NEUTRINO FLAVOR MIXING AND OSCILLATIONS IN FIELD THEORY

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The Lagrangian that is normally associated with Dirac neutrinos is analyzed in a complete and simple way through field theory. It is found that the elements of the neutrino mass matrix are field strength renormalization constants and that the flavor fields can be applied directly to the one-particle energy states through these rescaling factors. Moreover this Lagrangian describes neutrinos which are in a state of mixed flavor at any space-time point and therefore does not describe the phenomenology of neutrino oscillations properly, except in the ultra-relativistic limit where such a description is possible.

1 Introduction

Our discussion in the present paper will employ the following Lagrangian

$$\mathcal{L} = \bar{\psi}_e (i \gamma \cdot \partial - m_e) \psi_e + \bar{\psi}_\mu (i \gamma \cdot \partial - m_\mu) \psi_\mu - \delta (\bar{\psi}_e \psi_\mu + \bar{\psi}_\mu \psi_e),$$

(1.1)

which consist of two coupled Dirac neutrino fields $\psi_e, \psi_\mu$ with masses $m_e$ and $m_\mu$ respectively. The interaction between the two fields is provided through a lepton number violating term with a coupling constant $\delta$. The model allows for exact diagonalization. Neutrino and anti-neutrino flavor wave functions can be obtained as matrix elements of the quantized neutrino fields. For a general review about neutrino physics, see for example $2^{-7}$.

It is shown that the rotation matrix elements are field strength renormalization factors. Therefore the fields that makes sense to quantize are the free renormalized fields $\psi_1, \psi_2$ obtained by the diagonalization of Eq. (1.1). Only for those fields it is possible to define creation and annihilation operators and one-particle states. The fields $\psi_e$ and $\psi_\mu$ can be applied directly to the one-particle energy states through the rotation matrix.

Moreover, the conserved charge is the sum of the electron and muon charges which are not conserved separately. Therefore the above Lagrangian describes neutrinos which are in a state of mixed flavor at any space-time point and they do not describe the possibility to have only a given flavor at a given time, such as for example at production. Different authors came to the same conclusions through different approaches from the one shown here $8^{-10}$.
The problem may be a deep one and associated with the possibility that neutrinos violate the equivalence principle.

2 Field Theory Treatment

By diagonalizing the Lagrangian defined in Eq. (1.1) we obtain the renormalized masses

\[ m_{1,2} = \frac{1}{2}[(m_e + m_\mu) \pm R], \]

(2.1)

with

\[ R = \sqrt{(m_\mu - m_e)^2 + 4\delta^2}. \]

(2.2)

Corresponding to the positive energy solutions \( E_{1,2} = \sqrt{m_{1,2}^2 + p^2} \), we have the eigenfunctions

\[ \phi_{1,2}(x, t) = \left( \begin{array}{c} \frac{1}{\sqrt{1 + M_{1,2}}^2} \\ \frac{1}{M_{1,2}} \\ \frac{1}{\sqrt{2 E_{1,2}}} u_{1,2}(s, p) e^{i p \cdot x} e^{-i E_{1,2} t} \end{array} \right), \]

(2.3)

where \( s = 1, 2 \) is the spin index, \( u_{1,2}(s, p) \) are the Dirac spinors and

\[ M_{1,2} = \frac{m_\mu - m_e \pm R}{2\delta}. \]

(2.4)

The rotation matrix \( U \) between the flavor fields \( \psi_e, \psi_\mu \) and the free fields \( \psi_1, \psi_2 \) of renormalized masses \( m_1, m_2 \) respectively can be written in terms of the flavor vectors given by Eq. (2.3) as

\[ U = \left( \begin{array}{cc} \frac{1}{\sqrt{1 + M_1^2}} & \frac{M_1}{\sqrt{1 + M_1^2}} \\ \frac{1}{\sqrt{1 + M_2^2}} & -\frac{M_2}{\sqrt{1 + M_2^2}} \end{array} \right), \]

(2.5)

where we have used the fact that \( M_1 M_2 = -1 \). It is possible to see that if we write the electron and neutrino fields \( \psi_e, \psi_\mu \) in terms of the fields \( \psi_1, \psi_2 \) by means of the rotation matrix \( U \), the interacting Lagrangian given by Eq. (1.1) becomes uncoupled, i.e.,

\[ \mathcal{L}_D = \bar{\psi}_1 (i \gamma \cdot \partial - m_1) \psi_1 + \bar{\psi}_2 (i \gamma \cdot \partial - m_2) \psi_2. \]

(2.6)

The fields \( \psi_1 \) and \( \psi_2 \) are the dynamical variables which have to be quantized through the canonical anti-commutation relations. Only for them is it possible to define creation and annihilation operators and one-particle states. The fields
ψ_e and ψ_µ can be applied to the energy one-particle states using the rotation matrix defined in Eq. (2.5). For example, the neutrino wavefunction associated with the one-particle state of defined energy \( E_1 \) (\( |1_{ps}\rangle \)) is

\[
\psi_\nu(x, t) = \begin{pmatrix} \psi_e(x, t) \\ \psi_\mu(x, t) \end{pmatrix} = \frac{1}{\sqrt{V_1}} \left( \frac{1}{\sqrt{1 + M^2_1}} \right) \int \frac{d\mathbf{p}}{\sqrt{2E_1}} \psi_e(\mathbf{x}, t) e^{i\mathbf{p} \cdot \mathbf{x}} e^{-iE_1 t}.
\]

This, being a plane wave, gives a stationary probability of finding a neutrino at a given space-time point. However in any location inside the volume \( V \) there is a probability equal to \( \left( \frac{1}{1 + M^2_1} \right) \) of finding the neutrino in the electron flavor and probability equal to \( \left( \frac{M^2_1}{1 + M^2_1} \right) \) of finding it in the muon flavor.

The constants \( \left( \frac{1}{1 + M^2_1} \right) \) and \( \left( \frac{M^2_1}{1 + M^2_1} \right) \) are field-strength renormalization factors and give the probability amplitude for the field operator \( \psi_e (\psi_\mu) \) to create a one-particle eigenfunction of definite energy.

To be able to describe neutrinos which are in a superposition of different energies, we need a superposition of one-particle states. A general state of positive charge, momentum \( \mathbf{p} \) and spin \( s \) is given by

\[
|\phi_+\rangle = |A \phi_1^+(s, p) + B \phi_2(s, p)|0\rangle,
\]

where \( A \) and \( B \) specify the amount of each normal mode state of positive energy present in the state \( |\phi_+\rangle \) and

\[
|A|^2 + |B|^2 = 1.
\]

However it is possible to show that even the flavor wavefunctions associated with the state given by Eq. (2.8) describe neutrinos which are in a state of mixed flavor at any space-time point. Therefore they do not describe properly the neutrino oscillation phenomenology, where it is assumed that only one flavor is present at production. The standard neutrino oscillation probabilities can be recovered by making some ad hoc ultra-relativistic approximations in the wavefunctions.

The generalization of the model described here to two component Majorana neutrinos and to the CKM matrix for the quark sector can be found respectively in Ref.[13] and Ref. [14].
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