Vortex-state–mediated Josephson effect

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Abstract – The Josephson effect is a kind of macroscopic quantum phenomenon that supercurrent flows through a Josephson junction without any voltage applied. We predict a novel vortex-state–mediated Josephson effect in an SNS Josephson junction which hosts vortices. The vortex-state–mediated supercurrent is enhanced or reduced significantly in magnitude depending on the junction length, and exhibits several steps with the number of effective propagating channels in current-phase evolution at zero temperature. At finite temperatures, these supercurrent steps persist in the short junction limit, and develop into sawtooth oscillations if the junction length becomes comparable to the coherence length $\xi = \hbar v_F/\Delta$ of the superconductor. In the later case a supercurrent reversal can be observed. These findings may provide a smoking-gun signature of vortex bound states in superconductors and promise possible applications in future Josephson devices.

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The Josephson effect is a kind of macroscopic quantum phenomenon, first predicted by Brian Josephson, that Cooper pairs can tunnel through weakly coupled superconductors [1]. It also exists if superconductors are connected by a weak link of any physical nature [2,3].

In a clean SNS junction, the Josephson effect is remarkably different from the Josephson tunnel junctions. The underlying mechanism is the coherent Andreev reflection [4]. Interference between Andreev reflected electron-like and hole-like excitation wave functions in the quantum well formed by the pair potentials of the superconductors leads to the formation of Andreev bound states [5]. It is shown that a significant portion of supercurrent is carried by the discrete Andreev levels [6,7], and the critical current (the maximum current) decreases exponentially with temperature [5]. The introduction of impurities in the normal region would suppress the Josephson supercurrent [7]. If the normal region becomes a quantum point contact [8], the critical current may be an integer multiple of $e\Delta/\hbar$, where $2\Delta$ is the energy gap of the superconductor.

The existence of bound states inside a vortex, where the pair potential of the superconductor is zero, is predicted by Caroli, de Gennes, and Matricon [9] and confirmed with controversy [10] long after its discovery. The vortex bound states are still the Andreev bound states. Vortices in chiral $P$-wave superconductors or superfluids may even support zero-energy Majorana modes [11,12], one of the appealing candidates for topological quantum computation [13]. Tunneling between the Majorana zero modes at two vortices is investigated [14], demonstrating that the tunneling amplitude depends on the phase difference of the order parameters at the two vortices and decays exponentially with the distance between the vortices.

A question arises as to what happens when both the vortex bound states and Andreev bound states appear in an SNS junction. It is expected that coherent tunneling between the vortex bound states also transfers Cooper pairs and carries a portion of supercurrent, besides the Andreev bound states, at least in the short junction case. Therefore interference between these two pair-transfer channels may result in interesting Josephson effects. We anticipate that the Josephson supercurrent may exhibit some unique features in the presence of vortices. For this purpose, we propose an SNS junction that can support both the vortex bound states and the Andreev bound states, and investigate the supercurrent-phase characteristics in such a particular setup. The proposed SNS junction is very similar to the hybrid structures based on topological insulators [15,16]. It is known that the surface of a topological insulator can be described by a Dirac Hamiltonian [17]. In this sense, our proposed SNS Josephson junction can be considered as the Schrödinger version of the topological SNS junctions [15,16].
The SNS Josephson junction under consideration consists of two s-wave superconductive planar slabs, each with a hole, and a hollow cylindrical normal slab. The superconductive slabs are connected ideally by the normal slab (see fig. 1(a)). Here “ideally” means that neither barriers at the junctions nor Fermi velocity mismatch between the superconductor and the normal conductor are considered. Unlike the Dirac version of the junction based on topological insulators with insulating bulk and conducting surface, it may pose an experimental challenge to realize the proposed SNS junction. One possibility of realization is to prepare a shaped two-dimensional electron gas (2DEG) by heterostructure (GaAs/AlAs/Ga1−xAs) engineering, and then introduce superconductivity at the 2DEG planes by the proximity effect [12].

The surface superconductor in the presence of a vortex with a flux quantum \( \Phi_0 = hc/2e \) can be described by the Bogoliubov-de Gennes Hamiltonian [2] with an inhomogeneous pair potential \( \Delta(\rho) \) in the polar coordinate \( \rho = (\rho, \theta) \) with the origin at the vortex center

\[
H_S = \begin{bmatrix}
\mathcal{H}_e & \Delta(\rho) \\
\Delta^*(\rho) & \mathcal{H}_h
\end{bmatrix},
\]

where \( \mathcal{H}_e = -\hbar^2(e_\rho \partial_\rho + e_\theta \partial_\theta)/\rho + i e A_\phi e_\theta/hc) \) is the single-electron Hamiltonian.

