Interval Valued Fuzzy Ideals of Near-rings and its Anti-homomorphism

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Abstract: Aim of this study is to investigate anti-homomorphic images and pre-images of semiprime and primary ideals in interval valued fuzzy Near-rings. Further some results on f-invariant interval valued fuzzy ideal, f-invariant strongly primary interval valued fuzzy ideal and f-invariant semiprime interval valued fuzzy ideals of Near-rings are discussed.

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1 Introduction

The notion of a fuzzy set was introduced by Zadeh [13] in 1965, utilizing which Rosenfeld [11] has defined fuzzy subgroups. In 1975, Zadeh [16] investigated the notion of interval valued fuzzy subsets (in short i-v fuzzy subsets) where the values of the membership functions are closed intervals of numbers instead of single numbers. Liu introduced the concept of a fuzzy ideal of a near-ring in [8]. The concepts of prime fuzzy ideals, primary fuzzy ideals for ring were introduced in [9]. In 1991, Abou-Zaid [1] also exposed some results in fuzzy subnear-rings and fuzzy ideals in

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near-rings. Jun and Kim [4] and Davvaz [5] applied a few concepts of interval valued fuzzy subsets in near-rings. Sheikabdullah and Jeyaraman has discussed anti-homomorphic images and pre-images of prime fuzzy ideals and anti-homomorphic image of primary fuzzy ideals in a ring in [13]. The aim of this paper is to define and study i-v fuzzy primary ideals of a near ring $N$ and investigate anti-homomorphic images and pre-images of semi-prime, strongly primary i-v fuzzy ideals.

## 2 Preliminaries

**Definition 2.1.** [15] A non-empty set $N$ with two binary operations $+$ and $.$ is called a near-ring if:

i. $(N, +)$ is a group

ii. $(N, .)$ is a semigroup

iii. $x.(y + z) = x.y + x.z$ for all $x, y, z \in N.$

We will use the word Near-ring to mean left near-ring.

**Definition 2.2.** [15] Let $X$ be a non-empty universal set. A fuzzy subset $\mu$ of $X$ is a function $\mu : X \rightarrow [0, 1].$

**Example 2.3.** Let $N = \{a, b, c, d\}$ be the Klein’s four group. Define addition and multiplication in $N$ as follows.

|   | $a$ | $b$ | $c$ | $d$ |
|---|-----|-----|-----|-----|
| $+$ | $a$ | $b$ | $c$ | $d$ |
| $a$ | $a$ | $b$ | $c$ | $d$ |
| $b$ | $b$ | $a$ | $d$ | $c$ |
| $c$ | $c$ | $d$ | $b$ | $a$ |
| $d$ | $d$ | $c$ | $a$ | $b$ |

|   | $a$ | $b$ | $c$ | $d$ |
|---|-----|-----|-----|-----|
| $\cdot$ | $a$ | $a$ | $a$ | $a$ |
| $a$ | $a$ | $a$ | $a$ | $a$ |
| $b$ | $a$ | $a$ | $a$ | $a$ |
| $c$ | $a$ | $a$ | $a$ | $a$ |
| $d$ | $a$ | $a$ | $b$ | $b$ |

Here $(N, +, .)$ is a left near-ring. Define an interval valued fuzzy subset $\overline{\mu} : N \rightarrow D[0, 1]$ by

$\overline{\mu}(a) = [0.7, 0.8], \overline{\mu}(b) = [0.5, 0.6], \overline{\mu}(c) = [0.3, 0.4] = \overline{\mu}(d).$

It can be verified that $\overline{\mu}$ is an i-v fuzzy ideal of $N.$

**Definition 2.4.** [15] An interval number $\overline{a}$ on $[0, 1]$ is a closed subinterval of $[0, 1]$, that is, $\overline{a} = [a^-, a^+]$ such that $0 \leq a^- \leq a^+ \leq 1$ where $a^-$ and $a^+$ are the lower and upper end limits of $\overline{a}$ respectively. The set of all closed subintervals of $[0, 1]$ is
denoted by $D[0,1]$. We also identify the interval $[a, a]$ by the number $a \in [0,1]$. For any interval numbers $\tilde{a}_i = [a^-_i, a^+_i], \tilde{b}_i = [b^-_i, b^+_i] \in D[0,1], i \in I$, we define 
\begin{align*}
\text{max}^i\{\tilde{a}_i, \tilde{b}_i\} &= [\text{max}^i\{a^-_i, b^-_i\}, \text{max}^i\{a^+_i, b^+_i\}], \\
\text{min}^i\{\tilde{a}_i, \tilde{b}_i\} &= [\text{min}^i\{a^-_i, b^-_i\}, \text{min}^i\{a^+_i, b^+_i\}], \\
\text{inf}^i\tilde{a}_i &= \left[ \bigcap_{i \in I} a^-_i, \bigcap_{i \in I} a^+_i \right], \text{sup}^i\tilde{a}_i = \left[ \bigcup_{i \in I} a^-_i, \bigcup_{i \in I} a^+_i \right].
\end{align*}

