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Becchi-Rouet-Stora-Tyutin-Lagrangian Double Copy of Yang-Mills Theory

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We show that the double copy of gauge theory amplitudes to $\mathcal{N} = 0$ supergravity amplitudes extends from tree level to loop level. We first explain that color-kinematics duality is a condition for the Becchi-Rouet-Stora-Tyutin operator and the action of a field theory with cubic interaction terms to double copy to a consistent gauge theory. We then apply this argument to Yang-Mills theory, where color-kinematics duality is known to be satisfied on shell at the tree level. Finally, we show that the latter restriction can only lead to terms that can be absorbed in a sequence of field redefinitions, rendering the double copied action equivalent to $\mathcal{N} = 0$ supergravity.

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Introduction and summary.—Yang-Mills scattering amplitudes have been conjectured to satisfy a color-kinematics (CK) duality [1–3]: each amplitude can be written as a sum over purely trivalent graphs such that the kinematical numerators satisfy the same antisymmetry and Jacobi identities as the color contributions. CK duality has been shown to hold at the tree level [4–12]. If it holds, replacing the color contributions of a Yang-Mills amplitude with another copy of the kinematical contributions yields a gravity amplitude [3]. This is known as the double copy prescription, and it has far reaching consequences for our understanding of quantum gravity. For reviews and references see Refs. [13–15].

Explicit nontrivial examples [2,16–36] have suggested that the double copy extends to the loop level (i.e., to the integrands of loop amplitudes). In this Letter, we argue that this is indeed the case to any finite loop order.

Our approach builds on the ideas of manifestly CK-dual classical kinematic structure constants and Lagrangians [3,37–44]. A key ingredient in our argument is the Becchi-Rouet-Stora-Tyutin (BRST) formalism and its enlarged field space of external states [45]. We extend the idea that the BRST framework can be double copied [44,46–51] and double copy the complete BRST Lagrangian. See also Ref. [32] for a powerful approach to loop-level CK-dual amplitudes using the BRST invariance of the underlying pure spinor superstring.

We make the crucial observation that on-shell, CK duality violations due to longitudinal gluon modes can be compensated by harmless field redefinitions of the Nakanishi-Lautrup (NL) field. The Ward identities of the BRST symmetry then allow us to transfer CK duality from gluon amplitudes to those involving ghosts. Finally, on-shell treel level CK duality on the BRST-extended field space turns out to suffice to show that the BRST-Lagrangian double copied theory provides the loop integrands of a consistent perturbative quantization of $\mathcal{N} = 0$ supergravity. We stress that our results do not imply or rely on loop-level CK duality.

A longer paper giving explicit expressions for many of the steps discussed only abstractly in the following and explaining the origin of the double copy in terms of homotopy algebras, mathematical objects unifying scattering amplitudes and BRST Lagrangians, is in preparation [52]. There, we also intend to make a connection to the observed nontrivial modifications of CK duality at the loop level, cf., e.g., Refs. [53,54].

The BRST-Lagrangian double copy.—We start with an abstract perspective on the double copy. Any Lagrangian field theory is equivalent to a field theory with exclusively cubic interaction terms, by blowing up higher order vertices using auxiliary fields, cf. also Refs. [55,56]. A generic cubic action is

$$ S = \frac{1}{2} \Phi^I g_{IJ} \Phi^J + \frac{1}{3!} \Phi^I I_{JK} \Phi^J \Phi^K, $$

where the fields $\Phi^I$ are elements of some field space $\mathcal{F}$ and the DeWitt index $I$ encodes all field labels (including...
We require that fields split into “left” and “right” components (with independent left and right ghost numbers), but over a common space-time point. Consequently, we expand the DeWitt indices as

$$f_{\ldots} \equiv f_{\ldots}^{\alpha} \delta_{\ldots}^\alpha,$$

with $f_{\ldots}^{\alpha}$ and $\delta_{\ldots}^\alpha$ graded (with respect to the ghost numbers) symmetric, and $f^{\ldots}_{\ldots\ldots\ldots}$ etc., differential operators with constant coefficients. The indices $A$ and $\bar{A}$ range over the summands in $f_{\ldots\ldots}$. To simplify notation, we define

$$f_{\ldots\ldots} \equiv g_{\ldots} \tilde{g}_{\ldots} \tilde{f}_{\ldots} \delta_{\ldots}^\alpha \Phi^{\alpha\beta} \Phi^{\beta\gamma}.$$