The detailed matching procedure is given in [18]. The excitation spectrum can be obtained by matching the wave functions on the upper and lower superconductive surfaces with the wave function on the normal cylindrical surface at the boundaries \( \rho = R \). The detailed matching procedure is given in [18]. The excitation energy as a function of the superconducting phase difference \( \phi \) with a fixed angular momentum \( \mu = n + 1/2 \) is approximately determined by the equation

\[
(k_{F0}^{\mu} - k_{F0}^{\mu+1/2}) L = 4R \phi \pm (2n + 1)\pi.
\]

Compared with the Andreev spectrum equation given by eq. (16) in ref. [18] for the gedenkon junction, one observes that the vortex bound state effects are encoded in an additional phase term \(-4R_{1}(R)\). To estimate the value of the function \( R_{1}(R) \), we approximate the pair potential near the vortex center by \( \Delta(\rho) \approx \Delta(\rho^2 + 2\xi_{sc}^2)^{1/2} \), where \( \xi_{sc} = (D/2\Delta)^{1/2} \ll \xi \).
is the “dirty-limit” coherence length of the superconducting surfaces with diffusion constant $D$. In the regime of interest, $k_FR \gg 1, R \ll \xi, \Re_0(R) \approx \Re_0(0) \approx -E/2\Delta + \mu \ln(\xi/\xi_c)/k_F\xi$. The energy separation between the vortex bound states is then estimated to be $\Delta^2/E_F$, consistent with the result of Calori et al. [9].

The Josephson current is an equilibrium property of superconductors, and can be expressed by [18,19]

$$I = -\frac{2e}{h} \sum_{\mu} \tan \left( \frac{E_0}{2k_BT} \right) \frac{\partial E_{\mu}}{\partial \phi},$$

where the summation is over all the discrete subgap Andreev levels. At zero temperature, the Josephson supercurrent is $I = -\frac{2e}{h} \sum_{\mu,0 < E_\mu} \frac{\partial E_\mu}{\partial \phi}$.

Let us consider first the SNS junction without the cylinder connection. Two vortices can still host bound states. The wave functions of the vortex bound states may extend somewhat along the axial direction of the cylinder, since any physical slabs have a finite thickness. The bound state wave functions inside the vortex cores are given by $\psi_\mu(x) \propto e^{i(\mu-\frac{1}{2})|\mathbf{k}|}J_{\mu-\frac{1}{2}}(\frac{k_F|\mathbf{k}|}{\mu}) \times e^{-k_F|\mathbf{x}|}$, where $J_{\nu+\frac{1}{2}}$ are Bessel functions with arguments $(k_\perp \pm m_{\phi}/k_F^2)\rho$. $\Phi_\mu$ is a decaying function, $k_F^2 \approx k_F^2[(\mu/k_FR)^2 - 1]^{1/2}$. The amplitude for tunneling between the two vortices can be estimated from Bardeen’s well-known transfer Hamiltonian method [20] and is proportional to $\int d\rho d\phi \frac{\partial}{\partial \rho} \psi_{\mu}^\dagger \frac{\partial}{\partial \phi} \psi_{\mu} \propto \sin(\phi/2)e^{-k_F^2L}$. From this expression we see that only the vortex bound states with small angular momentum favor an effective tunneling. Adiabatic connection of the conductive cylindrical surface with the superconducting surfaces opens up conductive channels to transport Cooper pairs by the formation of the Andreev subbands. On the other hand, it leads to the hybridization between the vortex bound states and the Andreev subbands on the cylindrical surface. The hybridization becomes most pronounced when the corresponding angular momenta are pronounced.

