Photoproduction of $f_0(980)$ and $f_0(1500)$ resonances off a proton target

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We study the $\gamma p \rightarrow p f_0$ [$f_0(980)$ and $f_0(1500)$] reaction close to threshold within an effective Lagrangian approach. The production process is described by $s$-channel nucleon pole or $t$-channel $\rho$ and $\omega$ exchange. The $K^0\bar{K}^0$ invariant mass distributions of the $\gamma p \rightarrow f_0(980)p \rightarrow K^0\bar{K}^0$ and $\gamma p \rightarrow f_0(1500)p \rightarrow K^0\bar{K}^0\rho$ reactions are investigated, where the two kaons have been separated in $S$ wave decaying from $f_0(980)$ and $f_0(1500)$. It is shown that the $s$-channel process is favored for the production of $f_0(980)$, while for the $f_0(1500)$ production, the experimental measurements can be described quite well by the $t$-channel process. It is expected that the theoretical results can be tested by further experiments at CLAS.

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I. INTRODUCTION

The study of the structure of low-lying scalar mesons is a topic of high interest in hadronic physics and is attracting much attention [1,2]. For the scalar meson $f_0(980)$, it is now widely accepted that the simplest picture, where it is described as an orbital excitation of quark-antiquark pairs, is not compatible with the experimental observations on its decay modes. Thus, the $f_0(980)$ is thought to be a molecule state formed from the interaction of pseudoscalar mesons [3–10], and it couples strongly to the $K\bar{K}$ channel [11], which is its dominant component. The $f_0(1500)$, on the other hand, with a mass of 1504±6 MeV and a width of 109±7 MeV [12], is a candidate for having much glueball content [13,14]. Photoproduction of these scalar resonances provides a unique place to probe their nature.

On the experimental side, photoproduction of $f_0(980)$ meson on protons was measured by CLAS collaboration in Refs. [13,16] at the photon energy region of $E_\gamma = 3.0 - 3.8$ GeV, where $f_0(980)$ was detected via its decay in the $\pi^+\pi^-$ channel by performing a partial wave analysis of the reaction $\gamma p \rightarrow p\pi^+\pi^-$. However, the production rate of $f_0(980)$ is much smaller than the one for the $\rho$ meson. Very recently, a partial wave analysis is performed for the $\gamma p \rightarrow pK^+\bar{K}^-$ reaction by the CLAS collaboration [17], where the production amplitudes have been parametrized using a Regge-theory inspired model. There were also pioneering measurements [18,19] for the photoproduction of $K^+\bar{K}^-$ pairs. After that, there were several theoretical calculations about the scalar mesons production in the process of $\gamma p$ scattering. A combined analysis of $\pi\pi$ and $K\bar{K}$ photoproduction in $S$-wave is conducted in Ref. [20], while the $f_0(980)$ and $a_0(980)$ photoproduction for photon energies close to the $K\bar{K}$ production threshold was studied in Ref. [21] using tools of chiral unitary approach. In Ref. [22], within a model based on the Regge approach, a theoretical analysis of the data on photoproduction of the $f_0(980)$ was done, where it was shown that the radiative decay rate for $f_0(980)$ is important in the theoretical predictions. In Ref. [23] the $\gamma p \rightarrow a_0(980)p$ and $\gamma p \rightarrow f_0(980)p$ reactions were investigated with the main aim for studying the possibility of observing $a_0(980)$-$f_0(980)$ mixing in these processes. In Ref. [24], the $a_0(980)$ and $f_0(980)$ photoproduction was investigated by considering the Regge-cut effects which were fixed from $\pi^0$ photoproduction. With the Regge theory, the $f_0(1500)$ photoproduction was also studied in Ref. [25] at $E_\gamma = 9$ GeV.

