Zee Model Confronts SNO Data

Paul H. Frampton, Myoung C. Oh and Tadashi Yoshikawa

Institute of Field Physics, Department of Physics and Astronomy,
University of North Carolina, Chapel Hill, NC 27599-3255.

Abstract

We reexamine the solution of the minimal Zee model by comparing with the data of the SNO experiment, and conclude that the model is strongly disfavored but not yet excluded by the observations. Two extensions of the Zee model are briefly discussed both of which introduce additional freedom and can accommodate the data.
I. INTRODUCTION

To understand why neutrino masses are non-zero is one of the most important subjects in particle physics. Assuming only left-handed neutrinos $\nu_{aL}$ ($a = e, \mu, \tau$) the minimum standard model predicts $m(\nu_a) = 0$ since adding a bare Majorana mass term $M_{ab}\nu_{aL}^*\nu_{bL}$ with $M_{ab} = M_{ba}$ for these neutrinos violates gauge invariance (because of the concomitant $\bar{e}_{aL}e_{bL}$ - type terms) and hence renormalizability. Thus, further states must be added to accommodate the neutrino masses, and we look for the greatest economy and simplicity in doing this. For a review of the theory, see [1].

II. MINIMAL ZEE MODEL

The Zee model [2] is one of the most economical possible scenarios. In the Zee model, the Majorana neutrino masses are generated by a one loop diagram. The origin of the smallness of the masses come from this feature. Hence one of the present authors has discussed the comparison with the neutrino experimental data in Ref. [3]. And several other authors have also discussed it and shown similar results [4,5]. The solution of the neutrino mixing is bimaximal ($\theta_1 = \theta_3 = \pi/4, \theta_2 = 0$ with $\theta_i$ as defined below). This agrees with the atmospheric neutrino data. For the global analysis of solar neutrino data [6], this solution corresponds to the large-angle MSW (LMA) solution or the just-so vacuum oscillations (VAC). However the result from the Zee model does not agree so well with the LMA solution from the recent analysis [3] and the recent data of the super-Kamiokande [7] disfavors the VAC solution. So we need to reconsider the possibility of the LMA solution in the Zee model.

Recently the SNO group announced their experimental data of solar neutrino fluxes from $^8B$ decay measured by the charged current reaction rate, which is $\phi^{CC}(\nu_e) = 1.75 \pm 0.07^{+0.12}_{-0.11} \pm 0.05 \times 10^6 cm^{-2}s^{-1}$. By combining this with the data from the Super-Kamiokande and using the values of the total flux expected in the Standard Solar Model (SSM) [8], the
survival probability of $\nu_e$ was reported in ref. [9]. It is

$$P(\nu_e \to \nu_e) = \frac{\phi^{CC}}{\phi_{SSM}} = 0.347 \pm 0.029^{+0.056}_{-0.069}$$  \hspace{1cm} (1)$$

where the first error is from SNO and the second from the SSM (Solar Standard model) theoretical error. Using a combination of $\phi^{CC}(\nu_e)$ and $\phi^{ES}(\nu_x)$ from SNO and Super-Kamiokande leads to the estimate [9]

$$P(\nu_e \to \nu_e) = 0.322 \pm 0.076.$$  \hspace{1cm} (2)$$

This suggests the survival probability $P(\nu_e \to \nu_e)$ is nearer to $1/3$ than $1/2$ and this is new information for us to analyze.

In this paper, we rediscuss the compatibility of the answer from Zee model with the LMA solution [10] by comparing with the recent SNO data.