We present some examples of the excitation spectra for the junction with a fixed radius $k_FR = 2$ and different lengths $L$ in fig. 2. The excitation spectra for the gedenken junction are rather simple [21], which consists of three nearly degenerate Andreev subbands with decreasing slope as the junction length increases. When the vortex bound states are involved, the excitation spectra of the Andreev subbands become more intriguing. The degeneracy of the states are involved, the excitation spectra of the Andreev subbands is lifted in the presence of vortices, implying a hybridization between the vortex levels and the Andreev subbands with the same angular momentum. In the short junction case ($k_F R = 1$), there appear additional Andreev subbands corresponding to higher angular momenta besides the allowed numbers. However, these additional subbands with small angular momentum carry a significant supercurrent (we call them the effective transport channels), such as the $|\mu| = \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$ channels, the others with large angular momentum carry a trivial supercurrent. The formation of these additional transport channels is originated from the effective coherent tunneling between the vortex bound states with small angular momentum. As the junction length is increased, the tunneling between the vortex bound states becomes weaker and weaker, and finally no effective tunneling can be expected as the junction length $k_FR$ is about 2.6. As $k_FR = 2$, there exist only additional Andreev subbands with angular momenta $|\mu| = 5/2, 7/2, 9/2, 11/2$, and the channels with larger angular number $9/2, 11/2$ cannot carry supercurrent. If the junction length exceeds the critical value 2.6, no additional Andreev subbands can be formed, which is clearly demonstrated in fig. 2. However, the hybridization between the vortex bound states and the Andreev subband is still possible as long as their angular momenta are coincident, while the other vortex bound states with large angular momentum behave as impurities someway. The hybridization leads to the splitting of the almost degenerate Andreev subbands and modifies their phase difference dependence, especially for the subband $|\mu| = \frac{3}{2}$. We note that, due to the presence of vortex bound states, there also exist some unavoidable subband crossings.

From the observation of the interesting excitation spectra of the SNS junction with vortices, one can expect unusual Josephson supercurrent evolution with the superconducting phase difference. In fig. 3 we present the corresponding supercurrent-phase relation. The supercurrent of a Josephson junction exhibits a sinusoidal dependence on the superconducting phase difference. The evolution period of supercurrent with the phase difference is still $2\pi$ as in the usual SNS junction case. The $2\pi$-periodicity of the supercurrent evolution is originated from $2\pi$-periodicity of the energy spectra. At $\phi = \pm \pi$ the positive and negative Andreev modes have combined to form a standing wave with $\partial E/\partial \phi = 0$ [7], which are clearly shown in the excitation energy spectra in fig. 2. Therefore the Josephson current drops to zero at $\phi = \pm \pi$.  

Fig. 2: (Colour on-line) Excitation spectra of the junction with different cylinder lengths $k_FR$ indicated by the numbers in the panels. The other parameters are chosen as $k_FR = 2, k_F\xi = 10, k_F\xi_c = 4$. 

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A striking phenomenon is observed that the supercurrent develops step structures in response to the increase of the superconducting phase difference. The number of steps within one half-period is the same as the number of effective supercurrent-carrying modes, which is already confirmed from the excitation spectra in fig. 2. To the best of our knowledge, this observation has never been reported before. It can be considered as the fingerprint of the proposed junction. By comparing the supercurrent in the presence and in the absence of the vortices, we find that the vortex-state-mediated critical supercurrent is enhanced with an amount of about 33 percent in the short junction limit \( L = 0.1 \xi \), which drops rapidly (about 38 percent) as the cylinder length is doubled \( (L = 0.2 \xi) \). This observation confirms the tunneling characteristics between the bound states of the two vortices. In addition to the pronounced step structures, the supercurrent also declares a reversal as long as \( L \geq \xi \), with a significant suppression of the critical supercurrent when compared to the gedenken junction case.

The number of propagating channels on the cylindrical surface is given by \( 2k_F L - 1 \). There exist three nearly degenerate propagating channels \( \nu = 0, \pm 1 \) in the present case \( k_F R = 2 \), so the critical supercurrent through the gedenken junction can approach \( 3e\Delta/\hbar \) in the short junction limit. The significant enhancement of the supercurrent as \( k_F L = 1 \) in the presence of vortex bound states is due to the formation of additional transport channels.

The observed step structure can be well understood from the excitation spectra given in fig. 2. With the increase of the superconducting phase difference, more and more effective propagating channels are involved in the process of transporting Cooper pairs, resulting in a step increase of the supercurrent. If the cylinder length exceeds the critical value, no effective propagating channels can be formed from the vortex bound state tunneling.
effects, thus provide a smoking-gun evidence for the existence of vortex bound states in type-II superconductors.

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