On Supersymmetric $b-\tau$ Unification, Gauge Unification, and Fixed Points

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Abstract

The equality assumption of the $b$ and $\tau$ Yukawa couplings at the grand-unification scale can strongly constrain the allowed parameter space of supersymmetric models. We examine the constraints in the case that there is a discrepancy $>10\%$ in the gauge coupling unification assumption (which necessarily implies large perturbations at the grand scale). The constraints are shown to diminish in that case [most significantly so if $\alpha_s(M_Z) \approx 0.11$]. In particular, the requirement that the $t$ Yukawa coupling, $h_t$, is near its quasi-fixed point may not be necessary. We discuss the colored-triplet threshold as a simple example of a source for the discrepancies, and comment on its possible implications. In addition, we point out that supersymmetric (as well as unification-scale) threshold corrections to $h_t$ shift the fixed-point curve in the $m_t - \tan\beta$ plane. The implications for the prediction of the Higgs boson mass are briefly discussed.

I. INTRODUCTION

Unification of the $b$ and $\tau$ Yukawa couplings is known to be consistent with the assumption of low-energy supersymmetry. However, the allowed parameter space depends sensitively on the exact value of the strong coupling $\alpha_s(M_Z) = 0.12 \pm 0.01$ used in the calculation. In particular, using the results from gauge coupling unification to calculate the $b$ and $\tau$ Yukawa couplings, $h_b$ and $h_\tau$, respectively, strongly constrains the allowed range of the Higgs sector parameter $\tan\beta \equiv \langle H_2 \rangle / \langle H_1 \rangle$ to $\tan\beta \sim 1$ or $\tan\beta \gg 1$.

Gauge coupling unification (including low-energy threshold corrections but neglecting corrections at the grand-unification scale) generically implies
\[ \alpha_s(M_Z) \gtrsim 0.13 \text{ and } \alpha_s(M_G) \sim 0.04 \] (where \( M_G \) denotes the unification point). The one-loop expression for the weak-scale \( b \) to \( \tau \) mass ratio is

\[ \frac{m_b(M_Z)}{m_{\tau}(M_Z)} \sim 0.9 \left[ \frac{\alpha_s(M_Z)}{\alpha_s(M_G)} \right]^{3/8} \times Y, \quad (1) \]

where the 0.9 factor is from hypercharge renormalization, \( Y < 1 \) is a complicated function of the Yukawa couplings, which is important for large couplings, and \( m_{\tau}(M_Z) = 1.75 \text{ GeV} \). Eq. (1) and gauge unification imply (when neglecting \( Y \)) the prediction \( m_b(M_Z) \sim 4.5 \text{ GeV} \). In comparison, the allowed (one standard deviation) range is \( m_b(M_Z) \lesssim 3.2 \text{ GeV} \) (but because of low-energy renormalization the upper bound is a function of \( \alpha_s \)). The QCD corrections are thus too large and need to be compensated by either large Yukawa coupling which diminish \( Y \) (and also the prediction for \( \alpha_s \)) or finite one-loop supersymmetric threshold corrections to \( m_b \) (that are proportional to \( \tan \beta \)). Both mechanisms can be realized in the large \( \tan \beta \) regime. On the other hand, in the small \( \tan \beta \) regime only the former is relevant, and the allowed region is strongly constrained in \( \tan \beta \) by requiring for the top Yukawa coupling \( h_t(m_t) \gtrsim 1.1 \), i.e., that \( h_t \) is near its quasi-fixed point. It is interesting to note that for \( \tan \beta \sim 1 \) the Higgs sector imitates that of the Standard Model (SM) and contains a light SM-like Higgs boson, \( m_{h^0}^{\text{one-loop}} \lesssim 100 \text{ GeV} \), which is within reach of LEPII. Hence, in this minimal framework, Higgs boson searches contain information about Yukawa unification.

However, the large predicted values of \( \alpha_s(M_Z) \) (note that the prediction increases quadratically with \( m_t \)) are somewhat uncomfortable phenomenologically. Particularly so, if the \( Z \to b\bar{b} \) width is significantly larger than what is predicted in the SM, as is currently implied by experiment. (In that case, the predicted \( \alpha_s \) is typically subject to large and positive low-energy threshold corrections, which further aggravate the potential problem.) Low-energy corrections could have a large and negative contribution to the \( \alpha_s \) prediction only if (a) the low-energy spectrum is extremely heavy and degenerate, i.e., the correction parameters \( M_1, M_2 \) and \( M_3 \) defined in Ref.

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1 In our numerical calculations of gauge and Yukawa couplings we will follow the procedure of Ref. using two-loop renormalization group equations [three-loop equations for \( \alpha_s(Q < M_Z) \)]. The procedure is extended in a straightforward manner to include low-energy corrections to \( m_b \) (see below).

2 This is when considering finite QCD corrections (but see a discussion below) to \( m_t \) and resummation of leading logarithms, which are the two most important higher-order corrections. The formal one-loop bound does not account for these effects by definition, and is higher by \( 10 - 15 \text{ GeV} \) (for example, see [14]). I thank Howard Haber for the discussion of this point. See also [15].