In this notation $0 = [0,0]$ and $I = [1,1]$. For any interval numbers $\tilde{a} = [a^-, a^+]$ and $\tilde{b} = [b^-, b^+]$ on $[0,1]$, define
\begin{enumerate}
\item $\tilde{a} \leq \tilde{b}$ if and only if $a^- \leq b^-$ and $a^+ \leq b^+$.  
\item $\tilde{a} = \tilde{b}$ if and only if $a^- = b^-$ and $a^+ = b^+$.  
\item $\tilde{a} < \tilde{b}$ if and only if $\tilde{a} \leq \tilde{b}$ and $\tilde{a} \neq \tilde{b}$  
\item $k\tilde{a} = [ka^-, ka^+]$, whenever $0 \leq k \leq 1$.
\end{enumerate}

**Definition 2.5.** [15] Let $X$ be any set. A mapping $\overline{A} : X \rightarrow D[0,1]$ is called an interval-valued fuzzy subset (briefly, i-v fuzzy subset) of $X$ where $D[0,1]$ denotes the family of all closed subintervals of $[0,1]$ and $\overline{A}(x) = [A^-(x), A^+(x)]$ for all $x \in X$, where $A^-$ and $A^+$ are fuzzy subsets of $X$ such that $A^-(x) \leq A^+(x)$ for all $x \in X$.

Note that $\overline{A}(x)$ is an interval (a closed subset of $[0,1]$) and not a number from the interval $[0,1]$ as in the case of fuzzy subset.

**Definition 2.6.** [15] A mapping $\text{min}^i : D[0,1] \times D[0,1] \rightarrow D[0,1]$ defined by $\text{min}^i(\tilde{a}, \tilde{b}) = [\text{min}\{a^-, b^-\}, \text{min}\{a^+, b^+\}]$ for all $\tilde{a}, \tilde{b} \in D[0,1]$ is called an interval min-norm.

**Definition 2.7.** [15] A mapping $\text{max}^i : D[0,1] \times D[0,1] \rightarrow D[0,1]$ defined by $\text{max}^i(\tilde{a}, \tilde{b}) = [\text{max}\{a^-, b^-\}, \text{max}\{a^+, b^+\}]$ for all $\tilde{a}, \tilde{b} \in D[0,1]$ is called an interval max-norm.

**Definition 2.8.** [15] Let $N$ be a near-ring. An i-v fuzzy set $\overline{\mu}$ of $N$ is called an i-v fuzzy subnear-ring of $N$ if for all $x, y \in N$,  
\begin{enumerate}
\item $\overline{\mu}(x - y) \geq \text{min}^i\{\overline{\mu}(x), \overline{\mu}(y)\}$,
\item $\overline{\mu}(xy) \geq \text{min}^i\{\overline{\mu}(x), \overline{\mu}(y)\}$.
\end{enumerate}

**Definition 2.9.** [15] An i-v fuzzy subset $\overline{\mu}$ of a Near-ring $N$ is called an i-v fuzzy ideal of $N$ if $\overline{\mu}$ is an i-v fuzzy sub near-ring of $N$ and  
\begin{enumerate}
\item $\overline{\mu}(x) = \overline{\mu}(y + x - y)$,
\item $\overline{\mu}(xy) \geq \overline{\mu}(x)$,
\item $\overline{\mu}(x + i) - \overline{\mu}(y) \geq \overline{\mu}(i)$ for any $x, y, i \in N$.
\end{enumerate}
Proposition 2.10. [15] The anti-homomorphic image of an i-v fuzzy ideal of $N$ is an i-v fuzzy ideal of $N'$.

Proposition 2.11. [15] The homomorphic pre-image of an i-v fuzzy ideal of $N'$ is an i-v fuzzy ideal of $N$.

3 Main Results

Definition 3.1. An i-v fuzzy ideal $\mu$ of a near-ring $N$ is called an i-v prime fuzzy ideal if for any two i-v fuzzy ideals $\sigma$ and $\theta$ of $N$ the condition $\sigma \theta \subseteq \mu$ implies that $\sigma \subseteq \mu$ or $\theta \subseteq \mu$.

