Crack front échelon instability in mixed mode fracture of a strongly nonlinear elastic solid

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Abstract – In order to assess the role of elastic nonlinearity in gel fracture, we study the échelon instability in gelatin under mixed mode tensile and antiplane shear loading — i.e. the emergence of segmented crack front structures connected by steps. We evidence the existence of an energy-release-rate–dependent mode mixity threshold. We show that échelons appear via nucleation of localized helical front distortions, and that their emergence is the continuation of the cross-hatching instability of gels and rubbers under pure tensile loading, shifted by the biasing effect of the antiplane shear. This result, at odds with the direct bifurcation predicted by linear elastic fracture mechanics, can be assigned to the controlling role of elastic nonlinearity.

Owing to their high solvent content and to elastic properties mimicking those of biological tissues, hydrogels are the focus of intensive research as candidate materials for various biomedical purposes, e.g. tissue engineering or controlled drug delivery. The recent development of a wealth of high toughness gels [1,2] paves the way to the use of these soft solids as structural materials. Load-bearing applications, such as scaffolds for cartilage replacement, ask for the control of fracture properties under complex mechanical and environmental conditions in these materials which present a unique combination of ultra-high elastic deformability and toughness. This raises in particular the question of whether or not elastic nonlinearities at work in the tip region [3] play a crucial part in crack nucleation and path selection. For this purpose, in the spirit of the physics of dynamical systems, as shown by Bouchbinder [4], the study of crack front instabilities appears as a choice tool.

With this question in mind, we study here the échelon instability in (I + III) mixed mode fracture in a gelatin gel. While mode I (pure tension) crack surfaces are basically planar, they generally develop, under superimposed antiplane (mode III) shear loading [5], an “échelon” structure [6]. This can be roughly described as resulting from a crack front shape composed of a set of rotated segments connected by steep steps. Such échelon patterns are quite ubiquitous among materials: observations range from hard (glasses [7–9], rocks [10], metals, ...) to soft (gypsum, cheese [11]) solids.

Up to now, the few theoretical attempts [8,10,12] based on standard tools of linear elastic fracture mechanics (LEFM) and an additional heuristic ansatz (maximum energy release rate or local symmetry principles) have not been able to predict satisfactorily such structural features as the rotation of the front segments or their spatial extension. Besides, they left untouched the issue of the existence (reported by Sommer [7], but never confirmed since) of a finite threshold amount of mode mixity for the emergence of échelon cracks. In this respect, the recent work of Pons and Karma (PK) [13], based on a phase field model of brittle fracture, constitutes an important opening. They are able to describe their results on crack propagation as a linear instability of the straight front against helical deformations, which evolve via coarsening toward the facetted shape. This has led Leblond, Karma and Lazarus (LKL) [14] to perform a linear stability analysis in the framework of LEFM. On this basis, they predict the existence of a finite threshold ($K_{III}/K_I$), where $K_{I,III}$ are the stress intensity factors, i.e. the amplitudes of the LEFM square root singular asymptotic fields [5], imposed by the external loading) below which the planar crack remains stable. Moreover, the value of this threshold is fixed by that of the Poisson ratio only, and does not depend on the energy release rate. However, the absence
of any length scale in the LEFM framework results in a pathological feature of the corresponding bifurcation: for \((K_{III}/K_I) > (K_{III}/K_I)_c\), all wavelengths \(\lambda\) become simultaneously unstable, and the growth rate of the front distortion diverges as \(\lambda \to 0\). As pointed by LKL, tackling the associated regularization calls for identifying a small length cutoff, the physical origin and degree of material dependence of which remain open issues.

Our choice of gelatin to try and identify this length scale in highly deformable solids is dictated by its mechanical specificities. Indeed, in this physical hydrogel, fracture proceeds via stress-induced unzipping of (triple helix) crosslinks of the gelatin network, and subsequent dissipative pull-out of the unzipped polymer chains [15]. This demands that, in the crack tip vicinity, stresses build up to a level \(\sim 100E\), with \(E\) the small strain Young modulus. As discussed by Hui [16], this would be prohibited for linear elastic materials since, in this case, as the remote stress is increased, the crack tip radius grows, thus limiting tip stresses to values on the order of \(E\). However, in real materials, this “elastic crack blunting” mechanism is counteracted by elastic nonlinearities in the tip region. In soft polymer gels, since strain hardening is associated to the crossover between the coiled (entropic) and taut chains (enthalpic) elastic regimes, it is a huge effect which allows for the above-mentioned stress level.