3 The leading logarithm correction to \( \alpha^{-1}_s(M_Z) \) is given by \( (-\delta b_i/2\pi) \ln(M_i/M_Z) \) where \( \delta b_i = \)
are large and equal, or (b) \( M_2 \gg M_1, M_3 \) (see Figure 5a of Ref. \[20\]). The former mechanism is not very likely, as it implies a degeneracy between colored \((M_3)\) and non-colored (e.g., \(M_2\)) particles, contradictory to the different nature of the radiative corrections in both sectors. It was suggested, however, that the latter mechanism could be realized if the QCD gauge fermions (the gluinos) are much lighter than the weak gauge fermions (the winos) \[22\]. While possible, this would imply that supersymmetry breaking is transmitted to the observable sector at a much lower scale than the breaking of the grand-unified group: If the supersymmetry breaking is transmitted to the visible sector gravitationally at Planckian scales, then the ratio of the different gaugino masses is dictated by the grand-unified symmetry to be approximately equal to that of the respective gauge couplings. Such models \[24\] must contain new exotic matter beyond the minimal supersymmetric extension (MSSM), and are not discussed in this work (but see Ref. \[25\]).

Thus, if indeed \( \alpha_s(M_Z) \lesssim 0.12 \), then one expects (aside from the above mentioned caveat) significant perturbations to the naive grand-unification relations at the unification scale. This is a crucial point when discussing Yukawa unification. It is straightforward to show that low-energy corrections to the \( \alpha_s \) prediction constitute only a second-order perturbation in the \( m_b(M_Z) \) prediction \[1\] (but they could affect the \( M_Z - m_b \) renormalization). However, corrections at the unification scale are multiplied by a large logarithm and can, depending on the way in which they propagate into the \( m_b/m_\tau \) relation, correct the \( m_b \) prediction significantly.

In this note we investigate the possible implications of such a scenario to Yukawa unification. Our purpose is not to define the allowed parameter space with any high precision, but rather examine whether such a precision is possible beyond the minimal framework (which is not favored by the data). In Section II, we discuss two examples of corrections: nonrenormalizable operators (NRO’s) and colored triplet thresholds. (We also include in our numerical

\[25/10,25/6,4 \] for \( i = 1, 2, 3 \), respectively. \( \alpha_{1,2,3} \) denotes the hypercharge (normalized by \( 5/3 \)), weak and strong couplings, respectively.

\[4\] When including the radiative corrections, the leading-logarithm correction to the prediction is typically proportional to the supersymmetric Higgs mass \( \mu \) \[\bar{5}\] and is more likely to be positive. It is negative if \( \mu \) is very large. On the other hand, a large \( \mu \) typically implies large mixing between left- and right-handed scalars and possibly a light scalar. The inclusion of finite corrections results now in a positive shift of the one-loop correction \[6,7\]. Because of this anti-correlation between the finite and logarithmic corrections, it is very difficult to obtain a negative one-loop correction \[21\]. The Roszkowski-Shifman proposal described below does not affect the proportionality to \( \mu \), but only its coefficient \[\bar{5}\].

\[5\] If the gauge kinetic function is grossly non-minimal, then this relation, and also gauge coupling unification, can be altered \[23\].
analysis low-energy corrections to \( m_b \).\) We examine the allowed parameter space as a function of \( \alpha_s \) and of \( h_t \). The latter is a useful measure of the parameter space which is independent of the size of the low-energy corrections to \( m_t \), discussed in Section III. We find that the gap between the allowed small and large \( \tan \beta \) regions is a sensitive function of \( \alpha_s \), the low-energy corrections to \( m_b \) (and thus, the soft parameters), \( m_t \), and of the unification-scale perturbation to \( h_b/h_\tau \). Outside the minimal framework (which constrains \( \alpha_s \) and the perturbations), none of these parameters is significantly constrained and the range of the allowed \( \tan \beta \gg 1 \) region is ambiguous. In particular, the gap nearly vanishes if \( \alpha_s(M_Z) \sim 0.11 \), or if the unification scale perturbation is \( O(20\%) \). Even though one can, in general, distinguish two different branches, the distinction is less significant as the gap diminishes, undermining the motivation to consider one branch rather than the other. Thus, the strong constraints on \( b - \tau \) unification are intimately linked to the large values of \( \alpha_s \) predicted in the minimal framework. In Section III we discuss the sensitivity of the \( h_t \) fixed-point curve to different threshold and other corrections, and stress that one-loop supersymmetric corrections to \( h_t \) are as important as the standard QCD correction. We conclude in Section IV, where we also point out the implications to the prediction of the Higgs boson mass in Yukawa unified models.

II. GAUGE vs. YUKAWA UNIFICATION

Before discussing examples of possible unification-scale corrections to the \( \alpha_s \) prediction, it is important to realize the smallness of typical couplings at that scale and its implications:

- \( \alpha_s(M_G) \sim 0.04 \). Because of the QCD enhancement\(^6\) of small unification scale perturbations in the value of \( \alpha_s(M_G) \), the allowed \( \sim \pm 8\% \) range of \( \alpha_s(M_Z) = 0.12 \pm 0.01 \) corresponds to only \( a \sim \pm 3\% \) (or \( \sim \pm 0.0015 \)) range at the unification scale.