Definition 3.2. For an i-v fuzzy ideal $\mu$ of a near-ring, the i-v fuzzy radical of $\mu$, denoted by $\sqrt{\mu}$, is defined by $\sqrt{\mu} = \cap \{ \sigma : \sigma$ is an i-v fuzzy prime ideal of $N, \sigma \subseteq \mu, \sigma_* \subseteq \mu_* \}$. We denote $\mu_* = \{ x \in N : \mu(x) = \mu(0) \}$

Definition 3.3. An i-v fuzzy ideal $\mu$ of a near-ring $N$ is known as i-v fuzzy primary ideal if $\sigma \theta \subseteq \mu$, then either $\sigma \subseteq \mu$ or $\theta \subseteq \sqrt{\mu}$.

Definition 3.4. An i-v fuzzy ideal $\mu$ if a near-ring $N$ is called i-v strongly primary fuzzy ideal of a near-ring $N$ if $\mu$ is an i-v primary fuzzy ideal and $(\sqrt{\mu})^n \subseteq \mu$ for some $n \in N$.

Definition 3.5. An i-v fuzzy ideal $\mu$ of a near-ring $N$ is called i-v semi-prime if for any i-v fuzzy ideal $\sigma$ of $N$, $\sigma^2 \subseteq \mu$, then $\sigma \subseteq \mu$.

Definition 3.6. Let $X$ and $Y$ be two non-empty sets, $f : X \rightarrow Y$, $\mu$ and $\sigma$ be an i-v fuzzy subsets of $X$ and $Y$ respectively then $f(\mu)$, the image of $\mu$ under $f$ is an i-v fuzzy subset of $Y$ denoted by

$$f(\mu)(y) = \begin{cases} 
\sup(x) : f(x) = y & \text{if } f^{-1}(y) \neq \phi, \\
0 & \text{if } f^{-1}(y) = \phi.
\end{cases}$$

And $f^{-1}(\sigma)$, the pre-image of $\sigma$ under $f$ is an i-v fuzzy subset of $X$ defined by $f^{-1}(\sigma)(x) = \sigma(f(x)) \forall x \in X$.

Definition 3.7. If $\bar{\lambda}$ is an i-v fuzzy subset of $N$, then $\bar{\lambda}$ is said to have the sup property if for every subset $Y$ of $N$, there exists $y_0 \in Y$ such that $\bar{\lambda}(y_0) = \{ \lambda(y) | y \in Y \}$. 

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Definition 3.8. Let $I$ be a non-empty i-v fuzzy subset of $N$. Define a function $\overline{C}_I : N \rightarrow D[0, 1]$ by
\[
\overline{C}_I(x) = \begin{cases} 
1 & \text{if } x \in I, \\
0 & \text{otherwise} 
\end{cases}
\]
for all $x \in N$. Clearly $\overline{C}_I$ is an i-v fuzzy subset of $N$. $\overline{C}_I$ is called the i-v characteristic function of $I$. If the replace $I$ by $N$, $\overline{C}_N$ is the i-v characteristic function of $N$.

Definition 3.9. Let $N$ and $N'$ be two near-rings, a mapping $f : N \rightarrow N'$ is called an i-v fuzzy homomorphism if $f(\pi + \sigma) = f(\pi) + f(\sigma)$ and $f(\pi \sigma) = f(\pi)f(\sigma)$ where $\overline{\pi}$ and $\overline{\sigma}$ are i-v fuzzy ideals of $N$.

Definition 3.10. Let $N$ and $N'$ be two near-rings, a mapping $f : N \rightarrow N'$ is called an i-v fuzzy anti-homomorphism if $f(\pi + \sigma) = f(\pi) + f(\sigma)$ and $f(\pi \sigma) = f(\sigma)f(\pi)$ where $\overline{\pi}$ and $\overline{\sigma}$ are i-v fuzzy ideals of $N$.

Definition 3.11. Let $f : N \rightarrow N'$. An i-v fuzzy subset $\overline{\pi}$ of a near-ring is called $f$ -invariant if $f(x) = f(y)$ implies $\overline{\pi}(x) = \overline{\pi}(y)$, $x, y \in N$.

Definition 3.12. $N$ is called a fuzzy multiplication near-ring if for any two i-v fuzzy ideals $\overline{\pi}$ and $\overline{\sigma}$ of $N$ such that $\overline{\pi} \subseteq \overline{\sigma}$, there exists a fuzzy ideal $\overline{\iota}$ of $N$ such that $\overline{\sigma} = \overline{\pi} \circ \overline{\iota}$.

