Thermodynamic analysis and optimization of a vapor injection organic Rankine cycle system for low-grade heat recovery

D T Li*, Z L He, L T Ji and Z W Xing

School of Energy and Power Engineering, Xi’an Jiaotong University, 710049 Shaanxi, China.

* Corresponding author, Email: dantongli_xjtu@163.com

Abstract. The organic Rankine cycle (ORC) system has been widely used in waste heat recovery and geothermal utilization, but the system efficiency is limited due to the large exergy loss in the evaporation process. To improve the performance of ORC systems, in this paper, a vapor injection organic Rankine cycle (VIORC) system using positive-displacement expanders was proposed. The thermodynamic model of the proposed system was developed in views of energy and exergy. The effects of system operation parameters on system performance were investigated, and the system was optimized under different waste heat temperatures. Furthermore, the optimized systems were compared with basic ORC (BORC) systems. Results showed that compared with the BORC system, the proposed system increased the net output power by 3-8% and the exergy efficiency by 0.8-3.2% under the heat source temperature of 90-150 °C. The exergy loss analysis further showed that the reduction of the exergy loss in the evaporation process was the main contribution to the efficiency improvement of the proposed system.

Introduction

The organic Rankine cycle (ORC) system employing refrigerants as working fluids is widely used as a method of waste heat recovery due to its flexibility, low investment, and high safety [1]. However, its energy utilization efficiency is far lower than that of the Carrot’s cycle.

To improve the efficiency of the basic ORC (BORC) system, the optimization for the operating parameters and working fluids were first conducted [2-3]. Although system performance was improved, the exergy efficiency of the BORC system was still low [2]. A significant exergy loss in the evaporation process limited the efficiency improvement for the BORC system [4]. To reduce the exergy loss, one effective means was to improve the cycle architecture of the ORC system [5-6]. Therefore, many novel cycle architectures have been proposed such as improved single-pressure ORC, multiple-loop ORC (ML-ORC), and multiple-pressure ORC (MP-ORC).

ML-ORC system integrated some separate ORC systems to achieve the step-utilization of waste heat for the efficiency enhancement [7-9]. The related analysis showed that a dual-loop ORC system increased the system exergy efficiency by 11.2% compared with the single-loop ORC system [7]. MP-ORC system used multiple evaporators working at different pressures to achieve the step-utilization of waste heat [10-12]. A significant improvement for the system efficiency could be achieved by
reasonably configuring evaporators. However, the present ML-ORC and MP-ORC systems suffered from the high cost caused by the complex system lay-out and multiple components.

The improved single-pressure ORC system used an equivalent number of components to improve the system efficiency and thus had the advantages of low cost and high efficiency. The typical system included the organic flashing cycle (OFC) system [13,14] and the regenerative ORC (RORC) system [15-17]. The OFC system achieved an ideal heat absorption process from waste heat. However, a large throttle loss was inevitable, and thus the overall system efficiency was even lower than the BORC system [13]. The RORC system used a recuperator to reheat working fluid to increase the cycle average temperature. However, additional heat exchangers or turbines were needed in the RORC system resulting in the system’s worse economic performance [17,18]. Therefore, a high-efficiency ORC system with a low cost is still expected to further promote the development of the ORC technology. As a common technology in the fields of refrigeration and heat pumps, the vapor injection technology is considered as an effective method to enhance the system performance without increasing too much additional investment cost [19]. Until now, there is no available open literature to discuss the potential of the vapor injection technology in the ORC system.

To fill this gap, in this paper, a vapor injection organic Rankine cycle (VIORC) system using the positive-displacement expanders was proposed to investigate the improvement potential when the vapor injection technology is employed in an ORC system. The proposed system had the same number of main components (heat exchanger and expander) as the BORC system. To investigate the system performance, the thermodynamic models and performance evaluation models were established. The effect of the key operating parameters was investigated. The system performance was compared with the BORC system in views of energy and exergy. The exergy loss analysis revealed the advantages of the proposed system. The proposed VIORC system was promising to enhance the performance of traditional ORC systems with a limited capital investment difference.

