an enormous problem for Everett-style theories, and one which we will not make any attempt to solve here. We will instead make the (hopefully minimal) assumption that probabilities are attached in a basic way to worlds. For whichever part of a state we take to be a single world, the square of the associated coefficient is the probability given to that world. Beyond this, we keep silence.

The standard interpretation of a probability distribution in quantum mechanics is as the distribution of the outcomes of measurements on an ensemble of similarly prepared states. Given this, the relations (2) are statistical relations for an ensemble of states, and therefore give only a statistical relation for an individual system. However, in our neo-Evenettian interpretation we do not need to invoke an ensemble to make sense of our statistics as we have one within a single state { an ensemble of worlds and their associated probabilities. Probability distributions are therefore distributions of probability over worlds, and refer to single systems.

For example, take a single qubit system in the state $\frac{1}{2} (|i\rangle + |j\rangle)$. Standardly the probability distribution would be described as something like: "do measurements in the $|i\rangle$-$|j\rangle$ basis on a large number of similarly prepared systems, and roughly 50% will be up and 50% down." In the neo-Evenett theory it refers ontologically to the fact that within the single qubit system itself can be identified two worlds in the $|i\rangle$-$|j\rangle$ basis, both of which have associated probability 0.5. The former description is, of course, still how we would gain epistemic access to the distribution: we can of course only access one world at a time by measurement.

A probability distribution is therefore a set of worlds and their associated probabilities. The uncertainty relations (2) refer to two such sets that can be identified within the same state { the $|i\rangle$-worlds and the $|j\rangle$-worlds, both of which sets are found in the (single) state $|i\rangle$. The relations constrain what, given any set of $|i\rangle$-worlds, the spread of the associated set of $|j\rangle$-worlds is, and vice-versa. They are not in one sense fundamental: for any state we could look at the states of a group generator $A$ and the states of the group and work out $W(A)$ and we would find that they satisfy (2). However the relations do tell us more than that: that we would find this for any group and its generator in any state.

Fundamentally, the relations (2) for a neo-Evenettian say that it is impossible physically to have a state whose worlds corresponding to groups and their generators have probability distributions the spreads of which violate (2). Furthermore, this is the kind of interpretation that is given to all statistical statements in neo-Evenett theory: they are concerning the worlds of a single system and their associated probabilities, which is very

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4It is worth pointing out that in our neo-Evenett view there is no ontological classical probability: all such probability arises because of quantum probabilities. We therefore cannot say, for example, we are entangled with a given world but we do not know which one": if we cannot tell by any means which of a bunch of worlds we are entangled with then its relative state is a superposition of those worlds.
di erent from the standard interpretation. As we shall see below, this will lead to physical conclusions different from the standard interpretation, as probabilistic statements constrain individual systems and not just ensembles.

4 Measurements

4.1 Joint Measurement

The uncertainty principle refers to the degree of indeterminateness in the possible present knowledge of the simultaneous values of various quantities with which the quantum theory deals...

Ever since these fateful words of Heisenberg [6, p20] (and the subsequent notorious microscope exposition), the uncertainty relations have been regarded in large portions of the literature to be fundamentally statements about the joint measurement of non-commuting observables. \You can’t measure position and momentum simultaneously\" is a very common statement and, more generally, \you can’t simultaneously measure two non-commuting observables exactly\". Such conclusion are almost invariably drawn from inequalities such as [1] and pose a number of problems.

To begin with, such a statement is, in point of fact, false. It is fairly uncontroversial that if there is a simultaneous eigenstate of two quantities then on any reasonable theory of measurement we will be able to perform a measurement giving us these quantities. Non-commuting observables do not share complete sets of simultaneous eigenkets, but nothing stops them sharing some. For example [7, p33], a system with angular momentum \( L = 0 \) is in a simultaneous eigenstate of \( L_x = 0 \) and \( L_z = 0 \), even though they do not commute.

Furthermore, and fundamentally, the relations [1] (and indeed our relations [2]) are independent of a theory of measurement [3] (they are relations pertaining to statistical distributions. Until a connection is made between the statistics of any measurement and the state of a system in a singular, given measurement situation, these relations can tell us nothing about what is or is not allowed in one measurement. Indeed, on the \"standard\" view of the relations, the distributions referred to are gained by two separate sets of multiple measurements on similarly prepared states, there is not a joint measurement in sight.

Our neo-Everett theory does, however, give us the connection between statistics and singular states \( \) and a measurement theory \( \) (and thus we are allowed to try and draw

\[ R \text{e ady}_i (a_j + b_j) i \! \! R(0) i \! \! R\text{e ady}_i (a_j + b_j) i \! \! R(1) i \! \! R \]

In words: at the start of the experiment we identify a structure that is a measuring device set to \text{Ready}', and a quantum state in a superposition of 0 and 1. Afterwards we see two structures ('worlds'), one
some inferences from the uncertainty relations within it.

We concluded in the previous section that the uncertainty relations tell us what set of jai-worlds corresponds to a given set of j ai-worlds (and vice-versa), for the worlds of a single system. We can therefore see that the set of say, jai-worlds that corresponds to a single state j i cannot be an eigenstate of jai; that is, within the state in which we can identify a single j i-world we identify many jai-worlds. This is true for all single j i- and jai-worlds: there is never a structure in which we can identify only one j i-world and one jai-world.

The implications of this for the question of joint measurement are fairly straightforward. A measuring device cannot couple to a world which is both a given j i-world and an eigenvalue of $\hat{A}$ because such a world does not exist. Such a coupling would be what is meant in our neo-Everett interpretation by ‘measurement’, so we conclude that a joint measurement of j and $\hat{A}$ is not possible.

