Production of $\pi^0\rho^0$ Pair in Electron–Positron Annihilation in the Nambu–Jona–Lasinio Model

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INTRODUCTION

This work is devoted to the description of the process $e^+e^-\rightarrow\pi^0\rho^0$ recently measured in experiments of the years 2006 and 2011 [1, 2]. It is also concerned with a theoretical study of the process $e^+e^-\rightarrow\pi^0\rho^0$ within the $\omega$, $\omega'$, and $\rho$-mesons energy range. This process was recently measured at the CMD-2 detector at the VEPP-2M $e^+e^-$ collider [1–3].

The cross section of hadron production in the $e^+e^-$annihilation in the energy region $\sqrt{s} < 1.03$ GeV can be described in the vector meson dominance model (VDM) framework and is determined by the transitions of light vector mesons ($\omega$, $\omega'$, $\rho$) to the final states.

It is one of the series of works [4, 5], where the process $e^+e^-\rightarrow\pi^0\omega$, $\pi^0\omega'$ was described in the framework of the expanded NJL model [6, 7]. The results obtained were found to be in satisfactory agreement with the known experimental data [1, 2]. The main formalism including the $SU(2)\times SU(2)$ chiral NJL model coincides with one of the papers [4, 5]. The standard NJL Lagrangian which describes interactions of photons, pions, and vector $\rho$ and $\omega$ mesons with quarks is presented in [8, 9].

1. AMPLITUDE OF THE PROCESS $e^+e^-\rightarrow\omega, \omega', \phi \rightarrow \pi^0\rho^0$

The amplitude can be written down in the form:

$$T = \bar{\psi}_l e^+ e^-(\gamma^\mu\gamma^\nu\gamma^\lambda) \frac{\rho_\mu\rho_\nu}{m \cdot s} \{ B_{\gamma+\omega} + B_\omega + B_{\omega'} \} \epsilon_\lambda(\rho),$$

where $s = (p_1(\gamma^+\rho) + p_2(\gamma^-))^2$.

The quantity $B_{\gamma+\omega}$ to the contribution of the amplitude from the process with intermediate photons and $\omega$ mesons is as follows:

$$B_{\gamma+\omega} = \frac{M_\omega^2 + iM_\omega \Gamma_\omega}{M_\omega^2 - s + iM_\omega \Gamma_\omega} g_{\rho\gamma}(s),$$

where $\sin\theta_{\omega\phi} = -0.0523$.

The quantity $B_\omega$ to the contribution with $\phi$ meson in the intermediate state [7]:

$$B_\omega = \frac{s\sqrt{2}\sin\theta_{\omega\phi}}{s - M_\omega^2 + iM_\omega \Gamma_\omega} g_{\rho\gamma}(s),$$

where $\sin\theta_{\omega\phi}$ is taken from paper [10].

The quantity $B_{\omega'}$ to the contribution from the intermediate radial excitation of the $\omega$-meson state, $\omega' \rightarrow \pi^0\rho$, is taken from paper [10]

$$B_{\omega'} = \frac{s}{s - M_{\omega'}^2 \Gamma_{\omega'}} \left( \frac{\cos(\beta + \beta_0)}{\sin(2\beta_0)} \right)$$

$$- \Gamma \frac{\cos(\beta - \beta_0)}{\sin(2\beta_0)} \frac{1}{g_{\rho\gamma}} V_{\rho\omega'}(s),$$

where $\Gamma \approx 1/2$ will be specified below (see (8)) and

$$V_{\rho\pi^0\gamma}(s) = g_{\rho\gamma} \frac{\sin(\beta + \beta_0)\rho_{\gamma} I_3^{(3)}}{\sin(2\beta_0)}$$

$$+ \frac{\sin(\beta - \beta_0)\rho_{\gamma} I_3^{(3)}}{\sin(2\beta_0)} \frac{1}{g_{\rho\gamma}} V_{\rho\omega'}(s),$$

$$V_{\rho\pi^0\omega}(s) = -g_{\rho\omega} \frac{\cos(\beta + \beta_0)\rho_{\omega} I_3^{(3)}}{\sin(2\beta_0)}$$

$$+ \frac{\cos(\beta - \beta_0)\rho_{\omega} I_3^{(3)}}{\sin(2\beta_0)}.$$
In the amplitudes with excited mesons we have to take into account the mixing angle