Recently, the reaction $\gamma p \rightarrow pX \rightarrow pK^0\bar{K}^0$ was investigated by the CLAS Collaboration [26] with photon energies from 2.7–5.1 GeV, where it was found that the angular distributions of the data suggest that most of the $K^0\bar{K}^0$ decay is from scalar mesons in $S$ wave. In particular, a clear peak is seen at 1500 MeV in the invariant mass spectra of $K^0\bar{K}^0$, and the mass and width of this peak is consistent with that of the scalar meson $f_0(1500)$, while the enhancement close to $K^0\bar{K}^0$ threshold is due to the $f_0(980)$ decay. In addition, there is no clear signals for contributions from the baryon resonances.

In the present work, based on the new measurements of CLAS collaboration [24], we reanalyze the $\gamma p \rightarrow f_0(980)p \rightarrow K^0\bar{K}^0\rho$ and $\gamma p \rightarrow f_0(1500)p \rightarrow K^0\bar{K}^0\rho$ reactions 1 within an effective Lagrangian method near threshold. As in Refs. [22,23] we consider the contributions from $t$-channel $\rho^0$ and $\omega$ exchange. Since the cou-

1 We take $|K^0_S> = \frac{1}{\sqrt{2}}(|K^0_S > + |K^0_L >)$ and $|K^0_L > = \frac{1}{\sqrt{2}}(|K^0_S > - |K^0_L >)$, where we ignore the CP violation.
couplings of \( f_0 \) to \( V \gamma \) channel is scarce \cite{12}, we take these results obtained in Refs. \cite{27, 28}, where meson loops were considered, and the \( f_0(980) \) was taken as a dynamically generated state. On the other hand, possible \( s \)-channel proton pole process, which was not included in all these above theoretical calculations, is also investigated in this work. It is shown that the new measurements of Ref. \cite{26} may indicate the dominant \( s \)-channel contribution for the \( f_0(980) \) photoproduction. In this respect, we show in this work how the CLAS measurements could be used to determine the reaction mechanisms of the photoproduction of these scalar mesons.

In the next section, we will give the formalism and ingredients in this work, then numerical results and discussions are given in Sec. \[13\] A short summary is given in the last section.

**II. FORMALISM AND INGREDIENTS**

The effective Lagrangian method is widely used to calculate cross sections for different reactions in the resonance production region. In this section, we introduce theoretical formalism and ingredients to calculate the scalar mesons photoproduction off protons within the effective Lagrangian method.

**A. Interaction Lagrangian densities and scattering amplitudes**

We first consider the basic \( t \)-channel tree level diagram for the \( \gamma p \rightarrow pf_0 \) [\( f_0 \equiv f_0(980) \) or \( f_0(1500) \)] reaction as shown in Fig. \[1\] This includes the contributions from \( \rho^0 \) and \( \omega \) meson exchange terms.

\[
\gamma \quad p_1 \quad \rightarrow \quad \rho^0 \text{ or } \omega \quad \rightarrow \quad f_0
\]

**FIG. 1:** Schematic diagram of reaction mechanism for \( \gamma p \rightarrow pf_0 \) reaction with \( t \)-channel \( \rho^0 \) and \( \omega \) exchange. The definition of the kinematical variables \( (p_1, p_2, p_3, q) \) used in the present calculation are also shown.

Following Ref. \cite{28}, we can write down the amplitude for the \( f_0 \rightarrow V \gamma \) decay as

\[
T = -\frac{g_{f_0V\gamma}}{m_{f_0}} (k \cdot p_1 g^{\mu\nu} - k^\mu p_1^\nu) \varepsilon_V(k) \varepsilon_\gamma(p_1),
\]

from where, we can obtain the partial decay width of the \( f_0 \) meson into a vector meson and a photon,

\[
\Gamma_{f_0 \rightarrow V\gamma} = \frac{|k|}{8\pi m_{f_0}^2} \sum \sum |T|^2
\]

\[
= \frac{g_{Vf_0\gamma}^2 (M_{f_0}^2 - m_V^2)^3}{32\pi M_{f_0}^4}.
\]

With masses \( (M_{f_0(980)} = 990 \text{ MeV}, M_{f_0(1500)} = 1504 \text{ MeV} \), and \( m_\rho = m_\omega = m_V = 780 \text{ MeV} \), with the partial decay widths of the scalar \( f_0(980) \) and \( f_0(1500) \) mesons radiative decay into a vector meson and a photon as obtained in Refs. \cite{27, 28}, we obtain these coupling constants as list in Table \[1\]

