Impact of oscillations of shafts on machining accuracy using non-stationary machines

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Abstract. The solution of the problem of restoring parts and units of equipment of the large mass and size is possible on the basis of the development of the research base, including the development of models and theoretical relations, revealing complex reasons for causes of damage and equipment failure. This allows one to develop new effective technologies of maintenance and repair, implementation of which ensures the efficiency and durability of the machines. The development of new forms of technical maintenance and repair of equipment, based on a systematic evaluation of its technical condition with the help of modern diagnostic tools can significantly reduce the duration of the downtime.

1. Introduction

The preferred method of restoring surfaces and the function of the components depends on the type of the installed defect of the material, particularly its structural elements, technology of manufacture and assembly and operating conditions [1-6]. Technical and economic indicators, which have an important influence on the choice of the method of restoration parts and units of equipment, should also be taken into account.

2. The main part

Depending on design, features of large shafts can be analyzed as single-shaft, dual-mass shafts with several disks, shafts with evenly distributed mass, and others.

Processing non-stationary machines, knowing the system mass distribution of the shaft and setting the speed, it is possible to determine the size and the shape of the shaft deflection at the supports. Thus the trajectory of the cutting tool is set, in the case of the support module of rigid stationary, and the movement of the axis of rotation of the shaft treated is not tracked [7-11].

On the shaft there is a pronounced increase of the diameter, and if in this place the mass has considerably more weight of the main shaft (Fig. 1), the resulting system can be viewed as a single-mass.

During rotation, centrifugal force causes the bending axis of the shaft. The equation of motion of the mass centre can be represented as:

$$\frac{md^2 y}{dt^2} = -F$$

Elastic force $F$ will return the shaft to a neutral position and can be represented as:
\[ F = j_{\text{shaft}} y \]

where \( y \) – the amount of deflection, \( j_{\text{shaft}} \) – the stiffness of the shaft.

Thus, bringing the shaft to an equilibrium position can be represented as follows:

\[ \frac{md^2y}{dt^2} + j_{\text{shaft}} y = 0. \]

Circular frequency of the Flexural vibrations of the shaft can be set according to the equation:

\[ \omega = \frac{1}{\sqrt{j_{\text{shaft}} m}}. \]

This equation is valid for any single-mass system regardless of the location of the displaced mass, but in this case value \( j_{\text{shaft}} \) will change. Then the solution of the previous equation under given conditions allows one to set the offset of the center of mass at \( y \):

\[ y = y_0 \cos(\omega t + \varphi). \]

Thus, using this equation, it is possible to calculate the shape of the bending of the shaft to determine the trajectory of the cutting tool.

In practice, there are often stepped shafts, which in the processing can be considered as shafts with continuously distributed mass (Fig. 2).

Let us assume that \( m(x) \) - weight length of the single shaft in cross section \( X \). In this case, \( m(x) \) takes into account the weight of the shaft and the mass of its broadening. Usually stepped shafts of large units have multiple stages of approximately the same diameter. Then, in a curved position on the shaft, a distributed load operates:

\[ q(x) = \omega^2 m(x) y(x). \]

Bending moment in cross-section \( X \) will be [12, 13]:

\[ M(x) = \int_0^x Q(x_1)dx_1 = R_1x + \int_0^x g(x_2)dx_2dx_1. \]

where \( Q(x) \) – bending strength in the cross-section:

\[ Q(x) = R_1 + \int_0^x q(x_1)dx_1. \]

From the condition of equality to zero of the bending moment in cross-section \( x = l \), one obtains:

\[ M(l) = R_1l + \int_0^l g(x_2)dx_2dx_1 = 0 \]

i.e. bending moment at the hinged support beam under arbitrary plot \( g(x) \). The bending moment can be recorded as [12, 13]:

\[ M(x) = \int_0^l q(x_2)dx_2dx_1 - \frac{1}{2} \int_0^l q(x_2)dx_2dx_1. \]

Then the base equation of bending of the shaft is:

\[ \frac{d^2y}{dx^2} = \frac{M(x)}{E\mathcal{S}(x)}, \]

where \( E\mathcal{S}(x) \) - the stiffness of the shaft in bending.

Therefore, the equation expressing the magnitude of shaft deflection at an arbitrary distribution of bending moments is [12,13]:

\[ \frac{d^2y}{dx^2} = \frac{M(x)}{E\mathcal{S}(x)} \]
\[ y(x) = \int_0^x M(x_2) dx_2 dx_1 - \int_0^x \frac{M(x_2)}{E_3(x_2)} dx_2 dx_1. \]

Therefore, knowing \( m(x) \), \( y(x) \) and \( g(x) \), it is possible to determine the line of deflection of the rotating shaft and the automatic processing mode to set the program of motion of the cutting tool. It is possible to define the rotation of the cutter along the length of the shaft in a digital form.

