Methods of Electrodynamic Modelling for Reflector Antennas

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Abstract. The article discusses methods for calculating the electromagnetic characteristics of reflector antennas based on the finite integration method, the method of moments, and the hybrid method. In the course of the study, a performance comparison of these methods in the process of calculating the reflector antenna characteristics was made. Comparison of the calculation efficiency was carried out using four computing devices. On the basis of the calculations, the graphs of the scattering matrix and the directional diagrams of the antenna under study were obtained.

1. Introduction

The use of modern modelling tools makes it possible to determine the characteristics of complex microwave structures by solving Maxwell's equations. In the process of performing the simulation, an important stage is the choice of the optimal method for calculating the electrodynamic system characteristics. Most modern software products implement several methods for solving the assigned tasks.

The basic and most popular is the finite integration method, which was proposed by Thomas Weiland [1]. This method is especially effective when calculating the characteristics of small objects, since in larger geometrically problems there is a significant increase in the requirement for computing power, the amount of random access and read only memory.

When performing calculations of large and reflective objects, it is proposed to apply the method of moments, which was outlined in the works of Harrington, which is implemented on the integral equation of electric and magnetic fields with a surface structure division [2, 3].

An advanced solution is that combines a hybrid simulation system in which the feed is calculated using the finite integration method, and the reflector is calculated based on the method of moments. This type of electrodynamic modelling in theory makes it possible to achieve a reduction in the load on a computing device, it allows to simulate complex objects, for example, communication satellites with installed antenna systems and other carrier devices.

2. Finite integration technique (FIT)

The basic and most universal recommended for calculating objects up to 100λ, is the finite integration method proposed by Thomas Weiland [1]. This method for solving electrodynamic problems is based on Maxwell's equations in the integral form:
In the process of modelling the selected area is divided into cells consisting of two grids – the primary and secondary ones (the secondary grid is orthogonal to the primary); on the basis of which the Maxwell equations are discretized, after which the equations for each face of the cell are written:

\[ \oint_E d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} d\vec{s}, \quad \oint_H d\vec{l} = \int_S \left( \frac{\partial \vec{D}}{\partial t} + \vec{J} \right) d\vec{s} \]  

\[ \oint \vec{D} d\vec{s} = \int_V \rho dV, \quad \oint \vec{B} d\vec{s} = 0 \]  

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\[ \oint_E (\vec{r}, t) d\vec{s} = -\int_A \frac{\partial}{\partial t} \vec{B}(\vec{r}, t) d\vec{A} \quad \forall A \in \mathbb{R}^3 \]  

Figure 1 shows the cell that is used for calculations during the FIT implementation:

**Figure 1.** A cell with the indicated electric voltages $\hat{e}$ on the edges and magnetic fluxes $\hat{b}$ through the surface.

For each cell, equation (3) can be replaced by an ordinary differential equation:

\[ \hat{e}_x(i, j, k) + \hat{e}_y(i + 1, j, k) - \hat{e}_x(i, j + 1, k) - \hat{e}_y(i, j, k) = -\frac{d}{dt} \hat{b}_z(i, j, k) \]  

If this operation is carried out for each cell, then the calculation rule can be represented in matrix form: topological matrix $C$ as a discrete equivalent to the analytical rotor operator:

\[
\begin{pmatrix}
\hat{e}_{n1} & \cdots & \hat{e}_{n2} & \cdots & \hat{e}_{n3} & \cdots & \hat{e}_{n4}
\end{pmatrix}
\begin{pmatrix}
1 & \cdots & 1 & \cdots & -1 & \cdots & -1
\end{pmatrix}
\begin{pmatrix}
\hat{e}_{n1} \\
\hat{e}_{n2} \\
\hat{e}_{n3} \\
\hat{e}_{n4}
\end{pmatrix}
= -\frac{d}{dt}
\begin{pmatrix}
\hat{b}_z
\end{pmatrix}
\]  

If the proposed scheme is applied to the Ampere’s rule on the secondary grid, then the corresponding discrete circulation operator $\hat{C}$ will be obtained. In a similar way, discretizing the remaining divergence equations gives discrete operators $\hat{S}$ and $S$, which correspond to the flow...
belonging to the primary and secondary grids, respectively. These discrete matrix operators are composed of 0, 1, -1 only and represent topological information only. After applying the described method, we obtain a set of Maxwell grid equations:

\[ Ce = -\frac{\partial}{\partial t} b, \dot{\mathbf{C}}h = \frac{\partial}{\partial t} d + j \]  

(6)

\[ \dot{S}d = q, Sb = 0 \]  

(7)

Based on the obtained equations, the main characteristics of the electromagnetic system in the field of calculations are determined. However, the proposed technique is optimal for calculating small problems, since in the process of forming a grid partition at large sizes a very large number of cells are formed, which significantly increase the load on the computing device. To improve the method performance CST Studio Suite supports multithreaded computations, as well as compute unified device architecture (CUDA). This calculation method is implemented in the T-solver.