Under the Zee anzatz, the neutrino mass matrix in the flavor basis $(e, \mu, \tau)$ is

$$\mathcal{M} = \begin{pmatrix} 0 & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & 0 & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & 0 \end{pmatrix} = U \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^\dagger \hspace{1cm} (3)$$

where $m_1, m_2, m_3$ are the eigenvalues of $\mathcal{M}$ and $U$ is the unitary matrix to diagonalize it. $\mathcal{M}$ is real, traceless and symmetric. From the tracelessness condition,

$$m_1 + m_2 + m_3 = 0.$$  \hspace{1cm} (4)$$

This condition is a strong constraint. The mass pattern of exact solutions which satisfy the atmospheric neutrino data is [3]

$$m_1 = -m_2, \hspace{0.5cm} m_3 = 0,$$  \hspace{1cm} (5)$$

and the ratio between the two neutrino squared-mass differences is $r = \Delta_s/\Delta_a = |m_1^2 - m_2^2|/|m_1^2 - m_3^2| = 0$, where subscripts $s,a$ refer to solar, atmospheric respectively. We will examine $r > 0$ later.
With this situation, the allowed mixing matrix is the bimaximal one with $\theta_1 = \pi/4, \theta_2 = 0$ and $\theta_3 = \pi/4$, where the definition of the mixing angle is:

$$U = \begin{pmatrix}
c_2c_3 & c_2s_3 & s_2 \\
-c_1s_3 - s_1s_2c_3 & c_1c_3 - s_1s_2s_3 & s_1c_2 \\
s_1s_3 - c_1s_2c_3 & -s_1c_3 - c_1s_2s_3 & c_1c_2
\end{pmatrix}, \quad (6)$$

with $s_i$ and $c_i$ standing for sines and cosines of $\theta_i$ and the bimaximal mixing matrix is

$$U = \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
-\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\
\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}}
\end{pmatrix}. \quad (7)$$

To discuss the neutrino flux from the sun, we have to solve the neutrino propagation equation in the matter as follows:

$$i \frac{d}{dt} \begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} = \frac{1}{2E} M^2 \begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} = \frac{1}{2E} \left[ U \left( \begin{pmatrix} m_1^2 & 0 & 0 \\
0 & m_2^2 & 0 \\
0 & 0 & m_3^2 \end{pmatrix} \right) U^\dagger + \begin{pmatrix} A & 0 \\
0 & 0 \end{pmatrix} \right] \begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} \quad (8)$$

where $A = 2\sqrt{2} G_F N_e E$, $N_e$ is the density of electron neutrino in the sun, $E$ is the energy of the neutrino. On the condition of eq.(5), $|m_1| = |m_2|$ and $m_3 = 0$, $M^2 = \begin{pmatrix}
m_1^2 + A & 0 & 0 \\
0 & \frac{1}{2} m_1^2 & -\frac{1}{2} m_1^2 \\
0 & -\frac{1}{2} m_1^2 & \frac{1}{2} m_1^2
\end{pmatrix} \quad (9)$

Then, the rotation among the weak eigenstates and the mass eigenstates in the center of the Sun $(\nu_1^m, \nu_2^m, \nu_3^m)$ can be expressed as follows:

$$\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} = U_m \begin{pmatrix}
\nu_1^m \\
\nu_2^m \\
\nu_3^m
\end{pmatrix} = \begin{pmatrix}
0 & 1 & 0 \\
\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}
\end{pmatrix} \begin{pmatrix}
\nu_1^m \\
\nu_2^m \\
\nu_3^m
\end{pmatrix}, \quad (10)$$

\[1\text{There is no CP violation in Zee model.}\]
where we have taken the limit of large electron neutrino density, \((A \to \infty)\). At \(t = 0\) an electron neutrino is produced in the sun and it is composed mainly of the state \(\nu_2^m\) in the hierarchy \(^2\) we are considering in this work.

\[
|\nu_e(0) > = |\nu_2^m > .
\]

The time evolution of this state to time \(t\) is

\[
|\nu_e(t) > = e^{i \int_0^t \frac{\lambda_2}{\pi} dt} |\nu_2^m >
\]

where \(\lambda_2\) is the eigenvalue of \(M^2\) for \(\nu_2^m\) state. Since the neutrino is measured on the earth, the state of \(\nu_e\) is expressed by the mixing Eq.\((7)\). The amplitude is

\[
< \nu_e | \nu_e(t) > = \frac{1}{\sqrt{2}} e^{i \int \frac{\lambda_2}{\pi} dt}.
\]