**TABLE I:** Values of the coupling constants required for the estimation of the \( \gamma p \rightarrow pf_0 \) reaction.

| Decay channel | Partial decay width \( \Gamma_{f_0 \rightarrow V\gamma} \) (keV) | \( g_{Vf_0\gamma} \) |
|---------------|---------------------------------|----------------|
| \( f_0(980) \rightarrow \rho \gamma \) | 7.3 \pm 1.8 | 0.12 |
| \( f_0(980) \rightarrow \omega \gamma \) | 6.6 \pm 1.8 | 0.11 |
| \( f_0(1500) \rightarrow \rho \gamma \) | 77 \pm 8 | 0.11 |
| \( f_0(1500) \rightarrow \omega \gamma \) | 79 \pm 8 | 0.12 |

To compute the scattering amplitudes of the diagrams shown in Fig. \[1\] we need also the effective interactions for the \( pNN \) and \( \omega NN \) vertices. We take the interaction Lagrangian densities as used in Refs. \cite{29, 30}:

\[
\mathcal{L}_{\rho NN} = -g_{\rho NN}\bar{N}(\gamma^\mu - \frac{\kappa_\rho}{2m_N} \sigma^{\mu\nu} \partial_\nu) \gamma^\tau \cdot \bar{\rho}_\mu N, \quad (3)
\]

\[
\mathcal{L}_{\omega NN} = -g_{\omega NN}\bar{N}(\gamma^\mu - \frac{\kappa_\omega}{2m_N} \sigma^{\mu\nu} \partial_\nu) \omega^\mu N. \quad (4)
\]

We use the coupling constants \( g_{\rho NN} = 3.36, \kappa_\rho = 6.1, g_{\omega NN} = 15.85 \) and \( \kappa_\omega = 0 \) of Refs. \cite{31, 32}. Then we can write the \( \rho NN \) and \( \omega NN \) vertices as,

\[
-\mathcal{I}_{\rho NN} = ig_{\rho NN}(\gamma^\mu + i\frac{\kappa_\rho}{2m_N} \sigma^{\mu\nu} q^\nu) \varepsilon_\mu(\rho), \quad (5)
\]

\[
-\mathcal{I}_{\omega NN} = ig_{\omega NN}\gamma^\mu \varepsilon_\mu(\omega). \quad (6)
\]

**B. \( \gamma p \rightarrow pf_0 \) scattering amplitudes**

With ingredients given above, we can easily obtain the \( t \)-channel \( \gamma p \rightarrow pf_0 \) reaction invariant scattering amplitude:

\[
\mathcal{M}_V = -\bar{u}(p_3)\frac{g_{Vf_0\gamma}}{m_{f_0}} g_{\rho NN}(k \cdot p_1 g^{\mu\sigma} - k^\mu p_1^\sigma) G_{\mu\nu}
\]

\[
\times [\gamma_\mu + \frac{\kappa_V}{2m_\rho}(k_\mu - k g_\mu)] F_1 u(p_2, s_\rho) \varepsilon_\sigma(p_1),
\]
where $G_{\mu\nu}$ is the Feynman propagator of $\rho$ or $\omega$ meson which has the following form:
\[
G_{\mu\nu} = \frac{-i g_{\mu\nu} - k_\mu k_\nu/m_\rho^2}{k^2 - m_\rho^2},
\]
with $t = k^2$ and $\Lambda_c$ a free cut-off parameter.