Theorem 3.13. If $\overline{\sigma}$ is a prime i-v fuzzy ideal of a fuzzy multiplication near ring $N$ and $\overline{\pi}$ is any i-v fuzzy ideal of $N$ such that $\overline{\pi} \subseteq \overline{\sigma}$, then $\overline{\sigma} = \overline{\pi} \circ \overline{\sigma}$ and $\overline{\sigma} = \overline{\sigma}^\omega$ or $\overline{\pi} = \overline{\pi} \circ \overline{\sigma}^\omega$, where $\overline{\sigma}^\omega = \cap \{\overline{\sigma}^i | i \in N \{0\}\}$.

Proof. Since $\overline{\pi} \subseteq \overline{\sigma}$ and $N$ is an i-v fuzzy multiplication near-ring, there exists an i-v fuzzy ideal $\overline{\iota}$ of $N$ such that $\overline{\pi} = \overline{\pi} \circ \overline{\iota}$. Then since $\overline{\pi}$ is prime, $\overline{\pi} \supseteq \overline{\iota}$. Now $\overline{\pi} = \overline{\pi} \circ \overline{\iota} \subseteq \overline{\iota}$. Thus $\overline{\iota} = \overline{\iota}$ and hence $\overline{\pi} = \overline{\pi} \circ \overline{\iota}$. It now follows that $\overline{\pi} = \overline{\pi}^\omega$ or $\overline{\pi} = \overline{\pi} \circ \overline{\sigma}^\omega$.

Theorem 3.14. If $\sqrt{\overline{\pi}}$ is an i-v prime fuzzy ideal, then $\overline{\iota}$ is an i-v primary.

Proof. Let $\overline{\sigma} \subseteq \sqrt{\overline{\pi}}$. If $\overline{\sigma} = \overline{\sigma}_N$, then clearly $\overline{\iota}$ is an i-v primary. Assume $\overline{\sigma} \neq \overline{\sigma}_N$. Suppose that $f$ is not i-v primary. Then there exist i-v fuzzy points $\overline{x}_r$, $\overline{y}_i$ such that $\overline{x}_r \circ \overline{y}_i \subseteq \overline{\pi}$, $\overline{x}_r \subseteq \overline{\sigma}$, but $\overline{x}_r \not\subseteq \overline{\iota}$ and $\overline{y}_i^n \not\subseteq \overline{\iota}$ for all $n > 0$. Let $\overline{\sigma} = \overline{\pi} \cup \overline{\sigma} \circ (\overline{x}_r \circ \overline{\sigma}_N)$. Clearly, $\overline{\sigma}$ is an i-v fuzzy ideal of $N$. Suppose $\overline{x}_r \subseteq \overline{\sigma}$. Then since $\overline{x}_r \not\subseteq \overline{\iota}$, $\overline{x}_r \subseteq \overline{\sigma} \circ (\overline{x}_r \circ \overline{\sigma}_N)$. Thus $(\overline{g} \circ (\overline{x}_r \circ \overline{\sigma}_N))(x) \geq r$, $\overline{x}_r \cup \{g(a) \land (x_r \circ \overline{\sigma}_N)(b)|x = ab \geq r\}$.
Since $\mathcal{I}$ has the sup property, $\mathcal{G}$ also possesses the sup property. Hence, there exists $z \in S$ such that $\mathcal{G}(z) \geq \mathcal{I}$ and $x = z \times s = z \times s$. Thus $\mathcal{I}(z^n) \geq \mathcal{I}$, for some $n > 0$. Now $x = z^{n+1}s^n$ and since $\mathcal{I}$ is an $i$-$v$ fuzzy ideal, $\mathcal{I}(x) = \mathcal{I}(z^{n+1}s^n) \geq \mathcal{I}(z^n) \geq \mathcal{I}$, i.e. $\mathcal{I}_r \subseteq \mathcal{I}$, a contradiction. Hence $\mathcal{I}_r \not\subseteq \mathcal{I}$. Now, $\mathcal{I} \cup \mathcal{I}_r \circ \mathcal{N} \subseteq \mathcal{G}$. Thus there exists an $i$-$v$ fuzzy ideal $\mathcal{H}$ of $N$ such that $\mathcal{I} \cup \mathcal{I}_r \circ \mathcal{N} \subseteq \mathcal{G} \circ \mathcal{H}$. Again since $\mathcal{G} \not\subseteq \mathcal{G}$, $\mathcal{G} \subseteq \mathcal{G} \cup \mathcal{H}_r \circ \mathcal{N}$. Then by Theorem 3.13 $\mathcal{G} = \mathcal{G} \circ (\mathcal{H} \cup \mathcal{H}_r \circ \mathcal{N})$. Now $\mathcal{I} \cup \mathcal{I}_r \circ \mathcal{N} \subseteq \mathcal{G} \circ \mathcal{H} = \mathcal{G} \circ (\mathcal{G} \cup \mathcal{H}_r \circ \mathcal{N} \circ \mathcal{H} = \mathcal{G} \circ (\mathcal{G} \cup \mathcal{H}_r \circ \mathcal{N}) \subseteq \mathcal{G} \circ \mathcal{H}$. Hence $\mathcal{I}_r \subseteq \mathcal{I}$, a contradiction. Therefore, $\mathcal{I}$ is an $i$-$v$ primary.