Hence the survival probability is

\[
P(\nu_e \to \nu_e) = | < \nu_e | \nu_e(t) > |^2 = \frac{1}{2}.
\]

This is the result from the exact Zee anzatz with \(r = 0\) and is significantly disfavored by SNO data. We are led to consider a more realistic case, \(r \neq 0\) and \(|m_1| \neq |m_2|, m_3 \neq 0\).

For the more realistic case, we rewrite the mixing matrix of Eq.\((7)\) by using the following parameters which show the discrepancy from exact bimaximal mixing.

\[
c_1 = \cos \left(\frac{\pi}{4} - \xi_1\right) \sim \frac{1}{\sqrt{2}} (1 + \varepsilon_1 - \frac{1}{2} \varepsilon_1^2),
\]

\[
s_1 = \sin \left(\frac{\pi}{4} - \xi_1\right) \sim \frac{1}{\sqrt{2}} (1 + \varepsilon_1 - \frac{1}{2} \varepsilon_1^2),
\]

\[
c_2 = \cos (\xi_2) \sim (1 - \frac{1}{2} \varepsilon_2),
\]

\[
s_2 = \sin (\xi_2) \sim \varepsilon_2,
\]

\(^2\)As we show later, in the case \(m_3 \neq 0\), the mass hierarchy we need is \(m_2 > m_1 \gg m_3\). By the matter effect, the hierarchy in dense matter is \(m_2 \gg m_1 \gg m_3\). So we can define that the electron neutrino at the production point is composed mainly of the state \(\nu_2\).
\[ c_3 = \cos\left(\frac{\pi}{4} - \xi_3\right) \sim \frac{1}{\sqrt{2}}(1 + \varepsilon_3 - \frac{1}{2}\varepsilon_3^2), \]  
\[ s_3 = \sin\left(\frac{\pi}{4} - \xi_3\right) \sim \frac{1}{\sqrt{2}}(1 - \varepsilon_3 - \frac{1}{2}\varepsilon_3^2), \]

where we neglected $\varepsilon_4^x$ and $\xi_3$ is the difference of the angle from the bimaximal case and $\varepsilon_3 \equiv \sin \xi_3$. By using this expansion, we can find the following relations up to $O(\varepsilon^2)$ among the parameters from the conditions that the diagonal element of $M$ are zero,

\[ m_1 + m_2 = -2\varepsilon_3(m_1 - m_2), \]  
\[ \varepsilon_2 = 8\varepsilon_1\varepsilon_3. \]  

By these relation and the tracelessness condition $m_1 + m_2 + m_3 = 0$, we find the relation between $r = \Delta_s/\Delta_a$ and $\varepsilon_3$ to be

\[ r = \frac{|m_1^2 - m_2^2|}{|m_1^2 - m_3^2|} = \frac{8\varepsilon_3}{1 - 4\varepsilon_3^2 - 12\varepsilon_3^2}. \]  

The behavior of $r$ versus $\varepsilon_3$ is shown in Fig.1. The experimental data suggest that the ratio $r$ satisfies $r \leq 0.1$ at 90% confidence level. So to satisfy this upper bound on $r$, $\varepsilon_3$ cannot take too large values. From Fig.1, we find the magnitude of $\varepsilon_3$ is smaller than about 0.015.