### C. Differential cross section

The differential cross section for the $\gamma p \to pf_0$ reaction by the exchanged $\rho^0$ and $\omega$ mesons can be expressed as
\[
\frac{d\sigma}{dt} = \frac{1}{16\pi s} \frac{m_p^2}{|\mathbf{p}_1|^2} \left( \frac{1}{4} \sum |\mathcal{M}|^2 \right),
\]
where $s$ is the invariant mass square of the $\gamma p$ system, and $\mathbf{p}_1$ denotes the photon three momentum in the center of mass (c.m.) frame. The total invariant scattering amplitude $\mathcal{M}$ is given by
\[
\sum |\mathcal{M}|^2 = \sum |\mathcal{M}_\rho + \mathcal{M}_\omega|^2
\]
with
\[
\Gamma^\mu_V = \frac{g_{\gamma N N}g_{f_0 V}}{(t - m_V^2)}|\Gamma_{f_0}|^2\left[(1 + \kappa_V)p_1 \cdot k + (1 + \kappa_V)p_1 k^\mu + \frac{\kappa_V}{2M_N}p_1 \cdot (p_3 + p_2)k^\mu \right]
\]
and
\[
\Gamma_{f_0 \to K^0\bar{K}^0} = \Gamma_{f_0 \to K^0\bar{K}^0}^{\text{inv}} \left( \frac{M_{f_0}^2 - 4m_{K^0}^2}{M_{f_0}^2 - 4m_{K^0}^2 + M_{f_0}^2} \right),
\]
where $\Gamma_{f_0}$ is the total decay width and we take $\Gamma_{f_0} = 100$ MeV $^2$ and 109 MeV for $f_0(980)$ and $f_0(1500)$, respectively. $M_{\text{inv}}$ represents the invariant mass of $K^0\bar{K}^0$. For $f_0(1500)$, $\Gamma_{f_0 \to K^0\bar{K}^0}$ is given by
\[
\Gamma_{f_0 \to K^0\bar{K}^0}^{\text{inv}} = \Gamma_{f_0 \to K^0\bar{K}^0}^{\text{inv}} \left( \frac{M_{f_0}^2 - 4m_{K^0}^2}{M_{f_0}^2 - 4m_{K^0}^2 + M_{f_0}^2} \right),
\]
with $\Gamma_{f_0 \to K^0\bar{K}^0}^{\text{inv}} = 4.7$ MeV. While for the case of $f_0(980)$, we take
\[
\Gamma_{f_0 \to K^0\bar{K}^0} = \frac{g_{f_0 K\bar{K}}^2}{16\pi} \sqrt{M_{\text{inv}}^2 - 4m_{K^0}^2}/2M_{\text{inv}}^2,
\]
with $g_{f_0(980) K\bar{K}} = 3860$ MeV as in Refs. \[28\,34\,35\].

### III. NUMERICAL RESULTS AND DISCUSSION

In this section we will show the numerical results for the $\gamma p \to pf_0$ reaction. We firstly show the theoretical results for the case of $f_0(1500)$ photoproduction.

#### A. Invariant mass distributions for the $\gamma p \to pf_0(1500) \to pK^0\bar{K}^0$ reaction

We compare our theoretical calculations for the invariant $K^0\bar{K}^0$ mass distributions as a function of $M_{\text{inv}}$ with the recent CLAS data of Ref. \[26\]. The theoretical $d\sigma/dM_{\text{inv}}$ is calculated by
\[
\frac{d\sigma}{dM_{\text{inv}}} = \int_{E_{\gamma}^{\text{min}}}^{E_{\gamma}^{\text{max}}} dE_\gamma \int dt \frac{d^2\sigma}{dE_{\gamma} dt},
\]
where $E_{\gamma}^{\text{max}} = 5.1$ GeV and $E_{\gamma}^{\text{min}} = 2.7$ GeV, which are the photon energy region of Ref. \[26\].