We are able in this case, unlike with the relations [1], to be sure that our conclusion is universally valid; as [5] points out, [1] depends on the actual state being used, but [2] does not. This means we do not have the problem that [1] has (there is no possibility of joint eigenstates or of the relation being meaningless in some states.

The question now is how much of this comes from the uncertainty relations themselves. The simple fact that the states are not jointly measurable comes from their lack of shared eigenkets, and there will be many other states and observables that are not jointly measurable but which are not linked by uncertainty relations. What we can say is that the uncertainty relations relate a sub-set of states which are not jointly measurable to the underlying group structure that connects them. Without the relations they would be disparate phenomena, and the content of the relations is in relating them. Lack of joint measurement is a consequence (in a neo-Everettian framework) of the group structure, but not all states which are not jointly measurable are groups and their generators.

4.2 Repeated Measurements

We now move on to a situation that has been linked to the uncertainty relations (although to a lesser extent that joint measurement): repeated measurements. In this situation we perform a $\hat{A}$ measurement, then an a-m measurement, and then a $\hat{A}$ measurement. As is well known, we will find that the a-m measurement bears no relation to the initial. This is often taken to be a consequence of the uncertainty relations: that the two distributions cannot be equally sharp and so some of the information about the original measurement is lost when the second measurement takes place. In our neo-Everett picture we can see fairly clearly what is going on, which we have represented here in Figure 1.

A brief note is neededirst on the figure itself. The worlds represented are those where the readout of the device is '0' and the state of the system under measurement is $j_{11}$, and one where they are '1' and $j_{11}$.
Figure 1: The many worlds of a repeated measurement

\[
\begin{align*}
R(\cdot ij_1i) &= R(a_1ij_1i) \\
&= R(a_1i(j_1i + j_2i)@) & R(\cdot ij_2i) \\
R(\cdot ij_1i) &= R(a_2ij_1i) \\
&= R(a_2i(j_1i + j_2i)@) & R(\cdot ij_2i)
\end{align*}
\]
of the measuring device; as the figure makes clear, more than one other world can be identified in what are given here as single lines. The diagonal lines do not represent the splitting of worlds (a concept alien to neo-Everett theory); they can be thought of as, in a highly schematic way, showing the passage of time between the state in which we can identify one measuring device-structure, and when we can see two. Note further that we have simplified $j_i$ as $j_i$. Finally, normalization is omitted throughout.

We will assume for simplicity that we are dealing with the very restricted sets of observable outcomes $1_j, 2_j$; $a_1, a_2$. The quantities $1_j, 2_j$ on the diagram are the relative phases of the worlds. For example, the first decomposition comes from:

$$R (1j) i \cdot j i = X_{i} a_i h_{a_j} (i) j i$$
$$= h_{a_1 j} i_j a_i R (1) i + h_{a_2 j} i_j a_i R (2) i$$
$$= a_i R (1) i + a_i R (2) i$$

We therefore have:

$$= h_{a_1 j} i_j a_i = h_{a_2 j} i_j = h_1 a_i; = h_2 a_i; = h_1 a_i; = h_2 a_i.$$

We prepare our state by measuring. We then take this state as the start of our worlds-decomposition. The measuring device is in the state $R (1j) i$ and the system $j_i$. The state of the system can also be identified, structurally, as a superposition of the two $j_i$ states; therefore, when a subsequent a-m measurement is carried out we need that we can identify two states of the measuring device { $R (a_1) i$ and $R (a_2) i$}. We now have two measurement worlds, corresponding to the $j_i$ and $j_i$ worlds. Each of these worlds in turn is a superposition of the two $j_i$ states, and so we move to the last part of the diagram. Now if there were no decoherence at measurement, if the phase information between worlds is not lost to the correlations with the environment, then at this stage we would simply add up all four real states to see what our overall structure looks like. Not surprisingly, it all adds up to our initial state $R (1j) i j i$: if there is no decoherence, repeated measurement gets the initial answer the second time around. However, with decoherence the worlds at measurement become independent of each other (for all practical purposes) and cannot interfere. Thus we are left with four structures rather than one by the end of the experiment, and the information about the original state we prepared is lost; as this is carried in the phases of the worlds.

5 Conclusion

We have analyzed the uncertainty relations in terms of a neo-Everettian picture of quantum mechanics and found that they relate constraints on probability distributions over worlds for single systems. As a consequence we recover a view of the impossibility of joint measurement which is invalid on traditional arguments. We have also clearly analyzed a repeated measurement scenario in terms of neo-Everettian worlds. We have found that a single world in one basis is a whole set of worlds in another, and each way of decomposing into worlds-system is equally valid. When the bases are related as group and generator their probability distributions must satisfy the uncertainty
relations$^2$. We have seen that a neo-Everettian interpretation of such statistical relations is fundamentally different from that in the standard interpretation as they refer to single systems not ensembles. This has consequences far beyond the interpretation of the uncertainty relations. To give one example, the concept of quantum information is statistical, its definition using the density matrix of probabilities for the state, so it will have a very different physical interpretation in a neo-Everett theory from one with an ensemble theory of probability.

The impossibility of joint measurement and the loss of information in repeated measurements that we have seen are not fundamental consequences of the uncertainty relations. They are consequences of the state and how the different distributions are connected. Moreover, there are other pairs of observables not linked by uncertainty relations that are nevertheless not jointly measurable. However the relations do link together as a single set of phenomena that would otherwise have been disparate, by revealing consequences of the underlying group structure. They are not fundamental, but neither are they empty of interesting content.

Thanks

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