**Corollary 3.15.** Let $\mathcal{I}$ be an $i$-$v$ prime. Then for all positive integers $n$, $\mathcal{I}^n$ is an $i$-$v$ primary and its $i$-$v$ fuzzy radical is $\mathcal{I}$.

**Proof.** We first prove that $\sqrt[n]{\mathcal{I}} = \mathcal{I}$ for all $n > 0$. If $n = 1$, the result is obvious. Let $n > 1$. Then $\sqrt[n]{\mathcal{I}}^n(x) = \mathcal{I}(\mathcal{I}(x))^n \geq \mathcal{I}(x) \geq \mathcal{I}$ for all $x \in N$. Since $\mathcal{I}$ is an $i$-$v$ prime, $\mathcal{I}(x) = \mathcal{I}(x)$ and $\sqrt[n]{\mathcal{I}}^n(x) = \mathcal{I}(x)$ for all $x \in N$. Hence $\sqrt[n]{\mathcal{I}} = \mathcal{I}$. The desired result follows from Theorem 3.14

**Theorem 3.16.** Let $\mathcal{I}$ be an $i$-$v$ prime fuzzy ideal and $\mathcal{I}^n \neq \mathcal{I}^{n+1}$ for all $n > 0$. Then $\mathcal{I}^\omega$ is an $i$-$v$ prime fuzzy ideal.

**Proof.** Let $\mathcal{I}_l, \mathcal{I}_r$ an $i$-$v$ fuzzy points such that $\mathcal{I}_l \subseteq \mathcal{I}^\omega$ and $\mathcal{I}_r \subseteq \mathcal{I}^\omega$. We show that $\mathcal{I}_l \circ \mathcal{I}_r \subseteq \mathcal{I}^\omega$. When $\mathcal{I}_l \not\subseteq \mathcal{I}^\omega$, then since $\mathcal{I}$ is an $i$-$v$ prime, $\mathcal{I}_l \circ \mathcal{I}_r \not\subseteq \mathcal{I}^\omega$ and so $\mathcal{I}_l \circ \mathcal{I}_r \not\subseteq \mathcal{I}^\omega$. Suppose $\mathcal{I}_l \subseteq \mathcal{I}^\omega$ and $\mathcal{I}_r \subseteq \mathcal{I}^\omega$. Since $\mathcal{I}_l \subseteq \mathcal{I}^\omega$, there exists a positive integer $p$ such that $\mathcal{I}_l \subseteq \mathcal{I}^p$, $\mathcal{I}_r \subseteq \mathcal{I}^p$. By Corollary 3.15 $\mathcal{I}^p \subseteq \mathcal{I}$ is an $i$-$v$ primary fuzzy ideal with $i$-$v$ fuzzy radical $\mathcal{I}_r$, $\mathcal{I}_l \circ \mathcal{I}_r \subseteq \mathcal{I}^\omega$ and so $\mathcal{I}_l \circ \mathcal{I}_r \subseteq \mathcal{I}^\omega$. The case when $\mathcal{I}_l \subseteq \mathcal{I}^\omega$ is similar.