\[ \beta = 61.53° \] and \[ \gamma = 76.78° \], were defined in [10]. The standard value of the \( \phi - \omega \) mixing angle \( \theta_{\phi \omega} \approx -3° \) is used [9]. For the numerical calculations we use the values from the Particle Data Group [12]: \( \Gamma_{\omega} = 8.49 \text{ MeV} \), \( \Gamma_{\omega'} = 215 \text{ MeV} \), \( M_{\omega} = 782 \text{ MeV} \), \( M_{\omega'} = 775 \text{ MeV} \), \( M_{\rho} = 1420 \text{ MeV} \), \( M_{\phi} = 1020 \text{ MeV} \), \( \Gamma_{\phi} = 4.26 \text{ MeV} \). The \( \gamma - \phi \) transition differs from the above just by a factor 1/3 compared with \( \gamma - \rho \). In the amplitudes with excited mesons we have to take into account the \( \gamma - \rho_2 \) and \( \gamma - \bar{\omega}_2 \) transitions (\( \gamma - \omega_1(\rho) \)) transitions are the same as in the standard \( \gamma - \omega(\rho) \) cases) that can be expressed via the \( \gamma - \omega(\rho) \) transition with the additional factor [7, 10]

\[ \Gamma = \frac{I_\gamma}{\sqrt{I_\gamma I_\omega}} \approx 0.47. \]  

2. TOTAL CROSS SECTION

In (7), \( m \) is the constituent quark mass \( (m_u = m_d = 280 \text{ MeV}) \). For calculation of the total cross section of the process we use:

\[ \sigma(s) = \frac{3 \lambda^2 f_\rho^2}{32 \pi^3 \sqrt{f_\rho}} \lambda^{3/2}(s, M_{\rho}^2, M_{\phi}^2) \]

\[ \times |B_{\gamma + \omega} + B_{\rho + \phi}|^2, \]

where \( f_\rho = 93 \text{ MeV} \) is the pion decay constant, and \( \lambda(s, M_{\rho}^2, M_{\phi}^2) = (s - M_{\rho}^2 - M_{\phi}^2)^2 - 4 M_{\rho}^2 M_{\phi}^2, g_\rho \) is the vector meson coupling constant \( g_\rho \approx 6.14 \) corresponding to the standard relation \( g_\rho^2 \approx 3 \). The total cross section in the region \( 0.9 < \sqrt{s} < 2 \text{ GeV} \) is presented in the figure.

In the table the behavior of the cross section in the region \( m_u + m_p = \sqrt{s_{10}} < \sqrt{s} = 1.1 \text{ GeV} \) is presented. In this region the cross section has a resonance character. In conclusion, we would like to note the distinction between the \( \pi^0 \rho^0 \) and \( \pi^0 \omega \) processes, where the \( \phi \) resonance is not seen in the \( \pi^0 \omega \) process. A similar situation takes place in the process \( e^+e^- \rightarrow \pi^0 \gamma \) which was supported by experimental data.
The magnitude of the total cross section in the resonance region $0.915 < \sqrt{s} < 1.1$ GeV

| $\sqrt{s}$, GeV | $\sigma$, nb | $\sqrt{s}$, GeV | $\sigma$, nb | $\sqrt{s}$, GeV | $\sigma$, nb | $\sqrt{s}$, GeV | $\sigma$, nb |
|-----------------|-------------|-----------------|-------------|-----------------|-------------|-----------------|-------------|
| 0.915           | 0.0         | 0.95            | 3.4         | 1.02            | 796         | 1.054           | 0.13        |
| 0.916           | 0.022       | 0.956           | 4.27        | 1.026           | 58.2        | 1.056           | 0.2         |
| 0.918           | 0.11        | 0.962           | 5.2         | 1.03            | 15.8        | 1.06            | 0.38        |
| 0.922           | 0.38        | 0.972           | 7.1         | 1.04            | 1.03        | 1.07            | 0.88        |
| 0.926           | 0.71        | 0.98            | 9.11        | 1.048           | 0.05        | 1.08            | 1.36        |
| 0.932           | 1.3         | 1               | 22          | 1.05            | 0.04        | 1.09            | 1.78        |
| 0.944           | 2.2         | 1.01            | 58.6        | 1.052           | 0.07        | 1.1             | 2.14        |

As a by-product of our analysis we obtain the partial decay of the process $\phi \rightarrow \rho^0 \pi^0$, $\Gamma_{\phi \rightarrow \rho^0 \pi^0} \approx 0.5$ MeV which is in good agreement with PDG data [12].

CONCLUSIONS

The cross section of the process $e^+ e^- \rightarrow \pi^0 \rho^0$ was measured in the Spherical Neutral Detector (SND) experiment at the VEPP-2M collider in the energy region $\sqrt{s} = 980–1380$ MeV [1–3].

Our calculations for the process $e^+ e^- \rightarrow \pi^0 \rho^0$ showed the presence of two regions of enhancement of the cross section in the energy range below 1.020 and 1.4 GeV. The first one appears in the region of the $\phi$-meson mass and looks like a very high narrow peak. The second one is a smooth peak, it lies in the region of $\omega$-meson mass.

Notwithstanding, the process $e^+ e^- \rightarrow \pi^0 \rho^0$ is similar to the process $e^+ e^- \rightarrow \omega \pi^0$, but in our result in the $\phi$-meson mass region we have a very narrow peak which will be in agreement with experiment.

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