- \( h_\tau(M_G) \sim 1/100 \cos \beta \left( y_\tau = h_\tau^2/4\pi \sim 10^{-5} \right) \), and similarly \( h_b(M_G) \sim 0.01 \) (for \( \tan \beta \sim 1 \)). In extrapolating \( h_\tau \) we used the near flatness of its renormalization curve (for not too large \( \tan \beta \) ). Note also that when using the data as boundary conditions, \( h_b(M_G) < h_\tau(M_G) \) by \( O(10^{-3}) \). In Fig. 1 it is shown that typically [for \( \alpha_s(M_Z) = 0.12 \)] \( (h_b - h_\tau)/h_\tau \sim -0.2 \) at \( M_G \). The ratio is \( \sim -0.3 \) for \( \alpha_s(M_Z) = 0.13 \) and \( \sim -0.1 \) for \( \alpha_s(M_Z) = 0.11 \).

Hence, a small numerical perturbation constitutes a large percentile perturbation.

The smallness and near flatness of \( h_\tau \) is of particular importance in our case\(^{22} \). It implies that small shifts in \( h_\tau(M_G) \) correspond to an apparent

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\(^6\) This is similar to the scaling between the QCD and weak scales that drastically reduces large uncertainties in the \( \alpha_s \) measurements at \( O(1 \text{ GeV}) \) when propagated to \( M_Z \). (The smaller coupling is compensated in our case by a larger logarithm.)
large violation of $h_b - h_{\tau}$ unification. One can visualize this as shifting the initial point of a nearly flat line (the $h_{\tau}$ renormalization curve). A small shift can drastically change its intersection with the moderately sloped $h_b$ renormalization curve (the slope of the QCD renormalized $h_b$ curve decreases at high energies where the couplings are small), leading to an apparent (or effective) unification point which could be many orders of magnitude below $M_G$. (Recall that the renormalization curve is a function of $\ln Q$ and not $Q$.) One can control such shifts by requiring that the apparent Yukawa-unification scale is not more than two or three orders of magnitude below $M_G$ [4]. Such a constraint, however, is not motivated if one allows large shifts elsewhere [e.g., in $\alpha_s(M_G)$]. If one eliminates such (“no-conspiracy”) constraints, then there could be corrections of $\mathcal{O}(100\%)$ in the case that $h_b$ and $h_{\tau}$ are still numerically small (i.e., for $\tan \beta \sim 1$). On the other hand, from Fig. 1, one observes that already $\mathcal{O}(20\%)$ corrections remove many of the constraints. We return to this point below.

Next, we elaborate on possible corrections to $\alpha_s$. One mechanism that could possibly shift $\alpha_s(M_G)$ is gravitational smearing (i.e., gravitationally induced NRO’s), originally proposed as a non-perturbative mechanism [27,23] and later realized as an efficient perturbation (or smearing) to unification relations [28–30]. Requiring that the effect is perturbative typically constrains the coefficient of the (leading) operator such that the absolute value of the correction to the $\alpha_s(M_Z)$ prediction (which depends on the correlated shifts of all three gauge couplings) is $\lesssim 0.010 - 0.015$. (The exact number depends on the group theory structure.) One could argue for a larger correction, depending on the perturbativity criterion imposed. On the other hand, one typically expects a smaller correction, e.g., in Ref. [20] it was estimated that the absolute value of the correction is $\lesssim 0.006$. The correction can be propagated to the $m_b/m_{\tau}$ ratio as a constant shift in $\alpha_s(M_Z)$ [4] (see also Ref. [29]). In addition, other operators could now shift the boundary conditions of other couplings, e.g., $h_{\tau}(M_G)$, generating the perturbations discussed above.

A different mechanism for lowering the $\alpha_s$ prediction is by introducing an SU(5) breaking between (colored and non-colored) heavy chiral supermultiplet thresholds. In extended models many candidates could exist (for examples, see Refs. [20,31–35]). However, the most obvious candidate is the colored triplet Higgs, $T$, that has to be split from the light Higgs doublets (see also Ref. [36]). Indeed, the doublet-triplet splitting problem, even though solvable by fine-tuning of the superpotential and of the scalar potential, calls for non-generic solutions that may affect the properties of the triplet threshold [37]. We consider this generic threshold as an example only.

