Analysis of mathematical communication abilities of senior high school students in linear programming material

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Abstract. This article is the result of research that aims to analyze the students' mathematical communication abilities in mathematics to the subject of linear programming. The research was conducted using qualitative approach with case study method. The subjects were five high school students in Science program. Data communication students' mathematical ability is obtained by using tests and interviews. The results showed the ability of students' mathematical verbal communication is good because students can explain clearly and systematically to answer written in the worksheet. But the student's written mathematical communication ability are lacking in explaining the mathematical model, the use of symbols is less precise, and the writing is less systematic.

1. Introduction
Education is the most important in improving the quality of human resources. The development of human resources in the industrial revolution 4.0 is more focused on 21st century capabilities [1], one of which is the communication abilities. Communication abilities is a way of sharing ideas and classifying understanding through conversation, hearing, and writing. Communication abilities is also the most important part of mathematics and mathematics education [2]. There are two reasons that make a communication as an important part in mathematics learning, they are (a) mathematics as language; (b) mathematics learning as social activity [3].

At the formal education in Indonesia, students' mathematical communication abilities are still less, seen by the lack of activeness of students in expressing their opinions to the teacher and other students when learning takes place [4]. In addition, many students do not understand how to solve problems, use correct language and mathematical symbols. Students are very difficult to interpret the problems from symbols, tables, diagrams and pictures, student can not give meaning to the problems given so they can not find a way how to solve the problems. The evidence was revealed by Saparwadi [5], interpretation of tables, diagrams, symbols, graphs does not indicate that students have good and effective mathematical communicatio

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Linier programming is one of most important teaching material in mathematics and widely applied for making decision in various problems like economics, agriculture, healthy and many others. Although this material is closely related to daily life, many students still make errors in completing the problems given. The results of the study [6] show that linear program is one of the material in which many students make a completion error, especially on transformation and abilities in their respective processes and the lack of students' understanding of how to solve linear program problems with the correct steps. This
understanding is also strongly influenced by students' limited mathematical communication abilities [7]. The problems of Linear Program usually made in open-ended question [8]. Mathematical problems with open-ended problems are problems that have a high level of challenge and require problem formulation [9], so it can be seen how students communicate the description of the problem or formulate it in mathematical form to be solved.

This article focus on analyzing of mathematical communication abilities of high school students in linear program material. According Sumarmo [10], students with good mathematical communication abilities can have the ability to: (1) describe the problem situation and state the solution to the problem using images and algebraic presentation of the solution correctly; (2) use comprehensive representations to express mathematical concepts and solutions; (3) State the answers in written form appropriately and provide information in written form from the results obtained. This research aims to explore the mathematical communication abilities of high school students in linear program problems and determine the presence of difficulties and misunderstandings when dealing with linear program problems.

2. Methods
This study uses a qualitative approach with the case study method because the researcher emphasizes more on real cases, the close relationship between the researcher and the subject under study and the pressure of the situation that shapes the study [11]. The subjects of this study were five high school students. In this study the researchers conducted a three-step small-scale qualitative study involving five high school students in a science program. First, researchers designed three linear program problems to assess the ability of mathematical communication as shown in Table 1. The problem was designed in the form of open questions. Second, the researcher tests the problem with the research subject and then continues with in-depth interviews with students about the reasons for each step in solving the problem. Third, researchers analyzed students' answers based on indicators of mathematical communication abilities.

| No | Indicator | Problem |
|----|-----------|---------|
| 1  | The ability to describe the problem situation and state the solution to the problem using images and algebraic presentation of the solution correctly | A tour group of 240 people will rent hotel rooms for one night. The rooms available in that hotel are rooms for two people and for three people. The group will rent hotel rooms for at least 100 rooms. The cost of renting room for two people and three per night IDR 200,000.00 and IDR 250,000,00, respectively. Determine: a. Mathematical model of the problem b. Draw a mathematical model of the problem c. The objective function of the problem |
| 2  | The ability to use comprehensive representations to express mathematical concepts and solutions | The shaded area in the following figure is the set of resolving of inequality systems.... |
| 3  | The ability to state the answers in written form appropriately and provide information in written form from the results obtained | Mrs. Tika produces two kinds of cakes every day for sale. For the first kind of cake requires 30 grams of flour and 80 eggs, while the second kind of cake requires 90 grams of flour and 40 eggs. The ingredients available for both kind of cakes are 2,700 grams of flour and 3,200 eggs. The selling price for the first kind of cake is IDR 20,000 with a profit of 25% and the second of cake is IDR 50,000 with profit of 30%. If all kinds of cakes are sold out, a maximum profit from the sale of both kinds of cakes is.... |
3. Result and Discussion