System description

Fig. 1 shows the schematic diagram of the proposed VIORC system and its corresponding T-s diagram. The system consists of two working fluid pumps, two gas-liquid separators, an expander, an evaporator, a condenser, and a throttle valve. Different from the BORC system, the working fluid partially evaporates in the evaporator, and thus the heat transfer amount in the equal-temperature evaporation process reduces. As a result, the average heat transfer temperature difference of the evaporator in the proposed system is smaller than that in the BORC system. Therefore, the system is expected to have higher efficiency. At the same time, the difficulty on the system design and control is also increased due to the additional flow path for the proposed system. On the aspect of system design, the evaporator area should be precisely designed to keep the working fluid in Separator I at a suitable vapor quality. On the aspect of system control, the opening degrees of the throttle valve should be controlled reasonably to keep the suitable vapor injection pressure for the expander.

![Figure 1. A schematic diagram of the proposed VIORC system and its T-s diagram, (a) VIORC system layout, and (b) T-s diagram.](image)
The working process of the proposed VIORC system is as follows: the liquid working fluid is first pumped into a mixer to mix with the hot liquid in Separator II. Then the mixed working fluid flows into Evaporator to absorb thermal energy from waste heat and partially evaporates into the liquid-vapor mixture. Then the mixture flows into Separator I where it is separated into liquid and vapor streams. The vapor portion enters the expander to generate power. The liquid portion of the working fluid flows through a throttle valve to continue to generate vapor. The vapor is separated by Separator II and is injected into the expander through a supplemental inlet. The injected gas mixes with the original gas in the expander to raise the pressure in the working chamber. The mixed gas continues to expand to generate power. Finally, the exhausted working fluid from the expander rejects heat into the ambient through a condenser.

**Mathematical model**

1.1. Thermodynamic process

To establish the thermodynamic model of the proposed VIORC system, the following assumptions are made:

1) The system is running at a steady state.
2) The pressure drop in the system is ignored.
3) There is no subcooling at the condenser outlet.
4) Waste heat generates a fixed mass flow rate of hot water.
5) All heat transfer exchangers studied have the same minimum heat transfer temperature difference.
6) The vapor injection process is assumed as a constant volume compression process in a very short time.

Under the above assumptions, the working process of the expander is shown by a p-V diagram in Fig. 2. The points in the working process correspond to the state points in the T-s diagram in Fig. 1(b). When the vapor is injected in the expander, the state point is converted from point 6 to point 10 instantly, and the pressure at point 10 equals the vapor injection pressure of point 9. This instantaneous process is considered as the constant volume compression process due to the unchanging working chamber volume in the expander. The energy and mass conservation equations could be established for this process as shown in Fig. 2. For other components, the energy and mass balance equations are listed in Table 1. The state points of the working fluid in the cycle are shown in Fig. 1. Table 2 lists the exergy equations for each system component.

![Figure 2. The vapor injection process.](image)