Typically, one assumes $M_T \gtrsim M_G$ so that the loop-level (dimension-five) colored-Higgsino mediated proton decay [38] is sufficiently suppressed [39]. Nevertheless, the effectiveness of $M_T \sim 10^{-2} M_G$ in lowering the prediction for $\alpha_s$ may suggest a different mechanism for suppression of the dimension-
five proton decay operator. One possibility is that all Yukawa couplings of $T$ are suppressed, in which case the only correction to Yukawa unification is via the modification of $\alpha_s$. 

$$\Delta \alpha_s \sim \frac{9\alpha_s^2(M_Z)}{14\pi} \ln \frac{M_T}{M_G}$$

(2)

A different possibility is that some of the Yukawa couplings of $T$ to the third generation are not suppressed. This assumption is particularly motivated here, since naive Yukawa unification is successful only in the case of the third family, and thus, provides no information on the Yukawa couplings (and mixing angles) of the two light families. If these are the only couplings which are not suppressed, then proton decay constraints on $M_T$ are diminished. In addition to diminishing $\alpha_s(M_T)$, the triplet threshold in this case $(i)$ introduces a Yukawa coupling correction to $h_b/h_\tau$, $(ii)$ shifts the $h_t$ fixed point (see Section III), and $(iii)$ renormalizes the soft parameters (i.e., the scalar potential) corresponding to the third family, an effect which is particularly important for the mass of the scalar $\tau$, which could become too light or tachionic. From $(i)$ one has a correction factor to $Y$, which can be absorbed as a shift in the boundary conditions. (We will include it explicitly in the numerical integration, i.e., in the numerical calculation of $Y$.) From $(iii)$, there could be an enhancement of low-energy lepton-number violation processes, e.g., $\mu \to e\gamma$. In fact, both mechanisms, the operators and the triplet threshold, may be linked. Perturbations of some form or another are required in order to explain the failure of Yukawa unification for the two light families. One common mechanism to generate these perturbations is NRO’s which are either gravitational or higher-symmetry remnants. Such operators most probably shift also the third family Yukawa couplings, and could allow only extra suppressed couplings for the colored triplet.

7 Other possibilities involve suppression due to symmetries, group-theory, and the structure of the soft terms.

8 Ignoring proton decay constraints, one could entertain the idea that an intermediate scale triplet drives $\alpha_s(M_Z) < 0.11$, which is then corrected to $\alpha_s(M_Z) > 0.11$ by low-energy thresholds.

9 In the light triplet models of Ref. the correction is proportional to the logarithm of the triplet to (new) doublet mass ratio.

10 In principle, one could obtain a (model-dependent) lower bound on $M_T$, independent of proton decay and of the $\alpha_s$ prediction.

11 We find, for example, an enhancement as large as two orders of magnitude (for $M_T/M_G \gtrsim 10^{-3}$) to the $\mu \to e\gamma$ branching ratios of the models considered in Ref.
We examine the parameter space in Figs. 2–3, where we fixed $m_t^{pole} = 170$ GeV (consistent with direct [19] and indirect [19] determinations). In order to examine the smearing of the allowed $\tan \beta$ range for $\alpha_s(M_Z) = 0.12$, we require in Fig. 2 that $b - \tau$ unification at the $\alpha_1 - \alpha_2$ unification point ($M_G \approx 3 \times 10^{16}$ GeV) holds to a precision of either 5%, 15% or 25%. In practice, this would typically mean $h_b(M_G) \to 0.8h_\tau(M_G)$, leading to a better agreement with the data. For example, a perturbation of 15% [or $h_b(M_G) \sim 0.85h_\tau(M_G)$] corresponds in some cases to an apparent Yukawa-unification point as low as $10^{10}$ GeV. Low-energy corrections to $m_b$ [10] are also included and calculated explicitly assuming, for simplicity, “universal” boundary conditions to the soft parameters at the grand scale [14], and radiative symmetry breaking, agreement with experimental lower bounds on the masses (and an imposed upper bound of $\sim 2$ TeV), and using a monte-carlo scan of the parameter space (for further details, see [14]). We account for NRO’s (or other corrections whose main effect is to shift $\alpha_s$ at high energies) by fixing $\alpha_s(M_Z) = 0.120$ [and $\alpha_s(M_Z) = 0.110, 0.130$ in Fig. 3]. For comparison, we also show the respective allowed points when not including the low-energy corrections (diamonds).

As implied by Fig. 1, for a 25% perturbation, no constraints exist on small $\tan \beta$. It is interesting to note that it is extremely difficult to find very large $\tan \beta$ solutions. The exclusion of $\tan \beta \gtrsim 45 \sim m_t/m_b$ results from the simultaneous requirement of radiative symmetry breaking and acceptable threshold corrections to $m_b$ (and may be overcome by excessive tuning of parameters [51,52], in particular, in non-universal schemes [53,52]). When not including the low-energy corrections (diamonds), these points are again allowed, but the intermediate $\tan \beta$ range is excluded [unless there is a $\gtrsim \mathcal{O}(20\%)$ perturbation]. The extreme tuning (for small perturbations) of very small and very large $\tan \beta$ solutions (e.g., see diamonds in Fig. 2) may suggest that the allowed region of intermediate $\tan \beta$ solutions is preferred. However, one has to be cautious, as such solutions depend sensitively on the soft parameters [14]. In Fig. 4 we show the possible low-energy corrections to $m_b$, where points which constitute the 5% perturbation curve in Fig. 2 are indicated by bullets. Only a small fraction of points has the required $\sim -20\%$ correction. Therefore, for