When rotating large-sized long length shafts with low rigidity (tube mills, etc.) because of the oval shape of the polygon surfaces due to the displacement of the center of gravity, swings can occur. This is mainly transverse vibration; torsional vibrations are practically absent due to the fact that the shafts have large diameters and small speed of rotation. The cutting forces during the deformation of the shaft of the impact were extremely limited because they are minor compared to the weight of the shafts.

So that the housing of the grinding mill has a greater length, diameter and mass, and wall thickness, which is small relatively to its diameter, then the mass can be applied to the middle of the body, which is regarded as a rotor with one disk (Fig. 3).

Let us denote the mass of disc \( M \) and make that center \( O \) as a straight shaft when rotating with a frequency of \( \omega \) shifted to point \( O_1 \). Thus, there are two points associated with the rotor coordinate system. One is when the center of the disc is not shifted and the other - the center of the disc is offset. In the first case, when the center of gravity is not offset, the origin is at point \( O \), that is \( x \) coincides with the axis of rotation of the shaft, and other two axes \( Y \) and \( Z \) coincide with the principal axes of rigidity of the shaft with the unshifted center of gravity. The second system has the origin at point \( O_1 \) and axes \( X_1, Y_1, Z_1 \) coincide with the axes of inertia of the disc (Fig. 4).

**Figure 1.** The deflection of the rotating shaft with eccentric mass

\[ g(x) = \omega^2 m(x)y \]

\[ m(x) \]
Figure 2. Bending the rotating shaft with uniformly distributed masses

Figure 3. The position of coordinates on the rotating rotor

Figure 4. The offset of the axes of the shaft at the offset of the center of gravity

The moment of inertia of the disk relative to these coordinate systems can be written as:

\[ I_x = I_0; \quad I_y = I_1; \quad I_z = I_2. \]

The displacement of the center of gravity to point \( O_1 \) causes displacement of the axes, i.e. coordinate system \( O_1, X_1, Y_1, Z_1 \) shifts relative to coordinate system \( O, X, Y, Z \), linear value \( e_1 = e_1, e_x = e_2 \), and angular size \( \gamma_1 = \gamma_1, \gamma_2 = \gamma_2 \). These values characterize the magnitude of the inertial forces that develop due to the displacement of the center of gravity of the shaft.

If the center of gravity is not shifted, original coordinate system \( O_0, X_0, Y_0, Z_0 \) is located in the center of gravity and axis \( X \) goes through the centre of supports of the shaft. Let us set the rotation of a shaft rotating at the angle \( \Phi \); then axes \( O, X, Y, Z \) with respect to axes \( Y_0 \) and \( Z_0 \) occur at some value \( U_{YO} \) and \( U_{ZO} \), the rotation around these axes can be represented as \( \gamma_{YO} \) and \( \gamma_{ZO} \). In this case, axes
$X_0$ and $X$ coincide (Fig. 4).

For axes $O_1, X_1, Y_1, Z_1$, linear and angular displacement will be in the form: 

$$U_{1YO} \cdot \Phi_{1YO} \cdot \Phi_{1ZO}.$$

Therefore, when a small shaft is displaced, the relationship between displacement coordinates $O, X, Y, Z$ and $O_1, X_1, Y_1, Z_1$ can be represented as:

$$U_{1YO} = U_{YO} + e_1 \cdot \sin \varphi - e_2 \cdot \sin \varphi,$$

$$U_{1ZO} = U_{ZO} + e_1 \cdot \sin \varphi + e_2 \cdot \cos \varphi,$$

$$\Phi_{1YO} = \Phi_{YO} + \gamma_1 \cdot \cos \varphi - \gamma_2 \cdot \sin \varphi,$$

$$\Phi_{1ZO} = \Phi_{ZO} + \gamma_1 \cdot \sin \varphi + \gamma_2 \cdot \cos \varphi.$$

Vibrations along axis $X$ are not considered due to the fact that large trees have powerful support units, and the whole system of the grinding mill does not allow it to move in the longitudinal direction. In addition, one of the pillars captures the mill (shaft) of the longitudinal displacement.

As a result of vibration of the shaft, there is a linear shift of the center of gravity $E = \sqrt{e_1^2 + e_2^2}$, which leads to the shaft bending.

In connection with the bending of the shaft, the forces of friction are in horizontal (axis $Y$) and vertical (axis $Z$) planes [12]:

$$P_y = -Cu_{YO} - ku'_y,$$

$$P_z = -Cu_{ZO} - ku'_z - Mg,$$

where $C$ – elastic forces; $u$ – the amount of movement; $u'$ – the speed of movement; $k$ – the coefficient of proportionality.

3. Conclusion
As a result of applying the above-mentioned methodology to increase the period of operation of large rotating equipment, the authors determined that, if the shaft has an offset center of gravity, with the source data, it is possible to calculate the shaft deflection during processing and to adjust the movement of the cutting tool.

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