Finally, let $\mathcal{I}_l, \mathcal{I}_r \subseteq \mathcal{I}^\omega$. Then there exist positive integers $q, r$ such that $\mathcal{I}_l \subseteq \mathcal{I}^q$, $\mathcal{I}_r \subseteq \mathcal{I}^r$ and $\mathcal{I}_l \circ \mathcal{I}_r \subseteq \mathcal{I}^{q+r}$ and $\mathcal{I}_l \circ \mathcal{I}_r \subseteq \mathcal{I}^{q+r}$. Then $\mathcal{I}_l \circ \mathcal{I}_r \subseteq \mathcal{I}^p$, $\mathcal{I}_l \circ \mathcal{I}_r \subseteq \mathcal{I}^{p+1}$. Since $\mathcal{I}_l \circ \mathcal{I}_r \subseteq \mathcal{I}^\omega$, there exist $i$-$v$ fuzzy multiplication near $\mathcal{I}_l \circ \mathcal{I}_r$ of $N$ such that $\mathcal{I}_l \circ \mathcal{I}_r \circ \mathcal{I}_r \subseteq \mathcal{I}^\omega$. Now, if $\mathcal{I}_l \circ \mathcal{I}_r \subseteq \mathcal{I}^\omega$, then $\mathcal{I}_l \circ \mathcal{I}_r \circ \mathcal{I}_r \subseteq \mathcal{I}^\omega$ and $\mathcal{I}_l \circ \mathcal{I}_r \circ \mathcal{I}_r \subseteq \mathcal{I}^\omega$. Now, if $\mathcal{I}_l \circ \mathcal{I}_r \subseteq \mathcal{I}^\omega$, then $\mathcal{I}_l \circ \mathcal{I}_r \circ \mathcal{I}_r \subseteq \mathcal{I}^\omega$ and $\mathcal{I}_l \circ \mathcal{I}_r \circ \mathcal{I}_r \subseteq \mathcal{I}^\omega$. Since $\mathcal{I}^\omega$ is a $i$-$v$ primary fuzzy ideal with $i$-$v$ fuzzy radical $\mathcal{I}$ and $\mathcal{I}_l \circ \mathcal{I}_r \subseteq \mathcal{I}^\omega$ (since $\mathcal{I}$ is an $i$-$v$ prime), $\mathcal{I}^{q+r} \subseteq \mathcal{I}^{q+r+1}$. Thus $\mathcal{I}^{q+r} \subseteq \mathcal{I}^{q+r+1}$. Hence $\mathcal{I}_l \circ \mathcal{I}_r \subseteq \mathcal{I}^\omega$, i.e., $\mathcal{I}_l \circ \mathcal{I}_r \subseteq \mathcal{I}^\omega$. Thus $\mathcal{I}^\omega$ is an $i$-$v$ prime fuzzy ideal.

**Theorem 3.17.** If $\mathcal{I}$ is an $i$-$v$ primary fuzzy ideal, then $\mathcal{I} = \mathcal{I}^n$ for some positive integer $n$, where $\mathcal{I} = \sqrt[n]{\mathcal{I}}$. 


Proposition 3.18. If $\overline{f}$ is a proper prime i-v fuzzy ideal and $\overline{g}$ is an i-v fuzzy ideal of $N$ such that $\overline{g} \subseteq \overline{f}^n$ and $\overline{g} \not\subseteq \overline{f}^{n+1}$ for some $n > 0$, then $\overline{f}^n = \overline{g} : (\overline{y}_r \circ \overline{C}_N)$, where $\overline{y}_r \not\subseteq \overline{f}$.

Proof. Since $\overline{g} \subseteq \overline{f}^n$, there exists an i-v fuzzy ideal $\overline{h}$ of $N$ such that $\overline{g} = \overline{f}^n \circ \overline{h}$, $\overline{h} \not\subseteq \overline{f}$. Let $\overline{y}_k \subseteq \overline{h}$, $\overline{y}_r \not\subseteq \overline{f}$. Then $\overline{y}_r \circ \overline{C}_N \subseteq \overline{h}$ and $\overline{f}^n \circ (\overline{y}_r \circ \overline{C}_N) \subseteq \overline{f}^n \circ \overline{h} = \overline{g}$. Thus $\overline{f}^n \subseteq \overline{g} : (\overline{y}_r \circ \overline{C}_N)$. Now let $k$ be any i-v fuzzy ideal of $N$ such that $k \circ (\overline{y}_r \circ \overline{C}_N) \subseteq \overline{g}$. Then $k \circ (\overline{y}_r) \circ \overline{C}_N \subseteq \overline{f}^n$. Now since by Corollary 3.15, $\overline{f}^n$ is i-v primary with fuzzy radical $\overline{f}$ and $\overline{y}_r \circ \overline{C}_N \not\subseteq \overline{f}$, $k \subseteq \overline{f}^n$. Therefore, $\overline{g} : (\overline{y}_r \circ \overline{C}_N) \subseteq \overline{f}^n$ and hence $\overline{f}^n = \overline{g} : (\overline{y}_r \circ \overline{C}_N)$.