\begin{enumerate}
\item We require $4.00 \leq m_b(m_b) \leq 4.45$ GeV (e.g., see Ref. [8]). More points would be allowed had we imposed this constraint, but for $m_b(m_b^{pole} \sim 5$ GeV) rather than for $m_b(m_b)$. For a discussion, see also Refs. [8,10].
\item For simplicity, we do not include renormalization effects above $M_G$ [14].
\item There is also a correlation (which we do not treat in this work) between the $m_b$ correction and the size of the chargino loop contribution to $b \to s\gamma$, and a negative correction typically implies an enhancement of the $b \to s\gamma$ rate [51]. This effect is generally important for $\tan \beta \gtrsim 25 - 30$ and a too high $b \to s\gamma$ rate may exclude some of the allowed points in that region, depending on the charged Higgs mass.
\end{enumerate}
small perturbations, all solutions for Yukawa unification require some tuning.
(In principle, one could distinguish three allowed regions, but because of their
complimentary nature, we will keep identifying both the intermediate and the
very large tan \( \beta \) branches as the large tan \( \beta \) solution.)

We further examine solutions with small (5\%) perturbations in Fig. 3,
where we fix \( \alpha_s(M_Z) = 0.110, 0.130 \). The latter is roughly the value one would
get when requiring gauge coupling unification and \( M_T > M_G \), i.e., the minimal
framework. We also present curves requiring gauge coupling unification but
fixing \( M_T = 10^{15}, 10^{14} \) GeV \( \{\alpha_s(M_Z) \approx 0.118, 0.112, \text{ respectively}\} \). (The
triplet threshold is included numerically and the correlation between the shifts
in \( \alpha_s(M_Z), \alpha_s(M_G) \) and \( M_G \) is automatically accounted for.)

In the minimal framework, even when including the low-energy correc-
tions, the two branches, tan \( \beta \sim 1.3 \) and tan \( \beta \gtrsim 15 \) are clearly distinguished.
However, the small tan \( \beta \) solution is extremely tuned in this case because of
the large QCD correction (see Section III) and because of the
\( M_Z - m_b \) QCD renormalization. \([A \mathcal{O}(1 - 2\%) \text{ low-energy corrections can now exclude an otherwise consistent solution.} \]
While a significant gap remains in this case, it is smeared almost completely for \( \alpha_s \sim 0.110 \). It is worth stressing, however,
that some gap remains (for small perturbations) in all cases. Thus, one can
still distinguish two allowed branches, as in the minimal framework. This
is because of the fixed point relative insensitivity for corrections to \( \alpha_s \) and
the proportionality of the \( m_b \) corrections to tan \( \beta \), which lead to only negli-
gible smearing of the tan \( \beta \sim 1 \) branch. Nevertheless, smearing of the large
tan \( \beta \) branch down to tan \( \beta \sim 8(4) \) for \( \alpha_s(M_Z) \sim 0.120(0.110) \) significantly
diminishes the excluded region, as well as undermines arguments (based on
Yukawa unification) in favor of the tan \( \beta \sim 1 \) branch. Furthermore, as \( m_t^{\text{pole}} \)
increases, the \( h_t \) fixed-point curve is flatter in tan \( \beta \), further diminishing the
gap (see Fig. 5). Also, given the smallness of \( h_b \) and \( h_\tau \) for tan \( \beta \sim 1, \mathcal{O}(20\%) \)
perturbations are reasonable, as discussed above, and the tan \( \beta \sim 1 \) branch
could also be smeared (see Fig. 2).

In Fig. 5 we allow \( m_t^{\text{pole}} = 180\pm 12 \) GeV \([49]\) (with a Gaussian distribution)
and show the allowed values of the top Yukawa coupling \( h_t \) as a function of
tan \( \beta \) for \( \alpha_s(M_Z) = 0.120 \) and a 5\% perturbation. (Note that for large values
of \( m_t^{\text{pole}} \gtrsim 190 - 200 \) GeV, \( h_t \) could be near its fixed point for intermediate
values of tan \( \beta \).) The requirement \( h_t \gtrsim 1.1 \) holds for tan \( \beta \lesssim 8 \). This is a
reflection of the respective excluded region (the gap) in Fig. 2 where \( m_t^{\text{pole}} = 170 \) GeV \( \text{and } h_t \lesssim 1.1 \) for tan \( \beta \gtrsim 1.4 \). The fact that now there is no gap is
due to the higher values of \( m_t^{\text{pole}} \).

III. THE FIXED-POINT CURVE

Points near the \( h_t \) fixed-point were shown above to provide a solution to
\( b-\tau \) Yukawa unification. That solution is the least sensitive to either enhance-
ment or suppression of the low-energy corrections to \( m_b \) (the sensitivity grows
with \( \alpha_s \), as discussed above). However, the solution is a result of the large
numerical value of $h_t$ only, and because of the $h_t$ convergence to its fixed-point value this result is relatively insensitive to $\alpha_s(M_Z) = 0.12 \pm 0.01$. The translation of this value to a curve in the $m_t - \tan \beta$ plane contains a few ambiguities, which are worth recalling.