From the neutrino propagation equation, Eq.(8), after replacing $m_3$ by $m_3^2 = -2\varepsilon_3\Delta$, which comes from Eq.(21) and the traceless condition, we find

\[ M^2 = M_0^2 + M_1^2 + O(\varepsilon^2), \]  

where

\[ M_0^2 = \begin{pmatrix} \frac{1}{4}\Sigma + A & 0 & 0 \\ 0 & \frac{1}{4}\Sigma & -\frac{1}{4}\Sigma \\ 0 & -\frac{1}{4}\Sigma & \frac{1}{4}\Sigma \end{pmatrix} \]  

and

\[ M_1^2 = \begin{pmatrix} \varepsilon_3\Delta & -\frac{1}{2\sqrt{2}}(\Delta + \varepsilon_1\Delta + \varepsilon_2\Sigma) & \frac{1}{2\sqrt{2}}(\Delta - \varepsilon_1\Delta - \varepsilon_2\Sigma) \\ -\frac{1}{2\sqrt{2}}(\Delta + \varepsilon_1\Delta + \varepsilon_2\Sigma) & \frac{1}{2}(\varepsilon_1\Sigma - 3\varepsilon_3\Delta) & -\frac{1}{2}\varepsilon_3\Delta \\ \frac{1}{2\sqrt{2}}(\Delta - \varepsilon_1\Delta - \varepsilon_2\Sigma) & -\frac{1}{2}\varepsilon_3\Delta & \frac{1}{2}(\varepsilon_1\Sigma - 3\varepsilon_3\Delta) \end{pmatrix} \]
in which \( \Sigma = m_1^2 + m_2^2 \) and \( \Delta = m_1^2 - m_2^2 \). In the case of \( \varepsilon_3 \neq 0 \) and \( \varepsilon_1 = \varepsilon_2 = 0 \)\(^3\), the mixing matrix which diagonalizes \( M^2 \) in the matter is

\[
U_m = \frac{1}{\sqrt{2+K^2}} \begin{pmatrix}
\sqrt{2} & K & 0 \\
-K & 1 & \frac{\sqrt{2+K^2}}{\sqrt{2}} \\
K & -1 & \frac{\sqrt{2+K^2}}{\sqrt{2}}
\end{pmatrix}
\]  

(27)

where

\[
K = \sqrt{2-A} - 2\varepsilon_3\Delta - \sqrt{(A + 2\varepsilon_3\Delta)^2 + \Delta^2} 
\]  

(28)

This means that the mixing angle \( \theta^m_3 \) satisfies the following relation,

\[
\cos \theta^m_3 = \frac{\sqrt{2}}{\sqrt{2+K^2}}, \quad \sin \theta^m_3 = \frac{K}{\sqrt{2+K^2}}.
\]  

(29)

\(^3\)The atmospheric neutrino data favored \( \varepsilon_1 = 0 \) and from Eq.(22) \( \varepsilon_2 = 0 \) is also favored.
FIG. 1. $r$ as a function of $\varepsilon_3$.

FIG. 2. The survival probability of the electron neutrino. The horizontal lines show the bounds from the SNO experiment: the solid lines exclude the SSM theoretical error while the dotted lines include it. The diagonal line is from Eq. (30) with $P_C$ set to zero.
The production point of the neutrino from $^8B$ decay is in the central core of the Sun so that the electron density $A$ has a very large value. At the production point, $\sin^2 \theta_3^m \sim 1$. This means the electron neutrino at the production point is composed mainly of $\nu_2$ as for the earlier discussion on the case $m_3 = 0$. Then the survival probability is

$$< P(\nu_e \to \nu_e) >= \frac{1}{2} - \varepsilon_3 (1 - P_c)$$

(30)

where $P_c$ is the hopping probability at the resonance point and almost zero in the LMA region. The dependence of $\varepsilon_3$ on the survival probability is shown in Fig.2. The constraint from SNO experimental data is given in Eqs. (1) and (2). To satisfy this condition, the parameter, $\varepsilon_3$, would need to be larger than 0.1. But then the value of $r$ becomes significantly greater than 1 and this is very hard to accommodate within the data on $\Delta_a$ and $\Delta_s$. The data suggest $\Delta_a > 1.5 \times 10^{-3} eV^2$ [11] while $\Delta_S < 2 \times 10^{-4} eV^2$ [12,13] at 90% C.L.