In this study 5 students in senior high school of science program were asked to solving linear programming problem. The results are:

Figure 1 showed the variation of the answer the first problems that is a question to see the ability to describe the problem situation and state the solution to the problem using images and algebraic presentation of the solution correctly. For the problem “Mathematical model of the problem” (Problem 1a), there are three variation of answers that we found. The first answer see Figure 1(a), they have assumed correctly that x = many rooms for two people and y = many rooms for three people. But the written a mathematical model do not give a explanation or reason are 2x + 3y ≤ 240 and x + y ≥ 100. Other than, a mathematical model given is incorrect, can be seen ”A tour group of 240 people will rent a rooms for two people and three people for one night”, they write in a mathematical model that is 2x + 3y ≤ 240, even though they should be more careful to write it, for the people must be rent the rooms more than equals 240, because if it less than 240 as written by them there will be people from the group who do not get a room. The evidences were revealed by Niasih [12], that is still difficult for students to state daily activity problems in mathematical model. They do not write a condition “the rooms available are impossible zero or negative so x ≥ 0 and y ≥ 0. According them, the condition are definitely fulfilled, so they are not write anymore. Even though the condition must be write because it will describe a mathematical model of the problem given on relation to symmetry axis. The second answers, see Figure 1(b), they written the assumed incorrectly that is x = rooms for two people and y = rooms for three people without write a assumed with ”many rooms”, because in wiriting a mathematical model must be clear and precise. The third answer is more clear and complete, see Figure 1(c) although there are not explanation or reason related to written mathematical model. According interview that a written of explanation and reason a model no accustomed by the teacher when teaching linear program materials in class. In this answer a mathematical model is correct that 2x₁ + 3x₂ ≥ 240 and x₁ + x₂ ≥ 100 with x₁ = many rooms for two people and x₂ = many rooms for three people and a condition ”x ≥ 0 and y ≥ 0″ are written clearly.

(a)  
(b)  
(c)  

Figure 1. The students’ answers of Problem 1a

Figure 2 showed variation answers of problem 1b. First answer, see Figure 2(a) and 2(b), they do not give explanation about representation in the graph, they do no write the reason about how a graph can intersect the x-axis in (100,0) and intersect y-axis in (0,100). Other than, they do not give a reason how to determine the set settlement area on the graph. Although the graph is correctly but they can not write it in completely and systematically. Relevant to Wijaya [13] related to difficulties in the graph, shows that many students can understand how to draw a graph but are not focused on writing the graph along with how to get it and its properties, it can be seen in Figure 2(a) is not writing a condition that x ≥ 0 and y ≥ 0 but in the graph the set of solutions means that x ≥ 0 and y ≥ 0. Interview result that they drew a graphs according to the intersection they had calculated without writing them in answer sheet, because it reduced a lot of time. But we know that stating a solution with a graph must present the solution in algebra clearly and precisely. See Figure 2(c) they can explain the intersection point that will be present in the graph. They write the dividing line of graph is 2x₁+3x₂= 240 for looking for that intersect point.
the x-axis and y-axis to make it easy to draw a graph of $2x_1+3x_2 \geq 240$. After that write an intersect point of line to $x_1$ which means $x_2 = 0$ so that $2x_1+3(0) = 240$ and have a intersect point is (120,0). But they do not write a process about determine the set settlement area on the graph. According interview after the test, they understand about sign of inequality in the form “$\leq$” then the settlement set is to the left of the line and “$\geq$” the settlement set is to the right of line, but they can not writing it systematically with algebra expression.