In fact, this is only a quasi-fixed point (convergence from above). If the low-energy $h_t$ exceeds its fixed point value, then it becomes non-perturbative at some higher scale. In a consistent calculation the quasi fixed-point has to be defined numerically, e.g., that renormalization from two-loops is smaller than a certain fraction of that from one-loop. This leads, e.g., to the condition $h_t \lesssim 3$ at all scales below the cutoff scale $\Lambda$. Therefore, the cutoff scale for the calculation enters the definition. For example, using $10^{18}$ GeV rather than $M_G$ as a cutoff, leads [in SU(5)] to the requirement $h_t(M_G) \lesssim 2$, shifting the fixed point curve to slightly higher values of $\tan \beta$. (In fact, there may be another quasi-fixed point $h_t \sim 2$ at $M_G$.) In addition, the fixed-point value of $h_t$ depends on the other large couplings in the renormalization group equations, i.e., $\alpha_s$. The lower $\alpha_s$ is, the lower is that value, and again, the curve slides to slightly larger values of $\tan \beta$ (e.g., this can be seen in Fig. 3).

If there are other large couplings, i.e., new large Yukawa couplings (or a large number of new couplings), then the fixed-point value of $h_t$ also changes. The quasi-fixed point is reached by a cancellation of gauge and Yukawa terms. Since the size of the former is roughly fixed, any new Yukawa coupling modifies the upper bound on all other Yukawa couplings (that enter the same set of renormalization group equations). New Yukawa couplings could renormalize (i) $h_\tau$, (ii) $h_b$, and (iii) $h_t$. In most examples all three are relevant and a fixed-point value of $h_t < 1$ is possible (i.e., $\tan \beta$ slides to larger values) while still maintaining Yukawa unification. Some examples include (a) low-energy singlets, (b) fourth family, and (c) baryon and lepton number violating couplings.

A most interesting case is that of (d) an intermediate-scale right-handed neutrino where only (i) and (iii) occur. Before its decoupling at the intermediate-scale, the new Yukawa coupling, $h_\nu$, renormalizes $h_\tau$ in the same way that $h_t$ renormalizes $h_b$. The two Yukawa corrections roughly cancel in the ratio [assuming $h_\nu(M_G) \approx h_t(M_G)$], and the Yukawa correction function $Y$ in $\Gamma$ is closer to unity (depending on the right-handed neutrino scale), unless $h_b$, itself, is significantly large. The small $\tan \beta$ solution is excluded in this case, regardless of the exact location of the $h_t$ fixed point.

The generic heavy threshold corrections follow similar patterns. The adjoint field, like the singlet [case (a)], is coupled to the “Higgs-leg” of the Yukawa operators, and the effect cancels in the $h_b/h_\tau$ ratio. However, it also affects $h_t$, and hence, affects $h_b/h_\tau$ indirectly. However, unlike the low-energy singlet case, the indirect correction here is suppressed by a small logarithm. It could shift the fixed-point if its coupling to the Higgs doublets, which renormalizes $h_t$, is large [i.e., in SU(5) it is the case that the color triplet is heavy], and its self coupling (that determines its own mass) is small. The color triplet has lepto-quark couplings that unify with $h_t$, and is a special
example of (c). Because of its large mass (i.e., the small logarithm) the effect is again moderate. We find that for $M_T \gtrsim 10^{14}$ GeV the fixed-point value of $\tan \beta$ increases (including the modification of the $\alpha_s$ prediction) by less than 0.18 (and less than 0.06 for a fixed value of $\alpha_s$).

Lastly, supersymmetric threshold corrections to $m_b$ play a crucial rule in expanding the allowed parameter space: They generate the allowed intermediate $\tan \beta$ region in the case of small perturbations. Similar corrections have been shown to affect other parameters [59], an observation which is related to renewed interest [60,13] in (weak-scale) radiative fermion masses [11]. In fact, it is doubtful that one can consider predictions for the SM fermionic sector parameters independently from the supersymmetric spectrum parameters. The corrections that are relevant for our discussion are those for the $m_t^{pole}/m_t^{DR}$ ratio ($DR$ stands for the dimensional-reduction scheme),

$$h_t = \frac{m_t^{DR}}{174 \text{ GeV}} \frac{\sqrt{1 + \tan^2 \beta}}{\tan \beta}.$$  (3)

We defined the parameter $m_t^{DR}$ to absorb all threshold corrections, i.e., at one-loop

$$m_t^{DR} = m_t^{pole} \left[ 1 - \Delta_{QCD}^t - \Delta_{SUSY-QCD}^t - \Delta_{EW}^t \right],$$  (4)

where $\Delta_{QCD}^t = \frac{5 \alpha_s(m_t)}{3 \pi}$,  (5)

and $\Delta_{EW}^t$ includes electroweak and Yukawa contributions [11,63] that we neglect hereafter. $\Delta_{SUSY-QCD}^t$ includes new QCD contributions in the MSSM (which are only implicitly dependent on $\tan \beta$), that have been calculated using three- [11] and two-point [63] functions and shown to be potentially of the order of magnitude of $\Delta_{QCD}^t$. Recently, it has been further shown [44] that $\Delta_{SUSY-QCD}^t$ does not have a fixed sign [14] and introduces a significant ambiguity in the fixed-point curve. In particular, this correction can be more important than the $\sim 2\%$ two-loop QCD contribution to (5) that many authors include while neglecting supersymmetric loops.