Proposition 3.19. Let $f : N \to N'$ be a surjective near-ring anti-homomorphism and $\overline{\pi}'$ is an i-v fuzzy prime ideal of $N'$, then $f^{-1}(\overline{\pi}')$ is an i-v fuzzy prime ideal of $N$.

Proof. Let $\overline{\pi}$ and $\overline{\sigma}$ be two i-v fuzzy ideals of $N$ such that $\overline{\pi} \subseteq f^{-1}(\overline{\pi}')$.

If $f(\overline{\pi} \subseteq \overline{\pi}')$

Then $f(\overline{\pi} \subseteq \overline{\pi}')$.

Since $\overline{\pi}'$ is an i-v fuzzy prime ideal of $N'$

$f(\overline{\pi}) \subseteq \overline{\pi}'$ or $f(\overline{\pi}) \subseteq \overline{\pi}'$

Then $\overline{\sigma} \subseteq f^{-1}(\overline{\pi}')$ or $\overline{\pi} \subseteq f^{-1}(\overline{\pi}')$.

Therefore $f^{-1}(\overline{\pi}')$ is an i-v fuzzy prime ideal of $N$.

Proposition 3.20. Let $f : N \to N'$ be a surjective near ring anti- homomorphism and $\overline{\pi}'$ is an i-v primary fuzzy ideal of $N'$, then $f^{-1}(\overline{\pi}')$ is an i-v primary fuzzy ideal of $N$.
Proof. Let \( \overline{\mu} \) and \( \overline{\sigma} \) be two i-v fuzzy ideals of \( N \). Such that
\[
\overline{\mu} \overline{\sigma} \subseteq f^{-1}(\overline{\mu}') \quad \text{and} \quad \overline{\sigma} \nsubseteq f^{-1}(\overline{\mu}')
\]
\[
\Rightarrow f(\overline{\mu} \overline{\sigma}) \subseteq \overline{\mu}' \quad \text{and} \quad f(\overline{\sigma}) \nsubseteq \overline{\mu}'
\]
\[
\Rightarrow f(\overline{\sigma}) f(\overline{\mu}) \subseteq \overline{\mu}' \quad \text{and} \quad f(\overline{\sigma}) \nsubseteq \overline{\mu}'
\]
\[
\Rightarrow f(\overline{\mu}) \subseteq \sqrt{\overline{\mu}'} \quad \text{(Since} \overline{\mu}' \text{is an i-v primary fuzzy ideal)}
\]
\[
\Rightarrow \overline{\mu} \subseteq f^{-1}(\overline{\mu}')
\]
\[
\Rightarrow \overline{\mu} \subseteq \sqrt{f^{-1}(\overline{\mu}')}
\]
Therefore \( f^{-1}(\overline{\mu}') \) is an i-v primary fuzzy ideal of \( N \).

Lemma 3.21. If \( \overline{\mu} \) is an i-v primary fuzzy ideal of a near-ring \( N \), then \( \sqrt{\overline{\mu}} \) is an i-v prime fuzzy ideal of \( N \).

Proof. Let \( \overline{\sigma} \) and \( \overline{\theta} \) be two i-v fuzzy ideals of \( N \) such that \( \overline{\sigma} \overline{\theta} \subseteq \sqrt{\overline{\mu}} \) and \( \overline{\sigma} \nsubseteq \sqrt{\overline{\mu}} \)
\[
\Rightarrow \overline{\sigma} \overline{\theta} \subseteq \overline{\mu} \quad \text{and} \quad \overline{\sigma} \nsubseteq \overline{\mu}.
\]
Since \( \overline{\mu} \) is an i-v primary fuzzy ideal, \( \overline{\theta} \subseteq \sqrt{\overline{\mu}} \).
Therefore \( \sqrt{\overline{\mu}} \) is an i-v prime fuzzy ideal of \( N \).

Proposition 3.22. Let \( f : N \rightarrow N' \) be a surjective near-ring anti-homomorphism. If \( \overline{\mu} \) is an f-invariant i-v fuzzy ideal of \( N \) and \( \overline{\nu} \), an i-v fuzzy primary ideal of \( N \), then \( f(\overline{\mu}) \) is an i-v fuzzy primary ideal of \( N' \).