We conclude that the minimal Zee model is strongly disfavored but not yet fully excluded by the SNO/SuperKamiokande results. For the minimal Zee model to survive, one would need $\Delta_S$ to get larger than $\Delta_a$.

### III. EXTENSIONS OF THE MINIMAL ZEE MODEL

Before looking at specific extended Zee models we consider adding diagonal elements in the neutrino mass matrix and relaxing the tracelessness condition to the more general

$$m_1 + m_2 + m_3 = \delta \times m_1$$

(31)

where $\delta$ is the shift from 0 and normalized by $m_1$. Then, we request the $M_{ee} = 0$ in order to avoid the difficulty from double $\beta$ decay. We then find the constraints as follows:

$$m_1 + m_2 = -2\varepsilon_3 (m_1 - m_2),$$

(32)

$$m_3 = \frac{4\varepsilon_3 + \delta - 2\varepsilon_3 \delta}{1 - 2\varepsilon_3} m_1.$$

(33)
From these conditions,

\[ |m_1^2 - m_2^2| = \frac{8\varepsilon_3}{(1 - 2\varepsilon_3)^2}m_1^2, \quad (34) \]

\[ |m_1^2 - m_3^2| = \frac{(1 - \delta^2 - 4\varepsilon_3(1 - \delta)^2 - 4\varepsilon_3^2(3 - \delta)(1 - \delta))m_1^2}{(1 - 2\varepsilon_3)^2}. \quad (35) \]

To satisfy the survival probability result requires that \( \varepsilon_3 \) is greater than about 0.1, while \( r \) must be smaller than 0.1. However this condition is not satisfied unless we keep the mass relation \( m_1 > m_3 \). The correlation between the ratio \( r \) and \( m_3/m_1 \) when \( \varepsilon_3 = 0.1, 0.15 \) and 0.2 is shown in Fig.3. In this case, to satisfy the conditions from experiments, the mass hierarchy is \( |m_3| \sim |\delta m_1| = 2|M_{\mu\mu}| > |4m_1| > |m_2| > |m_1| \) and the parameter \( \delta \) has to be larger than about 4 from Fig.3. So a very large deviation from the tracelessness condition is needed.

![FIG. 3. The correlation between \( r \) and \( m_3 \)](image)

We consider the case that the all diagonal elements of \( M \) are not zero. Here we parameterize the diagonal elements as follows,

\[ M_{ee} = \delta_{ee} \times m_1, \quad (36) \]

\[ M_{\mu\mu} = M_{\tau\tau} = \delta_{\mu\mu} \times m_1. \quad (37) \]

Then, by the angles we defined in Eqs. (15) - (20), the relations Eqs. (31) and (32) are changed
to
\[ m_1 + m_2 + m_3 = (\delta_{ee} + 2\delta_{\mu\mu})m_1, \quad (38) \]
\[ m_1 + m_2 + 2\varepsilon_3(m_1 - m_2) = 2\delta_{ee} \times m_1. \quad (39) \]

From these relations,
\[ m_3 = 2\varepsilon_3(m_1 - m_2) - (\delta_{ee} - 2\delta_{\mu\mu})m_1. \quad (40) \]

And then,
\[ r = \frac{4(1 - \delta_{ee})(2\varepsilon_3 - \delta_{ee})}{(1 - 2\varepsilon_3)^2 - \{(1 - 2\varepsilon_3)(\delta_{ee} + 2\delta_{\mu\mu}) - 2(2\varepsilon_3 - \delta_{ee})\}^2}. \quad (41) \]

In Fig.4, \( \Delta_S = r\Delta_a \), with \( \Delta_a = 3 \times 10^{-3}eV^2 \) is plotted versus \( \delta_{ee} \) using Eq.(41). To reduce \( r \), \( \delta_{ee} \) should be near to \( 2\varepsilon_3 \) (or to 1), because the numerator of eq.(41) has the factor of \( (2\varepsilon_3 - \delta_{ee}) \). Hence, this condition will admit consistency with the LMA solution. However we have to note that \( \delta_{ee} \) is constrained by neutrinoless double \( \beta \) decay experiment. In this case \( (\delta_{\mu\mu} = 0) \), the mass pattern is
\[ m_1 \sim -m_2, \quad m_3 \sim M_{ee} \sim 2\varepsilon_3m_1, \quad (42) \]
and \( m_1 \sim O(10^{-2})eV \). This is not yet excluded by neutrinoless double \( \beta \) decay experiment.