In Fig. 6 we examine the corrections for the point $(m_t^{pole}, \tan \beta) = (170 \text{ GeV}, 1.4)$, i.e., in the vicinity of the “naive” fixed-point curve, and for $\alpha_s(M_Z) = 0.12$ (using the vertex formalism of Ref. [11] and imposing the same assumptions on the parameter space as above). By fixing $h_t$ to its fixed-point

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15 One also needs to include a $\Delta_{QCD}^b = [1/3][\alpha_s(M_Z)/\pi]$ when converting $m_b(M_Z)$ from its DR definition to its modified minimal-subtraction definition, which is the relevant one for $m_b(m_b)$. This correction is important, e.g., for $\alpha_s(M_Z) = 0.13$.

16 The leading logarithm terms agree in sign with $\Delta_{QCD}^t$, but the overall sign is model dependent.
value, the corrections are absorbed in the invariant combination $m_t^{pole}/\sin \beta$. (Note that the corrections, though represented by a mass parameter, are in fact corrections to the Yukawa coupling.) It is straightforward to absorb the corrections in $m_t^{pole}$ (vertical line), in which case the correction in our example is $-2% \lesssim \Delta_{\text{susy-QCD}}^{t} \lesssim 5\%$ or between $-3$ to $8$ GeV. (The asymmetry is due to the fixed sign of the leading logarithms). However, if $m_t^{pole}$ is known with high-precision, than the corrections are to be absorbed in $\sin \beta$ (horizontal line). [A similar procedure could be used to treat the uncertainty in $\alpha_s$ in (5).] The two-lines define a region in the parameter space that corresponds to one point on the “naive” fixed-point curve. Fig. 5 is insensitive to this ambiguity, but the interpretation of Figs. 2–3 is sensitive. The ambiguity in $m_t^{pole}/\sin \beta$ diminishes the required tuning of the $\tan \beta \sim 1$ solutions (at the price of dependence on the the soft term) in a similar way to the smearing of the large $\tan \beta$ solutions due to the corrections to $m_b$. The correction (absorbed in $m_t^{pole}$) is shown in Fig. 7 for any $\tan \beta$ for $\alpha_s(M_Z) = 0.12$ (and requiring $b-\tau$ unification with a 5% perturbation). The dependence on $\tan \beta$ is from the supersymmetric Higgs mass $\mu = \mu(\tan \beta, ...)$, the left-right $t$-scalar mixing, and a correlation between the $m_t$ and $m_b$ corrections (which we do not explore in detail in this work).

IV. CONCLUSIONS

To conclude, the increasing value of the $\alpha_s$ prediction significantly constrains the allowed parameter space for Yukawa unification. Yet, if $\alpha_s$ is significantly lower than predicted, there exists a significant perturbation at the grand scale, examples of which we discussed in Section II. Such a perturbation creates an ambiguity which removes many of the constraints on Yukawa unification. The constraints were shown to be a sensitive function of $\alpha_s$, unification-scale perturbations, and low-energy corrections to $m_b$ (and of $m_t$), and nearly vanish for $\alpha_s(M_Z) = 0.11$ or a $\mathcal{O}(20\%)$ low or high-scale correction. From our figures one can obtain a qualitative description of the excluded region (the gap) in terms of the lower bound on the large $\tan \beta$ branch (for $m_t^{pole} = 170$ GeV),

$$\tan \beta \gtrsim \frac{1}{2} \left\{ \frac{\alpha_s(M_Z) - 0.100}{0.001} + \frac{h_b(M_G) - h_\tau(M_G)}{0.010 \times h_\tau(M_G)} \right\}. \quad (6)$$

(We assume that the left-hand side of (6) is $\geq 1$, otherwise $\tan \beta \geq 1$.) Thus, the success of “simple” gauge unification [$\alpha_s(M_Z) > 0.12$] and the constraints on Yukawa unification are intimately linked, and the difference between the predicted and measured $\alpha_s$ values can be viewed as a sensitive measure of

\[17\] This is a similar procedure to absorbing radiative corrections in the weak angle rather than in $M_Z$. 

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the typical size of perturbation at the unification scale. We also pointed out the ambiguity in the location of the fixed point and demonstrated the need to consider threshold corrections to $m_t$ when discussing the fixed-point curve. This also affects the $h_t$-perturbativity lower bound on $\tan \beta$.

The required properties of the unification-scale perturbations, which we simply assumed when discussing examples, can, on the one hand, put severe constraints on model building and enhance the predictive power in the high-scale theory (see, for example, Ref. [35]). On the other hand, it implies loss of some predictive power in the low-energy theory, i.e., unlike the minimal framework, now there are no generic predictions but only model-dependent ones (which depend on additional parameters). The loss of low-energy predictive power may be compensated in some cases by the effects of threshold corrections (due to these perturbations) in the soft parameters on flavor changing neutral current processes, but these are again strongly model dependent.