Suppressing the position dependence, the Lagrangian of the theory becomes

$$\mathcal{L} = \frac{1}{2} \Phi^{\alpha\beta} g_{\ldots} \tilde{g}_{\ldots} \tilde{f}_{\ldots\ldots} \Phi^{\alpha\beta} + \frac{1}{3!} \Phi^{\alpha\beta\gamma} f_{\ldots\ldots\ldots\ldots} \Phi^{\alpha\beta\gamma},$$

(4)

where we used the shorthand $\tilde{f}_{\ldots\ldots} = f_{\ldots\ldots}$ for the evident expression in (3c). Analogously, we want the BRST operator to act on left and right indices separately, and we split $Q = Q_L + Q_R$ with

$$Q_L \Phi^{\alpha\beta} = q^{\alpha\gamma} g^{\beta\delta} \Phi^{\alpha\gamma} + \frac{1}{2} q_{\alpha\beta} \tilde{f}^{\alpha\gamma} \delta_{\beta}^\gamma \Phi^{\alpha\beta} + \frac{1}{3!} q_{\mu\alpha\beta} \tilde{f}^\mu_{\alpha\beta} \delta^{\gamma\delta} \Phi^{\alpha\beta} \Phi^{\gamma\delta},$$

(5)

where $\tilde{f}^{\alpha\beta}_{\ldots\ldots} = \frac{3}{2} \tilde{f}^{\alpha\beta}_{\ldots\ldots\ldots}$ and similarly for $Q_R \Phi$. As an example, consider the special case of cubic Yang-Mills theory, where the $q_{\alpha\beta}$ and $\tilde{f}^{\alpha\beta}_{\ldots\ldots}$ are the components of the Killing form and the structure constants of a gauge algebra, respectively, while $\tilde{g}_{\ldots}$ and $\tilde{f}^{\alpha\beta}_{\ldots\ldots}$ are the inner product and kinematical structure constants on the full BRST field space.

To double copy means to replace the left (or right) sector with a copy of the right (or left) sector of some, not necessarily the same, theory written in the form (4), (5). If the resulting action $S$ and BRST operator $Q$ satisfy again the relations $Q^2 = 0$, $QS = 0$, we obtain a consistently gauge-fixed theory ready for quantization.

It is not hard to see that $Q_{L/R}^2 = 0$ iff $Q_{L/R}^2 = 0$; the condition $Q_1, Q_0 + Q_0, Q_1 = 0$ may induce further conditions. For Yang-Mills theory, one readily computes that $C_K$ duality suffices for the condition $QS = 0$. If CK duality fails to hold up to certain terms, then $QS = 0$ also fails to hold up to the same terms, possibly multiplied by other fields and their derivatives. (Mathematically, the terms describing the failure of CK duality generate an ideal in the algebra of fields and their derivatives. The expressions $QS$ and $Q^2$ take values in this ideal.)

Preliminary observations.—We are interested in perturbative aspects and omit any nonperturbative issues. Also, we are interested in $n$-point amplitudes up to $\ell$ loops for $n$ and $\ell$ finite. Thus, there is always a number $N \in \mathbb{N}$ so that monomials of degree $m > N$ can be neglected in the Lagrangian. We always use the term “amplitude” for on-shell states and the term “correlator” for off-shell states.