The extension of the near-tip region where nonlinearities become relevant naturally provides a small scale cutoff \(\ell_{NL}\) below which the universal inverse square root LEFM stress divergence no longer holds, as directly demonstrated by Livne et al. [3]. Estimating this length as the distance where the LEFM stress reaches a value \(\sim E\) leads, in agreement with ref. [17], to \(\ell_{NL} \sim G/E\), with \(G\) the fracture energy. In hydrogels, \(\ell_{NL}\) typically lies in the 100 \(\mu\)m-1 mm range [18,19], much larger than both the process zone and network mesh sizes\(^1\). It can thus be expected to play a decisive role in crack path selection.

In this letter, on the basis of an extensive exploration of crack surface morphologies in the quasi-static regime, we bring direct evidence that the emergence of the \(\epsilon\)onchial structures does not follow the linear-instability-plus-coarsening PK picture. Rather, they develop via a mechanism akin to the emergence of the cross-hatching (CH) instability observed in mode I fracture of gels [18,20] and rubbers [21]—namely, the localized nucleation of steps, triggered by structural fluctuations and facilitated by shear. From this we conclude that, at least in soft and tough materials, the mixed mode fracture response is fully controlled by nonlinearities. We then argue that, more generally, the relevance of the LKL linear stability analysis should be correlated with the amplitude of mode I crack surface roughness.

**Experimental.**—Our experiments are performed, according to the protocol described in ref. [22], on gel slabs \((E = 12\) kPa\) made of 5 wt\% gelatin in a water-glycerol mixture. Sample sizes and compositions are listed in table 1. Mixed mode loading is introduced by notching the sample, previously stretched to the desired level, at an angle \(\theta_0\) (see fig. 1) from the plane of propagation of pure mode I cracks.

Profilometric analysis of crack surfaces is performed as follows. In order to circumvent sample evolution induced by solvent evaporation, we make replicas of the fresh fracture surfaces by casting and UV-curing thin layer of glue. The replicas are fixed on a two-axes \(x-y\) translation stage and scanned with the help of a confocal chromatic optical sensor (Micro Epsilon NCDT IFS 2401-1). The sensor itself is fixed to a \(z\) translation stage so as to extend its nominal 1 mm measuring range up to 2 cm. This home-built profilometer enables us to scan zones of typical areas \(1 \times 5\) cm\(^2\) with a vertical resolution of 100 nm and a lateral one of \(10 \times 10\) \(\mu\)m\(^2\).

We define the tilt angle \(\theta(x)\) as the inclination of the front line in the stretched sample with respect to the horizontal mid-plane. In order to identify the \(\theta = 0\) reference plane, all experimental runs are continued up to propagation distances such that the crack has returned to the

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\(^1\)Note that in this study, \(\ell_{NL}\) remains smaller than the external geometric scales, in particular the sample thickness.

**Table 1:** Gel sample characteristics. Slab dimensions (in cm) as defined in fig. 1. \(\eta\) (in mPa·s) is the viscosity of the water-glycerol solvent.

| Slab symbols | \(L\) | \(h\) | \(e\) | \(\eta\) |
|--------------|------|------|------|------|
| Diamonds     | 30   | 3    | 1    | 11   |
| Squares      | 30   | 3    | 1    | 3.6  |
| Circles      | 30   | 3    | 2    | 11   |
| Down triangles | 30   | 10   | 1    | 11   |
| Up triangles | 20   | 2    | 0.5  | 11   |

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**Fig. 1:** (Colour on-line) Sketch of the experimental setup: the gel slab (unstrained dimensions \(L \times h \times e\)), once stretched by \(\Delta h\), is notched at angle \(\theta_0\) by the blade \(B\).

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When $\mathcal{G} > \mathcal{G}^*$, as $m_0$ is increased the fracture behavior evolves as follows.

**Smooth regime.** At small notch angles ($m_0 < m_c$), the crack front remains quasi-linear. However, as it propagates, its tilt angle $\theta(x)$ decreases steadily until it returns to the mode I configuration. Indeed, we meet here with the difficulty inherent to mixed mode fracture experiments — namely, it is in practice impossible to impose a steady mode mixity amount. Here, as in refs. [8,9], we only impose its initial value. A crack surface typical of this smooth regime is shown in fig. 3(a).