In Fig. 3 we show the experimental results, $c_1 d\sigma/dM_{\text{inv}}$, for the $K^0\bar{K}^0$ invariant mass distributions for the $\gamma p \to pf_0(1500) \to pK^0\bar{K}^0$ reaction, comparing with the experimental measurements of Ref. \[26\], where $c_1 = 2.2$ and $\Lambda_c = 1.7$ GeV has been adjusted to the strength of the experimental data reported by the CLAS Collaboration \[26\] at its peak around $M_{\text{inv}} = 1500$ MeV. One can see that, we can describe quite well the experimental measurements for the $\gamma p \to pf_0(1500) \to pK^0\bar{K}^0$ reaction by considering the $t$-channel $\rho^0$ and $\omega$ exchange, especially for the case of $|t| < 1$ GeV$^2$. This may indicate that the $t$-channel process is dominant for the photoproduction of the $f_0(1500)$ resonance.

\[2\] We take a relative large value for the total decay width of the $f_0(980)$, which is favored by the new CLAS measurements \[24\].
B. Invariant mass distributions for the \( \gamma p \to pf_0(980) \to pK^0 \bar{K}^0 \) reaction

We first present the theoretical results for the \( \gamma p \to pf_0(980) \to pK^0 \bar{K}^0 \) reaction by including the t-channel \( \rho^0 \) and \( \omega \) exchange. The numerical results of the \( K^0 \bar{K}^0 \) invariant mass distributions obtained with \( c_1 = 0.9 \) and \( \Lambda_c = 1.07 \) GeV, are shown in Fig. 3. The peak of the \( K^0 \bar{K}^0 \) invariant mass distributions is around 1020 MeV, very close to the mass threshold (995 MeV) of \( K^0 \bar{K}^0 \).

One can see that the model cannot describe simultaneously both the experimental data for \( |t| < 1 \) GeV\(^2\) and \( |t| > 1 \) GeV\(^2\). At \( M_{\text{inv}} = 1020 \) MeV and \( E_\gamma = 3.9 \) GeV, the values of \( t \) is \(-5.36\) GeV\(^2\) < \( t < -0.02\) GeV\(^2\), from where we find that the phase space for \( |t| > 1 \) GeV\(^2\) is more than four times larger than the case of \( |t| < 1 \) GeV\(^2\). However, the t-channel form factor \( F_1 = (\frac{\Lambda_c^2 - m_N^2}{2\Lambda_c})^2 \) with \( \Lambda_c \sim 1.07 \) GeV will contribute a suppression with factor about 14 for the case of \( |t| > 1 \) GeV\(^2\) than that of \( |t| < 1 \) GeV\(^2\). Hence, it is expected that, with the values of \( c_1 = 0.9 \) and \( \Lambda_c = 1.07 \) GeV, the results for \( |t| > 1 \) GeV\(^2\) should be much smaller than the ones for \( |t| < 1 \) GeV\(^2\). But, the experimental data of Ref. [26] tell us that the values for \( |t| > 1 \) GeV\(^2\) are even larger than those for \( |t| < 1 \) GeV\(^2\). This may indicate that the t-channel exchange mechanism is not enough to explain the experimental measurements of CLAS Collaboration [26].

Next, we study another kind of reaction mechanism for \( \gamma p \to pf_0(980) \to pK^0 \bar{K}^0 \) reaction, which is depicted in Fig. 4, where we have considered the contribution from the s-channel nucleon pole term. To compute the contribution of this term, the interaction Lagrangian densities for \( \gamma pp \) and \( f_0(980)pp \) vertexes are needed. We take them as used in Refs. [23, 30]:

\[
\mathcal{L}_{\gamma pp} = -e\bar{p}[A - \frac{\kappa_p}{2m_N}\sigma^{\mu\nu}(\partial_{\nu}A_{\mu})]p, \tag{16}
\]

\[
\mathcal{L}_{f_0(980)pp} = g_{f_0pp}\bar{p}pf_0, \tag{17}
\]

where \( \kappa_p = 1.5 \).
Then one can easily write down the corresponding amplitude for $s$-channel nucleon pole term as,

$$\mathcal{M}_s = g_{f_0pp} F_2 \bar{u}(p_3) \frac{q_1 + m_p}{q_1^2 - m_p^2} \times (\gamma^\mu - \Gamma_c^\mu) = \frac{\kappa p}{2m_p} \gamma^\mu p \bar{u}(p_2) \epsilon_\mu(p_1),$$

(18)

with

$$F_2 = \frac{\Lambda_4^4}{\Lambda_4^4 + (q_1^2 - m_p^2)^2}$$

(19)

and

$$\Gamma_c^\mu = -\frac{p_1 \cdot p_2}{p_1 \cdot p_2} p_2^\mu,$$

(20)

which is obtained from a contact term and for keeping the scattering amplitude $\mathcal{M}_s$ gauge invariant \[36, 37\].