Although the quantization of Yang-Mills theory does not require it, it is convenient to start from the BV form [57] of the Yang-Mills Lagrangian on Minkowski space, using canonical notation for all fields,

$$\mathcal{L}_{YM} := -\frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} + A_\mu^{\alpha\beta} (\nabla^a c)^\alpha + \frac{g}{2} f_{\mu\nu\rho} c^\mu c^\nu c^\rho + b^\alpha c^\alpha,$$

(6)

with $g$ the Yang-Mills coupling constant. We use the gauge fixing fermion $\Psi := \Psi_0 + \Psi_1$ with

$$\Psi_0 := \int d^4x \bar{c}^a \left( \frac{\xi}{2} b^a - \partial^a A_\mu^{\alpha\beta} \frac{\xi}{2} b^a A_\mu^{\alpha\beta} \right), \quad \Psi_1 := \int d^4x \bar{c}^a \psi^a,$$

(7)

where $\psi^a$ is of ghost number zero and depends at least quadratically on the fields and their derivatives. We obtain the gauge-fixed Lagrangian

$$\mathcal{L}_{YM}^g = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{c}^a \partial\alpha (\nabla^a c)^\alpha + \frac{\xi}{2} (b^a)^2 - b^\alpha \partial^a A_\mu^{\alpha\beta} A_\mu^{\alpha\beta} + \frac{\delta \Psi_1}{\delta A_\mu^{\alpha\beta}} (\nabla^a c)^\alpha + \frac{\xi}{2} (b^a)^2 - b^\alpha \partial^a A_\mu^{\alpha\beta} A_\mu^{\alpha\beta},$$

(8)

For $\psi^a = 0$, we recover the $R_\xi$ gauges. The BRST transformations are
satisfying $Q_{\text{YM}}^2 = 0$ off shell.

The nonphysical fields enlarge the one-particle field space of asymptotic on-shell states by four types of states: the two unphysical polarizations of the gluon, called forward and backward and denoted by $A^1$ and $A^1$, and the ghost and antighost states [45]. All amplitudes will be built from the $n$-particle form of this BRST-extended on-shell field space, which carries an action of the linearization of $Q_{\text{YM}}$. The physical polarizations are singlets, $Q_{\text{YM}}^n A_{\text{phys}} = 0$, and we have two more doublets:

$$A^1 \xrightarrow{\mathcal{O}_1^{\text{lin}}} c \quad \text{and} \quad \bar{c} \xrightarrow{\mathcal{O}_n^{\text{lin}}} b = \frac{1}{\xi} \partial^\mu A^\mu_n + \cdots,$$

where the ellipsis indicates terms that arise from $\Psi_1$.

**Observation 1:** The set of connected correlation functions is BRST invariant because they can be written as linear combinations of products of correlation functions.

Crucial to our discussion are the supersymmetric Ward identities generated by the BRST operator. We start with the on-shell form, see, e.g., Refs. [58,59]. Since the free vacuum is invariant under the action of $Q_{\text{YM}}^\text{lin}$, we have the following on-shell Ward identities:

$$0 = \langle 0 | Q_{\text{YM}}^\text{lin} \mathcal{O}_1 \cdots \mathcal{O}_n | 0 \rangle.$$

We now consider the on-shell Ward identity for $\mathcal{O}_1 \cdots \mathcal{O}_n = A^1 \bar{c}(c\bar{c})^k A^{n-2k-2}_\text{phys}$ and obtain

$$0 \langle (c\bar{c})^{k+1} A^{n-2k-2}_\text{phys} | 0 \rangle \sim 0 | A^1 (c\bar{c})^k b A^{n-2k-2}_\text{phys} | 0 \rangle.$$

Thus,

**Observation 2:** Any amplitude with $k + 1$ ghost-antighost pairs and all gluons transversely polarized is given by a sum of amplitudes with $k$ ghost pairs.

From the construction of amplitudes via Feynman diagrams, it follows that we also have the following on-shell Ward identity for an approximate BRST symmetry.

**Observation 3:** Suppose that $QS = 0$ and $Q^2 = 0$ only on shell. Then, we still have Eq. (11) together with a corresponding identification of amplitudes with $k + 1$ ghost-antighost pairs and all gluons transversely polarized and a sum of amplitudes with $k$ ghost pairs.