Proof. Let \( \overline{\sigma}' \) and \( \overline{\theta}' \) be two i-v fuzzy ideals of \( N' \) such that \( \overline{\sigma}' \overline{\theta}' \subseteq f(\overline{\mu}) \) and \( \overline{\sigma}' \nsubseteq f(\overline{\mu}) \)
\[
\Rightarrow f^{-1}(\overline{\sigma}' \overline{\theta}') \subseteq f^{-1}(f(\overline{\mu})) = f^{-1}(\overline{\mu}')
\]
\[
\Rightarrow f^{-1}(\overline{\sigma}' \overline{\theta}') \subseteq \overline{\mu}' \quad \text{and} \quad f^{-1}(\overline{\sigma}') \nsubseteq \overline{\mu}'
\]
\[
f^{-1}(\overline{\sigma}') f^{-1}(\overline{\theta}') \subseteq \overline{\mu}' \quad \text{and} \quad f^{-1}(\overline{\sigma}') \nsubseteq \overline{\mu}'
\]
\[
\Rightarrow f^{-1}(\overline{\theta}') \subseteq \sqrt{\overline{\mu}'} \quad \text{(Since} \overline{\mu}' \text{is an i-v primary fuzzy ideal)}
\]
\[
\Rightarrow \overline{\theta}' \subseteq \sqrt{f(\overline{\mu})}.
\]
Therefore \( f(\overline{\mu}) \) is an i-v fuzzy primary ideal of \( N' \).

Proposition 3.23. For a surjective near-ring anti-homomorphism \( f : N \rightarrow N' \), if \( \overline{\mu} \) is an f-invariant i-v strongly primary fuzzy ideal of \( N \) then \( f(\overline{\mu}) \) is an i-v strongly primary fuzzy ideal of \( N' \).

Proof. Let \( \overline{\mu} \) be an \( f \)-invariant i-v strongly primary fuzzy ideal of \( N \).
\[
\Rightarrow \overline{\mu} \text{is an i-v primary fuzzy ideal and} \quad (\sqrt{\overline{\mu}})^n \subseteq \overline{\mu} \quad \text{for some} \quad n \in N
\]
\[
\Rightarrow f(\overline{\mu}) \text{is an i-v primary fuzzy ideal of} \quad N'.
\]
Since \( f(\overline{\mu}) \) is an i-v primary fuzzy ideal of \( N' \), \( \sqrt{f(\overline{\mu})} \) is an i-v prime fuzzy ideal of \( N' \). (By Lemma 3.21)
Since \( \sqrt{f(\overline{\mu})} = \wedge \{f(\overline{\sigma}), f(\overline{\tau}) \} \text{is an i-v fuzzy prime ideal of} \ N', f(\overline{\sigma}) \subseteq f(\overline{\mu}) \}.\]
Therefore \((\sqrt{f(\mu)})^n \subset f(\mu)\) for some \(N \in N\).
Hence \(f(\mu)\) is an \(i,v\) strongly primary fuzzy ideal of \(N\).

**Proposition 3.24.** For a surjective near-ring homomorphism \(f : N \rightarrow N'\), if \(\overline{\mu}\) is an \(i,v\) semi prime fuzzy ideal of \(N'\), then \(f^{-1}(\overline{\mu}')\) is an \(i,v\) semi prime fuzzy ideal of \(N\).

**Proof.** Given \(\overline{\mu}'\) is an \(i,v\) semi prime fuzzy ideal of \(N'\).
\(\Rightarrow\) \(\mu\) is an \(i,v\) fuzzy ideal of \(N'\) and \(\mu^2(x) = \mu(x)\) for all \(x \in N\).
\(\Rightarrow\) \(f^{-1}(\overline{\mu}')\) is an \(i,v\) fuzzy ideal of \(N\).
Let \(f^{-1}(\overline{\mu}') = \mu \Rightarrow (\overline{\mu}') = f(\mu)\)
Now \(\overline{\mu}' = \overline{\mu}' \overline{\mu}' = f(\mu)' = f(\mu) \overline{\mu} = f(\mu^2)\)
\(\Rightarrow\) \(\overline{\mu}'^2 = f^{-1}(\overline{\mu}') = \mu \Rightarrow [f^{-1}(\overline{\mu}')]^2(x) = f^{-1}(\overline{\mu}')(x)\) for all \(x \in N\).

**Conclusion**

In this article it is shown that for an \(i,v\) primary fuzzy ideal of a near ring \(N\), \(\sqrt{\mu}\) is an \(i,v\) prime fuzzy ideal. Further it has been proved for a \(f\)-invariant and \(i,v\) fuzzy primary ideal \(\mu\) of \(N\), \(f(\mu)\) is also an \(i,v\) fuzzy primary ideal.

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