In this discussion, we found the element \( M_{ee} \) is important to realize the LMA solution from the model based on the Zee model.
FIG. 4. $\Delta_S$ plotted versus $\delta_{ee}$ using Eq. (41) for a fixed $\Delta_a = 3 \times 10^{-3} eV^2$.

In the light of these remarks we now consider briefly two extensions which introduce more freedom and can accommodate all the present data: (1) More than one independent Higgs doublet coupling to charged leptons, enabling off-diagonal flavor vertices for those doublets without a vacuum value [14]. (2) Addition of a singlet doubly-charged scalar [15–17].

The neutrino Majorana mass in extended model (1) arises from the graph in Fig. 5.

FIG. 5. Majorana neutrino mass for the minimal Zee model or for the extended model (1)

FIG. 6. $\mu \rightarrow e\gamma$ in extended model (1)
The Yukawa couplings in this model are

\[ g_{ij}^{\Psi_L \phi_1 l_R} + g_{ij}^{\Psi_L \phi_2 l_R} + h.c. \]  \hspace{1cm} (43)

where \( \Psi_L \) is left-handed lepton doublet and \( l_R \) is right-handed charged lepton. The \( \phi_i \) are Higgs doublets. After SU(2) symmetry breaking the remaining charged Higgs coupling is

\[ \nu_L \left[ \sqrt{2} \frac{v^2_i}{v_1} m^i \delta^{ij} \right] l_R \frac{1}{\sqrt{v_1^2 + v_2^2}} \left( \sin \theta H_1^+ - \cos \theta H_2^+ \right) \]  \hspace{1cm} (44)

where \( \theta \) is mixing angle between the charged Higgs and the singlet Zee scalar, \( m^i \) is the mass of charged lepton, \( v_i \) are the vacuum expectation values for the Higgs bosons and \( H_i^+ \) are mass eigenstates. The \( g_2 \) has flavor off-diagonal elements and by this feature diagonal neutrino masses arise from Fig. 5.

The off-diagonal couplings also contain a \( \mu \to e\gamma \) flavor violating decay \[18\] arising from the diagram of Fig. 6. The branching ratio \[18\] is

\[ BR(\mu \to e\gamma) \propto \frac{a}{48\pi G_F^2} \left( \frac{v^2_1 + v^2_2}{v_1^2} \right) \left| g_2^{\nu \mu \gamma} \right|^2 \frac{\sin^2 \theta}{M^2_1} \hspace{1cm} (45) \]

where \( g_2 \) is the Yukawa coupling with flavor off-diagonal elements. The experimental bound is \( BR(\mu \to e\gamma) < 1.2 \times 10^{-11} \). From this bound,

\[ \frac{\left| g_2^{\nu \mu \gamma} \right|^2}{M^4} < 2 \times 10^{-17} (GeV)^{-4}, \]  \hspace{1cm} (46)

where \( \frac{1}{M^2} = \frac{\sin^2 \theta}{M^2_1} + \frac{\cos^2 \theta}{M^2_2} \). If we take \( M \sim 100 GeV \), this implies

\[ g_2 < (4 \sim 7) \times 10^{-3}. \]  \hspace{1cm} (47)