We shall also need the off-shell form of these Ward identities,

$$\partial^\mu \langle j_\mu(x) \mathcal{O}_1(x_1) \cdots \mathcal{O}_n(x_n) \rangle = \sum_{i=1}^n \delta(x-x_i) \langle (QS \mathcal{O}_i(x_i)) \Pi_{j\neq i} \mathcal{O}_j(x_j) \rangle.$$

Next, we make the following three general observations:

**Observation 4:** The on-shell relations between tree amplitudes from observation 2 induced by Eq. (11) extend to (off-shell) tree-level connected correlators. For example,

$$\langle A_\mu(x_1) b(x_2) A_\nu(x_3) \rangle = \langle \partial_\mu c(x_1) \bar{c}(x_2) A_\nu(x_3) \rangle + \langle A_\mu(x_1) \bar{c}(x_2) \partial_\nu c(x_3) \rangle.$$

If $X^a$ is independent of the NL field $b^a$, this modification preserves the theory at the quantum level by observation 7. Furthermore, if $X^a$ is at least quadratic in the fields, this transformation preserves the action of $Q_{\text{YM}}^\text{lin}$ on the BRST-extended on-shell field space.

Consider now the special case $\psi^a = 0$ and $X^a$ independent of $b^a$ and fix $Y^a$ iteratively such that the terms linear in $b^a$ of Eq. (15) vanish:

$$\xi X^a + \frac{\delta \Xi_1}{\delta \bar{c}^a} = \xi X^a + Y^a + \bar{c}^b \partial X^a + \cdots = 0.$$

This leads to the following observation:
Observation 8: Terms in the Lagrangian of the form $(\partial^\mu A_\mu)^a X^a$ with $X^a$ at least quadratic in the fields and their derivatives but independent of the NL field can be removed in $R_\xi$ gauges by shifting the NL field. This creates additional terms (15) which are at least of fourth order and preserve the amplitudes by observation 7.

Observation 9: Terms in the action that are proportional to a NL field can be absorbed by choosing a suitable term $\psi^a$. This leaves the physical sector invariant but it may modify the ghost sector. Because NL fields appear as trivial pairs in the BV action, it is not hard to see that this extends to general gauge theories, e.g., with several NL fields and ghosts for ghosts.

We also make the following three observations regarding the double copy.

Observation 10: The tree amplitudes of Yang-Mills theory can be written in CK-dual form [4–12].

Observation 11: For amplitudes in CK-dual form, there is a corresponding local, cubic, and physically equivalent Lagrangian whose partial amplitudes produce the kinematical numerators [39].

Observation 12: Double copying the Yang-Mills tree amplitudes in CK-dual form yields the tree amplitudes of $\mathcal{N} = 0$ supergravity [1–3].

CK-dual Yang-Mills theory.—In order to BRST-Lagrangian double copy Yang-Mills theory, we first must bring its action into the normalized form (4). Our goal will be to construct abstractly a Lagrangian that guarantees tree-level CK duality for the BRST-extended on-shell field space.

CK duality of the Feynman diagrams for the field space of physical gluons can be guaranteed by adding terms to the Lagrangian [3,39] and subsequently strictifying these, i.e., introducing a set of auxiliary fields such that all interaction vertices are cubic. This strictification is mostly determined by the color and momentum structure of the additional terms in the Lagrangian.

It remains to ensure CK duality for tree amplitudes involving ghosts or backward polarized gluon states, which we do by introducing compensating terms, preserving quantum equivalence. (Forward polarized gluons can be absorbed by residual gauge transformations and therefore do not appear in the Lagrangian. Thus, they cannot contribute to CK duality violations.)

We implement the necessary changes iteratively for $n$-point amplitudes, starting with $n = 4$. We can compensate for CK duality violations due to backward polarized gluons, which can be done by introducing terms of the form $(\partial^\mu A_\mu)^a X^a$. By observation 8, we can produce such terms, preserving quantum equivalence, and we immediately compensate for the additional terms linear in the NL field using observation 9. Since we perform all shifts at the level of the BV action and the gauge fixing fermion, the resulting action is automatically BRST invariant and its amplitudes are CK dual for external legs of ghost number 0.

