Joint Tomography with Seismic First-break And Reflection under Rugged Topographical Conditions

Tianzhi Guo*, Chun Xu, Liang Song
Chang’an University, Xi’an 710054, China
*Corresponding author e-mail: 466287221@qq.com

Abstract. Seismic travel-time tomography is one of method which can inverse some information of underground medium such as structure, velocity distribution and so on. In order to solve the problem about seismic tomography under rugged topographical conditions. The author develops a new way for seismic tomography by combine LTI method based on hybrid meshes which has high fitting degree for the variable structure underground with LSQR algorithm. Thus, the joint tomography with seismic first-break and reflection under rugged topographical conditions has been achieved. The stability and effectiveness is verified by theoretical model test.

1. Introduction
Travel-time tomography of seismic waves is a technique that uses travel-time of seismic waves to retrieve the relevant parameters of the model. Although tomography technology is becoming more and more mature nowadays, many methods such as body wave imaging, full waveform inversion, surface wave imaging and so on are developing continuously, but there are still some technical difficulties. How to reconstruct the velocity and interface morphology of underground media using tomography technology is also a problem worth studying when seismic exploration is carried out in areas such as mountains and sand dunes which are near the surface of the earth and have complex underground structures.

2. Principles
2.1. Hybrid grid for rugged interface
The joint tomography of first break wave and reflection wave on undulating surface discussed in this paper includes two aspects: forward calculation of seismic travel time and inverse calculation of corresponding velocity. In forward modeling, irregular quadrilateral mesh and rectangular mesh are combined to carry out model generation, which can not only be closer to the actual surface, but also have good computational efficiency [1]. Hybrid meshing has three cases as shown in Figure 1.
2.2. Fundamental Principles of Inversion

After partitioning the whole model area with mixed grids, N grids are formed according to the order of ray passing. The slowness of each grid is constant, which is expressed as a constant sequence \( \varepsilon_j (j \in [1, N]) \). For each \( j (j \in [1, N]) \) basic function is defined, which is expressed as follows:

\[
g_j(x, z) = \begin{cases} 
1, & (x, z) \in j \text{th grid} \\
0, & (x, z) \notin j \text{th grid}
\end{cases}
\]  

Thus, a linear combination of \( s_j \) and \( g_j(x, z) \) \( j \in (1, N) \) can be used to represent the seismic wave slowness function \( s(x, z) \) in medium.

\[
s(x, z) = \sum_{j=1}^{N} s_j g_j(x, z) \]  

It is assumed that in a seismic exploration, a total of I-channel records are actually recorded. Each record corresponds to a ray path of seismic wave from the point of shooting to the point of receiving, which is expressed by \( L_i (i \in [1, I]) \) and at the same time, each first arrival record corresponds to a travel time, which is expressed by \( t_i (i \in [1, I]) \). \( t_i \) can be considered as the result of curve integral of the slowness function \( s(x, z) \) of underground medium along the ray path. That is to say, \( t_i \) is a generalized Radon positive transformation of \( s(x, z) \), and its process can be characterized by the following formula.

\[
t_i = \int_{L_i} s(x, z) ds = \int_{L_i} \sum_{j=1}^{N} s_j g_j(x, z) ds = \sum_{j=1}^{N} s_j \int_{L_i} g_j(x, z) ds \quad i \in [1, I]
\]  

The expression \( \int_{L_i} g_j(x, z) ds \) can be expressed by \( a_{ij} (i \in [1, I], j \in [1, N]) \), then the formula (3) can be expressed by a matrix as follows:

\[
\begin{bmatrix}
a_{11} & a_{12} & \ldots & a_{1j} & \ldots & a_{1N} \\
a_{21} & a_{22} & \ldots & a_{2j} & \ldots & a_{2N} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
a_{i1} & a_{i2} & \ldots & a_{ij} & \ldots & a_{iN} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
a_{I1} & a_{I2} & \ldots & a_{Ij} & \ldots & a_{IN}
\end{bmatrix}
\begin{bmatrix}
s_1 \\
s_2 \\
\vdots \\
s_j \\
\vdots \\
s_N
\end{bmatrix}
= 
\begin{bmatrix}
t_1 \\
t_2 \\
\vdots \\
t_j \\
\vdots \\
t_I
\end{bmatrix}
\]  

It can be simplified as:
In formula $A = \left( a_{ij} \right)$ is a matrix whose elements are the length of each ray path in each grid element in the whole forward process; $S = \left( s_1, s_2, \cdots, s_N \right)^T$ represents a vector consisting of the slowness of seismic wave of medium in each grid element; $T = \left( t_1, t_2, \cdots, t_N \right)^T$ represents the vector formed by each seismic wave when it travels. Travel time tomography of seismic waves is the process of solving the equation (5). Here we use LSQR algorithm to solve the tomographic equations and obtain the medium slowness vector [2-4].

2.3. Problem and Processing Technology

Seismic travel time tomography inversion is a mixed problem, which is mainly caused by the uneven coverage of ray and data. According to the number and angle of ray coverage in the underground grid, the grid can be divided into under-determined space, over-determined space and zero space. There are two reasons for this confusion: one is external factors, such as observation mode, velocity structure, shot detection point interval and so on. Secondly, the internal reason is that the ray at the shot point is relatively dense, while the distance is relatively sparse, and there is no ray passing through some boundary locations. Figure 2 reflects this phenomenon perfectly.