Regarding the light Higgs boson mass $m_{h^0}$, its lightness is due to the accidental proximity of the $h_t$ fixed-point curve to the flat direction in the Higgs scalar potential for $\tan \beta = 1$. The latter implies $m_{h^0}^{\text{Tree}} < M_Z |\cos 2\beta| \to 0$ near the fixed point curve. If the curve slides to larger values of $\tan \beta$, $m_{h^0}^{\text{Tree}}$ increases. However, unless new Yukawa couplings are introduced [e.g., examples (a)-(c) above], the increase is $\lesssim 10$ GeV, and since the mass $m_{h^0}^{\text{one-loop}}$ is a sum in quadrature of tree and loop terms, it has no significant ambiguity. The ambiguity due to $\Delta_{\text{SUSY-QCD}}^t$ is more of an interpretational ambiguity, since $h_t$ (or $m_t^{\text{DR}}$) is the relevant parameter for the calculation of the loop correction in $m_{h^0}^{\text{one-loop}}$. (As commented above, this is actually one of the more important higher-order refinements of the calculation.) The prediction of $m_{h^0}^{\text{one-loop}}$ is thus insensitive to the corrections (if absorbed in $m_t^{\text{pole}}$). However, the correspondence between $m_t^{\text{pole}}$ and $m_{h^0}^{\text{one-loop}}$ is now ambiguous. We thus conclude that, indeed, Higgs searches can probe the MSSM fixed-point region. However, while this region may be motivated by various reasons (not the least, the existence of a fixed-point structure)\footnote{For example, it was recently suggested that the only possibility to reconcile the $Z \to b\bar{b}$ discrepancy, mentioned above, with supersymmetric extensions is if $\tan \beta \sim 1$ \cite{64}. See also \cite{44}.}, the diminished gap between the two allowed branches for Yukawa unification undermines the uniqueness of the $\tan \beta \sim 1$ branch and the motivation to consider this region based on $b - \tau$ unification, unless $\alpha_s$ is large and unification-scale perturbations are small.

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FIG. 1. The unification-scale difference $h_b - h_\tau$ is shown in $h_\tau$ units for $m_b(M_Z) = 3 \text{ GeV}$, $\alpha_s(M_Z) = 0.12$, $m_t^{\text{pole}} = 170 \text{ GeV}$ and as a function of $\tan \beta$. For comparison, we also show the difference for $m_b(M_Z) = 3.1 \text{ GeV}$ (which for $\alpha_s(M_Z) = 0.12$ is inconsistent with $m_b(m_b) < 4.45 \text{ GeV}$). Note the rapid change near the (naive) small and large $\tan \beta$ solutions, which is a measure of the required tuning.
FIG. 2. The MSSM points which are consistent with \( b - \tau \) unification for \( \alpha_s(M_Z) = 0.12 \) and \( m_t^{\text{pole}} = 170 \text{ GeV} \) are shown as a function of \( \tan \beta \) when including (bullets) and when omitting (diamonds) low-energy corrections to \( m_b \). The different curves correspond to \( h_b/h_\tau = 1 \pm 0.05, 1 \pm 0.15, 1 \pm 0.25, \) at the unification scale, respectively. (The two upper curves correspond to \( 1 \pm 0.25 \).)
FIG. 3. The MSSM points which are consistent with $b - \tau$ unification for $m_T^\text{pole} = 170$ GeV and when requiring $h_b/h_\tau = 1 \pm 0.05$ are shown as a function of $\tan \beta$ (including low-energy corrections to $m_b$). The upper and lower curves correspond to $\alpha_s(M_Z) = 0.13, 0.11$, respectively. In the two middle curves $\alpha_s(M_Z)$ is predicted when a colored triplet threshold at $M_T = 10^{15}, 10^{14}$ GeV (with Yukawa couplings to the third family) is assumed (and accounted for in the numerical integration).
FIG. 4. The low-energy threshold corrections to $m_b$ for the MSSM points considered in Fig. 2. Only the points indicated by bullets correspond to the $1 \pm 0.05$ curve in Fig. 2.
FIG. 5. The MSSM points which are consistent with $b - \tau$ unification for $\alpha_s(M_Z) = 0.12$, $m_t^{pole} = 180 \pm 12$ GeV and when requiring $h_b/h_\tau = 1 \pm 0.05$, are shown as a function of $\tan \beta$ and of the Yukawa coupling $h_t$ (which is calculated including only its QCD correction). Because of the larger values of $m_t^{pole}$, larger values of $h_t$ (and thus, solutions for $b - \tau$ unification) are obtained for $\tan \beta > 2$. The correspondence between $h_t$ and $m_t^{pole}$ could change when including the SUSY-QCD corrections of section III.
FIG. 6. The SUSY-QCD corrections to $h_t$ are absorbed in $m_{t}^{pole}$ (vertical line) and in $\tan \beta$ (horizontal line), smearing the naive point $m_{t}^{pole} = 170$ GeV and $\tan \beta = 1.4$ (assuming a fixed $h_t$ value and $\alpha_s(M_Z) = 0.12$). $b - \tau$ unification is not required.
FIG. 7. The SUSY-QCD corrections to $h_t$ are absorbed in $m_t^{\text{pole}}$ for the points of the $1 \pm 0.05$ curve in Fig. 2.