3. Method of Moments (MoM)

The method of moments (MoM) is an indirect method for solving problems of electrodynamics, since in the process of solving only some key problem is considered. In mathematics this solution is called Green's function. So, this method is effective only in the case when the function is determined in an analytically simple form, which occurs in problems where only the surface of the structure is required to discretize, and not the entire space, which makes it possible to achieve a large gain in modelling reflective structures. The sizes of supported objects for MoM are from 1\(\lambda\) to 500\(\lambda\) with the greatest efficiency being achieved when simulating reflector antennas.

In order to describe the principle of MoM operation for problems of electrodynamics, it is important to take into account that from a mathematical point of view an operator problem is solved:

\[ L(f) = g \]  

(8)

Where \(L(f)\) is the operator, which is specified in a certain space of functions; \(g\) is an unknown function.

When the solution of electrodynamics occurs, the integral operator is considered (therefore the calculator is called the integral operator). This method is optimal for solving two-dimensional problems, since in these cases it is required to consider two groups: E and H-waves. If the fields do not depend on the \(z\) component, then the E-waves have the following components: \(E_z, H_x, H_y\), and the H-waves: \(H_z, E_x, E_y\).

In order to obtain a two-dimensional integral operator for E-waves, it is required to consider an ideally conducting body which is excited by a plane wave, and in accordance with the set task it is required to determine the electromagnetic field scattered by the body, which satisfies the Maxwell equations in free space and the conditions:

1) boundary conditions on the metal surface: \(E_t = 0\);

2) radiation at infinity.

To solve the problem, it is required to consider the problem of the free space excitation by an electric current that will flow over the metal surface (S). This problem is well studied in electrodynamics and the Green's function of free space is used to solve it:

\[ A^e_z(V) = \int s^e_s (s')G(V', V)ds' \]  

(9)

Where \(s\) is the coordinates of the integration point located on the metal surface S; \(V\) is the coordinates of the observation point located outside the metal body; \(A^e_z(V)\) is the \(z\)-th vector potential component;
$G(V',V)$ is the Green's function of free space:

$$G(V',V) = -\frac{i}{4} H_0^{(2)}(kR)$$

(10)

$$R = \sqrt{(x-x')^2 + (y-y')^2}$$

(11)

Where $k$ is the free space wavenumber; $H_0^{(2)}(x)$ is the Hankel function of the second kind, zero order.

Thus, all field components can be expressed in terms of the vector potential, so the $E_z$ component:

$$E_z = -i\omega\mu_0 A_z$$

(12)

where $\omega$ is the circular frequency; $\mu_0$ is the absolute magnetic permeability of free space.

The resulting field has the following properties:
1) Satisfies Maxwell's equations in free space (based on the definition of Green's functions);
2) Satisfies the condition of radiation at infinity;
3) Boundary condition: $E_z + E_{zi} = 0$ on the surface $S$.

The required integral equation for the $E$-field:

$$\frac{\omega\mu_0}{4} \int_{S} J_z^{e}(s') H_0^{(2)}(kR) ds' = E_{zi} \quad \text{on } S$$

(13)

The resulting equation is fulfilled on the surface $S$;
$J_z^{e}(s')$ is the unknown function to be determined;
$H_0^{(2)}(kR)$ is the integral equation kernel.

For the magnetic field:

$$A_z^e(V) = \int_{S} J_z^{e}(s') G(V',V) ds'$$

(14)

$$E_z = -i\omega\mu_0 A_z^e + \frac{1}{io\varepsilon_0} \frac{\partial^2 A_z^e}{\partial \tau^2}$$

(15)

The required integral equation for the $H$-field:

$$\left(k^2 + \frac{\partial^2}{\partial \tau^2}\right) \int_{S} J_z^{h}(s') G(V',V) ds' = -\frac{\partial H_{zi}}{\partial n}$$

(16)

Thus, the basic equations for the characteristics of the field are obtained when solving an electrodynamic problem by the method of moments.