Wang and Braile [5] have proposed that if the problem of non-uniform coverage is neglected, the calibration amount of the inversion model will be proportional to the degree of ray coverage, that is to say, if the degree of ray coverage is high in the grid, it will be over-calibrated, if the degree of ray coverage is low, it will be inadequate or even not corrected. Based on the above principles, the tomographic inversion effect will be improved if the correlation between the calibration amount of the model and the degree of ray coverage is reduced. To solve this problem, this paper adopts the multi-scale progressive inversion strategy [6-8]. At the same time, the model parameters are weighted and regularized in inversion, and then LSQR iteration is used to solve the tomographic equations.

3. Model test

Fig. 3 is a layered model of the combination of undulating surface, undulating interface and fault interface. The size of the model is 400 x 200. The surface is undulating. The maximum elevation drop is 50 m in the range of 400 m. There is an undulating interface in the depth of 20-30 m below the surface. There is a fault velocity interface at 120 m below the ground. The upper, middle and lower layers of the model are set to 800 m/s, 1500 m/s and 2000 m/s respectively. The observation system is set up as unilateral firing. The horizontal component of the minimum offset is 10m and the total number of artillery is 124. Each shot has 72 channels. The distance between shot and detector is 2m, and the coordinate of the first shot is (0, 0).
Figure 3. Theoretical model in Rugged Topographical Conditions

Figure 4. Initial model of tomographic inversion

Fig. 5 ~ Fig. 7 is the result of multi-scale progressive inversion using joint tomography of first break and reflection waves. The forward mesh size is 1 m x 1m. Fig. 5 is based on Fig. 4 as the initial model, and the inversion mesh size is 20m x 20m. The results are obtained by two iterations. The initial velocity is smoothed to facilitate the subsequent inversion. Fig. 6 is based on Fig. 5 as the initial model, and the results of two iterations when the inversion mesh size is 10m x 10m. From the graph, we can see that the first undulating velocity interface and the second fault interface have already appeared, and the velocity has begun to converge, but the interface shape is blurred. Fig. 7 is based on Fig. 6 as the initial model, and the inversion mesh size is 2m x 2m. Compared with Fig. 6, it can be seen that the first undulating velocity interface and the second fault interface are better displayed. The inversion effect is slightly affected by the less or no ray coverage in the left part of the fault and the boundary of the interface, but on the whole, both the shape of the underground interface and the velocity of each layer are in good agreement with the velocity model shown in Fig. 3.
Figure 5. The result of tomographic inversion in grid 20m x 20m

Figure 6. The result of tomographic inversion in grid 10m x 10m
Fig. 7. The result of tomographic inversion in grid 2m x 2m

Fig. 8 is the travel time residuals corresponding to the final inversion result Fig. 7 at different offsets. The total travel time residuals of 23263 rays are shown in the figure. From the figure, it can be seen that more than 99% of the travel time residuals of rays are within ±5ms. From this, it can be seen that the convergence effect of the travel time errors of the final inversion result of the model trial calculation is better.

Figure 8. Tomographic inversion results travel residual

4. Conclusion
In seismic travel time tomography inversion, it is often better to use multiple seismic wave information to invert at the same time. In this paper, the principle of seismic travel time tomography under undulating surface is briefly described in combination with the characteristics of undulating surface. LSQR algorithm, which is widely used at present, is used to solve the tomographic equation group, and the joint tomography of first arrival wave and reflection wave on undulating surface is realized. The inversion results of model trial calculation show that not only the travel time residuals converge well,
but also the resolution of tomographic inversion is improved by using multi-scale progressive inversion strategy reasonably, which provides a new idea for inversion of medium under undulating surface.

References

[1] Wang Qi, Zhu Pan, Ye Pei, Li Qin, Li Qing-chun. Traveltime calculation based on linear interpolation in hybrid meshes for rugged topographical conditions [J]. Oil Geophysical Prospecting, 2018, 53 (01): 35 - 46.

[2] Christopher C. Paige and Michael A. Saunders. LSQR: Sparse Linear Equations and Least Squares Problems [J]. ACM Transactions on Mathematical Software, 1982, 8 (2): 195 - 209.

[3] Menke W. Geophysical Data Analysis: Discrete Inverse Theory. San Diego, California: Academic Press Inc, 1984.

[4] Paige CC, Saunders M A. LSQR: Sparse linear equations and least squares problems. ACM Transactions on Mathematical Software, 1982, 8 (2): 195 – 209.

[5] Wang B., Braile L. W. Effective approaches to handling non-uniform data coverage problem for wide-aperture refraction/reflection profiling. The 65th Ann. Int. Meeting SEG, Expanded abstracts, 1995, 659 – 662.

[6] Li Qing-chun, Ye Pei. Joint Tomography of Frist Break And Reflection Traveltime With Multi-scale Netsize method [J]. Oil Geophysical Prospecting, 2013, 48 (4): 536 - 544.

[7] Huang G J, Bai C Y. Simultaneous Inversion With multiple Traveltimes Within 2-D Complex Layered Media. Chinese J Geophys (in Chinese), 2010, 53 (12):2972 – 2981.

[8] Yang Shu-qing. Velocity Inversion From Reflection Travel-Time And Gradient Of Travel-Time By Tomography [J]. Petroleum Geophysics, 2008, Vol, 6 (1), 11 - 20.