The theoretical results of $K^0\bar{K}^0$ invariant mass distributions of the $\gamma p \to f_0(980)p \to K^0\bar{K}^0p$ reaction with the contribution from $s$-channel nucleon pole are shown in Fig. 6 from where we can see that we can explain the experimental measurements for both $|t| < 1$ GeV$^2$ and $|t| > 1$ GeV$^2$ cases quite well, since there is no so strong $t$ dependence factor $F_1$ in the $s$-channel process. The theoretical numerical results shown in Fig. 6 are obtained with $c_1 g_{f_0pp} = 7.5$ and $\Lambda_s = 1.1$ GeV.

One might think that the inclusion of higher nucleon excitations might improve the situation, since they have large mass and will give large contributions. However, at one certain photon energy $E_\gamma$, the propagator of the $s$-channel process is then just a constant. The estimation of the $K^0\bar{K}^0$ invariant mass distributions in our model is only sensitive to the production rate of the $f_0(980)$, and the nucleon pole term is sufficient for this purpose. By neglecting contribution from other $N^*$ resonances, we can present a more general picture of the $s$-channel $f_0(980)$ production processes, though our results are more general than this would suggest.

On the other hand, we calculate $d\sigma/dt$ for $\gamma p \to pf_0(980)$ reaction with the above two different reaction mechanisms at the photon energy $E_\gamma = 3.4$ GeV. The numerical results \[4\] are shown in Fig. 7 comparing with the experimental data taken from Refs. \[15, 17\]. The solid and dashed lines represent the result from $t$- and $|t| > 1$ GeV$^2$ cases quite well, since there is no so strong $t$ dependence factor $F_1$ in the $s$-channel process. The theoretical results are obtained with contribution from the $s$-channel nucleon pole.

\[4\] The results for the $s$-channel process are obtained with $g_{f_0pp} = 4.32$.\[4\]
s-channel process, respectively. One can see that both t-channel mechanism and s-channel process can describe fairly well the current experimental data. However, the line shapes of these two different reaction mechanisms are sizably different. The slope of the results for the s-channel process is more flat than the case of t-channel $\rho^0$ and $\omega$ exchange. We hope that this feature may be used to determine the reaction mechanism of $\gamma p \rightarrow pf_0(980)$ reaction.

IV. SUMMARY

In this work, we have studied the $\gamma p \rightarrow pf_0[f_0(980), f_0(1500)] \rightarrow pK^0\bar{K}^0$ reactions near threshold within an effective Lagrangian approach. The $K^0\bar{K}^0$ invariant mass distributions are evaluated, where the two kaons have been separated in $S$ wave decaying from the scalar mesons $f_0(980)$ and $f_0(1500)$. It is shown that the t-channel $\rho^0$ and $\omega$ exchange processes can describe the experimental data on the $\gamma p \rightarrow pf_0(1500) \rightarrow pK^0\bar{K}^0$ reaction, while the s-channel process is favored for the $\gamma p \rightarrow pf_0(980) \rightarrow pK^0\bar{K}^0$ reaction, since the t-channel mechanism for the $f_0(980)$ photoproduction fails to reproduce the experimental measurements. Furthermore, it is found that the theoretical numerical results for the $\gamma p \rightarrow pf_0(980)$ differential cross section, $d\sigma/dt$, of the two different reaction mechanisms are sizeably different. It is expected that the theoretical results can be tested by further experimental measurements at CLAS.

Finally, we would like to stress that, thanks to the important role played by the non t-channel process in the $\gamma p \rightarrow pf_0(980)$ reaction, accurate data for this reaction can be used to improve our knowledge about the reaction mechanism of this reaction and also the nature of $f_0(980)$. This work constitutes a first step in this direction.

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