![Graph of linear inequality](image)

**Figure 2.** The students’ answers of Problem 1b

The third problem (Problem 2) is to see the ability to use comprehensive representations to express mathematical concepts and solutions. See Figure 3, for the problem “The shaded area in the following figure is the set of resolving of inequality systems” (Problem 2). There are three variation answers in this problem. First, see Figure 3(a) the answer “$3x + 4y \leq 12$ and $5x + 2y \geq 10$” is wrong. They do not give explanation how determine an inequality. According them, because 5 is intersect point on x-axis and 2 is intersect point on y-axis, so it is written $5x + 2y$ and the settlement area is on the right line it is mean $\geq$. So the concept of linear program written is wrong, must be a concern of students in learning process that there is not misconception. See Figure 3(b) and 3(c), there is no different answer from the sample. All of sample give same answer that is $4x + 3y \leq 12$ and $2x + 5y \geq 10$, but there is a differences in writing of sample. See Figure 3(b) they write an intersect point (3,0) (0,4) and think of with (a,0) (0,b) and then substitution in $bx + ay = ab$ so $4x + 3y = 12$ 12. According them, they got the formula $bx + ay = ab$ from them tutor. But they do not understand correctly whether the formula is correct or not. The last answer See Figure 3(c) they write an intersect point (3,0) (0,4) and substitution in $\frac{x}{3} + \frac{y}{4} = 1$. Then
match a denominator \( \frac{4x + 3y}{12} = 1 \) so \( 4x + 3y = 12 \). There is not explanation how determine a settlement area in sign of inequality. Samples understand about it, but they are difficult to communicate in written form.

\[
\begin{align*}
2. & \quad 3x + 4y \leq 12 \\
& \quad 5x + 2y \leq 12 \quad 10
\end{align*}
\]

(a)

(b)

(c)

Figure 3. The students’ answers of Problem 2

Problem 3 is to see the ability to state the answers in written form appropriately and provide information in written form from the results obtained. There is no different answer from sample. All sample give a same answer; *maximum profit from the sale of both kinds of cakes is IDR 450.000*, but there were two variations written in a worksheet. First, see Figure 4(a) the answer is correct, but they do not explain the symbols written like “\( \rightarrow \)” so it is make the reader confused. Other than, there is no explanation about a written in worksheet. According interviewed, they can explain about mathematical model written very good in verbally, but a problem in systematic writing. They do not answer the question of the problem given, only give a sign for the answer written. Second, see Figure 4(b) can be represented in table form to make easy a mathematic model, but there is error in filling data in the table that is variables \( x \) and \( y \) without a assumed clearly. Actually, both of them (Figure 4a and 4b) wrote a mathematical models and graph in worksheet correctly, it is just that everything was written without explanation. There was not conclusion to answer the problem “If all kinds of cakes are sold out, a maximum profit from the sale of both kinds of cakes is....”. The third answer is more appropriate because it can explain and provide information in written form from the results obtained although there is a little less complete. See Figure 4(c), they written the assumed incomplete that is \( x = \) cake type I and \( y = \) cake type II without write a assumed with ”*many cakes “, because in writing a mathematical model must be clear and pricise. A mathematical model written is correctly, they write \( 30x + 90y \leq 2.700 \) then divide it by 30 so get a simple inequality is \( x + 3y \leq 90 \) in order to make easy to calculation and represent a mathematical model to the graph precisely and explain it. there are three critical points for finding a maximum profit from a problem given that were (40,0) (0,30) and intersect point of the line. They find that point by substituting and eliminating two equations and get an intersect point was (30,20). Next, substitute the three critical points into objective function was \( f(x,y) = 5.000x + 15.000y \) obtained from profit the type of cakes I is 20.000 multiplied by 25% and profit the type of cakes II is 50.000 multiplied by 30%. The most important, they wirte a conclusions very cleary.
After samples finish the test, we interview about the test given. As a result, samples find it difficult to work on all problems, but the samples still able to solve it. Samples can explain how mathematical models are obtained in verbally, but it is very difficult to write in worksheet clearly, precisely, and systematically.

4. Conclusion

The results showed that verbal mathematical communication abilities of students included in both categories because students could explain clearly and systematically for each answer written in a worksheet. However, students’ written mathematical communication abilities are still lacking because there are still students who have not been able to explain mathematical models in writing perfectly, there are still inappropriate use of symbols, and less systematic writing.

So students already have good mathematical communication abilities but still experience difficulties in communicating them in a written form that is correct and complete. Based on the results of this study, researchers suggest that students are more often taught to communicate problems in written form correctly and completely so that their mathematical communication abilities will increase optimally.

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