4. Hybrid computing

In modern computational problems it is very often necessary to deal with a complex simulation system, for example when it is necessary to perform a simulation of an installed antenna on a ship or aircraft. This problem cannot be solved using only the finite integration method, since the feature of the grid partition will lead to a serious load on computing power, so the most optimal way is to use hybrid modelling, when the antenna characteristics are found using the FIT, and then the obtained characteristics are transmitted for modelling using other methods of solving problems: MoM or asymptotic methods (SBR technique).
One of such tasks is reflex antennas. Since the reflector usually has large geometric dimensions, is made of a good conductor, so to increase the accuracy of modelling and reduce the load on the computing device, even at the cost of increasing the modelling time, the following modelling system is proposed:

1) Calculate the reflector irradiator using the finite simulation method;
2) Transmit the obtained results using the field source to the project with the installed reflector and performing the simulation using the method of moments;
3) Transfer the obtained results back to the irradiator simulation project to account for the influence of the reflector on the field characteristics.

This algorithm continues until the desired number of iterations which are set during the preparation of the simulation is reached. This technique is advanced and promising in the process of solving electrodynamic problems as it allows you to achieve a gain in the amount of operational memory used, processor load, as well as the division of tasks into small projects that are optimal for modelling large projects.

5. Analysis of the studied problem
In the course of the study of the modelling methods characteristics, the analysis of the reflex antenna which is configured for a frequency of 10 GHz, directive gain = 30 dBi was performed; the geometry of the structure is shown in Figure 2.

Figure 2. The type of the structure under study in the process of studying the effectiveness of methods for calculating the EM field characteristics.
The dimensions of the structure under consideration are given in the table 1. In the process of modelling a 3D structure which is modelled by the methods described above is formed.

**Table 1.** Dimensions of the structure under consideration.

| F  | D  | S  | Wg | Hg | Lg | Wa | Ha | Lf |
|----|----|----|----|----|----|----|----|----|
| 246 | 358.1 | 246 | 23.53 | 11.77 | 29.98 | 45 | 34.5 | 79.5 |

6. **Modelling of the considered structure**

In the process of preparing the simulation a grid partition occurs for the considered reflector antenna; the type of grid partition is shown in Figure 3.

![Grid partitioning for various methods of calculating the characteristics of EM problems: (a) - grid partitioning for the finite integration technique (FIT); (b) - partition using the Rao-Wilton-Glisson element [3] for the method of moments (MoM); (c) - grid subdivision of the irradiator in hybrid modelling; (d) - grid division of the reflector in hybrid modelling; (e) - combining the reflector and the irradiator (the irradiator is replaced by the field source).](image-url)
When performing the calculations, the frequency range under consideration was from 9 to 15 GHz, while the radiation patterns of the structure were plotted for the entire range in increments of 1 GHz (9, 10, 11...15 GHz).

The analysis of the results obtained in the simulation process will begin with the characteristics of the scattering matrix ($S_{11}$ system parameters).

![Figure 4](image)

**Figure 4.** The obtained dependences of the $S_{11}$ system parameters: line $S_{1,1}$ FIT—the finite integration technique; $S_{1, 1}$ MoM—the method of moments; $S_{1, [Source1]}$ Horn only—the graph of the $S_{11}$ parameter for the irradiator using the hybrid method (calculation using the FIT); $S_{[Source1],[Source1]}$ Hybrid—the final characteristics of the system when modelling using the hybrid method.

As can be seen from the obtained picture of the reflection coefficients, the moment method calculates these parameters only at the frequencies of the directivity diagram construction, the remaining frequencies are skipped during the modelling process. When calculating by the finite integration technique, the characteristic is determined over the entire frequency range, which increases the load during calculations. The parameters of the system when performing the hybrid calculation strongly depend on the parameters of the irradiator, which are determined using the FIT.

Thus, if it is necessary to determine the parameters of the scattering matrix in a wide frequency range, it is recommended to use the finite integration technique, which produces the construction regardless of the number of required radiation patterns over the entire frequency range. If it is necessary to determine the characteristics only at certain (target) frequencies, then it is possible to use the method of moments or hybrid (the advantage of hybrid is the ability to determine the characteristics of the irradiator in a wide frequency range).

Let us analyse the diagrams of directivity patterns for the considered situation at a frequency of 10 GHz, the theoretical calculated value is 30 dBi; the results obtained are shown in Figure 5, 6.
Figure 5. Directional patterns (f = 10 GHz, \( \phi = 0^\circ \)) obtained in the calculations: FIT - Finite integration technique; MoM - Method of moments; Hybrid - Hybrid computing.

Figure 6. Directional patterns (f = 10 GHz, \( \phi = 90^\circ \)) obtained in the calculations: FIT - Finite integration technique; MoM - Method of moments; Hybrid - Hybrid computing.

To analyse the results obtained we use table 2.

Based on the results obtained, it is clear that the main characteristics of the directional patterns coincide with high accuracy when using all the proposed methods, only the level of the side lobes when using hybrid modelling differs from the values obtained using FIT and MoM, since the number of iterations was limited to 1 in the simulation to speed up the calculations.