| Components | Working process | Energy equations | Equation number |
|------------|----------------|------------------|-----------------|
|            |                |                  |                 |
\begin{align*}
\text{Pump I} & \quad \text{Compression process 1-2} \quad h_2 - h_1 = \left( h_{2,2s} - h_1 \right) / \eta_p = W_f / \left( x_4 + x_6 (1 - x_4) \right) m_{\text{sat}} \quad \text{Eq. (1)} \\
\text{Mixing process Liquid mixing 13-3, 2-3} & \quad h_3 = (x_4 + x_6 (1 - x_4) + (1 - x_4)(1 - x_4)) h_3 \quad \text{Eq. (2)} \\
\text{Evaporator} & \quad \text{Constant pressure heat transfer process 3-4, 14-15} \quad h_4 - h_3 = Q_{\text{eva}} / m_{\text{sat}} = m_{\text{sat}} \left( h_{\text{sat},14} - h_{\text{sat},15} \right) / m_{\text{sat}} \quad \text{Eq. (3)} \\
\text{Separator I} & \quad \text{Gas separation 4-7, 4-5} \quad m_{\text{sat}} = m_{\text{sat},5} + m_{\text{sat},7} \quad \text{Eq. (4)} \\
& \quad x_4 = m_{\text{sat},5} / m_{\text{sat}} \quad \text{Eq. (5)} \\
\text{Expander} & \quad \text{Expansion process 5-6} \quad h_5 - h_4 = (h_5 - h_{\text{sat},5}) \eta_{\text{exp}} \quad \text{Eq. (6)} \\
\text{Throttle valve} & \quad \text{Throttling process 7-8} \quad h_6 - h_5 = 0 \quad \text{Eq. (7)} \\
\text{Separator II} & \quad \text{Gas separation 8-9, 8-12} \quad m_{\text{sat},7} = m_{\text{sat},12} + m_{\text{sat},9} \quad \text{Eq. (8)} \\
& \quad x_4 = m_{\text{sat},9} / m_{\text{sat},7} \quad \text{Eq. (9)} \\
\text{Mixing process Gas mixing 6-10, 9-10} & \quad (x_4 + x_6 (1 - x_4) + (1 - x_4)(1 - x_4)) m_0 = x_4 q_0 + (x_4 (1 - x_4)) h_3 \quad \text{Eq. (10)} \\
& \quad p_0 = (x_4 + x_6 (1 - x_4) + (1 - x_4)(1 - x_4)) \quad \text{Eq. (11)} \\
\text{Expander} & \quad \text{Expansion process 10-11} \quad h_{10} - h_{11} = (h_{10} - h_{\text{sat},11}) \eta_{\text{exp}} \quad \text{Eq. (12)} \\
\text{Pump II} & \quad \text{Compression process 12-13} \quad h_{13} - h_{12} = (h_{13} - h_{\text{sat},12}) / \eta_p = W_f / ((1 - x_4)(1 - x_4)) m_{\text{sat}} \quad \text{Eq. (13)} \\
\text{Condenser} & \quad \text{Constant pressure transfer 11-1,16-17} \quad h_1 - h_0 = Q_{\text{con}} / m_{\text{sat},11} = m_{\text{sat}} (h_7 - h_{\text{sat},11}) / m_{\text{sat},11} \quad \text{Eq. (14)}
\end{align*}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|l|l|}
\hline
Components & Working process & Exergy equations & Equation number \\
\hline
\hline
\text{Pump I} & 1-2 & \dot{I}_{p,1} = T_s (x_4 + x_6 (1 - x_4) m_{\text{sat}} (s_2 - s_1)) & Eq. (15) \\
\text{Mixing process} 13-3, 2-3 & & \dot{I}_{\text{Mixing},1} = T_s \left[ (x_4 + x_6 (1 - x_4) m_{\text{sat}}) (s_3 - s_2) + ((1 - x_4)(1 - x_4)) m_{\text{sat}} \right] (s_3 - s_3) & Eq. (16) \\
\text{Evaporator} & 3-4, 14-15 & \dot{I}_{\text{eva}} = T_s \left[ m_{\text{sat}} (s_4 - s_3) + m_{\text{sat}} (s_{15} - s_{14}) \right] & Eq. (17) \\
\text{Separator I} & 4-7, 4-5 & \dot{I}_{\text{sep}} = 0 & Eq. (18) \\
\text{Expander} & 5-6 & \dot{I}_{\text{exp},1} = T_s m_{\text{sat},5} (s_6 - s_5) & Eq. (19) \\
\text{Throttle valve} & 7-8 & \dot{I}_{\text{exp}} = T_s \left[ m_{\text{sat},7} (s_6 - s_5) \right] & Eq. (20) \\
\text{Separator II} & 8-9, 8-12 & \dot{I}_{\text{sep}} = 0 & Eq. (21) \\
\text{Mixing process} 6-10, 9-10 & & \dot{I}_{\text{Mixing},11} = T_s \left[ x_4 (1 - x_4) m_{\text{sat}} (s_{10} - s_9) + x_6 m_{\text{sat}} (s_{10} - s_9) \right] & Eq. (22) \\
\text{Expander} & 10-11 & \dot{I}_{\text{exp},11} = T_s \left[ (x_4 + x_6 (1 - x_4)) m_{\text{sat}} (s_{11} - s_{10}) \right] & Eq. (23) \\
\text{Pump II} & 12-13 & \dot{I}_{p,11} = T_s \left[ (1 - x_4)(1 - x_4) m_{\text{sat}} \right] (s_{13} - s_{12}) & Eq. (24) \\
\text{Condenser} & 11-1, 16-17 & \dot{I}_{\text{con}} = Q_{\text{con}} - (x_4 + x_6 (1 - x_4)) m_{\text{sat}} (s_{11} - s_1) & Eq. (25) \\
\hline
\end{tabular}
\caption{Exergy equations for the system component.}
\end{table}
1.2. Thermodynamic performance

System net power output, thermal and exergy efficiencies are used to evaluate the system performance and expressed below:

The net power output is calculated by:

\[ W_{\text{net}} = W_{\text{exp}} - W_{p,I} - W_{p,\text{II}} \]  \hspace{1cm} (26)