Figure 4(a) shows sections $\zeta(y)$ of a “smooth” fracture surface by planes perpendicular to the propagation direction $x$. As analyzed in ref. [8], finite slab thickness induces a mode II (in-plane shear) loading component which decreases gradually from the edges and vanishes on the midplane $y = 0$. However, due to the need for glue trimming along the replica edges, the scanning zone only extends up to a distance of 1 mm from each edge. Inspite of this limitation, one clearly distinguishes in fig. 4(a) the bending trend expected to result from this effect. We choose to define the tilt angle from the slope of the profile at $y = 0$. We then compute with the help of eq. (1) the local mode mixity indicator $m(x) = \tan\theta(x)$.

Such profile analysis, performed on samples with various $\theta_0$ and $c$ values shows that, as displayed in fig. 4(b), the tilt angle relaxation curves obey the scaling law

$$m(x) = m_0 f(x/e),$$

where $f$ exhibits an exponential-like decay over a range $x/e \approx 1$. In agreement with Saint-Venant’s principle, the memory of initial conditions is constrained by the sample geometry. Finally, closer examination of the profiles reveals, on top of the global smooth profile, a roughness of micrometric r.m.s. amplitude with the same characteristics as that of mode I crack surfaces (see inset of fig. 4(b)).

**Echelon regime.** For $m_0 \geq m_c(\mathcal{G})$ (fig. 3(b)), the fracture surfaces exhibit highly localized steps, oriented so as to reduce the tilt of the facets which join them. They nucleate at the very tip of the notch and grow, as the crack propagates, up to heights $\sim 100 \mu m$. Moreover, we find that they systematically drift along the front, in a seemingly random direction. As $m_0$ gets larger (fig. 3(c)), the step density increases and we observe step “collisions” which result in their merging. Thanks to the drift, the steps successively exit from the slab, thereby relaxing the average tilt angle and finally leaving a smoothed crack surface.

When putting this set of results in regard to the instability scenario emerging from refs. [13,14], several important discrepancies are noticeable:

- At variance with their prediction of a $\mathcal{G}$-independent finite $m_c$ value ($\approx 0.23$ for our quasi-incompressible gel)

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\[\text{Observation by H. Henry (private communication) of the same scaling behavior in phase field simulations of brittle fracture confirms its purely geometric origin.}\]
Fig. 3: (Colour on-line) Crack surface morphologies for parameter values as labeled in fig. 2. (a) Surface profile of a smooth crack (area $6 \times 24 \text{mm}^2$). (b), (c): surface profiles of cracks close beyond threshold (b), and deeper into the échelon regime (c) (areas $8 \times 10 \text{mm}^2$, vertical bar $500 \mu\text{m}$). $N$ indicates the notching line.

Fig. 4: (Colour on-line) (a) Sections of the smooth fracture surface shown in fig. 3(a), by planes perpendicular to the propagation direction $x$ and evenly spaced ($\Delta x = 2 \text{mm}$) from the initial straight notch, growing in the direction of the arrow. The hatched stripes indicate the edge regions of the $10 \text{mm}$ wide sample inaccessible to scanning. (b) Scaled mode mixity parameter ($m/m_0$) vs. scaled propagation distance ($x/e$) from the notch line, for: $\{e = 1 \text{ cm}, m_0 = 0.27, 0.36, 0.58, 0.84\}$ and $\{e = 2 \text{ cm}, m_0 = 0.36\}$. Inset: micro-roughness of crack (a) as revealed by the shadow pattern under illumination at the grazing angle $3 \text{ deg}$, and directly measured to be in this case $3.8 \mu\text{m} \text{ r.m.s.} \ (7.5 \mu\text{m} \text{ peak-to-peak})$.

i) we evidence the existence of a low-$G$ thresholdless domain; ii) for $G > G^\ast$, $m_c$, which is now finite, exhibits a tenfold increase when $G$ grows by a factor $\sim 2$. Note that, since we find that $m_c(G)$ is independent of the sample thickness, this dependence cannot be assigned to a mere finite-size effect.

PK suggest that échelon patterns emerge via a linear instability as a quasi-sinusoidal front modulation which then coarsens as it amplifies, according to a direct bifurcation picture. We observe a markedly different behavior, namely nucleation and growth of highly localized front distortions.