A particularly important point when performing simulations is their efficiency and resource intensity. To determine the dependence, modelling using different computing devices was performed. During the simulation, the number of threads in multithreaded modelling was limited to 8, which allowed us to determine the speed of calculations when using different devices. The results are shown in table 3.
Table 2. Analysis of directivity diagrams characteristics.

| Analysed characteristic | Finite integration technique | Method of moments | Hybrid method |
|-------------------------|------------------------------|------------------|--------------|
| Gain $f=10$ GHz, $\varphi=0^\circ$, dBi | 30.1 | 30.2 | 30.2 |
| Radiation direction $f = 10$ GHz, $\varphi = 0^\circ$, $^\circ$ | 0 | 0 | 0 |
| Main lobe width at 3 dB $f = 10$ GHz, $\varphi = 0^\circ$, $^\circ$ | 5.1 | 5.3 | 5.6 |
| Side lobe level $10$ GHz, $\varphi = 0^\circ$, dB | -19.7 | -19.6 | -25.2 |
| Directive gain $f=10$ GHz, $\varphi = 90^\circ$, dBi | 30.1 | 30.2 | 30.2 |
| Radiation direction $f=10$ GHz, $\varphi=90^\circ$, $^\circ$ | 0 | 0 | 0 |
| The width of the main lobe at the level of 3 dB $f=10$ GHz, $\varphi=90^\circ$, $^\circ$ | 5.5 | 5.5 | 5.7 |
| Side lobe level $10$ GHz, $\varphi=90^\circ$, dB | -22.7 | -21.7 | -25.5 |

Table 3. Study of the calculations effectiveness.

| Wake Chi Sqr. ($N=15$, df=1) | Finite integration technique | Method of moments | Hybrid method |
|------------------------------|------------------------------|------------------|--------------|
| Number of meshcells in the simulation | 10 221 120 | ---- | 1 233 362 |
| The number of surfaces in the simulation | ---- | 19 480 | 15 750 |
| Final project volume after calculations, MB | 856 | 120 | 309 |
| The use of RAM from 8 GB, GB | 5.4 | 4.3 | 3.5 |
| Calculation speed (8 threads), min:sec | Intel Core i9-9880h | 10:40 | 06:14 | 16:43 |
| Calculation speed | AMD FX-8320E | 17:13 | 12:38 | 34:36 |
| Calculation speed (8 threads), min:sec | Intel Xeon Gold 5118 | 07:05 | 06:05 | 19:09 |
| Calculation speed (8 threads), min:sec | Intel Xeon Gold 5118 + 2x Nvidia Tesla T4 | 04:38 | 04:22 | 18:10 |

Based on the results obtained, it is clear that the final integration method has high performance when using graphics accelerators, while it has the highest use of RAM, and the final project volume is much higher than that of the projects when using other methods. Effective use of the finite integration technique requires a high-performance CPU, a large number of RAM, and preferably a professional graphics accelerator.

The method of moments is the fastest when performing simulations using only the CPU, and has the minimum volume of the final project. All these features are achieved through the use of a grid
partitioning of the structure based on the Rao-Wilton-Glisson element in the calculation process [3]. The use of a graphics accelerator allows you to reduce the simulation time, but due to the better optimization of the FIT for the use of graphic acceleration, the calculation time of the MoM and FIT practically does not differ.

Hybrid simulation is the most time-consuming, but requires the least amount of RAM, which has a positive effect when calculating large projects. You can also notice that the minimum simulation time is observed when simulating using a computer based on Intel Core i9-9880h. Thus, to improve the calculation time, high performance per flow is required. High speed of the drive is also required as there is frequent transfer of information from one project to another. The use of graphics acceleration practically does not speed up the calculation process.

7. Conclusion
The use of modern calculation methods makes it possible to determine the main parameters of microwave devices at the design stage. The use of the finite integration technique makes it possible to determine the parameters of any electromagnetic structure due to the peculiarities of dividing space into cells in each of which the Maxwell equations are calculated, which leads to large computational loads.

To determine the characteristics of reflector antennas the method of moments is optimal. It can significantly reduce the simulation time, since it is based on a surface grid partition. Calculation based on a two-dimensional problem of electrodynamics makes it possible to achieve high results accuracy with a significant decrease in the load on the computing device in comparison with the FIT.

Hybrid modelling is promising for solving complex problems of electrodynamics, such as installing antennas on the body of large objects, optimizing calculations by applying several calculation methods. So, in addition to the hybrid interaction of the finite integration method and the method of moments it is possible to use asymptotic modelling methods in the modelling project which are based on the methods of physical optics and wave diffraction.

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