By observation 2, these amplitudes fully determine all amplitudes with ghosts and antighosts on external legs. Moreover, the CK-dual form of the former can be copied over to the latter, by literally copying trivalent Feynman diagrams for the gluon modes linked by the BRST symmetry to the ghost-antighost pairs. We do this iteratively in the number of ghost-antighost pairs. The consistency of the copying process is guaranteed by the full BRST symmetry of the action. We then use observation 12 to turn these CK-dual amplitudes for arbitrary ghost number into a local, cubic, and BRST-invariant Lagrangian.

The resulting Lagrangian $\mathcal{L}_{\text{YM}}^{\text{CK}}$ is of the form (4) and quantum equivalent to the Lagrangian $\mathcal{L}_{\text{YM}}$ given in Eq. (8).

The BRST-Lagrangian double copy of Yang-Mills Theory.—We now turn to the $\mathcal{N} = 0$ supergravity side. The gauge-fixed BRST Lagrangian $\mathcal{L}_{\mathcal{N}=0}^{\text{BRST}}$ of this theory is readily constructed. The following two diagrams concisely summarize the theory’s field content from the perspective of the double copy, describing the symmetrized and antisymmetrized tensor products of two copies of the BRST Yang-Mills fields:

Here, the physical fields of ghost number 0 are $h_{\mu \nu}$ (containing the metric perturbation about the Minkowski vacuum and the dilaton) and $B_{\mu \nu}$ (the Kalb-Ramond two-form). Ghost number increases by column from left to right, and all vector or form indices are made explicit. The arrows indicate factorization relations between the various fields [52]. In addition to the expected BRST field content, we have two trivial BV pairs $(\delta, \beta)$ and $(\bar{\beta}, \pi)$, see, e.g., Ref. [64] for the same fields in a different context. For more details, see Ref. [52] as well as Refs. [44,46–49].

The double copy of $Q_{\text{YM}}^\text{lin}$ and $\mathcal{L}_{\text{YM}}^{\text{CK}}$ yields a BRST operator $Q$ which satisfies $Q^2 = 0$ on shell and a Lagrangian $\mathcal{L}$ for the field content (17). The latter is quantum equivalent to the manifestly CK dual, cubic or strict form $\mathcal{L}_{\mathcal{N}=0}^\text{st}$ of $\mathcal{N} = 0$ supergravity obtained from observation 11: (i) Kinematic equivalence.—The two kinematic Lagrangians are equivalent and linked by evident suitable field redefinitions [52]. The existence of such a field redefinition is ensured by the linear double copy BRST operator $Q_{\text{YM}}^\text{lin}$ [44,48], which is equivalent to the linear BRST operator $Q_{\mathcal{N}=0}^\text{st,lin}$ and annihilates the quadratic double copy Lagrangian [52]. We implement the field redefinition on $\mathcal{L}_{\mathcal{N}=0}^\text{st}$, obtaining $\mathcal{L}_{\mathcal{N}=0}^{\text{st,1}}$. (ii) Ghost
number 0, partly.— Since the classical Yang-Mills action was written in a form with purely cubic, local interactions with manifest CK duality to all points, the tree amplitudes of $L$ for physical fields match those of $L_{N=0}^\text{st}$, cf. observation 12. The amplitudes for auxiliaries of ghost number 0 are determined by collinear limits of amplitudes of physical fields and thus also agree between the theories. By observation 5, we can implement a field redefinition $L_{N=0}^\text{st} \rightarrow L_{N=0}^\text{st}$ such that the interaction vertices of $L_{N=0}^\text{st}$ and $L$ agree for physical and auxiliary fields of ghost number 0 to any finite order. For these fields, also the tree-level correlators agree, too. By construction, this agreement extends to individual onshell Feynman diagrams, between the theories. By observation 7 then implies that both theories are quantum equivalent.

No additional research data beyond the data presented and cited in this work are needed to validate the research findings in this work.

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