The system thermal efficiency is calculated by:

\[ \eta_{\text{th}} = \frac{W_{\text{net}}}{Q_{\text{eva}}} \]  \hspace{1cm} (27)

The system exergy efficiency can be calculated by:

\[ \eta_{\text{exg}} = \frac{W_{\text{net}}}{W_{\text{net}} + I_{\text{tot}}} \]  \hspace{1cm} (28)

where \( I_{\text{tot}} \) is the total irreversible loss in the system, which can be expressed by:

\[ I_{\text{tot}} = \sum I_{\text{comp}} + I_{\text{Mixing}} + I_{\text{Waste}} \]  \hspace{1cm} (29)

where the term \( I_{\text{comp}} \) represents the exergy loss rate of the components in the system, the term \( I_{\text{Waste}} \) represents the waste exergy in the heating water at the evaporator outlet.

Results and discussion

1.3. Thermodynamic performance

Table 3 shows the operating parameters of the proposed system according to the previous studies [20-22]. These parameters are determined by the user’s settings. Table 4 shows the independent parameters of the proposed system. These parameters could be optimized under different optimization objectives. In this section, the effects of independent parameters on the system performance indices are first investigated.

| Parameter (Unit)                     | Symbol | Value       |
|--------------------------------------|--------|-------------|
| Heating water input temperature (°C)| \( T_{\text{hwt,in}} \) | 90–150      |
| Heating water flow rate (kg/s)       | \( \dot{m}_{\text{hwt}} \) | 5           |
| Heating water pressure (bar)         | \( p_{\text{hwt}} \) | 10          |
| Ambient temperature (°C)             | \( T_a \) | 25          |
| Expander isentropic efficiency (%)   | \( \eta_{\text{exp}} \) | 70          |
| Pump isentropic efficiency (%)       | \( \eta_p \) | 80          |
| Pinch temperature difference (°C)    | \( T_{\text{pinch}} \) | 5           |
| Working fluid (-)                    | -      | R245fa      |

Table 4. System independent parameters.

| Parameter (Unit)                     | Symbol |
|--------------------------------------|--------|
| Vapor quality at Separator I(-)      | \( x \) |
Evaporation temperature in Evaporator (℃) \( T_4 \)
The saturation temperature at the pressure of point 6 \( T_6 \)

1.3.1. Effect of vapor quality. Fig. 3 shows the effect of vapor quality at point 4 on system performance indices, where \( T_{\text{HWT,OUT}} \) represents the temperature of the heating water at the evaporation outlet. It is found that the system thermal efficiency increases with the increase in vapor quality. This is because the higher vapor quality causes a higher average evaporation temperature. However, maximum net output power and exergy efficiency exist for the system. It is because the increasing vapor quality also causes a reduction in the mass flow rate of the working fluid. With the increase in the vapor quality at point 4, the thermal efficiency is improved but the mass flow rate of working fluid reduces so that there is an optimized vapor quality to maximize the system’s net output power. It should be noted that since the total exergy in the heating water is constant, the system exergy efficiency has a linear relationship with the net output power.

![Figure 3. Effect of vapor quality at point 4 on system performance.](image)

In Fig. 3, it can be seen that the optimal system has the minimum heating water temperature at the evaporator outlet. It illustrates that the optimal system uses as much thermal energy from the heating water as possible. Fig. 4 shows the effect of vapor quality on the heat transfer process in the evaporator. It is found that for the system with a low vapor quality \((x=0.15)\), the minimum temperature difference \((T_{\text{pitch,0.15}})\) in the evaporator occurs at point 3 \((3_{0.15})\), while for the system with a high vapor quality \((x=0.5)\) the minimum temperature difference \((T_{\text{pitch,0.5}})\) occurs at the point 7 \((7_0.5)\). A suitable vapor quality \((x=0.32)\) makes the heat transfer temperature difference more uniform during the liquid heat transfer process \((3_{0.32} \text{ to } 7_{0.32})\). In this case, the heating water temperature at the evaporator outlet is lower so that the system has a higher net output power.

![Figure 4. Effect of vapor quality on the heat transfer process in evaporator.](image)
1.3.2. Effect of evaporation temperature. Fig. 5 shows the effect of evaporation temperature on system performance. It can be seen that as the evaporation temperature increases the system thermal efficiency and the heating water temperature at the evaporator outlet increase at the same time. It illustrates that with the increase in evaporation temperature the energy utilization efficiency increases while the energy inputting amount decreases. When the evaporation temperature is at around 85℃, the heating water temperature at the evaporator outlet begins to quickly increase. This increase causes a serious reduction in the net output power and system exergy efficiency so that an optimal evaporation temperature exists for the proposed system.