While, the mass elements are

\[ \delta_{ee} m_1 = M_{ee} \propto \frac{v^2_1 + v^2_2}{v_1} \mu g_2 f_{\mu \mu} m_\mu, \]  \hspace{1cm} (48)

\[ \frac{m_1}{\sqrt{2}} \sim M_{e\mu} \propto \frac{v_2}{v_1} \mu f_{e\mu} m^2_\mu. \]  \hspace{1cm} (49)

\[ ^4\text{We neglected the contribution from the first term in Eq.(44) in this discussion because it is already smaller than the } g_2 \text{ term if } g_2 \sim O(10^{-3}). \]
where $f_{e\mu}$ is the Zee scalar coupling and we neglected the term of $f_{e\tau}m_{\tau}$ because $f_{e\tau}/f_{e\nu} \propto m_{\mu}/m_{\tau}$ from $|M_{e\mu}| \sim |M_{e\tau}|$. The ratio is

$$\sqrt{2}\delta_{ee} \sim \frac{M_{ee}}{M_{e\mu}} \propto \frac{v_1^2 + v_2^2}{v_2m_{\mu}}g_2.$$ (50)

We need this ratio to be near to $2\sqrt{2}\epsilon_3$. So we find:

$$g_2 \sim 2\sqrt{2}\epsilon_3m_{\mu}\frac{v_2}{v_1^2 + v_2^2} \sim O(10^{-4}).$$ (51)

estimated for the case $v_1 = v_2$. This condition will then comply with the bound from $\mu \to e\gamma$. Then extended model (1) is consistent with all the data.

Finally we consider extended model (2) which contains new couplings with a doubly-charged scalar singlet $k^{++}$:

$$h_{XX} l_R^X CL_R^X k^{++} + h.c. + \kappa(h^+h^-k^{--} + h^-h^+k^{++}),$$ (52)

where $k^{++}$ is a doubly charged $SU(2)$ singlet scalar which couples to only right handed charged leptons. The contribution to the Majorana neutrino mass now comes from the two loop diagram of Fig.7.

![FIG. 7. Majorana neutrino mass from two loop diagram in the extended Model (2).](image)

The diagonal mass element is

$$M_{ee} \propto \frac{\kappa}{(4\pi)^4}[h_{\mu\mu} \frac{m_{\mu}^2}{m_k^2} f_{e\mu}^2 + h_{\tau\tau} \frac{m_{\tau}^2}{m_k^2} f_{e\tau}^2]F \sim \frac{\kappa}{(4\pi)^4}h_{\mu\mu} \frac{m_{\mu}^2}{m_k^2} f_{e\mu}^2.$$ (53)
where $m_X (X = \mu, \tau)$ is the charged lepton mass, $m_k$ is the mass of doubly charged scalar and $F$ show some log function. Here we neglected the term $f_{e\tau}^2$ for the same reason we neglected it in Eq.(48). To get the required mass element, $M_{ee}$ has to be around $2\epsilon_3 m_1$. Namely, the ratio between the diagonal and off-diagonal elements of the neutrino mass term should be near to $2\epsilon_3$.

$$\frac{M_{ee}}{M_{\mu\mu}} \sim \frac{1}{16\pi^2} \frac{\kappa v_1 m_h^2}{\mu v_2 m_k^2} h_{\mu\mu} f_{e\mu}$$

We may always realize this condition which depends on unknown parameters, and hence the extended model (2) is consistent with all the neutrino data.

**IV. DISCUSSION**

The minimal Zee model is very economical as a simple way to introduce neutrino mass into the standard model. However, it is seen to be barely consistent with the combination of SuperKamiokande and SNO data. It would need $\Delta_S > \Delta_a$ which looks like a considerable stretch from the observations at hand.

On the other hand, if we enrich the model by either (1) adding further Higgs doublets coupling to the charged leptons and to the singlet charged scalar or (2) by adding a doubly-charged singlet scalar, there is enough freedom to accommodate the SNO data.

**Acknowledgments.** This work was supported in part by the US Department of Energy under Grant No. DE-FG02-97ER-41036.
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