**Step nucleation.** These discrepancies raise an obvious issue: could nucleation, in our experiments, be heterogeneous, i.e. triggered by defects imprinted by the notching blade? In order to address this question, we have performed the following “mechanical quench” experiment: we initiate a crack in the smooth regime (point $q_1$ in fig. 2), let it propagate over a distance of $1.2 \text{ mm}$, then suddenly reduce the remote tensile loading, thus bringing the crack into the échelon regime (point $q_2$). Figure 5(a) shows the corresponding surface profile. Unambiguously, the steps nucleate quasi-simultaneously and without measurable delay on the quench line $Q$. Moreover, as illustrated on the blow-up fig. 5(c), they emerge from ridges of the micro-roughness already present in the pre-quench, nominally smooth, $NQ$ region. On the other hand, we find (see fig. 5(b)) that this micro-roughness exhibits no discernible correlation with the blade-generated grooves. We therefore conclude that the échelon instability develops via “homogeneous” (nonextrinsic) nucleation of localized structures.

This phenomenology is highly reminiscent of that of the Cross-Hatching (CH) instability previously observed in mode I fracture of highly deformable materials and analyzed, on the case of gelatin, in ref. [18]. This latter instability is characterized by the emergence, below a critical energy release rate $g_{\text{CH}}$, at random locations across the slab width, of narrow steps (see fig. 5(d)) drifting in either direction. We have interpreted them as the response of the crack front to transient pinning by a toughness fluctuation: this gives rise to a local $(I + II + III)$ mixed mode configuration, hence to a helicoidal self-amplifying...
front deformation evolving into two connected half-cracks. The resulting, symmetry-breaking, local morphology is thus akin to the (I + III) échelon one, while the global symmetry imposed by the absence of remote shear loading reflects into the random sign of the facet tilt angles.

As immediately appears when comparing figs. 5(c), (d), échelon and CH steps develop in a strikingly similar way. This suggests that, in our soft elastic system, the échelon instability might be the continuation of the CH one, that would be simply shifted under the biasing effect of shear loading. If such is the case, the value $G^*$ of $G$ for which the threshold $m_c$ vanishes must coincide with $G_{CH}$. We have therefore performed a set of experiments under pure mode I loading (see data points on the horizontal axis in fig. 2) and measured, for the gels studied here, $G_{CH} = 4.6 \pm 0.3 \text{J} \cdot \text{m}^{-2}$. As seen in fig. 2, this value is fully compatible with that of $G^*$. Let us moreover note that $m_c(G)$ turns out to be insensitive to a threefold variation of the solvent viscosity $\eta$, in agreement with the $\eta$-independence of $G_{CH}$ [18].

Discussion. – In summary, we are able to assert that, as far as the échelon instability is concerned, the effect of a finite mode mixity amount is merely to facilitate the local development of helical defects generated on front pinning tough sites provided by the intrinsic disorder of the random polymer gel network.

This conclusion may at first sight appear quite puzzling. Indeed, it points to the irrelevance to our system of the linear instability-plus-coarsening scenario. If such is the case, why is the (formally indisputable) LKL linear analysis not valid here or, in other words, what is the criterion for its validity?

With regard to this question, it is important to note that, as shown in [18], the in- and out-of-plane front deviation amplitudes for a fully developed CH defect turn out to be on the order of the cutoff length $\ell_{NL}$ associated with elastic nonlinearities. In other words, the nucleation and initial development of such defects take place in the elastically blunted, fully strain-hardened, tip vicinity. Hence the fact that LEFM cannot account for these processes.

Nevertheless, as discussed above, the LKL approach should be legitimate on length scales larger than $\ell_{NL}$, and the spatially extended destabilization which it predicts might compete with the biased CH process. Let us stress, however, that this extended response must be understood as driven by the noise renormalized up to the coarse-graining scale. Now, as the LEFM front problem is itself nonlinear through its free boundary nature, the front deformation as calculated by LKL is the first term of an amplitude expansion, and truncation to first order is valid only for noise amplitudes below some cutoff $A_{max}$ fixed by the precise expression of next-order terms. Note that $A_{max}$ should exhibit a universal dependence on the small strain elastic coefficients. On the other hand, the coarse-grained noise amplitude scale $\bar{A}(\ell_{NL})$, which can in principle be evaluated from the micro-roughness spectrum of mode I cracks, certainly depends on nonlinear (elastic, plastic) material properties. We believe our strongly nonlinear elastic system to correspond to the regime $\bar{A}(\ell_{NL}) > A_{max}$ in which the extended instability is irrelevant. These remarks clearly suggest the validity of the LKL scenario to be material dependent and, hence, the nature of the échelon instability itself to be a non-universal feature.