1.3.3. Effect of saturated temperature at the pressure of point 6. Fig. 6 shows the effect of the saturated temperature at the pressure of point 6 (T6). It should be noted that since the vapor injection process has a constant working chamber volume and the vapor injection mass is determined by the vapor quality, the parameter T6 directly affects the vapor injection pressure for the system. It is found that with the increase in temperature T6 both the system thermal efficiency and the heating water temperature at the evaporator outlet increase. As a result, an optimal temperature of T6 exists to maximize the system’s net output power.

1.4. System optimization and performance comparison
The above analysis reveals that all independent operating parameters in the system have the optimal values to maximize the system’s net output power. Therefore, in this section, a particle swarm optimization (PSO) algorithm is used to optimize the VIORC system with the maximum net output power as the optimization objective. Fig. 7 shows the optimized operating parameters under different waste heat temperatures. With the increase in waste heat temperature, the optimal vapor quality, evaporation temperature, and temperature T6 increase synchronously. To compare the performance of the proposed VIORC system with the basic ORC (BORC) system, the BORC systems are also
optimized under the same setting parameters in Table 3. Fig. 8 shows the schematic diagram of the BORC system.

**Figure 7.** Optimized system-independent parameters under different waste heat temperatures.

**Figure 8.** Schematic diagram of the basic ORC (BORC) system, (a) BORC system layout (b) $T$-$s$ diagram.

The optimal performance of the VIORC and BORC systems is shown in Fig. 9. It is found that the net output power of the VIORC system is always higher than that of the BORC system. As the waste heat temperature increases, the increment rate becomes more significant, which changes from 3% to 8%. The heating water temperature at the evaporator outlet keeps a limited change for the VIORC system, while the outlet temperature in the BORC system continually increases. It shows that the VIORC systems have a higher utilization degree of the thermal energy in heating water and thus achieve a larger net output power.

**Figure 9.** Optimal system performance under different waste heat temperatures.

Fig. 10 shows the efficiencies of the optimal systems under different waste heat temperatures. It is found that the thermal efficiency of the VIORC system is always lower than that of the BORC system. This is because that the VIORC systems could absorb more energies from the heating water so that the average heat absorption temperature from the heating water is lower. However, the system exergy
efficiency increases by 0.8 to 3.2% for the proposed system compared to the BORC system. This is attributed to the higher energy utilization degree of waste heat.

![Figure 10. Efficiencies of the optimal system under different waste heat temperatures.](image)

1.5. Exergy loss analysis

To further investigate the reason for the efficiency improvement of VIORC systems, the exergy loss distribution diagram is drawn for the VIORC and BORC systems under the waste heat temperature of 120°C as shown in Fig. 11, where the waste exergy represents the remained exergy in the heating water at the evaporator outlet. It is found that the evaporator exergy loss is reduced and exergy waste is improved for the proposed VIORC system. These could be attributed to the partial evaporation process in the evaporator.

![Figure 11. Exergy loss distribution of the proposed VIORC system and the BORC system.](image)

Conclusions

In this study, a vapor injection organic Rankine cycle (VIORC) system using positive-displacement expanders was proposed to improve the cycle efficiency of the traditional ORC system. The models for thermodynamic analysis and performance evaluation of the proposed system were established. The effects of key operating parameters on the system performance were investigated. The performance of the proposed VIORC system was compared with that of the BORC system. The specific conclusions are detailed below:

1. All of the three independent operating parameters (vapor quality, evaporation temperature, and saturated temperature at the pressure of point 6) in the VIORC system had an optimal value to maximize the system’s net output power.

2. Under the waste heat temperature of 90 to 150 °C, in comparison with the BORC system, the VIORC system achieved a net output power boost of 3-8%. As the waste heat temperature increased, the improvement effect became more significant.

3. The partial evaporation process in the evaporator had the main contribution to the performance improvement for the VIORC system.
It should be noted that in this paper the expander efficiency is simplified as a constant to compare the performance between the basic ORC system and the proposed system. In practice, the efficiency is largely affected by the operation conditions of expanders and ORC systems especially for the vapor injection expander. Further investigation is necessary to evaluate the proposed VIORC system’s performance on the real operation conditions.

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