We believe that our above conclusion about the effect of a finite mode mixity amount on the emergence of échelons applies to all materials which exhibit cross-hatching fractographic features under pure tensile loading. Beyond our physical hydrogel, these include chemical gels [20] and elastomers [21]. Whether this behavior extends to even stiffer solids (e.g., polymer glasses [23]) will remain an open issue pending characterization of crack surface morphologies obtained under steady and quasi-static conditions.

On the other hand, we have shown that, in gels, the échelon instability is controlled by the nonlinear elastic behavior relevant to the mesoscopic tip region limited by the characteristic length $\ell_{NL}$. This length has also been shown to govern the oscillatory instability in the rapid fracture of brittle gel films [4,19] and the crack branching response of gelatin gels to a solvent-induced environmental shock [24]. On this basis, we conclude that predicting fracture properties of soft-and-tough materials asks for the extension of the prevailing fracture mechanics approach so as to include nonlinear elasticity, a task for which phase field methods appear as very promising candidates.

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REFERENCES

[1] Gong J. P., Soft Matter, 6 (2010) 2583; Sun T.-L., Kurokawa T., Kuroda S., Bin Ihsan A., Akasaka T., Sato K., Haque A., Nakajima T. and Gong J.-P., Nat. Mater., 12 (2013) 932.
[2] Sun J.-Y., Zhao X., Iillerperuma W. R. K., Chaudhuri O., Oh K. W., Mooney D., Vlassak J. J. and Suo Z., Nature, 489 (2012) 133.
[3] Livne A., Bouchbinder E., Svetlisky I. and Fineberg J., Science, 327 (2010) 1359.
[4] Bouchbinder E., Phys. Rev. Lett., 103 (2009) 164301.
[5] Lawn B., Fracture of Brittle Solids, 2nd edition (Cambridge University Press) 1993.
[6] Hull D., Fractography (Cambridge University Press) 1999.
[7] Sommer E., Eng. Fract. Mech., 1 (1969) 539.
[8] Bisen Lin, Mear M. E. and Ravi-Chandar K., Int. J. Fract., 165 (2010) 175.
[9] Lazarus V., Buchholz F.-G., Fulland M. and Wiebesiek J., Int. J. Fract., 153 (2008) 141 and references therein.
[10] Cooke M. L. and Pollard D. D, J. Geophys. Res., 101 (1996) 3387.
[11] Goldstein R. V. and Osipenko N. M., Dokl. Phys., 57 (2012) 281.
[12] Lazarus V., Leblond J. B. and Mouchrif S. E., J. Mech. Phys. Solids, 49 (2001) 1421.
[13] Pons A. J. and Karma A., Nature, 464 (2010) 85.
[14] Leblond J. B., Karma A. and Lazarus V., J. Mech. Phys. Solids, 59 (2011) 1872.
[15] Baumberger T., Caroli C., Martina D., Nat. Mater., 5 (2006) 552.
[16] Hui C. Y., Jagota A., Bennison S. J. and Londono J. D., Proc. R. Soc. London, Ser. A, 459 (2003) 1489.
[17] Geubelle P. H. and Knauss W. G., J. Elast., 35 (1994) 61.
[18] Baumberger T., Caroli C., Martina D. and Ronsin O., Phys. Rev. Lett., 100 (2008) 178303.
[19] Goldman T., Harpaz R., Bouchbinder E. and Fineberg J., Phys. Rev. Lett., 108 (2012) 104303.
[20] Tanaka Y., Fukao K., Miyamoto Y. and Sekimoto K., Europhys. Lett., 43 (1998) 664.
[21] Gent A. N. and Pulford C. T. R., J. Mater. Sci., 19 (1984) 3612.
[22] Baumberger T., Caroli C. and Martina D., Eur. Phys. J. E, 21 (2006) 81.
[23] Dalmas D., Guerrero C., Scheibert J. and Bonamy D., Int. J. Fract., 184 (2013) 93.
[24] Baumberger T. and Ronsin O., Eur. Phys. J. E, 31 (2010) 51.